A Solution Manual For
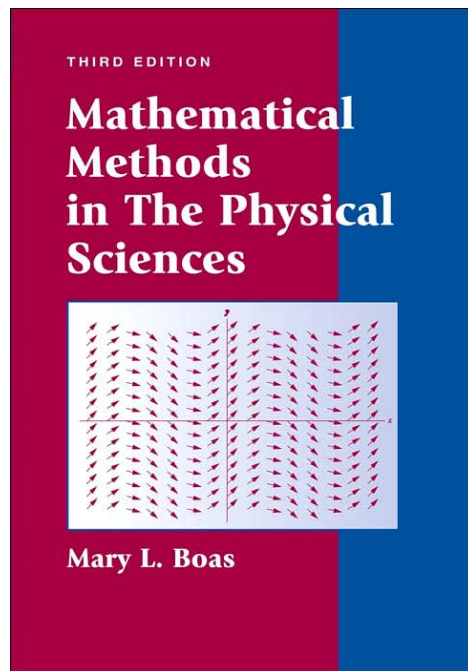
Mathematical Methods in the Physical Sciences. third edition. Mary L. Boas. John Wiley. 2006

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May 16, 2024

Contents

1 Chapter 8, Ordinary differential equations. Section 1. Introduction. page 394

1.1 problem 1

CAS Maple gives this as type [_quadrature]

$$y' - y = 0$$

✓ Solution by Maple
Time used: 0.0 (sec). Leaf size: 8

```
dsolve(diff(y(x),x)=y(x),y(x), singsol=all)
```

$$y(x) = c_1 e^x$$

✓ Solution by Mathematica
Time used: 0.012 (sec). Leaf size: 20

```
DSolve[y''[x]==y[x],y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to c_1 e^x + c_2 e^{-x}$$

2 Chapter 8, Ordinary differential equations. Section 2. Separable equations. page 398

2.1 problem 1

Internal problem ID [4749]

Book: Mathematical Methods in the Physical Sciences. third edition. Mary L. Boas. John Wiley. 2006

Section: Chapter 8, Ordinary differential equations. Section 2. Separable equations. page 398

Problem number: 1.

ODE order: 1.

ODE degree: 1.

CAS Maple gives this as type **[_separable]**

$$-y + xy' = 0$$

With initial conditions

$$[y(2) = 3]$$

✓ Solution by Maple
Time used: 0.0 (sec). Leaf size: 7

```
dsolve([x*diff(y(x),x)=y(x),y(2) = 3],y(x), singsol=all)
```

$$y(x) = \frac{3x}{2}$$

✓ Solution by Mathematica
Time used: 0.022 (sec). Leaf size: 10

```
DSolve[{x*y'[x]==y[x],{y[2]==3}},y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to \frac{3x}{2}$$

2.2 problem 2

Internal problem ID [4750]

Book: Mathematical Methods in the Physical Sciences. third edition. Mary L. Boas. John Wiley. 2006

Section: Chapter 8, Ordinary differential equations. Section 2. Separable equations. page 398

Problem number: 2.

ODE order: 1.

ODE degree: 1.

CAS Maple gives this as type [_separable]

$$ x\sqrt{1 - y^2} + y\sqrt{-x^2 + 1}\, y' = 0 $$

With initial conditions

$$ \left[y\left(\frac{1}{2}\right) = \frac{1}{2} \right] $$

✓ Solution by Maple
Time used: 0.36 (sec). Leaf size: 26

```
dsolve([x*sqrt(1-y(x)^2)+y(x)*sqrt(1-x^2)*diff(y(x),x)=0,y(1/2) = 1/2],y(x), singsol=all)
```

$$ y(x) = \sqrt{2\sqrt{3}\,\sqrt{-x^2 + 1} + x^2 - 3} $$

✓ Solution by Mathematica
Time used: 3.578 (sec). Leaf size: 38

```
DSolve[{x*Sqrt[1-y[x]^2]+y[x]*Sqrt[1-x^2]*y'[x]==0,{y[1/2]==1/2}},y[x],x,IncludeSingularSolu
```

$$ y(x) \to \sqrt{x^2} $$

$$ y(x) \to \sqrt{x^2 + 2\sqrt{3 - 3x^2} - 3} $$

2.3 problem 3

Internal problem ID [4751]

Book: Mathematical Methods in the Physical Sciences. third edition. Mary L. Boas. John Wiley. 2006

Section: Chapter 8, Ordinary differential equations. Section 2. Separable equations. page 398

Problem number: 3.

ODE order: 1.

ODE degree: 1.

CAS Maple gives this as type [_separable]

$$y' \sin(x) - y \ln(y) = 0$$

With initial conditions

$$\left[y\left(\frac{\pi}{3}\right) = e \right]$$

✔ Solution by Maple
Time used: 0.516 (sec). Leaf size: 17

```
dsolve([diff(y(x),x)*sin(x)=y(x)*ln(y(x)),y(1/3*Pi) = exp(1)],y(x), singsol=all)
```

$$y(x) = e^{-(\cot(x)-\csc(x))\sqrt{3}}$$

✔ Solution by Mathematica
Time used: 0.226 (sec). Leaf size: 19

```
DSolve[{y'[x]*Sin[x]==y[x]*Log[y[x]],{y[Pi/3]==Exp[1]}},y[x],x,IncludeSingularSolutions -> T
```

$$y(x) \to e^{e^{\operatorname{arctanh}\left(\frac{1}{2}\right) - \operatorname{arctanh}(\cos(x))}}$$

2.4   problem 4

Internal problem ID [4752]

Book: Mathematical Methods in the Physical Sciences. third edition. Mary L. Boas. John Wiley. 2006

Section: Chapter 8, Ordinary differential equations. Section 2. Separable equations. page 398

Problem number: 4.

ODE order: 1.

ODE degree: 1.

CAS Maple gives this as type [_separable]

$$y^2 + xyy' = -1$$

With initial conditions

$$[y(5) = 0]$$

✓   Solution by Maple
Time used: 0.031 (sec). Leaf size: 34

```
dsolve([(1+y(x)^2)+x*y(x)*diff(y(x),x)=0,y(5) = 0],y(x), singsol=all)
```

$$y(x) = \frac{\sqrt{-x^2 + 25}}{x}$$
$$y(x) = -\frac{\sqrt{-x^2 + 25}}{x}$$

✔   Solution by Mathematica
Time used: 0.329 (sec). Leaf size: 40

```
DSolve[{(1+y[x]^2)+x*y[x]*y'[x]==0,{y[5]==0}},y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to -\frac{\sqrt{25 - x^2}}{x}$$
$$y(x) \to \frac{\sqrt{25 - x^2}}{x}$$

2.5 problem 5

Internal problem ID [4753]

Book: Mathematical Methods in the Physical Sciences. third edition. Mary L. Boas. John Wiley. 2006
Section: Chapter 8, Ordinary differential equations. Section 2. Separable equations. page 398
Problem number: 5.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type [_quadrature]

$$ xyy' - xy - y = 0 $$

With initial conditions

$$ [y(1) = 1] $$

✓ Solution by Maple
Time used: 0.015 (sec). Leaf size: 8

```
dsolve([x*y(x)*diff(y(x),x)-x*y(x)=y(x),y(1) = 1],y(x), singsol=all)
```

$$ y(x) = x + \ln(x) $$

✓ Solution by Mathematica
Time used: 0.003 (sec). Leaf size: 9

```
DSolve[{x*y[x]*y'[x]-x*y[x]==y[x],{y[1]==1}},y[x],x,IncludeSingularSolutions -> True]
```

$$ y(x) \rightarrow x + \log(x) $$

2.6 problem 6

Internal problem ID [4754]

Book: Mathematical Methods in the Physical Sciences. third edition. Mary L. Boas. John Wiley. 2006

Section: Chapter 8, Ordinary differential equations. Section 2. Separable equations. page 398

Problem number: 6.

ODE order: 1.

ODE degree: 1.

CAS Maple gives this as type `[_separable]`

$$ y' - \frac{2xy^2 + x}{yx^2 - y} = 0 $$

With initial conditions

$$ \left[y\left(\sqrt{2}\right) = 0 \right] $$

✔ Solution by Maple
Time used: 0.031 (sec). Leaf size: 31

```
dsolve([diff(y(x),x)=(2*x*y(x)^2+x)/(x^2*y(x)-y(x)),y(2^(1/2)) = 0],y(x), singsol=all)
```

$$ y(x) = -\frac{\sqrt{2x^2 - 4}\,x}{2} $$

$$ y(x) = \frac{\sqrt{2x^2 - 4}\,x}{2} $$

✔ Solution by Mathematica
Time used: 3.88 (sec). Leaf size: 48

```
DSolve[{y'[x]==(2*x*y[x]^2+x)/(x^2*y[x]-y[x]),{y[Sqrt[2]]==0}},y[x],x,IncludeSingularSolutio
```

$$ y(x) \to -\frac{\sqrt{x^2\left(x^2 - 2\right)}}{\sqrt{2}} $$

$$ y(x) \to \frac{\sqrt{x^2\left(x^2 - 2\right)}}{\sqrt{2}} $$

2.7 problem 7

Internal problem ID [4755]

Book: Mathematical Methods in the Physical Sciences. third edition. Mary L. Boas. John Wiley. 2006
Section: Chapter 8, Ordinary differential equations. Section 2. Separable equations. page 398
Problem number: 7.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type [_separable]

$$ y'y + xy^2 = 8x $$

With initial conditions

$$ [y(1) = 3] $$

✓ Solution by Maple
Time used: 0.063 (sec). Leaf size: 17

```
dsolve([y(x)*diff(y(x),x)+(x*y(x)^2-8*x)=0,y(1) = 3],y(x), singsol=all)
```

$$ y(x) = \sqrt{e^{-(x-1)(1+x)} + 8} $$

✓ Solution by Mathematica
Time used: 1.924 (sec). Leaf size: 39

```
DSolve[{y[x]*y'[x]+(x*y[x]^2-8*x)==0,{y[1]==3}},y[x],x,IncludeSingularSolutions -> True]
```

$$ y(x) \to \sqrt{e^{1-x^2} + 8} $$
$$ y(x) \to \sqrt{e^{1-x^2} + 8} $$

2.8   problem 8

Internal problem ID [4756]

Book: Mathematical Methods in the Physical Sciences. third edition. Mary L. Boas. John Wiley. 2006
Section: Chapter 8, Ordinary differential equations. Section 2. Separable equations. page 398
Problem number: 8.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type [_separable]

$$y' + 2xy^2 = 0$$

With initial conditions

$$[y(2) = 1]$$

✔ Solution by Maple
Time used: 0.11 (sec). Leaf size: 11

```
dsolve([diff(y(x),x)+2*x*y(x)^2=0,y(2) = 1],y(x), singsol=all)
```

$$y(x) = \frac{1}{x^2 - 3}$$

✔ Solution by Mathematica
Time used: 0.121 (sec). Leaf size: 12

```
DSolve[{y'[x]+2*x*y[x]^2==0,{y[2]==1}},y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to \frac{1}{x^2 - 3}$$

2.9 problem 9

Internal problem ID [4757]

Book: Mathematical Methods in the Physical Sciences. third edition. Mary L. Boas. John Wiley. 2006

Section: Chapter 8, Ordinary differential equations. Section 2. Separable equations. page 398

Problem number: 9.

ODE order: 1.

ODE degree: 1.

CAS Maple gives this as type [_quadrature]

$$(1+y)\,y' - y = 0$$

With initial conditions

$$[y(1) = 1]$$

✓ Solution by Maple
Time used: 0.078 (sec). Leaf size: 7

```
dsolve([(1+y(x))*diff(y(x),x)=y(x),y(1) = 1],y(x), singsol=all)
```

$$y(x) = \mathrm{LambertW}\left(e^{x}\right)$$

✓ Solution by Mathematica
Time used: 2.162 (sec). Leaf size: 9

```
DSolve[{(1+y[x])*y'[x]==y[x],{y[1]==1}},y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to W(e^{x})$$

2.10 problem 10

Internal problem ID [4758]

Book: Mathematical Methods in the Physical Sciences. third edition. Mary L. Boas. John Wiley. 2006
Section: Chapter 8, Ordinary differential equations. Section 2. Separable equations. page 398
Problem number: 10.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type [_separable]

$$ y' - xy = x $$

With initial conditions

$$ [y(0) = 1] $$

✓ Solution by Maple
Time used: 0.015 (sec). Leaf size: 14

```
dsolve([diff(y(x),x)-x*y(x)=x,y(0) = 1],y(x), singsol=all)
```

$$ y(x) = -1 + 2\,\mathrm{e}^{\frac{x^2}{2}} $$

✓ Solution by Mathematica
Time used: 0.043 (sec). Leaf size: 20

```
DSolve[{y'[x]-x*y[x]==x,{y[1]==1}},y[x],x,IncludeSingularSolutions -> True]
```

$$ y(x) \to 2e^{\frac{1}{2}(x^2-1)} - 1 $$

2.11 problem 11

Internal problem ID [4759]

Book: Mathematical Methods in the Physical Sciences. third edition. Mary L. Boas. John Wiley. 2006

Section: Chapter 8, Ordinary differential equations. Section 2. Separable equations. page 398

Problem number: 11.

ODE order: 1.

ODE degree: 1.

CAS Maple gives this as type `[_quadrature]`

$$2y' - 3(y - 2)^{\frac{1}{3}} = 0$$

With initial conditions

$$[y(1) = 3]$$

✔ Solution by Maple
Time used: 0.016 (sec). Leaf size: 9

```
dsolve([2*diff(y(x),x)=3*(y(x)-2)^(1/3),y(1) = 3],y(x), singsol=all)
```

$$y(x) = 2 + x^{\frac{3}{2}}$$

✔ Solution by Mathematica
Time used: 0.007 (sec). Leaf size: 12

```
DSolve[{2*y'[x]==3*(y[x]-2)^(1/3),{y[1]==3}},y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to x^{3/2} + 2$$

2.12 problem 12

Internal problem ID [4760]

Book: Mathematical Methods in the Physical Sciences. third edition. Mary L. Boas. John Wiley. 2006
Section: Chapter 8, Ordinary differential equations. Section 2. Separable equations. page 398
Problem number: 12.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type [_separable]

$$(xy + x) y' + y = 0$$

With initial conditions

$$[y(1) = 1]$$

✓ Solution by Maple
Time used: 0.078 (sec). Leaf size: 11

```
dsolve([(x+x*y(x))*diff(y(x),x)+y(x)=0,y(1) = 1],y(x), singsol=all)
```

$$y(x) = \text{LambertW}\left(\frac{e}{x}\right)$$

✓ Solution by Mathematica
Time used: 2.09 (sec). Leaf size: 11

```
DSolve[{(x+x*y[x])*y'[x]+y[x]==0,{y[1]==1}},y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \rightarrow W\left(\frac{e}{x}\right)$$

3 Chapter 8, Ordinary differential equations. Section 3. Linear First-Order Equations. page 403

3.1 problem 1

Internal problem ID [4761]

Book: Mathematical Methods in the Physical Sciences. third edition. Mary L. Boas. John Wiley. 2006
Section: Chapter 8, Ordinary differential equations. Section 3. Linear First-Order Equations. page 403
Problem number: 1.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[[_linear, 'class A']]`

$$y' + y = e^x$$

✓ Solution by Maple
Time used: 0.0 (sec). Leaf size: 15

```
dsolve(diff(y(x),x)+y(x)=exp(x),y(x), singsol=all)
```

$$y(x) = \frac{e^x}{2} + e^{-x}c_1$$

✓ Solution by Mathematica
Time used: 0.039 (sec). Leaf size: 21

```
DSolve[y'[x]+y[x]==Exp[x],y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \rightarrow \frac{e^x}{2} + c_1 e^{-x}$$

3.2 problem 2

Internal problem ID [4762]

Book: Mathematical Methods in the Physical Sciences. third edition. Mary L. Boas. John Wiley. 2006
Section: Chapter 8, Ordinary differential equations. Section 3. Linear First-Order Equations. page 403
Problem number: 2.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type [_linear]

$$x^2 y' + 3xy = 1$$

✔ Solution by Maple
Time used: 0.0 (sec). Leaf size: 16

```
dsolve(x^2*diff(y(x),x)+3*x*y(x)=1,y(x), singsol=all)
```

$$y(x) = \frac{x^2 + 2c_1}{2x^3}$$

✔ Solution by Mathematica
Time used: 0.025 (sec). Leaf size: 20

```
DSolve[x^2*y'[x]+3*x*y[x]==1,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to \frac{x^2 + 2c_1}{2x^3}$$

3.3 problem 3

Internal problem ID [4763]

Book: Mathematical Methods in the Physical Sciences. third edition. Mary L. Boas. John Wiley. 2006
Section: Chapter 8, Ordinary differential equations. Section 3. Linear First-Order Equations. page 403
Problem number: 3.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type [_linear]

$$ y' + 2xy = x\,e^{-x^2} $$

✔ Solution by Maple
Time used: 0.0 (sec). Leaf size: 19

```
dsolve(diff(y(x),x)+2*x*y(x)-x*exp(-x^2)=0,y(x), singsol=all)
```

$$ y(x) = \frac{\left(x^2 + 2c_1\right)e^{-x^2}}{2} $$

✔ Solution by Mathematica
Time used: 0.052 (sec). Leaf size: 24

```
DSolve[y'[x]+2*x*y[x]-x*Exp[-x^2]==0,y[x],x,IncludeSingularSolutions -> True]
```

$$ y(x) \to \frac{1}{2}e^{-x^2}\left(x^2 + 2c_1\right) $$

3.4 problem 4

Internal problem ID [4764]

Book: Mathematical Methods in the Physical Sciences. third edition. Mary L. Boas. John Wiley. 2006
Section: Chapter 8, Ordinary differential equations. Section 3. Linear First-Order Equations. page 403
Problem number: 4.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type [_linear]

$$2xy' + y = 2x^{\frac{5}{2}}$$

✔ Solution by Maple
Time used: 0.0 (sec). Leaf size: 16

```
dsolve(2*x*diff(y(x),x)+y(x)=2*x^(5/2),y(x), singsol=all)
```

$$y(x) = \frac{x^3 + 3c_1}{3\sqrt{x}}$$

✔ Solution by Mathematica
Time used: 0.034 (sec). Leaf size: 22

```
DSolve[2*x*y'[x]+y[x]==2*x^(5/2),y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to \frac{x^3 + 3c_1}{3\sqrt{x}}$$

3.5 problem 5

Internal problem ID [4765]

Book: Mathematical Methods in the Physical Sciences. third edition. Mary L. Boas. John Wiley. 2006

Section: Chapter 8, Ordinary differential equations. Section 3. Linear First-Order Equations. page 403

Problem number: 5.

ODE order: 1.

ODE degree: 1.

CAS Maple gives this as type [_linear]

$$\cos(x) y' + y = \cos(x)^2$$

✓ Solution by Maple
Time used: 0.0 (sec). Leaf size: 28

```
dsolve(diff(y(x),x)*cos(x)+y(x)=cos(x)^2,y(x), singsol=all)
```

$$y(x) = \frac{(x - \cos(x) + c_1)(\cos(x) - \sin(x) + 1)}{\sin(x) + \cos(x) + 1}$$

✓ Solution by Mathematica
Time used: 0.079 (sec). Leaf size: 25

```
DSolve[y'[x]*Cos[x]+y[x]==Cos[x]^2,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to e^{-2\operatorname{arctanh}\left(\tan\left(\frac{x}{2}\right)\right)}\left(x - \cos(x) + c_1\right)$$

3.6 problem 6

Internal problem ID [4766]

Book: Mathematical Methods in the Physical Sciences. third edition. Mary L. Boas. John Wiley. 2006

Section: Chapter 8, Ordinary differential equations. Section 3. Linear First-Order Equations. page 403

Problem number: 6.

ODE order: 1.

ODE degree: 1.

CAS Maple gives this as type [_linear]

$$ y' + \frac{y}{\sqrt{x^2+1}} = \frac{1}{x+\sqrt{x^2+1}} $$

✓ Solution by Maple
Time used: 0.016 (sec). Leaf size: 19

```
dsolve(diff(y(x),x)+y(x)/sqrt(x^2+1)=1/(x+sqrt(x^2+1)),y(x), singsol=all)
```

$$ y(x) = \frac{x + c_1}{x + \sqrt{x^2+1}} $$

✓ Solution by Mathematica
Time used: 0.121 (sec). Leaf size: 23

```
DSolve[y'[x]+y[x]/Sqrt[x^2+1]==1/(x+Sqrt[x^2+1]),y[x],x,IncludeSingularSolutions -> True]
```

$$ y(x) \to \left(\sqrt{x^2+1} - x \right) (x + c_1) $$

3.7 problem 7

Internal problem ID [4767]

Book: Mathematical Methods in the Physical Sciences. third edition. Mary L. Boas. John Wiley. 2006
Section: Chapter 8, Ordinary differential equations. Section 3. Linear First-Order Equations. page 403
Problem number: 7.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type [_linear]

$$
(1 + e^x)\, y' + 2\, e^x y = (1 + e^x)\, e^x
$$

✓ Solution by Maple
Time used: 0.0 (sec). Leaf size: 30

```
dsolve((1+exp(x))*diff(y(x),x)+2*exp(x)*y(x)=(1+exp(x))*exp(x),y(x), singsol=all)
```

$$
y(x) = \frac{e^{3x} + 3\, e^{2x} + 3\, e^x + 3c_1}{3\,(1 + e^x)^2}
$$

✓ Solution by Mathematica
Time used: 0.08 (sec). Leaf size: 25

```
DSolve[(1+Exp[x])*y'[x]+2*Exp[x]*y[x]==(1+Exp[x])*Exp[x],y[x],x,IncludeSingularSolutions ->
```

$$
y(x) \to \frac{1}{3}(e^x + 1) + \frac{c_1}{(e^x + 1)^2}
$$

3.8 problem 8

Internal problem ID [4768]

Book: Mathematical Methods in the Physical Sciences. third edition. Mary L. Boas. John Wiley. 2006

Section: Chapter 8, Ordinary differential equations. Section 3. Linear First-Order Equations. page 403

Problem number: 8.

ODE order: 1.

ODE degree: 1.

CAS Maple gives this as type [_linear]

$$\boxed{x \ln (x) \, y' + y = \ln (x)}$$

✔ Solution by Maple

Time used: 0.0 (sec). Leaf size: 15

```
dsolve((x*ln(x))*diff(y(x),x)+y(x)=ln(x),y(x), singsol=all)
```

$$y(x) = \frac{\ln (x)}{2} + \frac{c_1}{\ln (x)}$$

✔ Solution by Mathematica

Time used: 0.029 (sec). Leaf size: 19

```
DSolve[(x*Log[x])*y'[x]+y[x]==Log[x],y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to \frac{\log(x)}{2} + \frac{c_1}{\log(x)}$$

3.9 problem 9

Internal problem ID [4769]

