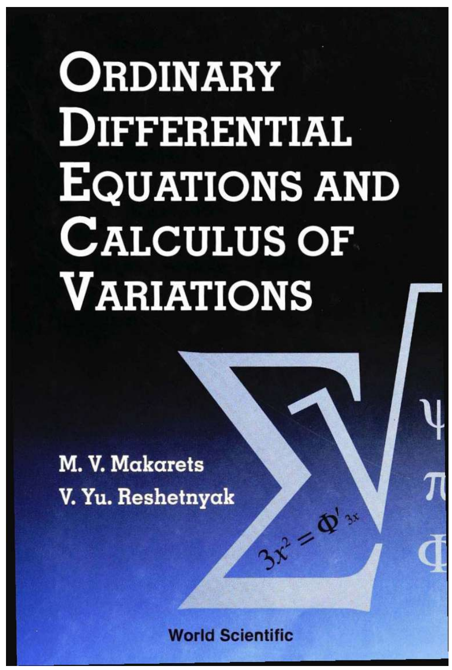
A Solution Manual For

Ordinary differential equations and calculus of variations. Makarets and Reshetnyak. Wold Scientific. Singapore. 1995



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1.1 problem 1

Internal problem ID [5714]

Book: Ordinary differential equations and calculus of variations. Makarets and Reshetnyak. Wold Scientific. Singapore. 1995
Section: Chapter 1. First order differential equations. Section 1.1 Separable equations problems. page 7
Problem number: 1.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_separable]`

$$\boxed{y' - \frac{x^2}{y} = 0}$$

✓ Solution by Maple
Time used: 0.0 (sec). Leaf size: 33

```
dsolve(diff(y(x),x)=x^2/y(x),y(x), singsol=all)
```

$$y(x) = -\frac{\sqrt{6x^3 + 9c_1}}{3}$$
$$y(x) = \frac{\sqrt{6x^3 + 9c_1}}{3}$$

✓ Solution by Mathematica
Time used: 0.084 (sec). Leaf size: 50

```
DSolve[y'[x]==x^2/y[x],y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to -\sqrt{\frac{2}{3}}\sqrt{x^3 + 3c_1}$$
$$y(x) \to \sqrt{\frac{2}{3}}\sqrt{x^3 + 3c_1}$$

1.2 problem 2

Internal problem ID [5715]

Book: Ordinary differential equations and calculus of variations. Makarets and Reshetnyak. Wold Scientific. Singapore. 1995
Section: Chapter 1. First order differential equations. Section 1.1 Separable equations problems. page 7
Problem number: 2.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_separable]`

$$\boxed{y' - \frac{x^2}{y(x^3+1)} = 0}$$

✓ Solution by Maple
Time used: 0.0 (sec). Leaf size: 39

```
dsolve(diff(y(x),x)=x^2/(y(x)*(1+x^3)),y(x), singsol=all)
```

$$y(x) = -\frac{\sqrt{6\ln(x^3+1)+9c_1}}{3}$$
$$y(x) = \frac{\sqrt{6\ln(x^3+1)+9c_1}}{3}$$

✓ Solution by Mathematica
Time used: 0.091 (sec). Leaf size: 56

```
DSolve[y'[x]==x^2/(y[x]*(1+x^3)),y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to -\sqrt{\frac{2}{3}}\sqrt{\log(x^3+1)+3c_1}$$
$$y(x) \to \sqrt{\frac{2}{3}}\sqrt{\log(x^3+1)+3c_1}$$

1.3 problem 3

Internal problem ID [5716]

Book: Ordinary differential equations and calculus of variations. Makarets and Reshetnyak. Wold Scientific. Singapore. 1995
Section: Chapter 1. First order differential equations. Section 1.1 Separable equations problems. page 7
Problem number: 3.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_separable]`

$$\boxed{y' - \sin(x)\, y = 0}$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 11

```
dsolve(diff(y(x),x)=y(x)*sin(x),y(x), singsol=all)
```

$$y(x) = c_1 \mathrm{e}^{-\cos(x)}$$

✓ Solution by Mathematica

Time used: 0.03 (sec). Leaf size: 19

```
DSolve[y'[x]==y[x]*Sin[x],y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to c_1 e^{-\cos(x)}$$
$$y(x) \to 0$$

1.4 problem 4

Internal problem ID [5717]

Book: Ordinary differential equations and calculus of variations. Makarets and Reshetnyak. Wold Scientific. Singapore. 1995
Section: Chapter 1. First order differential equations. Section 1.1 Separable equations problems. page 7
Problem number: 4.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_separable]`

$$\boxed{xy' - \sqrt{1 - y^2} = 0}$$

✓ Solution by Maple
Time used: 0.016 (sec). Leaf size: 9

```
dsolve(x*diff(y(x),x)=sqrt(1-y(x)^2),y(x), singsol=all)
```

$$y(x) = \sin(\ln(x) + c_1)$$

✓ Solution by Mathematica
Time used: 0.219 (sec). Leaf size: 29

```
DSolve[x*y'[x]==Sqrt[1-y[x]^2],y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to \cos(\log(x) + c_1)$$
$$y(x) \to -1$$
$$y(x) \to 1$$
$$y(x) \to \text{Interval}[\{-1, 1\}]$$

1.5 problem 5

Internal problem ID [5718]

Book: Ordinary differential equations and calculus of variations. Makarets and Reshetnyak. Wold Scientific. Singapore. 1995
Section: Chapter 1. First order differential equations. Section 1.1 Separable equations problems. page 7
Problem number: 5.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_separable]`

$$\boxed{y' - \frac{x^2}{1+y^2} = 0}$$

✓ Solution by Maple
Time used: 0.015 (sec). Leaf size: 268

```
dsolve(diff(y(x),x)=x^2/(1+y(x)^2),y(x), singsol=all)
```

$$y(x) = \frac{\left(4x^3 + 12c_1 + 4\sqrt{x^6 + 6c_1x^3 + 9c_1^2 + 4}\right)^{\frac{2}{3}} - 4}{2\left(4x^3 + 12c_1 + 4\sqrt{x^6 + 6c_1x^3 + 9c_1^2 + 4}\right)^{\frac{1}{3}}}$$

$$y(x) = -\frac{(1 + i\sqrt{3})\left(4x^3 + 12c_1 + 4\sqrt{x^6 + 6c_1x^3 + 9c_1^2 + 4}\right)^{\frac{2}{3}} + 4i\sqrt{3} - 4}{4\left(4x^3 + 12c_1 + 4\sqrt{x^6 + 6c_1x^3 + 9c_1^2 + 4}\right)^{\frac{1}{3}}}$$

$$y(x) = \frac{i\left(4x^3 + 12c_1 + 4\sqrt{x^6 + 6c_1x^3 + 9c_1^2 + 4}\right)^{\frac{2}{3}}\sqrt{3} + 4i\sqrt{3} - \left(4x^3 + 12c_1 + 4\sqrt{x^6 + 6c_1x^3 + 9c_1^2 + 4}\right)^{\frac{2}{3}} + 4}{4\left(4x^3 + 12c_1 + 4\sqrt{x^6 + 6c_1x^3 + 9c_1^2 + 4}\right)^{\frac{1}{3}}}$$

✓ Solution by Mathematica

Time used: 2.179 (sec). Leaf size: 307

```
DSolve[y'[x]==x^2/(1+y[x]^2),y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to \frac{-2+\sqrt[3]{2}\left(x^3+\sqrt{x^6+6c_1x^3+4+9{c_1}^2}+3c_1\right)^{2/3}}{2^{2/3}\sqrt[3]{x^3+\sqrt{x^6+6c_1x^3+4+9{c_1}^2}+3c_1}}$$

$$y(x) \to \frac{i(\sqrt{3}+i)\sqrt[3]{x^3+\sqrt{x^6+6c_1x^3+4+9{c_1}^2}+3c_1}}{2\sqrt[3]{2}} + \frac{1+i\sqrt{3}}{2^{2/3}\sqrt[3]{x^3+\sqrt{x^6+6c_1x^3+4+9{c_1}^2}+3c_1}}$$

$$y(x) \to \frac{1-i\sqrt{3}}{2^{2/3}\sqrt[3]{x^3+\sqrt{x^6+6c_1x^3+4+9{c_1}^2}+3c_1}} - \frac{(1+i\sqrt{3})\sqrt[3]{x^3+\sqrt{x^6+6c_1x^3+4+9{c_1}^2}+3c_1}}{2\sqrt[3]{2}}$$

1.6 problem 6

Internal problem ID [5719]

Book: Ordinary differential equations and calculus of variations. Makarets and Reshetnyak. Wold Scientific. Singapore. 1995

Section: Chapter 1. First order differential equations. Section 1.1 Separable equations problems. page 7

Problem number: 6.

ODE order: 1.

ODE degree: 1.

CAS Maple gives this as type `[_separable]`

$$\boxed{xyy' - \sqrt{1+y^2} = 0}$$

✓ Solution by Maple

Time used: 0.015 (sec). Leaf size: 17

```
dsolve(x*y(x)*diff(y(x),x)=sqrt(1+y(x)^2),y(x), singsol=all)
```

$$\ln(x) - \sqrt{1+y(x)^2} + c_1 = 0$$

✓ Solution by Mathematica

Time used: 0.229 (sec). Leaf size: 65

```
DSolve[x*y[x]*y'[x]==Sqrt[1+y[x]^2],y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to -\sqrt{\log^2(x) + 2c_1\log(x) - 1 + c_1{}^2}$$
$$y(x) \to \sqrt{\log^2(x) + 2c_1\log(x) - 1 + c_1{}^2}$$
$$y(x) \to -i$$
$$y(x) \to i$$

1.7 problem 7

Internal problem ID [5720]

Book: Ordinary differential equations and calculus of variations. Makarets and Reshetnyak. Wold Scientific. Singapore. 1995
Section: Chapter 1. First order differential equations. Section 1.1 Separable equations problems. page 7
Problem number: 7.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_separable]`

$$\boxed{(x^2-1)\,y'+2xy^2=0}$$

With initial conditions

$$[y(0)=1]$$

✓ Solution by Maple
Time used: 0.062 (sec). Leaf size: 20

```
dsolve([(x^2-1)*diff(y(x),x)+2*x*y(x)^2=0,y(0) = 1],y(x), singsol=all)
```

$$y(x)=\frac{1}{-i\pi+\ln(x-1)+\ln(x+1)+1}$$

✓ Solution by Mathematica
Time used: 0.162 (sec). Leaf size: 26

```
DSolve[{(x^2-1)*y'[x]+2*x*y[x]^2==0,{y[0]==1}},y[x],x,IncludeSingularSolutions -> True]
```

$$y(x)\to\frac{i}{i\log(x^2-1)+\pi+i}$$

1.8 problem 8

Internal problem ID [5721]

Book: Ordinary differential equations and calculus of variations. Makarets and Reshetnyak. Wold Scientific. Singapore. 1995
Section: Chapter 1. First order differential equations. Section 1.1 Separable equations problems. page 7
Problem number: 8.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_quadrature]`

$$\boxed{y' - 3y^{\frac{2}{3}} = 0}$$

With initial conditions

$$[y(2) = 0]$$

✓ Solution by Maple
Time used: 0.0 (sec). Leaf size: 5

```
dsolve([diff(y(x),x)=3*y(x)^(2/3),y(2) = 0],y(x), singsol=all)
```

$$y(x) = 0$$

✓ Solution by Mathematica
Time used: 0.002 (sec). Leaf size: 6

```
DSolve[{y'[x]==3*y[x]^(2/3),{y[2]==0}},y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to 0$$

1.9 problem 9

Internal problem ID [5722]

Book: Ordinary differential equations and calculus of variations. Makarets and Reshetnyak. Wold Scientific. Singapore. 1995
Section: Chapter 1. First order differential equations. Section 1.1 Separable equations problems. page 7
Problem number: 9.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_separable]`

$$\boxed{xy' + y - y^2 = 0}$$

With initial conditions

$$\left[y(1) = \frac{1}{2}\right]$$

✓ Solution by Maple
Time used: 0.078 (sec). Leaf size: 9

```
dsolve([x*diff(y(x),x)+y(x)=y(x)^2,y(1) = 1/2],y(x), singsol=all)
```

$$y(x) = \frac{1}{x+1}$$

✓ Solution by Mathematica
Time used: 0.252 (sec). Leaf size: 10

```
DSolve[{x*y'[x]+y[x]==y[x]^2,{y[1]==1/2}},y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to \frac{1}{x+1}$$

1.10 problem 10

Internal problem ID [5723]

Book: Ordinary differential equations and calculus of variations. Makarets and Reshetnyak. Wold Scientific. Singapore. 1995
Section: Chapter 1. First order differential equations. Section 1.1 Separable equations problems. page 7
Problem number: 10.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_separable]`

$$\boxed{2yx^2y' + y^2 = 2}$$

✓ Solution by Maple
Time used: 0.0 (sec). Leaf size: 29

```
dsolve(2*x^2*y(x)*diff(y(x),x)+y(x)^2=2,y(x), singsol=all)
```

$$y(x) = \sqrt{e^{\frac{1}{x}} c_1 + 2}$$
$$y(x) = -\sqrt{e^{\frac{1}{x}} c_1 + 2}$$

✓ Solution by Mathematica
Time used: 0.289 (sec). Leaf size: 70

```
DSolve[2*x*y[x]*y'[x]+y[x]^2==2,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to -\frac{\sqrt{2x + e^{2c_1}}}{\sqrt{x}}$$
$$y(x) \to \frac{\sqrt{2x + e^{2c_1}}}{\sqrt{x}}$$
$$y(x) \to -\sqrt{2}$$
$$y(x) \to \sqrt{2}$$

1.11 problem 11

Internal problem ID [5724]

Book: Ordinary differential equations and calculus of variations. Makarets and Reshetnyak. Wold Scientific. Singapore. 1995
Section: Chapter 1. First order differential equations. Section 1.1 Separable equations problems. page 7
Problem number: 11.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_separable]`

$$\boxed{y' - xy^2 - 2xy = 0}$$

✓ Solution by Maple
Time used: 0.015 (sec). Leaf size: 19

```
dsolve(diff(y(x),x)-x*y(x)^2=2*x*y(x),y(x), singsol=all)
```

$$y(x) = \frac{2}{-1 + 2\,\mathrm{e}^{-x^2} c_1}$$

✓ Solution by Mathematica
Time used: 0.276 (sec). Leaf size: 37

```
DSolve[y'[x]-2*x*y[x]^2==2*x*y[x],y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to -\frac{e^{x^2+c_1}}{-1 + e^{x^2+c_1}}$$
$$y(x) \to -1$$
$$y(x) \to 0$$

1.12 problem 12

Internal problem ID [5725]

Book: Ordinary differential equations and calculus of variations. Makarets and Reshetnyak. Wold Scientific. Singapore. 1995
Section: Chapter 1. First order differential equations. Section 1.1 Separable equations problems. page 7
Problem number: 12.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_quadrature]`

$$\boxed{(1+z')\,\mathrm{e}^{-z}=1}$$

✓ Solution by Maple
Time used: 0.031 (sec). Leaf size: 15

```
dsolve((1+diff(z(t),t))*exp(-z(t))=1,z(t), singsol=all)
```

$$z(t)=\ln\left(-\frac{1}{c_1\mathrm{e}^t-1}\right)$$

✓ Solution by Mathematica
Time used: 0.722 (sec). Leaf size: 28

```
DSolve[(1+z'[t])*Exp[-z[t]]==1,z[t],t,IncludeSingularSolutions -> True]
```

$$z(t)\to\log\left(\frac{1}{2}\left(1-\tanh\left(\frac{t+c_1}{2}\right)\right)\right)$$
$$z(t)\to 0$$

1.13 problem 13

Internal problem ID [5726]

Book: Ordinary differential equations and calculus of variations. Makarets and Reshetnyak. Wold Scientific. Singapore. 1995
Section: Chapter 1. First order differential equations. Section 1.1 Separable equations problems. page 7
Problem number: 13.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_separable]`

$$\boxed{y' - \frac{3x^2 + 4x + 2}{2y - 2} = 0}$$

With initial conditions

$$[y(0) = -1]$$

✓ Solution by Maple
Time used: 0.094 (sec). Leaf size: 19

```
dsolve([diff(y(x),x)=(3*x^2+4*x+2)/(2*(y(x)-1)),y(0) = -1],y(x), singsol=all)
```

$$y(x) = 1 - \sqrt{(x+2)(x^2+2)}$$

✓ Solution by Mathematica
Time used: 0.132 (sec). Leaf size: 26

```
DSolve[{y'[x]==(3*x^2+4*x+2)/(2*(y[x]-1)),{y[0]==-1}},y[x],x,IncludeSingularSolutions -> Tru
```

$$y(x) \to 1 - \sqrt{x^3 + 2x^2 + 2x + 4}$$

1.14 problem 14

Internal problem ID [5727]

Book: Ordinary differential equations and calculus of variations. Makarets and Reshetnyak. Wold Scientific. Singapore. 1995
Section: Chapter 1. First order differential equations. Section 1.1 Separable equations problems. page 7
Problem number: 14.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_separable]`

$$\boxed{-(1+e^x)\, yy' = -e^x}$$

With initial conditions

$$[y(0) = 1]$$

✓ Solution by Maple
Time used: 0.219 (sec). Leaf size: 19

```
dsolve([exp(x)-(1+exp(x))*y(x)*diff(y(x),x)=0,y(0) = 1],y(x), singsol=all)
```

$$y(x) = \sqrt{2\ln\left(e^x+1\right) - 2\ln\left(2\right) + 1}$$

✓ Solution by Mathematica
Time used: 0.182 (sec). Leaf size: 23

```
DSolve[{Exp[x]-(1+Exp[x])*y[x]*y'[x]==0,{y[0]==1}},y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to \sqrt{2\log\left(e^x+1\right) + 1 - \log(4)}$$

1.15 problem 15

Internal problem ID [5728]

Book: Ordinary differential equations and calculus of variations. Makarets and Reshetnyak. Wold Scientific. Singapore. 1995
Section: Chapter 1. First order differential equations. Section 1.1 Separable equations problems. page 7
Problem number: 15.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_separable]`

$$\boxed{\frac{y}{x-1}+\frac{xy'}{1+y}=0}$$

✓ Solution by Maple
Time used: 0.016 (sec). Leaf size: 15

```
dsolve(y(x)/(x-1)+x/(y(x)+1)*diff(y(x),x)=0,y(x), singsol=all)
```

$$y(x)=\frac{x}{-1+c_1(x-1)}$$

✓ Solution by Mathematica
Time used: 0.417 (sec). Leaf size: 33

```
DSolve[y[x]/(x-1)+x/(y[x]+1)*y'[x]==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x)\rightarrow -\frac{e^{c_1}x}{x+e^{c_1}x-1}$$
$$y(x)\rightarrow -1$$
$$y(x)\rightarrow 0$$

1.16 problem 16

Internal problem ID [5729]

Book: Ordinary differential equations and calculus of variations. Makarets and Reshetnyak. Wold Scientific. Singapore. 1995
Section: Chapter 1. First order differential equations. Section 1.1 Separable equations problems. page 7
Problem number: 16.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_separable]`

$$\boxed{(2y^3 + y)\, y' = -2x^3 - x}$$

✓ Solution by Maple
Time used: 0.016 (sec). Leaf size: 113

```
dsolve((x+2*x^3)+(y(x)+2*y(x)^3)*diff(y(x),x)=0,y(x), singsol=all)
```

$$y(x) = -\frac{\sqrt{-2 - 2\sqrt{-4x^4 - 4x^2 - 8c_1 - 1}}}{2}$$
$$y(x) = \frac{\sqrt{-2 - 2\sqrt{-4x^4 - 4x^2 - 8c_1 - 1}}}{2}$$
$$y(x) = -\frac{\sqrt{-2 + 2\sqrt{-4x^4 - 4x^2 - 8c_1 - 1}}}{2}$$
$$y(x) = \frac{\sqrt{-2 + 2\sqrt{-4x^4 - 4x^2 - 8c_1 - 1}}}{2}$$

