A Solution Manual For

Own collection of miscellaneous problems

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Contents

1 section 1.0

1.1 problem 1

Internal problem ID [7045]

Book: Own collection of miscellaneous problems
Section: section 1.0
Problem number: 1.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_separable]`

$$\boxed{y' - \frac{\cos(y)\sec(x)}{x} = 0}$$

✓ Solution by Maple
Time used: 0.11 (sec). Leaf size: 73

```
dsolve(diff(y(x),x) = cos(y(x))*sec(x)/x,y(x), singsol=all)
```

$$y(x) = \arctan\left(\frac{e^{2\left(\int \frac{\sec(x)}{x}dx\right)}c_1^2 - 1}{e^{2\left(\int \frac{\sec(x)}{x}dx\right)}c_1^2 + 1}, \frac{2\,e^{\int \frac{\sec(x)}{x}dx}c_1}{e^{2\left(\int \frac{\sec(x)}{x}dx\right)}c_1^2 + 1}\right)$$

✓ Solution by Mathematica
Time used: 5.307 (sec). Leaf size: 49

```
DSolve[y'[x]== Cos[y[x]]*Sec[x]/x,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to 2\arctan\left(\tanh\left(\frac{1}{2}\left(\int_1^x \frac{\sec(K[1])}{K[1]}dK[1] + c_1\right)\right)\right)$$
$$y(x) \to -\frac{\pi}{2}$$
$$y(x) \to \frac{\pi}{2}$$

1.2 problem 2

Internal problem ID [7046]

Book: Own collection of miscellaneous problems
Section: section 1.0
Problem number: 2.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_separable]`

$$\boxed{y' - x(\cos(y) + y) = 0}$$

✓ Solution by Maple
Time used: 0.0 (sec). Leaf size: 23

```
dsolve(diff(y(x),x) = x*(cos(y(x))+y(x)),y(x), singsol=all)
```

$$\frac{x^2}{2} - \left(\int^{y(x)} \frac{1}{\cos(\_a) + \_a} d\_a\right) + c_1 = 0$$

✓ Solution by Mathematica
Time used: 0.71 (sec). Leaf size: 33

```
DSolve[y'[x] == x*(Cos[y[x]]+y[x]),y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to \text{InverseFunction}\left[\int_1^{\#1} \frac{1}{\cos(K[1]) + K[1]} dK[1]\&\right]\left[\frac{x^2}{2} + c_1\right]$$

1.3 problem 3

Internal problem ID [7047]

Book: Own collection of miscellaneous problems
Section: section 1.0
Problem number: 3.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_separable]`

$$\boxed{y' - \frac{\sec(x)(\sin(y) + y)}{x} = 0}$$

✓ Solution by Maple
Time used: 0.0 (sec). Leaf size: 26

```
dsolve(diff(y(x),x) = sec(x)*(sin(y(x))+y(x))/x,y(x), singsol=all)
```

$$\int \frac{\sec(x)}{x}dx - \left(\int^{y(x)} \frac{1}{\sin(\_a) + \_a}d\_a\right) + c_1 = 0$$

✓ Solution by Mathematica
Time used: 1.312 (sec). Leaf size: 41

```
DSolve[y'[x]== Sec[x]*(Sin[y[x]]+y[x])/x,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to \text{InverseFunction}\left[\int_1^{\#1} \frac{1}{K[1] + \sin(K[1])}dK[1]\&\right]\left[\int_1^x \frac{\sec(K[2])}{K[2]}dK[2] + c_1\right]$$

1.4 problem 4

Internal problem ID [7048]

Book: Own collection of miscellaneous problems
Section: section 1.0
Problem number: 4.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_separable]`

$$y' - \left(5 + \frac{\sec(x)}{x}\right)(\sin(y) + y) = 0$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 30

```
dsolve(diff(y(x),x) = (5+sec(x)/x)*(sin(y(x))+y(x)),y(x), singsol=all)
```

$$\int \frac{5x + \sec(x)}{x} dx - \left(\int^{y(x)} \frac{1}{\sin(\_a) + \_a} d\_a\right) + c_1 = 0$$

✓ Solution by Mathematica

Time used: 19.938 (sec). Leaf size: 168

```
DSolve[y'[x] == (5+Sec[x]/x)*(Sin[y[x]]+y[x]),y[x],x,IncludeSingularSolutions -> True]
```

$$\text{Solve}\left[\int_1^x \left(-\frac{2\sec(K[1])}{K[1]} - \frac{5(-\sec(K[1])\sin(K[1]-y(x)) + \sec(K[1])\sin(K[1]+y(x)) + 2y(x))}{\sin(y(x)) + y(x)}\right) dK[1] + \int_1^{y(x)} \left(\frac{2}{K[2]+\sin(K[2])} - \int_1^x \left(\frac{5(\cos(K[2])+1)(2K[2] - \sec(K[1])\sin(K[1]-K[2]) + \sec(K[1])\sin(K[1]+K[2]))}{(K[2]+\sin(K[2]))^2} - \frac{5(\cos(K[1]}{}\right.\right.\right.$$

1.5 problem 5

Internal problem ID [7049]

Book: Own collection of miscellaneous problems
Section: section 1.0
Problem number: 5.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_quadrature]`

$$\boxed{y' - y = 1}$$

✓ Solution by Maple
Time used: 0.0 (sec). Leaf size: 10

```
dsolve(diff(y(x),x) = y(x)+1,y(x), singsol=all)
```

$$y(x) = -1 + e^{x} c_1$$

✓ Solution by Mathematica
Time used: 0.025 (sec). Leaf size: 18

```
DSolve[y'[x] == y[x]+1,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to -1 + c_1 e^{x}$$
$$y(x) \to -1$$

1.6 problem 6

Internal problem ID [7050]

Book: Own collection of miscellaneous problems
Section: section 1.0
Problem number: 6.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_quadrature]`

$$\boxed{y' = 1 + x}$$

✓ Solution by Maple
Time used: 0.0 (sec). Leaf size: 12

```
dsolve(diff(y(x),x) = 1+x,y(x), singsol=all)
```

$$y(x) = \frac{1}{2}x^2 + x + c_1$$

✓ Solution by Mathematica
Time used: 0.002 (sec). Leaf size: 16

```
DSolve[y'[x]== 1+x,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to \frac{x^2}{2} + x + c_1$$

1.7 problem 7

Internal problem ID [7051]

Book: Own collection of miscellaneous problems
Section: section 1.0
Problem number: 7.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_quadrature]`

$$\boxed{y' = x}$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 11

