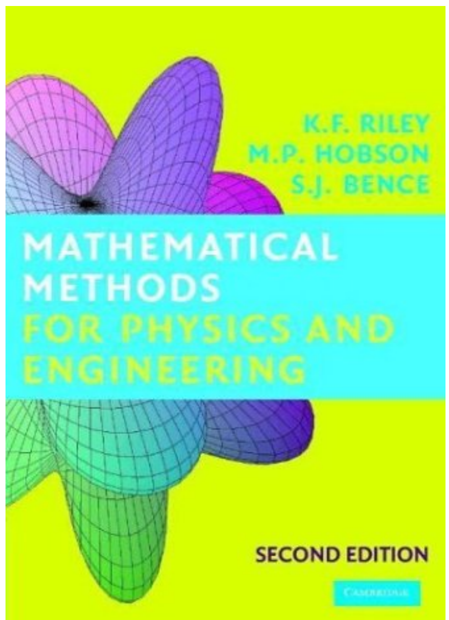
A Solution Manual For

Mathematical methods for physics and engineering, Riley, Hobson, Bence, second edition, 2002



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May 16, 2024

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1 Chapter 14, First order ordinary differential equations. 14.4 Exercises, page 490

1.1 problem Problem 14.2 (a)

Internal problem ID [2486]

Book: Mathematical methods for physics and engineering, Riley, Hobson, Bence, second edition, 2002
Section: Chapter 14, First order ordinary differential equations. 14.4 Exercises, page 490
Problem number: Problem 14.2 (a).
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type **[_separable]**

$$\boxed{y' - x y^3 = 0}$$

✓ Solution by Maple
Time used: 0.0 (sec). Leaf size: 27

```
dsolve(diff(y(x),x)-x*y(x)^3=0,y(x), singsol=all)
```

$$\begin{aligned} y(x) &= \frac{1}{\sqrt{-x^2 + c_1}} \\ y(x) &= -\frac{1}{\sqrt{-x^2 + c_1}} \end{aligned}$$

✓ Solution by Mathematica
Time used: 0.17 (sec). Leaf size: 44

```
DSolve[y'[x]-x*y[x]^3==0,y[x],x,IncludeSingularSolutions -> True]
```

$$\begin{aligned} y(x) &\to -\frac{1}{\sqrt{-x^2 - 2c_1}} \\ y(x) &\to \frac{1}{\sqrt{-x^2 - 2c_1}} \\ y(x) &\to 0 \end{aligned}$$

1.2 problem Problem 14.2 (b)

Internal problem ID [2487]

Book: Mathematical methods for physics and engineering, Riley, Hobson, Bence, second edition, 2002
Section: Chapter 14, First order ordinary differential equations. 14.4 Exercises, page 490
Problem number: Problem 14.2 (b).
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_separable]`

$$\boxed{\frac{y'}{\tan(x)} - \frac{y}{x^2+1} = 0}$$

✓ Solution by Maple
Time used: 0.0 (sec). Leaf size: 19

```
dsolve(diff(y(x),x)/tan(x)-y(x)/(1+x^2)=0,y(x), singsol=all)
```

$$y(x) = c_1 e^{\int \frac{\tan(x)}{x^2+1} dx}$$

✓ Solution by Mathematica
Time used: 9.987 (sec). Leaf size: 34

```
DSolve[y'[x]/Tan[x]-y[x]/(1+x^2)==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to c_1 \exp\left(\int_1^x \frac{\tan(K[1])}{K[1]^2+1} dK[1]\right)$$
$$y(x) \to 0$$

1.3 problem Problem 14.2 (c)

Internal problem ID [2488]

Book: Mathematical methods for physics and engineering, Riley, Hobson, Bence, second edition, 2002
Section: Chapter 14, First order ordinary differential equations. 14.4 Exercises, page 490
Problem number: Problem 14.2 (c).
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_separable]`

$$\boxed{y'x^2 + y^2x - 4y^2 = 0}$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 17

```
dsolve(x^2*diff(y(x),x)+x*y(x)^2=4*y(x)^2,y(x), singsol=all)
```

$$y(x) = \frac{x}{4 + x\ln(x) + c_1x}$$

✓ Solution by Mathematica

Time used: 0.146 (sec). Leaf size: 25

```
DSolve[y'[x]+x*y[x]^2==4*y[x]^2,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to \frac{2}{x^2 - 8x - 2c_1}$$
$$y(x) \to 0$$

1.4 problem Problem 14.3 (a)

Internal problem ID [2489]

Book: Mathematical methods for physics and engineering, Riley, Hobson, Bence, second edition, 2002
Section: Chapter 14, First order ordinary differential equations. 14.4 Exercises, page 490
Problem number: Problem 14.3 (a).
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_exact, _rational, [_1st_order, '_with_symmetry_[F(x)*G(y),0]`

$$y(2y^2x^2+1)\,y'+x(y^4+1)=0$$

✓ Solution by Maple
Time used: 0.015 (sec). Leaf size: 119

```
dsolve(y(x)*(2*x^2*y(x)^2+1)*diff(y(x),x)+x*(y(x)^4+1)=0,y(x), singsol=all)
```

$$y(x)=-\frac{\sqrt{-2-2\sqrt{-4x^4-8c_1x^2+1}}}{2x}$$
$$y(x)=\frac{\sqrt{-2-2\sqrt{-4x^4-8c_1x^2+1}}}{2x}$$
$$y(x)=-\frac{\sqrt{2}\,\sqrt{-1+\sqrt{-4x^4-8c_1x^2+1}}}{2x}$$
$$y(x)=\frac{\sqrt{2}\,\sqrt{-1+\sqrt{-4x^4-8c_1x^2+1}}}{2x}$$

✓ Solution by Mathematica

Time used: 10.416 (sec). Leaf size: 197

```
DSolve[y[x]*(2*x^2*y[x]^2+1)*y'[x]+x*(y[x]^4+1)==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to -\frac{\sqrt{-\frac{1+\sqrt{-4x^4+8c_1x^2+1}}{x^2}}}{\sqrt{2}}$$
$$y(x) \to \frac{\sqrt{-\frac{1+\sqrt{-4x^4+8c_1x^2+1}}{x^2}}}{\sqrt{2}}$$
$$y(x) \to -\frac{\sqrt{\frac{-1+\sqrt{-4x^4+8c_1x^2+1}}{x^2}}}{\sqrt{2}}$$
$$y(x) \to \frac{\sqrt{\frac{-1+\sqrt{-4x^4+8c_1x^2+1}}{x^2}}}{\sqrt{2}}$$
$$y(x) \to -\sqrt[4]{-1}$$
$$y(x) \to \sqrt[4]{-1}$$
$$y(x) \to -(-1)^{3/4}$$
$$y(x) \to (-1)^{3/4}$$

1.5 problem Problem 14.3 (b)

Internal problem ID [2490]

Book: Mathematical methods for physics and engineering, Riley, Hobson, Bence, second edition, 2002
Section: Chapter 14, First order ordinary differential equations. 14.4 Exercises, page 490
Problem number: Problem 14.3 (b).
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_linear]`

$$\boxed{2xy' + y = -3x}$$

✓ Solution by Maple
Time used: 0.016 (sec). Leaf size: 13

```
dsolve(2*x*diff(y(x),x)+3*x+y(x)=0,y(x), singsol=all)
```

$$y(x) = -x + \frac{c_1}{\sqrt{x}}$$

✓ Solution by Mathematica
Time used: 0.027 (sec). Leaf size: 17

```
DSolve[2*x*y'[x]+3*x+y[x]==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to -x + \frac{c_1}{\sqrt{x}}$$

1.6 problem Problem 14.3 (c)

Internal problem ID [2491]

Book: Mathematical methods for physics and engineering, Riley, Hobson, Bence, second edition, 2002
Section: Chapter 14, First order ordinary differential equations. 14.4 Exercises, page 490
Problem number: Problem 14.3 (c).
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[[_1st_order, '_with_symmetry_[F(x)*G(y),0]'], [_Abel, '2nd ty`

$$\boxed{\left(\cos\left(x\right)^{2}+y\sin\left(2x\right)\right)y'+y^{2}=0}$$

✓ Solution by Maple
Time used: 0.016 (sec). Leaf size: 14

```
dsolve((cos(x)^2+y(x)*sin(2*x))*diff(y(x),x)+y(x)^2=0,y(x), singsol=all)
```

$$c_1+y(x)^2\tan\left(x\right)+y(x)=0$$

✓ Solution by Mathematica
Time used: 23.536 (sec). Leaf size: 170

```
DSolve[(Cos[x]^2+y[x]*Sin[2*x])*y'[x]+y[x]^2==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x)\to -\frac{\cot(x)}{2} - \frac{\csc(2x)\sqrt{e^{-\operatorname{arctanh}(\cos(2x))}\left(4c_1\sin(2x)e^{\operatorname{arctanh}(\cos(2x))}+\csc(2x)+(\cos(2x)+2)\cot(2x)\right)}}{2\sqrt{\csc(2x)e^{-\operatorname{arctanh}(\cos(2x))}}}$$

$$y(x)\to -\frac{\cot(x)}{2} + \frac{\csc(2x)\sqrt{e^{-\operatorname{arctanh}(\cos(2x))}\left(4c_1\sin(2x)e^{\operatorname{arctanh}(\cos(2x))}+\csc(2x)+(\cos(2x)+2)\cot(2x)\right)}}{2\sqrt{\csc(2x)e^{-\operatorname{arctanh}(\cos(2x))}}}$$

$$y(x)\to 0$$

1.7 problem Problem 14.5 (a)

Internal problem ID [2492]

Book: Mathematical methods for physics and engineering, Riley, Hobson, Bence, second edition, 2002
Section: Chapter 14, First order ordinary differential equations. 14.4 Exercises, page 490
Problem number: Problem 14.5 (a).
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_linear]`

$$\left(-x^2+1\right)y'+4yx=\left(-x^2+1\right)^{\frac{3}{2}}$$

✓ Solution by Maple
Time used: 0.0 (sec). Leaf size: 42

```
dsolve((1-x^2)*diff(y(x),x)+2*x*y(x)+2*x*y(x)=(1-x^2)^(3/2),y(x), singsol=all)
```

$$y(x)=c_1x^4-x^3\sqrt{-x^2+1}-2c_1x^2+x\sqrt{-x^2+1}+c_1$$

✓ Solution by Mathematica
Time used: 0.116 (sec). Leaf size: 29

```
DSolve[(1-x^2)*y'[x]+2*x*y[x]+2*x*y[x]==(1-x^2)^(3/2),y[x],x,IncludeSingularSolutions -> Tru
```

$$y(x)\to\left(x^2-1\right)^2\left(\frac{x}{\sqrt{1-x^2}}+c_1\right)$$

1.8 problem Problem 14.5 (b)

Internal problem ID [2493]

Book: Mathematical methods for physics and engineering, Riley, Hobson, Bence, second edition, 2002
Section: Chapter 14, First order ordinary differential equations. 14.4 Exercises, page 490
Problem number: Problem 14.5 (b).
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_linear]`

$$y' - y \cot(x) = -\frac{1}{\sin(x)}$$

✓ Solution by Maple
Time used: 0.015 (sec). Leaf size: 11

```
dsolve(diff(y(x),x)-y(x)*cot(x)+1/sin(x)=0,y(x), singsol=all)
```

$$y(x) = c_1 \sin(x) + \cos(x)$$

✓ Solution by Mathematica
Time used: 0.051 (sec). Leaf size: 13