Book: Mathematical Methods in the Physical Sciences. third edition. Mary L. Boas. John Wiley. 2006
Section: Chapter 8, Ordinary differential equations. Section 3. Linear First-Order Equations. page 403
Problem number: 9.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type [_linear]

$$\left(-x^2 + 1\right) y' - xy = 2\sqrt{-x^2 + 1}\, x$$

✔ Solution by Maple
Time used: 0.0 (sec). Leaf size: 30

```
dsolve((1-x^2)*diff(y(x),x)=x*y(x)+2*x*sqrt(1-x^2),y(x), singsol=all)
```

$$y(x) = \frac{x^2}{\sqrt{-x^2 + 1}} + \frac{c_1}{\sqrt{x - 1}\sqrt{1 + x}}$$

✔ Solution by Mathematica
Time used: 0.062 (sec). Leaf size: 33

```
DSolve[(1-x^2)*y'[x]==x*y[x]+2*x*Sqrt[1-x^2],y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to \frac{x^2}{\sqrt{1 - x^2}} + \frac{c_1}{\sqrt{x^2 - 1}}$$

3.10 problem 10

Internal problem ID [4770]

Book: Mathematical Methods in the Physical Sciences. third edition. Mary L. Boas. John Wiley. 2006
Section: Chapter 8, Ordinary differential equations. Section 3. Linear First-Order Equations. page 403
Problem number: 10.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type [_linear]

$$y' + y \tanh(x) = 2\,e^x$$

✔ Solution by Maple
Time used: 0.0 (sec). Leaf size: 15

```
dsolve(diff(y(x),x)+y(x)*tanh(x)=2*exp(x),y(x), singsol=all)
```

$$y(x) = (x + c_1)\operatorname{sech}(x) + \cosh(x) + \sinh(x)$$

✔ Solution by Mathematica
Time used: 0.077 (sec). Leaf size: 29

```
DSolve[y'[x]+y[x]*Tanh[x]==2*Exp[x],y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to \frac{e^x\left(2x + e^{2x} + c_1\right)}{e^{2x} + 1}$$

3.11 problem 11

Internal problem ID [4771]

Book: Mathematical Methods in the Physical Sciences. third edition. Mary L. Boas. John Wiley. 2006
Section: Chapter 8, Ordinary differential equations. Section 3. Linear First-Order Equations. page 403
Problem number: 11.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type [_linear]

$$y' + y\cos(x) = \sin(2x)$$

✔ Solution by Maple
Time used: 0.0 (sec). Leaf size: 17

```
dsolve(diff(y(x),x)+y(x)*cos(x)=sin(2*x),y(x), singsol=all)
```

$$y(x) = 2\sin(x) - 2 + e^{-\sin(x)}c_1$$

✔ Solution by Mathematica
Time used: 0.052 (sec). Leaf size: 20

```
DSolve[y'[x]+y[x]*Cos[x]==Sin[2*x],y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to 2\sin(x) + c_1 e^{-\sin(x)} - 2$$

3.12 problem 12

Internal problem ID [4772]

Book: Mathematical Methods in the Physical Sciences. third edition. Mary L. Boas. John Wiley. 2006
Section: Chapter 8, Ordinary differential equations. Section 3. Linear First-Order Equations. page 403
Problem number: 12.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type [_linear]

$$x' + x \tan(y) = \cos(y)$$

✔ Solution by Maple
Time used: 0.0 (sec). Leaf size: 10

```
dsolve(diff(x(y),y)=cos(y)-x(y)*tan(y),x(y), singsol=all)
```

$$x(y) = (y + c_1) \cos(y)$$

✔ Solution by Mathematica
Time used: 0.055 (sec). Leaf size: 12

```
DSolve[x'[y]==Cos[y]-x[y]*Tan[y],x[y],y,IncludeSingularSolutions -> True]
```

$$x(y) \rightarrow (y + c_1) \cos(y)$$

3.13 problem 13

Internal problem ID [4773]

Book: Mathematical Methods in the Physical Sciences. third edition. Mary L. Boas. John Wiley. 2006

Section: Chapter 8, Ordinary differential equations. Section 3. Linear First-Order Equations. page 403

Problem number: 13.

ODE order: 1.

ODE degree: 1.

CAS Maple gives this as type `[[_linear, 'class A']]`

$$x' + x = e^y$$

✓ Solution by Maple
Time used: 0.0 (sec). Leaf size: 15

```
dsolve(diff(x(y),y)+(x(y)-exp(y))=0,x(y), singsol=all)
```

$$x(y) = \frac{e^y}{2} + e^{-y} c_1$$

✓ Solution by Mathematica
Time used: 0.039 (sec). Leaf size: 21

```
DSolve[x'[y]+(x[y]-Exp[y])==0,x[y],y,IncludeSingularSolutions -> True]
```

$$x(y) \to \frac{e^y}{2} + c_1 e^{-y}$$

3.14 problem 14

Internal problem ID [4774]

Book: Mathematical Methods in the Physical Sciences. third edition. Mary L. Boas. John Wiley. 2006

Section: Chapter 8, Ordinary differential equations. Section 3. Linear First-Order Equations. page 403

Problem number: 14.

ODE order: 1.

ODE degree: 1.

CAS Maple gives this as type [_linear]

$$x' - \frac{3y^{\frac{2}{3}} - x}{3y} = 0$$

✔ Solution by Maple
Time used: 0.0 (sec). Leaf size: 11

```
dsolve(diff(x(y),y)=(3*y^(2/3)-x(y))/(3*y),x(y), singsol=all)
```

$$x(y) = \frac{y + c_1}{y^{\frac{1}{3}}}$$

✔ Solution by Mathematica
Time used: 0.033 (sec). Leaf size: 15

```
DSolve[x'[y]==(3*y^(2/3)-x[y])/(3*y),x[y],y,IncludeSingularSolutions -> True]
```

$$x(y) \to \frac{y + c_1}{\sqrt[3]{y}}$$

4 Chapter 8, Ordinary differential equations. Section 4. OTHER METHODS FOR FIRST-ORDER EQUATIONS. page 406

4.1 problem 1

Internal problem ID [4775]

Book: Mathematical Methods in the Physical Sciences. third edition. Mary L. Boas. John Wiley. 2006
Section: Chapter 8, Ordinary differential equations. Section 4. OTHER METHODS FOR FIRST-ORDER EQUATIONS. page 406
Problem number: 1.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type [_Bernoulli]

$$ y' + y - xy^{\frac{2}{3}} = 0 $$

✔ Solution by Maple
Time used: 0.0 (sec). Leaf size: 19

```
dsolve(diff(y(x),x)+y(x)=x*y(x)^(2/3),y(x), singsol=all)
```

$$ -x + 3 - e^{-\frac{x}{3}} c_1 + y(x)^{\frac{1}{3}} = 0 $$

✔ Solution by Mathematica
Time used: 0.167 (sec). Leaf size: 27

```
DSolve[y'[x]+y[x]==x*y[x]^(2/3),y[x],x,IncludeSingularSolutions -> True]
```

$$ y(x) \to e^{-x} \left(e^{x/3}(x-3) + c_1 \right)^3 $$

4.2   problem 2

Internal problem ID [4776]

Book: Mathematical Methods in the Physical Sciences. third edition. Mary L. Boas. John Wiley. 2006
Section: Chapter 8, Ordinary differential equations. Section 4. OTHER METHODS FOR FIRST-ORDER EQUATIONS. page 406
Problem number: 2.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[[_homogeneous, 'class G'], _rational, _Bernoulli]`

$$y' + \frac{y}{x} - 2x^{\frac{3}{2}}\sqrt{y} = 0$$

✔ Solution by Maple
Time used: 0.0 (sec). Leaf size: 20

```
dsolve(diff(y(x),x)+1/x*y(x)=2*x^(3/2)*y(x)^(1/2),y(x), singsol=all)
```

$$\sqrt{y(x)} - \frac{x^3 + 3c_1}{3\sqrt{x}} = 0$$

✔ Solution by Mathematica
Time used: 0.162 (sec). Leaf size: 22

```
DSolve[y'[x]+1/x*y[x]==2*x^(3/2)*y[x]^(1/2),y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \rightarrow \frac{(x^3 + 3c_1)^2}{9x}$$

4.3 problem 3

Internal problem ID [4777]

Book: Mathematical Methods in the Physical Sciences. third edition. Mary L. Boas. John Wiley. 2006
Section: Chapter 8, Ordinary differential equations. Section 4. OTHER METHODS FOR FIRST-ORDER EQUATIONS. page 406
Problem number: 3.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type [_separable]

$$3xy^2y' + 3y^3 = 1$$

✓ Solution by Maple
Time used: 0.0 (sec). Leaf size: 74

```
dsolve(3*x*y(x)^2*diff(y(x),x)+3*y(x)^3=1,y(x), singsol=all)
```

$$y(x) = \frac{(9x^3 + 27c_1)^{\frac{1}{3}}}{3x}$$

$$y(x) = -\frac{(9x^3 + 27c_1)^{\frac{1}{3}} \left(1 + i\sqrt{3}\right)}{6x}$$

$$y(x) = \frac{(9x^3 + 27c_1)^{\frac{1}{3}} \left(i\sqrt{3} - 1\right)}{6x}$$

✓ Solution by Mathematica

Time used: 0.282 (sec). Leaf size: 195

```
DSolve[3*x*y[x]^2*y'[x]+3*y[x]^3==1,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to -\frac{\sqrt[3]{-\frac{1}{3}}\sqrt[3]{x^3 + e^{9c_1}}}{x}$$

$$y(x) \to \frac{\sqrt[3]{x^3 + e^{9c_1}}}{\sqrt[3]{3}x}$$

$$y(x) \to \frac{(-1)^{2/3}\sqrt[3]{x^3 + e^{9c_1}}}{\sqrt[3]{3}x}$$

$$y(x) \to -\sqrt[3]{-\frac{1}{3}}$$

$$y(x) \to \frac{1}{\sqrt[3]{3}}$$

$$y(x) \to \frac{(-1)^{2/3}}{\sqrt[3]{3}}$$

$$y(x) \to -\frac{\sqrt[3]{-\frac{1}{3}}\sqrt[3]{x^3}}{x}$$

$$y(x) \to \frac{\sqrt[3]{x^3}}{\sqrt[3]{3}x}$$

$$y(x) \to \frac{(-1)^{2/3}\sqrt[3]{x^3}}{\sqrt[3]{3}x}$$

4.4 problem 4

Internal problem ID [4778]

Book: Mathematical Methods in the Physical Sciences. third edition. Mary L. Boas. John Wiley. 2006

Section: Chapter 8, Ordinary differential equations. Section 4. OTHER METHODS FOR FIRST-ORDER EQUATIONS. page 406

Problem number: 4.

ODE order: 1.

ODE degree: 1.

CAS Maple gives this as type [_exact]

$$2x\,\mathrm{e}^{3y} + \left(3x^2\mathrm{e}^{3y} - y^2\right)y' = -\mathrm{e}^x$$

✓ Solution by Maple
Time used: 0.016 (sec). Leaf size: 22

```
dsolve((2*x*exp(3*y(x))+exp(x))+(3*x^2*exp(3*y(x))-y(x)^2)*diff(y(x),x)=0,y(x), singsol=all)
```

$$x^2\mathrm{e}^{3y(x)} + \mathrm{e}^x - \frac{y(x)^3}{3} + c_1 = 0$$

✓ Solution by Mathematica
Time used: 0.262 (sec). Leaf size: 28

```
DSolve[(2*x*Exp[3*y[x]]+Exp[x])+(3*x^2*Exp[3*y[x]]-y[x]^2)*y'[x]==0,y[x],x,IncludeSingularSo
```

$$\mathrm{Solve}\left[x^2e^{3y(x)} - \frac{1}{3}y(x)^3 + e^x = c_1, y(x)\right]$$

4.5 problem 5

Internal problem ID [4779]

Book: Mathematical Methods in the Physical Sciences. third edition. Mary L. Boas. John Wiley. 2006
Section: Chapter 8, Ordinary differential equations. Section 4. OTHER METHODS FOR FIRST-ORDER EQUATIONS. page 406
Problem number: 5.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type [[_homogeneous, 'class C'], _exact, _rational, [_Abel, '2nd ty

$$(x - y)\, y' + y = -1 - x$$

✓ Solution by Maple
Time used: 0.11 (sec). Leaf size: 30

```
dsolve((x-y(x))*diff(y(x),x)+(y(x)+x+1)=0,y(x), singsol=all)
```

$$y(x) = \frac{2c_1 x - \sqrt{1 + 8\left(x + \frac{1}{2}\right)^2 c_1^2}}{2c_1}$$

✓ Solution by Mathematica
Time used: 0.112 (sec). Leaf size: 55

```
DSolve[(x-y[x])*y'[x]+(y[x]+x+1)==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to x - i\sqrt{-2x^2 - 2x - c_1}$$
$$y(x) \to x + i\sqrt{-2x^2 - 2x - c_1}$$

4.6 problem 6

Internal problem ID [4780]

Book: Mathematical Methods in the Physical Sciences. third edition. Mary L. Boas. John Wiley. 2006

Section: Chapter 8, Ordinary differential equations. Section 4. OTHER METHODS FOR FIRST-ORDER EQUATIONS. page 406

Problem number: 6.

ODE order: 1.

ODE degree: 1.

CAS Maple gives this as type `unknown`

$$\cos\left(x\right)\cos\left(y\right) - \left(\sin\left(x\right)\sin\left(y\right) + \cos\left(y\right)^2\right)y' = -\sin\left(x\right)^2$$

✓ Solution by Maple
Time used: 0.312 (sec). Leaf size: 35

```
dsolve((cos(x)*cos(y(x))+sin(x)^2)-(sin(x)*sin(y(x))+cos(y(x))^2)*diff(y(x),x)=0,y(x), sings
```

$$c_1 + x - y(x) - \frac{\sin\left(2x\right)}{2} + \sin\left(y(x) + x\right) + \sin\left(-y(x) + x\right) - \frac{\sin\left(2y(x)\right)}{2} = 0$$

✓ Solution by Mathematica
Time used: 0.375 (sec). Leaf size: 43

```
DSolve[(Cos[x]*Cos[y[x]]+Sin[x]^2)-(Sin[x]*Sin[y[x]]+Cos[y[x]]^2)*y'[x]==0,y[x],x,IncludeSin
```

$$\text{Solve}\left[2\left(\frac{y(x)}{2} + \frac{1}{4}\sin(2y(x))\right) - 2\sin(x)\cos(y(x)) - x + \frac{1}{2}\sin(2x) = c_1, y(x)\right]$$

4.7 problem 7

CAS Maple gives this as type `[[_homogeneous, 'class A'], _rational, _Bernoulli]`

$$ x^2 y' + y^2 - xy = 0 $$

✓ Solution by Maple
Time used: 0.016 (sec). Leaf size: 12

```
dsolve(x^2*diff(y(x),x)+(y(x)^2-x*y(x))=0,y(x), singsol=all)
```

$$ y(x) = \frac{x}{\ln(x) + c_1} $$

✓ Solution by Mathematica
Time used: 0.132 (sec). Leaf size: 19

```
DSolve[x^2*y'[x]+(y[x]^2-x*y[x])==0,y[x],x,IncludeSingularSolutions -> True]
```

$$ y(x) \to \frac{x}{\log(x) + c_1} $$
$$ y(x) \to 0 $$

4.8 problem 8

Internal problem ID [4782]

Book: Mathematical Methods in the Physical Sciences. third edition. Mary L. Boas. John Wiley. 2006

Section: Chapter 8, Ordinary differential equations. Section 4. OTHER METHODS FOR FIRST-ORDER EQUATIONS. page 406

Problem number: 8.

ODE order: 1.

ODE degree: 1.

CAS Maple gives this as type `[[_homogeneous, 'class A'], _rational, _dAlembert]`

$$ y'y - \sqrt{x^2 + y^2} = -x $$

✓ Solution by Maple
Time used: 0.047 (sec). Leaf size: 27

```
dsolve(y(x)*diff(y(x),x)=-x+sqrt(x^2+y(x)^2),y(x), singsol=all)
```

$$ \frac{-c_1 y(x)^2 + \sqrt{x^2 + y(x)^2} + x}{y(x)^2} = 0 $$

✓ Solution by Mathematica
Time used: 0.378 (sec). Leaf size: 57

```
DSolve[y[x]*y'[x]==-x+Sqrt[x^2+y[x]^2],y[x],x,IncludeSingularSolutions -> True]
```

$$ y(x) \to -e^{\frac{c_1}{2}} \sqrt{2x + e^{c_1}} $$
$$ y(x) \to e^{\frac{c_1}{2}} \sqrt{2x + e^{c_1}} $$
$$ y(x) \to 0 $$

4.9 problem 9

Internal problem ID [4783]

Book: Mathematical Methods in the Physical Sciences. third edition. Mary L. Boas. John Wiley. 2006
Section: Chapter 8, Ordinary differential equations. Section 4. OTHER METHODS FOR FIRST-ORDER EQUATIONS. page 406
Problem number: 9.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type [[_homogeneous, 'class A'], _rational, _dAlembert]

$$ xy + \left(y^2 - x^2 \right) y' = 0 $$

✓ Solution by Maple
Time used: 0.016 (sec). Leaf size: 19

```
dsolve(x*y(x)+(y(x)^2-x^2)*diff(y(x),x)=0,y(x), singsol=all)
```

$$ y(x) = \sqrt{-\frac{1}{\text{LambertW}\left(-c_1 x^2\right)}}\, x $$

✓ Solution by Mathematica
Time used: 8.102 (sec). Leaf size: 56

```
DSolve[x*y[x]+(y[x]^2-x^2)*y'[x]==0,y[x],x,IncludeSingularSolutions -> True]
```

$$ y(x) \rightarrow -\frac{ix}{\sqrt{W\left(-e^{-2c_1}x^2\right)}} $$
$$ y(x) \rightarrow \frac{ix}{\sqrt{W\left(-e^{-2c_1}x^2\right)}} $$
$$ y(x) \rightarrow 0 $$

4.10 problem 10

Internal problem ID [4784]

Book: Mathematical Methods in the Physical Sciences. third edition. Mary L. Boas. John
Wiley. 2006
Section: Chapter 8, Ordinary differential equations. Section 4. OTHER METHODS FOR
FIRST-ORDER EQUATIONS. page 406
Problem number: 10.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[[_homogeneous, 'class A'], _rational, [_Abel, '2nd type', 'cl`

$$y^2 - xy + (xy + x^2) y' = 0$$

✓ Solution by Maple
Time used: 0.015 (sec). Leaf size: 14

```
dsolve((y(x)^2-x*y(x))+(x^2+x*y(x))*diff(y(x),x)=0,y(x), singsol=all)
```

$$y(x) = \frac{x}{\text{LambertW}\left(c_1 x^2\right)}$$

✓ Solution by Mathematica
Time used: 4.24 (sec). Leaf size: 25

```
DSolve[(y[x]^2-x*y[x])+(x^2+x*y[x])*y'[x]==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to \frac{x}{W\left(e^{-c_1} x^2\right)}$$
$$y(x) \to 0$$

4.11 problem 11

Internal problem ID [4785]

Book: Mathematical Methods in the Physical Sciences. third edition. Mary L. Boas. John Wiley. 2006
Section: Chapter 8, Ordinary differential equations. Section 4. OTHER METHODS FOR FIRST-ORDER EQUATIONS. page 406
Problem number: 11.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type [[_homogeneous, 'class C'], _dAlembert]

$$y' - \cos(x + y) = 0$$

✔ Solution by Maple
Time used: 0.016 (sec). Leaf size: 16

```
dsolve(diff(y(x),x)=cos(x+y(x)),y(x), singsol=all)
```

$$y(x) = -x - 2\arctan(-x + c_1)$$

✔ Solution by Mathematica
Time used: 0.933 (sec). Leaf size: 59

```
DSolve[y'[x]==Cos[x+y[x]],y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to -x + 2\arctan\left(x + \frac{c_1}{2}\right)$$
$$y(x) \to -x + 2\arctan\left(x + \frac{c_1}{2}\right)$$
$$y(x) \to -x - \pi$$
$$y(x) \to \pi - x$$

4.12 problem 12

Internal problem ID [4786]

Book: Mathematical Methods in the Physical Sciences. third edition. Mary L. Boas. John Wiley. 2006
Section: Chapter 8, Ordinary differential equations. Section 4. OTHER METHODS FOR FIRST-ORDER EQUATIONS. page 406
Problem number: 12.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[[_homogeneous, 'class A'], _dAlembert]`

$$ y' - \frac{y}{x} + \tan\left(\frac{y}{x}\right) = 0 $$

✓ Solution by Maple
Time used: 0.0 (sec). Leaf size: 14

```
dsolve(diff(y(x),x)=y(x)/x- tan(y(x)/x),y(x), singsol=all)
```

$$ y(x) = x \arcsin\left(\frac{1}{xc_1}\right) $$

✓ Solution by Mathematica
Time used: 12.97 (sec). Leaf size: 21

```
DSolve[y'[x]==y[x]/x- Tan[y[x]/x],y[x],x,IncludeSingularSolutions -> True]
```

$$ y(x) \to x \arcsin\left(\frac{e^{c_1}}{x}\right) $$
$$ y(x) \to 0 $$

4.13 problem 13

Internal problem ID [4787]

Book: Mathematical Methods in the Physical Sciences. third edition. Mary L. Boas. John Wiley. 2006
Section: Chapter 8, Ordinary differential equations. Section 4. OTHER METHODS FOR FIRST-ORDER EQUATIONS. page 406
Problem number: 13.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type [_linear]

$$(x - 1)\, y' + y = \frac{1}{x^2} - \frac{2}{x^3}$$

✓ Solution by Maple
Time used: 0.0 (sec). Leaf size: 17

```
dsolve((x-1)*diff(y(x),x)+y(x)-1/x^2+2/x^3=0,y(x), singsol=all)
```

$$y(x) = \frac{c_1}{x - 1} - \frac{1}{x^2}$$

✓ Solution by Mathematica
Time used: 0.034 (sec). Leaf size: 24

```
DSolve[(x-1)*y'[x]+y[x]-1/x^2+2/x^3==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to -\frac{c_1 x^2 + x - 1}{(x - 1)x^2}$$

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4.14 problem 25 part (a)

Internal problem ID [4788]

Book: Mathematical Methods in the Physical Sciences. third edition. Mary L. Boas. John Wiley. 2006
Section: Chapter 8, Ordinary differential equations. Section 4. OTHER METHODS FOR FIRST-ORDER EQUATIONS. page 406
Problem number: 25 part (a).
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[[_homogeneous, 'class G'], _rational, _Riccati]`

$$
\boxed{y' - xy^2 + \frac{2y}{x} = -\frac{1}{x^3}}
$$

✓ Solution by Maple
Time used: 0.0 (sec). Leaf size: 15

```
dsolve(diff(y(x),x)= x*y(x)^2-2/x*y(x)-1/x^3,y(x), singsol=all)
```

$$
y(x) = \frac{\tanh\left(-\ln\left(x\right) + c_1\right)}{x^2}
$$

✓ Solution by Mathematica
Time used: 1.188 (sec). Leaf size: 63

```
DSolve[y'[x]== x*y[x]^2-2/x*y[x]-1/x^3,y[x],x,IncludeSingularSolutions -> True]
```

$$
y(x) \to \frac{i \tan(i \log(x) + c_1)}{x^2}
$$
$$
y(x) \to \frac{-x^2 + e^{2i\,\mathrm{Interval}[\{0,\pi\}]}}{x^4 + x^2 e^{2i\,\mathrm{Interval}[\{0,\pi\}]}}
$$

4.15 problem 25 part (b)

Internal problem ID [4789]

Book: Mathematical Methods in the Physical Sciences. third edition. Mary L. Boas. John Wiley. 2006
Section: Chapter 8, Ordinary differential equations. Section 4. OTHER METHODS FOR FIRST-ORDER EQUATIONS. page 406
Problem number: 25 part (b).
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[[_homogeneous, 'class D'], _rational, _Riccati]`

$$ y' - \frac{2y^2}{x} - \frac{y}{x} = -2x $$

✓ Solution by Maple
Time used: 0.0 (sec). Leaf size: 15

