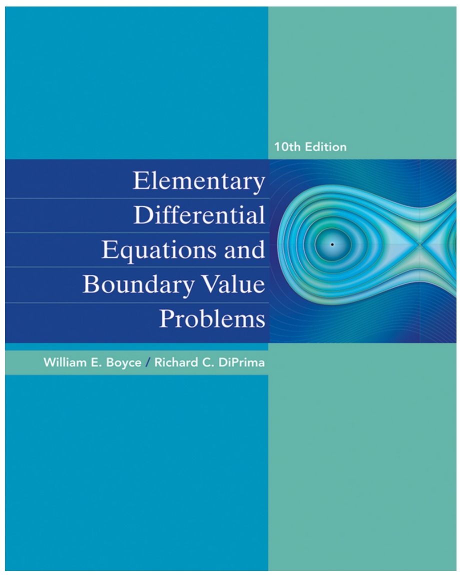
A Solution Manual For

Elementary differential equations and boundary value problems, 10th ed., Boyce and DiPrima



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May 16, 2024

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1.1 problem 1

Internal problem ID [448]

Book: Elementary differential equations and boundary value problems, 10th ed., Boyce and DiPrima
Section: Section 2.1. Page 40
Problem number: 1.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[[_linear, 'class A']]`

$$\boxed{3y + y' = e^{-2t} + t}$$

✓ Solution by Maple
Time used: 0.0 (sec). Leaf size: 19

```
dsolve(3*y(t)+diff(y(t),t) = exp(-2*t)+t,y(t), singsol=all)
```

$$y(t) = \frac{t}{3} - \frac{1}{9} + e^{-2t} + c_1 e^{-3t}$$

✓ Solution by Mathematica
Time used: 0.086 (sec). Leaf size: 27

```
DSolve[3*y[t]+y'[t] == Exp[-2*t]+t,y[t],t,IncludeSingularSolutions -> True]
```

$$y(t) \to \frac{t}{3} + e^{-2t} + c_1 e^{-3t} - \frac{1}{9}$$

1.2 problem 2

Internal problem ID [449]

Book: Elementary differential equations and boundary value problems, 10th ed., Boyce and DiPrima
Section: Section 2.1. Page 40
Problem number: 2.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[[_linear, 'class A']]`

$$\boxed{-2y + y' = e^{2t}t^2}$$

✓ Solution by Maple
Time used: 0.0 (sec). Leaf size: 17

```
dsolve(-2*y(t)+diff(y(t),t) = exp(2*t)*t^2,y(t), singsol=all)
```

$$y(t) = \frac{(t^3 + 3c_1)\, e^{2t}}{3}$$

✓ Solution by Mathematica
Time used: 0.043 (sec). Leaf size: 22

```
DSolve[-2*y[t]+y'[t]== Exp[2*t]*t^2,y[t],t,IncludeSingularSolutions -> True]
```

$$y(t) \to \frac{1}{3} e^{2t} (t^3 + 3c_1)$$

1.3 problem 3

Internal problem ID [450]

Book: Elementary differential equations and boundary value problems, 10th ed., Boyce and DiPrima
Section: Section 2.1. Page 40
Problem number: 3.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[[_linear, ‘class A‘]]`

$$\boxed{y + y' = 1 + t\,\mathrm{e}^{-t}}$$

✓ Solution by Maple
Time used: 0.0 (sec). Leaf size: 19

```
dsolve(y(t)+diff(y(t),t) = 1+t/exp(t),y(t), singsol=all)
```

$$y(t) = 1 + \frac{(t^2 + 2c_1)\,\mathrm{e}^{-t}}{2}$$

✓ Solution by Mathematica
Time used: 0.052 (sec). Leaf size: 27

```
DSolve[y[t]+y'[t] == 1+t/Exp[t],y[t],t,IncludeSingularSolutions -> True]
```

$$y(t) \to \frac{1}{2}e^{-t}\left(t^2 + 2e^t + 2c_1\right)$$

1.4 problem 4

Internal problem ID [451]

Book: Elementary differential equations and boundary value problems, 10th ed., Boyce and DiPrima
Section: Section 2.1. Page 40
Problem number: 4.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_linear]`

$$\boxed{\frac{y}{t} + y' = 3\cos(2t)}$$

✓ Solution by Maple
Time used: 0.0 (sec). Leaf size: 26

```
dsolve(y(t)/t+diff(y(t),t) = 3*cos(2*t),y(t), singsol=all)
```

$$y(t) = \frac{4c_1 + 6\sin(2t)\,t + 3\cos(2t)}{4t}$$

✓ Solution by Mathematica
Time used: 0.073 (sec). Leaf size: 30

```
DSolve[y[t]/t+y'[t] == 3*Cos[2*t],y[t],t,IncludeSingularSolutions -> True]
```

$$y(t) \to \frac{6t\sin(2t) + 3\cos(2t) + 4c_1}{4t}$$

1.5 problem 5

Internal problem ID [452]

Book: Elementary differential equations and boundary value problems, 10th ed., Boyce and DiPrima
Section: Section 2.1. Page 40
Problem number: 5.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[[_linear, 'class A']]`

$$\boxed{-2y + y' = 3\,e^t}$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 15

```
dsolve(-2*y(t)+diff(y(t),t) = 3*exp(t),y(t), singsol=all)
```

$$y(t) = -3\,e^t + c_1 e^{2t}$$

✓ Solution by Mathematica

Time used: 0.046 (sec). Leaf size: 17

```
DSolve[-2*y[t]+y'[t] == 3*Exp[t],y[t],t,IncludeSingularSolutions -> True]
```

$$y(t) \to e^t\left(-3 + c_1 e^t\right)$$

1.6 problem 6

Internal problem ID [453]

Book: Elementary differential equations and boundary value problems, 10th ed., Boyce and DiPrima
Section: Section 2.1. Page 40
Problem number: 6.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_linear]`

$$\boxed{2y + ty' = \sin(t)}$$

✓ Solution by Maple
Time used: 0.0 (sec). Leaf size: 17

```
dsolve(2*y(t)+t*diff(y(t),t) = sin(t),y(t), singsol=all)
```

$$y(t) = \frac{\sin(t) - \cos(t)\, t + c_1}{t^2}$$

✓ Solution by Mathematica
Time used: 0.039 (sec). Leaf size: 19

```
DSolve[2*y[t]+t*y'[t]== Sin[t],y[t],t,IncludeSingularSolutions -> True]
```

$$y(t) \to \frac{\sin(t) - t\cos(t) + c_1}{t^2}$$

1.7 problem 7

Internal problem ID [454]

Book: Elementary differential equations and boundary value problems, 10th ed., Boyce and DiPrima
Section: Section 2.1. Page 40
Problem number: 7.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_linear]`

$$\boxed{2yt + y' = 2t\,\mathrm{e}^{-t^2}}$$

✓ Solution by Maple

Time used: 0.016 (sec). Leaf size: 16

```
dsolve(2*t*y(t)+diff(y(t),t) = 2*t/exp(t^2),y(t), singsol=all)
```

$$y(t) = (t^2 + c_1)\,\mathrm{e}^{-t^2}$$

✓ Solution by Mathematica

Time used: 0.055 (sec). Leaf size: 19

```
DSolve[2*t*y[t]+y'[t] == 2*t/Exp[t^2],y[t],t,IncludeSingularSolutions -> True]
```

$$y(t) \to e^{-t^2}(t^2 + c_1)$$

1.8 problem 8

Internal problem ID [455]

Book: Elementary differential equations and boundary value problems, 10th ed., Boyce and DiPrima
Section: Section 2.1. Page 40
Problem number: 8.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_linear]`

$$\boxed{4yt + (t^2 + 1)\, y' = \frac{1}{(t^2 + 1)^2}}$$

✓ Solution by Maple
Time used: 0.015 (sec). Leaf size: 16

```
dsolve(4*t*y(t)+(t^2+1)*diff(y(t),t) = 1/(t^2+1)^2,y(t), singsol=all)
```

$$y(t) = \frac{\arctan(t) + c_1}{(t^2 + 1)^2}$$

✓ Solution by Mathematica
Time used: 0.041 (sec). Leaf size: 18

```
DSolve[4*t*y[t]+(t^2+1)*y'[t] == 1/(t^2+1)^2,y[t],t,IncludeSingularSolutions -> True]
```

$$y(t) \to \frac{\arctan(t) + c_1}{(t^2 + 1)^2}$$

1.9 problem 9

Internal problem ID [456]

Book: Elementary differential equations and boundary value problems, 10th ed., Boyce and DiPrima
Section: Section 2.1. Page 40
Problem number: 9.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[[_linear, ‘class A‘]]`

$$\boxed{y + 2y' = 3t}$$

✓ Solution by Maple
Time used: 0.0 (sec). Leaf size: 15

```
dsolve(y(t)+2*diff(y(t),t) = 3*t,y(t), singsol=all)
```

$$y(t) = 3t - 6 + e^{-\frac{t}{2}} c_1$$

✓ Solution by Mathematica
Time used: 0.045 (sec). Leaf size: 20

```
DSolve[y[t]+2*y'[t] == 3*t,y[t],t,IncludeSingularSolutions -> True]
```

$$y(t) \to 3t + c_1 e^{-t/2} - 6$$

1.10 problem 10

Internal problem ID [457]

Book: Elementary differential equations and boundary value problems, 10th ed., Boyce and DiPrima
Section: Section 2.1. Page 40
Problem number: 10.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_linear]`

$$\boxed{ty' - y = t^2 \mathrm{e}^{-t}}$$

✓ Solution by Maple
Time used: 0.0 (sec). Leaf size: 14

```
dsolve(-y(t)+t*diff(y(t),t) = t^2/exp(t),y(t), singsol=all)
```

$$y(t) = \left(-\mathrm{e}^{-t} + c_1\right) t$$

✓ Solution by Mathematica
Time used: 0.053 (sec). Leaf size: 17

```
DSolve[-y[t]+t*y'[t] == t^2/Exp[t],y[t],t,IncludeSingularSolutions -> True]
```

$$y(t) \to t\left(-e^{-t} + c_1\right)$$

1.11 problem 11

Internal problem ID [458]

Book: Elementary differential equations and boundary value problems, 10th ed., Boyce and DiPrima
Section: Section 2.1. Page 40
Problem number: 11.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[[_linear, 'class A']]`

$$\boxed{y + y' = 5\sin(2t)}$$

✓ Solution by Maple
Time used: 0.0 (sec). Leaf size: 21

```
dsolve(y(t)+diff(y(t),t) = 5*sin(2*t),y(t), singsol=all)
```

$$y(t) = \sin(2t) - 2\cos(2t) + e^{-t}c_1$$

✓ Solution by Mathematica
Time used: 0.087 (sec). Leaf size: 24

```
DSolve[y[t]+y'[t] == 5*Sin[2*t],y[t],t,IncludeSingularSolutions -> True]
```

$$y(t) \to \sin(2t) - 2\cos(2t) + c_1 e^{-t}$$

1.12 problem 12

Internal problem ID [459]

Book: Elementary differential equations and boundary value problems, 10th ed., Boyce and DiPrima
Section: Section 2.1. Page 40
Problem number: 12.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[[_linear, ‘class A‘]]`

$$\boxed{y + 2y' = 3t^2}$$

✓ Solution by Maple
Time used: 0.016 (sec). Leaf size: 20

```
dsolve(y(t)+2*diff(y(t),t) = 3*t^2,y(t), singsol=all)
```

$$y(t) = 3t^2 - 12t + 24 + e^{-\frac{t}{2}}c_1$$

✓ Solution by Mathematica
Time used: 0.047 (sec). Leaf size: 25

```
DSolve[y[t]+2*y'[t] == 3*t^2,y[t],t,IncludeSingularSolutions -> True]
```

$$y(t) \to 3t^2 - 12t + c_1 e^{-t/2} + 24$$

1.13 problem 13

Internal problem ID [460]

Book: Elementary differential equations and boundary value problems, 10th ed., Boyce and DiPrima
Section: Section 2.1. Page 40
Problem number: 13.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[[_linear, 'class A']]`

$$\boxed{-y + y' = 2\,e^{2t}t}$$

With initial conditions

$$[y(0) = 1]$$

✓ Solution by Maple
Time used: 0.016 (sec). Leaf size: 16

```
dsolve([-y(t)+diff(y(t),t) = 2*exp(2*t)*t,y(0) = 1],y(t), singsol=all)
```

$$y(t) = (2t - 2)\,e^{2t} + 3\,e^{t}$$

✓ Solution by Mathematica
Time used: 0.053 (sec). Leaf size: 19

```
DSolve[{-y[t]+y'[t] == 2*Exp[2*t]*t,y[0]==1},y[t],t,IncludeSingularSolutions -> True]
```

$$y(t) \to e^{t}\left(2e^{t}(t-1)+3\right)$$

1.14 problem 14

Internal problem ID [461]

Book: Elementary differential equations and boundary value problems, 10th ed., Boyce and DiPrima
Section: Section 2.1. Page 40
Problem number: 14.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[[_linear, ‘class A‘]]`

$$\boxed{2y + y' = t\,\mathrm{e}^{-2t}}$$

With initial conditions

$$[y(1) = 0]$$

✓ Solution by Maple
Time used: 0.0 (sec). Leaf size: 15

```
dsolve([2*y(t)+diff(y(t),t) = t/exp(2*t),y(1) = 0],y(t), singsol=all)
```

$$y(t) = \frac{(t^2 - 1)\,\mathrm{e}^{-2t}}{2}$$

✓ Solution by Mathematica
Time used: 0.054 (sec). Leaf size: 19

```
DSolve[{2*y[t]+y'[t] == t/Exp[2*t],y[1]==0},y[t],t,IncludeSingularSolutions -> True]
```

$$y(t) \to \frac{1}{2} e^{-2t} (t^2 - 1)$$

1.15 problem 15

Internal problem ID [462]

Book: Elementary differential equations and boundary value problems, 10th ed., Boyce and DiPrima
Section: Section 2.1. Page 40
Problem number: 15.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_linear]`

$$\boxed{2y + ty' = t^2 - t + 1}$$

With initial conditions

$$\left[y(1) = \frac{1}{2}\right]$$

✓ Solution by Maple
Time used: 0.0 (sec). Leaf size: 19

```
dsolve([2*y(t)+t*diff(y(t),t) = t^2-t+1,y(1) = 1/2],y(t), singsol=all)
```

$$y(t) = \frac{t^2}{4} - \frac{t}{3} + \frac{1}{2} + \frac{1}{12t^2}$$

✓ Solution by Mathematica
Time used: 0.028 (sec). Leaf size: 22

```
DSolve[{2*y[t]+t*y'[t] == t^2-t+1,y[1]==1/2},y[t],t,IncludeSingularSolutions -> True]
```

$$y(t) \to \frac{1}{12}\left(3t^2 + \frac{1}{t^2} - 4t + 6\right)$$

1.16 problem 16

Internal problem ID [463]

Book: Elementary differential equations and boundary value problems, 10th ed., Boyce and DiPrima
Section: Section 2.1. Page 40
Problem number: 16.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_linear]`

$$\boxed{y' + \frac{2y}{t} = \frac{\cos(t)}{t^2}}$$

With initial conditions

$$[y(\pi) = 0]$$

✓ Solution by Maple
Time used: 0.016 (sec). Leaf size: 10

```
dsolve([2*y(t)/t+diff(y(t),t) = cos(t)/t^2,y(Pi) = 0],y(t), singsol=all)
```

$$y(t) = \frac{\sin(t)}{t^2}$$

✓ Solution by Mathematica
Time used: 0.041 (sec). Leaf size: 11

```
DSolve[{2*y[t]/t+y'[t] == Cos[t]/t^2,y[Pi]==0},y[t],t,IncludeSingularSolutions -> True]
```

$$y(t) \to \frac{\sin(t)}{t^2}$$

1.17 problem 17

Internal problem ID [464]

Book: Elementary differential equations and boundary value problems, 10th ed., Boyce and DiPrima
Section: Section 2.1. Page 40
Problem number: 17.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[[_linear, 'class A']]`

$$\boxed{-2y + y' = e^{2t}}$$

With initial conditions

$$[y(0) = 2]$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 12

```
dsolve([-2*y(t)+diff(y(t),t) = exp(2*t),y(0) = 2],y(t), singsol=all)
```

$$y(t) = (2 + t)\, e^{2t}$$

✓ Solution by Mathematica

Time used: 0.039 (sec). Leaf size: 14

```
DSolve[{-2*y[t]+y'[t] == Exp[2*t],y[0]==2},y[t],t,IncludeSingularSolutions -> True]
```

$$y(t) \to e^{2t}(t + 2)$$

1.18 problem 18

Internal problem ID [465]

Book: Elementary differential equations and boundary value problems, 10th ed., Boyce and DiPrima
Section: Section 2.1. Page 40
Problem number: 18.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_linear]`

$$\boxed{2y + ty' = \sin(t)}$$

With initial conditions

$$\left[y\left(\frac{\pi}{2}\right) = 1\right]$$

✓ Solution by Maple
Time used: 0.016 (sec). Leaf size: 22

```
dsolve([2*y(t)+t*diff(y(t),t) = sin(t),y(1/2*Pi) = 1],y(t), singsol=all)
```

$$y(t) = \frac{\sin(t) - \cos(t)\,t + \frac{\pi^2}{4} - 1}{t^2}$$

✓ Solution by Mathematica
Time used: 0.034 (sec). Leaf size: 26

```
DSolve[{2*y[t]+t*y'[t] == Sin[t],y[Pi/2]==1},y[t],t,IncludeSingularSolutions -> True]
```

$$y(t) \to \frac{4\sin(t) - 4t\cos(t) + \pi^2 - 4}{4t^2}$$

1.19 problem 19

Internal problem ID [466]

Book: Elementary differential equations and boundary value problems, 10th ed., Boyce and DiPrima
Section: Section 2.1. Page 40
Problem number: 19.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_linear]`

$$\boxed{4yt^2 + y't^3 = \mathrm{e}^{-t}}$$

With initial conditions

$$[y(-1) = 0]$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 16

```
dsolve([4*t^2*y(t)+t^3*diff(y(t),t) = exp(-t),y(-1) = 0],y(t), singsol=all)
```

$$y(t) = -\frac{(t+1)\,\mathrm{e}^{-t}}{t^4}$$

✓ Solution by Mathematica

Time used: 0.067 (sec). Leaf size: 18

```
DSolve[{4*t^2*y[t]+t^3*y'[t] == Exp[-t],y[-1]==0},y[t],t,IncludeSingularSolutions -> True]
```

$$y(t) \to -\frac{e^{-t}(t+1)}{t^4}$$

1.20 problem 20

Internal problem ID [467]

Book: Elementary differential equations and boundary value problems, 10th ed., Boyce and DiPrima
Section: Section 2.1. Page 40
Problem number: 20.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_linear]`

$$\boxed{(t+1)\,y + ty' = t}$$

With initial conditions

$$[y(\ln(2)) = 1]$$

✓ Solution by Maple

Time used: 0.016 (sec). Leaf size: 17

```
dsolve([(1+t)*y(t)+t*diff(y(t),t) = t,y(ln(2)) = 1],y(t), singsol=all)
```

$$y(t) = \frac{t - 1 + 2\,\mathrm{e}^{-t}}{t}$$

✓ Solution by Mathematica

Time used: 0.034 (sec). Leaf size: 23

```
DSolve[{(1+t)*y[t]+t*y'[t]== t,y[Log[2]]==1},y[t],t,IncludeSingularSolutions -> True]
```

$$y(t) \to \frac{e^{-t}(e^t(t-1)+2)}{t}$$

1.21 problem 21

Internal problem ID [468]

Book: Elementary differential equations and boundary value problems, 10th ed., Boyce and DiPrima
Section: Section 2.1. Page 40
Problem number: 21.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type **[[_linear, 'class A']]**

$$-\frac{y}{2} + y' = 2\cos(t)$$

With initial conditions

$$[y(0) = a]$$

✓ Solution by Maple
Time used: 0.016 (sec). Leaf size: 21

```
dsolve([-1/2*y(t)+diff(y(t),t) = 2*cos(t),y(0) = a],y(t), singsol=all)
```

$$y(t) = -\frac{4\cos(t)}{5} + \frac{8\sin(t)}{5} + e^{\frac{t}{2}}a + \frac{4\,e^{\frac{t}{2}}}{5}$$

✓ Solution by Mathematica
Time used: 0.054 (sec). Leaf size: 31

```
DSolve[{-1/2*y[t]+y'[t] == 2*Cos[t],y[0]==a},y[t],t,IncludeSingularSolutions -> True]
```

$$y(t) \to \frac{1}{5}\left((5a+4)e^{t/2} + 8\sin(t) - 4\cos(t)\right)$$

1.22 problem 22

Internal problem ID [469]

Book: Elementary differential equations and boundary value problems, 10th ed., Boyce and DiPrima
Section: Section 2.1. Page 40
Problem number: 22.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[[_linear, 'class A']]`

$$\boxed{-y + 2y' = e^{\frac{t}{3}}}$$

With initial conditions

$$[y(0) = a]$$

✓ Solution by Maple

Time used: 0.016 (sec). Leaf size: 18

```
dsolve([-y(t)+2*diff(y(t),t) = exp(1/3*t),y(0) = a],y(t), singsol=all)
```

$$y(t) = e^{\frac{t}{3}}\left(-3 + (a+3)\,e^{\frac{t}{6}}\right)$$

✓ Solution by Mathematica

Time used: 0.052 (sec). Leaf size: 26

```
DSolve[{-y[t]+2*y'[t] == Exp[1/3*t],y[0]==a},y[t],t,IncludeSingularSolutions -> True]
```

$$y(t) \to e^{t/3}\left((a+3)e^{t/6} - 3\right)$$

1.23 problem 23

Internal problem ID [470]

Book: Elementary differential equations and boundary value problems, 10th ed., Boyce and DiPrima
Section: Section 2.1. Page 40
Problem number: 23.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type **[[_linear, ‘class A‘]]**

$$\boxed{-2y + 3y' = \mathrm{e}^{-\frac{\pi t}{2}}}$$

With initial conditions

$$[y(0) = a]$$

✓ Solution by Maple

Time used: 0.016 (sec). Leaf size: 35

```
dsolve([-2*y(t)+3*diff(y(t),t) = exp(-1/2*Pi*t),y(0) = a],y(t), singsol=all)
```

$$y(t) = \frac{\left(3\pi a - 2\,\mathrm{e}^{t\left(-\frac{\pi}{2}-\frac{2}{3}\right)} + 4a + 2\right)\mathrm{e}^{\frac{2t}{3}}}{3\pi + 4}$$

✓ Solution by Mathematica

Time used: 0.083 (sec). Leaf size: 43

```
DSolve[{-2*y[t]+3*y'[t] == Exp[-1/2*Pi*t],y[0]==a},y[t],t,IncludeSingularSolutions -> True]
```

$$y(t) \to \frac{e^{2t/3}\left((4+3\pi)a - 2e^{-\frac{1}{6}(4+3\pi)t} + 2\right)}{4+3\pi}$$

1.24 problem 24

Internal problem ID [471]

Book: Elementary differential equations and boundary value problems, 10th ed., Boyce and DiPrima
Section: Section 2.1. Page 40
Problem number: 24.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_linear]`

$$\boxed{(t+1)\,y+ty'=2t\,\mathrm{e}^{-t}}$$

With initial conditions

$$[y(1)=a]$$

✓ Solution by Maple
Time used: 0.031 (sec). Leaf size: 21

```
dsolve([(1+t)*y(t)+t*diff(y(t),t) = 2*t/exp(t),y(1) = a],y(t), singsol=all)
```

$$y(t)=\frac{(t^2+a\,\mathrm{e}-1)\,\mathrm{e}^{-t}}{t}$$

✓ Solution by Mathematica
Time used: 0.065 (sec). Leaf size: 22

```
DSolve[{(1+t)*y[t]+t*y'[t] == 2*t/Exp[t],y[1]==a},y[t],t,IncludeSingularSolutions -> True]
```

$$y(t)\to\frac{e^{-t}(ea+t^2-1)}{t}$$

1.25 problem 25

Internal problem ID [472]

Book: Elementary differential equations and boundary value problems, 10th ed., Boyce and DiPrima
Section: Section 2.1. Page 40
Problem number: 25.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_linear]`

$$\boxed{2y + ty' = \frac{\sin(t)}{t}}$$

With initial conditions

$$\left[y\left(-\frac{\pi}{2}\right) = a\right]$$

✓ Solution by Maple
Time used: 0.016 (sec). Leaf size: 19

```
dsolve([2*y(t)+t*diff(y(t),t) = sin(t)/t,y(-1/2*Pi) = a],y(t), singsol=all)
```

$$y(t) = \frac{-\cos(t) + \frac{a\,\pi^2}{4}}{t^2}$$

✓ Solution by Mathematica
Time used: 0.042 (sec). Leaf size: 22

```
DSolve[{2*y[t]+t*y'[t] == Sin[t]/t,y[-Pi/2]==a},y[t],t,IncludeSingularSolutions -> True]
```

$$y(t) \to \frac{\pi^2 a - 4\cos(t)}{4t^2}$$

1.26 problem 26

Internal problem ID [473]

Book: Elementary differential equations and boundary value problems, 10th ed., Boyce and DiPrima
Section: Section 2.1. Page 40
Problem number: 26.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_linear]`

$$\boxed{\cos(t) y + \sin(t) y' = e^t}$$

With initial conditions

$$[y(1) = a]$$

✓ Solution by Maple
Time used: 0.031 (sec). Leaf size: 18

```
dsolve([cos(t)*y(t)+sin(t)*diff(y(t),t) = exp(t),y(1) = a],y(t), singsol=all)
```

$$y(t) = \csc(t)\left(e^t + a\sin(1) - e\right)$$

✓ Solution by Mathematica
Time used: 0.062 (sec). Leaf size: 19

```
DSolve[{Cos[t]*y[t]+Sin[t]*y'[t] == Exp[t],y[1]==a},y[t],t,IncludeSingularSolutions -> True]
```

$$y(t) \to \csc(t)\left(a\sin(1) + e^t - e\right)$$

1.27 problem 27

Internal problem ID [474]

Book: Elementary differential equations and boundary value problems, 10th ed., Boyce and DiPrima
Section: Section 2.1. Page 40
Problem number: 27.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[[_linear, 'class A']]`

$$\boxed{y' + \frac{y}{2} = 2\cos(t)}$$

With initial conditions

$$[y(0) = -1]$$

✓ Solution by Maple
Time used: 0.0 (sec). Leaf size: 19

```
dsolve([1/2*y(t)+diff(y(t),t) = 2*cos(t),y(0) = -1],y(t), singsol=all)
```

$$y(t) = \frac{4\cos(t)}{5} + \frac{8\sin(t)}{5} - \frac{9\,\mathrm{e}^{-\frac{t}{2}}}{5}$$

✓ Solution by Mathematica
Time used: 0.049 (sec). Leaf size: 27

```
DSolve[{1/2*y[t]+y'[t] == 2*Cos[t],y[0]==-1},y[t],t,IncludeSingularSolutions -> True]
```

$$y(t) \to \frac{1}{5}\left(-9e^{-t/2} + 8\sin(t) + 4\cos(t)\right)$$

1.28 problem 28

Internal problem ID [475]

Book: Elementary differential equations and boundary value problems, 10th ed., Boyce and DiPrima
Section: Section 2.1. Page 40
Problem number: 28.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[[_linear, 'class A']]`

$$\boxed{\frac{2y}{3} + y' = -\frac{t}{2} + 1}$$

✓ Solution by Maple

Time used: 0.016 (sec). Leaf size: 15

```
dsolve(2/3*y(t)+diff(y(t),t) = 1-1/2*t,y(t), singsol=all)
```

$$y(t) = -\frac{3t}{4} + \frac{21}{8} + \mathrm{e}^{-\frac{2t}{3}} c_1$$

✓ Solution by Mathematica

Time used: 0.061 (sec). Leaf size: 24

```
DSolve[2/3*y[t]+y'[t] == 1-1/2*t,y[t],t,IncludeSingularSolutions -> True]
```

$$y(t) \to -\frac{3t}{4} + c_1 e^{-2t/3} + \frac{21}{8}$$

1.29 problem 29

Internal problem ID [476]

Book: Elementary differential equations and boundary value problems, 10th ed., Boyce and DiPrima
Section: Section 2.1. Page 40
Problem number: 29.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[[_linear, 'class A']]`

$$\boxed{\frac{y}{4} + y' = 3 + 2\cos(2t)}$$

With initial conditions

$$[y(0) = 0]$$

✓ Solution by Maple
Time used: 0.0 (sec). Leaf size: 24

```
dsolve([1/4*y(t)+diff(y(t),t) = 3+2*cos(2*t),y(0) = 0],y(t), singsol=all)
```

$$y(t) = 12 + \frac{8\cos(2t)}{65} + \frac{64\sin(2t)}{65} - \frac{788\,\mathrm{e}^{-\frac{t}{4}}}{65}$$

✓ Solution by Mathematica
Time used: 0.158 (sec). Leaf size: 32

```
DSolve[{1/4*y[t]+y'[t] == 3+2*Cos[2*t],y[0]==0},y[t],t,IncludeSingularSolutions -> True]
```

$$y(t) \to \frac{4}{65}\left(-197e^{-t/4} + 16\sin(2t) + 2\cos(2t) + 195\right)$$

1.30 problem 30

Internal problem ID [477]

Book: Elementary differential equations and boundary value problems, 10th ed., Boyce and DiPrima
Section: Section 2.1. Page 40
Problem number: 30.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[[_linear, 'class A']]`

$$\boxed{-y + y' = 1 + 3\sin(t)}$$

✓ Solution by Maple
Time used: 0.0 (sec). Leaf size: 18

```
dsolve(-y(t)+diff(y(t),t) = 1+3*sin(t),y(t), singsol=all)
```

$$y(t) = -1 - \frac{3\cos(t)}{2} - \frac{3\sin(t)}{2} + e^t c_1$$

✓ Solution by Mathematica
Time used: 0.072 (sec). Leaf size: 25

```
DSolve[-y[t]+y'[t] == 1+3*Sin[t],y[t],t,IncludeSingularSolutions -> True]
```

$$y(t) \to -\frac{3\sin(t)}{2} - \frac{3\cos(t)}{2} + c_1 e^t - 1$$

1.31 problem 31

Internal problem ID [478]

Book: Elementary differential equations and boundary value problems, 10th ed., Boyce and DiPrima
Section: Section 2.1. Page 40
Problem number: 31.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[[_linear, 'class A']]`

$$\boxed{-\frac{3y}{2} + y' = 2\,\mathrm{e}^{t} + 3t}$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 19

```
dsolve(-3/2*y(t)+diff(y(t),t) = 2*exp(t)+3*t,y(t), singsol=all)
```

$$y(t) = -2t - \frac{4}{3} - 4\,\mathrm{e}^{t} + \mathrm{e}^{\frac{3t}{2}} c_1$$

✓ Solution by Mathematica

Time used: 0.1 (sec). Leaf size: 27

```
DSolve[-3/2*y[t]+y'[t] == 2*Exp[t]+3*t,y[t],t,IncludeSingularSolutions -> True]
```

$$y(t) \to -2t - 4e^{t} + c_1 e^{3t/2} - \frac{4}{3}$$

2 Section 2.2. Page 48

2.1 problem 1

Internal problem ID [479]

Book: Elementary differential equations and boundary value problems, 10th ed., Boyce and DiPrima
Section: Section 2.2. Page 48
Problem number: 1.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_separable]`

$$\boxed{y' - \frac{x^2}{y} = 0}$$

✓ Solution by Maple
Time used: 0.0 (sec). Leaf size: 33

```
dsolve(diff(y(x),x) = x^2/y(x),y(x), singsol=all)
```

$$y(x) = -\frac{\sqrt{6x^3 + 9c_1}}{3}$$
$$y(x) = \frac{\sqrt{6x^3 + 9c_1}}{3}$$

✓ Solution by Mathematica
Time used: 0.093 (sec). Leaf size: 50

```
DSolve[y'[x] == x^2/y[x],y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to -\sqrt{\frac{2}{3}}\sqrt{x^3 + 3c_1}$$
$$y(x) \to \sqrt{\frac{2}{3}}\sqrt{x^3 + 3c_1}$$

2.2 problem 2

Internal problem ID [480]

Book: Elementary differential equations and boundary value problems, 10th ed., Boyce and DiPrima
Section: Section 2.2. Page 48
Problem number: 2.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_separable]`

$$\boxed{y' - \frac{x^2}{(x^3+1)\,y} = 0}$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 39

```
dsolve(diff(y(x),x) = x^2/(x^3+1)/y(x),y(x), singsol=all)
```

$$y(x) = -\frac{\sqrt{6\ln(x^3+1)+9c_1}}{3}$$
$$y(x) = \frac{\sqrt{6\ln(x^3+1)+9c_1}}{3}$$

✓ Solution by Mathematica

Time used: 0.12 (sec). Leaf size: 56

```
DSolve[y'[x] == x^2/(x^3+1)/y[x],y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to -\sqrt{\frac{2}{3}}\sqrt{\log(x^3+1)+3c_1}$$
$$y(x) \to \sqrt{\frac{2}{3}}\sqrt{\log(x^3+1)+3c_1}$$

2.3 problem 3

Internal problem ID [481]

Book: Elementary differential equations and boundary value problems, 10th ed., Boyce and DiPrima
Section: Section 2.2. Page 48
Problem number: 3.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_separable]`

$$\boxed{\sin(x)\, y^2 + y' = 0}$$

✓ Solution by Maple
Time used: 0.0 (sec). Leaf size: 12

```
dsolve(sin(x)*y(x)^2+diff(y(x),x) = 0,y(x), singsol=all)
```

$$y(x) = \frac{1}{-\cos(x) + c_1}$$

✓ Solution by Mathematica
Time used: 0.133 (sec). Leaf size: 19

```
DSolve[Sin[x]*y[x]^2+y'[x]== 0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to -\frac{1}{\cos(x) + c_1}$$
$$y(x) \to 0$$

2.4 problem 4

Internal problem ID [482]

Book: Elementary differential equations and boundary value problems, 10th ed., Boyce and DiPrima
Section: Section 2.2. Page 48
Problem number: 4.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_separable]`

$$\boxed{y' - \frac{3x^2 - 1}{3 + 2y} = 0}$$

✓ Solution by Maple
Time used: 0.0 (sec). Leaf size: 45

```
dsolve(diff(y(x),x) = (3*x^2-1)/(3+2*y(x)),y(x), singsol=all)
```

$$y(x) = -\frac{3}{2} - \frac{\sqrt{4x^3 + 4c_1 - 4x + 9}}{2}$$
$$y(x) = -\frac{3}{2} + \frac{\sqrt{4x^3 + 4c_1 - 4x + 9}}{2}$$

✓ Solution by Mathematica
Time used: 0.121 (sec). Leaf size: 59

```
DSolve[y'[x] == (3*x^2-1)/(3+2*y[x]),y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to \frac{1}{2}\left(-3 - \sqrt{4x^3 - 4x + 9 + 4c_1}\right)$$
$$y(x) \to \frac{1}{2}\left(-3 + \sqrt{4x^3 - 4x + 9 + 4c_1}\right)$$

2.5 problem 5

Internal problem ID [483]

Book: Elementary differential equations and boundary value problems, 10th ed., Boyce and DiPrima
Section: Section 2.2. Page 48
Problem number: 5.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_separable]`

$$\boxed{y' - \cos(x)^2 \cos(2y)^2 = 0}$$

✓ Solution by Maple

Time used: 0.015 (sec). Leaf size: 18

```
dsolve(diff(y(x),x) = cos(x)^2*cos(2*y(x))^2,y(x), singsol=all)
```

$$y(x) = \frac{\arctan\left(x + 2c_1 + \frac{\sin(2x)}{2}\right)}{2}$$

✓ Solution by Mathematica

Time used: 1.312 (sec). Leaf size: 63

```
DSolve[y'[x] == Cos[x]^2*Cos[2*y[x]]^2,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to \frac{1}{2}\arctan\left(x + \sin(x)\cos(x) + \frac{c_1}{4}\right)$$
$$y(x) \to \frac{1}{2}\arctan\left(x + \sin(x)\cos(x) + \frac{c_1}{4}\right)$$
$$y(x) \to -\frac{\pi}{4}$$
$$y(x) \to \frac{\pi}{4}$$

2.6 problem 6

Internal problem ID [484]

Book: Elementary differential equations and boundary value problems, 10th ed., Boyce and DiPrima
Section: Section 2.2. Page 48
Problem number: 6.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_separable]`

$$\boxed{y'x - \sqrt{1-y^2} = 0}$$

✓ Solution by Maple
Time used: 0.0 (sec). Leaf size: 9

```
dsolve(x*diff(y(x),x) = (1-y(x)^2)^(1/2),y(x), singsol=all)
```

$$y(x) = \sin(\ln(x) + c_1)$$

✓ Solution by Mathematica
Time used: 0.206 (sec). Leaf size: 29

```
DSolve[x*y'[x] == (1-y[x]^2)^(1/2),y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to \cos(\log(x) + c_1)$$
$$y(x) \to -1$$
$$y(x) \to 1$$
$$y(x) \to \text{Interval}[\{-1, 1\}]$$

2.7 problem 7

Internal problem ID [485]

Book: Elementary differential equations and boundary value problems, 10th ed., Boyce and DiPrima
Section: Section 2.2. Page 48
Problem number: 7.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[‘y=_G(x,y’)‘]`

$$\boxed{y' - \frac{-\mathrm{e}^{-x} + x}{\mathrm{e}^{y} + x} = 0}$$

✗ Solution by Maple

```
dsolve(diff(y(x),x) = (-exp(-x)+x)/(exp(y(x))+x),y(x), singsol=all)
```

No solution found

✗ Solution by Mathematica

Time used: 0.0 (sec). Leaf size: 0

```
DSolve[y'[x] == (-Exp[-x]+x)/(Exp[y[x]]+x),y[x],x,IncludeSingularSolutions -> True]
```

Not solved

2.8 problem 8

Internal problem ID [486]

Book: Elementary differential equations and boundary value problems, 10th ed., Boyce and DiPrima
Section: Section 2.2. Page 48
Problem number: 8.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_separable]`

$$\boxed{y' - \frac{x^2}{1+y^2} = 0}$$

✓ Solution by Maple
Time used: 0.016 (sec). Leaf size: 268

```
dsolve(diff(y(x),x) = x^2/(1+y(x)^2),y(x), singsol=all)
```

$$y(x) = \frac{\left(4x^3+12c_1+4\sqrt{x^6+6c_1x^3+9c_1^2+4}\right)^{\frac{2}{3}}-4}{2\left(4x^3+12c_1+4\sqrt{x^6+6c_1x^3+9c_1^2+4}\right)^{\frac{1}{3}}}$$

$$y(x) = -\frac{\left(1+i\sqrt{3}\right)\left(4x^3+12c_1+4\sqrt{x^6+6c_1x^3+9c_1^2+4}\right)^{\frac{2}{3}}+4i\sqrt{3}-4}{4\left(4x^3+12c_1+4\sqrt{x^6+6c_1x^3+9c_1^2+4}\right)^{\frac{1}{3}}}$$

$$y(x) = \frac{i\left(4x^3+12c_1+4\sqrt{x^6+6c_1x^3+9c_1^2+4}\right)^{\frac{2}{3}}\sqrt{3}+4i\sqrt{3}-\left(4x^3+12c_1+4\sqrt{x^6+6c_1x^3+9c_1^2+4}\right)^{\frac{2}{3}}+4}{4\left(4x^3+12c_1+4\sqrt{x^6+6c_1x^3+9c_1^2+4}\right)^{\frac{1}{3}}}$$

✓ Solution by Mathematica

Time used: 2.246 (sec). Leaf size: 307

```
DSolve[y'[x]== x^2/(1+y[x]^2),y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to \frac{-2+\sqrt[3]{2}\left(x^3+\sqrt{x^6+6c_1x^3+4+9{c_1}^2}+3c_1\right)^{2/3}}{2^{2/3}\sqrt[3]{x^3+\sqrt{x^6+6c_1x^3+4+9{c_1}^2}+3c_1}}$$

$$y(x) \to \frac{i\left(\sqrt{3}+i\right)\sqrt[3]{x^3+\sqrt{x^6+6c_1x^3+4+9{c_1}^2}+3c_1}}{2\sqrt[3]{2}} + \frac{1+i\sqrt{3}}{2^{2/3}\sqrt[3]{x^3+\sqrt{x^6+6c_1x^3+4+9{c_1}^2}+3c_1}}$$

$$y(x) \to \frac{1-i\sqrt{3}}{2^{2/3}\sqrt[3]{x^3+\sqrt{x^6+6c_1x^3+4+9{c_1}^2}+3c_1}} - \frac{\left(1+i\sqrt{3}\right)\sqrt[3]{x^3+\sqrt{x^6+6c_1x^3+4+9{c_1}^2}+3c_1}}{2\sqrt[3]{2}}$$

2.9 problem 9

Internal problem ID [487]

Book: Elementary differential equations and boundary value problems, 10th ed., Boyce and DiPrima
Section: Section 2.2. Page 48
Problem number: 9.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_separable]`

$$\boxed{y' - (1 - 2x)\, y^2 = 0}$$

With initial conditions

$$\left[y(0) = -\frac{1}{6}\right]$$

✓ Solution by Maple
Time used: 0.062 (sec). Leaf size: 14

```
dsolve([diff(y(x),x) = (1-2*x)*y(x)^2,y(0) = -1/6],y(x), singsol=all)
```

$$y(x) = \frac{1}{x^2 - x - 6}$$

✓ Solution by Mathematica
Time used: 0.131 (sec). Leaf size: 15

```
DSolve[{y'[x] == (1-2*x)*y[x]^2,y[0]==-1/6},y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to \frac{1}{x^2 - x - 6}$$

2.10 problem 10

Internal problem ID [488]

Book: Elementary differential equations and boundary value problems, 10th ed., Boyce and DiPrima
Section: Section 2.2. Page 48
Problem number: 10.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_separable]`

$$\boxed{y' - \frac{1-2x}{y} = 0}$$

With initial conditions

$$[y(1) = -2]$$

✓ Solution by Maple
Time used: 0.047 (sec). Leaf size: 18

```
dsolve([diff(y(x),x) = (1-2*x)/y(x),y(1) = -2],y(x), singsol=all)
```

$$y(x) = -\sqrt{-2x^2 + 2x + 4}$$

✓ Solution by Mathematica
Time used: 0.094 (sec). Leaf size: 24

```
DSolve[{y'[x] == (1-2*x)/y[x],y[1]==-2},y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to -\sqrt{2}\sqrt{-x^2 + x + 2}$$

2.11 problem 11

Internal problem ID [489]

Book: Elementary differential equations and boundary value problems, 10th ed., Boyce and DiPrima
Section: Section 2.2. Page 48
Problem number: 11.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_separable]`

$$\boxed{y' e^{-x} y = -x}$$

With initial conditions

$$[y(0) = 1]$$

✓ Solution by Maple
Time used: 0.078 (sec). Leaf size: 17

```
dsolve([x+y(x)*diff(y(x),x)/exp(x) = 0,y(0) = 1],y(x), singsol=all)
```

$$y(x) = \sqrt{-1 - 2x\,e^{x} + 2\,e^{x}}$$

✓ Solution by Mathematica
Time used: 1.763 (sec). Leaf size: 19

```
DSolve[{x+y[x]*y'[x]/Exp[x] == 0,y[0]==1},y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to \sqrt{-2e^{x}(x-1) - 1}$$

2.12 problem 12

Internal problem ID [490]

Book: Elementary differential equations and boundary value problems, 10th ed., Boyce and DiPrima
Section: Section 2.2. Page 48
Problem number: 12.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_separable]`

$$\boxed{r' - \frac{r^2}{x} = 0}$$

With initial conditions

$$[r(1) = 2]$$

✓ Solution by Maple
Time used: 0.015 (sec). Leaf size: 14

```
dsolve([diff(r(x),x) = r(x)^2/x,r(1) = 2],r(x), singsol=all)
```

$$r(x) = -\frac{2}{2\ln(x) - 1}$$

✓ Solution by Mathematica
Time used: 0.128 (sec). Leaf size: 15

```
DSolve[{r'[x] == r[x]^2/x,r[1]==2},r[x],x,IncludeSingularSolutions -> True]
```

$$r(x) \to \frac{2}{1 - 2\log(x)}$$

2.13 problem 13

Internal problem ID [491]

Book: Elementary differential equations and boundary value problems, 10th ed., Boyce and DiPrima
Section: Section 2.2. Page 48
Problem number: 13.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_separable]`

$$\boxed{y' - \frac{2x}{y + x^2 y} = 0}$$

With initial conditions

$$[y(0) = -2]$$

✓ Solution by Maple
Time used: 0.031 (sec). Leaf size: 18

```
dsolve([diff(y(x),x) = 2*x/(y(x)+x^2*y(x)),y(0) = -2],y(x), singsol=all)
```

$$y(x) = -\sqrt{2 \ln (x^2 + 1) + 4}$$

✓ Solution by Mathematica
Time used: 0.091 (sec). Leaf size: 24

```
DSolve[{y'[x] == 2*x/(y[x]+x^2*y[x]),y[0]==-2},y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to -\sqrt{2}\sqrt{\log (x^2 + 1) + 2}$$

2.14 problem 14

Internal problem ID [492]

Book: Elementary differential equations and boundary value problems, 10th ed., Boyce and DiPrima
Section: Section 2.2. Page 48
Problem number: 14.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_separable]`

$$\boxed{y' - \frac{x y^2}{\sqrt{x^2+1}} = 0}$$

With initial conditions

$$[y(0) = 1]$$

✓ Solution by Maple

Time used: 0.032 (sec). Leaf size: 17

```
dsolve([diff(y(x),x) = x*y(x)^2/(x^2+1)^(1/2),y(0) = 1],y(x), singsol=all)
```

$$y(x) = -\frac{1}{\sqrt{x^2+1}-2}$$

✓ Solution by Mathematica

Time used: 0.183 (sec). Leaf size: 20

```
DSolve[{y'[x] == x*y[x]^2/(x^2+1)^(1/2),y[0]==1},y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to \frac{1}{2-\sqrt{x^2+1}}$$

2.15 problem 15

Internal problem ID [493]

Book: Elementary differential equations and boundary value problems, 10th ed., Boyce and DiPrima
Section: Section 2.2. Page 48
Problem number: 15.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_separable]`

$$\boxed{y' - \frac{2x}{1+2y} = 0}$$

With initial conditions

$$[y(2) = 0]$$

✓ Solution by Maple
Time used: 0.047 (sec). Leaf size: 17

```
dsolve([diff(y(x),x) = 2*x/(1+2*y(x)),y(2) = 0],y(x), singsol=all)
```

$$y(x) = -\frac{1}{2} + \frac{\sqrt{4x^2-15}}{2}$$

✓ Solution by Mathematica
Time used: 0.115 (sec). Leaf size: 22

```
DSolve[{y'[x] == 2*x/(1+2*y[x]),y[2]==0},y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to \frac{1}{2}\left(\sqrt{4x^2-15}-1\right)$$

2.16 problem 16

Internal problem ID [494]

Book: Elementary differential equations and boundary value problems, 10th ed., Boyce and DiPrima
Section: Section 2.2. Page 48
Problem number: 16.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_separable]`

$$\boxed{y' - \frac{x(x^2+1)}{4y^3} = 0}$$

With initial conditions

$$\left[y(0) = -\frac{\sqrt{2}}{2}\right]$$

✓ Solution by Maple
Time used: 0.078 (sec). Leaf size: 15

```
dsolve([diff(y(x),x) = 1/4*x*(x^2+1)/y(x)^3,y(0) = -1/2*2^(1/2)],y(x), singsol=all)
```

$$y(x) = -\frac{\sqrt{2x^2+2}}{2}$$

✓ Solution by Mathematica
Time used: 0.23 (sec). Leaf size: 23

```
DSolve[{y'[x] == 1/4*x*(x^2+1)/y[x]^3,y[0]==-(1/Sqrt[2])},y[x],x,IncludeSingularSolutions ->
```

$$y(x) \to -\frac{\sqrt[4]{(x^2+1)^2}}{\sqrt{2}}$$

2.17 problem 17

Internal problem ID [495]

Book: Elementary differential equations and boundary value problems, 10th ed., Boyce and DiPrima
Section: Section 2.2. Page 48
Problem number: 17.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_separable]`

$$\boxed{y' - \frac{-e^x + 3x^2}{-5 + 2y} = 0}$$

With initial conditions

$$[y(0) = 1]$$

✓ Solution by Maple
Time used: 0.172 (sec). Leaf size: 21

```
dsolve([diff(y(x),x) = (-exp(x)+3*x^2)/(-5+2*y(x)),y(0) = 1],y(x), singsol=all)
```

$$y(x) = \frac{5}{2} - \frac{\sqrt{13 + 4x^3 - 4\,e^x}}{2}$$

✓ Solution by Mathematica
Time used: 0.891 (sec). Leaf size: 29

```
DSolve[{y'[x] == (-Exp[x]+3*x^2)/(-5+2*y[x]),y[0]==1},y[x],x,IncludeSingularSolutions -> Tru
```

$$y(x) \to \frac{1}{2}\left(5 - \sqrt{4x^3 - 4e^x + 13}\right)$$

2.18 problem 18

Internal problem ID [496]

Book: Elementary differential equations and boundary value problems, 10th ed., Boyce and DiPrima
Section: Section 2.2. Page 48
Problem number: 18.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_separable]`

$$\boxed{y' - \frac{-\mathrm{e}^{x} + \mathrm{e}^{-x}}{3 + 4y} = 0}$$

With initial conditions

$$[y(0) = 1]$$

✓ Solution by Maple

Time used: 0.235 (sec). Leaf size: 29

```
dsolve([diff(y(x),x) = (exp(-x)-exp(x))/(3+4*y(x)),y(0) = 1],y(x), singsol=all)
```

$$y(x) = -\frac{3}{4} + \frac{\sqrt{\mathrm{e}^{x} \left(-8\,\mathrm{e}^{2x} + 65\,\mathrm{e}^{x} - 8\right)}\,\mathrm{e}^{-x}}{4}$$

✓ Solution by Mathematica

Time used: 1.347 (sec). Leaf size: 29

```
DSolve[{y'[x] == (Exp[-x]-Exp[x])/(3+4*y[x]),y[0]==1},y[x],x,IncludeSingularSolutions -> Tru
```

$$y(x) \to \frac{1}{4}\left(\sqrt{-8e^{-x} - 8e^{x} + 65} - 3\right)$$

2.19 problem 19

Internal problem ID [497]

Book: Elementary differential equations and boundary value problems, 10th ed., Boyce and DiPrima
Section: Section 2.2. Page 48
Problem number: 19.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_separable]`

$$\boxed{\cos(3y)\, y' = -\sin(2x)}$$

With initial conditions

$$\left[y\left(\frac{\pi}{2}\right) = 0\right]$$

✓ Solution by Maple
Time used: 0.297 (sec). Leaf size: 15

```
dsolve([sin(2*x)+cos(3*y(x))*diff(y(x),x) = 0,y(1/2*Pi) = 0],y(x), singsol=all)
```

$$y(x) = \frac{\arcsin\left(\frac{3}{2} + \frac{3\cos(2x)}{2}\right)}{3}$$

✓ Solution by Mathematica
Time used: 0.614 (sec). Leaf size: 16

```
DSolve[{Sin[2*x]+Cos[3*y[x]]*y'[x] == 0,y[Pi/2]==0},y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to \frac{1}{3}\arcsin\left(3\cos^2(x)\right)$$

2.20 problem 20

Internal problem ID [498]

Book: Elementary differential equations and boundary value problems, 10th ed., Boyce and DiPrima
Section: Section 2.2. Page 48
Problem number: 20.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type **[_separable]**

$$\boxed{\sqrt{-x^2+1}\,y^2y' = \arcsin(x)}$$

With initial conditions

$$[y(0) = 1]$$

✓ Solution by Maple
Time used: 0.109 (sec). Leaf size: 16

```
dsolve([(-x^2+1)^(1/2)*y(x)^2*diff(y(x),x) = arcsin(x),y(0) = 1],y(x), singsol=all)
```

$$y(x) = \frac{\left(8+12\arcsin(x)^2\right)^{\frac{1}{3}}}{2}$$

✓ Solution by Mathematica
Time used: 0.527 (sec). Leaf size: 19

```
DSolve[{(-x^2+1)^(1/2)*y[x]^2*y'[x] == ArcSin[x],y[0]==1},y[x],x,IncludeSingularSolutions ->
```

$$y(x) \to \sqrt[3]{\frac{3\arcsin(x)^2}{2}+1}$$

2.21 problem 21

Internal problem ID [499]

Book: Elementary differential equations and boundary value problems, 10th ed., Boyce and DiPrima
Section: Section 2.2. Page 48
Problem number: 21.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_separable]`

$$\boxed{y' - \frac{3x^2+1}{-6y+3y^2} = 0}$$

With initial conditions

$$[y(0) = 1]$$

✓ Solution by Maple
Time used: 0.11 (sec). Leaf size: 109

```
dsolve([diff(y(x),x) = (3*x^2+1)/(-6*y(x)+3*y(x)^2),y(0) = 1],y(x), singsol=all)
```

$$y(x) = -\frac{\left(1+i\sqrt{3}\right)\left(4x^3+4x+4\sqrt{x^6+2x^4+x^2-4}\right)^{\frac{2}{3}} - 4i\sqrt{3} - 4\left(4x^3+4x+4\sqrt{x^6+2x^4+x^2-4}\right)^{\frac{1}{3}} + 4}{4\left(4x^3+4x+4\sqrt{x^6+2x^4+x^2-4}\right)^{\frac{1}{3}}}$$

✓ Solution by Mathematica
Time used: 4.019 (sec). Leaf size: 158

```
DSolve[{y'[x] == (3*x^2+1)/(-6*y[x]+3*y[x]^2),y[0]==1},y[x],x,IncludeSingularSolutions -> Tr
```

$$y(x) \to \frac{-i2^{2/3}\sqrt{3}\left(x^3+\sqrt{x^6+2x^4+x^2-4}+x\right)^{2/3} - 2^{2/3}\left(x^3+\sqrt{x^6+2x^4+x^2-4}+x\right)^{2/3} + 4\sqrt[3]{x^3+\sqrt{x^6+}}}{4\sqrt[3]{x^3+\sqrt{x^6+2x^4+x^2-4}+x}}$$

2.22 problem 22

Internal problem ID [500]

Book: Elementary differential equations and boundary value problems, 10th ed., Boyce and DiPrima
Section: Section 2.2. Page 48
Problem number: 22.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_separable]`

$$\boxed{y' - \frac{3x^2}{-4+3y^2} = 0}$$

With initial conditions

$$[y(1) = 0]$$

✓ Solution by Maple

Time used: 0.094 (sec). Leaf size: 73

```
dsolve([diff(y(x),x) = 3*x^2/(-4+3*y(x)^2),y(1) = 0],y(x), singsol=all)
```

$$y(x) = -\frac{\left(1+i\sqrt{3}\right)\left(-108+108x^3+12\sqrt{81x^6-162x^3-687}\right)^{\frac{2}{3}} - 48i\sqrt{3}+48}{12\left(-108+108x^3+12\sqrt{81x^6-162x^3-687}\right)^{\frac{1}{3}}}$$

✓ Solution by Mathematica

Time used: 9.526 (sec). Leaf size: 137

```
DSolve[{y'[x]== 3*x^2/(-4+3*y[x]^2),y[1]==0},y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to \frac{-i\sqrt[3]{2}3^{2/3}\left(9x^3+\sqrt{81x^6-162x^3-687}-9\right)^{2/3} - \sqrt[3]{2}\sqrt[6]{3}\left(9x^3+\sqrt{81x^6-162x^3-687}-9\right)^{2/3} - 8\sqrt{3}+}{2\,2^{2/3}3^{5/6}\sqrt[3]{9x^3+\sqrt{81x^6-162x^3-687}-9}}$$

2.23 problem 23

Internal problem ID [501]

Book: Elementary differential equations and boundary value problems, 10th ed., Boyce and DiPrima
Section: Section 2.2. Page 48
Problem number: 23.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_separable]`

$$\boxed{y' - 2y^2 - xy^2 = 0}$$

With initial conditions

$$[y(0) = 1]$$

✓ Solution by Maple
Time used: 0.063 (sec). Leaf size: 16

```
dsolve([diff(y(x),x) = 2*y(x)^2+x*y(x)^2,y(0) = 1],y(x), singsol=all)
```

$$y(x) = -\frac{2}{x^2 + 4x - 2}$$

✓ Solution by Mathematica
Time used: 0.127 (sec). Leaf size: 17

```
DSolve[{y'[x] == 2*y[x]^2+x*y[x]^2,y[0]==1},y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to -\frac{2}{x^2 + 4x - 2}$$

2.24 problem 24

Internal problem ID [502]

Book: Elementary differential equations and boundary value problems, 10th ed., Boyce and DiPrima
Section: Section 2.2. Page 48
Problem number: 24.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_separable]`

$$\boxed{y' - \frac{2 - e^x}{3 + 2y} = 0}$$

With initial conditions

$$[y(0) = 0]$$

✓ Solution by Maple
Time used: 0.141 (sec). Leaf size: 19

```
dsolve([diff(y(x),x) = (2-exp(x))/(3+2*y(x)),y(0) = 0],y(x), singsol=all)
```

$$y(x) = -\frac{3}{2} + \frac{\sqrt{13 - 4\,e^x + 8x}}{2}$$

✓ Solution by Mathematica
Time used: 0.737 (sec). Leaf size: 25

```
DSolve[{y'[x] == (2-Exp[x])/(3+2*y[x]),y[0]==0},y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to \frac{1}{2}\left(\sqrt{8x - 4e^x + 13} - 3\right)$$

2.25 problem 25

Internal problem ID [503]

Book: Elementary differential equations and boundary value problems, 10th ed., Boyce and DiPrima
Section: Section 2.2. Page 48
Problem number: 25.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_separable]`

$$\boxed{y' - \frac{2\cos(2x)}{3+2y} = 0}$$

With initial conditions

$$[y(0) = -1]$$

✓ Solution by Maple
Time used: 0.25 (sec). Leaf size: 18

```
dsolve([diff(y(x),x) = 2*cos(2*x)/(3+2*y(x)),y(0) = -1],y(x), singsol=all)
```

$$y(x) = -\frac{3}{2} + \frac{\sqrt{1+4\sin(2x)}}{2}$$

✓ Solution by Mathematica
Time used: 0.175 (sec). Leaf size: 23

```
DSolve[{y'[x] == 2*Cos[2*x]/(3+2*y[x]),y[0]==-1},y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to \frac{1}{2}\left(\sqrt{4\sin(2x)+1} - 3\right)$$

2.26 problem 26

Internal problem ID [504]