```
DSolve[y'[x]-y[x]*Cot[x]+1/Sin[x]==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to \cos(x) + c_1 \sin(x)$$

1.9 problem Problem 14.5 (c)

Internal problem ID [2494]

Book: Mathematical methods for physics and engineering, Riley, Hobson, Bence, second edition, 2002
Section: Chapter 14, First order ordinary differential equations. 14.4 Exercises, page 490
Problem number: Problem 14.5 (c).
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[[_homogeneous, ‘class G‘], _rational]`

$$(x + y^3)\, y' - y = 0$$

✓ Solution by Maple

Time used: 0.015 (sec). Leaf size: 224

```
dsolve((x+y(x)^3)*diff(y(x),x)=y(x),y(x), singsol=all)
```

$$y(x) = \frac{\left(27x + 3\sqrt{24c_1^3 + 81x^2}\right)^{\frac{2}{3}} - 6c_1}{3\left(27x + 3\sqrt{24c_1^3 + 81x^2}\right)^{\frac{1}{3}}}$$

$$y(x) = -\frac{i\sqrt{3}\left(27x + 3\sqrt{24c_1^3 + 81x^2}\right)^{\frac{2}{3}} + 6i\sqrt{3}\,c_1 + \left(27x + 3\sqrt{24c_1^3 + 81x^2}\right)^{\frac{2}{3}} - 6c_1}{6\left(27x + 3\sqrt{24c_1^3 + 81x^2}\right)^{\frac{1}{3}}}$$

$$y(x) = \frac{i\sqrt{3}\left(27x + 3\sqrt{24c_1^3 + 81x^2}\right)^{\frac{2}{3}} + 6i\sqrt{3}\,c_1 - \left(27x + 3\sqrt{24c_1^3 + 81x^2}\right)^{\frac{2}{3}} + 6c_1}{6\left(27x + 3\sqrt{24c_1^3 + 81x^2}\right)^{\frac{1}{3}}}$$

✓ Solution by Mathematica

Time used: 1.757 (sec). Leaf size: 263

```
DSolve[(x+y[x]^3)*y'[x]==y[x],y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to \frac{2\ 3^{2/3} c_1 - \sqrt[3]{3}\left(-9x + \sqrt{81x^2 + 24c_1{}^3}\right)^{2/3}}{3\sqrt[3]{-9x + \sqrt{81x^2 + 24c_1{}^3}}}$$

$$y(x) \to \frac{\sqrt[3]{3}\left(1 - i\sqrt{3}\right)\left(-9x + \sqrt{81x^2 + 24c_1{}^3}\right)^{2/3} - 2\sqrt[6]{3}\left(\sqrt{3} + 3i\right)c_1}{6\sqrt[3]{-9x + \sqrt{81x^2 + 24c_1{}^3}}}$$

$$y(x) \to \frac{\sqrt[3]{3}\left(1 + i\sqrt{3}\right)\left(-9x + \sqrt{81x^2 + 24c_1{}^3}\right)^{2/3} - 2\sqrt[6]{3}\left(\sqrt{3} - 3i\right)c_1}{6\sqrt[3]{-9x + \sqrt{81x^2 + 24c_1{}^3}}}$$

$$y(x) \to 0$$

1.10 problem Problem 14.6

Internal problem ID [2495]

Book: Mathematical methods for physics and engineering, Riley, Hobson, Bence, second edition, 2002
Section: Chapter 14, First order ordinary differential equations. 14.4 Exercises, page 490
Problem number: Problem 14.6.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_rational, _Bernoulli]`

$$\boxed{y' + \frac{2x^2 + y^2 + x}{yx} = 0}$$

✓ Solution by Maple
Time used: 0.016 (sec). Leaf size: 49

```
dsolve(diff(y(x),x) = - (2*x^2+y(x)^2+x)/(x*y(x)),y(x), singsol=all)
```

$$y(x) = -\frac{\sqrt{-9x^4 - 6x^3 + 9c_1}}{3x}$$
$$y(x) = \frac{\sqrt{-9x^4 - 6x^3 + 9c_1}}{3x}$$

✓ Solution by Mathematica
Time used: 0.251 (sec). Leaf size: 56

```
DSolve[y'[x] == - (2*x^2+y[x]^2+x)/(x*y[x]),y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to -\frac{\sqrt{-x^4 - \frac{2x^3}{3} + c_1}}{x}$$
$$y(x) \to \frac{\sqrt{-x^4 - \frac{2x^3}{3} + c_1}}{x}$$

1.11 problem Problem 14.11

Internal problem ID [2496]

Book: Mathematical methods for physics and engineering, Riley, Hobson, Bence, second edition, 2002
Section: Chapter 14, First order ordinary differential equations. 14.4 Exercises, page 490
Problem number: Problem 14.11.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[[_homogeneous, ‘class A‘], _rational, [_Abel, ‘2nd type‘, ‘cl`

$$\boxed{(y-x)\,y'+3y=-2x}$$

✓ Solution by Maple
Time used: 0.016 (sec). Leaf size: 26

```
dsolve((y(x)-x)*diff(y(x),x)+2*x+3*y(x)=0,y(x), singsol=all)
```

$$y(x)=x\left(-1+\tan\left(\operatorname{RootOf}\left(-4\_Z+\ln\left(\sec\left(\_Z\right)^{2}\right)+2\ln\left(x\right)+2c_{1}\right)\right)\right)$$

✓ Solution by Mathematica
Time used: 0.037 (sec). Leaf size: 45

```
DSolve[(y[x]-x)*y'[x]+2*x+3*y[x]==0,y[x],x,IncludeSingularSolutions -> True]
```

$$\text{Solve}\left[\frac{1}{2}\log\left(\frac{y(x)^2}{x^2}+\frac{2y(x)}{x}+2\right)-2\arctan\left(\frac{y(x)}{x}+1\right)=-\log(x)+c_1,y(x)\right]$$

1.12 problem Problem 14.14

Internal problem ID [2497]

Book: Mathematical methods for physics and engineering, Riley, Hobson, Bence, second edition, 2002
Section: Chapter 14, First order ordinary differential equations. 14.4 Exercises, page 490
Problem number: Problem 14.14.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[[_homogeneous, ‘class C‘], [_Abel, ‘2nd type‘, ‘class C‘], _d`

$$\boxed{y' - \frac{1}{x + 2y + 1} = 0}$$

✓ Solution by Maple
Time used: 0.016 (sec). Leaf size: 21

```
dsolve(diff(y(x),x) = 1/(x+2*y(x)+1),y(x), singsol=all)
```

$$y(x) = -\operatorname{LambertW}\left(-\frac{c_1 \mathrm{e}^{-\frac{x}{2}-\frac{3}{2}}}{2}\right) - \frac{x}{2} - \frac{3}{2}$$

✓ Solution by Mathematica
Time used: 60.047 (sec). Leaf size: 34

```
DSolve[y'[x] == 1/(x+2*y[x]+1),y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to \frac{1}{2}\left(-2W\left(-\frac{1}{2}c_1 e^{-\frac{x}{2}-\frac{3}{2}}\right) - x - 3\right)$$

1.13 problem Problem 14.15

Internal problem ID [2498]

Book: Mathematical methods for physics and engineering, Riley, Hobson, Bence, second edition, 2002
Section: Chapter 14, First order ordinary differential equations. 14.4 Exercises, page 490
Problem number: Problem 14.15.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[[_homogeneous, ‘class C‘], _rational, [_Abel, ‘2nd type‘, ‘cl`

$$\boxed{y' + \frac{y+x}{3x+3y-4} = 0}$$

✓ Solution by Maple
Time used: 0.031 (sec). Leaf size: 21

```
dsolve(diff(y(x),x) = - (x+y(x))/(3*x+3*y(x)-4),y(x), singsol=all)
```

$$y(x) = \frac{2\,\mathrm{LambertW}\left(\frac{3\,e^{x-3-c_1}}{2}\right)}{3} - x + 2$$

✓ Solution by Mathematica
Time used: 3.788 (sec). Leaf size: 33

```
DSolve[y'[x] == - (x+y[x])/(3*x+3*y[x]-4),y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to \frac{2}{3}W\left(-e^{x-1+c_1}\right) - x + 2$$
$$y(x) \to 2 - x$$

1.14 problem Problem 14.16

Internal problem ID [2499]

Book: Mathematical methods for physics and engineering, Riley, Hobson, Bence, second edition, 2002
Section: Chapter 14, First order ordinary differential equations. 14.4 Exercises, page 490
Problem number: Problem 14.16.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_separable]`

$$\boxed{y' - \tan(x)\cos(y)(\cos(y) + \sin(y)) = 0}$$

✓ Solution by Maple
Time used: 0.125 (sec). Leaf size: 11

```
dsolve(diff(y(x),x) = tan(x)*cos(y(x))*( cos(y(x)) + sin(y(x)) ),y(x), singsol=all)
```

$$y(x) = \arctan(-1 + \sec(x) c_1)$$

✓ Solution by Mathematica
Time used: 60.547 (sec). Leaf size: 143

```
DSolve[y'[x]==Tan[x]*Cos[y[x]]*( Cos[y[x]] + Sin[y[x]] ),y[x],x,IncludeSingularSolutions ->
```

$$y(x) \to -\arccos\left(-\frac{\cos(x)}{\sqrt{\cos(2x) - 2e^{\frac{c_1}{2}}\cos(x) + 1 + e^{c_1}}}\right)$$
$$y(x) \to \arccos\left(-\frac{\cos(x)}{\sqrt{\cos(2x) - 2e^{\frac{c_1}{2}}\cos(x) + 1 + e^{c_1}}}\right)$$
$$y(x) \to -\arccos\left(\frac{\cos(x)}{\sqrt{\cos(2x) - 2e^{\frac{c_1}{2}}\cos(x) + 1 + e^{c_1}}}\right)$$
$$y(x) \to \arccos\left(\frac{\cos(x)}{\sqrt{\cos(2x) - 2e^{\frac{c_1}{2}}\cos(x) + 1 + e^{c_1}}}\right)$$

1.15 problem Problem 14.17

Internal problem ID [2500]

Book: Mathematical methods for physics and engineering, Riley, Hobson, Bence, second edition, 2002
Section: Chapter 14, First order ordinary differential equations. 14.4 Exercises, page 490
Problem number: Problem 14.17.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[[_homogeneous, ‘class G‘], _exact, _rational, [_Abel, ‘2nd ty`

$$\boxed{x(1-2x^2y)\,y'+y-3y^2x^2=0}$$

With initial conditions

$$\left[y(1)=\frac{1}{2}\right]$$

✓ Solution by Maple
Time used: 0.094 (sec). Leaf size: 35

```
dsolve([x*(1-2*x^2*y(x))*diff(y(x),x) +y(x) = 3*x^2*y(x)^2,y(1) = 1/2],y(x), singsol=all)
```

$$y(x)=\frac{1-\sqrt{1-x}}{2x^2}$$
$$y(x)=\frac{1+\sqrt{1-x}}{2x^2}$$

✓ Solution by Mathematica
Time used: 0.599 (sec). Leaf size: 53

```
DSolve[{x*(1-2*x^2*y[x])*y'[x] +y[x] == 3*x^2*y[x]^2,y[1]==1/2},y[x],x,IncludeSingularSoluti
```

$$y(x)\to\frac{x-\sqrt{-((x-1)x^2)}}{2x^3}$$
$$y(x)\to\frac{\sqrt{-((x-1)x^2)}+x}{2x^3}$$

