A Solution Manual For

First order enumerated odes

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Contents

1 section 1

1.1 problem 1

Internal problem ID [7317]

Book: First order enumerated odes
Section: section 1
Problem number: 1.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_quadrature]`

$$\boxed{y' = 0}$$

✓ Solution by Maple
Time used: 0.0 (sec). Leaf size: 5

```
dsolve(diff(y(x),x)=0,y(x), singsol=all)
```

$$y(x) = c_1$$

✓ Solution by Mathematica
Time used: 0.002 (sec). Leaf size: 7

```
DSolve[y'[x]==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to c_1$$

1.2 problem 2

Internal problem ID [7318]

Book: First order enumerated odes
Section: section 1
Problem number: 2.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_quadrature]`

$$\boxed{y' = a}$$

✓ Solution by Maple
Time used: 0.0 (sec). Leaf size: 9

```
dsolve(diff(y(x),x)=a,y(x), singsol=all)
```

$$y(x) = ax + c_1$$

✓ Solution by Mathematica
Time used: 0.002 (sec). Leaf size: 11

```
DSolve[y'[x]==a,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to ax + c_1$$

1.3 problem 3

Internal problem ID [7319]

Book: First order enumerated odes
Section: section 1
Problem number: 3.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_quadrature]`

$$\boxed{y' = x}$$

✓ Solution by Maple
Time used: 0.0 (sec). Leaf size: 11

```
dsolve(diff(y(x),x)=x,y(x), singsol=all)
```

$$y(x) = \frac{x^2}{2} + c_1$$

✓ Solution by Mathematica
Time used: 0.002 (sec). Leaf size: 15

```
DSolve[y'[x]==x,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to \frac{x^2}{2} + c_1$$

1.4 problem 4

Internal problem ID [7320]

Book: First order enumerated odes
Section: section 1
Problem number: 4.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_quadrature]`

$$\boxed{y' = 1}$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 7

```
dsolve(diff(y(x),x)=1,y(x), singsol=all)
```

$$y(x) = x + c_1$$

✓ Solution by Mathematica

Time used: 0.002 (sec). Leaf size: 9

```
DSolve[y'[x]==1,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to x + c_1$$

1.5 problem 5

Internal problem ID [7321]

Book: First order enumerated odes
Section: section 1
Problem number: 5.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_quadrature]`

$$\boxed{y' = xa}$$

✓ Solution by Maple
Time used: 0.0 (sec). Leaf size: 12

```
dsolve(diff(y(x),x)=a*x,y(x), singsol=all)
```

$$y(x) = \frac{a\,x^2}{2} + c_1$$

✓ Solution by Mathematica
Time used: 0.002 (sec). Leaf size: 16

```
DSolve[y'[x]==a*x,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to \frac{ax^2}{2} + c_1$$

1.6 problem 6

Internal problem ID [7322]

Book: First order enumerated odes
Section: section 1
Problem number: 6.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_separable]`

$$\boxed{y' - axy = 0}$$

✓ Solution by Maple
Time used: 0.0 (sec). Leaf size: 13

```
dsolve(diff(y(x),x)=a*x*y(x),y(x), singsol=all)
```

$$y(x) = c_1 \mathrm{e}^{\frac{a\,x^2}{2}}$$

✓ Solution by Mathematica
Time used: 0.032 (sec). Leaf size: 23

```
DSolve[y'[x]==a*x*y[x],y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to c_1 e^{\frac{ax^2}{2}}$$
$$y(x) \to 0$$

1.7 problem 7

Internal problem ID [7323]

Book: First order enumerated odes
Section: section 1
Problem number: 7.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[[_linear, ‘class A‘]]`

$$\boxed{y' - y = xa}$$

✓ Solution by Maple
Time used: 0.0 (sec). Leaf size: 15

```
dsolve(diff(y(x),x)=a*x+y(x),y(x), singsol=all)
```

$$y(x) = c_1 e^x - a(x+1)$$

✓ Solution by Mathematica
Time used: 0.03 (sec). Leaf size: 18

```
DSolve[y'[x]==a*x+y[x],y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to -a(x+1) + c_1 e^x$$

1.8 problem 8

Internal problem ID [7324]

Book: First order enumerated odes
Section: section 1
Problem number: 8.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[[_linear, ‘class A‘]]`

$$\boxed{y' - by = xa}$$

✓ Solution by Maple
Time used: 0.0 (sec). Leaf size: 26

```
dsolve(diff(y(x),x)=a*x+b*y(x),y(x), singsol=all)
```

$$y(x) = \frac{e^{bx} c_1 b^2 - axb - a}{b^2}$$

✓ Solution by Mathematica
Time used: 0.054 (sec). Leaf size: 25

```
DSolve[y'[x]==a*x+b*y[x],y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to -\frac{abx + a}{b^2} + c_1 e^{bx}$$

1.9 problem 9

Internal problem ID [7325]

Book: First order enumerated odes
Section: section 1
Problem number: 9.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_quadrature]`

$$\boxed{y' - y = 0}$$

✓ Solution by Maple
Time used: 0.0 (sec). Leaf size: 8

```
dsolve(diff(y(x),x)=y(x),y(x), singsol=all)
```

$$y(x) = c_1 e^x$$

✓ Solution by Mathematica
Time used: 0.022 (sec). Leaf size: 16

```
DSolve[y'[x]==y[x],y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to c_1 e^x$$
$$y(x) \to 0$$

1.10 problem 10

Internal problem ID [7326]

Book: First order enumerated odes
Section: section 1
Problem number: 10.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_quadrature]`

$$\boxed{y' - by = 0}$$

✓ Solution by Maple
Time used: 0.0 (sec). Leaf size: 10

```
dsolve(diff(y(x),x)=b*y(x),y(x), singsol=all)
```

$$y(x) = e^{bx} c_1$$

✓ Solution by Mathematica
Time used: 0.03 (sec). Leaf size: 18

```
DSolve[y'[x]==b*y[x],y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to c_1 e^{bx}$$
$$y(x) \to 0$$

1.11 problem 11

Internal problem ID [7327]

Book: First order enumerated odes
Section: section 1
Problem number: 11.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[[_Riccati, _special]]`

$$\boxed{y' - by^2 = xa}$$

✓ Solution by Maple
Time used: 0.0 (sec). Leaf size: 59

```
dsolve(diff(y(x),x)=a*x+b*y(x)^2,y(x), singsol=all)
```

$$y(x) = \frac{(ab)^{\frac{1}{3}} \left(\mathrm{AiryAi}\left(1, -(ab)^{\frac{1}{3}} x\right) c_1 + \mathrm{AiryBi}\left(1, -(ab)^{\frac{1}{3}} x\right)\right)}{b \left(c_1 \,\mathrm{AiryAi}\left(-(ab)^{\frac{1}{3}} x\right) + \mathrm{AiryBi}\left(-(ab)^{\frac{1}{3}} x\right)\right)}$$

✓ Solution by Mathematica
Time used: 0.163 (sec). Leaf size: 331

```
DSolve[y'[x]==a*x+b*y[x]^2,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \rightarrow \frac{\sqrt{a}\sqrt{b}x^{3/2}\left(-2\,\mathrm{BesselJ}\left(-\frac{2}{3}, \frac{2}{3}\sqrt{a}\sqrt{b}x^{3/2}\right) + c_1\left(\mathrm{BesselJ}\left(\frac{2}{3}, \frac{2}{3}\sqrt{a}\sqrt{b}x^{3/2}\right) - \mathrm{BesselJ}\left(-\frac{4}{3}, \frac{2}{3}\sqrt{a}\sqrt{b}x^{3/2}\right)\right.\right.}{2bx\left(\mathrm{BesselJ}\left(\frac{1}{3}, \frac{2}{3}\sqrt{a}\sqrt{b}x^{3/2}\right) + c_1\,\mathrm{BesselJ}\left(-\frac{1}{3}, \frac{2}{3}\sqrt{a}\sqrt{b}x^{3/2}\right)\right)}$$

$$y(x) \rightarrow -\frac{\sqrt{a}\sqrt{b}x^{3/2}\,\mathrm{BesselJ}\left(-\frac{4}{3}, \frac{2}{3}\sqrt{a}\sqrt{b}x^{3/2}\right) - \sqrt{a}\sqrt{b}x^{3/2}\,\mathrm{BesselJ}\left(\frac{2}{3}, \frac{2}{3}\sqrt{a}\sqrt{b}x^{3/2}\right) + \mathrm{BesselJ}\left(-\frac{1}{3}, \frac{2}{3}\sqrt{a}\sqrt{b}x\right.}{2bx\,\mathrm{BesselJ}\left(-\frac{1}{3}, \frac{2}{3}\sqrt{a}\sqrt{b}x^{3/2}\right)}$$

1.12 problem 12

Internal problem ID [7328]

Book: First order enumerated odes
Section: section 1
Problem number: 12.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_quadrature]`

$$\boxed{y'c = 0}$$

✓ Solution by Maple
Time used: 0.0 (sec). Leaf size: 5

```
dsolve(c*diff(y(x),x)=0,y(x), singsol=all)
```

$$y(x) = c_1$$

✓ Solution by Mathematica
Time used: 0.002 (sec). Leaf size: 7

```
DSolve[c*y'[x]==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to c_1$$

1.13 problem 13

Internal problem ID [7329]

Book: First order enumerated odes
Section: section 1
Problem number: 13.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_quadrature]`

$$\boxed{y'c = a}$$

✓ Solution by Maple
Time used: 0.015 (sec). Leaf size: 12

```
dsolve(c*diff(y(x),x)=a,y(x), singsol=all)
```

$$y(x) = \frac{ax}{c} + c_1$$

✓ Solution by Mathematica
Time used: 0.002 (sec). Leaf size: 14

```
DSolve[c*y'[x]==a,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to \frac{ax}{c} + c_1$$

1.14 problem 14

Internal problem ID [7330]

Book: First order enumerated odes
Section: section 1
Problem number: 14.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_quadrature]`

$$\boxed{y'c = xa}$$

✓ Solution by Maple
Time used: 0.0 (sec). Leaf size: 15

```
dsolve(c*diff(y(x),x)=a*x,y(x), singsol=all)
```

$$y(x) = \frac{a\,x^2}{2c} + c_1$$

✓ Solution by Mathematica
Time used: 0.002 (sec). Leaf size: 19