```
dsolve(diff(y(x),x) = x,y(x), singsol=all)
```

$$y(x) = \frac{x^2}{2} + c_1$$

✓ Solution by Mathematica

Time used: 0.002 (sec). Leaf size: 15

```
DSolve[y'[x] == x,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to \frac{x^2}{2} + c_1$$

1.8 problem 8

Internal problem ID [7052]

Book: Own collection of miscellaneous problems
Section: section 1.0
Problem number: 8.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_quadrature]`

$$\boxed{y' - y = 0}$$

✓ Solution by Maple
Time used: 0.0 (sec). Leaf size: 8

```
dsolve(diff(y(x),x) = y(x),y(x), singsol=all)
```

$$y(x) = e^x c_1$$

✓ Solution by Mathematica
Time used: 0.028 (sec). Leaf size: 16

```
DSolve[y'[x] == y[x],y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to c_1 e^x$$
$$y(x) \to 0$$

1.9 problem 9

Internal problem ID [7053]

Book: Own collection of miscellaneous problems
Section: section 1.0
Problem number: 9.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_quadrature]`

$$\boxed{y' = 0}$$

✓ Solution by Maple
Time used: 0.0 (sec). Leaf size: 5

```
dsolve(diff(y(x),x) = 0,y(x), singsol=all)
```

$$y(x) = c_1$$

✓ Solution by Mathematica
Time used: 0.002 (sec). Leaf size: 7

```
DSolve[y'[x] == 0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to c_1$$

1.10 problem 10

Internal problem ID [7054]

Book: Own collection of miscellaneous problems
Section: section 1.0
Problem number: 10.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_quadrature]`

$$y' = 1 + \frac{\sec(x)}{x}$$

✓ Solution by Maple

Time used: 0.016 (sec). Leaf size: 15

```
dsolve(diff(y(x),x) = 1+sec(x)/x,y(x), singsol=all)
```

$$y(x) = \int \frac{\sec(x)}{x}dx + x + c_1$$

✓ Solution by Mathematica

Time used: 0.833 (sec). Leaf size: 25

```
DSolve[y'[x] == 1+Sec[x]/x,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to \int_1^x \left(\frac{\sec(K[1])}{K[1]} + 1\right) dK[1] + c_1$$

1.11 problem 11

Internal problem ID [7055]

Book: Own collection of miscellaneous problems
Section: section 1.0
Problem number: 11.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_linear]`

$$\boxed{y' - \frac{\sec(x)\, y}{x} = x}$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 31

```
dsolve(diff(y(x),x) = x+sec(x)*y(x)/x,y(x), singsol=all)
```

$$y(x) = \left(\int x\, \mathrm{e}^{-\left(\int \frac{\sec(x)}{x}dx\right)}dx + c_1\right) \mathrm{e}^{\int \frac{\sec(x)}{x}dx}$$

✓ Solution by Mathematica

Time used: 0.483 (sec). Leaf size: 56

```
DSolve[y'[x] == x+Sec[x]*y[x]/x,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to \exp\left(\int_1^x \frac{\sec(K[1])}{K[1]}dK[1]\right) \left(\int_1^x \exp\left(-\int_1^{K[2]} \frac{\sec(K[1])}{K[1]}dK[1]\right) K[2]dK[2] + c_1\right)$$

1.12 problem 12

Internal problem ID [7056]

Book: Own collection of miscellaneous problems
Section: section 1.0
Problem number: 12.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_separable]`

$$\boxed{y' - \frac{2y}{x} = 0}$$

With initial conditions

$$[y(0) = 0]$$

✓ Solution by Maple
Time used: 0.0 (sec). Leaf size: 9

```
dsolve([diff(y(x),x) = 2*y(x)/x,y(0) = 0],y(x), singsol=all)
```

$$y(x) = c_1 x^2$$

✓ Solution by Mathematica
Time used: 0.001 (sec). Leaf size: 6

```
DSolve[{y'[x] == 2*y[x]/x,y[0]==0},y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to 0$$

1.13 problem 13

Internal problem ID [7057]

Book: Own collection of miscellaneous problems
Section: section 1.0
Problem number: 13.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_separable]`

$$\boxed{y' - \frac{2y}{x} = 0}$$

✓ Solution by Maple
Time used: 0.0 (sec). Leaf size: 9

```
dsolve(diff(y(x),x) = 2*y(x)/x,y(x), singsol=all)
```

$$y(x) = c_1 x^2$$

✓ Solution by Mathematica
Time used: 0.024 (sec). Leaf size: 16

```
DSolve[y'[x] == 2*y[x]/x,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to c_1 x^2$$
$$y(x) \to 0$$

1.14 problem 14

Internal problem ID [7058]

Book: Own collection of miscellaneous problems
Section: section 1.0
Problem number: 14.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_separable]`

$$\boxed{y' - \frac{\ln(1+y^2)}{\ln(x^2+1)} = 0}$$

✓ Solution by Maple
Time used: 0.016 (sec). Leaf size: 30

```
dsolve(diff(y(x),x)=ln(y(x)^2+1)/ln(x^2+1),y(x), singsol=all)
```

$$\int \frac{1}{\ln(x^2+1)}dx - \left(\int^{y(x)} \frac{1}{\ln(\_a^2+1)}d\_a\right) + c_1 = 0$$

✓ Solution by Mathematica
Time used: 0.64 (sec). Leaf size: 48

```
DSolve[y'[x] == Log[1+y[x]^2]/Log[1+x^2],y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to \text{InverseFunction}\left[\int_1^{\#1} \frac{1}{\log(K[1]^2+1)}dK[1]\&\right]\left[\int_1^x \frac{1}{\log(K[2]^2+1)}dK[2] + c_1\right]$$
$$y(x) \to 0$$

1.15 problem 15

Internal problem ID [7059]

Book: Own collection of miscellaneous problems
Section: section 1.0
Problem number: 15.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_quadrature]`

$$\boxed{y' = \frac{1}{x}}$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 8

```
dsolve(diff(y(x),x)=1/x,y(x), singsol=all)
```

$$y(x) = \ln(x) + c_1$$

✓ Solution by Mathematica

Time used: 0.002 (sec). Leaf size: 10

```
DSolve[y'[x] == 1/x,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to \log(x) + c_1$$

1.16 problem 16

Internal problem ID [7060]