1.16 problem Problem 14.23 (a)

Internal problem ID [2501]

Book: Mathematical methods for physics and engineering, Riley, Hobson, Bence, second edition, 2002
Section: Chapter 14, First order ordinary differential equations. 14.4 Exercises, page 490
Problem number: Problem 14.23 (a) .
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_linear]`

$$\boxed{y' + \frac{xy}{a^2 + x^2} = x}$$

✓ Solution by Maple
Time used: 0.016 (sec). Leaf size: 26

```
dsolve(diff(y(x),x)+ (x*y(x))/(a^2+x^2)=x,y(x), singsol=all)
```

$$y(x) = \frac{a^2}{3} + \frac{x^2}{3} + \frac{c_1}{\sqrt{a^2 + x^2}}$$

✓ Solution by Mathematica
Time used: 0.043 (sec). Leaf size: 31

```
DSolve[y'[x]+ (x*y[x])/(a^2+x^2)==x,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to \frac{1}{3}\left(a^2 + x^2\right) + \frac{c_1}{\sqrt{a^2 + x^2}}$$

1.17 problem Problem 14.23 (b)

Internal problem ID [2502]

Book: Mathematical methods for physics and engineering, Riley, Hobson, Bence, second edition, 2002
Section: Chapter 14, First order ordinary differential equations. 14.4 Exercises, page 490
Problem number: Problem 14.23 (b) .
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_separable]`

$$\boxed{y' - \frac{4y^2}{x^2} + y^2 = 0}$$

✓ Solution by Maple
Time used: 0.0 (sec). Leaf size: 16

```
dsolve(diff(y(x),x)= 4*y(x)^2/x^2 - y(x)^2,y(x), singsol=all)
```

$$y(x) = \frac{x}{c_1 x + x^2 + 4}$$

✓ Solution by Mathematica
Time used: 0.15 (sec). Leaf size: 24

```
DSolve[y'[x]== 4*y[x]^2/x^2 - y[x]^2,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to \frac{x}{x^2 - c_1 x + 4}$$
$$y(x) \to 0$$

1.18 problem Problem 14.24 (a)

Internal problem ID [2503]

Book: Mathematical methods for physics and engineering, Riley, Hobson, Bence, second edition, 2002
Section: Chapter 14, First order ordinary differential equations. 14.4 Exercises, page 490
Problem number: Problem 14.24 (a) .
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_linear]`

$$\boxed{y' - \frac{y}{x} = 1}$$

With initial conditions

$$[y(1) = -1]$$

✓ Solution by Maple
Time used: 0.0 (sec). Leaf size: 10

```
dsolve([diff(y(x),x)-y(x)/x=1,y(1) = -1],y(x), singsol=all)
```

$$y(x) = x(-1 + \ln(x))$$

✓ Solution by Mathematica
Time used: 0.025 (sec). Leaf size: 11

```
DSolve[{y'[x]-y[x]/x==1,y[1]==-1},y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to x(\log(x) - 1)$$

1.19 problem Problem 14.24 (b)

Internal problem ID [2504]

Book: Mathematical methods for physics and engineering, Riley, Hobson, Bence, second edition, 2002
Section: Chapter 14, First order ordinary differential equations. 14.4 Exercises, page 490
Problem number: Problem 14.24 (b) .
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_linear]`

$$\boxed{y' - y \tan(x) = 1}$$

With initial conditions

$$\left[y\left(\frac{\pi}{4}\right) = 3\right]$$

✓ Solution by Maple
Time used: 0.015 (sec). Leaf size: 13

```
dsolve([diff(y(x),x)-y(x)*tan(x)=1,y(1/4*Pi) = 3],y(x), singsol=all)
```

$$y(x) = \tan(x) + \sec(x)\sqrt{2}$$

✓ Solution by Mathematica
Time used: 0.049 (sec). Leaf size: 16

```
DSolve[{y'[x]-y[x]*Tan[x]==1,y[Pi/4]==3},y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to \left(\sin(x) + \sqrt{2}\right)\sec(x)$$

1.20 problem Problem 14.24 (c)

Internal problem ID [2505]

Book: Mathematical methods for physics and engineering, Riley, Hobson, Bence, second edition, 2002
Section: Chapter 14, First order ordinary differential equations. 14.4 Exercises, page 490
Problem number: Problem 14.24 (c) .
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[[_homogeneous, ‘class A‘], _rational, _Riccati]`

$$\boxed{y' - \frac{y^2}{x^2} = \frac{1}{4}}$$

With initial conditions

$$[y(1) = 1]$$

✓ Solution by Maple
Time used: 0.032 (sec). Leaf size: 17

```
dsolve([diff(y(x),x)-y(x)^2/x^2=1/4,y(1) = 1],y(x), singsol=all)
```

$$y(x) = \frac{x(\ln(x) - 4)}{2\ln(x) - 4}$$

✓ Solution by Mathematica
Time used: 0.132 (sec). Leaf size: 20

```
DSolve[{y'[x]-y[x]^2/x^2==1/4,y[1]==1},y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to \frac{x(\log(x) - 4)}{2(\log(x) - 2)}$$

1.21 problem Problem 14.24 (d)

Internal problem ID [2506]

Book: Mathematical methods for physics and engineering, Riley, Hobson, Bence, second edition, 2002
Section: Chapter 14, First order ordinary differential equations. 14.4 Exercises, page 490
Problem number: Problem 14.24 (d) .
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[[_homogeneous, ‘class A‘], _rational, _Riccati]`

$$\boxed{y' - \frac{y^2}{x^2} = \frac{1}{4}}$$

✓ Solution by Maple
Time used: 0.0 (sec). Leaf size: 21

```
dsolve(diff(y(x),x)-y(x)^2/x^2=1/4,y(x), singsol=all)
```

$$y(x) = \frac{x(\ln(x) + c_1 - 2)}{2\ln(x) + 2c_1}$$

✓ Solution by Mathematica
Time used: 0.096 (sec). Leaf size: 36

```
DSolve[y'[x]-y[x]^2/x^2==1/4,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to \frac{x(\log(x) - 2 + 4c_1)}{2(\log(x) + 4c_1)}$$
$$y(x) \to \frac{x}{2}$$

1.22 problem Problem 14.26

Internal problem ID [2507]

Book: Mathematical methods for physics and engineering, Riley, Hobson, Bence, second edition, 2002
Section: Chapter 14, First order ordinary differential equations. 14.4 Exercises, page 490
Problem number: Problem 14.26.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_linear]`

$$\boxed{y' \sin(x) + 2\cos(x)\, y = 1}$$

With initial conditions

$$\left[y\left(\frac{\pi}{2}\right) = 1\right]$$

✓ Solution by Maple
Time used: 0.015 (sec). Leaf size: 10

```
dsolve([sin(x)*diff(y(x),x)+2*y(x)*cos(x)=1,y(1/2*Pi) = 1],y(x), singsol=all)
```

$$y(x) = \frac{1}{\cos(x) + 1}$$

✓ Solution by Mathematica
Time used: 0.046 (sec). Leaf size: 14

```
DSolve[{Sin[x]*y'[x]+2*y[x]*Cos[x]==1,y[Pi/2]==1},y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to \tan\left(\frac{x}{2}\right)\csc(x)$$

1.23 problem Problem 14.28

Internal problem ID [2508]

Book: Mathematical methods for physics and engineering, Riley, Hobson, Bence, second edition, 2002
Section: Chapter 14, First order ordinary differential equations. 14.4 Exercises, page 490
Problem number: Problem 14.28.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[[_homogeneous, ‘class C‘], _rational, [_Abel, ‘2nd type‘, ‘cl`

$$\boxed{(5x+y-7)\,y'-3y=3x+3}$$

✓ Solution by Maple
Time used: 0.609 (sec). Leaf size: 217

```
dsolve((5*x+y(x)-7)*diff(y(x),x)=3*(x+y(x)+1),y(x), singsol=all)
```

$y(x)$

$$= \frac{(x-5)\left(i\sqrt{3}-1\right)\left(216\sqrt{c_1\left(-2+x\right)^2\left(-\frac{1}{108}+\left(-2+x\right)^2c_1\right)}+1-216(-2+x)^2c_1}{i\sqrt{3}\left(216\sqrt{c_1\left(-2+x\right)^2\left(-\frac{1}{108}+\left(-2+x\right)^2c_1\right)}+1-216\left(-2+x\right)^2c_1\right)^{\frac{2}{3}}-i\sqrt{3}-\left(216\sqrt{c_1\left(-2+x\right)}\right.}$$

✓ Solution by Mathematica
Time used: 60.172 (sec). Leaf size: 1626

```
DSolve[(5*x+y[x]-7)*y'[x]==3*(x+y[x]+1),y[x],x,IncludeSingularSolutions -> True]
```

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1.24 problem Problem 14.29

Internal problem ID [2509]

Book: Mathematical methods for physics and engineering, Riley, Hobson, Bence, second edition, 2002
Section: Chapter 14, First order ordinary differential equations. 14.4 Exercises, page 490
Problem number: Problem 14.29.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[[_homogeneous, ‘class G‘], _rational, _Bernoulli]`

$$\boxed{xy' + y - \frac{y^2}{x^{\frac{3}{2}}} = 0}$$

With initial conditions

$$[y(1) = 1]$$

✓ Solution by Maple

Time used: 0.047 (sec). Leaf size: 18

```
dsolve([x*diff(y(x),x)+y(x)-y(x)^2/x^(3/2)=0,y(1) = 1],y(x), singsol=all)
```

$$y(x) = \frac{5x^{\frac{3}{2}}}{3x^{\frac{5}{2}} + 2}$$

✓ Solution by Mathematica

Time used: 0.162 (sec). Leaf size: 23

```
DSolve[{x*y'[x]+y[x]-y[x]^2/x^(3/2)==0,y[1]==1},y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to \frac{5x^{3/2}}{3x^{5/2} + 2}$$

1.25 problem Problem 14.30 (a)

Internal problem ID [2510]

Book: Mathematical methods for physics and engineering, Riley, Hobson, Bence, second edition, 2002
Section: Chapter 14, First order ordinary differential equations. 14.4 Exercises, page 490
Problem number: Problem 14.30 (a).
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[[_1st_order, '_with_symmetry_[F(x)*G(y),0]']]`

$$\boxed{(2\sin(y) - x) y' - \tan(y) = 0}$$

With initial conditions

$$[y(0) = 0]$$

✓ Solution by Maple
Time used: 0.015 (sec). Leaf size: 5

```
dsolve([(2*sin(y(x))-x)*diff(y(x),x)=tan(y(x)),y(0) = 0],y(x), singsol=all)
```

$$y(x) = 0$$

✓ Solution by Mathematica
Time used: 0.004 (sec). Leaf size: 6

```
DSolve[{(2*Sin[y[x]]-x)*y'[x]==Tan[y[x]],y[0]==0},y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to 0$$

1.26 problem Problem 14.30 (b)

Internal problem ID [2511]

Book: Mathematical methods for physics and engineering, Riley, Hobson, Bence, second edition, 2002
Section: Chapter 14, First order ordinary differential equations. 14.4 Exercises, page 490
Problem number: Problem 14.30 (b).
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[[_1st_order, '_with_symmetry_[F(x)*G(y),0]']]`

$$\boxed{(2\sin(y)-x)\,y'-\tan(y)=0}$$

With initial conditions

$$\left[y(0)=\frac{\pi}{2}\right]$$

✓ Solution by Maple

Time used: 10.359 (sec). Leaf size: 18