Book: Elementary differential equations and boundary value problems, 10th ed., Boyce and DiPrima
Section: Section 2.2. Page 48
Problem number: 26.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_separable]`

$$\boxed{y' - 2(x+1)\left(1+y^2\right) = 0}$$

With initial conditions

$$[y(0) = 0]$$

✓ Solution by Maple

Time used: 0.031 (sec). Leaf size: 12

```
dsolve([diff(y(x),x) = 2*(1+x)*(1+y(x)^2),y(0) = 0],y(x), singsol=all)
```

$$y(x) = \tan\left(x^2 + 2x\right)$$

✓ Solution by Mathematica

Time used: 0.199 (sec). Leaf size: 11

```
DSolve[{y'[x] == 2*(1+x)*(1+y[x]^2),y[0]==0},y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to \tan(x(x+2))$$

2.27 problem 27

Internal problem ID [505]

Book: Elementary differential equations and boundary value problems, 10th ed., Boyce and DiPrima
Section: Section 2.2. Page 48
Problem number: 27.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_separable]`

$$\boxed{y' - \frac{t(4-y)\,y}{3} = 0}$$

✓ Solution by Maple
Time used: 0.016 (sec). Leaf size: 19

```
dsolve(diff(y(t),t) = 1/3*t*(4-y(t))*y(t),y(t), singsol=all)
```

$$y(t) = \frac{4}{1 + 4\,\mathrm{e}^{-\frac{2t^2}{3}} c_1}$$

✓ Solution by Mathematica
Time used: 0.248 (sec). Leaf size: 44

```
DSolve[y'[t]== 1/3*t*(4-y[t])*y[t],y[t],t,IncludeSingularSolutions -> True]
```

$$y(t) \to \frac{4e^{\frac{2t^2}{3}}}{e^{\frac{2t^2}{3}} + e^{4c_1}}$$
$$y(t) \to 0$$
$$y(t) \to 4$$

2.28 problem 28

Internal problem ID [506]

Book: Elementary differential equations and boundary value problems, 10th ed., Boyce and DiPrima
Section: Section 2.2. Page 48
Problem number: 28.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_separable]`

$$\boxed{y' - \frac{ty(4-y)}{t+1} = 0}$$

✓ Solution by Maple
Time used: 0.0 (sec). Leaf size: 22

```
dsolve(diff(y(t),t) = t*y(t)*(4-y(t))/(1+t),y(t), singsol=all)
```

$$y(t) = \frac{4}{1 + 4\,\mathrm{e}^{-4t}\,(t+1)^4\,c_1}$$

✓ Solution by Mathematica
Time used: 1.337 (sec). Leaf size: 42

```
DSolve[y'[t] == t*y[t]*(4-y[t])/(1+t),y[t],t,IncludeSingularSolutions -> True]
```

$$y(t) \to \frac{4e^{4t}}{e^{4t} + e^{4c_1}(t+1)^4}$$
$$y(t) \to 0$$
$$y(t) \to 4$$

2.29 problem 29

Internal problem ID [507]

Book: Elementary differential equations and boundary value problems, 10th ed., Boyce and DiPrima
Section: Section 2.2. Page 48
Problem number: 29.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_quadrature]`

$$\boxed{y' - \frac{b + ay}{d + cy} = 0}$$

✓ Solution by Maple
Time used: 0.016 (sec). Leaf size: 61

```
dsolve(diff(y(x),x) = (b+a*y(x))/(d+c*y(x)),y(x), singsol=all)
```

$$y(x) = \frac{(ad - bc)\,\mathrm{LambertW}\left(\frac{c\,e^{\frac{(c_1+x)a^2+bc}{ad-bc}}}{ad-bc}\right) - bc}{ac}$$

✓ Solution by Mathematica
Time used: 16.166 (sec). Leaf size: 83

```
DSolve[y'[x] == (b+a*y[x])/(d+c*y[x]),y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to \frac{-bc + (ad - bc)W\left(-\frac{c\left(e^{-1-\frac{a^2(x+c_1)}{bc}}\right)^{\frac{bc}{bc-ad}}}{bc-ad}\right)}{ac}$$

$$y(x) \to -\frac{b}{a}$$

2.30 problem 31

Internal problem ID [508]

Book: Elementary differential equations and boundary value problems, 10th ed., Boyce and DiPrima
Section: Section 2.2. Page 48
Problem number: 31.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[[_homogeneous, ‘class A‘], _rational, _Riccati]`

$$\boxed{y' - \frac{x^2 + yx + y^2}{x^2} = 0}$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 11

```
dsolve(diff(y(x),x) = (x^2+x*y(x)+y(x)^2)/x^2,y(x), singsol=all)
```

$$y(x) = \tan\left(\ln\left(x\right) + c_1\right) x$$

✓ Solution by Mathematica

Time used: 0.199 (sec). Leaf size: 13

```
DSolve[y'[x] == (x^2+x*y[x]+y[x]^2)/x^2,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to x\tan(\log(x) + c_1)$$

2.31 problem 32

Internal problem ID [509]

Book: Elementary differential equations and boundary value problems, 10th ed., Boyce and DiPrima
Section: Section 2.2. Page 48
Problem number: 32.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[[_homogeneous, ‘class A‘], _rational, _Bernoulli]`

$$\boxed{y' - \frac{x^2 + 3y^2}{2xy} = 0}$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 26

```
dsolve(diff(y(x),x) = (x^2+3*y(x)^2)/(2*x*y(x)),y(x), singsol=all)
```

$$y(x) = \sqrt{c_1 x - 1}\, x$$
$$y(x) = -\sqrt{c_1 x - 1}\, x$$

✓ Solution by Mathematica

Time used: 0.251 (sec). Leaf size: 34

```
DSolve[y'[x] == (x^2+3*y[x]^2)/(2*x*y[x]),y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to -x\sqrt{-1 + c_1 x}$$
$$y(x) \to x\sqrt{-1 + c_1 x}$$

2.32 problem 33

Internal problem ID [510]

Book: Elementary differential equations and boundary value problems, 10th ed., Boyce and DiPrima
Section: Section 2.2. Page 48
Problem number: 33.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[[_homogeneous, ‘class A‘], _rational, [_Abel, ‘2nd type‘, ‘cl`

$$\boxed{y' - \frac{4y - 3x}{2x - y} = 0}$$

✓ Solution by Maple
Time used: 0.375 (sec). Leaf size: 26

```
dsolve(diff(y(x),x) = (4*y(x)-3*x)/(2*x-y(x)),y(x), singsol=all)
```

$$y(x) = x\left(-3 + \operatorname{RootOf}\left(\_Z^{20}c_1x^4 - \_Z^4 + 4\right)^4\right)$$

✓ Solution by Mathematica

Time used: 3.328 (sec). Leaf size: 336

```
DSolve[y'[x] == (4*y[x]-3*x)/(2*x-y[x]),y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to \text{Root}\left[\#1^5 + 15\#1^4x + 90\#1^3x^2 + 270\#1^2x^3 + \#1\left(405x^4 - e^{4c_1}\right) + 243x^5 + e^{4c_1}x\&, 1\right]$$

$$y(x) \to \text{Root}\left[\#1^5 + 15\#1^4x + 90\#1^3x^2 + 270\#1^2x^3 + \#1\left(405x^4 - e^{4c_1}\right) + 243x^5 + e^{4c_1}x\&, 2\right]$$

$$y(x) \to \text{Root}\left[\#1^5 + 15\#1^4x + 90\#1^3x^2 + 270\#1^2x^3 + \#1\left(405x^4 - e^{4c_1}\right) + 243x^5 + e^{4c_1}x\&, 3\right]$$

$$y(x) \to \text{Root}\left[\#1^5 + 15\#1^4x + 90\#1^3x^2 + 270\#1^2x^3 + \#1\left(405x^4 - e^{4c_1}\right) + 243x^5 + e^{4c_1}x\&, 4\right]$$

$$y(x) \to \text{Root}\left[\#1^5 + 15\#1^4x + 90\#1^3x^2 + 270\#1^2x^3 + \#1\left(405x^4 - e^{4c_1}\right) + 243x^5 + e^{4c_1}x\&, 5\right]$$

2.33 problem 34

Internal problem ID [511]

Book: Elementary differential equations and boundary value problems, 10th ed., Boyce and DiPrima
Section: Section 2.2. Page 48
Problem number: 34.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[[_homogeneous, ‘class A‘], _rational, [_Abel, ‘2nd type‘, ‘cl`

$$\boxed{y' + \frac{4x + 3y}{2x + y} = 0}$$

✓ Solution by Maple

Time used: 0.125 (sec). Leaf size: 1228

```
dsolve(diff(y(x),x) = - (4*x+3*y(x))/(2*x+y(x)),y(x), singsol=all)
```

$$y(x) = \frac{\frac{\left(-4c_1x^3+\left(4c_1x^3+4\sqrt{x^6c_1^2(4c_1x^3+1)}\right)^{\frac{2}{3}}\right)^2}{4\left(4c_1x^3+4\sqrt{x^6c_1^2(4c_1x^3+1)}\right)^{\frac{2}{3}}c_1} - x^3}{x^2}$$

$$y(x) = \frac{-3x^3\left(4c_1x^3+4\sqrt{4c_1^3x^9+x^6c_1^2}\right)^{\frac{2}{3}}c_1 + \left(c_1x^3+\sqrt{4c_1^3x^9+x^6c_1^2}\right)\left(4c_1x^3+4\sqrt{4c_1^3x^9+x^6c_1^2}\right)^{\frac{1}{3}} + 4x^6c_1^2}{\left(4c_1x^3+4\sqrt{4c_1^3x^9+x^6c_1^2}\right)^{\frac{2}{3}}c_1x^2}$$

$$y(x) = \frac{-3x^3\left(4c_1x^3+4\sqrt{4c_1^3x^9+x^6c_1^2}\right)^{\frac{2}{3}}c_1 + \left(c_1x^3+\sqrt{4c_1^3x^9+x^6c_1^2}\right)\left(4c_1x^3+4\sqrt{4c_1^3x^9+x^6c_1^2}\right)^{\frac{1}{3}} + 4x^6c_1^2}{\left(4c_1x^3+4\sqrt{4c_1^3x^9+x^6c_1^2}\right)^{\frac{2}{3}}c_1x^2}$$

$$y(x) = -\frac{\frac{\left(4\sqrt{3}\,c_1x^3+\sqrt{3}\left(4c_1x^3+4\sqrt{x^6c_1^2(4c_1x^3+1)}\right)^{\frac{2}{3}}+4ic_1x^3-i\left(4c_1x^3+4\sqrt{x^6c_1^2(4c_1x^3+1)}\right)^{\frac{2}{3}}\right)^2}{16\left(4c_1x^3+4\sqrt{x^6c_1^2(4c_1x^3+1)}\right)^{\frac{2}{3}}c_1} + x^3}{x^2}$$

$$y(x) = -\frac{\frac{\left(4\sqrt{3}\,c_1x^3+\sqrt{3}\left(4c_1x^3+4\sqrt{x^6c_1^2(4c_1x^3+1)}\right)^{\frac{2}{3}}-4ic_1x^3+i\left(4c_1x^3+4\sqrt{x^6c_1^2(4c_1x^3+1)}\right)^{\frac{2}{3}}\right)^2}{16\left(4c_1x^3+4\sqrt{x^6c_1^2(4c_1x^3+1)}\right)^{\frac{2}{3}}c_1} + x^3}{x^2}$$

$$y(x) = -\frac{2\left(\frac{3x^3\left(4c_1x^3+4\sqrt{4c_1^3x^9+x^6c_1^2}\right)^{\frac{2}{3}}c_1}{2} - \frac{\left(c_1x^3+\sqrt{4c_1^3x^9+x^6c_1^2}\right)\left(i\sqrt{3}-1\right)\left(4c_1x^3+4\sqrt{4c_1^3x^9+x^6c_1^2}\right)^{\frac{1}{3}}}{4} + x^6\left(1+i\sqrt{3}\right)c_1^2\right)}{\left(4c_1x^3+4\sqrt{4c_1^3x^9+x^6c_1^2}\right)^{\frac{2}{3}}c_1x^2}$$

$$y(x) = \frac{-3x^3\left(4c_1x^3+4\sqrt{4c_1^3x^9+x^6c_1^2}\right)^{\frac{2}{3}}c_1 - \frac{\left(c_1x^3+\sqrt{4c_1^3x^9+x^6c_1^2}\right)\left(1+i\sqrt{3}\right)\left(4c_1x^3+4\sqrt{4c_1^3x^9+x^6c_1^2}\right)^{\frac{1}{3}}}{2} + 2x^6\left(i\sqrt{3}-1\right)c}{\left(4c_1x^3+4\sqrt{4c_1^3x^9+x^6c_1^2}\right)^{\frac{2}{3}}c_1x^2}$$

$$y(x) = -\frac{2\left(\frac{3x^3\left(4c_1x^3+4\sqrt{4c_1^3x^9+x^6c_1^2}\right)^{\frac{2}{3}}c_1}{2} - \frac{\left(c_1x^3+\sqrt{4c_1^3x^9+x^6c_1^2}\right)\left(i\sqrt{3}-1\right)\left(4c_1x^3+4\sqrt{4c_1^3x^9+x^6c_1^2}\right)^{\frac{1}{3}}}{4} + x^6\left(1+i\sqrt{3}\right)c_1^2\right)}{\left(4c_1x^3+4\sqrt{4c_1^3x^9+x^6c_1^2}\right)^{\frac{2}{3}}c_1x^2}$$

✓ Solution by Mathematica

Time used: 20.375 (sec). Leaf size: 484

```
DSolve[y'[x] == - (4*x+3*y[x])/(2*x+y[x]),y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to \frac{\sqrt[3]{2x^3+\sqrt{4e^{3c_1}x^3+e^{6c_1}}+e^{3c_1}}}{\sqrt[3]{2}}+\frac{\sqrt[3]{2}x^2}{\sqrt[3]{2x^3+\sqrt{4e^{3c_1}x^3+e^{6c_1}}+e^{3c_1}}}-3x$$

$$y(x) \to \frac{i(\sqrt{3}+i)\sqrt[3]{2x^3+\sqrt{4e^{3c_1}x^3+e^{6c_1}}+e^{3c_1}}}{2\sqrt[3]{2}} - \frac{(1+i\sqrt{3})x^2}{2^{2/3}\sqrt[3]{2x^3+\sqrt{4e^{3c_1}x^3+e^{6c_1}}+e^{3c_1}}}-3x$$

$$y(x) \to -\frac{(1+i\sqrt{3})\sqrt[3]{2x^3+\sqrt{4e^{3c_1}x^3+e^{6c_1}}+e^{3c_1}}}{2\sqrt[3]{2}} + \frac{i(\sqrt{3}+i)x^2}{2^{2/3}\sqrt[3]{2x^3+\sqrt{4e^{3c_1}x^3+e^{6c_1}}+e^{3c_1}}}-3x$$

$$y(x) \to \sqrt[3]{x^3}+\frac{(x^3)^{2/3}}{x}-3x$$

$$y(x) \to \frac{1}{2}\left(i\left(\sqrt{3}+i\right)\sqrt[3]{x^3}+\frac{(-1-i\sqrt{3})(x^3)^{2/3}}{x}-6x\right)$$

$$y(x) \to \frac{1}{2}\left(\left(-1-i\sqrt{3}\right)\sqrt[3]{x^3}+\frac{i(\sqrt{3}+i)(x^3)^{2/3}}{x}-6x\right)$$

2.34 problem 35

Internal problem ID [512]

Book: Elementary differential equations and boundary value problems, 10th ed., Boyce and DiPrima
Section: Section 2.2. Page 48
Problem number: 35.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[[_homogeneous, ‘class A‘], _rational, [_Abel, ‘2nd type‘, ‘cl`

$$\boxed{y' - \frac{x+3y}{x-y} = 0}$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 21

```
dsolve(diff(y(x),x) = (x+3*y(x))/(x-y(x)),y(x), singsol=all)
```

$$y(x) = -\frac{x(\text{LambertW}\left(-2c_1x\right)+2)}{\text{LambertW}\left(-2c_1x\right)}$$

✓ Solution by Mathematica

Time used: 0.113 (sec). Leaf size: 33

```
DSolve[y'[x] == (x+3*y[x])/(x-y[x]),y[x],x,IncludeSingularSolutions -> True]
```

$$\text{Solve}\left[\frac{2}{\frac{y(x)}{x}+1}+\log\left(\frac{y(x)}{x}+1\right) = -\log(x)+c_1, y(x)\right]$$

2.35 problem 36

Internal problem ID [513]

Book: Elementary differential equations and boundary value problems, 10th ed., Boyce and DiPrima
Section: Section 2.2. Page 48
Problem number: 36.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[[_homogeneous, ‘class A‘], _rational, _Riccati]`

$$\boxed{3yx + y^2 - y'x^2 = -x^2}$$

✓ Solution by Maple
Time used: 0.0 (sec). Leaf size: 18

```
dsolve((x^2+3*x*y(x)+y(x)^2)-x^2* diff(y(x),x)=0,y(x), singsol=all)
```

$$y(x) = -\frac{x(\ln(x) + c_1 + 1)}{\ln(x) + c_1}$$

✓ Solution by Mathematica
Time used: 0.141 (sec). Leaf size: 28

```
DSolve[(x^2+3*x*y[x]+y[x]^2)-x^2* y'[x]==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to -\frac{x(\log(x) + 1 + c_1)}{\log(x) + c_1}$$
$$y(x) \to -x$$

2.36 problem 37

Internal problem ID [514]

Book: Elementary differential equations and boundary value problems, 10th ed., Boyce and DiPrima
Section: Section 2.2. Page 48
Problem number: 37.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[[_homogeneous, ‘class A‘], _rational, _Bernoulli]`

$$\boxed{y' - \frac{x^2 - 3y^2}{2yx} = 0}$$

✓ Solution by Maple
Time used: 0.015 (sec). Leaf size: 45

```
dsolve(diff(y(x),x) = (x^2-3*y(x)^2)/(2*x*y(x)),y(x), singsol=all)
```

$$y(x) = -\frac{\sqrt{5}\,\sqrt{x\,(x^5+5c_1)}}{5x^2}$$
$$y(x) = \frac{\sqrt{5}\,\sqrt{x\,(x^5+5c_1)}}{5x^2}$$

✓ Solution by Mathematica
Time used: 0.211 (sec). Leaf size: 50

```
DSolve[y'[x] == (x^2-3*y[x]^2)/(2*x*y[x]),y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to -\frac{\sqrt{\frac{x^5}{5}+c_1}}{x^{3/2}}$$
$$y(x) \to \frac{\sqrt{\frac{x^5}{5}+c_1}}{x^{3/2}}$$

2.37 problem 38

Internal problem ID [515]

Book: Elementary differential equations and boundary value problems, 10th ed., Boyce and DiPrima
Section: Section 2.2. Page 48
Problem number: 38.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[[_homogeneous, ‘class A‘], _rational, _Bernoulli]`

$$\boxed{y' - \frac{3y^2 - x^2}{2yx} = 0}$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 26

```
dsolve(diff(y(x),x) = (3*y(x)^2-x^2)/(2*x*y(x)),y(x), singsol=all)
```

$$y(x) = \sqrt{c_1 x + 1}\, x$$
$$y(x) = -\sqrt{c_1 x + 1}\, x$$

✓ Solution by Mathematica

Time used: 0.209 (sec). Leaf size: 34

```
DSolve[y'[x] == (3*y[x]^2-x^2)/(2*x*y[x]),y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to -x\sqrt{1 + c_1 x}$$
$$y(x) \to x\sqrt{1 + c_1 x}$$

3 Section 2.4. Page 76

3.1 problem 1

Internal problem ID [516]

Book: Elementary differential equations and boundary value problems, 10th ed., Boyce and DiPrima
Section: Section 2.4. Page 76
Problem number: 1.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_linear]`

$$\boxed{\ln(t)\, y + (t-3)\, y' = 2t}$$

✓ Solution by Maple
Time used: 0.0 (sec). Leaf size: 57

```
dsolve(ln(t)*y(t)+(-3+t)*diff(y(t),t) = 2*t,y(t), singsol=all)
```

$$y(t) = e^{\ln(3)^2+\mathrm{dilog}\left(\frac{t}{3}\right)}(-t+3)^{-\ln(3)}\left(-2\left(\int t(-t+3)^{-1+\ln(3)}\, e^{-\ln(3)^2-\mathrm{dilog}\left(\frac{t}{3}\right)}dt\right)+c_1\right)$$

✓ Solution by Mathematica
Time used: 0.188 (sec). Leaf size: 69

```
DSolve[Log[t]*y[t]+(-3+t)*y'[t] == 2*t,y[t],t,IncludeSingularSolutions -> True]
```

$$y(t) \to e^{\mathrm{PolyLog}\left(2,1-\frac{t}{3}\right)-\log(3)\log(t-3)}\left(\int_1^t \frac{2e^{\log(3)\log(K[1]-3)-\mathrm{PolyLog}\left(2,1-\frac{K[1]}{3}\right)}K[1]}{K[1]-3}dK[1]+c_1\right)$$

3.2 problem 2

Internal problem ID [517]

Book: Elementary differential equations and boundary value problems, 10th ed., Boyce and DiPrima
Section: Section 2.4. Page 76
Problem number: 2.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_separable]`

$$\boxed{y + (t-4)\,ty' = 0}$$

With initial conditions

$$[y(2) = 1]$$

✓ Solution by Maple

Time used: 0.015 (sec). Leaf size: 19

```
dsolve([y(t)+(-4+t)*t*diff(y(t),t) = 0,y(2) = 1],y(t), singsol=all)
```

$$y(t) = \frac{\left(\frac{1}{2}+\frac{i}{2}\right)\sqrt{2}\,t^{\frac{1}{4}}}{(-4+t)^{\frac{1}{4}}}$$

✓ Solution by Mathematica

Time used: 0.033 (sec). Leaf size: 20

```
DSolve[{y[t]+(-4+t)*t*y'[t] == 0,y[2]==1},y[t],t,IncludeSingularSolutions -> True]
```

$$y(t) \to \frac{\sqrt[4]{t}}{\sqrt[4]{4-t}}$$

3.3 problem 3

Internal problem ID [518]

Book: Elementary differential equations and boundary value problems, 10th ed., Boyce and DiPrima
Section: Section 2.4. Page 76
Problem number: 3.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_linear]`

$$\boxed{\tan(t)\, y + y' = \sin(t)}$$

With initial conditions

$$[y(\pi) = 0]$$

✓ Solution by Maple
Time used: 0.016 (sec). Leaf size: 17

```
dsolve([tan(t)*y(t)+diff(y(t),t) = sin(t),y(Pi) = 0],y(t), singsol=all)
```

$$y(t) = (-\ln(\cos(t)) + i\pi)\cos(t)$$

✓ Solution by Mathematica
Time used: 0.073 (sec). Leaf size: 20

```
DSolve[{Tan[t]*y[t]+y'[t] == Sin[t],y[Pi]==0},y[t],t,IncludeSingularSolutions -> True]
```

$$y(t) \to i\cos(t)(\pi + i\log(\cos(t)))$$

3.4 problem 4

Internal problem ID [519]

Book: Elementary differential equations and boundary value problems, 10th ed., Boyce and DiPrima
Section: Section 2.4. Page 76
Problem number: 4.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_linear]`

$$\boxed{2yt + \left(-t^2 + 4\right) y' = 3t^2}$$

With initial conditions

$$[y(-3) = 1]$$

✓ Solution by Maple

Time used: 0.032 (sec). Leaf size: 42

```
dsolve([2*t*y(t)+(-t^2+4)*diff(y(t),t) = 3*t^2,y(-3) = 1],y(t), singsol=all)
```

$$y(t) = \frac{3t}{2} + \frac{3\ln(2+t)\,t^2}{8} - \frac{3\ln(2+t)}{2} - \frac{3\ln(t-2)\,t^2}{8} + \frac{3\ln(t-2)}{2} + \frac{11t^2}{10} - \frac{22}{5} + \frac{3\ln(5)\,t^2}{8} - \frac{3\ln(5)}{2}$$

✓ Solution by Mathematica

Time used: 0.049 (sec). Leaf size: 67

```
DSolve[{2*t*y[t]+(-t^2+4)*y'[t] == 3*t^2,y[-3]==1},y[t],t,IncludeSingularSolutions -> True]
```

$$y(t) \to \frac{1}{40}\left(-15i\pi t^2 + 44t^2 + 15t^2\log(5) - 15\left(t^2-4\right)\log(2-t) + 15\left(t^2-4\right)\log(t+2) + 60t + 60i\pi - 176 - 60\log(5)\right)$$

3.5 problem 5

Internal problem ID [520]

Book: Elementary differential equations and boundary value problems, 10th ed., Boyce and DiPrima
Section: Section 2.4. Page 76
Problem number: 5.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_linear]`

$$\boxed{2yt + \left(-t^2+4\right) y' = 3t^2}$$

With initial conditions

$$[y(1) = -3]$$

✓ Solution by Maple
Time used: 0.047 (sec). Leaf size: 46

```
dsolve([2*t*y(t)+(-t^2+4)*diff(y(t),t) = 3*t^2,y(1) = -3],y(t), singsol=all)
```

$$y(t) = -6 + \frac{3(t^2-4)\ln(2+t)}{8} + \frac{3i\pi\, t^2}{8} - \frac{3\ln(3)\, t^2}{8} - \frac{3\ln(t-2)\, t^2}{8} - \frac{3i\pi}{2} + \frac{3t^2}{2} + \frac{3t}{2} + \frac{3\ln(3)}{2} + \frac{3\ln(t-2)}{2}$$

✓ Solution by Mathematica
Time used: 0.043 (sec). Leaf size: 52

```
DSolve[{2*t*y[t]+(-t^2+4)*y'[t] == 3*t^2,y[1]==-3},y[t],t,IncludeSingularSolutions -> True]
```

$$y(t) \to -\frac{3}{8}\left(-4t^2+t^2\log(3)+\left(t^2-4\right)\log(2-t)-\left(t^2-4\right)\log(t+2)-4t+16-4\log(3)\right)$$

3.6 problem 6

Internal problem ID [521]

Book: Elementary differential equations and boundary value problems, 10th ed., Boyce and DiPrima
Section: Section 2.4. Page 76
Problem number: 6.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_linear]`

$$\boxed{y + \ln(t)\, y' = \cot(t)}$$

✓ Solution by Maple
Time used: 0.015 (sec). Leaf size: 32

```
dsolve(y(t)+ln(t)*diff(y(t),t) = cot(t),y(t), singsol=all)
```

$$y(t) = \left(\int \frac{\cot(t)\, \mathrm{e}^{-\operatorname{expIntegral}_1(-\ln(t))}}{\ln(t)} dt + c_1\right) \mathrm{e}^{\operatorname{expIntegral}_1(-\ln(t))}$$

✓ Solution by Mathematica
Time used: 0.142 (sec). Leaf size: 36

```
DSolve[y[t]+Log[t]*y'[t] == Cot[t],y[t],t,IncludeSingularSolutions -> True]
```

$$y(t) \to e^{-\operatorname{LogIntegral}(t)} \left(\int_1^t \frac{e^{\operatorname{LogIntegral}(K[1])} \cot(K[1])}{\log(K[1])} dK[1] + c_1\right)$$

3.7 problem 11

Internal problem ID [522]

Book: Elementary differential equations and boundary value problems, 10th ed., Boyce and DiPrima
Section: Section 2.4. Page 76
Problem number: 11.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_separable]`

$$\boxed{y' - \frac{t^2+1}{3y-y^2} = 0}$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 444

```
dsolve(diff(y(t),t) = (t^2+1)/(3*y(t)-y(t)^2),y(t), singsol=all)
```

$$y(t) = \frac{\left(27-4t^3-12c_1-12t+2\sqrt{4t^6+24c_1t^3+24t^4-54t^3+36c_1^2+72c_1t+36t^2-162c_1-162t}\right)^{\frac{1}{3}}}{2} + \frac{9}{2\left(27-4t^3-12c_1-12t+2\sqrt{4t^6+24c_1t^3+24t^4-54t^3+36c_1^2+72c_1t+36t^2-162c_1-162t}\right)^{\frac{1}{3}}} + \frac{3}{2}$$

$$y(t) = -\frac{\left(1+i\sqrt{3}\right)\left(27-4t^3-12c_1-12t+2\sqrt{4}\sqrt{\left(t^3+3t+3c_1-\frac{27}{2}\right)\left(t^3+3c_1+3t\right)}\right)^{\frac{2}{3}} - 9i\sqrt{3} - 6\left(27-\cdots\right.}{4\left(27-4t^3-12c_1-12t+2\sqrt{4}\sqrt{\left(t^3+3t+3c_1-\frac{2}{2}\cdots\right.}\right.}$$

$$y(t) = \frac{\left(i\sqrt{3}-1\right)\left(27-4t^3-12c_1-12t+2\sqrt{4}\sqrt{\left(t^3+3t+3c_1-\frac{27}{2}\right)\left(t^3+3c_1+3t\right)}\right)^{\frac{2}{3}} - 9i\sqrt{3} + 6\left(27-4t^3\cdots\right.}{4\left(27-4t^3-12c_1-12t+2\sqrt{4}\sqrt{\left(t^3+3t+3c_1-\frac{27}{2}\right)\cdots}\right.}$$

✓ Solution by Mathematica

Time used: 3.185 (sec). Leaf size: 343

```
DSolve[y'[t] == (t^2+1)/(3*y[t]-y[t]^2),y[t],t,IncludeSingularSolutions -> True]
```

$$y(t)\to \frac{1}{2}\left(\sqrt[3]{-4t^3+\sqrt{-729+(4t^3+12t-3(9+4c_1))^2}-12t+27+12c_1}+\frac{9}{\sqrt[3]{-4t^3+\sqrt{-729+(4t^3+12t-3(9+4c_1))^2}-12t+27+12c_1}}+3\right)$$

$$y(t)\to \frac{1}{4}\left(i\left(\sqrt{3}+i\right)\sqrt[3]{-4t^3+\sqrt{-729+(4t^3+12t-3(9+4c_1))^2}-12t+27+12c_1}-\frac{9\left(1+i\sqrt{3}\right)}{\sqrt[3]{-4t^3+\sqrt{-729+(4t^3+12t-3(9+4c_1))^2}-12t+27+12c_1}}+6\right)$$

$$y(t)\to \frac{1}{4}\left(-\left(\left(1+i\sqrt{3}\right)\sqrt[3]{-4t^3+\sqrt{-729+(4t^3+12t-3(9+4c_1))^2}-12t+27+12c_1}\right)+\frac{9i\left(\sqrt{3}+i\right)}{\sqrt[3]{-4t^3+\sqrt{-729+(4t^3+12t-3(9+4c_1))^2}-12t+27+12c_1}}+6\right)$$

3.8 problem 12

Internal problem ID [523]

Book: Elementary differential equations and boundary value problems, 10th ed., Boyce and DiPrima
Section: Section 2.4. Page 76
Problem number: 12.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_separable]`

$$\boxed{y' - \frac{\cot(t)\, y}{1+y} = 0}$$

✓ Solution by Maple
Time used: 0.032 (sec). Leaf size: 9

```
dsolve(diff(y(t),t) = cot(t)*y(t)/(1+y(t)),y(t), singsol=all)
```

$$y(t) = \text{LambertW}\left(c_1 \sin\left(t\right)\right)$$

✓ Solution by Mathematica
Time used: 1.602 (sec). Leaf size: 18

```
DSolve[y'[t] == Cot[t]*y[t]/(1+y[t]),y[t],t,IncludeSingularSolutions -> True]
```

$$y(t) \to W(e^{c_1} \sin(t))$$
$$y(t) \to 0$$

3.9 problem 13

Internal problem ID [524]

Book: Elementary differential equations and boundary value problems, 10th ed., Boyce and DiPrima
Section: Section 2.4. Page 76
Problem number: 13.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type **[_separable]**

$$\boxed{y' + \frac{4t}{y} = 0}$$

✓ Solution by Maple
Time used: 0.0 (sec). Leaf size: 27

```
dsolve(diff(y(t),t) = -4*t/y(t),y(t), singsol=all)
```

$$y(t) = \sqrt{-4t^2 + c_1}$$
$$y(t) = -\sqrt{-4t^2 + c_1}$$

✓ Solution by Mathematica
Time used: 0.087 (sec). Leaf size: 46

```
DSolve[y'[t]== -4*t/y[t],y[t],t,IncludeSingularSolutions -> True]
```

$$y(t) \to -\sqrt{2}\sqrt{-2t^2 + c_1}$$
$$y(t) \to \sqrt{2}\sqrt{-2t^2 + c_1}$$

3.10 problem 14

Internal problem ID [525]

Book: Elementary differential equations and boundary value problems, 10th ed., Boyce and DiPrima
Section: Section 2.4. Page 76
Problem number: 14.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_separable]`

$$\boxed{y' - 2ty^2 = 0}$$

✓ Solution by Maple
Time used: 0.0 (sec). Leaf size: 13

```
dsolve(diff(y(t),t) = 2*t*y(t)^2,y(t), singsol=all)
```

$$y(t) = \frac{1}{-t^2 + c_1}$$

✓ Solution by Mathematica
Time used: 0.111 (sec). Leaf size: 20

```
DSolve[y'[t] == 2*t*y[t]^2,y[t],t,IncludeSingularSolutions -> True]
```

$$y(t) \to -\frac{1}{t^2 + c_1}$$
$$y(t) \to 0$$

3.11 problem 15

Internal problem ID [526]

Book: Elementary differential equations and boundary value problems, 10th ed., Boyce and DiPrima
Section: Section 2.4. Page 76
Problem number: 15.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_quadrature]`

$$\boxed{y^3 + y' = 0}$$

✓ Solution by Maple
Time used: 0.0 (sec). Leaf size: 23

```
dsolve(y(t)^3+diff(y(t),t) = 0,y(t), singsol=all)
```

$$y(t) = \frac{1}{\sqrt{2t + c_1}}$$
$$y(t) = -\frac{1}{\sqrt{2t + c_1}}$$

✓ Solution by Mathematica
Time used: 0.124 (sec). Leaf size: 40

```
DSolve[y[t]^3+y'[t] == 0,y[t],t,IncludeSingularSolutions -> True]
```

$$y(t) \to -\frac{1}{\sqrt{2t - 2c_1}}$$
$$y(t) \to \frac{1}{\sqrt{2t - 2c_1}}$$
$$y(t) \to 0$$

3.12 problem 16

Internal problem ID [527]

Book: Elementary differential equations and boundary value problems, 10th ed., Boyce and DiPrima
Section: Section 2.4. Page 76
Problem number: 16.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_separable]`

$$\boxed{y' - \frac{t^2}{(t^3+1)\, y} = 0}$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 39

```
dsolve(diff(y(t),t) = t^2/(t^3+1)/y(t),y(t), singsol=all)
```

$$y(t) = -\frac{\sqrt{6\ln(t^3+1) + 9c_1}}{3}$$
$$y(t) = \frac{\sqrt{6\ln(t^3+1) + 9c_1}}{3}$$

✓ Solution by Mathematica

Time used: 0.123 (sec). Leaf size: 56

```
DSolve[y'[t] == t^2/(t^3+1)/y[t],y[t],t,IncludeSingularSolutions -> True]
```

$$y(t) \to -\sqrt{\frac{2}{3}}\sqrt{\log(t^3+1) + 3c_1}$$
$$y(t) \to \sqrt{\frac{2}{3}}\sqrt{\log(t^3+1) + 3c_1}$$

3.13 problem 17

Internal problem ID [528]

Book: Elementary differential equations and boundary value problems, 10th ed., Boyce and DiPrima
Section: Section 2.4. Page 76
Problem number: 17.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_separable]`

$$\boxed{y' - t(3-y)\,y = 0}$$

✓ Solution by Maple
Time used: 0.0 (sec). Leaf size: 19

```
dsolve(diff(y(t),t) = t*(3-y(t))*y(t),y(t), singsol=all)
```

$$y(t) = \frac{3}{1 + 3\,\mathrm{e}^{-\frac{3t^2}{2}} c_1}$$

✓ Solution by Mathematica
Time used: 0.266 (sec). Leaf size: 44

```
DSolve[y'[t] == t*(3-y[t])*y[t],y[t],t,IncludeSingularSolutions -> True]
```

$$y(t) \to \frac{3e^{\frac{3t^2}{2}}}{e^{\frac{3t^2}{2}} + e^{3c_1}}$$
$$y(t) \to 0$$
$$y(t) \to 3$$

3.14 problem 18

Internal problem ID [529]

Book: Elementary differential equations and boundary value problems, 10th ed., Boyce and DiPrima
Section: Section 2.4. Page 76
Problem number: 18.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_Bernoulli]`

$$\boxed{y' - y(3 - yt) = 0}$$

✓ Solution by Maple
Time used: 0.0 (sec). Leaf size: 20

```
dsolve(diff(y(t),t) = y(t)*(3-t*y(t)),y(t), singsol=all)
```

$$y(t) = \frac{9}{-1 + 9c_1 e^{-3t} + 3t}$$

✓ Solution by Mathematica
Time used: 0.118 (sec). Leaf size: 35

```
DSolve[y'[t] == y[t]*(3-t*y[t]),y[t],t,IncludeSingularSolutions -> True]
```

$$y(t) \to \frac{9e^{3t}}{e^{3t}(3t - 1) + 9c_1}$$
$$y(t) \to 0$$

3.15 problem 19

Internal problem ID [530]

Book: Elementary differential equations and boundary value problems, 10th ed., Boyce and DiPrima
Section: Section 2.4. Page 76
Problem number: 19.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_Bernoulli]`

$$\boxed{y' + y(3 - yt) = 0}$$

✓ Solution by Maple
Time used: 0.0 (sec). Leaf size: 20

```
dsolve(diff(y(t),t) = -y(t)*(3-t*y(t)),y(t), singsol=all)
```

$$y(t) = \frac{9}{1 + 9c_1 e^{3t} + 3t}$$

✓ Solution by Mathematica
Time used: 0.126 (sec). Leaf size: 28

```
DSolve[y'[t] == -y[t]*(3-t*y[t]),y[t],t,IncludeSingularSolutions -> True]
```

$$y(t) \to \frac{9}{3t + 9c_1 e^{3t} + 1}$$
$$y(t) \to 0$$

3.16 problem 20

Internal problem ID [531]

Book: Elementary differential equations and boundary value problems, 10th ed., Boyce and DiPrima
Section: Section 2.4. Page 76
Problem number: 20.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_Riccati]`

$$\boxed{y' + y^2 = -1 + t}$$

✓ Solution by Maple
Time used: 0.0 (sec). Leaf size: 31

```
dsolve(diff(y(t),t) = t-1-y(t)^2,y(t), singsol=all)
```

$$y(t) = \frac{\operatorname{AiryAi}(1, t-1)\, c_1 + \operatorname{AiryBi}(1, t-1)}{\operatorname{AiryAi}(t-1)\, c_1 + \operatorname{AiryBi}(t-1)}$$

✓ Solution by Mathematica
Time used: 0.123 (sec). Leaf size: 47

```
DSolve[y'[t] == t-1-y[t]^2,y[t],t,IncludeSingularSolutions -> True]
```

$$y(t) \to \frac{\text{AiryBiPrime}(t-1) + c_1 \text{AiryAiPrime}(t-1)}{\text{AiryBi}(t-1) + c_1 \text{AiryAi}(t-1)}$$
$$y(t) \to \frac{\text{AiryAiPrime}(t-1)}{\text{AiryAi}(t-1)}$$

4 Section 2.5. Page 88

4.1 problem 1

Internal problem ID [532]

Book: Elementary differential equations and boundary value problems, 10th ed., Boyce and DiPrima
Section: Section 2.5. Page 88
Problem number: 1.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_quadrature]`

$$\boxed{y' - ay - by^2 = 0}$$

✓ Solution by Maple
Time used: 0.0 (sec). Leaf size: 20

```
dsolve(diff(y(x),x) = a*y(x)+b*y(x)^2,y(x), singsol=all)
```

$$y(x) = \frac{a}{\mathrm{e}^{-ax} c_1 a - b}$$

✓ Solution by Mathematica
Time used: 0.888 (sec). Leaf size: 45

```
DSolve[y'[x]== a*y[x]+b*y[x]^2,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to -\frac{a e^{a(x+c_1)}}{-1 + b e^{a(x+c_1)}}$$
$$y(x) \to 0$$
$$y(x) \to -\frac{a}{b}$$

4.2 problem 3

Internal problem ID [533]

Book: Elementary differential equations and boundary value problems, 10th ed., Boyce and DiPrima
Section: Section 2.5. Page 88
Problem number: 3.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_quadrature]`

$$\boxed{y' - y(-2+y)(-1+y) = 0}$$

✓ Solution by Maple
Time used: 0.125 (sec). Leaf size: 75

```
dsolve(diff(y(t),t) = y(t)*(-2+y(t))*(-1+y(t)),y(t), singsol=all)
```

$$y(t) = \frac{e^{2t}c_1}{\left(-1-\sqrt{-c_1 e^{2t}+1}\right)\sqrt{-c_1 e^{2t}+1}}$$
$$y(t) = \frac{e^{2t}c_1}{\left(1-\sqrt{-c_1 e^{2t}+1}\right)\sqrt{-c_1 e^{2t}+1}}$$

✓ Solution by Mathematica
Time used: 11.055 (sec). Leaf size: 100

```
DSolve[y'[t] == y[t]*(-2+y[t])*(-1+y[t]),y[t],t,IncludeSingularSolutions -> True]
```

$$y(t) \to \frac{-\sqrt{1+e^{2(t+c_1)}}+e^{2(t+c_1)}+1}{1+e^{2(t+c_1)}}$$
$$y(t) \to \frac{\sqrt{1+e^{2(t+c_1)}}+e^{2(t+c_1)}+1}{1+e^{2(t+c_1)}}$$
$$y(t) \to 0$$
$$y(t) \to 1$$
$$y(t) \to 2$$

4.3 problem 4

Internal problem ID [534]

Book: Elementary differential equations and boundary value problems, 10th ed., Boyce and DiPrima
Section: Section 2.5. Page 88
Problem number: 4.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_quadrature]`

$$\boxed{y' - e^y = -1}$$

✓ Solution by Maple
Time used: 0.0 (sec). Leaf size: 15

```
dsolve(diff(y(t),t) = -1+exp(y(t)),y(t), singsol=all)
```

$$y(t) = \ln\left(-\frac{1}{e^t c_1 - 1}\right)$$

✓ Solution by Mathematica
Time used: 0.817 (sec). Leaf size: 28

```
DSolve[y'[t]== -1+Exp[y[t]],y[t],t,IncludeSingularSolutions -> True]
```

$$y(t) \to \log\left(\frac{1}{2}\left(1 - \tanh\left(\frac{t + c_1}{2}\right)\right)\right)$$
$$y(t) \to 0$$

4.4 problem 5

Internal problem ID [535]

Book: Elementary differential equations and boundary value problems, 10th ed., Boyce and DiPrima
Section: Section 2.5. Page 88
Problem number: 5.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_quadrature]`

$$\boxed{y' - \mathrm{e}^{-y} = -1}$$

✓ Solution by Maple
Time used: 0.016 (sec). Leaf size: 18

```
dsolve(diff(y(t),t) = -1+exp(-y(t)),y(t), singsol=all)
```

$$y(t) = -t + \ln\left(\mathrm{e}^{t+c_1} - 1\right) - c_1$$

✓ Solution by Mathematica
Time used: 0.853 (sec). Leaf size: 21

```
DSolve[y'[t] == -1+Exp[-y[t]],y[t],t,IncludeSingularSolutions -> True]
```

$$y(t) \rightarrow \log\left(1 + e^{-t+c_1}\right)$$
$$y(t) \rightarrow 0$$

4.5 problem 6

Internal problem ID [536]

Book: Elementary differential equations and boundary value problems, 10th ed., Boyce and DiPrima
Section: Section 2.5. Page 88
Problem number: 6.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_quadrature]`

$$\boxed{y' + \frac{2\arctan(y)}{1+y^2} = 0}$$

✓ Solution by Maple
Time used: 0.0 (sec). Leaf size: 23

```
dsolve(diff(y(t),t) = -2*arctan(y(t))/(1+y(t)^2),y(t), singsol=all)
```

$$t + \frac{\left(\int^{y(t)} \frac{\_a^2+1}{\arctan(\_a)} d\_a\right)}{2} + c_1 = 0$$

✓ Solution by Mathematica
Time used: 1.013 (sec). Leaf size: 38

```
DSolve[y'[t] == -2*ArcTan[y[t]]/(1+y[t]^2),y[t],t,IncludeSingularSolutions -> True]
```

$$y(t) \to \text{InverseFunction}\left[\int_1^{\#1} \frac{K[1]^2+1}{\arctan(K[1])} dK[1]\&\right][-2t + c_1]$$
$$y(t) \to 0$$

4.6 problem 7

Internal problem ID [537]

Book: Elementary differential equations and boundary value problems, 10th ed., Boyce and DiPrima
Section: Section 2.5. Page 88
Problem number: 7.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_quadrature]`

$$\boxed{y' + k(-1 + y)^2 = 0}$$

✓ Solution by Maple
Time used: 0.0 (sec). Leaf size: 20

```
dsolve(diff(y(t),t) = -k*(-1+y(t))^2,y(t), singsol=all)
```

$$y(t) = \frac{1 + k(t + c_1)}{k(t + c_1)}$$

✓ Solution by Mathematica
Time used: 0.139 (sec). Leaf size: 30

```
DSolve[y'[t]== -k*(-1+y[t])^2,y[t],t,IncludeSingularSolutions -> True]
```

$$y(t) \to \frac{kt + 1 - c_1}{kt - c_1}$$
$$y(t) \to 1$$

4.7 problem 9

Internal problem ID [538]

Book: Elementary differential equations and boundary value problems, 10th ed., Boyce and DiPrima
Section: Section 2.5. Page 88
Problem number: 9.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_quadrature]`

$$\boxed{y' - y^2(y^2 - 1) = 0}$$

✓ Solution by Maple
Time used: 0.282 (sec). Leaf size: 47

```
dsolve(diff(y(t),t) = y(t)^2*(y(t)^2-1),y(t), singsol=all)
```

$$y(t) = e^{\text{RootOf}\left(-\ln\left(e^{-\_Z}-2\right)e^{-\_Z}+2c_1e^{-\_Z}+\_Z\,e^{-\_Z}+2t\,e^{-\_Z}+\ln\left(e^{-\_Z}-2\right)-2c_1-\_Z-2t-2\right)} - 1$$

✓ Solution by Mathematica
Time used: 0.246 (sec). Leaf size: 51

```
DSolve[y'[t] == y[t]^2*(y[t]^2-1),y[t],t,IncludeSingularSolutions -> True]
```

$$y(t) \to \text{InverseFunction}\left[\frac{1}{\#1} + \frac{1}{2}\log(1-\#1) - \frac{1}{2}\log(\#1+1)\&\right][t + c_1]$$
$$y(t) \to -1$$
$$y(t) \to 0$$
$$y(t) \to 1$$

4.8 problem 10

Internal problem ID [539]

Book: Elementary differential equations and boundary value problems, 10th ed., Boyce and DiPrima
Section: Section 2.5. Page 88
Problem number: 10.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_quadrature]`

$$\boxed{y' - y(1 - y^2) = 0}$$

✓ Solution by Maple
Time used: 0.0 (sec). Leaf size: 29

```
dsolve(diff(y(t),t) = y(t)*(1-y(t)^2),y(t), singsol=all)
```

$$y(t) = \frac{1}{\sqrt{c_1 e^{-2t} + 1}}$$
$$y(t) = -\frac{1}{\sqrt{c_1 e^{-2t} + 1}}$$

✓ Solution by Mathematica

Time used: 0.676 (sec). Leaf size: 100

```
DSolve[y'[t]== y[t]*(1-y[t]^2),y[t],t,IncludeSingularSolutions -> True]
```

$$y(t) \to -\frac{e^t}{\sqrt{e^{2t} + e^{2c_1}}}$$
$$y(t) \to \frac{e^t}{\sqrt{e^{2t} + e^{2c_1}}}$$
$$y(t) \to -1$$
$$y(t) \to 0$$
$$y(t) \to 1$$
$$y(t) \to -\frac{e^t}{\sqrt{e^{2t}}}$$
$$y(t) \to \frac{e^t}{\sqrt{e^{2t}}}$$

4.9 problem 11

Internal problem ID [540]

Book: Elementary differential equations and boundary value problems, 10th ed., Boyce and DiPrima
Section: Section 2.5. Page 88
Problem number: 11.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_quadrature]`

$$\boxed{y' + b\sqrt{y} - ay = 0}$$

✓ Solution by Maple
Time used: 0.015 (sec). Leaf size: 26

```
dsolve(diff(y(t),t) = -b*y(t)^(1/2)+a*y(t),y(t), singsol=all)
```

$$\frac{-\mathrm{e}^{\frac{at}{2}} c_1 a + \sqrt{y(t)}\, a - b}{a} = 0$$

✓ Solution by Mathematica
Time used: 0.844 (sec). Leaf size: 55

```
DSolve[y'[t] == -b*y[t]^(1/2)+a*y[t],y[t],t,IncludeSingularSolutions -> True]
```

$$y(t) \to \frac{e^{-ac_1}\left(e^{\frac{at}{2}} - be^{\frac{ac_1}{2}}\right)^2}{a^2}$$
$$y(t) \to 0$$
$$y(t) \to \frac{b^2}{a^2}$$

4.10 problem 12

Internal problem ID [541]

Book: Elementary differential equations and boundary value problems, 10th ed., Boyce and DiPrima
Section: Section 2.5. Page 88
Problem number: 12.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_quadrature]`

$$\boxed{y' - y^2(4 - y^2) = 0}$$

✓ Solution by Maple
Time used: 0.032 (sec). Leaf size: 49

```
dsolve(diff(y(t),t) = y(t)^2*(4-y(t)^2),y(t), singsol=all)
```

$$y(t) = e^{\mathrm{RootOf}\left(\ln\left(e^{-Z}-4\right)e^{-Z}+16c_1e^{-Z}-\_Ze^{-Z}+16t\,e^{-Z}-2\ln\left(e^{-Z}-4\right)-32c_1+2\_Z-32t+4\right)} - 2$$

✓ Solution by Mathematica
Time used: 0.247 (sec). Leaf size: 57

```
DSolve[y'[t] == y[t]^2*(4-y[t]^2),y[t],t,IncludeSingularSolutions -> True]
```

$$y(t) \to \text{InverseFunction}\left[\frac{1}{4\#1} + \frac{1}{16}\log(2 - \#1) - \frac{1}{16}\log(\#1 + 2)\&\right][-t + c_1]$$
$$y(t) \to -2$$
$$y(t) \to 0$$
$$y(t) \to 2$$

4.11 problem 13

Internal problem ID [542]

Book: Elementary differential equations and boundary value problems, 10th ed., Boyce and DiPrima
Section: Section 2.5. Page 88
Problem number: 13.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_quadrature]`

$$y' - (1-y)^2 y^2 = 0$$

✓ Solution by Maple

Time used: 0.016 (sec). Leaf size: 66

```
dsolve(diff(y(t),t) = (1-y(t))^2*y(t)^2,y(t), singsol=all)
```

$$y(t) = e^{\text{RootOf}\left(-2\ln\left(e^{\_Z}+1\right)e^{2\_Z}+c_1e^{2\_Z}+2\_Ze^{2\_Z}+t\,e^{2\_Z}-2\ln\left(e^{\_Z}+1\right)e^{\_Z}+c_1e^{\_Z}+2\_Ze^{\_Z}+t\,e^{\_Z}+2\,e^{\_Z}+1\right)} + 1$$

✓ Solution by Mathematica

Time used: 0.365 (sec). Leaf size: 50

```
DSolve[y'[t] == (1-y[t])^2*y[t]^2,y[t],t,IncludeSingularSolutions -> True]
```

$$y(t) \to \text{InverseFunction}\left[-\frac{1}{\#1-1} - \frac{1}{\#1} - 2\log(1-\#1) + 2\log(\#1)\&\right][t+c_1]$$
$$y(t) \to 0$$
$$y(t) \to 1$$

5 Section 2.6. Page 100

5.1 problem 1

Internal problem ID [543]

Book: Elementary differential equations and boundary value problems, 10th ed., Boyce and DiPrima
Section: Section 2.6. Page 100
Problem number: 1.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type **[_separable]**

$$\boxed{(-2+2y)\,y' = -3-2x}$$

✓ Solution by Maple
Time used: 0.0 (sec). Leaf size: 43

```
dsolve(3+2*x+(-2+2*y(x))*diff(y(x),x) = 0,y(x), singsol=all)
```

$$y(x) = 1-\sqrt{-x^2-c_1-3x+1}$$
$$y(x) = 1+\sqrt{-x^2-c_1-3x+1}$$

✓ Solution by Mathematica
Time used: 0.115 (sec). Leaf size: 51

```
DSolve[3+2*x+(-2+2*y[x])*y'[x] == 0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to 1-\sqrt{-x^2-3x+1+2c_1}$$
$$y(x) \to 1+\sqrt{-x^2-3x+1+2c_1}$$

5.2 problem 2

Internal problem ID [544]

Book: Elementary differential equations and boundary value problems, 10th ed., Boyce and DiPrima
Section: Section 2.6. Page 100
Problem number: 2.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[[_homogeneous, ‘class A‘], _rational, [_Abel, ‘2nd type‘, ‘cl`

$$\boxed{4y + (2x - 2y)\, y' = -2x}$$

✓ Solution by Maple

Time used: 0.14 (sec). Leaf size: 55

```
dsolve(2*x+4*y(x)+(2*x-2*y(x))*diff(y(x),x) = 0,y(x), singsol=all)
```

$$-\frac{\ln\left(\frac{-x^2-3xy(x)+y(x)^2}{x^2}\right)}{2} + \frac{\sqrt{13}\,\operatorname{arctanh}\left(\frac{(2y(x)-3x)\sqrt{13}}{13x}\right)}{13} - \ln(x) - c_1 = 0$$

✓ Solution by Mathematica

Time used: 0.065 (sec). Leaf size: 63

```
DSolve[2*x+4*y[x]+(2*x-2*y[x])*y'[x]== 0,y[x],x,IncludeSingularSolutions -> True]
```

$$\text{Solve}\left[\frac{1}{26}\left(\left(13+\sqrt{13}\right)\log\left(-\frac{2y(x)}{x}+\sqrt{13}+3\right) - \left(\sqrt{13}-13\right)\log\left(\frac{2y(x)}{x}+\sqrt{13}-3\right)\right) = -\log(x)+c_1, y(x)\right]$$

5.3 problem 3

Internal problem ID [545]

Book: Elementary differential equations and boundary value problems, 10th ed., Boyce and DiPrima
Section: Section 2.6. Page 100
Problem number: 3.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_exact, _rational]`

$$-2yx + (3 - x^2 + 6y^2)\, y' = -3x^2 - 2$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 372

```
dsolve(2+3*x^2-2*x*y(x)+(3-x^2+6*y(x)^2)*diff(y(x),x) = 0,y(x), singsol=all)
```

$$y(x) = \frac{\left(-54x^3 - 54c_1 - 108x + 6\sqrt{75x^6 + 162c_1x^3 + 378x^4 + 81c_1^2 + 324c_1x + 162x^2 + 162}\right)^{\frac{2}{3}} + 6x^2 - 18}{6\left(-54x^3 - 54c_1 - 108x + 6\sqrt{75x^6 + 162c_1x^3 + 378x^4 + 81c_1^2 + 324c_1x + 162x^2 + 162}\right)^{\frac{1}{3}}}$$

$$y(x) = \frac{\left(-1 - i\sqrt{3}\right)\left(-54x^3 - 54c_1 - 108x + 6\sqrt{75x^6 + 162c_1x^3 + 378x^4 + 81c_1^2 + 324c_1x + 162x^2 + 162}\right)^{\frac{1}{3}}}{12} + \frac{(x^2 - 3)\left(i\sqrt{3} - 1\right)}{2\left(-54x^3 - 54c_1 - 108x + 6\sqrt{75x^6 + 162c_1x^3 + 378x^4 + 81c_1^2 + 324c_1x + 162x^2 + 162}\right)^{\frac{1}{3}}}$$

$$y(x) = \frac{\frac{\left(i\sqrt{3}-1\right)\left(-54x^3-54c_1-108x+6\sqrt{75x^6+162c_1x^3+378x^4+81c_1^2+324c_1x+162x^2+162}\right)^{\frac{2}{3}}}{6} + \left(-1 - i\sqrt{3}\right)(x^2 - 3)}{2\left(-54x^3 - 54c_1 - 108x + 6\sqrt{75x^6 + 162c_1x^3 + 378x^4 + 81c_1^2 + 324c_1x + 162x^2 + 162}\right)^{\frac{1}{3}}}$$

✓ Solution by Mathematica

Time used: 8.724 (sec). Leaf size: 421

```
DSolve[2+3*x^2-2*x*y[x]+(3-x^2+6*y[x]^2)*y'[x] == 0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to -\frac{x^2-3}{\sqrt[3]{6}\sqrt[3]{9x^3+\sqrt{3}\sqrt{-2\left(x^2-3\right)^3+27\left(x^3+2x+c_1\right)^2}+18x+9c_1}} - \frac{\sqrt[3]{9x^3+\sqrt{3}\sqrt{-2\left(x^2-3\right)^3+27\left(x^3+2x+c_1\right)^2}+18x+9c_1}}{6^{2/3}}$$

$$y(x) \to \frac{\sqrt[3]{6}\left(1+i\sqrt{3}\right)\left(x^2-3\right)+\left(1-i\sqrt{3}\right)\left(9x^3+\sqrt{3}\sqrt{-2\left(x^2-3\right)^3+27\left(x^3+2x+c_1\right)^2}+18x+9c_1\right)^{2/3}}{2\ 6^{2/3}\sqrt[3]{9x^3+\sqrt{3}\sqrt{-2\left(x^2-3\right)^3+27\left(x^3+2x+c_1\right)^2}+18x+9c_1}}$$

$$y(x) \to \frac{\sqrt[3]{6}\left(1-i\sqrt{3}\right)\left(x^2-3\right)+\left(1+i\sqrt{3}\right)\left(9x^3+\sqrt{3}\sqrt{-2\left(x^2-3\right)^3+27\left(x^3+2x+c_1\right)^2}+18x+9c_1\right)^{2/3}}{2\ 6^{2/3}\sqrt[3]{9x^3+\sqrt{3}\sqrt{-2\left(x^2-3\right)^3+27\left(x^3+2x+c_1\right)^2}+18x+9c_1}}$$

5.4 problem 4

Internal problem ID [546]

Book: Elementary differential equations and boundary value problems, 10th ed., Boyce and DiPrima
Section: Section 2.6. Page 100
Problem number: 4.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_separable]`

$$\boxed{2y + 2xy^2 + (2x + 2x^2y)\, y' = 0}$$

✓ Solution by Maple
Time used: 0.0 (sec). Leaf size: 31

```
dsolve(2*y(x)+2*x*y(x)^2+(2*x+2*x^2*y(x))*diff(y(x),x) = 0,y(x), singsol=all)
```

$$y(x) = -\frac{1}{x}$$
$$y(x) = \frac{-1 - c_1}{x}$$
$$y(x) = \frac{c_1 - 1}{x}$$