✓ Solution by Mathematica

Time used: 2.086 (sec). Leaf size: 151

```
DSolve[(x+2*x^3)+(y[x]+2*y[x]^3)*y'[x]==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to -\frac{\sqrt{-1-\sqrt{-4x^4-4x^2+1+8c_1}}}{\sqrt{2}}$$
$$y(x) \to \frac{\sqrt{-1-\sqrt{-4x^4-4x^2+1+8c_1}}}{\sqrt{2}}$$
$$y(x) \to -\frac{\sqrt{-1+\sqrt{-4x^4-4x^2+1+8c_1}}}{\sqrt{2}}$$
$$y(x) \to \frac{\sqrt{-1+\sqrt{-4x^4-4x^2+1+8c_1}}}{\sqrt{2}}$$

1.17 problem 17

Internal problem ID [5730]

Book: Ordinary differential equations and calculus of variations. Makarets and Reshetnyak. Wold Scientific. Singapore. 1995
Section: Chapter 1. First order differential equations. Section 1.1 Separable equations problems. page 7
Problem number: 17.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_separable]`

$$\boxed{\frac{y'}{\sqrt{y}} = -\frac{1}{\sqrt{x}}}$$

✓ Solution by Maple
Time used: 0.0 (sec). Leaf size: 14

```
dsolve(1/sqrt(x)+diff(y(x),x)/sqrt(y(x))=0,y(x), singsol=all)
```

$$\sqrt{y(x)} + \sqrt{x} - c_1 = 0$$

✓ Solution by Mathematica
Time used: 0.125 (sec). Leaf size: 21

```
DSolve[1/Sqrt[x]+y'[x]/Sqrt[y[x]]==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to \frac{1}{4}\left(-2\sqrt{x} + c_1\right)^2$$

1.18 problem 18

Internal problem ID [5731]

Book: Ordinary differential equations and calculus of variations. Makarets and Reshetnyak. Wold Scientific. Singapore. 1995
Section: Chapter 1. First order differential equations. Section 1.1 Separable equations problems. page 7
Problem number: 18.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_separable]`

$$\boxed{\frac{y'}{\sqrt{1-y^2}} = -\frac{1}{\sqrt{-x^2+1}}}$$

✓ Solution by Maple
Time used: 0.016 (sec). Leaf size: 11

```
dsolve(1/sqrt(1-x^2)+diff(y(x),x)/sqrt(1-y(x)^2)=0,y(x), singsol=all)
```

$$y(x) = -\sin(\arcsin(x) + c_1)$$

✓ Solution by Mathematica
Time used: 0.288 (sec). Leaf size: 37

```
DSolve[1/Sqrt[1-x^2]+y'[x]/Sqrt[1-y[x]^2]==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to \cos\left(2\arctan\left(\frac{\sqrt{1-x^2}}{x+1}\right) + c_1\right)$$
$$y(x) \to \text{Interval}[\{-1, 1\}]$$

1.19 problem 19

Internal problem ID [5732]

Book: Ordinary differential equations and calculus of variations. Makarets and Reshetnyak. Wold Scientific. Singapore. 1995
Section: Chapter 1. First order differential equations. Section 1.1 Separable equations problems. page 7
Problem number: 19.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_separable]`

$$\boxed{2x\sqrt{1-y^2}+y'y=0}$$

✓ Solution by Maple
Time used: 0.0 (sec). Leaf size: 27

```
dsolve(2*x*sqrt(1-y(x)^2)+y(x)*diff(y(x),x)=0,y(x), singsol=all)
```

$$c_1+x^2+\frac{(y(x)-1)\,(y(x)+1)}{\sqrt{1-y\left(x\right)^2}}=0$$

✓ Solution by Mathematica
Time used: 0.288 (sec). Leaf size: 69

```
DSolve[2*x*Sqrt[1-y[x]^2]+y[x]*y'[x]==0,y[x],x,IncludeSingularSolutions -> True]
```

$$\begin{aligned}
y(x) &\to -\sqrt{-x^4+2c_1x^2+1-c_1{}^2} \\
y(x) &\to \sqrt{-x^4+2c_1x^2+1-c_1{}^2} \\
y(x) &\to -1 \\
y(x) &\to 1
\end{aligned}$$

1.20 problem 20

Internal problem ID [5733]

Book: Ordinary differential equations and calculus of variations. Makarets and Reshetnyak. Wold Scientific. Singapore. 1995
Section: Chapter 1. First order differential equations. Section 1.1 Separable equations problems. page 7
Problem number: 20.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_separable]`

$$\boxed{y' - (y - 1)(1 + x) = 0}$$

✓ Solution by Maple
Time used: 0.0 (sec). Leaf size: 15

```
dsolve(diff(y(x),x)=(y(x)-1)*(x+1),y(x), singsol=all)
```

$$y(x) = 1 + c_1 e^{\frac{x(x+2)}{2}}$$

✓ Solution by Mathematica
Time used: 0.033 (sec). Leaf size: 25

```
DSolve[y'[x]==(y[x]-1)*(x+1),y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to 1 + c_1 e^{\frac{1}{2}x(x+2)}$$
$$y(x) \to 1$$

1.21 problem 21

Internal problem ID [5734]

Book: Ordinary differential equations and calculus of variations. Makarets and Reshetnyak. Wold Scientific. Singapore. 1995
Section: Chapter 1. First order differential equations. Section 1.1 Separable equations problems. page 7
Problem number: 21.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_separable]`

$$\boxed{y' - e^{x-y} = 0}$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 9

```
dsolve(diff(y(x),x)=exp(x-y(x)),y(x), singsol=all)
```

$$y(x) = \ln\left(e^x + c_1\right)$$

✓ Solution by Mathematica

Time used: 0.743 (sec). Leaf size: 12

```
DSolve[y'[x]==Exp[x-y[x]],y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to \log\left(e^x + c_1\right)$$

1.22 problem 22

Internal problem ID [5735]

Book: Ordinary differential equations and calculus of variations. Makarets and Reshetnyak. Wold Scientific. Singapore. 1995
Section: Chapter 1. First order differential equations. Section 1.1 Separable equations problems. page 7
Problem number: 22.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_separable]`

$$\boxed{y' - \frac{\sqrt{y}}{\sqrt{x}} = 0}$$

✓ Solution by Maple
Time used: 0.016 (sec). Leaf size: 16

```
dsolve(diff(y(x),x)=sqrt(y(x))/sqrt(x),y(x), singsol=all)
```

$$\sqrt{y\left(x\right)} - \sqrt{x} - c_1 = 0$$

✓ Solution by Mathematica
Time used: 0.14 (sec). Leaf size: 26

```
DSolve[y'[x]==Sqrt[y[x]]/Sqrt[x],y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to \frac{1}{4}\left(2\sqrt{x} + c_1\right)^2$$
$$y(x) \to 0$$

1.23 problem 23

Internal problem ID [5736]

Book: Ordinary differential equations and calculus of variations. Makarets and Reshetnyak. Wold Scientific. Singapore. 1995
Section: Chapter 1. First order differential equations. Section 1.1 Separable equations problems. page 7
Problem number: 23.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_separable]`

$$\boxed{y' - \frac{\sqrt{y}}{x} = 0}$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 15

```
dsolve(diff(y(x),x)=sqrt(y(x))/x,y(x), singsol=all)
```

$$\sqrt{y(x)} - \frac{\ln(x)}{2} - c_1 = 0$$

✓ Solution by Mathematica

Time used: 0.111 (sec). Leaf size: 21

```
DSolve[y'[x]==Sqrt[y[x]]/x,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to \frac{1}{4}(\log(x) + c_1)^2$$
$$y(x) \to 0$$

1.24 problem 24

Internal problem ID [5737]

Book: Ordinary differential equations and calculus of variations. Makarets and Reshetnyak. Wold Scientific. Singapore. 1995
Section: Chapter 1. First order differential equations. Section 1.1 Separable equations problems. page 7
Problem number: 24.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_separable]`

$$\boxed{z' - 10^{x+z} = 0}$$

✓ Solution by Maple
Time used: 0.015 (sec). Leaf size: 29

```
dsolve(diff(z(x),x)=10^(x+z(x)),z(x), singsol=all)
```

$$z(x) = \frac{\ln\left(-\frac{1}{c_1 \ln(2)+c_1 \ln(5)+10^x}\right)}{\ln(2) + \ln(5)}$$

✓ Solution by Mathematica
Time used: 0.93 (sec). Leaf size: 24

```
DSolve[z'[x]==10^(x+z[x]),z[x],x,IncludeSingularSolutions -> True]
```

$$z(x) \to -\frac{\log(-10^x + c_1(-\log(10)))}{\log(10)}$$

1.25 problem 25

Internal problem ID [5738]

Book: Ordinary differential equations and calculus of variations. Makarets and Reshetnyak. Wold Scientific. Singapore. 1995
Section: Chapter 1. First order differential equations. Section 1.1 Separable equations problems. page 7
Problem number: 25.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_quadrature]`

$$\boxed{x' = -t + 1}$$

✓ Solution by Maple
Time used: 0.0 (sec). Leaf size: 12

```
dsolve(diff(x(t),t)+t=1,x(t), singsol=all)
```

$$x(t) = -\frac{1}{2}t^2 + t + c_1$$

✓ Solution by Mathematica
Time used: 0.002 (sec). Leaf size: 16

```
DSolve[x'[t]+t==1,x[t],t,IncludeSingularSolutions -> True]
```

$$x(t) \to -\frac{t^2}{2} + t + c_1$$

1.26 problem 26

Internal problem ID [5739]

Book: Ordinary differential equations and calculus of variations. Makarets and Reshetnyak. Wold Scientific. Singapore. 1995
Section: Chapter 1. First order differential equations. Section 1.1 Separable equations problems. page 7
Problem number: 26.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[[_homogeneous, ‘class C‘], _dAlembert]`

$$\boxed{y' - \cos(x - y) = 0}$$

✓ Solution by Maple
Time used: 0.031 (sec). Leaf size: 14

```
dsolve(diff(y(x),x)=cos(y(x)-x),y(x), singsol=all)
```

$$y(x) = x - 2 \operatorname{arccot}(-x + c_1)$$

✓ Solution by Mathematica
Time used: 0.439 (sec). Leaf size: 40

```
DSolve[y'[x]==Cos[y[x]-x],y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to x + 2\cot^{-1}\left(x - \frac{c_1}{2}\right)$$
$$y(x) \to x + 2\cot^{-1}\left(x - \frac{c_1}{2}\right)$$
$$y(x) \to x$$

1.27 problem 27

Internal problem ID [5740]

Book: Ordinary differential equations and calculus of variations. Makarets and Reshetnyak. Wold Scientific. Singapore. 1995
Section: Chapter 1. First order differential equations. Section 1.1 Separable equations problems. page 7
Problem number: 27.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[[_linear, ‘class A‘]]`

$$\boxed{y' - y = 2x - 3}$$

✓ Solution by Maple
Time used: 0.0 (sec). Leaf size: 13

```
dsolve(diff(y(x),x)-y(x)=2*x-3,y(x), singsol=all)
```

$$y(x) = -2x + 1 + \mathrm{e}^{x} c_1$$

✓ Solution by Mathematica
Time used: 0.073 (sec). Leaf size: 16

```
DSolve[y'[x]-y[x]==2*x-3,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to -2x + c_1 e^x + 1$$

1.28 problem 28

Internal problem ID [5741]

Book: Ordinary differential equations and calculus of variations. Makarets and Reshetnyak. Wold Scientific. Singapore. 1995
Section: Chapter 1. First order differential equations. Section 1.1 Separable equations problems. page 7
Problem number: 28.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[[_homogeneous, ‘class C‘], [_Abel, ‘2nd type‘, ‘class C‘], _d`

$$\boxed{(2y + x)\, y' = 1}$$

With initial conditions

$$[y(0) = -1]$$

✓ Solution by Maple
Time used: 0.032 (sec). Leaf size: 9

```
dsolve([(x+2*y(x))*diff(y(x),x)=1,y(0) = -1],y(x), singsol=all)
```

$$y(x) = -\frac{x}{2} - 1$$

✓ Solution by Mathematica
Time used: 0.019 (sec). Leaf size: 12

```
DSolve[{(x+2*y[x])*y'[x]==1,{y[0]==-1}},y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \rightarrow -\frac{x}{2} - 1$$

1.29 problem 29

Internal problem ID [5742]

Book: Ordinary differential equations and calculus of variations. Makarets and Reshetnyak. Wold Scientific. Singapore. 1995
Section: Chapter 1. First order differential equations. Section 1.1 Separable equations problems. page 7
Problem number: 29.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[[_linear, ‘class A‘]]`

$$\boxed{y + y' = 1 + 2x}$$

✓ Solution by Maple
Time used: 0.0 (sec). Leaf size: 15

```
dsolve(diff(y(x),x)+y(x)=2*x+1,y(x), singsol=all)
```

$$y(x) = 2x - 1 + c_1 \mathrm{e}^{-x}$$

✓ Solution by Mathematica
Time used: 0.057 (sec). Leaf size: 18

```
DSolve[y'[x]+y[x]==2*x+1,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to 2x + c_1 e^{-x} - 1$$

1.30 problem 30

Internal problem ID [5743]

Book: Ordinary differential equations and calculus of variations. Makarets and Reshetnyak. Wold Scientific. Singapore. 1995
Section: Chapter 1. First order differential equations. Section 1.1 Separable equations problems. page 7
Problem number: 30.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[[_homogeneous, ‘class C‘], _dAlembert]`

$$\boxed{y' - \cos(x - y - 1) = 0}$$

✓ Solution by Maple
Time used: 0.031 (sec). Leaf size: 15

```
dsolve(diff(y(x),x)=cos(x-y(x)-1),y(x), singsol=all)
```

$$y(x) = x - 1 - 2 \operatorname{arccot}(-x + c_1)$$

✓ Solution by Mathematica
Time used: 0.551 (sec). Leaf size: 50

```
DSolve[y'[x]==Cos[x-y[x]-1],y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to x - 2\cot^{-1}\left(-x + 1 + \frac{c_1}{2}\right) - 1$$
$$y(x) \to x - 2\cot^{-1}\left(-x + 1 + \frac{c_1}{2}\right) - 1$$
$$y(x) \to x - 1$$

1.31 problem 31

Internal problem ID [5744]

Book: Ordinary differential equations and calculus of variations. Makarets and Reshetnyak. Wold Scientific. Singapore. 1995
Section: Chapter 1. First order differential equations. Section 1.1 Separable equations problems. page 7
Problem number: 31.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[[_homogeneous, ‘class C‘], _dAlembert]`

$$\boxed{y' + \sin(x+y)^2 = 0}$$

✓ Solution by Maple
Time used: 0.031 (sec). Leaf size: 16

```
dsolve(diff(y(x),x)+sin(x+y(x))^2=0,y(x), singsol=all)
```

$$y(x) = -x - \arctan(-x + c_1)$$

✓ Solution by Mathematica
Time used: 0.195 (sec). Leaf size: 27

```
DSolve[y'[x]+Sin[x+y[x]]^2==0,y[x],x,IncludeSingularSolutions -> True]
```

$$\text{Solve}[2(\tan(y(x)+x) - \arctan(\tan(y(x)+x))) + 2y(x) = c_1, y(x)]$$

1.32 problem 32

Internal problem ID [5745]

Book: Ordinary differential equations and calculus of variations. Makarets and Reshetnyak. Wold Scientific. Singapore. 1995
Section: Chapter 1. First order differential equations. Section 1.1 Separable equations problems. page 7
Problem number: 32.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[[_homogeneous, ‘class C‘], _dAlembert]`

$$\boxed{y' - 2\sqrt{2x + y + 1} = 0}$$

✓ Solution by Maple
Time used: 0.031 (sec). Leaf size: 56

```
dsolve(diff(y(x),x)=2*sqrt(2*x+y(x)+1),y(x), singsol=all)
```

$$x - \sqrt{2x + y(x) + 1} - \frac{\ln\left(-1 + \sqrt{2x + y(x) + 1}\right)}{2} + \frac{\ln\left(\sqrt{2x + y(x) + 1} + 1\right)}{2} + \frac{\ln\left(y(x) + 2x\right)}{2} - c_1 = 0$$

✓ Solution by Mathematica
Time used: 11.43 (sec). Leaf size: 48

```
DSolve[y'[x]==2*Sqrt[2*x+y[x]+1],y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to W\left(-e^{-x-\frac{3}{2}+c_1}\right)^2 + 2W\left(-e^{-x-\frac{3}{2}+c_1}\right) - 2x$$
$$y(x) \to -2x$$

1.33 problem 33

Internal problem ID [5746]

Book: Ordinary differential equations and calculus of variations. Makarets and Reshetnyak. Wold Scientific. Singapore. 1995
Section: Chapter 1. First order differential equations. Section 1.1 Separable equations problems. page 7
Problem number: 33.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[[_homogeneous, ‘class C‘], _Riccati]`

$$\boxed{y' - (y + x + 1)^2 = 0}$$

✓ Solution by Maple
Time used: 0.0 (sec). Leaf size: 17

```
dsolve(diff(y(x),x)=(x+y(x)+1)^2,y(x), singsol=all)
```

$$y(x) = -x - 1 - \tan(-x + c_1)$$

✓ Solution by Mathematica
Time used: 0.498 (sec). Leaf size: 15

```
DSolve[y'[x]==(x+y[x]+1)^2,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to -x + \tan(x + c_1) - 1$$

1.34 problem 34

Internal problem ID [5747]

Book: Ordinary differential equations and calculus of variations. Makarets and Reshetnyak. Wold Scientific. Singapore. 1995
Section: Chapter 1. First order differential equations. Section 1.1 Separable equations problems. page 7
Problem number: 34.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_separable]`

$$\boxed{y^2 + xy^2 + (x^2 - yx^2)\, y' = 0}$$

✓ Solution by Maple
Time used: 0.016 (sec). Leaf size: 35

```
dsolve((y(x)^2+x*y(x)^2)+(x^2-x^2*y(x))*diff(y(x),x)=0,y(x), singsol=all)
```

$$y(x) = x\, \mathrm{e}^{\frac{\operatorname{LambertW}\left(-\frac{\mathrm{e}^{\frac{-c_1 x+1}{x}}}{x}\right)x + c_1 x - 1}{x}}$$

✓ Solution by Mathematica
Time used: 5.623 (sec). Leaf size: 30

```
DSolve[(y[x]^2+x*y[x]^2)+(x^2-x^2*y[x])*y'[x]==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to -\frac{1}{W\left(-\frac{e^{\frac{1}{x}-c_1}}{x}\right)}$$
$$y(x) \to 0$$

1.35 problem 35

Internal problem ID [5748]

Book: Ordinary differential equations and calculus of variations. Makarets and Reshetnyak. Wold Scientific. Singapore. 1995
Section: Chapter 1. First order differential equations. Section 1.1 Separable equations problems. page 7
Problem number: 35.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_separable]`

$$\boxed{(1+y^2)\left(e^{2x}-y'e^{y}\right)-(1+y)\,y'=0}$$

✓ Solution by Maple
Time used: 0.093 (sec). Leaf size: 30

```
dsolve((1+y(x)^2)*(exp(2*x)-exp(y(x))*diff(y(x),x))-(1+y(x))*diff(y(x),x)=0,y(x), singsol=al
```

$$\frac{e^{2x}}{2}-\arctan\left(y(x)\right)-\frac{\ln\left(1+y(x)^2\right)}{2}-e^{y(x)}+c_1=0$$

✓ Solution by Mathematica
Time used: 0.696 (sec). Leaf size: 70

```
DSolve[(1+y[x]^2)*(Exp[2*x]-Exp[y[x]]*y'[x])-(1+y[x])*y'[x]==0,y[x],x,IncludeSingularSolutio
```

$$y(x)\to \text{InverseFunction}\left[e^{\#1}+\left(\frac{1}{2}-\frac{i}{2}\right)\log(-\#1+i)+\left(\frac{1}{2}+\frac{i}{2}\right)\log(\#1+i)\&\right]\left[\frac{e^{2x}}{2}+c_1\right]$$
$$y(x)\to -i$$
$$y(x)\to i$$

2 Chapter 1. First order differential equations. Section 1.2 Homogeneous equations problems. page 12

2.1 problem 1

Internal problem ID [5749]

Book: Ordinary differential equations and calculus of variations. Makarets and Reshetnyak. Wold Scientific. Singapore. 1995
Section: Chapter 1. First order differential equations. Section 1.2 Homogeneous equations problems. page 12
Problem number: 1.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[[_homogeneous, ‘class A‘], _rational, [_Abel, ‘2nd type‘, ‘cl`

$$-y + (x + y)\, y' = -x$$

✓ Solution by Maple
Time used: 0.016 (sec). Leaf size: 24