```
dsolve(diff(y(x),x)= 2/x*y(x)^2+1/x*y(x)-2*x,y(x), singsol=all)
```

$$ y(x) = -\tanh\left(2x + 2c_1\right)x $$

✓ Solution by Mathematica
Time used: 0.716 (sec). Leaf size: 47

```
DSolve[y'[x]== 2/x*y[x]^2+1/x*y[x]-2*x,y[x],x,IncludeSingularSolutions -> True]
```

$$ y(x) \to \frac{x - xe^{4x+2c_1}}{1 + e^{4x+2c_1}} $$
$$ y(x) \to -x $$
$$ y(x) \to x $$

4.16 problem 25 part (c)

Internal problem ID [4790]

Book: Mathematical Methods in the Physical Sciences. third edition. Mary L. Boas. John Wiley. 2006
Section: Chapter 8, Ordinary differential equations. Section 4. OTHER METHODS FOR FIRST-ORDER EQUATIONS. page 406
Problem number: 25 part (c).
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[[_1st_order, _with_linear_symmetries], _Riccati]`

$$ y' - e^{-x} y^2 - y = -e^x $$

✔ Solution by Maple
Time used: 0.016 (sec). Leaf size: 16

```
dsolve(diff(y(x),x)= exp(-x)*y(x)^2+y(x)-exp(x),y(x), singsol=all)
```

$$ y(x) = i \tan(ix + c_1) e^x $$

✔ Solution by Mathematica
Time used: 0.302 (sec). Leaf size: 19

```
DSolve[y'[x]== Exp[-x]*y[x]^2+y[x]-Exp[x],y[x],x,IncludeSingularSolutions -> True]
```

$$ y(x) \to -e^x \tanh(x - ic_1) $$

5 Chapter 8, Ordinary differential equations. Section 5. SECOND-ORDER LINEAR EQUATIONSWITH CONSTANT COEFFICIENTS AND ZERO RIGHT-HAND SIDE. page 414

5.1 problem 1

Internal problem ID [4791]

Book: Mathematical Methods in the Physical Sciences. third edition. Mary L. Boas. John Wiley. 2006

Section: Chapter 8, Ordinary differential equations. Section 5. SECOND-ORDER LINEAR EQUATIONSWITH CONSTANT COEFFICIENTS AND ZERO RIGHT-HAND SIDE. page 414

Problem number: 1.

ODE order: 2.

ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _missing_x]]`

$$y'' + y' - 2y = 0$$

✓ Solution by Maple
Time used: 0.0 (sec). Leaf size: 17

```
dsolve(diff(y(x),x$2)+diff(y(x),x)-2*y(x)=0,y(x), singsol=all)
```

$$y(x) = \left(c_1 e^{3x} + c_2\right) e^{-2x}$$

✓ Solution by Mathematica
Time used: 0.013 (sec). Leaf size: 20

```
DSolve[y''[x]+y'[x]-2*y[x]==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to c_1 e^{-2x} + c_2 e^{x}$$

5.2 problem 2

Internal problem ID [4792]

Book: Mathematical Methods in the Physical Sciences. third edition. Mary L. Boas. John Wiley. 2006

Section: Chapter 8, Ordinary differential equations. Section 5. SECOND-ORDER LINEAR EQUATIONSWITH CONSTANT COEFFICIENTS AND ZERO RIGHT-HAND SIDE. page 414

Problem number: 2.

ODE order: 2.

ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _missing_x]]`

$$y'' - 4y' + 4y = 0$$

✓ Solution by Maple
Time used: 0.0 (sec). Leaf size: 14

```
dsolve(diff(y(x),x$2)-4*diff(y(x),x)+4*y(x)=0,y(x), singsol=all)
```

$$y(x) = e^{2x}(c_2 x + c_1)$$

✓ Solution by Mathematica
Time used: 0.015 (sec). Leaf size: 18

```
DSolve[y''[x]-4*y'[x]+4*y[x]==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to e^{2x}(c_2 x + c_1)$$

5.3 problem 3

Internal problem ID [4793]

Book: Mathematical Methods in the Physical Sciences. third edition. Mary L. Boas. John Wiley. 2006
Section: Chapter 8, Ordinary differential equations. Section 5. SECOND-ORDER LINEAR EQUATIONSWITH CONSTANT COEFFICIENTS AND ZERO RIGHT-HAND SIDE. page 414
Problem number: 3.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _missing_x]]`

$$y'' + 9y' = 0$$

✓ Solution by Maple
Time used: 0.0 (sec). Leaf size: 12

```
dsolve(diff(y(x),x$2)+9*diff(y(x),x)=0,y(x), singsol=all)
```

$$y(x) = c_1 + c_2 \mathrm{e}^{-9x}$$

✓ Solution by Mathematica
Time used: 0.015 (sec). Leaf size: 19

```
DSolve[y''[x]+9*y'[x]==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to c_2 - \frac{1}{9}c_1 e^{-9x}$$

5.4 problem 4

Internal problem ID [4794]

Book: Mathematical Methods in the Physical Sciences. third edition. Mary L. Boas. John Wiley. 2006
Section: Chapter 8, Ordinary differential equations. Section 5. SECOND-ORDER LINEAR EQUATIONSWITH CONSTANT COEFFICIENTS AND ZERO RIGHT-HAND SIDE. page 414
Problem number: 4.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type [[_2nd_order, _missing_x]]

$$y'' + 2y' + 2y = 0$$

✓ Solution by Maple
Time used: 0.0 (sec). Leaf size: 18

```
dsolve(diff(y(x),x$2)+2*diff(y(x),x)+2*y(x)=0,y(x), singsol=all)
```

$$y(x) = \mathrm{e}^{-x}(c_1 \sin(x) + c_2 \cos(x))$$

✓ Solution by Mathematica
Time used: 0.016 (sec). Leaf size: 22

```
DSolve[y''[x]+2*y'[x]+2*y[x]==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to e^{-x}(c_2 \cos(x) + c_1 \sin(x))$$

5.5 problem 5

Internal problem ID [4795]

Book: Mathematical Methods in the Physical Sciences. third edition. Mary L. Boas. John Wiley. 2006
Section: Chapter 8, Ordinary differential equations. Section 5. SECOND-ORDER LINEAR EQUATIONSWITH CONSTANT COEFFICIENTS AND ZERO RIGHT-HAND SIDE. page 414
Problem number: 5.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type [[_2nd_order, _missing_x]]

$$y'' - 2y' + 6y = 0$$

✓ Solution by Maple
Time used: 0.0 (sec). Leaf size: 24

```
dsolve(diff(y(x),x$2)-2*diff(y(x),x)+6*y(x)=0,y(x), singsol=all)
```

$$y(x) = e^x \left(c_1 \sin \left(\sqrt{5}\, x \right) + c_2 \cos \left(\sqrt{5}\, x \right) \right)$$

✓ Solution by Mathematica
Time used: 0.021 (sec). Leaf size: 32

```
DSolve[y''[x]-2*y'[x]+6*y[x]==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to e^x \left(c_2 \cos \left(\sqrt{5}x \right) + c_1 \sin \left(\sqrt{5}x \right) \right)$$

5.6 problem 6

Internal problem ID [4796]

Book: Mathematical Methods in the Physical Sciences. third edition. Mary L. Boas. John Wiley. 2006

Section: Chapter 8, Ordinary differential equations. Section 5. SECOND-ORDER LINEAR EQUATIONSWITH CONSTANT COEFFICIENTS AND ZERO RIGHT-HAND SIDE. page 414

Problem number: 6.

ODE order: 2.

ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _missing_x]]`

$$y'' + 16y = 0$$

✓ Solution by Maple
Time used: 0.0 (sec). Leaf size: 17

```
dsolve(diff(y(x),x$2)+16*y(x)=0,y(x), singsol=all)
```

$$y(x) = c_1 \sin(4x) + c_2 \cos(4x)$$

✓ Solution by Mathematica
Time used: 0.015 (sec). Leaf size: 20

```
DSolve[y''[x]+16*y[x]==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to c_1 \cos(4x) + c_2 \sin(4x)$$

5.7 problem 7

Internal problem ID [4797]

Book: Mathematical Methods in the Physical Sciences. third edition. Mary L. Boas. John Wiley. 2006
Section: Chapter 8, Ordinary differential equations. Section 5. SECOND-ORDER LINEAR EQUATIONSWITH CONSTANT COEFFICIENTS AND ZERO RIGHT-HAND SIDE. page 414
Problem number: 7.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _missing_x]]`

$$y'' - 5y' + 6y = 0$$

✓ Solution by Maple
Time used: 0.0 (sec). Leaf size: 17

```
dsolve(diff(y(x),x$2)-5*diff(y(x),x)+6*y(x)=0,y(x), singsol=all)
```

$$y(x) = \mathrm{e}^{2x}c_1 + c_2\mathrm{e}^{3x}$$

✓ Solution by Mathematica
Time used: 0.014 (sec). Leaf size: 20

```
DSolve[y''[x]-5*y'[x]+6*y[x]==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to e^{2x}(c_2 e^x + c_1)$$

5.8 problem 8

Internal problem ID [4798]

Book: Mathematical Methods in the Physical Sciences. third edition. Mary L. Boas. John Wiley. 2006

Section: Chapter 8, Ordinary differential equations. Section 5. SECOND-ORDER LINEAR EQUATIONSWITH CONSTANT COEFFICIENTS AND ZERO RIGHT-HAND SIDE. page 414

Problem number: 8.

ODE order: 2.

ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _missing_x]]`

$$y'' + 5y' = 0$$

✓ Solution by Maple
Time used: 0.016 (sec). Leaf size: 12

```
dsolve(diff(y(x),x$2)+5*diff(y(x),x)=0,y(x), singsol=all)
```

$$y(x) = c_1 + c_2 \mathrm{e}^{-5x}$$

✓ Solution by Mathematica
Time used: 0.01 (sec). Leaf size: 19

```
DSolve[y''[x]+5*y'[x]==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to c_2 - \frac{1}{5}c_1 e^{-5x}$$

5.9 problem 9

Internal problem ID [4799]

Book: Mathematical Methods in the Physical Sciences. third edition. Mary L. Boas. John Wiley. 2006

Section: Chapter 8, Ordinary differential equations. Section 5. SECOND-ORDER LINEAR EQUATIONSWITH CONSTANT COEFFICIENTS AND ZERO RIGHT-HAND SIDE. page 414

Problem number: 9.

ODE order: 2.

ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _missing_x]]`

$$y'' - 4y' + 13y = 0$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 22

```
dsolve(diff(y(x),x$2)-4*diff(y(x),x)+13*y(x)=0,y(x), singsol=all)
```

$$y(x) = e^{2x}(c_1 \sin(3x) + c_2 \cos(3x))$$

✓ Solution by Mathematica

Time used: 0.019 (sec). Leaf size: 26

```
DSolve[y''[x]-4*y'[x]+13*y[x]==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to e^{2x}(c_2 \cos(3x) + c_1 \sin(3x))$$

5.10 problem 12

Internal problem ID [4800]

Book: Mathematical Methods in the Physical Sciences. third edition. Mary L. Boas. John Wiley. 2006
Section: Chapter 8, Ordinary differential equations. Section 5. SECOND-ORDER LINEAR EQUATIONSWITH CONSTANT COEFFICIENTS AND ZERO RIGHT-HAND SIDE. page 414
Problem number: 12.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type [[_2nd_order, _missing_x]]

$$2y'' + y' - y = 0$$

✓ Solution by Maple
Time used: 0.0 (sec). Leaf size: 17

```
dsolve(2*diff(y(x),x$2)+diff(y(x),x)-y(x)=0,y(x), singsol=all)
```

$$y(x) = \left(c_1 e^{\frac{3x}{2}} + c_2\right) e^{-x}$$

✓ Solution by Mathematica
Time used: 0.013 (sec). Leaf size: 24

```
DSolve[2*y''[x]+y'[x]-y[x]==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to e^{-x}\left(c_1 e^{3x/2} + c_2\right)$$

5.11 problem 19

Internal problem ID [4801]

Book: Mathematical Methods in the Physical Sciences. third edition. Mary L. Boas. John Wiley. 2006

Section: Chapter 8, Ordinary differential equations. Section 5. SECOND-ORDER LINEAR EQUATIONSWITH CONSTANT COEFFICIENTS AND ZERO RIGHT-HAND SIDE. page 414

Problem number: 19.

ODE order: 2.

ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _missing_x]]`

$$y'' + (1 + 2i)\, y' + (-1 + i)\, y = 0$$

✔ Solution by Maple
Time used: 0.0 (sec). Leaf size: 20

```
dsolve(diff(y(x),x$2)+(1+2*I)*diff(y(x),x)+(I-1)*y(x)=0,y(x), singsol=all)
```

$$y(x) = c_1 \mathrm{e}^{(-1-i)x} + c_2 \mathrm{e}^{-ix}$$

✔ Solution by Mathematica
Time used: 0.015 (sec). Leaf size: 22

```
DSolve[y''[x]+(1+2*I)*y'[x]+(I-1)*y[x]==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to e^{(-1-i)x}(c_2 e^x + c_1)$$

5.12 problem 20

Internal problem ID [4802]

Book: Mathematical Methods in the Physical Sciences. third edition. Mary L. Boas. John Wiley. 2006
Section: Chapter 8, Ordinary differential equations. Section 5. SECOND-ORDER LINEAR EQUATIONSWITH CONSTANT COEFFICIENTS AND ZERO RIGHT-HAND SIDE. page 414
Problem number: 20.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _missing_x]]`

$$y'' + (1 + 2i)\, y' + (-1 + i)\, y = 0$$

✓   Solution by Maple
Time used: 0.0 (sec). Leaf size: 20

```
dsolve(diff(y(x),x$2)+(1+2*I)*diff(y(x),x)+(I-1)*y(x)=0,y(x), singsol=all)
```

$$y(x) = c_1 \mathrm{e}^{(-1-i)x} + c_2 \mathrm{e}^{-ix}$$

✓   Solution by Mathematica
Time used: 0.015 (sec). Leaf size: 22

```
DSolve[y''[x]+(1+2*I)*y'[x]+(I-1)*y[x]==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to e^{(-1-i)x}(c_2 e^x + c_1)$$

5.13 problem 24

Internal problem ID [4803]

Book: Mathematical Methods in the Physical Sciences. third edition. Mary L. Boas. John Wiley. 2006
Section: Chapter 8, Ordinary differential equations. Section 5. SECOND-ORDER LINEAR EQUATIONSWITH CONSTANT COEFFICIENTS AND ZERO RIGHT-HAND SIDE. page 414
Problem number: 24.
ODE order: 3.
ODE degree: 1.

CAS Maple gives this as type [[_3rd_order, _missing_x]]

$$y''' + y = 0$$

✓ Solution by Maple
Time used: 0.0 (sec). Leaf size: 37

```
dsolve(diff(y(x),x$3)+y(x)=0,y(x), singsol=all)
```

$$y(x) = \left(c_2 e^{\frac{3x}{2}} \sin\left(\frac{\sqrt{3}\,x}{2}\right) + c_3 e^{\frac{3x}{2}} \cos\left(\frac{\sqrt{3}\,x}{2}\right) + c_1 \right) e^{-x}$$

✓ Solution by Mathematica
Time used: 0.003 (sec). Leaf size: 56

```
DSolve[y'''[x]+y[x]==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to e^{-x} \left(c_3 e^{3x/2} \cos\left(\frac{\sqrt{3}x}{2}\right) + c_2 e^{3x/2} \sin\left(\frac{\sqrt{3}x}{2}\right) + c_1 \right)$$

5.14 problem 25

Internal problem ID [4804]

Book: Mathematical Methods in the Physical Sciences. third edition. Mary L. Boas. John Wiley. 2006

Section: Chapter 8, Ordinary differential equations. Section 5. SECOND-ORDER LINEAR EQUATIONSWITH CONSTANT COEFFICIENTS AND ZERO RIGHT-HAND SIDE. page 414

Problem number: 25.

ODE order: 3.

ODE degree: 1.

CAS Maple gives this as type `[[_3rd_order, _missing_x]]`

$$y''' + y'' - 6y' = 0$$

✓ Solution by Maple
Time used: 0.0 (sec). Leaf size: 23

```
dsolve(diff(y(x),x$3)+diff(y(x),x$2)-6*diff(y(x),x)=0,y(x), singsol=all)
```

$$y(x) = \left(c_2 e^{5x} + c_1 e^{3x} + c_3\right) e^{-3x}$$

✓ Solution by Mathematica
Time used: 0.034 (sec). Leaf size: 30

```
DSolve[y'''[x]+y''[x]-6*y'[x]==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to -\frac{1}{3} c_1 e^{-3x} + \frac{1}{2} c_2 e^{2x} + c_3$$

5.15 problem 26

Internal problem ID [4805]

Book: Mathematical Methods in the Physical Sciences. third edition. Mary L. Boas. John Wiley. 2006

Section: Chapter 8, Ordinary differential equations. Section 5. SECOND-ORDER LINEAR EQUATIONSWITH CONSTANT COEFFICIENTS AND ZERO RIGHT-HAND SIDE. page 414

Problem number: 26.

ODE order: 3.

ODE degree: 1.

CAS Maple gives this as type [[_3rd_order, _missing_x]]

$$y''' + 3y'' - 9y' - 5y = 0$$

✓ Solution by Maple
Time used: 0.016 (sec). Leaf size: 105

```
dsolve(diff(y(x),x$3)+3*diff(y(x),x$2)-9*diff(y(x),x)-5*y(x)=0,y(x), singsol=all)
```

$$
y(x) = c_1 e^{\left(-1-2\sin\left(\frac{\arctan\left(\frac{\sqrt{55}}{3}\right)}{3}+\frac{\pi}{6}\right)+2\sqrt{3}\cos\left(\frac{\arctan\left(\frac{\sqrt{55}}{3}\right)}{3}+\frac{\pi}{6}\right)\right)x}
$$
$$
+ c_2 e^{-2\left(\sqrt{3}\cos\left(\frac{\arctan\left(\frac{\sqrt{55}}{3}\right)}{3}+\frac{\pi}{6}\right)+\sin\left(\frac{\arctan\left(\frac{\sqrt{55}}{3}\right)}{3}+\frac{\pi}{6}\right)+\frac{1}{2}\right)x}
$$
$$
+ c_3 e^{\left(4\sin\left(\frac{\arctan\left(\frac{\sqrt{55}}{3}\right)}{3}+\frac{\pi}{6}\right)-1\right)x}
$$

✓ Solution by Mathematica
Time used: 0.003 (sec). Leaf size: 87

```
DSolve[y'''[x]+3*y''[x]-9*y'[x]-5*y[x]==0,y[x],x,IncludeSingularSolutions -> True]
```

$$
y(x) \to c_2 \exp\left(x\,\text{Root}\left[\#1^3 + 3\#1^2 - 9\#1 - 5\&, 2\right]\right)
$$
$$
+ c_3 \exp\left(x\,\text{Root}\left[\#1^3 + 3\#1^2 - 9\#1 - 5\&, 3\right]\right)
$$
$$
+ c_1 \exp\left(x\,\text{Root}\left[\#1^3 + 3\#1^2 - 9\#1 - 5\&, 1\right]\right)
$$

5.16 problem 28

Internal problem ID [4806]

Book: Mathematical Methods in the Physical Sciences. third edition. Mary L. Boas. John Wiley. 2006

Section: Chapter 8, Ordinary differential equations. Section 5. SECOND-ORDER LINEAR EQUATIONSWITH CONSTANT COEFFICIENTS AND ZERO RIGHT-HAND SIDE. page 414

Problem number: 28.

ODE order: 4.

ODE degree: 1.

CAS Maple gives this as type `[[_high_order, _missing_x]]`

$$\boxed{y'''' + 4y = 0}$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 33

```
dsolve(diff(y(x),x$4)+4*y(x)=0,y(x), singsol=all)
```

$$y(x) = c_1 \sin(x) \, \mathrm{e}^{-x} + c_2 \cos(x) \, \mathrm{e}^{-x} + c_3 \mathrm{e}^{x} \sin(x) + c_4 \mathrm{e}^{x} \cos(x)$$

✓ Solution by Mathematica

Time used: 0.004 (sec). Leaf size: 40

```
DSolve[y''''[x]+4*y[x]==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to e^{-x} \left(\left(c_4 e^{2x} + c_1 \right) \cos(x) + \left(c_3 e^{2x} + c_2 \right) \sin(x) \right)$$

6 Chapter 8, Ordinary differential equations. Section 6. SECOND-ORDER LINEAR EQUATIONSWITH CONSTANT COEFFICIENTS AND RIGHT-HAND SIDE NOT ZERO. page 422

6.1 problem 1

Internal problem ID [4807]

Book: Mathematical Methods in the Physical Sciences. third edition. Mary L. Boas. John Wiley. 2006

Section: Chapter 8, Ordinary differential equations. Section 6. SECOND-ORDER LINEAR EQUATIONSWITH CONSTANT COEFFICIENTS AND RIGHT-HAND SIDE NOT ZERO. page 422

Problem number: 1.

ODE order: 2.

ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _missing_x]]`

$$y'' - 4y' = 10$$

✔ Solution by Maple
Time used: 0.0 (sec). Leaf size: 16

```
dsolve(diff(y(x),x$2)-4*diff(y(x),x)=10,y(x), singsol=all)
```

$$y(x) = \frac{e^{4x}c_1}{4} - \frac{5x}{2} + c_2$$

✔ Solution by Mathematica
Time used: 0.015 (sec). Leaf size: 24

```
DSolve[y''[x]-4*y'[x]==10,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to -\frac{5x}{2} + \frac{1}{4}c_1 e^{4x} + c_2$$

6.2 problem 2

Internal problem ID [4808]

Book: Mathematical Methods in the Physical Sciences. third edition. Mary L. Boas. John Wiley. 2006

Section: Chapter 8, Ordinary differential equations. Section 6. SECOND-ORDER LINEAR EQUATIONSWITH CONSTANT COEFFICIENTS AND RIGHT-HAND SIDE NOT ZERO. page 422

Problem number: 2.

ODE order: 2.

ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _missing_x]]`

$$\boxed{y'' - 4y' + 4y = 16}$$

✔ Solution by Maple

Time used: 0.0 (sec). Leaf size: 16

```
dsolve(diff(y(x),x$2)-4*diff(y(x),x)+4*y(x)=16,y(x), singsol=all)
```

$$y(x) = 4 + (c_1 x + c_2) \, \mathrm{e}^{2x}$$

✔ Solution by Mathematica

Time used: 0.014 (sec). Leaf size: 20

```
DSolve[y''[x]-4*y'[x]+4*y[x]==16,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to 4 + e^{2x}(c_2 x + c_1)$$

6.3   problem 3

Internal problem ID [4809]

Book: Mathematical Methods in the Physical Sciences. third edition. Mary L. Boas. John Wiley. 2006
Section: Chapter 8, Ordinary differential equations. Section 6. SECOND-ORDER LINEAR EQUATIONSWITH CONSTANT COEFFICIENTS AND RIGHT-HAND SIDE NOT ZERO. page 422
Problem number: 3.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type [[_2nd_order, _with_linear_symmetries]]

$$y'' + y' - 2y = e^{2x}$$

✓ Solution by Maple
Time used: 0.015 (sec). Leaf size: 25

```
dsolve(diff(y(x),x$2)+diff(y(x),x)-2*y(x)=exp(2*x),y(x), singsol=all)
```

$$y(x) = \frac{\left(e^{4x} + 4c_2 e^{3x} + 4c_1\right) e^{-2x}}{4}$$

✓ Solution by Mathematica
Time used: 0.023 (sec). Leaf size: 29

```
DSolve[y''[x]+y'[x]-2*y[x]==Exp[2*x],y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to \frac{e^{2x}}{4} + c_1 e^{-2x} + c_2 e^{x}$$

6.4 problem 4

Internal problem ID [4810]

Book: Mathematical Methods in the Physical Sciences. third edition. Mary L. Boas. John Wiley. 2006

Section: Chapter 8, Ordinary differential equations. Section 6. SECOND-ORDER LINEAR EQUATIONSWITH CONSTANT COEFFICIENTS AND RIGHT-HAND SIDE NOT ZERO. page 422

Problem number: 4.

ODE order: 2.

ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$y'' - 2y' - 3y = 24\,e^{-3x}$$

✓ Solution by Maple
Time used: 0.0 (sec). Leaf size: 23

```
dsolve(diff(y(x),x$2)-2*diff(y(x),x)-3*y(x)=24*exp(-3*x),y(x), singsol=all)
```

$$y(x) = \left(e^{6x}c_1 + c_2 e^{2x} + 2\right) e^{-3x}$$

✓ Solution by Mathematica
Time used: 0.02 (sec). Leaf size: 29

```
DSolve[y''[x]-2*y'[x]-3*y[x]==24*Exp[-3*x],y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to e^{-3x}\left(c_1 e^{2x} + c_2 e^{6x} + 2\right)$$

6.5 problem 5

Internal problem ID [4811]

Book: Mathematical Methods in the Physical Sciences. third edition. Mary L. Boas. John Wiley. 2006
Section: Chapter 8, Ordinary differential equations. Section 6. SECOND-ORDER LINEAR EQUATIONSWITH CONSTANT COEFFICIENTS AND RIGHT-HAND SIDE NOT ZERO. page 422
Problem number: 5.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type [[_2nd_order, _with_linear_symmetries]]

$$y'' + y = 2\,\mathrm{e}^x$$

✓ Solution by Maple
Time used: 0.0 (sec). Leaf size: 15

```
dsolve(diff(y(x),x$2)+y(x)=2*exp(x),y(x), singsol=all)
```

$$y(x) = c_2 \sin(x) + \cos(x) c_1 + \mathrm{e}^x$$

✓ Solution by Mathematica
Time used: 0.015 (sec). Leaf size: 19

```
DSolve[y''[x]+y[x]==2*Exp[x],y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to e^x + c_1 \cos(x) + c_2 \sin(x)$$

6.6 problem 6

Internal problem ID [4812]

Book: Mathematical Methods in the Physical Sciences. third edition. Mary L. Boas. John Wiley. 2006

Section: Chapter 8, Ordinary differential equations. Section 6. SECOND-ORDER LINEAR EQUATIONSWITH CONSTANT COEFFICIENTS AND RIGHT-HAND SIDE NOT ZERO. page 422

Problem number: 6.

ODE order: 2.

ODE degree: 1.

CAS Maple gives this as type [[_2nd_order, _with_linear_symmetries]]

$$y'' + 6y' + 9y = 12\,e^{-x}$$

✓ Solution by Maple
Time used: 0.016 (sec). Leaf size: 21

```
dsolve(diff(y(x),x$2)+6*diff(y(x),x)+9*y(x)=12*exp(-x),y(x), singsol=all)
```

$$y(x) = (c_1 x + c_2)\,e^{-3x} + 3\,e^{-x}$$

✓ Solution by Mathematica
Time used: 0.019 (sec). Leaf size: 25

```
DSolve[y''[x]+6*y'[x]+9*y[x]==12*Exp[-x],y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to e^{-3x}\left(3e^{2x} + c_2 x + c_1\right)$$

6.7 problem 7

Internal problem ID [4813]

Book: Mathematical Methods in the Physical Sciences. third edition. Mary L. Boas. John Wiley. 2006

Section: Chapter 8, Ordinary differential equations. Section 6. SECOND-ORDER LINEAR EQUATIONSWITH CONSTANT COEFFICIENTS AND RIGHT-HAND SIDE NOT ZERO. page 422

Problem number: 7.

ODE order: 2.

ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$y'' - y' - 2y = 3\,e^{2x}$$

✔ Solution by Maple
Time used: 0.015 (sec). Leaf size: 19

```
dsolve(diff(y(x),x$2)-diff(y(x),x)-2*y(x)=3*exp(2*x),y(x), singsol=all)
```

$$y(x) = (c_2 + x)\,e^{2x} + e^{-x} c_1$$

✔ Solution by Mathematica
Time used: 0.023 (sec). Leaf size: 27

```
DSolve[y''[x]-y'[x]-2*y[x]==3*Exp[2*x],y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to c_1 e^{-x} + e^{2x}\left(x - \frac{1}{3} + c_2\right)$$

6.8 problem 8

Internal problem ID [4814]

Book: Mathematical Methods in the Physical Sciences. third edition. Mary L. Boas. John Wiley. 2006

Section: Chapter 8, Ordinary differential equations. Section 6. SECOND-ORDER LINEAR EQUATIONSWITH CONSTANT COEFFICIENTS AND RIGHT-HAND SIDE NOT ZERO.
page 422

Problem number: 8.

ODE order: 2.

ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$ y'' - 16y = 40\,\mathrm{e}^{4x} $$

✓ Solution by Maple
Time used: 0.0 (sec). Leaf size: 21

```
dsolve(diff(y(x),x$2)-16*y(x)=40*exp(4*x),y(x), singsol=all)
```

$$ y(x) = (5x + c_2)\,\mathrm{e}^{4x} + \mathrm{e}^{-4x}c_1 $$

✓ Solution by Mathematica
Time used: 0.029 (sec). Leaf size: 29

```
DSolve[y''[x]-16*y[x]==40*Exp[4*x],y[x],x,IncludeSingularSolutions -> True]
```

$$ y(x) \to e^{4x}\left(5x - \frac{5}{8} + c_1\right) + c_2 e^{-4x} $$

6.9 problem 9

Internal problem ID [4815]

Book: Mathematical Methods in the Physical Sciences. third edition. Mary L. Boas. John Wiley. 2006

Section: Chapter 8, Ordinary differential equations. Section 6. SECOND-ORDER LINEAR EQUATIONSWITH CONSTANT COEFFICIENTS AND RIGHT-HAND SIDE NOT ZERO. page 422

Problem number: 9.

ODE order: 2.

ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$y'' + 2y' + y = 2\,e^{-x}$$

✓ Solution by Maple
Time used: 0.015 (sec). Leaf size: 17

```
dsolve(diff(y(x),x$2)+2*diff(y(x),x)+y(x)=2*exp(-x),y(x), singsol=all)
```

$$y(x) = e^{-x}\left(c_1 x + x^2 + c_2\right)$$

✓ Solution by Mathematica
Time used: 0.024 (sec). Leaf size: 21

```
DSolve[y''[x]+2*y'[x]+y[x]==2*Exp[-x],y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to e^{-x}\left(x^2 + c_2 x + c_1\right)$$

6.10 problem 10

Internal problem ID [4816]

Book: Mathematical Methods in the Physical Sciences. third edition. Mary L. Boas. John Wiley. 2006

Section: Chapter 8, Ordinary differential equations. Section 6. SECOND-ORDER LINEAR EQUATIONSWITH CONSTANT COEFFICIENTS AND RIGHT-HAND SIDE NOT ZERO. page 422

Problem number: 10.

ODE order: 2.

ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$y'' - 6y' + 9y = 6\,\mathrm{e}^{3x}$$

✓ Solution by Maple
Time used: 0.015 (sec). Leaf size: 19

```
dsolve(diff(y(x),x$2)-6*diff(y(x),x)+9*y(x)=6*exp(3*x),y(x), singsol=all)
```

$$y(x) = \mathrm{e}^{3x}\left(c_1 x + 3x^2 + c_2\right)$$

✓ Solution by Mathematica
Time used: 0.024 (sec). Leaf size: 23

```
DSolve[y''[x]-6*y'[x]+9*y[x]==6*Exp[3*x],y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to e^{3x}\left(3x^2 + c_2 x + c_1\right)$$

6.11 problem 11

Internal problem ID [4817]

Book: Mathematical Methods in the Physical Sciences. third edition. Mary L. Boas. John Wiley. 2006
Section: Chapter 8, Ordinary differential equations. Section 6. SECOND-ORDER LINEAR EQUATIONSWITH CONSTANT COEFFICIENTS AND RIGHT-HAND SIDE NOT ZERO. page 422
Problem number: 11.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type [[_2nd_order, _linear, _nonhomogeneous]]

$$y'' + 2y' + 10y = 100\cos{(4x)}$$

✓ Solution by Maple
Time used: 0.0 (sec). Leaf size: 37

```
dsolve(diff(y(x),x$2)+2*diff(y(x),x)+10*y(x)=100*cos(4*x),y(x), singsol=all)
```

$$y(x) = \mathrm{e}^{-x}\sin{(3x)}\,c_2 + \mathrm{e}^{-x}\cos{(3x)}\,c_1 + 8\sin{(4x)} - 6\cos{(4x)}$$

✓ Solution by Mathematica
Time used: 0.025 (sec). Leaf size: 42

```
DSolve[y''[x]+2*y'[x]+10*y[x]==100*Cos[4*x],y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to 8\sin(4x) - 6\cos(4x) + c_2 e^{-x}\cos(3x) + c_1 e^{-x}\sin(3x)$$

6.12 problem 12

Internal problem ID [4818]

Book: Mathematical Methods in the Physical Sciences. third edition. Mary L. Boas. John Wiley. 2006

Section: Chapter 8, Ordinary differential equations. Section 6. SECOND-ORDER LINEAR EQUATIONSWITH CONSTANT COEFFICIENTS AND RIGHT-HAND SIDE NOT ZERO. page 422

Problem number: 12.

ODE order: 2.

ODE degree: 1.

CAS Maple gives this as type [[_2nd_order, _linear, _nonhomogeneous]]

$$ y'' + 4y' + 12y = 80\sin(2x) $$

✓ Solution by Maple
Time used: 0.0 (sec). Leaf size: 43

```
dsolve(diff(y(x),x$2)+4*diff(y(x),x)+12*y(x)=80*sin(2*x),y(x), singsol=all)
```

$$ y(x) = e^{-2x}\sin\left(2x\sqrt{2}\right)c_2 + e^{-2x}\cos\left(2x\sqrt{2}\right)c_1 + 5\sin(2x) - 5\cos(2x) $$

✓ Solution by Mathematica
Time used: 0.035 (sec). Leaf size: 52

```
DSolve[y''[x]+4*y'[x]+12*y[x]==80*Sin[2*x],y[x],x,IncludeSingularSolutions -> True]
```

$$ y(x) \to 5\sin(2x) - 5\cos(2x) + c_2 e^{-2x}\cos\left(2\sqrt{2}x\right) + c_1 e^{-2x}\sin\left(2\sqrt{2}x\right) $$

6.13 problem 13

Internal problem ID [4819]

Book: Mathematical Methods in the Physical Sciences. third edition. Mary L. Boas. John Wiley. 2006

Section: Chapter 8, Ordinary differential equations. Section 6. SECOND-ORDER LINEAR EQUATIONSWITH CONSTANT COEFFICIENTS AND RIGHT-HAND SIDE NOT ZERO. page 422

Problem number: 13.

ODE order: 2.

ODE degree: 1.

CAS Maple gives this as type [[_2nd_order, _linear, _nonhomogeneous]]

$$y'' - 2y' + y = 2\cos(x)$$

✔ Solution by Maple
Time used: 0.0 (sec). Leaf size: 17

```
dsolve(diff(y(x),x$2)-2*diff(y(x),x)+y(x)=2*cos(x),y(x), singsol=all)
```

$$y(x) = (c_1 x + c_2)\, \mathrm{e}^x - \sin(x)$$

✔ Solution by Mathematica
Time used: 0.018 (sec). Leaf size: 21

```
DSolve[y''[x]-2*y'[x]+y[x]==2*Cos[x],y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to -\sin(x) + e^x(c_2 x + c_1)$$

6.14 problem 14

Internal problem ID [4820]

Book: Mathematical Methods in the Physical Sciences. third edition. Mary L. Boas. John Wiley. 2006

Section: Chapter 8, Ordinary differential equations. Section 6. SECOND-ORDER LINEAR EQUATIONSWITH CONSTANT COEFFICIENTS AND RIGHT-HAND SIDE NOT ZERO. page 422

Problem number: 14.

ODE order: 2.

ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _linear, _nonhomogeneous]]`

$$y'' + 8y' + 25y = 120 \sin(5x)$$

✓ Solution by Maple
Time used: 0.0 (sec). Leaf size: 31

```
dsolve(diff(y(x),x$2)+8*diff(y(x),x)+25*y(x)=120*sin(5*x),y(x), singsol=all)
```

$$y(x) = \mathrm{e}^{-4x} \sin(3x) c_2 + \mathrm{e}^{-4x} \cos(3x) c_1 - 3 \cos(5x)$$

✓ Solution by Mathematica
Time used: 0.025 (sec). Leaf size: 36

```
DSolve[y''[x]+8*y'[x]+25*y[x]==120*Sin[5*x],y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to -3 \cos(5x) + c_2 e^{-4x} \cos(3x) + c_1 e^{-4x} \sin(3x)$$

6.15   problem 15

Internal problem ID [4821]

Book: Mathematical Methods in the Physical Sciences. third edition. Mary L. Boas. John Wiley. 2006

Section: Chapter 8, Ordinary differential equations. Section 6. SECOND-ORDER LINEAR EQUATIONSWITH CONSTANT COEFFICIENTS AND RIGHT-HAND SIDE NOT ZERO.

page 422

Problem number: 15.

ODE order: 2.

ODE degree: 1.

CAS Maple gives this as type [[_2nd_order, _linear, _nonhomogeneous]]

$$5y'' + 12y' + 20y = 120 \sin(2x)$$

✓   Solution by Maple

Time used: 0.0 (sec). Leaf size: 31

```
dsolve(5*diff(y(x),x$2)+12*diff(y(x),x)+20*y(x)=120*sin(2*x),y(x), singsol=all)
```

$$y(x) = e^{-\frac{6x}{5}} \sin\left(\frac{8x}{5}\right) c_2 + e^{-\frac{6x}{5}} \cos\left(\frac{8x}{5}\right) c_1 - 5\cos(2x)$$

✓   Solution by Mathematica

Time used: 0.027 (sec). Leaf size: 44

```
DSolve[5*y''[x]+12*y'[x]+20*y[x]==120*Sin[2*x],y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to -5\cos(2x) + c_2 e^{-6x/5} \cos\left(\frac{8x}{5}\right) + c_1 e^{-6x/5} \sin\left(\frac{8x}{5}\right)$$

6.16   problem 16

Internal problem ID [4822]

Book: Mathematical Methods in the Physical Sciences. third edition. Mary L. Boas. John Wiley. 2006

Section: Chapter 8, Ordinary differential equations. Section 6. SECOND-ORDER LINEAR EQUATIONSWITH CONSTANT COEFFICIENTS AND RIGHT-HAND SIDE NOT ZERO. page 422

Problem number: 16.

ODE order: 2.

ODE degree: 1.

CAS Maple gives this as type [[_2nd_order, _linear, _nonhomogeneous]]

$$y'' + 9y = 30\sin(3x)$$

✓ Solution by Maple
Time used: 0.0 (sec). Leaf size: 21

```
dsolve(diff(y(x),x$2)+9*y(x)=30*sin(3*x),y(x), singsol=all)
```

$$y(x) = (-5x + c_1)\cos(3x) + c_2\sin(3x)$$

✓ Solution by Mathematica
Time used: 0.033 (sec). Leaf size: 31

```
DSolve[y''[x]+9*y[x]==30*Sin[3*x],y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to (-5x + c_1)\cos(3x) + \frac{1}{6}(5 + 6c_2)\sin(3x)$$

6.17 problem 17

Internal problem ID [4823]

Book: Mathematical Methods in the Physical Sciences. third edition. Mary L. Boas. John Wiley. 2006
Section: Chapter 8, Ordinary differential equations. Section 6. SECOND-ORDER LINEAR EQUATIONSWITH CONSTANT COEFFICIENTS AND RIGHT-HAND SIDE NOT ZERO. page 422
Problem number: 17.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type [[_2nd_order, _linear, _nonhomogeneous]]

$$y'' + 16y = 16\cos(4x)$$

✓ Solution by Maple
Time used: 0.015 (sec). Leaf size: 29

```
dsolve(diff(y(x),x$2)+16*y(x)=16*cos(4*x),y(x), singsol=all)
```

$$y(x) = \frac{(4x + 2c_2)\sin(4x)}{2} + \frac{(2c_1 + 1)\cos(4x)}{2}$$

✓ Solution by Mathematica
Time used: 0.105 (sec). Leaf size: 28

```
DSolve[y''[x]+16*y[x]==16*Cos[4*x],y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to \left(\frac{1}{4} + c_1\right)\cos(4x) + (2x + c_2)\sin(4x)$$

6.18 problem 18

Internal problem ID [4824]

Book: Mathematical Methods in the Physical Sciences. third edition. Mary L. Boas. John Wiley. 2006

Section: Chapter 8, Ordinary differential equations. Section 6. SECOND-ORDER LINEAR EQUATIONSWITH CONSTANT COEFFICIENTS AND RIGHT-HAND SIDE NOT ZERO. page 422

Problem number: 18.

ODE order: 2.

ODE degree: 1.

CAS Maple gives this as type [[_2nd_order, _linear, _nonhomogeneous]]

$$y'' + 2y' + 17y = 60\,\mathrm{e}^{-4x}\sin(5x)$$

✓ Solution by Maple
Time used: 0.0 (sec). Leaf size: 35

```
dsolve(diff(y(x),x$2)+2*diff(y(x),x)+17*y(x)=60*exp(-4*x)*sin(5*x),y(x), singsol=all)
```

$$y(x) = \mathrm{e}^{-x}\sin(4x)\,c_2 + \mathrm{e}^{-x}\cos(4x)\,c_1 + 2\,\mathrm{e}^{-4x}\cos(5x)$$

✓ Solution by Mathematica
Time used: 0.033 (sec). Leaf size: 42

```
DSolve[y''[x]+2*y'[x]+17*y[x]==60*Exp[-4*x]*Sin[5*x],y[x],x,IncludeSingularSolutions -> True
```

$$y(x) \to e^{-4x}\left(2\cos(5x) + c_2 e^{3x}\cos(4x) + c_1 e^{3x}\sin(4x)\right)$$

6.19 problem 19

Internal problem ID [4825]

Book: Mathematical Methods in the Physical Sciences. third edition. Mary L. Boas. John Wiley. 2006

Section: Chapter 8, Ordinary differential equations. Section 6. SECOND-ORDER LINEAR EQUATIONSWITH CONSTANT COEFFICIENTS AND RIGHT-HAND SIDE NOT ZERO. page 422

Problem number: 19.

ODE order: 2.

ODE degree: 1.

CAS Maple gives this as type [[_2nd_order, _linear, _nonhomogeneous]]

$$4y'' + 4y' + 5y = 40\,e^{-\frac{3x}{2}}\sin(2x)$$

✓ Solution by Maple
Time used: 0.0 (sec). Leaf size: 47

```
dsolve(4*diff(y(x),x$2)+4*diff(y(x),x)+5*y(x)=40*exp(-3*x/2)*sin(2*x),y(x), singsol=all)
```

$$y(x) = 4\cos(x)^2 e^{-\frac{3x}{2}} - 2\,e^{-\frac{3x}{2}}\cos(x)\sin(x) + e^{-\frac{x}{2}}\cos(x)c_1 + e^{-\frac{x}{2}}\sin(x)c_2 - 2\,e^{-\frac{3x}{2}}$$

✓ Solution by Mathematica
Time used: 0.034 (sec). Leaf size: 42

```
DSolve[4*y''[x]+4*y'[x]+5*y[x]==40*Exp[-3*x/2]*Sin[2*x],y[x],x,IncludeSingularSolutions -> T
```

$$y(x) \to e^{-3x/2}\left(2\cos(2x) + c_1 e^x \sin(x) + \cos(x)\left(-2\sin(x) + c_2 e^x\right)\right)$$

6.20 problem 20

Internal problem ID [4826]

Book: Mathematical Methods in the Physical Sciences. third edition. Mary L. Boas. John Wiley. 2006

Section: Chapter 8, Ordinary differential equations. Section 6. SECOND-ORDER LINEAR EQUATIONSWITH CONSTANT COEFFICIENTS AND RIGHT-HAND SIDE NOT ZERO.

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Problem number: 20.

ODE order: 2.

ODE degree: 1.

CAS Maple gives this as type [[_2nd_order, _linear, _nonhomogeneous]]

$$y'' + 4y' + 8y = 30\,\mathrm{e}^{-\frac{x}{2}}\cos\left(\frac{5x}{2}\right)$$

✓ Solution by Maple
Time used: 0.016 (sec). Leaf size: 35

```
dsolve(diff(y(x),x$2)+4*diff(y(x),x)+8*y(x)=30*exp(-x/2)*cos(5/2*x),y(x), singsol=all)
```

$$y(x) = \mathrm{e}^{-2x}\sin\left(2x\right)c_2 + \mathrm{e}^{-2x}\cos\left(2x\right)c_1 + 4\,\mathrm{e}^{-\frac{x}{2}}\sin\left(\frac{5x}{2}\right)$$

✓ Solution by Mathematica
Time used: 0.035 (sec). Leaf size: 41

```
DSolve[y''[x]+4*y'[x]+8*y[x]==30*Exp[-x/2]*Cos[5/2*x],y[x],x,IncludeSingularSolutions -> Tru
```

$$y(x) \to e^{-2x}\left(4e^{3x/2}\sin\left(\frac{5x}{2}\right) + c_2\cos(2x) + c_1\sin(2x)\right)$$

88

6.21 problem 21

Internal problem ID [4827]

Book: Mathematical Methods in the Physical Sciences. third edition. Mary L. Boas. John Wiley. 2006

Section: Chapter 8, Ordinary differential equations. Section 6. SECOND-ORDER LINEAR EQUATIONSWITH CONSTANT COEFFICIENTS AND RIGHT-HAND SIDE NOT ZERO. page 422

Problem number: 21.

ODE order: 2.

ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$5y'' + 6y' + 2y = x^2 + 6x$$

✓ Solution by Maple
Time used: 0.0 (sec). Leaf size: 31

```
dsolve(5*diff(y(x),x$2)+6*diff(y(x),x)+2*y(x)=x^2+6*x,y(x), singsol=all)
```

$$y(x) = e^{-\frac{3x}{5}} \sin\left(\frac{x}{5}\right) c_2 + e^{-\frac{3x}{5}} \cos\left(\frac{x}{5}\right) c_1 + \frac{x^2}{2} - \frac{5}{2}$$

✓ Solution by Mathematica
Time used: 0.025 (sec). Leaf size: 47

```
DSolve[5*y''[x]+6*y'[x]+2*y[x]==x^2+6*x,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to \frac{1}{2}\left(x^2 - 5\right) + c_2 e^{-3x/5} \cos\left(\frac{x}{5}\right) + c_1 e^{-3x/5} \sin\left(\frac{x}{5}\right)$$

6.22 problem 22

Internal problem ID [4828]

Book: Mathematical Methods in the Physical Sciences. third edition. Mary L. Boas. John Wiley. 2006

Section: Chapter 8, Ordinary differential equations. Section 6. SECOND-ORDER LINEAR EQUATIONSWITH CONSTANT COEFFICIENTS AND RIGHT-HAND SIDE NOT ZERO. page 422

Problem number: 22.