✓ Solution by Mathematica
Time used: 0.023 (sec). Leaf size: 29

```
DSolve[2*y[x]+2*x*y[x]^2+(2*x+2*x^2*y[x])*y'[x] == 0,y[x],x,IncludeSingularSolutions -> True
```

$$y(x) \to -\frac{1}{x}$$
$$y(x) \to \frac{c_1}{x}$$
$$y(x) \to -\frac{1}{x}$$

5.5 problem 5

Internal problem ID [547]

Book: Elementary differential equations and boundary value problems, 10th ed., Boyce and DiPrima
Section: Section 2.6. Page 100
Problem number: 5.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[[_homogeneous, ‘class A‘], _rational, [_Abel, ‘2nd type‘, ‘cl`

$$\boxed{y' - \frac{-ax - by}{bx + cy} = 0}$$

✓ Solution by Maple
Time used: 0.016 (sec). Leaf size: 77

```
dsolve(diff(y(x),x) = (-a*x-b*y(x))/(b*x+c*y(x)),y(x), singsol=all)
```

$$y(x) = \frac{-bxc_1 + \sqrt{-x^2(ac - b^2)c_1^2 + c}}{cc_1}$$
$$y(x) = \frac{-bxc_1 - \sqrt{-x^2(ac - b^2)c_1^2 + c}}{cc_1}$$

✓ Solution by Mathematica
Time used: 17.783 (sec). Leaf size: 139

```
DSolve[y'[x]== (-a*x-b*y[x])/(b*x+c*y[x]),y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to -\frac{bx + \sqrt{-acx^2 + b^2x^2 + ce^{2c_1}}}{c}$$
$$y(x) \to \frac{-bx + \sqrt{b^2x^2 + c(-ax^2 + e^{2c_1})}}{c}$$
$$y(x) \to -\frac{\sqrt{x^2(b^2 - ac)} + bx}{c}$$
$$y(x) \to \frac{\sqrt{x^2(b^2 - ac)} - bx}{c}$$

5.6 problem 6

Internal problem ID [548]

Book: Elementary differential equations and boundary value problems, 10th ed., Boyce and DiPrima
Section: Section 2.6. Page 100
Problem number: 6.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[[_homogeneous, ‘class A‘], _rational, [_Abel, ‘2nd type‘, ‘cl`

$$y' - \frac{-ax + by}{bx - cy} = 0$$

✓ Solution by Maple

Time used: 0.422 (sec). Leaf size: 47

```
dsolve(diff(y(x),x) = (-a*x+b*y(x))/(b*x-c*y(x)),y(x), singsol=all)
```

$$y(x) = \operatorname{RootOf}\left(c\_Z^2 - a - e^{\operatorname{RootOf}\left(e^{\_Z}\cosh\left(\frac{\sqrt{ac}\,(2c_1+\_Z+2\ln(x))}{2b}\right)^2+a\right)}\right)x$$

✓ Solution by Mathematica

Time used: 0.072 (sec). Leaf size: 58

```
DSolve[y'[x] == (-a*x+b*y[x])/(b*x-c*y[x]),y[x],x,IncludeSingularSolutions -> True]
```

$$\text{Solve}\left[-\frac{b\operatorname{arctanh}\left(\frac{\sqrt{c}y(x)}{\sqrt{a}x}\right)}{\sqrt{a}\sqrt{c}} - \frac{1}{2}\log\left(\frac{cy(x)^2}{x^2} - a\right) = \log(x) + c_1, y(x)\right]$$

5.7 problem 7

Internal problem ID [549]

Book: Elementary differential equations and boundary value problems, 10th ed., Boyce and DiPrima
Section: Section 2.6. Page 100
Problem number: 7.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_exact]`

$$\boxed{e^{x} \sin(y) - 2\sin(x) y + (2\cos(x) + e^{x} \cos(y)) y' = 0}$$

✓ Solution by Maple

Time used: 0.015 (sec). Leaf size: 17

```
dsolve(exp(x)*sin(y(x))-2*sin(x)*y(x)+(2*cos(x)+exp(x)*cos(y(x)))*diff(y(x),x) = 0,y(x), sin
```

$$e^{x} \sin(y(x)) + 2\cos(x) y(x) + c_1 = 0$$

✓ Solution by Mathematica

Time used: 0.282 (sec). Leaf size: 20

```
DSolve[Exp[x]*Sin[y[x]]-2*Sin[x]*y[x]+(2*Cos[x]+Exp[x]*Cos[y[x]])*y'[x] == 0,y[x],x,IncludeS
```

$$\text{Solve}[e^x \sin(y(x)) + 2y(x)\cos(x) = c_1, y(x)]$$

5.8 problem 8

Internal problem ID [550]

Book: Elementary differential equations and boundary value problems, 10th ed., Boyce and DiPrima
Section: Section 2.6. Page 100
Problem number: 8.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[‘x=_G(y,y’)‘]`

$$\boxed{e^x \sin(y) + 3y - (3x - e^x \sin(y))\, y' = 0}$$

✗ Solution by Maple

```
dsolve(exp(x)*sin(y(x))+3*y(x)-(3*x-exp(x)*sin(y(x)))*diff(y(x),x) = 0,y(x), singsol=all)
```

No solution found

✗ Solution by Mathematica

Time used: 0.0 (sec). Leaf size: 0

```
DSolve[Exp[x]*Sin[y[x]]+3*y[x]-(3*x-Exp[x]*Sin[y[x]])*y'[x] == 0,y[x],x,IncludeSingularSolut
```

Not solved

5.9 problem 9

Internal problem ID [551]

Book: Elementary differential equations and boundary value problems, 10th ed., Boyce and DiPrima
Section: Section 2.6. Page 100
Problem number: 9.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_exact]`

$$-2\,\mathrm{e}^{yx}\sin(2x)+\mathrm{e}^{yx}\cos(2x)\,y+\left(-3+\mathrm{e}^{yx}x\cos(2x)\right)y'=-2x$$

✓ Solution by Maple
Time used: 0.031 (sec). Leaf size: 35

```
dsolve(2*x-2*exp(x*y(x))*sin(2*x)+exp(x*y(x))*cos(2*x)*y(x)+(-3+exp(x*y(x))*x*cos(2*x))*diff
```

$$y(x)=\frac{x^3+c_1x-3\,\mathrm{LambertW}\left(-\frac{x\cos(2x)\,\mathrm{e}^{\frac{x\left(x^2+c_1\right)}{3}}}{3}\right)}{3x}$$

✓ Solution by Mathematica
Time used: 5.197 (sec). Leaf size: 48

```
DSolve[2*x-2*Exp[x*y[x]]*Sin[2*x]+Exp[x*y[x]]*Cos[2*x]*y[x]+(-3+Exp[x*y[x]]*x*Cos[2*x])*y'[x
```

$$y(x)\to\frac{-3W\left(-\frac{1}{3}xe^{\frac{1}{3}x\left(x^2-c_1\right)}\cos(2x)\right)+x^3-c_1x}{3x}$$

5.10 problem 10

Internal problem ID [552]

Book: Elementary differential equations and boundary value problems, 10th ed., Boyce and DiPrima
Section: Section 2.6. Page 100
Problem number: 10.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_linear]`

$$\boxed{\frac{y}{x} + (\ln(x) - 2)\, y' = -6x}$$

✓ Solution by Maple
Time used: 0.219 (sec). Leaf size: 18

```
dsolve((y(x)/x+6*x)+(ln(x)-2)*diff(y(x),x) = 0,y(x), singsol=all)
```

$$y(x) = \frac{-3x^2 + c_1}{\ln(x) - 2}$$

✓ Solution by Mathematica
Time used: 0.079 (sec). Leaf size: 20

```
DSolve[(y[x]/x+6*x)+(Log[x]-2)*y'[x] == 0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to \frac{-3x^2 + c_1}{\log(x) - 2}$$

5.11 problem 11

Internal problem ID [553]

Book: Elementary differential equations and boundary value problems, 10th ed., Boyce and DiPrima
Section: Section 2.6. Page 100
Problem number: 11.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[[_Abel, '2nd type', 'class B']]`

$$\boxed{yx + (\ln(x) y + yx) y' = -x\ln(x)}$$

✗ Solution by Maple

```
dsolve((x*ln(x)+x*y(x))+(y(x)*ln(x)+x*y(x))*diff(y(x),x) = 0,y(x), singsol=all)
```

No solution found

✗ Solution by Mathematica

Time used: 0.0 (sec). Leaf size: 0

```
DSolve[(x*Log[x]+x*y[x])+(y[x]*Log[x]+x*y[x])*y'[x] == 0,y[x],x,IncludeSingularSolutions ->
```

Not solved

5.12 problem 12

Internal problem ID [554]

Book: Elementary differential equations and boundary value problems, 10th ed., Boyce and DiPrima
Section: Section 2.6. Page 100
Problem number: 12.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_separable]`

$$\frac{x}{(x^2+y^2)^{\frac{3}{2}}}+\frac{yy'}{(x^2+y^2)^{\frac{3}{2}}}=0$$

✓ Solution by Maple
Time used: 0.0 (sec). Leaf size: 27

```
dsolve(x/(x^2+y(x)^2)^(3/2)+y(x)*diff(y(x),x)/(x^2+y(x)^2)^(3/2) = 0,y(x), singsol=all)
```

$$y(x)=\sqrt{-x^2+c_1}$$
$$y(x)=-\sqrt{-x^2+c_1}$$

✓ Solution by Mathematica
Time used: 0.09 (sec). Leaf size: 39

```
DSolve[x/(x^2+y[x]^2)^(3/2)+y[x]*y'[x]/(x^2+y[x]^2)^(3/2) == 0,y[x],x,IncludeSingularSolutio
```

$$y(x)\to -\sqrt{-x^2+2c_1}$$
$$y(x)\to \sqrt{-x^2+2c_1}$$

5.13 problem 13

Internal problem ID [555]

Book: Elementary differential equations and boundary value problems, 10th ed., Boyce and DiPrima
Section: Section 2.6. Page 100
Problem number: 13.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[[_homogeneous, ‘class A‘], _exact, _rational, [_Abel, ‘2nd ty`

$$\boxed{-y + (-x + 2y)\, y' = -2x}$$

With initial conditions

$$[y(1) = 3]$$

✓ Solution by Maple
Time used: 0.094 (sec). Leaf size: 19

```
dsolve([2*x-y(x)+(-x+2*y(x))*diff(y(x),x) = 0,y(1) = 3],y(x), singsol=all)
```

$$y(x) = \frac{x}{2} + \frac{\sqrt{-3x^2 + 28}}{2}$$

✓ Solution by Mathematica
Time used: 0.456 (sec). Leaf size: 22

```
DSolve[{2*x-y[x]+(-x+2*y[x])*y'[x] == 0,y[1]==3},y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to \frac{1}{2}\left(\sqrt{28 - 3x^2} + x\right)$$

5.14 problem 14

Internal problem ID [556]

Book: Elementary differential equations and boundary value problems, 10th ed., Boyce and DiPrima
Section: Section 2.6. Page 100
Problem number: 14.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_exact, _rational, [_1st_order, '_with_symmetry_[F(x),G(x)]']`

$$\boxed{y + (x - 4y)\, y' = -9x^2 + 1}$$

With initial conditions

$$[y(1) = 0]$$

✓ Solution by Maple
Time used: 0.047 (sec). Leaf size: 25

```
dsolve([-1+9*x^2+y(x)+(x-4*y(x))*diff(y(x),x) = 0,y(1) = 0],y(x), singsol=all)
```

$$y(x) = \frac{x}{4} - \frac{\sqrt{24x^3 + x^2 - 8x - 16}}{4}$$

✓ Solution by Mathematica
Time used: 0.148 (sec). Leaf size: 34

```
DSolve[{-1+9*x^2+y[x]+(x-4*y[x])*y'[x] == 0,y[1]==0},y[x],x,IncludeSingularSolutions -> True
```

$$y(x) \to \frac{1}{4}\left(x + i\sqrt{-24x^3 - x^2 + 8x + 16}\right)$$

5.15 problem 19

Internal problem ID [557]

Book: Elementary differential equations and boundary value problems, 10th ed., Boyce and DiPrima
Section: Section 2.6. Page 100
Problem number: 19.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_separable]`

$$\boxed{y^3x^2 + x(1+y^2)\, y' = 0}$$

✓ Solution by Maple

Time used: 0.015 (sec). Leaf size: 37

```
dsolve(x^2*y(x)^3+x*(1+y(x)^2)*diff(y(x),x) = 0,y(x), singsol=all)
```

$$y(x) = \mathrm{e}^{-\frac{x^2}{2}-c_1}\sqrt{\frac{\mathrm{e}^{x^2+2c_1}}{\mathrm{LambertW}\left(\mathrm{e}^{x^2+2c_1}\right)}}$$

✓ Solution by Mathematica

Time used: 4.095 (sec). Leaf size: 46

```
DSolve[x^2*y[x]^3+x*(1+y[x]^2)*y'[x] == 0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to -\frac{1}{\sqrt{W\left(e^{x^2-2c_1}\right)}}$$
$$y(x) \to \frac{1}{\sqrt{W\left(e^{x^2-2c_1}\right)}}$$
$$y(x) \to 0$$

5.16 problem 21

Internal problem ID [558]

Book: Elementary differential equations and boundary value problems, 10th ed., Boyce and DiPrima
Section: Section 2.6. Page 100
Problem number: 21.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[[_1st_order, '_with_symmetry_[F(x)*G(y),0]']]`

$$\boxed{y + (2x - e^y y)\, y' = 0}$$

✓ Solution by Maple
Time used: 0.031 (sec). Leaf size: 34

```
dsolve(y(x)+(2*x-exp(y(x))*y(x))*diff(y(x),x) = 0,y(x), singsol=all)
```

$$\frac{\left(-y(x)^2 + 2y(x) - 2\right) e^{y(x)} + x y(x)^2 - c_1}{y\left(x\right)^2} = 0$$

✓ Solution by Mathematica
Time used: 0.225 (sec). Leaf size: 32

```
DSolve[y[x]+(2*x-Exp[y[x]]*y[x])*y'[x] == 0,y[x],x,IncludeSingularSolutions -> True]
```

$$\text{Solve}\left[x = \frac{e^{y(x)}(y(x)^2 - 2y(x) + 2)}{y(x)^2} + \frac{c_1}{y(x)^2}, y(x)\right]$$

5.17 problem 22

Internal problem ID [559]

Book: Elementary differential equations and boundary value problems, 10th ed., Boyce and DiPrima
Section: Section 2.6. Page 100
Problem number: 22.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type **[_separable]**

$$\boxed{(2+x)\sin(y) + x\cos(y)\, y' = 0}$$

✓ Solution by Maple

Time used: 0.016 (sec). Leaf size: 16

```
dsolve((2+x)*sin(y(x))+x*cos(y(x))*diff(y(x),x) = 0,y(x), singsol=all)
```

$$y(x) = \arcsin\left(\frac{e^{-x}}{c_1 x^2}\right)$$

✓ Solution by Mathematica

Time used: 51.022 (sec). Leaf size: 23

```
DSolve[(2+x)*Sin[y[x]]+x*Cos[y[x]]*y'[x] == 0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to \arcsin\left(\frac{e^{-x+c_1}}{x^2}\right)$$
$$y(x) \to 0$$

5.18 problem 25

Internal problem ID [560]

Book: Elementary differential equations and boundary value problems, 10th ed., Boyce and DiPrima
Section: Section 2.6. Page 100
Problem number: 25.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[[_homogeneous, ‘class D‘], _rational]`

$$2yx + 3x^2y + y^3 + (x^2 + y^2)\, y' = 0$$

✓ Solution by Maple
Time used: 0.015 (sec). Leaf size: 297

```
dsolve(2*x*y(x)+3*x^2*y(x)+y(x)^3+(x^2+y(x)^2)*diff(y(x),x) = 0,y(x), singsol=all)
```

$$y(x) = -\frac{\left(x^2 e^{6x} c_1^2 - \frac{2^{\frac{1}{3}}\left(\left(1+\sqrt{4x^6 e^{6x} c_1^2+1}\right) e^{6x} c_1^2\right)^{\frac{2}{3}}}{2}\right) 2^{\frac{1}{3}} e^{-3x}}{\left(\left(1+\sqrt{4x^6 e^{6x} c_1^2+1}\right) e^{6x} c_1^2\right)^{\frac{1}{3}} c_1}$$

$$y(x) = -\frac{e^{-3x} 2^{\frac{1}{3}} \left(2x^2 (i\sqrt{3}-1) e^{6x} c_1^2 + 2^{\frac{1}{3}} (1+i\sqrt{3}) \left(\left(1+\sqrt{4x^6 e^{6x} c_1^2+1}\right) e^{6x} c_1^2\right)^{\frac{2}{3}}\right)}{4\left(\left(1+\sqrt{4x^6 e^{6x} c_1^2+1}\right) e^{6x} c_1^2\right)^{\frac{1}{3}} c_1}$$

$$y(x) = \frac{\left(2x^2 (1+i\sqrt{3}) e^{6x} c_1^2 + 2^{\frac{1}{3}} (i\sqrt{3}-1) \left(\left(1+\sqrt{4x^6 e^{6x} c_1^2+1}\right) e^{6x} c_1^2\right)^{\frac{2}{3}}\right) e^{-3x} 2^{\frac{1}{3}}}{4\left(\left(1+\sqrt{4x^6 e^{6x} c_1^2+1}\right) e^{6x} c_1^2\right)^{\frac{1}{3}} c_1}$$

✓ Solution by Mathematica

Time used: 60.305 (sec). Leaf size: 383

```
DSolve[2*x*y[x]+3*x^2*y[x]+y[x]^3+(x^2+y[x]^2)*y'[x] == 0,y[x],x,IncludeSingularSolutions ->
```

$$y(x) \to \frac{e^{-3x}\left(-2e^{6x}x^2 + \sqrt[3]{2}\left(\sqrt{4e^{18x}x^6 + e^{6(2x+c_1)}} + e^{6x+3c_1}\right)^{2/3}\right)}{2^{2/3}\sqrt[3]{\sqrt{4e^{18x}x^6 + e^{6(2x+c_1)}} + e^{6x+3c_1}}}$$

$$y(x) \to \frac{i(\sqrt{3}+i)\,e^{-3x}\sqrt[3]{\sqrt{4e^{18x}x^6 + e^{6(2x+c_1)}} + e^{6x+3c_1}}}{2\sqrt[3]{2}} + \frac{(1+i\sqrt{3})\,e^{3x}x^2}{2^{2/3}\sqrt[3]{\sqrt{4e^{18x}x^6 + e^{6(2x+c_1)}} + e^{6x+3c_1}}}$$

$$y(x) \to \frac{(1-i\sqrt{3})\,e^{3x}x^2}{2^{2/3}\sqrt[3]{\sqrt{4e^{18x}x^6 + e^{6(2x+c_1)}} + e^{6x+3c_1}}} - \frac{(1+i\sqrt{3})\,e^{-3x}\sqrt[3]{\sqrt{4e^{18x}x^6 + e^{6(2x+c_1)}} + e^{6x+3c_1}}}{2\sqrt[3]{2}}$$

5.19 problem 26

Internal problem ID [561]

Book: Elementary differential equations and boundary value problems, 10th ed., Boyce and DiPrima
Section: Section 2.6. Page 100
Problem number: 26.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[[_linear, 'class A']]`

$$\boxed{y' - y = -1 + e^{2x}}$$

✓ Solution by Maple
Time used: 0.0 (sec). Leaf size: 14

```
dsolve(diff(y(x),x) = -1+exp(2*x)+y(x),y(x), singsol=all)
```

$$y(x) = e^{2x} + 1 + e^{x}c_1$$

✓ Solution by Mathematica
Time used: 0.047 (sec). Leaf size: 18

```
DSolve[y'[x] == -1+Exp[2*x]+y[x],y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to e^{2x} + c_1e^{x} + 1$$

5.20 problem 27

Internal problem ID [562]

Book: Elementary differential equations and boundary value problems, 10th ed., Boyce and DiPrima
Section: Section 2.6. Page 100
Problem number: 27.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[[_1st_order, '_with_symmetry_[F(x)*G(y),0]']]`

$$\boxed{\left(-\sin\left(y\right)+\frac{x}{y}\right)y'=-1}$$

✓ Solution by Maple
Time used: 0.078 (sec). Leaf size: 25

```
dsolve(1+(-sin(y(x))+x/y(x))*diff(y(x),x) = 0,y(x), singsol=all)
```

$$x+\frac{y(x)\cos\left(y(x)\right)-\sin\left(y(x)\right)-c_1}{y\left(x\right)}=0$$

✓ Solution by Mathematica
Time used: 0.143 (sec). Leaf size: 29

```
DSolve[1+(-Sin[y[x]]+x/y[x])*y'[x] == 0,y[x],x,IncludeSingularSolutions -> True]
```

$$\text{Solve}\left[x=\frac{\sin(y(x))-y(x)\cos(y(x))}{y(x)}+\frac{c_1}{y(x)},y(x)\right]$$

5.21 problem 28

Internal problem ID [563]

Book: Elementary differential equations and boundary value problems, 10th ed., Boyce and DiPrima
Section: Section 2.6. Page 100
Problem number: 28.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[[_1st_order, _with_exponential_symmetries]]`

$$\boxed{y + \left(-\mathrm{e}^{-2y} + 2yx\right) y' = 0}$$

✓ Solution by Maple
Time used: 0.031 (sec). Leaf size: 24

```
dsolve(y(x)+(-exp(-2*y(x))+2*x*y(x))*diff(y(x),x) = 0,y(x), singsol=all)
```

$$y(x) = \mathrm{e}^{\mathrm{RootOf}\left(c_1 \mathrm{e}^{-2\,\mathrm{e}^{\_Z}} + \_Z\,\mathrm{e}^{-2\,\mathrm{e}^{\_Z}} - x\right)}$$

✓ Solution by Mathematica
Time used: 0.243 (sec). Leaf size: 25

```
DSolve[y[x]+(-Exp[-2*y[x]]+2*x*y[x])*y'[x] == 0,y[x],x,IncludeSingularSolutions -> True]
```

$$\mathrm{Solve}\left[x = e^{-2y(x)} \log(y(x)) + c_1 e^{-2y(x)}, y(x)\right]$$

5.22 problem 29

Internal problem ID [564]

Book: Elementary differential equations and boundary value problems, 10th ed., Boyce and DiPrima
Section: Section 2.6. Page 100
Problem number: 29.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[[_1st_order, '_with_symmetry_[F(x)*G(y),0]']]`

$$\boxed{(e^x \cot(y) + 2\csc(y)\, y)\, y' = -e^x}$$

✓ Solution by Maple

Time used: 0.016 (sec). Leaf size: 15

```
dsolve(exp(x)+(exp(x)*cot(y(x))+2*csc(y(x))*y(x))*diff(y(x),x) = 0,y(x), singsol=all)
```

$$e^x \sin(y(x)) + y(x)^2 + c_1 = 0$$

✓ Solution by Mathematica

Time used: 0.312 (sec). Leaf size: 18

```
DSolve[Exp[x]+(Exp[x]*Cot[y[x]]+2*Csc[y[x]]*y[x])*y'[x] == 0,y[x],x,IncludeSingularSolutions
```

$$\text{Solve}\left[y(x)^2 + e^x \sin(y(x)) = c_1, y(x)\right]$$

5.23 problem 30

Internal problem ID [565]

Book: Elementary differential equations and boundary value problems, 10th ed., Boyce and DiPrima
Section: Section 2.6. Page 100
Problem number: 30.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_rational]`

$$\boxed{\frac{4x^3}{y^2} + \frac{3}{y} + \left(\frac{3x}{y^2} + 4y\right) y' = 0}$$

✓ Solution by Maple
Time used: 0.0 (sec). Leaf size: 17

```
dsolve(4*x^3/y(x)^2+3/y(x)+(3*x/y(x)^2+4*y(x))*diff(y(x),x) = 0,y(x), singsol=all)
```

$$x^4 + y(x)^4 + 3xy(x) + c_1 = 0$$

✓ Solution by Mathematica

Time used: 60.148 (sec). Leaf size: 1181

```
DSolve[4*x^3/y[x]^2+3/y[x]+(3*x/y[x]^2+4*y[x])*y'[x]== 0,y[x],x,IncludeSingularSolutions ->
```

$$y(x) \to -\frac{1}{2}\sqrt{\frac{6x}{\sqrt{\frac{4\sqrt[3]{2}(x^4-c_1)}{\sqrt[3]{243x^2+\sqrt{59049x^4-6912(x^4-c_1)^3}}}+\frac{\sqrt[3]{243x^2+\sqrt{59049x^4-6912(x^4-c_1)^3}}}{3\sqrt[3]{2}}}} - \frac{\quad}{\sqrt[3]{243 \cdots}}} - \frac{1}{2}\sqrt{\frac{4\sqrt[3]{2}\left(x^4-c_1\right)}{\sqrt[3]{243x^2+\sqrt{59049x^4-6912\left(x^4-c_1\right)^3}}}+\frac{\sqrt[3]{243x^2+\sqrt{59049x^4-6912\left(x^4-c_1\right)^3}}}{3\sqrt[3]{2}}}$$

$$y(x) \to \frac{1}{2}\sqrt{\frac{6x}{\sqrt{\frac{4\sqrt[3]{2}(x^4-c_1)}{\sqrt[3]{243x^2+\sqrt{59049x^4-6912(x^4-c_1)^3}}}+\frac{\sqrt[3]{243x^2+\sqrt{59049x^4-6912(x^4-c_1)^3}}}{3\sqrt[3]{2}}}} - \frac{\quad}{\sqrt[3]{243x^2 \cdots}}} - \frac{1}{2}\sqrt{\frac{4\sqrt[3]{2}\left(x^4-c_1\right)}{\sqrt[3]{243x^2+\sqrt{59049x^4-6912\left(x^4-c_1\right)^3}}}+\frac{\sqrt[3]{243x^2+\sqrt{59049x^4-6912\left(x^4-c_1\right)^3}}}{3\sqrt[3]{2}}}$$

$$y(x) \to \frac{1}{2}\sqrt{\frac{4\sqrt[3]{2}\left(x^4-c_1\right)}{\sqrt[3]{243x^2+\sqrt{59049x^4-6912\left(x^4-c_1\right)^3}}}+\frac{\sqrt[3]{243x^2+\sqrt{59049x^4-6912\left(x^4-c_1\right)^3}}}{3\sqrt[3]{2}}} - \frac{1}{2}\sqrt{-\frac{6x}{\sqrt{\frac{4\sqrt[3]{2}(x^4-c_1)}{\sqrt[3]{243x^2+\sqrt{59049x^4-6912(x^4-c_1)^3}}}+\frac{\sqrt[3]{243x^2+\sqrt{59049x^4-6912(x^4-c_1)^3}}}{3\sqrt[3]{2}}}} - \frac{\quad}{\sqrt[3]{24 \cdots}}}$$

$$y(x) \to \frac{1}{2}\sqrt{-\frac{6x}{\sqrt{\frac{4\sqrt[3]{2}(x^4-c_1)}{\sqrt[3]{243x^2+\sqrt{59049x^4-6912(x^4-c_1)^3}}}+\frac{\sqrt[3]{243x^2+\sqrt{59049x^4-6912(x^4-c_1)^3}}}{3\sqrt[3]{2}}}} - \frac{\quad}{\sqrt[3]{243 \cdots}}} + \frac{1}{2}\sqrt{\frac{4\sqrt[3]{2}\left(x^4-c_1\right)}{\sqrt[3]{243x^2+\sqrt{59049x^4-6912\left(x^4-c_1\right)^3}}}+\frac{\sqrt[3]{243x^2+\sqrt{59049x^4-6912\left(x^4-c_1\right)^3}}}{3\sqrt[3]{2}}}$$

5.24 problem 30

Internal problem ID [566]

Book: Elementary differential equations and boundary value problems, 10th ed., Boyce and DiPrima
Section: Section 2.6. Page 100
Problem number: 30.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_rational]`

$$\boxed{\frac{6}{y}+\left(\frac{x^2}{y}+\frac{3y}{x}\right)y'=-3x}$$

✓ Solution by Maple
Time used: 0.0 (sec). Leaf size: 326

```
dsolve(3*x+6/y(x)+(x^2/y(x)+3*y(x)/x)*diff(y(x),x) = 0,y(x), singsol=all)
```

$$y(x)=\frac{-12x^3+\left(-324x^2-108c_1+12\sqrt{12x^9+729x^4+486c_1x^2+81c_1^2}\right)^{\frac{2}{3}}}{6\left(-324x^2-108c_1+12\sqrt{12x^9+729x^4+486c_1x^2+81c_1^2}\right)^{\frac{1}{3}}}$$

$$y(x)=-\frac{(1+i\sqrt{3})\left(-324x^2-108c_1+12\sqrt{12x^9+729x^4+486c_1x^2+81c_1^2}\right)^{\frac{1}{3}}}{12}-\frac{x^3(i\sqrt{3}-1)}{\left(-324x^2-108c_1+12\sqrt{12x^9+729x^4+486c_1x^2+81c_1^2}\right)^{\frac{1}{3}}}$$

$$y(x)=\frac{12i\sqrt{3}\,x^3+i\sqrt{3}\left(-324x^2-108c_1+12\sqrt{12x^9+729x^4+486c_1x^2+81c_1^2}\right)^{\frac{2}{3}}+12x^3-\left(-324x^2-108c_1\right.}{12\left(-324x^2-108c_1+12\sqrt{12x^9+729x^4+486c_1x^2+81c_1^2}\right)^{\frac{1}{3}}}$$

✓ Solution by Mathematica

Time used: 4.558 (sec). Leaf size: 331

```
DSolve[3*x+6/y[x]+(x^2/y[x]+3*y[x]/x)*y'[x] == 0,y[x],x,IncludeSingularSolutions -> True]
```

$$
\begin{aligned}
y(x) \to & \frac{\sqrt[3]{-81x^2+\sqrt{108x^9+729(-3x^2+c_1)^2}+27c_1}}{3\sqrt[3]{2}} \\
& - \frac{\sqrt[3]{2}x^3}{\sqrt[3]{-81x^2+\sqrt{108x^9+729(-3x^2+c_1)^2}+27c_1}} \\
y(x) \to & \frac{(-1+i\sqrt{3})\sqrt[3]{-81x^2+\sqrt{108x^9+729(-3x^2+c_1)^2}+27c_1}}{6\sqrt[3]{2}} \\
& + \frac{(1+i\sqrt{3})x^3}{2^{2/3}\sqrt[3]{-81x^2+\sqrt{108x^9+729(-3x^2+c_1)^2}+27c_1}} \\
y(x) \to & \frac{(1-i\sqrt{3})x^3}{2^{2/3}\sqrt[3]{-81x^2+\sqrt{108x^9+729(-3x^2+c_1)^2}+27c_1}} \\
& - \frac{(1+i\sqrt{3})\sqrt[3]{-81x^2+\sqrt{108x^9+729(-3x^2+c_1)^2}+27c_1}}{6\sqrt[3]{2}}
\end{aligned}
$$

5.25 problem 32

Internal problem ID [567]

Book: Elementary differential equations and boundary value problems, 10th ed., Boyce and DiPrima
Section: Section 2.6. Page 100
Problem number: 32.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[[_homogeneous, ‘class A‘], _rational, [_Abel, ‘2nd type‘, ‘cl`

$$\boxed{3yx + y^2 + (x^2 + yx)\, y' = 0}$$

✓ Solution by Maple

Time used: 0.031 (sec). Leaf size: 59

```
dsolve(3*x*y(x)+y(x)^2+(x^2+x*y(x))*diff(y(x),x) = 0,y(x), singsol=all)
```

$$y(x) = \frac{-c_1 x^2 - \sqrt{c_1^2 x^4 + 1}}{c_1 x}$$
$$y(x) = \frac{-c_1 x^2 + \sqrt{c_1^2 x^4 + 1}}{c_1 x}$$

✓ Solution by Mathematica

Time used: 0.661 (sec). Leaf size: 93

```
DSolve[3*x*y[x]+y[x]^2+(x^2+x*y[x])*y'[x]== 0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to -\frac{x^2 + \sqrt{x^4 + e^{2c_1}}}{x}$$
$$y(x) \to -x + \frac{\sqrt{x^4 + e^{2c_1}}}{x}$$
$$y(x) \to -\frac{\sqrt{x^4} + x^2}{x}$$
$$y(x) \to \frac{\sqrt{x^4}}{x} - x$$

6 Miscellaneous problems, end of chapter 2. Page 133

6.1 problem 1

Internal problem ID [568]

Book: Elementary differential equations and boundary value problems, 10th ed., Boyce and DiPrima
Section: Miscellaneous problems, end of chapter 2. Page 133
Problem number: 1.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_linear]`

$$\boxed{y' - \frac{x^3 - 2y}{x} = 0}$$

✓ Solution by Maple
Time used: 0.0 (sec). Leaf size: 16

```
dsolve(diff(y(x),x) = (x^3-2*y(x))/x,y(x), singsol=all)
```

$$y(x) = \frac{x^5 + 5c_1}{5x^2}$$

✓ Solution by Mathematica
Time used: 0.024 (sec). Leaf size: 19

```
DSolve[y'[x]== (x^3-2*y[x])/x,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to \frac{x^3}{5} + \frac{c_1}{x^2}$$

6.2 problem 2

Internal problem ID [569]

Book: Elementary differential equations and boundary value problems, 10th ed., Boyce and DiPrima
Section: Miscellaneous problems, end of chapter 2. Page 133
Problem number: 2.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_separable]`

$$\boxed{y' - \frac{\cos(x) + 1}{2 - \sin(y)} = 0}$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 17

```
dsolve(diff(y(x),x) = (1+cos(x))/(2-sin(y(x))),y(x), singsol=all)
```

$$x + \sin(x) - 2y(x) - \cos(y(x)) + c_1 = 0$$

✓ Solution by Mathematica

Time used: 0.372 (sec). Leaf size: 27

```
DSolve[y'[x] == (1+Cos[x])/(2-Sin[y[x]]),y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to \text{InverseFunction}[-2\#1 - \cos(\#1)\&][-x - \sin(x) + c_1]$$

6.3 problem 3

Internal problem ID [570]

Book: Elementary differential equations and boundary value problems, 10th ed., Boyce and DiPrima
Section: Miscellaneous problems, end of chapter 2. Page 133
Problem number: 3.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_rational]`

$$\boxed{y' - \frac{2x + y}{3 - x + 3y^2} = 0}$$

With initial conditions

$$[y(0) = 0]$$

✓ Solution by Maple
Time used: 0.078 (sec). Leaf size: 75

```
dsolve([diff(y(x),x) = (2*x+y(x))/(3-x+3*y(x)^2),y(0) = 0],y(x), singsol=all)
```

$$y(x) = \frac{\left(108x^2 + 12\sqrt{81x^4 - 12x^3 + 108x^2 - 324x + 324}\right)^{\frac{2}{3}} + 12x - 36}{6\left(108x^2 + 12\sqrt{81x^4 - 12x^3 + 108x^2 - 324x + 324}\right)^{\frac{1}{3}}}$$

✓ Solution by Mathematica
Time used: 5.408 (sec). Leaf size: 114

```
DSolve[{y'[x] == (2*x+y[x])/(3-x+3*y[x]^2),y[0]==0},y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to -\frac{\sqrt[3]{2}\left(\sqrt{3}\sqrt{27x^4 - 4x^3 + 36x^2 - 108x + 108} - 9x^2\right)^{2/3} + 2\sqrt[3]{3}x - 6\sqrt[3]{3}}{6^{2/3}\sqrt[3]{\sqrt{3}\sqrt{27x^4 - 4x^3 + 36x^2 - 108x + 108} - 9x^2}}$$

6.4 problem 4

Internal problem ID [571]

Book: Elementary differential equations and boundary value problems, 10th ed., Boyce and DiPrima
Section: Miscellaneous problems, end of chapter 2. Page 133
Problem number: 4.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_separable]`

$$\boxed{y' - y + 2yx = -6x + 3}$$

✓ Solution by Maple
Time used: 0.016 (sec). Leaf size: 15

```
dsolve(diff(y(x),x) = 3-6*x+y(x)-2*x*y(x),y(x), singsol=all)
```

$$y(x) = -3 + \mathrm{e}^{-x(x-1)} c_1$$

✓ Solution by Mathematica
Time used: 0.048 (sec). Leaf size: 24

```
DSolve[y'[x] == 3-6*x+y[x]-2*x*y[x],y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to -3 + c_1 e^{x-x^2}$$
$$y(x) \to -3$$

6.5 problem 5

Internal problem ID [572]

Book: Elementary differential equations and boundary value problems, 10th ed., Boyce and DiPrima
Section: Miscellaneous problems, end of chapter 2. Page 133
Problem number: 5.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_rational, [_Abel, '2nd type', 'class B']]`

$$\boxed{y' - \frac{-1 - 2yx - y^2}{x^2 + 2yx} = 0}$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 59

```
dsolve(diff(y(x),x) = (-1-2*x*y(x)-y(x)^2)/(x^2+2*x*y(x)),y(x), singsol=all)
```

$$y(x) = \frac{-x^2 + \sqrt{x\,(x^3 - 4c_1 - 4x)}}{2x}$$
$$y(x) = \frac{-x^2 - \sqrt{x\,(x^3 - 4c_1 - 4x)}}{2x}$$

✓ Solution by Mathematica

Time used: 0.502 (sec). Leaf size: 67

```
DSolve[y'[x] == (-1-2*x*y[x]-y[x]^2)/(x^2+2*x*y[x]),y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to -\frac{x^2 + \sqrt{x\,(x^3 - 4x + 4c_1)}}{2x}$$
$$y(x) \to \frac{-x^2 + \sqrt{x\,(x^3 - 4x + 4c_1)}}{2x}$$

6.6 problem 6

Internal problem ID [573]

Book: Elementary differential equations and boundary value problems, 10th ed., Boyce and DiPrima
Section: Miscellaneous problems, end of chapter 2. Page 133
Problem number: 6.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_linear]`

$$\boxed{yx + y'x + y = 1}$$

With initial conditions

$$[y(1) = 0]$$

✓ Solution by Maple
Time used: 0.016 (sec). Leaf size: 18

```
dsolve([x*y(x)+x*diff(y(x),x) = 1-y(x),y(1) = 0],y(x), singsol=all)
```

$$y(x) = \frac{1 - e^{1-x}}{x}$$

✓ Solution by Mathematica
Time used: 0.03 (sec). Leaf size: 20

```
DSolve[{x*y[x]+x*y'[x] == 1-y[x],y[1]==0},y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to \frac{1 - e^{1-x}}{x}$$

6.7 problem 7

Internal problem ID [574]

Book: Elementary differential equations and boundary value problems, 10th ed., Boyce and DiPrima
Section: Miscellaneous problems, end of chapter 2. Page 133
Problem number: 7.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_separable]`

$$y' - \frac{4x^3 + 1}{y(2 + 3y)} = 0$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 382

```
dsolve(diff(y(x),x) = (4*x^3+1)/(y(x)*(2+3*y(x))),y(x), singsol=all)
```

$$y(x) = \frac{\left(-8 + 108x^4 + 108c_1 + 108x + 12\sqrt{81}\sqrt{(x^4 + c_1 + x)\left(x^4 + c_1 + x - \frac{4}{27}\right)}\right)^{\frac{2}{3}} - 2\left(-8 + 108x^4 + 108c_1 \right.}{6\left(-8 + 108x^4 + 108c_1 + 108x + 12\sqrt{81}\sqrt{(x^4 + c_1 + x)(x^4 \right.}$$

$$y(x) = -\frac{\left(1 + i\sqrt{3}\right)\left(-8 + 108x^4 + 108c_1 + 108x + 12\sqrt{81}\sqrt{(x^4 + c_1 + x)\left(x^4 + c_1 + x - \frac{4}{27}\right)}\right)^{\frac{2}{3}} - 4i\sqrt{3} + 4\left(\right.}{12\left(-8 + 108x^4 + 108c_1 + 108x + 12\sqrt{81}\sqrt{(x^4 + \right.}$$

$$y(x) = \frac{\left(i\sqrt{3} - 1\right)\left(-8 + 108x^4 + 108c_1 + 108x + 12\sqrt{81}\sqrt{(x^4 + c_1 + x)\left(x^4 + c_1 + x - \frac{4}{27}\right)}\right)^{\frac{2}{3}} - 4i\sqrt{3} - 4\left(\right.}{12\left(-8 + 108x^4 + 108c_1 + 108x + 12\sqrt{81}\sqrt{(x^4 + c_1 \right.}$$

✓ Solution by Mathematica

Time used: 4.502 (sec). Leaf size: 356

```
DSolve[y'[x]== (4*x^3+1)/(y[x]*(2+3*y[x])),y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to \frac{1}{6}\left(2^{2/3}\sqrt[3]{27x^4+\sqrt{-4+(27x^4+27x-2+27c_1)^2}+27x-2+27c_1} + \frac{2\sqrt[3]{2}}{\sqrt[3]{27x^4+\sqrt{-4+(27x^4+27x-2+27c_1)^2}+27x-2+27c_1}}-2\right)$$

$$y(x) \to \frac{1}{12}\left(i2^{2/3}\left(\sqrt{3}+i\right)\sqrt[3]{27x^4+\sqrt{-4+(27x^4+27x-2+27c_1)^2}+27x-2+27c_1} - \frac{2\sqrt[3]{2}(1+i\sqrt{3})}{\sqrt[3]{27x^4+\sqrt{-4+(27x^4+27x-2+27c_1)^2}+27x-2+27c_1}}-4\right)$$

$$y(x) \to \frac{1}{12}\left(-2^{2/3}\left(1+i\sqrt{3}\right)\sqrt[3]{27x^4+\sqrt{-4+(27x^4+27x-2+27c_1)^2}+27x-2+27c_1} + \frac{2i\sqrt[3]{2}(\sqrt{3}+i)}{\sqrt[3]{27x^4+\sqrt{-4+(27x^4+27x-2+27c_1)^2}+27x-2+27c_1}}-4\right)$$

6.8 problem 8

Internal problem ID [575]

Book: Elementary differential equations and boundary value problems, 10th ed., Boyce and DiPrima
Section: Miscellaneous problems, end of chapter 2. Page 133
Problem number: 8.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_linear]`

$$\boxed{2y + y'x = \frac{\sin(x)}{x}}$$

With initial conditions

$$[y(2) = 1]$$

✓ Solution by Maple
Time used: 0.015 (sec). Leaf size: 16

```
dsolve([2*y(x)+x*diff(y(x),x) = sin(x)/x,y(2) = 1],y(x), singsol=all)
```

$$y(x) = \frac{-\cos(x) + 4 + \cos(2)}{x^2}$$

✓ Solution by Mathematica
Time used: 0.039 (sec). Leaf size: 17

```
DSolve[{2*y[x]+x*y'[x] == Sin[x]/x,y[2]==1},y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to \frac{-\cos(x) + 4 + \cos(2)}{x^2}$$

6.9 problem 9

Internal problem ID [576]

Book: Elementary differential equations and boundary value problems, 10th ed., Boyce and DiPrima
Section: Miscellaneous problems, end of chapter 2. Page 133
Problem number: 9.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_rational, [_1st_order, '_with_symmetry_[F(x),G(x)]'], [_Abel`

$$\boxed{y' - \frac{-1 - 2yx}{x^2 + 2y} = 0}$$

✓ Solution by Maple
Time used: 0.0 (sec). Leaf size: 47

```
dsolve(diff(y(x),x) = (-1-2*x*y(x))/(x^2+2*y(x)),y(x), singsol=all)
```

$$y(x) = -\frac{x^2}{2} - \frac{\sqrt{x^4 - 4c_1 - 4x}}{2}$$
$$y(x) = -\frac{x^2}{2} + \frac{\sqrt{x^4 - 4c_1 - 4x}}{2}$$

✓ Solution by Mathematica
Time used: 0.15 (sec). Leaf size: 61

```
DSolve[y'[x]== (-1-2*x*y[x])/(x^2+2*y[x]),y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to \frac{1}{2}\left(-x^2 - \sqrt{x^4 - 4x + 4c_1}\right)$$
$$y(x) \to \frac{1}{2}\left(-x^2 + \sqrt{x^4 - 4x + 4c_1}\right)$$

6.10 problem 10

Internal problem ID [577]

Book: Elementary differential equations and boundary value problems, 10th ed., Boyce and DiPrima
Section: Miscellaneous problems, end of chapter 2. Page 133
Problem number: 10.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_separable]`

$$\boxed{\frac{yy'}{y-2} = -\frac{-x^2+x+1}{x^2}}$$

✓ Solution by Maple
Time used: 0.016 (sec). Leaf size: 26

```
dsolve((-x^2+x+1)/x^2+y(x)*diff(y(x),x)/(-2+y(x)) = 0,y(x), singsol=all)
```

$$y(x) = 2\,\mathrm{LambertW}\left(\frac{c_1 \mathrm{e}^{\frac{(x-1)^2}{2x}}}{2\sqrt{x}}\right) + 2$$

✓ Solution by Mathematica
Time used: 60.036 (sec). Leaf size: 63

```
DSolve[(-x^2+x+1)/x^2+y[x]*y'[x]/(-2+y[x]) == 0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to 2\left(1 + W\left(-\frac{1}{2}\sqrt{\frac{e^{x+\frac{1}{x}-2+c_1}}{x}}\right)\right)$$
$$y(x) \to 2\left(1 + W\left(\frac{1}{2}\sqrt{\frac{e^{x+\frac{1}{x}-2+c_1}}{x}}\right)\right)$$

6.11 problem 11

Internal problem ID [578]

Book: Elementary differential equations and boundary value problems, 10th ed., Boyce and DiPrima
Section: Miscellaneous problems, end of chapter 2. Page 133
Problem number: 11.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_exact]`

$$\boxed{y + (\mathrm{e}^{y} + x)\, y' = -x^{2}}$$

✓ Solution by Maple

Time used: 0.015 (sec). Leaf size: 39

```
dsolve(x^2+y(x)+(exp(y(x))+x)*diff(y(x),x) = 0,y(x), singsol=all)
```

$$y(x) = \frac{-x^{3} - 3x\,\mathrm{LambertW}\left(\frac{\mathrm{e}^{-\frac{x^{3}+3c_{1}}{3x}}}{x}\right) - 3c_{1}}{3x}$$

✓ Solution by Mathematica

Time used: 3.877 (sec). Leaf size: 42

```
DSolve[x^2+y[x]+(Exp[y[x]]+x)*y'[x] == 0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to -W\left(\frac{e^{-\frac{x^{2}}{3}+\frac{c_{1}}{x}}}{x}\right) - \frac{x^{2}}{3} + \frac{c_{1}}{x}$$

6.12 problem 12

Internal problem ID [579]

Book: Elementary differential equations and boundary value problems, 10th ed., Boyce and DiPrima
Section: Miscellaneous problems, end of chapter 2. Page 133
Problem number: 12.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_linear]`

$$\boxed{y' + y = \frac{1}{1 + \mathrm{e}^{x}}}$$

✓ Solution by Maple
Time used: 0.0 (sec). Leaf size: 16

```
dsolve(y(x)+diff(y(x),x) = 1/(1+exp(x)),y(x), singsol=all)
```

$$y(x) = (\ln(1 + \mathrm{e}^{x}) + c_1)\,\mathrm{e}^{-x}$$

✓ Solution by Mathematica
Time used: 0.067 (sec). Leaf size: 20

```
DSolve[y[x]+y'[x] == 1/(1+Exp[x]),y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to e^{-x}(\log(e^{x} + 1) + c_1)$$

6.13 problem 13

Internal problem ID [580]

Book: Elementary differential equations and boundary value problems, 10th ed., Boyce and DiPrima
Section: Miscellaneous problems, end of chapter 2. Page 133
Problem number: 13.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_separable]`

$$\boxed{y' - y^2 - 2xy^2 = 1 + 2x}$$

✓ Solution by Maple
Time used: 0.0 (sec). Leaf size: 11

```
dsolve(diff(y(x),x) = 1+2*x+y(x)^2+2*x*y(x)^2,y(x), singsol=all)
```

$$y(x) = \tan\left(x^2 + c_1 + x\right)$$

✓ Solution by Mathematica
Time used: 0.178 (sec). Leaf size: 13

```
DSolve[y'[x] == 1+2*x+y[x]^2+2*x*y[x]^2,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to \tan\left(x^2 + x + c_1\right)$$

6.14 problem 14

Internal problem ID [581]

Book: Elementary differential equations and boundary value problems, 10th ed., Boyce and DiPrima
Section: Miscellaneous problems, end of chapter 2. Page 133
Problem number: 14.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[[_homogeneous, ‘class A‘], _exact, _rational, [_Abel, ‘2nd ty`

$$\boxed{y + (x + 2y)\, y' = -x}$$

With initial conditions

$$[y(2) = 3]$$

✓ Solution by Maple

Time used: 0.093 (sec). Leaf size: 19

```
dsolve([x+y(x)+(x+2*y(x))*diff(y(x),x) = 0,y(2) = 3],y(x), singsol=all)
```

$$y(x) = -\frac{x}{2} + \frac{\sqrt{-x^2 + 68}}{2}$$

✓ Solution by Mathematica

Time used: 0.458 (sec). Leaf size: 24

```
DSolve[{x+y[x]+(x+2*y[x])*y'[x] == 0,y[2]==3},y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to \frac{1}{2}\left(\sqrt{68 - x^2} - x\right)$$

6.15 problem 15

Internal problem ID [582]

Book: Elementary differential equations and boundary value problems, 10th ed., Boyce and DiPrima
Section: Miscellaneous problems, end of chapter 2. Page 133
Problem number: 15.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_separable]`

$$\boxed{(1+\mathrm{e}^{x})\, y' - y + \mathrm{e}^{x} y = 0}$$

✓ Solution by Maple
Time used: 0.0 (sec). Leaf size: 14

```
dsolve((1+exp(x))*diff(y(x),x) = y(x)-exp(x)*y(x),y(x), singsol=all)
```

$$y(x) = \frac{c_1 \mathrm{e}^{x}}{(1+\mathrm{e}^{x})^2}$$

✓ Solution by Mathematica
Time used: 0.077 (sec). Leaf size: 23

```
DSolve[(1+Exp[x])*y'[x]== y[x]-Exp[x]*y[x],y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to \frac{c_1 e^x}{(e^x+1)^2}$$
$$y(x) \to 0$$

6.16 problem 16

Internal problem ID [583]

Book: Elementary differential equations and boundary value problems, 10th ed., Boyce and DiPrima
Section: Miscellaneous problems, end of chapter 2. Page 133
Problem number: 16.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[NONE]`

$$
\boxed{y' - \frac{-e^{2y}\cos(x) + \cos(y)\, e^{-x}}{2\, e^{2y}\sin(x) - \sin(y)\, e^{-x}} = 0}
$$

✓ Solution by Maple
Time used: 0.047 (sec). Leaf size: 21

```
dsolve(diff(y(x),x) = (-exp(2*y(x))*cos(x)+cos(y(x))/exp(x))/(2*exp(2*y(x))*sin(x)-sin(y(x))
```

$$
c_1 + \cos(y(x))\, e^{-x} + e^{2y(x)} \sin(x) = 0
$$

✓ Solution by Mathematica
Time used: 0.473 (sec). Leaf size: 25

```
DSolve[y'[x] == (-Exp[2*y[x]]*Cos[x]+Cos[y[x]]/Exp[x])/(2*Exp[2*y[x]]*Sin[x]-Sin[y[x]]/Exp[x
```

$$
\text{Solve}\left[e^{2y(x)} \sin(x) + e^{-x} \cos(y(x)) = c_1, y(x)\right]
$$

6.17 problem 17

Internal problem ID [584]

Book: Elementary differential equations and boundary value problems, 10th ed., Boyce and DiPrima
Section: Miscellaneous problems, end of chapter 2. Page 133
Problem number: 17.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[[_linear, ‘class A‘]]`

$$\boxed{y' - 3y = \mathrm{e}^{2x}}$$

✓ Solution by Maple
Time used: 0.0 (sec). Leaf size: 15

```
dsolve(diff(y(x),x) = exp(2*x)+3*y(x),y(x), singsol=all)
```

$$y(x) = (\mathrm{e}^{x} c_1 - 1)\,\mathrm{e}^{2x}$$

✓ Solution by Mathematica
Time used: 0.04 (sec). Leaf size: 19

```
DSolve[y'[x]== Exp[2*x]+3*y[x],y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to e^{2x}(-1 + c_1 e^{x})$$

6.18 problem 18

Internal problem ID [585]

Book: Elementary differential equations and boundary value problems, 10th ed., Boyce and DiPrima
Section: Miscellaneous problems, end of chapter 2. Page 133
Problem number: 18.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[[_linear, 'class A']]`

$$\boxed{2y + y' = \mathrm{e}^{-x^2-2x}}$$

✓ Solution by Maple
Time used: 0.0 (sec). Leaf size: 20

```
dsolve(2*y(x)+diff(y(x),x) = exp(-x^2-2*x),y(x), singsol=all)
```

$$y(x) = \frac{\left(\sqrt{\pi}\, \mathrm{erf}\left(x\right) + 2c_1\right) \mathrm{e}^{-2x}}{2}$$

✓ Solution by Mathematica
Time used: 0.09 (sec). Leaf size: 27

```
DSolve[2*y[x]+y'[x] == Exp[-x^2-2*x],y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to \frac{1}{2} e^{-2x} \left(\sqrt{\pi}\mathrm{erf}(x) + 2c_1\right)$$

6.19 problem 19

Internal problem ID [586]

Book: Elementary differential equations and boundary value problems, 10th ed., Boyce and DiPrima
Section: Miscellaneous problems, end of chapter 2. Page 133
Problem number: 19.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_rational]`

$$\boxed{y' - \frac{3x^2 - 2y - y^3}{2x + 3xy^2} = 0}$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 409

```
dsolve(diff(y(x),x) = (3*x^2-2*y(x)-y(x)^3)/(2*x+3*x*y(x)^2),y(x), singsol=all)
```

$$y(x) = -\frac{12^{\frac{1}{3}}\left(x^2 12^{\frac{1}{3}} - \frac{\left(\left(9x^3+\sqrt{3}\,\sqrt{27x^6-54c_1x^3+27c_1^2+32x^2}-9c_1\right)x^2\right)^{\frac{2}{3}}}{2}\right)}{3\left(\left(9x^3+\sqrt{3}\,\sqrt{27x^6-54c_1x^3+27c_1^2+32x^2}-9c_1\right)x^2\right)^{\frac{1}{3}}x}$$

$$y(x) = -\frac{2^{\frac{2}{3}}3^{\frac{1}{3}}\left(2i2^{\frac{2}{3}}3^{\frac{5}{6}}x^2 - 2x^2 2^{\frac{2}{3}}3^{\frac{1}{3}} + i\left(\left(9x^3+\sqrt{3}\,\sqrt{27x^6-54c_1x^3+27c_1^2+32x^2}-9c_1\right)x^2\right)^{\frac{2}{3}}\sqrt{3} + \left(\left(9x^3\right.\right.\right.}{12x\left(\left(9x^3+\sqrt{3}\,\sqrt{27x^6-54c_1x^3+27c_1^2+32x^2}-9c_1\right.\right.}$$

$$y(x) = \frac{2^{\frac{2}{3}}3^{\frac{1}{3}}\left(2\left(i3^{\frac{5}{6}}+3^{\frac{1}{3}}\right)x^2 2^{\frac{2}{3}} + \left(i\sqrt{3}-1\right)\left(\left(9x^3+\sqrt{3}\,\sqrt{27x^6-54c_1x^3+27c_1^2+32x^2}-9c_1\right)x^2\right)^{\frac{2}{3}}\right)}{12\left(\left(9x^3+\sqrt{3}\,\sqrt{27x^6-54c_1x^3+27c_1^2+32x^2}-9c_1\right)x^2\right)^{\frac{1}{3}}x}$$

✓ Solution by Mathematica

Time used: 32.075 (sec). Leaf size: 358

```
DSolve[y'[x] == (3*x^2-2*y[x]-y[x]^3)/(2*x+3*x*y[x]^2),y[x],x,IncludeSingularSolutions -> Tr
```

$$y(x) \to \frac{\sqrt[3]{27x^5 + 27c_1x^2 + \sqrt{864x^6 + 729x^4(x^3 + c_1)^2}}}{3\sqrt[3]{2}x} - \frac{2\sqrt[3]{2}x}{\sqrt[3]{27x^5 + 27c_1x^2 + \sqrt{864x^6 + 729x^4(x^3 + c_1)^2}}}$$

$$y(x) \to \frac{\sqrt[3]{2}(1 + i\sqrt{3})\,x}{\sqrt[3]{27x^5 + 27c_1x^2 + \sqrt{864x^6 + 729x^4(x^3 + c_1)^2}}} - \frac{(1 - i\sqrt{3})\sqrt[3]{27x^5 + 27c_1x^2 + \sqrt{864x^6 + 729x^4(x^3 + c_1)^2}}}{6\sqrt[3]{2}x}$$

$$y(x) \to \frac{\sqrt[3]{2}(1 - i\sqrt{3})\,x}{\sqrt[3]{27x^5 + 27c_1x^2 + \sqrt{864x^6 + 729x^4(x^3 + c_1)^2}}} - \frac{(1 + i\sqrt{3})\sqrt[3]{27x^5 + 27c_1x^2 + \sqrt{864x^6 + 729x^4(x^3 + c_1)^2}}}{6\sqrt[3]{2}x}$$

6.20 problem 20

Internal problem ID [587]

Book: Elementary differential equations and boundary value problems, 10th ed., Boyce and DiPrima
Section: Miscellaneous problems, end of chapter 2. Page 133
Problem number: 20.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_separable]`

$$\boxed{y' - e^{x+y} = 0}$$

✓ Solution by Maple
Time used: 0.0 (sec). Leaf size: 13

```
dsolve(diff(y(x),x) = exp(x+y(x)),y(x), singsol=all)
```

$$y(x) = \ln\left(-\frac{1}{e^x + c_1}\right)$$

✓ Solution by Mathematica
Time used: 0.752 (sec). Leaf size: 18

```
DSolve[y'[x] == Exp[x+y[x]],y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to -\log\left(-e^x - c_1\right)$$

6.21 problem 21

Internal problem ID [588]

Book: Elementary differential equations and boundary value problems, 10th ed., Boyce and DiPrima
Section: Miscellaneous problems, end of chapter 2. Page 133
Problem number: 21.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_rational]`

$$\boxed{\frac{-4 + 6yx + 2y^2}{3x^2 + 4yx + 3y^2} + y' = 0}$$

✓ Solution by Maple

Time used: 0.015 (sec). Leaf size: 517