Book: Own collection of miscellaneous problems
Section: section 1.0
Problem number: 16.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[[_homogeneous, ‘class G‘], _rational, [_Abel, ‘2nd type‘, ‘cl`

$$y' - \frac{-yx - 1}{4yx^3 - 2x^2} = 0$$

✓ Solution by Maple
Time used: 0.562 (sec). Leaf size: 37

```
dsolve(diff(y(x),x)=(-x*y(x)-1)/(4*x^3*y(x)-2*x^2),y(x), singsol=all)
```

$$y(x) = \frac{\mathrm{RootOf}\left(\_Z^{25}c_1 - 10\_Z^{20}c_1 + 25\_Z^{15}c_1 - 16x^5\right)^5 - 1}{4x}$$

✓ Solution by Mathematica
Time used: 15.76 (sec). Leaf size: 391

```
DSolve[y'[x] == (-x*y[x]-1)/(4*x^3*y[x]-2*x^2),y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to \mathrm{Root}\left[64\#1^5{c_1}^5x^5 - 80\#1^4{c_1}^5x^4 - 20\#1^3{c_1}^5x^3 + 25\#1^2{c_1}^5x^2 + 10\#1{c_1}^5x - x^5 + {c_1}^5\&, 1\right]$$
$$y(x) \to \mathrm{Root}\left[64\#1^5{c_1}^5x^5 - 80\#1^4{c_1}^5x^4 - 20\#1^3{c_1}^5x^3 + 25\#1^2{c_1}^5x^2 + 10\#1{c_1}^5x - x^5 + {c_1}^5\&, 2\right]$$
$$y(x) \to \mathrm{Root}\left[64\#1^5{c_1}^5x^5 - 80\#1^4{c_1}^5x^4 - 20\#1^3{c_1}^5x^3 + 25\#1^2{c_1}^5x^2 + 10\#1{c_1}^5x - x^5 + {c_1}^5\&, 3\right]$$
$$y(x) \to \mathrm{Root}\left[64\#1^5{c_1}^5x^5 - 80\#1^4{c_1}^5x^4 - 20\#1^3{c_1}^5x^3 + 25\#1^2{c_1}^5x^2 + 10\#1{c_1}^5x - x^5 + {c_1}^5\&, 4\right]$$
$$y(x) \to \mathrm{Root}\left[64\#1^5{c_1}^5x^5 - 80\#1^4{c_1}^5x^4 - 20\#1^3{c_1}^5x^3 + 25\#1^2{c_1}^5x^2 + 10\#1{c_1}^5x - x^5 + {c_1}^5\&, 5\right]$$

1.17 problem 17

Internal problem ID [7061]

Book: Own collection of miscellaneous problems
Section: section 1.0
Problem number: 17.
ODE order: 1.
ODE degree: 2.

CAS Maple gives this as type `[[_1st_order, _with_linear_symmetries], _Clairaut]`

$$\boxed{\frac{y'^2}{4} - xy' + y = 0}$$

✓ Solution by Maple
Time used: 0.062 (sec). Leaf size: 18

```
dsolve((1/4)*diff(y(x),x)^2-x*diff(y(x),x)+y(x)=0,y(x), singsol=all)
```

$$y(x) = x^2$$
$$y(x) = -\frac{c_1(c_1 - 4x)}{4}$$

✓ Solution by Mathematica
Time used: 0.011 (sec). Leaf size: 25

```
DSolve[(1/4)*(y'[x])^2-x*y'[x]+y[x]==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to c_1x - \frac{c_1{}^2}{4}$$
$$y(x) \to x^2$$

1.18 problem 18

Internal problem ID [7062]

Book: Own collection of miscellaneous problems
Section: section 1.0
Problem number: 18.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_quadrature]`

$$\boxed{y' - \sqrt{\frac{y+1}{y^2}} = 0}$$

With initial conditions

$$[y(0) = 1]$$

✓ Solution by Maple
Time used: 0.469 (sec). Leaf size: 148

```
dsolve([diff(y(x),x)=sqrt( (1+y(x))/y(x)^2),y(0) = 1],y(x), singsol=all)
```

$$y(x) = -\frac{\left(1+i\sqrt{3}\right)\left(-12\sqrt{2}\,x+9x^2+\sqrt{\left(-12\sqrt{2}\,x+9x^2-8\right)\left(3x-2\sqrt{2}\right)^2}\right)^{\frac{2}{3}} - 4i\sqrt{3} - 4\left(-12\sqrt{2}\,x+9x^2 \right.}{4\left(-12\sqrt{2}\,x+9x^2+\sqrt{\left(-12\sqrt{2}\,x+9x^2-8\right)\left(3x-2\sqrt{2}\right.}\right.}$$

✓ Solution by Mathematica

Time used: 0.097 (sec). Leaf size: 123

```
DSolve[{y'[x]==Sqrt[ (1+y[x])/y[x]^2],y[0]==1},y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to -\frac{1}{4}\left(1+i\sqrt{3}\right)\sqrt[3]{9x^2+\sqrt{81x^4-216\sqrt{2}x^3+288x^2-64}-12\sqrt{2}x} + \frac{i(\sqrt{3}+i)}{\sqrt[3]{9x^2+\sqrt{81x^4-216\sqrt{2}x^3+288x^2-64}-12\sqrt{2}x}} + 1$$

1.19 problem 19

Internal problem ID [7063]

Book: Own collection of miscellaneous problems
Section: section 1.0
Problem number: 19.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[‘y=_G(x,y’)‘]`

$$y' - \sqrt{1 - x^2 - y^2} = 0$$

✗ Solution by Maple

```
dsolve(diff(y(x),x)=sqrt( 1-x^2-y(x)^2),y(x), singsol=all)
```

No solution found

✗ Solution by Mathematica
Time used: 0.0 (sec). Leaf size: 0

```
DSolve[y'[x]==Sqrt[ 1-x^2-y[x]^2],y[x],x,IncludeSingularSolutions -> True]
```

Not solved

1.20 problem 20

Internal problem ID [7064]

Book: Own collection of miscellaneous problems
Section: section 1.0
Problem number: 20.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_Bernoulli]`

$$\boxed{y' + \frac{y}{3} - \frac{(1-2x)\, y^4}{3} = 0}$$

✓ Solution by Maple
Time used: 0.0 (sec). Leaf size: 63

