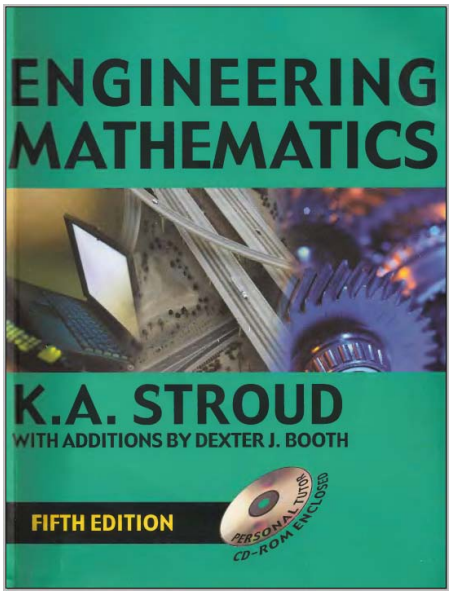
A Solution Manual For

Engineering Mathematics. By K. A. Stroud. 5th edition. Industrial press Inc. NY. 2001



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May 16, 2024

Contents

1 Program 24. First order differential equations. Test excercise 24. page 1067

1.1 problem 1

Internal problem ID [5075]

Book: Engineering Mathematics. By K. A. Stroud. 5th edition. Industrial press Inc. NY. 2001
Section: Program 24. First order differential equations. Test excercise 24. page 1067
Problem number: 1.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_quadrature]`

$$\boxed{xy' = x^2 + 2x - 3}$$

✓ Solution by Maple
Time used: 0.0 (sec). Leaf size: 18

```
dsolve(x*diff(y(x),x)=x^2+2*x-3,y(x), singsol=all)
```

$$y(x) = \frac{x^2}{2} + 2x - 3\ln(x) + c_1$$

✓ Solution by Mathematica
Time used: 0.003 (sec). Leaf size: 22

```
DSolve[x*y'[x]==x^2+2*x-3,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to \frac{x^2}{2} + 2x - 3\log(x) + c_1$$

1.2 problem 2

Internal problem ID [5076]

Book: Engineering Mathematics. By K. A. Stroud. 5th edition. Industrial press Inc. NY. 2001
Section: Program 24. First order differential equations. Test excercise 24. page 1067
Problem number: 2.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_separable]`

$$\boxed{(x+1)^2 y' - y^2 = 1}$$

✓ Solution by Maple
Time used: 0.016 (sec). Leaf size: 18

```
dsolve((1+x)^2*diff(y(x),x)=1+y(x)^2,y(x), singsol=all)
```

$$y(x) = \tan\left(\frac{-1 + c_1(x+1)}{x+1}\right)$$

✓ Solution by Mathematica
Time used: 0.264 (sec). Leaf size: 32

```
DSolve[(1+x)^2*y'[x]==1+y[x]^2,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to -\tan\left(\frac{1}{x+1} - c_1\right)$$
$$y(x) \to -i$$
$$y(x) \to i$$

1.3 problem 3

Internal problem ID [5077]

Book: Engineering Mathematics. By K. A. Stroud. 5th edition. Industrial press Inc. NY. 2001
Section: Program 24. First order differential equations. Test excercise 24. page 1067
Problem number: 3.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[[_linear, 'class A']]`

$$\boxed{y' + 2y = \mathrm{e}^{3x}}$$

✓ Solution by Maple
Time used: 0.0 (sec). Leaf size: 18

```
dsolve(diff(y(x),x)+2*y(x)=exp(3*x),y(x), singsol=all)
```

$$y(x) = \frac{(\mathrm{e}^{5x} + 5c_1)\,\mathrm{e}^{-2x}}{5}$$

✓ Solution by Mathematica
Time used: 0.043 (sec). Leaf size: 23

```
DSolve[y'[x]+2*y[x]==Exp[3*x],y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to \frac{e^{3x}}{5} + c_1 e^{-2x}$$

1.4 problem 4

Internal problem ID [5078]

Book: Engineering Mathematics. By K. A. Stroud. 5th edition. Industrial press Inc. NY. 2001
Section: Program 24. First order differential equations. Test excercise 24. page 1067
Problem number: 4.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_linear]`

$$\boxed{-y + xy' = x^2}$$

✓ Solution by Maple
Time used: 0.0 (sec). Leaf size: 9

```
dsolve(x*diff(y(x),x)-y(x)=x^2,y(x), singsol=all)
```

$$y(x) = (x + c_1)\, x$$

✓ Solution by Mathematica
Time used: 0.026 (sec). Leaf size: 11

```
DSolve[x*y'[x]-y[x]==x^2,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to x(x + c_1)$$

1.5 problem 5

Internal problem ID [5079]

Book: Engineering Mathematics. By K. A. Stroud. 5th edition. Industrial press Inc. NY. 2001
Section: Program 24. First order differential equations. Test excercise 24. page 1067
Problem number: 5.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_quadrature]`

$$\boxed{x^2 y' = x^3 \sin(3x) + 4}$$

✓ Solution by Maple
Time used: 0.016 (sec). Leaf size: 24

```
dsolve(x^2*diff(y(x),x)=x^3*sin(3*x)+4,y(x), singsol=all)
```

$$y(x) = \frac{\sin(3x)}{9} - \frac{x\cos(3x)}{3} - \frac{4}{x} + c_1$$

✓ Solution by Mathematica
Time used: 0.01 (sec). Leaf size: 30

```
DSolve[x^2*y'[x]==x^3*Sin[3*x]+4,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to -\frac{4}{x} + \frac{1}{9}\sin(3x) - \frac{1}{3}x\cos(3x) + c_1$$

1.6 problem 6

Internal problem ID [5080]

Book: Engineering Mathematics. By K. A. Stroud. 5th edition. Industrial press Inc. NY. 2001
Section: Program 24. First order differential equations. Test excercise 24. page 1067
Problem number: 6.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_separable]`

$$\boxed{x \cos(y) y' - \sin(y) = 0}$$

✓ Solution by Maple
Time used: 0.015 (sec). Leaf size: 8

```
dsolve(x*cos(y(x))*diff(y(x),x)-sin(y(x))=0,y(x), singsol=all)
```

$$y(x) = \arcsin(c_1 x)$$

✓ Solution by Mathematica
Time used: 9.024 (sec). Leaf size: 17

```
DSolve[x*Cos[y[x]]*y'[x]-Sin[y[x]]==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to \arcsin(e^{c_1} x)$$
$$y(x) \to 0$$

1.7 problem 7

Internal problem ID [5081]

Book: Engineering Mathematics. By K. A. Stroud. 5th edition. Industrial press Inc. NY. 2001
Section: Program 24. First order differential equations. Test excercise 24. page 1067
Problem number: 7.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[[_homogeneous, 'class A'], _rational, _dAlembert]`

$$\boxed{\left(x^3+xy^2\right)y'-2y^3=0}$$

✓ Solution by Maple
Time used: 0.063 (sec). Leaf size: 44

```
dsolve((x^3+x*y(x)^2)*diff(y(x),x)=2*y(x)^3,y(x), singsol=all)
```

$$y(x)=-\frac{\left(-c_1x+\sqrt{c_1^2x^2+4}\right)x}{2}$$
$$y(x)=\frac{\left(c_1x+\sqrt{c_1^2x^2+4}\right)x}{2}$$

✓ Solution by Mathematica
Time used: 1.2 (sec). Leaf size: 83

```
DSolve[(x^3+x*y[x]^2)*y'[x]==2*y[x]^3,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x)\to -\frac{1}{2}x\left(\sqrt{4+e^{2c_1}x^2}+e^{c_1}x\right)$$
$$y(x)\to \frac{1}{2}x\left(\sqrt{4+e^{2c_1}x^2}-e^{c_1}x\right)$$
$$y(x)\to 0$$
$$y(x)\to -x$$
$$y(x)\to x$$

1.8 problem 8

Internal problem ID [5082]

Book: Engineering Mathematics. By K. A. Stroud. 5th edition. Industrial press Inc. NY. 2001
Section: Program 24. First order differential equations. Test excercise 24. page 1067
Problem number: 8.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_separable]`

$$\boxed{\left(x^2-1\right)y'+2xy=x}$$

✓ Solution by Maple
Time used: 0.016 (sec). Leaf size: 21

```
dsolve((x^2-1)*diff(y(x),x)+2*x*y(x)=x,y(x), singsol=all)
```

$$y(x)=\frac{x^2+2c_1}{2x^2-2}$$

✓ Solution by Mathematica
Time used: 0.03 (sec). Leaf size: 31

```
DSolve[(x^2-1)*y'[x]+2*x*y[x]==x,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x)\to\frac{x^2+2c_1}{2\left(x^2-1\right)}$$
$$y(x)\to\frac{1}{2}$$

1.9 problem 9

Internal problem ID [5083]

Book: Engineering Mathematics. By K. A. Stroud. 5th edition. Industrial press Inc. NY. 2001
Section: Program 24. First order differential equations. Test excercise 24. page 1067
Problem number: 9.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_linear]`

$$\boxed{y' + y \tanh(x) = 2 \sinh(x)}$$

✓ Solution by Maple
Time used: 0.0 (sec). Leaf size: 14

```
dsolve(diff(y(x),x)+y(x)*tanh(x)=2*sinh(x),y(x), singsol=all)
```

$$y(x) = \left(\cosh(x)^2 - \frac{1}{2} + c_1 \right) \operatorname{sech}(x)$$

✓ Solution by Mathematica
Time used: 0.098 (sec). Leaf size: 20

```
DSolve[y'[x]+y[x]*Tanh[x]==2*Sinh[x],y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to \frac{1}{2}\operatorname{sech}(x)(\cosh(2x) + 2c_1)$$

1.10 problem 10

Internal problem ID [5084]

Book: Engineering Mathematics. By K. A. Stroud. 5th edition. Industrial press Inc. NY. 2001
Section: Program 24. First order differential equations. Test excercise 24. page 1067
Problem number: 10.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_linear]`

$$\boxed{xy' - 2y = \cos(x)\, x^3}$$

✓ Solution by Maple
Time used: 0.0 (sec). Leaf size: 12

```
dsolve(x*diff(y(x),x)-2*y(x)=x^3*cos(x),y(x), singsol=all)
```

$$y(x) = (\sin(x) + c_1)\, x^2$$

✓ Solution by Mathematica
Time used: 0.037 (sec). Leaf size: 14

```
DSolve[x*y'[x]-2*y[x]==x^3*Cos[x],y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to x^2(\sin(x) + c_1)$$

1.11 problem 11

Internal problem ID [5085]

Book: Engineering Mathematics. By K. A. Stroud. 5th edition. Industrial press Inc. NY. 2001
Section: Program 24. First order differential equations. Test excercise 24. page 1067
Problem number: 11.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[[_homogeneous, 'class G'], _rational, _Bernoulli]`

$$\boxed{y' + \frac{y}{x} - y^3 = 0}$$

✓ Solution by Maple
Time used: 0.0 (sec). Leaf size: 27

```
dsolve(diff(y(x),x)+y(x)/x=y(x)^3,y(x), singsol=all)
```

$$y(x) = \frac{1}{\sqrt{x\left(c_1 x + 2\right)}}$$
$$y(x) = -\frac{1}{\sqrt{x\left(c_1 x + 2\right)}}$$

✓ Solution by Mathematica
Time used: 0.375 (sec). Leaf size: 40

```
DSolve[y'[x]+y[x]/x==y[x]^3,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to -\frac{1}{\sqrt{x(2 + c_1 x)}}$$
$$y(x) \to \frac{1}{\sqrt{x(2 + c_1 x)}}$$
$$y(x) \to 0$$

1.12 problem 12

Internal problem ID [5086]

Book: Engineering Mathematics. By K. A. Stroud. 5th edition. Industrial press Inc. NY. 2001
Section: Program 24. First order differential equations. Test excercise 24. page 1067
Problem number: 12.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[[_homogeneous, 'class G'], _rational, _Bernoulli]`

$$\boxed{xy' + 3y - y^2x^2 = 0}$$

✓ Solution by Maple
Time used: 0.0 (sec). Leaf size: 15

```
dsolve(x*diff(y(x),x)+3*y(x)=x^2*y(x)^2,y(x), singsol=all)
```

$$y(x) = \frac{1}{x^2\left(c_1 x + 1\right)}$$

✓ Solution by Mathematica
Time used: 0.137 (sec). Leaf size: 22

```
DSolve[x*y'[x]+3*y[x]==x^2*y[x]^2,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to \frac{1}{x^2 + c_1 x^3}$$
$$y(x) \to 0$$

2 Program 24. First order differential equations. Further problems 24. page 1068

2.1 problem 1

Internal problem ID [5087]

Book: Engineering Mathematics. By K. A. Stroud. 5th edition. Industrial press Inc. NY. 2001
Section: Program 24. First order differential equations. Further problems 24. page 1068
Problem number: 1.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_separable]`

$$\boxed{x(y-3)\,y'-4y=0}$$

✓ Solution by Maple
Time used: 0.015 (sec). Leaf size: 16

```
dsolve(x*(y(x)-3)*diff(y(x),x)=4*y(x),y(x), singsol=all)
```

$$y(x)=-3\,\mathrm{LambertW}\left(-\frac{\mathrm{e}^{-\frac{4c_1}{3}}}{3x^{\frac{4}{3}}}\right)$$

✓ Solution by Mathematica
Time used: 13.068 (sec). Leaf size: 94

```
DSolve[x*(y[x]-3)*y'[x]==4*y[x],y[x],x,IncludeSingularSolutions -> True]
```

$$y(x)\to -3W\left(\frac{1}{3}\sqrt[3]{-\frac{e^{-c_1}}{x^4}}\right)$$
$$y(x)\to -3W\left(-\frac{1}{3}\sqrt[3]{-1}\sqrt[3]{-\frac{e^{-c_1}}{x^4}}\right)$$
$$y(x)\to -3W\left(\frac{1}{3}(-1)^{2/3}\sqrt[3]{-\frac{e^{-c_1}}{x^4}}\right)$$
$$y(x)\to 0$$