```
dsolve((x-y(x))+(x+y(x))*diff(y(x),x)=0,y(x), singsol=all)
```

$$y(x) = \tan\left(\text{RootOf}\left(2\_Z + \ln\left(\sec\left(\_Z\right)^2\right) + 2\ln\left(x\right) + 2c_1\right)\right) x$$

✓ Solution by Mathematica
Time used: 0.035 (sec). Leaf size: 34

```
DSolve[(x-y[x])+(x+y[x])*y'[x]==0,y[x],x,IncludeSingularSolutions -> True]
```

$$\text{Solve}\left[\arctan\left(\frac{y(x)}{x}\right) + \frac{1}{2}\log\left(\frac{y(x)^2}{x^2} + 1\right) = -\log(x) + c_1, y(x)\right]$$

2.2 problem 2

Internal problem ID [5750]

Book: Ordinary differential equations and calculus of variations. Makarets and Reshetnyak. Wold Scientific. Singapore. 1995
Section: Chapter 1. First order differential equations. Section 1.2 Homogeneous equations problems. page 12
Problem number: 2.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_separable]`

$$\boxed{y - 2xy + x^2y' = 0}$$

✓ Solution by Maple
Time used: 0.0 (sec). Leaf size: 13

```
dsolve((y(x)-2*x*y(x))+x^2*diff(y(x),x)=0,y(x), singsol=all)
```

$$y(x) = c_1 \mathrm{e}^{\frac{1}{x}} x^2$$

✓ Solution by Mathematica
Time used: 0.031 (sec). Leaf size: 21

```
DSolve[(y[x]-2*x*y[x])+x^2*y'[x]==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to c_1 e^{\frac{1}{x}} x^2$$
$$y(x) \to 0$$

2.3 problem 3

Internal problem ID [5751]

Book: Ordinary differential equations and calculus of variations. Makarets and Reshetnyak. Wold Scientific. Singapore. 1995
Section: Chapter 1. First order differential equations. Section 1.2 Homogeneous equations problems. page 12
Problem number: 3.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_rational, _Bernoulli]`

$$\boxed{2xy' - y(2x^2 - y^2) = 0}$$

✓ Solution by Maple
Time used: 0.015 (sec). Leaf size: 83

```
dsolve(2*x*diff(y(x),x)=y(x)*(2*x^2-y(x)^2),y(x), singsol=all)
```

$$y(x) = \frac{\sqrt{2}\,\sqrt{(2c_1 - \text{expIntegral}_1\left(-x^2\right))\,e^{x^2}}}{-2c_1 + \text{expIntegral}_1\left(-x^2\right)}$$
$$y(x) = \frac{\sqrt{2}\,\sqrt{(2c_1 - \text{expIntegral}_1\left(-x^2\right))\,e^{x^2}}}{2c_1 - \text{expIntegral}_1\left(-x^2\right)}$$

✓ Solution by Mathematica
Time used: 0.269 (sec). Leaf size: 65

```
DSolve[2*x*y'[x]==y[x]*(2*x^2-y[x]^2),y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to -\frac{e^{\frac{x^2}{2}}}{\sqrt{\frac{\text{ExpIntegralEi}(x^2)}{2} + c_1}}$$
$$y(x) \to \frac{e^{\frac{x^2}{2}}}{\sqrt{\frac{\text{ExpIntegralEi}(x^2)}{2} + c_1}}$$
$$y(x) \to 0$$

2.4 problem 4

Internal problem ID [5752]

Book: Ordinary differential equations and calculus of variations. Makarets and Reshetnyak. Wold Scientific. Singapore. 1995
Section: Chapter 1. First order differential equations. Section 1.2 Homogeneous equations problems. page 12
Problem number: 4.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[[_homogeneous, ‘class A‘], _rational, [_Abel, ‘2nd type‘, ‘cl`

$$\boxed{y^2 + x^2 y' - xyy' = 0}$$

✓ Solution by Maple
Time used: 0.031 (sec). Leaf size: 17

```
dsolve(y(x)^2+x^2*diff(y(x),x)=x*y(x)*diff(y(x),x),y(x), singsol=all)
```

$$y(x) = -x \operatorname{LambertW}\left(-\frac{e^{-c_1}}{x}\right)$$

✓ Solution by Mathematica
Time used: 2.289 (sec). Leaf size: 25

```
DSolve[y[x]^2+x^2*y'[x]==x*y[x]*y'[x],y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to -xW\left(-\frac{e^{-c_1}}{x}\right)$$
$$y(x) \to 0$$

2.5 problem 5

Internal problem ID [5753]

Book: Ordinary differential equations and calculus of variations. Makarets and Reshetnyak. Wold Scientific. Singapore. 1995
Section: Chapter 1. First order differential equations. Section 1.2 Homogeneous equations problems. page 12
Problem number: 5.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[[_homogeneous, 'class A'], _rational, _dAlembert]`

$$\boxed{(x^2 + y^2)\, y' - 2xy = 0}$$

✓ Solution by Maple
Time used: 0.094 (sec). Leaf size: 47

```
dsolve((x^2+y(x)^2)*diff(y(x),x)=2*x*y(x),y(x), singsol=all)
```

$$y(x) = \frac{1 - \sqrt{4c_1^2x^2 + 1}}{2c_1}$$
$$y(x) = \frac{1 + \sqrt{4c_1^2x^2 + 1}}{2c_1}$$

✓ Solution by Mathematica
Time used: 0.931 (sec). Leaf size: 70

```
DSolve[(x^2+y[x]^2)*y'[x]==2*x*y[x],y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to \frac{1}{2}\left(-\sqrt{4x^2 + e^{2c_1}} - e^{c_1}\right)$$
$$y(x) \to \frac{1}{2}\left(\sqrt{4x^2 + e^{2c_1}} - e^{c_1}\right)$$
$$y(x) \to 0$$

2.6 problem 6

Internal problem ID [5754]

Book: Ordinary differential equations and calculus of variations. Makarets and Reshetnyak. Wold Scientific. Singapore. 1995
Section: Chapter 1. First order differential equations. Section 1.2 Homogeneous equations problems. page 12
Problem number: 6.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[[_homogeneous, 'class A'], _dAlembert]`

$$\boxed{-y + xy' - \tan\left(\frac{y}{x}\right)x = 0}$$

✓ Solution by Maple
Time used: 0.015 (sec). Leaf size: 10

```
dsolve(x*diff(y(x),x)-y(x)=x*tan(y(x)/x),y(x), singsol=all)
```

$$y(x) = \arcsin(c_1 x)\, x$$

✓ Solution by Mathematica
Time used: 6.102 (sec). Leaf size: 19

```
DSolve[x*y'[x]-y[x]==x*Tan[y[x]/x],y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to x \arcsin\left(e^{c_1} x\right)$$
$$y(x) \to 0$$

2.7 problem 7

Internal problem ID [5755]

Book: Ordinary differential equations and calculus of variations. Makarets and Reshetnyak. Wold Scientific. Singapore. 1995
Section: Chapter 1. First order differential equations. Section 1.2 Homogeneous equations problems. page 12
Problem number: 7.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[[_homogeneous, ‘class A‘], _dAlembert]`

$$\boxed{xy' - y + x\,\mathrm{e}^{\frac{y}{x}} = 0}$$

✓ Solution by Maple
Time used: 0.0 (sec). Leaf size: 12

```
dsolve(x*diff(y(x),x)=y(x)-x*exp(y(x)/x),y(x), singsol=all)
```

$$y(x) = -\ln\left(\ln\left(x\right) + c_1\right) x$$

✓ Solution by Mathematica
Time used: 0.348 (sec). Leaf size: 16

```
DSolve[x*y'[x]==y[x]-x*Exp[y[x]/x],y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to -x\log(\log(x) - c_1)$$

2.8 problem 8

Internal problem ID [5756]

Book: Ordinary differential equations and calculus of variations. Makarets and Reshetnyak. Wold Scientific. Singapore. 1995
Section: Chapter 1. First order differential equations. Section 1.2 Homogeneous equations problems. page 12
Problem number: 8.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[[_homogeneous, ‘class A‘], _dAlembert]`

$$\boxed{-y + xy' - (x+y)\ln\left(\frac{x+y}{x}\right) = 0}$$

✓ Solution by Maple
Time used: 0.016 (sec). Leaf size: 12

```
dsolve(x*diff(y(x),x)-y(x)=(x+y(x))*ln((x+y(x))/x),y(x), singsol=all)
```

$$y(x) = x(-1 + e^{c_1 x})$$

✓ Solution by Mathematica
Time used: 0.406 (sec). Leaf size: 24

```
DSolve[x*y'[x]-y[x]==(x+y[x])*Log[ (x+y[x])/x],y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to x\left(-1 + e^{e^{-c_1 x}}\right)$$
$$y(x) \to 0$$

2.9 problem 9

Internal problem ID [5757]

Book: Ordinary differential equations and calculus of variations. Makarets and Reshetnyak. Wold Scientific. Singapore. 1995
Section: Chapter 1. First order differential equations. Section 1.2 Homogeneous equations problems. page 12
Problem number: 9.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[[_homogeneous, ‘class A‘], _dAlembert]`

$$\boxed{xy' - y\cos\left(\frac{y}{x}\right) = 0}$$

✓ Solution by Maple
Time used: 0.0 (sec). Leaf size: 27

```
dsolve(x*diff(y(x),x)=y(x)*cos(y(x)/x),y(x), singsol=all)
```

$$y(x) = \operatorname{RootOf}\left(\ln(x) + c_1 - \left(\int^{\_Z} \frac{1}{\_a(-1+\cos(\_a))} d\_a\right)\right) x$$

✓ Solution by Mathematica
Time used: 2.086 (sec). Leaf size: 33

```
DSolve[x*y'[x]==y[x]*Cos[y[x]/x],y[x],x,IncludeSingularSolutions -> True]
```

$$\text{Solve}\left[\int_1^{\frac{y(x)}{x}} \frac{1}{(\cos(K[1])-1)K[1]} dK[1] = \log(x) + c_1, y(x)\right]$$

2.10 problem 10

Internal problem ID [5758]

Book: Ordinary differential equations and calculus of variations. Makarets and Reshetnyak. Wold Scientific. Singapore. 1995
Section: Chapter 1. First order differential equations. Section 1.2 Homogeneous equations problems. page 12
Problem number: 10.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[[_homogeneous, ‘class A‘], _rational, _dAlembert]`

$$\boxed{y + \sqrt{xy} - xy' = 0}$$

✓ Solution by Maple
Time used: 0.015 (sec). Leaf size: 21

```
dsolve((y(x)+sqrt(x*y(x)))-x*diff(y(x),x)=0,y(x), singsol=all)
```

$$-\frac{y(x)}{\sqrt{xy\left(x\right)}} + \frac{\ln\left(x\right)}{2} - c_1 = 0$$

✓ Solution by Mathematica
Time used: 0.183 (sec). Leaf size: 17

```
DSolve[(y[x]+Sqrt[x*y[x]])-x*y'[x]==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to \frac{1}{4}x(\log(x) + c_1)^2$$

2.11 problem 11

Internal problem ID [5759]

Book: Ordinary differential equations and calculus of variations. Makarets and Reshetnyak. Wold Scientific. Singapore. 1995
Section: Chapter 1. First order differential equations. Section 1.2 Homogeneous equations problems. page 12
Problem number: 11.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[[_homogeneous, 'class A'], _rational, _dAlembert]`

$$\boxed{xy' - \sqrt{x^2 - y^2} - y = 0}$$

✓ Solution by Maple
Time used: 0.0 (sec). Leaf size: 27

```
dsolve(x*diff(y(x),x)-sqrt(x^2-y(x)^2)-y(x)=0,y(x), singsol=all)
```

$$-\arctan\left(\frac{y(x)}{\sqrt{x^2 - y(x)^2}}\right) + \ln(x) - c_1 = 0$$

✓ Solution by Mathematica
Time used: 0.243 (sec). Leaf size: 18

```
DSolve[x*y'[x]-Sqrt[x^2-y[x]^2]-y[x]==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to -x\cosh(i\log(x) + c_1)$$

2.12 problem 12

Internal problem ID [5760]

Book: Ordinary differential equations and calculus of variations. Makarets and Reshetnyak. Wold Scientific. Singapore. 1995
Section: Chapter 1. First order differential equations. Section 1.2 Homogeneous equations problems. page 12
Problem number: 12.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[[_homogeneous, ‘class A‘], _rational, [_Abel, ‘2nd type‘, ‘cl`

$$\boxed{y-(x-y)\,y'=-x}$$

✓ Solution by Maple
Time used: 0.016 (sec). Leaf size: 24

```
dsolve((x+y(x))-(x-y(x))*diff(y(x),x)=0,y(x), singsol=all)
```

$$y(x)=\tan\left(\mathrm{RootOf}\left(-2\_Z+\ln\left(\sec\left(\_Z\right)^{2}\right)+2\ln\left(x\right)+2c_{1}\right)\right)x$$

✓ Solution by Mathematica
Time used: 0.034 (sec). Leaf size: 36

```
DSolve[(x+y[x])-(x-y[x])*y'[x]==0,y[x],x,IncludeSingularSolutions -> True]
```

$$\mathrm{Solve}\left[\frac{1}{2}\log\left(\frac{y(x)^2}{x^2}+1\right)-\arctan\left(\frac{y(x)}{x}\right)=-\log(x)+c_1,y(x)\right]$$

2.13 problem 13

Internal problem ID [5761]

Book: Ordinary differential equations and calculus of variations. Makarets and Reshetnyak. Wold Scientific. Singapore. 1995
Section: Chapter 1. First order differential equations. Section 1.2 Homogeneous equations problems. page 12
Problem number: 13.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[[_homogeneous, ‘class A‘], _rational, _dAlembert]`

$$\boxed{2xy - y^2 + \left(y^2 + 2xy - x^2\right) y' = -x^2}$$

With initial conditions

$$[y(1) = -1]$$

✓ Solution by Maple
Time used: 0.203 (sec). Leaf size: 7

```
dsolve([(x^2+2*x*y(x)-y(x)^2)+(y(x)^2+2*x*y(x)-x^2)*diff(y(x),x)=0,y(1) = -1],y(x), singsol=
```

$$y(x) = -x$$

✗ Solution by Mathematica
Time used: 0.0 (sec). Leaf size: 0

```
DSolve[{(x^2+2*x*y[x]-y[x]^2)+(y[x]^2+2*x*y[x]-x^2)*y'[x]==0,{y[1]==-1}},y[x],x,IncludeSingu
```

{}

2.14 problem 14

Internal problem ID [5762]

Book: Ordinary differential equations and calculus of variations. Makarets and Reshetnyak. Wold Scientific. Singapore. 1995
Section: Chapter 1. First order differential equations. Section 1.2 Homogeneous equations problems. page 12
Problem number: 14.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[[_homogeneous, ‘class A‘], _rational, [_Abel, ‘2nd type‘, ‘cl`

$$\boxed{-y + xy' - y'y = 0}$$

✓ Solution by Maple

Time used: 0.015 (sec). Leaf size: 17

```
dsolve(x*diff(y(x),x)-y(x)=y(x)*diff(y(x),x),y(x), singsol=all)
```

$$y(x) = -\frac{x}{\mathrm{LambertW}\left(-x\,\mathrm{e}^{-c_1}\right)}$$

✓ Solution by Mathematica

Time used: 3.949 (sec). Leaf size: 25

```
DSolve[x*y'[x]-y[x]==y[x]*y'[x],y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to -\frac{x}{W\left(-e^{-c_1}x\right)}$$
$$y(x) \to 0$$

2.15 problem 15

Internal problem ID [5763]

Book: Ordinary differential equations and calculus of variations. Makarets and Reshetnyak. Wold Scientific. Singapore. 1995
Section: Chapter 1. First order differential equations. Section 1.2 Homogeneous equations problems. page 12
Problem number: 15.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[[_homogeneous, 'class A'], _rational, [_Abel, '2nd type', 'cl`

$$y^2 + (x^2 - xy)\, y' = 0$$

✓ Solution by Maple
Time used: 0.156 (sec). Leaf size: 17

```
dsolve(y(x)^2+(x^2-x*y(x))*diff(y(x),x)=0,y(x), singsol=all)
```

$$y(x) = -x \operatorname{LambertW}\left(-\frac{e^{-c_1}}{x}\right)$$

✓ Solution by Mathematica
Time used: 2.172 (sec). Leaf size: 25

```
DSolve[y[x]^2+(x^2-x*y[x])*y'[x]==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to -xW\left(-\frac{e^{-c_1}}{x}\right)$$
$$y(x) \to 0$$

2.16 problem 16

Internal problem ID [5764]

Book: Ordinary differential equations and calculus of variations. Makarets and Reshetnyak. Wold Scientific. Singapore. 1995
Section: Chapter 1. First order differential equations. Section 1.2 Homogeneous equations problems. page 12
Problem number: 16.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[[_homogeneous, ‘class A‘], _rational, _Riccati]`

$$\boxed{xy + y^2 - x^2y' = -x^2}$$

✓ Solution by Maple
Time used: 0.016 (sec). Leaf size: 11

```
dsolve((x^2+x*y(x)+y(x)^2)=x^2*diff(y(x),x),y(x), singsol=all)
```

$$y(x) = \tan(\ln(x) + c_1)\, x$$

✓ Solution by Mathematica
Time used: 0.188 (sec). Leaf size: 13

```
DSolve[(x^2+x*y[x]+y[x]^2)==x^2*y'[x],y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to x\tan(\log(x) + c_1)$$

2.17 problem 17

Internal problem ID [5765]

Book: Ordinary differential equations and calculus of variations. Makarets and Reshetnyak. Wold Scientific. Singapore. 1995
Section: Chapter 1. First order differential equations. Section 1.2 Homogeneous equations problems. page 12
Problem number: 17.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[[_homogeneous, ‘class A‘], _rational, _dAlembert]`

$$\boxed{\frac{1}{x^2 - xy + y^2} - \frac{y'}{2y^2 - xy} = 0}$$

✓ Solution by Maple
Time used: 5.532 (sec). Leaf size: 40

```
dsolve(1/(x^2-x*y(x)+y(x)^2)=1/(2*y(x)^2-x*y(x))*diff(y(x),x),y(x), singsol=all)
```

$$y(x) = \left(\operatorname{RootOf}\left(\_Z^8 c_1 x^2 + 2\_Z^6 c_1 x^2 - \_Z^4 - 2\_Z^2 - 1\right)^2 + 2\right) x$$

✓ Solution by Mathematica

Time used: 60.201 (sec). Leaf size: 1805

```
DSolve[1/(x^2-x*y[x]+y[x]^2)==1/(2*y[x]^2-x*y[x])*y'[x],y[x],x,IncludeSingularSolutions -> T
```

$y(x)$

$$\rightarrow \frac{1}{6}\left(-\sqrt{3}\sqrt{\sqrt[3]{54e^{2c_1}x^4+6\sqrt{3}\sqrt{e^{4c_1}x^4\left(27x^4+e^{4c_1}\right)}+e^{6c_1}}+\frac{e^{4c_1}}{\sqrt[3]{54e^{2c_1}x^4+6\sqrt{3}\sqrt{e^{4c_1}x^4\left(27x^4+e^{4c_1}\right)}+}}\right.$$

$$-\sqrt{3}\sqrt{-\sqrt[3]{54e^{2c_1}x^4+6\sqrt{3}\sqrt{e^{4c_1}x^4\left(27x^4+e^{4c_1}\right)}+e^{6c_1}}-\frac{e^{4c_1}}{\sqrt[3]{54e^{2c_1}x^4+6\sqrt{3}\sqrt{e^{4c_1}x^4\left(27x^4+e^{4c_1}\right)}+e^{6}}}}$$

$$\left.+9x\right)$$

$y(x)$

$$\rightarrow \frac{1}{6}\left(-\sqrt{3}\sqrt{\sqrt[3]{54e^{2c_1}x^4+6\sqrt{3}\sqrt{e^{4c_1}x^4\left(27x^4+e^{4c_1}\right)}+e^{6c_1}}+\frac{e^{4c_1}}{\sqrt[3]{54e^{2c_1}x^4+6\sqrt{3}\sqrt{e^{4c_1}x^4\left(27x^4+e^{4c_1}\right)}+}}\right.$$

$$+\sqrt{3}\sqrt{-\sqrt[3]{54e^{2c_1}x^4+6\sqrt{3}\sqrt{e^{4c_1}x^4\left(27x^4+e^{4c_1}\right)}+e^{6c_1}}-\frac{e^{4c_1}}{\sqrt[3]{54e^{2c_1}x^4+6\sqrt{3}\sqrt{e^{4c_1}x^4\left(27x^4+e^{4c_1}\right)}+e^{6}}}}$$

$$\left.+9x\right)$$

$y(x)$

$$\frac{e^{4c_1}}{}$$

2.18 problem 18

Internal problem ID [5766]

Book: Ordinary differential equations and calculus of variations. Makarets and Reshetnyak. Wold Scientific. Singapore. 1995
Section: Chapter 1. First order differential equations. Section 1.2 Homogeneous equations problems. page 12
Problem number: 18.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[[_homogeneous, 'class A'], _rational, _dAlembert]`

$$\boxed{y' - \frac{2xy}{3x^2 - y^2} = 0}$$

✓ Solution by Maple
Time used: 0.031 (sec). Leaf size: 317