```
DSolve[c*y'[x]==a*x,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to \frac{ax^2}{2c} + c_1$$

1.15 problem 15

Internal problem ID [7331]

Book: First order enumerated odes
Section: section 1
Problem number: 15.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[[_linear, ‘class A‘]]`

$$\boxed{y'c - y = xa}$$

✓ Solution by Maple
Time used: 0.016 (sec). Leaf size: 19

```
dsolve(c*diff(y(x),x)=a*x+y(x),y(x), singsol=all)
```

$$y(x) = \mathrm{e}^{\frac{x}{c}} c_1 - a(c + x)$$

✓ Solution by Mathematica
Time used: 0.05 (sec). Leaf size: 22

```
DSolve[c*y'[x]==a*x+y[x],y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to -a(c + x) + c_1 e^{\frac{x}{c}}$$

1.16 problem 16

Internal problem ID [7332]

Book: First order enumerated odes
Section: section 1
Problem number: 16.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[[_linear, ‘class A‘]]`

$$\boxed{y'c - by = xa}$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 29

```
dsolve(c*diff(y(x),x)=a*x+b*y(x),y(x), singsol=all)
```

$$y(x) = \frac{\mathrm{e}^{\frac{bx}{c}} c_1 b^2 - a(bx + c)}{b^2}$$

✓ Solution by Mathematica

Time used: 0.055 (sec). Leaf size: 28

```
DSolve[c*y'[x]==a*x+b*y[x],y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to -\frac{a(bx + c)}{b^2} + c_1 e^{\frac{bx}{c}}$$

1.17 problem 17

Internal problem ID [7333]

Book: First order enumerated odes
Section: section 1
Problem number: 17.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_quadrature]`

$$\boxed{y'c - y = 0}$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 12

```
dsolve(c*diff(y(x),x)=y(x),y(x), singsol=all)
```

$$y(x) = e^{\frac{x}{c}} c_1$$

✓ Solution by Mathematica

Time used: 0.023 (sec). Leaf size: 20

```
DSolve[c*y'[x]==y[x],y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to c_1 e^{\frac{x}{c}}$$
$$y(x) \to 0$$

1.18 problem 18

Internal problem ID [7334]

Book: First order enumerated odes
Section: section 1
Problem number: 18.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_quadrature]`

$$\boxed{y'c - by = 0}$$

✓ Solution by Maple
Time used: 0.0 (sec). Leaf size: 13

```
dsolve(c*diff(y(x),x)=b*y(x),y(x), singsol=all)
```

$$y(x) = \mathrm{e}^{\frac{bx}{c}} c_1$$

✓ Solution by Mathematica
Time used: 0.025 (sec). Leaf size: 21

```
DSolve[c*y'[x]==b*y[x],y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to c_1 e^{\frac{bx}{c}}$$
$$y(x) \to 0$$

1.19 problem 19

Internal problem ID [7335]

Book: First order enumerated odes
Section: section 1
Problem number: 19.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[[_Riccati, _special]]`

$$\boxed{y'c - by^2 = xa}$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 75

```
dsolve(c*diff(y(x),x)=a*x+b*y(x)^2,y(x), singsol=all)
```

$$y(x) = \frac{\left(\frac{ba}{c^2}\right)^{\frac{1}{3}} \left(\operatorname{AiryAi}\left(1, -\left(\frac{ba}{c^2}\right)^{\frac{1}{3}} x\right) c_1 + \operatorname{AiryBi}\left(1, -\left(\frac{ba}{c^2}\right)^{\frac{1}{3}} x\right)\right) c}{b \left(c_1 \operatorname{AiryAi}\left(-\left(\frac{ba}{c^2}\right)^{\frac{1}{3}} x\right) + \operatorname{AiryBi}\left(-\left(\frac{ba}{c^2}\right)^{\frac{1}{3}} x\right)\right)}$$

✓ Solution by Mathematica

Time used: 0.21 (sec). Leaf size: 628

```
DSolve[c*y'[x]==a*x+b*y[x]^2,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to \frac{c\left(x^{3/2}\sqrt{\frac{a}{c}}\sqrt{\frac{b}{c}}\left(-2\,\text{BesselJ}\left(-\frac{2}{3},\frac{2}{3}\sqrt{\frac{a}{c}}\sqrt{\frac{b}{c}}x^{3/2}\right)+c_1\left(\text{BesselJ}\left(\frac{2}{3},\frac{2}{3}\sqrt{\frac{a}{c}}\sqrt{\frac{b}{c}}x^{3/2}\right)-\text{BesselJ}\left(-\frac{4}{3},\frac{2}{3}\sqrt{\frac{a}{c}}\sqrt{\right.\right.\right.\right.}{2bx\left(\text{BesselJ}\left(\frac{1}{3},\frac{2}{3}\sqrt{\frac{a}{c}}\sqrt{\frac{b}{c}}x^{3/2}\right)+c_1\,\text{BesselJ}\left(-\frac{1}{3},\frac{2}{3}\sqrt{\frac{a}{c}}\sqrt{\frac{b}{c}}x^{3/\right.\right.}$$

$$y(x) \to -\frac{c\left(x^{3/2}\sqrt{\frac{a}{c}}\sqrt{\frac{b}{c}}\,\text{BesselJ}\left(-\frac{4}{3},\frac{2}{3}\sqrt{\frac{a}{c}}\sqrt{\frac{b}{c}}x^{3/2}\right)-x^{3/2}\sqrt{\frac{a}{c}}\sqrt{\frac{b}{c}}\,\text{BesselJ}\left(\frac{2}{3},\frac{2}{3}\sqrt{\frac{a}{c}}\sqrt{\frac{b}{c}}x^{3/2}\right)+\text{BesselJ}\left(-\frac{1}{3},\frac{2}{3}\right.\right.}{2bx\,\text{BesselJ}\left(-\frac{1}{3},\frac{2}{3}\sqrt{\frac{a}{c}}\sqrt{\frac{b}{c}}x^{3/2}\right)}$$

$$y(x) \to -\frac{c\left(x^{3/2}\sqrt{\frac{a}{c}}\sqrt{\frac{b}{c}}\,\text{BesselJ}\left(-\frac{4}{3},\frac{2}{3}\sqrt{\frac{a}{c}}\sqrt{\frac{b}{c}}x^{3/2}\right)-x^{3/2}\sqrt{\frac{a}{c}}\sqrt{\frac{b}{c}}\,\text{BesselJ}\left(\frac{2}{3},\frac{2}{3}\sqrt{\frac{a}{c}}\sqrt{\frac{b}{c}}x^{3/2}\right)+\text{BesselJ}\left(-\frac{1}{3},\frac{2}{3}\right.\right.}{2bx\,\text{BesselJ}\left(-\frac{1}{3},\frac{2}{3}\sqrt{\frac{a}{c}}\sqrt{\frac{b}{c}}x^{3/2}\right)}$$

1.20 problem 20

Internal problem ID [7336]

Book: First order enumerated odes
Section: section 1
Problem number: 20.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[[_Riccati, _special]]`

$$\boxed{y'c - \frac{xa + by^2}{r} = 0}$$

✓ Solution by Maple

Time used: 0.016 (sec). Leaf size: 91

```
dsolve(c*diff(y(x),x)=(a*x+b*y(x)^2)/r,y(x), singsol=all)
```

$$y(x) = \frac{\left(\frac{ba}{r^2c^2}\right)^{\frac{1}{3}} \left(\operatorname{AiryAi}\left(1, -\left(\frac{ba}{r^2c^2}\right)^{\frac{1}{3}} x\right) c_1 + \operatorname{AiryBi}\left(1, -\left(\frac{ba}{r^2c^2}\right)^{\frac{1}{3}} x\right)\right) rc}{b \left(c_1 \operatorname{AiryAi}\left(-\left(\frac{ba}{r^2c^2}\right)^{\frac{1}{3}} x\right) + \operatorname{AiryBi}\left(-\left(\frac{ba}{r^2c^2}\right)^{\frac{1}{3}} x\right)\right)}$$

✓ Solution by Mathematica

Time used: 0.222 (sec). Leaf size: 517

```
DSolve[c*y'[x]==(a*x+b*y[x]^2)/r,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to \frac{cr\left(x^{3/2}\sqrt{\frac{a}{cr}}\sqrt{\frac{b}{cr}}\left(-2\operatorname{BesselJ}\left(-\frac{2}{3}, \frac{2}{3}\sqrt{\frac{a}{cr}}\sqrt{\frac{b}{cr}}x^{3/2}\right) + c_1\left(\operatorname{BesselJ}\left(\frac{2}{3}, \frac{2}{3}\sqrt{\frac{a}{cr}}\sqrt{\frac{b}{cr}}x^{3/2}\right) - \operatorname{BesselJ}\left(-\frac{4}{3}, \frac{2}{3}\right.\right.\right.\right.}{2bx\left(\operatorname{BesselJ}\left(\frac{1}{3}, \frac{2}{3}\sqrt{\frac{a}{cr}}\sqrt{\frac{b}{cr}}x^{3/2}\right) + c_1\operatorname{BesselJ}\left(-\frac{1}{3}, \frac{2}{3}\sqrt{\frac{a}{cr}}\right.\right.}$$

$$y(x) \to -\frac{cr\left(x^{3/2}\sqrt{\frac{a}{cr}}\sqrt{\frac{b}{cr}}\operatorname{BesselJ}\left(-\frac{4}{3}, \frac{2}{3}\sqrt{\frac{a}{cr}}\sqrt{\frac{b}{cr}}x^{3/2}\right) - x^{3/2}\sqrt{\frac{a}{cr}}\sqrt{\frac{b}{cr}}\operatorname{BesselJ}\left(\frac{2}{3}, \frac{2}{3}\sqrt{\frac{a}{cr}}\sqrt{\frac{b}{cr}}x^{3/2}\right) + \operatorname{BesselJ}\right.}{2bx\operatorname{BesselJ}\left(-\frac{1}{3}, \frac{2}{3}\sqrt{\frac{a}{cr}}\sqrt{\frac{b}{cr}}x^{3/2}\right)}$$

1.21 problem 21

Internal problem ID [7337]

