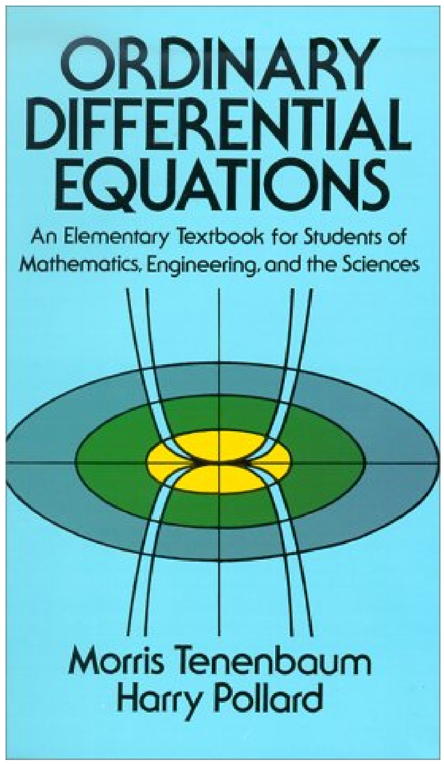
A Solution Manual For

Ordinary Differential Equations, By Tenenbaum and Pollard. Dover, NY 1963



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Contents

1 Chapter 2. Special types of differential equations of the first kind. Lesson 7

1.1 problem First order with homogeneous Coefficients. Exercise 7.2, page 61

Internal problem ID [4427]

Book: Ordinary Differential Equations, By Tenenbaum and Pollard. Dover, NY 1963
Section: Chapter 2. Special types of differential equations of the first kind. Lesson 7
Problem number: First order with homogeneous Coefficients. Exercise 7.2, page 61.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[[_homogeneous, ‘class A‘], _exact, _rational, _dAlembert]`

$$\boxed{2xy + \left(x^2+y^2\right) y' = 0}$$

✓ Solution by Maple

Time used: 0.032 (sec). Leaf size: 209

```
dsolve(2*x*y(x)+(x^2+y(x)^2)*diff(y(x),x)=0,y(x), singsol=all)
```

$$y(x) = -\frac{2\left(c_1x^2 - \frac{\left(4+4\sqrt{4c_1^3x^6+1}\right)^{\frac{2}{3}}}{4}\right)}{\left(4+4\sqrt{4c_1^3x^6+1}\right)^{\frac{1}{3}}\sqrt{c_1}}$$

$$y(x) = -\frac{\left(1+i\sqrt{3}\right)\left(4+4\sqrt{4c_1^3x^6+1}\right)^{\frac{1}{3}}}{4\sqrt{c_1}} - \frac{\sqrt{c_1}\left(i\sqrt{3}-1\right)x^2}{\left(4+4\sqrt{4c_1^3x^6+1}\right)^{\frac{1}{3}}}$$

$$y(x) = \frac{4i\sqrt{3}\,c_1x^2 + i\left(4+4\sqrt{4c_1^3x^6+1}\right)^{\frac{2}{3}}\sqrt{3} + 4c_1x^2 - \left(4+4\sqrt{4c_1^3x^6+1}\right)^{\frac{2}{3}}}{4\left(4+4\sqrt{4c_1^3x^6+1}\right)^{\frac{1}{3}}\sqrt{c_1}}$$

✓ Solution by Mathematica

Time used: 15.191 (sec). Leaf size: 401

```
DSolve[2*x*y[x]+(x^2+y[x]^2)*y'[x]==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x)\to \frac{\sqrt[3]{\sqrt{4x^6+e^{6c_1}}+e^{3c_1}}}{\sqrt[3]{2}}-\frac{\sqrt[3]{2}x^2}{\sqrt[3]{\sqrt{4x^6+e^{6c_1}}+e^{3c_1}}}$$

$$y(x)\to \frac{i2^{2/3}\left(\sqrt{3}+i\right)\left(\sqrt{4x^6+e^{6c_1}}+e^{3c_1}\right)^{2/3}+\sqrt[3]{2}\left(2+2i\sqrt{3}\right)x^2}{4\sqrt[3]{\sqrt{4x^6+e^{6c_1}}+e^{3c_1}}}$$

$$y(x)\to \frac{\left(1-i\sqrt{3}\right)x^2}{2^{2/3}\sqrt[3]{\sqrt{4x^6+e^{6c_1}}+e^{3c_1}}}-\frac{\left(1+i\sqrt{3}\right)\sqrt[3]{\sqrt{4x^6+e^{6c_1}}+e^{3c_1}}}{2\sqrt[3]{2}}$$

$$y(x)\to 0$$

$$y(x)\to \frac{1}{2}\sqrt[6]{x^6}\left(\frac{\left(1-i\sqrt{3}\right)\left(x^6\right)^{2/3}}{x^4}-i\sqrt{3}-1\right)$$

$$y(x)\to \frac{1}{2}\sqrt[6]{x^6}\left(\frac{\left(1+i\sqrt{3}\right)\left(x^6\right)^{2/3}}{x^4}+i\sqrt{3}-1\right)$$

$$y(x)\to \sqrt[6]{x^6}-\frac{\left(x^6\right)^{5/6}}{x^4}$$

1.2 problem First order with homogeneous Coefficients. Exercise 7.3, page 61

Internal problem ID [4428]

Book: Ordinary Differential Equations, By Tenenbaum and Pollard. Dover, NY 1963
Section: Chapter 2. Special types of differential equations of the first kind. Lesson 7
Problem number: First order with homogeneous Coefficients. Exercise 7.3, page 61.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[[_homogeneous, 'class A'], _rational, _dAlembert]`

$$\boxed{\left(x+\sqrt{y^2-xy}\right)y'-y=0}$$

✓ Solution by Maple
Time used: 0.015 (sec). Leaf size: 33

```
dsolve((x+sqrt(y(x)^2-x*y(x)))*diff(y(x),x)-y(x)=0,y(x), singsol=all)
```

$$\frac{\ln\left(y(x)\right)y(x)-c_1y(x)+2\sqrt{y\left(x\right)\left(y\left(x\right)-x\right)}}{y\left(x\right)}=0$$

✓ Solution by Mathematica
Time used: 0.291 (sec). Leaf size: 43

```
DSolve[(x+Sqrt[y[x]^2-x*y[x]])*y'[x]-y[x]==0,y[x],x,IncludeSingularSolutions -> True]
```

$$\text{Solve}\left[\frac{2\sqrt{\frac{y(x)}{x}-1}}{\sqrt{\frac{y(x)}{x}}}+\log\left(\frac{y(x)}{x}\right)=-\log(x)+c_1,y(x)\right]$$

1.3 problem First order with homogeneous Coefficients. Exercise 7.4, page 61

Internal problem ID [4429]

Book: Ordinary Differential Equations, By Tenenbaum and Pollard. Dover, NY 1963
Section: Chapter 2. Special types of differential equations of the first kind. Lesson 7
Problem number: First order with homogeneous Coefficients. Exercise 7.4, page 61.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[[_homogeneous, ‘class A‘], _rational, [_Abel, ‘2nd type‘, ‘cl`

$$\boxed{y - (x - y)\, y' = -x}$$

✓ Solution by Maple
Time used: 0.016 (sec). Leaf size: 24

```
dsolve((x+y(x))-(x-y(x))*diff(y(x),x)=0,y(x), singsol=all)
```

$$y(x) = \tan\left(\mathrm{RootOf}\left(-2\_Z + \ln\left(\sec\left(\_Z\right)^2\right) + 2\ln\left(x\right) + 2c_1\right)\right) x$$

✓ Solution by Mathematica
Time used: 0.032 (sec). Leaf size: 36

```
DSolve[(x+y[x])-(x-y[x])*y'[x]==0,y[x],x,IncludeSingularSolutions -> True]
```

$$\mathrm{Solve}\left[\frac{1}{2}\log\left(\frac{y(x)^2}{x^2} + 1\right) - \arctan\left(\frac{y(x)}{x}\right) = -\log(x) + c_1, y(x)\right]$$

1.4 problem First order with homogeneous Coefficients. Exercise 7.5, page 61

Internal problem ID [4430]

Book: Ordinary Differential Equations, By Tenenbaum and Pollard. Dover, NY 1963
Section: Chapter 2. Special types of differential equations of the first kind. Lesson 7
Problem number: First order with homogeneous Coefficients. Exercise 7.5, page 61.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[[_homogeneous, ‘class A‘], _dAlembert]`

$$\boxed{xy' - y - x\sin\left(\frac{y}{x}\right) = 0}$$

✓ Solution by Maple

Time used: 0.015 (sec). Leaf size: 44

```
dsolve(x*diff(y(x),x)-y(x)-x*sin(y(x)/x)=0,y(x), singsol=all)
```

$$y(x) = \arctan\left(\frac{2xc_1}{x^2c_1^2+1}, \frac{-x^2c_1^2+1}{x^2c_1^2+1}\right)x$$

✓ Solution by Mathematica

Time used: 0.325 (sec). Leaf size: 52

```
DSolve[x*y'[x]-y[x]-x*Sin[y[x]/x]==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to -x\arccos(-\tanh(\log(x)+c_1))$$
$$y(x) \to x\arccos(-\tanh(\log(x)+c_1))$$
$$y(x) \to 0$$
$$y(x) \to -\pi x$$
$$y(x) \to \pi x$$

1.5 problem First order with homogeneous Coefficients. Exercise 7.6, page 61

Internal problem ID [4431]

Book: Ordinary Differential Equations, By Tenenbaum and Pollard. Dover, NY 1963
Section: Chapter 2. Special types of differential equations of the first kind. Lesson 7
Problem number: First order with homogeneous Coefficients. Exercise 7.6, page 61.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[[_homogeneous, 'class A'], _rational, _dAlembert]`

$$\boxed{2yx^2 + y^3 + (xy^2 - 2x^3)\, y' = 0}$$

✓ Solution by Maple
Time used: 0.0 (sec). Leaf size: 22

```
dsolve((2*x^2*y(x)+y(x)^3)+(x*y(x)^2-2*x^3)*diff(y(x),x)=0,y(x), singsol=all)
```

$$y(x) = \sqrt{2}\sqrt{-\frac{1}{\mathrm{LambertW}\left(-2c_1x^4\right)}}\, x$$

✓ Solution by Mathematica
Time used: 5.64 (sec). Leaf size: 66

```
DSolve[(2*x^2*y[x]+y[x]^3)+(x*y[x]^2-2*x^3)*y'[x]==0,y[x],x,IncludeSingularSolutions -> True
```

$$y(x) \to -\frac{i\sqrt{2}x}{\sqrt{W\left(-2e^{-2c_1}x^4\right)}}$$
$$y(x) \to \frac{i\sqrt{2}x}{\sqrt{W\left(-2e^{-2c_1}x^4\right)}}$$
$$y(x) \to 0$$

1.6 problem First order with homogeneous Coefficients. Exercise 7.7, page 61

Internal problem ID [4432]

Book: Ordinary Differential Equations, By Tenenbaum and Pollard. Dover, NY 1963
Section: Chapter 2. Special types of differential equations of the first kind. Lesson 7
Problem number: First order with homogeneous Coefficients. Exercise 7.7, page 61.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[[_homogeneous, ‘class G‘], _dAlembert]`

$$\boxed{y^2 + \left(x\sqrt{y^2 - x^2} - xy\right) y' = 0}$$

✓ Solution by Maple
Time used: 0.015 (sec). Leaf size: 32

```
dsolve(y(x)^2+(x*sqrt(y(x)^2-x^2)-x*y(x))*diff(y(x),x)=0,y(x), singsol=all)
```

$$\frac{-c_1 x y(x) + y(x) + \sqrt{y\left(x\right)^2 - x^2}}{x y\left(x\right)} = 0$$

✓ Solution by Mathematica
Time used: 2.247 (sec). Leaf size: 111

```
DSolve[y[x]^2+(x*Sqrt[y[x]^2-x^2]-x*y[x])*y'[x]==0,y[x],x,IncludeSingularSolutions -> True]
```

$$\text{Solve}\left[-\frac{\sqrt{\frac{y(x)^2}{x^2}-1}\left(\log\left(\sqrt{\frac{y(x)}{x}+1}-1\right)+\log\left(\sqrt{\frac{y(x)}{x}+1}+1\right)\right)}{\sqrt{\frac{y(x)}{x}-1}\sqrt{\frac{y(x)}{x}+1}} - 2\log\left(\sqrt{\frac{y(x)}{x}-1}-\sqrt{\frac{y(x)}{x}+1}\right) = \log(x)+c_1, y(x)\right]$$

1.7 problem First order with homogeneous Coefficients. Exercise 7.8, page 61

Internal problem ID [4433]

Book: Ordinary Differential Equations, By Tenenbaum and Pollard. Dover, NY 1963
Section: Chapter 2. Special types of differential equations of the first kind. Lesson 7
Problem number: First order with homogeneous Coefficients. Exercise 7.8, page 61.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[[_homogeneous, 'class A'], _dAlembert]`

$$\boxed{\frac{y\cos\left(\frac{y}{x}\right)}{x}-\left(\frac{x\sin\left(\frac{y}{x}\right)}{y}+\cos\left(\frac{y}{x}\right)\right)y'=0}$$

✓ Solution by Maple
Time used: 0.031 (sec). Leaf size: 15

```
dsolve(y(x)/x*cos(y(x)/x)-(x/y(x)*sin(y(x)/x)+cos(y(x)/x))*diff(y(x),x)=0,y(x), singsol=all)
```

$$y(x)=\operatorname{RootOf}\left(\_Zxc_1\sin\left(\_Z\right)-1\right)x$$

✓ Solution by Mathematica
Time used: 0.247 (sec). Leaf size: 27

```
DSolve[y[x]/x*Cos[y[x]/x]-(x/y[x]*Sin[y[x]/x]+Cos[y[x]/x])*y'[x]==0,y[x],x,IncludeSingularSo
```

$$\text{Solve}\left[\log\left(\frac{y(x)}{x}\right)+\log\left(\sin\left(\frac{y(x)}{x}\right)\right)=-\log(x)+c_1,y(x)\right]$$

1.8 problem First order with homogeneous Coefficients. Exercise 7.9, page 61

Internal problem ID [4434]

Book: Ordinary Differential Equations, By Tenenbaum and Pollard. Dover, NY 1963
Section: Chapter 2. Special types of differential equations of the first kind. Lesson 7
Problem number: First order with homogeneous Coefficients. Exercise 7.9, page 61.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[[_homogeneous, ‘class A‘], _dAlembert]`

$$\boxed{y + x\ln\left(\frac{y}{x}\right)y' - 2xy' = 0}$$

✓ Solution by Maple
Time used: 0.016 (sec). Leaf size: 16

```
dsolve(y(x)+x*ln(y(x)/x)*diff(y(x),x)-2*x*diff(y(x),x)=0,y(x), singsol=all)
```

$$y(x) = -\frac{\text{LambertW}\left(-\mathrm{e}xc_1\right)}{c_1}$$

✓ Solution by Mathematica
Time used: 5.502 (sec). Leaf size: 35

```
DSolve[y[x]+x*Log[y[x]/x]*y'[x]-2*x*y'[x]==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to -e^{c_1} W\left(-e^{1-c_1}x\right)$$
$$y(x) \to 0$$
$$y(x) \to ex$$

1.9 problem First order with homogeneous Coefficients. Exercise 7.10, page 61

Internal problem ID [4435]

Book: Ordinary Differential Equations, By Tenenbaum and Pollard. Dover, NY 1963
Section: Chapter 2. Special types of differential equations of the first kind. Lesson 7
Problem number: First order with homogeneous Coefficients. Exercise 7.10, page 61.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[[_homogeneous, 'class A'], _dAlembert]`

$$\boxed{2y\,\mathrm{e}^{\frac{x}{y}} + \left(y - 2x\,\mathrm{e}^{\frac{x}{y}}\right)y' = 0}$$

✓ Solution by Maple

Time used: 0.031 (sec). Leaf size: 21

```
dsolve(2*y(x)*exp(x/y(x))+(y(x)-2*x*exp(x/y(x)))*diff(y(x),x)=0,y(x), singsol=all)
```

$$y(x) = \frac{x}{\operatorname{RootOf}\left(-\_Z\mathrm{e}^{-2\,\mathrm{e}^{\_Z}} + c_1 x\right)}$$

✓ Solution by Mathematica

Time used: 0.247 (sec). Leaf size: 29

```
DSolve[2*y[x]*Exp[x/y[x]]+(y[x]-2*x*Exp[x/y[x]])*y'[x]==0,y[x],x,IncludeSingularSolutions ->
```

$$\text{Solve}\left[-2e^{\frac{x}{y(x)}} - \log\left(\frac{y(x)}{x}\right) = \log(x) + c_1, y(x)\right]$$

1.10 problem First order with homogeneous Coefficients. Exercise 7.11, page 61

Internal problem ID [4436]

Book: Ordinary Differential Equations, By Tenenbaum and Pollard. Dover, NY 1963
Section: Chapter 2. Special types of differential equations of the first kind. Lesson 7
Problem number: First order with homogeneous Coefficients. Exercise 7.11, page 61.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[[_homogeneous, ‘class A‘], _dAlembert]`

$$x\,\mathrm{e}^{\frac{y}{x}} - y\sin\left(\frac{y}{x}\right) + x\sin\left(\frac{y}{x}\right)y' = 0$$

✓ Solution by Maple
Time used: 0.032 (sec). Leaf size: 63

```
dsolve((x*exp(y(x)/x)-y(x)*sin(y(x)/x))+x*sin(y(x)/x)*diff(y(x),x)=0,y(x), singsol=all)
```

$$y(x) = \operatorname{RootOf}\left(\mathrm{e}^{2\_Z}\left(4\ln(x)^2\mathrm{e}^{2\_Z} + 8\ln(x)\,\mathrm{e}^{2\_Z}c_1 + 4\,\mathrm{e}^{2\_Z}c_1^2 - 4\ln(x)\sin(\_Z)\,\mathrm{e}^{\_Z} - 4\sin(\_Z)\,\mathrm{e}^{\_Z}c_1 + 2\sin(\_Z)^2 - 1\right)\right)x$$

✓ Solution by Mathematica
Time used: 0.328 (sec). Leaf size: 39

```
DSolve[(x*Exp[y[x]/x]-y[x]*Sin[y[x]/x])+x*Sin[y[x]/x]*y'[x]==0,y[x],x,IncludeSingularSolutio
```

$$\text{Solve}\left[-\frac{1}{2}e^{-\frac{y(x)}{x}}\left(\sin\left(\frac{y(x)}{x}\right) + \cos\left(\frac{y(x)}{x}\right)\right) = -\log(x) + c_1, y(x)\right]$$

1.11 problem First order with homogeneous Coefficients. Exercise 7.12, page 61

Internal problem ID [4437]

Book: Ordinary Differential Equations, By Tenenbaum and Pollard. Dover, NY 1963
Section: Chapter 2. Special types of differential equations of the first kind. Lesson 7
Problem number: First order with homogeneous Coefficients. Exercise 7.12, page 61.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[[_homogeneous, ‘class A‘], _rational, _Bernoulli]`

$$\boxed{y^2 - 2xyy' = -x^2}$$

With initial conditions

$$[y(-1) = 0]$$

✓ Solution by Maple
Time used: 0.062 (sec). Leaf size: 23

```
dsolve([(x^2+y(x)^2)=2*x*y(x)*diff(y(x),x),y(-1) = 0],y(x), singsol=all)
```

$$y(x) = \sqrt{x\,(1+x)}$$
$$y(x) = -\sqrt{x\,(1+x)}$$

✓ Solution by Mathematica
Time used: 0.19 (sec). Leaf size: 36

```
DSolve[{(x^2+y[x]^2)==2*x*y[x]*y'[x],y[-1]==0},y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to -\sqrt{x}\sqrt{x+1}$$
$$y(x) \to \sqrt{x}\sqrt{x+1}$$

1.12 problem First order with homogeneous Coefficients. Exercise 7.13, page 61

Internal problem ID [4438]

Book: Ordinary Differential Equations, By Tenenbaum and Pollard. Dover, NY 1963
Section: Chapter 2. Special types of differential equations of the first kind. Lesson 7
Problem number: First order with homogeneous Coefficients. Exercise 7.13, page 61.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[[_homogeneous, ‘class A‘], _dAlembert]`

$$\boxed{x\,\mathrm{e}^{\frac{y}{x}}+y-xy'=0}$$

With initial conditions

$$[y(1)=0]$$

✓ Solution by Maple
Time used: 0.031 (sec). Leaf size: 15

```
dsolve([(x*exp(y(x)/x)+y(x))=x*diff(y(x),x),y(1) = 0],y(x), singsol=all)
```

$$y(x)=\ln\left(-\frac{1}{\ln(x)-1}\right)x$$

✓ Solution by Mathematica
Time used: 0.316 (sec). Leaf size: 15

```
DSolve[{(x*Exp[y[x]/x]+y[x])==x*y'[x],y[1]==0},y[x],x,IncludeSingularSolutions -> True]
```

$$y(x)\to -x\log(1-\log(x))$$

1.13 problem First order with homogeneous Coefficients. Exercise 7.14, page 61

Internal problem ID [4439]

Book: Ordinary Differential Equations, By Tenenbaum and Pollard. Dover, NY 1963
Section: Chapter 2. Special types of differential equations of the first kind. Lesson 7
Problem number: First order with homogeneous Coefficients. Exercise 7.14, page 61.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[[_homogeneous, ‘class A‘], _dAlembert]`

$$\boxed{y' - \frac{y}{x} + \csc\left(\frac{y}{x}\right) = 0}$$

With initial conditions

$$[y(1) = 0]$$

✓ Solution by Maple

Time used: 0.047 (sec). Leaf size: 22

```
dsolve([diff(y(x),x)-y(x)/x+csc(y(x)/x)=0,y(1) = 0],y(x), singsol=all)
```

$$y(x) = \arccos(\ln(x) + 1)\, x$$
$$y(x) = -\arccos(\ln(x) + 1)\, x$$

✓ Solution by Mathematica

Time used: 0.394 (sec). Leaf size: 24

```
DSolve[{y'[x]-y[x]/x+Csc[y[x]/x]==0,y[1]==0},y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to -x \arccos(\log(x) + 1)$$
$$y(x) \to x \arccos(\log(x) + 1)$$

1.14 problem First order with homogeneous Coefficients. Exercise 7.15, page 61

Internal problem ID [4440]

Book: Ordinary Differential Equations, By Tenenbaum and Pollard. Dover, NY 1963
Section: Chapter 2. Special types of differential equations of the first kind. Lesson 7
Problem number: First order with homogeneous Coefficients. Exercise 7.15, page 61.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[[_homogeneous, ‘class A‘], _rational, _Bernoulli]`

$$\boxed{xy - y^2 - x^2y' = 0}$$

With initial conditions

$$[y(1) = 1]$$

✓ Solution by Maple
Time used: 0.015 (sec). Leaf size: 12

```
dsolve([(x*y(x)-y(x)^2)-x^2*diff(y(x),x)=0,y(1) = 1],y(x), singsol=all)
```

$$y(x) = \frac{x}{\ln(x) + 1}$$

✓ Solution by Mathematica
Time used: 0.139 (sec). Leaf size: 13

```
DSolve[{(x*y[x]-y[x]^2)-x^2*y'[x]==0,y[1]==1},y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to \frac{x}{\log(x) + 1}$$

2 Chapter 2. Special types of differential equations of the first kind. Lesson 8

2.1 problem Differential equations with Linear Coefficients. Exercise 8.1, page 69

Internal problem ID [4441]

Book: Ordinary Differential Equations, By Tenenbaum and Pollard. Dover, NY 1963
Section: Chapter 2. Special types of differential equations of the first kind. Lesson 8
Problem number: Differential equations with Linear Coefficients. Exercise 8.1, page 69.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[[_homogeneous, ‘class C‘], _rational, [_Abel, ‘2nd type‘, ‘cl`

$$\boxed{2y - (2x - 4y)\, y' = -x + 4}$$

✓ Solution by Maple
Time used: 0.015 (sec). Leaf size: 31

```
dsolve((x+2*y(x)-4)-(2*x-4*y(x))*diff(y(x),x)=0,y(x), singsol=all)
```

$$y(x) = 1 - \frac{\tan\left(\operatorname{RootOf}\left(2\_Z + \ln\left(\sec\left(\_Z\right)^2\right) + 2\ln\left(x-2\right) + 2c_1\right)\right)\left(x-2\right)}{2}$$

✓ Solution by Mathematica
Time used: 0.057 (sec). Leaf size: 63

```
DSolve[(x+2*y[x]-4)-(2*x-4*y[x])*y'[x]==0,y[x],x,IncludeSingularSolutions -> True]
```

$$\text{Solve}\left[2\arctan\left(\frac{-2y(x) - x + 4}{x - 2y(x)}\right) + \log\left(\frac{x^2 + 4y(x)^2 - 8y(x) - 4x + 8}{2(x-2)^2}\right) + 2\log(x-2) + c_1 = 0, y(x)\right]$$

2.2 problem Differential equations with Linear Coefficients. Exercise 8.2, page 69

Internal problem ID [4442]

Book: Ordinary Differential Equations, By Tenenbaum and Pollard. Dover, NY 1963
Section: Chapter 2. Special types of differential equations of the first kind. Lesson 8
Problem number: Differential equations with Linear Coefficients. Exercise 8.2, page 69.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[[_homogeneous, ‘class C‘], _rational, [_Abel, ‘2nd type‘, ‘cl`

$$\boxed{2y - (3x + 2y - 1)\,y' = -3x - 1}$$

✓ Solution by Maple

Time used: 0.016 (sec). Leaf size: 21

```
dsolve((3*x+2*y(x)+1)-(3*x+2*y(x)-1)*diff(y(x),x)=0,y(x), singsol=all)
```

$$y(x) = -\frac{3x}{2} - \frac{2\,\mathrm{LambertW}\left(-\frac{c_1 \mathrm{e}^{\frac{1}{4}-\frac{25x}{4}}}{4}\right)}{5} + \frac{1}{10}$$

✓ Solution by Mathematica

Time used: 4.816 (sec). Leaf size: 43

```
DSolve[(3*x+2*y[x]+1)-(3*x+2*y[x]-1)*y'[x]==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to \frac{1}{10}\left(-4W\left(-e^{-\frac{25x}{4}-1+c_1}\right) - 15x + 1\right)$$
$$y(x) \to \frac{1}{10} - \frac{3x}{2}$$

2.3 problem Differential equations with Linear Coefficients. Exercise 8.3, page 69

Internal problem ID [4443]

Book: Ordinary Differential Equations, By Tenenbaum and Pollard. Dover, NY 1963
Section: Chapter 2. Special types of differential equations of the first kind. Lesson 8
Problem number: Differential equations with Linear Coefficients. Exercise 8.3, page 69.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_quadrature]`

$$\boxed{y + (2x + 2y + 2)\, y' = -1 - x}$$

✓ Solution by Maple
Time used: 0.0 (sec). Leaf size: 17

```
dsolve((x+y(x)+1)+(2*x+2*y(x)+2)*diff(y(x),x)=0,y(x), singsol=all)
```

$$y(x) = -1 - x$$
$$y(x) = -\frac{x}{2} + c_1$$

✓ Solution by Mathematica
Time used: 0.002 (sec). Leaf size: 22

```
DSolve[(x+y[x]+1)+(2*x+2*y[x]+2)*y'[x]==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to -x - 1$$
$$y(x) \to -\frac{x}{2} + c_1$$

2.4 problem Differential equations with Linear Coefficients. Exercise 8.4, page 69

Internal problem ID [4444]

Book: Ordinary Differential Equations, By Tenenbaum and Pollard. Dover, NY 1963
Section: Chapter 2. Special types of differential equations of the first kind. Lesson 8
Problem number: Differential equations with Linear Coefficients. Exercise 8.4, page 69.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[[_homogeneous, ‘class C‘], _rational, [_Abel, ‘2nd type‘, ‘cl`

$$\boxed{y + (2x + 2y - 3)\, y' = 1 - x}$$

✓ Solution by Maple

Time used: 0.016 (sec). Leaf size: 21

```
dsolve((x+y(x)-1)+(2*x+2*y(x)-3)*diff(y(x),x)=0,y(x), singsol=all)
```

$$y(x) = \frac{\text{LambertW}\left(2\,\mathrm{e}^{x-4-c_1}\right)}{2} + 2 - x$$

✓ Solution by Mathematica

Time used: 4.725 (sec). Leaf size: 33

```
DSolve[(x+y[x]-1)+(2*x+2*y[x]-3)*y'[x]==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to \frac{1}{2}\left(W\left(-e^{x-1+c_1}\right) - 2x + 4\right)$$
$$y(x) \to 2 - x$$

2.5 problem Differential equations with Linear Coefficients. Exercise 8.5, page 69

Internal problem ID [4445]

Book: Ordinary Differential Equations, By Tenenbaum and Pollard. Dover, NY 1963
Section: Chapter 2. Special types of differential equations of the first kind. Lesson 8
Problem number: Differential equations with Linear Coefficients. Exercise 8.5, page 69.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[[_homogeneous, 'class C'], _rational, [_Abel, '2nd type', 'cl`

$$\boxed{y-(x-y-1)\,y'=1-x}$$

✓ Solution by Maple
Time used: 0.015 (sec). Leaf size: 30

```
dsolve((x+y(x)-1)-(x-y(x)-1)*diff(y(x),x)=0,y(x), singsol=all)
```

$$y(x)=\tan\left(\operatorname{RootOf}\left(2\_Z+\ln\left(\sec\left(\_Z\right)^2\right)+2\ln\left(x-1\right)+2c_1\right)\right)(1-x)$$

✓ Solution by Mathematica
Time used: 0.057 (sec). Leaf size: 48

```
DSolve[(x+y[x]-1)-(x-y[x]-1)*y'[x]==0,y[x],x,IncludeSingularSolutions -> True]
```

$$\text{Solve}\left[2\arctan\left(\frac{y(x)+x-1}{-y(x)+x-1}\right)=\log\left(\frac{1}{2}\left(\frac{y(x)^2}{(x-1)^2}+1\right)\right)+2\log(x-1)+c_1,y(x)\right]$$

2.6 problem Differential equations with Linear Coefficients. Exercise 8.6, page 69

Internal problem ID [4446]

Book: Ordinary Differential Equations, By Tenenbaum and Pollard. Dover, NY 1963
Section: Chapter 2. Special types of differential equations of the first kind. Lesson 8
Problem number: Differential equations with Linear Coefficients. Exercise 8.6, page 69.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[[_homogeneous, ‘class C‘], _rational, [_Abel, ‘2nd type‘, ‘cl`

$$\boxed{y + (2x + 2y - 1)\, y' = -x}$$

✓ Solution by Maple
Time used: 0.031 (sec). Leaf size: 21

```
dsolve((x+y(x))+(2*x+2*y(x)-1)*diff(y(x),x)=0,y(x), singsol=all)
```

$$y(x) = \frac{\operatorname{LambertW}\left(2\,e^{x-2-c_1}\right)}{2} - x + 1$$

✓ Solution by Mathematica
Time used: 1.056 (sec). Leaf size: 33

```
DSolve[(x+y[x])+(2*x+2*y[x]-1)*y'[x]==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to \frac{1}{2}\left(W\left(-e^{x-1+c_1}\right) - 2x + 2\right)$$
$$y(x) \to 1 - x$$

2.7 problem Differential equations with Linear Coefficients. Exercise 8.7, page 69

Internal problem ID [4447]

Book: Ordinary Differential Equations, By Tenenbaum and Pollard. Dover, NY 1963
Section: Chapter 2. Special types of differential equations of the first kind. Lesson 8
Problem number: Differential equations with Linear Coefficients. Exercise 8.7, page 69.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_separable]`

$$\boxed{7y + (2x+1)\, y' = 3}$$

✓ Solution by Maple
Time used: 0.0 (sec). Leaf size: 15

```
dsolve((7*y(x)-3)+(2*x+1)*diff(y(x),x)=0,y(x), singsol=all)
```

$$y(x) = \frac{3}{7} + \frac{c_1}{(1+2x)^{\frac{7}{2}}}$$

✓ Solution by Mathematica
Time used: 0.038 (sec). Leaf size: 28

```
DSolve[(7*y[x]-3)+(2*x+1)*y'[x]==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to \frac{3}{7} + \frac{c_1}{(2x+1)^{7/2}}$$
$$y(x) \to \frac{3}{7}$$

2.8 problem Differential equations with Linear Coefficients. Exercise 8.8, page 69

Internal problem ID [4448]

Book: Ordinary Differential Equations, By Tenenbaum and Pollard. Dover, NY 1963
Section: Chapter 2. Special types of differential equations of the first kind. Lesson 8
Problem number: Differential equations with Linear Coefficients. Exercise 8.8, page 69.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[[_homogeneous, ‘class C‘], _rational, [_Abel, ‘2nd type‘, ‘cl`

$$\boxed{2y + (3x + 6y + 3)\, y' = -x}$$

✓ Solution by Maple
Time used: 0.016 (sec). Leaf size: 23

```
dsolve((x+2*y(x))+(3*x+6*y(x)+3)*diff(y(x),x)=0,y(x), singsol=all)
```

$$y(x) = -\operatorname{LambertW}\left(-\frac{e^{-\frac{3}{2}-\frac{x}{6}+\frac{c_1}{6}}}{2}\right) - \frac{3}{2} - \frac{x}{2}$$

✓ Solution by Mathematica
Time used: 4.834 (sec). Leaf size: 43

```
DSolve[(x+2*y[x])+(3*x+6*y[x]+3)*y'[x]==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to \frac{1}{2}\left(-2W\left(-e^{-\frac{x}{6}-1+c_1}\right) - x - 3\right)$$
$$y(x) \to \frac{1}{2}(-x - 3)$$

2.9 problem Differential equations with Linear Coefficients. Exercise 8.9, page 69

Internal problem ID [4449]

Book: Ordinary Differential Equations, By Tenenbaum and Pollard. Dover, NY 1963
Section: Chapter 2. Special types of differential equations of the first kind. Lesson 8
Problem number: Differential equations with Linear Coefficients. Exercise 8.9, page 69.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[[_homogeneous, ‘class C‘], _rational, [_Abel, ‘2nd type‘, ‘cl`

$$2y + (y-1)\,y' = -x$$

✓ Solution by Maple

Time used: 0.063 (sec). Leaf size: 30

```
dsolve((x+2*y(x))+(y(x)-1)*diff(y(x),x)=0,y(x), singsol=all)
```

$$y(x) = \frac{(-1-x)\,\mathrm{LambertW}\left(c_1(2+x)\right) - 2 - x}{\mathrm{LambertW}\left(c_1\,(2+x)\right)}$$

✓ Solution by Mathematica

Time used: 1.178 (sec). Leaf size: 143

```
DSolve[(x+2*y[x])+(y[x]-1)*y'[x]==0,y[x],x,IncludeSingularSolutions -> True]
```

$$\text{Solve}\left[-\frac{(-2)^{2/3}\left(-\left((x+1)\log\left(-\frac{3(-2)^{2/3}(x+2)}{y(x)-1}\right)\right) + x\log\left(\frac{3(-2)^{2/3}(y(x)+x+1)}{y(x)-1}\right) + \log\left(\frac{3(-2)^{2/3}(y(x)+x+1)}{y(x)-1}\right) + \right.}{9(y(x)+x+1)}\right.$$

2.10 problem Differential equations with Linear Coefficients. Exercise 8.10, page 69

Internal problem ID [4450]

Book: Ordinary Differential Equations, By Tenenbaum and Pollard. Dover, NY 1963
Section: Chapter 2. Special types of differential equations of the first kind. Lesson 8
Problem number: Differential equations with Linear Coefficients. Exercise 8.10, page 69.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[[_homogeneous, ‘class C‘], _exact, _rational, [_Abel, ‘2nd ty`

$$\boxed{-2y-(2x+7y-1)\,y'=-3x-4}$$

✓ Solution by Maple
Time used: 0.109 (sec). Leaf size: 33

```
dsolve((3*x-2*y(x)+4)-(2*x+7*y(x)-1)*diff(y(x),x)=0,y(x), singsol=all)
```

$$y(x)=\frac{-\sqrt{7+15625\left(x+\frac{26}{25}\right)^2 c_1^2}+(-50x+25)\,c_1}{175c_1}$$

✓ Solution by Mathematica
Time used: 0.126 (sec). Leaf size: 65

```
DSolve[(3*x-2*y[x]+4)-(2*x+7*y[x]-1)*y'[x]==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x)\to\frac{1}{7}\left(-\sqrt{25x^2+52x+1+49c_1}-2x+1\right)$$
$$y(x)\to\frac{1}{7}\left(\sqrt{25x^2+52x+1+49c_1}-2x+1\right)$$

2.11 problem Differential equations with Linear Coefficients. Exercise 8.11, page 69

Internal problem ID [4451]

Book: Ordinary Differential Equations, By Tenenbaum and Pollard. Dover, NY 1963
Section: Chapter 2. Special types of differential equations of the first kind. Lesson 8
Problem number: Differential equations with Linear Coefficients. Exercise 8.11, page 69.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[[_homogeneous, ‘class C‘], _rational, [_Abel, ‘2nd type‘, ‘cl`

$$\boxed{y + (3x + 3y - 4)\, y' = -x}$$

With initial conditions

$$[y(1) = 0]$$

✓ Solution by Maple
Time used: 0.156 (sec). Leaf size: 19

```
dsolve([(x+y(x))+(3*x+3*y(x)-4)*diff(y(x),x)=0,y(1) = 0],y(x), singsol=all)
```

$$y(x) = \frac{2\,\mathrm{LambertW}\left(-1, -\frac{3\,\mathrm{e}^{-\frac{5}{2}+x}}{2}\right)}{3} + 2 - x$$

✗ Solution by Mathematica
Time used: 0.0 (sec). Leaf size: 0

```
DSolve[{(x+y[x])+(3*x+3*y[x]-4)*y'[x]==0,y[1]==0},y[x],x,IncludeSingularSolutions -> True]
```

{}

2.12 problem Differential equations with Linear Coefficients. Exercise 8.12, page 69

Internal problem ID [4452]

Book: Ordinary Differential Equations, By Tenenbaum and Pollard. Dover, NY 1963
Section: Chapter 2. Special types of differential equations of the first kind. Lesson 8
Problem number: Differential equations with Linear Coefficients. Exercise 8.12, page 69.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[[_homogeneous, ‘class C‘], _rational, [_Abel, ‘2nd type‘, ‘cl`

$$\boxed{2y - (x + 2y - 1)\, y' = -3x - 3}$$

✓ Solution by Maple

Time used: 0.187 (sec). Leaf size: 93

```
dsolve((3*x+2*y(x)+3)-(x+2*y(x)-1)*diff(y(x),x)=0,y(x), singsol=all)
```

$$y(x) = \frac{(-2-x)\operatorname{RootOf}\left(-1+(16c_1x^5+160c_1x^4+640c_1x^3+1280c_1x^2+1280c_1x+512c_1)\_Z^{25}+(-80c_1x^5\right.}{2} + \frac{3x}{2} + \frac{9}{2}$$

✓ Solution by Mathematica

Time used: 60.094 (sec). Leaf size: 3081

```
DSolve[(3*x+2*y[x]+3)-(x+2*y[x]-1)*y'[x]==0,y[x],x,IncludeSingularSolutions -> True]
```

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2.13 problem Differential equations with Linear Coefficients. Exercise 8.13, page 69

Internal problem ID [4453]

Book: Ordinary Differential Equations, By Tenenbaum and Pollard. Dover, NY 1963
Section: Chapter 2. Special types of differential equations of the first kind. Lesson 8
Problem number: Differential equations with Linear Coefficients. Exercise 8.13, page 69.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[[_homogeneous, ‘class C‘], _rational, [_Abel, ‘2nd type‘, ‘cl`

$$y+(2x+y+3)\,y'=-7$$

With initial conditions

$$[y(0)=1]$$

✓ Solution by Maple
Time used: 0.14 (sec). Leaf size: 87

```
dsolve([(y(x)+7)+(2*x+y(x)+3)*diff(y(x),x)=0,y(0) = 1],y(x), singsol=all)
```

$$y(x)=\left(-x^3+6x^2-12x+72+8\sqrt{-2x^3+12x^2-24x+80}\right)^{\frac{1}{3}}+\frac{(x-2)^2}{\left(-x^3+6x^2-12x+72+8\sqrt{-2x^3+12x^2-24x+80}\right)^{\frac{1}{3}}}-x-5$$

✓ Solution by Mathematica
Time used: 6.783 (sec). Leaf size: 198

```
DSolve[{(y[x]+7)+(2*x+y[x]+3)*y'[x]==0,y[0]==1},y[x],x,IncludeSingularSolutions -> True]
```

$$y(x)\to\frac{x^2-\left(\sqrt[3]{-x^3+6x^2+8\sqrt{2}\sqrt{-x^3+6x^2-12x+40}-12x+72}+4\right)x+\left(-x^3+6x^2+8\sqrt{2}\sqrt{-x^3+6\right.}{\sqrt[3]{-x^3+6x^2+8\sqrt{2}\sqrt{-x^3+6x^2}}}$$

2.14 problem Differential equations with Linear Coefficients. Exercise 8.14, page 69

Internal problem ID [4454]

Book: Ordinary Differential Equations, By Tenenbaum and Pollard. Dover, NY 1963
Section: Chapter 2. Special types of differential equations of the first kind. Lesson 8
Problem number: Differential equations with Linear Coefficients. Exercise 8.14, page 69.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[[_homogeneous, ‘class C‘], _rational, [_Abel, ‘2nd type‘, ‘cl`

$$\boxed{y-(x-y-4)\,y'=-x-2}$$

✓ Solution by Maple

Time used: 0.204 (sec). Leaf size: 31

```
dsolve((x+y(x)+2)-(x-y(x)-4)*diff(y(x),x)=0,y(x), singsol=all)
```

$$y(x)=-3-\tan\left(\operatorname{RootOf}\left(2\_Z+\ln\left(\sec\left(\_Z\right)^{2}\right)+2\ln\left(x-1\right)+2c_{1}\right)\right)(x-1)$$

✓ Solution by Mathematica

Time used: 0.056 (sec). Leaf size: 58

```
DSolve[(x+y[x]+2)-(x-y[x]-4)*y'[x]==0,y[x],x,IncludeSingularSolutions -> True]
```

$$\text{Solve}\left[2\arctan\left(\frac{y(x)+x+2}{y(x)-x+4}\right)+\log\left(\frac{x^2+y(x)^2+6y(x)-2x+10}{2(x-1)^2}\right)+2\log(x-1)+c_1=0,y(x)\right]$$

3 Chapter 2. Special types of differential equations of the first kind. Lesson 9

3.1 problem Exact Differential equations. Exercise 9.4, page 79

Internal problem ID [4455]

Book: Ordinary Differential Equations, By Tenenbaum and Pollard. Dover, NY 1963
Section: Chapter 2. Special types of differential equations of the first kind. Lesson 9
Problem number: Exact Differential equations. Exercise 9.4, page 79.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_exact, _rational]`

$$3yx^2 + 8xy^2 + \left(x^3 + 8yx^2 + 12y^2\right) y' = 0$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 475

```
dsolve((3*x^2*y(x)+8*x*y(x)^2)+(x^3+8*x^2*y(x)+12*y(x)^2)*diff(y(x),x)=0,y(x), singsol=all)
```

$$y(x) = \frac{\left(9x^5 - 27c_1 - 8x^6 + 3\sqrt{-3x^{10} + 3x^9 + 48c_1x^6 - 54c_1x^5 + 81c_1^2}\right)^{\frac{1}{3}}}{6} + \frac{x^3(-3+4x)}{6\left(9x^5 - 27c_1 - 8x^6 + 3\sqrt{-3x^{10} + 3x^9 + 48c_1x^6 - 54c_1x^5 + 81c_1^2}\right)^{\frac{1}{3}}} - \frac{x^2}{3}$$

$$y(x) = \frac{\frac{\left(-i\sqrt{3}-1\right)\left(9x^5-27c_1-8x^6+3\sqrt{-3x^{10}+3x^9+48c_1x^6-54c_1x^5+81c_1^2}\right)^{\frac{2}{3}}}{4} + \left(-\left(9x^5 - 27c_1 - 8x^6 + 3\sqrt{-3x^{10} + 3x^9 + 48\right.\right.}{3\left(9x^5 - 27c_1 - 8x^6 + 3\sqrt{-3x^{10} + 3x^9 + 48c_1x^6 - 54c_1x^5\right.}}$$

$$y(x) = -\frac{\left(-\frac{i\sqrt{3}}{4} + \frac{1}{4}\right)\left(9x^5 - 27c_1 - 8x^6 + 3\sqrt{-3x^{10} + 3x^9 + 48c_1x^6 - 54c_1x^5 + 81c_1^2}\right)^{\frac{2}{3}} + \left(\left(9x^5 - 27c_1 - 8x\right.\right.}{3\left(9x^5 - 27c_1 - 8x^6 + 3\sqrt{-3x^{10} + 3x^9 + 48c\right.}}$$

✓ Solution by Mathematica

Time used: 1.703 (sec). Leaf size: 474