```
dsolve(diff(y(x),x)+y(x)/3= (1-2*x)/3*y(x)^4,y(x), singsol=all)
```

$$y(x) = \frac{1}{(e^x c_1 - 2x - 1)^{\frac{1}{3}}}$$
$$y(x) = -\frac{1 + i\sqrt{3}}{2\,(e^x c_1 - 2x - 1)^{\frac{1}{3}}}$$
$$y(x) = \frac{i\sqrt{3} - 1}{2\,(e^x c_1 - 2x - 1)^{\frac{1}{3}}}$$

✓ Solution by Mathematica

Time used: 4.53 (sec). Leaf size: 76

```
DSolve[y'[x]+y[x]/3== (1-2*x)/3*y[x]^4,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to \frac{1}{\sqrt[3]{-2x + c_1 e^x - 1}}$$
$$y(x) \to -\frac{\sqrt[3]{-1}}{\sqrt[3]{-2x + c_1 e^x - 1}}$$
$$y(x) \to \frac{(-1)^{2/3}}{\sqrt[3]{-2x + c_1 e^x - 1}}$$
$$y(x) \to 0$$

1.21 problem 21

Internal problem ID [7065]

Book: Own collection of miscellaneous problems
Section: section 1.0
Problem number: 21.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[[_1st_order, _with_linear_symmetries], _Chini]`

$$\boxed{y' - \sqrt{y} = x}$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 69

```
dsolve(diff(y(x),x)=sqrt(y(x))+x,y(x), singsol=all)
```

$$\frac{4 \operatorname{arctanh}\left(\sqrt{\frac{y(x)}{x^2}}\right)}{3} - \frac{2 \operatorname{arctanh}\left(2\sqrt{\frac{y(x)}{x^2}}\right)}{3} - \frac{\ln\left(\frac{-x^2+4y(x)}{x^2}\right)}{3} - \frac{2\ln(2)}{3} - \frac{2\ln\left(\frac{y(x)-x^2}{x^2}\right)}{3} - 2\ln(x) + c_1 = 0$$

✓ Solution by Mathematica

Time used: 47.265 (sec). Leaf size: 716

```
DSolve[y'[x]==Sqrt[y[x]]+x,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to \frac{1}{4}\left(3x^2 + \frac{e^{3c_1}x(8+e^{3c_1}x^3)}{\sqrt[3]{-e^{18c_1}x^6+20e^{15c_1}x^3+8\sqrt{-e^{24c_1}\left(-1+e^{3c_1}x^3\right)^3}+8e^{12c_1}}} + e^{-6c_1}\sqrt[3]{-e^{18c_1}x^6+20e^{15c_1}x^3+8\sqrt{-e^{24c_1}\left(-1+e^{3c_1}x^3\right)^3}+8e^{12c_1}}\right)$$

$$y(x) \to \frac{1}{72}\left(54x^2 - \frac{9i\left(\sqrt{3}-i\right)e^{3c_1}x(8+e^{3c_1}x^3)}{\sqrt[3]{-e^{18c_1}x^6+20e^{15c_1}x^3+8\sqrt{-e^{24c_1}\left(-1+e^{3c_1}x^3\right)^3}+8e^{12c_1}}} + 9i\left(\sqrt{3}+i\right)e^{-6c_1}\sqrt[3]{-e^{18c_1}x^6+20e^{15c_1}x^3+8\sqrt{-e^{24c_1}\left(-1+e^{3c_1}x^3\right)^3}+8e^{12c_1}}\right)$$

$$y(x) \to \frac{1}{72}\left(54x^2 + \frac{9i\left(\sqrt{3}+i\right)e^{3c_1}x(8+e^{3c_1}x^3)}{\sqrt[3]{-e^{18c_1}x^6+20e^{15c_1}x^3+8\sqrt{-e^{24c_1}\left(-1+e^{3c_1}x^3\right)^3}+8e^{12c_1}}} - 9\left(1+i\sqrt{3}\right)e^{-6c_1}\sqrt[3]{-e^{18c_1}x^6+20e^{15c_1}x^3+8\sqrt{-e^{24c_1}\left(-1+e^{3c_1}x^3\right)^3}+8e^{12c_1}}\right)$$

$$y(x) \to \frac{-(-x^6)^{2/3}+3x^4+\sqrt[3]{-x^6}x^2}{4x^2}$$

$$y(x) \to \frac{\left(1+i\sqrt{3}\right)(-x^6)^{2/3}+6x^4+i\left(\sqrt{3}+i\right)\sqrt[3]{-x^6}x^2}{8x^2}$$

$$y(x) \to \frac{1}{8}x^2\left(\frac{\left(1+i\sqrt{3}\right)x^4}{(-x^6)^{2/3}} + \frac{i\left(\sqrt{3}+i\right)x^2}{\sqrt[3]{-x^6}} + 6\right)$$

1.22 problem 23

Internal problem ID [7066]

Book: Own collection of miscellaneous problems
Section: section 1.0
Problem number: 23.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[[_homogeneous, ‘class A‘], _rational, [_Abel, ‘2nd type‘, ‘cl`

$$\boxed{x^2y' + y^2 - xyy' = 0}$$

✓ Solution by Maple
Time used: 0.047 (sec). Leaf size: 17

```
dsolve(x^2*diff(y(x),x)+y(x)^2=x*y(x)*diff(y(x),x),y(x), singsol=all)
```

$$y(x) = -x \operatorname{LambertW}\left(-\frac{e^{-c_1}}{x}\right)$$

✓ Solution by Mathematica
Time used: 2.396 (sec). Leaf size: 25

```
DSolve[x^2*y'[x]+y[x]^2==x*y[x]*y'[x],y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to -xW\left(-\frac{e^{-c_1}}{x}\right)$$
$$y(x) \to 0$$

1.23 problem 24

Internal problem ID [7067]

Book: Own collection of miscellaneous problems
Section: section 1.0
Problem number: 24.
ODE order: 1.
ODE degree: 2.