```
dsolve((-4+6*x*y(x)+2*y(x)^2)/(3*x^2+4*x*y(x)+3*y(x)^2)+diff(y(x),x)=0,y(x), singsol=all)
```

$$y(x) = \frac{\left(152x^3 - 108c_1 + 432x + 12\sqrt{216x^6 - 228c_1x^3 + 912x^4 + 81c_1^2 - 648c_1x + 1296x^2}\right)^{\frac{1}{3}}}{6} - \frac{10x^2}{3\left(152x^3 - 108c_1 + 432x + 12\sqrt{216x^6 - 228c_1x^3 + 912x^4 + 81c_1^2 - 648c_1x + 1296x^2}\right)^{\frac{1}{3}}} - \frac{2x}{3}$$

$$y(x) = -\frac{\left(1 + i\sqrt{3}\right)\left(152x^3 - 108c_1 + 432x + 12\sqrt{216x^6 - 228c_1x^3 + 912x^4 + 81c_1^2 - 648c_1x + 1296x^2}\right)^{\frac{1}{3}}}{12} - \frac{5x\left(i\sqrt{3}\,x - x + \frac{2\left(152x^3 - 108c_1 + 432x + 12\sqrt{216x^6 - 228c_1x^3 + 912x^4 + 81c_1^2 - 648c_1x + 1296x^2}\right)^{\frac{1}{3}}}{5}\right)}{3\left(152x^3 - 108c_1 + 432x + 12\sqrt{216x^6 - 228c_1x^3 + 912x^4 + 81c_1^2 - 648c_1x + 1296x^2}\right)^{\frac{1}{3}}}$$

$$y(x) = \frac{i\left(152x^3 - 108c_1 + 432x + 12\sqrt{216x^6 - 228c_1x^3 + 912x^4 + 81c_1^2 - 648c_1x + 1296x^2}\right)^{\frac{2}{3}}\sqrt{3} + 20i\sqrt{3}\,x^2 \cdots}{12 \cdots}$$

✓ Solution by Mathematica

Time used: 4.748 (sec). Leaf size: 383

```
DSolve[(-4+6*x*y[x]+2*y[x]^2)/(3*x^2+4*x*y[x]+3*y[x]^2)+y'[x]==0,y[x],x,IncludeSingularSolut
```

$$y(x) \to \frac{1}{6}\left(2^{2/3}\sqrt[3]{38x^3+\sqrt{500x^6+(38x^3+108x+27c_1)^2}+108x+27c_1} - \frac{10\sqrt[3]{2}x^2}{\sqrt[3]{38x^3+\sqrt{500x^6+(38x^3+108x+27c_1)^2}+108x+27c_1}} - 4x\right)$$

$$y(x) \to \frac{1}{12}\left(i2^{2/3}\left(\sqrt{3}+i\right)\sqrt[3]{38x^3+\sqrt{500x^6+(38x^3+108x+27c_1)^2}+108x+27c_1} + \frac{10\sqrt[3]{2}\left(1+i\sqrt{3}\right)x^2}{\sqrt[3]{38x^3+\sqrt{500x^6+(38x^3+108x+27c_1)^2}+108x+27c_1}} - 8x\right)$$

$$y(x) \to \frac{1}{12}\left(-2^{2/3}\left(1+i\sqrt{3}\right)\sqrt[3]{38x^3+\sqrt{500x^6+(38x^3+108x+27c_1)^2}+108x+27c_1} + \frac{10\sqrt[3]{2}\left(1-i\sqrt{3}\right)x^2}{\sqrt[3]{38x^3+\sqrt{500x^6+(38x^3+108x+27c_1)^2}+108x+27c_1}} - 8x\right)$$

6.22 problem 22

Internal problem ID [589]

Book: Elementary differential equations and boundary value problems, 10th ed., Boyce and DiPrima
Section: Miscellaneous problems, end of chapter 2. Page 133
Problem number: 22.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_separable]`

$$\boxed{y' - \frac{x^2-1}{1+y^2} = 0}$$

With initial conditions

$$[y(-1) = 1]$$

✓ Solution by Maple

Time used: 0.11 (sec). Leaf size: 87

```
dsolve([diff(y(x),x) = (x^2-1)/(1+y(x)^2),y(-1) = 1],y(x), singsol=all)
```

$$y(x) = \frac{\left(8+4x^3-12x+4\sqrt{x^6-6x^4+4x^3+9x^2-12x+8}\right)^{\frac{2}{3}}-4}{2\left(8+4x^3-12x+4\sqrt{x^6-6x^4+4x^3+9x^2-12x+8}\right)^{\frac{1}{3}}}$$

✓ Solution by Mathematica

Time used: 2.95 (sec). Leaf size: 97

```
DSolve[{y'[x]== (x^2-1)/(1+y[x]^2),y[-1]==1},y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to \frac{\sqrt[3]{2}\left(x^3+\sqrt{x^6-6x^4+4x^3+9x^2-12x+8}-3x+2\right)^{2/3}-2}{2^{2/3}\sqrt[3]{x^3+\sqrt{x^6-6x^4+4x^3+9x^2-12x+8}-3x+2}}$$

6.23 problem 23

Internal problem ID [590]

Book: Elementary differential equations and boundary value problems, 10th ed., Boyce and DiPrima
Section: Miscellaneous problems, end of chapter 2. Page 133
Problem number: 23.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_linear]`

$$\boxed{(t+1)\,y+ty' = \mathrm{e}^{2t}}$$

✓ Solution by Maple
Time used: 0.0 (sec). Leaf size: 21

```
dsolve((1+t)*y(t)+t*diff(y(t),t) = exp(2*t),y(t), singsol=all)
```

$$y(t) = \frac{\mathrm{e}^{2t}+3\,\mathrm{e}^{-t}c_1}{3t}$$

✓ Solution by Mathematica
Time used: 0.053 (sec). Leaf size: 27

```
DSolve[(1+t)*y[t]+t*y'[t] == Exp[2*t],y[t],t,IncludeSingularSolutions -> True]
```

$$y(t)\to \frac{e^{2t}+3c_1e^{-t}}{3t}$$

6.24 problem 24

Internal problem ID [591]

Book: Elementary differential equations and boundary value problems, 10th ed., Boyce and DiPrima
Section: Miscellaneous problems, end of chapter 2. Page 133
Problem number: 24.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_separable]`

$$2\cos(x)\sin(x)\sin(y) + \cos(y)\sin(x)^2 y' = 0$$

✓ Solution by Maple
Time used: 0.328 (sec). Leaf size: 18

```
dsolve(2*cos(x)*sin(x)*sin(y(x))+cos(y(x))*sin(x)^2*diff(y(x),x) = 0,y(x), singsol=all)
```

$$y(x) = -\arcsin\left(\frac{2c_1}{-1+\cos(2x)}\right)$$

✓ Solution by Mathematica
Time used: 5.176 (sec). Leaf size: 21

```
DSolve[2*Cos[x]*Sin[x]*Sin[y[x]]+Cos[y[x]]*Sin[x]^2*y'[x] == 0,y[x],x,IncludeSingularSolutio
```

$$y(x) \to \arcsin\left(\frac{1}{2}c_1\csc^2(x)\right)$$
$$y(x) \to 0$$

6.25 problem 25

Internal problem ID [592]

Book: Elementary differential equations and boundary value problems, 10th ed., Boyce and DiPrima
Section: Miscellaneous problems, end of chapter 2. Page 133
Problem number: 25.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_exact, _rational]`

$$\boxed{\frac{2x}{y} - \frac{y}{x^2+y^2} + \left(-\frac{x^2}{y^2} + \frac{x}{x^2+y^2}\right) y' = 0}$$

✓ Solution by Maple
Time used: 0.016 (sec). Leaf size: 17

```
dsolve(2*x/y(x)-y(x)/(x^2+y(x)^2)+(-x^2/y(x)^2+x/(x^2+y(x)^2))*diff(y(x),x) = 0,y(x), singso
```

$$y(x) = \cot\left(\mathrm{RootOf}\left(-\_Z + x\tan\left(\_Z\right) + c_1\right)\right) x$$

✓ Solution by Mathematica
Time used: 0.255 (sec). Leaf size: 23

```
DSolve[2*x/y[x]-y[x]/(x^2+y[x]^2)+(-x^2/y[x]^2+x/(x^2+y[x]^2))*y'[x] == 0,y[x],x,IncludeSing
```

$$\mathrm{Solve}\left[\arctan\left(\frac{x}{y(x)}\right) - \frac{x^2}{y(x)} = c_1, y(x)\right]$$

6.26 problem 26

Internal problem ID [593]

Book: Elementary differential equations and boundary value problems, 10th ed., Boyce and DiPrima
Section: Miscellaneous problems, end of chapter 2. Page 133
Problem number: 26.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[[_homogeneous, 'class A'], _dAlembert]`

$$\boxed{y'x - e^{\frac{y}{x}}x - y = 0}$$

✓ Solution by Maple
Time used: 0.0 (sec). Leaf size: 15

```
dsolve(x*diff(y(x),x) = exp(y(x)/x)*x+y(x),y(x), singsol=all)
```

$$y(x) = \ln\left(-\frac{1}{\ln(x) + c_1}\right)x$$

✓ Solution by Mathematica
Time used: 0.316 (sec). Leaf size: 18

```
DSolve[x*y'[x] == Exp[y[x]/x]*x+y[x],y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to -x\log(-\log(x) - c_1)$$

6.27 problem 27

Internal problem ID [594]

Book: Elementary differential equations and boundary value problems, 10th ed., Boyce and DiPrima
Section: Miscellaneous problems, end of chapter 2. Page 133
Problem number: 27.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_rational, [_1st_order, '_with_symmetry_[F(x)*G(y),0]']]`

$$\boxed{y' - \frac{x}{x^2 + y + y^3} = 0}$$

✓ Solution by Maple

Time used: 0.016 (sec). Leaf size: 35

```
dsolve(diff(y(x),x) = x/(x^2+y(x)+y(x)^3),y(x), singsol=all)
```

$$\frac{\left(-4y(x)^3 - 4x^2 - 6y(x)^2 - 10y(x) - 5\right) \mathrm{e}^{-2y(x)}}{4} + c_1 = 0$$

✓ Solution by Mathematica

Time used: 0.173 (sec). Leaf size: 48

```
DSolve[y'[x] == x/(x^2+y[x]+y[x]^3),y[x],x,IncludeSingularSolutions -> True]
```

$$\text{Solve}\left[-\frac{1}{2}x^2 e^{-2y(x)} - \frac{1}{8}e^{-2y(x)}\left(4y(x)^3 + 6y(x)^2 + 10y(x) + 5\right) = c_1, y(x)\right]$$

6.28 problem 28

Internal problem ID [595]

Book: Elementary differential equations and boundary value problems, 10th ed., Boyce and DiPrima
Section: Miscellaneous problems, end of chapter 2. Page 133
Problem number: 28.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_linear]`

$$\boxed{2y + ty' = -3t}$$

✓ Solution by Maple
Time used: 0.0 (sec). Leaf size: 13

```
dsolve(3*t+2*y(t) = -t*diff(y(t),t),y(t), singsol=all)
```

$$y(t) = -t + \frac{c_1}{t^2}$$

✓ Solution by Mathematica
Time used: 0.023 (sec). Leaf size: 15

```
DSolve[3*t+2*y[t] == -t*y'[t],y[t],t,IncludeSingularSolutions -> True]
```

$$y(t) \to -t + \frac{c_1}{t^2}$$

6.29 problem 29

Internal problem ID [596]

Book: Elementary differential equations and boundary value problems, 10th ed., Boyce and DiPrima
Section: Miscellaneous problems, end of chapter 2. Page 133
Problem number: 29.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[[_homogeneous, 'class A'], _rational, [_Abel, '2nd type', 'cl`

$$\boxed{y' - \frac{x+y}{x-y} = 0}$$

✓ Solution by Maple
Time used: 0.016 (sec). Leaf size: 24

```
dsolve(diff(y(x),x) = (x+y(x))/(x-y(x)),y(x), singsol=all)
```

$$y(x) = \tan\left(\mathrm{RootOf}\left(-2\_Z + \ln\left(\sec\left(\_Z\right)^2\right) + 2\ln\left(x\right) + 2c_1\right)\right) x$$

✓ Solution by Mathematica
Time used: 0.033 (sec). Leaf size: 36

```
DSolve[y'[x] == (x+y[x])/(x-y[x]),y[x],x,IncludeSingularSolutions -> True]
```

$$\mathrm{Solve}\left[\frac{1}{2}\log\left(\frac{y(x)^2}{x^2}+1\right) - \arctan\left(\frac{y(x)}{x}\right) = -\log(x) + c_1, y(x)\right]$$

6.30 problem 30

Internal problem ID [597]

Book: Elementary differential equations and boundary value problems, 10th ed., Boyce and DiPrima
Section: Miscellaneous problems, end of chapter 2. Page 133
Problem number: 30.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[[_homogeneous, ‘class A‘], _rational, [_Abel, ‘2nd type‘, ‘cl`

$$2yx + 3y^2 - (x^2 + 2yx)\, y' = 0$$

✓ Solution by Maple

Time used: 0.015 (sec). Leaf size: 33

```
dsolve(2*x*y(x)+3*y(x)^2-(x^2+2*x*y(x))*diff(y(x),x) = 0,y(x), singsol=all)
```

$$y(x) = -\frac{\left(1 + \sqrt{4c_1x + 1}\right)x}{2}$$
$$y(x) = \frac{\left(-1 + \sqrt{4c_1x + 1}\right)x}{2}$$

✓ Solution by Mathematica

Time used: 0.408 (sec). Leaf size: 61

```
DSolve[2*x*y[x]+3*y[x]^2-(x^2+2*x*y[x])*y'[x] == 0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to -\frac{1}{2}x\left(1 + \sqrt{1 + 4e^{c_1}x}\right)$$
$$y(x) \to \frac{1}{2}x\left(-1 + \sqrt{1 + 4e^{c_1}x}\right)$$
$$y(x) \to 0$$
$$y(x) \to -x$$

6.31 problem 31

Internal problem ID [598]

Book: Elementary differential equations and boundary value problems, 10th ed., Boyce and DiPrima
Section: Miscellaneous problems, end of chapter 2. Page 133
Problem number: 31.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[[_homogeneous, ‘class G‘], _rational, [_Abel, ‘2nd type‘, ‘cl`

$$\boxed{y' - \frac{-3x^2y - y^2}{2x^3 + 3yx} = 0}$$

With initial conditions

$$[y(1) = -2]$$

✓ Solution by Maple
Time used: 1.266 (sec). Leaf size: 111

```
dsolve([diff(y(x),x) = (-3*x^2*y(x)-y(x)^2)/(2*x^3+3*x*y(x)),y(1) = -2],y(x), singsol=all)
```

$$y(x) = \frac{\left(i\sqrt{3}-1\right)\left(-\left(x^7-6\sqrt{3}\,\sqrt{x^7+27}+54\right)x^2\right)^{\frac{2}{3}} - x^3\left(i\sqrt{3}\,x^3+x^3+2\left(-\left(x^7-6\sqrt{3}\,\sqrt{x^7+27}+54\right)x^2\right)^{\frac{1}{3}}\right.}{6\left(-\left(x^7-6\sqrt{3}\,\sqrt{x^7+27}+54\right)x^2\right)^{\frac{1}{3}}x}$$

✓ Solution by Mathematica
Time used: 40.923 (sec). Leaf size: 136

```
DSolve[{y'[x]== (-3*x^2*y[x]-y[x]^2)/(2*x^3+3*x*y[x]),y[1]==-2},y[x],x,IncludeSingularSoluti
```

$$y(x) \to \frac{i\left(\left(\sqrt{3}+i\right)x^3 - \left(\sqrt{3}-i\right)x^3 + \left(\sqrt{3}+i\right)\sqrt[3]{-x^9-54x^2+6\sqrt{3}\sqrt{x^4\left(x^7+27\right)}} - \frac{\left(\sqrt{3}-i\right)x^6}{\sqrt[3]{-x^9-54x^2+6\sqrt{3}}}\right.}{6x}$$

7 Chapter 3, Second order linear equations, 3.1 Homogeneous Equations with Constant Coefficients, page 144

7.1 problem 1

Internal problem ID [599]

Book: Elementary differential equations and boundary value problems, 10th ed., Boyce and DiPrima
Section: Chapter 3, Second order linear equations, 3.1 Homogeneous Equations with Constant Coefficients, page 144
Problem number: 1.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _missing_x]]`

$$\boxed{y'' + 2y' - 3y = 0}$$

✓ Solution by Maple
Time used: 0.016 (sec). Leaf size: 17

```
dsolve(diff(y(x),x$2) +2*diff(y(x),x)-3*y(x) = 0,y(x), singsol=all)
```

$$y(x) = \left(c_1 e^{4x} + c_2\right) e^{-3x}$$

✓ Solution by Mathematica
Time used: 0.011 (sec). Leaf size: 20

```
DSolve[y''[x]+2*y'[x]-3*y[x]==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to c_1 e^{-3x} + c_2 e^x$$

7.2 problem 2

Internal problem ID [600]

Book: Elementary differential equations and boundary value problems, 10th ed., Boyce and DiPrima
Section: Chapter 3, Second order linear equations, 3.1 Homogeneous Equations with Constant Coefficients, page 144
Problem number: 2.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _missing_x]]`

$$\boxed{y'' + 3y' + 2y = 0}$$

✓ Solution by Maple
Time used: 0.0 (sec). Leaf size: 17

```
dsolve(diff(y(x),x$2) +3*diff(y(x),x)+2*y(x) = 0,y(x), singsol=all)
```

$$y(x) = \mathrm{e}^{-2x}c_1 + c_2\mathrm{e}^{-x}$$

✓ Solution by Mathematica
Time used: 0.013 (sec). Leaf size: 20

```
DSolve[y''[x]+3*y'[x]+2*y[x]==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to e^{-2x}(c_2e^x + c_1)$$

7.3 problem 3

Internal problem ID [601]

Book: Elementary differential equations and boundary value problems, 10th ed., Boyce and DiPrima
Section: Chapter 3, Second order linear equations, 3.1 Homogeneous Equations with Constant Coefficients, page 144
Problem number: 3.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _missing_x]]`

$$\boxed{6y'' - y' - y = 0}$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 17

```
dsolve(6*diff(y(x),x$2) -diff(y(x),x)-y(x) = 0,y(x), singsol=all)
```

$$y(x) = \left(c_1 e^{\frac{5x}{6}} + c_2\right) e^{-\frac{x}{3}}$$

✓ Solution by Mathematica

Time used: 0.014 (sec). Leaf size: 26

```
DSolve[6*y''[x]-y'[x]-y[x]==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to e^{-x/3}\left(c_2 e^{5x/6} + c_1\right)$$

7.4 problem 4

Internal problem ID [602]

Book: Elementary differential equations and boundary value problems, 10th ed., Boyce and DiPrima
Section: Chapter 3, Second order linear equations, 3.1 Homogeneous Equations with Constant Coefficients, page 144
Problem number: 4.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _missing_x]]`

$$\boxed{2y'' - 3y' + y = 0}$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 15

```
dsolve(2*diff(y(x),x$2) -3*diff(y(x),x)+y(x) = 0,y(x), singsol=all)
```

$$y(x) = e^{x} c_1 + c_2 e^{\frac{x}{2}}$$

✓ Solution by Mathematica

Time used: 0.021 (sec). Leaf size: 35

```
DSolve[y''[x]-3*y'[x]+y[x]==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to e^{-\frac{1}{2}\left(\sqrt{5}-3\right)x}\left(c_2 e^{\sqrt{5}x} + c_1\right)$$

7.5 problem 5

Internal problem ID [603]

Book: Elementary differential equations and boundary value problems, 10th ed., Boyce and DiPrima
Section: Chapter 3, Second order linear equations, 3.1 Homogeneous Equations with Constant Coefficients, page 144
Problem number: 5.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _missing_x]]`

$$\boxed{y'' + 5y' = 0}$$

✓ Solution by Maple
Time used: 0.0 (sec). Leaf size: 12

```
dsolve(diff(y(x),x$2) +5*diff(y(x),x) = 0,y(x), singsol=all)
```

$$y(x) = c_1 + c_2 e^{-5x}$$

✓ Solution by Mathematica
Time used: 0.01 (sec). Leaf size: 19

```
DSolve[y''[x]+5*y'[x]==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to c_2 - \frac{1}{5} c_1 e^{-5x}$$

7.6 problem 6

Internal problem ID [604]

Book: Elementary differential equations and boundary value problems, 10th ed., Boyce and DiPrima
Section: Chapter 3, Second order linear equations, 3.1 Homogeneous Equations with Constant Coefficients, page 144
Problem number: 6.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _missing_x]]`

$$\boxed{4y'' - 9y = 0}$$

✓ Solution by Maple
Time used: 0.0 (sec). Leaf size: 17

```
dsolve(4*diff(y(x),x$2) -9*y(x) = 0,y(x), singsol=all)
```

$$y(x) = c_1 e^{-\frac{3x}{2}} + c_2 e^{\frac{3x}{2}}$$

✓ Solution by Mathematica
Time used: 0.013 (sec). Leaf size: 24

```
DSolve[4*y''[x]-9*y[x]==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to e^{-3x/2}(c_1 e^{3x} + c_2)$$

7.7 problem 7

Internal problem ID [605]

Book: Elementary differential equations and boundary value problems, 10th ed., Boyce and DiPrima
Section: Chapter 3, Second order linear equations, 3.1 Homogeneous Equations with Constant Coefficients, page 144
Problem number: 7.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _missing_x]]`

$$\boxed{y'' - 9y' + 9y = 0}$$

✓ Solution by Maple
Time used: 0.016 (sec). Leaf size: 27

```
dsolve(diff(y(x),x$2) -9*diff(y(x),x)+9*y(x) = 0,y(x), singsol=all)
```

$$y(x) = c_1 \mathrm{e}^{\frac{3(3+\sqrt{5})x}{2}} + c_2 \mathrm{e}^{-\frac{3(\sqrt{5}-3)x}{2}}$$

✓ Solution by Mathematica
Time used: 0.022 (sec). Leaf size: 36

```
DSolve[y''[x]-9*y'[x]+9*y[x]==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to e^{-\frac{3}{2}(\sqrt{5}-3)x}\left(c_2 e^{3\sqrt{5}x} + c_1\right)$$

7.8 problem 8

Internal problem ID [606]

Book: Elementary differential equations and boundary value problems, 10th ed., Boyce and DiPrima
Section: Chapter 3, Second order linear equations, 3.1 Homogeneous Equations with Constant Coefficients, page 144
Problem number: 8.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _missing_x]]`

$$\boxed{y'' - 2y' - 2y = 0}$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 26

```
dsolve(diff(y(x),x$2) -2*diff(y(x),x)-2*y(x) = 0,y(x), singsol=all)
```

$$y(x) = c_1 \mathrm{e}^{\left(1+\sqrt{3}\right)x} + c_2 \mathrm{e}^{-\left(\sqrt{3}-1\right)x}$$

✓ Solution by Mathematica

Time used: 0.019 (sec). Leaf size: 34

```
DSolve[y''[x]-2*y'[x]-2*y[x]==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to e^{x-\sqrt{3}x}\left(c_2 e^{2\sqrt{3}x} + c_1\right)$$

7.9 problem 9

Internal problem ID [607]

Book: Elementary differential equations and boundary value problems, 10th ed., Boyce and DiPrima
Section: Chapter 3, Second order linear equations, 3.1 Homogeneous Equations with Constant Coefficients, page 144
Problem number: 9.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _missing_x]]`

$$\boxed{y'' + y' - 2y = 0}$$

With initial conditions

$$[y(0) = 0, y'(0) = 1]$$

✓ Solution by Maple
Time used: 0.016 (sec). Leaf size: 16

```
dsolve([diff(y(x),x$2) +diff(y(x),x)-2*y(x) = 0,y(0) = 0, D(y)(0) = 1],y(x), singsol=all)
```

$$y(x) = \frac{(e^{3x} - 1)\, e^{-2x}}{3}$$

✓ Solution by Mathematica
Time used: 0.013 (sec). Leaf size: 21

```
DSolve[{y''[x]+y'[x]-2*y[x]==0,{y[0]==0,y'[0]==1}},y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to \frac{1}{3} e^{-2x} \left(e^{3x} - 1\right)$$

7.10 problem 10

Internal problem ID [608]

Book: Elementary differential equations and boundary value problems, 10th ed., Boyce and DiPrima
Section: Chapter 3, Second order linear equations, 3.1 Homogeneous Equations with Constant Coefficients, page 144
Problem number: 10.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _missing_x]]`

$$\boxed{y'' + 4y' + 3y = 0}$$

With initial conditions

$$[y(0) = 2, y'(0) = -1]$$

✓ Solution by Maple
Time used: 0.016 (sec). Leaf size: 17

```
dsolve([diff(y(x),x$2) +4*diff(y(x),x)+3*y(x) = 0,y(0) = 2, D(y)(0) = -1],y(x), singsol=all)
```

$$y(x) = -\frac{e^{-3x}}{2} + \frac{5\,e^{-x}}{2}$$

✓ Solution by Mathematica
Time used: 0.013 (sec). Leaf size: 23

```
DSolve[{y''[x]+4*y'[x]+3*y[x]==0,{y[0]==2,y'[0]==-1}},y[x],x,IncludeSingularSolutions -> Tru
```

$$y(x) \to \frac{1}{2}e^{-3x}\left(5e^{2x} - 1\right)$$

7.11 problem 11

Internal problem ID [609]

Book: Elementary differential equations and boundary value problems, 10th ed., Boyce and DiPrima
Section: Chapter 3, Second order linear equations, 3.1 Homogeneous Equations with Constant Coefficients, page 144
Problem number: 11.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _missing_x]]`

$$\boxed{6y'' - 5y' + y = 0}$$

With initial conditions

$$[y(0) = 4, y'(0) = 0]$$

✓ Solution by Maple
Time used: 0.016 (sec). Leaf size: 17

```
dsolve([6*diff(y(x),x$2) -5*diff(y(x),x)+y(x) = 0,y(0) = 4, D(y)(0) = 0],y(x), singsol=all)
```

$$y(x) = -8\,\mathrm{e}^{\frac{x}{2}} + 12\,\mathrm{e}^{\frac{x}{3}}$$

✓ Solution by Mathematica
Time used: 0.025 (sec). Leaf size: 48

```
DSolve[{6*y''[x]-5*y'[x]+2*y[x]==0,{y[0]==4,y'[0]==0}},y[x],x,IncludeSingularSolutions -> Tr
```

$$y(x) \to \frac{4}{23}e^{5x/12}\left(23\cos\left(\frac{\sqrt{23}x}{12}\right) - 5\sqrt{23}\sin\left(\frac{\sqrt{23}x}{12}\right)\right)$$

7.12 problem 12

Internal problem ID [610]

Book: Elementary differential equations and boundary value problems, 10th ed., Boyce and DiPrima
Section: Chapter 3, Second order linear equations, 3.1 Homogeneous Equations with Constant Coefficients, page 144
Problem number: 12.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _missing_x]]`

$$\boxed{y'' + 3y' = 0}$$

With initial conditions

$$[y(0) = -2, y'(0) = 3]$$

✓ Solution by Maple
Time used: 0.016 (sec). Leaf size: 12

```
dsolve([diff(y(x),x$2) +3*diff(y(x),x) = 0,y(0) = -2, D(y)(0) = 3],y(x), singsol=all)
```

$$y(x) = -1 - e^{-3x}$$

✓ Solution by Mathematica
Time used: 0.014 (sec). Leaf size: 14

```
DSolve[{y''[x]+3*y'[x]==0,{y[0]==-2,y'[0]==3}},y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to -e^{-3x} - 1$$

7.13 problem 13

Internal problem ID [611]

Book: Elementary differential equations and boundary value problems, 10th ed., Boyce and DiPrima
Section: Chapter 3, Second order linear equations, 3.1 Homogeneous Equations with Constant Coefficients, page 144
Problem number: 13.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _missing_x]]`

$$\boxed{y'' + 5y' + 3y = 0}$$

With initial conditions

$$[y(0) = 1, y'(0) = 0]$$

✓ Solution by Maple
Time used: 0.047 (sec). Leaf size: 39

```
dsolve([diff(y(x),x$2) +5*diff(y(x),x)+3*y(x) = 0,y(0) = 1, D(y)(0) = 0],y(x), singsol=all)
```

$$y(x) = \frac{(13 + 5\sqrt{13})\,\mathrm{e}^{\frac{(-5+\sqrt{13})x}{2}}}{26} + \frac{(13 - 5\sqrt{13})\,\mathrm{e}^{-\frac{(5+\sqrt{13})x}{2}}}{26}$$

✓ Solution by Mathematica
Time used: 0.023 (sec). Leaf size: 51

```
DSolve[{y''[x]+5*y'[x]+3*y[x]==0,{y[0]==1,y'[0]==0}},y[x],x,IncludeSingularSolutions -> True
```

$$y(x) \to \frac{1}{26} e^{-\frac{1}{2}\left(5+\sqrt{13}\right)x}\left(\left(13 + 5\sqrt{13}\right) e^{\sqrt{13}x} + 13 - 5\sqrt{13}\right)$$

7.14 problem 14

Internal problem ID [612]

Book: Elementary differential equations and boundary value problems, 10th ed., Boyce and DiPrima
Section: Chapter 3, Second order linear equations, 3.1 Homogeneous Equations with Constant Coefficients, page 144
Problem number: 14.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _missing_x]]`

$$\boxed{2y'' + y' - 4y = 0}$$

With initial conditions

$$[y(0) = 0, y'(0) = 1]$$

✓ Solution by Maple
Time used: 0.047 (sec). Leaf size: 30

```
dsolve([2*diff(y(x),x$2) +diff(y(x),x)-4*y(x) = 0,y(0) = 0, D(y)(0) = 1],y(x), singsol=all)
```

$$y(x) = \frac{2\left(\mathrm{e}^{\frac{\left(-1+\sqrt{33}\right)x}{4}} - \mathrm{e}^{-\frac{\left(1+\sqrt{33}\right)x}{4}}\right)\sqrt{33}}{33}$$

✓ Solution by Mathematica
Time used: 0.027 (sec). Leaf size: 40

```
DSolve[{2*y''[x]+y'[x]-4*y[x]==0,{y[0]==0,y'[0]==1}},y[x],x,IncludeSingularSolutions -> True
```

$$y(x) \to \frac{2e^{-\frac{1}{4}\left(1+\sqrt{33}\right)x}\left(e^{\frac{\sqrt{33}x}{2}} - 1\right)}{\sqrt{33}}$$

7.15 problem 15

Internal problem ID [613]

Book: Elementary differential equations and boundary value problems, 10th ed., Boyce and DiPrima
Section: Chapter 3, Second order linear equations, 3.1 Homogeneous Equations with Constant Coefficients, page 144
Problem number: 15.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _missing_x]]`

$$\boxed{y'' + 8y' - 9y = 0}$$

With initial conditions

$$[y(1) = 1, y'(1) = 0]$$

✓ Solution by Maple
Time used: 0.031 (sec). Leaf size: 19

```
dsolve([diff(y(x),x$2) +8*diff(y(x),x)-9*y(x) = 0,y(1) = 1, D(y)(1) = 0],y(x), singsol=all)
```

$$y(x) = \frac{e^{9-9x}}{10} + \frac{9\,e^{x-1}}{10}$$

✓ Solution by Mathematica
Time used: 0.013 (sec). Leaf size: 26

```
DSolve[{y''[x]+8*y'[x]-9*y[x]==0,{y[1]==1,y'[1]==0}},y[x],x,IncludeSingularSolutions -> True
```

$$y(x) \to \frac{1}{10}e^{9-9x} + \frac{9e^{x-1}}{10}$$

7.16 problem 16

Internal problem ID [614]

Book: Elementary differential equations and boundary value problems, 10th ed., Boyce and DiPrima
Section: Chapter 3, Second order linear equations, 3.1 Homogeneous Equations with Constant Coefficients, page 144
Problem number: 16.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _missing_x]]`

$$\boxed{4y'' - y = 0}$$

With initial conditions

$$[y(-2) = 1, y'(-2) = -1]$$

✓ Solution by Maple
Time used: 0.031 (sec). Leaf size: 21

```
dsolve([4*diff(y(x),x$2) -y(x) = 0,y(-2) = 1, D(y)(-2) = -1],y(x), singsol=all)
```

$$y(x) = -\frac{e^{1+\frac{x}{2}}}{2} + \frac{3\,e^{-1-\frac{x}{2}}}{2}$$

✓ Solution by Mathematica
Time used: 0.012 (sec). Leaf size: 25

```
DSolve[{4*y''[x]-y[x]==0,{y[-2]==1,y'[-2]==-1}},y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to -\frac{1}{2}e^{-\frac{x}{2}-1}\left(e^{x+2} - 3\right)$$

7.17 problem 19

Internal problem ID [615]

Book: Elementary differential equations and boundary value problems, 10th ed., Boyce and DiPrima
Section: Chapter 3, Second order linear equations, 3.1 Homogeneous Equations with Constant Coefficients, page 144
Problem number: 19.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _missing_x]]`

$$\boxed{y'' - y = 0}$$

With initial conditions

$$\left[y(0) = \frac{5}{4}, y'(0) = -\frac{3}{4}\right]$$

✓ Solution by Maple
Time used: 0.015 (sec). Leaf size: 13

```
dsolve([diff(y(x),x$2) -y(x) = 0,y(0) = 5/4, D(y)(0) = -3/4],y(x), singsol=all)
```

$$y(x) = \frac{e^{x}}{4} + e^{-x}$$

✓ Solution by Mathematica
Time used: 0.013 (sec). Leaf size: 18

```
DSolve[{y''[x]-y[x]==0,{y[0]==5/4,y'[0]==-3/4}},y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to e^{-x} + \frac{e^{x}}{4}$$

7.18 problem 20

Internal problem ID [616]

Book: Elementary differential equations and boundary value problems, 10th ed., Boyce and DiPrima
Section: Chapter 3, Second order linear equations, 3.1 Homogeneous Equations with Constant Coefficients, page 144
Problem number: 20.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _missing_x]]`

$$\boxed{2y'' - 3y' + y = 0}$$

With initial conditions

$$\left[y(0) = 2, y'(0) = \frac{1}{2}\right]$$

✓ Solution by Maple
Time used: 0.016 (sec). Leaf size: 15

```
dsolve([2*diff(y(x),x$2) -3*diff(y(x),x)+y(x) = 0,y(0) = 2, D(y)(0) = 1/2],y(x), singsol=all
```

$$y(x) = -\mathrm{e}^{x} + 3\,\mathrm{e}^{\frac{x}{2}}$$

✓ Solution by Mathematica
Time used: 0.012 (sec). Leaf size: 20

```
DSolve[{2*y''[x]-3*y'[x]+y[x]==0,{y[0]==2,y'[0]==1/2}},y[x],x,IncludeSingularSolutions -> Tr
```

$$y(x) \rightarrow 3e^{x/2} - e^{x}$$

7.19 problem 21

Internal problem ID [617]

Book: Elementary differential equations and boundary value problems, 10th ed., Boyce and DiPrima
Section: Chapter 3, Second order linear equations, 3.1 Homogeneous Equations with Constant Coefficients, page 144
Problem number: 21.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _missing_x]]`

$$\boxed{y'' - y' - 2y = 0}$$

With initial conditions

$$[y(0) = \alpha, y'(0) = 2]$$

✓ Solution by Maple

Time used: 0.015 (sec). Leaf size: 25

```
dsolve([diff(y(x),x$2) -diff(y(x),x)-2*y(x) = 0,y(0) = alpha, D(y)(0) = 2],y(x), singsol=all
```

$$y(x) = \frac{(2\alpha - 2)\,\mathrm{e}^{-x}}{3} + \frac{\mathrm{e}^{2x}(\alpha + 2)}{3}$$

✓ Solution by Mathematica

Time used: 0.013 (sec). Leaf size: 29

```
DSolve[{y''[x]-y'[x]-2*y[x]==0,{y[0]==\[Alpha],y'[0]==2}},y[x],x,IncludeSingularSolutions ->
```

$$y(x) \to \frac{1}{3}e^{-x}\left(2(\alpha - 1) + (\alpha + 2)e^{3x}\right)$$

7.20 problem 22

Internal problem ID [618]

Book: Elementary differential equations and boundary value problems, 10th ed., Boyce and DiPrima
Section: Chapter 3, Second order linear equations, 3.1 Homogeneous Equations with Constant Coefficients, page 144
Problem number: 22.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _missing_x]]`

$$\boxed{4y'' - y = 0}$$

With initial conditions

$$[y(0) = 2, y'(0) = \beta]$$

✓ Solution by Maple
Time used: 0.016 (sec). Leaf size: 22

```
dsolve([4*diff(y(x),x$2) -y(x) = 0,y(0) = 2, D(y)(0) = beta],y(x), singsol=all)
```

$$y(x) = (1 + \beta)\,\mathrm{e}^{\frac{x}{2}} - (\beta - 1)\,\mathrm{e}^{-\frac{x}{2}}$$

✓ Solution by Mathematica
Time used: 0.012 (sec). Leaf size: 25

```
DSolve[{4*y''[x]-y[x]==0,{y[0]==2,y'[0]==\[Beta]}},y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to e^{-x/2}(-\beta + (\beta + 1)e^{x} + 1)$$

7.21 problem 23

Internal problem ID [619]

Book: Elementary differential equations and boundary value problems, 10th ed., Boyce and DiPrima
Section: Chapter 3, Second order linear equations, 3.1 Homogeneous Equations with Constant Coefficients, page 144
Problem number: 23.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _missing_x]]`

$$\boxed{y'' - (2\alpha - 1)\, y' + \alpha(\alpha - 1)\, y = 0}$$

✓ Solution by Maple
Time used: 0.0 (sec). Leaf size: 19

```
dsolve(diff(y(x),x$2) -(2*alpha-1)*diff(y(x),x)+alpha*(alpha-1)*y(x) = 0,y(x), singsol=all)
```

$$y(x) = c_1 e^{\alpha x} + c_2 e^{(\alpha-1)x}$$

✓ Solution by Mathematica
Time used: 0.015 (sec). Leaf size: 24

```
DSolve[y''[x]-(2*\[Alpha]-1)*y'[x]+\[Alpha]*(\[Alpha]-1)*y[x]==0,y[x],x,IncludeSingularSolut
```

$$y(x) \to c_1 e^{(\alpha-1)x} + c_2 e^{\alpha x}$$

7.22 problem 24

Internal problem ID [620]

Book: Elementary differential equations and boundary value problems, 10th ed., Boyce and DiPrima
Section: Chapter 3, Second order linear equations, 3.1 Homogeneous Equations with Constant Coefficients, page 144
Problem number: 24.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _missing_x]]`

$$\boxed{y'' + (3-\alpha)\, y' - 2(\alpha-1)\, y = 0}$$

✓ Solution by Maple
Time used: 0.016 (sec). Leaf size: 19

```
dsolve(diff(y(x),x$2) +(3-alpha)*diff(y(x),x)-2*(alpha-1)*y(x) = 0,y(x), singsol=all)
```

$$y(x) = \mathrm{e}^{-2x} c_1 + c_2 \mathrm{e}^{(\alpha-1)x}$$

✓ Solution by Mathematica
Time used: 0.014 (sec). Leaf size: 24

```
DSolve[y''[x]+(3-\[Alpha])*y'[x]-2*(\[Alpha]-1)*y[x]==0,y[x],x,IncludeSingularSolutions -> T
```

$$y(x) \to e^{-2x}\left(c_1 e^{\alpha x + x} + c_2\right)$$

7.23 problem 25

Internal problem ID [621]

Book: Elementary differential equations and boundary value problems, 10th ed., Boyce and DiPrima
Section: Chapter 3, Second order linear equations, 3.1 Homogeneous Equations with Constant Coefficients, page 144
Problem number: 25.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _missing_x]]`

$$\boxed{2y'' + 3y' - 2y = 0}$$

With initial conditions

$$[y(0) = 1, y'(0) = -\beta]$$

✓ Solution by Maple
Time used: 0.016 (sec). Leaf size: 28

```
dsolve([2*diff(y(x),x$2) +3*diff(y(x),x)-2*y(x) = 0,y(0) = 1, D(y)(0) = -beta],y(x), singsol
```

$$y(x) = -\frac{\left(2\,\mathrm{e}^{\frac{5x}{2}}\beta - 4\,\mathrm{e}^{\frac{5x}{2}} - 2\beta - 1\right)\mathrm{e}^{-2x}}{5}$$

✓ Solution by Mathematica
Time used: 0.024 (sec). Leaf size: 67

```
DSolve[{y''[x]+3*y'[x]-2*y[x]==0,{y[0]==1,y'[0]==-\[Beta]}},y[x],x,IncludeSingularSolutions
```

$$y(x) \to \frac{1}{34}e^{-\frac{1}{2}\left(3+\sqrt{17}\right)x}\left(2\sqrt{17}\beta + \left(-2\sqrt{17}\beta + 3\sqrt{17} + 17\right)e^{\sqrt{17}x} - 3\sqrt{17} + 17\right)$$

7.24 problem 26

Internal problem ID [622]

Book: Elementary differential equations and boundary value problems, 10th ed., Boyce and DiPrima
Section: Chapter 3, Second order linear equations, 3.1 Homogeneous Equations with Constant Coefficients, page 144
Problem number: 26.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _missing_x]]`

$$\boxed{y'' + 5y' + 6y = 0}$$

With initial conditions

$$[y(0) = 2, y'(0) = \beta]$$

✓ Solution by Maple
Time used: 0.016 (sec). Leaf size: 23

```
dsolve([diff(y(x),x$2) +5*diff(y(x),x)+6*y(x) = 0,y(0) = 2, D(y)(0) = beta],y(x), singsol=al
```

$$y(x) = \mathrm{e}^{-2x}(6 + \beta) + (-\beta - 4)\,\mathrm{e}^{-3x}$$

✓ Solution by Mathematica
Time used: 0.013 (sec). Leaf size: 23

```
DSolve[{y''[x]+5*y'[x]+6*y[x]==0,{y[0]==2,y'[0]==\[Beta]}},y[x],x,IncludeSingularSolutions -
```

$$y(x) \to e^{-3x}(-\beta + (\beta + 6)e^{x} - 4)$$

8 Chapter 3, Second order linear equations, 3.3 Complex Roots of the Characteristic Equation , page 164

8.1 problem 7

Internal problem ID [623]

Book: Elementary differential equations and boundary value problems, 10th ed., Boyce and DiPrima
Section: Chapter 3, Second order linear equations, 3.3 Complex Roots of the Characteristic Equation , page 164
Problem number: 7.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _missing_x]]`

$$\boxed{y'' - 2y' + 2y = 0}$$

✓ Solution by Maple
Time used: 0.0 (sec). Leaf size: 16

```
dsolve(diff(y(x),x$2) -2*diff(y(x),x)+2*y(x) = 0,y(x), singsol=all)
```

$$y(x) = e^{x}(c_1 \sin (x) + c_2 \cos (x))$$

✓ Solution by Mathematica
Time used: 0.013 (sec). Leaf size: 20

```
DSolve[y''[x]-2*y'[x]+2*y[x]==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to e^{x}(c_2 \cos(x) + c_1 \sin(x))$$

8.2 problem 8

Internal problem ID [624]

Book: Elementary differential equations and boundary value problems, 10th ed., Boyce and DiPrima
Section: Chapter 3, Second order linear equations, 3.3 Complex Roots of the Characteristic Equation , page 164
Problem number: 8.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _missing_x]]`

$$\boxed{y'' - 2y' + 6y = 0}$$

✓ Solution by Maple
Time used: 0.0 (sec). Leaf size: 24

```
dsolve(diff(y(x),x$2) -2*diff(y(x),x)+6*y(x) = 0,y(x), singsol=all)
```

$$y(x) = e^x \left(c_1 \sin\left(\sqrt{5}\, x\right) + c_2 \cos\left(\sqrt{5}\, x\right)\right)$$

✓ Solution by Mathematica
Time used: 0.02 (sec). Leaf size: 32

```
DSolve[y''[x]-2*y'[x]+6*y[x]==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to e^x \left(c_2 \cos\left(\sqrt{5}x\right) + c_1 \sin\left(\sqrt{5}x\right)\right)$$

8.3 problem 9

Internal problem ID [625]

Book: Elementary differential equations and boundary value problems, 10th ed., Boyce and DiPrima
Section: Chapter 3, Second order linear equations, 3.3 Complex Roots of the Characteristic Equation , page 164
Problem number: 9.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _missing_x]]`

$$\boxed{y'' + 2y' - 8y = 0}$$

✓ Solution by Maple
Time used: 0.016 (sec). Leaf size: 17

```
dsolve(diff(y(x),x$2) +2*diff(y(x),x)-8*y(x) = 0,y(x), singsol=all)
```

$$y(x) = \left(\mathrm{e}^{6x}c_1 + c_2\right)\mathrm{e}^{-4x}$$

✓ Solution by Mathematica
Time used: 0.012 (sec). Leaf size: 22

```
DSolve[y''[x]+2*y'[x]-8*y[x]==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to e^{-4x}\left(c_2e^{6x} + c_1\right)$$

8.4 problem 10

Internal problem ID [626]

Book: Elementary differential equations and boundary value problems, 10th ed., Boyce and DiPrima
Section: Chapter 3, Second order linear equations, 3.3 Complex Roots of the Characteristic Equation , page 164
Problem number: 10.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _missing_x]]`

$$\boxed{y'' + 2y' + 2y = 0}$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 18

```
dsolve(diff(y(x),x$2) +2*diff(y(x),x)+2*y(x) = 0,y(x), singsol=all)
```

$$y(x) = \mathrm{e}^{-x}(c_1 \sin(x) + c_2 \cos(x))$$

✓ Solution by Mathematica

Time used: 0.015 (sec). Leaf size: 22

```
DSolve[y''[x]+2*y'[x]+2*y[x]==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \rightarrow e^{-x}(c_2 \cos(x) + c_1 \sin(x))$$

8.5 problem 11

Internal problem ID [627]

Book: Elementary differential equations and boundary value problems, 10th ed., Boyce and DiPrima
Section: Chapter 3, Second order linear equations, 3.3 Complex Roots of the Characteristic Equation , page 164
Problem number: 11.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _missing_x]]`

$$\boxed{y'' + 6y' + 13y = 0}$$

✓ Solution by Maple
Time used: 0.0 (sec). Leaf size: 22

```
dsolve(diff(y(x),x$2) +6*diff(y(x),x)+13*y(x) = 0,y(x), singsol=all)
```

$$y(x) = e^{-3x}(\sin(2x) c_1 + c_2 \cos(2x))$$

✓ Solution by Mathematica
Time used: 0.018 (sec). Leaf size: 26

```
DSolve[y''[x]+6*y'[x]+13*y[x]==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \rightarrow e^{-3x}(c_2 \cos(2x) + c_1 \sin(2x))$$

8.6 problem 12

Internal problem ID [628]

Book: Elementary differential equations and boundary value problems, 10th ed., Boyce and DiPrima
Section: Chapter 3, Second order linear equations, 3.3 Complex Roots of the Characteristic Equation , page 164
Problem number: 12.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _missing_x]]`

$$\boxed{4y'' + 9y = 0}$$

✓ Solution by Maple
Time used: 0.0 (sec). Leaf size: 17

```
dsolve(4*diff(y(x),x$2) +9*y(x) = 0,y(x), singsol=all)
```

$$y(x) = c_1 \sin\left(\frac{3x}{2}\right) + c_2 \cos\left(\frac{3x}{2}\right)$$

✓ Solution by Mathematica
Time used: 0.013 (sec). Leaf size: 20

```
DSolve[y''[x]+9*y[x]==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to c_1 \cos(3x) + c_2 \sin(3x)$$

8.7 problem 13

Internal problem ID [629]

Book: Elementary differential equations and boundary value problems, 10th ed., Boyce and DiPrima
Section: Chapter 3, Second order linear equations, 3.3 Complex Roots of the Characteristic Equation , page 164
Problem number: 13.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _missing_x]]`

$$\boxed{y'' + 2y' + \frac{5y}{4} = 0}$$

✓ Solution by Maple
Time used: 0.016 (sec). Leaf size: 22

```
dsolve(diff(y(x),x$2) +2*diff(y(x),x)+125/100*y(x) = 0,y(x), singsol=all)
```

$$y(x) = \mathrm{e}^{-x}\left(c_1 \sin\left(\frac{x}{2}\right) + c_2 \cos\left(\frac{x}{2}\right)\right)$$

✓ Solution by Mathematica
Time used: 0.018 (sec). Leaf size: 30

```
DSolve[y''[x]+2*y'[x]+125/100*y[x]==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to e^{-x}\left(c_2 \cos\left(\frac{x}{2}\right) + c_1 \sin\left(\frac{x}{2}\right)\right)$$

8.8 problem 14

Internal problem ID [630]

Book: Elementary differential equations and boundary value problems, 10th ed., Boyce and DiPrima
Section: Chapter 3, Second order linear equations, 3.3 Complex Roots of the Characteristic Equation , page 164
Problem number: 14.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _missing_x]]`

$$\boxed{9y'' + 9y' - 4y = 0}$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 17

```
dsolve(9*diff(y(x),x$2) +9*diff(y(x),x)-4*y(x) = 0,y(x), singsol=all)
```

$$y(x) = \left(c_2 e^{\frac{5x}{3}} + c_1\right) e^{-\frac{4x}{3}}$$

✓ Solution by Mathematica

Time used: 0.013 (sec). Leaf size: 26

```
DSolve[9*y''[x]+9*y'[x]-4*y[x]==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to e^{-4x/3}\left(c_2 e^{5x/3} + c_1\right)$$

8.9 problem 15

Internal problem ID [631]

Book: Elementary differential equations and boundary value problems, 10th ed., Boyce and DiPrima
Section: Chapter 3, Second order linear equations, 3.3 Complex Roots of the Characteristic Equation , page 164
Problem number: 15.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _missing_x]]`

$$\boxed{y'' + y' + \frac{5y}{4} = 0}$$

✓ Solution by Maple
Time used: 0.0 (sec). Leaf size: 18

```
dsolve(diff(y(x),x$2) +diff(y(x),x)+125/100*y(x) = 0,y(x), singsol=all)
```

$$y(x) = e^{-\frac{x}{2}}(c_1 \sin(x) + c_2 \cos(x))$$

✓ Solution by Mathematica
Time used: 0.015 (sec). Leaf size: 24

```
DSolve[y''[x]+y'[x]+125/100*y[x]==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to e^{-x/2}(c_2 \cos(x) + c_1 \sin(x))$$

8.10 problem 16

Internal problem ID [632]

Book: Elementary differential equations and boundary value problems, 10th ed., Boyce and DiPrima
Section: Chapter 3, Second order linear equations, 3.3 Complex Roots of the Characteristic Equation , page 164
Problem number: 16.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _missing_x]]`

$$\boxed{y'' + 4y' + \frac{25y}{4} = 0}$$

✓ Solution by Maple
Time used: 0.016 (sec). Leaf size: 22

```
dsolve(diff(y(x),x$2)+ 4*diff(y(x),x)+625/100*y(x) = 0,y(x), singsol=all)
```

$$y(x) = \mathrm{e}^{-2x}\left(c_1 \sin\left(\frac{3x}{2}\right) + c_2 \cos\left(\frac{3x}{2}\right)\right)$$

✓ Solution by Mathematica
Time used: 0.018 (sec). Leaf size: 30

```
DSolve[y''[x]+4*y'[x]+625/100*y[x]==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \rightarrow e^{-2x}\left(c_2 \cos\left(\frac{3x}{2}\right) + c_1 \sin\left(\frac{3x}{2}\right)\right)$$

8.11 problem 17

Internal problem ID [633]

Book: Elementary differential equations and boundary value problems, 10th ed., Boyce and DiPrima
Section: Chapter 3, Second order linear equations, 3.3 Complex Roots of the Characteristic Equation , page 164
Problem number: 17.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _missing_x]]`

$$\boxed{y'' + 4y = 0}$$

With initial conditions

$$[y(0) = 0, y'(0) = 1]$$

✓ Solution by Maple
Time used: 0.0 (sec). Leaf size: 10

```
dsolve([diff(y(x),x$2)+ 4*y(x) = 0,y(0) = 0, D(y)(0) = 1],y(x), singsol=all)
```

$$y(x) = \frac{\sin(2x)}{2}$$

✓ Solution by Mathematica
Time used: 0.012 (sec). Leaf size: 10

```
DSolve[{y''[x]+4*y[x]==0,{y[0]==0,y'[0]==1}},y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to \sin(x)\cos(x)$$

8.12 problem 18

Internal problem ID [634]

Book: Elementary differential equations and boundary value problems, 10th ed., Boyce and DiPrima
Section: Chapter 3, Second order linear equations, 3.3 Complex Roots of the Characteristic Equation , page 164
Problem number: 18.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _missing_x]]`

$$\boxed{y'' + 4y' + 5y = 0}$$

With initial conditions

$$[y(0) = 1, y'(0) = 0]$$

✓ Solution by Maple

Time used: 0.015 (sec). Leaf size: 16

```
dsolve([diff(y(x),x$2)+ 4*diff(y(x),x)+5*y(x) = 0,y(0) = 1, D(y)(0) = 0],y(x), singsol=all)
```

$$y(x) = \mathrm{e}^{-2x}(2\sin(x) + \cos(x))$$

✓ Solution by Mathematica

Time used: 0.015 (sec). Leaf size: 18

```
DSolve[{y''[x]+4*y'[x]+5*y[x]==0,{y[0]==1,y'[0]==0}},y[x],x,IncludeSingularSolutions -> True
```

$$y(x) \to e^{-2x}(2\sin(x) + \cos(x))$$

8.13 problem 19

Internal problem ID [635]

Book: Elementary differential equations and boundary value problems, 10th ed., Boyce and DiPrima
Section: Chapter 3, Second order linear equations, 3.3 Complex Roots of the Characteristic Equation , page 164
Problem number: 19.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _missing_x]]`

$$\boxed{y'' - 2y' + 5y = 0}$$

With initial conditions

$$\left[y\left(\frac{\pi}{2}\right) = 0, y'\left(\frac{\pi}{2}\right) = 2\right]$$

✓ Solution by Maple
Time used: 0.031 (sec). Leaf size: 16

```
dsolve([diff(y(x),x$2)- 2*diff(y(x),x)+5*y(x) = 0,y(1/2*Pi) = 0, D(y)(1/2*Pi) = 2],y(x), sin
```

$$y(x) = -\sin(2x)\,\mathrm{e}^{-\frac{\pi}{2}+x}$$

✓ Solution by Mathematica
Time used: 0.015 (sec). Leaf size: 20

```
DSolve[{y''[x]-2*y'[x]+5*y[x]==0,{y[Pi/2]==0,y'[Pi/2]==2}},y[x],x,IncludeSingularSolutions -
```

$$y(x) \to -e^{x-\frac{\pi}{2}}\sin(2x)$$

8.14 problem 20

Internal problem ID [636]

Book: Elementary differential equations and boundary value problems, 10th ed., Boyce and DiPrima
Section: Chapter 3, Second order linear equations, 3.3 Complex Roots of the Characteristic Equation , page 164
Problem number: 20.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _missing_x]]`

$$\boxed{y'' + y = 0}$$

With initial conditions

$$\left[y\left(\frac{\pi}{3}\right) = 2, y'\left(\frac{\pi}{3}\right) = -4\right]$$

✓ Solution by Maple
Time used: 0.015 (sec). Leaf size: 23

```
dsolve([diff(y(x),x$2)+y(x) = 0,y(1/3*Pi) = 2, D(y)(1/3*Pi) = -4],y(x), singsol=all)
```

$$y(x) = (\sin(x) + 2\cos(x))\sqrt{3} + \cos(x) - 2\sin(x)$$

✓ Solution by Mathematica
Time used: 0.012 (sec). Leaf size: 28

```
DSolve[{y''[x]+y[x]==0,{y[Pi/3]==2,y'[Pi/3]==-4}},y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to \left(\sqrt{3} - 2\right)\sin(x) + \left(1 + 2\sqrt{3}\right)\cos(x)$$

8.15 problem 21

Internal problem ID [637]

Book: Elementary differential equations and boundary value problems, 10th ed., Boyce and DiPrima
Section: Chapter 3, Second order linear equations, 3.3 Complex Roots of the Characteristic Equation , page 164
Problem number: 21.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _missing_x]]`

$$\boxed{y'' + y' + \frac{5y}{4} = 0}$$

With initial conditions

$$[y(0) = 3, y'(0) = 1]$$

✓ Solution by Maple
Time used: 0.016 (sec). Leaf size: 19

```
dsolve([diff(y(x),x$2)+ diff(y(x),x)+125/100*y(x) = 0,y(0) = 3, D(y)(0) = 1],y(x), singsol=a
```

$$y(x) = \frac{e^{-\frac{x}{2}}(5\sin(x) + 6\cos(x))}{2}$$

✓ Solution by Mathematica
Time used: 0.016 (sec). Leaf size: 25

```
DSolve[{y''[x]+y'[x]+125/100*y[x]==0,{y[0]==3,y'[0]==1}},y[x],x,IncludeSingularSolutions ->
```

$$y(x) \to \frac{1}{2}e^{-x/2}(5\sin(x) + 6\cos(x))$$

8.16 problem 22

Internal problem ID [638]

Book: Elementary differential equations and boundary value problems, 10th ed., Boyce and DiPrima
Section: Chapter 3, Second order linear equations, 3.3 Complex Roots of the Characteristic Equation , page 164
Problem number: 22.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _missing_x]]`

$$\boxed{y'' + 2y' + 2y = 0}$$

With initial conditions

$$\left[y\left(\frac{\pi}{4}\right) = 2, y'\left(\frac{\pi}{4}\right) = -2\right]$$

✓ Solution by Maple
Time used: 0.047 (sec). Leaf size: 21

```
dsolve([diff(y(x),x$2)+ 2*diff(y(x),x)+2*y(x) = 0,y(1/4*Pi) = 2, D(y)(1/4*Pi) = -2],y(x), si
```

$$y(x) = \sqrt{2}\,\mathrm{e}^{-x+\frac{\pi}{4}}(\sin(x) + \cos(x))$$

✓ Solution by Mathematica
Time used: 0.016 (sec). Leaf size: 27

```
DSolve[{y''[x]+2*y'[x]+2*y[x]==0,{y[Pi/4]==2,y'[Pi/4]==-2}},y[x],x,IncludeSingularSolutions
```

$$y(x) \to \sqrt{2}e^{\frac{\pi}{4}-x}(\sin(x) + \cos(x))$$

8.17 problem 23

Internal problem ID [639]

Book: Elementary differential equations and boundary value problems, 10th ed., Boyce and DiPrima
Section: Chapter 3, Second order linear equations, 3.3 Complex Roots of the Characteristic Equation , page 164
Problem number: 23.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _missing_x]]`

$$\boxed{u'' - u' + 2u = 0}$$

With initial conditions

$$[u(0) = 2, u'(0) = 0]$$

✓ Solution by Maple
Time used: 0.047 (sec). Leaf size: 31

```
dsolve([diff(u(x),x$2)- diff(u(x),x)+2*u(x) = 0,u(0) = 2, D(u)(0) = 0],u(x), singsol=all)
```

$$u(x) = -\frac{2\,\mathrm{e}^{\frac{x}{2}}\left(\sqrt{7}\,\sin\left(\frac{\sqrt{7}\,x}{2}\right) - 7\cos\left(\frac{\sqrt{7}\,x}{2}\right)\right)}{7}$$

✓ Solution by Mathematica
Time used: 0.015 (sec). Leaf size: 19