2.2 problem 2

Internal problem ID [5088]

Book: Engineering Mathematics. By K. A. Stroud. 5th edition. Industrial press Inc. NY. 2001
Section: Program 24. First order differential equations. Further problems 24. page 1068
Problem number: 2.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_separable]`

$$\boxed{\left(x^3+1\right)y' - yx^2 = 0}$$

With initial conditions

$$[y(1) = 2]$$

✓ Solution by Maple
Time used: 0.015 (sec). Leaf size: 15

```
dsolve([(1+x^3)*diff(y(x),x)=x^2*y(x),y(1) = 2],y(x), singsol=all)
```

$$y(x) = 2^{\frac{2}{3}}\left(x^3+1\right)^{\frac{1}{3}}$$

✓ Solution by Mathematica
Time used: 0.034 (sec). Leaf size: 20

```
DSolve[{(1+x^3)*y'[x]==x^2*y[x],{y[1]==2}},y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to 2^{2/3}\sqrt[3]{x^3+1}$$

2.3 problem 3

Internal problem ID [5089]

Book: Engineering Mathematics. By K. A. Stroud. 5th edition. Industrial press Inc. NY. 2001
Section: Program 24. First order differential equations. Further problems 24. page 1068
Problem number: 3.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_separable]`

$$\boxed{(1+y)^2 y' = -x^3}$$

✓ Solution by Maple
Time used: 0.0 (sec). Leaf size: 89

```
dsolve(x^3+(y(x)+1)^2*diff(y(x),x)=0,y(x), singsol=all)
```

$$y(x) = \frac{(-6x^4-24c_1)^{\frac{1}{3}}}{2} - 1$$
$$y(x) = -\frac{(-6x^4-24c_1)^{\frac{1}{3}}}{4} - \frac{i\sqrt{3}\,(-6x^4-24c_1)^{\frac{1}{3}}}{4} - 1$$
$$y(x) = -\frac{(-6x^4-24c_1)^{\frac{1}{3}}}{4} + \frac{i\sqrt{3}\,(-6x^4-24c_1)^{\frac{1}{3}}}{4} - 1$$

✓ Solution by Mathematica
Time used: 0.483 (sec). Leaf size: 110

```
DSolve[x^3+(y[x]+1)^2*y'[x]==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to -1 + \frac{\sqrt[3]{-3x^4+4+12c_1}}{2^{2/3}}$$
$$y(x) \to -1 + \frac{i\left(\sqrt{3}+i\right)\sqrt[3]{-3x^4+4+12c_1}}{2\,2^{2/3}}$$
$$y(x) \to -1 - \frac{\left(1+i\sqrt{3}\right)\sqrt[3]{-3x^4+4+12c_1}}{2\,2^{2/3}}$$

2.4 problem 4

Internal problem ID [5090]

Book: Engineering Mathematics. By K. A. Stroud. 5th edition. Industrial press Inc. NY. 2001
Section: Program 24. First order differential equations. Further problems 24. page 1068
Problem number: 4.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_separable]`

$$\boxed{\cos(y) + \left(1 + \mathrm{e}^{-x}\right) \sin(y)\, y' = 0}$$

With initial conditions

$$\left[y(0) = \frac{\pi}{4}\right]$$

✓ Solution by Maple
Time used: 0.344 (sec). Leaf size: 14

```
dsolve([cos(y(x))+(1+exp(-x))*sin(y(x))*diff(y(x),x)=0,y(0) = 1/4*Pi],y(x), singsol=all)
```

$$y(x) = \arccos\left(\frac{\sqrt{2}\left(\mathrm{e}^{x} + 1\right)}{4}\right)$$

✓ Solution by Mathematica
Time used: 50.086 (sec). Leaf size: 20

```
DSolve[{Cos[y[x]]+(1+Exp[-x])*Sin[y[x]]*y'[x]==0,{y[0]==Pi/4}},y[x],x,IncludeSingularSolutio
```

$$y(x) \to \arccos\left(\frac{e^{x} + 1}{2\sqrt{2}}\right)$$

2.5 problem 5

Internal problem ID [5091]

Book: Engineering Mathematics. By K. A. Stroud. 5th edition. Industrial press Inc. NY. 2001
Section: Program 24. First order differential equations. Further problems 24. page 1068
Problem number: 5.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_separable]`

$$\boxed{x^2(1+y)+y^2(x-1)\,y'=0}$$

✓ Solution by Maple
Time used: 0.016 (sec). Leaf size: 30

```
dsolve(x^2*(y(x)+1)+y(x)^2*(x-1)*diff(y(x),x)=0,y(x), singsol=all)
```

$$\frac{x^2}{2}+x+\ln(x-1)+\frac{y(x)^2}{2}-y(x)+\ln(y(x)+1)+c_1=0$$

✓ Solution by Mathematica
Time used: 0.42 (sec). Leaf size: 56

```
DSolve[x^2*(y[x]+1)+y[x]^2*(x-1)*y'[x]==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x)\to \text{InverseFunction}\left[\frac{1}{2}(\#1+1)^2-2(\#1+1)+\log(\#1+1)\&\right]\left[-\frac{x^2}{2}-x-\log(x-1)+\frac{3}{2}+c_1\right]$$

$$y(x)\to -1$$

2.6 problem 6

Internal problem ID [5092]

Book: Engineering Mathematics. By K. A. Stroud. 5th edition. Industrial press Inc. NY. 2001
Section: Program 24. First order differential equations. Further problems 24. page 1068
Problem number: 6.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[[_homogeneous, 'class A'], _exact, _rational, [_Abel, '2nd ty`

$$\boxed{(2y-x)\,y'-y=2x}$$

✓ Solution by Maple
Time used: 0.047 (sec). Leaf size: 51

```
dsolve((2*y(x)-x)*diff(y(x),x)=2*x+y(x),y(x), singsol=all)
```

$$y(x)=\frac{c_1x-\sqrt{5c_1^2x^2+4}}{2c_1}$$
$$y(x)=\frac{c_1x+\sqrt{5c_1^2x^2+4}}{2c_1}$$

✓ Solution by Mathematica
Time used: 0.454 (sec). Leaf size: 102

```
DSolve[(2*y[x]-x)*y'[x]==2*x+y[x],y[x],x,IncludeSingularSolutions -> True]
```

$$y(x)\to\frac{1}{2}\left(x-\sqrt{5x^2-4e^{c_1}}\right)$$
$$y(x)\to\frac{1}{2}\left(x+\sqrt{5x^2-4e^{c_1}}\right)$$
$$y(x)\to\frac{1}{2}\left(x-\sqrt{5}\sqrt{x^2}\right)$$
$$y(x)\to\frac{1}{2}\left(\sqrt{5}\sqrt{x^2}+x\right)$$

2.7 problem 7

Internal problem ID [5093]

Book: Engineering Mathematics. By K. A. Stroud. 5th edition. Industrial press Inc. NY. 2001
Section: Program 24. First order differential equations. Further problems 24. page 1068
Problem number: 7.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[[_homogeneous, 'class A'], _rational, [_Abel, '2nd type', 'cl`

$$\boxed{xy + y^2 + (x^2 - xy)\, y' = 0}$$

✓ Solution by Maple
Time used: 0.032 (sec). Leaf size: 17

```
dsolve((x*y(x)+y(x)^2)+(x^2-x*y(x))*diff(y(x),x)=0,y(x), singsol=all)
```

$$y(x) = -\operatorname{LambertW}\left(-\frac{\mathrm{e}^{-2c_1}}{x^2}\right) x$$

✓ Solution by Mathematica
Time used: 2.801 (sec). Leaf size: 25

```
DSolve[(x*y[x]+y[x]^2)+(x^2-x*y[x])*y'[x]==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to -xW\left(-\frac{e^{-c_1}}{x^2}\right)$$
$$y(x) \to 0$$

2.8 problem 8

Internal problem ID [5094]

Book: Engineering Mathematics. By K. A. Stroud. 5th edition. Industrial press Inc. NY. 2001
Section: Program 24. First order differential equations. Further problems 24. page 1068
Problem number: 8.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[[_homogeneous, 'class A'], _rational, _Bernoulli]`

$$\boxed{y^3 - 3y'y^2x = -x^3}$$

✓ Solution by Maple
Time used: 0.0 (sec). Leaf size: 74

```
dsolve((x^3+y(x)^3)=3*x*y(x)^2*diff(y(x),x),y(x), singsol=all)
```

$$y(x) = \frac{2^{\frac{2}{3}}(x(x^2+2c_1))^{\frac{1}{3}}}{2}$$
$$y(x) = -\frac{2^{\frac{2}{3}}(x(x^2+2c_1))^{\frac{1}{3}}\left(1+i\sqrt{3}\right)}{4}$$
$$y(x) = \frac{2^{\frac{2}{3}}(x(x^2+2c_1))^{\frac{1}{3}}\left(i\sqrt{3}-1\right)}{4}$$

✓ Solution by Mathematica
Time used: 0.21 (sec). Leaf size: 90

```
DSolve[(x^3+y[x]^3)==3*x*y[x]^2*y'[x],y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to -\sqrt[3]{-\frac{1}{2}}\sqrt[3]{x}\sqrt[3]{x^2+2c_1}$$
$$y(x) \to \frac{\sqrt[3]{x}\sqrt[3]{x^2+2c_1}}{\sqrt[3]{2}}$$
$$y(x) \to \frac{(-1)^{2/3}\sqrt[3]{x}\sqrt[3]{x^2+2c_1}}{\sqrt[3]{2}}$$

2.9 problem 9

Internal problem ID [5095]

Book: Engineering Mathematics. By K. A. Stroud. 5th edition. Industrial press Inc. NY. 2001
Section: Program 24. First order differential equations. Further problems 24. page 1068
Problem number: 9.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[[_homogeneous, 'class A'], _rational, [_Abel, '2nd type', 'cl`

$$\boxed{y + (4y + 3x)\, y' = 3x}$$

✓ Solution by Maple
Time used: 0.36 (sec). Leaf size: 278

```
dsolve(y(x)-3*x+(4*y(x)+3*x)*diff(y(x),x)=0,y(x), singsol=all)
```

$$y(x) = \frac{-3x^8 c_1 \operatorname{RootOf}\left(\_Z^{64} c_1 x^8 + 12\_Z^{56} c_1 x^8 + 48\_Z^{48} c_1 x^8 + 64\_Z^{40} c_1 x^8 - 1\right)^{56} - 24x^8 c_1 \operatorname{RootOf}\left(\_Z^{64} c_1 x^8 + \right.}{2c_1 x^7 \operatorname{RootOf}\left(\_Z^{64} c_1 x^8 + 12\_Z^{56} c_1 x^8 + 48\_Z^{48} c_1 x^8 + 64\_Z^{40} c_1 x^8 - 1\right)^{40} \left(\operatorname{RootOf}\left(\_Z^{64} c_1 x^8 + \right.}$$

✓ Solution by Mathematica

Time used: 5.296 (sec). Leaf size: 673

```
DSolve[y[x]-3*x+(4*y[x]+3*x)*y'[x]==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to \text{Root}\left[256\#1^8+512\#1^7x-512\#1^6x^2-896\#1^5x^3+800\#1^4x^4+352\#1^3x^5-576\#1^2x^6+216\#1x^7-27x^8+e^{8c_1}\&,1\right]$$
$$y(x) \to \text{Root}\left[256\#1^8+512\#1^7x-512\#1^6x^2-896\#1^5x^3+800\#1^4x^4+352\#1^3x^5-576\#1^2x^6+216\#1x^7-27x^8+e^{8c_1}\&,2\right]$$
$$y(x) \to \text{Root}\left[256\#1^8+512\#1^7x-512\#1^6x^2-896\#1^5x^3+800\#1^4x^4+352\#1^3x^5-576\#1^2x^6+216\#1x^7-27x^8+e^{8c_1}\&,3\right]$$
$$y(x) \to \text{Root}\left[256\#1^8+512\#1^7x-512\#1^6x^2-896\#1^5x^3+800\#1^4x^4+352\#1^3x^5-576\#1^2x^6+216\#1x^7-27x^8+e^{8c_1}\&,4\right]$$
$$y(x) \to \text{Root}\left[256\#1^8+512\#1^7x-512\#1^6x^2-896\#1^5x^3+800\#1^4x^4+352\#1^3x^5-576\#1^2x^6+216\#1x^7-27x^8+e^{8c_1}\&,5\right]$$
$$y(x) \to \text{Root}\left[256\#1^8+512\#1^7x-512\#1^6x^2-896\#1^5x^3+800\#1^4x^4+352\#1^3x^5-576\#1^2x^6+216\#1x^7-27x^8+e^{8c_1}\&,6\right]$$
$$y(x) \to \text{Root}\left[256\#1^8+512\#1^7x-512\#1^6x^2-896\#1^5x^3+800\#1^4x^4+352\#1^3x^5-576\#1^2x^6+216\#1x^7-27x^8+e^{8c_1}\&,7\right]$$
$$y(x) \to \text{Root}\left[256\#1^8+512\#1^7x-512\#1^6x^2-896\#1^5x^3+800\#1^4x^4+352\#1^3x^5-576\#1^2x^6+216\#1x^7-27x^8+e^{8c_1}\&,8\right]$$

2.10 problem 10

Internal problem ID [5096]

