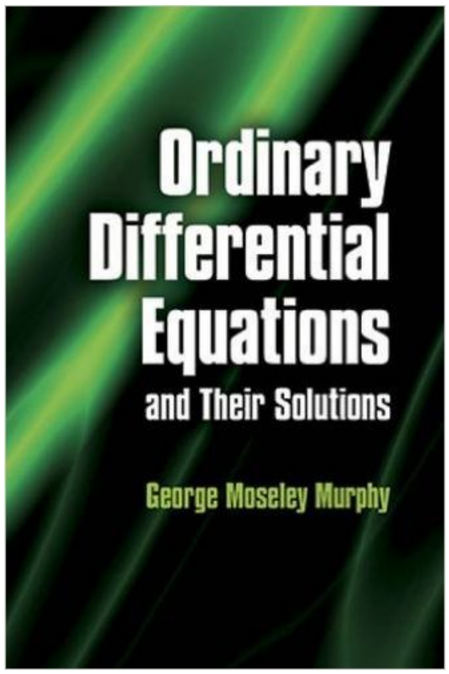
A Solution Manual For

Ordinary differential equations and their solutions. By George Moseley Murphy. 1960



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1 Various 1

1.1 problem 0

Internal problem ID [3264]

Book: Ordinary differential equations and their solutions. By George Moseley Murphy. 1960
Section: Various 1
Problem number: 0.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_quadrature]`

$$\boxed{y' = af(x)}$$

✓ Solution by Maple
Time used: 0.0 (sec). Leaf size: 12

```
dsolve(diff(y(x),x) = a*f(x),y(x), singsol=all)
```

$$y(x) = a\left(\int f(x)\,dx\right) + c_1$$

✓ Solution by Mathematica
Time used: 0.004 (sec). Leaf size: 20

```
DSolve[y'[x]==a*f[x],y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to \int_1^x af(K[1])dK[1] + c_1$$

1.2 problem 1

Internal problem ID [3265]

Book: Ordinary differential equations and their solutions. By George Moseley Murphy. 1960
Section: Various 1
Problem number: 1.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[[_linear, 'class A']]`

$$\boxed{y' - y = x + \sin(x)}$$

✓ Solution by Maple
Time used: 0.0 (sec). Leaf size: 21

```
dsolve(diff(y(x),x) = x+sin(x)+y(x),y(x), singsol=all)
```

$$y(x) = -x - 1 - \frac{\cos(x)}{2} - \frac{\sin(x)}{2} + e^x c_1$$

✓ Solution by Mathematica
Time used: 0.079 (sec). Leaf size: 28

```
DSolve[y'[x]==x+Sin[x]+y[x],y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to -x - \frac{\sin(x)}{2} - \frac{\cos(x)}{2} + c_1 e^x - 1$$

1.3 problem 2

Internal problem ID [3266]

Book: Ordinary differential equations and their solutions. By George Moseley Murphy. 1960
Section: Various 1
Problem number: 2.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[[_linear, 'class A']]`

$$\boxed{y' - 2y = x^2 + 3\cosh(x)}$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 58

```
dsolve(diff(y(x),x) = x^2+3*cosh(x)+2*y(x),y(x), singsol=all)
```

$$y(x) = -\frac{e^{2x}\left(\left(x^2+x+\frac{1}{2}\right)\cosh(2x)+\left(-x^2-x-\frac{1}{2}\right)\sinh(2x)-2c_1+3\cosh(x)-3\sinh(x)+\cosh(3x)-\mathrm{s}\right.}{2}$$

✓ Solution by Mathematica

Time used: 0.092 (sec). Leaf size: 46

```
DSolve[y'[x]==x^2+3*Cosh[x]+2*y[x],y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to -\frac{1}{4}e^{-x}\left(e^x\left(2x^2+2x+1\right)+6e^{2x}+2\right)+c_1e^{2x}$$

1.4 problem 3

Internal problem ID [3267]

Book: Ordinary differential equations and their solutions. By George Moseley Murphy. 1960
Section: Various 1
Problem number: 3.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[[_linear, 'class A']]`

$$\boxed{y' - cy = bx + a}$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 31

```
dsolve(diff(y(x),x) = a+b*x+c*y(x),y(x), singsol=all)
```

$$y(x) = \frac{\mathrm{e}^{cx} c_1 c^2 + (-bx - a)\, c - b}{c^2}$$

✓ Solution by Mathematica

Time used: 0.106 (sec). Leaf size: 28

```
DSolve[y'[x]==a+b*x+c*y[x],y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to -\frac{ac + bcx + b}{c^2} + c_1 e^{cx}$$

1.5 problem 4

Internal problem ID [3268]

Book: Ordinary differential equations and their solutions. By George Moseley Murphy. 1960
Section: Various 1
Problem number: 4.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[[_linear, 'class A']]`

$$\boxed{y' - ky = a\cos(bx + c)}$$

✓ Solution by Maple
Time used: 0.016 (sec). Leaf size: 47

```
dsolve(diff(y(x),x) = a*cos(b*x+c)+k*y(x),y(x), singsol=all)
```

$$y(x) = \frac{-\cos(bx+c)\,ak + \sin(bx+c)\,ab + e^{kx}c_1(b^2+k^2)}{b^2+k^2}$$

✓ Solution by Mathematica
Time used: 0.128 (sec). Leaf size: 43

```
DSolve[y'[x]==a*Cos[b*x+c]+k*y[x],y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to \frac{a(b\sin(bx+c) - k\cos(bx+c))}{b^2+k^2} + c_1e^{kx}$$

1.6 problem 5

Internal problem ID [3269]

Book: Ordinary differential equations and their solutions. By George Moseley Murphy. 1960
Section: Various 1
Problem number: 5.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[[_linear, ‘class A‘]]`

$$\boxed{y' - ky = a\sin(bx + c)}$$

✓ Solution by Maple
Time used: 0.0 (sec). Leaf size: 48

```
dsolve(diff(y(x),x) = a*sin(b*x+c)+k*y(x),y(x), singsol=all)
```

$$y(x) = \frac{-\cos(bx + c)\,ab - \sin(bx + c)\,ak + e^{kx}c_1(b^2 + k^2)}{b^2 + k^2}$$

✓ Solution by Mathematica
Time used: 0.123 (sec). Leaf size: 43

```
DSolve[y'[x]==a*Sin[b*x+c]+k*y[x],y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to -\frac{a(k\sin(bx + c) + b\cos(bx + c))}{b^2 + k^2} + c_1e^{kx}$$

1.7 problem 6

Internal problem ID [3270]

Book: Ordinary differential equations and their solutions. By George Moseley Murphy. 1960
Section: Various 1
Problem number: 6.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[[_linear, 'class A']]`

$$\boxed{y' - cy = a + b\,\mathrm{e}^{kx}}$$

✓ Solution by Maple
Time used: 0.0 (sec). Leaf size: 44

```
dsolve(diff(y(x),x) = a+b*exp(k*x)+c*y(x),y(x), singsol=all)
```

$$y(x) = \frac{c_1 c(c-k)\,\mathrm{e}^{cx} - b\,\mathrm{e}^{kx} c - a(c-k)}{c\,(c-k)}$$

✓ Solution by Mathematica
Time used: 0.166 (sec). Leaf size: 47

```
DSolve[y'[x]==a+b*Exp[k*x]+c*y[x],y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to \frac{a(k-c) - bce^{kx} + cc_1(c-k)e^{cx}}{c(c-k)}$$

1.8 problem 7

Internal problem ID [3271]

Book: Ordinary differential equations and their solutions. By George Moseley Murphy. 1960
Section: Various 1
Problem number: 7.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_linear]`

$$\boxed{y' - x(x^2 - y) = 0}$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 17

```
dsolve(diff(y(x),x) = x*(x^2-y(x)),y(x), singsol=all)
```

$$y(x) = x^2 - 2 + e^{-\frac{x^2}{2}} c_1$$

✓ Solution by Mathematica

Time used: 0.067 (sec). Leaf size: 22

```
DSolve[y'[x]==x*(x^2-y[x]),y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to x^2 + c_1 e^{-\frac{x^2}{2}} - 2$$

1.9 problem 8

Internal problem ID [3272]

Book: Ordinary differential equations and their solutions. By George Moseley Murphy. 1960
Section: Various 1
Problem number: 8.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_linear]`

$$\boxed{y' - x\left(\mathrm{e}^{-x^2} + ya\right) = 0}$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 34

```
dsolve(diff(y(x),x) = x*(exp(-x^2)+a*y(x)),y(x), singsol=all)
```

$$y(x) = \frac{\mathrm{e}^{\frac{a\,x^2}{2}}\left(-\mathrm{e}^{-\frac{x^2(2+a)}{2}} + c_1(2+a)\right)}{2+a}$$

✓ Solution by Mathematica

Time used: 0.122 (sec). Leaf size: 42

```
DSolve[y'[x]==x*(Exp[-x^2]+a*y[x]),y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to \frac{e^{\frac{ax^2}{2}}\left(-e^{-\frac{1}{2}(a+2)x^2} + (a+2)c_1\right)}{a+2}$$

1.10 problem 9

Internal problem ID [3273]

Book: Ordinary differential equations and their solutions. By George Moseley Murphy. 1960
Section: Various 1
Problem number: 9.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_linear]`

$$\boxed{y' - x^2(a\,x^3 + yb) = 0}$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 31

```
dsolve(diff(y(x),x) = x^2*(a*x^3+b*y(x)),y(x), singsol=all)
```

$$y(x) = \frac{-a\,x^3 b + \mathrm{e}^{\frac{b\,x^3}{3}} c_1 b^2 - 3a}{b^2}$$

✓ Solution by Mathematica

Time used: 0.088 (sec). Leaf size: 32

```
DSolve[y'[x]==x^2*(a*x^3+b*y[x]),y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to -\frac{a(bx^3+3)}{b^2} + c_1 e^{\frac{bx^3}{3}}$$

1.11 problem 10

Internal problem ID [3274]

Book: Ordinary differential equations and their solutions. By George Moseley Murphy. 1960
Section: Various 1
Problem number: 10.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_separable]`

$$\boxed{y' - a\,x^n y = 0}$$

✓ Solution by Maple

Time used: 0.016 (sec). Leaf size: 19

```
dsolve(diff(y(x),x) = a*x^n*y(x),y(x), singsol=all)
```

$$y(x) = c_1 \mathrm{e}^{\frac{a\,x^{n+1}}{n+1}}$$

✓ Solution by Mathematica

Time used: 0.049 (sec). Leaf size: 27

```
DSolve[y'[x]==a*x^n*y[x],y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to c_1 e^{\frac{ax^{n+1}}{n+1}}$$
$$y(x) \to 0$$

1.12 problem 11

Internal problem ID [3275]

Book: Ordinary differential equations and their solutions. By George Moseley Murphy. 1960
Section: Various 1
Problem number: 11.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_linear]`

$$\boxed{y' - \cos(x)\, y = \sin(x)\cos(x)}$$

✓ Solution by Maple
Time used: 0.0 (sec). Leaf size: 15

```
dsolve(diff(y(x),x) = cos(x)*sin(x)+y(x)*cos(x),y(x), singsol=all)
```

$$y(x) = -\sin(x) - 1 + e^{\sin(x)} c_1$$

✓ Solution by Mathematica
Time used: 0.054 (sec). Leaf size: 18

```
DSolve[y'[x]==Cos[x]*Sin[x]+y[x]*Cos[x],y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to -\sin(x) + c_1 e^{\sin(x)} - 1$$

1.13 problem 12

Internal problem ID [3276]

Book: Ordinary differential equations and their solutions. By George Moseley Murphy. 1960
Section: Various 1
Problem number: 12.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_linear]`

$$y' - \cos(x) y = e^{\sin(x)}$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 11

```
dsolve(diff(y(x),x) = exp(sin(x))+y(x)*cos(x),y(x), singsol=all)
```

$$y(x) = (c_1 + x) e^{\sin(x)}$$

✓ Solution by Mathematica

Time used: 0.124 (sec). Leaf size: 14

```
DSolve[y'[x]==Exp[Sin[x]]+y[x]*Cos[x],y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to (x + c_1)e^{\sin(x)}$$

1.14 problem 13

Internal problem ID [3277]

Book: Ordinary differential equations and their solutions. By George Moseley Murphy. 1960
Section: Various 1
Problem number: 13.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_separable]`

$$\boxed{y' - y \cot(x) = 0}$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 8

```
dsolve(diff(y(x),x) = y(x)*cot(x),y(x), singsol=all)
```

$$y(x) = c_1 \sin(x)$$

✓ Solution by Mathematica

Time used: 0.036 (sec). Leaf size: 15

```
DSolve[y'[x]==y[x]*Cot[x],y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to c_1 \sin(x)$$
$$y(x) \to 0$$

1.15 problem 14

Internal problem ID [3278]

Book: Ordinary differential equations and their solutions. By George Moseley Murphy. 1960
Section: Various 1
Problem number: 14.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_linear]`

$$\boxed{y' + y\cot(x) = 1}$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 13

```
dsolve(diff(y(x),x) = 1-y(x)*cot(x),y(x), singsol=all)
```

$$y(x) = -\cot(x) + \csc(x)\, c_1$$

✓ Solution by Mathematica

Time used: 0.04 (sec). Leaf size: 15

```
DSolve[y'[x]==1-y[x]*Cot[x],y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to -\cot(x) + c_1 \csc(x)$$

1.16 problem 15

Internal problem ID [3279]

Book: Ordinary differential equations and their solutions. By George Moseley Murphy. 1960
Section: Various 1
Problem number: 15.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_linear]`

$$\boxed{y' + y\cot(x) = \csc(x)\,x}$$

✓ Solution by Maple
Time used: 0.015 (sec). Leaf size: 14

```
dsolve(diff(y(x),x) = x*csc(x)-y(x)*cot(x),y(x), singsol=all)
```

$$y(x) = \csc(x)\left(\frac{x^2}{2} + c_1\right)$$

✓ Solution by Mathematica
Time used: 0.039 (sec). Leaf size: 19

```
DSolve[y'[x]==x*Csc[x]-y[x]*Cot[x],y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to \frac{1}{2}(x^2 + 2c_1)\csc(x)$$

1.17 problem 16

Internal problem ID [3280]

Book: Ordinary differential equations and their solutions. By George Moseley Murphy. 1960
Section: Various 1
Problem number: 16.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_separable]`

$$\boxed{y' - (2\csc(2x) + \cot(x))\, y = 0}$$

✓ Solution by Maple
Time used: 0.0 (sec). Leaf size: 10

```
dsolve(diff(y(x),x) = (2*csc(2*x)+cot(x))*y(x),y(x), singsol=all)
```

$$y(x) = \sin(x)\tan(x)\, c_1$$

✓ Solution by Mathematica
Time used: 0.068 (sec). Leaf size: 32

```
DSolve[y'[x]==(2*Csc[2*x]+Cot[x])*y[x],y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to c_1\sqrt{\sin(2x)}e^{-\frac{3}{2}\operatorname{arctanh}(\cos(2x))}$$
$$y(x) \to 0$$

1.18 problem 17

Internal problem ID [3281]

Book: Ordinary differential equations and their solutions. By George Moseley Murphy. 1960
Section: Various 1
Problem number: 17.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_linear]`

$$\boxed{y' + y\cot(x) = \sec(x)}$$

✓ Solution by Maple
Time used: 0.016 (sec). Leaf size: 14

```
dsolve(diff(y(x),x) = sec(x)-y(x)*cot(x),y(x), singsol=all)
```

$$y(x) = \csc(x)(-\ln(\cos(x)) + c_1)$$

✓ Solution by Mathematica
Time used: 0.05 (sec). Leaf size: 16

```
DSolve[y'[x]==Sec[x]-y[x]*Cot[x],y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \rightarrow \csc(x)(-\log(\cos(x)) + c_1)$$

1.19 problem 18

Internal problem ID [3282]

Book: Ordinary differential equations and their solutions. By George Moseley Murphy. 1960
Section: Various 1
Problem number: 18.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_linear]`

$$\boxed{y' - y\cot(x) = e^x \sin(x)}$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 11

```
dsolve(diff(y(x),x) = exp(x)*sin(x)+y(x)*cot(x),y(x), singsol=all)
```

$$y(x) = (e^x + c_1)\sin(x)$$

✓ Solution by Mathematica

Time used: 0.082 (sec). Leaf size: 14

```
DSolve[y'[x]==Exp[x]*Sin[x]+y[x]*Cot[x],y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to (e^x + c_1)\sin(x)$$

1.20 problem 19

Internal problem ID [3283]

Book: Ordinary differential equations and their solutions. By George Moseley Murphy. 1960
Section: Various 1
Problem number: 19.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_linear]`

$$\boxed{y' + 2y\cot(x) = -\csc(x)}$$

✓ Solution by Maple
Time used: 0.0 (sec). Leaf size: 13

```
dsolve(diff(y(x),x)+csc(x)+2*y(x)*cot(x) = 0,y(x), singsol=all)
```

$$y(x) = \csc(x)^2 (\cos(x) + c_1)$$

✓ Solution by Mathematica
Time used: 0.038 (sec). Leaf size: 15

```
DSolve[y'[x]+Csc[x]+2*y[x]*Cot[x]==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to \csc^2(x)(\cos(x) + c_1)$$

1.21 problem 20

Internal problem ID [3284]

Book: Ordinary differential equations and their solutions. By George Moseley Murphy. 1960
Section: Various 1
Problem number: 20.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_linear]`

$$y' + 2y\cot(2x) = 4\csc(x)\,x\sec(x)^2$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 94

```
dsolve(diff(y(x),x) = 4*csc(x)*x*sec(x)^2-2*y(x)*cot(2*x),y(x), singsol=all)
```

$$y(x) = 16\ \mathrm{csgn}(\csc(2x))\left(\sqrt{-\frac{e^{4ix}}{(e^{4ix}-1)^2}}\left(i\,\mathrm{dilog}\left(1+ie^{ix}\right)-i\,\mathrm{dilog}\left(1-ie^{ix}\right)\right.\right.$$
$$\left.\left.-x\ln\left(1+ie^{ix}\right)+x\ln\left(1-ie^{ix}\right)\right)+\frac{\csc(2x)\,c_1}{16}\right)$$

✓ Solution by Mathematica

Time used: 0.089 (sec). Leaf size: 60

```
DSolve[y'[x]==2*Csc[x]*2*x*Sec[x]^2-2*y[x]*Cot[2*x],y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to \csc(x)\sec(x)\left(-8ix\arctan\left(e^{ix}\right)+4i\,\mathrm{PolyLog}\left(2,-ie^{ix}\right)-4i\,\mathrm{PolyLog}\left(2,ie^{ix}\right)\right.$$
$$\left.+c_1\right)$$

1.22 problem 21

Internal problem ID [3285]

Book: Ordinary differential equations and their solutions. By George Moseley Murphy. 1960
Section: Various 1
Problem number: 21.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_linear]`

$$y' + 2y \csc(2x) = 2 \cot(x)^2 \cos(2x)$$

✓ Solution by Maple
Time used: 0.0 (sec). Leaf size: 20

```
dsolve(diff(y(x),x) = 2*cot(x)^2*cos(2*x)-2*y(x)*csc(2*x),y(x), singsol=all)
```

$$y(x) = \cot(x) \left(2 \ln(\sin(x)) + 2 \cos(x)^2 + c_1\right)$$

✓ Solution by Mathematica
Time used: 0.088 (sec). Leaf size: 21

```
DSolve[y'[x]==2*(Cot[x]^2*Cos[2*x]-y[x]*Csc[2*x]),y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to \cot(x)(\cos(2x) + 2\log(\sin(x)) - 1 + c_1)$$

1.23 problem 22

Internal problem ID [3286]

Book: Ordinary differential equations and their solutions. By George Moseley Murphy. 1960
Section: Various 1
Problem number: 22.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_linear]`

$$y' - 4\csc(x)\,x\left(\sin(x)^3 + y\right) = 0$$

✓ Solution by Maple

Time used: 0.031 (sec). Leaf size: 118

```
dsolve(diff(y(x),x) = 4*csc(x)*x*(sin(x)^3+y(x)),y(x), singsol=all)
```

$$y(x) = -2\left(1-e^{ix}\right)^{4x}\left(e^{ix}+1\right)^{-4x} e^{4i\left(\operatorname{dilog}\left(e^{ix}+1\right)-\operatorname{dilog}\left(1-e^{ix}\right)\right)}\left(-\frac{c_1}{2} + \int x\left(1-e^{ix}\right)^{-4x}\left(e^{ix}+1\right)^{4x} e^{-4i\left(\operatorname{dilog}\left(e^{ix}+1\right)-\operatorname{dilog}\left(1-e^{ix}\right)\right)}\left(-1+\cos(2x)\right)dx\right)$$

✓ Solution by Mathematica

Time used: 8.396 (sec). Leaf size: 148

```
DSolve[y'[x]==2*Csc[x]*2*x(Sin[x]^3+y[x]),y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to \exp\left(4i\,\operatorname{PolyLog}\left(2,-e^{ix}\right) - 4i\,\operatorname{PolyLog}\left(2,e^{ix}\right) + 4x\left(\log\left(1-e^{ix}\right) - \log\left(1+e^{ix}\right)\right)\right)\left(\int_1^x 4\exp\left(4K[1]\left(\log\left(1+e^{iK[1]}\right) - \log\left(1-e^{iK[1]}\right)\right) - 4i\,\operatorname{PolyLog}\left(2,-e^{iK[1]}\right) + 4i\,\operatorname{PolyLog}\left(2,e^{iK[1]}\right)\right)K[1]\sin^2(K[1])dK[1] + c_1\right)$$

1.24 problem 23

Internal problem ID [3287]

Book: Ordinary differential equations and their solutions. By George Moseley Murphy. 1960
Section: Various 1
Problem number: 23.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_linear]`

$$y' - 4\csc(x)\,x\left(1 - \tan(x)^2 + y\right) = 0$$

✓ Solution by Maple
Time used: 0.032 (sec). Leaf size: 120

```
dsolve(diff(y(x),x) = 4*csc(x)*x*(1-tan(x)^2+y(x)),y(x), singsol=all)
```

$$y(x) = -4\,e^{4i(\operatorname{dilog}(e^{ix}+1)-\operatorname{dilog}(1-e^{ix}))}\left(1-e^{ix}\right)^{4x}\left(\left(\int \csc(x)\left(\sec(x)^2-2\right)x\left(1 - e^{ix}\right)^{-4x}\left(e^{ix}+1\right)^{4x}e^{-4i(\operatorname{dilog}(e^{ix}+1)-\operatorname{dilog}(1-e^{ix}))}dx - \frac{c_1}{4}\right)\left(e^{ix}+1\right)^{-4x}\right)$$

✓ Solution by Mathematica
Time used: 11.321 (sec). Leaf size: 156

```
DSolve[y'[x]==2*Csc[x]*2*x*(1-Tan[x]^2+y[x]),y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to \exp\left(4i\operatorname{PolyLog}\left(2,-e^{ix}\right) - 4i\operatorname{PolyLog}\left(2,e^{ix}\right) + 4x\left(\log\left(1-e^{ix}\right) - \log\left(1+e^{ix}\right)\right)\right)\left(\int_1^x 4\exp\left(4K[1]\left(\log\left(1+e^{iK[1]}\right) - \log\left(1-e^{iK[1]}\right)\right) - 4i\operatorname{PolyLog}\left(2,-e^{iK[1]}\right) + 4i\operatorname{PolyLog}\left(2,e^{iK[1]}\right)\right)\cos(2K[1])\csc(K[1])K[1]\sec^2(K[1])dK[1] + c_1\right)$$

1.25 problem 24

Internal problem ID [3288]

Book: Ordinary differential equations and their solutions. By George Moseley Murphy. 1960
Section: Various 1
Problem number: 24.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_separable]`

$$\boxed{y' - y \sec(x) = 0}$$

✓ Solution by Maple
Time used: 0.0 (sec). Leaf size: 11

```
dsolve(diff(y(x),x) = y(x)*sec(x),y(x), singsol=all)
```

$$y(x) = c_1(\sec(x) + \tan(x))$$

✓ Solution by Mathematica
Time used: 0.036 (sec). Leaf size: 24

```
DSolve[y'[x]==y[x]*Sec[x],y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to c_1 e^{2\operatorname{arctanh}\left(\tan\left(\frac{x}{2}\right)\right)}$$
$$y(x) \to 0$$

1.26 problem 25

Internal problem ID [3289]

Book: Ordinary differential equations and their solutions. By George Moseley Murphy. 1960
Section: Various 1
Problem number: 25.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_linear]`

$$\boxed{y' - (1-y)\sec(x) = -\tan(x)}$$

✓ Solution by Maple
Time used: 0.016 (sec). Leaf size: 15

```
dsolve(diff(y(x),x)+tan(x) = (1-y(x))*sec(x),y(x), singsol=all)
```

$$y(x) = \frac{c_1 + x}{\sec(x) + \tan(x)}$$

✓ Solution by Mathematica
Time used: 0.792 (sec). Leaf size: 21

```
DSolve[y'[x]+Tan[x]==(1-y[x])*Sec[x],y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to (x + c_1)e^{-2\operatorname{arctanh}\left(\tan\left(\frac{x}{2}\right)\right)}$$

2 Various 2

2.1 problem 26

Internal problem ID [3290]

Book: Ordinary differential equations and their solutions. By George Moseley Murphy. 1960
Section: Various 2
Problem number: 26.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_separable]`

$$\boxed{y' - y\tan(x) = 0}$$

✓ Solution by Maple
Time used: 0.015 (sec). Leaf size: 8

```
dsolve(diff(y(x),x) = y(x)*tan(x),y(x), singsol=all)
```

$$y(x) = \sec(x)\, c_1$$

✓ Solution by Mathematica
Time used: 0.035 (sec). Leaf size: 15

```
DSolve[y'[x]==y[x]*Tan[x],y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to c_1 \sec(x)$$
$$y(x) \to 0$$

2.2 problem 27

Internal problem ID [3291]

Book: Ordinary differential equations and their solutions. By George Moseley Murphy. 1960
Section: Various 2
Problem number: 27.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_linear]`

$$\boxed{y' - y\tan(x) = \cos(x)}$$

✓ Solution by Maple
Time used: 0.0 (sec). Leaf size: 18

```
dsolve(diff(y(x),x) = cos(x)+y(x)*tan(x),y(x), singsol=all)
```

$$y(x) = \frac{(x + 2c_1)\sec(x)}{2} + \frac{\sin(x)}{2}$$

✓ Solution by Mathematica
Time used: 0.045 (sec). Leaf size: 21

```
DSolve[y'[x]==Cos[x]+y[x]*Tan[x],y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to \frac{1}{2}(\sin(x) + (x + 2c_1)\sec(x))$$

2.3 problem 28

Internal problem ID [3292]

Book: Ordinary differential equations and their solutions. By George Moseley Murphy. 1960
Section: Various 2
Problem number: 28.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_linear]`

$$\boxed{y' + y \tan(x) = \cos(x)}$$

✓ Solution by Maple
Time used: 0.016 (sec). Leaf size: 10

```
dsolve(diff(y(x),x) = cos(x)-y(x)*tan(x),y(x), singsol=all)
```

$$y(x) = (c_1 + x)\cos(x)$$

✓ Solution by Mathematica
Time used: 0.054 (sec). Leaf size: 12

```
DSolve[y'[x]==Cos[x]-y[x]*Tan[x],y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to (x + c_1)\cos(x)$$

2.4 problem 29

Internal problem ID [3293]

Book: Ordinary differential equations and their solutions. By George Moseley Murphy. 1960
Section: Various 2
Problem number: 29.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_linear]`

$$\boxed{y' + y\tan(x) = \sec(x)}$$

✓ Solution by Maple
Time used: 0.0 (sec). Leaf size: 11

```
dsolve(diff(y(x),x) = sec(x)-y(x)*tan(x),y(x), singsol=all)
```

$$y(x) = \cos(x)\, c_1 + \sin(x)$$

✓ Solution by Mathematica
Time used: 0.042 (sec). Leaf size: 13

```
DSolve[y'[x]==Sec[x]-y[x]*Tan[x],y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to \sin(x) + c_1 \cos(x)$$

2.5 problem 30

Internal problem ID [3294]

Book: Ordinary differential equations and their solutions. By George Moseley Murphy. 1960
Section: Various 2
Problem number: 30.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_linear]`

$$\boxed{y' - y\tan(x) = \sin(2x)}$$

✓ Solution by Maple
Time used: 0.0 (sec). Leaf size: 15

```
dsolve(diff(y(x),x) = sin(2*x)+y(x)*tan(x),y(x), singsol=all)
```

$$y(x) = -\frac{2\cos(x)^2}{3} + \sec(x)\,c_1$$

✓ Solution by Mathematica
Time used: 0.046 (sec). Leaf size: 19

```
DSolve[y'[x]==Sin[2*x]+y[x]*Tan[x],y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to -\frac{2\cos^2(x)}{3} + c_1\sec(x)$$

2.6 problem 31

Internal problem ID [3295]

Book: Ordinary differential equations and their solutions. By George Moseley Murphy. 1960
Section: Various 2
Problem number: 31.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_linear]`

$$\boxed{y' + y\tan(x) = \sin(2x)}$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 13

```
dsolve(diff(y(x),x) = sin(2*x)-y(x)*tan(x),y(x), singsol=all)
```

$$y(x) = (-2\cos(x) + c_1)\cos(x)$$

✓ Solution by Mathematica

Time used: 0.046 (sec). Leaf size: 15

```
DSolve[y'[x]==Sin[2*x]-y[x]*Tan[x],y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to \cos(x)(-2\cos(x) + c_1)$$

2.7 problem 32

Internal problem ID [3296]

Book: Ordinary differential equations and their solutions. By George Moseley Murphy. 1960
Section: Various 2
Problem number: 32.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_linear]`

$$\boxed{y' - 2y\tan(x) = \sin(x)}$$

✓ Solution by Maple

Time used: 0.015 (sec). Leaf size: 15

```
dsolve(diff(y(x),x) = sin(x)+2*y(x)*tan(x),y(x), singsol=all)
```

$$y(x) = -\frac{\cos(x)}{3} + \sec(x)^2 c_1$$

✓ Solution by Mathematica

Time used: 0.045 (sec). Leaf size: 19

```
DSolve[y'[x]==Sin[x]+2*y[x]*Tan[x],y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to -\frac{\cos(x)}{3} + c_1\sec^2(x)$$

2.8 problem 33

Internal problem ID [3297]

Book: Ordinary differential equations and their solutions. By George Moseley Murphy. 1960
Section: Various 2
Problem number: 33.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_linear]`

$$\boxed{y' - 2y\tan(2x) = 2\sec(2x) + 2}$$

✓ Solution by Maple
Time used: 0.0 (sec). Leaf size: 24

```
dsolve(diff(y(x),x) = 2+2*sec(2*x)+2*y(x)*tan(2*x),y(x), singsol=all)
```

$$y(x) = \sec(2x)\left(\operatorname{csgn}(\sec(2x))\, c_1 + \sin(2x) + 2x\right)$$

✓ Solution by Mathematica
Time used: 0.078 (sec). Leaf size: 20

```
DSolve[y'[x]==2*(1+Sec[2 x]+y[x] Tan[2 x]),y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to \sec(2x)(2x + \sin(2x) + c_1)$$

2.9 problem 34

Internal problem ID [3298]

Book: Ordinary differential equations and their solutions. By George Moseley Murphy. 1960
Section: Various 2
Problem number: 34.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_linear]`

$$\boxed{y' - 3y \tan(x) = \csc(x)}$$

✓ Solution by Maple
Time used: 0.016 (sec). Leaf size: 20

```
dsolve(diff(y(x),x) = csc(x)+3*y(x)*tan(x),y(x), singsol=all)
```

$$y(x) = \sec(x)^3 \left(\frac{\cos(x)^2}{2} + \ln(\sin(x)) + c_1\right)$$

✓ Solution by Mathematica
Time used: 0.06 (sec). Leaf size: 24

```
DSolve[y'[x]==Csc[x]+3 y[x] Tan[x],y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to \sec^3(x)\left(-\frac{1}{2}\sin^2(x) + \log(\sin(x)) + c_1\right)$$

2.10 problem 35

Internal problem ID [3299]

Book: Ordinary differential equations and their solutions. By George Moseley Murphy. 1960
Section: Various 2
Problem number: 35.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_separable]`

$$\boxed{y' - (a + \cos(\ln(x)) + \sin(\ln(x)))\, y = 0}$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 14

```
dsolve(diff(y(x),x) = (a+cos(ln(x))+sin(ln(x)))*y(x),y(x), singsol=all)
```

$$y(x) = c_1 e^{x(\sin(\ln(x))+a)}$$

✓ Solution by Mathematica

Time used: 0.062 (sec). Leaf size: 22

```
DSolve[y'[x]==(a+Cos[Log[x]]+Sin[Log[x]]) y[x],y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to c_1 e^{x(a+\sin(\log(x)))}$$
$$y(x) \to 0$$

2.11 problem 36

Internal problem ID [3300]

Book: Ordinary differential equations and their solutions. By George Moseley Murphy. 1960
Section: Various 2
Problem number: 36.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_linear]`

$$\boxed{y' + y\tanh(x) = 6\,\mathrm{e}^{2x}}$$

✓ Solution by Maple
Time used: 0.0 (sec). Leaf size: 25

```
dsolve(diff(y(x),x) = 6*exp(2*x)-y(x)*tanh(x),y(x), singsol=all)
```

$$y(x) = 2\tanh(x) + 4\sinh(x)\cosh(x) + 4\cosh(x)^2 + \operatorname{sech}(x)\,c_1$$

✓ Solution by Mathematica
Time used: 0.148 (sec). Leaf size: 33

```
DSolve[y'[x]==6 Exp[2 x]- y[x] Tanh[x],y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to \frac{e^x(6e^x + 2e^{3x} + c_1)}{e^{2x} + 1}$$

2.12 problem 37

Internal problem ID [3301]

Book: Ordinary differential equations and their solutions. By George Moseley Murphy. 1960
Section: Various 2
Problem number: 37.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_linear]`

$$\boxed{y' - f'(x)\, y = f(x)\, f'(x)}$$

✓ Solution by Maple
Time used: 0.0 (sec). Leaf size: 15

```
dsolve(diff(y(x),x) = f(x)*diff(f(x),x)+diff(f(x),x)*y(x),y(x), singsol=all)
```

$$y(x) = -f(x) - 1 + e^{f(x)} c_1$$

✓ Solution by Mathematica
Time used: 0.052 (sec). Leaf size: 18

```
DSolve[y'[x]==f[x] f'[x] + f'[x] y[x],y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to -f(x) + c_1 e^{f(x)} - 1$$

2.13 problem 38

Internal problem ID [3302]

Book: Ordinary differential equations and their solutions. By George Moseley Murphy. 1960
Section: Various 2
Problem number: 38.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_linear]`

$$\boxed{y' - g(x) y = f(x)}$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 24

```
dsolve(diff(y(x),x) = f(x)+g(x)*y(x),y(x), singsol=all)
```

$$y(x) = \left(\int f(x) e^{-\left(\int g(x)dx\right)} dx + c_1\right) e^{\int g(x)dx}$$

✓ Solution by Mathematica

Time used: 0.054 (sec). Leaf size: 47

```
DSolve[y'[x]==f[x] + g[x] y[x],y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to \exp\left(\int_1^x g(K[1])dK[1]\right)\left(\int_1^x \exp\left(-\int_1^{K[2]} g(K[1])dK[1]\right) f(K[2])dK[2] + c_1\right)$$

2.14 problem 39

Internal problem ID [3303]

Book: Ordinary differential equations and their solutions. By George Moseley Murphy. 1960
Section: Various 2
Problem number: 39.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_Riccati]`

$$\boxed{y' + y^2 = x^2}$$

✓ Solution by Maple

Time used: 0.016 (sec). Leaf size: 44

```
dsolve(diff(y(x),x) = x^2-y(x)^2,y(x), singsol=all)
```

$$y(x) = \frac{x\left(\text{BesselI}\left(-\frac{3}{4}, \frac{x^2}{2}\right) c_1 - \text{BesselK}\left(\frac{3}{4}, \frac{x^2}{2}\right)\right)}{c_1 \,\text{BesselI}\left(\frac{1}{4}, \frac{x^2}{2}\right) + \text{BesselK}\left(\frac{1}{4}, \frac{x^2}{2}\right)}$$

✓ Solution by Mathematica

Time used: 0.135 (sec). Leaf size: 197

```
DSolve[y'[x]==x^2 - y[x]^2,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to -\frac{-ix^2\left(2\,\text{BesselJ}\left(-\frac{3}{4}, \frac{ix^2}{2}\right) + c_1\left(\text{BesselJ}\left(-\frac{5}{4}, \frac{ix^2}{2}\right) - \text{BesselJ}\left(\frac{3}{4}, \frac{ix^2}{2}\right)\right)\right) - c_1\,\text{BesselJ}\left(-\frac{1}{4}, \frac{ix^2}{2}\right)}{2x\left(\text{BesselJ}\left(\frac{1}{4}, \frac{ix^2}{2}\right) + c_1\,\text{BesselJ}\left(-\frac{1}{4}, \frac{ix^2}{2}\right)\right)}$$

$$y(x) \to \frac{ix^2\,\text{BesselJ}\left(-\frac{5}{4}, \frac{ix^2}{2}\right) - ix^2\,\text{BesselJ}\left(\frac{3}{4}, \frac{ix^2}{2}\right) + \text{BesselJ}\left(-\frac{1}{4}, \frac{ix^2}{2}\right)}{2x\,\text{BesselJ}\left(-\frac{1}{4}, \frac{ix^2}{2}\right)}$$

2.15 problem 40

Internal problem ID [3304]

Book: Ordinary differential equations and their solutions. By George Moseley Murphy. 1960
Section: Various 2
Problem number: 40.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_Riccati]`

$$\boxed{y' - y^2 = -f(x)^2 + f'(x)}$$

✓ Solution by Maple
Time used: 0.0 (sec). Leaf size: 45

```
dsolve(diff(y(x),x)+f(x)^2 = diff(f(x),x)+y(x)^2,y(x), singsol=all)
```

$$y(x) = \frac{-f(x)\left(\int e^{2\left(\int f(x)dx\right)}dx\right) + f(x)\, c_1 + e^{2\left(\int f(x)dx\right)}}{c_1 - \left(\int e^{2\left(\int f(x)dx\right)}dx\right)}$$

✗ Solution by Mathematica
Time used: 0.0 (sec). Leaf size: 0

```
DSolve[y'[x]+f[x]^2==f'[x]+y[x]^2,y[x],x,IncludeSingularSolutions -> True]
```

Not solved

2.16 problem 41

Internal problem ID [3305]

Book: Ordinary differential equations and their solutions. By George Moseley Murphy. 1960
Section: Various 2
Problem number: 41.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_Riccati]`

$$\boxed{y' - y(y + x) = x - 1}$$

✓ Solution by Maple
Time used: 0.0 (sec). Leaf size: 66

```
dsolve(diff(y(x),x)+1-x = (x+y(x))*y(x),y(x), singsol=all)
```

$$y(x) = \frac{-i\sqrt{\pi}\,\mathrm{e}^{-2}\sqrt{2}\,\mathrm{erf}\left(\frac{i\sqrt{2}\,(-2+x)}{2}\right) + 2\,\mathrm{e}^{\frac{x(x-4)}{2}} - 2c_1}{i\sqrt{\pi}\,\mathrm{e}^{-2}\sqrt{2}\,\mathrm{erf}\left(\frac{i\sqrt{2}\,(-2+x)}{2}\right) + 2c_1}$$

✓ Solution by Mathematica
Time used: 0.179 (sec). Leaf size: 54

```
DSolve[y'[x]+1-x==(x+y[x])y[x],y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \rightarrow -1 + \frac{2e^{\frac{1}{2}(x-2)^2}}{-\sqrt{2\pi}\mathrm{erfi}\left(\frac{x-2}{\sqrt{2}}\right) + 2e^2c_1}$$

$$y(x) \rightarrow -1$$

2.17 problem 42

Internal problem ID [3306]

Book: Ordinary differential equations and their solutions. By George Moseley Murphy. 1960
Section: Various 2
Problem number: 42.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[[_homogeneous, ‘class C‘], _Riccati]`

$$\boxed{y' - (y + x)^2 = 0}$$

✓ Solution by Maple
Time used: 0.016 (sec). Leaf size: 16

```
dsolve(diff(y(x),x) = (x+y(x))^2,y(x), singsol=all)
```

$$y(x) = -x - \tan(c_1 - x)$$

✓ Solution by Mathematica
Time used: 0.603 (sec). Leaf size: 14

```
DSolve[y'[x]==(x+y[x])^2,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to -x + \tan(x + c_1)$$

2.18 problem 43

Internal problem ID [3307]

Book: Ordinary differential equations and their solutions. By George Moseley Murphy. 1960
Section: Various 2
Problem number: 43.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[[_homogeneous, ‘class C‘], _Riccati]`

$$\boxed{y' - (-y + x)^2 = 0}$$

✓ Solution by Maple

Time used: 0.032 (sec). Leaf size: 29

```
dsolve(diff(y(x),x) = (x-y(x))^2,y(x), singsol=all)
```

$$y(x) = \frac{c_1(x-1)\,e^{2x} - x - 1}{c_1 e^{2x} - 1}$$

✓ Solution by Mathematica

Time used: 0.126 (sec). Leaf size: 29

```
DSolve[y'[x]==(x-y[x])^2,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to x + \frac{1}{\frac{1}{2} + c_1 e^{2x}} - 1$$
$$y(x) \to x - 1$$

2.19 problem 44

Internal problem ID [3308]

Book: Ordinary differential equations and their solutions. By George Moseley Murphy. 1960
Section: Various 2
Problem number: 44.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[[_homogeneous, ‘class C‘], _Riccati]`

$$\boxed{y' - 3y - (-y + x)^2 = -3x + 3}$$

✓ Solution by Maple
Time used: 0.031 (sec). Leaf size: 25

```
dsolve(diff(y(x),x) = 3-3*x+3*y(x)+(x-y(x))^2,y(x), singsol=all)
```

$$y(x) = \frac{c_1(-2 + x)\,e^x + 1 - x}{e^x c_1 - 1}$$

✓ Solution by Mathematica
Time used: 0.174 (sec). Leaf size: 25

```
DSolve[y'[x]==3(1-x+y[x])+(x-y[x])^2,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to x + \frac{1}{1 + c_1 e^x} - 2$$
$$y(x) \to x - 2$$

2.20 problem 45

Internal problem ID [3309]

Book: Ordinary differential equations and their solutions. By George Moseley Murphy. 1960
Section: Various 2
Problem number: 45.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_Riccati]`

$$y' + (x^2 + 1)\, y - y^2 = 2x$$

✓ Solution by Maple
Time used: 0.0 (sec). Leaf size: 67

```
dsolve(diff(y(x),x) = 2*x-(x^2+1)*y(x)+y(x)^2,y(x), singsol=all)
```

$$y(x) = \frac{-x^2\left(\int \mathrm{e}^{\frac{x(x^2+3)}{3}}dx\right) + c_1x^2 + \mathrm{e}^{\frac{x(x^2+3)}{3}} - \left(\int \mathrm{e}^{\frac{x(x^2+3)}{3}}dx\right) + c_1}{c_1 - \left(\int \mathrm{e}^{\frac{x(x^2+3)}{3}}dx\right)}$$

✓ Solution by Mathematica
Time used: 0.337 (sec). Leaf size: 58

```
DSolve[y'[x]==2 x-(1+x^2)y[x]+y[x]^2,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to \frac{e^{\frac{x^3}{3}+x}}{-\int_1^x e^{\frac{K[1]^3}{3}+K[1]}dK[1] + c_1} + x^2 + 1$$
$$y(x) \to x^2 + 1$$

2.21 problem 46

Internal problem ID [3310]

Book: Ordinary differential equations and their solutions. By George Moseley Murphy. 1960
Section: Various 2
Problem number: 46.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[[_1st_order, _with_linear_symmetries], _Riccati]`

$$\boxed{y' + (2x^2 - y)\, y = x(x^3 + 2)}$$

✓ Solution by Maple
Time used: 0.015 (sec). Leaf size: 20

```
dsolve(diff(y(x),x) = x*(x^3+2)-(2*x^2-y(x))*y(x),y(x), singsol=all)
```

$$y(x) = \frac{c_1 x^2 + x^3 - 1}{c_1 + x}$$

✓ Solution by Mathematica
Time used: 0.134 (sec). Leaf size: 24

```
DSolve[y'[x]==x*(2+x^3)-(2*x^2-y[x])y[x],y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to x^2 + \frac{1}{-x + c_1}$$
$$y(x) \to x^2$$

2.22 problem 47

Internal problem ID [3311]

Book: Ordinary differential equations and their solutions. By George Moseley Murphy. 1960
Section: Various 2
Problem number: 47.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[[_1st_order, _with_linear_symmetries], _Riccati]`

$$y' - (2x^2 - y)\, y = 1 + x(-x^3 + 2)$$

✓ Solution by Maple

Time used: 0.031 (sec). Leaf size: 33

```
dsolve(diff(y(x),x) = 1+x*(-x^3+2)+(2*x^2-y(x))*y(x),y(x), singsol=all)
```

$$y(x) = \frac{(x^2 + 1)\, c_1 e^{2x} - x^2 + 1}{c_1 e^{2x} - 1}$$

✓ Solution by Mathematica

Time used: 0.125 (sec). Leaf size: 34

```
DSolve[y'[x]==1+x*(2-x^3)+(2*x^2-y[x])y[x],y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to x^2 - \frac{2}{1 + 2c_1 e^{2x}} + 1$$
$$y(x) \to x^2 + 1$$

2.23 problem 48

Internal problem ID [3312]

Book: Ordinary differential equations and their solutions. By George Moseley Murphy. 1960
Section: Various 2
Problem number: 48.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_Riccati]`

$$\boxed{y' + (\sin(x) - y)\, y = \cos(x)}$$

✓ Solution by Maple
Time used: 0.0 (sec). Leaf size: 38

```
dsolve(diff(y(x),x) = cos(x)-(sin(x)-y(x))*y(x),y(x), singsol=all)
```

$$y(x) = \frac{\sin(x)\left(\int \mathrm{e}^{-\cos(x)}dx\right) + c_1 \sin(x) - \mathrm{e}^{-\cos(x)}}{c_1 + \int \mathrm{e}^{-\cos(x)}dx}$$

✓ Solution by Mathematica
Time used: 42.807 (sec). Leaf size: 158

```
DSolve[y'[x]==Cos[x]-(Sin[x]-y[x])y[x],y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \rightarrow \frac{c_1 \sin(x) \int_1^x e^{-\cos(K[1])} dK[1] + \sin(x) + c_1 \left(-e^{-\cos(x)}\right)}{1 + c_1 \int_1^x e^{-\cos(K[1])} dK[1]}$$
$$y(x) \rightarrow \sin(x)$$
$$y(x) \rightarrow \frac{\sin^3(x) e^{\cos(x)} \int_1^{\cos(x)} \frac{e^{-K[1]}K[1]}{(1-K[1]^2)^{3/2}} dK[1]}{\sin^2(x) e^{\cos(x)} \int_1^{\cos(x)} \frac{e^{-K[1]}K[1]}{(1-K[1]^2)^{3/2}} dK[1] - \sqrt{\sin^2(x)}}$$

2.24 problem 49

Internal problem ID [3313]

Book: Ordinary differential equations and their solutions. By George Moseley Murphy. 1960
Section: Various 2
Problem number: 49.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_Riccati]`

$$\boxed{y' - (\sin(2x) + y)\, y = \cos(2x)}$$

✓ Solution by Maple
Time used: 0.0 (sec). Leaf size: 96

```
dsolve(diff(y(x),x) = cos(2*x)+(sin(2*x)+y(x))*y(x),y(x), singsol=all)
```

$$y(x) = \frac{\sin(x)\left(\operatorname{HeunC}\left(1, \frac{1}{2}, -\frac{1}{2}, -1, \frac{7}{8}, \frac{\cos(2x)}{2} + \frac{1}{2}\right) c_1 + 2\cos(x)\left(\cos(x)\operatorname{HeunCPrime}\left(1, \frac{1}{2}, -\frac{1}{2}, -1, \frac{7}{8}, \frac{\cos(2x)}{2}\right.\right.\right.}{c_1\cos(x)\operatorname{HeunC}\left(1, \frac{1}{2}, -\frac{1}{2}, -1, \frac{7}{8}, \frac{\cos(2x)}{2} + \frac{1}{2}\right) + \operatorname{HeunC}\left(1, -\frac{1}{2}, -\right.}$$

✓ Solution by Mathematica
Time used: 2.305 (sec). Leaf size: 111

```
DSolve[y'[x]==Cos[2 x]+(Sin[2 x]+y[x])y[x],y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to \frac{\sec(x)\left(\sin(x)\int_1^{\cos(x)} \frac{e^{-K[1]^2}}{K[1]^2\sqrt{K[1]^2-1}}dK[1] + c_1\sin(x) + \frac{e^{-\cos^2(x)}\tan(x)}{\sqrt{-\sin^2(x)}}\right)}{\int_1^{\cos(x)} \frac{e^{-K[1]^2}}{K[1]^2\sqrt{K[1]^2-1}}dK[1] + c_1}$$

$$y(x) \to \tan(x)$$

2.25 problem 50

Internal problem ID [3314]

Book: Ordinary differential equations and their solutions. By George Moseley Murphy. 1960
Section: Various 2
Problem number: 50.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_Riccati]`

$$\boxed{y' - xf(x)\, y - y^2 = f(x)}$$

✓ Solution by Maple

Time used: 0.015 (sec). Leaf size: 69

```
dsolve(diff(y(x),x) = f(x)+x*f(x)*y(x)+y(x)^2,y(x), singsol=all)
```

$$y(x) = \frac{e^{\int \frac{f(x)x^2-2}{x}dx}x + \int e^{\int \frac{f(x)x^2-2}{x}dx}dx - c_1}{\left(c_1 - \left(\int e^{\int \frac{f(x)x^2-2}{x}dx}dx\right)\right)x}$$

✓ Solution by Mathematica

Time used: 0.756 (sec). Leaf size: 111

```
DSolve[y'[x]==f[x]+x f[x] y[x]+y[x]^2,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to -\frac{\exp\left(-\int_1^x -f(K[1])K[1]dK[1]\right) + x\int_1^x \frac{\exp\left(-\int_1^{K[2]} -f(K[1])K[1]dK[1]\right)}{K[2]^2}dK[2] + c_1 x}{x^2\left(\int_1^x \frac{\exp\left(-\int_1^{K[2]} -f(K[1])K[1]dK[1]\right)}{K[2]^2}dK[2] + c_1\right)}$$

$$y(x) \to -\frac{1}{x}$$

2.26 problem 51

Internal problem ID [3315]

Book: Ordinary differential equations and their solutions. By George Moseley Murphy. 1960
Section: Various 2
Problem number: 51.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[[_homogeneous, ‘class C‘], _Riccati]`

$$\boxed{y' - (3 + x - 4y)^2 = 0}$$

✓ Solution by Maple

Time used: 0.032 (sec). Leaf size: 32

```
dsolve(diff(y(x),x) = (3+x-4*y(x))^2,y(x), singsol=all)
```

$$y(x) = \frac{c_1(5 + 2x)\,e^{4x} - 2x - 7}{8\,e^{4x}c_1 - 8}$$

✓ Solution by Mathematica

Time used: 0.148 (sec). Leaf size: 41

```
DSolve[y'[x]==(3+x-4 y[x])^2,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to \frac{1}{16}\left(4x + \frac{1}{\frac{1}{4} + c_1 e^{4x}} + 10\right)$$
$$y(x) \to \frac{1}{8}(2x + 5)$$

2.27 problem 52

Internal problem ID [3316]

Book: Ordinary differential equations and their solutions. By George Moseley Murphy. 1960
Section: Various 2
Problem number: 52.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[[_homogeneous, ‘class C‘], _Riccati]`

$$\boxed{y' - (1 + 4x + 9y)^2 = 0}$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 19

```
dsolve(diff(y(x),x) = (1+4*x+9*y(x))^2,y(x), singsol=all)
```

$$y(x) = -\frac{4x}{9} - \frac{1}{9} - \frac{2\tan(-6x + 6c_1)}{27}$$

✓ Solution by Mathematica

Time used: 0.165 (sec). Leaf size: 49

```
DSolve[y'[x]==(1+4 x+9 y[x])^2,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to \frac{1}{81}\left(-36x + \frac{1}{c_1 e^{12ix} - \frac{i}{12}} - (9 + 6i)\right)$$
$$y(x) \to \frac{1}{27}(-12x - (3 + 2i))$$

2.28 problem 53

Internal problem ID [3317]

Book: Ordinary differential equations and their solutions. By George Moseley Murphy. 1960
Section: Various 2
Problem number: 53.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_Riccati]`

$$\boxed{y' - 3by^2 = 3bx + 3a}$$

✓ Solution by Maple
Time used: 0.032 (sec). Leaf size: 80

```
dsolve(diff(y(x),x) = 3*a+3*b*x+3*b*y(x)^2,y(x), singsol=all)
```

$$y(x) = \frac{\left(\operatorname{AiryAi}\left(1, -\frac{3^{\frac{2}{3}}(bx+a)}{b^{\frac{1}{3}}}\right)c_1 + \operatorname{AiryBi}\left(1, -\frac{3^{\frac{2}{3}}(bx+a)}{b^{\frac{1}{3}}}\right)\right)3^{\frac{2}{3}}}{b^{\frac{1}{3}}\left(3c_1 \operatorname{AiryAi}\left(-\frac{3^{\frac{2}{3}}(bx+a)}{b^{\frac{1}{3}}}\right) + 3\operatorname{AiryBi}\left(-\frac{3^{\frac{2}{3}}(bx+a)}{b^{\frac{1}{3}}}\right)\right)}$$

✓ Solution by Mathematica
Time used: 0.219 (sec). Leaf size: 191

```
DSolve[y'[x]==3*(a+b*x+ b*y[x]^2),y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to \frac{b\left(\operatorname{AiryBiPrime}\left(-\frac{3^{2/3}b(a+bx)}{(-b^2)^{2/3}}\right) + c_1 \operatorname{AiryAiPrime}\left(-\frac{3^{2/3}b(a+bx)}{(-b^2)^{2/3}}\right)\right)}{\sqrt[3]{3}\,(-b^2)^{2/3}\left(\operatorname{AiryBi}\left(-\frac{3^{2/3}b(a+bx)}{(-b^2)^{2/3}}\right) + c_1 \operatorname{AiryAi}\left(-\frac{3^{2/3}b(a+bx)}{(-b^2)^{2/3}}\right)\right)}$$

$$y(x) \to \frac{b \operatorname{AiryAiPrime}\left(-\frac{3^{2/3}b(a+bx)}{(-b^2)^{2/3}}\right)}{\sqrt[3]{3}\,(-b^2)^{2/3} \operatorname{AiryAi}\left(-\frac{3^{2/3}b(a+bx)}{(-b^2)^{2/3}}\right)}$$

2.29 problem 54

Internal problem ID [3318]

Book: Ordinary differential equations and their solutions. By George Moseley Murphy. 1960
Section: Various 2
Problem number: 54.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_quadrature]`

$$\boxed{y' - by^2 = a}$$

✓ Solution by Maple
Time used: 0.016 (sec). Leaf size: 23

```
dsolve(diff(y(x),x) = a+b*y(x)^2,y(x), singsol=all)
```

$$y(x) = \frac{\sqrt{ab}\,\tan\left(\sqrt{ab}\,(c_1 + x)\right)}{b}$$

✓ Solution by Mathematica
Time used: 9.091 (sec). Leaf size: 68

```
DSolve[y'[x]==a+b y[x]^2,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to \frac{\sqrt{a}\tan\left(\sqrt{a}\sqrt{b}(x + c_1)\right)}{\sqrt{b}}$$
$$y(x) \to -\frac{i\sqrt{a}}{\sqrt{b}}$$
$$y(x) \to \frac{i\sqrt{a}}{\sqrt{b}}$$

3 Various 3

3.1 problem 55

Internal problem ID [3319]

Book: Ordinary differential equations and their solutions. By George Moseley Murphy. 1960
Section: Various 3
Problem number: 55.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[[_Riccati, _special]]`

$$\boxed{y' - by^2 = ax}$$

✓ Solution by Maple
Time used: 0.0 (sec). Leaf size: 59

```
dsolve(diff(y(x),x) = a*x+b*y(x)^2,y(x), singsol=all)
```

$$y(x) = \frac{(ab)^{\frac{1}{3}} \left(\mathrm{AiryAi}\left(1, -(ab)^{\frac{1}{3}}\, x\right) c_1 + \mathrm{AiryBi}\left(1, -(ab)^{\frac{1}{3}}\, x\right)\right)}{b \left(c_1 \,\mathrm{AiryAi}\left(-\,(ab)^{\frac{1}{3}}\, x\right) + \mathrm{AiryBi}\left(-\,(ab)^{\frac{1}{3}}\, x\right)\right)}$$

✓ Solution by Mathematica
Time used: 0.155 (sec). Leaf size: 331

```
DSolve[y'[x]==a x+b y[x]^2,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to \frac{\sqrt{a}\sqrt{b}x^{3/2}\left(-2\,\mathrm{BesselJ}\left(-\frac{2}{3}, \frac{2}{3}\sqrt{a}\sqrt{b}x^{3/2}\right) + c_1\left(\mathrm{BesselJ}\left(\frac{2}{3}, \frac{2}{3}\sqrt{a}\sqrt{b}x^{3/2}\right) - \mathrm{BesselJ}\left(-\frac{4}{3}, \frac{2}{3}\sqrt{a}\sqrt{b}x^{3/2}\right)\right.\right.}{2bx\left(\mathrm{BesselJ}\left(\frac{1}{3}, \frac{2}{3}\sqrt{a}\sqrt{b}x^{3/2}\right) + c_1\,\mathrm{BesselJ}\left(-\frac{1}{3}, \frac{2}{3}\sqrt{a}\sqrt{b}x^{3/2}\right)\right)}$$

$$y(x) \to -\frac{\sqrt{a}\sqrt{b}x^{3/2}\,\mathrm{BesselJ}\left(-\frac{4}{3}, \frac{2}{3}\sqrt{a}\sqrt{b}x^{3/2}\right) - \sqrt{a}\sqrt{b}x^{3/2}\,\mathrm{BesselJ}\left(\frac{2}{3}, \frac{2}{3}\sqrt{a}\sqrt{b}x^{3/2}\right) + \mathrm{BesselJ}\left(-\frac{1}{3}, \frac{2}{3}\sqrt{a}\sqrt{b}x\right.}{2bx\,\mathrm{BesselJ}\left(-\frac{1}{3}, \frac{2}{3}\sqrt{a}\sqrt{b}x^{3/2}\right)}$$

3.2 problem 56

Internal problem ID [3320]

Book: Ordinary differential equations and their solutions. By George Moseley Murphy. 1960
Section: Various 3
Problem number: 56.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_Riccati]`

$$\boxed{y' - cy^2 = bx + a}$$

✓ Solution by Maple
Time used: 0.015 (sec). Leaf size: 85

```
dsolve(diff(y(x),x) = a+b*x+c*y(x)^2,y(x), singsol=all)
```

$$y(x) = \frac{\left(\frac{b}{\sqrt{c}}\right)^{\frac{1}{3}} \left(\mathrm{AiryAi}\left(1, -\frac{bx+a}{\left(\frac{b}{\sqrt{c}}\right)^{\frac{2}{3}}}\right) c_1 + \mathrm{AiryBi}\left(1, -\frac{bx+a}{\left(\frac{b}{\sqrt{c}}\right)^{\frac{2}{3}}}\right)\right)}{\sqrt{c}\left(c_1 \,\mathrm{AiryAi}\left(-\frac{bx+a}{\left(\frac{b}{\sqrt{c}}\right)^{\frac{2}{3}}}\right) + \mathrm{AiryBi}\left(-\frac{bx+a}{\left(\frac{b}{\sqrt{c}}\right)^{\frac{2}{3}}}\right)\right)}$$

✓ Solution by Mathematica
Time used: 0.199 (sec). Leaf size: 143

```
DSolve[y'[x]==a+b x+c y[x]^2,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to \frac{b\left(\mathrm{AiryBiPrime}\left(-\frac{c(a+bx)}{(-bc)^{2/3}}\right) + c_1 \,\mathrm{AiryAiPrime}\left(-\frac{c(a+bx)}{(-bc)^{2/3}}\right)\right)}{(-bc)^{2/3}\left(\mathrm{AiryBi}\left(-\frac{c(a+bx)}{(-bc)^{2/3}}\right) + c_1 \,\mathrm{AiryAi}\left(-\frac{c(a+bx)}{(-bc)^{2/3}}\right)\right)}$$

$$y(x) \to \frac{b \,\mathrm{AiryAiPrime}\left(-\frac{c(a+bx)}{(-bc)^{2/3}}\right)}{(-bc)^{2/3}\,\mathrm{AiryAi}\left(-\frac{c(a+bx)}{(-bc)^{2/3}}\right)}$$

3.3 problem 57

Internal problem ID [3321]

Book: Ordinary differential equations and their solutions. By George Moseley Murphy. 1960
Section: Various 3
Problem number: 57.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_Riccati]`

$$y' - cy^2 = a\,x^{n-1} + b\,x^{2n}$$

✓ Solution by Maple
Time used: 0.0 (sec). Leaf size: 354

```
dsolve(diff(y(x),x) = a*x^(n-1)+b*x^(2*n)+c*y(x)^2,y(x), singsol=all)
```

$$y(x) = -\frac{\left((2+n)\sqrt{b} - i\sqrt{c}\,a\right) \mathrm{WhittakerM}\left(-\frac{(-2n-2)\sqrt{b}+i\sqrt{c}\,a}{\sqrt{b}\,(2n+2)}, \frac{1}{2n+2}, \frac{2i\sqrt{b}\,\sqrt{c}\,x^n x}{n+1}\right) - 2c_1\sqrt{b}\,(n+1)\,\mathrm{WhittakerW}}{2\sqrt{b}\,\left(\mathrm{WhittakerW}\right.}$$

✓ Solution by Mathematica

Time used: 1.115 (sec). Leaf size: 982

```
DSolve[y'[x]==a x^(n-1)+b x^(2 n)+c y[x]^2,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to -\frac{x^n\left(\sqrt{b}c_1(n+1)\sqrt{-(n+1)^2}\,\text{HypergeometricU}\left(\frac{1}{2}\left(\frac{\sqrt{c}a}{\sqrt{b}\sqrt{-(n+1)^2}}+\frac{n}{n+1}\right),\frac{n}{n+1},\frac{2\sqrt{b}\sqrt{c}x^{n+1}}{\sqrt{-(n+1)^2}}\right)+c_1\left(a\sqrt{c}(n\right.\right.}{\sqrt{c}(n+1)^2}$$

$$y(x) \to \frac{x^n\left(-\frac{\left(a\sqrt{c}(n+1)+\sqrt{b}\sqrt{-(n+1)^2}n\right)\text{HypergeometricU}\left(\frac{1}{2}\left(\frac{\sqrt{c}a}{\sqrt{b}\sqrt{-(n+1)^2}}+\frac{n}{n+1}+2\right),\frac{n}{n+1}+1,\frac{2\sqrt{b}\sqrt{c}x^{n+1}}{\sqrt{-(n+1)^2}}\right)}{\text{HypergeometricU}\left(\frac{1}{2}\left(\frac{\sqrt{c}a}{\sqrt{b}\sqrt{-(n+1)^2}}+\frac{n}{n+1}\right),\frac{n}{n+1},\frac{2\sqrt{b}\sqrt{c}x^{n+1}}{\sqrt{-(n+1)^2}}\right)}-\sqrt{b}\sqrt{-(n+1)^2}(n\right.}{\sqrt{c}(n+1)^2}$$

$$y(x) \to \frac{x^n\left(-\frac{\left(a\sqrt{c}(n+1)+\sqrt{b}\sqrt{-(n+1)^2}n\right)\text{HypergeometricU}\left(\frac{1}{2}\left(\frac{\sqrt{c}a}{\sqrt{b}\sqrt{-(n+1)^2}}+\frac{n}{n+1}+2\right),\frac{n}{n+1}+1,\frac{2\sqrt{b}\sqrt{c}x^{n+1}}{\sqrt{-(n+1)^2}}\right)}{\text{HypergeometricU}\left(\frac{1}{2}\left(\frac{\sqrt{c}a}{\sqrt{b}\sqrt{-(n+1)^2}}+\frac{n}{n+1}\right),\frac{n}{n+1},\frac{2\sqrt{b}\sqrt{c}x^{n+1}}{\sqrt{-(n+1)^2}}\right)}-\sqrt{b}\sqrt{-(n+1)^2}(n\right.}{\sqrt{c}(n+1)^2}$$

3.4 problem 58

Internal problem ID [3322]

Book: Ordinary differential equations and their solutions. By George Moseley Murphy. 1960
Section: Various 3
Problem number: 58.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[[_Riccati, _special]]`

$$\boxed{y' - by^2 = x^2 a}$$

✓ Solution by Maple
Time used: 0.016 (sec). Leaf size: 73

```
dsolve(diff(y(x),x) = a*x^2+b*y(x)^2,y(x), singsol=all)
```

$$y(x) = \frac{\left(-\operatorname{BesselJ}\left(-\frac{3}{4}, \frac{\sqrt{ab}\,x^2}{2}\right)c_1 - \operatorname{BesselY}\left(-\frac{3}{4}, \frac{\sqrt{ab}\,x^2}{2}\right)\right)\sqrt{ab}\,x}{b\left(c_1 \operatorname{BesselJ}\left(\frac{1}{4}, \frac{\sqrt{ab}\,x^2}{2}\right) + \operatorname{BesselY}\left(\frac{1}{4}, \frac{\sqrt{ab}\,x^2}{2}\right)\right)}$$

✓ Solution by Mathematica
Time used: 0.143 (sec). Leaf size: 305

```
DSolve[y'[x]==a x^2+b y[x]^2,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to \frac{\sqrt{a}\sqrt{b}x^2\left(-2\operatorname{BesselJ}\left(-\frac{3}{4}, \frac{1}{2}\sqrt{a}\sqrt{b}x^2\right) + c_1\left(\operatorname{BesselJ}\left(\frac{3}{4}, \frac{1}{2}\sqrt{a}\sqrt{b}x^2\right) - \operatorname{BesselJ}\left(-\frac{5}{4}, \frac{1}{2}\sqrt{a}\sqrt{b}x^2\right)\right)\right) - c_1}{2bx\left(\operatorname{BesselJ}\left(\frac{1}{4}, \frac{1}{2}\sqrt{a}\sqrt{b}x^2\right) + c_1 \operatorname{BesselJ}\left(-\frac{1}{4}, \frac{1}{2}\sqrt{a}\sqrt{b}x^2\right)\right)}$$

$$y(x) \to -\frac{\sqrt{a}\sqrt{b}x^2 \operatorname{BesselJ}\left(-\frac{5}{4}, \frac{1}{2}\sqrt{a}\sqrt{b}x^2\right) - \sqrt{a}\sqrt{b}x^2 \operatorname{BesselJ}\left(\frac{3}{4}, \frac{1}{2}\sqrt{a}\sqrt{b}x^2\right) + \operatorname{BesselJ}\left(-\frac{1}{4}, \frac{1}{2}\sqrt{a}\sqrt{b}x^2\right)}{2bx \operatorname{BesselJ}\left(-\frac{1}{4}, \frac{1}{2}\sqrt{a}\sqrt{b}x^2\right)}$$

3.5 problem 59

Internal problem ID [3323]

Book: Ordinary differential equations and their solutions. By George Moseley Murphy. 1960
Section: Various 3
Problem number: 59.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_quadrature]`

$$\boxed{y' - \mathrm{a1}\, y - \mathrm{a2}\, y^2 = \mathrm{a0}}$$

✓ Solution by Maple
Time used: 0.016 (sec). Leaf size: 44

```
dsolve(diff(y(x),x) = a0+a1*y(x)+a2*y(x)^2,y(x), singsol=all)
```

$$y(x) = \frac{-\mathrm{a1} + \tan\left(\frac{\sqrt{4\,\mathrm{a0}\,\mathrm{a2} - \mathrm{a1}^2}\,(c_1 + x)}{2}\right)\sqrt{4\,\mathrm{a0}\,\mathrm{a2} - \mathrm{a1}^2}}{2\,\mathrm{a2}}$$

✓ Solution by Mathematica
Time used: 32.049 (sec). Leaf size: 106

```
DSolve[y'[x]==a0+a1 y[x]+ a2 y[x]^2,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to \frac{-\mathrm{a1} + \sqrt{4\mathrm{a0}\mathrm{a2} - \mathrm{a1}^2}\tan\left(\frac{1}{2}(x + c_1)\sqrt{4\mathrm{a0}\mathrm{a2} - \mathrm{a1}^2}\right)}{2\mathrm{a2}}$$
$$y(x) \to \frac{\sqrt{\mathrm{a1}^2 - 4\mathrm{a0}\mathrm{a2}} - \mathrm{a1}}{2\mathrm{a2}}$$
$$y(x) \to -\frac{\sqrt{\mathrm{a1}^2 - 4\mathrm{a0}\mathrm{a2}} + \mathrm{a1}}{2\mathrm{a2}}$$

3.6 problem 60

Internal problem ID [3324]

Book: Ordinary differential equations and their solutions. By George Moseley Murphy. 1960
Section: Various 3
Problem number: 60.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_Riccati]`

$$\boxed{y' - ya - by^2 = f(x)}$$

✗ Solution by Maple

```
dsolve(diff(y(x),x) = f(x)+a*y(x)+b*y(x)^2,y(x), singsol=all)
```

No solution found

✗ Solution by Mathematica
Time used: 0.0 (sec). Leaf size: 0

```
DSolve[y'[x]==f[x]+a y[x]+b y[x]^2,y[x],x,IncludeSingularSolutions -> True]
```

Not solved

3.7 problem 61

Internal problem ID [3325]

Book: Ordinary differential equations and their solutions. By George Moseley Murphy. 1960
Section: Various 3
Problem number: 61.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_Riccati]`

$$\boxed{y' - a(-y + x)\, y = 1}$$

✓ Solution by Maple
Time used: 0.047 (sec). Leaf size: 73

```
dsolve(diff(y(x),x) = 1+a*(x-y(x))*y(x),y(x), singsol=all)
```

$$y(x) = \frac{2a^{\frac{3}{2}}c_1x + \sqrt{2}\,\mathrm{erf}\left(\frac{\sqrt{2}\,\sqrt{a}\,x}{2}\right)\sqrt{\pi}\,ax + 2\sqrt{a}\,\mathrm{e}^{-\frac{a\,x^2}{2}}}{a\left(\sqrt{\pi}\,\sqrt{2}\,\mathrm{erf}\left(\frac{\sqrt{2}\,\sqrt{a}\,x}{2}\right) + 2c_1\sqrt{a}\right)}$$

✓ Solution by Mathematica
Time used: 2.067 (sec). Leaf size: 93

```
DSolve[y'[x]==1+a(x-y[x])y[x],y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to \frac{\sqrt{2\pi}c_1x\mathrm{erf}\left(\frac{\sqrt{a}x}{\sqrt{2}}\right) + \frac{2\left(ax+c_1e^{-\frac{ax^2}{2}}\right)}{\sqrt{a}}}{2\sqrt{a} + \sqrt{2\pi}c_1\mathrm{erf}\left(\frac{\sqrt{a}x}{\sqrt{2}}\right)}$$

$$y(x) \to x$$

3.8 problem 62

Internal problem ID [3326]

Book: Ordinary differential equations and their solutions. By George Moseley Murphy. 1960
Section: Various 3
Problem number: 62.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_Riccati]`

$$y' - g(x)\, y - ay^2 = f(x)$$

✗ Solution by Maple

```
dsolve(diff(y(x),x) = f(x)+g(x)*y(x)+a*y(x)^2,y(x), singsol=all)
```

No solution found

✗ Solution by Mathematica
Time used: 0.0 (sec). Leaf size: 0

```
DSolve[y'[x]==f[x]+g[x] y[x]+a y[x]^2,y[x],x,IncludeSingularSolutions -> True]
```

Not solved

3.9 problem 63

Internal problem ID [3327]

Book: Ordinary differential equations and their solutions. By George Moseley Murphy. 1960
Section: Various 3
Problem number: 63.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_separable]`

$$\boxed{y' - xy(y+3) = 0}$$

✓ Solution by Maple
Time used: 0.0 (sec). Leaf size: 19

```
dsolve(diff(y(x),x) = x*y(x)*(3+y(x)),y(x), singsol=all)
```

$$y(x) = \frac{3}{-1 + 3\,\mathrm{e}^{-\frac{3x^2}{2}} c_1}$$

✓ Solution by Mathematica
Time used: 0.227 (sec). Leaf size: 49

```
DSolve[y'[x]==x*y[x](3+y[x]),y[x],x,IncludeSingularSolutions -> True]
```

$$\begin{aligned} y(x) &\to -\frac{3e^{\frac{3x^2}{2}+3c_1}}{-1 + e^{\frac{3x^2}{2}+3c_1}} \\ y(x) &\to -3 \\ y(x) &\to 0 \end{aligned}$$

3.10 problem 64

Internal problem ID [3328]

Book: Ordinary differential equations and their solutions. By George Moseley Murphy. 1960
Section: Various 3
Problem number: 64.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_Riccati]`

$$\boxed{y' - \left(2x^2+1\right) y + y^2 x = -x^3 - x + 1}$$

✓ Solution by Maple
Time used: 0.0 (sec). Leaf size: 56

```
dsolve(diff(y(x),x) = 1-x-x^3+(2*x^2+1)*y(x)-x*y(x)^2,y(x), singsol=all)
```

$$y(x) = \frac{c_1(x^2 - x + 1)\,\mathrm{e}^{\frac{x\left(x^2+3\right)}{3}} + \mathrm{e}^{\frac{x^3}{3}} x}{c_1 \mathrm{e}^{\frac{x\left(x^2+3\right)}{3}} (x-1) + \mathrm{e}^{\frac{x^3}{3}}}$$

✓ Solution by Mathematica
Time used: 0.233 (sec). Leaf size: 40

```
DSolve[y'[x]==1-x-x^3+(1+2 x^2)y[x]-x y[x]^2,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to \frac{e^x(x^2 - x + 1) + c_1 x}{e^x(x-1) + c_1}$$
$$y(x) \to x$$

3.11 problem 65

Internal problem ID [3329]

Book: Ordinary differential equations and their solutions. By George Moseley Murphy. 1960
Section: Various 3
Problem number: 65.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_Riccati]`

$$\boxed{y' - x(2 + x^2 y - y^2) = 0}$$

✓ Solution by Maple
Time used: 0.0 (sec). Leaf size: 51

```
dsolve(diff(y(x),x) = x*(2+x^2*y(x)-y(x)^2),y(x), singsol=all)
```

$$y(x) = \frac{\operatorname{erf}\left(\frac{x^2}{2}\right)\sqrt{\pi}\, c_1 x^2 + x^2\sqrt{\pi} + 2\,\mathrm{e}^{-\frac{x^4}{4}} c_1}{\sqrt{\pi}\,\left(\operatorname{erf}\left(\frac{x^2}{2}\right) c_1 + 1\right)}$$

✓ Solution by Mathematica
Time used: 0.298 (sec). Leaf size: 70

```
DSolve[y'[x]==x*(2+x^2*y[x]-y[x]^2),y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to \frac{\sqrt{\pi}x^2\operatorname{erf}\left(\frac{x^2}{2}\right) + 2e^{-\frac{x^4}{4}} + 2c_1x^2}{\sqrt{\pi}\operatorname{erf}\left(\frac{x^2}{2}\right) + 2c_1}$$
$$y(x) \to x^2$$

3.12 problem 66

Internal problem ID [3330]

Book: Ordinary differential equations and their solutions. By George Moseley Murphy. 1960
Section: Various 3
Problem number: 66.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_Riccati]`

$$\boxed{y' - (1-2x)\,y + (1-x)\,y^2 = x}$$

✓ Solution by Maple
Time used: 0.0 (sec). Leaf size: 35

```
dsolve(diff(y(x),x) = x+(1-2*x)*y(x)-(1-x)*y(x)^2,y(x), singsol=all)
```

$$y(x) = \frac{2x\,\mathrm{e}^{-x} + 2\,\mathrm{e}^{-x} - c_1}{2x\,\mathrm{e}^{-x} - c_1}$$

✓ Solution by Mathematica
Time used: 0.172 (sec). Leaf size: 30

```
DSolve[y'[x]==x+(1-2 x)y[x]-(1-x)y[x]^2,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to \frac{x + c_1 e^x + 1}{x + c_1 e^x}$$
$$y(x) \to 1$$

3.13 problem 67

Internal problem ID [3331]

Book: Ordinary differential equations and their solutions. By George Moseley Murphy. 1960
Section: Various 3
Problem number: 67.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_separable]`

$$\boxed{y' - axy^2 = 0}$$

✓ Solution by Maple
Time used: 0.0 (sec). Leaf size: 17

```
dsolve(diff(y(x),x) = a*x*y(x)^2,y(x), singsol=all)
```

$$y(x) = -\frac{2}{a\,x^2 - 2c_1}$$

✓ Solution by Mathematica
Time used: 0.128 (sec). Leaf size: 24

```
DSolve[y'[x]==a x y[x]^2,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to -\frac{2}{ax^2 + 2c_1}$$
$$y(x) \to 0$$

3.14 problem 68

Internal problem ID [3332]

Book: Ordinary differential equations and their solutions. By George Moseley Murphy. 1960
Section: Various 3
Problem number: 68.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_separable]`

$$\boxed{y' - x^n(a + by^2) = 0}$$

✓ Solution by Maple

Time used: 0.016 (sec). Leaf size: 36

```
dsolve(diff(y(x),x) = x^n*(a+b*y(x)^2),y(x), singsol=all)
```

$$y(x) = \frac{\tan\left(\frac{\sqrt{ab}\,(x^{n+1}+(n+1)c_1)}{n+1}\right)\sqrt{ab}}{b}$$

✓ Solution by Mathematica

Time used: 0.306 (sec). Leaf size: 78

```
DSolve[y'[x]==x^n(a + b y[x]^2),y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to \frac{\sqrt{a}\tan\left(\sqrt{a}\sqrt{b}\left(\frac{x^{n+1}}{n+1} + c_1\right)\right)}{\sqrt{b}}$$
$$y(x) \to -\frac{i\sqrt{a}}{\sqrt{b}}$$
$$y(x) \to \frac{i\sqrt{a}}{\sqrt{b}}$$

3.15 problem 69

Internal problem ID [3333]

Book: Ordinary differential equations and their solutions. By George Moseley Murphy. 1960
Section: Various 3
Problem number: 69.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_Riccati]`

$$y' - y^2 x^n b = a\,x^m$$

✓ Solution by Maple
Time used: 0.0 (sec). Leaf size: 170

```
dsolve(diff(y(x),x) = a*x^m+b*x^n*y(x)^2,y(x), singsol=all)
```

$$y(x) = \frac{x^{-\frac{n}{2}+\frac{m}{2}}\sqrt{ab}\left(\text{BesselY}\left(\frac{m+1}{m+n+2}, \frac{2\sqrt{ab}\,x^{\frac{m}{2}+\frac{n}{2}+1}}{m+n+2}\right)c_1 + \text{BesselJ}\left(\frac{m+1}{m+n+2}, \frac{2\sqrt{ab}\,x^{\frac{m}{2}+\frac{n}{2}+1}}{m+n+2}\right)\right)}{b\left(\text{BesselY}\left(\frac{-1-n}{m+n+2}, \frac{2\sqrt{ab}\,x^{\frac{m}{2}+\frac{n}{2}+1}}{m+n+2}\right)c_1 + \text{BesselJ}\left(\frac{-1-n}{m+n+2}, \frac{2\sqrt{ab}\,x^{\frac{m}{2}+\frac{n}{2}+1}}{m+n+2}\right)\right)}$$

✓ Solution by Mathematica

Time used: 1.822 (sec). Leaf size: 1805

```
DSolve[y'[x]==a x^m+ b x^n y[x]^2,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to -\frac{a^{-\frac{n+1}{2(m+n+2)}} b^{-\frac{2m+3n+5}{2(m+n+2)}} (m+n+1)^{\frac{n+1}{m+n+2}} \left((m+n+1)^2\right)^{\frac{n+1}{m+n+2}-\frac{1}{2}} x^{-n-1} \left(x^{m+n+1}\right)^{-\frac{n+1}{2(m+n+1)}} \left(a^{\frac{n+1}{2(m+n+2)}} b^{\frac{}{2(}}\right.}{}$$

$$y(x) \to \frac{x^{-n-1}\left(\sqrt{a}\sqrt{b}(m+n+1)\left(x^{m+n+1}\right)^{\frac{1}{2}\left(\frac{1}{m+n+1}+1\right)} \text{BesselJ}\left(\frac{m+1}{m+n+2}, \frac{2\sqrt{a}\sqrt{b}(m+n+1)\left(x^{m+n+1}\right)^{\frac{1}{2}\left(1+\frac{1}{m+n+1}\right)}}{\sqrt{(m+n+1)^2}(m+n+2)}\right) - \sqrt{}\right.}{2}$$

$$y(x) \to \frac{x^{-n-1}\left(\sqrt{a}\sqrt{b}(m+n+1)\left(x^{m+n+1}\right)^{\frac{1}{2}\left(\frac{1}{m+n+1}+1\right)} \text{BesselJ}\left(\frac{m+1}{m+n+2}, \frac{2\sqrt{a}\sqrt{b}(m+n+1)\left(x^{m+n+1}\right)^{\frac{1}{2}\left(1+\frac{1}{m+n+1}\right)}}{\sqrt{(m+n+1)^2}(m+n+2)}\right) - \sqrt{}\right.}{2}$$

3.16 problem 70

Internal problem ID [3334]

Book: Ordinary differential equations and their solutions. By George Moseley Murphy. 1960
Section: Various 3
Problem number: 70.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_Bernoulli]`

$$y' - (a + by\cos(kx))\,y = 0$$

✓ Solution by Maple

Time used: 0.015 (sec). Leaf size: 45

```
dsolve(diff(y(x),x) = (a+b*y(x)*cos(k*x))*y(x),y(x), singsol=all)
```

$$y(x) = \frac{a^2 + k^2}{c_1\,(a^2 + k^2)\,\mathrm{e}^{-ax} - b\,(\cos(kx)\,a + k\sin(kx))}$$

✓ Solution by Mathematica

Time used: 0.237 (sec). Leaf size: 62

```
DSolve[y'[x]==(a+b y[x] Cos[k x])y[x],y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to -\frac{(a^2 + k^2)\,e^{ax}}{-(c_1\,(a^2 + k^2)) + bke^{ax}\sin(kx) + abe^{ax}\cos(kx)}$$
$$y(x) \to 0$$

3.17 problem 71

Internal problem ID [3335]

Book: Ordinary differential equations and their solutions. By George Moseley Murphy. 1960
Section: Various 3
Problem number: 71.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_linear]`

$$\boxed{y' - \sin(x)\left(2\sec(x)^2 - y\right) = 0}$$

✓ Solution by Maple
Time used: 0.0 (sec). Leaf size: 23

```
dsolve(diff(y(x),x) = sin(x)*(2*sec(x)^2-y(x)),y(x), singsol=all)
```

$$y(x) = -2\ \mathrm{expIntegral}_1(\cos(x))\, \mathrm{e}^{\cos(x)} + \mathrm{e}^{\cos(x)} c_1 + 2\sec(x)$$

✓ Solution by Mathematica
Time used: 0.13 (sec). Leaf size: 28

```
DSolve[y'[x]==Sin[x](2 Sec[x]^2-y[x]),y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to 2e^{\cos(x)}\,\mathrm{ExpIntegralEi}(-\cos(x)) + 2\sec(x) + c_1 e^{\cos(x)}$$

3.18 problem 72

Internal problem ID [3336]

Book: Ordinary differential equations and their solutions. By George Moseley Murphy. 1960
Section: Various 3
Problem number: 72.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_Riccati]`

$$y' - (3 - \cot(x))\, y - y^2 \sin(x) = -4\csc(x)$$

✓ Solution by Maple

Time used: 0.016 (sec). Leaf size: 74

```
dsolve(diff(y(x),x)+4*csc(x) = (3-cot(x))*y(x)+y(x)^2*sin(x),y(x), singsol=all)
```

$$y(x) = -\frac{3\csc(x)\left(c_1\left(\operatorname{csgn}(\sin(x))+\frac{5}{3}\right)(\cos(x)+i\sin(x))^{-\frac{5i}{2}}+(\cos(x)+i\sin(x))^{\frac{5i}{2}}\left(\operatorname{csgn}(\sin(x))-\frac{5}{3}\right)\right)}{2c_1(\cos(x)+i\sin(x))^{-\frac{5i}{2}}+2(\cos(x)+i\sin(x))^{\frac{5i}{2}}}$$

✓ Solution by Mathematica

Time used: 0.264 (sec). Leaf size: 32

```
DSolve[y'[x]+4 Csc[x]==(3-Cot[x])y[x]+y[x]^2 Sin[x],y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to \left(-4+\frac{1}{\frac{1}{5}+c_1e^{5x}}\right)\csc(x)$$
$$y(x) \to -4\csc(x)$$

3.19 problem 73

Internal problem ID [3337]

Book: Ordinary differential equations and their solutions. By George Moseley Murphy. 1960
Section: Various 3
Problem number: 73.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_linear]`

$$\boxed{y' - y \sec(x) = (\sin(x) - 1)^2}$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 35

```
dsolve(diff(y(x),x) = y(x)*sec(x)+(sin(x)-1)^2,y(x), singsol=all)
```

$$y(x) = \left(-3\sin(x) + 4\ln(\sec(x) + \tan(x)) + 4\ln(\cos(x)) - \frac{\cos(2x)}{4} + c_1\right)(\sec(x) + \tan(x))$$

✓ Solution by Mathematica

Time used: 12.2 (sec). Leaf size: 50

```
DSolve[y'[x]==y[x] Sec[x]+(Sin[x]-1)^2,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to -\frac{1}{4}e^{2\operatorname{arctanh}\left(\tan\left(\frac{x}{2}\right)\right)}\left(\cos(2x) - 4\left(-3\sin(x) + 8\log\left(\sin\left(\frac{x}{2}\right) + \cos\left(\frac{x}{2}\right)\right) + c_1\right)\right)$$

3.20 problem 74

Internal problem ID [3338]

Book: Ordinary differential equations and their solutions. By George Moseley Murphy. 1960
Section: Various 3
Problem number: 74.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_separable]`

$$\boxed{y' + \tan(x)\left(1 - y^2\right) = 0}$$

✓ Solution by Maple
Time used: 0.0 (sec). Leaf size: 14

```
dsolve(diff(y(x),x)+tan(x)*(1-y(x)^2) = 0,y(x), singsol=all)
```

$$y(x) = -\tanh\left(-\ln\left(\cos\left(x\right)\right) + c_1\right)$$

✓ Solution by Mathematica
Time used: 0.656 (sec). Leaf size: 45

```
DSolve[y'[x]+Tan[x] (1-y[x]^2)==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to \frac{1 - e^{2c_1}\sec^2(x)}{1 + e^{2c_1}\sec^2(x)}$$
$$y(x) \to -1$$
$$y(x) \to 1$$

3.21 problem 75

Internal problem ID [3339]

Book: Ordinary differential equations and their solutions. By George Moseley Murphy. 1960
Section: Various 3
Problem number: 75.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_Riccati]`

$$\boxed{y' - g(x)\, y - h(x)\, y^2 = f(x)}$$

✗ Solution by Maple

```
dsolve(diff(y(x),x) = f(x)+g(x)*y(x)+h(x)*y(x)^2,y(x), singsol=all)
```

No solution found

✗ Solution by Mathematica
Time used: 0.0 (sec). Leaf size: 0

```
DSolve[y'[x]==f[x]+g[x] y[x]+h[x] y[x]^2,y[x],x,IncludeSingularSolutions -> True]
```

Not solved

3.22 problem 76

Internal problem ID [3340]

Book: Ordinary differential equations and their solutions. By George Moseley Murphy. 1960
Section: Various 3
Problem number: 76.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_separable]`

$$y' - \left(a + yb + cy^2\right) f(x) = 0$$

✓ Solution by Maple
Time used: 0.047 (sec). Leaf size: 47

```
dsolve(diff(y(x),x) = (a+b*y(x)+c*y(x)^2)*f(x),y(x), singsol=all)
```

$$y(x) = \frac{-b + \tan\left(\frac{\sqrt{4ac-b^2}\,\left(\int f(x)dx + c_1\right)}{2}\right)\sqrt{4ac - b^2}}{2c}$$

✓ Solution by Mathematica
Time used: 0.261 (sec). Leaf size: 115

```
DSolve[y'[x]==(a+b y[x]+c y[x]^2)f[x],y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to \frac{-b + \sqrt{4ac - b^2}\tan\left(\frac{1}{2}\sqrt{4ac - b^2}\left(\int_1^x f(K[1])dK[1] + c_1\right)\right)}{2c}$$
$$y(x) \to -\frac{\sqrt{b^2 - 4ac} + b}{2c}$$
$$y(x) \to \frac{\sqrt{b^2 - 4ac} - b}{2c}$$

3.23 problem 77

Internal problem ID [3341]

Book: Ordinary differential equations and their solutions. By George Moseley Murphy. 1960
Section: Various 3
Problem number: 77.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_Abel]`

$$\boxed{y' + (ax + y)\, y^2 = 0}$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 71

```
dsolve(diff(y(x),x)+(a*x+y(x))*y(x)^2=0,y(x), singsol=all)
```

$$y(x) = \frac{2a}{x^2a^2 + 2\,\mathrm{RootOf}\left(2^{\frac{1}{3}}\left(-a^2\right)^{\frac{1}{3}} \mathrm{AiryBi}\left(\_Z\right) c_1 x + 2^{\frac{1}{3}}\left(-a^2\right)^{\frac{1}{3}} x\, \mathrm{AiryAi}\left(\_Z\right) + 2\,\mathrm{AiryBi}\left(1, \_Z\right) c_1 + 2\,\mathrm{Airy}}$$

✓ Solution by Mathematica

Time used: 0.247 (sec). Leaf size: 195

```
DSolve[y'[x]+(a x+y[x])y[x]^2==0,y[x],x,IncludeSingularSolutions -> True]
```

$$\text{Solve}\left[\frac{\text{AiryAiPrime}\left(\frac{\sqrt[3]{-\frac{1}{2}}\sqrt[3]{a}}{y(x)}-\frac{1}{2}\sqrt[3]{-\frac{1}{2}}a^{4/3}x^2\right)-\left(-\frac{1}{2}\right)^{2/3}a^{2/3}x\,\text{AiryAi}\left(\frac{\sqrt[3]{-\frac{1}{2}}\sqrt[3]{a}}{y(x)}-\frac{1}{2}\sqrt[3]{-\frac{1}{2}}a^{4/3}x^2\right)}{\text{AiryBiPrime}\left(\frac{\sqrt[3]{-\frac{1}{2}}\sqrt[3]{a}}{y(x)}-\frac{1}{2}\sqrt[3]{-\frac{1}{2}}a^{4/3}x^2\right)-\left(-\frac{1}{2}\right)^{2/3}a^{2/3}x\,\text{AiryBi}\left(\frac{\sqrt[3]{-\frac{1}{2}}\sqrt[3]{a}}{y(x)}-\frac{1}{2}\sqrt[3]{-\frac{1}{2}}a^{4/3}x^2\right)}\right.$$

$$\left.+c_1=0,y(x)\right]$$

3.24 problem 78

Internal problem ID [3342]

Book: Ordinary differential equations and their solutions. By George Moseley Murphy. 1960
Section: Various 3
Problem number: 78.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_Abel]`

$$y' - (a\,\mathrm{e}^{x} + y)\,y^2 = 0$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 62

```
dsolve(diff(y(x),x) = (a*exp(x)+y(x))*y(x)^2,y(x), singsol=all)
```

$$\frac{a \,\mathrm{erf}\left(\frac{(a\,\mathrm{e}^{x} y(x)+1)\sqrt{2}}{2y(x)}\right)\sqrt{2}\,\sqrt{\pi}+2c_1 a+2\,\mathrm{e}^{-x-\frac{(a\,\mathrm{e}^{x} y(x)+1)^2}{2y(x)^2}}}{2a}=0$$

✓ Solution by Mathematica

Time used: 0.702 (sec). Leaf size: 78

```
DSolve[y'[x]==(a Exp[x]+y[x])y[x]^2,y[x],x,IncludeSingularSolutions -> True]
```

$$\text{Solve}\left[-iae^x = \frac{2e^{\frac{1}{2}\left(-iae^x-\frac{i}{y(x)}\right)^2}}{\sqrt{2\pi}\,\text{erfi}\left(\frac{-iae^x-\frac{i}{y(x)}}{\sqrt{2}}\right)+2c_1}, y(x)\right]$$

3.25 problem 79

Internal problem ID [3343]

Book: Ordinary differential equations and their solutions. By George Moseley Murphy. 1960
Section: Various 3
Problem number: 79.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_Abel]`

$$\boxed{y' + 3a(y + 2x)\, y^2 = 0}$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 57

```
dsolve(diff(y(x),x)+3*a*(2*x+y(x))*y(x)^2 = 0,y(x), singsol=all)
```

$$y(x) = \frac{1}{3a\, x^2 + \mathrm{RootOf}\left(3^{\frac{1}{3}} \left(-a\right)^{\frac{1}{3}} \mathrm{AiryBi}\left(\_Z\right) c_1 x + 3^{\frac{1}{3}} \left(-a\right)^{\frac{1}{3}} x\, \mathrm{AiryAi}\left(\_Z\right) + \mathrm{AiryBi}\left(1, \_Z\right) c_1 + \mathrm{AiryAi}\left(1,\_\right.}$$

✓ Solution by Mathematica

Time used: 0.313 (sec). Leaf size: 185

```
DSolve[y'[x]+3 a(2 x + y[x])y[x]^2==0,y[x],x,IncludeSingularSolutions -> True]
```

$$\mathrm{Solve}\left[\frac{\sqrt[3]{-3}\sqrt[3]{a}x\, \mathrm{AiryAi}\left((-3)^{2/3}a^{2/3}x^2 - \frac{(-1)^{2/3}}{\sqrt[3]{3}\sqrt[3]{a}y(x)}\right) + \mathrm{AiryAiPrime}\left((-3)^{2/3}a^{2/3}x^2 - \frac{(-1)^{2/3}}{\sqrt[3]{3}\sqrt[3]{a}y(x)}\right)}{\sqrt[3]{-3}\sqrt[3]{a}x\, \mathrm{AiryBi}\left((-3)^{2/3}a^{2/3}x^2 - \frac{(-1)^{2/3}}{\sqrt[3]{3}\sqrt[3]{a}y(x)}\right) + \mathrm{AiryBiPrime}\left((-3)^{2/3}a^{2/3}x^2 - \frac{(-1)^{2/3}}{\sqrt[3]{3}\sqrt[3]{a}y(x)}\right)} + c_1 = 0, y(x)\right]$$

3.26 problem 80

Internal problem ID [3344]

Book: Ordinary differential equations and their solutions. By George Moseley Murphy. 1960
Section: Various 3
Problem number: 80.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_quadrature]`

$$\boxed{y' - y(a + b y^2) = 0}$$

✓ Solution by Maple
Time used: 0.016 (sec). Leaf size: 70

```
dsolve(diff(y(x),x) = y(x)*(a+b*y(x)^2),y(x), singsol=all)
```

$$y(x) = \frac{\sqrt{(c_1 a\,\mathrm{e}^{-2ax} - b)\,a}}{c_1 a\,\mathrm{e}^{-2ax} - b}$$
$$y(x) = -\frac{\sqrt{(c_1 a\,\mathrm{e}^{-2ax} - b)\,a}}{c_1 a\,\mathrm{e}^{-2ax} - b}$$

✓ Solution by Mathematica
Time used: 1.914 (sec). Leaf size: 118

```
DSolve[y'[x]==y[x](a+b y[x]^2),y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to -\frac{i\sqrt{a}e^{a(x+c_1)}}{\sqrt{-1 + be^{2a(x+c_1)}}}$$
$$y(x) \to \frac{i\sqrt{a}e^{a(x+c_1)}}{\sqrt{-1 + be^{2a(x+c_1)}}}$$
$$y(x) \to 0$$
$$y(x) \to -\frac{i\sqrt{a}}{\sqrt{b}}$$
$$y(x) \to \frac{i\sqrt{a}}{\sqrt{b}}$$

3.27 problem 81

Internal problem ID [3345]

Book: Ordinary differential equations and their solutions. By George Moseley Murphy. 1960
Section: Various 3
Problem number: 81.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_quadrature]`

$$\boxed{y' - \mathrm{a1}\, y - \mathrm{a2}\, y^2 - \mathrm{a3}\, y^3 = \mathrm{a0}}$$

✓ Solution by Maple
Time used: 0.0 (sec). Leaf size: 30

```
dsolve(diff(y(x),x) = a0+a1*y(x)+a2*y(x)^2+a3*y(x)^3,y(x), singsol=all)
```

$$x - \left(\int^{y(x)} \frac{1}{\_a^3\, \mathrm{a3} + \_a^2\, \mathrm{a2} + \_a\, \mathrm{a1} + \mathrm{a0}} d\_a\right) + c_1 = 0$$

✓ Solution by Mathematica
Time used: 0.112 (sec). Leaf size: 54

```
DSolve[y'[x]==a0+a1 y[x]+a2 y[x]^2+ a3 y[x]^3,y[x],x,IncludeSingularSolutions -> True]
```

$$\mathrm{Solve}\left[\mathrm{RootSum}\left[\#1^3\mathrm{a3} + \#1^2\mathrm{a2} + \#1\mathrm{a1} + \mathrm{a0}\&, \frac{\log(y(x) - \#1)}{3\#1^2\mathrm{a3} + 2\#1\mathrm{a2} + \mathrm{a1}}\&\right] = x + c_1, y(x)\right]$$

3.28 problem 82

Internal problem ID [3346]

Book: Ordinary differential equations and their solutions. By George Moseley Murphy. 1960
Section: Various 3
Problem number: 82.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_separable]`

$$\boxed{y' - x y^3 = 0}$$

✓ Solution by Maple
Time used: 0.0 (sec). Leaf size: 27

```
dsolve(diff(y(x),x) = x*y(x)^3,y(x), singsol=all)
```

$$y(x) = \frac{1}{\sqrt{-x^2 + c_1}}$$
$$y(x) = -\frac{1}{\sqrt{-x^2 + c_1}}$$

✓ Solution by Mathematica
Time used: 0.162 (sec). Leaf size: 44

```
DSolve[y'[x]==x y[x]^3,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to -\frac{1}{\sqrt{-x^2 - 2c_1}}$$
$$y(x) \to \frac{1}{\sqrt{-x^2 - 2c_1}}$$
$$y(x) \to 0$$

3.29 problem 83

Internal problem ID [3347]

Book: Ordinary differential equations and their solutions. By George Moseley Murphy. 1960
Section: Various 3
Problem number: 83.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_Bernoulli]`

$$\boxed{y' + y(1 - y^2 x) = 0}$$

✓ Solution by Maple
Time used: 0.0 (sec). Leaf size: 39

```
dsolve(diff(y(x),x)+y(x)*(1-x*y(x)^2) = 0,y(x), singsol=all)
```

$$y(x) = -\frac{2}{\sqrt{2 + 4c_1 e^{2x} + 4x}}$$
$$y(x) = \frac{2}{\sqrt{2 + 4c_1 e^{2x} + 4x}}$$

✓ Solution by Mathematica
Time used: 2.822 (sec). Leaf size: 50

```
DSolve[y'[x]+y[x](1-x y[x]^2)==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to -\frac{1}{\sqrt{x + c_1 e^{2x} + \frac{1}{2}}}$$
$$y(x) \to \frac{1}{\sqrt{x + c_1 e^{2x} + \frac{1}{2}}}$$
$$y(x) \to 0$$

4 Various 4

4.1 problem 84

Internal problem ID [3348]

Book: Ordinary differential equations and their solutions. By George Moseley Murphy. 1960
Section: Various 4
Problem number: 84.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[[_homogeneous, ‘class G‘], _Abel]`

$$\boxed{y' - (a + bxy)\, y^2 = 0}$$

✓ Solution by Maple

Time used: 0.078 (sec). Leaf size: 103

```
dsolve(diff(y(x),x) = (a+b*x*y(x))*y(x)^2,y(x), singsol=all)
```

$$y(x) = \frac{e^{\operatorname{RootOf}\left(2\sqrt{a^2-4b}\,a \operatorname{arctanh}\left(\frac{2b\,e^{-Z}+a}{\sqrt{a^2-4b}}\right)-\ln\left(x^2\left(b\,e^{2\_Z}+a\,e^{\_Z}+1\right)\right)a^2+2c_1a^2+2\_Z a^2+4\ln\left(x^2\left(b\,e^{2\_Z}+a\,e^{\_Z}+1\right)\right)b-8c_1b-8\_Zb\right)}}{x}$$

✓ Solution by Mathematica

Time used: 0.214 (sec). Leaf size: 94

```
DSolve[y'[x]==(a+b x y[x])y[x]^2,y[x],x,IncludeSingularSolutions -> True]
```

$$\text{Solve}\left[\frac{a^2\left(-\frac{2\arctan\left(\frac{a+2bxy(x)}{a\sqrt{\frac{4b}{a^2}-1}}\right)}{\sqrt{\frac{4b}{a^2}-1}}-\log\left(\frac{bxy(x)(a+bxy(x))+b}{b^2x^2y(x)^2}\right)\right)}{2b}=\frac{a^2\log(x)}{b}+c_1,y(x)\right]$$

4.2 problem 87

Internal problem ID [3349]

Book: Ordinary differential equations and their solutions. By George Moseley Murphy. 1960
Section: Various 4
Problem number: 87.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_Bernoulli]`

$$\boxed{y' + 2xy(1 + axy^2) = 0}$$

✓ Solution by Maple

Time used: 0.032 (sec). Leaf size: 75

```
dsolve(diff(y(x),x)+2*x*y(x)*(1+a*x*y(x)^2) = 0,y(x), singsol=all)
```

$$y(x) = -\frac{2}{\sqrt{\left(a\,\mathrm{erf}\left(\sqrt{2}\,x\right)\sqrt{\pi}\,\sqrt{2} + 4c_1\right)\mathrm{e}^{2x^2} - 4ax}}$$
$$y(x) = \frac{2}{\sqrt{\left(a\,\mathrm{erf}\left(\sqrt{2}\,x\right)\sqrt{\pi}\,\sqrt{2} + 4c_1\right)\mathrm{e}^{2x^2} - 4ax}}$$

✓ Solution by Mathematica

Time used: 7.561 (sec). Leaf size: 106

```
DSolve[y'[x]+2 x y[x](1+ a x y[x]^2)==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to -\frac{2}{\sqrt{\sqrt{2\pi}ae^{2x^2}\mathrm{erf}\left(\sqrt{2}x\right) - 4ax + 4c_1e^{2x^2}}}$$
$$y(x) \to \frac{2}{\sqrt{\sqrt{2\pi}ae^{2x^2}\mathrm{erf}\left(\sqrt{2}x\right) - 4ax + 4c_1e^{2x^2}}}$$
$$y(x) \to 0$$

4.3 problem 90

Internal problem ID [3350]

Book: Ordinary differential equations and their solutions. By George Moseley Murphy. 1960
Section: Various 4
Problem number: 90.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_Bernoulli]`

$$\boxed{y' + (\tan(x) + y^2 \sec(x))\, y = 0}$$

✓ Solution by Maple
Time used: 0.031 (sec). Leaf size: 30

```
dsolve(diff(y(x),x)+(tan(x)+y(x)^2*sec(x))*y(x) = 0,y(x), singsol=all)
```

$$y(x) = \frac{\cos(x)}{\sqrt{2\sin(x) + c_1}}$$
$$y(x) = -\frac{\cos(x)}{\sqrt{2\sin(x) + c_1}}$$

✓ Solution by Mathematica
Time used: 3.746 (sec). Leaf size: 48

```
DSolve[y'[x]+(Tan[x]+y[x]^2 Sec[x])y[x]==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to -\frac{1}{\sqrt{\sec^2(x)(2\sin(x) + c_1)}}$$
$$y(x) \to \frac{1}{\sqrt{\sec^2(x)(2\sin(x) + c_1)}}$$
$$y(x) \to 0$$

4.4 problem 91

Internal problem ID [3351]

Book: Ordinary differential equations and their solutions. By George Moseley Murphy. 1960
Section: Various 4
Problem number: 91.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_separable]`

$$\boxed{y' + \tan(x)\sec(x)\, y^3 = 0}$$

✓ Solution by Maple
Time used: 0.0 (sec). Leaf size: 48

```
dsolve(diff(y(x),x)+y(x)^3*sec(x)*tan(x) = 0,y(x), singsol=all)
```

$$y(x) = \frac{\sqrt{(\cos(x)\, c_1 + 2)\cos(x)}}{\cos(x)\, c_1 + 2}$$
$$y(x) = -\frac{\sqrt{(\cos(x)\, c_1 + 2)\cos(x)}}{\cos(x)\, c_1 + 2}$$

✓ Solution by Mathematica
Time used: 0.386 (sec). Leaf size: 49

```
DSolve[y'[x]+y[x]^3 Sec[x] Tan[x]==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to -\frac{1}{\sqrt{2}\sqrt{\sec(x) - c_1}}$$
$$y(x) \to \frac{1}{\sqrt{2}\sqrt{\sec(x) - c_1}}$$
$$y(x) \to 0$$

4.5 problem 92

Internal problem ID [3352]

Book: Ordinary differential equations and their solutions. By George Moseley Murphy. 1960
Section: Various 4
Problem number: 92.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_Abel]`

$$y' - \mathrm{f1}(x)\, y - \mathrm{f2}(x)\, y^2 - \mathrm{f3}(x)\, y^3 = \mathrm{f0}(x)$$

✗ Solution by Maple

```
dsolve(diff(y(x),x) = f0(x)+f1(x)*y(x)+f2(x)*y(x)^2+f3(x)*y(x)^3,y(x), singsol=all)
```

No solution found

✗ Solution by Mathematica

Time used: 0.0 (sec). Leaf size: 0

```
DSolve[y'[x]==f0[x]+f1[x]y[x]+f2[x] y[x]^2+f3[x]y[x]^3,y[x],x,IncludeSingularSolutions -> Tr
```

Not solved

4.6 problem 94

Internal problem ID [3353]

Book: Ordinary differential equations and their solutions. By George Moseley Murphy. 1960
Section: Various 4
Problem number: 94.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[[_homogeneous, ‘class G‘], _Chini]`

$$\boxed{y' - b y^n = a\, x^{\frac{n}{-n+1}}}$$

✓ Solution by Maple

Time used: 0.203 (sec). Leaf size: 65

```
dsolve(diff(y(x),x) = a*x^(n/(1-n))+b*y(x)^n,y(x), singsol=all)
```

$$x^{\frac{n}{n-1}} \left(\int_{\_b}^{y(x)} \frac{1}{\_a^n b (n-1)\, x^{\frac{2n-1}{n-1}} + x^{\frac{n}{n-1}} \_a + a x (n-1)} d\_a\right) - c_1 = 0$$

✓ Solution by Mathematica

Time used: 0.307 (sec). Leaf size: 117

```
DSolve[y'[x]==a*x^(n/(1-n))+b*y[x]^n,y[x],x,IncludeSingularSolutions -> True]
```

$$\text{Solve}\left[\int_1^{\left(\frac{b x^{-\frac{n}{1-n}}}{a}\right)^{\frac{1}{n}} y(x)} \frac{1}{K[1]^n - \left(\frac{(-1)^n a^{1-n} (n-1)^{-n}}{b}\right)^{\frac{1}{n}} K[1] + 1} dK[1] = \int_1^x a K[2]^{\frac{n}{1-n}} \left(\frac{b K[2]^{-\frac{n}{1-n}}}{a}\right)^{\frac{1}{n}} dK[2 \right.$$
$$\left. + c_1, y(x)\right]$$

4.7 problem 95

Internal problem ID [3354]

Book: Ordinary differential equations and their solutions. By George Moseley Murphy. 1960
Section: Various 4
Problem number: 95.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_Bernoulli]`

$$\boxed{y' - f(x)\, y - g(x)\, y^k = 0}$$

✓ Solution by Maple
Time used: 0.078 (sec). Leaf size: 51

```
dsolve(diff(y(x),x) = f(x)*y(x)+g(x)*y(x)^k,y(x), singsol=all)
```

$$y(x) = e^{\int f(x)dx}\left(-k\left(\int g(x)\, e^{(k-1)\left(\int f(x)dx\right)}dx\right)+c_1+\int g(x)\, e^{(k-1)\left(\int f(x)dx\right)}dx\right)^{-\frac{1}{k-1}}$$

✓ Solution by Mathematica
Time used: 11.421 (sec). Leaf size: 129

```
DSolve[y'[x]==f[x] y[x]+g[x]y[x]^k,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to \left(\exp\left(-\left((k-1)\int_1^x f(K[1])dK[1]\right)\right)\left(-(k-1)\int_1^x \exp\left((k-1)\int_1^{K[2]} f(K[1])dK[1]\right) g(K[2])dK[2] + c_1\right)\right)^{\frac{1}{1-k}}$$

$$y(x) \to \left((k-1)\left(-\exp\left(-\left((k-1)\int_1^x f(K[1])dK[1]\right)\right)\right)\int_1^x \exp\left((k-1)\int_1^{K[2]} f(K[1])dK[1]\right) g(K[2])dK[2]\right)$$

4.8 problem 96

Internal problem ID [3355]

Book: Ordinary differential equations and their solutions. By George Moseley Murphy. 1960
Section: Various 4
Problem number: 96.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_Chini]`

$$\boxed{y' - g(x)\, y - h(x)\, y^n = f(x)}$$

✗ Solution by Maple

```
dsolve(diff(y(x),x) = f(x)+g(x)*y(x)+h(x)*y(x)^n,y(x), singsol=all)
```

No solution found

✗ Solution by Mathematica
Time used: 0.0 (sec). Leaf size: 0

```
DSolve[y'[x]==f[x]+g[x]y[x]+h[x]y[x]^n,y[x],x,IncludeSingularSolutions -> True]
```

Not solved

4.9 problem 98

Internal problem ID [3356]

Book: Ordinary differential equations and their solutions. By George Moseley Murphy. 1960
Section: Various 4
Problem number: 98.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_quadrature]`

$$\boxed{y' - \sqrt{|y|} = 0}$$

✓ Solution by Maple
Time used: 0.0 (sec). Leaf size: 29

```
dsolve(diff(y(x),x) = sqrt(abs(y(x))),y(x), singsol=all)
```

$$x + 2\left(\begin{cases} \sqrt{-y(x)} & y(x) \leq 0 \\ -\sqrt{y(x)} & 0 < y(x) \end{cases}\right) + c_1 = 0$$

✓ Solution by Mathematica
Time used: 0.164 (sec). Leaf size: 31

```
DSolve[y'[x]==Sqrt[Abs[y[x]]],y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to \text{InverseFunction}\left[\int_1^{\#1} \frac{1}{\sqrt{|K[1]|}} dK[1]\&\right][x + c_1]$$
$$y(x) \to 0$$

4.10 problem 99

Internal problem ID [3357]

Book: Ordinary differential equations and their solutions. By George Moseley Murphy. 1960
Section: Various 4
Problem number: 99.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_quadrature]`

$$\boxed{y' - yb - \sqrt{\mathrm{A0} + \mathrm{B0}\, y} = a}$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 27

```
dsolve(diff(y(x),x) = a+b*y(x)+sqrt(A0+B0*y(x)),y(x), singsol=all)
```

$$x - \left(\int^{y(x)} \frac{1}{a + b\_a + \sqrt{\mathrm{B0}\_a + \mathrm{A0}}} d\_a\right) + c_1 = 0$$

✓ Solution by Mathematica

Time used: 0.693 (sec). Leaf size: 172

```
DSolve[y'[x]==a+b y[x]+Sqrt[A0+B0 y[x]],y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to \text{InverseFunction}\left[\frac{\log\left(-\text{B0}\left(\sqrt{\#1\text{B0}+\text{A0}}+\#1b+a\right)\right)-\frac{2\text{B0}\arctan\left(\frac{2b\sqrt{\#1\text{B0}+\text{A0}}+\text{B0}}{\sqrt{\text{B0}(4ab-\text{B0})-4\text{A0}b^2}}\right)}{\sqrt{\text{B0}(4ab-\text{B0})-4\text{A0}b^2}}}{b}\&\right][x+c_1]$$

$$y(x) \to -\frac{\sqrt{-4ab\text{B0}+4\text{A0}b^2+\text{B0}^2}+2ab-\text{B0}}{2b^2}$$

$$y(x) \to \frac{\sqrt{-4ab\text{B0}+4\text{A0}b^2+\text{B0}^2}-2ab+\text{B0}}{2b^2}$$

4.11 problem 100

Internal problem ID [3358]

Book: Ordinary differential equations and their solutions. By George Moseley Murphy. 1960
Section: Various 4
Problem number: 100.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[[_homogeneous, ‘class G‘], _Chini]`

$$\boxed{y' - b\sqrt{y} = ax}$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 68

```
dsolve(diff(y(x),x) = a*x+b*sqrt(y(x)),y(x), singsol=all)
```

$$-\frac{\ln\left(\sqrt{y\left(x\right)}\, bx + a\, x^{2} - 2y(x)\right)}{2} + \frac{b\sqrt{y\left(x\right)}\ \operatorname{arctanh}\left(\frac{b\sqrt{y(x)}+2ax}{\sqrt{y(x)(b^{2}+8a)}}\right)}{\sqrt{y\left(x\right)\left(b^{2}+8a\right)}} + c_{1} = 0$$

✓ Solution by Mathematica

Time used: 0.267 (sec). Leaf size: 119

```
DSolve[y'[x]==a x+b Sqrt[y[x]],y[x],x,IncludeSingularSolutions -> True]
```

$$\text{Solve}\left[\frac{b^2\left(-\frac{2b\,\text{arctanh}\left(\frac{b^2-4a\sqrt{\frac{b^2y(x)}{a^2x^2}}}{b\sqrt{8a+b^2}}\right)}{\sqrt{8a+b^2}}-\log\left(b^2\left(\sqrt{\frac{b^2y(x)}{a^2x^2}}+1\right)-\frac{2b^2y(x)}{ax^2}\right)\right)}{2a}=\frac{b^2\log(x)}{a}+c_1,y(x)\right]$$

4.12 problem 101

Internal problem ID [3359]

Book: Ordinary differential equations and their solutions. By George Moseley Murphy. 1960
Section: Various 4
Problem number: 101.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[[_1st_order, _with_linear_symmetries]]`

$$\boxed{y' - x\sqrt{x^4 + 4y} = -x^3}$$

✓ Solution by Maple
Time used: 0.016 (sec). Leaf size: 47

```
dsolve(diff(y(x),x)+x^3 = x*sqrt(x^4+4*y(x)),y(x), singsol=all)
```

$$\frac{(y(x) - c_1)\sqrt{x^4 + 4y(x)} - x^2(c_1 + y(x))}{x^2 + \sqrt{x^4 + 4y(x)}} = 0$$

✓ Solution by Mathematica
Time used: 0.583 (sec). Leaf size: 30

```
DSolve[y'[x]+x^3==x Sqrt[x^4+4 y[x]],y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to 2e^{2c_1}\left(x^2 + 2e^{2c_1}\right)$$
$$y(x) \to 0$$

4.13 problem 102

Internal problem ID [3360]

Book: Ordinary differential equations and their solutions. By George Moseley Murphy. 1960
Section: Various 4
Problem number: 102.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_Bernoulli]`

$$\boxed{y' + 2y(1 - \sqrt{y}\, x) = 0}$$

✓ Solution by Maple
Time used: 0.0 (sec). Leaf size: 23

```
dsolve(diff(y(x),x)+2*y(x)*(1-x*sqrt(y(x))) = 0,y(x), singsol=all)
```

$$-\frac{-1 + (e^{x} c_1 + x + 1)\sqrt{y(x)}}{\sqrt{y(x)}} = 0$$

✓ Solution by Mathematica
Time used: 0.147 (sec). Leaf size: 21

```
DSolve[y'[x]+2 y[x] (1-x Sqrt[y[x]])==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to \frac{1}{(x + c_1 e^{x} + 1)^2}$$
$$y(x) \to 0$$

4.14 problem 103

Internal problem ID [3361]

Book: Ordinary differential equations and their solutions. By George Moseley Murphy. 1960
Section: Various 4
Problem number: 103.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type [_quadrature]

$$\boxed{y' - \sqrt{a + b y^2} = 0}$$

✓ Solution by Maple
Time used: 0.0 (sec). Leaf size: 35

```
dsolve(diff(y(x),x) = sqrt(a+b*y(x)^2),y(x), singsol=all)
```

$$\frac{(c_1 + x)\sqrt{b} - \ln\left(y(x)\sqrt{b} + \sqrt{a + b y(x)^2}\right)}{\sqrt{b}} = 0$$

✓ Solution by Mathematica
Time used: 60.157 (sec). Leaf size: 82

```
DSolve[y'[x]==Sqrt[a+b y[x]^2],y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to -\frac{\sqrt{a}\tanh\left(\sqrt{b}(x + c_1)\right)}{\sqrt{b\text{sech}^2\left(\sqrt{b}(x + c_1)\right)}}$$

$$y(x) \to \frac{\sqrt{a}\tanh\left(\sqrt{b}(x + c_1)\right)}{\sqrt{b\text{sech}^2\left(\sqrt{b}(x + c_1)\right)}}$$

4.15 problem 104

Internal problem ID [3362]

Book: Ordinary differential equations and their solutions. By George Moseley Murphy. 1960
Section: Various 4
Problem number: 104.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_quadrature]`

$$\boxed{y' - y\sqrt{yb + a} = 0}$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 24

```
dsolve(diff(y(x),x) = y(x)*sqrt(a+b*y(x)),y(x), singsol=all)
```

$$x + \frac{2 \operatorname{arctanh}\left(\frac{\sqrt{a+by(x)}}{\sqrt{a}}\right)}{\sqrt{a}} + c_1 = 0$$

✓ Solution by Mathematica

Time used: 19.061 (sec). Leaf size: 42

```
DSolve[y'[x]==y[x] Sqrt[a+b y[x]],y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to -\frac{a \operatorname{sech}^2\left(\frac{1}{2}\sqrt{a}(x+c_1)\right)}{b}$$
$$y(x) \to 0$$
$$y(x) \to -\frac{a}{b}$$

4.16 problem 105

Internal problem ID [3363]

Book: Ordinary differential equations and their solutions. By George Moseley Murphy. 1960
Section: Various 4
Problem number: 105.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[‘y=_G(x,y’)‘]`

$$\boxed{y' + (f(x) - y)\, g(x)\, \sqrt{(y-a)\,(y-b)} = 0}$$

✗ Solution by Maple

```
dsolve(diff(y(x),x)+(f(x)-y(x))*g(x)*sqrt((y(x)-a)*(y(x)-b)) = 0,y(x), singsol=all)
```

No solution found

✗ Solution by Mathematica

Time used: 0.0 (sec). Leaf size: 0

```
DSolve[y'[x]+(f[x]-y[x])g[x] Sqrt[(y[x]-a)(y[x]-b)]==0,y[x],x,IncludeSingularSolutions -> Tr
```

Not solved

4.17 problem 106

Internal problem ID [3364]

Book: Ordinary differential equations and their solutions. By George Moseley Murphy. 1960
Section: Various 4
Problem number: 106.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_quadrature]`

$$\boxed{y' - \sqrt{XY} = 0}$$

✓ Solution by Maple
Time used: 0.0 (sec). Leaf size: 13

```
dsolve(diff(y(x),x) = sqrt(X*Y),y(x), singsol=all)
```

$$y(x) = \sqrt{XY}\, x + c_1$$

✓ Solution by Mathematica
Time used: 0.003 (sec). Leaf size: 17

```
DSolve[y'[x]==Sqrt[ X Y],y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to x\sqrt{XY} + c_1$$

4.18 problem 107

Internal problem ID [3365]

Book: Ordinary differential equations and their solutions. By George Moseley Murphy. 1960
Section: Various 4
Problem number: 107.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_separable]`

$$\boxed{y' - \cos(x)^2 \cos(y) = 0}$$

✓ Solution by Maple

Time used: 0.187 (sec). Leaf size: 69

```
dsolve(diff(y(x),x) = cos(x)^2*cos(y(x)),y(x), singsol=all)
```

$$y(x) = \arctan\left(\frac{c_1^2 e^{x+\frac{\sin(2x)}{2}} - 1}{c_1^2 e^{x+\frac{\sin(2x)}{2}} + 1}, \frac{2c_1 e^{\frac{x}{2}+\frac{\sin(2x)}{4}}}{c_1^2 e^{x+\frac{\sin(2x)}{2}} + 1}\right)$$

✓ Solution by Mathematica

Time used: 1.035 (sec). Leaf size: 41

```
DSolve[y'[x]==Cos[x]^2 Cos[y[x]],y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to 2\arctan\left(\tanh\left(\frac{1}{8}(2x + \sin(2x) + c_1)\right)\right)$$
$$y(x) \to -\frac{\pi}{2}$$
$$y(x) \to \frac{\pi}{2}$$

4.19 problem 108

Internal problem ID [3366]

Book: Ordinary differential equations and their solutions. By George Moseley Murphy. 1960
Section: Various 4
Problem number: 108.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_separable]`

$$y' - \sec(x)^2 \cot(y) \cos(y) = 0$$

✓ Solution by Maple
Time used: 0.015 (sec). Leaf size: 11

```
dsolve(diff(y(x),x) = sec(x)^2*cot(y(x))*cos(y(x)),y(x), singsol=all)
```

$$y(x) = \arccos\left(\frac{1}{\tan(x) + c_1}\right)$$

✓ Solution by Mathematica
Time used: 0.823 (sec). Leaf size: 45

```
DSolve[y'[x]==Sec[x]^2 Cot[y[x]] Cos[y[x]],y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to -\sec^{-1}(\tan(x) + 2c_1)$$
$$y(x) \to \sec^{-1}(\tan(x) + 2c_1)$$
$$y(x) \to -\frac{\pi}{2}$$
$$y(x) \to \frac{\pi}{2}$$

4.20 problem 109

Internal problem ID [3367]

Book: Ordinary differential equations and their solutions. By George Moseley Murphy. 1960
Section: Various 4
Problem number: 109.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[[_homogeneous, ‘class C‘], _dAlembert]`

$$y' - b\cos(Ax + By) = a$$

✓ Solution by Maple
Time used: 0.062 (sec). Leaf size: 74

```
dsolve(diff(y(x),x) = a+b*cos(A*x+B*y(x)),y(x), singsol=all)
```

$$y(x) = \frac{-Ax - 2\arctan\left(\frac{\tan\left(\frac{\sqrt{(A+(a+b)B)(A+(a-b)B)}\,(c_1-x)}{2}\right)\sqrt{(A+(a+b)B)(A+(a-b)B)}}{A+(a-b)B}\right)}{B}$$

✓ Solution by Mathematica
Time used: 60.721 (sec). Leaf size: 102

```
DSolve[y'[x]==a+b Cos[A x+ B y[x]],y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to -\frac{Ax + 2\arctan\left(\frac{(B(a+b)+A)\tanh\left(\frac{(x-c_1)\left(B^2\left(a^2-b^2\right)+2aAB+A^2\right)}{2\sqrt{-((B(a-b)+A)(B(a+b)+A))}}\right)}{\sqrt{-((B(a-b)+A)(B(a+b)+A))}}\right)}{B}$$

4.21 problem 110

Internal problem ID [3368]

Book: Ordinary differential equations and their solutions. By George Moseley Murphy. 1960
Section: Various 4
Problem number: 110.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[‘y=_G(x,y’)‘]`

$$\boxed{y' + g(x)\sin(ya) + h(x)\cos(ya) = -f(x)}$$

✗ Solution by Maple

```
dsolve(diff(y(x),x)+f(x)+g(x)*sin(a*y(x))+h(x)*cos(a*y(x)) = 0,y(x), singsol=all)
```

No solution found

✗ Solution by Mathematica
Time used: 0.0 (sec). Leaf size: 0

```
DSolve[y'[x]+f[x]+g[x]Sin[a y[x]]+h[x] Cos[a y[x]]==0,y[x],x,IncludeSingularSolutions -> Tru
```

Not solved

4.22 problem 111

Internal problem ID [3369]

Book: Ordinary differential equations and their solutions. By George Moseley Murphy. 1960
Section: Various 4
Problem number: 111.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_quadrature]`

$$\boxed{y' - b\cos(y) = a}$$

✓ Solution by Maple
Time used: 0.015 (sec). Leaf size: 43

```
dsolve(diff(y(x),x) = a+b*cos(y(x)),y(x), singsol=all)
```

$$y(x) = 2\arctan\left(\frac{\tan\left(\frac{\sqrt{a^2-b^2}\,(c_1+x)}{2}\right)\sqrt{a^2-b^2}}{a-b}\right)$$

✓ Solution by Mathematica
Time used: 60.13 (sec). Leaf size: 47

```
DSolve[y'[x]==a+b Cos[y[x]],y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to 2\arctan\left(\frac{(a+b)\tanh\left(\frac{1}{2}\sqrt{b^2-a^2}(x+c_1)\right)}{\sqrt{b^2-a^2}}\right)$$

4.23 problem 112

Internal problem ID [3370]

Book: Ordinary differential equations and their solutions. By George Moseley Murphy. 1960
Section: Various 4
Problem number: 112.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[‘y=_G(x,y’)‘]`

$$y' + x\left(\sin(2y) - x^2 \cos(y)^2\right) = 0$$

✓ Solution by Maple

Time used: 0.015 (sec). Leaf size: 21

```
dsolve(diff(y(x),x)+x*(sin(2*y(x))-x^2*cos(y(x))^2) = 0,y(x), singsol=all)
```

$$y(x) = \arctan\left(\frac{c_1 \mathrm{e}^{-x^2}}{2} + \frac{x^2}{2} - \frac{1}{2}\right)$$

✓ Solution by Mathematica

Time used: 19.133 (sec). Leaf size: 105

```
DSolve[y'[x]+x*(Sin[2*y[x]]-x^2*Cos[y[x]]^2)==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to \tan^{-1}\left(\frac{1}{2}\left(x^2 - 8c_1 e^{-x^2} - 1\right)\right)$$
$$y(x) \to -\tan^{-1}\left(-\frac{x^2}{2} + 4c_1 e^{-x^2} + \frac{1}{2}\right)$$
$$y(x) \to -\frac{1}{2}\pi e^{x^2}\sqrt{e^{-2x^2}}$$
$$y(x) \to \frac{1}{2}\pi e^{x^2}\sqrt{e^{-2x^2}}$$

4.24 problem 113

Internal problem ID [3371]

Book: Ordinary differential equations and their solutions. By George Moseley Murphy. 1960
Section: Various 4
Problem number: 113.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_separable]`

$$\boxed{y' + \tan(x)\sec(x)\cos(y)^2 = 0}$$

✓ Solution by Maple
Time used: 0.0 (sec). Leaf size: 11

```
dsolve(diff(y(x),x)+tan(x)*sec(x)*cos(y(x))^2 = 0,y(x), singsol=all)
```

$$y(x) = -\arctan(\sec(x) + c_1)$$

✓ Solution by Mathematica
Time used: 1.593 (sec). Leaf size: 31

```
DSolve[y'[x]+Tan[x] Sec[x] Cos[y[x]]^2==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to \arctan(-\sec(x) + c_1)$$
$$y(x) \to -\frac{\pi}{2}$$
$$y(x) \to \frac{\pi}{2}$$

4.25 problem 114

Internal problem ID [3372]

Book: Ordinary differential equations and their solutions. By George Moseley Murphy. 1960
Section: Various 4
Problem number: 114.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_separable]`

$$\boxed{y' - \cot(x)\cot(y) = 0}$$

✓ Solution by Maple
Time used: 0.157 (sec). Leaf size: 11

```
dsolve(diff(y(x),x) = cot(x)*cot(y(x)),y(x), singsol=all)
```

$$y(x) = \arccos\left(\frac{\csc(x)}{c_1}\right)$$

✓ Solution by Mathematica
Time used: 5.785 (sec). Leaf size: 47

```
DSolve[y'[x]==Cot[x] Cot[y[x]],y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to -\arccos\left(-\frac{1}{2}c_1\csc(x)\right)$$
$$y(x) \to \arccos\left(-\frac{1}{2}c_1\csc(x)\right)$$
$$y(x) \to -\frac{\pi}{2}$$
$$y(x) \to \frac{\pi}{2}$$

4.26 problem 115

Internal problem ID [3373]

Book: Ordinary differential equations and their solutions. By George Moseley Murphy. 1960
Section: Various 4
Problem number: 115.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_separable]`

$$\boxed{y' + \cot(x)\cot(y) = 0}$$

✓ Solution by Maple
Time used: 0.156 (sec). Leaf size: 9

```
dsolve(diff(y(x),x)+cot(x)*cot(y(x)) = 0,y(x), singsol=all)
```

$$y(x) = \arccos(c_1 \sin(x))$$

✓ Solution by Mathematica
Time used: 5.682 (sec). Leaf size: 47

```
DSolve[y'[x]+Cot[x] Cot[y[x]]==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to -\arccos\left(-\frac{1}{2}c_1\sin(x)\right)$$
$$y(x) \to \arccos\left(-\frac{1}{2}c_1\sin(x)\right)$$
$$y(x) \to -\frac{\pi}{2}$$
$$y(x) \to \frac{\pi}{2}$$

5 Various 5

5.1 problem 116

Internal problem ID [3374]

Book: Ordinary differential equations and their solutions. By George Moseley Murphy. 1960
Section: Various 5
Problem number: 116.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_separable]`

$$\boxed{y' - \sin(x)(\csc(y) - \cot(y)) = 0}$$

✓ Solution by Maple

Time used: 0.141 (sec). Leaf size: 14

```
dsolve(diff(y(x),x) = sin(x)*(csc(y(x))-cot(y(x))),y(x), singsol=all)
```

$$y(x) = \arccos\left(\mathrm{e}^{-\cos(x)} c_1 + 1\right)$$

✓ Solution by Mathematica

Time used: 0.429 (sec). Leaf size: 70

```
DSolve[y'[x]==Sin[x](Csc[y[x]]-Cot[y[x]]),y[x],x,IncludeSingularSolutions -> True]
```

$$\text{Solve}\left[2\cos(x)\tan\left(\frac{y(x)}{2}\right)e^{\operatorname{arctanh}(\cos(y(x)))} - \sqrt{\sin^2(y(x))}\csc\left(\frac{y(x)}{2}\right)\sec\left(\frac{y(x)}{2}\right)\left(\log\left(\sec^2\left(\frac{y(x)}{2}\right)\right) - 2\log\left(\tan\left(\frac{y(x)}{2}\right)\right)\right) = c_1, y(x)\right]$$

5.2 problem 117

Internal problem ID [3375]

Book: Ordinary differential equations and their solutions. By George Moseley Murphy. 1960
Section: Various 5
Problem number: 117.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_separable]`

$$\boxed{y' - \tan(x)\cot(y) = 0}$$

✓ Solution by Maple

Time used: 0.157 (sec). Leaf size: 11

```
dsolve(diff(y(x),x) = tan(x)*cot(y(x)),y(x), singsol=all)
```

$$y(x) = \arccos\left(\frac{\cos(x)}{c_1}\right)$$

✓ Solution by Mathematica

Time used: 5.198 (sec). Leaf size: 47

```
DSolve[y'[x]==Tan[x] Cot[y[x]],y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to -\arccos\left(\frac{1}{2}c_1\cos(x)\right)$$
$$y(x) \to \arccos\left(\frac{1}{2}c_1\cos(x)\right)$$
$$y(x) \to -\frac{\pi}{2}$$
$$y(x) \to \frac{\pi}{2}$$

5.3 problem 118

Internal problem ID [3376]

Book: Ordinary differential equations and their solutions. By George Moseley Murphy. 1960
Section: Various 5
Problem number: 118.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_separable]`

$$\boxed{y' + \tan(x)\cot(y) = 0}$$

✓ Solution by Maple

Time used: 0.156 (sec). Leaf size: 9

```
dsolve(diff(y(x),x)+tan(x)*cot(y(x)) = 0,y(x), singsol=all)
```

$$y(x) = \arccos(\sec(x) c_1)$$

✓ Solution by Mathematica

Time used: 6.177 (sec). Leaf size: 47

```
DSolve[y'[x]+Tan[x] Cot[y[x]]==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to -\arccos\left(\frac{1}{2}c_1\sec(x)\right)$$
$$y(x) \to \arccos\left(\frac{1}{2}c_1\sec(x)\right)$$
$$y(x) \to -\frac{\pi}{2}$$
$$y(x) \to \frac{\pi}{2}$$

5.4 problem 119

Internal problem ID [3377]

Book: Ordinary differential equations and their solutions. By George Moseley Murphy. 1960
Section: Various 5
Problem number: 119.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_separable]`

$$\boxed{y' + \sin(2x)\csc(2y) = 0}$$

✓ Solution by Maple
Time used: 0.0 (sec). Leaf size: 17

```
dsolve(diff(y(x),x)+sin(2*x)*csc(2*y(x)) = 0,y(x), singsol=all)
```

$$y(x) = \frac{\arccos(-\cos(2x) + 4c_1)}{2}$$

✓ Solution by Mathematica
Time used: 0.554 (sec). Leaf size: 41

```
DSolve[y'[x]+Sin[2 x]Csc[2 y[x]]==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to -\frac{1}{2}\arccos(-\cos(2x) - 2c_1)$$
$$y(x) \to \frac{1}{2}\arccos(-\cos(2x) - 2c_1)$$

5.5 problem 120

Internal problem ID [3378]

Book: Ordinary differential equations and their solutions. By George Moseley Murphy. 1960
Section: Various 5
Problem number: 120.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[‘y=_G(x,y’)‘]`

$$y' - \tan(x)(\tan(y) + \sec(x)\sec(y)) = 0$$

✓ Solution by Maple
Time used: 0.015 (sec). Leaf size: 15

```
dsolve(diff(y(x),x) = tan(x)*(tan(y(x))+sec(x)*sec(y(x))),y(x), singsol=all)
```

$$y(x) = \arcsin(\sec(x)(-\ln(\cos(x)) + c_1))$$

✓ Solution by Mathematica
Time used: 9.719 (sec). Leaf size: 20

```
DSolve[y'[x]==Tan[x] (Tan[y[x]]+ Sec[x] Sec[y[x]]),y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to \arcsin\left(\frac{1}{4}\sec(x)(-4\log(\cos(x)) + c_1)\right)$$

5.6 problem 121

Internal problem ID [3379]

Book: Ordinary differential equations and their solutions. By George Moseley Murphy. 1960
Section: Various 5
Problem number: 121.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_separable]`

$$\boxed{y' - \cos(x)\sec(y)^2 = 0}$$

✓ Solution by Maple
Time used: 0.078 (sec). Leaf size: 22

```
dsolve(diff(y(x),x) = cos(x)*sec(y(x))^2,y(x), singsol=all)
```

$$y(x) = \frac{\text{RootOf}\left(-\_Z + 4c_1 + 4\sin(x) - \sin(\_Z)\right)}{2}$$

✓ Solution by Mathematica
Time used: 0.326 (sec). Leaf size: 32

```
DSolve[y'[x]==Cos[x] Sec[y[x]]^2,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to \text{InverseFunction}\left[2\left(\frac{\#1}{2} + \frac{1}{4}\sin(2\#1)\right)\&\right][2\sin(x) + c_1]$$

5.7 problem 122

Internal problem ID [3380]

Book: Ordinary differential equations and their solutions. By George Moseley Murphy. 1960
Section: Various 5
Problem number: 122.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_separable]`

$$y' - \sec(x)^2 \sec(y)^3 = 0$$

✓ Solution by Maple
Time used: 0.078 (sec). Leaf size: 74

```
dsolve(diff(y(x),x) = sec(x)^2*sec(y(x))^3,y(x), singsol=all)
```

$$y(x) = \arctan\left(\frac{3c_1 + 3\tan(x)}{\operatorname{RootOf}\left(\_Z^6 + 3\_Z^4 + 9c_1^2 + 18c_1\tan(x) + 9\tan(x)^2 - 4\right)^2 + 2}, \operatorname{RootOf}\left(\_Z^6 + 3\_Z^4 + 9c_1^2 + 18c_1\tan(x) + 9\tan(x)^2 - 4\right)\right)$$

✓ Solution by Mathematica

Time used: 24.108 (sec). Leaf size: 478

```
DSolve[y'[x]==Sec[x]^2 Sec[y[x]]^3,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to \arcsin\left(\frac{\sqrt[3]{-3\tan(x)+\sqrt{9\tan^2(x)+18c_1\tan(x)-4+9{c_1}^2}-3c_1}}{\sqrt[3]{2}} + \frac{\sqrt[3]{2}}{\sqrt[3]{-3\tan(x)+\sqrt{9\tan^2(x)+18c_1\tan(x)-4+9{c_1}^2}-3c_1}}\right)$$

$$y(x) \to -\arcsin\left(\frac{(1-i\sqrt{3})\sqrt[3]{-3\tan(x)+\sqrt{9\tan^2(x)+18c_1\tan(x)-4+9{c_1}^2}-3c_1}}{2\sqrt[3]{2}} + \frac{1+i\sqrt{3}}{2^{2/3}\sqrt[3]{-3\tan(x)+\sqrt{9\tan^2(x)+18c_1\tan(x)-4+9{c_1}^2}-3c_1}}\right)$$

$$y(x) \to -\arcsin\left(\frac{(1+i\sqrt{3})\sqrt[3]{-3\tan(x)+\sqrt{9\tan^2(x)+18c_1\tan(x)-4+9{c_1}^2}-3c_1}}{2\sqrt[3]{2}} + \frac{1-i\sqrt{3}}{2^{2/3}\sqrt[3]{-3\tan(x)+\sqrt{9\tan^2(x)+18c_1\tan(x)-4+9{c_1}^2}-3c_1}}\right)$$

$$y(x) \to \arcsin\left(\frac{\sqrt[3]{\sqrt{9\tan^2(x)-4}-3\tan(x)}}{\sqrt[3]{2}} + \frac{\sqrt[3]{2}}{\sqrt[3]{\sqrt{9\tan^2(x)-4}-3\tan(x)}}\right)$$

$$y(x) \to -\arcsin\left(\frac{(1+i\sqrt{3})\sqrt[3]{\sqrt{9\tan^2(x)-4}-3\tan(x)}}{2\sqrt[3]{2}} + \frac{1-i\sqrt{3}}{2^{2/3}\sqrt[3]{\sqrt{9\tan^2(x)-4}-3\tan(x)}}\right)$$

5.8 problem 123

Internal problem ID [3381]

Book: Ordinary differential equations and their solutions. By George Moseley Murphy. 1960
Section: Various 5
Problem number: 123.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_quadrature]`

$$\boxed{y' - b\sin(y) = a}$$

✓ Solution by Maple
Time used: 0.015 (sec). Leaf size: 44

```
dsolve(diff(y(x),x) = a+b*sin(y(x)),y(x), singsol=all)
```

$$y(x) = 2\arctan\left(\frac{-b + \tan\left(\frac{\sqrt{a^2-b^2}\,(c_1+x)}{2}\right)\sqrt{a^2-b^2}}{a}\right)$$

✓ Solution by Mathematica
Time used: 60.173 (sec). Leaf size: 52

```
DSolve[y'[x]==a+b Sin[y[x]],y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to 2\arctan\left(\frac{-b + \sqrt{a^2-b^2}\tan\left(\frac{1}{2}\sqrt{a^2-b^2}(x+c_1)\right)}{a}\right)$$

5.9 problem 125

Internal problem ID [3382]

Book: Ordinary differential equations and their solutions. By George Moseley Murphy. 1960
Section: Various 5
Problem number: 125.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[‘y=_G(x,y’)‘]`

$$y' - (\sin(y)\cos(x) + 1)\tan(y) = 0$$

✗ Solution by Maple

```
dsolve(diff(y(x),x) = (1+cos(x)*sin(y(x)))*tan(y(x)),y(x), singsol=all)
```

No solution found

✓ Solution by Mathematica

Time used: 1.965 (sec). Leaf size: 58

```
DSolve[y'[x]==(1+Cos[x] Sin[y[x]])Tan[y[x]],y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to \csc^{-1}\left(\frac{1}{2}(-\sin(x) - \cos(x) - 2c_1e^{-x})\right)$$
$$y(x) \to -\csc^{-1}\left(\frac{1}{2}(\sin(x) + \cos(x) + 2c_1e^{-x})\right)$$
$$y(x) \to 0$$

5.10 problem 126

Internal problem ID [3383]

Book: Ordinary differential equations and their solutions. By George Moseley Murphy. 1960
Section: Various 5
Problem number: 126.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_separable]`

$$\boxed{y' + \csc(2x)\sin(2y) = 0}$$

✓ Solution by Maple
Time used: 0.406 (sec). Leaf size: 80

```
dsolve(diff(y(x),x)+csc(2*x)*sin(2*y(x)) = 0,y(x), singsol=all)
```

$$y(x) = \frac{\arctan\left(-\frac{2\sin(2x)c_1}{c_1^2\cos(2x)-c_1^2-\cos(2x)-1}, \frac{c_1^2\cos(2x)-c_1^2+\cos(2x)+1}{c_1^2\cos(2x)-c_1^2-\cos(2x)-1}\right)}{2}$$

✓ Solution by Mathematica
Time used: 0.456 (sec). Leaf size: 68

```
DSolve[y'[x]+Csc[2 x] Sin[2 y[x]]==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to -\frac{1}{2}\arccos(-\tanh(\operatorname{arctanh}(\cos(2x)) + 2c_1))$$
$$y(x) \to \frac{1}{2}\arccos(-\tanh(\operatorname{arctanh}(\cos(2x)) + 2c_1))$$
$$y(x) \to 0$$
$$y(x) \to -\frac{\pi}{2}$$
$$y(x) \to \frac{\pi}{2}$$

5.11 problem 127

Internal problem ID [3384]

Book: Ordinary differential equations and their solutions. By George Moseley Murphy. 1960
Section: Various 5
Problem number: 127.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[‘y=_G(x,y’)‘]`

$$\boxed{y' + g(x)\tan(y) = -f(x)}$$

✗ Solution by Maple

```
dsolve(diff(y(x),x)+f(x)+g(x)*tan(y(x)) = 0,y(x), singsol=all)
```

No solution found

✗ Solution by Mathematica

Time used: 0.0 (sec). Leaf size: 0

```
DSolve[y'[x]+f[x]+g[x] Tan[ y[x]]==0,y[x],x,IncludeSingularSolutions -> True]
```

Not solved

5.12 problem 128

Internal problem ID [3385]

Book: Ordinary differential equations and their solutions. By George Moseley Murphy. 1960
Section: Various 5
Problem number: 128.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_quadrature]`

$$y' - \sqrt{a + b\cos(y)} = 0$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 21

```
dsolve(diff(y(x),x) = sqrt(a+b*cos(y(x))),y(x), singsol=all)
```

$$x - \left(\int^{y(x)} \frac{1}{\sqrt{a + b\cos(\_a)}} d\_a\right) + c_1 = 0$$

✓ Solution by Mathematica

Time used: 0.83 (sec). Leaf size: 55

```
DSolve[y'[x]==Sqrt[a+b Cos[ y[x]]],y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to 2\,\text{JacobiAmplitude}\left(\frac{1}{2}\sqrt{a+b}(x + c_1), \frac{2b}{a+b}\right)$$
$$y(x) \to -\arccos\left(-\frac{a}{b}\right)$$
$$y(x) \to \arccos\left(-\frac{a}{b}\right)$$

5.13 problem 129

Internal problem ID [3386]

Book: Ordinary differential equations and their solutions. By George Moseley Murphy. 1960
Section: Various 5
Problem number: 129.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[[_1st_order, '_with_symmetry_[F(x),G(x)]']]`

$$\boxed{y' - e^y = x}$$

✓ Solution by Maple
Time used: 0.016 (sec). Leaf size: 36

```
dsolve(diff(y(x),x) = x+exp(y(x)),y(x), singsol=all)
```

$$y(x) = \frac{x^2}{2} + \ln(2) - \ln\left(i\sqrt{\pi}\,\sqrt{2}\,\operatorname{erf}\left(\frac{i\sqrt{2}\,x}{2}\right) - 2c_1\right)$$

✓ Solution by Mathematica
Time used: 0.491 (sec). Leaf size: 40

```
DSolve[y'[x]==x+Exp[y[x]],y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to \frac{1}{2}\left(x^2 - 2\log\left(-\sqrt{\frac{\pi}{2}}\operatorname{erfi}\left(\frac{x}{\sqrt{2}}\right) - c_1\right)\right)$$

5.14 problem 130

Internal problem ID [3387]

Book: Ordinary differential equations and their solutions. By George Moseley Murphy. 1960
Section: Various 5
Problem number: 130.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_separable]`

$$\boxed{y' - e^{y+x} = 0}$$

✓ Solution by Maple
Time used: 0.0 (sec). Leaf size: 13

```
dsolve(diff(y(x),x) = exp(x+y(x)),y(x), singsol=all)
```

$$y(x) = \ln\left(-\frac{1}{e^x + c_1}\right)$$

✓ Solution by Mathematica
Time used: 0.773 (sec). Leaf size: 18

```
DSolve[y'[x]==Exp[x+y[x]],y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to -\log\left(-e^x - c_1\right)$$

5.15 problem 131

Internal problem ID [3388]

Book: Ordinary differential equations and their solutions. By George Moseley Murphy. 1960
Section: Various 5
Problem number: 131.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_separable]`

$$\boxed{y' - e^{x}\left(a + b\,e^{-y}\right) = 0}$$

✓ Solution by Maple
Time used: 0.156 (sec). Leaf size: 22

```
dsolve(diff(y(x),x) = exp(x)*(a+b*exp(-y(x))),y(x), singsol=all)
```

$$y(x) = -\ln\left(\frac{a}{e^{(e^{x}+c_1)a} - b}\right)$$

✓ Solution by Mathematica
Time used: 1.186 (sec). Leaf size: 24

```
DSolve[y'[x]==Exp[x](a+b Exp[-y[x]]),y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to \log\left(\frac{-b + e^{a(e^{x}+c_1)}}{a}\right)$$

5.16 problem 132

Internal problem ID [3389]

Book: Ordinary differential equations and their solutions. By George Moseley Murphy. 1960
Section: Various 5
Problem number: 132.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_separable]`

$$\boxed{y' + y\ln(x)\ln(y) = 0}$$

✓ Solution by Maple
Time used: 0.156 (sec). Leaf size: 16

```
dsolve(diff(y(x),x)+y(x)*ln(x)*ln(y(x)) = 0,y(x), singsol=all)
```

$$y(x) = \mathrm{e}^{\frac{x^{-x}\mathrm{e}^{x}}{c_1}}$$

✓ Solution by Mathematica
Time used: 0.224 (sec). Leaf size: 24

```
DSolve[y'[x]+y[x] Log[x] Log[y[x]]==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to e^{x^{-x}e^{x+c_1}}$$
$$y(x) \to 1$$

5.17 problem 133

Internal problem ID [3390]

Book: Ordinary differential equations and their solutions. By George Moseley Murphy. 1960
Section: Various 5
Problem number: 133.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[[_1st_order, '_with_symmetry_[F(x),G(y)]']]`

$$\boxed{y' - x^{m-1} y^{-n+1} f(a\,x^m + b y^n) = 0}$$

✓ Solution by Maple

Time used: 0.5 (sec). Leaf size: 174

```
dsolve(diff(y(x),x) = x^(m-1)*y(x)^(1-n)*f(a*x^m+b*y(x)^n),y(x), singsol=all)
```

$$y(x) = \left(- \frac{- \operatorname{RootOf}\left(\left(\left(\int^{\_Z} \frac{1}{\left(m^{\frac{1}{m}}\right)^m f\left(a\left(m^{\frac{1}{m}}\right)^m + b\left(\left(\frac{b\_a - am}{b}\right)^{\frac{1}{n}}\right)^n\right)\left(\left(\frac{b\_a - am}{b}\right)^{\frac{1}{n}}\right)^{-n} bn\_a - \left(m^{\frac{1}{m}}\right)^m f\left(a\left(m^{\frac{1}{m}}\right)^m + b\left(\left(\frac{b\_}{}\right.\right.\right.}\right.\right.\right.}{b}\right.$$

✓ Solution by Mathematica

Time used: 0.509 (sec). Leaf size: 242

```
DSolve[y'[x]==x^(m-1) y[x]^(1-n) f[a x^m + b y[x]^n],y[x],x,IncludeSingularSolutions -> True
```

$$\text{Solve}\left[\int_1^{y(x)}\left(-\frac{amK[2]^{n-1}}{am+bnf\left(ax^m+bK[2]^n\right)}\right.\right.$$
$$-\int_1^x\left(\frac{abmnK[1]^{m-1}K[2]^{n-1}f'(aK[1]^m+bK[2]^n)}{am+bnf\left(aK[1]^m+bK[2]^n\right)}-\frac{ab^2mn^2f(aK[1]^m+bK[2]^n)\,K[1]^{m-1}K[2]^{n-1}f'(aK[}{\left(am+bnf\left(aK[1]^m+bK[2]^n\right)\right)^2}\right.$$
$$\left.+\int_1^x\frac{amf(aK[1]^m+by(x)^n)\,K[1]^{m-1}}{am+bnf\left(aK[1]^m+by(x)^n\right)}dK[1]=c_1,y(x)\right]$$

5.18 problem 134

Internal problem ID [3391]

Book: Ordinary differential equations and their solutions. By George Moseley Murphy. 1960
Section: Various 5
Problem number: 134.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_quadrature]`

$$\boxed{y' - af(y) = 0}$$

✓ Solution by Maple
Time used: 0.0 (sec). Leaf size: 20

```
dsolve(diff(y(x),x) = a*f(y(x)),y(x), singsol=all)
```

$$x - \frac{\int^{y(x)} \frac{1}{f(\_a)} d\_a}{a} + c_1 = 0$$

✓ Solution by Mathematica
Time used: 0.295 (sec). Leaf size: 35

```
DSolve[y'[x]==a f[y[x]],y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to \text{InverseFunction}\left[\int_1^{\#1} \frac{1}{f(K[1])} dK[1]\&\right][ax + c_1]$$
$$y(x) \to f^{(-1)}(0)$$

5.19 problem 135

Internal problem ID [3392]

Book: Ordinary differential equations and their solutions. By George Moseley Murphy. 1960
Section: Various 5
Problem number: 135.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[[_homogeneous, ‘class C‘], _dAlembert]`

$$\boxed{y' - f(a + bx + cy) = 0}$$

✓ Solution by Maple

Time used: 0.047 (sec). Leaf size: 39

```
dsolve(diff(y(x),x) = f(a+b*x+c*y(x)),y(x), singsol=all)
```

$$y(x) = \frac{\operatorname{RootOf}\left(\left(\int^{\_Z} \frac{1}{f(\_ac+a)c+b} d\_a\right) c - x + c_1\right) c - bx}{c}$$

✓ Solution by Mathematica

Time used: 0.225 (sec). Leaf size: 262

```
DSolve[y'[x]==f[a+b x +c y[x]],y[x],x,IncludeSingularSolutions -> True]
```

$$\text{Solve}\left[\int_1^{y(x)} - \frac{f(a+bx+cK[2]) \int_1^x \left(\frac{c^2 f'(a+bK[1]+cK[2])}{b+cf(a+bK[1]+cK[2])} - \frac{c^3 f(a+bK[1]+cK[2]) f'(a+bK[1]+cK[2])}{(b+cf(a+bK[1]+cK[2]))^2}\right) dK[1] c + c + b \int_1^x \left(\frac{c^2 f'(a+b\ldots}{b+cf(a+b\ldots}\right.}{b+cf(a+bx+cK[2])} + \int_1^x \frac{cf(a+bK[1]+cy(x))}{b+cf(a+bK[1]+cy(x))} dK[1] = c_1, y(x)\right]$$

5.20 problem 136

Internal problem ID [3393]

Book: Ordinary differential equations and their solutions. By George Moseley Murphy. 1960
Section: Various 5
Problem number: 136.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_separable]`

$$\boxed{y' - f(x)\,g(y) = 0}$$

✓ Solution by Maple
Time used: 0.016 (sec). Leaf size: 20

```
dsolve(diff(y(x),x) = f(x)*g(y(x)),y(x), singsol=all)
```

$$\int f(x)\,dx - \left(\int^{y(x)} \frac{1}{g(\_a)} d\_a\right) + c_1 = 0$$

✓ Solution by Mathematica
Time used: 0.294 (sec). Leaf size: 42

```
DSolve[y'[x]==f[x] g[y[x]],y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to \text{InverseFunction}\left[\int_1^{\#1} \frac{1}{g(K[1])} dK[1]\&\right]\left[\int_1^x f(K[2])dK[2] + c_1\right]$$
$$y(x) \to g^{(-1)}(0)$$

5.21 problem 137

Internal problem ID [3394]

Book: Ordinary differential equations and their solutions. By George Moseley Murphy. 1960
Section: Various 5
Problem number: 137.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_linear]`

$$y' - y \sec (x) \operatorname{Csx}(x) = \sec (x)^2$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 32

```
dsolve(diff(y(x),x) = sec(x)^2+y(x)*sec(x)*Csx(x),y(x), singsol=all)
```

$$y(x) = \left(\int \sec (x)^2 \, \mathrm{e}^{-\left(\int \sec(x) \operatorname{Csx}(x)dx\right)}dx + c_1\right) \mathrm{e}^{\int \sec(x) \operatorname{Csx}(x)dx}$$

✓ Solution by Mathematica

Time used: 0.135 (sec). Leaf size: 57

```
DSolve[y'[x]==Sec[x]^2+y[x] Sec[x]Csx[x],y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to \exp \left(\int_1^x \operatorname{Csx}(K[1]) \sec(K[1])dK[1]\right) \left(\int_1^x \exp \left(-\int_1^{K[2]} \operatorname{Csx}(K[1]) \sec(K[1])dK[1]\right) \sec^2(K[2])dK[2] + c_1\right)$$

5.22 problem 139

Internal problem ID [3395]

Book: Ordinary differential equations and their solutions. By George Moseley Murphy. 1960
Section: Various 5
Problem number: 139.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[‘y=_G(x,y’)‘]`

$$2y' - 2\sin(y)^2\tan(y) + x\sin(2y) = 0$$

✗ Solution by Maple

```
dsolve(2*diff(y(x),x) = 2*sin(y(x))^2*tan(y(x))-x*sin(2*y(x)),y(x), singsol=all)
```

No solution found

✓ Solution by Mathematica

Time used: 60.375 (sec). Leaf size: 61

```
DSolve[2 y'[x]==2 Sin[y[x]]^2 Tan[y[x]]- x Sin[2 y[x]],y[x],x,IncludeSingularSolutions -> Tr
```

$$y(x) \to -\cot^{-1}\left(\sqrt{e^{x^2}\left(-\sqrt{\pi}\text{erf}(x) + 4c_1\right)}\right)$$
$$y(x) \to \cot^{-1}\left(\sqrt{e^{x^2}\left(-\sqrt{\pi}\text{erf}(x) + 4c_1\right)}\right)$$

5.23 problem 140

Internal problem ID [3396]

Book: Ordinary differential equations and their solutions. By George Moseley Murphy. 1960
Section: Various 5
Problem number: 140.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[[_homogeneous, ‘class G‘]]`

$$2y' - \sqrt{a^2x^2 - 4b\,x^2 - 4cy} = -ax$$

✓ Solution by Maple

Time used: 0.078 (sec). Leaf size: 432

```
dsolve(2*diff(y(x),x)+a*x = sqrt(a^2*x^2-4*b*x^2-4*c*y(x)),y(x), singsol=all)
```

$$-\left(\int_{\_b}^{x} \frac{-a\_a + \sqrt{(a^2-4b)\_a^2 - 4cy(x)}}{-a\_a^2 + \_a\sqrt{(a^2-4b)\_a^2 - 4cy(x)} - 4y(x)} d\_a\right)$$
$$-2\left(\int^{y(x)} \frac{2\left(\int_{\_b}^{x} -\frac{-\sqrt{(a^2-4b)\_a^2-4\_fc}\,\_aa+(a^2-4b)\_a^2-2\_fc}{\sqrt{(a^2-4b)\_a^2-4\_fc}\left(a\_a^2-\_a\sqrt{(a^2-4b)\_a^2-4\_fc}+4\_f\right)^2} d\_a\right) a\,x^2 - 2\left(\int_{\_b}^{x} -\frac{-\sqrt{(a^2 \cdots}}{\sqrt{(a^2-4b)\_a^2 \cdots}} \cdots\right.}{\cdots}\right.$$
$$+ c_1 = 0$$

✓ Solution by Mathematica

Time used: 0.586 (sec). Leaf size: 542

```
DSolve[2 y'[x]+a x==Sqrt[a^2 x^2-4 b x^2 -4 c y[x]],y[x],x,IncludeSingularSolutions -> True]
```

$$\text{Solve}\left[\text{RootSum}\left[\#1^4+2\#1^3c-2\#1^2a^2-4\#1^2ac+8\#1^2b+2\#1a^2c-8\#1bc+a^4-8a^2b\right.\right.$$

$$+16b^2\&,\frac{\#1^3\log\left(\#1x-\sqrt{x^2\left(a^2-4b\right)-4cy(x)}+2\sqrt{-cy(x)}\right)+\#1^3(-\log(x))+\#1^2c\log\left(\#1x-\sqrt{x\right.}}{}$$

$$\left.-\log\left(\sqrt{-cy(x)}\sqrt{x^2\left(a^2-4b\right)-4cy(x)}+2cy(x)\right)+\frac{1}{2}\log(y(x))+2\log(x)=c_1,y(x)\right]$$

5.24 problem 141

Internal problem ID [3397]

Book: Ordinary differential equations and their solutions. By George Moseley Murphy. 1960
Section: Various 5
Problem number: 141.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[[_1st_order, _with_linear_symmetries], _dAlembert]`

$$\boxed{3y' - \sqrt{x^2 - 3y} = x}$$

✓ Solution by Maple
Time used: 0.0 (sec). Leaf size: 108

```
dsolve(3*diff(y(x),x) = x+sqrt(x^2-3*y(x)),y(x), singsol=all)
```

$$\frac{2(x^2-3y(x))^{\frac{3}{2}}\left(c_1 y(x)^2 x^2 - 4c_1 y(x)^3 + 1\right) + 2\left(c_1 y(x)^2 x^2 - 4c_1 y(x)^3 - 1\right)\left(x^2 - \frac{9y(x)}{2}\right)x}{\left(x^2 - 4y\left(x\right)\right) y\left(x\right)^2 \left(x + \sqrt{x^2 - 3y\left(x\right)}\right)^2 \left(-2\sqrt{x^2 - 3y\left(x\right)} + x\right)} = 0$$

✓ Solution by Mathematica

Time used: 60.169 (sec). Leaf size: 499

```
DSolve[3 y'[x]==x+Sqrt[x^2-3 y[x]],y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to \frac{1}{12}\left(x^2 + \frac{x(x^3+216e^{3c_1})}{\sqrt[3]{x^6 - 540e^{3c_1}x^3 + 24\sqrt{3}\sqrt{e^{3c_1}\left(-x^3+27e^{3c_1}\right)^3} - 5832e^{6c_1}}} + \sqrt[3]{x^6 - 540e^{3c_1}x^3 + 24\sqrt{3}\sqrt{e^{3c_1}\left(-x^3+27e^{3c_1}\right)^3} - 5832e^{6c_1}}\right)$$

$$y(x) \to \frac{1}{24}\left(2x^2 - \frac{i\left(\sqrt{3}-i\right)x(x^3+216e^{3c_1})}{\sqrt[3]{x^6 - 540e^{3c_1}x^3 + 24\sqrt{3}\sqrt{e^{3c_1}\left(-x^3+27e^{3c_1}\right)^3} - 5832e^{6c_1}}} + i\left(\sqrt{3}+i\right)\sqrt[3]{x^6 - 540e^{3c_1}x^3 + 24\sqrt{3}\sqrt{e^{3c_1}\left(-x^3+27e^{3c_1}\right)^3} - 5832e^{6c_1}}\right)$$

$$y(x) \to \frac{1}{24}\left(2x^2 + \frac{i\left(\sqrt{3}+i\right)x(x^3+216e^{3c_1})}{\sqrt[3]{x^6 - 540e^{3c_1}x^3 + 24\sqrt{3}\sqrt{e^{3c_1}\left(-x^3+27e^{3c_1}\right)^3} - 5832e^{6c_1}}} - \left(1+i\sqrt{3}\right)\sqrt[3]{x^6 - 540e^{3c_1}x^3 + 24\sqrt{3}\sqrt{e^{3c_1}\left(-x^3+27e^{3c_1}\right)^3} - 5832e^{6c_1}}\right)$$

5.25 problem 142

Internal problem ID [3398]

Book: Ordinary differential equations and their solutions. By George Moseley Murphy. 1960
Section: Various 5
Problem number: 142.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_quadrature]`

$$\boxed{xy' = \sqrt{a^2 - x^2}}$$

✓ Solution by Maple
Time used: 0.0 (sec). Leaf size: 51

```
dsolve(x*diff(y(x),x) = sqrt(a^2-x^2),y(x), singsol=all)
```

$$y(x) = -a\ \mathrm{csgn}\,(a) \ln \left(\frac{a\left(\mathrm{csgn}\,(a)\sqrt{a^2 - x^2} + a\right)}{x}\right) - a\ \mathrm{csgn}\,(a) \ln(2) + \sqrt{a^2 - x^2} + c_1$$

✓ Solution by Mathematica
Time used: 0.029 (sec). Leaf size: 42

```
DSolve[x y'[x]==Sqrt[a^2-x^2],y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to -a\mathrm{arctanh}\left(\frac{\sqrt{a^2 - x^2}}{a}\right) + \sqrt{a^2 - x^2} + c_1$$

5.26 problem 143

Internal problem ID [3399]

Book: Ordinary differential equations and their solutions. By George Moseley Murphy. 1960
Section: Various 5
Problem number: 143.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_linear]`

$$\boxed{xy' + y = -x}$$

✓ Solution by Maple
Time used: 0.0 (sec). Leaf size: 13

```
dsolve(x*diff(y(x),x)+x+y(x) = 0,y(x), singsol=all)
```

$$y(x) = -\frac{x}{2} + \frac{c_1}{x}$$

✓ Solution by Mathematica
Time used: 0.024 (sec). Leaf size: 17

```
DSolve[x y'[x]+x + y[x]==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to -\frac{x}{2} + \frac{c_1}{x}$$

5.27 problem 144

Internal problem ID [3400]

Book: Ordinary differential equations and their solutions. By George Moseley Murphy. 1960
Section: Various 5
Problem number: 144.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_linear]`

$$\boxed{xy' - y = -x^2}$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 11

```
dsolve(x*diff(y(x),x)+x^2-y(x) = 0,y(x), singsol=all)
```

$$y(x) = (c_1 - x)\, x$$

✓ Solution by Mathematica

Time used: 0.027 (sec). Leaf size: 13

```
DSolve[x y'[x]+x^2-y[x]==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to x(-x + c_1)$$

5.28 problem 145

Internal problem ID [3401]

Book: Ordinary differential equations and their solutions. By George Moseley Murphy. 1960
Section: Various 5
Problem number: 145.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_linear]`

$$\boxed{xy' + y = x^3}$$

✓ Solution by Maple
Time used: 0.0 (sec). Leaf size: 16

```
dsolve(x*diff(y(x),x) = x^3-y(x),y(x), singsol=all)
```

$$y(x) = \frac{x^4 + 4c_1}{4x}$$

✓ Solution by Mathematica
Time used: 0.028 (sec). Leaf size: 19

```
DSolve[x y'[x]==x^3-y[x],y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to \frac{x^3}{4} + \frac{c_1}{x}$$

5.29 problem 146

Internal problem ID [3402]

Book: Ordinary differential equations and their solutions. By George Moseley Murphy. 1960
Section: Various 5
Problem number: 146.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_linear]`

$$\boxed{xy' - y = x^3 + 1}$$

✓ Solution by Maple
Time used: 0.016 (sec). Leaf size: 14

```
dsolve(x*diff(y(x),x) = 1+x^3+y(x),y(x), singsol=all)
```

$$y(x) = \frac{1}{2}x^3 - 1 + c_1 x$$

✓ Solution by Mathematica
Time used: 0.027 (sec). Leaf size: 18

```
DSolve[x y'[x]==1+x^3+y[x],y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to \frac{x^3}{2} + c_1 x - 1$$

6 Various 6

6.1 problem 147

Internal problem ID [3403]

Book: Ordinary differential equations and their solutions. By George Moseley Murphy. 1960
Section: Various 6
Problem number: 147.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_linear]`

$$\boxed{xy' - y = x^m}$$

✓ Solution by Maple
Time used: 0.0 (sec). Leaf size: 17

```
dsolve(x*diff(y(x),x) = x^m+y(x),y(x), singsol=all)
```

$$y(x) = \frac{x^m}{m-1} + c_1 x$$

✓ Solution by Mathematica
Time used: 0.042 (sec). Leaf size: 19

```
DSolve[x y'[x]==x^m+y[x],y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to \frac{x^m}{m-1} + c_1 x$$

6.2 problem 148

Internal problem ID [3404]

Book: Ordinary differential equations and their solutions. By George Moseley Murphy. 1960
Section: Various 6
Problem number: 148.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_linear]`

$$\boxed{xy' + y = x \sin(x)}$$

✓ Solution by Maple
Time used: 0.0 (sec). Leaf size: 17

```
dsolve(x*diff(y(x),x) = x*sin(x)-y(x),y(x), singsol=all)
```

$$y(x) = \frac{-x\cos(x) + \sin(x) + c_1}{x}$$

✓ Solution by Mathematica
Time used: 0.036 (sec). Leaf size: 19

```
DSolve[x y'[x]==x Sin[x]-y[x],y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to \frac{\sin(x) - x\cos(x) + c_1}{x}$$

6.3 problem 149

Internal problem ID [3405]

Book: Ordinary differential equations and their solutions. By George Moseley Murphy. 1960
Section: Various 6
Problem number: 149.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_linear]`

$$\boxed{xy' - y = x^2 \sin(x)}$$

✓ Solution by Maple
Time used: 0.0 (sec). Leaf size: 12

```
dsolve(x*diff(y(x),x) = x^2*sin(x)+y(x),y(x), singsol=all)
```

$$y(x) = (-\cos(x) + c_1)\, x$$

✓ Solution by Mathematica
Time used: 0.034 (sec). Leaf size: 14

```
DSolve[x y'[x]==x^2 Sin[x]+y[x],y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to x(-\cos(x) + c_1)$$

6.4 problem 150

Internal problem ID [3406]

Book: Ordinary differential equations and their solutions. By George Moseley Murphy. 1960
Section: Various 6
Problem number: 150.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_linear]`

$$\boxed{xy' + y = x^n \ln(x)}$$

✓ Solution by Maple
Time used: 0.0 (sec). Leaf size: 34

```
dsolve(x*diff(y(x),x) = x^n*ln(x)-y(x),y(x), singsol=all)
```

$$y(x) = \frac{x(-1 + (n+1)\ln(x))\,x^n + c_1(n+1)^2}{(n+1)^2\,x}$$

✓ Solution by Mathematica
Time used: 0.083 (sec). Leaf size: 29

```
DSolve[x y'[x]==x^n Log[x]-y[x],y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to \frac{x^n((n+1)\log(x) - 1)}{(n+1)^2} + \frac{c_1}{x}$$

6.5 problem 151

Internal problem ID [3407]

Book: Ordinary differential equations and their solutions. By George Moseley Murphy. 1960
Section: Various 6
Problem number: 151.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_linear]`

$$\boxed{xy' + 2y = \sin(x)}$$

✓ Solution by Maple
Time used: 0.0 (sec). Leaf size: 17

```
dsolve(x*diff(y(x),x) = sin(x)-2*y(x),y(x), singsol=all)
```

$$y(x) = \frac{-x\cos(x) + \sin(x) + c_1}{x^2}$$

✓ Solution by Mathematica
Time used: 0.034 (sec). Leaf size: 19

```
DSolve[x y'[x]==Sin[x]-2 y[x],y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to \frac{\sin(x) - x\cos(x) + c_1}{x^2}$$

6.6 problem 152

Internal problem ID [3408]

Book: Ordinary differential equations and their solutions. By George Moseley Murphy. 1960
Section: Various 6
Problem number: 152.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_separable]`

$$\boxed{xy' - ya = 0}$$

✓ Solution by Maple
Time used: 0.0 (sec). Leaf size: 9

```
dsolve(x*diff(y(x),x) = a*y(x),y(x), singsol=all)
```

$$y(x) = c_1 x^a$$

✓ Solution by Mathematica
Time used: 0.024 (sec). Leaf size: 16

```
DSolve[x y'[x]==a y[x],y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to c_1 x^a$$
$$y(x) \to 0$$

6.7 problem 153

Internal problem ID [3409]

Book: Ordinary differential equations and their solutions. By George Moseley Murphy. 1960
Section: Various 6
Problem number: 153.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_linear]`

$$\boxed{xy' - ya = x + 1}$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 31

```
dsolve(x*diff(y(x),x) = 1+x+a*y(x),y(x), singsol=all)
```

$$y(x) = \left(-\frac{x^{-a}(ax + a - 1)}{a\,(a-1)} + c_1\right) x^a$$

✓ Solution by Mathematica

Time used: 0.081 (sec). Leaf size: 28

```
DSolve[x y'[x]==1+x+a y[x],y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to -\frac{ax + a - 1}{(a-1)a} + c_1 x^a$$

6.8 problem 154

Internal problem ID [3410]

Book: Ordinary differential equations and their solutions. By George Moseley Murphy. 1960
Section: Various 6
Problem number: 154.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_linear]`

$$\boxed{xy' - yb = ax}$$

✓ Solution by Maple
Time used: 0.0 (sec). Leaf size: 19

```
dsolve(x*diff(y(x),x) = a*x+b*y(x),y(x), singsol=all)
```

$$y(x) = -\frac{ax}{b-1} + x^b c_1$$

✓ Solution by Mathematica
Time used: 0.035 (sec). Leaf size: 22

```
DSolve[x y'[x]==a x + b y[x],y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to \frac{ax}{1-b} + c_1 x^b$$

6.9 problem 155

Internal problem ID [3411]

Book: Ordinary differential equations and their solutions. By George Moseley Murphy. 1960
Section: Various 6
Problem number: 155.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_linear]`

$$\boxed{xy' - yb = x^2 a}$$

✓ Solution by Maple
Time used: 0.0 (sec). Leaf size: 21

```
dsolve(x*diff(y(x),x) = a*x^2+b*y(x),y(x), singsol=all)
```

$$y(x) = -\frac{a\,x^2}{b-2} + x^b c_1$$

✓ Solution by Mathematica
Time used: 0.04 (sec). Leaf size: 24

```
DSolve[x y'[x]==a x^2+b y[x],y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to \frac{ax^2}{2-b} + c_1 x^b$$

6.10 problem 156

Internal problem ID [3412]

Book: Ordinary differential equations and their solutions. By George Moseley Murphy. 1960
Section: Various 6
Problem number: 156.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_linear]`

$$\boxed{xy' - cy = b\,x^n + a}$$

✓ Solution by Maple
Time used: 0.0 (sec). Leaf size: 29

```
dsolve(x*diff(y(x),x) = a+b*x^n+c*y(x),y(x), singsol=all)
```

$$y(x) = -\frac{x^n b}{c-n} - \frac{a}{c} + x^c c_1$$

✓ Solution by Mathematica
Time used: 0.116 (sec). Leaf size: 31

```
DSolve[x y'[x]==a+b x^n+c y[x],y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to -\frac{a}{c} - \frac{bx^n}{c-n} + c_1 x^c$$

6.11 problem 157

Internal problem ID [3413]

Book: Ordinary differential equations and their solutions. By George Moseley Murphy. 1960
Section: Various 6
Problem number: 157.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_linear]`

$$\boxed{xy' + (-x+3)\, y = -2}$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 22

```
dsolve(x*diff(y(x),x)+2+(3-x)*y(x) = 0,y(x), singsol=all)
```

$$y(x) = \frac{e^{x} c_1 + 2x^2 + 4x + 4}{x^3}$$

✓ Solution by Mathematica

Time used: 0.041 (sec). Leaf size: 25

```
DSolve[x y'[x]+2+(3-x)y[x]==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to \frac{2x^2 + 4x + c_1 e^x + 4}{x^3}$$

6.12 problem 158

Internal problem ID [3414]

Book: Ordinary differential equations and their solutions. By George Moseley Murphy. 1960
Section: Various 6
Problem number: 158.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_linear]`

$$\boxed{xy' + (ax + 2)\, y = -x}$$

✓ Solution by Maple
Time used: 0.016 (sec). Leaf size: 35

```
dsolve(x*diff(y(x),x)+x+(a*x+2)*y(x) = 0,y(x), singsol=all)
```

$$y(x) = \frac{e^{-ax} c_1 a^3 - x^2 a^2 + 2ax - 2}{a^3 x^2}$$

✓ Solution by Mathematica
Time used: 0.06 (sec). Leaf size: 37

```
DSolve[x y'[x]+x+(2+a x)y[x]==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to \frac{-\frac{a^2x^2-2ax+2}{a^3} + c_1 e^{-ax}}{x^2}$$

6.13 problem 159

Internal problem ID [3415]

Book: Ordinary differential equations and their solutions. By George Moseley Murphy. 1960
Section: Various 6
Problem number: 159.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_separable]`

$$\boxed{xy' + (bx + a)\,y = 0}$$

✓ Solution by Maple
Time used: 0.0 (sec). Leaf size: 16

```
dsolve(x*diff(y(x),x)+(b*x+a)*y(x) = 0,y(x), singsol=all)
```

$$y(x) = c_1 \mathrm{e}^{-bx} x^{-a}$$

✓ Solution by Mathematica
Time used: 0.034 (sec). Leaf size: 24

```
DSolve[x y'[x]+(a+ b x)y[x]==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to c_1 x^{-a} e^{-bx}$$
$$y(x) \to 0$$

6.14 problem 160

Internal problem ID [3416]

Book: Ordinary differential equations and their solutions. By George Moseley Murphy. 1960
Section: Various 6
Problem number: 160.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_linear]`

$$\boxed{xy' - (-2x^2 + 1)\, y = x^3}$$

✓ Solution by Maple

Time used: 0.015 (sec). Leaf size: 17

```
dsolve(x*diff(y(x),x) = x^3+(-2*x^2+1)*y(x),y(x), singsol=all)
```

$$y(x) = \frac{x}{2} + \mathrm{e}^{-x^2} x c_1$$

✓ Solution by Mathematica

Time used: 0.033 (sec). Leaf size: 21

```
DSolve[x y'[x]==x^3+(1-2 x^2)y[x],y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to x\left(\frac{1}{2} + c_1 e^{-x^2}\right)$$

6.15 problem 161

Internal problem ID [3417]

Book: Ordinary differential equations and their solutions. By George Moseley Murphy. 1960
Section: Various 6
Problem number: 161.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_linear]`

$$\boxed{xy' + \left(-b\,x^2 + 1\right) y = ax}$$

✓ Solution by Maple
Time used: 0.015 (sec). Leaf size: 25

```
dsolve(x*diff(y(x),x) = a*x-(-b*x^2+1)*y(x),y(x), singsol=all)
```

$$y(x) = \frac{\mathrm{e}^{\frac{b\,x^2}{2}} c_1 b - a}{bx}$$

✓ Solution by Mathematica
Time used: 0.075 (sec). Leaf size: 30

```
DSolve[x y'[x]==a x-(1-b x^2)y[x],y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to \frac{-a + bc_1 e^{\frac{bx^2}{2}}}{bx}$$

6.16 problem 162

Internal problem ID [3418]

Book: Ordinary differential equations and their solutions. By George Moseley Murphy. 1960
Section: Various 6
Problem number: 162.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_linear]`

$$\boxed{xy' + \left(-x^2 a + 2\right) y = -x}$$

✓ Solution by Maple
Time used: 0.015 (sec). Leaf size: 57

```
dsolve(x*diff(y(x),x)+x+(-a*x^2+2)*y(x) = 0,y(x), singsol=all)
```

$$y(x) = \frac{-e^{\frac{a x^2}{2}} \operatorname{erf}\left(\frac{\sqrt{2}\,\sqrt{a}\,x}{2}\right) \sqrt{2}\,\sqrt{\pi} + 2\,e^{\frac{a x^2}{2}} c_1 a^{\frac{3}{2}} + 2x\sqrt{a}}{2a^{\frac{3}{2}} x^2}$$

✓ Solution by Mathematica
Time used: 0.1 (sec). Leaf size: 70

```
DSolve[x y'[x]+x+(2-a x^2)y[x]==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to \frac{-\frac{\sqrt{2\pi} e^{\frac{ax^2}{2}} \operatorname{erf}\left(\frac{\sqrt{a}x}{\sqrt{2}}\right)}{a^{3/2}} + 2c_1 e^{\frac{ax^2}{2}} + \frac{2x}{a}}{2x^2}$$

6.17 problem 163

Internal problem ID [3419]

Book: Ordinary differential equations and their solutions. By George Moseley Murphy. 1960
Section: Various 6
Problem number: 163.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_rational, _Riccati]`

$$\boxed{xy' + y^2 = -x^2}$$

✓ Solution by Maple
Time used: 0.016 (sec). Leaf size: 27

```
dsolve(x*diff(y(x),x)+x^2+y(x)^2 = 0,y(x), singsol=all)
```

$$y(x) = -\frac{(c_1 \operatorname{BesselY}(1, x) + \operatorname{BesselJ}(1, x))\, x}{c_1 \operatorname{BesselY}(0, x) + \operatorname{BesselJ}(0, x)}$$

✓ Solution by Mathematica
Time used: 0.163 (sec). Leaf size: 45

```
DSolve[x y'[x]+x^2+y[x]^2==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to -\frac{x(\operatorname{BesselY}(1, x) + c_1 \operatorname{BesselJ}(1, x))}{\operatorname{BesselY}(0, x) + c_1 \operatorname{BesselJ}(0, x)}$$
$$y(x) \to -\frac{x \operatorname{BesselJ}(1, x)}{\operatorname{BesselJ}(0, x)}$$

6.18 problem 164

Internal problem ID [3420]

Book: Ordinary differential equations and their solutions. By George Moseley Murphy. 1960
Section: Various 6
Problem number: 164.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[[_homogeneous, ‘class D‘], _rational, _Riccati]`

$$\boxed{xy' - y(y+1) = x^2}$$

✓ Solution by Maple
Time used: 0.0 (sec). Leaf size: 10

```
dsolve(x*diff(y(x),x) = x^2+y(x)*(1+y(x)),y(x), singsol=all)
```

$$y(x) = \tan(c_1 + x)\, x$$

✓ Solution by Mathematica
Time used: 0.191 (sec). Leaf size: 12

```
DSolve[x y'[x]==x^2+y[x](1+y[x]),y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to x \tan(x + c_1)$$

6.19 problem 165

Internal problem ID [3421]

Book: Ordinary differential equations and their solutions. By George Moseley Murphy. 1960
Section: Various 6
Problem number: 165.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_rational, _Riccati]`

$$\boxed{xy' - y + y^2 = x^{\frac{2}{3}}}$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 72

```
dsolve(x*diff(y(x),x)-y(x)+y(x)^2 = x^(2/3),y(x), singsol=all)
```

$$y(x) = \frac{x^{\frac{1}{3}} \left(c_1 e^{6x^{\frac{1}{3}}} \operatorname{abs}\left(1, 3x^{\frac{1}{3}} - 1\right) + c_1 e^{6x^{\frac{1}{3}}} \left|3x^{\frac{1}{3}} - 1\right| - 3x^{\frac{1}{3}}\right)}{c_1 e^{6x^{\frac{1}{3}}} \left|3x^{\frac{1}{3}} - 1\right| + 3x^{\frac{1}{3}} + 1}$$

✓ Solution by Mathematica

Time used: 0.203 (sec). Leaf size: 131

```
DSolve[x y'[x]-y[x]+y[x]^2==x^(2/3),y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to \frac{3x^{2/3} \left(c_1 \cosh\left(3\sqrt[3]{x}\right) - i \sinh\left(3\sqrt[3]{x}\right)\right)}{\left(-3i\sqrt[3]{x} - c_1\right) \cosh\left(3\sqrt[3]{x}\right) + \left(3c_1\sqrt[3]{x} + i\right) \sinh\left(3\sqrt[3]{x}\right)}$$

$$y(x) \to \frac{3x^{2/3} \cosh\left(3\sqrt[3]{x}\right)}{3\sqrt[3]{x} \sinh\left(3\sqrt[3]{x}\right) - \cosh\left(3\sqrt[3]{x}\right)}$$

6.20 problem 166

Internal problem ID [3422]

Book: Ordinary differential equations and their solutions. By George Moseley Murphy. 1960
Section: Various 6
Problem number: 166.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_separable]`

$$\boxed{xy' - by^2 = a}$$

✓ Solution by Maple
Time used: 0.0 (sec). Leaf size: 24

```
dsolve(x*diff(y(x),x) = a+b*y(x)^2,y(x), singsol=all)
```

$$y(x) = \frac{\sqrt{ab}\,\tan\left(\sqrt{ab}\,(\ln(x) + c_1)\right)}{b}$$

✓ Solution by Mathematica
Time used: 10.673 (sec). Leaf size: 69

```
DSolve[x y'[x]==a+b y[x]^2,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to \frac{\sqrt{a}\tan\left(\sqrt{a}\sqrt{b}(\log(x) + c_1)\right)}{\sqrt{b}}$$
$$y(x) \to -\frac{i\sqrt{a}}{\sqrt{b}}$$
$$y(x) \to \frac{i\sqrt{a}}{\sqrt{b}}$$

6.21 problem 167

Internal problem ID [3423]

Book: Ordinary differential equations and their solutions. By George Moseley Murphy. 1960
Section: Various 6
Problem number: 167.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[[_homogeneous, 'class D'], _rational, _Riccati]`

$$\boxed{xy' - y - by^2 = x^2a}$$

✓ Solution by Maple
Time used: 0.016 (sec). Leaf size: 24

```
dsolve(x*diff(y(x),x) = a*x^2+y(x)+b*y(x)^2,y(x), singsol=all)
```

$$y(x) = \frac{\tan\left(\sqrt{ab}\,(c_1 + x)\right) x\sqrt{ab}}{b}$$

✓ Solution by Mathematica
Time used: 17.546 (sec). Leaf size: 33

```
DSolve[x y'[x]==a x^2+y[x]+b y[x]^2,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to \frac{\sqrt{a}x\tan\left(\sqrt{a}\sqrt{b}(x + c_1)\right)}{\sqrt{b}}$$

6.22 problem 168

Internal problem ID [3424]

Book: Ordinary differential equations and their solutions. By George Moseley Murphy. 1960
Section: Various 6
Problem number: 168.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_rational, _Riccati]`

$$\boxed{xy' - (n + yb)\, y = a\, x^{2n}}$$

✓ Solution by Maple
Time used: 0.031 (sec). Leaf size: 34

```
dsolve(x*diff(y(x),x) = a*x^(2*n)+(n+b*y(x))*y(x),y(x), singsol=all)
```

$$y(x) = \frac{\tan\left(\frac{x^n \sqrt{a}\, \sqrt{b} - c_1 n}{n}\right) \sqrt{a}\, x^n}{\sqrt{b}}$$

✓ Solution by Mathematica
Time used: 0.336 (sec). Leaf size: 139

```
DSolve[x y'[x]==a x^(2 n)+(n+b y[x])y[x],y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to \frac{\sqrt{a}x^n \left(-\cos\left(\frac{\sqrt{a}\sqrt{b}x^n}{n}\right) + c_1 \sin\left(\frac{\sqrt{a}\sqrt{b}x^n}{n}\right)\right)}{\sqrt{b}\left(\sin\left(\frac{\sqrt{a}\sqrt{b}x^n}{n}\right) + c_1 \cos\left(\frac{\sqrt{a}\sqrt{b}x^n}{n}\right)\right)}$$

$$y(x) \to \frac{\sqrt{a}x^n \tan\left(\frac{\sqrt{a}\sqrt{b}x^n}{n}\right)}{\sqrt{b}}$$

6.23 problem 169

Internal problem ID [3425]

Book: Ordinary differential equations and their solutions. By George Moseley Murphy. 1960
Section: Various 6
Problem number: 169.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_rational, _Riccati]`

$$xy' - yb - cy^2 = a\,x^n$$

✓ Solution by Maple
Time used: 0.0 (sec). Leaf size: 164

```
dsolve(x*diff(y(x),x) = a*x^n+b*y(x)+c*y(x)^2,y(x), singsol=all)
```

$$y(x) = \frac{\left(\text{BesselY}\left(\frac{b+n}{n}, \frac{2\sqrt{ac}\,x^{\frac{n}{2}}}{n}\right) c_1 + \text{BesselJ}\left(\frac{b+n}{n}, \frac{2\sqrt{ac}\,x^{\frac{n}{2}}}{n}\right)\right)\sqrt{ac}\,x^{\frac{n}{2}} - b\left(\text{BesselY}\left(\frac{b}{n}, \frac{2\sqrt{ac}\,x^{\frac{n}{2}}}{n}\right) c_1 + \text{BesselJ}\left(\right.\right.}{c\left(\text{BesselY}\left(\frac{b}{n}, \frac{2\sqrt{ac}\,x^{\frac{n}{2}}}{n}\right) c_1 + \text{BesselJ}\left(\frac{b}{n}, \frac{2\sqrt{ac}\,x^{\frac{n}{2}}}{n}\right)\right)}$$

✓ Solution by Mathematica
Time used: 0.307 (sec). Leaf size: 402

```
DSolve[x y'[x]==a x^n+b y[x]+c y[x]^2,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to \frac{\sqrt{a}\sqrt{c}x^{n/2}\left(-2\,\text{BesselJ}\left(\frac{b}{n} - 1, \frac{2\sqrt{a}\sqrt{c}x^{n/2}}{n}\right) + c_1\left(\text{BesselJ}\left(1 - \frac{b}{n}, \frac{2\sqrt{a}\sqrt{c}x^{n/2}}{n}\right) - \text{BesselJ}\left(-\frac{b+n}{n}, \frac{2\sqrt{a}\sqrt{c}x^{n/2}}{n}\right.\right.\right.}{2c\left(\text{BesselJ}\left(\frac{b}{n}, \frac{2\sqrt{a}\sqrt{c}x^{n/2}}{n}\right) + c_1\,\text{BesselJ}\left(-\frac{b}{n}, \frac{2\sqrt{a}\sqrt{c}x^{n/2}}{n}\right)\right)}$$

$$y(x) \to -\frac{-\sqrt{a}\sqrt{c}x^{n/2}\,\text{BesselJ}\left(1 - \frac{b}{n}, \frac{2\sqrt{a}\sqrt{c}x^{n/2}}{n}\right) + \sqrt{a}\sqrt{c}x^{n/2}\,\text{BesselJ}\left(-\frac{b+n}{n}, \frac{2\sqrt{a}\sqrt{c}x^{n/2}}{n}\right) + b\,\text{BesselJ}\left(-\frac{b}{n}, \frac{2\sqrt{a}}{}\right.}{2c\,\text{BesselJ}\left(-\frac{b}{n}, \frac{2\sqrt{a}\sqrt{c}x^{n/2}}{n}\right)}$$

6.24 problem 170

Internal problem ID [3426]

Book: Ordinary differential equations and their solutions. By George Moseley Murphy. 1960
Section: Various 6
Problem number: 170.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_rational, _Riccati]`

$$\boxed{xy' - yb - cy^2 = k + a\,x^n}$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 231

```
dsolve(x*diff(y(x),x) = k+a*x^n+b*y(x)+c*y(x)^2,y(x), singsol=all)
```

$$y(x) = \frac{2\left(\operatorname{BesselY}\left(\frac{\sqrt{b^2-4ck}}{n}+1, \frac{2\sqrt{ac}\,x^{\frac{n}{2}}}{n}\right)c_1 + \operatorname{BesselJ}\left(\frac{\sqrt{b^2-4ck}}{n}+1, \frac{2\sqrt{ac}\,x^{\frac{n}{2}}}{n}\right)\right)\sqrt{ac}\,x^{\frac{n}{2}} - \left(\operatorname{BesselY}\left(\frac{\sqrt{b^2-4ck}}{n}, 2\sqrt{}\right.\right.}{2c\left(\operatorname{BesselY}\left(\frac{\sqrt{b^2-4ck}}{n}, \frac{2\sqrt{ac}\,x^{\frac{n}{2}}}{n}\right)c_1 + \operatorname{BesselJ}\left(\frac{\sqrt{b^2-4ck}}{n}, 2\sqrt{}\right.\right.}$$

✓ Solution by Mathematica

Time used: 0.812 (sec). Leaf size: 806

```
DSolve[x y'[x]==k +a x^n+b y[x]+c y[x]^2,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to -\frac{\sqrt{a}\sqrt{c}x^n\,\mathrm{Gamma}\left(\frac{n+\sqrt{b^2-4ck}}{n}\right)\mathrm{BesselJ}\left(\frac{\sqrt{b^2-4ck}}{n}-1, \frac{2\sqrt{a}\sqrt{c}\sqrt{x^n}}{n}\right) - \sqrt{a}\sqrt{c}x^n\,\mathrm{Gamma}\left(\frac{n+\sqrt{b^2-4ck}}{n}\right)\mathrm{Bess}}{}$$

$$y(x) \to -\frac{-\sqrt{a}\sqrt{c}\sqrt{x^n}\,\mathrm{BesselJ}\left(1-\frac{\sqrt{b^2-4ck}}{n}, \frac{2\sqrt{a}\sqrt{c}\sqrt{x^n}}{n}\right) + b\,\mathrm{BesselJ}\left(-\frac{\sqrt{b^2-4ck}}{n}, \frac{2\sqrt{a}\sqrt{c}\sqrt{x^n}}{n}\right) + \sqrt{a}\sqrt{c}\sqrt{x^n}\,\mathrm{BesselJ}}{2c\,\mathrm{BesselJ}\left(-\frac{\sqrt{b^2-4ck}}{n}, \frac{2\sqrt{a}\sqrt{c}\sqrt{x^n}}{n}\right)}$$

6.25 problem 171

Internal problem ID [3427]

Book: Ordinary differential equations and their solutions. By George Moseley Murphy. 1960
Section: Various 6
Problem number: 171.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_rational, [_Riccati, _special]]`

$$\boxed{xy' + y^2x = -a}$$

✓ Solution by Maple
Time used: 0.0 (sec). Leaf size: 59

```
dsolve(x*diff(y(x),x)+a+x*y(x)^2 = 0,y(x), singsol=all)
```

$$y(x) = \frac{\sqrt{a}\left(\text{BesselJ}\left(0, 2\sqrt{a}\,\sqrt{x}\right)c_1 + \text{BesselY}\left(0, 2\sqrt{a}\,\sqrt{x}\right)\right)}{\sqrt{x}\,\left(c_1\,\text{BesselJ}\left(1, 2\sqrt{a}\,\sqrt{x}\right) + \text{BesselY}\left(1, 2\sqrt{a}\,\sqrt{x}\right)\right)}$$

✓ Solution by Mathematica
Time used: 0.244 (sec). Leaf size: 289

```
DSolve[x y'[x]+a+x y[x]^2==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \rightarrow \frac{2\sqrt{a}\sqrt{x}\,\text{BesselY}\left(0, 2\sqrt{a}\sqrt{x}\right) + 2\,\text{BesselY}\left(1, 2\sqrt{a}\sqrt{x}\right) - 2\sqrt{a}\sqrt{x}\,\text{BesselY}\left(2, 2\sqrt{a}\sqrt{x}\right) - i\sqrt{a}c_1\sqrt{x}\,\text{Bess}}{4x\,\text{BesselY}\left(1, 2\sqrt{a}\sqrt{x}\right) - 2ic_1x\,\text{BesselJ}}$$

$$y(x) \rightarrow \frac{\sqrt{a}\sqrt{x}\,\text{BesselJ}\left(0, 2\sqrt{a}\sqrt{x}\right) + \text{BesselJ}\left(1, 2\sqrt{a}\sqrt{x}\right) - \sqrt{a}\sqrt{x}\,\text{BesselJ}\left(2, 2\sqrt{a}\sqrt{x}\right)}{2x\,\text{BesselJ}\left(1, 2\sqrt{a}\sqrt{x}\right)}$$

6.26 problem 172

Internal problem ID [3428]

Book: Ordinary differential equations and their solutions. By George Moseley Murphy. 1960
Section: Various 6
Problem number: 172.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[[_homogeneous, ‘class G‘], _rational, _Bernoulli]`

$$\boxed{xy' + (-yx + 1)\, y = 0}$$

✓ Solution by Maple
Time used: 0.0 (sec). Leaf size: 16

```
dsolve(x*diff(y(x),x)+(1-x*y(x))*y(x) = 0,y(x), singsol=all)
```

$$y(x) = \frac{1}{(-\ln(x) + c_1)\, x}$$

✓ Solution by Mathematica
Time used: 0.128 (sec). Leaf size: 22

```
DSolve[x y'[x]+(1-x y[x])y[x]==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to \frac{1}{-x\log(x) + c_1 x}$$
$$y(x) \to 0$$

6.27 problem 173

Internal problem ID [3429]

Book: Ordinary differential equations and their solutions. By George Moseley Murphy. 1960
Section: Various 6
Problem number: 173.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[[_homogeneous, 'class D'], _rational, _Bernoulli]`

$$\boxed{xy' - (-yx + 1)\, y = 0}$$

✓ Solution by Maple
Time used: 0.016 (sec). Leaf size: 16

```
dsolve(x*diff(y(x),x) = (1-x*y(x))*y(x),y(x), singsol=all)
```

$$y(x) = \frac{2x}{x^2 + 2c_1}$$

✓ Solution by Mathematica
Time used: 0.134 (sec). Leaf size: 23

```
DSolve[x y'[x]==(1-x y[x])y[x],y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to \frac{2x}{x^2 + 2c_1}$$
$$y(x) \to 0$$

6.28 problem 174

Internal problem ID [3430]

Book: Ordinary differential equations and their solutions. By George Moseley Murphy. 1960
Section: Various 6
Problem number: 174.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[[_homogeneous, 'class D'], _rational, _Bernoulli]`

$$\boxed{xy' - (1 + yx)\, y = 0}$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 16

```
dsolve(x*diff(y(x),x) = (1+x*y(x))*y(x),y(x), singsol=all)
```

$$y(x) = -\frac{2x}{x^2 - 2c_1}$$

✓ Solution by Mathematica

Time used: 0.131 (sec). Leaf size: 23

```
DSolve[x y'[x]==(1+x y[x])y[x],y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to -\frac{2x}{x^2 - 2c_1}$$
$$y(x) \to 0$$

6.29 problem 175

Internal problem ID [3431]

Book: Ordinary differential equations and their solutions. By George Moseley Murphy. 1960
Section: Various 6
Problem number: 175.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_Bernoulli]`

$$\boxed{xy' - a\,x^3(-yx+1)\,y = 0}$$

✓ Solution by Maple
Time used: 0.015 (sec). Leaf size: 95

```
dsolve(x*diff(y(x),x) = a*x^3*(1-x*y(x))*y(x),y(x), singsol=all)
```

$$y(x) = -\frac{3\Gamma\left(\frac{2}{3}\right)\left(-a\,x^3\right)^{\frac{1}{3}} 3^{\frac{2}{3}}}{-3\Gamma\left(\frac{2}{3}\right) \mathrm{e}^{-\frac{a\,x^3}{3}} 3^{\frac{2}{3}} c_1 \left(-a\,x^3\right)^{\frac{1}{3}} - 3\Gamma\left(\frac{2}{3}\right) 3^{\frac{2}{3}} x \left(-a\,x^3\right)^{\frac{1}{3}} + 2\pi\sqrt{3}\,\mathrm{e}^{-\frac{a\,x^3}{3}} x - 3\,\mathrm{e}^{-\frac{a\,x^3}{3}} \Gamma\left(\frac{2}{3}\right) \Gamma\left(\frac{1}{3}, -\frac{a\,x^3}{3}\right) x}$$

✓ Solution by Mathematica
Time used: 0.224 (sec). Leaf size: 66

```
DSolve[x y'[x]==a x^3(1-x y[x])y[x],y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to \frac{e^{\frac{ax^3}{3}} \sqrt[3]{-ax^3}}{\sqrt[3]{3} x \Gamma\left(\frac{4}{3}, -\frac{ax^3}{3}\right) + c_1 \sqrt[3]{-ax^3}}$$
$$y(x) \to 0$$

7 Various 7

7.1 problem 176

Internal problem ID [3432]

Book: Ordinary differential equations and their solutions. By George Moseley Murphy. 1960
Section: Various 7
Problem number: 176.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[[_homogeneous, ‘class D‘], _rational, _Riccati]`

$$x y' - \left(2 x^2 + 1\right) y - y^2 x = x^3$$

✓ Solution by Maple
Time used: 0.0 (sec). Leaf size: 24

```
dsolve(x*diff(y(x),x) = x^3+(2*x^2+1)*y(x)+x*y(x)^2,y(x), singsol=all)
```

$$y(x) = -\frac{x(x^2 + 2c_1 + 2)}{x^2 + 2c_1}$$

✓ Solution by Mathematica
Time used: 0.159 (sec). Leaf size: 34

```
DSolve[x y'[x]==x^3+(1+2 x^2)y[x]+x y[x]^2,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to -\frac{x(x^2 + 2 + 2c_1)}{x^2 + 2c_1}$$
$$y(x) \to -x$$

7.2 problem 177

Internal problem ID [3433]

Book: Ordinary differential equations and their solutions. By George Moseley Murphy. 1960
Section: Various 7
Problem number: 177.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[[_homogeneous, ‘class D‘], _rational, _Bernoulli]`

$$\boxed{xy' - y(2yx + 1) = 0}$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 15

```
dsolve(x*diff(y(x),x) = y(x)*(1+2*x*y(x)),y(x), singsol=all)
```

$$y(x) = \frac{x}{-x^2 + c_1}$$

✓ Solution by Mathematica

Time used: 0.133 (sec). Leaf size: 23

```
DSolve[x y'[x]==y[x](1+2 x y[x]),y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to -\frac{x}{x^2 - c_1}$$
$$y(x) \to 0$$

7.3 problem 178

Internal problem ID [3434]

Book: Ordinary differential equations and their solutions. By George Moseley Murphy. 1960
Section: Various 7
Problem number: 178.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_rational, [_1st_order, '_with_symmetry_[F(x),G(x)]'], _Ricca`

$$xy' + (2 + axy)\,y = -bx$$

✓ Solution by Maple
Time used: 0.016 (sec). Leaf size: 86

```
dsolve(x*diff(y(x),x)+b*x+(2+a*x*y(x))*y(x) = 0,y(x), singsol=all)
```

$$y(x) = \frac{-2abc_1x - i\sqrt{a}\,\mathrm{e}^{-2i\sqrt{a}\,\sqrt{b}\,x}\sqrt{b}\,x - 2ic_1\sqrt{a}\,\sqrt{b} - \mathrm{e}^{-2i\sqrt{a}\,\sqrt{b}\,x}}{xa\left(2ic_1\sqrt{a}\,\sqrt{b} + \mathrm{e}^{-2i\sqrt{a}\,\sqrt{b}\,x}\right)}$$

✓ Solution by Mathematica
Time used: 3.304 (sec). Leaf size: 43

```
DSolve[x y'[x]+b x+(2+a x y[x])y[x]==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to -\frac{1}{ax} - \sqrt{\frac{b}{a}}\tan\left(ax\sqrt{\frac{b}{a}} - c_1\right)$$

7.4 problem 179

Internal problem ID [3435]

Book: Ordinary differential equations and their solutions. By George Moseley Murphy. 1960
Section: Various 7
Problem number: 179.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_rational, _Riccati]`

$$\boxed{xy' + (\mathrm{a2} + \mathrm{a3}\, xy)\, y = -\mathrm{a1}\, x - \mathrm{a0}}$$

✓ Solution by Maple

Time used: 0.031 (sec). Leaf size: 403

```
dsolve(x*diff(y(x),x)+a0+a1*x+(a2+a3*x*y(x))*y(x) = 0,y(x), singsol=all)
```

$y(x) =$

$$-\frac{4\,\mathrm{a1}\left(\mathrm{a1}^3\,\mathrm{a3}\left(\mathrm{a3}\,\mathrm{a0} - \mathrm{a2}\sqrt{-\mathrm{a1}\,\mathrm{a3}}\right)\mathrm{KummerM}\left(\frac{\sqrt{-\mathrm{a1}\,\mathrm{a3}}\;\mathrm{a0} + \mathrm{a1}(\mathrm{a2}+2)}{2\,\mathrm{a1}}, \mathrm{a2}+1, 2x\sqrt{-\mathrm{a1}\,\mathrm{a3}}\right.\right.}{4\,\mathrm{a1}^3\,\mathrm{a3}^2\left(\sqrt{-\mathrm{a1}\,\mathrm{a3}}\;\mathrm{a0} + \mathrm{a1}\,\mathrm{a2}\right)\mathrm{KummerM}\left(\frac{\sqrt{-\mathrm{a1}\,\mathrm{a3}}\;\mathrm{a0} + \mathrm{a1}(\mathrm{a2}+2)}{2\,\mathrm{a1}}, \mathrm{a2}+1, 2x\sqrt{-\mathrm{a1}\,\mathrm{a3}}\right) - c_1\sqrt{-\mathrm{a1}\,\mathrm{a3}}\,(}$$

✓ Solution by Mathematica

Time used: 0.472 (sec). Leaf size: 421

```
DSolve[x y'[x]+a0+a1 x+(a2+a3 x y[x])y[x]==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to -\frac{i\left(\sqrt{\mathrm{a1}}c_1 \operatorname{HypergeometricU}\left(\frac{1}{2}\left(\frac{i\sqrt{\mathrm{a3}}\mathrm{a0}}{\sqrt{\mathrm{a1}}}+\mathrm{a2}\right), \mathrm{a2}, 2i\sqrt{\mathrm{a1}}\sqrt{\mathrm{a3}}x\right)+c_1\left(\sqrt{\mathrm{a1}}\mathrm{a2}+i\mathrm{a0}\sqrt{\mathrm{a3}}\right) \operatorname{Hypergeom}\right.}{\sqrt{\mathrm{a3}}\left(c_1 \operatorname{HypergeometricU}\left(\frac{1}{2}\left(\frac{i\sqrt{\mathrm{a3}}}{\sqrt{\mathrm{a}}}\right.\right.\right.}$$

$$y(x) \to \frac{\frac{\left(\mathrm{a0}\sqrt{\mathrm{a3}}-i\sqrt{\mathrm{a1}}\mathrm{a2}\right) \operatorname{HypergeometricU}\left(\frac{1}{2}\left(\frac{i\sqrt{\mathrm{a3}}\mathrm{a0}}{\sqrt{\mathrm{a1}}}+\mathrm{a2}+2\right),\mathrm{a2}+1,2i\sqrt{\mathrm{a1}}\sqrt{\mathrm{a3}}x\right)}{\operatorname{HypergeometricU}\left(\frac{1}{2}\left(\frac{i\sqrt{\mathrm{a3}}\mathrm{a0}}{\sqrt{\mathrm{a1}}}+\mathrm{a2}\right),\mathrm{a2},2i\sqrt{\mathrm{a1}}\sqrt{\mathrm{a3}}x\right)}-i\sqrt{\mathrm{a1}}}{\sqrt{\mathrm{a3}}}$$

7.5 problem 180

Internal problem ID [3436]

Book: Ordinary differential equations and their solutions. By George Moseley Murphy. 1960
Section: Various 7
Problem number: 180.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_rational, _Riccati]`

$$\boxed{xy' + a\,x^2y^2 + 2y = b}$$

✓ Solution by Maple
Time used: 0.0 (sec). Leaf size: 68

```
dsolve(x*diff(y(x),x)+a*x^2*y(x)^2+2*y(x) = b,y(x), singsol=all)
```

$$y(x) = \frac{\left(-\operatorname{BesselY}\left(1, \sqrt{-ab}\,x\right)c_1 - \operatorname{BesselJ}\left(1, \sqrt{-ab}\,x\right)\right)\sqrt{-ab}}{ax\left(c_1 \operatorname{BesselY}\left(0, \sqrt{-ab}\,x\right) + \operatorname{BesselJ}\left(0, \sqrt{-ab}\,x\right)\right)}$$

✓ Solution by Mathematica
Time used: 0.255 (sec). Leaf size: 158

```
DSolve[x y'[x]+a x^2 y[x]^2+2 y[x]==b,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to \frac{i\sqrt{b}\left(\operatorname{BesselY}\left(1, -i\sqrt{a}\sqrt{b}x\right) - c_1 \operatorname{BesselJ}\left(1, i\sqrt{a}\sqrt{b}x\right)\right)}{\sqrt{a}x\left(\operatorname{BesselY}\left(0, -i\sqrt{a}\sqrt{b}x\right) + c_1 \operatorname{BesselJ}\left(0, i\sqrt{a}\sqrt{b}x\right)\right)}$$

$$y(x) \to -\frac{i\sqrt{b}\operatorname{BesselJ}\left(1, i\sqrt{a}\sqrt{b}x\right)}{\sqrt{a}x \operatorname{BesselJ}\left(0, i\sqrt{a}\sqrt{b}x\right)}$$

7.6 problem 181

Internal problem ID [3437]

Book: Ordinary differential equations and their solutions. By George Moseley Murphy. 1960
Section: Various 7
Problem number: 181.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_rational, _Riccati]`

$$\boxed{xy' + \frac{(-m+n)\,y}{2} + x^n y^2 = -x^m}$$

✓ Solution by Maple

Time used: 0.015 (sec). Leaf size: 39

```
dsolve(x*diff(y(x),x)+x^m+1/2*(n-m)*y(x)+x^n*y(x)^2 = 0,y(x), singsol=all)
```

$$y(x) = -\tan\left(\frac{2x^{\frac{n}{2}+\frac{m}{2}} + c_1(n+m)}{n+m}\right) x^{-\frac{n}{2}+\frac{m}{2}}$$

✓ Solution by Mathematica

Time used: 0.578 (sec). Leaf size: 40

```
DSolve[x y'[x]+x^m+((n-m)/2) y[x]+x^n y[x]^2==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to -x^{\frac{m-n}{2}} \tan\left(\frac{2x^{\frac{m+n}{2}}}{m+n} - c_1\right)$$

7.7 problem 182

Internal problem ID [3438]

Book: Ordinary differential equations and their solutions. By George Moseley Murphy. 1960
Section: Various 7
Problem number: 182.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[[_homogeneous, ‘class G‘], _rational, _Bernoulli]`

$$\boxed{xy' + (a + b\,x^n y)\,y = 0}$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 29

```
dsolve(x*diff(y(x),x)+(a+b*x^n*y(x))*y(x) = 0,y(x), singsol=all)
```

$$y(x) = \frac{a-n}{c_1\,(a-n)\,x^a - b\,x^n}$$

✓ Solution by Mathematica

Time used: 0.284 (sec). Leaf size: 36

```
DSolve[x y'[x]+(a+b x^n y[x])y[x]==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to \frac{a-n}{-bx^n + c_1(a-n)x^a}$$
$$y(x) \to 0$$

7.8 problem 183

Internal problem ID [3439]

Book: Ordinary differential equations and their solutions. By George Moseley Murphy. 1960
Section: Various 7
Problem number: 183.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_rational, _Riccati]`

$$\boxed{xy' + yb + c\,x^n y^2 = a\,x^m}$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 166

```
dsolve(x*diff(y(x),x) = a*x^m-b*y(x)-c*x^n*y(x)^2,y(x), singsol=all)
```

$$y(x) = \frac{x^{-\frac{n}{2}+\frac{m}{2}}\sqrt{-ac}\left(-\mathrm{BesselY}\left(\frac{b+m}{n+m}, \frac{2\sqrt{-ac}\,x^{\frac{n}{2}+\frac{m}{2}}}{n+m}\right)c_1 - \mathrm{BesselJ}\left(\frac{b+m}{n+m}, \frac{2\sqrt{-ac}\,x^{\frac{n}{2}+\frac{m}{2}}}{n+m}\right)\right)}{c\left(\mathrm{BesselY}\left(\frac{b-n}{n+m}, \frac{2\sqrt{-ac}\,x^{\frac{n}{2}+\frac{m}{2}}}{n+m}\right)c_1 + \mathrm{BesselJ}\left(\frac{b-n}{n+m}, \frac{2\sqrt{-ac}\,x^{\frac{n}{2}+\frac{m}{2}}}{n+m}\right)\right)}$$

✓ Solution by Mathematica

Time used: 0.973 (sec). Leaf size: 1549

```
DSolve[x y'[x]==a x^m-b y[x]-c x^n y[x]^2,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to \frac{x^{-n}\left((-1)^{\frac{n}{m+n}}\sqrt{a}\sqrt{c}m(m+n)^{\frac{2n}{m+n}}x^{m+n}\,\mathrm{BesselI}\left(-\frac{b+m}{m+n}, \frac{2\sqrt{a}\sqrt{c}\sqrt{x^{m+n}}}{\sqrt{(m+n)^2}}\right)\mathrm{Gamma}\left(\frac{-b+m+2n}{m+n}\right)\left((m+n)^2\right)^{\frac{b}{m-}}\cdots\right.}{\cdots}$$

$$y(x) \to \frac{x^{-n}\left(\sqrt{a}\sqrt{c}(m+n)\sqrt{x^{m+n}}\,\mathrm{BesselI}\left(\frac{b+m}{m+n}, \frac{2\sqrt{a}\sqrt{c}\sqrt{x^{m+n}}}{\sqrt{(m+n)^2}}\right) + (n-b)\sqrt{(m+n)^2}\,\mathrm{BesselI}\left(\frac{b-n}{m+n}, \frac{2\sqrt{a}\sqrt{c}\sqrt{x^{m+n}}}{\sqrt{(m+n)^2}}\cdots\right.\right.}{2c\sqrt{(m+n)^2}\,\mathrm{BesselI}\left(\frac{b-n}{m+n}, \frac{2\sqrt{a}\sqrt{c}\sqrt{x^{m+n}}}{\sqrt{(m+n)^2}}\right)}$$

7.9 problem 184

Internal problem ID [3440]

Book: Ordinary differential equations and their solutions. By George Moseley Murphy. 1960
Section: Various 7
Problem number: 184.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[[_1st_order, _with_linear_symmetries], _rational, _Riccati]`

$$xy' + y - a\,x^n(-y+x)^2 = 2x$$

✓ Solution by Maple
Time used: 0.016 (sec). Leaf size: 34

```
dsolve(x*diff(y(x),x) = 2*x-y(x)+a*x^n*(x-y(x))^2,y(x), singsol=all)
```

$$y(x) = \frac{ax\,x^n - c_1x^2 - n + 1}{a\,x^n - c_1x}$$

✓ Solution by Mathematica
Time used: 1.064 (sec). Leaf size: 164

```
DSolve[x y'[x]==2 x -y[x]+a x^n(x-y[x])^2,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to \frac{x^{-n}\left(2ax^{n+\sqrt{(n-1)^2}+1} + 2ac_1\sqrt{(n-1)^2}x^{n+1} - \left(n+\sqrt{(n-1)^2}-1\right)x^{\sqrt{(n-1)^2}} - c_1\left(-n+\sqrt{(n-1)^2}+\right.\right.}{2a\left(x^{\sqrt{(n-1)^2}} + c_1\sqrt{(n-1)^2}\right)}$$

$$y(x) \to \frac{x^{-n}\left(2ax^{n+1} - n + \sqrt{(n-1)^2} + 1\right)}{2a}$$

7.10 problem 185

Internal problem ID [3441]

Book: Ordinary differential equations and their solutions. By George Moseley Murphy. 1960
Section: Various 7
Problem number: 185.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_Bernoulli]`

$$\boxed{xy' + (1 - ay\ln(x))\,y = 0}$$

✓ Solution by Maple
Time used: 0.0 (sec). Leaf size: 15

```
dsolve(x*diff(y(x),x)+(1-a*y(x)*ln(x))*y(x) = 0,y(x), singsol=all)
```

$$y(x) = \frac{1}{a\ln(x) + c_1x + a}$$

✓ Solution by Mathematica
Time used: 0.154 (sec). Leaf size: 22

```
DSolve[x y'[x]+(1-a y[x] Log[x])y[x]==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to \frac{1}{a\log(x) + a + c_1x}$$
$$y(x) \to 0$$

7.11 problem 186

Internal problem ID [3442]

Book: Ordinary differential equations and their solutions. By George Moseley Murphy. 1960
Section: Various 7
Problem number: 186.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[[_homogeneous, ‘class D‘], _Riccati]`

$$\boxed{xy' - y - (x^2 - y^2) f(x) = 0}$$

✓ Solution by Maple

Time used: 0.031 (sec). Leaf size: 13

```
dsolve(x*diff(y(x),x) = y(x)+(x^2-y(x)^2)*f(x),y(x), singsol=all)
```

$$y(x) = \tanh\left(\int f(x)\,dx + c_1\right)x$$

✓ Solution by Mathematica

Time used: 0.384 (sec). Leaf size: 65

```
DSolve[x y'[x]==y[x]+(x^2-y[x]^2)f[x],y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to \frac{x - x\exp\left(2\left(\int_1^x -f(K[1])dK[1] + c_1\right)\right)}{1 + \exp\left(2\left(\int_1^x -f(K[1])dK[1] + c_1\right)\right)}$$
$$y(x) \to -x$$
$$y(x) \to x$$

7.12 problem 187

Internal problem ID [3443]

Book: Ordinary differential equations and their solutions. By George Moseley Murphy. 1960
Section: Various 7
Problem number: 187.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_separable]`

$$xy' - y(y^2+1) = 0$$

✓ Solution by Maple
Time used: 0.0 (sec). Leaf size: 30

```
dsolve(x*diff(y(x),x) = y(x)*(1+y(x)^2),y(x), singsol=all)
```

$$y(x) = \frac{x}{\sqrt{-x^2+c_1}}$$
$$y(x) = -\frac{x}{\sqrt{-x^2+c_1}}$$

✓ Solution by Mathematica
Time used: 0.681 (sec). Leaf size: 110

```
DSolve[x y'[x]==y[x](1+y[x]^2),y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to -\frac{ie^{c_1}x}{\sqrt{-1+e^{2c_1}x^2}}$$
$$y(x) \to \frac{ie^{c_1}x}{\sqrt{-1+e^{2c_1}x^2}}$$
$$y(x) \to 0$$
$$y(x) \to -i$$
$$y(x) \to i$$
$$y(x) \to -\frac{ix}{\sqrt{x^2}}$$
$$y(x) \to \frac{ix}{\sqrt{x^2}}$$

7.13 problem 188

Internal problem ID [3444]

Book: Ordinary differential equations and their solutions. By George Moseley Murphy. 1960
Section: Various 7
Problem number: 188.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[[_homogeneous, ‘class G‘], _rational, _Bernoulli]`

$$\boxed{xy' + y(1 - y^2 x) = 0}$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 27

```
dsolve(x*diff(y(x),x)+(1-x*y(x)^2)*y(x) = 0,y(x), singsol=all)
```

$$y(x) = \frac{1}{\sqrt{x(c_1 x + 2)}}$$
$$y(x) = -\frac{1}{\sqrt{x(c_1 x + 2)}}$$

✓ Solution by Mathematica

Time used: 0.423 (sec). Leaf size: 40

```
DSolve[x y'[x]+(1-x y[x]^2)y[x]==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to -\frac{1}{\sqrt{x(2 + c_1 x)}}$$
$$y(x) \to \frac{1}{\sqrt{x(2 + c_1 x)}}$$
$$y(x) \to 0$$

7.14 problem 189

Internal problem ID [3445]

Book: Ordinary differential equations and their solutions. By George Moseley Murphy. 1960
Section: Various 7
Problem number: 189.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_rational, _Bernoulli]`

$$\boxed{xy' + y - a(x^2+1)\, y^3 = 0}$$

✓ Solution by Maple
Time used: 0.0 (sec). Leaf size: 43

```
dsolve(x*diff(y(x),x)+y(x) = a*(x^2+1)*y(x)^3,y(x), singsol=all)
```

$$y(x) = \frac{1}{\sqrt{-2x^2 \ln(x)\, a + c_1 x^2 + a}}$$
$$y(x) = -\frac{1}{\sqrt{-2x^2 \ln(x)\, a + c_1 x^2 + a}}$$

✓ Solution by Mathematica
Time used: 0.545 (sec). Leaf size: 56

```
DSolve[x y'[x]+y[x]==a(1+x^2)y[x]^3,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to -\frac{1}{\sqrt{-2ax^2 \log(x) + a + c_1 x^2}}$$
$$y(x) \to \frac{1}{\sqrt{-2ax^2 \log(x) + a + c_1 x^2}}$$
$$y(x) \to 0$$