```
DSolve[{u''[x]+4*u'[x]+5*u[x]==0,{u[0]==2,u'[0]==0}},u[x],x,IncludeSingularSolutions -> True
```

$$u(x) \to 2e^{-2x}(2\sin(x) + \cos(x))$$

8.18 problem 24

Internal problem ID [640]

Book: Elementary differential equations and boundary value problems, 10th ed., Boyce and DiPrima
Section: Chapter 3, Second order linear equations, 3.3 Complex Roots of the Characteristic Equation , page 164
Problem number: 24.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _missing_x]]`

$$\boxed{5u'' + 2u' + 7u = 0}$$

With initial conditions

$$[u(0) = 2, u'(0) = 1]$$

✓ Solution by Maple
Time used: 0.047 (sec). Leaf size: 32

```
dsolve([5*diff(u(x),x$2)+ 2*diff(u(x),x)+7*u(x) = 0,u(0) = 2, D(u)(0) = 1],u(x), singsol=all
```

$$u(x) = \frac{e^{-\frac{x}{5}} \left(7\sqrt{34}\,\sin\left(\frac{\sqrt{34}\,x}{5}\right) + 68\cos\left(\frac{\sqrt{34}\,x}{5}\right)\right)}{34}$$

✓ Solution by Mathematica
Time used: 0.025 (sec). Leaf size: 48

```
DSolve[{5*u''[x]+2*u'[x]+7*u[x]==0,{u[0]==2,u'[0]==1}},u[x],x,IncludeSingularSolutions -> Tr
```

$$u(x) \to \frac{1}{34} e^{-x/5} \left(7\sqrt{34}\sin\left(\frac{\sqrt{34}x}{5}\right) + 68\cos\left(\frac{\sqrt{34}x}{5}\right)\right)$$

8.19 problem 25

Internal problem ID [641]

Book: Elementary differential equations and boundary value problems, 10th ed., Boyce and DiPrima
Section: Chapter 3, Second order linear equations, 3.3 Complex Roots of the Characteristic Equation , page 164
Problem number: 25.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _missing_x]]`

$$\boxed{y'' + 2y' + 6y = 0}$$

With initial conditions

$$[y(0) = 2, y'(0) = \alpha]$$

✓ Solution by Maple
Time used: 0.047 (sec). Leaf size: 32

```
dsolve([diff(y(x),x$2)+ 2*diff(y(x),x)+6*y(x) = 0,y(0) = 2, D(y)(0) = alpha],y(x), singsol=a
```

$$y(x) = \frac{e^{-x}\left(\sqrt{5}\,(\alpha+2)\sin\left(\sqrt{5}\,x\right) + 10\cos\left(\sqrt{5}\,x\right)\right)}{5}$$

✓ Solution by Mathematica
Time used: 0.021 (sec). Leaf size: 42

```
DSolve[{y''[x]+2*y'[x]+6*y[x]==0,{y[0]==2,y'[0]==\[Alpha]}},y[x],x,IncludeSingularSolutions
```

$$y(x) \to \frac{1}{5}e^{-x}\left(\sqrt{5}(\alpha+2)\sin\left(\sqrt{5}x\right) + 10\cos\left(\sqrt{5}x\right)\right)$$

8.20 problem 26

Internal problem ID [642]

Book: Elementary differential equations and boundary value problems, 10th ed., Boyce and DiPrima
Section: Chapter 3, Second order linear equations, 3.3 Complex Roots of the Characteristic Equation , page 164
Problem number: 26.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _missing_x]]`

$$\boxed{y'' + 2ay' + (a^2 + 1)\, y = 0}$$

With initial conditions

$$[y(0) = 1, y'(0) = 0]$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 17

```
dsolve([diff(y(x),x$2)+ 2*a*diff(y(x),x)+(a^2+1)*y(x) = 0,y(0) = 1, D(y)(0) = 0],y(x), sings
```

$$y(x) = \mathrm{e}^{-ax}(a \sin (x) + \cos (x))$$

✓ Solution by Mathematica

Time used: 0.031 (sec). Leaf size: 94

```
DSolve[{y''[x]+2*a*y'[x]+(a^1+1)*y[x]==0,{y[0]==1,y'[0]==0}},y[x],x,IncludeSingularSolutions
```

$$y(x) \to \frac{e^{-\left(\left(\sqrt{a^2-a-1}+a\right)x\right)}\left(a\left(e^{2\sqrt{a^2-a-1}x} - 1\right) + \sqrt{a^2 - a - 1}\left(e^{2\sqrt{a^2-a-1}x} + 1\right)\right)}{2\sqrt{a^2 - a - 1}}$$

8.21 problem 35

Internal problem ID [643]

Book: Elementary differential equations and boundary value problems, 10th ed., Boyce and DiPrima
Section: Chapter 3, Second order linear equations, 3.3 Complex Roots of the Characteristic Equation , page 164
Problem number: 35.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_Emden, _Fowler], [_2nd_order, _linear, '_with_symmetry_[0,F`

$$\boxed{t^2y'' + ty' + y = 0}$$

✓ Solution by Maple
Time used: 0.016 (sec). Leaf size: 15

```
dsolve(t^2*diff(y(t),t$2)+ t*diff(y(t),t)+y(t) = 0,y(t), singsol=all)
```

$$y(t) = c_1 \sin(\ln(t)) + c_2 \cos(\ln(t))$$

✓ Solution by Mathematica
Time used: 0.017 (sec). Leaf size: 18

```
DSolve[t^2*y''[t]+t*y'[t]+y[t]==0,y[t],t,IncludeSingularSolutions -> True]
```

$$y(t) \to c_1 \cos(\log(t)) + c_2 \sin(\log(t))$$

8.22 problem 36

Internal problem ID [644]

Book: Elementary differential equations and boundary value problems, 10th ed., Boyce and DiPrima
Section: Chapter 3, Second order linear equations, 3.3 Complex Roots of the Characteristic Equation , page 164
Problem number: 36.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _exact, _linear, _homogeneous]]`

$$\boxed{t^2y'' + 4ty' + 2y = 0}$$

✓ Solution by Maple
Time used: 0.0 (sec). Leaf size: 13

```
dsolve(t^2*diff(y(t),t$2)+ 4*t*diff(y(t),t)+2*y(t) = 0,y(t), singsol=all)
```

$$y(t) = \frac{c_2t + c_1}{t^2}$$

✓ Solution by Mathematica
Time used: 0.016 (sec). Leaf size: 34

```
DSolve[t^2*y''[t]+4*t*y'[t]+y[t]==0,y[t],t,IncludeSingularSolutions -> True]
```

$$y(t) \to t^{-\frac{3}{2}-\frac{\sqrt{5}}{2}}\left(c_2t^{\sqrt{5}} + c_1\right)$$

8.23 problem 37

Internal problem ID [645]

Book: Elementary differential equations and boundary value problems, 10th ed., Boyce and DiPrima
Section: Chapter 3, Second order linear equations, 3.3 Complex Roots of the Characteristic Equation , page 164
Problem number: 37.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_Emden, _Fowler]]`

$$\boxed{t^2y'' + 3ty' + \frac{5y}{4} = 0}$$

✓ Solution by Maple
Time used: 0.0 (sec). Leaf size: 23

```
dsolve(t^2*diff(y(t),t$2)+ 3*t*diff(y(t),t)+125/100*y(t) = 0,y(t), singsol=all)
```

$$y(t) = \frac{c_1 \sin\left(\frac{\ln(t)}{2}\right) + c_2 \cos\left(\frac{\ln(t)}{2}\right)}{t}$$

✓ Solution by Mathematica
Time used: 0.025 (sec). Leaf size: 30

```
DSolve[t^2*y''[t]+3*t*y'[t]+125/100*y[t]==0,y[t],t,IncludeSingularSolutions -> True]
```

$$y(t) \to \frac{c_2 \cos\left(\frac{\log(t)}{2}\right) + c_1 \sin\left(\frac{\log(t)}{2}\right)}{t}$$

8.24 problem 38

Internal problem ID [646]

Book: Elementary differential equations and boundary value problems, 10th ed., Boyce and DiPrima
Section: Chapter 3, Second order linear equations, 3.3 Complex Roots of the Characteristic Equation , page 164
Problem number: 38.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _exact, _linear, _homogeneous]]`

$$\boxed{t^2y'' - 4ty' - 6y = 0}$$

✓ Solution by Maple
Time used: 0.0 (sec). Leaf size: 15

```
dsolve(t^2*diff(y(t),t$2)- 4*t*diff(y(t),t)-6*y(t) = 0,y(t), singsol=all)
```

$$y(t) = \frac{c_1 t^7 + c_2}{t}$$

✓ Solution by Mathematica
Time used: 0.01 (sec). Leaf size: 18

```
DSolve[t^2*y''[t]-4*t*y'[t]-6*y[t]==0,y[t],t,IncludeSingularSolutions -> True]
```

$$y(t) \to \frac{c_2 t^7 + c_1}{t}$$

8.25 problem 39

Internal problem ID [647]

Book: Elementary differential equations and boundary value problems, 10th ed., Boyce and DiPrima
Section: Chapter 3, Second order linear equations, 3.3 Complex Roots of the Characteristic Equation , page 164
Problem number: 39.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_Emden, _Fowler], [_2nd_order, _linear, '_with_symmetry_[0,F`

$$\boxed{t^2y'' - 4ty' + 6y = 0}$$

✓ Solution by Maple
Time used: 0.0 (sec). Leaf size: 13

```
dsolve(t^2*diff(y(t),t$2)-4*t*diff(y(t),t)+6*y(t) = 0,y(t), singsol=all)
```

$$y(t) = t^2(c_2t + c_1)$$

✓ Solution by Mathematica
Time used: 0.01 (sec). Leaf size: 16

```
DSolve[t^2*y''[t]-4*t*y'[t]+6*y[t]==0,y[t],t,IncludeSingularSolutions -> True]
```

$$y(t) \to t^2(c_2t + c_1)$$

8.26 problem 40

Internal problem ID [648]

Book: Elementary differential equations and boundary value problems, 10th ed., Boyce and DiPrima
Section: Chapter 3, Second order linear equations, 3.3 Complex Roots of the Characteristic Equation , page 164
Problem number: 40.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_Emden, _Fowler]]`

$$\boxed{t^2y'' - ty' + 5y = 0}$$

✓ Solution by Maple
Time used: 0.016 (sec). Leaf size: 21

```
dsolve(t^2*diff(y(t),t$2)- t*diff(y(t),t)+5*y(t) = 0,y(t), singsol=all)
```

$$y(t) = t(c_1 \sin (2 \ln (t)) + c_2 \cos (2 \ln (t)))$$

✓ Solution by Mathematica
Time used: 0.023 (sec). Leaf size: 24

```
DSolve[t^2*y''[t]-t*y'[t]+5*y[t]==0,y[t],t,IncludeSingularSolutions -> True]
```

$$y(t) \to t(c_2 \cos(2 \log(t)) + c_1 \sin(2 \log(t)))$$

8.27 problem 41

Internal problem ID [649]

Book: Elementary differential equations and boundary value problems, 10th ed., Boyce and DiPrima
Section: Chapter 3, Second order linear equations, 3.3 Complex Roots of the Characteristic Equation , page 164
Problem number: 41.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_Emden, _Fowler]]`

$$\boxed{t^2y'' + 3ty' - 3y = 0}$$

✓ Solution by Maple
Time used: 0.0 (sec). Leaf size: 15

```
dsolve(t^2*diff(y(t),t$2)+ 3*t*diff(y(t),t)-3*y(t) = 0,y(t), singsol=all)
```

$$y(t) = \frac{c_1 t^4 + c_2}{t^3}$$

✓ Solution by Mathematica
Time used: 0.011 (sec). Leaf size: 16

```
DSolve[t^2*y''[t]+3*t*y'[t]-3*y[t]==0,y[t],t,IncludeSingularSolutions -> True]
```

$$y(t) \to \frac{c_1}{t^3} + c_2 t$$

8.28 problem 42

Internal problem ID [650]

Book: Elementary differential equations and boundary value problems, 10th ed., Boyce and DiPrima
Section: Chapter 3, Second order linear equations, 3.3 Complex Roots of the Characteristic Equation , page 164
Problem number: 42.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_Emden, _Fowler]]`

$$\boxed{t^2y'' + 7ty' + 10y = 0}$$

✓ Solution by Maple
Time used: 0.0 (sec). Leaf size: 19

```
dsolve(t^2*diff(y(t),t$2)+ 7*t*diff(y(t),t)+10*y(t) = 0,y(t), singsol=all)
```

$$y(t) = \frac{c_1 \sin(\ln(t)) + c_2 \cos(\ln(t))}{t^3}$$

✓ Solution by Mathematica
Time used: 0.022 (sec). Leaf size: 22

```
DSolve[t^2*y''[t]+7*t*y'[t]+10*y[t]==0,y[t],t,IncludeSingularSolutions -> True]
```

$$y(t) \to \frac{c_2 \cos(\log(t)) + c_1 \sin(\log(t))}{t^3}$$

8.29 problem 44

Internal problem ID [651]

Book: Elementary differential equations and boundary value problems, 10th ed., Boyce and DiPrima
Section: Chapter 3, Second order linear equations, 3.3 Complex Roots of the Characteristic Equation , page 164
Problem number: 44.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries], [_2nd_order, _linear,`

$$y'' + ty' + e^{-t^2} y = 0$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 55

```
dsolve(diff(y(t),t$2)+ t*diff(y(t),t)+exp(-t^2)*y(t) = 0,y(t), singsol=all)
```

$$y(t) = c_1 \operatorname{csgn}\left(e^{\frac{t^2}{2}}\right) \sin\left(\frac{\sqrt{2}\,\sqrt{\pi}\ \operatorname{erf}\left(\frac{t\sqrt{2}}{2}\right)}{2}\right) + c_2 \cos\left(\frac{\sqrt{2}\ \operatorname{csgn}\left(e^{\frac{t^2}{2}}\right)\sqrt{\pi}\ \operatorname{erf}\left(\frac{t\sqrt{2}}{2}\right)}{2}\right)$$

✓ Solution by Mathematica

Time used: 0.059 (sec). Leaf size: 102

```
DSolve[y''[t]+t*y'[t]+exp(-t^2)*y[t]==0,y[t],t,IncludeSingularSolutions -> True]
```

$$y(t) \to e^{-\frac{1}{4}(\sqrt{4\exp+1}+1)t^2} \left(c_1 \operatorname{HermiteH}\left(-\frac{1}{2} - \frac{1}{2\sqrt{4\exp+1}}, \frac{\sqrt[4]{4\exp+1}t}{\sqrt{2}}\right) + c_2 \operatorname{Hypergeometric1F1}\left(\frac{1}{4}\left(1 + \frac{1}{\sqrt{4\exp+1}}\right), \frac{1}{2}, \frac{1}{2}\sqrt{4\exp+1}t^2\right)\right)$$

8.30 problem 46

Internal problem ID [652]

Book: Elementary differential equations and boundary value problems, 10th ed., Boyce and DiPrima
Section: Chapter 3, Second order linear equations, 3.3 Complex Roots of the Characteristic Equation , page 164
Problem number: 46.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$\boxed{ty'' + (t^2 - 1)\, y' + yt^3 = 0}$$

✓ Solution by Maple
Time used: 0.016 (sec). Leaf size: 34

```
dsolve(t*diff(y(t),t$2)+ (t^2-1)*diff(y(t),t)+t^3*y(t) = 0,y(t), singsol=all)
```

$$y(t) = \mathrm{e}^{-\frac{t^2}{4}} \left(c_1 \cos\left(\frac{t^2\sqrt{3}}{4}\right) + c_2 \sin\left(\frac{t^2\sqrt{3}}{4}\right)\right)$$

✓ Solution by Mathematica
Time used: 0.03 (sec). Leaf size: 48

```
DSolve[t*y''[t]+(t^2-1)*y'[t]+t^3*y[t]==0,y[t],t,IncludeSingularSolutions -> True]
```

$$y(t) \to e^{-\frac{t^2}{4}} \left(c_2 \cos\left(\frac{\sqrt{3}t^2}{4}\right) + c_1 \sin\left(\frac{\sqrt{3}t^2}{4}\right)\right)$$

9 Chapter 3, Second order linear equations, 3.4 Repeated roots, reduction of order , page 172

9.1 problem 1

Internal problem ID [653]

Book: Elementary differential equations and boundary value problems, 10th ed., Boyce and DiPrima
Section: Chapter 3, Second order linear equations, 3.4 Repeated roots, reduction of order , page 172
Problem number: 1.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _missing_x]]`

$$\boxed{y'' - 2y' + y = 0}$$

✓ Solution by Maple
Time used: 0.0 (sec). Leaf size: 12

```
dsolve(diff(y(x),x$2)-2*diff(y(x),x)+y(x) = 0,y(x), singsol=all)
```

$$y(x) = \mathrm{e}^{x}(c_2 x + c_1)$$

✓ Solution by Mathematica
Time used: 0.01 (sec). Leaf size: 16

```
DSolve[y''[x]-2*y'[x]+y[x]==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to e^{x}(c_2 x + c_1)$$

9.2 problem 2

Internal problem ID [654]

Book: Elementary differential equations and boundary value problems, 10th ed., Boyce and DiPrima
Section: Chapter 3, Second order linear equations, 3.4 Repeated roots, reduction of order , page 172
Problem number: 2.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _missing_x]]`

$$\boxed{9y'' + 6y' + y = 0}$$

✓ Solution by Maple
Time used: 0.0 (sec). Leaf size: 14

```
dsolve(9*diff(y(x),x$2)+6*diff(y(x),x)+y(x) = 0,y(x), singsol=all)
```

$$y(x) = \mathrm{e}^{-\frac{x}{3}}(c_2x + c_1)$$

✓ Solution by Mathematica
Time used: 0.013 (sec). Leaf size: 20

```
DSolve[9*y''[x]+6*y'[x]+y[x]==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to e^{-x/3}(c_2x + c_1)$$

9.3 problem 3

Internal problem ID [655]

Book: Elementary differential equations and boundary value problems, 10th ed., Boyce and DiPrima
Section: Chapter 3, Second order linear equations, 3.4 Repeated roots, reduction of order , page 172
Problem number: 3.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _missing_x]]`

$$\boxed{4y'' - 4y' - 3y = 0}$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 17

```
dsolve(4*diff(y(x),x$2)-4*diff(y(x),x)-3*y(x) = 0,y(x), singsol=all)
```

$$y(x) = c_1 e^{-\frac{x}{2}} + c_2 e^{\frac{3x}{2}}$$

✓ Solution by Mathematica

Time used: 0.013 (sec). Leaf size: 24

```
DSolve[4*y''[x]-4*y'[x]-3*y[x]==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to e^{-x/2}\left(c_2 e^{2x} + c_1\right)$$

9.4 problem 4

Internal problem ID [656]

Book: Elementary differential equations and boundary value problems, 10th ed., Boyce and DiPrima
Section: Chapter 3, Second order linear equations, 3.4 Repeated roots, reduction of order , page 172
Problem number: 4.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _missing_x]]`

$$\boxed{4y'' + 12y' + 9y = 0}$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 14

```
dsolve(4*diff(y(x),x$2)+12*diff(y(x),x)+9*y(x) = 0,y(x), singsol=all)
```

$$y(x) = \mathrm{e}^{-\frac{3x}{2}}(c_2x + c_1)$$

✓ Solution by Mathematica

Time used: 0.014 (sec). Leaf size: 20

```
DSolve[4*y''[x]+12*y'[x]+9*y[x]==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \rightarrow e^{-3x/2}(c_2x + c_1)$$

9.5 problem 5

Internal problem ID [657]

Book: Elementary differential equations and boundary value problems, 10th ed., Boyce and DiPrima
Section: Chapter 3, Second order linear equations, 3.4 Repeated roots, reduction of order , page 172
Problem number: 5.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _missing_x]]`

$$\boxed{y'' - 2y' + 10y = 0}$$

✓ Solution by Maple
Time used: 0.0 (sec). Leaf size: 20

```
dsolve(diff(y(x),x$2)-2*diff(y(x),x)+10*y(x) = 0,y(x), singsol=all)
```

$$y(x) = e^x(c_1 \sin(3x) + c_2 \cos(3x))$$

✓ Solution by Mathematica
Time used: 0.015 (sec). Leaf size: 24

```
DSolve[y''[x]-2*y'[x]+10*y[x]==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to e^x(c_2 \cos(3x) + c_1 \sin(3x))$$

9.6 problem 6

Internal problem ID [658]

Book: Elementary differential equations and boundary value problems, 10th ed., Boyce and DiPrima
Section: Chapter 3, Second order linear equations, 3.4 Repeated roots, reduction of order , page 172
Problem number: 6.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _missing_x]]`

$$\boxed{y'' - 6y' + 9y = 0}$$

✓ Solution by Maple
Time used: 0.0 (sec). Leaf size: 14

```
dsolve(diff(y(x),x$2)-6*diff(y(x),x)+9*y(x) = 0,y(x), singsol=all)
```

$$y(x) = \mathrm{e}^{3x}(c_2x + c_1)$$

✓ Solution by Mathematica
Time used: 0.013 (sec). Leaf size: 18

```
DSolve[y''[x]-6*y'[x]+9*y[x]==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to e^{3x}(c_2x + c_1)$$

9.7 problem 7

Internal problem ID [659]

Book: Elementary differential equations and boundary value problems, 10th ed., Boyce and DiPrima
Section: Chapter 3, Second order linear equations, 3.4 Repeated roots, reduction of order , page 172
Problem number: 7.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _missing_x]]`

$$\boxed{4y'' + 17y' + 4y = 0}$$

✓ Solution by Maple
Time used: 0.0 (sec). Leaf size: 17

```
dsolve(4*diff(y(x),x$2)+17*diff(y(x),x)+4*y(x) = 0,y(x), singsol=all)
```

$$y(x) = c_1 e^{-4x} + c_2 e^{-\frac{x}{4}}$$

✓ Solution by Mathematica
Time used: 0.012 (sec). Leaf size: 24

```
DSolve[4*y''[x]+17*y'[x]+4*y[x]==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to e^{-4x}\left(c_1 e^{15x/4} + c_2\right)$$

9.8 problem 8

Internal problem ID [660]

Book: Elementary differential equations and boundary value problems, 10th ed., Boyce and DiPrima
Section: Chapter 3, Second order linear equations, 3.4 Repeated roots, reduction of order , page 172
Problem number: 8.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _missing_x]]`

$$\boxed{16y'' + 24y' + 9y = 0}$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 14

```
dsolve(16*diff(y(x),x$2)+24*diff(y(x),x)+9*y(x) = 0,y(x), singsol=all)
```

$$y(x) = \mathrm{e}^{-\frac{3x}{4}}(c_2x + c_1)$$

✓ Solution by Mathematica

Time used: 0.014 (sec). Leaf size: 20

```
DSolve[16*y''[x]+24*y'[x]+9*y[x]==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to e^{-3x/4}(c_2x + c_1)$$

9.9 problem 9

Internal problem ID [661]

Book: Elementary differential equations and boundary value problems, 10th ed., Boyce and DiPrima
Section: Chapter 3, Second order linear equations, 3.4 Repeated roots, reduction of order , page 172
Problem number: 9.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _missing_x]]`

$$\boxed{25y'' - 20y' + 4y = 0}$$

✓ Solution by Maple
Time used: 0.0 (sec). Leaf size: 14

```
dsolve(25*diff(y(x),x$2)-20*diff(y(x),x)+4*y(x) = 0,y(x), singsol=all)
```

$$y(x) = \mathrm{e}^{\frac{2x}{5}}(c_2x + c_1)$$

✓ Solution by Mathematica
Time used: 0.015 (sec). Leaf size: 20

```
DSolve[25*y''[x]-20*y'[x]+4*y[x]==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to e^{2x/5}(c_2x + c_1)$$

9.10 problem 10

Internal problem ID [662]

Book: Elementary differential equations and boundary value problems, 10th ed., Boyce and DiPrima
Section: Chapter 3, Second order linear equations, 3.4 Repeated roots, reduction of order , page 172
Problem number: 10.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _missing_x]]`

$$\boxed{2y'' + 2y' + y = 0}$$

✓ Solution by Maple
Time used: 0.0 (sec). Leaf size: 22

```
dsolve(2*diff(y(x),x$2)+2*diff(y(x),x)+y(x) = 0,y(x), singsol=all)
```

$$y(x) = \mathrm{e}^{-\frac{x}{2}}\left(c_1 \sin\left(\frac{x}{2}\right) + c_2 \cos\left(\frac{x}{2}\right)\right)$$

✓ Solution by Mathematica
Time used: 0.017 (sec). Leaf size: 32

```
DSolve[2*y''[x]+2*y'[x]+y[x]==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to e^{-x/2}\left(c_2 \cos\left(\frac{x}{2}\right) + c_1 \sin\left(\frac{x}{2}\right)\right)$$

9.11 problem 11

Internal problem ID [663]

Book: Elementary differential equations and boundary value problems, 10th ed., Boyce and DiPrima
Section: Chapter 3, Second order linear equations, 3.4 Repeated roots, reduction of order , page 172
Problem number: 11.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _missing_x]]`

$$\boxed{9y'' - 12y' + 4y = 0}$$

With initial conditions

$$[y(0) = 2, y'(0) = -1]$$

✓ Solution by Maple
Time used: 0.015 (sec). Leaf size: 14

```
dsolve([9*diff(y(t),t$2)-12*diff(y(t),t)+4*y(t) = 0,y(0) = 2, D(y)(0) = -1],y(t), singsol=al
```

$$y(t) = -\frac{e^{\frac{2t}{3}}(-6 + 7t)}{3}$$

✓ Solution by Mathematica
Time used: 0.015 (sec). Leaf size: 15

```
DSolve[{9*y''[t]-12*y'[t]+4*y[t]==0,{y[0]==0,y'[0]==-1}},y[t],t,IncludeSingularSolutions ->
```

$$y(t) \to -e^{2t/3}t$$

9.12 problem 12

Internal problem ID [664]

Book: Elementary differential equations and boundary value problems, 10th ed., Boyce and DiPrima
Section: Chapter 3, Second order linear equations, 3.4 Repeated roots, reduction of order , page 172
Problem number: 12.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _missing_x]]`

$$\boxed{y'' - 6y' + 9y = 0}$$

With initial conditions

$$[y(0) = 0, y'(0) = 2]$$

✓ Solution by Maple
Time used: 0.016 (sec). Leaf size: 11

```
dsolve([diff(y(t),t$2)-6*diff(y(t),t)+9*y(t) = 0,y(0) = 0, D(y)(0) = 2],y(t), singsol=all)
```

$$y(t) = 2\,\mathrm{e}^{3t}t$$

✓ Solution by Mathematica
Time used: 0.014 (sec). Leaf size: 13

```
DSolve[{y''[t]-6*y'[t]+9*y[t]==0,{y[0]==0,y'[0]==2}},y[t],t,IncludeSingularSolutions -> True
```

$$y(t) \to 2e^{3t}t$$

9.13 problem 13

Internal problem ID [665]

Book: Elementary differential equations and boundary value problems, 10th ed., Boyce and DiPrima
Section: Chapter 3, Second order linear equations, 3.4 Repeated roots, reduction of order , page 172
Problem number: 13.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _missing_x]]`

$$\boxed{9y'' + 6y' + 82y = 0}$$

With initial conditions

$$[y(0) = -1, y'(0) = 2]$$

✓ Solution by Maple

Time used: 0.015 (sec). Leaf size: 23

```
dsolve([9*diff(y(t),t$2)+6*diff(y(t),t)+82*y(t) = 0,y(0) = -1, D(y)(0) = 2],y(t), singsol=al
```

$$y(t) = \frac{e^{-\frac{t}{3}}(5\sin(3t) - 9\cos(3t))}{9}$$

✓ Solution by Mathematica

Time used: 0.019 (sec). Leaf size: 29

```
DSolve[{9*y''[t]+6*y'[t]+82*y[t]==0,{y[0]==-1,y'[0]==2}},y[t],t,IncludeSingularSolutions ->
```

$$y(t) \to \frac{1}{9}e^{-t/3}(5\sin(3t) - 9\cos(3t))$$

9.14 problem 14

Internal problem ID [666]

Book: Elementary differential equations and boundary value problems, 10th ed., Boyce and DiPrima
Section: Chapter 3, Second order linear equations, 3.4 Repeated roots, reduction of order , page 172
Problem number: 14.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _missing_x]]`

$$\boxed{y'' + 4y' + 4y = 0}$$

With initial conditions

$$[y(-1) = 2, y'(-1) = 1]$$

✓ Solution by Maple
Time used: 0.031 (sec). Leaf size: 16

```
dsolve([diff(y(x),x$2)+4*diff(y(x),x)+4*y(x) = 0,y(-1) = 2, D(y)(-1) = 1],y(x), singsol=all)
```

$$y(x) = e^{-2x-2}(5x + 7)$$

✓ Solution by Mathematica
Time used: 0.014 (sec). Leaf size: 18

```
DSolve[{y''[x]+4*y'[x]+4*y[x]==0,{y[-1]==2,y'[-1]==1}},y[x],x,IncludeSingularSolutions -> Tr
```

$$y(x) \to e^{-2(x+1)}(5x + 7)$$

9.15 problem 15

Internal problem ID [667]

Book: Elementary differential equations and boundary value problems, 10th ed., Boyce and DiPrima
Section: Chapter 3, Second order linear equations, 3.4 Repeated roots, reduction of order , page 172
Problem number: 15.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _missing_x]]`

$$\boxed{4y'' + 12y' + 9y = 0}$$

With initial conditions

$$[y(0) = 1, y'(0) = -4]$$

✓ Solution by Maple

Time used: 0.015 (sec). Leaf size: 14

```
dsolve([4*diff(y(t),t$2)+12*diff(y(t),t)+9*y(t) = 0,y(0) = 1, D(y)(0) = -4],y(t), singsol=al
```

$$y(t) = -\frac{e^{-\frac{3t}{2}}(-2+5t)}{2}$$

✓ Solution by Mathematica

Time used: 0.015 (sec). Leaf size: 21

```
DSolve[{4*y''[t]+12*y'[t]+9*y[t]==0,{y[0]==1,y'[0]==-4}},y[t],t,IncludeSingularSolutions ->
```

$$y(t) \to \frac{1}{2}e^{-3t/2}(2-5t)$$

9.16 problem 16

Internal problem ID [668]

Book: Elementary differential equations and boundary value problems, 10th ed., Boyce and DiPrima
Section: Chapter 3, Second order linear equations, 3.4 Repeated roots, reduction of order , page 172
Problem number: 16.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _missing_x]]`

$$\boxed{y'' - y' + \frac{y}{4} = 0}$$

With initial conditions

$$[y(0) = 2, y'(0) = b]$$

✓ Solution by Maple
Time used: 0.016 (sec). Leaf size: 16

```
dsolve([diff(y(t),t$2)-diff(y(t),t)+25/100*y(t) = 0,y(0) = 2, D(y)(0) = b],y(t), singsol=all
```

$$y(t) = (2 + t(b - 1))\,\mathrm{e}^{\frac{t}{2}}$$

✓ Solution by Mathematica
Time used: 0.014 (sec). Leaf size: 20

```
DSolve[{y''[t]-y'[t]+25/100*y[t]==0,{y[0]==2,y'[0]==b}},y[t],t,IncludeSingularSolutions -> T
```

$$y(t) \to e^{t/2}((b - 1)t + 2)$$

9.17 problem 23

Internal problem ID [669]

Book: Elementary differential equations and boundary value problems, 10th ed., Boyce and DiPrima
Section: Chapter 3, Second order linear equations, 3.4 Repeated roots, reduction of order , page 172
Problem number: 23.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_Emden, _Fowler], [_2nd_order, _linear, '_with_symmetry_[0,F`

$$\boxed{t^2y'' - 4ty' + 6y = 0}$$

Given that one solution of the ode is

$$y_1 = t^2$$

✓ Solution by Maple
Time used: 0.016 (sec). Leaf size: 13

```
dsolve([t^2*diff(y(t),t$2)-4*t*diff(y(t),t)+6*y(t)=0,t^2],singsol=all)
```

$$y(t) = t^2(c_2t + c_1)$$

✓ Solution by Mathematica
Time used: 0.012 (sec). Leaf size: 16

```
DSolve[t^2*y''[t]-4*t*y'[t]+6*y[t]==0,y[t],t,IncludeSingularSolutions -> True]
```

$$y(t) \to t^2(c_2t + c_1)$$

9.18 problem 24

Internal problem ID [670]

Book: Elementary differential equations and boundary value problems, 10th ed., Boyce and DiPrima
Section: Chapter 3, Second order linear equations, 3.4 Repeated roots, reduction of order , page 172
Problem number: 24.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_Emden, _Fowler]]`

$$\boxed{t^2y'' + 2ty' - 2y = 0}$$

Given that one solution of the ode is

$$y_1 = t$$

✓ Solution by Maple
Time used: 0.0 (sec). Leaf size: 15

```
dsolve([t^2*diff(y(t),t$2)+2*t*diff(y(t),t)-2*y(t)=0,t],singsol=all)
```

$$y(t) = \frac{c_1 t^3 + c_2}{t^2}$$

✓ Solution by Mathematica
Time used: 0.01 (sec). Leaf size: 16

```
DSolve[t^2*y''[t]+2*t*y'[t]-2*y[t]==0,y[t],t,IncludeSingularSolutions -> True]
```

$$y(t) \to \frac{c_1}{t^2} + c_2 t$$

9.19 problem 25

Internal problem ID [671]

Book: Elementary differential equations and boundary value problems, 10th ed., Boyce and DiPrima
Section: Chapter 3, Second order linear equations, 3.4 Repeated roots, reduction of order , page 172
Problem number: 25.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _exact, _linear, _homogeneous]]`

$$\boxed{t^2y'' + 3ty' + y = 0}$$

Given that one solution of the ode is

$$y_1 = \frac{1}{t}$$

✓ Solution by Maple
Time used: 0.0 (sec). Leaf size: 14

```
dsolve([t^2*diff(y(t),t$2)+3*t*diff(y(t),t)+y(t)=0,1/t],singsol=all)
```

$$y(t) = \frac{c_2 \ln (t) + c_1}{t}$$

✓ Solution by Mathematica
Time used: 0.016 (sec). Leaf size: 17

```
DSolve[t^2*y''[t]+3*t*y'[t]+y[t]==0,y[t],t,IncludeSingularSolutions -> True]
```

$$y(t) \to \frac{c_2 \log(t) + c_1}{t}$$

9.20 problem 26

Internal problem ID [672]

Book: Elementary differential equations and boundary value problems, 10th ed., Boyce and DiPrima
Section: Chapter 3, Second order linear equations, 3.4 Repeated roots, reduction of order , page 172
Problem number: 26.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$\boxed{t^2y'' - t(2+t)\,y' + (2+t)\,y = 0}$$

Given that one solution of the ode is

$$y_1 = t$$

✓ Solution by Maple
Time used: 0.016 (sec). Leaf size: 12

```
dsolve([t^2*diff(y(t),t$2)-t*(t+2)*diff(y(t),t)+(t+2)*y(t)=0,t],singsol=all)
```

$$y(t) = t\left(c_1 + c_2 e^t\right)$$

✓ Solution by Mathematica
Time used: 0.025 (sec). Leaf size: 16

```
DSolve[t^2*y''[t]-t*(t+2)*y'[t]+(t+2)*y[t]==0,y[t],t,IncludeSingularSolutions -> True]
```

$$y(t) \to t\left(c_2 e^t + c_1\right)$$

9.21 problem 27

Internal problem ID [673]

Book: Elementary differential equations and boundary value problems, 10th ed., Boyce and DiPrima
Section: Chapter 3, Second order linear equations, 3.4 Repeated roots, reduction of order , page 172
Problem number: 27.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_Emden, _Fowler], [_2nd_order, _linear, '_with_symmetry_[0,F`

$$\boxed{xy'' - y' + 4yx^3 = 0}$$

Given that one solution of the ode is

$$y_1 = \sin(x^2)$$

✓ Solution by Maple
Time used: 0.0 (sec). Leaf size: 17

```
dsolve([x*diff(y(x),x$2)-diff(y(x),x)+4*x^3*y(x)=0,sin(x^2)],singsol=all)
```

$$y(x) = c_1 \sin(x^2) + c_2 \cos(x^2)$$

✓ Solution by Mathematica
Time used: 0.019 (sec). Leaf size: 20

```
DSolve[x*y''[x]-y'[x]+4*x^3*y[x]==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to c_1 \cos(x^2) + c_2 \sin(x^2)$$

9.22 problem 28

Internal problem ID [674]

Book: Elementary differential equations and boundary value problems, 10th ed., Boyce and DiPrima
Section: Chapter 3, Second order linear equations, 3.4 Repeated roots, reduction of order , page 172
Problem number: 28.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$\boxed{(x-1)\,y''-y'x+y=0}$$

Given that one solution of the ode is

$$y_1 = \mathrm{e}^{x}$$

✓ Solution by Maple
Time used: 0.015 (sec). Leaf size: 12

```
dsolve([(x-1)*diff(y(x),x$2)-x*diff(y(x),x)+y(x)=0,exp(x)],singsol=all)
```

$$y(x) = c_1x + \mathrm{e}^{x}c_2$$

✓ Solution by Mathematica
Time used: 0.031 (sec). Leaf size: 17

```
DSolve[(x-1)*y''[x]-x*y'[x]+y[x]==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to c_1e^x - c_2x$$

9.23 problem 29

Internal problem ID [675]

Book: Elementary differential equations and boundary value problems, 10th ed., Boyce and DiPrima
Section: Chapter 3, Second order linear equations, 3.4 Repeated roots, reduction of order , page 172
Problem number: 29.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$\boxed{x^2y'' - \left(x - \frac{3}{16}\right)y = 0}$$

Given that one solution of the ode is

$$y_1 = x^{\frac{1}{4}} e^{2\sqrt{x}}$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 25

```
dsolve([x^2*diff(y(x),x$2)-(x-1875/10000)*y(x)=0,x^(1/4)*exp(2*sqrt(x))],singsol=all)
```

$$y(x) = x^{\frac{1}{4}}\left(c_1 \sinh\left(2\sqrt{x}\right) + c_2 \cosh\left(2\sqrt{x}\right)\right)$$

✓ Solution by Mathematica

Time used: 0.047 (sec). Leaf size: 41

```
DSolve[x^2*y''[x]-(x-1875/10000)*y[x]==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to \frac{1}{2}e^{-2\sqrt{x}}\sqrt[4]{x}\left(2c_1e^{4\sqrt{x}} - c_2\right)$$

9.24 problem 30

Internal problem ID [676]

Book: Elementary differential equations and boundary value problems, 10th ed., Boyce and DiPrima
Section: Chapter 3, Second order linear equations, 3.4 Repeated roots, reduction of order , page 172
Problem number: 30.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$\boxed{x^2y'' + y'x + \left(x^2 - \frac{1}{4}\right)y = 0}$$

Given that one solution of the ode is

$$y_1 = \frac{\sin(x)}{\sqrt{x}}$$

✓ Solution by Maple

Time used: 0.016 (sec). Leaf size: 17

```
dsolve([x^2*diff(y(x),x$2)+x*diff(y(x),x)+(x^2-25/100)*y(x)=0,x^(-1/2)*sin(x)],singsol=all)
```

$$y(x) = \frac{c_1 \sin(x) + c_2 \cos(x)}{\sqrt{x}}$$

✓ Solution by Mathematica

Time used: 0.042 (sec). Leaf size: 39

```
DSolve[x^2*y''[x]+x*y'[x]+(x^2-25/100)*y[x]==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to \frac{e^{-ix}(2c_1 - ic_2e^{2ix})}{2\sqrt{x}}$$

9.25 problem 40

Internal problem ID [677]

Book: Elementary differential equations and boundary value problems, 10th ed., Boyce and DiPrima
Section: Chapter 3, Second order linear equations, 3.4 Repeated roots, reduction of order , page 172
Problem number: 40.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_Emden, _Fowler], [_2nd_order, _linear, '_with_symmetry_[0,F`

$$\boxed{t^2y'' - 3ty' + 4y = 0}$$

✓ Solution by Maple
Time used: 0.0 (sec). Leaf size: 14

```
dsolve(t^2*diff(y(t),t$2)-3*t*diff(y(t),t)+4*y(t)=0,y(t), singsol=all)
```

$$y(t) = t^2(c_2 \ln(t) + c_1)$$

✓ Solution by Mathematica
Time used: 0.016 (sec). Leaf size: 18

```
DSolve[t^2*y''[t]-3*t*y'[t]+4*y[t]==0,y[t],t,IncludeSingularSolutions -> True]
```

$$y(t) \to t^2(2c_2 \log(t) + c_1)$$

9.26 problem 41

Internal problem ID [678]

Book: Elementary differential equations and boundary value problems, 10th ed., Boyce and DiPrima
Section: Chapter 3, Second order linear equations, 3.4 Repeated roots, reduction of order , page 172
Problem number: 41.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_Emden, _Fowler]]`

$$\boxed{t^2y'' + 2ty' + \frac{y}{4} = 0}$$

✓ Solution by Maple
Time used: 0.0 (sec). Leaf size: 14

```
dsolve(t^2*diff(y(t),t$2)+2*t*diff(y(t),t)+25/100*y(t)=0,y(t), singsol=all)
```

$$y(t) = \frac{c_2 \ln(t) + c_1}{\sqrt{t}}$$

✓ Solution by Mathematica
Time used: 0.019 (sec). Leaf size: 24

```
DSolve[t^2*y''[t]+2*t*y'[t]+25/100*y[t]==0,y[t],t,IncludeSingularSolutions -> True]
```

$$y(t) \to \frac{c_2 \log(t) + 2c_1}{2\sqrt{t}}$$

9.27 problem 42

Internal problem ID [679]

Book: Elementary differential equations and boundary value problems, 10th ed., Boyce and DiPrima
Section: Chapter 3, Second order linear equations, 3.4 Repeated roots, reduction of order , page 172
Problem number: 42.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_Emden, _Fowler]]`

$$\boxed{2t^2y'' - 5ty' + 5y = 0}$$

✓ Solution by Maple
Time used: 0.0 (sec). Leaf size: 13

```
dsolve(2*t^2*diff(y(t),t$2)-5*t*diff(y(t),t)+5*y(t)=0,y(t), singsol=all)
```

$$y(t) = c_1 t + c_2 t^{\frac{5}{2}}$$

✓ Solution by Mathematica
Time used: 0.011 (sec). Leaf size: 18

```
DSolve[2*t^2*y''[t]-5*t*y'[t]+5*y[t]==0,y[t],t,IncludeSingularSolutions -> True]
```

$$y(t) \to t\left(c_2 t^{3/2} + c_1\right)$$

9.28 problem 43

Internal problem ID [680]

Book: Elementary differential equations and boundary value problems, 10th ed., Boyce and DiPrima
Section: Chapter 3, Second order linear equations, 3.4 Repeated roots, reduction of order , page 172
Problem number: 43.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _exact, _linear, _homogeneous]]`

$$\boxed{t^2y'' + 3ty' + y = 0}$$

✓ Solution by Maple
Time used: 0.015 (sec). Leaf size: 14

```
dsolve(t^2*diff(y(t),t$2)+3*t*diff(y(t),t)+y(t)=0,y(t), singsol=all)
```

$$y(t) = \frac{c_2 \ln(t) + c_1}{t}$$

✓ Solution by Mathematica
Time used: 0.015 (sec). Leaf size: 17

```
DSolve[t^2*y''[t]+3*t*y'[t]+y[t]==0,y[t],t,IncludeSingularSolutions -> True]
```

$$y(t) \to \frac{c_2 \log(t) + c_1}{t}$$

9.29 problem 44

Internal problem ID [681]

Book: Elementary differential equations and boundary value problems, 10th ed., Boyce and DiPrima
Section: Chapter 3, Second order linear equations, 3.4 Repeated roots, reduction of order , page 172
Problem number: 44.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_Emden, _Fowler]]`

$$\boxed{4t^2y'' - 8ty' + 9y = 0}$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 14

```
dsolve(4*t^2*diff(y(t),t$2)-8*t*diff(y(t),t)+9*y(t)=0,y(t), singsol=all)
```

$$y(t) = (c_2 \ln(t) + c_1) t^{\frac{3}{2}}$$

✓ Solution by Mathematica

Time used: 0.016 (sec). Leaf size: 25

```
DSolve[4*t^2*y''[t]-8*t*y'[t]+9*y[t]==0,y[t],t,IncludeSingularSolutions -> True]
```

$$y(t) \rightarrow \frac{1}{2}t^{3/2}(3c_2 \log(t) + 2c_1)$$

9.30 problem 45

Internal problem ID [682]

Book: Elementary differential equations and boundary value problems, 10th ed., Boyce and DiPrima
Section: Chapter 3, Second order linear equations, 3.4 Repeated roots, reduction of order , page 172
Problem number: 45.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_Emden, _Fowler]]`

$$\boxed{t^2y'' + 5ty' + 13y = 0}$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 23

```
dsolve(t^2*diff(y(t),t$2)+5*t*diff(y(t),t)+13*y(t)=0,y(t), singsol=all)
```

$$y(t) = \frac{c_1 \sin(3\ln(t)) + c_2 \cos(3\ln(t))}{t^2}$$

✓ Solution by Mathematica

Time used: 0.024 (sec). Leaf size: 26

```
DSolve[t^2*y''[t]+5*t*y'[t]+13*y[t]==0,y[t],t,IncludeSingularSolutions -> True]
```

$$y(t) \to \frac{c_2 \cos(3\log(t)) + c_1 \sin(3\log(t))}{t^2}$$

10 Chapter 3, Second order linear equations, section 3.6, Variation of Parameters. page 190

10.1 problem 1

Internal problem ID [683]

Book: Elementary differential equations and boundary value problems, 10th ed., Boyce and DiPrima
Section: Chapter 3, Second order linear equations, section 3.6, Variation of Parameters. page 190
Problem number: 1.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$\boxed{y'' - 5y' + 6y = 2\,\mathrm{e}^{t}}$$

✓ Solution by Maple
Time used: 0.0 (sec). Leaf size: 19

```
dsolve(diff(y(t),t$2)-5*diff(y(t),t)+6*y(t) = 2*exp(t),y(t), singsol=all)
```

$$y(t) = c_2\mathrm{e}^{2t} + c_1\mathrm{e}^{3t} + \mathrm{e}^{t}$$

✓ Solution by Mathematica
Time used: 0.015 (sec). Leaf size: 25

```
DSolve[y''[t]-5*y'[t]+6*y[t] == 2*Exp[t],y[t],t,IncludeSingularSolutions -> True]
```

$$y(t) \to e^{t}\left(c_1e^{t} + c_2e^{2t} + 1\right)$$

10.2 problem 2

Internal problem ID [684]

Book: Elementary differential equations and boundary value problems, 10th ed., Boyce and DiPrima
Section: Chapter 3, Second order linear equations, section 3.6, Variation of Parameters. page 190
Problem number: 2.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$\boxed{y'' - y' - 2y = 2\,\mathrm{e}^{-t}}$$

✓ Solution by Maple
Time used: 0.0 (sec). Leaf size: 24

```
dsolve(diff(y(t),t$2)-diff(y(t),t)-2*y(t) = 2*exp(-t),y(t), singsol=all)
```

$$y(t) = \frac{(-2t + 3c_1)\,\mathrm{e}^{-t}}{3} + c_2\mathrm{e}^{2t}$$

✓ Solution by Mathematica
Time used: 0.035 (sec). Leaf size: 32

```
DSolve[y''[t]-y'[t]-2*y[t] == 2*Exp[-t],y[t],t,IncludeSingularSolutions -> True]
```

$$y(t) \to \frac{1}{9}e^{-t}\left(-6t + 9c_2e^{3t} - 2 + 9c_1\right)$$

10.3 problem 3

Internal problem ID [685]

Book: Elementary differential equations and boundary value problems, 10th ed., Boyce and DiPrima
Section: Chapter 3, Second order linear equations, section 3.6, Variation of Parameters. page 190
Problem number: 3.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$\boxed{y'' + 2y' + y = 3\,\mathrm{e}^{-t}}$$

✓ Solution by Maple
Time used: 0.0 (sec). Leaf size: 19

```
dsolve(diff(y(t),t$2)+2*diff(y(t),t)+y(t) = 3*exp(-t),y(t), singsol=all)
```

$$y(t) = \mathrm{e}^{-t}\left(c_2 + c_1 t + \frac{3}{2}t^2\right)$$

✓ Solution by Mathematica
Time used: 0.024 (sec). Leaf size: 29

```
DSolve[y''[t]+2*y'[t]+y[t] == 3*Exp[-t],y[t],t,IncludeSingularSolutions -> True]
```

$$y(t) \to \frac{1}{2}e^{-t}\left(3t^2 + 2c_2 t + 2c_1\right)$$

10.4 problem 4

Internal problem ID [686]

Book: Elementary differential equations and boundary value problems, 10th ed., Boyce and DiPrima
Section: Chapter 3, Second order linear equations, section 3.6, Variation of Parameters. page 190
Problem number: 4.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$4y'' - 4y' + y = 16\,\mathrm{e}^{\frac{t}{2}}$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 19

```
dsolve(4*diff(y(t),t$2)-4*diff(y(t),t)+y(t) = 16*exp(t/2),y(t), singsol=all)
```

$$y(t) = \mathrm{e}^{\frac{t}{2}}\left(c_1 t + 2t^2 + c_2\right)$$

✓ Solution by Mathematica

Time used: 0.021 (sec). Leaf size: 25

```
DSolve[4*y''[t]-4*y'[t]+y[t]== 16*Exp[t/2],y[t],t,IncludeSingularSolutions -> True]
```

$$y(t) \to e^{t/2}\left(2t^2 + c_2 t + c_1\right)$$

10.5 problem 5

Internal problem ID [687]

Book: Elementary differential equations and boundary value problems, 10th ed., Boyce and DiPrima
Section: Chapter 3, Second order linear equations, section 3.6, Variation of Parameters. page 190
Problem number: 5.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _linear, _nonhomogeneous]]`

$$\boxed{y'' + y = \tan(t)}$$

✓ Solution by Maple
Time used: 0.016 (sec). Leaf size: 23

```
dsolve(diff(y(t),t$2)+y(t) = tan(t),y(t), singsol=all)
```

$$y(t) = c_2 \sin(t) + \cos(t) c_1 - \cos(t) \ln(\sec(t) + \tan(t))$$

✓ Solution by Mathematica
Time used: 0.04 (sec). Leaf size: 23

```
DSolve[y''[t]+y[t] == Tan[t],y[t],t,IncludeSingularSolutions -> True]
```

$$y(t) \to \cos(t)(-\operatorname{arctanh}(\sin(t))) + c_1 \cos(t) + c_2 \sin(t)$$

10.6 problem 6

Internal problem ID [688]

Book: Elementary differential equations and boundary value problems, 10th ed., Boyce and DiPrima
Section: Chapter 3, Second order linear equations, section 3.6, Variation of Parameters. page 190
Problem number: 6.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _linear, _nonhomogeneous]]`

$$\boxed{y'' + 9y = 9\sec(3t)^2}$$

✓ Solution by Maple
Time used: 0.0 (sec). Leaf size: 33

```
dsolve(diff(y(t),t$2)+9*y(t) = 9*sec(3*t)^2,y(t), singsol=all)
```

$$y(t) = c_2 \sin(3t) + c_1 \cos(3t) + \ln(\sec(3t) + \tan(3t)) \sin(3t) - 1$$

✓ Solution by Mathematica
Time used: 0.117 (sec). Leaf size: 31

```
DSolve[y''[t]+9*y[t] == 9*Sec[3*t]^2,y[t],t,IncludeSingularSolutions -> True]
```

$$y(t) \to c_1 \cos(3t) + \sin(3t) \coth^{-1}(\sin(3t)) + c_2 \sin(3t) - 1$$

10.7 problem 7

Internal problem ID [689]

Book: Elementary differential equations and boundary value problems, 10th ed., Boyce and DiPrima
Section: Chapter 3, Second order linear equations, section 3.6, Variation of Parameters. page 190
Problem number: 7.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _linear, _nonhomogeneous]]`

$$y'' + 4y' + 4y = \frac{e^{-2t}}{t^2}$$

✓ Solution by Maple
Time used: 0.0 (sec). Leaf size: 19

```
dsolve(diff(y(t),t$2)+4*diff(y(t),t)+4*y(t) = t^(-2)*exp(-2*t),y(t), singsol=all)
```

$$y(t) = e^{-2t}(-1 + c_1 t - \ln(t) + c_2)$$

✓ Solution by Mathematica
Time used: 0.028 (sec). Leaf size: 23

```
DSolve[y''[t]+4*y'[t]+4*y[t] == t^(-2)*Exp[-2*t],y[t],t,IncludeSingularSolutions -> True]
```

$$y(t) \to e^{-2t}(-\log(t) + c_2 t - 1 + c_1)$$

10.8 problem 8

Internal problem ID [690]

Book: Elementary differential equations and boundary value problems, 10th ed., Boyce and DiPrima
Section: Chapter 3, Second order linear equations, section 3.6, Variation of Parameters. page 190
Problem number: 8.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _linear, _nonhomogeneous]]`

$$\boxed{y'' + 4y = 3\csc(2t)}$$

✓ Solution by Maple
Time used: 0.0 (sec). Leaf size: 35

```
dsolve(diff(y(t),t$2)+4*y(t) = 3*csc(2*t),y(t), singsol=all)
```

$$y(t) = -\frac{3\ln(\csc(2t))\sin(2t)}{4} + \frac{(-6t + 4c_1)\cos(2t)}{4} + c_2\sin(2t)$$

✓ Solution by Mathematica
Time used: 0.036 (sec). Leaf size: 39

```
DSolve[y''[t]+4*y[t] ==3*Csc[2*t],y[t],t,IncludeSingularSolutions -> True]
```

$$y(t) \to \left(-\frac{3t}{2} + c_1\right)\cos(2t) + \frac{1}{4}\sin(2t)(3\log(\sin(2t)) + 4c_2)$$

10.9 problem 9

Internal problem ID [691]

Book: Elementary differential equations and boundary value problems, 10th ed., Boyce and DiPrima
Section: Chapter 3, Second order linear equations, section 3.6, Variation of Parameters. page 190
Problem number: 9.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _linear, _nonhomogeneous]]`

$$\boxed{y'' + y = 2\sec\left(\frac{t}{2}\right)}$$

✓ Solution by Maple
Time used: 0.0 (sec). Leaf size: 33

```
dsolve(diff(y(t),t$2)+y(t) = 2*sec(t/2),y(t), singsol=all)
```

$$y(t) = -4\sin(t)\ln\left(\sec\left(\frac{t}{2}\right) + \tan\left(\frac{t}{2}\right)\right) + c_2\sin(t) + \cos(t)\, c_1 + 8\cos\left(\frac{t}{2}\right)$$

✓ Solution by Mathematica
Time used: 0.114 (sec). Leaf size: 35

```
DSolve[y''[t]+y[t]== 2*Sec[t/2],y[t],t,IncludeSingularSolutions -> True]
```

$$y(t) \to -4\sin(t)\text{arctanh}\left(\sin\left(\frac{t}{2}\right)\right) + 8\cos\left(\frac{t}{2}\right) + c_1\cos(t) + c_2\sin(t)$$

10.10 problem 10

Internal problem ID [692]

Book: Elementary differential equations and boundary value problems, 10th ed., Boyce and DiPrima
Section: Chapter 3, Second order linear equations, section 3.6, Variation of Parameters. page 190
Problem number: 10.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _linear, _nonhomogeneous]]`

$$\boxed{y'' - 2y' + y = \frac{e^t}{t^2 + 1}}$$

✓ Solution by Maple
Time used: 0.015 (sec). Leaf size: 24

```
dsolve(diff(y(t),t$2)-2*diff(y(t),t)+y(t) = exp(t)/(1+t^2),y(t), singsol=all)
```

$$y(t) = e^t \left(c_2 + c_1 t - \frac{\ln(t^2 + 1)}{2} + t \arctan(t)\right)$$

✓ Solution by Mathematica
Time used: 0.029 (sec). Leaf size: 35

```
DSolve[y''[t]-2*y'[t]+y[t] == Exp[t]/(1+t^2),y[t],t,IncludeSingularSolutions -> True]
```

$$y(t) \to \frac{1}{2} e^t \left(2t \arctan(t) - \log\left(t^2 + 1\right) + 2(c_2 t + c_1)\right)$$

10.11 problem 11

Internal problem ID [693]

Book: Elementary differential equations and boundary value problems, 10th ed., Boyce and DiPrima
Section: Chapter 3, Second order linear equations, section 3.6, Variation of Parameters. page 190
Problem number: 11.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _linear, _nonhomogeneous]]`

$$\boxed{y'' - 5y' + 6y = g(t)}$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 46

```
dsolve(diff(y(t),t$2)-5*diff(y(t),t)+6*y(t) = g(t),y(t), singsol=all)
```

$$y(t) = c_2 e^{2t} + c_1 e^{3t} - \left(\int g(t)\,e^{-2t}dt\right)e^{2t} + \left(\int g(t)\,e^{-3t}dt\right)e^{3t}$$

✓ Solution by Mathematica

Time used: 0.064 (sec). Leaf size: 59

```
DSolve[y''[t]-5*y'[t]+6*y[t] == g[t],y[t],t,IncludeSingularSolutions -> True]
```

$$y(t) \to e^{2t}\left(\int_1^t -e^{-2K[1]}g(K[1])dK[1] + e^t \int_1^t e^{-3K[2]}g(K[2])dK[2] + c_2 e^t + c_1\right)$$

10.12 problem 12

Internal problem ID [694]

Book: Elementary differential equations and boundary value problems, 10th ed., Boyce and DiPrima
Section: Chapter 3, Second order linear equations, section 3.6, Variation of Parameters. page 190
Problem number: 12.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _linear, _nonhomogeneous]]`

$$\boxed{y'' + 4y = g(t)}$$

✓ Solution by Maple
Time used: 0.0 (sec). Leaf size: 47

```
dsolve(diff(y(t),t$2)+4*y(t) = g(t),y(t), singsol=all)
```

$$y(t) = c_2 \sin(2t) + c_1 \cos(2t) + \frac{\left(\int \cos(2t)\, g(t)\, dt\right) \sin(2t)}{2} - \frac{\left(\int \sin(2t)\, g(t)\, dt\right) \cos(2t)}{2}$$

✓ Solution by Mathematica
Time used: 0.093 (sec). Leaf size: 67

```
DSolve[y''[t]+4*y[t] == g[t],y[t],t,IncludeSingularSolutions -> True]
```

$$\begin{aligned} y(t) \to \cos(2t) &\int_1^t -\cos(K[1])g(K[1])\sin(K[1])dK[1] \\ &+ \sin(2t)\int_1^t \frac{1}{2}\cos(2K[2])g(K[2])dK[2] + c_1\cos(2t) + c_2\sin(2t) \end{aligned}$$

10.13 problem 13

Internal problem ID [695]

Book: Elementary differential equations and boundary value problems, 10th ed., Boyce and DiPrima
Section: Chapter 3, Second order linear equations, section 3.6, Variation of Parameters. page 190
Problem number: 13.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _exact, _linear, _nonhomogeneous]]`

$$\boxed{t^2y'' - 2y = 3t^2 - 1}$$

✓ Solution by Maple
Time used: 0.0 (sec). Leaf size: 22