Book: Engineering Mathematics. By K. A. Stroud. 5th edition. Industrial press Inc. NY. 2001
Section: Program 24. First order differential equations. Further problems 24. page 1068
Problem number: 10.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[[_homogeneous, 'class A'], _rational, _dAlembert]`

$$\boxed{\left(x^3+3xy^2\right)y'-y^3-3yx^2=0}$$

✓ Solution by Maple
Time used: 0.079 (sec). Leaf size: 23

```
dsolve((x^3+3*x*y(x)^2)*diff(y(x),x)=y(x)^3+3*x^2*y(x),y(x), singsol=all)
```

$$y(x)=\text{RootOf}\left(\_Z^4c_1x-c_1x-\_Z\right)^2x$$

✓ Solution by Mathematica

Time used: 60.142 (sec). Leaf size: 1659

```
DSolve[(x^3+3*x*y[x]^2)*y'[x]==y[x]^3+3*x^2*y[x],y[x],x,IncludeSingularSolutions -> True]
```

$$y(x)$$

$$\to \frac{1}{6}\left(-\sqrt{3}\sqrt{4x^2+\frac{16\sqrt[3]{2}x^4}{\sqrt[3]{128x^6+27e^{2c_1}x^2+3\sqrt{768e^{2c_1}x^8+81e^{4c_1}x^4}}}+\frac{\sqrt[3]{128x^6+27e^{2c_1}x^2+3\sqrt{768e^{2c_1}x^8}}}{\sqrt[3]{2}}}\right.$$

$$-3\sqrt{\frac{8x^2}{3}-\frac{16\sqrt[3]{2}x^4}{3\sqrt[3]{128x^6+27e^{2c_1}x^2+3\sqrt{768e^{2c_1}x^8+81e^{4c_1}x^4}}}-\frac{}{\sqrt{4x^2+\frac{16\sqrt[3]{2}x^4}{\sqrt[3]{128x^6+27e^{2c_1}x^2+3\sqrt{768e}}}}}}$$

$$y(x)$$

$$\to \frac{1}{6}\left(3\sqrt{\frac{8x^2}{3}-\frac{16\sqrt[3]{2}x^4}{3\sqrt[3]{128x^6+27e^{2c_1}x^2+3\sqrt{768e^{2c_1}x^8+81e^{4c_1}x^4}}}-\frac{}{\sqrt{4x^2+\frac{16\sqrt[3]{2}x^4}{\sqrt[3]{128x^6+27e^{2c_1}x^2+3\sqrt{768}}}}}}\right.$$

$$-\sqrt{3}\sqrt{4x^2+\frac{16\sqrt[3]{2}x^4}{\sqrt[3]{128x^6+27e^{2c_1}x^2+3\sqrt{768e^{2c_1}x^8+81e^{4c_1}x^4}}}+\frac{\sqrt[3]{128x^6+27e^{2c_1}x^2+3\sqrt{768e^{2c_1}x^8+8}}}{\sqrt[3]{2}}}$$

$$y(x)$$

$$\to \frac{1}{6}\left(\sqrt{3}\sqrt{4x^2+\frac{16\sqrt[3]{2}x^4}{\sqrt[3]{128x^6+27e^{2c_1}x^2+3\sqrt{768e^{2c_1}x^8+81e^{4c_1}x^4}}}+\frac{\sqrt[3]{128x^6+27e^{2c_1}x^2+3\sqrt{768e^{2c_1}x^8+}}}{\sqrt[3]{2}}}\right.$$

$$-3\sqrt{\frac{8x^2}{3}-\frac{16\sqrt[3]{2}x^4}{3\sqrt[3]{128x^6+27e^{2c_1}x^2+3\sqrt{768e^{2c_1}x^8+81e^{4c_1}x^4}}}+\frac{}{\sqrt{4x^2+\frac{16\sqrt[3]{2}x^4}{\sqrt[3]{128x^6+27e^{2c_1}x^2+3\sqrt{768e}}}}}}$$

2.11 problem 11

Internal problem ID [5097]

Book: Engineering Mathematics. By K. A. Stroud. 5th edition. Industrial press Inc. NY. 2001
Section: Program 24. First order differential equations. Further problems 24. page 1068
Problem number: 11.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_linear]`

$$\boxed{-y + xy' = x^3 + 3x^2 - 2x}$$

✓ Solution by Maple
Time used: 0.016 (sec). Leaf size: 21

```
dsolve(x*diff(y(x),x)-y(x)=x^3+3*x^2-2*x,y(x), singsol=all)
```

$$y(x) = \frac{\left(x^2 + 6x - 4\ln(x) + 2c_1\right)x}{2}$$

✓ Solution by Mathematica
Time used: 0.028 (sec). Leaf size: 24

```
DSolve[x*y'[x]-y[x]==x^3+3*x^2-2*x,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to x\left(\frac{x^2}{2} + 3x - 2\log(x) + c_1\right)$$

2.12 problem 12

Internal problem ID [5098]

Book: Engineering Mathematics. By K. A. Stroud. 5th edition. Industrial press Inc. NY. 2001
Section: Program 24. First order differential equations. Further problems 24. page 1068
Problem number: 12.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_linear]`

$$\boxed{y' + y \tan(x) = \sin(x)}$$

✓ Solution by Maple
Time used: 0.0 (sec). Leaf size: 14

```
dsolve(diff(y(x),x)+y(x)*tan(x)=sin(x),y(x), singsol=all)
```

$$y(x) = (-\ln(\cos(x)) + c_1)\cos(x)$$

✓ Solution by Mathematica
Time used: 0.058 (sec). Leaf size: 16

```
DSolve[y'[x]+y[x]*Tan[x]==Sin[x],y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to \cos(x)(-\log(\cos(x)) + c_1)$$

2.13 problem 13

Internal problem ID [5099]

Book: Engineering Mathematics. By K. A. Stroud. 5th edition. Industrial press Inc. NY. 2001
Section: Program 24. First order differential equations. Further problems 24. page 1068
Problem number: 13.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_linear]`

$$\boxed{-y + xy' = \cos(x)\, x^3}$$

With initial conditions

$$[y(\pi) = 0]$$

✓ Solution by Maple
Time used: 0.016 (sec). Leaf size: 14

```
dsolve([x*diff(y(x),x)-y(x)=x^3*cos(x),y(Pi) = 0],y(x), singsol=all)
```

$$y(x) = (\cos(x) + \sin(x)\, x + 1)\, x$$

✓ Solution by Mathematica
Time used: 0.042 (sec). Leaf size: 15

```
DSolve[{x*y'[x]-y[x]==x^3*Cos[x],{y[Pi]==0}},y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to x(x \sin(x) + \cos(x) + 1)$$

2.14 problem 14

Internal problem ID [5100]

Book: Engineering Mathematics. By K. A. Stroud. 5th edition. Industrial press Inc. NY. 2001
Section: Program 24. First order differential equations. Further problems 24. page 1068
Problem number: 14.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_separable]`

$$\boxed{\left(x^2+1\right)y' + 3xy = 5x}$$

With initial conditions

$$[y(1) = 2]$$

✓ Solution by Maple
Time used: 0.031 (sec). Leaf size: 18

```
dsolve([(1+x^2)*diff(y(x),x)+3*x*y(x)=5*x,y(1) = 2],y(x), singsol=all)
```

$$y(x) = \frac{5}{3} + \frac{2\sqrt{2}}{3\left(x^2+1\right)^{\frac{3}{2}}}$$

✓ Solution by Mathematica
Time used: 0.039 (sec). Leaf size: 27

```
DSolve[{(1+x^2)*y'[x]+3*x*y[x]==5*x,{y[1]==2}},y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to \frac{2\sqrt{2}}{3\left(x^2+1\right)^{3/2}} + \frac{5}{3}$$

2.15 problem 15

Internal problem ID [5101]

Book: Engineering Mathematics. By K. A. Stroud. 5th edition. Industrial press Inc. NY. 2001
Section: Program 24. First order differential equations. Further problems 24. page 1068
Problem number: 15.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_linear]`

$$\boxed{y' + y\cot(x) = 5\,\mathrm{e}^{\cos(x)}}$$

With initial conditions

$$\left[y\left(\frac{\pi}{2}\right) = -4\right]$$

✓ Solution by Maple
Time used: 0.016 (sec). Leaf size: 14

```
dsolve([diff(y(x),x)+y(x)*cot(x)=5*exp(cos(x)),y(1/2*Pi) = -4],y(x), singsol=all)
```

$$y(x) = -5\,\mathrm{e}^{\cos(x)}\csc(x) + \csc(x)$$

✓ Solution by Mathematica
Time used: 0.1 (sec). Leaf size: 16

```
DSolve[{y'[x]+y[x]*Cot[x]==5*Exp[Cos[x]],{y[Pi/2]==-4}},y[x],x,IncludeSingularSolutions -> T
```

$$y(x) \to \left(1 - 5e^{\cos(x)}\right)\csc(x)$$

2.16 problem 16

Internal problem ID [5102]

Book: Engineering Mathematics. By K. A. Stroud. 5th edition. Industrial press Inc. NY. 2001
Section: Program 24. First order differential equations. Further problems 24. page 1068
Problem number: 16.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[[_homogeneous, 'class C'], _rational, [_Abel, '2nd type', 'cl`

$$\boxed{(3x+3y-4)\,y'+y=-x}$$

✓ Solution by Maple
Time used: 0.031 (sec). Leaf size: 21

```
dsolve((3*x+3*y(x)-4)*diff(y(x),x)=-(x+y(x)),y(x), singsol=all)
```

$$y(x)=\frac{2\,\mathrm{LambertW}\left(\frac{3\,\mathrm{e}^{-3+x-c_1}}{2}\right)}{3}-x+2$$

✓ Solution by Mathematica
Time used: 3.675 (sec). Leaf size: 33

```
DSolve[(3*x+3*y[x]-4)*y'[x]==-(x+y[x]),y[x],x,IncludeSingularSolutions -> True]
```

$$y(x)\to\frac{2}{3}W\left(-e^{x-1+c_1}\right)-x+2$$
$$y(x)\to 2-x$$

2.17 problem 17

Internal problem ID [5103]

Book: Engineering Mathematics. By K. A. Stroud. 5th edition. Industrial press Inc. NY. 2001
Section: Program 24. First order differential equations. Further problems 24. page 1068
Problem number: 17.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_rational, [_1st_order, '_with_symmetry_[F(x)*G(y),0]'], [_Ab`

$$\boxed{-xy^2 - \left(x + yx^2\right) y' = -x}$$

✓ Solution by Maple
Time used: 0.0 (sec). Leaf size: 53

```
dsolve((x-x*y(x)^2)=(x+x^2*y(x))*diff(y(x),x),y(x), singsol=all)
```

$$x + \frac{\sqrt{y(x)^2 - 1}\,\ln\left(y(x) + \sqrt{y(x)^2 - 1}\right)}{(y(x) - 1)(y(x) + 1)} - \frac{c_1}{\sqrt{y(x) - 1}\,\sqrt{y(x) + 1}} = 0$$

✓ Solution by Mathematica
Time used: 0.127 (sec). Leaf size: 55

```
DSolve[(x-x*y[x]^2)==(x+x^2*y[x])*y'[x],y[x],x,IncludeSingularSolutions -> True]
```

$$\text{Solve}\left[x = -\frac{2\arctan\left(\frac{\sqrt{1-y(x)^2}}{y(x)+1}\right)}{\sqrt{1-y(x)^2}} + \frac{c_1}{\sqrt{1-y(x)^2}}, y(x)\right]$$

2.18 problem 18

Internal problem ID [5104]

Book: Engineering Mathematics. By K. A. Stroud. 5th edition. Industrial press Inc. NY. 2001
Section: Program 24. First order differential equations. Further problems 24. page 1068
Problem number: 18.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[[_homogeneous, 'class C'], _rational, [_Abel, '2nd type', 'cl`

$$\boxed{-y + (4y + x - 1)\, y' = 1 - x}$$

✓ Solution by Maple
Time used: 0.031 (sec). Leaf size: 29

```
dsolve((x-y(x)-1)+(4*y(x)+x-1)*diff(y(x),x)=0,y(x), singsol=all)
```

$$y(x) = -\frac{\tan\left(\text{RootOf}\left(\ln\left(\sec\left(\_Z\right)^2\right) - \_Z + 2\ln\left(x-1\right) + 2c_1\right)\right)(x-1)}{2}$$

✓ Solution by Mathematica
Time used: 0.059 (sec). Leaf size: 58

```
DSolve[(x-y[x]-1)+(4*y[x]+x-1)*y'[x]==0,y[x],x,IncludeSingularSolutions -> True]
```

$$\text{Solve}\left[2\arctan\left(\frac{2y(x) - 2x + 2}{4y(x) + x - 1}\right) + 2\log\left(\frac{4}{5}\left(\frac{4y(x)^2}{(x-1)^2} + 1\right)\right) + 4\log(x-1) + 5c_1 = 0, y(x)\right]$$

2.19 problem 19

Internal problem ID [5105]

Book: Engineering Mathematics. By K. A. Stroud. 5th edition. Industrial press Inc. NY. 2001
Section: Program 24. First order differential equations. Further problems 24. page 1068
Problem number: 19.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[[_homogeneous, ‘class C‘], _rational, [_Abel, ‘2nd type‘, ‘cl`

$$\boxed{3y + (7y - 3x + 3)\,y' = 7x - 7}$$

✓ Solution by Maple
Time used: 0.422 (sec). Leaf size: 1814

```
dsolve((3*y(x)-7*x+7)+(7*y(x)-3*x+3)*diff(y(x),x)=0,y(x), singsol=all)
```

Expression too large to display

✓ Solution by Mathematica
Time used: 60.706 (sec). Leaf size: 7785

```
DSolve[(3*y[x]-7*x+7)+(7*y[x]-3*x+3)*y'[x]==0,y[x],x,IncludeSingularSolutions -> True]
```

Too large to display

2.20 problem 20

Internal problem ID [5106]