```
dsolve(diff(y(x),x)=2*x*y(x)/(3*x^2-y(x)^2),y(x), singsol=all)
```

$$y(x) = \frac{1 + \frac{\left(12\sqrt{3}\,x\sqrt{27x^2c_1^2-4}\,c_1-108x^2c_1^2+8\right)^{\frac{1}{3}}}{2} + \frac{2}{\left(12\sqrt{3}\,x\sqrt{27x^2c_1^2-4}\,c_1-108x^2c_1^2+8\right)^{\frac{1}{3}}}}{3c_1}$$

$$y(x) = -\frac{\left(1+i\sqrt{3}\right)\left(12\sqrt{3}\,x\sqrt{27x^2c_1^2-4}\,c_1-108x^2c_1^2+8\right)^{\frac{2}{3}} - 4i\sqrt{3} - 4\left(12\sqrt{3}\,x\sqrt{27x^2c_1^2-4}\,c_1-108x^2c_1^2 \cdots\right.}{12\left(12\sqrt{3}\,x\sqrt{27x^2c_1^2-4}\,c_1-108x^2c_1^2+8\right)^{\frac{1}{3}}c_1}$$

$$y(x) = \frac{\left(i\sqrt{3}-1\right)\left(12\sqrt{3}\,x\sqrt{27x^2c_1^2-4}\,c_1-108x^2c_1^2+8\right)^{\frac{2}{3}} - 4i\sqrt{3} + 4\left(12\sqrt{3}\,x\sqrt{27x^2c_1^2-4}\,c_1-108x^2c_1^2+8 \cdots\right.}{12\left(12\sqrt{3}\,x\sqrt{27x^2c_1^2-4}\,c_1-108x^2c_1^2+8\right)^{\frac{1}{3}}c_1}$$

✓ Solution by Mathematica

Time used: 60.196 (sec). Leaf size: 458

```
DSolve[y'[x]==2*x*y[x]/(3*x^2-y[x]^2),y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to \frac{1}{3}\left(\frac{\sqrt[3]{27e^{c_1}x^2+3\sqrt{81e^{2c_1}x^4-12e^{4c_1}x^2}-2e^{3c_1}}}{\sqrt[3]{2}} + \frac{\sqrt[3]{2}e^{2c_1}}{\sqrt[3]{27e^{c_1}x^2+3\sqrt{81e^{2c_1}x^4-12e^{4c_1}x^2}-2e^{3c_1}}} - e^{c_1}\right)$$

$$y(x) \to \frac{i(\sqrt{3}+i)\sqrt[3]{27e^{c_1}x^2+3\sqrt{81e^{2c_1}x^4-12e^{4c_1}x^2}-2e^{3c_1}}}{6\sqrt[3]{2}} - \frac{i(\sqrt{3}-i)e^{2c_1}}{3\ 2^{2/3}\sqrt[3]{27e^{c_1}x^2+3\sqrt{81e^{2c_1}x^4-12e^{4c_1}x^2}-2e^{3c_1}}} - \frac{e^{c_1}}{3}$$

$$y(x) \to -\frac{i(\sqrt{3}-i)\sqrt[3]{27e^{c_1}x^2+3\sqrt{81e^{2c_1}x^4-12e^{4c_1}x^2}-2e^{3c_1}}}{6\sqrt[3]{2}} + \frac{i(\sqrt{3}+i)e^{2c_1}}{3\ 2^{2/3}\sqrt[3]{27e^{c_1}x^2+3\sqrt{81e^{2c_1}x^4-12e^{4c_1}x^2}-2e^{3c_1}}} - \frac{e^{c_1}}{3}$$

2.19 problem 19

Internal problem ID [5767]

Book: Ordinary differential equations and calculus of variations. Makarets and Reshetnyak. Wold Scientific. Singapore. 1995
Section: Chapter 1. First order differential equations. Section 1.2 Homogeneous equations problems. page 12
Problem number: 19.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[[_homogeneous, ‘class A‘], _rational, _Bernoulli]`

$$\boxed{y' - \frac{x}{y} - \frac{y}{x} = 0}$$

With initial conditions

$$[y(-1) = 0]$$

✓ Solution by Maple
Time used: 0.078 (sec). Leaf size: 34

```
dsolve([diff(y(x),x)=x/y(x)+y(x)/x,y(-1) = 0],y(x), singsol=all)
```

$$y(x) = \sqrt{2\ln(x) - 2i\pi}\, x$$
$$y(x) = -\sqrt{2\ln(x) - 2i\pi}\, x$$

✓ Solution by Mathematica
Time used: 0.19 (sec). Leaf size: 48

```
DSolve[{y'[x]==x/y[x]+y[x]/x,{y[-1]==0}},y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to -\sqrt{2}x\sqrt{\log(x) - i\pi}$$
$$y(x) \to \sqrt{2}x\sqrt{\log(x) - i\pi}$$

2.20 problem 20

Internal problem ID [5768]

Book: Ordinary differential equations and calculus of variations. Makarets and Reshetnyak. Wold Scientific. Singapore. 1995
Section: Chapter 1. First order differential equations. Section 1.2 Homogeneous equations problems. page 12
Problem number: 20.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[[_homogeneous, 'class A'], _rational, _dAlembert]`

$$\boxed{xy' - y - \sqrt{y^2 - x^2} = 0}$$

✓ Solution by Maple
Time used: 0.016 (sec). Leaf size: 28

```
dsolve(x*diff(y(x),x)=y(x)+sqrt(y(x)^2-x^2),y(x), singsol=all)
```

$$\frac{-c_1 x^2 + y(x) + \sqrt{y(x)^2 - x^2}}{x^2} = 0$$

✓ Solution by Mathematica
Time used: 0.366 (sec). Leaf size: 14

```
DSolve[x*y'[x]==y[x]+Sqrt[y[x]^2-x^2],y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to -x \cosh(\log(x) + c_1)$$

2.21 problem 21

Internal problem ID [5769]

Book: Ordinary differential equations and calculus of variations. Makarets and Reshetnyak. Wold Scientific. Singapore. 1995
Section: Chapter 1. First order differential equations. Section 1.2 Homogeneous equations problems. page 12
Problem number: 21.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[[_homogeneous, 'class A'], _dAlembert]`

$$\boxed{y + (2\sqrt{xy} - x)\, y' = 0}$$

✓ Solution by Maple
Time used: 0.0 (sec). Leaf size: 18

```
dsolve(y(x)+(2*sqrt(x*y(x))-x)*diff(y(x),x)=0,y(x), singsol=all)
```

$$\ln\left(y(x)\right) + \frac{x}{\sqrt{xy\left(x\right)}} - c_1 = 0$$

✓ Solution by Mathematica
Time used: 0.23 (sec). Leaf size: 33

```
DSolve[y[x]+(2*Sqrt[x*y[x]]-x)*y'[x]==0,y[x],x,IncludeSingularSolutions -> True]
```

$$\text{Solve}\left[\frac{2}{\sqrt{\frac{y(x)}{x}}} + 2\log\left(\frac{y(x)}{x}\right) = -2\log(x) + c_1, y(x)\right]$$

2.22 problem 22

Internal problem ID [5770]

Book: Ordinary differential equations and calculus of variations. Makarets and Reshetnyak. Wold Scientific. Singapore. 1995
Section: Chapter 1. First order differential equations. Section 1.2 Homogeneous equations problems. page 12
Problem number: 22.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[[_homogeneous, ‘class A‘], _dAlembert]`

$$\boxed{xy' - \ln\left(\frac{y}{x}\right) y = 0}$$

✓ Solution by Maple
Time used: 0.015 (sec). Leaf size: 12

```
dsolve(x*diff(y(x),x)=y(x)*ln(y(x)/x),y(x), singsol=all)
```

$$y(x) = \mathrm{e}^{c_1 x+1} x$$

✓ Solution by Mathematica
Time used: 0.199 (sec). Leaf size: 24

```
DSolve[x*y'[x]==y[x]*Log[y[x]/x],y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to xe^{1+e^{c_1}x}$$
$$y(x) \to ex$$

2.23 problem 23

Internal problem ID [5771]

Book: Ordinary differential equations and calculus of variations. Makarets and Reshetnyak. Wold Scientific. Singapore. 1995
Section: Chapter 1. First order differential equations. Section 1.2 Homogeneous equations problems. page 12
Problem number: 23.
ODE order: 1.
ODE degree: 2.

CAS Maple gives this as type `[_quadrature]`

$$\boxed{y'(y + y') - x(x + y) = 0}$$

With initial conditions

$$[y(0) = 0]$$

✓ Solution by Maple
Time used: 0.015 (sec). Leaf size: 9

```
dsolve([diff(y(x),x)*(diff(y(x),x)+y(x))=x*(x+y(x)),y(0) = 0],y(x), singsol=all)
```

$$y(x) = \frac{x^2}{2}$$

✓ Solution by Mathematica
Time used: 0.043 (sec). Leaf size: 28

```
DSolve[{y'[x]*(y'[x]+y[x])==x*(x+y[x]),{y[0]==0}},y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to \frac{x^2}{2}$$
$$y(x) \to -x - e^{-x} + 1$$

2.24 problem 24

Internal problem ID [5772]

Book: Ordinary differential equations and calculus of variations. Makarets and Reshetnyak. Wold Scientific. Singapore. 1995
Section: Chapter 1. First order differential equations. Section 1.2 Homogeneous equations problems. page 12
Problem number: 24.
ODE order: 1.
ODE degree: 2.

CAS Maple gives this as type `[[_homogeneous, ‘class A‘], _dAlembert]`

$$\boxed{(xy' + y)^2 - y'y^2 = 0}$$

✓ Solution by Maple
Time used: 0.078 (sec). Leaf size: 124

```
dsolve((x*diff(y(x),x)+y(x))^2=y(x)^2*diff(y(x),x),y(x), singsol=all)
```

$$y(x) = 4x$$
$$y(x) = 0$$
$$y(x) = -\frac{2c_1^2\left(-\sqrt{2}\, c_1 + x\right)}{-2c_1^2 + x^2}$$
$$y(x) = -\frac{2c_1^2\left(\sqrt{2}\, c_1 + x\right)}{-2c_1^2 + x^2}$$
$$y(x) = \frac{c_1^3\sqrt{2} - 2c_1^2 x}{-2c_1^2 + 4x^2}$$
$$y(x) = \frac{c_1^2\left(\sqrt{2}\, c_1 + 2x\right)}{2c_1^2 - 4x^2}$$

✓ <u>Solution by Mathematica</u>

Time used: 0.501 (sec). Leaf size: 62

```
DSolve[(x*y'[x]+y[x])^2==y[x]^2*y'[x],y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to -\frac{4e^{-2c_1}}{2 + e^{2c_1}x}$$
$$y(x) \to -\frac{e^{-2c_1}}{2 + 4e^{2c_1}x}$$
$$y(x) \to 0$$
$$y(x) \to 4x$$

2.25 problem 25

Internal problem ID [5773]

Book: Ordinary differential equations and calculus of variations. Makarets and Reshetnyak. Wold Scientific. Singapore. 1995
Section: Chapter 1. First order differential equations. Section 1.2 Homogeneous equations problems. page 12
Problem number: 25.
ODE order: 1.
ODE degree: 2.

CAS Maple gives this as type `[_separable]`

$$\boxed{x^2 {y'}^2 - 3xyy' + 2y^2 = 0}$$

✓ Solution by Maple
Time used: 0.0 (sec). Leaf size: 15

```
dsolve(x^2*diff(y(x),x)^2-3*x*y(x)*diff(y(x),x)+2*y(x)^2=0,y(x), singsol=all)
```

$$y(x) = c_1 x^2$$
$$y(x) = c_1 x$$

✓ Solution by Mathematica
Time used: 0.043 (sec). Leaf size: 24

```
DSolve[x^2*(y'[x])^2-3*x*y[x]*y'[x]+2*y[x]^2==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to c_1 x$$
$$y(x) \to c_1 x^2$$
$$y(x) \to 0$$

2.26 problem 26

Internal problem ID [5774]

Book: Ordinary differential equations and calculus of variations. Makarets and Reshetnyak. Wold Scientific. Singapore. 1995
Section: Chapter 1. First order differential equations. Section 1.2 Homogeneous equations problems. page 12
Problem number: 26.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[[_homogeneous, ‘class A‘], _rational, _dAlembert]`

$$\boxed{-y + xy' - \sqrt{x^2 + y^2} = 0}$$

✓ Solution by Maple
Time used: 0.016 (sec). Leaf size: 26

```
dsolve(x*diff(y(x),x)-y(x)=sqrt(x^2+y(x)^2),y(x), singsol=all)
```

$$\frac{-c_1x^2 + y(x) + \sqrt{x^2 + y(x)^2}}{x^2} = 0$$

✓ Solution by Mathematica
Time used: 0.331 (sec). Leaf size: 27

```
DSolve[x*y'[x]-y[x]==Sqrt[x^2+y[x]^2],y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to \frac{1}{2}e^{-c_1}\left(-1 + e^{2c_1}x^2\right)$$

2.27 problem 27

Internal problem ID [5775]

Book: Ordinary differential equations and calculus of variations. Makarets and Reshetnyak. Wold Scientific. Singapore. 1995
Section: Chapter 1. First order differential equations. Section 1.2 Homogeneous equations problems. page 12
Problem number: 27.
ODE order: 1.
ODE degree: 2.

CAS Maple gives this as type `[[_homogeneous, ‘class A‘], _rational, _dAlembert]`

$$\boxed{yy'^2 + 2xy' - y = 0}$$

✓ Solution by Maple
Time used: 0.078 (sec). Leaf size: 71

```
dsolve(y(x)*diff(y(x),x)^2+2*x*diff(y(x),x)-y(x)=0,y(x), singsol=all)
```

$$y(x) = -ix$$
$$y(x) = ix$$
$$y(x) = 0$$
$$y(x) = \sqrt{c_1 (c_1 - 2x)}$$
$$y(x) = \sqrt{c_1 (c_1 + 2x)}$$
$$y(x) = -\sqrt{c_1 (c_1 - 2x)}$$
$$y(x) = -\sqrt{c_1 (c_1 + 2x)}$$

✓ Solution by Mathematica

Time used: 0.451 (sec). Leaf size: 126

```
DSolve[y[x]*(y'[x])^2+2*x*y'[x]-y[x]==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to -e^{\frac{c_1}{2}}\sqrt{-2x+e^{c_1}}$$
$$y(x) \to e^{\frac{c_1}{2}}\sqrt{-2x+e^{c_1}}$$
$$y(x) \to -e^{\frac{c_1}{2}}\sqrt{2x+e^{c_1}}$$
$$y(x) \to e^{\frac{c_1}{2}}\sqrt{2x+e^{c_1}}$$
$$y(x) \to 0$$
$$y(x) \to -ix$$
$$y(x) \to ix$$

2.28 problem 28

Internal problem ID [5776]

Book: Ordinary differential equations and calculus of variations. Makarets and Reshetnyak. Wold Scientific. Singapore. 1995
Section: Chapter 1. First order differential equations. Section 1.2 Homogeneous equations problems. page 12
Problem number: 28.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_linear]`

$$y' + \frac{2y + x}{x} = 0$$

✓ Solution by Maple
Time used: 0.016 (sec). Leaf size: 13

```
dsolve(diff(y(x),x)+(x+2*y(x))/x=0,y(x), singsol=all)
```

$$y(x) = -\frac{x}{3} + \frac{c_1}{x^2}$$

✓ Solution by Mathematica
Time used: 0.025 (sec). Leaf size: 17

```
DSolve[y'[x]+(x+2*y[x])/x==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to -\frac{x}{3} + \frac{c_1}{x^2}$$

2.29 problem 29

Internal problem ID [5777]

Book: Ordinary differential equations and calculus of variations. Makarets and Reshetnyak. Wold Scientific. Singapore. 1995
Section: Chapter 1. First order differential equations. Section 1.2 Homogeneous equations problems. page 12
Problem number: 29.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[[_homogeneous, ‘class A‘], _rational, [_Abel, ‘2nd type‘, ‘cl`

$$\boxed{y' - \frac{y}{x+y} = 0}$$

✓ Solution by Maple
Time used: 0.016 (sec). Leaf size: 13

```
dsolve(diff(y(x),x)=y(x)/(x+y(x)),y(x), singsol=all)
```

$$y(x) = \frac{x}{\text{LambertW}\left(x\,e^{c_1}\right)}$$

✓ Solution by Mathematica
Time used: 3.517 (sec). Leaf size: 23

```
DSolve[y'[x]==y[x]/(x+y[x]),y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to \frac{x}{W\left(e^{-c_1}x\right)}$$
$$y(x) \to 0$$

2.30 problem 30

Internal problem ID [5778]

Book: Ordinary differential equations and calculus of variations. Makarets and Reshetnyak. Wold Scientific. Singapore. 1995
Section: Chapter 1. First order differential equations. Section 1.2 Homogeneous equations problems. page 12
Problem number: 30.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_linear]`

$$\boxed{xy' - \frac{y}{2} = x}$$

With initial conditions

$$[y(0) = 0]$$

✓ Solution by Maple
Time used: 0.047 (sec). Leaf size: 13

```
dsolve([x*diff(y(x),x)=x+1/2*y(x),y(0) = 0],y(x), singsol=all)
```

$$y(x) = 2x + \sqrt{x}\, c_1$$

✓ Solution by Mathematica
Time used: 0.046 (sec). Leaf size: 17

```
DSolve[{x*y'[x]==x+1/2*y[x],{y[0]==0}},y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to 2x + c_1\sqrt{x}$$

2.31 problem Example 3

Internal problem ID [5779]

Book: Ordinary differential equations and calculus of variations. Makarets and Reshetnyak. Wold Scientific. Singapore. 1995
Section: Chapter 1. First order differential equations. Section 1.2 Homogeneous equations problems. page 12
Problem number: Example 3.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[[_homogeneous, ‘class C‘], _rational, [_Abel, ‘2nd type‘, ‘cl`

$$\boxed{y' - \frac{x + y - 2}{y - x - 4} = 0}$$

✓ Solution by Maple
Time used: 0.265 (sec). Leaf size: 30

```
dsolve(diff(y(x),x)=(x+y(x)-2)/(y(x)-x-4),y(x), singsol=all)
```

$$y(x) = \frac{-\sqrt{2\left(x+1\right)^2 c_1^2 + 1} + \left(x+4\right) c_1}{c_1}$$

✓ Solution by Mathematica
Time used: 0.807 (sec). Leaf size: 59

```
DSolve[y'[x]==(x+y[x]-2)/(y[x]-x-4),y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to -i\sqrt{-2x^2 - 4x - 16 - c_1} + x + 4$$
$$y(x) \to i\sqrt{-2x^2 - 4x - 16 - c_1} + x + 4$$

2.32 problem Example 4

Internal problem ID [5780]

Book: Ordinary differential equations and calculus of variations. Makarets and Reshetnyak. Wold Scientific. Singapore. 1995
Section: Chapter 1. First order differential equations. Section 1.2 Homogeneous equations problems. page 12
Problem number: Example 4.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[[_homogeneous, ‘class C‘], _rational, [_Abel, ‘2nd type‘, ‘cl`

$$\boxed{-4y + (x + y - 2)\, y' = -2x - 6}$$

✓ Solution by Maple
Time used: 0.172 (sec). Leaf size: 198

```
dsolve((2*x-4*y(x)+6)+(x+y(x)-2)*diff(y(x),x)=0,y(x), singsol=all)
```

$$y(x) = -\frac{2\left(\left(\frac{i\sqrt{3}}{72} - \frac{1}{72}\right)\left(36\sqrt{3}\left(x - \frac{1}{3}\right)c_1^2\sqrt{\frac{243\left(x-\frac{1}{3}\right)^2 c_1 - 12x + 4}{c_1}} + 8 + 972\left(x - \frac{1}{3}\right)^2 c_1^2 + (-216x + 72)\, c_1\right)^{\frac{2}{3}} + \left(\frac{}{1}\right.\right.}{\left(36\sqrt{3}\,\left(x - \frac{1}{3}\right)c_1^2\sqrt{\frac{243(x}{}}\right.}$$