```
dsolve([(2*sin(y(x))-x)*diff(y(x),x)=tan(y(x)),y(0) = 1/2*Pi],y(x), singsol=all)
```

$$y(x)=\arcsin\left(\frac{x}{2}+\frac{\sqrt{x^2+4}}{2}\right)$$

✓ Solution by Mathematica

Time used: 18.018 (sec). Leaf size: 67

```
DSolve[{(2*Sin[y[x]]-x)*y'[x]==Tan[y[x]],y[0]==Pi/2},y[x],x,IncludeSingularSolutions -> True
```

$$y(x)\to\cot^{-1}\left(\sqrt{\frac{x^2}{2}-\frac{1}{2}\sqrt{x^4+4x^2}}\right)$$
$$y(x)\to\cot^{-1}\left(\frac{\sqrt{x^2+\sqrt{x^2(x^2+4)}}}{\sqrt{2}}\right)$$

1.27 problem Problem 14.31

Internal problem ID [2512]

Book: Mathematical methods for physics and engineering, Riley, Hobson, Bence, second edition, 2002
Section: Chapter 14, First order ordinary differential equations. 14.4 Exercises, page 490
Problem number: Problem 14.31.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _missing_x], _Liouville, [_2nd_order, _reducible`

$$\boxed{y'' + {y'}^2 + y' = 0}$$

With initial conditions

$$[y(0) = 0]$$

✓ Solution by Maple
Time used: 0.032 (sec). Leaf size: 18

```
dsolve([diff(y(x),x$2)+ (diff(y(x),x))^2+diff(y(x),x)=0,y(0) = 0],y(x), singsol=all)
```

$$y(x) = \ln\left(c_2 e^x - c_2 + 1\right) - x$$

✓ Solution by Mathematica
Time used: 0.395 (sec). Leaf size: 54

```
DSolve[{y''[x]+(y'[x])^2+y'[x]==0,y[0]==0},y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to \log\left(-e^x\right) - \log\left(e^x\right) - i\pi$$
$$y(x) \to -\log\left(e^x\right) + \log\left(-e^x + e^{c_1}\right) - \log\left(-1 + e^{c_1}\right)$$

2 Chapter 15, Higher order ordinary differential equations. 15.4 Exercises, page 523

2.1 problem Problem 15.1

Internal problem ID [2513]

Book: Mathematical methods for physics and engineering, Riley, Hobson, Bence, second edition, 2002
Section: Chapter 15, Higher order ordinary differential equations. 15.4 Exercises, page 523
Problem number: Problem 15.1.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _linear, _nonhomogeneous]]`

$$\boxed{x'' + \omega_0^2 x = a \cos(\omega t)}$$

With initial conditions

$$[x(0) = 0, x'(0) = 0]$$

✓ Solution by Maple
Time used: 0.032 (sec). Leaf size: 28

```
dsolve([diff(x(t),t$2)+ (omega__0)^2*x(t)=a*cos(omega*t),x(0) = 0, D(x)(0) = 0],x(t), singso
```

$$x(t) = \frac{a(\cos(\omega_0 t) - \cos(\omega t))}{\omega^2 - \omega_0^2}$$

✓ Solution by Mathematica
Time used: 0.371 (sec). Leaf size: 33

```
DSolve[{x''[t]+(Subscript[\[Omega],0])^2*x[t]==a*Cos[\[Omega]*t],{x[0]==0,x'[0]==0}},x[t],t,
```

$$x(t) \to \frac{a(\cos(t\omega_0) - \cos(t\omega))}{\omega^2 - \omega_0^2}$$

2.2 problem Problem 15.2(a)

Internal problem ID [2514]

Book: Mathematical methods for physics and engineering, Riley, Hobson, Bence, second edition, 2002
Section: Chapter 15, Higher order ordinary differential equations. 15.4 Exercises, page 523
Problem number: Problem 15.2(a).
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _missing_x]]`

$$\boxed{f'' + 2f' + 5f = 0}$$

With initial conditions

$$[f(0) = 1, f'(0) = 0]$$

✓ Solution by Maple
Time used: 0.016 (sec). Leaf size: 21

```
dsolve([diff(f(t),t$2)+2*diff(f(t),t)+5*f(t)=0,f(0) = 1, D(f)(0) = 0],f(t), singsol=all)
```

$$f(t) = \frac{e^{-t}(\sin(2t) + 2\cos(2t))}{2}$$

✓ Solution by Mathematica
Time used: 0.021 (sec). Leaf size: 25

```
DSolve[{f''[t]+2*f'[t]+5*f[t]==0,{f[0]==1,f'[0]==0}},f[t],t,IncludeSingularSolutions -> True
```

$$f(t) \to \frac{1}{2}e^{-t}(\sin(2t) + 2\cos(2t))$$

2.3 problem Problem 15.2(b)

Internal problem ID [2515]

Book: Mathematical methods for physics and engineering, Riley, Hobson, Bence, second edition, 2002
Section: Chapter 15, Higher order ordinary differential equations. 15.4 Exercises, page 523
Problem number: Problem 15.2(b).
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _linear, _nonhomogeneous]]`

$$\boxed{f'' + 2f' + 5f = e^{-t} \cos (3t)}$$

With initial conditions

$$[f(0) = 0, f'(0) = 0]$$

✓ Solution by Maple
Time used: 0.031 (sec). Leaf size: 25

```
dsolve([diff(f(t),t$2)+2*diff(f(t),t)+5*f(t)=exp(-t)*cos(3*t),f(0) = 0, D(f)(0) = 0],f(t), s
```

$$f(t) = -\frac{\left(-2\cos(t)^2 + 1 + 4\cos(t)^3 - 3\cos(t)\right) e^{-t}}{5}$$

✓ Solution by Mathematica
Time used: 0.118 (sec). Leaf size: 34

```
DSolve[{f''[t]+2*f'[t]+5*f[t]==Exp[-t]*Cos[3*t],{f[0]==0,f'[0]==0}},f[t],t,IncludeSingularSo
```

$$f(t) \to \frac{2}{5} e^{-t} \sin^2\left(\frac{t}{2}\right) (2\cos(t) + 2\cos(2t) + 1)$$

2.4 problem Problem 15.4

Internal problem ID [2516]

Book: Mathematical methods for physics and engineering, Riley, Hobson, Bence, second edition, 2002
Section: Chapter 15, Higher order ordinary differential equations. 15.4 Exercises, page 523
Problem number: Problem 15.4.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$\boxed{f'' + 6f' + 9f = \mathrm{e}^{-t}}$$

With initial conditions

$$[f(0) = 0, f'(0) = \lambda]$$

✓ Solution by Maple
Time used: 0.031 (sec). Leaf size: 26

```
dsolve([diff(f(t),t$2)+6*diff(f(t),t)+9*f(t)=exp(-t),f(0) = 0, D(f)(0) = lambda],f(t), sings
```

$$f(t) = \frac{(-1 + (4\lambda - 2)\, t)\, \mathrm{e}^{-3t}}{4} + \frac{\mathrm{e}^{-t}}{4}$$

✓ Solution by Mathematica
Time used: 0.034 (sec). Leaf size: 28

```
DSolve[{f''[t]+6*f'[t]+9*f[t]==Exp[-t],{f[0]==0,f'[0]==\[Lambda]}},f[t],t,IncludeSingularSol
```

$$f(t) \to \frac{1}{4} e^{-3t}\left((4\lambda - 2)t + e^{2t} - 1\right)$$

2.5 problem Problem 15.5(a)

Internal problem ID [2517]

Book: Mathematical methods for physics and engineering, Riley, Hobson, Bence, second edition, 2002
Section: Chapter 15, Higher order ordinary differential equations. 15.4 Exercises, page 523
Problem number: Problem 15.5(a).
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$\boxed{f'' + 8f' + 12f = 12\,\mathrm{e}^{-4t}}$$

With initial conditions

$$[f(0) = 0, f'(0) = 0]$$

✓ Solution by Maple
Time used: 0.016 (sec). Leaf size: 23

```
dsolve([diff(f(t),t$2)+8*diff(f(t),t)+12*f(t)=12*exp(-4*t),f(0) = 0, D(f)(0) = 0],f(t), sing
```

$$f(t) = \frac{3\,\mathrm{e}^{-2t}}{2} + \frac{3\,\mathrm{e}^{-6t}}{2} - 3\,\mathrm{e}^{-4t}$$

✓ Solution by Mathematica
Time used: 0.022 (sec). Leaf size: 23

```
DSolve[{f''[t]+8*f'[t]+12*f[t]==12*Exp[-4*t],{f[0]==0,f'[0]==0}},f[t],t,IncludeSingularSolut
```

$$f(t) \to \frac{3}{2}e^{-6t}\left(e^{2t} - 1\right)^2$$

2.6 problem Problem 15.5(b)

Internal problem ID [2518]

Book: Mathematical methods for physics and engineering, Riley, Hobson, Bence, second edition, 2002
Section: Chapter 15, Higher order ordinary differential equations. 15.4 Exercises, page 523
Problem number: Problem 15.5(b).
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$\boxed{f'' + 8f' + 12f = 12\,\mathrm{e}^{-4t}}$$

With initial conditions

$$[f(0) = 0, f'(0) = -2]$$

✓ Solution by Maple

Time used: 0.016 (sec). Leaf size: 21

```
dsolve([diff(f(t),t$2)+8*diff(f(t),t)+12*f(t)=12*exp(-4*t),f(0) = 0, D(f)(0) = -2],f(t), sin
```

$$f(t) = \mathrm{e}^{-2t} + 2\,\mathrm{e}^{-6t} - 3\,\mathrm{e}^{-4t}$$

✓ Solution by Mathematica

Time used: 0.019 (sec). Leaf size: 25

```
DSolve[{f''[t]+8*f'[t]+12*f[t]==12*Exp[-4*t],{f[0]==0,f'[0]==-2}},f[t],t,IncludeSingularSolu
```

$$f(t) \to e^{-6t}\left(-3e^{2t} + e^{4t} + 2\right)$$

2.7 problem Problem 15.7

Internal problem ID [2519]

Book: Mathematical methods for physics and engineering, Riley, Hobson, Bence, second edition, 2002
Section: Chapter 15, Higher order ordinary differential equations. 15.4 Exercises, page 523
Problem number: Problem 15.7.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$\boxed{y'' + 2y' + y = 4\,\mathrm{e}^{-x}}$$

✓ Solution by Maple
Time used: 0.0 (sec). Leaf size: 19

```
dsolve(diff(y(x),x$2)+2*diff(y(x),x)+y(x)=4*exp(-x),y(x), singsol=all)
```

$$y(x) = \mathrm{e}^{-x}\left(c_1 x + 2x^2 + c_2\right)$$

✓ Solution by Mathematica
Time used: 0.026 (sec). Leaf size: 23

```
DSolve[y''[x]+2*y'[x]+y[x]==4*Exp[-x],y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to e^{-x}\left(2x^2 + c_2 x + c_1\right)$$

2.8 problem Problem 15.9(a)

Internal problem ID [2520]

Book: Mathematical methods for physics and engineering, Riley, Hobson, Bence, second edition, 2002
Section: Chapter 15, Higher order ordinary differential equations. 15.4 Exercises, page 523
Problem number: Problem 15.9(a).
ODE order: 3.
ODE degree: 1.

CAS Maple gives this as type `[[_3rd_order, _with_linear_symmetries]]`

$$\boxed{y''' - 12y' + 16y = 32x - 8}$$

✓ Solution by Maple
Time used: 0.0 (sec). Leaf size: 31