7.15 problem 190

Internal problem ID [3446]

Book: Ordinary differential equations and their solutions. By George Moseley Murphy. 1960
Section: Various 7
Problem number: 190.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_rational, _Bernoulli]`

$$xy' - ya - b(x^2+1)\,y^3 = 0$$

✓ Solution by Maple
Time used: 0.031 (sec). Leaf size: 150

```
dsolve(x*diff(y(x),x) = a*y(x)+b*(x^2+1)*y(x)^3,y(x), singsol=all)
```

$$y(x) = -\frac{\sqrt{x^{2a}\left(-ab\,x^{2+2a}+(a+1)\left(c_1a-b\,x^{2a}\right)\right)a\,(a+1)}}{-ab\,x^{2+2a}+(a+1)\left(c_1a-b\,x^{2a}\right)}$$
$$y(x) = \frac{\sqrt{x^{2a}\left(-ab\,x^{2+2a}+(a+1)\left(c_1a-b\,x^{2a}\right)\right)a\,(a+1)}}{-ab\,x^{2+2a}+(a+1)\left(c_1a-b\,x^{2a}\right)}$$

✓ Solution by Mathematica
Time used: 3.908 (sec). Leaf size: 108

```
DSolve[x y'[x]==a y[x]+b(1+x^2)y[x]^3,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to -\frac{i\sqrt{a}\sqrt{a+1}x^a}{\sqrt{bx^{2a}\left(ax^2+a+1\right)-a(a+1)c_1}}$$
$$y(x) \to \frac{i\sqrt{a}\sqrt{a+1}x^a}{\sqrt{bx^{2a}\left(ax^2+a+1\right)-a(a+1)c_1}}$$
$$y(x) \to 0$$

7.16 problem 191

Internal problem ID [3447]

Book: Ordinary differential equations and their solutions. By George Moseley Murphy. 1960
Section: Various 7
Problem number: 191.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[[_homogeneous, ‘class G‘], _rational, _Bernoulli]`

$$\boxed{xy' + 2y - a\,x^{2k}y^k = 0}$$

✓ Solution by Maple
Time used: 0.016 (sec). Leaf size: 47

```
dsolve(x*diff(y(x),x)+2*y(x) = a*x^(2*k)*y(x)^k,y(x), singsol=all)
```

$$y(x) = \left(\frac{-a(k-1)\,x^2 + 2c_1}{x^2}\right)^{-\frac{1}{k-1}} x^{-\frac{2k}{k-1}} 2^{\frac{1}{k-1}}$$

✓ Solution by Mathematica
Time used: 16.14 (sec). Leaf size: 45

```
DSolve[x y'[x]+2 y[x]==a x^(2 k)y[x]^k,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to \left(\frac{1}{2}ax^{2k} - \frac{1}{2}akx^{2k} + c_1x^{2k-2}\right)^{\frac{1}{1-k}}$$

7.17 problem 192

Internal problem ID [3448]

Book: Ordinary differential equations and their solutions. By George Moseley Murphy. 1960
Section: Various 7
Problem number: 192.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_separable]`

$$\boxed{xy' - 4y + 4\sqrt{y} = 0}$$

✓ Solution by Maple
Time used: 0.0 (sec). Leaf size: 15

```
dsolve(x*diff(y(x),x) = 4*y(x)-4*sqrt(y(x)),y(x), singsol=all)
```

$$-c_1 x^2 + \sqrt{y(x)} - 1 = 0$$

✓ Solution by Mathematica
Time used: 0.186 (sec). Leaf size: 31

```
DSolve[x y'[x]==4(y[x]-Sqrt[y[x]]),y[x],x,IncludeSingularSolutions -> True]
```

$$\begin{aligned} y(x) &\to \left(1 + e^{\frac{c_1}{2}} x^2\right)^2 \\ y(x) &\to 0 \\ y(x) &\to 1 \end{aligned}$$

7.18 problem 193

Internal problem ID [3449]

Book: Ordinary differential equations and their solutions. By George Moseley Murphy. 1960
Section: Various 7
Problem number: 193.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_separable]`

$$\boxed{xy' + 2y - \sqrt{y^2 + 1} = 0}$$

✓ Solution by Maple
Time used: 0.0 (sec). Leaf size: 27

```
dsolve(x*diff(y(x),x)+2*y(x) = sqrt(1+y(x)^2),y(x), singsol=all)
```

$$\ln(x) - \left(\int^{y(x)} \frac{1}{-2\_a + \sqrt{\_a^2 + 1}} d\_a\right) + c_1 = 0$$

✓ Solution by Mathematica
Time used: 60.221 (sec). Leaf size: 2509

```
DSolve[x y'[x]+2 y[x]==Sqrt[1+y[x]^2],y[x],x,IncludeSingularSolutions -> True]
```

Too large to display

7.19 problem 194

Internal problem ID [3450]

Book: Ordinary differential equations and their solutions. By George Moseley Murphy. 1960
Section: Various 7
Problem number: 194.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[[_homogeneous, ‘class A‘], _rational, _dAlembert]`

$$\boxed{xy' - y - \sqrt{y^2 + x^2} = 0}$$

✓ Solution by Maple
Time used: 0.0 (sec). Leaf size: 26

```
dsolve(x*diff(y(x),x) = y(x)+sqrt(x^2+y(x)^2),y(x), singsol=all)
```

$$\frac{-c_1x^2 + \sqrt{x^2 + y(x)^2} + y(x)}{x^2} = 0$$

✓ Solution by Mathematica
Time used: 0.352 (sec). Leaf size: 27

```
DSolve[x y'[x]==y[x]+Sqrt[x^2+y[x]^2],y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to \frac{1}{2}e^{-c_1}\left(-1 + e^{2c_1}x^2\right)$$

7.20 problem 195

Internal problem ID [3451]

Book: Ordinary differential equations and their solutions. By George Moseley Murphy. 1960
Section: Various 7
Problem number: 195.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[[_homogeneous, ‘class A‘], _rational, _dAlembert]`

$$\boxed{xy' - y - \sqrt{x^2 - y^2} = 0}$$

✓ Solution by Maple
Time used: 0.016 (sec). Leaf size: 27

```
dsolve(x*diff(y(x),x) = y(x)+sqrt(x^2-y(x)^2),y(x), singsol=all)
```

$$-\arctan\left(\frac{y(x)}{\sqrt{x^2 - y(x)^2}}\right) + \ln(x) - c_1 = 0$$

✓ Solution by Mathematica
Time used: 0.238 (sec). Leaf size: 18

```
DSolve[x y'[x]==y[x]+Sqrt[x^2-y[x]^2],y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to -x\cosh(i\log(x) + c_1)$$

7.21 problem 196

Internal problem ID [3452]

Book: Ordinary differential equations and their solutions. By George Moseley Murphy. 1960
Section: Various 7
Problem number: 196.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[[_1st_order, '_with_symmetry_[F(x),G(x)*y+H(x)]']]`

$$\boxed{xy' - y - x\sqrt{y^2 + x^2} = 0}$$

✓ Solution by Maple
Time used: 0.813 (sec). Leaf size: 28

```
dsolve(x*diff(y(x),x) = y(x)+x*sqrt(x^2+y(x)^2),y(x), singsol=all)
```

$$\ln\left(y(x) + \sqrt{x^2 + y(x)^2}\right) - x - \ln(x) - c_1 = 0$$

✓ Solution by Mathematica
Time used: 0.282 (sec). Leaf size: 30

```
DSolve[x y'[x]==y[x]+x Sqrt[x^2+y[x]^2],y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to \frac{1}{2}xe^{-x-c_1}\left(-1 + e^{2(x+c_1)}\right)$$

7.22 problem 197

Internal problem ID [3453]

Book: Ordinary differential equations and their solutions. By George Moseley Murphy. 1960
Section: Various 7
Problem number: 197.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[[_1st_order, '_with_symmetry_[F(x),G(x)*y+H(x)]']]`

$$\boxed{xy' - y + x(-y + x)\sqrt{y^2 + x^2} = 0}$$

✓ Solution by Maple

Time used: 0.078 (sec). Leaf size: 50

```
dsolve(x*diff(y(x),x) = y(x)-x*(x-y(x))*sqrt(x^2+y(x)^2),y(x), singsol=all)
```

$$\ln(2) + \ln\left(\frac{x\left(\sqrt{2x^2 + 2y(x)^2} + y(x) + x\right)}{-x + y(x)}\right) + \frac{\sqrt{2}\,x^2}{2} - \ln(x) - c_1 = 0$$

✓ Solution by Mathematica

Time used: 1.529 (sec). Leaf size: 84

```
DSolve[x y'[x]==y[x]-x(x-y[x])Sqrt[x^2+y[x]^2],y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to \frac{x\tanh\left(\frac{x^2+2c_1}{2\sqrt{2}}\right)\left(2 + \sqrt{2}\tanh\left(\frac{x^2+2c_1}{2\sqrt{2}}\right)\right)}{\sqrt{2} + 2\tanh\left(\frac{x^2+2c_1}{2\sqrt{2}}\right)}$$

$$y(x) \to x$$

7.23 problem 198

Internal problem ID [3454]

Book: Ordinary differential equations and their solutions. By George Moseley Murphy. 1960
Section: Various 7
Problem number: 198.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[[_homogeneous, 'class A'], _dAlembert]`

$$\boxed{xy' - y - a\sqrt{y^2 + b^2x^2} = 0}$$

✓ Solution by Maple
Time used: 0.0 (sec). Leaf size: 39

```
dsolve(x*diff(y(x),x) = y(x)+a*sqrt(y(x)^2+b^2*x^2),y(x), singsol=all)
```

$$x^{-1-a}y(x) + x^{-1-a}\sqrt{y(x)^2 + b^2x^2} - c_1 = 0$$

✓ Solution by Mathematica
Time used: 0.402 (sec). Leaf size: 73

```
DSolve[x y'[x]==y[x]+a Sqrt[y[x]^2+b^2 x^2],y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to \frac{1}{2}be^{-c_1}\left(x^{1-a} - e^{2c_1}x^{a+1}\right)$$
$$y(x) \to \frac{1}{2}be^{-c_1}x^{1-a}\left(-1 + e^{2c_1}x^{2a}\right)$$

7.24 problem 199

Internal problem ID [3455]

Book: Ordinary differential equations and their solutions. By George Moseley Murphy. 1960
Section: Various 7
Problem number: 199.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[‘y=_G(x,y’)‘]`

$$\boxed{xy' + (\sin(y) - 3\cos(y)\,x^2)\cos(y) = 0}$$

✓ Solution by Maple

Time used: 0.016 (sec). Leaf size: 16

```
dsolve(x*diff(y(x),x)+(sin(y(x))-3*x^2*cos(y(x)))*cos(y(x)) = 0,y(x), singsol=all)
```

$$y(x) = \arctan\left(\frac{x^3 + 2c_1}{x}\right)$$

✓ Solution by Mathematica

Time used: 1.87 (sec). Leaf size: 53

```
DSolve[x y'[x]+(Sin[y[x]]-3 x^2 Cos[y[x]]) Cos[y[x]]==0,y[x],x,IncludeSingularSolutions -> T
```

$$y(x) \to \arctan\left(x^2 + \frac{c_1}{2x}\right)$$
$$y(x) \to -\frac{1}{2}\pi\sqrt{\frac{1}{x^2}}x$$
$$y(x) \to \frac{1}{2}\pi\sqrt{\frac{1}{x^2}}x$$

7.25 problem 200

Internal problem ID [3456]

Book: Ordinary differential equations and their solutions. By George Moseley Murphy. 1960
Section: Various 7
Problem number: 200.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[[_homogeneous, ‘class A‘], _dAlembert]`

$$\boxed{xy' - y + x\cos\left(\frac{y}{x}\right) = -x}$$

✓ Solution by Maple
Time used: 0.016 (sec). Leaf size: 12

```
dsolve(x*diff(y(x),x)+x-y(x)+x*cos(y(x)/x) = 0,y(x), singsol=all)
```

$$y(x) = -2\arctan(\ln(x) + c_1)\,x$$

✓ Solution by Mathematica
Time used: 0.435 (sec). Leaf size: 31

```
DSolve[x y'[x]+x -y[x]+x Cos[y[x]/x]==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to 2x\arctan(-\log(x) + c_1)$$
$$y(x) \to -\pi x$$
$$y(x) \to \pi x$$

7.26 problem 201

Internal problem ID [3457]

Book: Ordinary differential equations and their solutions. By George Moseley Murphy. 1960
Section: Various 7
Problem number: 201.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[[_homogeneous, ‘class A‘], _dAlembert]`

$$\boxed{xy' + x\cos\left(\frac{y}{x}\right)^2 - y = 0}$$

✓ Solution by Maple

Time used: 0.016 (sec). Leaf size: 12

```
dsolve(x*diff(y(x),x) = y(x)-x*cos(y(x)/x)^2,y(x), singsol=all)
```

$$y(x) = -\arctan\left(\ln\left(x\right) + c_1\right) x$$

✓ Solution by Mathematica

Time used: 0.492 (sec). Leaf size: 37

```
DSolve[x y'[x]==y[x]-x Cos[y[x]/x]^2,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to x\arctan(-\log(x) + 2c_1)$$
$$y(x) \to -\frac{\pi x}{2}$$
$$y(x) \to \frac{\pi x}{2}$$

7.27 problem 202

Internal problem ID [3458]

Book: Ordinary differential equations and their solutions. By George Moseley Murphy. 1960
Section: Various 7
Problem number: 202.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_separable]`

$$\boxed{xy' - (-2x^2 + 1)\cot(y)^2 = 0}$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 22

```
dsolve(x*diff(y(x),x) = (-2*x^2+1)*cot(y(x))^2,y(x), singsol=all)
```

$$x^2 + \frac{\pi}{2} - \ln(x) - y(x) + c_1 + \tan(y(x)) = 0$$

✓ Solution by Mathematica

Time used: 0.555 (sec). Leaf size: 55

```
DSolve[x y'[x]==(1-2 x^2)Cot[y[x]]^2,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to \text{InverseFunction}\left[\frac{1}{2}(\tan(\#1) - \arctan(\tan(\#1)))\&\right]\left[-\frac{x^2}{2} + \frac{\log(x)}{2} + c_1\right]$$
$$y(x) \to -\frac{\pi}{2}$$
$$y(x) \to \frac{\pi}{2}$$

7.28 problem 203

Internal problem ID [3459]

Book: Ordinary differential equations and their solutions. By George Moseley Murphy. 1960
Section: Various 7
Problem number: 203.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_separable]`

$$\boxed{xy' - y + \cot(y)^2 = 0}$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 22

```
dsolve(x*diff(y(x),x) = y(x)-cot(y(x))^2,y(x), singsol=all)
```

$$\ln(x) + c_1 + \int^{y(x)} \frac{1}{\cot(\_a)^2 - \_a} d\_a = 0$$

✓ Solution by Mathematica

Time used: 3.204 (sec). Leaf size: 49

```
DSolve[x y'[x]==y[x]-x Cot[y[x]]^2/x,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to \text{InverseFunction}\left[\int_1^{\#1} \frac{\cos(2K[1]) - 1}{K[1]\cos(2K[1]) + \cos(2K[1]) - K[1] + 1} dK[1]\&\right][\log(x) + c_1]$$

7.29 problem 204

Internal problem ID [3460]

Book: Ordinary differential equations and their solutions. By George Moseley Murphy. 1960
Section: Various 7
Problem number: 204.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[[_1st_order, '_with_symmetry_[F(x),G(x)*y+H(x)]']]`

$$\boxed{xy' + y + 2x\sec(yx) = 0}$$

✓ Solution by Maple

Time used: 0.016 (sec). Leaf size: 16

```
dsolve(x*diff(y(x),x)+y(x)+2*x*sec(x*y(x)) = 0,y(x), singsol=all)
```

$$y(x) = \frac{\arcsin(-x^2 + c_1)}{x}$$

✓ Solution by Mathematica

Time used: 0.465 (sec). Leaf size: 19

```
DSolve[x y'[x]+y[x]+2 x Sec[x y[x]]==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to -\frac{\arcsin(x^2 - c_1)}{x}$$

7.30 problem 205

Internal problem ID [3461]

Book: Ordinary differential equations and their solutions. By George Moseley Murphy. 1960
Section: Various 7
Problem number: 205.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[[_homogeneous, ‘class A‘], _dAlembert]`

$$xy' - y + x \sec\left(\frac{y}{x}\right) = 0$$

✓ Solution by Maple
Time used: 0.0 (sec). Leaf size: 12

```
dsolve(x*diff(y(x),x)-y(x)+x*sec(y(x)/x) = 0,y(x), singsol=all)
```

$$y(x) = -\arcsin\left(\ln\left(x\right) + c_1\right) x$$

✓ Solution by Mathematica
Time used: 0.408 (sec). Leaf size: 15

```
DSolve[x y'[x]-y[x]+x Sec[y[x]/x]==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to x \arcsin(-\log(x) + c_1)$$

8 Various 8

8.1 problem 206

Internal problem ID [3462]

Book: Ordinary differential equations and their solutions. By George Moseley Murphy. 1960
Section: Various 8
Problem number: 206.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[[_homogeneous, ‘class A‘], _dAlembert]`

$$xy' - y - x\sec\left(\frac{y}{x}\right)^2 = 0$$

✓ Solution by Maple

Time used: 0.062 (sec). Leaf size: 31

```
dsolve(x*diff(y(x),x) = y(x)+x*sec(y(x)/x)^2,y(x), singsol=all)
```

$$\frac{x\sin\left(\frac{2y(x)}{x}\right) + 2y(x)}{4x} - \ln(x) - c_1 = 0$$

✓ Solution by Mathematica

Time used: 0.255 (sec). Leaf size: 31

```
DSolve[x y'[x]==y[x]+x Sec[y[x]/x]^2,y[x],x,IncludeSingularSolutions -> True]
```

$$\text{Solve}\left[\frac{y(x)}{2x} + \frac{1}{4}\sin\left(\frac{2y(x)}{x}\right) = \log(x) + c_1, y(x)\right]$$

8.2 problem 207

Internal problem ID [3463]

Book: Ordinary differential equations and their solutions. By George Moseley Murphy. 1960
Section: Various 8
Problem number: 207.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[‘y=_G(x,y’)‘]`

$$\boxed{xy' - \sin(-y + x) = 0}$$

✗ Solution by Maple

```
dsolve(x*diff(y(x),x) = sin(x-y(x)),y(x), singsol=all)
```

No solution found

✗ Solution by Mathematica

Time used: 0.0 (sec). Leaf size: 0

```
DSolve[x y'[x]==Sin[x-y[x]],y[x],x,IncludeSingularSolutions -> True]
```

Not solved

8.3 problem 208

Internal problem ID [3464]

Book: Ordinary differential equations and their solutions. By George Moseley Murphy. 1960
Section: Various 8
Problem number: 208.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[[_homogeneous, ‘class A‘], _dAlembert]`

$$\boxed{xy' - y - \sin\left(\frac{y}{x}\right) x = 0}$$

✓ Solution by Maple

Time used: 0.032 (sec). Leaf size: 44

```
dsolve(x*diff(y(x),x) = y(x)+x*sin(y(x)/x),y(x), singsol=all)
```

$$y(x) = \arctan\left(\frac{2xc_1}{c_1^2x^2+1}, \frac{-c_1^2x^2+1}{c_1^2x^2+1}\right) x$$

✓ Solution by Mathematica

Time used: 0.325 (sec). Leaf size: 52

```
DSolve[x y'[x]==y[x]+x Sin[y[x]/x],y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to -x \arccos(-\tanh(\log(x) + c_1))$$
$$y(x) \to x \arccos(-\tanh(\log(x) + c_1))$$
$$y(x) \to 0$$
$$y(x) \to -\pi x$$
$$y(x) \to \pi x$$

8.4 problem 209

Internal problem ID [3465]

Book: Ordinary differential equations and their solutions. By George Moseley Murphy. 1960
Section: Various 8
Problem number: 209.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_separable]`

$$\boxed{xy' + \tan(y) = 0}$$

✓ Solution by Maple

Time used: 0.016 (sec). Leaf size: 12

```
dsolve(x*diff(y(x),x)+tan(y(x)) = 0,y(x), singsol=all)
```

$$y(x) = \arcsin\left(\frac{1}{c_1 x}\right)$$

✓ Solution by Mathematica

Time used: 14.286 (sec). Leaf size: 19

```
DSolve[x y'[x]+Tan[y[x]]==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to \arcsin\left(\frac{e^{c_1}}{x}\right)$$
$$y(x) \to 0$$

8.5 problem 210

Internal problem ID [3466]

Book: Ordinary differential equations and their solutions. By George Moseley Murphy. 1960
Section: Various 8
Problem number: 210.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[[_1st_order, _with_linear_symmetries]]`

$$\boxed{xy' + \tan(y + x) = -x}$$

✓ Solution by Maple

Time used: 0.531 (sec). Leaf size: 117

```
dsolve(x*diff(y(x),x)+x+tan(x+y(x)) = 0,y(x), singsol=all)
```

$$y(x) = \arctan\left(\frac{c_1}{x}, \frac{\sqrt{-c_1^2 + x^2}}{x}\right) - x$$
$$y(x) = \arctan\left(\frac{c_1}{x}, -\frac{\sqrt{-c_1^2 + x^2}}{x}\right) - x$$
$$y(x) = \arctan\left(-\frac{c_1}{x}, \frac{\sqrt{-c_1^2 + x^2}}{x}\right) - x$$
$$y(x) = \arctan\left(-\frac{c_1}{x}, -\frac{\sqrt{-c_1^2 + x^2}}{x}\right) - x$$

✓ Solution by Mathematica

Time used: 4.917 (sec). Leaf size: 16

```
DSolve[x y'[x]+x+Tan[x+y[x]]==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to -x + \arcsin\left(\frac{c_1}{x}\right)$$

8.6 problem 211

Internal problem ID [3467]

Book: Ordinary differential equations and their solutions. By George Moseley Murphy. 1960
Section: Various 8
Problem number: 211.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[[_homogeneous, ‘class A‘], _dAlembert]`

$$x y' - y + x \tan\left(\frac{y}{x}\right) = 0$$

✓ Solution by Maple
Time used: 0.016 (sec). Leaf size: 14

```
dsolve(x*diff(y(x),x) = y(x)-x*tan(y(x)/x),y(x), singsol=all)
```

$$y(x) = x \arcsin\left(\frac{1}{c_1 x}\right)$$

✓ Solution by Mathematica
Time used: 14.133 (sec). Leaf size: 21

```
DSolve[x y'[x]==y[x]-x Tan[y[x]/x],y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to x \arcsin\left(\frac{e^{c_1}}{x}\right)$$
$$y(x) \to 0$$

8.7 problem 212

Internal problem ID [3468]

Book: Ordinary differential equations and their solutions. By George Moseley Murphy. 1960
Section: Various 8
Problem number: 212.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[‘y=_G(x,y’)‘]`

$$\boxed{xy' - (y^2 + 1)\,(x^2 + \arctan(y)) = 0}$$

✓ Solution by Maple
Time used: 0.016 (sec). Leaf size: 10

```
dsolve(x*diff(y(x),x) = (1+y(x)^2)*(x^2+arctan(y(x))),y(x), singsol=all)
```

$$y(x) = \tan((c_1 + x)\,x)$$

✓ Solution by Mathematica
Time used: 0.315 (sec). Leaf size: 14

```
DSolve[x y'[x]==(1+y[x]^2)(x^2+ArcTan[y[x]]),y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to \tan(x(x + 2c_1))$$

8.8 problem 213

Internal problem ID [3469]

Book: Ordinary differential equations and their solutions. By George Moseley Murphy. 1960
Section: Various 8
Problem number: 213.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[[_homogeneous, 'class A'], _dAlembert]`

$$\boxed{xy' - y - x\,\mathrm{e}^{\frac{y}{x}} = 0}$$

✓ Solution by Maple

Time used: 0.015 (sec). Leaf size: 15

```
dsolve(x*diff(y(x),x) = y(x)+x*exp(y(x)/x),y(x), singsol=all)
```

$$y(x) = \ln\left(-\frac{1}{\ln(x) + c_1}\right)x$$

✓ Solution by Mathematica

Time used: 0.322 (sec). Leaf size: 18

```
DSolve[x y'[x]==y[x]+x Exp[y[x]/x],y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to -x\log(-\log(x) - c_1)$$

8.9 problem 214

Internal problem ID [3470]

Book: Ordinary differential equations and their solutions. By George Moseley Murphy. 1960
Section: Various 8
Problem number: 214.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[[_homogeneous, ‘class A‘], _dAlembert]`

$$\boxed{xy' - y - x\,\mathrm{e}^{\frac{y}{x}} = x}$$

✓ Solution by Maple
Time used: 0.016 (sec). Leaf size: 20

```
dsolve(x*diff(y(x),x) = x+y(x)+x*exp(y(x)/x),y(x), singsol=all)
```

$$y(x) = \left(\ln\left(-\frac{x}{x\,\mathrm{e}^{c_1} - 1}\right) + c_1\right) x$$

✓ Solution by Mathematica
Time used: 4.516 (sec). Leaf size: 38

```
DSolve[x y'[x]==x+y[x]+x Exp[y[x]/x],y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to x\log\left(\frac{1}{2}\left(-1 + \tanh\left(\frac{1}{2}(-\log(x) - c_1)\right)\right)\right)$$
$$y(x) \to i\pi x$$

8.10 problem 215

Internal problem ID [3471]

Book: Ordinary differential equations and their solutions. By George Moseley Murphy. 1960
Section: Various 8
Problem number: 215.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_separable]`

$$\boxed{xy' - y\ln(y) = 0}$$

✓ Solution by Maple
Time used: 0.015 (sec). Leaf size: 8

```
dsolve(x*diff(y(x),x) = y(x)*ln(y(x)),y(x), singsol=all)
```

$$y(x) = e^{c_1 x}$$

✓ Solution by Mathematica
Time used: 0.18 (sec). Leaf size: 18

```
DSolve[x y'[x]==y[x] Log[y[x]],y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to e^{e^{c_1} x}$$
$$y(x) \to 1$$

8.11 problem 216

Internal problem ID [3472]

Book: Ordinary differential equations and their solutions. By George Moseley Murphy. 1960
Section: Various 8
Problem number: 216.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[[_homogeneous, 'class A'], _dAlembert]`

$$\boxed{xy' - (\ln(x) - \ln(y) + 1)\, y = 0}$$

✓ Solution by Maple

Time used: 0.015 (sec). Leaf size: 12

```
dsolve(x*diff(y(x),x) = (1+ln(x)-ln(y(x)))*y(x),y(x), singsol=all)
```

$$y(x) = x\,\mathrm{e}^{\frac{c_1}{x}}$$

✓ Solution by Mathematica

Time used: 0.196 (sec). Leaf size: 22

```
DSolve[x y'[x]==(1+Log[x]-Log[y[x]])y[x],y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to x e^{\frac{e^{c_1}}{x}}$$
$$y(x) \to x$$

8.12 problem 217

Internal problem ID [3473]

Book: Ordinary differential equations and their solutions. By George Moseley Murphy. 1960
Section: Various 8
Problem number: 217.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[[_homogeneous, 'class G']]`

$$\boxed{xy' + (1 - \ln(x) - \ln(y))\, y = 0}$$

✓ Solution by Maple
Time used: 0.0 (sec). Leaf size: 12

```
dsolve(x*diff(y(x),x)+(1-ln(x)-ln(y(x)))*y(x) = 0,y(x), singsol=all)
```

$$y(x) = \frac{e^{c_1 x}}{x}$$

✓ Solution by Mathematica
Time used: 0.232 (sec). Leaf size: 26

```
DSolve[x y'[x]+(1-Log[x]-Log[y[x]])y[x]==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to \frac{e^{e^{-c_1} x}}{x}$$
$$y(x) \to \frac{1}{x}$$

8.13 problem 218

Internal problem ID [3474]

Book: Ordinary differential equations and their solutions. By George Moseley Murphy. 1960
Section: Various 8
Problem number: 218.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[[_homogeneous, ‘class A‘], _dAlembert]`

$$\boxed{xy' - y + 2x \tanh\left(\frac{y}{x}\right) = 0}$$

✓ Solution by Maple
Time used: 0.985 (sec). Leaf size: 34

```
dsolve(x*diff(y(x),x) = y(x)-2*x*tanh(y(x)/x),y(x), singsol=all)
```

$$y(x) = \operatorname{arctanh}\left(\frac{1}{\sqrt{-c_1 x^4 + 1}}\right) x$$
$$y(x) = -\operatorname{arctanh}\left(\frac{1}{\sqrt{-c_1 x^4 + 1}}\right) x$$

✓ Solution by Mathematica
Time used: 11.37 (sec). Leaf size: 21

```
DSolve[x y'[x]==y[x]-2 x Tanh[y[x]/x],y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to x \operatorname{arcsinh}\left(\frac{e^{c_1}}{x^2}\right)$$
$$y(x) \to 0$$

8.14 problem 219

Internal problem ID [3475]

Book: Ordinary differential equations and their solutions. By George Moseley Murphy. 1960
Section: Various 8
Problem number: 219.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[[_1st_order, '_with_symmetry_[F(x),G(x)*y+H(x)]']]`

$$\boxed{xy' + ny - f(x)\, g(x^n y) = 0}$$

✓ Solution by Maple

Time used: 0.14 (sec). Leaf size: 33

```
dsolve(x*diff(y(x),x)+n*y(x) = f(x)*g(x^n*y(x)),y(x), singsol=all)
```

$$y(x) = \text{RootOf}\left(-\left(\int x^{n-1} f(x)\, dx\right) + \int^{\_Z} \frac{1}{g(\_a)} d\_a + c_1\right) x^{-n}$$

✓ Solution by Mathematica

Time used: 1.899 (sec). Leaf size: 41

```
DSolve[x y'[x]+ n y[x]==f[x] g[x^n y[x]],y[x],x,IncludeSingularSolutions -> True]
```

$$\text{Solve}\left[\int_1^{x^n y(x)} \frac{1}{g(K[1])} dK[1] = \int_1^x f(K[2]) K[2]^{n-1} dK[2] + c_1, y(x)\right]$$

8.15 problem 220

Internal problem ID [3476]

Book: Ordinary differential equations and their solutions. By George Moseley Murphy. 1960
Section: Various 8
Problem number: 220.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[[_homogeneous, ‘class G‘]]`

$$\boxed{xy' - yf(x^m y^n) = 0}$$

✓ Solution by Maple

Time used: 0.125 (sec). Leaf size: 39

```
dsolve(x*diff(y(x),x) = y(x)*f(x^m*y(x)^n),y(x), singsol=all)
```

$$\int_{\_b}^{y(x)} \frac{1}{(f(x^m \_a^n) n + m) \_a} d\_a - \frac{\ln(x)}{n} - c_1 = 0$$

✓ Solution by Mathematica

Time used: 0.362 (sec). Leaf size: 186

```
DSolve[x y'[x]==y[x] f[x^m y[x]^n] ,y[x],x,IncludeSingularSolutions -> True]
```

$$\text{Solve}\left[\int_1^{y(x)} \left(-\frac{n}{(m + nf(x^m K[2]^n)) K[2]} - \int_1^x \left(\frac{n^2 K[1]^{m-1} K[2]^{n-1} f'(K[1]^m K[2]^n)}{m + nf(K[1]^m K[2]^n)} - \frac{n^3 f(K[1]^m K[2]^n) K[1]^{m-1} K[2]^{n-1} f'(K[1]^m K[2]^n)}{(m + nf(K[1]^m K[2]^n))^2}\right) dK[1]\right) dK[2] + \int_1^x \frac{nf(K[1]^m y(x)^n)}{(m + nf(K[1]^m y(x)^n)) K[1]} dK[1] = c_1, y(x)\right]$$

8.16 problem 221

Internal problem ID [3477]

Book: Ordinary differential equations and their solutions. By George Moseley Murphy. 1960
Section: Various 8
Problem number: 221.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_linear]`

$$\boxed{(x+1)\,y'-y=x^3(3x+4)}$$

✓ Solution by Maple
Time used: 0.0 (sec). Leaf size: 14

```
dsolve((1+x)*diff(y(x),x) = x^3*(4+3*x)+y(x),y(x), singsol=all)
```

$$y(x)=x^4+c_1x+c_1+x+1$$

✓ Solution by Mathematica
Time used: 0.046 (sec). Leaf size: 18

```
DSolve[(1+x) y'[x]==x^3(4+3 x)+y[x],y[x],x,IncludeSingularSolutions -> True]
```

$$y(x)\to x^4+(4+c_1)x+4+c_1$$

8.17 problem 222

Internal problem ID [3478]

Book: Ordinary differential equations and their solutions. By George Moseley Murphy. 1960
Section: Various 8
Problem number: 222.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_linear]`

$$(x+1)\,y' - 2y = (x+1)^4$$

✓ Solution by Maple
Time used: 0.0 (sec). Leaf size: 18

```
dsolve((1+x)*diff(y(x),x) = (1+x)^4+2*y(x),y(x), singsol=all)
```

$$y(x) = \left(\frac{1}{2}x^2 + x + c_1\right)(x+1)^2$$

✓ Solution by Mathematica
Time used: 0.032 (sec). Leaf size: 22

```
DSolve[(1+x) y'[x]==(1+x)^4+2 y[x],y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to (x+1)^2\left(\frac{x^2}{2} + x + c_1\right)$$

8.18 problem 223

Internal problem ID [3479]

Book: Ordinary differential equations and their solutions. By George Moseley Murphy. 1960
Section: Various 8
Problem number: 223.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_linear]`

$$\boxed{(x+1)\,y' - ny = \mathrm{e}^{x}(x+1)^{n+1}}$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 14

```
dsolve((1+x)*diff(y(x),x) = exp(x)*(1+x)^(n+1)+n*y(x),y(x), singsol=all)
```

$$y(x) = (\mathrm{e}^{x} + c_1)\,(x+1)^{n}$$

✓ Solution by Mathematica

Time used: 0.073 (sec). Leaf size: 17

```
DSolve[(1+x) y'[x]==Exp[x](1+x)^(n+1)+n y[x],y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to (e^{x} + c_1)\,(x+1)^{n}$$

8.19 problem 224

Internal problem ID [3480]

Book: Ordinary differential equations and their solutions. By George Moseley Murphy. 1960
Section: Various 8
Problem number: 224.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_rational, _Bernoulli]`

$$\boxed{(x+1)\,y' - ya - bxy^2 = 0}$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 31

```
dsolve((1+x)*diff(y(x),x) = a*y(x)+b*x*y(x)^2,y(x), singsol=all)
```

$$y(x) = \frac{(a+1)\,a}{ac_1\,(a+1)\,(x+1)^{-a} - bxa + b}$$

✓ Solution by Mathematica

Time used: 0.299 (sec). Leaf size: 44

```
DSolve[(1+x) y'[x]==a y[x]+b x y[x]^2,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to -\frac{a(a+1)(x+1)^a}{b(x+1)^a(ax-1) - a(a+1)c_1}$$
$$y(x) \to 0$$

8.20 problem 225

Internal problem ID [3481]

Book: Ordinary differential equations and their solutions. By George Moseley Murphy. 1960
Section: Various 8
Problem number: 225.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[[_1st_order, _with_linear_symmetries], _rational, _Bernoulli]`

$$\boxed{(x+1)\,y'+y+(x+1)^4\,y^3=0}$$

✓ Solution by Maple

Time used: 0.016 (sec). Leaf size: 40

```
dsolve((1+x)*diff(y(x),x)+y(x)+(1+x)^4*y(x)^3 = 0,y(x), singsol=all)
```

$$y(x) = \frac{1}{\sqrt{x^2+c_1+2x}\,(x+1)}$$
$$y(x) = -\frac{1}{\sqrt{x^2+c_1+2x}\,(x+1)}$$

✓ Solution by Mathematica

Time used: 0.555 (sec). Leaf size: 54

```
DSolve[(1+x) y'[x]+y[x]+(1+x)^4 y[x]^3==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to -\frac{1}{\sqrt{(x+1)^2\,(x^2+2x+c_1)}}$$
$$y(x) \to \frac{1}{\sqrt{(x+1)^2\,(x^2+2x+c_1)}}$$
$$y(x) \to 0$$

8.21 problem 226

Internal problem ID [3482]

Book: Ordinary differential equations and their solutions. By George Moseley Murphy. 1960
Section: Various 8
Problem number: 226.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_rational, _Bernoulli]`

$$\boxed{(x+1)\,y' - \left(1 - x y^3\right) y = 0}$$

✓ Solution by Maple

Time used: 0.015 (sec). Leaf size: 180

```
dsolve((1+x)*diff(y(x),x) = (1-x*y(x)^3)*y(x),y(x), singsol=all)
```

$$y(x) = \frac{2^{\frac{2}{3}}\left(\left(3x^4+8x^3+6x^2+4c_1\right)^2\right)^{\frac{1}{3}}(x+1)}{3x^4+8x^3+6x^2+4c_1}$$

$$y(x) = -\frac{\left(\left(3x^4+8x^3+6x^2+4c_1\right)^2\right)^{\frac{1}{3}}2^{\frac{2}{3}}\left(1+i\sqrt{3}\right)(x+1)}{6x^4+16x^3+12x^2+8c_1}$$

$$y(x) = \frac{\left(\left(3x^4+8x^3+6x^2+4c_1\right)^2\right)^{\frac{1}{3}}2^{\frac{2}{3}}\left(i\sqrt{3}-1\right)(x+1)}{6x^4+16x^3+12x^2+8c_1}$$

✓ Solution by Mathematica

Time used: 0.322 (sec). Leaf size: 124

```
DSolve[(1+x) y'[x]==(1-x y[x]^3)y[x],y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to -\frac{(-2)^{2/3}(x+1)}{\sqrt[3]{-3x^4-8x^3-6x^2-4c_1}}$$
$$y(x) \to -\frac{2^{2/3}(x+1)}{\sqrt[3]{-3x^4-8x^3-6x^2-4c_1}}$$
$$y(x) \to \frac{\sqrt[3]{-1}2^{2/3}(x+1)}{\sqrt[3]{-3x^4-8x^3-6x^2-4c_1}}$$
$$y(x) \to 0$$

8.22 problem 227

Internal problem ID [3483]

Book: Ordinary differential equations and their solutions. By George Moseley Murphy. 1960
Section: Various 8
Problem number: 227.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[[_1st_order, _with_linear_symmetries]]`

$$(x+1)\,y'-y-(x+1)\sqrt{y+1}=1$$

✓ Solution by Maple

Time used: 0.015 (sec). Leaf size: 81

```
dsolve((1+x)*diff(y(x),x) = 1+y(x)+(1+x)*sqrt(1+y(x)),y(x), singsol=all)
```

$$\frac{(-c_1 y(x)+1+c_1 x^2+(2c_1+1)\,x)\,\sqrt{y\,(x)+1}-(-c_1 y(x)-1+c_1 x^2+(2c_1-1)\,x)\,(x+1)}{(x^2+2x-y\,(x))\left(-\sqrt{y\,(x)+1}+x+1\right)} = 0$$

✓ Solution by Mathematica

Time used: 0.237 (sec). Leaf size: 60

```
DSolve[(1+x) y'[x]==(1+y[x])+(1+x)Sqrt[1+y[x]],y[x],x,IncludeSingularSolutions -> True]
```

$$\text{Solve}\left[\frac{2\sqrt{y(x)+1}\arctan\left(\frac{x+1}{\sqrt{-y(x)-1}}\right)}{\sqrt{-y(x)-1}}+\log\left(y(x)-(x+1)^2+1\right)-\log(x+1)=c_1, y(x)\right]$$

8.23 problem 228

Internal problem ID [3484]

Book: Ordinary differential equations and their solutions. By George Moseley Murphy. 1960
Section: Various 8
Problem number: 228.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_quadrature]`

$$\boxed{(x+a)\,y' = bx}$$

✓ Solution by Maple
Time used: 0.016 (sec). Leaf size: 17

```
dsolve((a+x)*diff(y(x),x) = b*x,y(x), singsol=all)
```

$$y(x) = -\ln(x+a)\,ab + bx + c_1$$

✓ Solution by Mathematica
Time used: 0.004 (sec). Leaf size: 19

```
DSolve[(a+x) y'[x]==b x,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to -ab\log(a+x) + bx + c_1$$

8.24 problem 229

Internal problem ID [3485]

Book: Ordinary differential equations and their solutions. By George Moseley Murphy. 1960
Section: Various 8
Problem number: 229.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_linear]`

$$\boxed{(x+a)\,y' - y = bx}$$

✓ Solution by Maple
Time used: 0.0 (sec). Leaf size: 22

```
dsolve((a+x)*diff(y(x),x) = b*x+y(x),y(x), singsol=all)
```

$$y(x) = b(x+a)\ln(x+a) + (b+c_1)\,a + c_1x$$

✓ Solution by Mathematica
Time used: 0.036 (sec). Leaf size: 26

```
DSolve[(a+x) y'[x]==b x+ y[x],y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to (a+x)\left(\frac{ab}{a+x} + b\log(a+x) + c_1\right)$$

8.25 problem 230

Internal problem ID [3486]

Book: Ordinary differential equations and their solutions. By George Moseley Murphy. 1960
Section: Various 8
Problem number: 230.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_linear]`

$$\boxed{(x+a)\,y'+y=-b\,x^2}$$

✓ Solution by Maple
Time used: 0.0 (sec). Leaf size: 24

```
dsolve((a+x)*diff(y(x),x)+b*x^2+y(x) = 0,y(x), singsol=all)
```

$$y(x)=\frac{-b\,x^3+3c_1}{3x+3a}$$

✓ Solution by Mathematica
Time used: 0.033 (sec). Leaf size: 25

```
DSolve[(a+x) y'[x]+b x^2+y[x]==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x)\to\frac{-bx^3+3c_1}{3(a+x)}$$

8.26 problem 231

Internal problem ID [3487]

Book: Ordinary differential equations and their solutions. By George Moseley Murphy. 1960
Section: Various 8
Problem number: 231.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_linear]`

$$\boxed{(x + a)\, y' - 3y = 2(x + a)^5}$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 19

```
dsolve((a+x)*diff(y(x),x) = 2*(a+x)^5+3*y(x),y(x), singsol=all)
```

$$y(x) = (2ax + x^2 + c_1)\,(x + a)^3$$

✓ Solution by Mathematica

Time used: 0.044 (sec). Leaf size: 21

```
DSolve[(a+x) y'[x]==2(a+x)^5+3 y[x],y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to (a + x)^3\,(2ax + x^2 + c_1)$$

8.27 problem 232

Internal problem ID [3488]

Book: Ordinary differential equations and their solutions. By George Moseley Murphy. 1960
Section: Various 8
Problem number: 232.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_separable]`

$$\boxed{(x+a)\,y' - cy = b}$$

✓ Solution by Maple
Time used: 0.0 (sec). Leaf size: 18

```
dsolve((a+x)*diff(y(x),x) = b+c*y(x),y(x), singsol=all)
```

$$y(x) = -\frac{b}{c} + (x+a)^{c}\, c_1$$

✓ Solution by Mathematica
Time used: 0.037 (sec). Leaf size: 30

```
DSolve[(a+x) y'[x]==(b+c y[x]),y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to -\frac{b}{c} + c_1(a+x)^{c}$$
$$y(x) \to -\frac{b}{c}$$

8.28 problem 233

Internal problem ID [3489]

Book: Ordinary differential equations and their solutions. By George Moseley Murphy. 1960
Section: Various 8
Problem number: 233.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_linear]`

$$\boxed{(x+a)\, y' - cy = bx}$$

✓ Solution by Maple
Time used: 0.0 (sec). Leaf size: 28

```
dsolve((a+x)*diff(y(x),x) = b*x+c*y(x),y(x), singsol=all)
```

$$y(x) = (x+a)^{c}\, c_1 - \frac{b(cx+a)}{c\,(c-1)}$$

✓ Solution by Mathematica
Time used: 0.083 (sec). Leaf size: 32

```
DSolve[(a+x) y'[x]==b x+c y[x],y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to \frac{ab+bcx}{c-c^2} + c_1(a+x)^c$$

8.29 problem 234

Internal problem ID [3490]

Book: Ordinary differential equations and their solutions. By George Moseley Murphy. 1960
Section: Various 8
Problem number: 234.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_separable]`

$$\boxed{(x + a)\, y' - y(1 - ya) = 0}$$

✓ Solution by Maple
Time used: 0.016 (sec). Leaf size: 15

```
dsolve((a+x)*diff(y(x),x) = y(x)*(1-a*y(x)),y(x), singsol=all)
```

$$y(x) = \frac{x + a}{ax + c_1}$$

✓ Solution by Mathematica
Time used: 0.719 (sec). Leaf size: 34

```
DSolve[(a+x) y'[x]==y[x](1-a y[x]),y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to \frac{a + x}{a^2 + ax + e^{c_1}}$$
$$y(x) \to 0$$
$$y(x) \to \frac{1}{a}$$

8.30 problem 235

Internal problem ID [3491]

Book: Ordinary differential equations and their solutions. By George Moseley Murphy. 1960
Section: Various 8
Problem number: 235.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_rational, _Bernoulli]`

$$\boxed{(-x+a)\,y'-y-(cx+b)\,y^3=0}$$

✓ Solution by Maple
Time used: 0.0 (sec). Leaf size: 59

```
dsolve((a-x)*diff(y(x),x) = y(x)+(c*x+b)*y(x)^3,y(x), singsol=all)
```

$$y(x)=\frac{1}{\sqrt{c_1a^2-2ac_1x+c_1x^2+ac-2cx-b}}$$
$$y(x)=-\frac{1}{\sqrt{(a-x)^2\,c_1+ac-2cx-b}}$$

✓ Solution by Mathematica
Time used: 0.469 (sec). Leaf size: 82

```
DSolve[(a-x) y'[x]==y[x]+(b+c x)y[x]^3,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x)\to-\frac{1}{\sqrt{a^2c_1+a(c-2c_1x)-b+x(-2c+c_1x)}}$$
$$y(x)\to\frac{1}{\sqrt{a^2c_1+a(c-2c_1x)-b+x(-2c+c_1x)}}$$
$$y(x)\to0$$

8.31 problem 236

Internal problem ID [3492]

Book: Ordinary differential equations and their solutions. By George Moseley Murphy. 1960
Section: Various 8
Problem number: 236.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_linear]`

$$\boxed{2xy' + y = 2x^3}$$

✓ Solution by Maple
Time used: 0.0 (sec). Leaf size: 15

```
dsolve(2*x*diff(y(x),x) = 2*x^3-y(x),y(x), singsol=all)
```

$$y(x) = \frac{2x^3}{7} + \frac{c_1}{\sqrt{x}}$$

✓ Solution by Mathematica
Time used: 0.029 (sec). Leaf size: 21

```
DSolve[2 x y'[x]==2 x^3-y[x],y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to \frac{2x^3}{7} + \frac{c_1}{\sqrt{x}}$$

8.32 problem 237

Internal problem ID [3493]

Book: Ordinary differential equations and their solutions. By George Moseley Murphy. 1960
Section: Various 8
Problem number: 237.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_rational, _Riccati]`

$$\boxed{2xy' - 4ixy - y^2 = -1}$$

✓ Solution by Maple
Time used: 0.031 (sec). Leaf size: 64

```
dsolve(2*x*diff(y(x),x)+1 = 4*I*x*y(x)+y(x)^2,y(x), singsol=all)
```

$$y(x) = \frac{i \,\mathrm{BesselJ}\left(1, x\right) - \mathrm{BesselK}\left(1, ix\right) c_1 + \mathrm{BesselK}\left(0, ix\right) c_1 + \mathrm{BesselJ}\left(0, x\right)}{i \,\mathrm{BesselJ}\left(1, x\right) - \mathrm{BesselK}\left(1, ix\right) c_1 - \mathrm{BesselK}\left(0, ix\right) c_1 - \mathrm{BesselJ}\left(0, x\right)}$$

✓ Solution by Mathematica
Time used: 0.538 (sec). Leaf size: 202

```
DSolve[2 x y'[x]+1==4 I x y[x]+y[x]^2,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to \frac{(1-i)c_1 e^{ix}\sqrt{x}((x-i)\,\mathrm{BesselJ}(0,x) - \mathrm{BesselJ}(1,x) + x\,\mathrm{BesselJ}(2,x)) - 4ixG_{1,2}^{2,0}\left(-2ix\middle|\begin{array}{c} -1 \\ -\frac{3}{2}, -\frac{1}{2} \end{array}\right)}{G_{1,2}^{2,0}\left(-2ix\middle|\begin{array}{c} 1 \\ -\frac{1}{2}, \frac{1}{2} \end{array}\right) + (1+i)c_1 e^{ix}\sqrt{x}(\mathrm{BesselJ}(0,x) - i\,\mathrm{BesselJ}(1,x))}$$

$$y(x) \to -\frac{i((x-i)\,\mathrm{BesselJ}(0,x) - \mathrm{BesselJ}(1,x) + x\,\mathrm{BesselJ}(2,x))}{\mathrm{BesselJ}(0,x) - i\,\mathrm{BesselJ}(1,x)}$$

$$y(x) \to -\frac{i((x-i)\,\mathrm{BesselJ}(0,x) - \mathrm{BesselJ}(1,x) + x\,\mathrm{BesselJ}(2,x))}{\mathrm{BesselJ}(0,x) - i\,\mathrm{BesselJ}(1,x)}$$

8.33 problem 238

Internal problem ID [3494]

Book: Ordinary differential equations and their solutions. By George Moseley Murphy. 1960
Section: Various 8
Problem number: 238.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_separable]`

$$\boxed{2xy' - y(y^2+1) = 0}$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 41

```
dsolve(2*x*diff(y(x),x) = y(x)*(1+y(x)^2),y(x), singsol=all)
```

$$y(x) = \frac{\sqrt{(c_1 - x)\,x}}{c_1 - x}$$
$$y(x) = \frac{\sqrt{(c_1 - x)\,x}}{-c_1 + x}$$

✓ Solution by Mathematica

Time used: 0.534 (sec). Leaf size: 82

```
DSolve[2 x y'[x]==y[x](1+y[x]^2),y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to -\frac{ie^{c_1}\sqrt{x}}{\sqrt{-1 + e^{2c_1}x}}$$
$$y(x) \to \frac{ie^{c_1}\sqrt{x}}{\sqrt{-1 + e^{2c_1}x}}$$
$$y(x) \to 0$$
$$y(x) \to -i$$
$$y(x) \to i$$

8.34 problem 239

Internal problem ID [3495]

Book: Ordinary differential equations and their solutions. By George Moseley Murphy. 1960
Section: Various 8
Problem number: 239.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_separable]`

$$\boxed{2xy' + y(y^2 + 1) = 0}$$

✓ Solution by Maple
Time used: 0.0 (sec). Leaf size: 23

```
dsolve(2*x*diff(y(x),x)+y(x)*(1+y(x)^2) = 0,y(x), singsol=all)
```

$$y(x) = \frac{1}{\sqrt{c_1 x - 1}}$$
$$y(x) = -\frac{1}{\sqrt{c_1 x - 1}}$$

✓ Solution by Mathematica
Time used: 0.442 (sec). Leaf size: 72

```
DSolve[2 x y'[x]+y[x](1+y[x]^2)==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to -\frac{ie^{c_1}}{\sqrt{-x + e^{2c_1}}}$$
$$y(x) \to \frac{ie^{c_1}}{\sqrt{-x + e^{2c_1}}}$$
$$y(x) \to 0$$
$$y(x) \to -i$$
$$y(x) \to i$$

8.35 problem 240

Internal problem ID [3496]

Book: Ordinary differential equations and their solutions. By George Moseley Murphy. 1960
Section: Various 8
Problem number: 240.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_rational, _Bernoulli]`

$$\boxed{2xy' - \left(1 + x - 6y^2\right) y = 0}$$

✓ Solution by Maple

Time used: 0.016 (sec). Leaf size: 54

```
dsolve(2*x*diff(y(x),x) = (1+x-6*y(x)^2)*y(x),y(x), singsol=all)
```

$$y(x) = \frac{\sqrt{\left(\mathrm{e}^{-x} c_1 + 6\right) x}}{\mathrm{e}^{-x} c_1 + 6}$$
$$y(x) = -\frac{\sqrt{\left(\mathrm{e}^{-x} c_1 + 6\right) x}}{\mathrm{e}^{-x} c_1 + 6}$$

✓ Solution by Mathematica

Time used: 0.671 (sec). Leaf size: 65

```
DSolve[2 x y'[x]==(1+x-6 y[x]^2)y[x],y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to -\frac{e^{x/2}\sqrt{x}}{\sqrt{6e^x + c_1}}$$
$$y(x) \to \frac{e^{x/2}\sqrt{x}}{\sqrt{6e^x + c_1}}$$
$$y(x) \to 0$$

9 Various 9

9.1 problem 241

Internal problem ID [3497]

Book: Ordinary differential equations and their solutions. By George Moseley Murphy. 1960
Section: Various 9
Problem number: 241.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_separable]`

$$\boxed{2xy' + 4y + \sqrt{a^2 - 4b - 4cy} = -a}$$

✓ Solution by Maple
Time used: 0.0 (sec). Leaf size: 34

```
dsolve(2*x*diff(y(x),x)+4*y(x)+a+sqrt(a^2-4*b-4*c*y(x)) = 0,y(x), singsol=all)
```

$$\ln(x) + 2\left(\int^{y(x)} \frac{1}{4\_a + a + \sqrt{-4\_ac + a^2 - 4b}} d\_a\right) + c_1 = 0$$

✓ Solution by Mathematica

Time used: 0.808 (sec). Leaf size: 177

```
DSolve[2 x y'[x]+4 y[x]+a +Sqrt[a^2-4 b- 4 c y[x]]==0,y[x],x,IncludeSingularSolutions -> Tru
```

$$y(x) \to \text{InverseFunction}\left[\frac{1}{4}\left(\log\left(c\left(\sqrt{a^2-4(\#1c+b)}+4\#1+a\right)\right) - \frac{2c\arctan\left(\frac{c-2\sqrt{a^2-4(\#1c+b)}}{\sqrt{-4a^2-4ac+16b-c^2}}\right)}{\sqrt{-4a^2-4ac+16b-c^2}}\right)\&\right]\left[-\frac{\log(x)}{2}+c_1\right]$$

$$y(x) \to \frac{1}{8}\left(-\sqrt{(2a+c)^2-16b}-2a-c\right)$$

$$y(x) \to \frac{1}{8}\left(\sqrt{(2a+c)^2-16b}-2a-c\right)$$

9.2 problem 242

Internal problem ID [3498]

Book: Ordinary differential equations and their solutions. By George Moseley Murphy. 1960
Section: Various 9
Problem number: 242.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_linear]`

$$\boxed{(1-2x)\, y' + 6y = 32x + 16}$$

✓ Solution by Maple
Time used: 0.0 (sec). Leaf size: 18

```
dsolve((1-2*x)*diff(y(x),x) = 16+32*x-6*y(x),y(x), singsol=all)
```

$$y(x) = \frac{4}{3} + 8x + (2x-1)^3 c_1$$

✓ Solution by Mathematica
Time used: 0.037 (sec). Leaf size: 22

```
DSolve[(1-2 x)y'[x]==2(8+16 x-3 y[x]),y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to 8x + c_1(2x-1)^3 + \frac{4}{3}$$

9.3 problem 243

Internal problem ID [3499]

Book: Ordinary differential equations and their solutions. By George Moseley Murphy. 1960
Section: Various 9
Problem number: 243.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_separable]`

$$\boxed{(1+2x)\, y' - 4\,\mathrm{e}^{-y} = -2}$$

✓ Solution by Maple
Time used: 0.031 (sec). Leaf size: 31

```
dsolve((1+2*x)*diff(y(x),x) = 4*exp(-y(x))-2,y(x), singsol=all)
```

$$y(x) = -\ln\left(\frac{2x+1}{-1+(4x+2)\,\mathrm{e}^{2c_1}}\right) - 2c_1$$

✓ Solution by Mathematica
Time used: 0.656 (sec). Leaf size: 26

```
DSolve[(1+2 x)y'[x]==4 Exp[-y[x]]-2,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to \log\left(2 + \frac{e^{c_1}}{2x+1}\right)$$
$$y(x) \to \log(2)$$

9.4 problem 244

Internal problem ID [3500]

Book: Ordinary differential equations and their solutions. By George Moseley Murphy. 1960
Section: Various 9
Problem number: 244.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_linear]`

$$\boxed{2(1-x)\,y'-y=4x\sqrt{1-x}}$$

✓ Solution by Maple
Time used: 0.0 (sec). Leaf size: 23

```
dsolve(2*(1-x)*diff(y(x),x) = 4*x*sqrt(1-x)+y(x),y(x), singsol=all)
```

$$y(x)=\frac{x^2}{\sqrt{1-x}}+\frac{c_1}{\sqrt{x-1}}$$

✓ Solution by Mathematica
Time used: 0.053 (sec). Leaf size: 32

```
DSolve[2(1-x)y'[x]==4 x Sqrt[1-x]+y[x],y[x],x,IncludeSingularSolutions -> True]
```

$$y(x)\to\frac{2x^2+\sqrt{2}c_1}{2\sqrt{1-x}}$$

9.5 problem 245

Internal problem ID [3501]

Book: Ordinary differential equations and their solutions. By George Moseley Murphy. 1960
Section: Various 9
Problem number: 245.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[[_1st_order, _with_linear_symmetries], _rational, _Bernoulli]`

$$2(x+1)\,y' + 2y + (x+1)^4\,y^3 = 0$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 49

```
dsolve(2*(1+x)*diff(y(x),x)+2*y(x)+(1+x)^4*y(x)^3 = 0,y(x), singsol=all)
```

$$y(x) = -\frac{2}{\sqrt{2x^2+4c_1+4x}\,(x+1)}$$
$$y(x) = \frac{2}{\sqrt{2x^2+4c_1+4x}\,(x+1)}$$

✓ Solution by Mathematica

Time used: 0.629 (sec). Leaf size: 69

```
DSolve[2(1+x)y'[x]+2 y[x]+(1+x)^4 y[x]^3==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to -\frac{\sqrt{2}}{\sqrt{(x+1)^2(x^2+2x+2c_1)}}$$
$$y(x) \to \frac{\sqrt{2}}{\sqrt{(x+1)^2(x^2+2x+2c_1)}}$$
$$y(x) \to 0$$

9.6 problem 246

Internal problem ID [3502]

Book: Ordinary differential equations and their solutions. By George Moseley Murphy. 1960
Section: Various 9
Problem number: 246.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_rational, _Riccati]`

$$3xy' - (1-3y)\,y = 3x^{\frac{2}{3}}$$

✓ Solution by Maple
Time used: 0.0 (sec). Leaf size: 19

```
dsolve(3*x*diff(y(x),x) = 3*x^(2/3)+(1-3*y(x))*y(x),y(x), singsol=all)
```

$$y(x) = i\tan\left(-3ix^{\frac{1}{3}} + c_1\right)x^{\frac{1}{3}}$$

✓ Solution by Mathematica
Time used: 0.181 (sec). Leaf size: 79

```
DSolve[3 x y'[x]==3 x^(2/3)+(1-3 y[x])y[x],y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to \frac{\sqrt[3]{x}\left(i\cosh\left(3\sqrt[3]{x}\right) + c_1\sinh\left(3\sqrt[3]{x}\right)\right)}{i\sinh\left(3\sqrt[3]{x}\right) + c_1\cosh\left(3\sqrt[3]{x}\right)}$$
$$y(x) \to \sqrt[3]{x}\tanh\left(3\sqrt[3]{x}\right)$$

9.7 problem 247

Internal problem ID [3503]

Book: Ordinary differential equations and their solutions. By George Moseley Murphy. 1960
Section: Various 9
Problem number: 247.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[[_homogeneous, ‘class G‘], _rational, _Bernoulli]`

$$\boxed{3xy' - (2 + xy^3)\, y = 0}$$

✓ Solution by Maple
Time used: 0.0 (sec). Leaf size: 115

```
dsolve(3*x*diff(y(x),x) = (2+x*y(x)^3)*y(x),y(x), singsol=all)
```

$$y(x) = -\frac{3^{\frac{1}{3}} \left(x^2 (x^3 - 3c_1)^2\right)^{\frac{1}{3}}}{x^3 - 3c_1}$$
$$y(x) = \frac{\left(x^2 (x^3 - 3c_1)^2\right)^{\frac{1}{3}} \left(i 3^{\frac{5}{6}} + 3^{\frac{1}{3}}\right)}{2x^3 - 6c_1}$$
$$y(x) = \frac{\left(x^2 (x^3 - 3c_1)^2\right)^{\frac{1}{3}} \left(i 3^{\frac{5}{6}} - 3^{\frac{1}{3}}\right)}{-2x^3 + 6c_1}$$

✓ Solution by Mathematica

Time used: 0.21 (sec). Leaf size: 89

```
DSolve[3 x y'[x]==(2+x y[x]^3)y[x],y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to -\frac{\sqrt[3]{-3}x^{2/3}}{\sqrt[3]{-x^3+3c_1}}$$
$$y(x) \to \frac{x^{2/3}}{\sqrt[3]{-\frac{x^3}{3}+c_1}}$$
$$y(x) \to \frac{(-1)^{2/3}x^{2/3}}{\sqrt[3]{-\frac{x^3}{3}+c_1}}$$
$$y(x) \to 0$$

9.8 problem 248

Internal problem ID [3504]

Book: Ordinary differential equations and their solutions. By George Moseley Murphy. 1960
Section: Various 9
Problem number: 248.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_Bernoulli]`

$$\boxed{3xy' - \left(1 + 3xy^3 \ln(x)\right) y = 0}$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 162

```
dsolve(3*x*diff(y(x),x) = (1+3*x*y(x)^3*ln(x))*y(x),y(x), singsol=all)
```

$$y(x) = \frac{2^{\frac{2}{3}} \left(-x(6 \ln(x) x^2 - 3x^2 - 4c_1)^2\right)^{\frac{1}{3}}}{6 \ln(x) x^2 - 3x^2 - 4c_1}$$

$$y(x) = -\frac{\left(1 + i\sqrt{3}\right) 2^{\frac{2}{3}} \left(-x(6 \ln(x) x^2 - 3x^2 - 4c_1)^2\right)^{\frac{1}{3}}}{12 \ln(x) x^2 - 6x^2 - 8c_1}$$

$$y(x) = \frac{2^{\frac{2}{3}} \left(-x(6 \ln(x) x^2 - 3x^2 - 4c_1)^2\right)^{\frac{1}{3}} \left(i\sqrt{3} - 1\right)}{12 \ln(x) x^2 - 6x^2 - 8c_1}$$

✓ Solution by Mathematica

Time used: 0.227 (sec). Leaf size: 120

```
DSolve[3 x y'[x]==(1+3 x y[x]^3 Log[x])y[x],y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to \frac{(-2)^{2/3}\sqrt[3]{x}}{\sqrt[3]{3x^2 - 6x^2\log(x) + 4c_1}}$$
$$y(x) \to \frac{2^{2/3}\sqrt[3]{x}}{\sqrt[3]{3x^2 - 6x^2\log(x) + 4c_1}}$$
$$y(x) \to -\frac{\sqrt[3]{-1}2^{2/3}\sqrt[3]{x}}{\sqrt[3]{3x^2 - 6x^2\log(x) + 4c_1}}$$
$$y(x) \to 0$$

9.9 problem 249

Internal problem ID [3505]

Book: Ordinary differential equations and their solutions. By George Moseley Murphy. 1960
Section: Various 9
Problem number: 249.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_separable]`

$$\boxed{y'x^2 + y = a}$$

✓ Solution by Maple
Time used: 0.016 (sec). Leaf size: 12

```
dsolve(x^2*diff(y(x),x) = a-y(x),y(x), singsol=all)
```

$$y(x) = a + c_1 e^{\frac{1}{x}}$$

✓ Solution by Mathematica
Time used: 0.03 (sec). Leaf size: 20

```
DSolve[x^2 y'[x]==a-y[x],y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to a + c_1 e^{\frac{1}{x}}$$
$$y(x) \to a$$

9.10 problem 250

Internal problem ID [3506]

Book: Ordinary differential equations and their solutions. By George Moseley Murphy. 1960
Section: Various 9
Problem number: 250.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_linear]`

$$\boxed{y'x^2 - yx = c\,x^2 + bx + a}$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 22

```
dsolve(x^2*diff(y(x),x) = a+b*x+c*x^2+x*y(x),y(x), singsol=all)
```

$$y(x) = xc\ln(x) - \frac{a}{2x} - b + c_1x$$

✓ Solution by Mathematica

Time used: 0.038 (sec). Leaf size: 26

```
DSolve[x^2 y'[x]==a+b x+c x^2+x y[x],y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to -\frac{a}{2x} - b + cx\log(x) + c_1x$$

9.11 problem 251

Internal problem ID [3507]

Book: Ordinary differential equations and their solutions. By George Moseley Murphy. 1960
Section: Various 9
Problem number: 251.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_linear]`

$$\boxed{y' x^2 + yx = c\,x^2 + bx + a}$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 22

```
dsolve(x^2*diff(y(x),x) = a+b*x+c*x^2-x*y(x),y(x), singsol=all)
```

$$y(x) = \frac{cx}{2} + b + \frac{a\ln(x)}{x} + \frac{c_1}{x}$$

✓ Solution by Mathematica

Time used: 0.035 (sec). Leaf size: 26

```
DSolve[x^2 y'[x]==a+b x+c x^2-x y[x],y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to \frac{a\log(x)}{x} + b + \frac{cx}{2} + \frac{c_1}{x}$$

9.12 problem 252

Internal problem ID [3508]

Book: Ordinary differential equations and their solutions. By George Moseley Murphy. 1960
Section: Various 9
Problem number: 252.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_linear]`

$$\boxed{y'x^2 + (1 - 2x)\, y = x^2}$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 16

```
dsolve(x^2*diff(y(x),x)+(1-2*x)*y(x) = x^2,y(x), singsol=all)
```

$$y(x) = x^2\left(1 + c_1 e^{\frac{1}{x}}\right)$$

✓ Solution by Mathematica

Time used: 0.035 (sec). Leaf size: 19

```
DSolve[x^2 y'[x]+(1-2 x)y[x]==x^2,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to x^2\left(1 + c_1 e^{\frac{1}{x}}\right)$$

9.13 problem 253

Internal problem ID [3509]

Book: Ordinary differential equations and their solutions. By George Moseley Murphy. 1960
Section: Various 9
Problem number: 253.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_linear]`

$$\boxed{y'x^2 - bxy = a}$$

✓ Solution by Maple
Time used: 0.0 (sec). Leaf size: 21

```
dsolve(x^2*diff(y(x),x) = a+b*x*y(x),y(x), singsol=all)
```

$$y(x) = -\frac{a}{x\,(b+1)} + x^b c_1$$

✓ Solution by Mathematica
Time used: 0.041 (sec). Leaf size: 22

```
DSolve[x^2 y'[x]==a+b x y[x],y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to -\frac{a}{bx+x} + c_1 x^b$$

9.14 problem 254

Internal problem ID [3510]

Book: Ordinary differential equations and their solutions. By George Moseley Murphy. 1960
Section: Various 9
Problem number: 254.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_separable]`

$$\boxed{y'x^2 - (bx + a)\, y = 0}$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 16

```
dsolve(x^2*diff(y(x),x) = (b*x+a)*y(x),y(x), singsol=all)
```

$$y(x) = c_1 \mathrm{e}^{-\frac{a}{x}} x^b$$

✓ Solution by Mathematica

Time used: 0.035 (sec). Leaf size: 24

```
DSolve[x^2 y'[x]==(a+b x)y[x],y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to c_1 e^{-\frac{a}{x}} x^b$$
$$y(x) \to 0$$

9.15 problem 255

Internal problem ID [3511]

Book: Ordinary differential equations and their solutions. By George Moseley Murphy. 1960
Section: Various 9
Problem number: 255.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_linear]`

$$\boxed{y'x^2 + x(x+2)\,y = x\left(1 - \mathrm{e}^{-2x}\right) - 2}$$

✓ Solution by Maple
Time used: 0.0 (sec). Leaf size: 27

```
dsolve(x^2*diff(y(x),x)+x*(2+x)*y(x) = x*(1-exp(-2*x))-2,y(x), singsol=all)
```

$$y(x) = \frac{\mathrm{e}^{-x}c_1 + \mathrm{e}^{-2x}x + \mathrm{e}^{-2x} + x - 3}{x^2}$$

✓ Solution by Mathematica
Time used: 0.173 (sec). Leaf size: 32

```
DSolve[x^2*y'[x]+x*(2+x)*y[x]==x*(1-Exp[-2*x])-2,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to \frac{e^{-2x}\left(e^{2x}(x-3) + x + c_1 e^x + 1\right)}{x^2}$$

9.16 problem 256

Internal problem ID [3512]

Book: Ordinary differential equations and their solutions. By George Moseley Murphy. 1960
Section: Various 9
Problem number: 256.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_linear]`

$$\boxed{y'x^2 + 2x(1-x)\,y = \mathrm{e}^x(2\,\mathrm{e}^x - 1)}$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 21

```
dsolve(x^2*diff(y(x),x)+2*x*(1-x)*y(x) = exp(x)*(2*exp(x)-1),y(x), singsol=all)
```

$$y(x) = \frac{(2x + c_1)\,\mathrm{e}^{2x} + \mathrm{e}^x}{x^2}$$

✓ Solution by Mathematica

Time used: 0.066 (sec). Leaf size: 24

```
DSolve[x^2 y'[x]+2 x(1-x)y[x]==Exp[x](2 Exp[x]-1),y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to \frac{e^x(1 + e^x(2x + c_1))}{x^2}$$

9.17 problem 257

Internal problem ID [3513]

Book: Ordinary differential equations and their solutions. By George Moseley Murphy. 1960
Section: Various 9
Problem number: 257.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[[_homogeneous, ‘class A‘], _rational, _Riccati]`

$$\boxed{y'x^2 + yx + y^2 = -x^2}$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 18

```
dsolve(x^2*diff(y(x),x)+x^2+x*y(x)+y(x)^2 = 0,y(x), singsol=all)
```

$$y(x) = -\frac{x(\ln(x) + c_1 - 1)}{\ln(x) + c_1}$$

✓ Solution by Mathematica

Time used: 0.144 (sec). Leaf size: 31

```
DSolve[x^2 y'[x]+x^2+x y[x]+y[x]^2==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to \frac{x(\log(x) - 1 - c_1)}{-\log(x) + c_1}$$
$$y(x) \to -x$$

9.18 problem 258

Internal problem ID [3514]

Book: Ordinary differential equations and their solutions. By George Moseley Murphy. 1960
Section: Various 9
Problem number: 258.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[[_homogeneous, 'class C'], _rational, _Riccati]`

$$\boxed{y'x^2 - (1 + 2x - y)^2 = 0}$$

✓ Solution by Maple
Time used: 0.719 (sec). Leaf size: 24

```
dsolve(x^2*diff(y(x),x) = (1+2*x-y(x))^2,y(x), singsol=all)
```

$$y(x) = 1 + \frac{x(c_1x^3 - 4)}{c_1x^3 - 1}$$

✓ Solution by Mathematica
Time used: 0.264 (sec). Leaf size: 41

```
DSolve[x^2 y'[x]==(1+2 x-y[x])^2,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to \frac{x^4 + x^3 + 12c_1x + 3c_1}{x^3 + 3c_1}$$
$$y(x) \to 4x + 1$$

9.19 problem 259

Internal problem ID [3515]

Book: Ordinary differential equations and their solutions. By George Moseley Murphy. 1960
Section: Various 9
Problem number: 259.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_separable]`

$$\boxed{y' x^2 - b y^2 = a}$$

✓ Solution by Maple
Time used: 0.0 (sec). Leaf size: 28

```
dsolve(x^2*diff(y(x),x) = a+b*y(x)^2,y(x), singsol=all)
```

$$y(x) = \frac{\tan\left(\frac{\sqrt{ab}\,(c_1 x-1)}{x}\right)\sqrt{ab}}{b}$$

✓ Solution by Mathematica
Time used: 0.184 (sec). Leaf size: 75

```
DSolve[x^2 y'[x]==a + b y[x]^2,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to -\frac{\sqrt{a}\tan\left(\frac{\sqrt{a}\sqrt{b}(1-c_1 x)}{x}\right)}{\sqrt{b}}$$
$$y(x) \to -\frac{i\sqrt{a}}{\sqrt{b}}$$
$$y(x) \to \frac{i\sqrt{a}}{\sqrt{b}}$$

9.20 problem 260

Internal problem ID [3516]

Book: Ordinary differential equations and their solutions. By George Moseley Murphy. 1960
Section: Various 9
Problem number: 260.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[[_homogeneous, ‘class A‘], _rational, _Bernoulli]`

$$y'x^2 - (ya + x)\,y = 0$$

✓ Solution by Maple
Time used: 0.0 (sec). Leaf size: 17

```
dsolve(x^2*diff(y(x),x) = (x+a*y(x))*y(x),y(x), singsol=all)
```

$$y(x) = -\frac{x}{a\ln(x) - c_1}$$

✓ Solution by Mathematica
Time used: 0.142 (sec). Leaf size: 22

```
DSolve[x^2 y'[x]==(x+a y[x])y[x],y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to \frac{x}{-a\log(x) + c_1}$$
$$y(x) \to 0$$

9.21 problem 261

Internal problem ID [3517]

Book: Ordinary differential equations and their solutions. By George Moseley Murphy. 1960
Section: Various 9
Problem number: 261.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[[_homogeneous, 'class A'], _rational, _Bernoulli]`

$$\boxed{y'x^2 - (ax + yb)\,y = 0}$$

✓ Solution by Maple

Time used: 0.015 (sec). Leaf size: 27

```
dsolve(x^2*diff(y(x),x) = (a*x+b*y(x))*y(x),y(x), singsol=all)
```

$$y(x) = \frac{x(a-1)}{(a-1)\,c_1 x^{-a+1} - b}$$

✓ Solution by Mathematica

Time used: 0.174 (sec). Leaf size: 36

```
DSolve[x^2 y'[x]==(a x+b y[x])y[x],y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to -\frac{(a-1)x^{a+1}}{bx^a - (a-1)c_1 x}$$
$$y(x) \to 0$$

9.22 problem 262

Internal problem ID [3518]

Book: Ordinary differential equations and their solutions. By George Moseley Murphy. 1960
Section: Various 9
Problem number: 262.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[[_homogeneous, ‘class A‘], _rational, _Riccati]`

$$\boxed{y'x^2 + bxy + cy^2 = -x^2 a}$$

✓ Solution by Maple
Time used: 0.0 (sec). Leaf size: 53

```
dsolve(x^2*diff(y(x),x)+a*x^2+b*x*y(x)+c*y(x)^2 = 0,y(x), singsol=all)
```

$$y(x) = -\frac{x\left(\sqrt{4ac-b^2-2b-1}\,\tan\left(\frac{\sqrt{4ac-b^2-2b-1}\,(\ln(x)+c_1)}{2}\right)+b+1\right)}{2c}$$

✓ Solution by Mathematica
Time used: 60.134 (sec). Leaf size: 66

```
DSolve[x^2 y'[x]+a x^2 +b x y[x]+c y[x]^2==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to -\frac{x\left(-\sqrt{4ac-b^2-2b-1}\tan\left(\frac{1}{2}\sqrt{4ac-b^2-2b-1}(-\log(x)+c_1)\right)+b+1\right)}{2c}$$

9.23 problem 263

Internal problem ID [3519]

Book: Ordinary differential equations and their solutions. By George Moseley Murphy. 1960
Section: Various 9
Problem number: 263.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_rational, _Riccati]`

$$\boxed{y'x^2 - y^2x^2 = b\,x^n + a}$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 196

```
dsolve(x^2*diff(y(x),x) = a+b*x^n+x^2*y(x)^2,y(x), singsol=all)
```

$$y(x) = \frac{2\sqrt{b}\left(\text{BesselY}\left(\frac{\sqrt{1-4a}}{n}+1, \frac{2\sqrt{b}\,x^{\frac{n}{2}}}{n}\right)c_1 + \text{BesselJ}\left(\frac{\sqrt{1-4a}}{n}+1, \frac{2\sqrt{b}\,x^{\frac{n}{2}}}{n}\right)\right)x^{\frac{n}{2}} - \left(\sqrt{1-4a}+1\right)\left(\text{BesselY}\left(\sqrt{\right.\right.}{2x\left(\text{BesselY}\left(\frac{\sqrt{1-4a}}{n}, \frac{2\sqrt{b}\,x^{\frac{n}{2}}}{n}\right)c_1 + \text{BesselJ}\left(\frac{\sqrt{1-4a}}{n}, \frac{2\sqrt{b}\,x^{\frac{n}{2}}}{n}\right)\right)}$$

✓ Solution by Mathematica

Time used: 0.976 (sec). Leaf size: 1434

```
DSolve[x^2 y'[x]==a+b x^n + x^2 y[x]^2,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to \frac{-n^{\frac{2\sqrt{(1-4a)n^2}}{n^2}+1}(x^n)^{\frac{i\sqrt{4a-1}}{n}+1}\text{BesselJ}\left(\frac{\sqrt{(1-4a)n^2}}{n^2}-1,\frac{2\sqrt{b}\sqrt{x^n}}{n}\right)\text{Gamma}\left(\frac{n+\sqrt{1-4a}}{n}\right)b^{\frac{i\sqrt{4a-1}}{n}+\frac{1}{2}}+n^{\frac{2\sqrt{(1-4a)n^2}}{n^2}}}{}$$

$$y(x) \to \frac{\frac{\sqrt{b}\sqrt{x^n}\left(\text{BesselJ}\left(1-\frac{\sqrt{(1-4a)n^2}}{n^2},\frac{2\sqrt{b}\sqrt{x^n}}{n}\right)-\text{BesselJ}\left(-\frac{\sqrt{(1-4a)n^2}}{n^2}-1,\frac{2\sqrt{b}\sqrt{x^n}}{n}\right)\right)}{\text{BesselJ}\left(-\frac{\sqrt{(1-4a)n^2}}{n^2},\frac{2\sqrt{b}\sqrt{x^n}}{n}\right)}-\frac{\sqrt{(1-4a)n^2}}{n}+i\sqrt{4a-1}-1}{2x}$$

9.24 problem 264

Internal problem ID [3520]

Book: Ordinary differential equations and their solutions. By George Moseley Murphy. 1960
Section: Various 9
Problem number: 264.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[[_homogeneous, ‘class G‘], _rational, _Riccati]`

$$\boxed{y'x^2 + xy(4 + yx) = -2}$$

✓ Solution by Maple
Time used: 0.0 (sec). Leaf size: 20

```
dsolve(x^2*diff(y(x),x)+2+x*y(x)*(4+x*y(x)) = 0,y(x), singsol=all)
```

$$y(x) = \frac{-2c_1 + x}{(c_1 - x)\,x}$$

✓ Solution by Mathematica
Time used: 0.16 (sec). Leaf size: 26

```
DSolve[x^2 y'[x]+2 + x y[x](4+x y[x])==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to -\frac{2}{x} + \frac{1}{x + c_1}$$
$$y(x) \to -\frac{2}{x}$$

9.25 problem 265

Internal problem ID [3521]

Book: Ordinary differential equations and their solutions. By George Moseley Murphy. 1960
Section: Various 9
Problem number: 265.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_rational, _Riccati]`

$$\boxed{y'x^2 + ax(-yx+1) - y^2x^2 = -2}$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 52

```
dsolve(x^2*diff(y(x),x)+2+a*x*(1-x*y(x))-x^2*y(x)^2 = 0,y(x), singsol=all)
```

$$y(x) = \frac{-(ax-1)(x^2a^2+2)\,\mathrm{e}^{ax} + c_1}{x\left((x^2a^2-2ax+2)\,\mathrm{e}^{ax} + c_1\right)}$$

✓ Solution by Mathematica

Time used: 0.366 (sec). Leaf size: 78

```
DSolve[x^2 y'[x]+2+a x(1-x y[x])-x^2 y[x]^2==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to \frac{e^{ax}(-a^3x^3 + a^2x^2 - 2ax + 2) + a^3c_1}{x\left(e^{ax}\left(a^2x^2 - 2ax + 2\right) + a^3c_1\right)}$$

$$y(x) \to \frac{1}{x}$$

9.26 problem 266

Internal problem ID [3522]

Book: Ordinary differential equations and their solutions. By George Moseley Murphy. 1960
Section: Various 9
Problem number: 266.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[[_homogeneous, ‘class G‘], _rational, [_Riccati, _special]]`

$$y'x^2 - b\,x^2y^2 = a$$

✓ Solution by Maple

Time used: 0.015 (sec). Leaf size: 40

```
dsolve(x^2*diff(y(x),x) = a+b*x^2*y(x)^2,y(x), singsol=all)
```

$$y(x) = \frac{-1 + \tan\left(\frac{\sqrt{4ab-1}\,(\ln(x)-c_1)}{2}\right)\sqrt{4ab-1}}{2bx}$$

✓ Solution by Mathematica

Time used: 0.188 (sec). Leaf size: 77

```
DSolve[x^2 y'[x]==a+b x^2 y[x]^2,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to \frac{-1 + \sqrt{1-4ab}\left(-1 + \frac{2c_1}{x^{\sqrt{1-4ab}}+c_1}\right)}{2bx}$$

$$y(x) \to \frac{\sqrt{1-4ab}-1}{2bx}$$

10 Various 10

10.1 problem 267

Internal problem ID [3523]

Book: Ordinary differential equations and their solutions. By George Moseley Murphy. 1960
Section: Various 10
Problem number: 267.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_rational, _Riccati]`

$$\boxed{y'x^2 - y^2 c\, x^2 = b\, x^n + a}$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 220

```
dsolve(x^2*diff(y(x),x) = a+b*x^n+c*x^2*y(x)^2,y(x), singsol=all)
```

$$y(x) = \frac{2\left(\text{BesselY}\left(\frac{\sqrt{-4ac+1}}{n}+1, \frac{2\sqrt{bc}\,x^{\frac{n}{2}}}{n}\right)c_1 + \text{BesselJ}\left(\frac{\sqrt{-4ac+1}}{n}+1, \frac{2\sqrt{bc}\,x^{\frac{n}{2}}}{n}\right)\right)\sqrt{bc}\,x^{\frac{n}{2}} - \left(\sqrt{-4ac+1}+1\right)\left(\text{B}\right.}{2xc\left(\text{BesselY}\left(\frac{\sqrt{-4ac+1}}{n}, \frac{2\sqrt{bc}\,x^{\frac{n}{2}}}{n}\right)c_1 + \text{BesselJ}\left(\frac{\sqrt{-4ac+1}}{n}\right.\right.}$$

✓ Solution by Mathematica

Time used: 1.136 (sec). Leaf size: 1779

```
DSolve[x^2 y'[x]==a+b x^n+c x^2 y[x]^2,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to \frac{-b^{\frac{i\sqrt{4ac-1}}{n}+\frac{1}{2}}n^{\frac{2\sqrt{(1-4ac)n^2}}{n^2}+1}(x^n)^{\frac{i\sqrt{4ac-1}}{n}+1}\text{BesselJ}\left(\frac{\sqrt{(1-4ac)n^2}}{n^2}-1,\frac{2\sqrt{b}\sqrt{c}\sqrt{x^n}}{n}\right)\text{Gamma}\left(\frac{n+\sqrt{1-4ac}}{n}\right)c^{\frac{i\sqrt{4ac-1}}{n}+\cdots}}{\cdots}$$

$$y(x) \to \frac{\frac{\sqrt{b}\sqrt{c}\sqrt{x^n}\left(\text{BesselJ}\left(1-\frac{\sqrt{(1-4ac)n^2}}{n^2},\frac{2\sqrt{b}\sqrt{c}\sqrt{x^n}}{n}\right)-\text{BesselJ}\left(-\frac{\sqrt{(1-4ac)n^2}}{n^2}-1,\frac{2\sqrt{b}\sqrt{c}\sqrt{x^n}}{n}\right)\right)}{\text{BesselJ}\left(-\frac{\sqrt{(1-4ac)n^2}}{n^2},\frac{2\sqrt{b}\sqrt{c}\sqrt{x^n}}{n}\right)}-\frac{\sqrt{n^2(1-4ac)}}{n}+i\sqrt{4ac-1}-1}{2cx}$$

10.2 problem 268

Internal problem ID [3524]

Book: Ordinary differential equations and their solutions. By George Moseley Murphy. 1960
Section: Various 10
Problem number: 268.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[[_homogeneous, ‘class G‘], _rational, _Riccati]`

$$\boxed{y' x^2 - bxy - y^2 c x^2 = a}$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 59

```
dsolve(x^2*diff(y(x),x) = a+b*x*y(x)+c*x^2*y(x)^2,y(x), singsol=all)
```

$$y(x) = \frac{-1 - b + \tan\left(\frac{\sqrt{4ac-b^2-2b-1}\,(\ln(x)-c_1)}{2}\right)\sqrt{4ac-b^2-2b-1}}{2cx}$$

✓ Solution by Mathematica

Time used: 0.272 (sec). Leaf size: 99

```
DSolve[x^2 y'[x]==a+b x y[x]+c x^2 y[x]^2,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to -\frac{\sqrt{-4ac+b^2+2b+1}\left(1 - \frac{2c_1}{x^{\sqrt{-4ac+b^2+2b+1}} + c_1}\right) + b + 1}{2cx}$$

$$y(x) \to -\frac{-\sqrt{-4ac+b^2+2b+1} + b + 1}{2cx}$$

10.3 problem 269

Internal problem ID [3525]

Book: Ordinary differential equations and their solutions. By George Moseley Murphy. 1960
Section: Various 10
Problem number: 269.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_rational, _Riccati]`

$$y'x^2 - bxy - c\,x^4y^2 = a$$

✓ Solution by Maple
Time used: 0.0 (sec). Leaf size: 111

```
dsolve(x^2*diff(y(x),x) = a+b*x*y(x)+c*x^4*y(x)^2,y(x), singsol=all)
```

$$y(x) = -\frac{a\left(\text{BesselY}\left(-\frac{1}{2}-\frac{b}{2}, x\sqrt{ac}\right)c_1 + \text{BesselJ}\left(-\frac{1}{2}-\frac{b}{2}, x\sqrt{ac}\right)\right)}{x\left(x\sqrt{ac}\left(c_1\,\text{BesselY}\left(\frac{1}{2}-\frac{b}{2}, x\sqrt{ac}\right) + \text{BesselJ}\left(\frac{1}{2}-\frac{b}{2}, x\sqrt{ac}\right)\right) + (b+1)\left(\text{BesselY}\left(-\frac{1}{2}-\frac{b}{2}, x\sqrt{ac}\right)c_1\right.\right.}$$

✓ Solution by Mathematica
Time used: 0.387 (sec). Leaf size: 394

```
DSolve[x^2 y'[x]==a+b x y[x]+c x^4 y[x]^2,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to -\frac{\sqrt{a}\sqrt{cx}\,\text{BesselY}\left(\frac{b+1}{2}, \sqrt{a}\sqrt{cx}\right) + (b+3)\,\text{BesselY}\left(\frac{b+3}{2}, \sqrt{a}\sqrt{cx}\right) - \sqrt{a}\sqrt{cx}\,\text{BesselY}\left(\frac{b+5}{2}, \sqrt{a}\sqrt{cx}\right) + }{2cx^3\left(\text{BesselY}\left(\frac{b+}{2}\right.\right.}$$

$$y(x) \to -\frac{\sqrt{a}\sqrt{cx}\,\text{BesselJ}\left(\frac{b+1}{2}, \sqrt{a}\sqrt{cx}\right) + (b+3)\,\text{BesselJ}\left(\frac{b+3}{2}, \sqrt{a}\sqrt{cx}\right) - \sqrt{a}\sqrt{cx}\,\text{BesselJ}\left(\frac{b+5}{2}, \sqrt{a}\sqrt{cx}\right)}{2cx^3\,\text{BesselJ}\left(\frac{b+3}{2}, \sqrt{a}\sqrt{cx}\right)}$$

10.4 problem 270

Internal problem ID [3526]

Book: Ordinary differential equations and their solutions. By George Moseley Murphy. 1960
Section: Various 10
Problem number: 270.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[[_homogeneous, ‘class D‘], _rational, _Bernoulli]`

$$y'x^2 + (x^2 + y^2 - x)\, y = 0$$

✓ Solution by Maple

Time used: 0.016 (sec). Leaf size: 32

```
dsolve(x^2*diff(y(x),x)+(x^2+y(x)^2-x)*y(x) = 0,y(x), singsol=all)
```

$$y(x) = \frac{x}{\sqrt{c_1 e^{2x} - 1}}$$
$$y(x) = -\frac{x}{\sqrt{c_1 e^{2x} - 1}}$$

✓ Solution by Mathematica

Time used: 4.843 (sec). Leaf size: 47

```
DSolve[x^2 y'[x]+(x^2+y[x]^2-x)y[x]==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to -\frac{x}{\sqrt{-1 + c_1 e^{2x}}}$$
$$y(x) \to \frac{x}{\sqrt{-1 + c_1 e^{2x}}}$$
$$y(x) \to 0$$

10.5 problem 271

Internal problem ID [3527]

Book: Ordinary differential equations and their solutions. By George Moseley Murphy. 1960
Section: Various 10
Problem number: 271.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[[_homogeneous, ‘class G‘], _rational, _Bernoulli]`

$$y'x^2 - 2y(x - y^2) = 0$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 39

```
dsolve(x^2*diff(y(x),x) = 2*y(x)*(x-y(x)^2),y(x), singsol=all)
```

$$y(x) = -\frac{3x^2}{\sqrt{12x^3 + 9c_1}}$$
$$y(x) = \frac{3x^2}{\sqrt{12x^3 + 9c_1}}$$

✓ Solution by Mathematica

Time used: 0.238 (sec). Leaf size: 51

```
DSolve[x^2 y'[x]==2 y[x](x-y[x]^2),y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to -\frac{x^2}{\sqrt{\frac{4x^3}{3} + c_1}}$$
$$y(x) \to \frac{x^2}{\sqrt{\frac{4x^3}{3} + c_1}}$$
$$y(x) \to 0$$

10.6 problem 272

Internal problem ID [3528]

Book: Ordinary differential equations and their solutions. By George Moseley Murphy. 1960
Section: Various 10
Problem number: 272.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_rational, _Abel]`

$$\boxed{y'x^2 - a\,x^2y^2 + ay^3 = 0}$$

✓ Solution by Maple
Time used: 0.0 (sec). Leaf size: 148

```
dsolve(x^2*diff(y(x),x) = a*x^2*y(x)^2-a*y(x)^3,y(x), singsol=all)
```

$$y(x) = \frac{1}{-ax - 2^{\frac{2}{3}}\left(-a\right)^{\frac{2}{3}}\operatorname{RootOf}\left(\operatorname{AiryBi}\left(\frac{\left(\_Z^2 2^{\frac{1}{3}}(-a)^{\frac{1}{3}}x-1\right)2^{\frac{2}{3}}}{2(-a)^{\frac{1}{3}}x}\right)c_1\_Z + \_Z\operatorname{AiryAi}\left(\frac{\left(\_Z^2 2^{\frac{1}{3}}(-a)^{\frac{1}{3}}x-1\right)2^{\frac{2}{3}}}{2(-a)^{\frac{1}{3}}x}\right) + \mathrm{Ai}}$$

✓ Solution by Mathematica
Time used: 0.458 (sec). Leaf size: 267

```
DSolve[x^2 y'[x]==a x^2 y[x]^2-a y[x]^3,y[x],x,IncludeSingularSolutions -> True]
```

$$\text{Solve}\left[\frac{\left(-\frac{1}{2^{2/3}a^{2/3}y(x)} - \frac{\sqrt[3]{a}x}{2^{2/3}}\right)\operatorname{AiryAi}\left(\left(-\frac{\sqrt[3]{a}x}{2^{2/3}} - \frac{1}{2^{2/3}a^{2/3}y(x)}\right)^2 + \frac{1}{\sqrt[3]{2}\sqrt[3]{a}x}\right) + \operatorname{AiryAiPrime}\left(\left(-\frac{\sqrt[3]{a}x}{2^{2/3}}\right.\right.}{\left(-\frac{1}{2^{2/3}a^{2/3}y(x)} - \frac{\sqrt[3]{a}x}{2^{2/3}}\right)\operatorname{AiryBi}\left(\left(-\frac{\sqrt[3]{a}x}{2^{2/3}} - \frac{1}{2^{2/3}a^{2/3}y(x)}\right)^2 + \frac{1}{\sqrt[3]{2}\sqrt[3]{a}x}\right) + \operatorname{AiryBiPrime}\left(\left(-\frac{\sqrt[3]{a}x}{2^{2/3}}\right.\right.}\right.$$

$$\left. + c_1 = 0, y(x)\right]$$

10.7 problem 273

Internal problem ID [3529]

Book: Ordinary differential equations and their solutions. By George Moseley Murphy. 1960
Section: Various 10
Problem number: 273.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_rational, _Abel]`

$$\boxed{y'x^2 + ay^2 + b\,x^2y^3 = 0}$$

✓ Solution by Maple
Time used: 0.0 (sec). Leaf size: 178

```
dsolve(x^2*diff(y(x),x)+a*y(x)^2+b*x^2*y(x)^3 = 0,y(x), singsol=all)
```

$$y(x) =$$

$$-\frac{2^{\frac{1}{3}}abx}{2^{\frac{1}{3}}a^2b - 2\left(a^2b^2\right)^{\frac{2}{3}} \operatorname{RootOf}\left(\operatorname{AiryBi}\left(-\frac{b2^{\frac{2}{3}}x-2\_Z^2\left(a^2b^2\right)^{\frac{1}{3}}}{2\left(a^2b^2\right)^{\frac{1}{3}}}\right)c_1\_Z + \_Z\operatorname{AiryAi}\left(-\frac{b2^{\frac{2}{3}}x-2\_Z^2\left(a^2b^2\right)^{\frac{1}{3}}}{2\left(a^2b^2\right)^{\frac{1}{3}}}\right) + \right.}$$

✓ Solution by Mathematica
Time used: 0.609 (sec). Leaf size: 343

```
DSolve[x^2 y'[x]+a y[x]^2+b x^2 y[x]^3==0,y[x],x,IncludeSingularSolutions -> True]
```

$$\operatorname{Solve}\left[\frac{\left(\frac{a^{2/3}}{2^{2/3}\sqrt[3]{b}x}+\frac{1}{2^{2/3}\sqrt[3]{a}\sqrt[3]{b}y(x)}\right)\operatorname{AiryAi}\left(\left(\frac{a^{2/3}}{2^{2/3}\sqrt[3]{b}x}+\frac{1}{2^{2/3}\sqrt[3]{b}y(x)\sqrt[3]{a}}\right)^2-\frac{\sqrt[3]{b}x}{\sqrt[3]{2}a^{2/3}}\right)+\operatorname{AiryAiPrime}\left(\left(\right.\right.}{\left(\frac{a^{2/3}}{2^{2/3}\sqrt[3]{b}x}+\frac{1}{2^{2/3}\sqrt[3]{a}\sqrt[3]{b}y(x)}\right)\operatorname{AiryBi}\left(\left(\frac{a^{2/3}}{2^{2/3}\sqrt[3]{b}x}+\frac{1}{2^{2/3}\sqrt[3]{b}y(x)\sqrt[3]{a}}\right)^2-\frac{\sqrt[3]{b}x}{\sqrt[3]{2}a^{2/3}}\right)+\operatorname{AiryBiPrime}\left(\left(\right.\right.}\right.$$

$$\left. + c_1 = 0, y(x)\right]$$

10.8 problem 274

Internal problem ID [3530]

Book: Ordinary differential equations and their solutions. By George Moseley Murphy. 1960
Section: Various 10
Problem number: 274.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[[_homogeneous, ‘class G‘], _rational, _Bernoulli]`

$$\boxed{y'x^2 - (ax + by^3)\,y = 0}$$

✓ Solution by Maple
Time used: 0.016 (sec). Leaf size: 174

```
dsolve(x^2*diff(y(x),x) = (a*x+b*y(x)^3)*y(x),y(x), singsol=all)
```

$$y(x) = \frac{27^{\frac{1}{3}}\left(\left(a-\frac{1}{3}\right)x\left(c_1\left(a-\frac{1}{3}\right)x^{-3a+1}-b\right)^2\right)^{\frac{1}{3}}}{c_1\,(3a-1)\,x^{-3a+1}-3b}$$

$$y(x) = -\frac{27^{\frac{1}{3}}\left(\left(a-\frac{1}{3}\right)x\left(c_1\left(a-\frac{1}{3}\right)x^{-3a+1}-b\right)^2\right)^{\frac{1}{3}}\left(1+i\sqrt{3}\right)}{(6a-2)\,c_1x^{-3a+1}-6b}$$

$$y(x) = \frac{27^{\frac{1}{3}}\left(\left(a-\frac{1}{3}\right)x\left(c_1\left(a-\frac{1}{3}\right)x^{-3a+1}-b\right)^2\right)^{\frac{1}{3}}\left(i\sqrt{3}-1\right)}{(6a-2)\,c_1x^{-3a+1}-6b}$$

✓ Solution by Mathematica

Time used: 3.526 (sec). Leaf size: 149

```
DSolve[x^2 y'[x]==(a x+b y[x]^3)y[x],y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to \frac{\sqrt[3]{(1-3a)x^{3a+1}}}{\sqrt[3]{3bx^{3a}+(1-3a)c_1x}}$$
$$y(x) \to -\frac{\sqrt[3]{-1}\sqrt[3]{(1-3a)x^{3a+1}}}{\sqrt[3]{3bx^{3a}+(1-3a)c_1x}}$$
$$y(x) \to \frac{(-1)^{2/3}\sqrt[3]{(1-3a)x^{3a+1}}}{\sqrt[3]{3bx^{3a}+(1-3a)c_1x}}$$
$$y(x) \to 0$$

10.9 problem 275

Internal problem ID [3531]

Book: Ordinary differential equations and their solutions. By George Moseley Murphy. 1960
Section: Various 10
Problem number: 275.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[[_homogeneous, ‘class G‘], _rational, _Bernoulli]`

$$\boxed{y' x^2 + yx + \sqrt{y} = 0}$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 19

```
dsolve(x^2*diff(y(x),x)+x*y(x)+sqrt(y(x)) = 0,y(x), singsol=all)
```

$$\sqrt{y(x)} - \frac{1}{x} - \frac{c_1}{\sqrt{x}} = 0$$

✓ Solution by Mathematica

Time used: 0.151 (sec). Leaf size: 21

```
DSolve[x^2 y'[x]+x y[x]+Sqrt[y[x]]==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to \frac{(1 + c_1\sqrt{x})^2}{x^2}$$

10.10 problem 276

Internal problem ID [3532]

Book: Ordinary differential equations and their solutions. By George Moseley Murphy. 1960
Section: Various 10
Problem number: 276.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[‘y=_G(x,y’)‘]`

$$\boxed{y'x^2 - \sec(y) - 3x\tan(y) = 0}$$

✓ Solution by Maple
Time used: 0.015 (sec). Leaf size: 17

```
dsolve(x^2*diff(y(x),x) = sec(y(x))+3*x*tan(y(x)),y(x), singsol=all)
```

$$y(x) = \arcsin\left(\frac{c_1 x^4 - 1}{4x}\right)$$

✓ Solution by Mathematica
Time used: 10.03 (sec). Leaf size: 23

```
DSolve[x^2 y'[x]==Sec[y[x]]+3 x Tan[y[x]],y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to -\arcsin\left(\frac{1}{4x} + 3c_1 x^3\right)$$

10.11 problem 277

Internal problem ID [3533]

Book: Ordinary differential equations and their solutions. By George Moseley Murphy. 1960
Section: Various 10
Problem number: 277.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_linear]`

$$\boxed{\left(-x^2+1\right)y'-y=-x^2+1}$$

✓ Solution by Maple
Time used: 0.0 (sec). Leaf size: 30

```
dsolve((-x^2+1)*diff(y(x),x) = 1-x^2+y(x),y(x), singsol=all)
```

$$y(x)=\frac{\left(\sqrt{-x^2+1}+\arcsin\left(x\right)+c_1\right)\left(x+1\right)}{\sqrt{-x^2+1}}$$

✓ Solution by Mathematica
Time used: 0.142 (sec). Leaf size: 56

```
DSolve[(1-x^2)y'[x]==1-x^2+y[x],y[x],x,IncludeSingularSolutions -> True]
```

$$y(x)\to\frac{\sqrt{x+1}\left(-2\arctan\left(\frac{\sqrt{1-x^2}}{x-1}\right)+\sqrt{1-x^2}+c_1\right)}{\sqrt{1-x}}$$

10.12 problem 278

Internal problem ID [3534]

Book: Ordinary differential equations and their solutions. By George Moseley Murphy. 1960
Section: Various 10
Problem number: 278.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_linear]`

$$\boxed{(-x^2+1)\,y'-yx=-1}$$

✓ Solution by Maple
Time used: 0.0 (sec). Leaf size: 45

```
dsolve((-x^2+1)*diff(y(x),x)+1 = x*y(x),y(x), singsol=all)
```

$$y(x)=\frac{\sqrt{x^2-1}\,\ln\left(x+\sqrt{x^2-1}\right)}{(x-1)\,(x+1)}+\frac{c_1}{\sqrt{x-1}\,\sqrt{x+1}}$$

✓ Solution by Mathematica
Time used: 0.035 (sec). Leaf size: 54

```
DSolve[(1-x^2)y'[x]+1==x y[x],y[x],x,IncludeSingularSolutions -> True]
```

$$y(x)\to\frac{-\log\left(1-\frac{x}{\sqrt{x^2-1}}\right)+\log\left(\frac{x}{\sqrt{x^2-1}}+1\right)+2c_1}{2\sqrt{x^2-1}}$$

10.13 problem 279

Internal problem ID [3535]

Book: Ordinary differential equations and their solutions. By George Moseley Murphy. 1960
Section: Various 10
Problem number: 279.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_linear]`

$$\boxed{(-x^2+1)\, y' + yx = 5}$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 20

```
dsolve((-x^2+1)*diff(y(x),x) = 5-x*y(x),y(x), singsol=all)
```

$$y(x) = \sqrt{x-1}\,\sqrt{x+1}\,c_1 + 5x$$

✓ Solution by Mathematica

Time used: 0.071 (sec). Leaf size: 21

```
DSolve[(1-x^2)y'[x]==5 -x y[x],y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \rightarrow 5x + c_1\sqrt{x^2-1}$$

10.14 problem 280

Internal problem ID [3536]

Book: Ordinary differential equations and their solutions. By George Moseley Murphy. 1960
Section: Various 10
Problem number: 280.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_linear]`

$$\boxed{(x^2+1)\,y'+yx=-a}$$

✓ Solution by Maple
Time used: 0.0 (sec). Leaf size: 19

```
dsolve((x^2+1)*diff(y(x),x)+a+x*y(x) = 0,y(x), singsol=all)
```

$$y(x)=\frac{-a\,\operatorname{arcsinh}(x)+c_1}{\sqrt{x^2+1}}$$

✓ Solution by Mathematica
Time used: 0.033 (sec). Leaf size: 34

```
DSolve[(1+x^2)y'[x]+a+x y[x]==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x)\to\frac{a\log\left(\sqrt{x^2+1}-x\right)+c_1}{\sqrt{x^2+1}}$$

10.15 problem 281

Internal problem ID [3537]

Book: Ordinary differential equations and their solutions. By George Moseley Murphy. 1960
Section: Various 10
Problem number: 281.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_linear]`

$$\boxed{(x^2+1)\,y' - yx = -a}$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 18

```
dsolve((x^2+1)*diff(y(x),x)+a-x*y(x) = 0,y(x), singsol=all)
```

$$y(x) = \sqrt{x^2+1}\,c_1 - ax$$

✓ Solution by Mathematica

Time used: 0.066 (sec). Leaf size: 22

```
DSolve[(1+x^2)y'[x]+a-x y[x]==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to -ax + c_1\sqrt{x^2+1}$$

10.16 problem 282

Internal problem ID [3538]

Book: Ordinary differential equations and their solutions. By George Moseley Murphy. 1960
Section: Various 10
Problem number: 282.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_linear]`

$$\boxed{\left(-x^2+1\right) y' - yx = -a}$$

✓ Solution by Maple
Time used: 0.0 (sec). Leaf size: 46

```
dsolve((-x^2+1)*diff(y(x),x)+a-x*y(x) = 0,y(x), singsol=all)
```

$$y(x) = \frac{a\sqrt{x^2-1}\,\ln\left(x+\sqrt{x^2-1}\right)}{(x-1)(x+1)} + \frac{c_1}{\sqrt{x-1}\,\sqrt{x+1}}$$

✓ Solution by Mathematica
Time used: 0.033 (sec). Leaf size: 57

```
DSolve[(1-x^2)y'[x]+a-x y[x]==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to \frac{-a\log\left(1-\frac{x}{\sqrt{x^2-1}}\right)+a\log\left(\frac{x}{\sqrt{x^2-1}}+1\right)+2c_1}{2\sqrt{x^2-1}}$$

10.17 problem 283

Internal problem ID [3539]

Book: Ordinary differential equations and their solutions. By George Moseley Murphy. 1960
Section: Various 10
Problem number: 283.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_separable]`

$$\boxed{(-x^2+1)\, y' + yx = x}$$

✓ Solution by Maple
Time used: 0.0 (sec). Leaf size: 18

```
dsolve((-x^2+1)*diff(y(x),x)-x+x*y(x) = 0,y(x), singsol=all)
```

$$y(x) = \sqrt{x-1}\,\sqrt{x+1}\,c_1 + 1$$

✓ Solution by Mathematica
Time used: 0.034 (sec). Leaf size: 24

```
DSolve[(1-x^2)y'[x]-x +x y[x]==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to 1 + c_1\sqrt{x^2-1}$$
$$y(x) \to 1$$

10.18 problem 284

Internal problem ID [3540]

Book: Ordinary differential equations and their solutions. By George Moseley Murphy. 1960
Section: Various 10
Problem number: 284.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_linear]`

$$\boxed{(-x^2+1)\, y' + yx = x^2}$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 66

```
dsolve((-x^2+1)*diff(y(x),x)-x^2+x*y(x) = 0,y(x), singsol=all)
```

$$y(x) = \frac{\sqrt{x-1}\,\sqrt{x+1}\, c_1\sqrt{x^2-1} - \ln\left(x+\sqrt{x^2-1}\right) x^2 + \sqrt{x^2-1}\, x + \ln\left(x+\sqrt{x^2-1}\right)}{\sqrt{x^2-1}}$$

✓ Solution by Mathematica

Time used: 0.095 (sec). Leaf size: 43

```
DSolve[(1-x^2)y'[x]-x^2 +x y[x]==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to \sqrt{x^2-1}\log\left(\sqrt{x^2-1}-x\right) + c_1\sqrt{x^2-1} + x$$

10.19 problem 285

Internal problem ID [3541]

Book: Ordinary differential equations and their solutions. By George Moseley Murphy. 1960
Section: Various 10
Problem number: 285.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_linear]`

$$\boxed{(-x^2+1)\,y'+yx=-x^2}$$

✓ Solution by Maple
Time used: 0.016 (sec). Leaf size: 53

```
dsolve((-x^2+1)*diff(y(x),x)+x^2+x*y(x) = 0,y(x), singsol=all)
```

$$y(x)=\frac{(x^2-1)\ln\left(x+\sqrt{x^2-1}\right)-\sqrt{x^2-1}\left(-\sqrt{x-1}\,\sqrt{x+1}\,c_1+x\right)}{\sqrt{x^2-1}}$$

✓ Solution by Mathematica
Time used: 0.041 (sec). Leaf size: 46

```
DSolve[(1-x^2)y'[x]+x^2 +x y[x]==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x)\to -\sqrt{x^2-1}\log\left(\sqrt{x^2-1}-x\right)+c_1\sqrt{x^2-1}-x$$

10.20 problem 286

Internal problem ID [3542]

Book: Ordinary differential equations and their solutions. By George Moseley Murphy. 1960
Section: Various 10
Problem number: 286.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_linear]`

$$\boxed{(x^2+1)\, y'+yx=x(x^2+1)}$$

✓ Solution by Maple
Time used: 0.0 (sec). Leaf size: 20

```
dsolve((x^2+1)*diff(y(x),x) = x*(x^2+1)-x*y(x),y(x), singsol=all)
```

$$y(x)=\frac{x^2}{3}+\frac{1}{3}+\frac{c_1}{\sqrt{x^2+1}}$$

✓ Solution by Mathematica
Time used: 0.056 (sec). Leaf size: 27

```
DSolve[(1+x^2)*y'[x]==x*(1+x^2)-x*y[x],y[x],x,IncludeSingularSolutions -> True]
```

$$y(x)\to \frac{1}{3}(x^2+1)+\frac{c_1}{\sqrt{x^2+1}}$$

10.21 problem 287

Internal problem ID [3543]

Book: Ordinary differential equations and their solutions. By George Moseley Murphy. 1960
Section: Various 10
Problem number: 287.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_linear]`

$$\boxed{(x^2+1)\,y' - x(3x^2 - y) = 0}$$

✓ Solution by Maple
Time used: 0.0 (sec). Leaf size: 18

```
dsolve((x^2+1)*diff(y(x),x) = x*(3*x^2-y(x)),y(x), singsol=all)
```

$$y(x) = x^2 - 2 + \frac{c_1}{\sqrt{x^2+1}}$$

✓ Solution by Mathematica
Time used: 0.06 (sec). Leaf size: 22

```
DSolve[(1+x^2)*y'[x]==x*(3*x^2-y[x]),y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to x^2 + \frac{c_1}{\sqrt{x^2+1}} - 2$$

10.22 problem 288

Internal problem ID [3544]

Book: Ordinary differential equations and their solutions. By George Moseley Murphy. 1960
Section: Various 10
Problem number: 288.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_separable]`

$$\boxed{\left(-x^{2}+1\right) y^{\prime}+2 y x=0}$$

✓ Solution by Maple
Time used: 0.0 (sec). Leaf size: 13

```
dsolve((-x^2+1)*diff(y(x),x)+2*x*y(x) = 0,y(x), singsol=all)
```

$$y(x)=c_{1} x^{2}-c_{1}$$

✓ Solution by Mathematica
Time used: 0.029 (sec). Leaf size: 18

```
DSolve[(1-x^2)y'[x]+2 x y[x]==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x)\to c_1(x^2-1)$$
$$y(x)\to 0$$

10.23 problem 289

Internal problem ID [3545]

Book: Ordinary differential equations and their solutions. By George Moseley Murphy. 1960
Section: Various 10
Problem number: 289.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_linear]`

$$\boxed{(x^2+1)\,y'-2x(-y+x)=0}$$

✓ Solution by Maple
Time used: 0.0 (sec). Leaf size: 23

```
dsolve((x^2+1)*diff(y(x),x) = 2*x*(x-y(x)),y(x), singsol=all)
```

$$y(x)=\frac{2x^3+3c_1}{3x^2+3}$$

✓ Solution by Mathematica
Time used: 0.033 (sec). Leaf size: 25

```
DSolve[(1+x^2)y'[x]==2 x(x-y[x]),y[x],x,IncludeSingularSolutions -> True]
```

$$y(x)\to\frac{2x^3+3c_1}{3x^2+3}$$

10.24 problem 290

Internal problem ID [3546]

Book: Ordinary differential equations and their solutions. By George Moseley Murphy. 1960
Section: Various 10
Problem number: 290.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_linear]`

$$\boxed{(x^2+1)\,y' - 2yx = 2x(x^2+1)^2}$$

✓ Solution by Maple
Time used: 0.0 (sec). Leaf size: 15

```
dsolve((x^2+1)*diff(y(x),x) = 2*x*(x^2+1)^2+2*x*y(x),y(x), singsol=all)
```

$$y(x) = (x^2+c_1)\,(x^2+1)$$

✓ Solution by Mathematica
Time used: 0.033 (sec). Leaf size: 17

```
DSolve[(1+x^2)y'[x]==2 x(1+x^2)^2+2 x y[x],y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to (x^2+1)\,(x^2+c_1)$$

10.25 problem 291

Internal problem ID [3547]

Book: Ordinary differential equations and their solutions. By George Moseley Murphy. 1960
Section: Various 10
Problem number: 291.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_linear]`

$$\boxed{(-x^2+1)\, y' - 2yx = -\cos(x)}$$

✓ Solution by Maple
Time used: 0.0 (sec). Leaf size: 16

```
dsolve((-x^2+1)*diff(y(x),x)+cos(x) = 2*x*y(x),y(x), singsol=all)
```

$$y(x) = \frac{\sin(x) + c_1}{x^2 - 1}$$

✓ Solution by Mathematica
Time used: 0.038 (sec). Leaf size: 18

```
DSolve[(1-x^2)y'[x]+Cos[x]==2 x y[x],y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to \frac{\sin(x) + c_1}{x^2 - 1}$$

11 Various 11

11.1 problem 292

Internal problem ID [3548]

Book: Ordinary differential equations and their solutions. By George Moseley Murphy. 1960
Section: Various 11
Problem number: 292.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_linear]`

$$2yx + (x^2 + 1)\, y' = \tan(x)$$

✓ Solution by Maple
Time used: 0.0 (sec). Leaf size: 19

```
dsolve((x^2+1)*diff(y(x),x) = tan(x)-2*x*y(x),y(x), singsol=all)
```

$$y(x) = \frac{-\ln(\cos(x)) + c_1}{x^2 + 1}$$

✓ Solution by Mathematica
Time used: 0.051 (sec). Leaf size: 21

```
DSolve[(1+x^2)y'[x]==Tan[x]-2 x y[x],y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to \frac{-\log(\cos(x)) + c_1}{x^2 + 1}$$

11.2 problem 293

Internal problem ID [3549]

Book: Ordinary differential equations and their solutions. By George Moseley Murphy. 1960
Section: Various 11
Problem number: 293.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_linear]`

$$\boxed{(-x^2+1)\,y'-4yx=a}$$

✓ Solution by Maple
Time used: 0.0 (sec). Leaf size: 30

```
dsolve((-x^2+1)*diff(y(x),x) = a+4*x*y(x),y(x), singsol=all)
```

$$y(x)=\frac{-a\,x^3+3ax+3c_1}{3\,(x-1)^2\,(x+1)^2}$$

✓ Solution by Mathematica
Time used: 0.031 (sec). Leaf size: 30

```
DSolve[(1-x^2)y'[x]==a+4 x y[x],y[x],x,IncludeSingularSolutions -> True]
```

$$y(x)\to\frac{-ax(x^2-3)+3c_1}{3\,(x^2-1)^2}$$

11.3 problem 294

Internal problem ID [3550]

Book: Ordinary differential equations and their solutions. By George Moseley Murphy. 1960
Section: Various 11
Problem number: 294.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_separable]`

$$\boxed{(x^2+1)\,y' - (2bx+a)\,y = 0}$$

✓ Solution by Maple

Time used: 0.016 (sec). Leaf size: 18

```
dsolve((x^2+1)*diff(y(x),x) = (2*b*x+a)*y(x),y(x), singsol=all)
```

$$y(x) = c_1\left(x^2+1\right)^b \mathrm{e}^{a\arctan(x)}$$

✓ Solution by Mathematica

Time used: 0.046 (sec). Leaf size: 26

```
DSolve[(1+x^2)y'[x]==(a+2 b x)y[x],y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to c_1\left(x^2+1\right)^b e^{a\arctan(x)}$$
$$y(x) \to 0$$

11.4 problem 295

Internal problem ID [3551]

Book: Ordinary differential equations and their solutions. By George Moseley Murphy. 1960
Section: Various 11
Problem number: 295.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_separable]`

$$\boxed{(x^2+1)\,y'-y^2=1}$$

✓ Solution by Maple
Time used: 0.0 (sec). Leaf size: 9

```
dsolve((x^2+1)*diff(y(x),x) = 1+y(x)^2,y(x), singsol=all)
```

$$y(x)=\tan(\arctan(x)+c_1)$$

✓ Solution by Mathematica
Time used: 0.237 (sec). Leaf size: 25

```
DSolve[(1+x^2)y'[x]==(1+y[x]^2),y[x],x,IncludeSingularSolutions -> True]
```

$$y(x)\to\tan(\arctan(x)+c_1)$$
$$y(x)\to -i$$
$$y(x)\to i$$

11.5 problem 296

Internal problem ID [3552]

Book: Ordinary differential equations and their solutions. By George Moseley Murphy. 1960
Section: Various 11
Problem number: 296.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_separable]`

$$\boxed{(-x^2+1)\,y'+y^2=1}$$

✓ Solution by Maple
Time used: 0.0 (sec). Leaf size: 13

```
dsolve((-x^2+1)*diff(y(x),x) = 1-y(x)^2,y(x), singsol=all)
```

$$y(x) = -\tanh\left(-\operatorname{arctanh}(x)+c_1\right)$$

✓ Solution by Mathematica
Time used: 0.589 (sec). Leaf size: 47

```
DSolve[(1-x^2)y'[x]==(1-y[x]^2),y[x],x,IncludeSingularSolutions -> True]
```

$$\begin{aligned} y(x) &\to -\frac{x+e^{2c_1}(x-1)+1}{-x+e^{2c_1}(x-1)-1} \\ y(x) &\to -1 \\ y(x) &\to 1 \end{aligned}$$

11.6 problem 297

Internal problem ID [3553]

Book: Ordinary differential equations and their solutions. By George Moseley Murphy. 1960
Section: Various 11
Problem number: 297.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_rational, [_1st_order, '_with_symmetry_[F(x),G(x)]'], _Ricca`

$$\boxed{\left(-x^2+1\right) y' + (2x-y)\, y = 1}$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 14

```
dsolve((-x^2+1)*diff(y(x),x) = 1-(2*x-y(x))*y(x),y(x), singsol=all)
```

$$y(x) = x + \frac{1}{-\operatorname{arctanh}(x) + c_1}$$

✓ Solution by Mathematica

Time used: 0.213 (sec). Leaf size: 52

```
DSolve[(1-x^2)y'[x]==1-(2 x-y[x])y[x],y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to \frac{x\log(1-x) - x\log(x+1) + 2c_1x + 2}{\log(1-x) - \log(x+1) + 2c_1}$$

$$y(x) \to x$$

11.7 problem 298

Internal problem ID [3554]

Book: Ordinary differential equations and their solutions. By George Moseley Murphy. 1960
Section: Various 11
Problem number: 298.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type [_rational, _Riccati]

$$(-x^2+1)\,y' - n\left(y^2-2yx+1\right) = 0$$

✓ Solution by Maple
Time used: 0.016 (sec). Leaf size: 279

```
dsolve((-x^2+1)*diff(y(x),x) = n*(1-2*x*y(x)+y(x)^2),y(x), singsol=all)
```

$$y(x) = \frac{\left(\frac{x+1}{x-1}\right)^n \left(-\frac{x}{2}-\frac{1}{2}\right)^{2n} \left(16(x+1)^2 \left(\left(x-\frac{1}{2}\right)n+\frac{1}{2}-\frac{x}{2}\right) \text{hypergeom}\left(\left[-n+1,-n+1\right],\left[2-2n\right],-\frac{2}{x-1}\right) c_1 (\right.}{(x+1)^2 \left(8c_1 \text{h}\right.}$$

✓ Solution by Mathematica
Time used: 0.361 (sec). Leaf size: 47

```
DSolve[(1-x^2)*y'[x]==n*(1-2*x*y[x]+y[x]^2),y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to \frac{Q_n(x)+c_1 P_n(x)}{Q_{n-1}(x)+c_1 P_{n-1}(x)}$$
$$y(x) \to \frac{P_n(x)}{P_{n-1}(x)}$$

11.8 problem 299

Internal problem ID [3555]

Book: Ordinary differential equations and their solutions. By George Moseley Murphy. 1960
Section: Various 11
Problem number: 299.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_separable]`

$$\boxed{(x^2+1)\,y'+xy(1-y)=0}$$

✓ Solution by Maple
Time used: 0.0 (sec). Leaf size: 17

```
dsolve((x^2+1)*diff(y(x),x)+x*y(x)*(1-y(x)) = 0,y(x), singsol=all)
```

$$y(x)=\frac{1}{1+\sqrt{x^2+1}\,c_1}$$

✓ Solution by Mathematica
Time used: 2.27 (sec). Leaf size: 33

```
DSolve[(1+x^2)y'[x]+x y[x](1-y[x])==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x)\to\frac{1}{1+e^{c_1}\sqrt{x^2+1}}$$
$$y(x)\to 0$$
$$y(x)\to 1$$

11.9 problem 300

Internal problem ID [3556]

Book: Ordinary differential equations and their solutions. By George Moseley Murphy. 1960
Section: Various 11
Problem number: 300.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_separable]`

$$\boxed{(-x^2+1)\, y' - xy(1+ya) = 0}$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 22

```
dsolve((-x^2+1)*diff(y(x),x) = x*y(x)*(1+a*y(x)),y(x), singsol=all)
```

$$y(x) = \frac{1}{\sqrt{x-1}\,\sqrt{x+1}\,c_1 - a}$$

✓ Solution by Mathematica

Time used: 3.979 (sec). Leaf size: 47

```
DSolve[(1-x^2)y'[x]==x y[x](1+a y[x]),y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to -\frac{e^{c_1}}{-\sqrt{1-x^2}+ae^{c_1}}$$
$$y(x) \to 0$$
$$y(x) \to -\frac{1}{a}$$

11.10 problem 301

Internal problem ID [3557]

Book: Ordinary differential equations and their solutions. By George Moseley Murphy. 1960
Section: Various 11
Problem number: 301.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_rational, _Abel]`

$$\boxed{(x^2+1)\,y' - y^2 + 2xy(1+y^2) = 1}$$

✓ Solution by Maple
Time used: 0.0 (sec). Leaf size: 65

```
dsolve((x^2+1)*diff(y(x),x) = 1+y(x)^2-2*x*y(x)*(1+y(x)^2),y(x), singsol=all)
```

$$c_1 + \frac{x}{\left(\frac{(x^2+1)\left(y(x)^2+1\right)}{(-1+xy(x))^2}\right)^{\frac{1}{4}}} + \frac{(x+y(x))\,\mathrm{hypergeom}\left(\left[\frac{1}{2},\frac{5}{4}\right],\left[\frac{3}{2}\right],-\frac{(x+y(x))^2}{(-1+xy(x))^2}\right)}{2xy\,(x)-2} = 0$$

✓ Solution by Mathematica
Time used: 0.401 (sec). Leaf size: 203

```
DSolve[(1+x^2)y'[x]==1+y[x]^2-2 x y[x](1+y[x]^2),y[x],x,IncludeSingularSolutions -> True]
```

$$\mathrm{Solve}\left[c_1 = \frac{\frac{1}{2}\left(\frac{1}{\frac{ix}{x^2+1}-\frac{ix^2y(x)}{x^2+1}}+\frac{i}{x}\right)\sqrt[4]{1-\left(\frac{1}{\frac{ix}{x^2+1}-\frac{ix^2y(x)}{x^2+1}}+\frac{i}{x}\right)^2}\ \mathrm{Hypergeometric2F1}\left(\frac{1}{2},\frac{5}{4},\frac{3}{2},\left(\frac{1}{\frac{ix}{x^2+1}-\frac{ix^2y(x)}{x^2+1}}\right.\right.}{\sqrt[4]{-1+\left(\frac{1}{\frac{ix}{x^2+1}-\frac{ix^2y(x)}{x^2+1}}+\frac{i}{x}\right)^2}}\right.$$

11.11 problem 302

Internal problem ID [3558]

Book: Ordinary differential equations and their solutions. By George Moseley Murphy. 1960
Section: Various 11
Problem number: 302.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[‘y=_G(x,y’)‘]`

$$\boxed{(x^2+1)\,y'+\cos(y)\,x\sin(y)-x(x^2+1)\cos(y)^2=0}$$

✓ Solution by Maple

Time used: 0.047 (sec). Leaf size: 142

```
dsolve((x^2+1)*diff(y(x),x)+x*sin(y(x))*cos(y(x)) = x*(x^2+1)*cos(y(x))^2,y(x), singsol=all)
```

$$y(x)=\frac{\arctan\left(\frac{6\sqrt{x^2+1}\left(x^2\sqrt{x^2+1}+\sqrt{x^2+1}+3c_1\right)}{10+6c_1(x^2+1)^{\frac{3}{2}}+x^6+3x^4+12x^2+9c_1^2},\frac{8+6(-x^2-1)c_1\sqrt{x^2+1}-x^6-3x^4+6x^2-9c_1^2}{10+6c_1(x^2+1)^{\frac{3}{2}}+x^6+3x^4+12x^2+9c_1^2}\right)}{2}$$

✓ Solution by Mathematica

Time used: 8.716 (sec). Leaf size: 97

```
DSolve[(1+x^2)y'[x]+x Sin[y[x]] Cos[y[x]]==x(1+x^2) (Cos[y[x]])^2,y[x],x,IncludeSingularSolu
```

$$y(x)\to\arctan\left(\frac{x^4+2x^2-6c_1\sqrt{x^2+1}+1}{3x^2+3}\right)$$
$$y(x)\to-\frac{1}{2}\pi\sqrt{\frac{1}{x^2+1}}\sqrt{x^2+1}$$
$$y(x)\to\frac{1}{2}\pi\sqrt{\frac{1}{x^2+1}}\sqrt{x^2+1}$$

11.12 problem 303

Internal problem ID [3559]

Book: Ordinary differential equations and their solutions. By George Moseley Murphy. 1960
Section: Various 11
Problem number: 303.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_linear]`

$$\boxed{(x^2+1)\,y' + y \operatorname{arccot}(x) = x^2+1}$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 23

```
dsolve((x^2+1)*diff(y(x),x) = 1+x^2-y(x)*arccot(x),y(x), singsol=all)
```

$$y(x) = \left(\int \mathrm{e}^{-\frac{\operatorname{arccot}(x)^2}{2}}dx + c_1\right)\mathrm{e}^{\frac{\operatorname{arccot}(x)^2}{2}}$$

✓ Solution by Mathematica

Time used: 3.503 (sec). Leaf size: 37

```
DSolve[(1+x^2)y'[x]==(1+x^2)-y[x] ArcCot[x],y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to e^{\frac{1}{2}\cot^{-1}(x)^2}\left(\int_1^x e^{-\frac{1}{2}\cot^{-1}(K[1])^2}dK[1] + c_1\right)$$

11.13 problem 304

Internal problem ID [3560]

Book: Ordinary differential equations and their solutions. By George Moseley Murphy. 1960
Section: Various 11
Problem number: 304.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_rational, _Bernoulli]`

$$\boxed{(-x^2+4)\, y' + 4y - (x+2)\, y^2 = 0}$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 21

```
dsolve((-x^2+4)*diff(y(x),x)+4*y(x) = (2+x)*y(x)^2,y(x), singsol=all)
```

$$y(x) = \frac{-2+x}{(\ln(2+x)+c_1)(2+x)}$$

✓ Solution by Mathematica

Time used: 0.214 (sec). Leaf size: 32

```
DSolve[(4-x^2)y'[x]+4 y[x]==(2+x)y[x]^2,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to \frac{2-x}{(x+2)(-\log(x+2)+c_1)}$$
$$y(x) \to 0$$

11.14 problem 305

Internal problem ID [3561]

Book: Ordinary differential equations and their solutions. By George Moseley Murphy. 1960
Section: Various 11
Problem number: 305.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_linear]`

$$\boxed{(a^2 + x^2)\, y' - yx = b}$$

✓ Solution by Maple
Time used: 0.0 (sec). Leaf size: 26

```
dsolve((a^2+x^2)*diff(y(x),x) = b+x*y(x),y(x), singsol=all)
```

$$y(x) = \frac{\sqrt{a^2 + x^2}\, c_1 a^2 + bx}{a^2}$$

✓ Solution by Mathematica
Time used: 0.086 (sec). Leaf size: 26

```
DSolve[(a^2+x^2)y'[x]==b+x y[x],y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to \frac{bx}{a^2} + c_1\sqrt{a^2 + x^2}$$

11.15 problem 306

Internal problem ID [3562]

Book: Ordinary differential equations and their solutions. By George Moseley Murphy. 1960
Section: Various 11
Problem number: 306.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_separable]`

$$(a^2 + x^2)\, y' - (b + y)\left(x + \sqrt{a^2 + x^2}\right) = 0$$

✓ Solution by Maple
Time used: 0.0 (sec). Leaf size: 53

```
dsolve((a^2+x^2)*diff(y(x),x) = (b+y(x))*(x+sqrt(a^2+x^2)),y(x), singsol=all)
```

$$y(x) = \frac{\left(\sqrt{a^2 + x^2}\, c_1 a^2 + bx\right)\left(x\sqrt{a^2 + x^2} + a^2 + x^2\right)}{\sqrt{a^2 + x^2}\, a^2}$$

✓ Solution by Mathematica
Time used: 0.28 (sec). Leaf size: 81

```
DSolve[(a^2+x^2)y'[x]==(b+y[x])(x+Sqrt[a^2+x^2]),y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to \frac{\left(x\left(x - \sqrt{a^2 + x^2}\right) + a^2\right)\left(bx - c_1\sqrt{a^2 + x^2}\right)}{\sqrt{a^2 + x^2}\left(x - \sqrt{a^2 + x^2}\right)^2}$$
$$y(x) \to -b$$

11.16 problem 307

Internal problem ID [3563]

Book: Ordinary differential equations and their solutions. By George Moseley Murphy. 1960
Section: Various 11
Problem number: 307.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_rational, _Bernoulli]`

$$\boxed{(a^2 + x^2)\, y' + y(-y + x) = 0}$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 28

```
dsolve((a^2+x^2)*diff(y(x),x)+(x-y(x))*y(x) = 0,y(x), singsol=all)
```

$$y(x) = \frac{a^2}{\sqrt{a^2 + x^2}\, c_1 a^2 - x}$$

✓ Solution by Mathematica

Time used: 0.266 (sec). Leaf size: 37

```
DSolve[(x^2+a^2)y'[x]+(x-y[x])y[x]==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to \frac{a^2}{-x + a^2 c_1 \sqrt{a^2 + x^2}}$$
$$y(x) \to 0$$

11.17 problem 308

Internal problem ID [3564]

Book: Ordinary differential equations and their solutions. By George Moseley Murphy. 1960
Section: Various 11
Problem number: 308.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_rational, _Riccati]`

$$\boxed{(a^2 + x^2)\, y' - 3yx + 2y^2 = a^2}$$

✓ Solution by Maple

Time used: 0.047 (sec). Leaf size: 235

```
dsolve((a^2+x^2)*diff(y(x),x) = a^2+3*x*y(x)-2*y(x)^2,y(x), singsol=all)
```

$$y(x) = -\frac{2\left(\sqrt{2}\,\sqrt{\frac{ix-a}{a}}\,c_1 a^2(ia-x)\operatorname{HeunCPrime}\left(0,-\frac{1}{2},2,0,\frac{5}{4},\frac{-ia+x}{ia+x}\right)+\sqrt{\frac{ix+a}{a}}\,a^2(ia-x)\operatorname{HeunCPrime}\left(0,\frac{1}{2},\right.\right.}{\left(i\sqrt{2}\,\sqrt{\frac{ix-a}{a}}\,c_1 x+\frac{\sqrt{\frac{ix+a}{a}}\,(ix-a)}{2}\right)(ia}$$

✓ Solution by Mathematica

Time used: 1.077 (sec). Leaf size: 63

```
DSolve[(a^2+x^2)y'[x]==a^2+3 x y[x]-2 y[x]^2,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to \frac{a^2 c_1(-x)\sqrt{a^2+x^2}+a^2+2x^2}{2x-a^2 c_1\sqrt{a^2+x^2}}$$
$$y(x) \to x$$

11.18 problem 309

Internal problem ID [3565]

Book: Ordinary differential equations and their solutions. By George Moseley Murphy. 1960
Section: Various 11
Problem number: 309.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_separable]`

$$\boxed{(a^2 + x^2)\, y' + yx + bxy^2 = 0}$$

✓ Solution by Maple

Time used: 0.015 (sec). Leaf size: 21

```
dsolve((a^2+x^2)*diff(y(x),x)+x*y(x)+b*x*y(x)^2 = 0,y(x), singsol=all)
```

$$y(x) = \frac{1}{\sqrt{a^2 + x^2}\, c_1 - b}$$

✓ Solution by Mathematica

Time used: 3.985 (sec). Leaf size: 47

```
DSolve[(x^2+a^2)y'[x]+x y[x]+b x y[x]^2==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to -\frac{e^{c_1}}{-\sqrt{a^2 + x^2} + be^{c_1}}$$
$$y(x) \to 0$$
$$y(x) \to -\frac{1}{b}$$

11.19 problem 310

Internal problem ID [3566]

Book: Ordinary differential equations and their solutions. By George Moseley Murphy. 1960
Section: Various 11
Problem number: 310.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_linear]`

$$\boxed{x(1-x)\,y'-y(x+1)=a}$$

✓ Solution by Maple
Time used: 0.0 (sec). Leaf size: 23

```
dsolve(x*(1-x)*diff(y(x),x) = a+(1+x)*y(x),y(x), singsol=all)
```

$$y(x)=\frac{-ax\ln(x)+c_1x-a}{(x-1)^2}$$

✓ Solution by Mathematica
Time used: 0.043 (sec). Leaf size: 24

```
DSolve[x(1-x)y'[x]==a+(1+x)y[x],y[x],x,IncludeSingularSolutions -> True]
```

$$y(x)\to-\frac{ax\log(x)+a-c_1x}{(x-1)^2}$$

11.20 problem 311

Internal problem ID [3567]

Book: Ordinary differential equations and their solutions. By George Moseley Murphy. 1960
Section: Various 11
Problem number: 311.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_linear]`

$$\boxed{x(1-x)\,y'-2yx=2}$$

✓ Solution by Maple
Time used: 0.0 (sec). Leaf size: 19

```
dsolve(x*(1-x)*diff(y(x),x) = 2+2*x*y(x),y(x), singsol=all)
```

$$y(x)=\frac{-2x+2\ln(x)+c_1}{(x-1)^2}$$

✓ Solution by Mathematica
Time used: 0.029 (sec). Leaf size: 21

```
DSolve[x(1-x)y'[x]==2(1+x y[x]),y[x],x,IncludeSingularSolutions -> True]
```

$$y(x)\to\frac{-2x+2\log(x)+c_1}{(x-1)^2}$$

11.21 problem 312

Internal problem ID [3568]

Book: Ordinary differential equations and their solutions. By George Moseley Murphy. 1960
Section: Various 11
Problem number: 312.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_linear]`

$$\boxed{x(1-x)\,y'-2yx=-2}$$

✓ Solution by Maple
Time used: 0.0 (sec). Leaf size: 19

```
dsolve(x*(1-x)*diff(y(x),x) = 2*x*y(x)-2,y(x), singsol=all)
```

$$y(x)=\frac{2x-2\ln(x)+c_1}{(x-1)^2}$$

✓ Solution by Mathematica
Time used: 0.028 (sec). Leaf size: 21

```
DSolve[x(1-x)y'[x]==2(x y[x]-1),y[x],x,IncludeSingularSolutions -> True]
```

$$y(x)\to\frac{2x-2\log(x)+c_1}{(x-1)^2}$$

11.22 problem 313

Internal problem ID [3569]

Book: Ordinary differential equations and their solutions. By George Moseley Murphy. 1960
Section: Various 11
Problem number: 313.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_separable]`

$$\boxed{x(x+1)\,y'-(1-2x)\,y=0}$$

✓ Solution by Maple
Time used: 0.016 (sec). Leaf size: 12

```
dsolve(x*(1+x)*diff(y(x),x) = (1-2*x)*y(x),y(x), singsol=all)
```

$$y(x)=\frac{c_1x}{(x+1)^3}$$

✓ Solution by Mathematica
Time used: 0.033 (sec). Leaf size: 19

```
DSolve[x(1+x)y'[x]==(1-2 x)y[x],y[x],x,IncludeSingularSolutions -> True]
```

$$y(x)\to\frac{c_1x}{(x+1)^3}$$
$$y(x)\to 0$$

11.23 problem 314

Internal problem ID [3570]

Book: Ordinary differential equations and their solutions. By George Moseley Murphy. 1960
Section: Various 11
Problem number: 314.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_linear]`

$$x(1-x)\,y' + (1+2x)\,y = a$$

✓ Solution by Maple
Time used: 0.015 (sec). Leaf size: 19

```
dsolve(x*(1-x)*diff(y(x),x)+(1+2*x)*y(x) = a,y(x), singsol=all)
```

$$y(x) = \frac{3(x-1)^3 c_1 + a}{3x}$$

✓ Solution by Mathematica
Time used: 0.035 (sec). Leaf size: 23

```
DSolve[x(1-x)y'[x]+(1+2 x)y[x]==a,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to \frac{a - 3c_1(x-1)^3}{3x}$$

11.24 problem 315

Internal problem ID [3571]

Book: Ordinary differential equations and their solutions. By George Moseley Murphy. 1960
Section: Various 11
Problem number: 315.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_linear]`

$$\boxed{x(1-x)\,y'-2(-x+2)\,y=a}$$

✓ Solution by Maple
Time used: 0.0 (sec). Leaf size: 25

```
dsolve(x*(1-x)*diff(y(x),x) = a+2*(2-x)*y(x),y(x), singsol=all)
```

$$y(x)=\frac{12c_1x^4+4ax-3a}{12\left(x-1\right)^2}$$

✓ Solution by Mathematica
Time used: 0.042 (sec). Leaf size: 29

```
DSolve[x(1-x)y'[x]==a+2(2-x)y[x],y[x],x,IncludeSingularSolutions -> True]
```

$$y(x)\to\frac{a(4x-3)+12c_1x^4}{12(x-1)^2}$$

11.25 problem 316

Internal problem ID [3572]

Book: Ordinary differential equations and their solutions. By George Moseley Murphy. 1960
Section: Various 11
Problem number: 316.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_linear]`

$$x(1-x)\,y' - 3yx + y = -2$$

✓ Solution by Maple
Time used: 0.0 (sec). Leaf size: 21

```
dsolve(x*(1-x)*diff(y(x),x)+2-3*x*y(x)+y(x) = 0,y(x), singsol=all)
```

$$y(x) = \frac{x^2 + c_1 - 2x}{x\,(x-1)^2}$$

✓ Solution by Mathematica
Time used: 0.033 (sec). Leaf size: 23

```
DSolve[x(1-x)y'[x]+(2-3 x y[x]+y[x])==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to \frac{x^2 - 2x + c_1}{(x-1)^2 x}$$

11.26 problem 317

Internal problem ID [3573]

Book: Ordinary differential equations and their solutions. By George Moseley Murphy. 1960
Section: Various 11
Problem number: 317.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_linear]`

$$x(x+1)\,y' - \left(x^2+x-1\right)y = (x+1)\left(x^2-1\right)$$

✓ Solution by Maple
Time used: 0.0 (sec). Leaf size: 19

```
dsolve(x*(1+x)*diff(y(x),x) = (1+x)*(x^2-1)+(x^2+x-1)*y(x),y(x), singsol=all)
```

$$y(x) = -\frac{(x+1)\left(-\mathrm{e}^{x}c_1+x\right)}{x}$$

✓ Solution by Mathematica
Time used: 0.043 (sec). Leaf size: 22

```
DSolve[x(1+x)y'[x]==(x+1)(x^2-1)+(x^2+x-1)y[x],y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to -\frac{(x+1)\left(x-c_1e^{x}\right)}{x}$$

11.27 problem 318

Internal problem ID [3574]

Book: Ordinary differential equations and their solutions. By George Moseley Murphy. 1960
Section: Various 11
Problem number: 318.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_linear]`

$$\boxed{(x-2)\,(x-3)\,y' - 8y + 3yx = -x^2}$$

✓ Solution by Maple
Time used: 0.0 (sec). Leaf size: 27

```
dsolve((x-2)*(x-3)*diff(y(x),x)+x^2-8*y(x)+3*x*y(x) = 0,y(x), singsol=all)
```

$$y(x) = \frac{-\frac{1}{4}x^4 + \frac{2}{3}x^3 + c_1}{(x-3)\,(-2+x)^2}$$

✓ Solution by Mathematica
Time used: 0.042 (sec). Leaf size: 34

```
DSolve[(x-2)(x-3)y'[x]+x^2-8 y[x]+3 x y[x]==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to \frac{-3x^4 + 8x^3 - 12c_1}{12(x-3)(x-2)^2}$$

11.28 problem 319

Internal problem ID [3575]

Book: Ordinary differential equations and their solutions. By George Moseley Murphy. 1960
Section: Various 11
Problem number: 319.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_separable]`

$$x(x+a)\,y' - (b+cy)\,y = 0$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 32

```
dsolve(x*(a+x)*diff(y(x),x) = (b+c*y(x))*y(x),y(x), singsol=all)
```

$$y(x) = \frac{b}{(x+a)^{\frac{b}{a}}\,x^{-\frac{b}{a}}c_1b - c}$$

✓ Solution by Mathematica

Time used: 0.973 (sec). Leaf size: 65

```
DSolve[x(a+x)y'[x]==(b+c y[x])y[x],y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to -\frac{be^{bc_1}x^{\frac{b}{a}}}{-(a+x)^{\frac{b}{a}} + ce^{bc_1}x^{\frac{b}{a}}}$$
$$y(x) \to 0$$
$$y(x) \to -\frac{b}{c}$$

12 Various 12

12.1 problem 320

Internal problem ID [3576]

Book: Ordinary differential equations and their solutions. By George Moseley Murphy. 1960
Section: Various 12
Problem number: 320.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_separable]`

$$\boxed{(x+a)^2 y' - 2(x+a)(b+y) = 0}$$

✓ Solution by Maple
Time used: 0.0 (sec). Leaf size: 15

```
dsolve((a+x)^2*diff(y(x),x) = 2*(a+x)*(b+y(x)),y(x), singsol=all)
```

$$y(x) = -b + (x+a)^2 c_1$$

✓ Solution by Mathematica
Time used: 0.05 (sec). Leaf size: 24

```
DSolve[(a+x)^2 y'[x]==2(a+x)(b+y[x]),y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to -b + c_1(a+x)^2$$
$$y(x) \to -b$$

12.2 problem 321

Internal problem ID [3577]

Book: Ordinary differential equations and their solutions. By George Moseley Murphy. 1960
Section: Various 12
Problem number: 321.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[[_homogeneous, 'class C'], _rational, _Riccati]`

$$\boxed{(x-a)^2 y' + k(x+y-a)^2 + y^2 = 0}$$

✓ Solution by Maple
Time used: 0.297 (sec). Leaf size: 34

```
dsolve((x-a)^2*diff(y(x),x)+k*(x+y(x)-a)^2+y(x)^2 = 0,y(x), singsol=all)
```

$$y(x) = \frac{(c_1 k(a-x)-1)(a-x)}{-1+(k+1)(a-x)c_1}$$

✓ Solution by Mathematica
Time used: 0.209 (sec). Leaf size: 50

```
DSolve[(x-a)^2 y'[x]+k(x+y[x]-a)^2+y[x]^2==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to \frac{k(a-x)}{k+1} + \frac{1}{\frac{k+1}{a-x} + c_1}$$
$$y(x) \to \frac{k(a-x)}{k+1}$$

12.3 problem 322

Internal problem ID [3578]

Book: Ordinary differential equations and their solutions. By George Moseley Murphy. 1960
Section: Various 12
Problem number: 322.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_separable]`

$$\boxed{(x-a)(x-b)y'+ky=0}$$

✓ Solution by Maple
Time used: 0.0 (sec). Leaf size: 37

```
dsolve((x-a)*(x-b)*diff(y(x),x)+k*y(x) = 0,y(x), singsol=all)
```

$$y(x)=c_1(x-a)^{-\frac{k}{a-b}}(x-b)^{\frac{k}{a-b}}$$

✓ Solution by Mathematica
Time used: 0.052 (sec). Leaf size: 39

```
DSolve[(x-a)(x-b)y'[x]+k y[x]==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x)\to c_1e^{\frac{k(\log(x-b)-\log(x-a))}{a-b}}$$
$$y(x)\to 0$$

12.4 problem 323

Internal problem ID [3579]

Book: Ordinary differential equations and their solutions. By George Moseley Murphy. 1960
Section: Various 12
Problem number: 323.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_linear]`

$$\boxed{(x-a)\,(x-b)\,y'-(-a-b+2x)\,y=(x-a)\,(x-b)}$$

✓ Solution by Maple
Time used: 0.0 (sec). Leaf size: 44

```
dsolve((x-a)*(x-b)*diff(y(x),x) = (x-a)*(x-b)+(2*x-a-b)*y(x),y(x), singsol=all)
```

$$y(x)=\frac{(\ln(x-a)-\ln(x-b)+(a-b)\,c_1)\,(-x+b)\,(a-x)}{a-b}$$

✓ Solution by Mathematica
Time used: 0.064 (sec). Leaf size: 42

```
DSolve[(x-a)(x-b)y'[x]==(x-a)(x-b)+(2 x-a-b)y[x],y[x],x,IncludeSingularSolutions -> True]
```

$$y(x)\to(x-a)(x-b)\left(\frac{\log(x-a)-\log(x-b)}{a-b}+c_1\right)$$

12.5 problem 324

Internal problem ID [3580]

Book: Ordinary differential equations and their solutions. By George Moseley Murphy. 1960
Section: Various 12
Problem number: 324.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_separable]`

$$\boxed{(x-a)\,(x-b)\,y'-cy^2=0}$$

✓ Solution by Maple
Time used: 0.0 (sec). Leaf size: 37

```
dsolve((x-a)*(x-b)*diff(y(x),x) = c*y(x)^2,y(x), singsol=all)
```

$$y(x)=\frac{a-b}{-c\ln(x-a)+c\ln(x-b)+(a-b)\,c_1}$$

✓ Solution by Mathematica
Time used: 0.428 (sec). Leaf size: 44

```
DSolve[(x-a)(x-b)y'[x]==c y[x]^2,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x)\to\frac{b-a}{c_1(a-b)+c\log(x-a)-c\log(x-b)}$$
$$y(x)\to 0$$

12.6 problem 325

Internal problem ID [3581]

Book: Ordinary differential equations and their solutions. By George Moseley Murphy. 1960
Section: Various 12
Problem number: 325.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_separable]`

$$\boxed{(x-a)(x-b)\,y'+k(y-a)(y-b)=0}$$

✓ Solution by Maple
Time used: 0.14 (sec). Leaf size: 113

```
dsolve((x-a)*(x-b)*diff(y(x),x)+k*(y(x)-a)*(y(x)-b) = 0,y(x), singsol=all)
```

$$y(x)=-\frac{\left(b\left(\frac{-x+b}{a-x}\right)^{-k}\mathrm{e}^{c_1k(a-b)}+(x-a)^k\,(x-b)^{-k}\,(a-b)\,\mathrm{e}^{c_1k(a-b)}-b\right)\left(\frac{-x+b}{a-x}\right)^k}{\left(\frac{-x+b}{a-x}\right)^k-\mathrm{e}^{c_1k(a-b)}}$$

✓ Solution by Mathematica
Time used: 2.369 (sec). Leaf size: 80

```
DSolve[(x-a)(x-b)y'[x]+k(y[x]-a)(y[x]-b)==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x)\to\frac{ae^{bc_1}(x-a)^k-be^{ac_1}(x-b)^k}{e^{bc_1}(x-a)^k-e^{ac_1}(x-b)^k}$$
$$y(x)\to a$$
$$y(x)\to b$$

12.7 problem 326

Internal problem ID [3582]

Book: Ordinary differential equations and their solutions. By George Moseley Murphy. 1960
Section: Various 12
Problem number: 326.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_rational, [_1st_order, '_with_symmetry_[F(x),G(x)]'], _Ricca`

$$(x-a)(x-b)y'+k(x+y-a)(x+y-b)+y^2=0$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 54

```
dsolve((x-a)*(x-b)*diff(y(x),x)+k*(x+y(x)-a)*(x+y(x)-b)+y(x)^2 = 0,y(x), singsol=all)
```

$$y(x)=\frac{\left((-x+b)^{k+1}+c_1(a-x)^k(a-x)\right)k}{(k+1)\left(c_1(a-x)^k+(-x+b)^k\right)}$$

✓ Solution by Mathematica

Time used: 60.297 (sec). Leaf size: 99

```
DSolve[(x-a)(x-b)y'[x]+k(x+y[x]-a)(x+y[x]-b)+y[x]^2==0,y[x],x,IncludeSingularSolutions -> Tr
```

$$y(x)\to\frac{1}{2}\left(\frac{k(a+b-2x)}{k+1}+\sqrt{-\frac{k^2(a-b)^2}{(k+1)^2}}\tan\left(\frac{(k+1)\sqrt{-\frac{k^2(a-b)^2}{(k+1)^2}}(\log(x-b)-\log(x-a))}{2(a-b)}+c_1\right)\right)$$

12.8 problem 327

Internal problem ID [3583]

Book: Ordinary differential equations and their solutions. By George Moseley Murphy. 1960
Section: Various 12
Problem number: 327.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_separable]`

$$\boxed{2y'x^2 - y = 0}$$

✓ Solution by Maple
Time used: 0.0 (sec). Leaf size: 12

```
dsolve(2*x^2*diff(y(x),x) = y(x),y(x), singsol=all)
```

$$y(x) = c_1 e^{-\frac{1}{2x}}$$

✓ Solution by Mathematica
Time used: 0.025 (sec). Leaf size: 22

```
DSolve[2 x^2 y'[x]==y[x],y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to c_1 e^{-\frac{1}{2}/x}$$
$$y(x) \to 0$$

12.9 problem 328

Internal problem ID [3584]

Book: Ordinary differential equations and their solutions. By George Moseley Murphy. 1960
Section: Various 12
Problem number: 328.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_linear]`

$$\boxed{2y'x^2 + 2x^2 y \cot (x) = -\cot (x) x + 1}$$

✓ Solution by Maple
Time used: 0.0 (sec). Leaf size: 14

```
dsolve(2*x^2*diff(y(x),x)+x*cot(x)-1+2*x^2*y(x)*cot(x) = 0,y(x), singsol=all)
```

$$y(x) = -\frac{1}{2x} + \csc (x) c_1$$

✓ Solution by Mathematica
Time used: 0.068 (sec). Leaf size: 18

```
DSolve[2 x^2 y'[x]+x Cot[x]-1+2 x^2 y[x] Cot[x]==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to -\frac{1}{2x} + c_1 \csc(x)$$

12.10 problem 329

Internal problem ID [3585]

Book: Ordinary differential equations and their solutions. By George Moseley Murphy. 1960
Section: Various 12
Problem number: 329.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[[_homogeneous, ‘class G‘], _rational, _Riccati]`

$$\boxed{2y'x^2 + 2yx - y^2x^2 = -1}$$

✓ Solution by Maple
Time used: 0.0 (sec). Leaf size: 17

```
dsolve(2*x^2*diff(y(x),x)+1+2*x*y(x)-x^2*y(x)^2 = 0,y(x), singsol=all)
```

$$y(x) = \frac{\tanh\left(-\frac{\ln(x)}{2} + \frac{c_1}{2}\right)}{x}$$

✓ Solution by Mathematica
Time used: 1.03 (sec). Leaf size: 61

```
DSolve[2 x^2 y'[x]+1+2 x y[x]- x^2 y[x]^2==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to \frac{i\tan\left(\frac{1}{2}i\log(x) + c_1\right)}{x}$$
$$y(x) \to \frac{-x + e^{2i\text{Interval}[\{0,\pi\}]}}{x^2 + xe^{2i\text{Interval}[\{0,\pi\}]}}$$

12.11 problem 330

Internal problem ID [3586]

Book: Ordinary differential equations and their solutions. By George Moseley Murphy. 1960
Section: Various 12
Problem number: 330.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[[_homogeneous, ‘class D‘], _Riccati]`

$$2y'x^2 - 2yx - (-\cot(x)\,x + 1)\left(x^2 - y^2\right) = 0$$

✓ Solution by Maple
Time used: 0.0 (sec). Leaf size: 21

```
dsolve(2*x^2*diff(y(x),x) = 2*x*y(x)+(1-x*cot(x))*(x^2-y(x)^2),y(x), singsol=all)
```

$$y(x) = -\tanh\left(\frac{\ln(\sin(x))}{2} - \frac{\ln(x)}{2} + \frac{c_1}{2}\right)x$$

✓ Solution by Mathematica
Time used: 1.1 (sec). Leaf size: 44

```
DSolve[2 x^2 y'[x]==2 x y[x]+(1-x Cot[x])(x^2-y[x]^2),y[x],x,IncludeSingularSolutions -> Tru
```

$$y(x) \to \frac{x(x - e^{2c_1}\sin(x))}{x + e^{2c_1}\sin(x)}$$
$$y(x) \to -x$$
$$y(x) \to x$$

12.12 problem 331

Internal problem ID [3587]

Book: Ordinary differential equations and their solutions. By George Moseley Murphy. 1960
Section: Various 12
Problem number: 331.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_linear]`

$$2(-x^2+1)\,y' - y(x+1) = \sqrt{-x^2+1}$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 25

```
dsolve(2*(-x^2+1)*diff(y(x),x) = sqrt(-x^2+1)+(1+x)*y(x),y(x), singsol=all)
```

$$y(x) = \frac{c_1}{\sqrt{x-1}} + \frac{x+1}{\sqrt{-x^2+1}}$$

✓ Solution by Mathematica

Time used: 0.34 (sec). Leaf size: 40

```
DSolve[2(1-x^2)y'[x]==Sqrt[1-x^2]+(1+x)y[x],y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to -\frac{2\sqrt{1-x^2} + c_1\sqrt{2-2x}}{2(x-1)}$$

12.13 problem 332

Internal problem ID [3588]

Book: Ordinary differential equations and their solutions. By George Moseley Murphy. 1960
Section: Various 12
Problem number: 332.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_linear]`

$$\boxed{x(1-2x)\,y'+(1-4x)\,y=-1}$$

✓ Solution by Maple
Time used: 0.0 (sec). Leaf size: 18

```
dsolve(x*(1-2*x)*diff(y(x),x)+1+(1-4*x)*y(x) = 0,y(x), singsol=all)
```

$$y(x)=\frac{c_1+x}{(2x-1)\,x}$$

✓ Solution by Mathematica
Time used: 0.038 (sec). Leaf size: 22

```
DSolve[x(1-2 x)y'[x]+1+(1-4 x)y[x]==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x)\to -\frac{x-c_1}{x-2x^2}$$

12.14 problem 333

Internal problem ID [3589]

Book: Ordinary differential equations and their solutions. By George Moseley Murphy. 1960
Section: Various 12
Problem number: 333.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_rational, [_1st_order, '_with_symmetry_[F(x),G(x)]'], _Ricca`

$$x(1-2x)\,y' + (4x+1)\,y - y^2 = 4x$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 19

```
dsolve(x*(1-2*x)*diff(y(x),x) = 4*x-(1+4*x)*y(x)+y(x)^2,y(x), singsol=all)
```

$$y(x) = \frac{-2x^2 + c_1}{c_1 - x}$$

✓ Solution by Mathematica

Time used: 0.199 (sec). Leaf size: 27

```
DSolve[x(1-2 x)y'[x]==4 x -(1+4 x)y[x]+y[x]^2,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to 1 + \frac{x(2x-1)}{x - c_1}$$
$$y(x) \to 1$$

12.15 problem 334

Internal problem ID [3590]

Book: Ordinary differential equations and their solutions. By George Moseley Murphy. 1960
Section: Various 12
Problem number: 334.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type [_linear]

$$\boxed{2x(1-x)\,y'+(1-2x)\,y=-x}$$

✓ Solution by Maple
Time used: 0.0 (sec). Leaf size: 45

```
dsolve(2*x*(1-x)*diff(y(x),x)+x+(1-2*x)*y(x) = 0,y(x), singsol=all)
```

$$y(x)=\frac{2\sqrt{x\,(x-1)}-\ln\left(2\right)+\ln\left(-1+2x+2\sqrt{x\,(x-1)}\right)+4c_1}{4\sqrt{x\,(x-1)}}$$

✓ Solution by Mathematica
Time used: 0.123 (sec). Leaf size: 67

```
DSolve[2 x(1-x)y'[x]+x+(1-2 x)y[x]==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x)\to\frac{-x^2+x+\sqrt{x-1}\sqrt{x}\log\left(\sqrt{x-1}-\sqrt{x}\right)+2c_1\sqrt{-((x-1)x)}}{2x-2x^2}$$

12.16 problem 335

Internal problem ID [3591]

Book: Ordinary differential equations and their solutions. By George Moseley Murphy. 1960
Section: Various 12
Problem number: 335.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_rational, _Riccati]`

$$\boxed{2x(1-x)\,y' + (1-x)\,y^2 = -x}$$

✓ Solution by Maple

Time used: 0.093 (sec). Leaf size: 97

```
dsolve(2*x*(1-x)*diff(y(x),x)+x+(1-x)*y(x)^2 = 0,y(x), singsol=all)
```

$$y(x) = \frac{x\left(\text{LegendreQ}\left(-\frac{1}{2},1,\frac{2-x}{x}\right)c_1 - \text{LegendreQ}\left(\frac{1}{2},1,\frac{2-x}{x}\right)c_1 + \text{LegendreP}\left(-\frac{1}{2},1,\frac{2-x}{x}\right) - \text{LegendreP}\left(\frac{1}{2},1,2\right.\right.}{2\left(\text{LegendreQ}\left(-\frac{1}{2},1,\frac{2-x}{x}\right)c_1 + \text{LegendreP}\left(-\frac{1}{2},1,\frac{2-x}{x}\right)\right)(x-1)}$$

✓ Solution by Mathematica

Time used: 0.806 (sec). Leaf size: 77

```
DSolve[2 x(1-x)y'[x]+x+(1-x)y[x]^2==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to -\frac{2\left(\pi G_{2,2}^{2,0}\left(x\,\middle|\,\begin{array}{c}\frac{1}{2},\frac{3}{2}\\0,1\end{array}\right) + c_1(\text{EllipticK}(x) - \text{EllipticE}(x))\right)}{\pi G_{2,2}^{2,0}\left(x\,\middle|\,\begin{array}{c}\frac{1}{2},\frac{3}{2}\\0,0\end{array}\right) + 2c_1\,\text{EllipticE}(x)}$$

$$y(x) \to 1 - \frac{\text{EllipticK}(x)}{\text{EllipticE}(x)}$$

12.17 problem 336

Internal problem ID [3592]

Book: Ordinary differential equations and their solutions. By George Moseley Murphy. 1960
Section: Various 12
Problem number: 336.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_linear]`

$$\boxed{2(x^2+x+1)\,y'+(1+2x)\,y=8x^2+1}$$

✓ Solution by Maple
Time used: 0.016 (sec). Leaf size: 19

```
dsolve(2*(x^2+x+1)*diff(y(x),x) = 1+8*x^2-(1+2*x)*y(x),y(x), singsol=all)
```

$$y(x)=2x-3+\frac{c_1}{\sqrt{x^2+x+1}}$$

✓ Solution by Mathematica
Time used: 0.211 (sec). Leaf size: 23

```
DSolve[2(1+x+x^2)y'[x]==1+8 x^2-(1+2 x)y[x],y[x],x,IncludeSingularSolutions -> True]
```

$$y(x)\to\frac{c_1}{\sqrt{x^2+x+1}}+2x-3$$

12.18 problem 337

Internal problem ID [3593]

Book: Ordinary differential equations and their solutions. By George Moseley Murphy. 1960
Section: Various 12
Problem number: 337.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_linear]`

$$\boxed{4(x^2+1)\,y'-4yx=x^2}$$

✓ Solution by Maple
Time used: 0.0 (sec). Leaf size: 23

```
dsolve(4*(x^2+1)*diff(y(x),x)-4*x*y(x)-x^2 = 0,y(x), singsol=all)
```

$$y(x)=\frac{(4c_1+\operatorname{arcsinh}(x))\sqrt{x^2+1}}{4}-\frac{x}{4}$$

✓ Solution by Mathematica
Time used: 0.098 (sec). Leaf size: 50

```
DSolve[4(1+x^2)y'[x]-4 x y[x]-x^2==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x)\to -\frac{1}{4}\sqrt{x^2+1}\log\left(\sqrt{x^2+1}-x\right)+c_1\sqrt{x^2+1}-\frac{x}{4}$$

12.19 problem 338

Internal problem ID [3594]

Book: Ordinary differential equations and their solutions. By George Moseley Murphy. 1960
Section: Various 12
Problem number: 338.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[[_homogeneous, 'class A'], _rational, _Riccati]`

$$\boxed{a\,x^2 y' - axy - b^2 y^2 = x^2}$$

✓ Solution by Maple

Time used: 0.015 (sec). Leaf size: 19

```
dsolve(a*x^2*diff(y(x),x) = x^2+a*x*y(x)+b^2*y(x)^2,y(x), singsol=all)
```

$$y(x) = \frac{\tan\left(\frac{b(\ln(x)+c_1)}{a}\right)x}{b}$$

✓ Solution by Mathematica

Time used: 0.231 (sec). Leaf size: 23

```
DSolve[a x^2 y'[x]==x^2+a x y[x]+b^2 y[x]^2,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to \frac{x\tan\left(\frac{b(\log(x)+ac_1)}{a}\right)}{b}$$

12.20 problem 339

Internal problem ID [3595]

Book: Ordinary differential equations and their solutions. By George Moseley Murphy. 1960
Section: Various 12
Problem number: 339.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_separable]`

$$\boxed{\left(b\,x^2+a\right)y'-By^2=A}$$

✓ Solution by Maple

Time used: 0.015 (sec). Leaf size: 42

```
dsolve((b*x^2+a)*diff(y(x),x) = A+B*y(x)^2,y(x), singsol=all)
```

$$y(x)=\frac{\tan\left(\frac{\sqrt{AB}\left(c_1\sqrt{ab}+\arctan\left(\frac{xb}{\sqrt{ab}}\right)\right)}{\sqrt{ab}}\right)\sqrt{AB}}{B}$$

✓ Solution by Mathematica

Time used: 26.769 (sec). Leaf size: 91

```
DSolve[(a+b x^2)y'[x]==(A+B y[x]^2),y[x],x,IncludeSingularSolutions -> True]
```

$$y(x)\to\frac{\sqrt{A}\tan\left(\sqrt{A}\sqrt{B}\left(\frac{\arctan\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{\sqrt{a}\sqrt{b}}+c_1\right)\right)}{\sqrt{B}}$$
$$y(x)\to-\frac{i\sqrt{A}}{\sqrt{B}}$$
$$y(x)\to\frac{i\sqrt{A}}{\sqrt{B}}$$

12.21 problem 340

Internal problem ID [3596]

Book: Ordinary differential equations and their solutions. By George Moseley Murphy. 1960
Section: Various 12
Problem number: 340.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_separable]`

$$\boxed{(b\,x^2+a)\,y'-cxy\ln(y)=0}$$

✓ Solution by Maple

Time used: 0.031 (sec). Leaf size: 24

```
dsolve((b*x^2+a)*diff(y(x),x) = c*x*y(x)*ln(y(x)),y(x), singsol=all)
```

$$y(x)=\mathrm{e}^{\mathrm{e}^{c c_1}(b\,x^2+a)^{\frac{c}{2b}}}$$

✓ Solution by Mathematica

Time used: 0.374 (sec). Leaf size: 33

```
DSolve[(a+b x^2)y'[x]==c x y[x] Log[y[x]],y[x],x,IncludeSingularSolutions -> True]
```

$$y(x)\to e^{e^{c_1}(a+bx^2)^{\frac{c}{2b}}}$$
$$y(x)\to 1$$

12.22 problem 341

Internal problem ID [3597]

Book: Ordinary differential equations and their solutions. By George Moseley Murphy. 1960
Section: Various 12
Problem number: 341.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_separable]`

$$\boxed{x(ax+1)\,y'-y=-a}$$

✓ Solution by Maple
Time used: 0.0 (sec). Leaf size: 17

```
dsolve(x*(a*x+1)*diff(y(x),x)+a-y(x) = 0,y(x), singsol=all)
```

$$y(x)=\frac{c_1x+a}{ax+1}$$

✓ Solution by Mathematica
Time used: 0.036 (sec). Leaf size: 24

```
DSolve[x(1+a x)y'[x]+a-y[x]==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x)\to\frac{a+c_1x}{ax+1}$$
$$y(x)\to a$$

12.23 problem 342

Internal problem ID [3598]

Book: Ordinary differential equations and their solutions. By George Moseley Murphy. 1960
Section: Various 12
Problem number: 342.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_rational, _Abel]`

$$(bx+a)^2 y' + cy^2 + (bx+a) y^3 = 0$$

✓ Solution by Maple

Time used: 0.016 (sec). Leaf size: 126

```
dsolve((b*x+a)^2*diff(y(x),x)+c*y(x)^2+(b*x+a)*y(x)^3 = 0,y(x), singsol=all)
```

$$\frac{\left(\sqrt{b}\, a + b^{\frac{3}{2}} x\right) e^{-\frac{((bx+a+c)y(x)+b(bx+a))((-bx-a+c)y(x)+b(bx+a))}{2y(x)^2(bx+a)^2 b}} + \frac{c\sqrt{2}\,\sqrt{\pi}\, e^{\frac{1}{2b}} \operatorname{erf}\left(\frac{\sqrt{2}\,(cy(x)+b(bx+a))}{2\sqrt{b}\, y(x)(bx+a)}\right)}{2} + b^{\frac{3}{2}} c_1}{b^{\frac{3}{2}}} = 0$$

✓ Solution by Mathematica

Time used: 1.435 (sec). Leaf size: 149

```
DSolve[(a+b x)^2 y'[x]+c y[x]^2+(a+b x)y[x]^3==0,y[x],x,IncludeSingularSolutions -> True]
```

$$\text{Solve}\left[-\frac{c}{\sqrt{-b(a+bx)^2}} = \frac{2\exp\left(\frac{1}{2}\left(-\frac{c}{\sqrt{-b(a+bx)^2}} - \frac{(-b(a+bx)^2)^{3/2}}{by(x)(a+bx)^3}\right)^2\right)}{\sqrt{2\pi}\text{erfi}\left(\frac{-\frac{c}{\sqrt{-b(a+bx)^2}} - \frac{(-b(a+bx)^2)^{3/2}}{by(x)(a+bx)^3}}{\sqrt{2}}\right) + 2c_1}, y(x)\right]$$

12.24 problem 343

Internal problem ID [3599]

Book: Ordinary differential equations and their solutions. By George Moseley Murphy. 1960
Section: Various 12
Problem number: 343.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_linear]`

$$\boxed{y'x^3 - yb\,x^2 = a}$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 21

```
dsolve(x^3*diff(y(x),x) = a+b*x^2*y(x),y(x), singsol=all)
```

$$y(x) = -\frac{a}{x^2\,(2+b)} + x^b c_1$$

✓ Solution by Mathematica

Time used: 0.045 (sec). Leaf size: 23

```
DSolve[x^3 y'[x]==a + b x^2 y[x],y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to -\frac{a}{(b+2)x^2} + c_1 x^b$$

12.25 problem 344

Internal problem ID [3600]

Book: Ordinary differential equations and their solutions. By George Moseley Murphy. 1960
Section: Various 12
Problem number: 344.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_linear]`

$$\boxed{y'x^3 - x^2y = -x^2 + 3}$$

✓ Solution by Maple
Time used: 0.0 (sec). Leaf size: 14

```
dsolve(x^3*diff(y(x),x) = 3-x^2+x^2*y(x),y(x), singsol=all)
```

$$y(x) = -\frac{1}{x^2} + 1 + c_1x$$

✓ Solution by Mathematica
Time used: 0.033 (sec). Leaf size: 16

```
DSolve[x^3 y'[x]==3 -x^2+x^2 y[x],y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to -\frac{1}{x^2} + c_1x + 1$$

12.26 problem 345

Internal problem ID [3601]

Book: Ordinary differential equations and their solutions. By George Moseley Murphy. 1960
Section: Various 12
Problem number: 345.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[[_homogeneous, ‘class G‘], _rational, _Riccati]`

$$\boxed{y' x^3 - y^2 = x^4}$$

✓ Solution by Maple
Time used: 0.0 (sec). Leaf size: 23

```
dsolve(x^3*diff(y(x),x) = x^4+y(x)^2,y(x), singsol=all)
```

$$y(x) = \frac{x^2(\ln(x) - c_1 - 1)}{\ln(x) - c_1}$$

✓ Solution by Mathematica
Time used: 0.168 (sec). Leaf size: 29

```
DSolve[x^3 y'[x]==x^4+y[x]^2,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to \frac{x^2(\log(x) - 1 + c_1)}{\log(x) + c_1}$$
$$y(x) \to x^2$$

12.27 problem 346

Internal problem ID [3602]

Book: Ordinary differential equations and their solutions. By George Moseley Murphy. 1960
Section: Various 12
Problem number: 346.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[[_homogeneous, ‘class D‘], _rational, _Bernoulli]`

$$\boxed{y'x^3 - y(x^2 + y) = 0}$$

✓ Solution by Maple
Time used: 0.0 (sec). Leaf size: 15

```
dsolve(x^3*diff(y(x),x) = y(x)*(x^2+y(x)),y(x), singsol=all)
```

$$y(x) = \frac{x^2}{c_1 x + 1}$$

✓ Solution by Mathematica
Time used: 0.141 (sec). Leaf size: 22

```
DSolve[x^3 y'[x]==y[x](x^2+y[x]),y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to \frac{x^2}{1 + c_1 x}$$
$$y(x) \to 0$$

12.28 problem 347

Internal problem ID [3603]

Book: Ordinary differential equations and their solutions. By George Moseley Murphy. 1960
Section: Various 12
Problem number: 347.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[[_homogeneous, 'class D'], _rational, _Riccati]`

$$\boxed{y'x^3 - (y-1)\,x^2 - y^2 = 0}$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 17

```
dsolve(x^3*diff(y(x),x) = x^2*(y(x)-1)+y(x)^2,y(x), singsol=all)
```

$$y(x) = -\tanh\left(\frac{c_1 x - 1}{x}\right) x$$

✓ Solution by Mathematica

Time used: 0.713 (sec). Leaf size: 51

```
DSolve[x^3 y'[x]==x^2(y[x]-1)+y[x]^2,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to \frac{x\left(e^{2/x} - e^{2c_1}\right)}{e^{2/x} + e^{2c_1}}$$
$$y(x) \to -x$$
$$y(x) \to x$$

12.29 problem 348

Internal problem ID [3604]

Book: Ordinary differential equations and their solutions. By George Moseley Murphy. 1960
Section: Various 12
Problem number: 348.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_separable]`

$$\boxed{y'x^3 - (x+1)\,y^2 = 0}$$

✓ Solution by Maple
Time used: 0.0 (sec). Leaf size: 22

```
dsolve(x^3*diff(y(x),x) = (1+x)*y(x)^2,y(x), singsol=all)
```

$$y(x) = \frac{2x^2}{2c_1x^2 + 2x + 1}$$

✓ Solution by Mathematica
Time used: 0.134 (sec). Leaf size: 29

```
DSolve[x^3 y'[x]==(1+x)y[x]^2,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to \frac{2x^2}{-2c_1x^2 + 2x + 1}$$
$$y(x) \to 0$$

12.30 problem 349

Internal problem ID [3605]

Book: Ordinary differential equations and their solutions. By George Moseley Murphy. 1960
Section: Various 12
Problem number: 349.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[[_homogeneous, ‘class G‘], _rational, _Riccati]`

$$\boxed{y'x^3 + x^2y(1 - x^2y) = -20}$$

✓ Solution by Maple

Time used: 0.532 (sec). Leaf size: 26

```
dsolve(x^3*diff(y(x),x)+20+x^2*y(x)*(1-x^2*y(x)) = 0,y(x), singsol=all)
```

$$y(x) = \frac{5x^9 + 4c_1}{(-x^9 + c_1)\, x^2}$$

✓ Solution by Mathematica

Time used: 0.172 (sec). Leaf size: 36

```
DSolve[x^3 y'[x]+20+x^2 y[x](1-x^2 y[x])==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to \frac{-5x^9 + 4c_1}{x^2\,(x^9 + c_1)}$$
$$y(x) \to \frac{4}{x^2}$$

12.31 problem 350

Internal problem ID [3606]

Book: Ordinary differential equations and their solutions. By George Moseley Murphy. 1960
Section: Various 12
Problem number: 350.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_rational, _Riccati]`

$$\boxed{y'x^3 + (3 - 2x)\,x^2 y - x^6 y^2 = -3}$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 27

```
dsolve(x^3*diff(y(x),x)+3+(3-2*x)*x^2*y(x)-x^6*y(x)^2 = 0,y(x), singsol=all)
```

$$y(x) = \frac{-3\,e^{4x}c_1 - 3}{x^3\,(e^{4x}c_1 - 3)}$$

✓ Solution by Mathematica

Time used: 0.173 (sec). Leaf size: 34

```
DSolve[x^3 y'[x]+3+(3-2 x)x^2 y[x]-x^6 y[x]^2==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to \frac{-3 + \frac{1}{\frac{1}{4}+c_1 e^{4x}}}{x^3}$$
$$y(x) \to -\frac{3}{x^3}$$

12.32 problem 351

Internal problem ID [3607]

Book: Ordinary differential equations and their solutions. By George Moseley Murphy. 1960
Section: Various 12
Problem number: 351.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[[_homogeneous, ‘class A‘], _rational, _Bernoulli]`

$$\boxed{y'x^3 - \left(y^2 + 2x^2\right) y = 0}$$

✓ Solution by Maple
Time used: 0.016 (sec). Leaf size: 34

```
dsolve(x^3*diff(y(x),x) = (2*x^2+y(x)^2)*y(x),y(x), singsol=all)
```

$$y(x) = \frac{x^2}{\sqrt{-x^2 + c_1}}$$
$$y(x) = -\frac{x^2}{\sqrt{-x^2 + c_1}}$$

✓ Solution by Mathematica
Time used: 0.168 (sec). Leaf size: 47

```
DSolve[x^3 y'[x]==(2 x^2+y[x]^2)y[x],y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to -\frac{x^2}{\sqrt{-x^2 + c_1}}$$
$$y(x) \to \frac{x^2}{\sqrt{-x^2 + c_1}}$$
$$y(x) \to 0$$

12.33 problem 352

Internal problem ID [3608]

Book: Ordinary differential equations and their solutions. By George Moseley Murphy. 1960
Section: Various 12
Problem number: 352.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[‘y=_G(x,y’)‘]`

$$\boxed{y'x^3 - \cos(y)\left(\cos(y) - 2x^2\sin(y)\right) = 0}$$

✓ Solution by Maple

Time used: 0.015 (sec). Leaf size: 15

```
dsolve(x^3*diff(y(x),x) = cos(y(x))*(cos(y(x))-2*x^2*sin(y(x))),y(x), singsol=all)
```

$$y(x) = \arctan\left(\frac{\ln(x) - c_1}{x^2}\right)$$

✓ Solution by Mathematica

Time used: 5.952 (sec). Leaf size: 55

```
DSolve[x^3 y'[x]==Cos[y[x]](Cos[y[x]]-2 x^2 Sin[y[x]]),y[x],x,IncludeSingularSolutions -> Tr
```

$$y(x) \to \arctan\left(\frac{\log(x) + 4c_1}{x^2}\right)$$
$$y(x) \to -\frac{1}{2}\pi\sqrt{\frac{1}{x^4}}x^2$$
$$y(x) \to \frac{1}{2}\pi\sqrt{\frac{1}{x^4}}x^2$$

12.34 problem 353

Internal problem ID [3609]

Book: Ordinary differential equations and their solutions. By George Moseley Murphy. 1960
Section: Various 12
Problem number: 353.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_linear]`

$$x(x^2+1)\,y'-y=x^2a$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 19

```
dsolve(x*(x^2+1)*diff(y(x),x) = a*x^2+y(x),y(x), singsol=all)
```

$$y(x)=\frac{(a\,\mathrm{arcsinh}\,(x)+c_1)\,x}{\sqrt{x^2+1}}$$

✓ Solution by Mathematica

Time used: 0.042 (sec). Leaf size: 36

```
DSolve[x(1+x^2)y'[x]==a x^2+y[x],y[x],x,IncludeSingularSolutions -> True]
```

$$y(x)\to\frac{x\left(-a\log\left(\sqrt{x^2+1}-x\right)+c_1\right)}{\sqrt{x^2+1}}$$

12.35 problem 354

Internal problem ID [3610]

Book: Ordinary differential equations and their solutions. By George Moseley Murphy. 1960
Section: Various 12
Problem number: 354.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_linear]`

$$x(-x^2+1)\,y'-y=x^2a$$

✓ Solution by Maple
Time used: 0.0 (sec). Leaf size: 49

```
dsolve(x*(-x^2+1)*diff(y(x),x) = a*x^2+y(x),y(x), singsol=all)
```

$$y(x)=x\left(-\frac{a\sqrt{x^2-1}\,\ln\left(x+\sqrt{x^2-1}\right)}{(x-1)\,(x+1)}+\frac{c_1}{\sqrt{x-1}\,\sqrt{x+1}}\right)$$

✓ Solution by Mathematica
Time used: 0.046 (sec). Leaf size: 42

```
DSolve[x(1-x^2)y'[x]==a x^2+y[x],y[x],x,IncludeSingularSolutions -> True]
```

$$y(x)\to\frac{x\left(-2a\arctan\left(\frac{\sqrt{1-x^2}}{x+1}\right)+c_1\right)}{\sqrt{1-x^2}}$$

13 Various 13

13.1 problem 355

Internal problem ID [3611]

Book: Ordinary differential equations and their solutions. By George Moseley Murphy. 1960
Section: Various 13
Problem number: 355.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_linear]`

$$x(x^2+1)\,y'-y=a\,x^3$$

✓ Solution by Maple
Time used: 0.0 (sec). Leaf size: 17

```
dsolve(x*(x^2+1)*diff(y(x),x) = a*x^3+y(x),y(x), singsol=all)
```

$$y(x)=x\left(a+\frac{c_1}{\sqrt{x^2+1}}\right)$$

✓ Solution by Mathematica
Time used: 0.04 (sec). Leaf size: 21

```
DSolve[x(1+x^2)y'[x]==a x^3+y[x],y[x],x,IncludeSingularSolutions -> True]
```

$$y(x)\to x\left(a+\frac{c_1}{\sqrt{x^2+1}}\right)$$

13.2 problem 356

Internal problem ID [3612]

Book: Ordinary differential equations and their solutions. By George Moseley Murphy. 1960
Section: Various 13
Problem number: 356.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_linear]`

$$\boxed{x(x^2+1)\,y'+x^2y=a}$$

✓ Solution by Maple
Time used: 0.0 (sec). Leaf size: 25

```
dsolve(x*(x^2+1)*diff(y(x),x) = a-x^2*y(x),y(x), singsol=all)
```

$$y(x)=\frac{-a\ \operatorname{arctanh}\left(\frac{1}{\sqrt{x^2+1}}\right)+c_1}{\sqrt{x^2+1}}$$

✓ Solution by Mathematica
Time used: 0.063 (sec). Leaf size: 31

```
DSolve[x(1+x^2)y'[x]==a-x^2 y[x],y[x],x,IncludeSingularSolutions -> True]
```

$$y(x)\to\frac{-a\operatorname{arctanh}\left(\sqrt{x^2+1}\right)+c_1}{\sqrt{x^2+1}}$$

13.3 problem 357

Internal problem ID [3613]

Book: Ordinary differential equations and their solutions. By George Moseley Murphy. 1960
Section: Various 13
Problem number: 357.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_separable]`

$$\boxed{x(x^2+1)\,y' - (-x^2+1)\,y = 0}$$

✓ Solution by Maple
Time used: 0.0 (sec). Leaf size: 14

```
dsolve(x*(x^2+1)*diff(y(x),x) = (-x^2+1)*y(x),y(x), singsol=all)
```

$$y(x) = \frac{c_1 x}{x^2+1}$$

✓ Solution by Mathematica
Time used: 0.037 (sec). Leaf size: 21

```
DSolve[x(1+x^2)y'[x]==(1-x^2)y[x],y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to \frac{c_1 x}{x^2+1}$$
$$y(x) \to 0$$

13.4 problem 358

Internal problem ID [3614]

Book: Ordinary differential equations and their solutions. By George Moseley Murphy. 1960
Section: Various 13
Problem number: 358.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_separable]`

$$\boxed{x(-x^2+1)\,y'-(x^2-x+1)\,y=0}$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 17

```
dsolve(x*(-x^2+1)*diff(y(x),x) = (x^2-x+1)*y(x),y(x), singsol=all)
```

$$y(x)=\frac{c_1 x}{(x+1)^{\frac{3}{2}}\sqrt{x-1}}$$

✓ Solution by Mathematica

Time used: 0.042 (sec). Leaf size: 30

```
DSolve[x(1-x^2)y'[x]==(1-x+x^2)y[x],y[x],x,IncludeSingularSolutions -> True]
```

$$y(x)\to\frac{c_1 x}{(x+1)\sqrt{1-x^2}}$$
$$y(x)\to 0$$

13.5 problem 359

Internal problem ID [3615]

Book: Ordinary differential equations and their solutions. By George Moseley Murphy. 1960
Section: Various 13
Problem number: 359.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_linear]`

$$\boxed{x(-x^2+1)\,y' - (-2x^2+1)\,y = a\,x^3}$$

✓ Solution by Maple
Time used: 0.0 (sec). Leaf size: 20

```
dsolve(x*(-x^2+1)*diff(y(x),x) = a*x^3+(-2*x^2+1)*y(x),y(x), singsol=all)
```

$$y(x) = x\left(\sqrt{x-1}\,\sqrt{x+1}\,c_1 + a\right)$$

✓ Solution by Mathematica
Time used: 0.05 (sec). Leaf size: 23

```
DSolve[x(1-x^2)y'[x]==a x^3+(1-2 x^2)y[x],y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to x\left(a + c_1\sqrt{1-x^2}\right)$$

13.6 problem 360

Internal problem ID [3616]

Book: Ordinary differential equations and their solutions. By George Moseley Murphy. 1960
Section: Various 13
Problem number: 360.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_linear]`

$$x(-x^2+1)\, y' - (-2x^2+1)\, y = x^3(-x^2+1)$$

✓ Solution by Maple
Time used: 0.0 (sec). Leaf size: 24

```
dsolve(x*(-x^2+1)*diff(y(x),x) = x^3*(-x^2+1)+(-2*x^2+1)*y(x),y(x), singsol=all)
```

$$y(x) = \sqrt{x+1}\, x\sqrt{x-1}\, c_1 + x^3 - x$$

✓ Solution by Mathematica
Time used: 0.045 (sec). Leaf size: 26

```
DSolve[x(1-x^2)y'[x]==x^3(1-x^2)+(1-2 x^2)y[x],y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to x\left(x^2 + c_1\sqrt{1-x^2} - 1\right)$$

13.7 problem 361

Internal problem ID [3617]

Book: Ordinary differential equations and their solutions. By George Moseley Murphy. 1960
Section: Various 13
Problem number: 361.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_linear]`

$$\boxed{x(x^2+1)\,y'+4x^2y=2}$$

✓ Solution by Maple
Time used: 0.0 (sec). Leaf size: 21

```
dsolve(x*(x^2+1)*diff(y(x),x) = 2-4*x^2*y(x),y(x), singsol=all)
```

$$y(x)=\frac{x^2+2\ln(x)+c_1}{(x^2+1)^2}$$

✓ Solution by Mathematica
Time used: 0.032 (sec). Leaf size: 23

```
DSolve[x(1+x^2)y'[x]==2(1-2 x^2 y[x]),y[x],x,IncludeSingularSolutions -> True]
```

$$y(x)\to\frac{x^2+2\log(x)+c_1}{(x^2+1)^2}$$

13.8 problem 362

Internal problem ID [3618]

Book: Ordinary differential equations and their solutions. By George Moseley Murphy. 1960
Section: Various 13
Problem number: 362.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_linear]`

$$\boxed{x(x^2+1)\,y' + (5x^2+3)\,y = x}$$

✓ Solution by Maple
Time used: 0.016 (sec). Leaf size: 23

```
dsolve(x*(x^2+1)*diff(y(x),x) = x-(5*x^2+3)*y(x),y(x), singsol=all)
```

$$y(x) = \frac{x^4+4c_1}{4x^3\,(x^2+1)}$$

✓ Solution by Mathematica
Time used: 0.039 (sec). Leaf size: 26

```
DSolve[x(1+x^2)y'[x]==x-(3+5 x^2)y[x],y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to \frac{x^4+4c_1}{4\,(x^5+x^3)}$$

13.9 problem 363

Internal problem ID [3619]

Book: Ordinary differential equations and their solutions. By George Moseley Murphy. 1960
Section: Various 13
Problem number: 363.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_rational, _Riccati]`

$$\boxed{x(-x^2+1)\,y'+(-x^2+1)\,y^2=-x^2}$$

✓ Solution by Maple

Time used: 0.016 (sec). Leaf size: 30

```
dsolve(x*(-x^2+1)*diff(y(x),x)+x^2+(-x^2+1)*y(x)^2 = 0,y(x), singsol=all)
```

$$y(x)=\frac{c_1\,\text{EllipticCE}\,(x)+\text{EllipticE}\,(x)-\text{EllipticK}\,(x)}{c_1\,\text{EllipticCE}\,(x)-c_1\,\text{EllipticCK}\,(x)+\text{EllipticE}\,(x)}$$

✓ Solution by Mathematica

Time used: 0.971 (sec). Leaf size: 91

```
DSolve[x(1-x^2)y'[x]+x^2+(1-x^2)y[x]^2==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x)\to-\frac{2\left(\pi G_{2,2}^{2,0}\left(x^2\middle|\begin{array}{c}\frac{1}{2},\frac{3}{2}\\0,1\end{array}\right)+c_1(\text{EllipticK}\,(x^2)-\text{EllipticE}\,(x^2))\right)}{\pi G_{2,2}^{2,0}\left(x^2\middle|\begin{array}{c}\frac{1}{2},\frac{3}{2}\\0,0\end{array}\right)+2c_1\,\text{EllipticE}\,(x^2)}$$

$$y(x)\to1-\frac{\text{EllipticK}\,(x^2)}{\text{EllipticE}\,(x^2)}$$

13.10 problem 364

Internal problem ID [3620]

Book: Ordinary differential equations and their solutions. By George Moseley Murphy. 1960
Section: Various 13
Problem number: 364.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[[_homogeneous, 'class D'], _rational, _Bernoulli]`

$$\boxed{x^2(1-x)\,y'-(-x+2)\,xy+y^2=0}$$

✓ Solution by Maple
Time used: 0.0 (sec). Leaf size: 17

```
dsolve(x^2*(1-x)*diff(y(x),x) = (2-x)*x*y(x)-y(x)^2,y(x), singsol=all)
```

$$y(x)=\frac{x^2}{1+c_1\,(x-1)}$$

✓ Solution by Mathematica
Time used: 0.195 (sec). Leaf size: 25

```
DSolve[x^2(1-x)y'[x]==(2-x)x y[x]-y[x]^2,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x)\to\frac{x^2}{c_1(-x)+1+c_1}$$
$$y(x)\to 0$$

13.11 problem 365

Internal problem ID [3621]

Book: Ordinary differential equations and their solutions. By George Moseley Murphy. 1960
Section: Various 13
Problem number: 365.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[[_homogeneous, ‘class A‘], _rational, _Bernoulli]`

$$\boxed{2y'x^3 - \left(x^2 - y^2\right)y = 0}$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 26

```
dsolve(2*x^3*diff(y(x),x) = (x^2-y(x)^2)*y(x),y(x), singsol=all)
```

$$y(x) = \frac{x}{\sqrt{c_1x - 1}}$$
$$y(x) = -\frac{x}{\sqrt{c_1x - 1}}$$

✓ Solution by Mathematica

Time used: 0.307 (sec). Leaf size: 39

```
DSolve[2 x^3 y'[x]==(x^2-y[x]^2)y[x],y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to -\frac{x}{\sqrt{-1 + c_1x}}$$
$$y(x) \to \frac{x}{\sqrt{-1 + c_1x}}$$
$$y(x) \to 0$$

13.12 problem 366

Internal problem ID [3622]

Book: Ordinary differential equations and their solutions. By George Moseley Murphy. 1960
Section: Various 13
Problem number: 366.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[[_homogeneous, ‘class A‘], _rational, _Bernoulli]`

$$\boxed{2y'x^3 - (3x^2 + ay^2)\, y = 0}$$

✓ Solution by Maple

Time used: 0.016 (sec). Leaf size: 48

```
dsolve(2*x^3*diff(y(x),x) = (3*x^2+a*y(x)^2)*y(x),y(x), singsol=all)
```

$$y(x) = \frac{\sqrt{(-ax + c_1)\, x}\, x}{-ax + c_1}$$
$$y(x) = \frac{\sqrt{(-ax + c_1)\, x}\, x}{ax - c_1}$$

✓ Solution by Mathematica

Time used: 0.216 (sec). Leaf size: 49

```
DSolve[2 x^3 y'[x]==(3 x^2+a y[x]^2)y[x],y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to -\frac{x^{3/2}}{\sqrt{-ax + c_1}}$$
$$y(x) \to \frac{x^{3/2}}{\sqrt{-ax + c_1}}$$
$$y(x) \to 0$$

13.13 problem 367

Internal problem ID [3623]

Book: Ordinary differential equations and their solutions. By George Moseley Murphy. 1960
Section: Various 13
Problem number: 367.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_rational, _Bernoulli]`

$$\boxed{6y'x^3 - 4x^2y - (1 - 3x)\, y^4 = 0}$$

✓ Solution by Maple
Time used: 0.016 (sec). Leaf size: 129

```
dsolve(6*x^3*diff(y(x),x) = 4*x^2*y(x)+(1-3*x)*y(x)^4,y(x), singsol=all)
```

$$y(x) = \frac{2^{\frac{1}{3}}\left(-x^2(-3x + \ln(x) - 2c_1)^2\right)^{\frac{1}{3}}}{-3x + \ln(x) - 2c_1}$$

$$y(x) = \frac{2^{\frac{1}{3}}\left(-x^2(-3x + \ln(x) - 2c_1)^2\right)^{\frac{1}{3}}\left(1 + i\sqrt{3}\right)}{6x - 2\ln(x) + 4c_1}$$

$$y(x) = \frac{2^{\frac{1}{3}}\left(-x^2(-3x + \ln(x) - 2c_1)^2\right)^{\frac{1}{3}}\left(i\sqrt{3} - 1\right)}{-6x + 2\ln(x) - 4c_1}$$

✓ Solution by Mathematica

Time used: 0.21 (sec). Leaf size: 99

```
DSolve[6 x^3 y'[x]==4 x^2 y[x]+(1-3 x)y[x]^4,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to -\frac{\sqrt[3]{-2}x^{2/3}}{\sqrt[3]{3x - \log(x) + 2c_1}}$$
$$y(x) \to \frac{x^{2/3}}{\sqrt[3]{\frac{3x}{2} - \frac{\log(x)}{2} + c_1}}$$
$$y(x) \to \frac{(-1)^{2/3}x^{2/3}}{\sqrt[3]{\frac{3x}{2} - \frac{\log(x)}{2} + c_1}}$$
$$y(x) \to 0$$

13.14 problem 368

Internal problem ID [3624]

Book: Ordinary differential equations and their solutions. By George Moseley Murphy. 1960
Section: Various 13
Problem number: 368.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[[_homogeneous, ‘class D‘], _rational, _Riccati]`

$$x\left(c\,x^2+bx+a\right)y'-\left(c\,x^2+bx+a\right)y-y^2=-x^2$$

✓ Solution by Maple

Time used: 0.016 (sec). Leaf size: 58

```
dsolve(x*(c*x^2+b*x+a)*diff(y(x),x)+x^2-(c*x^2+b*x+a)*y(x) = y(x)^2,y(x), singsol=all)
```

$$y(x)=-\tanh\left(\frac{c_1\sqrt{4ac-b^2}+2\arctan\left(\frac{2cx+b}{\sqrt{4ac-b^2}}\right)}{\sqrt{4ac-b^2}}\right)x$$

✓ Solution by Mathematica

Time used: 1.182 (sec). Leaf size: 116

```
DSolve[x(a+b x +c x^2)y'[x]+x^2-(a+b x+c x^2)y[x]==y[x]^2,y[x],x,IncludeSingularSolutions ->
```

$$y(x)\rightarrow-\frac{x\left(-1+\exp\left(\frac{4\arctan\left(\frac{b+2cx}{\sqrt{4ac-b^2}}\right)}{\sqrt{4ac-b^2}}+2c_1\right)\right)}{1+\exp\left(\frac{4\arctan\left(\frac{b+2cx}{\sqrt{4ac-b^2}}\right)}{\sqrt{4ac-b^2}}+2c_1\right)}$$
$$y(x)\rightarrow-x$$
$$y(x)\rightarrow x$$

13.15 problem 369

Internal problem ID [3625]

Book: Ordinary differential equations and their solutions. By George Moseley Murphy. 1960
Section: Various 13
Problem number: 369.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[[_homogeneous, ‘class D‘], _rational, _Bernoulli]`

$$\boxed{y'x^4 - (y + x^3)\, y = 0}$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 19

```
dsolve(x^4*diff(y(x),x) = (x^3+y(x))*y(x),y(x), singsol=all)
```

$$y(x) = \frac{2x^3}{2c_1x^2 + 1}$$

✓ Solution by Mathematica

Time used: 0.141 (sec). Leaf size: 26

```
DSolve[x^4 y'[x]==(x^3+y[x])y[x],y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to \frac{2x^3}{1 + 2c_1x^2}$$
$$y(x) \to 0$$

13.16 problem 370

Internal problem ID [3626]

Book: Ordinary differential equations and their solutions. By George Moseley Murphy. 1960
Section: Various 13
Problem number: 370.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_rational, [_Riccati, _special]]`

$$\boxed{y' x^4 + y^2 x^4 = -a^2}$$

✓ Solution by Maple

Time used: 0.015 (sec). Leaf size: 24

```
dsolve(x^4*diff(y(x),x)+a^2+x^4*y(x)^2 = 0,y(x), singsol=all)
```

$$y(x) = \frac{-a\tan\left(\frac{a(c_1 x-1)}{x}\right)+x}{x^2}$$

✓ Solution by Mathematica

Time used: 0.693 (sec). Leaf size: 87

```
DSolve[x^4 y'[x]+a^2+x^4 y[x]^2==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to \frac{-2ia^2c_1e^{\frac{2ia}{x}}+2ac_1xe^{\frac{2ia}{x}}+a-ix}{x^2\left(2ac_1e^{\frac{2ia}{x}}-i\right)}$$
$$y(x) \to \frac{x-ia}{x^2}$$

13.17 problem 371

Internal problem ID [3627]

Book: Ordinary differential equations and their solutions. By George Moseley Murphy. 1960
Section: Various 13
Problem number: 371.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[[_1st_order, '_with_symmetry_[F(x),G(x)*y+H(x)]']]`

$$\boxed{y'x^4 + yx^3 + \csc(yx) = 0}$$

✓ Solution by Maple

Time used: 0.079 (sec). Leaf size: 26

```
dsolve(x^4*diff(y(x),x)+x^3*y(x)+csc(x*y(x)) = 0,y(x), singsol=all)
```

$$y(x) = \frac{\frac{\pi}{2} + \arcsin\left(\frac{2c_1x^2+1}{2x^2}\right)}{x}$$

✓ Solution by Mathematica

Time used: 5.347 (sec). Leaf size: 40

```
DSolve[x^4 y'[x]+x^3 y[x]+ Csc[x y[x]]==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to -\frac{\arccos\left(-\frac{1}{2x^2} + c_1\right)}{x}$$
$$y(x) \to \frac{\arccos\left(-\frac{1}{2x^2} + c_1\right)}{x}$$

13.18 problem 372

Internal problem ID [3628]

Book: Ordinary differential equations and their solutions. By George Moseley Murphy. 1960
Section: Various 13
Problem number: 372.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_separable]`

$$\boxed{\left(-x^4+1\right) y'-2x\left(1-y^2\right)=0}$$

✓ Solution by Maple
Time used: 0.0 (sec). Leaf size: 31

```
dsolve((-x^4+1)*diff(y(x),x) = 2*x*(1-y(x)^2),y(x), singsol=all)
```

$$y(x)=-\tanh\left(\frac{\ln(x+1)}{2}-\frac{\ln(x^2+1)}{2}+\frac{\ln(x-1)}{2}+2c_1\right)$$

✓ Solution by Mathematica
Time used: 0.818 (sec). Leaf size: 55

```
DSolve[(1-x^4)y'[x]==2 x(1-y[x]^2),y[x],x,IncludeSingularSolutions -> True]
```

$$\begin{aligned} y(x) &\to -\frac{x^2+e^{2c_1}(x^2-1)+1}{-x^2+e^{2c_1}(x^2-1)-1} \\ y(x) &\to -1 \\ y(x) &\to 1 \end{aligned}$$

13.19 problem 373

Internal problem ID [3629]

Book: Ordinary differential equations and their solutions. By George Moseley Murphy. 1960
Section: Various 13
Problem number: 373.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_linear]`

$$x\left(-x^3+1\right)y'+\left(-4x^3+1\right)y=2x$$

✓ Solution by Maple
Time used: 0.0 (sec). Leaf size: 21

```
dsolve(x*(-x^3+1)*diff(y(x),x) = 2*x-(-4*x^3+1)*y(x),y(x), singsol=all)
```

$$y(x)=\frac{-x^2+c_1}{x^4-x}$$

✓ Solution by Mathematica
Time used: 0.038 (sec). Leaf size: 21

```
DSolve[x(1-x^3)y'[x]==2 x-(1-4 x^3)y[x],y[x],x,IncludeSingularSolutions -> True]
```

$$y(x)\to\frac{x^2+c_1}{x-x^4}$$

13.20 problem 374

Internal problem ID [3630]

Book: Ordinary differential equations and their solutions. By George Moseley Murphy. 1960
Section: Various 13
Problem number: 374.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_rational, _Riccati]`

$$x\left(-x^3+1\right)y'-(1-2yx)\,y=x^2$$

✓ Solution by Maple
Time used: 0.016 (sec). Leaf size: 18

```
dsolve(x*(-x^3+1)*diff(y(x),x) = x^2+(1-2*x*y(x))*y(x),y(x), singsol=all)
```

$$y(x)=\frac{(c_1+x)\,x}{c_1x^2+1}$$

✓ Solution by Mathematica
Time used: 0.363 (sec). Leaf size: 31

```
DSolve[x(1-x^3)y'[x]==x^2+(1-2 x y[x])y[x],y[x],x,IncludeSingularSolutions -> True]
```

$$y(x)\to\frac{x(1+2c_1x)}{x^2+2c_1}$$
$$y(x)\to x^2$$

13.21 problem 375

Internal problem ID [3631]

Book: Ordinary differential equations and their solutions. By George Moseley Murphy. 1960
Section: Various 13
Problem number: 375.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_rational, _Bernoulli]`

$$x^2(-x^2+1)\,y' - (x - 3yx^3)\,y = 0$$

✓ Solution by Maple

Time used: 0.015 (sec). Leaf size: 72

```
dsolve(x^2*(-x^2+1)*diff(y(x),x) = (x-3*x^3*y(x))*y(x),y(x), singsol=all)
```

$$y(x) = \frac{\sqrt{x^2-1}\,x}{\sqrt{x-1}\,\sqrt{x+1}\,c_1\sqrt{x^2-1} - 3\ln\left(x+\sqrt{x^2-1}\right)x^2 + 3\sqrt{x^2-1}\,x + 3\ln\left(x+\sqrt{x^2-1}\right)}$$

✓ Solution by Mathematica

Time used: 0.302 (sec). Leaf size: 63

```
DSolve[x^2(1-x^2)y'[x]==(x-3 x^3 y[x])y[x],y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to \frac{x}{-6\sqrt{1-x^2}\arctan\left(\frac{x}{\sqrt{1-x^2}-1}\right) + c_1\sqrt{1-x^2} + 3x}$$
$$y(x) \to 0$$

13.22 problem 376

Internal problem ID [3632]

Book: Ordinary differential equations and their solutions. By George Moseley Murphy. 1960
Section: Various 13
Problem number: 376.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_separable]`

$$x\left(-2x^3+1\right)y'-2\left(-x^3+1\right)y=0$$

✓ Solution by Maple
Time used: 0.0 (sec). Leaf size: 18

```
dsolve(x*(-2*x^3+1)*diff(y(x),x) = 2*(-x^3+1)*y(x),y(x), singsol=all)
```

$$y(x)=\frac{c_1x^2}{(2x^3-1)^{\frac{1}{3}}}$$

✓ Solution by Mathematica
Time used: 0.045 (sec). Leaf size: 27

```
DSolve[x(1-2 x^3) y'[x]==2(1-x^3)y[x],y[x],x,IncludeSingularSolutions -> True]
```

$$y(x)\to\frac{c_1x^2}{\sqrt[3]{1-2x^3}}$$
$$y(x)\to 0$$

13.23 problem 377

Internal problem ID [3633]

Book: Ordinary differential equations and their solutions. By George Moseley Murphy. 1960
Section: Various 13
Problem number: 377.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_rational, _Riccati]`

$$\left(c\,x^{2}+bx+a\right)^{2}\left(y'+y^{2}\right)=-A$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 490

```
dsolve((c*x^2+b*x+a)^2*(diff(y(x),x)+y(x)^2)+A = 0,y(x), singsol=all)
```

$$y(x) = \frac{2c\left(c_1\left(i\sqrt{\frac{-4ac+b^2-4A}{c^2}}\,c\sqrt{4ac-b^2}-\sqrt{-4ac+b^2}\,(2cx+b)\right)\left(\frac{-b+i\sqrt{4ac-b^2}-2cx}{i\sqrt{4ac-b^2}+2cx+b}\right)^{-\frac{c\sqrt{\frac{-4ac+b^2-4A}{c^2}}}{2\sqrt{-4ac+b^2}}}-\left(i\sqrt{\frac{-4a}{}}\right.\right.}{\sqrt{-4ac+b^2}\,\left(i\sqrt{4ac-b^2}+2cx+b\right)\left(-b+i\sqrt{4ac-b^2}-2cx\right)\left(c_1\left(\frac{-b+i\sqrt{4ac-b^2}}{i\sqrt{4ac-b^2}+2c}\right.\right.}$$

✓ Solution by Mathematica

Time used: 3.457 (sec). Leaf size: 743

```
DSolve[(a+b x+c x^2)^2 (y'[x]+y[x]^2)+A==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to \frac{b^2 c_1 \left(-\exp\left(\frac{2\sqrt{4ac-b^2}\sqrt{1-\frac{4A}{b^2-4ac}}\arctan\left(\frac{b+2cx}{\sqrt{4ac-b^2}}\right)}{\sqrt{b^2-4ac}}\right)\right) + bc_1\sqrt{b^2-4ac}\sqrt{1-\frac{4A}{b^2-4ac}}\exp\left(\frac{2\sqrt{4ac-b^2}\sqrt{1-\frac{4A}{b^2-4ac}}\,\mathrm{ar}}{\sqrt{b^2-4a}}\right.}{}$$

$$y(x) \to \frac{2cx\sqrt{b^2-4ac}\sqrt{1-\frac{4A}{b^2-4ac}} + b\sqrt{b^2-4ac}\sqrt{1-\frac{4A}{b^2-4ac}} + 4ac + 4A - b^2}{2\sqrt{b^2-4ac}\sqrt{1-\frac{4A}{b^2-4ac}}(a+x(b+cx))}$$

13.24 problem 378

Internal problem ID [3634]

Book: Ordinary differential equations and their solutions. By George Moseley Murphy. 1960
Section: Various 13
Problem number: 378.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_linear]`

$$\boxed{x^5 y' + 3y x^4 = 1}$$

✓ Solution by Maple
Time used: 0.0 (sec). Leaf size: 13

```
dsolve(x^5*diff(y(x),x) = 1-3*x^4*y(x),y(x), singsol=all)
```

$$y(x) = \frac{c_1 x - 1}{x^4}$$

✓ Solution by Mathematica
Time used: 0.027 (sec). Leaf size: 15

```
DSolve[x^5 y'[x]==1-3 x^4 y[x],y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to \frac{-1 + c_1 x}{x^4}$$

13.25 problem 379

Internal problem ID [3635]

Book: Ordinary differential equations and their solutions. By George Moseley Murphy. 1960
Section: Various 13
Problem number: 379.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[[_homogeneous, ‘class D‘], _rational, _Riccati]`

$$x(-x^4+1)\,y' - 2x(x^2-y^2) - (-x^4+1)\,y = 0$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 32

```
dsolve(x*(-x^4+1)*diff(y(x),x) = 2*x*(x^2-y(x)^2)+(-x^4+1)*y(x),y(x), singsol=all)
```

$$y(x) = -\tanh\left(\frac{\ln(x+1)}{2} - \frac{\ln(x^2+1)}{2} + \frac{\ln(x-1)}{2} + 2c_1\right)x$$

✓ Solution by Mathematica

Time used: 0.329 (sec). Leaf size: 58

```
DSolve[x(1-x^4)y'[x]==2 x(x^2-y[x]^2)+(1-x^4) y[x],y[x],x,IncludeSingularSolutions -> True]
```

$$\begin{aligned} y(x) &\to -\frac{x(x^2+e^{2c_1}(x^2-1)+1)}{-x^2+e^{2c_1}(x^2-1)-1} \\ y(x) &\to -x \\ y(x) &\to x \end{aligned}$$

14 Various 14

14.1 problem 380

Internal problem ID [3636]

Book: Ordinary differential equations and their solutions. By George Moseley Murphy. 1960
Section: Various 14
Problem number: 380.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_rational, _Abel]`

$$\boxed{x^7y' + 5y^2x^3 + 2(x^2+1)\,y^3 = 0}$$

✓ Solution by Maple
Time used: 0.015 (sec). Leaf size: 78

```
dsolve(x^7*diff(y(x),x)+5*x^3*y(x)^2+2*(x^2+1)*y(x)^3 = 0,y(x), singsol=all)
```

$$c_1 + \frac{x}{\left(\frac{x^6+x^2y(x)^2+2x^3y(x)+y(x)^2}{x^2y(x)^2}\right)^{\frac{1}{4}}} + \frac{(x^3+y(x))\,\mathrm{hypergeom}\left(\left[\frac{1}{2},\frac{5}{4}\right],\left[\frac{3}{2}\right],-\frac{(x^3+y(x))^2}{x^2y(x)^2}\right)}{2y(x)\,x} = 0$$

✓ Solution by Mathematica
Time used: 0.313 (sec). Leaf size: 123

```
DSolve[x^7 y'[x]+5 x^3 y[x]^2+2(1+x^2)y[x]^3==0,y[x],x,IncludeSingularSolutions -> True]
```

$$\mathrm{Solve}\left[c_1 = \frac{\frac{1}{2}\sqrt[4]{1-\left(\frac{ix^2}{y(x)}+\frac{i}{x}\right)^2}\left(\frac{ix^2}{y(x)}+\frac{i}{x}\right)\mathrm{Hypergeometric2F1}\left(\frac{1}{2},\frac{5}{4},\frac{3}{2},\left(\frac{ix^2}{y(x)}+\frac{i}{x}\right)^2\right)+ix}{\sqrt[4]{-1+\left(\frac{ix^2}{y(x)}+\frac{i}{x}\right)^2}},y(x)\right]$$

14.2 problem 381

Internal problem ID [3637]

Book: Ordinary differential equations and their solutions. By George Moseley Murphy. 1960
Section: Various 14
Problem number: 381.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_linear]`

$$\boxed{y' x^n - b\, x^{n-1} y = a}$$

✓ Solution by Maple
Time used: 0.0 (sec). Leaf size: 26

```
dsolve(x^n*diff(y(x),x) = a+b*x^(n-1)*y(x),y(x), singsol=all)
```

$$y(x) = -\frac{x^{-n+1} a}{n+b-1} + x^b c_1$$

✓ Solution by Mathematica
Time used: 0.107 (sec). Leaf size: 28

```
DSolve[x^n y'[x]==a+b x^(n-1) y[x],y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to -\frac{a x^{1-n}}{b+n-1} + c_1 x^b$$

14.3 problem 382

Internal problem ID [3638]

Book: Ordinary differential equations and their solutions. By George Moseley Murphy. 1960
Section: Various 14
Problem number: 382.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_Riccati]`

$$\boxed{y'x^n + y^2 = x^{2n-1}}$$

✓ Solution by Maple
Time used: 0.0 (sec). Leaf size: 80

```
dsolve(x^n*diff(y(x),x) = x^(2*n-1)-y(x)^2,y(x), singsol=all)
```

$$y(x) = \frac{\left(\mathrm{BesselK}\left(n, 2\sqrt{x}\right) c_1 - \mathrm{BesselI}\left(n, 2\sqrt{x}\right)\right) x^n}{-\mathrm{BesselI}\left(n+1, 2\sqrt{x}\right)\sqrt{x} - \sqrt{x}\,\mathrm{BesselK}\left(n+1, 2\sqrt{x}\right) c_1 + n\left(\mathrm{BesselK}\left(n, 2\sqrt{x}\right) c_1 - \mathrm{BesselI}\left(n, 2\sqrt{x}\right)\right.}$$

✓ Solution by Mathematica
Time used: 0.366 (sec). Leaf size: 293

```
DSolve[x^n y'[x]==x^(2 n -1)-y[x]^2,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to \frac{x^{n-1}\left(-\left((n-1)\,\mathrm{Gamma}(2-n)\,\mathrm{BesselI}\left(1-n, 2\sqrt{x}\right)\right) + \sqrt{x}\,\mathrm{Gamma}(2-n)\,\mathrm{BesselI}\left(2-n, 2\sqrt{x}\right) + \sqrt{\right.}{}$$

$$y(x) \to \frac{x^{n-1}\left(\sqrt{x}\,\mathrm{BesselI}\left(n-2, 2\sqrt{x}\right) - (n-1)\,\mathrm{BesselI}\left(n-1, 2\sqrt{x}\right) + \sqrt{x}\,\mathrm{BesselI}\left(n, 2\sqrt{x}\right)\right)}{2\,\mathrm{BesselI}\left(n-1, 2\sqrt{x}\right)}$$

14.4 problem 384

Internal problem ID [3639]

Book: Ordinary differential equations and their solutions. By George Moseley Murphy. 1960
Section: Various 14
Problem number: 384.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_Riccati]`

$$y' x^n + y^2 = -x^{-2+2n} - (-n+1)\, x^{n-1}$$

✓ Solution by Maple
Time used: 0.015 (sec). Leaf size: 1225

```
dsolve(x^n*diff(y(x),x)+x^(2*n-2)+y(x)^2+(1-n)*x^(n-1) = 0,y(x), singsol=all)
```

Expression too large to display

✗ Solution by Mathematica
Time used: 0.0 (sec). Leaf size: 0

```
DSolve[x^n*y'[x]+x^(2*n-2)+y[x]^2+(1-n)*x^(n-1)==0,y[x],x,IncludeSingularSolutions -> True]
```

Not solved

14.5 problem 385

Internal problem ID [3640]

Book: Ordinary differential equations and their solutions. By George Moseley Murphy. 1960
Section: Various 14
Problem number: 385.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[[_homogeneous, ‘class G‘], _Riccati]`

$$y'x^n - b^2y^2 = a^2x^{-2+2n}$$

✓ Solution by Maple
Time used: 0.032 (sec). Leaf size: 67

```
dsolve(x^n*diff(y(x),x) = a^2*x^(2*n-2)+b^2*y(x)^2,y(x), singsol=all)
```

$$y(x) = \frac{x^{n-1}\left(n-1+\tan\left(\frac{\sqrt{4a^2b^2-n^2+2n-1}\,(\ln(x)-c_1)}{2}\right)\sqrt{4a^2b^2-n^2+2n-1}\right)}{2b^2}$$

✓ Solution by Mathematica
Time used: 0.541 (sec). Leaf size: 162

```
DSolve[x^n y'[x]==a^2 x^(2 n-2)+b^2 y[x]^2,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to \frac{x^{n-1}\left(\left(-ab\sqrt{\frac{(n-1)^2}{a^2b^2}-4}+n-1\right)x^{ab\sqrt{\frac{(n-1)^2}{a^2b^2}-4}}+c_1\left(ab\sqrt{\frac{(n-1)^2}{a^2b^2}-4}+n-1\right)\right)}{2b^2\left(x^{ab\sqrt{\frac{(n-1)^2}{a^2b^2}-4}}+c_1\right)}$$

$$y(x) \to \frac{x^{n-1}\left(ab\sqrt{\frac{(n-1)^2}{a^2b^2}-4}+n-1\right)}{2b^2}$$

14.6 problem 386

Internal problem ID [3641]

Book: Ordinary differential equations and their solutions. By George Moseley Murphy. 1960
Section: Various 14
Problem number: 386.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_rational, _Riccati]`

$$\boxed{y' x^n - x^{n-1}\left(a\, x^{2n} + ny - by^2\right) = 0}$$

✓ Solution by Maple

Time used: 0.016 (sec). Leaf size: 35

```
dsolve(x^n*diff(y(x),x) = x^(n-1)*(a*x^(2*n)+n*y(x)-b*y(x)^2),y(x), singsol=all)
```

$$y(x) = \frac{\tanh\left(\frac{x^n \sqrt{a}\,\sqrt{b}+ic_1 n}{n}\right)\sqrt{a}\, x^n}{\sqrt{b}}$$

✓ Solution by Mathematica

Time used: 0.322 (sec). Leaf size: 153

```
DSolve[x^n y'[x]==x^(n-1)(a x^(2 n)+n y[x]-b y[x]^2),y[x],x,IncludeSingularSolutions -> True
```

$$y(x) \to \frac{\sqrt{a}x^n\left(-\cos\left(\frac{\sqrt{a}\sqrt{-b}x^n}{n}\right) + c_1\sin\left(\frac{\sqrt{a}\sqrt{-b}x^n}{n}\right)\right)}{\sqrt{-b}\left(\sin\left(\frac{\sqrt{a}\sqrt{-b}x^n}{n}\right) + c_1\cos\left(\frac{\sqrt{a}\sqrt{-b}x^n}{n}\right)\right)}$$

$$y(x) \to \frac{\sqrt{a}x^n\tan\left(\frac{\sqrt{a}\sqrt{-b}x^n}{n}\right)}{\sqrt{-b}}$$

14.7 problem 388

Internal problem ID [3642]

Book: Ordinary differential equations and their solutions. By George Moseley Murphy. 1960
Section: Various 14
Problem number: 388.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_Chini]`

$$\boxed{x^k y' - b y^n = a\,x^m}$$

✗ Solution by Maple

```
dsolve(x^k*diff(y(x),x) = a*x^m+b*y(x)^n,y(x), singsol=all)
```

No solution found

✗ Solution by Mathematica

Time used: 0.0 (sec). Leaf size: 0

```
DSolve[x^k y'[x]==a x^m + b y[x]^n,y[x],x,IncludeSingularSolutions -> True]
```

Not solved

14.8 problem 389

Internal problem ID [3643]

Book: Ordinary differential equations and their solutions. By George Moseley Murphy. 1960
Section: Various 14
Problem number: 389.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_linear]`

$$\boxed{y'\sqrt{x^2+1}+y=2x}$$

✓ Solution by Maple
Time used: 0.0 (sec). Leaf size: 34

```
dsolve(diff(y(x),x)*sqrt(x^2+1) = 2*x-y(x),y(x), singsol=all)
```

$$y(x)=\frac{x^2+x\sqrt{x^2+1}-\operatorname{arcsinh}(x)+c_1}{x+\sqrt{x^2+1}}$$

✓ Solution by Mathematica
Time used: 0.066 (sec). Leaf size: 50

```
DSolve[y'[x] Sqrt[1+x^2]==2 x -y[x],y[x],x,IncludeSingularSolutions -> True]
```

$$y(x)\rightarrow\left(\sqrt{x^2+1}-x\right)\left(x^2+\sqrt{x^2+1}x+\log\left(\sqrt{x^2+1}-x\right)+c_1\right)$$

14.9 problem 390

Internal problem ID [3644]

Book: Ordinary differential equations and their solutions. By George Moseley Murphy. 1960
Section: Various 14
Problem number: 390.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_separable]`

$$\boxed{y'\sqrt{-x^2+1}-y^2=1}$$

✓ Solution by Maple
Time used: 0.0 (sec). Leaf size: 9

```
dsolve(diff(y(x),x)*sqrt(-x^2+1) = 1+y(x)^2,y(x), singsol=all)
```

$$y(x)=\tan(\arcsin(x)+c_1)$$

✓ Solution by Mathematica
Time used: 0.283 (sec). Leaf size: 47

```
DSolve[y'[x] Sqrt[1-x^2]==1+y[x]^2,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x)\to -\tan\left(2\arctan\left(\frac{\sqrt{1-x^2}}{x+1}\right)-c_1\right)$$
$$y(x)\to -i$$
$$y(x)\to i$$

14.10 problem 391

Internal problem ID [3645]

Book: Ordinary differential equations and their solutions. By George Moseley Murphy. 1960
Section: Various 14
Problem number: 391.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_separable]`

$$\left(-\sqrt{x^2+1}+x\right)y'-y-\sqrt{y^2+1}=0$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 39