CAS Maple gives this as type `[_separable]`

$$\boxed{y - xy' - x^2 {y'}^2 = 0}$$

✓ Solution by Maple
Time used: 0.047 (sec). Leaf size: 97

```
dsolve(y(x)=x*diff(y(x),x)+x^2*diff(y(x),x)^2,y(x), singsol=all)
```

$$\ln(x) - \sqrt{4y(x)+1} - \frac{\ln\left(-1+\sqrt{4y(x)+1}\right)}{2} + \frac{\ln\left(1+\sqrt{4y(x)+1}\right)}{2} - \frac{\ln(y(x))}{2} - c_1 = 0$$

$$\ln(x) + \sqrt{4y(x)+1} + \frac{\ln\left(-1+\sqrt{4y(x)+1}\right)}{2} - \frac{\ln\left(1+\sqrt{4y(x)+1}\right)}{2} - \frac{\ln(y(x))}{2} - c_1 = 0$$

✓ Solution by Mathematica
Time used: 22.779 (sec). Leaf size: 72

```
DSolve[y[x]==x*y'[x]+x^2*(y'[x])^2,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to \frac{1}{4} W\left(-e^{-1-2c_1}x\right)\left(2 + W\left(-e^{-1-2c_1}x\right)\right)$$
$$y(x) \to \frac{1}{4} W\left(e^{-1+2c_1}x\right)\left(2 + W\left(e^{-1+2c_1}x\right)\right)$$
$$y(x) \to 0$$

1.24 problem 25

Internal problem ID [7068]

Book: Own collection of miscellaneous problems
Section: section 1.0
Problem number: 25.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_quadrature]`

$$\boxed{(x + y)\, y' = 0}$$

✓ Solution by Maple
Time used: 0.0 (sec). Leaf size: 11

```
dsolve((x+y(x))*diff(y(x),x)=0,y(x), singsol=all)
```

$$y(x) = -x$$
$$y(x) = c_1$$

✓ Solution by Mathematica
Time used: 0.002 (sec). Leaf size: 14

```
DSolve[(x+y[x])*y'[x]==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to -x$$
$$y(x) \to c_1$$

1.25 problem 26

Internal problem ID [7069]

Book: Own collection of miscellaneous problems
Section: section 1.0
Problem number: 26.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_quadrature]`

$$\boxed{xy' = 0}$$

✓ Solution by Maple
Time used: 0.0 (sec). Leaf size: 5

```
dsolve(x*diff(y(x),x)=0,y(x), singsol=all)
```

$$y(x) = c_1$$

✓ Solution by Mathematica
Time used: 0.002 (sec). Leaf size: 7

```
DSolve[x*y'[x]==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to c_1$$

1.26 problem 27

Internal problem ID [7070]

Book: Own collection of miscellaneous problems
Section: section 1.0
Problem number: 27.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_quadrature]`

$$\boxed{\frac{y'}{x+y}=0}$$

✓ Solution by Maple
Time used: 0.0 (sec). Leaf size: 5

```
dsolve(1/(x+y(x))*diff(y(x),x)=0,y(x), singsol=all)
```

$$y(x)=c_1$$

✓ Solution by Mathematica
Time used: 0.002 (sec). Leaf size: 7

```
DSolve[1/(x+y[x])*y'[x]==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x)\to c_1$$

1.27 problem 28

Internal problem ID [7071]

Book: Own collection of miscellaneous problems
Section: section 1.0
Problem number: 28.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_quadrature]`

$$\boxed{\frac{y'}{x} = 0}$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 5

```
dsolve(1/x*diff(y(x),x)=0,y(x), singsol=all)
```

$$y(x) = c_1$$

✓ Solution by Mathematica

Time used: 0.003 (sec). Leaf size: 7

```
DSolve[1/x*y'[x]==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to c_1$$

1.28 problem 29

Internal problem ID [7072]

Book: Own collection of miscellaneous problems
Section: section 1.0
Problem number: 29.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_quadrature]`

$$\boxed{y' = 0}$$

✓ Solution by Maple
Time used: 0.0 (sec). Leaf size: 5

```
dsolve(diff(y(x),x)=0,y(x), singsol=all)
```

$$y(x) = c_1$$

✓ Solution by Mathematica
Time used: 0.002 (sec). Leaf size: 7

```
DSolve[y'[x]==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to c_1$$

1.29 problem 30

Internal problem ID [7073]

Book: Own collection of miscellaneous problems
Section: section 1.0
Problem number: 30.
ODE order: 1.
ODE degree: 2.

CAS Maple gives this as type `[[_homogeneous, ‘class C‘], _rational, _dAlembert]`

$$\boxed{y - x{y'}^2 - {y'}^2 = 0}$$

✓ Solution by Maple
Time used: 0.078 (sec). Leaf size: 53

```
dsolve(y(x)=x*diff(y(x),x)^2+diff(y(x),x)^2,y(x), singsol=all)
```

$$y(x) = 0$$
$$y(x) = \frac{\left(x+1+\sqrt{(x+1)\,(c_1+1)}\right)^2}{x+1}$$
$$y(x) = \frac{\left(-x-1+\sqrt{(x+1)\,(c_1+1)}\right)^2}{x+1}$$

✓ Solution by Mathematica
Time used: 0.069 (sec). Leaf size: 57

```
DSolve[y[x]==x*(y'[x])^2+(y'[x])^2,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to x - c_1\sqrt{x+1} + 1 + \frac{{c_1}^2}{4}$$
$$y(x) \to x + c_1\sqrt{x+1} + 1 + \frac{{c_1}^2}{4}$$
$$y(x) \to 0$$

1.30 problem 31

Internal problem ID [7074]

Book: Own collection of miscellaneous problems
Section: section 1.0
Problem number: 31.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[[_homogeneous, 'class A'], _rational, _Riccati]`

$$y' - \frac{5x^2 - yx + y^2}{x^2} = 0$$

✓ Solution by Maple
Time used: 0.0 (sec). Leaf size: 19

```
dsolve(diff(y(x),x)=(5*x^2-x*y(x)+y(x)^2)/x^2,y(x), singsol=all)
```

$$y(x) = x(1 + 2\tan(2\ln(x) + 2c_1))$$

✓ Solution by Mathematica
Time used: 0.789 (sec). Leaf size: 18

```
DSolve[y'[x]==(5*x^2-x*y[x]+y[x]^2)/x^2,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to x + 2x\tan(2(\log(x) + c_1))$$

1.31 problem 32

Internal problem ID [7075]