```
DSolve[(3*x^2*y[x]+8*x*y[x]^2)+(x^3+8*x^2*y[x]+12*y[x]^2)*y'[x]==0,y[x],x,IncludeSingularSol
```

$$y(x) \to \frac{1}{6}\left(-2x^2 + \sqrt[3]{-8x^6 + 9x^5 + 3\sqrt{3}\sqrt{-x^{10} + x^9 - 16c_1x^6 + 18c_1x^5 + 27c_1{}^2} + 27c_1} + \frac{(4x-3)x^3}{\sqrt[3]{-8x^6 + 9x^5 + 3\sqrt{3}\sqrt{-x^{10} + x^9 - 16c_1x^6 + 18c_1x^5 + 27c_1{}^2} + 27c_1}}\right)$$

$$y(x) \to \frac{1}{48}\left(-16x^2 + 4i\left(\sqrt{3} + i\right)\sqrt[3]{-8x^6 + 9x^5 + 3\sqrt{3}\sqrt{-x^{10} + x^9 - 16c_1x^6 + 18c_1x^5 + 27c_1{}^2} + 27c_1} - \frac{4i\left(\sqrt{3} - i\right)(4x-3)x^3}{\sqrt[3]{-8x^6 + 9x^5 + 3\sqrt{3}\sqrt{-x^{10} + x^9 - 16c_1x^6 + 18c_1x^5 + 27c_1{}^2} + 27c_1}}\right)$$

$$y(x) \to \frac{1}{48}\left(-16x^2 - 4\left(1 + i\sqrt{3}\right)\sqrt[3]{-8x^6 + 9x^5 + 3\sqrt{3}\sqrt{-x^{10} + x^9 - 16c_1x^6 + 18c_1x^5 + 27c_1{}^2} + 27c_1} + \frac{4i\left(\sqrt{3} + i\right)(4x-3)x^3}{\sqrt[3]{-8x^6 + 9x^5 + 3\sqrt{3}\sqrt{-x^{10} + x^9 - 16c_1x^6 + 18c_1x^5 + 27c_1{}^2} + 27c_1}}\right)$$

3.2 problem Exact Differential equations. Exercise 9.5, page 79

Internal problem ID [4456]

Book: Ordinary Differential Equations, By Tenenbaum and Pollard. Dover, NY 1963
Section: Chapter 2. Special types of differential equations of the first kind. Lesson 9
Problem number: Exact Differential equations. Exercise 9.5, page 79.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[[_homogeneous, ‘class D‘], _exact, _rational, [_Abel, ‘2nd ty`

$$\boxed{\frac{2xy+1}{y}+\frac{(-x+y)\,y'}{y^2}=0}$$

✓ Solution by Maple

Time used: 0.016 (sec). Leaf size: 18

```
dsolve((2*x*y(x)+1)/y(x)+(y(x)-x)/y(x)^2*diff(y(x),x)=0,y(x), singsol=all)
```

$$y(x)=-\frac{x}{\operatorname{LambertW}\left(-\mathrm{e}^{x^2}c_1x\right)}$$

✓ Solution by Mathematica

Time used: 5.208 (sec). Leaf size: 29

```
DSolve[(2*x*y[x]+1)/y[x]+(y[x]-x)/y[x]^2*y'[x]==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x)\to-\frac{x}{W\left(x\left(-e^{x^2-c_1}\right)\right)}$$
$$y(x)\to 0$$

3.3 problem Exact Differential equations. Exercise 9.6, page 79

Internal problem ID [4457]

Book: Ordinary Differential Equations, By Tenenbaum and Pollard. Dover, NY 1963
Section: Chapter 2. Special types of differential equations of the first kind. Lesson 9
Problem number: Exact Differential equations. Exercise 9.6, page 79.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[[_homogeneous, ‘class A‘], _exact, _rational, _dAlembert]`

$$\boxed{2xy + (x^2 + y^2)\, y' = 0}$$

✓ Solution by Maple
Time used: 0.031 (sec). Leaf size: 209

```
dsolve(2*x*y(x)+(x^2+y(x)^2)*diff(y(x),x)=0,y(x), singsol=all)
```

$$y(x) = -\frac{2\left(c_1 x^2 - \frac{\left(4+4\sqrt{4c_1^3 x^6+1}\right)^{\frac{2}{3}}}{4}\right)}{\left(4+4\sqrt{4c_1^3 x^6+1}\right)^{\frac{1}{3}}\sqrt{c_1}}$$

$$y(x) = -\frac{\left(1+i\sqrt{3}\right)\left(4+4\sqrt{4c_1^3 x^6+1}\right)^{\frac{1}{3}}}{4\sqrt{c_1}} - \frac{\sqrt{c_1}\left(i\sqrt{3}-1\right)x^2}{\left(4+4\sqrt{4c_1^3 x^6+1}\right)^{\frac{1}{3}}}$$

$$y(x) = \frac{4i\sqrt{3}\, c_1 x^2 + i\left(4+4\sqrt{4c_1^3 x^6+1}\right)^{\frac{2}{3}}\sqrt{3} + 4c_1 x^2 - \left(4+4\sqrt{4c_1^3 x^6+1}\right)^{\frac{2}{3}}}{4\left(4+4\sqrt{4c_1^3 x^6+1}\right)^{\frac{1}{3}}\sqrt{c_1}}$$

✓ Solution by Mathematica

Time used: 15.514 (sec). Leaf size: 401

```
DSolve[2*x*y[x]+(x^2+y[x]^2)*y'[x]==0,y[x],x,IncludeSingularSolutions -> True]
```

$$
\begin{aligned}
y(x) &\to \frac{\sqrt[3]{\sqrt{4x^6+e^{6c_1}}+e^{3c_1}}}{\sqrt[3]{2}} - \frac{\sqrt[3]{2}x^2}{\sqrt[3]{\sqrt{4x^6+e^{6c_1}}+e^{3c_1}}} \\
y(x) &\to \frac{i2^{2/3}\left(\sqrt{3}+i\right)\left(\sqrt{4x^6+e^{6c_1}}+e^{3c_1}\right)^{2/3}+\sqrt[3]{2}\left(2+2i\sqrt{3}\right)x^2}{4\sqrt[3]{\sqrt{4x^6+e^{6c_1}}+e^{3c_1}}} \\
y(x) &\to \frac{\left(1-i\sqrt{3}\right)x^2}{2^{2/3}\sqrt[3]{\sqrt{4x^6+e^{6c_1}}+e^{3c_1}}} - \frac{\left(1+i\sqrt{3}\right)\sqrt[3]{\sqrt{4x^6+e^{6c_1}}+e^{3c_1}}}{2\sqrt[3]{2}} \\
y(x) &\to 0 \\
y(x) &\to \frac{1}{2}\sqrt[6]{x^6}\left(\frac{\left(1-i\sqrt{3}\right)\left(x^6\right)^{2/3}}{x^4} - i\sqrt{3} - 1\right) \\
y(x) &\to \frac{1}{2}\sqrt[6]{x^6}\left(\frac{\left(1+i\sqrt{3}\right)\left(x^6\right)^{2/3}}{x^4} + i\sqrt{3} - 1\right) \\
y(x) &\to \sqrt[6]{x^6} - \frac{\left(x^6\right)^{5/6}}{x^4}
\end{aligned}
$$

3.4 problem Exact Differential equations. Exercise 9.7, page 79

Internal problem ID [4458]

Book: Ordinary Differential Equations, By Tenenbaum and Pollard. Dover, NY 1963
Section: Chapter 2. Special types of differential equations of the first kind. Lesson 9
Problem number: Exact Differential equations. Exercise 9.7, page 79.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_exact]`

$$\boxed{e^{x} \sin \left(y\right)+e^{-y}-\left(x \,e^{-y}-e^{x} \cos \left(y\right)\right) y^{\prime}=0}$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 18

```
dsolve((exp(x)*sin(y(x))+exp(-y(x)))-(x*exp(-y(x))-exp(x)*cos(y(x)))*diff(y(x),x)=0,y(x), si
```

$$e^{x} \sin \left(y(x)\right)+x \,e^{-y(x)}+c_1=0$$

✓ Solution by Mathematica

Time used: 0.389 (sec). Leaf size: 24

```
DSolve[(Exp[x]*Sin[y[x]]+Exp[-y[x]])-(x*Exp[-y[x]]-Exp[x]*Cos[y[x]])*y'[x]==0,y[x],x,Include
```

$$\text{Solve}\left[x\left(-e^{-y(x)}\right)-e^x\sin(y(x))=c_1,y(x)\right]$$

3.5 problem Exact Differential equations. Exercise 9.8, page 79

Internal problem ID [4459]

Book: Ordinary Differential Equations, By Tenenbaum and Pollard. Dover, NY 1963
Section: Chapter 2. Special types of differential equations of the first kind. Lesson 9
Problem number: Exact Differential equations. Exercise 9.8, page 79.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_exact, [_1st_order, ‘_with_symmetry_[F(x)*G(y),0]‘]]`

$$\boxed{\cos(y) - (x\sin(y) - y^2)\,y' = 0}$$

✓ Solution by Maple
Time used: 0.016 (sec). Leaf size: 18

```
dsolve(cos(y(x))-(x*sin(y(x))-y(x)^2)*diff(y(x),x)=0,y(x), singsol=all)
```

$$x + \frac{\sec(y(x))\,(y(x)^3 - 3c_1)}{3} = 0$$

✓ Solution by Mathematica
Time used: 0.124 (sec). Leaf size: 23

```
DSolve[Cos[y[x]]-(x*Sin[y[x]]-y[x]^2)*y'[x]==0,y[x],x,IncludeSingularSolutions -> True]
```

$$\text{Solve}\left[x = -\frac{1}{3}y(x)^3\sec(y(x)) + c_1\sec(y(x)), y(x)\right]$$

3.6 problem Exact Differential equations. Exercise 9.9, page 79

Internal problem ID [4460]

Book: Ordinary Differential Equations, By Tenenbaum and Pollard. Dover, NY 1963
Section: Chapter 2. Special types of differential equations of the first kind. Lesson 9
Problem number: Exact Differential equations. Exercise 9.9, page 79.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_exact]`

$$-2xy + e^y + (y - x^2 + x e^y) y' = -x$$

✓ Solution by Maple
Time used: 0.016 (sec). Leaf size: 28

```
dsolve((x-2*x*y(x)+exp(y(x)))+(y(x)-x^2+x*exp(y(x)))*diff(y(x),x)=0,y(x), singsol=all)
```

$$-y(x) x^2 + x e^{y(x)} + \frac{x^2}{2} + \frac{y(x)^2}{2} + c_1 = 0$$

✓ Solution by Mathematica
Time used: 0.315 (sec). Leaf size: 35

```
DSolve[(x-2*x*y[x]+Exp[y[x]])+(y[x]-x^2+x*Exp[y[x]])*y'[x]==0,y[x],x,IncludeSingularSolution
```

$$\text{Solve}\left[x^2(-y(x)) + \frac{x^2}{2} + xe^{y(x)} + \frac{y(x)^2}{2} = c_1, y(x)\right]$$

3.7 problem Exact Differential equations. Exercise 9.10, page 79

Internal problem ID [4461]

Book: Ordinary Differential Equations, By Tenenbaum and Pollard. Dover, NY 1963
Section: Chapter 2. Special types of differential equations of the first kind. Lesson 9
Problem number: Exact Differential equations. Exercise 9.10, page 79.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_exact]`

$$\boxed{y^2 - (e^y - 2xy)\, y' = -x^2 + x}$$

✓ Solution by Maple
Time used: 0.0 (sec). Leaf size: 26

```
dsolve((x^2-x+y(x)^2)-(exp(y(x))-2*x*y(x))*diff(y(x),x)=0,y(x), singsol=all)
```

$$\frac{x^3}{3} + xy(x)^2 - \frac{x^2}{2} - e^{y(x)} + c_1 = 0$$

✓ Solution by Mathematica
Time used: 0.198 (sec). Leaf size: 32

```
DSolve[(x^2-x+y[x]^2)-(Exp[y[x]]-2*x*y[x])*y'[x]==0,y[x],x,IncludeSingularSolutions -> True]
```

$$\text{Solve}\left[-\frac{x^3}{3} + \frac{x^2}{2} - xy(x)^2 + e^{y(x)} = c_1, y(x)\right]$$

3.8 problem Exact Differential equations. Exercise 9.11, page 79

Internal problem ID [4462]

Book: Ordinary Differential Equations, By Tenenbaum and Pollard. Dover, NY 1963
Section: Chapter 2. Special types of differential equations of the first kind. Lesson 9
Problem number: Exact Differential equations. Exercise 9.11, page 79.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_exact]`

$$y\cos(x) + (2y + \sin(x) - \sin(y))\, y' = -2x$$

✓ Solution by Maple
Time used: 0.0 (sec). Leaf size: 20

```
dsolve((2*x+y(x)*cos(x))+(2*y(x)+sin(x)-sin(y(x)))*diff(y(x),x)=0,y(x), singsol=all)
```

$$\sin(x)\, y(x) + x^2 + y(x)^2 + \cos(y(x)) + c_1 = 0$$

✓ Solution by Mathematica
Time used: 0.198 (sec). Leaf size: 22

```
DSolve[(2*x+y[x]*Cos[x])+(2*y[x]+Sin[x]-Sin[y[x]])*y'[x]==0,y[x],x,IncludeSingularSolutions
```

$$\text{Solve}\left[x^2 + y(x)^2 + y(x)\sin(x) + \cos(y(x)) = c_1, y(x)\right]$$

3.9 problem Exact Differential equations. Exercise 9.12, page 79

Internal problem ID [4463]

Book: Ordinary Differential Equations, By Tenenbaum and Pollard. Dover, NY 1963
Section: Chapter 2. Special types of differential equations of the first kind. Lesson 9
Problem number: Exact Differential equations. Exercise 9.12, page 79.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[[_homogeneous, ‘class A‘], _exact, _dAlembert]`

$$\boxed{x\sqrt{x^2+y^2}-\frac{x^2yy'}{y-\sqrt{x^2+y^2}}=0}$$

✓ Solution by Maple
Time used: 0.0 (sec). Leaf size: 19

```
dsolve(x*sqrt(x^2+y(x)^2)-(x^2*y(x))/(y(x)- sqrt(x^2+y(x)^2))*diff(y(x),x)=0,y(x), singsol=a
```

$$c_1+\left(x^2+y(x)^2\right)^{\frac{3}{2}}+y(x)^3=0$$

✓ Solution by Mathematica

Time used: 60.259 (sec). Leaf size: 2125

```
DSolve[x*Sqrt[x^2+y[x]^2]-(x^2*y[x])/(y[x]- Sqrt[x^2+y[x]^2])*y'[x]==0,y[x],x,IncludeSingula
```

$y(x) \to$

$$x^2\sqrt{\frac{e^{6c_1}}{x^4}-6x^2+\frac{3(5x^6-4e^{6c_1})}{\sqrt[3]{-11x^{12}+14e^{6c_1}x^6+2\sqrt{(-x^6+e^{6c_1})(x^6+e^{6c_1})^3}-2e^{12c_1}}}+\frac{3\sqrt[3]{-11x^{12}+14e^{6}\cdots}}{\cdots}}$$

$$-\overline{\qquad\qquad}$$

$y(x)$

$$x^2\left(-\sqrt{\frac{e^{6c_1}}{x^4}-6x^2+\frac{3(5x^6-4e^{6c_1})}{\sqrt[3]{-11x^{12}+14e^{6c_1}x^6+2\sqrt{(-x^6+e^{6c_1})(x^6+e^{6c_1})^3}-2e^{12c_1}}}+\frac{3\sqrt[3]{-11x^{12}+14\cdots}}{\cdots}}\right.$$

$$\to\overline{\qquad\qquad}$$

$y(x)$

$$x^2\sqrt{\frac{e^{6c_1}}{x^4}-6x^2+\frac{3(5x^6-4e^{6c_1})}{\sqrt[3]{-11x^{12}+14e^{6c_1}x^6+2\sqrt{(-x^6+e^{6c_1})(x^6+e^{6c_1})^3}-2e^{12c_1}}}+\frac{3\sqrt[3]{-11x^{12}+14e^{6c_1}\cdots}}{\cdots}}$$

$$\to\overline{\qquad\qquad}$$

$y(x)$

$$x^2\sqrt{\frac{e^{6c_1}}{x^4}-6x^2+\frac{3(5x^6-4e^{6c_1})}{\sqrt[3]{-11x^{12}+14e^{6c_1}x^6+2\sqrt{(-x^6+e^{6c_1})(x^6+e^{6c_1})^3}-2e^{12c_1}}}+\frac{3\sqrt[3]{-11x^{12}+14e^{6c_1}\cdots}}{\cdots}}$$

$$\to\overline{\qquad\qquad}$$

3.10 problem Exact Differential equations. Exercise 9.13, page 79

Internal problem ID [4464]

Book: Ordinary Differential Equations, By Tenenbaum and Pollard. Dover, NY 1963
Section: Chapter 2. Special types of differential equations of the first kind. Lesson 9
Problem number: Exact Differential equations. Exercise 9.13, page 79.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_exact]`

$$\boxed{y^3 - (y^2 + 1 - 3xy^2)\, y' = -4x^3 + \sin(x)}$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 658

```
dsolve((4*x^3-sin(x)+y(x)^3)-(y(x)^2+1-3*x*y(x)^2)*diff(y(x),x)=0,y(x), singsol=all)
```

$$y(x) = \frac{\left(2^{\frac{1}{3}}\left(\left(-3x^4 - 3\cos(x) + \sqrt{\frac{(27x-9)\cos(x)^2+54\left(x-\frac{1}{3}\right)(x^4+c_1)\cos(x)+27x^9-9x^8+54c_1x^5-18c_1x^4+27c_1^2x-9c_1^2-4}{3x-1}} - 3c_1\right)\right.\right.}{\left(\left(-3x^4 - 3\cos(x) + \sqrt{\frac{(27x-9)\cos(x)^2+54\left(x-\frac{1}{3}\right)(x^4+c_1)\cos(x)+27x^9-9x^8+54c_1x^5-18c_1x^4+27c_1^2x-9c_1^2-4}{3x-1}} - 3c_1\right)\right.}$$

$$y(x) = -\frac{\left(2^{\frac{1}{3}}(1+i\sqrt{3})\left(-\left(3x^4 + 3\cos(x) - \sqrt{\frac{(27x-9)\cos(x)^2+54\left(x-\frac{1}{3}\right)(x^4+c_1)\cos(x)+27x^9-9x^8+54c_1x^5-18c_1x^4+27c_1^2x-9}{3x-1}}\right.\right.\right.}{4\left(-\left(3x^4 + 3\cos(x) - \sqrt{\frac{(27x-9)\cos(x)^2+54\left(x-\frac{1}{3}\right)(x^4+c_1)\cos(x)+27x^9-9x^8+54c_1x^5-18c_1x^4+27c_1^2x}{3x-1}}\right.\right.}$$

$$y(x) = \frac{\left(2^{\frac{1}{3}}(i\sqrt{3}-1)\left(-\left(3x^4 + 3\cos(x) - \sqrt{\frac{(27x-9)\cos(x)^2+54\left(x-\frac{1}{3}\right)(x^4+c_1)\cos(x)+27x^9-9x^8+54c_1x^5-18c_1x^4+27c_1^2x-9c_1^2-}{3x-1}}\right.\right.\right.}{4\left(-\left(3x^4 + 3\cos(x) - \sqrt{\frac{(27x-9)\cos(x)^2+54\left(x-\frac{1}{3}\right)(x^4+c_1)\cos(x)+27x^9-9x^8+54c_1x^5-18c_1x^4+27c_1^2x-9}{3x-1}}\right.\right.}$$

✓ Solution by Mathematica

Time used: 60.207 (sec). Leaf size: 682

```
DSolve[(4*x^3-Sin[x]+y[x]^3)-(y[x]^2+1-3*x*y[x]^2)*y'[x]==0,y[x],x,IncludeSingularSolutions
```

$$y(x) \to \frac{\sqrt[3]{2}\left(-27x^6+18x^5-3x^4+\frac{1}{27}\sqrt{4(9-27x)^3+6561(1-3x)^4\left(x^4+\cos(x)-c_1\right)^2}-27x^2\cos(x)+27c_1\right.}{2^{2/3}(3x-1)\sqrt[3]{-27x^6+18x^5-3x^4+\frac{1}{27}\sqrt{4(9-27x)^3+6561(1-3x)^4\left(x^4+\cos(x)-c_1\right)^2}-27x^2\,c}}$$

$$y(x) \to \frac{9i\sqrt[3]{2}\left(\sqrt{3}+i\right)\left(-27x^6+18x^5-3x^4+\frac{1}{27}\sqrt{4(9-27x)^3+6561(1-3x)^4\left(x^4+\cos(x)-c_1\right)^2}-27x^2\,\mathrm{co}\right.}{18\;2^{2/3}(3x-1)\sqrt[3]{-27x^6+18x^5-3x^4+\frac{1}{27}\sqrt{4(9-27x)^3+6561(1-3x)^4\left(x^4+\cos(x)\right.}}}$$

$$y(x) \to \frac{i\left(\sqrt{3}+i\right)}{2^{2/3}\sqrt[3]{-27x^6+18x^5-3x^4+\frac{1}{27}\sqrt{4(9-27x)^3+6561(1-3x)^4\left(x^4+\cos(x)-c_1\right)^2}-27x^2\cos(x)+27}}$$

$$-\frac{\left(1+i\sqrt{3}\right)\sqrt[3]{-54x^6+36x^5-6x^4+\frac{2}{27}\sqrt{4(9-27x)^3+6561(1-3x)^4\left(x^4+\cos(x)-c_1\right)^2}-54x^2\cos}}{2\;2^{2/3}(3x-1)}$$

3.11 problem Exact Differential equations. Exercise 9.15, page 79

Internal problem ID [4465]

Book: Ordinary Differential Equations, By Tenenbaum and Pollard. Dover, NY 1963
Section: Chapter 2. Special types of differential equations of the first kind. Lesson 9
Problem number: Exact Differential equations. Exercise 9.15, page 79.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_exact, _Bernoulli]`

$$\boxed{e^{x}\left(y^{3}+y^{3}x+1\right)+3y^{2}\left(e^{x}x-6\right)y'=0}$$

With initial conditions

$$[y(0)=1]$$

✓ Solution by Maple

Time used: 0.14 (sec). Leaf size: 38

```
dsolve([exp(x)*(y(x)^3+x*y(x)^3+1)+3*y(x)^2*(x*exp(x)-6)*diff(y(x),x)=0,y(0) = 1],y(x), sing
```

$$y(x)=\frac{\left(i\sqrt{3}-1\right)\left(-\left(e^{x}+5\right)\left(x\,e^{x}-6\right)^{2}\right)^{\frac{1}{3}}}{2x\,e^{x}-12}$$

✓ Solution by Mathematica

Time used: 1.114 (sec). Leaf size: 28

```
DSolve[{Exp[x]*(y[x]^3+x*y[x]^3+1)+3*y[x]^2*(x*Exp[x]-6)*y'[x]==0,y[0]==1},y[x],x,IncludeSin
```

$$y(x)\to\frac{\sqrt[3]{-e^{x}-5}}{\sqrt[3]{e^{x}x-6}}$$

3.12 problem Exact Differential equations. Exercise 9.16, page 79

Internal problem ID [4466]

Book: Ordinary Differential Equations, By Tenenbaum and Pollard. Dover, NY 1963
Section: Chapter 2. Special types of differential equations of the first kind. Lesson 9
Problem number: Exact Differential equations. Exercise 9.16, page 79.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_separable]`

$$\boxed{\sin(x)\cos(y)+\cos(x)\sin(y)\,y'=0}$$

With initial conditions

$$\left[y\left(\frac{\pi}{4}\right)=\frac{\pi}{4}\right]$$

✓ Solution by Maple
Time used: 0.141 (sec). Leaf size: 9

```
dsolve([sin(x)*cos(y(x))+cos(x)*sin(y(x))*diff(y(x),x)=0,y(1/4*Pi) = 1/4*Pi],y(x), singsol=a
```

$$y(x)=\frac{\pi}{2}-\arcsin\left(\frac{\sec(x)}{2}\right)$$

✓ Solution by Mathematica
Time used: 6.111 (sec). Leaf size: 12

```
DSolve[{Sin[x]*Cos[y[x]]+Cos[x]*Sin[y[x]]*y'[x]==0,y[Pi/4]==Pi/4},y[x],x,IncludeSingularSolu
```

$$y(x)\to\arccos\left(\frac{\sec(x)}{2}\right)$$

3.13 problem Exact Differential equations. Exercise 9.17, page 79

Internal problem ID [4467]

Book: Ordinary Differential Equations, By Tenenbaum and Pollard. Dover, NY 1963
Section: Chapter 2. Special types of differential equations of the first kind. Lesson 9
Problem number: Exact Differential equations. Exercise 9.17, page 79.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_exact]`

$$\boxed{y^2 e^{x y^2} + \left(2xy\, e^{x y^2} - 3y^2\right) y' = -4x^3}$$

With initial conditions

$$[y(1) = 0]$$

✓ Solution by Maple
Time used: 0.078 (sec). Leaf size: 23

```
dsolve([(y(x)^2*exp(x*y(x)^2)+4*x^3)+(2*x*y(x)*exp(x*y(x)^2)-3*y(x)^2)*diff(y(x),x)=0,y(1) =
```

$$y(x) = \text{RootOf}\left(-e^{x\_Z^2} - x^4 + \_Z^3 + 2\right)$$

✓ Solution by Mathematica
Time used: 0.332 (sec). Leaf size: 23

```
DSolve[{(y[x]^2*Exp[x*y[x]^2]+4*x^3)+(2*x*y[x]*Exp[x*y[x]^2]-3*y[x]^2)*y'[x]==0,y[1]==0},y[x
```

$$\text{Solve}\left[x^4 + e^{x y(x)^2} - y(x)^3 = 2, y(x)\right]$$

4 Chapter 2. Special types of differential equations of the first kind. Lesson 10

4.1 problem Recognizable Exact Differential equations. Integrating factors. Example 10.51, page 90

Internal problem ID [4468]

Book: Ordinary Differential Equations, By Tenenbaum and Pollard. Dover, NY 1963
Section: Chapter 2. Special types of differential equations of the first kind. Lesson 10
Problem number: Recognizable Exact Differential equations. Integrating factors. Example 10.51, page 90.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_separable]`

$$\boxed{y^2 + y - xy' = 0}$$

✓ Solution by Maple
Time used: 0.0 (sec). Leaf size: 13

```
dsolve((y(x)^2+y(x))-x*diff(y(x),x)=0,y(x), singsol=all)
```

$$y(x) = \frac{x}{-x + c_1}$$

✓ Solution by Mathematica
Time used: 0.274 (sec). Leaf size: 32

```
DSolve[(y[x]^2+y[x])-x*y'[x]==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to \frac{e^{c_1}x}{1 - e^{c_1}x}$$
$$y(x) \to -1$$
$$y(x) \to 0$$

4.2 problem Recognizable Exact Differential equations. Integrating factors. Example 10.52, page 90

Internal problem ID [4469]

Book: Ordinary Differential Equations, By Tenenbaum and Pollard. Dover, NY 1963
Section: Chapter 2. Special types of differential equations of the first kind. Lesson 10
Problem number: Recognizable Exact Differential equations. Integrating factors. Example 10.52, page 90.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_separable]`

$$\boxed{y\sec(x)+\sin(x)\,y'=0}$$

✓ Solution by Maple
Time used: 0.0 (sec). Leaf size: 8

```
dsolve((y(x)*sec(x))+sin(x)*diff(y(x),x)=0,y(x), singsol=all)
```

$$y(x)=\cot(x)\,c_1$$

✓ Solution by Mathematica
Time used: 0.035 (sec). Leaf size: 15

```
DSolve[(y[x]*Sec[x])+Sin[x]*y'[x]==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x)\to c_1\cot(x)$$
$$y(x)\to 0$$

4.3 problem Recognizable Exact Differential equations. Integrating factors. Example 10.661, page 90

Internal problem ID [4470]

Book: Ordinary Differential Equations, By Tenenbaum and Pollard. Dover, NY 1963
Section: Chapter 2. Special types of differential equations of the first kind. Lesson 10
Problem number: Recognizable Exact Differential equations. Integrating factors. Example 10.661, page 90.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[‘y=_G(x,y’)‘]`

$$\boxed{-\sin(y) + \cos(y)\, y' = -\mathrm{e}^{x}}$$

✓ Solution by Maple
Time used: 0.0 (sec). Leaf size: 13

```
dsolve((exp(x)-sin(y(x)))+cos(y(x))*diff(y(x),x)=0,y(x), singsol=all)
```

$$y(x) = -\arcsin\left((x + c_1)\,\mathrm{e}^{x}\right)$$

✓ Solution by Mathematica
Time used: 11.754 (sec). Leaf size: 16

```
DSolve[(Exp[x]-Sin[y[x]])+Cos[y[x]]*y'[x]==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to -\arcsin\left(e^{x}(x + c_1)\right)$$

4.4 problem Recognizable Exact Differential equations. Integrating factors. Example 10.701, page 90

Internal problem ID [4471]

Book: Ordinary Differential Equations, By Tenenbaum and Pollard. Dover, NY 1963
Section: Chapter 2. Special types of differential equations of the first kind. Lesson 10
Problem number: Recognizable Exact Differential equations. Integrating factors. Example 10.701, page 90.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_separable]`

$$\boxed{xy + (x^2 + 1)\, y' = 0}$$

✓ Solution by Maple
Time used: 0.0 (sec). Leaf size: 13

```
dsolve((x*y(x))+(1+x^2)*diff(y(x),x)=0,y(x), singsol=all)
```

$$y(x) = \frac{c_1}{\sqrt{x^2 + 1}}$$

✓ Solution by Mathematica
Time used: 0.027 (sec). Leaf size: 22

```
DSolve[(x*y[x])+(1+x^2)*y'[x]==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to \frac{c_1}{\sqrt{x^2 + 1}}$$
$$y(x) \to 0$$

4.5 problem Recognizable Exact Differential equations. Integrating factors. Example 10.741, page 90

Internal problem ID [4472]

Book: Ordinary Differential Equations, By Tenenbaum and Pollard. Dover, NY 1963
Section: Chapter 2. Special types of differential equations of the first kind. Lesson 10
Problem number: Recognizable Exact Differential equations. Integrating factors. Example 10.741, page 90.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_rational, [_Abel, '2nd type', 'class C']]`

$$\boxed{y^3 + xy^2 + y + (x^3 + yx^2 + x)\, y' = 0}$$

✓ Solution by Maple
Time used: 0.0 (sec). Leaf size: 99

```
dsolve((y(x)^3+x*y(x)^2+y(x))+(x^3+x^2*y(x)+x)*diff(y(x),x)=0,y(x), singsol=all)
```

$$y(x) = \frac{x^2+1}{\left(\sqrt{x^2+1}\,\sqrt{\frac{-1+(x^4+x^2)c_1}{x^2(x^2+1)}} - 1\right)x}$$
$$y(x) = \frac{-x^2-1}{\left(\sqrt{x^2+1}\,\sqrt{\frac{-1+(x^4+x^2)c_1}{x^2(x^2+1)}} + 1\right)x}$$

✓ Solution by Mathematica

Time used: 3.726 (sec). Leaf size: 114

```
DSolve[(y[x]^3+x*y[x]^2+y[x])+(x^3+x^2*y[x]+x)*y'[x]==0,y[x],x,IncludeSingularSolutions -> T
```

$$
\begin{aligned}
y(x) &\to -\frac{\sqrt{\frac{1}{x^3}}x(x^2+1)}{\sqrt{\frac{1}{x^3}}x^2-\sqrt{c_1x^3-\frac{1}{x}+c_1x}} \\
y(x) &\to -\frac{\sqrt{\frac{1}{x^3}}x(x^2+1)}{\sqrt{\frac{1}{x^3}}x^2+\sqrt{c_1x^3-\frac{1}{x}+c_1x}} \\
y(x) &\to 0
\end{aligned}
$$

4.6 problem Recognizable Exact Differential equations. Integrating factors. Example 10.781, page 90

Internal problem ID [4473]

Book: Ordinary Differential Equations, By Tenenbaum and Pollard. Dover, NY 1963
Section: Chapter 2. Special types of differential equations of the first kind. Lesson 10
Problem number: Recognizable Exact Differential equations. Integrating factors. Example 10.781, page 90.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_separable]`

$$\boxed{3y - xy' = 0}$$

✓ Solution by Maple
Time used: 0.0 (sec). Leaf size: 9

```
dsolve((3*y(x))-(x)*diff(y(x),x)=0,y(x), singsol=all)
```

$$y(x) = c_1 x^3$$

✓ Solution by Mathematica
Time used: 0.022 (sec). Leaf size: 16

```
DSolve[(3*y[x])-(x)*y'[x]==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to c_1 x^3$$
$$y(x) \to 0$$

4.7 problem Recognizable Exact Differential equations. Integrating factors. Example 10.81, page 90

Internal problem ID [4474]

Book: Ordinary Differential Equations, By Tenenbaum and Pollard. Dover, NY 1963
Section: Chapter 2. Special types of differential equations of the first kind. Lesson 10
Problem number: Recognizable Exact Differential equations. Integrating factors. Example 10.81, page 90.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_separable]`

$$\boxed{y - 3xy' = 0}$$

✓ Solution by Maple
Time used: 0.0 (sec). Leaf size: 9

```
dsolve((y(x))-(3*x)*diff(y(x),x)=0,y(x), singsol=all)
```

$$y(x) = c_1 x^{\frac{1}{3}}$$

✓ Solution by Mathematica
Time used: 0.022 (sec). Leaf size: 18

```
DSolve[(y[x])-(3*x)*y'[x]==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to c_1 \sqrt[3]{x}$$
$$y(x) \to 0$$

4.8 problem Recognizable Exact Differential equations. Integrating factors. Example 10.83, page 90

Internal problem ID [4475]

Book: Ordinary Differential Equations, By Tenenbaum and Pollard. Dover, NY 1963
Section: Chapter 2. Special types of differential equations of the first kind. Lesson 10
Problem number: Recognizable Exact Differential equations. Integrating factors. Example 10.83, page 90.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[[_homogeneous, ‘class G‘], _rational]`

$$y(2y^3x^2+3)+x(y^3x^2-1)\,y'=0$$

✓ Solution by Maple
Time used: 0.031 (sec). Leaf size: 39

```
dsolve((y(x)*(2*x^2*y(x)^3+3))+(x*(x^2*y(x)^3-1))*diff(y(x),x)=0,y(x), singsol=all)
```

$$y(x)=\frac{e^{-\frac{11c_1}{3}}x^3}{\operatorname{RootOf}\left(11\,e^{11c_1}\_Z^{15}-e^{11c_1}\_Z^{11}+4x^{11}\right)^5}$$

✓ Solution by Mathematica

Time used: 10.635 (sec). Leaf size: 1081

```
DSolve[(y[x]*(2*x^2*y[x]^3+3))+(x*(x^2*y[x]^3-1))*y'[x]==0,y[x],x,IncludeSingularSolutions -
```

$$y(x) \to \text{Root}\left[1024\#1^{15}x^{22} + 14080\#1^{12}x^{20} + 77440\#1^9x^{18} + 212960\#1^6x^{16} - \#1^4e^{\frac{44c_1}{3}} + 292820\#1^3x^{14} + 161051x^{12}\&, 1\right]$$

$$y(x) \to \text{Root}\left[1024\#1^{15}x^{22} + 14080\#1^{12}x^{20} + 77440\#1^9x^{18} + 212960\#1^6x^{16} - \#1^4e^{\frac{44c_1}{3}} + 292820\#1^3x^{14} + 161051x^{12}\&, 2\right]$$

$$y(x) \to \text{Root}\left[1024\#1^{15}x^{22} + 14080\#1^{12}x^{20} + 77440\#1^9x^{18} + 212960\#1^6x^{16} - \#1^4e^{\frac{44c_1}{3}} + 292820\#1^3x^{14} + 161051x^{12}\&, 3\right]$$

$$y(x) \to \text{Root}\left[1024\#1^{15}x^{22} + 14080\#1^{12}x^{20} + 77440\#1^9x^{18} + 212960\#1^6x^{16} - \#1^4e^{\frac{44c_1}{3}} + 292820\#1^3x^{14} + 161051x^{12}\&, 4\right]$$

$$y(x) \to \text{Root}\left[1024\#1^{15}x^{22} + 14080\#1^{12}x^{20} + 77440\#1^9x^{18} + 212960\#1^6x^{16} - \#1^4e^{\frac{44c_1}{3}} + 292820\#1^3x^{14} + 161051x^{12}\&, 5\right]$$

$$y(x) \to \text{Root}\left[1024\#1^{15}x^{22} + 14080\#1^{12}x^{20} + 77440\#1^9x^{18} + 212960\#1^6x^{16} - \#1^4e^{\frac{44c_1}{3}} + 292820\#1^3x^{14} + 161051x^{12}\&, 6\right]$$

$$y(x) \to \text{Root}\left[1024\#1^{15}x^{22} + 14080\#1^{12}x^{20} + 77440\#1^9x^{18} + 212960\#1^6x^{16} - \#1^4e^{\frac{44c_1}{3}} + 292820\#1^3x^{14} + 161051x^{12}\&, 7\right]$$

$$y(x) \to \text{Root}\left[1024\#1^{15}x^{22} + 14080\#1^{12}x^{20} + 77440\#1^9x^{18} + 212960\#1^6x^{16} - \#1^4e^{\frac{44c_1}{3}} + 292820\#1^3x^{14} + 161051x^{12}\&, 8\right]$$

$$y(x) \to \text{Root}\left[1024\#1^{15}x^{22} + 14080\#1^{12}x^{20} + 77440\#1^9x^{18} + 212960\#1^6x^{16} - \#1^4e^{\frac{44c_1}{3}} + 292820\#1^3x^{14} + 161051x^{12}\&, 9\right]$$

$$y(x) \to \text{Root}\left[1024\#1^{15}x^{22} + 14080\#1^{12}x^{20} + 77440\#1^9x^{18} + 212960\#1^6x^{16} - \#1^4e^{\frac{44c_1}{3}} + 292820\#1^3x^{14} + 161051x^{12}\&, 10\right]$$

$$y(x) \to \text{Root}\left[1024\#1^{15}x^{22} + 14080\#1^{12}x^{20} + 77440\#1^9x^{18} + 212960\#1^6x^{16} - \#1^4e^{\frac{44c_1}{3}} + 292820\#1^3x^{14} + 161051x^{12}\&, 11\right]$$

$$y(x) \to \text{Root}\left[1024\#1^{15}x^{22} + 14080\#1^{12}x^{20} + 77440\#1^9x^{18} + 212960\#1^6x^{16} - \#1^4e^{\frac{44c_1}{3}} + 292820\#1^3x^{14} + 161051x^{12}\&, 12\right]$$

$$y(x) \to \text{Root}\left[1024\#1^{15}x^{22} + 14080\#1^{12}x^{20} + 77440\#1^9x^{18} + 212960\#1^6x^{16} - \#1^4e^{\frac{44c_1}{3}}\right.$$

4.9 problem Recognizable Exact Differential equations. Integrating factors. Exercise 10.1, page 90

Internal problem ID [4476]

Book: Ordinary Differential Equations, By Tenenbaum and Pollard. Dover, NY 1963
Section: Chapter 2. Special types of differential equations of the first kind. Lesson 10
Problem number: Recognizable Exact Differential equations. Integrating factors. Exercise 10.1, page 90.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[[_homogeneous, ‘class A‘], _exact, _rational, _dAlembert]`

$$\boxed{2xy + \left(x^2 + y^2\right) y' = -x^2}$$

✓ Solution by Maple

Time used: 0.031 (sec). Leaf size: 321

```
dsolve((2*x*y(x)+x^2)+(x^2+y(x)^2)*diff(y(x),x)=0,y(x), singsol=all)
```

$$y(x) = -\frac{2\left(c_1 x^2 - \frac{\left(4-4x^3 c_1^{\frac{3}{2}}+4\sqrt{5x^6 c_1^3-2x^3 c_1^{\frac{3}{2}}+1}\right)^{\frac{2}{3}}}{4}\right)}{\sqrt{c_1}\left(4-4x^3 c_1^{\frac{3}{2}}+4\sqrt{5x^6 c_1^3-2x^3 c_1^{\frac{3}{2}}+1}\right)^{\frac{1}{3}}}$$

$$y(x) = -\frac{\left(1+i\sqrt{3}\right)\left(4-4x^3 c_1^{\frac{3}{2}}+4\sqrt{5x^6 c_1^3-2x^3 c_1^{\frac{3}{2}}+1}\right)^{\frac{1}{3}}}{4\sqrt{c_1}} - \frac{\left(i\sqrt{3}-1\right)\sqrt{c_1}\, x^2}{\left(4-4x^3 c_1^{\frac{3}{2}}+4\sqrt{5x^6 c_1^3-2x^3 c_1^{\frac{3}{2}}+1}\right)^{\frac{1}{3}}}$$

$$y(x) = \frac{4i\sqrt{3}\, c_1 x^2 + i\left(4-4x^3 c_1^{\frac{3}{2}}+4\sqrt{5x^6 c_1^3-2x^3 c_1^{\frac{3}{2}}+1}\right)^{\frac{2}{3}}\sqrt{3} + 4c_1 x^2 - \left(4-4x^3 c_1^{\frac{3}{2}}+4\sqrt{5x^6 c_1^3-2x^3 c_1^{\frac{3}{2}}+ \cdots}\right.}{4\left(4-4x^3 c_1^{\frac{3}{2}}+4\sqrt{5x^6 c_1^3-2x^3 c_1^{\frac{3}{2}}+1}\right)^{\frac{1}{3}}\sqrt{c_1}}$$

✓ Solution by Mathematica

Time used: 23.867 (sec). Leaf size: 597

```
DSolve[(2*x*y[x]+x^2)+(x^2+y[x]^2)*y'[x]==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to \frac{\sqrt[3]{-x^3+\sqrt{5x^6-2e^{3c_1}x^3+e^{6c_1}}+e^{3c_1}}}{\sqrt[3]{2}} - \frac{\sqrt[3]{2}x^2}{\sqrt[3]{-x^3+\sqrt{5x^6-2e^{3c_1}x^3+e^{6c_1}}+e^{3c_1}}}$$

$$y(x) \to \frac{\sqrt[3]{2}\left(2+2i\sqrt{3}\right)x^2+i2^{2/3}\left(\sqrt{3}+i\right)\left(-x^3+\sqrt{5x^6-2e^{3c_1}x^3+e^{6c_1}}+e^{3c_1}\right)^{2/3}}{4\sqrt[3]{-x^3+\sqrt{5x^6-2e^{3c_1}x^3+e^{6c_1}}+e^{3c_1}}}$$

$$y(x) \to \frac{\left(1-i\sqrt{3}\right)x^2}{2^{2/3}\sqrt[3]{-x^3+\sqrt{5x^6-2e^{3c_1}x^3+e^{6c_1}}+e^{3c_1}}} - \frac{\left(1+i\sqrt{3}\right)\sqrt[3]{-x^3+\sqrt{5x^6-2e^{3c_1}x^3+e^{6c_1}}+e^{3c_1}}}{2\sqrt[3]{2}}$$

$$y(x) \to \frac{2\sqrt[3]{-2}x^2+(-2)^{2/3}\left(\sqrt{5}\sqrt{x^6}-x^3\right)^{2/3}}{2\sqrt[3]{\sqrt{5}\sqrt{x^6}-x^3}}$$

$$y(x) \to \frac{\left(2\sqrt{5}\sqrt{x^6}-2x^3\right)^{2/3}-2\sqrt[3]{2}x^2}{2\sqrt[3]{\sqrt{5}\sqrt{x^6}-x^3}}$$

$$y(x) \to \frac{\sqrt[3]{2}\left(2-2i\sqrt{3}\right)x^2+\left(-1-i\sqrt{3}\right)\left(2\sqrt{5}\sqrt{x^6}-2x^3\right)^{2/3}}{4\sqrt[3]{\sqrt{5}\sqrt{x^6}-x^3}}$$

4.10 problem Recognizable Exact Differential equations. Integrating factors. Exercise 10.2, page 90

Internal problem ID [4477]

Book: Ordinary Differential Equations, By Tenenbaum and Pollard. Dover, NY 1963
Section: Chapter 2. Special types of differential equations of the first kind. Lesson 10
Problem number: Recognizable Exact Differential equations. Integrating factors. Exercise 10.2, page 90.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_exact]`

$$\boxed{y\cos(x) + (y^3 + \sin(x))\, y' = -x^2}$$

✓ Solution by Maple
Time used: 0.015 (sec). Leaf size: 21