ODE order: 2.

ODE degree: 1.

CAS Maple gives this as type [[_2nd_order, _missing_y]]

$$2y'' + y' = 2x$$

✓   Solution by Maple
Time used: 0.0 (sec). Leaf size: 19

```
dsolve(2*diff(y(x),x$2)+diff(y(x),x)=2*x,y(x), singsol=all)
```

$$y(x) = -2\,\mathrm{e}^{-\frac{x}{2}}c_1 + x^2 - 4x + c_2$$

✓   Solution by Mathematica
Time used: 0.039 (sec). Leaf size: 23

```
DSolve[y''[x]+y'[x]==2*x,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to x^2 - 2x - c_1 e^{-x} + c_2$$

6.23   problem 23

Internal problem ID [4829]

Book: Mathematical Methods in the Physical Sciences. third edition. Mary L. Boas. John Wiley. 2006
Section: Chapter 8, Ordinary differential equations. Section 6. SECOND-ORDER LINEAR EQUATIONSWITH CONSTANT COEFFICIENTS AND RIGHT-HAND SIDE NOT ZERO. page 422
Problem number: 23.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type [[_2nd_order, _linear, _nonhomogeneous]]

$$y'' + y = 2\,\mathrm{e}^x x$$

✓   Solution by Maple
Time used: 0.0 (sec). Leaf size: 19

```
dsolve(diff(y(x),x$2)+y(x)=2*x*exp(x),y(x), singsol=all)
```

$$y(x) = c_2 \sin(x) + \cos(x)\, c_1 + (x-1)\,\mathrm{e}^x$$

✓   Solution by Mathematica
Time used: 0.015 (sec). Leaf size: 23

```
DSolve[y''[x]+y[x]==2*x*Exp[x],y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to \mathrm{e}^x (x-1) + c_1 \cos(x) + c_2 \sin(x)$$

6.24 problem 24

Internal problem ID [4830]

Book: Mathematical Methods in the Physical Sciences. third edition. Mary L. Boas. John Wiley. 2006

Section: Chapter 8, Ordinary differential equations. Section 6. SECOND-ORDER LINEAR EQUATIONSWITH CONSTANT COEFFICIENTS AND RIGHT-HAND SIDE NOT ZERO.

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Problem number: 24.

ODE order: 2.

ODE degree: 1.

CAS Maple gives this as type [[_2nd_order, _linear, _nonhomogeneous]]

$$y'' - 6y' + 9y = 12\,\mathrm{e}^{3x}x$$

✓ Solution by Maple
Time used: 0.0 (sec). Leaf size: 19

```
dsolve(diff(y(x),x$2)-6*diff(y(x),x)+9*y(x)=12*x*exp(3*x),y(x), singsol=all)
```

$$y(x) = \mathrm{e}^{3x}\left(2x^3 + c_1 x + c_2\right)$$

✓ Solution by Mathematica
Time used: 0.022 (sec). Leaf size: 23

```
DSolve[y''[x]-6y'[x]+9*y[x]==12*x*Exp[3*x],y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to e^{3x}\left(2x^3 + c_2 x + c_1\right)$$

6.25 problem 25

Internal problem ID [4831]

Book: Mathematical Methods in the Physical Sciences. third edition. Mary L. Boas. John Wiley. 2006

Section: Chapter 8, Ordinary differential equations. Section 6. SECOND-ORDER LINEAR EQUATIONSWITH CONSTANT COEFFICIENTS AND RIGHT-HAND SIDE NOT ZERO.
page 422

Problem number: 25.

ODE order: 2.

ODE degree: 1.

CAS Maple gives this as type [[_2nd_order, _linear, _nonhomogeneous]]

$$y'' - 2y' - 3y = 16x^2 \mathrm{e}^{-x}$$

✓ Solution by Maple
Time used: 0.0 (sec). Leaf size: 34

```
dsolve(diff(y(x),x$2)-2*diff(y(x),x)-3*y(x)=16*x^2*exp(-x),y(x), singsol=all)
```

$$y(x) = \frac{\left(-8x^3 - 6x^2 + 6c_2 - 3x\right)\mathrm{e}^{-x}}{6} + c_1 \mathrm{e}^{3x}$$

✓ Solution by Mathematica
Time used: 0.046 (sec). Leaf size: 37

```
DSolve[y''[x]-2*y'[x]-3*y[x]==16*x*Exp[-x],y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to \frac{1}{4}e^{-x}\left(-8x^2 - 4x + 4c_2 e^{4x} - 1 + 4c_1\right)$$

6.26 problem 26

Internal problem ID [4832]

Book: Mathematical Methods in the Physical Sciences. third edition. Mary L. Boas. John Wiley. 2006

Section: Chapter 8, Ordinary differential equations. Section 6. SECOND-ORDER LINEAR EQUATIONSWITH CONSTANT COEFFICIENTS AND RIGHT-HAND SIDE NOT ZERO.

page 422

Problem number: 26.

ODE order: 2.

ODE degree: 1.

CAS Maple gives this as type [[_2nd_order, _linear, _nonhomogeneous]]

$$y'' + y = 8\sin(x)\,x$$

✓ Solution by Maple

Time used: 0.016 (sec). Leaf size: 24

```
dsolve(diff(y(x),x$2)+y(x)=8*x*sin(x),y(x), singsol=all)
```

$$y(x) = \left(-2x^2 + c_1\right)\cos(x) + 2\sin(x)\left(x + \frac{c_2}{2}\right)$$

✓ Solution by Mathematica

Time used: 0.031 (sec). Leaf size: 27

```
DSolve[y''[x]+y[x]==8*x*Sin[x],y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to \left(-2x^2 + 1 + c_1\right)\cos(x) + (2x + c_2)\sin(x)$$

6.27 problem 33

Internal problem ID [4833]

Book: Mathematical Methods in the Physical Sciences. third edition. Mary L. Boas. John Wiley. 2006

Section: Chapter 8, Ordinary differential equations. Section 6. SECOND-ORDER LINEAR EQUATIONSWITH CONSTANT COEFFICIENTS AND RIGHT-HAND SIDE NOT ZERO.

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Problem number: 33.

ODE order: 2.

ODE degree: 1.

CAS Maple gives this as type [[_2nd_order, _linear, _nonhomogeneous]]

$$y'' + y = x^3 - 1 + 2\cos(x) + (-4x + 2)\,\mathrm{e}^x$$

✓ Solution by Maple
Time used: 0.0 (sec). Leaf size: 32

```
dsolve(diff(y(x),x$2)+y(x)=x^3-1+2*cos(x)+(2-4*x)*exp(x),y(x), singsol=all)
```

$$y(x) = (1 + c_1)\cos(x) + (-2x + 3)\,\mathrm{e}^x + \sin(x)(c_2 + x) + x^3 - 6x - 1$$

✓ Solution by Mathematica
Time used: 0.572 (sec). Leaf size: 40

```
DSolve[y''[x]+y[x]==x^3-1+2*Cos[x]+(2-4*x)*Exp[x],y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to x^3 - 2e^x x - 6x + 3e^x + \left(\frac{1}{2} + c_1\right)\cos(x) + (x + c_2)\sin(x) - 1$$

6.28 problem 34

Internal problem ID [4834]

Book: Mathematical Methods in the Physical Sciences. third edition. Mary L. Boas. John Wiley. 2006
Section: Chapter 8, Ordinary differential equations. Section 6. SECOND-ORDER LINEAR EQUATIONSWITH CONSTANT COEFFICIENTS AND RIGHT-HAND SIDE NOT ZERO. page 422
Problem number: 34.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type [[_2nd_order, _with_linear_symmetries]]

$$y'' - 5y' + 6y = 2\,\mathrm{e}^x + 6x - 5$$

✓ Solution by Maple
Time used: 0.0 (sec). Leaf size: 20

```
dsolve(diff(y(x),x$2)-5*diff(y(x),x)+6*y(x)=2*exp(x)+6*x-5,y(x), singsol=all)
```

$$y(x) = c_2 \mathrm{e}^{2x} + c_1 \mathrm{e}^{3x} + x + \mathrm{e}^x$$

✓ Solution by Mathematica
Time used: 0.191 (sec). Leaf size: 26

```
DSolve[y''[x]-5*y'[x]+6*y[x]==2*Exp[x]+6*x-5,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to x + e^x + c_1 e^{2x} + c_2 e^{3x}$$

6.29 problem 35

Internal problem ID [4835]

Book: Mathematical Methods in the Physical Sciences. third edition. Mary L. Boas. John Wiley. 2006

Section: Chapter 8, Ordinary differential equations. Section 6. SECOND-ORDER LINEAR EQUATIONSWITH CONSTANT COEFFICIENTS AND RIGHT-HAND SIDE NOT ZERO.

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Problem number: 35.

ODE order: 2.

ODE degree: 1.

CAS Maple gives this as type [[_2nd_order, _linear, _nonhomogeneous]]

$$y'' - y = \sinh(x)$$

✓ Solution by Maple
Time used: 0.0 (sec). Leaf size: 28

```
dsolve(diff(y(x),x$2)-y(x)=sinh(x),y(x), singsol=all)
```

$$y(x) = \frac{(2x + 8c_1)\, \mathrm{e}^{-x}}{8} + \frac{\left(x + 4c_2 - \frac{1}{2}\right) \mathrm{e}^{x}}{4}$$

✓ Solution by Mathematica
Time used: 0.052 (sec). Leaf size: 38

```
DSolve[y''[x]-y[x]==Sinh[x],y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to \frac{1}{8}e^{-x}\left(2x + e^{2x}(2x - 1 + 8c_1) + 1 + 8c_2\right)$$

6.30 problem 36

CAS Maple gives this as type [[_2nd_order, _linear, _nonhomogeneous]]

$$y'' + y = 2\sin(x) + 4\cos(x)\,x$$

✓ Solution by Maple
Time used: 0.0 (sec). Leaf size: 18

```
dsolve(diff(y(x),x$2)+y(x)=2*sin(x)+4*x*cos(x),y(x), singsol=all)
```

$$y(x) = \left(x^2 + c_2 - 1\right)\sin(x) + \cos(x)\,c_1$$

✓ Solution by Mathematica
Time used: 0.111 (sec). Leaf size: 28

```
DSolve[y''[x]+y[x]==2*Sin[x]+4*x*Cos[x],y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to \frac{1}{2}\left(2x^2 - 1 + 2c_2\right)\sin(x) + c_1\cos(x)$$

6.31 problem 37

Internal problem ID [4837]

Book: Mathematical Methods in the Physical Sciences. third edition. Mary L. Boas. John Wiley. 2006

Section: Chapter 8, Ordinary differential equations. Section 6. SECOND-ORDER LINEAR EQUATIONSWITH CONSTANT COEFFICIENTS AND RIGHT-HAND SIDE NOT ZERO. page 422

Problem number: 37.

ODE order: 2.

ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _linear, _nonhomogeneous]]`

$$y'' + 2y' + y = 4\,e^x + (1 - x)\left(e^{2x} - 1\right)$$

✓ Solution by Maple
Time used: 0.016 (sec). Leaf size: 30

```
dsolve(diff(y(x),x$2)+2*diff(y(x),x)+y(x)=4*exp(x)+(1-x)*(exp(2*x)-1),y(x), singsol=all)
```

$$y(x) = -3 + (c_1 x + c_2)\,e^{-x} + \frac{(-3x + 5)\,e^{2x}}{27} + x + e^x$$

✓ Solution by Mathematica
Time used: 0.539 (sec). Leaf size: 38

```
DSolve[y''[x]+2*y'[x]+y[x]==4*Exp[x]+(1-x)*(Exp[2*x]-1),y[x],x,IncludeSingularSolutions -> T
```

$$y(x) \to \frac{1}{27}e^{2x}(5 - 3x) + e^x + x + e^{-x}(c_2 x + c_1) - 3$$

6.32 problem 38

Internal problem ID [4838]

Book: Mathematical Methods in the Physical Sciences. third edition. Mary L. Boas. John Wiley. 2006
Section: Chapter 8, Ordinary differential equations. Section 6. SECOND-ORDER LINEAR EQUATIONSWITH CONSTANT COEFFICIENTS AND RIGHT-HAND SIDE NOT ZERO. page 422
Problem number: 38.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type [[_2nd_order, _missing_y]]

$$y'' - 2y' = 9x\,\mathrm{e}^{-x} - 6x^2 + 4\,\mathrm{e}^{2x}$$

✔ Solution by Maple
Time used: 0.0 (sec). Leaf size: 39

```
dsolve(diff(y(x),x$2)-2*diff(y(x),x)=9*x*exp(-x)-6*x^2+4*exp(2*x),y(x), singsol=all)
```

$$y(x) = \frac{(4x + c_1 - 2)\,\mathrm{e}^{2x}}{2} + (3x + 4)\,\mathrm{e}^{-x} + x^3 + \frac{3x^2}{2} + \frac{3x}{2} + c_2$$

✔ Solution by Mathematica
Time used: 0.492 (sec). Leaf size: 49

```
DSolve[y''[x]-2*y'[x]==9*x*Exp[-x]-6*x^2+4*Exp[2*x],y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to \frac{1}{2}\left(x\left(2x^2 + 3x + 3\right) + e^{-x}(6x + 8) + e^{2x}(4x - 2 + c_1)\right) + c_2$$

7 Chapter 8, Ordinary differential equations. Section 7. Other second-Order equations. page 435

7.1 problem 1 (a)

Internal problem ID [4839]

Book: Mathematical Methods in the Physical Sciences. third edition. Mary L. Boas. John Wiley. 2006
Section: Chapter 8, Ordinary differential equations. Section 7. Other second-Order equations. page 435
Problem number: 1 (a).
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _missing_x], [_2nd_order, _exact, _nonlinear],`

$$ y'' + y'y = 0 $$

With initial conditions

$$ [y(0) = 5, y'(0) = 0] $$

✔ Solution by Maple
Time used: 0.032 (sec). Leaf size: 5

```
dsolve([diff(y(x),x$2)+y(x)*diff(y(x),x)=0,y(0) = 5, D(y)(0) = 0],y(x), singsol=all)
```

$$ y(x) = 5 $$

✗ Solution by Mathematica
Time used: 0.0 (sec). Leaf size: 0

```
DSolve[{y''[x]+y[x]*y'[x]==0,{y[0]==5,y'[0]==0}},y[x],x,IncludeSingularSolutions -> True]
```

{}

7.2 problem 1 (b)

Internal problem ID [4840]

Book: Mathematical Methods in the Physical Sciences. third edition. Mary L. Boas. John Wiley. 2006
Section: Chapter 8, Ordinary differential equations. Section 7. Other second-Order equations. page 435
Problem number: 1 (b).
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _missing_x], [_2nd_order, _exact, _nonlinear], _`

$$ y'' + y'y = 0 $$

With initial conditions

$$ [y(0) = 2, y'(0) = -2] $$

✓ Solution by Maple
Time used: 0.016 (sec). Leaf size: 11

```
dsolve([diff(y(x),x$2)+y(x)*diff(y(x),x)=0,y(0) = 2, D(y)(0) = -2],y(x), singsol=all)
```

$$ y(x) = \frac{2}{1+x} $$

✗ Solution by Mathematica
Time used: 0.0 (sec). Leaf size: 0

```
DSolve[{y''[x]+y[x]*y'[x]==0,{y[0]==2,y'[0]==-2}},y[x],x,IncludeSingularSolutions -> True]
```

{}

7.3 problem 1 (c)

Internal problem ID [4841]

Book: Mathematical Methods in the Physical Sciences. third edition. Mary L. Boas. John Wiley. 2006
Section: Chapter 8, Ordinary differential equations. Section 7. Other second-Order equations. page 435
Problem number: 1 (c).
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _missing_x], [_2nd_order, _exact, _nonlinear],`

$$ y'' + y'y = 0 $$

With initial conditions

$$ [y(0) = 1, y'(0) = -1] $$

✓ Solution by Maple
Time used: 0.062 (sec). Leaf size: 12

```
dsolve([diff(y(x),x$2)+y(x)*diff(y(x),x)=0,y(0) = 1, D(y)(0) = -1],y(x), singsol=all)
```

$$ y(x) = \cot\left(\frac{x}{2} + \frac{\pi}{4}\right) $$

✗ Solution by Mathematica
Time used: 0.0 (sec). Leaf size: 0

```
DSolve[{y''[x]+y[x]*y'[x]==0,{y[0]==1,y'[0]==-1}},y[x],x,IncludeSingularSolutions -> True]
```

{}

7.4 problem 1 (d)

Internal problem ID [4842]

Book: Mathematical Methods in the Physical Sciences. third edition. Mary L. Boas. John Wiley. 2006
Section: Chapter 8, Ordinary differential equations. Section 7. Other second-Order equations. page 435
Problem number: 1 (d).
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type [[_2nd_order, _missing_x], [_2nd_order, _exact, _nonlinear],

$$y'' + y'y = 0$$

With initial conditions

$$[y(0) = 0, y'(0) = 2]$$

✓ Solution by Maple
Time used: 0.031 (sec). Leaf size: 8

```
dsolve([diff(y(x),x$2)+y(x)*diff(y(x),x)=0,y(0) = 0, D(y)(0) = 2],y(x), singsol=all)
```

$$y(x) = 2 \tanh(x)$$

✓ Solution by Mathematica
Time used: 10.835 (sec). Leaf size: 9

```
DSolve[{y''[x]+y[x]*y'[x]==0,{y[0]==0,y'[0]==2}},y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to 2 \tanh(x)$$

7.5 problem 2

Internal problem ID [4843]

Book: Mathematical Methods in the Physical Sciences. third edition. Mary L. Boas. John Wiley. 2006
Section: Chapter 8, Ordinary differential equations. Section 7. Other second-Order equations. page 435
Problem number: 2.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _missing_y]]`

$$ y'' + 2xy' = 0 $$

✔ Solution by Maple
Time used: 0.016 (sec). Leaf size: 10

```
dsolve(diff(y(x),x$2)+2*x*diff(y(x),x)=0,y(x), singsol=all)
```

$$ y(x) = c_1 + \operatorname{erf}(x) c_2 $$

✔ Solution by Mathematica
Time used: 0.012 (sec). Leaf size: 21

```
DSolve[y''[x]+2*x*y'[x]==0,y[x],x,IncludeSingularSolutions -> True]
```

$$ y(x) \to \frac{1}{2}\sqrt{\pi} c_1 \operatorname{erf}(x) + c_2 $$

7.6 problem 3

Internal problem ID [4844]

Book: Mathematical Methods in the Physical Sciences. third edition. Mary L. Boas. John Wiley. 2006
Section: Chapter 8, Ordinary differential equations. Section 7. Other second-Order equations. page 435
Problem number: 3.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _missing_x], _Liouville, [_2nd_order, _reducible`

$$\boxed{2yy'' - y'^2 = 0}$$

✓ Solution by Maple
Time used: 0.015 (sec). Leaf size: 17

```
dsolve(2*y(x)*diff(y(x),x$2)=(diff(y(x),x))^2,y(x), singsol=all)
```

$$y(x) = 0$$
$$y(x) = \frac{(c_1 x + c_2)^2}{4}$$

✓ Solution by Mathematica
Time used: 0.018 (sec). Leaf size: 29

```
DSolve[2*y[x]*y''[x]==(y'[x])^2,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to \frac{(c_1 x + 2c_2)^2}{4c_2}$$
$$y(x) \to \text{Indeterminate}$$

7.7 problem 4

Internal problem ID [4845]

Book: Mathematical Methods in the Physical Sciences. third edition. Mary L. Boas. John Wiley. 2006
Section: Chapter 8, Ordinary differential equations. Section 7. Other second-Order equations. page 435
Problem number: 4.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _missing_y], [_2nd_order, _reducible, _mu_y_y1]]`

$$xy'' - y' - y'^3 = 0$$

✓ Solution by Maple
Time used: 0.016 (sec). Leaf size: 31

```
dsolve(x*diff(y(x),x$2)=diff(y(x),x)+(diff(y(x),x))^3,y(x), singsol=all)
```

$$y(x) = -\sqrt{-x^2 + c_1} + c_2$$
$$y(x) = \sqrt{-x^2 + c_1} + c_2$$

✓ Solution by Mathematica
Time used: 1.486 (sec). Leaf size: 103

```
DSolve[x*y''[x]==y'[x]+(y'[x])^3,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to c_2 - ie^{-c_1}\sqrt{-1 + e^{2c_1}x^2}$$
$$y(x) \to ie^{-c_1}\sqrt{-1 + e^{2c_1}x^2} + c_2$$
$$y(x) \to c_2 - i\sqrt{x^2}$$
$$y(x) \to i\sqrt{x^2} + c_2$$

7.8 problem 5

Internal problem ID [4846]

Book: Mathematical Methods in the Physical Sciences. third edition. Mary L. Boas. John Wiley. 2006

Section: Chapter 8, Ordinary differential equations. Section 7. Other second-Order equations. page 435

Problem number: 5.

ODE order: 2.

ODE degree: 2.

CAS Maple gives this as type `[[_2nd_order, _missing_x]]`

$$ y''^2 - k^2 \left(1 + y'^2 \right) = 0 $$

✓ Solution by Maple
Time used: 0.5 (sec). Leaf size: 55

```
dsolve((diff(y(x),x$2))^2=k^2*(1+ (diff(y(x),x))^2),y(x), singsol=all)
```

$$ y(x) = -ix + c_1 $$
$$ y(x) = ix + c_1 $$
$$ y(x) = \frac{4c_2^2 e^{kx} k^2 + 4c_1 c_2 k^2 + e^{-kx}}{4c_2 k^2} $$

✓ Solution by Mathematica
Time used: 0.451 (sec). Leaf size: 71

```
DSolve[(y''[x])^2==k^2*(1+ (y'[x])^2),y[x],x,IncludeSingularSolutions -> True]
```

$$ y(x) \to -\frac{e^{kx-c_1} + e^{-kx+c_1} - 2c_2 k}{2k} $$
$$ y(x) \to \frac{e^{kx+c_1} \left(1 + e^{-2(kx+c_1)} \right)}{2k} + c_2 $$

7.9 problem 6

Internal problem ID [4847]

Book: Mathematical Methods in the Physical Sciences. third edition. Mary L. Boas. John Wiley. 2006

Section: Chapter 8, Ordinary differential equations. Section 7. Other second-Order equations. page 435

Problem number: 6.

ODE order: 2.

ODE degree: 2.

CAS Maple gives this as type `[[_2nd_order, _missing_x], [_2nd_order, _exact, _nonlinear]]`

$$ -\frac{y''}{(1+y')^{\frac{3}{2}}} = -k $$

✔ Solution by Maple
Time used: 0.031 (sec). Leaf size: 19

```
dsolve(k=diff(y(x),x$2)*(1+ (diff(y(x),x)))^(-3/2),y(x), singsol=all)
```

$$ y(x) = -x - \frac{4}{k^2\,(x+c_1)} + c_2 $$

✔ Solution by Mathematica
Time used: 0.515 (sec). Leaf size: 75