```
dsolve((x-sqrt(x^2+1))*diff(y(x),x) = y(x)+sqrt(1+y(x)^2),y(x), singsol=all)
```

$$c_1+x^2+x\sqrt{x^2+1}+\operatorname{arcsinh}(x)+y(x)\sqrt{y(x)^2+1}+\operatorname{arcsinh}(y(x))-y(x)^2=0$$

✓ Solution by Mathematica

Time used: 0.922 (sec). Leaf size: 84

```
DSolve[(x-Sqrt[1+x^2])y'[x]==y[x]+Sqrt[1+ y[x]^2],y[x],x,IncludeSingularSolutions -> True]
```

$$y(x)\to \text{InverseFunction}\left[\frac{1}{2}\left(\#1\left(\sqrt{\#1^2+1}-\#1\right)-\log\left(\sqrt{\#1^2+1}-\#1\right)\right)\&\right]\left[\frac{1}{2}\left(\log\left(\sqrt{x^2+1}-x\right)-x\left(\sqrt{x^2+1}+x\right)\right)+c_1\right]$$

14.11 problem 392

Internal problem ID [3646]

Book: Ordinary differential equations and their solutions. By George Moseley Murphy. 1960
Section: Various 14
Problem number: 392.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_linear]`

$$\boxed{y'\sqrt{a^2+x^2}+y=-x+\sqrt{a^2+x^2}}$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 36

```
dsolve(diff(y(x),x)*sqrt(a^2+x^2)+x+y(x) = sqrt(a^2+x^2),y(x), singsol=all)
```

$$y(x)=\frac{a^2\ln\left(x+\sqrt{a^2+x^2}\right)+c_1}{x+\sqrt{a^2+x^2}}$$

✓ Solution by Mathematica

Time used: 8.172 (sec). Leaf size: 103

```
DSolve[y'[x] Sqrt[a^2+x^2]+x+y[x]==Sqrt[a^2 + x^2],y[x],x,IncludeSingularSolutions -> True]
```

$$y(x)\to\frac{1}{2}\left(x-\sqrt{a^2+x^2}\right)\left(\log\left(1-\frac{x}{\sqrt{a^2+x^2}}\right)-\log\left(\frac{x}{\sqrt{a^2+x^2}}+1\right)\right)+\frac{c_1\sqrt{1-\frac{x}{\sqrt{a^2+x^2}}}}{\sqrt{\frac{x}{\sqrt{a^2+x^2}}+1}}$$

14.12 problem 393

Internal problem ID [3647]

Book: Ordinary differential equations and their solutions. By George Moseley Murphy. 1960
Section: Various 14
Problem number: 393.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_separable]`

$$y'\sqrt{b^2+x^2}-\sqrt{y^2+a^2}=0$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 33

```
dsolve(diff(y(x),x)*sqrt(b^2+x^2) = sqrt(y(x)^2+a^2),y(x), singsol=all)
```

$$\ln\left(x+\sqrt{b^2+x^2}\right)-\ln\left(y(x)+\sqrt{y(x)^2+a^2}\right)+c_1=0$$

✓ Solution by Mathematica

Time used: 11.551 (sec). Leaf size: 167

```
DSolve[y'[x] Sqrt[x^2+b^2]==Sqrt[y[x]^2+a^2],y[x],x,IncludeSingularSolutions -> True]
```

$$y(x)\to-\frac{e^{-c_1}\sqrt{a^2\left(2x\left(\left(-1+e^{4c_1}\right)\sqrt{b^2+x^2}+\left(1+e^{4c_1}\right)x\right)+b^2\left(-1+e^{2c_1}\right)^2\right)}}{2b}$$

$$y(x)\to\frac{e^{-c_1}\sqrt{a^2\left(2x\left(\left(-1+e^{4c_1}\right)\sqrt{b^2+x^2}+\left(1+e^{4c_1}\right)x\right)+b^2\left(-1+e^{2c_1}\right)^2\right)}}{2b}$$

$$y(x)\to-ia$$

$$y(x)\to ia$$

14.13 problem 394

Internal problem ID [3648]

Book: Ordinary differential equations and their solutions. By George Moseley Murphy. 1960
Section: Various 14
Problem number: 394.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_separable]`

$$y'\sqrt{b^2 - x^2} - \sqrt{a^2 - y^2} = 0$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 37

```
dsolve(diff(y(x),x)*sqrt(b^2-x^2) = sqrt(a^2-y(x)^2),y(x), singsol=all)
```

$$\arctan\left(\frac{x}{\sqrt{b^2 - x^2}}\right) - \arctan\left(\frac{y(x)}{\sqrt{a^2 - y(x)^2}}\right) + c_1 = 0$$

✓ Solution by Mathematica

Time used: 4.959 (sec). Leaf size: 118

```
DSolve[y'[x] Sqrt[b^2-x^2]==Sqrt[a^2-y[x]^2],y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to -\frac{a \tan\left(\arctan\left(\frac{x}{\sqrt{b^2-x^2}}\right) + c_1\right)}{\sqrt{\sec^2\left(\arctan\left(\frac{x}{\sqrt{b^2-x^2}}\right) + c_1\right)}}$$

$$y(x) \to \frac{a \tan\left(\arctan\left(\frac{x}{\sqrt{b^2-x^2}}\right) + c_1\right)}{\sqrt{\sec^2\left(\arctan\left(\frac{x}{\sqrt{b^2-x^2}}\right) + c_1\right)}}$$

$$y(x) \to -a$$

$$y(x) \to a$$

14.14 problem 395

Internal problem ID [3649]

Book: Ordinary differential equations and their solutions. By George Moseley Murphy. 1960
Section: Various 14
Problem number: 395.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_separable]`

$$xy'\sqrt{a^2+x^2} - y\sqrt{b^2+y^2} = 0$$

✓ Solution by Maple
Time used: 0.016 (sec). Leaf size: 79

```
dsolve(x*diff(y(x),x)*sqrt(a^2+x^2) = y(x)*sqrt(b^2+y^2),y(x), singsol=all)
```

$$\frac{\operatorname{csgn}(b)\,a\ln(2) - \operatorname{csgn}(a)\,b\ln(2) + \operatorname{csgn}(b)\,a\ln\left(\frac{b\left(\sqrt{b^2+y(x)^2}\,\operatorname{csgn}(b)+b\right)}{y(x)}\right) - \operatorname{csgn}(a)\,b\ln\left(\frac{a\left(\sqrt{a^2+x^2}\,\operatorname{csgn}(a)+a\right.}{x}\right.}{ab}$$
$$= 0$$

✓ Solution by Mathematica

Time used: 26.26 (sec). Leaf size: 274

```
DSolve[x y'[x] Sqrt[a^2+x^2]==y[x] Sqrt[b^2+y[x]^2],y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \rightarrow -\frac{2b^{3/2}e^{bc_1}\left(a\left(\sqrt{a^2+x^2}-a\right)\right)^{\frac{b}{2a}}\left(\sqrt{a^2+x^2}+a\right)^{\frac{b}{2a}}}{\sqrt{\left(-b\left(\sqrt{a^2+x^2}+a\right)^{\frac{b}{a}}+e^{2bc_1}\left(a\left(\sqrt{a^2+x^2}-a\right)\right)^{\frac{b}{a}}\right)^2}}$$

$$y(x) \rightarrow \frac{2b^{3/2}e^{bc_1}\left(a\left(\sqrt{a^2+x^2}-a\right)\right)^{\frac{b}{2a}}\left(\sqrt{a^2+x^2}+a\right)^{\frac{b}{2a}}}{\sqrt{\left(-b\left(\sqrt{a^2+x^2}+a\right)^{\frac{b}{a}}+e^{2bc_1}\left(a\left(\sqrt{a^2+x^2}-a\right)\right)^{\frac{b}{a}}\right)^2}}$$

$$y(x) \rightarrow 0$$

$$y(x) \rightarrow -ib$$

$$y(x) \rightarrow ib$$

14.15 problem 396

Internal problem ID [3650]

Book: Ordinary differential equations and their solutions. By George Moseley Murphy. 1960
Section: Various 14
Problem number: 396.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_separable]`

$$xy'\sqrt{-a^2+x^2} - y\sqrt{y^2-b^2} = 0$$

✓ Solution by Maple
Time used: 0.0 (sec). Leaf size: 135

```
dsolve(x*diff(y(x),x)*sqrt(-a^2+x^2) = y(x)*sqrt(y(x)^2-b^2),y(x), singsol=all)
```

$$\frac{c_1\sqrt{-a^2}\,\sqrt{-b^2} + \sqrt{-a^2}\,\ln(2) - \sqrt{-b^2}\,\ln(2) - \sqrt{-b^2}\,\ln\left(\frac{\sqrt{-a^2}\,\sqrt{-a^2+x^2}-a^2}{x}\right) + \sqrt{-a^2}\,\ln\left(\frac{\sqrt{-b^2}\,\sqrt{y(x)^2-b^2}-b^2}{y(x)}\right.}{\sqrt{-a^2}\,\sqrt{-b^2}}$$
$= 0$

✓ Solution by Mathematica
Time used: 18.348 (sec). Leaf size: 101

```
DSolve[x y'[x] Sqrt[x^2-a^2]==y[x] Sqrt[y[x]^2-b^2],y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to -b\sqrt{\sec^2\left(\frac{b\left(\arctan\left(\frac{\sqrt{x^2-a^2}}{a}\right)+ac_1\right)}{a}\right)}$$
$$y(x) \to b\sqrt{\sec^2\left(\frac{b\left(\arctan\left(\frac{\sqrt{x^2-a^2}}{a}\right)+ac_1\right)}{a}\right)}$$
$$y(x) \to 0$$
$$y(x) \to -b$$
$$y(x) \to b$$

14.16 problem 397

Internal problem ID [3651]

Book: Ordinary differential equations and their solutions. By George Moseley Murphy. 1960
Section: Various 14
Problem number: 397.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_quadrature]`

$$\boxed{y'\sqrt{X}+\sqrt{Y}=0}$$

✓ Solution by Maple
Time used: 0.0 (sec). Leaf size: 15

```
dsolve(diff(y(x),x)*sqrt(X)+sqrt(Y) = 0,y(x), singsol=all)
```

$$y(x) = -\frac{\sqrt{Y}\,x}{\sqrt{X}} + c_1$$

✓ Solution by Mathematica
Time used: 0.002 (sec). Leaf size: 21

```
DSolve[y'[x] Sqrt[X]+Sqrt[Y]==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to -\frac{x\sqrt{Y}}{\sqrt{X}} + c_1$$

14.17 problem 398

Internal problem ID [3652]

Book: Ordinary differential equations and their solutions. By George Moseley Murphy. 1960
Section: Various 14
Problem number: 398.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_quadrature]`

$$\boxed{y'\sqrt{X}-\sqrt{Y}=0}$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 14

```
dsolve(diff(y(x),x)*sqrt(X) = sqrt(Y),y(x), singsol=all)
```

$$y(x)=\frac{\sqrt{Y}\,x}{\sqrt{X}}+c_1$$

✓ Solution by Mathematica

Time used: 0.002 (sec). Leaf size: 20

```
DSolve[y'[x] Sqrt[X]==Sqrt[Y],y[x],x,IncludeSingularSolutions -> True]
```

$$y(x)\to\frac{x\sqrt{Y}}{\sqrt{X}}+c_1$$

14.18 problem 399

Internal problem ID [3653]

Book: Ordinary differential equations and their solutions. By George Moseley Murphy. 1960
Section: Various 14
Problem number: 399.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_rational, [_Riccati, _special]]`

$$\boxed{x^{\frac{3}{2}}y' - b\,x^{\frac{3}{2}}y^2 = a}$$

✓ Solution by Maple
Time used: 0.015 (sec). Leaf size: 119

```
dsolve(x^(3/2)*diff(y(x),x) = a+b*x^(3/2)*y(x)^2,y(x), singsol=all)
```

$$y(x) = -\frac{2a\left(\text{BesselJ}\left(1, 4\sqrt{a}\,\sqrt{b}\,x^{\frac{1}{4}}\right)c_1 + \text{BesselY}\left(1, 4\sqrt{a}\,\sqrt{b}\,x^{\frac{1}{4}}\right)\right)}{\sqrt{x}\,\left(-2\sqrt{a}\,x^{\frac{1}{4}}\,\text{BesselJ}\left(0, 4\sqrt{a}\,\sqrt{b}\,x^{\frac{1}{4}}\right)\sqrt{b}\,c_1 - 2\,\text{BesselY}\left(0, 4\sqrt{a}\,\sqrt{b}\,x^{\frac{1}{4}}\right)\sqrt{a}\,\sqrt{b}\,x^{\frac{1}{4}} + \text{BesselJ}\left(1, 4\sqrt{\right.\right.}}$$

✓ Solution by Mathematica
Time used: 0.252 (sec). Leaf size: 373

```
DSolve[x^(3/2) y'[x]==a+ b x^(3/2) y[x]^2,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to -\frac{\sqrt{a}\sqrt{b}\sqrt[4]{x}\,\text{BesselY}\left(1, 4\sqrt{a}\sqrt{b}\sqrt[4]{x}\right) + \text{BesselY}\left(2, 4\sqrt{a}\sqrt{b}\sqrt[4]{x}\right) - \sqrt{a}\sqrt{b}\sqrt[4]{x}\,\text{BesselY}\left(3, 4\sqrt{a}\sqrt{b}\sqrt[4]{x}\right) -}{2bx\,\text{BesselY}\left(2, 4\sqrt{a}\sqrt{b}\sqrt[4]{x}\right)}$$

$$y(x) \to -\frac{\sqrt{a}\sqrt{b}\sqrt[4]{x}\,\text{BesselJ}\left(1, 4\sqrt{a}\sqrt{b}\sqrt[4]{x}\right) + \text{BesselJ}\left(2, 4\sqrt{a}\sqrt{b}\sqrt[4]{x}\right) - \sqrt{a}\sqrt{b}\sqrt[4]{x}\,\text{BesselJ}\left(3, 4\sqrt{a}\sqrt{b}\sqrt[4]{x}\right)}{2bx\,\text{BesselJ}\left(2, 4\sqrt{a}\sqrt{b}\sqrt[4]{x}\right)}$$

14.19 problem 400

Internal problem ID [3654]

Book: Ordinary differential equations and their solutions. By George Moseley Murphy. 1960
Section: Various 14
Problem number: 400.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_separable]`

$$y'\sqrt{x^3+1}-\sqrt{1+y^3}=0$$

✓ Solution by Maple
Time used: 0.0 (sec). Leaf size: 28

```
dsolve(diff(y(x),x)*sqrt(x^3+1) = sqrt(1+y(x)^3),y(x), singsol=all)
```

$$\int \frac{1}{\sqrt{x^3+1}}dx-\left(\int^{y(x)}\frac{1}{\sqrt{\_a^3+1}}d\_a\right)+c_1=0$$

✓ Solution by Mathematica
Time used: 40.487 (sec). Leaf size: 71

```
DSolve[y'[x] Sqrt[1+x^3]==Sqrt[1+y[x]^3],y[x],x,IncludeSingularSolutions -> True]
```

$$y(x)\to \text{InverseFunction}\left[\#1\,\text{Hypergeometric2F1}\left(\frac{1}{3},\frac{1}{2},\frac{4}{3},-\#1^3\right)\&\right]\left[x\,\text{Hypergeometric2F1}\left(\frac{1}{3},\frac{1}{2},\frac{4}{3},-x^3\right)+c_1\right]$$
$$y(x)\to -1$$
$$y(x)\to \sqrt[3]{-1}$$
$$y(x)\to -(-1)^{2/3}$$

14.20 problem 401

Internal problem ID [3655]

Book: Ordinary differential equations and their solutions. By George Moseley Murphy. 1960
Section: Various 14
Problem number: 401.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_separable]`

$$y'\sqrt{x(1-x)(-ax+1)}-\sqrt{y(1-y)(1-ya)}=0$$

✓ Solution by Maple
Time used: 0.0 (sec). Leaf size: 38

```
dsolve(diff(y(x),x)*sqrt(x*(1-x)*(-a*x+1)) = sqrt(y(x)*(1-y(x))*(1-a*y(x))),y(x), singsol=al
```

$$\int \frac{1}{\sqrt{x(x-1)(ax-1)}}dx-\left(\int^{y(x)}\frac{1}{\sqrt{\_a(\_a-1)(a\_a-1)}}d\_a\right)+c_1=0$$

✓ Solution by Mathematica
Time used: 17.58 (sec). Leaf size: 117

```
DSolve[y'[x] Sqrt[x (1-x)(1-a x)]==Sqrt[y[x](1-y[x])(1-a y[x])],y[x],x,IncludeSingularSoluti
```

$$y(x)\to \operatorname{ns}\left(\frac{1}{2}i\sqrt{a}c_1-\operatorname{EllipticF}\left(i\operatorname{arcsinh}\left(\frac{1}{\sqrt{x-1}}\right),\frac{a-1}{a}\right)|\frac{a-1}{a}\right)^2\left(-1+\operatorname{sn}\left(\frac{1}{2}i\sqrt{a}c_1-\operatorname{EllipticF}\left(i\operatorname{arcsinh}\left(\frac{1}{\sqrt{x-1}}\right),\frac{a-1}{a}\right)|\frac{a-1}{a}\right)^2\right)$$
$$y(x)\to 0$$
$$y(x)\to 1$$
$$y(x)\to \frac{1}{a}$$

14.21 problem 402

Internal problem ID [3656]

Book: Ordinary differential equations and their solutions. By George Moseley Murphy. 1960
Section: Various 14
Problem number: 402.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_separable]`

$$\boxed{y'\sqrt{-x^4+1}-\sqrt{1-y^4}=0}$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 32

```
dsolve(diff(y(x),x)*sqrt(-x^4+1) = sqrt(1-y(x)^4),y(x), singsol=all)
```

$$\int \frac{1}{\sqrt{-x^4+1}}dx-\left(\int^{y(x)}\frac{1}{\sqrt{-\_a^4+1}}d\_a\right)+c_1=0$$

✓ Solution by Mathematica

Time used: 40.393 (sec). Leaf size: 38

```
DSolve[y'[x] Sqrt[1-x^4]==Sqrt[1-y[x]^4],y[x],x,IncludeSingularSolutions -> True]
```

$$y(x)\to \operatorname{sn}(c_1+\operatorname{EllipticF}(\arcsin(x),-1)|-1)$$
$$y(x)\to -1$$
$$y(x)\to -i$$
$$y(x)\to i$$
$$y(x)\to 1$$

14.22 problem 403

Internal problem ID [3657]

Book: Ordinary differential equations and their solutions. By George Moseley Murphy. 1960
Section: Various 14
Problem number: 403.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_separable]`

$$y'\sqrt{x^4+x^2+1}-\sqrt{1+y^2+y^4}=0$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 34

```
dsolve(diff(y(x),x)*sqrt(x^4+x^2+1) = sqrt(1+y(x)^2+y(x)^4),y(x), singsol=all)
```

$$\int \frac{1}{\sqrt{x^4+x^2+1}}dx-\left(\int^{y(x)}\frac{1}{\sqrt{\_a^4+\_a^2+1}}d\_a\right)+c_1=0$$

✓ Solution by Mathematica

Time used: 41.523 (sec). Leaf size: 189

```
DSolve[y'[x]Sqrt[1+x^2+x^4]==Sqrt[1+y[x]^2+y[x]^4],y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to \text{InverseFunction}\left[\frac{(-1)^{2/3}\sqrt{\sqrt[3]{-1}\#1^2+1}\sqrt{1-(-1)^{2/3}\#1^2}\,\text{EllipticF}\left(i\text{arcsinh}\left((-1)^{5/6}\#1\right),(-1)^{2/3}\right)}{\sqrt{\#1^4+\#1^2+1}}\&\right.$$

$$y(x) \to -\sqrt[3]{-1}$$
$$y(x) \to \sqrt[3]{-1}$$
$$y(x) \to -(-1)^{2/3}$$
$$y(x) \to (-1)^{2/3}$$

14.23 problem 404

Internal problem ID [3658]

Book: Ordinary differential equations and their solutions. By George Moseley Murphy. 1960
Section: Various 14
Problem number: 404.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_quadrature]`

$$\boxed{y'\sqrt{X} = 0}$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 5

```
dsolve(diff(y(x),x)*sqrt(X) = 0,y(x), singsol=all)
```

$$y(x) = c_1$$

✓ Solution by Mathematica

Time used: 0.002 (sec). Leaf size: 7

```
DSolve[y'[x] Sqrt[X]==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to c_1$$

14.24 problem 405

Internal problem ID [3659]

Book: Ordinary differential equations and their solutions. By George Moseley Murphy. 1960
Section: Various 14
Problem number: 405.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_quadrature]`

$$\boxed{y'\sqrt{X}+\sqrt{Y}=0}$$

✓ Solution by Maple
Time used: 0.0 (sec). Leaf size: 15

```
dsolve(diff(y(x),x)*sqrt(X)+sqrt(Y) = 0,y(x), singsol=all)
```

$$y(x) = -\frac{\sqrt{Y}\,x}{\sqrt{X}}+c_1$$

✓ Solution by Mathematica
Time used: 0.002 (sec). Leaf size: 21

```
DSolve[y'[x] Sqrt[X]+Sqrt[Y]==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to -\frac{x\sqrt{Y}}{\sqrt{X}}+c_1$$

14.25 problem 406

Internal problem ID [3660]

Book: Ordinary differential equations and their solutions. By George Moseley Murphy. 1960
Section: Various 14
Problem number: 406.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_quadrature]`

$$\boxed{y'\sqrt{X} - \sqrt{Y} = 0}$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 14

```
dsolve(diff(y(x),x)*sqrt(X) = sqrt(Y),y(x), singsol=all)
```

$$y(x) = \frac{\sqrt{Y}\,x}{\sqrt{X}} + c_1$$

✓ Solution by Mathematica

Time used: 0.003 (sec). Leaf size: 20

```
DSolve[y'[x] Sqrt[X]==Sqrt[Y],y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to \frac{x\sqrt{Y}}{\sqrt{X}} + c_1$$

14.26 problem 407

Internal problem ID [3661]

Book: Ordinary differential equations and their solutions. By George Moseley Murphy. 1960
Section: Various 14
Problem number: 407.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_separable]`

$$\boxed{y'\left(x^3+1\right)^{\frac{2}{3}}+\left(1+y^3\right)^{\frac{2}{3}}=0}$$

✓ Solution by Maple

Time used: 0.031 (sec). Leaf size: 119

```
dsolve(diff(y(x),x)*(x^3+1)^(2/3)+(1+y(x)^3)^(2/3) = 0,y(x), singsol=all)
```

$$c_1 + \frac{2\pi\sqrt{3}\left(y(x)\left(x^3+1\right)^{\frac{1}{3}}\left(-x^3\right)^{\frac{1}{6}}\operatorname{LegendreP}\left(-\frac{1}{3},-\frac{1}{3},\frac{-y(x)^3+1}{1+y(x)^3}\right)+\left(1+y(x)^3\right)^{\frac{1}{3}}\operatorname{LegendreP}\left(-\frac{1}{3},-\frac{1}{3},\frac{-x^3+1}{x^3+1}\right.\right.}{9\left(-x^3\right)^{\frac{1}{6}}\left(x^3+1\right)^{\frac{1}{3}}\left(-y\left(x\right)^3\right)^{\frac{1}{6}}\left(1+y\left(x\right)^3\right)^{\frac{1}{3}}\Gamma\left(\frac{2}{3}\right)} = 0$$

✓ Solution by Mathematica

Time used: 3.082 (sec). Leaf size: 221

```
DSolve[y'[x](1+x^3)^(2/3)+(1+y[x]^3)^(2/3)==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x)$$
$$\to \text{InverseFunction}\left[\frac{3\sqrt[3]{\frac{\sqrt[3]{-1}-\#1}{1+\sqrt[3]{-1}}}(\#1+1)\left(\frac{\#1+(-1)^{2/3}}{(-1)^{2/3}-1}\right)^{2/3}\text{Hypergeometric2F1}\left(\frac{1}{3},\frac{2}{3},\frac{4}{3},\frac{\sqrt[3]{-1}(\#1+1}{\left(-1+\sqrt[3]{-1}\right)\#}\right.}{\left(\#1^3+1\right)^{2/3}}\right.$$

$$y(x) \to -1$$
$$y(x) \to \sqrt[3]{-1}$$
$$y(x) \to -(-1)^{2/3}$$

14.27 problem 408

Internal problem ID [3662]

Book: Ordinary differential equations and their solutions. By George Moseley Murphy. 1960
Section: Various 14
Problem number: 408.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_separable]`

$$y'\left(4x^3 + \mathrm{a1}\,x + \mathrm{a0}\right)^{\frac{2}{3}} + \left(\mathrm{a0} + \mathrm{a1}\,y + 4y^3\right)^{\frac{2}{3}} = 0$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 36

```
dsolve(diff(y(x),x)*(4*x^3+a1*x+a0)^(2/3)+(a0+a1*y(x)+4*y(x)^3)^(2/3) = 0,y(x), singsol=all)
```

$$\int \frac{1}{\left(4x^3 + \mathrm{a1}\,x + \mathrm{a0}\right)^{\frac{2}{3}}}dx + \int^{y(x)} \frac{1}{\left(4\_a^3 + \_a\,\mathrm{a1} + \mathrm{a0}\right)^{\frac{2}{3}}}d\_a + c_1 = 0$$

✓ Solution by Mathematica

Time used: 0.581 (sec). Leaf size: 558

```
DSolve[y'[x](a0+a1 x+4 x^3)^(2/3)+(a0+a1 y[x]+4 y[x]^3)^(2/3)==0,y[x],x,IncludeSingularSolut
```

$$\text{Solve}\left[\frac{3(y(x)-\text{Root}[4\#1^3+\#1\text{a1}+\text{a0}\&,1])\left(\frac{y(x)-\text{Root}[4\#1^3+\#1\text{a1}+\text{a0}\&,2]}{\text{Root}[4\#1^3+\#1\text{a1}+\text{a0}\&,1]-\text{Root}[4\#1^3+\#1\text{a1}+\text{a0}\&,2]}\right)^{2/3}}{}\right.$$

$$-\frac{3(x-\text{Root}[4\#1^3+\#1\text{a1}+\text{a0}\&,1])\left(\frac{x-\text{Root}[4\#1^3+\#1\text{a1}+\text{a0}\&,2]}{\text{Root}[4\#1^3+\#1\text{a1}+\text{a0}\&,1]-\text{Root}[4\#1^3+\#1\text{a1}+\text{a0}\&,2]}\right)^{2/3}\sqrt[3]{\frac{}{\text{Root}}}}{}$$

$$\left.+c_1,y(x)\right]$$

15 Various 15

15.1 problem 409

Internal problem ID [3663]

Book: Ordinary differential equations and their solutions. By George Moseley Murphy. 1960
Section: Various 15
Problem number: 409.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_quadrature]`

$$\boxed{X^{\frac{2}{3}}y' - Y^{\frac{2}{3}} = 0}$$

✓ Solution by Maple

Time used: 0.016 (sec). Leaf size: 14

```
dsolve(X^(2/3)*diff(y(x),x) = Y^(2/3),y(x), singsol=all)
```

$$y(x) = \frac{Y^{\frac{2}{3}}x}{X^{\frac{2}{3}}} + c_1$$

✓ Solution by Mathematica

Time used: 0.003 (sec). Leaf size: 20

```
DSolve[X^(2/3) y'[x]== Y^(2/3),y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to \frac{xY^{2/3}}{X^{2/3}} + c_1$$

15.2 problem 410

Internal problem ID [3664]

Book: Ordinary differential equations and their solutions. By George Moseley Murphy. 1960
Section: Various 15
Problem number: 410.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_Bernoulli]`

$$y'\left(a+\cos\left(\frac{x}{2}\right)^2\right)-y\tan\left(\frac{x}{2}\right)\left(1+a+\cos\left(\frac{x}{2}\right)^2-y\right)=0$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 78

```
dsolve(diff(y(x),x)*(a+cos(1/2*x)^2) = y(x)*tan(1/2*x)*(1+a+cos(1/2*x)^2-y(x)),y(x), singsol
```

$$y(x)=\frac{\sec\left(\frac{x}{2}\right)^2\left(a+\cos\left(\frac{x}{2}\right)^2\right)^{\frac{1}{a}}\cos\left(\frac{x}{2}\right)^{-\frac{2}{a}}}{\int\tan\left(\frac{x}{2}\right)\sec\left(\frac{x}{2}\right)^2\cos\left(\frac{x}{2}\right)^{-\frac{2}{a}}\left(a+\cos\left(\frac{x}{2}\right)^2\right)^{\frac{-a+1}{a}}dx+c_1}$$

✓ Solution by Mathematica

Time used: 1.657 (sec). Leaf size: 74

```
DSolve[y'[x](a+Cos[x/2]^2)==y[x] Tan[x/2](1+a+Cos[x/2]^2-y[x]),y[x],x,IncludeSingularSolutio
```

$$y(x)\to\frac{(a+1)\left(a+\cos^2\left(\frac{x}{2}\right)\right)^{\frac{1}{a}}}{\sin^2\left(\frac{x}{2}\right)\left(a+\cos^2\left(\frac{x}{2}\right)\right)^{\frac{1}{a}}+(a+1)c_1\cos^{\frac{2}{a}+2}\left(\frac{x}{2}\right)}$$
$$y(x)\to 0$$

15.3 problem 411

Internal problem ID [3665]

Book: Ordinary differential equations and their solutions. By George Moseley Murphy. 1960
Section: Various 15
Problem number: 411.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_separable]`

$$\left(-4\cos\left(x\right)^{2}+1\right)y'-\tan\left(x\right)\left(1+4\cos\left(x\right)^{2}\right)y=0$$

✓ Solution by Maple
Time used: 0.0 (sec). Leaf size: 15

```
dsolve((1-4*cos(x)^2)*diff(y(x),x) = tan(x)*(1+4*cos(x)^2)*y(x),y(x), singsol=all)
```

$$y(x)=\left(4\cos\left(x\right)-\sec\left(x\right)\right)c_1$$

✓ Solution by Mathematica
Time used: 0.436 (sec). Leaf size: 23

```
DSolve[(1-4 Cos[x]^2)y'[x]==Tan[x](1+4 Cos[x]^2)y[x],y[x],x,IncludeSingularSolutions -> True
```

$$y(x)\to c_1(2\cos(2x)+1)\sec(x)$$
$$y(x)\to 0$$

15.4 problem 412

Internal problem ID [3666]

Book: Ordinary differential equations and their solutions. By George Moseley Murphy. 1960
Section: Various 15
Problem number: 412.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_separable]`

$$\boxed{(-\sin(x)+1)\,y'+\cos(x)\,y=0}$$

✓ Solution by Maple
Time used: 0.0 (sec). Leaf size: 10

```
dsolve((1-sin(x))*diff(y(x),x)+y(x)*cos(x) = 0,y(x), singsol=all)
```

$$y(x)=c_1(\sin(x)-1)$$

✓ Solution by Mathematica
Time used: 0.303 (sec). Leaf size: 18

```
DSolve[(1-Sin[x])y'[x]+y[x] Cos[x]==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x)\to -c_1(\sin(x)-1)$$
$$y(x)\to 0$$

15.5 problem 413

Internal problem ID [3667]

Book: Ordinary differential equations and their solutions. By George Moseley Murphy. 1960
Section: Various 15
Problem number: 413.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_separable]`

$$(\cos(x) - \sin(x))\, y' + y(\cos(x) + \sin(x)) = 0$$

✓ Solution by Maple
Time used: 0.0 (sec). Leaf size: 13

```
dsolve((cos(x)-sin(x))*diff(y(x),x)+y(x)*(cos(x)+sin(x)) = 0,y(x), singsol=all)
```

$$y(x) = c_1(-\sin(x) + \cos(x))$$

✓ Solution by Mathematica
Time used: 0.121 (sec). Leaf size: 20

```
DSolve[(Cos[x]-Sin[x])y'[x]+y[x](Cos[x]+Sin[x])==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to c_1(\cos(x) - \sin(x))$$
$$y(x) \to 0$$

15.6 problem 414

Internal problem ID [3668]

Book: Ordinary differential equations and their solutions. By George Moseley Murphy. 1960
Section: Various 15
Problem number: 414.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_linear]`

$$(\mathrm{a0} + \mathrm{a1}\sin(x)^2)\, y' + \mathrm{a1}\, y \sin(2x) = -\mathrm{a2}\, x(\mathrm{a3} + \mathrm{a1}\sin(x)^2)$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 52

```
dsolve((a0+a1*sin(x)^2)*diff(y(x),x)+a2*x*(a3+a1*sin(x)^2)+a1*y(x)*sin(2*x) = 0,y(x), singso
```

$$y(x) = \frac{\mathrm{a2}\,\mathrm{a1}\cos(2x) + 2\,\mathrm{a2}\,x\,\mathrm{a1}\sin(2x) - 2x^2(\mathrm{a1} + 2\,\mathrm{a3})\,\mathrm{a2} + 8c_1}{-4\,\mathrm{a1}\cos(2x) + 8\,\mathrm{a0} + 4\,\mathrm{a1}}$$

✓ Solution by Mathematica

Time used: 0.411 (sec). Leaf size: 58

```
DSolve[(a0+a1 Sin[x]^2)y'[x]+a2 x(a3+a1 Sin[x]^2)+a1 y[x] Sin[2 x]==0,y[x],x,IncludeSingular
```

$$y(x) \to \frac{-2\mathrm{a1a2}x^2 + 2\mathrm{a1a2}x\sin(2x) + \mathrm{a1a2}\cos(2x) - 4\mathrm{a2a3}x^2 + 4c_1}{4(2\mathrm{a0} - \mathrm{a1}\cos(2x) + \mathrm{a1})}$$

15.7 problem 415

Internal problem ID [3669]

Book: Ordinary differential equations and their solutions. By George Moseley Murphy. 1960
Section: Various 15
Problem number: 415.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_linear]`

$$(x - e^x)\, y' + (1 - e^x)\, y = -x\, e^x$$

✓ Solution by Maple
Time used: 0.0 (sec). Leaf size: 23

```
dsolve((x-exp(x))*diff(y(x),x)+x*exp(x)+(1-exp(x))*y(x) = 0,y(x), singsol=all)
```

$$y(x) = \frac{x\, e^x - e^x + c_1}{-x + e^x}$$

✓ Solution by Mathematica
Time used: 0.094 (sec). Leaf size: 25

```
DSolve[(x-Exp[x])y'[x]+x Exp[x]+(1-Exp[x])y[x]==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to \frac{e^x(x-1) + c_1}{e^x - x}$$

15.8 problem 416

Internal problem ID [3670]

Book: Ordinary differential equations and their solutions. By George Moseley Murphy. 1960
Section: Various 15
Problem number: 416.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_linear]`

$$\boxed{y'x\ln(x) + y = ax(\ln(x) + 1)}$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 14

```
dsolve(diff(y(x),x)*x*ln(x) = a*x*(1+ln(x))-y(x),y(x), singsol=all)
```

$$y(x) = ax + \frac{c_1}{\ln(x)}$$

✓ Solution by Mathematica

Time used: 0.04 (sec). Leaf size: 16

```
DSolve[y'[x] x Log[x]==a x(1+Log[x])-y[x],y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to ax + \frac{c_1}{\log(x)}$$

15.9 problem 417

Internal problem ID [3671]

Book: Ordinary differential equations and their solutions. By George Moseley Murphy. 1960
Section: Various 15
Problem number: 417.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_separable]`

$$\boxed{yy' = -x}$$

✓ Solution by Maple
Time used: 0.0 (sec). Leaf size: 27

```
dsolve(y(x)*diff(y(x),x)+x = 0,y(x), singsol=all)
```

$$y(x) = \sqrt{-x^2 + c_1}$$
$$y(x) = -\sqrt{-x^2 + c_1}$$

✓ Solution by Mathematica
Time used: 0.09 (sec). Leaf size: 39

```
DSolve[y[x] y'[x]+x==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to -\sqrt{-x^2 + 2c_1}$$
$$y(x) \to \sqrt{-x^2 + 2c_1}$$

15.10 problem 418

Internal problem ID [3672]

Book: Ordinary differential equations and their solutions. By George Moseley Murphy. 1960
Section: Various 15
Problem number: 418.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_separable]`

$$\boxed{y y' = -x \, e^{x^2}}$$

✓ Solution by Maple

Time used: 0.016 (sec). Leaf size: 29

```
dsolve(y(x)*diff(y(x),x)+x*exp(x^2) = 0,y(x), singsol=all)
```

$$y(x) = \sqrt{-e^{x^2} + c_1}$$
$$y(x) = -\sqrt{-e^{x^2} + c_1}$$

✓ Solution by Mathematica

Time used: 1.743 (sec). Leaf size: 43

```
DSolve[y[x] y'[x]+x Exp[x^2]==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to -\sqrt{-e^{x^2} + 2c_1}$$
$$y(x) \to \sqrt{-e^{x^2} + 2c_1}$$

15.11 problem 419

Internal problem ID [3673]

Book: Ordinary differential equations and their solutions. By George Moseley Murphy. 1960
Section: Various 15
Problem number: 419.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_rational, [_Abel, '2nd type', 'class A']]`

$$\boxed{yy' + y = -x^3}$$

✗ Solution by Maple

```
dsolve(y(x)*diff(y(x),x)+x^3+y(x) = 0,y(x), singsol=all)
```

No solution found

✗ Solution by Mathematica

Time used: 0.0 (sec). Leaf size: 0

```
DSolve[y[x] y'[x]+x^3+y[x]==0,y[x],x,IncludeSingularSolutions -> True]
```

Not solved

15.12 problem 420

Internal problem ID [3674]

Book: Ordinary differential equations and their solutions. By George Moseley Murphy. 1960
Section: Various 15
Problem number: 420.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[[_homogeneous, ‘class A‘], _rational, [_Abel, ‘2nd type‘, ‘cl`

$$yy' + yb = -ax$$

✓ Solution by Maple

Time used: 0.188 (sec). Leaf size: 62

```
dsolve(y(x)*diff(y(x),x)+a*x+b*y(x) = 0,y(x), singsol=all)
```

$$y(x) = \operatorname{RootOf}\left(\_Z^2 - e^{\operatorname{RootOf}\left(\left(\left(-4\,e^{-\_Z}\cosh\left(\frac{\sqrt{b^2-4a}\,(2c_1+\_Z+2\ln(x))}{2b}\right)\right)^2 - b^2+4a\right)x^2\right)} + a + \_Zb\right)x$$

✓ Solution by Mathematica

Time used: 0.119 (sec). Leaf size: 74

```
DSolve[y[x] y'[x]+a x+b y[x]==0,y[x],x,IncludeSingularSolutions -> True]
```

$$\operatorname{Solve}\left[\frac{1}{2}\log\left(a + \frac{by(x)}{x} + \frac{y(x)^2}{x^2}\right) - \frac{b\arctan\left(\frac{b+\frac{2y(x)}{x}}{\sqrt{4a-b^2}}\right)}{\sqrt{4a-b^2}} = -\log(x) + c_1, y(x)\right]$$

15.13 problem 421

Internal problem ID [3675]

Book: Ordinary differential equations and their solutions. By George Moseley Murphy. 1960
Section: Various 15
Problem number: 421.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_separable]`

$$\boxed{yy' + x\,\mathrm{e}^{-x}(y+1) = 0}$$

✓ Solution by Maple

Time used: 0.031 (sec). Leaf size: 25

```
dsolve(y(x)*diff(y(x),x)+x*exp(-x)*(1+y(x)) = 0,y(x), singsol=all)
```

$$y(x) = -\operatorname{LambertW}\left(-\mathrm{e}^{(-x-1)\mathrm{e}^{-x}+c_1-1}\right) - 1$$

✓ Solution by Mathematica

Time used: 4.963 (sec). Leaf size: 63

```
DSolve[y[x] y'[x]+x Exp[-x](1+y[x])==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to -1 - W\left(-e^{-e^{-x}(x+(1+c_1)e^x+1)}\right)$$
$$y(x) \to -1$$
$$y(x) \to -W\left(-e^{-e^{-x}(x+e^x+1)}\right) - 1$$

15.14 problem 422

Internal problem ID [3676]

Book: Ordinary differential equations and their solutions. By George Moseley Murphy. 1960
Section: Various 15
Problem number: 422.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[[_Abel, '2nd type', 'class A']]`

$$\boxed{yy' - g(x)\, y = -f(x)}$$

✗ Solution by Maple

```
dsolve(y(x)*diff(y(x),x)+f(x) = g(x)*y(x),y(x), singsol=all)
```

No solution found

✗ Solution by Mathematica
Time used: 0.0 (sec). Leaf size: 0

```
DSolve[y[x] y'[x]+f[x]==g[x] y[x],y[x],x,IncludeSingularSolutions -> True]
```

Not solved

15.15 problem 423

Internal problem ID [3677]

Book: Ordinary differential equations and their solutions. By George Moseley Murphy. 1960
Section: Various 15
Problem number: 423.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[[_homogeneous, 'class D'], _rational, _Bernoulli]`

$$\boxed{yy' + y^2 = -4x(x+1)}$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 37

```
dsolve(y(x)*diff(y(x),x)+4*(1+x)*x+y(x)^2 = 0,y(x), singsol=all)
```

$$y(x) = \sqrt{\mathrm{e}^{-2x}c_1 - 4x^2}$$
$$y(x) = -\sqrt{\mathrm{e}^{-2x}c_1 - 4x^2}$$

✓ Solution by Mathematica

Time used: 6.205 (sec). Leaf size: 47

```
DSolve[y[x]  y'[x]+4(1+x)x+y[x]^2==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to -\sqrt{-4x^2 + c_1 e^{-2x}}$$
$$y(x) \to \sqrt{-4x^2 + c_1 e^{-2x}}$$

15.16 problem 424

Internal problem ID [3678]

Book: Ordinary differential equations and their solutions. By George Moseley Murphy. 1960
Section: Various 15
Problem number: 424.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_rational, _Bernoulli]`

$$yy' - by^2 = ax$$

✓ Solution by Maple

Time used: 0.016 (sec). Leaf size: 61

```
dsolve(y(x)*diff(y(x),x) = a*x+b*y(x)^2,y(x), singsol=all)
```

$$y(x) = -\frac{\sqrt{4\,\mathrm{e}^{2bx}c_1b^2 - 4bxa - 2a}}{2b}$$
$$y(x) = \frac{\sqrt{4\,\mathrm{e}^{2bx}c_1b^2 - 4bxa - 2a}}{2b}$$

✓ Solution by Mathematica

Time used: 11.801 (sec). Leaf size: 77

```
DSolve[y[x] y'[x]==a x+b y[x]^2,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to -\frac{i\sqrt{a\left(bx + \frac{1}{2}\right) - b^2c_1e^{2bx}}}{b}$$
$$y(x) \to \frac{i\sqrt{a\left(bx + \frac{1}{2}\right) - b^2c_1e^{2bx}}}{b}$$

15.17 problem 425

Internal problem ID [3679]

Book: Ordinary differential equations and their solutions. By George Moseley Murphy. 1960
Section: Various 15
Problem number: 425.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_Bernoulli]`

$$\boxed{yy' - ay^2 = b\cos(x+c)}$$

✓ Solution by Maple
Time used: 0.031 (sec). Leaf size: 106

```
dsolve(y(x)*diff(y(x),x) = b*cos(x+c)+a*y(x)^2,y(x), singsol=all)
```

$$y(x) = \frac{\sqrt{16\left(a^2+\frac{1}{4}\right)^2 c_1 \mathrm{e}^{2ax} - 16\left(a^2+\frac{1}{4}\right) b \left(a\cos(x+c) - \frac{\sin(x+c)}{2}\right)}}{4a^2+1}$$
$$y(x) = -\frac{\sqrt{16\left(a^2+\frac{1}{4}\right)^2 c_1 \mathrm{e}^{2ax} - 16\left(a^2+\frac{1}{4}\right) b \left(a\cos(x+c) - \frac{\sin(x+c)}{2}\right)}}{4a^2+1}$$

✓ Solution by Mathematica
Time used: 4.692 (sec). Leaf size: 106

```
DSolve[y[x] y'[x]== b Cos[x+c]+a y[x]^2,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to -\frac{\sqrt{(4a^2+1)\, c_1 e^{2ax} - 4ab\cos(c+x) + 2b\sin(c+x)}}{\sqrt{4a^2+1}}$$
$$y(x) \to \frac{\sqrt{(4a^2+1)\, c_1 e^{2ax} - 4ab\cos(c+x) + 2b\sin(c+x)}}{\sqrt{4a^2+1}}$$

15.18 problem 426

Internal problem ID [3680]

Book: Ordinary differential equations and their solutions. By George Moseley Murphy. 1960
Section: Various 15
Problem number: 426.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_quadrature]`

$$yy' - \mathrm{a1}\,y - \mathrm{a2}\,y^2 = \mathrm{a0}$$

✓ Solution by Maple

Time used: 0.563 (sec). Leaf size: 222

```
dsolve(y(x)*diff(y(x),x) = a0+a1*y(x)+a2*y(x)^2,y(x), singsol=all)
```

$$y(x) = \frac{4\tan\left(\mathrm{RootOf}\left(2c_1\,\mathrm{a2}\sqrt{4\,\mathrm{a0}\,\mathrm{a2}-\mathrm{a1}^2}+2x\,\mathrm{a2}\sqrt{4\,\mathrm{a0}\,\mathrm{a2}-\mathrm{a1}^2}+2\sqrt{4\,\mathrm{a0}\,\mathrm{a2}-\mathrm{a1}^2}\ln(2)-\sqrt{4\,\mathrm{a0}\,\mathrm{a2}-\mathrm{a1}\right.\right.}{}$$

✓ Solution by Mathematica

Time used: 0.385 (sec). Leaf size: 123

```
DSolve[y[x] y'[x]==a0+a1 y[x]+a2 y[x]^2,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to \mathrm{InverseFunction}\left[\frac{\log(\#1(\#1\mathrm{a2}+\mathrm{a1})+\mathrm{a0})-\frac{2\mathrm{a1}\arctan\left(\frac{2\#1\mathrm{a2}+\mathrm{a1}}{\sqrt{4\mathrm{a0a2}-\mathrm{a1}^2}}\right)}{\sqrt{4\mathrm{a0a2}-\mathrm{a1}^2}}}{2\mathrm{a2}}\&\right][x+c_1]$$

$$y(x) \to \frac{\sqrt{\mathrm{a1}^2-4\mathrm{a0a2}}-\mathrm{a1}}{2\mathrm{a2}}$$

$$y(x) \to -\frac{\sqrt{\mathrm{a1}^2-4\mathrm{a0a2}}+\mathrm{a1}}{2\mathrm{a2}}$$

15.19 problem 427

Internal problem ID [3681]

Book: Ordinary differential equations and their solutions. By George Moseley Murphy. 1960
Section: Various 15
Problem number: 427.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_separable]`

$$\boxed{yy' - bxy^2 = ax}$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 50

```
dsolve(y(x)*diff(y(x),x) = a*x+b*x*y(x)^2,y(x), singsol=all)
```

$$y(x) = \frac{\sqrt{-b\left(-\mathrm{e}^{b\,x^2}c_1 b + a\right)}}{b}$$
$$y(x) = -\frac{\sqrt{-b\left(-\mathrm{e}^{b\,x^2}c_1 b + a\right)}}{b}$$

✓ Solution by Mathematica

Time used: 0.972 (sec). Leaf size: 98

```
DSolve[y[x] y'[x]==a x+b x y[x]^2,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to -\frac{\sqrt{-a + e^{b(x^2+2c_1)}}}{\sqrt{b}}$$
$$y(x) \to \frac{\sqrt{-a + e^{b(x^2+2c_1)}}}{\sqrt{b}}$$
$$y(x) \to -\frac{i\sqrt{a}}{\sqrt{b}}$$
$$y(x) \to \frac{i\sqrt{a}}{\sqrt{b}}$$

15.20 problem 428

Internal problem ID [3682]

Book: Ordinary differential equations and their solutions. By George Moseley Murphy. 1960
Section: Various 15
Problem number: 428.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_Bernoulli]`

$$\boxed{yy' + y^2 \cot(x) = \csc(x)^2}$$

✓ Solution by Maple

Time used: 0.032 (sec). Leaf size: 28

```
dsolve(y(x)*diff(y(x),x) = csc(x)^2-y(x)^2*cot(x),y(x), singsol=all)
```

$$y(x) = \csc(x)\sqrt{2x + c_1}$$
$$y(x) = -\csc(x)\sqrt{2x + c_1}$$

✓ Solution by Mathematica

Time used: 0.489 (sec). Leaf size: 36

```
DSolve[y[x] y'[x]==Csc[x]^2- y[x]^2 Cot[x],y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to -\sqrt{2x + c_1}\csc(x)$$
$$y(x) \to \sqrt{2x + c_1}\csc(x)$$

15.21 problem 429

Internal problem ID [3683]

Book: Ordinary differential equations and their solutions. By George Moseley Murphy. 1960
Section: Various 15
Problem number: 429.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_quadrature]`

$$\boxed{yy' - \sqrt{y^2 + a^2} = 0}$$

✓ Solution by Maple
Time used: 0.0 (sec). Leaf size: 18

```
dsolve(y(x)*diff(y(x),x) = sqrt(y(x)^2+a^2),y(x), singsol=all)
```

$$x - \sqrt{y(x)^2 + a^2} + c_1 = 0$$

✓ Solution by Mathematica
Time used: 0.223 (sec). Leaf size: 61

```
DSolve[y[x]  y'[x]==Sqrt[a^2+y[x]^2],y[x],x,IncludeSingularSolutions -> True]
```

$$\begin{aligned} y(x) &\to -\sqrt{-a^2 + (x + c_1)^2} \\ y(x) &\to \sqrt{-a^2 + (x + c_1)^2} \\ y(x) &\to -ia \\ y(x) &\to ia \end{aligned}$$

15.22 problem 430

Internal problem ID [3684]

Book: Ordinary differential equations and their solutions. By George Moseley Murphy. 1960
Section: Various 15
Problem number: 430.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_quadrature]`

$$\boxed{yy' - \sqrt{y^2 - a^2} = 0}$$

✓ Solution by Maple
Time used: 0.0 (sec). Leaf size: 29

```
dsolve(y(x)*diff(y(x),x) = sqrt(y(x)^2-a^2),y(x), singsol=all)
```

$$x + \frac{(-y(x) + a)(y(x) + a)}{\sqrt{y(x)^2 - a^2}} + c_1 = 0$$

✓ Solution by Mathematica
Time used: 0.24 (sec). Leaf size: 51

```
DSolve[y[x]  y'[x]==Sqrt[y[x]^2-a^2],y[x],x,IncludeSingularSolutions -> True]
```

$$\begin{aligned} y(x) &\to -\sqrt{a^2 + (x + c_1)^2} \\ y(x) &\to \sqrt{a^2 + (x + c_1)^2} \\ y(x) &\to -a \\ y(x) &\to a \end{aligned}$$

15.23 problem 431

Internal problem ID [3685]

Book: Ordinary differential equations and their solutions. By George Moseley Murphy. 1960
Section: Various 15
Problem number: 431.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[NONE]`

$$yy' + f(y^2 + x^2)\, g(x) = -x$$

✓ Solution by Maple

Time used: 0.016 (sec). Leaf size: 30

```
dsolve(y(x)*diff(y(x),x)+x+f(x^2+y(x)^2)*g(x) = 0,y(x), singsol=all)
```

$$\int_{\_b}^{y(x)} \frac{\_a}{f(\_a^2 + x^2)} d\_a + \int g(x)\, dx - c_1 = 0$$

✓ Solution by Mathematica

Time used: 0.254 (sec). Leaf size: 95

```
DSolve[y[x] y'[x]+x+f[x^2+y[x]^2] g[x]==0,y[x],x,IncludeSingularSolutions -> True]
```

$$\text{Solve}\left[\int_1^{y(x)} \left(\frac{K[2]}{f(x^2 + K[2]^2)} - \int_1^x -\frac{2K[1]K[2]f'(K[1]^2 + K[2]^2)}{f(K[1]^2 + K[2]^2)^2} dK[1]\right) dK[2] + \int_1^x \left(g(K[1]) + \frac{K[1]}{f(K[1]^2 + y(x)^2)}\right) dK[1] = c_1, y(x)\right]$$

15.24 problem 432

Internal problem ID [3686]

Book: Ordinary differential equations and their solutions. By George Moseley Murphy. 1960
Section: Various 15
Problem number: 432.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[[_homogeneous, ‘class C‘], _rational, [_Abel, ‘2nd type‘, ‘cl`

$$\boxed{(y+1)\,y'-y=x}$$

✓ Solution by Maple

Time used: 1.281 (sec). Leaf size: 66

```
dsolve((1+y(x))*diff(y(x),x) = x+y(x),y(x), singsol=all)
```

$$-\frac{\ln\left(\frac{y(x)^2+(-x+3)y(x)-x^2+x+1}{(x-1)^2}\right)}{2}-\frac{\sqrt{5}\,\operatorname{arctanh}\left(\frac{(-2y(x)-3+x)\sqrt{5}}{5x-5}\right)}{5}-\ln(x-1)-c_1=0$$

✓ Solution by Mathematica

Time used: 0.108 (sec). Leaf size: 71

```
DSolve[(1+y[x])y'[x]==x+y[x],y[x],x,IncludeSingularSolutions -> True]
```

$$\text{Solve}\left[\frac{1}{2}\log\left(\frac{x^2-y(x)^2+(x-3)y(x)-x-1}{(x-1)^2}\right)+\log(1-x)=\frac{\operatorname{arctanh}\left(\frac{y(x)+2x-1}{\sqrt{5}(y(x)+1)}\right)}{\sqrt{5}}+c_1,y(x)\right]$$

15.25 problem 433

Internal problem ID [3687]

Book: Ordinary differential equations and their solutions. By George Moseley Murphy. 1960
Section: Various 15
Problem number: 433.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_separable]`

$$
\boxed{(y+1)\,y' - x^2(1-y) = 0}
$$

✓ Solution by Maple

Time used: 0.015 (sec). Leaf size: 20

```
dsolve((1+y(x))*diff(y(x),x) = x^2*(1-y(x)),y(x), singsol=all)
```

$$
y(x) = 2\,\mathrm{LambertW}\left(\frac{c_1 \mathrm{e}^{-\frac{x^3}{6}-\frac{1}{2}}}{2}\right) + 1
$$

✓ Solution by Mathematica

Time used: 30.295 (sec). Leaf size: 66

```
DSolve[(1+y[x])y'[x]==x^2(1-y[x]),y[x],x,IncludeSingularSolutions -> True]
```

$$
\begin{aligned}
y(x) &\to 1 + 2W\left(-\frac{1}{2}\sqrt{e^{-\frac{x^3}{3}-1+c_1}}\right) \\
y(x) &\to 1 + 2W\left(\frac{1}{2}\sqrt{e^{-\frac{x^3}{3}-1+c_1}}\right) \\
y(x) &\to 1
\end{aligned}
$$

15.26 problem 434

Internal problem ID [3688]

Book: Ordinary differential equations and their solutions. By George Moseley Murphy. 1960
Section: Various 15
Problem number: 434.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[[_homogeneous, ‘class A‘], _exact, _rational, [_Abel, ‘2nd ty`

$$(y + x)\, y' + y = 0$$

✓ Solution by Maple
Time used: 0.015 (sec). Leaf size: 35

```
dsolve((x+y(x))*diff(y(x),x)+y(x) = 0,y(x), singsol=all)
```

$$y(x) = -x - \sqrt{x^2 + 2c_1}$$
$$y(x) = -x + \sqrt{x^2 + 2c_1}$$

✓ Solution by Mathematica
Time used: 0.463 (sec). Leaf size: 84

```
DSolve[(x+y[x])y'[x]+y[x]==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to -x - \sqrt{x^2 + e^{2c_1}}$$
$$y(x) \to -x + \sqrt{x^2 + e^{2c_1}}$$
$$y(x) \to 0$$
$$y(x) \to -\sqrt{x^2} - x$$
$$y(x) \to \sqrt{x^2} - x$$

15.27 problem 435

Internal problem ID [3689]

Book: Ordinary differential equations and their solutions. By George Moseley Murphy. 1960
Section: Various 15
Problem number: 435.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[[_homogeneous, ‘class A‘], _rational, [_Abel, ‘2nd type‘, ‘cl`

$$\boxed{(-y+x)\,y'-y=0}$$

✓ Solution by Maple
Time used: 0.016 (sec). Leaf size: 17

```
dsolve((x-y(x))*diff(y(x),x) = y(x),y(x), singsol=all)
```

$$y(x)=-\frac{x}{\mathrm{LambertW}\left(-x\,\mathrm{e}^{-c_1}\right)}$$

✓ Solution by Mathematica
Time used: 4.105 (sec). Leaf size: 25

```
DSolve[(x-y[x])y'[x]==y[x],y[x],x,IncludeSingularSolutions -> True]
```

$$y(x)\to -\frac{x}{W\left(-e^{-c_1}x\right)}$$
$$y(x)\to 0$$

15.28 problem 436

Internal problem ID [3690]

Book: Ordinary differential equations and their solutions. By George Moseley Murphy. 1960
Section: Various 15
Problem number: 436.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[[_homogeneous, ‘class A‘], _rational, [_Abel, ‘2nd type‘, ‘cl`

$$\boxed{(y+x)\,y'-y=-x}$$

✓ Solution by Maple
Time used: 0.015 (sec). Leaf size: 24

```
dsolve((x+y(x))*diff(y(x),x)+x-y(x) = 0,y(x), singsol=all)
```

$$y(x)=\tan\left(\operatorname{RootOf}\left(2\_Z+\ln\left(\sec\left(\_Z\right)^2\right)+2\ln\left(x\right)+2c_1\right)\right)x$$

✓ Solution by Mathematica
Time used: 0.033 (sec). Leaf size: 34

```
DSolve[(x+y[x])y'[x]+(x-y[x])==0,y[x],x,IncludeSingularSolutions -> True]
```

$$\text{Solve}\left[\arctan\left(\frac{y(x)}{x}\right)+\frac{1}{2}\log\left(\frac{y(x)^2}{x^2}+1\right)=-\log(x)+c_1,y(x)\right]$$

15.29 problem 437

Internal problem ID [3691]

Book: Ordinary differential equations and their solutions. By George Moseley Murphy. 1960
Section: Various 15
Problem number: 437.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[[_homogeneous, ‘class A‘], _exact, _rational, [_Abel, ‘2nd ty`

$$(y + x) y' + y = x$$

✓ Solution by Maple
Time used: 0.047 (sec). Leaf size: 51

```
dsolve((x+y(x))*diff(y(x),x) = x-y(x),y(x), singsol=all)
```

$$y(x) = \frac{-c_1 x - \sqrt{2c_1^2 x^2 + 1}}{c_1}$$
$$y(x) = \frac{-c_1 x + \sqrt{2c_1^2 x^2 + 1}}{c_1}$$

✓ Solution by Mathematica
Time used: 0.488 (sec). Leaf size: 94

```
DSolve[(x+y[x])y'[x]==x-y[x],y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to -x - \sqrt{2x^2 + e^{2c_1}}$$
$$y(x) \to -x + \sqrt{2x^2 + e^{2c_1}}$$
$$y(x) \to -\sqrt{2}\sqrt{x^2} - x$$
$$y(x) \to \sqrt{2}\sqrt{x^2} - x$$

15.30 problem 438

Internal problem ID [3692]

Book: Ordinary differential equations and their solutions. By George Moseley Murphy. 1960
Section: Various 15
Problem number: 438.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[[_linear, 'class A']]`

$$\boxed{-y' - y = x - 1}$$

✓ Solution by Maple
Time used: 0.0 (sec). Leaf size: 15

```
dsolve(1-diff(y(x),x) = x+y(x),y(x), singsol=all)
```

$$y(x) = -x + 2 + \mathrm{e}^{-x} c_1$$

✓ Solution by Mathematica
Time used: 0.041 (sec). Leaf size: 18

```
DSolve[1-y'[x]==x+y[x],y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to -x + c_1 e^{-x} + 2$$

15.31 problem 439

Internal problem ID [3693]

Book: Ordinary differential equations and their solutions. By George Moseley Murphy. 1960
Section: Various 15
Problem number: 439.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[[_homogeneous, ‘class D‘], _rational, [_Abel, ‘2nd type‘, ‘cl`

$$\boxed{(-y+x)\,y'-(2yx+1)\,y=0}$$

✓ Solution by Maple
Time used: 0.047 (sec). Leaf size: 18

```
dsolve((x-y(x))*diff(y(x),x) = (1+2*x*y(x))*y(x),y(x), singsol=all)
```

$$y(x)=-\frac{x}{\mathrm{LambertW}\left(-\mathrm{e}^{x^2}c_1x\right)}$$

✓ Solution by Mathematica
Time used: 5.983 (sec). Leaf size: 29

```
DSolve[(x-y[x])y'[x]==(1+2 x y[x])y[x],y[x],x,IncludeSingularSolutions -> True]
```

$$y(x)\to -\frac{x}{W\left(x\left(-e^{x^2-c_1}\right)\right)}$$
$$y(x)\to 0$$

15.32 problem 440

Internal problem ID [3694]

Book: Ordinary differential equations and their solutions. By George Moseley Murphy. 1960
Section: Various 15
Problem number: 440.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[[_1st_order, ‘_with_symmetry_[F(x)*G(y),0]‘]]`

$$\boxed{(y + x) y' + \tan(y) = 0}$$

✓ Solution by Maple
Time used: 0.234 (sec). Leaf size: 16

```
dsolve((x+y(x))*diff(y(x),x)+tan(y(x)) = 0,y(x), singsol=all)
```

$$y(x) + x + \cot(y(x)) - \csc(y(x)) c_1 = 0$$

✓ Solution by Mathematica
Time used: 0.196 (sec). Leaf size: 29

```
DSolve[(x+y[x])y'[x]+Tan[y[x]]==0,y[x],x,IncludeSingularSolutions -> True]
```

$$\text{Solve}[x = \csc(y(x))(-y(x)\sin(y(x)) - \cos(y(x))) + c_1 \csc(y(x)), y(x)]$$

15.33 problem 441

Internal problem ID [3695]

Book: Ordinary differential equations and their solutions. By George Moseley Murphy. 1960
Section: Various 15
Problem number: 441.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[[_homogeneous, 'class A'], _dAlembert]`

$$(-y+x)\,y'-\left(\mathrm{e}^{-\frac{x}{y}}+1\right)y=0$$

✓ Solution by Maple

Time used: 0.047 (sec). Leaf size: 20

```
dsolve((x-y(x))*diff(y(x),x) = (exp(-x/y(x))+1)*y(x),y(x), singsol=all)
```

$$y(x)=-\frac{x}{\mathrm{LambertW}\left(\frac{xc_1}{c_1x-1}\right)}$$

✓ Solution by Mathematica

Time used: 1.348 (sec). Leaf size: 34

```
DSolve[(x-y[x])y'[x]==(Exp[-x/y[x]]+1)y[x],y[x],x,IncludeSingularSolutions -> True]
```

$$y(x)\to -\frac{x}{W\left(\frac{x}{x-e^{c_1}}\right)}$$
$$y(x)\to -\frac{x}{W(1)}$$

15.34 problem 442

Internal problem ID [3696]

Book: Ordinary differential equations and their solutions. By George Moseley Murphy. 1960
Section: Various 15
Problem number: 442.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[[_homogeneous, ‘class C‘], _rational, [_Abel, ‘2nd type‘, ‘cl`

$$\boxed{(x+y+1)\,y'+3y=-1-4x}$$

✓ Solution by Maple

Time used: 0.312 (sec). Leaf size: 29

```
dsolve((1+x+y(x))*diff(y(x),x)+1+4*x+3*y(x) = 0,y(x), singsol=all)
```

$$y(x)=-3-\frac{(-2+x)\,(2\,\mathrm{LambertW}\,(c_1(-2+x))+1)}{\mathrm{LambertW}\,(c_1\,(-2+x))}$$

✓ Solution by Mathematica

Time used: 1.292 (sec). Leaf size: 159

```
DSolve[(1+x+y[x])y'[x]+1+4 x+3 y[x]==0,y[x],x,IncludeSingularSolutions -> True]
```

$$\text{Solve}\left[\frac{(-2)^{2/3}\left(-2x\log\left(\frac{3(-2)^{2/3}(y(x)+2x-1)}{y(x)+x+1}\right)+(2x-1)\log\left(-\frac{3(-2)^{2/3}(x-2)}{y(x)+x+1}\right)+\log\left(\frac{3(-2)^{2/3}(y(x)+2x-1)}{y(x)+x+1}\right)+\cdots\right.}{9(y(x)+2x-1)}\right.$$

15.35 problem 443

Internal problem ID [3697]

Book: Ordinary differential equations and their solutions. By George Moseley Murphy. 1960
Section: Various 15
Problem number: 443.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[[_homogeneous, ‘class C‘], _exact, _rational, [_Abel, ‘2nd ty`

$$\boxed{(x+y+2)\,y'+y=1-x}$$

✓ Solution by Maple

Time used: 0.016 (sec). Leaf size: 39

```
dsolve((2+x+y(x))*diff(y(x),x) = 1-x-y(x),y(x), singsol=all)
```

$$y(x)=-x-2-\sqrt{-6c_1+6x+4}$$
$$y(x)=-x-2+\sqrt{-6c_1+6x+4}$$

✓ Solution by Mathematica

Time used: 0.126 (sec). Leaf size: 43

```
DSolve[(2+x+y[x])y'[x]==1-x-y[x],y[x],x,IncludeSingularSolutions -> True]
```

$$y(x)\to -x-\sqrt{6x+4+c_1}-2$$
$$y(x)\to -x+\sqrt{6x+4+c_1}-2$$

16 Various 16

16.1 problem 444

Internal problem ID [3698]

Book: Ordinary differential equations and their solutions. By George Moseley Murphy. 1960
Section: Various 16
Problem number: 444.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[[_homogeneous, ‘class C‘], _rational, [_Abel, ‘2nd type‘, ‘cl`

$$\boxed{(3-x-y)\,y'+3y=x+1}$$

✓ Solution by Maple
Time used: 0.313 (sec). Leaf size: 30

```
dsolve((3-x-y(x))*diff(y(x),x) = 1+x-3*y(x),y(x), singsol=all)
```

$$y(x)=\frac{(x-1)\,\text{LambertW}\left(-2c_1(-2+x)\right)-4+2x}{\text{LambertW}\left(-2c_1\left(-2+x\right)\right)}$$

✓ Solution by Mathematica
Time used: 1.073 (sec). Leaf size: 159

```
DSolve[(3-x-y[x])y'[x]==1+x-3 y[x],y[x],x,IncludeSingularSolutions -> True]
```

$$\text{Solve}\left[\frac{2^{2/3}\left(x\left(-\log\left(-\frac{3\ 2^{2/3}(-y(x)+x-1)}{y(x)+x-3}\right)\right)+(x-1)\log\left(\frac{6\ 2^{2/3}(x-2)}{y(x)+x-3}\right)+\log\left(-\frac{3\ 2^{2/3}(-y(x)+x-1)}{y(x)+x-3}\right)+y(x)\left(\right.\right.}{9(-y(x)+x-1)}\right.$$

16.2 problem 445

Internal problem ID [3699]

Book: Ordinary differential equations and their solutions. By George Moseley Murphy. 1960
Section: Various 16
Problem number: 445.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[[_homogeneous, ‘class C‘], _rational, [_Abel, ‘2nd type‘, ‘cl`

$$\boxed{(3-x+y)\,y'-3y=11-4x}$$

✓ Solution by Maple
Time used: 0.312 (sec). Leaf size: 30

```
dsolve((3-x+y(x))*diff(y(x),x) = 11-4*x+3*y(x),y(x), singsol=all)
```

$$y(x)=\frac{(2x-5)\,\mathrm{LambertW}\left(-c_1(-2+x)\right)-2+x}{\mathrm{LambertW}\left(-c_1\left(-2+x\right)\right)}$$

✓ Solution by Mathematica
Time used: 1.395 (sec). Leaf size: 179

```
DSolve[(3-x+y[x])y'[x]==11-4 x+3 y[x],y[x],x,IncludeSingularSolutions -> True]
```

$$\mathrm{Solve}\left[\frac{(-2)^{2/3}\left(-2x\log\left(\frac{3(-2)^{2/3}(-y(x)+2x-5)}{-y(x)+x-3}\right)+(2x-5)\log\left(-\frac{3(-2)^{2/3}(x-2)}{-y(x)+x-3}\right)+5\log\left(\frac{3(-2)^{2/3}(-y(x)+2x-5)}{-y(x)+x-3}\right.\right.}{9(-y(x)+2x-5)}\right.$$

16.3 problem 446

Internal problem ID [3700]

Book: Ordinary differential equations and their solutions. By George Moseley Murphy. 1960
Section: Various 16
Problem number: 446.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[[_homogeneous, ‘class A‘], _rational, [_Abel, ‘2nd type‘, ‘cl`

$$(y + 2x)\, y' - 2y = -x$$

✓ Solution by Maple

Time used: 0.016 (sec). Leaf size: 24

```
dsolve((2*x+y(x))*diff(y(x),x)+x-2*y(x) = 0,y(x), singsol=all)
```

$$y(x) = \tan\left(\mathrm{RootOf}\left(4\_Z + \ln\left(\sec\left(\_Z\right)^2\right) + 2\ln\left(x\right) + 2c_1\right)\right) x$$

✓ Solution by Mathematica

Time used: 0.034 (sec). Leaf size: 36

```
DSolve[(2 x+y[x])y'[x]+(x-2 y[x])==0,y[x],x,IncludeSingularSolutions -> True]
```

$$\mathrm{Solve}\left[2\arctan\left(\frac{y(x)}{x}\right) + \frac{1}{2}\log\left(\frac{y(x)^2}{x^2} + 1\right) = -\log(x) + c_1, y(x)\right]$$

16.4 problem 447

Internal problem ID [3701]

Book: Ordinary differential equations and their solutions. By George Moseley Murphy. 1960
Section: Various 16
Problem number: 447.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[[_homogeneous, ‘class C‘], _rational, [_Abel, ‘2nd type‘, ‘cl`

$$\boxed{(2x - y + 2)\, y' - 3y = -3 - 6x}$$

✓ Solution by Maple

Time used: 0.015 (sec). Leaf size: 21

```
dsolve((2+2*x-y(x))*diff(y(x),x)+3+6*x-3*y(x) = 0,y(x), singsol=all)
```

$$y(x) = 2x - \frac{3\,\mathrm{LambertW}\left(-\frac{c_1 \mathrm{e}^{\frac{25x}{3}+\frac{7}{3}}}{3}\right)}{5} + \frac{7}{5}$$

✓ Solution by Mathematica

Time used: 3.774 (sec). Leaf size: 41

```
DSolve[(2+2 x-y[x])y'[x]+3(1+2 x- y[x])==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to -\frac{3}{5} W\left(-e^{\frac{25x}{3}-1+c_1}\right) + 2x + \frac{7}{5}$$
$$y(x) \to 2x + \frac{7}{5}$$

16.5 problem 448

Internal problem ID [3702]

Book: Ordinary differential equations and their solutions. By George Moseley Murphy. 1960
Section: Various 16
Problem number: 448.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[[_homogeneous, ‘class C‘], [_Abel, ‘2nd type‘, ‘class C‘], _d`

$$\boxed{(2x - y + 3)\, y' = -2}$$

✓ Solution by Maple
Time used: 0.016 (sec). Leaf size: 19

```
dsolve((3+2*x-y(x))*diff(y(x),x)+2 = 0,y(x), singsol=all)
```

$$y(x) = \text{LambertW}\left(-2c_1 \mathrm{e}^{-2x-4}\right) + 2x + 4$$

✓ Solution by Mathematica
Time used: 0.034 (sec). Leaf size: 22

```
DSolve[(3+2 x-y[x])y'[x]+2==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to W\left(-2c_1 e^{-2(x+2)}\right) + 2x + 4$$

16.6 problem 449

Internal problem ID [3703]

Book: Ordinary differential equations and their solutions. By George Moseley Murphy. 1960
Section: Various 16
Problem number: 449.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[[_homogeneous, ‘class C‘], _rational, [_Abel, ‘2nd type‘, ‘cl`

$$(4+2x-y)\,y'-2y=-x-5$$

✓ Solution by Maple
Time used: 0.734 (sec). Leaf size: 117

```
dsolve((4+2*x-y(x))*diff(y(x),x)+5+x-2*y(x) = 0,y(x), singsol=all)
```

$$y(x) = \frac{\frac{1}{2}+\frac{\left(1-i\sqrt{3}\right)\left(27c_1(x+1)+3\sqrt{3}\sqrt{27c_1^2(x+1)^2-1}\right)^{\frac{2}{3}}}{6}+\frac{i\sqrt{3}}{2}-\left(3\sqrt{3}\sqrt{27c_1^2\left(x+1\right)^2-1}+27c_1x+27c_1\right)^{\frac{1}{3}}(x-1)\,c}{\left(27c_1\left(x+1\right)+3\sqrt{3}\sqrt{27c_1^2\left(x+1\right)^2-1}\right)^{\frac{1}{3}}c_1}$$

✓ Solution by Mathematica
Time used: 60.172 (sec). Leaf size: 1601

```
DSolve[(4+2 x-y[x])y'[x]+5+x-2 y[x]==0,y[x],x,IncludeSingularSolutions -> True]
```

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16.7 problem 450

Internal problem ID [3704]

Book: Ordinary differential equations and their solutions. By George Moseley Murphy. 1960
Section: Various 16
Problem number: 450.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[[_homogeneous, ‘class C‘], _exact, _rational, [_Abel, ‘2nd ty`

$$\boxed{(5-2x-y)\,y'-2y=x-4}$$

✓ Solution by Maple
Time used: 0.485 (sec). Leaf size: 32

```
dsolve((5-2*x-y(x))*diff(y(x),x)+4-x-2*y(x) = 0,y(x), singsol=all)
```

$$y(x)=\frac{-\sqrt{3\left(-2+x\right)^{2} c_1^2+1}+\left(-2x+5\right)c_1}{c_1}$$

✓ Solution by Mathematica
Time used: 0.172 (sec). Leaf size: 53

```
DSolve[(5-2 x-y[x])y'[x]+4-x-2 y[x]==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x)\to-\sqrt{3x^2-12x+25+c_1}-2x+5$$
$$y(x)\to\sqrt{3x^2-12x+25+c_1}-2x+5$$

16.8 problem 451

Internal problem ID [3705]

Book: Ordinary differential equations and their solutions. By George Moseley Murphy. 1960
Section: Various 16
Problem number: 451.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[[_homogeneous, ‘class C‘], _rational, [_Abel, ‘2nd type‘, ‘cl`

$$(1-3x+y)\,y'+2y=2x$$

✓ Solution by Maple

Time used: 1.172 (sec). Leaf size: 47

```
dsolve((1-3*x+y(x))*diff(y(x),x) = 2*x-2*y(x),y(x), singsol=all)
```

$$y(x)=\frac{1}{2}+\frac{(2x-1)\left(-1-\operatorname{RootOf}\left(-3+\left(8c_1x^3-12c_1x^2+6c_1x-c_1\right)\_Z^4-\_Z\right)\right)}{2}$$

✓ Solution by Mathematica

Time used: 60.161 (sec). Leaf size: 4937

```
DSolve[(1-3 x+y[x])y'[x]==2(x-y[x]),y[x],x,IncludeSingularSolutions -> True]
```

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16.9 problem 452

Internal problem ID [3706]

Book: Ordinary differential equations and their solutions. By George Moseley Murphy. 1960
Section: Various 16
Problem number: 452.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[[_homogeneous, ‘class C‘], _exact, _rational, [_Abel, ‘2nd ty`

$$(2-3x+y)\,y'-3y=-5+2x$$

✓ Solution by Maple
Time used: 6.031 (sec). Leaf size: 32

```
dsolve((2-3*x+y(x))*diff(y(x),x)+5-2*x-3*y(x) = 0,y(x), singsol=all)
```

$$y(x)=\frac{-\sqrt{11\,(x-1)^2\,c_1^2+1}+(-2+3x)\,c_1}{c_1}$$

✓ Solution by Mathematica
Time used: 0.149 (sec). Leaf size: 63

```
DSolve[(2-3 x+y[x])y'[x]+5-2 x-3 y[x]==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x)\to -i\sqrt{-11x^2+22x-4-c_1}+3x-2$$
$$y(x)\to i\sqrt{-11x^2+22x-4-c_1}+3x-2$$

16.10 problem 453

Internal problem ID [3707]

Book: Ordinary differential equations and their solutions. By George Moseley Murphy. 1960
Section: Various 16
Problem number: 453.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[[_homogeneous, ‘class A‘], _rational, [_Abel, ‘2nd type‘, ‘cl`

$$\boxed{(4x-y)\,y'-5y=-2x}$$

✓ Solution by Maple
Time used: 0.015 (sec). Leaf size: 47

```
dsolve((4*x-y(x))*diff(y(x),x)+2*x-5*y(x) = 0,y(x), singsol=all)
```

$$y(x)=\frac{-4c_1x-\sqrt{-12c_1x+1}+1}{2c_1}$$
$$y(x)=\frac{-4c_1x+1+\sqrt{-12c_1x+1}}{2c_1}$$

✓ Solution by Mathematica
Time used: 1.387 (sec). Leaf size: 80

```
DSolve[(4 x-y[x])y'[x]+2 x-5 y[x]==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x)\to\frac{1}{2}\left(-4x-e^{\frac{c_1}{2}}\sqrt{12x+e^{c_1}}-e^{c_1}\right)$$
$$y(x)\to\frac{1}{2}\left(-4x+e^{\frac{c_1}{2}}\sqrt{12x+e^{c_1}}-e^{c_1}\right)$$

16.11 problem 454

Internal problem ID [3708]

Book: Ordinary differential equations and their solutions. By George Moseley Murphy. 1960
Section: Various 16
Problem number: 454.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[[_homogeneous, ‘class C‘], _rational, [_Abel, ‘2nd type‘, ‘cl`

$$(6-4x-y)\,y'+y=2x$$

✓ Solution by Maple
Time used: 0.188 (sec). Leaf size: 198

```
dsolve((6-4*x-y(x))*diff(y(x),x) = 2*x-y(x),y(x), singsol=all)
```

$$y(x) = \frac{\frac{\left(1-i\sqrt{3}\right)\left(12\sqrt{3}\,c_1^2(x-1)\sqrt{\frac{27(x-1)^2c_1-4x+4}{c_1}}+8+108(x-1)^2c_1^2+(-72x+72)c_1\right)^{\frac{2}{3}}}{12}-\left(\frac{1}{3}+(x-3)\,c_1\right)\left(12\sqrt{3}\,c_1^2(x-1)\sqrt{\frac{27(x}{}}\right.}{\left(12\sqrt{3}\,c_1^2\,(x-1)\sqrt{\frac{27(x-1)^2c_1-4x+4}{c_1}}+8+108\,(x\right.}$$

✓ Solution by Mathematica
Time used: 60.097 (sec). Leaf size: 2581

```
DSolve[(6-4 x-y[x])y'[x]==2 x -y[x],y[x],x,IncludeSingularSolutions -> True]
```

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16.12 problem 455

Internal problem ID [3709]

Book: Ordinary differential equations and their solutions. By George Moseley Murphy. 1960
Section: Various 16
Problem number: 455.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[[_homogeneous, ‘class C‘], _rational, [_Abel, ‘2nd type‘, ‘cl`

$$\boxed{(1+5x-y)\,y'-5y=-x-5}$$

✓ Solution by Maple

Time used: 0.188 (sec). Leaf size: 243

```
dsolve((1+5*x-y(x))*diff(y(x),x)+5+x-5*y(x) = 0,y(x), singsol=all)
```

$$y(x) = \frac{i\left(1-\left(6\sqrt{3}\,x\sqrt{\frac{27c_1x^2+2x}{c_1}}\,c_1^2+54c_1^2x^2+18c_1x+1\right)^{\frac{2}{3}}+12c_1x\right)\sqrt{3}+6\left(\left(2-\left(6\sqrt{3}\,x\sqrt{\frac{27c_1x^2+2x}{c_1}}\,c_1^2+5\right.\right.\right.}{6c_1\left(6\sqrt{3}\right.}$$

✓ Solution by Mathematica

Time used: 60.045 (sec). Leaf size: 925

```
DSolve[(1+5 x-y[x])y'[x]+5+x-5 y[x]==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to -\frac{}{\text{Root}\left[\#1^6\left(186624x^4+186624e^{\frac{12c_1}{25}}x^6\right)+\#1^5\left(-186624x^3-186624e^{\frac{12c_1}{25}}x^5\right)+\#1^4\left(69984x^2+77\right.\right.} + 5x + 1$$

$$y(x) \to -\frac{}{\text{Root}\left[\#1^6\left(186624x^4+186624e^{\frac{12c_1}{25}}x^6\right)+\#1^5\left(-186624x^3-186624e^{\frac{12c_1}{25}}x^5\right)+\#1^4\left(69984x^2+77\right.\right.} + 5x + 1$$

$$y(x) \to -\frac{}{\text{Root}\left[\#1^6\left(186624x^4+186624e^{\frac{12c_1}{25}}x^6\right)+\#1^5\left(-186624x^3-186624e^{\frac{12c_1}{25}}x^5\right)+\#1^4\left(69984x^2+77\right.\right.} + 5x + 1$$

$$y(x) \to -\frac{}{\text{Root}\left[\#1^6\left(186624x^4+186624e^{\frac{12c_1}{25}}x^6\right)+\#1^5\left(-186624x^3-186624e^{\frac{12c_1}{25}}x^5\right)+\#1^4\left(69984x^2+77\right.\right.} + 5x + 1$$

$$y(x) \to -\frac{}{\text{Root}\left[\#1^6\left(186624x^4+186624e^{\frac{12c_1}{25}}x^6\right)+\#1^5\left(-186624x^3-186624e^{\frac{12c_1}{25}}x^5\right)+\#1^4\left(69984x^2+77\right.\right.} + 5x + 1$$

$$y(x) \to -\frac{}{\text{Root}\left[\#1^6\left(186624x^4+186624e^{\frac{12c_1}{25}}x^6\right)+\#1^5\left(-186624x^3-186624e^{\frac{12c_1}{25}}x^5\right)+\#1^4\left(69984x^2+77\right.\right.} + 5x + 1$$

16.13 problem 456

Internal problem ID [3710]

Book: Ordinary differential equations and their solutions. By George Moseley Murphy. 1960
Section: Various 16
Problem number: 456.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[[_homogeneous, ‘class C‘], _rational, [_Abel, ‘2nd type‘, ‘cl`

$$\boxed{(a+bx+y)\,y'-y=bx-a}$$

✓ Solution by Maple

Time used: 0.063 (sec). Leaf size: 63

```
dsolve((a+b*x+y(x))*diff(y(x),x)+a-b*x-y(x) = 0,y(x), singsol=all)
```

$$y(x)=\frac{2\,\text{LambertW}\left(\frac{e^{\frac{-c_1(b+1)^2+(b-1)a+x(b+1)^2}{2a}}}{2a}\right)a-b^2x+(-a-x)\,b+a}{b+1}$$

✓ Solution by Mathematica

Time used: 5.745 (sec). Leaf size: 118

```
DSolve[(a+b x+y[x])y'[x]+a-b x-y[x]==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x)\to\frac{2aW\left(-e^{\frac{(b+1)^2x}{2a}-1+c_1}\right)+a(-b)+a-b(b+1)x}{b+1}$$
$$y(x)\to\frac{a(-b)+a-b(b+1)x}{b+1}$$
$$y(x)\to\frac{2aW\left(-e^{\frac{(b+1)^2x}{2a}-1}\right)+a(-b)+a-b(b+1)x}{b+1}$$

16.14 problem 457

Internal problem ID [3711]

Book: Ordinary differential equations and their solutions. By George Moseley Murphy. 1960
Section: Various 16
Problem number: 457.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_rational, [_1st_order, '_with_symmetry_[F(x)*G(y),0]'], [_Ab`

$$\boxed{(x^2 - y)\, y' = -x}$$

✓ Solution by Maple
Time used: 0.0 (sec). Leaf size: 23

```
dsolve((x^2-y(x))*diff(y(x),x)+x = 0,y(x), singsol=all)
```

$$y(x) = x^2 + \frac{\operatorname{LambertW}\left(4c_1 \mathrm{e}^{-2x^2-1}\right)}{2} + \frac{1}{2}$$

✓ Solution by Mathematica
Time used: 5.024 (sec). Leaf size: 40

```
DSolve[(x^2-y[x])y'[x]+x==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to x^2 + \frac{1}{2}\left(1 + W\left(-e^{-2x^2-1+c_1}\right)\right)$$
$$y(x) \to x^2 + \frac{1}{2}$$

16.15 problem 458

Internal problem ID [3712]

Book: Ordinary differential equations and their solutions. By George Moseley Murphy. 1960
Section: Various 16
Problem number: 458.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[[_homogeneous, ‘class G‘], _rational, [_Abel, ‘2nd type‘, ‘cl`

$$\boxed{(x^2 - y)\, y' - 4yx = 0}$$

✓ Solution by Maple

Time used: 0.094 (sec). Leaf size: 57

```
dsolve((x^2-y(x))*diff(y(x),x) = 4*x*y(x),y(x), singsol=all)
```

$$y(x) = -\frac{c_1\sqrt{c_1^2 - 4x^2}}{2} + \frac{c_1^2}{2} - x^2$$
$$y(x) = \frac{c_1\sqrt{c_1^2 - 4x^2}}{2} + \frac{c_1^2}{2} - x^2$$

✓ Solution by Mathematica

Time used: 2.966 (sec). Leaf size: 246

```
DSolve[(x^2-y[x])y'[x]==4 x y[x],y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to x^2 \left(1 + \frac{2-2i}{\frac{i\sqrt{2}}{\sqrt{x^2 \cosh\left(\frac{2c_1}{9}\right) + x^2 \sinh\left(\frac{2c_1}{9}\right) - i}} - (1-i)}\right)$$

$$y(x) \to x^2 \left(1 + \frac{2-2i}{(-1+i) - \frac{i\sqrt{2}}{\sqrt{x^2 \cosh\left(\frac{2c_1}{9}\right) + x^2 \sinh\left(\frac{2c_1}{9}\right) - i}}}\right)$$

$$y(x) \to x^2 \left(1 + \frac{2-2i}{(-1+i) - \frac{\sqrt{2}}{\sqrt{x^2 \cosh\left(\frac{2c_1}{9}\right) + x^2 \sinh\left(\frac{2c_1}{9}\right) + i}}}\right)$$

$$y(x) \to x^2 \left(1 + \frac{2-2i}{\frac{\sqrt{2}}{\sqrt{x^2 \cosh\left(\frac{2c_1}{9}\right) + x^2 \sinh\left(\frac{2c_1}{9}\right) + i}} - (1-i)}\right)$$

$$y(x) \to 0$$

$$y(x) \to -x^2$$

16.16 problem 459

Internal problem ID [3713]

Book: Ordinary differential equations and their solutions. By George Moseley Murphy. 1960
Section: Various 16
Problem number: 459.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[[_Abel, '2nd type', 'class A']]`

$$(y - \csc(x)\cot(x))\,y' + \csc(x)\,(1 + \cos(x)\,y)\,y = 0$$

✓ Solution by Maple
Time used: 0.016 (sec). Leaf size: 37

```
dsolve((y(x)-cot(x)*csc(x))*diff(y(x),x)+csc(x)*(1+y(x)*cos(x))*y(x) = 0,y(x), singsol=all)
```

$$y(x) = \csc(x)\left(-\sqrt{c_1 + \cot(x)^2} + \cot(x)\right)$$
$$y(x) = \csc(x)\left(\sqrt{c_1 + \cot(x)^2} + \cot(x)\right)$$

✓ Solution by Mathematica
Time used: 1.639 (sec). Leaf size: 85

```
DSolve[(y[x]-Cot[x] Csc[x])y'[x]+Csc[x](1+y[x] Cos[x])y[x]==0,y[x],x,IncludeSingularSolution
```

$$y(x) \to \cot(x)\csc(x) - \frac{i\csc^2(x)\sqrt{(-1+c_1)\cos(2x) - 1 - c_1}}{\sqrt{2}}$$
$$y(x) \to \cot(x)\csc(x) + \frac{i\csc^2(x)\sqrt{(-1+c_1)\cos(2x) - 1 - c_1}}{\sqrt{2}}$$

17 Various 17

17.1 problem 460

Internal problem ID [3714]

Book: Ordinary differential equations and their solutions. By George Moseley Murphy. 1960
Section: Various 17
Problem number: 460.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[[_homogeneous, ‘class D‘], _rational, _Bernoulli]`

$$2yy' + y^2 = -x^2 - 2x$$

✓ Solution by Maple
Time used: 0.0 (sec). Leaf size: 37

```
dsolve(2*y(x)*diff(y(x),x)+2*x+x^2+y(x)^2 = 0,y(x), singsol=all)
```

$$y(x) = \sqrt{e^{-x}c_1 - x^2}$$
$$y(x) = -\sqrt{e^{-x}c_1 - x^2}$$

✓ Solution by Mathematica
Time used: 6.093 (sec). Leaf size: 47

```
DSolve[2 y[x] y'[x]+2 x+x^2+y[x]^2==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to -\sqrt{-x^2 + c_1 e^{-x}}$$
$$y(x) \to \sqrt{-x^2 + c_1 e^{-x}}$$

17.2 problem 461

Internal problem ID [3715]

Book: Ordinary differential equations and their solutions. By George Moseley Murphy. 1960
Section: Various 17
Problem number: 461.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_rational, _Bernoulli]`

$$\boxed{2yy' - y^2x = x^3}$$

✓ Solution by Maple
Time used: 0.015 (sec). Leaf size: 43

```
dsolve(2*y(x)*diff(y(x),x) = x*y(x)^2+x^3,y(x), singsol=all)
```

$$y(x) = \sqrt{e^{\frac{x^2}{2}} c_1 - x^2 - 2}$$
$$y(x) = -\sqrt{e^{\frac{x^2}{2}} c_1 - x^2 - 2}$$

✓ Solution by Mathematica
Time used: 7.301 (sec). Leaf size: 57

```
DSolve[2 y[x] y'[x]==x y[x]^2+x^3,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to -\sqrt{-x^2 + c_1 e^{\frac{x^2}{2}} - 2}$$
$$y(x) \to \sqrt{-x^2 + c_1 e^{\frac{x^2}{2}} - 2}$$

17.3 problem 462

Internal problem ID [3716]

Book: Ordinary differential equations and their solutions. By George Moseley Murphy. 1960
Section: Various 17
Problem number: 462.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[[_homogeneous, ‘class A‘], _rational, [_Abel, ‘2nd type‘, ‘cl`

$$\boxed{(x - 2y)\, y' - y = 0}$$

✓ Solution by Maple

Time used: 0.016 (sec). Leaf size: 17

```
dsolve((x-2*y(x))*diff(y(x),x) = y(x),y(x), singsol=all)
```

$$y(x) = -\frac{x}{2\,\mathrm{LambertW}\left(-\frac{x\,\mathrm{e}^{-\frac{c_1}{2}}}{2}\right)}$$

✓ Solution by Mathematica

Time used: 4.87 (sec). Leaf size: 31

```
DSolve[(x-2 y[x])y'[x]==y[x],y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to -\frac{x}{2W\left(-\frac{1}{2}e^{-\frac{c_1}{2}}x\right)}$$
$$y(x) \to 0$$

17.4 problem 463

Internal problem ID [3717]

Book: Ordinary differential equations and their solutions. By George Moseley Murphy. 1960
Section: Various 17
Problem number: 463.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[[_homogeneous, ‘class A‘], _rational, [_Abel, ‘2nd type‘, ‘cl`

$$(x+2y)\,y'-y=-2x$$

✓ Solution by Maple
Time used: 0.032 (sec). Leaf size: 22

```
dsolve((x+2*y(x))*diff(y(x),x)+2*x-y(x) = 0,y(x), singsol=all)
```

$$y(x)=\tan\left(\operatorname{RootOf}\left(\ln\left(\sec\left(\_Z\right)^2\right)+\_Z+2\ln\left(x\right)+2c_1\right)\right)x$$

✓ Solution by Mathematica
Time used: 0.033 (sec). Leaf size: 30

```
DSolve[(x+2 y[x])y'[x]+2 x -y[x]==0,y[x],x,IncludeSingularSolutions -> True]
```

$$\text{Solve}\left[\arctan\left(\frac{y(x)}{x}\right)+\log\left(\frac{y(x)^2}{x^2}+1\right)=-2\log(x)+c_1,y(x)\right]$$

17.5 problem 464

Internal problem ID [3718]

Book: Ordinary differential equations and their solutions. By George Moseley Murphy. 1960
Section: Various 17
Problem number: 464.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[[_homogeneous, ‘class A‘], _exact, _rational, [_Abel, ‘2nd ty`

$$\boxed{(x-2y)\,y'+y=-2x}$$

✓ Solution by Maple

Time used: 0.047 (sec). Leaf size: 51

```
dsolve((x-2*y(x))*diff(y(x),x)+2*x+y(x) = 0,y(x), singsol=all)
```

$$y(x)=\frac{c_1x-\sqrt{5c_1^2x^2+4}}{2c_1}$$
$$y(x)=\frac{c_1x+\sqrt{5c_1^2x^2+4}}{2c_1}$$

✓ Solution by Mathematica

Time used: 0.511 (sec). Leaf size: 102

```
DSolve[(x-2 y[x])y'[x]+2 x+y[x]==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x)\to\frac{1}{2}\left(x-\sqrt{5x^2-4e^{c_1}}\right)$$
$$y(x)\to\frac{1}{2}\left(x+\sqrt{5x^2-4e^{c_1}}\right)$$
$$y(x)\to\frac{1}{2}\left(x-\sqrt{5}\sqrt{x^2}\right)$$
$$y(x)\to\frac{1}{2}\left(\sqrt{5}\sqrt{x^2}+x\right)$$

17.6 problem 465

Internal problem ID [3719]

Book: Ordinary differential equations and their solutions. By George Moseley Murphy. 1960
Section: Various 17
Problem number: 465.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[[_homogeneous, ‘class C‘], _exact, _rational, [_Abel, ‘2nd ty`

$$\boxed{(1 + x - 2y)\, y' + y = 1 + 2x}$$

✓ Solution by Maple
Time used: 0.469 (sec). Leaf size: 33

```
dsolve((1+x-2*y(x))*diff(y(x),x) = 1+2*x-y(x),y(x), singsol=all)
```

$$y(x) = \frac{-\sqrt{4 - 27\left(x + \frac{1}{3}\right)^2 c_1^2} + (3x + 3)\, c_1}{6c_1}$$

✓ Solution by Mathematica
Time used: 0.143 (sec). Leaf size: 67

```
DSolve[(1+x-2 y[x])y'[x]==1+2 x-y[x],y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to \frac{1}{2}\left(-i\sqrt{3x^2 + 2x - 1 - 4c_1} + x + 1\right)$$
$$y(x) \to \frac{1}{2}\left(i\sqrt{3x^2 + 2x - 1 - 4c_1} + x + 1\right)$$

17.7 problem 466

Internal problem ID [3720]

Book: Ordinary differential equations and their solutions. By George Moseley Murphy. 1960
Section: Various 17
Problem number: 466.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[[_homogeneous, ‘class C‘], _rational, [_Abel, ‘2nd type‘, ‘cl`

$$\boxed{(x+2y+1)\,y'-2y=x-1}$$

✓ Solution by Maple
Time used: 0.032 (sec). Leaf size: 21

```
dsolve((1+x+2*y(x))*diff(y(x),x)+1-x-2*y(x) = 0,y(x), singsol=all)
```

$$y(x)=-\frac{x}{2}+\frac{2\,\mathrm{LambertW}\left(\frac{c_1 e^{\frac{9x}{4}-\frac{1}{4}}}{4}\right)}{3}+\frac{1}{6}$$

✓ Solution by Mathematica
Time used: 5.086 (sec). Leaf size: 43

```
DSolve[(1+x+2 y[x])y'[x]+1-x-2 y[x]==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x)\to\frac{1}{6}\left(4W\left(-e^{\frac{9x}{4}-1+c_1}\right)-3x+1\right)$$
$$y(x)\to\frac{1}{6}(1-3x)$$

17.8 problem 467

Internal problem ID [3721]

Book: Ordinary differential equations and their solutions. By George Moseley Murphy. 1960
Section: Various 17
Problem number: 467.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[[_homogeneous, ‘class C‘], _rational, [_Abel, ‘2nd type‘, ‘cl`

$$\boxed{(x+2y+1)\,y'-4y=-x-7}$$

✓ Solution by Maple

Time used: 0.172 (sec). Leaf size: 198

```
dsolve((1+x+2*y(x))*diff(y(x),x)+7+x-4*y(x) = 0,y(x), singsol=all)
```

$$y(x) = -\frac{4\left(\left(\frac{i\sqrt{3}}{48}-\frac{1}{48}\right)\left(12\sqrt{3}\,c_1^2(x+3)\sqrt{\frac{27(x+3)^2c_1-32x-96}{c_1}}+512+108(x+3)^2\,c_1^2+(-576x-1728)\,c_1\right)^{\frac{2}{3}}+\cdots\right.}{\left(12\sqrt{3}\,c_1^2\,(x+3)\sqrt{\frac{27(x+3\cdots}{\cdots}}\cdots\right.}$$

✓ Solution by Mathematica

Time used: 60.098 (sec). Leaf size: 2617

```
DSolve[(1+x+2 y[x])y'[x]+7+x-4 y[x]==0,y[x],x,IncludeSingularSolutions -> True]
```

Too large to display

17.9 problem 468

Internal problem ID [3722]

Book: Ordinary differential equations and their solutions. By George Moseley Murphy. 1960
Section: Various 17
Problem number: 468.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_exact, _rational, [_1st_order, '_with_symmetry_[F(x),G(x)]']`

$$\boxed{2(y+x)\,y'+2y=-x^2}$$

✓ Solution by Maple
Time used: 0.016 (sec). Leaf size: 51

```
dsolve(2*(x+y(x))*diff(y(x),x)+x^2+2*y(x) = 0,y(x), singsol=all)
```

$$y(x)=-x-\frac{\sqrt{-3x^3+9x^2-9c_1}}{3}$$
$$y(x)=-x+\frac{\sqrt{-3x^3+9x^2-9c_1}}{3}$$

✓ Solution by Mathematica
Time used: 0.16 (sec). Leaf size: 53

```
DSolve[2(x+y[x])y'[x]+x^2+2 y[x]==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x)\to -x-\sqrt{-\frac{x^3}{3}+x^2+c_1}$$
$$y(x)\to -x+\sqrt{-\frac{x^3}{3}+x^2+c_1}$$

17.10 problem 469

Internal problem ID [3723]

Book: Ordinary differential equations and their solutions. By George Moseley Murphy. 1960
Section: Various 17
Problem number: 469.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[[_homogeneous, ‘class C‘], _exact, _rational, [_Abel, ‘2nd ty`

$$\boxed{(3+2x-2y)\, y'+2y=6x+1}$$

✓ Solution by Maple
Time used: 0.484 (sec). Leaf size: 33

```
dsolve((3+2*x-2*y(x))*diff(y(x),x) = 1+6*x-2*y(x),y(x), singsol=all)
```

$$y(x)=\frac{-\sqrt{1-8\left(x-\frac{1}{2}\right)^{2} c_1^{2}}+(3+2x)\, c_1}{2c_1}$$

✓ Solution by Mathematica
Time used: 0.154 (sec). Leaf size: 67

```
DSolve[(3+2 x-2 y[x])y'[x]==1+6 x-2 y[x],y[x],x,IncludeSingularSolutions -> True]
```

$$y(x)\to -\frac{1}{2}i\sqrt{8x^2-8x-9-4c_1}+x+\frac{3}{2}$$
$$y(x)\to \frac{1}{2}i\sqrt{8x^2-8x-9-4c_1}+x+\frac{3}{2}$$

17.11 problem 470

Internal problem ID [3724]

Book: Ordinary differential equations and their solutions. By George Moseley Murphy. 1960
Section: Various 17
Problem number: 470.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[[_homogeneous, ‘class C‘], _rational, [_Abel, ‘2nd type‘, ‘cl`

$$\boxed{(1-4x-2y)\,y'+y=-2x}$$

✓ Solution by Maple
Time used: 0.031 (sec). Leaf size: 23

```
dsolve((1-4*x-2*y(x))*diff(y(x),x)+2*x+y(x) = 0,y(x), singsol=all)
```

$$y(x)=-\frac{\mathrm{LambertW}\left(-2\,\mathrm{e}^{4-25x+25c_1}\right)}{10}+\frac{2}{5}-2x$$

✓ Solution by Mathematica
Time used: 3.953 (sec). Leaf size: 39

```
DSolve[(1-4 x-2 y[x])y'[x]+2 x+y[x]==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x)\to -\frac{1}{10}W\left(-e^{-25x-1+c_1}\right)-2x+\frac{2}{5}$$
$$y(x)\to \frac{2}{5}-2x$$

17.12 problem 471

Internal problem ID [3725]

Book: Ordinary differential equations and their solutions. By George Moseley Murphy. 1960
Section: Various 17
Problem number: 471.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[[_homogeneous, ‘class C‘], _rational, [_Abel, ‘2nd type‘, ‘cl`

$$(6x - 2y)\, y' + y = 2 + 3x$$

✓ Solution by Maple

Time used: 0.031 (sec). Leaf size: 23

```
dsolve((6*x-2*y(x))*diff(y(x),x) = 2+3*x-y(x),y(x), singsol=all)
```

$$y(x) = -\frac{2\,\mathrm{LambertW}\left(-\frac{\mathrm{e}^{\frac{25x}{4}-1-\frac{25c_1}{4}}}{2}\right)}{5} + 3x - \frac{2}{5}$$

✓ Solution by Mathematica

Time used: 4.044 (sec). Leaf size: 40

```
DSolve[(6 x-2 y[x])y'[x]==2+3 x-y[x],y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to 3x - \frac{2}{5}\left(1 + W\left(-e^{\frac{25x}{4}-1+c_1}\right)\right)$$
$$y(x) \to 3x - \frac{2}{5}$$

17.13 problem 472

Internal problem ID [3726]

Book: Ordinary differential equations and their solutions. By George Moseley Murphy. 1960
Section: Various 17
Problem number: 472.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[[_homogeneous, 'class C'], _rational, [_Abel, '2nd type', 'cl`

$$\boxed{(19 + 9x + 2y)\, y' - 6y = -18 + 2x}$$

✓ Solution by Maple

Time used: 3.281 (sec). Leaf size: 29

```
dsolve((19+9*x+2*y(x))*diff(y(x),x)+18-2*x-6*y(x) = 0,y(x), singsol=all)
```

$$y(x) = \frac{\sqrt{1 + (-40x - 120)\, c_1} - 1 + (4x + 44)\, c_1}{8c_1}$$

✓ Solution by Mathematica

Time used: 16.36 (sec). Leaf size: 276

```
DSolve[(19+9 x+2 y[x])y'[x]+18-2 x-6 y[x]==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to -\frac{9x}{2} + \frac{(5-5i)(x+3)}{\frac{i\sqrt{2}}{\sqrt{(x+3)\cosh\left(\frac{2c_1}{9}\right)+(x+3)\sinh\left(\frac{2c_1}{9}\right)-i}} + (1-i)} - \frac{19}{2}$$

$$y(x) \to -\frac{9x}{2} + \frac{(5-5i)(x+3)}{(1-i) - \frac{i\sqrt{2}}{\sqrt{(x+3)\cosh\left(\frac{2c_1}{9}\right)+(x+3)\sinh\left(\frac{2c_1}{9}\right)-i}}} - \frac{19}{2}$$

$$y(x) \to -\frac{9x}{2} + \frac{(5-5i)(x+3)}{(1-i) - \frac{\sqrt{2}}{\sqrt{(x+3)\cosh\left(\frac{2c_1}{9}\right)+(x+3)\sinh\left(\frac{2c_1}{9}\right)+i}}} - \frac{19}{2}$$

$$y(x) \to -\frac{9x}{2} + \frac{(5-5i)(x+3)}{\frac{\sqrt{2}}{\sqrt{(x+3)\cosh\left(\frac{2c_1}{9}\right)+(x+3)\sinh\left(\frac{2c_1}{9}\right)+i}} + (1-i)} - \frac{19}{2}$$

$$y(x) \to -2(x+1)$$

$$y(x) \to \frac{x+11}{2}$$

17.14 problem 473

Internal problem ID [3727]

Book: Ordinary differential equations and their solutions. By George Moseley Murphy. 1960
Section: Various 17
Problem number: 473.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_exact, _rational, [_1st_order, '_with_symmetry_[F(x),G(x)]']`

$$\boxed{(x^3 + 2y)\, y' - 3x(2 - yx) = 0}$$

✓ Solution by Maple

Time used: 0.016 (sec). Leaf size: 51

```
dsolve((x^3+2*y(x))*diff(y(x),x) = 3*x*(2-x*y(x)),y(x), singsol=all)
```

$$y(x) = -\frac{x^3}{2} - \frac{\sqrt{x^6 + 12x^2 - 4c_1}}{2}$$
$$y(x) = -\frac{x^3}{2} + \frac{\sqrt{x^6 + 12x^2 - 4c_1}}{2}$$

✓ Solution by Mathematica

Time used: 0.161 (sec). Leaf size: 65

```
DSolve[(x^3+2 y[x])y'[x]==3 x(2 - x y[x]),y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to \frac{1}{2}\left(-x^3 - \sqrt{x^6 + 12x^2 + 4c_1}\right)$$
$$y(x) \to \frac{1}{2}\left(-x^3 + \sqrt{x^6 + 12x^2 + 4c_1}\right)$$

17.15 problem 474

Internal problem ID [3728]

Book: Ordinary differential equations and their solutions. By George Moseley Murphy. 1960
Section: Various 17
Problem number: 474.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[[_Abel, '2nd type', 'class A']]`

$$(\sec(x)\tan(x) - 2y)\, y' + \sec(x)\,(1 + 2y\sin(x)) = 0$$

✗ Solution by Maple

```
dsolve((tan(x)*sec(x)-2*y(x))*diff(y(x),x)+sec(x)*(1+2*y(x)*sin(x)) = 0,y(x), singsol=all)
```

No solution found

✗ Solution by Mathematica

Time used: 0.0 (sec). Leaf size: 0

```
DSolve[(Tan[x] Sec[x]-2 y[x])y'[x]+Sec[x](1+2 y[x] Sin[x])==0,y[x],x,IncludeSingularSolution
```

Not solved

17.16 problem 475

Internal problem ID [3729]

Book: Ordinary differential equations and their solutions. By George Moseley Murphy. 1960
Section: Various 17
Problem number: 475.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[[_Abel, '2nd type', 'class B']]`

$$\left(x\,\mathrm{e}^{-x}-2y\right)y'+\left(\mathrm{e}^{-x}+x\,\mathrm{e}^{-x}-2y\right)y=2x\,\mathrm{e}^{-2x}$$

✓ Solution by Maple

Time used: 0.015 (sec). Leaf size: 63

```
dsolve((x*exp(-x)-2*y(x))*diff(y(x),x) = 2*x*exp(-2*x)-(exp(-x)+x*exp(-x)-2*y(x))*y(x),y(x),
```

$$y(x)=\frac{\left(x\,\mathrm{e}^{x}-\sqrt{\mathrm{e}^{2x}\left(-3x^{2}+4c_{1}\right)}\right)\mathrm{e}^{-2x}}{2}$$
$$y(x)=\frac{\left(x\,\mathrm{e}^{x}+\sqrt{\mathrm{e}^{2x}\left(-3x^{2}+4c_{1}\right)}\right)\mathrm{e}^{-2x}}{2}$$

✓ Solution by Mathematica

Time used: 33.003 (sec). Leaf size: 81

```
DSolve[(x Exp[-x]-2 y[x])y'[x]==2 x Exp[-2 x]-(Exp[-x]+x Exp[-x]-2 y[x])y[x],y[x],x,IncludeS
```

$$y(x)\to\frac{1}{2}e^{-2x}\left(e^{x}x-\sqrt{e^{2x}\left(-3x^{2}+4c_{1}\right)}\right)$$
$$y(x)\to\frac{1}{2}e^{-2x}\left(e^{x}x+\sqrt{e^{2x}\left(-3x^{2}+4c_{1}\right)}\right)$$

17.17 problem 476

Internal problem ID [3730]

Book: Ordinary differential equations and their solutions. By George Moseley Murphy. 1960
Section: Various 17
Problem number: 476.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_separable]`

$$\boxed{3yy' + 5\cot(x)\cot(y)\cos(y)^2 = 0}$$

✓ Solution by Maple
Time used: 0.438 (sec). Leaf size: 22

```
dsolve(3*y(x)*diff(y(x),x)+5*cot(x)*cot(y(x))*cos(y(x))^2 = 0,y(x), singsol=all)
```

$$\ln(\sin(x)) + c_1 - \frac{3\tan(y(x))}{10} + \frac{3\sec(y(x))^2 y(x)}{10} = 0$$

✓ Solution by Mathematica
Time used: 0.488 (sec). Leaf size: 30

```
DSolve[3 y[x] y'[x]+5 Cot[x] Cot[y[x]] Cos[y[x]]^2==0,y[x],x,IncludeSingularSolutions -> Tru
```

$$\text{Solve}\left[40\sin(x)e^{\frac{3}{10}(y(x)\sec^2(y(x))-\tan(y(x)))} = c_1, y(x)\right]$$

18 Various 18

18.1 problem 477

Internal problem ID [3731]

Book: Ordinary differential equations and their solutions. By George Moseley Murphy. 1960
Section: Various 18
Problem number: 477.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_separable]`

$$\boxed{3(2-y)\,y'+yx=0}$$

✓ Solution by Maple
Time used: 0.016 (sec). Leaf size: 19

```
dsolve(3*(2-y(x))*diff(y(x),x)+x*y(x) = 0,y(x), singsol=all)
```

$$y(x)=-2\,\mathrm{LambertW}\left(-\frac{\mathrm{e}^{-\frac{x^2}{12}-\frac{c_1}{6}}}{2}\right)$$

✓ Solution by Mathematica
Time used: 24.428 (sec). Leaf size: 64

```
DSolve[3(2-y[x])y'[x]+x y[x]==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x)\to -2W\left(-\frac{1}{2}\sqrt{e^{-\frac{x^2}{6}-c_1}}\right)$$
$$y(x)\to -2W\left(\frac{1}{2}\sqrt{e^{-\frac{x^2}{6}-c_1}}\right)$$
$$y(x)\to 0$$

18.2 problem 478

Internal problem ID [3732]

Book: Ordinary differential equations and their solutions. By George Moseley Murphy. 1960
Section: Various 18
Problem number: 478.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[[_homogeneous, ‘class C‘], _rational, [_Abel, ‘2nd type‘, ‘cl`

$$\boxed{(x-3y)\,y'-y=-3x-4}$$

✓ Solution by Maple

Time used: 1.875 (sec). Leaf size: 227

```
dsolve((x-3*y(x))*diff(y(x),x)+4+3*x-y(x) = 0,y(x), singsol=all)
```

$$y(x) = \frac{i\left(-36864\left(x+\frac{3}{2}\right)^6 c_1^2+\left(864c_1x^3+3888c_1x^2+5832c_1x+12\sqrt{3}\,\sqrt{-16384c_1^2\left(x+\frac{3}{2}\right)^6\left(-\frac{27}{256}+\left(x+\frac{3}{2}\right.\right.}}\right.\right.}{}$$

✓ Solution by Mathematica

Time used: 60.044 (sec). Leaf size: 793

```
DSolve[(x-3 y[x])y'[x]+4+3 x-y[x]==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to \frac{x}{3}$$

$$-\frac{}{3\text{Root}\left[\#1^6\left(1024x^6+9216x^5+34560x^4+69120x^3+77760x^2+46656x+11664+16e^{12c_1}\right)+\#1^4\left(\right.\right.}$$

$$y(x) \to \frac{x}{3}$$

$$-\frac{}{3\text{Root}\left[\#1^6\left(1024x^6+9216x^5+34560x^4+69120x^3+77760x^2+46656x+11664+16e^{12c_1}\right)+\#1^4\left(\right.\right.}$$

$$y(x) \to \frac{x}{3}$$

$$-\frac{}{3\text{Root}\left[\#1^6\left(1024x^6+9216x^5+34560x^4+69120x^3+77760x^2+46656x+11664+16e^{12c_1}\right)+\#1^4\left(\right.\right.}$$

$$y(x) \to \frac{x}{3}$$

$$-\frac{}{3\text{Root}\left[\#1^6\left(1024x^6+9216x^5+34560x^4+69120x^3+77760x^2+46656x+11664+16e^{12c_1}\right)+\#1^4\left(\right.\right.}$$

$$y(x) \to \frac{x}{3}$$

$$-\frac{}{3\text{Root}\left[\#1^6\left(1024x^6+9216x^5+34560x^4+69120x^3+77760x^2+46656x+11664+16e^{12c_1}\right)+\#1^4\left(\right.\right.}$$

$$y(x) \to \frac{x}{3}$$

$$-\frac{}{3\text{Root}\left[\#1^6\left(1024x^6+9216x^5+34560x^4+69120x^3+77760x^2+46656x+11664+16e^{12c_1}\right)+\#1^4\left(\right.\right.}$$

18.3 problem 479

Internal problem ID [3733]

Book: Ordinary differential equations and their solutions. By George Moseley Murphy. 1960
Section: Various 18
Problem number: 479.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[[_homogeneous, ‘class C‘], _rational, [_Abel, ‘2nd type‘, ‘cl`

$$\boxed{(4 - x - 3y)\, y' - 3y = x - 3}$$

✓ Solution by Maple

Time used: 0.031 (sec). Leaf size: 21

```
dsolve((4-x-3*y(x))*diff(y(x),x)+3-x-3*y(x) = 0,y(x), singsol=all)
```

$$y(x) = -\frac{x}{3} - \frac{\mathrm{LambertW}\left(-\frac{c_1 \mathrm{e}^{\frac{4x}{3}+\frac{5}{3}}}{3}\right)}{2} + \frac{5}{6}$$

✓ Solution by Mathematica

Time used: 5.005 (sec). Leaf size: 43

```
DSolve[(4-x-3 y[x])y'[x]+3-x-3 y[x]==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to \frac{1}{6}\left(-3W\left(-e^{\frac{4x}{3}-1+c_1}\right) - 2x + 5\right)$$
$$y(x) \to \frac{1}{6}(5 - 2x)$$

18.4 problem 480

Internal problem ID [3734]

Book: Ordinary differential equations and their solutions. By George Moseley Murphy. 1960
Section: Various 18
Problem number: 480.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[[_homogeneous, ‘class C‘], _rational, [_Abel, ‘2nd type‘, ‘cl`

$$(2 + 2x + 3y)\, y' + 3y = 1 - 2x$$

✓ Solution by Maple

Time used: 0.031 (sec). Leaf size: 21

```
dsolve((2+2*x+3*y(x))*diff(y(x),x) = 1-2*x-3*y(x),y(x), singsol=all)
```

$$y(x) = -\frac{2x}{3} + 3\,\mathrm{LambertW}\left(\frac{c_1 \mathrm{e}^{-\frac{x}{9}-\frac{7}{9}}}{9}\right) + \frac{7}{3}$$

✓ Solution by Mathematica

Time used: 5.023 (sec). Leaf size: 43

```
DSolve[(2+2 x+3 y[x])y'[x]==1-2 x-3 y[x],y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to \frac{1}{3}\left(9W\left(-e^{-\frac{x}{9}-1+c_1}\right) - 2x + 7\right)$$
$$y(x) \to \frac{1}{3}(7 - 2x)$$

18.5 problem 481

Internal problem ID [3735]

Book: Ordinary differential equations and their solutions. By George Moseley Murphy. 1960
Section: Various 18
Problem number: 481.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[[_homogeneous, ‘class C‘], _rational, [_Abel, ‘2nd type‘, ‘cl`

$$\boxed{(5-2x-3y)\,y'-3y=2x-1}$$

✓ Solution by Maple
Time used: 0.016 (sec). Leaf size: 21

```
dsolve((5-2*x-3*y(x))*diff(y(x),x)+1-2*x-3*y(x) = 0,y(x), singsol=all)
```

$$y(x)=-\frac{2x}{3}-4\,\mathrm{LambertW}\left(-\frac{c_1 \mathrm{e}^{\frac{x}{12}-\frac{7}{12}}}{12}\right)-\frac{7}{3}$$

✓ Solution by Mathematica
Time used: 3.981 (sec). Leaf size: 43

```
DSolve[(5-2 x-3 y[x])y'[x]+1-2 x -3 y[x]==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x)\to -4W\left(-e^{\frac{x}{12}-1+c_1}\right)-\frac{2x}{3}-\frac{7}{3}$$
$$y(x)\to \frac{1}{3}(-2x-7)$$

18.6 problem 482

Internal problem ID [3736]

Book: Ordinary differential equations and their solutions. By George Moseley Murphy. 1960
Section: Various 18
Problem number: 482.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[[_homogeneous, ‘class C‘], _rational, [_Abel, ‘2nd type‘, ‘cl`

$$\boxed{(1+9x-3y)\,y'-y=-3x-2}$$

✓ Solution by Maple
Time used: 0.047 (sec). Leaf size: 23

```
dsolve((1+9*x-3*y(x))*diff(y(x),x)+2+3*x-y(x) = 0,y(x), singsol=all)
```

$$y(x)=\frac{\operatorname{LambertW}\left(3\,\mathrm{e}^{-20x-3+20c_1}\right)}{6}+3x+\frac{1}{2}$$

✓ Solution by Mathematica
Time used: 5.372 (sec). Leaf size: 37

```
DSolve[(1+9 x-3 y[x])y'[x]+2+3 x-y[x]==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x)\to\frac{1}{6}\left(W\left(-e^{-20x-1+c_1}\right)+18x+3\right)$$
$$y(x)\to 3x+\frac{1}{2}$$

18.7 problem 483

Internal problem ID [3737]

Book: Ordinary differential equations and their solutions. By George Moseley Murphy. 1960
Section: Various 18
Problem number: 483.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[[_homogeneous, ‘class A‘], _rational, [_Abel, ‘2nd type‘, ‘cl`

$$(4y+x)\,y'-y=-4x$$

✓ Solution by Maple

Time used: 0.016 (sec). Leaf size: 24

```
dsolve((x+4*y(x))*diff(y(x),x)+4*x-y(x) = 0,y(x), singsol=all)
```

$$y(x)=\tan\left(\operatorname{RootOf}\left(2\ln\left(\sec\left(\_Z\right)^2\right)+\_Z+4\ln\left(x\right)+4c_1\right)\right)x$$

✓ Solution by Mathematica

Time used: 0.034 (sec). Leaf size: 32

```
DSolve[(x+4 y[x])y'[x]+4 x-y[x]==0,y[x],x,IncludeSingularSolutions -> True]
```

$$\text{Solve}\left[\arctan\left(\frac{y(x)}{x}\right)+2\log\left(\frac{y(x)^2}{x^2}+1\right)=-4\log(x)+c_1,y(x)\right]$$

18.8 problem 484

Internal problem ID [3738]

Book: Ordinary differential equations and their solutions. By George Moseley Murphy. 1960
Section: Various 18
Problem number: 484.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[[_homogeneous, ‘class C‘], _rational, [_Abel, ‘2nd type‘, ‘cl`

$$(3 + 2x + 4y)\, y' - 2y = x + 1$$

✓ Solution by Maple
Time used: 0.031 (sec). Leaf size: 20

```
dsolve((3+2*x+4*y(x))*diff(y(x),x) = 1+x+2*y(x),y(x), singsol=all)
```

$$y(x) = -\frac{x}{2} + \frac{\operatorname{LambertW}\left(c_1 \mathrm{e}^{5+8x}\right)}{8} - \frac{5}{8}$$

✓ Solution by Mathematica
Time used: 4.873 (sec). Leaf size: 39

```
DSolve[(3+2 x+4 y[x])y'[x]==1+x+2 y[x],y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to \frac{1}{8}\left(W\left(-e^{8x-1+c_1}\right) - 4x - 5\right)$$
$$y(x) \to \frac{1}{8}(-4x - 5)$$

18.9 problem 485

Internal problem ID [3739]

Book: Ordinary differential equations and their solutions. By George Moseley Murphy. 1960
Section: Various 18
Problem number: 485.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[[_homogeneous, ‘class C‘], _exact, _rational, [_Abel, ‘2nd ty`

$$(5 + 2x - 4y)\, y' + 2y = x + 3$$

✓ Solution by Maple
Time used: 0.016 (sec). Leaf size: 41

```
dsolve((5+2*x-4*y(x))*diff(y(x),x) = 3+x-2*y(x),y(x), singsol=all)
```

$$y(x) = \frac{x}{2} + \frac{5}{4} - \frac{\sqrt{4c_1 - 4x + 25}}{4}$$
$$y(x) = \frac{x}{2} + \frac{5}{4} + \frac{\sqrt{4c_1 - 4x + 25}}{4}$$

✓ Solution by Mathematica
Time used: 0.111 (sec). Leaf size: 61

```
DSolve[(5+2 x-4 y[x])y'[x]==3+x-2 y[x],y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to \frac{1}{4}\left(2x - i\sqrt{4x - 25 - 16c_1} + 5\right)$$
$$y(x) \to \frac{1}{4}\left(2x + i\sqrt{4x - 25 - 16c_1} + 5\right)$$

18.10 problem 486

Internal problem ID [3740]

Book: Ordinary differential equations and their solutions. By George Moseley Murphy. 1960
Section: Various 18
Problem number: 486.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[[_homogeneous, ‘class C‘], _exact, _rational, [_Abel, ‘2nd ty`

$$\boxed{(5 + 3x - 4y)\, y' + 3y = 7x + 2}$$

✓ Solution by Maple
Time used: 0.468 (sec). Leaf size: 33

```
dsolve((5+3*x-4*y(x))*diff(y(x),x) = 2+7*x-3*y(x),y(x), singsol=all)
```

$$y(x) = \frac{-\sqrt{4 - 6859 \left(x - \frac{7}{19}\right)^2 c_1^2} + (57x + 95)\, c_1}{76 c_1}$$

✓ Solution by Mathematica
Time used: 0.145 (sec). Leaf size: 71

```
DSolve[(5+3 x-4 y[x])y'[x]==2+7 x-3 y[x],y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to \frac{1}{4}\left(-i\sqrt{19x^2 - 14x - 25 - 16c_1} + 3x + 5\right)$$
$$y(x) \to \frac{1}{4}\left(i\sqrt{19x^2 - 14x - 25 - 16c_1} + 3x + 5\right)$$

18.11 problem 487

Internal problem ID [3741]

Book: Ordinary differential equations and their solutions. By George Moseley Murphy. 1960
Section: Various 18
Problem number: 487.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[[_homogeneous, ‘class C‘], _rational, [_Abel, ‘2nd type‘, ‘cl`

$$4(-y-x+1)\,y' = x-2$$

✓ Solution by Maple
Time used: 0.328 (sec). Leaf size: 28

```
dsolve(4*(1-x-y(x))*diff(y(x),x)+2-x = 0,y(x), singsol=all)
```

$$y(x) = \frac{-x\,\mathrm{LambertW}\left(-c_1(-2+x)\right)+x-2}{2\,\mathrm{LambertW}\left(-c_1\left(-2+x\right)\right)}$$

✓ Solution by Mathematica
Time used: 3.485 (sec). Leaf size: 109

```
DSolve[4(1-x-y[x])y'[x]+2-x==0,y[x],x,IncludeSingularSolutions -> True]
```

$$\text{Solve}\left[\frac{2^{2/3}\left(x\log\left(\frac{x-2}{y(x)+x-1}\right)-x\log\left(\frac{2y(x)+x}{y(x)+x-1}\right)+2y(x)\left(\log\left(\frac{x-2}{y(x)+x-1}\right)-\log\left(\frac{2y(x)+x}{y(x)+x-1}\right)+1\right)+2x-2\right)}{9(2y(x)+x)}\right.$$

18.12 problem 488

Internal problem ID [3742]

Book: Ordinary differential equations and their solutions. By George Moseley Murphy. 1960
Section: Various 18
Problem number: 488.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[[_homogeneous, ‘class C‘], _rational, [_Abel, ‘2nd type‘, ‘cl`

$$\boxed{(11 - 11x - 4y)\, y' + 25y = 62 - 8x}$$

✓ Solution by Maple

Time used: 0.75 (sec). Leaf size: 218

```
dsolve((11-11*x-4*y(x))*diff(y(x),x) = 62-8*x-25*y(x),y(x), singsol=all)
```

$y(x)$

$$= \frac{4\left(x+\frac{1}{2}\right)\left(i\sqrt{3}-1\right)\left(708588\sqrt{\left(-\frac{32}{177147}+\left(x-\frac{1}{9}\right)^{2} c_{1}\right) c_{1}\left(x-\frac{1}{9}\right)^{2}}+64-70858\right.}{i\sqrt{3}\left(708588\sqrt{\left(-\frac{32}{177147}+\left(x-\frac{1}{9}\right)^{2} c_{1}\right) c_{1}\left(x-\frac{1}{9}\right)^{2}}+64-708588\left(x-\frac{1}{9}\right)^{2} c_{1}\right)^{\frac{2}{3}}-16i\sqrt{3}-\left(708588\right.}$$

✓ Solution by Mathematica

Time used: 60.174 (sec). Leaf size: 1677

```
DSolve[(11-11 x-4 y[x])y'[x]==62-8x -25 y[x],y[x],x,IncludeSingularSolutions -> True]
```

Too large to display

18.13 problem 489

Internal problem ID [3743]

Book: Ordinary differential equations and their solutions. By George Moseley Murphy. 1960
Section: Various 18
Problem number: 489.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[[_homogeneous, ‘class C‘], _rational, [_Abel, ‘2nd type‘, ‘cl`

$$\boxed{(6+3x+5y)\,y'-7y=x+2}$$

✓ Solution by Maple

Time used: 1.61 (sec). Leaf size: 46

```
dsolve((6+3*x+5*y(x))*diff(y(x),x) = 2+x+7*y(x),y(x), singsol=all)
```

$$y(x)=(-2-x)\operatorname{RootOf}\left(6+\left(c_1x^3+6c_1x^2+12c_1x+8c_1\right)\_Z^{12}-5\_Z^3\right)^3+2+x$$

✓ Solution by Mathematica

Time used: 60.159 (sec). Leaf size: 4977

```
DSolve[(6+3 x+5 y[x])y'[x]==2 + x+7 y[x],y[x],x,IncludeSingularSolutions -> True]
```

Too large to display

18.14 problem 490

Internal problem ID [3744]

Book: Ordinary differential equations and their solutions. By George Moseley Murphy. 1960
Section: Various 18
Problem number: 490.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[[_homogeneous, ‘class A‘], _rational, [_Abel, ‘2nd type‘, ‘cl`

$$(7x+5y)\,y'+8y=-10x$$

✓ Solution by Maple

Time used: 6.422 (sec). Leaf size: 38

```
dsolve((7*x+5*y(x))*diff(y(x),x)+10*x+8*y(x) = 0,y(x), singsol=all)
```

$$y(x)=x\left(\operatorname{RootOf}\left(\_Z^{25}c_1x^5-2\_Z^{20}c_1x^5+\_Z^{15}c_1x^5-1\right)^5-2\right)$$

✓ Solution by Mathematica

Time used: 2.325 (sec). Leaf size: 276

```
DSolve[(7 x+5 y[x])y'[x]+10 x+8 y[x]==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x)\to \operatorname{Root}\left[\#1^5+8\#1^4x+25\#1^3x^2+38\#1^2x^3+28\#1x^4+8x^5-e^{c_1}\&,1\right]$$
$$y(x)\to \operatorname{Root}\left[\#1^5+8\#1^4x+25\#1^3x^2+38\#1^2x^3+28\#1x^4+8x^5-e^{c_1}\&,2\right]$$
$$y(x)\to \operatorname{Root}\left[\#1^5+8\#1^4x+25\#1^3x^2+38\#1^2x^3+28\#1x^4+8x^5-e^{c_1}\&,3\right]$$
$$y(x)\to \operatorname{Root}\left[\#1^5+8\#1^4x+25\#1^3x^2+38\#1^2x^3+28\#1x^4+8x^5-e^{c_1}\&,4\right]$$
$$y(x)\to \operatorname{Root}\left[\#1^5+8\#1^4x+25\#1^3x^2+38\#1^2x^3+28\#1x^4+8x^5-e^{c_1}\&,5\right]$$

18.15 problem 491

Internal problem ID [3745]

Book: Ordinary differential equations and their solutions. By George Moseley Murphy. 1960
Section: Various 18
Problem number: 491.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_rational, [_Abel, '2nd type', 'class A']]`

$$(x + 4x^3 + 5y)\, y' + 3x^2 y + 4y = -7x^3$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 588

```
dsolve((x+4*x^3+5*y(x))*diff(y(x),x)+7*x^3+3*x^2*y(x)+4*y(x) = 0,y(x), singsol=all)
```

$$y(x) = \frac{16(-x^{15} + 5x^{13} - 10x^{11} + 10x^9 - 5x^7 + x^5 + 48c_1)\,\mathrm{RootOf}\left((-2x^{15} + 10x^{13} - 20x^{11} + 20x^9 - 10x^7 + 2\right.}{}$$

✓ Solution by Mathematica

Time used: 60.408 (sec). Leaf size: 3641

```
DSolve[(x+4 x^3+5 y[x])y'[x]+7 x^3+3 x^2 y[x]+4 y[x]==0,y[x],x,IncludeSingularSolutions -> T
```

Too large to display

18.16 problem 492

Internal problem ID [3746]

Book: Ordinary differential equations and their solutions. By George Moseley Murphy. 1960
Section: Various 18
Problem number: 492.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[[_homogeneous, ‘class C‘], _rational, [_Abel, ‘2nd type‘, ‘cl`

$$\boxed{(5 - x + 6y)\, y' - 4y = -x + 3}$$

✓ Solution by Maple
Time used: 3.328 (sec). Leaf size: 31

```
dsolve((5-x+6*y(x))*diff(y(x),x) = 3-x+4*y(x),y(x), singsol=all)
```

$$y(x) = \frac{-\sqrt{9 + (-8x - 8)\, c_1} - 3 + (4x - 4)\, c_1}{8c_1}$$

✓ Solution by Mathematica

Time used: 60.101 (sec). Leaf size: 1177

```
DSolve[(5-x+6 y[x])y'[x]==3-x+4 y[x],y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to \frac{1}{6}\left(x + \frac{2(x+\cdots}{x\sqrt{\frac{3}{(x+1)^2} - \frac{3(x+1)^2\cosh\left(\frac{4c_1}{9}\right)+3(x+1)^2\sinh\left(\frac{4c_1}{9}\right)+2}{(x+1)^2\left((x+1)^2\cosh\left(\frac{4c_1}{9}\right)+(x+1)^2\sinh\left(\frac{4c_1}{9}\right)+1\right)} - \sqrt{-\frac{\cosh\left(\frac{4c_1}{9}\right)+\sinh\left(\frac{4c_1}{9}\right)}{(x+1)^2\left((x+1)^2\cosh\left(\frac{4c_1}{9}\right)+(x+1)^2\sinh\left(\frac{4c_1}{9}\right)+1\right)^2}} + \cdots}} - 5\right)$$

$$y(x) \to \frac{1}{6}\left(x - \frac{2(x+\cdots}{x\sqrt{\frac{3}{(x+1)^2} - \frac{3(x+1)^2\cosh\left(\frac{4c_1}{9}\right)+3(x+1)^2\sinh\left(\frac{4c_1}{9}\right)+2}{(x+1)^2\left((x+1)^2\cosh\left(\frac{4c_1}{9}\right)+(x+1)^2\sinh\left(\frac{4c_1}{9}\right)+1\right)} - \sqrt{-\frac{\cosh\left(\frac{4c_1}{9}\right)+\sinh\left(\frac{4c_1}{9}\right)}{(x+1)^2\left((x+1)^2\cosh\left(\frac{4c_1}{9}\right)+(x+1)^2\sinh\left(\frac{4c_1}{9}\right)+1\right)^2}} + \cdots}} - 5\right)$$

$$y(x) \to \frac{1}{6}\left(x + \frac{2(x+\cdots}{\sqrt{\frac{3}{\cdots} - \frac{3(x+1)^2\cosh\left(\frac{4c_1}{9}\right)+3(x+1)^2\sinh\left(\frac{4c_1}{9}\right)+2}{\cdots} - \sqrt{\frac{\cosh\left(\frac{4c_1}{9}\right)+\sinh\left(\frac{4c_1}{9}\right)}{\cdots}}}}\right.$$

18.17 problem 493

Internal problem ID [3747]

Book: Ordinary differential equations and their solutions. By George Moseley Murphy. 1960
Section: Various 18
Problem number: 493.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[[_homogeneous, ‘class C‘], _rational, [_Abel, ‘2nd type‘, ‘cl`

$$\boxed{3(x+2y)\,y'+2y=1-x}$$

✓ Solution by Maple
Time used: 0.031 (sec). Leaf size: 23

```
dsolve(3*(x+2*y(x))*diff(y(x),x) = 1-x-2*y(x),y(x), singsol=all)
```

$$y(x)=-\operatorname{LambertW}\left(-\mathrm{e}^{-1-\frac{x}{6}+\frac{c_1}{6}}\right)-1-\frac{x}{2}$$

✓ Solution by Mathematica
Time used: 4.134 (sec). Leaf size: 39

```
DSolve[3(x+2 y[x])y'[x]==1-x-2 y[x],y[x],x,IncludeSingularSolutions -> True]
```

$$y(x)\to -W\left(-e^{-\frac{x}{6}-1+c_1}\right)-\frac{x}{2}-1$$
$$y(x)\to -\frac{x}{2}-1$$

18.18 problem 494

Internal problem ID [3748]

Book: Ordinary differential equations and their solutions. By George Moseley Murphy. 1960
Section: Various 18
Problem number: 494.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[[_homogeneous, ‘class C‘], _rational, [_Abel, ‘2nd type‘, ‘cl`

$$\boxed{3y + (7y - 3x + 3)\, y' = 7x - 7}$$

✓ Solution by Maple
Time used: 0.75 (sec). Leaf size: 1814

```
dsolve((3-3*x+7*y(x))*diff(y(x),x)+7-7*x+3*y(x) = 0,y(x), singsol=all)
```

Expression too large to display

✓ Solution by Mathematica
Time used: 60.781 (sec). Leaf size: 7785

```
DSolve[(3-3 x+7 y[x])y'[x]+7-7 x+3 y[x]==0,y[x],x,IncludeSingularSolutions -> True]
```

Too large to display

18.19 problem 495

Internal problem ID [3749]

Book: Ordinary differential equations and their solutions. By George Moseley Murphy. 1960
Section: Various 18
Problem number: 495.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[[_homogeneous, ‘class C‘], _rational, [_Abel, ‘2nd type‘, ‘cl`

$$(1 + x + 9y)\, y' + 5y = -x - 1$$

✓ Solution by Maple

Time used: 0.328 (sec). Leaf size: 29

```
dsolve((1+x+9*y(x))*diff(y(x),x)+1+x+5*y(x) = 0,y(x), singsol=all)
```

$$y(x) = -\frac{(x+1)\left(2 + 3\,\mathrm{LambertW}\left(\frac{2c_1(x+1)}{3}\right)\right)}{9\,\mathrm{LambertW}\left(\frac{2c_1(x+1)}{3}\right)}$$

✓ Solution by Mathematica

Time used: 1.908 (sec). Leaf size: 145

```
DSolve[(1+x+9 y[x])y'[x]+1+x+5 y[x]==0,y[x],x,IncludeSingularSolutions -> True]
```

$$\mathrm{Solve}\left[\frac{(-2)^{2/3}\left((x+1)\left(3\log\left(-\frac{6(-2)^{2/3}(x+1)}{9y(x)+x+1}\right) - 3\log\left(\frac{9(-2)^{2/3}(3y(x)+x+1)}{9y(x)+x+1}\right) + 1\right) + 9y(x)\left(\log\left(-\frac{6(-2)^{2/3}(x}{9y(x)+x}\right.\right.\right.}{27(3y(x)+x+1)}\right.$$

18.20 problem 496

Internal problem ID [3750]

Book: Ordinary differential equations and their solutions. By George Moseley Murphy. 1960
Section: Various 18
Problem number: 496.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[[_homogeneous, ‘class C‘], _exact, _rational, [_Abel, ‘2nd ty`

$$\boxed{(8+5x-12y)\,y'+5y=2x+3}$$

✓ Solution by Maple
Time used: 0.453 (sec). Leaf size: 32

```
dsolve((8+5*x-12*y(x))*diff(y(x),x) = 3+2*x-5*y(x),y(x), singsol=all)
```

$$y(x)=\frac{-\sqrt{\left(x+4\right)^{2}c_{1}^{2}+24}+\left(5x+8\right)c_{1}}{12c_{1}}$$

✓ Solution by Mathematica
Time used: 0.143 (sec). Leaf size: 77

```
DSolve[(8+5 x-12 y[x])y'[x]==3+2 x-5 y[x],y[x],x,IncludeSingularSolutions -> True]
```

$$y(x)\to\frac{1}{12}\left(-i\sqrt{-x^2-8x-16(4+9c_1)}+5x+8\right)$$
$$y(x)\to\frac{1}{12}\left(i\sqrt{-x^2-8x-16(4+9c_1)}+5x+8\right)$$

18.21 problem 497

Internal problem ID [3751]

Book: Ordinary differential equations and their solutions. By George Moseley Murphy. 1960
Section: Various 18
Problem number: 497.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[[_homogeneous, 'class C'], _rational, [_Abel, '2nd type', 'cl`

$$(140 + 7x - 16y)\, y' + y = -25 - 8x$$

✓ Solution by Maple
Time used: 0.812 (sec). Leaf size: 1151

```
dsolve((140+7*x-16*y(x))*diff(y(x),x)+25+8*x+y(x) = 0,y(x), singsol=all)
```

Expression too large to display

✓ Solution by Mathematica
Time used: 60.061 (sec). Leaf size: 1673

```
DSolve[(140+7 x-16 y[x])y'[x]+25+8 x+y[x]==0,y[x],x,IncludeSingularSolutions -> True]
```

Too large to display

18.22 problem 498

Internal problem ID [3752]

Book: Ordinary differential equations and their solutions. By George Moseley Murphy. 1960
Section: Various 18
Problem number: 498.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[[_homogeneous, ‘class C‘], _rational, [_Abel, ‘2nd type‘, ‘cl`

$$(3 + 9x + 21y)\, y' + 5y = 45 + 7x$$

✓ Solution by Maple

Time used: 0.687 (sec). Leaf size: 212

```
dsolve((3+9*x+21*y(x))*diff(y(x),x) = 45+7*x-5*y(x),y(x), singsol=all)
```

$$y(x) = \frac{(-x-5)\operatorname{RootOf}\left(-27 + \left(c_1x^7 + 35c_1x^6 + 525c_1x^5 + 4375c_1x^4 + 21875c_1x^3 + 65625c_1x^2 + 109375c_1x\right.\right.}{} + \frac{11}{3} + \frac{x}{3}$$

✓ Solution by Mathematica

Time used: 60.806 (sec). Leaf size: 7785

```
DSolve[(3+9 x+21 y[x])y'[x]==45 +7 x-5 y[x],y[x],x,IncludeSingularSolutions -> True]
```

Too large to display

18.23 problem 499

Internal problem ID [3753]

Book: Ordinary differential equations and their solutions. By George Moseley Murphy. 1960
Section: Various 18
Problem number: 499.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[[_homogeneous, ‘class A‘], _rational, [_Abel, ‘2nd type‘, ‘cl`

$$\boxed{(ax+yb)\,y' = -x}$$

✓ Solution by Maple
Time used: 0.172 (sec). Leaf size: 63

```
dsolve((a*x+b*y(x))*diff(y(x),x)+x = 0,y(x), singsol=all)
```

$$y(x) = \operatorname{RootOf}\left(\_Z^2 b - \mathrm{e}^{\operatorname{RootOf}\left(\left(4\,\mathrm{e}^{\_Z} b \cosh\left(\frac{\sqrt{a^2-4b}\,(2c_1+\_Z+2\ln(x))}{2a}\right)^2 + a^2 - 4b\right)x^2\right)} + 1 + a\_Z\right) x$$

✓ Solution by Mathematica
Time used: 0.117 (sec). Leaf size: 75

```
DSolve[(a x+b y[x])y'[x]+x==0,y[x],x,IncludeSingularSolutions -> True]
```

$$\text{Solve}\left[\frac{a\arctan\left(\frac{a+\frac{2by(x)}{x}}{\sqrt{4b-a^2}}\right)}{\sqrt{4b-a^2}} + \frac{1}{2}\log\left(\frac{ay(x)}{x} + \frac{by(x)^2}{x^2} + 1\right) = -\log(x) + c_1, y(x)\right]$$

18.24 problem 500

Internal problem ID [3754]

Book: Ordinary differential equations and their solutions. By George Moseley Murphy. 1960
Section: Various 18
Problem number: 500.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[[_homogeneous, 'class A'], _rational, [_Abel, '2nd type', 'cl`

$$\boxed{(ax + yb)\, y' + y = 0}$$

✓ Solution by Maple

Time used: 0.063 (sec). Leaf size: 23

```
dsolve((a*x+b*y(x))*diff(y(x),x)+y(x) = 0,y(x), singsol=all)
```

$$x + \frac{y(x)\, b}{a+1} - y(x)^{-a}\, c_1 = 0$$

✓ Solution by Mathematica

Time used: 0.041 (sec). Leaf size: 38

```
DSolve[(a x+b y[x])y'[x]+y[x]==0,y[x],x,IncludeSingularSolutions -> True]
```

$$\text{Solve}\left[\frac{\log\left(a + \frac{by(x)}{x} + 1\right) + a\log\left(\frac{y(x)}{x}\right)}{a+1} = -\log(x) + c_1, y(x)\right]$$

18.25 problem 501

Internal problem ID [3755]

Book: Ordinary differential equations and their solutions. By George Moseley Murphy. 1960
Section: Various 18
Problem number: 501.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[[_homogeneous, ‘class A‘], _exact, _rational, [_Abel, ‘2nd ty`

$$(ax + yb)\, y' + ya = -bx$$

✓ Solution by Maple

Time used: 0.063 (sec). Leaf size: 75

```
dsolve((a*x+b*y(x))*diff(y(x),x)+b*x+a*y(x) = 0,y(x), singsol=all)
```

$$y(x) = \frac{-c_1 a x + \sqrt{x^2 (a^2 - b^2)\, c_1^2 + b}}{b c_1}$$
$$y(x) = \frac{-c_1 a x - \sqrt{x^2 (a^2 - b^2)\, c_1^2 + b}}{c_1 b}$$

✓ Solution by Mathematica

Time used: 15.858 (sec). Leaf size: 143

```
DSolve[(a x+b y[x])y'[x]+b x+a y[x]==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to -\frac{a x + \sqrt{a^2 x^2 - b^2 x^2 + b e^{2c_1}}}{b}$$
$$y(x) \to \frac{-a x + \sqrt{a^2 x^2 - b^2 x^2 + b e^{2c_1}}}{b}$$
$$y(x) \to -\frac{\sqrt{x^2 (a^2 - b^2)} + a x}{b}$$
$$y(x) \to \frac{\sqrt{x^2 (a^2 - b^2)} - a x}{b}$$

18.26 problem 502

Internal problem ID [3756]

Book: Ordinary differential equations and their solutions. By George Moseley Murphy. 1960
Section: Various 18
Problem number: 502.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[[_homogeneous, ‘class A‘], _rational, [_Abel, ‘2nd type‘, ‘cl`

$$\boxed{(ax + yb)\, y' - ya = bx}$$

✓ Solution by Maple

Time used: 0.156 (sec). Leaf size: 48

```
dsolve((a*x+b*y(x))*diff(y(x),x) = b*x+a*y(x),y(x), singsol=all)
```

$$y(x) = x\left(1 + \mathrm{e}^{\mathrm{RootOf}\left(\mathrm{e}^{\_Z} - x^{\frac{2b}{a-b}}\mathrm{e}^{\frac{a\_Z + \_Zb + 2bc_1}{a-b}} + 2\right)}\right)$$

✓ Solution by Mathematica

Time used: 0.048 (sec). Leaf size: 48

```
DSolve[(a x+b y[x])y'[x]==b x+a y[x],y[x],x,IncludeSingularSolutions -> True]
```

$$\mathrm{Solve}\left[\frac{1}{2}(a+b)\log\left(1 - \frac{y(x)}{x}\right) + \frac{1}{2}(b-a)\log\left(\frac{y(x)}{x} + 1\right) = -b\log(x) + c_1, y(x)\right]$$

18.27 problem 505

Internal problem ID [3757]

Book: Ordinary differential equations and their solutions. By George Moseley Murphy. 1960
Section: Various 18
Problem number: 505.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_separable]`

$$\boxed{xyy' + y^2 = -1}$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 34

```
dsolve(x*y(x)*diff(y(x),x)+1+y(x)^2 = 0,y(x), singsol=all)
```

$$y(x) = \frac{\sqrt{-x^2 + c_1}}{x}$$
$$y(x) = -\frac{\sqrt{-x^2 + c_1}}{x}$$

✓ Solution by Mathematica

Time used: 0.347 (sec). Leaf size: 96

```
DSolve[x y[x] y'[x]+1+y[x]^2==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to -\frac{\sqrt{-x^2 + e^{2c_1}}}{x}$$
$$y(x) \to \frac{\sqrt{-x^2 + e^{2c_1}}}{x}$$
$$y(x) \to -i$$
$$y(x) \to i$$
$$y(x) \to \frac{x}{\sqrt{-x^2}}$$
$$y(x) \to \frac{\sqrt{-x^2}}{x}$$

18.28 problem 506

Internal problem ID [3758]

Book: Ordinary differential equations and their solutions. By George Moseley Murphy. 1960
Section: Various 18
Problem number: 506.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[[_homogeneous, ‘class D‘], _rational, _Bernoulli]`

$$\boxed{xyy' - y^2 = x}$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 27

```
dsolve(x*y(x)*diff(y(x),x) = x+y(x)^2,y(x), singsol=all)
```

$$y(x) = \sqrt{x\,(c_1x - 2)}$$
$$y(x) = -\sqrt{x\,(c_1x - 2)}$$

✓ Solution by Mathematica

Time used: 0.31 (sec). Leaf size: 42

```
DSolve[x y[x] y'[x]==x+y[x]^2,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to -\sqrt{x}\sqrt{-2 + c_1x}$$
$$y(x) \to \sqrt{x}\sqrt{-2 + c_1x}$$

18.29 problem 507

Internal problem ID [3759]

Book: Ordinary differential equations and their solutions. By George Moseley Murphy. 1960
Section: Various 18
Problem number: 507.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[[_homogeneous, ‘class A‘], _rational, _Bernoulli]`

$$\boxed{xyy' + y^2 = -x^2}$$

✓ Solution by Maple
Time used: 0.015 (sec). Leaf size: 39

```
dsolve(x*y(x)*diff(y(x),x)+x^2+y(x)^2 = 0,y(x), singsol=all)
```

$$y(x) = -\frac{\sqrt{-2x^4 + 4c_1}}{2x}$$
$$y(x) = \frac{\sqrt{-2x^4 + 4c_1}}{2x}$$

✓ Solution by Mathematica
Time used: 0.222 (sec). Leaf size: 46

```
DSolve[x y[x] y'[x]+x^2+y[x]^2==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to -\frac{\sqrt{-\frac{x^4}{2} + c_1}}{x}$$
$$y(x) \to \frac{\sqrt{-\frac{x^4}{2} + c_1}}{x}$$

18.30 problem 508

Internal problem ID [3760]

Book: Ordinary differential equations and their solutions. By George Moseley Murphy. 1960
Section: Various 18
Problem number: 508.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[[_homogeneous, ‘class D‘], _rational, _Bernoulli]`

$$\boxed{xyy' - y^2 = -x^4}$$

✓ Solution by Maple
Time used: 0.0 (sec). Leaf size: 30

```
dsolve(x*y(x)*diff(y(x),x)+x^4-y(x)^2 = 0,y(x), singsol=all)
```

$$y(x) = \sqrt{-x^2 + c_1}\, x$$
$$y(x) = -\sqrt{-x^2 + c_1}\, x$$

✓ Solution by Mathematica
Time used: 0.425 (sec). Leaf size: 43

```
DSolve[x y[x] y'[x]+x^4-y[x]^2==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to -\sqrt{-x^4 + c_1 x^2}$$
$$y(x) \to \sqrt{-x^4 + c_1 x^2}$$

18.31 problem 509

Internal problem ID [3761]

Book: Ordinary differential equations and their solutions. By George Moseley Murphy. 1960
Section: Various 18
Problem number: 509.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[[_homogeneous, 'class D'], _Bernoulli]`

$$
xyy' - y^2 = a\,x^3 \cos(x)
$$

✓ Solution by Maple

Time used: 0.015 (sec). Leaf size: 30

```
dsolve(x*y(x)*diff(y(x),x) = a*x^3*cos(x)+y(x)^2,y(x), singsol=all)
```

$$
\begin{aligned}
y(x) &= \sqrt{2a\sin(x) + c_1}\,x \\
y(x) &= -\sqrt{2a\sin(x) + c_1}\,x
\end{aligned}
$$

✓ Solution by Mathematica

Time used: 0.377 (sec). Leaf size: 38

```
DSolve[x y[x] y'[x]==a x^3 Cos[x]+y[x]^2,y[x],x,IncludeSingularSolutions -> True]
```

$$
\begin{aligned}
y(x) &\to -x\sqrt{2a\sin(x) + c_1} \\
y(x) &\to x\sqrt{2a\sin(x) + c_1}
\end{aligned}
$$

18.32 problem 510

Internal problem ID [3762]

Book: Ordinary differential equations and their solutions. By George Moseley Murphy. 1960
Section: Various 18
Problem number: 510.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[[_homogeneous, 'class A'], _rational, [_Abel, '2nd type', 'cl`

$$xyy' + yx - y^2 = x^2$$

✓ Solution by Maple

Time used: 0.032 (sec). Leaf size: 19

```
dsolve(x*y(x)*diff(y(x),x) = x^2-x*y(x)+y(x)^2,y(x), singsol=all)
```

$$y(x) = x\left(1 + \text{LambertW}\left(\frac{e^{-c_1-1}}{x}\right)\right)$$

✓ Solution by Mathematica

Time used: 3.803 (sec). Leaf size: 25

```
DSolve[x y[x] y'[x]==x^2-x y[x]+y[x]^2,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to x\left(1 + W\left(\frac{e^{-1+c_1}}{x}\right)\right)$$
$$y(x) \to x$$

18.33 problem 511

Internal problem ID [3763]

Book: Ordinary differential equations and their solutions. By George Moseley Murphy. 1960
Section: Various 18
Problem number: 511.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[[_homogeneous, ‘class A‘], _rational, [_Abel, ‘2nd type‘, ‘cl`

$$\boxed{xyy' - 2yx - y^2 = -2x^2}$$

✓ Solution by Maple
Time used: 0.046 (sec). Leaf size: 19

```
dsolve(x*y(x)*diff(y(x),x)+2*x^2-2*x*y(x)-y(x)^2 = 0,y(x), singsol=all)
```

$$y(x) = x\left(1 + \text{LambertW}\left(\mathrm{e}^{2c_1-1}x^2\right)\right)$$

✓ Solution by Mathematica
Time used: 3.881 (sec). Leaf size: 25

```
DSolve[x y[x] y'[x]+2 x^2-2 x y[x]-y[x]^2==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to x\left(1 + W\left(e^{-1+c_1}x^2\right)\right)$$
$$y(x) \to x$$

18.34 problem 512

Internal problem ID [3764]

Book: Ordinary differential equations and their solutions. By George Moseley Murphy. 1960
Section: Various 18
Problem number: 512.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_separable]`

$$\boxed{xyy' - by^2 = a}$$

✓ Solution by Maple

Time used: 0.016 (sec). Leaf size: 48

```
dsolve(x*y(x)*diff(y(x),x) = a+b*y(x)^2,y(x), singsol=all)
```

$$y(x) = \frac{\sqrt{-b\left(-x^{2b}c_1b + a\right)}}{b}$$
$$y(x) = -\frac{\sqrt{-b\left(-x^{2b}c_1b + a\right)}}{b}$$

✓ Solution by Mathematica

Time used: 1.6 (sec). Leaf size: 94

```
DSolve[x y[x] y'[x]==a+b y[x]^2,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to -\frac{\sqrt{-a + e^{2b(\log(x)+c_1)}}}{\sqrt{b}}$$
$$y(x) \to \frac{\sqrt{-a + e^{2b(\log(x)+c_1)}}}{\sqrt{b}}$$
$$y(x) \to -\frac{i\sqrt{a}}{\sqrt{b}}$$
$$y(x) \to \frac{i\sqrt{a}}{\sqrt{b}}$$

18.35 problem 513

Internal problem ID [3765]

Book: Ordinary differential equations and their solutions. By George Moseley Murphy. 1960
Section: Various 18
Problem number: 513.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[[_homogeneous, 'class G'], _rational, _Bernoulli]`

$$xyy' - by^2 = a\,x^n$$

✓ Solution by Maple
Time used: 0.015 (sec). Leaf size: 84

```
dsolve(x*y(x)*diff(y(x),x) = a*x^n+b*y(x)^2,y(x), singsol=all)
```

$$y(x) = \frac{\sqrt{-4\left(b-\frac{n}{2}\right)\left(-c_1\left(b-\frac{n}{2}\right)x^{2b}+a\,x^n\right)}}{2b-n}$$
$$y(x) = -\frac{\sqrt{-4\left(b-\frac{n}{2}\right)\left(-c_1\left(b-\frac{n}{2}\right)x^{2b}+a\,x^n\right)}}{2b-n}$$

✓ Solution by Mathematica
Time used: 4.461 (sec). Leaf size: 86

```
DSolve[x y[x] y'[x]==a x^n+b y[x]^2,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to -\frac{\sqrt{-2ax^n + c_1(2b-n)x^{2b}}}{\sqrt{2b-n}}$$
$$y(x) \to \frac{\sqrt{-2ax^n + c_1(2b-n)x^{2b}}}{\sqrt{2b-n}}$$

19 Various 19

19.1 problem 514

Internal problem ID [3766]

Book: Ordinary differential equations and their solutions. By George Moseley Murphy. 1960
Section: Various 19
Problem number: 514.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_separable]`

$$\boxed{xyy' - (x^2+1)(1-y^2) = 0}$$

✓ Solution by Maple
Time used: 0.015 (sec). Leaf size: 44

```
dsolve(x*y(x)*diff(y(x),x) = (x^2+1)*(1-y(x)^2),y(x), singsol=all)
```

$$y(x) = \frac{\sqrt{c_1 e^{-x^2} + x^2}}{x}$$
$$y(x) = -\frac{\sqrt{c_1 e^{-x^2} + x^2}}{x}$$

✓ Solution by Mathematica
Time used: 5.562 (sec). Leaf size: 99

```
DSolve[x y[x] y'[x]==(1+x^2)(1-y[x]^2),y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to -\frac{\sqrt{x^2 + e^{-x^2+2c_1}}}{x}$$
$$y(x) \to \frac{\sqrt{x^2 + e^{-x^2+2c_1}}}{x}$$
$$y(x) \to -1$$
$$y(x) \to 1$$
$$y(x) \to -\frac{\sqrt{x^2}}{x}$$
$$y(x) \to \frac{\sqrt{x^2}}{x}$$

19.2 problem 515

Internal problem ID [3767]

Book: Ordinary differential equations and their solutions. By George Moseley Murphy. 1960
Section: Various 19
Problem number: 515.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[[_homogeneous, ‘class A‘], _dAlembert]`

$$\boxed{xyy' + x^2 \operatorname{arccot}\left(\frac{y}{x}\right) - y^2 = 0}$$

✓ Solution by Maple

Time used: 0.015 (sec). Leaf size: 21

```
dsolve(x*y(x)*diff(y(x),x)+x^2*arccot(y(x)/x)-y(x)^2 = 0,y(x), singsol=all)
```

$$y(x) = \operatorname{RootOf}\left(\int^{\_Z} \frac{\_a}{\operatorname{arccot}(\_a)} d\_a + \ln(x) + c_1\right) x$$

✓ Solution by Mathematica

Time used: 0.596 (sec). Leaf size: 31

```
DSolve[x y[x] y'[x]+x^2 ArcCot[y[x]/x]-y[x]^2==0,y[x],x,IncludeSingularSolutions -> True]
```

$$\text{Solve}\left[\int_1^{\frac{y(x)}{x}} \frac{K[1]}{\cot^{-1}(K[1])} dK[1] = -\log(x) + c_1, y(x)\right]$$

19.3 problem 516

Internal problem ID [3768]

Book: Ordinary differential equations and their solutions. By George Moseley Murphy. 1960
Section: Various 19
Problem number: 516.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[[_homogeneous, ‘class A‘], _dAlembert]`

$$\boxed{xyy' + x^2 e^{-\frac{2y}{x}} - y^2 = 0}$$

✓ Solution by Maple
Time used: 0.016 (sec). Leaf size: 18

```
dsolve(x*y(x)*diff(y(x),x)+x^2*exp(-2*y(x)/x)-y(x)^2 = 0,y(x), singsol=all)
```

$$y(x) = \frac{(\operatorname{LambertW}(-4(\ln(x) + c_1) e^{-1}) + 1) x}{2}$$

✓ Solution by Mathematica
Time used: 60.225 (sec). Leaf size: 25

```
DSolve[x y[x] y'[x]+x^2 Exp[(-2 y[x])/x]-y[x]^2==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to \frac{1}{2} x \left(1 + W\left(\frac{4(-\log(x) + c_1)}{e}\right)\right)$$

19.4 problem 517

Internal problem ID [3769]

Book: Ordinary differential equations and their solutions. By George Moseley Murphy. 1960
Section: Various 19
Problem number: 517.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[[_homogeneous, ‘class G‘], _rational, [_Abel, ‘2nd type‘, ‘cl`

$$\boxed{(1 + yx)\, y' + y^2 = 0}$$

✓ Solution by Maple
Time used: 0.015 (sec). Leaf size: 13

```
dsolve((1+x*y(x))*diff(y(x),x)+y(x)^2 = 0,y(x), singsol=all)
```

$$y(x) = \frac{\text{LambertW}\left(x\, e^{c_1}\right)}{x}$$

✓ Solution by Mathematica
Time used: 1.751 (sec). Leaf size: 21

```
DSolve[(1+x y[x])y'[x]+y[x]^2==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to \frac{W(e^{c_1} x)}{x}$$
$$y(x) \to 0$$

19.5 problem 518

Internal problem ID [3770]

Book: Ordinary differential equations and their solutions. By George Moseley Murphy. 1960
Section: Various 19
Problem number: 518.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_separable]`

$$\boxed{x(y+1)\,y' - (1-x)\,y = 0}$$

✓ Solution by Maple
Time used: 0.016 (sec). Leaf size: 14

```
dsolve(x*(1+y(x))*diff(y(x),x)-(1-x)*y(x) = 0,y(x), singsol=all)
```

$$y(x) = \text{LambertW}\left(\frac{e^{-x}x}{c_1}\right)$$

✓ Solution by Mathematica
Time used: 4.467 (sec). Leaf size: 21

```
DSolve[x(1+y[x])y'[x]-(1-x)y[x]==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to W\left(xe^{-x+c_1}\right)$$
$$y(x) \to 0$$

19.6 problem 519

Internal problem ID [3771]

Book: Ordinary differential equations and their solutions. By George Moseley Murphy. 1960
Section: Various 19
Problem number: 519.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_separable]`

$$\boxed{x(1-y)\,y'+y(x+1)=0}$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 19

```
dsolve(x*(1-y(x))*diff(y(x),x)+(1+x)*y(x) = 0,y(x), singsol=all)
```

$$y(x)=-\operatorname{LambertW}\left(-\frac{\mathrm{e}^{-x}}{c_1 x}\right)$$

✓ Solution by Mathematica

Time used: 3.925 (sec). Leaf size: 28

```
DSolve[x(1-y[x])y'[x]+(1+x)y[x]==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x)\to -W\left(-\frac{e^{-x-c_1}}{x}\right)$$
$$y(x)\to 0$$

19.7 problem 520

Internal problem ID [3772]

Book: Ordinary differential equations and their solutions. By George Moseley Murphy. 1960
Section: Various 19
Problem number: 520.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_separable]`

$$\boxed{x(1-y)\,y'+(1-x)\,y=0}$$

✓ Solution by Maple
Time used: 0.016 (sec). Leaf size: 15

```
dsolve(x*(1-y(x))*diff(y(x),x)+(1-x)*y(x) = 0,y(x), singsol=all)
```

$$y(x) = -\operatorname{LambertW}\left(-\frac{\mathrm{e}^{x} c_{1}}{x}\right)$$

✓ Solution by Mathematica
Time used: 3.957 (sec). Leaf size: 26

```
DSolve[x(1-y[x])y'[x]+(1-x)y[x]==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to -W\left(-\frac{e^{x-c_1}}{x}\right)$$
$$y(x) \to 0$$

19.8 problem 521

Internal problem ID [3773]

Book: Ordinary differential equations and their solutions. By George Moseley Murphy. 1960
Section: Various 19
Problem number: 521.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_quadrature]`

$$\boxed{x(y+2)\,y' = -ax}$$

✓ Solution by Maple

Time used: 0.015 (sec). Leaf size: 39

```
dsolve(x*(2+y(x))*diff(y(x),x)+a*x = 0,y(x), singsol=all)
```

$$y(x) = -2 - \sqrt{4 + (-2x - 2c_1)\,a}$$
$$y(x) = -2 + \sqrt{4 + (-2x - 2c_1)\,a}$$

✓ Solution by Mathematica

Time used: 0.161 (sec). Leaf size: 50

```
DSolve[x(2+y[x])y'[x]+a x==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to -2 - \sqrt{2}\sqrt{-ax + 2 + c_1}$$
$$y(x) \to -2 + \sqrt{2}\sqrt{-ax + 2 + c_1}$$

19.9 problem 522

Internal problem ID [3774]

Book: Ordinary differential equations and their solutions. By George Moseley Murphy. 1960
Section: Various 19
Problem number: 522.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_rational, [_1st_order, '_with_symmetry_[F(x)*G(y),0]'], [_Ab`

$$(2+3x-yx)\,y'+y=0$$

✓ Solution by Maple

Time used: 0.062 (sec). Leaf size: 50

```
dsolve((2+3*x-x*y(x))*diff(y(x),x)+y(x) = 0,y(x), singsol=all)
```

$$\frac{y(x)^{3} c_{1}x-2y(x)^{2} c_{1}-4c_{1}y(x)+e^{y(x)}-4c_{1}}{xy\left(x\right)^{3}-2y\left(x\right)^{2}-4y\left(x\right)-4}=0$$

✓ Solution by Mathematica

Time used: 0.085 (sec). Leaf size: 35

```
DSolve[(2+3 x-x y[x])y'[x]+y[x]==0,y[x],x,IncludeSingularSolutions -> True]
```

$$\text{Solve}\left[x=-\frac{2(-y(x)^2-2y(x)-2)}{y(x)^3}+\frac{c_1e^{y(x)}}{y(x)^3},y(x)\right]$$

19.10 problem 523

Internal problem ID [3775]

Book: Ordinary differential equations and their solutions. By George Moseley Murphy. 1960
Section: Various 19
Problem number: 523.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_rational, [_Abel, '2nd type', 'class B']]`

$$x(y+4)\,y' - 2y - y^2 = 2x$$

✓ Solution by Maple
Time used: 0.0 (sec). Leaf size: 121

```
dsolve(x*(4+y(x))*diff(y(x),x) = 2*x+2*y(x)+y(x)^2,y(x), singsol=all)
```

$$y(x) = \frac{-\sqrt{x+4}\,\sqrt{\frac{(x+4)c_1-4}{x+4}}\,x - 4\sqrt{x}}{-\sqrt{x+4}\,\sqrt{\frac{(x+4)c_1-4}{x+4}} + \sqrt{x}}$$

$$y(x) = \frac{\sqrt{x+4}\,\sqrt{\frac{(x+4)c_1-4}{x+4}}\,x - 4\sqrt{x}}{\sqrt{x+4}\,\sqrt{\frac{(x+4)c_1-4}{x+4}} + \sqrt{x}}$$

✓ Solution by Mathematica
Time used: 1.118 (sec). Leaf size: 89

```
DSolve[x(4+y[x])y'[x]==2 x+2 y[x]+y[x]^2,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to -4 + \frac{1}{\frac{1}{x+4} - \frac{\sqrt{x}}{(x+4)^{3/2}\sqrt{-\frac{4}{x+4}+c_1}}}$$

$$y(x) \to -4 + \frac{1}{\frac{1}{x+4} + \frac{\sqrt{x}}{(x+4)^{3/2}\sqrt{-\frac{4}{x+4}+c_1}}}$$

$$y(x) \to x$$

19.11 problem 524

Internal problem ID [3776]

Book: Ordinary differential equations and their solutions. By George Moseley Murphy. 1960
Section: Various 19
Problem number: 524.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_rational, [_Abel, '2nd type', 'class B']]`

$$\boxed{x(y+a)\,y'+cy=-bx}$$

✗ Solution by Maple

```
dsolve(x*(a+y(x))*diff(y(x),x)+b*x+c*y(x) = 0,y(x), singsol=all)
```

No solution found

✗ Solution by Mathematica
Time used: 0.0 (sec). Leaf size: 0

```
DSolve[x(a+y[x])y'[x]+b x+c y[x]==0,y[x],x,IncludeSingularSolutions -> True]
```

Not solved

19.12 problem 525

Internal problem ID [3777]

Book: Ordinary differential equations and their solutions. By George Moseley Murphy. 1960
Section: Various 19
Problem number: 525.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_separable]`

$$\boxed{x(y+a)\,y'-y(Bx+A)=0}$$

✓ Solution by Maple
Time used: 0.016 (sec). Leaf size: 47

```
dsolve(x*(a+y(x))*diff(y(x),x) = y(x)*(B*x+A),y(x), singsol=all)
```

$$y(x)=x^{\frac{A}{a}}\mathrm{e}^{\frac{Bx-a\,\mathrm{LambertW}\left(\frac{x^{\frac{A}{a}}\mathrm{e}^{\frac{Bx+c_1}{a}}}{a}\right)+c_1}{a}}$$

✓ Solution by Mathematica
Time used: 1.06 (sec). Leaf size: 36

```
DSolve[x(a+y[x])y'[x]==y[x](A+B x),y[x],x,IncludeSingularSolutions -> True]
```

$$y(x)\to aW\left(\frac{x^{\frac{A}{a}}e^{\frac{Bx+c_1}{a}}}{a}\right)$$
$$y(x)\to 0$$

19.13 problem 526

Internal problem ID [3778]

Book: Ordinary differential equations and their solutions. By George Moseley Murphy. 1960
Section: Various 19
Problem number: 526.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[[_homogeneous, ‘class A‘], _rational, [_Abel, ‘2nd type‘, ‘cl`

$$x(y+x)\,y'+y^2=0$$

✓ Solution by Maple

Time used: 0.078 (sec). Leaf size: 45

```
dsolve(x*(x+y(x))*diff(y(x),x)+y(x)^2 = 0,y(x), singsol=all)
```

$$y(x)=\frac{1+\sqrt{c_1x^2+1}}{c_1x}$$
$$y(x)=\frac{1-\sqrt{c_1x^2+1}}{c_1x}$$

✓ Solution by Mathematica

Time used: 2.81 (sec). Leaf size: 80

```
DSolve[x(x+y[x])y'[x]+y[x]^2==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x)\to\frac{e^{2c_1}-\sqrt{e^{2c_1}\left(x^2+e^{2c_1}\right)}}{x}$$
$$y(x)\to\frac{\sqrt{e^{2c_1}\left(x^2+e^{2c_1}\right)}+e^{2c_1}}{x}$$
$$y(x)\to 0$$

19.14 problem 527

Internal problem ID [3779]

Book: Ordinary differential equations and their solutions. By George Moseley Murphy. 1960
Section: Various 19
Problem number: 527.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[[_homogeneous, ‘class A‘], _rational, [_Abel, ‘2nd type‘, ‘cl`

$$\boxed{x(-y+x)\,y'+y^2=0}$$

✓ Solution by Maple
Time used: 0.031 (sec). Leaf size: 17

```
dsolve(x*(x-y(x))*diff(y(x),x)+y(x)^2 = 0,y(x), singsol=all)
```

$$y(x) = -x\,\mathrm{LambertW}\left(-\frac{\mathrm{e}^{-c_1}}{x}\right)$$

✓ Solution by Mathematica
Time used: 2.271 (sec). Leaf size: 25

```
DSolve[x(x-y[x])y'[x]+y[x]^2==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to -xW\left(-\frac{e^{-c_1}}{x}\right)$$
$$y(x) \to 0$$

19.15 problem 528

Internal problem ID [3780]

Book: Ordinary differential equations and their solutions. By George Moseley Murphy. 1960
Section: Various 19
Problem number: 528.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[[_homogeneous, ‘class A‘], _rational, [_Abel, ‘2nd type‘, ‘cl`

$$\boxed{x(y+x)\,y'-y^2=x^2}$$

✓ Solution by Maple
Time used: 0.032 (sec). Leaf size: 22

```
dsolve(x*(x+y(x))*diff(y(x),x) = x^2+y(x)^2,y(x), singsol=all)
```

$$y(x)=x\left(2\,\mathrm{LambertW}\left(\frac{\mathrm{e}^{-\frac{1}{2}-\frac{c_1}{2}}}{2\sqrt{x}}\right)+1\right)$$

✓ Solution by Mathematica
Time used: 7.26 (sec). Leaf size: 35

```
DSolve[x(x+y[x])y'[x]==x^2+y[x]^2,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x)\to x+2xW\left(\frac{e^{\frac{-1+c_1}{2}}}{2\sqrt{x}}\right)$$
$$y(x)\to x$$

19.16 problem 529

Internal problem ID [3781]

Book: Ordinary differential equations and their solutions. By George Moseley Murphy. 1960
Section: Various 19
Problem number: 529.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[[_homogeneous, ‘class A‘], _rational, [_Abel, ‘2nd type‘, ‘cl`

$$x(-y+x)\,y'+3yx-y^2=-2x^2$$

✓ Solution by Maple

Time used: 0.062 (sec). Leaf size: 59

```
dsolve(x*(x-y(x))*diff(y(x),x)+2*x^2+3*x*y(x)-y(x)^2 = 0,y(x), singsol=all)
```

$$y(x)=\frac{c_1x^2-\sqrt{2c_1^2x^4+1}}{c_1x}$$
$$y(x)=\frac{c_1x^2+\sqrt{2c_1^2x^4+1}}{c_1x}$$

✓ Solution by Mathematica

Time used: 0.724 (sec). Leaf size: 99

```
DSolve[x(x-y[x])y'[x]+2 x^2+3 x y[x]-y[x]^2==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x)\to x-\frac{\sqrt{2x^4+e^{2c_1}}}{x}$$
$$y(x)\to x+\frac{\sqrt{2x^4+e^{2c_1}}}{x}$$
$$y(x)\to x-\frac{\sqrt{2}\sqrt{x^4}}{x}$$
$$y(x)\to \frac{\sqrt{2}\sqrt{x^4}}{x}+x$$

19.17 problem 530

Internal problem ID [3782]

Book: Ordinary differential equations and their solutions. By George Moseley Murphy. 1960
Section: Various 19
Problem number: 530.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[[_homogeneous, 'class A'], _dAlembert]`

$$x(y+x)\,y' - y(y+x) + x\sqrt{x^2-y^2} = 0$$

✓ Solution by Maple

Time used: 0.015 (sec). Leaf size: 48

```
dsolve(x*(x+y(x))*diff(y(x),x)-y(x)*(x+y(x))+x*sqrt(x^2-y(x)^2) = 0,y(x), singsol=all)
```

$$\frac{\arctan\left(\frac{y(x)}{\sqrt{x^2-y(x)^2}}\right)x + x\ln(x) - c_1 x - \sqrt{x^2-y(x)^2}}{x} = 0$$

✓ Solution by Mathematica

Time used: 0.348 (sec). Leaf size: 109

```
DSolve[x(x+y[x])y'[x]-y[x](x+y[x])+x Sqrt[x^2-y[x]^2]==0,y[x],x,IncludeSingularSolutions ->
```

$$\text{Solve}\left[\frac{2\sqrt{\frac{y(x)}{x}-1}\text{arctanh}\left(\frac{1}{\sqrt{\frac{\frac{y(x)}{x}-1}{\frac{y(x)}{x}+1}}}\right)+\left(\frac{y(x)}{x}-1\right)\sqrt{\frac{y(x)}{x}+1}}{\sqrt{\frac{\frac{y(x)}{x}-1}{\frac{y(x)}{x}+1}}\sqrt{\frac{y(x)}{x}+1}} = c_1 - i\log(x), y(x)\right]$$

19.18 problem 531

Internal problem ID [3783]

Book: Ordinary differential equations and their solutions. By George Moseley Murphy. 1960
Section: Various 19
Problem number: 531.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_rational, [_Abel, '2nd type', 'class B']]`

$$(a + x(y + x))\, y' - b(y + x)\, y = 0$$

✗ Solution by Maple

```
dsolve((a+x*(x+y(x)))*diff(y(x),x) = b*(x+y(x))*y(x),y(x), singsol=all)
```

No solution found

✗ Solution by Mathematica
Time used: 0.0 (sec). Leaf size: 0

```
DSolve[(a+x*(x+y[x]))*y'[x]==b*(x+y[x])*y[x],y[x],x,IncludeSingularSolutions -> True]
```

Not solved

19.19 problem 532

Internal problem ID [3784]

Book: Ordinary differential equations and their solutions. By George Moseley Murphy. 1960
Section: Various 19
Problem number: 532.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[[_homogeneous, ‘class A‘], _rational, [_Abel, ‘2nd type‘, ‘cl`

$$x(y+2x)\,y' - yx + y^2 = x^2$$

✓ Solution by Maple

Time used: 0.61 (sec). Leaf size: 59

```
dsolve(x*(2*x+y(x))*diff(y(x),x) = x^2+x*y(x)-y(x)^2,y(x), singsol=all)
```

$$y(x) = \frac{x\left(\operatorname{RootOf}\left(3\_Z^{15} + \_Z^9 - 2c_1x^3\right)^9 + c_1x^3\right)}{-\operatorname{RootOf}\left(3\_Z^{15} + \_Z^9 - 2c_1x^3\right)^9 + 2c_1x^3}$$

✓ Solution by Mathematica

Time used: 4.814 (sec). Leaf size: 431

```
DSolve[x(2 x+y[x])y'[x]==x^2+x y[x]-y[x]^2,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to \text{Root}\left[32\#1^5 - 80\#1^4x + 80\#1^3x^2 + \#1^2\left(-40x^3 + \frac{e^{6c_1}}{x^3}\right) + \#1\left(10x^4 + \frac{2e^{6c_1}}{x^2}\right) - x^5 + \frac{e^{6c_1}}{x}\&, 1\right]$$

$$y(x) \to \text{Root}\left[32\#1^5 - 80\#1^4x + 80\#1^3x^2 + \#1^2\left(-40x^3 + \frac{e^{6c_1}}{x^3}\right) + \#1\left(10x^4 + \frac{2e^{6c_1}}{x^2}\right) - x^5 + \frac{e^{6c_1}}{x}\&, 2\right]$$

$$y(x) \to \text{Root}\left[32\#1^5 - 80\#1^4x + 80\#1^3x^2 + \#1^2\left(-40x^3 + \frac{e^{6c_1}}{x^3}\right) + \#1\left(10x^4 + \frac{2e^{6c_1}}{x^2}\right) - x^5 + \frac{e^{6c_1}}{x}\&, 3\right]$$

$$y(x) \to \text{Root}\left[32\#1^5 - 80\#1^4x + 80\#1^3x^2 + \#1^2\left(-40x^3 + \frac{e^{6c_1}}{x^3}\right) + \#1\left(10x^4 + \frac{2e^{6c_1}}{x^2}\right) - x^5 + \frac{e^{6c_1}}{x}\&, 4\right]$$

$$y(x) \to \text{Root}\left[32\#1^5 - 80\#1^4x + 80\#1^3x^2 + \#1^2\left(-40x^3 + \frac{e^{6c_1}}{x^3}\right) + \#1\left(10x^4 + \frac{2e^{6c_1}}{x^2}\right) - x^5 + \frac{e^{6c_1}}{x}\&, 5\right]$$

19.20 problem 533

Internal problem ID [3785]

Book: Ordinary differential equations and their solutions. By George Moseley Murphy. 1960
Section: Various 19
Problem number: 533.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[[_homogeneous, ‘class A‘], _rational, [_Abel, ‘2nd type‘, ‘cl`

$$\boxed{x(4x-y)\,y'-6yx-y^2=-4x^2}$$

✓ Solution by Maple

Time used: 0.078 (sec). Leaf size: 69

```
dsolve(x*(4*x-y(x))*diff(y(x),x)+4*x^2-6*x*y(x)-y(x)^2 = 0,y(x), singsol=all)
```

$$y(x)=\frac{-4c_1^2x^2+\sqrt{-12c_1^2x^2+1}+1}{2x\,c_1^2}$$
$$y(x)=\frac{-4c_1^2x^2-\sqrt{-12c_1^2x^2+1}+1}{2x\,c_1^2}$$

✓ Solution by Mathematica

Time used: 1.469 (sec). Leaf size: 90

```
DSolve[x(4 x -y[x])y'[x]+4 x^2-6 x y[x]-y[x]^2==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x)\to -\frac{4x^2+e^{\frac{c_1}{2}}\sqrt{12x^2+e^{c_1}}+e^{c_1}}{2x}$$
$$y(x)\to -\frac{4x^2-e^{\frac{c_1}{2}}\sqrt{12x^2+e^{c_1}}+e^{c_1}}{2x}$$

19.21 problem 534

Internal problem ID [3786]

Book: Ordinary differential equations and their solutions. By George Moseley Murphy. 1960
Section: Various 19
Problem number: 534.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[[_homogeneous, ‘class G‘], _rational, [_Abel, ‘2nd type‘, ‘cl`

$$x(y + x^3)\, y' - (-y + x^3)\, y = 0$$

✓ Solution by Maple

Time used: 0.594 (sec). Leaf size: 41

```
dsolve(x*(x^3+y(x))*diff(y(x),x) = (x^3-y(x))*y(x),y(x), singsol=all)
```

$$y(x) = \frac{c_1\left(c_1 - \sqrt{x^4 + c_1^2}\right)}{x}$$
$$y(x) = \frac{c_1\left(c_1 + \sqrt{x^4 + c_1^2}\right)}{x}$$

✓ Solution by Mathematica

Time used: 0.778 (sec). Leaf size: 73

```
DSolve[x(x^3+y[x])y'[x]==(x^3-y[x])y[x],y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to \frac{x^4}{-x + \frac{\sqrt{1+c_1x^4}}{\sqrt{\frac{1}{x^2}}}}$$
$$y(x) \to -\frac{x^4}{x + \frac{\sqrt{1+c_1x^4}}{\sqrt{\frac{1}{x^2}}}}$$
$$y(x) \to 0$$

19.22 problem 535

Internal problem ID [3787]

Book: Ordinary differential equations and their solutions. By George Moseley Murphy. 1960
Section: Various 19
Problem number: 535.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[[_homogeneous, ‘class G‘], _rational, [_Abel, ‘2nd type‘, ‘cl`

$$x(2x^3 + y)\, y' - (2x^3 - y)\, y = 0$$

✓ Solution by Maple

Time used: 0.609 (sec). Leaf size: 47

```
dsolve(x*(2*x^3+y(x))*diff(y(x),x) = (2*x^3-y(x))*y(x),y(x), singsol=all)
```

$$y(x) = \frac{c_1\left(\sqrt{4x^4 + c_1^2} + c_1\right)}{2x}$$
$$y(x) = -\frac{c_1\left(-c_1 + \sqrt{4x^4 + c_1^2}\right)}{2x}$$

✓ Solution by Mathematica

Time used: 0.772 (sec). Leaf size: 76

```
DSolve[x(2 x^3+y[x])y'[x]==(2 x^3-y[x])y[x],y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to \frac{2x^4}{-x + \frac{\sqrt{1+4c_1x^4}}{\sqrt{\frac{1}{x^2}}}}$$
$$y(x) \to -\frac{2x^4}{x + \frac{\sqrt{1+4c_1x^4}}{\sqrt{\frac{1}{x^2}}}}$$
$$y(x) \to 0$$

19.23 problem 536

Internal problem ID [3788]

Book: Ordinary differential equations and their solutions. By George Moseley Murphy. 1960
Section: Various 19
Problem number: 536.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[[_homogeneous, ‘class G‘], _rational, [_Abel, ‘2nd type‘, ‘cl`

$$x(2x^3+y)\,y'-6y^2=0$$

✓ Solution by Maple
Time used: 0.672 (sec). Leaf size: 193

```
dsolve(x*(2*x^3+y(x))*diff(y(x),x) = 6*y(x)^2,y(x), singsol=all)
```

$$y(x) = -\frac{x^3\left(-x^3+\sqrt{x^3\,(x^3+8c_1)}-4c_1\right)}{2c_1}$$
$$y(x) = \frac{x^3\left(x^3+\sqrt{x^3\,(x^3+8c_1)}+4c_1\right)}{2c_1}$$
$$y(x) = -\frac{x^3\left(-x^3+\sqrt{x^3\,(x^3+8c_1)}-4c_1\right)}{2c_1}$$
$$y(x) = \frac{x^3\left(x^3+\sqrt{x^3\,(x^3+8c_1)}+4c_1\right)}{2c_1}$$
$$y(x) = -\frac{x^3\left(-x^3+\sqrt{x^3\,(x^3+8c_1)}-4c_1\right)}{2c_1}$$
$$y(x) = \frac{x^3\left(x^3+\sqrt{x^3\,(x^3+8c_1)}+4c_1\right)}{2c_1}$$

✓ Solution by Mathematica

Time used: 1.392 (sec). Leaf size: 123

```
DSolve[x(2 x^3+y[x])y'[x]==6 y[x]^2,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to 2x^3 \left(-1 + \frac{2}{1 - \frac{4x^{3/2}}{\sqrt{16x^3+c_1}}}\right)$$
$$y(x) \to 2x^3 \left(-1 + \frac{2}{1 + \frac{4x^{3/2}}{\sqrt{16x^3+c_1}}}\right)$$
$$y(x) \to 0$$
$$y(x) \to 2x^3$$
$$y(x) \to \frac{2\left((x^3)^{3/2} - x^{9/2}\right)}{x^{3/2} + \sqrt{x^3}}$$

19.24 problem 537

Internal problem ID [3789]

Book: Ordinary differential equations and their solutions. By George Moseley Murphy. 1960
Section: Various 19
Problem number: 537.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_separable]`

$$\boxed{y(1-x)\,y'+x(1-y)=0}$$

✓ Solution by Maple
Time used: 0.0 (sec). Leaf size: 22

```
dsolve(y(x)*(1-x)*diff(y(x),x)+x*(1-y(x)) = 0,y(x), singsol=all)
```

$$y(x)=\operatorname{LambertW}\left(\frac{\mathrm{e}^{-x-1}}{c_1\,(x-1)}\right)+1$$

✓ Solution by Mathematica
Time used: 6.836 (sec). Leaf size: 28

```
DSolve[y[x]*(1-x)*y'[x]+x*(1-y[x])==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x)\to 1+W\left(\frac{e^{-x-1+c_1}}{x-1}\right)$$
$$y(x)\to 1$$

19.25 problem 538

Internal problem ID [3790]

Book: Ordinary differential equations and their solutions. By George Moseley Murphy. 1960
Section: Various 19
Problem number: 538.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_separable]`

$$\boxed{(x+a)(x+b)y'-yx=0}$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 33

```
dsolve((a+x)*(b+x)*diff(y(x),x) = x*y(x),y(x), singsol=all)
```

$$y(x)=c_1(x+b)^{-\frac{b}{a-b}}(x+a)^{\frac{a}{a-b}}$$

✓ Solution by Mathematica

Time used: 0.054 (sec). Leaf size: 37

```
DSolve[(a+x)(b+x)y'[x]==x y[x],y[x],x,IncludeSingularSolutions -> True]
```

$$y(x)\to c_1e^{\frac{a\log(a+x)-b\log(b+x)}{a-b}}$$
$$y(x)\to 0$$

19.26 problem 539

Internal problem ID [3791]

Book: Ordinary differential equations and their solutions. By George Moseley Murphy. 1960
Section: Various 19
Problem number: 539.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_rational, _Bernoulli]`

$$\boxed{2xyy' - y^2 = 2x^3 - 1}$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 29

```
dsolve(2*x*y(x)*diff(y(x),x)+1-2*x^3-y(x)^2 = 0,y(x), singsol=all)
```

$$y(x) = \sqrt{x^3 + c_1 x + 1}$$
$$y(x) = -\sqrt{x^3 + c_1 x + 1}$$

✓ Solution by Mathematica

Time used: 0.332 (sec). Leaf size: 37

```
DSolve[2 x y[x] y'[x]+1-2 x^3-y[x]^2==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to -\sqrt{x^3 + c_1 x + 1}$$
$$y(x) \to \sqrt{x^3 + c_1 x + 1}$$

19.27 problem 540

Internal problem ID [3792]

Book: Ordinary differential equations and their solutions. By George Moseley Murphy. 1960
Section: Various 19
Problem number: 540.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_separable]`

$$\boxed{2xyy' + y^2 = -a}$$

✓ Solution by Maple
Time used: 0.016 (sec). Leaf size: 36

```
dsolve(2*x*y(x)*diff(y(x),x)+a+y(x)^2 = 0,y(x), singsol=all)
```

$$y(x) = \frac{\sqrt{(-ax + c_1)\,x}}{x}$$
$$y(x) = -\frac{\sqrt{(-ax + c_1)\,x}}{x}$$

✓ Solution by Mathematica
Time used: 0.358 (sec). Leaf size: 115

```
DSolve[2 x y[x] y'[x]+a+y[x]^2==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to -\frac{\sqrt{-ax + e^{2c_1}}}{\sqrt{x}}$$
$$y(x) \to \frac{\sqrt{-ax + e^{2c_1}}}{\sqrt{x}}$$
$$y(x) \to -i\sqrt{a}$$
$$y(x) \to i\sqrt{a}$$
$$y(x) \to \frac{a\sqrt{x}}{\sqrt{-ax}}$$
$$y(x) \to \frac{\sqrt{-ax}}{\sqrt{x}}$$

19.28 problem 541

Internal problem ID [3793]

Book: Ordinary differential equations and their solutions. By George Moseley Murphy. 1960
Section: Various 19
Problem number: 541.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[[_homogeneous, ‘class G‘], _rational, _Bernoulli]`

$$2xyy' - y^2 = ax$$

✓ Solution by Maple
Time used: 0.015 (sec). Leaf size: 29

```
dsolve(2*x*y(x)*diff(y(x),x) = a*x+y(x)^2,y(x), singsol=all)
```

$$y(x) = \sqrt{(a \ln (x) + c_1)\, x}$$
$$y(x) = -\sqrt{(a \ln (x) + c_1)\, x}$$

✓ Solution by Mathematica
Time used: 0.242 (sec). Leaf size: 44

```
DSolve[2 x y[x] y'[x]==a x +y[x]^2,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to -\sqrt{x}\sqrt{a \log(x) + c_1}$$
$$y(x) \to \sqrt{x}\sqrt{a \log(x) + c_1}$$

19.29 problem 542

Internal problem ID [3794]

Book: Ordinary differential equations and their solutions. By George Moseley Murphy. 1960
Section: Various 19
Problem number: 542.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[[_homogeneous, 'class A'], _exact, _rational, _Bernoulli]`

$$\boxed{2xyy' + y^2 = -x^2}$$

✓ Solution by Maple

Time used: 0.016 (sec). Leaf size: 47

```
dsolve(2*x*y(x)*diff(y(x),x)+x^2+y(x)^2 = 0,y(x), singsol=all)
```

$$y(x) = -\frac{\sqrt{3}\,\sqrt{-x\,(x^3 - 3c_1)}}{3x}$$
$$y(x) = \frac{\sqrt{3}\,\sqrt{-x\,(x^3 - 3c_1)}}{3x}$$

✓ Solution by Mathematica

Time used: 0.207 (sec). Leaf size: 60

```
DSolve[2 x y[x] y'[x]+x^2+y[x]^2==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to -\frac{\sqrt{-x^3 + 3c_1}}{\sqrt{3}\sqrt{x}}$$
$$y(x) \to \frac{\sqrt{-x^3 + 3c_1}}{\sqrt{3}\sqrt{x}}$$

19.30 problem 543

Internal problem ID [3795]

Book: Ordinary differential equations and their solutions. By George Moseley Murphy. 1960
Section: Various 19
Problem number: 543.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[[_homogeneous, ‘class A‘], _rational, _Bernoulli]`

$$\boxed{2xyy' - y^2 = x^2}$$

✓ Solution by Maple
Time used: 0.015 (sec). Leaf size: 23

```
dsolve(2*x*y(x)*diff(y(x),x) = x^2+y(x)^2,y(x), singsol=all)
```

$$y(x) = \sqrt{(c_1 + x)\,x}$$
$$y(x) = -\sqrt{(c_1 + x)\,x}$$

✓ Solution by Mathematica
Time used: 0.173 (sec). Leaf size: 38

```
DSolve[2 x y[x] y'[x]==x^2+y[x]^2,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to -\sqrt{x}\sqrt{x + c_1}$$
$$y(x) \to \sqrt{x}\sqrt{x + c_1}$$

19.31 problem 544

Internal problem ID [3796]

Book: Ordinary differential equations and their solutions. By George Moseley Murphy. 1960
Section: Various 19
Problem number: 544.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_rational, _Bernoulli]`

$$2xyy' - y^2 = 4x^2(1 + 2x)$$

✓ Solution by Maple

Time used: 0.015 (sec). Leaf size: 37

```
dsolve(2*x*y(x)*diff(y(x),x) = 4*x^2*(1+2*x)+y(x)^2,y(x), singsol=all)
```

$$y(x) = \sqrt{(4x^2 + c_1 + 4x)\,x}$$
$$y(x) = -\sqrt{(4x^2 + c_1 + 4x)\,x}$$

✓ Solution by Mathematica

Time used: 0.202 (sec). Leaf size: 52

```
DSolve[2 x y[x] y'[x]==4 x^2(1+2 x)+y[x]^2,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to -\sqrt{x}\sqrt{4x^2 + 4x + c_1}$$
$$y(x) \to \sqrt{x}\sqrt{4x^2 + 4x + c_1}$$

19.32 problem 545

Internal problem ID [3797]

Book: Ordinary differential equations and their solutions. By George Moseley Murphy. 1960
Section: Various 19
Problem number: 545.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_rational, _Bernoulli]`

$$2xyy' - 6y^2 = -x^2(a\,x^3 + 1)$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 45

```
dsolve(2*x*y(x)*diff(y(x),x)+x^2*(a*x^3+1) = 6*y(x)^2,y(x), singsol=all)
```

$$y(x) = -\frac{\sqrt{4c_1x^4 + 4a\,x^3 + 1}\,x}{2}$$
$$y(x) = \frac{\sqrt{4c_1x^4 + 4a\,x^3 + 1}\,x}{2}$$

✓ Solution by Mathematica

Time used: 0.775 (sec). Leaf size: 59

```
DSolve[2 x y[x] y'[x]+x^2(1+a x^3)==6 y[x]^2,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to -\frac{1}{2}\sqrt{4ax^5 + 4c_1x^6 + x^2}$$
$$y(x) \to \frac{1}{2}\sqrt{4ax^5 + 4c_1x^6 + x^2}$$

20 Various 20

20.1 problem 546

Internal problem ID [3798]

Book: Ordinary differential equations and their solutions. By George Moseley Murphy. 1960
Section: Various 20
Problem number: 546.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_exact, _rational, [_Abel, '2nd type', 'class B']]`

$$(3 - x + 2yx)\, y' - y + y^2 = -3x^2$$

✓ Solution by Maple

Time used: 0.016 (sec). Leaf size: 63

```
dsolve((3-x+2*x*y(x))*diff(y(x),x)+3*x^2-y(x)+y(x)^2 = 0,y(x), singsol=all)
```

$$y(x) = \frac{x - 3 + \sqrt{9 - 4x^4 + x^2 + (-4c_1 - 6)\, x}}{2x}$$
$$y(x) = \frac{x - 3 - \sqrt{9 - 4x^4 + x^2 + (-4c_1 - 6)\, x}}{2x}$$

✓ Solution by Mathematica

Time used: 0.555 (sec). Leaf size: 75

```
DSolve[(3-x+2 x y[x])y'[x]+3 x^2-y[x]+y[x]^2==0 ,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to -\frac{\sqrt{-4x^4 + x^2 - 6x + 4c_1x + 9} - x + 3}{2x}$$
$$y(x) \to \frac{\sqrt{-4x^4 + x^2 + (-6 + 4c_1)x + 9} + x - 3}{2x}$$

20.2 problem 547

Internal problem ID [3799]

Book: Ordinary differential equations and their solutions. By George Moseley Murphy. 1960
Section: Various 20
Problem number: 547.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[[_homogeneous, ‘class A‘], _rational, [_Abel, ‘2nd type‘, ‘cl`

$$\boxed{x(x-2y)\,y'+y^2=0}$$

✓ Solution by Maple
Time used: 0.031 (sec). Leaf size: 47

```
dsolve(x*(x-2*y(x))*diff(y(x),x)+y(x)^2 = 0,y(x), singsol=all)
```

$$y(x)=\frac{c_1x-\sqrt{c_1x\,(c_1x+4)}}{2c_1}$$
$$y(x)=\frac{c_1x+\sqrt{c_1x\,(c_1x+4)}}{2c_1}$$

✓ Solution by Mathematica
Time used: 5.122 (sec). Leaf size: 92

```
DSolve[x(x-2 y[x])y'[x]+y[x]^2==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x)\to\frac{1}{2}\left(x-\sqrt{x\,(x-4e^{c_1})}\right)$$
$$y(x)\to\frac{1}{2}\left(x+\sqrt{x\,(x-4e^{c_1})}\right)$$
$$y(x)\to 0$$
$$y(x)\to\frac{1}{2}\left(x-\sqrt{x^2}\right)$$
$$y(x)\to\frac{1}{2}\left(\sqrt{x^2}+x\right)$$

20.3 problem 548

Internal problem ID [3800]

Book: Ordinary differential equations and their solutions. By George Moseley Murphy. 1960
Section: Various 20
Problem number: 548.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[[_homogeneous, ‘class A‘], _rational, [_Abel, ‘2nd type‘, ‘cl`

$$\boxed{x(x+2y)\,y'+(2x-y)\,y=0}$$

✓ Solution by Maple

Time used: 0.625 (sec). Leaf size: 33

```
dsolve(x*(x+2*y(x))*diff(y(x),x)+(2*x-y(x))*y(x) = 0,y(x), singsol=all)
```

$$y(x)=\frac{\operatorname{RootOf}\left(\_Z^{18}+3\_Z^{3}c_{1}x^{3}-c_{1}x^{3}\right)^{15}}{c_{1}x^{2}}$$

✓ Solution by Mathematica

Time used: 3.255 (sec). Leaf size: 385

```
DSolve[x(x+2 y[x])y'[x]+(2 x-y[x])y[x]==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x)\to \operatorname{Root}\left[\#1^{6}+15\#1^{5}x+90\#1^{4}x^{2}+270\#1^{3}x^{3}+405\#1^{2}x^{4}+243\#1x^{5}-e^{3c_{1}}x^{3}\&,1\right]$$
$$y(x)\to \operatorname{Root}\left[\#1^{6}+15\#1^{5}x+90\#1^{4}x^{2}+270\#1^{3}x^{3}+405\#1^{2}x^{4}+243\#1x^{5}-e^{3c_{1}}x^{3}\&,2\right]$$
$$y(x)\to \operatorname{Root}\left[\#1^{6}+15\#1^{5}x+90\#1^{4}x^{2}+270\#1^{3}x^{3}+405\#1^{2}x^{4}+243\#1x^{5}-e^{3c_{1}}x^{3}\&,3\right]$$
$$y(x)\to \operatorname{Root}\left[\#1^{6}+15\#1^{5}x+90\#1^{4}x^{2}+270\#1^{3}x^{3}+405\#1^{2}x^{4}+243\#1x^{5}-e^{3c_{1}}x^{3}\&,4\right]$$
$$y(x)\to \operatorname{Root}\left[\#1^{6}+15\#1^{5}x+90\#1^{4}x^{2}+270\#1^{3}x^{3}+405\#1^{2}x^{4}+243\#1x^{5}-e^{3c_{1}}x^{3}\&,5\right]$$
$$y(x)\to \operatorname{Root}\left[\#1^{6}+15\#1^{5}x+90\#1^{4}x^{2}+270\#1^{3}x^{3}+405\#1^{2}x^{4}+243\#1x^{5}-e^{3c_{1}}x^{3}\&,6\right]$$

20.4 problem 549

Internal problem ID [3801]

Book: Ordinary differential equations and their solutions. By George Moseley Murphy. 1960
Section: Various 20
Problem number: 549.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[[_homogeneous, ‘class A‘], _exact, _rational, [_Abel, ‘2nd ty`

$$x(x-2y)\,y' + (2x-y)\,y = 0$$

✓ Solution by Maple
Time used: 0.047 (sec). Leaf size: 69

```
dsolve(x*(x-2*y(x))*diff(y(x),x)+(2*x-y(x))*y(x) = 0,y(x), singsol=all)
```

$$y(x) = \frac{c_1^2x^2 - \sqrt{c_1x\,(x^3c_1^3+4)}}{2x\,c_1^2}$$
$$y(x) = \frac{c_1^2x^2 + \sqrt{c_1x\,(x^3c_1^3+4)}}{2x\,c_1^2}$$

✓ Solution by Mathematica
Time used: 0.71 (sec). Leaf size: 114

```
DSolve[x(x-2 y[x])y'[x]+(2 x - y[x])y[x]==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to \frac{1}{2}\left(x - \frac{\sqrt{x^3-4e^{c_1}}}{\sqrt{x}}\right)$$
$$y(x) \to \frac{1}{2}\left(x + \frac{\sqrt{x^3-4e^{c_1}}}{\sqrt{x}}\right)$$
$$y(x) \to \frac{x}{2} - \frac{\sqrt{x^3}}{2\sqrt{x}}$$
$$y(x) \to \frac{x^{3/2}+\sqrt{x^3}}{2\sqrt{x}}$$

20.5 problem 550

Internal problem ID [3802]

Book: Ordinary differential equations and their solutions. By George Moseley Murphy. 1960
Section: Various 20
Problem number: 550.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_rational, [_Abel, '2nd type', 'class B']]`

$$\boxed{x(1 + x - 2y)\, y' + (1 - 2x + y)\, y = 0}$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 391

```
dsolve(x*(1+x-2*y(x))*diff(y(x),x)+(1-2*x+y(x))*y(x) = 0,y(x), singsol=all)
```

$$y(x) = \frac{3\,5^{\frac{1}{3}}\left(x\left(\sqrt{5}\,\sqrt{\frac{80c_1x^2+160c_1x+80c_1-x}{c_1}}-20x-20\right)c_1^2\right)^{\frac{1}{3}}}{40c_1} + \frac{3x5^{\frac{2}{3}}}{40\left(x\left(\sqrt{5}\,\sqrt{\frac{80c_1x^2+160c_1x+80c_1-x}{c_1}}-20x-20\right)c_1^2\right)^{\frac{1}{3}}} - x - 1$$

$$y(x) = \frac{\frac{3\,5^{\frac{1}{3}}\left(-1-i\sqrt{3}\right)\left(-20\left(-\frac{\sqrt{5}\,\sqrt{\frac{80(x+1)^2c_1-x}{c_1}}}{20}+x+1\right)c_1^2x\right)^{\frac{2}{3}}}{80} + \frac{3c_1\left(\frac{80(-x-1)\left(-20\left(-\frac{\sqrt{5}\,\sqrt{\frac{80(x+1)^2c_1-x}{c_1}}}{20}+x+1\right)c_1^2x\right)^{\frac{1}{3}}}{3}+5^{\frac{2}{3}}\left(i\sqrt{3}-1\right)x\right)}{80}}{\left(-20\left(-\frac{\sqrt{5}\,\sqrt{\frac{80(x+1)^2c_1-x}{c_1}}}{20}+x+1\right)c_1^2x\right)^{\frac{1}{3}}c_1}$$

$$y(x) = -\frac{3\left(5^{\frac{1}{3}}\left(1-i\sqrt{3}\right)\left(-20\left(-\frac{\sqrt{5}\,\sqrt{\frac{80(x+1)^2c_1-x}{c_1}}}{20}+x+1\right)c_1^2x\right)^{\frac{2}{3}} + c_1\left(\frac{80(x+1)\left(-20\left(-\frac{\sqrt{5}\,\sqrt{\frac{80(x+1)^2c_1-x}{c_1}}}{20}+x+1\right)c}{3}\right.\right.}{80\left(-20\left(-\frac{\sqrt{5}\,\sqrt{\frac{80(x+1)^2c_1-x}{c_1}}}{20}+x+1\right)c_1^2x\right)^{\frac{1}{3}}c_1}$$

✓ Solution by Mathematica

Time used: 44.02 (sec). Leaf size: 471

```
DSolve[x(1+x-2 y[x])y'[x]+(1-2 x+y[x])y[x]==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to -\frac{\sqrt[3]{2}x}{\sqrt[3]{27c_1{}^2x^2 + \sqrt{(27c_1{}^2x^2 + 27c_1{}^2x)^2 - 108c_1{}^3x^3} + 27c_1{}^2x}} - \frac{\sqrt[3]{27c_1{}^2x^2 + \sqrt{(27c_1{}^2x^2 + 27c_1{}^2x)^2 - 108c_1{}^3x^3} + 27c_1{}^2x}}{3\sqrt[3]{2}c_1} - x - 1$$

$$y(x) \to \frac{\left(1 + i\sqrt{3}\right)x}{2^{2/3}\sqrt[3]{27c_1{}^2x^2 + \sqrt{(27c_1{}^2x^2 + 27c_1{}^2x)^2 - 108c_1{}^3x^3} + 27c_1{}^2x}} + \frac{\left(1 - i\sqrt{3}\right)\sqrt[3]{27c_1{}^2x^2 + \sqrt{(27c_1{}^2x^2 + 27c_1{}^2x)^2 - 108c_1{}^3x^3} + 27c_1{}^2x}}{6\sqrt[3]{2}c_1} - x - 1$$

$$y(x) \to \frac{\left(1 - i\sqrt{3}\right)x}{2^{2/3}\sqrt[3]{27c_1{}^2x^2 + \sqrt{(27c_1{}^2x^2 + 27c_1{}^2x)^2 - 108c_1{}^3x^3} + 27c_1{}^2x}} + \frac{\left(1 + i\sqrt{3}\right)\sqrt[3]{27c_1{}^2x^2 + \sqrt{(27c_1{}^2x^2 + 27c_1{}^2x)^2 - 108c_1{}^3x^3} + 27c_1{}^2x}}{6\sqrt[3]{2}c_1} - x - 1$$

$$y(x) \to \text{Indeterminate}$$

$$y(x) \to -x - 1$$

20.6 problem 551

Internal problem ID [3803]

Book: Ordinary differential equations and their solutions. By George Moseley Murphy. 1960
Section: Various 20
Problem number: 551.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_rational, [_Abel, '2nd type', 'class B']]`

$$\boxed{x(1-x-2y)\,y'+(2x+y+1)\,y=0}$$

✓ Solution by Maple
Time used: 0.016 (sec). Leaf size: 389

```
dsolve(x*(1-x-2*y(x))*diff(y(x),x)+(1+2*x+y(x))*y(x) = 0,y(x), singsol=all)
```

$$y(x)=\frac{3\,5^{\frac{1}{3}}\left(x\left(\sqrt{5}\,\sqrt{\frac{80c_1x^2-160c_1x+80c_1-x}{c_1}}+20x-20\right)c_1^2\right)^{\frac{1}{3}}}{40c_1}+\frac{3x5^{\frac{2}{3}}}{40\left(x\left(\sqrt{5}\,\sqrt{\frac{80c_1x^2-160c_1x+80c_1-x}{c_1}}+20x-20\right)c_1^2\right)^{\frac{1}{3}}}+x-1$$

$$y(x)=\frac{\frac{3\,5^{\frac{1}{3}}\left(-1-i\sqrt{3}\right)\left(x\left(\sqrt{5}\,\sqrt{\frac{80(x-1)^2c_1-x}{c_1}}+20x-20\right)c_1^2\right)^{\frac{2}{3}}}{80}+\frac{3c_1\left(\frac{80(x-1)\left(x\left(\sqrt{5}\,\sqrt{\frac{80(x-1)^2c_1-x}{c_1}}+20x-20\right)c_1^2\right)^{\frac{1}{3}}}{3}+5^{\frac{2}{3}}\left(i\sqrt{3}-1\right)x\right)}{80}}{c_1\left(x\left(\sqrt{5}\,\sqrt{\frac{80(x-1)^2c_1-x}{c_1}}+20x-20\right)c_1^2\right)^{\frac{1}{3}}}$$

$$y(x)=-\frac{3\left(5^{\frac{1}{3}}\left(1-i\sqrt{3}\right)\left(x\left(\sqrt{5}\,\sqrt{\frac{80(x-1)^2c_1-x}{c_1}}+20x-20\right)c_1^2\right)^{\frac{2}{3}}+c_1\left(\frac{80(1-x)\left(x\left(\sqrt{5}\,\sqrt{\frac{80(x-1)^2c_1-x}{c_1}}+20x-20\right)c_1^2\right)}{3}\right.\right.}{80\left(x\left(\sqrt{5}\,\sqrt{\frac{80(x-1)^2c_1-x}{c_1}}+20x-20\right)c_1^2\right)^{\frac{1}{3}}c_1}$$

✓ Solution by Mathematica

Time used: 39.917 (sec). Leaf size: 463

```
DSolve[x(1-x-2 y[x])y'[x]+(1+2 x+y[x])y[x]==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to -\frac{\sqrt[3]{2}x}{\sqrt[3]{-27c_1{}^2x^2+\sqrt{108c_1{}^3x^3+\left(27c_1{}^2x-27c_1{}^2x^2\right){}^2}+27c_1{}^2x}} + \frac{\sqrt[3]{-27c_1{}^2x^2+\sqrt{108c_1{}^3x^3+\left(27c_1{}^2x-27c_1{}^2x^2\right){}^2}+27c_1{}^2x}}{3\sqrt[3]{2}c_1} + x - 1$$

$$y(x) \to \frac{\left(1+i\sqrt{3}\right)x}{2^{2/3}\sqrt[3]{-27c_1{}^2x^2+\sqrt{108c_1{}^3x^3+\left(27c_1{}^2x-27c_1{}^2x^2\right){}^2}+27c_1{}^2x}} - \frac{\left(1-i\sqrt{3}\right)\sqrt[3]{-27c_1{}^2x^2+\sqrt{108c_1{}^3x^3+\left(27c_1{}^2x-27c_1{}^2x^2\right){}^2}+27c_1{}^2x}}{6\sqrt[3]{2}c_1} + x - 1$$

$$y(x) \to \frac{\left(1-i\sqrt{3}\right)x}{2^{2/3}\sqrt[3]{-27c_1{}^2x^2+\sqrt{108c_1{}^3x^3+\left(27c_1{}^2x-27c_1{}^2x^2\right){}^2}+27c_1{}^2x}} - \frac{\left(1+i\sqrt{3}\right)\sqrt[3]{-27c_1{}^2x^2+\sqrt{108c_1{}^3x^3+\left(27c_1{}^2x-27c_1{}^2x^2\right){}^2}+27c_1{}^2x}}{6\sqrt[3]{2}c_1} + x - 1$$

$y(x) \to \text{Indeterminate}$

$y(x) \to x - 1$

20.7 problem 552

Internal problem ID [3804]

Book: Ordinary differential equations and their solutions. By George Moseley Murphy. 1960
Section: Various 20
Problem number: 552.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[[_homogeneous, ‘class G‘], _exact, _rational, [_Abel, ‘2nd ty`

$$2x(2x^2+y)\,y'+(12x^2+y)\,y=0$$

✓ Solution by Maple

Time used: 0.079 (sec). Leaf size: 51

```
dsolve(2*x*(2*x^2+y(x))*diff(y(x),x)+(12*x^2+y(x))*y(x) = 0,y(x), singsol=all)
```

$$y(x)=\frac{-2x^3+\sqrt{4x^6+c_1x}}{x}$$
$$y(x)=\frac{-2x^3-\sqrt{4x^6+c_1x}}{x}$$

✓ Solution by Mathematica

Time used: 0.472 (sec). Leaf size: 58

```
DSolve[2 x(2 x^2+y[x])y'[x]+(12 x^2+y[x])y[x]==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x)\to -\frac{2x^3+\sqrt{x\,(4x^5+c_1)}}{x}$$
$$y(x)\to \frac{-2x^3+\sqrt{x\,(4x^5+c_1)}}{x}$$

20.8 problem 553

Internal problem ID [3805]

Book: Ordinary differential equations and their solutions. By George Moseley Murphy. 1960
Section: Various 20
Problem number: 553.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_exact, _rational, _Bernoulli]`

$$2(x+1)\,yy' + y^2 = 3x^2 - 2x$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 52

```
dsolve(2*(1+x)*y(x)*diff(y(x),x)+2*x-3*x^2+y(x)^2 = 0,y(x), singsol=all)
```

$$y(x) = \frac{\sqrt{(x+1)\,(x^3 - x^2 + c_1)}}{x+1}$$
$$y(x) = -\frac{\sqrt{(x+1)\,(x^3 - x^2 + c_1)}}{x+1}$$

✓ Solution by Mathematica

Time used: 0.363 (sec). Leaf size: 56

```
DSolve[2(1+x)y[x] y'[x]+2 x-3 x^2+y[x]^2==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to -\frac{\sqrt{x^3 - x^2 + c_1}}{\sqrt{x+1}}$$
$$y(x) \to \frac{\sqrt{x^3 - x^2 + c_1}}{\sqrt{x+1}}$$

20.9 problem 554

Internal problem ID [3806]

Book: Ordinary differential equations and their solutions. By George Moseley Murphy. 1960
Section: Various 20
Problem number: 554.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[[_homogeneous, 'class A'], _rational, [_Abel, '2nd type', 'cl`

$$x(2x+3y)\,y'-y^2=0$$

✓ Solution by Maple

Time used: 0.094 (sec). Leaf size: 451

```
dsolve(x*(2*x+3*y(x))*diff(y(x),x) = y(x)^2,y(x), singsol=all)
```

$$y(x)=\frac{\frac{\left(108c_1x-8x^3c_1^3+12\sqrt{3}\,\sqrt{-c_1^2x^2\left(4c_1^2x^2-27\right)}\right)^{\frac{1}{3}}}{2}+\frac{2x^2c_1^2}{\left(108c_1x-8x^3c_1^3+12\sqrt{3}\,\sqrt{-c_1^2x^2\left(4c_1^2x^2-27\right)}\right)^{\frac{1}{3}}}-c_1x}{3c_1}$$

$$y(x)=\frac{4i\sqrt{3}\,c_1^2x^2-i\left(108c_1x-8x^3c_1^3+12\sqrt{3}\,\sqrt{-4\left(c_1^2x^2-\frac{27}{4}\right)c_1^2x^2}\right)^{\frac{2}{3}}\sqrt{3}-4c_1^2x^2-4\left(108c_1x-8x^3c_1^3+12\sqrt{\right.}}{12\left(108c_1x-8x^3c_1^3+12\sqrt{3}\,\sqrt{-4\left(c_1^2\right.}\right.}$$

$$y(x)=\frac{i\left(-4c_1^2x^2+\left(108c_1x-8x^3c_1^3+12\sqrt{3}\,\sqrt{-4\left(c_1^2x^2-\frac{27}{4}\right)c_1^2x^2}\right)^{\frac{2}{3}}\right)\sqrt{3}-\left(2c_1x+\left(108c_1x-8x^3c_1^3+12\sqrt{\right.}\right.}{12\left(108c_1x-8x^3c_1^3+12\sqrt{3}\,\sqrt{-4\left(c_1^2x^2-\frac{27}{4}\right)c_1^2x^2}\right)^{\frac{1}{3}}c_1}$$

✓ Solution by Mathematica

Time used: 60.159 (sec). Leaf size: 413

```
DSolve[x(2 x+3 y[x])y'[x]==y[x]^2,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to \frac{1}{3}\left(\frac{x^2}{\sqrt[3]{-x^3+\frac{3}{2}\sqrt{3}\sqrt{e^{c_1}x^2\left(-4x^2+27e^{c_1}\right)}+\frac{27e^{c_1}x}{2}}}+\sqrt[3]{-x^3+\frac{3}{2}\sqrt{3}\sqrt{e^{c_1}x^2\left(-4x^2+27e^{c_1}\right)}+\frac{27e^{c_1}x}{2}}-x\right)$$

$$y(x) \to \frac{1}{12}\left(-\frac{2\left(1+i\sqrt{3}\right)x^2}{\sqrt[3]{-x^3+\frac{3}{2}\sqrt{3}\sqrt{e^{c_1}x^2\left(-4x^2+27e^{c_1}\right)}+\frac{27e^{c_1}x}{2}}}+i2^{2/3}\left(\sqrt{3}+i\right)\sqrt[3]{-2x^3+3\sqrt{3}\sqrt{e^{c_1}x^2\left(-4x^2+27e^{c_1}\right)}+27e^{c_1}x}-4x\right)$$

$$y(x) \to \frac{1}{12}\left(\frac{2i\left(\sqrt{3}+i\right)x^2}{\sqrt[3]{-x^3+\frac{3}{2}\sqrt{3}\sqrt{e^{c_1}x^2\left(-4x^2+27e^{c_1}\right)}+\frac{27e^{c_1}x}{2}}}-2^{2/3}\left(1+i\sqrt{3}\right)\sqrt[3]{-2x^3+3\sqrt{3}\sqrt{e^{c_1}x^2\left(-4x^2+27e^{c_1}\right)}+27e^{c_1}x}-4x\right)$$

20.10 problem 555

Internal problem ID [3807]

Book: Ordinary differential equations and their solutions. By George Moseley Murphy. 1960
Section: Various 20
Problem number: 555.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[[_homogeneous, ‘class A‘], _rational, [_Abel, ‘2nd type‘, ‘cl`

$$x(2x+3y)\,y'+3(y+x)^2=0$$

✓ Solution by Maple
Time used: 0.063 (sec). Leaf size: 63

```
dsolve(x*(2*x+3*y(x))*diff(y(x),x)+3*(x+y(x))^2 = 0,y(x), singsol=all)
```

$$y(x)=\frac{-4c_1x^2-\sqrt{-2c_1^2x^4+6}}{6c_1x}$$
$$y(x)=\frac{-4c_1x^2+\sqrt{-2c_1^2x^4+6}}{6c_1x}$$

✓ Solution by Mathematica
Time used: 1.82 (sec). Leaf size: 135

```
DSolve[x(2 x+3 y[x])y'[x]+3(x+y[x])^2==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x)\to -\frac{4x^2+\sqrt{-2x^4+6e^{4c_1}}}{6x}$$
$$y(x)\to \frac{-4x^2+\sqrt{-2x^4+6e^{4c_1}}}{6x}$$
$$y(x)\to -\frac{\sqrt{2}\sqrt{-x^4}+4x^2}{6x}$$
$$y(x)\to \frac{\sqrt{2}\sqrt{-x^4}-4x^2}{6x}$$

20.11 problem 556

Internal problem ID [3808]

Book: Ordinary differential equations and their solutions. By George Moseley Murphy. 1960
Section: Various 20
Problem number: 556.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_exact, _rational, [_Abel, '2nd type', 'class B']]`

$$(3 + 6yx + x^2)\, y' + 2yx + 3y^2 = -2x$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 75

```
dsolve((3+6*x*y(x)+x^2)*diff(y(x),x)+2*x+2*x*y(x)+3*y(x)^2 = 0,y(x), singsol=all)
```

$$y(x) = \frac{-x^2 - 3 + \sqrt{x^4 - 12x^3 - 12c_1x + 6x^2 + 9}}{6x}$$
$$y(x) = \frac{-x^2 - 3 - \sqrt{x^4 - 12x^3 - 12c_1x + 6x^2 + 9}}{6x}$$

✓ Solution by Mathematica

Time used: 0.531 (sec). Leaf size: 83