Book: Engineering Mathematics. By K. A. Stroud. 5th edition. Industrial press Inc. NY. 2001
Section: Program 24. First order differential equations. Further problems 24. page 1068
Problem number: 20.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[[_homogeneous, 'class G'], _rational]`

$$\boxed{y(xy+1)+x\left(1+xy+y^2x^2\right)y'=0}$$

✓ Solution by Maple
Time used: 0.047 (sec). Leaf size: 38

```
dsolve(y(x)*(x*y(x)+1)+x*(1+x*y(x)+x^2*y(x)^2)*diff(y(x),x)=0,y(x), singsol=all)
```

$$y(x)=\frac{\mathrm{e}^{\mathrm{RootOf}\left(-2\ln(x)\mathrm{e}^{2\_Z}+2c_1\mathrm{e}^{2\_Z}+2\_Z\mathrm{e}^{2\_Z}-2\,\mathrm{e}^{\_Z}-1\right)}}{x}$$

✓ Solution by Mathematica
Time used: 0.11 (sec). Leaf size: 30

```
DSolve[y[x]*(x*y[x]+1)+x*(1+x*y[x]+x^2*y[x]^2)*y'[x]==0,y[x],x,IncludeSingularSolutions -> T
```

$$\text{Solve}\left[\frac{-\frac{1}{2x^2}-\frac{y(x)}{x}}{y(x)^2}+\log(y(x))=c_1,y(x)\right]$$

2.21 problem 21

Internal problem ID [5107]

Book: Engineering Mathematics. By K. A. Stroud. 5th edition. Industrial press Inc. NY. 2001
Section: Program 24. First order differential equations. Further problems 24. page 1068
Problem number: 21.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_Bernoulli]`

$$\boxed{y + y' - y^3 x = 0}$$

✓ Solution by Maple
Time used: 0.0 (sec). Leaf size: 39

```
dsolve(diff(y(x),x)+y(x)=x*y(x)^3,y(x), singsol=all)
```

$$y(x) = -\frac{2}{\sqrt{2 + 4\,\mathrm{e}^{2x} c_1 + 4x}}$$
$$y(x) = \frac{2}{\sqrt{2 + 4\,\mathrm{e}^{2x} c_1 + 4x}}$$

✓ Solution by Mathematica
Time used: 2.704 (sec). Leaf size: 50

```
DSolve[y'[x]+y[x]==x*y[x]^3,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to -\frac{1}{\sqrt{x + c_1 e^{2x} + \frac{1}{2}}}$$
$$y(x) \to \frac{1}{\sqrt{x + c_1 e^{2x} + \frac{1}{2}}}$$
$$y(x) \to 0$$

2.22 problem 22

Internal problem ID [5108]

Book: Engineering Mathematics. By K. A. Stroud. 5th edition. Industrial press Inc. NY. 2001
Section: Program 24. First order differential equations. Further problems 24. page 1068
Problem number: 22.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[[_1st_order, _with_linear_symmetries], _Bernoulli]`

$$\boxed{y + y' - y^4 \mathrm{e}^x = 0}$$

✓ Solution by Maple
Time used: 0.016 (sec). Leaf size: 138

```
dsolve(diff(y(x),x)+y(x)=y(x)^4*exp(x),y(x), singsol=all)
```

$$y(x) = \frac{2^{\frac{1}{3}} \left(\mathrm{e}^{2x} (2 \mathrm{e}^{2x} c_1 + 3)^2 \right)^{\frac{1}{3}} \mathrm{e}^{-x}}{2 \mathrm{e}^{2x} c_1 + 3}$$

$$y(x) = -\frac{\left(1 + i\sqrt{3}\right) 2^{\frac{1}{3}} \left(\mathrm{e}^{2x} (2 \mathrm{e}^{2x} c_1 + 3)^2 \right)^{\frac{1}{3}} \mathrm{e}^{-x}}{4 \mathrm{e}^{2x} c_1 + 6}$$

$$y(x) = \frac{2^{\frac{1}{3}} \left(\mathrm{e}^{2x} (2 \mathrm{e}^{2x} c_1 + 3)^2 \right)^{\frac{1}{3}} \left(i\sqrt{3} - 1 \right) \mathrm{e}^{-x}}{4 \mathrm{e}^{2x} c_1 + 6}$$

✓ Solution by Mathematica

Time used: 4.751 (sec). Leaf size: 90

```
DSolve[y'[x]+y[x]==y[x]^4*Exp[x],y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to -\frac{\sqrt[3]{-2}}{\sqrt[3]{e^x\left(3+2c_1e^{2x}\right)}}$$

$$y(x) \to \frac{1}{\sqrt[3]{\frac{3e^x}{2}+c_1e^{3x}}}$$

$$y(x) \to \frac{(-1)^{2/3}}{\sqrt[3]{\frac{3e^x}{2}+c_1e^{3x}}}$$

$$y(x) \to 0$$

2.23 problem 23

Internal problem ID [5109]

Book: Engineering Mathematics. By K. A. Stroud. 5th edition. Industrial press Inc. NY. 2001
Section: Program 24. First order differential equations. Further problems 24. page 1068
Problem number: 23.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_Bernoulli]`

$$\boxed{2y' + y - y^3(x-1) = 0}$$

✓ Solution by Maple
Time used: 0.0 (sec). Leaf size: 25

```
dsolve(2*diff(y(x),x)+y(x)=y(x)^3*(x-1),y(x), singsol=all)
```

$$y(x) = \frac{1}{\sqrt{e^x c_1 + x}}$$
$$y(x) = -\frac{1}{\sqrt{e^x c_1 + x}}$$

✓ Solution by Mathematica
Time used: 2.721 (sec). Leaf size: 40

```
DSolve[2*y'[x]+y[x]==y[x]^3*(x-1),y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to -\frac{1}{\sqrt{x + c_1 e^x}}$$
$$y(x) \to \frac{1}{\sqrt{x + c_1 e^x}}$$
$$y(x) \to 0$$

2.24 problem 24

Internal problem ID [5110]

Book: Engineering Mathematics. By K. A. Stroud. 5th edition. Industrial press Inc. NY. 2001
Section: Program 24. First order differential equations. Further problems 24. page 1068
Problem number: 24.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_Bernoulli]`

$$\boxed{y' - 2y\tan(x) - \tan(x)^2 y^2 = 0}$$

✓ Solution by Maple
Time used: 0.0 (sec). Leaf size: 20

```
dsolve(diff(y(x),x)-2*y(x)*tan(x)=y(x)^2*tan(x)^2,y(x), singsol=all)
```

$$y(x) = -\frac{3\sec(x)^2}{\tan(x)^3 - 3c_1}$$

✓ Solution by Mathematica
Time used: 0.519 (sec). Leaf size: 31

```
DSolve[y'[x]-2*y[x]*Tan[x]==y[x]^2*Tan[x]^2,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to \frac{3}{-\sin^2(x)\tan(x) + 3c_1\cos^2(x)}$$
$$y(x) \to 0$$

2.25 problem 25

Internal problem ID [5111]

Book: Engineering Mathematics. By K. A. Stroud. 5th edition. Industrial press Inc. NY. 2001
Section: Program 24. First order differential equations. Further problems 24. page 1068
Problem number: 25.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_Bernoulli]`

$$\boxed{y' + y\tan(x) - y^3\sec(x)^4 = 0}$$

✓ Solution by Maple
Time used: 0.032 (sec). Leaf size: 68

```
dsolve(diff(y(x),x)+y(x)*tan(x)=y(x)^3*sec(x)^4,y(x), singsol=all)
```

$$y(x) = \frac{\sqrt{\cos(x)^5\left(\cos(x)c_1 - 2\sin(x)\right)}\sec(x)}{-\cos(x)c_1 + 2\sin(x)}$$
$$y(x) = \frac{\sqrt{\cos(x)^5\left(\cos(x)c_1 - 2\sin(x)\right)}\sec(x)}{\cos(x)c_1 - 2\sin(x)}$$

✓ Solution by Mathematica
Time used: 4.061 (sec). Leaf size: 48

```
DSolve[y'[x]+y[x]*Tan[x]==y[x]^3*Sec[x]^4,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to -\frac{1}{\sqrt{\sec^2(x)(-2\tan(x) + c_1)}}$$
$$y(x) \to \frac{1}{\sqrt{\sec^2(x)(-2\tan(x) + c_1)}}$$
$$y(x) \to 0$$

2.26 problem 26

Internal problem ID [5112]

Book: Engineering Mathematics. By K. A. Stroud. 5th edition. Industrial press Inc. NY. 2001
Section: Program 24. First order differential equations. Further problems 24. page 1068
Problem number: 26.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_linear]`

$$\boxed{\left(-x^2+1\right)y' - xy = 1}$$

✓ Solution by Maple
Time used: 0.0 (sec). Leaf size: 46

```
dsolve((1-x^2)*diff(y(x),x)=1+x*y(x),y(x), singsol=all)
```

$$y(x) = -\frac{\sqrt{x^2-1}\,\ln\left(x+\sqrt{x^2-1}\right)}{(x-1)(x+1)} + \frac{c_1}{\sqrt{x-1}\,\sqrt{x+1}}$$

✓ Solution by Mathematica
Time used: 0.032 (sec). Leaf size: 54

```
DSolve[(1-x^2)*y'[x]==1+x*y[x],y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to \frac{\log\left(1-\frac{x}{\sqrt{x^2-1}}\right) - \log\left(\frac{x}{\sqrt{x^2-1}}+1\right) + 2c_1}{2\sqrt{x^2-1}}$$

2.27 problem 27

Internal problem ID [5113]

Book: Engineering Mathematics. By K. A. Stroud. 5th edition. Industrial press Inc. NY. 2001
Section: Program 24. First order differential equations. Further problems 24. page 1068
Problem number: 27.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_separable]`

$$\boxed{xyy' - (x+1)\sqrt{y-1} = 0}$$

✓ Solution by Maple
Time used: 0.0 (sec). Leaf size: 22

```
dsolve(x*y(x)*diff(y(x),x)-(1+x)*sqrt(y(x)-1)=0,y(x), singsol=all)
```

$$\frac{(-2y(x)-4)\sqrt{y(x)-1}}{3} + x + c_1 + \ln(x) = 0$$

✓ Solution by Mathematica

Time used: 5.614 (sec). Leaf size: 582

```
DSolve[x*y[x]*y'[x]-(1+x)*Sqrt[y[x]-1]==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x)$$

$$\to \frac{1}{2}\sqrt[3]{9x^2+3\sqrt{(x+\log(x)+c_1)^2\left(9x^2+9\log^2(x)+18c_1x+18(x+c_1)\log(x)+16+9{c_1}^2\right)}+9\log^2(x)$$

$$+\frac{2}{\sqrt[3]{9x^2+3\sqrt{(x+\log(x)+c_1)^2\left(9x^2+9\log^2(x)+18c_1x+18(x+c_1)\log(x)+16+9{c_1}^2\right)}+9\log^2(x)}$$

$$-1$$

$$y(x)\to \frac{1}{4}i\Big(\sqrt{3}$$

$$+i\Big)\sqrt[3]{9x^2+3\sqrt{(x+\log(x)+c_1)^2\left(9x^2+9\log^2(x)+18c_1x+18(x+c_1)\log(x)+16+9{c_1}^2\right)}+9\log^2($$

$$+\frac{-1-i\sqrt{3}}{\sqrt[3]{9x^2+3\sqrt{(x+\log(x)+c_1)^2\left(9x^2+9\log^2(x)+18c_1x+18(x+c_1)\log(x)+16+9{c_1}^2\right)}+9\log^2(x)}$$

$$-1$$

$$y(x)\to -\frac{1}{4}i\Big(\sqrt{3}$$

$$-i\Big)\sqrt[3]{9x^2+3\sqrt{(x+\log(x)+c_1)^2\left(9x^2+9\log^2(x)+18c_1x+18(x+c_1)\log(x)+16+9{c_1}^2\right)}+9\log^2($$

$$+\frac{-1+i\sqrt{3}}{\sqrt[3]{9x^2+3\sqrt{(x+\log(x)+c_1)^2\left(9x^2+9\log^2(x)+18c_1x+18(x+c_1)\log(x)+16+9{c_1}^2\right)}+9\log^2(x)}$$

$$-1$$

$$y(x)\to 1$$

2.28 problem 28

Internal problem ID [5114]

Book: Engineering Mathematics. By K. A. Stroud. 5th edition. Industrial press Inc. NY. 2001
Section: Program 24. First order differential equations. Further problems 24. page 1068
Problem number: 28.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[[_homogeneous, 'class A'], _rational, _dAlembert]`

$$\boxed{-2xy + 5y^2 - \left(x^2 + 2xy + y^2\right) y' = -x^2}$$

✓ Solution by Maple
Time used: 0.031 (sec). Leaf size: 35

```
dsolve((x^2-2*x*y(x)+5*y(x)^2)=(x^2+2*x*y(x)+y(x)^2)*diff(y(x),x),y(x), singsol=all)
```

$$y(x) = x\left(1 + \mathrm{e}^{\mathrm{RootOf}\left(\ln(x)\mathrm{e}^{2\_Z} + c_1\mathrm{e}^{2\_Z} + \_Z\mathrm{e}^{2\_Z} - 4\,\mathrm{e}^{\_Z} - 2\right)}\right)$$

✓ Solution by Mathematica
Time used: 0.343 (sec). Leaf size: 41

```
DSolve[(x^2-2*x*y[x]+5*y[x]^2)==(x^2+2*x*y[x]+y[x]^2)*y'[x],y[x],x,IncludeSingularSolutions
```

$$\text{Solve}\left[\frac{2 - \frac{4y(x)}{x}}{\left(\frac{y(x)}{x} - 1\right)^2} + \log\left(\frac{y(x)}{x} - 1\right) = -\log(x) + c_1, y(x)\right]$$

2.29 problem 29

Internal problem ID [5115]

Book: Engineering Mathematics. By K. A. Stroud. 5th edition. Industrial press Inc. NY. 2001
Section: Program 24. First order differential equations. Further problems 24. page 1068
Problem number: 29.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_Bernoulli]`

$$\boxed{y' - y\cot(x) - y^2\sec(x)^2 = 0}$$

With initial conditions

$$\left[y\left(\frac{\pi}{4}\right) = -1\right]$$

✓ Solution by Maple
Time used: 0.89 (sec). Leaf size: 18