```
dsolve((x^2+y(x)*cos(x))+(y(x)^3+sin(x))*diff(y(x),x)=0,y(x), singsol=all)
```

$$\frac{x^3}{3} + \sin(x)\, y(x) + \frac{y(x)^4}{4} + c_1 = 0$$

✓ Solution by Mathematica

Time used: 60.198 (sec). Leaf size: 1119

```
DSolve[(x^2+y[x]*Cos[x])+(y[x]^3+Sin[x])*y'[x]==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to \frac{\sqrt{\frac{4x^3+\left(27\sin^2(x)+\sqrt{729\sin^4(x)-64(x^3-3c_1)^3}\right)^{2/3}-12c_1}{\sqrt[3]{27\sin^2(x)+\sqrt{729\sin^4(x)-64\left(x^3-3c_1\right)^3}}}}}{\sqrt{6}}$$

$$-\frac{1}{2}\sqrt{-\frac{8\left(x^3-3c_1\right)}{3\sqrt[3]{27\sin^2(x)+\sqrt{729\sin^4(x)-64\left(x^3-3c_1\right)^3}}}-\frac{2}{3}\sqrt[3]{27\sin^2(x)+\sqrt{729\sin^4(x)-64\left(x^3-3c_1\right.}}}$$

$$y(x) \to \frac{\sqrt{\frac{4x^3+\left(27\sin^2(x)+\sqrt{729\sin^4(x)-64(x^3-3c_1)^3}\right)^{2/3}-12c_1}{\sqrt[3]{27\sin^2(x)+\sqrt{729\sin^4(x)-64\left(x^3-3c_1\right)^3}}}}}{\sqrt{6}}$$

$$+\frac{1}{2}\sqrt{-\frac{8\left(x^3-3c_1\right)}{3\sqrt[3]{27\sin^2(x)+\sqrt{729\sin^4(x)-64\left(x^3-3c_1\right)^3}}}-\frac{2}{3}\sqrt[3]{27\sin^2(x)+\sqrt{729\sin^4(x)-64\left(x^3-3c_1\right.}}}$$

$$y(x) \to -\frac{\sqrt{\frac{4x^3+\left(27\sin^2(x)+\sqrt{729\sin^4(x)-64(x^3-3c_1)^3}\right)^{2/3}-12c_1}{\sqrt[3]{27\sin^2(x)+\sqrt{729\sin^4(x)-64\left(x^3-3c_1\right)^3}}}}}{\sqrt{6}}$$

$$-\frac{1}{2}\sqrt{-\frac{8\left(x^3-3c_1\right)}{3\sqrt[3]{27\sin^2(x)+\sqrt{729\sin^4(x)-64\left(x^3-3c_1\right)^3}}}-\frac{2}{3}\sqrt[3]{27\sin^2(x)+\sqrt{729\sin^4(x)-64\left(x^3-3c_1\right.}}}$$

$$y(x)$$

$$\to \frac{1}{2}\sqrt{-\frac{8\left(x^3-3c_1\right)}{3\sqrt[3]{27\sin^2(x)+\sqrt{729\sin^4(x)-64\left(x^3-3c_1\right)^3}}}-\frac{2}{3}\sqrt[3]{27\sin^2(x)+\sqrt{729\sin^4(x)-64\left(x^3-3c_1\right)}}}$$

$$\sqrt{\frac{4x^3+\left(27\sin^2(x)+\sqrt{729\sin^4(x)-64(x^3-3c_1)^3}\right)^{2/3}-12c_1}{\sqrt[3]{27\sin^2(x)+\sqrt{729\sin^4(x)-64\left(x^3-3c_1\right)^3}}}}$$

4.11 problem Recognizable Exact Differential equations. Integrating factors. Exercise 10.3, page 90

Internal problem ID [4478]

Book: Ordinary Differential Equations, By Tenenbaum and Pollard. Dover, NY 1963
Section: Chapter 2. Special types of differential equations of the first kind. Lesson 10
Problem number: Recognizable Exact Differential equations. Integrating factors. Exercise 10.3, page 90.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_rational, _Bernoulli]`

$$\boxed{y^2 + xyy' = -x^2 - x}$$

✓ Solution by Maple
Time used: 0.016 (sec). Leaf size: 49

```
dsolve((x^2+y(x)^2+x)+(x*y(x))*diff(y(x),x)=0,y(x), singsol=all)
```

$$y(x) = -\frac{\sqrt{-18x^4 - 24x^3 + 36c_1}}{6x}$$
$$y(x) = \frac{\sqrt{-18x^4 - 24x^3 + 36c_1}}{6x}$$

✓ Solution by Mathematica
Time used: 0.242 (sec). Leaf size: 60

```
DSolve[(x^2+y[x]^2+x)+(x*y[x])*y'[x]==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to -\frac{\sqrt{-\frac{x^4}{2} - \frac{2x^3}{3} + c_1}}{x}$$
$$y(x) \to \frac{\sqrt{-\frac{x^4}{2} - \frac{2x^3}{3} + c_1}}{x}$$

4.12 problem Recognizable Exact Differential equations. Integrating factors. Exercise 10.4, page 90

Internal problem ID [4479]

Book: Ordinary Differential Equations, By Tenenbaum and Pollard. Dover, NY 1963
Section: Chapter 2. Special types of differential equations of the first kind. Lesson 10
Problem number: Recognizable Exact Differential equations. Integrating factors. Exercise 10.4, page 90.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_exact]`

$$-2xy + e^y + (y - x^2 + x e^y) y' = -x$$

✓ Solution by Maple
Time used: 0.0 (sec). Leaf size: 28

```
dsolve((x-2*x*y(x)+exp(y(x)))+(y(x)-x^2+x*exp(y(x)))*diff(y(x),x)=0,y(x), singsol=all)
```

$$-y(x) x^2 + x e^{y(x)} + \frac{x^2}{2} + \frac{y(x)^2}{2} + c_1 = 0$$

✓ Solution by Mathematica
Time used: 0.316 (sec). Leaf size: 35

```
DSolve[(x-2*x*y[x]+Exp[y[x]])+(y[x]-x^2+x*Exp[y[x]])*y'[x]==0,y[x],x,IncludeSingularSolution
```

$$\text{Solve}\left[x^2(-y(x)) + \frac{x^2}{2} + xe^{y(x)} + \frac{y(x)^2}{2} = c_1, y(x)\right]$$

4.13 problem Recognizable Exact Differential equations. Integrating factors. Exercise 10.5, page 90

Internal problem ID [4480]

Book: Ordinary Differential Equations, By Tenenbaum and Pollard. Dover, NY 1963
Section: Chapter 2. Special types of differential equations of the first kind. Lesson 10
Problem number: Recognizable Exact Differential equations. Integrating factors. Exercise 10.5, page 90.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_exact]`

$$e^{x} \sin \left(y\right)+e^{-y}-\left(x \,e^{-y}-e^{x} \cos \left(y\right)\right) y^{\prime}=0$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 18

```
dsolve((exp(x)*sin(y(x))+exp(-y(x)))-(x*exp(-y(x))-exp(x)*cos(y(x)))*diff(y(x),x)=0,y(x), si
```

$$e^{x} \sin \left(y(x)\right)+x \,e^{-y(x)}+c_{1}=0$$

✓ Solution by Mathematica

Time used: 0.377 (sec). Leaf size: 24

```
DSolve[(Exp[x]*Sin[y[x]]+Exp[-y[x]])-(x*Exp[-y[x]]-Exp[x]*Cos[y[x]])*y'[x]==0,y[x],x,Include
```

$$\text{Solve}\left[x\left(-e^{-y(x)}\right)-e^{x}\sin(y(x))=c_1,y(x)\right]$$

4.14 problem Recognizable Exact Differential equations. Integrating factors. Exercise 10.6, page 90

Internal problem ID [4481]

Book: Ordinary Differential Equations, By Tenenbaum and Pollard. Dover, NY 1963
Section: Chapter 2. Special types of differential equations of the first kind. Lesson 10
Problem number: Recognizable Exact Differential equations. Integrating factors. Exercise 10.6, page 90.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[[_1st_order, _with_linear_symmetries], _rational]`

$$\boxed{-y^2 - y - (x^2 - y^2 - x)\, y' = -x^2}$$

✓ Solution by Maple
Time used: 0.047 (sec). Leaf size: 28

```
dsolve((x^2-y(x)^2-y(x))-(x^2-y(x)^2-x)*diff(y(x),x)=0,y(x), singsol=all)
```

$$2y(x) - \ln(y(x) + x) + \ln(y(x) - x) - 2x - c_1 = 0$$

✓ Solution by Mathematica
Time used: 0.242 (sec). Leaf size: 32

```
DSolve[(x^2-y[x]^2-y[x])-(x^2-y[x]^2-x)*y'[x]==0,y[x],x,IncludeSingularSolutions -> True]
```

$$\text{Solve}\left[-\frac{e^{2x-2y(x)}(y(x)+x)}{2(x-y(x))} = c_1, y(x)\right]$$

4.15 problem Recognizable Exact Differential equations. Integrating factors. Exercise 10.7, page 90

Internal problem ID [4482]

Book: Ordinary Differential Equations, By Tenenbaum and Pollard. Dover, NY 1963
Section: Chapter 2. Special types of differential equations of the first kind. Lesson 10
Problem number: Recognizable Exact Differential equations. Integrating factors. Exercise 10.7, page 90.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_rational]`

$$\boxed{y^2x^4 - y + \left(y^4x^2 - x\right) y' = 0}$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 25

```
dsolve((x^4*y(x)^2-y(x))+(x^2*y(x)^4-x)*diff(y(x),x)=0,y(x), singsol=all)
```

$$-\frac{x^3}{3} - \frac{1}{y(x)\, x} - \frac{y(x)^3}{3} + c_1 = 0$$

✓ Solution by Mathematica

Time used: 60.131 (sec). Leaf size: 1507

```
DSolve[(x^4*y[x]^2-y[x])+(x^2*y[x]^4-x)*y'[x]==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to \frac{1}{4}\left(\sqrt{2}\sqrt{\frac{8\sqrt[3]{2}x + 2^{2/3}\left(x^9 - 6c_1x^6 + 9c_1{}^2x^3 + \sqrt{x^2\left(-256x + (x^4 - 3c_1x)^4\right)}\right)^{2/3}}{x\sqrt[3]{x^9 - 6c_1x^6 + 9c_1{}^2x^3 + \sqrt{x^2\left(-256x + (x^4 - 3c_1x)^4\right)}}}}\right.$$

$$-2\sqrt{-\frac{\sqrt[3]{x\left(x^4 - 3c_1x\right)^2 + \sqrt{x^2\left(-256x + (x^4 - 3c_1x)^4\right)}}}{\sqrt[3]{2}x} - \frac{2\sqrt{2}\left(x^3 - 3c_1\right)}{\sqrt{\frac{8\sqrt[3]{2}x + 2^{2/3}\left(x^9 - 6c_1x^6 + 9c_1{}^2x^3 + \sqrt{x^2(-256x + \cdots}\right.}{x\sqrt[3]{x^9 - 6c_1x^6 + 9c_1{}^2x^3 + \sqrt{x^2(-256x \cdots}}}}}}$$

$$y(x) \to \frac{1}{4}\left(\sqrt{2}\sqrt{\frac{8\sqrt[3]{2}x + 2^{2/3}\left(x^9 - 6c_1x^6 + 9c_1{}^2x^3 + \sqrt{x^2\left(-256x + (x^4 - 3c_1x)^4\right)}\right)^{2/3}}{x\sqrt[3]{x^9 - 6c_1x^6 + 9c_1{}^2x^3 + \sqrt{x^2\left(-256x + (x^4 - 3c_1x)^4\right)}}}}\right.$$

$$+2\sqrt{-\frac{\sqrt[3]{x\left(x^4 - 3c_1x\right)^2 + \sqrt{x^2\left(-256x + (x^4 - 3c_1x)^4\right)}}}{\sqrt[3]{2}x} - \frac{2\sqrt{2}\left(x^3 - 3c_1\right)}{\sqrt{\frac{8\sqrt[3]{2}x + 2^{2/3}\left(x^9 - 6c_1x^6 + 9c_1{}^2x^3 + \sqrt{x^2(-256x + \cdots}\right.}{x\sqrt[3]{x^9 - 6c_1x^6 + 9c_1{}^2x^3 + \sqrt{x^2(-256x \cdots}}}}}}$$

$$y(x) \to \frac{1}{4}\left(-\sqrt{2}\sqrt{\frac{8\sqrt[3]{2}x + 2^{2/3}\left(x^9 - 6c_1x^6 + 9c_1{}^2x^3 + \sqrt{x^2\left(-256x + (x^4 - 3c_1x)^4\right)}\right)^{2/3}}{x\sqrt[3]{x^9 - 6c_1x^6 + 9c_1{}^2x^3 + \sqrt{x^2\left(-256x + (x^4 - 3c_1x)^4\right)}}}}\right.$$

$$-2\sqrt{-\frac{\sqrt[3]{x\left(x^4 - 3c_1x\right)^2 + \sqrt{x^2\left(-256x + (x^4 - 3c_1x)^4\right)}}}{\sqrt[3]{2}x} + \frac{2\sqrt{2}\left(x^3 - 3c_1\right)}{\sqrt{\frac{8\sqrt[3]{2}x + 2^{2/3}\left(x^9 - 6c_1x^6 + 9c_1{}^2x^3 + \sqrt{x^2(-256x + \cdots}\right.}{x\sqrt[3]{x^9 - 6c_1x^6 + 9c_1{}^2x^3 + \sqrt{x^2(-256x \cdots}}}}}}$$

$y(x)$

4.16 problem Recognizable Exact Differential equations. Integrating factors. Exercise 10.8, page 90

Internal problem ID [4483]

Book: Ordinary Differential Equations, By Tenenbaum and Pollard. Dover, NY 1963
Section: Chapter 2. Special types of differential equations of the first kind. Lesson 10
Problem number: Recognizable Exact Differential equations. Integrating factors. Exercise 10.8, page 90.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[[_homogeneous, 'class G'], _rational]`

$$\boxed{y(2x+y^3)-x(2x-y^3)\,y'=0}$$

✓ Solution by Maple
Time used: 0.047 (sec). Leaf size: 330

```
dsolve((y(x)*(2*x+y(x)^3))-(x*(2*x-y(x)^3))*diff(y(x),x)=0,y(x), singsol=all)
```

$$y(x)=\frac{\frac{\left(-108x^4+12\sqrt{81x^4-12c_1^3}\,x^2+8c_1^3\right)^{\frac{1}{3}}}{2}+\frac{2c_1^2}{\left(-108x^4+12\sqrt{81x^4-12c_1^3}\,x^2+8c_1^3\right)^{\frac{1}{3}}}+c_1}{3x}$$

$$y(x)=\frac{\left(-i\sqrt{3}-1\right)\left(-108x^4+12\sqrt{81x^4-12c_1^3}\,x^2+8c_1^3\right)^{\frac{2}{3}}+4\Big(ic_1\sqrt{3}-c_1+\Big(-108x^4+12\sqrt{81x^4-12c_1^3}\,x^2}{12\left(-108x^4+12\sqrt{81x^4-12c_1^3}\,x^2+8c_1^3\right)^{\frac{1}{3}}x}$$

$$y(x)=\frac{\left(i\sqrt{3}-1\right)\left(-108x^4+12\sqrt{81x^4-12c_1^3}\,x^2+8c_1^3\right)^{\frac{2}{3}}+4\Big(-ic_1\sqrt{3}-c_1+\Big(-108x^4+12\sqrt{81x^4-12c_1^3}\,x^2}{12\left(-108x^4+12\sqrt{81x^4-12c_1^3}\,x^2+8c_1^3\right)^{\frac{1}{3}}x}$$

✓ Solution by Mathematica

Time used: 11.386 (sec). Leaf size: 371

```
DSolve[(y[x]*(2*x+y[x]^3))-(x*(2*x-y[x]^3))*y'[x]==0,y[x],x,IncludeSingularSolutions -> True
```

$$y(x) \to -\frac{\frac{2\sqrt[3]{2}c_1{}^2}{\sqrt[3]{27x^4+3\sqrt{81x^8+12c_1{}^3x^4}+2c_1{}^3}}+2^{2/3}\sqrt[3]{27x^4+3\sqrt{81x^8+12c_1{}^3x^4}+2c_1{}^3}+2c_1}{6x}$$

$$y(x) \to \frac{\frac{2\sqrt[3]{2}\left(1+i\sqrt{3}\right)c_1{}^2}{\sqrt[3]{27x^4+3\sqrt{81x^8+12c_1{}^3x^4}+2c_1{}^3}}+2^{2/3}\left(1-i\sqrt{3}\right)\sqrt[3]{27x^4+3\sqrt{81x^8+12c_1{}^3x^4}+2c_1{}^3}-4c_1}{12x}$$

$$y(x) \to \frac{\frac{2\sqrt[3]{2}\left(1-i\sqrt{3}\right)c_1{}^2}{\sqrt[3]{27x^4+3\sqrt{81x^8+12c_1{}^3x^4}+2c_1{}^3}}+2^{2/3}\left(1+i\sqrt{3}\right)\sqrt[3]{27x^4+3\sqrt{81x^8+12c_1{}^3x^4}+2c_1{}^3}-4c_1}{12x}$$

$$y(x) \to 0$$

4.17 problem Recognizable Exact Differential equations. Integrating factors. Exercise 10.9, page 90

Internal problem ID [4484]

Book: Ordinary Differential Equations, By Tenenbaum and Pollard. Dover, NY 1963
Section: Chapter 2. Special types of differential equations of the first kind. Lesson 10
Problem number: Recognizable Exact Differential equations. Integrating factors. Exercise 10.9, page 90.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_exact]`

$$\arctan(xy) + \frac{xy - 2xy^2}{y^2x^2+1} + \frac{(x^2 - 2yx^2)\, y'}{y^2x^2+1} = 0$$

✓ Solution by Maple

Time used: 0.047 (sec). Leaf size: 22

```
dsolve((arctan(x*y(x))+(x*y(x)-2*x*y(x)^2)/(1+x^2*y(x)^2))+((x^2-2*x^2*y(x))/(1+x^2*y(x)^2))
```

$$y(x) = \frac{\tan\left(\text{RootOf}\left(x\_Z - \ln\left(\sec(\_Z)^2\right) + c_1\right)\right)}{x}$$

✓ Solution by Mathematica

Time used: 0.173 (sec). Leaf size: 26

```
DSolve[(ArcTan[x*y[x]]+(x*y[x]-2*x*y[x]^2)/(1+x^2*y[x]^2))+((x^2-2*x^2*y[x])/(1+x^2*y[x]^2))
```

$$\text{Solve}\left[\log\left(x^2y(x)^2+1\right) - x\arctan(xy(x)) = c_1, y(x)\right]$$

4.18 problem Recognizable Exact Differential equations. Integrating factors. Exercise 10.10, page 90

Internal problem ID [4485]

Book: Ordinary Differential Equations, By Tenenbaum and Pollard. Dover, NY 1963
Section: Chapter 2. Special types of differential equations of the first kind. Lesson 10
Problem number: Recognizable Exact Differential equations. Integrating factors. Exercise 10.10, page 90.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[‘y=_G(x,y’)‘]`

$$\boxed{(e^y y - e^x x)\, y' = -e^x(x+1)}$$

✓ Solution by Maple
Time used: 0.015 (sec). Leaf size: 20

```
dsolve((exp(x)*(x+1))+(y(x)*exp(y(x))-x*exp(x))*diff(y(x),x)=0,y(x), singsol=all)
```

$$x\, e^{-y(x)+x} + \frac{y(x)^2}{2} + c_1 = 0$$

✓ Solution by Mathematica
Time used: 0.291 (sec). Leaf size: 26

```
DSolve[(Exp[x]*(x+1))+(y[x]*Exp[y[x]]-x*Exp[x])*y'[x]==0,y[x],x,IncludeSingularSolutions ->
```

$$\text{Solve}\left[-\frac{1}{2}y(x)^2 - xe^{x-y(x)} = c_1, y(x)\right]$$

4.19 problem Recognizable Exact Differential equations. Integrating factors. Exercise 10.11, page 90

Internal problem ID [4486]

Book: Ordinary Differential Equations, By Tenenbaum and Pollard. Dover, NY 1963
Section: Chapter 2. Special types of differential equations of the first kind. Lesson 10
Problem number: Recognizable Exact Differential equations. Integrating factors. Exercise 10.11, page 90.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[[_homogeneous, ‘class D‘], _exact, _rational, [_Abel, ‘2nd ty`

$$\boxed{\frac{xy+1}{y}+\frac{(-x+2y)\,y'}{y^2}=0}$$

✓ Solution by Maple
Time used: 0.016 (sec). Leaf size: 20

```
dsolve(((x*y(x)+1)/y(x))+((2*y(x)-x)/y(x)^2)*diff(y(x),x)=0,y(x), singsol=all)
```

$$y(x)=-\frac{x}{2\,\mathrm{LambertW}\left(-\frac{\mathrm{e}^{\frac{x^2}{4}}c_1x}{2}\right)}$$

✓ Solution by Mathematica
Time used: 3.618 (sec). Leaf size: 37

```
DSolve[((x*y[x]+1)/y[x])+((2*y[x]-x)/y[x]^2)*y'[x]==0,y[x],x,IncludeSingularSolutions -> Tru
```

$$y(x)\to-\frac{x}{2W\left(-\frac{1}{2}xe^{\frac{1}{4}(x^2-2c_1)}\right)}$$
$$y(x)\to 0$$

4.20 problem Recognizable Exact Differential equations. Integrating factors. Exercise 10.12, page 90

Internal problem ID [4487]

Book: Ordinary Differential Equations, By Tenenbaum and Pollard. Dover, NY 1963
Section: Chapter 2. Special types of differential equations of the first kind. Lesson 10
Problem number: Recognizable Exact Differential equations. Integrating factors. Exercise 10.12, page 90.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[[_homogeneous, ‘class A‘], _rational, [_Abel, ‘2nd type‘, ‘cl`

$$y^2 - 3xy + (xy - x^2)\, y' = 2x^2$$

✓ Solution by Maple
Time used: 0.031 (sec). Leaf size: 59

```
dsolve((y(x)^2-3*x*y(x)-2*x^2)+(x*y(x)-x^2)*diff(y(x),x)=0,y(x), singsol=all)
```

$$y(x) = \frac{c_1 x^2 - \sqrt{2c_1^2 x^4 + 1}}{c_1 x}$$
$$y(x) = \frac{c_1 x^2 + \sqrt{2c_1^2 x^4 + 1}}{c_1 x}$$

✓ Solution by Mathematica
Time used: 0.657 (sec). Leaf size: 99

```
DSolve[(y[x]^2-3*x*y[x]-2*x^2)+(x*y[x]-x^2)*y'[x]==0,y[x],x,IncludeSingularSolutions -> True
```

$$y(x) \to x - \frac{\sqrt{2x^4 + e^{2c_1}}}{x}$$
$$y(x) \to x + \frac{\sqrt{2x^4 + e^{2c_1}}}{x}$$
$$y(x) \to x - \frac{\sqrt{2}\sqrt{x^4}}{x}$$
$$y(x) \to \frac{\sqrt{2}\sqrt{x^4}}{x} + x$$

4.21 problem Recognizable Exact Differential equations. Integrating factors. Exercise 10.13, page 90

Internal problem ID [4488]

Book: Ordinary Differential Equations, By Tenenbaum and Pollard. Dover, NY 1963
Section: Chapter 2. Special types of differential equations of the first kind. Lesson 10
Problem number: Recognizable Exact Differential equations. Integrating factors. Exercise 10.13, page 90.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_rational, [_Abel, '2nd type', 'class B']]`

$$\boxed{y(y+2x+1)-x(x+2y-1)\,y'=0}$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 389

```
dsolve((y(x)*(y(x)+2*x+1))-(x*(2*y(x)+x-1))*diff(y(x),x)=0,y(x), singsol=all)
```

$$y(x) = \frac{3\,5^{\frac{1}{3}}\left(x\left(\sqrt{5}\,\sqrt{\frac{80c_1x^2-160c_1x+80c_1-x}{c_1}}+20x-20\right)c_1^2\right)^{\frac{1}{3}}}{40c_1} + \frac{3x5^{\frac{2}{3}}}{40\left(x\left(\sqrt{5}\,\sqrt{\frac{80c_1x^2-160c_1x+80c_1-x}{c_1}}+20x-20\right)c_1^2\right)^{\frac{1}{3}}} + x - 1$$

$$y(x) = \frac{\frac{3\,5^{\frac{1}{3}}\left(-i\sqrt{3}-1\right)\left(x\left(\sqrt{5}\,\sqrt{\frac{80(x-1)^2c_1-x}{c_1}}+20x-20\right)c_1^2\right)^{\frac{2}{3}}}{80}+\frac{3c_1\left(\frac{80(x-1)\left(x\left(\sqrt{5}\,\sqrt{\frac{80(x-1)^2c_1-x}{c_1}}+20x-20\right)c_1^2\right)^{\frac{1}{3}}}{3}+\left(i\sqrt{3}-1\right)5^{\frac{2}{3}}x\right)}{80}}{c_1\left(x\left(\sqrt{5}\,\sqrt{\frac{80(x-1)^2c_1-x}{c_1}}+20x-20\right)c_1^2\right)^{\frac{1}{3}}}$$

$$y(x) = \frac{\frac{3\left(i\sqrt{3}-1\right)5^{\frac{1}{3}}\left(x\left(\sqrt{5}\,\sqrt{\frac{80(x-1)^2c_1-x}{c_1}}+20x-20\right)c_1^2\right)^{\frac{2}{3}}}{80}+\frac{3\left(-\frac{80(1-x)\left(x\left(\sqrt{5}\,\sqrt{\frac{80(x-1)^2c_1-x}{c_1}}+20x-20\right)c_1^2\right)^{\frac{1}{3}}}{3}+\left(-i\sqrt{3}-1\right)5^{\frac{2}{3}}x\right)c_1}{80}}{c_1\left(x\left(\sqrt{5}\,\sqrt{\frac{80(x-1)^2c_1-x}{c_1}}+20x-20\right)c_1^2\right)^{\frac{1}{3}}}$$

✓ Solution by Mathematica

Time used: 41.715 (sec). Leaf size: 463

```
DSolve[(y[x]*(y[x]+2*x+1))-(x*(2*y[x]+x-1))*y'[x]==0,y[x],x,IncludeSingularSolutions -> True
```

$$y(x) \to -\frac{\sqrt[3]{2}x}{\sqrt[3]{-27c_1{}^2x^2+\sqrt{108c_1{}^3x^3+\left(27c_1{}^2x-27c_1{}^2x^2\right)^2}+27c_1{}^2x}} + \frac{\sqrt[3]{-27c_1{}^2x^2+\sqrt{108c_1{}^3x^3+\left(27c_1{}^2x-27c_1{}^2x^2\right)^2}+27c_1{}^2x}}{3\sqrt[3]{2}c_1}+x-1$$

$$y(x) \to \frac{\left(1+i\sqrt{3}\right)x}{2^{2/3}\sqrt[3]{-27c_1{}^2x^2+\sqrt{108c_1{}^3x^3+\left(27c_1{}^2x-27c_1{}^2x^2\right)^2}+27c_1{}^2x}} - \frac{\left(1-i\sqrt{3}\right)\sqrt[3]{-27c_1{}^2x^2+\sqrt{108c_1{}^3x^3+\left(27c_1{}^2x-27c_1{}^2x^2\right)^2}+27c_1{}^2x}}{6\sqrt[3]{2}c_1}+x-1$$

$$y(x) \to \frac{\left(1-i\sqrt{3}\right)x}{2^{2/3}\sqrt[3]{-27c_1{}^2x^2+\sqrt{108c_1{}^3x^3+\left(27c_1{}^2x-27c_1{}^2x^2\right)^2}+27c_1{}^2x}} - \frac{\left(1+i\sqrt{3}\right)\sqrt[3]{-27c_1{}^2x^2+\sqrt{108c_1{}^3x^3+\left(27c_1{}^2x-27c_1{}^2x^2\right)^2}+27c_1{}^2x}}{6\sqrt[3]{2}c_1}+x-1$$

$$y(x) \to \text{Indeterminate}$$

$$y(x) \to x-1$$

4.22 problem Recognizable Exact Differential equations. Integrating factors. Exercise 10.14, page 90

Internal problem ID [4489]

Book: Ordinary Differential Equations, By Tenenbaum and Pollard. Dover, NY 1963
Section: Chapter 2. Special types of differential equations of the first kind. Lesson 10
Problem number: Recognizable Exact Differential equations. Integrating factors. Exercise 10.14, page 90.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_rational, [_Abel, '2nd type', 'class B']]`

$$\boxed{y(2x - y - 1) + x(2y - x - 1)\, y' = 0}$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 391

```
dsolve((y(x)*(2*x-y(x)-1))+(x*(2*y(x)-x-1))*diff(y(x),x)=0,y(x), singsol=all)
```

$$y(x) = \frac{3\,5^{\frac{1}{3}}\left(x\left(\sqrt{5}\,\sqrt{\frac{80c_1x^2+160c_1x+80c_1-x}{c_1}}-20x-20\right)c_1^2\right)^{\frac{1}{3}}}{40c_1} + \frac{3x5^{\frac{2}{3}}}{40\left(x\left(\sqrt{5}\,\sqrt{\frac{80c_1x^2+160c_1x+80c_1-x}{c_1}}-20x-20\right)c_1^2\right)^{\frac{1}{3}}} - 1 - x$$

$$y(x) = \frac{\frac{3\,5^{\frac{1}{3}}\left(-i\sqrt{3}-1\right)\left(-20\left(-\frac{\sqrt{5}\,\sqrt{\frac{80(1+x)^2c_1-x}{c_1}}}{20}+x+1\right)c_1^2x\right)^{\frac{2}{3}}}{80} + \frac{3c_1\left(\frac{80(-1-x)\left(-20\left(-\frac{\sqrt{5}\,\sqrt{\frac{80(1+x)^2c_1-x}{c_1}}}{20}+x+1\right)c_1^2x\right)^{\frac{1}{3}}}{3}+\left(i\sqrt{3}-1\right)5^{\frac{2}{3}}x\right)}{80}}{\left(-20\left(-\frac{\sqrt{5}\,\sqrt{\frac{80(1+x)^2c_1-x}{c_1}}}{20}+x+1\right)c_1^2x\right)^{\frac{1}{3}}c_1}$$

$$y(x) = \frac{\frac{3\left(i\sqrt{3}-1\right)5^{\frac{1}{3}}\left(-20\left(-\frac{\sqrt{5}\,\sqrt{\frac{80(1+x)^2c_1-x}{c_1}}}{20}+x+1\right)c_1^2x\right)^{\frac{2}{3}}}{80} + \frac{3\left(-\frac{80(1+x)\left(-20\left(-\frac{\sqrt{5}\,\sqrt{\frac{80(1+x)^2c_1-x}{c_1}}}{20}+x+1\right)c_1^2x\right)^{\frac{1}{3}}}{3}+\left(-i\sqrt{3}-1\right)5^{\frac{2}{3}}x\right)c_1}{80}}{\left(-20\left(-\frac{\sqrt{5}\,\sqrt{\frac{80(1+x)^2c_1-x}{c_1}}}{20}+x+1\right)c_1^2x\right)^{\frac{1}{3}}c_1}$$

✓ Solution by Mathematica

Time used: 40.285 (sec). Leaf size: 471

```
DSolve[(y[x]*(2*x-y[x]-1))+(x*(2*y[x]-x-1))*y'[x]==0,y[x],x,IncludeSingularSolutions -> True
```

$$y(x) \to -\frac{\sqrt[3]{2}x}{\sqrt[3]{27c_1{}^2x^2+\sqrt{\left(27c_1{}^2x^2+27c_1{}^2x\right)^2-108c_1{}^3x^3}+27c_1{}^2x}} - \frac{\sqrt[3]{27c_1{}^2x^2+\sqrt{\left(27c_1{}^2x^2+27c_1{}^2x\right)^2-108c_1{}^3x^3}+27c_1{}^2x}}{3\sqrt[3]{2}c_1} - x - 1$$

$$y(x) \to \frac{\left(1+i\sqrt{3}\right)x}{2^{2/3}\sqrt[3]{27c_1{}^2x^2+\sqrt{\left(27c_1{}^2x^2+27c_1{}^2x\right)^2-108c_1{}^3x^3}+27c_1{}^2x}} + \frac{\left(1-i\sqrt{3}\right)\sqrt[3]{27c_1{}^2x^2+\sqrt{\left(27c_1{}^2x^2+27c_1{}^2x\right)^2-108c_1{}^3x^3}+27c_1{}^2x}}{6\sqrt[3]{2}c_1} - x - 1$$

$$y(x) \to \frac{\left(1-i\sqrt{3}\right)x}{2^{2/3}\sqrt[3]{27c_1{}^2x^2+\sqrt{\left(27c_1{}^2x^2+27c_1{}^2x\right)^2-108c_1{}^3x^3}+27c_1{}^2x}} + \frac{\left(1+i\sqrt{3}\right)\sqrt[3]{27c_1{}^2x^2+\sqrt{\left(27c_1{}^2x^2+27c_1{}^2x\right)^2-108c_1{}^3x^3}+27c_1{}^2x}}{6\sqrt[3]{2}c_1} - x - 1$$

$$y(x) \to \text{Indeterminate}$$

$$y(x) \to -x - 1$$

4.23 problem Recognizable Exact Differential equations. Integrating factors. Exercise 10.15, page 90

Internal problem ID [4490]

Book: Ordinary Differential Equations, By Tenenbaum and Pollard. Dover, NY 1963
Section: Chapter 2. Special types of differential equations of the first kind. Lesson 10
Problem number: Recognizable Exact Differential equations. Integrating factors. Exercise 10.15, page 90.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[[_homogeneous, ‘class G‘], _exact, _rational, [_Abel, ‘2nd ty`

$$\boxed{y^2 + 12yx^2 + (2xy + 4x^3)\, y' = 0}$$

✓ Solution by Maple
Time used: 0.031 (sec). Leaf size: 51

```
dsolve((y(x)^2+12*x^2*y(x))+(2*x*y(x)+4*x^3)*diff(y(x),x)=0,y(x), singsol=all)
```

$$y(x) = \frac{-2x^3 + \sqrt{4x^6 + c_1x}}{x}$$
$$y(x) = \frac{-2x^3 - \sqrt{4x^6 + c_1x}}{x}$$

✓ Solution by Mathematica
Time used: 0.431 (sec). Leaf size: 58

```
DSolve[(y[x]^2+12*x^2*y[x])+(2*x*y[x]+4*x^3)*y'[x]==0,y[x],x,IncludeSingularSolutions -> Tru
```

$$y(x) \to -\frac{2x^3 + \sqrt{x(4x^5 + c_1)}}{x}$$
$$y(x) \to \frac{-2x^3 + \sqrt{x(4x^5 + c_1)}}{x}$$

4.24 problem Recognizable Exact Differential equations. Integrating factors. Exercise 10.16, page 90

Internal problem ID [4491]

Book: Ordinary Differential Equations, By Tenenbaum and Pollard. Dover, NY 1963
Section: Chapter 2. Special types of differential equations of the first kind. Lesson 10
Problem number: Recognizable Exact Differential equations. Integrating factors. Exercise 10.16, page 90.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[[_homogeneous, ‘class A‘], _rational, [_Abel, ‘2nd type‘, ‘cl`

$$\boxed{3(x+y)^2 + x(3y+2x)\,y' = 0}$$

✓ Solution by Maple
Time used: 0.031 (sec). Leaf size: 63

```
dsolve((3*(y(x)+x)^2)+(x*(3*y(x)+2*x))*diff(y(x),x)=0,y(x), singsol=all)
```

$$y(x) = \frac{-4c_1x^2 - \sqrt{-2c_1^2x^4+6}}{6c_1x}$$
$$y(x) = \frac{-4c_1x^2 + \sqrt{-2c_1^2x^4+6}}{6c_1x}$$

✓ Solution by Mathematica
Time used: 1.741 (sec). Leaf size: 135

```
DSolve[(3*(y[x]+x)^2)+(x*(3*y[x]+2*x))*y'[x]==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to -\frac{4x^2 + \sqrt{-2x^4+6e^{4c_1}}}{6x}$$
$$y(x) \to \frac{-4x^2 + \sqrt{-2x^4+6e^{4c_1}}}{6x}$$
$$y(x) \to -\frac{\sqrt{2}\sqrt{-x^4}+4x^2}{6x}$$
$$y(x) \to \frac{\sqrt{2}\sqrt{-x^4}-4x^2}{6x}$$

4.25 problem Recognizable Exact Differential equations. Integrating factors. Exercise 10.17, page 90

Internal problem ID [4492]

Book: Ordinary Differential Equations, By Tenenbaum and Pollard. Dover, NY 1963
Section: Chapter 2. Special types of differential equations of the first kind. Lesson 10
Problem number: Recognizable Exact Differential equations. Integrating factors. Exercise 10.17, page 90.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_rational]`

$$\boxed{y - (x^2 + y^2 + x)\, y' = 0}$$

✓ Solution by Maple
Time used: 0.032 (sec). Leaf size: 40

```
dsolve((y(x))-(y(x)^2+x^2+x)*diff(y(x),x)=0,y(x), singsol=all)
```

$$\frac{\mathrm{e}^{-2iy(x)}(ix + y(x)) + 2(iy(x) + x)\, c_1}{2iy\,(x) + 2x} = 0$$

✓ Solution by Mathematica
Time used: 0.102 (sec). Leaf size: 18

```
DSolve[(y[x])-(y[x]^2+x^2+x)*y'[x]==0,y[x],x,IncludeSingularSolutions -> True]
```

$$\text{Solve}\left[y(x) - \arctan\left(\frac{x}{y(x)}\right) = c_1, y(x)\right]$$

4.26 problem Recognizable Exact Differential equations. Integrating factors. Exercise 10.18, page 90

Internal problem ID [4493]

Book: Ordinary Differential Equations, By Tenenbaum and Pollard. Dover, NY 1963
Section: Chapter 2. Special types of differential equations of the first kind. Lesson 10
Problem number: Recognizable Exact Differential equations. Integrating factors. Exercise 10.18, page 90.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_exact, _rational, [_1st_order, '_with_symmetry_[F(x)*G(y),0]`

$$\boxed{2xy + (x^2 + y^2 + a)\, y' = 0}$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 313

```
dsolve((2*x*y(x))+(x^2+y(x)^2+a)*diff(y(x),x)=0,y(x), singsol=all)
```

$$y(x) = \frac{\left(-12c_1 + 4\sqrt{4x^6 + 12a\,x^4 + 12x^2a^2 + 4a^3 + 9c_1^2}\right)^{\frac{2}{3}} - 4x^2 - 4a}{2\left(-12c_1 + 4\sqrt{4x^6 + 12a\,x^4 + 12x^2a^2 + 4a^3 + 9c_1^2}\right)^{\frac{1}{3}}}$$

$$y(x) = -\frac{\left(\frac{i\sqrt{3}}{4} + \frac{1}{4}\right)\left(-12c_1 + 4\sqrt{4x^6 + 12a\,x^4 + 12x^2a^2 + 4a^3 + 9c_1^2}\right)^{\frac{2}{3}} + (x^2 + a)\left(i\sqrt{3} - 1\right)}{\left(-12c_1 + 4\sqrt{4x^6 + 12a\,x^4 + 12x^2a^2 + 4a^3 + 9c_1^2}\right)^{\frac{1}{3}}}$$

$$y(x) = \frac{\frac{\left(i\sqrt{3}-1\right)\left(-12c_1+4\sqrt{4x^6+12a\,x^4+12x^2a^2+4a^3+9c_1^2}\right)^{\frac{2}{3}}}{4} + (x^2 + a)\left(1 + i\sqrt{3}\right)}{\left(-12c_1 + 4\sqrt{4x^6 + 12a\,x^4 + 12x^2a^2 + 4a^3 + 9c_1^2}\right)^{\frac{1}{3}}}$$

✓ Solution by Mathematica

Time used: 4.319 (sec). Leaf size: 299

```
DSolve[(2*x*y[x])+(x^2+y[x]^2+a)*y'[x]==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to \frac{\sqrt[3]{2}\left(\sqrt{4(a+x^2)^3+9c_1^2}+3c_1\right)^{2/3}-2a-2x^2}{2^{2/3}\sqrt[3]{\sqrt{4(a+x^2)^3+9c_1^2}+3c_1}}$$

$$y(x) \to \frac{(1+i\sqrt{3})(a+x^2)}{2^{2/3}\sqrt[3]{\sqrt{4(a+x^2)^3+9c_1^2}+3c_1}}+\frac{i(\sqrt{3}+i)\sqrt[3]{\sqrt{4(a+x^2)^3+9c_1^2}+3c_1}}{2\sqrt[3]{2}}$$

$$y(x) \to \frac{(1-i\sqrt{3})(a+x^2)}{2^{2/3}\sqrt[3]{\sqrt{4(a+x^2)^3+9c_1^2}+3c_1}}-\frac{i(\sqrt{3}-i)\sqrt[3]{\sqrt{4(a+x^2)^3+9c_1^2}+3c_1}}{2\sqrt[3]{2}}$$

$$y(x) \to 0$$

4.27 problem Recognizable Exact Differential equations. Integrating factors. Exercise 10.19, page 90

Internal problem ID [4494]

Book: Ordinary Differential Equations, By Tenenbaum and Pollard. Dover, NY 1963
Section: Chapter 2. Special types of differential equations of the first kind. Lesson 10
Problem number: Recognizable Exact Differential equations. Integrating factors. Exercise 10.19, page 90.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_exact, _rational]`

$$\boxed{2xy + (x^2 + y^2 + a)\, y' = -x^2 - b}$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 505

```
dsolve((2*x*y(x)+x^2+b)+(y(x)^2+x^2+a)*diff(y(x),x)=0,y(x), singsol=all)
```

$$y(x) = \frac{-4x^2 - 4a + \left(-4x^3 - 12xb - 12c_1 + 4\sqrt{5x^6 + 6(2a+b)x^4 + 6c_1x^3 + 3(4a^2+3b^2)x^2 + 18xbc_1 + 4a^3}\right.}{2\left(-4x^3 - 12xb - 12c_1 + 4\sqrt{5x^6 + 6(2a+b)x^4 + 6c_1x^3 + 3(4a^2+3b^2)x^2 + 18xbc_1 + 4a^3 + 9c_1^2}\right.}$$

$$y(x) = -\frac{\left(\frac{i\sqrt{3}}{4} + \frac{1}{4}\right)\left(-4x^3 - 12xb - 12c_1 + 4\sqrt{5x^6 + 6(2a+b)x^4 + 6c_1x^3 + 3(4a^2+3b^2)x^2 + 18xbc_1 + 4a^3}\right.}{\left(-4x^3 - 12xb - 12c_1 + 4\sqrt{5x^6 + 6(2a+b)x^4 + 6c_1x^3 + 3(4a^2+3b^2)x^2 + 18xbc_1}\right.}$$

$$y(x) = \frac{\frac{(i\sqrt{3}-1)\left(-4x^3-12xb-12c_1+4\sqrt{5x^6+6(2a+b)x^4+6c_1x^3+3(4a^2+3b^2)x^2+18xbc_1+4a^3+9c_1^2}\right)^{\frac{2}{3}}}{4} + (x^2+a)\left(1+i\sqrt{3}\right)}{\left(-4x^3 - 12xb - 12c_1 + 4\sqrt{5x^6 + 6(2a+b)x^4 + 6c_1x^3 + 3(4a^2+3b^2)x^2 + 18xbc_1 + 4a^3 + 9c_1^2}\right)^{\frac{1}{3}}}$$