✓ Solution by Mathematica
Time used: 60.144 (sec). Leaf size: 2563

```
DSolve[(2*x-4*y[x]+6)+(x+y[x]-2)*y'[x]==0,y[x],x,IncludeSingularSolutions -> True]
```

Too large to display

2.33 problem 31

Internal problem ID [5781]

Book: Ordinary differential equations and calculus of variations. Makarets and Reshetnyak. Wold Scientific. Singapore. 1995
Section: Chapter 1. First order differential equations. Section 1.2 Homogeneous equations problems. page 12
Problem number: 31.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[[_homogeneous, ‘class C‘], _rational, [_Abel, ‘2nd type‘, ‘cl`

$$\boxed{y' - \frac{2y - x + 5}{2x - y - 4} = 0}$$

✓ Solution by Maple
Time used: 0.391 (sec). Leaf size: 117

```
dsolve(diff(y(x),x)=(2*y(x)-x+5)/(2*x-y(x)-4),y(x), singsol=all)
```

$$y(x) = -\frac{(i\sqrt{3}-1)\left(27c_1(x-1)+3\sqrt{3}\sqrt{27(x-1)^2c_1^2-1}\right)^{\frac{2}{3}} - 3i\sqrt{3} - 3 + 6\left(3\sqrt{3}\sqrt{27(x-1)^2c_1^2-1}+2\right.}{6\left(27c_1(x-1)+3\sqrt{3}\sqrt{27(x-1)^2c_1^2-1}\right)^{\frac{1}{3}}c_1}$$

✓ Solution by Mathematica
Time used: 60.196 (sec). Leaf size: 1601

```
DSolve[y'[x]==(2*y[x]-x+5)/(2*x-y[x]-4),y[x],x,IncludeSingularSolutions -> True]
```

Too large to display

2.34 problem 32

Internal problem ID [5782]

Book: Ordinary differential equations and calculus of variations. Makarets and Reshetnyak. Wold Scientific. Singapore. 1995
Section: Chapter 1. First order differential equations. Section 1.2 Homogeneous equations problems. page 12
Problem number: 32.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[[_homogeneous, 'class C'], _rational, [_Abel, '2nd type', 'cl`

$$y' + \frac{4x + 3y + 15}{2x + y + 7} = 0$$

✓ Solution by Maple
Time used: 1.234 (sec). Leaf size: 227

```
dsolve(diff(y(x),x)=-(4*x+3*y(x)+15)/(2*x+y(x)+7),y(x), singsol=all)
```

$$y(x) = \frac{-24(x+3)^2 c_1 \left(x + \frac{10}{3}\right) \left(4\sqrt{-4\left(-\frac{1}{4} + (x+3)^3 c_1\right)(x+3)^6 c_1^2} + 4(x^3 + 9x^2 + 27x + 27) c_1\right)^{\frac{2}{3}} + i\Big(-1\cdots}{\cdots}$$

✓ Solution by Mathematica

Time used: 60.066 (sec). Leaf size: 763

```
DSolve[y'[x]==-(4*x+3*y[x]+15)/(2*x+y[x]+7),y[x],x,IncludeSingularSolutions -> True]
```

$y(x)$

$\rightarrow \overline{\text{Root}\left[\#1^6\left(16x^6+288x^5+2160x^4+8640x^3+19440x^2+23328x+11664+16e^{12c_1}\right)+\#1^4\left(-24x^4-\right.\right.}$

$-2x-7$

$y(x)$

$\rightarrow \overline{\text{Root}\left[\#1^6\left(16x^6+288x^5+2160x^4+8640x^3+19440x^2+23328x+11664+16e^{12c_1}\right)+\#1^4\left(-24x^4-\right.\right.}$

$-2x-7$

$y(x)$

$\rightarrow \overline{\text{Root}\left[\#1^6\left(16x^6+288x^5+2160x^4+8640x^3+19440x^2+23328x+11664+16e^{12c_1}\right)+\#1^4\left(-24x^4-\right.\right.}$

$-2x-7$

$y(x)$

$\rightarrow \overline{\text{Root}\left[\#1^6\left(16x^6+288x^5+2160x^4+8640x^3+19440x^2+23328x+11664+16e^{12c_1}\right)+\#1^4\left(-24x^4-\right.\right.}$

$-2x-7$

$y(x)$

$\rightarrow \overline{\text{Root}\left[\#1^6\left(16x^6+288x^5+2160x^4+8640x^3+19440x^2+23328x+11664+16e^{12c_1}\right)+\#1^4\left(-24x^4-\right.\right.}$

$-2x-7$

$y(x)$

$\rightarrow \overline{\text{Root}\left[\#1^6\left(16x^6+288x^5+2160x^4+8640x^3+19440x^2+23328x+11664+16e^{12c_1}\right)+\#1^4\left(-24x^4-\right.\right.}$

$-2x-7$

2.35 problem 33

Internal problem ID [5783]

Book: Ordinary differential equations and calculus of variations. Makarets and Reshetnyak. Wold Scientific. Singapore. 1995

Section: Chapter 1. First order differential equations. Section 1.2 Homogeneous equations problems. page 12

Problem number: 33.

ODE order: 1.

ODE degree: 1.

CAS Maple gives this as type `[[_homogeneous, ‘class C‘], _rational, [_Abel, ‘2nd type‘, ‘cl`

$$\boxed{y' - \frac{x+3y-5}{x-y-1} = 0}$$

✓ Solution by Maple

Time used: 0.188 (sec). Leaf size: 32

```
dsolve(diff(y(x),x)=(x+3*y(x)-5)/(x-y(x)-1),y(x), singsol=all)
```

$$y(x) = \frac{(-x+3)\,\mathrm{LambertW}\left(2c_1(-2+x)\right) - 2x + 4}{\mathrm{LambertW}\left(2c_1\left(-2+x\right)\right)}$$

✓ Solution by Mathematica

Time used: 1.041 (sec). Leaf size: 148

```
DSolve[y'[x]==(x+3*y[x]-5)/(x-y[x]-1),y[x],x,IncludeSingularSolutions -> True]
```

$$\text{Solve}\left[-\frac{2^{2/3}\left(x\log\left(-\frac{y(x)+x-3}{-y(x)+x-1}\right) - (x-3)\log\left(\frac{x-2}{-y(x)+x-1}\right) - 3\log\left(-\frac{y(x)+x-3}{-y(x)+x-1}\right) - y(x)\left(\log\left(\frac{x-2}{-y(x)+x-1}\right.\right.\right.}{9(y(x)+x-3)}\right.$$

2.36 problem 34

Internal problem ID [5784]

Book: Ordinary differential equations and calculus of variations. Makarets and Reshetnyak. Wold Scientific. Singapore. 1995
Section: Chapter 1. First order differential equations. Section 1.2 Homogeneous equations problems. page 12
Problem number: 34.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[[_homogeneous, ‘class C‘], _rational]`

$$y' - \frac{2(2+y)^2}{(y+x+1)^2} = 0$$

✓ Solution by Maple
Time used: 0.031 (sec). Leaf size: 25

```
dsolve(diff(y(x),x)=2*((y(x)+2)/(x+y(x)+1))^2,y(x), singsol=all)
```

$$y(x) = -2 - \tan\left(\text{RootOf}\left(-2\_Z + \ln\left(\tan\left(\_Z\right)\right) + \ln\left(x-1\right) + c_1\right)\right)\left(x-1\right)$$

✓ Solution by Mathematica
Time used: 0.138 (sec). Leaf size: 27

```
DSolve[y'[x]==2*((y[x]+2)/(x+y[x]+1))^2,y[x],x,IncludeSingularSolutions -> True]
```

$$\text{Solve}\left[2\arctan\left(\frac{1-x}{y(x)+2}\right) + \log(y(x)+2) = c_1, y(x)\right]$$

2.37 problem 35

Internal problem ID [5785]

Book: Ordinary differential equations and calculus of variations. Makarets and Reshetnyak. Wold Scientific. Singapore. 1995
Section: Chapter 1. First order differential equations. Section 1.2 Homogeneous equations problems. page 12
Problem number: 35.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[[_homogeneous, ‘class C‘], _rational, [_Abel, ‘2nd type‘, ‘cl`

$$\boxed{y - (4x + 2y - 3)\, y' = -1 - 2x}$$

✓ Solution by Maple
Time used: 0.031 (sec). Leaf size: 23

```
dsolve((2*x+y(x)+1)-(4*x+2*y(x)-3)*diff(y(x),x)=0,y(x), singsol=all)
```

$$y(x) = -\frac{\mathrm{LambertW}\left(-2\,\mathrm{e}^{-5x+2+5c_1}\right)}{2} - 2x + 1$$

✓ Solution by Mathematica
Time used: 11.239 (sec). Leaf size: 35

```
DSolve[(2*x+y[x]+1)-(4*x+2*y[x]-3)*y'[x]==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to -\frac{1}{2}W\left(-e^{-5x-1+c_1}\right) - 2x + 1$$
$$y(x) \to 1 - 2x$$

2.38 problem 36

Internal problem ID [5786]

Book: Ordinary differential equations and calculus of variations. Makarets and Reshetnyak. Wold Scientific. Singapore. 1995
Section: Chapter 1. First order differential equations. Section 1.2 Homogeneous equations problems. page 12
Problem number: 36.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[[_homogeneous, ‘class C‘], _exact, _rational, [_Abel, ‘2nd ty`

$$\boxed{-y + (y - x + 2)\, y' = 1 - x}$$

✓ Solution by Maple
Time used: 0.015 (sec). Leaf size: 35

```
dsolve((x-y(x)-1)+(y(x)-x+2)*diff(y(x),x)=0,y(x), singsol=all)
```

$$y(x) = x - 2 - \sqrt{2c_1 - 2x + 4}$$
$$y(x) = x - 2 + \sqrt{2c_1 - 2x + 4}$$

✓ Solution by Mathematica
Time used: 0.102 (sec). Leaf size: 49

```
DSolve[(x-y[x]-1)+(y[x]-x+2)*y'[x]==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to x - i\sqrt{2x - 4 - c_1} - 2$$
$$y(x) \to x + i\sqrt{2x - 4 - c_1} - 2$$

2.39 problem 37

Internal problem ID [5787]

Book: Ordinary differential equations and calculus of variations. Makarets and Reshetnyak. Wold Scientific. Singapore. 1995
Section: Chapter 1. First order differential equations. Section 1.2 Homogeneous equations problems. page 12
Problem number: 37.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[[_homogeneous, ‘class C‘], _rational, [_Abel, ‘2nd type‘, ‘cl`

$$\boxed{(x+4y)\,y'-3y=2x-5}$$

✓ Solution by Maple
Time used: 0.437 (sec). Leaf size: 186

```
dsolve((x+4*y(x))*diff(y(x),x)=2*x+3*y(x)-5,y(x), singsol=all)
```

$$y(x) = \frac{(x-5)\,\mathrm{RootOf}\left(\_Z^{36}+\left(3c_1x^6-72c_1x^5+720c_1x^4-3840c_1x^3+11520c_1x^2-18432c_1x+12288c_1\right)\_Z\right.}{\mathrm{RootOf}\left(\_Z^{36}+\left(3c_1x^6-72c_1x^5+720c_1x^4-3840c_1x^3+11520c_1x^2-18432c_1x+12288c_1\right)\_Z\right.}$$

✓ Solution by Mathematica

Time used: 60.076 (sec). Leaf size: 805

```
DSolve[(x+4*y[x])*y'[x]==2*x+3*y[x]-5,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to -\frac{x}{4}$$

$$+\frac{}{4\text{Root}\left[\#1^6\left(-3125x^6+75000x^5-750000x^4+4000000x^3-12000000x^2+19200000x-12800000\right.\right.}$$

$$y(x) \to -\frac{x}{4}$$

$$+\frac{}{4\text{Root}\left[\#1^6\left(-3125x^6+75000x^5-750000x^4+4000000x^3-12000000x^2+19200000x-12800000\right.\right.}$$

$$y(x) \to -\frac{x}{4}$$

$$+\frac{}{4\text{Root}\left[\#1^6\left(-3125x^6+75000x^5-750000x^4+4000000x^3-12000000x^2+19200000x-12800000\right.\right.}$$

$$y(x) \to -\frac{x}{4}$$

$$+\frac{}{4\text{Root}\left[\#1^6\left(-3125x^6+75000x^5-750000x^4+4000000x^3-12000000x^2+19200000x-12800000\right.\right.}$$

$$y(x) \to -\frac{x}{4}$$

$$+\frac{}{4\text{Root}\left[\#1^6\left(-3125x^6+75000x^5-750000x^4+4000000x^3-12000000x^2+19200000x-12800000\right.\right.}$$

$$y(x) \to -\frac{x}{4}$$

$$+\frac{}{4\text{Root}\left[\#1^6\left(-3125x^6+75000x^5-750000x^4+4000000x^3-12000000x^2+19200000x-12800000\right.\right.}$$

2.40 problem 38

Internal problem ID [5788]

Book: Ordinary differential equations and calculus of variations. Makarets and Reshetnyak. Wold Scientific. Singapore. 1995
Section: Chapter 1. First order differential equations. Section 1.2 Homogeneous equations problems. page 12
Problem number: 38.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[[_homogeneous, ‘class C‘], _rational, [_Abel, ‘2nd type‘, ‘cl`

$$\boxed{y - (2x + y - 4)\, y' = -2}$$

✓ Solution by Maple
Time used: 0.0 (sec). Leaf size: 49

```
dsolve(y(x)+2=(2*x+y(x)-4)*diff(y(x),x),y(x), singsol=all)
```

$$y(x) = \frac{-4c_1 + 1 + \sqrt{1 + 4(x-3)c_1}}{2c_1}$$
$$y(x) = \frac{-4c_1 + 1 - \sqrt{1 + 4(x-3)c_1}}{2c_1}$$

✓ Solution by Mathematica
Time used: 0.237 (sec). Leaf size: 82

```
DSolve[y[x]+2==(2*x+y[x]-4)*y'[x],y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to -\frac{\sqrt{1 + 4c_1(x-3)} - 1 + 4c_1}{2c_1}$$
$$y(x) \to \frac{\sqrt{1 + 4c_1(x-3)} + 1 - 4c_1}{2c_1}$$
$$y(x) \to -2$$
$$y(x) \to \text{Indeterminate}$$
$$y(x) \to 1 - x$$

2.41 problem 39

Internal problem ID [5789]

Book: Ordinary differential equations and calculus of variations. Makarets and Reshetnyak. Wold Scientific. Singapore. 1995
Section: Chapter 1. First order differential equations. Section 1.2 Homogeneous equations problems. page 12
Problem number: 39.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[[_homogeneous, ‘class C‘], _exact, _dAlembert]`

$$\boxed{(1+y')\ln\left(\frac{x+y}{x+3}\right)-\frac{x+y}{x+3}=0}$$

✓ Solution by Maple
Time used: 0.25 (sec). Leaf size: 40

```
dsolve((diff(y(x),x)+1)*ln((y(x)+x)/(x+3))=(y(x)+x)/(x+3),y(x), singsol=all)
```

$$y(x)=\frac{-x\,\mathrm{LambertW}\left(\frac{\mathrm{e}^{-1}}{(x+3)c_1}\right)c_1+1}{\mathrm{LambertW}\left(\frac{\mathrm{e}^{-1}}{(x+3)c_1}\right)c_1}$$

✓ Solution by Mathematica
Time used: 0.226 (sec). Leaf size: 30

```
DSolve[(y'[x]+1)*Log[(y[x]+x)/(x+3)]==(y[x]+x)/(x+3),y[x],x,IncludeSingularSolutions -> True
```

$$\text{Solve}\left[-y(x)+(y(x)+x)\log\left(\frac{y(x)+x}{x+3}\right)-x=c_1,y(x)\right]$$

2.42 problem 40

Internal problem ID [5790]

Book: Ordinary differential equations and calculus of variations. Makarets and Reshetnyak. Wold Scientific. Singapore. 1995
Section: Chapter 1. First order differential equations. Section 1.2 Homogeneous equations problems. page 12
Problem number: 40.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[[_homogeneous, 'class C'], _rational, [_Abel, '2nd type', 'cl`

$$y' - \frac{x-2y+5}{y-2x-4} = 0$$

✓ Solution by Maple
Time used: 0.375 (sec). Leaf size: 117

```
dsolve(diff(y(x),x)=(x-2*y(x)+5)/(y(x)-2*x-4),y(x), singsol=all)
```

$$y(x) = \frac{\frac{1}{2} + \frac{\left(1-i\sqrt{3}\right)\left(27(x+1)c_1+3\sqrt{3}\sqrt{27(x+1)^2c_1^2-1}\right)^{\frac{2}{3}}}{6} + \frac{i\sqrt{3}}{2} - \left(3\sqrt{3}\sqrt{27\left(x+1\right)^2c_1^2-1}+27c_1x+27c_1\right)^{\frac{1}{3}}\left(x-1\right)}{\left(27\left(x+1\right)c_1+3\sqrt{3}\sqrt{27\left(x+1\right)^2c_1^2-1}\right)^{\frac{1}{3}}c_1}$$

✓ Solution by Mathematica
Time used: 60.297 (sec). Leaf size: 1601

```
DSolve[y'[x]==(x-2*y[x]+5)/(y[x]-2*x-4),y[x],x,IncludeSingularSolutions -> True]
```

Too large to display

2.43 problem 41

Internal problem ID [5791]

Book: Ordinary differential equations and calculus of variations. Makarets and Reshetnyak. Wold Scientific. Singapore. 1995
Section: Chapter 1. First order differential equations. Section 1.2 Homogeneous equations problems. page 12
Problem number: 41.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[[_homogeneous, ‘class C‘], _rational, [_Abel, ‘2nd type‘, ‘cl`

$$y' - \frac{3x - y + 1}{2x + y + 4} = 0$$

✓ Solution by Maple
Time used: 0.797 (sec). Leaf size: 67

```
dsolve(diff(y(x),x)=(3*x-y(x)+1)/(2*x+y(x)+4),y(x), singsol=all)
```

$$-\frac{\ln\left(\frac{y(x)^2+(3x+7)y(x)-3x^2+7}{(x+1)^2}\right)}{2} + \frac{\sqrt{21}\,\operatorname{arctanh}\left(\frac{(2y(x)+7+3x)\sqrt{21}}{21x+21}\right)}{21} - \ln(x+1) - c_1 = 0$$

✓ Solution by Mathematica
Time used: 0.136 (sec). Leaf size: 79

```
DSolve[y'[x]==(3*x-y[x]+1)/(2*x+y[x]+4),y[x],x,IncludeSingularSolutions -> True]
```

$$\text{Solve}\left[2\sqrt{21}\operatorname{arctanh}\left(\frac{-\frac{10(x+1)}{y(x)+2(x+2)} - 1}{\sqrt{21}}\right) + 21\left(\log\left(-\frac{-3x^2 + y(x)^2 + (3x+7)y(x) + 7}{5(x+1)^2}\right) + 2\log(x+1) - 10c_1\right) = 0, y(x)\right]$$

2.44 problem Example 5

Internal problem ID [5792]

Book: Ordinary differential equations and calculus of variations. Makarets and Reshetnyak. Wold Scientific. Singapore. 1995
Section: Chapter 1. First order differential equations. Section 1.2 Homogeneous equations problems. page 12
Problem number: Example 5.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[[_homogeneous, ‘class G‘], _rational, _Bernoulli]`

$$\boxed{2xy' + \left(y^4x^2 + 1\right) y = 0}$$

✓ Solution by Maple
Time used: 0.032 (sec). Leaf size: 67

```
dsolve(2*x*diff(y(x),x)+(x^2*y(x)^4+1)*y(x)=0,y(x), singsol=all)
```

$$y(x) = \frac{1}{\sqrt{\sqrt{2\ln(x) + c_1}\, x}}$$
$$y(x) = \frac{1}{\sqrt{-\sqrt{2\ln(x) + c_1}\, x}}$$
$$y(x) = -\frac{1}{\sqrt{\sqrt{2\ln(x) + c_1}\, x}}$$
$$y(x) = -\frac{1}{\sqrt{-\sqrt{2\ln(x) + c_1}\, x}}$$