```
DSolve[(3+6 x y[x]+x^2)y'[x]+2 x+2 x y[x]+3 y[x]^2==0,y[x],x,IncludeSingularSolutions -> Tru
```

$$y(x) \to -\frac{x^2 + \sqrt{x^4 - 12x^3 + 6x^2 + 36c_1x + 9} + 3}{6x}$$
$$y(x) \to -\frac{x^2 - \sqrt{x^4 - 12x^3 + 6x^2 + 36c_1x + 9} + 3}{6x}$$

20.12 problem 557

Internal problem ID [3809]

Book: Ordinary differential equations and their solutions. By George Moseley Murphy. 1960
Section: Various 20
Problem number: 557.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_exact, _rational, [_Abel, '2nd type', 'class B']]`

$$3x(x+2y)\,y' + 3y(y+2x) = -x^3$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 72

```
dsolve(3*x*(x+2*y(x))*diff(y(x),x)+x^3+3*y(x)*(2*x+y(x)) = 0,y(x), singsol=all)
```

$$y(x) = \frac{-3x^2 + \sqrt{3}\,\sqrt{-x\,(x^4 - 3x^3 + 4c_1)}}{6x}$$
$$y(x) = \frac{-\sqrt{3}\,\sqrt{-x\,(x^4 - 3x^3 + 4c_1)} - 3x^2}{6x}$$

✓ Solution by Mathematica

Time used: 0.481 (sec). Leaf size: 75

```
DSolve[3 x(x+2 y[x])y'[x]+x^3+3 y[x](2 x+y[x])==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to -\frac{3x^2 + \sqrt{-3x^5 + 9x^4 + 36c_1x}}{6x}$$
$$y(x) \to \frac{-3x^2 + \sqrt{-3x^5 + 9x^4 + 36c_1x}}{6x}$$

20.13 problem 558

Internal problem ID [3810]

Book: Ordinary differential equations and their solutions. By George Moseley Murphy. 1960
Section: Various 20
Problem number: 558.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[[_homogeneous, ‘class A‘], _rational, _Bernoulli]`

$$\boxed{axyy' - y^2 = x^2}$$

✓ Solution by Maple
Time used: 0.015 (sec). Leaf size: 64

```
dsolve(a*x*y(x)*diff(y(x),x) = x^2+y(x)^2,y(x), singsol=all)
```

$$y(x) = \frac{\sqrt{\left(c_1 (a-1) x^{\frac{2}{a}} + x^2\right)(a-1)}}{a-1}$$
$$y(x) = -\frac{\sqrt{\left(c_1 (a-1) x^{\frac{2}{a}} + x^2\right)(a-1)}}{a-1}$$

✓ Solution by Mathematica
Time used: 4.315 (sec). Leaf size: 68

```
DSolve[a x y[x] y'[x]==x^2+y[x]^2,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to -\frac{\sqrt{x^2 + (a-1)c_1 x^{2/a}}}{\sqrt{a-1}}$$
$$y(x) \to \frac{\sqrt{x^2 + (a-1)c_1 x^{2/a}}}{\sqrt{a-1}}$$

20.14 problem 559

Internal problem ID [3811]

Book: Ordinary differential equations and their solutions. By George Moseley Murphy. 1960
Section: Various 20
Problem number: 559.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[[_homogeneous, ‘class A‘], _rational, _Bernoulli]`

$$a x y y' - y^2 = -x^2$$

✓ Solution by Maple
Time used: 0.016 (sec). Leaf size: 68

```
dsolve(a*x*y(x)*diff(y(x),x)+x^2-y(x)^2 = 0,y(x), singsol=all)
```

$$y(x) = \frac{\sqrt{\left(c_1 (a-1) x^{\frac{2}{a}} - x^2\right)(a-1)}}{a-1}$$
$$y(x) = -\frac{\sqrt{\left(c_1 (a-1) x^{\frac{2}{a}} - x^2\right)(a-1)}}{a-1}$$

✓ Solution by Mathematica
Time used: 4.177 (sec). Leaf size: 72

```
DSolve[a x y[x] y'[x]+x^2-y[x]^2==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to -\frac{\sqrt{-x^2 + (a-1)c_1 x^{2/a}}}{\sqrt{a-1}}$$
$$y(x) \to \frac{\sqrt{-x^2 + (a-1)c_1 x^{2/a}}}{\sqrt{a-1}}$$

20.15 problem 560

Internal problem ID [3812]

Book: Ordinary differential equations and their solutions. By George Moseley Murphy. 1960
Section: Various 20
Problem number: 560.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_separable]`

$$\boxed{x(yb+a)\,y'-cy=0}$$

✓ Solution by Maple

Time used: 0.031 (sec). Leaf size: 44

```
dsolve(x*(a+b*y(x))*diff(y(x),x) = c*y(x),y(x), singsol=all)
```

$$y(x)=x^{\frac{c}{a}}\mathrm{e}^{\frac{-a\,\mathrm{LambertW}\left(\frac{b\,x^{\frac{c}{a}}\mathrm{e}^{\frac{cc_1}{a}}}{a}\right)+cc_1}{a}}$$

✓ Solution by Mathematica

Time used: 0.954 (sec). Leaf size: 36

```
DSolve[x(a+b y[x])y'[x]==c y[x],y[x],x,IncludeSingularSolutions -> True]
```

$$y(x)\to\frac{aW\left(\frac{be^{\frac{c_1}{a}}x^{\frac{c}{a}}}{a}\right)}{b}$$
$$y(x)\to 0$$

20.16 problem 561

Internal problem ID [3813]

Book: Ordinary differential equations and their solutions. By George Moseley Murphy. 1960
Section: Various 20
Problem number: 561.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[[_homogeneous, ‘class A‘], _rational, [_Abel, ‘2nd type‘, ‘cl`

$$\boxed{x(x-ya)\,y'-y(y-ax)=0}$$

✓ Solution by Maple
Time used: 0.172 (sec). Leaf size: 60

```
dsolve(x*(x-a*y(x))*diff(y(x),x) = y(x)*(y(x)-a*x),y(x), singsol=all)
```

$$y(x)=x^{-a}\mathrm{e}^{(-a+1)\operatorname{RootOf}\left(x^{a+1}\mathrm{e}^{a\_Z+c_1a+c_1}+x^{a+1}\mathrm{e}^{a\_Z+c_1a-\_Z+c_1}-1\right)-c_1(a+1)}$$

✓ Solution by Mathematica
Time used: 0.159 (sec). Leaf size: 36

```
DSolve[x(x-a y[x])y'[x]==y[x](y[x]-a x),y[x],x,IncludeSingularSolutions -> True]
```

$$\text{Solve}\left[(a-1)\log\left(1-\frac{y(x)}{x}\right)+\log\left(\frac{y(x)}{x}\right)=-(a+1)\log(x)+c_1,y(x)\right]$$

20.17 problem 564

Internal problem ID [3814]

Book: Ordinary differential equations and their solutions. By George Moseley Murphy. 1960
Section: Various 20
Problem number: 564.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_rational, [_1st_order, '_with_symmetry_[F(x)*G(y),0]'], [_Ab`

$$x(x^n + ya)\, y' + (b + cy)\, y^2 = 0$$

✓ Solution by Maple

Time used: 0.266 (sec). Leaf size: 107

```
dsolve(x*(x^n+a*y(x))*diff(y(x),x)+(b+c*y(x))*y(x)^2 = 0,y(x), singsol=all)
```

$$y(x) = \frac{b}{\mathrm{RootOf}\left(-\_Z^{\frac{an}{b}} x^{-n} a^2 bn - \_Z^{\frac{an}{b}} x^{-n} a\, b^2 + c_1 a^2 n^2 + \_Z^{\frac{an}{b}} acn - \_Z^{\frac{an+b}{b}} anb + c_1 abn + \_Z^{\frac{an}{b}} bc\right) b - c}$$

✓ Solution by Mathematica

Time used: 1.357 (sec). Leaf size: 91

```
DSolve[x(x^n+a y[x])y'[x]+(b+c y[x])y[x]^2==0,y[x],x,IncludeSingularSolutions -> True]
```

$$\mathrm{Solve}\left[\frac{y(x)^{-\frac{an+b}{b}}(cy(x) - an)(b + cy(x))^{\frac{an}{b}}}{a^2 n^2 (an + b)} - \frac{x^{-n} e^{-\frac{an(\log(y(x)) - \log(b + cy(x)))}{b}}}{an^2} = c_1, y(x)\right]$$

20.18 problem 565

Internal problem ID [3815]

Book: Ordinary differential equations and their solutions. By George Moseley Murphy. 1960
Section: Various 20
Problem number: 565.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_exact, _rational, [_Abel, '2nd type', 'class B']]`

$$\boxed{(1-x^2y)\,y'-y^2x=-1}$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 51

```
dsolve((1-x^2*y(x))*diff(y(x),x)+1-x*y(x)^2 = 0,y(x), singsol=all)
```

$$y(x)=\frac{1+\sqrt{2c_1x^2+2x^3+1}}{x^2}$$
$$y(x)=\frac{1-\sqrt{2c_1x^2+2x^3+1}}{x^2}$$

✓ Solution by Mathematica

Time used: 0.528 (sec). Leaf size: 57

```
DSolve[(1-x^2 y[x])y'[x]+1-x y[x]^2==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x)\to\frac{1-\sqrt{2x^3+c_1x^2+1}}{x^2}$$
$$y(x)\to\frac{1+\sqrt{2x^3+c_1x^2+1}}{x^2}$$

20.19 problem 566

Internal problem ID [3816]

Book: Ordinary differential equations and their solutions. By George Moseley Murphy. 1960
Section: Various 20
Problem number: 566.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_rational, [_Abel, '2nd type', 'class B']]`

$$(1 - x^2 y)\, y' + y^2 x = 1$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 963

```
dsolve((1-x^2*y(x))*diff(y(x),x)-1+x*y(x)^2 = 0,y(x), singsol=all)
```

$$y(x) = \frac{4^{\frac{2}{3}}\left(c_1(-80+(c_1-80)\,x^6+160x^3)^2\left(\sqrt{5}\,\sqrt{-\frac{(x^3-1)^2}{c_1x^6-80x^6+160x^3-80}}-\frac{1}{4}\right)\right)^{\frac{2}{3}}+((-c_1+80)\,x^7-160x^4+8}{x^2 4^{\frac{2}{3}}\left(c_1\,(-80+(c_1-80)\,x^6+160x^3)^2\left(\sqrt{5}\,\sqrt{-\frac{(x^3-1)^2}{c_1x^6-80x^6+160x^3-80}}-\frac{1}{4}\right)\right)^{\frac{2}{3}}+\left(c_1x^4-4^{\frac{1}{3}}\right.}$$

$$y(x) = \frac{4^{\frac{2}{3}}\left(c_1(-80+(c_1-80)\,x^6+160x^3)^2\left(\sqrt{5}\,\sqrt{-\frac{(x^3-1)^2}{c_1x^6-80x^6+160x^3-80}}-\frac{1}{4}\right)\right)^{\frac{2}{3}}(\sqrt{3}+i)+\left(2i4^{\frac{1}{3}}\left(c_1(-80+\right.\right.}{x^2 4^{\frac{2}{3}}\left(c_1\,(-80+(c_1-80)\,x^6+160x^3)^2\left(\sqrt{5}\,\sqrt{-\frac{(x^3-1)^2}{c_1x^6-80x^6+160x^3-80}}-\frac{1}{4}\right)\right)^{\frac{2}{3}}(\sqrt{3}+i)+\left(2i4^{\frac{1}{3}}\left(c_1\,(-80\right.\right.}$$

$$y(x) = \frac{(i-\sqrt{3})\,4^{\frac{2}{3}}\left(c_1(-80+(c_1-80)\,x^6+160x^3)^2\left(\sqrt{5}\,\sqrt{-\frac{(x^3-1)^2}{c_1x^6-80x^6+160x^3-80}}-\frac{1}{4}\right)\right)^{\frac{2}{3}}+(-80+(c_1-80}{(i-\sqrt{3})\,x^2 4^{\frac{2}{3}}\left(c_1\,(-80+(c_1-80)\,x^6+160x^3)^2\left(\sqrt{5}\,\sqrt{-\frac{(x^3-1)^2}{c_1x^6-80x^6+160x^3-80}}-\frac{1}{4}\right)\right)^{\frac{2}{3}}+(-80+(c_1-8}$$

✓ Solution by Mathematica

Time used: 36.012 (sec). Leaf size: 506

```
DSolve[(1-x^2 y[x])y'[x]-1+x y[x]^2==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to \frac{\sqrt[3]{-(1-6c_1)^2x^3+\sqrt{(-1+6c_1)^3\left(6c_1x^6+(2-12c_1)x^3-1+6c_1\right)}+1+36{c_1}^2-12c_1}}{-1+6c_1} - \frac{x^2}{\sqrt[3]{-(1-6c_1)^2x^3+\sqrt{(-1+6c_1)^3\left(6c_1x^6+(2-12c_1)x^3-1+6c_1\right)}+1+36{c_1}^2-12c_1}} + x$$

$$y(x) \to \frac{i(\sqrt{3}+i)\sqrt[3]{-(1-6c_1)^2x^3+\sqrt{(-1+6c_1)^3\left(6c_1x^6+(2-12c_1)x^3-1+6c_1\right)}+1+36{c_1}^2-12c_1}}{-2+12c_1} + \frac{\left(1+i\sqrt{3}\right)x^2}{2\sqrt[3]{-(1-6c_1)^2x^3+\sqrt{(-1+6c_1)^3\left(6c_1x^6+(2-12c_1)x^3-1+6c_1\right)}+1+36{c_1}^2-12c_1}} + x$$

$$y(x) \to -\frac{i(\sqrt{3}-i)\sqrt[3]{-(1-6c_1)^2x^3+\sqrt{(-1+6c_1)^3\left(6c_1x^6+(2-12c_1)x^3-1+6c_1\right)}+1+36{c_1}^2-12c_1}}{-2+12c_1} + \frac{\left(1-i\sqrt{3}\right)x^2}{2\sqrt[3]{-(1-6c_1)^2x^3+\sqrt{(-1+6c_1)^3\left(6c_1x^6+(2-12c_1)x^3-1+6c_1\right)}+1+36{c_1}^2-12c_1}} + x$$

$$y(x) \to x$$

20.20 problem 567

Internal problem ID [3817]

Book: Ordinary differential equations and their solutions. By George Moseley Murphy. 1960
Section: Various 20
Problem number: 567.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[[_homogeneous, ‘class G‘], _rational, [_Abel, ‘2nd type‘, ‘cl`

$$x(-yx+1)\,y' + (1+yx)\,y = 0$$

✓ Solution by Maple
Time used: 0.032 (sec). Leaf size: 18

```
dsolve(x*(1-x*y(x))*diff(y(x),x)+(1+x*y(x))*y(x) = 0,y(x), singsol=all)
```

$$y(x) = -\frac{1}{\mathrm{LambertW}\left(-\frac{c_1}{x^2}\right)x}$$

✓ Solution by Mathematica
Time used: 6.024 (sec). Leaf size: 35

```
DSolve[x(1-x y[x])y'[x]+(1+x y[x])y[x]==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to -\frac{1}{xW\left(\frac{e^{-1+\frac{9c_1}{2^{2/3}}}}{x^2}\right)}$$

$$y(x) \to 0$$

20.21 problem 568

Internal problem ID [3818]

Book: Ordinary differential equations and their solutions. By George Moseley Murphy. 1960
Section: Various 20
Problem number: 568.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_exact, _rational, [_Abel, '2nd type', 'class B']]`

$$x(2+yx)\,y'+2y+y^2x=2x^3+3$$

✓ Solution by Maple
Time used: 0.0 (sec). Leaf size: 47

```
dsolve(x*(2+x*y(x))*diff(y(x),x) = 3+2*x^3-2*y(x)-x*y(x)^2,y(x), singsol=all)
```

$$y(x)=\frac{-2-\sqrt{x^4-2c_1+6x+4}}{x}$$
$$y(x)=\frac{-2+\sqrt{x^4-2c_1+6x+4}}{x}$$

✓ Solution by Mathematica
Time used: 0.633 (sec). Leaf size: 62

```
DSolve[x(2+x y[x])y'[x]==3+2 x^3-2 y[x]-x y[x]^2,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x)\to -\frac{2x+\sqrt{x^2\left(x^4+6x+4+c_1\right)}}{x^2}$$
$$y(x)\to \frac{-2x+\sqrt{x^2\left(x^4+6x+4+c_1\right)}}{x^2}$$

20.22 problem 569

Internal problem ID [3819]

Book: Ordinary differential equations and their solutions. By George Moseley Murphy. 1960
Section: Various 20
Problem number: 569.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[[_homogeneous, ‘class G‘], _rational, [_Abel, ‘2nd type‘, ‘cl`

$$x(2-yx)\,y'+2y-xy^2(1+yx)=0$$

✓ Solution by Maple
Time used: 0.016 (sec). Leaf size: 59

```
dsolve(x*(2-x*y(x))*diff(y(x),x)+2*y(x)-x*y(x)^2*(1+x*y(x)) = 0,y(x), singsol=all)
```

$$y(x)=\frac{-1+\sqrt{1-4\ln(x)+4c_1}}{2(-\ln(x)+c_1)\,x}$$
$$y(x)=\frac{1+\sqrt{1-4\ln(x)+4c_1}}{2(\ln(x)-c_1)\,x}$$

✓ Solution by Mathematica
Time used: 1.356 (sec). Leaf size: 86

```
DSolve[x(2-x y[x])y'[x]+2 y[x]-x y[x]^2(1+x y[x])==0,y[x],x,IncludeSingularSolutions -> True
```

$$y(x)\to\frac{2}{x+\sqrt{-\frac{1}{x^3}}x^2\sqrt{-x(-4\log(x)+1+4c_1)}}$$
$$y(x)\to\frac{2}{x+\left(-\frac{1}{x^3}\right)^{3/2}x^5\sqrt{-x(-4\log(x)+1+4c_1)}}$$
$$y(x)\to 0$$

20.23 problem 570

Internal problem ID [3820]

Book: Ordinary differential equations and their solutions. By George Moseley Murphy. 1960
Section: Various 20
Problem number: 570.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[[_homogeneous, ‘class G‘], _rational, [_Abel, ‘2nd type‘, ‘cl`

$$x(3 - yx)\, y' - y(yx - 1) = 0$$

✓ Solution by Maple
Time used: 0.328 (sec). Leaf size: 74

```
dsolve(x*(3-x*y(x))*diff(y(x),x) = y(x)*(x*y(x)-1),y(x), singsol=all)
```

$$y(x) = -\frac{3 \operatorname{LambertW}\left(\frac{(-x^2)^{\frac{1}{3}} c_1}{3}\right)}{x}$$
$$y(x) = -\frac{3 \operatorname{LambertW}\left(-\frac{(-x^2)^{\frac{1}{3}} c_1 (1+i\sqrt{3})}{6}\right)}{x}$$
$$y(x) = -\frac{3 \operatorname{LambertW}\left(\frac{(-x^2)^{\frac{1}{3}} c_1 (i\sqrt{3}-1)}{6}\right)}{x}$$

✓ Solution by Mathematica
Time used: 15.394 (sec). Leaf size: 35

```
DSolve[x(3-x y[x])y'[x]==y[x](x y[x]-1),y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to -\frac{3W\left(e^{-1+\frac{9c_1}{2^{2/3}}} x^{2/3}\right)}{x}$$
$$y(x) \to 0$$

20.24 problem 571

Internal problem ID [3821]

Book: Ordinary differential equations and their solutions. By George Moseley Murphy. 1960
Section: Various 20
Problem number: 571.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_separable]`

$$x^2(1-y)\,y' + (1-x)\,y = 0$$

✓ Solution by Maple

Time used: 0.031 (sec). Leaf size: 33

```
dsolve(x^2*(1-y(x))*diff(y(x),x)+(1-x)*y(x) = 0,y(x), singsol=all)
```

$$y(x) = x\,\mathrm{e}^{\frac{-\operatorname{LambertW}\left(-x\,\mathrm{e}^{\frac{c_1 x+1}{x}}\right)x+c_1 x+1}{x}}$$

✓ Solution by Mathematica

Time used: 4.497 (sec). Leaf size: 26

```
DSolve[x^2(1-y[x])y'[x]+(1-x)y[x]==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to -W\left(x\left(-e^{\frac{1}{x}-c_1}\right)\right)$$
$$y(x) \to 0$$

20.25 problem 572

Internal problem ID [3822]

Book: Ordinary differential equations and their solutions. By George Moseley Murphy. 1960
Section: Various 20
Problem number: 572.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_separable]`

$$\boxed{x^2(1-y)\,y' + (x+1)\,y^2 = 0}$$

✓ Solution by Maple
Time used: 0.031 (sec). Leaf size: 35

```
dsolve(x^2*(1-y(x))*diff(y(x),x)+(1+x)*y(x)^2 = 0,y(x), singsol=all)
```

$$y(x) = x\,\mathrm{e}^{\frac{\mathrm{LambertW}\left(-\frac{\mathrm{e}^{\frac{-c_1 x+1}{x}}}{x}\right)x+c_1 x-1}{x}}$$

✓ Solution by Mathematica
Time used: 6.17 (sec). Leaf size: 30

```
DSolve[x^2(1-y[x])y'[x]+(1+x)y[x]^2==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to -\frac{1}{W\left(-\frac{e^{\frac{1}{x}-c_1}}{x}\right)}$$
$$y(x) \to 0$$

20.26 problem 573

Internal problem ID [3823]

Book: Ordinary differential equations and their solutions. By George Moseley Murphy. 1960
Section: Various 20
Problem number: 573.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_separable]`

$$\boxed{(x^2+1)\, yy' + x(1-y^2) = 0}$$

✓ Solution by Maple
Time used: 0.0 (sec). Leaf size: 29

```
dsolve((x^2+1)*y(x)*diff(y(x),x)+x*(1-y(x)^2) = 0,y(x), singsol=all)
```

$$y(x) = \sqrt{c_1x^2 + c_1 + 1}$$
$$y(x) = -\sqrt{c_1x^2 + c_1 + 1}$$

✓ Solution by Mathematica
Time used: 1.033 (sec). Leaf size: 57

```
DSolve[(1+x^2)*y[x]*y'[x]+x*(1-y[x]^2)==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to -\sqrt{1 + e^{2c_1}\,(x^2+1)}$$
$$y(x) \to \sqrt{1 + e^{2c_1}\,(x^2+1)}$$
$$y(x) \to -1$$
$$y(x) \to 1$$

20.27 problem 574

Internal problem ID [3824]

Book: Ordinary differential equations and their solutions. By George Moseley Murphy. 1960
Section: Various 20
Problem number: 574.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_rational, _Bernoulli]`

$$\boxed{(-x^2+1)\, yy' + y^2x = -2x^2}$$

✓ Solution by Maple
Time used: 0.016 (sec). Leaf size: 87

```
dsolve((-x^2+1)*y(x)*diff(y(x),x)+2*x^2+x*y(x)^2 = 0,y(x), singsol=all)
```

$$y(x) = \sqrt{\ln(x-1)\, x^2 - \ln(x+1)\, x^2 + c_1x^2 - \ln(x-1) + \ln(x+1) - c_1 - 2x}$$
$$y(x) = -\sqrt{(x^2-1)\ln(x-1) + c_1x^2 - \ln(x+1)\, x^2 - 2x - c_1 + \ln(x+1)}$$

✓ Solution by Mathematica
Time used: 0.448 (sec). Leaf size: 93

```
DSolve[(1-x^2)y[x] y'[x]+2 x^2+x y[x]^2==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to -\sqrt{(x^2-1)\log(1-x) - (x^2-1)\log(x+1) + c_1x^2 - 2x - c_1}$$
$$y(x) \to \sqrt{(x^2-1)\log(1-x) - (x^2-1)\log(x+1) + c_1x^2 - 2x - c_1}$$

20.28 problem 575

Internal problem ID [3825]

Book: Ordinary differential equations and their solutions. By George Moseley Murphy. 1960
Section: Various 20
Problem number: 575.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[[_homogeneous, ‘class D‘], _rational, _Bernoulli]`

$$\boxed{2y'x^2y + y^2 = x^2(1 + 2x)}$$

✓ Solution by Maple

Time used: 0.016 (sec). Leaf size: 33

```
dsolve(2*x^2*y(x)*diff(y(x),x) = x^2*(1+2*x)-y(x)^2,y(x), singsol=all)
```

$$y(x) = \sqrt{c_1 e^{\frac{1}{x}} + x^2}$$
$$y(x) = -\sqrt{c_1 e^{\frac{1}{x}} + x^2}$$

✓ Solution by Mathematica

Time used: 7.192 (sec). Leaf size: 43

```
DSolve[2 x^2 y[x] y'[x]==x^2(1+2 x)-y[x]^2,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to -\sqrt{x^2 + c_1 e^{\frac{1}{x}}}$$
$$y(x) \to \sqrt{x^2 + c_1 e^{\frac{1}{x}}}$$

20.29 problem 576

Internal problem ID [3826]

Book: Ordinary differential equations and their solutions. By George Moseley Murphy. 1960
Section: Various 20
Problem number: 576.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[[_homogeneous, ‘class G‘], _rational, [_Abel, ‘2nd type‘, ‘cl`

$$x(1-2yx)\,y'+(2yx+1)\,y=0$$

✓ Solution by Maple

Time used: 0.047 (sec). Leaf size: 18

```
dsolve(x*(1-2*x*y(x))*diff(y(x),x)+(1+2*x*y(x))*y(x) = 0,y(x), singsol=all)
```

$$y(x)=-\frac{1}{2\,\mathrm{LambertW}\left(-\frac{c_1}{2x^2}\right)x}$$

✓ Solution by Mathematica

Time used: 5.96 (sec). Leaf size: 37

```
DSolve[x(1-2 x y[x])y'[x]+(1+2 x y[x])y[x]==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x)\to -\frac{1}{2xW\left(\frac{e^{-1+\frac{9c_1}{2^{2/3}}}}{x^2}\right)}$$

$$y(x)\to 0$$

21 Various 21

21.1 problem 577

Internal problem ID [3827]

Book: Ordinary differential equations and their solutions. By George Moseley Murphy. 1960
Section: Various 21
Problem number: 577.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[[_homogeneous, ‘class G‘], _rational, [_Abel, ‘2nd type‘, ‘cl`

$$x(2yx+1)\,y'+(2+3yx)\,y=0$$

✓ Solution by Maple

Time used: 0.031 (sec). Leaf size: 45

```
dsolve(x*(1+2*x*y(x))*diff(y(x),x)+(2+3*x*y(x))*y(x) = 0,y(x), singsol=all)
```

$$y(x)=\frac{-x+\sqrt{x\,(x+4c_1)}}{2x^2}$$
$$y(x)=\frac{-x-\sqrt{x\,(x+4c_1)}}{2x^2}$$

✓ Solution by Mathematica

Time used: 0.555 (sec). Leaf size: 69

```
DSolve[x(1+2 x y[x])y'[x]+(2+3 x y[x])y[x]==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x)\to -\frac{x^{3/2}+\sqrt{x^2(x+4c_1)}}{2x^{5/2}}$$
$$y(x)\to \frac{-x^{3/2}+\sqrt{x^2(x+4c_1)}}{2x^{5/2}}$$

21.2 problem 578

Internal problem ID [3828]

Book: Ordinary differential equations and their solutions. By George Moseley Murphy. 1960
Section: Various 21
Problem number: 578.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[[_homogeneous, ‘class G‘], _rational, [_Abel, ‘2nd type‘, ‘cl`

$$x(2yx+1)\,y' + \left(1+2yx-y^2x^2\right)y = 0$$

✓ Solution by Maple

Time used: 0.016 (sec). Leaf size: 59

```
dsolve(x*(1+2*x*y(x))*diff(y(x),x)+(1+2*x*y(x)-x^2*y(x)^2)*y(x) = 0,y(x), singsol=all)
```

$$y(x) = \frac{-2+\sqrt{4-2\ln(x)+2c_1}}{2(\ln(x)-c_1)\,x}$$
$$y(x) = \frac{2+\sqrt{4-2\ln(x)+2c_1}}{2(-\ln(x)+c_1)\,x}$$

✓ Solution by Mathematica

Time used: 0.786 (sec). Leaf size: 79

```
DSolve[x(1+2 x y[x])y'[x]+(1+2 x y[x]-x^2 y[x]^2)y[x]==0,y[x],x,IncludeSingularSolutions ->
```

$$y(x) \to \frac{x}{-2x^2+\frac{\sqrt{x(-2\log(x)+4+c_1)}}{\sqrt{\frac{1}{x^3}}}}$$
$$y(x) \to -\frac{x}{2x^2+\frac{\sqrt{x(-2\log(x)+4+c_1)}}{\sqrt{\frac{1}{x^3}}}}$$
$$y(x) \to 0$$

21.3 problem 579

Internal problem ID [3829]

Book: Ordinary differential equations and their solutions. By George Moseley Murphy. 1960
Section: Various 21
Problem number: 579.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[[_homogeneous, ‘class A‘], _rational, [_Abel, ‘2nd type‘, ‘cl`

$$\boxed{x^2(x-2y)\,y'+4y^2x-y^3=2x^3}$$

✓ Solution by Maple
Time used: 0.594 (sec). Leaf size: 65

```
dsolve(x^2*(x-2*y(x))*diff(y(x),x) = 2*x^3-4*x*y(x)^2+y(x)^3,y(x), singsol=all)
```

$$y(x)=\frac{x\left(2c_1x^2-\sqrt{3c_1x^2+1}\right)}{c_1x^2-1}$$
$$y(x)=\frac{x\left(2c_1x^2+\sqrt{3c_1x^2+1}\right)}{c_1x^2-1}$$

✓ Solution by Mathematica
Time used: 14.225 (sec). Leaf size: 132

```
DSolve[x^2(x-2 y[x])y'[x]==2 x^3-4 x y[x]^2+y[x]^3,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x)\to\frac{2x^3-\sqrt{e^{2c_1}x^2\left(-3x^2+e^{2c_1}\right)}}{x^2+e^{2c_1}}$$
$$y(x)\to\frac{2x^3+\sqrt{e^{2c_1}x^2\left(-3x^2+e^{2c_1}\right)}}{x^2+e^{2c_1}}$$
$$y(x)\to 2x$$
$$y(x)\to-\sqrt{x^2}$$
$$y(x)\to\sqrt{x^2}$$

21.4 problem 580

Internal problem ID [3830]

Book: Ordinary differential equations and their solutions. By George Moseley Murphy. 1960
Section: Various 21
Problem number: 580.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_separable]`

$$\boxed{2(x+1)\,xyy'-y^2=1}$$

✓ Solution by Maple
Time used: 0.0 (sec). Leaf size: 42

```
dsolve(2*(1+x)*x*y(x)*diff(y(x),x) = 1+y(x)^2,y(x), singsol=all)
```

$$y(x)=\frac{\sqrt{(x+1)\,(c_1x-1)}}{x+1}$$
$$y(x)=-\frac{\sqrt{(x+1)\,(c_1x-1)}}{x+1}$$

✓ Solution by Mathematica

Time used: 0.779 (sec). Leaf size: 115

```
DSolve[2(1+x)x y[x] y'[x]==1+y[x]^2,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to -\frac{\sqrt{-1+(-1+e^{2c_1})\,x}}{\sqrt{x+1}}$$
$$y(x) \to \frac{\sqrt{-1+(-1+e^{2c_1})\,x}}{\sqrt{x+1}}$$
$$y(x) \to -i$$
$$y(x) \to i$$
$$y(x) \to -\frac{\sqrt{-x-1}}{\sqrt{x+1}}$$
$$y(x) \to \frac{\sqrt{-x-1}}{\sqrt{x+1}}$$

21.5 problem 581

Internal problem ID [3831]

Book: Ordinary differential equations and their solutions. By George Moseley Murphy. 1960
Section: Various 21
Problem number: 581.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[[_homogeneous, ‘class G‘], _rational, _Bernoulli]`

$$\boxed{3y'x^2y + 2y^2x = -1}$$

✓ Solution by Maple
Time used: 0.0 (sec). Leaf size: 42

```
dsolve(3*x^2*y(x)*diff(y(x),x)+1+2*x*y(x)^2 = 0,y(x), singsol=all)
```

$$y(x) = \frac{\sqrt{x^{\frac{10}{3}}\left(-2x^{\frac{1}{3}} + c_1\right)}}{x^{\frac{7}{3}}}$$
$$y(x) = -\frac{\sqrt{x^{\frac{10}{3}}\left(-2x^{\frac{1}{3}} + c_1\right)}}{x^{\frac{7}{3}}}$$

✓ Solution by Mathematica
Time used: 3.776 (sec). Leaf size: 47

```
DSolve[3 x^2 y[x] y'[x]+1+2 x y[x]^2==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to -\sqrt{-\frac{2}{x} + \frac{c_1}{x^{4/3}}}$$
$$y(x) \to \sqrt{-\frac{2}{x} + \frac{c_1}{x^{4/3}}}$$

21.6 problem 582

Internal problem ID [3832]

Book: Ordinary differential equations and their solutions. By George Moseley Murphy. 1960
Section: Various 21
Problem number: 582.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[[_homogeneous, ‘class A‘], _rational, [_Abel, ‘2nd type‘, ‘cl`

$$x^2(4x-3y)\,y'-\left(6x^2-3yx+2y^2\right)y=0$$

✓ Solution by Maple

Time used: 0.109 (sec). Leaf size: 44

```
dsolve(x^2*(4*x-3*y(x))*diff(y(x),x) = (6*x^2-3*x*y(x)+2*y(x)^2)*y(x),y(x), singsol=all)
```

$$2\ln\left(\frac{y(x)}{x}\right)-\ln\left(\frac{x^2+y(x)^2}{x^2}\right)-\frac{3\arctan\left(\frac{y(x)}{x}\right)}{2}-\ln(x)-c_1=0$$

✓ Solution by Mathematica

Time used: 0.119 (sec). Leaf size: 43

```
DSolve[x^2(4 x-3 y[x])y'[x]==(6 x^2-3 x y[x]+2 y[x]^2)y[x],y[x],x,IncludeSingularSolutions -
```

$$\text{Solve}\left[3\arctan\left(\frac{y(x)}{x}\right)+2\log\left(\frac{y(x)^2}{x^2}+1\right)-4\log\left(\frac{y(x)}{x}\right)=-2\log(x)+c_1,y(x)\right]$$

21.7 problem 583

Internal problem ID [3833]

Book: Ordinary differential equations and their solutions. By George Moseley Murphy. 1960
Section: Various 21
Problem number: 583.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[[_homogeneous, ‘class G‘], _rational, [_Abel, ‘2nd type‘, ‘cl`

$$\boxed{(1 - yx^3)\, y' - y^2x^2 = 0}$$

✓ Solution by Maple

Time used: 0.594 (sec). Leaf size: 685

```
dsolve((1-x^3*y(x))*diff(y(x),x) = x^2*y(x)^2,y(x), singsol=all)
```

$$y(x) = \frac{3 + \frac{\left(\left(x^3+\sqrt{c_1^6+x^6}\right)^{\frac{2}{3}} - c_1^2\right)^2}{c_1^2\left(x^3+\sqrt{c_1^6+x^6}\right)^{\frac{2}{3}}}}{2x^3}$$

$$y(x) = \frac{\left(x^3+\sqrt{c_1^6+x^6}\right)^{\frac{4}{3}} + c_1^2\left(x^3+\sqrt{c_1^6+x^6}\right)^{\frac{2}{3}} + c_1^4}{2c_1^2\left(x^3+\sqrt{c_1^6+x^6}\right)^{\frac{2}{3}} x^3}$$

$$y(x) = \frac{\left(x^3+\sqrt{c_1^6+x^6}\right)^{\frac{4}{3}} + c_1^2\left(x^3+\sqrt{c_1^6+x^6}\right)^{\frac{2}{3}} + c_1^4}{2c_1^2\left(x^3+\sqrt{c_1^6+x^6}\right)^{\frac{2}{3}} x^3}$$

$$y(x) = \frac{2c_1^2\left(x^3+\sqrt{c_1^6+x^6}\right)^{\frac{2}{3}} + \left(x^3+\sqrt{c_1^6+x^6}\right)^{\frac{4}{3}}\left(i\sqrt{3}-1\right) - c_1^4\left(1+i\sqrt{3}\right)}{4\left(x^3+\sqrt{c_1^6+x^6}\right)^{\frac{2}{3}} c_1^2 x^3}$$

$$y(x) = -\frac{-2c_1^2\left(x^3+\sqrt{c_1^6+x^6}\right)^{\frac{2}{3}} + \left(1+i\sqrt{3}\right)\left(x^3+\sqrt{c_1^6+x^6}\right)^{\frac{4}{3}} - c_1^4\left(i\sqrt{3}-1\right)}{4\left(x^3+\sqrt{c_1^6+x^6}\right)^{\frac{2}{3}} c_1^2 x^3}$$

$$y(x) = \frac{2c_1^2\left(x^3+\sqrt{c_1^6+x^6}\right)^{\frac{2}{3}} + \left(x^3+\sqrt{c_1^6+x^6}\right)^{\frac{4}{3}}\left(i\sqrt{3}-1\right) - c_1^4\left(1+i\sqrt{3}\right)}{4\left(x^3+\sqrt{c_1^6+x^6}\right)^{\frac{2}{3}} c_1^2 x^3}$$

$$y(x) = -\frac{-2c_1^2\left(x^3+\sqrt{c_1^6+x^6}\right)^{\frac{2}{3}} + \left(1+i\sqrt{3}\right)\left(x^3+\sqrt{c_1^6+x^6}\right)^{\frac{4}{3}} - c_1^4\left(i\sqrt{3}-1\right)}{4\left(x^3+\sqrt{c_1^6+x^6}\right)^{\frac{2}{3}} c_1^2 x^3}$$

$$y(x) = \frac{2c_1^2\left(x^3+\sqrt{c_1^6+x^6}\right)^{\frac{2}{3}} + \left(x^3+\sqrt{c_1^6+x^6}\right)^{\frac{4}{3}}\left(i\sqrt{3}-1\right) - c_1^4\left(1+i\sqrt{3}\right)}{4\left(x^3+\sqrt{c_1^6+x^6}\right)^{\frac{2}{3}} c_1^2 x^3}$$

$$y(x) = -\frac{-2c_1^2\left(x^3+\sqrt{c_1^6+x^6}\right)^{\frac{2}{3}} + \left(1+i\sqrt{3}\right)\left(x^3+\sqrt{c_1^6+x^6}\right)^{\frac{4}{3}} - c_1^4\left(i\sqrt{3}-1\right)}{4\left(x^3+\sqrt{c_1^6+x^6}\right)^{\frac{2}{3}} c_1^2 x^3}$$

✓ Solution by Mathematica

Time used: 50.23 (sec). Leaf size: 331

```
DSolve[(1-x^3 y[x])y'[x]==x^2 y[x]^2,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to \frac{\sqrt[3]{12c_1x^6 + 2\sqrt{6}\sqrt{c_1x^6\left(1+6c_1x^6\right)}+1} + \frac{1}{\sqrt[3]{12c_1x^6 + 2\sqrt{6}\sqrt{c_1x^6\left(1+6c_1x^6\right)}+1}} + 1}{2x^3}$$

$$y(x) \to \frac{2i\left(\sqrt{3}+i\right)\sqrt[3]{12c_1x^6 + 2\sqrt{6}\sqrt{c_1x^6\left(1+6c_1x^6\right)}+1} - \frac{2\left(1+i\sqrt{3}\right)}{\sqrt[3]{12c_1x^6 + 2\sqrt{6}\sqrt{c_1x^6\left(1+6c_1x^6\right)}+1}} + 4}{8x^3}$$

$$y(x) \to \frac{-2\left(1+i\sqrt{3}\right)\sqrt[3]{12c_1x^6 + 2\sqrt{6}\sqrt{c_1x^6\left(1+6c_1x^6\right)}+1} + \frac{2i\left(\sqrt{3}+i\right)}{\sqrt[3]{12c_1x^6 + 2\sqrt{6}\sqrt{c_1x^6\left(1+6c_1x^6\right)}+1}} + 4}{8x^3}$$

$$y(x) \to 0$$

21.8 problem 584

Internal problem ID [3834]

Book: Ordinary differential equations and their solutions. By George Moseley Murphy. 1960
Section: Various 21
Problem number: 584.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[[_homogeneous, ‘class G‘], _exact, _rational, _Bernoulli]`

$$\boxed{2yx^3y' + 3y^2x^2 = -a}$$

✓ Solution by Maple
Time used: 0.016 (sec). Leaf size: 36

```
dsolve(2*x^3*y(x)*diff(y(x),x)+a+3*x^2*y(x)^2 = 0,y(x), singsol=all)
```

$$y(x) = \frac{\sqrt{(-ax + c_1)\,x}}{x^2}$$
$$y(x) = -\frac{\sqrt{(-ax + c_1)\,x}}{x^2}$$

✓ Solution by Mathematica
Time used: 0.311 (sec). Leaf size: 44

```
DSolve[2 x^3 y[x] y'[x]+a+3 x^2 y[x]^2 ==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to -\frac{\sqrt{-ax + c_1}}{x^{3/2}}$$
$$y(x) \to \frac{\sqrt{-ax + c_1}}{x^{3/2}}$$

21.9 problem 585

Internal problem ID [3835]

Book: Ordinary differential equations and their solutions. By George Moseley Murphy. 1960
Section: Various 21
Problem number: 585.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_exact, _rational, [_Abel, ‘2nd type‘, ‘class B‘]]`

$$x(3-2x^2y)\,y'+3y-3y^2x^2=4x$$

✓ Solution by Maple

Time used: 0.015 (sec). Leaf size: 49

```
dsolve(x*(3-2*x^2*y(x))*diff(y(x),x) = 4*x-3*y(x)+3*x^2*y(x)^2,y(x), singsol=all)
```

$$y(x)=\frac{3+\sqrt{-8x^3+4c_1x+9}}{2x^2}$$
$$y(x)=\frac{3-\sqrt{-8x^3+4c_1x+9}}{2x^2}$$

✓ Solution by Mathematica

Time used: 0.783 (sec). Leaf size: 71

```
DSolve[x(3-2 x^2 y[x])y'[x]==4 x-3 y[x]+3 x^2 y[x]^2,y[x],x,IncludeSingularSolutions -> True
```

$$y(x)\to -\frac{-3x+\sqrt{x^2(-8x^3+4c_1x+9)}}{2x^3}$$
$$y(x)\to \frac{3x+\sqrt{x^2(-8x^3+4c_1x+9)}}{2x^3}$$

21.10 problem 586

Internal problem ID [3836]

Book: Ordinary differential equations and their solutions. By George Moseley Murphy. 1960
Section: Various 21
Problem number: 586.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[[_homogeneous, ‘class G‘], _rational, [_Abel, ‘2nd type‘, ‘cl`

$$x\left(3+2x^2y\right)y'+\left(4+3x^2y\right)y=0$$

✓ Solution by Maple
Time used: 1.562 (sec). Leaf size: 39

```
dsolve(x*(3+2*x^2*y(x))*diff(y(x),x)+(4+3*x^2*y(x))*y(x) = 0,y(x), singsol=all)
```

$$y(x)=\frac{\operatorname{RootOf}\left(x^2\_Z^8-2\_Z^2c_1-c_1\right)^6x^2-2c_1}{x^2c_1}$$

✓ Solution by Mathematica

Time used: 60.296 (sec). Leaf size: 1769

```
DSolve[x(3+2 x^2 y[x])y'[x]+(4+3 x^2 y[x])y[x]==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to -\frac{1}{2x^2} + \frac{\sqrt{\frac{3}{x^4} - \frac{2\ 6^{2/3}e^{-2c_1}}{\sqrt[3]{e^{-6c_1}\left(\sqrt{48e^{6c_1}x^{18}+81e^{8c_1}x^{16}}-9e^{4c_1}x^8\right)}} + \frac{\sqrt[3]{6}\sqrt[3]{e^{-6c_1}\left(\sqrt{48e^{6c_1}x^{18}+81e^{8c_1}x^{16}}-9e^{4c_1}x^{8}\right)}}{x^6}}}{2\sqrt{3}}$$

$$-\frac{1}{2}\sqrt{\frac{2}{x^4} + \frac{2\ 2^{2/3}e^{-2c_1}}{\sqrt[3]{3}\sqrt[3]{e^{-6c_1}\left(\sqrt{48e^{6c_1}x^{18}+81e^{8c_1}x^{16}}-9e^{4c_1}x^8\right)}} - \frac{\sqrt[3]{2}\sqrt[3]{e^{-6c_1}\left(\sqrt{48e^{6c_1}x^{18}+81e^{8c_1}x^{16}}-9e\right)}}{3^{2/3}x^6}}$$

$$y(x) \to -\frac{1}{2x^2} + \frac{\sqrt{\frac{3}{x^4} - \frac{2\ 6^{2/3}e^{-2c_1}}{\sqrt[3]{e^{-6c_1}\left(\sqrt{48e^{6c_1}x^{18}+81e^{8c_1}x^{16}}-9e^{4c_1}x^8\right)}} + \frac{\sqrt[3]{6}\sqrt[3]{e^{-6c_1}\left(\sqrt{48e^{6c_1}x^{18}+81e^{8c_1}x^{16}}-9e^{4c_1}x^{8}\right)}}{x^6}}}{2\sqrt{3}}$$

$$+\frac{1}{2}\sqrt{\frac{2}{x^4} + \frac{2\ 2^{2/3}e^{-2c_1}}{\sqrt[3]{3}\sqrt[3]{e^{-6c_1}\left(\sqrt{48e^{6c_1}x^{18}+81e^{8c_1}x^{16}}-9e^{4c_1}x^8\right)}} - \frac{\sqrt[3]{2}\sqrt[3]{e^{-6c_1}\left(\sqrt{48e^{6c_1}x^{18}+81e^{8c_1}x^{16}}-9e\right)}}{3^{2/3}x^6}}$$

$$y(x) \to -\frac{1}{2x^2} - \frac{\sqrt{\frac{3}{x^4} - \frac{2\ 6^{2/3}e^{-2c_1}}{\sqrt[3]{e^{-6c_1}\left(\sqrt{48e^{6c_1}x^{18}+81e^{8c_1}x^{16}}-9e^{4c_1}x^8\right)}} + \frac{\sqrt[3]{6}\sqrt[3]{e^{-6c_1}\left(\sqrt{48e^{6c_1}x^{18}+81e^{8c_1}x^{16}}-9e^{4c_1}x^{8}\right)}}{x^6}}}{2\sqrt{3}}$$

$$-\frac{1}{2}\sqrt{\frac{2}{x^4} + \frac{2\ 2^{2/3}e^{-2c_1}}{\sqrt[3]{3}\sqrt[3]{e^{-6c_1}\left(\sqrt{48e^{6c_1}x^{18}+81e^{8c_1}x^{16}}-9e^{4c_1}x^8\right)}} - \frac{\sqrt[3]{2}\sqrt[3]{e^{-6c_1}\left(\sqrt{48e^{6c_1}x^{18}+81e^{8c_1}x^{16}}-9e\right)}}{3^{2/3}x^6}}$$

21.11 problem 587

Internal problem ID [3837]

Book: Ordinary differential equations and their solutions. By George Moseley Murphy. 1960
Section: Various 21
Problem number: 587.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[[_homogeneous, ‘class A‘], _rational, _dAlembert]`

$$\boxed{8yx^3y' - 6y^2x^2 - y^4 = -3x^4}$$

✓ Solution by Maple
Time used: 0.015 (sec). Leaf size: 54

```
dsolve(8*x^3*y(x)*diff(y(x),x)+3*x^4-6*x^2*y(x)^2-y(x)^4 = 0,y(x), singsol=all)
```

$$y(x) = \frac{x\sqrt{-(c_1x-1)(c_1x+3)}}{c_1x-1}$$
$$y(x) = -\frac{x\sqrt{-(c_1x-1)(c_1x+3)}}{c_1x-1}$$

✓ Solution by Mathematica

Time used: 5.038 (sec). Leaf size: 160

```
DSolve[8 x^3 y[x] y'[x]+3 x^4 -6 x^2 y[x]^2 -y[x]^4==0,y[x],x,IncludeSingularSolutions -> Tr
```

$$y(x) \to -\frac{\sqrt{-x^2 (3 + e^{8c_1} x)}}{\sqrt{-1 + e^{8c_1} x}}$$
$$y(x) \to \frac{\sqrt{-x^2 (3 + e^{8c_1} x)}}{\sqrt{-1 + e^{8c_1} x}}$$
$$y(x) \to -i\sqrt{3}\sqrt{-x^2}$$
$$y(x) \to i\sqrt{3}\sqrt{-x^2}$$
$$y(x) \to \frac{x^{5/2}}{\sqrt{-x^3}}$$
$$y(x) \to \frac{\sqrt{-x^3}}{\sqrt{x}}$$

21.12 problem 588

Internal problem ID [3838]

Book: Ordinary differential equations and their solutions. By George Moseley Murphy. 1960
Section: Various 21
Problem number: 588.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_separable]`

$$
\boxed{xy(b\,x^{2}+a)\,y'-By^{2}=A}
$$

✓ Solution by Maple
Time used: 0.015 (sec). Leaf size: 82

```
dsolve(x*y(x)*(b*x^2+a)*diff(y(x),x) = A+B*y(x)^2,y(x), singsol=all)
```

$$
\begin{aligned}
y(x) &= \frac{\sqrt{-B\left(-x^{\frac{2B}{a}}\,(b\,x^{2}+a)^{-\frac{B}{a}}\,c_1B+A\right)}}{B} \\
y(x) &= -\frac{\sqrt{-B\left(-x^{\frac{2B}{a}}\,(b\,x^{2}+a)^{-\frac{B}{a}}\,c_1B+A\right)}}{B}
\end{aligned}
$$

✓ Solution by Mathematica

Time used: 1.98 (sec). Leaf size: 134

```
DSolve[x y[x] (a+b x^2)y'[x]==A+B y[x]^2,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to -\frac{\sqrt{-A + e^{2Bc_1} x^{\frac{2B}{a}} \left(a + bx^2\right)^{-\frac{B}{a}}}}{\sqrt{B}}$$

$$y(x) \to \frac{\sqrt{-A + e^{2Bc_1} x^{\frac{2B}{a}} \left(a + bx^2\right)^{-\frac{B}{a}}}}{\sqrt{B}}$$

$$y(x) \to -\frac{i\sqrt{A}}{\sqrt{B}}$$

$$y(x) \to \frac{i\sqrt{A}}{\sqrt{B}}$$

21.13 problem 589

Internal problem ID [3839]

Book: Ordinary differential equations and their solutions. By George Moseley Murphy. 1960
Section: Various 21
Problem number: 589.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[[_homogeneous, 'class G'], _rational, _Bernoulli]`

$$\boxed{3yx^4y' + 2y^2x^3 = 1}$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 51

```
dsolve(3*x^4*y(x)*diff(y(x),x) = 1-2*x^3*y(x)^2,y(x), singsol=all)
```

$$y(x) = -\frac{\sqrt{5}\,\sqrt{x^{\frac{17}{3}}\left(-2+5x^{\frac{5}{3}}c_1\right)}}{5x^{\frac{13}{3}}}$$
$$y(x) = \frac{\sqrt{5}\,\sqrt{x^{\frac{17}{3}}\left(-2+5x^{\frac{5}{3}}c_1\right)}}{5x^{\frac{13}{3}}}$$

✓ Solution by Mathematica

Time used: 3.675 (sec). Leaf size: 51

```
DSolve[3 x^4 y[x] y'[x]==1-2 x^3 y[x]^2,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to -\sqrt{-\frac{2}{5x^3} + \frac{c_1}{x^{4/3}}}$$
$$y(x) \to \sqrt{-\frac{2}{5x^3} + \frac{c_1}{x^{4/3}}}$$

21.14 problem 590

Internal problem ID [3840]

Book: Ordinary differential equations and their solutions. By George Moseley Murphy. 1960
Section: Various 21
Problem number: 590.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_rational, [_Abel, '2nd type', 'class B']]`

$$x^7yy' - 5yx^3 = 2x^2 + 2$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 120

```
dsolve(x^7*y(x)*diff(y(x),x) = 2*x^2+2+5*x^3*y(x),y(x), singsol=all)
```

$$-\frac{\left(y(x)\,\mathrm{hypergeom}\left(\left[\frac{1}{2},\frac{5}{4}\right],\left[\frac{3}{2}\right],-\frac{\left(x^3y(x)+1\right)^2}{x^2}\right)x^3 - c_1x + \mathrm{hypergeom}\left(\left[\frac{1}{2},\frac{5}{4}\right],\left[\frac{3}{2}\right],-\frac{\left(x^3y(x)+1\right)^2}{x^2}\right)\right)\left(\frac{x^6y(x)^2\cdots}{\cdots}\right.}{\left(\frac{x^6y(x)^2+2x^3y(x)+x^2+1}{x^2}\right)^{\frac{1}{4}}x}$$
$$= 0$$

✓ Solution by Mathematica

Time used: 0.381 (sec). Leaf size: 98

```
DSolve[x^7 y[x] y'[x]==2(1+x^2)+5 x^3 y[x],y[x],x,IncludeSingularSolutions -> True]
```

$$\mathrm{Solve}\left[c_1 = \frac{\frac{i\left(x^3y(x)+1\right)\sqrt[4]{x^4y(x)^2+\frac{1}{x^2}+2xy(x)+1}\,\mathrm{Hypergeometric2F1}\left(\frac{1}{2},\frac{5}{4},\frac{3}{2},-\frac{\left(y(x)x^3+1\right)^2}{x^2}\right)}{2x}+ix}{\sqrt[4]{-\frac{\left(x^3y(x)+1\right)^2}{x^2}-1}}, y(x)\right]$$

21.15 problem 591

Internal problem ID [3841]

Book: Ordinary differential equations and their solutions. By George Moseley Murphy. 1960
Section: Various 21
Problem number: 591.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_separable]`

$$yy'\sqrt{x^2+1}+x\sqrt{y^2+1}=0$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 20

```
dsolve(y(x)*diff(y(x),x)*sqrt(x^2+1)+x*sqrt(1+y(x)^2) = 0,y(x), singsol=all)
```

$$\sqrt{x^2+1}+\sqrt{y(x)^2+1}+c_1=0$$

✓ Solution by Mathematica

Time used: 0.288 (sec). Leaf size: 75

```
DSolve[y[x] y'[x]Sqrt[1+x^2]+x Sqrt[1+y[x]^2]==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x)\to -\sqrt{x^2+c_1\left(-2\sqrt{x^2+1}+c_1\right)}$$
$$y(x)\to \sqrt{x^2+c_1\left(-2\sqrt{x^2+1}+c_1\right)}$$
$$y(x)\to -i$$
$$y(x)\to i$$

21.16 problem 592

Internal problem ID [3842]

Book: Ordinary differential equations and their solutions. By George Moseley Murphy. 1960
Section: Various 21
Problem number: 592.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_separable]`

$$\boxed{(y+1)\,y'\sqrt{x^2+1}-y^3=0}$$

✓ Solution by Maple
Time used: 0.016 (sec). Leaf size: 57

```
dsolve((1+y(x))*diff(y(x),x)*sqrt(x^2+1) = y(x)^3,y(x), singsol=all)
```

$$y(x)=\frac{-1+\sqrt{1-2c_1-2\operatorname{arcsinh}(x)}}{2c_1+2\operatorname{arcsinh}(x)}$$
$$y(x)=\frac{-1-\sqrt{1-2c_1-2\operatorname{arcsinh}(x)}}{2c_1+2\operatorname{arcsinh}(x)}$$

✓ Solution by Mathematica
Time used: 0.621 (sec). Leaf size: 120

```
DSolve[(1+y[x])y'[x]Sqrt[1+x^2]==y[x]^3,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x)\to -\frac{1+\sqrt{2\log\left(\sqrt{x^2+1}-x\right)+1-2c_1}}{-2\log\left(\sqrt{x^2+1}-x\right)+2c_1}$$
$$y(x)\to \frac{-1+\sqrt{2\log\left(\sqrt{x^2+1}-x\right)+1-2c_1}}{2\left(-\log\left(\sqrt{x^2+1}-x\right)+c_1\right)}$$
$$y(x)\to 0$$

21.17 problem 593

Internal problem ID [3843]

Book: Ordinary differential equations and their solutions. By George Moseley Murphy. 1960
Section: Various 21
Problem number: 593.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[[_Abel, ‘2nd type‘, ‘class C‘]]`

$$(\mathrm{g0}(x) + y\,\mathrm{g1}(x))\,y' - \mathrm{f1}(x)\,y - \mathrm{f2}(x)\,y^2 - \mathrm{f3}(x)\,y^3 = \mathrm{f0}(x)$$

✗ Solution by Maple

```
dsolve((g0(x)+y(x)*g1(x))*diff(y(x),x) = f0(x)+f1(x)*y(x)+f2(x)*y(x)^2+f3(x)*y(x)^3,y(x), si
```

No solution found

✗ Solution by Mathematica

Time used: 0.0 (sec). Leaf size: 0

```
DSolve[(g0[x]+y[x] g1[x])y'[x]==f0[x]+f1[x] y[x]+f2[x] y[x]^2+f3[x] y[x]^3,y[x],x,IncludeSin
```

Timed out

21.18 problem 594

Internal problem ID [3844]

Book: Ordinary differential equations and their solutions. By George Moseley Murphy. 1960
Section: Various 21
Problem number: 594.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_separable]`

$$\boxed{y^2y' + x(2 - y) = 0}$$

✓ Solution by Maple

Time used: 0.016 (sec). Leaf size: 27

```
dsolve(y(x)^2*diff(y(x),x)+x*(2-y(x)) = 0,y(x), singsol=all)
```

$$\frac{x^2}{2} - \frac{y(x)^2}{2} - 2y(x) - 4\ln\left(y(x) - 2\right) + c_1 = 0$$

✓ Solution by Mathematica

Time used: 0.37 (sec). Leaf size: 43

```
DSolve[y[x]^2*y'[x]+x*(2-y[x])==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to \text{InverseFunction}\left[\frac{\#1^2}{2} + 2\#1 + 4\log(\#1 - 2) - 6\&\right]\left[\frac{x^2}{2} + c_1\right]$$
$$y(x) \to 2$$

21.19 problem 595

Internal problem ID [3845]

Book: Ordinary differential equations and their solutions. By George Moseley Murphy. 1960
Section: Various 21
Problem number: 595.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_separable]`

$$\boxed{y^2y' - x(y^2+1) = 0}$$

✓ Solution by Maple

Time used: 0.031 (sec). Leaf size: 22

```
dsolve(y(x)^2*diff(y(x),x) = x*(1+y(x)^2),y(x), singsol=all)
```

$$y(x) = -\tan\left(\text{RootOf}\left(x^2 + 2\tan\left(\_Z\right) + 2c_1 - 2\_Z\right)\right)$$

✓ Solution by Mathematica

Time used: 0.214 (sec). Leaf size: 39

```
DSolve[y[x]^2*y'[x]==x*(1+y[x]^2),y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to \text{InverseFunction}\left[\#1 - \tan^{-1}(\#1)\&\right]\left[\frac{x^2}{2} + c_1\right]$$
$$y(x) \to -i$$
$$y(x) \to i$$

21.20 problem 596

Internal problem ID [3846]

Book: Ordinary differential equations and their solutions. By George Moseley Murphy. 1960
Section: Various 21
Problem number: 596.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_exact, _rational]`

$$\left(x+y^{2}\right) y^{\prime}+y=b x+a$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 584

```
dsolve((x+y(x)^2)*diff(y(x),x)+y(x) = b*x+a,y(x), singsol=all)
```

$$y(x) = \frac{\left(6b\,x^{2}+12ax-12c_{1}+2\sqrt{9b^{2}x^{4}+36a\,x^{3}b+36x^{2}a^{2}-36bc_{1}x^{2}-72c_{1}ax+16x^{3}+36c_{1}^{2}}\right)^{\frac{2}{3}}-4x}{2\left(6b\,x^{2}+12ax-12c_{1}+2\sqrt{9b^{2}x^{4}+36a\,x^{3}b+36x^{2}a^{2}-36bc_{1}x^{2}-72c_{1}ax+16x^{3}+36c_{1}^{2}}\right)^{\frac{1}{3}}}$$

$$y(x) = -\frac{i\sqrt{3}\left(6b\,x^{2}+12ax-12c_{1}+2\sqrt{9b^{2}x^{4}+36a\,x^{3}b+36x^{2}a^{2}-36bc_{1}x^{2}-72c_{1}ax+16x^{3}+36c_{1}^{2}}\right)^{\frac{2}{3}}+4i}{4\left(6b\,x^{2}+12ax-12c_{1}+2\sqrt{9b^{2}x^{4}+36a\,x^{3}b}\right.}$$

$$y(x) = \frac{i\sqrt{3}\left(6b\,x^{2}+12ax-12c_{1}+2\sqrt{9b^{2}x^{4}+36a\,x^{3}b+36x^{2}a^{2}-36bc_{1}x^{2}-72c_{1}ax+16x^{3}+36c_{1}^{2}}\right)^{\frac{2}{3}}+4i\sqrt{3}}{4\left(6b\,x^{2}+12ax-12c_{1}+2\sqrt{9b^{2}x^{4}+36a\,x^{3}b+}\right.}$$

✓ Solution by Mathematica

Time used: 5.463 (sec). Leaf size: 420

```
DSolve[(x+y[x]^2)y'[x]+y[x]==a+b x,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to \frac{-2\ 2^{2/3}x + \sqrt[3]{2}\left(\sqrt{16x^3 + 9\left(2ax + bx^2 + 2c_1\right)^2} + 6ax + 3bx^2 + 6c_1\right)^{2/3}}{2\sqrt[3]{\sqrt{16x^3 + 9\left(2ax + bx^2 + 2c_1\right)^2} + 6ax + 3bx^2 + 6c_1}}$$

$$y(x) \to \frac{i\sqrt[3]{2}\left(\sqrt{3}+i\right)\left(\sqrt{36a^2x^2 + 36abx^3 + 72ac_1x + 9b^2x^4 + 36bc_1x^2 + 16x^3 + 36c_1^2} + 6ax + 3bx^2 + 6c_1\right)^{2/3}}{4\sqrt[3]{\sqrt{16x^3 + 9\left(2ax + bx^2 + 2c_1\right)^2} + 6ax + 3bx^2 + 6c_1}}$$

$$y(x) \to \frac{x - i\sqrt{3}x}{\sqrt[3]{2}\sqrt[3]{\sqrt{16x^3 + 9\left(2ax + bx^2 + 2c_1\right)^2} + 6ax + 3bx^2 + 6c_1}} - \frac{i\left(\sqrt{3}-i\right)\sqrt[3]{\sqrt{16x^3 + 9\left(2ax + bx^2 + 2c_1\right)^2} + 6ax + 3bx^2 + 6c_1}}{2\ 2^{2/3}}$$

21.21 problem 597

Internal problem ID [3847]

Book: Ordinary differential equations and their solutions. By George Moseley Murphy. 1960
Section: Various 21
Problem number: 597.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_exact, _rational]`

$$\boxed{(x - y^2)\, y' + y = x^2}$$

✓ Solution by Maple

Time used: 0.016 (sec). Leaf size: 318

```
dsolve((x-y(x)^2)*diff(y(x),x) = x^2-y(x),y(x), singsol=all)
```

$$y(x) = \frac{\left(-4x^3 + 12c_1 + 4\sqrt{x^6 + (-6c_1 - 4)\, x^3 + 9c_1^2}\right)^{\frac{2}{3}} + 4x}{2\left(-4x^3 + 12c_1 + 4\sqrt{x^6 + (-6c_1 - 4)\, x^3 + 9c_1^2}\right)^{\frac{1}{3}}}$$

$$y(x) = \frac{i\left(-\left(-4x^3 + 12c_1 + 4\sqrt{x^6 + (-6c_1 - 4)\, x^3 + 9c_1^2}\right)^{\frac{2}{3}} + 4x\right)\sqrt{3} - \left(-4x^3 + 12c_1 + 4\sqrt{x^6 + (-6c_1 - 4)}\right.}{4\left(-4x^3 + 12c_1 + 4\sqrt{x^6 + (-6c_1 - 4)\, x^3 + 9c_1^2}\right)^{\frac{1}{3}}}$$

$$y(x) = \frac{i\left(\left(-4x^3 + 12c_1 + 4\sqrt{x^6 + (-6c_1 - 4)\, x^3 + 9c_1^2}\right)^{\frac{2}{3}} - 4x\right)\sqrt{3} - \left(-4x^3 + 12c_1 + 4\sqrt{x^6 + (-6c_1 - 4)\, x^3}\right.}{4\left(-4x^3 + 12c_1 + 4\sqrt{x^6 + (-6c_1 - 4)\, x^3 + 9c_1^2}\right)^{\frac{1}{3}}}$$

✓ Solution by Mathematica

Time used: 3.81 (sec). Leaf size: 326

```
DSolve[(x-y[x]^2)y'[x]==x^2-y[x],y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to -\frac{2x + \sqrt[3]{2}\left(x^3 + \sqrt{x^6 + (-4+6c_1)x^3 + 9{c_1}^2} + 3c_1\right)^{2/3}}{2^{2/3}\sqrt[3]{x^3 + \sqrt{x^6 + (-4+6c_1)x^3 + 9{c_1}^2} + 3c_1}}$$

$$y(x) \to \frac{2^{2/3}\left(1 - i\sqrt{3}\right)\left(x^3 + \sqrt{x^6 + (-4+6c_1)x^3 + 9{c_1}^2} + 3c_1\right)^{2/3} + \sqrt[3]{2}\left(2 + 2i\sqrt{3}\right)x}{4\sqrt[3]{x^3 + \sqrt{x^6 + (-4+6c_1)x^3 + 9{c_1}^2} + 3c_1}}$$

$$y(x) \to \frac{2^{2/3}\left(1 + i\sqrt{3}\right)\left(x^3 + \sqrt{x^6 + (-4+6c_1)x^3 + 9{c_1}^2} + 3c_1\right)^{2/3} + \sqrt[3]{2}\left(2 - 2i\sqrt{3}\right)x}{4\sqrt[3]{x^3 + \sqrt{x^6 + (-4+6c_1)x^3 + 9{c_1}^2} + 3c_1}}$$

21.22 problem 598

Internal problem ID [3848]

Book: Ordinary differential equations and their solutions. By George Moseley Murphy. 1960
Section: Various 21
Problem number: 598.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[[_homogeneous, 'class A'], _rational, _dAlembert]`

$$\boxed{(y^2+x^2)\,y'+yx=0}$$

✓ Solution by Maple
Time used: 0.828 (sec). Leaf size: 221

```
dsolve((x^2+y(x)^2)*diff(y(x),x)+x*y(x) = 0,y(x), singsol=all)
```

$$y(x)=\frac{\sqrt{x^2c_1\left(c_1x^2+\sqrt{c_1^2x^4+1}\right)}}{x\left(c_1x^2+\sqrt{c_1^2x^4+1}\right)c_1}$$
$$y(x)=\frac{\sqrt{x^2c_1\left(c_1x^2-\sqrt{c_1^2x^4+1}\right)}}{x\left(c_1x^2-\sqrt{c_1^2x^4+1}\right)c_1}$$
$$y(x)=-\frac{\sqrt{x^2c_1\left(c_1x^2+\sqrt{c_1^2x^4+1}\right)}}{x\left(c_1x^2+\sqrt{c_1^2x^4+1}\right)c_1}$$
$$y(x)=\frac{\sqrt{x^2c_1\left(c_1x^2-\sqrt{c_1^2x^4+1}\right)}}{x\left(-c_1x^2+\sqrt{c_1^2x^4+1}\right)c_1}$$

✓ Solution by Mathematica

Time used: 13.959 (sec). Leaf size: 218

```
DSolve[(x^2+y[x]^2)y'[x]+x y[x]==0,y[x],x,IncludeSingularSolutions -> True]
```

$$
\begin{aligned}
y(x) &\to -\sqrt{-x^2-\sqrt{x^4+e^{4c_1}}} \\
y(x) &\to \sqrt{-x^2-\sqrt{x^4+e^{4c_1}}} \\
y(x) &\to -\sqrt{-x^2+\sqrt{x^4+e^{4c_1}}} \\
y(x) &\to \sqrt{-x^2+\sqrt{x^4+e^{4c_1}}} \\
y(x) &\to 0 \\
y(x) &\to -\sqrt{-\sqrt{x^4}-x^2} \\
y(x) &\to \sqrt{-\sqrt{x^4}-x^2} \\
y(x) &\to -\sqrt{\sqrt{x^4}-x^2} \\
y(x) &\to \sqrt{\sqrt{x^4}-x^2}
\end{aligned}
$$

21.23 problem 599

Internal problem ID [3849]

Book: Ordinary differential equations and their solutions. By George Moseley Murphy. 1960
Section: Various 21
Problem number: 599.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[[_homogeneous, ‘class A‘], _rational, _dAlembert]`

$$\boxed{(y^2 + x^2)\, y' - yx = 0}$$

✓ Solution by Maple
Time used: 0.031 (sec). Leaf size: 16

```
dsolve((x^2+y(x)^2)*diff(y(x),x) = x*y(x),y(x), singsol=all)
```

$$y(x) = \sqrt{\frac{1}{\text{LambertW}\,(c_1 x^2)}}\, x$$

✓ Solution by Mathematica
Time used: 7.386 (sec). Leaf size: 49

```
DSolve[(x^2+y[x]^2)y'[x]==x y[x],y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to -\frac{x}{\sqrt{W\,(e^{-2c_1}x^2)}}$$
$$y(x) \to \frac{x}{\sqrt{W\,(e^{-2c_1}x^2)}}$$
$$y(x) \to 0$$

21.24 problem 600

Internal problem ID [3850]

Book: Ordinary differential equations and their solutions. By George Moseley Murphy. 1960
Section: Various 21
Problem number: 600.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[[_homogeneous, ‘class A‘], _rational, _dAlembert]`

$$\boxed{(x^2 - y^2)\, y' - 2yx = 0}$$

✓ Solution by Maple
Time used: 0.047 (sec). Leaf size: 47

```
dsolve((x^2-y(x)^2)*diff(y(x),x) = 2*x*y(x),y(x), singsol=all)
```

$$y(x) = \frac{1 - \sqrt{-4c_1^2x^2 + 1}}{2c_1}$$
$$y(x) = \frac{1 + \sqrt{-4c_1^2x^2 + 1}}{2c_1}$$

✓ Solution by Mathematica
Time used: 1.088 (sec). Leaf size: 66

```
DSolve[(x^2-y[x]^2)y'[x]==2 x y[x],y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to \frac{1}{2}\left(e^{c_1} - \sqrt{-4x^2 + e^{2c_1}}\right)$$
$$y(x) \to \frac{1}{2}\left(\sqrt{-4x^2 + e^{2c_1}} + e^{c_1}\right)$$
$$y(x) \to 0$$

21.25 problem 601

Internal problem ID [3851]

Book: Ordinary differential equations and their solutions. By George Moseley Murphy. 1960
Section: Various 21
Problem number: 601.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[[_homogeneous, ‘class A‘], _exact, _rational, _dAlembert]`

$$(x^2-y^2)\,y'+x(x+2y)=0$$

✓ Solution by Maple

Time used: 0.063 (sec). Leaf size: 320

```
dsolve((x^2-y(x)^2)*diff(y(x),x)+x*(x+2*y(x)) = 0,y(x), singsol=all)
```

$$y(x)=\frac{4c_1x^2+\left(4+4x^3c_1^{\frac{3}{2}}+4\sqrt{-3x^6c_1^3+2x^3c_1^{\frac{3}{2}}+1}\right)^{\frac{2}{3}}}{2\left(4+4x^3c_1^{\frac{3}{2}}+4\sqrt{-3x^6c_1^3+2x^3c_1^{\frac{3}{2}}+1}\right)^{\frac{1}{3}}\sqrt{c_1}}$$

$$y(x)=\frac{4i\sqrt{3}\,c_1x^2-i\left(4+4x^3c_1^{\frac{3}{2}}+4\sqrt{-3x^6c_1^3+2x^3c_1^{\frac{3}{2}}+1}\right)^{\frac{2}{3}}\sqrt{3}-4c_1x^2-\left(4+4x^3c_1^{\frac{3}{2}}+4\sqrt{-3x^6c_1^3+2x^3c_1^{\cdots}}\right)\cdots}{4\left(4+4x^3c_1^{\frac{3}{2}}+4\sqrt{-3x^6c_1^3+2x^3c_1^{\frac{3}{2}}+1}\right)^{\frac{1}{3}}\sqrt{c_1}}$$

$$y(x)=\frac{\left(4+4x^3c_1^{\frac{3}{2}}+4\sqrt{-3x^6c_1^3+2x^3c_1^{\frac{3}{2}}+1}\right)^{\frac{1}{3}}\left(i\sqrt{3}-1\right)}{4\sqrt{c_1}}-\frac{\sqrt{c_1}\left(1+i\sqrt{3}\right)x^2}{\left(4+4x^3c_1^{\frac{3}{2}}+4\sqrt{-3x^6c_1^3+2x^3c_1^{\frac{3}{2}}+1}\right)^{\frac{1}{3}}}$$

✓ Solution by Mathematica

Time used: 42.021 (sec). Leaf size: 611

```
DSolve[(x^2-y[x]^2)*y'[x]+x*(x+2*y[x])==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x)\to \frac{\sqrt[3]{x^3+\sqrt{-3x^6+2e^{3c_1}x^3+e^{6c_1}}+e^{3c_1}}}{\sqrt[3]{2}}+\frac{\sqrt[3]{2}x^2}{\sqrt[3]{x^3+\sqrt{-3x^6+2e^{3c_1}x^3+e^{6c_1}}+e^{3c_1}}}$$

$$y(x)\to \frac{i\left(\sqrt[3]{2}\left(\sqrt{3}+i\right)\left(x^3+\sqrt{-3x^6+2e^{3c_1}x^3+e^{6c_1}}+e^{3c_1}\right)^{2/3}-2\left(\sqrt{3}-i\right)x^2\right)}{2\ 2^{2/3}\sqrt[3]{x^3+\sqrt{-3x^6+2e^{3c_1}x^3+e^{6c_1}}+e^{3c_1}}}$$

$$y(x)\to \frac{2i\sqrt[3]{2}\left(\sqrt{3}+i\right)x^2+2^{2/3}\left(-1-i\sqrt{3}\right)\left(x^3+\sqrt{-3x^6+2e^{3c_1}x^3+e^{6c_1}}+e^{3c_1}\right)^{2/3}}{4\sqrt[3]{x^3+\sqrt{-3x^6+2e^{3c_1}x^3+e^{6c_1}}+e^{3c_1}}}$$

$$y(x)\to \frac{\sqrt[3]{\sqrt{3}\sqrt{-x^6}+x^3}}{\sqrt[3]{2}}+\frac{\sqrt[3]{2}x^2}{\sqrt[3]{\sqrt{3}\sqrt{-x^6}+x^3}}$$

$$y(x)\to \frac{\left(-2-2i\sqrt{3}\right)x^2+i\sqrt[3]{2}\left(\sqrt{3}+i\right)\left(\sqrt{3}\sqrt{-x^6}+x^3\right)^{2/3}}{2\ 2^{2/3}\sqrt[3]{\sqrt{3}\sqrt{-x^6}+x^3}}$$

$$y(x)\to \frac{2i\sqrt[3]{2}\left(\sqrt{3}+i\right)x^2+2^{2/3}\left(-1-i\sqrt{3}\right)\left(\sqrt{3}\sqrt{-x^6}+x^3\right)^{2/3}}{4\sqrt[3]{\sqrt{3}\sqrt{-x^6}+x^3}}$$

21.26 problem 602

Internal problem ID [3852]

Book: Ordinary differential equations and their solutions. By George Moseley Murphy. 1960
Section: Various 21
Problem number: 602.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[[_homogeneous, ‘class A‘], _exact, _rational, _dAlembert]`

$$\left(y^2+x^2\right) y'+2x(y+2x)=0$$

✓ Solution by Maple

Time used: 0.062 (sec). Leaf size: 321

```
dsolve((x^2+y(x)^2)*diff(y(x),x)+2*x*(2*x+y(x)) = 0,y(x), singsol=all)
```

$$y(x)=-\frac{2\left(c_1x^2-\frac{\left(4-16x^3c_1^{\frac{3}{2}}+4\sqrt{20x^6c_1^3-8x^3c_1^{\frac{3}{2}}+1}\right)^{\frac{2}{3}}}{4}\right)}{\left(4-16x^3c_1^{\frac{3}{2}}+4\sqrt{20x^6c_1^3-8x^3c_1^{\frac{3}{2}}+1}\right)^{\frac{1}{3}}\sqrt{c_1}}$$

$$y(x)=-\frac{\left(1+i\sqrt{3}\right)\left(4-16x^3c_1^{\frac{3}{2}}+4\sqrt{20x^6c_1^3-8x^3c_1^{\frac{3}{2}}+1}\right)^{\frac{1}{3}}}{4\sqrt{c_1}}-\frac{\sqrt{c_1}\left(i\sqrt{3}-1\right)x^2}{\left(4-16x^3c_1^{\frac{3}{2}}+4\sqrt{20x^6c_1^3-8x^3c_1^{\frac{3}{2}}+1}\right)^{\frac{1}{3}}}$$

$$y(x)=\frac{4i\sqrt{3}\,c_1x^2+i\left(4-16x^3c_1^{\frac{3}{2}}+4\sqrt{20x^6c_1^3-8x^3c_1^{\frac{3}{2}}+1}\right)^{\frac{2}{3}}\sqrt{3}+4c_1x^2-\left(4-16x^3c_1^{\frac{3}{2}}+4\sqrt{20x^6c_1^3-8x^3c\right.}{4\left(4-16x^3c_1^{\frac{3}{2}}+4\sqrt{20x^6c_1^3-8x^3c_1^{\frac{3}{2}}+1}\right)^{\frac{1}{3}}\sqrt{c_1}}$$

✓ Solution by Mathematica

Time used: 20.751 (sec). Leaf size: 593

```
DSolve[(x^2+y[x]^2)y'[x]+2 x(2 x+y[x])==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to \frac{\sqrt[3]{-4x^3+\sqrt{20x^6-8e^{3c_1}x^3+e^{6c_1}}+e^{3c_1}}}{\sqrt[3]{2}} - \frac{\sqrt[3]{2}x^2}{\sqrt[3]{-4x^3+\sqrt{20x^6-8e^{3c_1}x^3+e^{6c_1}}+e^{3c_1}}}$$

$$y(x) \to \frac{\sqrt[3]{2}\left(2+2i\sqrt{3}\right)x^2+i2^{2/3}\left(\sqrt{3}+i\right)\left(-4x^3+\sqrt{20x^6-8e^{3c_1}x^3+e^{6c_1}}+e^{3c_1}\right)^{2/3}}{4\sqrt[3]{-4x^3+\sqrt{20x^6-8e^{3c_1}x^3+e^{6c_1}}+e^{3c_1}}}$$

$$y(x) \to \frac{\left(1-i\sqrt{3}\right)x^2}{2^{2/3}\sqrt[3]{-4x^3+\sqrt{20x^6-8e^{3c_1}x^3+e^{6c_1}}+e^{3c_1}}} - \frac{\left(1+i\sqrt{3}\right)\sqrt[3]{-4x^3+\sqrt{20x^6-8e^{3c_1}x^3+e^{6c_1}}+e^{3c_1}}}{2\sqrt[3]{2}}$$

$$y(x) \to \sqrt[3]{\sqrt{5}\sqrt{x^6}-2x^3} - \frac{x^2}{\sqrt[3]{\sqrt{5}\sqrt{x^6}-2x^3}}$$

$$y(x) \to \frac{\left(1-i\sqrt{3}\right)x^2+\left(-1-i\sqrt{3}\right)\left(\sqrt{5}\sqrt{x^6}-2x^3\right)^{2/3}}{2\sqrt[3]{\sqrt{5}\sqrt{x^6}-2x^3}}$$

$$y(x) \to \frac{\left(1+i\sqrt{3}\right)x^2+i\left(\sqrt{3}+i\right)\left(\sqrt{5}\sqrt{x^6}-2x^3\right)^{2/3}}{2\sqrt[3]{\sqrt{5}\sqrt{x^6}-2x^3}}$$

21.27 problem 603

Internal problem ID [3853]

Book: Ordinary differential equations and their solutions. By George Moseley Murphy. 1960
Section: Various 21
Problem number: 603.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[[_1st_order, _with_linear_symmetries], _rational]`

$$\boxed{(1 - x^2 + y^2)\, y' + y^2 = x^2 + 1}$$

✓ Solution by Maple
Time used: 0.016 (sec). Leaf size: 28

```
dsolve((1-x^2+y(x)^2)*diff(y(x),x) = 1+x^2-y(x)^2,y(x), singsol=all)
```

$$y(x)^2 + 2xy(x) + x^2 + 2\ln(-x + y(x)) - c_1 = 0$$

✓ Solution by Mathematica
Time used: 0.349 (sec). Leaf size: 25

```
DSolve[(1-x^2+y[x]^2)y'[x]==1+x^2-y[x]^2,y[x],x,IncludeSingularSolutions -> True]
```

$$\text{Solve}\left[e^{\frac{1}{2}(y(x)+x)^2}(x - y(x)) = c_1, y(x)\right]$$

21.28 problem 604

Internal problem ID [3854]

Book: Ordinary differential equations and their solutions. By George Moseley Murphy. 1960
Section: Various 21
Problem number: 604.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_exact, _rational, [_1st_order, '_with_symmetry_[F(x)*G(y),0]`

$$\left(a^2+x^2+y^2\right)y'+2yx=0$$

✓ Solution by Maple

Time used: 0.015 (sec). Leaf size: 329

```
dsolve((a^2+x^2+y(x)^2)*diff(y(x),x)+2*x*y(x) = 0,y(x), singsol=all)
```

$$y(x) = -\frac{2\left(a^2+x^2-\frac{\left(-12c_1+4\sqrt{4a^6+12a^4x^2+12a^2x^4+4x^6+9c_1^2}\right)^{\frac{2}{3}}}{4}\right)}{\left(-12c_1+4\sqrt{4a^6+12a^4x^2+12a^2x^4+4x^6+9c_1^2}\right)^{\frac{1}{3}}}$$

$$y(x) = -\frac{\left(\frac{1}{4}+\frac{i\sqrt{3}}{4}\right)\left(-12c_1+4\sqrt{4a^6+12a^4x^2+12a^2x^4+4x^6+9c_1^2}\right)^{\frac{2}{3}}+\left(i\sqrt{3}-1\right)\left(a^2+x^2\right)}{\left(-12c_1+4\sqrt{4a^6+12a^4x^2+12a^2x^4+4x^6+9c_1^2}\right)^{\frac{1}{3}}}$$

$$y(x) = \frac{\frac{\left(i\sqrt{3}-1\right)\left(-12c_1+4\sqrt{4a^6+12a^4x^2+12a^2x^4+4x^6+9c_1^2}\right)^{\frac{2}{3}}}{4}+\left(1+i\sqrt{3}\right)\left(a^2+x^2\right)}{\left(-12c_1+4\sqrt{4a^6+12a^4x^2+12a^2x^4+4x^6+9c_1^2}\right)^{\frac{1}{3}}}$$

✓ Solution by Mathematica

Time used: 5.48 (sec). Leaf size: 317

```
DSolve[(a^2+x^2+y[x]^2)y'[x]+2 x y[x]==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to \frac{\sqrt[3]{2}\left(\sqrt{4\left(a^2+x^2\right)^3+9c_1{}^2}+3c_1\right)^{2/3}-2a^2-2x^2}{2^{2/3}\sqrt[3]{\sqrt{4\left(a^2+x^2\right)^3+9c_1{}^2}+3c_1}}$$

$$y(x) \to \frac{\left(1+i\sqrt{3}\right)\left(a^2+x^2\right)}{2^{2/3}\sqrt[3]{\sqrt{4\left(a^2+x^2\right)^3+9c_1{}^2}+3c_1}}+\frac{i\left(\sqrt{3}+i\right)\sqrt[3]{\sqrt{4\left(a^2+x^2\right)^3+9c_1{}^2}+3c_1}}{2\sqrt[3]{2}}$$

$$y(x) \to \frac{\left(1-i\sqrt{3}\right)\left(a^2+x^2\right)}{2^{2/3}\sqrt[3]{\sqrt{4\left(a^2+x^2\right)^3+9c_1{}^2}+3c_1}}-\frac{i\left(\sqrt{3}-i\right)\sqrt[3]{\sqrt{4\left(a^2+x^2\right)^3+9c_1{}^2}+3c_1}}{2\sqrt[3]{2}}$$

$$y(x) \to 0$$

21.29 problem 605

Internal problem ID [3855]

Book: Ordinary differential equations and their solutions. By George Moseley Murphy. 1960
Section: Various 21
Problem number: 605.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_exact, _rational]`

$$\boxed{(a^2 + x^2 + y^2)\, y' + 2yx = -b^2 - x^2}$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 569

```
dsolve((a^2+x^2+y(x)^2)*diff(y(x),x)+b^2+x^2+2*x*y(x) = 0,y(x), singsol=all)
```

$$y(x) = -\frac{2\left(a^2 + x^2 - \frac{\left(-12b^2x-4x^3-12c_1+4\sqrt{4a^6+12a^4x^2+12a^2x^4+9b^4x^2+6b^2x^4+5x^6+18b^2c_1x+6c_1x^3+9c_1^2}\right)^{\frac{2}{3}}}{4}\right)}{\left(-12b^2x - 4x^3 - 12c_1 + 4\sqrt{4a^6 + 12a^4x^2 + 12a^2x^4 + 9b^4x^2 + 6b^2x^4 + 5x^6 + 18b^2c_1x + 6c_1x^3 + 9c_1^2}\right)}$$

$$y(x) = -\frac{\left(\frac{1}{4} + \frac{i\sqrt{3}}{4}\right)\left(-12b^2x - 4x^3 - 12c_1 + 4\sqrt{4a^6 + 12a^4x^2 + 12a^2x^4 + 9b^4x^2 + 6b^2x^4 + 5x^6 + 18b^2c_1x + 6c}\right.}{\left(-12b^2x - 4x^3 - 12c_1 + 4\sqrt{4a^6 + 12a^4x^2 + 12a^2x^4 + 9b^4x^2 + 6b^2x^4 + 5x^6 + 18b^2c_1}\right.}$$

$$y(x) = \frac{\frac{\left(i\sqrt{3}-1\right)\left(-12b^2x-4x^3-12c_1+4\sqrt{4a^6+12a^4x^2+12a^2x^4+9b^4x^2+6b^2x^4+5x^6+18b^2c_1x+6c_1x^3+9c_1^2}\right)^{\frac{2}{3}}}{4} + \left(1 + i\sqrt{3}\right)\left(a^2 + x^2\right)}{\left(-12b^2x - 4x^3 - 12c_1 + 4\sqrt{4a^6 + 12a^4x^2 + 12a^2x^4 + 9b^4x^2 + 6b^2x^4 + 5x^6 + 18b^2c_1x + 6c_1x^3 + 9c_1^2}\right)^{\frac{1}{3}}}$$

✓ Solution by Mathematica

Time used: 7.229 (sec). Leaf size: 438

```
DSolve[(a^2+x^2+y[x]^2)y'[x]+b^2+x^2+2 x y[x]==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x)\to \frac{\sqrt[3]{2}\left(\sqrt{4\left(a^2+x^2\right)^3+\left(3b^2x+x^3-3c_1\right)^2}-3b^2x-x^3+3c_1\right)^{2/3}-2a^2-2x^2}{2^{2/3}\sqrt[3]{\sqrt{4\left(a^2+x^2\right)^3+\left(3b^2x+x^3-3c_1\right)^2}-3b^2x-x^3+3c_1}}$$

$$y(x)\to \frac{\left(1+i\sqrt{3}\right)\left(a^2+x^2\right)}{2^{2/3}\sqrt[3]{\sqrt{4\left(a^2+x^2\right)^3+\left(3b^2x+x^3-3c_1\right)^2}-3b^2x-x^3+3c_1}}+\frac{i\left(\sqrt{3}+i\right)\sqrt[3]{\sqrt{4\left(a^2+x^2\right)^3+\left(3b^2x+x^3-3c_1\right)^2}-3b^2x-x^3+3c_1}}{2\sqrt[3]{2}}$$

$$y(x)\to \frac{\left(1-i\sqrt{3}\right)\left(a^2+x^2\right)}{2^{2/3}\sqrt[3]{\sqrt{4\left(a^2+x^2\right)^3+\left(3b^2x+x^3-3c_1\right)^2}-3b^2x-x^3+3c_1}}-\frac{i\left(\sqrt{3}-i\right)\sqrt[3]{\sqrt{4\left(a^2+x^2\right)^3+\left(3b^2x+x^3-3c_1\right)^2}-3b^2x-x^3+3c_1}}{2\sqrt[3]{2}}$$

21.30 problem 606

Internal problem ID [3856]

Book: Ordinary differential equations and their solutions. By George Moseley Murphy. 1960
Section: Various 21
Problem number: 606.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_rational]`

$$\boxed{(x + x^2 + y^2)\, y' - y = 0}$$

✓ Solution by Maple
Time used: 0.078 (sec). Leaf size: 40

```
dsolve((x+x^2+y(x)^2)*diff(y(x),x) = y(x),y(x), singsol=all)
```

$$\frac{e^{-2iy(x)}(ix + y(x)) + 2(x + iy(x))\, c_1}{2iy\,(x) + 2x} = 0$$

✓ Solution by Mathematica
Time used: 0.117 (sec). Leaf size: 18

```
DSolve[(x+x^2+y[x]^2)y'[x]==y[x],y[x],x,IncludeSingularSolutions -> True]
```

$$\text{Solve}\left[y(x) - \arctan\left(\frac{x}{y(x)}\right) = c_1, y(x)\right]$$

22 Various 22

22.1 problem 607

Internal problem ID [3857]

Book: Ordinary differential equations and their solutions. By George Moseley Murphy. 1960
Section: Various 22
Problem number: 607.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[[_homogeneous, 'class A'], _rational, _dAlembert]`

$$\boxed{(3x^2 - y^2)\, y' - 2yx = 0}$$

✓ Solution by Maple

Time used: 0.031 (sec). Leaf size: 317

```
dsolve((3*x^2-y(x)^2)*diff(y(x),x) = 2*x*y(x),y(x), singsol=all)
```

$$y(x) = \frac{1 + \frac{\left(12\sqrt{3}\, x\sqrt{27c_1^2x^2-4}\, c_1 - 108c_1^2x^2+8\right)^{\frac{1}{3}}}{2} + \frac{2}{\left(12\sqrt{3}\, x\sqrt{27c_1^2x^2-4}\, c_1 - 108c_1^2x^2+8\right)^{\frac{1}{3}}}}{3c_1}$$

$$y(x) = -\frac{\left(1+i\sqrt{3}\right)\left(12\sqrt{3}\, x\sqrt{27c_1^2x^2-4}\, c_1 - 108c_1^2x^2+8\right)^{\frac{2}{3}} - 4i\sqrt{3} - 4\left(12\sqrt{3}\, x\sqrt{27c_1^2x^2-4}\, c_1 - 108c_1^2x^2 \right.}{12\left(12\sqrt{3}\, x\sqrt{27c_1^2x^2-4}\, c_1 - 108c_1^2x^2+8\right)^{\frac{1}{3}} c_1}$$

$$y(x) = \frac{\left(i\sqrt{3}-1\right)\left(12\sqrt{3}\, x\sqrt{27c_1^2x^2-4}\, c_1 - 108c_1^2x^2+8\right)^{\frac{2}{3}} - 4i\sqrt{3} + 4\left(12\sqrt{3}\, x\sqrt{27c_1^2x^2-4}\, c_1 - 108c_1^2x^2 + 8 \right.}{12\left(12\sqrt{3}\, x\sqrt{27c_1^2x^2-4}\, c_1 - 108c_1^2x^2+8\right)^{\frac{1}{3}} c_1}$$

✓ Solution by Mathematica

Time used: 60.21 (sec). Leaf size: 458

```
DSolve[(3 x^2-y[x]^2)y'[x]==2 x y[x],y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to \frac{1}{3}\left(\frac{\sqrt[3]{27e^{c_1}x^2+3\sqrt{81e^{2c_1}x^4-12e^{4c_1}x^2}-2e^{3c_1}}}{\sqrt[3]{2}} + \frac{\sqrt[3]{2}e^{2c_1}}{\sqrt[3]{27e^{c_1}x^2+3\sqrt{81e^{2c_1}x^4-12e^{4c_1}x^2}-2e^{3c_1}}} - e^{c_1}\right)$$

$$y(x) \to \frac{i(\sqrt{3}+i)\sqrt[3]{27e^{c_1}x^2+3\sqrt{81e^{2c_1}x^4-12e^{4c_1}x^2}-2e^{3c_1}}}{6\sqrt[3]{2}} - \frac{i(\sqrt{3}-i)e^{2c_1}}{3\ 2^{2/3}\sqrt[3]{27e^{c_1}x^2+3\sqrt{81e^{2c_1}x^4-12e^{4c_1}x^2}-2e^{3c_1}}} - \frac{e^{c_1}}{3}$$

$$y(x) \to -\frac{i(\sqrt{3}-i)\sqrt[3]{27e^{c_1}x^2+3\sqrt{81e^{2c_1}x^4-12e^{4c_1}x^2}-2e^{3c_1}}}{6\sqrt[3]{2}} + \frac{i(\sqrt{3}+i)e^{2c_1}}{3\ 2^{2/3}\sqrt[3]{27e^{c_1}x^2+3\sqrt{81e^{2c_1}x^4-12e^{4c_1}x^2}-2e^{3c_1}}} - \frac{e^{c_1}}{3}$$

22.2 problem 608

Internal problem ID [3858]

Book: Ordinary differential equations and their solutions. By George Moseley Murphy. 1960
Section: Various 22
Problem number: 608.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[[_homogeneous, ‘class G‘], _rational]`

$$\boxed{(x^4+y^2)\,y'-4yx^3=0}$$

✓ Solution by Maple

Time used: 0.109 (sec). Leaf size: 41

```
dsolve((x^4+y(x)^2)*diff(y(x),x) = 4*x^3*y(x),y(x), singsol=all)
```

$$y(x) = -\frac{\sqrt{4x^4+c_1^2}}{2}+\frac{c_1}{2}$$
$$y(x) = \frac{\sqrt{4x^4+c_1^2}}{2}+\frac{c_1}{2}$$

✓ Solution by Mathematica

Time used: 0.341 (sec). Leaf size: 58

```
DSolve[(x^4+y[x]^2)y'[x]==4 x^3 y[x],y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to \frac{1}{2}\left(c_1-\sqrt{4x^4+c_1{}^2}\right)$$
$$y(x) \to \frac{1}{2}\left(\sqrt{4x^4+c_1{}^2}+c_1\right)$$
$$y(x) \to 0$$

22.3 problem 609

Internal problem ID [3859]

Book: Ordinary differential equations and their solutions. By George Moseley Murphy. 1960
Section: Various 22
Problem number: 609.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_separable]`

$$y(y+1)\,y' = x(x+1)$$

✓ Solution by Maple

Time used: 0.016 (sec). Leaf size: 498

```
dsolve(y(x)*(1+y(x))*diff(y(x),x) = x*(1+x),y(x), singsol=all)
```

$$y(x) = \frac{\left(-1+4x^3+6x^2+12c_1+2\sqrt{4x^6+12x^5+24c_1x^3+9x^4+36c_1x^2-2x^3+36c_1^2-3x^2-6c_1}\right)^{\frac{1}{3}}}{2} + \frac{1}{2\left(-1+4x^3+6x^2+12c_1+2\sqrt{4x^6+12x^5+24c_1x^3+9x^4+36c_1x^2-2x^3+36c_1^2-3x^2-6c_1}\right)^{\frac{1}{3}}} - \frac{1}{2}$$

$$y(x) = -\frac{(1+i\sqrt{3})\left(4x^3+6x^2+2\sqrt{(2x^3+3x^2+6c_1)(2x^3+3x^2+6c_1-1)}+12c_1-1\right)^{\frac{2}{3}} - i\sqrt{3}+2\left(4x^3+\right.}{4\left(4x^3+6x^2+2\sqrt{(2x^3+3x^2+6c_1)(2x^3+3x^2+6c\right.}}$$

$$y(x) = \frac{(i\sqrt{3}-1)\left(4x^3+6x^2+2\sqrt{(2x^3+3x^2+6c_1)(2x^3+3x^2+6c_1-1)}+12c_1-1\right)^{\frac{2}{3}} - i\sqrt{3}-2\left(4x^3+6x\right.}{4\left(4x^3+6x^2+2\sqrt{(2x^3+3x^2+6c_1)(2x^3+3x^2+6c_1\right.}}$$

✓ Solution by Mathematica

Time used: 4.291 (sec). Leaf size: 346

```
DSolve[y[x]*(1+y[x])*y'[x]==x*(1+x),y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to \frac{1}{2}\left(\sqrt[3]{4x^3+6x^2+\sqrt{-1+(4x^3+6x^2-1+12c_1)^2}-1+12c_1} + \frac{1}{\sqrt[3]{4x^3+6x^2+\sqrt{-1+(4x^3+6x^2-1+12c_1)^2}-1+12c_1}}-1\right)$$

$$y(x) \to \frac{1}{8}\left(2i\left(\sqrt{3}+i\right)\sqrt[3]{4x^3+6x^2+\sqrt{-1+(4x^3+6x^2-1+12c_1)^2}-1+12c_1} + \frac{-2-2i\sqrt{3}}{\sqrt[3]{4x^3+6x^2+\sqrt{-1+(4x^3+6x^2-1+12c_1)^2}-1+12c_1}}-4\right)$$

$$y(x) \to \frac{1}{8}\left(-2\left(1+i\sqrt{3}\right)\sqrt[3]{4x^3+6x^2+\sqrt{-1+(4x^3+6x^2-1+12c_1)^2}-1+12c_1} + \frac{2i\left(\sqrt{3}+i\right)}{\sqrt[3]{4x^3+6x^2+\sqrt{-1+(4x^3+6x^2-1+12c_1)^2}-1+12c_1}}-4\right)$$

22.4 problem 610

Internal problem ID [3860]

Book: Ordinary differential equations and their solutions. By George Moseley Murphy. 1960
Section: Various 22
Problem number: 610.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_rational]`

$$(x+2y+y^2)\,y'+y(y+1)+(y+x)^2\,y^2=0$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 110

```
dsolve((x+2*y(x)+y(x)^2)*diff(y(x),x)+y(x)*(1+y(x))+(x+y(x))^2*y(x)^2 = 0,y(x), singsol=all)
```

$$y(x)=\frac{x^2-c_1x+\sqrt{x^4-2c_1x^3+(c_1^2-2)\,x^2+(4+2c_1)\,x-4c_1+1}-1}{-2x+2c_1}$$
$$y(x)=\frac{-x^2+c_1x+\sqrt{x^4-2c_1x^3+(c_1^2-2)\,x^2+(4+2c_1)\,x-4c_1+1}+1}{2x-2c_1}$$

✓ Solution by Mathematica

Time used: 2.323 (sec). Leaf size: 146

```
DSolve[(x+2 y[x]+y[x]^2)y'[x]+y[x](1+y[x])+(x+y[x])^2 y[x]^2==0,y[x],x,IncludeSingularSoluti
```

$$y(x)\to -\frac{x^2+\sqrt{(-x^2+c_1x+1)^2+4(x-c_1)}-c_1x-1}{2(x-c_1)}$$
$$y(x)\to \frac{-x^2+\sqrt{(-x^2+c_1x+1)^2+4(x-c_1)}+c_1x+1}{2(x-c_1)}$$
$$y(x)\to \frac{1}{2}\left(-\sqrt{x^2}-x\right)$$
$$y(x)\to \frac{1}{2}\left(\sqrt{x^2}-x\right)$$

22.5 problem 611

Internal problem ID [3861]

Book: Ordinary differential equations and their solutions. By George Moseley Murphy. 1960
Section: Various 22
Problem number: 611.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_rational, [_1st_order, '_with_symmetry_[F(x)*G(y),0]']]`

$$\boxed{(x^2 + 2y + y^2)\, y' = -2x}$$

✓ Solution by Maple

Time used: 0.016 (sec). Leaf size: 17

```
dsolve((x^2+2*y(x)+y(x)^2)*diff(y(x),x)+2*x = 0,y(x), singsol=all)
```

$$(x^2 + y(x)^2)\, e^{y(x)} + c_1 = 0$$

✓ Solution by Mathematica

Time used: 0.156 (sec). Leaf size: 24

```
DSolve[(x^2+2 y[x]+y[x]^2)y'[x]+2 x==0,y[x],x,IncludeSingularSolutions -> True]
```

$$\text{Solve}\left[x^2 e^{y(x)} + e^{y(x)} y(x)^2 = c_1, y(x)\right]$$

22.6 problem 612

Internal problem ID [3862]

Book: Ordinary differential equations and their solutions. By George Moseley Murphy. 1960
Section: Various 22
Problem number: 612.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_exact, _rational, [_1st_order, '_with_symmetry_[F(x)*G(y),0]`

$$\left(x^3 + 2y - y^2\right) y' + 3x^2 y = 0$$

✓ Solution by Maple

Time used: 0.016 (sec). Leaf size: 445

```
dsolve((x^3+2*y(x)-y(x)^2)*diff(y(x),x)+3*x^2*y(x) = 0,y(x), singsol=all)
```

$$y(x) = \frac{\left(12x^3 + 12c_1 + 8 + 4\sqrt{-4x^9 - 3x^6 + 18c_1x^3 + 9c_1^2 + 12c_1}\right)^{\frac{1}{3}}}{2} + \frac{2x^3 + 2}{\left(12x^3 + 12c_1 + 8 + 4\sqrt{-4x^9 - 3x^6 + 18c_1x^3 + 9c_1^2 + 12c_1}\right)^{\frac{1}{3}}} + 1$$

$$y(x) = -\frac{\left(12x^3 + 12c_1 + 8 + 4\sqrt{-4x^9 - 3x^6 + 18c_1x^3 + 9c_1^2 + 12c_1}\right)^{\frac{1}{3}}}{4} + \frac{-x^3 - 1}{\left(12x^3 + 12c_1 + 8 + 4\sqrt{-4x^9 - 3x^6 + 18c_1x^3 + 9c_1^2 + 12c_1}\right)^{\frac{1}{3}}} + 1 + \frac{i\sqrt{3}\left(x^3 - \frac{\left(12x^3+12c_1+8+4\sqrt{-4x^9-3x^6+18c_1x^3+9c_1^2+12c_1}\right)^{\frac{2}{3}}}{4} + 1\right)}{\left(12x^3 + 12c_1 + 8 + 4\sqrt{-4x^9 - 3x^6 + 18c_1x^3 + 9c_1^2 + 12c_1}\right)^{\frac{1}{3}}}$$

$$y(x) = \frac{\frac{\left(i\sqrt{3}-1\right)\left(12x^3+12c_1+8+4\sqrt{-4x^9-3x^6+18c_1x^3+9c_1^2+12c_1}\right)^{\frac{2}{3}}}{4} - 1 + \left(12x^3 + 12c_1 + 8 + 4\sqrt{-4x^9 - 3x^6 + 18c_1x^3 + \right.}{\left(12x^3 + 12c_1 + 8 + 4\sqrt{-4x^9 - 3x^6 + 18c_1x^3 + 9c_1^2 + 12c_1}\right)^{\frac{1}{3}}}$$

✓ Solution by Mathematica

Time used: 5.508 (sec). Leaf size: 409

```
DSolve[(x^3+2 y[x]-y[x]^2)y'[x]+3 x^2 y[x]==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to -\frac{\sqrt[3]{2}(x^3+1)}{\sqrt[3]{-3x^3+\sqrt{-4x^9-3x^6-18c_1x^3+3c_1(-4+3c_1)}-2+3c_1}} - \frac{\sqrt[3]{-3x^3+\sqrt{-4x^9-3x^6-18c_1x^3+3c_1(-4+3c_1)}-2+3c_1}}{\sqrt[3]{2}}+1$$

$$y(x) \to \frac{\left(1+i\sqrt{3}\right)(x^3+1)}{2^{2/3}\sqrt[3]{-3x^3+\sqrt{-4x^9-3x^6-18c_1x^3+3c_1(-4+3c_1)}-2+3c_1}} + \frac{\left(1-i\sqrt{3}\right)\sqrt[3]{-3x^3+\sqrt{-4x^9-3x^6-18c_1x^3+3c_1(-4+3c_1)}-2+3c_1}}{2\sqrt[3]{2}}+1$$

$$y(x) \to \frac{\left(1-i\sqrt{3}\right)(x^3+1)}{2^{2/3}\sqrt[3]{-3x^3+\sqrt{-4x^9-3x^6-18c_1x^3+3c_1(-4+3c_1)}-2+3c_1}} + \frac{\left(1+i\sqrt{3}\right)\sqrt[3]{-3x^3+\sqrt{-4x^9-3x^6-18c_1x^3+3c_1(-4+3c_1)}-2+3c_1}}{2\sqrt[3]{2}}+1$$

$$y(x) \to 0$$

22.7 problem 613

Internal problem ID [3863]

Book: Ordinary differential equations and their solutions. By George Moseley Murphy. 1960
Section: Various 22
Problem number: 613.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[[_1st_order, _with_linear_symmetries], _rational]`

$$(1 + y + yx + y^2)\, y' + y = -1$$

✓ Solution by Maple
Time used: 0.047 (sec). Leaf size: 19

```
dsolve((1+y(x)+x*y(x)+y(x)^2)*diff(y(x),x)+1+y(x) = 0,y(x), singsol=all)
```

$$-c_1(y(x) + 1)\, \mathrm{e}^{-y(x)} + x + y(x) = 0$$

✓ Solution by Mathematica
Time used: 0.143 (sec). Leaf size: 23

```
DSolve[(1+y[x]+x y[x]+y[x]^2)y'[x]+1+y[x]==0,y[x],x,IncludeSingularSolutions -> True]
```

$$\text{Solve}\left[x = -y(x) + c_1 e^{-y(x)}(y(x) + 1), y(x)\right]$$

22.8 problem 614

Internal problem ID [3864]

Book: Ordinary differential equations and their solutions. By George Moseley Murphy. 1960
Section: Various 22
Problem number: 614.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[[_homogeneous, ‘class C‘], _dAlembert]`

$$\boxed{(y+x)^2 y' = a^2}$$

✓ Solution by Maple
Time used: 0.047 (sec). Leaf size: 24

```
dsolve((x+y(x))^2*diff(y(x),x) = a^2,y(x), singsol=all)
```

$$y(x) = a \operatorname{RootOf}(\tan(\_Z)\, a - a\_Z + c_1 - x) - c_1$$

✓ Solution by Mathematica
Time used: 0.112 (sec). Leaf size: 21

```
DSolve[(x+y[x])^2 y'[x]==a^2,y[x],x,IncludeSingularSolutions -> True]
```

$$\text{Solve}\left[y(x) - a \arctan\left(\frac{y(x)+x}{a}\right) = c_1, y(x)\right]$$

22.9 problem 615

Internal problem ID [3865]

Book: Ordinary differential equations and their solutions. By George Moseley Murphy. 1960
Section: Various 22
Problem number: 615.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[[_homogeneous, ‘class C‘], _dAlembert]`

$$\boxed{(-y+x)^2 y' = a^2}$$

✓ Solution by Maple
Time used: 0.063 (sec). Leaf size: 36

```
dsolve((x-y(x))^2*diff(y(x),x) = a^2,y(x), singsol=all)
```

$$y(x) = \mathrm{e}^{\mathrm{RootOf}\left(a\ln\left(\mathrm{e}^{\_Z}+2a\right)-a\_Z-2\,\mathrm{e}^{\_Z}+2c_1-2a-2x\right)} + a + x$$

✓ Solution by Mathematica
Time used: 0.174 (sec). Leaf size: 49

```
DSolve[(x-y[x])^2 y'[x]==a^2,y[x],x,IncludeSingularSolutions -> True]
```

$$\text{Solve}\left[-\left(a^2\left(\frac{\log(a-y(x)+x)}{2a}-\frac{\log(-a-y(x)+x)}{2a}\right)\right)-y(x)=c_1,y(x)\right]$$

22.10 problem 616

Internal problem ID [3866]

Book: Ordinary differential equations and their solutions. By George Moseley Murphy. 1960
Section: Various 22
Problem number: 616.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[[_homogeneous, ‘class A‘], _rational, _dAlembert]`

$$\left(x^2+2yx-y^2\right)y'-2yx+y^2=-x^2$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 37

```
dsolve((x^2+2*x*y(x)-y(x)^2)*diff(y(x),x)+x^2-2*x*y(x)+y(x)^2 = 0,y(x), singsol=all)
```

$$y(x)=\text{RootOf}\left(\int^{\_Z}\frac{\_a^2-2\_a-1}{\_a^3-3\_a^2+\_a-1}d\_a+\ln(x)+c_1\right)x$$

✓ Solution by Mathematica

Time used: 0.132 (sec). Leaf size: 91

```
DSolve[(x^2+2 x y[x]-y[x]^2)y'[x]+x^2-2 x y[x]+y[x]^2==0,y[x],x,IncludeSingularSolutions ->
```

$$\text{Solve}\left[\text{RootSum}\left[\#1^3-3\#1^2+\#1-1\&,\frac{\#1^2\log\left(\frac{y(x)}{x}-\#1\right)-2\#1\log\left(\frac{y(x)}{x}-\#1\right)-\log\left(\frac{y(x)}{x}-\#1\right)}{3\#1^2-6\#1+1}\&\right]=-\log(x)+c_1,y(x)\right]$$

22.11 problem 619

Internal problem ID [3867]

Book: Ordinary differential equations and their solutions. By George Moseley Murphy. 1960
Section: Various 22
Problem number: 619.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[[_homogeneous, 'class A'], _rational, _dAlembert]`

$$\boxed{(y+x)^2 y' + 2yx - 5y^2 = x^2}$$

✓ Solution by Maple

Time used: 0.032 (sec). Leaf size: 35

```
dsolve((x+y(x))^2*diff(y(x),x) = x^2-2*x*y(x)+5*y(x)^2,y(x), singsol=all)
```

$$y(x) = x\left(1 + e^{\mathrm{RootOf}\left(e^{2\_Z}\ln(x)+c_1e^{2\_Z}+\_Ze^{2\_Z}-4\,e^{\_Z}-2\right)}\right)$$

✓ Solution by Mathematica

Time used: 0.34 (sec). Leaf size: 41

```
DSolve[(x+y[x])^2 y'[x]==x^2-2 x y[x]+5 y[x]^2,y[x],x,IncludeSingularSolutions -> True]
```

$$\mathrm{Solve}\left[\frac{2-\frac{4y(x)}{x}}{\left(\frac{y(x)}{x}-1\right)^2} + \log\left(\frac{y(x)}{x}-1\right) = -\log(x) + c_1, y(x)\right]$$

22.12 problem 620

Internal problem ID [3868]

Book: Ordinary differential equations and their solutions. By George Moseley Murphy. 1960
Section: Various 22
Problem number: 620.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[[_homogeneous, 'class C'], _rational]`

$$(a+b+x+y)^2 y' - 2(y+a)^2 = 0$$

✓ Solution by Maple
Time used: 0.016 (sec). Leaf size: 30

```
dsolve((a+b+x+y(x))^2*diff(y(x),x) = 2*(a+y(x))^2,y(x), singsol=all)
```

$$y(x) = -a + \tan\left(\mathrm{RootOf}\left(-2\_Z + \ln\left(\tan\left(\_Z\right)\right) + \ln\left(x+b\right) + c_1\right)\right)\left(-x-b\right)$$

✓ Solution by Mathematica
Time used: 0.151 (sec). Leaf size: 25

```
DSolve[(a+b+x+y[x])^2 y'[x]==2(a+y[x])^2,y[x],x,IncludeSingularSolutions -> True]
```

$$\mathrm{Solve}\left[\log(a+y(x)) - 2\arctan\left(\frac{b+x}{a+y(x)}\right) = c_1, y(x)\right]$$

22.13 problem 621

Internal problem ID [3869]

Book: Ordinary differential equations and their solutions. By George Moseley Murphy. 1960
Section: Various 22
Problem number: 621.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[[_homogeneous, 'class A'], _exact, _rational, _dAlembert]`

$$\left(2x^{2}+4yx-y^{2}\right)y'+4yx+2y^{2}=x^{2}$$

✓ Solution by Maple

Time used: 0.063 (sec). Leaf size: 439

```
dsolve((2*x^2+4*x*y(x)-y(x)^2)*diff(y(x),x) = x^2-4*x*y(x)-2*y(x)^2,y(x), singsol=all)
```

$$y(x)=\frac{\frac{\left(108x^{3}c_{1}^{3}+4+4\sqrt{-135c_{1}^{6}x^{6}+54x^{3}c_{1}^{3}+1}\right)^{\frac{1}{3}}}{2}+\frac{12x^{2}c_{1}^{2}}{\left(108x^{3}c_{1}^{3}+4+4\sqrt{-135c_{1}^{6}x^{6}+54x^{3}c_{1}^{3}+1}\right)^{\frac{1}{3}}}+2c_{1}x}{c_{1}}$$

$$y(x)=\frac{-\frac{\left(108x^{3}c_{1}^{3}+4+4\sqrt{-135c_{1}^{6}x^{6}+54x^{3}c_{1}^{3}+1}\right)^{\frac{1}{3}}}{4}-\frac{6x^{2}c_{1}^{2}}{\left(108x^{3}c_{1}^{3}+4+4\sqrt{-135c_{1}^{6}x^{6}+54x^{3}c_{1}^{3}+1}\right)^{\frac{1}{3}}}+2c_{1}x-\frac{i\sqrt{3}\left(-24c_{1}^{2}x^{2}+\left(108x^{3}c_{1}^{3}+4+4\cdots\right.\right.}{4\left(108x^{3}c_{1}^{3}+4+4\sqrt{-13\cdots}\right.}}{c_{1}}$$

$$y(x)=-\frac{24i\sqrt{3}\,c_{1}^{2}x^{2}-i\sqrt{3}\left(108x^{3}c_{1}^{3}+4+4\sqrt{-135c_{1}^{6}x^{6}+54x^{3}c_{1}^{3}+1}\right)^{\frac{2}{3}}+24c_{1}^{2}x^{2}-8c_{1}x\left(108x^{3}c_{1}^{3}+4+4\sqrt{\cdots}\right.}{4\left(108x^{3}c_{1}^{3}+4+4\sqrt{-135c_{1}^{6}x^{6}+54x^{\cdots}}\right.}$$

✓ Solution by Mathematica

Time used: 29.679 (sec). Leaf size: 781

```
DSolve[(2 x^2+4 x y[x]-y[x]^2)y'[x]==x^2-4 x y[x]-2 y[x]^2,y[x],x,IncludeSingularSolutions -
```

$$y(x) \to \frac{\sqrt[3]{27x^3 + \sqrt{-135x^6 + 54e^{3c_1}x^3 + e^{6c_1}} + e^{3c_1}}}{\sqrt[3]{2}} + \frac{6\sqrt[3]{2}x^2}{\sqrt[3]{27x^3 + \sqrt{-135x^6 + 54e^{3c_1}x^3 + e^{6c_1}} + e^{3c_1}}} + 2x$$

$$y(x) \to -\frac{\left(1 - i\sqrt{3}\right)\sqrt[3]{27x^3 + \sqrt{-135x^6 + 54e^{3c_1}x^3 + e^{6c_1}} + e^{3c_1}}}{2\sqrt[3]{2}} - \frac{3\sqrt[3]{2}\left(1 + i\sqrt{3}\right)x^2}{\sqrt[3]{27x^3 + \sqrt{-135x^6 + 54e^{3c_1}x^3 + e^{6c_1}} + e^{3c_1}}} + 2x$$

$$y(x) \to -\frac{\left(1 + i\sqrt{3}\right)\sqrt[3]{27x^3 + \sqrt{-135x^6 + 54e^{3c_1}x^3 + e^{6c_1}} + e^{3c_1}}}{2\sqrt[3]{2}} - \frac{3\sqrt[3]{2}\left(1 - i\sqrt{3}\right)x^2}{\sqrt[3]{27x^3 + \sqrt{-135x^6 + 54e^{3c_1}x^3 + e^{6c_1}} + e^{3c_1}}} + 2x$$

$$y(x) \to \frac{4\sqrt[3]{2}3^{2/3}x^2 + 4\sqrt[3]{\sqrt{15}\sqrt{-x^6} + 9x^3}x + 2^{2/3}\sqrt[3]{3}\left(\sqrt{15}\sqrt{-x^6} + 9x^3\right)^{2/3}}{2\sqrt[3]{\sqrt{15}\sqrt{-x^6} + 9x^3}}$$

$$y(x) \to \frac{-4\sqrt[3]{2}3^{2/3}x^2 + 12i\sqrt[3]{2}\sqrt[6]{3}x^2 + 8\sqrt[3]{\sqrt{15}\sqrt{-x^6} + 9x^3}x - i2^{2/3}3^{5/6}\left(\sqrt{15}\sqrt{-x^6} + 9x^3\right)^{2/3} - 2^{2/3}\sqrt[3]{3}\left(\sqrt{15}\sqrt{\right.}{4\sqrt[3]{\sqrt{15}\sqrt{-x^6} + 9x^3}}$$

$$y(x) \to \frac{\sqrt[3]{3}\left(\sqrt{15}\sqrt{-x^6} + 9x^3\right)^{2/3}\text{Root}\left[2\#1^3 - 1\&, 3\right] - 2\sqrt[3]{-2}3^{2/3}x^2 + 2\sqrt[3]{\sqrt{15}\sqrt{-x^6} + 9x^3}x}{\sqrt[3]{\sqrt{15}\sqrt{-x^6} + 9x^3}}$$

22.14 problem 622

Internal problem ID [3870]

Book: Ordinary differential equations and their solutions. By George Moseley Murphy. 1960
Section: Various 22
Problem number: 622.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[[_homogeneous, ‘class A‘], _rational, _dAlembert]`

$$\boxed{(y+3x)^2 y' - 4(3x+2y)\,y = 0}$$

✓ Solution by Maple
Time used: 0.047 (sec). Leaf size: 44

```
dsolve((3*x+y(x))^2*diff(y(x),x) = 4*(3*x+2*y(x))*y(x),y(x), singsol=all)
```

$$-3\ln\left(\frac{-3x+y(x)}{x}\right) - \ln\left(\frac{x+y(x)}{x}\right) + 3\ln\left(\frac{y(x)}{x}\right) - \ln(x) - c_1 = 0$$

✓ Solution by Mathematica

Time used: 60.159 (sec). Leaf size: 747

```
DSolve[(3 x+y[x])^2 y'[x]==4(3 x+2 y[x])y[x],y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to \frac{1}{4}\left(-\sqrt{12\sqrt[3]{-e^{c_1}x^4\left(-16x+e^{c_1}\right)}+16x^2-16e^{c_1}x+e^{2c_1}} - \sqrt{2}\sqrt{-6\sqrt[3]{-e^{c_1}x^4\left(-16x+e^{c_1}\right)}-48x^2+\frac{\left(-8x+e^{c_1}\right)^3-72x^2\left(-8x+e^{c_1}\right)}{\sqrt{12\sqrt[3]{-e^{c_1}x^4\left(-16x+e^{c_1}\right)}+16x^2-16e^{c_1}x+e^{2c_1}}}+(-8} + 8x - e^{c_1}\right)$$

$$y(x) \to \frac{1}{4}\left(-\sqrt{12\sqrt[3]{-e^{c_1}x^4\left(-16x+e^{c_1}\right)}+16x^2-16e^{c_1}x+e^{2c_1}} + \sqrt{2}\sqrt{-6\sqrt[3]{-e^{c_1}x^4\left(-16x+e^{c_1}\right)}-48x^2+\frac{\left(-8x+e^{c_1}\right)^3-72x^2\left(-8x+e^{c_1}\right)}{\sqrt{12\sqrt[3]{-e^{c_1}x^4\left(-16x+e^{c_1}\right)}+16x^2-16e^{c_1}x+e^{2c_1}}}+(-8} + 8x - e^{c_1}\right)$$

$$y(x) \to \frac{1}{4}\left(\sqrt{12\sqrt[3]{-e^{c_1}x^4\left(-16x+e^{c_1}\right)}+16x^2-16e^{c_1}x+e^{2c_1}} - \sqrt{2}\sqrt{-6\sqrt[3]{-e^{c_1}x^4\left(-16x+e^{c_1}\right)}-48x^2+\frac{72x^2\left(-8x+e^{c_1}\right)-\left(-8x+e^{c_1}\right)^3}{\sqrt{12\sqrt[3]{-e^{c_1}x^4\left(-16x+e^{c_1}\right)}+16x^2-16e^{c_1}x+e^{2c_1}}}+(-8} + 8x - e^{c_1}\right)$$

$$y(x) \to \frac{1}{4}\left(\sqrt{12\sqrt[3]{-e^{c_1}x^4\left(-16x+e^{c_1}\right)}+16x^2-16e^{c_1}x+e^{2c_1}} + \sqrt{2}\sqrt{-6\sqrt[3]{-e^{c_1}x^4\left(-16x+e^{c_1}\right)}-48x^2+\frac{72x^2\left(-8x+e^{c_1}\right)-\left(-8x+e^{c_1}\right)^3}{\sqrt{12\sqrt[3]{-e^{c_1}x^4\left(-16x+e^{c_1}\right)}+16x^2-16e^{c_1}x+e^{2c_1}}}+(-8} + 8x - e^{c_1}\right)$$

22.15 problem 623

Internal problem ID [3871]

Book: Ordinary differential equations and their solutions. By George Moseley Murphy. 1960
Section: Various 22
Problem number: 623.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[[_homogeneous, ‘class C‘], _rational]`

$$\boxed{(1-3x-y)^2 y'-(1-2y)(3-6x-4y)=0}$$

✓ Solution by Maple

Time used: 0.313 (sec). Leaf size: 75

```
dsolve((1-3*x-y(x))^2*diff(y(x),x) = (1-2*y(x))*(3-6*x-4*y(x)),y(x), singsol=all)
```

$$3\ln\left(\frac{1-2y(x)}{6x-1}\right)-4\ln(2)-3\ln\left(\frac{3x-y(x)}{6x-1}\right)$$
$$-\ln\left(\frac{-3y(x)+2-3x}{6x-1}\right)-\ln(6x-1)-c_1=0$$

✓ Solution by Mathematica

Time used: 60.194 (sec). Leaf size: 1089

```
DSolve[(1-3 x-y[x])^2 y'[x]==(1-2 y[x])(3-6 x-4 y[x]),y[x],x,IncludeSingularSolutions -> Tru
```

$$y(x) \to \frac{1}{6}\left(-\sqrt{36x^2-12x+16e^{c_1}(6x-1)+3\,2^{2/3}\sqrt[3]{-e^{c_1}(6x-1)^4(6x-1+e^{c_1})}+1+16e^{2c_1}} -\frac{1}{2}\sqrt{-\frac{8\left(-(6x-1)^3+96e^{2c_1}(6x-1)+30e^{c_1}(1-6x)^2+64e^{3c_1}\right)}{\sqrt{36x^2-12x+16e^{c_1}(6x-1)+3\,2^{2/3}\sqrt[3]{-e^{c_1}(6x-1)^4(6x-1+e^{c_1})}+1+16e^{2c_1}}}+8(12x+} +12x+1+4e^{c_1}\right)$$

$$y(x) \to \frac{1}{6}\left(-\sqrt{36x^2-12x+16e^{c_1}(6x-1)+3\,2^{2/3}\sqrt[3]{-e^{c_1}(6x-1)^4(6x-1+e^{c_1})}+1+16e^{2c_1}} +\frac{1}{2}\sqrt{-\frac{8\left(-(6x-1)^3+96e^{2c_1}(6x-1)+30e^{c_1}(1-6x)^2+64e^{3c_1}\right)}{\sqrt{36x^2-12x+16e^{c_1}(6x-1)+3\,2^{2/3}\sqrt[3]{-e^{c_1}(6x-1)^4(6x-1+e^{c_1})}+1+16e^{2c_1}}}+8(12x+} +12x+1+4e^{c_1}\right)$$

$$y(x) \to \frac{1}{6}\left(\sqrt{36x^2-12x+16e^{c_1}(6x-1)+3\,2^{2/3}\sqrt[3]{-e^{c_1}(6x-1)^4(6x-1+e^{c_1})}+1+16e^{2c_1}} -\frac{1}{2}\sqrt{\frac{8\left(-(6x-1)^3+96e^{2c_1}(6x-1)+30e^{c_1}(1-6x)^2+64e^{3c_1}\right)}{\sqrt{36x^2-12x+16e^{c_1}(6x-1)+3\,2^{2/3}\sqrt[3]{-e^{c_1}(6x-1)^4(6x-1+e^{c_1})}+1+16e^{2c_1}}}+8(12x+1} +12x+1+4e^{c_1}\right)$$

$$y(x) \to \frac{1}{6}\left(\sqrt{36x^2-12x+16e^{c_1}(6x-1)+3\,2^{2/3}\sqrt[3]{-e^{c_1}(6x-1)^4(6x-1+e^{c_1})}+1+16e^{2c_1}} +\frac{1}{2}\sqrt{\frac{8\left(-(6x-1)^3+96e^{2c_1}(6x-1)+30e^{c_1}(1-6x)^2+64e^{3c_1}\right)}{\sqrt{36x^2-12x+16e^{c_1}(6x-1)+3\,2^{2/3}\sqrt[3]{-e^{c_1}(6x-1)^4(6x-1+e^{c_1})}+1+16e^{2c_1}}}+8(12x+1}\right.$$

22.16 problem 624

Internal problem ID [3872]

Book: Ordinary differential equations and their solutions. By George Moseley Murphy. 1960
Section: Various 22
Problem number: 624.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[‘y=_G(x,y’)‘]`

$$(\cot (x)-2y^2)\, y'-y^3\csc (x)\sec (x)=0$$

✗ Solution by Maple

```
dsolve((cot(x)-2*y(x)^2)*diff(y(x),x) = y(x)^3*csc(x)*sec(x),y(x), singsol=all)
```

No solution found

✓ Solution by Mathematica

Time used: 7.946 (sec). Leaf size: 74

```
DSolve[(Cot[x]-2 y[x]^2)y'[x]==y[x]^3 Csc[x] Sec[x],y[x],x,IncludeSingularSolutions -> True]
```

$$y(x)\to -\frac{i\sqrt{\cot(x)}\sqrt{W\left(-2e^{-8c_1}\tan(x)\right)}}{\sqrt{2}}$$

$$y(x)\to \frac{i\sqrt{\cot(x)}\sqrt{W\left(-2e^{-8c_1}\tan(x)\right)}}{\sqrt{2}}$$

$$y(x)\to 0$$

22.17 problem 625

Internal problem ID [3873]

Book: Ordinary differential equations and their solutions. By George Moseley Murphy. 1960
Section: Various 22
Problem number: 625.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_rational, _Bernoulli]`

$$3y^2y' - ay^3 = x + 1$$

✓ Solution by Maple
Time used: 0.016 (sec). Leaf size: 106

```
dsolve(3*y(x)^2*diff(y(x),x) = 1+x+a*y(x)^3,y(x), singsol=all)
```

$$y(x) = \frac{\left(\left(c_1 e^{ax} a^2 - 1 + (-x-1)\,a\right) a\right)^{\frac{1}{3}}}{a}$$
$$y(x) = -\frac{\left(\left(c_1 e^{ax} a^2 - 1 + (-x-1)\,a\right) a\right)^{\frac{1}{3}} \left(1 + i\sqrt{3}\right)}{2a}$$
$$y(x) = \frac{\left(\left(c_1 e^{ax} a^2 - 1 + (-x-1)\,a\right) a\right)^{\frac{1}{3}} \left(i\sqrt{3} - 1\right)}{2a}$$

✓ Solution by Mathematica
Time used: 15.792 (sec). Leaf size: 111

```
DSolve[3 y[x]^2 y'[x]==1+x+a y[x]^3,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to \frac{\sqrt[3]{a^2 c_1 e^{ax} - a(x+1) - 1}}{a^{2/3}}$$
$$y(x) \to -\frac{\sqrt[3]{-1}\sqrt[3]{a^2 c_1 e^{ax} - a(x+1) - 1}}{a^{2/3}}$$
$$y(x) \to \frac{(-1)^{2/3}\sqrt[3]{a^2 c_1 e^{ax} - a(x+1) - 1}}{a^{2/3}}$$

22.18 problem 626

Internal problem ID [3874]

Book: Ordinary differential equations and their solutions. By George Moseley Murphy. 1960
Section: Various 22
Problem number: 626.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_exact, _rational]`

$$\boxed{(x^2 - 3y^2)\, y' + 2yx = -1}$$

✓ Solution by Maple
Time used: 0.0 (sec). Leaf size: 298

```
dsolve((x^2-3*y(x)^2)*diff(y(x),x)+1+2*x*y(x) = 0,y(x), singsol=all)
```

$$y(x) = \frac{\left(108x + 108c_1 + 12\sqrt{-12x^6 + 81c_1^2 + 162c_1x + 81x^2}\right)^{\frac{2}{3}} + 12x^2}{6\left(108x + 108c_1 + 12\sqrt{-12x^6 + 81c_1^2 + 162c_1x + 81x^2}\right)^{\frac{1}{3}}}$$

$$y(x) = \frac{12i\sqrt{3}\,x^2 - i\left(108x + 108c_1 + 12\sqrt{-12x^6 + 81c_1^2 + 162c_1x + 81x^2}\right)^{\frac{2}{3}}\sqrt{3} - 12x^2 - \left(108x + 108c_1 + 12\right.}{12\left(108x + 108c_1 + 12\sqrt{-12x^6 + 81c_1^2 + 162c_1x + 81x^2}\right)^{\frac{1}{3}}}$$

$$y(x) = \frac{\left(108x + 108c_1 + 12\sqrt{-12x^6 + 81c_1^2 + 162c_1x + 81x^2}\right)^{\frac{1}{3}}(i\sqrt{3} - 1)}{12} - \frac{(1 + i\sqrt{3})\, x^2}{\left(108x + 108c_1 + 12\sqrt{-12x^6 + 81c_1^2 + 162c_1x + 81x^2}\right)^{\frac{1}{3}}}$$

✓ Solution by Mathematica

Time used: 4.724 (sec). Leaf size: 307

```
DSolve[(x^2-3 y[x]^2)y'[x]+1+2 x y[x]==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to -\frac{\sqrt[3]{\sqrt{-108x^6+729(x-c_1)^2}-27x+27c_1}}{3\sqrt[3]{2}} - \frac{\sqrt[3]{2}x^2}{\sqrt[3]{\sqrt{-108x^6+729(x-c_1)^2}-27x+27c_1}}$$

$$y(x) \to \frac{\left(1-i\sqrt{3}\right)\sqrt[3]{\sqrt{-108x^6+729(x-c_1)^2}-27x+27c_1}}{6\sqrt[3]{2}} + \frac{\left(1+i\sqrt{3}\right)x^2}{2^{2/3}\sqrt[3]{\sqrt{-108x^6+729(x-c_1)^2}-27x+27c_1}}$$

$$y(x) \to \frac{\left(1+i\sqrt{3}\right)\sqrt[3]{\sqrt{-108x^6+729(x-c_1)^2}-27x+27c_1}}{6\sqrt[3]{2}} + \frac{\left(1-i\sqrt{3}\right)x^2}{2^{2/3}\sqrt[3]{\sqrt{-108x^6+729(x-c_1)^2}-27x+27c_1}}$$

22.19 problem 627

Internal problem ID [3875]

Book: Ordinary differential equations and their solutions. By George Moseley Murphy. 1960
Section: Various 22
Problem number: 627.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[[_homogeneous, ‘class A‘], _rational, _dAlembert]`

$$\boxed{(2x^2 + 3y^2)\, y' + x(y + 3x) = 0}$$

✓ Solution by Maple
Time used: 0.0 (sec). Leaf size: 35

```
dsolve((2*x^2+3*y(x)^2)*diff(y(x),x)+x*(3*x+y(x)) = 0,y(x), singsol=all)
```

$$y(x) = \text{RootOf}\left(\int^{\_Z} \frac{3\_a^2 + 2}{\_a^3 + \_a + 1} d\_a + 3\ln(x) + 3c_1\right) x$$

✓ Solution by Mathematica
Time used: 0.133 (sec). Leaf size: 66

```
DSolve[(2*x^2+3*y[x]^2)*y'[x]+x*(3*x+y[x])==0,y[x],x,IncludeSingularSolutions -> True]
```

$$\text{Solve}\left[\text{RootSum}\left[\#1^3 + \#1 + 1\&, \frac{3\#1^2 \log\left(\frac{y(x)}{x} - \#1\right) + 2\log\left(\frac{y(x)}{x} - \#1\right)}{3\#1^2 + 1}\&\right] = -3\log(x) + c_1, y(x)\right]$$

22.20 problem 628

Internal problem ID [3876]

Book: Ordinary differential equations and their solutions. By George Moseley Murphy. 1960
Section: Various 22
Problem number: 628.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[‘y=_G(x,y’)‘]`

$$3(x^2-y^2)\,y'+6(x+1)\,xy-2y^3=-3\,\mathrm{e}^{x}$$

✓ Solution by Maple

Time used: 0.015 (sec). Leaf size: 345

```
dsolve(3*(x^2-y(x)^2)*diff(y(x),x)+3*exp(x)+6*x*y(x)*(1+x)-2*y(x)^3 = 0,y(x), singsol=all)
```

$$y(x)=\frac{2^{\frac{1}{3}}\left(2\,\mathrm{e}^{4x}x^2+2^{\frac{1}{3}}\left(\left(\mathrm{e}^{3x}+c_1+\sqrt{-4x^6\mathrm{e}^{4x}+\mathrm{e}^{6x}+2c_1\mathrm{e}^{3x}+c_1^2}\right)\mathrm{e}^{4x}\right)^{\frac{2}{3}}\right)\mathrm{e}^{-2x}}{2\left(\left(\mathrm{e}^{3x}+c_1+\sqrt{-4x^6\mathrm{e}^{4x}+\mathrm{e}^{6x}+2c_1\mathrm{e}^{3x}+c_1^2}\right)\mathrm{e}^{4x}\right)^{\frac{1}{3}}}$$

$$y(x)=-\frac{\left(-2\,\mathrm{e}^{4x}\left(i\sqrt{3}-1\right)x^2+\left(1+i\sqrt{3}\right)2^{\frac{1}{3}}\left(\left(\mathrm{e}^{3x}+c_1+\sqrt{-4x^6\mathrm{e}^{4x}+\mathrm{e}^{6x}+2c_1\mathrm{e}^{3x}+c_1^2}\right)\mathrm{e}^{4x}\right)^{\frac{2}{3}}\right)\mathrm{e}^{-2x}2^{\frac{1}{3}}}{4\left(\left(\mathrm{e}^{3x}+c_1+\sqrt{-4x^6\mathrm{e}^{4x}+\mathrm{e}^{6x}+2c_1\mathrm{e}^{3x}+c_1^2}\right)\mathrm{e}^{4x}\right)^{\frac{1}{3}}}$$

$$y(x)=\frac{\mathrm{e}^{-2x}2^{\frac{1}{3}}\left(-2\,\mathrm{e}^{4x}\left(1+i\sqrt{3}\right)x^2+\left(i\sqrt{3}-1\right)2^{\frac{1}{3}}\left(\left(\mathrm{e}^{3x}+c_1+\sqrt{-4x^6\mathrm{e}^{4x}+\mathrm{e}^{6x}+2c_1\mathrm{e}^{3x}+c_1^2}\right)\mathrm{e}^{4x}\right)^{\frac{2}{3}}\right)}{4\left(\left(\mathrm{e}^{3x}+c_1+\sqrt{-4x^6\mathrm{e}^{4x}+\mathrm{e}^{6x}+2c_1\mathrm{e}^{3x}+c_1^2}\right)\mathrm{e}^{4x}\right)^{\frac{1}{3}}}$$

✓ Solution by Mathematica

Time used: 60.281 (sec). Leaf size: 497

```
DSolve[3(x^2-y[x]^2)y'[x]+3 Exp[x]+6 x y[x](1+x)-2 y[x]^3==0,y[x],x,IncludeSingularSolutions
```

$$
\begin{aligned}
y(x) \to & -\frac{e^{-2x}\sqrt[3]{\sqrt{e^{8x}\left(-4e^{4x}x^6+e^{6x}-2c_1e^{3x}+c_1{}^2\right)}-e^{7x}+c_1e^{4x}}}{\sqrt[3]{2}} \\
& -\frac{\sqrt[3]{2}e^{2x}x^2}{\sqrt[3]{\sqrt{e^{8x}\left(-4e^{4x}x^6+e^{6x}-2c_1e^{3x}+c_1{}^2\right)}-e^{7x}+c_1e^{4x}}}
\end{aligned}
$$

$$
\begin{aligned}
y(x) \to & \frac{\left(1-i\sqrt{3}\right)e^{-2x}\sqrt[3]{\sqrt{e^{8x}\left(-4e^{4x}x^6+e^{6x}-2c_1e^{3x}+c_1{}^2\right)}-e^{7x}+c_1e^{4x}}}{2\sqrt[3]{2}} \\
& +\frac{\left(1+i\sqrt{3}\right)e^{2x}x^2}{2^{2/3}\sqrt[3]{\sqrt{e^{8x}\left(-4e^{4x}x^6+e^{6x}-2c_1e^{3x}+c_1{}^2\right)}-e^{7x}+c_1e^{4x}}}
\end{aligned}
$$

$$
\begin{aligned}
y(x) \to & \frac{\left(1+i\sqrt{3}\right)e^{-2x}\sqrt[3]{\sqrt{e^{8x}\left(-4e^{4x}x^6+e^{6x}-2c_1e^{3x}+c_1{}^2\right)}-e^{7x}+c_1e^{4x}}}{2\sqrt[3]{2}} \\
& +\frac{\left(1-i\sqrt{3}\right)e^{2x}x^2}{2^{2/3}\sqrt[3]{\sqrt{e^{8x}\left(-4e^{4x}x^6+e^{6x}-2c_1e^{3x}+c_1{}^2\right)}-e^{7x}+c_1e^{4x}}}
\end{aligned}
$$

22.21 problem 629

Internal problem ID [3877]

Book: Ordinary differential equations and their solutions. By George Moseley Murphy. 1960
Section: Various 22
Problem number: 629.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[[_homogeneous, 'class A'], _exact, _rational, _dAlembert]`

$$(3x^2+2yx+4y^2)\,y'+6yx+y^2=-2x^2$$

✓ Solution by Maple

Time used: 0.063 (sec). Leaf size: 432

```
dsolve((3*x^2+2*x*y(x)+4*y(x)^2)*diff(y(x),x)+2*x^2+6*x*y(x)+y(x)^2 = 0,y(x), singsol=all)
```

$$y(x)=\frac{\left(x^3c_1^3+8+2\sqrt{333c_1^6x^6+4x^3c_1^3+16}\right)^{\frac{1}{3}}-\frac{11x^2c_1^2}{\left(x^3c_1^3+8+2\sqrt{333c_1^6x^6+4x^3c_1^3+16}\right)^{\frac{1}{3}}}-c_1x}{4c_1}$$

$$y(x)=-\frac{11i\sqrt{3}\,c_1^2x^2+i\left(x^3c_1^3+8+2\sqrt{333c_1^6x^6+4x^3c_1^3+16}\right)^{\frac{2}{3}}\sqrt{3}-11c_1^2x^2+2c_1x\left(x^3c_1^3+8+2\sqrt{333c_1^6x^6-}\right.}{8\left(x^3c_1^3+8+2\sqrt{333c_1^6x^6+4x^3c_1^3+16}\right)^{\frac{1}{3}}c}$$

$$y(x)=\frac{11i\sqrt{3}\,c_1^2x^2+i\left(x^3c_1^3+8+2\sqrt{333c_1^6x^6+4x^3c_1^3+16}\right)^{\frac{2}{3}}\sqrt{3}+11c_1^2x^2-2c_1x\left(x^3c_1^3+8+2\sqrt{333c_1^6x^6+4}\right.}{8\left(x^3c_1^3+8+2\sqrt{333c_1^6x^6+4x^3c_1^3+16}\right)^{\frac{1}{3}}c_1}$$

✓ Solution by Mathematica

Time used: 41.45 (sec). Leaf size: 612

```
DSolve[(3 x^2+2 x y[x]+4 y[x]^2)y'[x]+2 x^2+6 x y[x]+y[x]^2==0,y[x],x,IncludeSingularSolutio
```

$$y(x) \to \frac{1}{4}\left(\sqrt[3]{x^3+2\sqrt{333x^6+4e^{3c_1}x^3+16e^{6c_1}}+8e^{3c_1}} - \frac{11x^2}{\sqrt[3]{x^3+2\sqrt{333x^6+4e^{3c_1}x^3+16e^{6c_1}}+8e^{3c_1}}}-x\right)$$

$$y(x) \to \frac{1}{16}\left(2i\left(\sqrt{3}+i\right)\sqrt[3]{x^3+2\sqrt{333x^6+4e^{3c_1}x^3+16e^{6c_1}}+8e^{3c_1}} + \frac{22\left(1+i\sqrt{3}\right)x^2}{\sqrt[3]{x^3+2\sqrt{333x^6+4e^{3c_1}x^3+16e^{6c_1}}+8e^{3c_1}}}-4x\right)$$

$$y(x) \to \frac{1}{16}\left(-2\left(1+i\sqrt{3}\right)\sqrt[3]{x^3+2\sqrt{333x^6+4e^{3c_1}x^3+16e^{6c_1}}+8e^{3c_1}} + \frac{22\left(1-i\sqrt{3}\right)x^2}{\sqrt[3]{x^3+2\sqrt{333x^6+4e^{3c_1}x^3+16e^{6c_1}}+8e^{3c_1}}}-4x\right)$$

$$y(x) \to \frac{1}{4}\left(\sqrt[3]{6\sqrt{37}\sqrt{x^6}+x^3}-\frac{11x^2}{\sqrt[3]{6\sqrt{37}\sqrt{x^6}+x^3}}-x\right)$$

$$y(x) \to \frac{1}{8}\left(\left(-1-i\sqrt{3}\right)\sqrt[3]{6\sqrt{37}\sqrt{x^6}+x^3}+\frac{11\left(1-i\sqrt{3}\right)x^2}{\sqrt[3]{6\sqrt{37}\sqrt{x^6}+x^3}}-2x\right)$$

$$y(x) \to \frac{1}{8}\left(i\left(\sqrt{3}+i\right)\sqrt[3]{6\sqrt{37}\sqrt{x^6}+x^3}+\frac{11\left(1+i\sqrt{3}\right)x^2}{\sqrt[3]{6\sqrt{37}\sqrt{x^6}+x^3}}-2x\right)$$

22.22 problem 630

Internal problem ID [3878]

Book: Ordinary differential equations and their solutions. By George Moseley Murphy. 1960
Section: Various 22
Problem number: 630.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[[_homogeneous, ‘class C‘], _rational]`

$$\boxed{(1-3x+2y)^2 y'-(4+2x-3y)^2=0}$$

✓ Solution by Maple
Time used: 2.437 (sec). Leaf size: 1337

```
dsolve((1-3*x+2*y(x))^2*diff(y(x),x) = (4+2*x-3*y(x))^2,y(x), singsol=all)
```

Expression too large to display

✓ Solution by Mathematica
Time used: 60.209 (sec). Leaf size: 3501

```
DSolve[(1-3 x+2 y[x])^2 y'[x]==(4+2 x-3 y[x])^2,y[x],x,IncludeSingularSolutions -> True]
```

Too large to display

23 Various 23

23.1 problem 631

Internal problem ID [3879]

Book: Ordinary differential equations and their solutions. By George Moseley Murphy. 1960
Section: Various 23
Problem number: 631.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_exact, _rational]`

$$(1 - 3x^2y + 6y^2)\, y' - 3y^2x = -x^2$$

✓ Solution by Maple
Time used: 0.0 (sec). Leaf size: 771

```
dsolve((1-3*x^2*y(x)+6*y(x)^2)*diff(y(x),x)+x^2-3*x*y(x)^2 = 0,y(x), singsol=all)
```

$$y(x) = \frac{\left(-108x^2 - 144x^3 - 432c_1 + 27x^6 + 12\sqrt{-54x^9 - 162c_1x^6 + 144x^6 + 216x^5 + 864c_1x^3 - 27x^4 + 648c_1}\right.}{12} + \frac{3x^4 - 8}{4\left(-108x^2 - 144x^3 - 432c_1 + 27x^6 + 12\sqrt{-54x^9 - 162c_1x^6 + 144x^6 + 216x^5 + 864c_1x^3 - 27x^4 + 64}\right.} + \frac{x^2}{4}$$

$$y(x) = \frac{24 + i\left(-24 + 9x^4 - \left(-108x^2 - 144x^3 - 432c_1 + 27x^6 + 12\sqrt{-54x^9 + (-162c_1 + 144)\, x^6 + 216x^5 - 2}\right.\right.}{}$$

$$y(x) = \frac{24 + i\left(-9x^4 + \left(-108x^2 - 144x^3 - 432c_1 + 27x^6 + 12\sqrt{-54x^9 + (-162c_1 + 144)\, x^6 + 216x^5 - 27x^4 +}\right.\right.}{}$$

✓ Solution by Mathematica

Time used: 7.652 (sec). Leaf size: 570

```
DSolve[(1-3 x^2 y[x]+6 y[x]^2)y'[x]+x^2-3 x y[x]^2==0,y[x],x,IncludeSingularSolutions -> Tru
```

$$y(x) \to \frac{x^2}{4} - \frac{\sqrt[3]{-\frac{27x^6}{4} + 36x^3 + 27x^2 + \sqrt{4\left(6 - \frac{9x^4}{4}\right)^3 + \left(-\frac{27x^6}{4} + 36x^3 + 27x^2 + 108c_1\right)^2} + 108c_1}}{6\sqrt[3]{2}} + \frac{6 - \frac{9x^4}{4}}{3\ 2^{2/3}\sqrt[3]{-\frac{27x^6}{4} + 36x^3 + 27x^2 + \sqrt{4\left(6 - \frac{9x^4}{4}\right)^3 + \left(-\frac{27x^6}{4} + 36x^3 + 27x^2 + 108c_1\right)^2} + 108c_1}}$$

$$y(x) \to \frac{x^2}{4} + \frac{\left(1 - i\sqrt{3}\right)\sqrt[3]{-\frac{27x^6}{4} + 36x^3 + 27x^2 + \sqrt{4\left(6 - \frac{9x^4}{4}\right)^3 + \left(-\frac{27x^6}{4} + 36x^3 + 27x^2 + 108c_1\right)^2} + 108c_1}}{12\sqrt[3]{2}} - \frac{\left(1 + i\sqrt{3}\right)\left(6 - \frac{9x^4}{4}\right)}{6\ 2^{2/3}\sqrt[3]{-\frac{27x^6}{4} + 36x^3 + 27x^2 + \sqrt{4\left(6 - \frac{9x^4}{4}\right)^3 + \left(-\frac{27x^6}{4} + 36x^3 + 27x^2 + 108c_1\right)^2} + 108c_1}}$$

$$y(x) \to \frac{x^2}{4} + \frac{\left(1 + i\sqrt{3}\right)\sqrt[3]{-\frac{27x^6}{4} + 36x^3 + 27x^2 + \sqrt{4\left(6 - \frac{9x^4}{4}\right)^3 + \left(-\frac{27x^6}{4} + 36x^3 + 27x^2 + 108c_1\right)^2} + 108c_1}}{12\sqrt[3]{2}} - \frac{\left(1 - i\sqrt{3}\right)\left(6 - \frac{9x^4}{4}\right)}{6\ 2^{2/3}\sqrt[3]{-\frac{27x^6}{4} + 36x^3 + 27x^2 + \sqrt{4\left(6 - \frac{9x^4}{4}\right)^3 + \left(-\frac{27x^6}{4} + 36x^3 + 27x^2 + 108c_1\right)^2} + 108c_1}}$$

23.2 problem 632

Internal problem ID [3880]

Book: Ordinary differential equations and their solutions. By George Moseley Murphy. 1960
Section: Various 23
Problem number: 632.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_exact, _rational, [_1st_order, '_with_symmetry_[F(x),G(x)]']`

$$\boxed{(x-6y)^2 y' + 2yx - 6y^2 = -a}$$

✓ Solution by Maple

Time used: 0.016 (sec). Leaf size: 115

```
dsolve((x-6*y(x))^2*diff(y(x),x)+a+2*x*y(x)-6*y(x)^2 = 0,y(x), singsol=all)
```

$$y(x) = \frac{(-x^3-18ax-18c_1)^{\frac{1}{3}}}{6} + \frac{x}{6}$$
$$y(x) = -\frac{(-x^3-18ax-18c_1)^{\frac{1}{3}}}{12} - \frac{i\sqrt{3}\,(-x^3-18ax-18c_1)^{\frac{1}{3}}}{12} + \frac{x}{6}$$
$$y(x) = -\frac{(-x^3-18ax-18c_1)^{\frac{1}{3}}}{12} + \frac{i\sqrt{3}\,(-x^3-18ax-18c_1)^{\frac{1}{3}}}{12} + \frac{x}{6}$$

✓ Solution by Mathematica

Time used: 0.663 (sec). Leaf size: 115

```
DSolve[(x-6 y[x])^2 y'[x]+a+2 x y[x]-6 y[x]^2==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to \frac{1}{6}\left(x + \sqrt[3]{-18ax - x^3 + 18c_1}\right)$$
$$y(x) \to \frac{x}{6} + \frac{1}{12}i\left(\sqrt{3}+i\right)\sqrt[3]{-18ax - x^3 + 18c_1}$$
$$y(x) \to \frac{x}{6} - \frac{1}{12}\left(1 + i\sqrt{3}\right)\sqrt[3]{-18ax - x^3 + 18c_1}$$

23.3 problem 633

Internal problem ID [3881]

Book: Ordinary differential equations and their solutions. By George Moseley Murphy. 1960
Section: Various 23
Problem number: 633.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[[_homogeneous, 'class A'], _rational, _dAlembert]`

$$\boxed{(x^2 + ay^2)\, y' - yx = 0}$$

✓ Solution by Maple
Time used: 0.031 (sec). Leaf size: 23

```
dsolve((x^2+a*y(x)^2)*diff(y(x),x) = x*y(x),y(x), singsol=all)
```

$$y(x) = \sqrt{\frac{1}{a\,\mathrm{LambertW}\left(\frac{c_1 x^2}{a}\right)}}\, x$$

✓ Solution by Mathematica
Time used: 13.5 (sec). Leaf size: 71

```
DSolve[(x^2+a y[x]^2)y'[x]==x y[x],y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to -\frac{x}{\sqrt{a}\sqrt{W\left(\frac{x^2 e^{-\frac{2c_1}{a}}}{a}\right)}}$$
$$y(x) \to \frac{x}{\sqrt{a}\sqrt{W\left(\frac{x^2 e^{-\frac{2c_1}{a}}}{a}\right)}}$$
$$y(x) \to 0$$

23.4 problem 634

Internal problem ID [3882]

Book: Ordinary differential equations and their solutions. By George Moseley Murphy. 1960
Section: Various 23
Problem number: 634.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[[_homogeneous, 'class A'], _rational, _dAlembert]`

$$\boxed{(x^2 + yx + ay^2)\, y' - yx - y^2 = x^2 a}$$

✓ Solution by Maple

Time used: 0.156 (sec). Leaf size: 105

```
dsolve((x^2+x*y(x)+a*y(x)^2)*diff(y(x),x) = a*x^2+x*y(x)+y(x)^2,y(x), singsol=all)
```

$$y(x) = x^{\frac{-2a+2}{2+a}} \mathrm{e}^{\frac{(-a+1)\operatorname{RootOf}\left(\mathrm{e}^{\_Z} - x^{-\frac{6a}{2+a}} \mathrm{e}^{-\frac{2(a\_Z+3c_1a-\_Z)}{2+a}} - 3x^{-\frac{3a}{2+a}} \mathrm{e}^{-\frac{a\_Z+3c_1a-\_Z}{2+a}} - 3\right) - 3c_1a}{2+a}} + x$$

✓ Solution by Mathematica

Time used: 0.193 (sec). Leaf size: 54

```
DSolve[(x^2+x y[x]+a y[x]^2)y'[x]==a x^2+x y[x]+y[x]^2,y[x],x,IncludeSingularSolutions -> Tr
```

$$\text{Solve}\left[\frac{1}{3}(a-1)\log\left(\frac{y(x)^2}{x^2} + \frac{y(x)}{x} + 1\right) + \frac{1}{3}(a+2)\log\left(1 - \frac{y(x)}{x}\right) = -a\log(x) + c_1, y(x)\right]$$

23.5 problem 635

Internal problem ID [3883]

Book: Ordinary differential equations and their solutions. By George Moseley Murphy. 1960
Section: Various 23
Problem number: 635.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[[_homogeneous, 'class A'], _rational, _dAlembert]`

$$\boxed{(x^2a + 2yx - ay^2)\, y' - 2axy - y^2 = -x^2}$$

✓ Solution by Maple

Time used: 0.047 (sec). Leaf size: 59

```
dsolve((a*x^2+2*x*y(x)-a*y(x)^2)*diff(y(x),x)+x^2-2*a*x*y(x)-y(x)^2 = 0,y(x), singsol=all)
```

$$y(x) = \frac{a - \sqrt{-4c_1^2x^2 + a^2 - 4c_1x}}{2c_1}$$
$$y(x) = \frac{a + \sqrt{-4c_1^2x^2 + a^2 - 4c_1x}}{2c_1}$$

✓ Solution by Mathematica

Time used: 4.359 (sec). Leaf size: 87

```
DSolve[(a x^2+2 x y[x]-a y[x]^2)y'[x]+x^2-2 a x y[x]-y[x]^2==0,y[x],x,IncludeSingularSolutio
```

$$y(x) \to \frac{1}{2}\left(a(-e^{c_1}) - \sqrt{a^2e^{2c_1} + 4x\left(-x + e^{c_1}\right)}\right)$$
$$y(x) \to \frac{1}{2}\left(\sqrt{a^2e^{2c_1} + 4x\left(-x + e^{c_1}\right)} - ae^{c_1}\right)$$

23.6 problem 637

Internal problem ID [3884]

Book: Ordinary differential equations and their solutions. By George Moseley Murphy. 1960
Section: Various 23
Problem number: 637.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[[_homogeneous, 'class A'], _exact, _rational, _dAlembert]`

$$\left(x^2 a+2 b x y+c y^2\right) y'+2 a x y+b y^2=-k\, x^2$$

✓ Solution by Maple

Time used: 0.078 (sec). Leaf size: 1111

```
dsolve((a*x^2+2*b*x*y(x)+c*y(x)^2)*diff(y(x),x)+k*x^2+2*a*x*y(x)+b*y(x)^2 = 0,y(x), singsol=
```

$$y(x) = \frac{\frac{\left(12a\,x^3c_1^3bc-8b^3x^3c_1^3-4c_1^3c^2k\,x^3+4\sqrt{4a^3c\,c_1^6x^6-3a^2b^2c_1^6x^6-6abc\,c_1^6k\,x^6+4b^3c_1^6k\,x^6+c^2c_1^6k^2x^6+6a\,x^3c_1^3bc-4b^3x^3c_1^3-2c_1^3c^2k\,x^3+c^2}\,c+4c^2\right)^{\frac{1}{3}}}{2}}{}$$

$$y(x) = -\frac{\left(\frac{1}{4}+\frac{i\sqrt{3}}{4}\right)\left(4\sqrt{4\left(\frac{c^2k^2}{4}+\left(a^3-\frac{3}{2}abk\right)c-\frac{3a^2b^2}{4}+b^3k\right)x^6c_1^6+6\left(abc-\frac{2}{3}b^3-\frac{1}{3}c^2k\right)x^3c_1^3+c^2}\,c+(-4c}{\left(4\sqrt{4\left(\frac{c^2k^2}{4}+\right.}\right.}$$

$$y(x) = \frac{\frac{\left(i\sqrt{3}-1\right)\left(4\sqrt{4\left(\frac{c^2k^2}{4}+\left(a^3-\frac{3}{2}abk\right)c-\frac{3a^2b^2}{4}+b^3k\right)x^6c_1^6+6\left(abc-\frac{2}{3}b^3-\frac{1}{3}c^2k\right)x^3c_1^3+c^2}\,c+\left(-4c_1^3k\,x^3+4\right)c^2+12a\,x^3c_1^3bc-8b^3x^3c_1^3\right)^{\frac{2}{3}}}{4}+c_1x\,\Big(}{\left(4\sqrt{4\left(\frac{c^2k^2}{4}+\left(a^3-\frac{3}{2}abk\right)c-\frac{3a^2b^2}{4}+\right.}\right.}$$

✓ Solution by Mathematica

Time used: 60.354 (sec). Leaf size: 744

```
DSolve[(a x^2+2 b x y[x]+c y[x]^2)y'[x]+k x^2+2 a x y[x]+b y[x]^2==0,y[x],x,IncludeSingularS
```

$$y(x) \to \frac{2^{2/3}\sqrt[3]{\sqrt{-4x^6\left(b^2-ac\right)^3+\left(3abcx^3-2b^3x^3+c^2\left(-kx^3+e^{3c_1}\right)\right)^2}+3abcx^3-2b^3x^3-c^2kx^3+c^2e^{3c_1}+}}{2}$$

$$y(x) \to \frac{9i2^{2/3}\left(\sqrt{3}+i\right)\sqrt[3]{\sqrt{-4x^6\left(b^2-ac\right)^3+\left(3abcx^3-2b^3x^3+c^2\left(-kx^3+e^{3c_1}\right)\right)^2}+3abcx^3-2b^3x^3-c^2kx^3}}{}$$

$$y(x) \to \frac{-9\ 2^{2/3}\left(1+i\sqrt{3}\right)\sqrt[3]{\sqrt{-4x^6\left(b^2-ac\right)^3+\left(3abcx^3-2b^3x^3+c^2\left(-kx^3+e^{3c_1}\right)\right)^2}+3abcx^3-2b^3x^3-c^2k}}{}$$

23.7 problem 638

Internal problem ID [3885]

Book: Ordinary differential equations and their solutions. By George Moseley Murphy. 1960
Section: Various 23
Problem number: 638.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_separable]`

$$\boxed{x(1-y^2)\, y' - (x^2+1)\, y = 0}$$

✓ Solution by Maple
Time used: 0.015 (sec). Leaf size: 21

```
dsolve(x*(1-y(x)^2)*diff(y(x),x) = (x^2+1)*y(x),y(x), singsol=all)
```

$$y(x) = \frac{1}{\sqrt{-\frac{1}{\text{LambertW}\left(-e^{x^2} c_1 x^2\right)}}}$$

✓ Solution by Mathematica
Time used: 5.963 (sec). Leaf size: 62

```
DSolve[x(1-y[x]^2)y'[x]==(1+x^2)y[x],y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to -i\sqrt{W\left(x^2\left(-e^{x^2-2c_1}\right)\right)}$$
$$y(x) \to i\sqrt{W\left(x^2\left(-e^{x^2-2c_1}\right)\right)}$$
$$y(x) \to 0$$

23.8 problem 639

Internal problem ID [3886]

Book: Ordinary differential equations and their solutions. By George Moseley Murphy. 1960
Section: Various 23
Problem number: 639.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[[_homogeneous, ‘class G‘], _rational]`

$$x(3x - y^2)\, y' + (5x - 2y^2)\, y = 0$$

✓ Solution by Maple
Time used: 0.094 (sec). Leaf size: 35

```
dsolve(x*(3*x-y(x)^2)*diff(y(x),x)+(5*x-2*y(x)^2)*y(x) = 0,y(x), singsol=all)
```

$$\ln(x) - c_1 - \frac{2\ln\left(\frac{5y(x)^2 - 13x}{x}\right)}{65} + \frac{6\ln\left(\frac{y(x)}{\sqrt{x}}\right)}{13} = 0$$

✓ Solution by Mathematica

Time used: 7.068 (sec). Leaf size: 661

```
DSolve[x(3 x-y[x]^2)y'[x]+(5 x-2 y[x]^2)y[x]==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to \text{Root}\left[-\#1^{15} - \frac{25\#1^2 e^{\frac{65c_1}{2}}}{x^{26}} + \frac{65e^{\frac{65c_1}{2}}}{x^{25}}\&, 1\right]$$

$$y(x) \to \text{Root}\left[-\#1^{15} - \frac{25\#1^2 e^{\frac{65c_1}{2}}}{x^{26}} + \frac{65e^{\frac{65c_1}{2}}}{x^{25}}\&, 2\right]$$

$$y(x) \to \text{Root}\left[-\#1^{15} - \frac{25\#1^2 e^{\frac{65c_1}{2}}}{x^{26}} + \frac{65e^{\frac{65c_1}{2}}}{x^{25}}\&, 3\right]$$

$$y(x) \to \text{Root}\left[-\#1^{15} - \frac{25\#1^2 e^{\frac{65c_1}{2}}}{x^{26}} + \frac{65e^{\frac{65c_1}{2}}}{x^{25}}\&, 4\right]$$

$$y(x) \to \text{Root}\left[-\#1^{15} - \frac{25\#1^2 e^{\frac{65c_1}{2}}}{x^{26}} + \frac{65e^{\frac{65c_1}{2}}}{x^{25}}\&, 5\right]$$

$$y(x) \to \text{Root}\left[-\#1^{15} - \frac{25\#1^2 e^{\frac{65c_1}{2}}}{x^{26}} + \frac{65e^{\frac{65c_1}{2}}}{x^{25}}\&, 6\right]$$

$$y(x) \to \text{Root}\left[-\#1^{15} - \frac{25\#1^2 e^{\frac{65c_1}{2}}}{x^{26}} + \frac{65e^{\frac{65c_1}{2}}}{x^{25}}\&, 7\right]$$

$$y(x) \to \text{Root}\left[-\#1^{15} - \frac{25\#1^2 e^{\frac{65c_1}{2}}}{x^{26}} + \frac{65e^{\frac{65c_1}{2}}}{x^{25}}\&, 8\right]$$

$$y(x) \to \text{Root}\left[-\#1^{15} - \frac{25\#1^2 e^{\frac{65c_1}{2}}}{x^{26}} + \frac{65e^{\frac{65c_1}{2}}}{x^{25}}\&, 9\right]$$

$$y(x) \to \text{Root}\left[-\#1^{15} - \frac{25\#1^2 e^{\frac{65c_1}{2}}}{x^{26}} + \frac{65e^{\frac{65c_1}{2}}}{x^{25}}\&, 10\right]$$

$$y(x) \to \text{Root}\left[-\#1^{15} - \frac{25\#1^2 e^{\frac{65c_1}{2}}}{x^{26}} + \frac{65e^{\frac{65c_1}{2}}}{x^{25}}\&, 11\right]$$

$$y(x) \to \text{Root}\left[-\#1^{15} - \frac{25\#1^2 e^{\frac{65c_1}{2}}}{x^{26}} + \frac{65e^{\frac{65c_1}{2}}}{x^{25}}\&, 12\right]$$

$$y(x) \to \text{Root}\left[-\#1^{15} - \frac{25\#1^2 e^{\frac{65c_1}{2}}}{x^{26}} + \frac{65e^{\frac{65c_1}{2}}}{x^{25}}\&, 13\right]$$

$$y(x) \to \text{Root}\left[-\#1^{15} - \frac{25\#1^2 e^{\frac{65c_1}{2}}}{x^{26}} + \frac{65e^{\frac{65c_1}{2}}}{x^{25}}\&, 14\right]$$

$$y(x) \to \text{Root}\left[-\#1^{15} - \frac{25\#1^2 e^{\frac{65c_1}{2}}}{x^{26}} + \frac{65e^{\frac{65c_1}{2}}}{x^{25}}\&, 15\right]$$

23.9 problem 640

Internal problem ID [3887]

Book: Ordinary differential equations and their solutions. By George Moseley Murphy. 1960
Section: Various 23
Problem number: 640.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[[_homogeneous, ‘class D‘], _rational]`

$$x(y^2 + x^2)\, y' - (x^2 + x^4 + y^2)\, y = 0$$

✓ Solution by Maple
Time used: 0.031 (sec). Leaf size: 36

```
dsolve(x*(x^2+y(x)^2)*diff(y(x),x) = (x^2+x^4+y(x)^2)*y(x),y(x), singsol=all)
```

$$y(x) = \frac{e^{\frac{x^2}{2}+c_1} x}{\sqrt{\frac{e^{x^2+2c_1}}{\operatorname{LambertW}\left(e^{x^2+2c_1}\right)}}}$$

✓ Solution by Mathematica
Time used: 5.133 (sec). Leaf size: 49

```
DSolve[x(x^2+y[x]^2)y'[x]==(x^2+x^4+y[x]^2)y[x],y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to -x\sqrt{W\left(e^{x^2+2c_1}\right)}$$
$$y(x) \to x\sqrt{W\left(e^{x^2+2c_1}\right)}$$
$$y(x) \to 0$$

23.10 problem 641

Internal problem ID [3888]

Book: Ordinary differential equations and their solutions. By George Moseley Murphy. 1960
Section: Various 23
Problem number: 641.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_rational, [_1st_order, '_with_symmetry_[F(x),G(x)*y+H(x)]']]`

$$\boxed{x(1-x^2+y^2)\,y'+(1+x^2-y^2)\,y=0}$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 128

```
dsolve(x*(1-x^2+y(x)^2)*diff(y(x),x)+(1+x^2-y(x)^2)*y(x) = 0,y(x), singsol=all)
```

$$\frac{y(x)^2\,(x^2-1)}{x^2-y(x)^2-1}=-\frac{\sqrt{x+1}\,x\sqrt{x-1}}{\sqrt{\frac{c_1x^2-c_1+4}{x^2-1}}}-\frac{x^2}{2}+\frac{1}{2}$$
$$\frac{y(x)^2\,(x^2-1)}{x^2-y(x)^2-1}=\frac{\sqrt{x+1}\,x\sqrt{x-1}}{\sqrt{\frac{c_1x^2-c_1+4}{x^2-1}}}-\frac{x^2}{2}+\frac{1}{2}$$

✓ Solution by Mathematica

Time used: 1.518 (sec). Leaf size: 106

```
DSolve[x(1-x^2+y[x]^2)y'[x]+(1+x^2-y[x]^2)y[x]==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x)\to\frac{-\sqrt{x^2-4c_1x^2+4c_1{}^2}+x-2c_1x}{2c_1}$$
$$y(x)\to\frac{\sqrt{x^2-4c_1x^2+4c_1{}^2}+x-2c_1x}{2c_1}$$
$$y(x)\to\text{Indeterminate}$$
$$y(x)\to -x-1$$
$$y(x)\to 1-x$$

23.11 problem 642

Internal problem ID [3889]

Book: Ordinary differential equations and their solutions. By George Moseley Murphy. 1960
Section: Various 23
Problem number: 642.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_rational, [_1st_order, '_with_symmetry_[F(x),G(x)*y+H(x)]']]`

$$x(a-x^2-y^2)\,y'+(a+x^2+y^2)\,y=0$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 148

```
dsolve(x*(a-x^2-y(x)^2)*diff(y(x),x)+(a+x^2+y(x)^2)*y(x) = 0,y(x), singsol=all)
```

$$\frac{y(x)^2(-x^2+a)}{a-x^2-y(x)^2}=-\frac{\sqrt{x^2-a}\,x}{\sqrt{\frac{-c_1x^2+c_1a-4a}{-x^2+a}}}+\frac{x^2}{2}-\frac{a}{2}$$
$$\frac{y(x)^2(-x^2+a)}{a-x^2-y(x)^2}=\frac{\sqrt{x^2-a}\,x}{\sqrt{\frac{-c_1x^2+c_1a-4a}{-x^2+a}}}+\frac{x^2}{2}-\frac{a}{2}$$

✓ Solution by Mathematica

Time used: 1.014 (sec). Leaf size: 65

```
DSolve[x(a-x^2-y[x]^2)y'[x]+(a+x^2+y[x]^2)y[x]==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x)\to\frac{1}{2}\left(c_1x-\sqrt{-4a+(4+c_1{}^2)\,x^2}\right)$$
$$y(x)\to\frac{1}{2}\left(\sqrt{-4a+(4+c_1{}^2)\,x^2}+c_1x\right)$$

23.12 problem 643

Internal problem ID [3890]

Book: Ordinary differential equations and their solutions. By George Moseley Murphy. 1960
Section: Various 23
Problem number: 643.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[[_homogeneous, ‘class A‘], _rational, _dAlembert]`

$$x\left(y^2+2x^2\right)y'-\left(2x^2+3y^2\right)y=0$$

✓ Solution by Maple

Time used: 0.032 (sec). Leaf size: 37

```
dsolve(x*(2*x^2+y(x)^2)*diff(y(x),x) = (2*x^2+3*y(x)^2)*y(x),y(x), singsol=all)
```

$$y(x)=\mathrm{e}^{2c_1}\sqrt{2}\,\sqrt{\frac{\mathrm{e}^{-4c_1}}{x^4\,\mathrm{LambertW}\left(\frac{2\,\mathrm{e}^{-4c_1}}{x^4}\right)}}\,x^3$$

✓ Solution by Mathematica

Time used: 7.668 (sec). Leaf size: 61

```
DSolve[x(2 x^2+y[x]^2)y'[x]==(2 x^2+3 y[x]^2)y[x],y[x],x,IncludeSingularSolutions -> True]
```

$$y(x)\to-\frac{\sqrt{2}x}{\sqrt{W\left(\frac{2e^{-2c_1}}{x^4}\right)}}$$
$$y(x)\to\frac{\sqrt{2}x}{\sqrt{W\left(\frac{2e^{-2c_1}}{x^4}\right)}}$$
$$y(x)\to 0$$

23.13 problem 644

Internal problem ID [3891]

Book: Ordinary differential equations and their solutions. By George Moseley Murphy. 1960
Section: Various 23
Problem number: 644.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[[_1st_order, _with_linear_symmetries], _rational]`

$$\boxed{(x(a-x^2-y^2)+y)\,y'-(a-x^2-y^2)\,y=-x}$$

✓ Solution by Maple

Time used: 0.5 (sec). Leaf size: 37

```
dsolve((x*(a-x^2-y(x)^2)+y(x))*diff(y(x),x)+x-(a-x^2-y(x)^2)*y(x) = 0,y(x), singsol=all)
```

$$y(x)=\cot\left(\operatorname{RootOf}\left(2c_1a-2a\_Z+\ln\left(-\frac{x^2}{a\sin(\_Z)^2-x^2}\right)\right)\right)x$$

✓ Solution by Mathematica

Time used: 0.181 (sec). Leaf size: 47

```
DSolve[(x*(a-x^2-y[x]^2)+y[x])*y'[x]+x-(a-x^2-y[x]^2)*y[x]==0,y[x],x,IncludeSingularSolution
```

$$\text{Solve}\left[\frac{\log(-a+x^2+y(x)^2)-2a\tan^{-1}\left(\frac{y(x)}{x}\right)-\log(x^2+y(x)^2)}{2a}=c_1,y(x)\right]$$

23.14 problem 645

Internal problem ID [3892]

Book: Ordinary differential equations and their solutions. By George Moseley Murphy. 1960
Section: Various 23
Problem number: 645.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_separable]`

$$\boxed{x(y+a)^2 y' - b y^2 = 0}$$

✓ Solution by Maple
Time used: 0.093 (sec). Leaf size: 33

```
dsolve(x*(a+y(x))^2*diff(y(x),x) = b*y(x)^2,y(x), singsol=all)
```

$$y(x) = e^{\operatorname{RootOf}\left(\ln(x) b\, e^{\_Z} + c_1 b\, e^{\_Z} - 2\_Z a\, e^{\_Z} - e^{2\_Z} + a^2\right)}$$

✓ Solution by Mathematica
Time used: 0.425 (sec). Leaf size: 37

```
DSolve[x(a+y[x])^2 y'[x]==b y[x]^2,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to \text{InverseFunction}\left[-\frac{a^2}{\#1} + 2a\log(\#1) + \#1\&\right][b\log(x) + c_1]$$
$$y(x) \to 0$$

23.15 problem 646

Internal problem ID [3893]

Book: Ordinary differential equations and their solutions. By George Moseley Murphy. 1960
Section: Various 23
Problem number: 646.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[[_homogeneous, ‘class A‘], _rational, _dAlembert]`

$$\boxed{x(x^2 - yx + y^2)\,y' + (x^2 + yx + y^2)\,y = 0}$$

✓ Solution by Maple

Time used: 0.016 (sec). Leaf size: 22

```
dsolve(x*(x^2-x*y(x)+y(x)^2)*diff(y(x),x)+(x^2+x*y(x)+y(x)^2)*y(x) = 0,y(x), singsol=all)
```

$$y(x) = \tan\left(\mathrm{RootOf}\left(\ln\left(\tan\left(\_Z\right)\right) - \_Z + 2\ln\left(x\right) + 2c_1\right)\right)x$$

✓ Solution by Mathematica

Time used: 0.13 (sec). Leaf size: 28

```
DSolve[x(x^2-x y[x]+y[x]^2)y'[x]+(x^2+x y[x]+y[x]^2)y[x]==0,y[x],x,IncludeSingularSolutions
```

$$\text{Solve}\left[\log\left(\frac{y(x)}{x}\right) - \arctan\left(\frac{y(x)}{x}\right) = -2\log(x) + c_1, y(x)\right]$$

23.16 problem 647

Internal problem ID [3894]

Book: Ordinary differential equations and their solutions. By George Moseley Murphy. 1960
Section: Various 23
Problem number: 647.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[[_homogeneous, 'class A'], _rational, _dAlembert]`

$$x\left(x^{2}-yx-y^{2}\right)y'-\left(x^{2}+yx-y^{2}\right)y=0$$

✓ Solution by Maple

Time used: 0.235 (sec). Leaf size: 29

```
dsolve(x*(x^2-x*y(x)-y(x)^2)*diff(y(x),x) = (x^2+x*y(x)-y(x)^2)*y(x),y(x), singsol=all)
```

$$y(x)=\mathrm{e}^{\mathrm{RootOf}\left(2\,\mathrm{e}^{\_Z}\ln(x)+\mathrm{e}^{2\_Z}+2c_1\mathrm{e}^{\_Z}+\_Z\mathrm{e}^{\_Z}+1\right)}x$$

✓ Solution by Mathematica

Time used: 0.172 (sec). Leaf size: 31

```
DSolve[x(x^2-x y[x]-y[x]^2)y'[x]==(x^2+x y[x]-y[x]^2)y[x],y[x],x,IncludeSingularSolutions ->
```

$$\mathrm{Solve}\left[\frac{x}{y(x)}+\frac{y(x)}{x}+\log\left(\frac{y(x)}{x}\right)=-2\log(x)+c_1,y(x)\right]$$

23.17 problem 648

Internal problem ID [3895]

Book: Ordinary differential equations and their solutions. By George Moseley Murphy. 1960
Section: Various 23
Problem number: 648.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[[_homogeneous, 'class A'], _rational, _dAlembert]`

$$x(x^2 + axy + y^2)\,y' - (x^2 + bxy + y^2)\,y = 0$$

✓ Solution by Maple

Time used: 0.047 (sec). Leaf size: 43

```
dsolve(x*(x^2+a*x*y(x)+y(x)^2)*diff(y(x),x) = (x^2+b*x*y(x)+y(x)^2)*y(x),y(x), singsol=all)
```

$$y(x) = e^{\mathrm{RootOf}\left(e^{-\_Z} a \ln(x) - \ln(x) b\, e^{-\_Z} + e^{-\_Z} c_1 a - c_1 b\, e^{-\_Z} + \_Z a\, e^{-\_Z} + e^{2\_Z} - 1\right)} x$$

✓ Solution by Mathematica

Time used: 0.265 (sec). Leaf size: 38

```
DSolve[x(x^2+a x y[x]+y[x]^2)y'[x]==(x^2+b x y[x]+y[x]^2)y[x],y[x],x,IncludeSingularSolution
```

$$\mathrm{Solve}\left[a \log\left(\frac{y(x)}{x}\right) - \frac{x}{y(x)} + \frac{y(x)}{x} = (b-a)\log(x) + c_1, y(x)\right]$$

23.18 problem 649

Internal problem ID [3896]

Book: Ordinary differential equations and their solutions. By George Moseley Murphy. 1960
Section: Various 23
Problem number: 649.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[[_homogeneous, ‘class A‘], _rational, _dAlembert]`

$$\boxed{x(x^2 - 2y^2)\, y' - (-y^2 + 2x^2)\, y = 0}$$

✓ Solution by Maple

Time used: 0.562 (sec). Leaf size: 1765

```
dsolve(x*(x^2-2*y(x)^2)*diff(y(x),x) = (2*x^2-y(x)^2)*y(x),y(x), singsol=all)
```

$$y(x) = -\frac{6x^2c_1\sqrt{3}\,\sqrt{2}\left(8-108c_1^2x^2+12\sqrt{3}\,\sqrt{27c_1^4x^4-\cdots}\right.}{\left(\left(8-108c_1^2x^2+12\sqrt{3}\,\sqrt{27c_1^4x^4-4c_1^2x^2}\right)^{\frac{2}{3}}+2\left(8-108c_1^2x^2+12\sqrt{3}\,\sqrt{27c_1^4x^4-4c_1^2x^2}\right)^{\frac{1}{3}}+4\right)\sqrt{\cdots}}$$

$$y(x) = \frac{6x^2c_1\sqrt{3}\,\sqrt{2}\left(8-108c_1^2x^2+12\sqrt{3}\,\sqrt{27c_1^4x^4-4c\cdots}\right.}{\left(\left(8-108c_1^2x^2+12\sqrt{3}\,\sqrt{27c_1^4x^4-4c_1^2x^2}\right)^{\frac{2}{3}}+2\left(8-108c_1^2x^2+12\sqrt{3}\,\sqrt{27c_1^4x^4-4c_1^2x^2}\right)^{\frac{1}{3}}+4\right)\sqrt{\frac{\left(8-\cdots\right.}{\cdots}}}$$

$$y(x) = \frac{\cdots}{\left(\left(8-108c_1^2x^2+12\sqrt{3}\,\sqrt{27c_1^4x^4-4c_1^2x^2}\right)^{\frac{1}{3}}-2\right)\sqrt{\frac{\left(-\left(8-108c_1^2x^2+12\sqrt{3}\,\sqrt{27c_1^4x^4-4c_1^2x^2}\right)^{\frac{1}{3}}+2\right)\left(-2+i\left(\left(8-108c\cdots\right.\right.\right.}{\left(8-108c_1^2x^2\cdots\right.}}}$$

$$y(x) = \frac{\cdots}{\left(-\left(8-108c_1^2x^2+12\sqrt{3}\,\sqrt{27c_1^4x^4-4c_1^2x^2}\right)^{\frac{1}{3}}+2\right)\sqrt{\frac{\left(-\left(8-108c_1^2x^2+12\sqrt{3}\,\sqrt{27c_1^4x^4-4c_1^2x^2}\right)^{\frac{1}{3}}+2\right)\left(-2+i\left(\left(8-10\cdots\right.\right.\right.}{\left(8-108c\cdots\right.}}}$$

$$y(x) = -\frac{24x^2c_1\sqrt{3}\left(\cdots\right.}{\sqrt{\frac{\left(\left(8-108c_1^2x^2+12\sqrt{3}\,\sqrt{27c_1^4x^4-4c_1^2x^2}\right)^{\frac{1}{3}}-2\right)\left(2+i\left(\left(8-108c_1^2x^2+12\sqrt{3}\,\sqrt{27c_1^4x^4-4c_1^2x^2}\right)^{\frac{1}{3}}+2\right)\sqrt{3}-\left(8-108c_1^2x^2+12\sqrt{3}\,\sqrt{27c_1^4\cdots}\right.\right.}{\left(8-108c_1^2x^2+12\sqrt{3}\,\sqrt{27c_1^4x^4-4c_1^2x^2}\right)^{\frac{1}{3}}}}}$$

$$y(x) = \frac{24x^2c_1\sqrt{3}\left(8\cdots\right.}{\sqrt{\frac{\left(\left(8-108c_1^2x^2+12\sqrt{3}\,\sqrt{27c_1^4x^4-4c_1^2x^2}\right)^{\frac{1}{3}}-2\right)\left(2+i\left(\left(8-108c_1^2x^2+12\sqrt{3}\,\sqrt{27c_1^4x^4-4c_1^2x^2}\right)^{\frac{1}{3}}+2\right)\sqrt{3}-\left(8-108c_1^2x^2+12\sqrt{3}\,\sqrt{27c_1^4x^4-\cdots}\right.\right.}{\left(8-108c_1^2x^2+12\sqrt{3}\,\sqrt{27c_1^4x^4-4c_1^2x^2}\right)^{\frac{1}{3}}}}}$$

✓ Solution by Mathematica

Time used: 60.285 (sec). Leaf size: 873

```
DSolve[x(x^2-2 y[x]^2)y'[x]==(2 x^2-y[x]^2)y[x],y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to -\sqrt{-x^2 + \frac{\sqrt[3]{\sqrt{81e^{4c_1}x^8 - 12e^{6c_1}x^6} - 9e^{2c_1}x^4}}{\sqrt[3]{2}3^{2/3}} + \frac{\sqrt[3]{\frac{2}{3}}e^{2c_1}x^2}{\sqrt[3]{\sqrt{81e^{4c_1}x^8 - 12e^{6c_1}x^6} - 9e^{2c_1}x^4}}}$$

$$y(x) \to \sqrt{-x^2 + \frac{\sqrt[3]{\sqrt{81e^{4c_1}x^8 - 12e^{6c_1}x^6} - 9e^{2c_1}x^4}}{\sqrt[3]{2}3^{2/3}} + \frac{\sqrt[3]{\frac{2}{3}}e^{2c_1}x^2}{\sqrt[3]{\sqrt{81e^{4c_1}x^8 - 12e^{6c_1}x^6} - 9e^{2c_1}x^4}}}$$

$$y(x) \to -\frac{1}{2}\sqrt{-4x^2 + \left(\frac{2}{3}\right)^{2/3}\left(-1 - i\sqrt{3}\right)\sqrt[3]{\sqrt{81e^{4c_1}x^8 - 12e^{6c_1}x^6} - 9e^{2c_1}x^4} - \frac{2\sqrt[3]{2}\left(\sqrt{3} - 3i\right)e^{2c_1}x^2}{3^{5/6}\sqrt[3]{\sqrt{81e^{4c_1}x^8 - 12e^{6c_1}x^6} -}}}$$

$$y(x) \to \frac{1}{2}\sqrt{-4x^2 + \left(\frac{2}{3}\right)^{2/3}\left(-1 - i\sqrt{3}\right)\sqrt[3]{\sqrt{81e^{4c_1}x^8 - 12e^{6c_1}x^6} - 9e^{2c_1}x^4} - \frac{2\sqrt[3]{2}\left(\sqrt{3} - 3i\right)e^{2c_1}x^2}{3^{5/6}\sqrt[3]{\sqrt{81e^{4c_1}x^8 - 12e^{6c_1}x^6} - 9}}}$$

$$y(x) \to -\frac{1}{2}\sqrt{-4x^2 + i\left(\frac{2}{3}\right)^{2/3}\left(\sqrt{3} + i\right)\sqrt[3]{\sqrt{81e^{4c_1}x^8 - 12e^{6c_1}x^6} - 9e^{2c_1}x^4} - \frac{2\sqrt[3]{2}\left(\sqrt{3} + 3i\right)e^{2c_1}x^2}{3^{5/6}\sqrt[3]{\sqrt{81e^{4c_1}x^8 - 12e^{6c_1}x^6} - 9}}}$$

$$y(x) \to \frac{1}{2}\sqrt{-4x^2 + i\left(\frac{2}{3}\right)^{2/3}\left(\sqrt{3} + i\right)\sqrt[3]{\sqrt{81e^{4c_1}x^8 - 12e^{6c_1}x^6} - 9e^{2c_1}x^4} - \frac{2\sqrt[3]{2}\left(\sqrt{3} + 3i\right)e^{2c_1}x^2}{3^{5/6}\sqrt[3]{\sqrt{81e^{4c_1}x^8 - 12e^{6c_1}x^6} - 9e^2}}}$$

23.19 problem 650

Internal problem ID [3897]

Book: Ordinary differential equations and their solutions. By George Moseley Murphy. 1960
Section: Various 23
Problem number: 650.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[[_homogeneous, ‘class A‘], _rational, _dAlembert]`

$$x(x^2 + 2y^2) y' - (2x^2 + 3y^2) y = 0$$

✓ Solution by Maple

Time used: 0.656 (sec). Leaf size: 89

```
dsolve(x*(x^2+2*y(x)^2)*diff(y(x),x) = (2*x^2+3*y(x)^2)*y(x),y(x), singsol=all)
```

$$y(x) = -\frac{\sqrt{-2 - 2\sqrt{4c_1x^2 + 1}}\,x}{2}$$
$$y(x) = \frac{\sqrt{-2 - 2\sqrt{4c_1x^2 + 1}}\,x}{2}$$
$$y(x) = -\frac{\sqrt{-2 + 2\sqrt{4c_1x^2 + 1}}\,x}{2}$$
$$y(x) = \frac{\sqrt{-2 + 2\sqrt{4c_1x^2 + 1}}\,x}{2}$$

✓ Solution by Mathematica

Time used: 42.486 (sec). Leaf size: 277

```
DSolve[x(x^2+2 y[x]^2)y'[x]==(2 x^2+3 y[x]^2)y[x],y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to -\frac{\sqrt{-x^2 - \sqrt{x^4 + 4e^{2c_1}x^6}}}{\sqrt{2}}$$
$$y(x) \to \frac{\sqrt{-x^2 - \sqrt{x^4 + 4e^{2c_1}x^6}}}{\sqrt{2}}$$
$$y(x) \to -\frac{\sqrt{-x^2 + \sqrt{x^4 + 4e^{2c_1}x^6}}}{\sqrt{2}}$$
$$y(x) \to \sqrt{-\frac{x^2}{2} + \frac{1}{2}\sqrt{x^4 + 4e^{2c_1}x^6}}$$
$$y(x) \to -\frac{\sqrt{-\sqrt{x^4} - x^2}}{\sqrt{2}}$$
$$y(x) \to \frac{\sqrt{-\sqrt{x^4} - x^2}}{\sqrt{2}}$$
$$y(x) \to -\frac{\sqrt{\sqrt{x^4} - x^2}}{\sqrt{2}}$$
$$y(x) \to \frac{\sqrt{\sqrt{x^4} - x^2}}{\sqrt{2}}$$

23.20 problem 651

Internal problem ID [3898]

Book: Ordinary differential equations and their solutions. By George Moseley Murphy. 1960
Section: Various 23
Problem number: 651.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[[_homogeneous, ‘class A‘], _rational, _dAlembert]`

$$\boxed{2x\left(5x^2+y^2\right)y'-x^2y+y^3=0}$$

✓ Solution by Maple
Time used: 0.688 (sec). Leaf size: 29

```
dsolve(2*x*(5*x^2+y(x)^2)*diff(y(x),x) = x^2*y(x)-y(x)^3,y(x), singsol=all)
```

$$y(x)=\operatorname{RootOf}\left(\_Z^{45}c_1x^9-\_Z^{18}-6\_Z^9-9\right)^{\frac{9}{2}}x$$

✓ Solution by Mathematica
Time used: 2.771 (sec). Leaf size: 216

```
DSolve[2 x(5 x^2+y[x]^2)y'[x]==x^2 y[x]-y[x]^3,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x)\to \operatorname{Root}\left[-\#1^5+\frac{\#1^2e^{3c_1}}{x^{3/2}}+3e^{3c_1}\sqrt{x}\&,1\right]$$
$$y(x)\to \operatorname{Root}\left[-\#1^5+\frac{\#1^2e^{3c_1}}{x^{3/2}}+3e^{3c_1}\sqrt{x}\&,2\right]$$
$$y(x)\to \operatorname{Root}\left[-\#1^5+\frac{\#1^2e^{3c_1}}{x^{3/2}}+3e^{3c_1}\sqrt{x}\&,3\right]$$
$$y(x)\to \operatorname{Root}\left[-\#1^5+\frac{\#1^2e^{3c_1}}{x^{3/2}}+3e^{3c_1}\sqrt{x}\&,4\right]$$
$$y(x)\to \operatorname{Root}\left[-\#1^5+\frac{\#1^2e^{3c_1}}{x^{3/2}}+3e^{3c_1}\sqrt{x}\&,5\right]$$

23.21 problem 652

Internal problem ID [3899]

Book: Ordinary differential equations and their solutions. By George Moseley Murphy. 1960
Section: Various 23
Problem number: 652.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[[_homogeneous, 'class A'], _rational, _dAlembert]`

$$x\left(x^2+axy+2y^2\right)y'-(ax+2y)\,y^2=0$$

✓ Solution by Maple
Time used: 0.078 (sec). Leaf size: 21

```
dsolve(x*(x^2+a*x*y(x)+2*y(x)^2)*diff(y(x),x) = (a*x+2*y(x))*y(x)^2,y(x), singsol=all)
```

$$y(x)=\mathrm{e}^{\mathrm{RootOf}\left(\mathrm{e}^{2\_Z}+a\,\mathrm{e}^{\_Z}+c_1+\_Z+\ln(x)\right)}x$$

✓ Solution by Mathematica
Time used: 0.171 (sec). Leaf size: 34

```
DSolve[x(x^2+a x y[x]+2 y[x]^2)y'[x]==(a x+2 y[x])y[x]^2,y[x],x,IncludeSingularSolutions ->
```

$$\mathrm{Solve}\left[\frac{ay(x)}{x}+\frac{y(x)^2}{x^2}+\log\left(\frac{y(x)}{x}\right)=-\log(x)+c_1,y(x)\right]$$

23.22 problem 653

Internal problem ID [3900]

Book: Ordinary differential equations and their solutions. By George Moseley Murphy. 1960
Section: Various 23
Problem number: 653.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[[_homogeneous, ‘class G‘], _exact, _rational, _Bernoulli]`

$$3xy^2y' + y^3 = 2x$$

✓ Solution by Maple
Time used: 0.016 (sec). Leaf size: 73

```
dsolve(3*x*y(x)^2*diff(y(x),x) = 2*x-y(x)^3,y(x), singsol=all)
```

$$y(x) = \frac{\left(\left(x^2+c_1\right)x^2\right)^{\frac{1}{3}}}{x}$$
$$y(x) = -\frac{\left(\left(x^2+c_1\right)x^2\right)^{\frac{1}{3}}\left(1+i\sqrt{3}\right)}{2x}$$
$$y(x) = \frac{\left(\left(x^2+c_1\right)x^2\right)^{\frac{1}{3}}\left(i\sqrt{3}-1\right)}{2x}$$

✓ Solution by Mathematica
Time used: 0.22 (sec). Leaf size: 72

```
DSolve[3 x y[x]^2 y'[x]==2 x-y[x]^3,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to \frac{\sqrt[3]{x^2+c_1}}{\sqrt[3]{x}}$$
$$y(x) \to -\frac{\sqrt[3]{-1}\sqrt[3]{x^2+c_1}}{\sqrt[3]{x}}$$
$$y(x) \to \frac{(-1)^{2/3}\sqrt[3]{x^2+c_1}}{\sqrt[3]{x}}$$

23.23 problem 654

Internal problem ID [3901]

Book: Ordinary differential equations and their solutions. By George Moseley Murphy. 1960
Section: Various 23
Problem number: 654.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_rational, [_1st_order, '_with_symmetry_[F(x)*G(y),0]']]`

$$\boxed{(1 - 4x + 3y^2x)\, y' - (2 - y^2)\, y = 0}$$

✓ Solution by Maple

Time used: 0.016 (sec). Leaf size: 24

```
dsolve((1-4*x+3*x*y(x)^2)*diff(y(x),x) = (2-y(x)^2)*y(x),y(x), singsol=all)
```

$$x + \frac{1}{y(x)^2} - \frac{c_1}{\sqrt{y(x)^2 - 2}\, y(x)^2} = 0$$

✓ Solution by Mathematica

Time used: 60.163 (sec). Leaf size: 2348

```
DSolve[(1-4 x+3 x y[x]^2)y'[x]==(2-y[x]^2)y[x],y[x],x,IncludeSingularSolutions -> True]
```

Too large to display

23.24 problem 655

Internal problem ID [3902]

Book: Ordinary differential equations and their solutions. By George Moseley Murphy. 1960
Section: Various 23
Problem number: 655.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[[_homogeneous, 'class G'], _exact, _rational]`

$$x\left(-3y^2+x\right)y'+\left(2x-y^2\right)y=0$$

✓ Solution by Maple

Time used: 0.062 (sec). Leaf size: 242

```
dsolve(x*(x-3*y(x)^2)*diff(y(x),x)+(2*x-y(x)^2)*y(x) = 0,y(x), singsol=all)
```

$$y(x)=\frac{12^{\frac{1}{3}}\left(x^3 12^{\frac{1}{3}}+\left(\left(\sqrt{-12x^5+81c_1^2}+9c_1\right)x^2\right)^{\frac{2}{3}}\right)}{6x\left(\left(\sqrt{-12x^5+81c_1^2}+9c_1\right)x^2\right)^{\frac{1}{3}}}$$

$$y(x)=\frac{3^{\frac{1}{3}}2^{\frac{2}{3}}\left(\left(-1-i\sqrt{3}\right)\left(\left(\sqrt{-12x^5+81c_1^2}+9c_1\right)x^2\right)^{\frac{2}{3}}+\left(i3^{\frac{5}{6}}-3^{\frac{1}{3}}\right)2^{\frac{2}{3}}x^3\right)}{12\left(\left(\sqrt{-12x^5+81c_1^2}+9c_1\right)x^2\right)^{\frac{1}{3}}x}$$

$$y(x)=-\frac{3^{\frac{1}{3}}2^{\frac{2}{3}}\left(\left(1-i\sqrt{3}\right)\left(\left(\sqrt{-12x^5+81c_1^2}+9c_1\right)x^2\right)^{\frac{2}{3}}+\left(i3^{\frac{5}{6}}+3^{\frac{1}{3}}\right)2^{\frac{2}{3}}x^3\right)}{12\left(\left(\sqrt{-12x^5+81c_1^2}+9c_1\right)x^2\right)^{\frac{1}{3}}x}$$

✓ Solution by Mathematica

Time used: 33.937 (sec). Leaf size: 328

```
DSolve[x(x-3 y[x]^2)y'[x]+(2 x-y[x]^2)y[x]==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to -\frac{2\sqrt[3]{3}x^3 + \sqrt[3]{2}\left(9c_1x^2 + \sqrt{-12x^9 + 81c_1{}^2x^4}\right)^{2/3}}{6^{2/3}x\sqrt[3]{9c_1x^2 + \sqrt{-12x^9 + 81c_1{}^2x^4}}}$$

$$y(x) \to \frac{2\sqrt[3]{2}\sqrt[6]{3}\left(\sqrt{3} + 3i\right)x^3 + \sqrt[3]{3}\left(1 - i\sqrt{3}\right)\left(18c_1x^2 + 2\sqrt{-12x^9 + 81c_1{}^2x^4}\right)^{2/3}}{12x\sqrt[3]{9c_1x^2 + \sqrt{-12x^9 + 81c_1{}^2x^4}}}$$

$$y(x) \to \frac{2\sqrt[3]{2}\sqrt[6]{3}\left(\sqrt{3} - 3i\right)x^3 + \sqrt[3]{3}\left(1 + i\sqrt{3}\right)\left(18c_1x^2 + 2\sqrt{-12x^9 + 81c_1{}^2x^4}\right)^{2/3}}{12x\sqrt[3]{9c_1x^2 + \sqrt{-12x^9 + 81c_1{}^2x^4}}}$$

23.25 problem 656

Internal problem ID [3903]

Book: Ordinary differential equations and their solutions. By George Moseley Murphy. 1960
Section: Various 23
Problem number: 656.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_rational]`

$$\boxed{3x(x+y^2)\,y'-3yx-2y^3=-x^3}$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 360

```
dsolve(3*x*(x+y(x)^2)*diff(y(x),x)+x^3-3*x*y(x)-2*y(x)^3 = 0,y(x), singsol=all)
```

$$y(x)=\frac{\left(-4c_1x^2-4x^3+4\sqrt{x^3\left(c_1^2x+2c_1x^2+x^3+4\right)}\right)^{\frac{2}{3}}-4x}{2\left(-4c_1x^2-4x^3+4\sqrt{x^3\left(c_1^2x+2c_1x^2+x^3+4\right)}\right)^{\frac{1}{3}}}$$

$$y(x)=-\frac{i\sqrt{3}\left(-4c_1x^2-4x^3+4\sqrt{x^3\left(c_1^2x+2c_1x^2+x^3+4\right)}\right)^{\frac{2}{3}}+4i\sqrt{3}\,x+\left(-4c_1x^2-4x^3+4\sqrt{x^3\left(c_1^2x+2c\right.}\right.}{4\left(-4c_1x^2-4x^3+4\sqrt{x^3\left(c_1^2x+2c_1x^2+x^3+4\right)}\right)^{\frac{1}{3}}}$$

$$y(x)=\frac{i\sqrt{3}\left(-4c_1x^2-4x^3+4\sqrt{x^3\left(c_1^2x+2c_1x^2+x^3+4\right)}\right)^{\frac{2}{3}}+4i\sqrt{3}\,x-\left(-4c_1x^2-4x^3+4\sqrt{x^3\left(c_1^2x+2c_1x^2\right.}\right.}{4\left(-4c_1x^2-4x^3+4\sqrt{x^3\left(c_1^2x+2c_1x^2+x^3+4\right)}\right)^{\frac{1}{3}}}$$

✓ Solution by Mathematica

Time used: 28.201 (sec). Leaf size: 362

```
DSolve[3 x(x+y[x]^2)y'[x]+x^3-3 x y[x]-2 y[x]^3==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to \frac{\sqrt[3]{-x^3+c_1x^2+\sqrt{x^3\left(x^3-2c_1x^2+c_1{}^2x+4\right)}}}{\sqrt[3]{2}} - \frac{\sqrt[3]{2}x}{\sqrt[3]{-x^3+c_1x^2+\sqrt{x^3\left(x^3-2c_1x^2+c_1{}^2x+4\right)}}}$$

$$y(x) \to \frac{i2^{2/3}\left(\sqrt{3}+i\right)\left(-x^3+c_1x^2+\sqrt{x^3\left(x^3-2c_1x^2+c_1{}^2x+4\right)}\right)^{2/3}+\sqrt[3]{2}\left(2+2i\sqrt{3}\right)x}{4\sqrt[3]{-x^3+c_1x^2+\sqrt{x^3\left(x^3-2c_1x^2+c_1{}^2x+4\right)}}}$$

$$y(x) \to \frac{\sqrt[3]{2}\left(2-2i\sqrt{3}\right)x-i2^{2/3}\left(\sqrt{3}-i\right)\left(-x^3+c_1x^2+\sqrt{x^3\left(x^3-2c_1x^2+c_1{}^2x+4\right)}\right)^{2/3}}{4\sqrt[3]{-x^3+c_1x^2+\sqrt{x^3\left(x^3-2c_1x^2+c_1{}^2x+4\right)}}}$$

23.26 problem 657

Internal problem ID [3904]

Book: Ordinary differential equations and their solutions. By George Moseley Murphy. 1960
Section: Various 23
Problem number: 657.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_rational, [_1st_order, '_with_symmetry_[F(x)*G(y),0]']]`

$$x(x^3 - 3yx^3 + 4y^2)\, y' - 6y^3 = 0$$

✓ Solution by Maple
Time used: 0.046 (sec). Leaf size: 31

```
dsolve(x*(x^3-3*x^3*y(x)+4*y(x)^2)*diff(y(x),x) = 6*y(x)^3,y(x), singsol=all)
```

$$y(x) = e^{\mathrm{RootOf}\left(-3x^3 e^{\_Z} + 6c_1 x^3 + x^3\_Z + 2\,e^{2\_Z}\right)}$$

✓ Solution by Mathematica
Time used: 0.155 (sec). Leaf size: 27

```
DSolve[x(x^3-3 x^3 y[x]+4 y[x]^2)y'[x]==6 y[x]^3,y[x],x,IncludeSingularSolutions -> True]
```

$$\mathrm{Solve}\left[\frac{y(x)^2}{x^3} + \frac{1}{2}(\log(y(x)) - 3y(x)) = c_1, y(x)\right]$$

23.27 problem 658

Internal problem ID [3905]

Book: Ordinary differential equations and their solutions. By George Moseley Murphy. 1960
Section: Various 23
Problem number: 658.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[[_homogeneous, ‘class G‘], _exact, _rational, _Bernoulli]`

$$\boxed{6xy^2y' + 2y^3 = -x}$$

✓ Solution by Maple
Time used: 0.016 (sec). Leaf size: 92

```
dsolve(6*x*y(x)^2*diff(y(x),x)+x+2*y(x)^3 = 0,y(x), singsol=all)
```

$$y(x) = \frac{2^{\frac{1}{3}}\left(-\left(x^2-4c_1\right)x^2\right)^{\frac{1}{3}}}{2x}$$
$$y(x) = -\frac{2^{\frac{1}{3}}\left(-\left(x^2-4c_1\right)x^2\right)^{\frac{1}{3}}\left(1+i\sqrt{3}\right)}{4x}$$
$$y(x) = \frac{2^{\frac{1}{3}}\left(-\left(x^2-4c_1\right)x^2\right)^{\frac{1}{3}}\left(i\sqrt{3}-1\right)}{4x}$$

✓ Solution by Mathematica
Time used: 0.241 (sec). Leaf size: 99

```
DSolve[6 x y[x]^2 y'[x]+x+2 y[x]^3==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to \frac{\sqrt[3]{-x^2+4c_1}}{2^{2/3}\sqrt[3]{x}}$$
$$y(x) \to -\frac{\sqrt[3]{-1}\sqrt[3]{-x^2+4c_1}}{2^{2/3}\sqrt[3]{x}}$$
$$y(x) \to \frac{(-1)^{2/3}\sqrt[3]{-x^2+4c_1}}{2^{2/3}\sqrt[3]{x}}$$

23.28 problem 659

Internal problem ID [3906]

Book: Ordinary differential equations and their solutions. By George Moseley Murphy. 1960
Section: Various 23
Problem number: 659.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[[_homogeneous, ‘class G‘], _rational]`

$$\boxed{x(x+6y^2)\,y'+yx-3y^3=0}$$

✓ Solution by Maple

Time used: 0.063 (sec). Leaf size: 38

```
dsolve(x*(x+6*y(x)^2)*diff(y(x),x)+x*y(x)-3*y(x)^3 = 0,y(x), singsol=all)
```

$$y(x)=\frac{\mathrm{e}^{\frac{3c_1}{2}}\sqrt{6}}{6x\sqrt{\frac{\mathrm{e}^{3c_1}}{x^3\,\mathrm{LambertW}\left(\frac{6\,\mathrm{e}^{3c_1}}{x^3}\right)}}}$$

✓ Solution by Mathematica

Time used: 4.288 (sec). Leaf size: 69

```
DSolve[x(x+6 y[x]^2)y'[x]+x y[x]-3 y[x]^3==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x)\to -\frac{\sqrt{x}\sqrt{W\left(\frac{6e^{3c_1}}{x^3}\right)}}{\sqrt{6}}$$
$$y(x)\to \frac{\sqrt{x}\sqrt{W\left(\frac{6e^{3c_1}}{x^3}\right)}}{\sqrt{6}}$$
$$y(x)\to 0$$

23.29 problem 660

Internal problem ID [3907]

Book: Ordinary differential equations and their solutions. By George Moseley Murphy. 1960
Section: Various 23
Problem number: 660.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[[_homogeneous, 'class A'], _rational, _dAlembert]`

$$\boxed{x(x^2 - 6y^2)\, y' - 4(3y^2 + x^2)\, y = 0}$$

✓ Solution by Maple

Time used: 0.688 (sec). Leaf size: 53

```
dsolve(x*(x^2-6*y(x)^2)*diff(y(x),x) = 4*(x^2+3*y(x)^2)*y(x),y(x), singsol=all)
```

$$y(x) = -\frac{c_1\left(-1+\sqrt{\frac{-24x^6+c_1^2}{c_1^2}}\right)}{12x^2}$$
$$y(x) = \frac{c_1\left(1+\sqrt{\frac{-24x^6+c_1^2}{c_1^2}}\right)}{12x^2}$$

✓ Solution by Mathematica

Time used: 1.071 (sec). Leaf size: 67

```
DSolve[x(x^2-6 y[x]^2)y'[x]==4(x^2+3 y[x]^2)y[x],y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to \frac{e^{c_1} - \sqrt{-24x^6 + e^{2c_1}}}{12x^2}$$
$$y(x) \to \frac{\sqrt{-24x^6 + e^{2c_1}} + e^{c_1}}{12x^2}$$

23.30 problem 661

Internal problem ID [3908]

Book: Ordinary differential equations and their solutions. By George Moseley Murphy. 1960
Section: Various 23
Problem number: 661.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[[_homogeneous, ‘class G‘], _rational]`

$$x(3x-7y^2)\,y'+(5x-3y^2)\,y=0$$

✓ Solution by Maple

Time used: 8.578 (sec). Leaf size: 49

```
dsolve(x*(3*x-7*y(x)^2)*diff(y(x),x)+(5*x-3*y(x)^2)*y(x) = 0,y(x), singsol=all)
```

$$y(x)=\text{RootOf}\left(x^{\frac{3}{2}}\_Z^7-x^{\frac{5}{2}}\_Z^3-c_1\right)^2$$
$$y(x)=\text{RootOf}\left(x^{\frac{3}{2}}\_Z^7-x^{\frac{5}{2}}\_Z^3+c_1\right)^2$$

✓ Solution by Mathematica

Time used: 4.798 (sec). Leaf size: 288

```
DSolve[x(3 x-7 y[x]^2)y'[x]+(5  x-3 y[x]^2)y[x]==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x)\to\text{Root}\left[4\#1^7x^3-8\#1^5x^4+4\#1^3x^5-c_1{}^2\&,1\right]$$
$$y(x)\to\text{Root}\left[4\#1^7x^3-8\#1^5x^4+4\#1^3x^5-c_1{}^2\&,2\right]$$
$$y(x)\to\text{Root}\left[4\#1^7x^3-8\#1^5x^4+4\#1^3x^5-c_1{}^2\&,3\right]$$
$$y(x)\to\text{Root}\left[4\#1^7x^3-8\#1^5x^4+4\#1^3x^5-c_1{}^2\&,4\right]$$
$$y(x)\to\text{Root}\left[4\#1^7x^3-8\#1^5x^4+4\#1^3x^5-c_1{}^2\&,5\right]$$
$$y(x)\to\text{Root}\left[4\#1^7x^3-8\#1^5x^4+4\#1^3x^5-c_1{}^2\&,6\right]$$
$$y(x)\to\text{Root}\left[4\#1^7x^3-8\#1^5x^4+4\#1^3x^5-c_1{}^2\&,7\right]$$

23.31 problem 662

Internal problem ID [3909]

Book: Ordinary differential equations and their solutions. By George Moseley Murphy. 1960
Section: Various 23
Problem number: 662.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_separable]`

$$\boxed{y^2 y' x^2 = -x^3 + x - 1}$$

✓ Solution by Maple
Time used: 0.016 (sec). Leaf size: 113

```
dsolve(x^2*y(x)^2*diff(y(x),x)+1-x+x^3 = 0,y(x), singsol=all)
```

$$y(x) = \frac{2^{\frac{2}{3}} \left(-3x^2 \left(x^3 - \frac{2c_1 x}{3} - 2x \ln(x) - 2\right)\right)^{\frac{1}{3}}}{2x}$$
$$y(x) = -\frac{2^{\frac{2}{3}} \left(-3x^2 \left(x^3 - \frac{2c_1 x}{3} - 2x \ln(x) - 2\right)\right)^{\frac{1}{3}} \left(1 + i\sqrt{3}\right)}{4x}$$
$$y(x) = \frac{2^{\frac{2}{3}} \left(-3x^2 \left(x^3 - \frac{2c_1 x}{3} - 2x \ln(x) - 2\right)\right)^{\frac{1}{3}} \left(i\sqrt{3} - 1\right)}{4x}$$

✓ Solution by Mathematica

Time used: 0.341 (sec). Leaf size: 111

```
DSolve[x^2 y[x]^2 y'[x]+1-x+x^3==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to -\frac{\sqrt[3]{-\frac{3}{2}}\sqrt[3]{-x^3 + 2x\log(x) + 2c_1x + 2}}{\sqrt[3]{x}}$$
$$y(x) \to \frac{\sqrt[3]{-\frac{3x^3}{2} + 3x\log(x) + 3c_1x + 3}}{\sqrt[3]{x}}$$
$$y(x) \to \frac{(-1)^{2/3}\sqrt[3]{-\frac{3x^3}{2} + 3x\log(x) + 3c_1x + 3}}{\sqrt[3]{x}}$$

24 Various 24

24.1 problem 663

Internal problem ID [3910]

Book: Ordinary differential equations and their solutions. By George Moseley Murphy. 1960
Section: Various 24
Problem number: 663.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[[_homogeneous, 'class G'], _rational]`

$$\boxed{(1 - y^2x^2)\, y' - xy^3 = 0}$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 32

```
dsolve((1-x^2*y(x)^2)*diff(y(x),x) = x*y(x)^3,y(x), singsol=all)
```

$$y(x) = \frac{e^{-c_1}}{\sqrt{-\frac{e^{-2c_1}x^2}{\text{LambertW}(-e^{-2c_1}x^2)}}}$$

✓ Solution by Mathematica

Time used: 5.286 (sec). Leaf size: 60

```
DSolve[(1-x^2 y[x]^2)y'[x]==x y[x]^3,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to -\frac{i\sqrt{W(-e^{-2c_1}x^2)}}{x}$$
$$y(x) \to \frac{i\sqrt{W(-e^{-2c_1}x^2)}}{x}$$
$$y(x) \to 0$$

24.2 problem 664

Internal problem ID [3911]

Book: Ordinary differential equations and their solutions. By George Moseley Murphy. 1960
Section: Various 24
Problem number: 664.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[[_homogeneous, 'class G'], _rational, [_Abel, '2nd type', 'cl`

$$\boxed{(1 - y^2x^2)\, y' - (1 + yx)\, y^2 = 0}$$

✓ Solution by Maple
Time used: 0.015 (sec). Leaf size: 25

```
dsolve((1-x^2*y(x)^2)*diff(y(x),x) = (1+x*y(x))*y(x)^2,y(x), singsol=all)
```

$$y(x) = -\frac{1}{x}$$
$$y(x) = -\frac{\mathrm{LambertW}\left(-x\,\mathrm{e}^{-c_1}\right)}{x}$$

✓ Solution by Mathematica
Time used: 2.179 (sec). Leaf size: 43

```
DSolve[(1-x^2 y[x]^2)y'[x]==(1+x y[x])y[x]^2,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to -\frac{1}{x}$$
$$y(x) \to -\frac{W(-e^{-c_1}x)}{x}$$
$$y(x) \to 0$$
$$y(x) \to -\frac{1}{x}$$

24.3 problem 665

Internal problem ID [3912]

Book: Ordinary differential equations and their solutions. By George Moseley Murphy. 1960
Section: Various 24
Problem number: 665.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[[_homogeneous, 'class G'], _rational]`

$$x(1+y^2x)\,y'+y=0$$

✓ Solution by Maple
Time used: 0.188 (sec). Leaf size: 137

```
dsolve(x*(1+x*y(x)^2)*diff(y(x),x)+y(x) = 0,y(x), singsol=all)
```

$$y(x) = -\frac{\sqrt{2}\,\sqrt{xc_1\left(2c_1+x-\sqrt{x\,(x+4c_1)}\right)}}{2c_1x}$$
$$y(x) = \frac{\sqrt{2}\,\sqrt{xc_1\left(2c_1+x-\sqrt{x\,(x+4c_1)}\right)}}{2c_1x}$$
$$y(x) = -\frac{\sqrt{2}\,\sqrt{xc_1\left(2c_1+x+\sqrt{x\,(x+4c_1)}\right)}}{2c_1x}$$
$$y(x) = \frac{\sqrt{2}\,\sqrt{xc_1\left(2c_1+x+\sqrt{x\,(x+4c_1)}\right)}}{2c_1x}$$

✓ Solution by Mathematica

Time used: 0.329 (sec). Leaf size: 65

```
DSolve[x(1+x y[x]^2)y'[x]+y[x]==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to \frac{1}{2}\left(c_1 - \frac{\sqrt{4 + c_1{}^2 x}}{\sqrt{x}}\right)$$
$$y(x) \to \frac{1}{2}\left(\frac{\sqrt{4 + c_1{}^2 x}}{\sqrt{x}} + c_1\right)$$
$$y(x) \to 0$$

24.4 problem 666

Internal problem ID [3913]

Book: Ordinary differential equations and their solutions. By George Moseley Murphy. 1960
Section: Various 24
Problem number: 666.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[[_homogeneous, 'class G'], _rational]`

$$x(1+y^2x)\,y'-(2-3y^2x)\,y=0$$

✓ Solution by Maple

Time used: 0.093 (sec). Leaf size: 45

```
dsolve(x*(1+x*y(x)^2)*diff(y(x),x) = (2-3*x*y(x)^2)*y(x),y(x), singsol=all)
```

$$y(x)=\frac{c_1+\sqrt{4x^5+c_1^2}}{2x^3}$$
$$y(x)=\frac{c_1-\sqrt{4x^5+c_1^2}}{2x^3}$$

✓ Solution by Mathematica

Time used: 1.286 (sec). Leaf size: 75

```
DSolve[x(1+x y[x]^2)y'[x]==(2-3 x y[x]^2)y[x],y[x],x,IncludeSingularSolutions -> True]
```

$$y(x)\to-\frac{\sqrt{4x^5+e^{5c_1}}+e^{\frac{5c_1}{2}}}{2x^3}$$
$$y(x)\to\frac{\sqrt{4x^5+e^{5c_1}}-e^{\frac{5c_1}{2}}}{2x^3}$$

24.5 problem 667

Internal problem ID [3914]

Book: Ordinary differential equations and their solutions. By George Moseley Murphy. 1960
Section: Various 24
Problem number: 667.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_separable]`

$$x^2(y+a)^2 y' - (x^2+1)(y^2+a^2) = 0$$

✓ Solution by Maple
Time used: 0.016 (sec). Leaf size: 92

```
dsolve(x^2*(a+y(x))^2*diff(y(x),x) = (x^2+1)*(y(x)^2+a^2),y(x), singsol=all)
```

$$y(x) = \frac{-ax \operatorname{RootOf}\left(\_Z^2 a^2 x^2 - 2c_1\_Za\, x^2 - 2\_Za\, x^3 + c_1^2 x^2 + 2c_1 x^3 + x^2 a^2 + x^4 - x^2 \mathrm{e}^{\_Z} + 2ax\_Z - 2c_1 x -}{x}$$

✓ Solution by Mathematica
Time used: 0.504 (sec). Leaf size: 48

```
DSolve[x^2 (a+y[x])^2 y'[x]==(1+x^2)(a^2+y[x]^2),y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to \text{InverseFunction}\left[a \log \left(\#1^2 + a^2\right) + \#1\&\right]\left[x - \frac{1}{x} + c_1\right]$$
$$y(x) \to -ia$$
$$y(x) \to ia$$

24.6 problem 668

Internal problem ID [3915]

Book: Ordinary differential equations and their solutions. By George Moseley Murphy. 1960
Section: Various 24
Problem number: 668.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_separable]`

$$\boxed{(x^2+1)(y^2+1)y'+2xy(1-y^2)=0}$$

✓ Solution by Maple

Time used: 0.047 (sec). Leaf size: 61

```
dsolve((x^2+1)*(1+y(x)^2)*diff(y(x),x)+2*x*y(x)*(1-y(x)^2) = 0,y(x), singsol=all)
```

$$y(x) = \frac{c_1 x^2}{2} + \frac{c_1}{2} - \frac{\sqrt{4+(x^2+1)^2 c_1^2}}{2}$$
$$y(x) = \frac{c_1 x^2}{2} + \frac{c_1}{2} + \frac{\sqrt{4+(x^2+1)^2 c_1^2}}{2}$$

✓ Solution by Mathematica

Time used: 7.907 (sec). Leaf size: 98

```
DSolve[(1+x^2)(1+y[x]^2)y'[x]+2 x y[x](1-y[x]^2)==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to \frac{1}{2}\left(-e^{c_1}(x^2+1) - \sqrt{4+e^{2c_1}(x^2+1)^2}\right)$$
$$y(x) \to \frac{1}{2}\left(\sqrt{4+e^{2c_1}(x^2+1)^2} - e^{c_1}(x^2+1)\right)$$
$$y(x) \to -1$$
$$y(x) \to 0$$
$$y(x) \to 1$$

24.7 problem 669

Internal problem ID [3916]

Book: Ordinary differential equations and their solutions. By George Moseley Murphy. 1960
Section: Various 24
Problem number: 669.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_separable]`

$$\boxed{(x^2+1)\,(y^2+1)\,y' + 2xy(1-y)^2 = 0}$$

✓ Solution by Maple
Time used: 0.047 (sec). Leaf size: 40