✓ Solution by Mathematica

Time used: 6.558 (sec). Leaf size: 396

```
DSolve[(2*x*y[x]+x^2+b)+(y[x]^2+x^2+a)*y'[x]==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to \frac{\sqrt[3]{2}\left(\sqrt{4\left(a+x^2\right)^3+\left(3bx+x^3-3c_1\right)^2}-3bx-x^3+3c_1\right)^{2/3}-2a-2x^2}{2^{2/3}\sqrt[3]{\sqrt{4\left(a+x^2\right)^3+\left(3bx+x^3-3c_1\right)^2}-3bx-x^3+3c_1}}$$

$$y(x) \to \frac{\left(1+i\sqrt{3}\right)\left(a+x^2\right)}{2^{2/3}\sqrt[3]{\sqrt{4\left(a+x^2\right)^3+\left(3bx+x^3-3c_1\right)^2}-3bx-x^3+3c_1}} + \frac{i\left(\sqrt{3}+i\right)\sqrt[3]{\sqrt{4\left(a+x^2\right)^3+\left(3bx+x^3-3c_1\right)^2}-3bx-x^3+3c_1}}{2\sqrt[3]{2}}$$

$$y(x) \to \frac{\left(1-i\sqrt{3}\right)\left(a+x^2\right)}{2^{2/3}\sqrt[3]{\sqrt{4\left(a+x^2\right)^3+\left(3bx+x^3-3c_1\right)^2}-3bx-x^3+3c_1}} - \frac{i\left(\sqrt{3}-i\right)\sqrt[3]{\sqrt{4\left(a+x^2\right)^3+\left(3bx+x^3-3c_1\right)^2}-3bx-x^3+3c_1}}{2\sqrt[3]{2}}$$

5 Chapter 2. Special types of differential equations of the first kind. Lesson 11, Bernoulli Equations

5.1 problem Exercise 11.1, page 97

Internal problem ID [4495]

Book: Ordinary Differential Equations, By Tenenbaum and Pollard. Dover, NY 1963
Section: Chapter 2. Special types of differential equations of the first kind. Lesson 11, Bernoulli Equations
Problem number: Exercise 11.1, page 97.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_linear]`

$$\boxed{xy' + y = x^3}$$

✓ Solution by Maple
Time used: 0.0 (sec). Leaf size: 16

```
dsolve(x*diff(y(x),x)+y(x)=x^3,y(x), singsol=all)
```

$$y(x) = \frac{x^4 + 4c_1}{4x}$$

✓ Solution by Mathematica
Time used: 0.025 (sec). Leaf size: 19

```
DSolve[x*y'[x]+y[x]==x^3,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to \frac{x^3}{4} + \frac{c_1}{x}$$

5.2 problem Exercise 11.2, page 97

Internal problem ID [4496]

Book: Ordinary Differential Equations, By Tenenbaum and Pollard. Dover, NY 1963
Section: Chapter 2. Special types of differential equations of the first kind. Lesson 11, Bernoulli Equations
Problem number: Exercise 11.2, page 97.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_quadrature]`

$$\boxed{y' + ya = b}$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 18

```
dsolve(diff(y(x),x)+a*y(x)=b,y(x), singsol=all)
```

$$y(x) = \frac{e^{-ax} c_1 a + b}{a}$$

✓ Solution by Mathematica

Time used: 0.035 (sec). Leaf size: 29

```
DSolve[y'[x]+a*y[x]==b,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to \frac{b}{a} + c_1 e^{-ax}$$
$$y(x) \to \frac{b}{a}$$

5.3 problem Exercise 11.3, page 97

Internal problem ID [4497]

Book: Ordinary Differential Equations, By Tenenbaum and Pollard. Dover, NY 1963
Section: Chapter 2. Special types of differential equations of the first kind. Lesson 11, Bernoulli Equations
Problem number: Exercise 11.3, page 97.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_Bernoulli]`

$$\boxed{xy' + y - \ln(x)\, y^2 = 0}$$

✓ Solution by Maple
Time used: 0.0 (sec). Leaf size: 13

```
dsolve(x*diff(y(x),x)+y(x)=y(x)^2*ln(x),y(x), singsol=all)
```

$$y(x) = \frac{1}{1 + c_1 x + \ln(x)}$$

✓ Solution by Mathematica
Time used: 0.143 (sec). Leaf size: 20

```
DSolve[x*y'[x]+y[x]==y[x]^2*Log[x],y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to \frac{1}{\log(x) + c_1 x + 1}$$
$$y(x) \to 0$$

5.4 problem Exercise 11.4, page 97

Internal problem ID [4498]

Book: Ordinary Differential Equations, By Tenenbaum and Pollard. Dover, NY 1963
Section: Chapter 2. Special types of differential equations of the first kind. Lesson 11, Bernoulli Equations
Problem number: Exercise 11.4, page 97.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_linear]`

$$\boxed{x' + 2yx = \mathrm{e}^{-y^2}}$$

✓ Solution by Maple
Time used: 0.0 (sec). Leaf size: 14

```
dsolve(diff(x(y),y)+2*y*x(y)=exp(-y^2),x(y), singsol=all)
```

$$x(y) = (y + c_1)\,\mathrm{e}^{-y^2}$$

✓ Solution by Mathematica
Time used: 0.054 (sec). Leaf size: 17

```
DSolve[x'[y]+2*y*x[y]==Exp[-y^2],x[y],y,IncludeSingularSolutions -> True]
```

$$x(y) \to e^{-y^2}(y + c_1)$$

5.5 problem Exercise 11.5, page 97

Internal problem ID [4499]

Book: Ordinary Differential Equations, By Tenenbaum and Pollard. Dover, NY 1963
Section: Chapter 2. Special types of differential equations of the first kind. Lesson 11, Bernoulli Equations
Problem number: Exercise 11.5, page 97.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_linear]`

$$\boxed{r' - (r + \mathrm{e}^{-\theta}) \tan(\theta) = 0}$$

✓ Solution by Maple
Time used: 0.0 (sec). Leaf size: 21

```
dsolve(diff(r(theta),theta)=(r(theta)+exp(-theta))*tan(theta),r(theta), singsol=all)
```

$$r(\theta) = \frac{(-\tan(\theta) - 1)\,\mathrm{e}^{-\theta}}{2} + \sec(\theta)\, c_1$$

✓ Solution by Mathematica
Time used: 0.093 (sec). Leaf size: 24

```
DSolve[r'[\[Theta]]==(r[\[Theta]]+Exp[-\[Theta]])*Tan[\[Theta]],r[\[Theta]],\[Theta],Include
```

$$r(\theta) \to -\frac{1}{2} e^{-\theta} (\tan(\theta) + 1) + c_1 \sec(\theta)$$

5.6 problem Exercise 11.6, page 97

Internal problem ID [4500]

Book: Ordinary Differential Equations, By Tenenbaum and Pollard. Dover, NY 1963
Section: Chapter 2. Special types of differential equations of the first kind. Lesson 11, Bernoulli Equations
Problem number: Exercise 11.6, page 97.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_linear]`

$$\boxed{y' - \frac{2xy}{x^2+1} = 1}$$

✓ Solution by Maple
Time used: 0.0 (sec). Leaf size: 14

```
dsolve(diff(y(x),x)-(2*x*y(x))/(x^2+1)=1,y(x), singsol=all)
```

$$y(x) = (\arctan(x) + c_1)(x^2 + 1)$$

✓ Solution by Mathematica
Time used: 0.034 (sec). Leaf size: 16

```
DSolve[y'[x]-2*x*y[x]/(x^2+1)==1,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to (x^2 + 1)(\arctan(x) + c_1)$$

5.7 problem Exercise 11.7, page 97

Internal problem ID [4501]

Book: Ordinary Differential Equations, By Tenenbaum and Pollard. Dover, NY 1963
Section: Chapter 2. Special types of differential equations of the first kind. Lesson 11, Bernoulli Equations
Problem number: Exercise 11.7, page 97.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_Bernoulli]`

$$\boxed{y' + y - y^3 x = 0}$$

✓ Solution by Maple
Time used: 0.0 (sec). Leaf size: 39

```
dsolve(diff(y(x),x)+y(x)=x*y(x)^3,y(x), singsol=all)
```

$$y(x) = -\frac{2}{\sqrt{2 + 4\,e^{2x} c_1 + 4x}}$$
$$y(x) = \frac{2}{\sqrt{2 + 4\,e^{2x} c_1 + 4x}}$$

✓ Solution by Mathematica
Time used: 2.606 (sec). Leaf size: 50

```
DSolve[y'[x]+y[x]==x*y[x]^3,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to -\frac{1}{\sqrt{x + c_1 e^{2x} + \frac{1}{2}}}$$
$$y(x) \to \frac{1}{\sqrt{x + c_1 e^{2x} + \frac{1}{2}}}$$
$$y(x) \to 0$$

5.8 problem Exercise 11.8, page 97

Internal problem ID [4502]

Book: Ordinary Differential Equations, By Tenenbaum and Pollard. Dover, NY 1963
Section: Chapter 2. Special types of differential equations of the first kind. Lesson 11, Bernoulli Equations
Problem number: Exercise 11.8, page 97.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_rational, _Bernoulli]`

$$\boxed{(-x^3+1)\, y' - 2(x+1)\, y - y^{\frac{5}{2}} = 0}$$

✓ Solution by Maple
Time used: 0.016 (sec). Leaf size: 38

```
dsolve((1-x^3)*diff(y(x),x)-2*(1+x)*y(x)=y(x)^(5/2),y(x), singsol=all)
```

$$-\frac{(x-1)^2 c_1}{x^2+x+1} + \frac{1}{y(x)^{\frac{3}{2}}} + \frac{3}{4x^2+4x+4} = 0$$

✓ Solution by Mathematica
Time used: 3.024 (sec). Leaf size: 41

```
DSolve[(1-x^3)*y'[x]-2*(1+x)*y[x]==y[x]^(5/2),y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to \frac{2\sqrt[3]{2}}{\left(\frac{-3+4c_1(x-1)^2}{x^2+x+1}\right)^{2/3}}$$
$$y(x) \to 0$$

5.9 problem Exercise 11.9, page 97

Internal problem ID [4503]

Book: Ordinary Differential Equations, By Tenenbaum and Pollard. Dover, NY 1963
Section: Chapter 2. Special types of differential equations of the first kind. Lesson 11, Bernoulli Equations
Problem number: Exercise 11.9, page 97.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_linear]`

$$\tan(\theta) r' - r = \tan(\theta)^2$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 15

```
dsolve(tan(theta)*diff(r(theta),theta)-r(theta)=tan(theta)^2,r(theta), singsol=all)
```

$$r(\theta) = (\ln(\sec(\theta) + \tan(\theta)) + c_1) \sin(\theta)$$

✓ Solution by Mathematica

Time used: 0.051 (sec). Leaf size: 14

```
DSolve[Tan[\[Theta]]*r'[\[Theta]]-r[\[Theta]]==Tan[\[Theta]]^2,r[\[Theta]],\[Theta],IncludeS
```

$$r(\theta) \to \sin(\theta) \left(\coth^{-1}(\sin(\theta)) + c_1\right)$$

5.10 problem Exercise 11.11, page 97

Internal problem ID [4504]

Book: Ordinary Differential Equations, By Tenenbaum and Pollard. Dover, NY 1963
Section: Chapter 2. Special types of differential equations of the first kind. Lesson 11, Bernoulli Equations
Problem number: Exercise 11.11, page 97.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[[_linear, ‘class A‘]]`

$$\boxed{y' + 2y = 3\,\mathrm{e}^{-2x}}$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 14

```
dsolve(diff(y(x),x)+2*y(x)=3*exp(-2*x),y(x), singsol=all)
```

$$y(x) = (3x + c_1)\,\mathrm{e}^{-2x}$$

✓ Solution by Mathematica

Time used: 0.051 (sec). Leaf size: 17

```
DSolve[y'[x]+2*y[x]==3*Exp[-2*x],y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to e^{-2x}(3x + c_1)$$

5.11 problem Exercise 11.12, page 97

Internal problem ID [4505]

Book: Ordinary Differential Equations, By Tenenbaum and Pollard. Dover, NY 1963
Section: Chapter 2. Special types of differential equations of the first kind. Lesson 11, Bernoulli Equations
Problem number: Exercise 11.12, page 97.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[[_linear, 'class A']]`

$$\boxed{y' + 2y = \frac{3\,\mathrm{e}^{-2x}}{4}}$$

✓ Solution by Maple
Time used: 0.0 (sec). Leaf size: 17

```
dsolve(diff(y(x),x)+2*y(x)=3/4*exp(-2*x),y(x), singsol=all)
```

$$y(x) = \frac{(3x + 4c_1)\,\mathrm{e}^{-2x}}{4}$$

✓ Solution by Mathematica
Time used: 0.05 (sec). Leaf size: 22

```
DSolve[y'[x]+2*y[x]==3/4*Exp[-2*x],y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to \frac{1}{4}e^{-2x}(3x + 4c_1)$$

5.12 problem Exercise 11.11, page 97

Internal problem ID [4506]

Book: Ordinary Differential Equations, By Tenenbaum and Pollard. Dover, NY 1963
Section: Chapter 2. Special types of differential equations of the first kind. Lesson 11, Bernoulli Equations
Problem number: Exercise 11.11, page 97.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[[_linear, 'class A']]`

$$\boxed{y' + 2y = \sin(x)}$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 19

```
dsolve(diff(y(x),x)+2*y(x)=sin(x),y(x), singsol=all)
```

$$y(x) = -\frac{\cos(x)}{5} + \frac{2\sin(x)}{5} + \mathrm{e}^{-2x}c_1$$

✓ Solution by Mathematica

Time used: 0.042 (sec). Leaf size: 26

```
DSolve[y'[x]+2*y[x]==Sin[x],y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to \frac{2\sin(x)}{5} - \frac{\cos(x)}{5} + c_1e^{-2x}$$

5.13 problem Exercise 11.14, page 97

Internal problem ID [4507]

Book: Ordinary Differential Equations, By Tenenbaum and Pollard. Dover, NY 1963
Section: Chapter 2. Special types of differential equations of the first kind. Lesson 11, Bernoulli Equations
Problem number: Exercise 11.14, page 97.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_linear]`

$$\boxed{y' + y\cos(x) = \mathrm{e}^{2x}}$$

✓ Solution by Maple
Time used: 0.016 (sec). Leaf size: 21

```
dsolve(diff(y(x),x)+y(x)*cos(x)=exp(2*x),y(x), singsol=all)
```

$$y(x) = \left(\int \mathrm{e}^{2x+\sin(x)}dx + c_1\right)\mathrm{e}^{-\sin(x)}$$

✓ Solution by Mathematica
Time used: 0.735 (sec). Leaf size: 32

```
DSolve[y'[x]+y[x]*Cos[x]==Exp[2*x],y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to e^{-\sin(x)}\left(\int_1^x e^{2K[1]+\sin(K[1])}dK[1] + c_1\right)$$

5.14 problem Exercise 11.15, page 97

Internal problem ID [4508]

Book: Ordinary Differential Equations, By Tenenbaum and Pollard. Dover, NY 1963
Section: Chapter 2. Special types of differential equations of the first kind. Lesson 11, Bernoulli Equations
Problem number: Exercise 11.15, page 97.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_linear]`

$$y' + y\cos(x) = \frac{\sin(2x)}{2}$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 15

```
dsolve(diff(y(x),x)+y(x)*cos(x)=1/2*sin(2*x),y(x), singsol=all)
```

$$y(x) = \sin(x) - 1 + \mathrm{e}^{-\sin(x)}c_1$$

✓ Solution by Mathematica

Time used: 0.05 (sec). Leaf size: 18

```
DSolve[y'[x]+y[x]*Cos[x]==1/2*Sin[2*x],y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to \sin(x) + c_1 e^{-\sin(x)} - 1$$

5.15 problem Exercise 11.16, page 97

Internal problem ID [4509]

Book: Ordinary Differential Equations, By Tenenbaum and Pollard. Dover, NY 1963
Section: Chapter 2. Special types of differential equations of the first kind. Lesson 11, Bernoulli Equations
Problem number: Exercise 11.16, page 97.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_linear]`

$$\boxed{xy' + y = \sin(x)\, x}$$

✓ Solution by Maple
Time used: 0.0 (sec). Leaf size: 17

```
dsolve(x*diff(y(x),x)+y(x)=x*sin(x),y(x), singsol=all)
```

$$y(x) = \frac{-x\cos(x) + \sin(x) + c_1}{x}$$

✓ Solution by Mathematica
Time used: 0.035 (sec). Leaf size: 19

```
DSolve[x*y'[x]+y[x]==x*Sin[x],y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to \frac{\sin(x) - x\cos(x) + c_1}{x}$$

5.16 problem Exercise 11.17, page 97

Internal problem ID [4510]

Book: Ordinary Differential Equations, By Tenenbaum and Pollard. Dover, NY 1963
Section: Chapter 2. Special types of differential equations of the first kind. Lesson 11, Bernoulli Equations
Problem number: Exercise 11.17, page 97.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_linear]`

$$\boxed{-y + xy' = x^2 \sin(x)}$$

✓ Solution by Maple
Time used: 0.0 (sec). Leaf size: 12

```
dsolve(x*diff(y(x),x)-y(x)=x^2*sin(x),y(x), singsol=all)
```

$$y(x) = (-\cos(x) + c_1)\, x$$

✓ Solution by Mathematica
Time used: 0.037 (sec). Leaf size: 14

```
DSolve[x*y'[x]-y[x]==x^2*Sin[x],y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to x(-\cos(x) + c_1)$$

5.17 problem Exercise 11.18, page 97

Internal problem ID [4511]

Book: Ordinary Differential Equations, By Tenenbaum and Pollard. Dover, NY 1963
Section: Chapter 2. Special types of differential equations of the first kind. Lesson 11, Bernoulli Equations
Problem number: Exercise 11.18, page 97.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[[_homogeneous, ‘class D‘], _rational, _Bernoulli]`

$$\boxed{xy' + xy^2 - y = 0}$$

✓ Solution by Maple
Time used: 0.0 (sec). Leaf size: 16

```
dsolve(x*diff(y(x),x)+x*y(x)^2-y(x)=0,y(x), singsol=all)
```

$$y(x) = \frac{2x}{x^2 + 2c_1}$$

✓ Solution by Mathematica
Time used: 0.149 (sec). Leaf size: 23

```
DSolve[x*y'[x]+x*y[x]^2-y[x]==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to \frac{2x}{x^2 + 2c_1}$$
$$y(x) \to 0$$

5.18 problem Exercise 11.19, page 97

Internal problem ID [4512]

Book: Ordinary Differential Equations, By Tenenbaum and Pollard. Dover, NY 1963
Section: Chapter 2. Special types of differential equations of the first kind. Lesson 11, Bernoulli Equations
Problem number: Exercise 11.19, page 97.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_Bernoulli]`

$$\boxed{xy' - y(2\ln(x)\, y - 1) = 0}$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 15

```
dsolve(x*diff(y(x),x)-y(x)*(2*y(x)*ln(x)-1)=0,y(x), singsol=all)
```

$$y(x) = \frac{1}{2 + c_1 x + 2\ln(x)}$$

✓ Solution by Mathematica

Time used: 0.14 (sec). Leaf size: 22

```
DSolve[x*y'[x]-y[x]*(2*y[x]*Log[x]-1)==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to \frac{1}{2\log(x) + c_1 x + 2}$$
$$y(x) \to 0$$

5.19 problem Exercise 11.20, page 97

Internal problem ID [4513]

Book: Ordinary Differential Equations, By Tenenbaum and Pollard. Dover, NY 1963
Section: Chapter 2. Special types of differential equations of the first kind. Lesson 11, Bernoulli Equations
Problem number: Exercise 11.20, page 97.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[[_homogeneous, ‘class D‘], _rational, _Bernoulli]`

$$\boxed{x^2(x-1)\,y' - y^2 - x(-2+x)\,y = 0}$$

✓ Solution by Maple
Time used: 0.0 (sec). Leaf size: 17

```
dsolve(x^2*(x-1)*diff(y(x),x)-y(x)^2-x*(x-2)*y(x)=0,y(x), singsol=all)
```

$$y(x) = \frac{x^2}{1 + c_1\,(x-1)}$$

✓ Solution by Mathematica
Time used: 0.191 (sec). Leaf size: 25

```
DSolve[x^2*(x-1)*y'[x]-y[x]^2-x*(x-2)*y[x]==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to \frac{x^2}{c_1(-x) + 1 + c_1}$$
$$y(x) \to 0$$

5.20 problem Exercise 11.21, page 97

Internal problem ID [4514]

Book: Ordinary Differential Equations, By Tenenbaum and Pollard. Dover, NY 1963
Section: Chapter 2. Special types of differential equations of the first kind. Lesson 11, Bernoulli Equations
Problem number: Exercise 11.21, page 97.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type **[[_linear, ‘class A‘]]**

$$\boxed{y' - y = \mathrm{e}^x}$$

With initial conditions

$$[y(0) = 1]$$

✓ Solution by Maple
Time used: 0.016 (sec). Leaf size: 10

```
dsolve([diff(y(x),x)-y(x)=exp(x),y(0) = 1],y(x), singsol=all)
```

$$y(x) = \mathrm{e}^x(1 + x)$$

✓ Solution by Mathematica
Time used: 0.039 (sec). Leaf size: 12

```
DSolve[{y'[x]-y[x]==Exp[x],{y[0]==1}},y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to e^x(x + 1)$$

5.21 problem Exercise 11.22, page 97

Internal problem ID [4515]

Book: Ordinary Differential Equations, By Tenenbaum and Pollard. Dover, NY 1963
Section: Chapter 2. Special types of differential equations of the first kind. Lesson 11, Bernoulli Equations
Problem number: Exercise 11.22, page 97.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_separable]`

$$\boxed{y' + \frac{y}{x} - \frac{y^2}{x} = 0}$$

With initial conditions

$$[y(-1) = 1]$$

✓ Solution by Maple
Time used: 0.0 (sec). Leaf size: 5

```
dsolve([diff(y(x),x)+y(x)/x=y(x)^2/x,y(-1) = 1],y(x), singsol=all)
```

$$y(x) = 1$$

✓ Solution by Mathematica
Time used: 0.001 (sec). Leaf size: 6

```
DSolve[{y'[x]+y[x]/x==y[x]^2/x,{y[-1]==1}},y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to 1$$

5.22 problem Exercise 11.23, page 97

Internal problem ID [4516]

Book: Ordinary Differential Equations, By Tenenbaum and Pollard. Dover, NY 1963
Section: Chapter 2. Special types of differential equations of the first kind. Lesson 11, Bernoulli Equations
Problem number: Exercise 11.23, page 97.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_Bernoulli]`

$$\boxed{2\cos(x)\, y' - \sin(x)\, y + y^3 = 0}$$

With initial conditions

$$[y(0) = 1]$$

✓ Solution by Maple

Time used: 0.578 (sec). Leaf size: 33

```
dsolve([2*cos(x)*diff(y(x),x)=y(x)*sin(x)-y(x)^3,y(0) = 1],y(x), singsol=all)
```

$$y(x) = \frac{\sqrt{(2\cos(x)^2 - 1)(-\sin(x) + \cos(x))}}{2\cos(x)^2 - 1}$$

✓ Solution by Mathematica

Time used: 0.369 (sec). Leaf size: 14

```
DSolve[{2*Cos[x]*y'[x]==y[x]*Sin[x]-y[x]^3,{y[0]==1}},y[x],x,IncludeSingularSolutions -> Tru
```

$$y(x) \to \frac{1}{\sqrt{\sin(x) + \cos(x)}}$$

5.23 problem Exercise 11.24, page 97

Internal problem ID [4517]

Book: Ordinary Differential Equations, By Tenenbaum and Pollard. Dover, NY 1963
Section: Chapter 2. Special types of differential equations of the first kind. Lesson 11, Bernoulli Equations
Problem number: Exercise 11.24, page 97.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[[_1st_order, '_with_symmetry_[F(x)*G(y),0]']]`

$$\boxed{(x - \cos(y))\, y' + \tan(y) = 0}$$

With initial conditions

$$\left[y(1) = \frac{\pi}{6}\right]$$

✓ Solution by Maple

Time used: 1.172 (sec). Leaf size: 29

```
dsolve([(x-cos(y(x)))*diff(y(x),x)+tan(y(x))=0,y(1) = 1/6*Pi],y(x), singsol=all)
```

$$y(x) = \text{RootOf}\left(24\sin(\_Z)\, x - 6\sin(2\_Z) + 2\pi + 3\sqrt{3} - 12\_Z - 12\right)$$

✓ Solution by Mathematica

Time used: 0.216 (sec). Leaf size: 45

```
DSolve[{(x-Cos[y[x]])*y'[x]+Tan[y[x]]==0,{y[1]==Pi/6}},y[x],x,IncludeSingularSolutions -> Tr
```

$$\text{Solve}\left[x = \frac{1}{24}\left(12 - 3\sqrt{3} - 2\pi\right)\csc(y(x)) + \left(\frac{y(x)}{2} + \frac{1}{4}\sin(2y(x))\right)\csc(y(x)), y(x)\right]$$

5.24 problem Exercise 11.26, page 97

Internal problem ID [4518]

Book: Ordinary Differential Equations, By Tenenbaum and Pollard. Dover, NY 1963
Section: Chapter 2. Special types of differential equations of the first kind. Lesson 11, Bernoulli Equations
Problem number: Exercise 11.26, page 97.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_rational, _Riccati]`

$$\boxed{y' - \frac{2y}{x} + \frac{y^2}{x} = x^3}$$

✓ Solution by Maple
Time used: 0.0 (sec). Leaf size: 19

```
dsolve(diff(y(x),x)=x^3+2/x*y(x)-1/x*y(x)^2,y(x), singsol=all)
```

$$y(x) = i \tan\left(-\frac{i x^2}{2} + c_1\right) x^2$$

✓ Solution by Mathematica
Time used: 0.162 (sec). Leaf size: 75

```
DSolve[y'[x]==x^3+2/x*y[x]-1/x*y[x]^2,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to \frac{x^2\left(i \cosh\left(\frac{x^2}{2}\right) + c_1 \sinh\left(\frac{x^2}{2}\right)\right)}{i \sinh\left(\frac{x^2}{2}\right) + c_1 \cosh\left(\frac{x^2}{2}\right)}$$
$$y(x) \to x^2 \tanh\left(\frac{x^2}{2}\right)$$

5.25 problem Exercise 11.27, page 97

Internal problem ID [4519]

Book: Ordinary Differential Equations, By Tenenbaum and Pollard. Dover, NY 1963
Section: Chapter 2. Special types of differential equations of the first kind. Lesson 11, Bernoulli Equations
Problem number: Exercise 11.27, page 97.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_Riccati]`

$$\boxed{y' + \sin(x)\, y^2 = 2\sec(x)\tan(x)}$$

✓ Solution by Maple
Time used: 0.0 (sec). Leaf size: 25

```
dsolve(diff(y(x),x)=2*tan(x)*sec(x)-y(x)^2*sin(x),y(x), singsol=all)
```

$$y(x) = \frac{-2c_1\cos(x)^2 + \sec(x)}{c_1\cos(x)^3 + 1}$$

✓ Solution by Mathematica
Time used: 0.88 (sec). Leaf size: 32

```
DSolve[y'[x]==2*Tan[x]*Sec[x]-y[x]^2*Sin[x],y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to \frac{\sec(x)\left(-2\cos^3(x) + c_1\right)}{\cos^3(x) + c_1}$$
$$y(x) \to \sec(x)$$

5.26 problem Exercise 11.28, page 97

Internal problem ID [4520]

Book: Ordinary Differential Equations, By Tenenbaum and Pollard. Dover, NY 1963
Section: Chapter 2. Special types of differential equations of the first kind. Lesson 11, Bernoulli Equations
Problem number: Exercise 11.28, page 97.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[[_homogeneous, ‘class G‘], _rational, _Riccati]`

$$\boxed{y' + \frac{y}{x} + y^2 = \frac{1}{x^2}}$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 16

```
dsolve(diff(y(x),x)=1/x^2-y(x)/x-y(x)^2,y(x), singsol=all)
```

$$y(x) = -\frac{\tanh(-\ln(x) + c_1)}{x}$$

✓ Solution by Mathematica

Time used: 1.192 (sec). Leaf size: 62

```
DSolve[y'[x]==1/x^2-y[x]/x-y[x]^2,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to \frac{i\tan(c_1 - i\log(x))}{x}$$
$$y(x) \to -\frac{-x^2 + e^{2i\text{Interval}[\{0,\pi\}]}}{x^3 + xe^{2i\text{Interval}[\{0,\pi\}]}}$$

5.27 problem Exercise 11.29, page 97

Internal problem ID [4521]

Book: Ordinary Differential Equations, By Tenenbaum and Pollard. Dover, NY 1963
Section: Chapter 2. Special types of differential equations of the first kind. Lesson 11, Bernoulli Equations
Problem number: Exercise 11.29, page 97.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[[_homogeneous, 'class A'], _rational, _Riccati]`

$$\boxed{y' - \frac{y}{x} + \frac{y^2}{x^2} = 1}$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 11

```
dsolve(diff(y(x),x)=1+y(x)/x-y(x)^2/x^2,y(x), singsol=all)
```

$$y(x) = \tanh\left(\ln\left(x\right) + c_1\right) x$$

✓ Solution by Mathematica

Time used: 0.539 (sec). Leaf size: 43

```
DSolve[y'[x]==1+y[x]/x-y[x]^2/x^2,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to \frac{x(x^2 - e^{2c_1})}{x^2 + e^{2c_1}}$$
$$y(x) \to -x$$
$$y(x) \to x$$

6 Chapter 2. Special types of differential equations of the first kind. Lesson 12, Miscellaneous Methods

6.1 problem Exercise 12.1, page 103

Internal problem ID [4522]

Book: Ordinary Differential Equations, By Tenenbaum and Pollard. Dover, NY 1963
Section: Chapter 2. Special types of differential equations of the first kind. Lesson 12, Miscellaneous Methods
Problem number: Exercise 12.1, page 103.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_Bernoulli]`

$$\boxed{2xyy' + y^2(x+1) = \mathrm{e}^x}$$

✓ Solution by Maple
Time used: 0.0 (sec). Leaf size: 59

```
dsolve(2*x*y(x)*diff(y(x),x)+(1+x)*y(x)^2=exp(x),y(x), singsol=all)
```

$$y(x) = -\frac{\sqrt{2}\,\sqrt{x\,\mathrm{e}^x\,(\mathrm{e}^{2x}+2c_1)}\,\mathrm{e}^{-x}}{2x}$$
$$y(x) = \frac{\sqrt{2}\,\sqrt{x\,\mathrm{e}^x\,(\mathrm{e}^{2x}+2c_1)}\,\mathrm{e}^{-x}}{2x}$$

✓ Solution by Mathematica
Time used: 7.324 (sec). Leaf size: 66

```
DSolve[2*x*y[x]*y'[x]+(1+x)*y[x]^2==Exp[x],y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to -\frac{\sqrt{e^x + 2c_1 e^{-x}}}{\sqrt{2}\sqrt{x}}$$
$$y(x) \to \frac{\sqrt{e^x + 2c_1 e^{-x}}}{\sqrt{2}\sqrt{x}}$$

6.2 problem Exercise 12.2, page 103

Internal problem ID [4523]

Book: Ordinary Differential Equations, By Tenenbaum and Pollard. Dover, NY 1963
Section: Chapter 2. Special types of differential equations of the first kind. Lesson 12, Miscellaneous Methods
Problem number: Exercise 12.2, page 103.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[‘y=_G(x,y’)‘]`

$$\boxed{\cos(y)\, y' + \sin(y) = x^2}$$

✓ Solution by Maple
Time used: 0.0 (sec). Leaf size: 23

```
dsolve(cos(y(x))*diff(y(x),x)+sin(y(x))=x^2,y(x), singsol=all)
```

$$y(x) = -\arcsin\left(-x^2 + 2x - 2 + e^{-x}c_1\right)$$

✓ Solution by Mathematica
Time used: 14.047 (sec). Leaf size: 23

```
DSolve[Cos[y[x]]*y'[x]+Sin[y[x]]==x^2,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to \arcsin\left(x^2 - 2x - 2c_1e^{-x} + 2\right)$$

6.3 problem Exercise 12.3, page 103

Internal problem ID [4524]

Book: Ordinary Differential Equations, By Tenenbaum and Pollard. Dover, NY 1963
Section: Chapter 2. Special types of differential equations of the first kind. Lesson 12, Miscellaneous Methods
Problem number: Exercise 12.3, page 103.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[[_1st_order, _with_linear_symmetries]]`

$$\boxed{(x+1)\,y'-y-(x+1)\,\sqrt{1+y}=1}$$

✓ Solution by Maple
Time used: 0.0 (sec). Leaf size: 81

```
dsolve((x+1)*diff(y(x),x)-(y(x)+1)=(x+1)*sqrt(y(x)+1),y(x), singsol=all)
```

$$\frac{(-c_1y(x)+1+c_1x^2+(2c_1+1)\,x)\,\sqrt{y\,(x)+1}-(1+x)\,(-c_1y(x)-1+c_1x^2+(2c_1-1)\,x)}{(x^2+2x-y\,(x))\left(-\sqrt{y\,(x)+1}+1+x\right)} = 0$$

✓ Solution by Mathematica
Time used: 0.244 (sec). Leaf size: 60

```
DSolve[(x+1)*y'[x]-(y[x]+1)==(x+1)*Sqrt[y[x]+1],y[x],x,IncludeSingularSolutions -> True]
```

$$\text{Solve}\left[\frac{2\sqrt{y(x)+1}\arctan\left(\frac{x+1}{\sqrt{-y(x)-1}}\right)}{\sqrt{-y(x)-1}}+\log\left(y(x)-(x+1)^2+1\right)-\log(x+1)=c_1,y(x)\right]$$

6.4 problem Exercise 12.4, page 103

Internal problem ID [4525]

Book: Ordinary Differential Equations, By Tenenbaum and Pollard. Dover, NY 1963
Section: Chapter 2. Special types of differential equations of the first kind. Lesson 12, Miscellaneous Methods
Problem number: Exercise 12.4, page 103.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[[_homogeneous, ‘class C‘], _dAlembert]`

$$\boxed{e^{y}(1+y') = e^{x}}$$

✓ Solution by Maple
Time used: 0.031 (sec). Leaf size: 19

```
dsolve(exp(y(x))*(diff(y(x),x)+1)=exp(x),y(x), singsol=all)
```

$$y(x) = x - \ln(2) + \ln\left(1 + e^{-2x}c_1\right)$$

✓ Solution by Mathematica
Time used: 1.32 (sec). Leaf size: 22

```
DSolve[Exp[y[x]]*(y'[x]+1)==Exp[x],y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to -x + \log\left(\frac{e^{2x}}{2} + c_1\right)$$

6.5 problem Exercise 12.5, page 103

Internal problem ID [4526]

Book: Ordinary Differential Equations, By Tenenbaum and Pollard. Dover, NY 1963
Section: Chapter 2. Special types of differential equations of the first kind. Lesson 12, Miscellaneous Methods
Problem number: Exercise 12.5, page 103.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_separable]`

$$\boxed{y' \sin(y) + \sin(x) \cos(y) = \sin(x)}$$

✓ Solution by Maple

Time used: 0.031 (sec). Leaf size: 14

```
dsolve(diff(y(x),x)*sin(y(x))+sin(x)*cos(y(x))=sin(x),y(x), singsol=all)
```

$$y(x) = \arccos\left(e^{-\cos(x)} c_1 + 1\right)$$

✓ Solution by Mathematica

Time used: 0.792 (sec). Leaf size: 81

```
DSolve[y'[x]*Sin[y[x]]+Sin[x]*Cos[y[x]]==Sin[x],y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to 0$$

$$\text{Solve}\left[2\cos(x)\tan\left(\frac{y(x)}{2}\right)e^{\operatorname{arctanh}(\cos(y(x)))} - \sqrt{\sin^2(y(x))}\csc\left(\frac{y(x)}{2}\right)\sec\left(\frac{y(x)}{2}\right)\left(\log\left(\sec^2\left(\frac{y(x)}{2}\right)\right) - 2\log\left(\tan\left(\frac{y(x)}{2}\right)\right)\right) = c_1, y(x)\right]$$

$$y(x) \to 0$$

6.6 problem Exercise 12.6, page 103

Internal problem ID [4527]

Book: Ordinary Differential Equations, By Tenenbaum and Pollard. Dover, NY 1963
Section: Chapter 2. Special types of differential equations of the first kind. Lesson 12, Miscellaneous Methods
Problem number: Exercise 12.6, page 103.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[[_homogeneous, ‘class C‘], _dAlembert]`

$$\boxed{(x-y)^2 y' = 4}$$

✓ Solution by Maple
Time used: 0.063 (sec). Leaf size: 27

```
dsolve((x-y(x))^2*diff(y(x),x)=4,y(x), singsol=all)
```

$$y(x) + \ln(y(x) - x - 2) - \ln(y(x) - x + 2) - c_1 = 0$$

✓ Solution by Mathematica
Time used: 0.202 (sec). Leaf size: 36

```
DSolve[(x-y[x])^2*y'[x]==4,y[x],x,IncludeSingularSolutions -> True]
```

$$\text{Solve}\left[y(x) - 4\left(\frac{1}{4}\log(y(x) - x + 2) - \frac{1}{4}\log(-y(x) + x + 2)\right) = c_1, y(x)\right]$$

6.7 problem Exercise 12.7, page 103

Internal problem ID [4528]

Book: Ordinary Differential Equations, By Tenenbaum and Pollard. Dover, NY 1963
Section: Chapter 2. Special types of differential equations of the first kind. Lesson 12, Miscellaneous Methods
Problem number: Exercise 12.7, page 103.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[[_homogeneous, ‘class A‘], _rational, _dAlembert]`

$$-y + xy' - \sqrt{x^2 + y^2} = 0$$

✓ Solution by Maple
Time used: 0.016 (sec). Leaf size: 26

```
dsolve(x*diff(y(x),x)-y(x)=sqrt(x^2+y(x)^2),y(x), singsol=all)
```

$$\frac{-c_1x^2 + \sqrt{x^2 + y(x)^2} + y(x)}{x^2} = 0$$

✓ Solution by Mathematica
Time used: 0.337 (sec). Leaf size: 27

```
DSolve[x*y'[x]-y[x]==Sqrt[x^2+y[x]^2],y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to \frac{1}{2}e^{-c_1}\left(-1 + e^{2c_1}x^2\right)$$

6.8 problem Exercise 12.8, page 103

Internal problem ID [4529]

Book: Ordinary Differential Equations, By Tenenbaum and Pollard. Dover, NY 1963
Section: Chapter 2. Special types of differential equations of the first kind. Lesson 12, Miscellaneous Methods
Problem number: Exercise 12.8, page 103.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[[_homogeneous, ‘class C‘], _exact, _rational, [_Abel, ‘2nd ty`

$$(3x + 2y + 1)\, y' + 3y = -4x - 2$$

✓ Solution by Maple
Time used: 0.11 (sec). Leaf size: 32

```
dsolve((3*x+2*y(x)+1)*diff(y(x),x)+(4*x+3*y(x)+2)=0,y(x), singsol=all)
```

$$y(x) = \frac{-\sqrt{(x-1)^2 c_1^2 + 4} + (-3x - 1)\, c_1}{2c_1}$$

✓ Solution by Mathematica
Time used: 0.123 (sec). Leaf size: 61

```
DSolve[(3*x+2*y[x]+1)*y'[x]+(4*x+3*y[x]+2)==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to \frac{1}{2}\left(-\sqrt{x^2 - 2x + 1 + 4c_1} - 3x - 1\right)$$
$$y(x) \to \frac{1}{2}\left(\sqrt{x^2 - 2x + 1 + 4c_1} - 3x - 1\right)$$

6.9 problem Exercise 12.9, page 103

Internal problem ID [4530]

Book: Ordinary Differential Equations, By Tenenbaum and Pollard. Dover, NY 1963
Section: Chapter 2. Special types of differential equations of the first kind. Lesson 12, Miscellaneous Methods
Problem number: Exercise 12.9, page 103.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[[_homogeneous, ‘class A‘], _rational, _dAlembert]`

$$\boxed{(x^2 - y^2)\, y' - 2xy = 0}$$

✓ Solution by Maple
Time used: 0.016 (sec). Leaf size: 47

```
dsolve((x^2-y(x)^2)*diff(y(x),x)=2*x*y(x),y(x), singsol=all)
```

$$y(x) = \frac{1 - \sqrt{-4x^2c_1^2 + 1}}{2c_1}$$
$$y(x) = \frac{1 + \sqrt{-4x^2c_1^2 + 1}}{2c_1}$$

✓ Solution by Mathematica
Time used: 0.982 (sec). Leaf size: 66

```
DSolve[(x^2-y[x]^2)*y'[x]==2*x*y[x],y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to \frac{1}{2}\left(e^{c_1} - \sqrt{-4x^2 + e^{2c_1}}\right)$$
$$y(x) \to \frac{1}{2}\left(\sqrt{-4x^2 + e^{2c_1}} + e^{c_1}\right)$$
$$y(x) \to 0$$

6.10 problem Exercise 12.10, page 103

Internal problem ID [4531]

Book: Ordinary Differential Equations, By Tenenbaum and Pollard. Dover, NY 1963
Section: Chapter 2. Special types of differential equations of the first kind. Lesson 12, Miscellaneous Methods
Problem number: Exercise 12.10, page 103.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[[_1st_order, _with_linear_symmetries]]`

$$\boxed{y + (1 + e^{2x}y^2)\, y' = 0}$$

✓ Solution by Maple
Time used: 0.031 (sec). Leaf size: 18

```
dsolve(y(x)+(1+y(x)^2*exp(2*x))*diff(y(x),x)=0,y(x), singsol=all)
```

$$y(x) = \frac{e^{-x}}{\sqrt{\text{LambertW}\left(e^{-2x}c_1\right)}}$$

✓ Solution by Mathematica
Time used: 3.33 (sec). Leaf size: 57

```
DSolve[y[x]+(1+y[x]^2*Exp[2*x])*y'[x]==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to -\frac{e^{-x}}{\sqrt{W\left(e^{-2x+2c_1}\right)}}$$
$$y(x) \to \frac{e^{-x}}{\sqrt{W\left(e^{-2x+2c_1}\right)}}$$
$$y(x) \to 0$$

6.11 problem Exercise 12.11, page 103

Internal problem ID [4532]

Book: Ordinary Differential Equations, By Tenenbaum and Pollard. Dover, NY 1963
Section: Chapter 2. Special types of differential equations of the first kind. Lesson 12, Miscellaneous Methods
Problem number: Exercise 12.11, page 103.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[[_homogeneous, ‘class G‘], _rational, _Bernoulli]`

$$\boxed{yx^2 + y^2 + y'x^3 = 0}$$

✓ Solution by Maple
Time used: 0.0 (sec). Leaf size: 19

```
dsolve((x^2*y(x)+y(x)^2)+x^3*diff(y(x),x)=0,y(x), singsol=all)
```

$$y(x) = \frac{3x^2}{3c_1x^3 - 1}$$

✓ Solution by Mathematica
Time used: 0.143 (sec). Leaf size: 26

```
DSolve[(x^2*y[x]+y[x]^2)+x^3*y'[x]==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to \frac{3x^2}{-1 + 3c_1x^3}$$
$$y(x) \to 0$$

6.12 problem Exercise 12.12, page 103

Internal problem ID [4533]

Book: Ordinary Differential Equations, By Tenenbaum and Pollard. Dover, NY 1963
Section: Chapter 2. Special types of differential equations of the first kind. Lesson 12, Miscellaneous Methods
Problem number: Exercise 12.12, page 103.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_exact]`

$$y^2 e^{xy^2} + \left(2xy\, e^{xy^2} - 3y^2\right) y' = -4x^3$$

✓ Solution by Maple
Time used: 0.016 (sec). Leaf size: 21

```
dsolve((y(x)^2*exp(x*y(x)^2)+4*x^3)+(2*x*y(x)*exp(x*y(x)^2)-3*y(x)^2)*diff(y(x),x)=0,y(x), s
```

$$e^{xy(x)^2} + x^4 - y(x)^3 + c_1 = 0$$

✓ Solution by Mathematica
Time used: 0.279 (sec). Leaf size: 24

```
DSolve[(y[x]^2*Exp[x*y[x]^2]+4*x^3)+(2*x*y[x]*Exp[x*y[x]^2]-3*y[x]^2)*y'[x]==0,y[x],x,Includ
```

$$\text{Solve}\left[x^4 + e^{xy(x)^2} - y(x)^3 = c_1, y(x)\right]$$

6.13 problem Exercise 12.13, page 103

Internal problem ID [4534]