Book: First order enumerated odes
Section: section 1
Problem number: 21.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_rational, _Riccati]`

$$y'c - \frac{xa + by^2}{rx} = 0$$

✓ Solution by Maple
Time used: 0.016 (sec). Leaf size: 94

```
dsolve(c*diff(y(x),x)=(a*x+b*y(x)^2)/(r*x),y(x), singsol=all)
```

$$y(x) = \frac{\sqrt{\frac{xba}{r^2c^2}}\, cr \left(\operatorname{BesselY}\left(1, 2\sqrt{\frac{xba}{r^2c^2}}\right) c_1 + \operatorname{BesselJ}\left(1, 2\sqrt{\frac{xba}{r^2c^2}}\right)\right)}{b\left(c_1 \operatorname{BesselY}\left(0, 2\sqrt{\frac{xba}{r^2c^2}}\right) + \operatorname{BesselJ}\left(0, 2\sqrt{\frac{xba}{r^2c^2}}\right)\right)}$$

✓ Solution by Mathematica
Time used: 0.295 (sec). Leaf size: 207

```
DSolve[c*y'[x]==(a*x+b*y[x]^2)/(r*x),y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to \frac{\sqrt{a}\sqrt{x}\left(2\operatorname{BesselY}\left(1, \frac{2\sqrt{a}\sqrt{b}\sqrt{x}}{cr}\right) + c_1 \operatorname{BesselJ}\left(1, \frac{2\sqrt{a}\sqrt{b}\sqrt{x}}{cr}\right)\right)}{\sqrt{b}\left(2\operatorname{BesselY}\left(0, \frac{2\sqrt{a}\sqrt{b}\sqrt{x}}{cr}\right) + c_1 \operatorname{BesselJ}\left(0, \frac{2\sqrt{a}\sqrt{b}\sqrt{x}}{cr}\right)\right)}$$

$$y(x) \to \frac{\sqrt{a}\sqrt{x}\operatorname{BesselJ}\left(1, \frac{2\sqrt{a}\sqrt{b}\sqrt{x}}{cr}\right)}{\sqrt{b}\operatorname{BesselJ}\left(0, \frac{2\sqrt{a}\sqrt{b}\sqrt{x}}{cr}\right)}$$

1.22 problem 22

Internal problem ID [7338]

Book: First order enumerated odes
Section: section 1
Problem number: 22.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_rational, _Riccati]`

$$\boxed{y'c - \frac{xa + by^2}{r\,x^2} = 0}$$

✓ Solution by Maple
Time used: 0.0 (sec). Leaf size: 106

```
dsolve(c*diff(y(x),x)=(a*x+b*y(x)^2)/(r*x^2),y(x), singsol=all)
```

$$y(x) = \frac{a\left(\mathrm{BesselY}\left(0, 2\sqrt{\frac{ba}{c^2r^2x}}\right)c_1 + \mathrm{BesselJ}\left(0, 2\sqrt{\frac{ba}{c^2r^2x}}\right)\right)}{cr\sqrt{\frac{ba}{c^2r^2x}}\left(c_1\,\mathrm{BesselY}\left(1, 2\sqrt{\frac{ba}{c^2r^2x}}\right) + \mathrm{BesselJ}\left(1, 2\sqrt{\frac{ba}{c^2r^2x}}\right)\right)}$$

✓ Solution by Mathematica

Time used: 0.358 (sec). Leaf size: 492

```
DSolve[c*y'[x]==(a*x+b*y[x]^2)/(r*x^2),y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to \frac{2\sqrt{a}\sqrt{b}\,\text{BesselY}\left(0,\frac{2\sqrt{a}\sqrt{b}\sqrt{\frac{1}{x}}}{cr}\right)+\frac{2cr\,\text{BesselY}\left(1,\frac{2\sqrt{a}\sqrt{b}\sqrt{\frac{1}{x}}}{cr}\right)}{\sqrt{\frac{1}{x}}}-2\sqrt{a}\sqrt{b}\,\text{BesselY}\left(2,\frac{2\sqrt{a}\sqrt{b}\sqrt{\frac{1}{x}}}{cr}\right)-i\sqrt{a}\sqrt{b}c_1\,\text{Be}}{2b\sqrt{\frac{1}{x}}\left(2\,\text{BesselY}\left(1,\frac{2\sqrt{a}\sqrt{b}\sqrt{\frac{1}{x}}}{cr}\right)-ic_1\,\text{Bess}\right.}$$

$$y(x) \to \frac{x\left(\sqrt{a}\sqrt{b}\sqrt{\frac{1}{x}}\,\text{BesselJ}\left(0,\frac{2\sqrt{a}\sqrt{b}\sqrt{\frac{1}{x}}}{cr}\right)+cr\,\text{BesselJ}\left(1,\frac{2\sqrt{a}\sqrt{b}\sqrt{\frac{1}{x}}}{cr}\right)-\sqrt{a}\sqrt{b}\sqrt{\frac{1}{x}}\,\text{BesselJ}\left(2,\frac{2\sqrt{a}\sqrt{b}\sqrt{\frac{1}{x}}}{cr}\right)\right)}{2b\,\text{BesselJ}\left(1,\frac{2\sqrt{a}\sqrt{b}\sqrt{\frac{1}{x}}}{cr}\right)}$$

1.23 problem 23

Internal problem ID [7339]

Book: First order enumerated odes
Section: section 1
Problem number: 23.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_rational, _Bernoulli]`

$$\boxed{y'c - \frac{xa + by^2}{y} = 0}$$

✓ Solution by Maple
Time used: 0.016 (sec). Leaf size: 69

```
dsolve(c*diff(y(x),x)=(a*x+b*y(x)^2)/y(x),y(x), singsol=all)
```

$$y(x) = -\frac{\sqrt{4\,\mathrm{e}^{\frac{2bx}{c}} c_1 b^2 - 4axb - 2ac}}{2b}$$
$$y(x) = \frac{\sqrt{4\,\mathrm{e}^{\frac{2bx}{c}} c_1 b^2 - 4axb - 2ac}}{2b}$$

✓ Solution by Mathematica
Time used: 5.371 (sec). Leaf size: 85

```
DSolve[c*y'[x]==(a*x+b*y[x]^2)/y[x],y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to -\frac{i\sqrt{abx + \frac{ac}{2} + b^2 c_1 \left(-e^{\frac{2bx}{c}}\right)}}{b}$$
$$y(x) \to \frac{i\sqrt{abx + \frac{ac}{2} + b^2 c_1 \left(-e^{\frac{2bx}{c}}\right)}}{b}$$

1.24 problem 24

Internal problem ID [7340]

Book: First order enumerated odes
Section: section 1
Problem number: 24.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_quadrature]`

$$\boxed{a \sin(x)\, y x y' = 0}$$

✓ Solution by Maple
Time used: 0.016 (sec). Leaf size: 9

```
dsolve(a*sin(x)*y(x)*x*diff(y(x),x)=0,y(x), singsol=all)
```

$$y(x) = 0$$
$$y(x) = c_1$$

✓ Solution by Mathematica
Time used: 0.003 (sec). Leaf size: 12

```
DSolve[a*Sin[x]*y[x]*x*y'[x]==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to 0$$
$$y(x) \to c_1$$

1.25 problem 25

Internal problem ID [7341]

Book: First order enumerated odes
Section: section 1
Problem number: 25.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_quadrature]`

$$\boxed{f(x)\sin(x)\,yxy'\pi = 0}$$

✓ Solution by Maple
Time used: 0.0 (sec). Leaf size: 9

```
dsolve(f(x)*sin(x)*y(x)*x*diff(y(x),x)*Pi=0,y(x), singsol=all)
```

$$y(x) = 0$$
$$y(x) = c_1$$

✓ Solution by Mathematica
Time used: 0.004 (sec). Leaf size: 12

```
DSolve[f(x)*Sin[x]*y[x]*x*y'[x]*Pi==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to 0$$
$$y(x) \to c_1$$

1.26 problem 26

Internal problem ID [7342]

Book: First order enumerated odes
Section: section 1
Problem number: 26.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[[_linear, ‘class A‘]]`

$$y' - y = \sin(x)$$

✓ Solution by Maple
Time used: 0.015 (sec). Leaf size: 17

```
dsolve(diff(y(x),x)=sin(x)+y(x),y(x), singsol=all)
```

$$y(x) = -\frac{\cos(x)}{2} - \frac{\sin(x)}{2} + c_1 e^x$$

✓ Solution by Mathematica
Time used: 0.04 (sec). Leaf size: 24

```
DSolve[y'[x]==Sin[x]+y[x],y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to -\frac{\sin(x)}{2} - \frac{\cos(x)}{2} + c_1 e^x$$

1.27 problem 27

Internal problem ID [7343]

Book: First order enumerated odes
Section: section 1
Problem number: 27.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_Riccati]`

$$\boxed{y' - y^2 = \sin(x)}$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 59

```
dsolve(diff(y(x),x)=sin(x)+y(x)^2,y(x), singsol=all)
```

$$y(x) = \frac{-c_1 \operatorname{MathieuSPrime}\left(0, -2, -\frac{\pi}{4} + \frac{x}{2}\right) - \operatorname{MathieuCPrime}\left(0, -2, -\frac{\pi}{4} + \frac{x}{2}\right)}{2c_1 \operatorname{MathieuS}\left(0, -2, -\frac{\pi}{4} + \frac{x}{2}\right) + 2 \operatorname{MathieuC}\left(0, -2, -\frac{\pi}{4} + \frac{x}{2}\right)}$$

✓ Solution by Mathematica

Time used: 0.208 (sec). Leaf size: 105

```
DSolve[y'[x]==Sin[x]+y[x]^2,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to \frac{-\text{MathieuSPrime}\left[0, -2, \frac{1}{4}(\pi - 2x)\right] + c_1 \text{MathieuCPrime}\left[0, -2, \frac{1}{4}(\pi - 2x)\right]}{2\left(\text{MathieuS}\left[0, -2, \frac{1}{4}(2x - \pi)\right] + c_1 \text{MathieuC}\left[0, -2, \frac{1}{4}(\pi - 2x)\right]\right)}$$

$$y(x) \to \frac{\text{MathieuCPrime}\left[0, -2, \frac{1}{4}(\pi - 2x)\right]}{2\text{MathieuC}\left[0, -2, \frac{1}{4}(\pi - 2x)\right]}$$

1.28 problem 28

Internal problem ID [7344]

Book: First order enumerated odes
Section: section 1
Problem number: 28.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_linear]`

$$\boxed{y' - \frac{y}{x} = \cos(x)}$$

✓ Solution by Maple
Time used: 0.0 (sec). Leaf size: 10