```
DSolve[k==y''[x]*(1+ (y'[x])^2)^(-3/2),y[x],x,IncludeSingularSolutions -> True]
```

$$ y(x) \to c_2 - \frac{i\sqrt{k^2 x^2 + 2c_1 k x - 1 + c_1{}^2}}{k} $$
$$ y(x) \to \frac{i\sqrt{k^2 x^2 + 2c_1 k x - 1 + c_1{}^2}}{k} + c_2 $$

110

7.10 problem 16 (a)

Internal problem ID [4848]

Book: Mathematical Methods in the Physical Sciences. third edition. Mary L. Boas. John Wiley. 2006
Section: Chapter 8, Ordinary differential equations. Section 7. Other second-Order equations. page 435
Problem number: 16 (a).
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_Emden, _Fowler]]`

$$ x^2 y'' + 3xy' - 3y = 0 $$

✓ Solution by Maple
Time used: 0.0 (sec). Leaf size: 15

```
dsolve(x^2*diff(y(x),x$2)+3*x*diff(y(x),x)-3*y(x)=0,y(x), singsol=all)
```

$$ y(x) = \frac{c_1 x^4 + c_2}{x^3} $$

✓ Solution by Mathematica
Time used: 0.011 (sec). Leaf size: 16

```
DSolve[x^2*y''[x]+3*x*y'[x]-3*y[x]==0,y[x],x,IncludeSingularSolutions -> True]
```

$$ y(x) \to \frac{c_1}{x^3} + c_2 x $$

7.11 problem 16 (b)

Internal problem ID [4849]

Book: Mathematical Methods in the Physical Sciences. third edition. Mary L. Boas. John Wiley. 2006
Section: Chapter 8, Ordinary differential equations. Section 7. Other second-Order equations. page 435
Problem number: 16 (b).
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_Emden, _Fowler], [_2nd_order, _linear, '_with_symmetry_[0,F`

$$x^2 y'' + xy' - 4y = 0$$

✔ Solution by Maple
Time used: 0.0 (sec). Leaf size: 15

```
dsolve(x^2*diff(y(x),x$2)+x*diff(y(x),x)-4*y(x)=0,y(x), singsol=all)
```

$$y(x) = \frac{c_2 x^4 + c_1}{x^2}$$

✔ Solution by Mathematica
Time used: 0.011 (sec). Leaf size: 18

```
DSolve[x^2*y''[x]+x*y'[x]-4*y[x]==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to \frac{c_2 x^4 + c_1}{x^2}$$

7.12 problem 16 (c)

Internal problem ID [4850]

Book: Mathematical Methods in the Physical Sciences. third edition. Mary L. Boas. John Wiley. 2006
Section: Chapter 8, Ordinary differential equations. Section 7. Other second-Order equations. page 435
Problem number: 16 (c).
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_Emden, _Fowler]]`

$$x^2 y'' + 7xy' + 9y = 0$$

✔ Solution by Maple
Time used: 0.0 (sec). Leaf size: 14

```
dsolve(x^2*diff(y(x),x$2)+7*x*diff(y(x),x)+9*y(x)=0,y(x), singsol=all)
```

$$y(x) = \frac{c_2 \ln(x) + c_1}{x^3}$$

✔ Solution by Mathematica
Time used: 0.018 (sec). Leaf size: 18

```
DSolve[x^2*y''[x]+7*x*y'[x]+9*y[x]==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to \frac{3c_2 \log(x) + c_1}{x^3}$$

7.13 problem 16 (d)

Internal problem ID [4851]

Book: Mathematical Methods in the Physical Sciences. third edition. Mary L. Boas. John Wiley. 2006
Section: Chapter 8, Ordinary differential equations. Section 7. Other second-Order equations. page 435
Problem number: 16 (d).
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_Emden, _Fowler]]`

$$x^2 y'' - xy' + 6y = 0$$

✔ Solution by Maple
Time used: 0.0 (sec). Leaf size: 25

```
dsolve(x^2*diff(y(x),x$2)-x*diff(y(x),x)+6*y(x)=0,y(x), singsol=all)
```

$$y(x) = x\left(c_1 \sin\left(\sqrt{5}\ln(x)\right) + c_2 \cos\left(\sqrt{5}\ln(x)\right)\right)$$

✔ Solution by Mathematica
Time used: 0.031 (sec). Leaf size: 32

```
DSolve[x^2*y''[x]-x*y'[x]+6*y[x]==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to x\left(c_2 \cos\left(\sqrt{5}\log(x)\right) + c_1 \sin\left(\sqrt{5}\log(x)\right)\right)$$

7.14 problem 17

Internal problem ID [4852]

Book: Mathematical Methods in the Physical Sciences. third edition. Mary L. Boas. John Wiley. 2006
Section: Chapter 8, Ordinary differential equations. Section 7. Other second-Order equations. page 435
Problem number: 17.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$x^2 y'' + xy' - 16y = 8x^4$$

✔ Solution by Maple
Time used: 0.0 (sec). Leaf size: 29

```
dsolve(x^2*diff(y(x),x$2)+x*diff(y(x),x)-16*y(x)=8*x^4,y(x), singsol=all)
```

$$y(x) = \frac{8x^8 \ln(x) + (8c_2 - 1) x^8 + 8c_1}{8x^4}$$

✔ Solution by Mathematica
Time used: 0.016 (sec). Leaf size: 28

```
DSolve[x^2*y''[x]+x*y'[x]-16*y[x]==8*x^4,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to x^4 \log(x) + \left(-\frac{1}{8} + c_2\right) x^4 + \frac{c_1}{x^4}$$

7.15 problem 18

Internal problem ID [4853]

Book: Mathematical Methods in the Physical Sciences. third edition. Mary L. Boas. John Wiley. 2006
Section: Chapter 8, Ordinary differential equations. Section 7. Other second-Order equations. page 435
Problem number: 18.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type [[_2nd_order, _exact, _linear, _nonhomogeneous]]

$$
x^2 y'' + xy' - y = x - \frac{1}{x}
$$

✓ Solution by Maple
Time used: 0.0 (sec). Leaf size: 31

```
dsolve(x^2*diff(y(x),x$2)+x*diff(y(x),x)-y(x)=x-1/x,y(x), singsol=all)
```

$$
y(x) = \frac{2 \ln(x) x^2 + 4c_2 x^2 + 2 \ln(x) + 4c_1 + 1}{4x}
$$

✓ Solution by Mathematica
Time used: 0.023 (sec). Leaf size: 37

```
DSolve[x^2*y''[x]+x*y'[x]-y[x]==x-1/x,y[x],x,IncludeSingularSolutions -> True]
```

$$
y(x) \to \frac{2(x^2 + 1)\log(x) + (-1 + 4c_2)x^2 + 1 + 4c_1}{4x}
$$

7.16 problem 19

Internal problem ID [4854]

Book: Mathematical Methods in the Physical Sciences. third edition. Mary L. Boas. John Wiley. 2006
Section: Chapter 8, Ordinary differential equations. Section 7. Other second-Order equations. page 435
Problem number: 19.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type [[_2nd_order, _with_linear_symmetries]]

$$x^2 y'' - 5xy' + 9y = 2x^3$$

✔ Solution by Maple
Time used: 0.0 (sec). Leaf size: 18

```
dsolve(x^2*diff(y(x),x$2)-5*x*diff(y(x),x)+9*y(x)=2*x^3,y(x), singsol=all)
```

$$y(x) = x^3 \left(c_2 + c_1 \ln(x) + \ln(x)^2 \right)$$

✔ Solution by Mathematica
Time used: 0.02 (sec). Leaf size: 22

```
DSolve[x^2*y''[x]-5*x*y'[x]+9*y[x]==2*x^3,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to x^3 \left(\log^2(x) + 3c_2 \log(x) + c_1 \right)$$

7.17 problem 20

Internal problem ID [4855]

Book: Mathematical Methods in the Physical Sciences. third edition. Mary L. Boas. John Wiley. 2006
Section: Chapter 8, Ordinary differential equations. Section 7. Other second-Order equations. page 435
Problem number: 20.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type [[_2nd_order, _linear, _nonhomogeneous]]

$$x^2 y'' - 3xy' + 4y = 6\ln(x)\, x^2$$

✓ Solution by Maple
Time used: 0.0 (sec). Leaf size: 18

```
dsolve(x^2*diff(y(x),x$2)-3*x*diff(y(x),x)+4*y(x)=6*x^2*ln(x),y(x), singsol=all)
```

$$y(x) = x^2 \left(c_2 + c_1 \ln(x) + \ln(x)^3 \right)$$

✓ Solution by Mathematica
Time used: 0.021 (sec). Leaf size: 22

```
DSolve[x^2*y''[x]-3*x*y'[x]+4*y[x]==6*x^2*Log[x],y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to x^2 \left(\log^3(x) + 2c_2 \log(x) + c_1 \right)$$

7.18 problem 21

Internal problem ID [4856]

Book: Mathematical Methods in the Physical Sciences. third edition. Mary L. Boas. John Wiley. 2006
Section: Chapter 8, Ordinary differential equations. Section 7. Other second-Order equations. page 435
Problem number: 21.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type [[_2nd_order, _with_linear_symmetries]]

$$x^2 y'' + y = 3x^2$$

✓ Solution by Maple
Time used: 0.0 (sec). Leaf size: 34

```
dsolve(x^2*diff(y(x),x$2)+y(x)=3*x^2,y(x), singsol=all)
```

$$y(x) = \sqrt{x} \sin\left(\frac{\sqrt{3} \ln(x)}{2}\right) c_2 + \sqrt{x} \cos\left(\frac{\sqrt{3} \ln(x)}{2}\right) c_1 + x^2$$

✓ Solution by Mathematica
Time used: 0.144 (sec). Leaf size: 47

```
DSolve[x^2*y''[x]+y[x]==3*x^2,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to \sqrt{x}\left(x^{3/2} + c_1 \cos\left(\frac{1}{2}\sqrt{3}\log(x)\right) + c_2 \sin\left(\frac{1}{2}\sqrt{3}\log(x)\right)\right)$$

7.19 problem 22

Internal problem ID [4857]

Book: Mathematical Methods in the Physical Sciences. third edition. Mary L. Boas. John Wiley. 2006
Section: Chapter 8, Ordinary differential equations. Section 7. Other second-Order equations. page 435
Problem number: 22.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$x^2 y'' + xy' + y = 2x$$

✔ Solution by Maple
Time used: 0.0 (sec). Leaf size: 16

```
dsolve(x^2*diff(y(x),x$2)+x*diff(y(x),x)+y(x)=2*x,y(x), singsol=all)
```

$$y(x) = \sin\left(\ln\left(x\right)\right) c_2 + \cos\left(\ln\left(x\right)\right) c_1 + x$$

✔ Solution by Mathematica
Time used: 0.027 (sec). Leaf size: 19

```
DSolve[x^2*y''[x]+x*y'[x]+y[x]==2*x,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to x + c_1 \cos(\log(x)) + c_2 \sin(\log(x))$$

7.20 problem 25

Internal problem ID [4858]

Book: Mathematical Methods in the Physical Sciences. third edition. Mary L. Boas. John Wiley. 2006

Section: Chapter 8, Ordinary differential equations. Section 7. Other second-Order equations. page 435

Problem number: 25.

ODE order: 2.

ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$x^2(2-x)\,y'' + 2xy' - 2y = 0$$

Given that one solution of the ode is

$$y_1 = x$$

✓ Solution by Maple
Time used: 0.0 (sec). Leaf size: 19

```
dsolve([x^2*(2-x)*diff(y(x),x$2)+2*x*diff(y(x),x)-2*y(x)=0,x],singsol=all)
```

$$y(x) = \frac{c_1 x^2 + c_2(x-1)}{x}$$

✓ Solution by Mathematica
Time used: 0.044 (sec). Leaf size: 24

```
DSolve[x^2*(2-x)*y''[x]+2*x*y'[x]-2*y[x]==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to \frac{c_1(x-2)^2 + c_2(x-1)}{x}$$

7.21 problem 26

Internal problem ID [4859]

Book: Mathematical Methods in the Physical Sciences. third edition. Mary L. Boas. John Wiley. 2006
Section: Chapter 8, Ordinary differential equations. Section 7. Other second-Order equations. page 435
Problem number: 26.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$\left(x^2 + 1\right) y'' - 2xy' + 2y = 0$$

Given that one solution of the ode is

$$y_1 = x$$

✓ Solution by Maple
Time used: 0.0 (sec). Leaf size: 16

```
dsolve([(x^2+1)*diff(y(x),x$2)-2*x*diff(y(x),x)+2*y(x)=0,x],singsol=all)
```

$$y(x) = c_2 x^2 + c_1 x - c_2$$

✓ Solution by Mathematica
Time used: 0.043 (sec). Leaf size: 21

```
DSolve[(x^2+1)*y''[x]-2*x*y'[x]+2*y[x]==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to c_2 x - c_1 (x - i)^2$$

7.22 problem 27

Internal problem ID [4860]

Book: Mathematical Methods in the Physical Sciences. third edition. Mary L. Boas. John Wiley. 2006
Section: Chapter 8, Ordinary differential equations. Section 7. Other second-Order equations. page 435
Problem number: 27.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$xy'' - 2(x+1)\,y' + (x+2)\,y = 0$$

Given that one solution of the ode is

$$y_1 = e^x$$

✓ Solution by Maple
Time used: 0.0 (sec). Leaf size: 14

```
dsolve([x*diff(y(x),x$2)-2*(x+1)*diff(y(x),x)+(x+2)*y(x)=0,exp(x)],singsol=all)
```

$$y(x) = e^x\left(c_2 x^3 + c_1\right)$$

✓ Solution by Mathematica
Time used: 0.024 (sec). Leaf size: 23

```
DSolve[x*y''[x]-2*(x+1)*y'[x]+(x+2)*y[x]==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to \frac{1}{3}e^x\left(c_2 x^3 + 3c_1\right)$$

7.23 problem 28

Internal problem ID [4861]

Book: Mathematical Methods in the Physical Sciences. third edition. Mary L. Boas. John Wiley. 2006

Section: Chapter 8, Ordinary differential equations. Section 7. Other second-Order equations. page 435

Problem number: 28.

ODE order: 2.

ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$3xy'' - 2(3x - 1)\, y' + (3x - 2)\, y = 0$$

Given that one solution of the ode is

$$y_1 = \mathrm{e}^x$$

✓ Solution by Maple
Time used: 0.0 (sec). Leaf size: 14

```
dsolve([3*x*diff(y(x),x$2)-2*(3*x-1)*diff(y(x),x)+(3*x-2)*y(x)=0,exp(x)],singsol=all)
```

$$y(x) = \mathrm{e}^x \left(c_1 + x^{\frac{1}{3}} c_2 \right)$$

✓ Solution by Mathematica
Time used: 0.026 (sec). Leaf size: 21

```
DSolve[3*x*y''[x]-2*(3*x-1)*y'[x]+(3*x-2)*y[x]==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to \mathrm{e}^x \left(3c_2 \sqrt[3]{x} + c_1 \right)$$

7.24 problem 29

Internal problem ID [4862]

Book: Mathematical Methods in the Physical Sciences. third edition. Mary L. Boas. John Wiley. 2006

Section: Chapter 8, Ordinary differential equations. Section 7. Other second-Order equations. page 435

Problem number: 29.

ODE order: 2.

ODE degree: 1.

CAS Maple gives this as type [[_2nd_order, _exact, _linear, _homogeneous]]

$$x^2 y'' + (x+1) y' - y = 0$$

Given that one solution of the ode is

$$y_1 = x + 1$$

✓ Solution by Maple
Time used: 0.0 (sec). Leaf size: 16

```
dsolve([x^2*diff(y(x),x$2)+(x+1)*diff(y(x),x)-y(x)=0,x+1],singsol=all)
```

$$y(x) = c_2 e^{\frac{1}{x}} x + c_1 x + c_1$$

✓ Solution by Mathematica
Time used: 0.077 (sec). Leaf size: 21

```
DSolve[x^2*y''[x]+(x+1)*y'[x]-y[x]==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to c_1 e^{\frac{1}{x}} x + c_2(x+1)$$

7.25 problem 30

Internal problem ID [4863]

Book: Mathematical Methods in the Physical Sciences. third edition. Mary L. Boas. John Wiley. 2006
Section: Chapter 8, Ordinary differential equations. Section 7. Other second-Order equations. page 435
Problem number: 30.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$x(x+1)\, y'' - (x-1)\, y' + y = 0$$

Given that one solution of the ode is

$$y_1 = x - 1$$

✓ Solution by Maple
Time used: 0.015 (sec). Leaf size: 20

```
dsolve([x*(x+1)*diff(y(x),x$2)-(x-1)*diff(y(x),x)+y(x)=0,x-1],singsol=all)
```

$$y(x) = (x-1)\, c_2 \ln(x) - 4c_2 + c_1(x-1)$$

✓ Solution by Mathematica
Time used: 0.052 (sec). Leaf size: 23

```
DSolve[x*(x+1)*y''[x]-(x-1)*y'[x]+y[x]==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to c_1(x-1) + c_2((x-1)\log(x) - 4)$$

8 Chapter 8, Ordinary differential equations. Section 13. Miscellaneous problems. page 466

8.1 problem 1

Internal problem ID [4864]

Book: Mathematical Methods in the Physical Sciences. third edition. Mary L. Boas. John Wiley. 2006
Section: Chapter 8, Ordinary differential equations. Section 13. Miscellaneous problems. page 466
Problem number: 1.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type [_linear]

$$ x^2 y' - xy = \frac{1}{x} $$

✓ Solution by Maple
Time used: 0.0 (sec). Leaf size: 13

```
dsolve(x^2*diff(y(x),x)-x*y(x)=1/x,y(x), singsol=all)
```

$$ y(x) = \left(-\frac{1}{3x^3} + c_1 \right) x $$

✓ Solution by Mathematica
Time used: 0.03 (sec). Leaf size: 17

```
DSolve[x^2*y'[x]-x*y[x]==1/x,y[x],x,IncludeSingularSolutions -> True]
```

$$ y(x) \to -\frac{1}{3x^2} + c_1 x $$

8.2 problem 2

Internal problem ID [4865]

Book: Mathematical Methods in the Physical Sciences. third edition. Mary L. Boas. John Wiley. 2006

Section: Chapter 8, Ordinary differential equations. Section 13. Miscellaneous problems. page 466

Problem number: 2.

ODE order: 1.

ODE degree: 1.

CAS Maple gives this as type [_separable]

$$x \ln (y) \, y' - \ln (x) \, y = 0$$

✔ Solution by Maple
Time used: 0.016 (sec). Leaf size: 31

```
dsolve(x*ln(y(x))*diff(y(x),x)-y(x)*ln(x)=0,y(x), singsol=all)
```

$$y(x) = e^{\sqrt{\ln(x)^2 + 2c_1}}$$
$$y(x) = e^{-\sqrt{\ln(x)^2 + 2c_1}}$$

✔ Solution by Mathematica
Time used: 0.815 (sec). Leaf size: 60

```
DSolve[x*Log[y[x]]*y'[x]-y[x]*Log[x]==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to e^{-\sqrt{\log^2(x) + 2c_1}}$$
$$y(x) \to e^{\sqrt{\log^2(x) + 2c_1}}$$
$$y(x) \to 0$$
$$y(x) \to e^{2i\text{Interval}[\{0,\pi\}]}$$

8.3 problem 3

Internal problem ID [4866]

Book: Mathematical Methods in the Physical Sciences. third edition. Mary L. Boas. John Wiley. 2006

Section: Chapter 8, Ordinary differential equations. Section 13. Miscellaneous problems. page 466

Problem number: 3.

ODE order: 3.

ODE degree: 1.

CAS Maple gives this as type `[[_3rd_order, _missing_x]]`

$$y''' + 2y'' + 2y' = 0$$

✔ Solution by Maple
Time used: 0.0 (sec). Leaf size: 22

```
dsolve(diff(y(x),x$3)+2*diff(y(x),x$2)+2*diff(y(x),x)=0,y(x), singsol=all)
```

$$y(x) = c_1 + \mathrm{e}^{-x} \sin(x) c_2 + c_3 \cos(x) \mathrm{e}^{-x}$$

✔ Solution by Mathematica
Time used: 0.113 (sec). Leaf size: 37

```
DSolve[y'''[x]+2*y''[x]+2*y'[x]==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to \frac{1}{2} e^{-x} ((c_2 - c_1) \sin(x) - (c_1 + c_2) \cos(x)) + c_3$$

8.4 problem 4

Internal problem ID [4867]

Book: Mathematical Methods in the Physical Sciences. third edition. Mary L. Boas. John Wiley. 2006
Section: Chapter 8, Ordinary differential equations. Section 13. Miscellaneous problems. page 466
Problem number: 4.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _missing_x]]`

$$r'' - 6r' + 9r = 0$$

✔ Solution by Maple
Time used: 0.0 (sec). Leaf size: 14

```
dsolve(diff(r(t),t$2)-6*diff(r(t),t)+9*r(t)=0,r(t), singsol=all)
```

$$r(t) = e^{3t}(c_2 t + c_1)$$

✔ Solution by Mathematica
Time used: 0.015 (sec). Leaf size: 18

```
DSolve[r''[t]-6*r'[t]+9*r[t]==0,r[t],t,IncludeSingularSolutions -> True]
```

$$r(t) \to e^{3t}(c_2 t + c_1)$$

8.5 problem 5

Internal problem ID [4868]

Book: Mathematical Methods in the Physical Sciences. third edition. Mary L. Boas. John
Wiley. 2006
Section: Chapter 8, Ordinary differential equations. Section 13. Miscellaneous problems.
page 466
Problem number: 5.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type **[_exact, [_1st_order, '_with_symmetry_[F(x),G(x)]'], [_Abel, '**

$$
-y \sin (2x) - \left(\sin (x)^2 - 2y \right) y' = -2x
$$

✔ Solution by Maple
Time used: 0.016 (sec). Leaf size: 75

```
dsolve(2*x-y(x)*sin(2*x)=(sin(x)^2-2*y(x))*diff(y(x),x),y(x), singsol=all)
```

$$
y(x) = \frac{1}{4} - \frac{\cos (2x)}{4} - \frac{\sqrt{\cos (2x)^2 - 16x^2 - 2\cos (2x) - 16c_1 + 1}}{4}
$$
$$
y(x) = \frac{1}{4} - \frac{\cos (2x)}{4} + \frac{\sqrt{\cos (2x)^2 - 16x^2 - 2\cos (2x) - 16c_1 + 1}}{4}
$$

✔ Solution by Mathematica
Time used: 0.259 (sec). Leaf size: 89

```
DSolve[2*x-y[x]*Sin[2*x]==(Sin[x]^2-2*y[x])*y'[x],y[x],x,IncludeSingularSolutions -> True]
```

$$
y(x) \to \frac{1}{4} \left(-\sqrt{-16x^2 + \cos^2(2x) - 2\cos(2x) + 1 + 16c_1} - \cos(2x) + 1 \right)
$$
$$
y(x) \to \frac{1}{4} \left(\sqrt{-16x^2 + \cos^2(2x) - 2\cos(2x) + 1 + 16c_1} - \cos(2x) + 1 \right)
$$

8.6 problem 6

Internal problem ID [4869]

Book: Mathematical Methods in the Physical Sciences. third edition. Mary L. Boas. John Wiley. 2006
Section: Chapter 8, Ordinary differential equations. Section 13. Miscellaneous problems. page 466
Problem number: 6.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type [[_2nd_order, _linear, _nonhomogeneous]]

$$y'' + 2y' + 2y = 10\,\mathrm{e}^x + 6\,\mathrm{e}^{-x}\cos(x)$$

✓ Solution by Maple
Time used: 0.0 (sec). Leaf size: 30

```
dsolve(diff(y(x),x$2)+2*diff(y(x),x)+2*y(x)=10*exp(x)+6*exp(-x)*cos(x),y(x), singsol=all)
```

$$y(x) = \left((c_1 + 3)\cos(x) + 3\left(x + \frac{c_2}{3}\right)\sin(x)\right)\mathrm{e}^{-x} + 2\,\mathrm{e}^x$$

✓ Solution by Mathematica
Time used: 0.212 (sec). Leaf size: 41

```
DSolve[y''[x]+2*y'[x]+2*y[x]==10*Exp[x]+6*Exp[-x]*Cos[x],y[x],x,IncludeSingularSolutions ->
```

$$y(x) \to \frac{1}{2}e^{-x}\left(4e^{2x} + (3 + 2c_2)\cos(x) + 2(3x + c_1)\sin(x)\right)$$

8.7 problem 7

Internal problem ID [4870]

Book: Mathematical Methods in the Physical Sciences. third edition. Mary L. Boas. John Wiley. 2006
Section: Chapter 8, Ordinary differential equations. Section 13. Miscellaneous problems. page 466
Problem number: 7.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[[_homogeneous, 'class G'], _rational, _Bernoulli]`

$$3x^3y^2y' - y^3x^2 = 1$$

✔ Solution by Maple
Time used: 0.0 (sec). Leaf size: 85

```
dsolve(3*x^3*y(x)^2*diff(y(x),x)-x^2*y(x)^3=1,y(x), singsol=all)
```

$$y(x) = \frac{3^{\frac{2}{3}}\left(3c_1x^4 - x\right)^{\frac{1}{3}}}{3x}$$

$$y(x) = -\frac{3^{\frac{2}{3}}\left(3c_1x^4 - x\right)^{\frac{1}{3}}\left(1 + i\sqrt{3}\right)}{6x}$$

$$y(x) = -\frac{\left(3^{\frac{2}{3}} - 3i3^{\frac{1}{6}}\right)\left(3c_1x^4 - x\right)^{\frac{1}{3}}}{6x}$$

Solution by Mathematica

Time used: 0.518 (sec). Leaf size: 85

```
DSolve[3*x^3*y[x]^2*y'[x]-x^2*y[x]^3==1,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to -\frac{\sqrt[3]{-\frac{1}{3}}\sqrt[3]{-1+3c_1x^3}}{x^{2/3}}$$

$$y(x) \to \frac{\sqrt[3]{-\frac{1}{3}+c_1x^3}}{x^{2/3}}$$

$$y(x) \to \frac{(-1)^{2/3}\sqrt[3]{-\frac{1}{3}+c_1x^3}}{x^{2/3}}$$

8.8 problem 8

Internal problem ID [4871]

Book: Mathematical Methods in the Physical Sciences. third edition. Mary L. Boas. John Wiley. 2006

Section: Chapter 8, Ordinary differential equations. Section 13. Miscellaneous problems. page 466

Problem number: 8.