```
dsolve((x^2+1)*(1+y(x)^2)*diff(y(x),x)+2*x*y(x)*(1-y(x))^2 = 0,y(x), singsol=all)
```

$$y(x) = \mathrm{e}^{\mathrm{RootOf}\left(\ln(x^2+1)\mathrm{e}^{\_Z}+2c_1\mathrm{e}^{\_Z}+\_Z\mathrm{e}^{\_Z}-\ln(x^2+1)-2c_1-\_Z-2\right)}$$

✓ Solution by Mathematica
Time used: 0.324 (sec). Leaf size: 40

```
DSolve[(1+x^2)(1+y[x]^2)y'[x]+2 x y[x](1-y[x])^2==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to \text{InverseFunction}\left[\log(\#1) - \frac{2}{\#1-1}\&\right]\left[-\log\left(x^2+1\right)+c_1\right]$$
$$y(x) \to 0$$
$$y(x) \to 1$$

24.8 problem 670

Internal problem ID [3917]

Book: Ordinary differential equations and their solutions. By George Moseley Murphy. 1960
Section: Various 24
Problem number: 670.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_exact, _rational]`

$$(1 - x^3 + 6y^2x^2)\, y' - (6 + 3yx - 4y^3)\, x = 0$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 490

```
dsolve((1-x^3+6*x^2*y(x)^2)*diff(y(x),x) = (6+3*x*y(x)-4*y(x)^3)*x,y(x), singsol=all)
```

$$y(x) = \frac{6x^3 + \left(162x^3 + 6\sqrt{3}\,\sqrt{-2x^9 + 249x^6 - 162c_1x^4 + 27c_1^2x^2 - 6x^3 + 2} - 54c_1x\right)^{\frac{2}{3}} - 6}{6x\left(162x^3 + 6\sqrt{3}\,\sqrt{-2x^9 + 249x^6 - 162c_1x^4 + 27c_1^2x^2 - 6x^3 + 2} - 54c_1x\right)^{\frac{1}{3}}}$$

$$y(x) = \frac{6i\sqrt{3}\,x^3 - i\left(162x^3 + 6\sqrt{3}\,\sqrt{-2x^9 + 249x^6 - 162c_1x^4 + 27c_1^2x^2 - 6x^3 + 2} - 54c_1x\right)^{\frac{2}{3}}\sqrt{3} - 6x^3 - 6i\sqrt{3}}{12x\left(162x^3 + 6\sqrt{3}\,\sqrt{-2x^9 + 249x^6 - 162c_1x^4 + 2}\right.}$$

$$y(x) = -\frac{6i\sqrt{3}\,x^3 - i\left(162x^3 + 6\sqrt{3}\,\sqrt{-2x^9 + 249x^6 - 162c_1x^4 + 27c_1^2x^2 - 6x^3 + 2} - 54c_1x\right)^{\frac{2}{3}}\sqrt{3} + 6x^3 - 6i}{12x\left(162x^3 + 6\sqrt{3}\,\sqrt{-2x^9 + 249x^6 - 162c_1x^4 +}\right.}$$

✓ Solution by Mathematica

Time used: 54.173 (sec). Leaf size: 424

```
DSolve[(1-x^3+6 x^2 y[x]^2)y'[x]==(6+3 x y[x]-4 y[x]^3)x,y[x],x,IncludeSingularSolutions ->
```

$$y(x) \to -\frac{\sqrt[3]{2}(x^3-1)}{\sqrt[3]{-324x^6+108c_1x^4+\sqrt{-864x^6(x^3-1)^3+(-324x^6+108c_1x^4)^2}}} - \frac{\sqrt[3]{-324x^6+108c_1x^4+\sqrt{-864x^6(x^3-1)^3+(-324x^6+108c_1x^4)^2}}}{6\sqrt[3]{2}x^2}$$

$$y(x) \to \frac{(1+i\sqrt{3})(x^3-1)}{2^{2/3}\sqrt[3]{-324x^6+108c_1x^4+\sqrt{-864x^6(x^3-1)^3+(-324x^6+108c_1x^4)^2}}} + \frac{(1-i\sqrt{3})\sqrt[3]{-324x^6+108c_1x^4+\sqrt{-864x^6(x^3-1)^3+(-324x^6+108c_1x^4)^2}}}{12\sqrt[3]{2}x^2}$$

$$y(x) \to \frac{(1-i\sqrt{3})(x^3-1)}{2^{2/3}\sqrt[3]{-324x^6+108c_1x^4+\sqrt{-864x^6(x^3-1)^3+(-324x^6+108c_1x^4)^2}}} + \frac{(1+i\sqrt{3})\sqrt[3]{-324x^6+108c_1x^4+\sqrt{-864x^6(x^3-1)^3+(-324x^6+108c_1x^4)^2}}}{12\sqrt[3]{2}x^2}$$

24.9 problem 671

Internal problem ID [3918]

Book: Ordinary differential equations and their solutions. By George Moseley Murphy. 1960
Section: Various 24
Problem number: 671.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_exact, _rational]`

$$x\left(3+5x-12y^2x+4x^2y\right)y'+\left(3+10x-8y^2x+6x^2y\right)y=0$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 739

```
dsolve(x*(3+5*x-12*x*y(x)^2+4*x^2*y(x))*diff(y(x),x)+(3+10*x-8*x*y(x)^2+6*x^2*y(x))*y(x) = 0
```

$$y(x) = \frac{2^{\frac{2}{3}}\left(\left(2x^5+45x^3+3\sqrt{3}\,\sqrt{8c_1x^5-25x^6-30x^5+180c_1x^3-509x^4+108c_1x^2-900x^3+108c_1^2-540x^2}\right.\right.}{12x} + \frac{\left(x^3+15x+9\right)2^{\frac{1}{3}}}{6\left(\left(2x^5+45x^3+3\sqrt{3}\,\sqrt{8c_1x^5-25x^6-30x^5+180c_1x^3-509x^4+108c_1x^2-900x^3+108c_1^2-540x^2}\right.\right.} + \frac{x}{6}$$

$$y(x) = -\frac{-4x^2\left(2x^6+45x^4+27x^3+3x\sqrt{3}\,\sqrt{-25x^6+2\left(-15+4c_1\right)x^5-509x^4+180\left(c_1-5\right)x^3+108\left(c_1-\right.}\right.}{24\left(2x^6+4\right.}$$

$$y(x) = \frac{4x^2\left(2x^6+45x^4+27x^3+3x\sqrt{3}\,\sqrt{-25x^6+2\left(-15+4c_1\right)x^5-509x^4+180\left(c_1-5\right)x^3+108\left(c_1-5\right)x}\right.}{24\left(2x^6+45x^4\right.}$$

✓ Solution by Mathematica

Time used: 60.181 (sec). Leaf size: 660

```
DSolve[x(3+5 x-12 x y[x]^2+4 x^2 y[x])y'[x]+(3+10 x-8 x y[x]^2+6 x^2 y[x])y[x]==0,y[x],x,Inc
```

$$y(x) \to -\frac{\sqrt[3]{-16x^9 - 360x^7 - 216x^6 + 432c_1x^4 + 8\sqrt{-4x^9\left(x^3+15x+9\right)^3 + \left(2x^9+45x^7+27x^6-54c_1x^4\right)^2}}}{12\sqrt[3]{2}x^2} - \frac{\left(x^3+15x+9\right)x}{3\ 2^{2/3}\sqrt[3]{-2x^9-45x^7-27x^6+54c_1x^4+3\sqrt{3}\sqrt{-x^8\left(25x^6+(30+8c_1)x^5+509x^4+180(5+c_1)x^3+\right.}}} + \frac{x}{6}$$

$$y(x) \to \frac{\left(1-i\sqrt{3}\right)\sqrt[3]{-16x^9-360x^7-216x^6+432c_1x^4+8\sqrt{-4x^9\left(x^3+15x+9\right)^3+\left(2x^9+45x^7+27x^6-54\right.}}}{24\sqrt[3]{2}x^2} + \frac{\left(1+i\sqrt{3}\right)\left(x^3+15x+9\right)x}{6\ 2^{2/3}\sqrt[3]{-2x^9-45x^7-27x^6+54c_1x^4+3\sqrt{3}\sqrt{-x^8\left(25x^6+(30+8c_1)x^5+509x^4+180(5+c_1)x^3+\right.}}} + \frac{x}{6}$$

$$y(x) \to \frac{\left(1+i\sqrt{3}\right)\sqrt[3]{-16x^9-360x^7-216x^6+432c_1x^4+8\sqrt{-4x^9\left(x^3+15x+9\right)^3+\left(2x^9+45x^7+27x^6-54\right.}}}{24\sqrt[3]{2}x^2} + \frac{\left(1-i\sqrt{3}\right)\left(x^3+15x+9\right)x}{6\ 2^{2/3}\sqrt[3]{-2x^9-45x^7-27x^6+54c_1x^4+3\sqrt{3}\sqrt{-x^8\left(25x^6+(30+8c_1)x^5+509x^4+180(5+c_1)x^3+\right.}}} + \frac{x}{6}$$

24.10 problem 672

Internal problem ID [3919]

Book: Ordinary differential equations and their solutions. By George Moseley Murphy. 1960
Section: Various 24
Problem number: 672.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_separable]`

$$\boxed{x^3(y^2+1)\,y'+3x^2y=0}$$

✓ Solution by Maple

Time used: 0.016 (sec). Leaf size: 14

```
dsolve(x^3*(1+y(x)^2)*diff(y(x),x)+3*x^2*y(x) = 0,y(x), singsol=all)
```

$$y(x)=\frac{1}{\sqrt{\frac{1}{\operatorname{LambertW}\left(\frac{c_1}{x^6}\right)}}}$$

✓ Solution by Mathematica

Time used: 4.067 (sec). Leaf size: 46

```
DSolve[x^3(1+y[x]^2)y'[x]+3 x^2 y[x]==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x)\to -\sqrt{W\left(\frac{e^{2c_1}}{x^6}\right)}$$
$$y(x)\to \sqrt{W\left(\frac{e^{2c_1}}{x^6}\right)}$$
$$y(x)\to 0$$

24.11 problem 673

Internal problem ID [3920]

Book: Ordinary differential equations and their solutions. By George Moseley Murphy. 1960
Section: Various 24
Problem number: 673.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[[_homogeneous, 'class G'], _rational]`

$$\boxed{x(-yx+1)^2 y' + (1+y^2x^2)\, y = 0}$$

✓ Solution by Maple
Time used: 0.172 (sec). Leaf size: 34

```
dsolve(x*(1-x*y(x))^2*diff(y(x),x)+(1+x^2*y(x)^2)*y(x) = 0,y(x), singsol=all)
```

$$y(x) = \frac{e^{\mathrm{RootOf}\left(-e^{2\_Z}-2\,e^{\_Z}\ln(x)+2c_1e^{\_Z}+2\_Z e^{\_Z}+1\right)}}{x}$$

✓ Solution by Mathematica
Time used: 0.1 (sec). Leaf size: 25

```
DSolve[x(1-x y[x])^2 y'[x]+(1+x^2 y[x]^2)y[x]==0,y[x],x,IncludeSingularSolutions -> True]
```

$$\mathrm{Solve}\left[xy(x) - \frac{1}{xy(x)} - 2\log(y(x)) = c_1, y(x)\right]$$

24.12 problem 674

Internal problem ID [3921]

Book: Ordinary differential equations and their solutions. By George Moseley Murphy. 1960
Section: Various 24
Problem number: 674.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[[_homogeneous, 'class G'], _rational]`

$$\boxed{(1-y^2x^4)\,y'-y^3x^3=0}$$

✓ Solution by Maple
Time used: 0.64 (sec). Leaf size: 157

```
dsolve((1-x^4*y(x)^2)*diff(y(x),x) = x^3*y(x)^3,y(x), singsol=all)
```

$$y(x)=\frac{\sqrt{-c_1-\sqrt{c_1\,(x^4+c_1)}}\left(c_1-\sqrt{c_1\,(x^4+c_1)}\right)}{c_1x^4}$$
$$y(x)=\frac{\sqrt{-c_1+\sqrt{c_1\,(x^4+c_1)}}\left(c_1+\sqrt{c_1\,(x^4+c_1)}\right)}{c_1x^4}$$
$$y(x)=\frac{\sqrt{-c_1-\sqrt{c_1\,(x^4+c_1)}}\left(-c_1+\sqrt{c_1\,(x^4+c_1)}\right)}{c_1x^4}$$
$$y(x)=\frac{\left(-c_1-\sqrt{c_1\,(x^4+c_1)}\right)\sqrt{-c_1+\sqrt{c_1\,(x^4+c_1)}}}{c_1x^4}$$

✓ Solution by Mathematica

Time used: 11.613 (sec). Leaf size: 122

```
DSolve[(1-x^4 y[x]^2)y'[x]==x^3 y[x]^3,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to -\sqrt{\frac{1-\sqrt{1+4c_1x^4}}{x^4}}$$
$$y(x) \to \sqrt{\frac{1-\sqrt{1+4c_1x^4}}{x^4}}$$
$$y(x) \to -\sqrt{\frac{1+\sqrt{1+4c_1x^4}}{x^4}}$$
$$y(x) \to \sqrt{\frac{1+\sqrt{1+4c_1x^4}}{x^4}}$$
$$y(x) \to 0$$

24.13 problem 675

Internal problem ID [3922]

Book: Ordinary differential equations and their solutions. By George Moseley Murphy. 1960
Section: Various 24
Problem number: 675.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_exact, _rational]`

$$\boxed{(3x - y^3)\, y' + 3y = x^2}$$

✓ Solution by Maple
Time used: 0.0 (sec). Leaf size: 21

```
dsolve((3*x-y(x)^3)*diff(y(x),x) = x^2-3*y(x),y(x), singsol=all)
```

$$-\frac{x^3}{3} + 3xy(x) - \frac{y(x)^4}{4} + c_1 = 0$$

✓ Solution by Mathematica

Time used: 60.166 (sec). Leaf size: 1211

```
DSolve[(3 x-y[x]^3)y'[x]==x^2-3 y[x],y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to -\frac{\sqrt{\frac{4x^3+\left(243x^2-\frac{1}{432}\sqrt{11019960576x^4-4(144x^3+432c_1)^3}\right)^{2/3}+12c_1}{\sqrt[3]{243x^2-\frac{1}{432}\sqrt{11019960576x^4-4\left(144x^3+432c_1\right)^3}}}}}{\sqrt{6}} - \frac{1}{2}\sqrt{-\frac{12\sqrt{6}x}{\sqrt{\frac{4x^3+\left(243x^2-\frac{1}{432}\sqrt{11019960576x^4-4(144x^3+432c_1)^3}\right)^{2/3}+12c_1}{\sqrt[3]{243x^2-\frac{1}{432}\sqrt{11019960576x^4-4\left(144x^3+432c_1\right)^3}}}}} - \frac{2}{3}\sqrt[3]{243x^2-\frac{1}{432}\sqrt{11019960576x\cdots}}}$$

$$y(x) \to \frac{1}{2}\sqrt{-\frac{12\sqrt{6}x}{\sqrt{\frac{4x^3+\left(243x^2-\frac{1}{432}\sqrt{11019960576x^4-4(144x^3+432c_1)^3}\right)^{2/3}+12c_1}{\sqrt[3]{243x^2-\frac{1}{432}\sqrt{11019960576x^4-4\left(144x^3+432c_1\right)^3}}}}} - \frac{2}{3}\sqrt[3]{243x^2-\frac{1}{432}\sqrt{11019960576x^4\cdots}}} - \frac{\sqrt{\frac{4x^3+\left(243x^2-\frac{1}{432}\sqrt{11019960576x^4-4(144x^3+432c_1)^3}\right)^{2/3}+12c_1}{\sqrt[3]{243x^2-\frac{1}{432}\sqrt{11019960576x^4-4\left(144x^3+432c_1\right)^3}}}}}{\sqrt{6}}$$

$$y(x) \to \frac{\sqrt{\frac{4x^3+\left(243x^2-\frac{1}{432}\sqrt{11019960576x^4-4(144x^3+432c_1)^3}\right)^{2/3}+12c_1}{\sqrt[3]{243x^2-\frac{1}{432}\sqrt{11019960576x^4-4\left(144x^3+432c_1\right)^3}}}}}{\sqrt{6}} - \frac{1}{2}\sqrt{\frac{12\sqrt{6}x}{\sqrt{\frac{4x^3+\left(243x^2-\frac{1}{432}\sqrt{11019960576x^4-4(144x^3+432c_1)^3}\right)^{2/3}+12c_1}{\sqrt[3]{243x^2-\frac{1}{432}\sqrt{11019960576x^4-4\left(144x^3+432c_1\right)^3}}}}} - \frac{2}{3}\sqrt[3]{243x^2-\frac{1}{432}\sqrt{11019960576x^4\cdots}}}$$

$$y(x) \to \frac{\sqrt{\frac{4x^3+\left(243x^2-\frac{1}{432}\sqrt{11019960576x^4-4(144x^3+432c_1)^3}\right)^{2/3}+12c_1}{\sqrt[3]{243x^2-\frac{1}{432}\sqrt{11019960576x^4-4\left(144x^3+432c_1\right)^3}}}}}{\sqrt{6}} + \frac{1}{2}\sqrt{\frac{12\sqrt{6}x}{\sqrt{\frac{4x^3+\left(243x^2-\frac{1}{432}\sqrt{11019960576x^4-4(144x^3+432c_1)^3}\right)^{2/3}+12c_1}{\sqrt[3]{243x^2-\frac{1}{432}\sqrt{11019960576x^4-4\left(144x^3+432c_1\right)^3}}}}} - \frac{2}{3}\sqrt[3]{243x^2-\frac{1}{432}\sqrt{11019960576x^4\cdots}}}$$

24.14 problem 676

Internal problem ID [3923]

Book: Ordinary differential equations and their solutions. By George Moseley Murphy. 1960
Section: Various 24
Problem number: 676.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[[_homogeneous, ‘class A‘], _rational, _dAlembert]`

$$\boxed{(x^3 - y^3)\, y' + x^2 y = 0}$$

✓ Solution by Maple

Time used: 0.844 (sec). Leaf size: 389

```
dsolve((x^3-y(x)^3)*diff(y(x),x)+x^2*y(x) = 0,y(x), singsol=all)
```

$$y(x) = \frac{x}{\left(-\left(c_1x^3+\sqrt{c_1^2x^6+1}\right)x^3c_1\right)^{\frac{1}{3}}}$$

$$y(x) = \frac{x}{\left(c_1\left(-c_1x^3+\sqrt{c_1^2x^6+1}\right)x^3\right)^{\frac{1}{3}}}$$

$$y(x) = \frac{4x}{\left(1+i\sqrt{3}\right)^2\left(-\left(c_1x^3+\sqrt{c_1^2x^6+1}\right)x^3c_1\right)^{\frac{1}{3}}}$$

$$y(x) = \frac{4x}{\left(c_1\left(-c_1x^3+\sqrt{c_1^2x^6+1}\right)x^3\right)^{\frac{1}{3}}\left(1+i\sqrt{3}\right)^2}$$

$$y(x) = \frac{4x}{\left(i\sqrt{3}-1\right)^2\left(-\left(c_1x^3+\sqrt{c_1^2x^6+1}\right)x^3c_1\right)^{\frac{1}{3}}}$$

$$y(x) = \frac{4x}{\left(c_1\left(-c_1x^3+\sqrt{c_1^2x^6+1}\right)x^3\right)^{\frac{1}{3}}\left(i\sqrt{3}-1\right)^2}$$

$$y(x) = \frac{4x}{\left(i\sqrt{3}-1\right)^2\left(-\left(c_1x^3+\sqrt{c_1^2x^6+1}\right)x^3c_1\right)^{\frac{1}{3}}}$$

$$y(x) = \frac{4x}{\left(c_1\left(-c_1x^3+\sqrt{c_1^2x^6+1}\right)x^3\right)^{\frac{1}{3}}\left(i\sqrt{3}-1\right)^2}$$

$$y(x) = \frac{4x}{\left(1+i\sqrt{3}\right)^2\left(-\left(c_1x^3+\sqrt{c_1^2x^6+1}\right)x^3c_1\right)^{\frac{1}{3}}}$$

$$y(x) = \frac{4x}{\left(c_1\left(-c_1x^3+\sqrt{c_1^2x^6+1}\right)x^3\right)^{\frac{1}{3}}\left(1+i\sqrt{3}\right)^2}$$

✓ Solution by Mathematica

Time used: 6.658 (sec). Leaf size: 352

```
DSolve[(x^3-y[x]^3)y'[x]+x^2 y[x]==0,y[x],x,IncludeSingularSolutions -> True]
```

$$
\begin{aligned}
y(x) &\to \sqrt[3]{x^3-\sqrt{x^6-e^{6c_1}}}\\
y(x) &\to -\sqrt[3]{-1}\sqrt[3]{x^3-\sqrt{x^6-e^{6c_1}}}\\
y(x) &\to (-1)^{2/3}\sqrt[3]{x^3-\sqrt{x^6-e^{6c_1}}}\\
y(x) &\to \sqrt[3]{x^3+\sqrt{x^6-e^{6c_1}}}\\
y(x) &\to -\sqrt[3]{-1}\sqrt[3]{x^3+\sqrt{x^6-e^{6c_1}}}\\
y(x) &\to (-1)^{2/3}\sqrt[3]{x^3+\sqrt{x^6-e^{6c_1}}}\\
y(x) &\to 0\\
y(x) &\to \sqrt[3]{x^3-\sqrt{x^6}}\\
y(x) &\to -\sqrt[3]{-1}\sqrt[3]{x^3-\sqrt{x^6}}\\
y(x) &\to (-1)^{2/3}\sqrt[3]{x^3-\sqrt{x^6}}\\
y(x) &\to \sqrt[3]{\sqrt{x^6}+x^3}\\
y(x) &\to -\sqrt[3]{-1}\sqrt[3]{\sqrt{x^6}+x^3}\\
y(x) &\to (-1)^{2/3}\sqrt[3]{\sqrt{x^6}+x^3}
\end{aligned}
$$

24.15 problem 677

Internal problem ID [3924]

Book: Ordinary differential equations and their solutions. By George Moseley Murphy. 1960
Section: Various 24
Problem number: 677.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[[_homogeneous, ‘class A‘], _exact, _rational, _dAlembert]`

$$\boxed{(x^3+y^3)\,y'+x^2(ax+3y)=0}$$

✓ Solution by Maple
Time used: 0.063 (sec). Leaf size: 29

```
dsolve((x^3+y(x)^3)*diff(y(x),x)+x^2*(a*x+3*y(x)) = 0,y(x), singsol=all)
```

$$y(x)=\frac{\operatorname{RootOf}\left(a\,x^4c_1^{\frac{4}{3}}+4x^3c_1\_Z+\_Z^4-1\right)}{c_1^{\frac{1}{3}}}$$

✓ Solution by Mathematica

Time used: 60.172 (sec). Leaf size: 1430

```
DSolve[(x^3+y[x]^3)y'[x]+x^2(a x+3 y[x])==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to \frac{\sqrt{\frac{\sqrt[3]{3}ax^4+\left(9x^6+\sqrt{3}\sqrt{27x^{12}+(-ax^4+e^{4c_1})^3}\right)^{2/3}-\sqrt[3]{3}e^{4c_1}}{\sqrt[3]{9x^6+\sqrt{3}\sqrt{27x^{12}+(-ax^4+e^{4c_1})^3}}}}-\sqrt{-\sqrt[3]{9x^6+\sqrt{3}\sqrt{27x^{12}+(-ax^4+e^{4c_1})^3}}+\frac{\cdots}{\sqrt[3]{9x^6 \cdots}}}}{\sqrt{2}\sqrt[3]{3}}$$

$$y(x) \to \frac{\sqrt{\frac{\sqrt[3]{3}ax^4+\left(9x^6+\sqrt{3}\sqrt{27x^{12}+(-ax^4+e^{4c_1})^3}\right)^{2/3}-\sqrt[3]{3}e^{4c_1}}{\sqrt[3]{9x^6+\sqrt{3}\sqrt{27x^{12}+(-ax^4+e^{4c_1})^3}}}}+\sqrt{-\sqrt[3]{9x^6+\sqrt{3}\sqrt{27x^{12}+(-ax^4+e^{4c_1})^3}}+\frac{\cdots}{\sqrt[3]{9x^6 \cdots}}}}{\sqrt{2}\sqrt[3]{3}}$$

$$y(x) \to -\frac{\sqrt{\frac{\sqrt[3]{3}ax^4+\left(9x^6+\sqrt{3}\sqrt{27x^{12}+(-ax^4+e^{4c_1})^3}\right)^{2/3}-\sqrt[3]{3}e^{4c_1}}{\sqrt[3]{9x^6+\sqrt{3}\sqrt{27x^{12}+(-ax^4+e^{4c_1})^3}}}}+\sqrt{-\sqrt[3]{9x^6+\sqrt{3}\sqrt{27x^{12}+(-ax^4+e^{4c_1})^3}}+\frac{\cdots}{\sqrt[3]{9x \cdots}}}}{\sqrt{2}\sqrt[3]{3}}$$

$$y(x) \to \frac{\sqrt{-\sqrt[3]{9x^6+\sqrt{3}\sqrt{27x^{12}+(-ax^4+e^{4c_1})^3}}+\frac{\sqrt[3]{3}(-ax^4+e^{4c_1})}{\sqrt[3]{9x^6+\sqrt{3}\sqrt{27x^{12}+(-ax^4+e^{4c_1})^3}}}+\frac{\cdots}{\sqrt{\frac{\sqrt[3]{3}ax^4+\left(9x^6+\sqrt{3} \cdots\right.}{\sqrt[3]{9x^6+\sqrt{3} \cdots}}}}}}{\sqrt{2}\sqrt[3]{3}}$$

24.16 problem 678

Internal problem ID [3925]

Book: Ordinary differential equations and their solutions. By George Moseley Murphy. 1960
Section: Various 24
Problem number: 678.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_exact, _rational]`

$$\boxed{(x - x^2y - y^3)\, y' + y - y^2x = x^3}$$

✓ Solution by Maple
Time used: 0.016 (sec). Leaf size: 29

```
dsolve((x-x^2*y(x)-y(x)^3)*diff(y(x),x) = x^3-y(x)+x*y(x)^2,y(x), singsol=all)
```

$$-\frac{x^4}{4} - \frac{x^2 y(x)^2}{2} + x y(x) - \frac{y(x)^4}{4} + c_1 = 0$$

✓ Solution by Mathematica

Time used: 60.188 (sec). Leaf size: 1807

```
DSolve[(x-x^2 y[x]-y[x]^3)y'[x]==x^3-y[x]+x y[x]^2,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to -\frac{\sqrt{-2x^2 + \sqrt[3]{-8x^6 + 9(3+4c_1)x^2 + 3\sqrt{3}\sqrt{-16x^8 + (27 - 16c_1{}^2 + 72c_1)\,x^4 + 64c_1{}^3}} + \frac{\cdots}{\sqrt[3]{-8x^6 + 9(3 \cdots}}}}{\sqrt{6}} - \frac{1}{2}\sqrt{-\frac{8x^2}{3} - \frac{4\sqrt{6}x}{\sqrt{-2x^2 + \sqrt[3]{-8x^6 + 9(3+4c_1)x^2 + 3\sqrt{3}\sqrt{-16x^8 + (27 - 16c_1{}^2 + 72c_1)\,x^4 + 64c_1{}^3}} + \cdots}}\cdots}$$

$$y(x) \to \frac{1}{2}\sqrt{-\frac{8x^2}{3} - \frac{4\sqrt{6}x}{\sqrt{-2x^2 + \sqrt[3]{-8x^6 + 9(3+4c_1)x^2 + 3\sqrt{3}\sqrt{-16x^8 + (27 - 16c_1{}^2 + 72c_1)\,x^4 + 64c_1{}^3}} + \cdots}}\cdots} - \frac{\sqrt{-2x^2 + \sqrt[3]{-8x^6 + 9(3+4c_1)x^2 + 3\sqrt{3}\sqrt{-16x^8 + (27 - 16c_1{}^2 + 72c_1)\,x^4 + 64c_1{}^3}} + \frac{\cdots}{\sqrt[3]{-8x^6 + 9(3 \cdots}}}}{\sqrt{6}}$$

$$y(x) \to \frac{\sqrt{-2x^2 + \sqrt[3]{-8x^6 + 9(3+4c_1)x^2 + 3\sqrt{3}\sqrt{-16x^8 + (27 - 16c_1{}^2 + 72c_1)\,x^4 + 64c_1{}^3}} + \frac{\cdots}{\sqrt[3]{-8x^6 + 9(3 + \cdots}}}}{\sqrt{6}} - \frac{1}{2}\sqrt{-\frac{8x^2}{3} + \frac{4\sqrt{6}x}{\sqrt{-2x^2 + \sqrt[3]{-8x^6 + 9(3+4c_1)x^2 + 3\sqrt{3}\sqrt{-16x^8 + (27 - 16c_1{}^2 + 72c_1)\,x^4 + 64c_1{}^3}} + \cdots}}\cdots}$$

$$y(x) \to \frac{\sqrt{-2x^2 + \sqrt[3]{-8x^6 + 9(3+4c_1)x^2 + 3\sqrt{3}\sqrt{-16x^8 + (27 - 16c_1{}^2 + 72c_1)\,x^4 + 64c_1{}^3}} + \frac{\cdots}{\sqrt[3]{-8x^6 + 9(3 + \cdots}}}}{\sqrt{6}} + \frac{1}{2}\sqrt{-\frac{8x^2}{3} + \frac{4\sqrt{6}x}{\sqrt{-2x^2 + \sqrt[3]{-8x^6 + 9(3+4c_1)x^2 + 3\sqrt{3}\sqrt{-16x^8 + (27 - 16c_1{}^2 + 72c_1)\,x^4 + 64c_1{}^3}} + \cdots}}\cdots}$$

24.17 problem 679

Internal problem ID [3926]

Book: Ordinary differential equations and their solutions. By George Moseley Murphy. 1960
Section: Various 24
Problem number: 679.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_rational]`

$$(x\,a^2 + y(x^2 - y^2))\,y' + x(x^2 - y^2) - ya^2 = 0$$

✗ Solution by Maple

```
dsolve((a^2*x+y(x)*(x^2-y(x)^2))*diff(y(x),x)+x*(x^2-y(x)^2) = a^2*y(x),y(x), singsol=all)
```

No solution found

✓ Solution by Mathematica

Time used: 0.294 (sec). Leaf size: 48

```
DSolve[(a^2*x+y[x]*(x^2-y[x]^2))*y'[x]+x*(x^2-y[x]^2)==a^2*y[x],y[x],x,IncludeSingularSoluti
```

$$\text{Solve}\left[-\frac{1}{2}a^2\log(x-y(x))+\frac{1}{2}a^2\log(y(x)+x)+\frac{x^2}{2}+\frac{y(x)^2}{2}=c_1,y(x)\right]$$

24.18 problem 680

Internal problem ID [3927]

Book: Ordinary differential equations and their solutions. By George Moseley Murphy. 1960
Section: Various 24
Problem number: 680.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_exact, _rational]`

$$\left(a+x^2+y^2\right)yy'-x\left(a-x^2-y^2\right)=0$$

✓ Solution by Maple

Time used: 0.031 (sec). Leaf size: 113

```
dsolve((a+x^2+y(x)^2)*y(x)*diff(y(x),x) = x*(a-x^2-y(x)^2),y(x), singsol=all)
```

$$y(x)=\sqrt{-x^2-a-2\sqrt{a\,x^2-c_1}}$$
$$y(x)=\sqrt{-x^2-a+2\sqrt{a\,x^2-c_1}}$$
$$y(x)=-\sqrt{-x^2-a-2\sqrt{a\,x^2-c_1}}$$
$$y(x)=-\sqrt{-x^2-a+2\sqrt{a\,x^2-c_1}}$$

✓ Solution by Mathematica

Time used: 7.635 (sec). Leaf size: 149

```
DSolve[(a+x^2+y[x]^2)*y[x]*y'[x]==x*(a-x^2-y[x]^2),y[x],x,IncludeSingularSolutions -> True]
```

$$y(x)\to-\sqrt{-\sqrt{a^2+4ax^2+4c_1}-a-x^2}$$
$$y(x)\to\sqrt{-\sqrt{a^2+4ax^2+4c_1}-a-x^2}$$
$$y(x)\to-\sqrt{\sqrt{a^2+4ax^2+4c_1}-a-x^2}$$
$$y(x)\to\sqrt{\sqrt{a^2+4ax^2+4c_1}-a-x^2}$$

24.19 problem 681

Internal problem ID [3928]

Book: Ordinary differential equations and their solutions. By George Moseley Murphy. 1960
Section: Various 24
Problem number: 681.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[[_homogeneous, ‘class A‘], _exact, _rational, _dAlembert]`

$$\boxed{(y^2+3x^2)\,yy'+x(3y^2+x^2)=0}$$

✓ Solution by Maple

Time used: 0.11 (sec). Leaf size: 119

```
dsolve((3*x^2+y(x)^2)*y(x)*diff(y(x),x)+x*(x^2+3*y(x)^2) = 0,y(x), singsol=all)
```

$$y(x)=\frac{\sqrt{-3c_1x^2-\sqrt{8c_1^2x^4+1}}}{\sqrt{c_1}}$$
$$y(x)=\frac{\sqrt{-3c_1x^2+\sqrt{8c_1^2x^4+1}}}{\sqrt{c_1}}$$
$$y(x)=-\frac{\sqrt{-3c_1x^2-\sqrt{8c_1^2x^4+1}}}{\sqrt{c_1}}$$
$$y(x)=-\frac{\sqrt{-3c_1x^2+\sqrt{8c_1^2x^4+1}}}{\sqrt{c_1}}$$

✓ Solution by Mathematica

Time used: 8.874 (sec). Leaf size: 245

```
DSolve[(3*x^2+y[x]^2)*y[x]*y'[x]+x*(x^2+3*y[x]^2)==0,y[x],x,IncludeSingularSolutions -> True
```

$$y(x) \to -\sqrt{-3x^2 - \sqrt{8x^4 + e^{4c_1}}}$$
$$y(x) \to \sqrt{-3x^2 - \sqrt{8x^4 + e^{4c_1}}}$$
$$y(x) \to -\sqrt{-3x^2 + \sqrt{8x^4 + e^{4c_1}}}$$
$$y(x) \to \sqrt{-3x^2 + \sqrt{8x^4 + e^{4c_1}}}$$
$$y(x) \to -\sqrt{-2\sqrt{2}\sqrt{x^4} - 3x^2}$$
$$y(x) \to \sqrt{-2\sqrt{2}\sqrt{x^4} - 3x^2}$$
$$y(x) \to -\sqrt{2\sqrt{2}\sqrt{x^4} - 3x^2}$$
$$y(x) \to \sqrt{2\sqrt{2}\sqrt{x^4} - 3x^2}$$

24.20 problem 682

Internal problem ID [3929]

Book: Ordinary differential equations and their solutions. By George Moseley Murphy. 1960
Section: Various 24
Problem number: 682.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_rational]`

$$\left(a-3x^2-y^2\right)yy'+x\left(-x^2+y^2+a\right)=0$$

✓ Solution by Maple

Time used: 0.016 (sec). Leaf size: 122

```
dsolve((a-3*x^2-y(x)^2)*y(x)*diff(y(x),x)+x*(a-x^2+y(x)^2) = 0,y(x), singsol=all)
```

$$y(x)=\frac{\sqrt{-\operatorname{LambertW}\left(-\left(-2x^2+a\right)\mathrm{e}^2c_1\right)\left(x^2\operatorname{LambertW}\left(-\left(-2x^2+a\right)\mathrm{e}^2c_1\right)-2x^2+a\right)}}{\operatorname{LambertW}\left(-\left(-2x^2+a\right)\mathrm{e}^2c_1\right)}$$

$$y(x)=-\frac{\sqrt{-\operatorname{LambertW}\left(-\left(-2x^2+a\right)\mathrm{e}^2c_1\right)\left(x^2\operatorname{LambertW}\left(-\left(-2x^2+a\right)\mathrm{e}^2c_1\right)-2x^2+a\right)}}{\operatorname{LambertW}\left(-\left(-2x^2+a\right)\mathrm{e}^2c_1\right)}$$

✓ Solution by Mathematica

Time used: 0.336 (sec). Leaf size: 39

```
DSolve[(a-3*x^2-y[x]^2)*y[x]*y'[x]+x*(a-x^2+y[x]^2)==0,y[x],x,IncludeSingularSolutions -> Tr
```

$$\text{Solve}\left[\frac{1}{2}\left(\frac{a+2y(x)^2}{x^2+y(x)^2}+\log\left(x^2+y(x)^2\right)\right)=c_1,y(x)\right]$$

24.21 problem 683

Internal problem ID [3930]

Book: Ordinary differential equations and their solutions. By George Moseley Murphy. 1960
Section: Various 24
Problem number: 683.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[[_homogeneous, ‘class A‘], _rational, _dAlembert]`

$$\boxed{2y^3y' + y^2x = x^3}$$

✓ Solution by Maple

Time used: 0.734 (sec). Leaf size: 649

```
dsolve(2*y(x)^3*diff(y(x),x) = x^3-x*y(x)^2,y(x), singsol=all)
```

$$y(x) = -\frac{\sqrt{2}\,\sqrt{\frac{c_1^2x^4-c_1x^2\left(2+x^6c_1^3+2\sqrt{x^6c_1^3+1}\right)^{\frac{1}{3}}+\left(2+x^6c_1^3+2\sqrt{x^6c_1^3+1}\right)^{\frac{2}{3}}}{\left(2+x^6c_1^3+2\sqrt{x^6c_1^3+1}\right)^{\frac{1}{3}}}}}{2\sqrt{c_1}}$$

$$y(x) = \frac{\sqrt{2}\,\sqrt{\frac{c_1^2x^4-c_1x^2\left(2+x^6c_1^3+2\sqrt{x^6c_1^3+1}\right)^{\frac{1}{3}}+\left(2+x^6c_1^3+2\sqrt{x^6c_1^3+1}\right)^{\frac{2}{3}}}{\left(2+x^6c_1^3+2\sqrt{x^6c_1^3+1}\right)^{\frac{1}{3}}}}}{2\sqrt{c_1}}$$

$$y(x) = -\frac{\sqrt{\frac{\left(c_1x^2+\left(2+x^6c_1^3+2\sqrt{x^6c_1^3+1}\right)^{\frac{1}{3}}\right)\left(\left(-1-i\sqrt{3}\right)\left(2+x^6c_1^3+2\sqrt{x^6c_1^3+1}\right)^{\frac{1}{3}}+\left(i\sqrt{3}-1\right)x^2c_1\right)}{\left(2+x^6c_1^3+2\sqrt{x^6c_1^3+1}\right)^{\frac{1}{3}}}}}{2\sqrt{c_1}}$$

$$y(x) = \frac{\sqrt{\frac{\left(c_1x^2+\left(2+x^6c_1^3+2\sqrt{x^6c_1^3+1}\right)^{\frac{1}{3}}\right)\left(\left(-1-i\sqrt{3}\right)\left(2+x^6c_1^3+2\sqrt{x^6c_1^3+1}\right)^{\frac{1}{3}}+\left(i\sqrt{3}-1\right)x^2c_1\right)}{\left(2+x^6c_1^3+2\sqrt{x^6c_1^3+1}\right)^{\frac{1}{3}}}}}{2\sqrt{c_1}}$$

$$y(x) = -\frac{\sqrt{\frac{\left(\left(2+x^6c_1^3+2\sqrt{x^6c_1^3+1}\right)^{\frac{1}{3}}\left(i\sqrt{3}-1\right)+\left(-1-i\sqrt{3}\right)x^2c_1\right)\left(c_1x^2+\left(2+x^6c_1^3+2\sqrt{x^6c_1^3+1}\right)^{\frac{1}{3}}\right)}{\left(2+x^6c_1^3+2\sqrt{x^6c_1^3+1}\right)^{\frac{1}{3}}}}}{2\sqrt{c_1}}$$

$$y(x) = \frac{\sqrt{\frac{\left(\left(2+x^6c_1^3+2\sqrt{x^6c_1^3+1}\right)^{\frac{1}{3}}\left(i\sqrt{3}-1\right)+\left(-1-i\sqrt{3}\right)x^2c_1\right)\left(c_1x^2+\left(2+x^6c_1^3+2\sqrt{x^6c_1^3+1}\right)^{\frac{1}{3}}\right)}{\left(2+x^6c_1^3+2\sqrt{x^6c_1^3+1}\right)^{\frac{1}{3}}}}}{2\sqrt{c_1}}$$

✓ Solution by Mathematica

Time used: 60.202 (sec). Leaf size: 714

```
DSolve[2*y[x]^3*y'[x]==x^3-x*y[x]^2,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x)\to -\frac{\sqrt{\sqrt[3]{x^6+2\sqrt{e^{24c_1}-e^{12c_1}x^6}-2e^{12c_1}}-x^2+\frac{x^4}{\sqrt[3]{x^6+2\sqrt{e^{24c_1}-e^{12c_1}x^6}-2e^{12c_1}}}}}{\sqrt{2}}$$

$$y(x)\to \frac{\sqrt{\sqrt[3]{x^6+2\sqrt{e^{24c_1}-e^{12c_1}x^6}-2e^{12c_1}}-x^2+\frac{x^4}{\sqrt[3]{x^6+2\sqrt{e^{24c_1}-e^{12c_1}x^6}-2e^{12c_1}}}}}{\sqrt{2}}$$

$$y(x)\to -\frac{1}{2}\sqrt{\left(-1-i\sqrt{3}\right)\sqrt[3]{x^6+2\sqrt{e^{24c_1}-e^{12c_1}x^6}-2e^{12c_1}}-2x^2+\frac{i\left(\sqrt{3}+i\right)x^4}{\sqrt[3]{x^6+2\sqrt{e^{24c_1}-e^{12c_1}x^6}-2e^{12c_1}}}}$$

$$y(x)\to \frac{1}{2}\sqrt{\left(-1-i\sqrt{3}\right)\sqrt[3]{x^6+2\sqrt{e^{24c_1}-e^{12c_1}x^6}-2e^{12c_1}}-2x^2+\frac{i\left(\sqrt{3}+i\right)x^4}{\sqrt[3]{x^6+2\sqrt{e^{24c_1}-e^{12c_1}x^6}-2e^{12c_1}}}}$$

$$y(x)\to -\frac{1}{2}\sqrt{i\left(\sqrt{3}+i\right)\sqrt[3]{x^6+2\sqrt{e^{24c_1}-e^{12c_1}x^6}-2e^{12c_1}}-2x^2+\frac{\left(-1-i\sqrt{3}\right)x^4}{\sqrt[3]{x^6+2\sqrt{e^{24c_1}-e^{12c_1}x^6}-2e^{12c_1}}}}$$

$$y(x)\to \frac{1}{2}\sqrt{i\left(\sqrt{3}+i\right)\sqrt[3]{x^6+2\sqrt{e^{24c_1}-e^{12c_1}x^6}-2e^{12c_1}}-2x^2+\frac{\left(-1-i\sqrt{3}\right)x^4}{\sqrt[3]{x^6+2\sqrt{e^{24c_1}-e^{12c_1}x^6}-2e^{12c_1}}}}$$

24.22 problem 684

Internal problem ID [3931]

Book: Ordinary differential equations and their solutions. By George Moseley Murphy. 1960
Section: Various 24
Problem number: 684.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_separable]`

$$y(1+2y^2)\,y' = x(2x^2+1)$$

✓ Solution by Maple

Time used: 0.015 (sec). Leaf size: 113

```
dsolve(y(x)*(1+2*y(x)^2)*diff(y(x),x) = x*(2*x^2+1),y(x), singsol=all)
```

$$y(x) = -\frac{\sqrt{-2-2\sqrt{4x^4+4x^2+8c_1+1}}}{2}$$
$$y(x) = \frac{\sqrt{-2-2\sqrt{4x^4+4x^2+8c_1+1}}}{2}$$
$$y(x) = -\frac{\sqrt{-2+2\sqrt{4x^4+4x^2+8c_1+1}}}{2}$$
$$y(x) = \frac{\sqrt{-2+2\sqrt{4x^4+4x^2+8c_1+1}}}{2}$$

✓ Solution by Mathematica

Time used: 2.474 (sec). Leaf size: 151

```
DSolve[y[x]*(1+2*y[x]^2)*y'[x]==x*(1+2*x^2),y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to -\frac{\sqrt{-1-\sqrt{4x^4+4x^2+1+8c_1}}}{\sqrt{2}}$$
$$y(x) \to \frac{\sqrt{-1-\sqrt{4x^4+4x^2+1+8c_1}}}{\sqrt{2}}$$
$$y(x) \to -\frac{\sqrt{-1+\sqrt{4x^4+4x^2+1+8c_1}}}{\sqrt{2}}$$
$$y(x) \to \frac{\sqrt{-1+\sqrt{4x^4+4x^2+1+8c_1}}}{\sqrt{2}}$$

24.23 problem 685

Internal problem ID [3932]

Book: Ordinary differential equations and their solutions. By George Moseley Murphy. 1960
Section: Various 24
Problem number: 685.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[[_homogeneous, 'class A'], _rational, _dAlembert]`

$$\boxed{(3x^2 + 2y^2)\, yy' = -x^3}$$

✓ Solution by Maple

Time used: 0.656 (sec). Leaf size: 137

```
dsolve((3*x^2+2*y(x)^2)*y(x)*diff(y(x),x)+x^3 = 0,y(x), singsol=all)
```

$$y(x) = -\frac{\sqrt{-8c_1^2x^2 - 2\sqrt{8c_1^2x^2 + 1} + 2}}{4c_1}$$
$$y(x) = \frac{\sqrt{-8c_1^2x^2 - 2\sqrt{8c_1^2x^2 + 1} + 2}}{4c_1}$$
$$y(x) = -\frac{\sqrt{-8c_1^2x^2 + 2\sqrt{8c_1^2x^2 + 1} + 2}}{4c_1}$$
$$y(x) = \frac{\sqrt{-8c_1^2x^2 + 2\sqrt{8c_1^2x^2 + 1} + 2}}{4c_1}$$

✓ Solution by Mathematica

Time used: 22.078 (sec). Leaf size: 253

```
DSolve[(3*x^2+2*y[x]^2)*y[x]*y'[x]+x^3==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to -\frac{\sqrt{-4x^2 - \sqrt{8e^{2c_1}x^2 + e^{4c_1}} + e^{2c_1}}}{2\sqrt{2}}$$
$$y(x) \to \frac{\sqrt{-4x^2 - \sqrt{8e^{2c_1}x^2 + e^{4c_1}} + e^{2c_1}}}{2\sqrt{2}}$$
$$y(x) \to -\frac{\sqrt{-4x^2 + \sqrt{8e^{2c_1}x^2 + e^{4c_1}} + e^{2c_1}}}{2\sqrt{2}}$$
$$y(x) \to \frac{\sqrt{-4x^2 + \sqrt{8e^{2c_1}x^2 + e^{4c_1}} + e^{2c_1}}}{2\sqrt{2}}$$
$$y(x) \to \text{Undefined}$$
$$y(x) \to -\frac{\sqrt{-x^2}}{\sqrt{2}}$$
$$y(x) \to \frac{\sqrt{-x^2}}{\sqrt{2}}$$

24.24 problem 686

Internal problem ID [3933]

Book: Ordinary differential equations and their solutions. By George Moseley Murphy. 1960
Section: Various 24
Problem number: 686.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[[_homogeneous, ‘class A‘], _exact, _rational, _dAlembert]`

$$\boxed{(5x^2 + 2y^2)\, yy' + x(x^2 + 5y^2) = 0}$$

✓ Solution by Maple

Time used: 0.109 (sec). Leaf size: 125

```
dsolve((5*x^2+2*y(x)^2)*y(x)*diff(y(x),x)+x*(x^2+5*y(x)^2) = 0,y(x), singsol=all)
```

$$y(x) = -\frac{\sqrt{-10c_1x^2 - 2\sqrt{23c_1^2x^4 + 2}}}{2\sqrt{c_1}}$$
$$y(x) = \frac{\sqrt{-10c_1x^2 - 2\sqrt{23c_1^2x^4 + 2}}}{2\sqrt{c_1}}$$
$$y(x) = -\frac{\sqrt{-10c_1x^2 + 2\sqrt{23c_1^2x^4 + 2}}}{2\sqrt{c_1}}$$
$$y(x) = \frac{\sqrt{-10c_1x^2 + 2\sqrt{23c_1^2x^4 + 2}}}{2\sqrt{c_1}}$$

✓ Solution by Mathematica

Time used: 20.296 (sec). Leaf size: 295

```
DSolve[(5*x^2+2*y[x]^2)*y[x]*y'[x]+x*(x^2+5*y[x]^2)==0,y[x],x,IncludeSingularSolutions -> Tr
```

$$y(x) \to -\frac{\sqrt{-5x^2 - \sqrt{23x^4 + 2e^{4c_1}}}}{\sqrt{2}}$$
$$y(x) \to \frac{\sqrt{-5x^2 - \sqrt{23x^4 + 2e^{4c_1}}}}{\sqrt{2}}$$
$$y(x) \to -\frac{\sqrt{-5x^2 + \sqrt{23x^4 + 2e^{4c_1}}}}{\sqrt{2}}$$
$$y(x) \to \frac{\sqrt{-5x^2 + \sqrt{23x^4 + 2e^{4c_1}}}}{\sqrt{2}}$$
$$y(x) \to -\frac{\sqrt{-\sqrt{23}\sqrt{x^4} - 5x^2}}{\sqrt{2}}$$
$$y(x) \to \frac{\sqrt{-\sqrt{23}\sqrt{x^4} - 5x^2}}{\sqrt{2}}$$
$$y(x) \to -\frac{\sqrt{\sqrt{23}\sqrt{x^4} - 5x^2}}{\sqrt{2}}$$
$$y(x) \to \frac{\sqrt{\sqrt{23}\sqrt{x^4} - 5x^2}}{\sqrt{2}}$$

24.25 problem 687

Internal problem ID [3934]

Book: Ordinary differential equations and their solutions. By George Moseley Murphy. 1960
Section: Various 24
Problem number: 687.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_rational]`

$$\left(x^{2}-x^{3}+3y^{2}x+2y^{3}\right)y'+3x^{2}y+y^{2}-y^{3}=-2x^{3}$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 409

```
dsolve((x^2-x^3+3*x*y(x)^2+2*y(x)^3)*diff(y(x),x)+2*x^3+3*x^2*y(x)+y(x)^2-y(x)^3 = 0,y(x), s
```

$$y(x)=\frac{\left(-108x^{3}-108c_{1}x+12\sqrt{81x^{6}+162c_{1}x^{4}+12x^{3}+\left(81c_{1}^{2}+36c_{1}\right)x^{2}+36x\,c_{1}^{2}+12c_{1}^{3}}\right)^{\frac{2}{3}}-12c_{1}-12x}{6\left(-108x^{3}-108c_{1}x+12\sqrt{81x^{6}+162c_{1}x^{4}+12x^{3}+\left(81c_{1}^{2}+36c_{1}\right)x^{2}+36x\,c_{1}^{2}+12c_{1}^{3}}\right)^{\frac{1}{3}}}$$

$$y(x)=-\frac{\left(\frac{i\sqrt{3}}{12}+\frac{1}{12}\right)\left(-108x^{3}-108c_{1}x+12\sqrt{81x^{6}+162c_{1}x^{4}+12x^{3}+\left(81c_{1}^{2}+36c_{1}\right)x^{2}+36x\,c_{1}^{2}+12c_{1}^{3}}\right)^{\frac{2}{3}}+}{\left(-108x^{3}-108c_{1}x+12\sqrt{81x^{6}+162c_{1}x^{4}+12x^{3}+\left(81c_{1}^{2}+36c_{1}\right)x^{2}+36x\,c_{1}^{2}+12c_{1}^{3}}\right.}$$

$$y(x)=\frac{\frac{\left(i\sqrt{3}-1\right)\left(-108x^{3}-108c_{1}x+12\sqrt{81x^{6}+162c_{1}x^{4}+12x^{3}+\left(81c_{1}^{2}+36c_{1}\right)x^{2}+36x\,c_{1}^{2}+12c_{1}^{3}}\right)^{\frac{2}{3}}}{12}+\left(c_{1}+x\right)\left(1+i\sqrt{3}\right)}{\left(-108x^{3}-108c_{1}x+12\sqrt{81x^{6}+162c_{1}x^{4}+12x^{3}+\left(81c_{1}^{2}+36c_{1}\right)x^{2}+36x\,c_{1}^{2}+12c_{1}^{3}}\right)^{\frac{1}{3}}}$$

✓ Solution by Mathematica

Time used: 8.541 (sec). Leaf size: 368

```
DSolve[(x^2-x^3+3 x y[x]^2+2 y[x]^3)y'[x]+2 x^3+3 x^2 y[x]+y[x]^2-y[x]^3==0,y[x],x,IncludeSi
```

$$y(x) \to \frac{\sqrt[3]{2}(x+c_1)}{\sqrt[3]{27x^3+\sqrt{729\left(x^3+c_1x\right)^2+108(x+c_1)^3}+27c_1x}} - \frac{\sqrt[3]{27x^3+\sqrt{729\left(x^3+c_1x\right)^2+108(x+c_1)^3}+27c_1x}}{3\sqrt[3]{2}}$$

$$y(x) \to \frac{2^{2/3}\left(1-i\sqrt{3}\right)\left(27x^3+\sqrt{729\left(x^3+c_1x\right)^2+108(x+c_1)^3}+27c_1x\right)^{2/3} - 6i\sqrt[3]{2}\left(\sqrt{3}-i\right)(x+c_1)}{12\sqrt[3]{27x^3+\sqrt{729\left(x^3+c_1x\right)^2+108(x+c_1)^3}+27c_1x}}$$

$$y(x) \to \frac{2^{2/3}\left(1+i\sqrt{3}\right)\left(27x^3+\sqrt{729\left(x^3+c_1x\right)^2+108(x+c_1)^3}+27c_1x\right)^{2/3} + 6i\sqrt[3]{2}\left(\sqrt{3}+i\right)(x+c_1)}{12\sqrt[3]{27x^3+\sqrt{729\left(x^3+c_1x\right)^2+108(x+c_1)^3}+27c_1x}}$$

$$y(x) \to -x$$

24.26 problem 688

Internal problem ID [3935]

Book: Ordinary differential equations and their solutions. By George Moseley Murphy. 1960
Section: Various 24
Problem number: 688.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[[_homogeneous, ‘class A‘], _exact, _rational, _dAlembert]`

$$(3x^3 + 6x^2y - 3y^2x + 20y^3)\, y' + 9x^2y + 6y^2x - y^3 = -4x^3$$

✓ Solution by Maple

Time used: 0.063 (sec). Leaf size: 50

```
dsolve((3*x^3+6*x^2*y(x)-3*x*y(x)^2+20*y(x)^3)*diff(y(x),x)+4*x^3+9*x^2*y(x)+6*x*y(x)^2-y(x)
```

$$y(x) = \frac{\operatorname{RootOf}\left(c_1^4x^4 + 3\_Z c_1^3x^3 + 3\_Z^2c_1^2x^2 - \_Z^3c_1x + 5\_Z^4 - 1\right)}{c_1}$$

✓ Solution by Mathematica

Time used: 60.176 (sec). Leaf size: 2201

```
DSolve[(3 x^3+6 x^2 y[x]-3 x y[x]^2+20 y[x]^3)y'[x]+4 x^3+9 x^2 y[x]+6 x y[x]^2-y[x]^3==0,y[
```

$$y(x) \to \frac{1}{2}\sqrt{-\frac{39x^2}{100}+\frac{\sqrt[3]{99x^6+351e^{c_1}x^2+\sqrt{3}\sqrt{-67037x^{12}+185406e^{c_1}x^8-83733e^{2c_1}x^4+32000e^{3c_1}}}}{5\sqrt[3]{2}3^{2/3}}+\frac{}{5\sqrt[3]{99\cdots}}}$$

$$-\frac{1}{2}\sqrt{-\frac{39x^2}{50}-\frac{\sqrt[3]{99x^6+351e^{c_1}x^2+\sqrt{3}\sqrt{-67037x^{12}+185406e^{c_1}x^8-83733e^{2c_1}x^4+32000e^{3c_1}}}}{5\sqrt[3]{2}3^{2/3}}+\frac{}{5\sqrt[3]{\cdots}}}$$

$$+\frac{x}{20}$$

$$y(x) \to \frac{1}{2}\sqrt{-\frac{39x^2}{100}+\frac{\sqrt[3]{99x^6+351e^{c_1}x^2+\sqrt{3}\sqrt{-67037x^{12}+185406e^{c_1}x^8-83733e^{2c_1}x^4+32000e^{3c_1}}}}{5\sqrt[3]{2}3^{2/3}}+\frac{}{5\sqrt[3]{99\cdots}}}$$

$$+\frac{1}{2}\sqrt{-\frac{39x^2}{50}-\frac{\sqrt[3]{99x^6+351e^{c_1}x^2+\sqrt{3}\sqrt{-67037x^{12}+185406e^{c_1}x^8-83733e^{2c_1}x^4+32000e^{3c_1}}}}{5\sqrt[3]{2}3^{2/3}}+\frac{}{5\sqrt[3]{\cdots}}}$$

$$+\frac{x}{20}$$

$$y(x) \to$$

$$-\frac{1}{2}\sqrt{-\frac{39x^2}{100}+\frac{\sqrt[3]{99x^6+351e^{c_1}x^2+\sqrt{3}\sqrt{-67037x^{12}+185406e^{c_1}x^8-83733e^{2c_1}x^4+32000e^{3c_1}}}}{5\sqrt[3]{2}3^{2/3}}+\frac{}{5\sqrt[3]{\cdots}}}$$

$$-\frac{1}{2}\sqrt{-\frac{39x^2}{50}-\frac{\sqrt[3]{99x^6+351e^{c_1}x^2+\sqrt{3}\sqrt{-67037x^{12}+185406e^{c_1}x^8-83733e^{2c_1}x^4+32000e^{3c_1}}}}{5\sqrt[3]{2}3^{2/3}}+\frac{}{5\sqrt[3]{\cdots}}}$$

$$+\frac{x}{20}$$

$$y(x) \to$$

24.27 problem 689

Internal problem ID [3936]

Book: Ordinary differential equations and their solutions. By George Moseley Murphy. 1960
Section: Various 24
Problem number: 689.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[[_homogeneous, ‘class A‘], _rational, _dAlembert]`

$$\boxed{(x^3 + ay^3)\, y' - x^2 y = 0}$$

✓ Solution by Maple

Time used: 0.031 (sec). Leaf size: 23

```
dsolve((x^3+a*y(x)^3)*diff(y(x),x) = x^2*y(x),y(x), singsol=all)
```

$$y(x) = \left(\frac{1}{a \operatorname{LambertW}\left(\frac{x^3 c_1}{a}\right)}\right)^{\frac{1}{3}} x$$

✓ Solution by Mathematica

Time used: 18.61 (sec). Leaf size: 113

```
DSolve[(x^3+a y[x]^3)y'[x]==x^2 y[x],y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to \frac{x}{\sqrt[3]{a}\sqrt[3]{W\left(\frac{x^3 e^{-\frac{3c_1}{a}}}{a}\right)}}$$

$$y(x) \to -\frac{\sqrt[3]{-1}x}{\sqrt[3]{a}\sqrt[3]{W\left(\frac{x^3 e^{-\frac{3c_1}{a}}}{a}\right)}}$$

$$y(x) \to \frac{(-1)^{2/3}x}{\sqrt[3]{a}\sqrt[3]{W\left(\frac{x^3 e^{-\frac{3c_1}{a}}}{a}\right)}}$$

$$y(x) \to 0$$

24.28 problem 691

Internal problem ID [3937]

Book: Ordinary differential equations and their solutions. By George Moseley Murphy. 1960
Section: Various 24
Problem number: 691.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_separable]`

$$\boxed{xy^3y' - (-x^2+1)(y^2+1) = 0}$$

✓ Solution by Maple
Time used: 0.359 (sec). Leaf size: 29

```
dsolve(x*y(x)^3*diff(y(x),x) = (-x^2+1)*(1+y(x)^2),y(x), singsol=all)
```

$$\frac{x^2}{2} - \ln(x) + \frac{y(x)^2}{2} - \frac{\ln(y(x)^2+1)}{2} + c_1 = 0$$

✓ Solution by Mathematica
Time used: 60.095 (sec). Leaf size: 61

```
DSolve[x y[x]^3 y'[x]==(1-x^2)(1+y[x]^2),y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to -\sqrt{-1 - W\left(-\frac{e^{x^2-1-2c_1}}{x^2}\right)}$$
$$y(x) \to \sqrt{-1 - W\left(-\frac{e^{x^2-1-2c_1}}{x^2}\right)}$$

24.29 problem 692

Internal problem ID [3938]

Book: Ordinary differential equations and their solutions. By George Moseley Murphy. 1960
Section: Various 24
Problem number: 692.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[[_homogeneous, 'class G'], _rational]`

$$x(x-y^3)\,y'-(3x+y^3)\,y=0$$

✓ Solution by Maple

Time used: 0.11 (sec). Leaf size: 272

```
dsolve(x*(x-y(x)^3)*diff(y(x),x) = (3*x+y(x)^3)*y(x),y(x), singsol=all)
```

$$y(x)=\frac{\left(-27x^4+3\sqrt{81x^8-3\,\mathrm{e}^{8c_1}}\right)^{\frac{2}{3}}+3\,\mathrm{e}^{\frac{8c_1}{3}}}{3x\left(-27x^4+3\sqrt{81x^8-3\,\mathrm{e}^{8c_1}}\right)^{\frac{1}{3}}}$$

$$y(x)=\frac{-i\sqrt{3}\left(-27x^4+3\sqrt{81x^8-3\,\mathrm{e}^{8c_1}}\right)^{\frac{2}{3}}+3i\sqrt{3}\,\mathrm{e}^{\frac{8c_1}{3}}-\left(-27x^4+3\sqrt{81x^8-3\,\mathrm{e}^{8c_1}}\right)^{\frac{2}{3}}-3\,\mathrm{e}^{\frac{8c_1}{3}}}{6x\left(-27x^4+3\sqrt{81x^8-3\,\mathrm{e}^{8c_1}}\right)^{\frac{1}{3}}}$$

$$y(x)=-\frac{-i\sqrt{3}\left(-27x^4+3\sqrt{81x^8-3\,\mathrm{e}^{8c_1}}\right)^{\frac{2}{3}}+3i\sqrt{3}\,\mathrm{e}^{\frac{8c_1}{3}}+\left(-27x^4+3\sqrt{81x^8-3\,\mathrm{e}^{8c_1}}\right)^{\frac{2}{3}}+3\,\mathrm{e}^{\frac{8c_1}{3}}}{6x\left(-27x^4+3\sqrt{81x^8-3\,\mathrm{e}^{8c_1}}\right)^{\frac{1}{3}}}$$

✓ Solution by Mathematica

Time used: 60.326 (sec). Leaf size: 356

```
DSolve[x(x-y[x]^3)y'[x]==(3 x+y[x]^3)y[x],y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to \frac{e^{\frac{8c_1}{3}}}{\sqrt[3]{-27x^7+3\sqrt{3}\sqrt{-x^6\left(-27x^8+e^{8c_1}\right)}}}+\frac{\sqrt[3]{-9x^7+\sqrt{3}\sqrt{-x^6\left(-27x^8+e^{8c_1}\right)}}}{3^{2/3}x^2}$$

$$y(x) \to \frac{\frac{i\sqrt[6]{3}\left(\sqrt{3}+i\right)\left(-9x^7+\sqrt{3}\sqrt{-x^6\left(-27x^8+e^{8c_1}\right)}\right)^{2/3}}{x^2}-\left(\sqrt{3}+3i\right)e^{\frac{8c_1}{3}}}{2\ 3^{5/6}\sqrt[3]{-9x^7+\sqrt{3}\sqrt{-x^6\left(-27x^8+e^{8c_1}\right)}}}$$

$$y(x) \to \frac{\frac{\left(-1-i\sqrt{3}\right)\left(-9x^7+\sqrt{3}\sqrt{-x^6\left(-27x^8+e^{8c_1}\right)}\right)^{2/3}}{x^2}+i\sqrt[3]{3}\left(\sqrt{3}+i\right)e^{\frac{8c_1}{3}}}{2\ 3^{2/3}\sqrt[3]{-9x^7+\sqrt{3}\sqrt{-x^6\left(-27x^8+e^{8c_1}\right)}}}$$

24.30 problem 693

Internal problem ID [3939]

Book: Ordinary differential equations and their solutions. By George Moseley Murphy. 1960
Section: Various 24
Problem number: 693.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[[_homogeneous, ‘class A‘], _rational, _dAlembert]`

$$x(2x^3+y^3)\,y' - (2x^3 - x^2y + y^3)\,y = 0$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 285

```
dsolve(x*(2*x^3+y(x)^3)*diff(y(x),x) = (2*x^3-x^2*y(x)+y(x)^3)*y(x),y(x), singsol=all)
```

$$y(x) = -\frac{\left(-\left(54+6\sqrt{6c_1^3+18c_1^2\ln(x)+18c_1\ln(x)^2+6\ln(x)^3+81}\right)^{\frac{2}{3}}+6\ln(x)+6c_1\right)x}{3\left(54+6\sqrt{6c_1^3+18c_1^2\ln(x)+18c_1\ln(x)^2+6\ln(x)^3+81}\right)^{\frac{1}{3}}}$$

$$y(x) = -\frac{\left(\left(\frac{i\sqrt{3}}{6}+\frac{1}{6}\right)\left(54+6\sqrt{6c_1^3+18c_1^2\ln(x)+18c_1\ln(x)^2+6\ln(x)^3+81}\right)^{\frac{2}{3}}+\left(i\sqrt{3}-1\right)\left(\ln(x)+c_1\right)\right)}{\left(54+6\sqrt{6c_1^3+18c_1^2\ln(x)+18c_1\ln(x)^2+6\ln(x)^3+81}\right)^{\frac{1}{3}}}$$

$$y(x) = \frac{\left(\frac{\left(i\sqrt{3}-1\right)\left(54+6\sqrt{6c_1^3+18c_1^2\ln(x)+18c_1\ln(x)^2+6\ln(x)^3+81}\right)^{\frac{2}{3}}}{6}+\left(\ln(x)+c_1\right)\left(1+i\sqrt{3}\right)\right)x}{\left(54+6\sqrt{6c_1^3+18c_1^2\ln(x)+18c_1\ln(x)^2+6\ln(x)^3+81}\right)^{\frac{1}{3}}}$$

✓ Solution by Mathematica

Time used: 60.176 (sec). Leaf size: 362

```
DSolve[x(2 x^3+y[x]^3)y'[x]==(2 x^3-x^2 y[x]+y[x]^3)y[x],y[x],x,IncludeSingularSolutions ->
```

$$y(x) \to \frac{-6^{2/3}x^2\log(x)+6^{2/3}c_1x^2+\sqrt[3]{6}\left(9x^3+\sqrt{3}\sqrt{x^6\left(27+2(\log(x)-c_1)^3\right)}\right)^{2/3}}{3\sqrt[3]{9x^3+\sqrt{3}\sqrt{x^6\left(27+2(\log(x)-c_1)^3\right)}}}$$

$$y(x) \to \frac{i\left(\sqrt{3}+i\right)\sqrt[3]{9x^3+\sqrt{3}\sqrt{x^6\left(27+2(\log(x)-c_1)^3\right)}}}{6^{2/3}} + \frac{\left(1+i\sqrt{3}\right)x^2(\log(x)-c_1)}{\sqrt[3]{6}\sqrt[3]{9x^3+\sqrt{3}\sqrt{x^6\left(27+2(\log(x)-c_1)^3\right)}}}$$

$$y(x) \to \frac{i\left(\sqrt{3}+i\right)x^2(-\log(x)+c_1)}{\sqrt[3]{6}\sqrt[3]{9x^3+\sqrt{3}\sqrt{x^6\left(27+2(\log(x)-c_1)^3\right)}}} - \frac{\left(1+i\sqrt{3}\right)\sqrt[3]{9x^3+\sqrt{3}\sqrt{x^6\left(27+2(\log(x)-c_1)^3\right)}}}{6^{2/3}}$$

24.31 problem 694

Internal problem ID [3940]

Book: Ordinary differential equations and their solutions. By George Moseley Murphy. 1960
Section: Various 24
Problem number: 694.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[[_homogeneous, 'class A'], _rational, _dAlembert]`

$$x\left(2x^3 - y^3\right) y' - \left(x^3 - 2y^3\right) y = 0$$

✓ Solution by Maple

Time used: 0.016 (sec). Leaf size: 327

```
dsolve(x*(2*x^3-y(x)^3)*diff(y(x),x) = (x^3-2*y(x)^3)*y(x),y(x), singsol=all)
```

$$y(x) = \frac{\left(\frac{\left(-108+8x^3c_1^3+12\sqrt{-12x^3c_1^3+81}\right)^{\frac{1}{3}}}{2} + \frac{2x^2c_1^2}{\left(-108+8x^3c_1^3+12\sqrt{-12x^3c_1^3+81}\right)^{\frac{1}{3}}} + c_1x\right)x}{3}$$

$$y(x) = -\frac{\left(-4i\sqrt{3}\,c_1^2x^2 + i\sqrt{3}\left(-108+8x^3c_1^3+12\sqrt{-12x^3c_1^3+81}\right)^{\frac{2}{3}} + 4c_1^2x^2 - 4c_1x\left(-108+8x^3c_1^3+12\sqrt{-\right.\right.}{12\left(-108+8x^3c_1^3+12\sqrt{-12x^3c_1^3+81}\right)^{\frac{1}{3}}}$$

$$y(x) = \frac{\left(-108+8x^3c_1^3+12\sqrt{-12x^3c_1^3+81}\right)^{\frac{1}{3}}\left(i\sqrt{3}-1\right)x}{12} - \frac{c_1x^2\left(ixc_1\sqrt{3}+c_1x-\left(-108+8x^3c_1^3+12\sqrt{-12x^3c_1^3+81}\right)^{\frac{1}{3}}\right)}{3\left(-108+8x^3c_1^3+12\sqrt{-12x^3c_1^3+81}\right)^{\frac{1}{3}}}$$

✓ Solution by Mathematica

Time used: 57.083 (sec). Leaf size: 542

```
DSolve[x(2 x^3-y[x]^3)y'[x]==(x^3-2 y[x]^3)y[x],y[x],x,IncludeSingularSolutions -> True]
```

$$y(x)\to \frac{1}{3}\left(e^{c_1}x^2+\frac{\sqrt[3]{2e^{3c_1}x^6-27x^3+3\sqrt{81x^6-12e^{3c_1}x^9}}}{\sqrt[3]{2}}+\frac{\sqrt[3]{2}e^{2c_1}x^4}{\sqrt[3]{2e^{3c_1}x^6-27x^3+3\sqrt{81x^6-12e^{3c_1}x^9}}}\right)$$

$$y(x)\to \frac{e^{c_1}x^2}{3}+\frac{i\left(\sqrt{3}+i\right)\sqrt[3]{2e^{3c_1}x^6-27x^3+3\sqrt{81x^6-12e^{3c_1}x^9}}}{6\sqrt[3]{2}}-\frac{i\left(\sqrt{3}-i\right)e^{2c_1}x^4}{3\ 2^{2/3}\sqrt[3]{2e^{3c_1}x^6-27x^3+3\sqrt{81x^6-12e^{3c_1}x^9}}}$$

$$y(x)\to \frac{e^{c_1}x^2}{3}-\frac{i\left(\sqrt{3}-i\right)\sqrt[3]{2e^{3c_1}x^6-27x^3+3\sqrt{81x^6-12e^{3c_1}x^9}}}{6\sqrt[3]{2}}+\frac{i\left(\sqrt{3}+i\right)e^{2c_1}x^4}{3\ 2^{2/3}\sqrt[3]{2e^{3c_1}x^6-27x^3+3\sqrt{81x^6-12e^{3c_1}x^9}}}$$

$$y(x)\to \frac{\sqrt[3]{\sqrt{x^6}-x^3}}{\sqrt[3]{2}}$$

$$y(x)\to -\frac{i\left(\sqrt{3}-i\right)\sqrt[3]{\sqrt{x^6}-x^3}}{2\sqrt[3]{2}}$$

$$y(x)\to \frac{i\left(\sqrt{3}+i\right)\sqrt[3]{\sqrt{x^6}-x^3}}{2\sqrt[3]{2}}$$

24.32 problem 695

Internal problem ID [3941]

Book: Ordinary differential equations and their solutions. By George Moseley Murphy. 1960
Section: Various 24
Problem number: 695.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[[_homogeneous, ‘class A‘], _rational, _dAlembert]`

$$x(x^3 + 3x^2y + y^3)\, y' - (y^2 + 3x^2)\, y^2 = 0$$

✓ Solution by Maple

Time used: 0.079 (sec). Leaf size: 27

```
dsolve(x*(x^3+3*x^2*y(x)+y(x)^3)*diff(y(x),x) = (3*x^2+y(x)^2)*y(x)^2,y(x), singsol=all)
```

$$y(x) = \mathrm{e}^{\mathrm{RootOf}\left(\mathrm{e}^{3\_Z}+9\,\mathrm{e}^{\_Z}+3c_1+3\_Z+3\ln(x)\right)} x$$

✓ Solution by Mathematica

Time used: 0.236 (sec). Leaf size: 37

```
DSolve[x(x^3+3 x^2 y[x]+y[x]^3)y'[x]==(3 x^2+y[x]^2)y[x]^2,y[x],x,IncludeSingularSolutions -
```

$$\mathrm{Solve}\left[\frac{y(x)^3}{3x^3} + \frac{3y(x)}{x} + \log\left(\frac{y(x)}{x}\right) = -\log(x) + c_1, y(x)\right]$$

24.33 problem 696

Internal problem ID [3942]

Book: Ordinary differential equations and their solutions. By George Moseley Murphy. 1960
Section: Various 24
Problem number: 696.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[[_homogeneous, ‘class A‘], _rational, _dAlembert]`

$$\boxed{x(x^3-2y^3)\,y'-(2x^3-y^3)\,y=0}$$

✓ Solution by Maple

Time used: 0.031 (sec). Leaf size: 317

```
dsolve(x*(x^3-2*y(x)^3)*diff(y(x),x) = (2*x^3-y(x)^3)*y(x),y(x), singsol=all)
```

$$y(x)=\frac{12^{\frac{1}{3}}\left(x12^{\frac{1}{3}}c_1+\left(x\left(-9c_1x^2+\sqrt{3}\,\sqrt{\frac{27c_1^3x^4-4x}{c_1}}\right)c_1^2\right)^{\frac{2}{3}}\right)}{6c_1\left(x\left(-9c_1x^2+\sqrt{3}\,\sqrt{\frac{27c_1^3x^4-4x}{c_1}}\right)c_1^2\right)^{\frac{1}{3}}}$$

$$y(x)=\frac{3^{\frac{1}{3}}2^{\frac{2}{3}}\left(\left(-1-i\sqrt{3}\right)\left(x\left(-9c_1x^2+\sqrt{3}\,\sqrt{\frac{27c_1^3x^4-4x}{c_1}}\right)c_1^2\right)^{\frac{2}{3}}+\left(i3^{\frac{5}{6}}-3^{\frac{1}{3}}\right)c_12^{\frac{2}{3}}x\right)}{12\left(x\left(-9c_1x^2+\sqrt{3}\,\sqrt{\frac{27c_1^3x^4-4x}{c_1}}\right)c_1^2\right)^{\frac{1}{3}}c_1}$$

$$y(x)=-\frac{3^{\frac{1}{3}}\left(\left(1-i\sqrt{3}\right)\left(x\left(-9c_1x^2+\sqrt{3}\,\sqrt{\frac{27c_1^3x^4-4x}{c_1}}\right)c_1^2\right)^{\frac{2}{3}}+c_12^{\frac{2}{3}}x\left(i3^{\frac{5}{6}}+3^{\frac{1}{3}}\right)\right)2^{\frac{2}{3}}}{12\left(x\left(-9c_1x^2+\sqrt{3}\,\sqrt{\frac{27c_1^3x^4-4x}{c_1}}\right)c_1^2\right)^{\frac{1}{3}}c_1}$$

✓ Solution by Mathematica

Time used: 60.353 (sec). Leaf size: 331

```
DSolve[x(x^3-2 y[x]^3)y'[x]==(2 x^3-y[x]^3)y[x],y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to \frac{\sqrt[3]{2}\left(-9x^3+\sqrt{81x^6-12e^{3c_1}x^3}\right)^{2/3}+2\sqrt[3]{3}e^{c_1}x}{6^{2/3}\sqrt[3]{-9x^3+\sqrt{81x^6-12e^{3c_1}x^3}}}$$

$$y(x) \to \frac{i\sqrt[3]{2}\sqrt[6]{3}\left(\sqrt{3}+i\right)\left(-9x^3+\sqrt{81x^6-12e^{3c_1}x^3}\right)^{2/3}-2\left(\sqrt{3}+3i\right)e^{c_1}x}{2\,2^{2/3}3^{5/6}\sqrt[3]{-9x^3+\sqrt{81x^6-12e^{3c_1}x^3}}}$$

$$y(x) \to \frac{\sqrt[3]{2}\sqrt[6]{3}\left(-1-i\sqrt{3}\right)\left(-9x^3+\sqrt{81x^6-12e^{3c_1}x^3}\right)^{2/3}-2\left(\sqrt{3}-3i\right)e^{c_1}x}{2\,2^{2/3}3^{5/6}\sqrt[3]{-9x^3+\sqrt{81x^6-12e^{3c_1}x^3}}}$$

24.34 problem 697

Internal problem ID [3943]

Book: Ordinary differential equations and their solutions. By George Moseley Murphy. 1960
Section: Various 24
Problem number: 697.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[[_homogeneous, ‘class G‘], _rational]`

$$x(x^4 - 2y^3)\, y' + (2x^4 + y^3)\, y = 0$$

✓ Solution by Maple

Time used: 0.734 (sec). Leaf size: 28

```
dsolve(x*(x^4-2*y(x)^3)*diff(y(x),x)+(2*x^4+y(x)^3)*y(x) = 0,y(x), singsol=all)
```

$$\ln(x) - c_1 + \frac{3\ln\left(\frac{y(x)\left(-2x^4+y(x)^3\right)}{x^{\frac{16}{3}}}\right)}{10} = 0$$

✓ Solution by Mathematica

Time used: 60.195 (sec). Leaf size: 1139

```
DSolve[x(x^4-2 y[x]^3)y'[x]+(2 x^4+y[x]^3)y[x]==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to \frac{1}{6}\left(-\sqrt[6]{2}3^{2/3}\sqrt{\frac{4\sqrt[3]{6}c_1x^2 + \left(9x^8 - \sqrt{81x^{16} - 384c_1{}^3x^6}\right)^{2/3}}{\sqrt[3]{9x^8 - \sqrt{81x^{16} - 384c_1{}^3x^6}}}}\right.$$

$$-3\sqrt{-\frac{\sqrt[3]{18x^8 - 2\sqrt{81x^{16} - 384c_1{}^3x^6}}}{3^{2/3}} - \frac{4\,2^{2/3}c_1x^2}{\sqrt[3]{27x^8 - 3\sqrt{81x^{16} - 384c_1{}^3x^6}}} - \frac{4\sqrt{3}x^4}{\sqrt{\frac{4\,6^{2/3}c_1x^2+\sqrt[3]{6}\left(9x^8-\sqrt{81x^{16}\ldots\right.}{\sqrt[3]{9x^8 - \sqrt{81x^{16} - 3\ldots}}}}}}$$

$$y(x) \to \frac{1}{6}\left(3\sqrt{-\frac{\sqrt[3]{18x^8 - 2\sqrt{81x^{16} - 384c_1{}^3x^6}}}{3^{2/3}} - \frac{4\,2^{2/3}c_1x^2}{\sqrt[3]{27x^8 - 3\sqrt{81x^{16} - 384c_1{}^3x^6}}} - \frac{4\sqrt{3}x^4}{\sqrt{\frac{4\,6^{2/3}c_1x^2+\sqrt[3]{6}\left(9x^8-\sqrt{81x\ldots\right.}{\sqrt[3]{9x^8 - \sqrt{81x^{16} - \ldots}}}}}}\right.$$

$$\left. - \sqrt[6]{2}3^{2/3}\sqrt{\frac{4\sqrt[3]{6}c_1x^2 + \left(9x^8 - \sqrt{81x^{16} - 384c_1{}^3x^6}\right)^{2/3}}{\sqrt[3]{9x^8 - \sqrt{81x^{16} - 384c_1{}^3x^6}}}}\right)$$

$$y(x) \to \frac{1}{6}\left(\sqrt[6]{2}3^{2/3}\sqrt{\frac{4\sqrt[3]{6}c_1x^2 + \left(9x^8 - \sqrt{81x^{16} - 384c_1{}^3x^6}\right)^{2/3}}{\sqrt[3]{9x^8 - \sqrt{81x^{16} - 384c_1{}^3x^6}}}}\right.$$

$$-3\sqrt{-\frac{\sqrt[3]{18x^8 - 2\sqrt{81x^{16} - 384c_1{}^3x^6}}}{3^{2/3}} - \frac{4\,2^{2/3}c_1x^2}{\sqrt[3]{27x^8 - 3\sqrt{81x^{16} - 384c_1{}^3x^6}}} + \frac{4\sqrt{3}x^4}{\sqrt{\frac{4\,6^{2/3}c_1x^2+\sqrt[3]{6}\left(9x^8-\sqrt{81x^{16}\ldots\right.}{\sqrt[3]{9x^8 - \sqrt{81x^{16} - 3\ldots}}}}}}$$

$$y(x) \to \frac{1}{6}\left(\sqrt[6]{2}3^{2/3}\sqrt{\frac{4\sqrt[3]{6}c_1x^2 + \left(9x^8 - \sqrt{81x^{16} - 384c_1{}^3x^6}\right)^{2/3}}{\ldots}}\right.$$

25 Various 25

25.1 problem 698

Internal problem ID [3944]

Book: Ordinary differential equations and their solutions. By George Moseley Murphy. 1960
Section: Various 25
Problem number: 698.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_rational]`

$$\boxed{x\left(x+y+2y^{3}\right)y'-y(-y+x)=0}$$

✓ Solution by Maple
Time used: 0.016 (sec). Leaf size: 29

```
dsolve(x*(x+y(x)+2*y(x)^3)*diff(y(x),x) = (x-y(x))*y(x),y(x), singsol=all)
```

$$y(x)=\mathrm{e}^{\mathrm{RootOf}\left(-\mathrm{e}^{3\_Z}-\mathrm{e}^{\_Z}\ln(x)+c_1\mathrm{e}^{\_Z}-\_Z\mathrm{e}^{\_Z}+x\right)}$$

✓ Solution by Mathematica
Time used: 0.322 (sec). Leaf size: 23

```
DSolve[x(x+y[x]+2 y[x]^3)y'[x]==(x-y[x])y[x],y[x],x,IncludeSingularSolutions -> True]
```

$$\mathrm{Solve}\left[y(x)^2-\frac{x}{y(x)}+\log(y(x))+\log(x)=c_1,y(x)\right]$$

25.2 problem 699

Internal problem ID [3945]

Book: Ordinary differential equations and their solutions. By George Moseley Murphy. 1960
Section: Various 25
Problem number: 699.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_rational, [_1st_order, '_with_symmetry_[F(x)*G(y),0]']]`

$$\boxed{(5x - y - 7xy^3)\, y' + 5y - y^4 = 0}$$

✓ Solution by Maple

Time used: 0.016 (sec). Leaf size: 34

```
dsolve((5*x-y(x)-7*x*y(x)^3)*diff(y(x),x)+5*y(x)-y(x)^4 = 0,y(x), singsol=all)
```

$$x + \frac{\frac{y(x)^5}{5} - \frac{5y(x)^2}{2} - c_1}{y(x)\left(y(x)^3 - 5\right)^2} = 0$$

✓ Solution by Mathematica

Time used: 60.185 (sec). Leaf size: 302

```
DSolve[(5 x-y[x]-7 x y[x]^3)y'[x]+5 y[x]-y[x]^4==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to \text{Root}\left[10\#1^7x + 2\#1^5 - 100\#1^4x - 25\#1^2 + 250\#1x - 10c_1\&, 1\right]$$
$$y(x) \to \text{Root}\left[10\#1^7x + 2\#1^5 - 100\#1^4x - 25\#1^2 + 250\#1x - 10c_1\&, 2\right]$$
$$y(x) \to \text{Root}\left[10\#1^7x + 2\#1^5 - 100\#1^4x - 25\#1^2 + 250\#1x - 10c_1\&, 3\right]$$
$$y(x) \to \text{Root}\left[10\#1^7x + 2\#1^5 - 100\#1^4x - 25\#1^2 + 250\#1x - 10c_1\&, 4\right]$$
$$y(x) \to \text{Root}\left[10\#1^7x + 2\#1^5 - 100\#1^4x - 25\#1^2 + 250\#1x - 10c_1\&, 5\right]$$
$$y(x) \to \text{Root}\left[10\#1^7x + 2\#1^5 - 100\#1^4x - 25\#1^2 + 250\#1x - 10c_1\&, 6\right]$$
$$y(x) \to \text{Root}\left[10\#1^7x + 2\#1^5 - 100\#1^4x - 25\#1^2 + 250\#1x - 10c_1\&, 7\right]$$

25.3 problem 700

Internal problem ID [3946]

Book: Ordinary differential equations and their solutions. By George Moseley Murphy. 1960
Section: Various 25
Problem number: 700.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_rational]`

$$\boxed{x\left(1-2x y^{3}\right) y'+\left(1-2y x^{3}\right) y=0}$$

✓ Solution by Maple
Time used: 0.0 (sec). Leaf size: 361

```
dsolve(x*(1-2*x*y(x)^3)*diff(y(x),x)+(1-2*x^3*y(x))*y(x) = 0,y(x), singsol=all)
```

$$y(x) = -\frac{12^{\frac{1}{3}}\left(-\left(\left(-9+\sqrt{12x^{8}-36c_{1}x^{6}+36c_{1}^{2}x^{4}-12c_{1}^{3}x^{2}+81}\right)x^{2}\right)^{\frac{2}{3}}+12^{\frac{1}{3}}x^{2}(x^{2}-c_{1})\right)}{6\left(\left(-9+\sqrt{12x^{8}-36c_{1}x^{6}+36c_{1}^{2}x^{4}-12c_{1}^{3}x^{2}+81}\right)x^{2}\right)^{\frac{1}{3}}x}$$

$$y(x) = -\frac{3^{\frac{1}{3}}\left(\left(1+i\sqrt{3}\right)\left(\left(-9+\sqrt{12x^{8}-36c_{1}x^{6}+36c_{1}^{2}x^{4}-12c_{1}^{3}x^{2}+81}\right)x^{2}\right)^{\frac{2}{3}}+\left(i3^{\frac{5}{6}}-3^{\frac{1}{3}}\right)2^{\frac{2}{3}}\left(x^{2}-c_{1}\right)x^{2}}{12\left(\left(-9+\sqrt{12x^{8}-36c_{1}x^{6}+36c_{1}^{2}x^{4}-12c_{1}^{3}x^{2}+81}\right)x^{2}\right)^{\frac{1}{3}}x}$$

$$y(x) = \frac{3^{\frac{1}{3}}\left(\left(i\sqrt{3}-1\right)\left(\left(-9+\sqrt{12x^{8}-36c_{1}x^{6}+36c_{1}^{2}x^{4}-12c_{1}^{3}x^{2}+81}\right)x^{2}\right)^{\frac{2}{3}}+2^{\frac{2}{3}}\left(x^{2}-c_{1}\right)x^{2}\left(i3^{\frac{5}{6}}+3^{\frac{1}{3}}\right)\right)2}{12\left(\left(-9+\sqrt{12x^{8}-36c_{1}x^{6}+36c_{1}^{2}x^{4}-12c_{1}^{3}x^{2}+81}\right)x^{2}\right)^{\frac{1}{3}}x}$$

✓ Solution by Mathematica

Time used: 50.301 (sec). Leaf size: 358

```
DSolve[x(1-2 x y[x]^3)y'[x]+(1-2 x^3 y[x])y[x]==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to \frac{\sqrt[3]{2}(-x^3+c_1x)}{\sqrt[3]{-27x^2+\sqrt{729x^4+108x^3(x^3-c_1x)^3}}} + \frac{\sqrt[3]{-27x^2+\sqrt{729x^4+108x^3(x^3-c_1x)^3}}}{3\sqrt[3]{2}x}$$

$$y(x) \to \frac{\left(1+i\sqrt{3}\right)(x^3-c_1x)}{2^{2/3}\sqrt[3]{-27x^2+\sqrt{729x^4+108x^3(x^3-c_1x)^3}}} - \frac{\left(1-i\sqrt{3}\right)\sqrt[3]{-27x^2+\sqrt{729x^4+108x^3(x^3-c_1x)^3}}}{6\sqrt[3]{2}x}$$

$$y(x) \to \frac{\left(1-i\sqrt{3}\right)(x^3-c_1x)}{2^{2/3}\sqrt[3]{-27x^2+\sqrt{729x^4+108x^3(x^3-c_1x)^3}}} - \frac{\left(1+i\sqrt{3}\right)\sqrt[3]{-27x^2+\sqrt{729x^4+108x^3(x^3-c_1x)^3}}}{6\sqrt[3]{2}x}$$

25.4 problem 701

Internal problem ID [3947]

Book: Ordinary differential equations and their solutions. By George Moseley Murphy. 1960
Section: Various 25
Problem number: 701.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_rational, [_1st_order, '_with_symmetry_[F(x)*G(y),0]']]`

$$\boxed{x(2 - y^2 x - 2xy^3)\, y' + 2y = -1}$$

✓ Solution by Maple
Time used: 0.015 (sec). Leaf size: 46

```
dsolve(x*(2-x*y(x)^2-2*x*y(x)^3)*diff(y(x),x)+1+2*y(x) = 0,y(x), singsol=all)
```

$$y(x) = -\frac{1}{2}$$
$$y(x) = \frac{e^{\mathrm{RootOf}\left(x\,e^{3\_Z} - 4x\,e^{2\_Z} + 8c_1 x\,e^{\_Z} + 2\_Z e^{\_Z} x + 3\,e^{\_Z} x + 16\right)}}{2} - \frac{1}{2}$$

✓ Solution by Mathematica
Time used: 0.483 (sec). Leaf size: 47

```
DSolve[x(2-x y[x]^2-2 x y[x]^3)y'[x]+1+2 y[x]==0,y[x],x,IncludeSingularSolutions -> True]
```

$$\mathrm{Solve}\left[\frac{1}{64}\left(-4y(x)^2 + 4y(x) - 2\log(8y(x)+4) + 3\right) - \frac{1}{4x(2y(x)+1)} = c_1, y(x)\right]$$

25.5 problem 702

Internal problem ID [3948]

Book: Ordinary differential equations and their solutions. By George Moseley Murphy. 1960
Section: Various 25
Problem number: 702.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_exact, _rational, [_1st_order, '_with_symmetry_[F(x)*G(y),0]`

$$\boxed{(2 - 10y^3x^2 + 3y^2)\, y' - x(1 + 5y^4) = 0}$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 27

```
dsolve((2-10*x^2*y(x)^3+3*y(x)^2)*diff(y(x),x) = x*(1+5*y(x)^4),y(x), singsol=all)
```

$$-\frac{5y(x)^4\, x^2}{2} - \frac{x^2}{2} + y(x)^3 + 2y(x) + c_1 = 0$$

✓ Solution by Mathematica

Time used: 60.262 (sec). Leaf size: 2097

```
DSolve[(2-10*x^2*y[x]^3+3*y[x]^2)*y'[x]==x*(1+5*y[x]^4),y[x],x,IncludeSingularSolutions -> T
```

$y(x) \rightarrow$

$$-\frac{\sqrt{3}x^2\sqrt{\frac{5\sqrt[3]{6}x^2\sqrt[3]{189x^2+\sqrt{3}\sqrt{27\left(21x^2-2c_1\right)^2-16\left(5x^4-10c_1x^2-2\right)^3}-18c_1}+\frac{\cdots}{\sqrt[3]{189x^2+\sqrt{3}\sqrt{2\cdots}}}}{x^4}}}{\cdots}$$

$y(x)$

$$\rightarrow \frac{-\sqrt{3}x^2\sqrt{\frac{5\sqrt[3]{6}x^2\sqrt[3]{189x^2+\sqrt{3}\sqrt{27\left(21x^2-2c_1\right)^2-16\left(5x^4-10c_1x^2-2\right)^3}-18c_1}+\frac{\cdots}{\sqrt[3]{189x^2+\sqrt{3}\sqrt{2\cdots}}}}{x^4}}}{\cdots}$$

$y(x)$

$$\rightarrow \frac{\sqrt{3}x^2\sqrt{\frac{5\sqrt[3]{6}x^2\sqrt[3]{189x^2+\sqrt{3}\sqrt{27\left(21x^2-2c_1\right)^2-16\left(5x^4-10c_1x^2-2\right)^3}-18c_1}+\frac{\cdots}{\sqrt[3]{189x^2+\sqrt{3}\sqrt{27\cdots}}}}{x^4}}}{\cdots}$$

$y(x)$

$$\rightarrow \frac{\sqrt{3}x^2\sqrt{\frac{5\sqrt[3]{6}x^2\sqrt[3]{189x^2+\sqrt{3}\sqrt{27\left(21x^2-2c_1\right)^2-16\left(5x^4-10c_1x^2-2\right)^3}-18c_1}+\frac{\cdots}{\sqrt[3]{189x^2+\sqrt{3}\sqrt{27\cdots}}}}{x^4}}}{\cdots}$$

25.6 problem 703

Internal problem ID [3949]

Book: Ordinary differential equations and their solutions. By George Moseley Murphy. 1960
Section: Various 25
Problem number: 703.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_rational]`

$$x\left(a+bxy^3\right)y'+\left(a+c\,x^3y\right)y=0$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 492

```
dsolve(x*(a+b*x*y(x)^3)*diff(y(x),x)+(a+c*x^3*y(x))*y(x) = 0,y(x), singsol=all)
```

$$y(x) = \frac{3^{\frac{1}{3}}\left(-x^2b(c\,x^2-2c_1)\,3^{\frac{1}{3}}+\left(\left(9a+\sqrt{\frac{3c^3x^8-18c^2c_1x^6+36c\,c_1^2x^4-24c_1^3x^2+81a^2b}{b}}\right)b^2x^2\right)^{\frac{2}{3}}\right)}{3\left(\left(9a+\sqrt{\frac{3c^3x^8-18c^2c_1x^6+36c\,c_1^2x^4-24c_1^3x^2+81a^2b}{b}}\right)b^2x^2\right)^{\frac{1}{3}}bx}$$

$$y(x) = -\frac{\left(\left(1+i\sqrt{3}\right)\left(\left(9a+\sqrt{\frac{3c^3x^8-18c^2c_1x^6+36c\,c_1^2x^4-24c_1^3x^2+81a^2b}{b}}\right)b^2x^2\right)^{\frac{2}{3}}+x^2\left(i3^{\frac{5}{6}}-3^{\frac{1}{3}}\right)b(c\,x^2-2c_1)\right)3^{\frac{1}{3}}}{6\left(\left(9a+\sqrt{\frac{3c^3x^8-18c^2c_1x^6+36c\,c_1^2x^4-24c_1^3x^2+81a^2b}{b}}\right)b^2x^2\right)^{\frac{1}{3}}bx}$$

$$y(x) = \frac{3^{\frac{1}{3}}\left(\left(i\sqrt{3}-1\right)\left(\left(9a+\sqrt{\frac{3c^3x^8-18c^2c_1x^6+36c\,c_1^2x^4-24c_1^3x^2+81a^2b}{b}}\right)b^2x^2\right)^{\frac{2}{3}}+x^2b\left(i3^{\frac{5}{6}}+3^{\frac{1}{3}}\right)(c\,x^2-2c_1)\right)}{6\left(\left(9a+\sqrt{\frac{3c^3x^8-18c^2c_1x^6+36c\,c_1^2x^4-24c_1^3x^2+81a^2b}{b}}\right)b^2x^2\right)^{\frac{1}{3}}bx}$$

✓ Solution by Mathematica

Time used: 60.307 (sec). Leaf size: 484

```
DSolve[x(a+b x y[x]^3)y'[x]+(a+c x^3 y[x])y[x]==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to \frac{x(-cx^2+2c_1)}{\sqrt[3]{3}\sqrt[3]{9ab^2x^2+\sqrt{3}\sqrt{b^3x^4\left(27a^2b+x^2\left(cx^2-2c_1\right)^3\right)}}} + \frac{\sqrt[3]{9ab^2x^2+\sqrt{3}\sqrt{b^3x^4\left(27a^2b+x^2\left(cx^2-2c_1\right)^3\right)}}}{3^{2/3}bx}$$

$$y(x) \to \frac{i\sqrt[3]{3}(\sqrt{3}+i)\left(9ab^2x^2+\sqrt{3}\sqrt{b^3x^4\left(27a^2b+x^2\left(cx^2-2c_1\right)^3\right)}\right)^{2/3}+\sqrt[6]{3}(\sqrt{3}+3i)\,bx^2(cx^2-2c_1)}{6bx\sqrt[3]{9ab^2x^2+\sqrt{3}\sqrt{b^3x^4\left(27a^2b+x^2\left(cx^2-2c_1\right)^3\right)}}}$$

$$y(x) \to \frac{\sqrt[6]{3}(\sqrt{3}-3i)\,bx^2(cx^2-2c_1)-i\sqrt[3]{3}(\sqrt{3}-i)\left(9ab^2x^2+\sqrt{3}\sqrt{b^3x^4\left(27a^2b+x^2\left(cx^2-2c_1\right)^3\right)}\right)^{2/3}}{6bx\sqrt[3]{9ab^2x^2+\sqrt{3}\sqrt{b^3x^4\left(27a^2b+x^2\left(cx^2-2c_1\right)^3\right)}}}$$

25.7 problem 704

Internal problem ID [3950]

Book: Ordinary differential equations and their solutions. By George Moseley Murphy. 1960
Section: Various 25
Problem number: 704.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_rational]`

$$x(1-2y^3x^2)\,y' + (1-2y^2x^3)\,y = 0$$

✓ Solution by Maple
Time used: 0.0 (sec). Leaf size: 811

```
dsolve(x*(1-2*x^2*y(x)^3)*diff(y(x),x)+(1-2*x^3*y(x)^2)*y(x) = 0,y(x), singsol=all)
```

$$y(x) = \frac{\left(\left(c_1^3x^2 - 6c_1^2x^3 + 12c_1x^4 - 8x^5 + 3\sqrt{-6c_1^3x^2 + 36c_1^2x^3 - 72c_1x^4 + 48x^5 + 81} - 27\right)x\right)^{\frac{1}{3}}}{6x} + \frac{(-2x+c_1)^2\,x}{6\left(\left(c_1^3x^2 - 6c_1^2x^3 + 12c_1x^4 - 8x^5 + 3\sqrt{-6c_1^3x^2 + 36c_1^2x^3 - 72c_1x^4 + 48x^5 + 81} - 27\right)x\right)^{\frac{1}{3}}} - \frac{x}{3} + \frac{c_1}{6}$$

$$y(x) = \frac{-2(-c_1x + 2x^2)\left(\left(c_1^3x^2 - 6c_1^2x^3 + 12c_1x^4 - 8x^5 + 3\sqrt{-6c_1^3x^2 + 36c_1^2x^3 - 72c_1x^4 + 48x^5 + 81} - 27\right)x\right)}{}$$

$$y(x) = \frac{2(c_1x - 2x^2)\left(\left(c_1^3x^2 - 6c_1^2x^3 + 12c_1x^4 - 8x^5 + 3\sqrt{-6c_1^3x^2 + 36c_1^2x^3 - 72c_1x^4 + 48x^5 + 81} - 27\right)x\right)^{\frac{1}{3}} +}{}$$

✓ Solution by Mathematica

Time used: 60.157 (sec). Leaf size: 672

```
DSolve[x(1-2 x^2 y[x]^3)y'[x]+(1-2 x^3 y[x]^2)y[x]==0,y[x],x,IncludeSingularSolutions -> Tru
```

$$y(x) \to \frac{-2x^3 + c_1x^2 + \frac{x^4(-2x+c_1)^2}{\sqrt[3]{-8x^9 + 12c_1x^8 - 6c_1{}^2x^7 + c_1{}^3x^6 - 27x^4 + 3\sqrt{3}\sqrt{x^8\left(16x^5 - 24c_1x^4 + 12c_1{}^2x^3 - 2c_1{}^3 \right.}}}}{}$$

$$y(x) \to \frac{2x^2(-2x + c_1) - \frac{i\left(\sqrt{3}-i\right)x^4(-2x+c_1)^2}{\sqrt[3]{-8x^9 + 12c_1x^8 - 6c_1{}^2x^7 + c_1{}^3x^6 - 27x^4 + 3\sqrt{3}\sqrt{x^8\left(16x^5 - 24c_1x^4 + 12c_1{}^2x^3 - 2c \right.}}}}{}$$

$$y(x) \to \frac{2x^2(-2x + c_1) + \frac{i\left(\sqrt{3}+i\right)x^4(-2x+c_1)^2}{\sqrt[3]{-8x^9 + 12c_1x^8 - 6c_1{}^2x^7 + c_1{}^3x^6 - 27x^4 + 3\sqrt{3}\sqrt{x^8\left(16x^5 - 24c_1x^4 + 12c_1{}^2x^3 - 2c \right.}}}}{}$$

25.8 problem 705

Internal problem ID [3951]

Book: Ordinary differential equations and their solutions. By George Moseley Murphy. 1960
Section: Various 25
Problem number: 705.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[[_homogeneous, ‘class G‘], _rational]`

$$x(-yx+1)\left(1-y^2x^2\right)y'+(1+yx)\left(1+y^2x^2\right)y=0$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 42

```
dsolve(x*(1-x*y(x))*(1-x^2*y(x)^2)*diff(y(x),x)+(1+x*y(x))*(1+x^2*y(x)^2)*y(x) = 0,y(x), sin
```

$$y(x)=-\frac{1}{x}$$
$$y(x)=\frac{e^{\mathrm{RootOf}\left(-e^{2\_Z}-2\,e^{\_Z}\ln(x)+2c_1e^{\_Z}+2\_Ze^{\_Z}+1\right)}}{x}$$

✓ Solution by Mathematica

Time used: 0.342 (sec). Leaf size: 35

```
DSolve[x(1-x y[x])(1-x^2 y[x]^2)y'[x]+(1+x y[x])(1+x^2 y[x]^2)y[x]==0,y[x],x,IncludeSingular
```

$$y(x)\to -\frac{1}{x}$$
$$\mathrm{Solve}\left[xy(x)-\frac{1}{xy(x)}-2\log(y(x))=c_1,y(x)\right]$$

25.9 problem 706

Internal problem ID [3952]

Book: Ordinary differential equations and their solutions. By George Moseley Murphy. 1960
Section: Various 25
Problem number: 706.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[[_homogeneous, ‘class G‘], _rational]`

$$\boxed{(x^2 - y^4)\, y' - yx = 0}$$

✓ Solution by Maple

Time used: 0.141 (sec). Leaf size: 97

```
dsolve((x^2-y(x)^4)*diff(y(x),x) = x*y(x),y(x), singsol=all)
```

$$y(x) = -\frac{\sqrt{2c_1 - 2\sqrt{c_1^2 - 4x^2}}}{2}$$
$$y(x) = \frac{\sqrt{2c_1 - 2\sqrt{c_1^2 - 4x^2}}}{2}$$
$$y(x) = -\frac{\sqrt{2c_1 + 2\sqrt{c_1^2 - 4x^2}}}{2}$$
$$y(x) = \frac{\sqrt{2c_1 + 2\sqrt{c_1^2 - 4x^2}}}{2}$$

✓ Solution by Mathematica

Time used: 2.529 (sec). Leaf size: 122

```
DSolve[(x^2-y[x]^4)y'[x]==x y[x],y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to -\sqrt{-\sqrt{-x^2 + c_1{}^2} - c_1}$$
$$y(x) \to \sqrt{-\sqrt{-x^2 + c_1{}^2} - c_1}$$
$$y(x) \to -\sqrt{\sqrt{-x^2 + c_1{}^2} - c_1}$$
$$y(x) \to \sqrt{\sqrt{-x^2 + c_1{}^2} - c_1}$$
$$y(x) \to 0$$

25.10 problem 707

Internal problem ID [3953]

Book: Ordinary differential equations and their solutions. By George Moseley Murphy. 1960
Section: Various 25
Problem number: 707.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[[_homogeneous, 'class G'], _rational]`

$$\boxed{(x^3 - y^4)\, y' - 3x^2 y = 0}$$

✓ Solution by Maple

Time used: 0.125 (sec). Leaf size: 25

```
dsolve((x^3-y(x)^4)*diff(y(x),x) = 3*x^2*y(x),y(x), singsol=all)
```

$$y(x) = \operatorname{RootOf}\left(x^9\_Z^4 + 3 - e^{\frac{9c_1}{4}}\_Z\right) x^3$$

✓ Solution by Mathematica

Time used: 60.209 (sec). Leaf size: 1021

```
DSolve[(x^3-y[x]^4)y'[x]==3 x^2 y[x],y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to \frac{1}{2}\sqrt{\frac{\sqrt[3]{9c_1{}^2-\sqrt{-256x^9+81c_1{}^4}}}{\sqrt[3]{2}}+\frac{4\sqrt[3]{2}x^3}{\sqrt[3]{9c_1{}^2-\sqrt{-256x^9+81c_1{}^4}}}}$$

$$-\frac{1}{2}\sqrt{-\frac{\sqrt[3]{9c_1{}^2-\sqrt{-256x^9+81c_1{}^4}}}{\sqrt[3]{2}}-\frac{4\sqrt[3]{2}x^3}{\sqrt[3]{9c_1{}^2-\sqrt{-256x^9+81c_1{}^4}}}-\frac{6c_1}{\sqrt{\frac{\sqrt[3]{9c_1{}^2-\sqrt{-256x^9+81c_1{}^4}}}{\sqrt[3]{2}}+}}}$$

$$y(x) \to \frac{1}{2}\sqrt{\frac{\sqrt[3]{9c_1{}^2-\sqrt{-256x^9+81c_1{}^4}}}{\sqrt[3]{2}}+\frac{4\sqrt[3]{2}x^3}{\sqrt[3]{9c_1{}^2-\sqrt{-256x^9+81c_1{}^4}}}}$$

$$+\frac{1}{2}\sqrt{-\frac{\sqrt[3]{9c_1{}^2-\sqrt{-256x^9+81c_1{}^4}}}{\sqrt[3]{2}}-\frac{4\sqrt[3]{2}x^3}{\sqrt[3]{9c_1{}^2-\sqrt{-256x^9+81c_1{}^4}}}-\frac{6c_1}{\sqrt{\frac{\sqrt[3]{9c_1{}^2-\sqrt{-256x^9+81c_1{}^4}}}{\sqrt[3]{2}}+}}}$$

$$y(x) \to -\frac{1}{2}\sqrt{\frac{\sqrt[3]{9c_1{}^2-\sqrt{-256x^9+81c_1{}^4}}}{\sqrt[3]{2}}+\frac{4\sqrt[3]{2}x^3}{\sqrt[3]{9c_1{}^2-\sqrt{-256x^9+81c_1{}^4}}}}$$

$$-\frac{1}{2}\sqrt{-\frac{\sqrt[3]{9c_1{}^2-\sqrt{-256x^9+81c_1{}^4}}}{\sqrt[3]{2}}-\frac{4\sqrt[3]{2}x^3}{\sqrt[3]{9c_1{}^2-\sqrt{-256x^9+81c_1{}^4}}}+\frac{6c_1}{\sqrt{\frac{\sqrt[3]{9c_1{}^2-\sqrt{-256x^9+81c_1{}^4}}}{\sqrt[3]{2}}+}}}$$

$$y(x)$$

$$\to \frac{1}{2}\sqrt{-\frac{\sqrt[3]{9c_1{}^2-\sqrt{-256x^9+81c_1{}^4}}}{\sqrt[3]{2}}-\frac{4\sqrt[3]{2}x^3}{\sqrt[3]{9c_1{}^2-\sqrt{-256x^9+81c_1{}^4}}}+\frac{6c_1}{\sqrt{\frac{\sqrt[3]{9c_1{}^2-\sqrt{-256x^9+81c_1{}^4}}}{\sqrt[3]{2}}+}}}$$

$$-\frac{1}{2}\sqrt{\frac{\sqrt[3]{9c_1{}^2-\sqrt{-256x^9+81c_1{}^4}}}{\sqrt[3]{2}}+\frac{4\sqrt[3]{2}x^3}{\sqrt[3]{9c_1{}^2-\sqrt{-256x^9+81c_1{}^4}}}}$$

25.11 problem 708

Internal problem ID [3954]

Book: Ordinary differential equations and their solutions. By George Moseley Murphy. 1960
Section: Various 25
Problem number: 708.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_rational]`

$$\boxed{\left(a^2x^2+\left(y^2+x^2\right)^2\right)y'-y a^2 x=0}$$

✓ Solution by Maple

Time used: 0.032 (sec). Leaf size: 197

```
dsolve((a^2*x^2+(x^2+y(x)^2)^2)*diff(y(x),x) = a^2*x*y(x),y(x), singsol=all)
```

$$y(x)=-\frac{\sqrt{-2a^2-2x^2-2\sqrt{x^4+(2a^2-2c_1)\,x^2+(a^2+c_1)^2}-2c_1}}{2}$$
$$y(x)=\frac{\sqrt{-2a^2-2x^2-2\sqrt{x^4+(2a^2-2c_1)\,x^2+(a^2+c_1)^2}-2c_1}}{2}$$
$$y(x)=-\frac{\sqrt{-2a^2-2x^2+2\sqrt{x^4+(2a^2-2c_1)\,x^2+(a^2+c_1)^2}-2c_1}}{2}$$
$$y(x)=\frac{\sqrt{-2a^2-2x^2+2\sqrt{x^4+(2a^2-2c_1)\,x^2+(a^2+c_1)^2}-2c_1}}{2}$$

✓ Solution by Mathematica

Time used: 7.125 (sec). Leaf size: 272

```
DSolve[(a^2 x^2+(x^2+y[x]^2)^2)y'[x]==a^2 x y[x],y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to -\frac{\sqrt{-\sqrt{(a^2+x^2-c_1{}^2)^2+4c_1{}^2x^2}-a^2-x^2+c_1{}^2}}{\sqrt{2}}$$
$$y(x) \to \frac{\sqrt{-\sqrt{(a^2+x^2-c_1{}^2)^2+4c_1{}^2x^2}-a^2-x^2+c_1{}^2}}{\sqrt{2}}$$
$$y(x) \to -\frac{\sqrt{\sqrt{(a^2+x^2-c_1{}^2)^2+4c_1{}^2x^2}-a^2-x^2+c_1{}^2}}{\sqrt{2}}$$
$$y(x) \to \frac{\sqrt{\sqrt{(a^2+x^2-c_1{}^2)^2+4c_1{}^2x^2}-a^2-x^2+c_1{}^2}}{\sqrt{2}}$$
$$y(x) \to 0$$
$$y(x) \to -\sqrt{-x^2}$$
$$y(x) \to \sqrt{-x^2}$$

25.12 problem 709

Internal problem ID [3955]

Book: Ordinary differential equations and their solutions. By George Moseley Murphy. 1960
Section: Various 25
Problem number: 709.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[[_homogeneous, ‘class G‘], _rational]`

$$\boxed{2(x - y^4)\, y' - y = 0}$$

✓ Solution by Maple
Time used: 0.015 (sec). Leaf size: 89

```
dsolve(2*(x-y(x)^4)*diff(y(x),x) = y(x),y(x), singsol=all)
```

$$y(x) = -\frac{\sqrt{-2\sqrt{c_1^2 - 4x} + 2c_1}}{2}$$
$$y(x) = \frac{\sqrt{-2\sqrt{c_1^2 - 4x} + 2c_1}}{2}$$
$$y(x) = -\frac{\sqrt{2\sqrt{c_1^2 - 4x} + 2c_1}}{2}$$
$$y(x) = \frac{\sqrt{2\sqrt{c_1^2 - 4x} + 2c_1}}{2}$$

✓ Solution by Mathematica

Time used: 2.408 (sec). Leaf size: 128

```
DSolve[2(x-y[x]^4)y'[x]==y[x],y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to -\frac{\sqrt{c_1 - \sqrt{-4x + c_1{}^2}}}{\sqrt{2}}$$

$$y(x) \to \frac{\sqrt{c_1 - \sqrt{-4x + c_1{}^2}}}{\sqrt{2}}$$

$$y(x) \to -\frac{\sqrt{\sqrt{-4x + c_1{}^2} + c_1}}{\sqrt{2}}$$

$$y(x) \to \frac{\sqrt{\sqrt{-4x + c_1{}^2} + c_1}}{\sqrt{2}}$$

$$y(x) \to 0$$

25.13 problem 710

Internal problem ID [3956]

Book: Ordinary differential equations and their solutions. By George Moseley Murphy. 1960
Section: Various 25
Problem number: 710.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_rational, [_1st_order, '_with_symmetry_[F(x)*G(y),0]']]`

$$\boxed{\left(4x - x y^3 - 2 y^4\right) y' - \left(2 + y^3\right) y = 0}$$

✓ Solution by Maple

Time used: 0.016 (sec). Leaf size: 27

```
dsolve((4*x-x*y(x)^3-2*y(x)^4)*diff(y(x),x) = (2+y(x)^3)*y(x),y(x), singsol=all)
```

$$x - \frac{\left(-y(x)^2 + c_1\right) y(x)^2}{2 + y\left(x\right)^3} = 0$$

✓ Solution by Mathematica

Time used: 60.285 (sec). Leaf size: 2021

```
DSolve[(4 x-x y[x]^3-2 y[x]^4)y'[x]==(2+y[x]^3)y[x],y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to -\frac{1}{2}\sqrt{\frac{\sqrt[3]{54x^3+\sqrt{(54x^3+144c_1x-2c_1{}^3)^2-4(24x+c_1{}^2)^3}+144c_1x-2c_1{}^3}}{3\sqrt[3]{2}}+\frac{}{3\sqrt[3]{54x^3+\sqrt{(54x^3+}}}} - \frac{1}{2}\sqrt{-\frac{\sqrt[3]{54x^3+\sqrt{(54x^3+144c_1x-2c_1{}^3)^2-4(24x+c_1{}^2)^3}+144c_1x-2c_1{}^3}}{3\sqrt[3]{2}}-\frac{}{3\sqrt[3]{54x^3+\sqrt{(54x^3-}}}} - \frac{x}{4}$$

$$y(x) \to -\frac{1}{2}\sqrt{\frac{\sqrt[3]{54x^3+\sqrt{(54x^3+144c_1x-2c_1{}^3)^2-4(24x+c_1{}^2)^3}+144c_1x-2c_1{}^3}}{3\sqrt[3]{2}}+\frac{}{3\sqrt[3]{54x^3+\sqrt{(54x^3+}}}} + \frac{1}{2}\sqrt{-\frac{\sqrt[3]{54x^3+\sqrt{(54x^3+144c_1x-2c_1{}^3)^2-4(24x+c_1{}^2)^3}+144c_1x-2c_1{}^3}}{3\sqrt[3]{2}}-\frac{}{3\sqrt[3]{54x^3+\sqrt{(54x^3-}}}} - \frac{x}{4}$$

$$y(x) \to \frac{1}{2}\sqrt{\frac{\sqrt[3]{54x^3+\sqrt{(54x^3+144c_1x-2c_1{}^3)^2-4(24x+c_1{}^2)^3}+144c_1x-2c_1{}^3}}{3\sqrt[3]{2}}+\frac{}{3\sqrt[3]{54x^3+\sqrt{(54x^3+14}}}} - \frac{1}{2}\sqrt{-\frac{\sqrt[3]{54x^3+\sqrt{(54x^3+144c_1x-2c_1{}^3)^2-4(24x+c_1{}^2)^3}+144c_1x-2c_1{}^3}}{3\sqrt[3]{2}}-\frac{}{3\sqrt[3]{54x^3+\sqrt{(54x^3-}}}} - \frac{x}{4}$$

$$y(x) \to \frac{1}{2}\sqrt{\frac{\sqrt[3]{54x^3+\sqrt{(54x^3+144c_1x-2c_1{}^3)^2-4(24x+c_1{}^2)^3}+144c_1x-2c_1{}^3}}{3\sqrt[3]{2}}+\frac{}{3\sqrt[3]{54x^3+\sqrt{(54x^3+14}}}}$$

25.14 problem 711

Internal problem ID [3957]

Book: Ordinary differential equations and their solutions. By George Moseley Murphy. 1960
Section: Various 25
Problem number: 711.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[[_homogeneous, 'class A'], _rational, _dAlembert]`

$$\left(a\,x^3+(ax+yb)^3\right)yy'+x\left((ax+yb)^3+by^3\right)=0$$

✓ Solution by Maple

Time used: 0.672 (sec). Leaf size: 160

```
dsolve((a*x^3+(a*x+b*y(x))^3)*y(x)*diff(y(x),x)+x*((a*x+b*y(x))^3+b*y(x)^3) = 0,y(x), singso
```

$$y(x)=\frac{x\left(c_1x-a\,\mathrm{RootOf}\left(a^2\_Z^4-2axc_1\_Z^3+\left(a^2c_1^2x^2+b^2c_1^2x^2+c_1^2x^2-b^2\right)\_Z^2-2a\,x^3c_1^3\_Z+c_1^4x^4\right)\right)}{b\,\mathrm{RootOf}\left(a^2\_Z^4-2axc_1\_Z^3+\left(a^2c_1^2x^2+b^2c_1^2x^2+c_1^2x^2-b^2\right)\_Z^2-2a\,x^3c_1^3\_Z+c_1^4x^4\right)}$$

✓ Solution by Mathematica

Time used: 61.456 (sec). Leaf size: 13289

```
DSolve[(a*x^3+(a*x+b*y[x])^3)*y[x]*y'[x]+x*((a*x+b*y[x])^3+b*y[x]^3)==0,y[x],x,IncludeSingul
```

Too large to display

25.15 problem 712

Internal problem ID [3958]

Book: Ordinary differential equations and their solutions. By George Moseley Murphy. 1960
Section: Various 25
Problem number: 712.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_rational]`

$$\boxed{\left(x+2y+2y^{3}x^{2}+y^{4}x\right)y'+\left(y^{4}+1\right)y=0}$$

✓ Solution by Maple

Time used: 0.016 (sec). Leaf size: 633

```
dsolve((x+2*y(x)+2*x^2*y(x)^3+x*y(x)^4)*diff(y(x),x)+(1+y(x)^4)*y(x) = 0,y(x), singsol=all)
```

$$y(x) = \frac{-1+\frac{\left(108c_1^3x^2+12\sqrt{3}\,\sqrt{27c_1^4x^2+4c_1x^4+18c_1^2x^2-x^2-4c_1}\,xc_1+36c_1x^2-8\right)^{\frac{1}{3}}}{2}-\frac{2(3c_1x^2-1)}{\left(108c_1^3x^2+12\sqrt{3}\,\sqrt{27c_1^4x^2+4c_1x^4+18c_1^2x^2-x^2-4c_1}\,xc_1+\cdots\right.}}{3c_1x}$$

$$y(x) = \frac{i\left(4-12c_1x^2-\left(108c_1^3x^2+12\sqrt{3}\,\sqrt{27c_1^4x^2+18c_1^2x^2+\left(4x^4-4\right)c_1-x^2}\,xc_1+36c_1x^2-8\right)^{\frac{2}{3}}\right)\sqrt{3}+12\cdots}{12\left(108c_1^3x^2+12\sqrt{3}\,\sqrt{27c_1^4x^2+18c_1^2x^2+\left(4x^4\cdots\right.\right.}}$$

$$y(x) = \frac{12i\sqrt{3}\,c_1x^2+i\left(108c_1^3x^2+12\sqrt{3}\,\sqrt{27c_1^4x^2+18c_1^2x^2+\left(4x^4-4\right)c_1-x^2}\,xc_1+36c_1x^2-8\right)^{\frac{2}{3}}\sqrt{3}+12c_1x\cdots}{12c_1x\cdots}$$

✓ Solution by Mathematica

Time used: 14.749 (sec). Leaf size: 675

```
DSolve[(x+2 y[x]+2 x^2 y[x]^3+x y[x]^4)y'[x]+(1+y[x]^4)y[x]==0,y[x],x,IncludeSingularSolutio
```

$$y(x) \to \frac{\frac{2c_1(3x^2+c_1)}{\sqrt[3]{\frac{9}{2}(3+c_1^2)x^2+\frac{3}{2}\sqrt{3}\sqrt{-4c_1^3x^6+(27-c_1^4+18c_1^2)x^4+4c_1^3x^2}+c_1^3}} + 2^{2/3}\sqrt[3]{9(3+c_1^2)x^2+3\sqrt{3}}}{6x}$$

$$y(x) \to \frac{-\frac{2i(\sqrt{3}-i)c_1(3x^2+c_1)}{\sqrt[3]{\frac{9}{2}(3+c_1^2)x^2+\frac{3}{2}\sqrt{3}\sqrt{-4c_1^3x^6+(27-c_1^4+18c_1^2)x^4+4c_1^3x^2}+c_1^3}} + i2^{2/3}(\sqrt{3}+i)\sqrt[3]{9(3+c}}{12x}$$

$$y(x) \to \frac{\frac{2i(\sqrt{3}+i)c_1(3x^2+c_1)}{\sqrt[3]{\frac{9}{2}(3+c_1^2)x^2+\frac{3}{2}\sqrt{3}\sqrt{-4c_1^3x^6+(27-c_1^4+18c_1^2)x^4+4c_1^3x^2}+c_1^3}} - 2^{2/3}(1+i\sqrt{3})\sqrt[3]{9(3+c_1}}{12x}$$

$$y(x) \to 0$$

$$y(x) \to -\sqrt[4]{-1}$$

$$y(x) \to \sqrt[4]{-1}$$

$$y(x) \to -(-1)^{3/4}$$

$$y(x) \to (-1)^{3/4}$$

$$y(x) \to \frac{1}{2}x\left(-1+\frac{ix^2}{\sqrt{-x^4}}\right)$$

$$y(x) \to -\frac{x}{2}+\frac{i\sqrt{-x^4}}{2x}$$

25.16 problem 713

Internal problem ID [3959]

Book: Ordinary differential equations and their solutions. By George Moseley Murphy. 1960
Section: Various 25
Problem number: 713.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[[_homogeneous, ‘class G‘], _rational]`

$$\boxed{2x\left(x^3+y^4\right)y'-\left(x^3+2y^4\right)y=0}$$

✓ Solution by Maple

Time used: 0.25 (sec). Leaf size: 285

```
dsolve(2*x*(x^3+y(x)^4)*diff(y(x),x) = (x^3+2*y(x)^4)*y(x),y(x), singsol=all)
```

$$y(x) = -\frac{2^{\frac{3}{4}}\left(\left(2c_1+x-\sqrt{x\left(x+4c_1\right)}\right)x^3c_1^3\right)^{\frac{1}{4}}}{2c_1}$$

$$y(x) = \frac{2^{\frac{3}{4}}\left(\left(2c_1+x-\sqrt{x\left(x+4c_1\right)}\right)x^3c_1^3\right)^{\frac{1}{4}}}{2c_1}$$

$$y(x) = -\frac{2^{\frac{3}{4}}\left(\left(2c_1+x+\sqrt{x\left(x+4c_1\right)}\right)x^3c_1^3\right)^{\frac{1}{4}}}{2c_1}$$

$$y(x) = \frac{2^{\frac{3}{4}}\left(\left(2c_1+x+\sqrt{x\left(x+4c_1\right)}\right)x^3c_1^3\right)^{\frac{1}{4}}}{2c_1}$$

$$y(x) = -\frac{i2^{\frac{3}{4}}\left(\left(2c_1+x-\sqrt{x\left(x+4c_1\right)}\right)x^3c_1^3\right)^{\frac{1}{4}}}{2c_1}$$

$$y(x) = -\frac{i2^{\frac{3}{4}}\left(\left(2c_1+x+\sqrt{x\left(x+4c_1\right)}\right)x^3c_1^3\right)^{\frac{1}{4}}}{2c_1}$$

$$y(x) = \frac{i2^{\frac{3}{4}}\left(\left(2c_1+x-\sqrt{x\left(x+4c_1\right)}\right)x^3c_1^3\right)^{\frac{1}{4}}}{2c_1}$$

$$y(x) = \frac{i2^{\frac{3}{4}}\left(\left(2c_1+x+\sqrt{x\left(x+4c_1\right)}\right)x^3c_1^3\right)^{\frac{1}{4}}}{2c_1}$$

✓ Solution by Mathematica

Time used: 4.15 (sec). Leaf size: 166

```
DSolve[2 x(x^3+y[x]^4)y'[x]==(x^3+2 y[x]^4)y[x],y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to -\frac{\sqrt{c_1x^2 - x^{3/2}\sqrt{4 + c_1{}^2x}}}{\sqrt{2}}$$
$$y(x) \to \frac{\sqrt{c_1x^2 - x^{3/2}\sqrt{4 + c_1{}^2x}}}{\sqrt{2}}$$
$$y(x) \to -\frac{\sqrt{x^{3/2}\sqrt{4 + c_1{}^2x} + c_1x^2}}{\sqrt{2}}$$
$$y(x) \to \frac{\sqrt{x^{3/2}\sqrt{4 + c_1{}^2x} + c_1x^2}}{\sqrt{2}}$$
$$y(x) \to 0$$

25.17 problem 714

Internal problem ID [3960]

Book: Ordinary differential equations and their solutions. By George Moseley Murphy. 1960
Section: Various 25
Problem number: 714.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[[_homogeneous, ‘class G‘], _rational]`

$$\boxed{x\left(1-y^{4}x^{2}\right)y'+y=0}$$

✓ Solution by Maple

Time used: 0.141 (sec). Leaf size: 133

```
dsolve(x*(1-x^2*y(x)^4)*diff(y(x),x)+y(x) = 0,y(x), singsol=all)
```

$$y(x) = -\frac{\sqrt{2}\,\sqrt{x c_1 \left(x-\sqrt{-4c_1^2+x^2}\right)}}{2c_1 x}$$
$$y(x) = \frac{\sqrt{2}\,\sqrt{x c_1 \left(x-\sqrt{-4c_1^2+x^2}\right)}}{2c_1 x}$$
$$y(x) = -\frac{\sqrt{2}\,\sqrt{x c_1 \left(x+\sqrt{-4c_1^2+x^2}\right)}}{2x c_1}$$
$$y(x) = \frac{\sqrt{2}\,\sqrt{x c_1 \left(x+\sqrt{-4c_1^2+x^2}\right)}}{2x c_1}$$

✓ Solution by Mathematica

Time used: 13.903 (sec). Leaf size: 172

```
DSolve[x(1-x^2 y[x]^4)y'[x]+y[x]==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to -\sqrt{c_1 - \frac{\sqrt{x^2\left(-1 + c_1{}^2 x^2\right)}}{x^2}}$$
$$y(x) \to \sqrt{c_1 - \frac{\sqrt{x^2\left(-1 + c_1{}^2 x^2\right)}}{x^2}}$$
$$y(x) \to -\sqrt{\frac{\sqrt{x^2\left(-1 + c_1{}^2 x^2\right)}}{x^2} + c_1}$$
$$y(x) \to \sqrt{\frac{\sqrt{x^2\left(-1 + c_1{}^2 x^2\right)}}{x^2} + c_1}$$
$$y(x) \to 0$$
$$y(x) \to -\frac{1}{\sqrt[4]{-x^2}}$$
$$y(x) \to \frac{1}{\sqrt[4]{-x^2}}$$

25.18 problem 715

Internal problem ID [3961]

Book: Ordinary differential equations and their solutions. By George Moseley Murphy. 1960
Section: Various 25
Problem number: 715.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[[_homogeneous, ‘class G‘], _rational]`

$$\boxed{(x^2 - y^5)\, y' - 2yx = 0}$$

✓ Solution by Maple

Time used: 0.11 (sec). Leaf size: 25

```
dsolve((x^2-y(x)^5)*diff(y(x),x) = 2*x*y(x),y(x), singsol=all)
```

$$y(x) = \operatorname{RootOf}\left(x^8\_Z^5 + 4 - \mathrm{e}^{\frac{8c_1}{5}}\_Z\right) x^2$$

✓ Solution by Mathematica

Time used: 2.91 (sec). Leaf size: 121

```
DSolve[(x^2-y[x]^5)y'[x]==2 x y[x],y[x],x,IncludeSingularSolutions -> True]
```

$$\begin{aligned}
y(x) &\to \operatorname{Root}\left[\#1^5 + 4\#1c_1 + 4x^2\&, 1\right] \\
y(x) &\to \operatorname{Root}\left[\#1^5 + 4\#1c_1 + 4x^2\&, 2\right] \\
y(x) &\to \operatorname{Root}\left[\#1^5 + 4\#1c_1 + 4x^2\&, 3\right] \\
y(x) &\to \operatorname{Root}\left[\#1^5 + 4\#1c_1 + 4x^2\&, 4\right] \\
y(x) &\to \operatorname{Root}\left[\#1^5 + 4\#1c_1 + 4x^2\&, 5\right] \\
y(x) &\to 0
\end{aligned}$$

25.19 problem 716

Internal problem ID [3962]

Book: Ordinary differential equations and their solutions. By George Moseley Murphy. 1960
Section: Various 25
Problem number: 716.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[[_homogeneous, 'class G'], _rational]`

$$x(x^3+y^5)\,y'-(x^3-y^5)\,y=0$$

✓ Solution by Maple
Time used: 0.078 (sec). Leaf size: 37

```
dsolve(x*(x^3+y(x)^5)*diff(y(x),x) = (x^3-y(x)^5)*y(x),y(x), singsol=all)
```

$$\ln(x)-c_1+\frac{5\ln\left(\frac{4y(x)^5-x^3}{x^3}\right)}{8}-\frac{5\ln\left(\frac{y(x)}{x^{\frac{3}{5}}}\right)}{2}=0$$

✓ Solution by Mathematica
Time used: 2.72 (sec). Leaf size: 141

```
DSolve[x(x^3+y[x]^5)y'[x]==(x^3-y[x]^5)y[x],y[x],x,IncludeSingularSolutions -> True]
```

$$y(x)\to \text{Root}\left[4\#1^5x-4\#1^4c_1-x^4\&,1\right]$$
$$y(x)\to \text{Root}\left[4\#1^5x-4\#1^4c_1-x^4\&,2\right]$$
$$y(x)\to \text{Root}\left[4\#1^5x-4\#1^4c_1-x^4\&,3\right]$$
$$y(x)\to \text{Root}\left[4\#1^5x-4\#1^4c_1-x^4\&,4\right]$$
$$y(x)\to \text{Root}\left[4\#1^5x-4\#1^4c_1-x^4\&,5\right]$$

25.20 problem 717

Internal problem ID [3963]

Book: Ordinary differential equations and their solutions. By George Moseley Murphy. 1960
Section: Various 25
Problem number: 717.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_rational]`

$$x^3(1+5x^3y^7)\,y' + (3y^5x^5-1)\,y^3 = 0$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 25

```
dsolve(x^3*(1+5*x^3*y(x)^7)*diff(y(x),x)+(3*x^5*y(x)^5-1)*y(x)^3 = 0,y(x), singsol=all)
```

$$-x^3y(x)^5 - \frac{1}{2x^2} + \frac{1}{2y(x)^2} + c_1 = 0$$

✓ Solution by Mathematica

Time used: 7.805 (sec). Leaf size: 253

```
DSolve[x^3(1+5 x^3 y[x]^7)y'[x]+(3 x^5 y[x]^5-1)y[x]^3==0,y[x],x,IncludeSingularSolutions ->
```

$$\begin{aligned}
y(x) &\to \text{Root}\left[2\#1^7x^5 + \#1^2(1-2c_1x^2) - x^2\&, 1\right]\\
y(x) &\to \text{Root}\left[2\#1^7x^5 + \#1^2(1-2c_1x^2) - x^2\&, 2\right]\\
y(x) &\to \text{Root}\left[2\#1^7x^5 + \#1^2(1-2c_1x^2) - x^2\&, 3\right]\\
y(x) &\to \text{Root}\left[2\#1^7x^5 + \#1^2(1-2c_1x^2) - x^2\&, 4\right]\\
y(x) &\to \text{Root}\left[2\#1^7x^5 + \#1^2(1-2c_1x^2) - x^2\&, 5\right]\\
y(x) &\to \text{Root}\left[2\#1^7x^5 + \#1^2(1-2c_1x^2) - x^2\&, 6\right]\\
y(x) &\to \text{Root}\left[2\#1^7x^5 + \#1^2(1-2c_1x^2) - x^2\&, 7\right]
\end{aligned}$$

25.21 problem 718

Internal problem ID [3964]

Book: Ordinary differential equations and their solutions. By George Moseley Murphy. 1960
Section: Various 25
Problem number: 718.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[[_homogeneous, 'class C'], _dAlembert]`

$$(1+a(y+x))^n y' + a(y+x)^n = 0$$

✓ Solution by Maple

Time used: 0.109 (sec). Leaf size: 42

```
dsolve((1+a*(x+y(x)))^n*diff(y(x),x)+a*(x+y(x))^n = 0,y(x), singsol=all)
```

$$y(x) = -x + \operatorname{RootOf}\left(-x + \int^{\_Z} \frac{(a\_a+1)^n}{-a\_a^n + (a\_a+1)^n} d\_a + c_1\right)$$

✓ Solution by Mathematica

Time used: 7.033 (sec). Leaf size: 331

```
DSolve[(1+a*(x+y[x]))^n*y'[x]+a*(x+y[x])^n==0,y[x],x,IncludeSingularSolutions -> True]
```

$$\text{Solve}\left[\int_1^x \frac{a(K[1]+y(x))^n}{a(K[1]+y(x))^n - (a(K[1]+y(x))+1)^n} dK[1] + \int_1^{y(x)} \right.$$

$$-\frac{-a \int_1^x \left(\frac{an(K[1]+K[2])^{n-1}}{a(K[1]+K[2])^n-(a(K[1]+K[2])+1)^n} - \frac{a(K[1]+K[2])^n\left(an(K[1]+K[2])^{n-1}-an(a(K[1]+K[2])+1)^{n-1}\right)}{(a(K[1]+K[2])^n-(a(K[1]+K[2])+1)^n)^2}\right) dK[1](x+K[2]}{(}$$

25.22 problem 719

Internal problem ID [3965]

Book: Ordinary differential equations and their solutions. By George Moseley Murphy. 1960
Section: Various 25
Problem number: 719.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[[_homogeneous, 'class G'], _rational]`

$$x(a + xy^n)\, y' + yb = 0$$

✓ Solution by Maple
Time used: 0.282 (sec). Leaf size: 38

```
dsolve(x*(a+x*y(x)^n)*diff(y(x),x)+b*y(x) = 0,y(x), singsol=all)
```

$$(xy(x)^n - bn + a)^{bn} \left(y(x)^n\right)^{-a} x^{-bn} - c_1 = 0$$

✓ Solution by Mathematica
Time used: 0.41 (sec). Leaf size: 61

```
DSolve[x(a+x y[x]^n)y'[x]+b y[x]==0,y[x],x,IncludeSingularSolutions -> True]
```

$$\text{Solve}\left[-\frac{an\log(ay(x) - bny(x))}{a - bn} - \frac{bn(\log(x) - \log\left(a - bn + xy(x)^n\right))}{a - bn} = c_1, y(x)\right]$$

25.23 problem 720

Internal problem ID [3966]

Book: Ordinary differential equations and their solutions. By George Moseley Murphy. 1960
Section: Various 25
Problem number: 720.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_Bernoulli]`

$$\boxed{f(x)\, y^m y' + g(x)\, y^{m+1} + h(x)\, y^n = 0}$$

✓ Solution by Maple

Time used: 0.109 (sec). Leaf size: 61

```
dsolve(f(x)*y(x)^m*diff(y(x),x)+g(x)*y(x)^(m+1)+h(x)*y(x)^n = 0,y(x), singsol=all)
```

$$y(x) = \mathrm{e}^{-\left(\int \frac{g(x)}{f(x)}dx\right)} \left((n-m-1)\left(\int \frac{h(x)\, \mathrm{e}^{(-n+m+1)\left(\int \frac{g(x)}{f(x)}dx\right)}}{f\left(x\right)}dx\right) + c_1\right)^{\frac{1}{-n+m+1}}$$

✓ Solution by Mathematica

Time used: 14.159 (sec). Leaf size: 187

```
DSolve[f[x] y[x]^m y'[x]+ g[x] y[x]^(m+1)+ h[x] y[x]^n==0,y[x],x,IncludeSingularSolutions ->
```

$$y(x) \to \left(\exp\left((m-n+1)\int_1^x -\frac{g(K[1])}{f(K[1])}dK[1]\right)\left((m-n+1)\int_1^x -\frac{\exp\left(-\left((m-n+1)\int_1^{K[2]} -\frac{g(K[1])}{f(K[1])}dK[1]\right)\right)h(K[2])}{f(K[2])}dK[2]+c_1\right)\right)^{\frac{1}{m-n+1}}$$

$$y(x) \to \left((m-n+1)\exp\left((m-n+1)\int_1^x -\frac{g(K[1])}{f(K[1])}dK[1]\right)\int_1^x -\frac{\exp\left(-\left((m-n+1)\int_1^{K[2]} -\frac{g(K[1])}{f(K[1])}dK[1]\right)\right)h(K[2])}{f(K[2])}dK[2]\right)^{\frac{1}{m-n+1}}$$

25.24 problem 721

Internal problem ID [3967]

Book: Ordinary differential equations and their solutions. By George Moseley Murphy. 1960
Section: Various 25
Problem number: 721.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_separable]`

$$y'\sqrt{b^2+y^2} = \sqrt{a^2+x^2}$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 67

```
dsolve(diff(y(x),x)*sqrt(b^2+y(x)^2) = sqrt(a^2+x^2),y(x), singsol=all)
```

$$\frac{x\sqrt{a^2+x^2}}{2} + \frac{a^2\ln\left(x+\sqrt{a^2+x^2}\right)}{2} - \frac{y(x)\sqrt{b^2+y(x)^2}}{2} - \frac{b^2\ln\left(y(x)+\sqrt{b^2+y(x)^2}\right)}{2} + c_1 = 0$$

✓ Solution by Mathematica

Time used: 1.428 (sec). Leaf size: 93

```
DSolve[y'[x] Sqrt[y[x]^2+b^2]==Sqrt[x^2+a^2],y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to \text{InverseFunction}\left[\frac{1}{2}\#1\sqrt{\#1^2+b^2} - \frac{1}{2}b^2\log\left(\sqrt{\#1^2+b^2}-\#1\right)\&\right]\left[\frac{1}{2}x\sqrt{a^2+x^2} - \frac{1}{2}a^2\log\left(\sqrt{a^2+x^2}-x\right)+c_1\right]$$

25.25 problem 722

Internal problem ID [3968]

Book: Ordinary differential equations and their solutions. By George Moseley Murphy. 1960
Section: Various 25
Problem number: 722.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_separable]`

$$y'\sqrt{b^2-y^2} = \sqrt{a^2-x^2}$$

✓ Solution by Maple
Time used: 0.0 (sec). Leaf size: 75

```
dsolve(diff(y(x),x)*sqrt(b^2-y(x)^2) = sqrt(a^2-x^2),y(x), singsol=all)
```

$$\frac{x\sqrt{a^2-x^2}}{2}+\frac{a^2\arctan\left(\frac{x}{\sqrt{a^2-x^2}}\right)}{2}-\frac{y(x)\sqrt{b^2-y(x)^2}}{2}-\frac{b^2\arctan\left(\frac{y(x)}{\sqrt{b^2-y(x)^2}}\right)}{2}+c_1=0$$

✓ Solution by Mathematica
Time used: 1.636 (sec). Leaf size: 97

```
DSolve[y'[x] Sqrt[b^2-y[x]^2]==Sqrt[a^2-x^2],y[x],x,IncludeSingularSolutions -> True]
```

$$y(x)\to \text{InverseFunction}\left[\frac{1}{2}b^2\arctan\left(\frac{\#1}{\sqrt{b^2-\#1^2}}\right)+\frac{1}{2}\#1\sqrt{b^2-\#1^2}\&\right]\left[\frac{1}{2}\left(a^2\arctan\left(\frac{x}{\sqrt{a^2-x^2}}\right)+x\sqrt{a^2-x^2}+2c_1\right)\right]$$

25.26 problem 723

Internal problem ID [3969]

Book: Ordinary differential equations and their solutions. By George Moseley Murphy. 1960
Section: Various 25
Problem number: 723.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_separable]`

$$y'\sqrt{y} = \sqrt{x}$$

✓ Solution by Maple
Time used: 0.0 (sec). Leaf size: 16

```
dsolve(diff(y(x),x)*sqrt(y(x)) = sqrt(x),y(x), singsol=all)
```

$$y(x)^{\frac{3}{2}} - x^{\frac{3}{2}} - c_1 = 0$$

✓ Solution by Mathematica
Time used: 0.004 (sec). Leaf size: 20

```
DSolve[y'[x] Sqrt[Y]==Sqrt[X],y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to \frac{x\sqrt{X}}{\sqrt{Y}} + c_1$$

25.27 problem 724

Internal problem ID [3970]

Book: Ordinary differential equations and their solutions. By George Moseley Murphy. 1960
Section: Various 25
Problem number: 724.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[[_homogeneous, ‘class C‘], _dAlembert]`

$$\boxed{\left(1+\sqrt{y+x}\right) y' = -1}$$

✓ Solution by Maple
Time used: 0.016 (sec). Leaf size: 19

```
dsolve((1+sqrt(x+y(x)))*diff(y(x),x)+1 = 0,y(x), singsol=all)
```

$$-y(x) - 2\sqrt{x+y\left(x\right)} - c_1 = 0$$

✓ Solution by Mathematica
Time used: 0.146 (sec). Leaf size: 39

```
DSolve[(1+Sqrt[x+y[x]])y'[x]+1==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to -2\sqrt{x+1+c_1} + 2 + c_1$$
$$y(x) \to 2\sqrt{x+1+c_1} + 2 + c_1$$

25.28 problem 725

Internal problem ID [3971]

Book: Ordinary differential equations and their solutions. By George Moseley Murphy. 1960
Section: Various 25
Problem number: 725.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[[_homogeneous, ‘class A‘], _rational, _dAlembert]`

$$y'\sqrt{yx} - y - \sqrt{yx} = -x$$

✓ Solution by Maple

Time used: 0.016 (sec). Leaf size: 84

```
dsolve(diff(y(x),x)*sqrt(x*y(x))+x-y(x) = sqrt(x*y(x)),y(x), singsol=all)
```

$$\frac{\left(3x - 3\sqrt{xy\left(x\right)}\right)\ln\left(-x + \sqrt{xy\left(x\right)}\right) + \left(x - \sqrt{xy\left(x\right)}\right)\ln\left(\sqrt{xy\left(x\right)} + x\right) + \left(2\ln\left(x\right) + c_1\right)\sqrt{xy\left(x\right)} - x}{x - \sqrt{xy\left(x\right)}}$$
$$= 0$$

✓ Solution by Mathematica

Time used: 0.235 (sec). Leaf size: 62

```
DSolve[y'[x] Sqrt[x y[x]]+x -y[x]==Sqrt[x y[x]],y[x],x,IncludeSingularSolutions -> True]
```

$$\text{Solve}\left[\frac{1}{1-\sqrt{\frac{y(x)}{x}}} + \frac{3}{2}\log\left(\sqrt{\frac{y(x)}{x}} - 1\right) + \frac{1}{2}\log\left(\sqrt{\frac{y(x)}{x}} + 1\right) = -\log(x) + c_1, y(x)\right]$$

25.29 problem 726

Internal problem ID [3972]

Book: Ordinary differential equations and their solutions. By George Moseley Murphy. 1960
Section: Various 25
Problem number: 726.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[[_homogeneous, 'class A'], _dAlembert]`

$$\boxed{(x - 2\sqrt{yx})\, y' - y = 0}$$

✓ Solution by Maple

Time used: 0.016 (sec). Leaf size: 18

```
dsolve((x-2*sqrt(x*y(x)))*diff(y(x),x) = y(x),y(x), singsol=all)
```

$$\ln (y(x)) + \frac{x}{\sqrt{xy(x)}} - c_1 = 0$$

✓ Solution by Mathematica

Time used: 0.354 (sec). Leaf size: 33

```
DSolve[(x-2 Sqrt[x y[x]])y'[x]==y[x],y[x],x,IncludeSingularSolutions -> True]
```

$$\text{Solve}\left[\frac{2}{\sqrt{\frac{y(x)}{x}}} + 2\log\left(\frac{y(x)}{x}\right) = -2\log(x) + c_1, y(x)\right]$$

25.30 problem 727

Internal problem ID [3973]

Book: Ordinary differential equations and their solutions. By George Moseley Murphy. 1960
Section: Various 25
Problem number: 727.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_separable]`

$$\left(y+\sqrt{y^2+1}\right)\left(x^2+1\right)^{\frac{3}{2}} y'-y^2=1$$

✓ Solution by Maple

Time used: 0.157 (sec). Leaf size: 28

```
dsolve((y(x)+sqrt(1+y(x)^2))*(x^2+1)^(3/2)*diff(y(x),x) = 1+y(x)^2,y(x), singsol=all)
```

$$\frac{x}{\sqrt{x^2+1}}-\operatorname{arcsinh}(y(x))-\frac{\ln\left(y(x)^2+1\right)}{2}+c_1=0$$

✓ Solution by Mathematica

Time used: 16.191 (sec). Leaf size: 115

```
DSolve[(y[x]+Sqrt[1+y[x]^2])(1+x^2)^(3/2) y'[x]==1+y[x]^2,y[x],x,IncludeSingularSolutions ->
```

$$y(x)\to -\frac{i\left(1+e^{\frac{x}{\sqrt{x^2+1}}+c_1}\right)}{\sqrt{1+2e^{\frac{x}{\sqrt{x^2+1}}+c_1}}}$$
$$y(x)\to \frac{i\left(1+e^{\frac{x}{\sqrt{x^2+1}}+c_1}\right)}{\sqrt{1+2e^{\frac{x}{\sqrt{x^2+1}}+c_1}}}$$
$$y(x)\to -i$$
$$y(x)\to i$$

25.31 problem 728

Internal problem ID [3974]

Book: Ordinary differential equations and their solutions. By George Moseley Murphy. 1960
Section: Various 25
Problem number: 728.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_separable]`

$$\left(y+\sqrt{y^2+1}\right)\left(x^2+1\right)^{\frac{3}{2}} y'-y^2=1$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 28

```
dsolve((y(x)+sqrt(1+y(x)^2))*(x^2+1)^(3/2)*diff(y(x),x) = 1+y(x)^2,y(x), singsol=all)
```

$$\frac{x}{\sqrt{x^2+1}}-\operatorname{arcsinh}\left(y(x)\right)-\frac{\ln\left(y(x)^2+1\right)}{2}+c_1=0$$

✓ Solution by Mathematica

Time used: 0.775 (sec). Leaf size: 115

```
DSolve[(1+x^2)^(3/2) (y[x]+Sqrt[1+y[x]^2])y'[x]==1+y[x]^2,y[x],x,IncludeSingularSolutions ->
```

$$y(x)\to -\frac{i\left(1+e^{\frac{x}{\sqrt{x^2+1}}+c_1}\right)}{\sqrt{1+2e^{\frac{x}{\sqrt{x^2+1}}+c_1}}}$$
$$y(x)\to \frac{i\left(1+e^{\frac{x}{\sqrt{x^2+1}}+c_1}\right)}{\sqrt{1+2e^{\frac{x}{\sqrt{x^2+1}}+c_1}}}$$
$$y(x)\to -i$$
$$y(x)\to i$$

25.32 problem 729

Internal problem ID [3975]

Book: Ordinary differential equations and their solutions. By George Moseley Murphy. 1960
Section: Various 25
Problem number: 729.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[[_homogeneous, ‘class A‘], _rational, _dAlembert]`

$$\boxed{\left(-\sqrt{y^2+x^2}+x\right)y'-y=0}$$

✓ Solution by Maple

Time used: 0.094 (sec). Leaf size: 18

```
dsolve((x-sqrt(x^2+y(x)^2))*diff(y(x),x) = y(x),y(x), singsol=all)
```

$$-c_1+\sqrt{x^2+y(x)^2}+x=0$$

✓ Solution by Mathematica

Time used: 0.818 (sec). Leaf size: 57

```
DSolve[(x-Sqrt[x^2+y[x]^2])y'[x]==y[x],y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to -e^{\frac{c_1}{2}}\sqrt{-2x+e^{c_1}}$$
$$y(x) \to e^{\frac{c_1}{2}}\sqrt{-2x+e^{c_1}}$$
$$y(x) \to 0$$

25.33 problem 730

Internal problem ID [3976]

Book: Ordinary differential equations and their solutions. By George Moseley Murphy. 1960
Section: Various 25
Problem number: 730.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[‘y=_G(x,y’)‘]`

$$\boxed{x\left(1-\sqrt{x^2-y^2}\right)y'-y=0}$$

✓ Solution by Maple
Time used: 0.031 (sec). Leaf size: 27

```
dsolve(x*(1-sqrt(x^2-y(x)^2))*diff(y(x),x) = y(x),y(x), singsol=all)
```

$$y(x)-\arctan\left(\frac{y(x)}{\sqrt{x^2-y(x)^2}}\right)-c_1=0$$

✓ Solution by Mathematica
Time used: 0.84 (sec). Leaf size: 29

```
DSolve[x(1-Sqrt[x^2-y[x]^2])y'[x]==y[x],y[x],x,IncludeSingularSolutions -> True]
```

$$\text{Solve}\left[\arctan\left(\frac{\sqrt{x^2-y(x)^2}}{y(x)}\right)+y(x)=c_1,y(x)\right]$$

25.34 problem 731

Internal problem ID [3977]

Book: Ordinary differential equations and their solutions. By George Moseley Murphy. 1960
Section: Various 25
Problem number: 731.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[[_homogeneous, ‘class G‘], _dAlembert]`

$$\boxed{x\left(x+\sqrt{y^2+x^2}\right) y'+\sqrt{y^2+x^2}\, y=0}$$

✓ Solution by Maple

Time used: 0.031 (sec). Leaf size: 130

```
dsolve(x*(x+sqrt(x^2+y(x)^2))*diff(y(x),x)+y(x)*sqrt(x^2+y(x)^2) = 0,y(x), singsol=all)
```

$$-\left(\int_{\_b}^{x}\frac{\sqrt{\_a^2+y(x)^2}}{\_a\left(2\sqrt{\_a^2+y(x)^2}+\_a\right)}d\_a\right)$$
$$+\int^{y(x)}\frac{\_f^2\left(2\sqrt{\_f^2+x^2}+x\right)\left(\int_{\_b}^{x}\frac{1}{\sqrt{\_a^2+\_f^2}\left(2\sqrt{\_a^2+\_f^2}+\_a\right)^2}d\_a\right)-x-\sqrt{\_f^2+x^2}}{\_f\left(2\sqrt{\_f^2+x^2}+x\right)}d\_f$$
$$+c_1=0$$

✓ Solution by Mathematica

Time used: 60.36 (sec). Leaf size: 1457

```
DSolve[x(x+Sqrt[x^2+y[x]^2])y'[x] +y[x] Sqrt[x^2+y[x]^2]==0,y[x],x,IncludeSingularSolutions
```

$$y(x) \to -\frac{1}{2}\sqrt{\frac{x^6 - x^4\sqrt[3]{\frac{-x^{12}+20e^{6c_1}x^6+8\sqrt{e^{6c_1}\left(-x^6+e^{6c_1}\right)^3}+8e^{12c_1}}{x^6}} + x^2\left(\frac{-x^{12}+20e^{6c_1}x^6+8\sqrt{e^{6c_1}\left(-x^6+e^{6c_1}\right)^3}+\cdots}{x^6}\right.\cdots}{x^2\sqrt[3]{\frac{-x^{12}+20e^{6c_1}x^6+8\sqrt{e^{6c_1}\left(-x^6+e^{6c_1}\right)^3}+8e^{12c_1}}{x^6}}}}$$

$$y(x) \to \frac{1}{2}\sqrt{\frac{x^6 - x^4\sqrt[3]{\frac{-x^{12}+20e^{6c_1}x^6+8\sqrt{e^{6c_1}\left(-x^6+e^{6c_1}\right)^3}+8e^{12c_1}}{x^6}} + x^2\left(\frac{-x^{12}+20e^{6c_1}x^6+8\sqrt{e^{6c_1}\left(-x^6+e^{6c_1}\right)^3}+8e\cdots}{x^6}\right.\cdots}{x^2\sqrt[3]{\frac{-x^{12}+20e^{6c_1}x^6+8\sqrt{e^{6c_1}\left(-x^6+e^{6c_1}\right)^3}+8e^{12c_1}}{x^6}}}}$$

$$y(x) \to -\frac{\sqrt{\frac{i\left(\left(\sqrt{3}+i\right)x^6+2ix^4\sqrt[3]{\frac{-x^{12}+20e^{6c_1}x^6+8\sqrt{e^{6c_1}\left(-x^6+e^{6c_1}\right)^3}+8e^{12c_1}}{x^6}}-\left(\sqrt{3}-i\right)x^2\left(\frac{-x^{12}+20e^{6c_1}x^6+8\sqrt{e^{6c_1}\left(-x\cdots\right.}}{x^6}\right.\cdots\right.}{x^2\sqrt[3]{\frac{-x^{12}+20e^{6c_1}x^6+8\sqrt{e^{6c_1}\left(-x^6+e^{6c_1}\right)^3}+8e^{12c_1}}{x^6}}}}}{2\sqrt{2}}$$

$$y(x) \to \frac{\sqrt{\frac{i\left(\left(\sqrt{3}+i\right)x^6+2ix^4\sqrt[3]{\frac{-x^{12}+20e^{6c_1}x^6+8\sqrt{e^{6c_1}\left(-x^6+e^{6c_1}\right)^3}+8e^{12c_1}}{x^6}}-\left(\sqrt{3}-i\right)x^2\left(\frac{-x^{12}+20e^{6c_1}x^6+8\sqrt{e^{6c_1}\left(-x^6+\cdots\right.}}{x^6}\right.\cdots\right.}{x^2\sqrt[3]{\frac{-x^{12}+20e^{6c_1}x^6+8\sqrt{e^{6c_1}\left(-x^6+e^{6c_1}\right)^3}+8e^{12c_1}}{x^6}}}}}{2\sqrt{2}}$$

$$y(x) \to -\frac{\sqrt{\frac{i\left(x^2\left(x^2+\sqrt[3]{\frac{-x^{12}+20e^{6c_1}x^6+8\sqrt{e^{6c_1}\left(-x^6+e^{6c_1}\right)^3}+8e^{12c_1}}{x^6}}\right)\left(\left(\sqrt{3}+i\right)\sqrt[3]{\frac{-x^{12}+20e^{6c_1}x^6+8\sqrt{e^{6c}\cdots}}{x^6}}\right.\cdots\right.}{x^2\sqrt[3]{\frac{-x^{12}+20e^{6c_1}x^6+8\sqrt{e^{6c_1}\left(-x^6+e^{6c_1}\right)^3}+8e^{12\cdots}}{x^6}}}}}{2\sqrt{2}}$$

$$y(x) \cdots \sqrt{\frac{i\left(x^2\left(x^2+\sqrt[3]{\frac{-x^{12}+20e^{6c_1}x^6+8\sqrt{e^{6c_1}\left(-x^6+e^{6c_1}\right)^3}+8e^{12c_1}}{x^6}}\right)\left(\left(\sqrt{3}+i\right)\sqrt[3]{\frac{-x^{12}+20e^{6c_1}x^6+8\sqrt{e^{6c_1}\cdots}}{x^6}}\right.\cdots\right.}{\cdots}}$$

26 Various 26

26.1 problem 732

Internal problem ID [3978]

Book: Ordinary differential equations and their solutions. By George Moseley Murphy. 1960
Section: Various 26
Problem number: 732.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[[_1st_order, _with_linear_symmetries], _dAlembert]`

$$xy\left(x+\sqrt{x^2-y^2}\right)y'-y^2x+\left(x^2-y^2\right)^{\frac{3}{2}}=0$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 40

```
dsolve(x*y(x)*(x+sqrt(x^2-y(x)^2))*diff(y(x),x) = x*y(x)^2-(x^2-y(x)^2)^(3/2),y(x), singsol=
```

$$\frac{2\ln(x)\,x^2-c_1x^2+y(x)^2-2x\sqrt{x^2-y(x)^2}}{x^2}=0$$

✓ Solution by Mathematica

Time used: 26.912 (sec). Leaf size: 385

```
DSolve[x y[x](x+Sqrt[x^2-y[x]^2])y'[x]==x y[x]^2-(x^2-y[x]^2)^(3/2),y[x],x,IncludeSingularSo
```

$$y(x) \to -\sqrt{-2\sqrt{-x^4(-2\log(x)-1+2c_1)}-2x^2\log(x)+(-1+2c_1)x^2}$$
$$y(x) \to \sqrt{-2\sqrt{-x^4(-2\log(x)-1+2c_1)}-2x^2\log(x)+(-1+2c_1)x^2}$$
$$y(x) \to -\sqrt{2\sqrt{-x^4(-2\log(x)-1+2c_1)}-2x^2\log(x)+(-1+2c_1)x^2}$$
$$y(x) \to \sqrt{2\sqrt{-x^4(-2\log(x)-1+2c_1)}-2x^2\log(x)+(-1+2c_1)x^2}$$
$$y(x) \to -\sqrt{-2\sqrt{x^4(-2\log(x)+1+2c_1)}+2x^2\log(x)-((1+2c_1)x^2)}$$
$$y(x) \to \sqrt{-2\sqrt{x^4(-2\log(x)+1+2c_1)}+2x^2\log(x)-((1+2c_1)x^2)}$$
$$y(x) \to -\sqrt{2\sqrt{x^4(-2\log(x)+1+2c_1)}+2x^2\log(x)-((1+2c_1)x^2)}$$
$$y(x) \to \sqrt{2\sqrt{x^4(-2\log(x)+1+2c_1)}+2x^2\log(x)-((1+2c_1)x^2)}$$

26.2 problem 734

Internal problem ID [3979]

Book: Ordinary differential equations and their solutions. By George Moseley Murphy. 1960
Section: Various 26
Problem number: 734.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[[_1st_order, _with_linear_symmetries]]`

$$\left(x\sqrt{1+x^2+y^2}-y\left(y^2+x^2\right)\right)y'-x\left(y^2+x^2\right)-y\sqrt{1+x^2+y^2}=0$$

✓ Solution by Maple

Time used: 0.094 (sec). Leaf size: 25

```
dsolve((x*sqrt(1+x^2+y(x)^2)-y(x)*(x^2+y(x)^2))*diff(y(x),x) = x*(x^2+y(x)^2)+y(x)*sqrt(1+x^
```

$$\arctan\left(\frac{x}{y(x)}\right)+\sqrt{1+x^2+y(x)^2}-c_1=0$$

✓ Solution by Mathematica

Time used: 0.27 (sec). Leaf size: 27

```
DSolve[(x*Sqrt[1+x^2+y[x]^2]-y[x]*(x^2+y[x]^2))*y'[x]==x*(x^2+y[x]^2)+y[x]*Sqrt[1+x^2+y[x]^2
```

$$\text{Solve}\left[\sqrt{x^2+y(x)^2+1}+\tan^{-1}\left(\frac{x}{y(x)}\right)=c_1,y(x)\right]$$

26.3 problem 736

Internal problem ID [3980]

Book: Ordinary differential equations and their solutions. By George Moseley Murphy. 1960
Section: Various 26
Problem number: 736.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `unknown`

$$y' \cos(y) (\cos(y) - \sin(A) \sin(x)) + \cos(x) (\cos(x) - \sin(A) \sin(y)) = 0$$

✓ Solution by Maple
Time used: 0.125 (sec). Leaf size: 33

```
dsolve(diff(y(x),x)*cos(y(x))*(cos(y(x))-sin(A)*sin(x))+cos(x)*(cos(x)-sin(A)*sin(y(x))) = 0
```

$$\frac{(-2\sin(A)\sin(x) + \cos(y(x)))\sin(y(x))}{2} + \frac{\cos(x)\sin(x)}{2} + \frac{x}{2} + c_1 + \frac{y(x)}{2} = 0$$

✓ Solution by Mathematica
Time used: 0.649 (sec). Leaf size: 43

```
DSolve[y'[x] Cos[y[x]](Cos[y[x]]- Sin[A] Sin[x])+Cos[x](Cos[x]-Sin[A]Sin[y[x]])==0,y[x],x,In
```

$$\text{Solve}\left[4\sin(A)\sin(x)\sin(y(x)) - 4\left(\frac{y(x)}{2} + \frac{1}{4}\sin(2y(x))\right) - 2x - \sin(2x) = c_1, y(x)\right]$$

26.4 problem 737

Internal problem ID [3981]

Book: Ordinary differential equations and their solutions. By George Moseley Murphy. 1960
Section: Various 26
Problem number: 737.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_exact]`

$$(a\cos(bx+ya)-b\sin(ax+yb))\,y'+b\cos(bx+ya)-a\sin(ax+yb)=0$$

✓ Solution by Maple

Time used: 0.031 (sec). Leaf size: 43

```
dsolve((a*cos(b*x+a*y(x))-b*sin(a*x+b*y(x)))*diff(y(x),x)+b*cos(b*x+a*y(x))-a*sin(a*x+b*y(x)
```

$$y(x)=\frac{-bx+\operatorname{RootOf}\left(2x\,a^2-2b^2x-\pi a-2\arcsin\left(\sin\left(\_Z\right)+c_1\right)a+2\_Zb\right)}{a}$$

✓ Solution by Mathematica

Time used: 1.106 (sec). Leaf size: 50

```
DSolve[(a Cos[b x+a y[x]]-b Sin[a x+ b y[x]])y'[x]+b Cos[b x+a y[x]]-a Sin[a x+b y[x]]==0,y[
```

$$\text{Solve}[\sin(ax)\sin(by(x))-\cos(ax)\cos(by(x))\\ -\sin(bx)\cos(ay(x))-\cos(bx)\sin(ay(x))=c_1,y(x)]$$

26.5 problem 739

Internal problem ID [3982]

Book: Ordinary differential equations and their solutions. By George Moseley Murphy. 1960
Section: Various 26
Problem number: 739.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[NONE]`

$$(x + \cos(x)\sec(y))\,y' + \tan(y) - y\sin(x)\sec(y) = 0$$

✓ Solution by Maple

Time used: 0.031 (sec). Leaf size: 15

```
dsolve((x+cos(x)*sec(y(x)))*diff(y(x),x)+tan(y(x))-y(x)*sin(x)*sec(y(x)) = 0,y(x), singsol=a
```

$$\cos(x)\,y(x) + \sin(y(x))\,x + c_1 = 0$$

✓ Solution by Mathematica

Time used: 0.234 (sec). Leaf size: 17

```
DSolve[(x+Cos[x] Sec[y[x]])y'[x]+Tan[y[x]]-y[x] Sin[x] Sec[y[x]]==0,y[x],x,IncludeSingularSo
```

$$\text{Solve}[x\sin(y(x)) + y(x)\cos(x) = c_1, y(x)]$$

26.6 problem 742

Internal problem ID [3983]

Book: Ordinary differential equations and their solutions. By George Moseley Murphy. 1960
Section: Various 26
Problem number: 742.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[[_1st_order, _with_linear_symmetries]]`

$$\boxed{(1+\tan(y)(y+x))y'=-1}$$

✓ Solution by Maple

Time used: 0.016 (sec). Leaf size: 13

```
dsolve((1+(x+y(x))*tan(y(x)))*diff(y(x),x)+1 = 0,y(x), singsol=all)
```

$$x-\cos(y(x))c_1+y(x)=0$$

✓ Solution by Mathematica

Time used: 0.473 (sec). Leaf size: 66

```
DSolve[(1+(x+y[x]) Tan[y[x]])y'[x]+1==0,y[x],x,IncludeSingularSolutions -> True]
```

$$\text{Solve}\left[x=\cos(y(x))\left(-y(x)\sec(y(x))-\coth^{-1}(\sin(y(x)))-\log\left(\cos\left(\frac{y(x)}{2}\right)-\sin\left(\frac{y(x)}{2}\right)\right)+\log\left(\sin\left(\frac{y(x)}{2}\right)+\cos\left(\frac{y(x)}{2}\right)\right)\right)+c_1\cos(y(x)),y(x)\right]$$

26.7 problem 743

Internal problem ID [3984]

Book: Ordinary differential equations and their solutions. By George Moseley Murphy. 1960
Section: Various 26
Problem number: 743.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[[_homogeneous, ‘class A‘], _dAlembert]`

$$x\left(x - y\tan\left(\frac{y}{x}\right)\right)y' + \left(x + y\tan\left(\frac{y}{x}\right)\right)y = 0$$

✓ Solution by Maple

Time used: 0.187 (sec). Leaf size: 18

```
dsolve(x*(x-y(x)*tan(y(x)/x))*diff(y(x),x)+(x+y(x)*tan(y(x)/x))*y(x) = 0,y(x), singsol=all)
```

$$y(x) = x\,\mathrm{RootOf}\left(\_Z\cos\left(\_Z\right)x^2 - c_1\right)$$

✓ Solution by Mathematica

Time used: 0.54 (sec). Leaf size: 31

```
DSolve[x(x-y[x] Tan[y[x]/x])y'[x]+(x+y[x] Tan[y[x]/x])y[x]==0,y[x],x,IncludeSingularSolution
```

$$\text{Solve}\left[-\log\left(\frac{y(x)}{x}\right) - \log\left(\cos\left(\frac{y(x)}{x}\right)\right) = 2\log(x) + c_1, y(x)\right]$$

26.8 problem 744

Internal problem ID [3985]

Book: Ordinary differential equations and their solutions. By George Moseley Murphy. 1960
Section: Various 26
Problem number: 744.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_exact]`

$$\boxed{(e^{x}+x\,e^{y})\,y'+e^{x}y+e^{y}=0}$$

✓ Solution by Maple
Time used: 0.047 (sec). Leaf size: 29

```
dsolve((exp(x)+x*exp(y(x)))*diff(y(x),x)+y(x)*exp(x)+exp(y(x)) = 0,y(x), singsol=all)
```

$$y(x)=-\operatorname{LambertW}\left(x\,e^{-x-e^{-x}c_1}\right)-e^{-x}c_1$$

✓ Solution by Mathematica
Time used: 3.489 (sec). Leaf size: 33

```
DSolve[(Exp[x]+x Exp[y[x]])y'[x]+y[x] Exp[x]+Exp[y[x]]==0,y[x],x,IncludeSingularSolutions ->
```

$$y(x)\to c_1e^{-x}-W\left(xe^{-x+c_1e^{-x}}\right)$$

26.9 problem 745

Internal problem ID [3986]

Book: Ordinary differential equations and their solutions. By George Moseley Murphy. 1960
Section: Various 26
Problem number: 745.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[[_1st_order, _with_linear_symmetries]]`

$$\boxed{(1-2x-\ln(y))\,y'+2y=0}$$

✓ Solution by Maple
Time used: 0.016 (sec). Leaf size: 17

```
dsolve((1-2*x-ln(y(x)))*diff(y(x),x)+2*y(x) = 0,y(x), singsol=all)
```

$$y(x)=-\frac{\mathrm{LambertW}\left(-2\,\mathrm{e}^{-2x}c_1\right)}{2c_1}$$

✓ Solution by Mathematica
Time used: 60.223 (sec). Leaf size: 23

```
DSolve[(1-2 x -Log[y[x]])y'[x]+2 y[x]==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x)\to-\frac{W(-2c_1e^{-2x})}{2c_1}$$

26.10 problem 746

Internal problem ID [3987]

Book: Ordinary differential equations and their solutions. By George Moseley Murphy. 1960
Section: Various 26
Problem number: 746.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_exact]`

$$\boxed{(\sinh(x) + x\cosh(y))\,y' + y\cosh(x) + \sinh(y) = 0}$$

✓ Solution by Maple

Time used: 0.25 (sec). Leaf size: 180

```
dsolve((sinh(x)+x*cosh(y(x)))*diff(y(x),x)+y(x)*cosh(x)+sinh(y(x)) = 0,y(x), singsol=all)
```

$$y(x) = -\frac{\left(2c_1 \mathrm{e}^{\mathrm{RootOf}\left(\_Z \mathrm{e}^{2x+\_Z} - x\,\mathrm{e}^{2x+\_Z} + x\,\mathrm{e}^{2\_Z} + 2c_1 \mathrm{e}^{x+\_Z} - \mathrm{e}^{2x} x - \_Z \mathrm{e}^{\_Z} + \mathrm{e}^{\_Z} x\right)+x} + x\left(\mathrm{e}^{2\,\mathrm{RootOf}\left(\_Z \mathrm{e}^{2x+\_Z} - x\,\mathrm{e}^{2x+\_Z} + x\,\mathrm{e}^{2\_Z} + 2\right.}\right.\right.}{\mathrm{e}^{2x} - 1}$$

✓ Solution by Mathematica

Time used: 0.349 (sec). Leaf size: 17

```
DSolve[(Sinh[x]+x Cosh[y[x]])y'[x]+y[x] Cosh[x]+Sinh[y[x]]==0,y[x],x,IncludeSingularSolution
```

$$\mathrm{Solve}[x\sinh(y(x)) + y(x)\sinh(x) = c_1, y(x)]$$

26.11 problem 747

Internal problem ID [3988]

Book: Ordinary differential equations and their solutions. By George Moseley Murphy. 1960
Section: Various 26
Problem number: 747.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_separable]`

$$y'(1+\sinh(x))\sinh(y)+\cosh(x)(\cosh(y)-1)=0$$

✓ Solution by Maple

Time used: 0.047 (sec). Leaf size: 123

```
dsolve(diff(y(x),x)*(1+sinh(x))*sinh(y(x))+cosh(x)*(cosh(y(x))-1) = 0,y(x), singsol=all)
```

$$y(x)=-2\,\mathrm{arctanh}\left(\frac{c_1\sqrt{2}\,\sqrt{\frac{-c_1 e^x e^{2x}+(-2c_1+2)e^{2x}+e^x c_1}{c_1^2}}}{c_1 e^{2x}+(2c_1-2)\,e^x-c_1}\right)$$
$$y(x)=2\,\mathrm{arctanh}\left(\frac{c_1\sqrt{2}\,\sqrt{\frac{-c_1 e^x e^{2x}+(-2c_1+2)e^{2x}+e^x c_1}{c_1^2}}}{c_1 e^{2x}+(2c_1-2)\,e^x-c_1}\right)$$

✓ Solution by Mathematica

Time used: 10.351 (sec). Leaf size: 32

```
DSolve[y'[x](1+Sinh[x])Sinh[y[x]]+Cosh[x](Cosh[y[x]]-1)==0,y[x],x,IncludeSingularSolutions -
```

$$y(x)\to 0$$
$$y(x)\to 2\mathrm{arcsinh}\left(\frac{c_1}{4\sqrt{\sinh(x)+1}}\right)$$
$$y(x)\to 0$$

26.12 problem 748

Internal problem ID [3989]

Book: Ordinary differential equations and their solutions. By George Moseley Murphy. 1960
Section: Various 26
Problem number: 748.
ODE order: 1.
ODE degree: 2.

CAS Maple gives this as type `[_quadrature]`

$$y'^2 = a\,x^n$$

✓ Solution by Maple

Time used: 0.078 (sec). Leaf size: 51

```
dsolve(diff(y(x),x)^2 = a*x^n,y(x), singsol=all)
```

$$y(x) = \frac{2x\sqrt{a\,x^n} + c_1(2+n)}{2+n}$$
$$y(x) = \frac{-2x\sqrt{a\,x^n} + c_1(2+n)}{2+n}$$

✓ Solution by Mathematica

Time used: 0.008 (sec). Leaf size: 57

```
DSolve[(y'[x])^2 == a x^n,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to -\frac{2\sqrt{a}x^{\frac{n}{2}+1}}{n+2} + c_1$$
$$y(x) \to \frac{2\sqrt{a}x^{\frac{n}{2}+1}}{n+2} + c_1$$

26.13 problem 749

Internal problem ID [3990]

Book: Ordinary differential equations and their solutions. By George Moseley Murphy. 1960
Section: Various 26
Problem number: 749.
ODE order: 1.
ODE degree: 2.

CAS Maple gives this as type `[_quadrature]`

$$\boxed{y'^2 - y = 0}$$

✓ Solution by Maple

Time used: 0.078 (sec). Leaf size: 17

```
dsolve(diff(y(x),x)^2 = y(x),y(x), singsol=all)
```

$$y(x) = 0$$
$$y(x) = \frac{(-c_1 + x)^2}{4}$$

✓ Solution by Mathematica

Time used: 0.061 (sec). Leaf size: 36

```
DSolve[(y'[x])^2 == y[x],y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to \frac{1}{4}(x - c_1)^2$$
$$y(x) \to \frac{1}{4}(x + c_1)^2$$
$$y(x) \to 0$$

26.14 problem 750

Internal problem ID [3991]

Book: Ordinary differential equations and their solutions. By George Moseley Murphy. 1960
Section: Various 26
Problem number: 750.
ODE order: 1.
ODE degree: 2.

CAS Maple gives this as type `[[_homogeneous, 'class C'], _dAlembert]`

$$y'^2 + y = x$$

✓ Solution by Maple

Time used: 0.078 (sec). Leaf size: 31

```
dsolve(diff(y(x),x)^2 = x-y(x),y(x), singsol=all)
```

$$y(x) = -\operatorname{LambertW}\left(c_1 e^{-1-\frac{x}{2}}\right)^2 - 2\operatorname{LambertW}\left(c_1 e^{-1-\frac{x}{2}}\right) + x - 1$$

✓ Solution by Mathematica

Time used: 27.515 (sec). Leaf size: 98

```
DSolve[(y'[x])^2==x-y[x],y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to -W\left(e^{-\frac{x}{2}-1-\frac{c_1}{2}}\right)^2 - 2W\left(e^{-\frac{x}{2}-1-\frac{c_1}{2}}\right) + x - 1$$
$$y(x) \to -W\left(-e^{\frac{1}{2}(-x-2+c_1)}\right)^2 - 2W\left(-e^{\frac{1}{2}(-x-2+c_1)}\right) + x - 1$$
$$y(x) \to x - 1$$

26.15 problem 751

Internal problem ID [3992]

Book: Ordinary differential equations and their solutions. By George Moseley Murphy. 1960
Section: Various 26
Problem number: 751.
ODE order: 1.
ODE degree: 2.

CAS Maple gives this as type `[[_homogeneous, ‘class G‘]]`

$$\boxed{{y'}^2 - y = x^2}$$

✓ Solution by Maple

Time used: 0.109 (sec). Leaf size: 271

```
dsolve(diff(y(x),x)^2 = x^2+y(x),y(x), singsol=all)
```

$$-17\ln\left(-x^4-x^2y(x)+4y(x)^2\right)-17\ln\left(-\sqrt{y(x)+x^2}\,x+2y(x)\right)+17\ln\left(\sqrt{y(x)+x^2}\,x+2y(x)\right)+\left(2\,\mathrm{arctanh}\left(\frac{\left(4\sqrt{y(x)+x^2}+x\right)\sqrt{17}}{17x}\right)-2\,\mathrm{arctanh}\left(\frac{\left(x-4\sqrt{y(x)+x^2}\right)\sqrt{17}}{17x}\right)-2\,\mathrm{arctanh}\left(\frac{\left(x^2-8y(x)\right)\sqrt{17}}{17x^2}\right)\right)\sqrt{17}-c_1=0$$

$$17\ln\left(-x^4-x^2y(x)+4y(x)^2\right)-17\ln\left(-\sqrt{y(x)+x^2}\,x+2y(x)\right)+17\ln\left(\sqrt{y(x)+x^2}\,x+2y(x)\right)+\left(2\,\mathrm{arctanh}\left(\frac{\left(4\sqrt{y(x)+x^2}+x\right)\sqrt{17}}{17x}\right)-2\,\mathrm{arctanh}\left(\frac{\left(x-4\sqrt{y(x)+x^2}\right)\sqrt{17}}{17x}\right)+2\,\mathrm{arctanh}\left(\frac{\left(x^2-8y(x)\right)\sqrt{17}}{17x^2}\right)\right)\sqrt{17}-c_1=0$$

✓ Solution by Mathematica

Time used: 1.538 (sec). Leaf size: 215

```
DSolve[(y'[x])^2==x^2+y[x],y[x],x,IncludeSingularSolutions -> True]
```

$$\text{Solve}\left[\frac{1}{34}\left(-34\log\left(\sqrt{x^2+y(x)}-x\right)\right.\right.$$
$$-\left(\sqrt{17}-17\right)\log\left(2x\sqrt{x^2+y(x)}-2x^2-\sqrt{17}y(x)+3y(x)\right)$$
$$\left.\left.+\left(17+\sqrt{17}\right)\log\left(2x\sqrt{x^2+y(x)}-2x^2+\left(3+\sqrt{17}\right)y(x)\right)\right)=c_1,y(x)\right]$$

$$\text{Solve}\left[\frac{1}{34}\left(-34\log\left(\sqrt{x^2+y(x)}-x\right)\right.\right.$$
$$+\left(17+\sqrt{17}\right)\log\left(2x\sqrt{x^2+y(x)}-2x^2+\left(\sqrt{17}-5\right)y(x)\right)$$
$$\left.\left.-\left(\sqrt{17}-17\right)\log\left(2x\sqrt{x^2+y(x)}-2x^2-\left(5+\sqrt{17}\right)y(x)\right)\right)=c_1,y(x)\right]$$

26.16 problem 752

Internal problem ID [3993]

Book: Ordinary differential equations and their solutions. By George Moseley Murphy. 1960
Section: Various 26
Problem number: 752.
ODE order: 1.
ODE degree: 2.

CAS Maple gives this as type `[[_homogeneous, ‘class G‘]]`

$$\boxed{y'^2 - 4y = -x^2}$$

✓ Solution by Maple

Time used: 0.093 (sec). Leaf size: 136

```
dsolve(diff(y(x),x)^2+x^2 = 4*y(x),y(x), singsol=all)
```

$$y(x) = \frac{x^2 \left(2 \operatorname{LambertW}\left(\frac{x\sqrt{2}\,e^{\frac{c_1}{2}}}{2}\right)^2 + 2 \operatorname{LambertW}\left(\frac{x\sqrt{2}\,e^{\frac{c_1}{2}}}{2}\right) + 1\right)}{4 \operatorname{LambertW}\left(\frac{x\sqrt{2}\,e^{\frac{c_1}{2}}}{2}\right)^2}$$

$$y(x) = \frac{x^2 \left(2 \operatorname{LambertW}\left(-\frac{\sqrt{2}\,c_1 x}{2}\right)^2 + 2 \operatorname{LambertW}\left(-\frac{\sqrt{2}\,c_1 x}{2}\right) + 1\right)}{4 \operatorname{LambertW}\left(-\frac{\sqrt{2}\,c_1 x}{2}\right)^2}$$

$$y(x) = \frac{x^2 \left(2 \operatorname{LambertW}\left(\frac{\sqrt{2}\,c_1 x}{2}\right)^2 + 2 \operatorname{LambertW}\left(\frac{\sqrt{2}\,c_1 x}{2}\right) + 1\right)}{4 \operatorname{LambertW}\left(\frac{\sqrt{2}\,c_1 x}{2}\right)^2}$$

✓ Solution by Mathematica

Time used: 2.887 (sec). Leaf size: 162

```
DSolve[(y'[x])^2+x^2==4 y[x],y[x],x,IncludeSingularSolutions -> True]
```

$$\text{Solve}\left[\operatorname{arctanh}\left(\frac{x}{\sqrt{4y(x)-x^2}}\right)+\frac{x\left(-\sqrt{4y(x)-x^2}\right)+(x^2-2y(x))\log(2y(x)-x^2)+2y(x)}{2(x^2-2y(x))}=c_1,y(x)\right]$$

$$\text{Solve}\left[\frac{x\sqrt{4y(x)-x^2}+(x^2-2y(x))\log(2y(x)-x^2)+2y(x)}{2(x^2-2y(x))}-\operatorname{arctanh}\left(\frac{x}{\sqrt{4y(x)-x^2}}\right)=c_1,y(x)\right]$$

26.17 problem 753

Internal problem ID [3994]

Book: Ordinary differential equations and their solutions. By George Moseley Murphy. 1960
Section: Various 26
Problem number: 753.
ODE order: 1.
ODE degree: 2.

CAS Maple gives this as type `[[_homogeneous, 'class G']]`

$$\boxed{y'^2 - 8y = -3x^2}$$

✓ Solution by Maple
Time used: 0.094 (sec). Leaf size: 153

```
dsolve(diff(y(x),x)^2+3*x^2 = 8*y(x),y(x), singsol=all)
```

$$y(x) = \frac{3x^2}{8} + \frac{\operatorname{RootOf}\left(\_Z^6 - 18x\_Z^5 + 135x^2\_Z^4 - 540x^3\_Z^3 + (1215x^4 - 16c_1)\_Z^2 + (-1458x^5 + 32c_1x)\_Z -}{8}$$

$$y(x) = \frac{3x^2}{8} + \frac{\operatorname{RootOf}\left(\_Z^6 + 18x\_Z^5 + 135x^2\_Z^4 + 540x^3\_Z^3 + (1215x^4 - 16c_1)\_Z^2 + (1458x^5 - 32c_1x)\_Z + 7\right.}{8}$$

✓ Solution by Mathematica

Time used: 61.018 (sec). Leaf size: 1865

```
DSolve[(y'[x])^2+3 x^2==8 y[x],y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to \frac{1}{96}\Bigg(144x^2$$

$$-8\,2^{2/3}\sqrt[3]{-729x^4\cosh(2c_1) - 729x^4\sinh(2c_1) - 270x^2\cosh(4c_1) - 270x^2\sinh(4c_1) + 3\sqrt{3}\sqrt{x^2(\cosh(7c}$$

$$-\frac{16\sqrt[3]{2}(54x^2\cosh(2c_1) + 54x^2\sinh(2c_1) +}{\sqrt[3]{-729x^4\cosh(2c_1) - 729x^4\sinh(2c_1) - 270x^2\cosh(4c_1) - 270x^2\sinh(4c_1) + 3\sqrt{3}\sqrt{x^2(\cosh(7c_1) +}}}$$

$$+ 32\cosh(2c_1) + 32\sinh(2c_1)\Bigg)$$

$$y(x) \to \frac{1}{192}\Bigg(288x^2$$

$$+8\,2^{2/3}\left(1-i\sqrt{3}\right)\sqrt[3]{-729x^4\cosh(2c_1) - 729x^4\sinh(2c_1) - 270x^2\cosh(4c_1) - 270x^2\sinh(4c_1) + 3\sqrt{3}\sqrt{}}$$

$$+\frac{16\sqrt[3]{2}(1+i\sqrt{3})\,(54x^2\cosh(2c_1) + 54x^2\sinh(2c}{\sqrt[3]{-729x^4\cosh(2c_1) - 729x^4\sinh(2c_1) - 270x^2\cosh(4c_1) - 270x^2\sinh(4c_1) + 3\sqrt{3}\sqrt{x^2(\cosh(7c_1) +}}}$$

$$+ 64\cosh(2c_1) + 64\sinh(2c_1)\Bigg)$$

$$y(x) \to \frac{1}{192}\Bigg(288x^2$$

$$+8\,2^{2/3}\left(1+i\sqrt{3}\right)\sqrt[3]{-729x^4\cosh(2c_1) - 729x^4\sinh(2c_1) - 270x^2\cosh(4c_1) - 270x^2\sinh(4c_1) + 3\sqrt{3}\sqrt{}}$$

$$+\frac{16\sqrt[3]{2}(1-i\sqrt{3})\,(54x^2\cosh(2c_1) + 54x^2\sinh(2c}{\sqrt[3]{-729x^4\cosh(2c_1) - 729x^4\sinh(2c_1) - 270x^2\cosh(4c_1) - 270x^2\sinh(4c_1) + 3\sqrt{3}\sqrt{x^2(\cosh(7c_1) +}}}$$

$$+ 64\cosh(2c_1) + 64\sinh(2c_1)\Bigg)$$

$$y(x) \to \frac{1}{96}\Bigg(144x^2$$

$$-8\,2^{2/3}\sqrt[3]{729x^4\cosh(2c_1) + 729x^4\sinh(2c_1) - 270x^2\cosh(4c_1) - 270x^2\sinh(4c_1) + 3\sqrt{3}\sqrt{x^2(\cosh(7c_1)}}$$

$$+\frac{16\sqrt[3]{2}(54x^2\cosh(2c_1) + 54x^2\sinh(2c_1) - c}{\sqrt[3]{729x^4\cosh(2c_1) + 729x^4\sinh(2c_1) - 270x^2\cosh(4c_1) - 270x^2\sinh(4c_1) + 3\sqrt{3}\sqrt{x^2(\cosh(7c_1) + \sin}}}$$

26.18 problem 754

Internal problem ID [3995]

Book: Ordinary differential equations and their solutions. By George Moseley Murphy. 1960
Section: Various 26
Problem number: 754.
ODE order: 1.
ODE degree: 2.

CAS Maple gives this as type `[[_homogeneous, ‘class G‘]]`

$$\boxed{{y'}^2 + yb = -x^2 a}$$

✗ Solution by Maple

```
dsolve(diff(y(x),x)^2+a*x^2+b*y(x) = 0,y(x), singsol=all)
```

No solution found

✓ Solution by Mathematica

Time used: 1.739 (sec). Leaf size: 581

```
DSolve[(y'[x])^2+a x^2+b y[x]==0,y[x],x,IncludeSingularSolutions -> True]
```

$$\text{Solve}\Bigg[\text{RootSum}\Bigg[\#1^4 - \#1^3 b + 2\#1^2 a + \#1 ab + a^2\&, \frac{2\#1^3 \log\left(\#1x - \sqrt{-ax^2 - by(x)} + \sqrt{-by(x)}\right) - 2\#1^3 \log(x) - \#1^2 b \log\left(\#1x - \sqrt{-ax^2 - by(x}\right.}{}$$

$$- \log\left(\sqrt{-by(x)}\sqrt{-ax^2 - by(x)} + by(x)\right) + \frac{1}{2}\log(y(x)) + 2\log(x) = c_1, y(x)\Bigg]$$

$$\text{Solve}\Bigg[\text{RootSum}\Bigg[\#1^4 + \#1^3 b + 2\#1^2 a - \#1 ab + a^2\&, \frac{-2\#1^3 \log\left(\#1x - \sqrt{-ax^2 - by(x)} + \sqrt{-by(x)}\right) + 2\#1^3 \log(x) - \#1^2 b \log\left(\#1x - \sqrt{-ax^2 - by(\right.}{}$$

$$- \log\left(\sqrt{-by(x)}\sqrt{-ax^2 - by(x)} + by(x)\right) + \frac{1}{2}\log(y(x)) + 2\log(x) = c_1, y(x)\Bigg]$$

26.19 problem 755

Internal problem ID [3996]

Book: Ordinary differential equations and their solutions. By George Moseley Murphy. 1960
Section: Various 26
Problem number: 755.
ODE order: 1.
ODE degree: 2.

CAS Maple gives this as type `[_quadrature]`

$$\boxed{y'^2 - y^2 = 1}$$

✓ Solution by Maple
Time used: 0.047 (sec). Leaf size: 31

```
dsolve(diff(y(x),x)^2 = 1+y(x)^2,y(x), singsol=all)
```

$$y(x) = -i$$
$$y(x) = i$$
$$y(x) = -\sinh(c_1 - x)$$
$$y(x) = \sinh(c_1 - x)$$

✓ Solution by Mathematica
Time used: 0.284 (sec). Leaf size: 69

```
DSolve[(y'[x])^2==1+y[x]^2,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to \frac{1}{2}\left(e^{-x+c_1} - e^{x-c_1}\right)$$
$$y(x) \to \frac{1}{2}\left(e^{x+c_1} - e^{-x-c_1}\right)$$
$$y(x) \to -i$$
$$y(x) \to i$$

26.20 problem 756

Internal problem ID [3997]

Book: Ordinary differential equations and their solutions. By George Moseley Murphy. 1960
Section: Various 26
Problem number: 756.
ODE order: 1.
ODE degree: 2.

CAS Maple gives this as type `[_quadrature]`

$$\boxed{y'^2 + y^2 = 1}$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 29

```
dsolve(diff(y(x),x)^2 = 1-y(x)^2,y(x), singsol=all)
```

$$y(x) = -1$$
$$y(x) = 1$$
$$y(x) = -\sin(c_1 - x)$$
$$y(x) = \sin(c_1 - x)$$

✓ Solution by Mathematica

Time used: 0.203 (sec). Leaf size: 39

```
DSolve[(y'[x])^2==1-y[x]^2,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to \cos(x + c_1)$$
$$y(x) \to \cos(x - c_1)$$
$$y(x) \to -1$$
$$y(x) \to 1$$
$$y(x) \to \text{Interval}[\{-1, 1\}]$$

26.21 problem 757

Internal problem ID [3998]

Book: Ordinary differential equations and their solutions. By George Moseley Murphy. 1960
Section: Various 26
Problem number: 757.
ODE order: 1.
ODE degree: 2.

CAS Maple gives this as type `[_quadrature]`

$$y'^2 + y^2 = a^2$$

✓ Solution by Maple
Time used: 0.047 (sec). Leaf size: 60

```
dsolve(diff(y(x),x)^2 = a^2-y(x)^2,y(x), singsol=all)
```

$$y(x) = -a$$
$$y(x) = a$$
$$y(x) = -\tan(c_1 - x)\sqrt{\cos(c_1 - x)^2 a^2}$$
$$y(x) = \tan(c_1 - x)\sqrt{\cos(c_1 - x)^2 a^2}$$

✓ Solution by Mathematica

Time used: 5.305 (sec). Leaf size: 111

```
DSolve[(y'[x])^2==a^2-y[x]^2,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to -\frac{a\tan(x-c_1)}{\sqrt{\sec^2(x-c_1)}}$$
$$y(x) \to \frac{a\tan(x-c_1)}{\sqrt{\sec^2(x-c_1)}}$$
$$y(x) \to -\frac{a\tan(x+c_1)}{\sqrt{\sec^2(x+c_1)}}$$
$$y(x) \to \frac{a\tan(x+c_1)}{\sqrt{\sec^2(x+c_1)}}$$
$$y(x) \to -a$$
$$y(x) \to a$$

26.22 problem 758

Internal problem ID [3999]

Book: Ordinary differential equations and their solutions. By George Moseley Murphy. 1960
Section: Various 26
Problem number: 758.
ODE order: 1.
ODE degree: 2.

CAS Maple gives this as type `[_quadrature]`

$$\boxed{y'^2 - y^2 a^2 = 0}$$

✓ Solution by Maple
Time used: 0.016 (sec). Leaf size: 20

```
dsolve(diff(y(x),x)^2 = a^2*y(x)^2,y(x), singsol=all)
```

$$y(x) = c_1 e^{ax}$$
$$y(x) = c_1 e^{-ax}$$

✓ Solution by Mathematica
Time used: 0.069 (sec). Leaf size: 31

```
DSolve[(y'[x])^2==a^2 y[x]^2,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to c_1 e^{-ax}$$
$$y(x) \to c_1 e^{ax}$$
$$y(x) \to 0$$

26.23 problem 759

Internal problem ID [4000]

Book: Ordinary differential equations and their solutions. By George Moseley Murphy. 1960
Section: Various 26
Problem number: 759.
ODE order: 1.
ODE degree: 2.

CAS Maple gives this as type `[_quadrature]`

$$\boxed{y'^2 - by^2 = a}$$

✓ Solution by Maple
Time used: 0.047 (sec). Leaf size: 99

```
dsolve(diff(y(x),x)^2 = a+b*y(x)^2,y(x), singsol=all)
```

$$y(x) = \frac{\sqrt{-ab}}{b}$$
$$y(x) = -\frac{\sqrt{-ab}}{b}$$
$$y(x) = \frac{e^{-\sqrt{b}\,(c_1+x)}\left(-a\,e^{2c_1\sqrt{b}} + e^{2x\sqrt{b}}\right)}{2\sqrt{b}}$$
$$y(x) = -\frac{e^{-\sqrt{b}\,(c_1+x)}\left(a\,e^{2x\sqrt{b}} - e^{2c_1\sqrt{b}}\right)}{2\sqrt{b}}$$

✓ Solution by Mathematica

Time used: 60.109 (sec). Leaf size: 171

```
DSolve[(y'[x])^2==a+b y[x]^2,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to -\frac{\sqrt{a}\tanh\left(\sqrt{b}(x-c_1)\right)}{\sqrt{b\text{sech}^2\left(\sqrt{b}(x-c_1)\right)}}$$
$$y(x) \to \frac{\sqrt{a}\tanh\left(\sqrt{b}(x-c_1)\right)}{\sqrt{b\text{sech}^2\left(\sqrt{b}(x-c_1)\right)}}$$
$$y(x) \to -\frac{\sqrt{a}\tanh\left(\sqrt{b}(x+c_1)\right)}{\sqrt{b\text{sech}^2\left(\sqrt{b}(x+c_1)\right)}}$$
$$y(x) \to \frac{\sqrt{a}\tanh\left(\sqrt{b}(x+c_1)\right)}{\sqrt{b\text{sech}^2\left(\sqrt{b}(x+c_1)\right)}}$$

26.24 problem 760

Internal problem ID [4001]

Book: Ordinary differential equations and their solutions. By George Moseley Murphy. 1960
Section: Various 26
Problem number: 760.
ODE order: 1.
ODE degree: 2.

CAS Maple gives this as type `[_separable]`

$$\boxed{y'^2 - y^2x^2 = 0}$$

✓ Solution by Maple
Time used: 0.0 (sec). Leaf size: 23

```
dsolve(diff(y(x),x)^2 = x^2*y(x)^2,y(x), singsol=all)
```

$$y(x) = \mathrm{e}^{\frac{x^2}{2}} c_1$$
$$y(x) = \mathrm{e}^{-\frac{x^2}{2}} c_1$$

✓ Solution by Mathematica
Time used: 0.065 (sec). Leaf size: 38

```
DSolve[(y'[x])^2==x^2 y[x]^2,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to c_1 e^{-\frac{x^2}{2}}$$
$$y(x) \to c_1 e^{\frac{x^2}{2}}$$
$$y(x) \to 0$$

26.25 problem 761

Internal problem ID [4002]

Book: Ordinary differential equations and their solutions. By George Moseley Murphy. 1960
Section: Various 26
Problem number: 761.
ODE order: 1.
ODE degree: 2.

CAS Maple gives this as type `[_quadrature]`

$$y'^2 - (y-1)\,y^2 = 0$$

✓ Solution by Maple

Time used: 0.062 (sec). Leaf size: 22

```
dsolve(diff(y(x),x)^2 = (y(x)-1)*y(x)^2,y(x), singsol=all)
```

$$y(x) = 1$$
$$y(x) = 0$$
$$y(x) = \sec\left(\frac{c_1}{2} - \frac{x}{2}\right)^2$$

✓ Solution by Mathematica

Time used: 1.652 (sec). Leaf size: 45

```
DSolve[(y'[x])^2==(y[x]-1)y[x]^2,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to \sec^2\left(\frac{x-c_1}{2}\right)$$
$$y(x) \to 1 + \tan^2\left(\frac{x+c_1}{2}\right)$$
$$y(x) \to 0$$
$$y(x) \to 1$$

26.26 problem 762

Internal problem ID [4003]

Book: Ordinary differential equations and their solutions. By George Moseley Murphy. 1960
Section: Various 26
Problem number: 762.
ODE order: 1.
ODE degree: 2.

CAS Maple gives this as type `[_quadrature]`

$$\boxed{y'^2 - (y-a)(y-b)(y-c) = 0}$$

✓ Solution by Maple
Time used: 0.078 (sec). Leaf size: 75

```
dsolve(diff(y(x),x)^2 = (y(x)-a)*(y(x)-b)*(y(x)-c),y(x), singsol=all)
```

$$y(x) = a$$
$$y(x) = b$$
$$y(x) = c$$
$$x - \left(\int^{y(x)} \frac{1}{\sqrt{(-a+\_a)(\_a-b)(\_a-c)}} d\_a\right) - c_1 = 0$$
$$x + \int^{y(x)} \frac{1}{\sqrt{(-a+\_a)(\_a-b)(\_a-c)}} d\_a - c_1 = 0$$

✓ Solution by Mathematica

Time used: 48.497 (sec). Leaf size: 188

```
DSolve[(y'[x])^2==(y[x]-a)(y[x]-b)(y[x]-c),y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to \operatorname{ns}\left(\frac{1}{2}\sqrt{a-b}(c_1 - ix)|\frac{a-c}{a-b}\right)^2 \left(a\operatorname{sn}\left(\frac{1}{2}\sqrt{a-b}(c_1 - ix)|\frac{a-c}{a-b}\right)^2 - a + b\right)$$
$$y(x) \to \operatorname{ns}\left(\frac{1}{2}\sqrt{a-b}(ix + c_1)|\frac{a-c}{a-b}\right)^2 \left(a\operatorname{sn}\left(\frac{1}{2}\sqrt{a-b}(ix + c_1)|\frac{a-c}{a-b}\right)^2 - a + b\right)$$
$$y(x) \to a$$
$$y(x) \to b$$
$$y(x) \to c$$

26.27 problem 763

Internal problem ID [4004]

Book: Ordinary differential equations and their solutions. By George Moseley Murphy. 1960
Section: Various 26
Problem number: 763.
ODE order: 1.
ODE degree: 2.

CAS Maple gives this as type `[_quadrature]`

$$y'^2 - a^2 y^n = 0$$

✓ Solution by Maple
Time used: 0.141 (sec). Leaf size: 72

```
dsolve(diff(y(x),x)^2 = a^2*y(x)^n,y(x), singsol=all)
```

$$y(x) = 4^{\frac{1}{n-2}} \left(-\frac{1}{a\left(-c_1 + x\right)\left(n-2\right)}\right)^{\frac{2}{n-2}}$$
$$y(x) = 4^{\frac{1}{n-2}} \left(\frac{1}{a\left(-c_1 + x\right)\left(n-2\right)}\right)^{\frac{2}{n-2}}$$

✓ Solution by Mathematica
Time used: 3.27 (sec). Leaf size: 77

```
DSolve[(y'[x])^2==a^2 y[x]^n,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to 2^{\frac{2}{n-2}}(-((n-2)(ax+c_1)))^{-\frac{2}{n-2}}$$
$$y(x) \to 2^{\frac{2}{n-2}}((n-2)(ax-c_1))^{-\frac{2}{n-2}}$$
$$y(x) \to 0^{\frac{1}{n}}$$

26.28 problem 764

Internal problem ID [4005]

Book: Ordinary differential equations and their solutions. By George Moseley Murphy. 1960
Section: Various 26
Problem number: 764.
ODE order: 1.
ODE degree: 2.

CAS Maple gives this as type `[_quadrature]`

$$y'^2 - a^2(1 - \ln(y)^2)\, y^2 = 0$$

✓ Solution by Maple
Time used: 0.078 (sec). Leaf size: 45

```
dsolve(diff(y(x),x)^2 = a^2*(1-ln(y(x))^2)*y(x)^2,y(x), singsol=all)
```

$$y(x) = \mathrm{e}^{\mathrm{RootOf}\left(a^2 \mathrm{e}^{2\_Z}(\_Z^2-1)\right)}$$
$$y(x) = \mathrm{e}^{-\sin((c_1-x)a)}$$
$$y(x) = \mathrm{e}^{\sin((c_1-x)a)}$$

✓ Solution by Mathematica
Time used: 16.422 (sec). Leaf size: 197

```
DSolve[(y'[x])^2==a^2(1-Log[y[x]]^2)y[x]^2,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to \exp\left(-\frac{1}{2}\sqrt{-e^{2iax-2c_1} - e^{2c_1-2iax} + 2}\right)$$
$$y(x) \to \exp\left(\frac{1}{2}\sqrt{-e^{2iax-2c_1} - e^{2c_1-2iax} + 2}\right)$$
$$y(x) \to \exp\left(-\frac{1}{2}\sqrt{-e^{-2iax-2c_1}\left(-1 + e^{2iax+2c_1}\right)^2}\right)$$
$$y(x) \to \exp\left(\frac{1}{2}\sqrt{-e^{-2iax-2c_1}\left(-1 + e^{2iax+2c_1}\right)^2}\right)$$
$$y(x) \to \frac{1}{e}$$
$$y(x) \to e$$

26.29 problem 765

Internal problem ID [4006]

Book: Ordinary differential equations and their solutions. By George Moseley Murphy. 1960
Section: Various 26
Problem number: 765.
ODE order: 1.
ODE degree: 2.

CAS Maple gives this as type `[[_1st_order, '_with_symmetry_[F(x),G(x)*y+H(x)]']]`

$$\boxed{y'^2 + f(x)(y-a)(y-b) = 0}$$

✓ Solution by Maple

Time used: 0.125 (sec). Leaf size: 220

```
dsolve(diff(y(x),x)^2+f(x)*(y(x)-a)*(y(x)-b) = 0,y(x), singsol=all)
```

$$\frac{\sqrt{(y(x)-a)(y(x)-b)}\left(-\ln(2)+\ln\left(-a-b+2y(x)+2\sqrt{(y(x)-a)(y(x)-b)}\right)\right)}{\sqrt{y(x)-b}\,\sqrt{y(x)-a}} - \frac{\int^{x}\sqrt{-f(\_a)(y(x)-a)(y(x)-b)}d\_a}{\sqrt{y(x)-b}\,\sqrt{y(x)-a}} + c_1 = 0$$

$$\frac{\sqrt{(y(x)-a)(y(x)-b)}\left(-\ln(2)+\ln\left(-a-b+2y(x)+2\sqrt{(y(x)-a)(y(x)-b)}\right)\right)}{\sqrt{y(x)-b}\,\sqrt{y(x)-a}} + \frac{\int^{x}\sqrt{-f(\_a)(y(x)-a)(y(x)-b)}d\_a}{\sqrt{y(x)-b}\,\sqrt{y(x)-a}} + c_1 = 0$$

✓ Solution by Mathematica

Time used: 4.208 (sec). Leaf size: 89

```
DSolve[(y'[x])^2+ f[x](y[x]-a)(y[x]-b)==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to \frac{1}{2}\left((b-a)\cosh\left(\int_1^x -i\sqrt{f(K[2])}dK[2] + c_1\right) + a + b\right)$$
$$y(x) \to \frac{1}{2}\left((b-a)\cosh\left(\int_1^x i\sqrt{f(K[3])}dK[3] + c_1\right) + a + b\right)$$
$$y(x) \to a$$
$$y(x) \to b$$

27 Various 27

27.1 problem 766

Internal problem ID [4007]

Book: Ordinary differential equations and their solutions. By George Moseley Murphy. 1960
Section: Various 27
Problem number: 766.
ODE order: 1.
ODE degree: 2.

CAS Maple gives this as type `[[_1st_order, '_with_symmetry_[F(x),G(x)*y+H(x)]']]`

$$
{y'}^2 + f(x)\left(y-a\right)^2\left(y-b\right) = 0
$$

✓ Solution by Maple

Time used: 0.156 (sec). Leaf size: 112

```
dsolve(diff(y(x),x)^2+f(x)*(y(x)-a)^2*(y(x)-b) = 0,y(x), singsol=all)
```

$$
\frac{2\arctan\left(\frac{\sqrt{y(x)-b}}{\sqrt{-a+b}}\right)}{\sqrt{-a+b}} + \frac{\int^{x}\sqrt{-f\left(\_a\right)\left(y\left(x\right)-b\right)}d\_a}{\sqrt{y\left(x\right)-b}} + c_1 = 0
$$

$$
\frac{2\arctan\left(\frac{\sqrt{y(x)-b}}{\sqrt{-a+b}}\right)}{\sqrt{-a+b}} - \frac{\int^{x}\sqrt{-f\left(\_a\right)\left(y\left(x\right)-b\right)}d\_a}{\sqrt{y\left(x\right)-b}} + c_1 = 0
$$

✓ Solution by Mathematica

Time used: 60.157 (sec). Leaf size: 93

```
DSolve[(y'[x])^2+f[x](y[x]-a)^2 (y[x]-b)==0,y[x],x,IncludeSingularSolutions -> True]
```

$$
y(x) \to b + (b-a)\tan^2\left(\frac{1}{2}\sqrt{a-b}\left(\int_1^x -\sqrt{f(K[1])}dK[1] + c_1\right)\right)
$$

$$
y(x) \to b + (b-a)\tan^2\left(\frac{1}{2}\sqrt{a-b}\left(\int_1^x \sqrt{f(K[2])}dK[2] + c_1\right)\right)
$$

27.2 problem 767

Internal problem ID [4008]

Book: Ordinary differential equations and their solutions. By George Moseley Murphy. 1960
Section: Various 27
Problem number: 767.
ODE order: 1.
ODE degree: 2.

CAS Maple gives this as type `[[_1st_order, '_with_symmetry_[F(x),G(x)*y+H(x)]']]`

$$y'^2 + f(x)(y-a)(y-b)(y-c) = 0$$

✓ Solution by Maple

Time used: 0.375 (sec). Leaf size: 156

```
dsolve(diff(y(x),x)^2+f(x)*(y(x)-a)*(y(x)-b)*(y(x)-c) = 0,y(x), singsol=all)
```

$$\int^{y(x)} \frac{1}{\sqrt{(-a+\_a)(\_a-b)(\_a-c)}} d\_a - \frac{\int^{x} \sqrt{-f(\_a)(y(x)-c)(y(x)-b)(y(x)-a)}d\_a}{\sqrt{(y(x)-a)(y(x)-b)(y(x)-c)}} + c_1 = 0$$

$$\int^{y(x)} \frac{1}{\sqrt{(-a+\_a)(\_a-b)(\_a-c)}} d\_a + \frac{\int^{x} \sqrt{-f(\_a)(y(x)-c)(y(x)-b)(y(x)-a)}d\_a}{\sqrt{(y(x)-a)(y(x)-b)(y(x)-c)}} + c_1 = 0$$

✓ Solution by Mathematica

Time used: 39.221 (sec). Leaf size: 228

```
DSolve[(y'[x])^2+f[x](y[x]-a)(y[x]-b)(y[x]-c)==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to \text{ns}\left(\frac{1}{2}\sqrt{a-b}\left(c_1 + \int_1^x -\sqrt{f(K[1])}dK[1]\right) | \frac{a-c}{a-b}\right)^2 \left(a\,\text{sn}\left(\frac{1}{2}\sqrt{a-b}\left(c_1+\int_1^x -\sqrt{f(K[1])}dK[1]\right) | \frac{a-c}{a-b}\right)^2 - a + b\right)$$

$$y(x) \to \text{ns}\left(\frac{1}{2}\sqrt{a-b}\left(c_1 + \int_1^x \sqrt{f(K[2])}dK[2]\right) | \frac{a-c}{a-b}\right)^2 \left(a\,\text{sn}\left(\frac{1}{2}\sqrt{a-b}\left(c_1+\int_1^x \sqrt{f(K[2])}dK[2]\right) | \frac{a-c}{a-b}\right)^2 - a + b\right)$$

$$y(x) \to a$$

$$y(x) \to b$$

$$y(x) \to c$$

27.3 problem 768

Internal problem ID [4009]

Book: Ordinary differential equations and their solutions. By George Moseley Murphy. 1960
Section: Various 27
Problem number: 768.
ODE order: 1.
ODE degree: 2.

CAS Maple gives this as type `[[_1st_order, ‘_with_symmetry_[F(x),G(x)*y+H(x)]‘]]`

$$y'^2 + f(x)(y-a)^2(y-b)(y-c) = 0$$

✓ Solution by Maple

Time used: 0.125 (sec). Leaf size: 342

```
dsolve(diff(y(x),x)^2+f(x)*(y(x)-a)^2*(y(x)-b)*(y(x)-c) = 0,y(x), singsol=all)
```

$$-\frac{\ln\left(\frac{-2\sqrt{(y(x)-b)(y(x)-c)}\,\sqrt{(a-b)(a-c)}+(-2a+b+c)y(x)+(b+c)a-2bc}{-y(x)+a}\right)\sqrt{a^2-ab-ac+bc}\,\sqrt{y(x)-b}\,\sqrt{y(x)-c}}{(a-c)(a-b)\sqrt{bc-cy(x)-by(x)+y(x)^2}} + \frac{\int^x \sqrt{(b-y(x))(y(x)-c)f(\_a)}d\_a}{\sqrt{y(x)-c}\,\sqrt{y(x)-b}} + c_1 = 0$$

$$-\frac{\ln\left(\frac{-2\sqrt{(y(x)-b)(y(x)-c)}\,\sqrt{(a-b)(a-c)}+(-2a+b+c)y(x)+(b+c)a-2bc}{-y(x)+a}\right)\sqrt{a^2-ab-ac+bc}\,\sqrt{y(x)-b}\,\sqrt{y(x)-c}}{(a-c)(a-b)\sqrt{bc-cy(x)-by(x)+y(x)^2}} - \frac{\int^x \sqrt{(b-y(x))(y(x)-c)f(\_a)}d\_a}{\sqrt{y(x)-c}\,\sqrt{y(x)-b}} + c_1 = 0$$

✓ Solution by Mathematica

Time used: 60.323 (sec). Leaf size: 251

```
DSolve[(y'[x])^2+f[x](y[x]-a)^2 (y[x]-b) (y[x]-c)==0,y[x],x,IncludeSingularSolutions -> True
```

$$y(x) \to \frac{c(a-b) + b(a-c)\tan^2\left(\frac{1}{2}\sqrt{a-b}\sqrt{c-a}\left(\int_1^x -i\sqrt{f(K[1])}dK[1] + c_1\right)\right)}{(a-c)\tan^2\left(\frac{1}{2}\sqrt{a-b}\sqrt{c-a}\left(\int_1^x -i\sqrt{f(K[1])}dK[1] + c_1\right)\right) + a - b}$$

$$y(x) \to \frac{c(a-b) + b(a-c)\tan^2\left(\frac{1}{2}\sqrt{a-b}\sqrt{c-a}\left(\int_1^x i\sqrt{f(K[2])}dK[2] + c_1\right)\right)}{(a-c)\tan^2\left(\frac{1}{2}\sqrt{a-b}\sqrt{c-a}\left(\int_1^x i\sqrt{f(K[2])}dK[2] + c_1\right)\right) + a - b}$$

27.4 problem 770

Internal problem ID [4010]

Book: Ordinary differential equations and their solutions. By George Moseley Murphy. 1960
Section: Various 27
Problem number: 770.
ODE order: 1.
ODE degree: 2.

CAS Maple gives this as type `[_separable]`

$$y'^2 - f(x)^2 (y-a)(y-b)(y-c)^2 = 0$$

✓ Solution by Maple

Time used: 1.157 (sec). Leaf size: 284

```
dsolve(diff(y(x),x)^2 = f(x)^2*(y(x)-a)*(y(x)-b)*(y(x)-c)^2,y(x), singsol=all)
```

$$y(x) = \frac{c\,\mathrm{e}^{2\left(\int f(x)dx+c_1\right)\sqrt{(a-c)(b-c)}} + ((4b-2c)\,a-2bc)\,\mathrm{e}^{\left(\int f(x)dx+c_1\right)\sqrt{(a-c)(b-c)}} + c(a-b)^2}{\mathrm{e}^{2\left(\int f(x)dx+c_1\right)\sqrt{(a-c)(b-c)}} + (2a+2b-4c)\,\mathrm{e}^{\left(\int f(x)dx+c_1\right)\sqrt{(a-c)(b-c)}} + a^2-2ab+b^2}$$

$$y(x) = \frac{((4b-2c)\,a-2bc)\,\mathrm{e}^{-\left(\int f(x)dx+c_1\right)\sqrt{(a-c)(b-c)}} + \left(\mathrm{e}^{-2\left(\int f(x)dx+c_1\right)\sqrt{(a-c)(b-c)}} + (a-b)^2\right)c}{(2a+2b-4c)\,\mathrm{e}^{-\left(\int f(x)dx+c_1\right)\sqrt{(a-c)(b-c)}} + a^2-2ab+b^2+\mathrm{e}^{-2\left(\int f(x)dx+c_1\right)\sqrt{(a-c)(b-c)}}}$$

✓ Solution by Mathematica

Time used: 60.31 (sec). Leaf size: 223

```
DSolve[(y'[x])^2==f[x]^2 (y[x]-a)(y[x]-b)(y[x]-c)^2,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to \frac{b(a-c)+a(b-c)\tan^2\left(\frac{1}{2}\sqrt{c-a}\sqrt{b-c}\left(\int_1^x -f(K[1])dK[1]+c_1\right)\right)}{(b-c)\tan^2\left(\frac{1}{2}\sqrt{c-a}\sqrt{b-c}\left(\int_1^x -f(K[1])dK[1]+c_1\right)\right)+a-c}$$

$$y(x) \to \frac{b(a-c)+a(b-c)\tan^2\left(\frac{1}{2}\sqrt{c-a}\sqrt{b-c}\left(\int_1^x f(K[2])dK[2]+c_1\right)\right)}{(b-c)\tan^2\left(\frac{1}{2}\sqrt{c-a}\sqrt{b-c}\left(\int_1^x f(K[2])dK[2]+c_1\right)\right)+a-c}$$

27.5 problem 771

Internal problem ID [4011]

Book: Ordinary differential equations and their solutions. By George Moseley Murphy. 1960
Section: Various 27
Problem number: 771.
ODE order: 1.
ODE degree: 2.

CAS Maple gives this as type `[‘y=_G(x,y’)‘]`

$$y'^2 - f(x)^2 (y - u(x))^2 (y - a) (y - b) = 0$$

✗ Solution by Maple

```
dsolve(diff(y(x),x)^2 = f(x)^2*(y(x)-u(x))^2*(y(x)-a)*(y(x)-b),y(x), singsol=all)
```

No solution found

✗ Solution by Mathematica
Time used: 0.0 (sec). Leaf size: 0

```
DSolve[(y'[x])^2==f[x]^2 (y[x]-u[x])^2 (y[x]-a)(y[x]-b),y[x],x,IncludeSingularSolutions -> T
```

Not solved

27.6 problem 772

Internal problem ID [4012]

Book: Ordinary differential equations and their solutions. By George Moseley Murphy. 1960
Section: Various 27
Problem number: 772.
ODE order: 1.
ODE degree: 2.

CAS Maple gives this as type `[_quadrature]`

$$\boxed{y'^2 + 2y' = -x}$$

✓ Solution by Maple
Time used: 0.062 (sec). Leaf size: 45

```
dsolve(diff(y(x),x)^2+2*diff(y(x),x)+x = 0,y(x), singsol=all)
```

$$y(x) = \frac{(-2x+2)\sqrt{1-x}}{3} - x + c_1$$
$$y(x) = \frac{(2x-2)\sqrt{1-x}}{3} - x + c_1$$

✓ Solution by Mathematica
Time used: 0.015 (sec). Leaf size: 47

```
DSolve[(y'[x])^2+2 y'[x]+x==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to -\frac{2}{3}(1-x)^{3/2} - x + c_1$$
$$y(x) \to \frac{2}{3}(1-x)^{3/2} - x + c_1$$

27.7 problem 773

Internal problem ID [4013]

Book: Ordinary differential equations and their solutions. By George Moseley Murphy. 1960
Section: Various 27
Problem number: 773.
ODE order: 1.
ODE degree: 2.

CAS Maple gives this as type `[[_homogeneous, 'class C'], _dAlembert]`

$$y'^2 - 2y' + a(-y + x) = 0$$

✓ Solution by Maple

Time used: 0.047 (sec). Leaf size: 28

```
dsolve(diff(y(x),x)^2-2*diff(y(x),x)+a*(x-y(x)) = 0,y(x), singsol=all)
```

$$y(x) = \frac{ax - 1}{a}$$
$$y(x) = \frac{(-c_1 + x)^2 a}{4} + c_1$$

✓ Solution by Mathematica

Time used: 0.304 (sec). Leaf size: 84

```
DSolve[(y'[x])^2-2*y'[x]+a*(x-y[x])==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to \frac{1}{4}a\left(x^2 - 2\sqrt{2}c_1x + 2c_1^2\right) - \frac{1}{a} + x$$
$$y(x) \to \frac{1}{4}a\left(x^2 + 2\sqrt{2}c_1x + 2c_1^2\right) - \frac{1}{a} + x$$
$$y(x) \to x - \frac{1}{a}$$

27.8 problem 774

Internal problem ID [4014]

Book: Ordinary differential equations and their solutions. By George Moseley Murphy. 1960
Section: Various 27
Problem number: 774.
ODE order: 1.
ODE degree: 2.

CAS Maple gives this as type `[_quadrature]`

$$y'^2 - 2y' - y^2 = 0$$

✓ Solution by Maple

Time used: 0.078 (sec). Leaf size: 66

```
dsolve(diff(y(x),x)^2-2*diff(y(x),x)-y(x)^2 = 0,y(x), singsol=all)
```

$$\frac{-\sqrt{y(x)^2+1}+\operatorname{arcsinh}(y(x))\,y(x)-1+(-c_1+x)\,y(x)}{y(x)}=0$$
$$\frac{\sqrt{y(x)^2+1}-\operatorname{arcsinh}(y(x))\,y(x)-1+(-c_1+x)\,y(x)}{y(x)}=0$$

✓ Solution by Mathematica

Time used: 1.359 (sec). Leaf size: 104

```
DSolve[(y'[x])^2-2 y'[x]-y[x]^2==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x)\to \text{InverseFunction}\left[-\frac{\sqrt{\#1^2+1}+\#1\log\left(\sqrt{\#1^2+1}-\#1\right)+1}{\#1}\&\right][-x+c_1]$$
$$y(x)\to \text{InverseFunction}\left[-\frac{\sqrt{\#1^2+1}}{\#1}-\log\left(\sqrt{\#1^2+1}-\#1\right)+\frac{1}{\#1}\&\right][x+c_1]$$
$$y(x)\to 0$$

27.9 problem 775

Internal problem ID [4015]

Book: Ordinary differential equations and their solutions. By George Moseley Murphy. 1960
Section: Various 27
Problem number: 775.
ODE order: 1.
ODE degree: 2.