```
dsolve(diff(y(x),x)=cos(x)+y(x)/x,y(x), singsol=all)
```

$$y(x) = (\operatorname{Ci}(x) + c_1)\, x$$

✓ Solution by Mathematica
Time used: 0.038 (sec). Leaf size: 12

```
DSolve[y'[x]==Cos[x]+y[x]/x,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to x(\text{CosIntegral}(x) + c_1)$$

1.29 problem 29

Internal problem ID [7345]

Book: First order enumerated odes
Section: section 1
Problem number: 29.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_Riccati]`

$$\boxed{y' - \frac{y^2}{x} = \cos(x)}$$

✗ Solution by Maple

```
dsolve(diff(y(x),x)=cos(x)+y(x)^2/x,y(x), singsol=all)
```

No solution found

✗ Solution by Mathematica
Time used: 0.0 (sec). Leaf size: 0

```
DSolve[y'[x]==Cos[x]+y[x]^2/x,y[x],x,IncludeSingularSolutions -> True]
```

Not solved

1.30 problem 30

Internal problem ID [7346]

Book: First order enumerated odes
Section: section 1
Problem number: 30.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_Riccati]`

$$\boxed{y' - y - by^2 = x}$$

✓ Solution by Maple

Time used: 0.015 (sec). Leaf size: 105

```
dsolve(diff(y(x),x)=x+y(x)+b*y(x)^2,y(x), singsol=all)
```

$$y(x) = \frac{2b^{\frac{1}{3}} \operatorname{AiryAi}\left(1, -\frac{4bx-1}{4b^{\frac{2}{3}}}\right) c_1 + 2 \operatorname{AiryBi}\left(1, -\frac{4bx-1}{4b^{\frac{2}{3}}}\right) b^{\frac{1}{3}} - \operatorname{AiryAi}\left(-\frac{4bx-1}{4b^{\frac{2}{3}}}\right) c_1 - \operatorname{AiryBi}\left(-\frac{4bx-1}{4b^{\frac{2}{3}}}\right)}{2b\left(\operatorname{AiryAi}\left(-\frac{4bx-1}{4b^{\frac{2}{3}}}\right) c_1 + \operatorname{AiryBi}\left(-\frac{4bx-1}{4b^{\frac{2}{3}}}\right)\right)}$$

✓ Solution by Mathematica

Time used: 0.222 (sec). Leaf size: 211

```
DSolve[y'[x]==x+y[x]+b*y[x]^2,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to -\frac{-(-b)^{2/3} \operatorname{AiryBi}\left(\frac{\frac{1}{4}-bx}{(-b)^{2/3}}\right) + 2b \operatorname{AiryBiPrime}\left(\frac{\frac{1}{4}-bx}{(-b)^{2/3}}\right) + c_1\left(2b \operatorname{AiryAiPrime}\left(\frac{\frac{1}{4}-bx}{(-b)^{2/3}}\right) - (-b)^{2/3} \operatorname{Airy}\right.}{2(-b)^{5/3}\left(\operatorname{AiryBi}\left(\frac{\frac{1}{4}-bx}{(-b)^{2/3}}\right) + c_1 \operatorname{AiryAi}\left(\frac{\frac{1}{4}-bx}{(-b)^{2/3}}\right)\right)}$$

$$y(x) \to -\frac{\frac{2\sqrt[3]{-b}\operatorname{AiryAiPrime}\left(\frac{\frac{1}{4}-bx}{(-b)^{2/3}}\right)}{\operatorname{AiryAi}\left(\frac{\frac{1}{4}-bx}{(-b)^{2/3}}\right)} + 1}{2b}$$

1.31 problem 31

Internal problem ID [7347]

Book: First order enumerated odes
Section: section 1
Problem number: 31.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_quadrature]`

$$\boxed{xy' = 0}$$

✓ Solution by Maple
Time used: 0.0 (sec). Leaf size: 5

```
dsolve(x*diff(y(x),x)=0,y(x), singsol=all)
```

$$y(x) = c_1$$

✓ Solution by Mathematica
Time used: 0.002 (sec). Leaf size: 7

```
DSolve[x*y'[x]==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to c_1$$

1.32 problem 32

Internal problem ID [7348]

Book: First order enumerated odes
Section: section 1
Problem number: 32.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_quadrature]`

$$\boxed{5y' = 0}$$

✓ Solution by Maple
Time used: 0.0 (sec). Leaf size: 5

```
dsolve(5*diff(y(x),x)=0,y(x), singsol=all)
```

$$y(x) = c_1$$

✓ Solution by Mathematica
Time used: 0.002 (sec). Leaf size: 7

```
DSolve[5*y'[x]==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to c_1$$

1.33 problem 33

Internal problem ID [7349]

Book: First order enumerated odes
Section: section 1
Problem number: 33.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_quadrature]`

$$\boxed{ey' = 0}$$

✓ Solution by Maple
Time used: 0.0 (sec). Leaf size: 5

```
dsolve(exp(1)*diff(y(x),x)=0,y(x), singsol=all)
```

$$y(x) = c_1$$

✓ Solution by Mathematica
Time used: 0.002 (sec). Leaf size: 7

```
DSolve[Exp[1]*y'[x]==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to c_1$$

1.34 problem 34

Internal problem ID [7350]

Book: First order enumerated odes
Section: section 1
Problem number: 34.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_quadrature]`

$$\boxed{\pi y' = 0}$$

✓ Solution by Maple
Time used: 0.0 (sec). Leaf size: 5

```
dsolve(Pi*diff(y(x),x)=0,y(x), singsol=all)
```

$$y(x) = c_1$$

✓ Solution by Mathematica
Time used: 0.002 (sec). Leaf size: 7

```
DSolve[Pi*y'[x]==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to c_1$$

1.35 problem 35

Internal problem ID [7351]

Book: First order enumerated odes
Section: section 1
Problem number: 35.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_quadrature]`

$$\boxed{y' \sin(x) = 0}$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 5

```
dsolve(sin(x)*diff(y(x),x)=0,y(x), singsol=all)
```

$$y(x) = c_1$$

✓ Solution by Mathematica

Time used: 0.002 (sec). Leaf size: 7

```
DSolve[Sin[x]*y'[x]==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to c_1$$

1.36 problem 36

Internal problem ID [7352]

Book: First order enumerated odes
Section: section 1
Problem number: 36.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_quadrature]`

$$\boxed{f(x)\,y' = 0}$$

✓ Solution by Maple
Time used: 0.0 (sec). Leaf size: 5

```
dsolve(f(x)*diff(y(x),x)=0,y(x), singsol=all)
```

$$y(x) = c_1$$

✓ Solution by Mathematica
Time used: 0.002 (sec). Leaf size: 7

```
DSolve[f[x]*y'[x]==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to c_1$$

1.37 problem 37

Internal problem ID [7353]

Book: First order enumerated odes
Section: section 1
Problem number: 37.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_quadrature]`

$$\boxed{xy' = 1}$$

✓ Solution by Maple
Time used: 0.0 (sec). Leaf size: 8

```
dsolve(x*diff(y(x),x)=1,y(x), singsol=all)
```

$$y(x) = \ln(x) + c_1$$

✓ Solution by Mathematica
Time used: 0.002 (sec). Leaf size: 10

```
DSolve[x*y'[x]==1,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to \log(x) + c_1$$

1.38 problem 38

Internal problem ID [7354]

Book: First order enumerated odes
Section: section 1
Problem number: 38.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_quadrature]`

$$\boxed{xy' = \sin(x)}$$

✓ Solution by Maple
Time used: 0.0 (sec). Leaf size: 8

```
dsolve(x*diff(y(x),x)=sin(x),y(x), singsol=all)
```

$$y(x) = \operatorname{Si}(x) + c_1$$

✓ Solution by Mathematica
Time used: 0.015 (sec). Leaf size: 10

```
DSolve[x*y'[x]==Sin[x],y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to \operatorname{Si}(x) + c_1$$

1.39 problem 39

Internal problem ID [7355]

Book: First order enumerated odes
Section: section 1
Problem number: 39.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_quadrature]`

$$\boxed{(x-1)\,y' = 0}$$

✓ Solution by Maple
Time used: 0.0 (sec). Leaf size: 5

```
dsolve((x-1)*diff(y(x),x)=0,y(x), singsol=all)
```

$$y(x) = c_1$$

✓ Solution by Mathematica
Time used: 0.002 (sec). Leaf size: 7

```
DSolve[(x-1)*y'[x]==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to c_1$$

1.40 problem 40

Internal problem ID [7356]

Book: First order enumerated odes
Section: section 1
Problem number: 40.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_quadrature]`

$$\boxed{yy' = 0}$$

✓ Solution by Maple
Time used: 0.0 (sec). Leaf size: 11

```
dsolve(y(x)*diff(y(x),x)=0,y(x), singsol=all)
```

$$y(x) = 0$$
$$y(x) = -c_1$$

✓ Solution by Mathematica
Time used: 0.002 (sec). Leaf size: 12

```
DSolve[y[x]*y'[x]==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to 0$$
$$y(x) \to c_1$$

1.41 problem 41

Internal problem ID [7357]

Book: First order enumerated odes
Section: section 1
Problem number: 41.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_quadrature]`

$$\boxed{xy'y = 0}$$

✓ Solution by Maple
Time used: 0.0 (sec). Leaf size: 9

```
dsolve(x*y(x)*diff(y(x),x)=0,y(x), singsol=all)
```

$$y(x) = 0$$
$$y(x) = c_1$$

✓ Solution by Mathematica
Time used: 0.002 (sec). Leaf size: 12

```
DSolve[x*y[x]*y'[x]==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to 0$$
$$y(x) \to c_1$$

1.42 problem 42

Internal problem ID [7358]

Book: First order enumerated odes
Section: section 1
Problem number: 42.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_quadrature]`

$$\boxed{xy \sin(x)\, y' = 0}$$

✓ Solution by Maple
Time used: 0.0 (sec). Leaf size: 9

```
dsolve(x*y(x)*sin(x)*diff(y(x),x)=0,y(x), singsol=all)
```

$$y(x) = 0$$
$$y(x) = c_1$$

✓ Solution by Mathematica
Time used: 0.002 (sec). Leaf size: 12

```
DSolve[x*y[x]*Sin[x]*y'[x]==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to 0$$
$$y(x) \to c_1$$

1.43 problem 43

Internal problem ID [7359]

Book: First order enumerated odes
Section: section 1
Problem number: 43.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_quadrature]`

$$\boxed{\pi y \sin(x) y' = 0}$$

✓ Solution by Maple
Time used: 0.016 (sec). Leaf size: 9