ODE order: 2.

ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$x^2 y'' - xy' + y = x$$

✓ Solution by Maple
Time used: 0.015 (sec). Leaf size: 18

```
dsolve(x^2*diff(y(x),x$2)-x*diff(y(x),x)+y(x)=x,y(x), singsol=all)
```

$$y(x) = x\left(c_2 + c_1 \ln(x) + \frac{\ln(x)^2}{2}\right)$$

✓ Solution by Mathematica
Time used: 0.019 (sec). Leaf size: 25

```
DSolve[x^2*y''[x]-x*y'[x]+y[x]==x,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to \frac{1}{2}x\left(\log^2(x) + 2c_2\log(x) + 2c_1\right)$$

8.9 problem 9

Internal problem ID [4872]

Book: Mathematical Methods in the Physical Sciences. third edition. Mary L. Boas. John Wiley. 2006

Section: Chapter 8, Ordinary differential equations. Section 13. Miscellaneous problems. page 466

Problem number: 9.

ODE order: 1.

ODE degree: 1.

CAS Maple gives this as type `[[_1st_order, _with_linear_symmetries], _Bernoulli]`

$$y' - 2y - y^2 \mathrm{e}^{3x} = 0$$

✓ Solution by Maple
Time used: 0.0 (sec). Leaf size: 20

```
dsolve(diff(y(x),x)-(2*y(x)+y(x)^2*exp(3*x))=0,y(x), singsol=all)
```

$$y(x) = -\frac{5\,\mathrm{e}^{2x}}{\mathrm{e}^{5x} - 5c_1}$$

✓ Solution by Mathematica
Time used: 0.223 (sec). Leaf size: 29

```
DSolve[y'[x]-(2*y[x]+y[x]^2*Exp[3*x])==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to -\frac{5e^{2x}}{e^{5x} - 5c_1}$$
$$y(x) \to 0$$

8.10 problem 10

Internal problem ID [4873]

Book: Mathematical Methods in the Physical Sciences. third edition. Mary L. Boas. John Wiley. 2006
Section: Chapter 8, Ordinary differential equations. Section 13. Miscellaneous problems. page 466
Problem number: 10.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type [_separable]

$$ u(1-v) + v^2(1-u)\,u' = 0 $$

✔ Solution by Maple
Time used: 0.015 (sec). Leaf size: 33

```
dsolve(u(v)*(1-v)+v^2*(1-u(v))*diff(u(v),v)=0,u(v), singsol=all)
```

$$ u(v) = v\,\mathrm{e}^{\dfrac{-\operatorname{LambertW}\left(-v\,\mathrm{e}^{\frac{c_1 v + 1}{v}}\right)v + c_1 v + 1}{v}} $$

✔ Solution by Mathematica
Time used: 2.999 (sec). Leaf size: 26

```
DSolve[u[v]*(1-v)+v^2*(1-u[v])*u'[v]==0,u[v],v,IncludeSingularSolutions -> True]
```

$$ u(v) \to -W\left(v\left(-e^{\frac{1}{v}-c_1}\right)\right) $$
$$ u(v) \to 0 $$

8.11 problem 11

Internal problem ID [4874]

Book: Mathematical Methods in the Physical Sciences. third edition. Mary L. Boas. John Wiley. 2006

Section: Chapter 8, Ordinary differential equations. Section 13. Miscellaneous problems. page 466

Problem number: 11.

ODE order: 1.

ODE degree: 1.

CAS Maple gives this as type [_linear]

$$-xy' + y = -2x$$

✔ Solution by Maple

Time used: 0.0 (sec). Leaf size: 12

```
dsolve((y(x)+2*x)-x*diff(y(x),x)=0,y(x), singsol=all)
```

$$y(x) = (2\ln(x) + c_1)x$$

✔ Solution by Mathematica

Time used: 0.024 (sec). Leaf size: 14

```
DSolve[(y[x]+2*x)-x*y'[x]==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to x(2\log(x) + c_1)$$

8.12   problem 12

Internal problem ID [4875]

Book: Mathematical Methods in the Physical Sciences. third edition. Mary L. Boas. John Wiley. 2006

Section: Chapter 8, Ordinary differential equations. Section 13. Miscellaneous problems. page 466

Problem number: 12.

ODE order: 2.

ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _missing_y]]`

$$xy'' + y' = 4x$$

✔ Solution by Maple
Time used: 0.0 (sec). Leaf size: 13

```
dsolve(x*diff(y(x),x$2)+diff(y(x),x)=4*x,y(x), singsol=all)
```

$$y(x) = x^2 + c_1 \ln(x) + c_2$$

✔ Solution by Mathematica
Time used: 0.026 (sec). Leaf size: 16

```
DSolve[x*y''[x]+y'[x]==4*x,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to x^2 + c_1 \log(x) + c_2$$

8.13 problem 13

Internal problem ID [4876]

Book: Mathematical Methods in the Physical Sciences. third edition. Mary L. Boas. John Wiley. 2006
Section: Chapter 8, Ordinary differential equations. Section 13. Miscellaneous problems. page 466
Problem number: 13.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$y'' + 4y' + 5y = 26\,e^{3x}$$

✔ Solution by Maple
Time used: 0.0 (sec). Leaf size: 22

```
dsolve(diff(y(x),x$2)+4*diff(y(x),x)+5*y(x)=26*exp(3*x),y(x), singsol=all)
```

$$y(x) = \left(e^{5x} + c_2 \sin(x) + \cos(x)\,c_1\right)e^{-2x}$$

✔ Solution by Mathematica
Time used: 0.02 (sec). Leaf size: 27

```
DSolve[y''[x]+4*y'[x]+5*y[x]==26*Exp[3*x],y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to e^{-2x}\left(e^{5x} + c_2 \cos(x) + c_1 \sin(x)\right)$$

8.14   problem 14

Internal problem ID [4877]

Book: Mathematical Methods in the Physical Sciences. third edition. Mary L. Boas. John Wiley. 2006

Section: Chapter 8, Ordinary differential equations. Section 13. Miscellaneous problems. page 466

Problem number: 14.

ODE order: 2.

ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _linear, _nonhomogeneous]]`

$$y'' + 4y' + 5y = 2\,\mathrm{e}^{-2x}\cos\left(x\right)$$

✓ Solution by Maple
Time used: 0.016 (sec). Leaf size: 20

```
dsolve(diff(y(x),x$2)+4*diff(y(x),x)+5*y(x)=2*exp(-2*x)*cos(x),y(x), singsol=all)
```

$$y(x) = \left(\sin\left(x\right)\left(c_2 + x\right) + \cos\left(x\right)c_1\right)\mathrm{e}^{-2x}$$

✓ Solution by Mathematica
Time used: 0.049 (sec). Leaf size: 26

```
DSolve[y''[x]+4*y'[x]+5*y[x]==2*Exp[-2*x]*Cos[x],y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to e^{-2x}\left((1 + c_2)\cos(x) + (x + c_1)\sin(x)\right)$$

8.15 problem 15

Internal problem ID [4878]

Book: Mathematical Methods in the Physical Sciences. third edition. Mary L. Boas. John Wiley. 2006
Section: Chapter 8, Ordinary differential equations. Section 13. Miscellaneous problems. page 466
Problem number: 15.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type [[_2nd_order, _with_linear_symmetries]]

$$ y'' - 4y' + 4y = 6\,e^{2x} $$

✔ Solution by Maple
Time used: 0.015 (sec). Leaf size: 19

```
dsolve(diff(y(x),x$2)-4*diff(y(x),x)+4*y(x)=6*exp(2*x),y(x), singsol=all)
```

$$ y(x) = e^{2x}\left(c_1 x + 3x^2 + c_2\right) $$

✔ Solution by Mathematica
Time used: 0.022 (sec). Leaf size: 23

```
DSolve[y''[x]-4*y'[x]+4*y[x]==6*Exp[2*x],y[x],x,IncludeSingularSolutions -> True]
```

$$ y(x) \to e^{2x}\left(3x^2 + c_2 x + c_1\right) $$

8.16   problem 16

Internal problem ID [4879]

Book: Mathematical Methods in the Physical Sciences. third edition. Mary L. Boas. John Wiley. 2006
Section: Chapter 8, Ordinary differential equations. Section 13. Miscellaneous problems. page 466
Problem number: 16.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$y'' - 5y' + 6y = e^{2x}$$

✔ Solution by Maple
Time used: 0.016 (sec). Leaf size: 21

```
dsolve(diff(y(x),x$2)-5*diff(y(x),x)+6*y(x)=exp(2*x),y(x), singsol=all)
```

$$y(x) = (-x + c_1)\, e^{2x} + c_2 e^{3x}$$

✔ Solution by Mathematica
Time used: 0.023 (sec). Leaf size: 24

```
DSolve[y''[x]-5*y'[x]+6*y[x]==Exp[2*x],y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to e^{2x}(-x + c_2 e^x - 1 + c_1)$$

8.17   problem 17

Internal problem ID [4880]

Book: Mathematical Methods in the Physical Sciences. third edition. Mary L. Boas. John
Wiley. 2006
Section: Chapter 8, Ordinary differential equations. Section 13. Miscellaneous problems.
page 466
Problem number: 17.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[[_homogeneous, 'class A'], _exact, _rational, [_Abel, '2nd ty`

$$(2x + y)\,y' + 2y = x$$

✔ Solution by Maple
Time used: 0.031 (sec). Leaf size: 51

```
dsolve((2*x+y(x))*diff(y(x),x)-(x-2*y(x))=0,y(x), singsol=all)
```

$$y(x) = \frac{-2c_1 x - \sqrt{5c_1^2 x^2 + 1}}{c_1}$$

$$y(x) = \frac{-2c_1 x + \sqrt{5c_1^2 x^2 + 1}}{c_1}$$

✔ Solution by Mathematica
Time used: 0.458 (sec). Leaf size: 94

```
DSolve[(2*x+y[x])*y'[x]-(x-2*y[x])==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to -2x - \sqrt{5x^2 + e^{2c_1}}$$
$$y(x) \to -2x + \sqrt{5x^2 + e^{2c_1}}$$
$$y(x) \to -\sqrt{5}\sqrt{x^2} - 2x$$
$$y(x) \to \sqrt{5}\sqrt{x^2} - 2x$$

8.18 problem 18

Internal problem ID [4881]

Book: Mathematical Methods in the Physical Sciences. third edition. Mary L. Boas. John Wiley. 2006
Section: Chapter 8, Ordinary differential equations. Section 13. Miscellaneous problems. page 466
Problem number: 18.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type [[_1st_order, '_with_symmetry_[F(x)*G(y),0]']]

$$\left(\cos\left(y\right)x - e^{-\sin(y)}\right)y' = -1$$

✓ Solution by Maple
Time used: 0.031 (sec). Leaf size: 20

```
dsolve((x*cos(y(x)) - exp(-sin(y(x))))*diff(y(x),x)+1=0,y(x), singsol=all)
```

$$\left(-y(x) - c_1\right)e^{-\sin(y(x))} + x = 0$$

✓ Solution by Mathematica
Time used: 0.734 (sec). Leaf size: 26

```
DSolve[(x*Cos[y[x]] - Exp[-Sin[y[x]]])*y'[x]+1==0,y[x],x,IncludeSingularSolutions -> True]
```

$$\text{Solve}\left[x = y(x)e^{-\sin(y(x))} + c_1 e^{-\sin(y(x))}, y(x)\right]$$

8.19 problem 19

Internal problem ID [4882]

Book: Mathematical Methods in the Physical Sciences. third edition. Mary L. Boas. John Wiley. 2006
Section: Chapter 8, Ordinary differential equations. Section 13. Miscellaneous problems. page 466
Problem number: 19.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type [_linear]

$$y' \sin(x)^2 + (x+y) \sin(2x) = -\sin(x)^2$$

✓ Solution by Maple
Time used: 0.016 (sec). Leaf size: 19

```
dsolve(sin(x)^2*diff(y(x),x)+(sin(x)^2+(x+y(x))*sin(2*x))=0,y(x), singsol=all)
```

$$y(x) = -\frac{2c_1}{-1 + \cos(2x)} - x$$

✓ Solution by Mathematica
Time used: 0.056 (sec). Leaf size: 27

```
DSolve[Sin[x]^2*y'[x]+(Sin[x]^2+(x+y[x])*Sin[2*x])==0,y[x],x,IncludeSingularSolutions -> Tru
```

$$y(x) \to \frac{1}{2} \csc^2(x)(-x + x\cos(2x) + 2c_1)$$

8.20 problem 20

Internal problem ID [4883]

Book: Mathematical Methods in the Physical Sciences. third edition. Mary L. Boas. John Wiley. 2006
Section: Chapter 8, Ordinary differential equations. Section 13. Miscellaneous problems. page 466
Problem number: 20.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type [[_2nd_order, _linear, _nonhomogeneous]]

$$y'' - 2y' + 5y = 5x + 4\,\mathrm{e}^x(1 + \sin(2x))$$

✔ Solution by Maple
Time used: 0.0 (sec). Leaf size: 36

```
dsolve(diff(y(x),x$2)-2*diff(y(x),x)+5*y(x)=5*x+4*exp(x)*(1+sin(2*x)),y(x), singsol=all)
```

$$y(x) = \frac{2}{5} - \mathrm{e}^x(x - c_1 - 1)\cos(2x) + \frac{(2c_2 + 1)\,\mathrm{e}^x\sin(2x)}{2} + x + \mathrm{e}^x$$

✔ Solution by Mathematica
Time used: 1.313 (sec). Leaf size: 45

```
DSolve[y''[x]-2*y'[x]+5*y[x]==5*x+4*Exp[x]*(1+Sin[2*x]),y[x],x,IncludeSingularSolutions -> T
```

$$y(x) \to x + e^x - e^x(x - c_2)\cos(2x) + \frac{1}{4}(1 + 4c_1)e^x\sin(2x) + \frac{2}{5}$$

8.21 problem 21

Internal problem ID [4884]

Book: Mathematical Methods in the Physical Sciences. third edition. Mary L. Boas. John Wiley. 2006
Section: Chapter 8, Ordinary differential equations. Section 13. Miscellaneous problems. page 466
Problem number: 21.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type [_separable]

$$\boxed{y' + xy - \frac{x}{y} = 0}$$

✔ Solution by Maple
Time used: 0.0 (sec). Leaf size: 33

```
dsolve(diff(y(x),x)+x*y(x)=x/y(x),y(x), singsol=all)
```

$$y(x) = \sqrt{e^{-x^2} c_1 + 1}$$
$$y(x) = -\sqrt{e^{-x^2} c_1 + 1}$$

✔ Solution by Mathematica
Time used: 1.922 (sec). Leaf size: 57

```
DSolve[y'[x]+x*y[x]==x/y[x],y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to -\sqrt{1 + e^{-x^2 + 2c_1}}$$
$$y(x) \to \sqrt{1 + e^{-x^2 + 2c_1}}$$
$$y(x) \to -1$$
$$y(x) \to 1$$

8.22 problem 22

Internal problem ID [4885]

Book: Mathematical Methods in the Physical Sciences. third edition. Mary L. Boas. John Wiley. 2006

Section: Chapter 8, Ordinary differential equations. Section 13. Miscellaneous problems. page 466

Problem number: 22.

ODE order: 4.

ODE degree: 1.

CAS Maple gives this as type `[[_high_order, _missing_x]]`

$$y'''' - 2y''' + 13y'' - 18y' + 36y = 0$$

✔ Solution by Maple
Time used: 0.015 (sec). Leaf size: 37

```
dsolve(diff(y(x),x$4)-2*diff(y(x),x$3)+13*diff(y(x),x$2)-18*diff(y(x),x)+36*y(x)=0,y(x), sin
```

$$y(x) = c_1 \mathrm{e}^x \sin\left(\sqrt{3}\,x\right) + c_2 \mathrm{e}^x \cos\left(\sqrt{3}\,x\right) + c_3 \sin\left(3x\right) + c_4 \cos\left(3x\right)$$

✔ Solution by Mathematica
Time used: 0.004 (sec). Leaf size: 48

```
DSolve[y''''[x]-2*y'''[x]+13*y''[x]-18*y'[x]+36*y[x]==0,y[x],x,IncludeSingularSolutions -> T
```

$$y(x) \to c_3 \cos(3x) + c_2 e^x \cos\left(\sqrt{3}x\right) + c_4 \sin(3x) + c_1 e^x \sin\left(\sqrt{3}x\right)$$

8.23 problem 23

Internal problem ID [4886]

Book: Mathematical Methods in the Physical Sciences. third edition. Mary L. Boas. John
Wiley. 2006
Section: Chapter 8, Ordinary differential equations. Section 13. Miscellaneous problems.
page 466
Problem number: 23.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type [_linear]

$$\sin(\theta)\cos(\theta)\,r' - r\cos(\theta)^2 = \sin(\theta)^2$$

✔ Solution by Maple
Time used: 0.0 (sec). Leaf size: 15

```
dsolve(sin(theta)*cos(theta)*diff(r(theta),theta)-sin(theta)^2=r(theta)*cos(theta)^2,r(theta
```

$$r(\theta) = (\ln(\sec(\theta) + \tan(\theta)) + c_1)\sin(\theta)$$

✔ Solution by Mathematica
Time used: 0.042 (sec). Leaf size: 14

```
DSolve[Sin[\[Theta]]*Cos[\[Theta]]*r'[\[Theta]]-Sin[\[Theta]]^2==r[\[Theta]]*Cos[\[Theta]]^2
```

$$r(\theta) \to \sin(\theta)\left(\coth^{-1}(\sin(\theta)) + c_1\right)$$

8.24 problem 24

Internal problem ID [4887]

Book: Mathematical Methods in the Physical Sciences. third edition. Mary L. Boas. John Wiley. 2006

Section: Chapter 8, Ordinary differential equations. Section 13. Miscellaneous problems. page 466

Problem number: 24.

ODE order: 2.

ODE degree: 1.

CAS Maple gives this as type [[_2nd_order, _exact, _nonlinear], _Liouville, [_2nd_order, _w

$$ x\left(yy'' + y'^2 \right) - y'y = 0 $$

✓ Solution by Maple
Time used: 0.0 (sec). Leaf size: 35

```
dsolve(x*(y(x)*diff(y(x),x$2) + diff(y(x),x)^2)= y(x)*diff(y(x),x),y(x), singsol=all)
```

$$ y(x) = 0 $$
$$ y(x) = \sqrt{c_1 x^2 + 2c_2} $$
$$ y(x) = -\sqrt{c_1 x^2 + 2c_2} $$

✓ Solution by Mathematica
Time used: 0.234 (sec). Leaf size: 18

```
DSolve[x*(y[x]*y''[x]+(y'[x])^2)==y[x]*y'[x],y[x],x,IncludeSingularSolutions -> True]
```

$$ y(x) \to c_2\sqrt{x^2 + c_1} $$

8.25 problem 25

Internal problem ID [4888]

Book: Mathematical Methods in the Physical Sciences. third edition. Mary L. Boas. John Wiley. 2006
Section: Chapter 8, Ordinary differential equations. Section 13. Miscellaneous problems. page 466
Problem number: 25.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type [_separable]

$$3yx^2 + y'x^3 = 0$$

With initial conditions

$$[y(1) = 2]$$

✔ Solution by Maple
Time used: 0.0 (sec). Leaf size: 9

```
dsolve([3*x^2*y(x)+x^3*diff(y(x),x)=0,y(1) = 2],y(x), singsol=all)
```

$$y(x) = \frac{2}{x^3}$$

✔ Solution by Mathematica
Time used: 0.035 (sec). Leaf size: 10

```
DSolve[{3*x^2*y[x]+x^3*y'[x]==0,{y[1]==2}},y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to \frac{2}{x^3}$$

8.26 problem 26

Internal problem ID [4889]

Book: Mathematical Methods in the Physical Sciences. third edition. Mary L. Boas. John Wiley. 2006

Section: Chapter 8, Ordinary differential equations. Section 13. Miscellaneous problems. page 466

Problem number: 26.

ODE order: 1.

ODE degree: 1.