```
dsolve([diff(y(x),x)-y(x)*cot(x)=y(x)^2*sec(x)^2,y(1/4*Pi) = -1],y(x), singsol=all)
```

$$y(x) = \frac{2\sin(x)}{\sqrt{2} - 2\sec(x)}$$

✓ Solution by Mathematica
Time used: 0.46 (sec). Leaf size: 22

```
DSolve[{y'[x]-y[x]*Cot[x]==y[x]^2*Sec[x]^2,{y[Pi/4]==-1}},y[x],x,IncludeSingularSolutions ->
```

$$y(x) \to \frac{\sin(2x)}{\sqrt{2}\cos(x) - 2}$$

2.30 problem 30

Internal problem ID [5116]

Book: Engineering Mathematics. By K. A. Stroud. 5th edition. Industrial press Inc. NY. 2001
Section: Program 24. First order differential equations. Further problems 24. page 1068
Problem number: 30.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_separable]`

$$\boxed{y + (x^2 - 4x)\, y' = 0}$$

✓ Solution by Maple
Time used: 0.0 (sec). Leaf size: 14

```
dsolve(y(x)+(x^2-4*x)*diff(y(x),x)=0,y(x), singsol=all)
```

$$y(x) = \frac{c_1 x^{\frac{1}{4}}}{(x-4)^{\frac{1}{4}}}$$

✓ Solution by Mathematica
Time used: 0.033 (sec). Leaf size: 27

```
DSolve[y[x]+(x^2-4*x)*y'[x]==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to \frac{c_1 \sqrt[4]{x}}{\sqrt[4]{4-x}}$$
$$y(x) \to 0$$

2.31 problem 31

Internal problem ID [5117]

Book: Engineering Mathematics. By K. A. Stroud. 5th edition. Industrial press Inc. NY. 2001
Section: Program 24. First order differential equations. Further problems 24. page 1068
Problem number: 31.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_linear]`

$$\boxed{y' - y\tan(x) = \cos(x) - 2\sin(x)\,x}$$

With initial conditions

$$\left[y\left(\frac{\pi}{6}\right) = 0\right]$$

✓ Solution by Maple
Time used: 0.015 (sec). Leaf size: 14

```
dsolve([diff(y(x),x)-y(x)*tan(x)=cos(x)-2*x*sin(x),y(1/6*Pi) = 0],y(x), singsol=all)
```

$$y(x) = \cos(x)\,x - \frac{\pi\sec(x)}{8}$$

✓ Solution by Mathematica
Time used: 0.073 (sec). Leaf size: 25

```
DSolve[{y'[x]-y[x]*Tan[x]==Cos[x]-2*x*Sin[x],{y[Pi/6]==0}},y[x],x,IncludeSingularSolutions -
```

$$y(x) \to \frac{1}{8}(4x + 4x\cos(2x) - \pi)\sec(x)$$

2.32 problem 32

Internal problem ID [5118]

Book: Engineering Mathematics. By K. A. Stroud. 5th edition. Industrial press Inc. NY. 2001
Section: Program 24. First order differential equations. Further problems 24. page 1068
Problem number: 32.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[[_homogeneous, ‘class A‘], _rational, [_Abel, ‘2nd type‘, ‘cl`

$$\boxed{y' - \frac{2xy + y^2}{x^2 + 2xy} = 0}$$

✓ Solution by Maple
Time used: 0.032 (sec). Leaf size: 356

```
dsolve(diff(y(x),x)=(2*x*y(x)+y(x)^2)/(x^2+2*x*y(x)),y(x), singsol=all)
```

$$y(x) = \frac{12^{\frac{1}{3}} \left(x \left(\sqrt{3} \sqrt{\frac{x(27c_1 x - 4)}{c_1}} + 9x \right) c_1^2 \right)^{\frac{1}{3}}}{6c_1} + \frac{x 12^{\frac{2}{3}}}{6 \left(x \left(\sqrt{3} \sqrt{\frac{x(27c_1 x - 4)}{c_1}} + 9x \right) c_1^2 \right)^{\frac{1}{3}}} + x$$

$$y(x) = \frac{-\frac{\left(i 3^{\frac{5}{6}} + 3^{\frac{1}{3}}\right) 2^{\frac{2}{3}} \left(x \left(\sqrt{3} \sqrt{\frac{27c_1 x^2 - 4x}{c_1}} + 9x \right) c_1^2 \right)^{\frac{2}{3}}}{6} + \left(2 \left(x \left(\sqrt{3} \sqrt{\frac{27c_1 x^2 - 4x}{c_1}} + 9x \right) c_1^2 \right)^{\frac{1}{3}} + 2^{\frac{1}{3}} \left(i 3^{\frac{1}{6}} - \frac{3^{\frac{2}{3}}}{3} \right) \right) x c_1}{2 \left(x \left(\sqrt{3} \sqrt{\frac{27c_1 x^2 - 4x}{c_1}} + 9x \right) c_1^2 \right)^{\frac{1}{3}} c_1}$$

$$y(x) = \frac{-\frac{\left(i 3^{\frac{5}{6}} - 3^{\frac{1}{3}}\right) 2^{\frac{2}{3}} \left(x \left(\sqrt{3} \sqrt{\frac{27c_1 x^2 - 4x}{c_1}} + 9x \right) c_1^2 \right)^{\frac{2}{3}}}{6} + \left(-2 \left(x \left(\sqrt{3} \sqrt{\frac{27c_1 x^2 - 4x}{c_1}} + 9x \right) c_1^2 \right)^{\frac{1}{3}} + 2^{\frac{1}{3}} \left(i 3^{\frac{1}{6}} + \frac{3^{\frac{2}{3}}}{3} \right) \right) x c_1}{2 \left(x \left(\sqrt{3} \sqrt{\frac{27c_1 x^2 - 4x}{c_1}} + 9x \right) c_1^2 \right)^{\frac{1}{3}} c_1}$$

✓ Solution by Mathematica

Time used: 56.42 (sec). Leaf size: 404

```
DSolve[y'[x]==(2*x*y[x]+y[x]^2)/(x^2+2*x*y[x]),y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to -\frac{\sqrt[3]{\frac{2}{3}} e^{c_1} x}{\sqrt[3]{\sqrt{3}\sqrt{e^{2c_1}x^3\left(27x+4e^{c_1}\right)}-9e^{c_1}x^2}}+\frac{\sqrt[3]{\sqrt{3}\sqrt{e^{2c_1}x^3\left(27x+4e^{c_1}\right)}-9e^{c_1}x^2}}{\sqrt[3]{2}3^{2/3}}+x$$

$$y(x) \to \frac{\left(1+i\sqrt{3}\right)e^{c_1}x}{2^{2/3}\sqrt[3]{3\sqrt{3}\sqrt{e^{2c_1}x^3\left(27x+4e^{c_1}\right)}-27e^{c_1}x^2}}+\frac{i\left(\sqrt{3}+i\right)\sqrt[3]{\sqrt{3}\sqrt{e^{2c_1}x^3\left(27x+4e^{c_1}\right)}-9e^{c_1}x^2}}{2\sqrt[3]{2}3^{2/3}}+x$$

$$y(x) \to \frac{\left(1-i\sqrt{3}\right)e^{c_1}x}{2^{2/3}\sqrt[3]{3\sqrt{3}\sqrt{e^{2c_1}x^3\left(27x+4e^{c_1}\right)}-27e^{c_1}x^2}}-\frac{\left(1+i\sqrt{3}\right)\sqrt[3]{\sqrt{3}\sqrt{e^{2c_1}x^3\left(27x+4e^{c_1}\right)}-9e^{c_1}x^2}}{2\sqrt[3]{2}3^{2/3}}+x$$

2.33 problem 33

Internal problem ID [5119]

Book: Engineering Mathematics. By K. A. Stroud. 5th edition. Industrial press Inc. NY. 2001
Section: Program 24. First order differential equations. Further problems 24. page 1068
Problem number: 33.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_separable]`

$$\boxed{\left(x^2+1\right)y' - x(1+y) = 0}$$

✓ Solution by Maple
Time used: 0.0 (sec). Leaf size: 15

```
dsolve((1+x^2)*diff(y(x),x)=x*(1+y(x)),y(x), singsol=all)
```

$$y(x) = \sqrt{x^2+1}\, c_1 - 1$$

✓ Solution by Mathematica
Time used: 0.031 (sec). Leaf size: 24

```
DSolve[(1+x^2)*y'[x]==x*(1+y[x]),y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to -1 + c_1\sqrt{x^2+1}$$
$$y(x) \to -1$$

2.34 problem 34

Internal problem ID [5120]

Book: Engineering Mathematics. By K. A. Stroud. 5th edition. Industrial press Inc. NY. 2001
Section: Program 24. First order differential equations. Further problems 24. page 1068
Problem number: 34.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_linear]`

$$\boxed{xy' + 2y = 3x - 1}$$

With initial conditions

$$[y(2) = 1]$$

✓ Solution by Maple
Time used: 0.016 (sec). Leaf size: 12

```
dsolve([x*diff(y(x),x)+2*y(x)=3*x-1,y(2) = 1],y(x), singsol=all)
```

$$y(x) = x - \frac{1}{2} - \frac{2}{x^2}$$

✓ Solution by Mathematica
Time used: 0.027 (sec). Leaf size: 15

```
DSolve[{x*y'[x]+2*y[x]==3*x-1,{y[2]==1}},y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to -\frac{2}{x^2} + x - \frac{1}{2}$$

2.35 problem 35

Internal problem ID [5121]

Book: Engineering Mathematics. By K. A. Stroud. 5th edition. Industrial press Inc. NY. 2001
Section: Program 24. First order differential equations. Further problems 24. page 1068
Problem number: 35.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[[_homogeneous, ‘class A‘], _rational, [_Abel, ‘2nd type‘, ‘cl`

$$\boxed{x^2y' - y^2 + xyy' = 0}$$

With initial conditions

$$[y(1) = 1]$$

✓ Solution by Maple
Time used: 0.375 (sec). Leaf size: 13

```
dsolve([x^2*diff(y(x),x)=y(x)^2-x*y(x)*diff(y(x),x),y(1) = 1],y(x), singsol=all)
```

$$y(x) = \text{LambertW}\left(\frac{\mathrm{e}}{x}\right) x$$

✓ Solution by Mathematica
Time used: 2.335 (sec). Leaf size: 13

```
DSolve[{x^2*y'[x]==y[x]^2-x*y[x]*y'[x],{y[1]==1}},y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to xW\left(\frac{e}{x}\right)$$

2.36 problem 36

Internal problem ID [5122]

Book: Engineering Mathematics. By K. A. Stroud. 5th edition. Industrial press Inc. NY. 2001
Section: Program 24. First order differential equations. Further problems 24. page 1068
Problem number: 36.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_separable]`

$$\boxed{y' - \mathrm{e}^{3x-2y} = 0}$$

With initial conditions

$$[y(0) = 0]$$

✓ Solution by Maple
Time used: 0.078 (sec). Leaf size: 15

```
dsolve([diff(y(x),x)=exp(3*x-2*y(x)),y(0) = 0],y(x), singsol=all)
```

$$y(x) = -\frac{\ln(3)}{2} + \frac{\ln(1 + 2\,\mathrm{e}^{3x})}{2}$$

✓ Solution by Mathematica
Time used: 0.881 (sec). Leaf size: 23

```
DSolve[{y'[x]==Exp[3*x-2*y[x]],{y[0]==0}},y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to \frac{1}{2}\log\left(\frac{1}{3}\left(2e^{3x} + 1\right)\right)$$

2.37 problem 37

Internal problem ID [5123]

Book: Engineering Mathematics. By K. A. Stroud. 5th edition. Industrial press Inc. NY. 2001
Section: Program 24. First order differential equations. Further problems 24. page 1068
Problem number: 37.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_linear]`

$$\boxed{y' + \frac{y}{x} = \sin(2x)}$$

With initial conditions

$$\left[y\left(\frac{\pi}{4}\right) = 2\right]$$

✓ Solution by Maple
Time used: 0.015 (sec). Leaf size: 25

```
dsolve([diff(y(x),x)+1/x*y(x)=sin(2*x),y(1/4*Pi) = 2],y(x), singsol=all)
```

$$y(x) = \frac{-2x\cos(2x) + 2\pi + \sin(2x) - 1}{4x}$$

✓ Solution by Mathematica
Time used: 0.042 (sec). Leaf size: 28

```
DSolve[{y'[x]+1/x*y[x]==Sin[2*x],{y[Pi/4]==2}},y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to \frac{\sin(2x) - 2x\cos(2x) + 2\pi - 1}{4x}$$

2.38 problem 38

Internal problem ID [5124]

Book: Engineering Mathematics. By K. A. Stroud. 5th edition. Industrial press Inc. NY. 2001
Section: Program 24. First order differential equations. Further problems 24. page 1068
Problem number: 38.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[[_homogeneous, 'class A'], _rational, [_Abel, '2nd type', 'cl`

$$\boxed{y^2 + x^2 y' - xyy' = 0}$$

✓ Solution by Maple
Time used: 0.016 (sec). Leaf size: 17

```
dsolve(y(x)^2+x^2*diff(y(x),x)=x*y(x)*diff(y(x),x),y(x), singsol=all)
```

$$y(x) = -x \operatorname{LambertW}\left(-\frac{\mathrm{e}^{-c_1}}{x}\right)$$

✓ Solution by Mathematica
Time used: 2.23 (sec). Leaf size: 25

```
DSolve[y[x]^2+x^2*y'[x]==x*y[x]*y'[x],y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to -xW\left(-\frac{e^{-c_1}}{x}\right)$$
$$y(x) \to 0$$

2.39 problem 39

Internal problem ID [5125]

Book: Engineering Mathematics. By K. A. Stroud. 5th edition. Industrial press Inc. NY. 2001
Section: Program 24. First order differential equations. Further problems 24. page 1068
Problem number: 39.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[[_homogeneous, 'class A'], _exact, _rational, _Bernoulli]`

$$\boxed{2xyy' + y^2 = x^2}$$

✓ Solution by Maple
Time used: 0.0 (sec). Leaf size: 45