```
dsolve(diff(y(x),x$3)-12*diff(y(x),x)+16*y(x)=32*x-8,y(x), singsol=all)
```

$$y(x) = \left((2x+1)\,\mathrm{e}^{4x} + (c_3 x + c_2)\,\mathrm{e}^{6x} + c_1\right)\mathrm{e}^{-4x}$$

✓ Solution by Mathematica
Time used: 0.004 (sec). Leaf size: 35

```
DSolve[y'''[x]-12*y'[x]+16*y[x]==32*x-8,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to c_1 e^{-4x} + c_2 e^{2x} + x\left(2 + c_3 e^{2x}\right) + 1$$

2.9 problem Problem 15.9(b)

Internal problem ID [2521]

Book: Mathematical methods for physics and engineering, Riley, Hobson, Bence, second edition, 2002
Section: Chapter 15, Higher order ordinary differential equations. 15.4 Exercises, page 523
Problem number: Problem 15.9(b).
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries], [_2nd_order, _reducibl`

$$0 = -\frac{y''}{y} + \frac{y'^2}{y^2} - \frac{2a\coth(2ax)\,y'}{y} + 2a^2$$

✓ Solution by Maple

Time used: 0.032 (sec). Leaf size: 53

```
dsolve(diff( 1/y(x)*diff(y(x),x),x)+(2*a*coth(2*a*x))*(1/y(x)*diff(y(x),x))=2*a^2,y(x), sing
```

$$y(x) = \mathrm{e}^{\frac{-x\,a^2+c_1\,\operatorname{arctanh}\left(\mathrm{e}^{2ax}\right)-c_2}{a}}\sqrt{\mathrm{e}^{ax}-1}\,\sqrt{\mathrm{e}^{ax}+1}\,\sqrt{\mathrm{e}^{2ax}+1}$$

✓ Solution by Mathematica

Time used: 60.504 (sec). Leaf size: 287

```
DSolve[D[1/y[x]*y'[x],x]+(2*a*Coth[1/y[x]*y'[x]])==2*a^2,y[x],x,IncludeSingularSolutions ->
```

$$y(x) \rightarrow c_2\exp\left(\frac{-\operatorname{PolyLog}\left(2, \frac{(a+1)\exp\left(-2\text{InverseFunction}\left[\frac{-((a+1)\log(1-\tanh(\#1)))+(a-1)\log(\tanh(\#1)+1)+2\log(1-a\tanh(\#1}{2\left(a^2-1\right)}\right.\right.}{a-1}\right.}{}\right.$$

2.10 problem Problem 15.21

Internal problem ID [2522]

Book: Mathematical methods for physics and engineering, Riley, Hobson, Bence, second edition, 2002
Section: Chapter 15, Higher order ordinary differential equations. 15.4 Exercises, page 523
Problem number: Problem 15.21.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$\boxed{x^2y'' - xy' + y = x}$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 18

```
dsolve(x^2*diff(y(x),x$2)-x*diff(y(x),x)+y(x)=x,y(x), singsol=all)
```

$$y(x) = x\left(c_2 + \ln(x)\,c_1 + \frac{\ln(x)^2}{2}\right)$$

✓ Solution by Mathematica

Time used: 0.019 (sec). Leaf size: 25

```
DSolve[x^2*y''[x]-x*y'[x]+y[x]==x,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to \frac{1}{2}x\left(\log^2(x) + 2c_2\log(x) + 2c_1\right)$$

2.11 problem Problem 15.22

Internal problem ID [2523]

Book: Mathematical methods for physics and engineering, Riley, Hobson, Bence, second edition, 2002
Section: Chapter 15, Higher order ordinary differential equations. 15.4 Exercises, page 523
Problem number: Problem 15.22.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _exact, _linear, _nonhomogeneous]]`

$$\boxed{(x+1)^2 y'' + 3(x+1) y' + y = x^2}$$

✓ Solution by Maple
Time used: 0.015 (sec). Leaf size: 39

```
dsolve((x+1)^2*diff(y(x),x$2)+3*(x+1)*diff(y(x),x)+y(x)=x^2,y(x), singsol=all)
```

$$y(x) = \frac{(18c_1 - 6)\ln(x+1) + 2x^3 - 3x^2 + 6x + 18c_2}{18x + 18}$$

✓ Solution by Mathematica
Time used: 0.047 (sec). Leaf size: 44

```
DSolve[(x+1)^2*y''[x]+3*(x+1)*y'[x]+y[x]==x^2,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to \frac{2x^3 - 3x^2 + 6x + 6(-1 + 3c_2)\log(x+1) + 18c_1}{18(x+1)}$$

2.12 problem Problem 15.23

Internal problem ID [2524]

Book: Mathematical methods for physics and engineering, Riley, Hobson, Bence, second edition, 2002
Section: Chapter 15, Higher order ordinary differential equations. 15.4 Exercises, page 523
Problem number: Problem 15.23.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries], [_2nd_order, _linear,`

$$(x-2)\,y''+3y'+\frac{4y}{x^2}=0$$

✓ Solution by Maple
Time used: 0.0 (sec). Leaf size: 26

```
dsolve((x-2)*diff(y(x),x$2)+3*diff(y(x),x)+4*y(x)/x^2=0,y(x), singsol=all)
```

$$y(x)=\frac{c_2x^3+3c_1x-4c_1}{x\,(-2+x)^2}$$

✓ Solution by Mathematica
Time used: 0.074 (sec). Leaf size: 45

```
DSolve[(x-2)*y''[x]+3*y'[x]+4*y[x]/x^2==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x)\to\frac{6c_1x^3+3c_2x-4c_2}{6\sqrt{2-x}(x-2)^{3/2}x}$$

2.13 problem Problem 15.24(a)

Internal problem ID [2525]

Book: Mathematical methods for physics and engineering, Riley, Hobson, Bence, second edition, 2002
Section: Chapter 15, Higher order ordinary differential equations. 15.4 Exercises, page 523
Problem number: Problem 15.24(a).
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _linear, _nonhomogeneous]]`

$$\boxed{y'' - y = x^n}$$

✓ Solution by Maple
Time used: 0.015 (sec). Leaf size: 85

```
dsolve(diff(y(x),x$2)-y(x)=x^n,y(x), singsol=all)
```

$$y(x) = -\frac{\left(-e^{\frac{3x}{2}} x^{\frac{n}{2}} \operatorname{WhittakerM}\left(\frac{n}{2}, \frac{n}{2}+\frac{1}{2}, x\right) + \left(x^n(n\Gamma(n,-x) - \Gamma(n+1))\left(-x\right)^{-n} - 2c_1 e^{2x} + e^x x^n - 2c_2\right)(n}{2n+2}$$

✓ Solution by Mathematica
Time used: 0.055 (sec). Leaf size: 58

```
DSolve[y''[x]-y[x]==x^n,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to -\frac{1}{2}e^{-x}x^n(-x)^{-n}\Gamma(n+1,-x) - \frac{1}{2}e^x\Gamma(n+1,x) + c_1e^x + c_2e^{-x}$$

2.14 problem Problem 15.24(b)

Internal problem ID [2526]

Book: Mathematical methods for physics and engineering, Riley, Hobson, Bence, second edition, 2002
Section: Chapter 15, Higher order ordinary differential equations. 15.4 Exercises, page 523
Problem number: Problem 15.24(b).
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _linear, _nonhomogeneous]]`

$$\boxed{y'' - 2y' + y = 2x\,\mathrm{e}^{x}}$$

✓ Solution by Maple
Time used: 0.016 (sec). Leaf size: 17

```
dsolve(diff(y(x),x$2)-2*diff(y(x),x)+y(x)=2*x*exp(x),y(x), singsol=all)
```

$$y(x) = \mathrm{e}^{x}\left(c_2 + c_1 x + \frac{1}{3}x^3\right)$$

✓ Solution by Mathematica
Time used: 0.023 (sec). Leaf size: 25

```
DSolve[y''[x]-2*y'[x]+y[x]==2*x*Exp[x],y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to \frac{1}{3}e^{x}\left(x^3 + 3c_2 x + 3c_1\right)$$

2.15 problem Problem 15.33

Internal problem ID [2527]

Book: Mathematical methods for physics and engineering, Riley, Hobson, Bence, second edition, 2002
Section: Chapter 15, Higher order ordinary differential equations. 15.4 Exercises, page 523
Problem number: Problem 15.33.
ODE order: 3.
ODE degree: 1.

CAS Maple gives this as type `[[_3rd_order, _exact, _nonlinear]]`

Solve

$$2yy''' + 2(y + 3y')\,y'' + 2(y')^2 - \sin(x) = 0$$

✓ Solution by Maple

Time used: 0.015 (sec). Leaf size: 81

```
dsolve(2*y(x)*diff(y(x),x$3)+2*(y(x)+3*diff(y(x),x))*diff(y(x),x$2)+2*(diff(y(x),x))^2=sin(x
```

$$y(x) = -\frac{\sqrt{2}\sqrt{-4\left(\left(-\frac{\cos(x)}{4} + \frac{\sin(x)}{4} + c_1(x-1) + c_3\right)e^x - c_2\right)e^x\,e^{-x}}}{2}$$

$$y(x) = \frac{\sqrt{2}\sqrt{-4\left(\left(-\frac{\cos(x)}{4} + \frac{\sin(x)}{4} + c_1(x-1) + c_3\right)e^x - c_2\right)e^x\,e^{-x}}}{2}$$

✓ Solution by Mathematica

Time used: 0.473 (sec). Leaf size: 88

```
DSolve[2*y[x]*y'''[x]+2*(y[x]+3*y'[x])*y''[x]+2*(y'[x])^2==Sin[x],y[x],x,IncludeSingularSolu
```

$$y(x) \to -\frac{\sqrt{-\sin(x) + \cos(x) + 2c_1x + 2c_3e^{-x} - 2c_1 - 4c_2}}{\sqrt{2}}$$

$$y(x) \to \frac{\sqrt{-\sin(x) + \cos(x) + 2c_1x + 2c_3e^{-x} - 2c_1 - 4c_2}}{\sqrt{2}}$$

2.16 problem Problem 15.34

Internal problem ID [2528]

Book: Mathematical methods for physics and engineering, Riley, Hobson, Bence, second edition, 2002
Section: Chapter 15, Higher order ordinary differential equations. 15.4 Exercises, page 523
Problem number: Problem 15.34.
ODE order: 3.
ODE degree: 1.

CAS Maple gives this as type `[[_3rd_order, _missing_y]]`

$$\boxed{xy''' + 2y'' = Ax}$$

✓ Solution by Maple
Time used: 0.0 (sec). Leaf size: 20