```
dsolve(Pi*y(x)*sin(x)*diff(y(x),x)=0,y(x), singsol=all)
```

$$y(x) = 0$$
$$y(x) = c_1$$

✓ Solution by Mathematica
Time used: 0.002 (sec). Leaf size: 12

```
DSolve[Pi*y[x]*Sin[x]*y'[x]==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to 0$$
$$y(x) \to c_1$$

1.44 problem 44

Internal problem ID [7360]

Book: First order enumerated odes
Section: section 1
Problem number: 44.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_quadrature]`

$$\boxed{\sin(x)\, y' x = 0}$$

✓ Solution by Maple
Time used: 0.0 (sec). Leaf size: 5

```
dsolve(x*sin(x)*diff(y(x),x)=0,y(x), singsol=all)
```

$$y(x) = c_1$$

✓ Solution by Mathematica
Time used: 0.003 (sec). Leaf size: 7

```
DSolve[x*Sin[x]*y'[x]==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to c_1$$

1.45 problem 45

Internal problem ID [7361]

Book: First order enumerated odes
Section: section 1
Problem number: 45.
ODE order: 1.
ODE degree: 2.

CAS Maple gives this as type `[_quadrature]`

$$\boxed{x \sin(x) {y'}^2 = 0}$$

✓ Solution by Maple
Time used: 0.015 (sec). Leaf size: 5

```
dsolve(x*sin(x)*diff(y(x),x)^2=0,y(x), singsol=all)
```

$$y(x) = c_1$$

✓ Solution by Mathematica
Time used: 0.002 (sec). Leaf size: 7

```
DSolve[x*Sin[x]*y'[x]^2==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to c_1$$

1.46 problem 46

Internal problem ID [7362]

Book: First order enumerated odes
Section: section 1
Problem number: 46.
ODE order: 1.
ODE degree: 2.

CAS Maple gives this as type `[_quadrature]`

$$\boxed{yy'^2 = 0}$$

✓ Solution by Maple
Time used: 0.0 (sec). Leaf size: 9

```
dsolve(y(x)*diff(y(x),x)^2=0,y(x), singsol=all)
```

$$y(x) = 0$$
$$y(x) = c_1$$

✓ Solution by Mathematica
Time used: 0.002 (sec). Leaf size: 12

```
DSolve[y[x]*(y'[x])^2==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to 0$$
$$y(x) \to c_1$$

1.47 problem 47

Internal problem ID [7363]

Book: First order enumerated odes
Section: section 1
Problem number: 47.
ODE order: 1.
ODE degree: 0.

CAS Maple gives this as type `[_quadrature]`

$$\boxed{y'^n = 0}$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 5

```
dsolve(diff(y(x),x)^n=0,y(x), singsol=all)
```

$$y(x) = c_1$$

✓ Solution by Mathematica

Time used: 0.003 (sec). Leaf size: 15

```
DSolve[(y'[x])^n==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to 0^{\frac{1}{n}} x + c_1$$

1.48 problem 48

Internal problem ID [7364]

Book: First order enumerated odes
Section: section 1
Problem number: 48.
ODE order: 1.
ODE degree: 0.

CAS Maple gives this as type `[_quadrature]`

$$\boxed{xy'^n = 0}$$

✓ Solution by Maple
Time used: 0.0 (sec). Leaf size: 5

```
dsolve(x*diff(y(x),x)^n=0,y(x), singsol=all)
```

$$y(x) = c_1$$

✓ Solution by Mathematica
Time used: 0.003 (sec). Leaf size: 15

```
DSolve[x*(y'[x])^n==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to 0^{\frac{1}{n}}x + c_1$$

1.49 problem 49

Internal problem ID [7365]

Book: First order enumerated odes
Section: section 1
Problem number: 49.
ODE order: 1.
ODE degree: 2.

CAS Maple gives this as type `[_quadrature]`

$$\boxed{y'^2 = x}$$

✓ Solution by Maple

Time used: 0.016 (sec). Leaf size: 21

```
dsolve(diff(y(x),x)^2=x,y(x), singsol=all)
```

$$y(x) = \frac{2x^{\frac{3}{2}}}{3} + c_1$$
$$y(x) = -\frac{2x^{\frac{3}{2}}}{3} + c_1$$

✓ Solution by Mathematica

Time used: 0.003 (sec). Leaf size: 33

```
DSolve[(y'[x])^2==x,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to -\frac{2x^{3/2}}{3} + c_1$$
$$y(x) \to \frac{2x^{3/2}}{3} + c_1$$

1.50 problem 50

Internal problem ID [7366]

Book: First order enumerated odes
Section: section 1
Problem number: 50.
ODE order: 1.
ODE degree: 2.

CAS Maple gives this as type `[[_homogeneous, 'class C'], _dAlembert]`

$$\boxed{y'^2 - y = x}$$

✓ Solution by Maple

Time used: 0.016 (sec). Leaf size: 33

```
dsolve(diff(y(x),x)^2=x+y(x),y(x), singsol=all)
```

$$y(x) = \operatorname{LambertW}\left(-c_1 \mathrm{e}^{-\frac{x}{2}-1}\right)^2 + 2\operatorname{LambertW}\left(-c_1 \mathrm{e}^{-\frac{x}{2}-1}\right) - x + 1$$

✓ Solution by Mathematica

Time used: 18.817 (sec). Leaf size: 100

```
DSolve[(y'[x])^2==x+y[x],y[x],x,IncludeSingularSolutions -> True]
```

$$\begin{aligned}
y(x) &\to W\left(-e^{-\frac{x}{2}-1-\frac{c_1}{2}}\right)^2 + 2W\left(-e^{-\frac{x}{2}-1-\frac{c_1}{2}}\right) - x + 1 \\
y(x) &\to W\left(e^{\frac{1}{2}(-x-2+c_1)}\right)^2 + 2W\left(e^{\frac{1}{2}(-x-2+c_1)}\right) - x + 1 \\
y(x) &\to 1 - x
\end{aligned}$$

1.51 problem 51

Internal problem ID [7367]

Book: First order enumerated odes
Section: section 1
Problem number: 51.
ODE order: 1.
ODE degree: 2.

CAS Maple gives this as type `[[_homogeneous, ‘class A‘], _rational, _dAlembert]`

$$\boxed{y'^2 - \frac{y}{x} = 0}$$

✓ Solution by Maple
Time used: 0.015 (sec). Leaf size: 39

```
dsolve(diff(y(x),x)^2=y(x)/x,y(x), singsol=all)
```

$$y(x) = 0$$
$$y(x) = \frac{\left(x + \sqrt{c_1 x}\right)^2}{x}$$
$$y(x) = \frac{\left(-x + \sqrt{c_1 x}\right)^2}{x}$$

✓ Solution by Mathematica
Time used: 0.048 (sec). Leaf size: 46

```
DSolve[(y'[x])^2==y[x]/x,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to \frac{1}{4}\left(-2\sqrt{x} + c_1\right)^2$$
$$y(x) \to \frac{1}{4}\left(2\sqrt{x} + c_1\right)^2$$
$$y(x) \to 0$$

1.52 problem 52

Internal problem ID [7368]

Book: First order enumerated odes
Section: section 1
Problem number: 52.
ODE order: 1.
ODE degree: 2.

CAS Maple gives this as type `[_separable]`

$$y'^2 - \frac{y^2}{x} = 0$$

✓ Solution by Maple
Time used: 0.031 (sec). Leaf size: 27

```
dsolve(diff(y(x),x)^2=y(x)^2/x,y(x), singsol=all)
```

$$y(x) = 0$$
$$y(x) = c_1 e^{-2\sqrt{x}}$$
$$y(x) = c_1 e^{2\sqrt{x}}$$

✓ Solution by Mathematica
Time used: 0.068 (sec). Leaf size: 38

```
DSolve[(y'[x])^2==y[x]^2/x,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \rightarrow c_1 e^{-2\sqrt{x}}$$
$$y(x) \rightarrow c_1 e^{2\sqrt{x}}$$
$$y(x) \rightarrow 0$$

1.53 problem 53

Internal problem ID [7369]

Book: First order enumerated odes
Section: section 1
Problem number: 53.
ODE order: 1.
ODE degree: 2.

CAS Maple gives this as type `[[_homogeneous, ‘class G‘]]`

$$y'^2 - \frac{y^3}{x} = 0$$

✓ Solution by Maple
Time used: 0.015 (sec). Leaf size: 27

```
dsolve(diff(y(x),x)^2=y(x)^3/x,y(x), singsol=all)
```

$$y(x) = 0$$
$$y(x) = \frac{\text{WeierstrassP}\,(1,0,0)\,2^{\frac{2}{3}}}{\left(\sqrt{x}\,2^{\frac{1}{3}} + c_1\right)^2}$$

✓ Solution by Mathematica
Time used: 0.073 (sec). Leaf size: 42

```
DSolve[(y'[x])^2==y[x]^3/x,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to \frac{4}{\left(-2\sqrt{x} + c_1\right)^2}$$
$$y(x) \to \frac{4}{\left(2\sqrt{x} + c_1\right)^2}$$
$$y(x) \to 0$$

1.54 problem 54

Internal problem ID [7370]

Book: First order enumerated odes
Section: section 1
Problem number: 54.
ODE order: 1.
ODE degree: 3.

CAS Maple gives this as type `[[_homogeneous, ‘class G‘], _rational]`

$$\boxed{y'^3 - \frac{y^2}{x} = 0}$$

✓ Solution by Maple

Time used: 0.047 (sec). Leaf size: 353