```
dsolve(2*x*y(x)*diff(y(x),x)=x^2-y(x)^2,y(x), singsol=all)
```

$$y(x) = -\frac{\sqrt{3}\sqrt{x\left(x^3+3c_1\right)}}{3x}$$
$$y(x) = \frac{\sqrt{3}\sqrt{x\left(x^3+3c_1\right)}}{3x}$$

✓ Solution by Mathematica
Time used: 0.2 (sec). Leaf size: 56

```
DSolve[2*x*y[x]*y'[x]==x^2-y[x]^2,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to -\frac{\sqrt{x^3+3c_1}}{\sqrt{3}\sqrt{x}}$$
$$y(x) \to \frac{\sqrt{x^3+3c_1}}{\sqrt{3}\sqrt{x}}$$

2.40 problem 40

Internal problem ID [5126]

Book: Engineering Mathematics. By K. A. Stroud. 5th edition. Industrial press Inc. NY. 2001
Section: Program 24. First order differential equations. Further problems 24. page 1068
Problem number: 40.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[[_homogeneous, 'class C'], _rational, [_Abel, '2nd type', 'cl`

$$\boxed{y' - \frac{-2y + x + 1}{2x - 4y} = 0}$$

With initial conditions

$$[y(1) = 1]$$

✓ Solution by Maple
Time used: 0.234 (sec). Leaf size: 17

```
dsolve([diff(y(x),x)=(x-2*y(x)+1)/(2*x-4*y(x)),y(1) = 1],y(x), singsol=all)
```

$$y(x) = \frac{x}{2} + \frac{\sqrt{-2x+3}}{2}$$

✓ Solution by Mathematica
Time used: 0.115 (sec). Leaf size: 24

```
DSolve[{y'[x]==(x-2*y[x]+1)/(2*x-4*y[x]),{y[1]==1}},y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to \frac{1}{2}\left(x - i\sqrt{2x-3}\right)$$

2.41 problem 41

Internal problem ID [5127]

Book: Engineering Mathematics. By K. A. Stroud. 5th edition. Industrial press Inc. NY. 2001
Section: Program 24. First order differential equations. Further problems 24. page 1068
Problem number: 41.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_linear]`

$$\boxed{\left(-x^3+1\right)y'+yx^2=x^2\left(-x^3+1\right)}$$

✓ Solution by Maple
Time used: 0.0 (sec). Leaf size: 20

```
dsolve((1-x^3)*diff(y(x),x)+x^2*y(x)=x^2*(1-x^3),y(x), singsol=all)
```

$$y(x)=\frac{x^3}{2}-\frac{1}{2}+\left(x^3-1\right)^{\frac{1}{3}}c_1$$

✓ Solution by Mathematica
Time used: 0.054 (sec). Leaf size: 27

```
DSolve[(1-x^3)*y'[x]+x^2*y[x]==x^2*(1-x^3),y[x],x,IncludeSingularSolutions -> True]
```

$$y(x)\to\frac{1}{2}\left(x^3+2c_1\sqrt[3]{x^3-1}-1\right)$$

2.42 problem 42

Internal problem ID [5128]

Book: Engineering Mathematics. By K. A. Stroud. 5th edition. Industrial press Inc. NY. 2001
Section: Program 24. First order differential equations. Further problems 24. page 1068
Problem number: 42.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_linear]`

$$\boxed{y' + \frac{y}{x} = \sin(x)}$$

With initial conditions

$$\left[y\left(\frac{\pi}{2}\right) = 0\right]$$

✓ Solution by Maple
Time used: 0.015 (sec). Leaf size: 17

```
dsolve([diff(y(x),x)+y(x)/x=sin(x),y(1/2*Pi) = 0],y(x), singsol=all)
```

$$y(x) = \frac{\sin(x) - \cos(x)\, x - 1}{x}$$

✓ Solution by Mathematica
Time used: 0.04 (sec). Leaf size: 18

```
DSolve[{y'[x]+y[x]/x==Sin[x],{y[Pi/2]==0}},y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to \frac{\sin(x) - x\cos(x) - 1}{x}$$

2.43 problem 43

Internal problem ID [5129]

Book: Engineering Mathematics. By K. A. Stroud. 5th edition. Industrial press Inc. NY. 2001
Section: Program 24. First order differential equations. Further problems 24. page 1068
Problem number: 43.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_separable]`

$$\boxed{y' + xy^2 = -x}$$

With initial conditions

$$[y(1) = 0]$$

✓ Solution by Maple
Time used: 0.047 (sec). Leaf size: 14

```
dsolve([diff(y(x),x)+x+x*y(x)^2=0,y(1) = 0],y(x), singsol=all)
```

$$y(x) = -\tan\left(\frac{x^2}{2} - \frac{1}{2}\right)$$

✓ Solution by Mathematica
Time used: 0.215 (sec). Leaf size: 17

```
DSolve[{y'[x]+x+x*y[x]^2==0,{y[1]==0}},y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to \tan\left(\frac{1}{2}(1 - x^2)\right)$$

2.44 problem 44

Internal problem ID [5130]

Book: Engineering Mathematics. By K. A. Stroud. 5th edition. Industrial press Inc. NY. 2001
Section: Program 24. First order differential equations. Further problems 24. page 1068
Problem number: 44.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_linear]`

$$\boxed{y' + \left(\frac{1}{x} - \frac{2x}{-x^2+1}\right) y = \frac{1}{-x^2+1}}$$

✓ Solution by Maple
Time used: 0.0 (sec). Leaf size: 25

```
dsolve(diff(y(x),x)+(1/x-(2*x)/(1-x^2))*y(x)=1/(1-x^2),y(x), singsol=all)
```

$$y(x) = \frac{-x^2 + 2c_1}{2x^3 - 2x}$$

✓ Solution by Mathematica
Time used: 0.038 (sec). Leaf size: 25

```
DSolve[y'[x]+(1/x-(2*x)/(1-x^2))*y[x]==1/(1-x^2),y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to \frac{x^2 + 2c_1}{2x - 2x^3}$$

2.45 problem 45

Internal problem ID [5131]

Book: Engineering Mathematics. By K. A. Stroud. 5th edition. Industrial press Inc. NY. 2001
Section: Program 24. First order differential equations. Further problems 24. page 1068
Problem number: 45.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_linear]`

$$\boxed{\left(x^2+1\right)y' + xy = \left(x^2+1\right)^{\frac{3}{2}}}$$

✓ Solution by Maple
Time used: 0.0 (sec). Leaf size: 23

```
dsolve((1+x^2)*diff(y(x),x)+x*y(x)=(1+x^2)^(3/2),y(x), singsol=all)
```

$$y(x) = \frac{x^3 + 3c_1 + 3x}{3\sqrt{x^2+1}}$$

✓ Solution by Mathematica
Time used: 0.065 (sec). Leaf size: 29

```
DSolve[(1+x^2)*y'[x]+x*y[x]==(1+x^2)^(3/2),y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to \frac{x^3 + 3x + 3c_1}{3\sqrt{x^2+1}}$$

2.46 problem 46

Internal problem ID [5132]

Book: Engineering Mathematics. By K. A. Stroud. 5th edition. Industrial press Inc. NY. 2001
Section: Program 24. First order differential equations. Further problems 24. page 1068
Problem number: 46.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_separable]`

$$\boxed{x(1+y^2)-y(x^2+1)y'=0}$$

✓ Solution by Maple
Time used: 0.016 (sec). Leaf size: 29

```
dsolve(x*(1+y(x)^2)-y(x)*(1+x^2)*diff(y(x),x)=0,y(x), singsol=all)
```

$$y(x)=\sqrt{c_1x^2+c_1-1}$$
$$y(x)=-\sqrt{c_1x^2+c_1-1}$$

✓ Solution by Mathematica
Time used: 0.499 (sec). Leaf size: 61

```
DSolve[x*(1+y[x]^2)-y[x]*(1+x^2)*y'[x]==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x)\to -\sqrt{-1+e^{2c_1}\left(x^2+1\right)}$$
$$y(x)\to \sqrt{-1+e^{2c_1}\left(x^2+1\right)}$$
$$y(x)\to -i$$
$$y(x)\to i$$

2.47 problem 47

Internal problem ID [5133]

Book: Engineering Mathematics. By K. A. Stroud. 5th edition. Industrial press Inc. NY. 2001
Section: Program 24. First order differential equations. Further problems 24. page 1068
Problem number: 47.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_separable]`

$$\boxed{\frac{r \tan(\theta) r'}{a^2 - r^2} = 1}$$

With initial conditions

$$\left[r\left(\frac{\pi}{4}\right) = 0\right]$$

✓ Solution by Maple
Time used: 0.188 (sec). Leaf size: 39

```
dsolve([r(theta)*tan(theta)/(a^2-r(theta)^2)*diff(r(theta),theta)=1,r(1/4*Pi) = 0],r(theta),
```

$$r(\theta) = -\frac{a\sqrt{2}\sqrt{-\cos(2\theta)}\csc(\theta)}{2}$$
$$r(\theta) = \frac{a\sqrt{2}\sqrt{-\cos(2\theta)}\csc(\theta)}{2}$$

✓ Solution by Mathematica
Time used: 0.149 (sec). Leaf size: 51

```
DSolve[{r[\[Theta]]*Tan[\[Theta]]/(a^2-r[\[Theta]]^2)*r'[\[Theta]]==1,{r[Pi/4]==0}},r[\[Thet
```

$$r(\theta) \to -\sqrt{\frac{a^2\cos(2\theta)}{\cos(2\theta) - 1}}$$
$$r(\theta) \to \sqrt{\frac{a^2\cos(2\theta)}{\cos(2\theta) - 1}}$$

2.48 problem 48

Internal problem ID [5134]

Book: Engineering Mathematics. By K. A. Stroud. 5th edition. Industrial press Inc. NY. 2001
Section: Program 24. First order differential equations. Further problems 24. page 1068
Problem number: 48.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_linear]`

$$\boxed{y' + y\cot(x) = \cos(x)}$$

With initial conditions

$$[y(0) = 0]$$

✓ Solution by Maple
Time used: 0.157 (sec). Leaf size: 8

```
dsolve([diff(y(x),x)+y(x)*cot(x)=cos(x),y(0) = 0],y(x), singsol=all)
```

$$y(x) = \frac{\sin(x)}{2}$$

✓ Solution by Mathematica
Time used: 0.103 (sec). Leaf size: 11

```
DSolve[{y'[x]+y[x]*Cot[x]==Cos[x],{y[0]==0}},y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to \frac{\sin(x)}{2}$$

2.49 problem 49

Internal problem ID [5135]

Book: Engineering Mathematics. By K. A. Stroud. 5th edition. Industrial press Inc. NY. 2001
Section: Program 24. First order differential equations. Further problems 24. page 1068
Problem number: 49.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[[_homogeneous, 'class G'], _rational, _Bernoulli]`

$$\boxed{y' + \frac{y}{x} - xy^2 = 0}$$

✓ Solution by Maple
Time used: 0.0 (sec). Leaf size: 15

```
dsolve(diff(y(x),x)+y(x)/x=x*y(x)^2,y(x), singsol=all)
```

$$y(x) = \frac{1}{(-x + c_1)\, x}$$

✓ Solution by Mathematica
Time used: 0.135 (sec). Leaf size: 23

```
DSolve[y'[x]+y[x]/x==x*y[x]^2,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to -\frac{1}{x^2 - c_1 x}$$
$$y(x) \to 0$$

3 Program 25. Second order differential equations. Test Excercise 25. page 1093

3.1 problem 1

Internal problem ID [5136]

Book: Engineering Mathematics. By K. A. Stroud. 5th edition. Industrial press Inc. NY. 2001
Section: Program 25. Second order differential equations. Test Excercise 25. page 1093
Problem number: 1.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _missing_x]]`

$$\boxed{y'' - y' - 2y = 8}$$

✓ Solution by Maple
Time used: 0.0 (sec). Leaf size: 18

```
dsolve(diff(y(x),x$2)-diff(y(x),x)-2*y(x)=8,y(x), singsol=all)
```

$$y(x) = c_2 \mathrm{e}^{-x} + \mathrm{e}^{2x} c_1 - 4$$

✓ Solution by Mathematica
Time used: 0.014 (sec). Leaf size: 23

```
DSolve[y''[x]-y'[x]-2*y[x]==8,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to c_1 e^{-x} + c_2 e^{2x} - 4$$

3.2 problem 2

Internal problem ID [5137]

Book: Engineering Mathematics. By K. A. Stroud. 5th edition. Industrial press Inc. NY. 2001
Section: Program 25. Second order differential equations. Test Excercise 25. page 1093
Problem number: 2.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$\boxed{y'' - 4y = 10\,\mathrm{e}^{3x}}$$

✓ Solution by Maple
Time used: 0.0 (sec). Leaf size: 23

```
dsolve(diff(y(x),x$2)-4*y(x)=10*exp(3*x),y(x), singsol=all)
```

$$y(x) = \left(2\,\mathrm{e}^{5x} + \mathrm{e}^{4x} c_1 + c_2\right) \mathrm{e}^{-2x}$$

✓ Solution by Mathematica
Time used: 0.017 (sec). Leaf size: 29

```
DSolve[y''[x]-4*y[x]==10*Exp[3*x],y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to e^{-2x}\left(2e^{5x} + c_1 e^{4x} + c_2\right)$$

3.3 problem 3

Internal problem ID [5138]

Book: Engineering Mathematics. By K. A. Stroud. 5th edition. Industrial press Inc. NY. 2001
Section: Program 25. Second order differential equations. Test Excercise 25. page 1093
Problem number: 3.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$\boxed{y'' + 2y' + y = \mathrm{e}^{-2x}}$$