Book: Ordinary Differential Equations, By Tenenbaum and Pollard. Dover, NY 1963
Section: Chapter 2. Special types of differential equations of the first kind. Lesson 12, Miscellaneous Methods
Problem number: Exercise 12.13, page 103.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[[_1st_order, _with_linear_symmetries]]`

$$\boxed{y' - \left(x^2 + 2y - 1\right)^{\frac{2}{3}} = -x}$$

✓ Solution by Maple
Time used: 0.0 (sec). Leaf size: 21

```
dsolve(diff(y(x),x)=(x^2+2*y(x)-1)^(2/3)-x,y(x), singsol=all)
```

$$x - \frac{3(x^2 + 2y(x) - 1)^{\frac{1}{3}}}{2} - c_1 = 0$$

✓ Solution by Mathematica
Time used: 0.214 (sec). Leaf size: 40

```
DSolve[y'[x]==(x^2+2*y[x]-1)^(2/3)-x,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to \frac{1}{54}\left(8x^3 - 3(9 + 8c_1)x^2 + 24{c_1}^2 x + 27 - 8{c_1}^3\right)$$

6.14 problem Exercise 12.14, page 103

Internal problem ID [4535]

Book: Ordinary Differential Equations, By Tenenbaum and Pollard. Dover, NY 1963
Section: Chapter 2. Special types of differential equations of the first kind. Lesson 12, Miscellaneous Methods
Problem number: Exercise 12.14, page 103.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_Bernoulli]`

$$xy' + y - x^2(1 + e^x)\, y^2 = 0$$

✓ Solution by Maple
Time used: 0.0 (sec). Leaf size: 18

```
dsolve(x*diff(y(x),x)+y(x)=x^2*(1+exp(x))*y(x)^2,y(x), singsol=all)
```

$$y(x) = -\frac{1}{(x + e^x - c_1)\, x}$$

✓ Solution by Mathematica
Time used: 0.249 (sec). Leaf size: 55

```
DSolve[x*y'[x]+y[x]==x^2*(1+exp[x])*y[x]^2,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to \frac{1}{-x \int_1^x (\exp(K[1]) + 1)dK[1] + c_1 x}$$
$$y(x) \to 0$$
$$y(x) \to -\frac{1}{x \int_1^x (\exp(K[1]) + 1)dK[1]}$$

6.15 problem Exercise 12.15, page 103

Internal problem ID [4536]

Book: Ordinary Differential Equations, By Tenenbaum and Pollard. Dover, NY 1963
Section: Chapter 2. Special types of differential equations of the first kind. Lesson 12, Miscellaneous Methods
Problem number: Exercise 12.15, page 103.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_separable]`

$$\boxed{2y - xy\ln(x) - 2x\ln(x)\,y' = 0}$$

✓ Solution by Maple
Time used: 0.016 (sec). Leaf size: 12

```
dsolve((2*y(x)-x*y(x)*ln(x))-2*x*ln(x)*diff(y(x),x)=0,y(x), singsol=all)
```

$$y(x) = c_1 \mathrm{e}^{-\frac{x}{2}} \ln(x)$$

✓ Solution by Mathematica
Time used: 0.035 (sec). Leaf size: 22

```
DSolve[(2*y[x]-x*y[x]*Log[x])-2*x*Log[x]*y'[x]==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to c_1 e^{-x/2} \log(x)$$
$$y(x) \to 0$$

6.16 problem Exercise 12.16, page 103

Internal problem ID [4537]

Book: Ordinary Differential Equations, By Tenenbaum and Pollard. Dover, NY 1963
Section: Chapter 2. Special types of differential equations of the first kind. Lesson 12, Miscellaneous Methods
Problem number: Exercise 12.16, page 103.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[[_linear, ‘class A‘]]`

$$\boxed{y' + ya = k\,\mathrm{e}^{bx}}$$

✓ Solution by Maple
Time used: 0.0 (sec). Leaf size: 29

```
dsolve(diff(y(x),x)+a*y(x)=k*exp(b*x),y(x), singsol=all)
```

$$y(x) = \frac{\left(k\,\mathrm{e}^{x(a+b)} + c_1(a+b)\right)\mathrm{e}^{-ax}}{a+b}$$

✓ Solution by Mathematica
Time used: 0.072 (sec). Leaf size: 33

```
DSolve[y'[x]+a*y[x]==k*Exp[b*x],y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to \frac{e^{-ax}\left(ke^{x(a+b)} + c_1(a+b)\right)}{a+b}$$

6.17 problem Exercise 12.17, page 103

Internal problem ID [4538]

Book: Ordinary Differential Equations, By Tenenbaum and Pollard. Dover, NY 1963
Section: Chapter 2. Special types of differential equations of the first kind. Lesson 12, Miscellaneous Methods
Problem number: Exercise 12.17, page 103.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[[_homogeneous, 'class C'], _Riccati]`

$$\boxed{y' - (x+y)^2 = 0}$$

✓ Solution by Maple
Time used: 0.0 (sec). Leaf size: 16

```
dsolve(diff(y(x),x)=(x+y(x))^2,y(x), singsol=all)
```

$$y(x) = -x - \tan(-x + c_1)$$

✓ Solution by Mathematica
Time used: 0.472 (sec). Leaf size: 14

```
DSolve[y'[x]==(x+y[x])^2,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to -x + \tan(x + c_1)$$

6.18 problem Exercise 12.18, page 103

Internal problem ID [4539]

Book: Ordinary Differential Equations, By Tenenbaum and Pollard. Dover, NY 1963
Section: Chapter 2. Special types of differential equations of the first kind. Lesson 12, Miscellaneous Methods
Problem number: Exercise 12.18, page 103.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_Bernoulli]`

$$\boxed{y' + 8x^3y^3 + 2xy = 0}$$

✓ Solution by Maple
Time used: 0.0 (sec). Leaf size: 43

```
dsolve(diff(y(x),x)+8*x^3*y(x)^3+2*x*y(x)=0,y(x), singsol=all)
```

$$y(x) = \frac{1}{\sqrt{e^{2x^2}c_1 - 4x^2 - 2}}$$
$$y(x) = -\frac{1}{\sqrt{e^{2x^2}c_1 - 4x^2 - 2}}$$

✓ Solution by Mathematica
Time used: 7.034 (sec). Leaf size: 58

```
DSolve[y'[x]+8*x^3*y[x]^3+2*x*y[x]==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to -\frac{1}{\sqrt{-4x^2 + c_1e^{2x^2} - 2}}$$
$$y(x) \to \frac{1}{\sqrt{-4x^2 + c_1e^{2x^2} - 2}}$$
$$y(x) \to 0$$

6.19 problem Exercise 12.19, page 103

Internal problem ID [4540]

Book: Ordinary Differential Equations, By Tenenbaum and Pollard. Dover, NY 1963
Section: Chapter 2. Special types of differential equations of the first kind. Lesson 12, Miscellaneous Methods
Problem number: Exercise 12.19, page 103.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[NONE]`

$$\left(xy\sqrt{x^2-y^2}+x\right)y'-y+x^2\sqrt{x^2-y^2}=0$$

✓ Solution by Maple
Time used: 0.031 (sec). Leaf size: 34

```
dsolve((x*y(x)*sqrt(x^2-y(x)^2)+x)*diff(y(x),x)=y(x)-x^2*sqrt(x^2-y(x)^2),y(x), singsol=all)
```

$$\frac{y(x)^2}{2}+\arctan\left(\frac{y(x)}{\sqrt{x^2-y(x)^2}}\right)+\frac{x^2}{2}-c_1=0$$

✓ Solution by Mathematica
Time used: 1.772 (sec). Leaf size: 44

```
DSolve[(x*y[x]*Sqrt[x^2-y[x]^2]+x)*y'[x]==y[x]-x^2*Sqrt[x^2-y[x]^2],y[x],x,IncludeSingularSo
```

$$\text{Solve}\left[-\arctan\left(\frac{\sqrt{x^2-y(x)^2}}{y(x)}\right)+\frac{x^2}{2}+\frac{y(x)^2}{2}=c_1,y(x)\right]$$

6.20 problem Exercise 12.20, page 103

Internal problem ID [4541]

Book: Ordinary Differential Equations, By Tenenbaum and Pollard. Dover, NY 1963
Section: Chapter 2. Special types of differential equations of the first kind. Lesson 12, Miscellaneous Methods
Problem number: Exercise 12.20, page 103.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[[_linear, 'class A']]`

$$\boxed{y' + ya = b\sin(kx)}$$

✓ Solution by Maple
Time used: 0.0 (sec). Leaf size: 45

```
dsolve(diff(y(x),x)+a*y(x)=b*sin(k*x),y(x), singsol=all)
```

$$y(x) = \frac{\mathrm{e}^{-ax}c_1(a^2+k^2) + b(\sin(kx)\,a - k\cos(kx))}{a^2+k^2}$$

✓ Solution by Mathematica
Time used: 0.083 (sec). Leaf size: 40

```
DSolve[y'[x]+a*y[x]==b*Sin[k*x],y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to \frac{b(a\sin(kx) - k\cos(kx))}{a^2+k^2} + c_1e^{-ax}$$

6.21 problem Exercise 12.21, page 103

Internal problem ID [4542]

Book: Ordinary Differential Equations, By Tenenbaum and Pollard. Dover, NY 1963
Section: Chapter 2. Special types of differential equations of the first kind. Lesson 12, Miscellaneous Methods
Problem number: Exercise 12.21, page 103.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_separable]`

$$\boxed{xy' - y^2 = -1}$$

✓ Solution by Maple
Time used: 0.0 (sec). Leaf size: 11

```
dsolve(x*diff(y(x),x)-y(x)^2+1=0,y(x), singsol=all)
```

$$y(x) = -\tanh(\ln(x) + c_1)$$

✓ Solution by Mathematica
Time used: 0.486 (sec). Leaf size: 43

```
DSolve[x*y'[x]-y[x]^2+1==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to \frac{1 - e^{2c_1}x^2}{1 + e^{2c_1}x^2}$$
$$y(x) \to -1$$
$$y(x) \to 1$$

6.22 problem Exercise 12.22, page 103

Internal problem ID [4543]

Book: Ordinary Differential Equations, By Tenenbaum and Pollard. Dover, NY 1963
Section: Chapter 2. Special types of differential equations of the first kind. Lesson 12, Miscellaneous Methods
Problem number: Exercise 12.22, page 103.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[[_1st_order, '_with_symmetry_[F(x)*G(y),0]']]`

$$\boxed{(y^2 + a\sin(x))\, y' = \cos(x)}$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 43

```
dsolve((y(x)^2+a*sin(x))*diff(y(x),x)=cos(x),y(x), singsol=all)
```

$$\frac{(-\sin(x)\, a^3 - y(x)^2\, a^2 - 2ay(x) - 2)\, \mathrm{e}^{-ay(x)} + c_1 a^3}{a^3} = 0$$

✓ Solution by Mathematica

Time used: 0.194 (sec). Leaf size: 45

```
DSolve[(y[x]^2+a*Sin[x])*y'[x]==Cos[x],y[x],x,IncludeSingularSolutions -> True]
```

$$\text{Solve}\left[\sin(x)\left(-e^{-ay(x)}\right) - \frac{e^{-ay(x)}(a^2y(x)^2 + 2ay(x) + 2)}{a^3} = c_1, y(x)\right]$$

6.23 problem Exercise 12.23, page 103

Internal problem ID [4544]

Book: Ordinary Differential Equations, By Tenenbaum and Pollard. Dover, NY 1963
Section: Chapter 2. Special types of differential equations of the first kind. Lesson 12, Miscellaneous Methods
Problem number: Exercise 12.23, page 103.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[[_homogeneous, ‘class A‘], _dAlembert]`

$$\boxed{xy' - x\,\mathrm{e}^{\frac{y}{x}} - y = x}$$

✓ Solution by Maple
Time used: 0.015 (sec). Leaf size: 20

```
dsolve(x*diff(y(x),x)=x*exp(y(x)/x)+x+y(x),y(x), singsol=all)
```

$$y(x) = \left(\ln\left(-\frac{x}{x\,\mathrm{e}^{c_1} - 1}\right) + c_1\right)x$$

✓ Solution by Mathematica
Time used: 4.512 (sec). Leaf size: 38

```
DSolve[x*y'[x]==x*Exp[y[x]/x]+x+y[x],y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to x\log\left(\frac{1}{2}\left(-1 + \tanh\left(\frac{1}{2}(-\log(x) - c_1)\right)\right)\right)$$
$$y(x) \to i\pi x$$

6.24 problem Exercise 12.24, page 103

Internal problem ID [4545]

Book: Ordinary Differential Equations, By Tenenbaum and Pollard. Dover, NY 1963
Section: Chapter 2. Special types of differential equations of the first kind. Lesson 12, Miscellaneous Methods
Problem number: Exercise 12.24, page 103.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_linear]`

$$y' + y\cos(x) = e^{-\sin(x)}$$

✓ Solution by Maple
Time used: 0.0 (sec). Leaf size: 13

```
dsolve(diff(y(x),x)+y(x)*cos(x)=exp(-sin(x)),y(x), singsol=all)
```

$$y(x) = (x + c_1)\,e^{-\sin(x)}$$

✓ Solution by Mathematica
Time used: 0.123 (sec). Leaf size: 16

```
DSolve[y'[x]+y[x]*Cos[x]==Exp[-Sin[x]],y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to (x + c_1)e^{-\sin(x)}$$

6.25 problem Exercise 12.25, page 103

Internal problem ID [4546]

Book: Ordinary Differential Equations, By Tenenbaum and Pollard. Dover, NY 1963
Section: Chapter 2. Special types of differential equations of the first kind. Lesson 12, Miscellaneous Methods
Problem number: Exercise 12.25, page 103.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[[_homogeneous, 'class G']]`

$$\boxed{xy' - y(\ln(xy) - 1) = 0}$$

✓ Solution by Maple
Time used: 0.015 (sec). Leaf size: 14

```
dsolve(x*diff(y(x),x)-y(x)*(ln(x*y(x))-1)=0,y(x), singsol=all)
```

$$y(x) = \frac{\mathrm{e}^{\frac{x}{c_1}}}{x}$$

✓ Solution by Mathematica
Time used: 0.186 (sec). Leaf size: 24

```
DSolve[x*y'[x]-y[x]*(Log[x*y[x]]-1)==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to \frac{e^{e^{c_1}x}}{x}$$
$$y(x) \to \frac{1}{x}$$

6.26 problem Exercise 12.26, page 103

Internal problem ID [4547]

Book: Ordinary Differential Equations, By Tenenbaum and Pollard. Dover, NY 1963
Section: Chapter 2. Special types of differential equations of the first kind. Lesson 12, Miscellaneous Methods
Problem number: Exercise 12.26, page 103.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[[_homogeneous, ‘class D‘], _rational, _Bernoulli]`

$$\boxed{y'x^3 - y^2 - yx^2 = 0}$$

✓ Solution by Maple
Time used: 0.0 (sec). Leaf size: 15

```
dsolve(x^3*diff(y(x),x)-y(x)^2-x^2*y(x)=0,y(x), singsol=all)
```

$$y(x) = \frac{x^2}{c_1 x + 1}$$

✓ Solution by Mathematica
Time used: 0.129 (sec). Leaf size: 22

```
DSolve[x^3*y'[x]-y[x]^2-x^2*y[x]==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to \frac{x^2}{1 + c_1 x}$$
$$y(x) \to 0$$

6.27 problem Exercise 12.27, page 103

Internal problem ID [4548]

Book: Ordinary Differential Equations, By Tenenbaum and Pollard. Dover, NY 1963
Section: Chapter 2. Special types of differential equations of the first kind. Lesson 12, Miscellaneous Methods
Problem number: Exercise 12.27, page 103.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_linear]`

$$\boxed{xy' + ya = -b\,x^n}$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 23

```
dsolve(x*diff(y(x),x)+a*y(x)+b*x^n=0,y(x), singsol=all)
```

$$y(x) = -\frac{x^n b}{a+n} + x^{-a} c_1$$

✓ Solution by Mathematica

Time used: 0.056 (sec). Leaf size: 25

```
DSolve[x*y'[x]+a*y[x]+b*x^n==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to -\frac{b x^n}{a+n} + c_1 x^{-a}$$

6.28 problem Exercise 12.28, page 103

Internal problem ID [4549]

Book: Ordinary Differential Equations, By Tenenbaum and Pollard. Dover, NY 1963
Section: Chapter 2. Special types of differential equations of the first kind. Lesson 12, Miscellaneous Methods
Problem number: Exercise 12.28, page 103.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[[_homogeneous, 'class A'], _dAlembert]`

$$\boxed{xy' - x\sin\left(\frac{y}{x}\right) - y = 0}$$

✓ Solution by Maple
Time used: 0.015 (sec). Leaf size: 44

```
dsolve(x*diff(y(x),x)-x*sin(y(x)/x)-y(x)=0,y(x), singsol=all)
```

$$y(x) = \arctan\left(\frac{2xc_1}{x^2c_1^2+1}, \frac{-x^2c_1^2+1}{x^2c_1^2+1}\right)x$$

✓ Solution by Mathematica
Time used: 0.321 (sec). Leaf size: 52

```
DSolve[x*y'[x]-x*Sin[y[x]/x]-y[x]==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to -x\arccos(-\tanh(\log(x)+c_1))$$
$$y(x) \to x\arccos(-\tanh(\log(x)+c_1))$$
$$y(x) \to 0$$
$$y(x) \to -\pi x$$
$$y(x) \to \pi x$$

6.29 problem Exercise 12.29, page 103

Internal problem ID [4550]

Book: Ordinary Differential Equations, By Tenenbaum and Pollard. Dover, NY 1963
Section: Chapter 2. Special types of differential equations of the first kind. Lesson 12, Miscellaneous Methods
Problem number: Exercise 12.29, page 103.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[[_homogeneous, ‘class A‘], _rational, [_Abel, ‘2nd type‘, ‘cl`

$$\boxed{y^2 - 3xy + (xy - x^2)\, y' = 2x^2}$$

✓ Solution by Maple
Time used: 0.016 (sec). Leaf size: 59

```
dsolve((x*y(x)-x^2)*diff(y(x),x)+y(x)^2-3*x*y(x)-2*x^2=0,y(x), singsol=all)
```

$$y(x) = \frac{c_1 x^2 - \sqrt{2c_1^2 x^4 + 1}}{c_1 x}$$
$$y(x) = \frac{c_1 x^2 + \sqrt{2c_1^2 x^4 + 1}}{c_1 x}$$

✓ Solution by Mathematica
Time used: 0.625 (sec). Leaf size: 99

```
DSolve[(x*y[x]-x^2)*y'[x]+y[x]^2-3*x*y[x]-2*x^2==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to x - \frac{\sqrt{2x^4 + e^{2c_1}}}{x}$$
$$y(x) \to x + \frac{\sqrt{2x^4 + e^{2c_1}}}{x}$$
$$y(x) \to x - \frac{\sqrt{2}\sqrt{x^4}}{x}$$
$$y(x) \to \frac{\sqrt{2}\sqrt{x^4}}{x} + x$$

6.30 problem Exercise 12.30, page 103

Internal problem ID [4551]

Book: Ordinary Differential Equations, By Tenenbaum and Pollard. Dover, NY 1963
Section: Chapter 2. Special types of differential equations of the first kind. Lesson 12, Miscellaneous Methods
Problem number: Exercise 12.30, page 103.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_exact, _rational, [_Abel, '2nd type', 'class B']]`

$$\boxed{(6xy + x^2 + 3)\, y' + 3y^2 + 2xy = -2x}$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 75

```
dsolve((6*x*y(x)+x^2+3)*diff(y(x),x)+3*y(x)^2+2*x*y(x)+2*x=0,y(x), singsol=all)
```

$$y(x) = \frac{-x^2 - 3 + \sqrt{x^4 - 12x^3 - 12c_1x + 6x^2 + 9}}{6x}$$
$$y(x) = \frac{-x^2 - 3 - \sqrt{x^4 - 12x^3 - 12c_1x + 6x^2 + 9}}{6x}$$

✓ Solution by Mathematica

Time used: 0.477 (sec). Leaf size: 83

```
DSolve[(6*x*y[x]+x^2+3)*y'[x]+3*y[x]^2+2*x*y[x]+2*x==0,y[x],x,IncludeSingularSolutions -> Tr
```

$$y(x) \to -\frac{x^2 + \sqrt{x^4 - 12x^3 + 6x^2 + 36c_1x + 9} + 3}{6x}$$
$$y(x) \to -\frac{x^2 - \sqrt{x^4 - 12x^3 + 6x^2 + 36c_1x + 9} + 3}{6x}$$

6.31 problem Exercise 12.31, page 103

Internal problem ID [4552]

Book: Ordinary Differential Equations, By Tenenbaum and Pollard. Dover, NY 1963
Section: Chapter 2. Special types of differential equations of the first kind. Lesson 12, Miscellaneous Methods
Problem number: Exercise 12.31, page 103.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[[_homogeneous, 'class A'], _rational, _Riccati]`

$$\boxed{x^2y' + y^2 + xy = -x^2}$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 18

```
dsolve(x^2*diff(y(x),x)+y(x)^2+x*y(x)+x^2=0,y(x), singsol=all)
```

$$y(x) = -\frac{x(\ln(x) + c_1 - 1)}{\ln(x) + c_1}$$

✓ Solution by Mathematica

Time used: 0.139 (sec). Leaf size: 31

```
DSolve[x^2*y'[x]+y[x]^2+x*y[x]+x^2==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to \frac{x(\log(x) - 1 - c_1)}{-\log(x) + c_1}$$
$$y(x) \to -x$$

6.32 problem Exercise 12.32, page 103

Internal problem ID [4553]

Book: Ordinary Differential Equations, By Tenenbaum and Pollard. Dover, NY 1963
Section: Chapter 2. Special types of differential equations of the first kind. Lesson 12, Miscellaneous Methods
Problem number: Exercise 12.32, page 103.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_linear]`

$$\boxed{(x^2-1)\,y'+2xy=\cos(x)}$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 16

```
dsolve((x^2-1)*diff(y(x),x)+2*x*y(x)-cos(x)=0,y(x), singsol=all)
```

$$y(x)=\frac{\sin(x)+c_1}{x^2-1}$$

✓ Solution by Mathematica

Time used: 0.037 (sec). Leaf size: 18

```
DSolve[(x^2-1)*y'[x]+2*x*y[x]-Cos[x]==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x)\to\frac{\sin(x)+c_1}{x^2-1}$$

6.33 problem Exercise 12.33, page 103

Internal problem ID [4554]

Book: Ordinary Differential Equations, By Tenenbaum and Pollard. Dover, NY 1963
Section: Chapter 2. Special types of differential equations of the first kind. Lesson 12, Miscellaneous Methods
Problem number: Exercise 12.33, page 103.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_exact, _rational, [_Abel, '2nd type', 'class B']]`

$$\boxed{(yx^2-1)\,y'+xy^2=1}$$

✓ Solution by Maple
Time used: 0.0 (sec). Leaf size: 51

```
dsolve((x^2*y(x)-1)*diff(y(x),x)+x*y(x)^2-1=0,y(x), singsol=all)
```

$$y(x)=\frac{1+\sqrt{-2c_1x^2+2x^3+1}}{x^2}$$
$$y(x)=\frac{1-\sqrt{-2c_1x^2+2x^3+1}}{x^2}$$

✓ Solution by Mathematica
Time used: 0.505 (sec). Leaf size: 57

```
DSolve[(x^2*y[x]-1)*y'[x]+x*y[x]^2-1==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x)\to\frac{1-\sqrt{2x^3+c_1x^2+1}}{x^2}$$
$$y(x)\to\frac{1+\sqrt{2x^3+c_1x^2+1}}{x^2}$$

6.34 problem Exercise 12.34, page 103

Internal problem ID [4555]

Book: Ordinary Differential Equations, By Tenenbaum and Pollard. Dover, NY 1963
Section: Chapter 2. Special types of differential equations of the first kind. Lesson 12, Miscellaneous Methods
Problem number: Exercise 12.34, page 103.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_separable]`

$$\boxed{(x^2-1)\, y' + xy - 3xy^2 = 0}$$

✓ Solution by Maple

Time used: 0.015 (sec). Leaf size: 20

```
dsolve((x^2-1)*diff(y(x),x)+x*y(x)-3*x*y(x)^2=0,y(x), singsol=all)
```

$$y(x) = \frac{1}{3+\sqrt{x-1}\,\sqrt{1+x}\, c_1}$$

✓ Solution by Mathematica

Time used: 2.214 (sec). Leaf size: 35

```
DSolve[(x^2-1)*y'[x]+x*y[x]-3*x*y[x]^2==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to \frac{1}{3+e^{c_1}\sqrt{x^2-1}}$$
$$y(x) \to 0$$
$$y(x) \to \frac{1}{3}$$

6.35 problem Exercise 12.35, page 103

Internal problem ID [4556]

Book: Ordinary Differential Equations, By Tenenbaum and Pollard. Dover, NY 1963
Section: Chapter 2. Special types of differential equations of the first kind. Lesson 12, Miscellaneous Methods
Problem number: Exercise 12.35, page 103.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_separable]`

$$\left(x^2-1\right) y'-2 x y \ln \left(y\right)=0$$

✓ Solution by Maple
Time used: 0.0 (sec). Leaf size: 13

```
dsolve((x^2-1)*diff(y(x),x)-2*x*y(x)*ln(y(x))=0,y(x), singsol=all)
```

$$y(x)=\mathrm{e}^{c_1(x-1)(1+x)}$$

✓ Solution by Mathematica
Time used: 0.223 (sec). Leaf size: 22

```
DSolve[(x^2-1)*y'[x]-2*x*y[x]*Log[y[x]]==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x)\to e^{e^{c_1}\left(x^2-1\right)}$$
$$y(x)\to 1$$

6.36 problem Exercise 12.36, page 103

Internal problem ID [4557]

Book: Ordinary Differential Equations, By Tenenbaum and Pollard. Dover, NY 1963
Section: Chapter 2. Special types of differential equations of the first kind. Lesson 12, Miscellaneous Methods
Problem number: Exercise 12.36, page 103.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_exact, _rational]`

$$\boxed{(1+x^2+y^2)\,y'+2xy=-x^2-3}$$

✓ Solution by Maple
Time used: 0.015 (sec). Leaf size: 370

```
dsolve((x^2+y(x)^2+1)*diff(y(x),x)+2*x*y(x)+x^2+3=0,y(x), singsol=all)
```

$$y(x) = \frac{\left(-4x^3-12c_1-36x+4\sqrt{5x^6+6c_1x^3+30x^4+9c_1^2+54c_1x+93x^2+4}\right)^{\frac{2}{3}}-4x^2-4}{2\left(-4x^3-12c_1-36x+4\sqrt{5x^6+6c_1x^3+30x^4+9c_1^2+54c_1x+93x^2+4}\right)^{\frac{1}{3}}}$$

$$y(x) = -\frac{\left(\frac{i\sqrt{3}}{4}+\frac{1}{4}\right)\left(-4x^3-12c_1-36x+4\sqrt{5x^6+6c_1x^3+30x^4+9c_1^2+54c_1x+93x^2+4}\right)^{\frac{2}{3}}+\left(i\sqrt{3}-1\right)(}{\left(-4x^3-12c_1-36x+4\sqrt{5x^6+6c_1x^3+30x^4+9c_1^2+54c_1x+93x^2+4}\right)^{\frac{1}{3}}}$$

$$y(x) = \frac{\left(i\sqrt{3}-1\right)\left(-4x^3-12c_1-36x+4\sqrt{5x^6+6c_1x^3+30x^4+9c_1^2+54c_1x+93x^2+4}\right)^{\frac{1}{3}}}{4} + \frac{\left(1+i\sqrt{3}\right)\left(x^2+1\right)}{\left(-4x^3-12c_1-36x+4\sqrt{5x^6+6c_1x^3+30x^4+9c_1^2+54c_1x+93x^2+4}\right)^{\frac{1}{3}}}$$

✓ Solution by Mathematica

Time used: 5.385 (sec). Leaf size: 411

```
DSolve[(x^2+y[x]^2+1)*y'[x]+2*x*y[x]+x^2+3==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to \frac{\sqrt[3]{-27x^3+\sqrt{4\left(9x^2+9\right)^3+729\left(x^3+9x-3c_1\right)^2}-243x+81c_1}}{3\sqrt[3]{2}} - \frac{3\sqrt[3]{2}(x^2+1)}{\sqrt[3]{-27x^3+\sqrt{4\left(9x^2+9\right)^3+729\left(x^3+9x-3c_1\right)^2}-243x+81c_1}}$$

$$y(x) \to \frac{3\left(1+i\sqrt{3}\right)\left(x^2+1\right)}{2^{2/3}\sqrt[3]{-27x^3+\sqrt{4\left(9x^2+9\right)^3+729\left(x^3+9x-3c_1\right)^2}-243x+81c_1}} + \frac{\left(-1+i\sqrt{3}\right)\sqrt[3]{-27x^3+\sqrt{4\left(9x^2+9\right)^3+729\left(x^3+9x-3c_1\right)^2}-243x+81c_1}}{6\sqrt[3]{2}}$$

$$y(x) \to \frac{3\left(1-i\sqrt{3}\right)\left(x^2+1\right)}{2^{2/3}\sqrt[3]{-27x^3+\sqrt{4\left(9x^2+9\right)^3+729\left(x^3+9x-3c_1\right)^2}-243x+81c_1}} - \frac{\left(1+i\sqrt{3}\right)\sqrt[3]{-27x^3+\sqrt{4\left(9x^2+9\right)^3+729\left(x^3+9x-3c_1\right)^2}-243x+81c_1}}{6\sqrt[3]{2}}$$

6.37 problem Exercise 12.37, page 103

Internal problem ID [4558]

Book: Ordinary Differential Equations, By Tenenbaum and Pollard. Dover, NY 1963
Section: Chapter 2. Special types of differential equations of the first kind. Lesson 12, Miscellaneous Methods
Problem number: Exercise 12.37, page 103.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_linear]`

$$\boxed{\cos(x)\, y' + y = -(1+\sin(x))\cos(x)}$$

✓ Solution by Maple
Time used: 0.0 (sec). Leaf size: 29

```
dsolve(diff(y(x),x)*cos(x)+y(x)+(1+sin(x))*cos(x)=0,y(x), singsol=all)
```

$$y(x) = \frac{-2\ln(\sec(x)+\tan(x)) + 2\ln(\cos(x)) + \sin(x) + c_1}{\sec(x)+\tan(x)}$$

✓ Solution by Mathematica
Time used: 0.671 (sec). Leaf size: 40

```
DSolve[y'[x]*Cos[x]+y[x]+(1+Sin[x])*Cos[x]==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to e^{-2\operatorname{arctanh}\left(\tan\left(\frac{x}{2}\right)\right)}\left(\sin(x) + 4\log\left(\cos\left(\frac{x}{2}\right) - \sin\left(\frac{x}{2}\right)\right) + c_1\right)$$

6.38 problem Exercise 12.38, page 103

Internal problem ID [4559]

Book: Ordinary Differential Equations, By Tenenbaum and Pollard. Dover, NY 1963
Section: Chapter 2. Special types of differential equations of the first kind. Lesson 12, Miscellaneous Methods
Problem number: Exercise 12.38, page 103.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[[_homogeneous, ‘class G‘], _exact, _rational, [_Abel, ‘2nd ty`

$$\boxed{(2xy + 4x^3)\, y' + y^2 + 12yx^2 = 0}$$

✓ Solution by Maple
Time used: 0.032 (sec). Leaf size: 51

```
dsolve((2*x*y(x)+4*x^3)*diff(y(x),x)+y(x)^2+12*x^2*y(x)=0,y(x), singsol=all)
```

$$y(x) = \frac{-2x^3 + \sqrt{4x^6 + c_1 x}}{x}$$
$$y(x) = \frac{-2x^3 - \sqrt{4x^6 + c_1 x}}{x}$$

✓ Solution by Mathematica
Time used: 0.441 (sec). Leaf size: 58

```
DSolve[(2*x*y[x]+4*x^3)*y'[x]+y[x]^2+12*x^2*y[x]==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to -\frac{2x^3 + \sqrt{x\,(4x^5 + c_1)}}{x}$$
$$y(x) \to \frac{-2x^3 + \sqrt{x\,(4x^5 + c_1)}}{x}$$

6.39 problem Exercise 12.39, page 103

Internal problem ID [4560]

Book: Ordinary Differential Equations, By Tenenbaum and Pollard. Dover, NY 1963
Section: Chapter 2. Special types of differential equations of the first kind. Lesson 12, Miscellaneous Methods
Problem number: Exercise 12.39, page 103.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_rational, [_1st_order, '_with_symmetry_[F(x)*G(y),0]'], [_Ab`

$$\boxed{(x^2 - y)\, y' = -x}$$

✓ Solution by Maple
Time used: 0.016 (sec). Leaf size: 23

```
dsolve((x^2-y(x))*diff(y(x),x)+x=0,y(x), singsol=all)
```

$$y(x) = x^2 + \frac{\operatorname{LambertW}\left(4c_1 \mathrm{e}^{-2x^2-1}\right)}{2} + \frac{1}{2}$$

✓ Solution by Mathematica
Time used: 5.105 (sec). Leaf size: 40

```
DSolve[(x^2-y[x])*y'[x]+x==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to x^2 + \frac{1}{2}\left(1 + W\left(-e^{-2x^2-1+c_1}\right)\right)$$
$$y(x) \to x^2 + \frac{1}{2}$$

6.40 problem Exercise 12.40, page 103

Internal problem ID [4561]

Book: Ordinary Differential Equations, By Tenenbaum and Pollard. Dover, NY 1963
Section: Chapter 2. Special types of differential equations of the first kind. Lesson 12, Miscellaneous Methods
Problem number: Exercise 12.40, page 103.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[[_homogeneous, ‘class G‘], _rational, [_Abel, ‘2nd type‘, ‘cl`

$$\boxed{(x^2 - y)\, y' - 4xy = 0}$$

✓ Solution by Maple
Time used: 0.047 (sec). Leaf size: 57

```
dsolve((x^2-y(x))*diff(y(x),x)-4*x*y(x)=0,y(x), singsol=all)
```

$$y(x) = -\frac{c_1\sqrt{c_1^2 - 4x^2}}{2} + \frac{c_1^2}{2} - x^2$$
$$y(x) = \frac{c_1\sqrt{c_1^2 - 4x^2}}{2} + \frac{c_1^2}{2} - x^2$$

✓ Solution by Mathematica

Time used: 2.441 (sec). Leaf size: 246

```
DSolve[(x^2-y[x])*y'[x]-4*x*y[x]==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to x^2 \left(1 + \frac{2-2i}{\frac{i\sqrt{2}}{\sqrt{x^2\cosh\left(\frac{2c_1}{9}\right)+x^2\sinh\left(\frac{2c_1}{9}\right)-i}} - (1-i)}\right)$$

$$y(x) \to x^2 \left(1 + \frac{2-2i}{(-1+i) - \frac{i\sqrt{2}}{\sqrt{x^2\cosh\left(\frac{2c_1}{9}\right)+x^2\sinh\left(\frac{2c_1}{9}\right)-i}}}\right)$$

$$y(x) \to x^2 \left(1 + \frac{2-2i}{(-1+i) - \frac{\sqrt{2}}{\sqrt{x^2\cosh\left(\frac{2c_1}{9}\right)+x^2\sinh\left(\frac{2c_1}{9}\right)+i}}}\right)$$

$$y(x) \to x^2 \left(1 + \frac{2-2i}{\frac{\sqrt{2}}{\sqrt{x^2\cosh\left(\frac{2c_1}{9}\right)+x^2\sinh\left(\frac{2c_1}{9}\right)+i}} - (1-i)}\right)$$

$$y(x) \to 0$$

$$y(x) \to -x^2$$

6.41 problem Exercise 12.41, page 103

Internal problem ID [4562]

Book: Ordinary Differential Equations, By Tenenbaum and Pollard. Dover, NY 1963
Section: Chapter 2. Special types of differential equations of the first kind. Lesson 12, Miscellaneous Methods
Problem number: Exercise 12.41, page 103.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[[_homogeneous, 'class A'], _rational, _Bernoulli]`

$$\boxed{xyy' + y^2 = -x^2}$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 39

```
dsolve(x*y(x)*diff(y(x),x)+x^2+y(x)^2=0,y(x), singsol=all)
```

$$y(x) = -\frac{\sqrt{-2x^4 + 4c_1}}{2x}$$
$$y(x) = \frac{\sqrt{-2x^4 + 4c_1}}{2x}$$

✓ Solution by Mathematica

Time used: 0.211 (sec). Leaf size: 46

```
DSolve[x*y[x]*y'[x]+x^2+y[x]^2==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to -\frac{\sqrt{-\frac{x^4}{2} + c_1}}{x}$$
$$y(x) \to \frac{\sqrt{-\frac{x^4}{2} + c_1}}{x}$$

6.42 problem Exercise 12.42, page 103

Internal problem ID [4563]

Book: Ordinary Differential Equations, By Tenenbaum and Pollard. Dover, NY 1963
Section: Chapter 2. Special types of differential equations of the first kind. Lesson 12, Miscellaneous Methods
Problem number: Exercise 12.42, page 103.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[[_homogeneous, ‘class A‘], _rational, _Bernoulli]`

$$\boxed{2xyy' - y^2 = -3x^2}$$

✓ Solution by Maple
Time used: 0.015 (sec). Leaf size: 29

```
dsolve(2*x*y(x)*diff(y(x),x)+3*x^2-y(x)^2=0,y(x), singsol=all)
```

$$y(x) = \sqrt{(-3x + c_1)\, x}$$
$$y(x) = -\sqrt{c_1 x - 3x^2}$$

✓ Solution by Mathematica
Time used: 0.306 (sec). Leaf size: 35

```
DSolve[2*x*y[x]*y'[x]+3*x^2-y[x]^2==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to -\sqrt{x(-3x + c_1)}$$
$$y(x) \to \sqrt{x(-3x + c_1)}$$

6.43 problem Exercise 12.43, page 103

Internal problem ID [4564]

Book: Ordinary Differential Equations, By Tenenbaum and Pollard. Dover, NY 1963
Section: Chapter 2. Special types of differential equations of the first kind. Lesson 12, Miscellaneous Methods
Problem number: Exercise 12.43, page 103.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[[_homogeneous, 'class A'], _rational, _dAlembert]`

$$\left(2y^3x - x^4\right)y' + 2yx^3 - y^4 = 0$$

✓ Solution by Maple
Time used: 0.015 (sec). Leaf size: 317

```
dsolve((2*x*y(x)^3-x^4)*diff(y(x),x)+2*x^3*y(x)-y(x)^4=0,y(x), singsol=all)
```

$$y(x) = \frac{12^{\frac{1}{3}}\left(x12^{\frac{1}{3}}c_1 + \left(x\left(-9c_1x^2 + \sqrt{3}\sqrt{\frac{27c_1^3x^4 - 4x}{c_1}}\right)c_1^2\right)^{\frac{2}{3}}\right)}{6c_1\left(x\left(-9c_1x^2 + \sqrt{3}\sqrt{\frac{27c_1^3x^4 - 4x}{c_1}}\right)c_1^2\right)^{\frac{1}{3}}}$$

$$y(x) = \frac{3^{\frac{1}{3}}\left(\left(-i\sqrt{3} - 1\right)\left(x\left(-9c_1x^2 + \sqrt{3}\sqrt{\frac{27c_1^3x^4 - 4x}{c_1}}\right)c_1^2\right)^{\frac{2}{3}} + \left(i3^{\frac{5}{6}} - 3^{\frac{1}{3}}\right)c_1 2^{\frac{2}{3}}x\right)2^{\frac{2}{3}}}{12\left(x\left(-9c_1x^2 + \sqrt{3}\sqrt{\frac{27c_1^3x^4 - 4x}{c_1}}\right)c_1^2\right)^{\frac{1}{3}}c_1}$$

$$y(x) = -\frac{3^{\frac{1}{3}}\left(\left(1 - i\sqrt{3}\right)\left(x\left(-9c_1x^2 + \sqrt{3}\sqrt{\frac{27c_1^3x^4 - 4x}{c_1}}\right)c_1^2\right)^{\frac{2}{3}} + \left(i3^{\frac{5}{6}} + 3^{\frac{1}{3}}\right)c_1 2^{\frac{2}{3}}x\right)2^{\frac{2}{3}}}{12\left(x\left(-9c_1x^2 + \sqrt{3}\sqrt{\frac{27c_1^3x^4 - 4x}{c_1}}\right)c_1^2\right)^{\frac{1}{3}}c_1}$$

✓ Solution by Mathematica

Time used: 60.224 (sec). Leaf size: 331

```
DSolve[(2*x*y[x]^3-x^4)*y'[x]+2*x^3*y[x]-y[x]^4==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to \frac{\sqrt[3]{2}\left(-9x^3+\sqrt{81x^6-12e^{3c_1}x^3}\right)^{2/3}+2\sqrt[3]{3}e^{c_1}x}{6^{2/3}\sqrt[3]{-9x^3+\sqrt{81x^6-12e^{3c_1}x^3}}}$$

$$y(x) \to \frac{i\sqrt[3]{2}\sqrt[6]{3}\left(\sqrt{3}+i\right)\left(-9x^3+\sqrt{81x^6-12e^{3c_1}x^3}\right)^{2/3}-2\left(\sqrt{3}+3i\right)e^{c_1}x}{2\ 2^{2/3}3^{5/6}\sqrt[3]{-9x^3+\sqrt{81x^6-12e^{3c_1}x^3}}}$$

$$y(x) \to \frac{\sqrt[3]{2}\sqrt[6]{3}\left(-1-i\sqrt{3}\right)\left(-9x^3+\sqrt{81x^6-12e^{3c_1}x^3}\right)^{2/3}-2\left(\sqrt{3}-3i\right)e^{c_1}x}{2\ 2^{2/3}3^{5/6}\sqrt[3]{-9x^3+\sqrt{81x^6-12e^{3c_1}x^3}}}$$

6.44 problem Exercise 12.44, page 103

Internal problem ID [4565]

Book: Ordinary Differential Equations, By Tenenbaum and Pollard. Dover, NY 1963
Section: Chapter 2. Special types of differential equations of the first kind. Lesson 12, Miscellaneous Methods
Problem number: Exercise 12.44, page 103.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[[_homogeneous, ‘class G‘], _rational]`

$$(xy-1)^2 xy' + (y^2x^2+1) y = 0$$

✓ Solution by Maple
Time used: 0.047 (sec). Leaf size: 34

```
dsolve((x*y(x)-1)^2*x*diff(y(x),x)+(x^2*y(x)^2+1)*y(x)=0,y(x), singsol=all)
```

$$y(x) = \frac{e^{\mathrm{RootOf}\left(-e^{2\_Z}-2\ln(x)e^{\_Z}+2c_1e^{\_Z}+2\_Ze^{\_Z}+1\right)}}{x}$$

✓ Solution by Mathematica
Time used: 0.108 (sec). Leaf size: 25

```
DSolve[(x*y[x]-1)^2*x*y'[x]+(x^2*y[x]^2+1)*y[x]==0,y[x],x,IncludeSingularSolutions -> True]
```

$$\mathrm{Solve}\left[xy(x) - \frac{1}{xy(x)} - 2\log(y(x)) = c_1, y(x)\right]$$

6.45 problem Exercise 12.45, page 103

Internal problem ID [4566]

Book: Ordinary Differential Equations, By Tenenbaum and Pollard. Dover, NY 1963
Section: Chapter 2. Special types of differential equations of the first kind. Lesson 12, Miscellaneous Methods
Problem number: Exercise 12.45, page 103.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[[_homogeneous, 'class A'], _exact, _rational, _dAlembert]`

$$\boxed{(x^2+y^2)\,y'+2x(2x+y)=0}$$

✓ Solution by Maple

Time used: 0.031 (sec). Leaf size: 321

```
dsolve((x^2+y(x)^2)*diff(y(x),x)+2*x*(2*x+y(x))=0,y(x), singsol=all)
```

$$y(x)=-\frac{2\left(c_1x^2-\frac{\left(4-16x^3c_1^{\frac{3}{2}}+4\sqrt{20x^6c_1^3-8x^3c_1^{\frac{3}{2}}+1}\right)^{\frac{2}{3}}}{4}\right)}{\sqrt{c_1}\left(4-16x^3c_1^{\frac{3}{2}}+4\sqrt{20x^6c_1^3-8x^3c_1^{\frac{3}{2}}+1}\right)^{\frac{1}{3}}}$$

$$y(x)=-\frac{(1+i\sqrt{3})\left(4-16x^3c_1^{\frac{3}{2}}+4\sqrt{20x^6c_1^3-8x^3c_1^{\frac{3}{2}}+1}\right)^{\frac{1}{3}}}{4\sqrt{c_1}}-\frac{\sqrt{c_1}\,(i\sqrt{3}-1)\,x^2}{\left(4-16x^3c_1^{\frac{3}{2}}+4\sqrt{20x^6c_1^3-8x^3c_1^{\frac{3}{2}}+1}\right)^{\frac{1}{3}}}$$

$$y(x)=\frac{4i\sqrt{3}\,c_1x^2+i\sqrt{3}\left(4-16x^3c_1^{\frac{3}{2}}+4\sqrt{20x^6c_1^3-8x^3c_1^{\frac{3}{2}}+1}\right)^{\frac{2}{3}}+4c_1x^2-\left(4-16x^3c_1^{\frac{3}{2}}+4\sqrt{20x^6c_1^3-8x^3c\right.}{4\left(4-16x^3c_1^{\frac{3}{2}}+4\sqrt{20x^6c_1^3-8x^3c_1^{\frac{3}{2}}+1}\right)^{\frac{1}{3}}\sqrt{c_1}}$$