Book: Own collection of miscellaneous problems
Section: section 1.0
Problem number: 32.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[[_homogeneous, ‘class C‘], _rational, [_Abel, ‘2nd type‘, ‘cl`

$$\boxed{3x + (x+2)\, x' = -2t}$$

✓ Solution by Maple
Time used: 2.813 (sec). Leaf size: 30

```
dsolve(2*t+3*x(t)+(x(t)+2)*diff(x(t),t)=0,x(t), singsol=all)
```

$$x(t) = \frac{-\sqrt{4\,(t-3)\,c_1+1}-1+(-4t+8)\,c_1}{2c_1}$$

✓ Solution by Mathematica

Time used: 60.104 (sec). Leaf size: 1165

```
DSolve[2*t+3*x[t]+(x[t]+2)*x'[t]==0,x[t],t,IncludeSingularSolutions -> True]
```

$$x(t) \to -2 - \frac{2(t-3}{t\sqrt{\frac{3}{(t-3)^2} - \frac{3(t-3)^2\cosh\left(\frac{4c_1}{9}\right)+3(t-3)^2\sinh\left(\frac{4c_1}{9}\right)+2}{(t-3)^2\left((t-3)^2\cosh\left(\frac{4c_1}{9}\right)+(t-3)^2\sinh\left(\frac{4c_1}{9}\right)+1\right)} - \sqrt{-\frac{\cosh\left(\frac{4c_1}{9}\right)+\sinh\left(\frac{4c_1}{9}\right)}{(t-3)^2\left((t-3)^2\cosh\left(\frac{4c_1}{9}\right)+(t-3)^2\sinh\left(\frac{4c_1}{9}\right)+1\right)^2}} - 3\sqrt{}}}$$

$$x(t) \to -2 + \frac{2(t-3}{t\sqrt{\frac{3}{(t-3)^2} - \frac{3(t-3)^2\cosh\left(\frac{4c_1}{9}\right)+3(t-3)^2\sinh\left(\frac{4c_1}{9}\right)+2}{(t-3)^2\left((t-3)^2\cosh\left(\frac{4c_1}{9}\right)+(t-3)^2\sinh\left(\frac{4c_1}{9}\right)+1\right)} - \sqrt{-\frac{\cosh\left(\frac{4c_1}{9}\right)+\sinh\left(\frac{4c_1}{9}\right)}{(t-3)^2\left((t-3)^2\cosh\left(\frac{4c_1}{9}\right)+(t-3)^2\sinh\left(\frac{4c_1}{9}\right)+1\right)^2}} - 3\sqrt{}}}$$

$$x(t) \to -2 - \frac{2(t-3}{t\sqrt{\frac{3}{(t-3)^2} - \frac{3(t-3)^2\cosh\left(\frac{4c_1}{9}\right)+3(t-3)^2\sinh\left(\frac{4c_1}{9}\right)+2}{(t-3)^2\left((t-3)^2\cosh\left(\frac{4c_1}{9}\right)+(t-3)^2\sinh\left(\frac{4c_1}{9}\right)+1\right)} + \sqrt{-\frac{\cosh\left(\frac{4c_1}{9}\right)+\sinh\left(\frac{4c_1}{9}\right)}{(t-3)^2\left((t-3)^2\cosh\left(\frac{4c_1}{9}\right)+(t-3)^2\sinh\left(\frac{4c_1}{9}\right)+1\right)^2}} - 3\sqrt{}}}$$

$$x(t) \to -2 + \frac{2(t-3}{t\sqrt{\frac{3}{(t-3)^2} - \frac{3(t-3)^2\cosh\left(\frac{4c_1}{9}\right)+3(t-3)^2\sinh\left(\frac{4c_1}{9}\right)+2}{(t-3)^2\left((t-3)^2\cosh\left(\frac{4c_1}{9}\right)+(t-3)^2\sinh\left(\frac{4c_1}{9}\right)+1\right)} + \sqrt{-\frac{\cosh\left(\frac{4c_1}{9}\right)+\sinh\left(\frac{4c_1}{9}\right)}{(t-3)^2\left((t-3)^2\cosh\left(\frac{4c_1}{9}\right)+(t-3)^2\sinh\left(\frac{4c_1}{9}\right)+1\right)^2}} - 3\sqrt{}}}$$

1.32 problem 33

Internal problem ID [7076]

Book: Own collection of miscellaneous problems
Section: section 1.0
Problem number: 33.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_quadrature]`

$$\boxed{y' - \frac{1}{1-y} = 0}$$

With initial conditions

$$[y(0) = 2]$$

✓ Solution by Maple
Time used: 0.031 (sec). Leaf size: 13

```
dsolve([diff(y(t),t)=1/(1-y(t)),y(0) = 2],y(t), singsol=all)
```

$$y(t) = 1 + \sqrt{1-2t}$$

✓ Solution by Mathematica
Time used: 0.003 (sec). Leaf size: 16

```
DSolve[{y'[t]==1/(1-y[t]),y[0]==2},y[t],t,IncludeSingularSolutions -> True]
```

$$y(t) \to \sqrt{1-2t} + 1$$

1.33 problem 34

Internal problem ID [7077]

Book: Own collection of miscellaneous problems
Section: section 1.0
Problem number: 34.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_quadrature]`

$$\boxed{p' - ap + bp^2 = 0}$$

With initial conditions

$$[p(\mathrm{t0}) = \mathrm{p0}]$$

✓ Solution by Maple
Time used: 0.062 (sec). Leaf size: 29

```
dsolve([diff(p(t),t)=a*p(t)-b*p(t)^2,p(t0) = p0],p(t), singsol=all)
```

$$p(t) = \frac{a\,\mathrm{p0}}{(-\,\mathrm{p0}\,b + a)\,\mathrm{e}^{-a(t-\mathrm{t0})} + \mathrm{p0}\,b}$$

✓ Solution by Mathematica
Time used: 0.865 (sec). Leaf size: 39

```
DSolve[{p'[t]==a*p[t]-b*p[t]^2,p[t0]==p0},p[t],t,IncludeSingularSolutions -> True]
```

$$p(t) \to \frac{a\mathrm{p0}e^{at}}{b\mathrm{p0}\left(e^{at} - e^{a\mathrm{t0}}\right) + ae^{a\mathrm{t0}}}$$

1.34 problem 35

Internal problem ID [7078]

Book: Own collection of miscellaneous problems
Section: section 1.0
Problem number: 35.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[[_homogeneous, 'class G'], _exact, _rational, _Bernoulli]`

$$\boxed{y^2 + 2xyy' = -\frac{2}{x}}$$

✓ Solution by Maple
Time used: 0.031 (sec). Leaf size: 36

```
dsolve((y(x)^2+2/x)+2*y(x)*x*diff(y(x),x)=0,y(x), singsol=all)
```

$$y(x) = \frac{\sqrt{x\left(-2\ln\left(x\right) + c_1\right)}}{x}$$
$$y(x) = -\frac{\sqrt{x\left(-2\ln\left(x\right) + c_1\right)}}{x}$$