✓ Solution by Mathematica

Time used: 1.552 (sec). Leaf size: 92

```
DSolve[2*x*y'[x]+(x^2*y[x]^4+1)*y[x]==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to -\frac{1}{\sqrt[4]{x^2(2\log(x) + c_1)}}$$
$$y(x) \to -\frac{i}{\sqrt[4]{x^2(2\log(x) + c_1)}}$$
$$y(x) \to \frac{i}{\sqrt[4]{x^2(2\log(x) + c_1)}}$$
$$y(x) \to \frac{1}{\sqrt[4]{x^2(2\log(x) + c_1)}}$$
$$y(x) \to 0$$

2.45 problem Example 6

Internal problem ID [5793]

Book: Ordinary differential equations and calculus of variations. Makarets and Reshetnyak. Wold Scientific. Singapore. 1995
Section: Chapter 1. First order differential equations. Section 1.2 Homogeneous equations problems. page 12
Problem number: Example 6.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[[_homogeneous, ‘class G‘], _rational]`

$$\boxed{2xy'(x - y^2) + y^3 = 0}$$

✓ Solution by Maple
Time used: 0.031 (sec). Leaf size: 28

```
dsolve(2*x*diff(y(x),x)*(x-y(x)^2)+y(x)^3=0,y(x), singsol=all)
```

$$y(x) = \frac{e^{\frac{c_1}{2}}}{\sqrt{-\frac{e^{c_1}}{x\,\text{LambertW}\left(-\frac{e^{c_1}}{x}\right)}}}$$

✓ Solution by Mathematica
Time used: 2.287 (sec). Leaf size: 60

```
DSolve[2*x*y'[x]*(x-y[x]^2)+y[x]^3==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to -i\sqrt{x}\sqrt{W\left(-\frac{e^{c_1}}{x}\right)}$$
$$y(x) \to i\sqrt{x}\sqrt{W\left(-\frac{e^{c_1}}{x}\right)}$$
$$y(x) \to 0$$

2.46 problem 42

Internal problem ID [5794]

Book: Ordinary differential equations and calculus of variations. Makarets and Reshetnyak. Wold Scientific. Singapore. 1995
Section: Chapter 1. First order differential equations. Section 1.2 Homogeneous equations problems. page 12
Problem number: 42.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[[_homogeneous, ‘class G‘], _rational, _Riccati]`

$$\boxed{x^3(y'-x)-y^2=0}$$

✓ Solution by Maple
Time used: 0.015 (sec). Leaf size: 23

```
dsolve(x^3*(diff(y(x),x)-x)=y(x)^2,y(x), singsol=all)
```

$$y(x)=\frac{x^2(\ln(x)-c_1-1)}{\ln(x)-c_1}$$

✓ Solution by Mathematica
Time used: 0.157 (sec). Leaf size: 29

```
DSolve[x^3*(y'[x]-x)==y[x]^2,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x)\to\frac{x^2(\log(x)-1+c_1)}{\log(x)+c_1}$$
$$y(x)\to x^2$$

2.47 problem 43

Internal problem ID [5795]

Book: Ordinary differential equations and calculus of variations. Makarets and Reshetnyak. Wold Scientific. Singapore. 1995
Section: Chapter 1. First order differential equations. Section 1.2 Homogeneous equations problems. page 12
Problem number: 43.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[[_homogeneous, ‘class G‘], _rational, _Bernoulli]`

$$\boxed{2x^2y' - y^3 - xy = 0}$$

✓ Solution by Maple
Time used: 0.016 (sec). Leaf size: 45

```
dsolve(2*x^2*diff(y(x),x)=y(x)^3+x*y(x),y(x), singsol=all)
```

$$y(x) = \frac{\sqrt{(-\ln(x) + c_1)\, x}}{\ln(x) - c_1}$$
$$y(x) = \frac{\sqrt{(-\ln(x) + c_1)\, x}}{-\ln(x) + c_1}$$

✓ Solution by Mathematica
Time used: 0.158 (sec). Leaf size: 49

```
DSolve[2*x^2*y'[x]==y[x]^3+x*y[x],y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to -\frac{\sqrt{x}}{\sqrt{-\log(x) + c_1}}$$
$$y(x) \to \frac{\sqrt{x}}{\sqrt{-\log(x) + c_1}}$$
$$y(x) \to 0$$

2.48 problem 44

Internal problem ID [5796]

Book: Ordinary differential equations and calculus of variations. Makarets and Reshetnyak. Wold Scientific. Singapore. 1995
Section: Chapter 1. First order differential equations. Section 1.2 Homogeneous equations problems. page 12
Problem number: 44.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[[_homogeneous, ‘class G‘], _rational, [_Abel, ‘2nd type‘, ‘cl`

$$\boxed{y + x(1 + 2xy)\, y' = 0}$$

✓ Solution by Maple
Time used: 0.031 (sec). Leaf size: 18

```
dsolve(y(x)+x*(2*x*y(x)+1)*diff(y(x),x)=0,y(x), singsol=all)
```

$$y(x) = \frac{1}{2\,\mathrm{LambertW}\left(\frac{c_1}{2x}\right) x}$$

✓ Solution by Mathematica
Time used: 60.506 (sec). Leaf size: 36

```
DSolve[y[x]+x*(2*x*y[x]+1)*y'[x]==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to \frac{1}{2xW\left(\frac{e^{\frac{1}{2}\left(-2-9\sqrt[3]{-2}c_1\right)}}{x}\right)}$$

2.49 problem 45

Internal problem ID [5797]

Book: Ordinary differential equations and calculus of variations. Makarets and Reshetnyak. Wold Scientific. Singapore. 1995
Section: Chapter 1. First order differential equations. Section 1.2 Homogeneous equations problems. page 12
Problem number: 45.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[[_1st_order, _with_linear_symmetries], _Chini]`

$$\boxed{2y' - 4\sqrt{y} = -x}$$

✓ Solution by Maple
Time used: 0.016 (sec). Leaf size: 100

```
dsolve(2*diff(y(x),x)+x=4*sqrt(y(x)),y(x), singsol=all)
```

$$\frac{(-x^2 + 4y(x)) \ln\left(\frac{x^2-4y(x)}{x^2}\right) + 2i(x^2 - 4y(x)) \arctan\left(2\sqrt{-\frac{y(x)}{x^2}}\right) - 4i\sqrt{-\frac{y(x)}{x^2}}\, x^2 + 4(-c_1 + 2\ln(x))\, y(x)}{x^2 - 4y(x)} = 0$$

✓ Solution by Mathematica
Time used: 0.104 (sec). Leaf size: 49

```
DSolve[2*y'[x]+x==4*Sqrt[y[x]],y[x],x,IncludeSingularSolutions -> True]
```

$$\text{Solve}\left[4\left(\frac{4}{4\sqrt{\frac{y(x)}{x^2}}+2} + 2\log\left(4\sqrt{\frac{y(x)}{x^2}}+2\right)\right) = -8\log(x) + c_1, y(x)\right]$$

2.50 problem 46

Internal problem ID [5798]

Book: Ordinary differential equations and calculus of variations. Makarets and Reshetnyak. Wold Scientific. Singapore. 1995
Section: Chapter 1. First order differential equations. Section 1.2 Homogeneous equations problems. page 12
Problem number: 46.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[[_homogeneous, ‘class G‘], _rational, [_Riccati, _special]]`

$$\boxed{y' - y^2 = -\frac{2}{x^2}}$$

✓ Solution by Maple
Time used: 0.328 (sec). Leaf size: 24

```
dsolve(diff(y(x),x)=y(x)^2-2/x^2,y(x), singsol=all)
```

$$y(x) = \frac{2x^3 + c_1}{x(-x^3 + c_1)}$$

✓ Solution by Mathematica
Time used: 0.14 (sec). Leaf size: 32

```
DSolve[y'[x]==y[x]^2-2/x^2,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to \frac{-2x^3 + c_1}{x(x^3 + c_1)}$$
$$y(x) \to \frac{1}{x}$$

2.51 problem 47

Internal problem ID [5799]

Book: Ordinary differential equations and calculus of variations. Makarets and Reshetnyak. Wold Scientific. Singapore. 1995
Section: Chapter 1. First order differential equations. Section 1.2 Homogeneous equations problems. page 12
Problem number: 47.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[[_homogeneous, ‘class G‘]]`

$$\boxed{2xy' + y - y^2\sqrt{x - y^2x^2} = 0}$$

✓ Solution by Maple
Time used: 0.0 (sec). Leaf size: 38

```
dsolve(2*x*diff(y(x),x)+y(x)=y(x)^2*sqrt(x-x^2*y(x)^2),y(x), singsol=all)
```

$$-\frac{-1 + xy(x)^2}{y(x)\sqrt{-x\left(-1 + xy(x)^2\right)}} + \frac{\ln(x)}{2} - c_1 = 0$$

✓ Solution by Mathematica
Time used: 1.852 (sec). Leaf size: 62

```
DSolve[2*x*y'[x]+y[x]==y[x]^2*Sqrt[x-x^2*y[x]^2],y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to -\frac{2}{\sqrt{x\left(\log^2(x) - 2c_1\log(x) + 4 + c_1^2\right)}}$$
$$y(x) \to \frac{2}{\sqrt{x\left(\log^2(x) - 2c_1\log(x) + 4 + c_1^2\right)}}$$
$$y(x) \to 0$$

2.52 problem 48

Internal problem ID [5800]

Book: Ordinary differential equations and calculus of variations. Makarets and Reshetnyak. Wold Scientific. Singapore. 1995
Section: Chapter 1. First order differential equations. Section 1.2 Homogeneous equations problems. page 12
Problem number: 48.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[[_homogeneous, 'class G']]`

$$\boxed{\frac{2xyy'}{3}-\sqrt{x^6-y^4}-y^2=0}$$

✓ Solution by Maple
Time used: 0.031 (sec). Leaf size: 102

```
dsolve(2/3*x*y(x)*diff(y(x),x)=sqrt(x^6-y(x)^4)+y(x)^2,y(x), singsol=all)
```

$$-\left(\int_{\_b}^{x}\frac{\sqrt{\_a^6-y(x)^4}+y(x)^2}{\sqrt{\_a^6-y(x)^4}\,\_a}d\_a\right)+\frac{2\left(\int^{y(x)}\frac{\_f\left(3\sqrt{x^6-\_f^4}\left(\int_{\_b}^{x}\frac{\_a^5}{\left(\_a^6-\_f^4\right)^{\frac{3}{2}}}d\_a\right)+1\right)}{\sqrt{x^6-\_f^4}}d\_f\right)}{3}+c_1=0$$

✓ Solution by Mathematica

Time used: 6.948 (sec). Leaf size: 128

```
DSolve[2/3*x*y[x]*y'[x]==Sqrt[x^6-y[x]^4]+y[x]^2,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to -\frac{x^{3/2}}{\sqrt[4]{\sec^2\left(-\frac{\log\left(x^6\right)}{2}+3c_1\right)}}$$
$$y(x) \to -\frac{ix^{3/2}}{\sqrt[4]{\sec^2\left(-\frac{\log\left(x^6\right)}{2}+3c_1\right)}}$$
$$y(x) \to \frac{ix^{3/2}}{\sqrt[4]{\sec^2\left(-\frac{\log\left(x^6\right)}{2}+3c_1\right)}}$$
$$y(x) \to \frac{x^{3/2}}{\sqrt[4]{\sec^2\left(-\frac{\log\left(x^6\right)}{2}+3c_1\right)}}$$

2.53 problem 49

Internal problem ID [5801]

Book: Ordinary differential equations and calculus of variations. Makarets and Reshetnyak. Wold Scientific. Singapore. 1995
Section: Chapter 1. First order differential equations. Section 1.2 Homogeneous equations problems. page 12
Problem number: 49.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[[_homogeneous, ‘class G‘], _rational, [_Abel, ‘2nd type‘, ‘cl`

$$\boxed{2y + (yx^2 + 1)\, xy' = 0}$$

✓ Solution by Maple
Time used: 0.031 (sec). Leaf size: 16

```
dsolve(2*y(x)+(x^2*y(x)+1)*x*diff(y(x),x)=0,y(x), singsol=all)
```

$$y(x) = \frac{1}{\operatorname{LambertW}\left(\frac{c_1}{x^2}\right) x^2}$$

✓ Solution by Mathematica
Time used: 60.405 (sec). Leaf size: 33

```
DSolve[2*y[x]+(x^2*y[x]+1)*x*y'[x]==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to \frac{1}{x^2 W\left(\frac{e^{\frac{1}{2}\left(-2-9\sqrt[3]{-2}c_1\right)}}{x^2}\right)}$$

2.54 problem 50

Internal problem ID [5802]

Book: Ordinary differential equations and calculus of variations. Makarets and Reshetnyak. Wold Scientific. Singapore. 1995
Section: Chapter 1. First order differential equations. Section 1.2 Homogeneous equations problems. page 12
Problem number: 50.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[[_homogeneous, ‘class G‘], _rational, [_Abel, ‘2nd type‘, ‘cl`

$$\boxed{y(xy+1)+(1-xy)\,xy'=0}$$

✓ Solution by Maple

Time used: 0.031 (sec). Leaf size: 18

```
dsolve(y(x)*(1+x*y(x))+(1-x*y(x))*x*diff(y(x),x)=0,y(x), singsol=all)
```

$$y(x)=-\frac{1}{\operatorname{LambertW}\left(-\frac{c_1}{x^2}\right)x}$$

✓ Solution by Mathematica

Time used: 6.096 (sec). Leaf size: 35

```
DSolve[y[x]*(1+x*y[x])+(1-x*y[x])*x*y'[x]==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x)\to -\frac{1}{xW\left(\frac{e^{-1+\frac{9c_1}{2^{2/3}}}}{x^2}\right)}$$
$$y(x)\to 0$$

2.55 problem 51

Internal problem ID [5803]

Book: Ordinary differential equations and calculus of variations. Makarets and Reshetnyak. Wold Scientific. Singapore. 1995
Section: Chapter 1. First order differential equations. Section 1.2 Homogeneous equations problems. page 12
Problem number: 51.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[[_homogeneous, ‘class G‘], _rational]`

$$\boxed{y\left(y^{2} x^{2}+1\right)+\left(y^{2} x^{2}-1\right) x y'=0}$$

✓ Solution by Maple
Time used: 0.032 (sec). Leaf size: 33

```
dsolve(y(x)*(x^2*y(x)^2+1)+(x^2*y(x)^2-1)*x*diff(y(x),x)=0,y(x), singsol=all)
```

$$y(x)=\frac{\mathrm{e}^{-2c_{1}} x}{\sqrt{-\frac{x^{4} \mathrm{e}^{-4c_{1}}}{\operatorname{LambertW}\left(-x^{4} \mathrm{e}^{-4c_{1}}\right)}}}$$

✓ Solution by Mathematica
Time used: 31.376 (sec). Leaf size: 60

```
DSolve[y[x]*(x^2*y[x]^2+1)+(x^2*y[x]^2-1)*x*y'[x]==0,y[x],x,IncludeSingularSolutions -> True
```

$$\begin{aligned}
y(x) &\to -\frac{i\sqrt{W\left(-e^{-2c_1}x^4\right)}}{x}\\
y(x) &\to \frac{i\sqrt{W\left(-e^{-2c_1}x^4\right)}}{x}\\
y(x) &\to 0
\end{aligned}$$

2.56 problem 52

Internal problem ID [5804]

Book: Ordinary differential equations and calculus of variations. Makarets and Reshetnyak. Wold Scientific. Singapore. 1995
Section: Chapter 1. First order differential equations. Section 1.2 Homogeneous equations problems. page 12
Problem number: 52.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[[_homogeneous, 'class G'], _rational]`

$$\boxed{(x^2 - y^4)\, y' - xy = 0}$$

✓ Solution by Maple
Time used: 0.125 (sec). Leaf size: 97

```
dsolve((x^2-y(x)^4)*diff(y(x),x)-x*y(x)=0,y(x), singsol=all)
```

$$y(x) = -\frac{\sqrt{-2\sqrt{c_1^2 - 4x^2} + 2c_1}}{2}$$
$$y(x) = \frac{\sqrt{-2\sqrt{c_1^2 - 4x^2} + 2c_1}}{2}$$
$$y(x) = -\frac{\sqrt{2\sqrt{c_1^2 - 4x^2} + 2c_1}}{2}$$
$$y(x) = \frac{\sqrt{2\sqrt{c_1^2 - 4x^2} + 2c_1}}{2}$$

✓ Solution by Mathematica

Time used: 5.14 (sec). Leaf size: 122

```
DSolve[(x^2-y[x]^4)*y'[x]-x*y[x]==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to -\sqrt{-\sqrt{-x^2 + c_1{}^2} - c_1}$$
$$y(x) \to \sqrt{-\sqrt{-x^2 + c_1{}^2} - c_1}$$
$$y(x) \to -\sqrt{\sqrt{-x^2 + c_1{}^2} - c_1}$$
$$y(x) \to \sqrt{\sqrt{-x^2 + c_1{}^2} - c_1}$$
$$y(x) \to 0$$

2.57 problem 53

Internal problem ID [5805]

Book: Ordinary differential equations and calculus of variations. Makarets and Reshetnyak. Wold Scientific. Singapore. 1995
Section: Chapter 1. First order differential equations. Section 1.2 Homogeneous equations problems. page 12
Problem number: 53.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[[_homogeneous, ‘class G‘]]`

$$y\left(1+\sqrt{y^4x^2-1}\right)+2xy'=0$$

✓ Solution by Maple
Time used: 0.016 (sec). Leaf size: 32

```
dsolve(y(x)*(1+sqrt(x^2*y(x)^4-1))+2*x*diff(y(x),x)=0,y(x), singsol=all)
```

$$y(x)=\frac{\operatorname{RootOf}\left(-\ln(x)+c_1-2\left(\int^{-Z}\frac{1}{\_a\sqrt{\_a^4-1}}d\_a\right)\right)}{\sqrt{x}}$$

✓ Solution by Mathematica
Time used: 0.188 (sec). Leaf size: 40

```
DSolve[y[x]*(1+Sqrt[x^2*y[x]^4-1])+2*x*y'[x]==0,y[x],x,IncludeSingularSolutions -> True]
```

$$\text{Solve}\left[\arctan\left(\sqrt{x^2y(x)^4-1}\right)+\frac{1}{2}\log\left(x^2y(x)^4\right)-2\log(y(x))=c_1,y(x)\right]$$

3 Chapter 1. First order differential equations. Section 1.3. Exact equations problems. page 24

3.1 problem 1

Internal problem ID [5806]

Book: Ordinary differential equations and calculus of variations. Makarets and Reshetnyak. Wold Scientific. Singapore. 1995
Section: Chapter 1. First order differential equations. Section 1.3. Exact equations problems. page 24
Problem number: 1.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_exact, _rational]`

$$\boxed{x(2-9xy^2)+y(4y^2-6x^3)\,y'=0}$$

✓ Solution by Maple
Time used: 0.031 (sec). Leaf size: 125

```
dsolve(x*(2-9*x*y(x)^2)+y(x)*(4*y(x)^2-6*x^3)*diff(y(x),x)=0,y(x), singsol=all)
```

$$y(x) = -\frac{\sqrt{6x^3-2\sqrt{9x^6-4x^2-4c_1}}}{2}$$
$$y(x) = \frac{\sqrt{6x^3-2\sqrt{9x^6-4x^2-4c_1}}}{2}$$
$$y(x) = -\frac{\sqrt{6x^3+2\sqrt{9x^6-4x^2-4c_1}}}{2}$$
$$y(x) = \frac{\sqrt{6x^3+2\sqrt{9x^6-4x^2-4c_1}}}{2}$$

✓ Solution by Mathematica

Time used: 5.767 (sec). Leaf size: 163