✓ Solution by Mathematica

Time used: 18.874 (sec). Leaf size: 593

```
DSolve[(x^2+y[x]^2)*y'[x]+2*x*(2*x+y[x])==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to \frac{\sqrt[3]{-4x^3+\sqrt{20x^6-8e^{3c_1}x^3+e^{6c_1}}+e^{3c_1}}}{\sqrt[3]{2}} - \frac{\sqrt[3]{2}x^2}{\sqrt[3]{-4x^3+\sqrt{20x^6-8e^{3c_1}x^3+e^{6c_1}}+e^{3c_1}}}$$

$$y(x) \to \frac{\sqrt[3]{2}\left(2+2i\sqrt{3}\right)x^2+i2^{2/3}\left(\sqrt{3}+i\right)\left(-4x^3+\sqrt{20x^6-8e^{3c_1}x^3+e^{6c_1}}+e^{3c_1}\right)^{2/3}}{4\sqrt[3]{-4x^3+\sqrt{20x^6-8e^{3c_1}x^3+e^{6c_1}}+e^{3c_1}}}$$

$$y(x) \to \frac{\left(1-i\sqrt{3}\right)x^2}{2^{2/3}\sqrt[3]{-4x^3+\sqrt{20x^6-8e^{3c_1}x^3+e^{6c_1}}+e^{3c_1}}} - \frac{\left(1+i\sqrt{3}\right)\sqrt[3]{-4x^3+\sqrt{20x^6-8e^{3c_1}x^3+e^{6c_1}}+e^{3c_1}}}{2\sqrt[3]{2}}$$

$$y(x) \to \sqrt[3]{\sqrt{5}\sqrt{x^6}-2x^3} - \frac{x^2}{\sqrt[3]{\sqrt{5}\sqrt{x^6}-2x^3}}$$

$$y(x) \to \frac{\left(1-i\sqrt{3}\right)x^2+\left(-1-i\sqrt{3}\right)\left(\sqrt{5}\sqrt{x^6}-2x^3\right)^{2/3}}{2\sqrt[3]{\sqrt{5}\sqrt{x^6}-2x^3}}$$

$$y(x) \to \frac{\left(1+i\sqrt{3}\right)x^2+i\left(\sqrt{3}+i\right)\left(\sqrt{5}\sqrt{x^6}-2x^3\right)^{2/3}}{2\sqrt[3]{\sqrt{5}\sqrt{x^6}-2x^3}}$$

6.46 problem Exercise 12.46, page 103

Internal problem ID [4567]

Book: Ordinary Differential Equations, By Tenenbaum and Pollard. Dover, NY 1963
Section: Chapter 2. Special types of differential equations of the first kind. Lesson 12, Miscellaneous Methods
Problem number: Exercise 12.46, page 103.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[[_homogeneous, 'class G'], _exact, _rational, _Bernoulli]`

$$\boxed{3xy^2y' + y^3 = 2x}$$

✓ Solution by Maple
Time used: 0.016 (sec). Leaf size: 73

```
dsolve(3*x*y(x)^2*diff(y(x),x)+y(x)^3-2*x=0,y(x), singsol=all)
```

$$y(x) = \frac{((x^2+c_1)\,x^2)^{\frac{1}{3}}}{x}$$
$$y(x) = -\frac{((x^2+c_1)\,x^2)^{\frac{1}{3}}\,(1+i\sqrt{3})}{2x}$$
$$y(x) = \frac{((x^2+c_1)\,x^2)^{\frac{1}{3}}\,(i\sqrt{3}-1)}{2x}$$

✓ Solution by Mathematica
Time used: 0.224 (sec). Leaf size: 72

```
DSolve[3*x*y[x]^2*y'[x]+y[x]^3-2*x==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to \frac{\sqrt[3]{x^2+c_1}}{\sqrt[3]{x}}$$
$$y(x) \to -\frac{\sqrt[3]{-1}\,\sqrt[3]{x^2+c_1}}{\sqrt[3]{x}}$$
$$y(x) \to \frac{(-1)^{2/3}\,\sqrt[3]{x^2+c_1}}{\sqrt[3]{x}}$$

6.47 problem Exercise 12.47, page 103

Internal problem ID [4568]

Book: Ordinary Differential Equations, By Tenenbaum and Pollard. Dover, NY 1963
Section: Chapter 2. Special types of differential equations of the first kind. Lesson 12, Miscellaneous Methods
Problem number: Exercise 12.47, page 103.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[[_homogeneous, 'class A'], _rational, _dAlembert]`

$$\boxed{2y^3y' + xy^2 = x^3}$$

✓ Solution by Maple

Time used: 0.422 (sec). Leaf size: 649

```
dsolve(2*y(x)^3*diff(y(x),x)+x*y(x)^2-x^3=0,y(x), singsol=all)
```

$$y(x) = -\frac{\sqrt{2}\,\sqrt{\frac{x^4c_1^2-c_1x^2\left(2+x^6c_1^3+2\sqrt{x^6c_1^3+1}\right)^{\frac{1}{3}}+\left(2+x^6c_1^3+2\sqrt{x^6c_1^3+1}\right)^{\frac{2}{3}}}{\left(2+x^6c_1^3+2\sqrt{x^6c_1^3+1}\right)^{\frac{1}{3}}}}}{2\sqrt{c_1}}$$

$$y(x) = \frac{\sqrt{2}\,\sqrt{\frac{x^4c_1^2-c_1x^2\left(2+x^6c_1^3+2\sqrt{x^6c_1^3+1}\right)^{\frac{1}{3}}+\left(2+x^6c_1^3+2\sqrt{x^6c_1^3+1}\right)^{\frac{2}{3}}}{\left(2+x^6c_1^3+2\sqrt{x^6c_1^3+1}\right)^{\frac{1}{3}}}}}{2\sqrt{c_1}}$$

$$y(x) = -\frac{\sqrt{\frac{\left(\left(-i\sqrt{3}-1\right)\left(2+x^6c_1^3+2\sqrt{x^6c_1^3+1}\right)^{\frac{1}{3}}+\left(i\sqrt{3}-1\right)x^2c_1\right)\left(c_1x^2+\left(2+x^6c_1^3+2\sqrt{x^6c_1^3+1}\right)^{\frac{1}{3}}\right)}{\left(2+x^6c_1^3+2\sqrt{x^6c_1^3+1}\right)^{\frac{1}{3}}}}}{2\sqrt{c_1}}$$

$$y(x) = \frac{\sqrt{\frac{\left(\left(-i\sqrt{3}-1\right)\left(2+x^6c_1^3+2\sqrt{x^6c_1^3+1}\right)^{\frac{1}{3}}+\left(i\sqrt{3}-1\right)x^2c_1\right)\left(c_1x^2+\left(2+x^6c_1^3+2\sqrt{x^6c_1^3+1}\right)^{\frac{1}{3}}\right)}{\left(2+x^6c_1^3+2\sqrt{x^6c_1^3+1}\right)^{\frac{1}{3}}}}}{2\sqrt{c_1}}$$

$$y(x) = -\frac{\sqrt{\frac{\left(\left(2+x^6c_1^3+2\sqrt{x^6c_1^3+1}\right)^{\frac{1}{3}}\left(i\sqrt{3}-1\right)+\left(-i\sqrt{3}-1\right)x^2c_1\right)\left(c_1x^2+\left(2+x^6c_1^3+2\sqrt{x^6c_1^3+1}\right)^{\frac{1}{3}}\right)}{\left(2+x^6c_1^3+2\sqrt{x^6c_1^3+1}\right)^{\frac{1}{3}}}}}{2\sqrt{c_1}}$$

$$y(x) = \frac{\sqrt{\frac{\left(\left(2+x^6c_1^3+2\sqrt{x^6c_1^3+1}\right)^{\frac{1}{3}}\left(i\sqrt{3}-1\right)+\left(-i\sqrt{3}-1\right)x^2c_1\right)\left(c_1x^2+\left(2+x^6c_1^3+2\sqrt{x^6c_1^3+1}\right)^{\frac{1}{3}}\right)}{\left(2+x^6c_1^3+2\sqrt{x^6c_1^3+1}\right)^{\frac{1}{3}}}}}{2\sqrt{c_1}}$$

✓ Solution by Mathematica

Time used: 60.13 (sec). Leaf size: 714

```
DSolve[2*y[x]^3*y'[x]+x*y[x]^2-x^3==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to -\frac{\sqrt{\sqrt[3]{x^6+2\sqrt{e^{24c_1}-e^{12c_1}x^6}-2e^{12c_1}}-x^2+\frac{x^4}{\sqrt[3]{x^6+2\sqrt{e^{24c_1}-e^{12c_1}x^6}-2e^{12c_1}}}}}{\sqrt{2}}$$

$$y(x) \to \frac{\sqrt{\sqrt[3]{x^6+2\sqrt{e^{24c_1}-e^{12c_1}x^6}-2e^{12c_1}}-x^2+\frac{x^4}{\sqrt[3]{x^6+2\sqrt{e^{24c_1}-e^{12c_1}x^6}-2e^{12c_1}}}}}{\sqrt{2}}$$

$$y(x) \to -\frac{1}{2}\sqrt{\left(-1-i\sqrt{3}\right)\sqrt[3]{x^6+2\sqrt{e^{24c_1}-e^{12c_1}x^6}-2e^{12c_1}}-2x^2+\frac{i\left(\sqrt{3}+i\right)x^4}{\sqrt[3]{x^6+2\sqrt{e^{24c_1}-e^{12c_1}x^6}-2e^{12c_1}}}}$$

$$y(x) \to \frac{1}{2}\sqrt{\left(-1-i\sqrt{3}\right)\sqrt[3]{x^6+2\sqrt{e^{24c_1}-e^{12c_1}x^6}-2e^{12c_1}}-2x^2+\frac{i\left(\sqrt{3}+i\right)x^4}{\sqrt[3]{x^6+2\sqrt{e^{24c_1}-e^{12c_1}x^6}-2e^{12c_1}}}}$$

$$y(x) \to -\frac{1}{2}\sqrt{i\left(\sqrt{3}+i\right)\sqrt[3]{x^6+2\sqrt{e^{24c_1}-e^{12c_1}x^6}-2e^{12c_1}}-2x^2+\frac{\left(-1-i\sqrt{3}\right)x^4}{\sqrt[3]{x^6+2\sqrt{e^{24c_1}-e^{12c_1}x^6}-2e^{12c_1}}}}$$

$$y(x) \to \frac{1}{2}\sqrt{i\left(\sqrt{3}+i\right)\sqrt[3]{x^6+2\sqrt{e^{24c_1}-e^{12c_1}x^6}-2e^{12c_1}}-2x^2+\frac{\left(-1-i\sqrt{3}\right)x^4}{\sqrt[3]{x^6+2\sqrt{e^{24c_1}-e^{12c_1}x^6}-2e^{12c_1}}}}$$

6.48 problem Exercise 12.48, page 103

Internal problem ID [4569]

Book: Ordinary Differential Equations, By Tenenbaum and Pollard. Dover, NY 1963
Section: Chapter 2. Special types of differential equations of the first kind. Lesson 12, Miscellaneous Methods
Problem number: Exercise 12.48, page 103.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_rational]`

$$(2y^3x + xy + x^2)\, y' - xy + y^2 = 0$$

✓ Solution by Maple
Time used: 0.0 (sec). Leaf size: 29

```
dsolve((2*x*y(x)^3+x*y(x)+x^2)*diff(y(x),x)-x*y(x)+y(x)^2=0,y(x), singsol=all)
```

$$y(x) = e^{\operatorname{RootOf}\left(-e^{3\_Z} - \ln(x) e^{-\_Z} + c_1 e^{-\_Z} - \_Z e^{-\_Z} + x\right)}$$

✓ Solution by Mathematica
Time used: 0.225 (sec). Leaf size: 23

```
DSolve[(2*x*y[x]^3+x*y[x]+x^2)*y'[x]-x*y[x]+y[x]^2==0,y[x],x,IncludeSingularSolutions -> Tru
```

$$\text{Solve}\left[y(x)^2 - \frac{x}{y(x)} + \log(y(x)) + \log(x) = c_1, y(x)\right]$$

6.49 problem Exercise 12.49, page 103

Internal problem ID [4570]

Book: Ordinary Differential Equations, By Tenenbaum and Pollard. Dover, NY 1963
Section: Chapter 2. Special types of differential equations of the first kind. Lesson 12, Miscellaneous Methods
Problem number: Exercise 12.49, page 103.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_separable]`

$$\boxed{(2y^3 + y)\, y' = 2x^3 + x}$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 113

```
dsolve((2*y(x)^3+y(x))*diff(y(x),x)-2*x^3-x=0,y(x), singsol=all)
```

$$y(x) = -\frac{\sqrt{-2 - 2\sqrt{4x^4 + 4x^2 + 8c_1 + 1}}}{2}$$
$$y(x) = \frac{\sqrt{-2 - 2\sqrt{4x^4 + 4x^2 + 8c_1 + 1}}}{2}$$
$$y(x) = -\frac{\sqrt{-2 + 2\sqrt{4x^4 + 4x^2 + 8c_1 + 1}}}{2}$$
$$y(x) = \frac{\sqrt{-2 + 2\sqrt{4x^4 + 4x^2 + 8c_1 + 1}}}{2}$$

✓ Solution by Mathematica

Time used: 2.313 (sec). Leaf size: 151

```
DSolve[(2*y[x]^3+y[x])*y'[x]-2*x^3-x==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to -\frac{\sqrt{-1-\sqrt{4x^4+4x^2+1+8c_1}}}{\sqrt{2}}$$
$$y(x) \to \frac{\sqrt{-1-\sqrt{4x^4+4x^2+1+8c_1}}}{\sqrt{2}}$$
$$y(x) \to -\frac{\sqrt{-1+\sqrt{4x^4+4x^2+1+8c_1}}}{\sqrt{2}}$$
$$y(x) \to \frac{\sqrt{-1+\sqrt{4x^4+4x^2+1+8c_1}}}{\sqrt{2}}$$

6.50 problem Exercise 12.50, page 103

Internal problem ID [4571]

Book: Ordinary Differential Equations, By Tenenbaum and Pollard. Dover, NY 1963
Section: Chapter 2. Special types of differential equations of the first kind. Lesson 12, Miscellaneous Methods
Problem number: Exercise 12.50, page 103.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_separable]`

$$\boxed{y' - e^{x-y} = -e^x}$$

✓ Solution by Maple
Time used: 0.047 (sec). Leaf size: 20

```
dsolve(diff(y(x),x)-exp(x-y(x))+exp(x)=0,y(x), singsol=all)
```

$$y(x) = -e^x + \ln\left(-1 + e^{e^x + c_1}\right) - c_1$$

✓ Solution by Mathematica
Time used: 2.135 (sec). Leaf size: 23

```
DSolve[y'[x]-Exp[x-y[x]]+Exp[x]==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to \log\left(1 + e^{-e^x + c_1}\right)$$
$$y(x) \to 0$$

7 Chapter 4. Higher order linear differential equations. Lesson 20. Constant coefficients

7.1 problem Exercise 20.1, page 220

Internal problem ID [4572]

Book: Ordinary Differential Equations, By Tenenbaum and Pollard. Dover, NY 1963
Section: Chapter 4. Higher order linear differential equations. Lesson 20. Constant coefficients
Problem number: Exercise 20.1, page 220.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _missing_x]]`

$$\boxed{y'' + 2y' = 0}$$

✓ Solution by Maple
Time used: 0.0 (sec). Leaf size: 12

```
dsolve(diff(y(x),x$2)+2*diff(y(x),x)=0,y(x), singsol=all)
```

$$y(x) = c_1 + c_2 e^{-2x}$$

✓ Solution by Mathematica
Time used: 0.01 (sec). Leaf size: 19

```
DSolve[y''[x]+2*y'[x]==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to c_2 - \frac{1}{2} c_1 e^{-2x}$$

7.2 problem Exercise 20.2, page 220

Internal problem ID [4573]

Book: Ordinary Differential Equations, By Tenenbaum and Pollard. Dover, NY 1963
Section: Chapter 4. Higher order linear differential equations. Lesson 20. Constant coefficients
Problem number: Exercise 20.2, page 220.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _missing_x]]`

$$\boxed{y'' - 3y' + 2y = 0}$$

✓ Solution by Maple
Time used: 0.015 (sec). Leaf size: 15

```
dsolve(diff(y(x),x$2)-3*diff(y(x),x)+2*y(x)=0,y(x), singsol=all)
```

$$y(x) = \mathrm{e}^{2x}c_1 + c_2\mathrm{e}^{x}$$

✓ Solution by Mathematica
Time used: 0.012 (sec). Leaf size: 18

```
DSolve[y''[x]-3*y'[x]+2*y[x]==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to e^{x}(c_2e^{x} + c_1)$$

7.3 problem Exercise 20.3, page 220

Internal problem ID [4574]

Book: Ordinary Differential Equations, By Tenenbaum and Pollard. Dover, NY 1963
Section: Chapter 4. Higher order linear differential equations. Lesson 20. Constant coefficients
Problem number: Exercise 20.3, page 220.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _missing_x]]`

$$\boxed{y'' - y = 0}$$

✓ Solution by Maple
Time used: 0.0 (sec). Leaf size: 15

```
dsolve(diff(y(x),x$2)-y(x)=0,y(x), singsol=all)
```

$$y(x) = c_1 e^x + c_2 e^{-x}$$

✓ Solution by Mathematica
Time used: 0.011 (sec). Leaf size: 20

```
DSolve[y''[x]-y[x]==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to c_1 e^x + c_2 e^{-x}$$

7.4 problem Exercise 20.5, page 220

Internal problem ID [4575]

Book: Ordinary Differential Equations, By Tenenbaum and Pollard. Dover, NY 1963
Section: Chapter 4. Higher order linear differential equations. Lesson 20. Constant coefficients
Problem number: Exercise 20.5, page 220.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _missing_x]]`

$$\boxed{6y'' - 11y' + 4y = 0}$$

✓ Solution by Maple
Time used: 0.0 (sec). Leaf size: 17

```
dsolve(6*diff(y(x),x$2)-11*diff(y(x),x)+4*y(x)=0,y(x), singsol=all)
```

$$y(x) = c_1 \mathrm{e}^{\frac{4x}{3}} + c_2 \mathrm{e}^{\frac{x}{2}}$$

✓ Solution by Mathematica
Time used: 0.028 (sec). Leaf size: 35

```
DSolve[y''[x]-11*y'[x]+4*y[x]==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to e^{-\frac{1}{2}\left(\sqrt{105}-11\right)x}\left(c_2 e^{\sqrt{105}x} + c_1\right)$$

7.5 problem Exercise 20.6, page 220

Internal problem ID [4576]

Book: Ordinary Differential Equations, By Tenenbaum and Pollard. Dover, NY 1963
Section: Chapter 4. Higher order linear differential equations. Lesson 20. Constant coefficients
Problem number: Exercise 20.6, page 220.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _missing_x]]`

$$\boxed{y'' + 2y' - y = 0}$$

✓ Solution by Maple

Time used: 0.016 (sec). Leaf size: 26

```
dsolve(diff(y(x),x$2)+2*diff(y(x),x)-y(x)=0,y(x), singsol=all)
```

$$y(x) = c_1 e^{\left(\sqrt{2}-1\right)x} + c_2 e^{-\left(1+\sqrt{2}\right)x}$$

✓ Solution by Mathematica

Time used: 0.019 (sec). Leaf size: 34

```
DSolve[y''[x]+2*y'[x]-y[x]==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to e^{-\left(\left(1+\sqrt{2}\right)x\right)}\left(c_2 e^{2\sqrt{2}x} + c_1\right)$$

7.6 problem Exercise 20.7, page 220

Internal problem ID [4577]

Book: Ordinary Differential Equations, By Tenenbaum and Pollard. Dover, NY 1963
Section: Chapter 4. Higher order linear differential equations. Lesson 20. Constant coefficients
Problem number: Exercise 20.7, page 220.
ODE order: 3.
ODE degree: 1.

CAS Maple gives this as type `[[_3rd_order, _missing_x]]`

$$\boxed{y''' + y'' - 10y' - 6y = 0}$$

✓ Solution by Maple

Time used: 0.015 (sec). Leaf size: 32

```
dsolve(diff(y(x),x$3)+diff(y(x),x$2)-10*diff(y(x),x)-6*y(x)=0,y(x), singsol=all)
```

$$y(x) = c_1 e^{3x} + c_2 e^{\left(-2+\sqrt{2}\right)x} + c_3 e^{-\left(2+\sqrt{2}\right)x}$$

✓ Solution by Mathematica

Time used: 0.004 (sec). Leaf size: 43

```
DSolve[y'''[x]+y''[x]-10*y'[x]-6*y[x]==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to c_1 e^{-\left(\left(2+\sqrt{2}\right)x\right)} + c_2 e^{\left(\sqrt{2}-2\right)x} + c_3 e^{3x}$$

7.7 problem Exercise 20.8, page 220

Internal problem ID [4578]

Book: Ordinary Differential Equations, By Tenenbaum and Pollard. Dover, NY 1963
Section: Chapter 4. Higher order linear differential equations. Lesson 20. Constant coefficients
Problem number: Exercise 20.8, page 220.
ODE order: 4.
ODE degree: 1.

CAS Maple gives this as type `[[_high_order, _missing_x]]`

$$\boxed{y'''' - y''' - 4y'' + 4y' = 0}$$

✓ Solution by Maple
Time used: 0.016 (sec). Leaf size: 29

```
dsolve(diff(y(x),x$4)-diff(y(x),x$3)-4*diff(y(x),x$2)+4*diff(y(x),x)=0,y(x), singsol=all)
```

$$y(x) = \left(c_2 e^{4x} + c_3 e^{3x} + e^{2x} c_1 + c_4\right) e^{-2x}$$

✓ Solution by Mathematica
Time used: 0.014 (sec). Leaf size: 36

```
DSolve[y''''[x]-y'''[x]-4*y''[x]+4*y'[x]==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to -\frac{1}{2} c_1 e^{-2x} + c_2 e^x + \frac{1}{2} c_3 e^{2x} + c_4$$

7.8 problem Exercise 20.9, page 220

Internal problem ID [4579]

Book: Ordinary Differential Equations, By Tenenbaum and Pollard. Dover, NY 1963
Section: Chapter 4. Higher order linear differential equations. Lesson 20. Constant coefficients
Problem number: Exercise 20.9, page 220.
ODE order: 4.
ODE degree: 1.

CAS Maple gives this as type `[[_high_order, _missing_x]]`

$$\boxed{y'''' + 4y''' + y'' - 4y' - 2y = 0}$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 36

```
dsolve(diff(y(x),x$4)+4*diff(y(x),x$3)+diff(y(x),x$2)-4*diff(y(x),x)-2*y(x)=0,y(x), singsol=
```

$$y(x) = c_1 e^{x} + c_2 e^{-x} + c_3 e^{\left(-2+\sqrt{2}\right)x} + c_4 e^{-\left(2+\sqrt{2}\right)x}$$

✓ Solution by Mathematica

Time used: 0.003 (sec). Leaf size: 49

```
DSolve[y''''[x]+4*y'''[x]+y''[x]-4*y'[x]-2*y[x]==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to c_1 e^{-\left(\left(2+\sqrt{2}\right)x\right)} + c_2 e^{\left(\sqrt{2}-2\right)x} + c_3 e^{-x} + c_4 e^{x}$$

7.9 problem Exercise 20.10, page 220

Internal problem ID [4580]

Book: Ordinary Differential Equations, By Tenenbaum and Pollard. Dover, NY 1963
Section: Chapter 4. Higher order linear differential equations. Lesson 20. Constant coefficients
Problem number: Exercise 20.10, page 220.
ODE order: 4.
ODE degree: 1.

CAS Maple gives this as type `[[_high_order, _missing_x]]`

$$\boxed{y'''' - y a^2 = 0}$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 38

```
dsolve(diff(y(x),x$4)-a^2*y(x)=0,y(x), singsol=all)
```

$$y(x) = c_1 e^{\sqrt{a}\, x} + c_2 e^{-\sqrt{a}\, x} + c_3 \sin\left(\sqrt{a}\, x\right) + c_4 \cos\left(\sqrt{a}\, x\right)$$

✓ Solution by Mathematica

Time used: 0.003 (sec). Leaf size: 53

```
DSolve[y''''[x]-a^2*y[x]==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to c_2 e^{-\sqrt{a}x} + c_4 e^{\sqrt{a}x} + c_1 \cos\left(\sqrt{a}x\right) + c_3 \sin\left(\sqrt{a}x\right)$$

7.10 problem Exercise 20.11, page 220

Internal problem ID [4581]

Book: Ordinary Differential Equations, By Tenenbaum and Pollard. Dover, NY 1963
Section: Chapter 4. Higher order linear differential equations. Lesson 20. Constant coefficients
Problem number: Exercise 20.11, page 220.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _missing_x]]`

$$\boxed{y'' - 2ky' - 2y = 0}$$

✓ Solution by Maple
Time used: 0.0 (sec). Leaf size: 35

```
dsolve(diff(y(x),x$2)-2*k*diff(y(x),x)-2*y(x)=0,y(x), singsol=all)
```

$$y(x) = c_1 \mathrm{e}^{\left(k+\sqrt{k^2+2}\right)x} + c_2 \mathrm{e}^{\left(k-\sqrt{k^2+2}\right)x}$$

✓ Solution by Mathematica
Time used: 0.022 (sec). Leaf size: 44

```
DSolve[y''[x]-2*k*y'[x]-2*y[x]==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to c_1 e^{\left(k-\sqrt{k^2+2}\right)x} + c_2 e^{\left(\sqrt{k^2+2}+k\right)x}$$

7.11 problem Exercise 20.12, page 220

Internal problem ID [4582]

Book: Ordinary Differential Equations, By Tenenbaum and Pollard. Dover, NY 1963
Section: Chapter 4. Higher order linear differential equations. Lesson 20. Constant coefficients
Problem number: Exercise 20.12, page 220.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _missing_x]]`

$$\boxed{y'' + 4ky' - 12k^2 y = 0}$$

✓ Solution by Maple
Time used: 0.0 (sec). Leaf size: 19

```
dsolve(diff(y(x),x$2)+4*k*diff(y(x),x)-12*k^2*y(x)=0,y(x), singsol=all)
```

$$y(x) = \left(c_1 e^{8kx} + c_2\right) e^{-6kx}$$

✓ Solution by Mathematica
Time used: 0.014 (sec). Leaf size: 24

```
DSolve[y''[x]+4*k*y'[x]-12*k^2*y[x]==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to e^{-6kx}\left(c_2 e^{8kx} + c_1\right)$$

7.12 problem Exercise 20.13, page 220

Internal problem ID [4583]

Book: Ordinary Differential Equations, By Tenenbaum and Pollard. Dover, NY 1963
Section: Chapter 4. Higher order linear differential equations. Lesson 20. Constant coefficients
Problem number: Exercise 20.13, page 220.
ODE order: 4.
ODE degree: 1.

CAS Maple gives this as type `[[_high_order, _quadrature]]`

$$\boxed{y'''' = 0}$$

✓ Solution by Maple
Time used: 0.016 (sec). Leaf size: 21

```
dsolve(diff(y(x),x$4)=0,y(x), singsol=all)
```

$$y(x) = \frac{1}{6}c_1x^3 + \frac{1}{2}c_2x^2 + c_3x + c_4$$

✓ Solution by Mathematica
Time used: 0.002 (sec). Leaf size: 22

```
DSolve[y''''[x]==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \rightarrow x(x(c_4x + c_3) + c_2) + c_1$$

7.13 problem Exercise 20.14, page 220

Internal problem ID [4584]

Book: Ordinary Differential Equations, By Tenenbaum and Pollard. Dover, NY 1963
Section: Chapter 4. Higher order linear differential equations. Lesson 20. Constant coefficients
Problem number: Exercise 20.14, page 220.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _missing_x]]`

$$\boxed{y'' + 4y' + 4y = 0}$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 14

```
dsolve(diff(y(x),x$2)+4*diff(y(x),x)+4*y(x)=0,y(x), singsol=all)
```

$$y(x) = \mathrm{e}^{-2x}(c_2x + c_1)$$

✓ Solution by Mathematica

Time used: 0.013 (sec). Leaf size: 18

```
DSolve[y''[x]+4*y'[x]+4*y[x]==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to e^{-2x}(c_2x + c_1)$$

7.14 problem Exercise 20.15, page 220

Internal problem ID [4585]

Book: Ordinary Differential Equations, By Tenenbaum and Pollard. Dover, NY 1963
Section: Chapter 4. Higher order linear differential equations. Lesson 20. Constant coefficients
Problem number: Exercise 20.15, page 220.
ODE order: 3.
ODE degree: 1.

CAS Maple gives this as type `[[_3rd_order, _missing_x]]`

$$\boxed{3y''' + 5y'' + y' - y = 0}$$

✓ Solution by Maple
Time used: 0.0 (sec). Leaf size: 20

```
dsolve(3*diff(y(x),x$3)+5*diff(y(x),x$2)+diff(y(x),x)-y(x)=0,y(x), singsol=all)
```

$$y(x) = \left(c_1 e^{\frac{4x}{3}} + c_3 x + c_2\right) e^{-x}$$

✓ Solution by Mathematica
Time used: 0.003 (sec). Leaf size: 28

```
DSolve[3*y'''[x]+5*y''[x]+y'[x]-y[x]==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to e^{-x}\left(c_1 e^{4x/3} + c_3 x + c_2\right)$$

7.15 problem Exercise 20.16, page 220

Internal problem ID [4586]

Book: Ordinary Differential Equations, By Tenenbaum and Pollard. Dover, NY 1963
Section: Chapter 4. Higher order linear differential equations. Lesson 20. Constant coefficients
Problem number: Exercise 20.16, page 220.
ODE order: 3.
ODE degree: 1.

CAS Maple gives this as type `[[_3rd_order, _missing_x]]`

$$\boxed{y''' - 6y'' + 12y' - 8y = 0}$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 19

```
dsolve(diff(y(x),x$3)-6*diff(y(x),x$2)+12*diff(y(x),x)-8*y(x)=0,y(x), singsol=all)
```

$$y(x) = e^{2x}(c_3x^2 + c_2x + c_1)$$

✓ Solution by Mathematica

Time used: 0.003 (sec). Leaf size: 23

```
DSolve[y'''[x]-6*y''[x]+12*y'[x]-8*y[x]==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to e^{2x}(x(c_3x + c_2) + c_1)$$

7.16 problem Exercise 20.17, page 220

Internal problem ID [4587]

Book: Ordinary Differential Equations, By Tenenbaum and Pollard. Dover, NY 1963
Section: Chapter 4. Higher order linear differential equations. Lesson 20. Constant coefficients
Problem number: Exercise 20.17, page 220.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _missing_x]]`

$$\boxed{y'' - 2ay' + ya^2 = 0}$$

✓ Solution by Maple
Time used: 0.0 (sec). Leaf size: 14

```
dsolve(diff(y(x),x$2)-2*a*diff(y(x),x)+a^2*y(x)=0,y(x), singsol=all)
```

$$y(x) = \mathrm{e}^{ax}(c_2x + c_1)$$

✓ Solution by Mathematica
Time used: 0.013 (sec). Leaf size: 18

```
DSolve[y''[x]-2*a*y'[x]+a^2*y[x]==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to e^{ax}(c_2x + c_1)$$

7.17 problem Exercise 20.18, page 220

Internal problem ID [4588]

Book: Ordinary Differential Equations, By Tenenbaum and Pollard. Dover, NY 1963
Section: Chapter 4. Higher order linear differential equations. Lesson 20. Constant coefficients
Problem number: Exercise 20.18, page 220.
ODE order: 4.
ODE degree: 1.

CAS Maple gives this as type `[[_high_order, _missing_x]]`

$$\boxed{y'''' + 3y''' = 0}$$

✓ Solution by Maple
Time used: 0.0 (sec). Leaf size: 20

```
dsolve(diff(y(x),x$4)+3*diff(y(x),x$3)=0,y(x), singsol=all)
```

$$y(x) = c_1 + c_2x + c_3x^2 + c_4\mathrm{e}^{-3x}$$

✓ Solution by Mathematica
Time used: 0.026 (sec). Leaf size: 28

```
DSolve[y''''[x]+3*y'''[x]==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to -\frac{1}{27}c_1e^{-3x} + x(c_4x + c_3) + c_2$$

7.18 problem Exercise 20.19, page 220

Internal problem ID [4589]

Book: Ordinary Differential Equations, By Tenenbaum and Pollard. Dover, NY 1963
Section: Chapter 4. Higher order linear differential equations. Lesson 20. Constant coefficients
Problem number: Exercise 20.19, page 220.
ODE order: 4.
ODE degree: 1.

CAS Maple gives this as type `[[_high_order, _missing_x]]`

$$\boxed{y'''' - 2y'' = 0}$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 26

```
dsolve(diff(y(x),x$4)-2*diff(y(x),x$2)=0,y(x), singsol=all)
```

$$y(x) = c_1 + c_2 x + c_3 \mathrm{e}^{x\sqrt{2}} + c_4 \mathrm{e}^{-x\sqrt{2}}$$

✓ Solution by Mathematica

Time used: 0.073 (sec). Leaf size: 42

```
DSolve[y''''[x]-2*y''[x]==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to \frac{1}{2} e^{-\sqrt{2}x} \left(c_1 e^{2\sqrt{2}x} + c_2\right) + c_4 x + c_3$$

7.19 problem Exercise 20.20, page 220

Internal problem ID [4590]

Book: Ordinary Differential Equations, By Tenenbaum and Pollard. Dover, NY 1963
Section: Chapter 4. Higher order linear differential equations. Lesson 20. Constant coefficients
Problem number: Exercise 20.20, page 220.
ODE order: 4.
ODE degree: 1.

CAS Maple gives this as type `[[_high_order, _missing_x]]`

$$y'''' + 2y''' - 11y'' - 12y' + 36y = 0$$

✓ Solution by Maple
Time used: 0.0 (sec). Leaf size: 24

```
dsolve(diff(y(x),x$4)+2*diff(y(x),x$3)-11*diff(y(x),x$2)-12*diff(y(x),x)+36*y(x)=0,y(x), sin
```

$$y(x) = \left((c_2x + c_1)\,\mathrm{e}^{5x} + xc_4 + c_3\right)\mathrm{e}^{-3x}$$

✓ Solution by Mathematica
Time used: 0.003 (sec). Leaf size: 35

```
DSolve[y''''[x]+2*y'''[x]-11*y''[x]-12*y'[x]+36*y[x]==0,y[x],x,IncludeSingularSolutions -> T
```

$$y(x) \to e^{-3x}\left(c_3e^{5x} + x\left(c_4e^{5x} + c_2\right) + c_1\right)$$

7.20 problem Exercise 20.21, page 220

Internal problem ID [4591]

Book: Ordinary Differential Equations, By Tenenbaum and Pollard. Dover, NY 1963
Section: Chapter 4. Higher order linear differential equations. Lesson 20. Constant coefficients
Problem number: Exercise 20.21, page 220.
ODE order: 4.
ODE degree: 1.

CAS Maple gives this as type `[[_high_order, _missing_x]]`

$$\boxed{36y'''' - 37y'' + 4y' + 5y = 0}$$

✓ Solution by Maple
Time used: 0.0 (sec). Leaf size: 29

```
dsolve(36*diff(y(x),x$4)-37*diff(y(x),x$2)+4*diff(y(x),x)+5*y(x)=0,y(x), singsol=all)
```

$$y(x) = \left(c_3 e^{\frac{11x}{6}} + c_1 e^{\frac{3x}{2}} + c_2 e^{\frac{2x}{3}} + c_4\right) e^{-x}$$

✓ Solution by Mathematica
Time used: 0.003 (sec). Leaf size: 44

```
DSolve[36*y''''[x]-37*y''[x]+4*y'[x]+5*y[x]==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to e^{-x}\left(c_1 e^{11x/6} + c_2 e^{2x/3} + c_3 e^{3x/2} + c_4\right)$$

7.21 problem Exercise 20.22, page 220

Internal problem ID [4592]

Book: Ordinary Differential Equations, By Tenenbaum and Pollard. Dover, NY 1963
Section: Chapter 4. Higher order linear differential equations. Lesson 20. Constant coefficients
Problem number: Exercise 20.22, page 220.
ODE order: 4.
ODE degree: 1.

CAS Maple gives this as type `[[_high_order, _missing_x]]`

$$\boxed{y'''' - 8y'' + 36y = 0}$$

✓ Solution by Maple

Time used: 0.016 (sec). Leaf size: 48

```
dsolve(diff(y(x),x$4)-8*diff(y(x),x$2)+36*y(x)=0,y(x), singsol=all)
```

$$y(x) = c_1 e^{\sqrt{5}\,x} \sin(x) - c_2 e^{-\sqrt{5}\,x} \sin(x) + c_3 e^{\sqrt{5}\,x} \cos(x) + c_4 e^{-\sqrt{5}\,x} \cos(x)$$

✓ Solution by Mathematica

Time used: 0.007 (sec). Leaf size: 142

```
DSolve[y''''[x]-8*y''[x]+36*y[x]==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to e^{-\sqrt{6}x\cos\left(\frac{1}{2}\arctan\left(\frac{\sqrt{5}}{2}\right)\right)}\left(\left(c_3 e^{2\sqrt{6}x\cos\left(\frac{1}{2}\arctan\left(\frac{\sqrt{5}}{2}\right)\right)} + c_2\right)\cos\left(\sqrt{6}x\sin\left(\frac{1}{2}\arctan\left(\frac{\sqrt{5}}{2}\right)\right)\right) + \sin\left(\sqrt{6}x\sin\left(\frac{1}{2}\arctan\left(\frac{\sqrt{5}}{2}\right)\right)\right)\left(c_1 e^{2\sqrt{6}x\cos\left(\frac{1}{2}\arctan\left(\frac{\sqrt{5}}{2}\right)\right)} + c_4\right)\right)$$

7.22 problem Exercise 20.23, page 220

Internal problem ID [4593]

Book: Ordinary Differential Equations, By Tenenbaum and Pollard. Dover, NY 1963
Section: Chapter 4. Higher order linear differential equations. Lesson 20. Constant coefficients
Problem number: Exercise 20.23, page 220.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _missing_x]]`

$$\boxed{y'' - 2y' + 5y = 0}$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 20

```
dsolve(diff(y(x),x$2)-2*diff(y(x),x)+5*y(x)=0,y(x), singsol=all)
```

$$y(x) = e^{x}(c_1 \sin(2x) + c_2 \cos(2x))$$

✓ Solution by Mathematica

Time used: 0.016 (sec). Leaf size: 24

```
DSolve[y''[x]-2*y'[x]+5*y[x]==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to e^{x}(c_2 \cos(2x) + c_1 \sin(2x))$$

7.23 problem Exercise 20.24, page 220

Internal problem ID [4594]

Book: Ordinary Differential Equations, By Tenenbaum and Pollard. Dover, NY 1963
Section: Chapter 4. Higher order linear differential equations. Lesson 20. Constant coefficients
Problem number: Exercise 20.24, page 220.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _missing_x]]`

$$\boxed{y'' - y' + y = 0}$$

✓ Solution by Maple
Time used: 0.0 (sec). Leaf size: 28

```
dsolve(diff(y(x),x$2)-diff(y(x),x)+y(x)=0,y(x), singsol=all)
```

$$y(x) = e^{\frac{x}{2}}\left(c_1 \sin\left(\frac{\sqrt{3}\,x}{2}\right) + c_2 \cos\left(\frac{\sqrt{3}\,x}{2}\right)\right)$$

✓ Solution by Mathematica
Time used: 0.021 (sec). Leaf size: 42

```
DSolve[y''[x]-y'[x]+y[x]==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to e^{x/2}\left(c_1 \cos\left(\frac{\sqrt{3}x}{2}\right) + c_2 \sin\left(\frac{\sqrt{3}x}{2}\right)\right)$$

7.24 problem Exercise 20.25, page 220

Internal problem ID [4595]

Book: Ordinary Differential Equations, By Tenenbaum and Pollard. Dover, NY 1963
Section: Chapter 4. Higher order linear differential equations. Lesson 20. Constant coefficients
Problem number: Exercise 20.25, page 220.
ODE order: 4.
ODE degree: 1.

CAS Maple gives this as type `[[_high_order, _missing_x]]`

$$\boxed{y'''' + 5y'' + 6y = 0}$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 37

```
dsolve(diff(y(x),x$4)+5*diff(y(x),x$2)+6*y(x)=0,y(x), singsol=all)
```

$$y(x) = c_1 \sin\left(\sqrt{3}\,x\right) + c_2 \cos\left(\sqrt{3}\,x\right) + c_3 \sin\left(x\sqrt{2}\right) + c_4 \cos\left(x\sqrt{2}\right)$$

✓ Solution by Mathematica

Time used: 0.003 (sec). Leaf size: 50

```
DSolve[y''''[x]+5*y''[x]+6*y[x]==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to c_3 \cos\left(\sqrt{2}x\right) + c_1 \cos\left(\sqrt{3}x\right) + c_4 \sin\left(\sqrt{2}x\right) + c_2 \sin\left(\sqrt{3}x\right)$$

7.25 problem Exercise 20.26, page 220

Internal problem ID [4596]

Book: Ordinary Differential Equations, By Tenenbaum and Pollard. Dover, NY 1963
Section: Chapter 4. Higher order linear differential equations. Lesson 20. Constant coefficients
Problem number: Exercise 20.26, page 220.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _missing_x]]`

$$y'' - 4y' + 20y = 0$$

✓ Solution by Maple
Time used: 0.0 (sec). Leaf size: 22

```
dsolve(diff(y(x),x$2)-4*diff(y(x),x)+20*y(x)=0,y(x), singsol=all)
```

$$y(x) = e^{2x}(c_1 \sin(4x) + c_2 \cos(4x))$$

✓ Solution by Mathematica
Time used: 0.017 (sec). Leaf size: 26

```
DSolve[y''[x]-4*y'[x]+20*y[x]==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to e^{2x}(c_2 \cos(4x) + c_1 \sin(4x))$$

7.26 problem Exercise 20.27, page 220

Internal problem ID [4597]

Book: Ordinary Differential Equations, By Tenenbaum and Pollard. Dover, NY 1963
Section: Chapter 4. Higher order linear differential equations. Lesson 20. Constant coefficients
Problem number: Exercise 20.27, page 220.
ODE order: 4.
ODE degree: 1.

CAS Maple gives this as type `[[_high_order, _missing_x]]`

$$\boxed{y'''' + 4y'' + 4y = 0}$$

✓ Solution by Maple
Time used: 0.0 (sec). Leaf size: 29

```
dsolve(diff(y(x),x$4)+4*diff(y(x),x$2)+4*y(x)=0,y(x), singsol=all)
```

$$y(x) = (xc_4 + c_2)\cos\left(x\sqrt{2}\right) + \sin\left(x\sqrt{2}\right)(c_3x + c_1)$$

✓ Solution by Mathematica
Time used: 0.003 (sec). Leaf size: 38

```
DSolve[y''''[x]+4*y''[x]+4*y[x]==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to (c_2x + c_1)\cos\left(\sqrt{2}x\right) + (c_4x + c_3)\sin\left(\sqrt{2}x\right)$$

7.27 problem Exercise 20.28, page 220

Internal problem ID [4598]

Book: Ordinary Differential Equations, By Tenenbaum and Pollard. Dover, NY 1963
Section: Chapter 4. Higher order linear differential equations. Lesson 20. Constant coefficients
Problem number: Exercise 20.28, page 220.
ODE order: 3.
ODE degree: 1.