✓ Solution by Mathematica
Time used: 0.207 (sec). Leaf size: 44

```
DSolve[(y[x]^2+2/x)+2*y[x]*x*y'[x]==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to -\frac{\sqrt{-2\log(x) + c_1}}{\sqrt{x}}$$
$$y(x) \to \frac{\sqrt{-2\log(x) + c_1}}{\sqrt{x}}$$

1.35 problem 36

Internal problem ID [7079]

Book: Own collection of miscellaneous problems
Section: section 1.0
Problem number: 36.
ODE order: 1.
ODE degree: 0.

CAS Maple gives this as type `[_Clairaut]`

$$x f' - f - \frac{f'^2 \left(1 - f'^{\lambda}\right)^2}{\lambda^2} = 0$$

✓ Solution by Maple

Time used: 0.468 (sec). Leaf size: 318

```
dsolve(x*diff(f(x),x)-f(x)=diff(f(x),x)^2/lambda^2*(1-diff(f(x),x)^lambda)^2,f(x), singsol=a
```

$$f(x) = 0$$

$$f(x) = \frac{\lambda^2 x^2 \left(2\lambda\, \mathrm{e}^{\mathrm{RootOf}\left(2\lambda\, \mathrm{e}^{-Z(2\lambda+1)}+2\,\mathrm{e}^{-Z(2\lambda+1)}-2\lambda\, \mathrm{e}^{-Z(\lambda+1)}-x\,\lambda^2-4\,\mathrm{e}^{-Z(\lambda+1)}+2\,\mathrm{e}^{-Z}\right)\lambda} + \mathrm{e}^{\mathrm{Roo}}\right.}{4\left(\lambda\, \mathrm{e}^{\mathrm{RootOf}\left(2\lambda\, \mathrm{e}^{-Z(2\lambda+1)}+2\,\mathrm{e}^{-Z(2\lambda+1)}-2\lambda\, \mathrm{e}^{-Z(\lambda+1)}-x\,\lambda^2-4\,\mathrm{e}^{-Z(\lambda+1)}+2\,\mathrm{e}^{-Z}\right)\lambda} + \mathrm{e}^{\mathrm{RootOf}\left(2\lambda\, \mathrm{e}^{-Z(2\lambda+1)}+2\,\mathrm{e}^{-Z(2\lambda+1)}-2\lambda\, \mathrm{e}^{-Z(\lambda+1)}-\right.}\right.}$$

$$f(x) = c_1 x - \frac{c_1^2 \left(-1 + c_1^{\lambda}\right)^2}{\lambda^2}$$

✓ Solution by Mathematica

Time used: 15.811 (sec). Leaf size: 30

```
DSolve[x*f'[x]-f[x]==f'[x]^2/\[Lambda]^2*(1-f'[x]^\[Lambda])^2,f[x],x,IncludeSingularSolutio
```

$$f(x) \to c_1 \left(x - \frac{c_1 \left(-1 + c_1{}^{\lambda}\right)^2}{\lambda^2}\right)$$
$$f(x) \to 0$$

1.36 problem 37

Internal problem ID [7080]

Book: Own collection of miscellaneous problems
Section: section 1.0
Problem number: 37.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_rational, _Riccati]`

$$\boxed{xy' - 2y + by^2 = c\,x^4}$$

✓ Solution by Maple
Time used: 0.015 (sec). Leaf size: 31

```
dsolve(x*diff(y(x),x)-2*y(x)+b*y(x)^2=c*x^4,y(x), singsol=all)
```

$$y(x) = \frac{i \tan \left(-\frac{i x^2 \sqrt{b}\, \sqrt{c}}{2} + c_1\right) x^2 \sqrt{c}}{\sqrt{b}}$$

✓ Solution by Mathematica
Time used: 0.251 (sec). Leaf size: 153

```
DSolve[x*y'[x]-2*y[x]+b*y[x]^2==c*x^4,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to \frac{\sqrt{c} x^2 \left(-\cos \left(\frac{1}{2}\sqrt{-b}\sqrt{c} x^2\right) + c_1 \sin \left(\frac{1}{2}\sqrt{-b}\sqrt{c} x^2\right)\right)}{\sqrt{-b} \left(\sin \left(\frac{1}{2}\sqrt{-b}\sqrt{c} x^2\right) + c_1 \cos \left(\frac{1}{2}\sqrt{-b}\sqrt{c} x^2\right)\right)}$$

$$y(x) \to \frac{\sqrt{c} x^2 \tan \left(\frac{1}{2}\sqrt{-b}\sqrt{c} x^2\right)}{\sqrt{-b}}$$

1.37 problem 38

Internal problem ID [7081]

Book: Own collection of miscellaneous problems
Section: section 1.0
Problem number: 38.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_rational, _Riccati]`

$$\boxed{xy' - y + y^2 = x^{\frac{2}{3}}}$$

✓ Solution by Maple
Time used: 0.0 (sec). Leaf size: 72

```
dsolve(x*diff(y(x),x)-y(x)+y(x)^2=x^(2/3),y(x), singsol=all)
```

$$y(x) = \frac{x^{\frac{1}{3}}\left(c_1 e^{6x^{\frac{1}{3}}} \operatorname{abs}\left(1, 3x^{\frac{1}{3}} - 1\right) + c_1 e^{6x^{\frac{1}{3}}} \left|3x^{\frac{1}{3}} - 1\right| - 3x^{\frac{1}{3}}\right)}{c_1 e^{6x^{\frac{1}{3}}} \left|3x^{\frac{1}{3}} - 1\right| + 3x^{\frac{1}{3}} + 1}$$

✓ Solution by Mathematica
Time used: 0.221 (sec). Leaf size: 131

```
DSolve[x*y'[x]-y[x]+y[x]^2==x^(2/3),y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to \frac{3x^{2/3}\left(c_1 \cosh\left(3\sqrt[3]{x}\right) - i \sinh\left(3\sqrt[3]{x}\right)\right)}{\left(-3i\sqrt[3]{x} - c_1\right)\cosh\left(3\sqrt[3]{x}\right) + \left(3c_1\sqrt[3]{x} + i\right)\sinh\left(3\sqrt[3]{x}\right)}$$

$$y(x) \to \frac{3x^{2/3}\cosh\left(3\sqrt[3]{x}\right)}{3\sqrt[3]{x}\sinh\left(3\sqrt[3]{x}\right) - \cosh\left(3\sqrt[3]{x}\right)}$$

1.38 problem 39

Internal problem ID [7082]

Book: Own collection of miscellaneous problems
Section: section 1.0
Problem number: 39.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_rational, _Riccati]`

$$\boxed{u' + u^2 = \frac{1}{x^{\frac{4}{5}}}}$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 46