```
dsolve(t^2*diff(y(t),t$2)-2*y(t) = 3*t^2-1,y(t), singsol=all)
```

$$y(t) = t^2c_2 + \frac{1}{2} + t^2 \ln(t) + \frac{c_1}{t}$$

✓ Solution by Mathematica
Time used: 0.015 (sec). Leaf size: 31

```
DSolve[t^2*y''[t]-2*y[t] == 3*t^2-1,y[t],t,IncludeSingularSolutions -> True]
```

$$y(t) \to t^2 \log(t) + \left(-\frac{1}{3} + c_2\right) t^2 + \frac{c_1}{t} + \frac{1}{2}$$

10.14 problem 14

Internal problem ID [696]

Book: Elementary differential equations and boundary value problems, 10th ed., Boyce and DiPrima
Section: Chapter 3, Second order linear equations, section 3.6, Variation of Parameters. page 190
Problem number: 14.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$\boxed{t^2 y'' - t(2+t)\, y' + (2+t)\, y = 2t^3}$$

✓ Solution by Maple
Time used: 0.281 (sec). Leaf size: 15

```
dsolve(t^2*diff(y(t),t$2)-t*(t+2)*diff(y(t),t)+(t+2)*y(t) = 2*t^3,y(t), singsol=all)
```

$$y(t) = t\left(e^t c_1 + c_2 - 2t\right)$$

✓ Solution by Mathematica
Time used: 0.025 (sec). Leaf size: 20

```
DSolve[t^2*y''[t]-t*(t+2)*y'[t]+(t+2)*y[t] == 2*t^3,y[t],t,IncludeSingularSolutions -> True]
```

$$y(t) \to t\left(-2t + c_2 e^t - 2 + c_1\right)$$

10.15 problem 15

Internal problem ID [697]

Book: Elementary differential equations and boundary value problems, 10th ed., Boyce and DiPrima
Section: Chapter 3, Second order linear equations, section 3.6, Variation of Parameters. page 190
Problem number: 15.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$\boxed{ty'' - (t+1)\, y' + y = e^{2t} t^2}$$

✓ Solution by Maple
Time used: 0.0 (sec). Leaf size: 23

```
dsolve(t*diff(y(t),t$2)-(1+t)*diff(y(t),t)+y(t) = t^2*exp(2*t),y(t), singsol=all)
```

$$y(t) = (t+1)\, c_2 + e^t c_1 + \frac{(t-1)\, e^{2t}}{2}$$

✓ Solution by Mathematica
Time used: 0.052 (sec). Leaf size: 31

```
DSolve[t*y''[t]-(1+t)*y'[t]+y[t] == t^2*Exp[2*t],y[t],t,IncludeSingularSolutions -> True]
```

$$y(t) \to \frac{1}{2} e^{2t}(t-1) + c_1 e^t - c_2(t+1)$$

10.16 problem 16

Internal problem ID [698]

Book: Elementary differential equations and boundary value problems, 10th ed., Boyce and DiPrima
Section: Chapter 3, Second order linear equations, section 3.6, Variation of Parameters. page 190
Problem number: 16.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$\boxed{(1-t)\, y'' + ty' - y = 2(-1+t)^2\, \mathrm{e}^{-t}}$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 25

```
dsolve((1-t)*diff(y(t),t$2)+t*diff(y(t),t)-y(t) = 2*(t-1)^2*exp(-t),y(t), singsol=all)
```

$$y(t) = c_2 t + \mathrm{e}^t c_1 - t\, \mathrm{e}^{-t} + \frac{\mathrm{e}^{-t}}{2}$$

✓ Solution by Mathematica

Time used: 0.07 (sec). Leaf size: 30

```
DSolve[(1-t)*y''[t]+t*y'[t]-y[t] == 2*(t-1)^2*Exp[-t],y[t],t,IncludeSingularSolutions -> Tru
```

$$y(t) \to e^{-t}\left(\frac{1}{2} - t\right) + c_1 e^t - c_2 t$$

10.17 problem 17

Internal problem ID [699]

Book: Elementary differential equations and boundary value problems, 10th ed., Boyce and DiPrima
Section: Chapter 3, Second order linear equations, section 3.6, Variation of Parameters. page 190
Problem number: 17.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _linear, _nonhomogeneous]]`

$$\boxed{x^2y'' - 3y'x + 4y = \ln(x)\, x^2}$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 20

```
dsolve(x^2*diff(y(x),x$2)-3*x*diff(y(x),x)+4*y(x) = x^2*ln(x),y(x), singsol=all)
```

$$y(x) = x^2\left(c_2 + \ln(x)\, c_1 + \frac{\ln(x)^3}{6}\right)$$

✓ Solution by Mathematica

Time used: 0.027 (sec). Leaf size: 27

```
DSolve[x^2*y''[x]-3*x*y'[x]+4*y[x] == x^2*Log[x],y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to \frac{1}{6}x^2\left(\log^3(x) + 12c_2\log(x) + 6c_1\right)$$

10.18 problem 20

Internal problem ID [700]

Book: Elementary differential equations and boundary value problems, 10th ed., Boyce and DiPrima
Section: Chapter 3, Second order linear equations, section 3.6, Variation of Parameters. page 190
Problem number: 20.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _linear, _nonhomogeneous]]`

$$x^2y'' + y'x + \left(x^2 - \frac{1}{4}\right)y = g(x)$$

✓ Solution by Maple
Time used: 0.015 (sec). Leaf size: 44

```
dsolve(x^2*diff(y(x),x$2)+x*diff(y(x),x)+(x^2-25/100)*y(x) = g(x),y(x), singsol=all)
```

$$y(x) = \frac{\sin(x)\, c_2 + \cos(x)\, c_1 + \left(\int \frac{\cos(x)g(x)}{x^{\frac{3}{2}}}dx\right)\sin(x) - \left(\int \frac{\sin(x)g(x)}{x^{\frac{3}{2}}}dx\right)\cos(x)}{\sqrt{x}}$$

✓ Solution by Mathematica
Time used: 0.203 (sec). Leaf size: 107

```
DSolve[x^2*y''[x]+x*y'[x]+(x^2-25/100)*y[x] == g[x],y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to \frac{e^{-ix}\left(2\int_1^x \frac{ie^{iK[1]}g(K[1])}{2K[1]^{3/2}}dK[1] - ie^{2ix}\int_1^x \frac{e^{-iK[2]}g(K[2])}{K[2]^{3/2}}dK[2] - ic_2e^{2ix} + 2c_1\right)}{2\sqrt{x}}$$

10.19 problem 29

Internal problem ID [701]

Book: Elementary differential equations and boundary value problems, 10th ed., Boyce and DiPrima
Section: Chapter 3, Second order linear equations, section 3.6, Variation of Parameters. page 190
Problem number: 29.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$\boxed{t^2y'' - 2ty' + 2y = 4t^2}$$

✓ Solution by Maple
Time used: 0.015 (sec). Leaf size: 18

```
dsolve(t^2*diff(y(t),t$2)-2*t*diff(y(t),t)+2*y(t) = 4*t^2,y(t), singsol=all)
```

$$y(t) = t(4t\ln(t) + (c_1 - 4)t + c_2)$$

✓ Solution by Mathematica
Time used: 0.013 (sec). Leaf size: 21

```
DSolve[t^2*y''[t]-2*t*y'[t]+2*y[t] ==4*t^2,y[t],t,IncludeSingularSolutions -> True]
```

$$y(t) \to t(4t\log(t) + (-4 + c_2)t + c_1)$$

10.20 problem 30

Internal problem ID [702]

Book: Elementary differential equations and boundary value problems, 10th ed., Boyce and DiPrima
Section: Chapter 3, Second order linear equations, section 3.6, Variation of Parameters. page 190
Problem number: 30.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _exact, _linear, _nonhomogeneous]]`

$$\boxed{t^2y'' + 7ty' + 5y = t}$$

✓ Solution by Maple
Time used: 0.0 (sec). Leaf size: 27

```
dsolve(t^2*diff(y(t),t$2)+7*t*diff(y(t),t)+5*y(t) = t,y(t), singsol=all)
```

$$y(t) = \frac{t^6 + 3c_1t^4 - 4c_1^3 + 12c_2}{12t^5}$$

✓ Solution by Mathematica
Time used: 0.015 (sec). Leaf size: 23

```
DSolve[t^2*y''[t]+7*t*y'[t]+5*y[t] ==t,y[t],t,IncludeSingularSolutions -> True]
```

$$y(t) \to \frac{c_1}{t^5} + \frac{t}{12} + \frac{c_2}{t}$$

10.21 problem 31

Internal problem ID [703]

Book: Elementary differential equations and boundary value problems, 10th ed., Boyce and DiPrima
Section: Chapter 3, Second order linear equations, section 3.6, Variation of Parameters. page 190
Problem number: 31.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$\boxed{ty'' - (t+1)\,y' + y = \mathrm{e}^{2t}t^2}$$

✓ Solution by Maple
Time used: 0.0 (sec). Leaf size: 23

```
dsolve(t*diff(y(t),t$2)-(1+t)*diff(y(t),t)+y(t) = t^2*exp(2*t),y(t), singsol=all)
```

$$y(t) = (t+1)\,c_2 + \mathrm{e}^t c_1 + \frac{(t-1)\,\mathrm{e}^{2t}}{2}$$

✓ Solution by Mathematica
Time used: 0.033 (sec). Leaf size: 31

```
DSolve[t*y''[t]-(1+t)*y'[t]+y[t] ==t^2*Exp[2*t],y[t],t,IncludeSingularSolutions -> True]
```

$$y(t) \to \frac{1}{2}e^{2t}(t-1) + c_1 e^t - c_2(t+1)$$

10.22 problem 32

Internal problem ID [704]

Book: Elementary differential equations and boundary value problems, 10th ed., Boyce and DiPrima
Section: Chapter 3, Second order linear equations, section 3.6, Variation of Parameters. page 190
Problem number: 32.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$\boxed{(1-t)\, y'' + t y' - y = 2(-1+t)\, \mathrm{e}^{-t}}$$

✓ Solution by Maple
Time used: 0.016 (sec). Leaf size: 39

```
dsolve((1-t)*diff(y(t),t$2)+t*diff(y(t),t)-y(t) = 2*(t-1)*exp(-t),y(t), singsol=all)
```

$$y(t) = -2\,\mathrm{e}^{-1}\,\mathrm{expIntegral}_1\,(t-1)\,t + 2\,\mathrm{expIntegral}_1\,(2t-2)\,\mathrm{e}^{t-2} + \mathrm{e}^{t}c_1 + c_2 t + \mathrm{e}^{-t}$$

✓ Solution by Mathematica
Time used: 0.187 (sec). Leaf size: 47

```
DSolve[(1-t)*y''[t]+t*y'[t]-y[t] ==2*(t-1)*Exp[-t],y[t],t,IncludeSingularSolutions -> True]
```

$$y(t) \to -2e^{t-2}\,\mathrm{ExpIntegralEi}(2-2t) + \frac{2t\,\mathrm{ExpIntegralEi}(1-t)}{e} + e^{-t} + c_1 e^{t} - c_2 t$$

11 Chapter 3, Second order linear equations, 3.7 Mechanical and Electrical Vibrations. page 203

11.1 problem 28

Internal problem ID [705]

Book: Elementary differential equations and boundary value problems, 10th ed., Boyce and DiPrima
Section: Chapter 3, Second order linear equations, 3.7 Mechanical and Electrical Vibrations. page 203
Problem number: 28.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _missing_x]]`

$$\boxed{u'' + 2u = 0}$$

✓ Solution by Maple
Time used: 0.0 (sec). Leaf size: 21

```
dsolve(diff(u(t),t$2)+2*u(t) = 0,u(t), singsol=all)
```

$$u(t) = c_1 \sin\left(t\sqrt{2}\right) + c_2 \cos\left(t\sqrt{2}\right)$$

✓ Solution by Mathematica
Time used: 0.017 (sec). Leaf size: 28

```
DSolve[u''[t]+2*u[t] ==0,u[t],t,IncludeSingularSolutions -> True]
```

$$u(t) \to c_1 \cos\left(\sqrt{2}t\right) + c_2 \sin\left(\sqrt{2}t\right)$$

11.2 problem 29

Internal problem ID [706]

Book: Elementary differential equations and boundary value problems, 10th ed., Boyce and DiPrima
Section: Chapter 3, Second order linear equations, 3.7 Mechanical and Electrical Vibrations. page 203
Problem number: 29.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _missing_x]]`

$$\boxed{u'' + \frac{u'}{4} + 2u = 0}$$

With initial conditions

$$[u(0) = 0, u'(0) = 2]$$

✓ Solution by Maple

Time used: 0.016 (sec). Leaf size: 20

```
dsolve([diff(u(t),t$2)+1/4*diff(u(t),t)+2*u(t) = 0,u(0) = 0, D(u)(0) = 2],u(t), singsol=all)
```

$$u(t) = \frac{16\sqrt{127}\,\mathrm{e}^{-\frac{t}{8}} \sin\left(\frac{\sqrt{127}\,t}{8}\right)}{127}$$

✓ Solution by Mathematica

Time used: 0.025 (sec). Leaf size: 30

```
DSolve[{u''[t]+1/4*u'[t]+2*u[t] ==0,{u[0]==0,u'[0]==2}},u[t],t,IncludeSingularSolutions -> T
```

$$u(t) \to \frac{16e^{-t/8} \sin\left(\frac{\sqrt{127}t}{8}\right)}{\sqrt{127}}$$

12 Chapter 3, Second order linear equations, 3.7 Forced Vibrations. page 217

12.1 problem 21

Internal problem ID [707]

Book: Elementary differential equations and boundary value problems, 10th ed., Boyce and DiPrima
Section: Chapter 3, Second order linear equations, 3.7 Forced Vibrations. page 217
Problem number: 21.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _linear, _nonhomogeneous]]`

$$u'' + \frac{u'}{8} + 4u = 3\cos\left(\frac{t}{4}\right)$$

With initial conditions

$$[u(0) = 2, u'(0) = 0]$$

✓ Solution by Maple
Time used: 0.031 (sec). Leaf size: 46

```
dsolve([diff(u(t),t$2)+125/1000*diff(u(t),t)+4*u(t) = 3*cos(t/4),u(0) = 2, D(u)(0) = 0],u(t)
```

$$u(t) = \frac{19274\,\mathrm{e}^{-\frac{t}{16}}\sqrt{1023}\,\sin\left(\frac{\sqrt{1023}\,t}{16}\right)}{16242171} + \frac{19658\,\mathrm{e}^{-\frac{t}{16}}\cos\left(\frac{\sqrt{1023}\,t}{16}\right)}{15877} + \frac{96\sin\left(\frac{t}{4}\right)}{15877} + \frac{12096\cos\left(\frac{t}{4}\right)}{15877}$$

✓ Solution by Mathematica
Time used: 0.039 (sec). Leaf size: 71

```
DSolve[{u''[t]+125/1000*u'[t]+4*u[t] ==3*Cos[t/4],{u[0]==0,u'[0]==0}},u[t],t,IncludeSingular
```

$$u(t) \to \frac{32\left(1023\sin\left(\frac{t}{4}\right) - 130\sqrt{1023}e^{-t/16}\sin\left(\frac{\sqrt{1023}t}{16}\right) + 128898\cos\left(\frac{t}{4}\right) - 128898e^{-t/16}\cos\left(\frac{\sqrt{1023}t}{16}\right)\right)}{5414057}$$

12.2 problem 22

Internal problem ID [708]

Book: Elementary differential equations and boundary value problems, 10th ed., Boyce and DiPrima
Section: Chapter 3, Second order linear equations, 3.7 Forced Vibrations. page 217
Problem number: 22.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _linear, _nonhomogeneous]]`

$$\boxed{u'' + \frac{u'}{8} + 4u = 3\cos(2t)}$$

With initial conditions

$$[u(0) = 2, u'(0) = 0]$$

✓ Solution by Maple
Time used: 0.032 (sec). Leaf size: 40

```
dsolve([diff(u(t),t$2)+125/1000*diff(u(t),t)+4*u(t) = 3*cos(2*t),u(0) = 2, D(u)(0) = 0],u(t)
```

$$u(t) = -\frac{382\,\mathrm{e}^{-\frac{t}{16}}\sqrt{1023}\,\sin\left(\frac{\sqrt{1023}\,t}{16}\right)}{1023} + 2\,\mathrm{e}^{-\frac{t}{16}}\cos\left(\frac{\sqrt{1023}\,t}{16}\right) + 12\sin(2t)$$

✓ Solution by Mathematica
Time used: 0.03 (sec). Leaf size: 39

```
DSolve[{u''[t]+125/1000*u'[t]+4*u[t] ==3*Cos[2*t],{u[0]==0,u'[0]==0}},u[t],t,IncludeSingular
```

$$u(t) \to 12\sin(2t) - 128\sqrt{\frac{3}{341}}e^{-t/16}\sin\left(\frac{\sqrt{1023}t}{16}\right)$$

12.3 problem 23

Internal problem ID [709]

Book: Elementary differential equations and boundary value problems, 10th ed., Boyce and DiPrima
Section: Chapter 3, Second order linear equations, 3.7 Forced Vibrations. page 217
Problem number: 23.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _linear, _nonhomogeneous]]`

$$\boxed{u'' + \frac{u'}{8} + 4u = 3\cos(6t)}$$

With initial conditions

$$[u(0) = 2, u'(0) = 0]$$

✓ Solution by Maple

Time used: 0.031 (sec). Leaf size: 46

```
dsolve([diff(u(t),t$2)+125/1000*diff(u(t),t)+4*u(t) = 3*cos(6*t),u(0) = 2, D(u)(0) = 0],u(t)
```

$$u(t) = \frac{2806\,\mathrm{e}^{-\frac{t}{16}}\sqrt{1023}\,\sin\left(\frac{\sqrt{1023}\,t}{16}\right)}{1524549} + \frac{34322\,\mathrm{e}^{-\frac{t}{16}}\cos\left(\frac{\sqrt{1023}\,t}{16}\right)}{16393} + \frac{36\sin(6t)}{16393} - \frac{1536\cos(6t)}{16393}$$

✓ Solution by Mathematica

Time used: 0.032 (sec). Leaf size: 74

```
DSolve[{u''[t]+125/1000*u'[t]+4*u[t] ==3*Cos[6*t],{u[0]==0,u'[0]==0}},u[t],t,IncludeSingular
```

$$u(t) \to -\frac{4e^{-t/16}\left(-3069e^{t/16}\sin(6t) + 160\sqrt{1023}\sin\left(\frac{\sqrt{1023}t}{16}\right) + 130944e^{t/16}\cos(6t) - 130944\cos\left(\frac{\sqrt{1023}t}{16}\right)\right)}{5590013}$$

12.4 problem 24

Internal problem ID [710]

Book: Elementary differential equations and boundary value problems, 10th ed., Boyce and DiPrima
Section: Chapter 3, Second order linear equations, 3.7 Forced Vibrations. page 217
Problem number: 24.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[NONE]`

$$\boxed{u'' + u' + \frac{u^3}{5} = \cos(t)}$$

With initial conditions

$$[u(0) = 2, u'(0) = 0]$$

✗ Solution by Maple

```
dsolve([diff(u(t),t$2)+diff(u(t),t)+1/5*u(t)^3 = cos(t),u(0) = 2, D(u)(0) = 0],u(t), singsol
```

No solution found

✗ Solution by Mathematica
Time used: 0.0 (sec). Leaf size: 0

```
DSolve[{u''[t]+u'[t]+1/5*u[t]^3 ==3*Cos[t],{u[0]==0,u'[0]==0}},u[t],t,IncludeSingularSolutio
```

Not solved

13 Chapter 5.2, Series Solutions Near an Ordinary Point, Part I. page 263

13.1 problem 1

Internal problem ID [711]

Book: Elementary differential equations and boundary value problems, 10th ed., Boyce and DiPrima
Section: Chapter 5.2, Series Solutions Near an Ordinary Point, Part I. page 263
Problem number: 1.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _missing_x]]`

$$\boxed{y'' - y = 0}$$

With the expansion point for the power series method at $x = 0$.

✓ Solution by Maple
Time used: 0.0 (sec). Leaf size: 34

```
Order:=6;
dsolve(diff(y(x),x$2)-y(x)=0,y(x),type='series',x=0);
```

$$y(x) = \left(1 + \frac{1}{2}x^2 + \frac{1}{24}x^4\right) y(0) + \left(x + \frac{1}{6}x^3 + \frac{1}{120}x^5\right) D(y)(0) + O(x^6)$$

✓ Solution by Mathematica
Time used: 0.001 (sec). Leaf size: 42

```
AsymptoticDSolveValue[y''[x]-y[x]==0,y[x],{x,0,5}]
```

$$y(x) \to c_2\left(\frac{x^5}{120} + \frac{x^3}{6} + x\right) + c_1\left(\frac{x^4}{24} + \frac{x^2}{2} + 1\right)$$

13.2 problem 2

Internal problem ID [712]

Book: Elementary differential equations and boundary value problems, 10th ed., Boyce and DiPrima
Section: Chapter 5.2, Series Solutions Near an Ordinary Point, Part I. page 263
Problem number: 2.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _exact, _linear, _homogeneous]]`

$$\boxed{y'' - y'x - y = 0}$$

With the expansion point for the power series method at $x = 0$.

✓ Solution by Maple
Time used: 0.015 (sec). Leaf size: 34

```
Order:=6;
dsolve(diff(y(x),x$2)-x*diff(y(x),x)-y(x)=0,y(x),type='series',x=0);
```

$$y(x) = \left(1 + \frac{1}{2}x^2 + \frac{1}{8}x^4\right) y(0) + \left(x + \frac{1}{3}x^3 + \frac{1}{15}x^5\right) D(y)(0) + O(x^6)$$

✓ Solution by Mathematica
Time used: 0.001 (sec). Leaf size: 42

```
AsymptoticDSolveValue[y''[x]-x*y'[x]-y[x]==0,y[x],{x,0,5}]
```

$$y(x) \to c_2\left(\frac{x^5}{15} + \frac{x^3}{3} + x\right) + c_1\left(\frac{x^4}{8} + \frac{x^2}{2} + 1\right)$$

13.3 problem 4

Internal problem ID [713]

Book: Elementary differential equations and boundary value problems, 10th ed., Boyce and DiPrima
Section: Chapter 5.2, Series Solutions Near an Ordinary Point, Part I. page 263
Problem number: 4.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_Emden, _Fowler]]`

$$
y'' + k^2x^2y = 0
$$

With the expansion point for the power series method at $x = 0$.

✓ Solution by Maple
Time used: 0.0 (sec). Leaf size: 30

```
Order:=6;
dsolve(diff(y(x),x$2)+k^2*x^2*y(x)=0,y(x),type='series',x=0);
```

$$
y(x) = \left(1 - \frac{k^2x^4}{12}\right) y(0) + \left(x - \frac{1}{20}k^2x^5\right) D(y)(0) + O(x^6)
$$

✓ Solution by Mathematica
Time used: 0.001 (sec). Leaf size: 34

```
AsymptoticDSolveValue[y''[x]+k^2*x^2*y[x]==0,y[x],{x,0,5}]
```

$$
y(x) \to c_2\left(x - \frac{k^2x^5}{20}\right) + c_1\left(1 - \frac{k^2x^4}{12}\right)
$$

13.4 problem 5

Internal problem ID [714]

Book: Elementary differential equations and boundary value problems, 10th ed., Boyce and DiPrima
Section: Chapter 5.2, Series Solutions Near an Ordinary Point, Part I. page 263
Problem number: 5.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_Emden, _Fowler]]`

$$\boxed{(1-x)\,y''+y=0}$$

With the expansion point for the power series method at $x=0$.

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 49

```
Order:=6;
dsolve((1-x)*diff(y(x),x$2)+y(x)=0,y(x),type='series',x=0);
```

$$y(x)=\left(1-\frac{1}{2}x^2-\frac{1}{6}x^3-\frac{1}{24}x^4-\frac{1}{60}x^5\right)y(0)+\left(x-\frac{1}{6}x^3-\frac{1}{12}x^4-\frac{1}{24}x^5\right)D(y)(0)+O\left(x^6\right)$$

✓ Solution by Mathematica

Time used: 0.001 (sec). Leaf size: 63

```
AsymptoticDSolveValue[(1-x)*y''[x]+y[x]==0,y[x],{x,0,5}]
```

$$y(x)\to c_2\left(-\frac{x^5}{24}-\frac{x^4}{12}-\frac{x^3}{6}+x\right)+c_1\left(-\frac{x^5}{60}-\frac{x^4}{24}-\frac{x^3}{6}-\frac{x^2}{2}+1\right)$$

13.5 problem 6

Internal problem ID [715]

Book: Elementary differential equations and boundary value problems, 10th ed., Boyce and DiPrima
Section: Chapter 5.2, Series Solutions Near an Ordinary Point, Part I. page 263
Problem number: 6.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$\boxed{(x^2+2)\, y'' - y'x + 4y = 0}$$

With the expansion point for the power series method at $x = 0$.

✓ Solution by Maple
Time used: 0.0 (sec). Leaf size: 34

```
Order:=6;
dsolve((2+x^2)*diff(y(x),x$2)-x*diff(y(x),x)+4*y(x)=0,y(x),type='series',x=0);
```

$$y(x) = \left(1 - x^2 + \frac{1}{6}x^4\right) y(0) + \left(x - \frac{1}{4}x^3 + \frac{7}{160}x^5\right) D(y)\,(0) + O(x^6)$$

✓ Solution by Mathematica
Time used: 0.001 (sec). Leaf size: 40

```
AsymptoticDSolveValue[(2+x^2)*y''[x]-x*y'[x]+4*y[x]==0,y[x],{x,0,5}]
```

$$y(x) \to c_2\left(\frac{7x^5}{160} - \frac{x^3}{4} + x\right) + c_1\left(\frac{x^4}{6} - x^2 + 1\right)$$

13.6 problem 7

Internal problem ID [716]

Book: Elementary differential equations and boundary value problems, 10th ed., Boyce and DiPrima
Section: Chapter 5.2, Series Solutions Near an Ordinary Point, Part I. page 263
Problem number: 7.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$\boxed{y'' + y'x + 2y = 0}$$

With the expansion point for the power series method at $x = 0$.

✓ Solution by Maple
Time used: 0.0 (sec). Leaf size: 34

```
Order:=6;
dsolve(diff(y(x),x$2)+x*diff(y(x),x)+2*y(x)=0,y(x),type='series',x=0);
```

$$y(x) = \left(1 - x^2 + \frac{1}{3}x^4\right) y(0) + \left(x - \frac{1}{2}x^3 + \frac{1}{8}x^5\right) D(y)(0) + O(x^6)$$

✓ Solution by Mathematica
Time used: 0.001 (sec). Leaf size: 40

```
AsymptoticDSolveValue[y''[x]+x*y'[x]+2*y[x]==0,y[x],{x,0,5}]
```

$$y(x) \to c_2\left(\frac{x^5}{8} - \frac{x^3}{2} + x\right) + c_1\left(\frac{x^4}{3} - x^2 + 1\right)$$

13.7 problem 9

Internal problem ID [717]

Book: Elementary differential equations and boundary value problems, 10th ed., Boyce and DiPrima
Section: Chapter 5.2, Series Solutions Near an Ordinary Point, Part I. page 263
Problem number: 9.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$\boxed{(x^2+1)\,y''-4y'x+6y=0}$$

With the expansion point for the power series method at $x=0$.

✓ Solution by Maple
Time used: 0.0 (sec). Leaf size: 25

```
Order:=6;
dsolve((1+x^2)*diff(y(x),x$2)-4*x*diff(y(x),x)+6*y(x)=0,y(x),type='series',x=0);
```

$$y(x)=y(0)+D(y)\,(0)\,x-3y(0)\,x^2-\frac{D(y)\,(0)\,x^3}{3}$$

✓ Solution by Mathematica
Time used: 0.001 (sec). Leaf size: 26

```
AsymptoticDSolveValue[(1+x^2)*y''[x]-4*x*y'[x]+6*y[x]==0,y[x],{x,0,5}]
```

$$y(x)\to c_2\left(x-\frac{x^3}{3}\right)+c_1(1-3x^2)$$

13.8 problem 10

Internal problem ID [718]

Book: Elementary differential equations and boundary value problems, 10th ed., Boyce and DiPrima
Section: Chapter 5.2, Series Solutions Near an Ordinary Point, Part I. page 263
Problem number: 10.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _exact, _linear, _homogeneous]]`

$$\boxed{(-x^2+4)\,y''+2y=0}$$

With the expansion point for the power series method at $x=0$.

✓ Solution by Maple
Time used: 0.0 (sec). Leaf size: 29

```
Order:=6;
dsolve((4-x^2)*diff(y(x),x$2)+2*y(x)=0,y(x),type='series',x=0);
```

$$y(x)=\left(1-\frac{x^2}{4}\right)y(0)+\left(x-\frac{1}{12}x^3-\frac{1}{240}x^5\right)D(y)(0)+O\left(x^6\right)$$

✓ Solution by Mathematica
Time used: 0.001 (sec). Leaf size: 35

```
AsymptoticDSolveValue[(4-x^2)*y''[x]+2*y[x]==0,y[x],{x,0,5}]
```

$$y(x)\to c_1\left(1-\frac{x^2}{4}\right)+c_2\left(-\frac{x^5}{240}-\frac{x^3}{12}+x\right)$$

13.9 problem 11

Internal problem ID [719]

Book: Elementary differential equations and boundary value problems, 10th ed., Boyce and DiPrima
Section: Chapter 5.2, Series Solutions Near an Ordinary Point, Part I. page 263
Problem number: 11.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _exact, _linear, _homogeneous]]`

$$\boxed{(-x^2+3)\,y''-3y'x-y=0}$$

With the expansion point for the power series method at $x=0$.

✓ Solution by Maple
Time used: 0.016 (sec). Leaf size: 34

```
Order:=6;
dsolve((3-x^2)*diff(y(x),x$2)-3*x*diff(y(x),x)-y(x)=0,y(x),type='series',x=0);
```

$$y(x)=\left(1+\frac{1}{6}x^2+\frac{1}{24}x^4\right)y(0)+\left(x+\frac{2}{9}x^3+\frac{8}{135}x^5\right)D(y)(0)+O\left(x^6\right)$$

✓ Solution by Mathematica
Time used: 0.001 (sec). Leaf size: 70

```
AsymptoticDSolveValue[(3-x^2)*y''[x]-3*y'[x]-y[x]==0,y[x],{x,0,5}]
```

$$y(x)\to c_1\left(\frac{13x^5}{1080}+\frac{x^4}{36}+\frac{x^3}{18}+\frac{x^2}{6}+1\right)+c_2\left(\frac{49x^5}{1080}+\frac{7x^4}{72}+\frac{2x^3}{9}+\frac{x^2}{2}+x\right)$$

13.10 problem 12

Internal problem ID [720]

Book: Elementary differential equations and boundary value problems, 10th ed., Boyce and DiPrima
Section: Chapter 5.2, Series Solutions Near an Ordinary Point, Part I. page 263
Problem number: 12.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$\boxed{(1-x)\,y''+y'x-y=0}$$

With the expansion point for the power series method at $x=0$.

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 34

```
Order:=6;
dsolve((1-x)*diff(y(x),x$2)+x*diff(y(x),x)-y(x)=0,y(x),type='series',x=0);
```

$$y(x)=\left(1+\frac{1}{2}x^2+\frac{1}{6}x^3+\frac{1}{24}x^4+\frac{1}{120}x^5\right)y(0)+D(y)\,(0)\,x+O\left(x^6\right)$$

✓ Solution by Mathematica

Time used: 0.001 (sec). Leaf size: 41

```
AsymptoticDSolveValue[(1-x)*y''[x]+x*y'[x]-y[x]==0,y[x],{x,0,5}]
```

$$y(x)\to c_1\left(\frac{x^5}{120}+\frac{x^4}{24}+\frac{x^3}{6}+\frac{x^2}{2}+1\right)+c_2x$$

13.11 problem 13

Internal problem ID [721]

Book: Elementary differential equations and boundary value problems, 10th ed., Boyce and DiPrima
Section: Chapter 5.2, Series Solutions Near an Ordinary Point, Part I. page 263
Problem number: 13.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$\boxed{2y'' + y'x + 3y = 0}$$

With the expansion point for the power series method at $x = 0$.

✓ Solution by Maple
Time used: 0.0 (sec). Leaf size: 34

```
Order:=6;
dsolve(2*diff(y(x),x$2)+x*diff(y(x),x)+3*y(x)=0,y(x),type='series',x=0);
```

$$y(x) = \left(1 - \frac{3}{4}x^2 + \frac{5}{32}x^4\right) y(0) + \left(x - \frac{1}{3}x^3 + \frac{1}{20}x^5\right) D(y)(0) + O(x^6)$$

✓ Solution by Mathematica
Time used: 0.001 (sec). Leaf size: 42

```
AsymptoticDSolveValue[2*y''[x]+x*y'[x]+3*y[x]==0,y[x],{x,0,5}]
```

$$y(x) \to c_2\left(\frac{x^5}{20} - \frac{x^3}{3} + x\right) + c_1\left(\frac{5x^4}{32} - \frac{3x^2}{4} + 1\right)$$

13.12 problem 15

Internal problem ID [722]

Book: Elementary differential equations and boundary value problems, 10th ed., Boyce and DiPrima
Section: Chapter 5.2, Series Solutions Near an Ordinary Point, Part I. page 263
Problem number: 15.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _exact, _linear, _homogeneous]]`

$$\boxed{y'' - y'x - y = 0}$$

With initial conditions

$$[y(0) = 2, y'(0) = 1]$$

With the expansion point for the power series method at $x = 0$.

✓ Solution by Maple
Time used: 0.0 (sec). Leaf size: 20

```
Order:=6;
dsolve([diff(y(x),x$2)-x*diff(y(x),x)-y(x)=0,y(0) = 2, D(y)(0) = 1],y(x),type='series',x=0);
```

$$y(x) = 2 + x + x^2 + \frac{1}{3}x^3 + \frac{1}{4}x^4 + \frac{1}{15}x^5 + \mathrm{O}\left(x^6\right)$$

✓ Solution by Mathematica
Time used: 0.001 (sec). Leaf size: 30

```
AsymptoticDSolveValue[{y''[x]-x*y'[x]-y[x]==0,{y[0]==2,y'[0]==1}},y[x],{x,0,5}]
```

$$y(x) \to \frac{x^5}{15} + \frac{x^4}{4} + \frac{x^3}{3} + x^2 + x + 2$$

13.13 problem 16

Internal problem ID [723]

Book: Elementary differential equations and boundary value problems, 10th ed., Boyce and DiPrima
Section: Chapter 5.2, Series Solutions Near an Ordinary Point, Part I. page 263
Problem number: 16.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$\boxed{(x^2+2)\, y'' - y'x + 4y = 0}$$

With initial conditions

$$[y(0) = -1, y'(0) = 3]$$

With the expansion point for the power series method at $x = 0$.

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 20

```
Order:=6;
dsolve([(2+x^2)*diff(y(x),x$2)-x*diff(y(x),x)+4*y(x)=0,y(0) = -1, D(y)(0) = 3],y(x),type='se
```

$$y(x) = -1 + 3x + x^2 - \frac{3}{4}x^3 - \frac{1}{6}x^4 + \frac{21}{160}x^5 + \mathrm{O}\left(x^6\right)$$

✓ Solution by Mathematica

Time used: 0.001 (sec). Leaf size: 32

```
AsymptoticDSolveValue[{(2+x^2)*y''[x]-x*y'[x]+4*y[x]==0,{y[0]==-1,y'[0]==3}},y[x],{x,0,5}]
```

$$y(x) \rightarrow \frac{21x^5}{160} - \frac{x^4}{6} - \frac{3x^3}{4} + x^2 + 3x - 1$$

13.14 problem 17

Internal problem ID [724]

Book: Elementary differential equations and boundary value problems, 10th ed., Boyce and DiPrima
Section: Chapter 5.2, Series Solutions Near an Ordinary Point, Part I. page 263
Problem number: 17.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type **[[_2nd_order, _with_linear_symmetries]]**

$$\boxed{y'' + y'x + 2y = 0}$$

With initial conditions

$$[y(0) = 4, y'(0) = -1]$$

With the expansion point for the power series method at $x = 0$.

✓ Solution by Maple
Time used: 0.016 (sec). Leaf size: 20

```
Order:=6;
dsolve([diff(y(x),x$2)+x*diff(y(x),x)+2*y(x)=0,y(0) = 4, D(y)(0) = -1],y(x),type='series',x=
```

$$y(x) = 4 - x - 4x^2 + \frac{1}{2}x^3 + \frac{4}{3}x^4 - \frac{1}{8}x^5 + \mathrm{O}\left(x^6\right)$$

✓ Solution by Mathematica
Time used: 0.001 (sec). Leaf size: 34

```
AsymptoticDSolveValue[{y''[x]+x*y'[x]+2*y[x]==0,{y[0]==4,y'[0]==-1}},y[x],{x,0,5}]
```

$$y(x) \to -\frac{x^5}{8} + \frac{4x^4}{3} + \frac{x^3}{2} - 4x^2 - x + 4$$

13.15 problem 18

Internal problem ID [725]

Book: Elementary differential equations and boundary value problems, 10th ed., Boyce and DiPrima
Section: Chapter 5.2, Series Solutions Near an Ordinary Point, Part I. page 263
Problem number: 18.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$\boxed{(1-x)\, y'' + y'x - y = 0}$$

With initial conditions

$$[y(0) = -3, y'(0) = 2]$$

With the expansion point for the power series method at $x = 0$.

✓ Solution by Maple
Time used: 0.0 (sec). Leaf size: 20

```
Order:=6;
dsolve([(1-x)*diff(y(x),x$2)+x*diff(y(x),x)-y(x)=0,y(0) = -3, D(y)(0) = 2],y(x),type='series
```

$$y(x) = -3 + 2x - \frac{3}{2}x^2 - \frac{1}{2}x^3 - \frac{1}{8}x^4 - \frac{1}{40}x^5 + \mathrm{O}\left(x^6\right)$$

✓ Solution by Mathematica
Time used: 0.001 (sec). Leaf size: 36

```
AsymptoticDSolveValue[{(1-x)*y''[x]+x*y'[x]-y[x]==0,{y[0]==-3,y'[0]==2}},y[x],{x,0,5}]
```

$$y(x) \to -\frac{x^5}{40} - \frac{x^4}{8} - \frac{x^3}{2} - \frac{3x^2}{2} + 2x - 3$$

13.16 problem 21

Internal problem ID [726]

Book: Elementary differential equations and boundary value problems, 10th ed., Boyce and DiPrima
Section: Chapter 5.2, Series Solutions Near an Ordinary Point, Part I. page 263
Problem number: 21.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$\boxed{y'' - 2y'x + \lambda y = 0}$$

With the expansion point for the power series method at $x = 0$.

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 63

```
Order:=6;
dsolve(diff(y(x),x$2)-2*x*diff(y(x),x)+lambda*y(x)=0,y(x),type='series',x=0);
```

$$y(x) = \left(1 - \frac{\lambda x^2}{2} + \frac{\lambda(\lambda-4)x^4}{24}\right) y(0) + \left(x - \frac{(\lambda-2)x^3}{6} + \frac{(\lambda-2)(-6+\lambda)x^5}{120}\right) D(y)(0) + O(x^6)$$

✓ Solution by Mathematica

Time used: 0.001 (sec). Leaf size: 80

```
AsymptoticDSolveValue[y''[x]-2*x*y'[x]+\[Lambda]*y[x]==0,y[x],{x,0,5}]
```

$$y(x) \to c_2\left(\frac{\lambda^2 x^5}{120} - \frac{\lambda x^5}{15} + \frac{x^5}{10} - \frac{\lambda x^3}{6} + \frac{x^3}{3} + x\right) + c_1\left(\frac{\lambda^2 x^4}{24} - \frac{\lambda x^4}{6} - \frac{\lambda x^2}{2} + 1\right)$$

13.17 problem 23

Internal problem ID [727]

Book: Elementary differential equations and boundary value problems, 10th ed., Boyce and DiPrima
Section: Chapter 5.2, Series Solutions Near an Ordinary Point, Part I. page 263
Problem number: 23.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _exact, _linear, _homogeneous]]`

$$\boxed{y'' - y'x - y = 0}$$

With initial conditions

$$[y(0) = 1, y'(0) = 0]$$

With the expansion point for the power series method at $x = 0$.

✓ Solution by Maple

Time used: 0.015 (sec). Leaf size: 14

```
Order:=6;
dsolve([diff(y(x),x$2)-x*diff(y(x),x)-y(x)=0,y(0) = 1, D(y)(0) = 0],y(x),type='series',x=0);
```

$$y(x) = 1 + \frac{1}{2}x^2 + \frac{1}{8}x^4 + \mathrm{O}\left(x^6\right)$$

✓ Solution by Mathematica

Time used: 0.001 (sec). Leaf size: 19

```
AsymptoticDSolveValue[{y''[x]-x*y'[x]-y[x]==0,{y[0]==1,y'[0]==0}},y[x],{x,0,5}]
```

$$y(x) \to \frac{x^4}{8} + \frac{x^2}{2} + 1$$

13.18 problem 24

Internal problem ID [728]

Book: Elementary differential equations and boundary value problems, 10th ed., Boyce and DiPrima
Section: Chapter 5.2, Series Solutions Near an Ordinary Point, Part I. page 263
Problem number: 24.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$\boxed{(x^2 + 2) y'' - y'x + 4y = 0}$$

With initial conditions

$$[y(0) = 1, y'(0) = 0]$$

With the expansion point for the power series method at $x = 0$.

✓ Solution by Maple
Time used: 0.0 (sec). Leaf size: 14

```
Order:=6;
dsolve([(2+x^2)*diff(y(x),x$2)-x*diff(y(x),x)+4*y(x)=0,y(0) = 1, D(y)(0) = 0],y(x),type='ser
```

$$y(x) = 1 - x^2 + \frac{1}{6}x^4 + O(x^6)$$

✓ Solution by Mathematica
Time used: 0.001 (sec). Leaf size: 17

```
AsymptoticDSolveValue[{(2+x^2)*y''[x]-x*y'[x]+4*y[x]==0,{y[0]==1,y'[0]==0}},y[x],{x,0,5}]
```

$$y(x) \to \frac{x^4}{6} - x^2 + 1$$

13.19 problem 25

Internal problem ID [729]

Book: Elementary differential equations and boundary value problems, 10th ed., Boyce and DiPrima
Section: Chapter 5.2, Series Solutions Near an Ordinary Point, Part I. page 263
Problem number: 25.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$\boxed{y'' + y'x + 2y = 0}$$

With initial conditions

$$[y(0) = 0, y'(0) = 1]$$

With the expansion point for the power series method at $x = 0$.

✓ Solution by Maple
Time used: 0.0 (sec). Leaf size: 14

```
Order:=6;
dsolve([diff(y(x),x$2)+x*diff(y(x),x)+2*y(x)=0,y(0) = 0, D(y)(0) = 1],y(x),type='series',x=0
```

$$y(x) = x - \frac{1}{2}x^3 + \frac{1}{8}x^5 + \mathrm{O}\left(x^6\right)$$

✓ Solution by Mathematica
Time used: 0.001 (sec). Leaf size: 19

```
AsymptoticDSolveValue[{y''[x]+x*y'[x]+2*y[x]==0,{y[0]==0,y'[0]==1}},y[x],{x,0,5}]
```

$$y(x) \to \frac{x^5}{8} - \frac{x^3}{2} + x$$

13.20 problem 26

Internal problem ID [730]

Book: Elementary differential equations and boundary value problems, 10th ed., Boyce and DiPrima
Section: Chapter 5.2, Series Solutions Near an Ordinary Point, Part I. page 263
Problem number: 26.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$\boxed{(-x^2+4)\,y''+y'x+2y=0}$$

With initial conditions

$$[y(0)=0, y'(0)=1]$$

With the expansion point for the power series method at $x=0$.

✓ Solution by Maple
Time used: 0.0 (sec). Leaf size: 14

```
Order:=6;
dsolve([(4-x^2)*diff(y(x),x$2)+x*diff(y(x),x)+2*y(x)=0,y(0) = 0, D(y)(0) = 1],y(x),type='ser
```

$$y(x)=x-\frac{1}{8}x^3-\frac{1}{640}x^5+\mathrm{O}\left(x^6\right)$$

✓ Solution by Mathematica
Time used: 0.001 (sec). Leaf size: 19

```
AsymptoticDSolveValue[{(4-x^2)*y''[x]+x*y'[x]+2*y[x]==0,{y[0]==0,y'[0]==1}},y[x],{x,0,5}]
```

$$y(x)\to -\frac{x^5}{640}-\frac{x^3}{8}+x$$

13.21 problem 27

Internal problem ID [731]

Book: Elementary differential equations and boundary value problems, 10th ed., Boyce and DiPrima
Section: Chapter 5.2, Series Solutions Near an Ordinary Point, Part I. page 263
Problem number: 27.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_Emden, _Fowler]]`

$$\boxed{y'' + x^2 y = 0}$$

With initial conditions

$$[y(0) = 1, y'(0) = 0]$$

With the expansion point for the power series method at $x = 0$.

✓ Solution by Maple
Time used: 0.282 (sec). Leaf size: 12

```
Order:=6;
dsolve([diff(y(x),x$2)+x^2*y(x)=0,y(0) = 1, D(y)(0) = 0],y(x),type='series',x=0);
```

$$y(x) = 1 - \frac{1}{12}x^4 + \mathrm{O}\left(x^6\right)$$

✓ Solution by Mathematica
Time used: 0.001 (sec). Leaf size: 12

```
AsymptoticDSolveValue[{y''[x]+x^2*y[x]==0,{y[0]==1,y'[0]==0}},y[x],{x,0,5}]
```

$$y(x) \to 1 - \frac{x^4}{12}$$

13.22 problem 28

Internal problem ID [732]

Book: Elementary differential equations and boundary value problems, 10th ed., Boyce and DiPrima
Section: Chapter 5.2, Series Solutions Near an Ordinary Point, Part I. page 263
Problem number: 28.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries], [_2nd_order, _linear,`

$$(1-x)\,y''+y'x-2y=0$$

With initial conditions

$$[y(0)=0, y'(0)=1]$$

With the expansion point for the power series method at $x=0$.

✓ Solution by Maple
Time used: 0.0 (sec). Leaf size: 16

```
Order:=6;
dsolve([(1-x)*diff(y(x),x$2)+x*diff(y(x),x)-2*y(x)=0,y(0) = 0, D(y)(0) = 1],y(x),type='serie
```

$$y(x)=x+\frac{1}{6}x^3+\frac{1}{12}x^4+\frac{1}{24}x^5+\mathrm{O}\left(x^6\right)$$

✓ Solution by Mathematica
Time used: 0.001 (sec). Leaf size: 26

```
AsymptoticDSolveValue[{(1-x)*y''[x]+x*y'[x]-2*y[x]==0,{y[0]==0,y'[0]==1}},y[x],{x,0,5}]
```

$$y(x)\to\frac{x^5}{24}+\frac{x^4}{12}+\frac{x^3}{6}+x$$

14 Chapter 5.3, Series Solutions Near an Ordinary Point, Part II. page 269

14.1 problem 1

Internal problem ID [733]

Book: Elementary differential equations and boundary value problems, 10th ed., Boyce and DiPrima
Section: Chapter 5.3, Series Solutions Near an Ordinary Point, Part II. page 269
Problem number: 1.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _exact, _linear, _homogeneous]]`

$$\boxed{y'' + y'x + y = 0}$$

With initial conditions

$$[y(0) = 1, y'(0) = 0]$$

With the expansion point for the power series method at $x = 0$.

✓ Solution by Maple
Time used: 0.0 (sec). Leaf size: 14

```
Order:=6;
dsolve([diff(y(x),x$2)+x*diff(y(x),x)+y(x)=0,y(0) = 1, D(y)(0) = 0],y(x),type='series',x=0);
```

$$y(x) = 1 - \frac{1}{2}x^2 + \frac{1}{8}x^4 + \mathrm{O}\left(x^6\right)$$

✓ Solution by Mathematica
Time used: 0.001 (sec). Leaf size: 19

```
AsymptoticDSolveValue[{y''[x]+x*y'[x]+y[x]==0,{y[0]==1,y'[0]==0}},y[x],{x,0,5}]
```

$$y(x) \to \frac{x^4}{8} - \frac{x^2}{2} + 1$$

14.2 problem 2

Internal problem ID [734]

Book: Elementary differential equations and boundary value problems, 10th ed., Boyce and DiPrima
Section: Chapter 5.3, Series Solutions Near an Ordinary Point, Part II. page 269
Problem number: 2.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _exact, _linear, _homogeneous]]`

$$\boxed{y'' + \sin(x)\, y' + \cos(x)\, y = 0}$$

With initial conditions

$$[y(0) = 0, y'(0) = 1]$$

With the expansion point for the power series method at $x = 0$.

✓ Solution by Maple
Time used: 0.0 (sec). Leaf size: 14

```
Order:=6;
dsolve([diff(y(x),x$2)+sin(x)*diff(y(x),x)+cos(x)*y(x)=0,y(0) = 0, D(y)(0) = 1],y(x),type='s
```

$$y(x) = x - \frac{1}{3}x^3 + \frac{1}{10}x^5 + \mathrm{O}\left(x^6\right)$$

✓ Solution by Mathematica
Time used: 0.001 (sec). Leaf size: 19

```
AsymptoticDSolveValue[{y''[x]+Sin[x]*y'[x]+Cos[x]*y[x]==0,{y[0]==0,y'[0]==1}},y[x],{x,0,5}]
```

$$y(x) \to \frac{x^5}{10} - \frac{x^3}{3} + x$$

14.3 problem 3

Internal problem ID [735]

Book: Elementary differential equations and boundary value problems, 10th ed., Boyce and DiPrima
Section: Chapter 5.3, Series Solutions Near an Ordinary Point, Part II. page 269
Problem number: 3.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$\boxed{x^2y'' + (x+1)\,y' + 3\ln(x)\,y = 0}$$

With initial conditions

$$[y(1) = 2, y'(1) = 0]$$

With the expansion point for the power series method at $x = 1$.

✓ Solution by Maple
Time used: 0.0 (sec). Leaf size: 16

```
Order:=6;
dsolve([x^2*diff(y(x),x$2)+(1+x)*diff(y(x),x)+3*ln(x)*y(x)=0,y(1) = 2, D(y)(1) = 0],y(x),typ
```

$$y(x) = 2 - (x-1)^3 + \frac{7}{4}(x-1)^4 - \frac{49}{20}(x-1)^5 + \mathrm{O}\left((x-1)^6\right)$$

✓ Solution by Mathematica
Time used: 0.001 (sec). Leaf size: 30

```
AsymptoticDSolveValue[{x^2*y''[x]+(1+x)*y'[x]+3*Log[x]*y[x]==0,{y[1]==2,y'[1]==0}},y[x],{x,1
```

$$y(x) \to -\frac{49}{20}(x-1)^5 + \frac{7}{4}(x-1)^4 - (x-1)^3 + 2$$

14.4 problem 4

Internal problem ID [736]

Book: Elementary differential equations and boundary value problems, 10th ed., Boyce and DiPrima
Section: Chapter 5.3, Series Solutions Near an Ordinary Point, Part II. page 269
Problem number: 4.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$\boxed{y'' + y'x^2 + \sin(x)\, y = 0}$$

With initial conditions

$$[y(0) = a_0, y'(0) = a_1]$$

With the expansion point for the power series method at $x = 0$.

✓ Solution by Maple
Time used: 0.0 (sec). Leaf size: 24

```
Order:=6;
dsolve([diff(y(x),x$2)+x^2*diff(y(x),x)+sin(x)*y(x)=0,y(0) = a__0, D(y)(0) = a__1],y(x),type
```

$$y(x) = a_0 + a_1 x - \frac{1}{6}a_0 x^3 - \frac{1}{6}a_1 x^4 + \frac{1}{120}a_0 x^5 + \mathrm{O}\left(x^6\right)$$

✓ Solution by Mathematica
Time used: 0.001 (sec). Leaf size: 32

```
AsymptoticDSolveValue[{y''[x]+x^2*y'[x]+Sin[x]*y[x]==0,{y[0]==a0,y'[0]==a1}},y[x],{x,0,5}]
```

$$y(x) \to \frac{\mathrm{a0}x^5}{120} - \frac{\mathrm{a0}x^3}{6} + \mathrm{a0} - \frac{\mathrm{a1}x^4}{6} + \mathrm{a1}x$$

14.5 problem 5. case $x_0 = 0$

Internal problem ID [737]

Book: Elementary differential equations and boundary value problems, 10th ed., Boyce and DiPrima
Section: Chapter 5.3, Series Solutions Near an Ordinary Point, Part II. page 269
Problem number: 5. case $x_0 = 0$.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$\boxed{y'' + 4y' + 6yx = 0}$$

With the expansion point for the power series method at $x = 0$.

✓ Solution by Maple
Time used: 0.016 (sec). Leaf size: 47

```
Order:=6;
dsolve(diff(y(x),x$2)+4*diff(y(x),x)+6*x*y(x)=0,y(x),type='series',x=0);
```

$$y(x) = \left(1 - x^3 + x^4 - \frac{4}{5}x^5\right) y(0) + \left(x - 2x^2 + \frac{8}{3}x^3 - \frac{19}{6}x^4 + \frac{47}{15}x^5\right) D(y)(0) + O(x^6)$$

✓ Solution by Mathematica
Time used: 0.001 (sec). Leaf size: 55

```
AsymptoticDSolveValue[y''[x]+4*y'[x]+6*x*y[x]==0,y[x],{x,0,5}]
```

$$y(x) \to c_1\left(-\frac{4x^5}{5} + x^4 - x^3 + 1\right) + c_2\left(\frac{47x^5}{15} - \frac{19x^4}{6} + \frac{8x^3}{3} - 2x^2 + x\right)$$

14.6 problem 5. case $x_0 = 4$

Internal problem ID [738]

Book: Elementary differential equations and boundary value problems, 10th ed., Boyce and DiPrima
Section: Chapter 5.3, Series Solutions Near an Ordinary Point, Part II. page 269
Problem number: 5. case $x_0 = 4$.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$\boxed{y'' + 4y' + 6yx = 0}$$

With the expansion point for the power series method at $x = 4$.

✓ Solution by Maple
Time used: 0.0 (sec). Leaf size: 54

```
Order:=6;
dsolve(diff(y(x),x$2)+4*diff(y(x),x)+6*x*y(x)=0,y(x),type='series',x=4);
```

$$y(x) = \left(1 - 12(x-4)^2 + 15(x-4)^3 + 9(x-4)^4 - \frac{108(x-4)^5}{5}\right) y(4) + \left(x - 4 - 2(x-4)^2 - \frac{4(x-4)^3}{3} + \frac{29(x-4)^4}{6} - \frac{5(x-4)^5}{3}\right) D(y)(4) + O(x^6)$$

✓ Solution by Mathematica
Time used: 0.001 (sec). Leaf size: 79

```
AsymptoticDSolveValue[y''[x]+4*y'[x]+6*x*y[x]==0,y[x],{x,4,5}]
```

$$y(x) \to c_1\left(-\frac{108}{5}(x-4)^5 + 9(x-4)^4 + 15(x-4)^3 - 12(x-4)^2 + 1\right) + c_2\left(-\frac{5}{3}(x-4)^5 + \frac{29}{6}(x-4)^4 - \frac{4}{3}(x-4)^3 - 2(x-4)^2 + x - 4\right)$$

14.7 problem 6. case $x_0 = 0$

Internal problem ID [739]

Book: Elementary differential equations and boundary value problems, 10th ed., Boyce and DiPrima
Section: Chapter 5.3, Series Solutions Near an Ordinary Point, Part II. page 269
Problem number: 6. case $x_0 = 0$.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$\boxed{(x^2 - 2x - 3)\, y'' + y'x + 4y = 0}$$

With the expansion point for the power series method at $x = 0$.

✓ Solution by Maple

Time used: 0.016 (sec). Leaf size: 49

```
Order:=6;
dsolve((x^2-2*x-3)*diff(y(x),x$2)+x*diff(y(x),x)+4*y(x)=0,y(x),type='series',x=0);
```

$$y(x) = \left(1 + \frac{2}{3}x^2 - \frac{4}{27}x^3 + \frac{16}{81}x^4 - \frac{1}{9}x^5\right) y(0) + \left(x + \frac{5}{18}x^3 - \frac{5}{54}x^4 + \frac{7}{72}x^5\right) D(y)(0) + O(x^6)$$

✓ Solution by Mathematica

Time used: 0.001 (sec). Leaf size: 63

```
AsymptoticDSolveValue[(x^2-2*x-3)*y''[x]+x*y'[x]+4*y[x]==0,y[x],{x,0,5}]
```

$$y(x) \to c_2\left(\frac{7x^5}{72} - \frac{5x^4}{54} + \frac{5x^3}{18} + x\right) + c_1\left(-\frac{x^5}{9} + \frac{16x^4}{81} - \frac{4x^3}{27} + \frac{2x^2}{3} + 1\right)$$

14.8 problem 6. case $x_0 = 4$ only

Internal problem ID [740]

Book: Elementary differential equations and boundary value problems, 10th ed., Boyce and DiPrima
Section: Chapter 5.3, Series Solutions Near an Ordinary Point, Part II. page 269
Problem number: 6. case $x_0 = 4$ only.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$\boxed{(x^2 - 2x - 3)\, y'' + y'x + 4y = 0}$$

With the expansion point for the power series method at $x = 4$.

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 54

```
Order:=6;
dsolve((x^2-2*x-3)*diff(y(x),x$2)+x*diff(y(x),x)+4*y(x)=0,y(x),type='series',x=4);
```

$$y(x) = \left(1 - \frac{2(x-4)^2}{5} + \frac{4(x-4)^3}{15} - \frac{4(x-4)^4}{25} + \frac{199(x-4)^5}{1875}\right) y(4) + \left(x - 4 - \frac{2(x-4)^2}{5} + \frac{(x-4)^3}{10} - \frac{2(x-4)^4}{75} + \frac{157(x-4)^5}{15000}\right) D(y)(4) + O(x^6)$$

✓ Solution by Mathematica

Time used: 0.001 (sec). Leaf size: 87

```
AsymptoticDSolveValue[(x^2-2*x-3)*y''[x]+x*y'[x]+4*y[x]==0,y[x],{x,4,5}]
```

$$y(x) \to c_1\left(\frac{199(x-4)^5}{1875} - \frac{4}{25}(x-4)^4 + \frac{4}{15}(x-4)^3 - \frac{2}{5}(x-4)^2 + 1\right) + c_2\left(\frac{157(x-4)^5}{15000} - \frac{2}{75}(x-4)^4 + \frac{1}{10}(x-4)^3 - \frac{2}{5}(x-4)^2 + x - 4\right)$$

14.9 problem 6. case $x_0 = -4$

Internal problem ID [741]

Book: Elementary differential equations and boundary value problems, 10th ed., Boyce and DiPrima
Section: Chapter 5.3, Series Solutions Near an Ordinary Point, Part II. page 269
Problem number: 6. case $x_0 = -4$.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$(x^2 - 2x - 3)\, y'' + y'x + 4y = 0$$

With the expansion point for the power series method at $x = -4$.

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 54