CAS Maple gives this as type `[[_3rd_order, _missing_x]]`

$$\boxed{y''' + 8y = 0}$$

✓ Solution by Maple
Time used: 0.0 (sec). Leaf size: 35

```
dsolve(diff(y(x),x$3)+8*y(x)=0,y(x), singsol=all)
```

$$y(x) = \left(c_2 e^{3x} \sin\left(\sqrt{3}\, x\right) + c_3 e^{3x} \cos\left(\sqrt{3}\, x\right) + c_1\right) e^{-2x}$$

✓ Solution by Mathematica
Time used: 0.003 (sec). Leaf size: 42

```
DSolve[y'''[x]+8*y[x]==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to c_1 e^{-2x} + c_3 e^x \cos\left(\sqrt{3}x\right) + c_2 e^x \sin\left(\sqrt{3}x\right)$$

7.28 problem Exercise 20.29, page 220

Internal problem ID [4599]

Book: Ordinary Differential Equations, By Tenenbaum and Pollard. Dover, NY 1963
Section: Chapter 4. Higher order linear differential equations. Lesson 20. Constant coefficients
Problem number: Exercise 20.29, page 220.
ODE order: 4.
ODE degree: 1.

CAS Maple gives this as type `[[_high_order, _missing_x]]`

$$\boxed{y'''' + 4y'' = 0}$$

✓ Solution by Maple
Time used: 0.0 (sec). Leaf size: 21

```
dsolve(diff(y(x),x$4)+4*diff(y(x),x$2)=0,y(x), singsol=all)
```

$$y(x) = c_1 + c_2 x + c_3 \sin(2x) + c_4 \cos(2x)$$

✓ Solution by Mathematica
Time used: 0.118 (sec). Leaf size: 32

```
DSolve[y''''[x]+4*y''[x]==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to c_4 x - \frac{1}{4} c_1 \cos(2x) - \frac{1}{4} c_2 \sin(2x) + c_3$$

7.29 problem Exercise 20.30, page 220

Internal problem ID [4600]

Book: Ordinary Differential Equations, By Tenenbaum and Pollard. Dover, NY 1963
Section: Chapter 4. Higher order linear differential equations. Lesson 20. Constant coefficients
Problem number: Exercise 20.30, page 220.
ODE order: 5.
ODE degree: 1.

CAS Maple gives this as type `[[_high_order, _missing_x]]`

$$\boxed{y^{(5)} + 2y''' + y' = 0}$$

✓ Solution by Maple
Time used: 0.0 (sec). Leaf size: 22

```
dsolve(diff(y(x),x$5)+2*diff(y(x),x$3)+diff(y(x),x)=0,y(x), singsol=all)
```

$$y(x) = (c_5x + c_3)\cos(x) + (xc_4 + c_2)\sin(x) + c_1$$

✓ Solution by Mathematica
Time used: 0.05 (sec). Leaf size: 35

```
DSolve[y'''''[x]+2*y'''[x]+y'[x]==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to (-c_4x + c_2 - c_3)\cos(x) + (c_2x + c_1 + c_4)\sin(x) + c_5$$

7.30 problem Exercise 20, problem 31, page 220

Internal problem ID [4601]

Book: Ordinary Differential Equations, By Tenenbaum and Pollard. Dover, NY 1963
Section: Chapter 4. Higher order linear differential equations. Lesson 20. Constant coefficients
Problem number: Exercise 20, problem 31, page 220.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _quadrature]]`

$$\boxed{y'' = 0}$$

With initial conditions

$$[y(1) = 2, y'(1) = -1]$$

✓ Solution by Maple
Time used: 0.0 (sec). Leaf size: 9

```
dsolve([diff(y(x),x$2)=0,y(1) = 2, D(y)(1) = -1],y(x), singsol=all)
```

$$y(x) = -x + 3$$

✓ Solution by Mathematica
Time used: 0.005 (sec). Leaf size: 10

```
DSolve[{y''[x]==0,{y[1]==2,y'[1]==-1}},y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to 3 - x$$

7.31 problem Exercise 20, problem 32, page 220

Internal problem ID [4602]

Book: Ordinary Differential Equations, By Tenenbaum and Pollard. Dover, NY 1963
Section: Chapter 4. Higher order linear differential equations. Lesson 20. Constant coefficients
Problem number: Exercise 20, problem 32, page 220.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _missing_x]]`

$$\boxed{y'' + 4y' + 4y = 0}$$

With initial conditions

$$[y(0) = 1, y'(0) = 1]$$

✓ Solution by Maple
Time used: 0.016 (sec). Leaf size: 14

```
dsolve([diff(y(x),x$2)+4*diff(y(x),x)+4*y(x)=0,y(0) = 1, D(y)(0) = 1],y(x), singsol=all)
```

$$y(x) = e^{-2x}(1 + 3x)$$

✓ Solution by Mathematica
Time used: 0.014 (sec). Leaf size: 16

```
DSolve[{y''[x]+4*y'[x]+4*y[x]==0,{y[0]==1,y'[0]==1}},y[x],x,IncludeSingularSolutions -> True
```

$$y(x) \to e^{-2x}(3x + 1)$$

7.32 problem Exercise 20, problem 33, page 220

Internal problem ID [4603]

Book: Ordinary Differential Equations, By Tenenbaum and Pollard. Dover, NY 1963
Section: Chapter 4. Higher order linear differential equations. Lesson 20. Constant coefficients
Problem number: Exercise 20, problem 33, page 220.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _missing_x]]`

$$\boxed{y'' - 2y' + 5y = 0}$$

With initial conditions

$$[y(0) = 2, y'(0) = 1]$$

✓ Solution by Maple

Time used: 0.016 (sec). Leaf size: 19

```
dsolve([diff(y(x),x$2)-2*diff(y(x),x)+5*y(x)=0,y(0) = 2, D(y)(0) = 1],y(x), singsol=all)
```

$$y(x) = -\frac{e^{x}(\sin(2x) - 4\cos(2x))}{2}$$

✓ Solution by Mathematica

Time used: 0.017 (sec). Leaf size: 25

```
DSolve[{y''[x]-2*y'[x]+5*y[x]==0,{y[0]==2,y'[0]==1}},y[x],x,IncludeSingularSolutions -> True
```

$$y(x) \to \frac{1}{2}e^{x}(4\cos(2x) - \sin(2x))$$

7.33 problem Exercise 20, problem 34, page 220

Internal problem ID [4604]

Book: Ordinary Differential Equations, By Tenenbaum and Pollard. Dover, NY 1963
Section: Chapter 4. Higher order linear differential equations. Lesson 20. Constant coefficients
Problem number: Exercise 20, problem 34, page 220.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _missing_x]]`

$$\boxed{y'' - 4y' + 20y = 0}$$

With initial conditions

$$\left[y\left(\frac{\pi}{2}\right) = 1, y'\left(\frac{\pi}{2}\right) = 1\right]$$

✓ Solution by Maple

Time used: 0.031 (sec). Leaf size: 25

```
dsolve([diff(y(x),x$2)-4*diff(y(x),x)+20*y(x)=0,y(1/2*Pi) = 1, D(y)(1/2*Pi) = 1],y(x), sings
```

$$y(x) = -\frac{(\sin(4x) - 4\cos(4x))\,\mathrm{e}^{-\pi+2x}}{4}$$

✓ Solution by Mathematica

Time used: 0.019 (sec). Leaf size: 31

```
DSolve[{y''[x]-4*y'[x]+20*y[x]==0,{y[Pi/2]==1,y'[Pi/2]==1}},y[x],x,IncludeSingularSolutions
```

$$y(x) \to \frac{1}{4}e^{2x-\pi}(4\cos(4x) - \sin(4x))$$

7.34 problem Exercise 20, problem 35, page 220

Internal problem ID [4605]

Book: Ordinary Differential Equations, By Tenenbaum and Pollard. Dover, NY 1963
Section: Chapter 4. Higher order linear differential equations. Lesson 20. Constant coefficients
Problem number: Exercise 20, problem 35, page 220.
ODE order: 3.
ODE degree: 1.

CAS Maple gives this as type `[[_3rd_order, _missing_x]]`

$$\boxed{3y''' + 5y'' + y' - y = 0}$$

With initial conditions

$$[y(0) = 0, y'(0) = 1, y''(0) = -1]$$

✓ Solution by Maple

Time used: 0.016 (sec). Leaf size: 21

```
dsolve([3*diff(y(x),x$3)+5*diff(y(x),x$2)+diff(y(x),x)-y(x)=0,y(0) = 0, D(y)(0) = 1, (D@@2)(
```

$$y(x) = \frac{\left(9\,\mathrm{e}^{\frac{4x}{3}} + 4x - 9\right)\mathrm{e}^{-x}}{16}$$

✓ Solution by Mathematica

Time used: 0.004 (sec). Leaf size: 28

```
DSolve[{3*y'''[x]+5*y''[x]+y'[x]-y[x]==0,{y[0]==0,y'[0]==1,y''[0]==-1}},y[x],x,IncludeSingul
```

$$y(x) \to \frac{1}{16}e^{-x}\left(4x + 9e^{4x/3} - 9\right)$$

8 Chapter 4. Higher order linear differential equations. Lesson 21. Undetermined Coefficients

8.1 problem Exercise 21.3, page 231

Internal problem ID [4606]

Book: Ordinary Differential Equations, By Tenenbaum and Pollard. Dover, NY 1963
Section: Chapter 4. Higher order linear differential equations. Lesson 21. Undetermined Coefficients
Problem number: Exercise 21.3, page 231.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _missing_x]]`

$$\boxed{y'' + 3y' + 2y = 4}$$

✓ Solution by Maple
Time used: 0.0 (sec). Leaf size: 19

```
dsolve(diff(y(x),x$2)+3*diff(y(x),x)+2*y(x)=4,y(x), singsol=all)
```

$$y(x) = -\mathrm{e}^{-2x} c_1 + c_2 \mathrm{e}^{-x} + 2$$

✓ Solution by Mathematica
Time used: 0.014 (sec). Leaf size: 23

```
DSolve[y''[x]+3*y'[x]+2*y[x]==4,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to c_1 e^{-2x} + c_2 e^{-x} + 2$$

8.2 problem Exercise 21.4, page 231

Internal problem ID [4607]

Book: Ordinary Differential Equations, By Tenenbaum and Pollard. Dover, NY 1963
Section: Chapter 4. Higher order linear differential equations. Lesson 21. Undetermined Coefficients
Problem number: Exercise 21.4, page 231.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$y'' + 3y' + 2y = 12\,\mathrm{e}^{x}$$

✓ Solution by Maple
Time used: 0.0 (sec). Leaf size: 23

```
dsolve(diff(y(x),x$2)+3*diff(y(x),x)+2*y(x)=12*exp(x),y(x), singsol=all)
```

$$y(x) = -\left(-2\,\mathrm{e}^{3x} - c_2\mathrm{e}^{x} + c_1\right)\mathrm{e}^{-2x}$$

✓ Solution by Mathematica
Time used: 0.016 (sec). Leaf size: 27

```
DSolve[y''[x]+3*y'[x]+2*y[x]==12*Exp[x],y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to e^{-2x}\left(2e^{3x} + c_2e^{x} + c_1\right)$$

8.3 problem Exercise 21.5, page 231

Internal problem ID [4608]

Book: Ordinary Differential Equations, By Tenenbaum and Pollard. Dover, NY 1963
Section: Chapter 4. Higher order linear differential equations. Lesson 21. Undetermined Coefficients
Problem number: Exercise 21.5, page 231.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$\boxed{y'' + 3y' + 2y = \mathrm{e}^{ix}}$$

✓ Solution by Maple
Time used: 0.0 (sec). Leaf size: 28

```
dsolve(diff(y(x),x$2)+3*diff(y(x),x)+2*y(x)=exp(I*x),y(x), singsol=all)
```

$$y(x) = \mathrm{e}^{-x}\left(\left(\frac{1}{10} - \frac{3i}{10}\right)\mathrm{e}^{(1+i)x} - \mathrm{e}^{-x}c_1 + c_2\right)$$

✓ Solution by Mathematica
Time used: 0.046 (sec). Leaf size: 37

```
DSolve[y''[x]+3*y'[x]+2*y[x]==Exp[I*x],y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to \left(\frac{1}{10} - \frac{3i}{10}\right)e^{ix} + c_1e^{-2x} + c_2e^{-x}$$

8.4 problem Exercise 21.6, page 231

Internal problem ID [4609]

Book: Ordinary Differential Equations, By Tenenbaum and Pollard. Dover, NY 1963
Section: Chapter 4. Higher order linear differential equations. Lesson 21. Undetermined Coefficients
Problem number: Exercise 21.6, page 231.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _linear, _nonhomogeneous]]`

$$\boxed{y'' + 3y' + 2y = \sin(x)}$$

✓ Solution by Maple
Time used: 0.0 (sec). Leaf size: 26

```
dsolve(diff(y(x),x$2)+3*diff(y(x),x)+2*y(x)=sin(x),y(x), singsol=all)
```

$$y(x) = -e^{-2x}c_1 - \frac{3\cos(x)}{10} + \frac{\sin(x)}{10} + c_2 e^{-x}$$

✓ Solution by Mathematica
Time used: 0.047 (sec). Leaf size: 32

```
DSolve[y''[x]+3*y'[x]+2*y[x]==Sin[x],y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to \frac{1}{10}\left(\sin(x) - 3\cos(x) + 10e^{-2x}(c_2 e^x + c_1)\right)$$

8.5 problem Exercise 21.7, page 231

Internal problem ID [4610]

Book: Ordinary Differential Equations, By Tenenbaum and Pollard. Dover, NY 1963
Section: Chapter 4. Higher order linear differential equations. Lesson 21. Undetermined Coefficients
Problem number: Exercise 21.7, page 231.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _linear, _nonhomogeneous]]`

$$\boxed{y'' + 3y' + 2y = \cos(x)}$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 26

```
dsolve(diff(y(x),x$2)+3*diff(y(x),x)+2*y(x)=cos(x),y(x), singsol=all)
```

$$y(x) = -e^{-2x}c_1 + \frac{\cos(x)}{10} + \frac{3\sin(x)}{10} + c_2e^{-x}$$

✓ Solution by Mathematica

Time used: 0.052 (sec). Leaf size: 32

```
DSolve[y''[x]+3*y'[x]+2*y[x]==Cos[x],y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to \frac{1}{10}\left(3\sin(x) + \cos(x) + 10e^{-2x}(c_2e^x + c_1)\right)$$

8.6 problem Exercise 21.8, page 231

Internal problem ID [4611]

Book: Ordinary Differential Equations, By Tenenbaum and Pollard. Dover, NY 1963
Section: Chapter 4. Higher order linear differential equations. Lesson 21. Undetermined Coefficients
Problem number: Exercise 21.8, page 231.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _linear, _nonhomogeneous]]`

$$\boxed{y'' + 3y' + 2y = 8 + 6\,e^{x} + 2\sin(x)}$$

✓ Solution by Maple
Time used: 0.0 (sec). Leaf size: 38

```
dsolve(diff(y(x),x$2)+3*diff(y(x),x)+2*y(x)=8+6*exp(x)+2*sin(x),y(x), singsol=all)
```

$$y(x) = -e^{-2x}\left(\left(-4 + \frac{3\cos(x)}{5} - \frac{\sin(x)}{5}\right)e^{2x} - c_2 e^{x} + c_1 - e^{3x}\right)$$

✓ Solution by Mathematica
Time used: 0.165 (sec). Leaf size: 38

```
DSolve[y''[x]+3*y'[x]+2*y[x]==8+6*Exp[x]+2*Sin[x],y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to e^{x} + \frac{\sin(x)}{5} - \frac{3\cos(x)}{5} + c_1 e^{-2x} + c_2 e^{-x} + 4$$

8.7 problem Exercise 21.9, page 231

Internal problem ID [4612]

Book: Ordinary Differential Equations, By Tenenbaum and Pollard. Dover, NY 1963
Section: Chapter 4. Higher order linear differential equations. Lesson 21. Undetermined Coefficients
Problem number: Exercise 21.9, page 231.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$\boxed{y'' + y' + y = x^2}$$

✓ Solution by Maple
Time used: 0.016 (sec). Leaf size: 37

```
dsolve(diff(y(x),x$2)+diff(y(x),x)+y(x)=x^2,y(x), singsol=all)
```

$$y(x) = \mathrm{e}^{-\frac{x}{2}} \sin\left(\frac{\sqrt{3}\,x}{2}\right) c_2 + \mathrm{e}^{-\frac{x}{2}} \cos\left(\frac{\sqrt{3}\,x}{2}\right) c_1 + x^2 - 2x$$

✓ Solution by Mathematica
Time used: 0.023 (sec). Leaf size: 54

```
DSolve[y''[x]+y'[x]+y[x]==x^2,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to e^{-x/2}\left(e^{x/2}(x-2)x + c_2 \cos\left(\frac{\sqrt{3}x}{2}\right) + c_1 \sin\left(\frac{\sqrt{3}x}{2}\right)\right)$$

8.8 problem Exercise 21.10, page 231

Internal problem ID [4613]

Book: Ordinary Differential Equations, By Tenenbaum and Pollard. Dover, NY 1963
Section: Chapter 4. Higher order linear differential equations. Lesson 21. Undetermined Coefficients
Problem number: Exercise 21.10, page 231.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _linear, _nonhomogeneous]]`

$$\boxed{y'' - 2y' - 8y = 9\,\mathrm{e}^{x}x + 10\,\mathrm{e}^{-x}}$$

✓ Solution by Maple

Time used: 0.015 (sec). Leaf size: 28

```
dsolve(diff(y(x),x$2)-2*diff(y(x),x)-8*y(x)=9*x*exp(x)+10*exp(-x),y(x), singsol=all)
```

$$y(x) = \left(\mathrm{e}^{6x}c_1 - \mathrm{e}^{3x}x - 2\,\mathrm{e}^{x} + c_2\right)\mathrm{e}^{-2x}$$

✓ Solution by Mathematica

Time used: 0.145 (sec). Leaf size: 35

```
DSolve[y''[x]-2*y'[x]-8*y[x]==9*x*Exp[x]+10*Exp[-x],y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to e^{-2x}\left(-e^{3x}x - 2e^{x} + c_2e^{6x} + c_1\right)$$

8.9 problem Exercise 21.11, page 231

Internal problem ID [4614]

Book: Ordinary Differential Equations, By Tenenbaum and Pollard. Dover, NY 1963
Section: Chapter 4. Higher order linear differential equations. Lesson 21. Undetermined Coefficients
Problem number: Exercise 21.11, page 231.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _missing_y]]`

$$y'' - 3y' = 2\sin(x)\,e^{2x}$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 28

```
dsolve(diff(y(x),x$2)-3*diff(y(x),x)=2*exp(2*x)*sin(x),y(x), singsol=all)
```

$$y(x) = \frac{e^{2x}(-\cos(x) - 3\sin(x))}{5} + \frac{c_1 e^{3x}}{3} + c_2$$

✓ Solution by Mathematica

Time used: 0.245 (sec). Leaf size: 33

```
DSolve[y''[x]-3*y'[x]==2*Exp[2*x]*Sin[x],y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to \frac{1}{15}e^{2x}(-9\sin(x) - 3\cos(x) + 5c_1 e^x) + c_2$$

8.10 problem Exercise 21.13, page 231

Internal problem ID [4615]

Book: Ordinary Differential Equations, By Tenenbaum and Pollard. Dover, NY 1963
Section: Chapter 4. Higher order linear differential equations. Lesson 21. Undetermined Coefficients
Problem number: Exercise 21.13, page 231.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _missing_y]]`

$$\boxed{y'' + y' = x^2 + 2x}$$

✓ Solution by Maple
Time used: 0.0 (sec). Leaf size: 18

```
dsolve(diff(y(x),x$2)+diff(y(x),x)=x^2+2*x,y(x), singsol=all)
```

$$y(x) = \frac{x^3}{3} - \mathrm{e}^{-x} c_1 + c_2$$

✓ Solution by Mathematica
Time used: 0.036 (sec). Leaf size: 24

```
DSolve[y''[x]+y'[x]==x^2+2*x,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to \frac{x^3}{3} - c_1 e^{-x} + c_2$$

8.11 problem Exercise 21.14, page 231

Internal problem ID [4616]

Book: Ordinary Differential Equations, By Tenenbaum and Pollard. Dover, NY 1963
Section: Chapter 4. Higher order linear differential equations. Lesson 21. Undetermined Coefficients
Problem number: Exercise 21.14, page 231.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _missing_y]]`

$$y'' + y' = x + \sin(2x)$$

✓ Solution by Maple
Time used: 0.0 (sec). Leaf size: 33

```
dsolve(diff(y(x),x$2)+diff(y(x),x)=x+sin(2*x),y(x), singsol=all)
```

$$y(x) = \frac{x^2}{2} - e^{-x}c_1 - \frac{\sin(2x)}{5} - \frac{\cos(2x)}{10} - x + c_2$$

✓ Solution by Mathematica
Time used: 0.359 (sec). Leaf size: 43

```
DSolve[y''[x]+y'[x]==x+Sin[2*x],y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to \frac{x^2}{2} - x - \frac{1}{5}\sin(2x) - \frac{1}{10}\cos(2x) - c_1e^{-x} + c_2$$

8.12 problem Exercise 21.15, page 231

Internal problem ID [4617]

Book: Ordinary Differential Equations, By Tenenbaum and Pollard. Dover, NY 1963
Section: Chapter 4. Higher order linear differential equations. Lesson 21. Undetermined Coefficients
Problem number: Exercise 21.15, page 231.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _linear, _nonhomogeneous]]`

$$\boxed{y'' + y = 4\sin(x)\, x}$$

✓ Solution by Maple
Time used: 0.015 (sec). Leaf size: 21

```
dsolve(diff(y(x),x$2)+y(x)=4*x*sin(x),y(x), singsol=all)
```

$$y(x) = (-x^2 + c_1)\cos(x) + \sin(x)(c_2 + x)$$

✓ Solution by Mathematica
Time used: 0.048 (sec). Leaf size: 27

```
DSolve[y''[x]+y[x]==4*x*Sin[x],y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to \left(-x^2 + \frac{1}{2} + c_1\right)\cos(x) + (x + c_2)\sin(x)$$

8.13 problem Exercise 21.16, page 231

Internal problem ID [4618]

Book: Ordinary Differential Equations, By Tenenbaum and Pollard. Dover, NY 1963
Section: Chapter 4. Higher order linear differential equations. Lesson 21. Undetermined Coefficients
Problem number: Exercise 21.16, page 231.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _linear, _nonhomogeneous]]`

$$\boxed{y'' + 4y = x \sin(2x)}$$

✓ Solution by Maple
Time used: 0.0 (sec). Leaf size: 31

```
dsolve(diff(y(x),x$2)+4*y(x)=x*sin(2*x),y(x), singsol=all)
```

$$y(x) = \frac{(-x^2 + 8c_1)\cos(2x)}{8} + \frac{\sin(2x)(16c_2 + x)}{16}$$

✓ Solution by Mathematica
Time used: 0.048 (sec). Leaf size: 38

```
DSolve[y''[x]+4*y[x]==x*Sin[2*x],y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to \frac{1}{64}\left((-8x^2 + 1 + 64c_1)\cos(2x) + 4(x + 16c_2)\sin(2x)\right)$$

8.14 problem Exercise 21.17, page 231

Internal problem ID [4619]

Book: Ordinary Differential Equations, By Tenenbaum and Pollard. Dover, NY 1963
Section: Chapter 4. Higher order linear differential equations. Lesson 21. Undetermined Coefficients
Problem number: Exercise 21.17, page 231.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _linear, _nonhomogeneous]]`

$$\boxed{y'' + 2y' + y = x^2 e^{-x}}$$

✓ Solution by Maple

Time used: 0.016 (sec). Leaf size: 19

```
dsolve(diff(y(x),x$2)+2*diff(y(x),x)+y(x)=x^2*exp(-x),y(x), singsol=all)
```

$$y(x) = e^{-x}\left(c_2 + c_1 x + \frac{1}{12}x^4\right)$$

✓ Solution by Mathematica

Time used: 0.025 (sec). Leaf size: 27

```
DSolve[y''[x]+2*y'[x]+y[x]==x^2*Exp[-x],y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to \frac{1}{12}e^{-x}\left(x^4 + 12c_2 x + 12c_1\right)$$

8.15 problem Exercise 21.19, page 231

Internal problem ID [4620]

Book: Ordinary Differential Equations, By Tenenbaum and Pollard. Dover, NY 1963
Section: Chapter 4. Higher order linear differential equations. Lesson 21. Undetermined Coefficients
Problem number: Exercise 21.19, page 231.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _linear, _nonhomogeneous]]`

$$\boxed{y'' + 3y' + 2y = e^{-2x} + x^2}$$

✓ Solution by Maple
Time used: 0.0 (sec). Leaf size: 33

```
dsolve(diff(y(x),x$2)+3*diff(y(x),x)+2*y(x)=exp(-2*x)+x^2,y(x), singsol=all)
```

$$y(x) = \frac{7}{4} + (-c_1 - x - 1)\,e^{-2x} + \frac{x^2}{2} + c_2 e^{-x} - \frac{3x}{2}$$

✓ Solution by Mathematica
Time used: 0.078 (sec). Leaf size: 41

```
DSolve[y''[x]+3*y'[x]+2*y[x]==Exp[-2*x]+x^2,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to \frac{1}{4}(2x^2 - 6x + 7) + e^{-2x}(-x - 1 + c_1) + c_2 e^{-x}$$

8.16 problem Exercise 21.20, page 231

Internal problem ID [4621]

Book: Ordinary Differential Equations, By Tenenbaum and Pollard. Dover, NY 1963
Section: Chapter 4. Higher order linear differential equations. Lesson 21. Undetermined Coefficients
Problem number: Exercise 21.20, page 231.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _linear, _nonhomogeneous]]`

$$\boxed{y'' - 3y' + 2y = x\,\mathrm{e}^{-x}}$$

✓ Solution by Maple
Time used: 0.0 (sec). Leaf size: 29

```
dsolve(diff(y(x),x$2)-3*diff(y(x),x)+2*y(x)=x*exp(-x),y(x), singsol=all)
```

$$y(x) = \frac{(36c_1\mathrm{e}^{3x} + 36c_2\mathrm{e}^{2x} + 6x + 5)\,\mathrm{e}^{-x}}{36}$$

✓ Solution by Mathematica
Time used: 0.019 (sec). Leaf size: 34

```
DSolve[y''[x]-3*y'[x]+2*y[x]==x*Exp[-x],y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to \frac{1}{36}e^{-x}(6x+5) + c_1e^x + c_2e^{2x}$$

8.17 problem Exercise 21.21, page 231

Internal problem ID [4622]

Book: Ordinary Differential Equations, By Tenenbaum and Pollard. Dover, NY 1963
Section: Chapter 4. Higher order linear differential equations. Lesson 21. Undetermined Coefficients
Problem number: Exercise 21.21, page 231.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$\boxed{y'' + y' - 6y = x + e^{2x}}$$

✓ Solution by Maple

Time used: 0.016 (sec). Leaf size: 35

```
dsolve(diff(y(x),x$2)+diff(y(x),x)-6*y(x)=x+exp(2*x),y(x), singsol=all)
```

$$y(x) = -\frac{\left(\left(-\frac{6x}{5} - 6c_2 + \frac{6}{25}\right) e^{5x} + \left(x + \frac{1}{6}\right) e^{3x} - 6c_1\right) e^{-3x}}{6}$$

✓ Solution by Mathematica

Time used: 0.085 (sec). Leaf size: 40

```
DSolve[y''[x]+y'[x]-6*y[x]==x+Exp[2*x],y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to \frac{1}{36}(-6x - 1) + c_1 e^{-3x} + e^{2x}\left(\frac{x}{5} - \frac{1}{25} + c_2\right)$$

8.18 problem Exercise 21.22, page 231

Internal problem ID [4623]

Book: Ordinary Differential Equations, By Tenenbaum and Pollard. Dover, NY 1963
Section: Chapter 4. Higher order linear differential equations. Lesson 21. Undetermined Coefficients
Problem number: Exercise 21.22, page 231.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _linear, _nonhomogeneous]]`

$$\boxed{y'' + y = \sin(x) + e^{-x}}$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 26

```
dsolve(diff(y(x),x$2)+y(x)=sin(x)+exp(-x),y(x), singsol=all)
```

$$y(x) = \frac{e^{-x}}{2} + \frac{(2c_1 - x)\cos(x)}{2} + c_2 \sin(x)$$

✓ Solution by Mathematica

Time used: 0.337 (sec). Leaf size: 36

```
DSolve[y''[x]+y[x]==Sin[x]+Exp[-x],y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to \frac{1}{4}\left(2e^{-x} + \sin(x) - 2x\cos(x) + 4c_1\cos(x) + 4c_2\sin(x)\right)$$

8.19 problem Exercise 21.24, page 231

Internal problem ID [4624]

Book: Ordinary Differential Equations, By Tenenbaum and Pollard. Dover, NY 1963
Section: Chapter 4. Higher order linear differential equations. Lesson 21. Undetermined Coefficients
Problem number: Exercise 21.24, page 231.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _linear, _nonhomogeneous]]`

$$\boxed{y'' + y = \sin(x)^2}$$

✓ Solution by Maple

Time used: 0.016 (sec). Leaf size: 20

```
dsolve(diff(y(x),x$2)+y(x)=sin(x)^2,y(x), singsol=all)
```

$$y(x) = c_2 \sin(x) + \cos(x) c_1 + \frac{\cos(x)^2}{3} + \frac{1}{3}$$

✓ Solution by Mathematica

Time used: 0.038 (sec). Leaf size: 27

```
DSolve[y''[x]+y[x]==Sin[x]^2,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to \frac{1}{6}(\cos(2x) + 6c_1 \cos(x) + 6c_2 \sin(x) + 3)$$

8.20 problem Exercise 21.27, page 231

Internal problem ID [4625]

Book: Ordinary Differential Equations, By Tenenbaum and Pollard. Dover, NY 1963
Section: Chapter 4. Higher order linear differential equations. Lesson 21. Undetermined Coefficients
Problem number: Exercise 21.27, page 231.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _linear, _nonhomogeneous]]`

$$\boxed{y'' + y = \sin(x)\sin(2x)}$$

✓ Solution by Maple

Time used: 0.016 (sec). Leaf size: 26

```
dsolve(diff(y(x),x$2)+y(x)=sin(2*x)*sin(x),y(x), singsol=all)
```

$$y(x) = -\frac{\sin(x)^2\cos(x)}{4} + \frac{(4c_2 + x)\sin(x)}{4} + \cos(x)\,c_1$$

✓ Solution by Mathematica

Time used: 0.039 (sec). Leaf size: 33

```
DSolve[y''[x]+y[x]==Sin[2*x]*Sin[x],y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to \frac{1}{16}(\cos(3x) + (-1 + 16c_1)\cos(x) + 4(x + 4c_2)\sin(x))$$

8.21 problem Exercise 21.28, page 231

Internal problem ID [4626]

Book: Ordinary Differential Equations, By Tenenbaum and Pollard. Dover, NY 1963
Section: Chapter 4. Higher order linear differential equations. Lesson 21. Undetermined Coefficients
Problem number: Exercise 21.28, page 231.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$\boxed{y'' - 5y' - 6y = e^{3x}}$$

With initial conditions

$$[y(0) = 2, y'(0) = 1]$$

✓ Solution by Maple
Time used: 0.016 (sec). Leaf size: 23

```
dsolve([diff(y(x),x$2)-5*diff(y(x),x)-6*y(x)=exp(3*x),y(0) = 2, D(y)(0) = 1],y(x), singsol=a
```

$$y(x) = \frac{45\,e^{-x}}{28} + \frac{10\,e^{6x}}{21} - \frac{e^{3x}}{12}$$

✓ Solution by Mathematica
Time used: 0.027 (sec). Leaf size: 30

```
DSolve[{y''[x]-5*y'[x]-6*y[x]==Exp[3*x],{y[0]==2,y'[0]==1}},y[x],x,IncludeSingularSolutions
```

$$y(x) \to \frac{1}{84}e^{-x}\left(-7e^{4x} + 40e^{7x} + 135\right)$$

8.22 problem Exercise 21.29, page 231

Internal problem ID [4627]

Book: Ordinary Differential Equations, By Tenenbaum and Pollard. Dover, NY 1963
Section: Chapter 4. Higher order linear differential equations. Lesson 21. Undetermined Coefficients
Problem number: Exercise 21.29, page 231.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _linear, _nonhomogeneous]]`

$$\boxed{y'' - y' - 2y = 5\sin(x)}$$

With initial conditions

$$[y(0) = 1, y'(0) = -1]$$

✓ Solution by Maple
Time used: 0.031 (sec). Leaf size: 25

```
dsolve([diff(y(x),x$2)-diff(y(x),x)-2*y(x)=5*sin(x),y(0) = 1, D(y)(0) = -1],y(x), singsol=al
```

$$y(x) = \frac{e^{-x}}{6} + \frac{e^{2x}}{3} + \frac{\cos(x)}{2} - \frac{3\sin(x)}{2}$$

✓ Solution by Mathematica
Time used: 0.022 (sec). Leaf size: 30

```
DSolve[{y''[x]-y'[x]-2*y[x]==5*Sin[x],{y[0]==1,y'[0]==-1}},y[x],x,IncludeSingularSolutions -
```

$$y(x) \to \frac{1}{6}\left(e^{-x} + 2e^{2x} - 9\sin(x) + 3\cos(x)\right)$$

8.23 problem Exercise 21.31, page 231

Internal problem ID [4628]

Book: Ordinary Differential Equations, By Tenenbaum and Pollard. Dover, NY 1963
Section: Chapter 4. Higher order linear differential equations. Lesson 21. Undetermined Coefficients
Problem number: Exercise 21.31, page 231.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _linear, _nonhomogeneous]]`

$$\boxed{y'' + 9y = 8\cos(x)}$$

With initial conditions

$$\left[y\left(\frac{\pi}{2}\right) = -1, y'\left(\frac{\pi}{2}\right) = 1\right]$$

✓ Solution by Maple
Time used: 0.031 (sec). Leaf size: 17

```
dsolve([diff(y(x),x$2)+9*y(x)=8*cos(x),y(1/2*Pi) = -1, D(y)(1/2*Pi) = 1],y(x), singsol=all)
```

$$y(x) = \sin(3x) + \frac{2\cos(3x)}{3} + \cos(x)$$

✓ Solution by Mathematica
Time used: 0.02 (sec). Leaf size: 20

```
DSolve[{y''[x]+9*y[x]==8*Cos[x],{y[Pi/2]==-1,y'[Pi/2]==1}},y[x],x,IncludeSingularSolutions -
```

$$y(x) \to \sin(3x) + \cos(x) + \frac{2}{3}\cos(3x)$$

8.24 problem Exercise 21.32, page 231

Internal problem ID [4629]

Book: Ordinary Differential Equations, By Tenenbaum and Pollard. Dover, NY 1963
Section: Chapter 4. Higher order linear differential equations. Lesson 21. Undetermined Coefficients
Problem number: Exercise 21.32, page 231.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _linear, _nonhomogeneous]]`

$$\boxed{y'' - 5y' + 6y = e^x(2x - 3)}$$

With initial conditions

$$[y(0) = 1, y'(0) = 3]$$

✓ Solution by Maple
Time used: 0.016 (sec). Leaf size: 13

```
dsolve([diff(y(x),x$2)-5*diff(y(x),x)+6*y(x)=exp(x)*(2*x-3),y(0) = 1, D(y)(0) = 3],y(x), sin
```

$$y(x) = e^{2x} + x\,e^{x}$$

✓ Solution by Mathematica
Time used: 0.02 (sec). Leaf size: 35

```
DSolve[{y''[x]-5*y'[x]-6*y[x]==Exp[x]*(2*x-3),{y[0]==1,y'[0]==3}},y[x],x,IncludeSingularSolu
```

$$y(x) \to \frac{1}{175}e^{-x}\left(-7e^{2x}(5x - 9) + 87e^{7x} + 25\right)$$

8.25 problem Exercise 21.33, page 231

Internal problem ID [4630]

Book: Ordinary Differential Equations, By Tenenbaum and Pollard. Dover, NY 1963
Section: Chapter 4. Higher order linear differential equations. Lesson 21. Undetermined Coefficients
Problem number: Exercise 21.33, page 231.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$\boxed{y'' - 3y' + 2y = \mathrm{e}^{-x}}$$

With initial conditions

$$[y(0) = 1, y'(0) = -1]$$

✓ Solution by Maple
Time used: 0.016 (sec). Leaf size: 21

```
dsolve([diff(y(x),x$2)-3*diff(y(x),x)+2*y(x)=exp(-x),y(0) = 1, D(y)(0) = -1],y(x), singsol=a
```

$$y(x) = -\frac{5\,\mathrm{e}^{2x}}{3} + \frac{5\,\mathrm{e}^{x}}{2} + \frac{\mathrm{e}^{-x}}{6}$$

✓ Solution by Mathematica
Time used: 0.024 (sec). Leaf size: 31

```
DSolve[{y''[x]-3*y'[x]+2*y[x]==Exp[-x],{y[0]==1,y'[0]==-1}},y[x],x,IncludeSingularSolutions
```

$$y(x) \to \frac{e^{-x}}{6} + \frac{5e^{x}}{2} - \frac{5e^{2x}}{3}$$

9 Chapter 4. Higher order linear differential equations. Lesson 22. Variation of Parameters

9.1 problem Exercise 22.1, page 240

Internal problem ID [4631]

Book: Ordinary Differential Equations, By Tenenbaum and Pollard. Dover, NY 1963
Section: Chapter 4. Higher order linear differential equations. Lesson 22. Variation of Parameters
Problem number: Exercise 22.1, page 240.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _linear, _nonhomogeneous]]`

$$\boxed{y'' + y = \sec(x)}$$

✓ Solution by Maple
Time used: 0.0 (sec). Leaf size: 22

```
dsolve(diff(y(x),x$2)+y(x)=sec(x),y(x), singsol=all)
```

$$y(x) = -\ln(\sec(x))\cos(x) + \cos(x)\, c_1 + \sin(x)\,(c_2 + x)$$

✓ Solution by Mathematica
Time used: 0.021 (sec). Leaf size: 22

```
DSolve[y''[x]+y[x]==Sec[x],y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to (x + c_2)\sin(x) + \cos(x)(\log(\cos(x)) + c_1)$$

9.2 problem Exercise 22.2, page 240

Internal problem ID [4632]

Book: Ordinary Differential Equations, By Tenenbaum and Pollard. Dover, NY 1963
Section: Chapter 4. Higher order linear differential equations. Lesson 22. Variation of Parameters
Problem number: Exercise 22.2, page 240.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _linear, _nonhomogeneous]]`

$$\boxed{y'' + y = \cot(x)}$$

✓ Solution by Maple
Time used: 0.0 (sec). Leaf size: 24

```
dsolve(diff(y(x),x$2)+y(x)=cot(x),y(x), singsol=all)
```

$$y(x) = c_2 \sin(x) + \cos(x) c_1 + \sin(x) \ln(\csc(x) - \cot(x))$$

✓ Solution by Mathematica
Time used: 0.033 (sec). Leaf size: 33

```
DSolve[y''[x]+y[x]==Cot[x],y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to c_1 \cos(x) + \sin(x) \left(\log\left(\sin\left(\frac{x}{2}\right)\right) - \log\left(\cos\left(\frac{x}{2}\right)\right) + c_2\right)$$

9.3 problem Exercise 22.3, page 240

Internal problem ID [4633]

Book: Ordinary Differential Equations, By Tenenbaum and Pollard. Dover, NY 1963
Section: Chapter 4. Higher order linear differential equations. Lesson 22. Variation of Parameters
Problem number: Exercise 22.3, page 240.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _linear, _nonhomogeneous]]`

$$\boxed{y'' + y = \sec(x)^2}$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 23

```
dsolve(diff(y(x),x$2)+y(x)=sec(x)^2,y(x), singsol=all)
```

$$y(x) = c_2 \sin(x) + \cos(x) c_1 + \ln(\sec(x) + \tan(x)) \sin(x) - 1$$

✓ Solution by Mathematica

Time used: 0.036 (sec). Leaf size: 28

```
DSolve[y''[x]+y[x]==Sec[x]^2,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to 2\sin(x)\text{arctanh}\left(\tan\left(\frac{x}{2}\right)\right) + c_1\cos(x) + c_2\sin(x) - 1$$

9.4 problem Exercise 22.4, page 240

Internal problem ID [4634]

Book: Ordinary Differential Equations, By Tenenbaum and Pollard. Dover, NY 1963
Section: Chapter 4. Higher order linear differential equations. Lesson 22. Variation of Parameters
Problem number: Exercise 22.4, page 240.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _linear, _nonhomogeneous]]`

$$\boxed{y'' - y = \sin(x)^2}$$

✓ Solution by Maple
Time used: 0.0 (sec). Leaf size: 22

```
dsolve(diff(y(x),x$2)-y(x)=sin(x)^2,y(x), singsol=all)
```

$$y(x) = c_2 e^x + e^{-x} c_1 + \frac{\cos(x)^2}{5} - \frac{3}{5}$$

✓ Solution by Mathematica
Time used: 0.052 (sec). Leaf size: 30

```
DSolve[y''[x]-y[x]==Sin[x]^2,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to \frac{1}{10}(\cos(2x) - 5) + c_1 e^x + c_2 e^{-x}$$

9.5 problem Exercise 22.5, page 240

Internal problem ID [4635]

Book: Ordinary Differential Equations, By Tenenbaum and Pollard. Dover, NY 1963
Section: Chapter 4. Higher order linear differential equations. Lesson 22. Variation of Parameters
Problem number: Exercise 22.5, page 240.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _linear, _nonhomogeneous]]`

$$\boxed{y'' + y = \sin(x)^2}$$

✓ Solution by Maple
Time used: 0.0 (sec). Leaf size: 20

```
dsolve(diff(y(x),x$2)+y(x)=sin(x)^2,y(x), singsol=all)
```

$$y(x) = c_2 \sin(x) + \cos(x) c_1 + \frac{\cos(x)^2}{3} + \frac{1}{3}$$

✓ Solution by Mathematica
Time used: 0.024 (sec). Leaf size: 27

```
DSolve[y''[x]+y[x]==Sin[x]^2,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to \frac{1}{6}(\cos(2x) + 6c_1 \cos(x) + 6c_2 \sin(x) + 3)$$

9.6 problem Exercise 22.6, page 240

Internal problem ID [4636]

Book: Ordinary Differential Equations, By Tenenbaum and Pollard. Dover, NY 1963
Section: Chapter 4. Higher order linear differential equations. Lesson 22. Variation of Parameters
Problem number: Exercise 22.6, page 240.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$\boxed{y'' + 3y' + 2y = 12\,\mathrm{e}^{x}}$$

✓ Solution by Maple
Time used: 0.0 (sec). Leaf size: 23

```
dsolve(diff(y(x),x$2)+3*diff(y(x),x)+2*y(x)=12*exp(x),y(x), singsol=all)
```

$$y(x) = -\left(-2\,\mathrm{e}^{3x} - c_2\mathrm{e}^{x} + c_1\right)\mathrm{e}^{-2x}$$

✓ Solution by Mathematica
Time used: 0.016 (sec). Leaf size: 27

```
DSolve[y''[x]+3*y'[x]+2*y[x]==12*Exp[x],y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to e^{-2x}\left(2e^{3x} + c_2e^{x} + c_1\right)$$

9.7 problem Exercise 22.7, page 240

Internal problem ID [4637]

Book: Ordinary Differential Equations, By Tenenbaum and Pollard. Dover, NY 1963
Section: Chapter 4. Higher order linear differential equations. Lesson 22. Variation of Parameters
Problem number: Exercise 22.7, page 240.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _linear, _nonhomogeneous]]`

$$\boxed{y'' + 2y' + y = x^2 e^{-x}}$$

✓ Solution by Maple
Time used: 0.016 (sec). Leaf size: 19

```
dsolve(diff(y(x),x$2)+2*diff(y(x),x)+y(x)=x^2*exp(-x),y(x), singsol=all)
```

$$y(x) = e^{-x}\left(c_2 + c_1 x + \frac{1}{12}x^4\right)$$

✓ Solution by Mathematica
Time used: 0.024 (sec). Leaf size: 27

```
DSolve[y''[x]+2*y'[x]+y[x]==x^2*Exp[-x],y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to \frac{1}{12}e^{-x}\left(x^4 + 12c_2 x + 12c_1\right)$$

9.8 problem Exercise 22.8, page 240

Internal problem ID [4638]

Book: Ordinary Differential Equations, By Tenenbaum and Pollard. Dover, NY 1963
Section: Chapter 4. Higher order linear differential equations. Lesson 22. Variation of Parameters
Problem number: Exercise 22.8, page 240.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _linear, _nonhomogeneous]]`

$$\boxed{y'' + y = 4\sin(x)\, x}$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 21