```
dsolve(diff(y(x),x)^3=y(x)^2/x,y(x), singsol=all)
```

$$y(x) = 0$$
$$y(x) = -\frac{3x^{\frac{4}{3}}c_1}{8} + \frac{3x^{\frac{2}{3}}c_1^2}{8} - \frac{c_1^3}{8} + \frac{x^2}{8}$$
$$y(x) = \frac{3\left(-i\sqrt{3}-1\right)c_1^2x^{\frac{2}{3}}}{16} + \frac{3c_1\left(1-i\sqrt{3}\right)x^{\frac{4}{3}}}{16} - \frac{c_1^3}{8} + \frac{x^2}{8}$$
$$y(x) = \frac{3\left(i\sqrt{3}-1\right)c_1^2x^{\frac{2}{3}}}{16} + \frac{3\left(1+i\sqrt{3}\right)c_1x^{\frac{4}{3}}}{16} - \frac{c_1^3}{8} + \frac{x^2}{8}$$
$$y(x) = \frac{3x^{\frac{4}{3}}c_1}{16} + \frac{3x^{\frac{2}{3}}c_1^2}{32} + \frac{c_1^3}{64} + \frac{x^2}{8}$$
$$y(x) = \frac{3\left(-i\sqrt{3}-1\right)c_1^2x^{\frac{2}{3}}}{64} + \frac{3\left(i\sqrt{3}-1\right)c_1x^{\frac{4}{3}}}{32} + \frac{c_1^3}{64} + \frac{x^2}{8}$$
$$y(x) = \frac{3\left(i\sqrt{3}-1\right)c_1^2x^{\frac{2}{3}}}{64} + \frac{3c_1\left(-i\sqrt{3}-1\right)x^{\frac{4}{3}}}{32} + \frac{c_1^3}{64} + \frac{x^2}{8}$$
$$y(x) = -\frac{3x^{\frac{4}{3}}c_1}{16} + \frac{3x^{\frac{2}{3}}c_1^2}{32} - \frac{c_1^3}{64} + \frac{x^2}{8}$$
$$y(x) = \frac{3\left(-i\sqrt{3}-1\right)c_1^2x^{\frac{2}{3}}}{64} + \frac{3c_1\left(1-i\sqrt{3}\right)x^{\frac{4}{3}}}{32} - \frac{c_1^3}{64} + \frac{x^2}{8}$$
$$y(x) = \frac{3\left(i\sqrt{3}-1\right)c_1^2x^{\frac{2}{3}}}{64} + \frac{3\left(1+i\sqrt{3}\right)c_1x^{\frac{4}{3}}}{32} - \frac{c_1^3}{64} + \frac{x^2}{8}$$

✓ Solution by Mathematica

Time used: 0.083 (sec). Leaf size: 152

```
DSolve[(y'[x])^3==y[x]^2/x,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to \frac{1}{216}\left(3x^{2/3} + 2c_1\right)^3$$
$$y(x) \to \frac{1}{216}\left(18i\left(\sqrt{3} + i\right)c_1{}^2x^{2/3} - 27i\left(\sqrt{3} - i\right)c_1x^{4/3} + 27x^2 + 8c_1{}^3\right)$$
$$y(x) \to \frac{1}{216}\left(-18i\left(\sqrt{3} - i\right)c_1{}^2x^{2/3} + 27i\left(\sqrt{3} + i\right)c_1x^{4/3} + 27x^2 + 8c_1{}^3\right)$$
$$y(x) \to 0$$

1.55 problem 55

Internal problem ID [7371]

Book: First order enumerated odes
Section: section 1
Problem number: 55.
ODE order: 1.
ODE degree: 2.

CAS Maple gives this as type `[[_homogeneous, ‘class G‘]]`

$$\boxed{y'^2 - \frac{1}{yx} = 0}$$

✓ Solution by Maple
Time used: 0.031 (sec). Leaf size: 51

```
dsolve(diff(y(x),x)^2=1/(y(x)*x),y(x), singsol=all)
```

$$\frac{y(x)\sqrt{xy(x)} - c_1\sqrt{x} - 3x}{\sqrt{x}} = 0$$
$$\frac{y(x)\sqrt{xy(x)} - c_1\sqrt{x} + 3x}{\sqrt{x}} = 0$$

✓ Solution by Mathematica
Time used: 3.748 (sec). Leaf size: 53

```
DSolve[(y'[x])^2==1/(y[x]*x),y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to \left(\frac{3}{2}\right)^{2/3} \left(-2\sqrt{x} + c_1\right)^{2/3}$$
$$y(x) \to \left(\frac{3}{2}\right)^{2/3} \left(2\sqrt{x} + c_1\right)^{2/3}$$

1.56 problem 56

Internal problem ID [7372]

Book: First order enumerated odes
Section: section 1
Problem number: 56.
ODE order: 1.
ODE degree: 2.

CAS Maple gives this as type `[[_homogeneous, ‘class G‘]]`

$$y'^2 - \frac{1}{y^3 x} = 0$$

✓ Solution by Maple
Time used: 0.046 (sec). Leaf size: 55

```
dsolve(diff(y(x),x)^2=1/(x*y(x)^3),y(x), singsol=all)
```

$$\frac{\sqrt{xy(x)}\, y(x)^2 - c_1\sqrt{x} - 5x}{\sqrt{x}} = 0$$
$$\frac{\sqrt{xy(x)}\, y(x)^2 - c_1\sqrt{x} + 5x}{\sqrt{x}} = 0$$

✓ Solution by Mathematica
Time used: 0.112 (sec). Leaf size: 53

```
DSolve[(y'[x])^2==1/(x*y[x]^3),y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to \left(\frac{5}{2}\right)^{2/5} \left(-2\sqrt{x} + c_1\right)^{2/5}$$
$$y(x) \to \left(\frac{5}{2}\right)^{2/5} \left(2\sqrt{x} + c_1\right)^{2/5}$$

1.57 problem 57

Internal problem ID [7373]

Book: First order enumerated odes
Section: section 1
Problem number: 57.
ODE order: 1.
ODE degree: 2.

CAS Maple gives this as type `[_separable]`

$$y'^2 - \frac{1}{y^3 x^2} = 0$$

✓ Solution by Maple
Time used: 0.047 (sec). Leaf size: 29

```
dsolve(diff(y(x),x)^2=1/(x^2*y(x)^3),y(x), singsol=all)
```

$$\ln(x) - \frac{2y(x)^{\frac{5}{2}}}{5} - c_1 = 0$$
$$\ln(x) + \frac{2y(x)^{\frac{5}{2}}}{5} - c_1 = 0$$

✓ Solution by Mathematica
Time used: 0.139 (sec). Leaf size: 45

```
DSolve[(y'[x])^2==1/(x^2*y[x]^3),y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to \left(\frac{5}{2}\right)^{2/5} (-\log(x) + c_1)^{2/5}$$
$$y(x) \to \left(\frac{5}{2}\right)^{2/5} (\log(x) + c_1)^{2/5}$$

1.58 problem 58

Internal problem ID [7374]

Book: First order enumerated odes
Section: section 1
Problem number: 58.
ODE order: 1.
ODE degree: 4.

CAS Maple gives this as type `[[_homogeneous, ‘class G‘], _rational]`

$$\boxed{y'^4 - \frac{1}{y^3 x} = 0}$$

✓ Solution by Maple

Time used: 0.063 (sec). Leaf size: 123

```
dsolve(diff(y(x),x)^4=1/(x*y(x)^3),y(x), singsol=all)
```

$$-\frac{7x^3 - 3y(x)\left(x^3 y(x)\right)^{\frac{3}{4}} + c_1 x^{\frac{9}{4}}}{x^{\frac{9}{4}}} = 0$$
$$\frac{-7x^3 + 3iy(x)\left(x^3 y(x)\right)^{\frac{3}{4}} - c_1 x^{\frac{9}{4}}}{x^{\frac{9}{4}}} = 0$$
$$\frac{7x^3 + 3iy(x)\left(x^3 y(x)\right)^{\frac{3}{4}} - c_1 x^{\frac{9}{4}}}{x^{\frac{9}{4}}} = 0$$
$$\frac{7x^3 + 3y(x)\left(x^3 y(x)\right)^{\frac{3}{4}} - c_1 x^{\frac{9}{4}}}{x^{\frac{9}{4}}} = 0$$

✓ Solution by Mathematica

Time used: 7.225 (sec). Leaf size: 129

```
DSolve[(y'[x])^4==1/(x*y[x]^3),y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to \frac{\left(-\frac{28x^{3/4}}{3}+7c_1\right)^{4/7}}{2\sqrt[7]{2}}$$
$$y(x) \to \frac{\left(7c_1-\frac{28}{3}ix^{3/4}\right)^{4/7}}{2\sqrt[7]{2}}$$
$$y(x) \to \frac{\left(\frac{28}{3}ix^{3/4}+7c_1\right)^{4/7}}{2\sqrt[7]{2}}$$
$$y(x) \to \frac{\left(\frac{28x^{3/4}}{3}+7c_1\right)^{4/7}}{2\sqrt[7]{2}}$$

1.59 problem 59

Internal problem ID [7375]

Book: First order enumerated odes
Section: section 1
Problem number: 59.
ODE order: 1.
ODE degree: 2.

CAS Maple gives this as type `[_separable]`

$$
\boxed{y'^2 - \frac{1}{x^3 y^4} = 0}
$$

✓ Solution by Maple
Time used: 0.047 (sec). Leaf size: 137

```
dsolve(diff(y(x),x)^2=1/(x^3*y(x)^4),y(x), singsol=all)
```

$$
y(x) = \left(\frac{c_1\sqrt{x}-6}{\sqrt{x}}\right)^{\frac{1}{3}}
$$
$$
y(x) = -\frac{\left(\frac{c_1\sqrt{x}-6}{\sqrt{x}}\right)^{\frac{1}{3}}\left(1+i\sqrt{3}\right)}{2}
$$
$$
y(x) = \frac{\left(\frac{c_1\sqrt{x}-6}{\sqrt{x}}\right)^{\frac{1}{3}}\left(i\sqrt{3}-1\right)}{2}
$$
$$
y(x) = \left(\frac{c_1\sqrt{x}+6}{\sqrt{x}}\right)^{\frac{1}{3}}
$$
$$
y(x) = -\frac{\left(\frac{c_1\sqrt{x}+6}{\sqrt{x}}\right)^{\frac{1}{3}}\left(1+i\sqrt{3}\right)}{2}
$$
$$
y(x) = \frac{\left(\frac{c_1\sqrt{x}+6}{\sqrt{x}}\right)^{\frac{1}{3}}\left(i\sqrt{3}-1\right)}{2}
$$