CAS Maple gives this as type [_linear]

$$-y + xy' = x^2$$

With initial conditions

$$[y(2) = 6]$$

✓ Solution by Maple
Time used: 0.0 (sec). Leaf size: 9

```
dsolve([x*diff(y(x),x)-y(x)=x^2,y(2) = 6],y(x), singsol=all)
```

$$y(x) = x(1 + x)$$

✓ Solution by Mathematica
Time used: 0.026 (sec). Leaf size: 10

```
DSolve[{x*y'[x]-y[x]==x^2,{y[2]==6}},y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \rightarrow x(x + 1)$$

8.27 problem 27

Internal problem ID [4890]

Book: Mathematical Methods in the Physical Sciences. third edition. Mary L. Boas. John Wiley. 2006
Section: Chapter 8, Ordinary differential equations. Section 13. Miscellaneous problems. page 466
Problem number: 27.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _missing_x]]`

$$y'' + y' - 6y = 6$$

With initial conditions

$$[y(0) = 1, y'(0) = 4]$$

✓ Solution by Maple
Time used: 0.016 (sec). Leaf size: 12

```
dsolve([diff(y(x),x$2)+diff(y(x),x)-6*y(x)=6,y(0) = 1, D(y)(0) = 4],y(x), singsol=all)
```

$$y(x) = 2\,e^{2x} - 1$$

✓ Solution by Mathematica
Time used: 0.015 (sec). Leaf size: 14

```
DSolve[{y''[x]+y'[x]-6*y[x]==6,{y[0]==1,y'[0]==4}},y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to 2e^{2x} - 1$$

8.28   problem 28

Internal problem ID [4891]

Book: Mathematical Methods in the Physical Sciences. third edition. Mary L. Boas. John Wiley. 2006

Section: Chapter 8, Ordinary differential equations. Section 13. Miscellaneous problems. page 466

Problem number: 28.

ODE order: 2.

ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _missing_x], [_2nd_order, _exact, _nonlinear], [`

$$yy'' + y'^2 = -4$$

With initial conditions

$$[y(1) = 3, y'(1) = 0]$$

✔ Solution by Maple
Time used: 0.891 (sec). Leaf size: 16

```
dsolve([y(x)*diff(y(x),x$2)+diff(y(x),x)^2+4=0,y(1) = 3, D(y)(1) = 0],y(x), singsol=all)
```

$$y(x) = \sqrt{-4x^2 + 8x + 5}$$

✔ Solution by Mathematica
Time used: 31.559 (sec). Leaf size: 19

```
DSolve[{y[x]*y''[x]+y'[x]^2+4==0,{y[1]==3,y'[1]==0}},y[x],x,IncludeSingularSolutions -> True
```

$$y(x) \to \sqrt{-4x^2 + 8x + 5}$$

9 Chapter 12, Series Solutions of Differential Equations. Section 1. Miscellaneous problems. page 564

9.1   problem 1, using series method

Internal problem ID [4892]

Book: Mathematical Methods in the Physical Sciences. third edition. Mary L. Boas. John Wiley. 2006
Section: Chapter 12, Series Solutions of Differential Equations. Section 1. Miscellaneous problems. page 564
Problem number: 1, using series method.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type [_separable]

$$ xy' - xy - y = 0 $$

With the expansion point for the power series method at $x = 0$.

✓   Solution by Maple
Time used: 0.0 (sec). Leaf size: 23

```
Order:=6;
dsolve(x*diff(y(x),x)=x*y(x)+y(x),y(x),type='series',x=0);
```

$$ y(x) = c_1 x \left(1 + x + \frac{1}{2}x^2 + \frac{1}{6}x^3 + \frac{1}{24}x^4 + \frac{1}{120}x^5 \right) + O(x^6) $$

✓   Solution by Mathematica
Time used: 0.002 (sec). Leaf size: 38

```
AsymptoticDSolveValue[x*y'[x]==x*y[x]+y[x],y[x],{x,0,5}]
```

$$ y(x) \to c_1 x \left(\frac{x^5}{120} + \frac{x^4}{24} + \frac{x^3}{6} + \frac{x^2}{2} + x + 1 \right) $$

9.2 problem 1, using elementary method

Internal problem ID [4893]

Book: Mathematical Methods in the Physical Sciences. third edition. Mary L. Boas. John Wiley. 2006
Section: Chapter 12, Series Solutions of Differential Equations. Section 1. Miscellaneous problems. page 564
Problem number: 1, using elementary method.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type [_separable]

$$\boxed{xy' - xy - y = 0}$$

✔ Solution by Maple
Time used: 0.0 (sec). Leaf size: 9

```
dsolve(x*diff(y(x),x)=x*y(x)+y(x),y(x), singsol=all)
```

$$y(x) = x\,\mathrm{e}^x c_1$$

✔ Solution by Mathematica
Time used: 0.026 (sec). Leaf size: 17

```
DSolve[x*y'[x]==x*y[x]+y[x],y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to c_1 e^x x$$
$$y(x) \to 0$$

9.3 problem 2, using series method

Internal problem ID [4894]

Book: Mathematical Methods in the Physical Sciences. third edition. Mary L. Boas. John Wiley. 2006
Section: Chapter 12, Series Solutions of Differential Equations. Section 1. Miscellaneous problems. page 564
Problem number: 2, using series method.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_separable]`

$$\boxed{y' - 3yx^2 = 0}$$

With the expansion point for the power series method at $x = 0$.

✓ Solution by Maple
Time used: 0.0 (sec). Leaf size: 14

```
Order:=6;
dsolve(diff(y(x),x)=3*x^2*y(x),y(x),type='series',x=0);
```

$$y(x) = (x^3 + 1) y(0) + O(x^6)$$

✓ Solution by Mathematica
Time used: 0.001 (sec). Leaf size: 11

```
AsymptoticDSolveValue[y'[x]==3*x^2*y[x],y[x],{x,0,5}]
```

$$y(x) \to c_1 (x^3 + 1)$$

9.4 problem 2, using elementary method

Internal problem ID [4895]

Book: Mathematical Methods in the Physical Sciences. third edition. Mary L. Boas. John Wiley. 2006
Section: Chapter 12, Series Solutions of Differential Equations. Section 1. Miscellaneous problems. page 564
Problem number: 2, using elementary method.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type [_separable]

$$\boxed{y' - 3yx^2 = 0}$$

✔ Solution by Maple
Time used: 0.0 (sec). Leaf size: 10

```
dsolve(diff(y(x),x)=3*x^2*y(x),y(x), singsol=all)
```

$$y(x) = c_1 e^{x^3}$$

✔ Solution by Mathematica
Time used: 0.023 (sec). Leaf size: 18

```
DSolve[y'[x]==3*x^2*y[x],y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to c_1 e^{x^3}$$
$$y(x) \to 0$$

9.5 problem 3, using series method

Internal problem ID [4896]

Book: Mathematical Methods in the Physical Sciences. third edition. Mary L. Boas. John Wiley. 2006
Section: Chapter 12, Series Solutions of Differential Equations. Section 1. Miscellaneous problems. page 564
Problem number: 3, using series method.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type [_separable]

$$-y + xy' = 0$$

With the expansion point for the power series method at $x = 0$.

✓ Solution by Maple
Time used: 0.0 (sec). Leaf size: 13

```
Order:=6;
dsolve(x*diff(y(x),x)=y(x),y(x),type='series',x=0);
```

$$y(x) = c_1 x + O(x^6)$$

✓ Solution by Mathematica
Time used: 0.001 (sec). Leaf size: 7

```
AsymptoticDSolveValue[x*y'[x]==y[x],y[x],{x,0,5}]
```

$$y(x) \rightarrow c_1 x$$

9.6   problem 3, using elementary method

Internal problem ID [4897]

Book: Mathematical Methods in the Physical Sciences. third edition. Mary L. Boas. John Wiley. 2006
Section: Chapter 12, Series Solutions of Differential Equations. Section 1. Miscellaneous problems. page 564
Problem number: 3, using elementary method.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type [_separable]

$$\boxed{-y + xy' = 0}$$

✔ Solution by Maple
Time used: 0.0 (sec). Leaf size: 7

```
dsolve(x*diff(y(x),x)=y(x),y(x), singsol=all)
```

$$y(x) = c_1 x$$

✔ Solution by Mathematica
Time used: 0.022 (sec). Leaf size: 14

```
DSolve[x*y'[x]==y[x],y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \rightarrow c_1 x$$
$$y(x) \rightarrow 0$$

9.7 problem 4, using series method

Internal problem ID [4898]

Book: Mathematical Methods in the Physical Sciences. third edition. Mary L. Boas. John Wiley. 2006

Section: Chapter 12, Series Solutions of Differential Equations. Section 1. Miscellaneous problems. page 564

Problem number: 4, using series method.

ODE order: 2.

ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _missing_x]]`

$$y'' + 4y = 0$$

With the expansion point for the power series method at $x = 0$.

✔ Solution by Maple
Time used: 0.0 (sec). Leaf size: 34

```
Order:=6;
dsolve(diff(y(x),x$2)=-4*y(x),y(x),type='series',x=0);
```

$$y(x) = \left(1 - 2x^2 + \frac{2}{3}x^4\right)y(0) + \left(x - \frac{2}{3}x^3 + \frac{2}{15}x^5\right)D(y)(0) + O(x^6)$$

✔ Solution by Mathematica
Time used: 0.001 (sec). Leaf size: 40

```
AsymptoticDSolveValue[y''[x]==-4*y[x],y[x],{x,0,5}]
```

$$y(x) \to c_2\left(\frac{2x^5}{15} - \frac{2x^3}{3} + x\right) + c_1\left(\frac{2x^4}{3} - 2x^2 + 1\right)$$

9.8 problem 4, using elementary method

Internal problem ID [4899]

Book: Mathematical Methods in the Physical Sciences. third edition. Mary L. Boas. John Wiley. 2006
Section: Chapter 12, Series Solutions of Differential Equations. Section 1. Miscellaneous problems. page 564
Problem number: 4, using elementary method.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _missing_x]]`

$$\boxed{y'' + 4y = 0}$$

✓ Solution by Maple
Time used: 0.0 (sec). Leaf size: 17

```
dsolve(diff(y(x),x$2)=-4*y(x),y(x), singsol=all)
```

$$y(x) = c_1 \sin(2x) + c_2 \cos(2x)$$

✓ Solution by Mathematica
Time used: 0.014 (sec). Leaf size: 20

```
DSolve[y''[x]==-4*y[x],y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to c_1 \cos(2x) + c_2 \sin(2x)$$

9.9 problem 5, using series method

Internal problem ID [4900]

Book: Mathematical Methods in the Physical Sciences. third edition. Mary L. Boas. John Wiley. 2006
Section: Chapter 12, Series Solutions of Differential Equations. Section 1. Miscellaneous problems. page 564
Problem number: 5, using series method.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type [[_2nd_order, _missing_x]]

$$y'' - y = 0$$

With the expansion point for the power series method at $x = 0$.

✓ Solution by Maple
Time used: 0.0 (sec). Leaf size: 34

```
Order:=6;
dsolve(diff(y(x),x$2)=y(x),y(x),type='series',x=0);
```

$$y(x) = \left(1 + \frac{1}{2}x^2 + \frac{1}{24}x^4\right)y(0) + \left(x + \frac{1}{6}x^3 + \frac{1}{120}x^5\right)D(y)(0) + O(x^6)$$

✓ Solution by Mathematica
Time used: 0.001 (sec). Leaf size: 42

```
AsymptoticDSolveValue[y''[x]==y[x],y[x],{x,0,5}]
```

$$y(x) \to c_2\left(\frac{x^5}{120} + \frac{x^3}{6} + x\right) + c_1\left(\frac{x^4}{24} + \frac{x^2}{2} + 1\right)$$

9.10 problem 5, using elementary method

Internal problem ID [4901]

Book: Mathematical Methods in the Physical Sciences. third edition. Mary L. Boas. John Wiley. 2006
Section: Chapter 12, Series Solutions of Differential Equations. Section 1. Miscellaneous problems. page 564
Problem number: 5, using elementary method.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _missing_x]]`

$$y'' - y = 0$$

✔ Solution by Maple
Time used: 0.0 (sec). Leaf size: 15

```
dsolve(diff(y(x),x$2)=y(x),y(x), singsol=all)
```

$$y(x) = c_1 \mathrm{e}^x + c_2 \mathrm{e}^{-x}$$

✔ Solution by Mathematica
Time used: 0.012 (sec). Leaf size: 20

```
DSolve[y''[x]==y[x],y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to c_1 e^x + c_2 e^{-x}$$

9.11 problem 6, using series method

Internal problem ID [4902]

Book: Mathematical Methods in the Physical Sciences. third edition. Mary L. Boas. John Wiley. 2006
Section: Chapter 12, Series Solutions of Differential Equations. Section 1. Miscellaneous problems. page 564
Problem number: 6, using series method.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type [[_2nd_order, _missing_x]]

$$ y'' - 2y' + y = 0 $$

With the expansion point for the power series method at $x = 0$.

✔ Solution by Maple
Time used: 0.016 (sec). Leaf size: 52

```
Order:=6;
dsolve(diff(y(x),x$2)-2*diff(y(x),x)+y(x)=0,y(x),type='series',x=0);
```

$$ y(x) = \left(1 - \frac{1}{2}x^2 - \frac{1}{3}x^3 - \frac{1}{8}x^4 - \frac{1}{30}x^5\right) y(0) $$
$$ + \left(x + x^2 + \frac{1}{2}x^3 + \frac{1}{6}x^4 + \frac{1}{24}x^5\right) D(y)(0) + O(x^6) $$

✔ Solution by Mathematica
Time used: 0.001 (sec). Leaf size: 66

```
AsymptoticDSolveValue[y''[x]-2*y'[x]+y[x]==0,y[x],{x,0,5}]
```

$$ y(x) \to c_1 \left(-\frac{x^5}{30} - \frac{x^4}{8} - \frac{x^3}{3} - \frac{x^2}{2} + 1\right) + c_2 \left(\frac{x^5}{24} + \frac{x^4}{6} + \frac{x^3}{2} + x^2 + x\right) $$

9.12 problem 6, using elementary method

Internal problem ID [4903]

Book: Mathematical Methods in the Physical Sciences. third edition. Mary L. Boas. John Wiley. 2006
Section: Chapter 12, Series Solutions of Differential Equations. Section 1. Miscellaneous problems. page 564
Problem number: 6, using elementary method.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _missing_x]]`

$$y'' - 2y' + y = 0$$

✓ Solution by Maple
Time used: 0.0 (sec). Leaf size: 12

```
dsolve(diff(y(x),x$2)-2*diff(y(x),x)+y(x)=0,y(x), singsol=all)
```

$$y(x) = e^x (c_2 x + c_1)$$

✓ Solution by Mathematica
Time used: 0.011 (sec). Leaf size: 16

```
DSolve[y''[x]-2*y'[x]+y[x]==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \rightarrow e^x (c_2 x + c_1)$$

9.13 problem 7, using series method

Internal problem ID [4904]

Book: Mathematical Methods in the Physical Sciences. third edition. Mary L. Boas. John Wiley. 2006
Section: Chapter 12, Series Solutions of Differential Equations. Section 1. Miscellaneous problems. page 564
Problem number: 7, using series method.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_Emden, _Fowler]]`

$$x^2 y'' - 3xy' + 3y = 0$$

With the expansion point for the power series method at $x = 0$.

✔ Solution by Maple
Time used: 0.0 (sec). Leaf size: 25

```
Order:=6;
dsolve(x^2*diff(y(x),x$2)-3*x*diff(y(x),x)+3*y(x)=0,y(x),type='series',x=0);
```

$$y(x) = c_1 x^3 \left(1 + O\left(x^6\right)\right) + c_2 x \left(-2 + O\left(x^6\right)\right)$$

✔ Solution by Mathematica
Time used: 0.004 (sec). Leaf size: 14

```
AsymptoticDSolveValue[x^2*y''[x]-3*x*y'[x]+3*y[x]==0,y[x],{x,0,5}]
```

$$y(x) \to c_2 x^3 + c_1 x$$

9.14 problem 7, using elementary method

Internal problem ID [4905]

Book: Mathematical Methods in the Physical Sciences. third edition. Mary L. Boas. John Wiley. 2006
Section: Chapter 12, Series Solutions of Differential Equations. Section 1. Miscellaneous problems. page 564
Problem number: 7, using elementary method.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type [[_Emden, _Fowler]]

$$x^2 y'' - 3xy' + 3y = 0$$

✔ Solution by Maple
Time used: 0.0 (sec). Leaf size: 13

```
dsolve(x^2*diff(y(x),x$2)-3*x*diff(y(x),x)+3*y(x)=0,y(x), singsol=all)
```

$$y(x) = x\left(c_2 x^2 + c_1\right)$$

✔ Solution by Mathematica
Time used: 0.01 (sec). Leaf size: 16

```
DSolve[x^2*y''[x]-3*x*y'[x]+3*y[x]==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to x\left(c_2 x^2 + c_1\right)$$

9.15 problem 8, using series method

Internal problem ID [4906]

Book: Mathematical Methods in the Physical Sciences. third edition. Mary L. Boas. John Wiley. 2006
Section: Chapter 12, Series Solutions of Differential Equations. Section 1. Miscellaneous problems. page 564
Problem number: 8, using series method.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$(x^2 + 2x)\, y'' - 2(x+1)\, y' + 2y = 0$$

With the expansion point for the power series method at $x = 0$.

✓ Solution by Maple
Time used: 0.016 (sec). Leaf size: 28

```
Order:=6;
dsolve((x^2+2*x)*diff(y(x),x$2)-2*(x+1)*diff(y(x),x)+2*y(x)=0,y(x),type='series',x=0);
```

$$y(x) = \left(1 + \mathrm{O}\left(x^6\right)\right) c_1 x^2 + c_2 \left(-2 - 2x - \frac{1}{2}x^2 + \mathrm{O}\left(x^6\right)\right)$$

✓ Solution by Mathematica
Time used: 0.045 (sec). Leaf size: 23

```
AsymptoticDSolveValue[(x^2+2*x)*y''[x]-2*(x+1)*y'[x]+2*y[x]==0,y[x],{x,0,5}]
```

$$y(x) \to c_2 x^2 + c_1 \left(\frac{x^2}{4} + x + 1\right)$$

9.16 problem 8, using elementary method

Internal problem ID [4907]

Book: Mathematical Methods in the Physical Sciences. third edition. Mary L. Boas. John Wiley. 2006
Section: Chapter 12, Series Solutions of Differential Equations. Section 1. Miscellaneous problems. page 564
Problem number: 8, using elementary method.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$(x^2 + 2x) y'' - 2(x + 1) y' + 2y = 0$$

✔ Solution by Maple
Time used: 0.0 (sec). Leaf size: 14

```
dsolve((x^2+2*x)*diff(y(x),x$2)-2*(x+1)*diff(y(x),x)+2*y(x)=0,y(x), singsol=all)
```

$$y(x) = c_1 x^2 + c_2 x + c_2$$

✔ Solution by Mathematica
Time used: 0.034 (sec). Leaf size: 19

```
DSolve[(x^2+2*x)*y''[x]-2*(x+1)*y'[x]+2*y[x]==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \rightarrow c_1 x^2 - c_2(x + 1)$$

9.17 problem 9, using series method

Internal problem ID [4908]

Book: Mathematical Methods in the Physical Sciences. third edition. Mary L. Boas. John Wiley. 2006

Section: Chapter 12, Series Solutions of Differential Equations. Section 1. Miscellaneous problems. page 564

Problem number: 9, using series method.

ODE order: 2.

ODE degree: 1.

CAS Maple gives this as type [[_2nd_order, _with_linear_symmetries]]

$$\left(x^2 + 1\right) y'' - 2xy' + 2y = 0$$

With the expansion point for the power series method at $x = 0$.

✔ Solution by Maple
Time used: 0.0 (sec). Leaf size: 18

```
Order:=6;
dsolve((x^2+1)*diff(y(x),x$2)-2*x*diff(y(x),x)+2*y(x)=0,y(x),type='series',x=0);
```

$$y(x) = y(0) + D(y)(0)\, x - y(0)\, x^2$$

✔ Solution by Mathematica
Time used: 0.001 (sec). Leaf size: 18

```
AsymptoticDSolveValue[(x^2+1)*y''[x]-2*x*y'[x]+2*y[x]==0,y[x],{x,0,5}]
```

$$y(x) \rightarrow c_1\left(1 - x^2\right) + c_2 x$$

9.18 problem 9, using elementary method

Internal problem ID [4909]

Book: Mathematical Methods in the Physical Sciences. third edition. Mary L. Boas. John Wiley. 2006
Section: Chapter 12, Series Solutions of Differential Equations. Section 1. Miscellaneous problems. page 564
Problem number: 9, using elementary method.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$\left(x^2 + 1\right) y'' - 2xy' + 2y = 0$$

✓ Solution by Maple
Time used: 0.0 (sec). Leaf size: 16

```
dsolve((x^2+1)*diff(y(x),x$2)-2*x*diff(y(x),x)+2*y(x)=0,y(x), singsol=all)
```

$$y(x) = c_2 x^2 + c_1 x - c_2$$

✓ Solution by Mathematica
Time used: 0.041 (sec). Leaf size: 21

```
DSolve[(x^2+1)*y''[x]-2*x*y'[x]+2*y[x]==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to c_2 x - c_1 (x - i)^2$$

9.19 problem 10, using series method

Internal problem ID [4910]

Book: Mathematical Methods in the Physical Sciences. third edition. Mary L. Boas. John Wiley. 2006
Section: Chapter 12, Series Solutions of Differential Equations. Section 1. Miscellaneous problems. page 564
Problem number: 10, using series method.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$ y'' - 4xy' + \left(4x^2 - 2\right) y = 0 $$

With the expansion point for the power series method at $x = 0$.

✔ Solution by Maple
Time used: 0.0 (sec). Leaf size: 30

```
Order:=6;
dsolve(diff(y(x),x$2)-4*x*diff(y(x),x)+(4*x^2-2)*y(x)=0,y(x),type='series',x=0);
```

$$ y(x) = \left(1 + x^2 + \frac{1}{2}x^4\right) y(0) + \left(x + x^3 + \frac{1}{2}x^5\right) D(y)(0) + O(x^6) $$

✔ Solution by Mathematica
Time used: 0.001 (sec). Leaf size: 34

```
AsymptoticDSolveValue[y''[x]-4*x*y'[x]+(4*x^2-2)*y[x]==0,y[x],{x,0,5}]
```

$$ y(x) \rightarrow c_2 \left(\frac{x^5}{2} + x^3 + x\right) + c_1 \left(\frac{x^4}{2} + x^2 + 1\right) $$

9.20 problem 10, using elementary method

Internal problem ID [4911]

Book: Mathematical Methods in the Physical Sciences. third edition. Mary L. Boas. John Wiley. 2006
Section: Chapter 12, Series Solutions of Differential Equations. Section 1. Miscellaneous problems. page 564
Problem number: 10, using elementary method.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$y'' - 4xy' + \left(4x^2 - 2\right)y = 0$$

✓ Solution by Maple
Time used: 0.0 (sec). Leaf size: 14

```
dsolve(diff(y(x),x$2)-4*x*diff(y(x),x)+(4*x^2-2)*y(x)=0,y(x), singsol=all)
```

$$y(x) = e^{x^2}(c_2 x + c_1)$$

✓ Solution by Mathematica
Time used: 0.02 (sec). Leaf size: 18

```
DSolve[y''[x]-4*x*y'[x]+(4*x^2-2)*y[x]==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to e^{x^2}(c_2 x + c_1)$$