```
Order:=6;
dsolve((x^2-2*x-3)*diff(y(x),x$2)+x*diff(y(x),x)+4*y(x)=0,y(x),type='series',x=-4);
```

$$y(x) = \left(1 - \frac{2(x+4)^2}{21} - \frac{4(x+4)^3}{189} - \frac{4(x+4)^4}{1323} - \frac{(x+4)^5}{3087}\right) y(-4) + \left(x+4+\frac{2(x+4)^2}{21} - \frac{(x+4)^3}{54} - \frac{11(x+4)^4}{1323} - \frac{157(x+4)^5}{74088}\right) D(y)(-4) + O(x^6)$$

✓ Solution by Mathematica

Time used: 0.001 (sec). Leaf size: 87

```
AsymptoticDSolveValue[(x^2-2*x-3)*y''[x]+x*y'[x]+4*y[x]==0,y[x],{x,-4,5}]
```

$$y(x) \to c_1\left(-\frac{(x+4)^5}{3087} - \frac{4(x+4)^4}{1323} - \frac{4}{189}(x+4)^3 - \frac{2}{21}(x+4)^2 + 1\right) + c_2\left(-\frac{157(x+4)^5}{74088} - \frac{11(x+4)^4}{1323} - \frac{1}{54}(x+4)^3 + \frac{2}{21}(x+4)^2 + x + 4\right)$$

14.10 problem 7. case $x_0 = 0$

Internal problem ID [742]

Book: Elementary differential equations and boundary value problems, 10th ed., Boyce and DiPrima
Section: Chapter 5.3, Series Solutions Near an Ordinary Point, Part II. page 269
Problem number: 7. case $x_0 = 0$.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$\boxed{(x^3+1)\,y'' + 4y'x + y = 0}$$

With the expansion point for the power series method at $x = 0$.

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 39

```
Order:=6;
dsolve((1+x^3)*diff(y(x),x$2)+4*x*diff(y(x),x)+y(x)=0,y(x),type='series',x=0);
```

$$y(x) = \left(1 - \frac{1}{2}x^2 + \frac{3}{8}x^4 + \frac{1}{20}x^5\right) y(0) + \left(x - \frac{5}{6}x^3 + \frac{13}{24}x^5\right) D(y)(0) + O(x^6)$$

✓ Solution by Mathematica

Time used: 0.001 (sec). Leaf size: 49

```
AsymptoticDSolveValue[(1+x^3)*y''[x]+4*x*y'[x]+y[x]==0,y[x],{x,0,5}]
```

$$y(x) \to c_2\left(\frac{13x^5}{24} - \frac{5x^3}{6} + x\right) + c_1\left(\frac{x^5}{20} + \frac{3x^4}{8} - \frac{x^2}{2} + 1\right)$$

14.11 problem 7. case $x_0 = 2$

Internal problem ID [743]

Book: Elementary differential equations and boundary value problems, 10th ed., Boyce and DiPrima
Section: Chapter 5.3, Series Solutions Near an Ordinary Point, Part II. page 269
Problem number: 7. case $x_0 = 2$.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$\boxed{(x^3 + 1)\, y'' + 4y'x + y = 0}$$

With the expansion point for the power series method at $x = 2$.

✓ Solution by Maple
Time used: 0.016 (sec). Leaf size: 54

```
Order:=6;
dsolve((1+x^3)*diff(y(x),x$2)+4*x*diff(y(x),x)+y(x)=0,y(x),type='series',x=2);
```

$$y(x) = \left(1 - \frac{(-2+x)^2}{18} + \frac{10(-2+x)^3}{243} - \frac{451(-2+x)^4}{17496} + \frac{1151(-2+x)^5}{78732}\right) y(2) + \left(-2 + x - \frac{4(-2+x)^2}{9} + \frac{115(-2+x)^3}{486} - \frac{271(-2+x)^4}{2187} + \frac{9713(-2+x)^5}{157464}\right) D(y)(2) + O(x^6)$$

✓ Solution by Mathematica

Time used: 0.001 (sec). Leaf size: 87

```
AsymptoticDSolveValue[(1+x^3)*y''[x]+4*x*y'[x]+y[x]==0,y[x],{x,2,5}]
```

$$y(x) \to c_1 \left(\frac{1151(x-2)^5}{78732} - \frac{451(x-2)^4}{17496} + \frac{10}{243}(x-2)^3 - \frac{1}{18}(x-2)^2 + 1 \right) + c_2 \left(\frac{9713(x-2)^5}{157464} - \frac{271(x-2)^4}{2187} + \frac{115}{486}(x-2)^3 - \frac{4}{9}(x-2)^2 + x - 2 \right)$$

14.12 problem 8

Internal problem ID [744]

Book: Elementary differential equations and boundary value problems, 10th ed., Boyce and DiPrima
Section: Chapter 5.3, Series Solutions Near an Ordinary Point, Part II. page 269
Problem number: 8.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_Emden, _Fowler]]`

$$\boxed{xy'' + y = 0}$$

With the expansion point for the power series method at $x = 1$.

✓ Solution by Maple
Time used: 0.0 (sec). Leaf size: 49

```
Order:=6;
dsolve(x*diff(y(x),x$2)+y(x)=0,y(x),type='series',x=1);
```

$$y(x) = \left(1 - \frac{(x-1)^2}{2} + \frac{(x-1)^3}{6} - \frac{(x-1)^4}{24} + \frac{(x-1)^5}{60}\right) y(1) + \left(x - 1 - \frac{(x-1)^3}{6} + \frac{(x-1)^4}{12} - \frac{(x-1)^5}{24}\right) D(y)(1) + O(x^6)$$

✓ Solution by Mathematica
Time used: 0.001 (sec). Leaf size: 78

```
AsymptoticDSolveValue[x*y''[x]+y[x]==0,y[x],{x,1,5}]
```

$$y(x) \to c_1\left(\frac{1}{60}(x-1)^5 - \frac{1}{24}(x-1)^4 + \frac{1}{6}(x-1)^3 - \frac{1}{2}(x-1)^2 + 1\right) + c_2\left(-\frac{1}{24}(x-1)^5 + \frac{1}{12}(x-1)^4 - \frac{1}{6}(x-1)^3 + x - 1\right)$$

14.13 problem 10

Internal problem ID [745]

Book: Elementary differential equations and boundary value problems, 10th ed., Boyce and DiPrima
Section: Chapter 5.3, Series Solutions Near an Ordinary Point, Part II. page 269
Problem number: 10.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[_Gegenbauer, [_2nd_order, _linear, '_with_symmetry_[0,F(x)]']]`

$$\boxed{(-x^2+1)\,y''-y'x+\alpha^2y=0}$$

With the expansion point for the power series method at $x=0$.

✓ Solution by Maple

Time used: 0.016 (sec). Leaf size: 71

```
Order:=6;
dsolve((1-x^2)*diff(y(x),x$2)-x*diff(y(x),x)+alpha^2*y(x)=0,y(x),type='series',x=0);
```

$$y(x)=\left(1-\frac{\alpha^2x^2}{2}+\frac{\alpha^2(\alpha^2-4)\,x^4}{24}\right)y(0)+\left(x-\frac{(\alpha^2-1)\,x^3}{6}+\frac{(\alpha^4-10\alpha^2+9)\,x^5}{120}\right)D(y)\,(0)+O(x^6)$$

✓ Solution by Mathematica

Time used: 0.001 (sec). Leaf size: 88

```
AsymptoticDSolveValue[(1-x^2)*y''[x]-x*y'[x]+\[Alpha]^2*y[x]==0,y[x],{x,0,5}]
```

$$y(x)\to c_2\left(\frac{\alpha^4x^5}{120}-\frac{\alpha^2x^5}{12}+\frac{3x^5}{40}-\frac{\alpha^2x^3}{6}+\frac{x^3}{6}+x\right)+c_1\left(\frac{\alpha^4x^4}{24}-\frac{\alpha^2x^4}{6}-\frac{\alpha^2x^2}{2}+1\right)$$

14.14 problem 16

Internal problem ID [746]

Book: Elementary differential equations and boundary value problems, 10th ed., Boyce and DiPrima
Section: Chapter 5.3, Series Solutions Near an Ordinary Point, Part II. page 269
Problem number: 16.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_quadrature]`

$$\boxed{y' - y = 0}$$

With the expansion point for the power series method at $x = 0$.

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 34

```
Order:=6;
dsolve(diff(y(x),x)-y(x)=0,y(x),type='series',x=0);
```

$$y(x) = \left(1 + x + \frac{1}{2}x^2 + \frac{1}{6}x^3 + \frac{1}{24}x^4 + \frac{1}{120}x^5\right) y(0) + O(x^6)$$

✓ Solution by Mathematica

Time used: 0.001 (sec). Leaf size: 37

```
AsymptoticDSolveValue[y'[x]-y[x]==0,y[x],{x,0,5}]
```

$$y(x) \to c_1 \left(\frac{x^5}{120} + \frac{x^4}{24} + \frac{x^3}{6} + \frac{x^2}{2} + x + 1\right)$$

14.15 problem 17

Internal problem ID [747]

Book: Elementary differential equations and boundary value problems, 10th ed., Boyce and DiPrima
Section: Chapter 5.3, Series Solutions Near an Ordinary Point, Part II. page 269
Problem number: 17.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_separable]`

$$\boxed{-yx + y' = 0}$$

With the expansion point for the power series method at $x = 0$.

✓ Solution by Maple
Time used: 0.0 (sec). Leaf size: 21

```
Order:=6;
dsolve(diff(y(x),x)-x*y(x)=0,y(x),type='series',x=0);
```

$$y(x) = \left(1 + \frac{1}{2}x^2 + \frac{1}{8}x^4\right) y(0) + O(x^6)$$

✓ Solution by Mathematica
Time used: 0.001 (sec). Leaf size: 22

```
AsymptoticDSolveValue[y'[x]-x*y[x]==0,y[x],{x,0,5}]
```

$$y(x) \to c_1 \left(\frac{x^4}{8} + \frac{x^2}{2} + 1\right)$$

14.16 problem 19

Internal problem ID [748]

Book: Elementary differential equations and boundary value problems, 10th ed., Boyce and DiPrima
Section: Chapter 5.3, Series Solutions Near an Ordinary Point, Part II. page 269
Problem number: 19.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_separable]`

$$\boxed{(1-x)\, y' - y = 0}$$

With the expansion point for the power series method at $x = 0$.

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 26

```
Order:=6;
dsolve((1-x)*diff(y(x),x)=y(x),y(x),type='series',x=0);
```

$$y(x) = (x^5 + x^4 + x^3 + x^2 + x + 1)\, y(0) + O(x^6)$$

✓ Solution by Mathematica

Time used: 0.001 (sec). Leaf size: 21

```
AsymptoticDSolveValue[(1-x)*y'[x]==y[x],y[x],{x,0,5}]
```

$$y(x) \to c_1(x^5 + x^4 + x^3 + x^2 + x + 1)$$

14.17 problem 22

Internal problem ID [749]

Book: Elementary differential equations and boundary value problems, 10th ed., Boyce and DiPrima
Section: Chapter 5.3, Series Solutions Near an Ordinary Point, Part II. page 269
Problem number: 22.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[_Gegenbauer]`

$$(-x^2+1)\,y''-2y'x+\alpha(\alpha+1)\,y=0$$

With the expansion point for the power series method at $x=0$.

✓ Solution by Maple
Time used: 0.016 (sec). Leaf size: 101

```
Order:=6;
dsolve((1-x^2)*diff(y(x),x$2)-2*x*diff(y(x),x)+alpha*(alpha+1)*y(x)=0,y(x),type='series',x=0
```

$$y(x)=\left(1-\frac{\alpha(1+\alpha)\,x^2}{2}+\frac{\alpha(\alpha^3+2\alpha^2-5\alpha-6)\,x^4}{24}\right)y(0)+\left(x-\frac{(\alpha^2+\alpha-2)\,x^3}{6}+\frac{(\alpha^4+2\alpha^3-13\alpha^2-14\alpha+24)\,x^5}{120}\right)D(y)\,(0)+O(x^6)$$

✓ Solution by Mathematica
Time used: 0.001 (sec). Leaf size: 127

```
AsymptoticDSolveValue[(1-x^2)*y''[x]-2*x*y'[x]+\[Alpha]*(\[Alpha]+1)*y[x]==0,y[x],{x,0,5}]
```

$$y(x)\to c_2\left(\frac{1}{60}(-\alpha^2-\alpha)\,x^5-\frac{1}{120}(-\alpha^2-\alpha)\,(\alpha^2+\alpha)\,x^5-\frac{1}{10}(\alpha^2+\alpha)\,x^5+\frac{x^5}{5}-\frac{1}{6}(\alpha^2+\alpha)\,x^3+\frac{x^3}{3}+x\right)+c_1\left(\frac{1}{24}(\alpha^2+\alpha)^2\,x^4-\frac{1}{4}(\alpha^2+\alpha)\,x^4-\frac{1}{2}(\alpha^2+\alpha)\,x^2+1\right)$$

15 Chapter 7.5, Homogeneous Linear Systems with Constant Coefficients. page 407

15.1 problem 30

Internal problem ID [750]

Book: Elementary differential equations and boundary value problems, 10th ed., Boyce and DiPrima
Section: Chapter 7.5, Homogeneous Linear Systems with Constant Coefficients. page 407
Problem number: 30.
ODE order: 1.
ODE degree: 1.

Solve

$$x_1'(t) = -\frac{x_1(t)}{10} + \frac{3x_2(t)}{40}$$
$$x_2'(t) = \frac{x_1(t)}{10} - \frac{x_2(t)}{5}$$

With initial conditions

$$[x_1(0) = -17, x_2(0) = -21]$$

✓ Solution by Maple

Time used: 0.031 (sec). Leaf size: 34

```
dsolve([diff(x__1(t),t) = -1/10*x__1(t)+3/40*x__2(t), diff(x__2(t),t) = 1/10*x__1(t)-1/5*x__
```

$$x_1(t) = -\frac{165\,\mathrm{e}^{-\frac{t}{20}}}{8} + \frac{29\,\mathrm{e}^{-\frac{t}{4}}}{8}$$
$$x_2(t) = -\frac{55\,\mathrm{e}^{-\frac{t}{20}}}{4} - \frac{29\,\mathrm{e}^{-\frac{t}{4}}}{4}$$

✓ Solution by Mathematica

Time used: 0.007 (sec). Leaf size: 52

```
DSolve[{x1'[t]==-1/10*x1[t]+3/40*x2[t],x2'[t]==1/10*x1[t]-1/5*x2[t]},{x1[0]==-17,x2[0]==-21}
```

$$\mathrm{x1}(t) \to \frac{1}{8}e^{-t/4}\left(29 - 165e^{t/5}\right)$$
$$\mathrm{x2}(t) \to -\frac{1}{4}e^{-t/4}\left(55e^{t/5} + 29\right)$$

16 Chapter 7.6, Complex Eigenvalues. page 417

16.1 problem 1

Internal problem ID [751]

Book: Elementary differential equations and boundary value problems, 10th ed., Boyce and DiPrima
Section: Chapter 7.6, Complex Eigenvalues. page 417
Problem number: 1.
ODE order: 1.
ODE degree: 1.

Solve

$$x_1'(t) = 3x_1(t) - 2x_2(t)$$
$$x_2'(t) = 4x_1(t) - x_2(t)$$

✓ Solution by Maple
Time used: 0.016 (sec). Leaf size: 56

```
dsolve([diff(x__1(t),t)=3*x__1(t)-2*x__2(t),diff(x__2(t),t)=4*x__1(t)-1*x__2(t)],singsol=all
```

$$x_1(t) = e^t(c_1 \sin(2t) + c_2 \cos(2t))$$
$$x_2(t) = -e^t(c_1 \cos(2t) - c_2 \cos(2t) - c_1 \sin(2t) - c_2 \sin(2t))$$

✓ Solution by Mathematica
Time used: 0.005 (sec). Leaf size: 58

```
DSolve[{x1'[t]==3*x1[t]-2*x2[t],x2'[t]==4*x1[t]-1*x2[t]},{x1[t],x2[t]},t,IncludeSingularSolu
```

$$\text{x1}(t) \to e^t(c_1 \cos(2t) + (c_1 - c_2)\sin(2t))$$
$$\text{x2}(t) \to e^t(c_2 \cos(2t) + (2c_1 - c_2)\sin(2t))$$

16.2 problem 2

Internal problem ID [752]

Book: Elementary differential equations and boundary value problems, 10th ed., Boyce and DiPrima
Section: Chapter 7.6, Complex Eigenvalues. page 417
Problem number: 2.
ODE order: 1.
ODE degree: 1.

Solve

$$x_1'(t) = -x_1(t) - 4x_2(t)$$
$$x_2'(t) = x_1(t) - x_2(t)$$

✓ Solution by Maple
Time used: 0.016 (sec). Leaf size: 46

```
dsolve([diff(x__1(t),t)=-1*x__1(t)-4*x__2(t),diff(x__2(t),t)=1*x__1(t)-1*x__2(t)],singsol=al
```

$$x_1(t) = e^{-t}(c_1 \sin(2t) + c_2 \cos(2t))$$
$$x_2(t) = -\frac{e^{-t}(c_1 \cos(2t) - c_2 \sin(2t))}{2}$$

✓ Solution by Mathematica
Time used: 0.005 (sec). Leaf size: 55

```
DSolve[{x1'[t]==-1*x1[t]-4*x2[t],x2'[t]==1*x1[t]-1*x2[t]},{x1[t],x2[t]},t,IncludeSingularSol
```

$$\text{x1}(t) \to e^{-t}(c_1 \cos(2t) - 2c_2 \sin(2t))$$
$$\text{x2}(t) \to \frac{1}{2}e^{-t}(2c_2 \cos(2t) + c_1 \sin(2t))$$

16.3 problem 3

Internal problem ID [753]

Book: Elementary differential equations and boundary value problems, 10th ed., Boyce and DiPrima
Section: Chapter 7.6, Complex Eigenvalues. page 417
Problem number: 3.
ODE order: 1.
ODE degree: 1.

Solve

$$x_1'(t) = 2x_1(t) - 5x_2(t)$$
$$x_2'(t) = x_1(t) - 2x_2(t)$$

✓ Solution by Maple

Time used: 0.015 (sec). Leaf size: 38

```
dsolve([diff(x__1(t),t)=2*x__1(t)-5*x__2(t),diff(x__2(t),t)=1*x__1(t)-2*x__2(t)],singsol=all
```

$$x_1(t) = c_1 \sin(t) + c_2 \cos(t)$$
$$x_2(t) = -\frac{c_1 \cos(t)}{5} + \frac{c_2 \sin(t)}{5} + \frac{2c_1 \sin(t)}{5} + \frac{2c_2 \cos(t)}{5}$$

✓ Solution by Mathematica

Time used: 0.005 (sec). Leaf size: 41

```
DSolve[{x1'[t]==2*x1[t]-5*x2[t],x2'[t]==1*x1[t]-2*x2[t]},{x1[t],x2[t]},t,IncludeSingularSolu
```

$$\text{x1}(t) \to c_1(2\sin(t) + \cos(t)) - 5c_2 \sin(t)$$
$$\text{x2}(t) \to c_2 \cos(t) + (c_1 - 2c_2) \sin(t)$$

16.4 problem 4

Internal problem ID [754]

Book: Elementary differential equations and boundary value problems, 10th ed., Boyce and DiPrima
Section: Chapter 7.6, Complex Eigenvalues. page 417
Problem number: 4.
ODE order: 1.
ODE degree: 1.

Solve

$$x_1'(t) = 2x_1(t) - \frac{5x_2(t)}{2}$$
$$x_2'(t) = \frac{9x_1(t)}{5} - x_2(t)$$

✓ Solution by Maple

Time used: 0.015 (sec). Leaf size: 58

```
dsolve([diff(x__1(t),t)=2*x__1(t)-5/2*x__2(t),diff(x__2(t),t)=9/5*x__1(t)-1*x__2(t)],singsol
```

$$x_1(t) = e^{\frac{t}{2}} \left(\sin\left(\frac{3t}{2}\right) c_1 + \cos\left(\frac{3t}{2}\right) c_2\right)$$
$$x_2(t) = \frac{3\,e^{\frac{t}{2}} \left(\sin\left(\frac{3t}{2}\right) c_1 + \sin\left(\frac{3t}{2}\right) c_2 - \cos\left(\frac{3t}{2}\right) c_1 + \cos\left(\frac{3t}{2}\right) c_2\right)}{5}$$

✓ Solution by Mathematica

Time used: 0.006 (sec). Leaf size: 84

```
DSolve[{x1'[t]==2*x1[t]-5/2*x2[t],x2'[t]==9/5*x1[t]-1*x2[t]},{x1[t],x2[t]},t,IncludeSingular
```

$$\text{x1}(t) \to \frac{1}{3}e^{t/2}\left(3c_1\cos\left(\frac{3t}{2}\right) + (3c_1 - 5c_2)\sin\left(\frac{3t}{2}\right)\right)$$
$$\text{x2}(t) \to \frac{1}{5}e^{t/2}\left(5c_2\cos\left(\frac{3t}{2}\right) + (6c_1 - 5c_2)\sin\left(\frac{3t}{2}\right)\right)$$

16.5 problem 5

Internal problem ID [755]

Book: Elementary differential equations and boundary value problems, 10th ed., Boyce and DiPrima
Section: Chapter 7.6, Complex Eigenvalues. page 417
Problem number: 5.
ODE order: 1.
ODE degree: 1.

Solve

$$
\begin{aligned}
x_1'(t) &= x_1(t) - x_2(t) \\
x_2'(t) &= 5x_1(t) - 3x_2(t)
\end{aligned}
$$

✓ Solution by Maple

Time used: 0.016 (sec). Leaf size: 48

```
dsolve([diff(x__1(t),t)=1*x__1(t)-1*x__2(t),diff(x__2(t),t)=5*x__1(t)-3*x__2(t)],singsol=all
```

$$
\begin{aligned}
x_1(t) &= e^{-t}(c_1 \sin(t) + c_2 \cos(t)) \\
x_2(t) &= -e^{-t}(c_1 \cos(t) - 2c_2 \cos(t) - 2c_1 \sin(t) - c_2 \sin(t))
\end{aligned}
$$

✓ Solution by Mathematica

Time used: 0.005 (sec). Leaf size: 56

```
DSolve[{x1'[t]==1*x1[t]-1*x2[t],x2'[t]==5*x1[t]-3*x2[t]},{x1[t],x2[t]},t,IncludeSingularSolu
```

$$
\begin{aligned}
\text{x1}(t) &\to e^{-t}(c_1 \cos(t) + (2c_1 - c_2)\sin(t)) \\
\text{x2}(t) &\to e^{-t}(c_2 \cos(t) + (5c_1 - 2c_2)\sin(t))
\end{aligned}
$$

16.6 problem 6

Internal problem ID [756]

Book: Elementary differential equations and boundary value problems, 10th ed., Boyce and DiPrima
Section: Chapter 7.6, Complex Eigenvalues. page 417
Problem number: 6.
ODE order: 1.
ODE degree: 1.

Solve

$$x_1'(t) = x_1(t) + 2x_2(t)$$
$$x_2'(t) = -5x_1(t) - x_2(t)$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 50

```
dsolve([diff(x__1(t),t)=1*x__1(t)+2*x__2(t),diff(x__2(t),t)=-5*x__1(t)-1*x__2(t)],singsol=al
```

$$x_1(t) = c_1 \sin(3t) + c_2 \cos(3t)$$
$$x_2(t) = \frac{3c_1 \cos(3t)}{2} - \frac{3c_2 \sin(3t)}{2} - \frac{c_1 \sin(3t)}{2} - \frac{c_2 \cos(3t)}{2}$$

✓ Solution by Mathematica

Time used: 0.004 (sec). Leaf size: 54

```
DSolve[{x1'[t]==1*x1[t]+2*x2[t],x2'[t]==-5*x1[t]-1*x2[t]},{x1[t],x2[t]},t,IncludeSingularSol
```

$$\text{x1}(t) \to c_1 \cos(3t) + \frac{1}{3}(c_1 + 2c_2)\sin(3t)$$
$$\text{x2}(t) \to c_2 \cos(3t) - \frac{1}{3}(5c_1 + c_2)\sin(3t)$$

16.7 problem 7

Internal problem ID [757]

Book: Elementary differential equations and boundary value problems, 10th ed., Boyce and DiPrima
Section: Chapter 7.6, Complex Eigenvalues. page 417
Problem number: 7.
ODE order: 1.
ODE degree: 1.

Solve

$$
\begin{aligned}
x_1'(t) &= x_1(t) \\
x_2'(t) &= 2x_1(t) + x_2(t) - 2x_3(t) \\
x_3'(t) &= 3x_1(t) + 2x_2(t) + x_3(t)
\end{aligned}
$$

✓ Solution by Maple

Time used: 0.109 (sec). Leaf size: 73

```
dsolve([diff(x__1(t),t)=1*x__1(t)+0*x__2(t)+0*x__3(t),diff(x__2(t),t)=2*x__1(t)+1*x__2(t)-2*
```

$$
\begin{aligned}
x_1(t) &= c_3 e^t \\
x_2(t) &= \frac{e^t(2c_1\cos(2t) - 3c_3\cos(2t) + 2c_2\sin(2t) - 3c_3)}{2} \\
x_3(t) &= -\frac{e^t(2c_2\cos(2t) - 2c_1\sin(2t) + 3c_3\sin(2t) - 2c_3)}{2}
\end{aligned}
$$

✓ Solution by Mathematica

Time used: 0.007 (sec). Leaf size: 95

```
DSolve[{x1'[t]==1*x1[t]+0*x2[t]+0*x3[t],x2'[t]==2*x1[t]+1*x2[t]-2*x3[t],x3'[t]==3*x1[t]+2*x2
```

$$
\begin{aligned}
\text{x1}(t) &\to c_1 e^t \\
\text{x2}(t) &\to \frac{1}{2}e^t((3c_1 + 2c_2)\cos(2t) + 2(c_1 - c_3)\sin(2t) - 3c_1) \\
\text{x3}(t) &\to \frac{1}{2}e^t(-2(c_1 - c_3)\cos(2t) + (3c_1 + 2c_2)\sin(2t) + 2c_1)
\end{aligned}
$$

16.8 problem 8

Internal problem ID [758]

Book: Elementary differential equations and boundary value problems, 10th ed., Boyce and DiPrima
Section: Chapter 7.6, Complex Eigenvalues. page 417
Problem number: 8.
ODE order: 1.
ODE degree: 1.

Solve

$$
\begin{aligned}
x_1'(t) &= -3x_1(t) + 2x_3(t)\\
x_2'(t) &= x_1(t) - x_2(t)\\
x_3'(t) &= -2x_1(t) - x_2(t)
\end{aligned}
$$

✓ Solution by Maple

Time used: 0.031 (sec). Leaf size: 146

```
dsolve([diff(x__1(t),t)=-3*x__1(t)+0*x__2(t)+2*x__3(t),diff(x__2(t),t)=1*x__1(t)-1*x__2(t)-0
```

$$
\begin{aligned}
x_1(t) &= c_1 e^{-2t} + c_2 e^{-t} \sin\left(\sqrt{2}\, t\right) + c_3 e^{-t} \cos\left(\sqrt{2}\, t\right)\\
x_2(t) &= -c_1 e^{-2t} - \frac{c_2 e^{-t} \sqrt{2}\, \cos\left(\sqrt{2}\, t\right)}{2} + \frac{c_3 e^{-t} \sqrt{2}\, \sin\left(\sqrt{2}\, t\right)}{2}\\
x_3(t) &= \frac{c_1 e^{-2t}}{2} + c_2 e^{-t} \sin\left(\sqrt{2}\, t\right) + \frac{c_2 e^{-t} \sqrt{2}\, \cos\left(\sqrt{2}\, t\right)}{2}\\
&\quad + c_3 e^{-t} \cos\left(\sqrt{2}\, t\right) - \frac{c_3 e^{-t} \sqrt{2}\, \sin\left(\sqrt{2}\, t\right)}{2}
\end{aligned}
$$

✓ Solution by Mathematica

Time used: 0.038 (sec). Leaf size: 235

```
DSolve[{x1'[t]==-3*x1[t]+0*x2[t]+2*x3[t],x2'[t]==1*x1[t]-1*x2[t]-0*x3[t],x3'[t]==-2*x1[t]-1*
```

$$x1(t) \to \frac{1}{3}e^{-2t}\left((c_1+2(c_2+c_3))e^t\cos\left(\sqrt{2}t\right)-\sqrt{2}(2c_1+c_2-2c_3)e^t\sin\left(\sqrt{2}t\right)+2(c_1-c_2-c_3)\right)$$

$$x2(t) \to \frac{1}{6}e^{-2t}\left(2(2c_1+c_2-2c_3)e^t\cos\left(\sqrt{2}t\right)+\sqrt{2}(c_1+2(c_2+c_3))e^t\sin\left(\sqrt{2}t\right)+4(-c_1+c_2+c_3)\right)$$

$$x3(t) \to \frac{1}{6}e^{-2t}\left(-2(c_1-c_2-4c_3)e^t\cos\left(\sqrt{2}t\right)-\sqrt{2}(5c_1+4c_2-2c_3)e^t\sin\left(\sqrt{2}t\right)+2(c_1-c_2-c_3)\right)$$

16.9 problem 9

Internal problem ID [759]

Book: Elementary differential equations and boundary value problems, 10th ed., Boyce and DiPrima
Section: Chapter 7.6, Complex Eigenvalues. page 417
Problem number: 9.
ODE order: 1.
ODE degree: 1.

Solve

$$x_1'(t) = x_1(t) - 5x_2(t)$$
$$x_2'(t) = x_1(t) - 3x_2(t)$$

With initial conditions

$$[x_1(0) = 1, x_2(0) = 1]$$

✓ Solution by Maple

Time used: 0.125 (sec). Leaf size: 35

```
dsolve([diff(x__1(t),t) = x__1(t)-5*x__2(t), diff(x__2(t),t) = x__1(t)-3*x__2(t), x__1(0) =
```

$$x_1(t) = e^{-t}(-3\sin(t) + \cos(t))$$
$$x_2(t) = \frac{e^{-t}(5\cos(t) - 5\sin(t))}{5}$$

✓ Solution by Mathematica

Time used: 0.006 (sec). Leaf size: 34

```
DSolve[{x1'[t]==1*x1[t]-5*x2[t],x2'[t]==1*x1[t]-3*x2[t]},{x1[0]==1,x2[0]==1},{x1[t],x2[t]},t
```

$$\text{x1}(t) \to e^{-t}(\cos(t) - 3\sin(t))$$
$$\text{x2}(t) \to e^{-t}(\cos(t) - \sin(t))$$

16.10 problem 10

Internal problem ID [760]

Book: Elementary differential equations and boundary value problems, 10th ed., Boyce and DiPrima
Section: Chapter 7.6, Complex Eigenvalues. page 417
Problem number: 10.
ODE order: 1.
ODE degree: 1.

Solve

$$x_1'(t) = -3x_1(t) + 2x_2(t)$$
$$x_2'(t) = -x_1(t) - x_2(t)$$

With initial conditions

$$[x_1(0) = 1, x_2(0) = -2]$$

✓ Solution by Maple

Time used: 0.015 (sec). Leaf size: 35

```
dsolve([diff(x__1(t),t) = -3*x__1(t)+2*x__2(t), diff(x__2(t),t) = -x__1(t)-x__2(t), x__1(0)
```

$$x_1(t) = e^{-2t}(-5\sin(t) + \cos(t))$$
$$x_2(t) = \frac{e^{-2t}(-6\sin(t) - 4\cos(t))}{2}$$

✓ Solution by Mathematica

Time used: 0.006 (sec). Leaf size: 27

```
DSolve[{x1'[t]==-3*x1[t]+2*x2[t],x2'[t]==-1*x1[t]-1*x2[t]},{x1[0]==1,x2[0]==1},{x1[t],x2[t]}
```

$$\text{x1}(t) \to e^{-2t}(\sin(t) + \cos(t))$$
$$\text{x2}(t) \to e^{-2t}\cos(t)$$

16.11 problem 11

Internal problem ID [761]

Book: Elementary differential equations and boundary value problems, 10th ed., Boyce and DiPrima
Section: Chapter 7.6, Complex Eigenvalues. page 417
Problem number: 11.
ODE order: 1.
ODE degree: 1.

Solve

$$x_1'(t) = \frac{3x_1(t)}{4} - 2x_2(t)$$
$$x_2'(t) = x_1(t) - \frac{5x_2(t)}{4}$$

✓ Solution by Maple
Time used: 0.016 (sec). Leaf size: 46

```
dsolve([diff(x__1(t),t)=3/4*x__1(t)-2*x__2(t),diff(x__2(t),t)=1*x__1(t)-5/4*x__2(t)],singsol
```

$$x_1(t) = \mathrm{e}^{-\frac{t}{4}}(c_1 \sin(t) + c_2 \cos(t))$$
$$x_2(t) = \frac{\mathrm{e}^{-\frac{t}{4}}(c_1 \sin(t) + c_2 \sin(t) - c_1 \cos(t) + c_2 \cos(t))}{2}$$

✓ Solution by Mathematica
Time used: 0.005 (sec). Leaf size: 56

```
DSolve[{x1'[t]==3/4*x1[t]-2*x2[t],x2'[t]==1*x1[t]-5/4*x2[t]},{x1[t],x2[t]},t,IncludeSingular
```

$$\mathrm{x1}(t) \to e^{-t/4}(c_1 \cos(t) + (c_1 - 2c_2) \sin(t))$$
$$\mathrm{x2}(t) \to e^{-t/4}(c_2 \cos(t) + (c_1 - c_2) \sin(t))$$

16.12 problem 12

Internal problem ID [762]

Book: Elementary differential equations and boundary value problems, 10th ed., Boyce and DiPrima
Section: Chapter 7.6, Complex Eigenvalues. page 417
Problem number: 12.
ODE order: 1.
ODE degree: 1.

Solve

$$x_1'(t) = -\frac{4x_1(t)}{5} + 2x_2(t)$$
$$x_2'(t) = -x_1(t) + \frac{6x_2(t)}{5}$$

✓ Solution by Maple

Time used: 0.016 (sec). Leaf size: 46

```
dsolve([diff(x__1(t),t)=-4/5*x__1(t)+2*x__2(t),diff(x__2(t),t)=-1*x__1(t)+6/5*x__2(t)],sings
```

$$x_1(t) = e^{\frac{t}{5}}(c_1 \sin(t) + c_2 \cos(t))$$
$$x_2(t) = \frac{e^{\frac{t}{5}}(c_1 \sin(t) - c_2 \sin(t) + c_1 \cos(t) + c_2 \cos(t))}{2}$$

✓ Solution by Mathematica

Time used: 0.005 (sec). Leaf size: 56

```
DSolve[{x1'[t]==-4/5*x1[t]+2*x2[t],x2'[t]==-1*x1[t]+6/5*x2[t]},{x1[t],x2[t]},t,IncludeSingul
```

$$\text{x1}(t) \to e^{t/5}(c_1 \cos(t) - (c_1 - 2c_2)\sin(t))$$
$$\text{x2}(t) \to e^{t/5}(c_2(\sin(t) + \cos(t)) - c_1 \sin(t))$$

16.13 problem 23

Internal problem ID [763]

Book: Elementary differential equations and boundary value problems, 10th ed., Boyce and DiPrima
Section: Chapter 7.6, Complex Eigenvalues. page 417
Problem number: 23.
ODE order: 1.
ODE degree: 1.

Solve

$$
\begin{aligned}
x_1'(t) &= -\frac{x_1(t)}{4} + x_2(t) \\
x_2'(t) &= -x_1(t) - \frac{x_2(t)}{4} \\
x_3'(t) &= -\frac{x_3(t)}{4}
\end{aligned}
$$

✓ Solution by Maple
Time used: 0.016 (sec). Leaf size: 47

```
dsolve([diff(x__1(t),t)=-1/4*x__1(t)+1*x__2(t)+0*x__3(t),diff(x__2(t),t)=-1*x__1(t)-1/4*x__2
```

$$
\begin{aligned}
x_1(t) &= \mathrm{e}^{-\frac{t}{4}}(c_1 \sin(t) + c_2 \cos(t)) \\
x_2(t) &= -\mathrm{e}^{-\frac{t}{4}}(c_2 \sin(t) - c_1 \cos(t)) \\
x_3(t) &= c_3 \mathrm{e}^{-\frac{t}{4}}
\end{aligned}
$$

✓ Solution by Mathematica
Time used: 0.023 (sec). Leaf size: 110

```
DSolve[{x1'[t]==-1/4*x1[t]+1*x2[t]+0*x3[t],x2'[t]==-1*x1[t]-1/4*x2[t]+0*x3[t],x3'[t]==0*x1[t
```

$$
\begin{aligned}
&\text{x1}(t) \to e^{-t/4}(c_1 \cos(t) + c_2 \sin(t)) \\
&\text{x2}(t) \to e^{-t/4}(c_2 \cos(t) - c_1 \sin(t)) \\
&\text{x3}(t) \to c_3 e^{-t/4} \\
&\text{x1}(t) \to e^{-t/4}(c_1 \cos(t) + c_2 \sin(t)) \\
&\text{x2}(t) \to e^{-t/4}(c_2 \cos(t) - c_1 \sin(t)) \\
&\text{x3}(t) \to 0
\end{aligned}
$$

16.14 problem 24

Internal problem ID [764]

Book: Elementary differential equations and boundary value problems, 10th ed., Boyce and DiPrima
Section: Chapter 7.6, Complex Eigenvalues. page 417
Problem number: 24.
ODE order: 1.
ODE degree: 1.

Solve

$$
\begin{aligned}
x_1'(t) &= -\frac{x_1(t)}{4} + x_2(t) \\
x_2'(t) &= -x_1(t) - \frac{x_2(t)}{4} \\
x_3'(t) &= \frac{x_3(t)}{10}
\end{aligned}
$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 47

```
dsolve([diff(x__1(t),t)=-1/4*x__1(t)+1*x__2(t)+0*x__3(t),diff(x__2(t),t)=-1*x__1(t)-1/4*x__2
```

$$
\begin{aligned}
x_1(t) &= e^{-\frac{t}{4}}(c_1 \sin(t) + c_2 \cos(t)) \\
x_2(t) &= -e^{-\frac{t}{4}}(c_2 \sin(t) - c_1 \cos(t)) \\
x_3(t) &= c_3 e^{\frac{t}{10}}
\end{aligned}
$$

✓ Solution by Mathematica

Time used: 0.021 (sec). Leaf size: 110

```
DSolve[{x1'[t]==-1/4*x1[t]+1*x2[t]+0*x3[t],x2'[t]==-1*x1[t]-1/4*x2[t]+0*x3[t],x3'[t]==0*x1[t
```

$$
\begin{aligned}
\text{x1}(t) &\to e^{-t/4}(c_1 \cos(t) + c_2 \sin(t)) \\
\text{x2}(t) &\to e^{-t/4}(c_2 \cos(t) - c_1 \sin(t)) \\
\text{x3}(t) &\to c_3 e^{t/10} \\
\text{x1}(t) &\to e^{-t/4}(c_1 \cos(t) + c_2 \sin(t)) \\
\text{x2}(t) &\to e^{-t/4}(c_2 \cos(t) - c_1 \sin(t)) \\
\text{x3}(t) &\to 0
\end{aligned}
$$

16.15 problem 25

Internal problem ID [765]

Book: Elementary differential equations and boundary value problems, 10th ed., Boyce and DiPrima
Section: Chapter 7.6, Complex Eigenvalues. page 417
Problem number: 25.
ODE order: 1.
ODE degree: 1.

Solve

$$x_1'(t) = -\frac{x_1(t)}{2} - \frac{x_2(t)}{8}$$
$$x_2'(t) = 2x_1(t) - \frac{x_2(t)}{2}$$

✓ Solution by Maple
Time used: 0.016 (sec). Leaf size: 46

```
dsolve([diff(x__1(t),t)=-1/2*x__1(t)-1/8*x__2(t),diff(x__2(t),t)=2*x__1(t)-1/2*x__2(t)],sing
```

$$x_1(t) = \mathrm{e}^{-\frac{t}{2}} \left(c_2 \cos\left(\frac{t}{2}\right) + c_1 \sin\left(\frac{t}{2}\right)\right)$$
$$x_2(t) = -4\,\mathrm{e}^{-\frac{t}{2}} \left(\cos\left(\frac{t}{2}\right) c_1 - \sin\left(\frac{t}{2}\right) c_2\right)$$

✓ Solution by Mathematica
Time used: 0.005 (sec). Leaf size: 68

```
DSolve[{x1'[t]==-1/2*x1[t]-1/8*x2[t],x2'[t]==2*x1[t]-1/2*x2[t]},{x1[t],x2[t]},t,IncludeSingu
```

$$\text{x1}(t) \to \frac{1}{4} e^{-t/2} \left(4c_1 \cos\left(\frac{t}{2}\right) - c_2 \sin\left(\frac{t}{2}\right)\right)$$
$$\text{x2}(t) \to e^{-t/2} \left(c_2 \cos\left(\frac{t}{2}\right) + 4c_1 \sin\left(\frac{t}{2}\right)\right)$$

17 Chapter 7.8, Repeated Eigenvalues. page 436

17.1 problem 1

Internal problem ID [766]

Book: Elementary differential equations and boundary value problems, 10th ed., Boyce and DiPrima
Section: Chapter 7.8, Repeated Eigenvalues. page 436
Problem number: 1.
ODE order: 1.
ODE degree: 1.

Solve

$$x_1'(t) = 3x_1(t) - 4x_2(t)$$
$$x_2'(t) = x_1(t) - x_2(t)$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 31

```
dsolve([diff(x__1(t),t)=3*x__1(t)-4*x__2(t),diff(x__2(t),t)=1*x__1(t)-1*x__2(t)],singsol=all
```

$$x_1(t) = e^t(c_2t + c_1)$$
$$x_2(t) = \frac{e^t(2c_2t + 2c_1 - c_2)}{4}$$

✓ Solution by Mathematica

Time used: 0.002 (sec). Leaf size: 41

```
DSolve[{x1'[t]==3*x1[t]-4*x2[t],x2'[t]==1*x1[t]-1*x2[t]},{x1[t],x2[t]},t,IncludeSingularSolu
```

$$\text{x1}(t) \to e^t(2c_1t - 4c_2t + c_1)$$
$$\text{x2}(t) \to e^t((c_1 - 2c_2)t + c_2)$$

17.2 problem 2

Internal problem ID [767]

Book: Elementary differential equations and boundary value problems, 10th ed., Boyce and DiPrima
Section: Chapter 7.8, Repeated Eigenvalues. page 436
Problem number: 2.
ODE order: 1.
ODE degree: 1.

Solve

$$x_1'(t) = 4x_1(t) - 2x_2(t)$$
$$x_2'(t) = 8x_1(t) - 4x_2(t)$$

✓ Solution by Maple
Time used: 0.016 (sec). Leaf size: 24

```
dsolve([diff(x__1(t),t)=4*x__1(t)-2*x__2(t),diff(x__2(t),t)=8*x__1(t)-4*x__2(t)],singsol=all
```

$$x_1(t) = c_1t + c_2$$
$$x_2(t) = -\frac{1}{2}c_1 + 2c_1t + 2c_2$$

✓ Solution by Mathematica
Time used: 0.002 (sec). Leaf size: 34

```
DSolve[{x1'[t]==4*x1[t]-2*x2[t],x2'[t]==8*x1[t]-4*x2[t]},{x1[t],x2[t]},t,IncludeSingularSolu
```

$$\text{x1}(t) \to 4c_1t - 2c_2t + c_1$$
$$\text{x2}(t) \to 8c_1t - 4c_2t + c_2$$

17.3 problem 3

Internal problem ID [768]

Book: Elementary differential equations and boundary value problems, 10th ed., Boyce and DiPrima
Section: Chapter 7.8, Repeated Eigenvalues. page 436
Problem number: 3.
ODE order: 1.
ODE degree: 1.

Solve

$$
\begin{aligned}
x_1'(t) &= -\frac{3x_1(t)}{2} + x_2(t) \\
x_2'(t) &= -\frac{x_1(t)}{4} - \frac{x_2(t)}{2}
\end{aligned}
$$

✓ Solution by Maple
Time used: 0.016 (sec). Leaf size: 32

```
dsolve([diff(x__1(t),t)=-3/2*x__1(t)+1*x__2(t),diff(x__2(t),t)=-1/4*x__1(t)-1/2*x__2(t)],sin
```

$$
\begin{aligned}
x_1(t) &= \mathrm{e}^{-t}(c_2t + c_1) \\
x_2(t) &= \frac{\mathrm{e}^{-t}(c_2t + c_1 + 2c_2)}{2}
\end{aligned}
$$

✓ Solution by Mathematica
Time used: 0.002 (sec). Leaf size: 54

```
DSolve[{x1'[t]==-3/2*x1[t]+1*x2[t],x2'[t]==-1/4*x1[t]-1/2*x2[t]},{x1[t],x2[t]},t,IncludeSing
```

$$
\begin{aligned}
\mathrm{x1}(t) &\to \frac{1}{2}e^{-t}(2c_2t - c_1(t-2)) \\
\mathrm{x2}(t) &\to \frac{1}{4}e^{-t}(c_1(-t) + 2c_2t + 4c_2)
\end{aligned}
$$

17.4 problem 4

Internal problem ID [769]

Book: Elementary differential equations and boundary value problems, 10th ed., Boyce and DiPrima
Section: Chapter 7.8, Repeated Eigenvalues. page 436
Problem number: 4.
ODE order: 1.
ODE degree: 1.

Solve

$$
\begin{aligned}
x_1'(t) &= -3x_1(t) + \frac{5x_2(t)}{2} \\
x_2'(t) &= -\frac{5x_1(t)}{2} + 2x_2(t)
\end{aligned}
$$

✓ Solution by Maple
Time used: 0.016 (sec). Leaf size: 35

```
dsolve([diff(x__1(t),t)=-3*x__1(t)+5/2*x__2(t),diff(x__2(t),t)=-5/2*x__1(t)+2*x__2(t)],sings
```

$$
\begin{aligned}
x_1(t) &= \mathrm{e}^{-\frac{t}{2}}(c_2 t + c_1) \\
x_2(t) &= \frac{\mathrm{e}^{-\frac{t}{2}}(5c_2 t + 5c_1 + 2c_2)}{5}
\end{aligned}
$$

✓ Solution by Mathematica
Time used: 0.002 (sec). Leaf size: 59

```
DSolve[{x1'[t]==-3*x1[t]+5/2*x2[t],x2'[t]==-5/2*x1[t]+2*x2[t]},{x1[t],x2[t]},t,IncludeSingul
```

$$
\begin{aligned}
\mathrm{x1}(t) &\to \frac{1}{2}e^{-t/2}(c_1(2-5t) + 5c_2 t) \\
\mathrm{x2}(t) &\to \frac{1}{2}e^{-t/2}(-5c_1 t + 5c_2 t + 2c_2)
\end{aligned}
$$

17.5 problem 5

Internal problem ID [770]

Book: Elementary differential equations and boundary value problems, 10th ed., Boyce and DiPrima
Section: Chapter 7.8, Repeated Eigenvalues. page 436
Problem number: 5.
ODE order: 1.
ODE degree: 1.

Solve

$$x_1'(t) = x_1(t) + x_2(t) + x_3(t)$$
$$x_2'(t) = 2x_1(t) + x_2(t) - x_3(t)$$
$$x_3'(t) = -x_2(t) + x_3(t)$$

✓ Solution by Maple

Time used: 0.031 (sec). Leaf size: 76

```
dsolve([diff(x__1(t),t)=1*x__1(t)+1*x__2(t)+1*x__3(t),diff(x__2(t),t)=2*x__1(t)+1*x__2(t)-1*
```

$$x_1(t) = -\frac{3\,e^{-t}c_1}{2} - c_3 e^{2t}$$
$$x_2(t) = 2\,e^{-t}c_1 - c_2 e^{2t} - e^{2t}c_3 t - c_3 e^{2t}$$
$$x_3(t) = e^{-t}c_1 + c_2 e^{2t} + e^{2t}c_3 t$$

✓ Solution by Mathematica

Time used: 0.01 (sec). Leaf size: 164

```
DSolve[{x1'[t]==1*x1[t]+1*x2[t]+1*x3[t],x2'[t]==2*x1[t]+1*x2[t]-1*x3[t],x3'[t]==0*x1[t]-1*x2
```

$$\text{x1}(t) \to \frac{1}{3}e^{-t}\left(c_1\left(2e^{3t}+1\right) + (c_2+c_3)\left(e^{3t}-1\right)\right)$$
$$\text{x2}(t) \to \frac{1}{9}e^{-t}\left(c_1\left(e^{3t}(6t+4)-4\right) + c_2\left(e^{3t}(3t+5)+4\right) + c_3\left(e^{3t}(3t-4)+4\right)\right)$$
$$\text{x3}(t) \to \frac{1}{9}e^{-t}\left(c_1\left(e^{3t}(2-6t)-2\right) + c_2\left(2-e^{3t}(3t+2)\right) - c_3\left(e^{3t}(3t-7)-2\right)\right)$$

17.6 problem 6

Internal problem ID [771]

Book: Elementary differential equations and boundary value problems, 10th ed., Boyce and DiPrima
Section: Chapter 7.8, Repeated Eigenvalues. page 436
Problem number: 6.
ODE order: 1.
ODE degree: 1.

Solve

$$x_1'(t) = x_2(t) + x_3(t)$$
$$x_2'(t) = x_1(t) + x_3(t)$$
$$x_3'(t) = x_1(t) + x_2(t)$$

✓ Solution by Maple
Time used: 0.031 (sec). Leaf size: 64

```
dsolve([diff(x__1(t),t)=0*x__1(t)+1*x__2(t)+1*x__3(t),diff(x__2(t),t)=1*x__1(t)+0*x__2(t)+1*
```

$$x_1(t) = c_2 e^{-t} + c_3 e^{2t}$$
$$x_2(t) = c_2 e^{-t} + c_3 e^{2t} + e^{-t} c_1$$
$$x_3(t) = -2c_2 e^{-t} + c_3 e^{2t} - e^{-t} c_1$$

✓ Solution by Mathematica
Time used: 0.005 (sec). Leaf size: 124

```
DSolve[{x1'[t]==0*x1[t]+1*x2[t]+1*x3[t],x2'[t]==1*x1[t]+0*x2[t]+1*x3[t],x3'[t]==1*x1[t]+1*x2
```

$$\text{x1}(t) \to \frac{1}{3} e^{-t} \left(c_1 \left(e^{3t} + 2\right) + (c_2 + c_3) \left(e^{3t} - 1\right)\right)$$
$$\text{x2}(t) \to \frac{1}{3} e^{-t} \left(c_1 \left(e^{3t} - 1\right) + c_2 \left(e^{3t} + 2\right) + c_3 \left(e^{3t} - 1\right)\right)$$
$$\text{x3}(t) \to \frac{1}{3} e^{-t} \left(c_1 \left(e^{3t} - 1\right) + c_2 \left(e^{3t} - 1\right) + c_3 \left(e^{3t} + 2\right)\right)$$

17.7 problem 7

Internal problem ID [772]

Book: Elementary differential equations and boundary value problems, 10th ed., Boyce and DiPrima
Section: Chapter 7.8, Repeated Eigenvalues. page 436
Problem number: 7.
ODE order: 1.
ODE degree: 1.

Solve

$$x_1'(t) = x_1(t) - 4x_2(t)$$
$$x_2'(t) = 4x_1(t) - 7x_2(t)$$

With initial conditions

$$[x_1(0) = 3, x_2(0) = 2]$$

✓ Solution by Maple

Time used: 0.015 (sec). Leaf size: 29

```
dsolve([diff(x__1(t),t) = x__1(t)-4*x__2(t), diff(x__2(t),t) = 4*x__1(t)-7*x__2(t), x__1(0)
```

$$x_1(t) = e^{-3t}(4t + 3)$$
$$x_2(t) = \frac{e^{-3t}(16t + 8)}{4}$$

✓ Solution by Mathematica

Time used: 0.005 (sec). Leaf size: 34

```
DSolve[{x1'[t]==1*x1[t]-4*x2[t],x2'[t]==1*x1[t]-4*x2[t]},{x1[0]==3,x2[0]==2},{x1[t],x2[t]},t
```

$$\text{x1}(t) \to \frac{5e^{-3t}}{3} + \frac{4}{3}$$
$$\text{x2}(t) \to \frac{5e^{-3t}}{3} + \frac{1}{3}$$

17.8 problem 8

Internal problem ID [773]

Book: Elementary differential equations and boundary value problems, 10th ed., Boyce and DiPrima
Section: Chapter 7.8, Repeated Eigenvalues. page 436
Problem number: 8.
ODE order: 1.
ODE degree: 1.

Solve

$$x_1'(t) = -\frac{5x_1(t)}{2} + \frac{3x_2(t)}{2}$$
$$x_2'(t) = -\frac{3x_1(t)}{2} + \frac{x_2(t)}{2}$$

With initial conditions

$$[x_1(0) = 3, x_2(0) = -1]$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 29

```
dsolve([diff(x__1(t),t) = -5/2*x__1(t)+3/2*x__2(t), diff(x__2(t),t) = -3/2*x__1(t)+1/2*x__2(
```

$$x_1(t) = e^{-t}(-6t + 3)$$
$$x_2(t) = \frac{e^{-t}(-18t - 3)}{3}$$

✓ Solution by Mathematica

Time used: 0.003 (sec). Leaf size: 31

```
DSolve[{x1'[t]==-5/2*x1[t]+3/2*x2[t],x2'[t]==-3/2*x1[t]+1/2*x2[t]},{x1[0]==3,x2[0]==-1},{x1[
```

$$\text{x1}(t) \to e^{-t}(3 - 6t)$$
$$\text{x2}(t) \to -e^{-t}(6t + 1)$$

17.9 problem 9

Internal problem ID [774]

Book: Elementary differential equations and boundary value problems, 10th ed., Boyce and DiPrima
Section: Chapter 7.8, Repeated Eigenvalues. page 436
Problem number: 9.
ODE order: 1.
ODE degree: 1.

Solve

$$
\begin{aligned}
x_1'(t) &= 2x_1(t) + \frac{3x_2(t)}{2} \\
x_2'(t) &= -\frac{3x_1(t)}{2} - x_2(t)
\end{aligned}
$$

With initial conditions

$$[x_1(0) = 3, x_2(0) = -2]$$

✓ Solution by Maple
Time used: 0.015 (sec). Leaf size: 29

```
dsolve([diff(x__1(t),t) = 2*x__1(t)+3/2*x__2(t), diff(x__2(t),t) = -3/2*x__1(t)-x__2(t), x__
```

$$
\begin{aligned}
x_1(t) &= e^{\frac{t}{2}}\left(\frac{3t}{2} + 3\right) \\
x_2(t) &= -\frac{e^{\frac{t}{2}}\left(\frac{9t}{2} + 6\right)}{3}
\end{aligned}
$$

✓ Solution by Mathematica
Time used: 0.003 (sec). Leaf size: 38

```
DSolve[{x1'[t]==2*x1[t]+3/2*x2[t],x2'[t]==-3/2*x1[t]-1*x2[t]},{x1[0]==3,x2[0]==-2},{x1[t],x2
```

$$
\begin{aligned}
\text{x1}(t) &\to \frac{3}{2}e^{t/2}(t+2) \\
\text{x2}(t) &\to -\frac{1}{2}e^{t/2}(3t+4)
\end{aligned}
$$

17.10 problem 10

Internal problem ID [775]

Book: Elementary differential equations and boundary value problems, 10th ed., Boyce and DiPrima
Section: Chapter 7.8, Repeated Eigenvalues. page 436
Problem number: 10.
ODE order: 1.
ODE degree: 1.

Solve

$$x_1'(t) = 3x_1(t) + 9x_2(t)$$
$$x_2'(t) = -x_1(t) - 3x_2(t)$$

With initial conditions

$$[x_1(0) = 2, x_2(0) = 4]$$

✓ Solution by Maple

Time used: 0.016 (sec). Leaf size: 18

```
dsolve([diff(x__1(t),t) = 3*x__1(t)+9*x__2(t), diff(x__2(t),t) = -x__1(t)-3*x__2(t), x__1(0)
```

$$x_1(t) = 42t + 2$$
$$x_2(t) = 4 - 14t$$

✓ Solution by Mathematica

Time used: 0.003 (sec). Leaf size: 18

```
DSolve[{x1'[t]==3*x1[t]+9*x2[t],x2'[t]==-1*x1[t]-3*x2[t]},{x1[0]==2,x2[0]==4},{x1[t],x2[t]},
```

$$\text{x1}(t) \to 42t + 2$$
$$\text{x2}(t) \to 4 - 14t$$

17.11 problem 11

Internal problem ID [776]

Book: Elementary differential equations and boundary value problems, 10th ed., Boyce and DiPrima
Section: Chapter 7.8, Repeated Eigenvalues. page 436
Problem number: 11.
ODE order: 1.
ODE degree: 1.

Solve

$$x_1'(t) = x_1(t)$$
$$x_2'(t) = -4x_1(t) + x_2(t)$$
$$x_3'(t) = 3x_1(t) + 6x_2(t) + 2x_3(t)$$

With initial conditions

$$[x_1(0) = -1, x_2(0) = 2, x_3(0) = -30]$$

✓ Solution by Maple

Time used: 0.032 (sec). Leaf size: 39

```
dsolve([diff(x__1(t),t) = x__1(t), diff(x__2(t),t) = -4*x__1(t)+x__2(t), diff(x__3(t),t) = 3
```

$$x_1(t) = -e^t$$
$$x_2(t) = (4t + 2)\,e^t$$
$$x_3(t) = -24\,e^t t - 33\,e^t + 3\,e^{2t}$$

✓ Solution by Mathematica

Time used: 0.007 (sec). Leaf size: 39

```
DSolve[{x1'[t]==1*x1[t]+0*x2[t]+0*x3[t],x2'[t]==-4*x1[t]+1*x2[t]+0*x3[t],x3'[t]==3*x1[t]+6*x
```

$$\text{x1}(t) \to -e^t$$
$$\text{x2}(t) \to 2e^t(2t + 1)$$
$$\text{x3}(t) \to 3e^t\left(-8t + e^t - 11\right)$$

17.12 problem 12

Internal problem ID [777]

Book: Elementary differential equations and boundary value problems, 10th ed., Boyce and DiPrima
Section: Chapter 7.8, Repeated Eigenvalues. page 436
Problem number: 12.
ODE order: 1.
ODE degree: 1.

Solve

$$
\begin{aligned}
x_1'(t) &= -\frac{5x_1(t)}{2} + x_2(t) + x_3(t) \\
x_2'(t) &= x_1(t) - \frac{5x_2(t)}{2} + x_3(t) \\
x_3'(t) &= x_1(t) + x_2(t) - \frac{5x_3(t)}{2}
\end{aligned}
$$

With initial conditions

$$[x_1(0) = 2, x_2(0) = 3, x_3(0) = -1]$$

✓ Solution by Maple
Time used: 0.031 (sec). Leaf size: 50