CAS Maple gives this as type `[_quadrature]`

$$\boxed{y'^2 - 5y' = -6}$$

✓ Solution by Maple
Time used: 0.0 (sec). Leaf size: 17

```
dsolve(diff(y(x),x)^2-5*diff(y(x),x)+6 = 0,y(x), singsol=all)
```

$$y(x) = 3x + c_1$$
$$y(x) = 2x + c_1$$

✓ Solution by Mathematica
Time used: 0.003 (sec). Leaf size: 21

```
DSolve[(y'[x])^2-5 y'[x]+6==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to 2x + c_1$$
$$y(x) \to 3x + c_1$$

27.10 problem 776

Internal problem ID [4016]

Book: Ordinary differential equations and their solutions. By George Moseley Murphy. 1960
Section: Various 27
Problem number: 776.
ODE order: 1.
ODE degree: 2.

CAS Maple gives this as type `[_quadrature]`

$$\boxed{y'^2 - 7y' = -12}$$

✓ Solution by Maple
Time used: 0.0 (sec). Leaf size: 17

```
dsolve(diff(y(x),x)^2-7*diff(y(x),x)+12 = 0,y(x), singsol=all)
```

$$y(x) = 4x + c_1$$
$$y(x) = 3x + c_1$$

✓ Solution by Mathematica
Time used: 0.003 (sec). Leaf size: 21

```
DSolve[(y'[x])^2-7 y'[x]+12==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to 3x + c_1$$
$$y(x) \to 4x + c_1$$

27.11 problem 777

Internal problem ID [4017]

Book: Ordinary differential equations and their solutions. By George Moseley Murphy. 1960
Section: Various 27
Problem number: 777.
ODE order: 1.
ODE degree: 2.

CAS Maple gives this as type `[_quadrature]`

$$\boxed{{y'}^2 + y'a = -b}$$

✓ Solution by Maple

Time used: 0.078 (sec). Leaf size: 43

```
dsolve(diff(y(x),x)^2+a*diff(y(x),x)+b = 0,y(x), singsol=all)
```

$$y(x) = -\frac{ax}{2} - \frac{x\sqrt{a^2-4b}}{2} + c_1$$
$$y(x) = -\frac{ax}{2} + \frac{x\sqrt{a^2-4b}}{2} + c_1$$

✓ Solution by Mathematica

Time used: 0.005 (sec). Leaf size: 59

```
DSolve[(y'[x])^2+a y'[x]+b==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to -\frac{1}{2}x\sqrt{a^2-4b} - \frac{ax}{2} + c_1$$
$$y(x) \to \frac{1}{2}x\sqrt{a^2-4b} - \frac{ax}{2} + c_1$$

27.12 problem 778

Internal problem ID [4018]

Book: Ordinary differential equations and their solutions. By George Moseley Murphy. 1960
Section: Various 27
Problem number: 778.
ODE order: 1.
ODE degree: 2.

CAS Maple gives this as type `[_quadrature]`

$$\boxed{y'^2 + y'a = -bx}$$

✓ Solution by Maple
Time used: 0.063 (sec). Leaf size: 70

```
dsolve(diff(y(x),x)^2+a*diff(y(x),x)+b*x = 0,y(x), singsol=all)
```

$$y(x) = \frac{(a^2 - 4bx)^{\frac{3}{2}} - 6b(ax - 2c_1)}{12b}$$
$$y(x) = \frac{(-a^2 + 4bx)\sqrt{a^2 - 4bx} - 6b(ax - 2c_1)}{12b}$$

✓ Solution by Mathematica
Time used: 0.027 (sec). Leaf size: 68

```
DSolve[(y'[x])^2+a y'[x]+b x==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to -\frac{(a^2 - 4bx)^{3/2} + 6abx}{12b} + c_1$$
$$y(x) \to \frac{1}{2}\left(\frac{(a^2 - 4bx)^{3/2}}{6b} - ax\right) + c_1$$

27.13 problem 779

Internal problem ID [4019]

Book: Ordinary differential equations and their solutions. By George Moseley Murphy. 1960
Section: Various 27
Problem number: 779.
ODE order: 1.
ODE degree: 2.

CAS Maple gives this as type `[_quadrature]`

$$\boxed{y'^2 + y'a + yb = 0}$$

✓ Solution by Maple

Time used: 0.062 (sec). Leaf size: 245

```
dsolve(diff(y(x),x)^2+a*diff(y(x),x)+b*y(x) = 0,y(x), singsol=all)
```

$$y(x) = -\frac{a^2\left(\text{LambertW}\left(-\frac{2\sqrt{-b}\,\mathrm{e}^{\frac{(c_1-x)b-a}{a}}}{a}\right)+2\right)\text{LambertW}\left(-\frac{2\sqrt{-b}\,\mathrm{e}^{\frac{(c_1-x)b-a}{a}}}{a}\right)}{4b}$$

$$y(x) = -\frac{a^2\left(\text{LambertW}\left(\frac{2\sqrt{-b}\,\mathrm{e}^{\frac{(c_1-x)b-a}{a}}}{a}\right)+2\right)\text{LambertW}\left(\frac{2\sqrt{-b}\,\mathrm{e}^{\frac{(c_1-x)b-a}{a}}}{a}\right)}{4b}$$

$$y(x) = \mathrm{e}^{\frac{-a\,\text{LambertW}\left(\frac{2\,\mathrm{e}^{\frac{(c_1-x)b-a}{a}}}{a\sqrt{-\frac{1}{b}}}\right)-a+(c_1-x)b}{a}}\left(a\sqrt{-\frac{1}{b}}+\mathrm{e}^{\frac{-a\,\text{LambertW}\left(\frac{2\,\mathrm{e}^{\frac{(c_1-x)b-a}{a}}}{a\sqrt{-\frac{1}{b}}}\right)-a+(c_1-x)b}{a}}\right)$$

✓ Solution by Mathematica

Time used: 1.204 (sec). Leaf size: 119

```
DSolve[(y'[x])^2+a y'[x]+b y[x]==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to \text{InverseFunction}\left[-\frac{\sqrt{a^2-4\#1b}+a\log\left(b\left(a-\sqrt{a^2-4\#1b}\right)\right)}{2b}\&\right]\left[\frac{x}{2}+c_1\right]$$

$$y(x) \to \text{InverseFunction}\left[-\frac{\sqrt{a^2-4\#1b}-a\log\left(b\left(\sqrt{a^2-4\#1b}+a\right)\right)}{2b}\&\right]\left[-\frac{x}{2}+c_1\right]$$

$$y(x) \to 0$$

27.14 problem 780

Internal problem ID [4020]

Book: Ordinary differential equations and their solutions. By George Moseley Murphy. 1960
Section: Various 27
Problem number: 780.
ODE order: 1.
ODE degree: 2.

CAS Maple gives this as type `[_quadrature]`

$$y'^2 + xy' = -1$$

✓ Solution by Maple

Time used: 0.062 (sec). Leaf size: 63

```
dsolve(diff(y(x),x)^2+x*diff(y(x),x)+1 = 0,y(x), singsol=all)
```

$$y(x) = -\frac{x^2}{4} - \frac{x\sqrt{x^2-4}}{4} + \ln\left(x + \sqrt{x^2-4}\right) + c_1$$
$$y(x) = \frac{x\sqrt{x^2-4}}{4} - \ln\left(x + \sqrt{x^2-4}\right) - \frac{x^2}{4} + c_1$$

✓ Solution by Mathematica

Time used: 0.049 (sec). Leaf size: 87

```
DSolve[(y'[x])^2+x y'[x]+1==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to -\frac{x^2}{4} - \frac{1}{4}\sqrt{x^2-4}x - \log\left(\sqrt{x^2-4} - x\right) + c_1$$
$$y(x) \to -\frac{x^2}{4} + \frac{1}{4}\sqrt{x^2-4}x + \log\left(\sqrt{x^2-4} - x\right) + c_1$$

27.15 problem 781

Internal problem ID [4021]

Book: Ordinary differential equations and their solutions. By George Moseley Murphy. 1960
Section: Various 27
Problem number: 781.
ODE order: 1.
ODE degree: 2.

CAS Maple gives this as type `[[_1st_order, _with_linear_symmetries], _Clairaut]`

$$\boxed{y'^2 + xy' - y = 0}$$

✓ Solution by Maple

Time used: 0.062 (sec). Leaf size: 17

```
dsolve(diff(y(x),x)^2+x*diff(y(x),x)-y(x) = 0,y(x), singsol=all)
```

$$y(x) = -\frac{x^2}{4}$$
$$y(x) = c_1(c_1 + x)$$

✓ Solution by Mathematica

Time used: 0.009 (sec). Leaf size: 23

```
DSolve[(y'[x])^2+x y'[x]-y[x]==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to c_1(x + c_1)$$
$$y(x) \to -\frac{x^2}{4}$$

27.16 problem 782

Internal problem ID [4022]

Book: Ordinary differential equations and their solutions. By George Moseley Murphy. 1960
Section: Various 27
Problem number: 782.
ODE order: 1.
ODE degree: 2.

CAS Maple gives this as type `[[_1st_order, _with_linear_symmetries], _Clairaut]`

$$y'^2 - xy' + y = 0$$

✓ Solution by Maple

Time used: 0.032 (sec). Leaf size: 19

```
dsolve(diff(y(x),x)^2-x*diff(y(x),x)+y(x) = 0,y(x), singsol=all)
```

$$y(x) = \frac{x^2}{4}$$
$$y(x) = c_1(-c_1 + x)$$

✓ Solution by Mathematica

Time used: 0.009 (sec). Leaf size: 25

```
DSolve[(y'[x])^2-x y'[x]+y[x]==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to c_1(x - c_1)$$
$$y(x) \to \frac{x^2}{4}$$

27.17 problem 783

Internal problem ID [4023]

Book: Ordinary differential equations and their solutions. By George Moseley Murphy. 1960
Section: Various 27
Problem number: 783.
ODE order: 1.
ODE degree: 2.

CAS Maple gives this as type `[[_1st_order, _with_linear_symmetries], _dAlembert]`

$$
\boxed{y'^2 - xy' - y = 0}
$$

✓ Solution by Maple

Time used: 0.047 (sec). Leaf size: 77

```
dsolve(diff(y(x),x)^2-x*diff(y(x),x)-y(x) = 0,y(x), singsol=all)
```

$$
\frac{c_1}{\sqrt{2x - 2\sqrt{x^2 + 4y(x)}}} + \frac{2x}{3} + \frac{\sqrt{x^2 + 4y(x)}}{3} = 0
$$

$$
\frac{c_1}{\sqrt{2x + 2\sqrt{x^2 + 4y(x)}}} + \frac{2x}{3} - \frac{\sqrt{x^2 + 4y(x)}}{3} = 0
$$

✓ Solution by Mathematica

Time used: 60.276 (sec). Leaf size: 1003

```
DSolve[(y'[x])^2-x y'[x]-y[x]==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to \frac{\left(x^2+\sqrt[3]{-x^6+20e^{3c_1}x^3+8\sqrt{e^{3c_1}\left(-x^3+e^{3c_1}\right)^3}+8e^{6c_1}}\right)^2+8e^{3c_1}x}{4\sqrt[3]{-x^6+20e^{3c_1}x^3+8\sqrt{e^{3c_1}\left(-x^3+e^{3c_1}\right)^3}+8e^{6c_1}}}$$

$$y(x) \to \frac{1}{8}\left(4x^2-\frac{i(\sqrt{3}-i)\,x(x^3+8e^{3c_1})}{\sqrt[3]{-x^6+20e^{3c_1}x^3+8\sqrt{e^{3c_1}\left(-x^3+e^{3c_1}\right)^3}+8e^{6c_1}}} + i\left(\sqrt{3}+i\right)\sqrt[3]{-x^6+20e^{3c_1}x^3+8\sqrt{e^{3c_1}\left(-x^3+e^{3c_1}\right)^3}+8e^{6c_1}}\right)$$

$$y(x) \to \frac{1}{8}\left(4x^2+\frac{i(\sqrt{3}+i)\,x(x^3+8e^{3c_1})}{\sqrt[3]{-x^6+20e^{3c_1}x^3+8\sqrt{e^{3c_1}\left(-x^3+e^{3c_1}\right)^3}+8e^{6c_1}}} - \left(1+i\sqrt{3}\right)\sqrt[3]{-x^6+20e^{3c_1}x^3+8\sqrt{e^{3c_1}\left(-x^3+e^{3c_1}\right)^3}+8e^{6c_1}}\right)$$

$$y(x) \to \frac{2\sqrt[3]{2}x^4+2^{2/3}\left(-2x^6-10e^{3c_1}x^3+\sqrt{e^{3c_1}\left(4x^3+e^{3c_1}\right)^3}+e^{6c_1}\right)^{2/3}+4x^2\sqrt[3]{-2x^6-10e^{3c_1}x^3+\sqrt{e^{3c_1}(4x \ldots}}{8\sqrt[3]{-2x^6-10e^{3c_1}x^3+\sqrt{e^{3c_1}\left(4x^3+e^{3c_1}\right)^3}+e^{6c_1}}}$$

$$y(x) \to \frac{1}{16}\left(8x^2+\frac{2\sqrt[3]{2}(1+i\sqrt{3})\,x(-x^3+2e^{3c_1})}{\sqrt[3]{-2x^6-10e^{3c_1}x^3+\sqrt{e^{3c_1}\left(4x^3+e^{3c_1}\right)^3}+e^{6c_1}}} + i2^{2/3}\left(\sqrt{3}+i\right)\sqrt[3]{-2x^6-10e^{3c_1}x^3+\sqrt{e^{3c_1}\left(4x^3+e^{3c_1}\right)^3}+e^{6c_1}}\right)$$

$$y(x) \to \frac{1}{16}\left(8x^2+\frac{2i\sqrt[3]{2}(\sqrt{3}+i)\,x(x^3-2e^{3c_1})}{\sqrt[3]{-2x^6-10e^{3c_1}x^3+\sqrt{e^{3c_1}\left(4x^3+e^{3c_1}\right)^3}+e^{6c_1}}} - 2^{2/3}\left(1+i\sqrt{3}\right)\sqrt[3]{-2x^6-10e^{3c_1}x^3+\sqrt{e^{3c_1}\left(4x^3+e^{3c_1}\right)^3}+e^{6c_1}}\right)$$

27.18 problem 784

Internal problem ID [4024]

Book: Ordinary differential equations and their solutions. By George Moseley Murphy. 1960
Section: Various 27
Problem number: 784.
ODE order: 1.
ODE degree: 2.

CAS Maple gives this as type `[[_1st_order, _with_linear_symmetries], _dAlembert]`

$$\boxed{y'^2 + xy' - y = -x}$$

✓ Solution by Maple

Time used: 0.047 (sec). Leaf size: 36

```
dsolve(diff(y(x),x)^2+x*diff(y(x),x)+x-y(x) = 0,y(x), singsol=all)
```

$$y(x) = x - \frac{x^2}{4} + \operatorname{LambertW}\left(\frac{c_1 \mathrm{e}^{-1+\frac{x}{2}}}{2}\right)^2 + 2\operatorname{LambertW}\left(\frac{c_1 \mathrm{e}^{-1+\frac{x}{2}}}{2}\right) + 1$$

✓ Solution by Mathematica

Time used: 3.196 (sec). Leaf size: 177

```
DSolve[(y'[x])^2+x y'[x]+x -y[x]==0,y[x],x,IncludeSingularSolutions -> True]
```

$$\begin{aligned} &\text{Solve}\Big[-\sqrt{x^2+4y(x)-4x} + 2\log\left(\sqrt{x^2+4y(x)-4x} - x + 2\right) \\ &\quad - 2\log\left(-x\sqrt{x^2+4y(x)-4x} + x^2 + 4y(x) - 2x - 4\right) + x = c_1, y(x)\Big] \\ &\text{Solve}\left[-4\text{arctanh}\left(\frac{(x-5)\sqrt{x^2+4y(x)-4x} - x^2 - 4y(x) + 7x - 6}{(x-3)\sqrt{x^2+4y(x)-4x} - x^2 - 4y(x) + 5x - 2}\right)\right. \\ &\quad \left. + \sqrt{x^2+4y(x)-4x} + x = c_1, y(x)\right] \end{aligned}$$

27.19 problem 785

Internal problem ID [4025]

Book: Ordinary differential equations and their solutions. By George Moseley Murphy. 1960
Section: Various 27
Problem number: 785.
ODE order: 1.
ODE degree: 2.

CAS Maple gives this as type `[[_1st_order, _with_linear_symmetries], _Clairaut]`

$$\boxed{y'^2 + (1-x)\, y' + y = 0}$$

✓ Solution by Maple

Time used: 0.047 (sec). Leaf size: 22

```
dsolve(diff(y(x),x)^2+(1-x)*diff(y(x),x)+y(x) = 0,y(x), singsol=all)
```

$$y(x) = \frac{(x-1)^2}{4}$$
$$y(x) = c_1(-c_1 + x - 1)$$

✓ Solution by Mathematica

Time used: 0.01 (sec). Leaf size: 28

```
DSolve[(y'[x])^2+(1-x)y'[x]+y[x]==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to c_1(x - 1 - c_1)$$
$$y(x) \to \frac{1}{4}(x-1)^2$$

27.20 problem 786

Internal problem ID [4026]

Book: Ordinary differential equations and their solutions. By George Moseley Murphy. 1960
Section: Various 27
Problem number: 786.
ODE order: 1.
ODE degree: 2.

CAS Maple gives this as type `[[_1st_order, _with_linear_symmetries], _Clairaut]`

$$\boxed{y'^2 - (x+1)\,y' + y = 0}$$

✓ Solution by Maple
Time used: 0.047 (sec). Leaf size: 22

```
dsolve(diff(y(x),x)^2-(1+x)*diff(y(x),x)+y(x) = 0,y(x), singsol=all)
```

$$y(x) = \frac{(x+1)^2}{4}$$
$$y(x) = c_1(x+1-c_1)$$

✓ Solution by Mathematica
Time used: 0.01 (sec). Leaf size: 28

```
DSolve[(y'[x])^2-(1+x)y'[x]+y[x]==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to c_1(x+1-c_1)$$
$$y(x) \to \frac{1}{4}(x+1)^2$$

27.21 problem 787

Internal problem ID [4027]

Book: Ordinary differential equations and their solutions. By George Moseley Murphy. 1960
Section: Various 27
Problem number: 787.
ODE order: 1.
ODE degree: 2.

CAS Maple gives this as type `[[_1st_order, _with_linear_symmetries], _Clairaut]`

$$\boxed{y'^2 - (-x+2)\,y' - y = -1}$$

✓ Solution by Maple
Time used: 0.062 (sec). Leaf size: 24

```
dsolve(diff(y(x),x)^2-(2-x)*diff(y(x),x)+1-y(x) = 0,y(x), singsol=all)
```

$$y(x) = x - \frac{1}{4}x^2$$
$$y(x) = 1 + c_1^2 + c_1(-2+x)$$

✓ Solution by Mathematica
Time used: 0.01 (sec). Leaf size: 29

```
DSolve[(y'[x])^2-(2-x)y'[x]+1-y[x]==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to c_1(x-2) + 1 + c_1{}^2$$
$$y(x) \to -\frac{1}{4}(x-4)x$$

27.22 problem 788

Internal problem ID [4028]

Book: Ordinary differential equations and their solutions. By George Moseley Murphy. 1960
Section: Various 27
Problem number: 788.
ODE order: 1.
ODE degree: 2.

CAS Maple gives this as type `[[_1st_order, _with_linear_symmetries], _Clairaut]`

$$y'^2 + (x + a)\, y' - y = 0$$

✓ Solution by Maple

Time used: 0.047 (sec). Leaf size: 20

```
dsolve(diff(y(x),x)^2+(a+x)*diff(y(x),x)-y(x) = 0,y(x), singsol=all)
```

$$y(x) = -\frac{(x + a)^2}{4}$$
$$y(x) = c_1(c_1 + a + x)$$

✓ Solution by Mathematica

Time used: 0.01 (sec). Leaf size: 26

```
DSolve[(y'[x])^2+(a+x)y'[x]-y[x]==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to c_1(a + x + c_1)$$
$$y(x) \to -\frac{1}{4}(a + x)^2$$

27.23 problem 789

Internal problem ID [4029]

Book: Ordinary differential equations and their solutions. By George Moseley Murphy. 1960
Section: Various 27
Problem number: 789.
ODE order: 1.
ODE degree: 2.

CAS Maple gives this as type `[_quadrature]`

$$\boxed{y'^2 - 2xy' = -1}$$

✓ Solution by Maple

Time used: 0.063 (sec). Leaf size: 65

```
dsolve(diff(y(x),x)^2-2*x*diff(y(x),x)+1 = 0,y(x), singsol=all)
```

$$y(x) = \frac{x^2}{2} - \frac{\sqrt{x^2-1}\,x}{2} + \frac{\ln\left(x+\sqrt{x^2-1}\right)}{2} + c_1$$
$$y(x) = \frac{x^2}{2} + \frac{\sqrt{x^2-1}\,x}{2} - \frac{\ln\left(x+\sqrt{x^2-1}\right)}{2} + c_1$$

✓ Solution by Mathematica

Time used: 0.078 (sec). Leaf size: 90

```
DSolve[(y'[x])^2-2 x y'[x]+1==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to \frac{1}{2}\left(-2\text{arctanh}\left(\frac{\sqrt{x^2-1}}{x-1}\right) + x^2 + \sqrt{x^2-1}x + 2c_1\right)$$
$$y(x) \to \text{arctanh}\left(\frac{\sqrt{x^2-1}}{x-1}\right) + \frac{x^2}{2} - \frac{1}{2}\sqrt{x^2-1}x + c_1$$

27.24 problem 790

Internal problem ID [4030]

Book: Ordinary differential equations and their solutions. By George Moseley Murphy. 1960
Section: Various 27
Problem number: 790.
ODE order: 1.
ODE degree: 2.

CAS Maple gives this as type `[_quadrature]`

$$y'^2 + 2xy' = 3x^2$$

✓ Solution by Maple
Time used: 0.0 (sec). Leaf size: 21

```
dsolve(diff(y(x),x)^2+2*x*diff(y(x),x)-3*x^2 = 0,y(x), singsol=all)
```

$$y(x) = \frac{x^2}{2} + c_1$$
$$y(x) = -\frac{3x^2}{2} + c_1$$

✓ Solution by Mathematica
Time used: 0.003 (sec). Leaf size: 29

```
DSolve[(y'[x])^2+2 x y'[x]-3 x^2==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to -\frac{3x^2}{2} + c_1$$
$$y(x) \to \frac{x^2}{2} + c_1$$

27.25 problem 791

Internal problem ID [4031]

Book: Ordinary differential equations and their solutions. By George Moseley Murphy. 1960
Section: Various 27
Problem number: 791.
ODE order: 1.
ODE degree: 2.

CAS Maple gives this as type `[[_1st_order, _with_linear_symmetries], _dAlembert]`

$$\boxed{{y'}^{2}+2xy'-y=0}$$

✓ Solution by Maple

Time used: 0.062 (sec). Leaf size: 650

```
dsolve(diff(y(x),x)^2+2*x*diff(y(x),x)-y(x) = 0,y(x), singsol=all)
```

$$y(x) = \frac{\left(x^{2}-x\left(-x^{3}+2\sqrt{3}\,\sqrt{-c_{1}\left(x^{3}-3c_{1}\right)}+6c_{1}\right)^{\frac{1}{3}}+\left(-x^{3}+2\sqrt{3}\,\sqrt{-c_{1}\left(x^{3}-3c_{1}\right)}+6c_{1}\right)^{\frac{2}{3}}\right)\left(x^{2}+3x\left(-\right.\right.}{4\left(-x^{3}+2\sqrt{3}\,\sqrt{-c_{1}\left(x^{3}-3c_{1}\right)}\right.}$$

$$y(x) = \frac{\left(i\sqrt{3}\left(-x^{3}+2\sqrt{3}\,\sqrt{-c_{1}\left(x^{3}-3c_{1}\right)}+6c_{1}\right)^{\frac{2}{3}}-i\sqrt{3}\,x^{2}+\left(-x^{3}+2\sqrt{3}\,\sqrt{-c_{1}\left(x^{3}-3c_{1}\right)}+6c_{1}\right)^{\frac{2}{3}}+2x\left(\right.\right.}{}$$

$$y(x) = \frac{\left(i\sqrt{3}\,x^{2}-i\sqrt{3}\left(-x^{3}+2\sqrt{3}\,\sqrt{-c_{1}\left(x^{3}-3c_{1}\right)}+6c_{1}\right)^{\frac{2}{3}}+x^{2}+2x\left(-x^{3}+2\sqrt{3}\,\sqrt{-c_{1}\left(x^{3}-3c_{1}\right)}+6c_{1}\right)\right.}{}$$

✓ Solution by Mathematica

Time used: 60.255 (sec). Leaf size: 931

```
DSolve[(y'[x])^2+2 x y'[x]-y[x]==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to \frac{1}{4}\left(-x^2+\frac{x(x^3+8e^{3c_1})}{\sqrt[3]{-x^6+20e^{3c_1}x^3+8\sqrt{e^{3c_1}\left(-x^3+e^{3c_1}\right)^3}+8e^{6c_1}}}+\sqrt[3]{-x^6+20e^{3c_1}x^3+8\sqrt{e^{3c_1}\left(-x^3+e^{3c_1}\right)^3}+8e^{6c_1}}\right)$$

$$y(x) \to \frac{1}{72}\left(-18x^2-\frac{9i(\sqrt{3}-i)\,x(x^3+8e^{3c_1})}{\sqrt[3]{-x^6+20e^{3c_1}x^3+8\sqrt{e^{3c_1}\left(-x^3+e^{3c_1}\right)^3}+8e^{6c_1}}}+9i\left(\sqrt{3}+i\right)\sqrt[3]{-x^6+20e^{3c_1}x^3+8\sqrt{e^{3c_1}\left(-x^3+e^{3c_1}\right)^3}+8e^{6c_1}}\right)$$

$$y(x) \to \frac{1}{72}\left(-18x^2+\frac{9i(\sqrt{3}+i)\,x(x^3+8e^{3c_1})}{\sqrt[3]{-x^6+20e^{3c_1}x^3+8\sqrt{e^{3c_1}\left(-x^3+e^{3c_1}\right)^3}+8e^{6c_1}}}-9\left(1+i\sqrt{3}\right)\sqrt[3]{-x^6+20e^{3c_1}x^3+8\sqrt{e^{3c_1}\left(-x^3+e^{3c_1}\right)^3}+8e^{6c_1}}\right)$$

$$y(x) \to \frac{1}{4}\left(-x^2+\frac{x(x^3-8e^{3c_1})}{\sqrt[3]{-x^6-20e^{3c_1}x^3+8\sqrt{e^{3c_1}\left(x^3+e^{3c_1}\right)^3}+8e^{6c_1}}}+\sqrt[3]{-x^6-20e^{3c_1}x^3+8\sqrt{e^{3c_1}\left(x^3+e^{3c_1}\right)^3}+8e^{6c_1}}\right)$$

$$y(x) \to \frac{1}{72}\left(-18x^2+\frac{9(1+i\sqrt{3})\,x(-x^3+8e^{3c_1})}{\sqrt[3]{-x^6-20e^{3c_1}x^3+8\sqrt{e^{3c_1}\left(x^3+e^{3c_1}\right)^3}+8e^{6c_1}}}+9i\left(\sqrt{3}+i\right)\sqrt[3]{-x^6-20e^{3c_1}x^3+8\sqrt{e^{3c_1}\left(x^3+e^{3c_1}\right)^3}+8e^{6c_1}}\right)$$

$$y(x) \to \frac{1}{72}\left(-18x^2+\frac{9i(\sqrt{3}+i)\,x(x^3-8e^{3c_1})}{\sqrt[3]{-x^6-20e^{3c_1}x^3+8\sqrt{e^{3c_1}\left(x^3+e^{3c_1}\right)^3}+8e^{6c_1}}}-9\left(1+i\sqrt{3}\right)\sqrt[3]{-x^6-20e^{3c_1}x^3+8\sqrt{e^{3c_1}\left(x^3+e^{3c_1}\right)^3}+8e^{6c_1}}\right)$$

27.26 problem 792

Internal problem ID [4032]

Book: Ordinary differential equations and their solutions. By George Moseley Murphy. 1960
Section: Various 27
Problem number: 792.
ODE order: 1.
ODE degree: 2.

CAS Maple gives this as type `[[_1st_order, _with_linear_symmetries], _dAlembert]`

$$y'^2 + 2xy' - y = 0$$

✓ Solution by Maple
Time used: 0.016 (sec). Leaf size: 650

```
dsolve(diff(y(x),x)^2+2*x*diff(y(x),x)-y(x) = 0,y(x), singsol=all)
```

$$y(x) = \frac{\left(x^2 - x\left(-x^3 + 2\sqrt{3}\,\sqrt{-c_1\,(x^3 - 3c_1)} + 6c_1\right)^{\frac{1}{3}} + \left(-x^3 + 2\sqrt{3}\,\sqrt{-c_1\,(x^3 - 3c_1)} + 6c_1\right)^{\frac{2}{3}}\right)\left(x^2 + 3x\left(-\right.\right.}{4\left(-x^3 + 2\sqrt{3}\,\sqrt{-c_1\,(x^3 - 3c_1)}\right.}$$

$$y(x) = \frac{\left(i\sqrt{3}\left(-x^3 + 2\sqrt{3}\,\sqrt{-c_1\,(x^3 - 3c_1)} + 6c_1\right)^{\frac{2}{3}} - i\sqrt{3}\,x^2 + \left(-x^3 + 2\sqrt{3}\,\sqrt{-c_1\,(x^3 - 3c_1)} + 6c_1\right)^{\frac{2}{3}} + 2x\left(\right.\right.}{}$$

$$y(x) = \frac{\left(i\sqrt{3}\,x^2 - i\sqrt{3}\left(-x^3 + 2\sqrt{3}\,\sqrt{-c_1\,(x^3 - 3c_1)} + 6c_1\right)^{\frac{2}{3}} + x^2 + 2x\left(-x^3 + 2\sqrt{3}\,\sqrt{-c_1\,(x^3 - 3c_1)} + 6c_1\right)^{\frac{1}{3}}\right.}{}$$

✓ Solution by Mathematica

Time used: 60.131 (sec). Leaf size: 931

```
DSolve[(y'[x])^2+2 x y'[x]-y[x]==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to \frac{1}{4}\left(-x^2 + \frac{x(x^3+8e^{3c_1})}{\sqrt[3]{-x^6+20e^{3c_1}x^3+8\sqrt{e^{3c_1}\left(-x^3+e^{3c_1}\right)^3}+8e^{6c_1}}} + \sqrt[3]{-x^6+20e^{3c_1}x^3+8\sqrt{e^{3c_1}\left(-x^3+e^{3c_1}\right)^3}+8e^{6c_1}}\right)$$

$$y(x) \to \frac{1}{72}\left(-18x^2 - \frac{9i(\sqrt{3}-i)\,x(x^3+8e^{3c_1})}{\sqrt[3]{-x^6+20e^{3c_1}x^3+8\sqrt{e^{3c_1}\left(-x^3+e^{3c_1}\right)^3}+8e^{6c_1}}} + 9i\left(\sqrt{3}+i\right)\sqrt[3]{-x^6+20e^{3c_1}x^3+8\sqrt{e^{3c_1}\left(-x^3+e^{3c_1}\right)^3}+8e^{6c_1}}\right)$$

$$y(x) \to \frac{1}{72}\left(-18x^2 + \frac{9i(\sqrt{3}+i)\,x(x^3+8e^{3c_1})}{\sqrt[3]{-x^6+20e^{3c_1}x^3+8\sqrt{e^{3c_1}\left(-x^3+e^{3c_1}\right)^3}+8e^{6c_1}}} - 9\left(1+i\sqrt{3}\right)\sqrt[3]{-x^6+20e^{3c_1}x^3+8\sqrt{e^{3c_1}\left(-x^3+e^{3c_1}\right)^3}+8e^{6c_1}}\right)$$

$$y(x) \to \frac{1}{4}\left(-x^2 + \frac{x(x^3-8e^{3c_1})}{\sqrt[3]{-x^6-20e^{3c_1}x^3+8\sqrt{e^{3c_1}\left(x^3+e^{3c_1}\right)^3}+8e^{6c_1}}} + \sqrt[3]{-x^6-20e^{3c_1}x^3+8\sqrt{e^{3c_1}\left(x^3+e^{3c_1}\right)^3}+8e^{6c_1}}\right)$$

$$y(x) \to \frac{1}{72}\left(-18x^2 + \frac{9(1+i\sqrt{3})\,x(-x^3+8e^{3c_1})}{\sqrt[3]{-x^6-20e^{3c_1}x^3+8\sqrt{e^{3c_1}\left(x^3+e^{3c_1}\right)^3}+8e^{6c_1}}} + 9i\left(\sqrt{3}+i\right)\sqrt[3]{-x^6-20e^{3c_1}x^3+8\sqrt{e^{3c_1}\left(x^3+e^{3c_1}\right)^3}+8e^{6c_1}}\right)$$

$$y(x) \to \frac{1}{72}\left(-18x^2 + \frac{9i(\sqrt{3}+i)\,x(x^3-8e^{3c_1})}{\sqrt[3]{-x^6-20e^{3c_1}x^3+8\sqrt{e^{3c_1}\left(x^3+e^{3c_1}\right)^3}+8e^{6c_1}}} - 9\left(1+i\sqrt{3}\right)\sqrt[3]{-x^6-20e^{3c_1}x^3+8\sqrt{e^{3c_1}\left(x^3+e^{3c_1}\right)^3}+8e^{6c_1}}\right)$$

27.27 problem 793

Internal problem ID [4033]

Book: Ordinary differential equations and their solutions. By George Moseley Murphy. 1960
Section: Various 27
Problem number: 793.
ODE order: 1.
ODE degree: 2.

CAS Maple gives this as type `[[_1st_order, _with_linear_symmetries], _Clairaut]`

$$\boxed{y'^2 - 2xy' + 2y = 0}$$

✓ Solution by Maple
Time used: 0.062 (sec). Leaf size: 20

```
dsolve(diff(y(x),x)^2-2*x*diff(y(x),x)+2*y(x) = 0,y(x), singsol=all)
```

$$y(x) = \frac{x^2}{2}$$
$$y(x) = -\frac{c_1(-2x + c_1)}{2}$$

✓ Solution by Mathematica
Time used: 0.01 (sec). Leaf size: 29

```
DSolve[(y'[x])^2-2 x y'[x]+2 y[x]==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to c_1 x - \frac{c_1{}^2}{2}$$
$$y(x) \to \frac{x^2}{2}$$

27.28 problem 794

Internal problem ID [4034]

Book: Ordinary differential equations and their solutions. By George Moseley Murphy. 1960
Section: Various 27
Problem number: 794.
ODE order: 1.
ODE degree: 2.

CAS Maple gives this as type `[_quadrature]`

$$y'^2 - (1+2x)\, y' = x(1-x)$$

✓ Solution by Maple

Time used: 0.063 (sec). Leaf size: 50

```
dsolve(diff(y(x),x)^2-(1+2*x)*diff(y(x),x)-x*(1-x) = 0,y(x), singsol=all)
```

$$y(x) = \frac{(-8x-1)\sqrt{8x+1}}{24} + \frac{x^2}{2} + \frac{x}{2} + c_1$$
$$y(x) = \frac{x}{2} + \frac{(8x+1)^{\frac{3}{2}}}{24} + \frac{x^2}{2} + c_1$$

✓ Solution by Mathematica

Time used: 0.014 (sec). Leaf size: 62

```
DSolve[(y'[x])^2-(1+2*x)*y'[x]-x*(1-x)==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to \frac{x^2}{2} + \frac{x}{2} - \frac{1}{24}(8x+1)^{3/2} + c_1$$
$$y(x) \to \frac{1}{2}\left(x^2 + x + \frac{1}{12}(8x+1)^{3/2}\right) + c_1$$

27.29 problem 795

Internal problem ID [4035]

Book: Ordinary differential equations and their solutions. By George Moseley Murphy. 1960
Section: Various 27
Problem number: 795.
ODE order: 1.
ODE degree: 2.

CAS Maple gives this as type `[[_1st_order, _with_linear_symmetries], _dAlembert]`

$$\boxed{y'^2 + 2(1-x)\,y' + 2y = 2x}$$

✓ Solution by Maple

Time used: 0.062 (sec). Leaf size: 32

```
dsolve(diff(y(x),x)^2+2*(1-x)*diff(y(x),x)-2*x+2*y(x) = 0,y(x), singsol=all)
```

$$y(x) = \frac{x^2}{2} - \frac{\mathrm{LambertW}\left(-\mathrm{e}^{-x} c_1\right)^2}{2} - \mathrm{LambertW}\left(-\mathrm{e}^{-x} c_1\right)$$

✓ Solution by Mathematica

Time used: 1.796 (sec). Leaf size: 171

```
DSolve[(y'[x])^2+2(1-x)y'[x]-2(x-y[x])==0,y[x],x,IncludeSingularSolutions -> True]
```

$$\text{Solve}\left[2\text{arctanh}\left(\frac{(x-2)\sqrt{x^2-2y(x)+1}-x^2+2y(x)+2x-1}{x\sqrt{x^2-2y(x)+1}-x^2+2y(x)-1}\right) - \sqrt{x^2-2y(x)+1}+x = c_1, y(x)\right]$$

$$\text{Solve}\left[2\text{arctanh}\left(\frac{x\sqrt{x^2-2y(x)+1}-x^2+2y(x)-1}{(x+2)\sqrt{x^2-2y(x)+1}-x^2+2y(x)-2x-1}\right) + \sqrt{x^2-2y(x)+1}+x = c_1, y(x)\right]$$

27.30 problem 796

Internal problem ID [4036]

Book: Ordinary differential equations and their solutions. By George Moseley Murphy. 1960
Section: Various 27
Problem number: 796.
ODE order: 1.
ODE degree: 2.

CAS Maple gives this as type `[[_1st_order, _with_linear_symmetries], _dAlembert]`

$$\boxed{y'^2 + 3xy' - y = 0}$$

✓ Solution by Maple

Time used: 0.063 (sec). Leaf size: 85

```
dsolve(diff(y(x),x)^2+3*x*diff(y(x),x)-y(x) = 0,y(x), singsol=all)
```

$$\frac{c_1}{\left(-6x - 2\sqrt{9x^2 + 4y(x)}\right)^{\frac{3}{2}}} + \frac{2x}{5} - \frac{\sqrt{9x^2 + 4y(x)}}{5} = 0$$

$$\frac{c_1}{\left(-6x + 2\sqrt{9x^2 + 4y(x)}\right)^{\frac{3}{2}}} + \frac{2x}{5} + \frac{\sqrt{9x^2 + 4y(x)}}{5} = 0$$

✓ Solution by Mathematica

Time used: 21.387 (sec). Leaf size: 776

```
DSolve[(y'[x])^2+3 x y'[x]-y[x]==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to \text{Root}\left[16\#1^5 + 40\#1^4x^2 + 25\#1^3x^4 + 160\#1^2e^{5c_1}x + 360\#1e^{5c_1}x^3 + 216e^{5c_1}x^5 - 64e^{10c_1}\&, 1\right]$$

$$y(x) \to \text{Root}\left[16\#1^5 + 40\#1^4x^2 + 25\#1^3x^4 + 160\#1^2e^{5c_1}x + 360\#1e^{5c_1}x^3 + 216e^{5c_1}x^5 - 64e^{10c_1}\&, 2\right]$$

$$y(x) \to \text{Root}\left[16\#1^5 + 40\#1^4x^2 + 25\#1^3x^4 + 160\#1^2e^{5c_1}x + 360\#1e^{5c_1}x^3 + 216e^{5c_1}x^5 - 64e^{10c_1}\&, 3\right]$$

$$y(x) \to \text{Root}\left[16\#1^5 + 40\#1^4x^2 + 25\#1^3x^4 + 160\#1^2e^{5c_1}x + 360\#1e^{5c_1}x^3 + 216e^{5c_1}x^5 - 64e^{10c_1}\&, 4\right]$$

$$y(x) \to \text{Root}\left[16\#1^5 + 40\#1^4x^2 + 25\#1^3x^4 + 160\#1^2e^{5c_1}x + 360\#1e^{5c_1}x^3 + 216e^{5c_1}x^5 - 64e^{10c_1}\&, 5\right]$$

$$y(x) \to \text{Root}\left[1024\#1^5 + 2560\#1^4x^2 + 1600\#1^3x^4 - 160\#1^2e^{5c_1}x - 360\#1e^{5c_1}x^3 - 216e^{5c_1}x^5 - e^{10c_1}\&, 1\right]$$

$$y(x) \to \text{Root}\left[1024\#1^5 + 2560\#1^4x^2 + 1600\#1^3x^4 - 160\#1^2e^{5c_1}x - 360\#1e^{5c_1}x^3 - 216e^{5c_1}x^5 - e^{10c_1}\&, 2\right]$$

$$y(x) \to \text{Root}\left[1024\#1^5 + 2560\#1^4x^2 + 1600\#1^3x^4 - 160\#1^2e^{5c_1}x - 360\#1e^{5c_1}x^3 - 216e^{5c_1}x^5 - e^{10c_1}\&, 3\right]$$

$$y(x) \to \text{Root}\left[1024\#1^5 + 2560\#1^4x^2 + 1600\#1^3x^4 - 160\#1^2e^{5c_1}x - 360\#1e^{5c_1}x^3 - 216e^{5c_1}x^5 - e^{10c_1}\&, 4\right]$$

$$y(x) \to \text{Root}\left[1024\#1^5 + 2560\#1^4x^2 + 1600\#1^3x^4 - 160\#1^2e^{5c_1}x - 360\#1e^{5c_1}x^3 - 216e^{5c_1}x^5 - e^{10c_1}\&, 5\right]$$

$$y(x) \to 0$$

27.31 problem 797

Internal problem ID [4037]

Book: Ordinary differential equations and their solutions. By George Moseley Murphy. 1960
Section: Various 27
Problem number: 797.
ODE order: 1.
ODE degree: 2.

CAS Maple gives this as type `[[_1st_order, _with_linear_symmetries], _Clairaut]`

$$y'^2 - 4(x+1)\,y' + 4y = 0$$

✓ Solution by Maple
Time used: 0.062 (sec). Leaf size: 21

```
dsolve(diff(y(x),x)^2-4*(1+x)*diff(y(x),x)+4*y(x) = 0,y(x), singsol=all)
```

$$y(x) = (x+1)^2$$
$$y(x) = -\frac{c_1(-4x + c_1 - 4)}{4}$$

✓ Solution by Mathematica
Time used: 0.012 (sec). Leaf size: 27

```
DSolve[(y'[x])^2-4(1+x)y'[x]+4 y[x]==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to -\frac{1}{4}c_1(-4x - 4 + c_1)$$
$$y(x) \to (x+1)^2$$

28 Various 28

28.1 problem 798

Internal problem ID [4038]

Book: Ordinary differential equations and their solutions. By George Moseley Murphy. 1960
Section: Various 28
Problem number: 798.
ODE order: 1.
ODE degree: 2.

CAS Maple gives this as type `[_quadrature]`

$$
\boxed{y'^2 + axy' = bc\,x^2}
$$

✓ Solution by Maple

Time used: 0.063 (sec). Leaf size: 53

```
dsolve(diff(y(x),x)^2+a*x*diff(y(x),x) = b*c*x^2,y(x), singsol=all)
```

$$
y(x) = \frac{x^2\sqrt{a^2+4bc}}{4} - \frac{a\,x^2}{4} + c_1
$$

$$
y(x) = -\frac{x^2\sqrt{a^2+4bc}}{4} - \frac{a\,x^2}{4} + c_1
$$

✓ Solution by Mathematica

Time used: 0.013 (sec). Leaf size: 59

```
DSolve[(y'[x])^2+a x y'[x]==b c x^2,y[x],x,IncludeSingularSolutions -> True]
```

$$
y(x) \to \frac{1}{4}x^2\left(\sqrt{a^2+4bc}-a\right) + c_1
$$

$$
y(x) \to -\frac{1}{4}x^2\left(\sqrt{a^2+4bc}+a\right) + c_1
$$

28.2 problem 799

Internal problem ID [4039]

Book: Ordinary differential equations and their solutions. By George Moseley Murphy. 1960
Section: Various 28
Problem number: 799.
ODE order: 1.
ODE degree: 2.

CAS Maple gives this as type `[[_1st_order, _with_linear_symmetries], _Clairaut]`

$$\boxed{y'^2 - axy' + ya = 0}$$

✓ Solution by Maple

Time used: 0.078 (sec). Leaf size: 25

```
dsolve(diff(y(x),x)^2-a*x*diff(y(x),x)+a*y(x) = 0,y(x), singsol=all)
```

$$y(x) = \frac{a\,x^2}{4}$$
$$y(x) = \frac{c_1(ax - c_1)}{a}$$

✓ Solution by Mathematica

Time used: 0.015 (sec). Leaf size: 29

```
DSolve[(y'[x])^2-a x y'[x]+a y[x]==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to c_1\left(x - \frac{c_1}{a}\right)$$
$$y(x) \to \frac{ax^2}{4}$$

28.3 problem 800

Internal problem ID [4040]

Book: Ordinary differential equations and their solutions. By George Moseley Murphy. 1960
Section: Various 28
Problem number: 800.
ODE order: 1.
ODE degree: 2.

CAS Maple gives this as type `[[_homogeneous, ‘class G‘]]`

$$\boxed{y'^2 + axy' + cy = -b\,x^2}$$

✗ Solution by Maple

```
dsolve(diff(y(x),x)^2+a*x*diff(y(x),x)+b*x^2+c*y(x) = 0,y(x), singsol=all)
```

No solution found

✓ Solution by Mathematica

Time used: 2.892 (sec). Leaf size: 1085

```
DSolve[(y'[x])^2+a x y'[x]+b x^2+c y[x]==0,y[x],x,IncludeSingularSolutions -> True]
```

$$\text{Solve}\left[\text{RootSum}\left[\#1^4-2\#1^3c-2\#1^2a^2-4\#1^2ac+8\#1^2b-2\#1a^2c+8\#1bc+a^4-8a^2b+16b^2\&,\frac{-\#1^3\log\left(\#1x-\sqrt{x^2\left(a^2-4b\right)-4cy(x)}+2\sqrt{-cy(x)}\right)+\#1^3\log(x)+\#1^2c\log\left(\#1x-\sqrt{x^2\right.}}{}\right.\right.$$

$$-\log\left(\sqrt{-cy(x)}\sqrt{x^2\left(a^2-4b\right)-4cy(x)}+2cy(x)\right)+\frac{1}{2}\log(y(x))+2\log(x)=c_1,y(x)\Big]$$

$$\text{Solve}\left[\text{RootSum}\left[\#1^4+2\#1^3c-2\#1^2a^2-4\#1^2ac+8\#1^2b+2\#1a^2c-8\#1bc+a^4-8a^2b+16b^2\&,\frac{\#1^3\log\left(\#1x-\sqrt{x^2\left(a^2-4b\right)-4cy(x)}+2\sqrt{-cy(x)}\right)+\#1^3(-\log(x))+\#1^2c\log\left(\#1x-\sqrt{x\right.}}{}\right.\right.$$

$$-\log\left(\sqrt{-cy(x)}\sqrt{x^2\left(a^2-4b\right)-4cy(x)}+2cy(x)\right)+\frac{1}{2}\log(y(x))+2\log(x)=c_1,y(x)\Big]$$

28.4 problem 801

Internal problem ID [4041]

Book: Ordinary differential equations and their solutions. By George Moseley Murphy. 1960
Section: Various 28
Problem number: 801.
ODE order: 1.
ODE degree: 2.

CAS Maple gives this as type `[[_1st_order, _with_linear_symmetries], _Clairaut]`

$$\boxed{{y'}^{2}+(bx+a)\,y'-yb=-c}$$

✓ Solution by Maple
Time used: 0.047 (sec). Leaf size: 50

```
dsolve(diff(y(x),x)^2+(b*x+a)*diff(y(x),x)+c = b*y(x),y(x), singsol=all)
```

$$y(x)=\frac{-b^{2}x^{2}-2bxa-a^{2}+4c}{4b}$$
$$y(x)=\frac{c_{1}^{2}+(bx+a)\,c_{1}+c}{b}$$

✓ Solution by Mathematica
Time used: 0.016 (sec). Leaf size: 51

```
DSolve[(y'[x])^2+(a+b x)y'[x]+c==b y[x],y[x],x,IncludeSingularSolutions -> True]
```

$$y(x)\to\frac{c+c_1(a+bx+c_1)}{b}$$
$$y(x)\to-\frac{a^2+2abx+b^2x^2-4c}{4b}$$

28.5 problem 802

Internal problem ID [4042]

Book: Ordinary differential equations and their solutions. By George Moseley Murphy. 1960
Section: Various 28
Problem number: 802.
ODE order: 1.
ODE degree: 2.

CAS Maple gives this as type `[_quadrature]`

$$\boxed{y'^2 - 2y'x^2 + 2xy' = 0}$$

✓ Solution by Maple
Time used: 0.0 (sec). Leaf size: 20

```
dsolve(diff(y(x),x)^2-2*x^2*diff(y(x),x)+2*x*diff(y(x),x) = 0,y(x), singsol=all)
```

$$y(x) = \frac{2}{3}x^3 - x^2 + c_1$$
$$y(x) = c_1$$

✓ Solution by Mathematica
Time used: 0.006 (sec). Leaf size: 26

```
DSolve[(y'[x])^2-2 x^2 y'[x]+2 x y'[x]==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to c_1$$
$$y(x) \to \frac{2x^3}{3} - x^2 + c_1$$

28.6 problem 804

Internal problem ID [4043]

Book: Ordinary differential equations and their solutions. By George Moseley Murphy. 1960
Section: Various 28
Problem number: 804.
ODE order: 1.
ODE degree: 2.

CAS Maple gives this as type `[[_1st_order, _with_linear_symmetries]]`

$$
\boxed{y'^2 + a\,x^3 y' - 2ya\,x^2 = 0}
$$

✓ Solution by Maple

Time used: 0.454 (sec). Leaf size: 27

```
dsolve(diff(y(x),x)^2+a*x^3*diff(y(x),x)-2*a*x^2*y(x) = 0,y(x), singsol=all)
```

$$
\begin{aligned}
y(x) &= -\frac{a\,x^4}{8} \\
y(x) &= \frac{c_1(a\,x^2 + 2c_1)}{a}
\end{aligned}
$$

✓ Solution by Mathematica

Time used: 1.03 (sec). Leaf size: 78

```
DSolve[(y'[x])^2+a x^3 y'[x]-2 a x^2 y[x]==0,y[x],x,IncludeSingularSolutions -> True]
```

$$
\begin{aligned}
y(x) &\to \frac{1}{8}e^{2c_1}\left(-2\sqrt{a}x^2 + e^{2c_1}\right) \\
y(x) &\to 2\sqrt{a}e^{2c_1}x^2 + 8e^{4c_1} \\
y(x) &\to 0 \\
y(x) &\to -\frac{ax^4}{8}
\end{aligned}
$$

28.7 problem 805

Internal problem ID [4044]

Book: Ordinary differential equations and their solutions. By George Moseley Murphy. 1960
Section: Various 28
Problem number: 805.
ODE order: 1.
ODE degree: 2.

CAS Maple gives this as type `[[_1st_order, _with_linear_symmetries]]`

$$\boxed{y'^2 - 2a\,x^3 y' + 4ya\,x^2 = 0}$$

✓ Solution by Maple

Time used: 0.454 (sec). Leaf size: 27

```
dsolve(diff(y(x),x)^2-2*a*x^3*diff(y(x),x)+4*a*x^2*y(x) = 0,y(x), singsol=all)
```

$$y(x) = \frac{a\,x^4}{4}$$
$$y(x) = \frac{c_1(a\,x^2 - c_1)}{a}$$

✓ Solution by Mathematica

Time used: 6.101 (sec). Leaf size: 262

```
DSolve[(y'[x])^2-2 a x^3 y'[x]+4 a x^2 y[x]==0,y[x],x,IncludeSingularSolutions -> True]
```

$$\text{Solve}\left[\frac{1}{4}\left(\frac{\sqrt{a}x\sqrt{ax^4-4y(x)}}{\sqrt{ax^2\left(ax^4-4y(x)\right)}}+1\right)\log(y(x))-\frac{\sqrt{a}x\sqrt{ax^4-4y(x)}\log\left(\sqrt{ax^4-4y(x)}+\sqrt{a}x^2\right)}{2\sqrt{ax^2\left(ax^4-4y(x)\right)}}=c_1,y(x)\right]$$

$$\text{Solve}\left[\frac{\sqrt{a}x\sqrt{ax^4-4y(x)}\log\left(\sqrt{ax^4-4y(x)}+\sqrt{a}x^2\right)}{2\sqrt{ax^2\left(ax^4-4y(x)\right)}}+\frac{1}{4}\left(1-\frac{\sqrt{a}x\sqrt{ax^4-4y(x)}}{\sqrt{ax^2\left(ax^4-4y(x)\right)}}\right)\log(y(x))=c_1,y(x)\right]$$

$$y(x)\to\frac{ax^4}{4}$$

28.8 problem 806

Internal problem ID [4045]

Book: Ordinary differential equations and their solutions. By George Moseley Murphy. 1960
Section: Various 28
Problem number: 806.
ODE order: 1.
ODE degree: 2.

CAS Maple gives this as type `[[_1st_order, _with_linear_symmetries]]`

$$y'^2 + 4x^5y' - 12yx^4 = 0$$

✓ Solution by Maple
Time used: 0.313 (sec). Leaf size: 23

```
dsolve(diff(y(x),x)^2+4*x^5*diff(y(x),x)-12*x^4*y(x) = 0,y(x), singsol=all)
```

$$y(x) = -\frac{x^6}{3}$$
$$y(x) = c_1x^3 + \frac{3}{4}c_1^2$$

✓ Solution by Mathematica

Time used: 2.23 (sec). Leaf size: 217

```
DSolve[(y'[x])^2+4 x^5 y'[x]-12 x^4 y[x]==0,y[x],x,IncludeSingularSolutions -> True]
```

$$\text{Solve}\left[\frac{1}{6}\left(\log(y(x))-\frac{x^2\sqrt{x^6+3y(x)}\log(y(x))}{\sqrt{x^4(x^6+3y(x))}}\right)+\frac{x^2\sqrt{x^6+3y(x)}\log\left(\sqrt{x^6+3y(x)}+x^3\right)}{3\sqrt{x^4(x^6+3y(x))}}=c_1,y(x)\right]$$

$$\text{Solve}\left[\frac{1}{6}\left(\frac{x^2\sqrt{x^6+3y(x)}\log(y(x))}{\sqrt{x^4(x^6+3y(x))}}+\log(y(x))\right)-\frac{x^2\sqrt{x^6+3y(x)}\log\left(\sqrt{x^6+3y(x)}+x^3\right)}{3\sqrt{x^4(x^6+3y(x))}}=c_1,y(x)\right]$$

$$y(x)\to -\frac{x^6}{3}$$

28.9 problem 807

Internal problem ID [4046]

Book: Ordinary differential equations and their solutions. By George Moseley Murphy. 1960
Section: Various 28
Problem number: 807.
ODE order: 1.
ODE degree: 2.

CAS Maple gives this as type `[_quadrature]`

$$y'^2 - 2y' \cosh(x) = -1$$

✓ Solution by Maple
Time used: 0.078 (sec). Leaf size: 19

```
dsolve(diff(y(x),x)^2-2*diff(y(x),x)*cosh(x)+1 = 0,y(x), singsol=all)
```

$$y(x) = -e^{-x} + c_1$$
$$y(x) = e^{x} + c_1$$

✓ Solution by Mathematica
Time used: 0.054 (sec). Leaf size: 25

```
DSolve[(y'[x])^2-2 y'[x] Cosh[x]+1==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to \sinh(x) - \cosh(x) + c_1$$
$$y(x) \to \sinh(x) + \cosh(x) + c_1$$

28.10 problem 808

Internal problem ID [4047]

Book: Ordinary differential equations and their solutions. By George Moseley Murphy. 1960
Section: Various 28
Problem number: 808.
ODE order: 1.
ODE degree: 2.

CAS Maple gives this as type `[_quadrature]`

$$y'^2 + yy' - x(y + x) = 0$$

✓ Solution by Maple
Time used: 0.0 (sec). Leaf size: 25

```
dsolve(diff(y(x),x)^2+y(x)*diff(y(x),x) = x*(x+y(x)),y(x), singsol=all)
```

$$y(x) = \frac{x^2}{2} + c_1$$
$$y(x) = 1 + \mathrm{e}^{-x} c_1 - x$$

✓ Solution by Mathematica
Time used: 0.057 (sec). Leaf size: 32

```
DSolve[(y'[x])^2+y[x]*y'[x]==x*(x+y[x]),y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to \frac{x^2}{2} + c_1$$
$$y(x) \to -x + c_1 e^{-x} + 1$$

28.11 problem 809

Internal problem ID [4048]

Book: Ordinary differential equations and their solutions. By George Moseley Murphy. 1960
Section: Various 28
Problem number: 809.
ODE order: 1.
ODE degree: 2.

CAS Maple gives this as type `[[_1st_order, _with_linear_symmetries]]`

$$\boxed{y'^2 - yy' = -\mathrm{e}^x}$$

✓ Solution by Maple

Time used: 0.609 (sec). Leaf size: 34

```
dsolve(diff(y(x),x)^2-y(x)*diff(y(x),x)+exp(x) = 0,y(x), singsol=all)
```

$$y(x) = -2\,\mathrm{e}^{\frac{x}{2}}$$
$$y(x) = 2\,\mathrm{e}^{\frac{x}{2}}$$
$$y(x) = \frac{c_1^2\mathrm{e}^x + 1}{c_1}$$

✓ Solution by Mathematica

Time used: 60.35 (sec). Leaf size: 59

```
DSolve[(y'[x])^2-y[x] y'[x]+Exp[x]==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to -\sqrt{-e^{-c_1}\left(-e^x + e^{c_1}\right)^2}$$
$$y(x) \to \sqrt{-e^{-c_1}\left(e^x - e^{c_1}\right)^2}$$

28.12 problem 810

Internal problem ID [4049]

Book: Ordinary differential equations and their solutions. By George Moseley Murphy. 1960
Section: Various 28
Problem number: 810.
ODE order: 1.
ODE degree: 2.

CAS Maple gives this as type `[_quadrature]`

$$\boxed{{y'}^2 + (y + x)\, y' + yx = 0}$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 20

```
dsolve(diff(y(x),x)^2+(x+y(x))*diff(y(x),x)+x*y(x) = 0,y(x), singsol=all)
```

$$y(x) = -\frac{x^2}{2} + c_1$$
$$y(x) = e^{-x} c_1$$

✓ Solution by Mathematica

Time used: 0.037 (sec). Leaf size: 32

```
DSolve[(y'[x])^2+(x+y[x])y'[x]+x y[x]==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to c_1 e^{-x}$$
$$y(x) \to -\frac{x^2}{2} + c_1$$
$$y(x) \to 0$$

28.13 problem 811

Internal problem ID [4050]

Book: Ordinary differential equations and their solutions. By George Moseley Murphy. 1960
Section: Various 28
Problem number: 811.
ODE order: 1.
ODE degree: 2.

CAS Maple gives this as type `[_dAlembert]`

$$\boxed{y'^2 - 2yy' = 2x}$$

✓ Solution by Maple
Time used: 0.063 (sec). Leaf size: 223

```
dsolve(diff(y(x),x)^2-2*y(x)*diff(y(x),x)-2*x = 0,y(x), singsol=all)
```

$$\frac{\frac{\left(y(x)-\sqrt{y(x)^2+2x}\right)\operatorname{arcsinh}\left(-y(x)+\sqrt{y(x)^2+2x}\right)}{2}+x\sqrt{2y\left(x\right)^2+2x-2y\left(x\right)\sqrt{y\left(x\right)^2+2x}+1}-2c_1y(x)+2c_1\sqrt{y}}{\sqrt{2y\left(x\right)^2+2x-2y\left(x\right)\sqrt{y\left(x\right)^2+2x}+1}}$$
$$=0$$

$$\frac{\frac{\left(-y(x)-\sqrt{y(x)^2+2x}\right)\operatorname{arcsinh}\left(y(x)+\sqrt{y(x)^2+2x}\right)}{2}+x\sqrt{2y\left(x\right)^2+2x+2y\left(x\right)\sqrt{y\left(x\right)^2+2x}+1}+2c_1y(x)+2c_1\sqrt{y}}{\sqrt{2y\left(x\right)^2+2x+2y\left(x\right)\sqrt{y\left(x\right)^2+2x}+1}}$$
$$=0$$

✓ Solution by Mathematica

Time used: 0.995 (sec). Leaf size: 74

```
DSolve[(y'[x])^2-2 y[x] y'[x]-2 x==0,y[x],x,IncludeSingularSolutions -> True]
```

$$\text{Solve}\left[\left\{x=-\frac{K[1]\log\left(\sqrt{K[1]^2+1}-K[1]\right)}{2\sqrt{K[1]^2+1}}+\frac{c_1K[1]}{\sqrt{K[1]^2+1}},y(x)=\frac{K[1]}{2}-\frac{x}{K[1]}\right\},\{y(x),K[1]\}\right]$$

28.14 problem 812

Internal problem ID [4051]

Book: Ordinary differential equations and their solutions. By George Moseley Murphy. 1960
Section: Various 28
Problem number: 812.
ODE order: 1.
ODE degree: 2.

CAS Maple gives this as type `[_quadrature]`

$$y'^2 + (2y+1)\, y' + y(y-1) = 0$$

✓ Solution by Maple

Time used: 0.047 (sec). Leaf size: 143

```
dsolve(diff(y(x),x)^2+(1+2*y(x))*diff(y(x),x)+y(x)*(y(x)-1) = 0,y(x), singsol=all)
```

$$x + \frac{3\ln(y(x)-1)}{2} - \frac{\ln(y(x))}{2} - \frac{3\ln\left(\sqrt{8y(x)+1}-3\right)}{2} - \frac{\ln\left(\sqrt{8y(x)+1}+1\right)}{2} + \frac{3\ln\left(\sqrt{8y(x)+1}+3\right)}{2} + \frac{\ln\left(\sqrt{8y(x)+1}-1\right)}{2} - c_1 = 0$$

$$x + \frac{3\ln(y(x)-1)}{2} - \frac{\ln(y(x))}{2} + \frac{3\ln\left(\sqrt{8y(x)+1}-3\right)}{2} + \frac{\ln\left(\sqrt{8y(x)+1}+1\right)}{2} - \frac{3\ln\left(\sqrt{8y(x)+1}+3\right)}{2} - \frac{\ln\left(\sqrt{8y(x)+1}-1\right)}{2} - c_1 = 0$$

✓ Solution by Mathematica

Time used: 60.121 (sec). Leaf size: 1373

```
DSolve[(y'[x])^2+(1+2 y[x])y'[x]+y[x](y[x]-1)==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to \frac{e^{-2x}\left(128e^{x}(12e^{x}+e^{2c_1})+64\sqrt[3]{24\sqrt{3}\sqrt{-e^{7x+4c_1}\left(-27e^{x}+e^{2c_1}\right)^3}+540e^{4(x+c_1)}+5832e^{5x+2c_1}-e^{3x+6c_1}}\right.}{1536}$$

$$y(x) \to \frac{e^{-2x}\left(256e^{x}(12e^{x}+e^{2c_1})+64i\left(\sqrt{3}+i\right)\sqrt[3]{24\sqrt{3}\sqrt{-e^{7x+4c_1}\left(-27e^{x}+e^{2c_1}\right)^3}+540e^{4(x+c_1)}+5832e^{5x+2c_1}}\right.}{3072}$$

$$y(x) \to \frac{e^{-2x}\left(256e^{x}(12e^{x}+e^{2c_1})-64\left(1+i\sqrt{3}\right)\sqrt[3]{24\sqrt{3}\sqrt{-e^{7x+4c_1}\left(-27e^{x}+e^{2c_1}\right)^3}+540e^{4(x+c_1)}+5832e^{5x+2}}\right.}{3072}$$

$$y(x) \to \frac{e^{-2(x+2c_1)}\left(128e^{x+2c_1}(1+12e^{x+2c_1})+64\sqrt[3]{24\sqrt{3}\sqrt{e^{7(x+2c_1)}\left(-1+27e^{x+2c_1}\right)^3}+5832e^{5(x+2c_1)}-e^{3x+6c_1}}\right.}{1536}$$

$$y(x) \to \frac{e^{-2(x+2c_1)}\left(256e^{x+2c_1}(1+12e^{x+2c_1})+64i\left(\sqrt{3}+i\right)\sqrt[3]{24\sqrt{3}\sqrt{e^{7(x+2c_1)}\left(-1+27e^{x+2c_1}\right)^3}+5832e^{5(x+2c_1)}}\right.}{30}$$

$$y(x) \to \frac{e^{-2(x+2c_1)}\left(256e^{x+2c_1}(1+12e^{x+2c_1})-64\left(1+i\sqrt{3}\right)\sqrt[3]{24\sqrt{3}\sqrt{e^{7(x+2c_1)}\left(-1+27e^{x+2c_1}\right)^3}+5832e^{5(x+2c_1)}}\right.}{30}$$

28.15 problem 813

Internal problem ID [4052]

Book: Ordinary differential equations and their solutions. By George Moseley Murphy. 1960
Section: Various 28
Problem number: 813.
ODE order: 1.
ODE degree: 2.

CAS Maple gives this as type `[_quadrature]`

$$\boxed{y'^2 - 2(-y + x)\, y' - 4yx = 0}$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 18

```
dsolve(diff(y(x),x)^2-2*(x-y(x))*diff(y(x),x)-4*x*y(x) = 0,y(x), singsol=all)
```

$$y(x) = x^2 + c_1$$
$$y(x) = \mathrm{e}^{-2x} c_1$$

✓ Solution by Mathematica

Time used: 0.037 (sec). Leaf size: 28

```
DSolve[(y'[x])^2-2(x-y[x])y'[x]-4 x y[x]==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to c_1 e^{-2x}$$
$$y(x) \to x^2 + c_1$$
$$y(x) \to 0$$

28.16 problem 814

Internal problem ID [4053]

Book: Ordinary differential equations and their solutions. By George Moseley Murphy. 1960
Section: Various 28
Problem number: 814.
ODE order: 1.
ODE degree: 2.

CAS Maple gives this as type `[_quadrature]`

$$
y'^2 - (4y + 1)\, y' + (4y + 1)\, y = 0
$$

✓ Solution by Maple
Time used: 0.063 (sec). Leaf size: 137

```
dsolve(diff(y(x),x)^2-(1+4*y(x))*diff(y(x),x)+(1+4*y(x))*y(x) = 0,y(x), singsol=all)
```

$$
\begin{aligned}
y(x) &= -\frac{1}{4} \\
y(x) &= -\frac{\sqrt{-\mathrm{e}^{-2x} c_1}\, \mathrm{e}^{2x} + c_1}{c_1 \sqrt{-\mathrm{e}^{-2x} c_1}} \\
y(x) &= \frac{-\sqrt{-\mathrm{e}^{-2x} c_1}\, \mathrm{e}^{2x} + c_1}{\sqrt{-\mathrm{e}^{-2x} c_1}\, c_1} \\
y(x) &= \frac{-\sqrt{-\mathrm{e}^{-2x} c_1}\, \mathrm{e}^{2x} + c_1}{\sqrt{-\mathrm{e}^{-2x} c_1}\, c_1} \\
y(x) &= -\frac{\sqrt{-\mathrm{e}^{-2x} c_1}\, \mathrm{e}^{2x} + c_1}{c_1 \sqrt{-\mathrm{e}^{-2x} c_1}}
\end{aligned}
$$

✓ Solution by Mathematica

Time used: 0.23 (sec). Leaf size: 67

```
DSolve[(y'[x])^2-(1+4 y[x])y'[x]+(1+4 y[x])y[x]==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to \frac{1}{4}e^{x-4c_1}\left(e^x + 2e^{2c_1}\right)$$
$$y(x) \to \frac{1}{4}e^{x+2c_1}\left(-2 + e^{x+2c_1}\right)$$
$$y(x) \to -\frac{1}{4}$$
$$y(x) \to 0$$

28.17 problem 815

Internal problem ID [4054]

Book: Ordinary differential equations and their solutions. By George Moseley Murphy. 1960
Section: Various 28
Problem number: 815.
ODE order: 1.
ODE degree: 2.

CAS Maple gives this as type `[_quadrature]`

$$y'^2 - 2(1-3y)\,y' - (4-9y)\,y = 0$$

✓ Solution by Maple

Time used: 0.062 (sec). Leaf size: 123

```
dsolve(diff(y(x),x)^2-2*(1-3*y(x))*diff(y(x),x)-(4-9*y(x))*y(x) = 0,y(x), singsol=all)
```

$$y(x) = \frac{4}{9}$$

$$y(x) = \frac{\mathrm{RootOf}\left(\_Z^8 \mathrm{e}^{24x} + 24\_Z^7 \mathrm{e}^{24x} + 240\_Z^6 \mathrm{e}^{24x} + 1280\_Z^5 \mathrm{e}^{24x} + \left(3840\,\mathrm{e}^{24x} - 1458\,\mathrm{e}^{12x} c_1\right)\_Z^4 + \left(6144\,\mathrm{e}^{2}\right.\right.}{9} + \frac{4}{9}$$

✓ Solution by Mathematica

Time used: 60.291 (sec). Leaf size: 4769

```
DSolve[(y'[x])^2-2(1-3 y[x])y'[x]-(4-9 y[x])y[x]==0,y[x],x,IncludeSingularSolutions -> True]
```

Too large to display

28.18 problem 816

Internal problem ID [4055]

Book: Ordinary differential equations and their solutions. By George Moseley Murphy. 1960
Section: Various 28
Problem number: 816.
ODE order: 1.
ODE degree: 2.

CAS Maple gives this as type `[_quadrature]`

$$y'^2 + (a + 6y)\, y' + y(3a + b + 9y) = 0$$

✓ Solution by Maple
Time used: 0.063 (sec). Leaf size: 349

```
dsolve(diff(y(x),x)^2+(a+6*y(x))*diff(y(x),x)+y(x)*(3*a+b+9*y(x)) = 0,y(x), singsol=all)
```

$$y(x) = -\frac{\mathrm{RootOf}\left(3a\ln\left(-\frac{b}{\left(3\_Z-2b\right)^2}\right)+2b\ln\left(-\frac{b}{\left(3\_Z-2b\right)^2}\right)+12a\ln(2)+4b\ln(2)-3a\ln\left(-\frac{\left(\_Z+2a\right)^2}{b}\right.}{}$$

$$y(x) = -\frac{\mathrm{e}^{\mathrm{RootOf}\left(-3a\ln\left(-\frac{\left(3\,\mathrm{e}^{\_Z}+6a+2b\right)^2}{b}\right)-2b\ln\left(-\frac{\left(3\,\mathrm{e}^{\_Z}+6a+2b\right)^2}{b}\right)-3a\ln\left(-\frac{1}{b}\right)+12a\ln(2)+4b\ln(2)+18c_1a+6c_1b-6a\_Z-18ax-6bx\right)}\left(\mathrm{e}\right.}{}$$

✓ Solution by Mathematica

Time used: 0.634 (sec). Leaf size: 175

```
DSolve[(y'[x])^2+(a+6 y[x])y'[x]+y[x](3 a+b+9 y[x])==0,y[x],x,IncludeSingularSolutions -> Tr
```

$$y(x) \to \text{InverseFunction}\left[\frac{(3a+2b)\log\left(-3\sqrt{a^2-4\#1b}+3a+2b\right)+3a\log\left(\sqrt{a^2-4\#1b}+a\right)}{6(3a+b)}\&\right]\left[-\frac{x}{2}+c_1\right]$$

$$y(x) \to \text{InverseFunction}\left[-\frac{3a\log\left(a-\sqrt{a^2-4\#1b}\right)+(3a+2b)\log\left(3\sqrt{a^2-4\#1b}+3a+2b\right)}{6(3a+b)}\&\right]\left[\frac{x}{2}+c_1\right]$$

$$y(x) \to 0$$

$$y(x) \to \frac{1}{9}(-3a-b)$$

28.19 problem 817

Internal problem ID [4056]

Book: Ordinary differential equations and their solutions. By George Moseley Murphy. 1960
Section: Various 28
Problem number: 817.
ODE order: 1.
ODE degree: 2.

CAS Maple gives this as type `[_dAlembert]`

$$y'^2 + ayy' = ax$$

✓ Solution by Maple

Time used: 0.078 (sec). Leaf size: 398

```
dsolve(diff(y(x),x)^2+a*y(x)*diff(y(x),x)-a*x = 0,y(x), singsol=all)
```

$$\frac{\left(-ay(x)+\sqrt{a\left(ay\left(x\right)^{2}+4x\right)}\right)c_1}{\sqrt{-2ay\left(x\right)+2\sqrt{a\left(ay\left(x\right)^{2}+4x\right)}+4}\sqrt{-2ay\left(x\right)+2\sqrt{a\left(ay\left(x\right)^{2}+4x\right)}-4}}+x$$
$$+\frac{\left(-ay(x)+\sqrt{a\left(ay\left(x\right)^{2}+4x\right)}\right)\left(-\ln\left(2\right)+\ln\left(-ay(x)+\sqrt{a\left(ay\left(x\right)^{2}+4x\right)}+\sqrt{2a^{2}y\left(x\right)^{2}-2\sqrt{a\left(a\right.}}\right.\right.}{a\sqrt{2a^{2}y\left(x\right)^{2}-2\sqrt{a\left(ay\left(x\right)^{2}+4x\right)}\,ay\left(x\right)+4ax-4}}$$
$$=0$$

$$\frac{\left(ay(x)+\sqrt{a\left(ay\left(x\right)^{2}+4x\right)}\right)c_1}{\sqrt{-2ay\left(x\right)-2\sqrt{a\left(ay\left(x\right)^{2}+4x\right)}+4}\sqrt{-2ay\left(x\right)-2\sqrt{a\left(ay\left(x\right)^{2}+4x\right)}-4}}+x$$
$$-\frac{\left(ay(x)+\sqrt{a\left(ay\left(x\right)^{2}+4x\right)}\right)\left(-\ln\left(2\right)+\ln\left(-ay(x)-\sqrt{a\left(ay\left(x\right)^{2}+4x\right)}+\sqrt{2a^{2}y\left(x\right)^{2}+2\sqrt{a\left(ay\right.}}\right.\right.}{a\sqrt{2a^{2}y\left(x\right)^{2}+2\sqrt{a\left(ay\left(x\right)^{2}+4x\right)}\,ay\left(x\right)+4ax-4}}$$
$$=0$$

✓ Solution by Mathematica

Time used: 0.633 (sec). Leaf size: 83

```
DSolve[(y'[x])^2+a y[x] y'[x]-a x==0,y[x],x,IncludeSingularSolutions -> True]
```

$$\text{Solve}\left[\left\{x=-\frac{2K[1]\arctan\left(\frac{\sqrt{1-K[1]^2}}{K[1]+1}\right)}{a\sqrt{1-K[1]^2}}+\frac{c_1K[1]}{\sqrt{1-K[1]^2}},y(x)=\frac{x}{K[1]}-\frac{K[1]}{a}\right\},\{y(x),K[1]\}\right]$$

28.20 problem 818

Internal problem ID [4057]

Book: Ordinary differential equations and their solutions. By George Moseley Murphy. 1960
Section: Various 28
Problem number: 818.
ODE order: 1.
ODE degree: 2.

CAS Maple gives this as type `[_dAlembert]`

$$\boxed{y'^2 - ayy' = ax}$$

✓ Solution by Maple

Time used: 0.078 (sec). Leaf size: 183

```
dsolve(diff(y(x),x)^2-a*y(x)*diff(y(x),x)-a*x = 0,y(x), singsol=all)
```

$$x + \frac{\left(-ay(x) + \sqrt{a\left(ay\left(x\right)^2 + 4x\right)}\right)\left(c_1 a + \operatorname{arcsinh}\left(\frac{ay(x)}{2} - \frac{\sqrt{a\left(ay(x)^2+4x\right)}}{2}\right)\right)}{\sqrt{2a^2y\left(x\right)^2 - 2\sqrt{a\left(ay\left(x\right)^2 + 4x\right)}\, ay\left(x\right) + 4ax + 4\,a}} = 0$$

$$x - \frac{\left(ay(x) + \sqrt{a\left(ay\left(x\right)^2 + 4x\right)}\right)\left(c_1 a + \operatorname{arcsinh}\left(\frac{ay(x)}{2} + \frac{\sqrt{a\left(ay(x)^2+4x\right)}}{2}\right)\right)}{\sqrt{2a^2y\left(x\right)^2 + 2\sqrt{a\left(ay\left(x\right)^2 + 4x\right)}\, ay\left(x\right) + 4ax + 4\,a}} = 0$$

✓ Solution by Mathematica

Time used: 0.91 (sec). Leaf size: 75

```
DSolve[(y'[x])^2-a y[x] y'[x]-a x==0,y[x],x,IncludeSingularSolutions -> True]
```

$$\text{Solve}\left[\left\{x=-\frac{K[1]\log\left(\sqrt{K[1]^2+1}-K[1]\right)}{a\sqrt{K[1]^2+1}}+\frac{c_1K[1]}{\sqrt{K[1]^2+1}},y(x)=\frac{K[1]}{a}-\frac{x}{K[1]}\right\},\{y(x),K[1]\}\right]$$

28.21 problem 819

Internal problem ID [4058]

Book: Ordinary differential equations and their solutions. By George Moseley Murphy. 1960
Section: Various 28
Problem number: 819.
ODE order: 1.
ODE degree: 2.

CAS Maple gives this as type `[_quadrature]`

$$y'^2 + (ax + yb)\, y' + abxy = 0$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 22

```
dsolve(diff(y(x),x)^2+(a*x+b*y(x))*diff(y(x),x)+a*b*x*y(x) = 0,y(x), singsol=all)
```

$$y(x) = -\frac{a\,x^2}{2} + c_1$$
$$y(x) = c_1 \mathrm{e}^{-bx}$$

✓ Solution by Mathematica

Time used: 0.04 (sec). Leaf size: 34

```
DSolve[(y'[x])^2+(a x+b y[x])y'[x]+a b x y[x]==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to c_1 e^{-bx}$$
$$y(x) \to -\frac{ax^2}{2} + c_1$$
$$y(x) \to 0$$

28.22 problem 820

Internal problem ID [4059]

Book: Ordinary differential equations and their solutions. By George Moseley Murphy. 1960
Section: Various 28
Problem number: 820.
ODE order: 1.
ODE degree: 2.

CAS Maple gives this as type `[[_1st_order, '_with_symmetry_[F(x),G(y)]']]`

$$y'^2 - xyy' + y^2 \ln(ya) = 0$$

✓ Solution by Maple

Time used: 0.203 (sec). Leaf size: 43

```
dsolve(diff(y(x),x)^2-x*diff(y(x),x)*y(x)+y(x)^2*ln(a*y(x)) = 0,y(x), singsol=all)
```

$$y(x) = \frac{e^{\frac{x^2}{4}}}{a}$$
$$y(x) = \frac{e^{c_1(-c_1+x)}}{a}$$
$$y(x) = \frac{e^{-c_1(c_1+x)}}{a}$$

✓ Solution by Mathematica

Time used: 0.324 (sec). Leaf size: 30

```
DSolve[(y'[x])^2-x y'[x] y[x]+y[x]^2 Log[a y[x]]==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to \frac{e^{\frac{1}{4}c_1(2x-c_1)}}{a}$$
$$y(x) \to 0$$

28.23 problem 821

Internal problem ID [4060]

Book: Ordinary differential equations and their solutions. By George Moseley Murphy. 1960
Section: Various 28
Problem number: 821.
ODE order: 1.
ODE degree: 2.

CAS Maple gives this as type `[_quadrature]`

$$y'^2 - (2yx+1)\, y' + 2yx = 0$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 16

```
dsolve(diff(y(x),x)^2-(1+2*x*y(x))*diff(y(x),x)+2*x*y(x) = 0,y(x), singsol=all)
```

$$y(x) = c_1 + x$$
$$y(x) = e^{x^2} c_1$$

✓ Solution by Mathematica

Time used: 0.01 (sec). Leaf size: 21

```
DSolve[(y'[x])^2-(1+2 x y[x])y'[x]+2 x y[x]==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to c_1 e^{x^2}$$
$$y(x) \to x + c_1$$

28.24 problem 822

Internal problem ID [4061]

Book: Ordinary differential equations and their solutions. By George Moseley Murphy. 1960
Section: Various 28
Problem number: 822.
ODE order: 1.
ODE degree: 2.

CAS Maple gives this as type `[_quadrature]`

$$y'^2 - (4 + y^2)\, y' + y^2 = -4$$

✓ Solution by Maple
Time used: 0.125 (sec). Leaf size: 77

```
dsolve(diff(y(x),x)^2-(4+y(x)^2)*diff(y(x),x)+4+y(x)^2 = 0,y(x), singsol=all)
```

$$y(x) = -2i$$
$$y(x) = 2i$$
$$x + 2\left(\int^{y(x)} \frac{1}{-\_a^2 + \sqrt{\_a^2 (\_a^2 + 4)} - 4} d\_a\right) - c_1 = 0$$
$$x - 2\left(\int^{y(x)} \frac{1}{\_a^2 + \sqrt{\_a^2 (\_a^2 + 4)} + 4} d\_a\right) - c_1 = 0$$

✓ Solution by Mathematica
Time used: 0.463 (sec). Leaf size: 73

```
DSolve[(y'[x])^2-(4+y[x]^2)y'[x]+4+y[x]^2==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to \frac{x^2 - 4c_1 x - 1 + 4{c_1}^2}{x - 2c_1}$$
$$y(x) \to \frac{x^2 + 4c_1 x - 1 + 4{c_1}^2}{x + 2c_1}$$
$$y(x) \to -2i$$
$$y(x) \to 2i$$

29 Various 29

29.1 problem 823

Internal problem ID [4062]

Book: Ordinary differential equations and their solutions. By George Moseley Murphy. 1960
Section: Various 29
Problem number: 823.
ODE order: 1.
ODE degree: 2.

CAS Maple gives this as type `[_separable]`

$$
\boxed{y'^2 - (-y + x)\, yy' - xy^3 = 0}
$$

✓ Solution by Maple
Time used: 0.0 (sec). Leaf size: 20

```
dsolve(diff(y(x),x)^2-(x-y(x))*y(x)*diff(y(x),x)-x*y(x)^3 = 0,y(x), singsol=all)
```

$$
y(x) = \frac{1}{c_1 + x}
$$
$$
y(x) = \mathrm{e}^{\frac{x^2}{2}} c_1
$$

✓ Solution by Mathematica
Time used: 0.136 (sec). Leaf size: 34

```
DSolve[(y'[x])^2-(x-y[x])y[x] y'[x]-x y[x]^3==0,y[x],x,IncludeSingularSolutions -> True]
```

$$
y(x) \to \frac{1}{x - c_1}
$$
$$
y(x) \to c_1 e^{\frac{x^2}{2}}
$$
$$
y(x) \to 0
$$

29.2 problem 824

Internal problem ID [4063]

Book: Ordinary differential equations and their solutions. By George Moseley Murphy. 1960
Section: Various 29
Problem number: 824.
ODE order: 1.
ODE degree: 2.

CAS Maple gives this as type `[[_homogeneous, 'class G']]`

$$\boxed{y'^2 + x y^2 y' + y^3 = 0}$$

✓ Solution by Maple

Time used: 0.296 (sec). Leaf size: 124

```
dsolve(diff(y(x),x)^2+x*y(x)^2*diff(y(x),x)+y(x)^3 = 0,y(x), singsol=all)
```

$$y(x) = \frac{4}{x^2}$$
$$y(x) = 0$$
$$y(x) = \frac{2\sqrt{2}\, x - 2c_1}{c_1 (c_1^2 - 2x^2)}$$
$$y(x) = \frac{-2\sqrt{2}\, x - 2c_1}{c_1 (c_1^2 - 2x^2)}$$
$$y(x) = -\frac{\left(\sqrt{2}\, c_1 x - 2\right) c_1^2}{2c_1^2 x^2 - 4}$$
$$y(x) = \frac{\left(\sqrt{2}\, c_1 x + 2\right) c_1^2}{2c_1^2 x^2 - 4}$$

✓ Solution by Mathematica

Time used: 0.914 (sec). Leaf size: 71

```
DSolve[(y'[x])^2+x y[x]^2 y'[x]+y[x]^3==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to \frac{\cosh(c_1) - \sinh(c_1)}{-ix + \cosh(c_1) + \sinh(c_1)}$$
$$y(x) \to \frac{\cosh(c_1) - \sinh(c_1)}{ix + \cosh(c_1) + \sinh(c_1)}$$
$$y(x) \to 0$$
$$y(x) \to \frac{4}{x^2}$$

29.3 problem 825

Internal problem ID [4064]

Book: Ordinary differential equations and their solutions. By George Moseley Murphy. 1960
Section: Various 29
Problem number: 825.
ODE order: 1.
ODE degree: 2.

CAS Maple gives this as type `[[_1st_order, _with_linear_symmetries]]`

$$y'^2 - 2y'y^2x^3 - 4y^3x^2 = 0$$

✓ Solution by Maple

Time used: 0.297 (sec). Leaf size: 132

```
dsolve(diff(y(x),x)^2-2*x^3*y(x)^2*diff(y(x),x)-4*x^2*y(x)^3 = 0,y(x), singsol=all)
```

$$y(x) = -\frac{4}{x^4}$$
$$y(x) = 0$$
$$y(x) = \frac{(\sqrt{2}\,x^2c_1 - 2)\,c_1^2}{2c_1^2x^4 - 4}$$
$$y(x) = -\frac{(\sqrt{2}\,x^2c_1 + 2)\,c_1^2}{2c_1^2x^4 - 4}$$
$$y(x) = \frac{-2\sqrt{2}\,x^2 + 2c_1}{c_1\,(-2x^4 + c_1^2)}$$
$$y(x) = \frac{2\sqrt{2}\,x^2 + 2c_1}{c_1\,(-2x^4 + c_1^2)}$$

✓ Solution by Mathematica

Time used: 1.481 (sec). Leaf size: 177

```
DSolve[(y'[x])^2-2 x^3 y[x]^2 y'[x]-4 x^2 y[x]^3==0,y[x],x,IncludeSingularSolutions -> True]
```

$$\text{Solve}\left[-\frac{x\sqrt{x^4y(x)+4}y(x)^{3/2}\log\left(\sqrt{x^4y(x)+4}+x^2\sqrt{y(x)}\right)}{2\sqrt{x^2y(x)^3\left(x^4y(x)+4\right)}}-\frac{1}{4}\log(y(x))=c_1,y(x)\right]$$

$$\text{Solve}\left[\frac{xy(x)^{3/2}\sqrt{x^4y(x)+4}\log\left(\sqrt{x^4y(x)+4}+x^2\sqrt{y(x)}\right)}{2\sqrt{x^2y(x)^3\left(x^4y(x)+4\right)}}-\frac{1}{4}\log(y(x))=c_1,y(x)\right]$$

$$y(x)\to 0$$

$$y(x)\to -\frac{4}{x^4}$$

29.4 problem 826

Internal problem ID [4065]

Book: Ordinary differential equations and their solutions. By George Moseley Murphy. 1960
Section: Various 29
Problem number: 826.
ODE order: 1.
ODE degree: 2.

CAS Maple gives this as type `[_separable]`

$$y'^2 - xy(y^2 + x^2)\, y' + y^4x^4 = 0$$

✓ Solution by Maple
Time used: 0.0 (sec). Leaf size: 38

```
dsolve(diff(y(x),x)^2-x*y(x)*(x^2+y(x)^2)*diff(y(x),x)+x^4*y(x)^4 = 0,y(x), singsol=all)
```

$$y(x) = \frac{1}{\sqrt{-x^2 + c_1}}$$
$$y(x) = -\frac{1}{\sqrt{-x^2 + c_1}}$$
$$y(x) = c_1 \mathrm{e}^{\frac{x^4}{4}}$$

✓ Solution by Mathematica
Time used: 0.188 (sec). Leaf size: 60

```
DSolve[(y'[x])^2-x y[x](x^2+y[x]^2)y'[x]+x^4 y[x]^4==0,y[x],x,IncludeSingularSolutions -> Tr
```

$$y(x) \to -\frac{1}{\sqrt{-x^2 - 2c_1}}$$
$$y(x) \to \frac{1}{\sqrt{-x^2 - 2c_1}}$$
$$y(x) \to c_1 e^{\frac{x^4}{4}}$$
$$y(x) \to 0$$

29.5 problem 827

Internal problem ID [4066]

Book: Ordinary differential equations and their solutions. By George Moseley Murphy. 1960
Section: Various 29
Problem number: 827.
ODE order: 1.
ODE degree: 2.

CAS Maple gives this as type `[[_homogeneous, ‘class G‘]]`

$$\boxed{y'^2 + 2xy^3y' + y^4 = 0}$$

✓ Solution by Maple
Time used: 0.109 (sec). Leaf size: 48

```
dsolve(diff(y(x),x)^2+2*x*y(x)^3*diff(y(x),x)+y(x)^4 = 0,y(x), singsol=all)
```

$$y(x) = -\frac{1}{x}$$
$$y(x) = \frac{1}{x}$$
$$y(x) = 0$$
$$y(x) = \frac{1}{\sqrt{-c_1(-2x + c_1)}}$$
$$y(x) = -\frac{1}{\sqrt{c_1(-c_1 + 2x)}}$$

✓ Solution by Mathematica

Time used: 0.852 (sec). Leaf size: 161

```
DSolve[(y'[x])^2+2 x y[x]^3 y'[x]+y[x]^4==0,y[x],x,IncludeSingularSolutions -> True]
```

$$\text{Solve}\left[-\frac{\sqrt{x^2y(x)^2-1}y(x)^2\text{arctanh}\left(\frac{xy(x)}{\sqrt{x^2y(x)^2-1}}\right)}{\sqrt{y(x)^4\left(x^2y(x)^2-1\right)}}-\log(y(x))=c_1,y(x)\right]$$

$$\text{Solve}\left[\frac{y(x)^2\sqrt{x^2y(x)^2-1}\text{arctanh}\left(\frac{xy(x)}{\sqrt{x^2y(x)^2-1}}\right)}{\sqrt{y(x)^4\left(x^2y(x)^2-1\right)}}-\log(y(x))=c_1,y(x)\right]$$

$$y(x)\to 0$$

$$y(x)\to -\frac{1}{x}$$

$$y(x)\to \frac{1}{x}$$

29.6 problem 828

Internal problem ID [4067]

Book: Ordinary differential equations and their solutions. By George Moseley Murphy. 1960
Section: Various 29
Problem number: 828.
ODE order: 1.
ODE degree: 2.

CAS Maple gives this as type `[_separable]`

$$\boxed{y'^2 + 2yy'\cot(x) - y^2 = 0}$$

✓ Solution by Maple

Time used: 0.125 (sec). Leaf size: 39

```
dsolve(diff(y(x),x)^2+2*y(x)*diff(y(x),x)*cot(x)-y(x)^2 = 0,y(x), singsol=all)
```

$$y(x) = 0$$
$$y(x) = \frac{\operatorname{csgn}(\sin(x))\, c_1}{\cos(x) + \operatorname{csgn}(\sec(x))}$$
$$y(x) = \csc(x)^2 (\cos(x) + \operatorname{csgn}(\sec(x)))\operatorname{csgn}(\sin(x))\, c_1$$

✓ Solution by Mathematica

Time used: 0.158 (sec). Leaf size: 36

```
DSolve[(y'[x])^2+2 y[x] y'[x] Cot[x]-y[x]^2==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to c_1 \csc^2\left(\frac{x}{2}\right)$$
$$y(x) \to c_1 \sec^2\left(\frac{x}{2}\right)$$
$$y(x) \to 0$$

29.7 problem 829

Internal problem ID [4068]

Book: Ordinary differential equations and their solutions. By George Moseley Murphy. 1960
Section: Various 29
Problem number: 829.
ODE order: 1.
ODE degree: 2.

CAS Maple gives this as type `[[_1st_order, _with_linear_symmetries]]`

$$\boxed{y'^2 - 3xy^{\frac{2}{3}}y' + 9y^{\frac{5}{3}} = 0}$$

✓ Solution by Maple

Time used: 0.297 (sec). Leaf size: 149

```
dsolve(diff(y(x),x)^2-3*x*y(x)^(2/3)*diff(y(x),x)+9*y(x)^(5/3) = 0,y(x), singsol=all)
```

$$y(x) = \frac{x^6}{64}$$
$$y(x) = 0$$
$$\ln(x) + \frac{\sqrt{-\frac{y(x)\left(\frac{y(x)}{x^6}\right)^{\frac{1}{3}}\left(4\left(\frac{y(x)}{x^6}\right)^{\frac{1}{3}}-1\right)}{x^6}}\,\operatorname{arctanh}\left(\sqrt{-4\left(\frac{y(x)}{x^6}\right)^{\frac{1}{3}}+1}\right)}{\left(\frac{y(x)}{x^6}\right)^{\frac{2}{3}}\sqrt{-4\left(\frac{y(x)}{x^6}\right)^{\frac{1}{3}}+1}} + \frac{\ln\left(\frac{64y(x)}{x^6}-1\right)}{6} - \frac{\ln\left(4\left(\frac{y(x)}{x^6}\right)^{\frac{1}{3}}-1\right)}{6} - \frac{\ln\left(16\left(\frac{y(x)}{x^6}\right)^{\frac{2}{3}}+4\left(\frac{y(x)}{x^6}\right)^{\frac{1}{3}}+1\right)}{6} + \frac{\ln\left(\frac{y(x)}{x^6}\right)}{6} - c_1 = 0$$

✓ Solution by Mathematica

Time used: 17.485 (sec). Leaf size: 701

```
DSolve[(y'[x])^2-3 x y[x]^(2/3) y'[x]+9 y[x]^(5/3)==0,y[x],x,IncludeSingularSolutions -> Tru
```

$$\text{Solve}\left[\frac{8x^2\log(y(x))-6\sqrt{x^4}\log\left(x^2\sqrt{x^2-4\sqrt[3]{y(x)}}\right)-3\sqrt{x^4}\log\left(4\sqrt[3]{y(x)}-x^2\right)+6\left(\sqrt{x^4}-x^2\right)\log\left(1\right.}{}\right.$$

$$\left.-\frac{\sqrt{\left(x^2-4\sqrt[3]{y(x)}\right)y(x)^{4/3}}\log\left(\sqrt{x^2-4\sqrt[3]{y(x)}}-x\right)}{\sqrt{x^2-4\sqrt[3]{y(x)}}\,y(x)^{2/3}}=c_1,y(x)\right]$$

$$\text{Solve}\left[\frac{\sqrt{\left(x^2-4\sqrt[3]{y(x)}\right)y(x)^{4/3}}\log\left(\sqrt{x^2-4\sqrt[3]{y(x)}}-x\right)}{\sqrt{x^2-4\sqrt[3]{y(x)}}\,y(x)^{2/3}}\right.$$

$$+\frac{8x^2\log(y(x))+6\sqrt{x^4}\log\left(x^2\sqrt{x^2-4\sqrt[3]{y(x)}}\right)+3\sqrt{x^4}\log\left(4\sqrt[3]{y(x)}-x^2\right)+6\left(x^2-\sqrt{x^4}\right)\log\left(16x^2\sqrt{\right.}{}$$

$$y(x)\to 0$$

29.8 problem 830

Internal problem ID [4069]

Book: Ordinary differential equations and their solutions. By George Moseley Murphy. 1960
Section: Various 29
Problem number: 830.
ODE order: 1.
ODE degree: 2.

CAS Maple gives this as type `[[_homogeneous, ‘class C‘], _dAlembert]`

$$\boxed{y'^2 - \mathrm{e}^{4x-2y}(y'-1) = 0}$$

✓ Solution by Maple
Time used: 1.969 (sec). Leaf size: 307

```
dsolve(diff(y(x),x)^2 = exp(4*x-2*y(x))*(diff(y(x),x)-1),y(x), singsol=all)
```

$$\frac{-\frac{\sqrt{-4\,\mathrm{e}^{-4y(x)+8x}\mathrm{e}^{-4x+2y(x)}+\mathrm{e}^{-4y(x)+8x}}\,\mathrm{e}^{-4x+2y(x)}\operatorname{arctanh}\left(\frac{1}{\sqrt{-4\,\mathrm{e}^{-4x+2y(x)}+1}}\right)}{2}+\sqrt{-4\,\mathrm{e}^{-4x+2y(x)}+1}\left(x-\frac{\ln\left(2\,\mathrm{e}^{-2x+y(x)}+1\right)}{4}\right.}{\sqrt{-4\,\mathrm{e}^{-4x+2y(x)}+1}}$$
$$= 0$$

$$\frac{\frac{\sqrt{-4\,\mathrm{e}^{-4y(x)+8x}\mathrm{e}^{-4x+2y(x)}+\mathrm{e}^{-4y(x)+8x}}\,\mathrm{e}^{-4x+2y(x)}\operatorname{arctanh}\left(\frac{1}{\sqrt{-4\,\mathrm{e}^{-4x+2y(x)}+1}}\right)}{2}+\sqrt{-4\,\mathrm{e}^{-4x+2y(x)}+1}\left(x-\frac{\ln\left(2\,\mathrm{e}^{-2x+y(x)}+1\right)}{4}\right.}{\sqrt{-4\,\mathrm{e}^{-4x+2y(x)}+1}}$$
$$= 0$$

✓ Solution by Mathematica

Time used: 2.551 (sec). Leaf size: 383

```
DSolve[(y'[x])^2==Exp[4 x -2 y[x]] (y'[x]-1),y[x],x,IncludeSingularSolutions -> True]
```

$$\text{Solve}\left[-\frac{e^{-2x}\sqrt{e^{8x}-4e^{2y(x)+4x}}\operatorname{arctanh}\left(\frac{-\sqrt{e^{4x}-4e^{2y(x)}}+e^{2x}+1}{\sqrt{e^{4x}-4e^{2y(x)}}-e^{2x}+1}\right)}{\sqrt{e^{4x}-4e^{2y(x)}}} - \frac{e^{-2x}\sqrt{e^{8x}-4e^{2y(x)+4x}}y(x)}{2\sqrt{e^{4x}-4e^{2y(x)}}} + \frac{y(x)}{2} = c_1, y(x)\right]$$

$$\text{Solve}\left[\frac{e^{-2x}\sqrt{e^{8x}-4e^{2y(x)+4x}}\operatorname{arctanh}\left(\frac{-\sqrt{e^{4x}-4e^{2y(x)}}+e^{2x}+1}{\sqrt{e^{4x}-4e^{2y(x)}}-e^{2x}+1}\right)}{\sqrt{e^{4x}-4e^{2y(x)}}} + \frac{\left(\sqrt{e^{4x}-4e^{2y(x)}}\sqrt{e^{8x}-4e^{2y(x)+4x}}-4e^{2(y(x)+x)}+e^{6x}\right)y(x)}{2e^{6x}-8e^{2(y(x)+x)}} = c_1, y(x)\right]$$

$$y(x) \to \frac{1}{2}\left(\log\left(\frac{e^{8x}}{4}\right)-4x\right)$$

29.9 problem 831

Internal problem ID [4070]

Book: Ordinary differential equations and their solutions. By George Moseley Murphy. 1960
Section: Various 29
Problem number: 831.
ODE order: 1.
ODE degree: 2.

CAS Maple gives this as type `[[_1st_order, _with_linear_symmetries], _dAlembert]`

$$\boxed{2y'^2 + xy' - 2y = 0}$$

✓ Solution by Maple

Time used: 0.063 (sec). Leaf size: 31

```
dsolve(2*diff(y(x),x)^2+x*diff(y(x),x)-2*y(x) = 0,y(x), singsol=all)
```

$$y(x) = \frac{x^2\left(1 + 2\,\mathrm{LambertW}\left(\frac{x\,\mathrm{e}^{\frac{c_1}{4}}}{4}\right)\right)}{16\,\mathrm{LambertW}\left(\frac{x\,\mathrm{e}^{\frac{c_1}{4}}}{4}\right)^2}$$

✓ Solution by Mathematica

Time used: 1.193 (sec). Leaf size: 130

```
DSolve[2 (y'[x])^2+x y'[x]-2 y[x]==0,y[x],x,IncludeSingularSolutions -> True]
```

$$\mathrm{Solve}\left[-\frac{\frac{1}{2}x\sqrt{x^2+16y(x)} - 8y(x)\log\left(\sqrt{x^2+16y(x)} - x\right) + \frac{x^2}{2}}{8y(x)} = c_1, y(x)\right]$$

$$\mathrm{Solve}\left[\frac{\frac{1}{2}x\sqrt{x^2+16y(x)} - 8y(x)\log\left(\sqrt{x^2+16y(x)} - x\right) - \frac{x^2}{2}}{8y(x)} + \log(y(x)) = c_1, y(x)\right]$$

$$y(x) \to 0$$

29.10 problem 832

Internal problem ID [4071]

Book: Ordinary differential equations and their solutions. By George Moseley Murphy. 1960
Section: Various 29
Problem number: 832.
ODE order: 1.
ODE degree: 2.

CAS Maple gives this as type `[[_1st_order, _with_linear_symmetries], _Clairaut]`

$$2{y'}^2 - (1-x)\,y' - y = 0$$

✓ Solution by Maple

Time used: 0.078 (sec). Leaf size: 22

```
dsolve(2*diff(y(x),x)^2-(1-x)*diff(y(x),x)-y(x) = 0,y(x), singsol=all)
```

$$y(x) = -\frac{(x-1)^2}{8}$$
$$y(x) = c_1(2c_1 + x - 1)$$

✓ Solution by Mathematica

Time used: 0.007 (sec). Leaf size: 28

```
DSolve[2 (y'[x])^2-(1-x)y'[x]-y[x]==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to c_1(x - 1 + 2c_1)$$
$$y(x) \to -\frac{1}{8}(x-1)^2$$

29.11 problem 833

Internal problem ID [4072]

Book: Ordinary differential equations and their solutions. By George Moseley Murphy. 1960
Section: Various 29
Problem number: 833.
ODE order: 1.
ODE degree: 2.

CAS Maple gives this as type `[[_homogeneous, ‘class G‘]]`

$$2y'^2 - 2y'x^2 + 3yx = 0$$

✓ Solution by Maple
Time used: 0.062 (sec). Leaf size: 77

```
dsolve(2*diff(y(x),x)^2-2*x^2*diff(y(x),x)+3*x*y(x) = 0,y(x), singsol=all)
```

$$y(x) = \frac{x^3}{6}$$
$$y(x) = \frac{\sqrt{6}\,\sqrt{-c_1 x}\,x}{3} + c_1$$
$$y(x) = -\frac{\sqrt{6}\,\sqrt{-c_1 x}\,x}{3} + c_1$$
$$y(x) = -\frac{\sqrt{6}\,\sqrt{-c_1 x}\,x}{3} + c_1$$
$$y(x) = \frac{\sqrt{6}\,\sqrt{-c_1 x}\,x}{3} + c_1$$

✓ Solution by Mathematica

Time used: 2.615 (sec). Leaf size: 213

```
DSolve[2 (y'[x])^2-2 x^2 y'[x]+3 x y[x]==0,y[x],x,IncludeSingularSolutions -> True]
```

$$\text{Solve}\left[\frac{1}{3}\left(1-\frac{\sqrt{x^4-6xy(x)}}{\sqrt{x}\sqrt{x^3-6y(x)}}\right)\log(y(x)) + \frac{2\sqrt{x^4-6xy(x)}\log\left(x^{3/2}+\sqrt{x^3-6y(x)}\right)}{3\sqrt{x}\sqrt{x^3-6y(x)}}=c_1,y(x)\right]$$

$$\text{Solve}\left[\frac{1}{3}\left(\frac{\sqrt{x^4-6xy(x)}}{\sqrt{x}\sqrt{x^3-6y(x)}}+1\right)\log(y(x)) - \frac{2\sqrt{x^4-6xy(x)}\log\left(x^{3/2}+\sqrt{x^3-6y(x)}\right)}{3\sqrt{x}\sqrt{x^3-6y(x)}}=c_1,y(x)\right]$$

$$y(x)\to\frac{x^3}{6}$$

29.12 problem 834

Internal problem ID [4073]

Book: Ordinary differential equations and their solutions. By George Moseley Murphy. 1960
Section: Various 29
Problem number: 834.
ODE order: 1.
ODE degree: 2.

CAS Maple gives this as type `[_quadrature]`

$$2{y'}^2 + 2(6y - 1)\, y' + 3y(6y - 1) = 0$$

✓ Solution by Maple

Time used: 0.063 (sec). Leaf size: 61

```
dsolve(2*diff(y(x),x)^2+2*(6*y(x)-1)*diff(y(x),x)+3*y(x)*(6*y(x)-1) = 0,y(x), singsol=all)
```

$$y(x) = \frac{1}{6}$$
$$y(x) = -\frac{\left(\sqrt{6}\, \mathrm{e}^{\frac{3x}{2}+\frac{3c_1}{2}} + 3\, \mathrm{e}^{3c_1}\right) \mathrm{e}^{-3x}}{3}$$
$$y(x) = \frac{\left(\sqrt{6}\, \mathrm{e}^{\frac{3x}{2}+\frac{3c_1}{2}} - 3\, \mathrm{e}^{3c_1}\right) \mathrm{e}^{-3x}}{3}$$

✓ Solution by Mathematica

Time used: 0.275 (sec). Leaf size: 81

```
DSolve[2 (y'[x])^2+2(6 y[x]-1)y'[x]+3 y[x](6  y[x]-1)==0,y[x],x,IncludeSingularSolutions ->
```

$$y(x) \to -\frac{1}{6} e^{-3x+3c_1} \left(2e^{3x/2} + e^{3c_1}\right)$$
$$y(x) \to \frac{1}{6} e^{-3(x+2c_1)} \left(-1 + 2e^{\frac{3x}{2}+3c_1}\right)$$
$$y(x) \to 0$$
$$y(x) \to \frac{1}{6}$$

29.13 problem 835

Internal problem ID [4074]

Book: Ordinary differential equations and their solutions. By George Moseley Murphy. 1960
Section: Various 29
Problem number: 835.
ODE order: 1.
ODE degree: 2.

CAS Maple gives this as type `[[_1st_order, _with_linear_symmetries], _dAlembert]`

$$\boxed{3y'^2 - 2xy' + y = 0}$$

✓ Solution by Maple

Time used: 0.062 (sec). Leaf size: 611

```
dsolve(3*diff(y(x),x)^2-2*x*diff(y(x),x)+y(x) = 0,y(x), singsol=all)
```

$$y(x) = -\frac{\left(x^2 + x\left(x^3 + 6\sqrt{3}\sqrt{-c_1\left(x^3 - 27c_1\right)} - 54c_1\right)^{\frac{1}{3}} + \left(x^3 + 6\sqrt{3}\sqrt{-c_1\left(x^3 - 27c_1\right)} - 54c_1\right)^{\frac{2}{3}}\right)\left(x^2 - 3x\right.}{12\left(x^3 + 6\sqrt{3}\sqrt{-c_1\left(x^3 - 27\right.\right.}}$$

$$y(x) = -\frac{\left(i\left(x^3 + 6\sqrt{3}\sqrt{-c_1\left(x^3 - 27c_1\right)} - 54c_1\right)^{\frac{2}{3}}\sqrt{3} - i\sqrt{3}\,x^2 + \left(x^3 + 6\sqrt{3}\sqrt{-c_1\left(x^3 - 27c_1\right)} - 54c_1\right)^{\frac{2}{3}} - 2\right.}{}$$

$$y(x) = -\frac{\left(i\sqrt{3}\,x^2 - i\left(x^3 + 6\sqrt{3}\sqrt{-c_1\left(x^3 - 27c_1\right)} - 54c_1\right)^{\frac{2}{3}}\sqrt{3} + x^2 - 2x\left(x^3 + 6\sqrt{3}\sqrt{-c_1\left(x^3 - 27c_1\right)} - 54c\right.\right.}{}$$

✓ Solution by Mathematica

Time used: 60.138 (sec). Leaf size: 995

```
DSolve[3 (y'[x])^2-2 x y'[x]+y[x]==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to \frac{1}{12}\left(x^2 + \frac{x(x^3+216e^{3c_1})}{\sqrt[3]{x^6-540e^{3c_1}x^3+24\sqrt{3}\sqrt{e^{3c_1}\left(-x^3+27e^{3c_1}\right)^3}-5832e^{6c_1}}} + \sqrt[3]{x^6-540e^{3c_1}x^3+24\sqrt{3}\sqrt{e^{3c_1}\left(-x^3+27e^{3c_1}\right)^3}-5832e^{6c_1}}\right)$$

$$y(x) \to \frac{1}{24}\left(2x^2 - \frac{i\left(\sqrt{3}-i\right)x(x^3+216e^{3c_1})}{\sqrt[3]{x^6-540e^{3c_1}x^3+24\sqrt{3}\sqrt{e^{3c_1}\left(-x^3+27e^{3c_1}\right)^3}-5832e^{6c_1}}} + i\left(\sqrt{3}+i\right)\sqrt[3]{x^6-540e^{3c_1}x^3+24\sqrt{3}\sqrt{e^{3c_1}\left(-x^3+27e^{3c_1}\right)^3}-5832e^{6c_1}}\right)$$

$$y(x) \to \frac{1}{24}\left(2x^2 + \frac{i\left(\sqrt{3}+i\right)x(x^3+216e^{3c_1})}{\sqrt[3]{x^6-540e^{3c_1}x^3+24\sqrt{3}\sqrt{e^{3c_1}\left(-x^3+27e^{3c_1}\right)^3}-5832e^{6c_1}}} - \left(1+i\sqrt{3}\right)\sqrt[3]{x^6-540e^{3c_1}x^3+24\sqrt{3}\sqrt{e^{3c_1}\left(-x^3+27e^{3c_1}\right)^3}-5832e^{6c_1}}\right)$$

$$y(x) \to \frac{x^4 + \left(x^6+20e^{3c_1}x^3+8\sqrt{e^{3c_1}\left(x^3+e^{3c_1}\right)^3}-8e^{6c_1}\right)^{2/3} + x^2\sqrt[3]{x^6+20e^{3c_1}x^3+8\sqrt{e^{3c_1}\left(x^3+e^{3c_1}\right)^3}-8}}{12\sqrt[3]{x^6+20e^{3c_1}x^3+8\sqrt{e^{3c_1}\left(x^3+e^{3c_1}\right)^3}-8e^{6c_1}}}$$

$$y(x) \to \frac{1}{24}\left(2x^2 + \frac{\left(1+i\sqrt{3}\right)x(-x^3+8e^{3c_1})}{\sqrt[3]{x^6+20e^{3c_1}x^3+8\sqrt{e^{3c_1}\left(x^3+e^{3c_1}\right)^3}-8e^{6c_1}}} + i\left(\sqrt{3}+i\right)\sqrt[3]{x^6+20e^{3c_1}x^3+8\sqrt{e^{3c_1}\left(x^3+e^{3c_1}\right)^3}-8e^{6c_1}}\right)$$

$$y(x) \to \frac{1}{24}\left(2x^2 + \frac{i\left(\sqrt{3}+i\right)x(x^3-8e^{3c_1})}{\sqrt[3]{x^6+20e^{3c_1}x^3+8\sqrt{e^{3c_1}\left(x^3+e^{3c_1}\right)^3}-8e^{6c_1}}} - \left(1+i\sqrt{3}\right)\sqrt[3]{x^6+20e^{3c_1}x^3+8\sqrt{e^{3c_1}\left(x^3+e^{3c_1}\right)^3}-8e^{6c_1}}\right)$$

29.14 problem 836

Internal problem ID [4075]

Book: Ordinary differential equations and their solutions. By George Moseley Murphy. 1960
Section: Various 29
Problem number: 836.
ODE order: 1.
ODE degree: 2.

CAS Maple gives this as type `[[_homogeneous, ‘class G‘]]`

$$3y'^2 + 4xy' - y = -x^2$$

✓ Solution by Maple

Time used: 0.047 (sec). Leaf size: 93

```
dsolve(3*diff(y(x),x)^2+4*x*diff(y(x),x)+x^2-y(x) = 0,y(x), singsol=all)
```

$$y(x) = -\frac{x^2}{3}$$
$$y(x) = -\frac{x^2}{4} + \frac{\sqrt{3}\, c_1 x}{6} + \frac{c_1^2}{4}$$
$$y(x) = -\frac{x^2}{4} - \frac{\sqrt{3}\, c_1 x}{6} + \frac{c_1^2}{4}$$
$$y(x) = -\frac{x^2}{4} - \frac{\sqrt{3}\, c_1 x}{6} + \frac{c_1^2}{4}$$
$$y(x) = -\frac{x^2}{4} + \frac{\sqrt{3}\, c_1 x}{6} + \frac{c_1^2}{4}$$

✓ Solution by Mathematica

Time used: 4.13 (sec). Leaf size: 121

```
DSolve[3 (y'[x])^2+4 x y'[x]+x^2-y[x]==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to \frac{1}{12}\left(-3x^2 + 2x - 2e^{c_1}(x+1) + 1 + e^{2c_1}\right)$$
$$y(x) \to \frac{-3x^2 - 3x^2\tanh^2\left(\frac{c_1}{2}\right) + 4x + 2(3x-2)x\tanh\left(\frac{c_1}{2}\right) + 4}{12\left(-1+\tanh\left(\frac{c_1}{2}\right)\right)^2}$$
$$y(x) \to -\frac{x^2}{3}$$
$$y(x) \to \frac{1}{12}\left(-3x^2 + 2x + 1\right)$$

29.15 problem 837

Internal problem ID [4076]

Book: Ordinary differential equations and their solutions. By George Moseley Murphy. 1960
Section: Various 29
Problem number: 837.
ODE order: 1.
ODE degree: 2.

CAS Maple gives this as type `[_quadrature]`

$$\boxed{4y'^2 = 9x}$$

✓ Solution by Maple

Time used: 0.062 (sec). Leaf size: 19

```
dsolve(4*diff(y(x),x)^2 = 9*x,y(x), singsol=all)
```

$$y(x) = -x^{\frac{3}{2}} + c_1$$
$$y(x) = x^{\frac{3}{2}} + c_1$$

✓ Solution by Mathematica

Time used: 0.004 (sec). Leaf size: 27

```
DSolve[4 (y'[x])^2==9 x,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to -x^{3/2} + c_1$$
$$y(x) \to x^{3/2} + c_1$$

29.16 problem 838

Internal problem ID [4077]

Book: Ordinary differential equations and their solutions. By George Moseley Murphy. 1960
Section: Various 29
Problem number: 838.
ODE order: 1.
ODE degree: 2.

CAS Maple gives this as type `[[_1st_order, _with_linear_symmetries]]`

$$4{y'}^2 + 2x\,\mathrm{e}^{-2y}y' - \mathrm{e}^{-2y} = 0$$

✓ Solution by Maple

Time used: 0.157 (sec). Leaf size: 103

```
dsolve(4*diff(y(x),x)^2+2*x*exp(-2*y(x))*diff(y(x),x)-exp(-2*y(x)) = 0,y(x), singsol=all)
```

$$y(x) = -\ln(2) - \frac{\ln\left(-\frac{1}{x^2}\right)}{2}$$

$$y(x) = c_1 - \operatorname{arctanh}\left(\frac{x}{\operatorname{RootOf}\left(\_Z^2 - x^2 - 4\,\mathrm{e}^{\operatorname{RootOf}\left(4\,\mathrm{e}^{\_Z}\sinh\left(-\frac{\_Z}{2}+c_1\right)^2 - x^2\right)}\right)}\right)$$

$$y(x) = c_1 + \operatorname{arctanh}\left(\frac{x}{\operatorname{RootOf}\left(\_Z^2 - x^2 - 4\,\mathrm{e}^{\operatorname{RootOf}\left(4\,\mathrm{e}^{\_Z}\sinh\left(-\frac{\_Z}{2}+c_1\right)^2 - x^2\right)}\right)}\right)$$

✓ Solution by Mathematica

Time used: 10.24 (sec). Leaf size: 119

```
DSolve[4 (y'[x])^2+2 x Exp[-2 y[x]] y'[x]-Exp[-2 y[x]]==0,y[x],x,IncludeSingularSolutions ->
```

$$y(x) \to \log\left(-e^{\frac{c_1}{2}}\sqrt{-x+e^{c_1}}\right)$$
$$y(x) \to \log\left(e^{\frac{c_1}{2}}\sqrt{-x+e^{c_1}}\right)$$
$$y(x) \to \log\left(-e^{\frac{c_1}{2}}\sqrt{x+e^{c_1}}\right)$$
$$y(x) \to \log\left(e^{\frac{c_1}{2}}\sqrt{x+e^{c_1}}\right)$$
$$y(x) \to \frac{1}{2}\log\left(-\frac{x^2}{4}\right)$$

29.17 problem 839

Internal problem ID [4078]

Book: Ordinary differential equations and their solutions. By George Moseley Murphy. 1960
Section: Various 29
Problem number: 839.
ODE order: 1.
ODE degree: 2.

CAS Maple gives this as type `[[_homogeneous, ‘class C‘], _dAlembert]`

$$4{y'}^2 + 2\,\mathrm{e}^{-2y+2x}y' - \mathrm{e}^{-2y+2x} = 0$$

✓ Solution by Maple

Time used: 2.485 (sec). Leaf size: 115

```
dsolve(4*diff(y(x),x)^2+2*exp(2*x-2*y(x))*diff(y(x),x)-exp(2*x-2*y(x)) = 0,y(x), singsol=all
```

$$y(x) = c_1 - \operatorname{arctanh}\left(\frac{1}{\operatorname{RootOf}\left(\_Z^2 - 4\,\mathrm{e}^{\operatorname{RootOf}\left(4\,\mathrm{e}^{\_Z}\cosh\left(-\frac{\_Z}{2}-x+c_1\right)^2+16\,\mathrm{e}^{2\_Z}\sinh\left(-\frac{\_Z}{2}-x+c_1\right)^2-8\,\mathrm{e}^{\_Z}-1\right)} - 1\right)}\right)$$

$$y(x) = c_1 + \operatorname{arctanh}\left(\frac{1}{\operatorname{RootOf}\left(\_Z^2 - 4\,\mathrm{e}^{\operatorname{RootOf}\left(4\,\mathrm{e}^{\_Z}\cosh\left(-\frac{\_Z}{2}-x+c_1\right)^2+16\,\mathrm{e}^{2\_Z}\sinh\left(-\frac{\_Z}{2}-x+c_1\right)^2-8\,\mathrm{e}^{\_Z}-1\right)} - 1\right)}\right)$$

✓ Solution by Mathematica

Time used: 1.709 (sec). Leaf size: 332

```
DSolve[4 (y'[x])^2+2 Exp[2 x-2 y[x]] y'[x]-Exp[2 x-2 y[x]]==0,y[x],x,IncludeSingularSolution
```

$$\text{Solve}\left[-\frac{2e^{-x}\sqrt{4e^{2(y(x)+x)}+e^{4x}}\operatorname{arctanh}\left(\frac{-\sqrt{4e^{2y(x)}+e^{2x}}+e^x+1}{\sqrt{4e^{2y(x)}+e^{2x}}-e^x+1}\right)}{\sqrt{4e^{2y(x)}+e^{2x}}} - \frac{e^{-x}\sqrt{4e^{2(y(x)+x)}+e^{4x}}y(x)}{\sqrt{4e^{2y(x)}+e^{2x}}}+y(x)=c_1,y(x)\right]$$

$$\text{Solve}\left[\frac{2e^{-x}\sqrt{4e^{2(y(x)+x)}+e^{4x}}\operatorname{arctanh}\left(\frac{-\sqrt{4e^{2y(x)}+e^{2x}}+e^x+1}{\sqrt{4e^{2y(x)}+e^{2x}}-e^x+1}\right)}{\sqrt{4e^{2y(x)}+e^{2x}}} + \frac{e^{-x}\sqrt{4e^{2(y(x)+x)}+e^{4x}}y(x)}{\sqrt{4e^{2y(x)}+e^{2x}}}+y(x)=c_1,y(x)\right]$$

$$y(x)\to\frac{1}{2}\left(\log\left(-\frac{e^{4x}}{4}\right)-2x\right)$$

29.18 problem 840

Internal problem ID [4079]

Book: Ordinary differential equations and their solutions. By George Moseley Murphy. 1960
Section: Various 29
Problem number: 840.
ODE order: 1.
ODE degree: 2.

CAS Maple gives this as type `[[_1st_order, _with_linear_symmetries], _dAlembert]`

$$5{y'}^2 + 3xy' - y = 0$$

✓ Solution by Maple

Time used: 0.063 (sec). Leaf size: 85

```
dsolve(5*diff(y(x),x)^2+3*x*diff(y(x),x)-y(x) = 0,y(x), singsol=all)
```

$$\frac{c_1}{\left(-30x - 10\sqrt{9x^2 + 20y(x)}\right)^{\frac{3}{2}}} + \frac{2x}{5} - \frac{\sqrt{9x^2 + 20y(x)}}{5} = 0$$
$$\frac{c_1}{\left(-30x + 10\sqrt{9x^2 + 20y(x)}\right)^{\frac{3}{2}}} + \frac{2x}{5} + \frac{\sqrt{9x^2 + 20y(x)}}{5} = 0$$

✓ Solution by Mathematica

Time used: 14.84 (sec). Leaf size: 771

```
DSolve[5 (y'[x])^2+3 x y'[x]-y[x]==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to \text{Root}\left[16\#1^5 + 8\#1^4x^2 + \#1^3x^4 + 4000\#1^2e^{5c_1}x + 1800\#1e^{5c_1}x^3 + 216e^{5c_1}x^5 - 200000e^{10c_1}\&, 1\right]$$
$$y(x) \to \text{Root}\left[16\#1^5 + 8\#1^4x^2 + \#1^3x^4 + 4000\#1^2e^{5c_1}x + 1800\#1e^{5c_1}x^3 + 216e^{5c_1}x^5 - 200000e^{10c_1}\&, 2\right]$$
$$y(x) \to \text{Root}\left[16\#1^5 + 8\#1^4x^2 + \#1^3x^4 + 4000\#1^2e^{5c_1}x + 1800\#1e^{5c_1}x^3 + 216e^{5c_1}x^5 - 200000e^{10c_1}\&, 3\right]$$
$$y(x) \to \text{Root}\left[16\#1^5 + 8\#1^4x^2 + \#1^3x^4 + 4000\#1^2e^{5c_1}x + 1800\#1e^{5c_1}x^3 + 216e^{5c_1}x^5 - 200000e^{10c_1}\&, 4\right]$$
$$y(x) \to \text{Root}\left[16\#1^5 + 8\#1^4x^2 + \#1^3x^4 + 4000\#1^2e^{5c_1}x + 1800\#1e^{5c_1}x^3 + 216e^{5c_1}x^5 - 200000e^{10c_1}\&, 5\right]$$
$$y(x) \to \text{Root}\left[3200000\#1^5 + 1600000\#1^4x^2 + 200000\#1^3x^4 - 4000\#1^2e^{5c_1}x - 1800\#1e^{5c_1}x^3 - 216e^{5c_1}x^5 - e^{10c_1}\&, 1\right]$$
$$y(x) \to \text{Root}\left[3200000\#1^5 + 1600000\#1^4x^2 + 200000\#1^3x^4 - 4000\#1^2e^{5c_1}x - 1800\#1e^{5c_1}x^3 - 216e^{5c_1}x^5 - e^{10c_1}\&, 2\right]$$
$$y(x) \to \text{Root}\left[3200000\#1^5 + 1600000\#1^4x^2 + 200000\#1^3x^4 - 4000\#1^2e^{5c_1}x - 1800\#1e^{5c_1}x^3 - 216e^{5c_1}x^5 - e^{10c_1}\&, 3\right]$$
$$y(x) \to \text{Root}\left[3200000\#1^5 + 1600000\#1^4x^2 + 200000\#1^3x^4 - 4000\#1^2e^{5c_1}x - 1800\#1e^{5c_1}x^3 - 216e^{5c_1}x^5 - e^{10c_1}\&, 4\right]$$
$$y(x) \to \text{Root}\left[3200000\#1^5 + 1600000\#1^4x^2 + 200000\#1^3x^4 - 4000\#1^2e^{5c_1}x - 1800\#1e^{5c_1}x^3 - 216e^{5c_1}x^5 - e^{10c_1}\&, 5\right]$$
$$y(x) \to 0$$

29.19 problem 841

Internal problem ID [4080]

Book: Ordinary differential equations and their solutions. By George Moseley Murphy. 1960
Section: Various 29
Problem number: 841.
ODE order: 1.
ODE degree: 2.

CAS Maple gives this as type `[[_1st_order, _with_linear_symmetries], _dAlembert]`

$$\boxed{5{y'}^2 + 6xy' - 2y = 0}$$

✓ Solution by Maple

Time used: 0.063 (sec). Leaf size: 85

```
dsolve(5*diff(y(x),x)^2+6*x*diff(y(x),x)-2*y(x) = 0,y(x), singsol=all)
```

$$\frac{c_1}{\left(-15x - 5\sqrt{9x^2 + 10y(x)}\right)^{\frac{3}{2}}} + \frac{2x}{5} - \frac{\sqrt{9x^2 + 10y(x)}}{5} = 0$$

$$\frac{c_1}{\left(-15x + 5\sqrt{9x^2 + 10y(x)}\right)^{\frac{3}{2}}} + \frac{2x}{5} + \frac{\sqrt{9x^2 + 10y(x)}}{5} = 0$$

✓ Solution by Mathematica

Time used: 15.032 (sec). Leaf size: 771

```
DSolve[5 (y'[x])^2+6 x y'[x]-2 y[x]==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to \text{Root}\left[4\#1^5 + 4\#1^4x^2 + \#1^3x^4 + 1000\#1^2e^{5c_1}x + 900\#1e^{5c_1}x^3 + 216e^{5c_1}x^5 - 25000e^{10c_1}\&, 1\right]$$
$$y(x) \to \text{Root}\left[4\#1^5 + 4\#1^4x^2 + \#1^3x^4 + 1000\#1^2e^{5c_1}x + 900\#1e^{5c_1}x^3 + 216e^{5c_1}x^5 - 25000e^{10c_1}\&, 2\right]$$
$$y(x) \to \text{Root}\left[4\#1^5 + 4\#1^4x^2 + \#1^3x^4 + 1000\#1^2e^{5c_1}x + 900\#1e^{5c_1}x^3 + 216e^{5c_1}x^5 - 25000e^{10c_1}\&, 3\right]$$
$$y(x) \to \text{Root}\left[4\#1^5 + 4\#1^4x^2 + \#1^3x^4 + 1000\#1^2e^{5c_1}x + 900\#1e^{5c_1}x^3 + 216e^{5c_1}x^5 - 25000e^{10c_1}\&, 4\right]$$
$$y(x) \to \text{Root}\left[4\#1^5 + 4\#1^4x^2 + \#1^3x^4 + 1000\#1^2e^{5c_1}x + 900\#1e^{5c_1}x^3 + 216e^{5c_1}x^5 - 25000e^{10c_1}\&, 5\right]$$
$$y(x) \to \text{Root}\left[100000\#1^5 + 100000\#1^4x^2 + 25000\#1^3x^4 - 1000\#1^2e^{5c_1}x - 900\#1e^{5c_1}x^3 - 216e^{5c_1}x^5 - e^{10c_1}\&, 1\right]$$
$$y(x) \to \text{Root}\left[100000\#1^5 + 100000\#1^4x^2 + 25000\#1^3x^4 - 1000\#1^2e^{5c_1}x - 900\#1e^{5c_1}x^3 - 216e^{5c_1}x^5 - e^{10c_1}\&, 2\right]$$
$$y(x) \to \text{Root}\left[100000\#1^5 + 100000\#1^4x^2 + 25000\#1^3x^4 - 1000\#1^2e^{5c_1}x - 900\#1e^{5c_1}x^3 - 216e^{5c_1}x^5 - e^{10c_1}\&, 3\right]$$
$$y(x) \to \text{Root}\left[100000\#1^5 + 100000\#1^4x^2 + 25000\#1^3x^4 - 1000\#1^2e^{5c_1}x - 900\#1e^{5c_1}x^3 - 216e^{5c_1}x^5 - e^{10c_1}\&, 4\right]$$
$$y(x) \to \text{Root}\left[100000\#1^5 + 100000\#1^4x^2 + 25000\#1^3x^4 - 1000\#1^2e^{5c_1}x - 900\#1e^{5c_1}x^3 - 216e^{5c_1}x^5 - e^{10c_1}\&, 5\right]$$
$$y(x) \to 0$$

29.20 problem 842

Internal problem ID [4081]

Book: Ordinary differential equations and their solutions. By George Moseley Murphy. 1960
Section: Various 29
Problem number: 842.
ODE order: 1.
ODE degree: 2.

CAS Maple gives this as type `[[_1st_order, _with_linear_symmetries]]`

$$9{y'}^2+3xy^4y'+y^5=0$$

✓ Solution by Maple
Time used: 0.265 (sec). Leaf size: 102

```
dsolve(9*diff(y(x),x)^2+3*x*y(x)^4*diff(y(x),x)+y(x)^5 = 0,y(x), singsol=all)
```

$$y(x)=\frac{2^{\frac{2}{3}}}{x^{\frac{2}{3}}}$$
$$y(x)=-\frac{2^{\frac{2}{3}}\left(1+i\sqrt{3}\right)}{2x^{\frac{2}{3}}}$$
$$y(x)=\frac{2^{\frac{2}{3}}\left(i\sqrt{3}-1\right)}{2x^{\frac{2}{3}}}$$
$$y(x)=0$$
$$y(x)=\frac{\operatorname{RootOf}\left(-2\ln\left(x\right)+3\left(\int^{\_Z}\frac{\_a^3+\sqrt{\_a^3\left(\_a^3-4\right)}-4}{\_a\left(\_a^3-4\right)}d\_a\right)+2c_1\right)}{x^{\frac{2}{3}}}$$

✓ Solution by Mathematica

Time used: 1.075 (sec). Leaf size: 212

```
DSolve[9 (y'[x])^2+3 x y[x]^4 y'[x]+y[x]^5==0,y[x],x,IncludeSingularSolutions -> True]
```

$$\text{Solve}\left[-\frac{\sqrt{x^2y(x)^3-4}\,y(x)^{5/2}\text{arctanh}\left(\frac{xy(x)^{3/2}}{\sqrt{x^2y(x)^3-4}}\right)}{\sqrt{y(x)^5\left(x^2y(x)^3-4\right)}}-\frac{3}{2}\log(y(x))=c_1,y(x)\right]$$

$$\text{Solve}\left[\frac{y(x)^{5/2}\sqrt{x^2y(x)^3-4}\,\text{arctanh}\left(\frac{xy(x)^{3/2}}{\sqrt{x^2y(x)^3-4}}\right)}{\sqrt{y(x)^5\left(x^2y(x)^3-4\right)}}-\frac{3}{2}\log(y(x))=c_1,y(x)\right]$$

$$y(x)\to 0$$

$$y(x)\to \frac{(-2)^{2/3}}{x^{2/3}}$$

$$y(x)\to \frac{2^{2/3}}{x^{2/3}}$$

$$y(x)\to -\frac{\sqrt[3]{-1}2^{2/3}}{x^{2/3}}$$

29.21 problem 843

Internal problem ID [4082]

Book: Ordinary differential equations and their solutions. By George Moseley Murphy. 1960
Section: Various 29
Problem number: 843.
ODE order: 1.
ODE degree: 2.

CAS Maple gives this as type `[_quadrature]`

$$\boxed{{y'}^2 x = a}$$

✓ Solution by Maple
Time used: 0.062 (sec). Leaf size: 25

```
dsolve(x*diff(y(x),x)^2 = a,y(x), singsol=all)
```

$$y(x) = 2\sqrt{ax} + c_1$$
$$y(x) = -2\sqrt{ax} + c_1$$

✓ Solution by Mathematica
Time used: 0.005 (sec). Leaf size: 39

```
DSolve[x (y'[x])^2==a,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to -2\sqrt{a}\sqrt{x} + c_1$$
$$y(x) \to 2\sqrt{a}\sqrt{x} + c_1$$

29.22 problem 844

Internal problem ID [4083]

Book: Ordinary differential equations and their solutions. By George Moseley Murphy. 1960
Section: Various 29
Problem number: 844.
ODE order: 1.
ODE degree: 2.

CAS Maple gives this as type `[_quadrature]`

$$\boxed{y'^2 x = -x^2 + a}$$

✓ Solution by Maple
Time used: 0.063 (sec). Leaf size: 47

```
dsolve(x*diff(y(x),x)^2 = -x^2+a,y(x), singsol=all)
```

$$y(x) = \int \frac{\sqrt{x\left(-x^2+a\right)}}{x} dx + c_1$$
$$y(x) = -\left(\int \frac{\sqrt{x\left(-x^2+a\right)}}{x} dx\right) + c_1$$

✓ Solution by Mathematica
Time used: 5.7 (sec). Leaf size: 113

```
DSolve[x (y'[x])^2==(a-x^2),y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \rightarrow -\frac{2\sqrt{x}\sqrt{a-x^2}\,\text{Hypergeometric2F1}\left(-\frac{1}{2},\frac{1}{4},\frac{5}{4},\frac{x^2}{a}\right)}{\sqrt{1-\frac{x^2}{a}}} + c_1$$
$$y(x) \rightarrow \frac{2\sqrt{x}\sqrt{a-x^2}\,\text{Hypergeometric2F1}\left(-\frac{1}{2},\frac{1}{4},\frac{5}{4},\frac{x^2}{a}\right)}{\sqrt{1-\frac{x^2}{a}}} + c_1$$

29.23 problem 845

Internal problem ID [4084]

Book: Ordinary differential equations and their solutions. By George Moseley Murphy. 1960
Section: Various 29
Problem number: 845.
ODE order: 1.
ODE degree: 2.

CAS Maple gives this as type `[[_homogeneous, 'class A'], _rational, _dAlembert]`

$$\boxed{y'^2x - y = 0}$$

✓ Solution by Maple
Time used: 0.062 (sec). Leaf size: 39

```
dsolve(x*diff(y(x),x)^2 = y(x),y(x), singsol=all)
```

$$y(x) = 0$$
$$y(x) = \frac{\left(x + \sqrt{c_1 x}\right)^2}{x}$$
$$y(x) = \frac{\left(-x + \sqrt{c_1 x}\right)^2}{x}$$

✓ Solution by Mathematica
Time used: 0.047 (sec). Leaf size: 46

```
DSolve[x (y'[x])^2==y[x],y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to \frac{1}{4}\left(-2\sqrt{x} + c_1\right)^2$$
$$y(x) \to \frac{1}{4}\left(2\sqrt{x} + c_1\right)^2$$
$$y(x) \to 0$$

29.24 problem 846

Internal problem ID [4085]

Book: Ordinary differential equations and their solutions. By George Moseley Murphy. 1960
Section: Various 29
Problem number: 846.
ODE order: 1.
ODE degree: 2.

CAS Maple gives this as type `[[_homogeneous, ‘class A‘], _rational, _dAlembert]`

$$\boxed{{y'}^2 x - 2y = -x}$$

✓ Solution by Maple

Time used: 0.078 (sec). Leaf size: 96

```
dsolve(x*diff(y(x),x)^2+x-2*y(x) = 0,y(x), singsol=all)
```

$$y(x) = \frac{\left(2\,\mathrm{LambertW}\left(\frac{\sqrt{c_1 x}}{c_1}\right)^2 + 2\,\mathrm{LambertW}\left(\frac{\sqrt{c_1 x}}{c_1}\right) + 1\right) x}{2\,\mathrm{LambertW}\left(\frac{\sqrt{c_1 x}}{c_1}\right)^2}$$
$$y(x) = \frac{\left(2\,\mathrm{LambertW}\left(-\frac{\sqrt{c_1 x}}{c_1}\right)^2 + 2\,\mathrm{LambertW}\left(-\frac{\sqrt{c_1 x}}{c_1}\right) + 1\right) x}{2\,\mathrm{LambertW}\left(-\frac{\sqrt{c_1 x}}{c_1}\right)^2}$$

✓ Solution by Mathematica

Time used: 0.642 (sec). Leaf size: 97

```
DSolve[x (y'[x])^2+x-2 y[x]==0,y[x],x,IncludeSingularSolutions -> True]
```

$$\mathrm{Solve}\left[\frac{2}{\sqrt{\frac{2y(x)}{x} - 1} - 1} - 2\log\left(\sqrt{\frac{2y(x)}{x} - 1} - 1\right) = \log(x) + c_1, y(x)\right]$$
$$\mathrm{Solve}\left[\frac{2}{\sqrt{\frac{2y(x)}{x} - 1} + 1} + 2\log\left(\sqrt{\frac{2y(x)}{x} - 1} + 1\right) = -\log(x) + c_1, y(x)\right]$$

29.25 problem 847

Internal problem ID [4086]

Book: Ordinary differential equations and their solutions. By George Moseley Murphy. 1960
Section: Various 29
Problem number: 847.
ODE order: 1.
ODE degree: 2.

CAS Maple gives this as type `[_rational, _dAlembert]`

$$\boxed{y'^2 x + y' - y = 0}$$

✓ Solution by Maple

Time used: 0.047 (sec). Leaf size: 59

```
dsolve(x*diff(y(x),x)^2+diff(y(x),x) = y(x),y(x), singsol=all)
```

$$y(x) = 2\,e^{\mathrm{RootOf}\left(-x\,e^{2\_Z}+2\,e^{\_Z}x+\_Z+c_1-x-e^{\_Z}\right)}x + \mathrm{RootOf}\left(-x\,e^{2\_Z}+2\,e^{\_Z}x+\_Z+c_1-x-e^{\_Z}\right)+c_1-x$$

✓ Solution by Mathematica

Time used: 0.918 (sec). Leaf size: 46

```
DSolve[x (y'[x])^2+y'[x]==y[x],y[x],x,IncludeSingularSolutions -> True]
```

$$\mathrm{Solve}\left[\left\{x=\frac{\log(K[1])-K[1]}{(K[1]-1)^2}+\frac{c_1}{(K[1]-1)^2},y(x)=xK[1]^2+K[1]\right\},\{y(x),K[1]\}\right]$$

29.26 problem 848

Internal problem ID [4087]

Book: Ordinary differential equations and their solutions. By George Moseley Murphy. 1960
Section: Various 29
Problem number: 848.
ODE order: 1.
ODE degree: 2.

CAS Maple gives this as type `[_rational, _dAlembert]`

$$\boxed{y'^2x + 2y' - y = 0}$$

✓ Solution by Maple

Time used: 0.062 (sec). Leaf size: 65

```
dsolve(x*diff(y(x),x)^2+2*diff(y(x),x)-y(x) = 0,y(x), singsol=all)
```

$$\begin{aligned} y(x) = {} & 2\,\mathrm{e}^{\mathrm{RootOf}\left(-x\,\mathrm{e}^{2\_Z}+2\,\mathrm{e}^{\_Z}x-2\,\mathrm{e}^{\_Z}+c_1+2\_Z-x\right)}x \\ & + 2\,\mathrm{RootOf}\left(-x\,\mathrm{e}^{2\_Z}+2\,\mathrm{e}^{\_Z}x-2\,\mathrm{e}^{\_Z}+c_1+2\_Z-x\right)+c_1-x \end{aligned}$$

✓ Solution by Mathematica

Time used: 13.72 (sec). Leaf size: 50

```
DSolve[x (y'[x])^2+2 y'[x]-y[x]==0,y[x],x,IncludeSingularSolutions -> True]
```

$$\mathrm{Solve}\left[\left\{x = \frac{2\log(K[1]) - 2K[1]}{(K[1]-1)^2} + \frac{c_1}{(K[1]-1)^2}, y(x) = xK[1]^2 + 2K[1]\right\}, \{y(x), K[1]\}\right]$$

29.27 problem 849

Internal problem ID [4088]

Book: Ordinary differential equations and their solutions. By George Moseley Murphy. 1960
Section: Various 29
Problem number: 849.
ODE order: 1.
ODE degree: 2.

CAS Maple gives this as type `[_rational, _dAlembert]`

$$\boxed{{y'}^2 x - 2y' - y = 0}$$

✓ Solution by Maple

Time used: 0.062 (sec). Leaf size: 65

```
dsolve(x*diff(y(x),x)^2-2*diff(y(x),x)-y(x) = 0,y(x), singsol=all)
```

$$y(x) = 2\,\mathrm{e}^{\mathrm{RootOf}\left(-x\,\mathrm{e}^{2\_Z}+2\,\mathrm{e}^{\_Z}x+2\,\mathrm{e}^{\_Z}+c_1-2\_Z-x\right)}x \\ -2\,\mathrm{RootOf}\left(-x\,\mathrm{e}^{2\_Z}+2\,\mathrm{e}^{\_Z}x+2\,\mathrm{e}^{\_Z}+c_1-2\_Z-x\right)+c_1-x$$

✓ Solution by Mathematica

Time used: 1.475 (sec). Leaf size: 50

```
DSolve[x (y'[x])^2-2 y'[x]-y[x]==0,y[x],x,IncludeSingularSolutions -> True]
```

$$\mathrm{Solve}\left[\left\{x = \frac{2K[1]-2\log(K[1])}{(K[1]-1)^2}+\frac{c_1}{(K[1]-1)^2}, y(x) = xK[1]^2-2K[1]\right\}, \{y(x), K[1]\}\right]$$

29.28 problem 850

Internal problem ID [4089]

Book: Ordinary differential equations and their solutions. By George Moseley Murphy. 1960
Section: Various 29
Problem number: 850.
ODE order: 1.
ODE degree: 2.

CAS Maple gives this as type `[_rational, _dAlembert]`

$$\boxed{y'^2x + 4y' - 2y = 0}$$

✓ Solution by Maple

Time used: 0.047 (sec). Leaf size: 67

```
dsolve(x*diff(y(x),x)^2+4*diff(y(x),x)-2*y(x) = 0,y(x), singsol=all)
```

$$y(x) = 2\,\mathrm{e}^{\mathrm{RootOf}\left(-x\,\mathrm{e}^{2\_Z}+4\,\mathrm{e}^{\_Z}x-4\,\mathrm{e}^{\_Z}+c_1+8\_Z-4x\right)}x + 4\,\mathrm{RootOf}\left(-x\,\mathrm{e}^{2\_Z}+4\,\mathrm{e}^{\_Z}x-4\,\mathrm{e}^{\_Z}+c_1+8\_Z-4x\right) + \frac{c_1}{2} - 2x$$

✓ Solution by Mathematica

Time used: 30.862 (sec). Leaf size: 90

```
DSolve[x (y'[x])^2+4 y'[x]-2 y[x]==0,y[x],x,IncludeSingularSolutions -> True]
```

$$\mathrm{Solve}\left[\left\{x = -\frac{2(2K[1]-y(K[1]))}{K[1]^2}, y(x) = 4\left(\frac{2}{K[1]}+\log(K[1])\right)\exp\left(-4\left(\frac{1}{2}\log(2-K[1])-\frac{1}{2}\log(K[1])\right)\right) + c_1\exp\left(-4\left(\frac{1}{2}\log(2-K[1])-\frac{1}{2}\log(K[1])\right)\right)\right\}, \{y(x), K[1]\}\right]$$

29.29 problem 851

Internal problem ID [4090]

Book: Ordinary differential equations and their solutions. By George Moseley Murphy. 1960
Section: Various 29
Problem number: 851.
ODE order: 1.
ODE degree: 2.

CAS Maple gives this as type `[[_homogeneous, ‘class A‘], _rational, _dAlembert]`

$$\boxed{{y'}^2 x + x y' - y = 0}$$

✓ Solution by Maple

Time used: 0.078 (sec). Leaf size: 65

```
dsolve(x*diff(y(x),x)^2+x*diff(y(x),x)-y(x) = 0,y(x), singsol=all)
```

$$y(x) = \frac{\left(1 + 2\,\mathrm{LambertW}\left(-\frac{1}{2\sqrt{\frac{c_1}{x}}}\right)\right) x}{4\,\mathrm{LambertW}\left(-\frac{1}{2\sqrt{\frac{c_1}{x}}}\right)^2}$$

$$y(x) = \frac{\left(1 + 2\,\mathrm{LambertW}\left(\frac{1}{2\sqrt{\frac{c_1}{x}}}\right)\right) x}{4\,\mathrm{LambertW}\left(\frac{1}{2\sqrt{\frac{c_1}{x}}}\right)^2}$$

✓ Solution by Mathematica

Time used: 0.58 (sec). Leaf size: 102

```
DSolve[x (y'[x])^2+x y'[x]-y[x]==0,y[x],x,IncludeSingularSolutions -> True]
```

$$\text{Solve}\left[\frac{1}{\sqrt{\frac{4y(x)}{x}+1}-1}-\log\left(\sqrt{\frac{4y(x)}{x}+1}-1\right)=\frac{\log(x)}{2}+c_1,y(x)\right]$$
$$\text{Solve}\left[\frac{1}{\sqrt{\frac{4y(x)}{x}+1}+1}+\log\left(\sqrt{\frac{4y(x)}{x}+1}+1\right)=-\frac{\log(x)}{2}+c_1,y(x)\right]$$
$$y(x)\to 0$$

29.30 problem 852

Internal problem ID [4091]

Book: Ordinary differential equations and their solutions. By George Moseley Murphy. 1960
Section: Various 29
Problem number: 852.
ODE order: 1.
ODE degree: 2.

CAS Maple gives this as type `[_quadrature]`

$$\boxed{y'^2 x - (x^2+1)\, y' = -x}$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 18

```
dsolve(x*diff(y(x),x)^2-(x^2+1)*diff(y(x),x)+x = 0,y(x), singsol=all)
```

$$y(x) = \ln(x) + c_1$$
$$y(x) = \frac{x^2}{2} + c_1$$

✓ Solution by Mathematica

Time used: 0.003 (sec). Leaf size: 24

```
DSolve[x (y'[x])^2-(1+x^2)y'[x]+x==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to \frac{x^2}{2} + c_1$$
$$y(x) \to \log(x) + c_1$$

29.31 problem 853

Internal problem ID [4092]

Book: Ordinary differential equations and their solutions. By George Moseley Murphy. 1960
Section: Various 29
Problem number: 853.
ODE order: 1.
ODE degree: 2.

CAS Maple gives this as type `[[_homogeneous, 'class G'], _dAlembert]`

$$
{y'}^2 x + y y' = -a
$$

✓ Solution by Maple

Time used: 0.078 (sec). Leaf size: 177

```
dsolve(x*diff(y(x),x)^2+y(x)*diff(y(x),x)+a = 0,y(x), singsol=all)
```

$$
-\frac{8\left(-\frac{3c_1\left(y(x)-\sqrt{-4ax+y(x)^2}\right)\sqrt{\frac{-y(x)+\sqrt{-4ax+y(x)^2}}{x}}}{8}+ax-\frac{3y(x)^2}{4}+\frac{3y(x)\sqrt{-4ax+y(x)^2}}{4}\right)x}{3\left(y(x)-\sqrt{-4ax+y(x)^2}\right)^2}=0
$$

$$
-\frac{8x\left(\frac{3c_1\left(y(x)+\sqrt{-4ax+y(x)^2}\right)\sqrt{\frac{-2y(x)-2\sqrt{-4ax+y(x)^2}}{x}}}{4}+ax-\frac{3y(x)^2}{4}-\frac{3y(x)\sqrt{-4ax+y(x)^2}}{4}\right)}{3\left(y(x)+\sqrt{-4ax+y(x)^2}\right)^2}=0
$$

✓ Solution by Mathematica

Time used: 60.29 (sec). Leaf size: 4845

```
DSolve[x (y'[x])^2+y[x] y'[x]+a==0,y[x],x,IncludeSingularSolutions -> True]
```

Too large to display

29.32 problem 854

Internal problem ID [4093]

Book: Ordinary differential equations and their solutions. By George Moseley Murphy. 1960
Section: Various 29
Problem number: 854.
ODE order: 1.
ODE degree: 2.

CAS Maple gives this as type `[[_homogeneous, ‘class G‘], _rational, _Clairaut]`

$$\boxed{{y'}^2 x - y y' = -a}$$

✓ Solution by Maple
Time used: 0.078 (sec). Leaf size: 35

```
dsolve(x*diff(y(x),x)^2-y(x)*diff(y(x),x)+a = 0,y(x), singsol=all)
```

$$y(x) = -2\sqrt{a x}$$
$$y(x) = 2\sqrt{a x}$$
$$y(x) = \frac{x c_1^2 + a}{c_1}$$

✓ Solution by Mathematica
Time used: 0.013 (sec). Leaf size: 53

```
DSolve[x (y'[x])^2-y[x] y'[x]+a==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to \frac{a}{c_1} + c_1 x$$
$$y(x) \to \text{Indeterminate}$$
$$y(x) \to -2\sqrt{a}\sqrt{x}$$
$$y(x) \to 2\sqrt{a}\sqrt{x}$$

29.33 problem 855

Internal problem ID [4094]

Book: Ordinary differential equations and their solutions. By George Moseley Murphy. 1960
Section: Various 29
Problem number: 855.
ODE order: 1.
ODE degree: 2.

CAS Maple gives this as type `[[_homogeneous, ‘class A‘], _rational, _dAlembert]`

$$\boxed{{y'}^2 x - y y' = -a x}$$

✓ Solution by Maple

Time used: 0.078 (sec). Leaf size: 50

```
dsolve(x*diff(y(x),x)^2-y(x)*diff(y(x),x)+a*x = 0,y(x), singsol=all)
```

$$y(x) = \left(-\operatorname{LambertW}\left(-\frac{x^2}{c_1^2 a}\right) + 1\right) a c_1 \sqrt{-\frac{x^2}{c_1^2 a \operatorname{LambertW}\left(-\frac{x^2}{c_1^2 a}\right)}}$$

✓ Solution by Mathematica

Time used: 0.924 (sec). Leaf size: 167

```
DSolve[x (y'[x])^2-y[x] y'[x]+a x==0,y[x],x,IncludeSingularSolutions -> True]
```

$$\text{Solve}\left[\frac{4a\arctan\left(\frac{y(x)}{x\sqrt{4a-\frac{y(x)^2}{x^2}}}\right)+\frac{y(x)\left(\sqrt{4a-\frac{y(x)^2}{x^2}}-\frac{iy(x)}{x}\right)}{x}}{8a}=\frac{1}{2}i\log(x)+c_1,y(x)\right]$$

$$\text{Solve}\left[\frac{4a\arctan\left(\frac{y(x)}{x\sqrt{4a-\frac{y(x)^2}{x^2}}}\right)+\frac{y(x)\left(\sqrt{4a-\frac{y(x)^2}{x^2}}+\frac{iy(x)}{x}\right)}{x}}{8a}=c_1-\frac{1}{2}i\log(x),y(x)\right]$$

29.34 problem 857

Internal problem ID [4095]

Book: Ordinary differential equations and their solutions. By George Moseley Murphy. 1960
Section: Various 29
Problem number: 857.
ODE order: 1.
ODE degree: 2.

CAS Maple gives this as type `[[_homogeneous, ‘class G‘]]`

$$\boxed{{y'}^2 x + y y' = -x^3}$$

✓ Solution by Maple

Time used: 0.125 (sec). Leaf size: 272

```
dsolve(x*diff(y(x),x)^2+y(x)*diff(y(x),x)+x^3 = 0,y(x), singsol=all)
```

$$-\left(\int_{\_b}^{x}\frac{y(x)-\sqrt{-4\_a^{4}+y(x)^{2}}}{\_a\left(5y(x)-\sqrt{-4\_a^{4}+y(x)^{2}}\right)}d\_a\right)-2\left(\int^{y(x)}\frac{1+\left(40\_f-8\sqrt{-4x^{4}+\_f^{2}}\right)\left(\int_{\_b}^{x}\frac{\_a^{3}}{\left(-5\_f+\sqrt{-4\_a^{4}+\_f^{2}}\right)^{2}\sqrt{-4\_a^{4}+\_f^{2}}}d\_a\right)}{5\_f-\sqrt{-4x^{4}+\_f^{2}}}d\_f\right)+c_{1}=0$$

$$-\left(\int_{\_b}^{x}\frac{y(x)+\sqrt{-4\_a^{4}+y(x)^{2}}}{\left(\sqrt{-4\_a^{4}+y(x)^{2}}+5y(x)\right)\_a}d\_a\right)+2\left(\int^{y(x)}\frac{-1+8\left(\sqrt{-4x^{4}+\_f^{2}}+5\_f\right)\left(\int_{\_b}^{x}\frac{\_a^{3}}{\left(\sqrt{-4\_a^{4}+\_f^{2}}+5\_f\right)^{2}\sqrt{-4\_a^{4}+\_f^{2}}}d\_a\right)}{\sqrt{-4x^{4}+\_f^{2}}+5\_f}d\_f\right)+c_{1}=0$$

✓ Solution by Mathematica

Time used: 0.825 (sec). Leaf size: 107

```
DSolve[x (y'[x])^2+y[x] y'[x]+x^3==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x)\to x^{2}\text{InverseFunction}\left[\int_{1}^{\#1}\frac{1}{5K[2]+\sqrt{K[2]^{2}-4}}dK[2]\&\right]\left[\int_{1}^{x}-\frac{1}{2K[3]}dK[3]+c_{1}\right]$$

$$y(x)\to x^{2}\text{InverseFunction}\left[\int_{1}^{\#1}\frac{1}{\sqrt{K[4]^{2}-4}-5K[4]}dK[4]\&\right]\left[\int_{1}^{x}\frac{1}{2K[5]}dK[5]+c_{1}\right]$$

29.35 problem 858

Internal problem ID [4096]

Book: Ordinary differential equations and their solutions. By George Moseley Murphy. 1960
Section: Various 29
Problem number: 858.
ODE order: 1.
ODE degree: 2.

CAS Maple gives this as type `[[_homogeneous, ‘class A‘], _rational, _dAlembert]`

$$\boxed{y'^2 x - y y' + y a = 0}$$

✓ Solution by Maple

Time used: 0.078 (sec). Leaf size: 42

```
dsolve(x*diff(y(x),x)^2-y(x)*diff(y(x),x)+a*y(x) = 0,y(x), singsol=all)
```

$$y(x) = 0$$
$$y(x) = -\frac{\left(\operatorname{LambertW}\left(-\frac{x\,\mathrm{e}}{c_1 a}\right) - 1\right)^2 a x}{\operatorname{LambertW}\left(-\frac{x\,\mathrm{e}}{c_1 a}\right)}$$

✓ Solution by Mathematica

Time used: 2.921 (sec). Leaf size: 173

```
DSolve[x (y'[x])^2-y[x] y'[x]+a y[x]==0,y[x],x,IncludeSingularSolutions -> True]
```

$$\text{Solve}\left[\frac{-\sqrt{\frac{y(x)}{x}}\sqrt{\frac{y(x)}{x}-4a}-4a\log\left(\sqrt{\frac{y(x)}{x}-4a}-\sqrt{\frac{y(x)}{x}}\right)+\frac{y(x)}{x}}{4a}=-\frac{\log(x)}{2}+c_1,y(x)\right]$$

$$\text{Solve}\left[-\frac{\sqrt{\frac{y(x)}{x}}\sqrt{\frac{y(x)}{x}-4a}+4a\log\left(\sqrt{\frac{y(x)}{x}-4a}-\sqrt{\frac{y(x)}{x}}\right)+\frac{y(x)}{x}}{4a}=\frac{\log(x)}{2}+c_1,y(x)\right]$$

$$y(x)\to 0$$

30 Various 30

30.1 problem 859

Internal problem ID [4097]

Book: Ordinary differential equations and their solutions. By George Moseley Murphy. 1960
Section: Various 30
Problem number: 859.
ODE order: 1.
ODE degree: 2.

CAS Maple gives this as type `[[_homogeneous, ‘class G‘]]`

$$y'^2 x + y y' - y^4 = 0$$

✓ Solution by Maple
Time used: 0.093 (sec). Leaf size: 89

```
dsolve(x*diff(y(x),x)^2+y(x)*diff(y(x),x)-y(x)^4 = 0,y(x), singsol=all)
```

$$y(x) = -\frac{1}{2\sqrt{-x}}$$
$$y(x) = \frac{1}{2\sqrt{-x}}$$
$$y(x) = 0$$
$$y(x) = -\frac{\coth\left(-\frac{\ln(x)}{2} + \frac{c_1}{2}\right)\sqrt{\operatorname{sech}\left(-\frac{\ln(x)}{2} + \frac{c_1}{2}\right)^2 x}}{2x}$$
$$y(x) = \frac{\coth\left(-\frac{\ln(x)}{2} + \frac{c_1}{2}\right)\sqrt{\operatorname{sech}\left(-\frac{\ln(x)}{2} + \frac{c_1}{2}\right)^2 x}}{2x}$$

✓ Solution by Mathematica

Time used: 0.63 (sec). Leaf size: 84

```
DSolve[x (y'[x])^2+y[x] y'[x]-y[x]^4==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to -\frac{2e^{\frac{c_1}{2}}}{-4x + e^{c_1}}$$

$$y(x) \to \frac{2e^{\frac{c_1}{2}}}{-4x + e^{c_1}}$$

$$y(x) \to 0$$

$$y(x) \to -\frac{i}{2\sqrt{x}}$$

$$y(x) \to \frac{i}{2\sqrt{x}}$$

30.2 problem 860

Internal problem ID [4098]

Book: Ordinary differential equations and their solutions. By George Moseley Murphy. 1960
Section: Various 30
Problem number: 860.
ODE order: 1.
ODE degree: 2.

CAS Maple gives this as type `[[_1st_order, _with_linear_symmetries], _rational, _Clairaut]`

$$\boxed{{y'}^2 x + (a - y)\, y' = -b}$$

✓ Solution by Maple

Time used: 0.094 (sec). Leaf size: 42

```
dsolve(x*diff(y(x),x)^2+(a-y(x))*diff(y(x),x)+b = 0,y(x), singsol=all)
```

$$y(x) = a - 2\sqrt{bx}$$
$$y(x) = a + 2\sqrt{bx}$$
$$y(x) = \frac{x\, c_1^2 + c_1 a + b}{c_1}$$

✓ Solution by Mathematica

Time used: 0.014 (sec). Leaf size: 58

```
DSolve[x (y'[x])^2+(a-y[x])y'[x]+b==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to a + \frac{b}{c_1} + c_1 x$$
$$y(x) \to \text{Indeterminate}$$
$$y(x) \to a - 2\sqrt{b}\sqrt{x}$$
$$y(x) \to a + 2\sqrt{b}\sqrt{x}$$

30.3 problem 861

Internal problem ID [4099]

Book: Ordinary differential equations and their solutions. By George Moseley Murphy. 1960
Section: Various 30
Problem number: 861.
ODE order: 1.
ODE degree: 2.

CAS Maple gives this as type `[[_1st_order, _with_linear_symmetries], _rational, _dAlembert]`

$$y'^2 x + (-y + x) y' - y = -1$$

✓ Solution by Maple
Time used: 0.094 (sec). Leaf size: 44

```
dsolve(x*diff(y(x),x)^2+(x-y(x))*diff(y(x),x)+1-y(x) = 0,y(x), singsol=all)
```

$$y(x) = -x - 2\sqrt{x}$$
$$y(x) = -x + 2\sqrt{x}$$
$$y(x) = \frac{x\, c_1^2 + c_1 x + 1}{c_1 + 1}$$

✓ Solution by Mathematica
Time used: 0.013 (sec). Leaf size: 46

```
DSolve[x (y'[x])^2+(x-y[x])y'[x]+1-y[x]==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to c_1 x + \frac{1}{1 + c_1}$$
$$y(x) \to -x - 2\sqrt{x}$$
$$y(x) \to 2\sqrt{x} - x$$

30.4 problem 862

Internal problem ID [4100]

Book: Ordinary differential equations and their solutions. By George Moseley Murphy. 1960
Section: Various 30
Problem number: 862.
ODE order: 1.
ODE degree: 2.

CAS Maple gives this as type `[[_1st_order, _with_linear_symmetries], _rational, _dAlembert]`

$$y'^2 x + (a + x - y) y' - y = 0$$

✓ Solution by Maple
Time used: 0.109 (sec). Leaf size: 49

```
dsolve(x*diff(y(x),x)^2+(a+x-y(x))*diff(y(x),x)-y(x) = 0,y(x), singsol=all)
```

$$y(x) = a - x - 2\sqrt{-ax}$$
$$y(x) = a - x + 2\sqrt{-ax}$$
$$y(x) = \frac{c_1(c_1x + a + x)}{c_1 + 1}$$

✓ Solution by Mathematica
Time used: 0.012 (sec). Leaf size: 60

```
DSolve[x (y'[x])^2+(a+x-y[x])y'[x]-y[x]==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to c_1\left(x + \frac{a}{1 + c_1}\right)$$
$$y(x) \to \left(\sqrt{a} - i\sqrt{x}\right)^2$$
$$y(x) \to \left(\sqrt{a} + i\sqrt{x}\right)^2$$

30.5 problem 863

Internal problem ID [4101]

Book: Ordinary differential equations and their solutions. By George Moseley Murphy. 1960
Section: Various 30
Problem number: 863.
ODE order: 1.
ODE degree: 2.

CAS Maple gives this as type `[[_homogeneous, ‘class A‘], _dAlembert]`

$$\boxed{{y'}^{2}x-(3x-y)\,y'+y=0}$$

✓ Solution by Maple

Time used: 0.078 (sec). Leaf size: 139

```
dsolve(x*diff(y(x),x)^2-(3*x-y(x))*diff(y(x),x)+y(x) = 0,y(x), singsol=all)
```

$$y(x)=x$$

$$\frac{c_1\left(-5x+y(x)-\sqrt{9x^2-10xy\left(x\right)+y\left(x\right)^2}\right)}{x\left(\frac{3x-y(x)+\sqrt{9x^2-10xy(x)+y(x)^2}}{x}\right)^{\frac{3}{2}}}+x=0$$

$$\frac{\left(-5x+y(x)+\sqrt{9x^2-10xy\left(x\right)+y\left(x\right)^2}\right)c_1\sqrt{2}}{4x\left(\frac{3x-y(x)-\sqrt{9x^2-10xy(x)+y(x)^2}}{x}\right)^{\frac{3}{2}}}+x=0$$

✓ Solution by Mathematica

Time used: 60.322 (sec). Leaf size: 1225

```
DSolve[x (y'[x])^2-(3 x-y[x])y'[x]+y[x]==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to \frac{1}{384}\left(\frac{\frac{4e^{8c_1}}{x^2}-6912e^{4c_1}}{\sqrt[3]{-\frac{-373248e^{4c_1}x^4+4320e^{8c_1}x^2-48\sqrt{6}x\sqrt{e^{8c_1}\left(216x^2+e^{4c_1}\right)^3}+e^{12c_1}}{x^3}}} + 4\sqrt[3]{\frac{373248e^{4c_1}x^4-4320e^{8c_1}x^2+48\sqrt{6}x\sqrt{e^{8c_1}\left(216x^2+e^{4c_1}\right)^3}-e^{12c_1}}{x^3}} - \frac{4e^{4c_1}}{x}\right)$$

$$y(x) \to \frac{1}{768}\left(\frac{\left(1+i\sqrt{3}\right)\left(6912e^{4c_1}-\frac{4e^{8c_1}}{x^2}\right)}{\sqrt[3]{-\frac{-373248e^{4c_1}x^4+4320e^{8c_1}x^2-48\sqrt{6}x\sqrt{e^{8c_1}\left(216x^2+e^{4c_1}\right)^3}+e^{12c_1}}{x^3}}} + 4i\left(\sqrt{3}+i\right)\sqrt[3]{-\frac{-373248e^{4c_1}x^4+4320e^{8c_1}x^2-48\sqrt{6}x\sqrt{e^{8c_1}\left(216x^2+e^{4c_1}\right)^3}+e^{12c_1}}{x^3}} - \frac{8e^{4c_1}}{x}\right)$$

$$y(x) \to \frac{1}{768}\left(\frac{\left(1-i\sqrt{3}\right)\left(6912e^{4c_1}-\frac{4e^{8c_1}}{x^2}\right)}{\sqrt[3]{-\frac{-373248e^{4c_1}x^4+4320e^{8c_1}x^2-48\sqrt{6}x\sqrt{e^{8c_1}\left(216x^2+e^{4c_1}\right)^3}+e^{12c_1}}{x^3}}} - 4\left(1+i\sqrt{3}\right)\sqrt[3]{-\frac{-373248e^{4c_1}x^4+4320e^{8c_1}x^2-48\sqrt{6}x\sqrt{e^{8c_1}\left(216x^2+e^{4c_1}\right)^3}+e^{12c_1}}{x^3}} - \frac{8e^{4c_1}}{x}\right)$$

30.6 problem 864

Internal problem ID [4102]

Book: Ordinary differential equations and their solutions. By George Moseley Murphy. 1960
Section: Various 30
Problem number: 864.
ODE order: 1.
ODE degree: 2.

CAS Maple gives this as type `[[_homogeneous, 'class C'], _rational, _dAlembert]`

$$\boxed{{y'}^2 x - y - yb = -bx - a}$$

✓ Solution by Maple
Time used: 0.125 (sec). Leaf size: 48

```
dsolve(x*diff(y(x),x)^2+a+b*x-y(x)-b*y(x) = 0,y(x), singsol=all)
```

$$y(x) = \frac{\left(\left(\operatorname{RootOf}\left(\_Z - \_Z^{\frac{1}{b}} \left(\frac{c_1}{x}\right)^{\frac{b-1}{2b}} - b + 1\right) + 1\right)^2 + b\right) x + a}{b+1}$$

✓ Solution by Mathematica

Time used: 91.9 (sec). Leaf size: 1197

```
DSolve[x (y'[x])^2+(a+b x-y[x])-b y[x]==0,y[x],x,IncludeSingularSolutions -> True]
```

$$\text{Solve}\left[\frac{2(b+1)\left(-\log\left(\sqrt{-a+by(x)-bx+y(x)}+\sqrt{x}\right)+b\log\left(\sqrt{-a+by(x)-bx+y(x)}+b\sqrt{x}\right)+\right.}{}\right.$$

$$-\frac{2(b+1)\left((b-1)\log\left(\sqrt{-a+by(x)+y(x)}\sqrt{-a+by(x)-bx+y(x)}+a-(b+1)y(x)\right)+\log\left(\sqrt{x}\sqrt{\right.\right.}}{}$$

$$\text{Solve}\left[\frac{2(b+1)\left(-\log\left(\sqrt{-a+by(x)-bx+y(x)}-\sqrt{x}\right)+b\log\left(\sqrt{-a+by(x)-bx+y(x)}-b\sqrt{x}\right)+\right.}{}\right.$$

$$-\frac{2(b+1)\left((b-1)\log\left(\sqrt{-a+by(x)+y(x)}\sqrt{-a+by(x)-bx+y(x)}+a-(b+1)y(x)\right)+\log\left(-\sqrt{x}\right.\right.}{}$$

30.7 problem 865

Internal problem ID [4103]

Book: Ordinary differential equations and their solutions. By George Moseley Murphy. 1960
Section: Various 30
Problem number: 865.
ODE order: 1.
ODE degree: 2.

CAS Maple gives this as type `[[_homogeneous, ‘class G‘], _rational, _dAlembert]`

$$\boxed{{y'}^2 x - 2yy' = -a}$$

✓ Solution by Maple

Time used: 0.079 (sec). Leaf size: 796

```
dsolve(x*diff(y(x),x)^2-2*y(x)*diff(y(x),x)+a = 0,y(x), singsol=all)
```

$$y(x) = \frac{\left(\frac{4x^2}{\left(-36c_1^2a+8x^3+12\sqrt{a\left(9c_1^2a-4x^3\right)}\,c_1\right)^{\frac{1}{3}}} + 2x + \left(-36c_1^2a+8x^3+12\sqrt{a\left(9c_1^2a-4x^3\right)}\,c_1\right)^{\frac{1}{3}}\right)x}{12c_1} + \frac{3ac_1\left(-36c_1^2a+8x^3+12\sqrt{a\left(9c_1^2a-4x^3\right)}\,c_1\right)^{\frac{1}{3}}}{\left(-36c_1^2a+8x^3+12\sqrt{a\left(9c_1^2a-4x^3\right)}\,c_1\right)^{\frac{2}{3}} + 2x\left(-36c_1^2a+8x^3+12\sqrt{a\left(9c_1^2a-4x^3\right)}\,c_1\right)^{\frac{1}{3}} + 4x^2}$$

$$y(x) = -\frac{x\left(\left(1+i\sqrt{3}\right)\left(-36c_1^2a+8x^3+12\sqrt{a\left(9c_1^2a-4x^3\right)}\,c_1\right)^{\frac{2}{3}} - 4x\left(-36c_1^2a+8x^3+12\sqrt{a\left(9c_1^2a-4x^3\right)}\,c_1\right.}{24\left(-36c_1^2a+8x^3+12\sqrt{a\left(9c_1^2a-4x^3\right)}\,c_1\right)^{\frac{1}{3}}c_1} + \frac{6ac_1\left(-36c_1^2a+8x^3+12\sqrt{a\left(9c_1^2a-4x^3\right)}\,c_1\right)}{4i\sqrt{3}\,x^2 - i\sqrt{3}\left(-36c_1^2a+8x^3+12\sqrt{a\left(9c_1^2a-4x^3\right)}\,c_1\right)^{\frac{2}{3}} - 4x^2 + 4x\left(-36c_1^2a+8x^3+12\sqrt{a\left(9c_1^2a-\right.\right.}}$$

$$y(x) = \frac{\left(\left(i\sqrt{3}-1\right)\left(-36c_1^2a+8x^3+12\sqrt{a\left(9c_1^2a-4x^3\right)}\,c_1\right)^{\frac{2}{3}} + 4x\left(-36c_1^2a+8x^3+12\sqrt{a\left(9c_1^2a-4x^3\right)}\,c_1\right)^{\frac{1}{3}}\right.}{24\left(-36c_1^2a+8x^3+12\sqrt{a\left(9c_1^2a-4x^3\right)}\,c_1\right)^{\frac{1}{3}}c_1} - \frac{6ac_1\left(-36c_1^2a+8x^3+12\sqrt{a\left(9c_1^2a-4x^3\right)}\,c_1\right)}{4i\sqrt{3}\,x^2 - i\sqrt{3}\left(-36c_1^2a+8x^3+12\sqrt{a\left(9c_1^2a-4x^3\right)}\,c_1\right)^{\frac{2}{3}} + 4x^2 - 4x\left(-36c_1^2a+8x^3+12\sqrt{a\left(9c_1^2a-\right.\right.}}$$

✓ Solution by Mathematica

Time used: 60.165 (sec). Leaf size: 1553

```
DSolve[x (y'[x])^2-2 y[x] y'[x]+a==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to \frac{e^{-\frac{3c_1}{2}}\left(a^4x^4+\left(-a^6x^6+20a^3e^{3c_1}x^3+8\sqrt{e^{3c_1}\left(-a^3x^3+e^{3c_1}\right)^3}+8e^{6c_1}\right)^{2/3}-a^2x^2\sqrt[3]{-a^6x^6+20a^3e^{3c_1}x}\right.}{4\sqrt[3]{-a^6x^6+20a^3e^{3c_1}x^3+8\sqrt{e^{3c_1}\left(-a^3x^3+e^{3c_1}\right)^3}+8e^{6c}}}$$

$$y(x) \to \frac{ie^{-\frac{3c_1}{2}}\left(-\left(\left(\sqrt{3}-i\right)a^4x^4\right)+\left(\sqrt{3}+i\right)\left(-a^6x^6+20a^3e^{3c_1}x^3+8\sqrt{e^{3c_1}\left(-a^3x^3+e^{3c_1}\right)^3}+8e^{6c_1}\right)^{2/3}+2a\right.}{8\sqrt[3]{-a^6x^6+20a^3e^{3c_1}x^3+8\sqrt{e^{3c_1}\left(-a^3\right.}}}$$

$$y(x) \to \frac{e^{-\frac{3c_1}{2}}\left(i\left(\sqrt{3}+i\right)a^4x^4-i\left(\sqrt{3}-i\right)\left(-a^6x^6+20a^3e^{3c_1}x^3+8\sqrt{e^{3c_1}\left(-a^3x^3+e^{3c_1}\right)^3}+8e^{6c_1}\right)^{2/3}-2a^2x\right.}{8\sqrt[3]{-a^6x^6+20a^3e^{3c_1}x^3+8\sqrt{e^{3c_1}\left(-a^3x\right.}}}$$

$$y(x) \to \frac{e^{-\frac{3c_1}{2}}\left(a^4x^4+\left(a^6x^6-20a^3e^{3c_1}x^3+8\sqrt{e^{3c_1}\left(-a^3x^3+e^{3c_1}\right)^3}-8e^{6c_1}\right)^{2/3}+a^2x^2\sqrt[3]{a^6x^6-20a^3e^{3c_1}x^3+}\right.}{4\sqrt[3]{a^6x^6-20a^3e^{3c_1}x^3+8\sqrt{e^{3c_1}\left(-a^3x^3+e^{3c_1}\right)^3}-8e^{6c_1}}}$$

$$y(x) \to \frac{e^{-\frac{3c_1}{2}}\left(\left(-1-i\sqrt{3}\right)a^4x^4+i\left(\sqrt{3}+i\right)\left(a^6x^6-20a^3e^{3c_1}x^3+8\sqrt{e^{3c_1}\left(-a^3x^3+e^{3c_1}\right)^3}-8e^{6c_1}\right)^{2/3}+2a^2x\right.}{8\sqrt[3]{a^6x^6-20a^3e^{3c_1}x^3+8\sqrt{e^{3c_1}\left(-a^3x^3\right.}}}$$

$$y(x) \to \frac{e^{-\frac{3c_1}{2}}\left(i\left(\sqrt{3}+i\right)a^4x^4-i\left(\sqrt{3}-i\right)\left(a^6x^6-20a^3e^{3c_1}x^3+8\sqrt{e^{3c_1}\left(-a^3x^3+e^{3c_1}\right)^3}-8e^{6c_1}\right)^{2/3}+2a^2x^2\right.}{8\sqrt[3]{a^6x^6-20a^3e^{3c_1}x^3+8\sqrt{e^{3c_1}\left(-a^3x^3+\right.}}}$$

30.8 problem 867

Internal problem ID [4104]

Book: Ordinary differential equations and their solutions. By George Moseley Murphy. 1960
Section: Various 30
Problem number: 867.
ODE order: 1.
ODE degree: 2.

CAS Maple gives this as type `[[_homogeneous, ‘class A‘], _rational, _dAlembert]`

$$\boxed{{y'}^2 x - 2yy' = -ax}$$

✓ Solution by Maple
Time used: 0.079 (sec). Leaf size: 33

```
dsolve(x*diff(y(x),x)^2-2*y(x)*diff(y(x),x)+a*x = 0,y(x), singsol=all)
```

$$y(x) = x\sqrt{a}$$
$$y(x) = -x\sqrt{a}$$
$$y(x) = \frac{\left(\frac{x^2}{c_1^2} + a\right) c_1}{2}$$

✓ Solution by Mathematica

Time used: 16.916 (sec). Leaf size: 400

```
DSolve[x (y'[x])^2-2 y[x] y'[x]+a x==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x)\to -\frac{\sqrt{a}x\tan(c_1-i\log(x))}{\sqrt{\sec^2(c_1-i\log(x))}}$$
$$y(x)\to \frac{\sqrt{a}x\tan(c_1-i\log(x))}{\sqrt{\sec^2(c_1-i\log(x))}}$$
$$y(x)\to -\frac{\sqrt{a}x\tan(i\log(x)+c_1)}{\sqrt{\sec^2(i\log(x)+c_1)}}$$
$$y(x)\to \frac{\sqrt{a}x\tan(i\log(x)+c_1)}{\sqrt{\sec^2(i\log(x)+c_1)}}$$
$$y(x)\to -\sqrt{a}x$$
$$y(x)\to \sqrt{a}x$$
$$y(x)\to \frac{i\sqrt{a}e^{2i\text{Interval}[\{0,\pi\}]}\left(e^{2i\text{Interval}[\{0,\pi\}]}\sqrt{\frac{x^2e^{2i\text{Interval}[\{0,\pi\}]}}{\left(x^2+e^{2i\text{Interval}[\{0,\pi\}]}\right)^2}}-x^4\sqrt{\frac{x^2e^{2i\text{Interval}[\{0,\pi\}]}}{\left(x^2+e^{2i\text{Interval}[\{0,\pi\}]}\right)^2}}\right)}{2x}$$
$$y(x)\to \frac{i\sqrt{a}e^{2i\text{Interval}[\{0,\pi\}]}\left(x^4\sqrt{\frac{x^2e^{2i\text{Interval}[\{0,\pi\}]}}{\left(x^2+e^{2i\text{Interval}[\{0,\pi\}]}\right)^2}}-e^{2i\text{Interval}[\{0,\pi\}]}\sqrt{\frac{x^2e^{2i\text{Interval}[\{0,\pi\}]}}{\left(x^2+e^{2i\text{Interval}[\{0,\pi\}]}\right)^2}}\right)}{2x}$$

30.9 problem 868

Internal problem ID [4105]

Book: Ordinary differential equations and their solutions. By George Moseley Murphy. 1960
Section: Various 30
Problem number: 868.
ODE order: 1.
ODE degree: 2.

CAS Maple gives this as type `[[_homogeneous, 'class A'], _rational, _dAlembert]`

$$\boxed{y'^2x - 2yy' + 2y = -x}$$

✓ Solution by Maple
Time used: 0.094 (sec). Leaf size: 44

```
dsolve(x*diff(y(x),x)^2-2*y(x)*diff(y(x),x)+x+2*y(x) = 0,y(x), singsol=all)
```

$$y(x) = \left(1-\sqrt{2}\right)x$$
$$y(x) = x\left(1+\sqrt{2}\right)$$
$$y(x) = \frac{2c_1^2 + 2c_1x + x^2}{2c_1}$$

✓ Solution by Mathematica
Time used: 0.247 (sec). Leaf size: 78

```
DSolve[x (y'[x])^2-2 y[x] y'[x]+x +2 y[x]==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to -\frac{1}{2}e^{-c_1}x^2 + x - e^{c_1}$$
$$y(x) \to -e^{c_1}x^2 + x - \frac{e^{-c_1}}{2}$$
$$y(x) \to x - \sqrt{2}x$$
$$y(x) \to \left(1+\sqrt{2}\right)x$$

30.10 problem 869

Internal problem ID [4106]

Book: Ordinary differential equations and their solutions. By George Moseley Murphy. 1960
Section: Various 30
Problem number: 869.
ODE order: 1.
ODE degree: 2.

CAS Maple gives this as type `[[_homogeneous, 'class G']]`

$$\boxed{y'^2 x - 3yy' = -9x^2}$$

✓ Solution by Maple

Time used: 0.094 (sec). Leaf size: 51

```
dsolve(x*diff(y(x),x)^2-3*y(x)*diff(y(x),x)+9*x^2 = 0,y(x), singsol=all)
```

$$y(x) = -2x^{\frac{3}{2}}$$
$$y(x) = 2x^{\frac{3}{2}}$$
$$y(x) = \frac{4x^3 + c_1^2}{2c_1}$$
$$y(x) = \frac{c_1^2 x^3 + 4}{2c_1}$$

✓ Solution by Mathematica

Time used: 0.31 (sec). Leaf size: 79

```
DSolve[x (y'[x])^2-3 y[x] y'[x]+9 x^2==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to -\frac{1}{2}e^{-\frac{3c_1}{2}}\left(4x^3 + e^{3c_1}\right)$$
$$y(x) \to \frac{1}{2}e^{-\frac{3c_1}{2}}\left(4x^3 + e^{3c_1}\right)$$
$$y(x) \to -2x^{3/2}$$
$$y(x) \to 2x^{3/2}$$

30.11 problem 870

Internal problem ID [4107]

Book: Ordinary differential equations and their solutions. By George Moseley Murphy. 1960
Section: Various 30
Problem number: 870.
ODE order: 1.
ODE degree: 2.

CAS Maple gives this as type `[_quadrature]`

$$
\boxed{y'^2 x - (2x + 3y)\, y' + 6y = 0}
$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 17

```
dsolve(x*diff(y(x),x)^2-(2*x+3*y(x))*diff(y(x),x)+6*y(x) = 0,y(x), singsol=all)
```

$$
\begin{aligned}
y(x) &= c_1 x^3 \\
y(x) &= 2x + c_1
\end{aligned}
$$

✓ Solution by Mathematica

Time used: 0.038 (sec). Leaf size: 26

```
DSolve[x (y'[x])^2-(2 x+3 y[x])y'[x]+6 y[x]==0,y[x],x,IncludeSingularSolutions -> True]
```

$$
\begin{aligned}
y(x) &\to c_1 x^3 \\
y(x) &\to 2x + c_1 \\
y(x) &\to 0
\end{aligned}
$$

30.12 problem 871

Internal problem ID [4108]

Book: Ordinary differential equations and their solutions. By George Moseley Murphy. 1960
Section: Various 30
Problem number: 871.
ODE order: 1.
ODE degree: 2.

CAS Maple gives this as type `[[_homogeneous, ‘class G‘], _rational, _dAlembert]`

$$\boxed{y'^2x - ayy' = -b}$$

✓ Solution by Maple

Time used: 0.079 (sec). Leaf size: 381

```
dsolve(x*diff(y(x),x)^2-a*y(x)*diff(y(x),x)+b = 0,y(x), singsol=all)
```

$$\frac{4\left(2^{-\frac{1}{a-1}}y(x)\left(a-\frac{1}{2}\right)^2 a\sqrt{a^2y\left(x\right)^2-4bx}-\frac{bx2^{\frac{a-2}{a-1}}}{4}+2^{-\frac{1}{a-1}}\left(\left(a-\frac{1}{2}\right)^2 ay(x)^2-2bx(a-1)\right)a\right)c_1\left(\frac{ay(x)+\sqrt{}}{}\right.}{(2a-1)\left(ay\left(x\right)+\sqrt{a^2y\left(x\right)^2-}\right.}$$

$$=0$$

$$\frac{-4c_1\left(-2^{-\frac{1}{a-1}}y(x)\left(a-\frac{1}{2}\right)^2 a\sqrt{a^2y\left(x\right)^2-4bx}-\frac{bx2^{\frac{a-2}{a-1}}}{4}+2^{-\frac{1}{a-1}}\left(\left(a-\frac{1}{2}\right)^2 ay(x)^2-2bx(a-1)\right)a\right)\left(\frac{ay(x}{}\right.}{(2a-1)\left(ay\left(x\right)-\sqrt{a^2y\left(x\right)^2}\right.}$$

$$=0$$

✓ Solution by Mathematica

Time used: 0.712 (sec). Leaf size: 143

```
DSolve[x (y'[x])^2-a y[x] y'[x]+b==0,y[x],x,IncludeSingularSolutions -> True]
```

$$\text{Solve}\left[\frac{2\left((a-1)\log\left(\sqrt{a^2y(x)^2-4bx}+(a-1)y(x)\right)+a\log\left(\sqrt{a^2y(x)^2-4bx}-ay(x)\right)\right)}{2a-1}=c_1,y(x)\right]$$

$$\text{Solve}\left[\frac{2\left((a-1)\log\left(\sqrt{a^2y(x)^2-4bx}-ay(x)+y(x)\right)+a\log\left(\sqrt{a^2y(x)^2-4bx}+ay(x)\right)\right)}{2a-1}=c_1,y(x)\right]$$

30.13 problem 872

Internal problem ID [4109]

Book: Ordinary differential equations and their solutions. By George Moseley Murphy. 1960
Section: Various 30
Problem number: 872.
ODE order: 1.
ODE degree: 2.

CAS Maple gives this as type `[[_homogeneous, 'class A'], _dAlembert]`

$$\boxed{{y'}^2 x + a y y' = -b x}$$

✓ Solution by Maple

Time used: 0.125 (sec). Leaf size: 217

```
dsolve(x*diff(y(x),x)^2+a*y(x)*diff(y(x),x)+b*x = 0,y(x), singsol=all)
```

$$\frac{-c_1 2^{\frac{2+a}{2+2a}} \left(a y(x) - \sqrt{a^2 y(x)^2 - 4b\,x^2}\right) \left(\frac{a\left(-y(x)(a+1)\sqrt{a^2 y(x)^2 - 4b\,x^2} + (a^2+a) y(x)^2 - 2b\,x^2\right)}{x^2}\right)^{\frac{-a-2}{2+2a}} + x^2}{x} = 0$$

$$\frac{c_1 2^{\frac{2+a}{2+2a}} \left(a y(x) + \sqrt{a^2 y(x)^2 - 4b\,x^2}\right) \left(\frac{a\left(y(x)(a+1)\sqrt{a^2 y(x)^2 - 4b\,x^2} + (a^2+a) y(x)^2 - 2b\,x^2\right)}{x^2}\right)^{\frac{-a-2}{2+2a}} + x^2}{x} = 0$$

✓ Solution by Mathematica

Time used: 2.082 (sec). Leaf size: 423

```
DSolve[x (y'[x])^2+a y[x] y'[x]+b x==0,y[x],x,IncludeSingularSolutions -> True]
```

$$\text{Solve}\left[-\frac{i\left(2\log\left(-i\sqrt{4b-\frac{a^2y(x)^2}{x^2}}+\frac{ay(x)}{x}+2i\sqrt{b}\right)+2(a+1)\log\left(i\sqrt{4b-\frac{a^2y(x)^2}{x^2}}+\frac{ay(x)}{x}-2i\sqrt{b}\right)-(a\right.}{4(a+1)}\right.$$

$$\left.-\frac{1}{2}i\log(x),y(x)\right]$$

$$\text{Solve}\left[\frac{i\left(2(a+1)\log\left(-i\sqrt{4b-\frac{a^2y(x)^2}{x^2}}+\frac{ay(x)}{x}+2i\sqrt{b}\right)+2\log\left(i\sqrt{4b-\frac{a^2y(x)^2}{x^2}}+\frac{ay(x)}{x}-2i\sqrt{b}\right)-(a+\right.}{4(a+1)}\right.$$

$$\left.+c_1,y(x)\right]$$

30.14 problem 873

Internal problem ID [4110]

Book: Ordinary differential equations and their solutions. By George Moseley Murphy. 1960
Section: Various 30
Problem number: 873.
ODE order: 1.
ODE degree: 2.

CAS Maple gives this as type `[_quadrature]`

$$y'^2 x - (1 + yx)\, y' + y = 0$$

✓ Solution by Maple
Time used: 0.0 (sec). Leaf size: 15

```
dsolve(x*diff(y(x),x)^2-(1+x*y(x))*diff(y(x),x)+y(x) = 0,y(x), singsol=all)
```

$$y(x) = \ln(x) + c_1$$
$$y(x) = e^x c_1$$

✓ Solution by Mathematica
Time used: 0.01 (sec). Leaf size: 20

```
DSolve[x (y'[x])^2-(1+x y[x])y'[x]+y[x]==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to c_1 e^x$$
$$y(x) \to \log(x) + c_1$$

30.15 problem 874

Internal problem ID [4111]

Book: Ordinary differential equations and their solutions. By George Moseley Murphy. 1960
Section: Various 30
Problem number: 874.
ODE order: 1.
ODE degree: 2.

CAS Maple gives this as type `[_quadrature]`

$$
{y'}^2 x + (1-x)\,y y' - y^2 = 0
$$

✓ Solution by Maple

Time used: 0.016 (sec). Leaf size: 16

```
dsolve(x*diff(y(x),x)^2+(1-x)*y(x)*diff(y(x),x)-y(x)^2 = 0,y(x), singsol=all)
```

$$
y(x) = \frac{c_1}{x}
$$
$$
y(x) = \mathrm{e}^x c_1
$$

✓ Solution by Mathematica

Time used: 0.044 (sec). Leaf size: 26

```
DSolve[x (y'[x])^2+(1-x)y[x] y'[x]-y[x]^2==0,y[x],x,IncludeSingularSolutions -> True]
```

$$
y(x) \to c_1 e^x
$$
$$
y(x) \to \frac{c_1}{x}
$$
$$
y(x) \to 0
$$

30.16 problem 875

Internal problem ID [4112]

Book: Ordinary differential equations and their solutions. By George Moseley Murphy. 1960
Section: Various 30
Problem number: 875.
ODE order: 1.
ODE degree: 2.

CAS Maple gives this as type `[_quadrature]`

$$\boxed{y'^2 x + (1 - x^2 y)\, y' - yx = 0}$$

✓ Solution by Maple

Time used: 0.015 (sec). Leaf size: 21

```
dsolve(x*diff(y(x),x)^2+(1-x^2*y(x))*diff(y(x),x)-x*y(x) = 0,y(x), singsol=all)
```

$$y(x) = \mathrm{e}^{\frac{x^2}{2}} c_1$$
$$y(x) = -\ln(x) + c_1$$

✓ Solution by Mathematica

Time used: 0.01 (sec). Leaf size: 28

```
DSolve[x (y'[x])^2+(1-x^2 y[x])y'[x]-x y[x]==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to c_1 e^{\frac{x^2}{2}}$$
$$y(x) \to -\log(x) + c_1$$

30.17 problem 876

Internal problem ID [4113]

Book: Ordinary differential equations and their solutions. By George Moseley Murphy. 1960
Section: Various 30
Problem number: 876.
ODE order: 1.
ODE degree: 2.

CAS Maple gives this as type `[[_homogeneous, ‘class C‘], _rational, _dAlembert]`

$$(x+1)\,y'^2-y=0$$

✓ Solution by Maple

Time used: 0.093 (sec). Leaf size: 53

```
dsolve((1+x)*diff(y(x),x)^2 = y(x),y(x), singsol=all)
```

$$y(x)=0$$
$$y(x)=\frac{\left(x+1+\sqrt{(x+1)\,(c_1+1)}\right)^2}{x+1}$$
$$y(x)=\frac{\left(-x-1+\sqrt{(x+1)\,(c_1+1)}\right)^2}{x+1}$$

✓ Solution by Mathematica

Time used: 0.063 (sec). Leaf size: 57

```
DSolve[(1+x) (y'[x])^2==y[x],y[x],x,IncludeSingularSolutions -> True]
```

$$y(x)\to x-c_1\sqrt{x+1}+1+\frac{{c_1}^2}{4}$$
$$y(x)\to x+c_1\sqrt{x+1}+1+\frac{{c_1}^2}{4}$$
$$y(x)\to 0$$

30.18 problem 877

Internal problem ID [4114]

Book: Ordinary differential equations and their solutions. By George Moseley Murphy. 1960
Section: Various 30
Problem number: 877.
ODE order: 1.
ODE degree: 2.

CAS Maple gives this as type `[[_1st_order, _with_linear_symmetries], _rational, _dAlembert]`

$$\boxed{(x+1)\, {y'}^{2}-(y+x)\, y'+y=0}$$

✓ Solution by Maple

Time used: 0.109 (sec). Leaf size: 45

```
dsolve((1+x)*diff(y(x),x)^2-(x+y(x))*diff(y(x),x)+y(x) = 0,y(x), singsol=all)
```

$$y(x) = x+2-2\sqrt{x+1}$$
$$y(x) = x+2+2\sqrt{x+1}$$
$$y(x) = \frac{c_1(c_1x+c_1-x)}{c_1-1}$$

✓ Solution by Mathematica

Time used: 0.014 (sec). Leaf size: 51

```
DSolve[(1+x) (y'[x])^2-(x+y[x])y'[x]+y[x]==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to c_1\left(x+\frac{c_1}{-1+c_1}\right)$$
$$y(x) \to x-2\sqrt{x+1}+2$$
$$y(x) \to x+2\sqrt{x+1}+2$$

30.19 problem 878

Internal problem ID [4115]

Book: Ordinary differential equations and their solutions. By George Moseley Murphy. 1960
Section: Various 30
Problem number: 878.
ODE order: 1.
ODE degree: 2.

CAS Maple gives this as type `[[_1st_order, _with_linear_symmetries], _Clairaut]`

$$(-x+a)\, {y'}^2 + y y' = b$$

✓ Solution by Maple

Time used: 0.141 (sec). Leaf size: 49

```
dsolve((a-x)*diff(y(x),x)^2+y(x)*diff(y(x),x)-b = 0,y(x), singsol=all)
```

$$y(x) = -2\sqrt{-b\,(a-x)}$$
$$y(x) = 2\sqrt{-b\,(a-x)}$$
$$y(x) = \frac{(x-a)\, c_1^2 + b}{c_1}$$

✓ Solution by Mathematica

Time used: 0.015 (sec). Leaf size: 59

```
DSolve[(a-x) (y'[x])^2+y[x] y'[x]-b==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to c_1(x-a) + \frac{b}{c_1}$$
$$y(x) \to \text{Indeterminate}$$
$$y(x) \to -2\sqrt{b(x-a)}$$
$$y(x) \to 2\sqrt{b(x-a)}$$

30.20 problem 880

Internal problem ID [4116]

Book: Ordinary differential equations and their solutions. By George Moseley Murphy. 1960
Section: Various 30
Problem number: 880.
ODE order: 1.
ODE degree: 2.

CAS Maple gives this as type `[_rational, _dAlembert]`

$$2{y'}^2x+(2x-y)\,y'-y=-1$$

✓ Solution by Maple

Time used: 0.062 (sec). Leaf size: 110

```
dsolve(2*x*diff(y(x),x)^2+(2*x-y(x))*diff(y(x),x)+1-y(x) = 0,y(x), singsol=all)
```

$$y(x)=-2\left(x\,\mathrm{e}^{\mathrm{RootOf}\left(-x\,\mathrm{e}^{3\_Z}+2x\,\mathrm{e}^{2\_Z}+c_1\mathrm{e}^{\_Z}+\_Z\mathrm{e}^{\_Z}-\mathrm{e}^{\_Z}x+1\right)}-\mathrm{e}^{2\,\mathrm{RootOf}\left(-x\,\mathrm{e}^{3\_Z}+2x\,\mathrm{e}^{2\_Z}+c_1\mathrm{e}^{\_Z}+\_Z\mathrm{e}^{\_Z}-\mathrm{e}^{\_Z}x+1\right)}x-\frac{1}{2}\right)\mathrm{e}^{-\,\mathrm{RootOf}\left(-x\,\mathrm{e}^{3\_Z}+2x\,\mathrm{e}^{2\_Z}+c_1\mathrm{e}^{\_Z}+\_Z\mathrm{e}^{\_Z}-\mathrm{e}^{\_Z}x+1\right)}$$

✓ Solution by Mathematica

Time used: 1.438 (sec). Leaf size: 49

```
DSolve[2 x (y'[x])^2+(2 x-y[x])y'[x]+1-y[x]==0,y[x],x,IncludeSingularSolutions -> True]
```

$$\mathrm{Solve}\left[\left\{x=\frac{\frac{1}{K[1]+1}+\log(K[1]+1)}{K[1]^2}+\frac{c_1}{K[1]^2},y(x)=2xK[1]+\frac{1}{K[1]+1}\right\},\{y(x),K[1]\}\right]$$

30.21 problem 881

Internal problem ID [4117]

Book: Ordinary differential equations and their solutions. By George Moseley Murphy. 1960
Section: Various 30
Problem number: 881.
ODE order: 1.
ODE degree: 2.

CAS Maple gives this as type `[[_homogeneous, 'class A'], _rational, _dAlembert]`

$$\boxed{3y'^2x - 6yy' + 2y = -x}$$

✓ Solution by Maple

Time used: 0.078 (sec). Leaf size: 32

```
dsolve(3*x*diff(y(x),x)^2-6*y(x)*diff(y(x),x)+x+2*y(x) = 0,y(x), singsol=all)
```

$$y(x) = x$$
$$y(x) = -\frac{x}{3}$$
$$y(x) = \frac{4c_1^2 + 2c_1x + x^2}{6c_1}$$

✓ Solution by Mathematica

Time used: 0.314 (sec). Leaf size: 67

```
DSolve[3 x (y'[x])^2- 6 y[x] y'[x]+x +2 y[x]==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to -\frac{1}{3}x\left(-1 + 2\cosh\left(-\log(x) + \sqrt{3}c_1\right)\right)$$
$$y(x) \to -\frac{1}{3}x\left(-1 + 2\cosh\left(\log(x) + \sqrt{3}c_1\right)\right)$$
$$y(x) \to -\frac{x}{3}$$
$$y(x) \to x$$

30.22 problem 882

Internal problem ID [4118]

Book: Ordinary differential equations and their solutions. By George Moseley Murphy. 1960
Section: Various 30
Problem number: 882.
ODE order: 1.
ODE degree: 2.

CAS Maple gives this as type `[[_1st_order, _with_linear_symmetries], _Clairaut]`

$$\boxed{(3x+1)\,y'^2-3(y+2)\,y'=-9}$$

✓ Solution by Maple

Time used: 0.125 (sec). Leaf size: 51

```
dsolve((1+3*x)*diff(y(x),x)^2-3*(2+y(x))*diff(y(x),x)+9 = 0,y(x), singsol=all)
```

$$y(x)=-2-2\sqrt{3x+1}$$
$$y(x)=-2+2\sqrt{3x+1}$$
$$y(x)=\frac{9+(3x+1)\,c_1^2-6c_1}{3c_1}$$

✓ Solution by Mathematica

Time used: 0.013 (sec). Leaf size: 60

```
DSolve[(1+3 x) (y'[x])^2-3(2+y[x])y'[x]+9==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x)\to c_1\left(x+\frac{1}{3}\right)-2+\frac{3}{c_1}$$
$$y(x)\to \text{Indeterminate}$$
$$y(x)\to -2\left(\sqrt{3x+1}+1\right)$$
$$y(x)\to 2\left(\sqrt{3x+1}-1\right)$$

30.23 problem 883

Internal problem ID [4119]

Book: Ordinary differential equations and their solutions. By George Moseley Murphy. 1960
Section: Various 30
Problem number: 883.
ODE order: 1.
ODE degree: 2.

CAS Maple gives this as type `[_rational, _dAlembert]`

$$(5+3x)\,{y'}^2-(3+3y)\,y'+y=0$$

✓ Solution by Maple

Time used: 0.078 (sec). Leaf size: 249

```
dsolve((5+3*x)*diff(y(x),x)^2-(3+3*y(x))*diff(y(x),x)+y(x) = 0,y(x), singsol=all)
```

$$\frac{-108\left(x-\frac{3y(x)}{2}-\frac{\sqrt{9+9y(x)^2+(-12x-2)y(x)}}{2}+\frac{1}{6}\right)\left(c_1-\frac{\operatorname{expIntegral}_1\left(\frac{-9y(x)-9-3\sqrt{9+9y(x)^2+(-12x-2)y(x)}}{10+6x}\right)}{2}\right)\mathrm{e}^{\frac{-9y(x)-9-3\cdots}{\cdots}}}{30+18x} = 0$$

$$\frac{108\left(x-\frac{3y(x)}{2}+\frac{\sqrt{9+9y(x)^2+(-12x-2)y(x)}}{2}+\frac{1}{6}\right)\left(c_1+\frac{\operatorname{expIntegral}_1\left(\frac{-9y(x)-9+3\sqrt{9+9y(x)^2+(-12x-2)y(x)}}{10+6x}\right)}{2}\right)\mathrm{e}^{\frac{-9y(x)-9+3\sqrt{9\cdots}}{\cdots}}}{30+18x} = 0$$

✓ Solution by Mathematica

Time used: 1.709 (sec). Leaf size: 106

```
DSolve[(5+3 x) (y'[x])^2-(3+3 y[x])y'[x]+y[x]==0,y[x],x,IncludeSingularSolutions -> True]
```

$$\text{Solve}\left[\left\{x=\frac{e^{-3K[1]}(3K[1]-1)\left((9-27K[1])\,\text{ExpIntegralEi}(3K[1])+4e^{3K[1]}\right)}{9K[1]-3}+c_1e^{-3K[1]}(3K[1]-1),y(x)=\frac{3xK[1]^2}{3K[1]-1}+\frac{5K[1]^2-3K[1]}{3K[1]-1}\right\},\{y(x),K[1]\}\right]$$

30.24 problem 884

Internal problem ID [4120]

Book: Ordinary differential equations and their solutions. By George Moseley Murphy. 1960
Section: Various 30
Problem number: 884.
ODE order: 1.
ODE degree: 2.

CAS Maple gives this as type `[_quadrature]`

$$\boxed{4{y'}^2 x = (a - 3x)^2}$$

✓ Solution by Maple

Time used: 0.062 (sec). Leaf size: 30

```
dsolve(4*x*diff(y(x),x)^2 = (a-3*x)^2,y(x), singsol=all)
```

$$y(x) = -\sqrt{x}\,(a - x) + c_1$$
$$y(x) = \sqrt{x}\,(a - x) + c_1$$

✓ Solution by Mathematica

Time used: 0.02 (sec). Leaf size: 37

```
DSolve[4 x (y'[x])^2==(a-3 x)^2,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to \sqrt{x}(a - x) + c_1$$
$$y(x) \to \sqrt{x}(x - a) + c_1$$

30.25 problem 885

Internal problem ID [4121]

Book: Ordinary differential equations and their solutions. By George Moseley Murphy. 1960
Section: Various 30
Problem number: 885.
ODE order: 1.
ODE degree: 2.

CAS Maple gives this as type `[[_homogeneous, ‘class A‘], _rational, _dAlembert]`

$$\boxed{4{y'}^2x + 2xy' - y = 0}$$

✓ Solution by Maple

Time used: 0.016 (sec). Leaf size: 35

```
dsolve(4*x*diff(y(x),x)^2+2*x*diff(y(x),x)-y(x) = 0,y(x), singsol=all)
```

$$y(x) = -\frac{x}{4}$$
$$y(x) = 4c_1 + 2\sqrt{c_1 x}$$
$$y(x) = 4c_1 - 2\sqrt{c_1 x}$$

✓ Solution by Mathematica

Time used: 0.138 (sec). Leaf size: 72

```
DSolve[4 x (y'[x])^2+2 x y'[x]-y[x]==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to \frac{1}{4}e^{2c_1}\left(-2\sqrt{x} + e^{2c_1}\right)$$
$$y(x) \to \frac{1}{4}e^{-4c_1}\left(1 + 2e^{2c_1}\sqrt{x}\right)$$
$$y(x) \to 0$$
$$y(x) \to -\frac{x}{4}$$

30.26 problem 886

Internal problem ID [4122]

Book: Ordinary differential equations and their solutions. By George Moseley Murphy. 1960
Section: Various 30
Problem number: 886.
ODE order: 1.
ODE degree: 2.

CAS Maple gives this as type `[[_homogeneous, 'class G'], _rational, _dAlembert]`

$$\boxed{4{y'}^2x - 3yy' = -3}$$

✓ Solution by Maple

Time used: 0.063 (sec). Leaf size: 123

```
dsolve(4*x*diff(y(x),x)^2-3*y(x)*diff(y(x),x)+3 = 0,y(x), singsol=all)
```

$$y(x) = -\frac{2x\left(6+\sqrt{16c_1x+9}\right)}{3\sqrt{x\left(3+\sqrt{16c_1x+9}\right)}}$$
$$y(x) = \frac{2x\left(6+\sqrt{16c_1x+9}\right)}{3\sqrt{x\left(3+\sqrt{16c_1x+9}\right)}}$$
$$y(x) = \frac{2x\left(-6+\sqrt{16c_1x+9}\right)}{3\sqrt{-x\left(-3+\sqrt{16c_1x+9}\right)}}$$
$$y(x) = -\frac{2x\left(-6+\sqrt{16c_1x+9}\right)}{3\sqrt{-x\left(-3+\sqrt{16c_1x+9}\right)}}$$

✓ Solution by Mathematica

Time used: 27.786 (sec). Leaf size: 187

```
DSolve[4 x (y'[x])^2-3 y[x] y'[x]+3==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to -\frac{\sqrt{432x - e^{-\frac{c_1}{2}} \left(-144x + e^{c_1}\right)^{3/2} + e^{c_1}}}{6\sqrt{3}}$$

$$y(x) \to \frac{\sqrt{432x - e^{-\frac{c_1}{2}} \left(-144x + e^{c_1}\right)^{3/2} + e^{c_1}}}{6\sqrt{3}}$$

$$y(x) \to -\frac{\sqrt{432x + e^{-\frac{c_1}{2}} \left(-144x + e^{c_1}\right)^{3/2} + e^{c_1}}}{6\sqrt{3}}$$

$$y(x) \to \frac{\sqrt{432x + e^{-\frac{c_1}{2}} \left(-144x + e^{c_1}\right)^{3/2} + e^{c_1}}}{6\sqrt{3}}$$

30.27 problem 887

Internal problem ID [4123]

Book: Ordinary differential equations and their solutions. By George Moseley Murphy. 1960
Section: Various 30
Problem number: 887.
ODE order: 1.
ODE degree: 2.

CAS Maple gives this as type `[[_homogeneous, ‘class G‘], _rational, _dAlembert]`

$$\boxed{4{y'}^2x + 4yy' = 1}$$

✓ Solution by Maple

Time used: 0.063 (sec). Leaf size: 149

```
dsolve(4*x*diff(y(x),x)^2+4*y(x)*diff(y(x),x) = 1,y(x), singsol=all)
```

$$\frac{2\left(\frac{3c_1\left(y(x)-\sqrt{x+y(x)^2}\right)\sqrt{\frac{-y(x)+\sqrt{x+y(x)^2}}{x}}}{2}+3y(x)^2-3y(x)\sqrt{x+y\left(x\right)^2}+x\right)x}{3\left(y\left(x\right)-\sqrt{x+y\left(x\right)^2}\right)^2} = 0$$

$$\frac{2\left(-3c_1\left(y(x)+\sqrt{x+y\left(x\right)^2}\right)\sqrt{\frac{-2y(x)-2\sqrt{x+y(x)^2}}{x}}+3y(x)^2+3y(x)\sqrt{x+y\left(x\right)^2}+x\right)x}{3\left(y\left(x\right)+\sqrt{x+y\left(x\right)^2}\right)^2} = 0$$

✓ Solution by Mathematica

Time used: 60.239 (sec). Leaf size: 4057

```
DSolve[4 x (y'[x])^2+4 y[x] y'[x]==1,y[x],x,IncludeSingularSolutions -> True]
```

Too large to display

30.28 problem 888

Internal problem ID [4124]

Book: Ordinary differential equations and their solutions. By George Moseley Murphy. 1960
Section: Various 30
Problem number: 888.
ODE order: 1.
ODE degree: 2.

CAS Maple gives this as type `[[_homogeneous, ‘class G‘]]`

$$\boxed{4{y'}^2x+4yy'-y^4=0}$$

✓ Solution by Maple

Time used: 0.078 (sec). Leaf size: 86

```
dsolve(4*x*diff(y(x),x)^2+4*y(x)*diff(y(x),x)-y(x)^4 = 0,y(x), singsol=all)
```

$$y(x)=\frac{1}{\sqrt{-x}}$$
$$y(x)=-\frac{1}{\sqrt{-x}}$$
$$y(x)=0$$
$$y(x)=\frac{\coth\left(-\frac{\ln(x)}{2}+\frac{c_1}{2}\right)\sqrt{\operatorname{sech}\left(-\frac{\ln(x)}{2}+\frac{c_1}{2}\right)^2 x}}{x}$$
$$y(x)=-\frac{\coth\left(-\frac{\ln(x)}{2}+\frac{c_1}{2}\right)\sqrt{\operatorname{sech}\left(-\frac{\ln(x)}{2}+\frac{c_1}{2}\right)^2 x}}{x}$$

✓ Solution by Mathematica

Time used: 0.61 (sec). Leaf size: 80

```
DSolve[4 x (y'[x])^2+4 y[x] y'[x]-y[x]^4==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to -\frac{2e^{\frac{c_1}{2}}}{-x+e^{c_1}}$$
$$y(x) \to \frac{2e^{\frac{c_1}{2}}}{-x+e^{c_1}}$$
$$y(x) \to 0$$
$$y(x) \to -\frac{i}{\sqrt{x}}$$
$$y(x) \to \frac{i}{\sqrt{x}}$$

30.29 problem 889

Internal problem ID [4125]

Book: Ordinary differential equations and their solutions. By George Moseley Murphy. 1960
Section: Various 30
Problem number: 889.
ODE order: 1.
ODE degree: 2.

CAS Maple gives this as type `[_quadrature]`

$$\boxed{4(-x+2)\,{y'}^2 = -1}$$

✓ Solution by Maple
Time used: 0.047 (sec). Leaf size: 23

```
dsolve(4*(2-x)*diff(y(x),x)^2+1 = 0,y(x), singsol=all)
```

$$y(x) = -\sqrt{-2+x}+c_1$$
$$y(x) = \sqrt{-2+x}+c_1$$

✓ Solution by Mathematica
Time used: 0.005 (sec). Leaf size: 31

```
DSolve[4(2-x) (y'[x])^2+1==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to -\sqrt{x-2}+c_1$$
$$y(x) \to \sqrt{x-2}+c_1$$

30.30 problem 890

Internal problem ID [4126]

Book: Ordinary differential equations and their solutions. By George Moseley Murphy. 1960
Section: Various 30
Problem number: 890.
ODE order: 1.
ODE degree: 2.

CAS Maple gives this as type `[[_homogeneous, ‘class G‘]]`

$$16{y'}^2x + 8yy' + y^6 = 0$$

✓ Solution by Maple
Time used: 0.078 (sec). Leaf size: 103

```
dsolve(16*x*diff(y(x),x)^2+8*y(x)*diff(y(x),x)+y(x)^6 = 0,y(x), singsol=all)
```

$$y(x) = \frac{1}{x^{\frac{1}{4}}}$$
$$y(x) = -\frac{1}{x^{\frac{1}{4}}}$$
$$y(x) = -\frac{i}{x^{\frac{1}{4}}}$$
$$y(x) = \frac{i}{x^{\frac{1}{4}}}$$
$$y(x) = 0$$
$$y(x) = \frac{\operatorname{RootOf}\left(-\ln(x) + c_1 + 4\left(\int^{\_Z} \frac{1}{\_a\sqrt{-\_a^4+1}}d\_a\right)\right)}{x^{\frac{1}{4}}}$$
$$y(x) = \frac{\operatorname{RootOf}\left(-\ln(x) + c_1 - 4\left(\int^{\_Z} \frac{1}{\_a\sqrt{-\_a^4+1}}d\_a\right)\right)}{x^{\frac{1}{4}}}$$

✓ Solution by Mathematica

Time used: 0.705 (sec). Leaf size: 171

```
DSolve[16 x(y'[x])^2+8 y[x] y'[x]+y[x]^6==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \rightarrow -\frac{\sqrt{2}e^{\frac{c_1}{4}}}{\sqrt{x+e^{c_1}}}$$
$$y(x) \rightarrow -\frac{i\sqrt{2}e^{\frac{c_1}{4}}}{\sqrt{x+e^{c_1}}}$$
$$y(x) \rightarrow \frac{i\sqrt{2}e^{\frac{c_1}{4}}}{\sqrt{x+e^{c_1}}}$$
$$y(x) \rightarrow \frac{\sqrt{2}e^{\frac{c_1}{4}}}{\sqrt{x+e^{c_1}}}$$
$$y(x) \rightarrow 0$$
$$y(x) \rightarrow -\frac{1}{\sqrt[4]{x}}$$
$$y(x) \rightarrow -\frac{i}{\sqrt[4]{x}}$$
$$y(x) \rightarrow \frac{i}{\sqrt[4]{x}}$$
$$y(x) \rightarrow \frac{1}{\sqrt[4]{x}}$$

30.31 problem 891

Internal problem ID [4127]

Book: Ordinary differential equations and their solutions. By George Moseley Murphy. 1960
Section: Various 30
Problem number: 891.
ODE order: 1.
ODE degree: 2.

CAS Maple gives this as type `[_quadrature]`

$$\boxed{{y'}^2 x^2 = a^2}$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 20

```
dsolve(x^2*diff(y(x),x)^2 = a^2,y(x), singsol=all)
```

$$y(x) = a \ln(x) + c_1$$
$$y(x) = -a \ln(x) + c_1$$

✓ Solution by Mathematica

Time used: 0.003 (sec). Leaf size: 24

```
DSolve[x^2 (y'[x])^2==a^2,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to -a \log(x) + c_1$$
$$y(x) \to a \log(x) + c_1$$

30.32 problem 892

Internal problem ID [4128]

Book: Ordinary differential equations and their solutions. By George Moseley Murphy. 1960
Section: Various 30
Problem number: 892.
ODE order: 1.
ODE degree: 2.

CAS Maple gives this as type `[_separable]`

$$\boxed{y'^2x^2 - y^2 = 0}$$

✓ Solution by Maple
Time used: 0.016 (sec). Leaf size: 15

```
dsolve(x^2*diff(y(x),x)^2 = y(x)^2,y(x), singsol=all)
```

$$y(x) = c_1x$$
$$y(x) = \frac{c_1}{x}$$

✓ Solution by Mathematica
Time used: 0.042 (sec). Leaf size: 24

```
DSolve[x^2 (y'[x])^2==y[x]^2,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to \frac{c_1}{x}$$
$$y(x) \to c_1x$$
$$y(x) \to 0$$

30.33 problem 893

Internal problem ID [4129]

Book: Ordinary differential equations and their solutions. By George Moseley Murphy. 1960
Section: Various 30
Problem number: 893.
ODE order: 1.
ODE degree: 2.

CAS Maple gives this as type `[[_homogeneous, ‘class A‘], _rational, _dAlembert]`

$$\boxed{{y'}^2x^2 - y^2 = -x^2}$$

✓ Solution by Maple
Time used: 0.078 (sec). Leaf size: 44

```
dsolve(x^2*diff(y(x),x)^2+x^2-y(x)^2 = 0,y(x), singsol=all)
```

$$y(x) = \frac{x(\operatorname{LambertW}(-\mathrm{e}c_1x^4) - 1)}{2\operatorname{LambertW}(-\mathrm{e}c_1x^4)\sqrt{-\frac{1}{\operatorname{LambertW}(-\mathrm{e}c_1x^4)}}}$$

✓ Solution by Mathematica

Time used: 2.792 (sec). Leaf size: 172

```
DSolve[x^2 (y'[x])^2+x^2-y[x]^2==0,y[x],x,IncludeSingularSolutions -> True]
```

$$\text{Solve}\left[\frac{1}{2}\left(-\frac{y(x)^2}{x^2}-\frac{\sqrt{\frac{y(x)}{x}-1}\sqrt{\frac{y(x)}{x}+1}y(x)}{x}\right.\right.$$
$$\left.\left.-2\log\left(\sqrt{\frac{y(x)}{x}-1}-\sqrt{\frac{y(x)}{x}+1}\right)+1\right)=\log(x)+c_1,y(x)\right]$$
$$\text{Solve}\left[\frac{1}{2}\left(\frac{y(x)^2}{x^2}-\frac{\sqrt{\frac{y(x)}{x}-1}\sqrt{\frac{y(x)}{x}+1}y(x)}{x}\right.\right.$$
$$\left.\left.-2\log\left(\sqrt{\frac{y(x)}{x}-1}-\sqrt{\frac{y(x)}{x}+1}\right)-1\right)=-\log(x)+c_1,y(x)\right]$$

30.34 problem 894

Internal problem ID [4130]

Book: Ordinary differential equations and their solutions. By George Moseley Murphy. 1960
Section: Various 30
Problem number: 894.
ODE order: 1.
ODE degree: 2.

CAS Maple gives this as type `[_linear]`

$$
{y'}^2 x^2 - (-y + x)^2 = 0
$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 24

```
dsolve(x^2*diff(y(x),x)^2 = (x-y(x))^2,y(x), singsol=all)
```

$$
y(x) = x(-\ln(x) + c_1)
$$
$$
y(x) = \frac{x}{2} + \frac{c_1}{x}
$$

✓ Solution by Mathematica

Time used: 0.075 (sec). Leaf size: 30

```
DSolve[x^2 (y'[x])^2==(x-y[x])^2,y[x],x,IncludeSingularSolutions -> True]
```

$$
y(x) \to \frac{x}{2} + \frac{c_1}{x}
$$
$$
y(x) \to x(-\log(x) + c_1)
$$

30.35 problem 895

Internal problem ID [4131]

Book: Ordinary differential equations and their solutions. By George Moseley Murphy. 1960
Section: Various 30
Problem number: 895.
ODE order: 1.
ODE degree: 2.

CAS Maple gives this as type `[_separable]`

$$y'^2x^2 + y^2 - y^4 = 0$$

✓ Solution by Maple
Time used: 0.156 (sec). Leaf size: 52

```
dsolve(x^2*diff(y(x),x)^2+y(x)^2-y(x)^4 = 0,y(x), singsol=all)
```

$$y(x) = -1$$
$$y(x) = 1$$
$$y(x) = 0$$
$$y(x) = \csc(-\ln(x) + c_1)\operatorname{csgn}(\sec(-\ln(x) + c_1))$$
$$y(x) = -\csc(-\ln(x) + c_1)\operatorname{csgn}(\sec(-\ln(x) + c_1))$$

✓ Solution by Mathematica
Time used: 1.724 (sec). Leaf size: 88

```
DSolve[x^2 (y'[x])^2+y[x]^2-y[x]^4==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to -\sqrt{\sec^2(-\log(x) + c_1)}$$
$$y(x) \to \sqrt{\sec^2(-\log(x) + c_1)}$$
$$y(x) \to -\sqrt{\sec^2(\log(x) + c_1)}$$
$$y(x) \to \sqrt{\sec^2(\log(x) + c_1)}$$
$$y(x) \to -1$$
$$y(x) \to 0$$
$$y(x) \to 1$$

30.36 problem 896

Internal problem ID [4132]

Book: Ordinary differential equations and their solutions. By George Moseley Murphy. 1960
Section: Various 30
Problem number: 896.
ODE order: 1.
ODE degree: 2.