```
DSolve[x*(2-9*x*y[x]^2)+y[x]*(4*y[x]^2-6*x^3)*y'[x]==0,y[x],x,IncludeSingularSolutions -> Tr
```

$$y(x) \to -\frac{\sqrt{3x^3 - \sqrt{9x^6 - 4x^2 + 4c_1}}}{\sqrt{2}}$$
$$y(x) \to \frac{\sqrt{3x^3 - \sqrt{9x^6 - 4x^2 + 4c_1}}}{\sqrt{2}}$$
$$y(x) \to -\frac{\sqrt{3x^3 + \sqrt{9x^6 - 4x^2 + 4c_1}}}{\sqrt{2}}$$
$$y(x) \to \frac{\sqrt{3x^3 + \sqrt{9x^6 - 4x^2 + 4c_1}}}{\sqrt{2}}$$

3.2 problem 2

Internal problem ID [5807]

Book: Ordinary differential equations and calculus of variations. Makarets and Reshetnyak. Wold Scientific. Singapore. 1995
Section: Chapter 1. First order differential equations. Section 1.3. Exact equations problems. page 24
Problem number: 2.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_exact, [_1st_order, '_with_symmetry_[F(x),G(y)]']]`

$$\boxed{\frac{y}{x} + \left(y^3 + \ln\left(x\right)\right) y' = 0}$$

✓ Solution by Maple
Time used: 0.016 (sec). Leaf size: 16

```
dsolve(y(x)/x+(y(x)^3+ln(x))*diff(y(x),x)=0,y(x), singsol=all)
```

$$\ln\left(x\right) y(x) + \frac{y(x)^4}{4} + c_1 = 0$$

✓ Solution by Mathematica

Time used: 60.188 (sec). Leaf size: 1025

```
DSolve[y[x]/x+(y[x]^3+Log[x])*y'[x]==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x)\to \frac{\sqrt{\frac{\sqrt[3]{3}\left(9\log^2(x)+\sqrt{81\log^4(x)+192c_1{}^3}\right)^{2/3}-4\ 3^{2/3}c_1}{\sqrt[3]{9\log^2(x)+\sqrt{81\log^4(x)+192c_1{}^3}}}}}{\sqrt{6}} -\frac{1}{2}\sqrt{\frac{8c_1}{\sqrt[3]{3}\sqrt[3]{9\log^2(x)+\sqrt{81\log^4(x)+192c_1{}^3}}}-\frac{2\sqrt[3]{9\log^2(x)+\sqrt{81\log^4(x)+192c_1{}^3}}}{3^{2/3}}-\frac{\cdots}{\sqrt{\frac{\sqrt[3]{3}\left(9\log^2(x)\cdots\right.}{\sqrt[3]{9\log^2(x\cdots}}}}\cdots}$$

$$y(x)\to \frac{1}{2}\left(\frac{\sqrt{\frac{2\sqrt[3]{3}\left(9\log^2(x)+\sqrt{81\log^4(x)+192c_1{}^3}\right)^{2/3}-8\ 3^{2/3}c_1}{\sqrt[3]{9\log^2(x)+\sqrt{81\log^4(x)+192c_1{}^3}}}}}{\sqrt{3}}\right.$$

$$\left. +\sqrt{\frac{8c_1}{\sqrt[3]{3}\sqrt[3]{9\log^2(x)+\sqrt{81\log^4(x)+192c_1{}^3}}}-\frac{2\sqrt[3]{9\log^2(x)+\sqrt{81\log^4(x)+192c_1{}^3}}}{3^{2/3}}-\frac{\cdots}{\sqrt{\frac{\sqrt[3]{3}\left(9\log^2(x)+\cdots\right.}{\sqrt[3]{9\log^2(x)\cdots}}}}\cdots}\right)$$

$$y(x)\to -\frac{\sqrt{\frac{\sqrt[3]{3}\left(9\log^2(x)+\sqrt{81\log^4(x)+192c_1{}^3}\right)^{2/3}-4\ 3^{2/3}c_1}{\sqrt[3]{9\log^2(x)+\sqrt{81\log^4(x)+192c_1{}^3}}}}}{\sqrt{6}} -\frac{1}{2}\sqrt{\frac{8c_1}{\sqrt[3]{3}\sqrt[3]{9\log^2(x)+\sqrt{81\log^4(x)+192c_1{}^3}}}-\frac{2\sqrt[3]{9\log^2(x)+\sqrt{81\log^4(x)+192c_1{}^3}}}{3^{2/3}}+\frac{\cdots}{\sqrt{\frac{\sqrt[3]{3}\left(9\log^2(x)\cdots\right.}{\sqrt[3]{9\log^2(x\cdots}}}}\cdots}$$

$y(x)$

3.3 problem 3

Internal problem ID [5808]

Book: Ordinary differential equations and calculus of variations. Makarets and Reshetnyak. Wold Scientific. Singapore. 1995
Section: Chapter 1. First order differential equations. Section 1.3. Exact equations problems. page 24
Problem number: 3.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_separable]`

$$\boxed{y'(2y-2)=-2x-3}$$

✓ Solution by Maple
Time used: 0.0 (sec). Leaf size: 43

```
dsolve((2*x+3)+(2*y(x)-2)*diff(y(x),x)=0,y(x), singsol=all)
```

$$y(x)=1-\sqrt{-x^2-c_1-3x+1}$$
$$y(x)=1+\sqrt{-x^2-c_1-3x+1}$$

✓ Solution by Mathematica
Time used: 0.159 (sec). Leaf size: 51

```
DSolve[(2*x+3)+(2*y[x]-2)*y'[x]==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x)\to 1-\sqrt{-x^2-3x+1+2c_1}$$
$$y(x)\to 1+\sqrt{-x^2-3x+1+2c_1}$$

3.4 problem 4

Internal problem ID [5809]

Book: Ordinary differential equations and calculus of variations. Makarets and Reshetnyak. Wold Scientific. Singapore. 1995
Section: Chapter 1. First order differential equations. Section 1.3. Exact equations problems. page 24
Problem number: 4.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[[_homogeneous, ‘class A‘], _rational, [_Abel, ‘2nd type‘, ‘cl`

$$\boxed{4y + (2x - 2y)\, y' = -2x}$$

✓ Solution by Maple
Time used: 0.328 (sec). Leaf size: 55

```
dsolve((2*x+4*y(x))+(2*x-2*y(x))*diff(y(x),x)=0,y(x), singsol=all)
```

$$-\frac{\ln\left(\frac{-x^2-3xy(x)+y(x)^2}{x^2}\right)}{2} + \frac{\sqrt{13}\,\operatorname{arctanh}\left(\frac{(2y(x)-3x)\sqrt{13}}{13x}\right)}{13} - \ln(x) - c_1 = 0$$

✓ Solution by Mathematica
Time used: 0.044 (sec). Leaf size: 51

```
DSolve[(2*x+3)+(2*y[x]-2)*y'[x]==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to 1 - \sqrt{-x^2 - 3x + 1 + 2c_1}$$
$$y(x) \to 1 + \sqrt{-x^2 - 3x + 1 + 2c_1}$$

4 Chapter 2. Linear homogeneous equations. Section 2.2 problems. page 95

4.1 problem 49

Internal problem ID [5810]

Book: Ordinary differential equations and calculus of variations. Makarets and Reshetnyak. Wold Scientific. Singapore. 1995
Section: Chapter 2. Linear homogeneous equations. Section 2.2 problems. page 95
Problem number: 49.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _missing_x]]`

$$\boxed{y'' + 2y' - y = 0}$$

✓ Solution by Maple
Time used: 0.0 (sec). Leaf size: 26

```
dsolve(diff(y(x),x$2)+2*diff(y(x),x)-y(x)=0,y(x), singsol=all)
```

$$y(x) = c_1 \mathrm{e}^{\left(\sqrt{2}-1\right)x} + c_2 \mathrm{e}^{-\left(1+\sqrt{2}\right)x}$$

✓ Solution by Mathematica
Time used: 0.02 (sec). Leaf size: 34

```
DSolve[y''[x]+2*y'[x]-y[x]==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to e^{-\left(\left(1+\sqrt{2}\right)x\right)}\left(c_2 e^{2\sqrt{2}x} + c_1\right)$$

4.2 problem 50

Internal problem ID [5811]

Book: Ordinary differential equations and calculus of variations. Makarets and Reshetnyak. Wold Scientific. Singapore. 1995
Section: Chapter 2. Linear homogeneous equations. Section 2.2 problems. page 95
Problem number: 50.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _exact, _linear, _homogeneous]]`

$$\boxed{y'' + \frac{y'}{x} - \frac{y}{x^2} = 0}$$

✓ Solution by Maple
Time used: 0.015 (sec). Leaf size: 15

```
dsolve(diff(y(x),x$2)+1/x*diff(y(x),x)-1/x^2*y(x)=0,y(x), singsol=all)
```

$$y(x) = \frac{c_2 x^2 + c_1}{x}$$

✓ Solution by Mathematica
Time used: 0.011 (sec). Leaf size: 16

```
DSolve[y''[x]+1/x*y'[x]-1/x^2*y[x]==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to \frac{c_1}{x} + c_2 x$$

4.3 problem 51

Internal problem ID [5812]

Book: Ordinary differential equations and calculus of variations. Makarets and Reshetnyak. Wold Scientific. Singapore. 1995
Section: Chapter 2. Linear homogeneous equations. Section 2.2 problems. page 95
Problem number: 51.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries], [_2nd_order, _linear,`

$$\boxed{(x^2+1)\,y''+xy'+y=0}$$

✓ Solution by Maple
Time used: 0.0 (sec). Leaf size: 15

```
dsolve((x^2+1)*diff(y(x),x$2)+x*diff(y(x),x)+y(x)=0,y(x), singsol=all)
```

$$y(x)=c_1\sin(\operatorname{arcsinh}(x))+c_2\cos(\operatorname{arcsinh}(x))$$

✓ Solution by Mathematica
Time used: 0.039 (sec). Leaf size: 43

```
DSolve[(x^2+1)*y''[x]+x*y'[x]+y[x]==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x)\to c_1\cos\left(\log\left(\sqrt{x^2+1}-x\right)\right)-c_2\sin\left(\log\left(\sqrt{x^2+1}-x\right)\right)$$

4.4 problem 52

Internal problem ID [5813]

Book: Ordinary differential equations and calculus of variations. Makarets and Reshetnyak. Wold Scientific. Singapore. 1995
Section: Chapter 2. Linear homogeneous equations. Section 2.2 problems. page 95
Problem number: 52.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$y'' - y' \cot(x) + y \cos(x) = 0$$

✓ Solution by Maple
Time used: 2.0 (sec). Leaf size: 49

```
dsolve(diff(y(x),x$2)-cot(x)*diff(y(x),x)+cos(x)*y(x)=0,y(x), singsol=all)
```

$$y(x) = (1+\cos(x))\,\mathrm{HeunC}\left(0,1,-1,-2,\frac{3}{2},\frac{\cos(x)}{2}+\frac{1}{2}\right)\left(c_1 + c_2\left(\int^{\cos(x)} \frac{1}{(\_a+1)^2\,\mathrm{HeunC}\left(0,1,-1,-2,\frac{3}{2},\frac{\_a}{2}+\frac{1}{2}\right)^2}d\_a\right)\right)$$

✗ Solution by Mathematica
Time used: 0.0 (sec). Leaf size: 0

```
DSolve[y''[x]-Cot[x]*y'[x]+Cos[x]*y[x]==0,y[x],x,IncludeSingularSolutions -> True]
```

Not solved

4.5 problem 53

Internal problem ID [5814]

Book: Ordinary differential equations and calculus of variations. Makarets and Reshetnyak. Wold Scientific. Singapore. 1995
Section: Chapter 2. Linear homogeneous equations. Section 2.2 problems. page 95
Problem number: 53.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_Emden, _Fowler]]`

$$\boxed{y'' + \frac{y'}{x} + yx^2 = 0}$$

✓ Solution by Maple
Time used: 0.0 (sec). Leaf size: 23

```
dsolve(diff(y(x),x$2)+1/x*diff(y(x),x)+x^2*y(x)=0,y(x), singsol=all)
```

$$y(x) = c_1 \operatorname{BesselJ}\left(0, \frac{x^2}{2}\right) + c_2 \operatorname{BesselY}\left(0, \frac{x^2}{2}\right)$$

✓ Solution by Mathematica
Time used: 0.088 (sec). Leaf size: 31

```
DSolve[y''[x]+1/x*y'[x]+x^2*y[x]==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to c_1 \operatorname{BesselJ}\left(0, \frac{x^2}{2}\right) + 2c_2 \operatorname{BesselY}\left(0, \frac{x^2}{2}\right)$$

4.6 problem 54

Internal problem ID [5815]

Book: Ordinary differential equations and calculus of variations. Makarets and Reshetnyak. Wold Scientific. Singapore. 1995
Section: Chapter 2. Linear homogeneous equations. Section 2.2 problems. page 95
Problem number: 54.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$x^2(-x^2+1)\,y''+2x(-x^2+1)\,y'-2y=0$$

✓ Solution by Maple
Time used: 0.0 (sec). Leaf size: 47

```
dsolve(x^2*(1-x^2)*diff(y(x),x$2)+2*x*(1-x^2)*diff(y(x),x)-2*y(x)=0,y(x), singsol=all)
```

$$y(x)=\frac{c_2(x^2-1)\ln(x-1)+(-x^2+1)\,c_2\ln(x+1)+2c_1x^2-2c_2x-2c_1}{2x^2}$$

✓ Solution by Mathematica
Time used: 0.06 (sec). Leaf size: 56

```
DSolve[x^2*(1-x^2)*y''[x]+2*x*(1-x^2)*y'[x]-2*y[x]==0,y[x],x,IncludeSingularSolutions -> Tru
```

$$y(x)\to\frac{-4c_1x^2-c_2(x^2-1)\log(1-x)+c_2(x^2-1)\log(x+1)+2c_2x+4c_1}{4x^2}$$

4.7 problem 55

Internal problem ID [5816]

Book: Ordinary differential equations and calculus of variations. Makarets and Reshetnyak. Wold Scientific. Singapore. 1995
Section: Chapter 2. Linear homogeneous equations. Section 2.2 problems. page 95
Problem number: 55.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _exact, _linear, _homogeneous]]`

$$\boxed{(-x^2+1)\, y'' - xy' + y = 0}$$

✓ Solution by Maple
Time used: 0.0 (sec). Leaf size: 20

```
dsolve((1-x^2)*diff(y(x),x$2)-x*diff(y(x),x)+y(x)=0,y(x), singsol=all)
```

$$y(x) = c_1 x + c_2 \sqrt{x-1}\,\sqrt{x+1}$$

✓ Solution by Mathematica
Time used: 0.193 (sec). Leaf size: 97

```
DSolve[(1-x^2)*y''[x]-x*y'[x]+y[x]==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to c_1 \cosh\left(\frac{2\sqrt{1-x^2}\arctan\left(\frac{\sqrt{1-x^2}}{x+1}\right)}{\sqrt{x^2-1}}\right) - ic_2 \sinh\left(\frac{2\sqrt{1-x^2}\arctan\left(\frac{\sqrt{1-x^2}}{x+1}\right)}{\sqrt{x^2-1}}\right)$$

4.8 problem 56

Internal problem ID [5817]

Book: Ordinary differential equations and calculus of variations. Makarets and Reshetnyak. Wold Scientific. Singapore. 1995
Section: Chapter 2. Linear homogeneous equations. Section 2.2 problems. page 95
Problem number: 56.
ODE order: 3.
ODE degree: 1.

CAS Maple gives this as type `[[_3rd_order, _with_linear_symmetries]]`

$$\boxed{y''' - 2y''x + 4x^2y' + 8yx^3 = 0}$$

✗ Solution by Maple

```
dsolve(diff(y(x),x$3)-2*x*diff(y(x),x$2)+4*x^2*diff(y(x),x)+8*x^3*y(x)=0,y(x), singsol=all)
```

No solution found

✗ Solution by Mathematica
Time used: 0.0 (sec). Leaf size: 0

```
DSolve[y'''[x]-2*x*y''[x]+4*x^2*y'[x]+8*x^3*y[x]==0,y[x],x,IncludeSingularSolutions -> True]
```

Not solved

4.9 problem 57

Internal problem ID [5818]

Book: Ordinary differential equations and calculus of variations. Makarets and Reshetnyak. Wold Scientific. Singapore. 1995
Section: Chapter 2. Linear homogeneous equations. Section 2.2 problems. page 95
Problem number: 57.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$\boxed{y'' + x(1-x)\,y' + \mathrm{e}^{x} y = 0}$$

✗ Solution by Maple

```
dsolve(diff(y(x),x$2)+x*(1-x)*diff(y(x),x)+exp(x)*y(x)=0,y(x), singsol=all)
```

No solution found

✗ Solution by Mathematica
Time used: 0.0 (sec). Leaf size: 0

```
DSolve[y''[x]+x*(1-x)*y'[x]+Exp[x]*y[x]==0,y[x],x,IncludeSingularSolutions -> True]
```

Not solved

4.10 problem 58

Internal problem ID [5819]

Book: Ordinary differential equations and calculus of variations. Makarets and Reshetnyak. Wold Scientific. Singapore. 1995
Section: Chapter 2. Linear homogeneous equations. Section 2.2 problems. page 95
Problem number: 58.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_Emden, _Fowler]]`

$$\boxed{x^2y'' + 2xy' + 4y = 0}$$

✓ Solution by Maple
Time used: 0.0 (sec). Leaf size: 29

```
dsolve(x^2*diff(y(x),x$2)+2*x*diff(y(x),x)+4*y(x)=0,y(x), singsol=all)
```

$$y(x) = \frac{c_1 \sin\left(\frac{\sqrt{15}\,\ln(x)}{2}\right) + c_2 \cos\left(\frac{\sqrt{15}\,\ln(x)}{2}\right)}{\sqrt{x}}$$

✓ Solution by Mathematica
Time used: 0.033 (sec). Leaf size: 42

```
DSolve[x^2*y''[x]+2*x*y'[x]+4*y[x]==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to \frac{c_2 \cos\left(\frac{1}{2}\sqrt{15}\log(x)\right) + c_1 \sin\left(\frac{1}{2}\sqrt{15}\log(x)\right)}{\sqrt{x}}$$

4.11 problem 59

Internal problem ID [5820]

Book: Ordinary differential equations and calculus of variations. Makarets and Reshetnyak. Wold Scientific. Singapore. 1995
Section: Chapter 2. Linear homogeneous equations. Section 2.2 problems. page 95
Problem number: 59.
ODE order: 4.
ODE degree: 1.