✓ Solution by Maple
Time used: 0.0 (sec). Leaf size: 19

```
dsolve(diff(y(x),x$2)+2*diff(y(x),x)+y(x)=exp(-2*x),y(x), singsol=all)
```

$$y(x) = (c_1 x + c_2)\,\mathrm{e}^{-x} + \mathrm{e}^{-2x}$$

✓ Solution by Mathematica
Time used: 0.029 (sec). Leaf size: 24

```
DSolve[y''[x]+2*y'[x]+y[x]==Exp[-2*x],y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to e^{-2x}(1 + e^{x}(c_2 x + c_1))$$

3.4 problem 4

Internal problem ID [5139]

Book: Engineering Mathematics. By K. A. Stroud. 5th edition. Industrial press Inc. NY. 2001
Section: Program 25. Second order differential equations. Test Excercise 25. page 1093
Problem number: 4.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$\boxed{y'' + 25y = 5x^2 + x}$$

✓ Solution by Maple
Time used: 0.0 (sec). Leaf size: 26

```
dsolve(diff(y(x),x$2)+25*y(x)=5*x^2+x,y(x), singsol=all)
```

$$y(x) = \sin(5x)\, c_2 + \cos(5x)\, c_1 + \frac{x^2}{5} + \frac{x}{25} - \frac{2}{125}$$

✓ Solution by Mathematica
Time used: 0.018 (sec). Leaf size: 34

```
DSolve[y''[x]+25*y[x]==5*x^2+x,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to \frac{1}{125}\left(25x^2 + 5x - 2\right) + c_1 \cos(5x) + c_2 \sin(5x)$$

3.5 problem 5

Internal problem ID [5140]

Book: Engineering Mathematics. By K. A. Stroud. 5th edition. Industrial press Inc. NY. 2001
Section: Program 25. Second order differential equations. Test Excercise 25. page 1093
Problem number: 5.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _linear, _nonhomogeneous]]`

$$\boxed{y'' - 2y' + y = 4\sin(x)}$$

✓ Solution by Maple
Time used: 0.0 (sec). Leaf size: 17

```
dsolve(diff(y(x),x$2)-2*diff(y(x),x)+y(x)=4*sin(x),y(x), singsol=all)
```

$$y(x) = (c_1 x + c_2)\,\mathrm{e}^{x} + 2\cos(x)$$

✓ Solution by Mathematica
Time used: 0.019 (sec). Leaf size: 21

```
DSolve[y''[x]-2*y'[x]+y[x]==4*Sin[x],y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to 2\cos(x) + e^{x}(c_2 x + c_1)$$

3.6 problem 6

Internal problem ID [5141]

Book: Engineering Mathematics. By K. A. Stroud. 5th edition. Industrial press Inc. NY. 2001
Section: Program 25. Second order differential equations. Test Excercise 25. page 1093
Problem number: 6.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$\boxed{y'' + 4y' + 5y = 2\,\mathrm{e}^{-2x}}$$

With initial conditions

$$[y(0) = 1, y'(0) = -2]$$

✓ Solution by Maple
Time used: 0.031 (sec). Leaf size: 14

```
dsolve([diff(y(x),x$2)+4*diff(y(x),x)+5*y(x)=2*exp(-2*x),y(0) = 1, D(y)(0) = -2],y(x), sings
```

$$y(x) = -\mathrm{e}^{-2x}(\cos(x) - 2)$$

✓ Solution by Mathematica
Time used: 0.023 (sec). Leaf size: 16

```
DSolve[{y''[x]+4*y'[x]+5*y[x]==2*Exp[-2*x],{y[0]==1,y'[0]==-2}},y[x],x,IncludeSingularSoluti
```

$$y(x) \to -e^{-2x}(\cos(x) - 2)$$

3.7 problem 7

Internal problem ID [5142]

Book: Engineering Mathematics. By K. A. Stroud. 5th edition. Industrial press Inc. NY. 2001
Section: Program 25. Second order differential equations. Test Excercise 25. page 1093
Problem number: 7.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$\boxed{3y'' - 2y' - y = 2x - 3}$$

✓ Solution by Maple
Time used: 0.0 (sec). Leaf size: 19

```
dsolve(3*diff(y(x),x$2)-2*diff(y(x),x)-y(x)=2*x-3,y(x), singsol=all)
```

$$y(x) = \mathrm{e}^{-\frac{x}{3}} c_2 + \mathrm{e}^{x} c_1 - 2x + 7$$

✓ Solution by Mathematica
Time used: 0.014 (sec). Leaf size: 26

```
DSolve[3*y''[x]-2*y'[x]-y[x]==2*x-3,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to -2x + c_1 e^{-x/3} + c_2 e^{x} + 7$$

3.8 problem 8

Internal problem ID [5143]

Book: Engineering Mathematics. By K. A. Stroud. 5th edition. Industrial press Inc. NY. 2001
Section: Program 25. Second order differential equations. Test Excercise 25. page 1093
Problem number: 8.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$\boxed{y'' - 6y' + 8y = 8\,\mathrm{e}^{4x}}$$

✓ Solution by Maple
Time used: 0.0 (sec). Leaf size: 25

```
dsolve(diff(y(x),x$2)-6*diff(y(x),x)+8*y(x)=8*exp(4*x),y(x), singsol=all)
```

$$y(x) = \frac{(8x + c_1 - 4)\,\mathrm{e}^{4x}}{2} + c_2\mathrm{e}^{2x}$$

✓ Solution by Mathematica
Time used: 0.027 (sec). Leaf size: 27

```
DSolve[y''[x]-6*y'[x]+8*y[x]==8*Exp[4*x],y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to c_1 e^{2x} + e^{4x}(4x - 2 + c_2)$$

4 Program 25. Second order differential equations. Further problems 25. page 1094

4.1 problem 1

Internal problem ID [5144]

Book: Engineering Mathematics. By K. A. Stroud. 5th edition. Industrial press Inc. NY. 2001
Section: Program 25. Second order differential equations. Further problems 25. page 1094
Problem number: 1.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$\boxed{2y'' - 7y' - 4y = \mathrm{e}^{3x}}$$

✓ Solution by Maple
Time used: 0.016 (sec). Leaf size: 23

```
dsolve(2*diff(y(x),x$2)-7*diff(y(x),x)-4*y(x)=exp(3*x),y(x), singsol=all)
```

$$y(x) = \mathrm{e}^{-\frac{x}{2}} c_2 + \mathrm{e}^{4x} c_1 - \frac{\mathrm{e}^{3x}}{7}$$

✓ Solution by Mathematica
Time used: 0.018 (sec). Leaf size: 33

```
DSolve[2*y''[x]-7*y'[x]-4*y[x]==Exp[3*x],y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to -\frac{e^{3x}}{7} + c_1 e^{-x/2} + c_2 e^{4x}$$

4.2 problem 2

Internal problem ID [5145]

Book: Engineering Mathematics. By K. A. Stroud. 5th edition. Industrial press Inc. NY. 2001
Section: Program 25. Second order differential equations. Further problems 25. page 1094
Problem number: 2.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$\boxed{y'' - 6y' + 9y = 54x + 18}$$

✓ Solution by Maple
Time used: 0.015 (sec). Leaf size: 19

```
dsolve(diff(y(x),x$2)-6*diff(y(x),x)+9*y(x)=54*x+18,y(x), singsol=all)
```

$$y(x) = (c_1 x + c_2)\,\mathrm{e}^{3x} + 6x + 6$$

✓ Solution by Mathematica
Time used: 0.015 (sec). Leaf size: 27

```
DSolve[y''[x]-6*y'[x]+9*y[x]==54*x+18,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to c_1 e^{3x} + x\left(6 + c_2 e^{3x}\right) + 6$$

4.3 problem 3

Internal problem ID [5146]

Book: Engineering Mathematics. By K. A. Stroud. 5th edition. Industrial press Inc. NY. 2001
Section: Program 25. Second order differential equations. Further problems 25. page 1094
Problem number: 3.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _linear, _nonhomogeneous]]`

$$\boxed{y'' - 5y' + 6y = 100 \sin(4x)}$$

✓ Solution by Maple
Time used: 0.015 (sec). Leaf size: 29

```
dsolve(diff(y(x),x$2)-5*diff(y(x),x)+6*y(x)=100*sin(4*x),y(x), singsol=all)
```

$$y(x) = \mathrm{e}^{3x} c_2 + \mathrm{e}^{2x} c_1 - 2\sin(4x) + 4\cos(4x)$$

✓ Solution by Mathematica
Time used: 0.022 (sec). Leaf size: 33

```
DSolve[y''[x]-5*y'[x]+6*y[x]==100*Sin[4*x],y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to -2\sin(4x) + 4\cos(4x) + e^{2x}(c_2 e^{x} + c_1)$$

4.4 problem 4

Internal problem ID [5147]

Book: Engineering Mathematics. By K. A. Stroud. 5th edition. Industrial press Inc. NY. 2001
Section: Program 25. Second order differential equations. Further problems 25. page 1094
Problem number: 4.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _linear, _nonhomogeneous]]`

$$\boxed{y'' + 2y' + y = 4\sinh(x)}$$

✓ Solution by Maple
Time used: 0.016 (sec). Leaf size: 32

```
dsolve(diff(y(x),x$2)+2*diff(y(x),x)+y(x)=4*sinh(x),y(x), singsol=all)
```

$$y(x) = \frac{\left(-2x^2 + (2c_1 + 2)\,x + 2c_2 + 1\right)\mathrm{e}^{-x}}{2} + \frac{\mathrm{e}^{x}}{2}$$

✓ Solution by Mathematica
Time used: 0.052 (sec). Leaf size: 31

```
DSolve[y''[x]+2*y'[x]+y[x]==4*Sinh[x],y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to \frac{e^x}{2} + e^{-x}\left(-x^2 + c_2 x + c_1\right)$$

4.5 problem 5

Internal problem ID [5148]

Book: Engineering Mathematics. By K. A. Stroud. 5th edition. Industrial press Inc. NY. 2001
Section: Program 25. Second order differential equations. Further problems 25. page 1094
Problem number: 5.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _linear, _nonhomogeneous]]`

$$\boxed{y'' + y' - 2y = 2\cosh(2x)}$$

✓ Solution by Maple
Time used: 0.015 (sec). Leaf size: 31

```
dsolve(diff(y(x),x$2)+diff(y(x),x)-2*y(x)=2*cosh(2*x),y(x), singsol=all)
```

$$y(x) = \frac{(9\,\mathrm{e}^{4x} + 36\,\mathrm{e}^{3x}c_2 + 36c_1 - 12x - 7)\,\mathrm{e}^{-2x}}{36}$$

✓ Solution by Mathematica
Time used: 0.047 (sec). Leaf size: 39

```
DSolve[y''[x]+y'[x]-2*y[x]==2*Cosh[2*x],y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to \frac{1}{36}e^{-2x}\left(-12x + 9e^{4x} + 36c_2e^{3x} - 4 + 36c_1\right)$$

4.6 problem 6

Internal problem ID [5149]

Book: Engineering Mathematics. By K. A. Stroud. 5th edition. Industrial press Inc. NY. 2001
Section: Program 25. Second order differential equations. Further problems 25. page 1094
Problem number: 6.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$\boxed{y'' - y' + 10y = 20 - \mathrm{e}^{2x}}$$

✓ Solution by Maple
Time used: 0.015 (sec). Leaf size: 38

```
dsolve(diff(y(x),x$2)-diff(y(x),x)+10*y(x)=20-exp(2*x),y(x), singsol=all)
```

$$y(x) = \mathrm{e}^{\frac{x}{2}} \sin\left(\frac{\sqrt{39}\,x}{2}\right) c_2 + \mathrm{e}^{\frac{x}{2}} \cos\left(\frac{\sqrt{39}\,x}{2}\right) c_1 + 2 - \frac{\mathrm{e}^{2x}}{12}$$

✓ Solution by Mathematica
Time used: 1.291 (sec). Leaf size: 58

```
DSolve[y''[x]-y'[x]+10*y[x]==20-Exp[2*x],y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to -\frac{e^{2x}}{12} + c_2 e^{x/2} \cos\left(\frac{\sqrt{39}x}{2}\right) + c_1 e^{x/2} \sin\left(\frac{\sqrt{39}x}{2}\right) + 2$$

4.7 problem 7

Internal problem ID [5150]

Book: Engineering Mathematics. By K. A. Stroud. 5th edition. Industrial press Inc. NY. 2001
Section: Program 25. Second order differential equations. Further problems 25. page 1094
Problem number: 7.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _linear, _nonhomogeneous]]`

$$\boxed{y'' + 4y' + 4y = 2\cos(x)^2}$$

✓ Solution by Maple
Time used: 0.015 (sec). Leaf size: 22

```
dsolve(diff(y(x),x$2)+4*diff(y(x),x)+4*y(x)=2*cos(x)^2,y(x), singsol=all)
```

$$y(x) = \frac{1}{4} + (c_1 x + c_2)\,\mathrm{e}^{-2x} + \frac{\sin(2x)}{8}$$

✓ Solution by Mathematica
Time used: 0.132 (sec). Leaf size: 29

```
DSolve[y''[x]+4*y'[x]+4*y[x]==2*Cos[x]^2,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to \frac{1}{8}\left(\sin(2x) + 8e^{-2x}(c_2 x + c_1) + 2\right)$$

4.8 problem 8

Internal problem ID [5151]

Book: Engineering Mathematics. By K. A. Stroud. 5th edition. Industrial press Inc. NY. 2001
Section: Program 25. Second order differential equations. Further problems 25. page 1094
Problem number: 8.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$\boxed{y'' - 4y' + 3y = x + \mathrm{e}^{2x}}$$