CAS Maple gives this as type `[_separable]`

$$\boxed{{y'}^2 x^2 - x y' + y(1 - y) = 0}$$

✓ Solution by Maple
Time used: 0.0 (sec). Leaf size: 17

```
dsolve(x^2*diff(y(x),x)^2-x*diff(y(x),x)+y(x)*(1-y(x)) = 0,y(x), singsol=all)
```

$$y(x) = c_1 x$$
$$y(x) = \frac{c_1 + x}{x}$$

✓ Solution by Mathematica
Time used: 0.044 (sec). Leaf size: 31

```
DSolve[x^2 (y'[x])^2-x y'[x]+y[x](1-y[x])==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to c_1 x$$
$$y(x) \to \frac{x + c_1}{x}$$
$$y(x) \to 0$$
$$y(x) \to 1$$

30.37 problem 897

Internal problem ID [4133]

Book: Ordinary differential equations and their solutions. By George Moseley Murphy. 1960
Section: Various 30
Problem number: 897.
ODE order: 1.
ODE degree: 2.

CAS Maple gives this as type `[_rational]`

$$\boxed{y'^2x^2 + 2axy' - 2ya = -a^2 - x^2}$$

✓ Solution by Maple

Time used: 0.063 (sec). Leaf size: 78

```
dsolve(x^2*diff(y(x),x)^2+2*a*x*diff(y(x),x)+a^2+x^2-2*a*y(x) = 0,y(x), singsol=all)
```

$$y(x) - \operatorname{RootOf}\left(-x\sqrt{\frac{a\left(-2\operatorname{RootOf}\left(-2ay\left(x\right)+a^2+x^2+2a\_Z+\_Z^2\right)+2\_Z-a\right)}{x^2}} - a\operatorname{arcsinh}\left(\frac{\operatorname{RootOf}\left(-2ay(x)+a^2+x^2+2a\_Z+\_Z^2\right)}{x}\right)+c_1\right) = 0$$

✓ Solution by Mathematica

Time used: 0.943 (sec). Leaf size: 82

```
DSolve[x^2 (y'[x])^2+2 a x y'[x]+a^2+x^2-2 a y[x]==0,y[x],x,IncludeSingularSolutions -> True
```

$$\text{Solve}\left[\left\{y(x) = \frac{2axK[1]+x^2K[1]^2+a^2+x^2}{2a}, x = \frac{a\log\left(\sqrt{K[1]^2+1}-K[1]\right)}{\sqrt{K[1]^2+1}} + \frac{c_1}{\sqrt{K[1]^2+1}}\right\}, \{y(x), K[1]\}\right]$$

30.38 problem 898

Internal problem ID [4134]

Book: Ordinary differential equations and their solutions. By George Moseley Murphy. 1960
Section: Various 30
Problem number: 898.
ODE order: 1.
ODE degree: 2.

CAS Maple gives this as type `[[_1st_order, _with_linear_symmetries], _rational]`

$$y'^2x^2 - 2xyy' + y(y+1) = x$$

✓ Solution by Maple
Time used: 0.125 (sec). Leaf size: 22

```
dsolve(x^2*diff(y(x),x)^2-2*x*diff(y(x),x)*y(x)-x+y(x)*(1+y(x)) = 0,y(x), singsol=all)
```

$$y(x) = x$$
$$y(x) = c_1\sqrt{x} - \frac{x\,c_1^2}{4} + x - 1$$

✓ Solution by Mathematica
Time used: 0.115 (sec). Leaf size: 55

```
DSolve[x^2 (y'[x])^2-2 x y[x] y'[x]-x+y[x](1+y[x])==0,y[x],x,IncludeSingularSolutions -> Tru
```

$$y(x) \to x + \frac{{c_1}^2 x}{4} - ic_1\sqrt{x} - 1$$
$$y(x) \to x + \frac{{c_1}^2 x}{4} + ic_1\sqrt{x} - 1$$

30.39 problem 899

Internal problem ID [4135]

Book: Ordinary differential equations and their solutions. By George Moseley Murphy. 1960
Section: Various 30
Problem number: 899.
ODE order: 1.
ODE degree: 2.

CAS Maple gives this as type `[_rational, [_1st_order, '_with_symmetry_[F(x),G(x)*y+H(x)]']]`

$$\boxed{{y'}^2 x^2 - 2xyy' + \left(-x^2+1\right) y^2 = x^4}$$

✓ Solution by Maple
Time used: 0.453 (sec). Leaf size: 58

```
dsolve(x^2*diff(y(x),x)^2-2*x*diff(y(x),x)*y(x)-x^4+(-x^2+1)*y(x)^2 = 0,y(x), singsol=all)
```

$$y(x) = -ix$$
$$y(x) = ix$$
$$y(x) = -\frac{x(\mathrm{e}^x - c_1^2\mathrm{e}^{-x})}{2c_1}$$
$$y(x) = \frac{x(c_1^2\mathrm{e}^x - \mathrm{e}^{-x})}{2c_1}$$

✓ Solution by Mathematica
Time used: 0.247 (sec). Leaf size: 60

```
DSolve[x^2 (y'[x])^2-2 x y[x] y'[x]-x^4+(1-x^2)y[x]^2==0,y[x],x,IncludeSingularSolutions ->
```

$$y(x) \to \frac{1}{2}xe^{-x-c_1}\left(-1+e^{2(x+c_1)}\right)$$
$$y(x) \to \frac{1}{2}\left(xe^{-x+c_1} - xe^{x-c_1}\right)$$

31 Various 31

31.1 problem 900

Internal problem ID [4136]

Book: Ordinary differential equations and their solutions. By George Moseley Murphy. 1960
Section: Various 31
Problem number: 900.
ODE order: 1.
ODE degree: 2.

CAS Maple gives this as type `[[_1st_order, _with_linear_symmetries], _rational, _Clairaut]`

$$\boxed{{y'}^2 x^2 - (2yx + 1) y' + y^2 = -1}$$

✓ Solution by Maple

Time used: 0.063 (sec). Leaf size: 42

```
dsolve(x^2*diff(y(x),x)^2-(1+2*x*y(x))*diff(y(x),x)+1+y(x)^2 = 0,y(x), singsol=all)
```

$$y(x) = \frac{4x^2 - 1}{4x}$$
$$y(x) = c_1 x - \sqrt{c_1 - 1}$$
$$y(x) = c_1 x + \sqrt{c_1 - 1}$$

✓ Solution by Mathematica

Time used: 1.522 (sec). Leaf size: 66

```
DSolve[x^2 (y'[x])^2-(1+2 x y[x])y'[x]+1+y[x]^2==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to x + e^{-2c_1} x + e^{-c_1}$$
$$y(x) \to x + \frac{1}{4} e^{-2c_1} x + \frac{e^{-c_1}}{2}$$
$$y(x) \to x$$
$$y(x) \to x - \frac{1}{4x}$$

31.2 problem 901

Internal problem ID [4137]

Book: Ordinary differential equations and their solutions. By George Moseley Murphy. 1960
Section: Various 31
Problem number: 901.
ODE order: 1.
ODE degree: 2.

CAS Maple gives this as type `[[_homogeneous, ‘class G‘], _rational, _Clairaut]`

$$\boxed{y'^2x^2 - (a + 2yx)\, y' + y^2 = 0}$$

✓ Solution by Maple

Time used: 0.078 (sec). Leaf size: 36

```
dsolve(x^2*diff(y(x),x)^2-(a+2*x*y(x))*diff(y(x),x)+y(x)^2 = 0,y(x), singsol=all)
```

$$y(x) = -\frac{a}{4x}$$
$$y(x) = c_1x - \sqrt{c_1a}$$
$$y(x) = c_1x + \sqrt{c_1a}$$

✓ Solution by Mathematica

Time used: 0.35 (sec). Leaf size: 64

```
DSolve[x^2 (y'[x])^2-(a+2 x y[x])y'[x]+y[x]^2==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to \frac{x - 2\sqrt{a}c_1}{4{c_1}^2}$$
$$y(x) \to \frac{x + 2\sqrt{a}c_1}{4{c_1}^2}$$
$$y(x) \to 0$$
$$y(x) \to -\frac{a}{4x}$$

31.3 problem 902

Internal problem ID [4138]

Book: Ordinary differential equations and their solutions. By George Moseley Murphy. 1960
Section: Various 31
Problem number: 902.
ODE order: 1.
ODE degree: 2.

CAS Maple gives this as type `[[_homogeneous, ‘class A‘], _rational, _dAlembert]`

$$\boxed{y'^2x^2 - x(x-2y)\,y' + y^2 = 0}$$

✓ Solution by Maple
Time used: 0.063 (sec). Leaf size: 32

```
dsolve(x^2*diff(y(x),x)^2-x*(x-2*y(x))*diff(y(x),x)+y(x)^2 = 0,y(x), singsol=all)
```

$$y(x) = \frac{x}{4}$$
$$y(x) = \frac{c_1(-c_1 + x)}{x}$$
$$y(x) = -\frac{c_1(c_1 + x)}{x}$$

✓ Solution by Mathematica
Time used: 0.217 (sec). Leaf size: 64

```
DSolve[x^2 (y'[x])^2-x(x-2 y[x])y'[x]+y[x]^2==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to \frac{e^{-4c_1} - 2ie^{-2c_1}x}{4x}$$
$$y(x) \to \frac{2ie^{-2c_1}x + e^{-4c_1}}{4x}$$
$$y(x) \to 0$$

31.4 problem 903

Internal problem ID [4139]

Book: Ordinary differential equations and their solutions. By George Moseley Murphy. 1960
Section: Various 31
Problem number: 903.
ODE order: 1.
ODE degree: 2.

CAS Maple gives this as type `[[_1st_order, '_with_symmetry_[F(x),G(y)]']]`

$$\boxed{y'^2x^2 + 2x(y + 2x)\, y' + y^2 = 4a}$$

✓ Solution by Maple

Time used: 0.078 (sec). Leaf size: 36

```
dsolve(x^2*diff(y(x),x)^2+2*x*(2*x+y(x))*diff(y(x),x)-4*a+y(x)^2 = 0,y(x), singsol=all)
```

$$y(x) = \frac{-x^2 - a}{x}$$
$$y(x) = \frac{c_1^2 + 4c_1x - 4a}{4x}$$

✓ Solution by Mathematica

Time used: 1.286 (sec). Leaf size: 44

```
DSolve[x^2 (y'[x])^2+2 x(2 x+y[x])y'[x]-4 a+y[x]^2==0,y[x],x,IncludeSingularSolutions -> Tru
```

$$y(x) \to \frac{-a + c_1(-2x + c_1)}{x}$$
$$y(x) \to -2\sqrt{a}$$
$$y(x) \to 2\sqrt{a}$$

31.5 problem 904

Internal problem ID [4140]

Book: Ordinary differential equations and their solutions. By George Moseley Murphy. 1960
Section: Various 31
Problem number: 904.
ODE order: 1.
ODE degree: 2.

CAS Maple gives this as type `[[_homogeneous, ‘class G‘], _rational]`

$$y'^2x^2 + x\left(-2y + x^3\right) y' - \left(2x^3 - y\right) y = 0$$

✓ Solution by Maple
Time used: 0.093 (sec). Leaf size: 49

```
dsolve(x^2*diff(y(x),x)^2+x*(x^3-2*y(x))*diff(y(x),x)-(2*x^3-y(x))*y(x) = 0,y(x), singsol=al
```

$$y(x) = -\frac{x^3}{4}$$
$$y(x) = c_1x(c_1 + x)$$
$$y(x) = c_1x(c_1 - x)$$
$$y(x) = c_1x(c_1 - x)$$
$$y(x) = c_1x(c_1 + x)$$

✓ Solution by Mathematica
Time used: 1.874 (sec). Leaf size: 58

```
DSolve[x^2 (y'[x])^2+x(x^3-2 y[x])y'[x]-(2 x^3-y[x])y[x]==0,y[x],x,IncludeSingularSolutions
```

$$y(x) \to -x(\cosh(c_1) + \sinh(c_1))(-ix + \cosh(c_1) + \sinh(c_1))$$
$$y(x) \to -x(\cosh(c_1) + \sinh(c_1))(ix + \cosh(c_1) + \sinh(c_1))$$
$$y(x) \to 0$$

31.6 problem 905

Internal problem ID [4141]

Book: Ordinary differential equations and their solutions. By George Moseley Murphy. 1960
Section: Various 31
Problem number: 905.
ODE order: 1.
ODE degree: 2.

CAS Maple gives this as type `[_separable]`

$$\boxed{{y'}^2 x^2 + 3xyy' + 2y^2 = 0}$$

✓ Solution by Maple
Time used: 0.0 (sec). Leaf size: 17

```
dsolve(x^2*diff(y(x),x)^2+3*x*diff(y(x),x)*y(x)+2*y(x)^2 = 0,y(x), singsol=all)
```

$$y(x) = \frac{c_1}{x}$$
$$y(x) = \frac{c_1}{x^2}$$

✓ Solution by Mathematica
Time used: 0.042 (sec). Leaf size: 26

```
DSolve[x^2 (y'[x])^2+3 x y[x] y'[x]+2 y[x]^2==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to \frac{c_1}{x^2}$$
$$y(x) \to \frac{c_1}{x}$$
$$y(x) \to 0$$

31.7 problem 906

Internal problem ID [4142]

Book: Ordinary differential equations and their solutions. By George Moseley Murphy. 1960
Section: Various 31
Problem number: 906.
ODE order: 1.
ODE degree: 2.

CAS Maple gives this as type `[[_homogeneous, ‘class G‘], _rational]`

$$\boxed{y'^2x^2 - 3xyy' + 2y^2 = -x^3}$$

✓ Solution by Maple

Time used: 0.109 (sec). Leaf size: 49

```
dsolve(x^2*diff(y(x),x)^2-3*x*diff(y(x),x)*y(x)+x^3+2*y(x)^2 = 0,y(x), singsol=all)
```

$$y(x) = -2x^{\frac{3}{2}}$$
$$y(x) = 2x^{\frac{3}{2}}$$
$$y(x) = \frac{x(c_1^2 + 4x)}{2c_1}$$
$$y(x) = \frac{x(x\,c_1^2 + 4)}{2c_1}$$

✓ Solution by Mathematica

Time used: 60.287 (sec). Leaf size: 961

```
DSolve[x^2 (y'[x])^2-3 x y[x] y'[x]+x^3+2 y[x]^2==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to \frac{e^{-\frac{3c_1}{2}}\left(2\sqrt[3]{2}e^{3c_1}x^3+\left(-4e^{3c_1}x^6-e^{6c_1}x^3+\sqrt{e^{6c_1}x^6\left(-4x^3+e^{3c_1}\right)^2}\right)^{2/3}\right)}{2^{2/3}\sqrt[3]{-4e^{3c_1}x^6-e^{6c_1}x^3+\sqrt{e^{6c_1}x^6\left(-4x^3+e^{3c_1}\right)^2}}}$$

$$y(x) \to \frac{ie^{-\frac{3c_1}{2}}\left(\left(\sqrt{3}+i\right)\left(-4e^{3c_1}x^6-e^{6c_1}x^3+\sqrt{e^{6c_1}x^6\left(-4x^3+e^{3c_1}\right)^2}\right)^{2/3}-2\sqrt[3]{2}\left(\sqrt{3}-i\right)e^{3c_1}x^3\right)}{2\ 2^{2/3}\sqrt[3]{-4e^{3c_1}x^6-e^{6c_1}x^3+\sqrt{e^{6c_1}x^6\left(-4x^3+e^{3c_1}\right)^2}}}$$

$$y(x) \to -\frac{ie^{-\frac{3c_1}{2}}\left(\left(\sqrt{3}-i\right)\left(-4e^{3c_1}x^6-e^{6c_1}x^3+\sqrt{e^{6c_1}x^6\left(-4x^3+e^{3c_1}\right)^2}\right)^{2/3}-2\sqrt[3]{2}\left(\sqrt{3}+i\right)e^{3c_1}x^3\right)}{2\ 2^{2/3}\sqrt[3]{-4e^{3c_1}x^6-e^{6c_1}x^3+\sqrt{e^{6c_1}x^6\left(-4x^3+e^{3c_1}\right)^2}}}$$

$$y(x) \to \frac{e^{-\frac{3c_1}{2}}\left(2\sqrt[3]{2}e^{3c_1}x^3+\left(4e^{3c_1}x^6+e^{6c_1}x^3+\sqrt{e^{6c_1}x^6\left(-4x^3+e^{3c_1}\right)^2}\right)^{2/3}\right)}{2^{2/3}\sqrt[3]{4e^{3c_1}x^6+e^{6c_1}x^3+\sqrt{e^{6c_1}x^6\left(-4x^3+e^{3c_1}\right)^2}}}$$

$$y(x) \to \frac{ie^{-\frac{3c_1}{2}}\left(\left(\sqrt{3}+i\right)\left(4e^{3c_1}x^6+e^{6c_1}x^3+\sqrt{e^{6c_1}x^6\left(-4x^3+e^{3c_1}\right)^2}\right)^{2/3}-2\sqrt[3]{2}\left(\sqrt{3}-i\right)e^{3c_1}x^3\right)}{2\ 2^{2/3}\sqrt[3]{4e^{3c_1}x^6+e^{6c_1}x^3+\sqrt{e^{6c_1}x^6\left(-4x^3+e^{3c_1}\right)^2}}}$$

$$y(x) \to -\frac{ie^{-\frac{3c_1}{2}}\left(\left(\sqrt{3}-i\right)\left(4e^{3c_1}x^6+e^{6c_1}x^3+\sqrt{e^{6c_1}x^6\left(-4x^3+e^{3c_1}\right)^2}\right)^{2/3}-2\sqrt[3]{2}\left(\sqrt{3}+i\right)e^{3c_1}x^3\right)}{2\ 2^{2/3}\sqrt[3]{4e^{3c_1}x^6+e^{6c_1}x^3+\sqrt{e^{6c_1}x^6\left(-4x^3+e^{3c_1}\right)^2}}}$$

31.8 problem 907

Internal problem ID [4143]

Book: Ordinary differential equations and their solutions. By George Moseley Murphy. 1960
Section: Various 31
Problem number: 907.
ODE order: 1.
ODE degree: 2.

CAS Maple gives this as type `[_separable]`

$$y'^2x^2 + 4xyy' - 5y^2 = 0$$

✓ Solution by Maple
Time used: 0.015 (sec). Leaf size: 15

```
dsolve(x^2*diff(y(x),x)^2+4*x*diff(y(x),x)*y(x)-5*y(x)^2 = 0,y(x), singsol=all)
```

$$y(x) = c_1x$$
$$y(x) = \frac{c_1}{x^5}$$

✓ Solution by Mathematica
Time used: 0.04 (sec). Leaf size: 24

```
DSolve[x^2 (y'[x])^2+4 x y[x] y'[x]-5 y[x]^2==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to \frac{c_1}{x^5}$$
$$y(x) \to c_1x$$
$$y(x) \to 0$$

31.9 problem 908

Internal problem ID [4144]

Book: Ordinary differential equations and their solutions. By George Moseley Murphy. 1960
Section: Various 31
Problem number: 908.
ODE order: 1.
ODE degree: 2.

CAS Maple gives this as type `[_separable]`

$$y'^2x^2 - 4x(y+2)\,y' + 4(y+2)\,y = 0$$

✓ Solution by Maple
Time used: 0.078 (sec). Leaf size: 137

```
dsolve(x^2*diff(y(x),x)^2-4*x*(2+y(x))*diff(y(x),x)+4*(2+y(x))*y(x) = 0,y(x), singsol=all)
```

$$y(x) = -2$$
$$y(x) = \frac{2\sqrt{2}\,\sqrt{c_1x^2} + x^2}{c_1}$$
$$y(x) = \frac{-2\sqrt{2}\,\sqrt{c_1x^2} + x^2}{c_1}$$
$$y(x) = \frac{\left(-8c_1^2 + x^2\right)\left(-2\sqrt{2}\,c_1 + x\right)x}{\left(-4\sqrt{2}\,c_1x + 8c_1^2 + x^2\right)c_1^2}$$
$$y(x) = \frac{\left(-8c_1^2 + x^2\right)\left(2\sqrt{2}\,c_1 + x\right)x}{\left(4\sqrt{2}\,c_1x + 8c_1^2 + x^2\right)c_1^2}$$

✓ Solution by Mathematica
Time used: 0.233 (sec). Leaf size: 69

```
DSolve[x^2 (y'[x])^2-4 x(2+y[x])y'[x]+4(2+y[x])y[x]==0,y[x],x,IncludeSingularSolutions -> Tr
```

$$y(x) \to e^{-c_1}x\left(x - 2\sqrt{2}e^{\frac{c_1}{2}}\right)$$
$$y(x) \to e^{c_1}x^2 - 2\sqrt{2}e^{\frac{c_1}{2}}x$$
$$y(x) \to -2$$
$$y(x) \to 0$$

31.10 problem 909

Internal problem ID [4145]

Book: Ordinary differential equations and their solutions. By George Moseley Murphy. 1960
Section: Various 31
Problem number: 909.
ODE order: 1.
ODE degree: 2.

CAS Maple gives this as type `[_separable]`

$$\boxed{y'^2x^2 - 5xyy' + 6y^2 = 0}$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 17

```
dsolve(x^2*diff(y(x),x)^2-5*x*y(x)*diff(y(x),x)+6*y(x)^2 = 0,y(x), singsol=all)
```

$$y(x) = c_1x^2$$
$$y(x) = c_1x^3$$

✓ Solution by Mathematica

Time used: 0.043 (sec). Leaf size: 26

```
DSolve[x^2 (y'[x])^2-5 x y[x] y'[x]+6 y[x]^2==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to c_1x^2$$
$$y(x) \to c_1x^3$$
$$y(x) \to 0$$

31.11 problem 910

Internal problem ID [4146]

Book: Ordinary differential equations and their solutions. By George Moseley Murphy. 1960
Section: Various 31
Problem number: 910.
ODE order: 1.
ODE degree: 2.

CAS Maple gives this as type `[_rational]`

$$
{y'}^2x^2+x\left(x^2+yx-2y\right)y'+(1-x)\left(x^2-y\right)y=0
$$

✗ Solution by Maple

```
dsolve(x^2*diff(y(x),x)^2+x*(x^2+x*y(x)-2*y(x))*diff(y(x),x)+(1-x)*(x^2-y(x))*y(x) = 0,y(x),
```

No solution found

✗ Solution by Mathematica
Time used: 0.0 (sec). Leaf size: 0

```
DSolve[x^2 (y'[x])^2+x(x^2+x y[x]-2 y[x])y'[x]+(1-x)(x^2-y[x])y[x]==0,y[x],x,IncludeSingular
```

Not solved

31.12 problem 911

Internal problem ID [4147]

Book: Ordinary differential equations and their solutions. By George Moseley Murphy. 1960
Section: Various 31
Problem number: 911.
ODE order: 1.
ODE degree: 2.

CAS Maple gives this as type `[[_homogeneous, 'class A'], _dAlembert]`

$$y'^2x^2 + (y + 2x)\, yy' + y^2 = 0$$

✓ Solution by Maple

Time used: 0.109 (sec). Leaf size: 124

```
dsolve(x^2*diff(y(x),x)^2+(2*x+y(x))*y(x)*diff(y(x),x)+y(x)^2 = 0,y(x), singsol=all)
```

$$y(x) = -4x$$
$$y(x) = 0$$
$$y(x) = \frac{2c_1^2\left(-\sqrt{2}\, c_1 + x\right)}{-2c_1^2 + x^2}$$
$$y(x) = \frac{2c_1^2\left(\sqrt{2}\, c_1 + x\right)}{-2c_1^2 + x^2}$$
$$y(x) = \frac{c_1^2\left(\sqrt{2}\, c_1 - 2x\right)}{2c_1^2 - 4x^2}$$
$$y(x) = \frac{c_1^3\sqrt{2} + 2x\, c_1^2}{-2c_1^2 + 4x^2}$$

✓ Solution by Mathematica

Time used: 0.698 (sec). Leaf size: 63

```
DSolve[x^2 (y'[x])^2+(2 x+y[x])y[x] y'[x]+y[x]^2==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to -\frac{e^{4c_1}}{-x+e^{2c_1}}$$
$$y(x) \to \frac{e^{4c_1}}{4\left(4x+e^{2c_1}\right)}$$
$$y(x) \to 0$$
$$y(x) \to -4x$$

31.13 problem 912

Internal problem ID [4148]

Book: Ordinary differential equations and their solutions. By George Moseley Murphy. 1960
Section: Various 31
Problem number: 912.
ODE order: 1.
ODE degree: 2.

CAS Maple gives this as type `[[_homogeneous, 'class A'], _dAlembert]`

$$y'^2 x^2 + (2x - y)\, y y' + y^2 = 0$$

✓ Solution by Maple
Time used: 0.094 (sec). Leaf size: 124

```
dsolve(x^2*diff(y(x),x)^2+(2*x-y(x))*y(x)*diff(y(x),x)+y(x)^2 = 0,y(x), singsol=all)
```

$$y(x) = 4x$$
$$y(x) = 0$$
$$y(x) = -\frac{2c_1^2\left(-\sqrt{2}\, c_1 + x\right)}{-2c_1^2 + x^2}$$
$$y(x) = -\frac{2c_1^2\left(\sqrt{2}\, c_1 + x\right)}{-2c_1^2 + x^2}$$
$$y(x) = \frac{c_1^3\sqrt{2} - 2x\, c_1^2}{-2c_1^2 + 4x^2}$$
$$y(x) = \frac{c_1^2\left(\sqrt{2}\, c_1 + 2x\right)}{2c_1^2 - 4x^2}$$

✓ Solution by Mathematica

Time used: 0.698 (sec). Leaf size: 62

```
DSolve[x^2 (y'[x])^2+(2 x-y[x])y[x] y'[x]+y[x]^2==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to -\frac{4e^{-2c_1}}{2 + e^{2c_1}x}$$
$$y(x) \to -\frac{e^{-2c_1}}{2 + 4e^{2c_1}x}$$
$$y(x) \to 0$$
$$y(x) \to 4x$$

31.14 problem 913

Internal problem ID [4149]

Book: Ordinary differential equations and their solutions. By George Moseley Murphy. 1960
Section: Various 31
Problem number: 913.
ODE order: 1.
ODE degree: 2.

CAS Maple gives this as type `[_quadrature]`

$$\boxed{y'^2 x^2 + (a + b\,x^2 y^3)\, y' + ab y^3 = 0}$$

✓ Solution by Maple
Time used: 0.0 (sec). Leaf size: 35

```
dsolve(x^2*diff(y(x),x)^2+(a+b*x^2*y(x)^3)*diff(y(x),x)+a*b*y(x)^3 = 0,y(x), singsol=all)
```

$$y(x) = \frac{a}{x} + c_1$$
$$y(x) = \frac{1}{\sqrt{2bx + c_1}}$$
$$y(x) = -\frac{1}{\sqrt{2bx + c_1}}$$

✓ Solution by Mathematica
Time used: 0.065 (sec). Leaf size: 49

```
DSolve[x^2 (y'[x])^2+(a+b x^2 y[x]^3)y'[x]+a b y[x]^3==0,y[x],x,IncludeSingularSolutions ->
```

$$y(x) \to -\frac{1}{\sqrt{2bx - 2c_1}}$$
$$y(x) \to \frac{1}{\sqrt{2bx - 2c_1}}$$
$$y(x) \to \frac{a}{x} + c_1$$

31.15 problem 914

Internal problem ID [4150]

Book: Ordinary differential equations and their solutions. By George Moseley Murphy. 1960
Section: Various 31
Problem number: 914.
ODE order: 1.
ODE degree: 2.

CAS Maple gives this as type `[_rational, [_1st_order, '_with_symmetry_[F(x),G(x)*y+H(x)]']]`

$$\left(-x^2+1\right) {y'}^2+y^2=1$$

✓ Solution by Maple

Time used: 0.125 (sec). Leaf size: 162

```
dsolve((-x^2+1)*diff(y(x),x)^2 = 1-y(x)^2,y(x), singsol=all)
```

$$y(x) = -1$$
$$y(x) = 1$$
$$\frac{\sqrt{y(x)^2-1}\,\ln\left(y(x)+\sqrt{y(x)^2-1}\right)}{\sqrt{y(x)-1}\,\sqrt{y(x)+1}}-\frac{\int^x \frac{\sqrt{(\_a^2-1)\left(y(x)^2-1\right)}}{\_a^2-1}d\_a}{\sqrt{y(x)-1}\,\sqrt{y(x)+1}}+c_1=0$$
$$\frac{\sqrt{y(x)^2-1}\,\ln\left(y(x)+\sqrt{y(x)^2-1}\right)}{\sqrt{y(x)-1}\,\sqrt{y(x)+1}}+\frac{\int^x \frac{\sqrt{(\_a^2-1)\left(y(x)^2-1\right)}}{\_a^2-1}d\_a}{\sqrt{y(x)-1}\,\sqrt{y(x)+1}}+c_1=0$$

✓ Solution by Mathematica

Time used: 5.283 (sec). Leaf size: 297

```
DSolve[(1-x^2) (y'[x])^2==1-y[x]^2,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to -\frac{1}{2}e^{-c_1}\sqrt{2x^2 - 2\sqrt{x^2-1}x + e^{4c_1}\left(2x^2 + 2\sqrt{x^2-1}x - 1\right) - 1 + 2e^{2c_1}}$$
$$y(x) \to \frac{1}{2}e^{-c_1}\sqrt{2x^2 - 2\sqrt{x^2-1}x + e^{4c_1}\left(2x^2 + 2\sqrt{x^2-1}x - 1\right) - 1 + 2e^{2c_1}}$$
$$y(x) \to -\frac{1}{2}\sqrt{e^{-2c_1}\left(2x^2 + 2\sqrt{x^2-1}x + e^{4c_1}\left(2x^2 - 2\sqrt{x^2-1}x - 1\right) - 1 + 2e^{2c_1}\right)}$$
$$y(x) \to \frac{1}{2}\sqrt{e^{-2c_1}\left(2x^2 + 2\sqrt{x^2-1}x + e^{4c_1}\left(2x^2 - 2\sqrt{x^2-1}x - 1\right) - 1 + 2e^{2c_1}\right)}$$
$$y(x) \to -1$$
$$y(x) \to 1$$

31.16 problem 915

Internal problem ID [4151]

Book: Ordinary differential equations and their solutions. By George Moseley Murphy. 1960
Section: Various 31
Problem number: 915.
ODE order: 1.
ODE degree: 2.

CAS Maple gives this as type `[[_1st_order, '_with_symmetry_[F(x),G(y)]']]`

$$\left(-x^{2}+1\right) {y'}^{2}+2 x y y' = -4 x^{2}$$

✓ Solution by Maple
Time used: 0.937 (sec). Leaf size: 46

```
dsolve((-x^2+1)*diff(y(x),x)^2+2*x*y(x)*diff(y(x),x)+4*x^2 = 0,y(x), singsol=all)
```

$$y(x) = -2\sqrt{-x^{2}+1}$$
$$y(x) = 2\sqrt{-x^{2}+1}$$
$$y(x) = -c_{1}+c_{1} x^{2}-\frac{1}{c_{1}}$$

✓ Solution by Mathematica
Time used: 0.35 (sec). Leaf size: 63

```
DSolve[(1-x^2) (y'[x])^2+2 x y[x] y'[x]+4 x^2==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to \frac{-4x^{2}+4+{c_{1}}^{2}}{2c_{1}}$$
$$y(x) \to \text{Indeterminate}$$
$$y(x) \to -2\sqrt{1-x^{2}}$$
$$y(x) \to 2\sqrt{1-x^{2}}$$

31.17 problem 916

Internal problem ID [4152]

Book: Ordinary differential equations and their solutions. By George Moseley Murphy. 1960
Section: Various 31
Problem number: 916.
ODE order: 1.
ODE degree: 2.

CAS Maple gives this as type `[_quadrature]`

$$\boxed{(a^2+x^2)\,y'^2=b^2}$$

✓ Solution by Maple
Time used: 0.078 (sec). Leaf size: 40

```
dsolve((a^2+x^2)*diff(y(x),x)^2 = b^2,y(x), singsol=all)
```

$$y(x)=b\ln\left(x+\sqrt{a^2+x^2}\right)+c_1$$
$$y(x)=-b\ln\left(x+\sqrt{a^2+x^2}\right)+c_1$$

✓ Solution by Mathematica
Time used: 0.005 (sec). Leaf size: 101

```
DSolve[(a^2+x^2) (y'[x])^2==b^2,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x)\to\frac{1}{2}b\log\left(1-\frac{x}{\sqrt{a^2+x^2}}\right)-\frac{1}{2}b\log\left(\frac{x}{\sqrt{a^2+x^2}}+1\right)+c_1$$
$$y(x)\to-\frac{1}{2}b\log\left(1-\frac{x}{\sqrt{a^2+x^2}}\right)+\frac{1}{2}b\log\left(\frac{x}{\sqrt{a^2+x^2}}+1\right)+c_1$$

31.18 problem 917

Internal problem ID [4153]

Book: Ordinary differential equations and their solutions. By George Moseley Murphy. 1960
Section: Various 31
Problem number: 917.
ODE order: 1.
ODE degree: 2.

CAS Maple gives this as type `[_quadrature]`

$$\boxed{(a^2 - x^2)\, y'^2 = -b^2}$$

✓ Solution by Maple
Time used: 0.078 (sec). Leaf size: 44

```
dsolve((a^2-x^2)*diff(y(x),x)^2+b^2 = 0,y(x), singsol=all)
```

$$y(x) = b\ln\left(x + \sqrt{-a^2 + x^2}\right) + c_1$$
$$y(x) = -b\ln\left(x + \sqrt{-a^2 + x^2}\right) + c_1$$

✓ Solution by Mathematica
Time used: 0.007 (sec). Leaf size: 109

```
DSolve[(a^2-x^2) (y'[x])^2+b^2==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to \frac{1}{2}b\log\left(1 - \frac{x}{\sqrt{x^2 - a^2}}\right) - \frac{1}{2}b\log\left(\frac{x}{\sqrt{x^2 - a^2}} + 1\right) + c_1$$
$$y(x) \to -\frac{1}{2}b\log\left(1 - \frac{x}{\sqrt{x^2 - a^2}}\right) + \frac{1}{2}b\log\left(\frac{x}{\sqrt{x^2 - a^2}} + 1\right) + c_1$$

31.19 problem 918

Internal problem ID [4154]

Book: Ordinary differential equations and their solutions. By George Moseley Murphy. 1960
Section: Various 31
Problem number: 918.
ODE order: 1.
ODE degree: 2.

CAS Maple gives this as type `[_quadrature]`

$$\boxed{(a^2 - x^2)\, y'^2 = b^2}$$

✓ Solution by Maple

Time used: 0.078 (sec). Leaf size: 44

```
dsolve((a^2-x^2)*diff(y(x),x)^2 = b^2,y(x), singsol=all)
```

$$y(x) = b \arctan\left(\frac{x}{\sqrt{a^2 - x^2}}\right) + c_1$$
$$y(x) = -b \arctan\left(\frac{x}{\sqrt{a^2 - x^2}}\right) + c_1$$

✓ Solution by Mathematica

Time used: 0.007 (sec). Leaf size: 52

```
DSolve[(a^2-x^2) (y'[x])^2==b^2,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to -b \arctan\left(\frac{x}{\sqrt{a^2 - x^2}}\right) + c_1$$
$$y(x) \to b \arctan\left(\frac{x}{\sqrt{a^2 - x^2}}\right) + c_1$$

31.20 problem 919

Internal problem ID [4155]

Book: Ordinary differential equations and their solutions. By George Moseley Murphy. 1960
Section: Various 31
Problem number: 919.
ODE order: 1.
ODE degree: 2.

CAS Maple gives this as type `[_quadrature]`

$$\boxed{(a^2 - x^2)\, y'^2 = x^2}$$

✓ Solution by Maple
Time used: 0.078 (sec). Leaf size: 52

```
dsolve((a^2-x^2)*diff(y(x),x)^2 = x^2,y(x), singsol=all)
```

$$y(x) = -\frac{(a-x)(x+a)}{\sqrt{a^2-x^2}} + c_1$$
$$y(x) = \frac{(a-x)(x+a)}{\sqrt{a^2-x^2}} + c_1$$

✓ Solution by Mathematica
Time used: 0.005 (sec). Leaf size: 43

```
DSolve[(a^2-x^2) (y'[x])^2==x^2,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to -\sqrt{a^2-x^2} + c_1$$
$$y(x) \to \sqrt{a^2-x^2} + c_1$$

31.21 problem 920

Internal problem ID [4156]

Book: Ordinary differential equations and their solutions. By George Moseley Murphy. 1960
Section: Various 31
Problem number: 920.
ODE order: 1.
ODE degree: 2.

CAS Maple gives this as type `[[_1st_order, '_with_symmetry_[F(x),G(y)]']]`

$$\boxed{(a^2 - x^2)\, y'^2 + 2xyy' = -x^2}$$

✓ Solution by Maple
Time used: 1.39 (sec). Leaf size: 51

```
dsolve((a^2-x^2)*diff(y(x),x)^2+2*x*y(x)*diff(y(x),x)+x^2 = 0,y(x), singsol=all)
```

$$y(x) = \sqrt{a^2 - x^2}$$
$$y(x) = -\sqrt{a^2 - x^2}$$
$$y(x) = c_1 x^2 - c_1 a^2 - \frac{1}{4c_1}$$

✓ Solution by Mathematica
Time used: 0.417 (sec). Leaf size: 67

```
DSolve[(a^2-x^2) (y'[x])^2+2 x y[x] y'[x]+x^2==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to \frac{a^2 - x^2 + c_1^2}{2c_1}$$
$$y(x) \to \text{Indeterminate}$$
$$y(x) \to -\sqrt{a^2 - x^2}$$
$$y(x) \to \sqrt{a^2 - x^2}$$

31.22 problem 921

Internal problem ID [4157]

Book: Ordinary differential equations and their solutions. By George Moseley Murphy. 1960
Section: Various 31
Problem number: 921.
ODE order: 1.
ODE degree: 2.

CAS Maple gives this as type `[_separable]`

$$\boxed{(a^2 - x^2)\, y'^2 - 2xyy' - y^2 = 0}$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 23

```
dsolve((a^2-x^2)*diff(y(x),x)^2-2*x*y(x)*diff(y(x),x)-y(x)^2 = 0,y(x), singsol=all)
```

$$y(x) = \frac{c_1}{a - x}$$
$$y(x) = \frac{c_1}{x + a}$$

✓ Solution by Mathematica

Time used: 0.048 (sec). Leaf size: 32

```
DSolve[(a^2-x^2) (y'[x])^2-2 x y[x] y'[x]-y[x]^2==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to \frac{c_1}{a - x}$$
$$y(x) \to \frac{c_1}{a + x}$$
$$y(x) \to 0$$

31.23 problem 922

Internal problem ID [4158]

Book: Ordinary differential equations and their solutions. By George Moseley Murphy. 1960
Section: Various 31
Problem number: 922.
ODE order: 1.
ODE degree: 2.

CAS Maple gives this as type `[[_1st_order, _with_linear_symmetries], _rational, _Clairaut]`

$$(a^2 + x^2)\, y'^2 - 2xyy' + y^2 = -b$$

✓ Solution by Maple

Time used: 0.14 (sec). Leaf size: 84

```
dsolve((a^2+x^2)*diff(y(x),x)^2-2*x*y(x)*diff(y(x),x)+b+y(x)^2 = 0,y(x), singsol=all)
```

$$y(x) = \frac{\sqrt{-b\,(a^2 + x^2)}}{a}$$
$$y(x) = -\frac{\sqrt{-b\,(a^2 + x^2)}}{a}$$
$$y(x) = c_1 x - \sqrt{-a^2 c_1^2 - b}$$
$$y(x) = c_1 x + \sqrt{-a^2 c_1^2 - b}$$

✓ Solution by Mathematica

Time used: 0.463 (sec). Leaf size: 100

```
DSolve[(a^2+x^2) (y'[x])^2-2 x y[x] y'[x]+b+y[x]^2==0,y[x],x,IncludeSingularSolutions -> Tru
```

$$y(x) \to c_1 x - \sqrt{-b - a^2 {c_1}^2}$$
$$y(x) \to \sqrt{-b - a^2 {c_1}^2} + c_1 x$$
$$y(x) \to -\frac{\sqrt{-b\,(a^2 + x^2)}}{a}$$
$$y(x) \to \frac{\sqrt{-b\,(a^2 + x^2)}}{a}$$

31.24 problem 924

Internal problem ID [4159]

Book: Ordinary differential equations and their solutions. By George Moseley Murphy. 1960
Section: Various 31
Problem number: 924.
ODE order: 1.
ODE degree: 2.

CAS Maple gives this as type `[_linear]`

$$4{y'}^2x^2-4xyy'+y^2=8x^3$$

✓ Solution by Maple
Time used: 0.079 (sec). Leaf size: 30

```
dsolve(4*x^2*diff(y(x),x)^2-4*x*y(x)*diff(y(x),x) = 8*x^3-y(x)^2,y(x), singsol=all)
```

$$y(x)=\left(-\sqrt{2}\,x+c_1\right)\sqrt{x}$$
$$y(x)=\left(\sqrt{2}\,x+c_1\right)\sqrt{x}$$

✓ Solution by Mathematica
Time used: 0.076 (sec). Leaf size: 42

```
DSolve[4 x^2 (y'[x])^2-4 x y[x] y'[x]==8 x^3 -y[x]^2,y[x],x,IncludeSingularSolutions -> True
```

$$y(x)\to\sqrt{x}\left(-\sqrt{2}x+c_1\right)$$
$$y(x)\to\sqrt{x}\left(\sqrt{2}x+c_1\right)$$

31.25 problem 925

Internal problem ID [4160]

Book: Ordinary differential equations and their solutions. By George Moseley Murphy. 1960
Section: Various 31
Problem number: 925.
ODE order: 1.
ODE degree: 2.

CAS Maple gives this as type `[[_homogeneous, ‘class A‘], _rational, _dAlembert]`

$$a\,x^2 {y'}^2 - 2axyy' + y^2 = -a(-a+1)\,x^2$$

✓ Solution by Maple

Time used: 0.172 (sec). Leaf size: 106

```
dsolve(a*x^2*diff(y(x),x)^2-2*a*x*y(x)*diff(y(x),x)+a*(1-a)*x^2+y(x)^2 = 0,y(x), singsol=all
```

$$y(x) = x\sqrt{-a}$$
$$y(x) = -x\sqrt{-a}$$
$$y(x) = \operatorname{RootOf}\left(-\ln(x) - \left(\int^{\_Z} \frac{\sqrt{(a-1)\left(\_a^2+a\right)a}}{(a-1)\left(\_a^2+a\right)} d\_a\right) + c_1\right) x$$
$$y(x) = \operatorname{RootOf}\left(-\ln(x) + \int^{\_Z} \frac{\sqrt{(a-1)\left(\_a^2+a\right)a}}{(a-1)\left(\_a^2+a\right)} d\_a + c_1\right) x$$

✓ Solution by Mathematica

Time used: 0.637 (sec). Leaf size: 241

```
DSolve[a x^2 (y'[x])^2-2 a x y[x] y'[x]+a(1-a)x^2+y[x]^2==0,y[x],x,IncludeSingularSolutions
```

$$y(x) \to \frac{1}{2}\sqrt{a}e^{-c_1}x^{1-\sqrt{\frac{a-1}{a}}}\left(x^{2\sqrt{\frac{a-1}{a}}} - e^{2c_1}\right)$$
$$y(x) \to \frac{1}{2}\sqrt{a}e^{-c_1}x^{1-\sqrt{\frac{a-1}{a}}}\left(-x^{2\sqrt{\frac{a-1}{a}}} + e^{2c_1}\right)$$
$$y(x) \to -\frac{1}{2}\sqrt{a}e^{-c_1}x^{1-\sqrt{\frac{a-1}{a}}}\left(-1 + e^{2c_1}x^{2\sqrt{\frac{a-1}{a}}}\right)$$
$$y(x) \to \frac{1}{2}\sqrt{a}e^{-c_1}x^{1-\sqrt{\frac{a-1}{a}}}\left(-1 + e^{2c_1}x^{2\sqrt{\frac{a-1}{a}}}\right)$$

31.26 problem 926

Internal problem ID [4161]

Book: Ordinary differential equations and their solutions. By George Moseley Murphy. 1960
Section: Various 31
Problem number: 926.
ODE order: 1.
ODE degree: 2.

CAS Maple gives this as type `[[_homogeneous, 'class A'], _dAlembert]`

$$\left(-a^2+1\right)x^2{y'}^2-2y'xy+y^2=a^2x^2$$

✓ Solution by Maple

Time used: 0.094 (sec). Leaf size: 229

```
dsolve((-a^2+1)*x^2*diff(y(x),x)^2-2*x*y(x)*diff(y(x),x)-a^2*x^2+y(x)^2 = 0,y(x), singsol=al
```

$$\frac{2a\ln(x)-2\sqrt{-a^2}\,\arctan\left(\frac{a^2y(x)}{\sqrt{-a^2}\sqrt{\frac{-x^2a^2+x^2+y(x)^2}{x^2}}\,x}\right)+\ln\left(\frac{x^2+y(x)^2}{x^2}\right)a-2c_1a+2\ln\left(\frac{\sqrt{\frac{-x^2a^2+x^2+y(x)^2}{x^2}}\,x+y(x)}{x}\right.}{2a}$$
$$=0$$

$$\frac{2a\ln(x)+2\sqrt{-a^2}\,\arctan\left(\frac{a^2y(x)}{\sqrt{-a^2}\sqrt{\frac{-x^2a^2+x^2+y(x)^2}{x^2}}\,x}\right)+\ln\left(\frac{x^2+y(x)^2}{x^2}\right)a-2c_1a-2\ln\left(\frac{\sqrt{\frac{-x^2a^2+x^2+y(x)^2}{x^2}}\,x+y(x)}{x}\right.}{2a}$$
$$=0$$

✓ Solution by Mathematica

Time used: 1.04 (sec). Leaf size: 223

```
DSolve[(1-a^2)x^2 (y'[x])^2-2 x y[x] y'[x]-a^2 x^2 + y[x]^2==0,y[x],x,IncludeSingularSolutio
```

$$\text{Solve}\left[\frac{2i\arctan\left(\frac{y(x)}{x\sqrt{a^2-\frac{y(x)^2}{x^2}-1}}\right)-2ia\arctan\left(\frac{ay(x)}{x\sqrt{a^2-\frac{y(x)^2}{x^2}-1}}\right)+a\log\left(\frac{y(x)^2}{x^2}+1\right)}{2a^2-2}=\frac{a\log\left(x-a^2x\right)}{1-a^2}\right.$$

$$\left.+c_1,y(x)\right]$$

$$\text{Solve}\left[\frac{-2i\arctan\left(\frac{y(x)}{x\sqrt{a^2-\frac{y(x)^2}{x^2}-1}}\right)+2ia\arctan\left(\frac{ay(x)}{x\sqrt{a^2-\frac{y(x)^2}{x^2}-1}}\right)+a\log\left(\frac{y(x)^2}{x^2}+1\right)}{2a^2-2}=\frac{a\log\left(x-a^2x\right)}{1-a^2}\right.$$

$$\left.+c_1,y(x)\right]$$

31.27 problem 927

Internal problem ID [4162]

Book: Ordinary differential equations and their solutions. By George Moseley Murphy. 1960
Section: Various 31
Problem number: 927.
ODE order: 1.
ODE degree: 2.

CAS Maple gives this as type `[_quadrature]`

$$\boxed{x^3 {y'}^2 = a}$$

✓ Solution by Maple
Time used: 0.094 (sec). Leaf size: 37

```
dsolve(x^3*diff(y(x),x)^2 = a,y(x), singsol=all)
```

$$y(x) = \frac{c_1 x - 2\sqrt{ax}}{x}$$
$$y(x) = \frac{c_1 x + 2\sqrt{ax}}{x}$$

✓ Solution by Mathematica
Time used: 0.004 (sec). Leaf size: 39

```
DSolve[x^3 (y'[x])^2==a,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to -\frac{2\sqrt{a}}{\sqrt{x}} + c_1$$
$$y(x) \to \frac{2\sqrt{a}}{\sqrt{x}} + c_1$$

31.28 problem 928

Internal problem ID [4163]

Book: Ordinary differential equations and their solutions. By George Moseley Murphy. 1960
Section: Various 31
Problem number: 928.
ODE order: 1.
ODE degree: 2.

CAS Maple gives this as type `[[_homogeneous, ‘class G‘], _rational]`

$$x^3y'^2 + xy' - y = 0$$

✗ Solution by Maple

```
dsolve(x^3*diff(y(x),x)^2+x*diff(y(x),x)-y(x) = 0,y(x), singsol=all)
```

No solution found

✓ Solution by Mathematica

Time used: 105.529 (sec). Leaf size: 7052

```
DSolve[x^3 (y'[x])^2+x y'[x]-y[x]==0,y[x],x,IncludeSingularSolutions -> True]
```

Too large to display

31.29 problem 929

Internal problem ID [4164]

Book: Ordinary differential equations and their solutions. By George Moseley Murphy. 1960
Section: Various 31
Problem number: 929.
ODE order: 1.
ODE degree: 2.

CAS Maple gives this as type `[[_homogeneous, ‘class G‘]]`

$$x^3 {y'}^2 + y' x^2 y = -a$$

✓ Solution by Maple

Time used: 0.36 (sec). Leaf size: 66

```
dsolve(x^3*diff(y(x),x)^2+x^2*y(x)*diff(y(x),x)+a = 0,y(x), singsol=all)
```

$$y(x) = -\frac{2\sqrt{ax}}{x}$$
$$y(x) = \frac{2\sqrt{ax}}{x}$$
$$y(x) = \frac{x\, c_1^2 + 4a}{2c_1 x}$$
$$y(x) = \frac{4ax + c_1^2}{2c_1 x}$$

✓ Solution by Mathematica

Time used: 0.851 (sec). Leaf size: 57

```
DSolve[x^3 (y'[x])^2+x^2 y[x] y'[x]+a==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to -\frac{e^{-\frac{c_1}{2}}(x + 4ae^{c_1})}{2x}$$
$$y(x) \to \frac{e^{-\frac{c_1}{2}}(x + 4ae^{c_1})}{2x}$$

31.30 problem 931

Internal problem ID [4165]

Book: Ordinary differential equations and their solutions. By George Moseley Murphy. 1960
Section: Various 31
Problem number: 931.
ODE order: 1.
ODE degree: 2.

CAS Maple gives this as type `[_rational, [_1st_order, '_with_symmetry_[F(x),G(x)*y+H(x)]']]`

$$x(-x^2+1)\,y'^2-2(-x^2+1)\,yy'+x(1-y^2)=0$$

✓ Solution by Maple

Time used: 0.407 (sec). Leaf size: 33

```
dsolve(x*(-x^2+1)*diff(y(x),x)^2-2*(-x^2+1)*y(x)*diff(y(x),x)+x*(1-y(x)^2) = 0,y(x), singsol
```

$$y(x)=-x$$
$$y(x)=x$$
$$y(x)=\sqrt{-c_1^2+1}+\sqrt{x^2-1}\,c_1$$

✓ Solution by Mathematica

Time used: 0.752 (sec). Leaf size: 75

```
DSolve[x*(1-x^2)*(y'[x])^2-2*(1-x^2)*y[x]*y'[x]+x*(1-y[x]^2)==0,y[x],x,IncludeSingularSoluti
```

$$y(x)\to -x\cos\left(2\tan^{-1}\left(\sqrt{\frac{x-1}{x+1}}\right)+ic_1\right)$$
$$y(x)\to -x\cos\left(2\tan^{-1}\left(\sqrt{\frac{x-1}{x+1}}\right)-ic_1\right)$$
$$y(x)\to -x$$
$$y(x)\to x$$

31.31 problem 932

Internal problem ID [4166]

Book: Ordinary differential equations and their solutions. By George Moseley Murphy. 1960
Section: Various 31
Problem number: 932.
ODE order: 1.
ODE degree: 2.

CAS Maple gives this as type `[_quadrature]`

$$4x(-x+a)(-x+b)\,y'^2 = \left(ab-2(a+b)\,x+2x^2\right)^2$$

✓ Solution by Maple

Time used: 0.11 (sec). Leaf size: 85

```
dsolve(4*x*(a-x)*(b-x)*diff(y(x),x)^2 = (a*b-2*x*(a+b)+2*x^2)^2,y(x), singsol=all)
```

$$y(x) = -\frac{\left(\int \frac{2x^2+(-2a-2b)x+ab}{\sqrt{x(-x+b)(a-x)}}dx\right)}{2} + c_1$$
$$y(x) = \frac{\left(\int \frac{2x^2+(-2a-2b)x+ab}{\sqrt{x(-x+b)(a-x)}}dx\right)}{2} + c_1$$

✓ Solution by Mathematica

Time used: 14.208 (sec). Leaf size: 375

```
DSolve[4 x(a-x)(b-x) (y'[x])^2==(a b-2 x(a+b)+2 x^2)^2,y[x],x,IncludeSingularSolutions -> Tr
```

$$y(x) \to c_1 - \frac{(a-x)\left(2(a^2-b^2)\sqrt{\frac{x}{a}}\sqrt{\frac{x-b}{a-b}}E\left(i\operatorname{arcsinh}\left(\sqrt{\frac{x}{a}-1}\right)|\frac{a}{a-b}\right)+b(a+2b)\sqrt{\frac{x}{a}}\sqrt{\frac{x-b}{a-b}}\operatorname{EllipticF}\left(i\operatorname{arcsinh}\left(\sqrt{\right.\right.}{3\sqrt{\frac{x}{a}-1}\sqrt{x(a-x)(x-b)}}$$

$$y(x) \to \frac{(a-x)\left(2(a^2-b^2)\sqrt{\frac{x}{a}}\sqrt{\frac{x-b}{a-b}}E\left(i\operatorname{arcsinh}\left(\sqrt{\frac{x}{a}-1}\right)|\frac{a}{a-b}\right)+b(a+2b)\sqrt{\frac{x}{a}}\sqrt{\frac{x-b}{a-b}}\operatorname{EllipticF}\left(i\operatorname{arcsinh}\left(\sqrt{\frac{x}{a}\right.\right.}{3\sqrt{\frac{x}{a}-1}\sqrt{x(a-x)(x-b)}} + c_1$$

31.32 problem 933

Internal problem ID [4167]

Book: Ordinary differential equations and their solutions. By George Moseley Murphy. 1960
Section: Various 31
Problem number: 933.
ODE order: 1.
ODE degree: 2.

CAS Maple gives this as type `[[_homogeneous, ‘class G‘], _rational]`

$$\boxed{x^4 {y'}^2 - x y' - y = 0}$$

✓ Solution by Maple

Time used: 0.078 (sec). Leaf size: 81

```
dsolve(x^4*diff(y(x),x)^2-x*diff(y(x),x)-y(x) = 0,y(x), singsol=all)
```

$$y(x) = -\frac{1}{4x^2}$$
$$y(x) = \frac{-c_1 i - x}{x\, c_1^2}$$
$$y(x) = \frac{c_1 i - x}{x\, c_1^2}$$
$$y(x) = \frac{c_1 i - x}{x\, c_1^2}$$
$$y(x) = \frac{-c_1 i - x}{x\, c_1^2}$$

✓ Solution by Mathematica

Time used: 0.517 (sec). Leaf size: 123

```
DSolve[x^4 (y'[x])^2-x y'[x]-y[x]==0,y[x],x,IncludeSingularSolutions -> True]
```

$$\text{Solve}\left[-\frac{x\sqrt{4x^2y(x)+1}\text{arctanh}\left(\sqrt{4x^2y(x)+1}\right)}{\sqrt{4x^4y(x)+x^2}}-\frac{1}{2}\log(y(x))=c_1,y(x)\right]$$
$$\text{Solve}\left[\frac{x\sqrt{4x^2y(x)+1}\text{arctanh}\left(\sqrt{4x^2y(x)+1}\right)}{\sqrt{4x^4y(x)+x^2}}-\frac{1}{2}\log(y(x))=c_1,y(x)\right]$$
$$y(x)\to 0$$

31.33 problem 934

Internal problem ID [4168]

Book: Ordinary differential equations and their solutions. By George Moseley Murphy. 1960
Section: Various 31
Problem number: 934.
ODE order: 1.
ODE degree: 2.

CAS Maple gives this as type `[[_homogeneous, ‘class G‘], _rational]`

$$x^4 {y'}^2 + 2x^3 y y' = 4$$

✓ Solution by Maple

Time used: 0.093 (sec). Leaf size: 49

```
dsolve(x^4*diff(y(x),x)^2+2*x^3*y(x)*diff(y(x),x)-4 = 0,y(x), singsol=all)
```

$$y(x) = -\frac{2i}{x}$$
$$y(x) = \frac{2i}{x}$$
$$y(x) = \frac{2\sinh(-\ln(x) + c_1)}{x}$$
$$y(x) = -\frac{2\sinh(-\ln(x) + c_1)}{x}$$

✓ Solution by Mathematica

Time used: 0.679 (sec). Leaf size: 71

```
DSolve[x^4 (y'[x])^2+2 x^3 y[x] y'[x]-4==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to \frac{4e^{c_1}}{x^2} - \frac{e^{-c_1}}{4}$$
$$y(x) \to \frac{e^{-c_1}}{4} - \frac{4e^{c_1}}{x^2}$$
$$y(x) \to -\frac{2i}{x}$$
$$y(x) \to \frac{2i}{x}$$

32 Various 32

32.1 problem 935

Internal problem ID [4169]

Book: Ordinary differential equations and their solutions. By George Moseley Murphy. 1960
Section: Various 32
Problem number: 935.
ODE order: 1.
ODE degree: 2.

CAS Maple gives this as type `[[_homogeneous, 'class G']]`

$$\boxed{x^4 {y'}^2 + x y^2 y' - y^3 = 0}$$

✓ Solution by Maple

Time used: 0.297 (sec). Leaf size: 129

```
dsolve(x^4*diff(y(x),x)^2+x*y(x)^2*diff(y(x),x)-y(x)^3 = 0,y(x), singsol=all)
```

$$y(x) = -4x^2$$
$$y(x) = 0$$
$$y(x) = \frac{\left(\sqrt{2}\, c_1 - 2x\right) c_1^2 x}{2c_1^2 - 4x^2}$$
$$y(x) = -\frac{\left(\sqrt{2}\, c_1 + 2x\right) c_1^2 x}{2c_1^2 - 4x^2}$$
$$y(x) = -\frac{2\left(-c_1 x + \sqrt{2}\right) x}{c_1 \left(c_1^2 x^2 - 2\right)}$$
$$y(x) = \frac{2\left(c_1 x + \sqrt{2}\right) x}{c_1 \left(c_1^2 x^2 - 2\right)}$$

✓ Solution by Mathematica

Time used: 0.844 (sec). Leaf size: 79

```
DSolve[x^4 (y'[x])^2+x y[x]^2 y'[x]-y[x]^3==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to -\frac{x(\cosh(2c_1) + \sinh(2c_1))}{x + i\cosh(c_1) + i\sinh(c_1)}$$
$$y(x) \to \frac{x(\cosh(2c_1) + \sinh(2c_1))}{-x + i\cosh(c_1) + i\sinh(c_1)}$$
$$y(x) \to 0$$

32.2 problem 936

Internal problem ID [4170]

Book: Ordinary differential equations and their solutions. By George Moseley Murphy. 1960
Section: Various 32
Problem number: 936.
ODE order: 1.
ODE degree: 2.

CAS Maple gives this as type `[_quadrature]`

$$x^2(a^2 - x^2)\, y'^2 = -1$$

✓ Solution by Maple
Time used: 0.109 (sec). Leaf size: 111

```
dsolve(x^2*(a^2-x^2)*diff(y(x),x)^2+1 = 0,y(x), singsol=all)
```

$$y(x) = \frac{c_1\sqrt{-a^2} - \ln(2) - \ln\left(\frac{\sqrt{-a^2}\,\sqrt{-a^2+x^2}-a^2}{x}\right)}{\sqrt{-a^2}}$$
$$y(x) = \frac{c_1\sqrt{-a^2} + \ln(2) + \ln\left(\frac{\sqrt{-a^2}\,\sqrt{-a^2+x^2}-a^2}{x}\right)}{\sqrt{-a^2}}$$

✓ Solution by Mathematica
Time used: 0.013 (sec). Leaf size: 120

```
DSolve[x^2(a^2-x^2) (y'[x])^2+1==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to -\frac{x\sqrt{x^2-a^2}\arctan\left(\frac{\sqrt{x^2-a^2}}{a}\right)}{a\sqrt{x^4-a^2x^2}} + c_1$$
$$y(x) \to \frac{x\sqrt{x^2-a^2}\arctan\left(\frac{\sqrt{x^2-a^2}}{a}\right)}{a\sqrt{x^4-a^2x^2}} + c_1$$

32.3 problem 937

Internal problem ID [4171]

Book: Ordinary differential equations and their solutions. By George Moseley Murphy. 1960
Section: Various 32
Problem number: 937.
ODE order: 1.
ODE degree: 2.

CAS Maple gives this as type `[_rational, [_1st_order, '_with_symmetry_[F(x),G(x)*y+H(x)]']]`

$$3x^4 {y'}^2 - yx - y = 0$$

✓ Solution by Maple

Time used: 0.109 (sec). Leaf size: 201

```
dsolve(3*x^4*diff(y(x),x)^2-x*y(x)-y(x) = 0,y(x), singsol=all)
```

$$y(x) = 0$$
$$y(x) = \frac{\left(\sqrt{3}\,\operatorname{arctanh}\left(\sqrt{x+1}\right) x\sqrt{x+1} + 3c_1 x\sqrt{x+1} + \sqrt{3}\,x + \sqrt{3}\right)^2}{36\,(x+1)\,x^2}$$
$$y(x) = \frac{\left(\sqrt{3}\,\operatorname{arctanh}\left(\sqrt{x+1}\right) x\sqrt{x+1} - 3c_1 x\sqrt{x+1} + \sqrt{3}\,x + \sqrt{3}\right)^2}{36\,(x+1)\,x^2}$$
$$y(x) = \frac{\left(\sqrt{3}\,\operatorname{arctanh}\left(\sqrt{x+1}\right) x\sqrt{x+1} - 3c_1 x\sqrt{x+1} + \sqrt{3}\,x + \sqrt{3}\right)^2}{36\,(x+1)\,x^2}$$
$$y(x) = \frac{\left(\sqrt{3}\,\operatorname{arctanh}\left(\sqrt{x+1}\right) x\sqrt{x+1} + 3c_1 x\sqrt{x+1} + \sqrt{3}\,x + \sqrt{3}\right)^2}{36\,(x+1)\,x^2}$$

✓ Solution by Mathematica

Time used: 0.153 (sec). Leaf size: 171

```
DSolve[3 x^4 (y'[x])^2-x y[x]-y[x]==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to \frac{x^2\text{arctanh}\left(\sqrt{x+1}\right)^2 + 2x\text{arctanh}\left(\sqrt{x+1}\right)\left(\sqrt{x+1} - \sqrt{3}c_1x\right) + 3c_1{}^2x^2 + x - 2\sqrt{3}c_1x\sqrt{x+1} + 1}{12x^2}$$

$$y(x) \to \frac{x^2\text{arctanh}\left(\sqrt{x+1}\right)^2 + 2x\text{arctanh}\left(\sqrt{x+1}\right)\left(\sqrt{x+1} + \sqrt{3}c_1x\right) + 3c_1{}^2x^2 + x + 2\sqrt{3}c_1x\sqrt{x+1} + 1}{12x^2}$$

$$y(x) \to 0$$

32.4 problem 938

Internal problem ID [4172]

Book: Ordinary differential equations and their solutions. By George Moseley Murphy. 1960
Section: Various 32
Problem number: 938.
ODE order: 1.
ODE degree: 2.

CAS Maple gives this as type `[[_homogeneous, ‘class G‘]]`

$$\boxed{4x^5 {y'}^2 + 12x^4 y y' = -9}$$

✓ Solution by Maple
Time used: 0.344 (sec). Leaf size: 53

```
dsolve(4*x^5*diff(y(x),x)^2+12*x^4*y(x)*diff(y(x),x)+9 = 0,y(x), singsol=all)
```

$$y(x) = \frac{1}{x^{\frac{3}{2}}}$$
$$y(x) = -\frac{1}{x^{\frac{3}{2}}}$$
$$y(x) = \frac{c_1^2 x^3 + 1}{2c_1 x^3}$$
$$y(x) = \frac{x^3 + c_1^2}{2c_1 x^3}$$

✓ Solution by Mathematica

Time used: 7.064 (sec). Leaf size: 75

```
DSolve[4 x^5 (y'[x])^2+12 x^4 y[x] y'[x]+9==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to -\frac{1}{\sqrt{x^3 \operatorname{sech}^2\left(\frac{3}{2}(-\log(x)+c_1)\right)}}$$
$$y(x) \to \frac{1}{\sqrt{x^3 \operatorname{sech}^2\left(\frac{3}{2}(-\log(x)+c_1)\right)}}$$
$$y(x) \to -\frac{1}{x^{3/2}}$$
$$y(x) \to \frac{1}{x^{3/2}}$$

32.5 problem 939

Internal problem ID [4173]

Book: Ordinary differential equations and their solutions. By George Moseley Murphy. 1960
Section: Various 32
Problem number: 939.
ODE order: 1.
ODE degree: 2.

CAS Maple gives this as type `[[_homogeneous, ‘class G‘], _rational]`

$$\boxed{x^6 {y'}^2 - 2xy' - 4y = 0}$$

✓ Solution by Maple

Time used: 0.062 (sec). Leaf size: 89

```
dsolve(x^6*diff(y(x),x)^2-2*x*diff(y(x),x)-4*y(x) = 0,y(x), singsol=all)
```

$$y(x) = -\frac{1}{4x^4}$$
$$y(x) = \frac{-c_1 i - x^2}{c_1^2 x^2}$$
$$y(x) = \frac{c_1 i - x^2}{x^2 c_1^2}$$
$$y(x) = \frac{c_1 i - x^2}{x^2 c_1^2}$$
$$y(x) = \frac{-c_1 i - x^2}{c_1^2 x^2}$$

✓ Solution by Mathematica

Time used: 0.549 (sec). Leaf size: 128

```
DSolve[x^6 (y'[x])^2-2 x y'[x]-4 y[x]==0,y[x],x,IncludeSingularSolutions -> True]
```

$$\text{Solve}\left[-\frac{x\sqrt{4x^4y(x)+1}\operatorname{arctanh}\left(\sqrt{4x^4y(x)+1}\right)}{2\sqrt{4x^6y(x)+x^2}}-\frac{1}{4}\log(y(x))=c_1,y(x)\right]$$
$$\text{Solve}\left[\frac{x\sqrt{4x^4y(x)+1}\operatorname{arctanh}\left(\sqrt{4x^4y(x)+1}\right)}{2\sqrt{4x^6y(x)+x^2}}-\frac{1}{4}\log(y(x))=c_1,y(x)\right]$$
$$y(x)\to 0$$

32.6 problem 940

Internal problem ID [4174]

Book: Ordinary differential equations and their solutions. By George Moseley Murphy. 1960
Section: Various 32
Problem number: 940.
ODE order: 1.
ODE degree: 2.

CAS Maple gives this as type `[[_homogeneous, 'class G']]`

$$\boxed{x^8y'^2 + 3xy' + 9y = 0}$$

✓ Solution by Maple
Time used: 0.079 (sec). Leaf size: 45

```
dsolve(x^8*diff(y(x),x)^2+3*x*diff(y(x),x)+9*y(x) = 0,y(x), singsol=all)
```

$$y(x) = \frac{1}{4x^6}$$
$$y(x) = \frac{-x^3 + c_1}{x^3 c_1^2}$$
$$y(x) = \frac{-x^3 - c_1}{c_1^2 x^3}$$

✓ Solution by Mathematica
Time used: 0.562 (sec). Leaf size: 130

```
DSolve[x^8 (y'[x])^2+3 x y'[x]+9 y[x]==0,y[x],x,IncludeSingularSolutions -> True]
```

$$\text{Solve}\left[\frac{x\sqrt{4x^6y(x)-1}\arctan\left(\sqrt{4x^6y(x)-1}\right)}{3\sqrt{x^2-4x^8y(x)}} - \frac{1}{6}\log(y(x)) = c_1, y(x)\right]$$
$$\text{Solve}\left[\frac{\sqrt{x^2-4x^8y(x)}\arctan\left(\sqrt{4x^6y(x)-1}\right)}{3x\sqrt{4x^6y(x)-1}} - \frac{1}{6}\log(y(x)) = c_1, y(x)\right]$$
$$y(x) \to 0$$

32.7 problem 941

Internal problem ID [4175]

Book: Ordinary differential equations and their solutions. By George Moseley Murphy. 1960
Section: Various 32
Problem number: 941.
ODE order: 1.
ODE degree: 2.

CAS Maple gives this as type `[_quadrature]`

$$
\boxed{y{y'}^2 = a}
$$

✓ Solution by Maple
Time used: 0.157 (sec). Leaf size: 173

```
dsolve(y(x)*diff(y(x),x)^2 = a,y(x), singsol=all)
```

$$
y(x) = \frac{12^{\frac{2}{3}}(a^2(-c_1+x))^{\frac{2}{3}}}{4a}
$$
$$
y(x) = \frac{12^{\frac{2}{3}}(a^2(-c_1+x))^{\frac{2}{3}}\left(1+i\sqrt{3}\right)^2}{16a}
$$
$$
y(x) = \frac{12^{\frac{2}{3}}(a^2(-c_1+x))^{\frac{2}{3}}\left(i\sqrt{3}-1\right)^2}{16a}
$$
$$
y(x) = \frac{12^{\frac{2}{3}}(a^2(c_1-x))^{\frac{2}{3}}}{4a}
$$
$$
y(x) = \frac{12^{\frac{2}{3}}(a^2(c_1-x))^{\frac{2}{3}}\left(1+i\sqrt{3}\right)^2}{16a}
$$
$$
y(x) = \frac{12^{\frac{2}{3}}(a^2(c_1-x))^{\frac{2}{3}}\left(i\sqrt{3}-1\right)^2}{16a}
$$

✓ Solution by Mathematica

Time used: 3.749 (sec). Leaf size: 54

```
DSolve[y[x] (y'[x])^2==a,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to \left(\frac{3}{2}\right)^{2/3} \left(-\sqrt{a}x + c_1\right)^{2/3}$$
$$y(x) \to \left(\frac{3}{2}\right)^{2/3} \left(\sqrt{a}x + c_1\right)^{2/3}$$

32.8 problem 942

Internal problem ID [4176]

Book: Ordinary differential equations and their solutions. By George Moseley Murphy. 1960
Section: Various 32
Problem number: 942.
ODE order: 1.
ODE degree: 2.

CAS Maple gives this as type `[[_homogeneous, 'class A'], _dAlembert]`

$$y{y'}^2 = x\,a^2$$

✓ Solution by Maple

Time used: 0.156 (sec). Leaf size: 78

```
dsolve(y(x)*diff(y(x),x)^2 = a^2*x,y(x), singsol=all)
```

$$x\left(1-\frac{c_1}{\left(-\frac{a^2\left(-ax\sqrt{xy(x)}+y(x)^2\right)}{y(x)^2}\right)^{\frac{2}{3}} y\left(x\right)}\right) = 0$$

$$x\left(1-\frac{c_1}{\left(-\frac{a^2\left(ax\sqrt{xy(x)}+y(x)^2\right)}{y(x)^2}\right)^{\frac{2}{3}} y\left(x\right)}\right) = 0$$

✓ Solution by Mathematica

Time used: 3.625 (sec). Leaf size: 46

```
DSolve[y[x] (y'[x])^2==a^2 x,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \rightarrow \left(-ax^{3/2}+\frac{3c_1}{2}\right)^{2/3}$$
$$y(x) \rightarrow \left(ax^{3/2}+\frac{3c_1}{2}\right)^{2/3}$$

32.9 problem 943

Internal problem ID [4177]

Book: Ordinary differential equations and their solutions. By George Moseley Murphy. 1960
Section: Various 32
Problem number: 943.
ODE order: 1.
ODE degree: 2.

CAS Maple gives this as type `[[_1st_order, _with_linear_symmetries]]`

$$\boxed{yy'^2 = \mathrm{e}^{2x}}$$

✓ Solution by Maple

Time used: 0.172 (sec). Leaf size: 67

```
dsolve(y(x)*diff(y(x),x)^2 = exp(2*x),y(x), singsol=all)
```

$$\frac{2y(x)^2 + 3c_1\sqrt{y(x)} - 3\sqrt{\mathrm{e}^{2x}y(x)}}{3\sqrt{y(x)}} = 0$$
$$\frac{2y(x)^2 + 3c_1\sqrt{y(x)} + 3\sqrt{\mathrm{e}^{2x}y(x)}}{3\sqrt{y(x)}} = 0$$

✓ Solution by Mathematica

Time used: 2.162 (sec). Leaf size: 47

```
DSolve[y[x] (y'[x])^2==Exp[2 x],y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to \left(\frac{3}{2}\right)^{2/3} \left(-e^x + c_1\right)^{2/3}$$
$$y(x) \to \left(\frac{3}{2}\right)^{2/3} \left(e^x + c_1\right)^{2/3}$$

32.10 problem 944

Internal problem ID [4178]

Book: Ordinary differential equations and their solutions. By George Moseley Murphy. 1960
Section: Various 32
Problem number: 944.
ODE order: 1.
ODE degree: 2.

CAS Maple gives this as type `[[_homogeneous, ‘class A‘], _rational, _dAlembert]`

$$y{y'}^2+2axy'-ya=0$$

✓ Solution by Maple
Time used: 0.172 (sec). Leaf size: 113

```
dsolve(y(x)*diff(y(x),x)^2+2*a*x*diff(y(x),x)-a*y(x) = 0,y(x), singsol=all)
```

$$y(x)=x\sqrt{-a}$$
$$y(x)=-x\sqrt{-a}$$
$$y(x)=0$$
$$y(x)=\mathrm{RootOf}\left(-\ln(x)-\left(\int^{\_Z}\frac{\_a^2+\sqrt{(\_a^2+a)\,a}+a}{\_a\,(\_a^2+a)}d\_a\right)+c_1\right)x$$
$$y(x)=\mathrm{RootOf}\left(-\ln(x)+\int^{\_Z}-\frac{\_a^2-\sqrt{(\_a^2+a)\,a}+a}{\_a\,(\_a^2+a)}d\_a+c_1\right)x$$

✓ Solution by Mathematica
Time used: 8.189 (sec). Leaf size: 88

```
DSolve[y[x] (y'[x])^2+2 a x y'[x]-a y[x]==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x)\to-\sqrt{e^{c_1}\left(-2\sqrt{a}x+e^{c_1}\right)}$$
$$y(x)\to\sqrt{e^{c_1}\left(-2\sqrt{a}x+e^{c_1}\right)}$$
$$y(x)\to 0$$
$$y(x)\to -i\sqrt{a}x$$
$$y(x)\to i\sqrt{a}x$$

32.11 problem 945

Internal problem ID [4179]

Book: Ordinary differential equations and their solutions. By George Moseley Murphy. 1960
Section: Various 32
Problem number: 945.
ODE order: 1.
ODE degree: 2.

CAS Maple gives this as type `[[_homogeneous, ‘class A‘], _dAlembert]`

$$\boxed{yy'^2 - 4a^2xy' + ya^2 = 0}$$

✓ Solution by Maple

Time used: 0.328 (sec). Leaf size: 122

```
dsolve(y(x)*diff(y(x),x)^2-4*a^2*x*diff(y(x),x)+a^2*y(x) = 0,y(x), singsol=all)
```

$$y(x) = 0$$
$$y(x) = \operatorname{RootOf}\left(-\ln(x) - \left(\int^{\_Z} \frac{\_a^2 - 2a^2 + \sqrt{-\_a^2a^2 + 4a^4}}{\_a\left(\_a^2 - 3a^2\right)}d\_a\right) + c_1\right)x$$
$$y(x) = \operatorname{RootOf}\left(-\ln(x) + \int^{\_Z} -\frac{\_a^2 - 2a^2 - \sqrt{-\_a^2a^2 + 4a^4}}{\_a\left(\_a^2 - 3a^2\right)}d\_a + c_1\right)x$$

✓ Solution by Mathematica

Time used: 8.731 (sec). Leaf size: 758

```
DSolve[y[x] (y'[x])^2-4 a^2 x y'[x]+a^2 y[x]==0,y[x],x,IncludeSingularSolutions -> True]
```

$$\text{Solve}\left[\frac{8\left(4a^2-\frac{y(x)^2}{x^2}\right)^{3/2}\text{arcsinh}\left(\frac{\sqrt{\frac{y(x)}{x}-2a}}{2\sqrt{a}}\right)+\sqrt{a}\sqrt{\frac{y(x)}{ax}+2}\left(\sqrt{-\left(\frac{y(x)}{x}-2a\right)^2}\sqrt{2a+\frac{y(x)}{x}}\sqrt{4a^2-\frac{y(x)^2}{x^2}}\left(\right.\right.}{}\right.$$

$$\left.-\log(x)+c_1,y(x)\right]$$

$$\text{Solve}\left[\frac{\sqrt{a}\sqrt{\frac{y(x)}{ax}+2}\left(\sqrt{-\left(\frac{y(x)}{x}-2a\right)^2}\sqrt{2a+\frac{y(x)}{x}}\sqrt{4a^2-\frac{y(x)^2}{x^2}}\left(\log\left(3a^2-\frac{y(x)^2}{x^2}\right)+8\arctan\left(\frac{\sqrt{2a-\frac{y(x)}{x}}}{\sqrt{2a+\frac{y(x)}{x}}}\right.\right.\right.}{}\right.$$

$$\left.-\log(x)+c_1,y(x)\right]$$

$$y(x)\to 0$$

32.12 problem 946

Internal problem ID [4180]

Book: Ordinary differential equations and their solutions. By George Moseley Murphy. 1960
Section: Various 32
Problem number: 946.
ODE order: 1.
ODE degree: 2.

CAS Maple gives this as type `[[_homogeneous, ‘class A‘], _rational, _dAlembert]`

$$\boxed{yy'^2 + axy' + yb = 0}$$

✓ Solution by Maple

Time used: 0.453 (sec). Leaf size: 108

```
dsolve(y(x)*diff(y(x),x)^2+a*x*diff(y(x),x)+b*y(x) = 0,y(x), singsol=all)
```

$$y(x) = 0$$
$$y(x) = \text{RootOf}\left(-2\ln(x) - \left(\int^{\_Z} \frac{2\_a^2 + \sqrt{-4\_a^2 b + a^2} + a}{\_a\,(\_a^2 + a + b)} d\_a\right) + 2c_1\right) x$$
$$y(x) = \text{RootOf}\left(-2\ln(x) + \int^{\_Z} -\frac{2\_a^2 + a - \sqrt{-4\_a^2 b + a^2}}{\_a\,(\_a^2 + a + b)} d\_a + 2c_1\right) x$$

✓ Solution by Mathematica

Time used: 0.617 (sec). Leaf size: 162

```
DSolve[y[x] (y'[x])^2+a x y'[x]+b y[x]==0,y[x],x,IncludeSingularSolutions -> True]
```

$$\text{Solve}\left[\frac{a\log\left(\sqrt{a^2-\frac{4by(x)^2}{x^2}}+a\right)+(a+2b)\log\left(\sqrt{a^2-\frac{4by(x)^2}{x^2}}-a-2b\right)}{4(a+b)}=-\frac{\log(x)}{2}+c_1,y(x)\right]$$

$$\text{Solve}\left[\frac{a\log\left(\sqrt{a^2-\frac{4by(x)^2}{x^2}}-a\right)+(a+2b)\log\left(\sqrt{a^2-\frac{4by(x)^2}{x^2}}+a+2b\right)}{4(a+b)}=-\frac{\log(x)}{2}+c_1,y(x)\right]$$

$$y(x)\to 0$$

32.13 problem 947

Internal problem ID [4181]

Book: Ordinary differential equations and their solutions. By George Moseley Murphy. 1960
Section: Various 32
Problem number: 947.
ODE order: 1.
ODE degree: 2.

CAS Maple gives this as type `[[_homogeneous, ‘class C‘], _rational, _dAlembert]`

$$y{y'}^2 - (-2bx + a)\, y' - yb = 0$$

✓ Solution by Maple

Time used: 0.406 (sec). Leaf size: 149

```
dsolve(y(x)*diff(y(x),x)^2-(-2*b*x+a)*diff(y(x),x)-b*y(x) = 0,y(x), singsol=all)
```

$$y(x) = -\frac{-2bx + a}{2\sqrt{-b}}$$
$$y(x) = \frac{-2bx + a}{2\sqrt{-b}}$$
$$y(x) = 0$$
$$y(x) = \sqrt{\frac{c_1 b + \sqrt{c_1 b\,(-2bx + a)^2}}{b}}$$
$$y(x) = \sqrt{-\frac{-c_1 b + \sqrt{c_1 b\,(-2bx + a)^2}}{b}}$$
$$y(x) = -\sqrt{\frac{c_1 b + \sqrt{c_1 b\,(-2bx + a)^2}}{b}}$$
$$y(x) = -\sqrt{\frac{c_1 b - \sqrt{c_1 b\,(-2bx + a)^2}}{b}}$$

✓ Solution by Mathematica

Time used: 1.067 (sec). Leaf size: 409

```
DSolve[y[x] (y'[x])^2-(a-2 b x)y'[x]-b y[x]==0,y[x],x,IncludeSingularSolutions -> True]
```

$$\text{Solve}\left[\frac{\left(b-\sqrt{b^2}\right)\log(y(x))}{b} - \frac{-b\log\left(\sqrt{a^2-4abx+4b\left(bx^2+y(x)^2\right)}-a-2\sqrt{b^2}x\right)+\sqrt{b^2}\log\left(b\left(\sqrt{a^2-4abx+4b\left(bx^2+y(x)^2\right)}-a\right.\right.}{2\sqrt{b^2}}\right.$$

$$\text{Solve}\left[\frac{-b\log\left(\sqrt{a^2-4abx+4b\left(bx^2+y(x)^2\right)}-a-2\sqrt{b^2}x\right)+\sqrt{b^2}\log\left(b\left(\sqrt{a^2-4abx+4b\left(bx^2+y(x)^2\right.}\right.\right.}{2\sqrt{b^2}}\right.$$
$$+\frac{\left(\sqrt{b^2}+b\right)\log(y(x))}{b}=c_1,y(x)\right]$$

$$y(x)\to -\frac{i(2bx-a)}{2\sqrt{b}}$$

$$y(x)\to \frac{i(2bx-a)}{2\sqrt{b}}$$

32.14 problem 948

Internal problem ID [4182]

Book: Ordinary differential equations and their solutions. By George Moseley Murphy. 1960
Section: Various 32
Problem number: 948.
ODE order: 1.
ODE degree: 2.

CAS Maple gives this as type `[[_1st_order, _with_linear_symmetries]]`

$$\boxed{yy'^2 + y'x^3 - x^2y = 0}$$

✓ Solution by Maple

Time used: 0.328 (sec). Leaf size: 89

```
dsolve(y(x)*diff(y(x),x)^2+x^3*diff(y(x),x)-x^2*y(x) = 0,y(x), singsol=all)
```

$$y(x) = -\frac{ix^2}{2}$$
$$y(x) = \frac{ix^2}{2}$$
$$y(x) = 0$$
$$y(x) = -\frac{\sqrt{c_1\left(-4x^2 + c_1\right)}}{4}$$
$$y(x) = \frac{\sqrt{c_1\left(-4x^2 + c_1\right)}}{4}$$
$$y(x) = -\frac{2\sqrt{c_1x^2 + 4}}{c_1}$$
$$y(x) = \frac{2\sqrt{c_1x^2 + 4}}{c_1}$$

✓ Solution by Mathematica

Time used: 1.28 (sec). Leaf size: 244

```
DSolve[y[x] (y'[x])^2+x^3 y'[x]-x^2 y[x]==0,y[x],x,IncludeSingularSolutions -> True]
```

$$\text{Solve}\left[\frac{\sqrt{x^6+4x^2y(x)^2}\log\left(\sqrt{x^4+4y(x)^2}+x^2\right)}{2x\sqrt{x^4+4y(x)^2}}+\frac{1}{2}\left(1-\frac{\sqrt{x^6+4x^2y(x)^2}}{x\sqrt{x^4+4y(x)^2}}\right)\log(y(x))=c_1,y(x)\right]$$

$$\text{Solve}\left[\frac{1}{2}\left(\frac{\sqrt{x^6+4x^2y(x)^2}}{x\sqrt{x^4+4y(x)^2}}+1\right)\log(y(x))-\frac{\sqrt{x^6+4x^2y(x)^2}\log\left(\sqrt{x^4+4y(x)^2}+x^2\right)}{2x\sqrt{x^4+4y(x)^2}}=c_1,y(x)\right]$$

$$y(x)\to -\frac{ix^2}{2}$$

$$y(x)\to \frac{ix^2}{2}$$

32.15 problem 949

Internal problem ID [4183]

Book: Ordinary differential equations and their solutions. By George Moseley Murphy. 1960
Section: Various 32
Problem number: 949.
ODE order: 1.
ODE degree: 2.

CAS Maple gives this as type `[_quadrature]`

$$yy'^2 + (-y + x)\, y' = x$$

✓ Solution by Maple
Time used: 0.0 (sec). Leaf size: 33

```
dsolve(y(x)*diff(y(x),x)^2+(x-y(x))*diff(y(x),x)-x = 0,y(x), singsol=all)
```

$$y(x) = \sqrt{-x^2 + c_1}$$
$$y(x) = -\sqrt{-x^2 + c_1}$$
$$y(x) = c_1 + x$$

✓ Solution by Mathematica
Time used: 0.146 (sec). Leaf size: 47

```
DSolve[y[x] (y'[x])^2+(x-y[x])y'[x]-x==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to x + c_1$$
$$y(x) \to -\sqrt{-x^2 + 2c_1}$$
$$y(x) \to \sqrt{-x^2 + 2c_1}$$

32.16 problem 950

Internal problem ID [4184]

Book: Ordinary differential equations and their solutions. By George Moseley Murphy. 1960
Section: Various 32
Problem number: 950.
ODE order: 1.
ODE degree: 2.

CAS Maple gives this as type `[[_homogeneous, ‘class A‘], _rational, _dAlembert]`

$$yy'^2 - (y + x)\, y' + y = 0$$

✓ Solution by Maple

Time used: 0.141 (sec). Leaf size: 176

```
dsolve(y(x)*diff(y(x),x)^2-(x+y(x))*diff(y(x),x)+y(x) = 0,y(x), singsol=all)
```

$$y(x) = x$$
$$y(x) = 0$$
$$\frac{-x\sqrt{\frac{(3y(x)+x)(x-y(x))}{x^2}} + 2y(x)\ln\left(\frac{y(x)}{x}\right) + \left(-2\,\mathrm{arctanh}\left(\frac{x+y(x)}{x\sqrt{\frac{(3y(x)+x)(x-y(x))}{x^2}}}\right) - 2c_1 + 2\ln(x)\right)y(x) - x}{2y(x)} = 0$$
$$\frac{x\sqrt{\frac{(3y(x)+x)(x-y(x))}{x^2}} + 2y(x)\ln\left(\frac{y(x)}{x}\right) + \left(2\,\mathrm{arctanh}\left(\frac{x+y(x)}{x\sqrt{\frac{(3y(x)+x)(x-y(x))}{x^2}}}\right) - 2c_1 + 2\ln(x)\right)y(x) - x}{2y(x)} = 0$$

✓ Solution by Mathematica

Time used: 4.229 (sec). Leaf size: 320

```
DSolve[y[x] (y'[x])^2-(x+y[x])y'[x]+y[x]==0,y[x],x,IncludeSingularSolutions -> True]
```

$$\text{Solve}\left[-\frac{x\left(-i\sqrt{\frac{y(x)}{x}-1}\sqrt{\frac{3y(x)}{x}+1}+\frac{4y(x)\log\left(\sqrt{\frac{3y(x)}{x}-3}-\sqrt{\frac{3y(x)}{x}+1}\right)}{x}-\frac{4y(x)\log\left(-i\left(\frac{3y(x)}{x}+1\right)+i\sqrt{\frac{3y(x)}{x}-3}\sqrt{\frac{3y(x)}{x}+1}+\cdots\right.}{x}\cdots\right.}{4y(x)}\right.$$

$$\left.-\frac{\log(x)}{2}+c_1,y(x)\right]$$

$$\text{Solve}\left[-\frac{x\left(i\sqrt{\frac{y(x)}{x}-1}\sqrt{\frac{3y(x)}{x}+1}+\frac{4y(x)\log\left(\sqrt{\frac{3y(x)}{x}-3}-\sqrt{\frac{3y(x)}{x}+1}\right)}{x}-\frac{4y(x)\log\left(i\left(\frac{3y(x)}{x}+1\right)-i\sqrt{\frac{3y(x)}{x}-3}\sqrt{\frac{3y(x)}{x}+1}+\sqrt{2-\cdots}\right.}{x}\cdots\right.}{4y(x)}\right.$$

$$\left.-\frac{\log(x)}{2}+c_1,y(x)\right]$$

$$y(x)\to 0$$

32.17 problem 951

Internal problem ID [4185]

Book: Ordinary differential equations and their solutions. By George Moseley Murphy. 1960
Section: Various 32
Problem number: 951.
ODE order: 1.
ODE degree: 2.

CAS Maple gives this as type `[_quadrature]`

$$\boxed{yy'^2 - (1 + yx)\, y' = -x}$$

✓ Solution by Maple
Time used: 0.016 (sec). Leaf size: 33

```
dsolve(y(x)*diff(y(x),x)^2-(1+x*y(x))*diff(y(x),x)+x = 0,y(x), singsol=all)
```

$$y(x) = \sqrt{2x + c_1}$$
$$y(x) = -\sqrt{2x + c_1}$$
$$y(x) = \frac{x^2}{2} + c_1$$

✓ Solution by Mathematica
Time used: 0.1 (sec). Leaf size: 52

```
DSolve[y[x] (y'[x])^2-(1+x y[x])y'[x]+x==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to -\sqrt{2}\sqrt{x + c_1}$$
$$y(x) \to \sqrt{2}\sqrt{x + c_1}$$
$$y(x) \to \frac{x^2}{2} + c_1$$

32.18 problem 952

Internal problem ID [4186]

Book: Ordinary differential equations and their solutions. By George Moseley Murphy. 1960
Section: Various 32
Problem number: 952.
ODE order: 1.
ODE degree: 2.

CAS Maple gives this as type `[_quadrature]`

$$y{y'}^2 + (x - y^2)\, y' - yx = 0$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 34

```
dsolve(y(x)*diff(y(x),x)^2+(x-y(x)^2)*diff(y(x),x)-x*y(x) = 0,y(x), singsol=all)
```

$$y(x) = \sqrt{-x^2 + c_1}$$
$$y(x) = -\sqrt{-x^2 + c_1}$$
$$y(x) = \mathrm{e}^x c_1$$

✓ Solution by Mathematica

Time used: 0.098 (sec). Leaf size: 54

```
DSolve[y[x] (y'[x])^2+(x-y[x]^2)y'[x]-x y[x]==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to c_1 e^x$$
$$y(x) \to -\sqrt{-x^2 + 2c_1}$$
$$y(x) \to \sqrt{-x^2 + 2c_1}$$
$$y(x) \to 0$$

32.19 problem 953

Internal problem ID [4187]

Book: Ordinary differential equations and their solutions. By George Moseley Murphy. 1960
Section: Various 32
Problem number: 953.
ODE order: 1.
ODE degree: 2.

CAS Maple gives this as type `[_quadrature]`

$$y{y'}^2 + y = a$$

✓ Solution by Maple
Time used: 0.141 (sec). Leaf size: 339

```
dsolve(y(x)*diff(y(x),x)^2+y(x) = a,y(x), singsol=all)
```

$$y(x) = a$$

$$y(x) = \frac{(\text{RootOf}\left((\cos(\_Z)\,a + a\_Z + 2c_1 - 2x)\,(-\cos(\_Z)\,a + a\_Z + 2c_1 - 2x)\right)a - 2x + 2c_1)\tan(\text{RootOf}(}{2} + \frac{a}{2}$$

$$y(x) = \frac{(-\text{RootOf}\left((\cos(\_Z)\,a + a\_Z + 2c_1 - 2x)\,(-\cos(\_Z)\,a + a\_Z + 2c_1 - 2x)\right)a + 2x - 2c_1)\tan(\text{RootC}}{2} + \frac{a}{2}$$

$$y(x) = \frac{(\text{RootOf}\left((\cos(\_Z)\,a - a\_Z + 2c_1 - 2x)\,(-\cos(\_Z)\,a - a\_Z + 2c_1 - 2x)\right)a + 2x - 2c_1)\tan(\text{RootOf}(}{2} + \frac{a}{2}$$

$$y(x) = \frac{(-\text{RootOf}\left((\cos(\_Z)\,a - a\_Z + 2c_1 - 2x)\,(-\cos(\_Z)\,a - a\_Z + 2c_1 - 2x)\right)a - 2x + 2c_1)\tan(\text{RootC}}{2} + \frac{a}{2}$$

✓ Solution by Mathematica

Time used: 0.429 (sec). Leaf size: 106

```
DSolve[y[x] (y'[x])^2+y[x]==a,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to \text{InverseFunction}\left[a \arctan\left(\frac{\sqrt{\#1}}{\sqrt{a-\#1}}\right) - \sqrt{\#1}\sqrt{a-\#1}\&\right][-x+c_1]$$
$$y(x) \to \text{InverseFunction}\left[a \arctan\left(\frac{\sqrt{\#1}}{\sqrt{a-\#1}}\right) - \sqrt{\#1}\sqrt{a-\#1}\&\right][x+c_1]$$
$$y(x) \to a$$

32.20 problem 954

Internal problem ID [4188]

Book: Ordinary differential equations and their solutions. By George Moseley Murphy. 1960
Section: Various 32
Problem number: 954.
ODE order: 1.
ODE degree: 2.

CAS Maple gives this as type `[[_homogeneous, ‘class A‘], _rational, _dAlembert]`

$$(y+x)\, {y'}^2 + 2xy' - y = 0$$

✓ Solution by Maple

Time used: 0.297 (sec). Leaf size: 121

```
dsolve((x+y(x))*diff(y(x),x)^2+2*x*diff(y(x),x)-y(x) = 0,y(x), singsol=all)
```

$$y(x) = -\frac{\left(1+i\sqrt{3}\right)x}{2}$$
$$y(x) = \frac{\left(i\sqrt{3}-1\right)x}{2}$$
$$\ln(x) - \operatorname{arctanh}\left(\frac{y(x)+2x}{2x\sqrt{\frac{y(x)^2+xy(x)+x^2}{x^2}}}\right) + \ln\left(\frac{y(x)}{x}\right) - c_1 = 0$$
$$\ln(x) + \operatorname{arctanh}\left(\frac{y(x)+2x}{2x\sqrt{\frac{y(x)^2+xy(x)+x^2}{x^2}}}\right) + \ln\left(\frac{y(x)}{x}\right) - c_1 = 0$$

✓ Solution by Mathematica

Time used: 4.545 (sec). Leaf size: 166

```
DSolve[(x+y[x]) (y'[x])^2+2 x y'[x]-y[x]==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to -\frac{2}{3}\sqrt{e^{c_1}\left(-3x+e^{c_1}\right)}-\frac{e^{c_1}}{3}$$
$$y(x) \to \frac{2}{3}\sqrt{e^{c_1}\left(-3x+e^{c_1}\right)}-\frac{e^{c_1}}{3}$$
$$y(x) \to e^{c_1}-2\sqrt{e^{c_1}\left(x+e^{c_1}\right)}$$
$$y(x) \to 2\sqrt{e^{c_1}\left(x+e^{c_1}\right)}+e^{c_1}$$
$$y(x) \to 0$$
$$y(x) \to -\frac{1}{2}i\left(\sqrt{3}-i\right)x$$
$$y(x) \to \frac{1}{2}i\left(\sqrt{3}+i\right)x$$

32.21 problem 955

Internal problem ID [4189]

Book: Ordinary differential equations and their solutions. By George Moseley Murphy. 1960
Section: Various 32
Problem number: 955.
ODE order: 1.
ODE degree: 2.

CAS Maple gives this as type `[[_homogeneous, ‘class C‘], _dAlembert]`

$$(2x-y)\,{y'}^2-2(1-x)\,y'-y=-2$$

✓ Solution by Maple
Time used: 2.016 (sec). Leaf size: 71

```
dsolve((2*x-y(x))*diff(y(x),x)^2-2*(1-x)*diff(y(x),x)+2-y(x) = 0,y(x), singsol=all)
```

$$y(x)=-\sqrt{2}\,x+\sqrt{2}+x+1$$
$$y(x)=(x-1)\,\sqrt{2}+x+1$$
$$y(x)=2+\frac{c_1}{2}-\frac{\sqrt{c_1\,(-c_1+4x-4)}}{2}$$
$$y(x)=2+c_1-\sqrt{c_1\,(-c_1+2x-2)}$$

✓ Solution by Mathematica
Time used: 4.92 (sec). Leaf size: 187

```
DSolve[(2 x -y[x]) (y'[x])^2-2(1-x)y'[x]+2-y[x]==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x)\to-\frac{1}{2}\sqrt{-e^{c_1}\left(4x-4+e^{c_1}\right)}+2-\frac{e^{c_1}}{2}$$
$$y(x)\to\frac{1}{2}\left(\sqrt{-e^{c_1}\left(4x-4+e^{c_1}\right)}+4-e^{c_1}\right)$$
$$y(x)\to-\sqrt{-e^{c_1}\left(2x-2+e^{c_1}\right)}+2-e^{c_1}$$
$$y(x)\to\sqrt{-e^{c_1}\left(2x-2+e^{c_1}\right)}+2-e^{c_1}$$
$$y(x)\to2$$
$$y(x)\to x-\sqrt{2}\sqrt{(x-1)^2}+1$$
$$y(x)\to x+\sqrt{2}\sqrt{(x-1)^2}+1$$

32.22 problem 956

Internal problem ID [4190]

Book: Ordinary differential equations and their solutions. By George Moseley Murphy. 1960
Section: Various 32
Problem number: 956.
ODE order: 1.
ODE degree: 2.

CAS Maple gives this as type `[[_homogeneous, ‘class C‘], _rational, _dAlembert]`

$$2yy'^2 + (5-4x)\,y' + 2y = 0$$

✓ Solution by Maple

Time used: 0.25 (sec). Leaf size: 119

```
dsolve(2*y(x)*diff(y(x),x)^2+(5-4*x)*diff(y(x),x)+2*y(x) = 0,y(x), singsol=all)
```

$$y(x) = x - \frac{5}{4}$$
$$y(x) = -x + \frac{5}{4}$$
$$y(x) = 0$$
$$y(x) = \frac{\sqrt{4c_1 + 2\sqrt{-c_1\,(-5+4x)^2}}}{2}$$
$$y(x) = -\frac{\sqrt{4c_1 + 2\sqrt{-c_1\,(-5+4x)^2}}}{2}$$
$$y(x) = \frac{\sqrt{4c_1 - 2\sqrt{-c_1\,(-5+4x)^2}}}{2}$$
$$y(x) = -\frac{\sqrt{4c_1 - 2\sqrt{-c_1\,(-5+4x)^2}}}{2}$$

✓ Solution by Mathematica

Time used: 0.722 (sec). Leaf size: 160

```
DSolve[(2 y[x] (y'[x])^2)+(5-4 x)y'[x]+2 y[x]==0,y[x],x,IncludeSingularSolutions -> True]
```

$$
\begin{aligned}
y(x) &\to -i\sqrt{2}e^{\frac{c_1}{2}}\sqrt{4x-5+8e^{c_1}} \\
y(x) &\to i\sqrt{2}e^{\frac{c_1}{2}}\sqrt{4x-5+8e^{c_1}} \\
y(x) &\to -\frac{1}{4}ie^{\frac{c_1}{2}}\sqrt{8x-10+e^{c_1}} \\
y(x) &\to \frac{1}{4}ie^{\frac{c_1}{2}}\sqrt{8x-10+e^{c_1}} \\
y(x) &\to 0 \\
y(x) &\to \frac{5}{4}-x \\
y(x) &\to x-\frac{5}{4}
\end{aligned}
$$

32.23 problem 957

Internal problem ID [4191]

Book: Ordinary differential equations and their solutions. By George Moseley Murphy. 1960
Section: Various 32
Problem number: 957.
ODE order: 1.
ODE degree: 2.

CAS Maple gives this as type `[[_1st_order, _with_linear_symmetries]]`

$$9y{y'}^2 + 4y'x^3 - 4x^2y = 0$$

✓ Solution by Maple

Time used: 0.328 (sec). Leaf size: 89

```
dsolve(9*y(x)*diff(y(x),x)^2+4*x^3*diff(y(x),x)-4*x^2*y(x) = 0,y(x), singsol=all)
```

$$y(x) = -\frac{ix^2}{3}$$
$$y(x) = \frac{ix^2}{3}$$
$$y(x) = 0$$
$$y(x) = -\frac{2\sqrt{c_1x^2+9}}{c_1}$$
$$y(x) = \frac{2\sqrt{c_1x^2+9}}{c_1}$$
$$y(x) = -\frac{\sqrt{c_1(-4x^2+c_1)}}{6}$$
$$y(x) = \frac{\sqrt{c_1(-4x^2+c_1)}}{6}$$

✓ Solution by Mathematica

Time used: 1.295 (sec). Leaf size: 244

```
DSolve[9 y[x] (y'[x])^2+4 x^3 y'[x]-4 x^2 y[x]==0,y[x],x,IncludeSingularSolutions -> True]
```

$$\text{Solve}\left[\frac{\sqrt{x^6+9x^2y(x)^2}\log\left(\sqrt{x^4+9y(x)^2}+x^2\right)}{2x\sqrt{x^4+9y(x)^2}} + \frac{1}{2}\left(1-\frac{\sqrt{x^6+9x^2y(x)^2}}{x\sqrt{x^4+9y(x)^2}}\right)\log(y(x)) = c_1, y(x)\right]$$

$$\text{Solve}\left[\frac{1}{2}\left(\frac{\sqrt{x^6+9x^2y(x)^2}}{x\sqrt{x^4+9y(x)^2}}+1\right)\log(y(x)) - \frac{\sqrt{x^6+9x^2y(x)^2}\log\left(\sqrt{x^4+9y(x)^2}+x^2\right)}{2x\sqrt{x^4+9y(x)^2}} = c_1, y(x)\right]$$

$$y(x) \to -\frac{ix^2}{3}$$

$$y(x) \to \frac{ix^2}{3}$$

32.24 problem 958

Internal problem ID [4192]

Book: Ordinary differential equations and their solutions. By George Moseley Murphy. 1960
Section: Various 32
Problem number: 958.
ODE order: 1.
ODE degree: 2.

CAS Maple gives this as type `[_quadrature]`

$$(1 - ya)\, {y'}^2 - ya = 0$$

✓ Solution by Maple

Time used: 0.172 (sec). Leaf size: 399

```
dsolve((1-a*y(x))*diff(y(x),x)^2 = a*y(x),y(x), singsol=all)
```

$$y(x) = 0$$

$$y(x) = \frac{\operatorname{RootOf}\left(4a^2c_1^2 - 8x\, a^2c_1 + 4x^2a^2 - 4 \operatorname{csgn}(a)\, ac_1 \operatorname{RootOf}\left(-4 \operatorname{csgn}(a)\, ac_1\_Z + 4 \operatorname{csgn}(a)\, ax\_Z + 4a^2\right.\right.}{}$$

$$y(x) = \frac{\operatorname{RootOf}\left(4a^2c_1^2 - 8x\, a^2c_1 + 4x^2a^2 + 4 \operatorname{csgn}(a)\, ac_1 \operatorname{RootOf}\left(4 \operatorname{csgn}(a)\, ac_1\_Z - 4 \operatorname{csgn}(a)\, ax\_Z + 4a^2c_1^2\right.\right.}{}$$

✓ Solution by Mathematica

Time used: 0.571 (sec). Leaf size: 147

```
DSolve[(1-a y[x]) (y'[x])^2==a y[x],y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to \text{InverseFunction}\left[\frac{2\arctan\left(\frac{\sqrt{\#1}\sqrt{a}}{\sqrt{1-\#1a-1}}\right)}{\sqrt{a}} + \sqrt{\#1}\sqrt{1-\#1a}\&\right]\left[-\sqrt{a}x + c_1\right]$$

$$y(x) \to \text{InverseFunction}\left[\frac{2\arctan\left(\frac{\sqrt{\#1}\sqrt{a}}{\sqrt{1-\#1a-1}}\right)}{\sqrt{a}} + \sqrt{\#1}\sqrt{1-\#1a}\&\right]\left[\sqrt{a}x + c_1\right]$$

$$y(x) \to 0$$

32.25 problem 960

Internal problem ID [4193]

Book: Ordinary differential equations and their solutions. By George Moseley Murphy. 1960
Section: Various 32
Problem number: 960.
ODE order: 1.
ODE degree: 2.

CAS Maple gives this as type `[_quadrature]`

$$\left(x^2 - ya\right) {y'}^2 - 2xyy' = 0$$

✓ Solution by Maple
Time used: 0.031 (sec). Leaf size: 28

```
dsolve((x^2-a*y(x))*diff(y(x),x)^2-2*x*y(x)*diff(y(x),x) = 0,y(x), singsol=all)
```

$$y(x) = -\frac{x^2}{a \operatorname{LambertW}\left(-\frac{c_1 x^2}{a}\right)}$$
$$y(x) = c_1$$

✓ Solution by Mathematica

Time used: 8.11 (sec). Leaf size: 310

```
DSolve[(x^2-a y[x]) (y'[x])^2-2 x y[x] y'[x]==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to c_1$$

$$\text{Solve}\left[\frac{\left(2-\frac{2(2axy(x)+x^3)}{\sqrt[3]{x^3}(x^2-ay(x))}\right)\left(\frac{\frac{6x^3}{x^2-ay(x)}-4x}{\sqrt[3]{x^3}}+4\right)\left(\left(1-\frac{x(2ay(x)+x^2)}{\sqrt[3]{x^3}(x^2-ay(x))}\right)\log\left(\frac{2-\frac{2\left(2axy(x)+x^3\right)}{\sqrt[3]{x^3}\left(x^2-ay(x)\right)}}{\sqrt[3]{2}}\right)+\left(\frac{2axy(}{\sqrt[3]{x^3}(x}\right.\right.}{18\sqrt[3]{2}\left(-\frac{(2ay(x)+x^2)^3}{(x^2-ay(x))^3}+\frac{3(2axy(x)+x^3)}{\sqrt[3]{x^3}(x^2-ay(x))}-2\right)}\right.$$

$$\left.+c_1, y(x)\right]$$

$$y(x) \to 0$$

32.26 problem 961

Internal problem ID [4194]

Book: Ordinary differential equations and their solutions. By George Moseley Murphy. 1960
Section: Various 32
Problem number: 961.
ODE order: 1.
ODE degree: 2.

CAS Maple gives this as type `[_quadrature]`

$$\boxed{xyy'^2 + (y + x)\, y' = -1}$$

✓ Solution by Maple
Time used: 0.0 (sec). Leaf size: 32

```
dsolve(x*y(x)*diff(y(x),x)^2+(x+y(x))*diff(y(x),x)+1 = 0,y(x), singsol=all)
```

$$y(x) = -\ln(x) + c_1$$
$$y(x) = \sqrt{-2x + c_1}$$
$$y(x) = -\sqrt{-2x + c_1}$$

✓ Solution by Mathematica
Time used: 0.084 (sec). Leaf size: 53

```
DSolve[x y[x] (y'[x])^2+(x+y[x])y'[x]+1==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to -\sqrt{2}\sqrt{-x + c_1}$$
$$y(x) \to \sqrt{2}\sqrt{-x + c_1}$$
$$y(x) \to -\log(x) + c_1$$

32.27 problem 962

Internal problem ID [4195]

Book: Ordinary differential equations and their solutions. By George Moseley Murphy. 1960
Section: Various 32
Problem number: 962.
ODE order: 1.
ODE degree: 2.

CAS Maple gives this as type `[_separable]`

$$x y {y'}^2 + \left(y^2 + x^2\right) y' + y x = 0$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 35

```
dsolve(x*y(x)*diff(y(x),x)^2+(x^2+y(x)^2)*diff(y(x),x)+x*y(x) = 0,y(x), singsol=all)
```

$$y(x) = \frac{c_1}{x}$$
$$y(x) = \sqrt{-x^2 + c_1}$$
$$y(x) = -\sqrt{-x^2 + c_1}$$

✓ Solution by Mathematica

Time used: 0.142 (sec). Leaf size: 54

```
DSolve[x y[x] (y'[x])^2+(x^2 + y[x]^2)y'[x]+x y[x]==0,y[x],x,IncludeSingularSolutions -> Tru
```

$$y(x) \to \frac{c_1}{x}$$
$$y(x) \to -\sqrt{-x^2 + 2c_1}$$
$$y(x) \to \sqrt{-x^2 + 2c_1}$$
$$y(x) \to 0$$

33 Various 33

33.1 problem 963

Internal problem ID [4196]

Book: Ordinary differential equations and their solutions. By George Moseley Murphy. 1960
Section: Various 33
Problem number: 963.
ODE order: 1.
ODE degree: 2.

CAS Maple gives this as type `[_separable]`

$$xyy'^2 + (x^2 - y^2)\,y' - yx = 0$$

✓ Solution by Maple
Time used: 0.0 (sec). Leaf size: 33

```
dsolve(x*y(x)*diff(y(x),x)^2+(x^2-y(x)^2)*diff(y(x),x)-x*y(x) = 0,y(x), singsol=all)
```

$$y(x) = c_1 x$$
$$y(x) = \sqrt{-x^2 + c_1}$$
$$y(x) = -\sqrt{-x^2 + c_1}$$

✓ Solution by Mathematica
Time used: 0.06 (sec). Leaf size: 65

```
DSolve[x y[x] (y'[x])^2+(x^2-y[x]^2)y'[x]-x y[x]==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to c_1 x$$
$$y(x) \to -\sqrt{-x^2 + 2c_1}$$
$$y(x) \to \sqrt{-x^2 + 2c_1}$$
$$y(x) \to -ix$$
$$y(x) \to ix$$

33.2 problem 964

Internal problem ID [4197]

Book: Ordinary differential equations and their solutions. By George Moseley Murphy. 1960
Section: Various 33
Problem number: 964.
ODE order: 1.
ODE degree: 2.

CAS Maple gives this as type `[_separable]`

$$\boxed{xyy'^2 - (x^2 - y^2)\, y' - yx = 0}$$

✓ Solution by Maple
Time used: 0.0 (sec). Leaf size: 31

```
dsolve(x*y(x)*diff(y(x),x)^2-(x^2-y(x)^2)*diff(y(x),x)-x*y(x) = 0,y(x), singsol=all)
```

$$y(x) = \frac{c_1}{x}$$
$$y(x) = \sqrt{x^2 + c_1}$$
$$y(x) = -\sqrt{x^2 + c_1}$$

✓ Solution by Mathematica
Time used: 0.146 (sec). Leaf size: 50

```
DSolve[x y[x] (y'[x])^2-(x^2-y[x]^2)y'[x]-x y[x]==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to \frac{c_1}{x}$$
$$y(x) \to -\sqrt{x^2 + 2c_1}$$
$$y(x) \to \sqrt{x^2 + 2c_1}$$
$$y(x) \to 0$$

33.3 problem 965

Internal problem ID [4198]

Book: Ordinary differential equations and their solutions. By George Moseley Murphy. 1960
Section: Various 33
Problem number: 965.
ODE order: 1.
ODE degree: 2.

CAS Maple gives this as type `[_rational]`

$$x y {y'}^2 + \left(a + x^2 - y^2\right) y' - y x = 0$$

✗ Solution by Maple

```
dsolve(x*y(x)*diff(y(x),x)^2+(a+x^2-y(x)^2)*diff(y(x),x)-x*y(x) = 0,y(x), singsol=all)
```

No solution found

✓ Solution by Mathematica

Time used: 0.419 (sec). Leaf size: 112

```
DSolve[x y[x] (y'[x])^2+(a+x^2-y[x]^2)y'[x]-x y[x]==0,y[x],x,IncludeSingularSolutions -> Tru
```

$$y(x) \to \sqrt{c_1 \left(x^2 + \frac{a}{1 + c_1}\right)}$$
$$y(x) \to -\sqrt{\left(\sqrt{a} - ix\right)^2}$$
$$y(x) \to \sqrt{\left(\sqrt{a} - ix\right)^2}$$
$$y(x) \to -\sqrt{\left(\sqrt{a} + ix\right)^2}$$
$$y(x) \to \sqrt{\left(\sqrt{a} + ix\right)^2}$$

33.4 problem 966

Internal problem ID [4199]

Book: Ordinary differential equations and their solutions. By George Moseley Murphy. 1960
Section: Various 33
Problem number: 966.
ODE order: 1.
ODE degree: 2.

CAS Maple gives this as type `[_rational]`

$$x y {y'}^2 - \left(a - b\,x^2 + y^2\right) y' - b x y = 0$$

✗ Solution by Maple

```
dsolve(x*y(x)*diff(y(x),x)^2-(a-b*x^2+y(x)^2)*diff(y(x),x)-b*x*y(x) = 0,y(x), singsol=all)
```

No solution found

✓ Solution by Mathematica

Time used: 1.599 (sec). Leaf size: 131

```
DSolve[x y[x] (y'[x])^2-(a-b x^2+y[x]^2)y'[x]-b x y[x]==0,y[x],x,IncludeSingularSolutions ->
```

$$y(x) \to \sqrt{c_1 \left(x^2 - \frac{a}{b + c_1}\right)}$$
$$y(x) \to -\sqrt{-\left(\sqrt{a} + \sqrt{b}x\right)^2}$$
$$y(x) \to \sqrt{-\left(\sqrt{a} + \sqrt{b}x\right)^2}$$
$$y(x) \to -\sqrt{-\left(\sqrt{a} - \sqrt{b}x\right)^2}$$
$$y(x) \to \sqrt{-\left(\sqrt{a} - \sqrt{b}x\right)^2}$$

33.5 problem 967

Internal problem ID [4200]

Book: Ordinary differential equations and their solutions. By George Moseley Murphy. 1960
Section: Various 33
Problem number: 967.
ODE order: 1.
ODE degree: 2.

CAS Maple gives this as type `[_separable]`

$$xyy'^2 + (3x^2 - 2y^2)\, y' - 6yx = 0$$

✓ Solution by Maple

Time used: 0.016 (sec). Leaf size: 35

```
dsolve(x*y(x)*diff(y(x),x)^2+(3*x^2-2*y(x)^2)*diff(y(x),x)-6*x*y(x) = 0,y(x), singsol=all)
```

$$y(x) = c_1 x^2$$
$$y(x) = \sqrt{-3x^2 + c_1}$$
$$y(x) = -\sqrt{-3x^2 + c_1}$$

✓ Solution by Mathematica

Time used: 0.131 (sec). Leaf size: 54

```
DSolve[x y[x] (y'[x])^2+(3 x^2-2 y[x]^2)y'[x]-6 x y[x]==0,y[x],x,IncludeSingularSolutions ->
```

$$y(x) \to c_1 x^2$$
$$y(x) \to -\sqrt{-3x^2 + 2c_1}$$
$$y(x) \to \sqrt{-3x^2 + 2c_1}$$
$$y(x) \to 0$$

33.6 problem 968

Internal problem ID [4201]

Book: Ordinary differential equations and their solutions. By George Moseley Murphy. 1960
Section: Various 33
Problem number: 968.
ODE order: 1.
ODE degree: 2.

CAS Maple gives this as type `[[_homogeneous, ‘class A‘], _rational, _dAlembert]`

$$x(x-2y)\,{y'}^2-2xyy'-2yx+y^2=0$$

✓ Solution by Maple

Time used: 0.859 (sec). Leaf size: 103

```
dsolve(x*(x-2*y(x))*diff(y(x),x)^2-2*x*y(x)*diff(y(x),x)-2*x*y(x)+y(x)^2 = 0,y(x), singsol=a
```

$$y(x)=0$$
$$y(x)=\operatorname{RootOf}\left(-2\ln(x)-\left(\int^{\_Z}\frac{2\_a^2+\sqrt{2}\,\sqrt{\_a\,(\_a-1)^2}}{\_a\,(\_a^2+1)}d\_a\right)+2c_1\right)x$$
$$y(x)=\operatorname{RootOf}\left(-2\ln(x)+\int^{\_Z}\frac{\sqrt{2}\,\sqrt{\_a\,(\_a-1)^2}-2\_a^2}{\_a\,(\_a^2+1)}d\_a+2c_1\right)x$$

✓ Solution by Mathematica

Time used: 4.905 (sec). Leaf size: 167

```
DSolve[x(x-2 y[x]) (y'[x])^2-2 x y[x] y'[x]-2 x y[x]+y[x]^2==0,y[x],x,IncludeSingularSolutio
```

$$y(x) \to -\sqrt{-x\left(x+2e^{\frac{c_1}{2}}\right)}-e^{\frac{c_1}{2}}$$
$$y(x) \to \sqrt{-x\left(x+2e^{\frac{c_1}{2}}\right)}-e^{\frac{c_1}{2}}$$
$$y(x) \to e^{\frac{c_1}{2}}-\sqrt{x\left(-x+2e^{\frac{c_1}{2}}\right)}$$
$$y(x) \to \sqrt{x\left(-x+2e^{\frac{c_1}{2}}\right)}+e^{\frac{c_1}{2}}$$
$$y(x) \to -\sqrt{-x^2}$$
$$y(x) \to \sqrt{-x^2}$$

33.7 problem 969

Internal problem ID [4202]

Book: Ordinary differential equations and their solutions. By George Moseley Murphy. 1960
Section: Various 33
Problem number: 969.
ODE order: 1.
ODE degree: 2.

CAS Maple gives this as type `[[_homogeneous, ‘class A‘], _dAlembert]`

$$x(x-2y)\, {y'}^2+6y'xy-2yx+y^2=0$$

✓ Solution by Maple

Time used: 0.797 (sec). Leaf size: 115

```
dsolve(x*(x-2*y(x))*diff(y(x),x)^2+6*x*y(x)*diff(y(x),x)-2*x*y(x)+y(x)^2 = 0,y(x), singsol=a
```

$$y(x)=0$$
$$y(x)=\mathrm{RootOf}\left(-2\ln(x)-\left(\int^{\_Z}\frac{2\_a^2+\sqrt{2}\,\sqrt{\_a\,(\_a+1)^2}-4\_a}{\_a\,(\_a^2-4\_a+1)}d\_a\right)+2c_1\right)x$$
$$y(x)=\mathrm{RootOf}\left(-2\ln(x)+\int^{\_Z}\frac{\sqrt{2}\,\sqrt{\_a\,(\_a+1)^2}-2\_a^2+4\_a}{\_a\,(\_a^2-4\_a+1)}d\_a+2c_1\right)x$$

✓ Solution by Mathematica

Time used: 7.579 (sec). Leaf size: 196

```
DSolve[x(x-2 y[x]) (y'[x])^2+6 x y[x] y'[x]-2 x y[x]+y[x]^2==0,y[x],x,IncludeSingularSolutio
```

$$y(x) \to 2x - \sqrt{x\left(3x - 2e^{\frac{c_1}{2}}\right)} - e^{\frac{c_1}{2}}$$
$$y(x) \to 2x + \sqrt{x\left(3x - 2e^{\frac{c_1}{2}}\right)} - e^{\frac{c_1}{2}}$$
$$y(x) \to 2x - \sqrt{x\left(3x + 2e^{\frac{c_1}{2}}\right)} + e^{\frac{c_1}{2}}$$
$$y(x) \to 2x + \sqrt{x\left(3x + 2e^{\frac{c_1}{2}}\right)} + e^{\frac{c_1}{2}}$$
$$y(x) \to 2x - \sqrt{3}\sqrt{x^2}$$
$$y(x) \to \sqrt{3}\sqrt{x^2} + 2x$$

33.8 problem 970

Internal problem ID [4203]

Book: Ordinary differential equations and their solutions. By George Moseley Murphy. 1960
Section: Various 33
Problem number: 970.
ODE order: 1.
ODE degree: 2.

CAS Maple gives this as type `[_quadrature]`

$$y^2 {y'}^2 = a^2$$

✓ Solution by Maple
Time used: 0.016 (sec). Leaf size: 49

```
dsolve(y(x)^2*diff(y(x),x)^2 = a^2,y(x), singsol=all)
```

$$y(x) = \sqrt{2ax + c_1}$$
$$y(x) = -\sqrt{2ax + c_1}$$
$$y(x) = \sqrt{-2ax + c_1}$$
$$y(x) = -\sqrt{-2ax + c_1}$$

✓ Solution by Mathematica
Time used: 0.285 (sec). Leaf size: 85

```
DSolve[y[x]^2(y'[x])^2==a^2,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to -\sqrt{2}\sqrt{-ax + c_1}$$
$$y(x) \to \sqrt{2}\sqrt{-ax + c_1}$$
$$y(x) \to -\sqrt{2}\sqrt{ax + c_1}$$
$$y(x) \to \sqrt{2}\sqrt{ax + c_1}$$

33.9 problem 971

Internal problem ID [4204]

Book: Ordinary differential equations and their solutions. By George Moseley Murphy. 1960
Section: Various 33
Problem number: 971.
ODE order: 1.
ODE degree: 2.

CAS Maple gives this as type `[_quadrature]`

$$y^2 y'^2 + y^2 = a^2$$

✓ Solution by Maple

Time used: 0.734 (sec). Leaf size: 54

```
dsolve(y(x)^2*diff(y(x),x)^2-a^2+y(x)^2 = 0,y(x), singsol=all)
```

$$y(x) = -a$$
$$y(x) = a$$
$$y(x) = \sqrt{a^2 - c_1^2 + 2c_1 x - x^2}$$
$$y(x) = -\sqrt{(a + x - c_1)(c_1 + a - x)}$$

✓ Solution by Mathematica

Time used: 0.239 (sec). Leaf size: 101

```
DSolve[y[x]^2 (y'[x])^2-a^2 +y[x]^2==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to -\sqrt{a^2 - (x + c_1)^2}$$
$$y(x) \to \sqrt{a^2 - (x + c_1)^2}$$
$$y(x) \to -\sqrt{a^2 - (x - c_1)^2}$$
$$y(x) \to \sqrt{a^2 - (x - c_1)^2}$$
$$y(x) \to -a$$
$$y(x) \to a$$

33.10 problem 972

Internal problem ID [4205]

Book: Ordinary differential equations and their solutions. By George Moseley Murphy. 1960
Section: Various 33
Problem number: 972.
ODE order: 1.
ODE degree: 2.

CAS Maple gives this as type `[[_1st_order, _with_linear_symmetries], _rational]`

$$\boxed{y^2 {y'}^2 - 3xy' + y = 0}$$

✓ Solution by Maple

Time used: 0.516 (sec). Leaf size: 119

```
dsolve(y(x)^2*diff(y(x),x)^2-3*x*diff(y(x),x)+y(x) = 0,y(x), singsol=all)
```

$$y(x) = \frac{18^{\frac{1}{3}} (x^2)^{\frac{1}{3}}}{2}$$
$$y(x) = -\frac{2^{\frac{1}{3}} (x^2)^{\frac{1}{3}} \left(3i3^{\frac{1}{6}} + 3^{\frac{2}{3}}\right)}{4}$$
$$y(x) = \frac{2^{\frac{1}{3}} (x^2)^{\frac{1}{3}} \left(3i3^{\frac{1}{6}} - 3^{\frac{2}{3}}\right)}{4}$$
$$y(x) = 0$$
$$y(x) = \operatorname{RootOf}\left(-2\ln(x) - 3\left(\int^{\_Z} \frac{4\_a^3 + 3\sqrt{-4\_a^3 + 9} - 9}{\_a\,(4\_a^3 - 9)} d\_a\right) + 2c_1\right) x^{\frac{2}{3}}$$

✓ Solution by Mathematica

Time used: 0.623 (sec). Leaf size: 247

```
DSolve[y[x]^2 (y'[x])^2-3 x y'[x]+y[x]==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to e^{\frac{c_1}{3}} \sqrt[3]{e^{c_1} - 3ix}$$
$$y(x) \to -\sqrt[3]{-1} e^{\frac{c_1}{3}} \sqrt[3]{e^{c_1} - 3ix}$$
$$y(x) \to (-1)^{2/3} e^{\frac{c_1}{3}} \sqrt[3]{e^{c_1} - 3ix}$$
$$y(x) \to e^{\frac{c_1}{3}} \sqrt[3]{3ix + e^{c_1}}$$
$$y(x) \to -\sqrt[3]{-1} e^{\frac{c_1}{3}} \sqrt[3]{3ix + e^{c_1}}$$
$$y(x) \to (-1)^{2/3} e^{\frac{c_1}{3}} \sqrt[3]{3ix + e^{c_1}}$$
$$y(x) \to 0$$
$$y(x) \to \left(-\frac{3}{2}\right)^{2/3} x^{2/3}$$
$$y(x) \to \left(\frac{3}{2}\right)^{2/3} x^{2/3}$$
$$y(x) \to -\sqrt[3]{-1} \left(\frac{3}{2}\right)^{2/3} x^{2/3}$$

33.11 problem 973

Internal problem ID [4206]

Book: Ordinary differential equations and their solutions. By George Moseley Murphy. 1960
Section: Various 33
Problem number: 973.
ODE order: 1.
ODE degree: 2.

CAS Maple gives this as type `[[_1st_order, _with_linear_symmetries]]`

$$\boxed{y^2 {y'}^2 - 6y' x^3 + 4x^2 y = 0}$$

✓ Solution by Maple

Time used: 0.563 (sec). Leaf size: 107

```
dsolve(y(x)^2*diff(y(x),x)^2-6*x^3*diff(y(x),x)+4*x^2*y(x) = 0,y(x), singsol=all)
```

$$y(x) = \frac{18^{\frac{1}{3}} x^{\frac{4}{3}}}{2}$$
$$y(x) = -\frac{18^{\frac{1}{3}} x^{\frac{4}{3}} \left(1 + i\sqrt{3}\right)}{4}$$
$$y(x) = \frac{18^{\frac{1}{3}} x^{\frac{4}{3}} \left(i\sqrt{3} - 1\right)}{4}$$
$$y(x) = 0$$
$$y(x) = \text{RootOf}\left(-4\ln(x) - 3\left(\int^{\_Z} \frac{4\_a^3 + 3\sqrt{-4\_a^3 + 9} - 9}{\_a\left(4\_a^3 - 9\right)} d\_a\right) + 4c_1\right) x^{\frac{4}{3}}$$

✓ Solution by Mathematica

Time used: 2.556 (sec). Leaf size: 304

```
DSolve[y[x]^2 (y'[x])^2-6 x^3 y'[x]+4 x^2 y[x]==0,y[x],x,IncludeSingularSolutions -> True]
```

$$\text{Solve}\left[\frac{\sqrt{9x^6-4x^2y(x)^3}\log\left(\sqrt{9x^4-4y(x)^3}+3x^2\right)}{2x\sqrt{9x^4-4y(x)^3}} - \frac{3}{4}\left(\frac{\sqrt{9x^6-4x^2y(x)^3}\log(y(x))}{x\sqrt{9x^4-4y(x)^3}}-\log(y(x))\right)=c_1,y(x)\right]$$

$$\text{Solve}\left[\frac{3}{4}\left(\frac{\sqrt{9x^6-4x^2y(x)^3}\log(y(x))}{x\sqrt{9x^4-4y(x)^3}}+\log(y(x))\right) - \frac{\sqrt{9x^6-4x^2y(x)^3}\log\left(\sqrt{9x^4-4y(x)^3}+3x^2\right)}{2x\sqrt{9x^4-4y(x)^3}}=c_1,y(x)\right]$$

$$y(x)\to\left(-\frac{3}{2}\right)^{2/3}x^{4/3}$$

$$y(x)\to\left(\frac{3}{2}\right)^{2/3}x^{4/3}$$

$$y(x)\to-\sqrt[3]{-1}\left(\frac{3}{2}\right)^{2/3}x^{4/3}$$

33.12 problem 974

Internal problem ID [4207]

Book: Ordinary differential equations and their solutions. By George Moseley Murphy. 1960
Section: Various 33
Problem number: 974.
ODE order: 1.
ODE degree: 2.

CAS Maple gives this as type `[_rational, [_1st_order, '_with_symmetry_[F(x),G(y)]']]`

$$
y^2 {y'}^2 - 4ayy' + y^2 = -4a^2 + 4ax
$$

✓ Solution by Maple

Time used: 0.625 (sec). Leaf size: 72

```
dsolve(y(x)^2*diff(y(x),x)^2-4*a*y(x)*diff(y(x),x)+4*a^2-4*a*x+y(x)^2 = 0,y(x), singsol=all)
```

$$
\begin{aligned}
y(x) &= -2\sqrt{ax} \\
y(x) &= 2\sqrt{ax} \\
y(x) &= \sqrt{4ax - c_1^2 + 2c_1 x - x^2} \\
y(x) &= -\sqrt{-x^2 + (4a + 2c_1)\, x - c_1^2}
\end{aligned}
$$

✓ Solution by Mathematica

Time used: 0.75 (sec). Leaf size: 85

```
DSolve[y[x]^2 (y'[x])^2-4 a y[x] y'[x]+4 a^2-4 a x+y[x]^2==0,y[x],x,IncludeSingularSolutions
```

$$
\begin{aligned}
y(x) &\to -\frac{\sqrt{16a^3x - 4a^2x^2 - 4ac_1x - c_1{}^2}}{2a} \\
y(x) &\to \frac{\sqrt{16a^3x - 4a^2x^2 - 4ac_1x - c_1{}^2}}{2a}
\end{aligned}
$$

33.13 problem 975

Internal problem ID [4208]

Book: Ordinary differential equations and their solutions. By George Moseley Murphy. 1960
Section: Various 33
Problem number: 975.
ODE order: 1.
ODE degree: 2.

CAS Maple gives this as type `[_quadrature]`

$$y^2 {y'}^2 - (x+1)\, y y' = -x$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 45

```
dsolve(y(x)^2*diff(y(x),x)^2-(1+x)*y(x)*diff(y(x),x)+x = 0,y(x), singsol=all)
```

$$y(x) = \sqrt{2x + c_1}$$
$$y(x) = -\sqrt{2x + c_1}$$
$$y(x) = \sqrt{x^2 + c_1}$$
$$y(x) = -\sqrt{x^2 + c_1}$$

✓ Solution by Mathematica

Time used: 0.152 (sec). Leaf size: 72

```
DSolve[y[x]^2 (y'[x])^2-(1+x)y[x] y'[x]+x==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to -\sqrt{2}\sqrt{x + c_1}$$
$$y(x) \to \sqrt{2}\sqrt{x + c_1}$$
$$y(x) \to -\sqrt{x^2 + 2c_1}$$
$$y(x) \to \sqrt{x^2 + 2c_1}$$

33.14 problem 976

Internal problem ID [4209]

Book: Ordinary differential equations and their solutions. By George Moseley Murphy. 1960
Section: Various 33
Problem number: 976.
ODE order: 1.
ODE degree: 2.

CAS Maple gives this as type `[_separable]`

$$y^2 y'^2 + 2xyy' = -x^2$$

✓ Solution by Maple

Time used: 0.438 (sec). Leaf size: 31

```
dsolve(y(x)^2*diff(y(x),x)^2+2*x*y(x)*diff(y(x),x)+x^2 = 0,y(x), singsol=all)
```

$$y(x) = \sqrt{-x^2 + 2c_1}$$
$$y(x) = -\sqrt{-x^2 + 2c_1}$$

✓ Solution by Mathematica

Time used: 0.096 (sec). Leaf size: 39

```
DSolve[y[x]^2 (y'[x])^2+2 x y[x] y'[x]+x^2==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to -\sqrt{-x^2 + 2c_1}$$
$$y(x) \to \sqrt{-x^2 + 2c_1}$$

33.15 problem 977

Internal problem ID [4210]

Book: Ordinary differential equations and their solutions. By George Moseley Murphy. 1960
Section: Various 33
Problem number: 977.
ODE order: 1.
ODE degree: 2.

CAS Maple gives this as type `[_rational, [_1st_order, '_with_symmetry_[F(x),G(y)]']]`

$$y^2 {y'}^2 + 2xyy' - y^2 = -a$$

✓ Solution by Maple

Time used: 0.266 (sec). Leaf size: 57

```
dsolve(y(x)^2*diff(y(x),x)^2+2*x*y(x)*diff(y(x),x)+a-y(x)^2 = 0,y(x), singsol=all)
```

$$y(x) = \sqrt{-x^2 + a}$$
$$y(x) = -\sqrt{-x^2 + a}$$
$$y(x) = \sqrt{c_1^2 - 2c_1 x + a}$$
$$y(x) = -\sqrt{c_1^2 - 2c_1 x + a}$$

✓ Solution by Mathematica

Time used: 0.569 (sec). Leaf size: 61

```
DSolve[y[x]^2 (y'[x])^2+2 x y[x] y'[x]+a-y[x]^2==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to -\sqrt{a + c_1(-2x + c_1)}$$
$$y(x) \to \sqrt{a + c_1(-2x + c_1)}$$
$$y(x) \to -\sqrt{a}$$
$$y(x) \to \sqrt{a}$$

33.16 problem 978

Internal problem ID [4211]

Book: Ordinary differential equations and their solutions. By George Moseley Murphy. 1960
Section: Various 33
Problem number: 978.
ODE order: 1.
ODE degree: 2.

CAS Maple gives this as type `[[_homogeneous, ‘class A‘], _rational, _dAlembert]`

$$y^2{y'}^2 - 2xyy' + 2y^2 = x^2$$

✓ Solution by Maple

Time used: 0.047 (sec). Leaf size: 107

```
dsolve(y(x)^2*diff(y(x),x)^2-2*x*y(x)*diff(y(x),x)-x^2+2*y(x)^2 = 0,y(x), singsol=all)
```

$$y(x) = -x$$
$$y(x) = x$$
$$y(x) = \sqrt{-2c_1\sqrt{2}\,x - c_1^2 - x^2}$$
$$y(x) = \sqrt{2c_1\sqrt{2}\,x - c_1^2 - x^2}$$
$$y(x) = -\sqrt{-2c_1\sqrt{2}\,x - c_1^2 - x^2}$$
$$y(x) = -\sqrt{2c_1\sqrt{2}\,x - c_1^2 - x^2}$$

✓ Solution by Mathematica

Time used: 7.875 (sec). Leaf size: 233

```
DSolve[y[x]^2 (y'[x])^2-2 x y[x] y'[x]-x^2+2 y[x]^2==0,y[x],x,IncludeSingularSolutions -> Tr
```

$$y(x) \to -\sqrt{-x^2 - 4\sqrt{2}x\cosh(c_1) - 4\sqrt{2}x\sinh(c_1) - 4\cosh(2c_1) - 4\sinh(2c_1)}$$
$$y(x) \to \sqrt{-x^2 - 4\sqrt{2}x\cosh(c_1) - 4\sqrt{2}x\sinh(c_1) - 4\cosh(2c_1) - 4\sinh(2c_1)}$$
$$y(x) \to -\sqrt{-x^2 + 4\sqrt{2}x\cosh(c_1) + 4\sqrt{2}x\sinh(c_1) - 4\cosh(2c_1) - 4\sinh(2c_1)}$$
$$y(x) \to \sqrt{-x^2 + 4\sqrt{2}x\cosh(c_1) + 4\sqrt{2}x\sinh(c_1) - 4\cosh(2c_1) - 4\sinh(2c_1)}$$
$$y(x) \to -\sqrt{-x^2}$$
$$y(x) \to \sqrt{-x^2}$$

33.17 problem 979

Internal problem ID [4212]

Book: Ordinary differential equations and their solutions. By George Moseley Murphy. 1960
Section: Various 33
Problem number: 979.
ODE order: 1.
ODE degree: 2.

CAS Maple gives this as type `[_rational, [_1st_order, '_with_symmetry_[F(x),G(y)]']]`

$$y^2 {y'}^2 - 2xyy' + 2y^2 = x^2 - a$$

✓ Solution by Maple

Time used: 0.062 (sec). Leaf size: 83

```
dsolve(y(x)^2*diff(y(x),x)^2-2*x*y(x)*diff(y(x),x)+a-x^2+2*y(x)^2 = 0,y(x), singsol=all)
```

$$y(x) = -\frac{\sqrt{4x^2 - 2a}}{2}$$
$$y(x) = \frac{\sqrt{4x^2 - 2a}}{2}$$
$$y(x) = -\frac{\sqrt{-8c_1^2 + 16c_1 x - 4x^2 - 2a}}{2}$$
$$y(x) = \frac{\sqrt{-8c_1^2 + 16c_1 x - 4x^2 - 2a}}{2}$$

✓ Solution by Mathematica

Time used: 0.744 (sec). Leaf size: 63

```
DSolve[y[x]^2 (y'[x])^2-2 x y[x] y'[x]+a -x^2+2 y[x]^2==0,y[x],x,IncludeSingularSolutions ->
```

$$y(x) \to -\sqrt{-\frac{a}{2} - x^2 + 4c_1 x - 2{c_1}^2}$$
$$y(x) \to \sqrt{-\frac{a}{2} - x^2 + 4c_1 x - 2{c_1}^2}$$

33.18 problem 980

Internal problem ID [4213]

Book: Ordinary differential equations and their solutions. By George Moseley Murphy. 1960
Section: Various 33
Problem number: 980.
ODE order: 1.
ODE degree: 2.

CAS Maple gives this as type `[_rational, [_1st_order, '_with_symmetry_[F(x),G(y)]']]`

$$y^2{y'}^2 + 2axyy' + (-a+1)\,y^2 = -(a-1)\,b - x^2a$$

✓ Solution by Maple

Time used: 0.094 (sec). Leaf size: 88

```
dsolve(y(x)^2*diff(y(x),x)^2+2*a*x*y(x)*diff(y(x),x)+(a-1)*b+a*x^2+(1-a)*y(x)^2 = 0,y(x), si
```

$$y(x) = \sqrt{-a\,x^2+b}$$
$$y(x) = -\sqrt{-a\,x^2+b}$$
$$y(x) = \sqrt{a\,c_1^2 - 2ac_1x - c_1^2 + 2c_1x - x^2 + b}$$
$$y(x) = -\sqrt{(a-1)\,c_1^2 - 2x\,(a-1)\,c_1 - x^2 + b}$$

✓ Solution by Mathematica

Time used: 1.186 (sec). Leaf size: 65

```
DSolve[y[x]^2 (y'[x])^2+2 a x y[x] y'[x]+(a-1)b+a x^2+(1-a)y[x]^2==0,y[x],x,IncludeSingularS
```

$$y(x) \to -\sqrt{-2(a-1)c_1x + (a-1)c_1{}^2 + b - x^2}$$
$$y(x) \to \sqrt{-2(a-1)c_1x + (a-1)c_1{}^2 + b - x^2}$$

33.19 problem 981

Internal problem ID [4214]

Book: Ordinary differential equations and their solutions. By George Moseley Murphy. 1960
Section: Various 33
Problem number: 981.
ODE order: 1.
ODE degree: 2.

CAS Maple gives this as type `[_quadrature]`

$$\boxed{(1-y^2)\,y'^2=1}$$

✓ Solution by Maple

Time used: 0.016 (sec). Leaf size: 46

```
dsolve((1-y(x)^2)*diff(y(x),x)^2 = 1,y(x), singsol=all)
```

$$y(x)=\sin\left(\operatorname{RootOf}\left(\sin\left(\_Z\right)\operatorname{csgn}\left(\cos\left(\_Z\right)\right)\cos\left(\_Z\right)+\_Z+2c_1-2x\right)\right)$$
$$y(x)=\sin\left(\operatorname{RootOf}\left(-\sin\left(\_Z\right)\operatorname{csgn}\left(\cos\left(\_Z\right)\right)\cos\left(\_Z\right)-\_Z+2c_1-2x\right)\right)$$

✓ Solution by Mathematica

Time used: 0.06 (sec). Leaf size: 105

```
DSolve[(1-y[x]^2) (y'[x])^2==1,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x)\to\text{InverseFunction}\left[\frac{1}{2}\#1\sqrt{1-\#1^2}-\arctan\left(\frac{\sqrt{1-\#1^2}}{\#1+1}\right)\&\right][-x+c_1]$$
$$y(x)\to\text{InverseFunction}\left[\frac{1}{2}\#1\sqrt{1-\#1^2}-\arctan\left(\frac{\sqrt{1-\#1^2}}{\#1+1}\right)\&\right][x+c_1]$$

33.20 problem 982

Internal problem ID [4215]

Book: Ordinary differential equations and their solutions. By George Moseley Murphy. 1960
Section: Various 33
Problem number: 982.
ODE order: 1.
ODE degree: 2.

CAS Maple gives this as type `[_quadrature]`

$$
\boxed{(a^2 - y^2)\, y'^2 - y^2 = 0}
$$

✓ Solution by Maple
Time used: 0.031 (sec). Leaf size: 115

```
dsolve((a^2-y(x)^2)*diff(y(x),x)^2 = y(x)^2,y(x), singsol=all)
```

$$
y(x) = 0
$$

$$
a \operatorname{csgn}(a) \ln(2) + a \operatorname{csgn}(a) \ln\left(\frac{a\left(\operatorname{csgn}(a)\sqrt{a^2 - y(x)^2} + a\right)}{y(x)}\right) - \sqrt{a^2 - y(x)^2} - c_1 + x = 0
$$

$$
-a \operatorname{csgn}(a) \ln(2) - a \operatorname{csgn}(a) \ln\left(\frac{a\left(\operatorname{csgn}(a)\sqrt{a^2 - y(x)^2} + a\right)}{y(x)}\right) + \sqrt{a^2 - y(x)^2} - c_1 + x = 0
$$

✓ Solution by Mathematica

Time used: 0.337 (sec). Leaf size: 102

```
DSolve[(a^2-y[x]^2) (y'[x])^2==y[x]^2,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to \text{InverseFunction}\left[\sqrt{a^2 - \#1^2} - a\text{arctanh}\left(\frac{\sqrt{a^2 - \#1^2}}{a}\right) \&\right][-x + c_1]$$

$$y(x) \to \text{InverseFunction}\left[\sqrt{a^2 - \#1^2} - a\text{arctanh}\left(\frac{\sqrt{a^2 - \#1^2}}{a}\right) \&\right][x + c_1]$$

$$y(x) \to 0$$

33.21 problem 983

Internal problem ID [4216]

Book: Ordinary differential equations and their solutions. By George Moseley Murphy. 1960
Section: Various 33
Problem number: 983.
ODE order: 1.
ODE degree: 2.

CAS Maple gives this as type `[‘y=_G(x,y’)‘]`

$$\boxed{(a^2 - 2axy + y^2)\, y'^2 + 2ayy' + y^2 = 0}$$

✗ Solution by Maple

```
dsolve((a^2-2*a*x*y(x)+y(x)^2)*diff(y(x),x)^2+2*a*y(x)*diff(y(x),x)+y(x)^2 = 0,y(x), singsol
```

No solution found

✗ Solution by Mathematica
Time used: 0.0 (sec). Leaf size: 0

```
DSolve[(a^2-2 a x y[x]+y[x]^2) (y'[x])^2+2 a y[x] y'[x]+y[x]^2==0,y[x],x,IncludeSingularSolu
```

Not solved

33.22 problem 985

Internal problem ID [4217]

Book: Ordinary differential equations and their solutions. By George Moseley Murphy. 1960
Section: Various 33
Problem number: 985.
ODE order: 1.
ODE degree: 2.

CAS Maple gives this as type `[[_homogeneous, 'class A'], _rational, _dAlembert]`

$$((-a+1)\,x^2+y^2)\,{y'}^2+2axyy'+(-a+1)\,y^2=-x^2$$

✓ Solution by Maple

Time used: 0.094 (sec). Leaf size: 75

```
dsolve(((1-a)*x^2+y(x)^2)*diff(y(x),x)^2+2*a*x*y(x)*diff(y(x),x)+x^2+(1-a)*y(x)^2 = 0,y(x),
```

$$y(x)=-ix$$
$$y(x)=ix$$
$$y(x)=\tan\left(\text{RootOf}\left(-2\_Z\sqrt{a-1}-\ln\left(x^2\sec\left(\_Z\right)^2\right)+2c_1\right)\right)x$$
$$y(x)=\tan\left(\text{RootOf}\left(2\_Z\sqrt{a-1}-\ln\left(x^2\sec\left(\_Z\right)^2\right)+2c_1\right)\right)x$$

✓ Solution by Mathematica

Time used: 0.328 (sec). Leaf size: 101

```
DSolve[((1-a)x^2+y[x]^2)(y'[x])^2+2 a x y[x] y'[x]+x^2+(1-a)y[x]^2==0,y[x],x,IncludeSingular
```

$$\text{Solve}\left[\sqrt{a-1}\arctan\left(\frac{y(x)}{x}\right)-\frac{1}{2}\log\left(\frac{y(x)^2}{x^2}+1\right)=\log(x)+c_1,y(x)\right]$$
$$\text{Solve}\left[\sqrt{a-1}\arctan\left(\frac{y(x)}{x}\right)+\frac{1}{2}\log\left(\frac{y(x)^2}{x^2}+1\right)=-\log(x)+c_1,y(x)\right]$$
$$y(x)\to -ix$$
$$y(x)\to ix$$

33.23 problem 986

Internal problem ID [4218]

Book: Ordinary differential equations and their solutions. By George Moseley Murphy. 1960
Section: Various 33
Problem number: 986.
ODE order: 1.
ODE degree: 2.

CAS Maple gives this as type `[[_homogeneous, 'class A'], _dAlembert]`

$$\left(\left(-4a^2+1\right)x^2+y^2\right)y'^2-8a^2xyy'+\left(-4a^2+1\right)y^2=-x^2$$

✓ Solution by Maple
Time used: 0.094 (sec). Leaf size: 136

```
dsolve(((-4*a^2+1)*x^2+y(x)^2)*diff(y(x),x)^2-8*a^2*x*y(x)*diff(y(x),x)+x^2+(-4*a^2+1)*y(x)^
```

$$y(x)=\operatorname{RootOf}\left(-\ln(x)+\int^{\_Z}-\frac{\_a^3-8\_a\,a^2-\sqrt{(4a^2-1)(\_a^2+1)^2}+\_a}{\_a^4-16\_a^2a^2+2\_a^2+1}d\_a+c_1\right)x$$

$$y(x)=\operatorname{RootOf}\left(-\ln(x)-\left(\int^{\_Z}\frac{\_a^3-8\_a\,a^2+\sqrt{(4a^2-1)(\_a^2+1)^2}+\_a}{\_a^4-16\_a^2a^2+2\_a^2+1}d\_a\right)+c_1\right)x$$

✓ Solution by Mathematica

Time used: 1.499 (sec). Leaf size: 328

```
DSolve[((1-4 a^2)x^2+y[x]^2) (y'[x])^2 - 8 a^2 x y[x] y'[x]+x^2+(1-4 a^2)y[x]^2==0,y[x],x,In
```

$$\text{Solve}\left[\frac{1}{4}\left(-\frac{2\sqrt{2a-1}\sqrt{2a+1}\arctan\left(\frac{\frac{y(x)}{x}-2a}{\sqrt{1-4a^2}}\right)}{\sqrt{1-4a^2}} - \frac{2\sqrt{2a-1}\sqrt{2a+1}\arctan\left(\frac{2a+\frac{y(x)}{x}}{\sqrt{1-4a^2}}\right)}{\sqrt{1-4a^2}} + \log\left(-\frac{4ay(x)}{x}+\frac{y(x)^2}{x^2}+1\right) + \log\left(\frac{4ay(x)}{x}+\frac{y(x)^2}{x^2}+1\right)\right) = -\log(x)+c_1, y(x)\right]$$

$$\text{Solve}\left[-\frac{-2\sqrt{2a-1}\sqrt{2a+1}\arctan\left(\frac{\frac{y(x)}{x}-2a}{\sqrt{1-4a^2}}\right) - 2\sqrt{2a-1}\sqrt{2a+1}\arctan\left(\frac{2a+\frac{y(x)}{x}}{\sqrt{1-4a^2}}\right) - \sqrt{1-4a^2}\Big(\log\Big(-}{4\sqrt{1-4a^2}}\right.$$

$$\left. -\log(x)+c_1, y(x)\right]$$

33.24 problem 987

Internal problem ID [4219]

Book: Ordinary differential equations and their solutions. By George Moseley Murphy. 1960
Section: Various 33
Problem number: 987.
ODE order: 1.
ODE degree: 2.

CAS Maple gives this as type `[[_homogeneous, 'class A'], _rational, _dAlembert]`

$$
\left(\left(-a^{2}+1\right) x^{2}+y^{2}\right) {y'}^{2}+2 a^{2} x y y'+\left(-a^{2}+1\right) y^{2}=-x^{2}
$$

✓ Solution by Maple

Time used: 0.406 (sec). Leaf size: 79

```
dsolve((((-a^2+1)*x^2+y(x)^2)*diff(y(x),x)^2+2*a^2*x*y(x)*diff(y(x),x)+x^2+(-a^2+1)*y(x)^2 =
```

$$
\begin{aligned}
y(x) &= -ix \\
y(x) &= ix \\
y(x) &= \tan\left(\operatorname{RootOf}\left(-2\_Z\sqrt{a^{2}-1}-\ln\left(x^{2}\sec\left(\_Z\right)^{2}\right)+2c_{1}\right)\right) x \\
y(x) &= \tan\left(\operatorname{RootOf}\left(2\_Z\sqrt{a^{2}-1}-\ln\left(x^{2}\sec\left(\_Z\right)^{2}\right)+2c_{1}\right)\right) x
\end{aligned}
$$

✓ Solution by Mathematica

Time used: 0.381 (sec). Leaf size: 115

```
DSolve[((1-a^2)x^2+y[x]^2) (y'[x])^2 +2 a^2 x y[x] y'[x]+x^2+(1-a^2) y[x]^2==0,y[x],x,Includ
```

$$
\begin{aligned}
&\text{Solve}\left[\sqrt{a-1}\sqrt{a+1}\arctan\left(\frac{y(x)}{x}\right)-\frac{1}{2}\log\left(\frac{y(x)^{2}}{x^{2}}+1\right)=\log(x)+c_{1},y(x)\right] \\
&\text{Solve}\left[\sqrt{a-1}\sqrt{a+1}\arctan\left(\frac{y(x)}{x}\right)+\frac{1}{2}\log\left(\frac{y(x)^{2}}{x^{2}}+1\right)=-\log(x)+c_{1},y(x)\right] \\
&y(x)\to -ix \\
&y(x)\to ix
\end{aligned}
$$

33.25 problem 988

Internal problem ID [4220]

Book: Ordinary differential equations and their solutions. By George Moseley Murphy. 1960
Section: Various 33
Problem number: 988.
ODE order: 1.
ODE degree: 2.

CAS Maple gives this as type `[[_homogeneous, ‘class A‘], _rational, [_Abel, ‘2nd type‘, ‘cl`

$$(y+x)^2 y'^2 - y^2 = 0$$

✓ Solution by Maple

Time used: 0.015 (sec). Leaf size: 47

```
dsolve((x+y(x))^2*diff(y(x),x)^2 = y(x)^2,y(x), singsol=all)
```

$$y(x) = \frac{x}{\text{LambertW}\left(x\,\mathrm{e}^{c_1}\right)}$$
$$y(x) = -x - \sqrt{x^2 + 2c_1}$$
$$y(x) = -x + \sqrt{x^2 + 2c_1}$$

✓ Solution by Mathematica

Time used: 4.107 (sec). Leaf size: 101

```
DSolve[(x+y[x])^2 (y'[x])^2==y[x]^2,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to -x - \sqrt{x^2 + e^{2c_1}}$$
$$y(x) \to -x + \sqrt{x^2 + e^{2c_1}}$$
$$y(x) \to \frac{x}{W\left(e^{-c_1}x\right)}$$
$$y(x) \to 0$$
$$y(x) \to -\sqrt{x^2} - x$$
$$y(x) \to \sqrt{x^2} - x$$

33.26 problem 989

Internal problem ID [4221]

Book: Ordinary differential equations and their solutions. By George Moseley Murphy. 1960
Section: Various 33
Problem number: 989.
ODE order: 1.
ODE degree: 2.

CAS Maple gives this as type `[[_homogeneous, ‘class A‘], _rational, [_Abel, ‘2nd type‘, ‘cl`

$$(y+x)^2 y'^2 - (x^2 - yx - 2y^2)\, y' - y(-y+x) = 0$$

✓ Solution by Maple
Time used: 0.047 (sec). Leaf size: 85

```
dsolve((x+y(x))^2*diff(y(x),x)^2-(x^2-x*y(x)-2*y(x)^2)*diff(y(x),x)-(x-y(x))*y(x) = 0,y(x),
```

$$y(x) = -x - \sqrt{x^2 + 2c_1}$$
$$y(x) = -x + \sqrt{x^2 + 2c_1}$$
$$y(x) = \frac{-c_1 x - \sqrt{2c_1^2 x^2 + 1}}{c_1}$$
$$y(x) = \frac{-c_1 x + \sqrt{2c_1^2 x^2 + 1}}{c_1}$$

✓ Solution by Mathematica

Time used: 0.592 (sec). Leaf size: 172

```
DSolve[(x+y[x])^2 (y'[x])^2 -(x^2-x y[x]-2 y[x]^2) y'[x]-(x-y[x])y[x]==0,y[x],x,IncludeSingu
```

$$y(x) \to -x - \sqrt{x^2 + e^{2c_1}}$$
$$y(x) \to -x + \sqrt{x^2 + e^{2c_1}}$$
$$y(x) \to -x - \sqrt{2x^2 + e^{2c_1}}$$
$$y(x) \to -x + \sqrt{2x^2 + e^{2c_1}}$$
$$y(x) \to -\sqrt{x^2} - x$$
$$y(x) \to \sqrt{x^2} - x$$
$$y(x) \to -\sqrt{2}\sqrt{x^2} - x$$
$$y(x) \to \sqrt{2}\sqrt{x^2} - x$$

33.27 problem 990

Internal problem ID [4222]

Book: Ordinary differential equations and their solutions. By George Moseley Murphy. 1960
Section: Various 33
Problem number: 990.
ODE order: 1.
ODE degree: 2.

CAS Maple gives this as type `[[_homogeneous, ‘class C‘], _dAlembert]`

$$\left(a^{2}-\left(-y+x\right)^{2}\right) {y'}^{2}+2 y' a^{2}-\left(-y+x\right)^{2}=-a^{2}$$

✓ Solution by Maple

Time used: 0.281 (sec). Leaf size: 135

```
dsolve((a^2-(x-y(x))^2)*diff(y(x),x)^2+2*a^2*diff(y(x),x)+a^2-(x-y(x))^2 = 0,y(x), singsol=a
```

$$y(x) = x-\sqrt{2}\, a$$
$$y(x) = x+\sqrt{2}\, a$$
$$y(x) = x+\operatorname{RootOf}\left(-2x-\left(\int^{\_Z}\frac{\_a^{2}-2a^{2}+\sqrt{-\_a^{4}+2\_a^{2}a^{2}}}{\_a^{2}-2a^{2}}d\_a\right)+2c_{1}\right)$$
$$y(x) = x+\operatorname{RootOf}\left(-2x+\int^{\_Z}-\frac{-2a^{2}+\_a^{2}-\sqrt{-\_a^{4}+2\_a^{2}a^{2}}}{\_a^{2}-2a^{2}}d\_a+2c_{1}\right)$$

✓ Solution by Mathematica

Time used: 51.486 (sec). Leaf size: 18407

```
DSolve[(a^2-(x-y[x])^2)(y'[x])^2+2 a^2 y'[x]+a^2-(x-y[x])^2==0,y[x],x,IncludeSingularSolutio
```

Too large to display

33.28 problem 991

Internal problem ID [4223]

Book: Ordinary differential equations and their solutions. By George Moseley Murphy. 1960
Section: Various 33
Problem number: 991.
ODE order: 1.
ODE degree: 2.

CAS Maple gives this as type `[_rational, [_1st_order, '_with_symmetry_[F(x),G(y)]']]`

$$2y^2{y'}^2 + 2y'xy + y^2 = -x^2 + 1$$

✓ Solution by Maple

Time used: 0.109 (sec). Leaf size: 103

```
dsolve(2*y(x)^2*diff(y(x),x)^2+2*x*y(x)*diff(y(x),x)-1+x^2+y(x)^2 = 0,y(x), singsol=all)
```

$$y(x) = -\frac{\sqrt{-2x^2+4}}{2}$$
$$y(x) = \frac{\sqrt{-2x^2+4}}{2}$$
$$y(x) = \sqrt{\operatorname{RootOf}\left(-2\ln(x) + 2\operatorname{arctanh}\left(\sqrt{-2\_Z-1}\right) - \ln(\_Z+1) + 2c_1\right)x^2 + 1}$$
$$y(x) = -\sqrt{\operatorname{RootOf}\left(-2\ln(x) + 2\operatorname{arctanh}\left(\sqrt{-2\_Z-1}\right) - \ln(\_Z+1) + 2c_1\right)x^2 + 1}$$

✓ Solution by Mathematica

Time used: 0.576 (sec). Leaf size: 57

```
DSolve[2 y[x]^2 (y'[x])^2 +2 x y[x] y'[x]-1+x^2+y[x]^2==0,y[x],x,IncludeSingularSolutions ->
```

$$y(x) \to -\sqrt{-x^2 + c_1x + 1 - \frac{c_1{}^2}{2}}$$
$$y(x) \to \sqrt{-x^2 + c_1x + 1 - \frac{c_1{}^2}{2}}$$

33.29 problem 992

Internal problem ID [4224]

Book: Ordinary differential equations and their solutions. By George Moseley Murphy. 1960
Section: Various 33
Problem number: 992.
ODE order: 1.
ODE degree: 2.

CAS Maple gives this as type `[[_homogeneous, 'class A'], _dAlembert]`

$$3y^2y'^2 - 2xyy' + 4y^2 = x^2$$

✓ Solution by Maple

Time used: 0.047 (sec). Leaf size: 105

```
dsolve(3*y(x)^2*diff(y(x),x)^2-2*x*y(x)*diff(y(x),x)-x^2+4*y(x)^2 = 0,y(x), singsol=all)
```

$$y(x) = -\frac{\sqrt{3}\,x}{3}$$

$$y(x) = \frac{\sqrt{3}\,x}{3}$$

$$\ln(x) - \operatorname{arctanh}\left(\frac{\sqrt{\frac{x^2-3y(x)^2}{x^2}}}{2}\right) + \frac{\ln\left(\frac{x^2+y(x)^2}{x^2}\right)}{2} - c_1 = 0$$

$$\ln(x) + \operatorname{arctanh}\left(\frac{\sqrt{\frac{x^2-3y(x)^2}{x^2}}}{2}\right) + \frac{\ln\left(\frac{x^2+y(x)^2}{x^2}\right)}{2} - c_1 = 0$$

✓ Solution by Mathematica

Time used: 0.639 (sec). Leaf size: 179

```
DSolve[3  y[x]^2 (y'[x])^2 -2 x y[x] y'[x]-x^2+4 y[x]^2==0,y[x],x,IncludeSingularSolutions -
```

$$y(x) \to -\frac{\sqrt{-3x^2 - 4ie^{3c_1}x + e^{6c_1}}}{\sqrt{3}}$$
$$y(x) \to \frac{\sqrt{-3x^2 - 4ie^{3c_1}x + e^{6c_1}}}{\sqrt{3}}$$
$$y(x) \to -\frac{\sqrt{-3x^2 + 4ie^{3c_1}x + e^{6c_1}}}{\sqrt{3}}$$
$$y(x) \to \frac{\sqrt{-3x^2 + 4ie^{3c_1}x + e^{6c_1}}}{\sqrt{3}}$$
$$y(x) \to -\sqrt{-x^2}$$
$$y(x) \to \sqrt{-x^2}$$

33.30 problem 993

Internal problem ID [4225]

Book: Ordinary differential equations and their solutions. By George Moseley Murphy. 1960
Section: Various 33
Problem number: 993.
ODE order: 1.
ODE degree: 2.

CAS Maple gives this as type `[_separable]`

$$4y^2{y'}^2 + 2(3x+1)\,xyy' = -3x^3$$

✓ Solution by Maple
Time used: 0.016 (sec). Leaf size: 59

```
dsolve(4*y(x)^2*diff(y(x),x)^2+2*(1+3*x)*x*y(x)*diff(y(x),x)+3*x^3 = 0,y(x), singsol=all)
```

$$y(x) = -\frac{\sqrt{-2x^2+4c_1}}{2}$$
$$y(x) = \frac{\sqrt{-2x^2+4c_1}}{2}$$
$$y(x) = \sqrt{-x^3+c_1}$$
$$y(x) = -\sqrt{-x^3+c_1}$$

✓ Solution by Mathematica
Time used: 0.169 (sec). Leaf size: 81

```
DSolve[4 y[x]^2 (y'[x])^2 +2(1+3 x)x y[x] y'[x]+3 x^3==0,y[x],x,IncludeSingularSolutions ->
```

$$y(x) \to -\sqrt{-x^3+2c_1}$$
$$y(x) \to \sqrt{-x^3+2c_1}$$
$$y(x) \to -\sqrt{-\frac{x^2}{2}+2c_1}$$
$$y(x) \to \sqrt{-\frac{x^2}{2}+2c_1}$$

33.31 problem 994

Internal problem ID [4226]

Book: Ordinary differential equations and their solutions. By George Moseley Murphy. 1960
Section: Various 33
Problem number: 994.
ODE order: 1.
ODE degree: 2.

CAS Maple gives this as type `[[_homogeneous, ‘class A‘], _rational, [_Abel, ‘2nd type‘, ‘cl`

$$\boxed{(x^2-4y^2)\,y'^2+6xyy'+y^2=4x^2}$$

✓ Solution by Maple

Time used: 1.453 (sec). Leaf size: 93

```
dsolve((x^2-4*y(x)^2)*diff(y(x),x)^2+6*x*y(x)*diff(y(x),x)-4*x^2+y(x)^2 = 0,y(x), singsol=al
```

$$y(x)=\frac{x\left(-\operatorname{RootOf}\left(\_Z^{16}+2\_Z^{4}c_1x^4-c_1x^4\right)^4+1\right)}{\operatorname{RootOf}\left(\_Z^{16}+2\_Z^{4}c_1x^4-c_1x^4\right)^4}$$

$$y(x)=\frac{\frac{\operatorname{RootOf}\left(\_Z^{16}-2\_Z^{4}c_1x^4-c_1x^4\right)^{12}}{c_1}-x^4}{x^3}$$

✓ Solution by Mathematica

Time used: 60.117 (sec). Leaf size: 3017

```
DSolve[(x^2-4 y[x]^2) (y'[x])^2 +6 x y[x] y'[x]-4 x^2+y[x]^2==0,y[x],x,IncludeSingularSoluti
```

Too large to display

33.32 problem 995

Internal problem ID [4227]

Book: Ordinary differential equations and their solutions. By George Moseley Murphy. 1960
Section: Various 33
Problem number: 995.
ODE order: 1.
ODE degree: 2.

CAS Maple gives this as type `[[_1st_order, _with_linear_symmetries], _rational]`

$$\boxed{9y^2y'^2 - 3xy' + y = 0}$$

✓ Solution by Maple

Time used: 0.078 (sec). Leaf size: 111

```
dsolve(9*y(x)^2*diff(y(x),x)^2-3*x*diff(y(x),x)+y(x) = 0,y(x), singsol=all)
```

$$y(x) = \frac{2^{\frac{1}{3}}(x^2)^{\frac{1}{3}}}{2}$$
$$y(x) = -\frac{2^{\frac{1}{3}}(x^2)^{\frac{1}{3}}\left(1+i\sqrt{3}\right)}{4}$$
$$y(x) = \frac{2^{\frac{1}{3}}(x^2)^{\frac{1}{3}}\left(-1+i\sqrt{3}\right)}{4}$$
$$y(x) = 0$$
$$y(x) = \text{RootOf}\left(-2\ln(x) - 3\left(\int^{\_Z} \frac{4\_a^3 + \sqrt{-4\_a^3+1} - 1}{\_a\,(4\_a^3-1)}d\_a\right) + 2c_1\right)x^{\frac{2}{3}}$$

✓ Solution by Mathematica

Time used: 0.49 (sec). Leaf size: 243

```
DSolve[9 y[x]^2 (y'[x])^2 -3 x y'[x]+y[x]==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to e^{\frac{c_1}{3}} \sqrt[3]{e^{c_1} - ix}$$
$$y(x) \to -\sqrt[3]{-1} e^{\frac{c_1}{3}} \sqrt[3]{e^{c_1} - ix}$$
$$y(x) \to (-1)^{2/3} e^{\frac{c_1}{3}} \sqrt[3]{e^{c_1} - ix}$$
$$y(x) \to e^{\frac{c_1}{3}} \sqrt[3]{ix + e^{c_1}}$$
$$y(x) \to -\sqrt[3]{-1} e^{\frac{c_1}{3}} \sqrt[3]{ix + e^{c_1}}$$
$$y(x) \to (-1)^{2/3} e^{\frac{c_1}{3}} \sqrt[3]{ix + e^{c_1}}$$
$$y(x) \to 0$$
$$y(x) \to \left(-\frac{1}{2}\right)^{2/3} x^{2/3}$$
$$y(x) \to \frac{x^{2/3}}{2^{2/3}}$$
$$y(x) \to -\frac{\sqrt[3]{-1} x^{2/3}}{2^{2/3}}$$

34 Various 34

34.1 problem 996

Internal problem ID [4228]

Book: Ordinary differential equations and their solutions. By George Moseley Murphy. 1960
Section: Various 34
Problem number: 996.
ODE order: 1.
ODE degree: 2.

CAS Maple gives this as type `[_quadrature]`

$$\boxed{(2-3y)^2 y'^2 + 4y = 4}$$

✓ Solution by Maple

Time used: 0.047 (sec). Leaf size: 477

```
dsolve((2-3*y(x))^2*diff(y(x),x)^2 = 4-4*y(x),y(x), singsol=all)
```

$$y(x) = 1$$

$$y(x) = -\frac{\left(\left(108c_1 - 108x + 12\sqrt{81c_1^2 - 162c_1x + 81x^2 - 12}\right)^{\frac{1}{3}} + \frac{12}{\left(108c_1 - 108x + 12\sqrt{81c_1^2 - 162c_1x + 81x^2 - 12}\right)^{\frac{1}{3}}}\right)^2}{36} + 1$$

$$y(x) = 1 + \frac{\left(\left(i-\sqrt{3}\right)\left(108c_1 - 108x + 12\sqrt{81c_1^2 - 162c_1x + 81x^2 - 12}\right)^{\frac{2}{3}} + 12i + 12\sqrt{3}\right)^2}{144\left(108c_1 - 108x + 12\sqrt{81c_1^2 - 162c_1x + 81x^2 - 12}\right)^{\frac{2}{3}}}$$

$$y(x) = 1 + \frac{\left(\left(\sqrt{3}+i\right)\left(108c_1 - 108x + 12\sqrt{81c_1^2 - 162c_1x + 81x^2 - 12}\right)^{\frac{2}{3}} + 12i - 12\sqrt{3}\right)^2}{144\left(108c_1 - 108x + 12\sqrt{81c_1^2 - 162c_1x + 81x^2 - 12}\right)^{\frac{2}{3}}}$$

$$y(x) = -\frac{\left(\left(-108c_1 + 108x + 12\sqrt{81c_1^2 - 162c_1x + 81x^2 - 12}\right)^{\frac{1}{3}} + \frac{12}{\left(-108c_1 + 108x + 12\sqrt{81c_1^2 - 162c_1x + 81x^2 - 12}\right)^{\frac{1}{3}}}\right)^2}{36} + 1$$

$$y(x) = 1 + \frac{\left(\left(i-\sqrt{3}\right)\left(-108c_1 + 108x + 12\sqrt{81c_1^2 - 162c_1x + 81x^2 - 12}\right)^{\frac{2}{3}} + 12i + 12\sqrt{3}\right)^2}{144\left(-108c_1 + 108x + 12\sqrt{81c_1^2 - 162c_1x + 81x^2 - 12}\right)^{\frac{2}{3}}}$$

$$y(x) = 1 + \frac{\left(\left(\sqrt{3}+i\right)\left(-108c_1 + 108x + 12\sqrt{81c_1^2 - 162c_1x + 81x^2 - 12}\right)^{\frac{2}{3}} + 12i - 12\sqrt{3}\right)^2}{144\left(-108c_1 + 108x + 12\sqrt{81c_1^2 - 162c_1x + 81x^2 - 12}\right)^{\frac{2}{3}}}$$

✓ Solution by Mathematica

Time used: 4.408 (sec). Leaf size: 896

```
DSolve[(2-3 y[x])^2 (y'[x])^2 ==4(1-y[x]),y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to \frac{1}{12}\left(2\sqrt[3]{-108x^2+3\sqrt{3}\sqrt{(2x+c_1)^2(108x^2+108c_1x-16+27c_1{}^2)}-108c_1x+8-27c_1{}^2} + \frac{8}{\sqrt[3]{-108x^2+3\sqrt{3}\sqrt{(2x+c_1)^2(108x^2+108c_1x-16+27c_1{}^2)}-108c_1x+8-27c_1{}^2}} + 4\right)$$

$$y(x) \to \frac{1}{24}\left(2i\left(\sqrt{3}+i\right)\sqrt[3]{-108x^2+3\sqrt{3}\sqrt{(2x+c_1)^2(108x^2+108c_1x-16+27c_1{}^2)}-108c_1x+8-27c_1{}^2} - \frac{8(1+i\sqrt{3})}{\sqrt[3]{-108x^2+3\sqrt{3}\sqrt{(2x+c_1)^2(108x^2+108c_1x-16+27c_1{}^2)}-108c_1x+8-27c_1{}^2}} + 8\right)$$

$$y(x) \to \frac{1}{24}\left(-2\left(1+i\sqrt{3}\right)\sqrt[3]{-108x^2+3\sqrt{3}\sqrt{(2x+c_1)^2(108x^2+108c_1x-16+27c_1{}^2)}-108c_1x+8-27c_1{}^2} + \frac{8i(\sqrt{3}+i)}{\sqrt[3]{-108x^2+3\sqrt{3}\sqrt{(2x+c_1)^2(108x^2+108c_1x-16+27c_1{}^2)}-108c_1x+8-27c_1{}^2}} + 8\right)$$

$$y(x) \to \frac{1}{12}\left(2\sqrt[3]{-108x^2+3\sqrt{3}\sqrt{(-2x+c_1)^2(108x^2-108c_1x-16+27c_1{}^2)}+108c_1x+8-27c_1{}^2} + \frac{8}{\sqrt[3]{-108x^2+3\sqrt{3}\sqrt{(-2x+c_1)^2(108x^2-108c_1x-16+27c_1{}^2)}+108c_1x+8-27c_1{}^2}} + 4\right)$$

34.2 problem 997

Internal problem ID [4229]

Book: Ordinary differential equations and their solutions. By George Moseley Murphy. 1960
Section: Various 34
Problem number: 997.
ODE order: 1.
ODE degree: 2.

CAS Maple gives this as type `[[_homogeneous, ‘class A‘], _rational, _dAlembert]`

$$\left(-a^2+1\right) y^2 y'^2 - 3a^2 x y y' + y^2 = a^2 x^2$$

✓ Solution by Maple

Time used: 0.062 (sec). Leaf size: 198

```
dsolve((-a^2+1)*y(x)^2*diff(y(x),x)^2-3*a^2*x*y(x)*diff(y(x),x)-a^2*x^2+y(x)^2 = 0,y(x), sin
```

$$y(x) = \operatorname{RootOf}\left(-2\ln(x) - \left(\int^{\_Z} \frac{\left(2\_a^2 a^2 - 2\_a^2 + 3a^2 + \sqrt{4\_a^2 a^2 + 5a^4 - 4\_a^2 + 4a^2}\right)\_a}{a^2\_a^4 - \_a^4 + 3\_a^2 a^2 - \_a^2 + a^2} d\_a\right) + 2c_1\right) x$$

$$y(x) = \operatorname{RootOf}\left(-2\ln(x) + \int^{\_Z} -\frac{\left(2\_a^2 a^2 - 2\_a^2 + 3a^2 - \sqrt{4\_a^2 a^2 + 5a^4 - 4\_a^2 + 4a^2}\right)\_a}{a^2\_a^4 - \_a^4 + 3\_a^2 a^2 - \_a^2 + a^2} d\_a + 2c_1\right) x$$

✓ Solution by Mathematica

Time used: 1.255 (sec). Leaf size: 342

```
DSolve[(1-a^2)y[x]^2 (y'[x])^2 -2 a^2 x y[x] y'[x]-a^2 x y[x] y'[x]-a^2 x^2+y[x]^2==0,y[x],x
```

$$\text{Solve}\left[\frac{\log\left(-\left(a^2\left(\frac{2y(x)^2}{x^2}+3\right)\right)+\sqrt{5a^4+4a^2\left(\frac{y(x)^2}{x^2}+1\right)-\frac{4y(x)^2}{x^2}+\frac{2y(x)^2}{x^2}}\right)-\frac{2\arctan\left(\frac{1-\sqrt{5a^4+4a^2\left(\frac{y(x)^2}{x^2}+1\right)-\cdots}}{\sqrt{-5a^4+2a^2-1}}\cdots\right.}{\sqrt{-5a^4+2a^2-1}}\cdots}{4a^2-4}+c_1,y(x)\right]$$

$$\text{Solve}\left[\frac{\log\left(a^2\left(\frac{2y(x)^2}{x^2}+3\right)+\sqrt{5a^4+4a^2\left(\frac{y(x)^2}{x^2}+1\right)-\frac{4y(x)^2}{x^2}}-\frac{2y(x)^2}{x^2}\right)-\frac{2\arctan\left(\frac{\sqrt{5a^4+4a^2\left(\frac{y(x)^2}{x^2}+1\right)-\frac{4y(x)^2}{x^2}}+1\cdots}{\sqrt{-5a^4+2a^2-1}}\cdots\right.}{\sqrt{-5a^4+2a^2-1}}\cdots}{4a^2-4}+c_1,y(x)\right]$$

34.3 problem 998

Internal problem ID [4230]

Book: Ordinary differential equations and their solutions. By George Moseley Murphy. 1960
Section: Various 34
Problem number: 998.
ODE order: 1.
ODE degree: 2.

CAS Maple gives this as type `[_rational, [_1st_order, '_with_symmetry_[F(x),G(y)]']]`

$$\boxed{(-b+a)\, y^2 {y'}^2 - 2bxyy' + ay^2 = b\, x^2 + ab}$$

✓ Solution by Maple

Time used: 0.219 (sec). Leaf size: 765

```
dsolve((a-b)*y(x)^2*diff(y(x),x)^2-2*b*x*y(x)*diff(y(x),x)-a*b-b*x^2+a*y(x)^2 = 0,y(x), sing
```

$$y(x) = \frac{\sqrt{b\left(x^2+a-b\right)(a-b)}}{a-b}$$

$$y(x) = -\frac{\sqrt{b\left(x^2+a-b\right)(a-b)}}{a-b}$$

$$-\left(\int_{\_b}^{x} \frac{b\_a+\sqrt{a\left((-a+b)\,y\left(x\right)^{2}+b\left(\_a^{2}+a-b\right)\right)}}{\sqrt{a\left((-a+b)\,y\left(x\right)^{2}+b\left(\_a^{2}+a-b\right)\right)}\,\_a+(-a+b)\,y\left(x\right)^{2}+b\left(\_a^{2}+a-b\right)}d\_a\right)$$

$$+\int^{y(x)} \frac{\left(\left(\sqrt{a\left(-b^{2}+\left(\_f^{2}+x^{2}+a\right)b-a\_f^{2}\right)}\,x+(-a+b)\_f^{2}+b(x^{2}+a-b)\right)\left(\int_{\_b}^{x} \frac{(a-b)\left(2\right.}{\sqrt{a\left(-b^{2}+\left(\_a^{2}+\right.\right.}}\right.\right.}{\sqrt{a\left(-b^{2}+\left(\_f^{2}+x^{2}+a\right)b-a\_f^{2}\right)}\,x}$$

$$+c_1 = 0$$

$$-\left(\int_{\_b}^{x} \frac{b\_a-\sqrt{a\left((-a+b)\,y\left(x\right)^{2}+b\left(\_a^{2}+a-b\right)\right)}}{-\sqrt{a\left((-a+b)\,y\left(x\right)^{2}+b\left(\_a^{2}+a-b\right)\right)}\,\_a+(-a+b)\,y\left(x\right)^{2}+b\left(\_a^{2}+a-b\right)}d\_a\right)$$

$$+\int^{y(x)} \frac{\left(\left(-\sqrt{a\left(-b^{2}+\left(\_f^{2}+x^{2}+a\right)b-a\_f^{2}\right)}\,x+(-a+b)\_f^{2}+b(x^{2}+a-b)\right)\left(\int_{\_b}^{x} -\frac{(a-}{\sqrt{a\left(-b^{2}+\left(\_a^{2}\right.\right.}}\right.\right.}{-\sqrt{a\left(-b^{2}+\left(\_f^{2}+x^{2}+a\right)b-a\_f^{2}\right.}}$$

$$+c_1 = 0$$

✓ Solution by Mathematica

Time used: 1.39 (sec). Leaf size: 86

```
DSolve[(a-b) y[x]^2 (y'[x])^2 -2 b x y[x] y'[x]-a b -b x^2+a y[x]^2==0,y[x],x,IncludeSingula
```

$$y(x) \to -\frac{\sqrt{b\left(b-x^2\right)+a\left(-b+(x-c_1)^2\right)}}{\sqrt{b-a}}$$

$$y(x) \to \frac{\sqrt{b\left(b-x^2\right)+a\left(-b+(x-c_1)^2\right)}}{\sqrt{b-a}}$$

34.4 problem 999

Internal problem ID [4231]

Book: Ordinary differential equations and their solutions. By George Moseley Murphy. 1960
Section: Various 34
Problem number: 999.
ODE order: 1.
ODE degree: 2.

CAS Maple gives this as type `[[_homogeneous, 'class C'], _dAlembert]`

$$a^2\left(b^2-(cx-ya)^2\right)y'^2+2a\,b^2cy'+c^2\left(b^2-(cx-ya)^2\right)=0$$

✓ Solution by Maple

Time used: 0.218 (sec). Leaf size: 200

```
dsolve(a^2*(b^2-(c*x-a*y(x))^2)*diff(y(x),x)^2+2*a*b^2*c*diff(y(x),x)+c^2*(b^2-(c*x-a*y(x))^
```

$$y(x)=\frac{cx-\sqrt{2}\,b}{a}$$
$$y(x)=\frac{cx+\sqrt{2}\,b}{a}$$
$$y(x)=\frac{\operatorname{RootOf}\left(-a\left(\int^{\_Z}\frac{\_a^2a^2-2b^2+\sqrt{-a^2\_a^2\left(\_a^2a^2-2b^2\right)}}{\_a^2a^2-2b^2}d\_a\right)+2c_1c-2cx\right)a+cx}{a}$$
$$y(x)=\frac{\operatorname{RootOf}\left(a\left(\int^{\_Z}-\frac{\_a^2a^2-2b^2-\sqrt{-a^2\_a^2\left(\_a^2a^2-2b^2\right)}}{\_a^2a^2-2b^2}d\_a\right)+2c_1c-2cx\right)a+cx}{a}$$

✓ Solution by Mathematica

Time used: 2.249 (sec). Leaf size: 71

```
DSolve[a^2 ( b^2 -(c x-a y[x])^2 ) (y'[x])^2 +2 a b^2 c y'[x]+c^2(b^2-(c x-a y[x])^2)==0,y[x
```

$$y(x)\to\frac{cc_1-\sqrt{b^2-c^2(x-c_1)^2}}{a}$$
$$y(x)\to\frac{\sqrt{b^2-c^2(x-c_1)^2}+cc_1}{a}$$

34.5 problem 1000

Internal problem ID [4232]

Book: Ordinary differential equations and their solutions. By George Moseley Murphy. 1960
Section: Various 34
Problem number: 1000.
ODE order: 1.
ODE degree: 2.

CAS Maple gives this as type `[[_homogeneous, ‘class G‘], _rational]`

$$\boxed{xy^2y'^2 - y^3y' = -x\,a^2}$$

✓ Solution by Maple

Time used: 0.109 (sec). Leaf size: 152

```
dsolve(x*y(x)^2*diff(y(x),x)^2-y(x)^3*diff(y(x),x)+a^2*x = 0,y(x), singsol=all)
```

$$y(x) = \sqrt{2}\,\sqrt{-ax}$$
$$y(x) = -\sqrt{2}\,\sqrt{-ax}$$
$$y(x) = \sqrt{2}\,\sqrt{ax}$$
$$y(x) = -\sqrt{2}\,\sqrt{ax}$$
$$y(x) = \frac{\mathrm{e}^{\frac{c_1}{2}+\frac{\mathrm{RootOf}\left(16x\,a^2\mathrm{e}^{2\_Z+2c_1}+\mathrm{e}^{2\_Z}x^3-4\,\mathrm{e}^{2c_1+3\_Z}\right)}{2}}}{\sqrt{x}}$$
$$y(x) = \sqrt{x}\,\mathrm{e}^{-\frac{c_1}{2}+\frac{\mathrm{RootOf}\left(x^2\left(16a^2x^2\mathrm{e}^{2\_Z-2c_1}-4\,\mathrm{e}^{3\_Z-2c_1}x+\mathrm{e}^{2\_Z}\right)\right)}{2}}$$

✓ Solution by Mathematica

Time used: 22.383 (sec). Leaf size: 219

```
DSolve[x y[x]^2 (y'[x])^2 - y[x]^3 y'[x]+a^2 x==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to -\sqrt{-2a^2e^{-c_1}x^2 - \frac{e^{c_1}}{2}}$$
$$y(x) \to \sqrt{-2a^2e^{-c_1}x^2 - \frac{e^{c_1}}{2}}$$
$$y(x) \to -\frac{\sqrt{4a^2e^{-c_1}x^2 + e^{c_1}}}{\sqrt{2}}$$
$$y(x) \to \frac{\sqrt{4a^2e^{-c_1}x^2 + e^{c_1}}}{\sqrt{2}}$$
$$y(x) \to -\sqrt{2}\sqrt{a}\sqrt{x}$$
$$y(x) \to -i\sqrt{2}\sqrt{a}\sqrt{x}$$
$$y(x) \to i\sqrt{2}\sqrt{a}\sqrt{x}$$
$$y(x) \to \sqrt{2}\sqrt{a}\sqrt{x}$$

34.6 problem 1001

Internal problem ID [4233]

Book: Ordinary differential equations and their solutions. By George Moseley Murphy. 1960
Section: Various 34
Problem number: 1001.
ODE order: 1.
ODE degree: 2.

CAS Maple gives this as type `[_rational]`

$$
xy^2{y'}^2+\left(a-x^3-y^3\right)y'+x^2y=0
$$

✓ Solution by Maple

Time used: 0.125 (sec). Leaf size: 251

```
dsolve(x*y(x)^2*diff(y(x),x)^2+(a-x^3-y(x)^3)*diff(y(x),x)+x^2*y(x) = 0,y(x), singsol=all)
```

$$
\begin{aligned}
y(x) &= \left(x^3+a-2x\sqrt{ax}\right)^{\frac{1}{3}} \\
y(x) &= \left(x^3+a+2x\sqrt{ax}\right)^{\frac{1}{3}} \\
y(x) &= -\frac{\left(x^3+a-2x\sqrt{ax}\right)^{\frac{1}{3}}\left(1+i\sqrt{3}\right)}{2} \\
y(x) &= \frac{\left(x^3+a-2x\sqrt{ax}\right)^{\frac{1}{3}}\left(-1+i\sqrt{3}\right)}{2} \\
y(x) &= -\frac{\left(x^3+a+2x\sqrt{ax}\right)^{\frac{1}{3}}\left(1+i\sqrt{3}\right)}{2} \\
y(x) &= \frac{\left(x^3+a+2x\sqrt{ax}\right)^{\frac{1}{3}}\left(-1+i\sqrt{3}\right)}{2} \\
y(x) &= 0 \\
\int_{\_b}^{y(x)} &\frac{\_a^2}{\sqrt{\_a^6+\left(-2x^3-2a\right)\_a^3+\left(-x^3+a\right)^2}}d\_a+\frac{\ln\left(x\right)}{2}-c_1=0 \\
\int_{\_b}^{y(x)} &\frac{\_a^2}{\sqrt{\_a^6+\left(-2x^3-2a\right)\_a^3+\left(-x^3+a\right)^2}}d\_a-\frac{\ln\left(x\right)}{2}-c_1=0
\end{aligned}
$$

✓ Solution by Mathematica

Time used: 0.427 (sec). Leaf size: 194

```
DSolve[x y[x]^2 (y'[x])^2 +(a-x^3-y[x]^3) y'[x]+x^2 y[x]==0,y[x],x,IncludeSingularSolutions
```

$$y(x) \to \frac{\sqrt[3]{a+(-1+c_1)x^3}}{\sqrt[3]{1-\frac{1}{c_1}}}$$
$$y(x) \to 0$$
$$y(x) \to \sqrt[3]{\left(\sqrt{a}-x^{3/2}\right)^2}$$
$$y(x) \to -\sqrt[3]{-1}\sqrt[3]{\left(\sqrt{a}-x^{3/2}\right)^2}$$
$$y(x) \to (-1)^{2/3}\sqrt[3]{\left(\sqrt{a}-x^{3/2}\right)^2}$$
$$y(x) \to \sqrt[3]{\left(\sqrt{a}+x^{3/2}\right)^2}$$
$$y(x) \to -\sqrt[3]{-1}\sqrt[3]{\left(\sqrt{a}+x^{3/2}\right)^2}$$
$$y(x) \to (-1)^{2/3}\sqrt[3]{\left(\sqrt{a}+x^{3/2}\right)^2}$$

34.7 problem 1003

Internal problem ID [4234]

Book: Ordinary differential equations and their solutions. By George Moseley Murphy. 1960
Section: Various 34
Problem number: 1003.
ODE order: 1.
ODE degree: 2.

CAS Maple gives this as type `[[_homogeneous, 'class G'], _rational]`

$$2xy^2{y'}^2 - y^3y' = a$$

✓ Solution by Maple

Time used: 0.25 (sec). Leaf size: 159

```
dsolve(2*x*y(x)^2*diff(y(x),x)^2-y(x)^3*diff(y(x),x)-a = 0,y(x), singsol=all)
```

$$y(x) = 2^{\frac{3}{4}}(-ax)^{\frac{1}{4}}$$
$$y(x) = -2^{\frac{3}{4}}(-ax)^{\frac{1}{4}}$$
$$y(x) = -i2^{\frac{3}{4}}(-ax)^{\frac{1}{4}}$$
$$y(x) = i2^{\frac{3}{4}}(-ax)^{\frac{1}{4}}$$
$$y(x) = \frac{2^{\frac{1}{4}}\left(a(c_1-x)^2\, c_1^3\right)^{\frac{1}{4}}}{c_1}$$
$$y(x) = -\frac{2^{\frac{1}{4}}\left(a(c_1-x)^2\, c_1^3\right)^{\frac{1}{4}}}{c_1}$$
$$y(x) = -\frac{i2^{\frac{1}{4}}\left(a(c_1-x)^2\, c_1^3\right)^{\frac{1}{4}}}{c_1}$$
$$y(x) = \frac{i2^{\frac{1}{4}}\left(a(c_1-x)^2\, c_1^3\right)^{\frac{1}{4}}}{c_1}$$

✓ Solution by Mathematica

Time used: 1.666 (sec). Leaf size: 151

```
DSolve[2 x y[x]^2 (y'[x])^2 -y[x]^3 y'[x] -a ==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to -\frac{e^{-\frac{c_1}{4}}\sqrt{-8ax + e^{c_1}}}{\sqrt{2}}$$
$$y(x) \to \frac{e^{-\frac{c_1}{4}}\sqrt{-8ax + e^{c_1}}}{\sqrt{2}}$$
$$y(x) \to -(-2)^{3/4}\sqrt[4]{a}\sqrt[4]{x}$$
$$y(x) \to (-2)^{3/4}\sqrt[4]{a}\sqrt[4]{x}$$
$$y(x) \to (-1-i)\sqrt[4]{2}\sqrt[4]{a}\sqrt[4]{x}$$
$$y(x) \to (1+i)\sqrt[4]{2}\sqrt[4]{a}\sqrt[4]{x}$$

34.8 problem 1004

Internal problem ID [4235]

Book: Ordinary differential equations and their solutions. By George Moseley Murphy. 1960
Section: Various 34
Problem number: 1004.
ODE order: 1.
ODE degree: 2.

CAS Maple gives this as type `[[_homogeneous, ‘class A‘], _rational, _Bernoulli]`

$$4x^2y^2y'^2 - (y^2+x^2)^2 = 0$$

✓ Solution by Maple
Time used: 0.0 (sec). Leaf size: 69

```
dsolve(4*x^2*y(x)^2*diff(y(x),x)^2 = (x^2+y(x)^2)^2,y(x), singsol=all)
```

$$y(x) = \sqrt{(x+c_1)\,x}$$
$$y(x) = -\sqrt{(x+c_1)\,x}$$
$$y(x) = -\frac{\sqrt{3}\,\sqrt{-x\,(x^3-3c_1)}}{3x}$$
$$y(x) = \frac{\sqrt{3}\,\sqrt{-x\,(x^3-3c_1)}}{3x}$$

✓ Solution by Mathematica
Time used: 0.526 (sec). Leaf size: 97

```
DSolve[4 x^2 y[x]^2(y'[x])^2 ==(x^2+y[x]^2)^2,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to -\sqrt{x}\sqrt{x+c_1}$$
$$y(x) \to \sqrt{x}\sqrt{x+c_1}$$
$$y(x) \to -\frac{\sqrt{-x^3+3c_1}}{\sqrt{3}\sqrt{x}}$$
$$y(x) \to \frac{\sqrt{-x^3+3c_1}}{\sqrt{3}\sqrt{x}}$$

34.9 problem 1006

Internal problem ID [4236]

Book: Ordinary differential equations and their solutions. By George Moseley Murphy. 1960
Section: Various 34
Problem number: 1006.
ODE order: 1.
ODE degree: 2.

CAS Maple gives this as type `[[_1st_order, _with_linear_symmetries], _rational]`

$$4y^3{y'}^2 - 4xy' + y = 0$$

✓ Solution by Maple
Time used: 0.109 (sec). Leaf size: 82

```
dsolve(4*y(x)^3*diff(y(x),x)^2-4*x*diff(y(x),x)+y(x) = 0,y(x), singsol=all)
```

$$y(x) = \sqrt{-x}$$
$$y(x) = -\sqrt{-x}$$
$$y(x) = \sqrt{x}$$
$$y(x) = -\sqrt{x}$$
$$y(x) = 0$$
$$y(x) = \text{RootOf}\left(-\ln(x) - 2\left(\int^{\_Z} \frac{\_a^4 + \sqrt{-\_a^4 + 1} - 1}{\_a\left(\_a^4 - 1\right)} d\_a\right) + c_1\right)\sqrt{x}$$

✓ Solution by Mathematica

Time used: 0.55 (sec). Leaf size: 282

```
DSolve[4 y[x]^3 (y'[x])^2 -4 x y'[x]+y[x]==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to -e^{\frac{c_1}{4}} \sqrt[4]{e^{c_1} - 2ix}$$
$$y(x) \to -ie^{\frac{c_1}{4}} \sqrt[4]{e^{c_1} - 2ix}$$
$$y(x) \to ie^{\frac{c_1}{4}} \sqrt[4]{e^{c_1} - 2ix}$$
$$y(x) \to e^{\frac{c_1}{4}} \sqrt[4]{e^{c_1} - 2ix}$$
$$y(x) \to -e^{\frac{c_1}{4}} \sqrt[4]{2ix + e^{c_1}}$$
$$y(x) \to -ie^{\frac{c_1}{4}} \sqrt[4]{2ix + e^{c_1}}$$
$$y(x) \to ie^{\frac{c_1}{4}} \sqrt[4]{2ix + e^{c_1}}$$
$$y(x) \to e^{\frac{c_1}{4}} \sqrt[4]{2ix + e^{c_1}}$$
$$y(x) \to 0$$
$$y(x) \to -\sqrt{x}$$
$$y(x) \to -i\sqrt{x}$$
$$y(x) \to i\sqrt{x}$$
$$y(x) \to \sqrt{x}$$

34.10 problem 1012

Internal problem ID [4237]

Book: Ordinary differential equations and their solutions. By George Moseley Murphy. 1960
Section: Various 34
Problem number: 1012.
ODE order: 1.
ODE degree: 2.

CAS Maple gives this as type `[[_homogeneous, ‘class G‘], _rational]`

$$\boxed{3xy^4y'^2 - y^5y' = -1}$$

✓ Solution by Maple

Time used: 0.047 (sec). Leaf size: 297

```
dsolve(3*x*y(x)^4*diff(y(x),x)^2-y(x)^5*diff(y(x),x)+1 = 0,y(x), singsol=all)
```

$$y(x) = 2^{\frac{1}{3}} 3^{\frac{1}{6}} x^{\frac{1}{6}}$$
$$y(x) = -2^{\frac{1}{3}} 3^{\frac{1}{6}} x^{\frac{1}{6}}$$
$$y(x) = -\frac{\left(1+i\sqrt{3}\right) 3^{\frac{1}{6}} 2^{\frac{1}{3}} x^{\frac{1}{6}}}{2}$$
$$y(x) = \frac{\left(-1+i\sqrt{3}\right) 3^{\frac{1}{6}} 2^{\frac{1}{3}} x^{\frac{1}{6}}}{2}$$
$$y(x) = -\frac{\left(-1+i\sqrt{3}\right) 3^{\frac{1}{6}} 2^{\frac{1}{3}} x^{\frac{1}{6}}}{2}$$
$$y(x) = \frac{\left(1+i\sqrt{3}\right) 3^{\frac{1}{6}} 2^{\frac{1}{3}} x^{\frac{1}{6}}}{2}$$
$$y(x) = \frac{3^{\frac{1}{6}} \left(-\left(c_1-x\right)^2 c_1^5\right)^{\frac{1}{6}}}{c_1}$$
$$y(x) = -\frac{3^{\frac{1}{6}} \left(-\left(c_1-x\right)^2 c_1^5\right)^{\frac{1}{6}}}{c_1}$$
$$y(x) = -\frac{\left(1+i\sqrt{3}\right) 3^{\frac{1}{6}} \left(-\left(c_1-x\right)^2 c_1^5\right)^{\frac{1}{6}}}{2c_1}$$
$$y(x) = \frac{\left(i 3^{\frac{2}{3}}-3^{\frac{1}{6}}\right) \left(-\left(c_1-x\right)^2 c_1^5\right)^{\frac{1}{6}}}{2c_1}$$
$$y(x) = -\frac{\left(-1+i\sqrt{3}\right) 3^{\frac{1}{6}} \left(-\left(c_1-x\right)^2 c_1^5\right)^{\frac{1}{6}}}{2c_1}$$
$$y(x) = \frac{\left(i 3^{\frac{2}{3}}+3^{\frac{1}{6}}\right) \left(-\left(c_1-x\right)^2 c_1^5\right)^{\frac{1}{6}}}{2c_1}$$

✓ Solution by Mathematica

Time used: 1.332 (sec). Leaf size: 230

```
DSolve[3 x y[x]^4 (y'[x])^2 -y[x]^5 y'[x]+1==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to -\sqrt[3]{-\frac{1}{2}}e^{-\frac{c_1}{6}}\sqrt[3]{12x+e^{c_1}}$$
$$y(x) \to e^{-\frac{c_1}{6}}\sqrt[3]{6x+\frac{e^{c_1}}{2}}$$
$$y(x) \to (-1)^{2/3}e^{-\frac{c_1}{6}}\sqrt[3]{6x+\frac{e^{c_1}}{2}}$$
$$y(x) \to -\sqrt[3]{-2}\sqrt[6]{3}\sqrt[6]{x}$$
$$y(x) \to \sqrt[3]{-2}\sqrt[6]{3}\sqrt[6]{x}$$
$$y(x) \to -\sqrt[3]{2}\sqrt[6]{3}\sqrt[6]{x}$$
$$y(x) \to \sqrt[3]{2}\sqrt[6]{3}\sqrt[6]{x}$$
$$y(x) \to -(-1)^{2/3}\sqrt[3]{2}\sqrt[6]{3}\sqrt[6]{x}$$
$$y(x) \to (-1)^{2/3}\sqrt[3]{2}\sqrt[6]{3}\sqrt[6]{x}$$

34.11 problem 1013

Internal problem ID [4238]

Book: Ordinary differential equations and their solutions. By George Moseley Murphy. 1960
Section: Various 34
Problem number: 1013.
ODE order: 1.
ODE degree: 2.

CAS Maple gives this as type `[[_homogeneous, ‘class G‘], _rational]`

$$\boxed{9xy^4y'^2 - 3y^5y' = a}$$

✓ Solution by Maple

Time used: 0.047 (sec). Leaf size: 279

```
dsolve(9*x*y(x)^4*diff(y(x),x)^2-3*y(x)^5*diff(y(x),x)-a = 0,y(x), singsol=all)
```

$$y(x) = 2^{\frac{1}{3}}(-ax)^{\frac{1}{6}}$$
$$y(x) = -2^{\frac{1}{3}}(-ax)^{\frac{1}{6}}$$
$$y(x) = -\frac{(1+i\sqrt{3})\,2^{\frac{1}{3}}(-ax)^{\frac{1}{6}}}{2}$$
$$y(x) = \frac{(-1+i\sqrt{3})\,2^{\frac{1}{3}}(-ax)^{\frac{1}{6}}}{2}$$
$$y(x) = -\frac{(-1+i\sqrt{3})\,2^{\frac{1}{3}}(-ax)^{\frac{1}{6}}}{2}$$
$$y(x) = \frac{(1+i\sqrt{3})\,2^{\frac{1}{3}}(-ax)^{\frac{1}{6}}}{2}$$
$$y(x) = \frac{\left(a(c_1-x)^2\,c_1^5\right)^{\frac{1}{6}}}{c_1}$$
$$y(x) = -\frac{\left(a(c_1-x)^2\,c_1^5\right)^{\frac{1}{6}}}{c_1}$$
$$y(x) = -\frac{(1+i\sqrt{3})\left(a(c_1-x)^2\,c_1^5\right)^{\frac{1}{6}}}{2c_1}$$
$$y(x) = \frac{(-1+i\sqrt{3})\left(a(c_1-x)^2\,c_1^5\right)^{\frac{1}{6}}}{2c_1}$$
$$y(x) = -\frac{(-1+i\sqrt{3})\left(a(c_1-x)^2\,c_1^5\right)^{\frac{1}{6}}}{2c_1}$$
$$y(x) = \frac{(1+i\sqrt{3})\left(a(c_1-x)^2\,c_1^5\right)^{\frac{1}{6}}}{2c_1}$$

✓ Solution by Mathematica

Time used: 11.244 (sec). Leaf size: 358

```
DSolve[9 x y[x]^4  (y'[x])^2 -3 y[x]^5 y'[x]-a==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to -\sqrt[3]{-\frac{1}{2}}e^{-\frac{c_1}{6}}\sqrt[3]{-4ax+e^{c_1}}$$
$$y(x) \to \frac{e^{-\frac{c_1}{6}}\sqrt[3]{-4ax+e^{c_1}}}{\sqrt[3]{2}}$$
$$y(x) \to \frac{(-1)^{2/3}e^{-\frac{c_1}{6}}\sqrt[3]{-4ax+e^{c_1}}}{\sqrt[3]{2}}$$
$$y(x) \to -\sqrt[3]{-\frac{1}{2}}\sqrt[3]{-e^{-\frac{c_1}{2}}\left(-4ax+e^{c_1}\right)}$$
$$y(x) \to \frac{\sqrt[3]{e^{-\frac{c_1}{2}}\left(4ax-e^{c_1}\right)}}{\sqrt[3]{2}}$$
$$y(x) \to \frac{(-1)^{2/3}\sqrt[3]{-e^{-\frac{c_1}{2}}\left(-4ax+e^{c_1}\right)}}{\sqrt[3]{2}}$$
$$y(x) \to -i\sqrt[3]{2}\sqrt[6]{a}\sqrt[6]{x}$$
$$y(x) \to i\sqrt[3]{2}\sqrt[6]{a}\sqrt[6]{x}$$
$$y(x) \to -\sqrt[6]{-1}\sqrt[3]{2}\sqrt[6]{a}\sqrt[6]{x}$$
$$y(x) \to \sqrt[6]{-1}\sqrt[3]{2}\sqrt[6]{a}\sqrt[6]{x}$$
$$y(x) \to -(-1)^{5/6}\sqrt[3]{2}\sqrt[6]{a}\sqrt[6]{x}$$
$$y(x) \to (-1)^{5/6}\sqrt[3]{2}\sqrt[6]{a}\sqrt[6]{x}$$

34.12 problem 1014

Internal problem ID [4239]

Book: Ordinary differential equations and their solutions. By George Moseley Murphy. 1960
Section: Various 34
Problem number: 1014.
ODE order: 1.
ODE degree: 2.

CAS Maple gives this as type `[[_1st_order, '_with_symmetry_[F(x),G(x)*y+H(x)]']]`

$$9(-x^2+1)\,y^4{y'}^2+6xy^5y' = -4x^2$$

✓ Solution by Maple

Time used: 0.109 (sec). Leaf size: 231

```
dsolve(9*(-x^2+1)*y(x)^4*diff(y(x),x)^2+6*x*y(x)^5*diff(y(x),x)+4*x^2 = 0,y(x), singsol=all)
```

$$y(x) = 2^{\frac{1}{3}}\left(-x^2+1\right)^{\frac{1}{6}}$$
$$y(x) = -2^{\frac{1}{3}}\left(-x^2+1\right)^{\frac{1}{6}}$$
$$y(x) = -\frac{\left(1+i\sqrt{3}\right)2^{\frac{1}{3}}(-x^2+1)^{\frac{1}{6}}}{2}$$
$$y(x) = \frac{\left(-1+i\sqrt{3}\right)2^{\frac{1}{3}}(-x^2+1)^{\frac{1}{6}}}{2}$$
$$y(x) = -\frac{\left(-1+i\sqrt{3}\right)2^{\frac{1}{3}}(-x^2+1)^{\frac{1}{6}}}{2}$$
$$y(x) = \frac{\left(1+i\sqrt{3}\right)2^{\frac{1}{3}}(-x^2+1)^{\frac{1}{6}}}{2}$$
$$y(x) = \frac{2^{\frac{2}{3}}\left((-4c_1^2+x^2-1)\,c_1^2\right)^{\frac{1}{3}}}{2c_1}$$
$$y(x) = -\frac{2^{\frac{2}{3}}\left((-4c_1^2+x^2-1)\,c_1^2\right)^{\frac{1}{3}}\left(1+i\sqrt{3}\right)}{4c_1}$$
$$y(x) = \frac{2^{\frac{2}{3}}\left((-4c_1^2+x^2-1)\,c_1^2\right)^{\frac{1}{3}}\left(-1+i\sqrt{3}\right)}{4c_1}$$

✓ Solution by Mathematica

Time used: 0.415 (sec). Leaf size: 199

```
DSolve[9(1-x^2) y[x]^4 (y'[x])^2 +6 x y[x]^5 y'[x]+4 x^2==0,y[x],x,IncludeSingularSolutions
```

$$y(x) \to -\frac{\sqrt[3]{-\frac{1}{2}}\sqrt[3]{-4x^2+4+c_1{}^2}}{\sqrt[3]{c_1}}$$
$$y(x) \to -1$$
$$y(x) \to 0$$
$$y(x) \to \sqrt[3]{-\frac{1}{2}}$$
$$y(x) \to \text{Indeterminate}$$
$$y(x) \to -\sqrt[3]{-2}\sqrt[6]{1-x^2}$$
$$y(x) \to \sqrt[3]{-2}\sqrt[6]{1-x^2}$$
$$y(x) \to -\sqrt[3]{2}\sqrt[6]{1-x^2}$$
$$y(x) \to \sqrt[3]{2}\sqrt[6]{1-x^2}$$
$$y(x) \to -(-1)^{2/3}\sqrt[3]{2}\sqrt[6]{1-x^2}$$
$$y(x) \to (-1)^{2/3}\sqrt[3]{2}\sqrt[6]{1-x^2}$$

34.13 problem 1015

Internal problem ID [4240]

Book: Ordinary differential equations and their solutions. By George Moseley Murphy. 1960
Section: Various 34
Problem number: 1015.
ODE order: 1.
ODE degree: 3.

CAS Maple gives this as type `[_quadrature]`

$$y'^3 = bx + a$$

✓ Solution by Maple
Time used: 0.016 (sec). Leaf size: 90

```
dsolve(diff(y(x),x)^3 = b*x+a,y(x), singsol=all)
```

$$y(x) = \frac{(3bx+3a)(bx+a)^{\frac{1}{3}} + 4c_1 b}{4b}$$
$$y(x) = \frac{-3(bx+a)^{\frac{4}{3}}\left(1+i\sqrt{3}\right) + 8c_1 b}{8b}$$
$$y(x) = \frac{3(bx+a)^{\frac{4}{3}}\left(-1+i\sqrt{3}\right) + 8c_1 b}{8b}$$

✓ Solution by Mathematica
Time used: 0.008 (sec). Leaf size: 80

```
DSolve[(y'[x])^3 ==a+b x,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to \frac{3(a+bx)^{4/3}}{4b} + c_1$$
$$y(x) \to -\frac{3\sqrt[3]{-1}(a+bx)^{4/3}}{4b} + c_1$$
$$y(x) \to \frac{3(-1)^{2/3}(a+bx)^{4/3}}{4b} + c_1$$

34.14 problem 1016

Internal problem ID [4241]

Book: Ordinary differential equations and their solutions. By George Moseley Murphy. 1960
Section: Various 34
Problem number: 1016.
ODE order: 1.
ODE degree: 3.

CAS Maple gives this as type `[_quadrature]`

$$y'^3 = a\,x^n$$

✓ Solution by Maple
Time used: 0.015 (sec). Leaf size: 100

```
dsolve(diff(y(x),x)^3 = a*x^n,y(x), singsol=all)
```

$$y(x) = \frac{3x(a\,x^n)^{\frac{1}{3}} + c_1(n+3)}{n+3}$$
$$y(x) = \frac{\left(3i\sqrt{3}\,x - 3x\right)\left(a\,x^n\right)^{\frac{1}{3}} + 2c_1(n+3)}{2n+6}$$
$$y(x) = \frac{\left(-3i\sqrt{3}\,x - 3x\right)\left(a\,x^n\right)^{\frac{1}{3}} + 2c_1(n+3)}{2n+6}$$

✓ Solution by Mathematica
Time used: 0.008 (sec). Leaf size: 95

```
DSolve[(y'[x])^3 ==a x^n,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to \frac{3\sqrt[3]{a}x^{\frac{n}{3}+1}}{n+3} + c_1$$
$$y(x) \to -\frac{3\sqrt[3]{-1}\sqrt[3]{a}x^{\frac{n}{3}+1}}{n+3} + c_1$$
$$y(x) \to \frac{3(-1)^{2/3}\sqrt[3]{a}x^{\frac{n}{3}+1}}{n+3} + c_1$$

34.15 problem 1017

Internal problem ID [4242]

Book: Ordinary differential equations and their solutions. By George Moseley Murphy. 1960
Section: Various 34
Problem number: 1017.
ODE order: 1.
ODE degree: 3.

CAS Maple gives this as type `[[_homogeneous, ‘class C‘], _dAlembert]`

$$\boxed{y'^3 - y = -x}$$

✓ Solution by Maple
Time used: 0.015 (sec). Leaf size: 217

```
dsolve(diff(y(x),x)^3+x-y(x) = 0,y(x), singsol=all)
```

$$x - \frac{3(y(x)-x)^{\frac{2}{3}}}{2} - 3(y(x)-x)^{\frac{1}{3}} - 3\ln\left((y(x)-x)^{\frac{1}{3}} - 1\right) - c_1 = 0$$

$$x + \frac{3(y(x)-x)^{\frac{2}{3}}}{4} - \frac{3i\sqrt{3}\,(y(x)-x)^{\frac{2}{3}}}{4} + \frac{3(y(x)-x)^{\frac{1}{3}}}{2} + \frac{3i\sqrt{3}\,(y(x)-x)^{\frac{1}{3}}}{2} + 6\ln(2) - 3\ln\left(-4 - 2i\sqrt{3}\,(y(x)-x)^{\frac{1}{3}} - 2(y(x)-x)^{\frac{1}{3}}\right) - c_1 = 0$$

$$x + \frac{3(y(x)-x)^{\frac{2}{3}}}{4} + \frac{3i\sqrt{3}\,(y(x)-x)^{\frac{2}{3}}}{4} + \frac{3(y(x)-x)^{\frac{1}{3}}}{2} - \frac{3i\sqrt{3}\,(y(x)-x)^{\frac{1}{3}}}{2} + 6\ln(2) - 3\ln\left(2i\sqrt{3}\,(y(x)-x)^{\frac{1}{3}} - 2(y(x)-x)^{\frac{1}{3}} - 4\right) - c_1 = 0$$

✓ Solution by Mathematica

Time used: 10.999 (sec). Leaf size: 298

```
DSolve[(y'[x])^3 +x-y[x]==0 x,y[x],x,IncludeSingularSolutions -> True]
```

$$\text{Solve}\left[\frac{3}{2}(y(x)-x)^{2/3}+3\sqrt[3]{y(x)-x}+3\log\left(\sqrt[3]{y(x)-x}-1\right)-x=c_1,y(x)\right]$$

$$\text{Solve}\left[\frac{1}{2}\left(\frac{1}{2}\sqrt[3]{y(x)-x}\left(4i(y(x)-x)^{2/3}+3\sqrt{3}\sqrt[3]{y(x)-x}-3i\sqrt[3]{y(x)-x}-6\sqrt{3}-6i\right)+6i\log\left(\sqrt{2-2i\sqrt{3}}\right.\right.\right.$$

$$\left.-i(y(x)-x)=c_1,y(x)\right]$$

$$\text{Solve}\left[\frac{y(x)}{2}\right.$$

$$+\frac{1}{4}\left(-\frac{1}{2}\sqrt[3]{y(x)-x}\left(4(y(x)-x)^{2/3}+3i\sqrt{3}\sqrt[3]{y(x)-x}-3\sqrt[3]{y(x)-x}-6i\sqrt{3}-6\right)-6\log\left(2i\sqrt[3]{y(x)-x}+\sqrt{}\right.\right.$$

34.16 problem 1018

Internal problem ID [4243]

Book: Ordinary differential equations and their solutions. By George Moseley Murphy. 1960
Section: Various 34
Problem number: 1018.
ODE order: 1.
ODE degree: 3.

CAS Maple gives this as type `[[_1st_order, '_with_symmetry_[F(x),G(x)*y+H(x)]']]`

$$y'^3 - (a + yb + cy^2) f(x) = 0$$

✓ Solution by Maple

Time used: 0.031 (sec). Leaf size: 191

```
dsolve(diff(y(x),x)^3 = (a+b*y(x)+c*y(x)^2)*f(x),y(x), singsol=all)
```

$$\int^{y(x)} \frac{1}{(\_a^2c + b\_a + a)^{\frac{1}{3}}} d\_a - \frac{\int^x \left(\left(a + by(x) + cy(x)^2\right) f(\_a)\right)^{\frac{1}{3}} d\_a}{\left(a + by(x) + cy(x)^2\right)^{\frac{1}{3}}} + c_1 = 0$$

$$\int^{y(x)} \frac{1}{(\_a^2c + b\_a + a)^{\frac{1}{3}}} d\_a + \frac{(1 + i\sqrt{3}) \left(\int^x \left(\left(a + by(x) + cy(x)^2\right) f(\_a)\right)^{\frac{1}{3}} d\_a\right)}{2 \left(a + by(x) + cy(x)^2\right)^{\frac{1}{3}}} + c_1 = 0$$

$$\int^{y(x)} \frac{1}{(\_a^2c + b\_a + a)^{\frac{1}{3}}} d\_a - \frac{(-1 + i\sqrt{3}) \left(\int^x \left(\left(a + by(x) + cy(x)^2\right) f(\_a)\right)^{\frac{1}{3}} d\_a\right)}{2 \left(a + by(x) + cy(x)^2\right)^{\frac{1}{3}}} + c_1 = 0$$

✓ Solution by Mathematica

Time used: 21.19 (sec). Leaf size: 405

```
DSolve[(y'[x])^3 ==(a+b y[x]+c y[x]^2) f[x],y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to \text{InverseFunction}\left[\frac{(2\#1c+b)\sqrt[3]{\frac{c(\#1(\#1c+b)+a)}{4ac-b^2}}\,\text{Hypergeometric2F1}\left(\frac{1}{3},\frac{1}{2},\frac{3}{2},\frac{(b+2c\#1)^2}{b^2-4ac}\right)}{\sqrt[3]{2}c\sqrt[3]{\#1(\#1c+b)+a}}\&\right]\left[\int_1^x \sqrt[3]{}\right.$$
$$\left.+c_1\right]$$

$$y(x) \to \text{InverseFunction}\left[\frac{(2\#1c+b)\sqrt[3]{\frac{c(\#1(\#1c+b)+a)}{4ac-b^2}}\,\text{Hypergeometric2F1}\left(\frac{1}{3},\frac{1}{2},\frac{3}{2},\frac{(b+2c\#1)^2}{b^2-4ac}\right)}{\sqrt[3]{2}c\sqrt[3]{\#1(\#1c+b)+a}}\&\right]\left[\int_1^x\right.$$
$$\left.-\sqrt[3]{-1}\sqrt[3]{f(K[2])}dK[2]+c_1\right]$$

$$y(x) \to \text{InverseFunction}\left[\frac{(2\#1c+b)\sqrt[3]{\frac{c(\#1(\#1c+b)+a)}{4ac-b^2}}\,\text{Hypergeometric2F1}\left(\frac{1}{3},\frac{1}{2},\frac{3}{2},\frac{(b+2c\#1)^2}{b^2-4ac}\right)}{\sqrt[3]{2}c\sqrt[3]{\#1(\#1c+b)+a}}\&\right]\left[\int_1^x(-\right.$$
$$\left.+c_1\right]$$

$$y(x) \to -\frac{\sqrt{b^2-4ac}+b}{2c}$$
$$y(x) \to \frac{\sqrt{b^2-4ac}-b}{2c}$$

34.17 problem 1019

Internal problem ID [4244]

Book: Ordinary differential equations and their solutions. By George Moseley Murphy. 1960
Section: Various 34
Problem number: 1019.
ODE order: 1.
ODE degree: 3.