```
dsolve([diff(x__1(t),t) = -5/2*x__1(t)+x__2(t)+x__3(t), diff(x__2(t),t) = x__1(t)-5/2*x__2(t
```

$$
\begin{aligned}
x_1(t) &= \frac{2\,e^{-\frac{7t}{2}}}{3} + \frac{4\,e^{-\frac{t}{2}}}{3} \\
x_2(t) &= \frac{5\,e^{-\frac{7t}{2}}}{3} + \frac{4\,e^{-\frac{t}{2}}}{3} \\
x_3(t) &= -\frac{7\,e^{-\frac{7t}{2}}}{3} + \frac{4\,e^{-\frac{t}{2}}}{3}
\end{aligned}
$$

✓ Solution by Mathematica

Time used: 0.008 (sec). Leaf size: 71

```
DSolve[{x1'[t]==-5/2*x1[t]+1*x2[t]+1*x3[t],x2'[t]==1*x1[t]-5/2*x2[t]+1*x3[t],x3'[t]==1*x1[t]
```

$$
\begin{aligned}
\mathrm{x1}(t) &\to \frac{2}{3}e^{-7t/2}\left(2e^{3t}+1\right)\\
\mathrm{x2}(t) &\to \frac{1}{3}e^{-7t/2}\left(4e^{3t}+5\right)\\
\mathrm{x3}(t) &\to \frac{1}{3}e^{-7t/2}\left(4e^{3t}-7\right)
\end{aligned}
$$

18 Chapter 7.9, Nonhomogeneous Linear Systems. page 447

18.1 problem 1

Internal problem ID [778]

Book: Elementary differential equations and boundary value problems, 10th ed., Boyce and DiPrima
Section: Chapter 7.9, Nonhomogeneous Linear Systems. page 447
Problem number: 1.
ODE order: 1.
ODE degree: 1.

Solve

$$x_1'(t) = 2x_1(t) - x_2(t) + e^t$$
$$x_2'(t) = 3x_1(t) - 2x_2(t) + t$$

✓ Solution by Maple
Time used: 0.031 (sec). Leaf size: 54

```
dsolve([diff(x__1(t),t)=2*x__1(t)-1*x__2(t)+exp(t),diff(x__2(t),t)=3*x__1(t)-2*x__2(t)+t],si
```

$$x_1(t) = \frac{e^{-t}c_1}{3} + c_2e^t + \frac{3e^t t}{2} - \frac{e^t}{4} + t$$
$$x_2(t) = c_2e^t + e^{-t}c_1 + \frac{3e^t t}{2} - \frac{3e^t}{4} + 2t - 1$$

✓ Solution by Mathematica
Time used: 0.123 (sec). Leaf size: 97

```
DSolve[{x1'[t]==2*x1[t]-1*x2[t]+Exp[t],x2'[t]==3*x1[t]-2*x2[t]+t},{x1[t],x2[t]},t,IncludeSin
```

$$\text{x1}(t) \to \frac{1}{4}e^{-t}\left(4e^t t + e^{2t}(6t - 1 + 6c_1 - 2c_2) - 2c_1 + 2c_2\right)$$
$$\text{x2}(t) \to \frac{1}{4}e^{-t}\left(e^t(8t - 4) + e^{2t}(6t - 3 + 6c_1 - 2c_2) - 6c_1 + 6c_2\right)$$

18.2 problem 2

Internal problem ID [779]

Book: Elementary differential equations and boundary value problems, 10th ed., Boyce and DiPrima
Section: Chapter 7.9, Nonhomogeneous Linear Systems. page 447
Problem number: 2.
ODE order: 1.
ODE degree: 1.

Solve

$$x_1'(t) = x_1(t) + \sqrt{3}\, x_2(t) + \mathrm{e}^{t}$$
$$x_2'(t) = \sqrt{3}\, x_1(t) - x_2(t) + \sqrt{3}\, \mathrm{e}^{-t}$$

✓ Solution by Maple
Time used: 0.141 (sec). Leaf size: 71

```
dsolve([diff(x__1(t),t)=1*x__1(t)+sqrt(3)*x__2(t)+exp(t),diff(x__2(t),t)=sqrt(3)*x__1(t)-1*x
```

$$x_1(t) = \sinh(2t)\, c_2 + \cosh(2t)\, c_1 - \frac{5\cosh(t)}{3} + \frac{\sinh(t)}{3}$$

$$x_2(t) = -\frac{\sqrt{3}\,(\cosh(2t)\, c_1 - 2\cosh(2t)\, c_2 - 2\sinh(2t)\, c_1 + \sinh(2t)\, c_2 + \mathrm{e}^{t} + 2\sinh(t) - 2\cosh(t))}{3}$$

✓ Solution by Mathematica

Time used: 2.501 (sec). Leaf size: 313

```
DSolve[{x1'[t]==1*x1[t]+Sqrt[4]*x2[t]+Exp[t],x2'[t]==Sqrt[3]*x1[t]-1*x2[t]+Sqrt[3]*Exp[-t]},
```

$$x1(t) \to \frac{1}{6}\left(-6e^{-t} - \frac{2(6+\sqrt{3})\,e^t}{1+2\sqrt{3}} + \frac{\left(3\left(\sqrt{1+2\sqrt{3}}-1\right)c_1 - 6c_2\right)e^{-\sqrt{1+2\sqrt{3}}t}}{\sqrt{1+2\sqrt{3}}} + \frac{3\left(\left(1+\sqrt{1+2\sqrt{3}}\right)c_1 + 2c_2\right)e^{\sqrt{1+2\sqrt{3}}t}}{\sqrt{1+2\sqrt{3}}}\right)$$

$$x2(t) \to \frac{1}{4}\left(4e^{-t} - 2e^t + \frac{2\left((6+\sqrt{3})\,c_1 + (1+2\sqrt{3})\left(\sqrt{1+2\sqrt{3}}-1\right)c_2\right)e^{\sqrt{1+2\sqrt{3}}t}}{(1+2\sqrt{3})^{3/2}} + \frac{\left(2(1+2\sqrt{3})\left(1+\sqrt{1+2\sqrt{3}}\right)c_2 - 2(6+\sqrt{3})\,c_1\right)e^{-\sqrt{1+2\sqrt{3}}t}}{(1+2\sqrt{3})^{3/2}}\right)$$

18.3 problem 3

Internal problem ID [780]

Book: Elementary differential equations and boundary value problems, 10th ed., Boyce and DiPrima
Section: Chapter 7.9, Nonhomogeneous Linear Systems. page 447
Problem number: 3.
ODE order: 1.
ODE degree: 1.

Solve

$$x_1'(t) = 2x_1(t) - 5x_2(t) - \cos(t)$$
$$x_2'(t) = x_1(t) - 2x_2(t) + \sin(t)$$

✓ Solution by Maple
Time used: 0.063 (sec). Leaf size: 60

```
dsolve([diff(x__1(t),t)=2*x__1(t)-5*x__2(t)-cos(t),diff(x__2(t),t)=1*x__1(t)-2*x__2(t)+sin(t
```

$$x_1(t) = c_2 \sin(t) - \sin(t)\, t + c_1 \cos(t) + 2\cos(t)\, t - \cos(t)$$
$$x_2(t) = \frac{c_1 \sin(t)}{5} + \frac{2c_2 \sin(t)}{5} + \frac{2c_1 \cos(t)}{5} - \frac{c_2 \cos(t)}{5} + \cos(t)\, t - \cos(t)$$

✓ Solution by Mathematica
Time used: 0.069 (sec). Leaf size: 61

```
DSolve[{x1'[t]==2*x1[t]-5*x2[t]-Cos[t],x2'[t]==1*x1[t]-2*x2[t]+Sin[t]},{x1[t],x2[t]},t,Inclu
```

$$\text{x1}(t) \to \left(2t - \frac{1}{2} + c_1\right) \cos(t) - (t - 1 - 2c_1 + 5c_2)\sin(t)$$
$$\text{x2}(t) \to (t - 1 + c_2)\cos(t) + \frac{1}{2}(1 + 2c_1 - 4c_2)\sin(t)$$

18.4 problem 4

Internal problem ID [781]

Book: Elementary differential equations and boundary value problems, 10th ed., Boyce and DiPrima
Section: Chapter 7.9, Nonhomogeneous Linear Systems. page 447
Problem number: 4.
ODE order: 1.
ODE degree: 1.

Solve

$$x_1'(t) = x_1(t) + x_2(t) + \mathrm{e}^{-2t}$$
$$x_2'(t) = 4x_1(t) - 2x_2(t) - 2\,\mathrm{e}^{t}$$

✓ Solution by Maple
Time used: 0.016 (sec). Leaf size: 45

```
dsolve([diff(x__1(t),t)=1*x__1(t)+1*x__2(t)+exp(-2*t),diff(x__2(t),t)=4*x__1(t)-2*x__2(t)-2*
```

$$x_1(t) = c_2\mathrm{e}^{2t} - \frac{c_1\mathrm{e}^{-3t}}{4} + \frac{\mathrm{e}^{t}}{2}$$
$$x_2(t) = c_2\mathrm{e}^{2t} + c_1\mathrm{e}^{-3t} - \mathrm{e}^{-2t}$$

✓ Solution by Mathematica
Time used: 0.585 (sec). Leaf size: 84

```
DSolve[{x1'[t]==1*x1[t]+1*x2[t]+Exp[-2*t],x2'[t]==4*x1[t]-2*x2[t]-2*Exp[t]},{x1[t],x2[t]},t,
```

$$\mathrm{x1}(t) \to \frac{e^t}{2} + \frac{1}{5}(c_1 - c_2)e^{-3t} + \frac{1}{5}(4c_1 + c_2)e^{2t}$$
$$\mathrm{x2}(t) \to \frac{1}{5}e^{-3t}\left(-5e^t + (4c_1 + c_2)e^{5t} - 4c_1 + 4c_2\right)$$

18.5 problem 5

Internal problem ID [782]

Book: Elementary differential equations and boundary value problems, 10th ed., Boyce and DiPrima
Section: Chapter 7.9, Nonhomogeneous Linear Systems. page 447
Problem number: 5.
ODE order: 1.
ODE degree: 1.

Solve

$$x_1'(t) = 4x_1(t) - 2x_2(t) + \frac{1}{t^3}$$
$$x_2'(t) = 8x_1(t) - 4x_2(t) - \frac{1}{t^2}$$

✓ Solution by Maple
Time used: 0.031 (sec). Leaf size: 47

```
dsolve([diff(x__1(t),t)=4*x__1(t)-2*x__2(t)+1/(t^3),diff(x__2(t),t)=8*x__1(t)-4*x__2(t)-1/(t
```

$$x_1(t) = -\frac{1}{2t^2} + \frac{2}{t} - 2\ln(t) + c_1 t + c_2$$
$$x_2(t) = 2c_1 t - 4\ln(t) - \frac{c_1}{2} + 2c_2 + \frac{5}{t}$$

✓ Solution by Mathematica
Time used: 0.006 (sec). Leaf size: 61

```
DSolve[{x1'[t]==4*x1[t]-2*x2[t]+1/(t^3),x2'[t]==8*x1[t]-4*x2[t]-1/(t^2)},{x1[t],x2[t]},t,Inc
```

$$\text{x1}(t) \rightarrow -\frac{1}{2t^2} + \frac{2}{t} - 2\log(t) + 4c_1 t - 2c_2 t - 2 + c_1$$
$$\text{x2}(t) \rightarrow \frac{5}{t} - 4\log(t) + 8c_1 t - 4c_2 t - 4 + c_2$$

18.6 problem 6

Internal problem ID [783]

Book: Elementary differential equations and boundary value problems, 10th ed., Boyce and DiPrima
Section: Chapter 7.9, Nonhomogeneous Linear Systems. page 447
Problem number: 6.
ODE order: 1.
ODE degree: 1.

Solve

$$x_1'(t) = -4x_1(t) + 2x_2(t) + \frac{1}{t}$$
$$x_2'(t) = 2x_1(t) - x_2(t) + \frac{2}{t} + 4$$

✓ Solution by Maple

Time used: 0.031 (sec). Leaf size: 45

```
dsolve([diff(x__1(t),t)=-4*x__1(t)+2*x__2(t)+1/t,diff(x__2(t),t)=2*x__1(t)-1*x__2(t)+2/t+4],
```

$$x_1(t) = \ln(-5t) - \frac{c_1 e^{-5t}}{5} + \frac{8t}{5} + c_2$$
$$x_2(t) = \frac{c_1 e^{-5t}}{10} + 2\ln(-5t) + 2c_2 + \frac{16t}{5} + \frac{4}{5}$$

✓ Solution by Mathematica

Time used: 0.02 (sec). Leaf size: 86

```
DSolve[{x1'[t]==-4*x1[t]+2*x2[t]+1/t,x2'[t]==2*x1[t]-1*x2[t]+2/t+4},{x1[t],x2[t]},t,IncludeS
```

$$\text{x1}(t) \to \frac{1}{25}\left(40t + 25\log(t) + 20c_1e^{-5t} - 10c_2e^{-5t} - 8 + 5c_1 + 10c_2\right)$$
$$\text{x2}(t) \to \frac{1}{25}\left(80t + 50\log(t) - 10c_1e^{-5t} + 5c_2e^{-5t} + 4 + 10c_1 + 20c_2\right)$$

18.7 problem 7

Internal problem ID [784]

Book: Elementary differential equations and boundary value problems, 10th ed., Boyce and DiPrima
Section: Chapter 7.9, Nonhomogeneous Linear Systems. page 447
Problem number: 7.
ODE order: 1.
ODE degree: 1.

Solve

$$x_1'(t) = x_1(t) + x_2(t) + 2\,e^t$$
$$x_2'(t) = 4x_1(t) + x_2(t) - e^t$$

✓ Solution by Maple
Time used: 0.015 (sec). Leaf size: 44

```
dsolve([diff(x__1(t),t)=1*x__1(t)+1*x__2(t)+2*exp(t),diff(x__2(t),t)=4*x__1(t)+1*x__2(t)-exp
```

$$x_1(t) = c_2 e^{3t} + e^{-t} c_1 + \frac{e^t}{4}$$
$$x_2(t) = 2c_2 e^{3t} - 2\,e^{-t} c_1 - 2\,e^t$$

✓ Solution by Mathematica
Time used: 0.004 (sec). Leaf size: 80

```
DSolve[{x1'[t]==1*x1[t]+1*x2[t]+2*Exp[t],x2'[t]==4*x1[t]+1*x2[t]-Exp[t]},{x1[t],x2[t]},t,Inc
```

$$\text{x1}(t) \to \frac{1}{4} e^{-t} \left(e^{2t} + (2c_1 + c_2) e^{4t} + 2c_1 - c_2\right)$$
$$\text{x2}(t) \to \frac{1}{2} e^{-t} \left(-4e^{2t} + (2c_1 + c_2) e^{4t} - 2c_1 + c_2\right)$$

18.8 problem 8

Internal problem ID [785]

Book: Elementary differential equations and boundary value problems, 10th ed., Boyce and DiPrima
Section: Chapter 7.9, Nonhomogeneous Linear Systems. page 447
Problem number: 8.
ODE order: 1.
ODE degree: 1.

Solve

$$x_1'(t) = 2x_1(t) - x_2(t) + \mathrm{e}^t$$
$$x_2'(t) = 3x_1(t) - 2x_2(t) - \mathrm{e}^t$$

✓ Solution by Maple
Time used: 0.016 (sec). Leaf size: 45

```
dsolve([diff(x__1(t),t)=2*x__1(t)-1*x__2(t)+exp(t),diff(x__2(t),t)=3*x__1(t)-2*x__2(t)-exp(t
```

$$x_1(t) = c_2\mathrm{e}^t + \mathrm{e}^{-t}c_1 + 2\,\mathrm{e}^t t$$
$$x_2(t) = c_2\mathrm{e}^t + 3\,\mathrm{e}^{-t}c_1 + 2\,\mathrm{e}^t t - \mathrm{e}^t$$

✓ Solution by Mathematica
Time used: 0.009 (sec). Leaf size: 80

```
DSolve[{x1'[t]==2*x1[t]-1*x2[t]+Exp[t],x2'[t]==3*x1[t]-2*x2[t]-Exp[t]},{x1[t],x2[t]},t,Inclu
```

$$\mathrm{x1}(t) \to \frac{1}{2}e^{-t}\left(e^{2t}(4t - 1 + 3c_1 - c_2) - c_1 + c_2\right)$$
$$\mathrm{x2}(t) \to \frac{1}{2}e^{-t}\left(e^{2t}(4t - 3 + 3c_1 - c_2) - 3c_1 + 3c_2\right)$$

18.9 problem 9

Internal problem ID [786]

Book: Elementary differential equations and boundary value problems, 10th ed., Boyce and DiPrima
Section: Chapter 7.9, Nonhomogeneous Linear Systems. page 447
Problem number: 9.
ODE order: 1.
ODE degree: 1.

Solve

$$x_1'(t) = -\frac{5x_1(t)}{4} + \frac{3x_2(t)}{4} + 2t$$
$$x_2'(t) = \frac{3x_1(t)}{4} - \frac{5x_2(t)}{4} + e^t$$

✓ Solution by Maple
Time used: 0.031 (sec). Leaf size: 51

```
dsolve([diff(x__1(t),t)=-5/4*x__1(t)+3/4*x__2(t)+2*t,diff(x__2(t),t)=3/4*x__1(t)-5/4*x__2(t)
```

$$x_1(t) = c_2 e^{-2t} + c_1 e^{-\frac{t}{2}} - \frac{17}{4} + \frac{e^t}{6} + \frac{5t}{2}$$
$$x_2(t) = -c_2 e^{-2t} + c_1 e^{-\frac{t}{2}} + \frac{e^t}{2} - \frac{15}{4} + \frac{3t}{2}$$

✓ Solution by Mathematica
Time used: 0.349 (sec). Leaf size: 101

```
DSolve[{x1'[t]==-5/4*x1[t]+3/4*x2[t]+2*t,x2'[t]==3/4*x1[t]-5/4*x2[t]+Exp[t]},{x1[t],x2[t]},t
```

$$\text{x1}(t) \to \frac{1}{12}\left(30t + 2e^t + 6(c_1 - c_2)e^{-2t} + 6(c_1 + c_2)e^{-t/2} - 51\right)$$
$$\text{x2}(t) \to \frac{1}{4}e^{-2t}\left(3e^{2t}(2t - 5) + 2e^{3t} + 2(c_1 + c_2)e^{3t/2} - 2c_1 + 2c_2\right)$$

18.10 problem 10

Internal problem ID [787]

Book: Elementary differential equations and boundary value problems, 10th ed., Boyce and DiPrima
Section: Chapter 7.9, Nonhomogeneous Linear Systems. page 447
Problem number: 10.
ODE order: 1.
ODE degree: 1.

Solve

$$x_1'(t) = -3x_1(t) + \sqrt{2}\, x_2(t) + \mathrm{e}^{-t}$$
$$x_2'(t) = \sqrt{2}\, x_1(t) - 2x_2(t) - \mathrm{e}^{-t}$$

✓ Solution by Maple

Time used: 0.063 (sec). Leaf size: 90

```
dsolve([diff(x__1(t),t)=-3*x__1(t)+sqrt(2)*x__2(t)+exp(-t),diff(x__2(t),t)=sqrt(2)*x__1(t)-2
```

$$x_1(t) = c_2\mathrm{e}^{-t} + \mathrm{e}^{-4t}c_1 - \frac{t\,\mathrm{e}^{-t}\sqrt{2}}{3} + \frac{t\,\mathrm{e}^{-t}}{3}$$
$$x_2(t) = -\frac{2t\,\mathrm{e}^{-t}}{3} - \frac{\mathrm{e}^{-t}}{3} + \sqrt{2}\,\mathrm{e}^{-t}c_2 + \frac{t\,\mathrm{e}^{-t}\sqrt{2}}{3} - \frac{\sqrt{2}\,\mathrm{e}^{-4t}c_1}{2} - \frac{\sqrt{2}\,\mathrm{e}^{-t}}{3}$$

✓ Solution by Mathematica

Time used: 0.035 (sec). Leaf size: 128

```
DSolve[{x1'[t]==-3*x1[t]+Sqrt[2]*x2[t]+Exp[-t],x2'[t]==Sqrt[2]*x1[t]-2*x2[t]-Exp[-t]},{x1[t]
```

$$\text{x1}(t) \to \frac{1}{9}e^{-4t}\left(e^{3t}\left(-3\left(\sqrt{2}-1\right)t + \sqrt{2} + 2 + 3c_1 + 3\sqrt{2}c_2\right) + 6c_1 - 3\sqrt{2}c_2\right)$$
$$\text{x2}(t) \to \frac{1}{9}e^{-4t}\left(e^{3t}\left(3\left(\sqrt{2}-2\right)t - \sqrt{2} - 1 + 3\sqrt{2}c_1 + 6c_2\right) - 3\sqrt{2}c_1 + 3c_2\right)$$

18.11 problem 11

Internal problem ID [788]

Book: Elementary differential equations and boundary value problems, 10th ed., Boyce and DiPrima
Section: Chapter 7.9, Nonhomogeneous Linear Systems. page 447
Problem number: 11.
ODE order: 1.
ODE degree: 1.

Solve

$$x_1'(t) = 2x_1(t) - 5x_2(t)$$
$$x_2'(t) = x_1(t) - 2x_2(t) + \cos(t)$$

✓ Solution by Maple
Time used: 0.031 (sec). Leaf size: 57

```
dsolve([diff(x__1(t),t)=2*x__1(t)-5*x__2(t)+0,diff(x__2(t),t)=1*x__1(t)-2*x__2(t)+cos(t)],si
```

$$x_1(t) = c_2 \sin(t) + c_1 \cos(t) - \frac{5 \sin(t) t}{2}$$
$$x_2(t) = -\frac{c_2 \cos(t)}{5} + \frac{c_1 \sin(t)}{5} + \frac{\cos(t) t}{2} + \frac{\sin(t)}{2} + \frac{2c_2 \sin(t)}{5} + \frac{2c_1 \cos(t)}{5} - \sin(t) t$$

✓ Solution by Mathematica
Time used: 0.024 (sec). Leaf size: 60

```
DSolve[{x1'[t]==2*x1[t]-5*x2[t]+0,x2'[t]==1*x1[t]-2*x2[t]-Cos[t]},{x1[t],x2[t]},t,IncludeSin
```

$$\text{x1}(t) \to \left(\frac{5}{2} + c_1\right) \cos(t) + \frac{1}{2}(5t + 4c_1 - 10c_2) \sin(t)$$
$$\text{x2}(t) \to \left(-\frac{t}{2} + 1 + c_2\right) \cos(t) + (t + c_1 - 2c_2) \sin(t)$$

18.12 problem 12

Internal problem ID [789]

Book: Elementary differential equations and boundary value problems, 10th ed., Boyce and DiPrima
Section: Chapter 7.9, Nonhomogeneous Linear Systems. page 447
Problem number: 12.
ODE order: 1.
ODE degree: 1.

Solve

$$
\begin{aligned}
x_1'(t) &= 2x_1(t) - 5x_2(t) + \csc(t) \\
x_2'(t) &= x_1(t) - 2x_2(t) + \sec(t)
\end{aligned}
$$

✓ Solution by Maple

Time used: 0.235 (sec). Leaf size: 113

```
dsolve([diff(x__1(t),t)=2*x__1(t)-5*x__2(t)+csc(t),diff(x__2(t),t)=1*x__1(t)-2*x__2(t)+sec(t
```

$$
\begin{aligned}
x_1(t) &= \ln(\sin(t))\cos(t) - 5\cos(t)\ln(\cos(t)) + c_1\cos(t) - 2\cos(t)\,t \\
&\quad + 2\ln(\sin(t))\sin(t) + c_2\sin(t) - 4\sin(t)\,t + \cos(t) \\
x_2(t) &= -2\cos(t)\ln(\cos(t)) + \frac{2c_1\cos(t)}{5} - \frac{c_2\cos(t)}{5} \\
&\quad + \ln(\sin(t))\sin(t) - \sin(t)\ln(\cos(t)) + \frac{c_1\sin(t)}{5} \\
&\quad + \frac{2c_2\sin(t)}{5} - 2\sin(t)\,t - \frac{\cos(t)^2}{5\sin(t)} + \frac{2\cos(t)}{5} + \frac{\csc(t)}{5}
\end{aligned}
$$

✓ Solution by Mathematica

Time used: 0.018 (sec). Leaf size: 79

```
DSolve[{x1'[t]==2*x1[t]-5*x2[t]+Csc[t],x2'[t]==1*x1[t]-2*x2[t]+Sec[t]},{x1[t],x2[t]},t,Inclu
```

$$
\begin{aligned}
\text{x1}(t) &\to \sin(t)(-4t + 2\log(\sin(t)) + 2c_1 - 5c_2) \\
&\quad + \cos(t)(-2t + \log(\sin(t)) - 5\log(\cos(t)) + c_1) \\
\text{x2}(t) &\to \cos(t)(-2\log(\cos(t)) + c_2) + \sin(t)(-2t + \log(\sin(t)) - \log(\cos(t)) + c_1 - 2c_2)
\end{aligned}
$$

18.13 problem 13

Internal problem ID [790]

Book: Elementary differential equations and boundary value problems, 10th ed., Boyce and DiPrima
Section: Chapter 7.9, Nonhomogeneous Linear Systems. page 447
Problem number: 13.
ODE order: 1.
ODE degree: 1.

Solve

$$x_1'(t) = -\frac{x_1(t)}{2} - \frac{x_2(t)}{8} + \frac{e^{-\frac{t}{2}}}{2}$$
$$x_2'(t) = 2x_1(t) - \frac{x_2(t)}{2}$$

✓ Solution by Maple
Time used: 0.016 (sec). Leaf size: 47

```
dsolve([diff(x__1(t),t)=-1/2*x__1(t)-1/8*x__2(t)+1/2*exp(-t/2),diff(x__2(t),t)=2*x__1(t)-1/2
```

$$x_1(t) = \frac{e^{-\frac{t}{2}}\left(c_2\cos\left(\frac{t}{2}\right) - c_1\sin\left(\frac{t}{2}\right)\right)}{4}$$
$$x_2(t) = e^{-\frac{t}{2}}\left(4 + \cos\left(\frac{t}{2}\right)c_1 + \sin\left(\frac{t}{2}\right)c_2\right)$$

✓ Solution by Mathematica
Time used: 0.022 (sec). Leaf size: 69

```
DSolve[{x1'[t]==-1/2*x1[t]-1/8*x2[t]+1/2*Exp[-t/2],x2'[t]==2*x1[t]-1/2*x2[t]+0},{x1[t],x2[t]
```

$$\text{x1}(t) \to \frac{1}{4}e^{-t/2}\left(4c_1\cos\left(\frac{t}{2}\right) - c_2\sin\left(\frac{t}{2}\right)\right)$$
$$\text{x2}(t) \to e^{-t/2}\left(c_2\cos\left(\frac{t}{2}\right) + 4c_1\sin\left(\frac{t}{2}\right) + 4\right)$$

18.14 problem 18

Internal problem ID [791]

Book: Elementary differential equations and boundary value problems, 10th ed., Boyce and DiPrima
Section: Chapter 7.9, Nonhomogeneous Linear Systems. page 447
Problem number: 18.
ODE order: 1.
ODE degree: 1.

Solve

$$x_1'(t) = -2x_1(t) + x_2(t) + 2\,\mathrm{e}^{-t}$$
$$x_2'(t) = x_1(t) - 2x_2(t) + 3t$$

With initial conditions

$$[x_1(0) = \alpha_1, x_2(0) = \alpha_2]$$

✓ Solution by Maple

Time used: 0.031 (sec). Leaf size: 93

```
dsolve([diff(x__1(t),t) = -2*x__1(t)+x__2(t)+2*exp(-t), diff(x__2(t),t) = x__1(t)-2*x__2(t)+
```

$$x_1(t) = \left(\frac{3}{2} + \frac{\alpha_2}{2} + \frac{\alpha_1}{2}\right)\mathrm{e}^{-t} - \left(\frac{2}{3} + \frac{\alpha_2}{2} - \frac{\alpha_1}{2}\right)\mathrm{e}^{-3t} + \frac{\mathrm{e}^{-t}}{2} + t\,\mathrm{e}^{-t} - \frac{4}{3} + t$$
$$x_2(t) = \left(\frac{3}{2} + \frac{\alpha_2}{2} + \frac{\alpha_1}{2}\right)\mathrm{e}^{-t} + \left(\frac{2}{3} + \frac{\alpha_2}{2} - \frac{\alpha_1}{2}\right)\mathrm{e}^{-3t} + t\,\mathrm{e}^{-t} + 2t - \frac{5}{3} - \frac{\mathrm{e}^{-t}}{2}$$

✓ Solution by Mathematica

Time used: 0.061 (sec). Leaf size: 122

```
DSolve[{x1'[t]==-2*x1[t]+1*x2[t]+2*Exp[-t],x2'[t]==1*x1[t]-2*x2[t]+3*t},{x1[0]==a1,x2[0]==a2
```

$$\mathrm{x1}(t) \to \frac{1}{6}e^{-3t}\left(3\mathrm{a1}\left(e^{2t}+1\right) + 3\mathrm{a2}\left(e^{2t}-1\right) + 12e^{2t} - 8e^{3t} + 6e^{2t}t + 6e^{3t}t - 4\right)$$
$$\mathrm{x2}(t) \to \frac{1}{6}e^{-3t}\left(3\mathrm{a1}\left(e^{2t}-1\right) + 3\mathrm{a2}\left(e^{2t}+1\right) + 6e^{2t}(t+1) + 2e^{3t}(6t-5) + 4\right)$$

19 Chapter 9.1, The Phase Plane: Linear Systems. page 505

19.1 problem 1

Internal problem ID [792]

Book: Elementary differential equations and boundary value problems, 10th ed., Boyce and DiPrima
Section: Chapter 9.1, The Phase Plane: Linear Systems. page 505
Problem number: 1.
ODE order: 1.
ODE degree: 1.

Solve

$$x_1'(t) = 3x_1(t) - 2x_2(t)$$
$$x_2'(t) = 2x_1(t) - 2x_2(t)$$

✓ Solution by Maple
Time used: 0.0 (sec). Leaf size: 36

```
dsolve([diff(x__1(t),t)=3*x__1(t)-2*x__2(t),diff(x__2(t),t)=2*x__1(t)-2*x__2(t)],singsol=all
```

$$x_1(t) = \mathrm{e}^{-t} c_1 + c_2 \mathrm{e}^{2t}$$
$$x_2(t) = 2\,\mathrm{e}^{-t} c_1 + \frac{c_2 \mathrm{e}^{2t}}{2}$$

✓ Solution by Mathematica
Time used: 0.003 (sec). Leaf size: 73

```
DSolve[{x1'[t]==3*x1[t]-2*x2[t],x2'[t]==2*x1[t]-2*x2[t]},{x1[t],x2[t]},t,IncludeSingularSolu
```

$$\mathrm{x1}(t) \to \frac{1}{3} e^{-t} \left(c_1 \left(4e^{3t} - 1\right) - 2c_2 \left(e^{3t} - 1\right)\right)$$
$$\mathrm{x2}(t) \to \frac{1}{3} e^{-t} \left(2c_1 \left(e^{3t} - 1\right) - c_2 \left(e^{3t} - 4\right)\right)$$

19.2 problem 2

Internal problem ID [793]

Book: Elementary differential equations and boundary value problems, 10th ed., Boyce and DiPrima
Section: Chapter 9.1, The Phase Plane: Linear Systems. page 505
Problem number: 2.
ODE order: 1.
ODE degree: 1.

Solve

$$x_1'(t) = 5x_1(t) - x_2(t)$$
$$x_2'(t) = 3x_1(t) + x_2(t)$$

✓ Solution by Maple
Time used: 0.015 (sec). Leaf size: 35

```
dsolve([diff(x__1(t),t)=5*x__1(t)-1*x__2(t),diff(x__2(t),t)=3*x__1(t)+1*x__2(t)],singsol=all
```

$$x_1(t) = c_1 e^{4t} + c_2 e^{2t}$$
$$x_2(t) = c_1 e^{4t} + 3c_2 e^{2t}$$

✓ Solution by Mathematica
Time used: 0.003 (sec). Leaf size: 73

```
DSolve[{x1'[t]==5*x1[t]-1*x2[t],x2'[t]==3*x1[t]+1*x2[t]},{x1[t],x2[t]},t,IncludeSingularSolu
```

$$\text{x1}(t) \to \frac{1}{2}e^{2t}\left(c_1\left(3e^{2t} - 1\right) - c_2\left(e^{2t} - 1\right)\right)$$
$$\text{x2}(t) \to \frac{1}{2}e^{2t}\left(3c_1\left(e^{2t} - 1\right) - c_2\left(e^{2t} - 3\right)\right)$$

19.3 problem 3

Internal problem ID [794]

Book: Elementary differential equations and boundary value problems, 10th ed., Boyce and DiPrima
Section: Chapter 9.1, The Phase Plane: Linear Systems. page 505
Problem number: 3.
ODE order: 1.
ODE degree: 1.

Solve

$$
\begin{aligned}
x_1'(t) &= 2x_1(t) - x_2(t) \\
x_2'(t) &= 3x_1(t) - 2x_2(t)
\end{aligned}
$$

✓ Solution by Maple

Time used: 0.016 (sec). Leaf size: 31

```
dsolve([diff(x__1(t),t)=2*x__1(t)-1*x__2(t),diff(x__2(t),t)=3*x__1(t)-2*x__2(t)],singsol=all
```

$$
\begin{aligned}
x_1(t) &= c_1 e^t + c_2 e^{-t} \\
x_2(t) &= c_1 e^t + 3c_2 e^{-t}
\end{aligned}
$$

✓ Solution by Mathematica

Time used: 0.003 (sec). Leaf size: 73

```
DSolve[{x1'[t]==2*x1[t]-1*x2[t],x2'[t]==3*x1[t]-2*x2[t]},{x1[t],x2[t]},t,IncludeSingularSolu
```

$$
\begin{aligned}
\mathrm{x1}(t) &\to \frac{1}{2}e^{-t}\left(c_1\left(3e^{2t}-1\right)-c_2\left(e^{2t}-1\right)\right) \\
\mathrm{x2}(t) &\to \frac{1}{2}e^{-t}\left(3c_1\left(e^{2t}-1\right)-c_2\left(e^{2t}-3\right)\right)
\end{aligned}
$$

19.4 problem 4

Internal problem ID [795]

Book: Elementary differential equations and boundary value problems, 10th ed., Boyce and DiPrima
Section: Chapter 9.1, The Phase Plane: Linear Systems. page 505
Problem number: 4.
ODE order: 1.
ODE degree: 1.

Solve

$$x_1'(t) = x_1(t) - 4x_2(t)$$
$$x_2'(t) = 4x_1(t) - 7x_2(t)$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 35

```
dsolve([diff(x__1(t),t)=1*x__1(t)-4*x__2(t),diff(x__2(t),t)=4*x__1(t)-7*x__2(t)],singsol=all
```

$$x_1(t) = \mathrm{e}^{-3t}(c_2t + c_1)$$
$$x_2(t) = \frac{\mathrm{e}^{-3t}(4c_2t + 4c_1 - c_2)}{4}$$

✓ Solution by Mathematica

Time used: 0.002 (sec). Leaf size: 46

```
DSolve[{x1'[t]==1*x1[t]-4*x2[t],x2'[t]==4*x1[t]-7*x2[t]},{x1[t],x2[t]},t,IncludeSingularSolu
```

$$\text{x1}(t) \to e^{-3t}(4c_1t - 4c_2t + c_1)$$
$$\text{x2}(t) \to e^{-3t}(4(c_1 - c_2)t + c_2)$$

19.5 problem 5

Internal problem ID [796]

Book: Elementary differential equations and boundary value problems, 10th ed., Boyce and DiPrima
Section: Chapter 9.1, The Phase Plane: Linear Systems. page 505
Problem number: 5.
ODE order: 1.
ODE degree: 1.

Solve

$$x_1'(t) = x_1(t) - 5x_2(t)$$
$$x_2'(t) = x_1(t) - 3x_2(t)$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 48

```
dsolve([diff(x__1(t),t)=1*x__1(t)-5*x__2(t),diff(x__2(t),t)=1*x__1(t)-3*x__2(t)],singsol=all
```

$$x_1(t) = e^{-t}(c_1 \sin(t) + c_2 \cos(t))$$
$$x_2(t) = \frac{e^{-t}(-c_1 \cos(t) + c_2 \sin(t) + 2c_1 \sin(t) + 2c_2 \cos(t))}{5}$$

✓ Solution by Mathematica

Time used: 0.005 (sec). Leaf size: 54

```
DSolve[{x1'[t]==1*x1[t]-5*x2[t],x2'[t]==1*x1[t]-3*x2[t]},{x1[t],x2[t]},t,IncludeSingularSolu
```

$$\text{x1}(t) \rightarrow e^{-t}(c_1 \cos(t) + (2c_1 - 5c_2)\sin(t))$$
$$\text{x2}(t) \rightarrow e^{-t}(c_2 \cos(t) + (c_1 - 2c_2)\sin(t))$$

19.6 problem 6

Internal problem ID [797]

Book: Elementary differential equations and boundary value problems, 10th ed., Boyce and DiPrima
Section: Chapter 9.1, The Phase Plane: Linear Systems. page 505
Problem number: 6.
ODE order: 1.
ODE degree: 1.

Solve

$$x_1'(t) = 2x_1(t) - 5x_2(t)$$
$$x_2'(t) = x_1(t) - 2x_2(t)$$

✓ Solution by Maple
Time used: 0.016 (sec). Leaf size: 38

```
dsolve([diff(x__1(t),t)=2*x__1(t)-5*x__2(t),diff(x__2(t),t)=1*x__1(t)-2*x__2(t)],singsol=all
```

$$x_1(t) = c_1 \sin(t) + c_2 \cos(t)$$
$$x_2(t) = -\frac{c_1 \cos(t)}{5} + \frac{c_2 \sin(t)}{5} + \frac{2c_1 \sin(t)}{5} + \frac{2c_2 \cos(t)}{5}$$

✓ Solution by Mathematica
Time used: 0.004 (sec). Leaf size: 41

```
DSolve[{x1'[t]==2*x1[t]-5*x2[t],x2'[t]==1*x1[t]-2*x2[t]},{x1[t],x2[t]},t,IncludeSingularSolu
```

$$\text{x1}(t) \to c_1(2\sin(t) + \cos(t)) - 5c_2 \sin(t)$$
$$\text{x2}(t) \to c_2 \cos(t) + (c_1 - 2c_2) \sin(t)$$

19.7 problem 7

Internal problem ID [798]

Book: Elementary differential equations and boundary value problems, 10th ed., Boyce and DiPrima
Section: Chapter 9.1, The Phase Plane: Linear Systems. page 505
Problem number: 7.
ODE order: 1.
ODE degree: 1.

Solve

$$x_1'(t) = 3x_1(t) - 2x_2(t)$$
$$x_2'(t) = 4x_1(t) - x_2(t)$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 56

```
dsolve([diff(x__1(t),t)=3*x__1(t)-2*x__2(t),diff(x__2(t),t)=4*x__1(t)-1*x__2(t)],singsol=all
```

$$x_1(t) = e^t(c_1 \sin(2t) + c_2 \cos(2t))$$
$$x_2(t) = -e^t(c_1 \cos(2t) - c_2 \cos(2t) - c_1 \sin(2t) - c_2 \sin(2t))$$

✓ Solution by Mathematica

Time used: 0.005 (sec). Leaf size: 58

```
DSolve[{x1'[t]==3*x1[t]-2*x2[t],x2'[t]==4*x1[t]-1*x2[t]},{x1[t],x2[t]},t,IncludeSingularSolu
```

$$\text{x1}(t) \to e^t(c_1 \cos(2t) + (c_1 - c_2)\sin(2t))$$
$$\text{x2}(t) \to e^t(c_2 \cos(2t) + (2c_1 - c_2)\sin(2t))$$

19.8 problem 8

Internal problem ID [799]

Book: Elementary differential equations and boundary value problems, 10th ed., Boyce and DiPrima
Section: Chapter 9.1, The Phase Plane: Linear Systems. page 505
Problem number: 8.
ODE order: 1.
ODE degree: 1.

Solve

$$
\begin{aligned}
x_1'(t) &= -x_1(t) - x_2(t) \\
x_2'(t) &= -\frac{5x_2(t)}{2}
\end{aligned}
$$

✓ Solution by Maple

Time used: 0.016 (sec). Leaf size: 28

```
dsolve([diff(x__1(t),t)=-1*x__1(t)-1*x__2(t),diff(x__2(t),t)=0*x__1(t)-25/10*x__2(t)],singso
```

$$
\begin{aligned}
x_1(t) &= \frac{2c_2 \mathrm{e}^{-\frac{5t}{2}}}{3} + \mathrm{e}^{-t}c_1 \\
x_2(t) &= c_2 \mathrm{e}^{-\frac{5t}{2}}
\end{aligned}
$$

✓ Solution by Mathematica

Time used: 0.006 (sec). Leaf size: 47

```
DSolve[{x1'[t]==-1*x1[t]-1*x2[t],x2'[t]==0*x1[t]-25/10*x2[t]},{x1[t],x2[t]},t,IncludeSingula
```

$$
\begin{aligned}
\mathrm{x1}(t) &\rightarrow \left(c_1 - \frac{2c_2}{3}\right)e^{-t} + \frac{2}{3}c_2 e^{-5t/2} \\
\mathrm{x2}(t) &\rightarrow c_2 e^{-5t/2}
\end{aligned}
$$

19.9 problem 9

Internal problem ID [800]

Book: Elementary differential equations and boundary value problems, 10th ed., Boyce and DiPrima
Section: Chapter 9.1, The Phase Plane: Linear Systems. page 505
Problem number: 9.
ODE order: 1.
ODE degree: 1.

Solve

$$x_1'(t) = 3x_1(t) - 4x_2(t)$$
$$x_2'(t) = x_1(t) - x_2(t)$$

✓ Solution by Maple
Time used: 0.016 (sec). Leaf size: 31

```
dsolve([diff(x__1(t),t)=3*x__1(t)-4*x__2(t),diff(x__2(t),t)=1*x__1(t)-1*x__2(t)],singsol=all
```

$$x_1(t) = e^t(c_2t + c_1)$$
$$x_2(t) = \frac{e^t(2c_2t + 2c_1 - c_2)}{4}$$

✓ Solution by Mathematica
Time used: 0.002 (sec). Leaf size: 41

```
DSolve[{x1'[t]==3*x1[t]-4*x2[t],x2'[t]==1*x1[t]-1*x2[t]},{x1[t],x2[t]},t,IncludeSingularSolu
```

$$\text{x1}(t) \to e^t(2c_1t - 4c_2t + c_1)$$
$$\text{x2}(t) \to e^t((c_1 - 2c_2)t + c_2)$$

19.10 problem 10

Internal problem ID [801]

Book: Elementary differential equations and boundary value problems, 10th ed., Boyce and DiPrima
Section: Chapter 9.1, The Phase Plane: Linear Systems. page 505
Problem number: 10.
ODE order: 1.
ODE degree: 1.

Solve

$$x_1'(t) = x_1(t) + 2x_2(t)$$
$$x_2'(t) = -5x_1(t)$$

✓ Solution by Maple

Time used: 0.016 (sec). Leaf size: 84

```
dsolve([diff(x__1(t),t)=1*x__1(t)+2*x__2(t),diff(x__2(t),t)=-5*x__1(t)-0*x__2(t)],singsol=al
```

$$x_1(t) = \frac{e^{\frac{t}{2}} \left(\sin\left(\frac{\sqrt{39}\, t}{2}\right) \sqrt{39}\, c_2 - \cos\left(\frac{\sqrt{39}\, t}{2}\right) \sqrt{39}\, c_1 - \sin\left(\frac{\sqrt{39}\, t}{2}\right) c_1 - \cos\left(\frac{\sqrt{39}\, t}{2}\right) c_2\right)}{10}$$
$$x_2(t) = e^{\frac{t}{2}} \left(\sin\left(\frac{\sqrt{39}\, t}{2}\right) c_1 + \cos\left(\frac{\sqrt{39}\, t}{2}\right) c_2\right)$$

✓ Solution by Mathematica

Time used: 0.007 (sec). Leaf size: 54

```
DSolve[{x1'[t]==1*x1[t]+2*x2[t],x2'[t]==-5*x1[t]-1*x2[t]},{x1[t],x2[t]},t,IncludeSingularSol
```

$$\text{x1}(t) \to c_1 \cos(3t) + \frac{1}{3}(c_1 + 2c_2)\sin(3t)$$
$$\text{x2}(t) \to c_2 \cos(3t) - \frac{1}{3}(5c_1 + c_2)\sin(3t)$$

19.11 problem 11

Internal problem ID [802]

Book: Elementary differential equations and boundary value problems, 10th ed., Boyce and DiPrima
Section: Chapter 9.1, The Phase Plane: Linear Systems. page 505
Problem number: 11.
ODE order: 1.
ODE degree: 1.

Solve

$$x_1'(t) = -x_1(t)$$
$$x_2'(t) = -x_2(t)$$

✓ Solution by Maple

Time used: 0.016 (sec). Leaf size: 20

```
dsolve([diff(x__1(t),t)=-1*x__1(t)-0*x__2(t),diff(x__2(t),t)=0*x__1(t)-1*x__2(t)],singsol=al
```

$$x_1(t) = c_2 e^{-t}$$
$$x_2(t) = e^{-t} c_1$$

✓ Solution by Mathematica

Time used: 0.034 (sec). Leaf size: 65

```
DSolve[{x1'[t]==-1*x1[t]-0*x2[t],x2'[t]==0*x1[t]-1*x2[t]},{x1[t],x2[t]},t,IncludeSingularSol
```

$$\text{x1}(t) \to c_1 e^{-t}$$
$$\text{x2}(t) \to c_2 e^{-t}$$
$$\text{x1}(t) \to c_1 e^{-t}$$
$$\text{x2}(t) \to 0$$
$$\text{x1}(t) \to 0$$
$$\text{x2}(t) \to c_2 e^{-t}$$
$$\text{x1}(t) \to 0$$
$$\text{x2}(t) \to 0$$

19.12 problem 12

Internal problem ID [803]

Book: Elementary differential equations and boundary value problems, 10th ed., Boyce and DiPrima
Section: Chapter 9.1, The Phase Plane: Linear Systems. page 505
Problem number: 12.
ODE order: 1.
ODE degree: 1.

Solve

$$x_1'(t) = 2x_1(t) - \frac{5x_2(t)}{2}$$
$$x_2'(t) = \frac{9x_1(t)}{5} - x_2(t)$$

✓ Solution by Maple
Time used: 0.0 (sec). Leaf size: 58

```
dsolve([diff(x__1(t),t)=2*x__1(t)-5/2*x__2(t),diff(x__2(t),t)=9/5*x__1(t)-1*x__2(t)],singsol
```

$$x_1(t) = e^{\frac{t}{2}}\left(\sin\left(\frac{3t}{2}\right)c_1 + \cos\left(\frac{3t}{2}\right)c_2\right)$$
$$x_2(t) = \frac{3\,e^{\frac{t}{2}}\left(\sin\left(\frac{3t}{2}\right)c_1 + \sin\left(\frac{3t}{2}\right)c_2 - \cos\left(\frac{3t}{2}\right)c_1 + \cos\left(\frac{3t}{2}\right)c_2\right)}{5}$$

✓ Solution by Mathematica
Time used: 0.005 (sec). Leaf size: 84

```
DSolve[{x1'[t]==2*x1[t]-5/2*x2[t],x2'[t]==9/5*x1[t]-1*x2[t]},{x1[t],x2[t]},t,IncludeSingular
```

$$\text{x1}(t) \to \frac{1}{3}e^{t/2}\left(3c_1\cos\left(\frac{3t}{2}\right) + (3c_1 - 5c_2)\sin\left(\frac{3t}{2}\right)\right)$$
$$\text{x2}(t) \to \frac{1}{5}e^{t/2}\left(5c_2\cos\left(\frac{3t}{2}\right) + (6c_1 - 5c_2)\sin\left(\frac{3t}{2}\right)\right)$$

19.13 problem 13

Internal problem ID [804]

Book: Elementary differential equations and boundary value problems, 10th ed., Boyce and DiPrima
Section: Chapter 9.1, The Phase Plane: Linear Systems. page 505
Problem number: 13.
ODE order: 1.
ODE degree: 1.

Solve

$$x_1'(t) = x_1(t) + x_2(t) - 2$$
$$x_2'(t) = x_1(t) - x_2(t)$$

✓ Solution by Maple
Time used: 0.016 (sec). Leaf size: 72

```
dsolve([diff(x__1(t),t)=1*x__1(t)+1*x__2(t)-2,diff(x__2(t),t)=1*x__1(t)-1*x__2(t)],singsol=a
```

$$x_1(t) = e^{\sqrt{2}\,t}c_2 + e^{-\sqrt{2}\,t}c_1 + 1$$
$$x_2(t) = \sqrt{2}\,e^{\sqrt{2}\,t}c_2 - \sqrt{2}\,e^{-\sqrt{2}\,t}c_1 - e^{\sqrt{2}\,t}c_2 - e^{-\sqrt{2}\,t}c_1 + 1$$

✓ Solution by Mathematica
Time used: 0.097 (sec). Leaf size: 160

```
DSolve[{x1'[t]==1*x1[t]+1*x2[t]-2,x2'[t]==1*x1[t]-1*x2[t]},{x1[t],x2[t]},t,IncludeSingularSo
```

$$\text{x1}(t) \to \frac{1}{4}e^{-\sqrt{2}t}\left(4e^{\sqrt{2}t} + \left(\left(2+\sqrt{2}\right)c_1 + \sqrt{2}c_2\right)e^{2\sqrt{2}t} - \left(\left(\sqrt{2}-2\right)c_1\right) - \sqrt{2}c_2\right)$$
$$\text{x2}(t) \to \frac{1}{4}e^{-\sqrt{2}t}\left(4e^{\sqrt{2}t} + \left(\sqrt{2}c_1 - \left(\sqrt{2}-2\right)c_2\right)e^{2\sqrt{2}t} - \sqrt{2}c_1 + \left(2+\sqrt{2}\right)c_2\right)$$

19.14 problem 14

Internal problem ID [805]

Book: Elementary differential equations and boundary value problems, 10th ed., Boyce and DiPrima
Section: Chapter 9.1, The Phase Plane: Linear Systems. page 505
Problem number: 14.
ODE order: 1.
ODE degree: 1.

Solve

$$x_1'(t) = -2x_1(t) + x_2(t) - 2$$
$$x_2'(t) = x_1(t) - 2x_2(t) + 1$$

✓ Solution by Maple
Time used: 0.015 (sec). Leaf size: 36

```
dsolve([diff(x__1(t),t)=-2*x__1(t)+1*x__2(t)-2,diff(x__2(t),t)=1*x__1(t)-2*x__2(t)+1],singso
```

$$x_1(t) = c_2 e^{-t} + c_1 e^{-3t} - 1$$
$$x_2(t) = c_2 e^{-t} - c_1 e^{-3t}$$

✓ Solution by Mathematica
Time used: 0.022 (sec). Leaf size: 72

```
DSolve[{x1'[t]==-2*x1[t]+1*x2[t]-2,x2'[t]==1*x1[t]-2*x2[t]+1},{x1[t],x2[t]},t,IncludeSingula
```

$$\text{x1}(t) \to \frac{1}{2}e^{-3t}\left(-2e^{3t} + (c_1 + c_2)e^{2t} + c_1 - c_2\right)$$
$$\text{x2}(t) \to \frac{1}{2}e^{-3t}\left(c_1\left(e^{2t} - 1\right) + c_2\left(e^{2t} + 1\right)\right)$$

19.15 problem 15

Internal problem ID [806]

Book: Elementary differential equations and boundary value problems, 10th ed., Boyce and DiPrima
Section: Chapter 9.1, The Phase Plane: Linear Systems. page 505
Problem number: 15.
ODE order: 1.
ODE degree: 1.

Solve

$$x_1'(t) = -x_1(t) - x_2(t) - 1$$
$$x_2'(t) = 2x_1(t) - x_2(t) + 5$$

✓ Solution by Maple
Time used: 0.016 (sec). Leaf size: 61

```
dsolve([diff(x__1(t),t)=-1*x__1(t)-1*x__2(t)-1,diff(x__2(t),t)=2*x__1(t)-1*x__2(t)+5],singso
```

$$x_1(t) = -2 + e^{-t}\left(\cos\left(\sqrt{2}\,t\right) c_1 + c_2 \sin\left(\sqrt{2}\,t\right)\right)$$
$$x_2(t) = 1 - e^{-t}\sqrt{2}\left(c_2 \cos\left(\sqrt{2}\,t\right) - c_1 \sin\left(\sqrt{2}\,t\right)\right)$$

✓ Solution by Mathematica
Time used: 0.414 (sec). Leaf size: 85

```
DSolve[{x1'[t]==-1*x1[t]-1*x2[t]-1,x2'[t]==2*x1[t]-1*x2[t]+5},{x1[t],x2[t]},t,IncludeSingula
```

$$\text{x1}(t) \to c_1 e^{-t} \cos\left(\sqrt{2}t\right) - \frac{c_2 e^{-t} \sin\left(\sqrt{2}t\right)}{\sqrt{2}} - 2$$
$$\text{x2}(t) \to e^{-t}\left(e^t + c_2 \cos\left(\sqrt{2}t\right) + \sqrt{2}c_1 \sin\left(\sqrt{2}t\right)\right)$$

20 Chapter 9.2, Autonomous Systems and Stability. page 517

20.1 problem 1

Internal problem ID [807]

Book: Elementary differential equations and boundary value problems, 10th ed., Boyce and DiPrima
Section: Chapter 9.2, Autonomous Systems and Stability. page 517
Problem number: 1.
ODE order: 1.
ODE degree: 1.

Solve

$$x'(t) = -x(t)$$
$$y'(t) = -2y(t)$$

With initial conditions

$$[x(0) = 4, y(0) = 2]$$

✓ Solution by Maple

Time used: 0.016 (sec). Leaf size: 20

```
dsolve([diff(x(t),t) = -x(t), diff(y(t),t) = -2*y(t), x(0) = 4, y(0) = 2], singsol=all)
```

$$x(t) = 4\,\mathrm{e}^{-t}$$
$$y(t) = 2\,\mathrm{e}^{-2t}$$

✓ Solution by Mathematica

Time used: 0.038 (sec). Leaf size: 22

```
DSolve[{x'[t]==-1*x[t]+0*y[t],y'[t]==-2*y[t]},{x[0]==4,y[0]==2},{x[t],y[t]},t,IncludeSingula
```

$$x(t) \to 4e^{-t}$$
$$y(t) \to 2e^{-2t}$$

20.2 problem 2 part 1

Internal problem ID [808]

Book: Elementary differential equations and boundary value problems, 10th ed., Boyce and DiPrima
Section: Chapter 9.2, Autonomous Systems and Stability. page 517
Problem number: 2 part 1.
ODE order: 1.
ODE degree: 1.

Solve

$$x'(t) = -x(t)$$
$$y'(t) = 2y(t)$$

With initial conditions

$$[x(0) = 4, y(0) = 2]$$

✓ Solution by Maple
Time used: 0.016 (sec). Leaf size: 20

```
dsolve([diff(x(t),t) = -x(t), diff(y(t),t) = 2*y(t), x(0) = 4, y(0) = 2], singsol=all)
```

$$x(t) = 4\,e^{-t}$$
$$y(t) = 2\,e^{2t}$$

✓ Solution by Mathematica
Time used: 0.038 (sec). Leaf size: 22

```
DSolve[{x'[t]==-1*x[t]+0*y[t],y'[t]==0*x[t]+2*y[t]},{x[0]==4,y[0]==2},{x[t],y[t]},t,IncludeS
```

$$x(t) \to 4e^{-t}$$
$$y(t) \to 2e^{2t}$$

20.3 problem 2 part 2

Internal problem ID [809]

Book: Elementary differential equations and boundary value problems, 10th ed., Boyce and DiPrima
Section: Chapter 9.2, Autonomous Systems and Stability. page 517
Problem number: 2 part 2.
ODE order: 1.
ODE degree: 1.

Solve

$$x'(t) = -x(t)$$
$$y'(t) = 2y(t)$$

With initial conditions

$$[x(0) = 4, y(0) = 0]$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 15

```
dsolve([diff(x(t),t) = -x(t), diff(y(t),t) = 2*y(t), x(0) = 4, y(0) = 0], singsol=all)
```

$$x(t) = 4\,\mathrm{e}^{-t}$$
$$y(t) = 0$$

✓ Solution by Mathematica

Time used: 0.042 (sec). Leaf size: 16

```
DSolve[{x'[t]==-1*x[t]+0*y[t],y'[t]==0*x[t]+2*y[t]},{x[0]==4,y[0]==0},{x[t],y[t]},t,IncludeS
```

$$x(t) \to 4e^{-t}$$
$$y(t) \to 0$$

20.4 problem 3 part 1

Internal problem ID [810]

Book: Elementary differential equations and boundary value problems, 10th ed., Boyce and DiPrima
Section: Chapter 9.2, Autonomous Systems and Stability. page 517
Problem number: 3 part 1.
ODE order: 1.
ODE degree: 1.

Solve

$$x'(t) = -y(t)$$
$$y'(t) = x(t)$$

With initial conditions

$$[x(0) = 4, y(0) = 0]$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 16

```
dsolve([diff(x(t),t) = -y(t), diff(y(t),t) = x(t), x(0) = 4, y(0) = 0], singsol=all)
```

$$x(t) = 4\cos(t)$$
$$y(t) = 4\sin(t)$$

✓ Solution by Mathematica

Time used: 0.002 (sec). Leaf size: 16

```
DSolve[{x'[t]==-0*x[t]-1*y[t],y'[t]==1*x[t]+0*y[t]},{x[0]==4,y[0]==0},{x[t],y[t]},t,IncludeS
```

$$x(t) \to 4\cos(t)$$
$$y(t) \to 4\sin(t)$$

20.5 problem 3 part 2

Internal problem ID [811]

Book: Elementary differential equations and boundary value problems, 10th ed., Boyce and DiPrima
Section: Chapter 9.2, Autonomous Systems and Stability. page 517
Problem number: 3 part 2.
ODE order: 1.
ODE degree: 1.

Solve

$$x'(t) = -y(t)$$
$$y'(t) = x(t)$$

With initial conditions

$$[x(0) = 0, y(0) = 4]$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 16

```
dsolve([diff(x(t),t) = -y(t), diff(y(t),t) = x(t), x(0) = 0, y(0) = 4], singsol=all)
```

$$x(t) = -4\sin(t)$$
$$y(t) = 4\cos(t)$$

✓ Solution by Mathematica

Time used: 0.002 (sec). Leaf size: 16

```
DSolve[{x'[t]==-0*x[t]-1*y[t],y'[t]==1*x[t]+0*y[t]},{x[0]==0,y[0]==4},{x[t],y[t]},t,IncludeS
```

$$x(t) \to -4\sin(t)$$
$$y(t) \to 4\cos(t)$$