✓ Solution by Maple
Time used: 0.015 (sec). Leaf size: 25

```
dsolve(diff(y(x),x$2)-4*diff(y(x),x)+3*y(x)=x+exp(2*x),y(x), singsol=all)
```

$$y(x) = \mathrm{e}^{x} c_2 + \mathrm{e}^{3x} c_1 - \mathrm{e}^{2x} + \frac{x}{3} + \frac{4}{9}$$

✓ Solution by Mathematica
Time used: 0.129 (sec). Leaf size: 35

```
DSolve[y''[x]-4*y'[x]+3*y[x]==x+Exp[2*x],y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to \frac{x}{3} - e^{2x} + c_1 e^{x} + c_2 e^{3x} + \frac{4}{9}$$

4.9 problem 9

Internal problem ID [5152]

Book: Engineering Mathematics. By K. A. Stroud. 5th edition. Industrial press Inc. NY. 2001
Section: Program 25. Second order differential equations. Further problems 25. page 1094
Problem number: 9.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$\boxed{y'' - 2y' + 3y = x^2 - 1}$$

✓ Solution by Maple
Time used: 0.0 (sec). Leaf size: 34

```
dsolve(diff(y(x),x$2)-2*diff(y(x),x)+3*y(x)=x^2-1,y(x), singsol=all)
```

$$y(x) = \mathrm{e}^{x} \sin\left(\sqrt{2}\, x\right) c_2 + \mathrm{e}^{x} \cos\left(\sqrt{2}\, x\right) c_1 + \frac{x^2}{3} + \frac{4x}{9} - \frac{7}{27}$$

✓ Solution by Mathematica
Time used: 0.023 (sec). Leaf size: 48

```
DSolve[y''[x]-2*y'[x]+3*y[x]==x^2-1,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to \frac{1}{27}\left(9x^2 + 12x - 7\right) + c_2 e^{x} \cos\left(\sqrt{2}x\right) + c_1 e^{x} \sin\left(\sqrt{2}x\right)$$

4.10 problem 10

Internal problem ID [5153]

Book: Engineering Mathematics. By K. A. Stroud. 5th edition. Industrial press Inc. NY. 2001
Section: Program 25. Second order differential equations. Further problems 25. page 1094
Problem number: 10.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _linear, _nonhomogeneous]]`

$$\boxed{y'' - 9y = \mathrm{e}^{3x} + \sin(x)}$$

✓ Solution by Maple
Time used: 0.016 (sec). Leaf size: 29

```
dsolve(diff(y(x),x$2)-9*y(x)=exp(3*x)+sin(x),y(x), singsol=all)
```

$$y(x) = \frac{(-1 + 6x + 36c_2)\,\mathrm{e}^{3x}}{36} + \mathrm{e}^{-3x}c_1 - \frac{\sin(x)}{10}$$

✓ Solution by Mathematica
Time used: 0.126 (sec). Leaf size: 37

```
DSolve[y''[x]-9*y[x]==Exp[3*x]+Sin[x],y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to -\frac{\sin(x)}{10} + e^{3x}\left(\frac{x}{6} - \frac{1}{36} + c_1\right) + c_2 e^{-3x}$$

4.11 problem 12

Internal problem ID [5154]

Book: Engineering Mathematics. By K. A. Stroud. 5th edition. Industrial press Inc. NY. 2001
Section: Program 25. Second order differential equations. Further problems 25. page 1094
Problem number: 12.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$\boxed{x'' + 4x' + 3x = \mathrm{e}^{-3t}}$$

With initial conditions

$$\left[x(0) = \frac{1}{2}, x'(0) = -2\right]$$

✓ Solution by Maple
Time used: 0.031 (sec). Leaf size: 13

```
dsolve([diff(x(t),t$2)+4*diff(x(t),t)+3*x(t)=exp(-3*t),x(0) = 1/2, D(x)(0) = -2],x(t), sings
```

$$x(t) = -\frac{\mathrm{e}^{-3t}(t-1)}{2}$$

✓ Solution by Mathematica
Time used: 0.034 (sec). Leaf size: 17

```
DSolve[{x''[t]+4*x'[t]+3*x[t]==Exp[-3*t],{x[0]==1/2,x'[0]==-2}},x[t],t,IncludeSingularSoluti
```

$$x(t) \to -\frac{1}{2}e^{-3t}(t-1)$$

4.12 problem 13

Internal problem ID [5155]

Book: Engineering Mathematics. By K. A. Stroud. 5th edition. Industrial press Inc. NY. 2001
Section: Program 25. Second order differential equations. Further problems 25. page 1094
Problem number: 13.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _linear, _nonhomogeneous]]`

$$\boxed{y'' + 4y' + 5y = 6\sin(t)}$$

✓ Solution by Maple
Time used: 0.0 (sec). Leaf size: 29

```
dsolve(diff(y(t),t$2)+4*diff(y(t),t)+5*y(t)=6*sin(t),y(t), singsol=all)
```

$$y(t) = \mathrm{e}^{-2t}\sin(t)\,c_2 + \mathrm{e}^{-2t}\cos(t)\,c_1 - \frac{3\cos(t)}{4} + \frac{3\sin(t)}{4}$$

✓ Solution by Mathematica
Time used: 0.027 (sec). Leaf size: 36

```
DSolve[y''[t]+4*y'[t]+5*y[t]==6*Sin[t],y[t],t,IncludeSingularSolutions -> True]
```

$$y(t) \to \left(-\frac{3}{4} + c_2 e^{-2t}\right)\cos(t) + \left(\frac{3}{4} + c_1 e^{-2t}\right)\sin(t)$$

4.13 problem 14

Internal problem ID [5156]

Book: Engineering Mathematics. By K. A. Stroud. 5th edition. Industrial press Inc. NY. 2001
Section: Program 25. Second order differential equations. Further problems 25. page 1094
Problem number: 14.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _linear, _nonhomogeneous]]`

$$\boxed{x'' - 3x' + 2x = \sin(t)}$$

With initial conditions

$$[x(0) = 0, x'(0) = 0]$$

✓ Solution by Maple
Time used: 0.015 (sec). Leaf size: 23

```
dsolve([diff(x(t),t$2)-3*diff(x(t),t)+2*x(t)=sin(t),x(0) = 0, D(x)(0) = 0],x(t), singsol=all
```

$$x(t) = \frac{e^{2t}}{5} + \frac{3\cos(t)}{10} + \frac{\sin(t)}{10} - \frac{e^{t}}{2}$$

✓ Solution by Mathematica
Time used: 0.048 (sec). Leaf size: 27

```
DSolve[{x''[t]-3*x'[t]+2*x[t]==Sin[t],{x[0]==0,x'[0]==0}},x[t],t,IncludeSingularSolutions ->
```

$$x(t) \to \frac{1}{10}\left(e^{t}\left(2e^{t} - 5\right) + \sin(t) + 3\cos(t)\right)$$

4.14 problem 15

Internal problem ID [5157]

Book: Engineering Mathematics. By K. A. Stroud. 5th edition. Industrial press Inc. NY. 2001
Section: Program 25. Second order differential equations. Further problems 25. page 1094
Problem number: 15.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _linear, _nonhomogeneous]]`

$$\boxed{y'' + 3y' + 2y = 3\sin(x)}$$

With initial conditions

$$\left[y(0) = -\frac{9}{10}, y'(0) = -\frac{7}{10}\right]$$

✓ Solution by Maple
Time used: 0.031 (sec). Leaf size: 23

```
dsolve([diff(y(x),x$2)+3*diff(y(x),x)+2*y(x)=3*sin(x),y(0) = -9/10, D(y)(0) = -7/10],y(x), s
```

$$y(x) = \mathrm{e}^{-2x} - \frac{9\cos(x)}{10} + \frac{3\sin(x)}{10} - \mathrm{e}^{-x}$$

✓ Solution by Mathematica
Time used: 0.022 (sec). Leaf size: 30

```
DSolve[{y''[x]+3*y'[x]+2*y[x]==3*Sin[x],{y[0]==-9/10,y'[0]==-7/10}},y[x],x,IncludeSingularSo
```

$$y(x) \to -e^{-2x}(e^x - 1) + \frac{3\sin(x)}{10} - \frac{9\cos(x)}{10}$$

4.15 problem 16

Internal problem ID [5158]

Book: Engineering Mathematics. By K. A. Stroud. 5th edition. Industrial press Inc. NY. 2001
Section: Program 25. Second order differential equations. Further problems 25. page 1094
Problem number: 16.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$\boxed{y'' + 6y' + 10y = 50x}$$

✓ Solution by Maple
Time used: 0.016 (sec). Leaf size: 25

```
dsolve(diff(y(x),x$2)+6*diff(y(x),x)+10*y(x)=50*x,y(x), singsol=all)
```

$$y(x) = \mathrm{e}^{-3x} \sin(x)\, c_2 + \mathrm{e}^{-3x} \cos(x)\, c_1 + 5x - 3$$

✓ Solution by Mathematica
Time used: 0.017 (sec). Leaf size: 30

```
DSolve[y''[x]+6*y'[x]+10*y[x]==50*x,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to 5x + c_2 e^{-3x} \cos(x) + c_1 e^{-3x} \sin(x) - 3$$

4.16 problem 17

Internal problem ID [5159]

Book: Engineering Mathematics. By K. A. Stroud. 5th edition. Industrial press Inc. NY. 2001
Section: Program 25. Second order differential equations. Further problems 25. page 1094
Problem number: 17.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _linear, _nonhomogeneous]]`

$$\boxed{x'' + 2x' + 2x = 85\sin(3t)}$$

With initial conditions

$$[x(0) = 0, x'(0) = -20]$$

✓ Solution by Maple
Time used: 0.047 (sec). Leaf size: 33

```
dsolve([diff(x(t),t$2)+2*diff(x(t),t)+2*x(t)=85*sin(3*t),x(0) = 0, D(x)(0) = -20],x(t), sing
```

$$x(t) = (7\sin(t) + 6\cos(t))\,\mathrm{e}^{-t} - 6\cos(3t) - 7\sin(3t)$$

✓ Solution by Mathematica
Time used: 0.026 (sec). Leaf size: 36

```
DSolve[{x''[t]+2*x'[t]+2*x[t]==85*Sin[3*t],{x[0]==0,x'[0]==-20}},x[t],t,IncludeSingularSolut
```

$$x(t) \to 7e^{-t}\sin(t) - 7\sin(3t) + 6e^{-t}\cos(t) - 6\cos(3t)$$

4.17 problem 18

Internal problem ID [5160]

Book: Engineering Mathematics. By K. A. Stroud. 5th edition. Industrial press Inc. NY. 2001
Section: Program 25. Second order differential equations. Further problems 25. page 1094
Problem number: 18.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _linear, _nonhomogeneous]]`

$$\boxed{y'' + 4y = 3\sin(x)}$$

With initial conditions

$$\left[y(0) = 0, y'\left(\frac{\pi}{2}\right) = 1\right]$$

✓ Solution by Maple
Time used: 0.031 (sec). Leaf size: 13

```
dsolve([diff(y(x),x$2)=3*sin(x)-4*y(x),y(0) = 0, D(y)(1/2*Pi) = 1],y(x), singsol=all)
```

$$y(x) = -\frac{\sin(2x)}{2} + \sin(x)$$

✓ Solution by Mathematica
Time used: 0.021 (sec). Leaf size: 13

```
DSolve[{y''[x]==3*Sin[x]-4*y[x],{y[0]==0,y'[Pi/2]==1}},y[x],x,IncludeSingularSolutions -> Tr
```

$$y(x) \to -(\sin(x)(\cos(x) - 1))$$

4.18 problem 19

Internal problem ID [5161]

Book: Engineering Mathematics. By K. A. Stroud. 5th edition. Industrial press Inc. NY. 2001
Section: Program 25. Second order differential equations. Further problems 25. page 1094
Problem number: 19.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _missing_x]]`

$$\boxed{\frac{x''}{2} + 48x = 0}$$

With initial conditions

$$\left[x(0) = \frac{1}{6}, x'(0) = 0\right]$$

✓ Solution by Maple
Time used: 0.016 (sec). Leaf size: 13

```
dsolve([1/2*diff(x(t),t$2)=-48*x(t),x(0) = 1/6, D(x)(0) = 0],x(t), singsol=all)
```

$$x(t) = \frac{\cos\left(4\sqrt{6}\,t\right)}{6}$$

✓ Solution by Mathematica
Time used: 0.021 (sec). Leaf size: 18

```
DSolve[{1/2*x''[t]==-48*x[t],{x[0]==1/6,x'[0]==0}},x[t],t,IncludeSingularSolutions -> True]
```

$$x(t) \to \frac{1}{6}\cos\left(4\sqrt{6}t\right)$$

4.19 problem 20

Internal problem ID [5162]

Book: Engineering Mathematics. By K. A. Stroud. 5th edition. Industrial press Inc. NY. 2001
Section: Program 25. Second order differential equations. Further problems 25. page 1094
Problem number: 20.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _linear, _nonhomogeneous]]`

$$\boxed{x'' + 5x' + 6x = \cos(t)}$$

With initial conditions

$$\left[x(0) = \frac{1}{10}, x'(0) = 0\right]$$

✓ Solution by Maple
Time used: 0.015 (sec). Leaf size: 25

```
dsolve([diff(x(t),t$2)+5*diff(x(t),t)+6*x(t)=cos(t),x(0) = 1/10, D(x)(0) = 0],x(t), singsol=
```

$$x(t) = \frac{e^{-3t}}{10} - \frac{e^{-2t}}{10} + \frac{\cos(t)}{10} + \frac{\sin(t)}{10}$$

✓ Solution by Mathematica
Time used: 0.059 (sec). Leaf size: 26

```
DSolve[{x''[t]+5*x'[t]+6*x[t]==Cos[t],{x[0]==1/10,x'[0]==0}},x[t],t,IncludeSingularSolutions
```

$$x(t) \to \frac{1}{10}\left(e^{-3t} - e^{-2t} + \sin(t) + \cos(t)\right)$$