```
dsolve(diff(y(x),x$2)+y(x)=4*x*sin(x),y(x), singsol=all)
```

$$y(x) = (-x^2 + c_1)\cos(x) + \sin(x)(c_2 + x)$$

✓ Solution by Mathematica

Time used: 0.028 (sec). Leaf size: 27

```
DSolve[y''[x]+y[x]==4*x*Sin[x],y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to \left(-x^2 + \frac{1}{2} + c_1\right)\cos(x) + (x + c_2)\sin(x)$$

9.9 problem Exercise 22.9, page 240

Internal problem ID [4639]

Book: Ordinary Differential Equations, By Tenenbaum and Pollard. Dover, NY 1963
Section: Chapter 4. Higher order linear differential equations. Lesson 22. Variation of Parameters
Problem number: Exercise 22.9, page 240.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _linear, _nonhomogeneous]]`

$$\boxed{y'' + 2y' + y = \mathrm{e}^{-x} \ln (x)}$$

✓ Solution by Maple
Time used: 0.0 (sec). Leaf size: 30

```
dsolve(diff(y(x),x$2)+2*diff(y(x),x)+y(x)=exp(-x)*ln(x),y(x), singsol=all)
```

$$y(x) = \frac{\mathrm{e}^{-x}(2\ln (x)\, x^2 + 4c_1 x - 3x^2 + 4c_2)}{4}$$

✓ Solution by Mathematica
Time used: 0.026 (sec). Leaf size: 36

```
DSolve[y''[x]+2*y'[x]+y[x]==Exp[-x]*Log[x],y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to \frac{1}{4}e^{-x}\left(-3x^2 + 2x^2 \log(x) + 4c_2 x + 4c_1\right)$$

9.10 problem Exercise 22.10, page 240

Internal problem ID [4640]

Book: Ordinary Differential Equations, By Tenenbaum and Pollard. Dover, NY 1963
Section: Chapter 4. Higher order linear differential equations. Lesson 22. Variation of Parameters
Problem number: Exercise 22.10, page 240.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _linear, _nonhomogeneous]]`

$$\boxed{y'' + y = \csc(x)}$$

✓ Solution by Maple
Time used: 0.0 (sec). Leaf size: 24

```
dsolve(diff(y(x),x$2)+y(x)=csc(x),y(x), singsol=all)
```

$$y(x) = -\ln(\csc(x))\sin(x) + (-x + c_1)\cos(x) + c_2\sin(x)$$

✓ Solution by Mathematica
Time used: 0.022 (sec). Leaf size: 24

```
DSolve[y''[x]+y[x]==Csc[x],y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to (-x + c_1)\cos(x) + \sin(x)(\log(\sin(x)) + c_2)$$

9.11 problem Exercise 22.11, page 240

Internal problem ID [4641]

Book: Ordinary Differential Equations, By Tenenbaum and Pollard. Dover, NY 1963
Section: Chapter 4. Higher order linear differential equations. Lesson 22. Variation of Parameters
Problem number: Exercise 22.11, page 240.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _linear, _nonhomogeneous]]`

$$\boxed{y'' + y = \tan(x)^2}$$

✓ Solution by Maple

Time used: 0.016 (sec). Leaf size: 23

```
dsolve(diff(y(x),x$2)+y(x)=tan(x)^2,y(x), singsol=all)
```

$$y(x) = c_2 \sin(x) + \cos(x) c_1 - 2 + \ln(\sec(x) + \tan(x)) \sin(x)$$

✓ Solution by Mathematica

Time used: 0.109 (sec). Leaf size: 23

```
DSolve[y''[x]+y[x]==Tan[x]^2,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to \sin(x)\text{arctanh}(\sin(x)) + c_1 \cos(x) + c_2 \sin(x) - 2$$

9.12 problem Exercise 22.12, page 240

Internal problem ID [4642]

Book: Ordinary Differential Equations, By Tenenbaum and Pollard. Dover, NY 1963
Section: Chapter 4. Higher order linear differential equations. Lesson 22. Variation of Parameters
Problem number: Exercise 22.12, page 240.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _linear, _nonhomogeneous]]`

$$\boxed{y'' + 2y' + y = \frac{e^{-x}}{x}}$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 20

```
dsolve(diff(y(x),x$2)+2*diff(y(x),x)+y(x)=exp(-x)/x,y(x), singsol=all)
```

$$y(x) = e^{-x}(\ln(x)\,x + x(c_1 - 1) + c_2)$$

✓ Solution by Mathematica

Time used: 0.023 (sec). Leaf size: 24

```
DSolve[y''[x]+2*y'[x]+y[x]==Exp[-x]/x,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to e^{-x}(x\log(x) + (-1 + c_2)x + c_1)$$

9.13 problem Exercise 22.13, page 240

Internal problem ID [4643]

Book: Ordinary Differential Equations, By Tenenbaum and Pollard. Dover, NY 1963
Section: Chapter 4. Higher order linear differential equations. Lesson 22. Variation of Parameters
Problem number: Exercise 22.13, page 240.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _linear, _nonhomogeneous]]`

$$\boxed{y'' + y = \sec(x)\csc(x)}$$

✓ Solution by Maple
Time used: 0.0 (sec). Leaf size: 34

```
dsolve(diff(y(x),x$2)+y(x)=sec(x)*csc(x),y(x), singsol=all)
```

$$y(x) = c_2 \sin(x) + \cos(x)\, c_1 + \sin(x) \ln(\csc(x) - \cot(x)) - \ln(\sec(x) + \tan(x)) \cos(x)$$

✓ Solution by Mathematica
Time used: 0.027 (sec). Leaf size: 30

```
DSolve[y''[x]+y[x]==Sec[x]*Csc[x],y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to -\sin(x)\text{arctanh}(\cos(x)) + c_1 \cos(x) + c_2 \sin(x) + \cos(x)\left(-\coth^{-1}(\sin(x))\right)$$

9.14 problem Exercise 22.14, page 240

Internal problem ID [4644]

Book: Ordinary Differential Equations, By Tenenbaum and Pollard. Dover, NY 1963
Section: Chapter 4. Higher order linear differential equations. Lesson 22. Variation of Parameters
Problem number: Exercise 22.14, page 240.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _linear, _nonhomogeneous]]`

$$\boxed{y'' - 2y' + y = e^{x} \ln(x)}$$

✓ Solution by Maple

Time used: 0.015 (sec). Leaf size: 28

```
dsolve(diff(y(x),x$2)-2*diff(y(x),x)+y(x)=exp(x)*ln(x),y(x), singsol=all)
```

$$y(x) = \frac{e^{x}(2\ln(x)\, x^{2} + 4c_1 x - 3x^{2} + 4c_2)}{4}$$

✓ Solution by Mathematica

Time used: 0.021 (sec). Leaf size: 34

```
DSolve[y''[x]-2*y'[x]+y[x]==Exp[x]*Log[x],y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to \frac{1}{4}e^{x}\left(-3x^{2} + 2x^{2}\log(x) + 4c_2 x + 4c_1\right)$$

9.15 problem Exercise 22.15, page 240

Internal problem ID [4645]

Book: Ordinary Differential Equations, By Tenenbaum and Pollard. Dover, NY 1963
Section: Chapter 4. Higher order linear differential equations. Lesson 22. Variation of Parameters
Problem number: Exercise 22.15, page 240.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _linear, _nonhomogeneous]]`

$$\boxed{y'' - 3y' + 2y = \cos\left(\mathrm{e}^{-x}\right)}$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 24

```
dsolve(diff(y(x),x$2)-3*diff(y(x),x)+2*y(x)=cos(exp(-x)),y(x), singsol=all)
```

$$y(x) = \left(-\mathrm{e}^{x} \cos\left(\mathrm{e}^{-x}\right) + (c_1 - 1)\,\mathrm{e}^{x} + c_2\right) \mathrm{e}^{x}$$

✓ Solution by Mathematica

Time used: 0.082 (sec). Leaf size: 29

```
DSolve[y''[x]-3*y'[x]+2*y[x]==Cos[Exp[-x]],y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to e^{x}\left(-e^{x} \cos\left(e^{-x}\right) + c_2 e^{x} + c_1\right)$$

9.16 problem Exercise 22, problem 16, page 240

Internal problem ID [4646]

Book: Ordinary Differential Equations, By Tenenbaum and Pollard. Dover, NY 1963
Section: Chapter 4. Higher order linear differential equations. Lesson 22. Variation of Parameters
Problem number: Exercise 22, problem 16, page 240.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$\boxed{x^2y'' - xy' + y = x}$$

✓ Solution by Maple
Time used: 0.016 (sec). Leaf size: 18

```
dsolve(x^2*diff(y(x),x$2)-x*diff(y(x),x)+y(x)=x,y(x), singsol=all)
```

$$y(x) = x\left(c_2 + c_1 \ln(x) + \frac{\ln(x)^2}{2}\right)$$

✓ Solution by Mathematica
Time used: 0.019 (sec). Leaf size: 25

```
DSolve[x^2*y''[x]-x*y'[x]+y[x]==x,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to \frac{1}{2}x\left(\log^2(x) + 2c_2 \log(x) + 2c_1\right)$$

9.17 problem Exercise 22, problem 17, page 240

Internal problem ID [4647]

Book: Ordinary Differential Equations, By Tenenbaum and Pollard. Dover, NY 1963
Section: Chapter 4. Higher order linear differential equations. Lesson 22. Variation of Parameters
Problem number: Exercise 22, problem 17, page 240.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _exact, _linear, _nonhomogeneous]]`

$$\boxed{y'' - \frac{2y'}{x} + \frac{2y}{x^2} = \ln(x)\, x}$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 25

```
dsolve(diff(y(x),x$2)-2/x*diff(y(x),x)+2/x^2*y(x)=x*ln(x),y(x), singsol=all)
```

$$y(x) = \frac{\ln(x)\, x^3}{2} - \frac{3x^3}{4} + c_2 x^2 + c_1 x$$

✓ Solution by Mathematica

Time used: 0.014 (sec). Leaf size: 32

```
DSolve[y''[x]-2/x*y'[x]+2/x^2*y[x]==x*Log[x],y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to \frac{1}{4} x\left(-3x^2 + 2x^2 \log(x) + 4c_2 x + 4c_1\right)$$

9.18 problem Exercise 22, problem 18, page 240

Internal problem ID [4648]

Book: Ordinary Differential Equations, By Tenenbaum and Pollard. Dover, NY 1963
Section: Chapter 4. Higher order linear differential equations. Lesson 22. Variation of Parameters
Problem number: Exercise 22, problem 18, page 240.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$\boxed{x^2y'' + xy' - 4y = x^3}$$

✓ Solution by Maple
Time used: 0.0 (sec). Leaf size: 20

```
dsolve(x^2*diff(y(x),x$2)+x*diff(y(x),x)-4*y(x)=x^3,y(x), singsol=all)
```

$$y(x) = \frac{c_2}{x^2} + c_1 x^2 + \frac{x^3}{5}$$

✓ Solution by Mathematica
Time used: 0.014 (sec). Leaf size: 25

```
DSolve[x^2*y''[x]+x*y'[x]-4*y[x]==x^3,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to \frac{x^3}{5} + c_2 x^2 + \frac{c_1}{x^2}$$

9.19 problem Exercise 22, problem 19, page 240

Internal problem ID [4649]

Book: Ordinary Differential Equations, By Tenenbaum and Pollard. Dover, NY 1963
Section: Chapter 4. Higher order linear differential equations. Lesson 22. Variation of Parameters
Problem number: Exercise 22, problem 19, page 240.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _exact, _linear, _nonhomogeneous]]`

$$\boxed{x^2y'' + xy' - y = x^2 \mathrm{e}^{-x}}$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 25

```
dsolve(x^2*diff(y(x),x$2)+x*diff(y(x),x)-y(x)=x^2*exp(-x),y(x), singsol=all)
```

$$y(x) = \frac{c_2x^2 + \mathrm{e}^{-x}x + \mathrm{e}^{-x} + c_1}{x}$$

✓ Solution by Mathematica

Time used: 0.024 (sec). Leaf size: 27

```
DSolve[x^2*y''[x]+x*y'[x]-y[x]==x^2*Exp[-x],y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to \frac{c_2x^2 + e^{-x}(x+1) + c_1}{x}$$

9.20 problem Exercise 22, problem 20, page 240

Internal problem ID [4650]

Book: Ordinary Differential Equations, By Tenenbaum and Pollard. Dover, NY 1963
Section: Chapter 4. Higher order linear differential equations. Lesson 22. Variation of Parameters
Problem number: Exercise 22, problem 20, page 240.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _exact, _linear, _nonhomogeneous]]`

$$\boxed{2x^2y'' + 3xy' - y = \frac{1}{x}}$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 24

```
dsolve(2*x^2*diff(y(x),x$2)+3*x*diff(y(x),x)-y(x)=1/x,y(x), singsol=all)
```

$$y(x) = \frac{9x^{\frac{3}{2}}c_2 - 3\ln(x) + 9c_1 - 2}{9x}$$

✓ Solution by Mathematica

Time used: 0.018 (sec). Leaf size: 31

```
DSolve[2*x^2*y''[x]+3*x*y'[x]-y[x]==1/x,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to \frac{9c_2x^{3/2} - 3\log(x) - 2 + 9c_1}{9x}$$

10 Chapter 8. Special second order equations. Lesson 35. Independent variable x absent

10.1 problem Exercise 35.1, page 504

Internal problem ID [4651]

Book: Ordinary Differential Equations, By Tenenbaum and Pollard. Dover, NY 1963
Section: Chapter 8. Special second order equations. Lesson 35. Independent variable x absent
Problem number: Exercise 35.1, page 504.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _missing_x], [_2nd_order, _exact, _nonlinear], _`

$$\boxed{y'' - 2y'y = 0}$$

✓ Solution by Maple
Time used: 0.266 (sec). Leaf size: 16

```
dsolve(diff(y(x),x$2)=2*y(x)*diff(y(x),x),y(x), singsol=all)
```

$$y(x) = \frac{\tan\left(\frac{c_2+x}{c_1}\right)}{c_1}$$

✓ Solution by Mathematica
Time used: 9.872 (sec). Leaf size: 24

```
DSolve[y''[x]==2*y[x]*y'[x],y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to \sqrt{c_1} \tan\left(\sqrt{c_1}(x + c_2)\right)$$

10.2 problem Exercise 35.2, page 504

Internal problem ID [4652]

Book: Ordinary Differential Equations, By Tenenbaum and Pollard. Dover, NY 1963
Section: Chapter 8. Special second order equations. Lesson 35. Independent variable x absent
Problem number: Exercise 35.2, page 504.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _missing_x], [_2nd_order, _reducible, _mu_x_y1]]`

$$\boxed{y^3 y'' = k}$$

✓ Solution by Maple
Time used: 0.016 (sec). Leaf size: 46

```
dsolve(y(x)^3*diff(y(x),x$2)=k,y(x), singsol=all)
```

$$y(x) = \frac{\sqrt{\left((c_2 + x)^2 c_1^2 + k\right) c_1}}{c_1}$$
$$y(x) = -\frac{\sqrt{\left((c_2 + x)^2 c_1^2 + k\right) c_1}}{c_1}$$

✓ Solution by Mathematica
Time used: 2.878 (sec). Leaf size: 63

```
DSolve[y[x]^3*y''[x]==k,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to -\frac{\sqrt{k + c_1{}^2(x + c_2)^2}}{\sqrt{c_1}}$$
$$y(x) \to \frac{\sqrt{k + c_1{}^2(x + c_2)^2}}{\sqrt{c_1}}$$
$$y(x) \to \text{Indeterminate}$$

10.3 problem Exercise 35.3, page 504

Internal problem ID [4653]

Book: Ordinary Differential Equations, By Tenenbaum and Pollard. Dover, NY 1963
Section: Chapter 8. Special second order equations. Lesson 35. Independent variable x absent
Problem number: Exercise 35.3, page 504.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _missing_x], [_2nd_order, _reducible, _mu_x_y1]]`

$$yy'' - y'^2 = -1$$

✓ Solution by Maple

Time used: 0.016 (sec). Leaf size: 59

```
dsolve(y(x)*diff(y(x),x$2)=(diff(y(x),x))^2-1,y(x), singsol=all)
```

$$y(x) = \frac{c_1\left(-\mathrm{e}^{\frac{c_2+x}{c_1}} + \mathrm{e}^{\frac{-c_2-x}{c_1}}\right)}{2}$$
$$y(x) = -\frac{c_1\left(-\mathrm{e}^{\frac{c_2+x}{c_1}} + \mathrm{e}^{\frac{-c_2-x}{c_1}}\right)}{2}$$

✓ Solution by Mathematica

Time used: 60.201 (sec). Leaf size: 85

```
DSolve[y[x]*y''[x]==(y'[x])^2-1,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to -\frac{ie^{-c_1}\tanh\left(e^{c_1}(x+c_2)\right)}{\sqrt{-\mathrm{sech}^2\left(e^{c_1}(x+c_2)\right)}}$$
$$y(x) \to \frac{ie^{-c_1}\tanh\left(e^{c_1}(x+c_2)\right)}{\sqrt{-\mathrm{sech}^2\left(e^{c_1}(x+c_2)\right)}}$$

10.4 problem Exercise 35.4, page 504

Internal problem ID [4654]

Book: Ordinary Differential Equations, By Tenenbaum and Pollard. Dover, NY 1963
Section: Chapter 8. Special second order equations. Lesson 35. Independent variable x absent
Problem number: Exercise 35.4, page 504.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _missing_y]]`

$$\boxed{x^2 y'' + x y' = 1}$$

✓ Solution by Maple
Time used: 0.0 (sec). Leaf size: 16

```
dsolve(x^2*diff(y(x),x$2)+x*(diff(y(x),x))=1,y(x), singsol=all)
```

$$y(x) = c_2 + c_1 \ln(x) + \frac{\ln(x)^2}{2}$$

✓ Solution by Mathematica
Time used: 0.012 (sec). Leaf size: 21

```
DSolve[x^2*y''[x]+x*y'[x]==1,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to \frac{\log^2(x)}{2} + c_1 \log(x) + c_2$$

10.5 problem Exercise 35.5, page 504

Internal problem ID [4655]

Book: Ordinary Differential Equations, By Tenenbaum and Pollard. Dover, NY 1963
Section: Chapter 8. Special second order equations. Lesson 35. Independent variable x absent
Problem number: Exercise 35.5, page 504.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _missing_y]]`

$$\boxed{xy'' - y' = x^2}$$

✓ Solution by Maple
Time used: 0.0 (sec). Leaf size: 17

```
dsolve(x*diff(y(x),x$2)-diff(y(x),x)=x^2,y(x), singsol=all)
```

$$y(x) = \frac{1}{3}x^3 + \frac{1}{2}c_1x^2 + c_2$$

✓ Solution by Mathematica
Time used: 0.027 (sec). Leaf size: 24

```
DSolve[x*y''[x]-y'[x]==x^2,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to \frac{x^3}{3} + \frac{c_1x^2}{2} + c_2$$

10.6 problem Exercise 35.6, page 504

Internal problem ID [4656]

Book: Ordinary Differential Equations, By Tenenbaum and Pollard. Dover, NY 1963
Section: Chapter 8. Special second order equations. Lesson 35. Independent variable x absent
Problem number: Exercise 35.6, page 504.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _missing_x], _Liouville, [_2nd_order, _reducible`

$$(1+y)\,y'' - 3y'^2 = 0$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 59

```
dsolve((y(x)+1)*diff(y(x),x$2)=3*(diff(y(x),x))^2,y(x), singsol=all)
```

$$y(x) = -1$$
$$y(x) = -\frac{\sqrt{-2c_1x - 2c_2} - 1}{\sqrt{-2c_1x - 2c_2}}$$
$$y(x) = -\frac{\sqrt{-2c_1x - 2c_2} + 1}{\sqrt{-2c_1x - 2c_2}}$$

✓ Solution by Mathematica

Time used: 1.485 (sec). Leaf size: 107

```
DSolve[(y[x]+1)*y''[x]==3*(y'[x])^2,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to -\frac{2c_1x + \sqrt{2}\sqrt{-c_1(x+c_2)} + 2c_2c_1}{2c_1(x+c_2)}$$
$$y(x) \to \frac{-2c_1x + \sqrt{2}\sqrt{-c_1(x+c_2)} - 2c_2c_1}{2c_1(x+c_2)}$$
$$y(x) \to -1$$
$$y(x) \to \text{Indeterminate}$$

10.7 problem Exercise 35.7, page 504

Internal problem ID [4657]

Book: Ordinary Differential Equations, By Tenenbaum and Pollard. Dover, NY 1963
Section: Chapter 8. Special second order equations. Lesson 35. Independent variable x absent
Problem number: Exercise 35.7, page 504.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _missing_x], [_2nd_order, _reducible, _mu_x_y1]]`

$$\boxed{r'' + \frac{k}{r^2} = 0}$$

✓ Solution by Maple

Time used: 0.062 (sec). Leaf size: 369

```
dsolve(diff(r(t),t$2)=-k/(r(t)^2),r(t), singsol=all)
```

$$r(t) = \frac{c_1\left(c_1^2k^2 - 2kc_1 e^{\mathrm{RootOf}\left(\mathrm{csgn}\left(\frac{1}{c_1}\right)c_1^4k^2+2\_Zc_1^3k\,e^{\_Z}-\mathrm{csgn}\left(\frac{1}{c_1}\right)e^{2\_Z}c_1^2-2\,\mathrm{csgn}\left(\frac{1}{c_1}\right)e^{\_Z}c_2-2\,\mathrm{csgn}\left(\frac{1}{c_1}\right)e^{\_Z}t\right)} + e^{2\,\mathrm{RootOf}\left(\mathrm{csgn}\left(\right.\right.}\right.}{}$$

$$r(t) = \frac{c_1\left(c_1^2k^2 - 2kc_1 e^{\mathrm{RootOf}\left(\mathrm{csgn}\left(\frac{1}{c_1}\right)c_1^4k^2+2\_Zc_1^3k\,e^{\_Z}-\mathrm{csgn}\left(\frac{1}{c_1}\right)e^{2\_Z}c_1^2+2\,\mathrm{csgn}\left(\frac{1}{c_1}\right)e^{\_Z}c_2+2\,\mathrm{csgn}\left(\frac{1}{c_1}\right)e^{\_Z}t\right)} + e^{2\,\mathrm{RootOf}\left(\mathrm{csgn}\left(\right.\right.}\right.}{}$$

✓ Solution by Mathematica

Time used: 0.169 (sec). Leaf size: 65

```
DSolve[r''[t]==-k/(r[t]^2),r[t],t,IncludeSingularSolutions -> True]
```

$$\mathrm{Solve}\left[\left(\frac{r(t)\sqrt{\frac{2k}{r(t)}+c_1}}{c_1} - \frac{2k\,\mathrm{arctanh}\left(\frac{\sqrt{\frac{2k}{r(t)}+c_1}}{\sqrt{c_1}}\right)}{c_1{}^{3/2}}\right)^2 = (t+c_2)^2, r(t)\right]$$

10.8 problem Exercise 35.8, page 504

Internal problem ID [4658]

Book: Ordinary Differential Equations, By Tenenbaum and Pollard. Dover, NY 1963
Section: Chapter 8. Special second order equations. Lesson 35. Independent variable x absent
Problem number: Exercise 35.8, page 504.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _missing_x], [_2nd_order, _reducible, _mu_x_y1]]`

$$\boxed{y'' - \frac{3ky^2}{2} = 0}$$

✓ Solution by Maple
Time used: 0.015 (sec). Leaf size: 15

```
dsolve(diff(y(x),x$2)=3/2*k*y(x)^2,y(x), singsol=all)
```

$$y(x) = \frac{4\,\mathrm{WeierstrassP}\left(x + c_1, 0, c_2\right)}{k}$$

✗ Solution by Mathematica
Time used: 0.0 (sec). Leaf size: 0

```
DSolve[y''[x]==3/2*(k*y[x]^2),y[x],x,IncludeSingularSolutions -> True]
```

Not solved

10.9 problem Exercise 35.9, page 504

Internal problem ID [4659]

Book: Ordinary Differential Equations, By Tenenbaum and Pollard. Dover, NY 1963
Section: Chapter 8. Special second order equations. Lesson 35. Independent variable x absent
Problem number: Exercise 35.9, page 504.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _missing_x], [_2nd_order, _reducible, _mu_x_y1]]`

$$\boxed{y'' - 2k y^3 = 0}$$

✓ Solution by Maple
Time used: 0.0 (sec). Leaf size: 20

```
dsolve(diff(y(x),x$2)=2*k*y(x)^3,y(x), singsol=all)
```

$$y(x) = c_2 \,\mathrm{JacobiSN}\left(\left(\sqrt{-k}\, x + c_1\right) c_2, i\right)$$

✓ Solution by Mathematica
Time used: 61.304 (sec). Leaf size: 115

```
DSolve[y''[x]==2*k*y[x]^3,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to -\frac{i \operatorname{sn}\left((-1)^{3/4}\sqrt{\sqrt{k}\sqrt{c_1}(x+c_2)^2}\middle| -1\right)}{\sqrt{\frac{i\sqrt{k}}{\sqrt{c_1}}}}$$

$$y(x) \to \frac{i \operatorname{sn}\left((-1)^{3/4}\sqrt{\sqrt{k}\sqrt{c_1}(x+c_2)^2}\middle| -1\right)}{\sqrt{\frac{i\sqrt{k}}{\sqrt{c_1}}}}$$

10.10 problem Exercise 35.10, page 504

Internal problem ID [4660]

Book: Ordinary Differential Equations, By Tenenbaum and Pollard. Dover, NY 1963
Section: Chapter 8. Special second order equations. Lesson 35. Independent variable x absent
Problem number: Exercise 35.10, page 504.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _missing_x], [_2nd_order, _exact, _nonlinear], [`

$$yy'' + y'^2 - y' = 0$$

✓ Solution by Maple

Time used: 0.031 (sec). Leaf size: 34

```
dsolve(y(x)*diff(y(x),x$2)+(diff(y(x),x))^2-diff(y(x),x)=0,y(x), singsol=all)
```

$$y(x) = 0$$
$$y(x) = -c_1 \left(\text{LambertW}\left(-\frac{e^{\frac{-c_1-c_2-x}{c_1}}}{c_1}\right) + 1\right)$$

✓ Solution by Mathematica

Time used: 60.084 (sec). Leaf size: 32

```
DSolve[y[x]*y''[x]+(y'[x])^2-y'[x]==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to -c_1 \left(1 + W\left(-\frac{e^{-\frac{x+c_1+c_2}{c_1}}}{c_1}\right)\right)$$

10.11 problem Exercise 35.11, page 504

Internal problem ID [4661]

Book: Ordinary Differential Equations, By Tenenbaum and Pollard. Dover, NY 1963
Section: Chapter 8. Special second order equations. Lesson 35. Independent variable x absent
Problem number: Exercise 35.11, page 504.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _missing_x], [_2nd_order, _reducible, _mu_x_y1]]`

$$\boxed{r'' - \frac{h^2}{r^3} + \frac{k}{r^2} = 0}$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 441

```
dsolve(diff(r(t),t$2)= h^2/r(t)^3-k/r(t)^2,r(t), singsol=all)
```

$$r(t) = \frac{c_1\left(c_1^2k^2 - 2kc_1 e^{\mathrm{RootOf}\left(\mathrm{csgn}\left(\frac{1}{c_1}\right)c_1^4k^2+2\_Z c_1^3 k\,e^{\_Z}-\mathrm{csgn}\left(\frac{1}{c_1}\right)e^{2\_Z}c_1^2+\mathrm{csgn}\left(\frac{1}{c_1}\right)c_1^2h^2-2\,\mathrm{csgn}\left(\frac{1}{c_1}\right)e^{\_Z}c_2-2\,\mathrm{csgn}\left(\frac{1}{c_1}\right)e^{\_Z}t\right)} + e^{2}\right.}{}$$

$$r(t) = \frac{c_1\left(c_1^2k^2 - 2kc_1 e^{\mathrm{RootOf}\left(\mathrm{csgn}\left(\frac{1}{c_1}\right)c_1^4k^2+2\_Z c_1^3 k\,e^{\_Z}-\mathrm{csgn}\left(\frac{1}{c_1}\right)e^{2\_Z}c_1^2+\mathrm{csgn}\left(\frac{1}{c_1}\right)c_1^2h^2+2\,\mathrm{csgn}\left(\frac{1}{c_1}\right)e^{\_Z}c_2+2\,\mathrm{csgn}\left(\frac{1}{c_1}\right)e^{\_Z}t\right)} + e^{2}\right.}{}$$

✓ <u>Solution by Mathematica</u>

Time used: 1.099 (sec). Leaf size: 130

```
DSolve[r''[t]==h^2/r[t]^3-k/r[t]^2,r[t],t,IncludeSingularSolutions -> True]
```

$$\text{Solve}\left[\frac{\left(\sqrt{c_1}(-h^2+r(t)(2k+c_1r(t)))-k\sqrt{-h^2+r(t)(2k+c_1r(t))}\text{arctanh}\left(\frac{k+c_1r(t)}{\sqrt{c_1}\sqrt{-h^2+r(t)(2k+c_1r(t))}}\right)\right)^2}{c_1{}^3r(t)^2\left(-\frac{h^2}{r(t)^2}+\frac{2k}{r(t)}+c_1\right)}=\right.$$

$$\left.+c_2)^2,r(t)\right]$$

10.12 problem Exercise 35.12, page 504

Internal problem ID [4662]

Book: Ordinary Differential Equations, By Tenenbaum and Pollard. Dover, NY 1963
Section: Chapter 8. Special second order equations. Lesson 35. Independent variable x absent
Problem number: Exercise 35.12, page 504.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _missing_x], [_2nd_order, _reducible, _mu_x_y1],`

$$\boxed{yy'' + y'^3 - y'^2 = 0}$$

✓ Solution by Maple

Time used: 0.015 (sec). Leaf size: 36

```
dsolve(y(x)*diff(y(x),x$2)+(diff(y(x),x))^3-diff(y(x),x)^2=0,y(x), singsol=all)
```

$$y(x) = 0$$
$$y(x) = c_1$$
$$y(x) = \mathrm{e}^{\frac{-c_1 \operatorname{LambertW}\left(\frac{\mathrm{e}^{\frac{c_2+x}{c_1}}}{c_1}\right)+c_2+x}{c_1}}$$

✓ Solution by Mathematica

Time used: 22.229 (sec). Leaf size: 32

```
DSolve[y[x]*y''[x]+(y'[x])^3-(y'[x])^2==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to e^{c_1} W\left(e^{e^{-c_1}(x - e^{c_1} c_1 + c_2)}\right)$$

10.13 problem Exercise 35.13, page 504

Internal problem ID [4663]

Book: Ordinary Differential Equations, By Tenenbaum and Pollard. Dover, NY 1963
Section: Chapter 8. Special second order equations. Lesson 35. Independent variable x absent
Problem number: Exercise 35.13, page 504.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _missing_x], _Liouville, [_2nd_order, _reducible`

$$\boxed{yy'' - 3y'^2 = 0}$$

✓ Solution by Maple
Time used: 0.016 (sec). Leaf size: 33

```
dsolve(y(x)*diff(y(x),x$2)-3*(diff(y(x),x))^2=0,y(x), singsol=all)
```

$$y(x) = 0$$
$$y(x) = \frac{1}{\sqrt{-2c_1x - 2c_2}}$$
$$y(x) = -\frac{1}{\sqrt{-2c_1x - 2c_2}}$$

✓ Solution by Mathematica
Time used: 0.106 (sec). Leaf size: 14

```
DSolve[y[x]*y''[x]-(y'[x])^2==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to c_2e^{c_1x}$$

10.14 problem Exercise 35.14, page 504

Internal problem ID [4664]

Book: Ordinary Differential Equations, By Tenenbaum and Pollard. Dover, NY 1963
Section: Chapter 8. Special second order equations. Lesson 35. Independent variable x absent
Problem number: Exercise 35.14, page 504.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _missing_y], [_2nd_order, _reducible, _mu_y_y1]]`

$$\boxed{(x^2+1)\, y'' + y'^2 = -1}$$

✓ Solution by Maple
Time used: 0.016 (sec). Leaf size: 33

```
dsolve((1+x^2)*diff(y(x),x$2)+(diff(y(x),x))^2+1=0,y(x), singsol=all)
```

$$y(x) = \frac{\ln(c_1x-1)\, c_1^2 + c_2c_1^2 + c_1x + \ln(c_1x-1)}{c_1^2}$$

✓ Solution by Mathematica
Time used: 7.091 (sec). Leaf size: 33

```
DSolve[(1+x^2)*y''[x]+(y'[x])^2+1==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to -x\cot(c_1) + \csc^2(c_1)\log(-x\sin(c_1) - \cos(c_1)) + c_2$$

10.15 problem Exercise 35.15, page 504

Internal problem ID [4665]

Book: Ordinary Differential Equations, By Tenenbaum and Pollard. Dover, NY 1963
Section: Chapter 8. Special second order equations. Lesson 35. Independent variable x absent
Problem number: Exercise 35.15, page 504.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _missing_y]]`

$$\boxed{(x^2+1)\,y''+2x(1+y')=0}$$

✓ Solution by Maple
Time used: 0.0 (sec). Leaf size: 15

```
dsolve((1+x^2)*diff(y(x),x$2)+2*x*(diff(y(x),x)+1)=0,y(x), singsol=all)
```

$$y(x)=-x+(1+c_1)\arctan(x)+c_2$$

✓ Solution by Mathematica
Time used: 0.039 (sec). Leaf size: 18

```
DSolve[(1+x^2)*y''[x]+2*x*(y'[x]+1)==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x)\to(1+c_1)\arctan(x)-x+c_2$$

10.16 problem Exercise 35.16, page 504

Internal problem ID [4666]

Book: Ordinary Differential Equations, By Tenenbaum and Pollard. Dover, NY 1963
Section: Chapter 8. Special second order equations. Lesson 35. Independent variable x absent
Problem number: Exercise 35.16, page 504.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _missing_x], _Liouville, [_2nd_order, _reducible`

$$\boxed{(1+y)\,y'' - 3y'^2 = 0}$$

With initial conditions

$$\left[y(1) = 0, y'(1) = -\frac{1}{2}\right]$$

✓ Solution by Maple

Time used: 0.344 (sec). Leaf size: 15

```
dsolve([(y(x)+1)*diff(y(x),x$2)=3*(diff(y(x),x))^2,y(1) = 0, D(y)(1) = -1/2],y(x), singsol=a
```

$$y(x) = \frac{-x + \sqrt{x}}{x}$$

✓ Solution by Mathematica

Time used: 1.693 (sec). Leaf size: 572

```
DSolve[{(y[x]+1)*y''[x]==3*(y'[x])^2,{y[1]==0,y'[0]==-1/2}},y[x],x,IncludeSingularSolutions
```

$$y(x) \to \frac{6\left(\left(-12 + 3\ 2^{2/3}\sqrt[3]{27-3\sqrt{69}} - \sqrt[3]{2}\left(27-3\sqrt{69}\right)^{2/3} + 3\ 2^{2/3}\sqrt[3]{3\left(9+\sqrt{69}\right)} - \sqrt[3]{2}\left(3\left(9+\sqrt{69}\right)\right)^{2/3}\right)\right.}{}$$

10.17 problem Exercise 35.17, page 504

Internal problem ID [4667]

Book: Ordinary Differential Equations, By Tenenbaum and Pollard. Dover, NY 1963
Section: Chapter 8. Special second order equations. Lesson 35. Independent variable x absent
Problem number: Exercise 35.17, page 504.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _missing_x], [_2nd_order, _exact, _nonlinear], [`

$$\boxed{y'' - y' e^{y} = 0}$$

With initial conditions

$$[y(3) = 0, y'(3) = 1]$$

✓ Solution by Maple
Time used: 0.063 (sec). Leaf size: 12

```
dsolve([diff(y(x),x$2)=diff(y(x),x)*exp(y(x)),y(3) = 0, D(y)(3) = 1],y(x), singsol=all)
```

$$y(x) = -\ln(-x + 4)$$

✓ Solution by Mathematica
Time used: 7.673 (sec). Leaf size: 13

```
DSolve[{y''[x]==y'[x]*Exp[y[x]],{y[3]==0,y'[3]==1}},y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to -\log(4 - x)$$

10.18 problem Exercise 35.18, page 504

Internal problem ID [4668]

Book: Ordinary Differential Equations, By Tenenbaum and Pollard. Dover, NY 1963
Section: Chapter 8. Special second order equations. Lesson 35. Independent variable x absent
Problem number: Exercise 35.18, page 504.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _missing_x], [_2nd_order, _exact, _nonlinear], _`

$$\boxed{y'' - 2y'y = 0}$$

With initial conditions

$$[y(0) = 1, y'(0) = 2]$$

✓ Solution by Maple
Time used: 0.062 (sec). Leaf size: 10

```
dsolve([diff(y(x),x$2)=2*y(x)*diff(y(x),x),y(0) = 1, D(y)(0) = 2],y(x), singsol=all)
```

$$y(x) = \tan\left(x + \frac{\pi}{4}\right)$$

✗ Solution by Mathematica
Time used: 0.0 (sec). Leaf size: 0

```
DSolve[{y''[x]==2*y[x]*y'[x],{y[0]==1,y'[0]==2}},y[x],x,IncludeSingularSolutions -> True]
```

{}

10.19 problem Exercise 35.19, page 504

Internal problem ID [4669]

Book: Ordinary Differential Equations, By Tenenbaum and Pollard. Dover, NY 1963
Section: Chapter 8. Special second order equations. Lesson 35. Independent variable x absent
Problem number: Exercise 35.19, page 504.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _missing_x], [_2nd_order, _reducible, _mu_x_y1]]`

$$\boxed{2y'' - e^{y} = 0}$$

With initial conditions

$$[y(0) = 0, y'(0) = 1]$$

✓ Solution by Maple
Time used: 0.063 (sec). Leaf size: 15

```
dsolve([2*diff(y(x),x$2)=exp(y(x)),y(0) = 0, D(y)(0) = 1],y(x), singsol=all)
```

$$y(x) = 2\ln(2) + \ln\left(\frac{1}{(x-2)^2}\right)$$

✓ Solution by Mathematica
Time used: 0.048 (sec). Leaf size: 15

```
DSolve[{2*y''[x]==Exp[y[x]],{y[0]==0,y'[0]==1}},y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to -2\log\left(1 - \frac{x}{2}\right)$$

10.20 problem Exercise 35.20, page 504

Internal problem ID [4670]

Book: Ordinary Differential Equations, By Tenenbaum and Pollard. Dover, NY 1963
Section: Chapter 8. Special second order equations. Lesson 35. Independent variable x absent
Problem number: Exercise 35.20, page 504.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _missing_y]]`

$$\boxed{x^2 y'' + x y' = 1}$$

With initial conditions

$$[y(1) = 1, y'(1) = 2]$$

✓ Solution by Maple
Time used: 0.016 (sec). Leaf size: 16

```
dsolve([x^2*diff(y(x),x$2)+x*diff(y(x),x)=1,y(1) = 1, D(y)(1) = 2],y(x), singsol=all)
```

$$y(x) = 1 + 2\ln(x) + \frac{\ln(x)^2}{2}$$

✓ Solution by Mathematica
Time used: 0.013 (sec). Leaf size: 19

```
DSolve[{x^2*y''[x]+x*y'[x]==1,{y[1]==1,y'[1]==2}},y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to \frac{1}{2}\left(\log^2(x) + 4\log(x) + 2\right)$$

10.21 problem Exercise 35.21, page 504

Internal problem ID [4671]

Book: Ordinary Differential Equations, By Tenenbaum and Pollard. Dover, NY 1963
Section: Chapter 8. Special second order equations. Lesson 35. Independent variable x absent
Problem number: Exercise 35.21, page 504.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _missing_y]]`

$$\boxed{xy'' - y' = x^2}$$

With initial conditions

$$[y(1) = 0, y'(1) = -1]$$

✓ Solution by Maple
Time used: 0.0 (sec). Leaf size: 16

```
dsolve([x*diff(y(x),x$2)-diff(y(x),x)=x^2,y(1) = 0, D(y)(1) = -1],y(x), singsol=all)
```

$$y(x) = \frac{1}{3}x^3 - x^2 + \frac{2}{3}$$

✓ Solution by Mathematica
Time used: 0.028 (sec). Leaf size: 19

```
DSolve[{x*y''[x]-y'[x]==x^2,{y[1]==0,y'[1]==-1}},y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to \frac{1}{3}(x^3 - 3x^2 + 2)$$

10.22 problem Exercise 35.23(a), page 504

Internal problem ID [4672]

Book: Ordinary Differential Equations, By Tenenbaum and Pollard. Dover, NY 1963
Section: Chapter 8. Special second order equations. Lesson 35. Independent variable x absent
Problem number: Exercise 35.23(a), page 504.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[_Liouville, [_2nd_order, _with_linear_symmetries], [_2nd_orde`

$$xyy'' - 2xy'^2 + y'y = 0$$

✓ Solution by Maple

Time used: 0.016 (sec). Leaf size: 18

```
dsolve(x*y(x)*diff(y(x),x$2)-2*x*(diff(y(x),x))^2+y(x)*diff(y(x),x)=0,y(x), singsol=all)
```

$$y(x) = 0$$
$$y(x) = -\frac{1}{c_1 \ln(x) + c_2}$$

✓ Solution by Mathematica

Time used: 0.243 (sec). Leaf size: 22

```
DSolve[x*y[x]*y''[x]-2*x*(y'[x])^2+y[x]*y'[x]==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to \frac{c_2}{-\log(x) + c_1}$$
$$y(x) \to 0$$

10.23 problem Exercise 35.23(b), page 504

Internal problem ID [4673]

Book: Ordinary Differential Equations, By Tenenbaum and Pollard. Dover, NY 1963
Section: Chapter 8. Special second order equations. Lesson 35. Independent variable x absent
Problem number: Exercise 35.23(b), page 504.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _exact, _nonlinear], _Liouville, [_2nd_order, _w`

$$x y y'' + x y'^2 - y' y = 0$$

✓ Solution by Maple

Time used: 0.016 (sec). Leaf size: 35

```
dsolve(x*y(x)*diff(y(x),x$2)+x*(diff(y(x),x))^2-y(x)*diff(y(x),x)=0,y(x), singsol=all)
```

$$y(x) = 0$$
$$y(x) = \sqrt{c_1 x^2 + 2c_2}$$
$$y(x) = -\sqrt{c_1 x^2 + 2c_2}$$

✓ Solution by Mathematica

Time used: 0.241 (sec). Leaf size: 18

```
DSolve[x*y[x]*y''[x]+x*(y'[x])^2-y[x]*y'[x]==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to c_2 \sqrt{x^2 + c_1}$$

10.24 problem Exercise 35.23(c), page 504

Internal problem ID [4674]

Book: Ordinary Differential Equations, By Tenenbaum and Pollard. Dover, NY 1963
Section: Chapter 8. Special second order equations. Lesson 35. Independent variable x absent
Problem number: Exercise 35.23(c), page 504.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries], [_2nd_order, _reducibl`

$$xyy'' - 2xy'^2 + (1 + y)\, y' = 0$$

✓ Solution by Maple
Time used: 0.062 (sec). Leaf size: 22

```
dsolve(x*y(x)*diff(y(x),x$2)-2*x*(diff(y(x),x))^2+(1+y(x))*diff(y(x),x)=0,y(x), singsol=all)
```

$$y(x) = 0$$
$$y(x) = c_1 \tanh\left(\frac{\ln(x) - c_2}{2c_1}\right)$$

✓ Solution by Mathematica
Time used: 20.549 (sec). Leaf size: 52

```
DSolve[x*y[x]*y''[x]-2*x*(y'[x])^2+(1+y[x])*y'[x]==0,y[x],x,IncludeSingularSolutions -> True
```

$$y(x) \to \frac{\tan\left(\frac{\sqrt{c_1}(\log(x)-c_2)}{\sqrt{2}}\right)}{\sqrt{2}\sqrt{c_1}}$$
$$y(x) \to \frac{1}{2}(\log(x) - c_2)$$