```
dsolve(x*diff(y(x),x$3)+2*diff(y(x),x$2)=A*x,y(x), singsol=all)
```

$$y(x) = \frac{A\,x^3}{18} - \ln(x)\,c_1 + c_2 x + c_3$$

✓ Solution by Mathematica
Time used: 0.048 (sec). Leaf size: 26

```
DSolve[x*y'''[x]+2*y''[x]==A*x,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to \frac{Ax^3}{18} + c_3 x - c_1 \log(x) + c_2$$

2.17 problem Problem 15.35

Internal problem ID [2529]

Book: Mathematical methods for physics and engineering, Riley, Hobson, Bence, second edition, 2002
Section: Chapter 15, Higher order ordinary differential equations. 15.4 Exercises, page 523
Problem number: Problem 15.35.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _linear, _nonhomogeneous]]`

$$\boxed{y'' + 4xy' + (4x^2 + 6)\, y = \mathrm{e}^{-x^2} \sin(2x)}$$

✓ Solution by Maple
Time used: 0.016 (sec). Leaf size: 30

```
dsolve(diff(y(x),x$2)+4*x*diff(y(x),x)+(4*x^2+6)*y(x)=exp(-x^2)*sin(2*x),y(x), singsol=all)
```

$$y(x) = -\frac{((x - 4c_2)\cos(2x) - 4\sin(2x)\, c_1)\, \mathrm{e}^{-x^2}}{4}$$

✓ Solution by Mathematica
Time used: 0.13 (sec). Leaf size: 52

```
DSolve[y''[x]+4*x*y'[x]+(4*x^2+6)*y[x]==Exp[-x^2]*Sin[2*x],y[x],x,IncludeSingularSolutions -
```

$$y(x) \to \frac{1}{32} e^{-x(x+2i)} \left(-4x - e^{4ix}(4x + i + 8ic_2) + i + 32c_1\right)$$

3 Chapter 16, Series solutions of ODEs. Section 16.6 Exercises, page 550

3.1 problem Problem 16.1

Internal problem ID [2530]

Book: Mathematical methods for physics and engineering, Riley, Hobson, Bence, second edition, 2002
Section: Chapter 16, Series solutions of ODEs. Section 16.6 Exercises, page 550
Problem number: Problem 16.1.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[_Gegenbauer]`

$$\boxed{(-z^2+1)\,y''-3zy'+\lambda y=0}$$

With the expansion point for the power series method at $z=0$.

✓ Solution by Maple
Time used: 0.0 (sec). Leaf size: 63

```
Order:=6;
dsolve((1-z^2)*diff(y(z),z$2)-3*z*diff(y(z),z)+lambda*y(z)=0,y(z),type='series',z=0);
```

$$y(z)=\left(1-\frac{\lambda\,z^2}{2}+\frac{\lambda(\lambda-8)\,z^4}{24}\right)y(0)+\left(z-\frac{(\lambda-3)\,z^3}{6}+\frac{(\lambda-3)\,(\lambda-15)\,z^5}{120}\right)D(y)\,(0)+O(z^6)$$

✓ Solution by Mathematica
Time used: 0.001 (sec). Leaf size: 80

```
AsymptoticDSolveValue[(1-z^2)*y''[z]-3*z*y'[z]+\[Lambda]*y[z]==0,y[z],{z,0,5}]
```

$$y(z)\to c_2\left(\frac{\lambda^2z^5}{120}-\frac{3\lambda z^5}{20}+\frac{3z^5}{8}-\frac{\lambda z^3}{6}+\frac{z^3}{2}+z\right)+c_1\left(\frac{\lambda^2z^4}{24}-\frac{\lambda z^4}{3}-\frac{\lambda z^2}{2}+1\right)$$

3.2 problem Problem 16.2

Internal problem ID [2531]

Book: Mathematical methods for physics and engineering, Riley, Hobson, Bence, second edition, 2002
Section: Chapter 16, Series solutions of ODEs. Section 16.6 Exercises, page 550
Problem number: Problem 16.2.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$\boxed{4zy'' + 2(1-z)\, y' - y = 0}$$

With the expansion point for the power series method at $z = 0$.

✓ Solution by Maple

Time used: 0.016 (sec). Leaf size: 44

```
Order:=6;
dsolve(4*z*diff(y(z),z$2)+2*(1-z)*diff(y(z),z)-y(z)=0,y(z),type='series',z=0);
```

$$y(z) = c_1\sqrt{z}\left(1 + \frac{1}{3}z + \frac{1}{15}z^2 + \frac{1}{105}z^3 + \frac{1}{945}z^4 + \frac{1}{10395}z^5 + \mathrm{O}\left(z^6\right)\right) + c_2\left(1 + \frac{1}{2}z + \frac{1}{8}z^2 + \frac{1}{48}z^3 + \frac{1}{384}z^4 + \frac{1}{3840}z^5 + \mathrm{O}\left(z^6\right)\right)$$

✓ Solution by Mathematica

Time used: 0.003 (sec). Leaf size: 85

```
AsymptoticDSolveValue[4*z*y''[z]+2*(1-z)*y'[z]-y[z]==0,y[z],{z,0,5}]
```

$$y(z) \to c_1\sqrt{z}\left(\frac{z^5}{10395} + \frac{z^4}{945} + \frac{z^3}{105} + \frac{z^2}{15} + \frac{z}{3} + 1\right) + c_2\left(\frac{z^5}{3840} + \frac{z^4}{384} + \frac{z^3}{48} + \frac{z^2}{8} + \frac{z}{2} + 1\right)$$

3.3 problem Problem 16.3

Internal problem ID [2532]

Book: Mathematical methods for physics and engineering, Riley, Hobson, Bence, second edition, 2002
Section: Chapter 16, Series solutions of ODEs. Section 16.6 Exercises, page 550
Problem number: Problem 16.3.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_Emden, _Fowler], [_2nd_order, _linear, '_with_symmetry_[0,F`

$$zy'' - 2y' + 9z^5y = 0$$

With the expansion point for the power series method at $z = 0$.

✓ Solution by Maple
Time used: 0.016 (sec). Leaf size: 28

```
Order:=7;
dsolve(z*diff(y(z),z$2)-2*diff(y(z),z)+9*z^5*y(z)=0,y(z),type='series',z=0);
```

$$y(z) = c_1z^3\left(1 - \frac{1}{6}z^6 + \mathrm{O}\left(z^7\right)\right) + c_2\left(12 - 6z^6 + \mathrm{O}\left(z^7\right)\right)$$

✓ Solution by Mathematica
Time used: 0.004 (sec). Leaf size: 12

```
AsymptoticDSolveValue[z*y''[z]-2*y'[z]+9*z^5*y[z]==0,y[z],{z,0,6}]
```

$$y(z) \to c_2z^3 + c_1$$

3.4 problem Problem 16.4

Internal problem ID [2533]

Book: Mathematical methods for physics and engineering, Riley, Hobson, Bence, second edition, 2002
Section: Chapter 16, Series solutions of ODEs. Section 16.6 Exercises, page 550
Problem number: Problem 16.4.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$\boxed{f'' + 2(z-1)\, f' + 4f = 0}$$

With the expansion point for the power series method at $z = 0$.

✓ Solution by Maple

Time used: 0.016 (sec). Leaf size: 52

```
Order:=6;
dsolve(diff(f(z),z$2)+2*(z-1)*diff(f(z),z)+4*f(z)=0,f(z),type='series',z=0);
```

$$f(z) = \left(1-2z^2-\frac{4}{3}z^3+\frac{2}{3}z^4+\frac{14}{15}z^5\right) f(0)+\left(z+z^2-\frac{1}{3}z^3-\frac{5}{6}z^4-\frac{1}{6}z^5\right) D(f)(0)+O\left(z^6\right)$$

✓ Solution by Mathematica

Time used: 0.001 (sec). Leaf size: 127

```
AsymptoticDSolveValue[f''[z]+2*(z-a)*f'[z]+4*f[z]==0,f[z],{z,0,5}]
```

$$f(z) \to c_1\left(-\frac{4}{15}a^3z^5 - \frac{2a^2z^4}{3} + \frac{6az^5}{5} - \frac{4az^3}{3} + \frac{4z^4}{3} - 2z^2 + 1\right) + c_2\left(\frac{2a^4z^5}{15} + \frac{a^3z^4}{3} - \frac{4a^2z^5}{5} + \frac{2a^2z^3}{3} - \frac{7az^4}{6} + az^2 + \frac{z^5}{2} - z^3 + z\right)$$

3.5 problem Problem 16.6

Internal problem ID [2534]

Book: Mathematical methods for physics and engineering, Riley, Hobson, Bence, second edition, 2002
Section: Chapter 16, Series solutions of ODEs. Section 16.6 Exercises, page 550
Problem number: Problem 16.6.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$z^2y'' - \frac{3zy'}{2} + (z+1)\,y = 0$$

With the expansion point for the power series method at $z = 0$.

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 47

```
Order:=6;
dsolve(z^2*diff(y(z),z$2)-3/2*z*diff(y(z),z)+(1+z)*y(z)=0,y(z),type='series',z=0);
```

$$y(z) = c_1\sqrt{z}\left(1 + 2z - 2z^2 + \frac{4}{9}z^3 - \frac{2}{45}z^4 + \frac{4}{1575}z^5 + \mathrm{O}\left(z^6\right)\right) + c_2z^2\left(1 - \frac{2}{5}z + \frac{2}{35}z^2 - \frac{4}{945}z^3 + \frac{2}{10395}z^4 - \frac{4}{675675}z^5 + \mathrm{O}\left(z^6\right)\right)$$

✓ Solution by Mathematica

Time used: 0.003 (sec). Leaf size: 84

```
AsymptoticDSolveValue[z^2*y''[z]-3/2*z*y'[z]+(1+z)*y[z]==0,y[z],{z,0,5}]
```

$$y(z) \to c_1\left(-\frac{4z^5}{675675} + \frac{2z^4}{10395} - \frac{4z^3}{945} + \frac{2z^2}{35} - \frac{2z}{5} + 1\right)z^2 + c_2\left(\frac{4z^5}{1575} - \frac{2z^4}{45} + \frac{4z^3}{9} - 2z^2 + 2z + 1\right)\sqrt{z}$$

3.6 problem Problem 16.8

Internal problem ID [2535]

Book: Mathematical methods for physics and engineering, Riley, Hobson, Bence, second edition, 2002
Section: Chapter 16, Series solutions of ODEs. Section 16.6 Exercises, page 550
Problem number: Problem 16.8.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[_Lienard]`

$$zy'' - 2y' + zy = 0$$

With the expansion point for the power series method at $z = 0$.

✓ Solution by Maple
Time used: 0.016 (sec). Leaf size: 32

```
Order:=6;
dsolve(z*diff(y(z),z$2)-2*diff(y(z),z)+z*y(z)=0,y(z),type='series',z=0);
```

$$y(z) = c_1 z^3 \left(1 - \frac{1}{10}z^2 + \frac{1}{280}z^4 + O\left(z^6\right)\right) + c_2 \left(12 + 6z^2 - \frac{3}{2}z^4 + O\left(z^6\right)\right)$$

✓ Solution by Mathematica
Time used: 0.009 (sec). Leaf size: 44

```
AsymptoticDSolveValue[z*y''[z]-2*y'[z]+z*y[z]==0,y[z],{z,0,5}]
```

$$y(z) \to c_1 \left(-\frac{z^4}{8} + \frac{z^2}{2} + 1\right) + c_2 \left(\frac{z^7}{280} - \frac{z^5}{10} + z^3\right)$$

3.7 problem Problem 16.9

Internal problem ID [2536]

Book: Mathematical methods for physics and engineering, Riley, Hobson, Bence, second edition, 2002
Section: Chapter 16, Series solutions of ODEs. Section 16.6 Exercises, page 550
Problem number: Problem 16.9.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _exact, _linear, _homogeneous]]`

$$y'' - 2zy' - 2y = 0$$

With the expansion point for the power series method at $z = 0$.