✓ Solution by Mathematica

Time used: 3.775 (sec). Leaf size: 157

```
DSolve[(y'[x])^2==1/(x^3*y[x]^4),y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to -\sqrt[3]{-3}\sqrt[3]{-\frac{2}{\sqrt{x}}+c_1}$$
$$y(x) \to \sqrt[3]{3}\sqrt[3]{-\frac{2}{\sqrt{x}}+c_1}$$
$$y(x) \to (-1)^{2/3}\sqrt[3]{3}\sqrt[3]{-\frac{2}{\sqrt{x}}+c_1}$$
$$y(x) \to -\sqrt[3]{-3}\sqrt[3]{\frac{2}{\sqrt{x}}+c_1}$$
$$y(x) \to \sqrt[3]{3}\sqrt[3]{\frac{2}{\sqrt{x}}+c_1}$$
$$y(x) \to (-1)^{2/3}\sqrt[3]{3}\sqrt[3]{\frac{2}{\sqrt{x}}+c_1}$$

1.60 problem 60

Internal problem ID [7376]

Book: First order enumerated odes
Section: section 1
Problem number: 60.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[[_homogeneous, ‘class C‘], _dAlembert]`

$$\boxed{y' - \sqrt{1 + 6x + y} = 0}$$

✓ Solution by Maple
Time used: 0.015 (sec). Leaf size: 57

```
dsolve(diff(y(x),x)=(1+6*x+y(x))^(1/2),y(x), singsol=all)
```

$$x - 2\sqrt{1 + 6x + y(x)} + 6\ln\left(6 + \sqrt{1 + 6x + y(x)}\right) - 6\ln\left(-6 + \sqrt{1 + 6x + y(x)}\right) + 6\ln(-35 + y(x) + 6x) - c_1 = 0$$

✓ Solution by Mathematica
Time used: 13.35 (sec). Leaf size: 65

```
DSolve[y'[x]==(1+6*x+y[x])^(1/2),y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to 36W\left(-\frac{1}{6}e^{\frac{1}{72}(-6x-73+6c_1)}\right)^2 + 72W\left(-\frac{1}{6}e^{\frac{1}{72}(-6x-73+6c_1)}\right) - 6x + 35$$
$$y(x) \to 35 - 6x$$

1.61 problem 61

Internal problem ID [7377]

Book: First order enumerated odes
Section: section 1
Problem number: 61.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[[_homogeneous, 'class C'], _dAlembert]`

$$y' - (1 + 6x + y)^{\frac{1}{3}} = 0$$

✓ Solution by Maple

Time used: 0.015 (sec). Leaf size: 79

```
dsolve(diff(y(x),x)=(1+6*x+y(x))^(1/3),y(x), singsol=all)
```

$$x - \frac{3(1 + 6x + y(x))^{\frac{2}{3}}}{2} - 72 \ln \left(6 + (1 + 6x + y(x))^{\frac{1}{3}}\right) + 36 \ln \left((1 + 6x + y(x))^{\frac{2}{3}} - 6(1 + 6x + y(x))^{\frac{1}{3}} + 36\right) - 36 \ln (217 + y(x) + 6x) + 18(1 + 6x + y(x))^{\frac{1}{3}} - c_1 = 0$$

✓ Solution by Mathematica

Time used: 0.246 (sec). Leaf size: 66

```
DSolve[y'[x]==(1+6*x+y[x])^(1/3),y[x],x,IncludeSingularSolutions -> True]
```

$$\text{Solve}\left[\frac{1}{6}\left(y(x) - 9(y(x) + 6x + 1)^{2/3} + 108\sqrt[3]{y(x) + 6x + 1} - 648 \log \left(\sqrt[3]{y(x) + 6x + 1} + 6\right) + 6x + 1\right) - \frac{y(x)}{6} = c_1, y(x)\right]$$

1.62 problem 62

Internal problem ID [7378]

Book: First order enumerated odes
Section: section 1
Problem number: 62.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[[_homogeneous, 'class C'], _dAlembert]`

$$y' - (1 + 6x + y)^{\frac{1}{4}} = 0$$

✓ Solution by Maple

Time used: 0.015 (sec). Leaf size: 109

```
dsolve(diff(y(x),x)=(1+6*x+y(x))^(1/4),y(x), singsol=all)
```

$$\begin{aligned} &x + 216\ln(-y(x) - 6x + 1295) + 12\sqrt{1 + 6x + y(x)} \\ &+ 216\ln\left(\sqrt{1 + 6x + y(x)} - 36\right) - 216\ln\left(\sqrt{1 + 6x + y(x)} + 36\right) \\ &- 144(1 + 6x + y(x))^{\frac{1}{4}} + 432\ln\left(6 + (1 + 6x + y(x))^{\frac{1}{4}}\right) \\ &- 432\ln\left((1 + 6x + y(x))^{\frac{1}{4}} - 6\right) - \frac{4(1 + 6x + y(x))^{\frac{3}{4}}}{3} - c_1 = 0 \end{aligned}$$

✓ Solution by Mathematica

Time used: 2.535 (sec). Leaf size: 79

```
DSolve[y'[x]==(1+6*x+y[x])^(1/4),y[x],x,IncludeSingularSolutions -> True]
```

$$\text{Solve}\left[\frac{1}{6}\left(y(x) - 8(y(x) + 6x + 1)^{3/4} + 72\sqrt{y(x) + 6x + 1} - 864\sqrt[4]{y(x) + 6x + 1} + 5184\log\left(\sqrt[4]{y(x) + 6x + 1} + 6\right) + 6x + 1\right) - \frac{y(x)}{6} = c_1, y(x)\right]$$

1.63 problem 63

Internal problem ID [7379]

Book: First order enumerated odes
Section: section 1
Problem number: 63.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[[_homogeneous, ‘class C‘], _dAlembert]`

$$\boxed{y' - (a + xb + y)^4 = 0}$$

✓ Solution by Maple
Time used: 0.015 (sec). Leaf size: 49

```
dsolve(diff(y(x),x)=(a+b*x+y(x))^(4),y(x), singsol=all)
```

$$y(x) = -bx + \operatorname{RootOf}\left(-x + \int^{\_Z} \frac{1}{\_a^4 + 4\_a^3 a + 6\_a^2 a^2 + 4\_a\, a^3 + a^4 + b} d\_a + c_1\right)$$

✓ Solution by Mathematica
Time used: 0.429 (sec). Leaf size: 163

```
DSolve[y'[x]==(a+b*x+y[x])^(4),y[x],x,IncludeSingularSolutions -> True]
```

$$\text{Solve}\left[\frac{2\sqrt{2}\arctan\left(1 - \frac{\sqrt{2}(a+bx+y(x))}{\sqrt[4]{b}}\right) - 2\sqrt{2}\arctan\left(\frac{\sqrt{2}(a+bx+y(x))}{\sqrt[4]{b}} + 1\right) + \sqrt{2}\log\left((a + bx + y(x))^2 - \sqrt{\right.}{8b^3}\right.$$

1.64 problem 64

Internal problem ID [7380]

Book: First order enumerated odes
Section: section 1
Problem number: 64.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[[_homogeneous, ‘class C‘], _dAlembert]`

$$\boxed{y' - (\pi + x + 7y)^{\frac{7}{2}} = 0}$$

✓ Solution by Maple

Time used: 0.031 (sec). Leaf size: 33

```
dsolve(diff(y(x),x)=(Pi+x+7*y(x))^(7/2),y(x), singsol=all)
```

$$y(x) = -\frac{x}{7} + \text{RootOf}\left(-x + 7\left(\int^{\_Z} \frac{1}{1 + 7\left(\pi + 7\_a\right)^{\frac{7}{2}}} d\_a\right) + c_1\right)$$

✓ Solution by Mathematica

Time used: 30.556 (sec). Leaf size: 43

```
DSolve[y'[x]==(Pi+x+7*y[x])^(7/2),y[x],x,IncludeSingularSolutions -> True]
```

$$\text{Solve}\left[-(7y(x)+x+\pi)\left(\text{Hypergeometric2F1}\left(\frac{2}{7},1,\frac{9}{7},-7(x+7y(x)+\pi)^{7/2}\right)-1\right) - 7y(x) = c_1, y(x)\right]$$

1.65 problem 65

Internal problem ID [7381]

Book: First order enumerated odes
Section: section 1
Problem number: 65.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[[_homogeneous, ‘class C‘], _dAlembert]`

$$\boxed{y' - (a + xb + cy)^6 = 0}$$

✓ Solution by Maple

Time used: 0.016 (sec). Leaf size: 94

```
dsolve(diff(y(x),x)=(a+b*x+c*y(x))^6,y(x), singsol=all)
```

$$y(x) = \frac{\text{RootOf}\left(\left(\left(\int^{-Z} \frac{1}{c^7\_a^6+6\_a^5 a\, c^6+15\_a^4 a^2 c^5+20\_a^3 a^3 c^4+15\_a^2 a^4 c^3+6\_a\, a^5 c^2+a^6 c+b} d\_a\right) c - x + c_1\right) c - bx}{c}$$

✓ Solution by Mathematica

Time used: 1.941 (sec). Leaf size: 274

```
DSolve[y'[x]==(a+b*x+c*y[x])^6,y[x],x,IncludeSingularSolutions -> True]
```

$$\text{Solve}\left[\frac{-4\sqrt[6]{b}\arctan\left(\frac{\sqrt[6]{c}(a+bx+cy(x))}{\sqrt[6]{b}}\right) + 2\sqrt[6]{b}\arctan\left(\sqrt{3} - \frac{2\sqrt[6]{c}(a+bx+cy(x))}{\sqrt[6]{b}}\right) - 2\sqrt[6]{b}\arctan\left(\frac{2\sqrt[6]{c}(a+bx+cy}{\sqrt[6]{b}}\right.}{} \right.$$

$$\left. - \frac{cy(x)}{b} = c_1, y(x)\right]$$

1.66 problem 66

Internal problem ID [7382]

Book: First order enumerated odes
Section: section 1
Problem number: 66.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_separable]`

$$\boxed{y' - e^{x+y} = 0}$$

✓ Solution by Maple
Time used: 0.016 (sec). Leaf size: 13

```
dsolve(diff(y(x),x)=exp(x+y(x)),y(x), singsol=all)
```

$$y(x) = \ln\left(-\frac{1}{e^x + c_1}\right)$$

✓ Solution by Mathematica
Time used: 0.876 (sec). Leaf size: 18

```
DSolve[y'[x]==Exp[x+y[x]],y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to -\log\left(-e^x - c_1\right)$$

1.67 problem 67

Internal problem ID [7383]

Book: First order enumerated odes
Section: section 1
Problem number: 67.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[[_homogeneous, ‘class C‘], _dAlembert]`

$$\boxed{y' - e^{x+y} = 10}$$

✓ Solution by Maple
Time used: 0.031 (sec). Leaf size: 26