CAS Maple gives this as type `[[_high_order, _with_linear_symmetries]]`

$$\boxed{x^4y'''' - x^2y'' + y = 0}$$

✓ Solution by Maple
Time used: 0.0 (sec). Leaf size: 36

```
dsolve(x^4*diff(y(x),x$4)-x^2*diff(y(x),x$2)+y(x)=0,y(x), singsol=all)
```

$$y(x) = \sum_{\_a=1}^{4} x^{\mathrm{RootOf}\left(\_Z^4-6\_Z^3+10\_Z^2-5\_Z+1,\mathrm{index}=\_a\right)}\_C_{\_a}$$

✓ Solution by Mathematica
Time used: 0.004 (sec). Leaf size: 130

```
DSolve[x^4*y''''[x]-x^2*y''[x]+y[x]==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to c_4 x^{\mathrm{Root}\left[\#1^4-6\#1^3+10\#1^2-5\#1+1\&,4\right]} + c_3 x^{\mathrm{Root}\left[\#1^4-6\#1^3+10\#1^2-5\#1+1\&,3\right]} + c_1 x^{\mathrm{Root}\left[\#1^4-6\#1^3+10\#1^2-5\#1+1\&,1\right]} + c_2 x^{\mathrm{Root}\left[\#1^4-6\#1^3+10\#1^2-5\#1+1\&,2\right]}$$

4.12 problem 60

Internal problem ID [5821]

Book: Ordinary differential equations and calculus of variations. Makarets and Reshetnyak. Wold Scientific. Singapore. 1995
Section: Chapter 2. Linear homogeneous equations. Section 2.2 problems. page 95
Problem number: 60.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries], [_2nd_order, _linear,`

$$\boxed{(x^2+1)\, y''+xy'+y=0}$$

✓ Solution by Maple
Time used: 0.015 (sec). Leaf size: 15

```
dsolve((1+x^2)*diff(y(x),x$2)+x*diff(y(x),x)+y(x)=0,y(x), singsol=all)
```

$$y(x)=c_1\sin\left(\operatorname{arcsinh}\left(x\right)\right)+c_2\cos\left(\operatorname{arcsinh}\left(x\right)\right)$$

✓ Solution by Mathematica
Time used: 0.038 (sec). Leaf size: 43

```
DSolve[(1+x^2)*y''[x]+x*y'[x]+y[x]==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x)\to c_1\cos\left(\log\left(\sqrt{x^2+1}-x\right)\right)-c_2\sin\left(\log\left(\sqrt{x^2+1}-x\right)\right)$$

5 Chapter 2. Linear homogeneous equations. Section 2.3.4 problems. page 104

5.1 problem 1

Internal problem ID [5822]

Book: Ordinary differential equations and calculus of variations. Makarets and Reshetnyak. Wold Scientific. Singapore. 1995
Section: Chapter 2. Linear homogeneous equations. Section 2.3.4 problems. page 104
Problem number: 1.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _exact, _linear, _nonhomogeneous]]`

$$\boxed{y'' + xy' + y = 2x\,\mathrm{e}^{x} - 1}$$

✓ Solution by Maple
Time used: 0.015 (sec). Leaf size: 56

```
dsolve(diff(y(x),x$2)+x*diff(y(x),x)+y(x)=2*x*exp(x)-1,y(x), singsol=all)
```

$$y(x) = 2i\sqrt{2}\,\sqrt{\pi}\,\mathrm{e}^{-\frac{x^2}{2}-\frac{1}{2}}\,\mathrm{erf}\left(\frac{i\sqrt{2}\,(x+1)}{2}\right) + \left(c_1\,\mathrm{erf}\left(\frac{i\sqrt{2}\,x}{2}\right) + c_2\right)\mathrm{e}^{-\frac{x^2}{2}} + 2\,\mathrm{e}^{x} - 1$$

✓ Solution by Mathematica
Time used: 0.148 (sec). Leaf size: 53

```
DSolve[y''[x]+x*y'[x]+y[x]==2*x*Exp[x]-1,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to e^{-\frac{x^2}{2}}\left(\int_1^x e^{\frac{K[1]^2}{2}}\left(c_1 + 2e^{K[1]}(K[1]-1) - K[1]\right)dK[1] + c_2\right)$$

5.2 problem 2

Internal problem ID [5823]

Book: Ordinary differential equations and calculus of variations. Makarets and Reshetnyak. Wold Scientific. Singapore. 1995
Section: Chapter 2. Linear homogeneous equations. Section 2.3.4 problems. page 104
Problem number: 2.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$\boxed{y''x + xy' - y = x^2 + 2x}$$

✓ Solution by Maple

Time used: 0.016 (sec). Leaf size: 22

```
dsolve(x*diff(y(x),x$2)+x*diff(y(x),x)-y(x)=x^2+2*x,y(x), singsol=all)
```

$$y(x) = -c_2 \mathrm{e}^{-x} + x(c_2 \operatorname{expIntegral}_1(x) + x + c_1)$$

✓ Solution by Mathematica

Time used: 0.274 (sec). Leaf size: 31

```
DSolve[x*y''[x]+x*y'[x]-y[x]==x^2+2*x,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to -c_2 x \operatorname{ExpIntegralEi}(-x) + x^2 + c_1 x - c_2 e^{-x}$$

5.3 problem 3

Internal problem ID [5824]

Book: Ordinary differential equations and calculus of variations. Makarets and Reshetnyak. Wold Scientific. Singapore. 1995
Section: Chapter 2. Linear homogeneous equations. Section 2.3.4 problems. page 104
Problem number: 3.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _exact, _linear, _nonhomogeneous]]`

$$x^2y'' + xy' - y = x^2 + 2x$$

✓ Solution by Maple
Time used: 0.0 (sec). Leaf size: 22

```
dsolve(x^2*diff(y(x),x$2)+x*diff(y(x),x)-y(x)=x^2+2*x,y(x), singsol=all)
```

$$y(x) = \frac{c_1}{x} + c_2x + \frac{(x + 3\ln(x))\,x}{3}$$

✓ Solution by Mathematica
Time used: 0.017 (sec). Leaf size: 31

```
DSolve[x^2*y''[x]+x*y'[x]-y[x]==x^2+2*x,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to \frac{x^2}{3} + x\log(x) + \left(-\frac{1}{2} + c_2\right)x + \frac{c_1}{x}$$

5.4 problem 4

Internal problem ID [5825]

Book: Ordinary differential equations and calculus of variations. Makarets and Reshetnyak. Wold Scientific. Singapore. 1995
Section: Chapter 2. Linear homogeneous equations. Section 2.3.4 problems. page 104
Problem number: 4.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$\boxed{x^3 y'' + x y' - y = \cos\left(\frac{1}{x}\right)}$$

✓ Solution by Maple
Time used: 0.0 (sec). Leaf size: 26

```
dsolve(x^3*diff(y(x),x$2)+x*diff(y(x),x)-y(x)=cos(1/x),y(x), singsol=all)
```

$$y(x) = -\frac{x\left(-2\,\mathrm{e}^{\frac{1}{x}} c_2 + \cos\left(\frac{1}{x}\right) + \sin\left(\frac{1}{x}\right) - 2c_1\right)}{2}$$

✓ Solution by Mathematica
Time used: 0.272 (sec). Leaf size: 32

```
DSolve[x^3*y''[x]+x*y'[x]-y[x]==Cos[1/x],y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to -\frac{1}{2}x\left(\sin\left(\frac{1}{x}\right) + \cos\left(\frac{1}{x}\right) - 2\left(c_1 e^{\frac{1}{x}} + c_2\right)\right)$$

5.5 problem 5

Internal problem ID [5826]

Book: Ordinary differential equations and calculus of variations. Makarets and Reshetnyak. Wold Scientific. Singapore. 1995
Section: Chapter 2. Linear homogeneous equations. Section 2.3.4 problems. page 104
Problem number: 5.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _exact, _linear, _nonhomogeneous]]`

$$\boxed{x(1+x)\,y''+(x+2)\,y'-y=x+\frac{1}{x}}$$

✓ Solution by Maple
Time used: 0.0 (sec). Leaf size: 42

```
dsolve(x*(1+x)*diff(y(x),x$2)+(x+2)*diff(y(x),x)-y(x)=x+1/x,y(x), singsol=all)
```

$$y(x)=\frac{2\ln(x)\,x^2+4c_2x^2+4\ln(x)\,x+8c_2x+4c_1+4c_2+6x+5}{4x}$$

✓ Solution by Mathematica
Time used: 0.046 (sec). Leaf size: 37

```
DSolve[x*(1+x)*y''[x]+(x+2)*y'[x]-y[x]==x+1/x,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x)\to\frac{1}{2}(x+2)\log(x)+\frac{1+c_1}{x}+\frac{1}{4}(-1+2c_2)x+1+c_2$$

5.6 problem 6

Internal problem ID [5827]

Book: Ordinary differential equations and calculus of variations. Makarets and Reshetnyak. Wold Scientific. Singapore. 1995
Section: Chapter 2. Linear homogeneous equations. Section 2.3.4 problems. page 104
Problem number: 6.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$\boxed{2xy'' + (-2+x)\,y' - y = x^2 - 1}$$

✓ Solution by Maple

Time used: 0.015 (sec). Leaf size: 20

```
dsolve(2*x*diff(y(x),x$2)+(x-2)*diff(y(x),x)-y(x)=x^2-1,y(x), singsol=all)
```

$$y(x) = (-2+x)\,c_2 + c_1 \mathrm{e}^{-\frac{x}{2}} + x^2 + 1$$

✓ Solution by Mathematica

Time used: 0.256 (sec). Leaf size: 30

```
DSolve[2*x*y''[x]+(x-2)*y'[x]-y[x]==x^2-1,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to x^2 - 4x + c_1 e^{-x/2} + 2c_2(x-2) + 9$$

5.7 problem 7

Internal problem ID [5828]

Book: Ordinary differential equations and calculus of variations. Makarets and Reshetnyak. Wold Scientific. Singapore. 1995
Section: Chapter 2. Linear homogeneous equations. Section 2.3.4 problems. page 104
Problem number: 7.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _linear, _nonhomogeneous]]`

$$x^2(1+x)\,y''+x(4x+3)\,y'-y=x+\frac{1}{x}$$

✓ Solution by Maple
Time used: 0.141 (sec). Leaf size: 640

```
dsolve(x^2*(x+1)*diff(y(x),x$2)+x*(4*x+3)*diff(y(x),x)-y(x)=x+1/x,y(x), singsol=all)
```

$y(x)$

$$= \frac{-5x^{-\sqrt{2}}\left(\sqrt{2}-\frac{6}{5}\right)\text{hypergeom}\left(\left[2-\sqrt{2},-1-\sqrt{2}\right],\left[1-2\sqrt{2}\right],-x\right)\left(\int \frac{}{\left(-7\sqrt{2}\ \text{hypergeom}\left(\left[\sqrt{2}-1,\sqrt{2}-1\right],\left[1+2\right.\right.\right.}\right.}{}$$

✓ Solution by Mathematica

Time used: 7.882 (sec). Leaf size: 636

```
DSolve[x^2*(x+1)*y''[x]+x*(4*x+3)*y'[x]-y[x]==x+1/x,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to x^{-1-\sqrt{2}}\Bigg(x^{2\sqrt{2}}\,\text{Hypergeometric2F1}\left(-1+\sqrt{2},2+\sqrt{2},1+2\sqrt{2},\right.$$

$$\left.-x\right)\int_1^x \frac{}{(K[2]+1)\left(\left(4+\sqrt{2}\right)\text{Hypergeometric2F1}\left(-\sqrt{2},3-\sqrt{2},2-2\sqrt{2},-K[2]\right)\text{Hypergeometric2}\right.}$$

$$+\text{Hypergeometric2F1}\left(-1-\sqrt{2},2-\sqrt{2},1-2\sqrt{2},-x\right)\int_1^x$$

$$-\frac{}{(K[1]+1)\left(\left(4+\sqrt{2}\right)\text{Hypergeometric2F1}\left(-\sqrt{2},3-\sqrt{2},2-2\sqrt{2},-K[1]\right)\text{Hypergeometric2F1}\left(-1\right.\right.}$$

$$+c_2x^{2\sqrt{2}}\,\text{Hypergeometric2F1}\left(-1+\sqrt{2},2+\sqrt{2},1+2\sqrt{2},-x\right)$$

$$+c_1\,\text{Hypergeometric2F1}\left(-1-\sqrt{2},2-\sqrt{2},1-2\sqrt{2},-x\right)\Bigg)$$

5.8 problem 8

Internal problem ID [5829]

Book: Ordinary differential equations and calculus of variations. Makarets and Reshetnyak. Wold Scientific. Singapore. 1995
Section: Chapter 2. Linear homogeneous equations. Section 2.3.4 problems. page 104
Problem number: 8.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$\boxed{x^2(\ln(x)-1)\,y''-xy'+y=x(-\ln(x)+1)^2}$$

✓ Solution by Maple
Time used: 0.031 (sec). Leaf size: 25

```
dsolve(x^2*(ln(x)-1)*diff(y(x),x$2)-x*diff(y(x),x)+y(x)=x*(1-ln(x))^2,y(x), singsol=all)
```

$$y(x)=\frac{\ln(x)^2 x}{2}+(-x-c_1)\ln(x)+c_2x$$

✓ Solution by Mathematica
Time used: 0.105 (sec). Leaf size: 27

```
DSolve[x^2*(Log[x]-1)*y''[x]-x*y'[x]+y[x]==x*(1-Log[x])^2,y[x],x,IncludeSingularSolutions ->
```

$$y(x)\to\frac{1}{2}x\log^2(x)+c_1x-(x+c_2)\log(x)$$

5.9 problem 9

Internal problem ID [5830]

Book: Ordinary differential equations and calculus of variations. Makarets and Reshetnyak. Wold Scientific. Singapore. 1995
Section: Chapter 2. Linear homogeneous equations. Section 2.3.4 problems. page 104
Problem number: 9.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _linear, _nonhomogeneous]]`

$$\boxed{xy'' + 2y' + xy = \sec(x)}$$

✓ Solution by Maple

Time used: 0.016 (sec). Leaf size: 26

```
dsolve(x*diff(y(x),x$2)+2*diff(y(x),x)+x*y(x)=sec(x),y(x), singsol=all)
```

$$y(x) = \frac{-\ln(\sec(x))\cos(x) + \cos(x)\,c_1 + \sin(x)\,(x + c_2)}{x}$$

✓ Solution by Mathematica

Time used: 0.077 (sec). Leaf size: 65

```
DSolve[x*y''[x]+2*y'[x]+x*y[x]==Sec[x],y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to \frac{e^{-ix}(e^{2ix}\log(1 + e^{-2ix}) + \log(1 + e^{2ix}) - ic_2e^{2ix} + 2c_1)}{2x}$$

5.10 problem 10

Internal problem ID [5831]

Book: Ordinary differential equations and calculus of variations. Makarets and Reshetnyak. Wold Scientific. Singapore. 1995
Section: Chapter 2. Linear homogeneous equations. Section 2.3.4 problems. page 104
Problem number: 10.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _linear, _nonhomogeneous]]`

$$\left(-x^2+1\right) y''-x y'+\frac{y}{4}=-\frac{x^2}{2}+\frac{1}{2}$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 53

```
dsolve((1-x^2)*diff(y(x),x$2)-x*diff(y(x),x)+1/4*y(x)=1/2*(1-x^2),y(x), singsol=all)
```

$$y(x)=\frac{2(x^2+7)\sqrt{x+\sqrt{x^2-1}}+15c_1x+15c_1\sqrt{x^2-1}+15c_2}{15\sqrt{x+\sqrt{x^2-1}}}$$

✓ Solution by Mathematica

Time used: 19.346 (sec). Leaf size: 307

```
DSolve[(1-x^2)*y''[x]-x*y'[x]+1/4*y[x]==1/2*(1-x^2),y[x],x,IncludeSingularSolutions -> True]
```

$$
\begin{aligned}
y(x) &\\
\rightarrow \cosh\left(\frac{\sqrt{1-x^2}\arctan\left(\frac{\sqrt{1-x^2}}{x+1}\right)}{\sqrt{x^2-1}}\right)\int_1^x \sqrt{K[1]^2-1}\sinh\left(\frac{\arctan\left(\frac{\sqrt{1-K[1]^2}}{K[1]+1}\right)\sqrt{1-K[1]^2}}{\sqrt{K[1]^2-1}}\right)dK[1] &\\
- i\sinh\left(\frac{\sqrt{1-x^2}\arctan\left(\frac{\sqrt{1-x^2}}{x+1}\right)}{\sqrt{x^2-1}}\right)\int_1^x &\\
-i\cosh\left(\frac{\arctan\left(\frac{\sqrt{1-K[2]^2}}{K[2]+1}\right)\sqrt{1-K[2]^2}}{\sqrt{K[2]^2-1}}\right)\sqrt{K[2]^2-1}dK[2] &\\
+ c_1\cosh\left(\frac{\sqrt{1-x^2}\arctan\left(\frac{\sqrt{1-x^2}}{x+1}\right)}{\sqrt{x^2-1}}\right) - ic_2\sinh\left(\frac{\sqrt{1-x^2}\arctan\left(\frac{\sqrt{1-x^2}}{x+1}\right)}{\sqrt{x^2-1}}\right)
\end{aligned}
$$

5.11 problem 11

Internal problem ID [5832]

Book: Ordinary differential equations and calculus of variations. Makarets and Reshetnyak. Wold Scientific. Singapore. 1995
Section: Chapter 2. Linear homogeneous equations. Section 2.3.4 problems. page 104
Problem number: 11.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _linear, _nonhomogeneous]]`

$$(\cos(x)+\sin(x))\,y''-2\cos(x)\,y'+(\cos(x)-\sin(x))\,y=(\cos(x)+\sin(x))^2\,\mathrm{e}^{2x}$$

✓ Solution by Maple

Time used: 0.422 (sec). Leaf size: 322

```
dsolve((cos(x)+sin(x))*diff(y(x),x$2)-2*cos(x)*diff(y(x),x)+(cos(x)-sin(x))*y(x)=(cos(x)+sin
```

$$y(x)=-\cos(x)\left(\left(\left(\int \mathrm{e}^{\int \frac{(-\cot(x)+1)\cos(x)+2\sin(x)(\tan(x)+1)}{\cos(x)+\sin(x)}dx}\sin(x)\,dx\right)c_1\right.\right.$$
$$-\left(\int \mathrm{e}^{2x-2\left(\int \frac{\sin(x)}{\cos(x)+\sin(x)}dx\right)-2\left(\int \frac{\sin(x)\tan(x)}{\cos(x)+\sin(x)}dx\right)+\int \frac{\cos(x)\cot(x)}{\cos(x)+\sin(x)}dx-\left(\int \frac{\cos(x)}{\cos(x)+\sin(x)}dx\right)}(\csc(x)+\sec(x))\,dx\right)\left(\int \mathrm{e}^{2(\cdots}\right.$$
$$+\int \mathrm{e}^{2x-2\left(\int \frac{\sin(x)}{\cos(x)+\sin(x)}dx\right)-2\left(\int \frac{\sin(x)\tan(x)}{\cos(x)+\sin(x)}dx\right)+\int \frac{\cos(x)\cot(x)}{\cos(x)+\sin(x)}dx-\left(\int \frac{\cos(x)}{\cos(x)+\sin(x)}dx\right)}(\csc(x)$$
$$+\sec(x))\left(\int \mathrm{e}^{2\left(\int \frac{\sin(x)}{\cos(x)+\sin(x)}dx\right)+2\left(\int \frac{\sin(x)\tan(x)}{\cos(x)+\sin(x)}dx\right)-\left(\int \frac{\cos(x)\cot(x)}{\cos(x)+\sin(x)}dx\right)+\int \frac{\cos(x)}{\cos(x)+\sin(x)}dx}\sin(x)\,dx\right)dx$$
$$\left.-c_2\right)$$

✓ Solution by Mathematica

Time used: 4.817 (sec). Leaf size: 476

```
DSolve[(Cos[x]+Sin[x])*y''[x]-2*Cos[x]*y'[x]+(Cos[x]-Sin[x])*y[x]==(Cos[x]+Sin[x])^2*Exp[2*x
```

$y(x)$

$$\rightarrow \frac{\left(\frac{1}{4}+\frac{i}{4}\right)\left(e^{-2ix}\right)^{\frac{1}{2}-\frac{i}{2}}\left(e^{ix}\right)^{1-2i}\left(-\frac{i\left(-1+e^{2i\arctan\left(e^{-2ix}\right)}\right)}{1+e^{2i\arctan\left(e^{-2ix}\right)}}\right)^{-\frac{1}{2}-\frac{i}{2}}\left(-i\left(e^{-2ix}\right)^{i}\sqrt{1+e^{-4ix}}\sqrt{1+e^{4ix}}e^{2i(2x+\arctan\cdots}\right.}{\sqrt{-e^{4ix}}\sqrt{-(1+\cdots}}$$

$$+\frac{c_2e^{3ix}\left(e^{-2ix}\right)^{\frac{1}{2}+\frac{i}{2}}\sqrt{1+e^{-4ix}}\left(e^{2i\arctan\left(e^{-2ix}\right)}+i\right)\left(-\frac{i\left(-1+e^{2i\arctan\left(e^{-2ix}\right)}\right)}{1+e^{2i\arctan\left(e^{-2ix}\right)}}\right)^{\frac{1}{2}-\frac{i}{2}}}{\sqrt{1+e^{4ix}}\left(-1+e^{2i\arctan\left(e^{-2ix}\right)}\right)}$$

$$+c_1\left(e^{ix}\right)^{-i}$$

5.12 problem 12

Internal problem ID [5833]

Book: Ordinary differential equations and calculus of variations. Makarets and Reshetnyak. Wold Scientific. Singapore. 1995
Section: Chapter 2. Linear homogeneous equations. Section 2.3.4 problems. page 104
Problem number: 12.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _linear, _nonhomogeneous]]`

$$(\cos(x)-\sin(x))\, y''-2\sin(x)\, y'+(\cos(x)+\sin(x))\, y=(\cos(x)-\sin(x))^2$$

✗ Solution by Maple

```
dsolve((cos(x)-sin(x))*diff(y(x),x$2)-2*sin(x)*diff(y(x),x)+(cos(x)+sin(x))*y(x)=(cos(x)-sin
```

No solution found

✓ Solution by Mathematica
Time used: 15.918 (sec). Leaf size: 7186

```
DSolve[(Cos[x]-Sin[x])*y''[x]-2*Sin[x]*y'[x]+(Cos[x]+Sin[x])*y[x]==(Cos[x]-Sin[x])^2,y[x],x,
```

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