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 32

```
Order:=6;
dsolve(diff(y(z),z$2)-2*z*diff(y(z),z)-2*y(z)=0,y(z),type='series',z=0);
```

$$y(z) = \left(1 + z^2 + \frac{1}{2}z^4\right) y(0) + \left(z + \frac{2}{3}z^3 + \frac{4}{15}z^5\right) D(y)\,(0) + O\left(z^6\right)$$

✓ Solution by Mathematica

Time used: 0.001 (sec). Leaf size: 38

```
AsymptoticDSolveValue[y''[z]-2*z*y'[z]-2*y[z]==0,y[z],{z,0,5}]
```

$$y(z) \to c_2\left(\frac{4z^5}{15} + \frac{2z^3}{3} + z\right) + c_1\left(\frac{z^4}{2} + z^2 + 1\right)$$

3.8 problem Problem 16.10

Internal problem ID [2537]

Book: Mathematical methods for physics and engineering, Riley, Hobson, Bence, second edition, 2002
Section: Chapter 16, Series solutions of ODEs. Section 16.6 Exercises, page 550
Problem number: Problem 16.10.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[_Jacobi]`

$$\boxed{z(1-z)\,y''+(1-z)\,y'+\lambda y=0}$$

With the expansion point for the power series method at $z=0$.

✓ Solution by Maple
Time used: 0.016 (sec). Leaf size: 261

```
Order:=6;
dsolve(z*(1-z)*diff(y(z),z$2)+(1-z)*diff(y(z),z)+lambda*y(z)=0,y(z),type='series',z=0);
```

$$y(z)=\left(2\lambda z+\left(\frac{1}{4}\lambda-\frac{3}{4}\lambda^2\right)z^2+\left(-\frac{37}{108}\lambda^2+\frac{2}{27}\lambda+\frac{11}{108}\lambda^3\right)z^3+\left(\frac{139}{1728}\lambda^3-\frac{649}{3456}\lambda^2+\frac{1}{32}\lambda-\frac{25}{3456}\lambda^4\right)z^4+\left(-\frac{13}{1600}\lambda^4+\frac{8467}{144000}\lambda^3-\frac{2527}{21600}\lambda^2+\frac{2}{125}\lambda+\frac{137}{432000}\lambda^5\right)z^5+\mathrm{O}\left(z^6\right)\right)c_2+\left(1-\lambda z+\frac{1}{4}(-1+\lambda)\lambda z^2-\frac{1}{36}\lambda\left(\lambda^2-5\lambda+4\right)z^3+\frac{1}{576}\lambda\left(\lambda^3-14\lambda^2+49\lambda-36\right)z^4-\frac{1}{14400}\lambda(-1+\lambda)(\lambda-4)(\lambda-16)(\lambda-9)z^5+\mathrm{O}\left(z^6\right)\right)\left(c_2\ln(z)+c_1\right)$$

✓ Solution by Mathematica

Time used: 0.005 (sec). Leaf size: 940

```
AsymptoticDSolveValue[z*(1-z)*y''[z]+(1-z)*y'[z]+\[Lambda]*y[z]==0,y[z],{z,0,5}]
```

$$y(z) \to \left(\frac{1}{25}\left(\lambda^2 - \frac{1}{4}(\lambda^2-\lambda)\,\lambda - \frac{1}{9}\left(\lambda^2 - \frac{1}{4}(\lambda^2-\lambda)\,\lambda - \lambda\right)\lambda\right.\right.$$

$$-\frac{1}{16}\left(\lambda^2 - \frac{1}{4}(\lambda^2-\lambda)\,\lambda - \frac{1}{9}\left(\lambda^2 - \frac{1}{4}(\lambda^2-\lambda)\,\lambda - \lambda\right)\lambda - \lambda\right)\lambda - \lambda\Big)\, z^5$$

$$+\frac{1}{16}\left(\lambda^2 - \frac{1}{4}(\lambda^2-\lambda)\,\lambda - \frac{1}{9}\left(\lambda^2 - \frac{1}{4}(\lambda^2-\lambda)\,\lambda - \lambda\right)\lambda - \lambda\right) z^4$$

$$+\frac{1}{9}\left(\lambda^2 - \frac{1}{4}(\lambda^2-\lambda)\,\lambda - \lambda\right) z^3 + \frac{1}{4}(\lambda^2-\lambda)\, z^2 - \lambda z + 1\Big)\, c_1$$

$$+c_2\left(-\frac{2}{125}\left(\lambda^2 - \frac{1}{4}(\lambda^2-\lambda)\,\lambda - \frac{1}{9}\left(\lambda^2 - \frac{1}{4}(\lambda^2-\lambda)\,\lambda - \lambda\right)\lambda\right.\right.$$

$$-\frac{1}{16}\left(\lambda^2 - \frac{1}{4}(\lambda^2-\lambda)\,\lambda - \frac{1}{9}\left(\lambda^2 - \frac{1}{4}(\lambda^2-\lambda)\,\lambda - \lambda\right)\lambda - \lambda\right)\lambda - \lambda\Big)\, z^5 + \frac{1}{25}\left(\frac{\lambda^3}{2}\right.$$

$$-2\lambda^2 + \frac{1}{4}(\lambda^2-\lambda)\,\lambda + \frac{2}{27}\left(\lambda^2 - \frac{1}{4}(\lambda^2-\lambda)\,\lambda - \lambda\right)\lambda - \frac{1}{9}\left(\frac{\lambda^3}{2} - 2\lambda^2 + \frac{1}{4}(\lambda^2-\lambda)\,\lambda\right)\lambda$$

$$+\frac{1}{32}\left(\lambda^2 - \frac{1}{4}(\lambda^2-\lambda)\,\lambda - \frac{1}{9}\left(\lambda^2 - \frac{1}{4}(\lambda^2-\lambda)\,\lambda - \lambda\right)\lambda - \lambda\right)\lambda$$

$$-\frac{1}{16}\left(\frac{\lambda^3}{2} - 2\lambda^2 + \frac{1}{4}(\lambda^2-\lambda)\,\lambda + \frac{2}{27}\left(\lambda^2 - \frac{1}{4}(\lambda^2-\lambda)\,\lambda - \lambda\right)\lambda - \frac{1}{9}\left(\frac{\lambda^3}{2} - 2\lambda^2 + \frac{1}{4}(\lambda^2-\lambda)\,\lambda\right)\lambda\right)\lambda\Big)\, z^5$$

$$-\frac{1}{32}\left(\lambda^2 - \frac{1}{4}(\lambda^2-\lambda)\,\lambda - \frac{1}{9}\left(\lambda^2 - \frac{1}{4}(\lambda^2-\lambda)\,\lambda - \lambda\right)\lambda - \lambda\right) z^4 + \frac{1}{16}\left(\frac{\lambda^3}{2} - 2\lambda^2\right.$$

$$+\frac{1}{4}(\lambda^2-\lambda)\,\lambda + \frac{2}{27}\left(\lambda^2 - \frac{1}{4}(\lambda^2-\lambda)\,\lambda - \lambda\right)\lambda - \frac{1}{9}\left(\frac{\lambda^3}{2} - 2\lambda^2 + \frac{1}{4}(\lambda^2-\lambda)\,\lambda\right)\lambda\Big)\, z^4$$

$$-\frac{2}{27}\left(\lambda^2 - \frac{1}{4}(\lambda^2-\lambda)\,\lambda - \lambda\right) z^3 + \frac{1}{9}\left(\frac{\lambda^3}{2} - 2\lambda^2 + \frac{1}{4}(\lambda^2-\lambda)\,\lambda\right) z^3 - \frac{\lambda^2 z^2}{2}$$

$$-\frac{1}{4}(\lambda^2-\lambda)\, z^2 + 2\lambda z$$

$$+\left(\frac{1}{25}\left(\lambda^2-\frac{1}{4}(\lambda^2-\lambda)\,\lambda-\frac{1}{9}\left(\lambda^2-\frac{1}{4}(\lambda^2-\lambda)\,\lambda-\lambda\right)\lambda-\frac{1}{16}\left(\lambda^2-\frac{1}{4}(\lambda^2-\lambda)\,\lambda-\frac{1}{9}\left(\lambda^2-\frac{1}{4}(\lambda^2-\lambda)\,\lambda-\lambda\right)\right.\right.\right.$$

$$+\frac{1}{16}\left(\lambda^2 - \frac{1}{4}(\lambda^2-\lambda)\,\lambda - \frac{1}{9}\left(\lambda^2 - \frac{1}{4}(\lambda^2-\lambda)\,\lambda - \lambda\right)\lambda - \lambda\right) z^4$$

$$+\frac{1}{9}\left(\lambda^2 - \frac{1}{4}(\lambda^2-\lambda)\,\lambda - \lambda\right) z^3 + \frac{1}{4}(\lambda^2-\lambda)\, z^2 - \lambda z + 1\Big)\log(z)\Big)$$

3.9 problem Problem 16.11

Internal problem ID [2538]

Book: Mathematical methods for physics and engineering, Riley, Hobson, Bence, second edition, 2002
Section: Chapter 16, Series solutions of ODEs. Section 16.6 Exercises, page 550
Problem number: Problem 16.11.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$zy'' + (2z - 3)\, y' + \frac{4y}{z} = 0$$

With the expansion point for the power series method at $z = 0$.

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 69

```
Order:=6;
dsolve(z*diff(y(z),z$2)+(2*z-3)*diff(y(z),z)+4/z*y(z)=0,y(z),type='series',z=0);
```

$$y(z) = \left((c_2 \ln(z) + c_1) \left(1 - 4z + 6z^2 - \frac{16}{3}z^3 + \frac{10}{3}z^4 - \frac{8}{5}z^5 + \mathrm{O}\left(z^6\right)\right) + \left(6z - 13z^2 + \frac{124}{9}z^3 - \frac{173}{18}z^4 + \frac{374}{75}z^5 + \mathrm{O}\left(z^6\right)\right) c_2 \right) z^2$$

✓ Solution by Mathematica

Time used: 0.005 (sec). Leaf size: 116

```
AsymptoticDSolveValue[z*y''[z]+(2*z-3)*y'[z]+4/z*y[z]==0,y[z],{z,0,5}]
```

$$y(z) \to c_1 \left(-\frac{8z^5}{5} + \frac{10z^4}{3} - \frac{16z^3}{3} + 6z^2 - 4z + 1\right) z^2 + c_2 \left(\left(\frac{374z^5}{75} - \frac{173z^4}{18} + \frac{124z^3}{9} - 13z^2 + 6z\right) z^2 + \left(-\frac{8z^5}{5} + \frac{10z^4}{3} - \frac{16z^3}{3} + 6z^2 - 4z + 1\right) z^2 \log(z) \right)$$

3.10 problem Problem 16.12 (a)

Internal problem ID [2539]

Book: Mathematical methods for physics and engineering, Riley, Hobson, Bence, second edition, 2002
Section: Chapter 16, Series solutions of ODEs. Section 16.6 Exercises, page 550
Problem number: Problem 16.12 (a).
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$\boxed{(z^2 + 5z + 6)\, y'' + 2y = 0}$$

With the expansion point for the power series method at $z = 0$.