```
dsolve(diff(y(x),x)=10+exp(x+y(x)),y(x), singsol=all)
```

$$y(x) = -x + \ln(11) + \ln\left(\frac{e^{11x}}{-e^{11x} + c_1}\right)$$

✓ Solution by Mathematica
Time used: 3.4 (sec). Leaf size: 42

```
DSolve[y'[x]==10+Exp[x+y[x]],y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to \log\left(-\frac{11e^{10x+11c_1}}{-1 + e^{11(x+c_1)}}\right)$$
$$y(x) \to \log\left(-11e^{-x}\right)$$

1.68 problem 68

Internal problem ID [7384]

Book: First order enumerated odes
Section: section 1
Problem number: 68.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[[_1st_order, '_with_symmetry_[F(x),G(x)]']]`

$$\boxed{y' - 10\,\mathrm{e}^{x+y} = x^2}$$

✓ Solution by Maple
Time used: 0.016 (sec). Leaf size: 30

```
dsolve(diff(y(x),x)=10*exp(x+y(x))+x^2,y(x), singsol=all)
```

$$y(x) = \frac{x^3}{3} - \ln\left(-c_1 - 10\left(\int \mathrm{e}^{\frac{x(x^2+3)}{3}} dx\right)\right)$$

✓ Solution by Mathematica
Time used: 0.431 (sec). Leaf size: 115

```
DSolve[y'[x]==10*Exp[x+y[x]]+x^2,y[x],x,IncludeSingularSolutions -> True]
```

$$\text{Solve}\left[\int_1^{y(x)} -\frac{1}{10}e^{-K[2]}\left(10e^{K[2]}\int_1^x -\frac{1}{10}e^{\frac{K[1]^3}{3}-K[2]}K[1]^2dK[1] + e^{\frac{x^3}{3}}\right)dK[2] + \int_1^x \left(\frac{1}{10}e^{\frac{K[1]^3}{3}-y(x)}K[1]^2 + e^{\frac{K[1]^3}{3}+K[1]}\right)dK[1] = c_1, y(x)\right]$$

1.69 problem 69

Internal problem ID [7385]

Book: First order enumerated odes
Section: section 1
Problem number: 69.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[[_1st_order, '_with_symmetry_[F(x),G(x)]']]`

$$\boxed{y' - e^{x+y}x = \sin(x)}$$

✓ Solution by Maple

Time used: 0.016 (sec). Leaf size: 29

```
dsolve(diff(y(x),x)=x*exp(x+y(x))+sin(x),y(x), singsol=all)
```

$$y(x) = -\cos(x) - \ln\left(-c_1 - \left(\int x\,e^{x-\cos(x)}dx\right)\right)$$

✓ Solution by Mathematica

Time used: 3.93 (sec). Leaf size: 100

```
DSolve[y'[x]==x*Exp[x+y[x]]+Sin[x],y[x],x,IncludeSingularSolutions -> True]
```

$$\text{Solve}\left[\int_1^x \left(-e^{K[1]-\cos(K[1])}K[1] - e^{-\cos(K[1])-y(x)}\sin(K[1])\right)dK[1] + \int_1^{y(x)} -e^{-\cos(x)-K[2]}\left(e^{\cos(x)+K[2]}\int_1^x e^{-\cos(K[1])-K[2]}\sin(K[1])dK[1]-1\right)dK[2] = c_1, y(x)\right]$$

1.70 problem 70

Internal problem ID [7386]

Book: First order enumerated odes
Section: section 1
Problem number: 70.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[[_1st_order, '_with_symmetry_[F(x),G(x)]']]`

$$\boxed{y' - 5\,e^{x^2+20y} = \sin(x)}$$

✓ Solution by Maple
Time used: 0.016 (sec). Leaf size: 33

```
dsolve(diff(y(x),x)=5*exp(x^2+20*y(x))+sin(x),y(x), singsol=all)
```

$$y(x) = -\cos(x) - \frac{\ln(20)}{20} - \frac{\ln\left(-c_1 - 5\left(\int e^{x^2-20\cos(x)}dx\right)\right)}{20}$$

✓ Solution by Mathematica
Time used: 10.354 (sec). Leaf size: 140

```
DSolve[y'[x]==5*Exp[x^2+20*y[x]]+Sin[x],y[x],x,IncludeSingularSolutions -> True]
```

$$\text{Solve}\left[\int_1^x -\frac{1}{100}e^{-20\cos(K[1])-20y(x)}\left(\sin(K[1])+5e^{K[1]^2+20y(x)}\right)dK[1]+\int_1^{y(x)} -\frac{1}{100}e^{-20\cos(x)-20K[2]}\left(100e^{20\cos(x)+20K[2]}\int_1^x\left(\frac{1}{5}e^{-20\cos(K[1])-20K[2]}\left(\sin(K[1])+5e^{K[1]^2+20K[2]}\right)-e^{K[1]^2-20\cos\right.\right.\right.$$
$$\left.\left.-1\right)dK[2]=c_1,y(x)\right]$$

2 section 2 (system of first order ode's)

2.1 problem 1

Internal problem ID [7387]

Book: First order enumerated odes
Section: section 2 (system of first order ode's)
Problem number: 1.
ODE order: 1.
ODE degree: 1.

Solve

$$x'(t) + y'(t) = x(t) + y(t) + t$$
$$x'(t) + y'(t) = 2x(t) + 3y(t) + e^t$$

✓ Solution by Maple

Time used: 0.015 (sec). Leaf size: 31

```
dsolve([diff(x(t),t)+diff(y(t),t)-x(t)=y(t)+t,diff(x(t),t)+diff(y(t),t)=2*x(t)+3*y(t)+exp(t)
```

$$x(t) = -3t - 2 + c_1 e^t$$
$$y(t) = 2t + 1 - \frac{c_1 e^t}{2} - \frac{e^t}{2}$$

✓ Solution by Mathematica

Time used: 0.042 (sec). Leaf size: 37

```
DSolve[{x'[t]+y'[t]-x[t]==y[t]+t,x'[t]+y'[t]==2*x[t]+3*y[t]+Exp[t]},{x[t],y[t]},t,IncludeSin
```

$$x(t) \to -3t + (1 + 2c_1)e^t - 2$$
$$y(t) \to 2t - (1 + c_1)e^t + 1$$

2.2 problem 2

Internal problem ID [7388]

Book: First order enumerated odes
Section: section 2 (system of first order ode's)
Problem number: 2.
ODE order: 1.
ODE degree: 1.

Solve

$$x'(t) = -x(t) - 2y(t) + t - e^t$$
$$y'(t) = 3x(t) + 5y(t) - t + 2e^t$$

✓ Solution by Maple

Time used: 0.015 (sec). Leaf size: 95

```
dsolve([2*diff(x(t),t)+diff(y(t),t)-x(t)=y(t)+t,diff(x(t),t)+diff(y(t),t)=2*x(t)+3*y(t)+exp(
```

$$x(t) = e^{(2+\sqrt{3})t} c_2 + e^{-(-2+\sqrt{3})t} c_1 - 3t - 11$$
$$y(t) = -\frac{e^{(2+\sqrt{3})t} c_2 \sqrt{3}}{2} + \frac{e^{-(-2+\sqrt{3})t} c_1 \sqrt{3}}{2} - \frac{3\,e^{(2+\sqrt{3})t} c_2}{2} - \frac{3\,e^{-(-2+\sqrt{3})t} c_1}{2} - \frac{e^t}{2} + 2t + 7$$

✓ Solution by Mathematica

Time used: 10.209 (sec). Leaf size: 174

```
DSolve[{2*x'[t]+y'[t]-x[t]==y[t]+t,x'[t]+y'[t]==2*x[t]+3*y[t]+Exp[t]},{x[t],y[t]},t,IncludeS
```

$$x(t) \to \frac{1}{6} e^{-((\sqrt{3}-2)t)} \left(-6e^{(\sqrt{3}-2)t}(3t+11) + \left(-3\left(\sqrt{3}-1\right)c_1 - 2\sqrt{3}c_2\right) e^{2\sqrt{3}t} + 3\left(1+\sqrt{3}\right)c_1 + 2\sqrt{3}c_2\right)$$
$$y(t) \to \frac{1}{2}\left(4t - e^t + \left(-\sqrt{3}c_1 - \sqrt{3}c_2 + c_2\right) e^{-((\sqrt{3}-2)t)} + \left(\sqrt{3}c_1 + \left(1+\sqrt{3}\right)c_2\right) e^{(2+\sqrt{3})t} + 14\right)$$

2.3 problem 3

Internal problem ID [7389]

Book: First order enumerated odes
Section: section 2 (system of first order ode's)
Problem number: 3.
ODE order: 1.
ODE degree: 1.

Solve

$$x'(t) + y'(t) = x(t) + y(t) + t + \sin(t) + \cos(t)$$
$$x'(t) + y'(t) = 2x(t) + 3y(t) + e^t$$

✓ Solution by Maple
Time used: 0.078 (sec). Leaf size: 45

```
dsolve([diff(x(t),t)+diff(y(t),t)-x(t)=y(t)+t+sin(t)+cos(t),diff(x(t),t)+diff(y(t),t)=2*x(t)
```

$$x(t) = -\sin(t) - 3\cos(t) + c_1 e^t - 3t - 2$$
$$y(t) = \sin(t) + 2\cos(t) - \frac{c_1 e^t}{2} + 2t + 1 - \frac{e^t}{2}$$

✓ Solution by Mathematica
Time used: 0.055 (sec). Leaf size: 54

```
DSolve[{x'[t]+y'[t]-x[t]==y[t]+t+Sin[t]+Cos[t],x'[t]+y'[t]==2*x[t]+3*y[t]+Exp[t]},{x[t],y[t]
```

$$x(t) \to -3t + e^t - \sin(t) - 3\cos(t) + 2c_1 e^t - 2$$
$$y(t) \to 2t - e^t + \sin(t) + 2\cos(t) - c_1 e^t + 1$$