CAS Maple gives this as type `[_quadrature]`

$$y'^3 - (y-a)^2 (y-b)^2 = 0$$

✓ Solution by Maple

Time used: 0.031 (sec). Leaf size: 146

```
dsolve(diff(y(x),x)^3 = (y(x)-a)^2*(y(x)-b)^2,y(x), singsol=all)
```

$$y(x) = a$$
$$y(x) = b$$
$$x - \left(\int^{y(x)} \frac{1}{\left((\_a-a)^2 (\_a-b)^2\right)^{\frac{1}{3}}} d\_a\right) - c_1 = 0$$
$$\frac{2\left(\int^{y(x)} \frac{1}{\left((\_a-a)^2(\_a-b)^2\right)^{\frac{1}{3}}} d\_a\right) + i(x-c_1)\sqrt{3} + x - c_1}{1+i\sqrt{3}} = 0$$
$$\frac{-2\left(\int^{y(x)} \frac{1}{\left((\_a-a)^2(\_a-b)^2\right)^{\frac{1}{3}}} d\_a\right) + i(x-c_1)\sqrt{3} - x + c_1}{-1+i\sqrt{3}} = 0$$

✓ Solution by Mathematica

Time used: 1.124 (sec). Leaf size: 246

```
DSolve[(y'[x])^3 ==(y[x]-a)^2 (y[x]-b)^2,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to \text{InverseFunction}\left[-\frac{3\sqrt[3]{a-\#1}\left(\frac{\#1-b}{a-b}\right)^{2/3}\text{Hypergeometric2F1}\left(\frac{1}{3},\frac{2}{3},\frac{4}{3},\frac{a-\#1}{a-b}\right)}{(b-\#1)^{2/3}}\&\right][x+c_1]$$

$$y(x) \to \text{InverseFunction}\left[-\frac{3\sqrt[3]{a-\#1}\left(\frac{\#1-b}{a-b}\right)^{2/3}\text{Hypergeometric2F1}\left(\frac{1}{3},\frac{2}{3},\frac{4}{3},\frac{a-\#1}{a-b}\right)}{(b-\#1)^{2/3}}\&\right]\left[-\sqrt[3]{-1}x+c_1\right]$$

$$y(x) \to \text{InverseFunction}\left[-\frac{3\sqrt[3]{a-\#1}\left(\frac{\#1-b}{a-b}\right)^{2/3}\text{Hypergeometric2F1}\left(\frac{1}{3},\frac{2}{3},\frac{4}{3},\frac{a-\#1}{a-b}\right)}{(b-\#1)^{2/3}}\&\right]\left[(-1)^{2/3}x+c_1\right]$$

$$y(x) \to a$$

$$y(x) \to b$$

34.18 problem 1020

Internal problem ID [4245]

Book: Ordinary differential equations and their solutions. By George Moseley Murphy. 1960
Section: Various 34
Problem number: 1020.
ODE order: 1.
ODE degree: 3.

CAS Maple gives this as type `[[_1st_order, `_with_symmetry_[F(x),G(x)*y+H(x)]`]]`

$$\boxed{y'^3 + f(x)(y-a)^2(y-b)^2 = 0}$$

✓ Solution by Maple

Time used: 0.047 (sec). Leaf size: 212

```
dsolve(diff(y(x),x)^3+f(x)*(y(x)-a)^2*(y(x)-b)^2 = 0,y(x), singsol=all)
```

$$\int^{y(x)} \frac{1}{((\_a-b)(\_a-a))^{\frac{2}{3}}} d\_a - \frac{\int^x \left(-f(\_a)(y(x)-a)^2 (y(x)-b)^2\right)^{\frac{1}{3}} d\_a}{((y(x)-b)(y(x)-a))^{\frac{2}{3}}} + c_1 = 0$$

$$\int^{y(x)} \frac{1}{((\_a-b)(\_a-a))^{\frac{2}{3}}} d\_a + \frac{(1+i\sqrt{3})\left(\int^x \left(-f(\_a)(y(x)-a)^2 (y(x)-b)^2\right)^{\frac{1}{3}} d\_a\right)}{2((y(x)-b)(y(x)-a))^{\frac{2}{3}}} + c_1 = 0$$

$$\int^{y(x)} \frac{1}{((\_a-b)(\_a-a))^{\frac{2}{3}}} d\_a - \frac{(-1+i\sqrt{3})\left(\int^x \left(-f(\_a)(y(x)-a)^2 (y(x)-b)^2\right)^{\frac{1}{3}} d\_a\right)}{2((y(x)-b)(y(x)-a))^{\frac{2}{3}}} + c_1 = 0$$

✓ Solution by Mathematica

Time used: 1.083 (sec). Leaf size: 287

```
DSolve[(y'[x])^3 +f[x] (y[x]-a)^2 (y[x]-b)^2==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to \text{InverseFunction}\left[-\frac{3\sqrt[3]{a-\#1}\left(\frac{\#1-b}{a-b}\right)^{2/3}\text{Hypergeometric2F1}\left(\frac{1}{3},\frac{2}{3},\frac{4}{3},\frac{a-\#1}{a-b}\right)}{(b-\#1)^{2/3}}\&\right]\left[\int_1^x -\sqrt[3]{f(K[1])}dK\right.$$

$$y(x) \to \text{InverseFunction}\left[-\frac{3\sqrt[3]{a-\#1}\left(\frac{\#1-b}{a-b}\right)^{2/3}\text{Hypergeometric2F1}\left(\frac{1}{3},\frac{2}{3},\frac{4}{3},\frac{a-\#1}{a-b}\right)}{(b-\#1)^{2/3}}\&\right]\left[\int_1^x \sqrt[3]{-1}\sqrt[3]{f(K[2]}\right.$$

$$y(x) \to \text{InverseFunction}\left[-\frac{3\sqrt[3]{a-\#1}\left(\frac{\#1-b}{a-b}\right)^{2/3}\text{Hypergeometric2F1}\left(\frac{1}{3},\frac{2}{3},\frac{4}{3},\frac{a-\#1}{a-b}\right)}{(b-\#1)^{2/3}}\&\right]\left[\int_1^x -(-1)^{2/3}\sqrt[3]{f(\right.$$

$y(x) \to a$

$y(x) \to b$

34.19 problem 1021

Internal problem ID [4246]

Book: Ordinary differential equations and their solutions. By George Moseley Murphy. 1960
Section: Various 34
Problem number: 1021.
ODE order: 1.
ODE degree: 3.

CAS Maple gives this as type `[[_1st_order, '_with_symmetry_[F(x),G(x)*y+H(x)]']]`

$$y'^3 + f(x)(y-a)^2(y-b)^2(y-c)^2 = 0$$

✓ Solution by Maple

Time used: 0.344 (sec). Leaf size: 269

```
dsolve(diff(y(x),x)^3+f(x)*(y(x)-a)^2*(y(x)-b)^2*(y(x)-c)^2 = 0,y(x), singsol=all)
```

$$\int^{y(x)} \frac{1}{((\_a-c)(\_a-b)(\_a-a))^{\frac{2}{3}}} d\_a - \frac{\int^x \left(-f(\_a)(y(x)-c)^2(y(x)-b)^2(y(x)-a)^2\right)^{\frac{1}{3}} d\_a}{((y(x)-c)(y(x)-b)(y(x)-a))^{\frac{2}{3}}} + c_1 = 0$$

$$\int^{y(x)} \frac{1}{((\_a-c)(\_a-b)(\_a-a))^{\frac{2}{3}}} d\_a + \frac{(1+i\sqrt{3})\left(\int^x \left(-f(\_a)(y(x)-c)^2(y(x)-b)^2(y(x)-a)^2\right)^{\frac{1}{3}} d\_a\right)}{2((y(x)-c)(y(x)-b)(y(x)-a))^{\frac{2}{3}}} + c_1 = 0$$

$$\int^{y(x)} \frac{1}{((\_a-c)(\_a-b)(\_a-a))^{\frac{2}{3}}} d\_a - \frac{(-1+i\sqrt{3})\left(\int^x \left(-f(\_a)(y(x)-c)^2(y(x)-b)^2(y(x)-a)^2\right)^{\frac{1}{3}} d\_a\right)}{2((y(x)-c)(y(x)-b)(y(x)-a))^{\frac{2}{3}}} + c_1 = 0$$

✓ Solution by Mathematica

Time used: 15.995 (sec). Leaf size: 421

```
DSolve[(y'[x])^3 +f[x](y[x]-a)^2 (y[x]-b)^2 (y[x]-c)^2==0,y[x],x,IncludeSingularSolutions ->
```

$$y(x) \to \text{InverseFunction}\left[\frac{3\sqrt[3]{a-\#1}\sqrt[3]{c-\#1}\left(\frac{(b-\#1)(a-c)}{(c-\#1)(a-b)}\right)^{2/3}\text{Hypergeometric2F1}\left(\frac{1}{3},\frac{2}{3},\frac{4}{3},\frac{(c-b)(a-\#1)}{(a-b)(c-\#1)}\right)}{(b-\#1)^{2/3}(a-c)}\&\right]$$

$$y(x) \to \text{InverseFunction}\left[\frac{3\sqrt[3]{a-\#1}\sqrt[3]{c-\#1}\left(\frac{(b-\#1)(a-c)}{(c-\#1)(a-b)}\right)^{2/3}\text{Hypergeometric2F1}\left(\frac{1}{3},\frac{2}{3},\frac{4}{3},\frac{(c-b)(a-\#1)}{(a-b)(c-\#1)}\right)}{(b-\#1)^{2/3}(a-c)}\&\right]$$

$$y(x) \to \text{InverseFunction}\left[\frac{3\sqrt[3]{a-\#1}\sqrt[3]{c-\#1}\left(\frac{(b-\#1)(a-c)}{(c-\#1)(a-b)}\right)^{2/3}\text{Hypergeometric2F1}\left(\frac{1}{3},\frac{2}{3},\frac{4}{3},\frac{(c-b)(a-\#1)}{(a-b)(c-\#1)}\right)}{(b-\#1)^{2/3}(a-c)}\&\right]$$

$$y(x) \to a$$
$$y(x) \to b$$
$$y(x) \to c$$

34.20 problem 1022

Internal problem ID [4247]

Book: Ordinary differential equations and their solutions. By George Moseley Murphy. 1960
Section: Various 34
Problem number: 1022.
ODE order: 1.
ODE degree: 3.

CAS Maple gives this as type `[_quadrature]`

$$y'^3 + y' = bx - a$$

✓ Solution by Maple

Time used: 0.016 (sec). Leaf size: 302

```
dsolve(diff(y(x),x)^3+diff(y(x),x)+a-b*x = 0,y(x), singsol=all)
```

$$y(x) = -\frac{\left(\int \frac{i\left(108bx-108a+12\sqrt{81b^2x^2-162abx+81a^2+12}\right)^{\frac{2}{3}}\sqrt{3}+\left(108bx-108a+12\sqrt{81b^2x^2-162abx+81a^2+12}\right)^{\frac{2}{3}}+12i\sqrt{3}-12}{\left(108bx-108a+12\sqrt{81b^2x^2-162abx+81a^2+12}\right)^{\frac{1}{3}}}dx\right)}{12} + c_1$$

$$y(x) = \frac{\left(\int \frac{\left(-1+i\sqrt{3}\right)\left(108bx-108a+12\sqrt{81b^2x^2-162abx+81a^2+12}\right)^{\frac{2}{3}}+12i\sqrt{3}+12}{\left(108bx-108a+12\sqrt{81b^2x^2-162abx+81a^2+12}\right)^{\frac{1}{3}}}dx\right)}{12} + c_1$$

$$y(x) = \frac{\left(\int \frac{\left(108bx-108a+12\sqrt{81b^2x^2-162abx+81a^2+12}\right)^{\frac{2}{3}}-12}{\left(108bx-108a+12\sqrt{81b^2x^2-162abx+81a^2+12}\right)^{\frac{1}{3}}}dx\right)}{6} + c_1$$

✓ Solution by Mathematica

Time used: 2.035 (sec). Leaf size: 811

```
DSolve[(y'[x])^3 +y'[x]+a-b x==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to \frac{1}{144}\left(\frac{2^{2/3}\sqrt[3]{3}\left(\sqrt{3}\sqrt{27a^2-54abx+27b^2x^2+4}-9a+9bx\right)^{4/3}}{b} - \frac{4\sqrt[3]{2}3^{2/3}\left(\sqrt{3}\sqrt{27a^2-54abx+27b^2x^2+4}-9a+9bx\right)^{2/3}}{b} - \frac{24\ 2^{2/3}\sqrt[3]{3}}{b\left(\sqrt{3}\sqrt{27a^2-54abx+27b^2x^2+4}-9a+9bx\right)^{2/3}} + \frac{24\sqrt[3]{2}3^{2/3}}{b\left(\sqrt{3}\sqrt{27a^2-54abx+27b^2x^2+4}-9a+9bx\right)^{4/3}} + 144c_1\right)$$

$$y(x) \to \frac{1}{288}\left(\frac{i2^{2/3}\sqrt[3]{3}(\sqrt{3}+i)\left(\sqrt{3}\sqrt{27a^2-54abx+27b^2x^2+4}-9a+9bx\right)^{4/3}}{b} + \frac{4\sqrt[3]{2}3^{2/3}(1+i\sqrt{3})\left(\sqrt{3}\sqrt{27a^2-54abx+27b^2x^2+4}-9a+9bx\right)^{2/3}}{b} + \frac{24\ 2^{2/3}\sqrt[3]{3}(1-i\sqrt{3})}{b\left(\sqrt{3}\sqrt{27a^2-54abx+27b^2x^2+4}-9a+9bx\right)^{2/3}} - \frac{24\sqrt[3]{2}3^{2/3}(1+i\sqrt{3})}{b\left(\sqrt{3}\sqrt{27a^2-54abx+27b^2x^2+4}-9a+9bx\right)^{4/3}} + 288c_1\right)$$

$$y(x) \to \frac{1}{288}\left(-\frac{2^{2/3}\sqrt[3]{3}(1+i\sqrt{3})\left(\sqrt{3}\sqrt{27a^2-54abx+27b^2x^2+4}-9a+9bx\right)^{4/3}}{b} + \frac{4\sqrt[3]{2}3^{2/3}(1-i\sqrt{3})\left(\sqrt{3}\sqrt{27a^2-54abx+27b^2x^2+4}-9a+9bx\right)^{2/3}}{b} + \frac{24\ 2^{2/3}\sqrt[3]{3}(1+i\sqrt{3})}{b\left(\sqrt{3}\sqrt{27a^2-54abx+27b^2x^2+4}-9a+9bx\right)^{2/3}} + \frac{24i\sqrt[3]{2}3^{2/3}(\sqrt{3}+i)}{b\left(\sqrt{3}\sqrt{27a^2-54abx+27b^2x^2+4}-9a+9bx\right)^{4/3}} + 288c_1\right)$$

34.21 problem 1023

Internal problem ID [4248]

Book: Ordinary differential equations and their solutions. By George Moseley Murphy. 1960
Section: Various 34
Problem number: 1023.
ODE order: 1.
ODE degree: 3.

CAS Maple gives this as type `[_quadrature]`

$$\boxed{y'^3 + y' - y = 0}$$

✓ Solution by Maple

Time used: 0.015 (sec). Leaf size: 221

```
dsolve(diff(y(x),x)^3+diff(y(x),x)-y(x) = 0,y(x), singsol=all)
```

$$x - 6\left(\int^{y(x)} \frac{\left(108\_a + 12\sqrt{81\_a^2 + 12}\right)^{\frac{1}{3}}}{\left(108\_a + 12\sqrt{81\_a^2 + 12}\right)^{\frac{2}{3}} - 12} d\_a\right) - c_1 = 0$$

$$\frac{-12\left(\int^{y(x)} \frac{\left(108\_a+12\sqrt{81\_a^2+12}\right)^{\frac{1}{3}}}{-6-6i\sqrt{3}-\left(108\_a+12\sqrt{81\_a^2+12}\right)^{\frac{2}{3}}} d\_a\right) + i(x - c_1)\sqrt{3} + x - c_1}{1 + i\sqrt{3}} = 0$$

$$\frac{12\left(\int^{y(x)} \frac{\left(108\_a+12\sqrt{81\_a^2+12}\right)^{\frac{1}{3}}}{-\left(108\_a+12\sqrt{81\_a^2+12}\right)^{\frac{2}{3}}+\left(\sqrt{3}+3i\right)^2} d\_a\right) + i(x - c_1)\sqrt{3} + c_1 - x}{-1 + i\sqrt{3}} = 0$$

✓ Solution by Mathematica

Time used: 0.35 (sec). Leaf size: 335

```
DSolve[(y'[x])^3 +y'[x]-y[x]==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to \text{InverseFunction}\left[\int \frac{\sqrt[3]{\sqrt{729\#1^2+108}-27\#1}}{2^{2/3}\left(\sqrt{729\#1^2+108}-27\#1\right)^{2/3}-6\sqrt[3]{2}}d\#1\&\right]\left[-\frac{x}{6}+c_1\right]$$

$$y(x) \to \text{InverseFunction}\left[\int \frac{\sqrt[3]{\sqrt{729\#1^2+108}-27\#1}}{-i2^{2/3}\sqrt{3}\left(\sqrt{729\#1^2+108}-27\#1\right)^{2/3}+2^{2/3}\left(\sqrt{729\#1^2+108}-27\#1\right)^{2/3}-6}\right.$$

$$y(x) \to \text{InverseFunction}\left[\int \frac{\sqrt[3]{\sqrt{729\#1^2+108}-27\#1}}{i2^{2/3}\sqrt{3}\left(\sqrt{729\#1^2+108}-27\#1\right)^{2/3}+2^{2/3}\left(\sqrt{729\#1^2+108}-27\#1\right)^{2/3}+6i}\right.$$

$$y(x) \to 0$$

34.22 problem 1024

Internal problem ID [4249]

Book: Ordinary differential equations and their solutions. By George Moseley Murphy. 1960
Section: Various 34
Problem number: 1024.
ODE order: 1.
ODE degree: 3.

CAS Maple gives this as type `[_quadrature]`

$$\boxed{y'^3 + y' - e^y = 0}$$

✓ Solution by Maple

Time used: 0.015 (sec). Leaf size: 233

```
dsolve(diff(y(x),x)^3+diff(y(x),x) = exp(y(x)),y(x), singsol=all)
```

$$x - 6\left(\int^{y(x)} \frac{\left(108\,e^{-\_a} + 12\sqrt{12 + 81\,e^{2\_a}}\right)^{\frac{1}{3}}}{\left(108\,e^{-\_a} + 12\sqrt{12 + 81\,e^{2\_a}}\right)^{\frac{2}{3}} - 12} d\_a\right) - c_1 = 0$$

$$\frac{-12\left(\int^{y(x)} \frac{\left(108\,e^{-\_a}+12\sqrt{12+81\,e^{2\_a}}\right)^{\frac{1}{3}}}{-\left(108\,e^{-\_a}+12\sqrt{12+81\,e^{2\_a}}\right)^{\frac{2}{3}}-6-6i\sqrt{3}} d\_a\right) + i(x - c_1)\sqrt{3} + x - c_1}{1 + i\sqrt{3}} = 0$$

$$\frac{12\left(\int^{y(x)} \frac{\left(108\,e^{-\_a}+12\sqrt{12+81\,e^{2\_a}}\right)^{\frac{1}{3}}}{-\left(108\,e^{-\_a}+12\sqrt{12+81\,e^{2\_a}}\right)^{\frac{2}{3}}+\left(\sqrt{3}+3i\right)^{2}} d\_a\right) + i(x - c_1)\sqrt{3} + c_1 - x}{-1 + i\sqrt{3}} = 0$$

✓ Solution by Mathematica

Time used: 168.19 (sec). Leaf size: 1244

```
DSolve[(y'[x])^3 +y'[x]==Exp[ y[x]],y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \rightarrow \text{InverseFunction}\left[\frac{1}{36}\left(\frac{e^{-\#1}\left(2^{2/3}\sqrt[3]{\sqrt{81e^{2\#1}+12}-9e^{\#1}}\sqrt{81e^{2\#1}+12}-9\ 2^{2/3}e^{\#1}\sqrt[3]{\sqrt{81e^{2\#1}+12}-\right.}{\left(\sqrt{81e^{2\#1}+12}-9e^{\#1}\right)^{2/3}}\right.\right.$$

$$\left.\left.-12\sqrt[3]{6}\arctan\left(\frac{6^{2/3}\sqrt[3]{\sqrt{81e^{2\#1}+12}-9e^{\#1}}}{\sqrt[3]{2}\left(\sqrt{81e^{2\#1}+12}-9e^{\#1}\right)^{2/3}-2\sqrt[3]{3}}\right)\right)+\frac{e^{-\#1}}{3\ 6^{2/3}}\&\right]\left[-\frac{x}{6^{2/3}}+c_1\right]$$

$$y(x) \rightarrow \text{InverseFunction}\left[-\frac{e^{-\#1}}{6\ 2^{2/3}3^{5/6}}\right.$$

$$-\frac{1}{144}i\left(\frac{e^{-\#1}\left(-12i\sqrt[3]{2}\sqrt[6]{3}e^{\#1}\left(\sqrt{81e^{2\#1}+12}-9e^{\#1}\right)^{2/3}\arctan\left(\frac{6^{2/3}\sqrt[3]{\sqrt{81e^{2\#1}+12}-9e^{\#1}}}{\sqrt[3]{2}\left(\sqrt{81e^{2\#1}+12}-9e^{\#1}\right)^{2/3}-2\sqrt[3]{3}}\right)-3\right.}{}\right.$$

$$y(x) \rightarrow \text{InverseFunction}\left[-\frac{e^{-\#1}}{6\ 2^{2/3}3^{5/6}}\right.$$

$$+\frac{1}{144}i\left(\frac{e^{-\#1}\left(12i\sqrt[3]{2}\sqrt[6]{3}e^{\#1}\left(\sqrt{81e^{2\#1}+12}-9e^{\#1}\right)^{2/3}\arctan\left(\frac{6^{2/3}\sqrt[3]{\sqrt{81e^{2\#1}+12}-9e^{\#1}}}{\sqrt[3]{2}\left(\sqrt{81e^{2\#1}+12}-9e^{\#1}\right)^{2/3}-2\sqrt[3]{3}}\right)+3i2\right.}{}\right.$$

34.23 problem 1025

Internal problem ID [4250]

Book: Ordinary differential equations and their solutions. By George Moseley Murphy. 1960
Section: Various 34
Problem number: 1025.
ODE order: 1.
ODE degree: 3.

CAS Maple gives this as type `[_quadrature]`

$$\boxed{y'^3 - 7y' = -6}$$

✓ Solution by Maple
Time used: 0.0 (sec). Leaf size: 23

```
dsolve(diff(y(x),x)^3-7*diff(y(x),x)+6 = 0,y(x), singsol=all)
```

$$y(x) = 2x + c_1$$
$$y(x) = x + c_1$$
$$y(x) = -3x + c_1$$

✓ Solution by Mathematica
Time used: 0.002 (sec). Leaf size: 29

```
DSolve[(y'[x])^3-7 y'[x]+6==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to -3x + c_1$$
$$y(x) \to x + c_1$$
$$y(x) \to 2x + c_1$$

34.24 problem 1026

Internal problem ID [4251]

Book: Ordinary differential equations and their solutions. By George Moseley Murphy. 1960
Section: Various 34
Problem number: 1026.
ODE order: 1.
ODE degree: 3.

CAS Maple gives this as type `[[_1st_order, _with_linear_symmetries], _dAlembert]`

$$\boxed{y'^3 - xy' + ya = 0}$$

✓ Solution by Maple

Time used: 0.078 (sec). Leaf size: 821

```
dsolve(diff(y(x),x)^3-x*diff(y(x),x)+a*y(x) = 0,y(x), singsol=all)
```

$$-\frac{48\left(6^{-\frac{1}{a-1}}c_1\left(-108y(x)\,a+12\sqrt{81y\left(x\right)^2a^2-12x^3}\right)^{\frac{2}{3}}\left(a-\frac{3}{2}\right)^2\left(\frac{\left(-108y(x)a+12\sqrt{81y(x)^2a^2-12x^3}\right)^{\frac{2}{3}}+12x}{\left(-108y(x)a+12\sqrt{81y(x)^2a^2-12x^3}\right)^{\frac{1}{3}}}\right)^{\frac{1}{a-}}\cdots}{\left(-108y\left(x\right)a+12\cdots\right.}$$

$$=0$$

$$\frac{192c_1\left(-108y(x)\,a+12\sqrt{81y\left(x\right)^2a^2-12x^3}\right)^{\frac{2}{3}}\left(a-\frac{3}{2}\right)^2 12^{-\frac{1}{a-1}}\left(\frac{i\left(-108y(x)a+12\sqrt{81y(x)^2a^2-12x^3}\right)^{\frac{2}{3}}\sqrt{3}-12i\sqrt{3}\,x\cdots}{\left(-108y(x)a+12\sqrt{81y(\cdots}\right.}\right.}{\cdots}$$

$$=0$$

$$-\frac{9\left(\frac{64c_1\left(-108y(x)a+12\sqrt{81y(x)^2a^2-12x^3}\right)^{\frac{2}{3}}\left(a-\frac{3}{2}\right)^2\left(-\frac{i\left(-108y(x)a+12\sqrt{81y(x)^2a^2-12x^3}\right)^{\frac{2}{3}}\sqrt{3}-12i\sqrt{3}\,x+\left(-108y(x)a+12\sqrt{81y(x)^2a^2-12x^3}\right)\cdots}{12\left(-108y(x)a+12\sqrt{81y(x)^2a^2-12x^3}\right)^{\frac{1}{3}}}\right.}{3}\right.}{\left(-\cdots\right.}$$

$$=0$$

✗ Solution by Mathematica

Time used: 0.0 (sec). Leaf size: 0

```
DSolve[(y'[x])^3 -x y'[x]+a y[x]==0,y[x],x,IncludeSingularSolutions -> True]
```

Timed out

34.25 problem 1027

Internal problem ID [4252]

Book: Ordinary differential equations and their solutions. By George Moseley Murphy. 1960
Section: Various 34
Problem number: 1027.
ODE order: 1.
ODE degree: 3.

CAS Maple gives this as type `[[_1st_order, _with_linear_symmetries], _dAlembert]`

$$\boxed{y'^3 + 2xy' - y = 0}$$

✓ Solution by Maple
Time used: 0.015 (sec). Leaf size: 141

```
dsolve(diff(y(x),x)^3+2*x*diff(y(x),x)-y(x) = 0,y(x), singsol=all)
```

$$y(x) = \frac{2\left(-2x + \sqrt{x^2 + 3c_1}\right)\sqrt{-6\sqrt{x^2 + 3c_1} - 6x}}{9}$$
$$y(x) = -\frac{2\left(-2x + \sqrt{x^2 + 3c_1}\right)\sqrt{-6\sqrt{x^2 + 3c_1} - 6x}}{9}$$
$$y(x) = -\frac{2\left(2x + \sqrt{x^2 + 3c_1}\right)\sqrt{6\sqrt{x^2 + 3c_1} - 6x}}{9}$$
$$y(x) = \frac{2\left(2x + \sqrt{x^2 + 3c_1}\right)\sqrt{6\sqrt{x^2 + 3c_1} - 6x}}{9}$$

✗ Solution by Mathematica
Time used: 0.0 (sec). Leaf size: 0

```
DSolve[(y'[x])^3 +2*x*y'[x]-y[x]==0,y[x],x,IncludeSingularSolutions -> True]
```

Timed out

34.26 problem 1028

Internal problem ID [4253]

Book: Ordinary differential equations and their solutions. By George Moseley Murphy. 1960
Section: Various 34
Problem number: 1028.
ODE order: 1.
ODE degree: 3.

CAS Maple gives this as type `[[_1st_order, _with_linear_symmetries], _dAlembert]`

$$\boxed{y'^3 - 2xy' - y = 0}$$

✓ Solution by Maple

Time used: 0.015 (sec). Leaf size: 450

```
dsolve(diff(y(x),x)^3-2*x*diff(y(x),x)-y(x) = 0,y(x), singsol=all)
```

$$-\frac{c_1}{\left(\frac{\left(108y(x)+12\sqrt{-96x^3+81y(x)^2}\right)^{\frac{2}{3}}+24x}{\left(108y(x)+12\sqrt{-96x^3+81y(x)^2}\right)^{\frac{1}{3}}}\right)^{\frac{2}{3}}}+x$$
$$-\frac{\left(\left(108y(x)+12\sqrt{-96x^3+81y(x)^2}\right)^{\frac{2}{3}}+24x\right)^2}{96\left(108y(x)+12\sqrt{-96x^3+81y(x)^2}\right)^{\frac{2}{3}}}=0$$

$$-\frac{c_1}{\left(\frac{i\sqrt{3}\left(108y(x)+12\sqrt{-96x^3+81y(x)^2}\right)^{\frac{2}{3}}-24i\sqrt{3}\,x-\left(108y(x)+12\sqrt{-96x^3+81y(x)^2}\right)^{\frac{2}{3}}-24x}{\left(108y(x)+12\sqrt{-96x^3+81y(x)^2}\right)^{\frac{1}{3}}}\right)^{\frac{2}{3}}}$$
$$+x+\frac{3\left(-\frac{\left(\sqrt{3}+i\right)\left(108y(x)+12\sqrt{-96x^3+81y(x)^2}\right)^{\frac{2}{3}}}{24}+x\left(-i+\sqrt{3}\right)\right)^2}{2\left(108y(x)+12\sqrt{-96x^3+81y(x)^2}\right)^{\frac{2}{3}}}=0$$

$$-\frac{12^{\frac{2}{3}}c_1}{\left(\frac{-i\sqrt{3}\left(108y(x)+12\sqrt{-96x^3+81y(x)^2}\right)^{\frac{2}{3}}+24i\sqrt{3}\,x-\left(108y(x)+12\sqrt{-96x^3+81y(x)^2}\right)^{\frac{2}{3}}-24x}{\left(108y(x)+12\sqrt{-96x^3+81y(x)^2}\right)^{\frac{1}{3}}}\right)^{\frac{2}{3}}}$$
$$+x+\frac{3\left(\frac{\left(i-\sqrt{3}\right)\left(108y(x)+12\sqrt{-96x^3+81y(x)^2}\right)^{\frac{2}{3}}}{24}+\left(\sqrt{3}+i\right)x\right)^2}{2\left(108y(x)+12\sqrt{-96x^3+81y(x)^2}\right)^{\frac{2}{3}}}=0$$

✗ Solution by Mathematica

Time used: 0.0 (sec). Leaf size: 0

```
DSolve[(y'[x])^3 -2*x*y'[x]-y[x]==0,y[x],x,IncludeSingularSolutions -> True]
```

Timed out

34.27 problem 1029

Internal problem ID [4254]

Book: Ordinary differential equations and their solutions. By George Moseley Murphy. 1960
Section: Various 34
Problem number: 1029.
ODE order: 1.
ODE degree: 3.

CAS Maple gives this as type `[_quadrature]`

$$\boxed{y'^3 - axy' = -x^3}$$

✓ Solution by Maple

Time used: 0.015 (sec). Leaf size: 252

```
dsolve(diff(y(x),x)^3-a*x*diff(y(x),x)+x^3 = 0,y(x), singsol=all)
```

$$y(x) = \frac{\left(\int \left(\left(-108x^3 + 12\sqrt{3}\,\sqrt{-4a^3x^3 + 27x^6}\right)^{\frac{1}{3}} \left(-1 + i\sqrt{3}\right) - \frac{12a\left(1+i\sqrt{3}\right)x}{\left(-108x^3+12\sqrt{3}\,\sqrt{-4a^3x^3+27x^6}\right)^{\frac{1}{3}}}\right) dx\right)}{12} + c_1$$

$$y(x) = -\frac{\left(\int \left(\left(1 + i\sqrt{3}\right) \left(-108x^3 + 12\sqrt{3}\,\sqrt{-4a^3x^3 + 27x^6}\right)^{\frac{1}{3}} - \frac{12a\left(-1+i\sqrt{3}\right)x}{\left(-108x^3+12\sqrt{3}\,\sqrt{-4a^3x^3+27x^6}\right)^{\frac{1}{3}}}\right) dx\right)}{12} + c_1$$

$$y(x) = \frac{\left(\int \frac{\left(-108x^3+12\sqrt{3}\,\sqrt{-4a^3x^3+27x^6}\right)^{\frac{2}{3}} + 12ax}{\left(-108x^3+12\sqrt{3}\,\sqrt{-4a^3x^3+27x^6}\right)^{\frac{1}{3}}} dx\right)}{6} + c_1$$

✓ Solution by Mathematica

Time used: 166.72 (sec). Leaf size: 349

```
DSolve[(y'[x])^3 -a*x*y'[x]+x^3==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to \int_1^x \frac{2\sqrt[3]{3}aK[1] + \sqrt[3]{2}\left(\sqrt{81K[1]^6 - 12a^3K[1]^3} - 9K[1]^3\right)^{2/3}}{6^{2/3}\sqrt[3]{\sqrt{81K[1]^6 - 12a^3K[1]^3} - 9K[1]^3}}dK[1] + c_1$$

$$y(x) \to \int_1^x \frac{i\sqrt[3]{3}(i+\sqrt{3})\left(2\sqrt{81K[2]^6 - 12a^3K[2]^3} - 18K[2]^3\right)^{2/3} - 2\sqrt[3]{2}\sqrt[6]{3}(3i+\sqrt{3})\,aK[2]}{12\sqrt[3]{\sqrt{81K[2]^6 - 12a^3K[2]^3} - 9K[2]^3}}dK[2] + c_1$$

$$y(x) \to \int_1^x \frac{\sqrt[3]{3}(-1-i\sqrt{3})\left(2\sqrt{81K[3]^6 - 12a^3K[3]^3} - 18K[3]^3\right)^{2/3} - 2\sqrt[3]{2}\sqrt[6]{3}(-3i+\sqrt{3})\,aK[3]}{12\sqrt[3]{\sqrt{81K[3]^6 - 12a^3K[3]^3} - 9K[3]^3}}dK[3] + c_1$$

35 Various 35

35.1 problem 1030

Internal problem ID [4255]

Book: Ordinary differential equations and their solutions. By George Moseley Murphy. 1960
Section: Various 35
Problem number: 1030.
ODE order: 1.
ODE degree: 3.

CAS Maple gives this as type `[[_1st_order, _with_linear_symmetries], _Clairaut]`

$$y'^3 + axy' - ya = 0$$

✓ Solution by Maple
Time used: 0.047 (sec). Leaf size: 46

```
dsolve(diff(y(x),x)^3+a*x*diff(y(x),x)-a*y(x) = 0,y(x), singsol=all)
```

$$y(x) = -\frac{2\sqrt{3}\,\sqrt{-ax}\,x}{9}$$
$$y(x) = \frac{2\sqrt{3}\,\sqrt{-ax}\,x}{9}$$
$$y(x) = \frac{c_1(ax + c_1^2)}{a}$$

✓ Solution by Mathematica
Time used: 0.011 (sec). Leaf size: 68

```
DSolve[(y'[x])^3 +a*x*y'[x]-a*y[x]==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to \frac{c_1{}^3}{a} + c_1 x$$
$$y(x) \to -\frac{2i\sqrt{a}x^{3/2}}{3\sqrt{3}}$$
$$y(x) \to \frac{2i\sqrt{a}x^{3/2}}{3\sqrt{3}}$$

35.2 problem 1031

Internal problem ID [4256]

Book: Ordinary differential equations and their solutions. By George Moseley Murphy. 1960
Section: Various 35
Problem number: 1031.
ODE order: 1.
ODE degree: 3.

CAS Maple gives this as type `[[_1st_order, _with_linear_symmetries], _Clairaut]`

$$y'^3 - (bx + a)\, y' + yb = 0$$

✓ Solution by Maple
Time used: 0.047 (sec). Leaf size: 65

```
dsolve(diff(y(x),x)^3-(b*x+a)*diff(y(x),x)+b*y(x) = 0,y(x), singsol=all)
```

$$y(x) = -\frac{2\sqrt{3bx+3a}\,(bx+a)}{9b}$$
$$y(x) = \frac{2\sqrt{3bx+3a}\,(bx+a)}{9b}$$
$$y(x) = \frac{c_1(bx - c_1^2 + a)}{b}$$

✓ Solution by Mathematica
Time used: 0.011 (sec). Leaf size: 72

```
DSolve[(y'[x])^3 -(a+b*x)y'[x]+b*y[x]==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to \frac{c_1(a + bx - c_1{}^2)}{b}$$
$$y(x) \to -\frac{2(a+bx)^{3/2}}{3\sqrt{3}b}$$
$$y(x) \to \frac{2(a+bx)^{3/2}}{3\sqrt{3}b}$$

35.3 problem 1034

Internal problem ID [4257]

Book: Ordinary differential equations and their solutions. By George Moseley Murphy. 1960
Section: Various 35
Problem number: 1034.
ODE order: 1.
ODE degree: 3.

CAS Maple gives this as type [_quadrature]

$$y'^3 - 2yy' + y^2 = 0$$

✓ Solution by Maple

Time used: 0.016 (sec). Leaf size: 295

```
dsolve(diff(y(x),x)^3-2*y(x)*diff(y(x),x)+y(x)^2 = 0,y(x), singsol=all)
```

$$y(x) = 0$$

$$-2^{\frac{2}{3}}\sqrt{3}\left(\int^{y(x)} \frac{\left(-3\sqrt{3}\_a^2 + \sqrt{27\_a^4 - 32\_a^3}\right)^{\frac{1}{3}}}{2^{\frac{1}{3}}\left(-3\sqrt{3}\_a^2 + \sqrt{27\_a^4 - 32\_a^3}\right)^{\frac{2}{3}} + 4\_a} d\_a\right) + x - c_1 = 0$$

$$\frac{2\,2^{\frac{2}{3}}\sqrt{3}\left(\int^{y(x)} \frac{\left(-3\sqrt{3}\_\_a^2+\sqrt{27\_\_a^4-32\_\_a^3}\right)^{\frac{1}{3}}}{2^{\frac{1}{3}}\left(-3\sqrt{3}\_\_a^2+\sqrt{27\_\_a^4-32\_\_a^3}\right)^{\frac{2}{3}}-2i\_\_a\sqrt{3}-2\_\_a} d\_a\right) + (x - c_1)\left(1 + i\sqrt{3}\right)}{1 + i\sqrt{3}} = 0$$

$$\frac{2i2^{\frac{2}{3}}\sqrt{3}\left(\int^{y(x)} \frac{\left(-3\sqrt{3}\_\_a^2+\sqrt{27\_\_a^4-32\_\_a^3}\right)^{\frac{1}{3}}}{2^{\frac{1}{3}}\left(-3\sqrt{3}\_\_a^2+\sqrt{27\_\_a^4-32\_\_a^3}\right)^{\frac{2}{3}}+2i\_\_a\sqrt{3}-2\_\_a} d\_a\right) + (x - c_1)\left(\sqrt{3} + i\right)}{\sqrt{3} + i} = 0$$

✓ Solution by Mathematica

Time used: 0.534 (sec). Leaf size: 427

```
DSolve[(y'[x])^3 -2*y[x]*y'[x]+y[x]^2==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to \text{InverseFunction}\left[\int \frac{\sqrt[3]{\sqrt{3}\sqrt{\#1^3(27\#1-32)}-9\#1^2}}{\sqrt[3]{2}\left(\sqrt{3}\sqrt{\#1^3(27\#1-32)}-9\#1^2\right)^{2/3}+4\sqrt[3]{3}\#1}d\#1\&\right]\left[\frac{x}{6^{2/3}}+c_1\right]$$

$$y(x) \to \text{InverseFunction}\left[\int \frac{\sqrt[3]{\sqrt{3}\sqrt{\#1^3(27\#1-32)}-9\#1^2}}{\sqrt[3]{2}3^{2/3}\left(\sqrt{3}\sqrt{\#1^3(27\#1-32)}-9\#1^2\right)^{2/3}-\sqrt[3]{2}\sqrt[6]{3}i\left(\sqrt{3}\sqrt{\#1^3(27\#1-32)}-9\right.}\right.$$

$$y(x) \to \text{InverseFunction}\left[\int \frac{\sqrt[3]{\sqrt{3}\sqrt{\#1^3(27\#1-32)}-9\#1^2}}{\sqrt[3]{2}3^{2/3}\left(\sqrt{3}\sqrt{\#1^3(27\#1-32)}-9\#1^2\right)^{2/3}+\sqrt[3]{2}\sqrt[6]{3}i\left(\sqrt{3}\sqrt{\#1^3(27\#1-32)}-9\right.}\right.$$

$$y(x) \to 0$$

35.4 problem 1035

Internal problem ID [4258]

Book: Ordinary differential equations and their solutions. By George Moseley Murphy. 1960
Section: Various 35
Problem number: 1035.
ODE order: 1.
ODE degree: 3.

CAS Maple gives this as type `[[_1st_order, _with_linear_symmetries]]`

$$y'^3 - axyy' + 2ay^2 = 0$$

✓ Solution by Maple
Time used: 0.094 (sec). Leaf size: 33

```
dsolve(diff(y(x),x)^3-a*x*y(x)*diff(y(x),x)+2*a*y(x)^2 = 0,y(x), singsol=all)
```

$$y(x) = \frac{a\,x^3}{27}$$
$$y(x) = 0$$
$$y(x) = \frac{(ac_1x-1)^2}{4c_1^3a^2}$$

✓ Solution by Mathematica
Time used: 146.625 (sec). Leaf size: 13176

```
DSolve[(y'[x])^3 -a*x*y[x]*y'[x]+2*a*y[x]^2==0,y[x],x,IncludeSingularSolutions -> True]
```

Too large to display

35.5 problem 1037

Internal problem ID [4259]

Book: Ordinary differential equations and their solutions. By George Moseley Murphy. 1960
Section: Various 35
Problem number: 1037.
ODE order: 1.
ODE degree: 3.

CAS Maple gives this as type `[[_1st_order, _with_linear_symmetries]]`

$$y'^3 - xy^4y' - y^5 = 0$$

✓ Solution by Maple

Time used: 0.11 (sec). Leaf size: 47

```
dsolve(diff(y(x),x)^3-x*y(x)^4*diff(y(x),x)-y(x)^5 = 0,y(x), singsol=all)
```

$$y(x) = -\frac{3\sqrt{3}}{2x^{\frac{3}{2}}}$$
$$y(x) = \frac{3\sqrt{3}}{2x^{\frac{3}{2}}}$$
$$y(x) = 0$$
$$y(x) = c_1\sqrt{\frac{c_1^{10}}{(c_1^4x-1)^2}}$$

✓ Solution by Mathematica

Time used: 0.032 (sec). Leaf size: 64

```
DSolve[(y'[x])^3 -x*y[x]^4*y'[x]- y[x]^5==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to \frac{1}{c_1 x - c_1{}^3}$$
$$y(x) \to 0$$
$$y(x) \to \text{Indeterminate}$$
$$y(x) \to -\frac{3\sqrt{3}}{2x^{3/2}}$$
$$y(x) \to \frac{3\sqrt{3}}{2x^{3/2}}$$

35.6 problem 1038

Internal problem ID [4260]

Book: Ordinary differential equations and their solutions. By George Moseley Murphy. 1960
Section: Various 35
Problem number: 1038.
ODE order: 1.
ODE degree: 3.

CAS Maple gives this as type `[[_homogeneous, ‘class C‘], _dAlembert]`

$$\boxed{y'^3 + e^{3x-2y}(y' - 1) = 0}$$

✓ Solution by Maple

Time used: 7.391 (sec). Leaf size: 944

```
dsolve(diff(y(x),x)^3+exp(3*x-2*y(x))*(diff(y(x),x)-1) = 0,y(x), singsol=all)
```

$$y(x) = \frac{3x}{2} + \operatorname{RootOf}\Bigg(x + 2\,2^{\frac{1}{3}}3^{\frac{2}{3}}\left(\int^{\_Z} \frac{e^{2\_a}\left(\left(\sqrt{3}\,\sqrt{\left(4+27\,e^{2\_a}\right)e^{-6\_a}}\,e^{2\_a}+9\right)e^{-2\_a}\right)^{\frac{1}{3}}}{3\,e^{2\_a}2^{\frac{1}{3}}3^{\frac{2}{3}}\left(\left(\sqrt{3}\,\sqrt{\left(4+27\,e^{2\_a}\right)e^{-6\_a}}\,e^{2\_a}+9\right)e^{-2\_a}\right)^{\frac{1}{3}} - 2\left(\left(\sqrt{3}\,\sqrt{\left(4+27\,e^{2\_a}\right)e^{-6}}\right.\right.} \right. - c_1\Bigg)$$

$$y(x) = \frac{3x}{2} + \operatorname{RootOf}\Bigg(-2\left(\int^{\_Z} \frac{e^{2\_a+3}\left(2\sqrt{}\right.}{3\,e^{2\_a+3}\left(2\sqrt{e^{-4\_a}\left(4\,e^{-2\_a}+27\right)}+6\sqrt{3}\,e^{-2\_a}\right)^{\frac{1}{3}}3^{\frac{5}{6}} + 4\,e^{2\_a+3}3^{\frac{1}{3}}\left(\left(\sqrt{e^{-4\_a}\left(4\,e\right.}\right.\right.}\right. + 6i3^{\frac{1}{3}}\left(\int^{\_Z} \frac{e^{2\_a+3}\left(2\sqrt{e^{-4\_a}\left(\right.}\right.}{3\,e^{2\_a+3}\left(2\sqrt{e^{-4\_a}\left(4\,e^{-2\_a}+27\right)}+6\sqrt{3}\,e^{-2\_a}\right)^{\frac{1}{3}}3^{\frac{5}{6}} + 4\,e^{2\_a+3}3^{\frac{1}{3}}\left(\left(\sqrt{e^{-4\_a}\left(4\,e^{-2\_a}+2\right.}\right.\right.}\right. + c_1 - x\Bigg)$$

$$y(x) = \frac{3x}{2} + \operatorname{RootOf}\Bigg(2\left(\int^{\_Z} \frac{e^{2\_a+3}\left(2\sqrt{}\right.}{-4\,e^{2\_a+3}3^{\frac{1}{3}}\left(\left(\sqrt{e^{-4\_a}\left(4\,e^{-2\_a}+27\right)}+3\sqrt{3}\,e^{-2\_a}\right)^{2}\right)^{\frac{1}{3}} - 9ie^{2\_a+3}3^{\frac{1}{3}}\left(2\sqrt{e^{-4\_a}\left(\right.}\right.}\right. + 6i3^{\frac{1}{3}}\left(\int^{\_Z} \frac{e^{2\_a+3}\left(2\sqrt{e^{-4\_a}\left(\right.}\right.}{-4\,e^{2\_a+3}3^{\frac{1}{3}}\left(\left(\sqrt{e^{-4\_a}\left(4\,e^{-2\_a}+27\right)}+3\sqrt{3}\,e^{-2\_a}\right)^{2}\right)^{\frac{1}{3}} - 9ie^{2\_a+3}3^{\frac{1}{3}}\left(2\sqrt{e^{-4\_a}\left(4\,e^{-2\_a}\right.}\right.}\right. + c_1 - x\Bigg)$$

✗ Solution by Mathematica

Time used: 0.0 (sec). Leaf size: 0

```
DSolve[(y'[x])^3 +Exp[3*x -2*y[x]]*(y'[x]-1)==0,y[x],x,IncludeSingularSolutions -> True]
```

Timed out

35.7 problem 1039

Internal problem ID [4261]

Book: Ordinary differential equations and their solutions. By George Moseley Murphy. 1960
Section: Various 35
Problem number: 1039.
ODE order: 1.
ODE degree: 3.

CAS Maple gives this as type `[‘y=_G(x,y’)‘]`

$$
{y'}^3+\mathrm{e}^{-2y}\left(\mathrm{e}^{2x}+\mathrm{e}^{3x}\right)y'-\mathrm{e}^{3x-2y}=0
$$

✓ Solution by Maple

Time used: 0.266 (sec). Leaf size: 28

```
dsolve(diff(y(x),x)^3+exp(-2*y(x))*(exp(2*x)+exp(3*x))*diff(y(x),x)-exp(3*x-2*y(x)) = 0,y(x)
```

$$
y(x)=x-\frac{\ln\left(-\frac{\mathrm{e}^{2x}}{(c_1+1)(-c_1+\mathrm{e}^{x})^2}\right)}{2}
$$

✗ Solution by Mathematica

Time used: 0.0 (sec). Leaf size: 0

```
DSolve[(y'[x])^3 +Exp[-2*y[x]]*(Exp[2*x]+Exp[3*x])(y'[x])-Exp[3*x-2*y[x]]==0,y[x],x,IncludeS
```

Timed out

35.8 problem 1040

Internal problem ID [4262]

Book: Ordinary differential equations and their solutions. By George Moseley Murphy. 1960
Section: Various 35
Problem number: 1040.
ODE order: 1.
ODE degree: 3.

CAS Maple gives this as type `[_quadrature]`

$$\boxed{y'^3 + y'^2 - y = 0}$$

✓ Solution by Maple

Time used: 0.032 (sec). Leaf size: 433

```
dsolve(diff(y(x),x)^3+diff(y(x),x)^2-y(x) = 0,y(x), singsol=all)
```

$$y(x) = 0$$

$$3\,2^{\frac{1}{3}}\sqrt{3}\left(\int^{y(x)} \frac{\left(9\sqrt{27\_a^2-4\_a}+(27\_a-2)\sqrt{3}\right)^{\frac{1}{3}}}{\sqrt{3}\,2^{\frac{1}{3}}\left(9\sqrt{27\_a^2-4\_a}+(27\_a-2)\sqrt{3}\right)^{\frac{1}{3}}-3^{\frac{1}{3}}\left(9\sqrt{27\_a^2-4\_a}+(27\_a-2)\sqrt{3}\right)^{\frac{2}{3}}-\right.$$
$$+x-c_1=0$$

$$\frac{12\,2^{\frac{1}{3}}\sqrt{3}\left(\int^{y(x)} -\frac{\left(9\sqrt{27\_a^2-4\_a}+(27\_a-2)\sqrt{3}\right)^{\frac{1}{3}}}{\left(2^{\frac{1}{3}}3^{\frac{1}{3}}+3^{\frac{1}{6}}\left(9\sqrt{27\_a^2-4\_a}+(27\_a-2)\sqrt{3}\right)^{\frac{1}{3}}\right)\left(i3^{\frac{5}{6}}2^{\frac{1}{3}}+2^{\frac{1}{3}}3^{\frac{1}{3}}-2\,3^{\frac{1}{6}}\left(9\sqrt{27\_a^2-4\_a}+(27\_a-2)\sqrt{3}\right)^{\frac{1}{3}}\right)}d\_\right.}{1+i\sqrt{3}}$$
$$=0$$

$$\frac{12i2^{\frac{1}{3}}\sqrt{3}\left(\int^{y(x)} \frac{\left(9\sqrt{27\_a^2-4\_a}+(27\_a-2)\sqrt{3}\right)^{\frac{1}{3}}}{\left(2^{\frac{1}{3}}3^{\frac{1}{3}}+3^{\frac{1}{6}}\left(9\sqrt{27\_a^2-4\_a}+(27\_a-2)\sqrt{3}\right)^{\frac{1}{3}}\right)\left(i3^{\frac{5}{6}}2^{\frac{1}{3}}+2\,3^{\frac{1}{6}}\left(9\sqrt{27\_a^2-4\_a}+(27\_a-2)\sqrt{3}\right)^{\frac{1}{3}}-2^{\frac{1}{3}}3^{\frac{1}{3}}\right)}d\_\right.}{\sqrt{3}+i}$$
$$=0$$

✓ Solution by Mathematica

Time used: 105.918 (sec). Leaf size: 515

```
DSolve[(y'[x])^3 + (y'[x])^2 -y[x]==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to \text{InverseFunction}\left[\int_1^{\#1} \frac{\sqrt[3]{-27K[1]+3\sqrt{3}\sqrt{K[1](27K[1]-4)}+2}}{2^{2/3}\left(-27K[1]+3\sqrt{3}\sqrt{K[1](27K[1]-4)}+2\right)^{2/3}+2\sqrt[3]{-27K[1]+3\sqrt{3}\sqrt{K[1]}}}\right.$$

$$y(x) \to \text{InverseFunction}\left[\int_1^{\#1} \frac{\sqrt[3]{-27K[2]+}}{-i2^{2/3}\sqrt{3}\left(-27K[2]+3\sqrt{3}\sqrt{K[2](27K[2]-4)}+2\right)^{2/3}+2^{2/3}\left(-27K[2]+3\sqrt{}\right.}\right.$$

$$y(x) \to \text{InverseFunction}\left[\int_1^{\#1} \frac{\sqrt[3]{-27K[3]+3}}{i2^{2/3}\sqrt{3}\left(-27K[3]+3\sqrt{3}\sqrt{K[3](27K[3]-4)}+2\right)^{2/3}+2^{2/3}\left(-27K[3]+3\sqrt{3}\right.}\right.$$

$$y(x) \to 0$$

35.9 problem 1041

Internal problem ID [4263]

Book: Ordinary differential equations and their solutions. By George Moseley Murphy. 1960
Section: Various 35
Problem number: 1041.
ODE order: 1.
ODE degree: 3.

CAS Maple gives this as type `[_quadrature]`

$$y'^3 - y'^2 + y^2 = 0$$

✓ Solution by Maple

Time used: 0.031 (sec). Leaf size: 465

```
dsolve(diff(y(x),x)^3-diff(y(x),x)^2+y(x)^2 = 0,y(x), singsol=all)
```

$$y(x) = 0$$

$$-3\,3^{\frac{5}{6}}2^{\frac{2}{3}}\left(\int^{y(x)} \frac{\left(-27\sqrt{3}\,\_a^2 + 2\sqrt{3} + 9\sqrt{27\_a^4 - 4\_a^2}\right)^{\frac{1}{3}}}{3^{\frac{5}{6}}2^{\frac{2}{3}}\left(-27\sqrt{3}\,\_a^2 + 2\sqrt{3} + 9\sqrt{27\_a^4 - 4\_a^2}\right)^{\frac{1}{3}} + 3^{\frac{2}{3}}2^{\frac{1}{3}}\left(-27\sqrt{3}\,\_a^2 + 2\sqrt{3} + 9\sqrt{27\_a^4}\right.}\right.$$

$$+x - c_1 = 0$$

$$\frac{36\,3^{\frac{5}{6}}2^{\frac{2}{3}}\left(\int^{y(x)} \frac{\left(-27\sqrt{3}\,\_a^2+2\sqrt{3}+9\sqrt{27\_a^4-4\_a^2}\right)^{\frac{1}{3}}}{\left(3i\sqrt{3}+3^{\frac{5}{6}}2^{\frac{2}{3}}\left(-27\sqrt{3}\,\_a^2+2\sqrt{3}+9\sqrt{27\_a^4-4\_a^2}\right)^{\frac{1}{3}}+3\right)\left(3^{\frac{5}{6}}2^{\frac{2}{3}}\left(-27\sqrt{3}\,\_a^2+2\sqrt{3}+9\sqrt{27\_a^4-4\_a^2}\right)^{\frac{1}{3}}-6\right)} d\_\right.}{1+i\sqrt{3}}$$

$$= 0$$

$$\frac{i(x-c_1)\sqrt{3} + 36\,3^{\frac{5}{6}}2^{\frac{2}{3}}\left(\int^{y(x)} \frac{\left(-27\sqrt{3}\,\_a^2+2\sqrt{3}+9\sqrt{27\_a^4-4\_a^2}\right)^{\frac{1}{3}}}{\left(-3^{\frac{5}{6}}2^{\frac{2}{3}}\left(-27\sqrt{3}\,\_a^2+2\sqrt{3}+9\sqrt{27\_a^4-4\_a^2}\right)^{\frac{1}{3}}+6\right)\left(-3i\sqrt{3}+3^{\frac{5}{6}}2^{\frac{2}{3}}\left(-27\sqrt{3}\,\_a^2+2\sqrt{3}+9\sqrt{2}\right.\right.}\right.}{-1+i\sqrt{3}}$$

$$= 0$$

✓ Solution by Mathematica

Time used: 47.889 (sec). Leaf size: 583

```
DSolve[(y'[x])^3 - (y'[x])^2 +y[x]^2==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to \text{InverseFunction}\left[\int_1^{\#1} \frac{\sqrt[3]{-27K[1]^2+3\sqrt{3}\sqrt{K[1]^2\left(27K[1]^2-4\right)}+2}\cdots}{2^{2/3}\left(-27K[1]^2+3\sqrt{3}\sqrt{K[1]^2\left(27K[1]^2-4\right)}+2\right)^{2/3}+2\sqrt[3]{-27K[1]^2+3\sqrt{3}\sqrt{\cdots}}\cdots}\right.$$

$$y(x) \to \text{InverseFunction}\left[\int_1^{\#1} \frac{\sqrt[3]{-27K[2\cdots}}{-i2^{2/3}\sqrt{3}\left(-27K[2]^2+3\sqrt{3}\sqrt{K[2]^2\left(27K[2]^2-4\right)}+2\right)^{2/3}-2^{2/3}\left(-27K[2]^2+\cdots\right.}\right.$$

$$y(x) \to \text{InverseFunction}\left[\int_1^{\#1} \frac{\sqrt[3]{-27K[3]\cdots}}{i2^{2/3}\sqrt{3}\left(-27K[3]^2+3\sqrt{3}\sqrt{K[3]^2\left(27K[3]^2-4\right)}+2\right)^{2/3}-2^{2/3}\left(-27K[3]^2+3\cdots\right.}\right.$$

$$y(x) \to 0$$

35.10 problem 1042

Internal problem ID [4264]

Book: Ordinary differential equations and their solutions. By George Moseley Murphy. 1960
Section: Various 35
Problem number: 1042.
ODE order: 1.
ODE degree: 3.

CAS Maple gives this as type `[[_1st_order, _with_linear_symmetries], _Clairaut]`

$$y'^3 - y'^2 + xy' - y = 0$$

✓ Solution by Maple

Time used: 0.047 (sec). Leaf size: 56

```
dsolve(diff(y(x),x)^3-diff(y(x),x)^2+x*diff(y(x),x)-y(x) = 0,y(x), singsol=all)
```

$$y(x) = \frac{x}{3} - \frac{2}{27} - \frac{2\sqrt{-(3x-1)^3}}{27}$$
$$y(x) = \frac{x}{3} - \frac{2}{27} + \frac{2\sqrt{-(3x-1)^3}}{27}$$
$$y(x) = c_1(c_1^2 - c_1 + x)$$

✓ Solution by Mathematica

Time used: 0.025 (sec). Leaf size: 74

```
DSolve[(y'[x])^3 - (y'[x])^2 +x*y'[x]-y[x]==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to c_1(x + (-1 + c_1)c_1)$$
$$y(x) \to \frac{1}{27}\left(9x - 2\left(\sqrt{-(3x-1)^3} + 1\right)\right)$$
$$y(x) \to \frac{1}{27}\left(9x + 2\sqrt{-(3x-1)^3} - 2\right)$$

35.11 problem 1043

Internal problem ID [4265]

Book: Ordinary differential equations and their solutions. By George Moseley Murphy. 1960
Section: Various 35
Problem number: 1043.
ODE order: 1.
ODE degree: 3.

CAS Maple gives this as type `[[_homogeneous, ‘class C‘], _dAlembert]`

$$\boxed{y'^3 - ay'^2 + by = -abx}$$

✓ Solution by Maple

Time used: 0.047 (sec). Leaf size: 132

```
dsolve(diff(y(x),x)^3-a*diff(y(x),x)^2+b*y(x)+a*b*x = 0,y(x), singsol=all)
```

$$y(x) = \frac{2a^3 - 5\,\mathrm{e}^{\mathrm{RootOf}\left(-10\_Z a^2 - 3\,\mathrm{e}^{2\_Z} + 16a\,\mathrm{e}^{\_Z} + 2c_1 b - 13a^2 - 2bx\right)} a^2 + 4\,\mathrm{e}^{2\,\mathrm{RootOf}\left(-10\_Z a^2 - 3\,\mathrm{e}^{2\_Z} + 16a\,\mathrm{e}^{\_Z} + 2c_1 b - 13a^2 - 2bx\right)} a -}{b}$$

✓ Solution by Mathematica

Time used: 0.63 (sec). Leaf size: 398

```
DSolve[(y'[x])^3 - a*(y'[x])^2 +b*y[x]+a*b*x==0,y[x],x,IncludeSingularSolutions -> True]
```

$$\text{Solve}\left[\left\{x=\frac{5a\left(\frac{\sqrt[3]{2a^3+\sqrt{(2a^3-27abx-27by(x))^2-4a^6}-27abx-27by(x)}}{3\sqrt[3]{2}}+\frac{\cdots}{3\sqrt[3]{2a^3+\sqrt{(2a^3-27\cdots}}}\right.}{\cdots}\right.\right.$$

$$\left.\left.+c_1\right\},y(x)\right]$$

35.12 problem 1044

Internal problem ID [4266]

Book: Ordinary differential equations and their solutions. By George Moseley Murphy. 1960
Section: Various 35
Problem number: 1044.
ODE order: 1.
ODE degree: 3.

CAS Maple gives this as type `[_quadrature]`

$$\boxed{y'^3 + \mathrm{a0}\, y'^2 + \mathrm{a1}\, y' + \mathrm{a3}\, y = -\,\mathrm{a2}}$$

✓ Solution by Maple

Time used: 0.047 (sec). Leaf size: 981

```
dsolve(diff(y(x),x)^3+a0*diff(y(x),x)^2+a1*diff(y(x),x)+a2+a3*y(x) = 0,y(x), singsol=all)
```

$$x$$

$$-6\left(\int^{y(x)} \frac{\left(36\,\mathrm{a1}\,\mathrm{a0} - \right.}{\left(36\,\mathrm{a1}\,\mathrm{a0} - 108\,\mathrm{a3}\_a - 108\,\mathrm{a2} - 8\,\mathrm{a0}^3 + 12\sqrt{12\left(\mathrm{a3}\_a + \mathrm{a2}\right)\mathrm{a0}^3 - 3\,\mathrm{a1}^2\,\mathrm{a0}^2 - 54\,\mathrm{a1}\left(\mathrm{a3}\_a + \mathrm{a2}\right.}\right.}\right.$$

$$-\,c_1 = 0$$

$$\frac{-12\left(\int^{y(x)} \frac{}{i\left(\mathrm{a0}\left(36\,\mathrm{a1}\,\mathrm{a0} - 108\,\mathrm{a3}\_\_a - 108\,\mathrm{a2} - 8\,\mathrm{a0}^3 + 12\sqrt{12(\mathrm{a3}\_\_a + \mathrm{a2})\,\mathrm{a0}^3 - 3\,\mathrm{a1}^2\,\mathrm{a0}^2 - 54\,\mathrm{a1}(\mathrm{a3}\_\_a + \mathrm{a2})\,\mathrm{a0} + 81\_\_a^2\,\mathrm{a3}^2 + 162\_\_a\,\mathrm{a2}}\right.\right.}\right.}{}$$

$$= 0$$

$$\frac{-12\left(\int^{y(x)} \frac{}{i\left(\mathrm{a0}\left(36\,\mathrm{a1}\,\mathrm{a0} - 108\,\mathrm{a3}\_\_a - 108\,\mathrm{a2} - 8\,\mathrm{a0}^3 + 12\sqrt{12(\mathrm{a3}\_\_a + \mathrm{a2})\,\mathrm{a0}^3 - 3\,\mathrm{a1}^2\,\mathrm{a0}^2 - 54\,\mathrm{a1}(\mathrm{a3}\_\_a + \mathrm{a2})\,\mathrm{a0} + 81\_\_a^2\,\mathrm{a3}^2 + 162\_\_a\,\mathrm{a2}}\right.\right.}\right.}{}$$

$$= 0$$

✗ Solution by Mathematica

Time used: 0.0 (sec). Leaf size: 0

```
DSolve[(y'[x])^3 + a0*(y'[x])^2 +a1*y'[x]+a2 +a3*y[x]==0,y[x],x,IncludeSingularSolutions ->
```

Timed out

35.13 problem 1046

Internal problem ID [4267]

Book: Ordinary differential equations and their solutions. By George Moseley Murphy. 1960
Section: Various 35
Problem number: 1046.
ODE order: 1.
ODE degree: 3.

CAS Maple gives this as type `[_quadrature]`

$$
{y'}^{3}+\left(1-3x\right){y'}^{2}-x\left(1-3x\right)y'=x^{3}+1
$$

✓ Solution by Maple

Time used: 0.031 (sec). Leaf size: 380

```
dsolve(diff(y(x),x)^3+(1-3*x)*diff(y(x),x)^2-x*(1-3*x)*diff(y(x),x)-1-x^3 = 0,y(x), singsol=
```

$$
y(x)=-\frac{\left(\int \frac{\left(1+i\sqrt{3}\right)\left(12\sqrt{3}\,\sqrt{4x^{3}-x^{2}+18x+23}+36x+100\right)^{\frac{2}{3}}+12\left(x-\frac{1}{3}\right)\left(i\sqrt{3}-\left(12\sqrt{3}\,\sqrt{4x^{3}-x^{2}+18x+23}+36x+100\right)^{\frac{1}{3}}-1\right)}{\left(12\sqrt{3}\,\sqrt{4x^{3}-x^{2}+18x+23}+36x+100\right)^{\frac{1}{3}}}dx\right)}{12}+c_{1}
$$

$$
y(x)=\frac{\left(\int \frac{\left(-1+i\sqrt{3}\right)\left(12\sqrt{3}\,\sqrt{4x^{3}-x^{2}+18x+23}+36x+100\right)^{\frac{2}{3}}+12\left(x-\frac{1}{3}\right)\left(i\sqrt{3}+\left(12\sqrt{3}\,\sqrt{4x^{3}-x^{2}+18x+23}+36x+100\right)^{\frac{1}{3}}+1\right)}{\left(12\sqrt{3}\,\sqrt{4x^{3}-x^{2}+18x+23}+36x+100\right)^{\frac{1}{3}}}dx\right)}{12}+c_{1}
$$

$$
y(x)=\frac{\left(\int \frac{4+6\left(-2+\left(12\sqrt{3}\,\sqrt{4x^{3}-x^{2}+18x+23}+36x+100\right)^{\frac{1}{3}}\right)x+\left(12\sqrt{3}\,\sqrt{4x^{3}-x^{2}+18x+23}+36x+100\right)^{\frac{2}{3}}-2\left(12\sqrt{3}\,\sqrt{4x^{3}-x^{2}+18x+23}+36x+10\right.}{\left(12\sqrt{3}\,\sqrt{4x^{3}-x^{2}+18x+23}+36x+100\right)^{\frac{1}{3}}}\right.}{6}+c_{1}
$$

✓ Solution by Mathematica

Time used: 110.523 (sec). Leaf size: 379

```
DSolve[(y'[x])^3+(1-3*x)(y'[x])^2-x*(1-3*x)*y'[x]-1 -x^3==0,y[x],x,IncludeSingularSolutions
```

$$y(x) \to \int_1^x \frac{1}{6}\left(6K[1] - 2^{2/3}\sqrt[3]{-9K[1] + 3\sqrt{12K[1]^3 - 3K[1]^2 + 54K[1] + 69} - 25} + \frac{2\sqrt[3]{2}(3K[1]-1)}{\sqrt[3]{-9K[1] + 3\sqrt{12K[1]^3 - 3K[1]^2 + 54K[1] + 69} - 25}} - 2\right) dK[1] + c_1$$

$$y(x) \to \int_1^x \frac{1}{12}\left(12K[2] + 2^{2/3}\left(1 - i\sqrt{3}\right)\sqrt[3]{-9K[2] + 3\sqrt{12K[2]^3 - 3K[2]^2 + 54K[2] + 69} - 25} - \frac{2i\sqrt[3]{2}\left(-i+\sqrt{3}\right)(3K[2]-1)}{\sqrt[3]{-9K[2] + 3\sqrt{12K[2]^3 - 3K[2]^2 + 54K[2] + 69} - 25}} - 4\right) dK[2] + c_1$$

$$y(x) \to \int_1^x \frac{1}{12}\left(12K[3] + 2^{2/3}\left(1 + i\sqrt{3}\right)\sqrt[3]{-9K[3] + 3\sqrt{12K[3]^3 - 3K[3]^2 + 54K[3] + 69} - 25} + \frac{2i\sqrt[3]{2}\left(i+\sqrt{3}\right)(3K[3]-1)}{\sqrt[3]{-9K[3] + 3\sqrt{12K[3]^3 - 3K[3]^2 + 54K[3] + 69} - 25}} - 4\right) dK[3] + c_1$$

35.14 problem 1047

Internal problem ID [4268]

Book: Ordinary differential equations and their solutions. By George Moseley Murphy. 1960
Section: Various 35
Problem number: 1047.
ODE order: 1.
ODE degree: 3.

CAS Maple gives this as type `[_quadrature]`

$$y'^3 - yy'^2 + y^2 = 0$$

✓ Solution by Maple

Time used: 0.015 (sec). Leaf size: 420

```
dsolve(diff(y(x),x)^3-y(x)*diff(y(x),x)^2+y(x)^2 = 0,y(x), singsol=all)
```

$$y(x) = 0$$

$$x - 6\left(\int^{y(x)} \frac{\left(8\_a^3 - 108\_a^2 + 12\sqrt{3}\,\sqrt{-4\_a^5 + 27\_a^4}\right)^{\frac{1}{3}}}{\left(8\_a^3 - 108\_a^2 + 12\sqrt{3}\,\sqrt{-4\_a^5 + 27\_a^4}\right)^{\frac{2}{3}} + 2\left(8\_a^3 - 108\_a^2 + 12\sqrt{3}\,\sqrt{-4\_a^5 + 27\_a^4}\right.}\right.$$

$$- c_1 = 0$$

$$\frac{12\left(\int^{y(x)} \frac{\left(8\_a^3-108\_a^2+12\sqrt{3}\,\sqrt{-4\_a^5+27\_a^4}\right)^{\frac{1}{3}}}{\left(i\_a\sqrt{3}+\left(8\_a^3-108\_a^2+12\sqrt{3}\,\sqrt{-4\_a^5+27\_a^4}\right)^{\frac{1}{3}}+\_a\right)\left(\left(8\_a^3-108\_a^2+12\sqrt{3}\,\sqrt{-4\_a^5+27\_a^4}\right)^{\frac{1}{3}}-2\_a\right)} d\_\right.}{1+i\sqrt{3}}$$

$$= 0$$

$$\frac{12\left(\int^{y(x)} \frac{\left(8\_a^3-108\_a^2+12\sqrt{3}\,\sqrt{-4\_a^5+27\_a^4}\right)^{\frac{1}{3}}}{\left(-i\_a\sqrt{3}+\left(8\_a^3-108\_a^2+12\sqrt{3}\,\sqrt{-4\_a^5+27\_a^4}\right)^{\frac{1}{3}}+\_a\right)\left(-\left(8\_a^3-108\_a^2+12\sqrt{3}\,\sqrt{-4\_a^5+27\_a^4}\right)^{\frac{1}{3}}+2\_a\right)}\right.}{-1+i\sqrt{3}}$$

$$= 0$$

✓ Solution by Mathematica

Time used: 56.7 (sec). Leaf size: 653

```
DSolve[(y'[x])^3 -y[x]*(y'[x])^2+y[x]^2==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to \text{InverseFunction}\left[\int_1^{\#1} \frac{\sqrt[3]{2K[1]^3 - 27K[1]^2 + 3\sqrt{3}\sqrt{-K[1]^4(4\ldots}}}{2\sqrt[3]{2}K[1]^2 + 2\sqrt[3]{2K[1]^3 - 27K[1]^2 + 3\sqrt{3}\sqrt{-K[1]^4(4K[1]-27)}}\,K[1] + 2^{2/3}\left(2\ldots\right.}\right.$$

$$y(x) \to \text{InverseFunction}\left[\int_1^{\#1} \frac{\ldots}{2i\sqrt[3]{2}\sqrt{3}K[2]^2 - 2\sqrt[3]{2}K[2]^2 + 4\sqrt[3]{2K[2]^3 - 27K[2]^2 + 3\sqrt{3}\sqrt{-K[2]^4(4K[2]-2\ldots}}}\right.$$

$$y(x) \to \text{InverseFunction}\left[\int_1^{\#1} \frac{\ldots}{-2i\sqrt[3]{2}\sqrt{3}K[3]^2 - 2\sqrt[3]{2}K[3]^2 + 4\sqrt[3]{2K[3]^3 - 27K[3]^2 + 3\sqrt{3}\sqrt{-K[3]^4(4K[3]-\ldots}}}\right.$$

$$y(x) \to 0$$

35.15 problem 1048

Internal problem ID [4269]

Book: Ordinary differential equations and their solutions. By George Moseley Murphy. 1960
Section: Various 35
Problem number: 1048.
ODE order: 1.
ODE degree: 3.

CAS Maple gives this as type `[_quadrature]`

$$y'^3 + (\cos(x)\cot(x) - y)\,y'^2 - (1 + y\cos(x)\cot(x))\,y' + y = 0$$

✓ Solution by Maple

Time used: 0.063 (sec). Leaf size: 32

```
dsolve(diff(y(x),x)^3+(cos(x)*cot(x)-y(x))*diff(y(x),x)^2-(1+y(x)*cos(x)*cot(x))*diff(y(x),x
```

$$y(x) = c_1 \mathrm{e}^x$$
$$y(x) = -\ln(\csc(x) - \cot(x)) + c_1$$
$$y(x) = -\cos(x) + c_1$$

✓ Solution by Mathematica

Time used: 0.024 (sec). Leaf size: 32

```
DSolve[(y'[x])^3 +(Cos[x]*Cot[x]-y[x])*(y'[x])^2-(1+y[x]*Cos[x]*Cot[x])*y'[x]+y[x]==0,y[x],x
```

$$y(x) \to c_1 e^x$$
$$y(x) \to \operatorname{arctanh}(\cos(x)) + c_1$$
$$y(x) \to -\cos(x) + c_1$$

35.16 problem 1049

Internal problem ID [4270]

Book: Ordinary differential equations and their solutions. By George Moseley Murphy. 1960
Section: Various 35
Problem number: 1049.
ODE order: 1.
ODE degree: 3.

CAS Maple gives this as type `[_quadrature]`

$$y'^3 + (2x - y^2) y'^2 - 2xy^2y' = 0$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 25

```
dsolve(diff(y(x),x)^3+(2*x-y(x)^2)*diff(y(x),x)^2-2*x*y(x)^2*diff(y(x),x) = 0,y(x), singsol=
```

$$y(x) = \frac{1}{c_1 - x}$$
$$y(x) = -x^2 + c_1$$
$$y(x) = c_1$$

✓ Solution by Mathematica

Time used: 0.056 (sec). Leaf size: 31

```
DSolve[(y'[x])^3 +(2*x-y[x]^2)*(y'[x])^2 -2*x*y[x]^2 y'[x]==0,y[x],x,IncludeSingularSolution
```

$$y(x) \to -\frac{1}{x + c_1}$$
$$y(x) \to c_1$$
$$y(x) \to -x^2 + c_1$$

35.17 problem 1050

Internal problem ID [4271]

Book: Ordinary differential equations and their solutions. By George Moseley Murphy. 1960
Section: Various 35
Problem number: 1050.
ODE order: 1.
ODE degree: 3.

CAS Maple gives this as type `[_quadrature]`

$$y'^3 - (y^2 + 2x)\, y'^2 + (x^2 - y^2 + 2y^2 x)\, y' - (x^2 - y^2)\, y^2 = 0$$

✓ Solution by Maple

Time used: 0.016 (sec). Leaf size: 35

```
dsolve(diff(y(x),x)^3-(2*x+y(x)^2)*diff(y(x),x)^2+(x^2-y(x)^2+2*x*y(x)^2)*diff(y(x),x)-(x^2-
```

$$y(x) = \frac{1}{c_1 - x}$$
$$y(x) = -x - 1 + c_1 e^{x}$$
$$y(x) = x - 1 + e^{-x} c_1$$

✓ Solution by Mathematica

Time used: 0.153 (sec). Leaf size: 48

```
DSolve[(y'[x])^3 -(2*x+y[x]^2)*(y'[x])^2 +(x^2 -y[x]^2+2* x* y[x]^2)* y'[x]-(x^2-y[x]^2)*y[x
```

$$y(x) \to -\frac{1}{x + c_1}$$
$$y(x) \to x + c_1 e^{-x} - 1$$
$$y(x) \to -x + c_1 e^{x} - 1$$
$$y(x) \to 0$$

35.18 problem 1051

Internal problem ID [4272]

Book: Ordinary differential equations and their solutions. By George Moseley Murphy. 1960
Section: Various 35
Problem number: 1051.
ODE order: 1.
ODE degree: 3.

CAS Maple gives this as type `[_quadrature]`

$$y'^3 - (x^2 + yx + y^2)\, y'^2 + xy(x^2 + yx + y^2)\, y' - y^3x^3 = 0$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 32

```
dsolve(diff(y(x),x)^3-(x^2+x*y(x)+y(x)^2)*diff(y(x),x)^2+x*y(x)*(x^2+x*y(x)+y(x)^2)*diff(y(x
```

$$y(x) = \frac{x^3}{3} + c_1$$
$$y(x) = \frac{1}{c_1 - x}$$
$$y(x) = c_1 \mathrm{e}^{\frac{x^2}{2}}$$

✓ Solution by Mathematica

Time used: 0.1 (sec). Leaf size: 48

```
DSolve[(y'[x])^3 -(x^2+x y[x]+ y[x]^2) (y'[x])^2 +x y[x](x^2 +x y[x]+ y[x]^2) y'[x]-x^3 y[x]
```

$$y(x) \to -\frac{1}{x + c_1}$$
$$y(x) \to c_1 e^{\frac{x^2}{2}}$$
$$y(x) \to \frac{x^3}{3} + c_1$$
$$y(x) \to 0$$

35.19 problem 1052

Internal problem ID [4273]

Book: Ordinary differential equations and their solutions. By George Moseley Murphy. 1960
Section: Various 35
Problem number: 1052.
ODE order: 1.
ODE degree: 3.

CAS Maple gives this as type `[_quadrature]`

$$y'^3 - (x^2 + y^2x + y^4)\, y'^2 + xy^2(x^2 + y^2x + y^4)\, y' - y^6x^3 = 0$$

✓ Solution by Maple
Time used: 0.016 (sec). Leaf size: 75

```
dsolve(diff(y(x),x)^3-(x^2+x*y(x)^2+y(x)^4)*diff(y(x),x)^2+x*y(x)^2*(x^2+x*y(x)^2+y(x)^4)*di
```

$$y(x) = \frac{x^3}{3} + c_1$$
$$y(x) = \frac{1}{(-3x + c_1)^{\frac{1}{3}}}$$
$$y(x) = -\frac{1 + i\sqrt{3}}{2\,(-3x + c_1)^{\frac{1}{3}}}$$
$$y(x) = \frac{-1 + i\sqrt{3}}{2\,(-3x + c_1)^{\frac{1}{3}}}$$
$$y(x) = -\frac{2}{x^2 - 2c_1}$$

✓ Solution by Mathematica

Time used: 0.241 (sec). Leaf size: 110

```
DSolve[(y'[x])^3 -(x^2+x y[x]^2+ y[x]^4) (y'[x])^2 +x y[x]^2(x^2 +x y[x]^2+ y[x]^4) y'[x]-x^
```

$$y(x) \to -\frac{\sqrt[3]{-\frac{1}{3}}}{\sqrt[3]{-x-c_1}}$$
$$y(x) \to \frac{1}{\sqrt[3]{3}\sqrt[3]{-x-c_1}}$$
$$y(x) \to \frac{(-1)^{2/3}}{\sqrt[3]{3}\sqrt[3]{-x-c_1}}$$
$$y(x) \to \frac{x^3}{3}+c_1$$
$$y(x) \to -\frac{2}{x^2+2c_1}$$
$$y(x) \to 0$$

35.20 problem 1053

Internal problem ID [4274]

Book: Ordinary differential equations and their solutions. By George Moseley Murphy. 1960
Section: Various 35
Problem number: 1053.
ODE order: 1.
ODE degree: 3.

CAS Maple gives this as type `[[_1st_order, _with_linear_symmetries], _dAlembert]`

$$\boxed{2y'^3 + xy' - 2y = 0}$$

✓ Solution by Maple
Time used: 0.016 (sec). Leaf size: 58

```
dsolve(2*diff(y(x),x)^3+x*diff(y(x),x)-2*y(x) = 0,y(x), singsol=all)
```

$$y(x) = \frac{(-c_1^2 - 24x)\sqrt{c_1^2 + 24x}}{432} - \frac{c_1^3}{432} - \frac{c_1 x}{12}$$
$$y(x) = \frac{(c_1^2 + 24x)^{\frac{3}{2}}}{432} - \frac{c_1^3}{432} - \frac{c_1 x}{12}$$

✗ Solution by Mathematica
Time used: 0.0 (sec). Leaf size: 0

```
DSolve[2 (y'[x])^3 +x y'[x]-2 y[x]==0,y[x],x,IncludeSingularSolutions -> True]
```

Timed out

35.21 problem 1054

Internal problem ID [4275]

Book: Ordinary differential equations and their solutions. By George Moseley Murphy. 1960
Section: Various 35
Problem number: 1054.
ODE order: 1.
ODE degree: 3.

CAS Maple gives this as type `[_quadrature]`

$$\boxed{2{y'}^3 + {y'}^2 - y = 0}$$

✓ Solution by Maple

Time used: 0.015 (sec). Leaf size: 393

```
dsolve(2*diff(y(x),x)^3+diff(y(x),x)^2-y(x) = 0,y(x), singsol=all)
```

$$y(x) = 0$$

$$6\sqrt{3}\left(\int^{y(x)} \frac{\left(18\sqrt{27\_a^2-\_a}+(54\_a-1)\sqrt{3}\right)^{\frac{1}{3}}}{-3^{\frac{1}{3}}\left(18\sqrt{27\_a^2-\_a}+(54\_a-1)\sqrt{3}\right)^{\frac{2}{3}}+\sqrt{3}\left(18\sqrt{27\_a^2-\_a}+(54\_a-1)\sqrt{3}\right)^{\frac{1}{3}}-3^{\frac{2}{3}}\right.$$

$$+x-c_1=0$$

$$\frac{24i\sqrt{3}\left(\int^{y(x)} \frac{\left(18\sqrt{27\_a^2-\_a}+(54\_a-1)\sqrt{3}\right)^{\frac{1}{3}}}{\left(3^{\frac{1}{3}}+3^{\frac{1}{6}}\left(18\sqrt{27\_a^2-\_a}+(54\_a-1)\sqrt{3}\right)^{\frac{1}{3}}\right)\left(i3^{\frac{5}{6}}+3^{\frac{1}{3}}-2\,3^{\frac{1}{6}}\left(18\sqrt{27\_a^2-\_a}+(54\_a-1)\sqrt{3}\right)^{\frac{1}{3}}\right)}d\_a\right)+(x}{-i+\sqrt{3}}$$

$$=0$$

$$\frac{24i\sqrt{3}\left(\int^{y(x)} \frac{\left(18\sqrt{27\_a^2-\_a}+(54\_a-1)\sqrt{3}\right)^{\frac{1}{3}}}{\left(i3^{\frac{5}{6}}-3^{\frac{1}{3}}+2\,3^{\frac{1}{6}}\left(18\sqrt{27\_a^2-\_a}+(54\_a-1)\sqrt{3}\right)^{\frac{1}{3}}\right)\left(3^{\frac{1}{3}}+3^{\frac{1}{6}}\left(18\sqrt{27\_a^2-\_a}+(54\_a-1)\sqrt{3}\right)^{\frac{1}{3}}\right)}d\_a\right)+(x}{\sqrt{3}+i}$$

$$=0$$

✗ Solution by Mathematica

Time used: 0.0 (sec). Leaf size: 0

```
DSolve[2 (y'[x])^3 + (y'[x])^2 - y[x]==0,y[x],x,IncludeSingularSolutions -> True]
```

Timed out

35.22 problem 1055

Internal problem ID [4276]

Book: Ordinary differential equations and their solutions. By George Moseley Murphy. 1960
Section: Various 35
Problem number: 1055.
ODE order: 1.
ODE degree: 3.

CAS Maple gives this as type `[[_1st_order, _with_linear_symmetries]]`

$$3y'^3 - x^4 y' + 2x^3 y = 0$$

✓ Solution by Maple

Time used: 0.047 (sec). Leaf size: 34

```
dsolve(3*diff(y(x),x)^3-x^4*diff(y(x),x)+2*x^3*y(x) = 0,y(x), singsol=all)
```

$$y(x) = -\frac{x^3}{9}$$
$$y(x) = \frac{x^3}{9}$$
$$y(x) = \frac{c_1^2 x^2 - 3}{2c_1^3}$$

✓ Solution by Mathematica

Time used: 87.281 (sec). Leaf size: 15992

```
DSolve[3 (y'[x])^3 - x^4 y'[x]+2 x^3 y[x]==0,y[x],x,IncludeSingularSolutions -> True]
```

Too large to display

35.23 problem 1056

Internal problem ID [4277]

Book: Ordinary differential equations and their solutions. By George Moseley Murphy. 1960
Section: Various 35
Problem number: 1056.
ODE order: 1.
ODE degree: 3.

CAS Maple gives this as type `[_quadrature]`

$$4{y'}^3 + 4y' = x$$

✓ Solution by Maple

Time used: 0.016 (sec). Leaf size: 183

```
dsolve(4*diff(y(x),x)^3+4*diff(y(x),x) = x,y(x), singsol=all)
```

$$y(x) = \frac{\left(\int \frac{\left(-1+i\sqrt{3}\right)\left(27x+3\sqrt{81x^2+192}\right)^{\frac{2}{3}}+12i\sqrt{3}+12}{\left(27x+3\sqrt{81x^2+192}\right)^{\frac{1}{3}}}dx\right)}{12} + c_1$$

$$y(x) = -\frac{\left(\int \frac{i\sqrt{3}\left(27x+3\sqrt{81x^2+192}\right)^{\frac{2}{3}}+12i\sqrt{3}+\left(27x+3\sqrt{81x^2+192}\right)^{\frac{2}{3}}-12}{\left(27x+3\sqrt{81x^2+192}\right)^{\frac{1}{3}}}dx\right)}{12} + c_1$$

$$y(x) = \frac{\left(\int \frac{\left(27x+3\sqrt{81x^2+192}\right)^{\frac{2}{3}}-12}{\left(27x+3\sqrt{81x^2+192}\right)^{\frac{1}{3}}}dx\right)}{6} + c_1$$

✓ Solution by Mathematica

Time used: 3.014 (sec). Leaf size: 376

```
DSolve[4 (y'[x])^3 +4 y'[x]==x,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to -\frac{i(\sqrt{3}-i)\left(-27x^2+3\sqrt{81x^2+192}x-16\right)}{2\sqrt[3]{3}\left(\sqrt{81x^2+192}-9x\right)^{4/3}} + \frac{(1-i\sqrt{3})\left(-27x^2+3\sqrt{81x^2+192}x+32\right)}{16\ 3^{2/3}\left(\sqrt{81x^2+192}-9x\right)^{2/3}} + c_1$$

$$y(x) \to \frac{i(\sqrt{3}+i)\left(-27x^2+3\sqrt{81x^2+192}x-16\right)}{2\sqrt[3]{3}\left(\sqrt{81x^2+192}-9x\right)^{4/3}} + \frac{(1+i\sqrt{3})\left(-27x^2+3\sqrt{81x^2+192}x+32\right)}{16\ 3^{2/3}\left(\sqrt{81x^2+192}-9x\right)^{2/3}} + c_1$$

$$y(x) \to \frac{\left(\sqrt{81x^2+192}-9x\right)^{4/3}}{48\ 3^{2/3}} - \frac{8}{3^{2/3}\left(\sqrt{81x^2+192}-9x\right)^{2/3}} + \frac{-27\ 3^{2/3}x^2+9\sqrt[6]{3}\sqrt{27x^2+64}x-16\ 3^{2/3}}{3\left(\sqrt{81x^2+192}-9x\right)^{4/3}} + c_1$$

35.24 problem 1057

Internal problem ID [4278]

Book: Ordinary differential equations and their solutions. By George Moseley Murphy. 1960
Section: Various 35
Problem number: 1057.
ODE order: 1.
ODE degree: 3.

CAS Maple gives this as type `[[_homogeneous, ‘class C‘], _dAlembert]`

$$\boxed{8y'^3 + 12y'^2 - 27y = 27x}$$

✓ Solution by Maple

Time used: 0.032 (sec). Leaf size: 43

```
dsolve(8*diff(y(x),x)^3+12*diff(y(x),x)^2 = 27*x+27*y(x),y(x), singsol=all)
```

$$y(x) = -x + \frac{4}{27}$$
$$y(x) = (c_1 - x)\sqrt{x - c_1} - c_1$$
$$y(x) = (x - c_1)^{\frac{3}{2}} - c_1$$

✗ Solution by Mathematica

Time used: 0.0 (sec). Leaf size: 0

```
DSolve[8 (y'[x])^3 + 12 (y'[x])^2 ==27(x+y[x]),y[x],x,IncludeSingularSolutions -> True]
```

Timed out

35.25 problem 1058

Internal problem ID [4279]

Book: Ordinary differential equations and their solutions. By George Moseley Murphy. 1960
Section: Various 35
Problem number: 1058.
ODE order: 1.
ODE degree: 3.

CAS Maple gives this as type `[[_1st_order, _with_linear_symmetries], _Clairaut]`

$$xy'^3 - yy'^2 = -a$$

✓ Solution by Maple
Time used: 0.031 (sec). Leaf size: 76

```
dsolve(x*diff(y(x),x)^3-y(x)*diff(y(x),x)^2+a = 0,y(x), singsol=all)
```

$$y(x) = \frac{3\, 2^{\frac{1}{3}} (a\, x^2)^{\frac{1}{3}}}{2}$$
$$y(x) = -\frac{3\, 2^{\frac{1}{3}} (a\, x^2)^{\frac{1}{3}} \left(1 + i\sqrt{3}\right)}{4}$$
$$y(x) = \frac{3\, 2^{\frac{1}{3}} (a\, x^2)^{\frac{1}{3}} \left(-1 + i\sqrt{3}\right)}{4}$$
$$y(x) = \frac{c_1^3 x + a}{c_1^2}$$

✓ Solution by Mathematica

Time used: 0.015 (sec). Leaf size: 89

```
DSolve[x (y'[x])^3 - y[x] (y'[x])^2 +a==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to \frac{a}{{c_1}^2} + c_1 x$$
$$y(x) \to \frac{3\sqrt[3]{a}x^{2/3}}{2^{2/3}}$$
$$y(x) \to -\frac{3\sqrt[3]{-1}\sqrt[3]{a}x^{2/3}}{2^{2/3}}$$
$$y(x) \to \frac{3(-1)^{2/3}\sqrt[3]{a}x^{2/3}}{2^{2/3}}$$

35.26 problem 1060

Internal problem ID [4280]

Book: Ordinary differential equations and their solutions. By George Moseley Murphy. 1960
Section: Various 35
Problem number: 1060.
ODE order: 1.
ODE degree: 3.

CAS Maple gives this as type `[_quadrature]`

$$xy'^3 - (x + x^2 + y)\, y'^2 + (x^2 + y + yx)\, y' - yx = 0$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 23

```
dsolve(x*diff(y(x),x)^3-(x+x^2+y(x))*diff(y(x),x)^2+(x^2+y(x)+x*y(x))*diff(y(x),x)-x*y(x) =
```

$$y(x) = c_1 x$$
$$y(x) = x + c_1$$
$$y(x) = \frac{x^2}{2} + c_1$$

✓ Solution by Mathematica

Time used: 0.044 (sec). Leaf size: 36

```
DSolve[x (y'[x])^3 - (x+x^2+y[x])(y'[x])^2 + (x^2+y[x]+x y[x]) y'[x]-x y[x]==0,y[x],x,Includ
```

$$y(x) \to c_1 x$$
$$y(x) \to x + c_1$$
$$y(x) \to \frac{x^2}{2} + c_1$$
$$y(x) \to 0$$

35.27 problem 1061

Internal problem ID [4281]

Book: Ordinary differential equations and their solutions. By George Moseley Murphy. 1960
Section: Various 35
Problem number: 1061.
ODE order: 1.
ODE degree: 3.

CAS Maple gives this as type `[[_1st_order, _with_linear_symmetries]]`

$$\boxed{xy'^3 - 2y'^2y = -4x^2}$$

✓ Solution by Maple

Time used: 0.156 (sec). Leaf size: 800

```
dsolve(x*diff(y(x),x)^3-2*y(x)*diff(y(x),x)^2+4*x^2 = 0,y(x), singsol=all)
```

$$y(x) = \frac{3x^{\frac{4}{3}}}{2}$$

$$y(x) = -\frac{3x^{\frac{4}{3}}\left(1+i\sqrt{3}\right)}{4}$$

$$y(x) = \frac{3x^{\frac{4}{3}}\left(-1+i\sqrt{3}\right)}{4}$$

$$y(x) = \frac{c_1^3+128x^2}{32c_1}$$

$$y(x) = \frac{c_1^3-128x^2}{32c_1}$$

$$y(x) = \frac{c_1\left(-1728x^2+c_1^3+24\sqrt{6}\,\sqrt{-x^2\left(c_1^3-864x^2\right)}\right)^{\frac{1}{3}}}{96} + \frac{c_1^3}{96\left(-1728x^2+c_1^3+24\sqrt{6}\,\sqrt{-x^2\left(c_1^3-864x^2\right)}\right)^{\frac{1}{3}}} + \frac{c_1^2}{96}$$

$$y(x) = \frac{c_1\left(c_1^3+24\sqrt{6}\,\sqrt{x^2\left(c_1^3+864x^2\right)}+1728x^2\right)^{\frac{1}{3}}}{96} + \frac{c_1^3}{96\left(c_1^3+24\sqrt{6}\,\sqrt{x^2\left(c_1^3+864x^2\right)}+1728x^2\right)^{\frac{1}{3}}} + \frac{c_1^2}{96}$$

$$y(x) = \frac{\left(c_1-\left(-1728x^2+c_1^3+24\sqrt{6}\,\sqrt{-c_1^3x^2+864x^4}\right)^{\frac{1}{3}}\right)c_1\left(i\left(\left(-1728x^2+c_1^3+24\sqrt{6}\,\sqrt{-c_1^3x^2+864x^4}\right)^{\frac{1}{3}}\right.\right.}{192\left(-1728x^2+c_1^3+24\sqrt{6}\,\sqrt{-c_1^3x^2+864x\right.}}$$

$$y(x) = \frac{\left(-1+i\sqrt{3}\right)c_1\left(-1728x^2+c_1^3+24\sqrt{3}\,\sqrt{2}\,\sqrt{-c_1^3x^2+864x^4}\right)^{\frac{1}{3}}}{192} - \frac{\left(i\sqrt{3}\,c_1+c_1-2\left(-1728x^2+c_1^3+24\sqrt{3}\,\sqrt{2}\,\sqrt{-c_1^3x^2+864x^4}\right)^{\frac{1}{3}}\right)c_1^2}{192\left(-1728x^2+c_1^3+24\sqrt{3}\,\sqrt{2}\,\sqrt{-c_1^3x^2+864x^4}\right)^{\frac{1}{3}}}$$

$$y(x) = \frac{\left(c_1-\left(c_1^3+24\sqrt{6}\,\sqrt{c_1^3x^2+864x^4}+1728x^2\right)^{\frac{1}{3}}\right)c_1\left(i\left(\left(c_1^3+24\sqrt{6}\,\sqrt{c_1^3x^2+864x^4}+1728x^2\right)^{\frac{1}{3}}+c_1\right)\right.}{192\left(c_1^3+24\sqrt{6}\,\sqrt{c_1^3x^2+864x^4}+1728x^2\right)^{\frac{1}{3}}}$$

$$y(x) = \frac{\left(-1+i\sqrt{3}\right)c_1\left(c_1^3+24\sqrt{3}\,\sqrt{2}\,\sqrt{c_1^3x^2+864x^4}+1728x^2\right)^{\frac{1}{3}}}{192}$$

✓ Solution by Mathematica

Time used: 169.538 (sec). Leaf size: 15120

```
DSolve[x (y'[x])^3 - 2 y[x](y'[x])^2 + 4 x^2==0,y[x],x,IncludeSingularSolutions -> True]
```

Too large to display

35.28 problem 1062

Internal problem ID [4282]

Book: Ordinary differential equations and their solutions. By George Moseley Murphy. 1960
Section: Various 35
Problem number: 1062.
ODE order: 1.
ODE degree: 3.

CAS Maple gives this as type `[[_1st_order, _with_linear_symmetries], _dAlembert]`

$$2xy'^3 - 3yy'^2 = x$$

✓ Solution by Maple

Time used: 0.078 (sec). Leaf size: 79

```
dsolve(2*x*diff(y(x),x)^3-3*y(x)*diff(y(x),x)^2-x = 0,y(x), singsol=all)
```

$$y(x) = -\frac{\left(-1+i\sqrt{3}\right)x}{2}$$
$$y(x) = \frac{\left(1+i\sqrt{3}\right)x}{2}$$
$$y(x) = -x$$
$$y(x) = \frac{2x\sqrt{c_1 x} - c_1^2}{3c_1}$$
$$y(x) = \frac{-c_1^2 - 2x\sqrt{c_1 x}}{3c_1}$$

✓ Solution by Mathematica

Time used: 28.499 (sec). Leaf size: 4317

```
DSolve[2 x (y'[x])^3 - 3 y[x] (y'[x])^2 -x==0,y[x],x,IncludeSingularSolutions -> True]
```

Too large to display

35.29 problem 1063

Internal problem ID [4283]

Book: Ordinary differential equations and their solutions. By George Moseley Murphy. 1960
Section: Various 35
Problem number: 1063.
ODE order: 1.
ODE degree: 3.

CAS Maple gives this as type `[[_1st_order, _with_linear_symmetries], _dAlembert]`

$$4xy'^3 - 6yy'^2 + 3y = x$$

✓ Solution by Maple
Time used: 0.047 (sec). Leaf size: 84

```
dsolve(4*x*diff(y(x),x)^3-6*y(x)*diff(y(x),x)^2-x+3*y(x) = 0,y(x), singsol=all)
```

$$y(x) = -\frac{(1+\sqrt{3})\, x}{2}$$
$$y(x) = \frac{(\sqrt{3}-1)\, x}{2}$$
$$y(x) = x$$
$$y(x) = \frac{-(x+c_1)\sqrt{2}\,\sqrt{c_1\,(x+c_1)} - c_1^2}{3c_1}$$
$$y(x) = \frac{(x+c_1)\sqrt{2}\,\sqrt{c_1\,(x+c_1)} - c_1^2}{3c_1}$$

✓ Solution by Mathematica
Time used: 1.317 (sec). Leaf size: 79

```
DSolve[4 x (y'[x])^3 -6 y[x] (y'[x])^2-x +3 y[x]==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to -\frac{\sqrt{2}\sqrt{c_1(x+c_1)^3} + c_1{}^2}{3c_1}$$
$$y(x) \to -\frac{c_1{}^2 - \sqrt{2}\sqrt{c_1(x+c_1)^3}}{3c_1}$$
$$y(x) \to \text{Indeterminate}$$

36 Various 36

36.1 problem 1064

Internal problem ID [4284]

Book: Ordinary differential equations and their solutions. By George Moseley Murphy. 1960
Section: Various 36
Problem number: 1064.
ODE order: 1.
ODE degree: 3.

CAS Maple gives this as type `[[_1st_order, _with_linear_symmetries], _dAlembert]`

$$8xy'^3 - 12yy'^2 + 9y = 0$$

✓ Solution by Maple

Time used: 0.032 (sec). Leaf size: 61

```
dsolve(8*x*diff(y(x),x)^3-12*y(x)*diff(y(x),x)^2+9*y(x) = 0,y(x), singsol=all)
```

$$y(x) = -\frac{3x}{2}$$
$$y(x) = \frac{3x}{2}$$
$$y(x) = 0$$
$$y(x) = -\frac{(3c_1 + x)\sqrt{c_1(3c_1 + x)}}{3c_1}$$
$$y(x) = \frac{(3c_1 + x)\sqrt{c_1(3c_1 + x)}}{3c_1}$$

✓ Solution by Mathematica

Time used: 0.453 (sec). Leaf size: 77

```
DSolve[8 x (y'[x])^3 -12 y[x] (y'[x])^2 + 9 y[x]==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to -\frac{(x+3c_1)^{3/2}}{3\sqrt{c_1}}$$
$$y(x) \to \frac{(x+3c_1)^{3/2}}{3\sqrt{c_1}}$$
$$y(x) \to 0$$
$$y(x) \to \text{Indeterminate}$$
$$y(x) \to -\frac{3x}{2}$$
$$y(x) \to \frac{3x}{2}$$

36.2 problem 1065

Internal problem ID [4285]

Book: Ordinary differential equations and their solutions. By George Moseley Murphy. 1960
Section: Various 36
Problem number: 1065.
ODE order: 1.
ODE degree: 3.

CAS Maple gives this as type `[[_1st_order, _with_linear_symmetries], _Clairaut]`

$$\boxed{x^2 {y'}^3 - 2xy{y'}^2 + y^2 y' = -1}$$

✓ Solution by Maple
Time used: 0.047 (sec). Leaf size: 82

```
dsolve(x^2*diff(y(x),x)^3-2*x*y(x)*diff(y(x),x)^2+y(x)^2*diff(y(x),x)+1 = 0,y(x), singsol=al
```

$$y(x) = \frac{3\, 2^{\frac{1}{3}} (-x)^{\frac{1}{3}}}{2}$$
$$y(x) = -\frac{3\, 2^{\frac{1}{3}} (-x)^{\frac{1}{3}} \left(1 + i\sqrt{3}\right)}{4}$$
$$y(x) = \frac{3\, 2^{\frac{1}{3}} (-x)^{\frac{1}{3}} \left(-1 + i\sqrt{3}\right)}{4}$$
$$y(x) = c_1 x - \frac{1}{\sqrt{-c_1}}$$
$$y(x) = c_1 x + \frac{1}{\sqrt{-c_1}}$$

✓ Solution by Mathematica
Time used: 65.79 (sec). Leaf size: 33909

```
DSolve[x^2 (y'[x])^3 -2 x y[x] (y'[x])^2 +  y[x]^2 y'[x]+1==0,y[x],x,IncludeSingularSolution
```

Too large to display

36.3 problem 1066

Internal problem ID [4286]

Book: Ordinary differential equations and their solutions. By George Moseley Murphy. 1960
Section: Various 36
Problem number: 1066.
ODE order: 1.
ODE degree: 3.

CAS Maple gives this as type `[_quadrature]`

$$(a^2 - x^2)\, y'^3 + bx(a^2 - x^2)\, y'^2 - y' = bx$$

✓ Solution by Maple

Time used: 0.015 (sec). Leaf size: 52

```
dsolve((a^2-x^2)*diff(y(x),x)^3+b*x*(a^2-x^2)*diff(y(x),x)^2-diff(y(x),x)-b*x = 0,y(x), sing
```

$$y(x) = -\frac{b\,x^2}{2} + c_1$$
$$y(x) = \arctan\left(\frac{x}{\sqrt{a^2 - x^2}}\right) + c_1$$
$$y(x) = -\arctan\left(\frac{x}{\sqrt{a^2 - x^2}}\right) + c_1$$

✓ Solution by Mathematica

Time used: 0.005 (sec). Leaf size: 64

```
DSolve[(a^2-x^2) (y'[x])^3 +b x (a^2-x^2) (y'[x])^2 -y'[x] -b x==0,y[x],x,IncludeSingularSol
```

$$y(x) \to -\frac{bx^2}{2} + c_1$$
$$y(x) \to -\arctan\left(\frac{x}{\sqrt{a^2 - x^2}}\right) + c_1$$
$$y(x) \to \arctan\left(\frac{x}{\sqrt{a^2 - x^2}}\right) + c_1$$

36.4 problem 1067

Internal problem ID [4287]

Book: Ordinary differential equations and their solutions. By George Moseley Murphy. 1960
Section: Various 36
Problem number: 1067.
ODE order: 1.
ODE degree: 3.

CAS Maple gives this as type `[‘y=_G(x,y’)‘]`

$$xy'^3 - 3x^2yy'^2 + x(x^5 + 3y^2)\, y' - 2yx^5 - y^3 = 0$$

✗ Solution by Maple

```
dsolve(x*diff(y(x),x)^3-3*x^2*y(x)*diff(y(x),x)^2+x*(x^5+3*y(x)^2)*diff(y(x),x)-2*x^5*y(x)-y
```

No solution found

✗ Solution by Mathematica

Time used: 0.0 (sec). Leaf size: 0

```
DSolve[x (y'[x])^3 -3 x^2 y[x] (y'[x])^2 +x(x^5+3 y[x]^2) y'[x]-2 x^5 y[x]- y[x]^3==0,y[x],x
```

Timed out

36.5 problem 1068

Internal problem ID [4288]

Book: Ordinary differential equations and their solutions. By George Moseley Murphy. 1960
Section: Various 36
Problem number: 1068.
ODE order: 1.
ODE degree: 3.

CAS Maple gives this as type `[[_homogeneous, ‘class G‘]]`

$$2x^3y'^3 + 6x^2yy'^2 - (1 - 6yx)\,yy' + 2y^3 = 0$$

✓ Solution by Maple

Time used: 3.766 (sec). Leaf size: 1755

```
dsolve(2*x^3*diff(y(x),x)^3+6*x^2*y(x)*diff(y(x),x)^2-(1-6*x*y(x))*y(x)*diff(y(x),x)+2*y(x)^
```

$y(x) = 0$
Expression too large to display
Expression too large to display
Expression too large to display

✓ Solution by Mathematica

Time used: 62.111 (sec). Leaf size: 179

```
DSolve[2 x^3 (y'[x])^3 +6 x^2 y[x] (y'[x])^2 -(1-6 x y[x])y[x] y'[x]+2 y[x]^3==0,y[x],x,Incl
```

$$y(x) \to \frac{\int_1^x \frac{\text{InverseFunction}\left[-\frac{2\sqrt{\#1^2-8\#1^3}\arctan\left(\sqrt{8\#1-1}\right)}{\#1\sqrt{8\#1-1}} - 14\log\left(\#1^2(8\#1-1)\right) + \log\left(\#1^{14}(8\#1-1)^{15/2}\left(\#1-\sqrt{\#1^2-8\#\right.\right.\right.}{K[1]}}{x}$$

36.6 problem 1070

Internal problem ID [4289]

Book: Ordinary differential equations and their solutions. By George Moseley Murphy. 1960
Section: Various 36
Problem number: 1070.
ODE order: 1.
ODE degree: 3.

CAS Maple gives this as type `[[_1st_order, _with_linear_symmetries]]`

$$x^4 {y'}^3 - x^3 y {y'}^2 - x^2 y^2 y' + x y^3 = 1$$

✓ Solution by Maple

Time used: 0.672 (sec). Leaf size: 698

```
dsolve(x^4*diff(y(x),x)^3-x^3*y(x)*diff(y(x),x)^2-x^2*y(x)^2*diff(y(x),x)+x*y(x)^3 = 1,y(x),
```

$$y(x) = \frac{3\, 2^{\frac{1}{3}} (x^2)^{\frac{1}{3}}}{4x}$$

$$y(x) = -\frac{3\, 2^{\frac{1}{3}} (x^2)^{\frac{1}{3}} \left(1 + i\sqrt{3}\right)}{8x}$$

$$y(x) = \frac{3\, 2^{\frac{1}{3}} (x^2)^{\frac{1}{3}} \left(-1 + i\sqrt{3}\right)}{8x}$$

$$y(x) = \frac{\mathrm{RootOf}\left(-\ln(x) + 6\left(\int^{\_Z} \frac{\left(-32\_\_a^3 + 6\sqrt{-96\_\_a^3 + 81} + 54\right)^{\frac{1}{3}}}{8\, 2^{\frac{2}{3}} \_\_a^2 + 2^{\frac{1}{3}} \left(-32\_\_a^3 + 6\sqrt{-96\_\_a^3 + 81} + 54\right)^{\frac{2}{3}} + 4\_\_a\left(-32\_\_a^3 + 6\sqrt{-96\_\_a^3 + 81} + 54\right)^{\frac{1}{3}}} d\_a\right.\right.}{x^{\frac{1}{3}}}$$

$$y(x) = \frac{\mathrm{RootOf}\left(3i\sqrt{3}\left(\int^{\_Z} \frac{\left(-32\_\_a^3 + 6\sqrt{-96\_\_a^3 + 81} + 54\right)^{\frac{1}{3}}}{4i\sqrt{3}\, 2^{\frac{2}{3}} \_\_a^2 - 2i\_\_a\sqrt{3}\left(-32\_\_a^3 + 6\sqrt{-96\_\_a^3 + 81} + 54\right)^{\frac{1}{3}} + 4\, 2^{\frac{2}{3}} \_\_a^2 - 2^{\frac{1}{3}} \left(-32\_\_a^3 + 6\sqrt{-96\_\_a^3 + 81} + 5\right.}\right.\right.}{}$$

$$y(x) = \frac{\mathrm{RootOf}\left(3i\sqrt{3}\left(\int^{\_Z} \frac{\left(-32\_\_a^3 + 6\sqrt{-96\_\_a^3 + 81} + 54\right)^{\frac{1}{3}}}{4i\sqrt{3}\, 2^{\frac{2}{3}} \_\_a^2 - 2i\_\_a\sqrt{3}\left(-32\_\_a^3 + 6\sqrt{-96\_\_a^3 + 81} + 54\right)^{\frac{1}{3}} - 4\, 2^{\frac{2}{3}} \_\_a^2 + 2^{\frac{1}{3}} \left(-32\_\_a^3 + 6\sqrt{-96\_\_a^3 + 81} + 5\right.}\right.\right.}{}$$

✓ Solution by Mathematica

Time used: 84.141 (sec). Leaf size: 67473

```
DSolve[x^4 (y'[x])^3 -x^3 y[x] (y'[x])^2 - x^2 y[x]^2 y'[x]+x y[x]^3==1,y[x],x,IncludeSingul
```

Too large to display

36.7 problem 1071

Internal problem ID [4290]

Book: Ordinary differential equations and their solutions. By George Moseley Murphy. 1960
Section: Various 36
Problem number: 1071.
ODE order: 1.
ODE degree: 3.

CAS Maple gives this as type `[[_1st_order, _with_linear_symmetries]]`

$$x^6 {y'}^3 - x y' - y = 0$$

✓ Solution by Maple

Time used: 0.125 (sec). Leaf size: 36

```
dsolve(x^6*diff(y(x),x)^3-x*diff(y(x),x)-y(x) = 0,y(x), singsol=all)
```

$$y(x) = -\frac{2\sqrt{3}}{9x^{\frac{3}{2}}}$$
$$y(x) = \frac{2\sqrt{3}}{9x^{\frac{3}{2}}}$$
$$y(x) = c_1^3 - \frac{c_1}{x}$$

✗ Solution by Mathematica

Time used: 0.0 (sec). Leaf size: 0

```
DSolve[x^6 (y'[x])^3 -x y'[x] - y[x]==0,y[x],x,IncludeSingularSolutions -> True]
```

Timed out

36.8 problem 1072

Internal problem ID [4291]

Book: Ordinary differential equations and their solutions. By George Moseley Murphy. 1960
Section: Various 36
Problem number: 1072.
ODE order: 1.
ODE degree: 3.

CAS Maple gives this as type `[[_1st_order, _with_linear_symmetries], _dAlembert]`

$$\boxed{yy'^3 - 3xy' + 3y = 0}$$

✓ Solution by Maple

Time used: 0.203 (sec). Leaf size: 607

```
dsolve(y(x)*diff(y(x),x)^3-3*x*diff(y(x),x)+3*y(x) = 0,y(x), singsol=all)
```

$$y(x) = 0$$

$$y(x) = \operatorname{RootOf}\left(-2\ln(x) + \int^{\_Z} \frac{-2\left(4\sqrt{\frac{9\_a^3-4}{\_a}}\,\_a^5+12\_a^6-24\_a^3+8\right)^{\frac{1}{3}}\_a^3-8\_a^3+\left(4\sqrt{\frac{9\_a^3-4}{\_a}}\,\_a^5+12\_a^6-24\_a^3\right.}{\_a^4\left(4\sqrt{\frac{9\_a^3-4}{\_a}}\,\_a^5+12\_a^6-24\_a^3+8\right.} + 2c_1\right)x$$

$$y(x) = \operatorname{RootOf}\left(-4\ln(x) + \int^{\_Z} \frac{8i\sqrt{3}\,\_a^3+i\sqrt{3}\left(4\sqrt{\frac{9\_a^3-4}{\_a}}\,\_a^5+12\_a^6-24\_a^3+8\right)^{\frac{2}{3}}-4\left(4\sqrt{\frac{9\_a^3-4}{\_a}}\,\_a^5+12\_a^6-24\_\right.}{\_a^4\left(4\sqrt{\right.}} + 4c_1\right)x$$

$$y(x) = \operatorname{RootOf}\left(-4\ln(x) - \left(\int^{\_Z} \frac{8i\sqrt{3}\,\_a^3+i\sqrt{3}\left(4\sqrt{\frac{9\_a^3-4}{\_a}}\,\_a^5+12\_a^6-24\_a^3+8\right)^{\frac{2}{3}}+4\left(4\sqrt{\frac{9\_a^3-4}{\_a}}\,\_a^5+12\_a^6-24\right.}{\_a^4\left(4\right.} + 4c_1\right)x$$

✓ Solution by Mathematica

Time used: 152.094 (sec). Leaf size: 8706

```
DSolve[y[x] (y'[x])^3 -3 x y'[x] + 3 y[x]==0,y[x],x,IncludeSingularSolutions -> True]
```

Too large to display

36.9 problem 1073

Internal problem ID [4292]

Book: Ordinary differential equations and their solutions. By George Moseley Murphy. 1960
Section: Various 36
Problem number: 1073.
ODE order: 1.
ODE degree: 3.

CAS Maple gives this as type `[[_1st_order, _with_linear_symmetries], _dAlembert]`

$$\boxed{2yy'^3 - 3xy' + 2y = 0}$$

✓ Solution by Maple

Time used: 0.078 (sec). Leaf size: 734

```
dsolve(2*y(x)*diff(y(x),x)^3-3*x*diff(y(x),x)+2*y(x) = 0,y(x), singsol=all)
```

$$y(x) = \frac{2^{\frac{2}{3}}x}{2}$$
$$y(x) = -\frac{2^{\frac{2}{3}}\left(1+i\sqrt{3}\right)x}{4}$$
$$y(x) = \frac{2^{\frac{2}{3}}\left(-1+i\sqrt{3}\right)x}{4}$$
$$y(x) = 0$$

$$y(x) = \mathrm{RootOf}\left(-\ln(x) + \int^{\_Z} -\frac{2\left(\left(\sqrt{2}\,\sqrt{\frac{1}{\_a(2\_a^3-1)}}\,\_a^2+1\right)(2\_a^3-1)^2\right)^{\frac{1}{3}}\_a^3 + 2\_a^3 - \left(\left(\sqrt{2}\,\sqrt{\frac{1}{\_a(2\_a^3-1)}}\,\_a^2+1\right)(2\_a^3 -\right.}{\_a\,(2\_a^3-1)\left(\left(\sqrt{2}\,\sqrt{\frac{1}{\_a(2\_a^3-1)}}\,\_a^2+1\right)(2\_a\right.} + c_1\right) x$$

$$y(x) = \mathrm{RootOf}\left(-2\ln(x) + \int^{\_Z} \frac{2i\sqrt{3}\_a^3 + i\sqrt{3}\left(\left(\sqrt{2}\,\sqrt{\frac{1}{\_a(2\_a^3-1)}}\,\_a^2+1\right)(2\_a^3-1)^2\right)^{\frac{2}{3}} - 4\left(\left(\sqrt{2}\,\sqrt{\frac{1}{\_a(2\_a^3-1)}}\,\_a^2+1\right.\right.}{\left(\left(\sqrt{2}\,\sqrt{\right.\right.} + 2c_1\right) x$$

$$y(x) = \mathrm{RootOf}\left(-2\ln(x) - \left(\int^{\_Z} \frac{2i\sqrt{3}\_a^3 + i\sqrt{3}\left(\left(\sqrt{2}\,\sqrt{\frac{1}{\_a(2\_a^3-1)}}\,\_a^2+1\right)(2\_a^3-1)^2\right)^{\frac{2}{3}} + 4\left(\left(\sqrt{2}\,\sqrt{\frac{1}{\_a(2\_a^3-1)}}\,\_a^2+\right.\right.}{\_a\,(2\_} \right.\right.$$

$$+ 2c_1\Big) x$$

✓ Solution by Mathematica

Time used: 172.826 (sec). Leaf size: 10331

```
DSolve[2 y[x] (y'[x])^3 -3 x y'[x]+2 y[x]==0,y[x],x,IncludeSingularSolutions -> True]
```

Too large to display

36.10 problem 1076

Internal problem ID [4293]

Book: Ordinary differential equations and their solutions. By George Moseley Murphy. 1960
Section: Various 36
Problem number: 1076.
ODE order: 1.
ODE degree: 3.

CAS Maple gives this as type `[_quadrature]`

$$(2y+x)\,{y'}^3+3(x+y)\,{y'}^2+(2x+y)\,y'=0$$

✓ Solution by Maple

Time used: 0.031 (sec). Leaf size: 65

```
dsolve((x+2*y(x))*diff(y(x),x)^3+3*(x+y(x))*diff(y(x),x)^2+(2*x+y(x))*diff(y(x),x) = 0,y(x),
```

$$y(x)=c_1-x$$
$$y(x)=\frac{-c_1x-\sqrt{-3c_1^2x^2+4}}{2c_1}$$
$$y(x)=\frac{-c_1x+\sqrt{-3c_1^2x^2+4}}{2c_1}$$
$$y(x)=c_1$$

✓ Solution by Mathematica

Time used: 0.471 (sec). Leaf size: 130

```
DSolve[(x+2 y[x])(y'[x])^3+3 (x+y[x]) (y'[x])^2+ (2 x+y[x]) y'[x] ==0,y[x],x,IncludeSingular
```

$$y(x) \to \frac{1}{2}\left(-x-\sqrt{-3x^2+4e^{c_1}}\right)$$
$$y(x) \to \frac{1}{2}\left(-x+\sqrt{-3x^2+4e^{c_1}}\right)$$
$$y(x) \to c_1$$
$$y(x) \to -x+c_1$$
$$y(x) \to \frac{1}{2}\left(-\sqrt{3}\sqrt{-x^2}-x\right)$$
$$y(x) \to \frac{1}{2}\left(\sqrt{3}\sqrt{-x^2}-x\right)$$

36.11 problem 1077

Internal problem ID [4294]

Book: Ordinary differential equations and their solutions. By George Moseley Murphy. 1960
Section: Various 36
Problem number: 1077.
ODE order: 1.
ODE degree: 3.

CAS Maple gives this as type `[[_1st_order, _with_linear_symmetries]]`

$$y^2 y'^3 - xy' + y = 0$$

✓ Solution by Maple

Time used: 0.11 (sec). Leaf size: 133

```
dsolve(y(x)^2*diff(y(x),x)^3-x*diff(y(x),x)+y(x) = 0,y(x), singsol=all)
```

$$y(x) = 0$$
$$y(x) = -\frac{2\sqrt{-24c_1^3 + 27c_1 x - 3\sqrt{(4c_1^2 - 3x)^3}}}{9}$$
$$y(x) = \frac{2\sqrt{-24c_1^3 + 27c_1 x - 3\sqrt{(4c_1^2 - 3x)^3}}}{9}$$
$$y(x) = -\frac{2\sqrt{-24c_1^3 + 27c_1 x + 3\sqrt{(4c_1^2 - 3x)^3}}}{9}$$
$$y(x) = \frac{2\sqrt{-24c_1^3 + 27c_1 x + 3\sqrt{(4c_1^2 - 3x)^3}}}{9}$$

✗ Solution by Mathematica

Time used: 0.0 (sec). Leaf size: 0

```
DSolve[y[x]^2 (y'[x])^3- x y'[x] + y[x]==0,y[x],x,IncludeSingularSolutions -> True]
```

Timed out

36.12 problem 1078

Internal problem ID [4295]

Book: Ordinary differential equations and their solutions. By George Moseley Murphy. 1960
Section: Various 36
Problem number: 1078.
ODE order: 1.
ODE degree: 3.

CAS Maple gives this as type `[[_1st_order, _with_linear_symmetries]]`

$$y^2 {y'}^3 + 2xy' - y = 0$$

✓ Solution by Maple

Time used: 0.094 (sec). Leaf size: 97

```
dsolve(y(x)^2*diff(y(x),x)^3+2*x*diff(y(x),x)-y(x) = 0,y(x), singsol=all)
```

$$y(x) = -\frac{2(-x^3)^{\frac{1}{4}} 6^{\frac{1}{4}}}{3}$$
$$y(x) = \frac{2(-x^3)^{\frac{1}{4}} 6^{\frac{1}{4}}}{3}$$
$$y(x) = -\frac{2i(-x^3)^{\frac{1}{4}} 6^{\frac{1}{4}}}{3}$$
$$y(x) = \frac{2i(-x^3)^{\frac{1}{4}} 6^{\frac{1}{4}}}{3}$$
$$y(x) = 0$$
$$y(x) = \sqrt{c_1 (c_1^2 + 2x)}$$
$$y(x) = -\sqrt{c_1 (c_1^2 + 2x)}$$

✓ Solution by Mathematica

Time used: 0.136 (sec). Leaf size: 119

```
DSolve[y[x]^2 (y'[x])^3+2 x y'[x] -y[x]==0,y[x],x,IncludeSingularSolutions -> True]
```

$$
\begin{aligned}
y(x) &\to -\sqrt{2c_1x + c_1{}^3} \\
y(x) &\to \sqrt{2c_1x + c_1{}^3} \\
y(x) &\to (-1-i)\left(\frac{2}{3}\right)^{3/4} x^{3/4} \\
y(x) &\to (1-i)\left(\frac{2}{3}\right)^{3/4} x^{3/4} \\
y(x) &\to (-1+i)\left(\frac{2}{3}\right)^{3/4} x^{3/4} \\
y(x) &\to (1+i)\left(\frac{2}{3}\right)^{3/4} x^{3/4}
\end{aligned}
$$

36.13 problem 1079

Internal problem ID [4296]

Book: Ordinary differential equations and their solutions. By George Moseley Murphy. 1960
Section: Various 36
Problem number: 1079.
ODE order: 1.
ODE degree: 3.

CAS Maple gives this as type `[[_1st_order, _with_linear_symmetries]]`

$$4y^2{y'}^3 - 2xy' + y = 0$$

✓ Solution by Maple

Time used: 0.094 (sec). Leaf size: 100

```
dsolve(4*y(x)^2*diff(y(x),x)^3-2*x*diff(y(x),x)+y(x) = 0,y(x), singsol=all)
```

$$y(x) = -\frac{2^{\frac{3}{4}}3^{\frac{1}{4}}x^{\frac{3}{4}}}{3}$$
$$y(x) = \frac{2^{\frac{3}{4}}3^{\frac{1}{4}}x^{\frac{3}{4}}}{3}$$
$$y(x) = -\frac{i2^{\frac{3}{4}}3^{\frac{1}{4}}x^{\frac{3}{4}}}{3}$$
$$y(x) = \frac{i2^{\frac{3}{4}}3^{\frac{1}{4}}x^{\frac{3}{4}}}{3}$$
$$y(x) = 0$$
$$y(x) = \sqrt{2}\sqrt{c_1(-2c_1^2 + x)}$$
$$y(x) = -\sqrt{2}\sqrt{c_1(-2c_1^2 + x)}$$

✓ Solution by Mathematica

Time used: 83.677 (sec). Leaf size: 11250

```
DSolve[4 y[x]^2 (y'[x])^3 - 2 x y'[x] +y[x]==0,y[x],x,IncludeSingularSolutions -> True]
```

Too large to display

36.14 problem 1080

Internal problem ID [4297]

Book: Ordinary differential equations and their solutions. By George Moseley Murphy. 1960
Section: Various 36
Problem number: 1080.
ODE order: 1.
ODE degree: 3.

CAS Maple gives this as type `[[_1st_order, _with_linear_symmetries]]`

$$16y^2{y'}^3 + 2xy' - y = 0$$

✓ Solution by Maple
Time used: 0.094 (sec). Leaf size: 104

```
dsolve(16*y(x)^2*diff(y(x),x)^3+2*x*diff(y(x),x)-y(x) = 0,y(x), singsol=all)
```

$$y(x) = -\frac{(-x^3)^{\frac{1}{4}} 6^{\frac{1}{4}}}{3}$$
$$y(x) = \frac{(-x^3)^{\frac{1}{4}} 6^{\frac{1}{4}}}{3}$$
$$y(x) = -\frac{i(-x^3)^{\frac{1}{4}} 6^{\frac{1}{4}}}{3}$$
$$y(x) = \frac{i(-x^3)^{\frac{1}{4}} 6^{\frac{1}{4}}}{3}$$
$$y(x) = 0$$
$$y(x) = \sqrt{2}\sqrt{c_1 (8c_1^2 + x)}$$
$$y(x) = -\sqrt{2}\sqrt{c_1 (8c_1^2 + x)}$$

✓ Solution by Mathematica

Time used: 0.137 (sec). Leaf size: 107

```
DSolve[16 y[x]^2 (y'[x])^3 +2 x y'[x] -y[x]==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to \sqrt{c_1 (x + 2c_1{}^2)}$$
$$y(x) \to -\frac{\sqrt[4]{-2}x^{3/4}}{3^{3/4}}$$
$$y(x) \to \frac{(1-i)x^{3/4}}{\sqrt[4]{2}3^{3/4}}$$
$$y(x) \to \frac{i\sqrt[4]{-2}x^{3/4}}{3^{3/4}}$$
$$y(x) \to \frac{\sqrt[4]{-2}x^{3/4}}{3^{3/4}}$$

36.15 problem 1081

Internal problem ID [4298]

Book: Ordinary differential equations and their solutions. By George Moseley Murphy. 1960
Section: Various 36
Problem number: 1081.
ODE order: 1.
ODE degree: 3.

CAS Maple gives this as type `[‘y=_G(x,y’)‘]`

$$x y^{2} {y'}^{3}-y^{3} {y'}^{2}+x \left(x^{2}+1\right) y'-y x^{2} = 0$$

✗ Solution by Maple

```
dsolve(x*y(x)^2*diff(y(x),x)^3-y(x)^3*diff(y(x),x)^2+x*(x^2+1)*diff(y(x),x)-x^2*y(x) = 0,y(x
```

No solution found

✓ Solution by Mathematica

Time used: 0.56 (sec). Leaf size: 399

```
DSolve[x y[x]^2 (y'[x])^3 -y[x]^3 (y'[x])^2 + x (1+x^2) y'[x] -x^2 y[x]==0,y[x],x,IncludeSin
```

$$y(x) \to -\sqrt{c_1\left(x^2+\frac{1}{1+{c_1}^2}\right)}$$

$$y(x) \to \sqrt{c_1\left(x^2+\frac{1}{1+{c_1}^2}\right)}$$

$$y(x) \to -\frac{\sqrt[4]{-8x^4+20x^2-\sqrt{-\left(8x^2-1\right)^3}+1}}{2^{3/4}}$$

$$y(x) \to -\frac{i\sqrt[4]{-8x^4+20x^2-\sqrt{-\left(8x^2-1\right)^3}+1}}{2^{3/4}}$$

$$y(x) \to \frac{i\sqrt[4]{-8x^4+20x^2-\sqrt{-\left(8x^2-1\right)^3}+1}}{2^{3/4}}$$

$$y(x) \to \frac{\sqrt[4]{-8x^4+20x^2-\sqrt{-\left(8x^2-1\right)^3}+1}}{2^{3/4}}$$

$$y(x) \to -\frac{\sqrt[4]{-8x^4+20x^2+\sqrt{-\left(8x^2-1\right)^3}+1}}{2^{3/4}}$$

$$y(x) \to -\frac{i\sqrt[4]{-8x^4+20x^2+\sqrt{-\left(8x^2-1\right)^3}+1}}{2^{3/4}}$$

$$y(x) \to \frac{i\sqrt[4]{-8x^4+20x^2+\sqrt{-\left(8x^2-1\right)^3}+1}}{2^{3/4}}$$

$$y(x) \to \frac{\sqrt[4]{-8x^4+20x^2+\sqrt{-\left(8x^2-1\right)^3}+1}}{2^{3/4}}$$

36.16 problem 1084

Internal problem ID [4299]

Book: Ordinary differential equations and their solutions. By George Moseley Murphy. 1960
Section: Various 36
Problem number: 1084.
ODE order: 1.
ODE degree: 3.

CAS Maple gives this as type `[‘y=_G(x,y’)‘]`

$$\boxed{y^3y'^3 - (1 - 3x)\, y^2y'^2 + 3x^2yy' - y^2 = -x^3}$$

✓ Solution by Maple

Time used: 0.188 (sec). Leaf size: 349

```
dsolve(y(x)^3*diff(y(x),x)^3-(1-3*x)*y(x)^2*diff(y(x),x)^2+3*x^2*y(x)*diff(y(x),x)+x^3-y(x)^
```

$$y(x) = -\frac{\sqrt{-6-81x^2-6\sqrt{-\left(6x-1\right)^3}+54x}}{9}$$
$$y(x) = \frac{\sqrt{-6-81x^2-6\sqrt{-\left(6x-1\right)^3}+54x}}{9}$$
$$y(x) = -\frac{\sqrt{-6-81x^2+6\sqrt{-\left(6x-1\right)^3}+54x}}{9}$$
$$y(x) = \frac{\sqrt{-6-81x^2+6\sqrt{-\left(6x-1\right)^3}+54x}}{9}$$
$$y(x) = \sqrt{-\left(c_1^3\right)^{\frac{2}{3}}+2c_1x+c_1^3-x^2}$$
$$y(x) = -\sqrt{-\left(c_1^3\right)^{\frac{2}{3}}+2c_1x+c_1^3-x^2}$$
$$y(x) = -\frac{\sqrt{\left(-2i\sqrt{3}+2\right)\left(c_1^3\right)^{\frac{2}{3}}-4i\sqrt{3}\,c_1x+4c_1^3-4x^2-4c_1x}}{2}$$
$$y(x) = \frac{\sqrt{\left(-2i\sqrt{3}+2\right)\left(c_1^3\right)^{\frac{2}{3}}-4i\sqrt{3}\,c_1x+4c_1^3-4x^2-4c_1x}}{2}$$
$$y(x) = -\frac{\sqrt{\left(2i\sqrt{3}+2\right)\left(c_1^3\right)^{\frac{2}{3}}+4i\sqrt{3}\,c_1x+4c_1^3-4x^2-4c_1x}}{2}$$
$$y(x) = \frac{\sqrt{\left(2i\sqrt{3}+2\right)\left(c_1^3\right)^{\frac{2}{3}}+4i\sqrt{3}\,c_1x+4c_1^3-4x^2-4c_1x}}{2}$$

✗ Solution by Mathematica

Time used: 0.0 (sec). Leaf size: 0

```
DSolve[y[x]^3 (y'[x])^3 -(1-3 x) y[x]^2 (y'[x])^2 +3 x^2 y[x] y'[x]+x^3 - y[x]^2==0,y[x],x,I
```

Timed out

36.17 problem 1085

Internal problem ID [4300]

Book: Ordinary differential equations and their solutions. By George Moseley Murphy. 1960
Section: Various 36
Problem number: 1085.
ODE order: 1.
ODE degree: 3.

CAS Maple gives this as type `[[_1st_order, _with_linear_symmetries]]`

$$y^4 {y'}^3 - 6xy' + 2y = 0$$

✓ Solution by Maple

Time used: 0.109 (sec). Leaf size: 167

```
dsolve(y(x)^4*diff(y(x),x)^3-6*x*diff(y(x),x)+2*y(x) = 0,y(x), singsol=all)
```

$$y(x) = \sqrt{x\left(-1 - i\sqrt{3}\right)}$$
$$y(x) = \sqrt{\left(-1 + i\sqrt{3}\right)x}$$
$$y(x) = -\sqrt{-\left(1 + i\sqrt{3}\right)x}$$
$$y(x) = -\sqrt{\left(-1 + i\sqrt{3}\right)x}$$
$$y(x) = \sqrt{2}\,\sqrt{x}$$
$$y(x) = -\sqrt{2}\,\sqrt{x}$$
$$y(x) = 0$$
$$y(x) = \frac{2^{\frac{2}{3}}(-c_1^3 + 6c_1x)^{\frac{1}{3}}}{2}$$
$$y(x) = -\frac{2^{\frac{2}{3}}(-c_1^3 + 6c_1x)^{\frac{1}{3}}\left(1 + i\sqrt{3}\right)}{4}$$
$$y(x) = \frac{2^{\frac{2}{3}}(-c_1^3 + 6c_1x)^{\frac{1}{3}}\left(-1 + i\sqrt{3}\right)}{4}$$

✓ Solution by Mathematica

Time used: 81.226 (sec). Leaf size: 22649

```
DSolve[y[x]^4 (y'[x])^3 -6 x y'[x] +2 y[x]==0,y[x],x,IncludeSingularSolutions -> True]
```

Too large to display

36.18 problem 1086

Internal problem ID [4301]

Book: Ordinary differential equations and their solutions. By George Moseley Murphy. 1960
Section: Various 36
Problem number: 1086.
ODE order: 1.
ODE degree: 4.

CAS Maple gives this as type `[_quadrature]`

$$\boxed{y'^4 - (y-a)^3 (y-b)^2 = 0}$$

✓ Solution by Maple
Time used: 0.047 (sec). Leaf size: 133

```
dsolve(diff(y(x),x)^4 = (y(x)-a)^3*(y(x)-b)^2,y(x), singsol=all)
```

$$y(x) = a$$
$$y(x) = b$$
$$x - \left(\int^{y(x)} \frac{1}{\left((\_a-a)^3 (\_a-b)^2\right)^{\frac{1}{4}}} d\_a\right) - c_1 = 0$$
$$x - i\left(\int^{y(x)} \frac{1}{\left((\_a-a)^3 (\_a-b)^2\right)^{\frac{1}{4}}} d\_a\right) - c_1 = 0$$
$$x + i\left(\int^{y(x)} \frac{1}{\left((\_a-a)^3 (\_a-b)^2\right)^{\frac{1}{4}}} d\_a\right) - c_1 = 0$$
$$x + \int^{y(x)} \frac{1}{\left((\_a-a)^3 (\_a-b)^2\right)^{\frac{1}{4}}} d\_a - c_1 = 0$$

✓ Solution by Mathematica

Time used: 2.363 (sec). Leaf size: 333

```
DSolve[(y'[x])^4 == (y[x]-a)^3 (y[x]-b)^2 ,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to \text{InverseFunction}\left[-\frac{4\sqrt[4]{a-\#1}\sqrt{\frac{\#1-b}{a-b}}\,\text{Hypergeometric2F1}\left(\frac{1}{4},\frac{1}{2},\frac{5}{4},\frac{a-\#1}{a-b}\right)}{\sqrt{b-\#1}}\&\right]\left[-\sqrt[4]{-1}x+c_1\right]$$

$$y(x) \to \text{InverseFunction}\left[-\frac{4\sqrt[4]{a-\#1}\sqrt{\frac{\#1-b}{a-b}}\,\text{Hypergeometric2F1}\left(\frac{1}{4},\frac{1}{2},\frac{5}{4},\frac{a-\#1}{a-b}\right)}{\sqrt{b-\#1}}\&\right]\left[\sqrt[4]{-1}x+c_1\right]$$

$$y(x) \to \text{InverseFunction}\left[-\frac{4\sqrt[4]{a-\#1}\sqrt{\frac{\#1-b}{a-b}}\,\text{Hypergeometric2F1}\left(\frac{1}{4},\frac{1}{2},\frac{5}{4},\frac{a-\#1}{a-b}\right)}{\sqrt{b-\#1}}\&\right]\left[-(-1)^{3/4}x+c_1\right]$$

$$y(x) \to \text{InverseFunction}\left[-\frac{4\sqrt[4]{a-\#1}\sqrt{\frac{\#1-b}{a-b}}\,\text{Hypergeometric2F1}\left(\frac{1}{4},\frac{1}{2},\frac{5}{4},\frac{a-\#1}{a-b}\right)}{\sqrt{b-\#1}}\&\right]\left[(-1)^{3/4}x+c_1\right]$$

$$y(x) \to a$$
$$y(x) \to b$$

36.19 problem 1087

Internal problem ID [4302]

Book: Ordinary differential equations and their solutions. By George Moseley Murphy. 1960
Section: Various 36
Problem number: 1087.
ODE order: 1.
ODE degree: 4.

CAS Maple gives this as type `[[_1st_order, '_with_symmetry_[F(x),G(x)*y+H(x)]']]`

$$y'^4 + f(x)(y-a)^3(y-b)^2 = 0$$

✓ Solution by Maple

Time used: 0.062 (sec). Leaf size: 274

```
dsolve(diff(y(x),x)^4+f(x)*(y(x)-a)^3*(y(x)-b)^2 = 0,y(x), singsol=all)
```

$$\int^{y(x)} \frac{1}{(\_a-a)^{\frac{3}{4}}\sqrt{\_a-b}}d\_a - \frac{\int^x \left(-f(\_a)(y(x)-a)^3(y(x)-b)^2\right)^{\frac{1}{4}}d\_a}{(y(x)-a)^{\frac{3}{4}}\sqrt{y(x)-b}} + c_1 = 0$$

$$\int^{y(x)} \frac{1}{(\_a-a)^{\frac{3}{4}}\sqrt{\_a-b}}d\_a + \frac{i\left(\int^x \left(-f(\_a)(y(x)-a)^3(y(x)-b)^2\right)^{\frac{1}{4}}d\_a\right)}{(y(x)-a)^{\frac{3}{4}}\sqrt{y(x)-b}} + c_1 = 0$$

$$\int^{y(x)} \frac{1}{(\_a-a)^{\frac{3}{4}}\sqrt{\_a-b}}d\_a - \frac{i\left(\int^x \left(-f(\_a)(y(x)-a)^3(y(x)-b)^2\right)^{\frac{1}{4}}d\_a\right)}{(y(x)-a)^{\frac{3}{4}}\sqrt{y(x)-b}} + c_1 = 0$$

$$\int^{y(x)} \frac{1}{(\_a-a)^{\frac{3}{4}}\sqrt{\_a-b}}d\_a + \frac{\int^x \left(-f(\_a)(y(x)-a)^3(y(x)-b)^2\right)^{\frac{1}{4}}d\_a}{(y(x)-a)^{\frac{3}{4}}\sqrt{y(x)-b}} + c_1 = 0$$

✓ Solution by Mathematica

Time used: 1.878 (sec). Leaf size: 369

```
DSolve[(y'[x])^4 +f[x] (y[x]-a)^3 (y[x]-b)^2==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to \text{InverseFunction}\left[-\frac{4\sqrt[4]{a-\#1}\sqrt{\frac{\#1-b}{a-b}}\,\text{Hypergeometric2F1}\left(\frac{1}{4},\frac{1}{2},\frac{5}{4},\frac{a-\#1}{a-b}\right)}{\sqrt{b-\#1}}\&\right]\left[\int_1^x -\sqrt[4]{f(K[1])}dK[1]+c_1\right]$$

$$y(x) \to \text{InverseFunction}\left[-\frac{4\sqrt[4]{a-\#1}\sqrt{\frac{\#1-b}{a-b}}\,\text{Hypergeometric2F1}\left(\frac{1}{4},\frac{1}{2},\frac{5}{4},\frac{a-\#1}{a-b}\right)}{\sqrt{b-\#1}}\&\right]\left[\int_1^x -i\sqrt[4]{f(K[2])}dK[2]+c_1\right]$$

$$y(x) \to \text{InverseFunction}\left[-\frac{4\sqrt[4]{a-\#1}\sqrt{\frac{\#1-b}{a-b}}\,\text{Hypergeometric2F1}\left(\frac{1}{4},\frac{1}{2},\frac{5}{4},\frac{a-\#1}{a-b}\right)}{\sqrt{b-\#1}}\&\right]\left[\int_1^x i\sqrt[4]{f(K[3])}dK[3]+c_1\right]$$

$$y(x) \to \text{InverseFunction}\left[-\frac{4\sqrt[4]{a-\#1}\sqrt{\frac{\#1-b}{a-b}}\,\text{Hypergeometric2F1}\left(\frac{1}{4},\frac{1}{2},\frac{5}{4},\frac{a-\#1}{a-b}\right)}{\sqrt{b-\#1}}\&\right]\left[\int_1^x \sqrt[4]{f(K[4])}dK[4]+c_1\right]$$

$$y(x) \to a$$
$$y(x) \to b$$

36.20 problem 1088

Internal problem ID [4303]

Book: Ordinary differential equations and their solutions. By George Moseley Murphy. 1960
Section: Various 36
Problem number: 1088.
ODE order: 1.
ODE degree: 4.

CAS Maple gives this as type `[[_1st_order, '_with_symmetry_[F(x),G(x)*y+H(x)]']]`

$$\boxed{y'^4 + f(x)\,(y-a)^3\,(y-b)^3 = 0}$$

✓ Solution by Maple

Time used: 0.078 (sec). Leaf size: 262

```
dsolve(diff(y(x),x)^4+f(x)*(y(x)-a)^3*(y(x)-b)^3 = 0,y(x), singsol=all)
```

$$\int^{y(x)} \frac{1}{\left((\_a-b)\,(\_a-a)\right)^{\frac{3}{4}}}d\_a - \frac{\int^x \left(-f(\_a)\,(y(x)-a)^3\,(y(x)-b)^3\right)^{\frac{1}{4}} d\_a}{\left((y(x)-b)\,(y(x)-a)\right)^{\frac{3}{4}}} + c_1 = 0$$

$$\int^{y(x)} \frac{1}{\left((\_a-b)\,(\_a-a)\right)^{\frac{3}{4}}}d\_a + \frac{i\left(\int^x \left(-f(\_a)\,(y(x)-a)^3\,(y(x)-b)^3\right)^{\frac{1}{4}} d\_a\right)}{\left((y(x)-b)\,(y(x)-a)\right)^{\frac{3}{4}}} + c_1 = 0$$

$$\int^{y(x)} \frac{1}{\left((\_a-b)\,(\_a-a)\right)^{\frac{3}{4}}}d\_a - \frac{i\left(\int^x \left(-f(\_a)\,(y(x)-a)^3\,(y(x)-b)^3\right)^{\frac{1}{4}} d\_a\right)}{\left((y(x)-b)\,(y(x)-a)\right)^{\frac{3}{4}}} + c_1 = 0$$

$$\int^{y(x)} \frac{1}{\left((\_a-b)\,(\_a-a)\right)^{\frac{3}{4}}}d\_a + \frac{\int^x \left(-f(\_a)\,(y(x)-a)^3\,(y(x)-b)^3\right)^{\frac{1}{4}} d\_a}{\left((y(x)-b)\,(y(x)-a)\right)^{\frac{3}{4}}} + c_1 = 0$$

✓ Solution by Mathematica

Time used: 2.126 (sec). Leaf size: 385

```
DSolve[(y'[x])^4 +f[x] (y[x]-a)^3 (y[x]-b)^3==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to \text{InverseFunction}\left[-\frac{4\sqrt[4]{a-\#1}\left(\frac{\#1-b}{a-b}\right)^{3/4}\text{Hypergeometric2F1}\left(\frac{1}{4},\frac{3}{4},\frac{5}{4},\frac{a-\#1}{a-b}\right)}{(b-\#1)^{3/4}}\&\right]\left[\int_1^x -\sqrt[4]{-1}\sqrt[4]{f(K[\right.$$

$$y(x) \to \text{InverseFunction}\left[-\frac{4\sqrt[4]{a-\#1}\left(\frac{\#1-b}{a-b}\right)^{3/4}\text{Hypergeometric2F1}\left(\frac{1}{4},\frac{3}{4},\frac{5}{4},\frac{a-\#1}{a-b}\right)}{(b-\#1)^{3/4}}\&\right]\left[\int_1^x \sqrt[4]{-1}\sqrt[4]{f(K[2]\right.$$

$$y(x) \to \text{InverseFunction}\left[-\frac{4\sqrt[4]{a-\#1}\left(\frac{\#1-b}{a-b}\right)^{3/4}\text{Hypergeometric2F1}\left(\frac{1}{4},\frac{3}{4},\frac{5}{4},\frac{a-\#1}{a-b}\right)}{(b-\#1)^{3/4}}\&\right]\left[\int_1^x -(-1)^{3/4}\sqrt[4]{f(\right.$$

$$y(x) \to \text{InverseFunction}\left[-\frac{4\sqrt[4]{a-\#1}\left(\frac{\#1-b}{a-b}\right)^{3/4}\text{Hypergeometric2F1}\left(\frac{1}{4},\frac{3}{4},\frac{5}{4},\frac{a-\#1}{a-b}\right)}{(b-\#1)^{3/4}}\&\right]\left[\int_1^x (-1)^{3/4}\sqrt[4]{f(K[\right.$$

$$y(x) \to a$$
$$y(x) \to b$$

36.21 problem 1089

Internal problem ID [4304]

Book: Ordinary differential equations and their solutions. By George Moseley Murphy. 1960
Section: Various 36
Problem number: 1089.
ODE order: 1.
ODE degree: 4.

CAS Maple gives this as type `[[_1st_order, '_with_symmetry_[F(x),G(x)*y+H(x)]']]`

$$
{y'}^4 + f(x)\left(y-a\right)^3\left(y-b\right)^3\left(y-c\right)^2 = 0
$$

✓ Solution by Maple

Time used: 0.187 (sec). Leaf size: 92

```
dsolve(diff(y(x),x)^4+f(x)*(y(x)-a)^3*(y(x)-b)^3*(y(x)-c)^2 = 0,y(x), singsol=all)
```

$$
\int^{y(x)} \frac{1}{\left(\_a-a\right)^{\frac{3}{4}} \sqrt{\_a-c}\,\left(\_a-b\right)^{\frac{3}{4}}} d\_a - \frac{\int^{x} \left(-f(\_a)\left(y(x)-c\right)^{2}\left(y(x)-b\right)^{3}\left(y(x)-a\right)^{3}\right)^{\frac{1}{4}} d\_a}{\left(y\left(x\right)-a\right)^{\frac{3}{4}} \sqrt{y\left(x\right)-c}\,\left(y\left(x\right)-b\right)^{\frac{3}{4}}} + c_1 = 0
$$

✓ Solution by Mathematica

Time used: 23.526 (sec). Leaf size: 562

```
DSolve[(y'[x])^4 +f[x] (y[x]-a)^3 (y[x]-b)^3 (y[x]-c)^2 ==0,y[x],x,IncludeSingularSolutions
```

$$y(x) \to \text{InverseFunction}\left[\frac{4\sqrt[4]{a-\#1}\sqrt{c-\#1}\left(\frac{(b-\#1)(a-c)}{(c-\#1)(a-b)}\right)^{3/4}\text{Hypergeometric2F1}\left(\frac{1}{4},\frac{3}{4},\frac{5}{4},\frac{(c-b)(a-\#1)}{(a-b)(c-\#1)}\right)}{(b-\#1)^{3/4}(a-c)}\&\right]\Big[$$

$$y(x) \to \text{InverseFunction}\left[\frac{4\sqrt[4]{a-\#1}\sqrt{c-\#1}\left(\frac{(b-\#1)(a-c)}{(c-\#1)(a-b)}\right)^{3/4}\text{Hypergeometric2F1}\left(\frac{1}{4},\frac{3}{4},\frac{5}{4},\frac{(c-b)(a-\#1)}{(a-b)(c-\#1)}\right)}{(b-\#1)^{3/4}(a-c)}\&\right]\Big[$$

$$y(x) \to \text{InverseFunction}\left[\frac{4\sqrt[4]{a-\#1}\sqrt{c-\#1}\left(\frac{(b-\#1)(a-c)}{(c-\#1)(a-b)}\right)^{3/4}\text{Hypergeometric2F1}\left(\frac{1}{4},\frac{3}{4},\frac{5}{4},\frac{(c-b)(a-\#1)}{(a-b)(c-\#1)}\right)}{(b-\#1)^{3/4}(a-c)}\&\right]\Big[$$

$$y(x) \to \text{InverseFunction}\left[\frac{4\sqrt[4]{a-\#1}\sqrt{c-\#1}\left(\frac{(b-\#1)(a-c)}{(c-\#1)(a-b)}\right)^{3/4}\text{Hypergeometric2F1}\left(\frac{1}{4},\frac{3}{4},\frac{5}{4},\frac{(c-b)(a-\#1)}{(a-b)(c-\#1)}\right)}{(b-\#1)^{3/4}(a-c)}\&\right]\Big[$$

$$y(x) \to a$$

$$y(x) \to b$$

$$y(x) \to c$$

36.22 problem 1090

Internal problem ID [4305]

Book: Ordinary differential equations and their solutions. By George Moseley Murphy. 1960
Section: Various 36
Problem number: 1090.
ODE order: 1.
ODE degree: 4.

CAS Maple gives this as type `[[_1st_order, _with_linear_symmetries], _dAlembert]`

$$\boxed{y'^4 + xy' - 3y = 0}$$

✓ Solution by Maple

Time used: 0.016 (sec). Leaf size: 34

```
dsolve(diff(y(x),x)^4+x*diff(y(x),x)-3*y(x) = 0,y(x), singsol=all)
```

$$\left[x(\_T) = \frac{\sqrt{\_T}\left(4\_T^{\frac{5}{2}} + 5c_1\right)}{5}, y(\_T) = \frac{3\_T^4}{5} + \frac{\_T^{\frac{3}{2}}c_1}{3}\right]$$

✗ Solution by Mathematica

Time used: 0.0 (sec). Leaf size: 0

```
DSolve[(y'[x])^4 +x y'[x]-3 y[x]==0,y[x],x,IncludeSingularSolutions -> True]
```

Timed out

36.23 problem 1092

Internal problem ID [4306]

Book: Ordinary differential equations and their solutions. By George Moseley Murphy. 1960
Section: Various 36
Problem number: 1092.
ODE order: 1.
ODE degree: 4.

CAS Maple gives this as type `[[_homogeneous, 'class G']]`

$$y'^4 - 4x^2yy'^2 + 16xy^2y' - 16y^3 = 0$$

✓ Solution by Maple

Time used: 0.078 (sec). Leaf size: 120

```
dsolve(diff(y(x),x)^4-4*x^2*y(x)*diff(y(x),x)^2+16*x*y(x)^2*diff(y(x),x)-16*y(x)^3 = 0,y(x),
```

$$y(x) = \frac{x^4}{16}$$
$$y(x) = 0$$
$$y(x)\left(\sqrt{x^2 - 4\sqrt{y(x)}} - x\right)^{-\frac{2\sqrt{x^2y(x)-4y(x)^{\frac{3}{2}}}}{\sqrt{x^2-4\sqrt{y(x)}}\,\sqrt{y(x)}}}\left(\sqrt{x^2 - 4\sqrt{y(x)}} + x\right)^{\frac{2\sqrt{x^2y(x)-4y(x)^{\frac{3}{2}}}}{\sqrt{x^2-4\sqrt{y(x)}}\,\sqrt{y(x)}}} - c_1 = 0$$

✓ Solution by Mathematica

Time used: 32.769 (sec). Leaf size: 519

```
DSolve[(y'[x])^4 -4 x^2 y[x] (y'[x])^2+16 x y[x]^2 y'[x]-16 y[x]^3==0,y[x],x,IncludeSingular
```

$$\text{Solve}\left[\frac{\sqrt{\left(x^2+4\sqrt{y(x)}\right)y(x)}\log\left(\sqrt{x^2+4\sqrt{y(x)}}-x\right)}{\sqrt{x^2+4\sqrt{y(x)}}\sqrt{y(x)}}+\frac{1}{4}\left(\log(y(x))-\frac{\sqrt{x^2+4\sqrt{y(x)}}\sqrt{y(x)}\log(y(x))}{\sqrt{\left(x^2+4\sqrt{y(x)}\right)y(x)}}\right)=c_1,y(x)\right]$$

$$\text{Solve}\left[\frac{1}{4}\left(\frac{\sqrt{x^2+4\sqrt{y(x)}}\sqrt{y(x)}\log(y(x))}{\sqrt{\left(x^2+4\sqrt{y(x)}\right)y(x)}}+\log(y(x))\right)-\frac{\sqrt{\left(x^2+4\sqrt{y(x)}\right)y(x)}\log\left(\sqrt{x^2+4\sqrt{y(x)}}-x\right)}{\sqrt{x^2+4\sqrt{y(x)}}\sqrt{y(x)}}=c_1,y(x)\right]$$

$$\text{Solve}\left[\frac{1}{2}\left(\frac{\sqrt{x^2-4\sqrt{y(x)}}\sqrt{y(x)}\log(y(x))}{2\sqrt{x^2y(x)-4y(x)^{3/2}}}+\frac{1}{2}\log(y(x))\right)-\frac{\sqrt{\left(x^2-4\sqrt{y(x)}\right)y(x)}\log\left(\sqrt{x^2-4\sqrt{y(x)}}-x\right)}{\sqrt{x^2-4\sqrt{y(x)}}\sqrt{y(x)}}=c_1,y(x)\right]$$

$$\text{Solve}\left[\frac{\sqrt{\left(x^2-4\sqrt{y(x)}\right)y(x)}\log\left(\sqrt{x^2-4\sqrt{y(x)}}-x\right)}{\sqrt{x^2-4\sqrt{y(x)}}\sqrt{y(x)}}+\left(\frac{1}{4}-\frac{\sqrt{x^2y(x)-4y(x)^{3/2}}}{4\sqrt{x^2-4\sqrt{y(x)}}\sqrt{y(x)}}\right)\log(y(x))=c_1,y(x)\right]$$

$$y(x)\to 0$$

$$y(x)\to\frac{x^4}{16}$$

36.24 problem 1093

Internal problem ID [4307]

Book: Ordinary differential equations and their solutions. By George Moseley Murphy. 1960
Section: Various 36
Problem number: 1093.
ODE order: 1.
ODE degree: 4.

CAS Maple gives this as type `[_quadrature]`

$$y'^4 + 4yy'^3 + 6y^2y'^2 - (1 - 4y^3)\, y' - (3 - y^3)\, y = 0$$

✓ Solution by Maple

Time used: 0.016 (sec). Leaf size: 204

```
dsolve(diff(y(x),x)^4+4*y(x)*diff(y(x),x)^3+6*y(x)^2*diff(y(x),x)^2-(1-4*y(x)^3)*diff(y(x),x
```

$$x + \frac{\ln\left(\left(-14640y(x)^6 - 93435y(x)^3 - 256\right)\operatorname{RootOf}\left(\_Z^4 + 4y(x)\_Z^3 + 6y(x)^2\_Z^2 + \left(4y(x)^3 - 1\right)\_Z + y\right.\right.}{}$$

$$- c_1 = 0$$

✓ Solution by Mathematica

Time used: 98.115 (sec). Leaf size: 2925

```
DSolve[(y'[x])^4 +4 y[x] (y'[x])^3+6 y[x]^2 (y'[x])^2-(1-4 y[x]^3) y'[x]- (3-y[x]^3) y[x]==0
```

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36.25 problem 1094

Internal problem ID [4308]

Book: Ordinary differential equations and their solutions. By George Moseley Murphy. 1960
Section: Various 36
Problem number: 1094.
ODE order: 1.
ODE degree: 4.

CAS Maple gives this as type `[_quadrature]`

$$\boxed{2y'^4 - yy' = 2}$$

✓ Solution by Maple
Time used: 0.031 (sec). Leaf size: 217

```
dsolve(2*diff(y(x),x)^4-y(x)*diff(y(x),x)-2 = 0,y(x), singsol=all)
```

$$y(x) = -\frac{\sqrt{-6\sqrt{(c_1^2 - 2c_1x + x^2 + 12)^3} - 6c_1^3 + 18c_1^2x + (-18x^2 + 216)\,c_1 + 6x^3 - 216x}}{9}$$

$$y(x) = \frac{\sqrt{-6\sqrt{(c_1^2 - 2c_1x + x^2 + 12)^3} - 6c_1^3 + 18c_1^2x + (-18x^2 + 216)\,c_1 + 6x^3 - 216x}}{9}$$

$$y(x) = -\frac{\sqrt{6\sqrt{(c_1^2 - 2c_1x + x^2 + 12)^3} - 6c_1^3 + 18c_1^2x + (-18x^2 + 216)\,c_1 + 6x^3 - 216x}}{9}$$

$$y(x) = \frac{\sqrt{6\sqrt{(c_1^2 - 2c_1x + x^2 + 12)^3} - 6c_1^3 + 18c_1^2x + (-18x^2 + 216)\,c_1 + 6x^3 - 216x}}{9}$$

✓ Solution by Mathematica
Time used: 116.271 (sec). Leaf size: 12753

```
DSolve[2 (y'[x])^4 -y[x] y'[x]-2 ==0,y[x],x,IncludeSingularSolutions -> True]
```

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36.26 problem 1095

Internal problem ID [4309]

Book: Ordinary differential equations and their solutions. By George Moseley Murphy. 1960
Section: Various 36
Problem number: 1095.
ODE order: 1.
ODE degree: 4.

CAS Maple gives this as type `[[_1st_order, _with_linear_symmetries]]`

$$xy'^4 - 2yy'^3 = -12x^3$$

✓ Solution by Maple

Time used: 0.797 (sec). Leaf size: 66

```
dsolve(x*diff(y(x),x)^4-2*y(x)*diff(y(x),x)^3+12*x^3 = 0,y(x), singsol=all)
```

$$y(x) = \frac{2\sqrt{6}\,(-x)^{\frac{3}{2}}}{3}$$
$$y(x) = -\frac{2\sqrt{6}\,(-x)^{\frac{3}{2}}}{3}$$
$$y(x) = -\frac{2\sqrt{6}\,x^{\frac{3}{2}}}{3}$$
$$y(x) = \frac{2\sqrt{6}\,x^{\frac{3}{2}}}{3}$$
$$y(x) = \frac{12c_1^4 + x^2}{2c_1}$$

✓ Solution by Mathematica

Time used: 42.244 (sec). Leaf size: 30947

```
DSolve[x (y'[x])^4 -2 y[x] (y'[x])^3+12 x^3==0,y[x],x,IncludeSingularSolutions -> True]
```

Too large to display

36.27 problem 1098

Internal problem ID [4310]

Book: Ordinary differential equations and their solutions. By George Moseley Murphy. 1960
Section: Various 36
Problem number: 1098.
ODE order: 1.
ODE degree: 5.

CAS Maple gives this as type `[_quadrature]`

$$\boxed{3y'^5 - yy' = -1}$$

✓ Solution by Maple

Time used: 0.016 (sec). Leaf size: 87

```
dsolve(3*diff(y(x),x)^5-y(x)*diff(y(x),x)+1 = 0,y(x), singsol=all)
```

$$y(x) = \frac{5\,\mathrm{RootOf}\left(1+8\_Z^5+(2c_1-2x)\_Z^2\right)^3+2c_1-2x}{2\,\mathrm{RootOf}\left(1+8\_Z^5+(2c_1-2x)\_Z^2\right)\left(4\,\mathrm{RootOf}\left(1+8\_Z^5+(2c_1-2x)\_Z^2\right)^3+c_1-x\right)}$$

✓ Solution by Mathematica

Time used: 0.13 (sec). Leaf size: 176

```
DSolve[3 (y'[x])^5 -y[x] y'[x]+1==0,y[x],x,IncludeSingularSolutions -> True]
```

$$\text{Solve}\left[\int_1^{y(x)} \frac{1}{\text{Root}\left[3\#1^5 - K[1]\#1 + 1\&, 1\right]} dK[1] = x + c_1, y(x)\right]$$
$$\text{Solve}\left[\int_1^{y(x)} \frac{1}{\text{Root}\left[3\#1^5 - K[2]\#1 + 1\&, 2\right]} dK[2] = x + c_1, y(x)\right]$$
$$\text{Solve}\left[\int_1^{y(x)} \frac{1}{\text{Root}\left[3\#1^5 - K[3]\#1 + 1\&, 3\right]} dK[3] = x + c_1, y(x)\right]$$
$$\text{Solve}\left[\int_1^{y(x)} \frac{1}{\text{Root}\left[3\#1^5 - K[4]\#1 + 1\&, 4\right]} dK[4] = x + c_1, y(x)\right]$$
$$\text{Solve}\left[\int_1^{y(x)} \frac{1}{\text{Root}\left[3\#1^5 - K[5]\#1 + 1\&, 5\right]} dK[5] = x + c_1, y(x)\right]$$

36.28 problem 1099

Internal problem ID [4311]

Book: Ordinary differential equations and their solutions. By George Moseley Murphy. 1960
Section: Various 36
Problem number: 1099.
ODE order: 1.
ODE degree: 6.

CAS Maple gives this as type `[_quadrature]`

$$\boxed{y'^6 - (y-a)^4 (y-b)^3 = 0}$$

✓ Solution by Maple

Time used: 0.047 (sec). Leaf size: 281

```
dsolve(diff(y(x),x)^6 = (y(x)-a)^4*(y(x)-b)^3,y(x), singsol=all)
```

$$y(x) = a$$
$$y(x) = b$$
$$x - \left(\int^{y(x)} \frac{1}{\left((\_a - a)^4 (\_a - b)^3\right)^{\frac{1}{6}}} d\_a\right) - c_1 = 0$$
$$\frac{2\left(\int^{y(x)} \frac{1}{\left((\_a-a)^4(\_a-b)^3\right)^{\frac{1}{6}}} d\_a\right) + i(x - c_1)\sqrt{3} - c_1 + x}{1 + i\sqrt{3}} = 0$$
$$\frac{-2\left(\int^{y(x)} \frac{1}{\left((\_a-a)^4(\_a-b)^3\right)^{\frac{1}{6}}} d\_a\right) + i(x - c_1)\sqrt{3} + c_1 - x}{-1 + i\sqrt{3}} = 0$$
$$\frac{2\left(\int^{y(x)} \frac{1}{\left((\_a-a)^4(\_a-b)^3\right)^{\frac{1}{6}}} d\_a\right) + i(x - c_1)\sqrt{3} + c_1 - x}{-1 + i\sqrt{3}} = 0$$
$$\frac{-2\left(\int^{y(x)} \frac{1}{\left((\_a-a)^4(\_a-b)^3\right)^{\frac{1}{6}}} d\_a\right) + i(x - c_1)\sqrt{3} - c_1 + x}{1 + i\sqrt{3}} = 0$$
$$x + \int^{y(x)} \frac{1}{\left((\_a - a)^4 (\_a - b)^3\right)^{\frac{1}{6}}} d\_a - c_1 = 0$$

✓ Solution by Mathematica

Time used: 2.108 (sec). Leaf size: 489

```
DSolve[(y'[x])^6 == (y[x]-a)^4 (y[x]-b)^3,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to \text{InverseFunction}\left[-\frac{3\sqrt[3]{a-\#1}\sqrt{\frac{\#1-b}{a-b}}\,\text{Hypergeometric2F1}\left(\frac{1}{3},\frac{1}{2},\frac{4}{3},\frac{a-\#1}{a-b}\right)}{\sqrt{b-\#1}}\&\right][c_1-ix]$$

$$y(x) \to \text{InverseFunction}\left[-\frac{3\sqrt[3]{a-\#1}\sqrt{\frac{\#1-b}{a-b}}\,\text{Hypergeometric2F1}\left(\frac{1}{3},\frac{1}{2},\frac{4}{3},\frac{a-\#1}{a-b}\right)}{\sqrt{b-\#1}}\&\right][ix+c_1]$$

$$y(x) \to \text{InverseFunction}\left[-\frac{3\sqrt[3]{a-\#1}\sqrt{\frac{\#1-b}{a-b}}\,\text{Hypergeometric2F1}\left(\frac{1}{3},\frac{1}{2},\frac{4}{3},\frac{a-\#1}{a-b}\right)}{\sqrt{b-\#1}}\&\right][-\sqrt[6]{-1}x+c_1]$$

$$y(x) \to \text{InverseFunction}\left[-\frac{3\sqrt[3]{a-\#1}\sqrt{\frac{\#1-b}{a-b}}\,\text{Hypergeometric2F1}\left(\frac{1}{3},\frac{1}{2},\frac{4}{3},\frac{a-\#1}{a-b}\right)}{\sqrt{b-\#1}}\&\right][\sqrt[6]{-1}x+c_1]$$

$$y(x) \to \text{InverseFunction}\left[-\frac{3\sqrt[3]{a-\#1}\sqrt{\frac{\#1-b}{a-b}}\,\text{Hypergeometric2F1}\left(\frac{1}{3},\frac{1}{2},\frac{4}{3},\frac{a-\#1}{a-b}\right)}{\sqrt{b-\#1}}\&\right][-(-1)^{5/6}x+c_1]$$

$$y(x) \to \text{InverseFunction}\left[-\frac{3\sqrt[3]{a-\#1}\sqrt{\frac{\#1-b}{a-b}}\,\text{Hypergeometric2F1}\left(\frac{1}{3},\frac{1}{2},\frac{4}{3},\frac{a-\#1}{a-b}\right)}{\sqrt{b-\#1}}\&\right][(-1)^{5/6}x+c_1]$$

$$y(x) \to a$$

$$y(x) \to b$$

36.29 problem 1100

Internal problem ID [4312]

Book: Ordinary differential equations and their solutions. By George Moseley Murphy. 1960
Section: Various 36
Problem number: 1100.
ODE order: 1.
ODE degree: 6.

CAS Maple gives this as type `[[_1st_order, '_with_symmetry_[F(x),G(x)*y+H(x)]']]`

$$\boxed{y'^6 + f(x)(y-a)^4 (y-b)^3 = 0}$$

✓ Solution by Maple

Time used: 0.078 (sec). Leaf size: 69

```
dsolve(diff(y(x),x)^6+f(x)*(y(x)-a)^4*(y(x)-b)^3 = 0,y(x), singsol=all)
```

$$\int^{y(x)} \frac{1}{(\_a-a)^{\frac{2}{3}}\sqrt{\_a-b}}d\_a - \frac{\int^{x} \left(-f(\_a)(y(x)-a)^4 (y(x)-b)^3\right)^{\frac{1}{6}} d\_a}{(y(x)-a)^{\frac{2}{3}}\sqrt{y(x)-b}} + c_1 = 0$$

✓ Solution by Mathematica

Time used: 1.867 (sec). Leaf size: 561

```
DSolve[(y'[x])^6 +f[x] (y[x]-a)^4 (y[x]-b)^3==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to \text{InverseFunction}\left[-\frac{3\sqrt[3]{a-\#1}\sqrt{\frac{\#1-b}{a-b}}\,\text{Hypergeometric2F1}\left(\frac{1}{3},\frac{1}{2},\frac{4}{3},\frac{a-\#1}{a-b}\right)}{\sqrt{b-\#1}}\&\right]\left[\int_1^x -\sqrt[6]{f(K[1])}dK[1]+c_1\right]$$

$$y(x) \to \text{InverseFunction}\left[-\frac{3\sqrt[3]{a-\#1}\sqrt{\frac{\#1-b}{a-b}}\,\text{Hypergeometric2F1}\left(\frac{1}{3},\frac{1}{2},\frac{4}{3},\frac{a-\#1}{a-b}\right)}{\sqrt{b-\#1}}\&\right]\left[\int_1^x \sqrt[6]{f(K[2])}dK[2]+c_1\right]$$

$$y(x) \to \text{InverseFunction}\left[-\frac{3\sqrt[3]{a-\#1}\sqrt{\frac{\#1-b}{a-b}}\,\text{Hypergeometric2F1}\left(\frac{1}{3},\frac{1}{2},\frac{4}{3},\frac{a-\#1}{a-b}\right)}{\sqrt{b-\#1}}\&\right]\left[\int_1^x -\sqrt[3]{-1}\sqrt[6]{f(K[3])}dK[3]+c_1\right]$$

$$y(x) \to \text{InverseFunction}\left[-\frac{3\sqrt[3]{a-\#1}\sqrt{\frac{\#1-b}{a-b}}\,\text{Hypergeometric2F1}\left(\frac{1}{3},\frac{1}{2},\frac{4}{3},\frac{a-\#1}{a-b}\right)}{\sqrt{b-\#1}}\&\right]\left[\int_1^x \sqrt[3]{-1}\sqrt[6]{f(K[4])}d + c_1\right]$$

$$y(x) \to \text{InverseFunction}\left[-\frac{3\sqrt[3]{a-\#1}\sqrt{\frac{\#1-b}{a-b}}\,\text{Hypergeometric2F1}\left(\frac{1}{3},\frac{1}{2},\frac{4}{3},\frac{a-\#1}{a-b}\right)}{\sqrt{b-\#1}}\&\right]\left[\int_1^x -(-1)^{2/3}\sqrt[6]{f(K[5])}dK[5]+c_1\right]$$

$$y(x) \to \text{InverseFunction}\left[-\frac{3\sqrt[3]{a-\#1}\sqrt{\frac{\#1-b}{a-b}}\,\text{Hypergeometric2F1}\left(\frac{1}{3},\frac{1}{2},\frac{4}{3},\frac{a-\#1}{a-b}\right)}{\sqrt{b-\#1}}\&\right]\left[\int_1^x (-1)^{2/3}\sqrt[6]{f(K[6])} + c_1\right]$$

36.30 problem 1101

Internal problem ID [4313]

Book: Ordinary differential equations and their solutions. By George Moseley Murphy. 1960
Section: Various 36
Problem number: 1101.
ODE order: 1.
ODE degree: 6.

CAS Maple gives this as type `[[_1st_order, ‘_with_symmetry_[F(x),G(x)*y+H(x)]‘]]`

$$\boxed{y'^6 + f(x)\,(y-a)^5\,(y-b)^3 = 0}$$

✓ Solution by Maple

Time used: 0.094 (sec). Leaf size: 69

```
dsolve(diff(y(x),x)^6+f(x)*(y(x)-a)^5*(y(x)-b)^3 = 0,y(x), singsol=all)
```

$$\int^{y(x)} \frac{1}{(\_a-a)^{\frac{5}{6}}\sqrt{\_a-b}}d\_a - \frac{\int^{x} \left(-f(\_a)\,(y(x)-b)^3\,(y(x)-a)^5\right)^{\frac{1}{6}}d\_a}{(y(x)-a)^{\frac{5}{6}}\sqrt{y(x)-b}} + c_1 = 0$$

✓ Solution by Mathematica

Time used: 1.867 (sec). Leaf size: 567

```
DSolve[(y'[x])^6 +f[x] (y[x]-a)^5 (y[x]-b)^3==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to \text{InverseFunction}\left[-\frac{6\sqrt[6]{a-\#1}\sqrt{\frac{\#1-b}{a-b}}\,\text{Hypergeometric2F1}\left(\frac{1}{6},\frac{1}{2},\frac{7}{6},\frac{a-\#1}{a-b}\right)}{\sqrt{b-\#1}}\&\right]\left[\int_1^x -i\sqrt[6]{f(K[1])}dK[1]+c_1\right]$$

$$y(x) \to \text{InverseFunction}\left[-\frac{6\sqrt[6]{a-\#1}\sqrt{\frac{\#1-b}{a-b}}\,\text{Hypergeometric2F1}\left(\frac{1}{6},\frac{1}{2},\frac{7}{6},\frac{a-\#1}{a-b}\right)}{\sqrt{b-\#1}}\&\right]\left[\int_1^x i\sqrt[6]{f(K[2])}dK[2]+c_1\right]$$

$$y(x) \to \text{InverseFunction}\left[-\frac{6\sqrt[6]{a-\#1}\sqrt{\frac{\#1-b}{a-b}}\,\text{Hypergeometric2F1}\left(\frac{1}{6},\frac{1}{2},\frac{7}{6},\frac{a-\#1}{a-b}\right)}{\sqrt{b-\#1}}\&\right]\left[\int_1^x -\sqrt[6]{-1}\sqrt[6]{f(K[3])}dK[3]+c_1\right]$$

$$y(x) \to \text{InverseFunction}\left[-\frac{6\sqrt[6]{a-\#1}\sqrt{\frac{\#1-b}{a-b}}\,\text{Hypergeometric2F1}\left(\frac{1}{6},\frac{1}{2},\frac{7}{6},\frac{a-\#1}{a-b}\right)}{\sqrt{b-\#1}}\&\right]\left[\int_1^x \sqrt[6]{-1}\sqrt[6]{f(K[4])}dK[4]+c_1\right]$$

$$y(x) \to \text{InverseFunction}\left[-\frac{6\sqrt[6]{a-\#1}\sqrt{\frac{\#1-b}{a-b}}\,\text{Hypergeometric2F1}\left(\frac{1}{6},\frac{1}{2},\frac{7}{6},\frac{a-\#1}{a-b}\right)}{\sqrt{b-\#1}}\&\right]\left[\int_1^x -(-1)^{5/6}\sqrt[6]{f(K[5])}dK[5]+c_1\right]$$

$$y(x) \to \text{InverseFunction}\left[-\frac{6\sqrt[6]{a-\#1}\sqrt{\frac{\#1-b}{a-b}}\,\text{Hypergeometric2F1}\left(\frac{1}{6},\frac{1}{2},\frac{7}{6},\frac{a-\#1}{a-b}\right)}{\sqrt{b-\#1}}\&\right]\left[\int_1^x (-1)^{5/6}\sqrt[6]{f(K[6])}dK[6]+c_1\right]$$

36.31 problem 1102

Internal problem ID [4314]

Book: Ordinary differential equations and their solutions. By George Moseley Murphy. 1960
Section: Various 36
Problem number: 1102.
ODE order: 1.
ODE degree: 6.

CAS Maple gives this as type `[[_1st_order, '_with_symmetry_[F(x),G(x)*y+H(x)]']]`

$$y'^6 + f(x)\,(y-a)^5\,(y-b)^4 = 0$$

✓ Solution by Maple

Time used: 0.094 (sec). Leaf size: 69

```
dsolve(diff(y(x),x)^6+f(x)*(y(x)-a)^5*(y(x)-b)^4 = 0,y(x), singsol=all)
```

$$\int^{y(x)} \frac{1}{(\_a-a)^{\frac{5}{6}}\,(\_a-b)^{\frac{2}{3}}} d\_a - \frac{\int^{x} \left(-f(\_a)\,(y(x)-b)^4\,(y(x)-a)^5\right)^{\frac{1}{6}} d\_a}{(y\,(x)-a)^{\frac{5}{6}}\,(y\,(x)-b)^{\frac{2}{3}}} + c_1 = 0$$

✓ Solution by Mathematica

Time used: 1.947 (sec). Leaf size: 561

```
DSolve[(y'[x])^6 +f[x] (y[x]-a)^5 (y[x]-b)^4==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to \text{InverseFunction}\left[-\frac{6\sqrt[6]{a-\#1}\left(\frac{\#1-b}{a-b}\right)^{2/3}\text{Hypergeometric2F1}\left(\frac{1}{6},\frac{2}{3},\frac{7}{6},\frac{a-\#1}{a-b}\right)}{(b-\#1)^{2/3}}\&\right]\left[\int_1^x -\sqrt[6]{f(K[1])}dK\right.$$

$$y(x) \to \text{InverseFunction}\left[-\frac{6\sqrt[6]{a-\#1}\left(\frac{\#1-b}{a-b}\right)^{2/3}\text{Hypergeometric2F1}\left(\frac{1}{6},\frac{2}{3},\frac{7}{6},\frac{a-\#1}{a-b}\right)}{(b-\#1)^{2/3}}\&\right]\left[\int_1^x \sqrt[6]{f(K[2])}dK[\right.$$

$$y(x) \to \text{InverseFunction}\left[-\frac{6\sqrt[6]{a-\#1}\left(\frac{\#1-b}{a-b}\right)^{2/3}\text{Hypergeometric2F1}\left(\frac{1}{6},\frac{2}{3},\frac{7}{6},\frac{a-\#1}{a-b}\right)}{(b-\#1)^{2/3}}\&\right]\left[\int_1^x -\sqrt[3]{-1}\sqrt[6]{f(K[\right.$$

$$y(x) \to \text{InverseFunction}\left[-\frac{6\sqrt[6]{a-\#1}\left(\frac{\#1-b}{a-b}\right)^{2/3}\text{Hypergeometric2F1}\left(\frac{1}{6},\frac{2}{3},\frac{7}{6},\frac{a-\#1}{a-b}\right)}{(b-\#1)^{2/3}}\&\right]\left[\int_1^x \sqrt[3]{-1}\sqrt[6]{f(K[4]\right.$$

$$y(x) \to \text{InverseFunction}\left[-\frac{6\sqrt[6]{a-\#1}\left(\frac{\#1-b}{a-b}\right)^{2/3}\text{Hypergeometric2F1}\left(\frac{1}{6},\frac{2}{3},\frac{7}{6},\frac{a-\#1}{a-b}\right)}{(b-\#1)^{2/3}}\&\right]\left[\int_1^x -(-1)^{2/3}\sqrt[6]{f(\right.$$

$$y(x) \to \text{InverseFunction}\left[-\frac{6\sqrt[6]{a-\#1}\left(\frac{\#1-b}{a-b}\right)^{2/3}\text{Hypergeometric2F1}\left(\frac{1}{6},\frac{2}{3},\frac{7}{6},\frac{a-\#1}{a-b}\right)}{(b-\#1)^{2/3}}\&\right]\left[\int_1^x (-1)^{2/3}\sqrt[6]{f(K[\right.$$

$$y(x) \to a$$
$$y(x) \to b$$

37 Various 37

37.1 problem 1104

Internal problem ID [4315]

Book: Ordinary differential equations and their solutions. By George Moseley Murphy. 1960
Section: Various 37
Problem number: 1104.
ODE order: 1.
ODE degree: 6.

CAS Maple gives this as type `[_rational]`

$$x^2\left(y'^6 + 3y^4 + 3y^2 + 1\right) = a^2$$

✗ Solution by Maple

```
dsolve(x^2*(diff(y(x),x)^6+3*y(x)^4+3*y(x)^2+1) = a^2,y(x), singsol=all)
```

No solution found

✗ Solution by Mathematica

Time used: 0.0 (sec). Leaf size: 0

```
DSolve[x^2 ( (y'[x])^6 +3 (y[x])^4 +3 (y[x])^2 + 1)==a^2,y[x],x,IncludeSingularSolutions ->
```

Not solved

37.2 problem 1115

Internal problem ID [4316]

Book: Ordinary differential equations and their solutions. By George Moseley Murphy. 1960
Section: Various 37
Problem number: 1115.
ODE order: 1.
ODE degree: 2.

CAS Maple gives this as type `[[_homogeneous, ‘class G‘], _Clairaut]`

$$\boxed{2\sqrt{ay'} + xy' - y = 0}$$

✓ Solution by Maple
Time used: 0.063 (sec). Leaf size: 15

```
dsolve(2*sqrt(a*diff(y(x),x))+x*diff(y(x),x)-y(x) = 0,y(x), singsol=all)
```

$$y(x) = 2\sqrt{c_1 a} + c_1 x$$

✓ Solution by Mathematica
Time used: 0.078 (sec). Leaf size: 25

```
DSolve[2 Sqrt[a y'[x]]+x y'[x]-y[x]==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to 2\sqrt{ac_1} + c_1 x$$
$$y(x) \to 0$$

37.3 problem 1116

Internal problem ID [4317]

Book: Ordinary differential equations and their solutions. By George Moseley Murphy. 1960
Section: Various 37
Problem number: 1116.
ODE order: 1.
ODE degree: 2.

CAS Maple gives this as type `[[_homogeneous, ‘class C‘], _dAlembert]`

$$\boxed{(x-y)\sqrt{y'}-a(1+y')=0}$$

✓ Solution by Maple
Time used: 0.063 (sec). Leaf size: 48

```
dsolve((x-y(x))*sqrt(diff(y(x),x)) = a*(1+diff(y(x),x)),y(x), singsol=all)
```

$$y(x)=x-2a$$
$$y(x)=x-\frac{a\left(a^2+(x-c_1)^2\right)}{\sqrt{\frac{a^2}{(c_1-x)^2}}\,(x-c_1)^2}$$

✓ Solution by Mathematica
Time used: 69.72 (sec). Leaf size: 9683

```
DSolve[(x-y[x])*Sqrt[y'[x]]== a*(1+y'[x]),y[x],x,IncludeSingularSolutions -> True]
```

Too large to display

37.4 problem 1117

Internal problem ID [4318]

Book: Ordinary differential equations and their solutions. By George Moseley Murphy. 1960
Section: Various 37
Problem number: 1117.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_separable]`

$$2(1+y)^{\frac{3}{2}} + 3xy' - 3y = 0$$

✓ Solution by Maple

Time used: 0.015 (sec). Leaf size: 31

```
dsolve(2*(1+y(x))^(3/2)+3*x*diff(y(x),x)-3*y(x) = 0,y(x), singsol=all)
```

$$\ln(x) + 3\left(\int^{y(x)} \frac{1}{(2\_a+2)\sqrt{\_a+1} - 3\_a} d\_a\right) + c_1 = 0$$

✓ Solution by Mathematica

Time used: 0.123 (sec). Leaf size: 55

```
DSolve[2 (1+y[x])^(3/2) + 3 x y'[x]-3 y[x]==0,y[x],x,IncludeSingularSolutions -> True]
```

$$\text{Solve}\left[\frac{1}{3}\text{RootSum}\left[2\#1^3 - 3\#1^2 + 3\&, \frac{\log\left(\sqrt{y(x)+1} - \#1\right)}{\#1 - 1}\&\right] = -\frac{\log(x)}{3} + c_1, y(x)\right]$$

37.5 problem 1118

Internal problem ID [4319]

Book: Ordinary differential equations and their solutions. By George Moseley Murphy. 1960
Section: Various 37
Problem number: 1118.
ODE order: 1.
ODE degree: 2.

CAS Maple gives this as type `[_quadrature]`

$$\boxed{\sqrt{1+y'^2}+ay'=x}$$

✓ Solution by Maple
Time used: 0.047 (sec). Leaf size: 113

```
dsolve(sqrt(1+diff(y(x),x)^2)+a*diff(y(x),x) = x,y(x), singsol=all)
```

$$y(x)=\frac{a\,x^2+x\sqrt{a^2+x^2-1}+(a-1)\left(\ln\left(x+\sqrt{a^2+x^2-1}\right)+2c_1\right)(a+1)}{2a^2-2}$$
$$y(x)=\frac{a\,x^2-x\sqrt{a^2+x^2-1}-(a-1)(a+1)\left(\ln\left(x+\sqrt{a^2+x^2-1}\right)-2c_1\right)}{2a^2-2}$$

✓ Solution by Mathematica
Time used: 0.066 (sec). Leaf size: 113

```
DSolve[Sqrt[1+(y'[x])^2]+ a y'[x]==x,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x)\to\frac{1}{2}\left(\frac{x\left(ax-\sqrt{a^2+x^2-1}\right)}{a^2-1}+\log\left(\sqrt{a^2+x^2-1}-x\right)\right)+c_1$$
$$y(x)\to\frac{1}{2}\left(\frac{x\left(\sqrt{a^2+x^2-1}+ax\right)}{a^2-1}-\log\left(\sqrt{a^2+x^2-1}-x\right)\right)+c_1$$

37.6 problem 1119

Internal problem ID [4320]

Book: Ordinary differential equations and their solutions. By George Moseley Murphy. 1960
Section: Various 37
Problem number: 1119.
ODE order: 1.
ODE degree: 2.

CAS Maple gives this as type `[_quadrature]`

$$\sqrt{1+y'^2}+ay'-y=0$$

✓ Solution by Maple
Time used: 0.031 (sec). Leaf size: 112

```
dsolve(sqrt(1+diff(y(x),x)^2)+a*diff(y(x),x) = y(x),y(x), singsol=all)
```

$$-\left(\int^{y(x)}\frac{1}{\_aa+\sqrt{\_a^2+a^2-1}}d\_a\right)a^2+\int^{y(x)}\frac{1}{\_aa+\sqrt{\_a^2+a^2-1}}d\_a-c_1+x=0$$

$$\left(\int^{y(x)}\frac{1}{-\_aa+\sqrt{\_a^2+a^2-1}}d\_a\right)a^2-\left(\int^{y(x)}\frac{1}{-\_aa+\sqrt{\_a^2+a^2-1}}d\_a\right)-c_1+x=0$$

✓ Solution by Mathematica

Time used: 0.833 (sec). Leaf size: 210

```
DSolve[Sqrt[1+(y'[x])^2]+ a y'[x]==y[x],y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \rightarrow \text{InverseFunction}\left[\frac{a\left(\log\left(\sqrt{\#1^2+a^2-1}-\#1-a+1\right)+\log\left(\sqrt{\#1^2+a^2-1}-\#1+a-1\right)\right)-(a}{a^2-1} + c_1\right]$$

$$y(x) \rightarrow \text{InverseFunction}\left[\frac{a\left(\log\left(\sqrt{\#1^2+a^2-1}-\#1-a-1\right)+\log\left(\sqrt{\#1^2+a^2-1}-\#1+a+1\right)\right)-(a}{a^2-1} + c_1\right]$$

$$y(x) \rightarrow 1$$

37.7 problem 1120

Internal problem ID [4321]

Book: Ordinary differential equations and their solutions. By George Moseley Murphy. 1960
Section: Various 37
Problem number: 1120.
ODE order: 1.
ODE degree: 2.

CAS Maple gives this as type `[_quadrature]`

$$\boxed{\sqrt{1+y'^2}-xy'=0}$$

✓ Solution by Maple
Time used: 0.031 (sec). Leaf size: 33

```
dsolve(sqrt(1+diff(y(x),x)^2) = x*diff(y(x),x),y(x), singsol=all)
```

$$y(x) = \ln\left(x+\sqrt{x^2-1}\right)+c_1$$
$$y(x) = -\ln\left(x+\sqrt{x^2-1}\right)+c_1$$

✓ Solution by Mathematica
Time used: 0.008 (sec). Leaf size: 89

```
DSolve[Sqrt[1+(y'[x])^2]==x y'[x],y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to \frac{1}{2}\left(\log\left(1-\frac{x}{\sqrt{x^2-1}}\right)-\log\left(\frac{x}{\sqrt{x^2-1}}+1\right)+2c_1\right)$$
$$y(x) \to \frac{1}{2}\left(-\log\left(1-\frac{x}{\sqrt{x^2-1}}\right)+\log\left(\frac{x}{\sqrt{x^2-1}}+1\right)+2c_1\right)$$

37.8 problem 1123

Internal problem ID [4322]

Book: Ordinary differential equations and their solutions. By George Moseley Murphy. 1960
Section: Various 37
Problem number: 1123.
ODE order: 1.
ODE degree: 2.

CAS Maple gives this as type `[[_1st_order, _with_linear_symmetries], _Clairaut]`

$$\boxed{\sqrt{a^2+b^2y'^2}+xy'-y=0}$$

✓ Solution by Maple

Time used: 0.312 (sec). Leaf size: 21

```
dsolve(sqrt(a^2+b^2*diff(y(x),x)^2)+x*diff(y(x),x)-y(x) = 0,y(x), singsol=all)
```

$$y(x)=\sqrt{b^2c_1^2+a^2}+c_1x$$

✓ Solution by Mathematica

Time used: 0.383 (sec). Leaf size: 37

```
DSolve[Sqrt[a^2+b^2 (y'[x])^2] +x y'[x] -y[x]==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x)\to\sqrt{a^2+b^2{c_1}^2}+c_1x$$
$$y(x)\to\sqrt{a^2}$$

37.9 problem 1125

Internal problem ID [4323]

Book: Ordinary differential equations and their solutions. By George Moseley Murphy. 1960
Section: Various 37
Problem number: 1125.
ODE order: 1.
ODE degree: 2.

CAS Maple gives this as type `[[_1st_order, _with_linear_symmetries], _Clairaut]`

$$\boxed{a\sqrt{1+y'^2}+xy'-y=0}$$

✓ Solution by Maple
Time used: 0.188 (sec). Leaf size: 17

```
dsolve(a*sqrt(1+diff(y(x),x)^2)+x*diff(y(x),x)-y(x) = 0,y(x), singsol=all)
```

$$y(x) = a\sqrt{c_1^2+1}+c_1x$$

✓ Solution by Mathematica
Time used: 0.059 (sec). Leaf size: 27

```
DSolve[a Sqrt[1+(y'[x])^2] + x y'[x] -y[x]==0 y'[x],y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to a\sqrt{1+c_1{}^2}+c_1x$$
$$y(x) \to a$$

37.10 problem 1126

Internal problem ID [4324]

Book: Ordinary differential equations and their solutions. By George Moseley Murphy. 1960
Section: Various 37
Problem number: 1126.
ODE order: 1.
ODE degree: 2.

CAS Maple gives this as type `[[_homogeneous, 'class A'], _dAlembert]`

$$ax\sqrt{1+y'^2}+xy'-y=0$$

✓ Solution by Maple

Time used: 0.047 (sec). Leaf size: 340

```
dsolve(a*x*sqrt(1+diff(y(x),x)^2)+x*diff(y(x),x)-y(x) = 0,y(x), singsol=all)
```

$$\frac{x\sqrt{\frac{-a^2x^2+y(x)^2a^2+2\sqrt{y(x)^2-a^2x^2+x^2}\,ay(x)+x^2+y(x)^2}{(a^2-1)^2x^2}}-\mathrm{e}^{\frac{\operatorname{arcsinh}\left(\frac{\sqrt{y(x)^2-a^2x^2+x^2}\,a+y(x)}{(a^2-1)x}\right)}{a}}c_1}{\sqrt{\frac{-a^2x^2+y(x)^2a^2+2\sqrt{y(x)^2-a^2x^2+x^2}\,ay(x)+x^2+y(x)^2}{(a^2-1)^2x^2}}}=0$$

$$\frac{x\sqrt{\frac{-a^2x^2+y(x)^2a^2-2\sqrt{y(x)^2-a^2x^2+x^2}\,ay(x)+x^2+y(x)^2}{(a^2-1)^2x^2}}-\mathrm{e}^{\frac{\operatorname{arcsinh}\left(\frac{-\sqrt{y(x)^2-a^2x^2+x^2}\,a+y(x)}{(a^2-1)x}\right)}{a}}c_1}{\sqrt{\frac{-a^2x^2+y(x)^2a^2-2\sqrt{y(x)^2-a^2x^2+x^2}\,ay(x)+x^2+y(x)^2}{(a^2-1)^2x^2}}}=0$$

✓ Solution by Mathematica

Time used: 0.992 (sec). Leaf size: 223

```
DSolve[a x Sqrt[1+(y'[x])^2]+x y'[x] -y[x]==0,y[x],x,IncludeSingularSolutions -> True]
```

$$\text{Solve}\left[\frac{2i\arctan\left(\frac{y(x)}{x\sqrt{a^2-\frac{y(x)^2}{x^2}-1}}\right)-2ia\arctan\left(\frac{ay(x)}{x\sqrt{a^2-\frac{y(x)^2}{x^2}-1}}\right)+a\log\left(\frac{y(x)^2}{x^2}+1\right)}{2a^2-2}=\frac{a\log\left(x-a^2x\right)}{1-a^2}\right.$$

$$\left.+c_1,y(x)\right]$$

$$\text{Solve}\left[\frac{-2i\arctan\left(\frac{y(x)}{x\sqrt{a^2-\frac{y(x)^2}{x^2}-1}}\right)+2ia\arctan\left(\frac{ay(x)}{x\sqrt{a^2-\frac{y(x)^2}{x^2}-1}}\right)+a\log\left(\frac{y(x)^2}{x^2}+1\right)}{2a^2-2}=\frac{a\log\left(x-a^2x\right)}{1-a^2}\right.$$

$$\left.+c_1,y(x)\right]$$

37.11 problem 1129

Internal problem ID [4325]

Book: Ordinary differential equations and their solutions. By George Moseley Murphy. 1960
Section: Various 37
Problem number: 1129.
ODE order: 1.
ODE degree: 2.

CAS Maple gives this as type `[[_homogeneous, ‘class A‘], _rational, _dAlembert]`

$$\boxed{\sqrt{(a\,x^2+y^2)\left(1+y'^2\right)}-y'y=ax}$$

✓ Solution by Maple

Time used: 1.203 (sec). Leaf size: 142

```
dsolve(((a*x^2+y(x)^2)*(1+diff(y(x),x)^2))^(1/2)-y(x)*diff(y(x),x)-a*x = 0,y(x), singsol=all
```

$$y(x)=\sqrt{-a}\,x$$
$$y(x)=-\sqrt{-a}\,x$$
$$y(x)=-\frac{a^2(a-1)\,x^{\frac{-\sqrt{a(a-1)}+a}{a}}-x^{\frac{a+\sqrt{a(a-1)}}{a}}c_1^2}{2\sqrt{a\,(a-1)}\,c_1}$$
$$y(x)=-\frac{-x^{\frac{-\sqrt{a(a-1)}+a}{a}}c_1^2+x^{\frac{a+\sqrt{a(a-1)}}{a}}a^2(a-1)}{2\sqrt{a\,(a-1)}\,c_1}$$

✓ Solution by Mathematica

Time used: 0.701 (sec). Leaf size: 241

```
DSolve[((a x^2+y[x]^2)(1+(y'[x])^2))^(1/2) -y[x] y'[x]-a x==0,y[x],x,IncludeSingularSolution
```

$$y(x) \to \frac{1}{2}\sqrt{a}e^{-c_1}x^{1-\sqrt{\frac{a-1}{a}}}\left(x^{2\sqrt{\frac{a-1}{a}}} - e^{2c_1}\right)$$
$$y(x) \to \frac{1}{2}\sqrt{a}e^{-c_1}x^{1-\sqrt{\frac{a-1}{a}}}\left(-x^{2\sqrt{\frac{a-1}{a}}} + e^{2c_1}\right)$$
$$y(x) \to -\frac{1}{2}\sqrt{a}e^{-c_1}x^{1-\sqrt{\frac{a-1}{a}}}\left(-1 + e^{2c_1}x^{2\sqrt{\frac{a-1}{a}}}\right)$$
$$y(x) \to \frac{1}{2}\sqrt{a}e^{-c_1}x^{1-\sqrt{\frac{a-1}{a}}}\left(-1 + e^{2c_1}x^{2\sqrt{\frac{a-1}{a}}}\right)$$

37.12 problem 1130

Internal problem ID [4326]

Book: Ordinary differential equations and their solutions. By George Moseley Murphy. 1960
Section: Various 37
Problem number: 1130.
ODE order: 1.
ODE degree: 3.

CAS Maple gives this as type `[_Clairaut]`

$$a\left(1+y'^3\right)^{\frac{1}{3}}+xy'-y=0$$

✓ Solution by Maple
Time used: 0.5 (sec). Leaf size: 17

```
dsolve(a*(1+diff(y(x),x)^3)^(1/3)+x*diff(y(x),x)-y(x) = 0,y(x), singsol=all)
```

$$y(x)=a\left(c_1^3+1\right)^{\frac{1}{3}}+c_1x$$

✓ Solution by Mathematica
Time used: 0.176 (sec). Leaf size: 27

```
DSolve[a (1+ (y'[x])^3)^(1/3) +x y'[x]-y[x]==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x)\to a\sqrt[3]{1+c_1{}^3}+c_1x$$
$$y(x)\to a$$

37.13 problem 1132

Internal problem ID [4327]

Book: Ordinary differential equations and their solutions. By George Moseley Murphy. 1960
Section: Various 37
Problem number: 1132.
ODE order: 1.
ODE degree: 0.

CAS Maple gives this as type `[_Clairaut]`

$$\boxed{\cos(y') + xy' - y = 0}$$

✓ Solution by Maple

Time used: 0.016 (sec). Leaf size: 27

```
dsolve(cos(diff(y(x),x))+x*diff(y(x),x) = y(x),y(x), singsol=all)
```

$$y(x) = \arcsin(x)\, x + \sqrt{-x^2 + 1}$$
$$y(x) = \cos(c_1) + c_1 x$$

✓ Solution by Mathematica

Time used: 0.051 (sec). Leaf size: 18

```
DSolve[Cos[y'[x]]+x*y'[x]==y[x],y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to c_1 x + \cos(c_1)$$
$$y(x) \to 1$$

37.14 problem 1133

Internal problem ID [4328]

Book: Ordinary differential equations and their solutions. By George Moseley Murphy. 1960
Section: Various 37
Problem number: 1133.
ODE order: 1.
ODE degree: 0.

CAS Maple gives this as type `[_quadrature]`

$$\boxed{a\cos(y') + by' = -x}$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 18

```
dsolve(a*cos(diff(y(x),x))+b*diff(y(x),x)+x = 0,y(x), singsol=all)
```

$$y(x) = \int \mathrm{RootOf}\left(a\cos(\_Z) + \_Zb + x\right)dx + c_1$$

✓ Solution by Mathematica

Time used: 0.07 (sec). Leaf size: 49

```
DSolve[a Cos[y'[x]] + b y'[x]+x ==0,y[x],x,IncludeSingularSolutions -> True]
```

$$\text{Solve}\left[\left\{y(x) = a\sin(K[1]) - aK[1]\cos(K[1]) - \frac{1}{2}bK[1]^2 + c_1, x = -a\cos(K[1]) - bK[1]\right\}, \{y(x), K[1]\}\right]$$

37.15 problem 1134

Internal problem ID [4329]

Book: Ordinary differential equations and their solutions. By George Moseley Murphy. 1960
Section: Various 37
Problem number: 1134.
ODE order: 1.
ODE degree: 0.

CAS Maple gives this as type `[_quadrature]`

$$\boxed{\sin(y') + y' = x}$$

✓ Solution by Maple
Time used: 0.016 (sec). Leaf size: 16

```
dsolve(sin(diff(y(x),x))+diff(y(x),x) = x,y(x), singsol=all)
```

$$y(x) = \int \text{RootOf}\left(\sin\left(\_Z\right) + \_Z - x\right) dx + c_1$$

✓ Solution by Mathematica
Time used: 0.036 (sec). Leaf size: 38

```
DSolve[Sin[y'[x]]+ y'[x]==x,y[x],x,IncludeSingularSolutions -> True]
```

$$\text{Solve}\left[\left\{x = K[1] + \sin(K[1]), y(x) = \frac{K[1]^2}{2} + K[1]\sin(K[1]) + \cos(K[1]) + c_1\right\}, \{y(x), K[1]\}\right]$$

37.16 problem 1135

Internal problem ID [4330]

Book: Ordinary differential equations and their solutions. By George Moseley Murphy. 1960
Section: Various 37
Problem number: 1135.
ODE order: 1.
ODE degree: 0.

CAS Maple gives this as type `[_quadrature]`

$$\boxed{y' \sin(y') + \cos(y') - y = 0}$$

✓ Solution by Maple

Time used: 0.171 (sec). Leaf size: 32

```
dsolve(diff(y(x),x)*sin(diff(y(x),x))+cos(diff(y(x),x)) = y(x),y(x), singsol=all)
```

$$y(x) = 1$$

$$x - \left(\int^{y(x)} \frac{1}{\text{RootOf}(\_Z \sin(\_Z) + \cos(\_Z) - \_a)} d\_a\right) - c_1 = 0$$

✓ Solution by Mathematica

Time used: 0.044 (sec). Leaf size: 28

```
DSolve[y'[x] Sin[y'[x]]+ Cos[y'[x]]==y[x],y[x],x,IncludeSingularSolutions -> True]
```

$$\text{Solve}[\{x = \sin(K[1]) + c_1, y(x) = K[1]\sin(K[1]) + \cos(K[1])\}, \{y(x), K[1]\}]$$

37.17 problem 1137

Internal problem ID [4331]

Book: Ordinary differential equations and their solutions. By George Moseley Murphy. 1960
Section: Various 37
Problem number: 1137.
ODE order: 1.
ODE degree: 0.

CAS Maple gives this as type `[_dAlembert]`

$$y'^2(x + \sin(y')) - y = 0$$

✓ Solution by Maple

Time used: 0.359 (sec). Leaf size: 56

```
dsolve(diff(y(x),x)^2*(x+sin(diff(y(x),x))) = y(x),y(x), singsol=all)
```

$$y(x) = 0$$

$$\left[x(\_T) = \frac{(-\_T^2 + \_T)\sin(\_T) - \cos(\_T) + c_1}{(\_T - 1)^2}, y(\_T) = -\frac{((\_T - 1)\sin(\_T) + \cos(\_T) - c_1)\_T^2}{(\_T - 1)^2}\right]$$

✓ Solution by Mathematica

Time used: 0.162 (sec). Leaf size: 61

```
DSolve[(y'[x])^2 (x+Sin[y'[x]])==y[x],y[x],x,IncludeSingularSolutions -> True]
```

$$\text{Solve}\left[\left\{x = \frac{-(K[1]-1)K[1]\sin(K[1]) - \cos(K[1])}{(K[1]-1)^2} + \frac{c_1}{(K[1]-1)^2}, y(x) = xK[1]^2 + K[1]^2\sin(K[1])\right\}, \{y(x), K[1]\}\right]$$

37.18 problem 1138

Internal problem ID [4332]

Book: Ordinary differential equations and their solutions. By George Moseley Murphy. 1960
Section: Various 37
Problem number: 1138.
ODE order: 1.
ODE degree: 0.

CAS Maple gives this as type `[_Clairaut]`

$$\left(1+y'^2\right)\sin\left(-y+xy'\right)^2=1$$

✓ Solution by Maple

Time used: 0.172 (sec). Leaf size: 139

```
dsolve((1+diff(y(x),x)^2)*sin(y(x)-x*diff(y(x),x))^2 = 1,y(x), singsol=all)
```

$$y(x)=-x\sqrt{\frac{1}{x}}\,\sqrt{1-x}-\arcsin\left(\frac{1}{\sqrt{\frac{1}{x}}}\right)$$
$$y(x)=x\sqrt{\frac{1}{x}}\,\sqrt{1-x}+\arcsin\left(\frac{1}{\sqrt{\frac{1}{x}}}\right)$$
$$y(x)=-x\sqrt{-\frac{1}{x}}\,\sqrt{x+1}+\arcsin\left(\frac{1}{\sqrt{-\frac{1}{x}}}\right)$$
$$y(x)=x\sqrt{-\frac{1}{x}}\,\sqrt{x+1}-\arcsin\left(\frac{1}{\sqrt{-\frac{1}{x}}}\right)$$
$$y(x)=c_1x-\arcsin\left(\frac{1}{\sqrt{c_1^2+1}}\right)$$
$$y(x)=c_1x+\arcsin\left(\frac{1}{\sqrt{c_1^2+1}}\right)$$

✓ Solution by Mathematica

Time used: 0.335 (sec). Leaf size: 77

```
DSolve[(1+(y'[x])^2) (Sin[y[x]-x y'[x]])^2==1,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to c_1 x - \frac{1}{2} \arccos\left(\frac{-1 + c_1{}^2}{1 + c_1{}^2}\right)$$
$$y(x) \to \frac{1}{2} \arccos\left(\frac{-1 + c_1{}^2}{1 + c_1{}^2}\right) + c_1 x$$
$$y(x) \to -\frac{\pi}{2}$$
$$y(x) \to \frac{\pi}{2}$$

37.19 problem 1140

Internal problem ID [4333]

Book: Ordinary differential equations and their solutions. By George Moseley Murphy. 1960
Section: Various 37
Problem number: 1140.
ODE order: 1.
ODE degree: 0.

CAS Maple gives this as type `[_quadrature]`

$$\left(1+{y'}^2\right)\left(\arctan\left(y'\right)+ax\right)+y'=0$$

✓ Solution by Maple
Time used: 0.0 (sec). Leaf size: 20

```
dsolve((1+diff(y(x),x)^2)*(arctan(diff(y(x),x))+a*x)+diff(y(x),x) = 0,y(x), singsol=all)
```

$$y(x)=\int \tan\left(\operatorname{RootOf}\left(ax+\sin\left(\_Z\right)\cos\left(\_Z\right)+\_Z\right)\right)dx+c_1$$

✓ Solution by Mathematica
Time used: 1.199 (sec). Leaf size: 58

```
DSolve[(1+(y'[x])^2)(ArcTan[y'[x]]+a x)+y'[x]==0,y[x],x,IncludeSingularSolutions -> True]
```

$$\text{Solve}\left[\left\{y(x)=\frac{1}{a\left(K[1]^2+1\right)}+c_1,x=\frac{K[1]^2(-\arctan(K[1]))-\arctan(K[1])-K[1]}{a\left(K[1]^2+1\right)}\right\},\{y(x),K[1]\}\right]$$

37.20 problem 1141

Internal problem ID [4334]

Book: Ordinary differential equations and their solutions. By George Moseley Murphy. 1960
Section: Various 37
Problem number: 1141.
ODE order: 1.
ODE degree: 0.

CAS Maple gives this as type `[_quadrature]`

$$-{y'}^2 = -e^{y'-y} - 1$$

✓ Solution by Maple
Time used: 0.016 (sec). Leaf size: 31

```
dsolve(exp(diff(y(x),x)-y(x))-diff(y(x),x)^2+1 = 0,y(x), singsol=all)
```

$$x - \left(\int^{y(x)} \frac{1}{\operatorname{RootOf}\left(-e^{\_Z-\_a} + \_Z^2 - 1\right)} d\_a\right) - c_1 = 0$$

✓ Solution by Mathematica
Time used: 0.132 (sec). Leaf size: 44

```
DSolve[Exp[y'[x]-y[x]]-(y'[x])^2+1==0,y[x],x,IncludeSingularSolutions -> True]
```

$$\text{Solve}\left[\{x = -\log(1 - K[1]) + \log(K[1]) + \log(K[1] + 1) + c_1, y(x) = K[1] - \log\left(K[1]^2 - 1\right)\}, \{y(x), K[1]\}\right]$$

37.21 problem 1143

Internal problem ID [4335]

Book: Ordinary differential equations and their solutions. By George Moseley Murphy. 1960
Section: Various 37
Problem number: 1143.
ODE order: 1.
ODE degree: 0.

CAS Maple gives this as type `[_quadrature]`

$$\boxed{\ln(y') + xy' = -a}$$

✓ Solution by Maple
Time used: 0.0 (sec). Leaf size: 24

```
dsolve(ln(diff(y(x),x))+x*diff(y(x),x)+a = 0,y(x), singsol=all)
```

$$y(x) = \frac{\operatorname{LambertW}(x\,\mathrm{e}^{-a})^2}{2} + \operatorname{LambertW}\left(x\,\mathrm{e}^{-a}\right) + c_1$$

✓ Solution by Mathematica
Time used: 0.034 (sec). Leaf size: 30

```
DSolve[Log[y'[x]]+x y'[x]+ a ==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to \frac{1}{2}W\left(e^{-a}x\right)^2 + W\left(e^{-a}x\right) + c_1$$

37.22 problem 1144

Internal problem ID [4336]

Book: Ordinary differential equations and their solutions. By George Moseley Murphy. 1960
Section: Various 37
Problem number: 1144.
ODE order: 1.
ODE degree: 0.

CAS Maple gives this as type `[[_1st_order, _with_linear_symmetries], _Clairaut]`

$$\boxed{\ln (y') + xy' - y = -a}$$

✓ Solution by Maple
Time used: 0.0 (sec). Leaf size: 23

```
dsolve(ln(diff(y(x),x))+x*diff(y(x),x)+a = y(x),y(x), singsol=all)
```

$$y(x) = \ln \left(-\frac{1}{x}\right) + a - 1$$
$$y(x) = \ln (c_1) + c_1 x + a$$

✓ Solution by Mathematica
Time used: 0.046 (sec). Leaf size: 27

```
DSolve[Log[y'[x]]+x y'[x]+ a ==y[x],y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to a + c_1 x + \log(c_1)$$
$$y(x) \to a + \log \left(-\frac{1}{x}\right) - 1$$

37.23 problem 1145

Internal problem ID [4337]

Book: Ordinary differential equations and their solutions. By George Moseley Murphy. 1960
Section: Various 37
Problem number: 1145.
ODE order: 1.
ODE degree: 0.

CAS Maple gives this as type `[[_1st_order, _with_linear_symmetries], _dAlembert]`

$$\boxed{\ln(y') + xy' + by = -a}$$

✓ Solution by Maple
Time used: 0.078 (sec). Leaf size: 73

```
dsolve(ln(diff(y(x),x))+x*diff(y(x),x)+a+b*y(x) = 0,y(x), singsol=all)
```

$$\frac{-\left(\left(\frac{\mathrm{LambertW}\left(x\,\mathrm{e}^{-by(x)-a}\right)}{x}\right)^{-\frac{1}{b+1}} c_1 - x\right) b\,\mathrm{LambertW}\left(x\,\mathrm{e}^{-by(x)-a}\right) - x}{b\,\mathrm{LambertW}\left(x\,\mathrm{e}^{-by(x)-a}\right)} = 0$$

✓ Solution by Mathematica
Time used: 0.139 (sec). Leaf size: 59

```
DSolve[Log[y'[x]]+x y'[x]+ a +b y[x]==0,y[x],x,IncludeSingularSolutions -> True]
```

$$\text{Solve}\left[b\left(\frac{(b+1)\log\left(1-bW\left(xe^{-a-by(x)}\right)\right)}{b^2} + \frac{W\left(xe^{-a-by(x)}\right)}{b}\right) + by(x) = c_1, y(x)\right]$$

37.24 problem 1146

Internal problem ID [4338]

Book: Ordinary differential equations and their solutions. By George Moseley Murphy. 1960
Section: Various 37
Problem number: 1146.
ODE order: 1.
ODE degree: 0.

CAS Maple gives this as type `[[_1st_order, _with_linear_symmetries], _dAlembert]`

$$\boxed{\ln(y') + 4xy' - 2y = 0}$$

✓ Solution by Maple
Time used: 0.015 (sec). Leaf size: 75

```
dsolve(ln(diff(y(x),x))+4*x*diff(y(x),x)-2*y(x) = 0,y(x), singsol=all)
```

$$y(x) = -\ln(2) + \frac{\ln\left(\frac{-1+\sqrt{16c_1x+1}}{x}\right)}{2} - \frac{1}{2} + \frac{\sqrt{16c_1x+1}}{2}$$
$$y(x) = -\ln(2) + \frac{\ln\left(\frac{-1-\sqrt{16c_1x+1}}{x}\right)}{2} - \frac{1}{2} - \frac{\sqrt{16c_1x+1}}{2}$$

✓ Solution by Mathematica
Time used: 0.093 (sec). Leaf size: 36

```
DSolve[Log[y'[x]]+4 x y'[x]-2 y[x]==0,y[x],x,IncludeSingularSolutions -> True]
```

$$\text{Solve}\left[W\left(4xe^{2y(x)}\right) - \log\left(W\left(4xe^{2y(x)}\right) + 2\right) - 2y(x) = c_1, y(x)\right]$$

37.25 problem 1147

Internal problem ID [4339]

Book: Ordinary differential equations and their solutions. By George Moseley Murphy. 1960
Section: Various 37
Problem number: 1147.
ODE order: 1.
ODE degree: 0.

CAS Maple gives this as type `[[_1st_order, _with_linear_symmetries], _Clairaut]`

$$\boxed{\ln(y') + a(-y + xy') = 0}$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 32

```
dsolve(ln(diff(y(x),x))+a*(x*diff(y(x),x)-y(x)) = 0,y(x), singsol=all)
```

$$y(x) = \frac{\ln\left(-\frac{1}{ax}\right) - 1}{a}$$
$$y(x) = c_1 x + \frac{\ln(c_1)}{a}$$

✓ Solution by Mathematica

Time used: 0.047 (sec). Leaf size: 36

```
DSolve[Log[y'[x]]+a*( x*y'[x]-y[x])==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to \frac{\log(c_1)}{a} + c_1 x$$
$$y(x) \to \frac{\log\left(-\frac{1}{ax}\right) - 1}{a}$$

37.26 problem 1148

Internal problem ID [4340]

Book: Ordinary differential equations and their solutions. By George Moseley Murphy. 1960
Section: Various 37
Problem number: 1148.
ODE order: 1.
ODE degree: 0.

CAS Maple gives this as type `[[_homogeneous, ‘class C‘], _dAlembert]`

$$\boxed{a(\ln(y') - y') + y = x}$$

✓ Solution by Maple
Time used: 0.0 (sec). Leaf size: 38

```
dsolve(a*(ln(diff(y(x),x))-diff(y(x),x))-x+y(x) = 0,y(x), singsol=all)
```

$$y(x) = a + x$$
$$y(x) = -a \ln\left(e^{\frac{x-c_1}{a}}\right) + a\, e^{\frac{x-c_1}{a}} + x$$

✓ Solution by Mathematica
Time used: 0.39 (sec). Leaf size: 22

```
DSolve[a*(Log[y'[x]]-y'[x])-x+y[x]==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to ae^{\frac{x-c_1}{a}} + c_1$$

37.27 problem 1149

Internal problem ID [4341]

Book: Ordinary differential equations and their solutions. By George Moseley Murphy. 1960
Section: Various 37
Problem number: 1149.
ODE order: 1.
ODE degree: 0.

CAS Maple gives this as type `[_separable]`

$$\boxed{y\ln(y') + y' - y\ln(y) - xy = 0}$$

✓ Solution by Maple
Time used: 0.063 (sec). Leaf size: 17

```
dsolve(y(x)*ln(diff(y(x),x))+diff(y(x),x)-y(x)*ln(y(x))-x*y(x) = 0,y(x), singsol=all)
```

$$y(x) = c_1 \mathrm{e}^{\frac{\mathrm{LambertW}(\mathrm{e}^{x})(\mathrm{LambertW}(\mathrm{e}^{x})+2)}{2}}$$

✓ Solution by Mathematica
Time used: 0.093 (sec). Leaf size: 24

```
DSolve[y[x] Log[y'[x]] + y'[x] -y[x] Log[y[x]] -x y[x]==0,y[x],x,IncludeSingularSolutions ->
```

$$y(x) \to c_1 e^{\frac{1}{2}W(e^x)(W(e^x)+2)}$$

37.28 problem 1150

Internal problem ID [4342]

Book: Ordinary differential equations and their solutions. By George Moseley Murphy. 1960
Section: Various 37
Problem number: 1150.
ODE order: 1.
ODE degree: 0.

CAS Maple gives this as type `[[_1st_order, _with_linear_symmetries], _Clairaut]`

$$\boxed{y' \ln(y') - (x+1)\, y' + y = 0}$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 18

```
dsolve(diff(y(x),x)*ln(diff(y(x),x))-(1+x)*diff(y(x),x)+y(x) = 0,y(x), singsol=all)
```

$$y(x) = e^x$$
$$y(x) = c_1(-\ln(c_1) + x + 1)$$

✓ Solution by Mathematica

Time used: 1.646 (sec). Leaf size: 21

```
DSolve[y'[x] Log[y'[x]] -(1+x) y'[x]+y[x]==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to c_1(x + 1 - \log(c_1))$$
$$y(x) \to 0$$

37.29 problem 1152

Internal problem ID [4343]

Book: Ordinary differential equations and their solutions. By George Moseley Murphy. 1960
Section: Various 37
Problem number: 1152.
ODE order: 1.
ODE degree: 0.

CAS Maple gives this as type `[_Clairaut]`

$$y' \ln \left(y' + \sqrt{a + y'^2}\right) - \sqrt{1 + y'^2} - xy' + y = 0$$

✗ Solution by Maple

```
dsolve(diff(y(x),x)*ln(diff(y(x),x)+sqrt(a+diff(y(x),x)^2))-sqrt(1+diff(y(x),x)^2)-x*diff(y(
```

No solution found

✓ Solution by Mathematica

Time used: 60.03 (sec). Leaf size: 38

```
DSolve[y'[x]*Log[y'[x]+Sqrt[a+(y'[x])^2]]-Sqrt[1+(y'[x])^2]-x*y'[x]+y[x]==0,y[x],x,IncludeSi
```

$$y(x) \to -c_1 \log \left(\sqrt{a + c_1^2} + c_1\right) + c_1 x + \sqrt{1 + c_1^2}$$

37.30 problem 1153

Internal problem ID [4344]

Book: Ordinary differential equations and their solutions. By George Moseley Murphy. 1960
Section: Various 37
Problem number: 1153.
ODE order: 1.
ODE degree: 0.

CAS Maple gives this as type `[_dAlembert]`

$$\boxed{\ln(\cos(y')) + y' \tan(y') - y = 0}$$

✓ Solution by Maple
Time used: 0.031 (sec). Leaf size: 33

```
dsolve(ln(cos(diff(y(x),x)))+diff(y(x),x)*tan(diff(y(x),x)) = y(x),y(x), singsol=all)
```

$$y(x) = 0$$
$$x - \left(\int^{y(x)} \frac{1}{\operatorname{RootOf}(\ln(\cos(\_Z)) + \_Z\tan(\_Z) - \_a)} d\_a\right) - c_1 = 0$$

✓ Solution by Mathematica
Time used: 0.073 (sec). Leaf size: 29

```
DSolve[Log[Cos[y'[x]]]+y'[x] Tan[y'[x]]==y[x],y[x],x,IncludeSingularSolutions -> True]
```

$$\text{Solve}[\{x = \tan(K[1]) + c_1, y(x) = K[1]\tan(K[1]) + \log(\cos(K[1]))\}, \{y(x), K[1]\}]$$