✓ Solution by Maple

Time used: 0.016 (sec). Leaf size: 49

```
Order:=6;
dsolve((z^2+5*z+6)*diff(y(z),z$2)+2*y(z)=0,y(z),type='series',z=0);
```

$$y(z) = \left(1 - \frac{1}{6}z^2 + \frac{5}{108}z^3 - \frac{13}{1296}z^4 + \frac{5}{2592}z^5\right) y(0) + \left(z - \frac{1}{18}z^3 + \frac{5}{216}z^4 - \frac{17}{2160}z^5\right) D(y)(0) + O(z^6)$$

✓ Solution by Mathematica

Time used: 0.001 (sec). Leaf size: 63

```
AsymptoticDSolveValue[(z^2+5*z+6)*y''[z]+2*y[z]==0,y[z],{z,0,5}]
```

$$y(z) \to c_2\left(-\frac{17z^5}{2160} + \frac{5z^4}{216} - \frac{z^3}{18} + z\right) + c_1\left(\frac{5z^5}{2592} - \frac{13z^4}{1296} + \frac{5z^3}{108} - \frac{z^2}{6} + 1\right)$$

3.11 problem Problem 16.12 (b)

Internal problem ID [2540]

Book: Mathematical methods for physics and engineering, Riley, Hobson, Bence, second edition, 2002
Section: Chapter 16, Series solutions of ODEs. Section 16.6 Exercises, page 550
Problem number: Problem 16.12 (b).
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_Emden, _Fowler]]`

$$\boxed{(z^2+5z+7)\,y''+2y=0}$$

With the expansion point for the power series method at $z=0$.

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 49

```
Order:=6;
dsolve((z^2+5*z+7)*diff(y(z),z$2)+2*y(z)=0,y(z),type='series',z=0);
```

$$y(z)=\left(1-\frac{1}{7}z^2+\frac{5}{147}z^3-\frac{11}{2058}z^4+\frac{5}{14406}z^5\right)y(0)+\left(z-\frac{1}{21}z^3+\frac{5}{294}z^4-\frac{47}{10290}z^5\right)D(y)(0)+O(z^6)$$

✓ Solution by Mathematica

Time used: 0.001 (sec). Leaf size: 63

```
AsymptoticDSolveValue[(z^2+5*z+7)*y''[z]+2*y[z]==0,y[z],{z,0,5}]
```

$$y(z)\to c_2\left(-\frac{47z^5}{10290}+\frac{5z^4}{294}-\frac{z^3}{21}+z\right)+c_1\left(\frac{5z^5}{14406}-\frac{11z^4}{2058}+\frac{5z^3}{147}-\frac{z^2}{7}+1\right)$$

3.12 problem Problem 16.13

Internal problem ID [2541]

Book: Mathematical methods for physics and engineering, Riley, Hobson, Bence, second edition, 2002
Section: Chapter 16, Series solutions of ODEs. Section 16.6 Exercises, page 550
Problem number: Problem 16.13.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_Emden, _Fowler]]`

$$\boxed{y'' + \frac{y}{z^3} = 0}$$

With the expansion point for the power series method at $z = 0$.

✗ Solution by Maple

```
Order:=6;
dsolve(diff(y(z),z$2)+1/z^3*y(z)=0,y(z),type='series',z=0);
```

No solution found

✓ Solution by Mathematica

Time used: 0.037 (sec). Leaf size: 222

```
AsymptoticDSolveValue[y''[z]+1/z^3*y[z]==0,y[z],{z,0,5}]
```

$$y(z) \to c_1 e^{-\frac{2i}{\sqrt{z}}} z^{3/4} \left(-\frac{468131288625 i z^{9/2}}{8796093022208} + \frac{66891825 i z^{7/2}}{4294967296} - \frac{72765 i z^{5/2}}{8388608} + \frac{105 i z^{3/2}}{8192} + \frac{33424574007825 z^5}{281474976710656} - \frac{14783093325 z^4}{549755813888} + \frac{2837835 z^3}{268435456} - \frac{4725 z^2}{524288} + \frac{15z}{512} - \frac{3i\sqrt{z}}{16} + 1 \right) + c_2 e^{\frac{2i}{\sqrt{z}}} z^{3/4} \left(\frac{468131288625 i z^{9/2}}{8796093022208} - \frac{66891825 i z^{7/2}}{4294967296} + \frac{72765 i z^{5/2}}{8388608} - \frac{105 i z^{3/2}}{8192} + \frac{33424574007825 z^5}{281474976710656} - \right.$$

3.13 problem Problem 16.14

Internal problem ID [2542]

Book: Mathematical methods for physics and engineering, Riley, Hobson, Bence, second edition, 2002
Section: Chapter 16, Series solutions of ODEs. Section 16.6 Exercises, page 550
Problem number: Problem 16.14.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[_Laguerre]`

$$\boxed{zy'' + (1-z)\,y' + \lambda y = 0}$$

With the expansion point for the power series method at $z = 0$.

✓ Solution by Maple
Time used: 0.016 (sec). Leaf size: 309

```
Order:=6;
dsolve(z*diff(y(z),z$2)+(1-z)*diff(y(z),z)+lambda*y(z)=0,y(z),type='series',z=0);
```

$$y(z) = \left((2\lambda+1)\,z + \left(\frac{1}{4}\lambda + \frac{1}{4} - \frac{3}{4}\lambda^2\right)z^2 + \left(-\frac{2}{9}\lambda^2 + \frac{1}{27}\lambda + \frac{1}{18} + \frac{11}{108}\lambda^3\right)z^3 + \left(\frac{7}{192}\lambda^3 - \frac{167}{3456}\lambda^2 + \frac{1}{192}\lambda + \frac{1}{96} - \frac{25}{3456}\lambda^4\right)z^4 + \left(\frac{1}{1500}\lambda - \frac{37}{4320}\lambda^2 + \frac{719}{86400}\lambda^3 + \frac{1}{600} - \frac{61}{21600}\lambda^4 + \frac{137}{432000}\lambda^5\right)z^5 + \mathrm{O}\left(z^6\right)\right)c_2 + \left(1 - \lambda z + \frac{1}{4}(-1+\lambda)\,\lambda z^2 - \frac{1}{36}(\lambda-2)\,(-1+\lambda)\,\lambda z^3 + \frac{1}{576}(\lambda-3)\,(\lambda-2)\,(-1+\lambda)\,\lambda z^4 - \frac{1}{14400}(\lambda-4)\,(\lambda-3)\,(\lambda-2)\,(-1+\lambda)\,\lambda z^5 + \mathrm{O}\left(z^6\right)\right)(c_2\ln(z) + c_1)$$

✓ Solution by Mathematica

Time used: 0.004 (sec). Leaf size: 415

```
AsymptoticDSolveValue[z*y''[z]+(1-z)*y'[z]+\[Lambda]*y[z]==0,y[z],{z,0,5}]
```

$$
\begin{aligned}
y(z) \to\ & c_1\left(-\frac{(\lambda-4)(\lambda-3)(\lambda-2)(\lambda-1)\lambda z^5}{14400}+\frac{1}{576}(\lambda-3)(\lambda-2)(\lambda-1)\lambda z^4\right. \\
& \left.-\frac{1}{36}(\lambda-2)(\lambda-1)\lambda z^3+\frac{1}{4}(\lambda-1)\lambda z^2-\lambda z+1\right) \\
& + c_2\left(\frac{(\lambda-4)(\lambda-3)(\lambda-2)(\lambda-1)z^5}{14400}+\frac{(\lambda-4)(\lambda-3)(\lambda-2)\lambda z^5}{14400}\right. \\
& +\frac{(\lambda-4)(\lambda-3)(\lambda-1)\lambda z^5}{14400}+\frac{(\lambda-4)(\lambda-2)(\lambda-1)\lambda z^5}{14400} \\
& +\frac{137(\lambda-4)(\lambda-3)(\lambda-2)(\lambda-1)\lambda z^5}{432000}+\frac{(\lambda-3)(\lambda-2)(\lambda-1)\lambda z^5}{14400} \\
& -\frac{1}{576}(\lambda-3)(\lambda-2)(\lambda-1)z^4-\frac{1}{576}(\lambda-3)(\lambda-2)\lambda z^4-\frac{1}{576}(\lambda-3)(\lambda-1)\lambda z^4 \\
& -\frac{25(\lambda-3)(\lambda-2)(\lambda-1)\lambda z^4}{3456}-\frac{1}{576}(\lambda-2)(\lambda-1)\lambda z^4+\frac{1}{36}(\lambda-2)(\lambda-1)z^3 \\
& +\frac{1}{36}(\lambda-2)\lambda z^3+\frac{11}{108}(\lambda-2)(\lambda-1)\lambda z^3+\frac{1}{36}(\lambda-1)\lambda z^3-\frac{1}{4}(\lambda-1)z^2-\frac{3}{4}(\lambda-1)\lambda z^2 \\
& -\frac{\lambda z^2}{4}+\left(-\frac{(\lambda-4)(\lambda-3)(\lambda-2)(\lambda-1)\lambda z^5}{14400}+\frac{1}{576}(\lambda-3)(\lambda-2)(\lambda-1)\lambda z^4\right. \\
& \left.\left.-\frac{1}{36}(\lambda-2)(\lambda-1)\lambda z^3+\frac{1}{4}(\lambda-1)\lambda z^2-\lambda z+1\right)\log(z)+2\lambda z+z\right)
\end{aligned}
$$

3.14 problem Problem 16.15

Internal problem ID [2543]

Book: Mathematical methods for physics and engineering, Riley, Hobson, Bence, second edition, 2002
Section: Chapter 16, Series solutions of ODEs. Section 16.6 Exercises, page 550
Problem number: Problem 16.15.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[_Gegenbauer, [_2nd_order, _linear, '_with_symmetry_[0,F(x)]']]`

$$\boxed{(-z^2+1)\, y'' - zy' + m^2 y = 0}$$

With the expansion point for the power series method at $z = 0$.

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 71

```
Order:=6;
dsolve((1-z^2)*diff(y(z),z$2)-z*diff(y(z),z)+m^2*y(z)=0,y(z),type='series',z=0);
```

$$y(z) = \left(1 - \frac{m^2 z^2}{2} + \frac{m^2(m^2-4)\, z^4}{24}\right) y(0) + \left(z - \frac{(m^2-1)\, z^3}{6} + \frac{(m^4-10m^2+9)\, z^5}{120}\right) D(y)\,(0) + O(z^6)$$

✓ Solution by Mathematica

Time used: 0.001 (sec). Leaf size: 88

```
AsymptoticDSolveValue[(1-z^2)*y''[z]-z*y'[z]+m^2*y[z]==0,y[z],{z,0,5}]
```

$$y(z) \to c_2\left(\frac{m^4 z^5}{120} - \frac{m^2 z^5}{12} - \frac{m^2 z^3}{6} + \frac{3z^5}{40} + \frac{z^3}{6} + z\right) + c_1\left(\frac{m^4 z^4}{24} - \frac{m^2 z^4}{6} - \frac{m^2 z^2}{2} + 1\right)$$