```
dsolve(diff(u(x),x)+u(x)^2=x^(-4/5),u(x), singsol=all)
```

$$u(x) = \frac{\operatorname{BesselI}\left(-\frac{1}{6}, \frac{5x^{\frac{3}{5}}}{3}\right) c_1 - \operatorname{BesselK}\left(\frac{1}{6}, \frac{5x^{\frac{3}{5}}}{3}\right)}{x^{\frac{2}{5}}\left(c_1 \operatorname{BesselI}\left(\frac{5}{6}, \frac{5x^{\frac{3}{5}}}{3}\right) + \operatorname{BesselK}\left(\frac{5}{6}, \frac{5x^{\frac{3}{5}}}{3}\right)\right)}$$

✓ Solution by Mathematica

Time used: 0.293 (sec). Leaf size: 286

```
DSolve[u'[x]+u[x]^2==x^(-4/5),u[x],x,IncludeSingularSolutions -> True]
```

$$u(x) \to \frac{(-1)^{5/6}x^{3/5} \operatorname{Gamma}\left(\frac{11}{6}\right) \operatorname{BesselI}\left(-\frac{1}{6}, \frac{5x^{3/5}}{3}\right) + (-1)^{5/6} \operatorname{Gamma}\left(\frac{11}{6}\right) \operatorname{BesselI}\left(\frac{5}{6}, \frac{5x^{3/5}}{3}\right) + (-1)^{5/6}x^{3/5} \mathrm{G}}{2x\left((-1)^{5/6} \mathrm{Gamn}\right.}$$

$$u(x) \to \frac{x^{3/5} \operatorname{BesselI}\left(-\frac{11}{6}, \frac{5x^{3/5}}{3}\right) + \operatorname{BesselI}\left(-\frac{5}{6}, \frac{5x^{3/5}}{3}\right) + x^{3/5} \operatorname{BesselI}\left(\frac{1}{6}, \frac{5x^{3/5}}{3}\right)}{2x \operatorname{BesselI}\left(-\frac{5}{6}, \frac{5x^{3/5}}{3}\right)}$$

1.39 problem 40

Internal problem ID [7083]

Book: Own collection of miscellaneous problems
Section: section 1.0
Problem number: 40.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[[_homogeneous, ‘class A‘], _rational, [_Abel, ‘2nd type‘, ‘cl`

$$\boxed{yy' - y = x}$$

✓ Solution by Maple

Time used: 0.359 (sec). Leaf size: 53

```
dsolve(y(x)*diff(y(x),x)-y(x)=x,y(x), singsol=all)
```

$$-\frac{\ln\left(\frac{-x^2-xy(x)+y(x)^2}{x^2}\right)}{2} - \frac{\sqrt{5}\operatorname{arctanh}\left(\frac{(-2y(x)+x)\sqrt{5}}{5x}\right)}{5} - \ln(x) - c_1 = 0$$

✓ Solution by Mathematica

Time used: 0.073 (sec). Leaf size: 63

```
DSolve[y[x]*y'[x] - y[x] == x,y[x],x,IncludeSingularSolutions -> True]
```

$$\text{Solve}\left[\frac{1}{10}\left(\left(5+\sqrt{5}\right)\log\left(-\frac{2y(x)}{x}+\sqrt{5}+1\right)-\left(\sqrt{5}-5\right)\log\left(\frac{2y(x)}{x}+\sqrt{5}-1\right)\right)= -\log(x)+c_1, y(x)\right]$$

1.40 problem 41

Internal problem ID [7084]

Book: Own collection of miscellaneous problems
Section: section 1.0
Problem number: 41.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _missing_x]]`

$$\boxed{y'' + 2y' + y = 0}$$

✓ Solution by Maple
Time used: 0.016 (sec). Leaf size: 14

```
dsolve(diff(y(x),x$2)+2*diff(y(x),x)+y(x)=0,y(x), singsol=all)
```

$$y(x) = e^{-x}(c_2 x + c_1)$$

✓ Solution by Mathematica
Time used: 0.014 (sec). Leaf size: 18

```
DSolve[y''[x]+2*y'[x]+y[x]==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to e^{-x}(c_2 x + c_1)$$

1.41 problem 41

Internal problem ID [7085]

Book: Own collection of miscellaneous problems
Section: section 1.0
Problem number: 41.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _missing_x]]`

$$\boxed{5y'' + 2y' + 4y = 0}$$

With initial conditions

$$[y(0) = 0, y'(0) = 5]$$

✓ Solution by Maple

Time used: 0.047 (sec). Leaf size: 20

```
dsolve([5*diff(y(x),x$2)+2*diff(y(x),x)+4*y(x)=0,y(0) = 0, D(y)(0) = 5],y(x), singsol=all)
```

$$y(x) = \frac{25\sqrt{19}\, e^{-\frac{x}{5}} \sin\left(\frac{\sqrt{19}\, x}{5}\right)}{19}$$

✓ Solution by Mathematica

Time used: 0.036 (sec). Leaf size: 6

```
DSolve[{5*y''[x]+2*y'[x]+4*y[x]==0,{y[0]==0,y'[0]==0}},y[x],x,IncludeSingularSolutions -> Tr
```

$$y(x) \to 0$$

1.42 problem 42

Internal problem ID [7086]

Book: Own collection of miscellaneous problems
Section: section 1.0
Problem number: 42.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _missing_x]]`

$$\boxed{y'' + y' + 4y = 1}$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 32

```
dsolve(diff(y(x),x$2)+diff(y(x),x)+4*y(x)=1,y(x), singsol=all)
```

$$y(x) = \mathrm{e}^{-\frac{x}{2}} \sin\left(\frac{\sqrt{15}\, x}{2}\right) c_2 + \mathrm{e}^{-\frac{x}{2}} \cos\left(\frac{\sqrt{15}\, x}{2}\right) c_1 + \frac{1}{4}$$

✓ Solution by Mathematica

Time used: 0.029 (sec). Leaf size: 51

```
DSolve[y''[x]+y'[x]+4*y[x]==1,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to c_2 e^{-x/2} \cos\left(\frac{\sqrt{15}x}{2}\right) + c_1 e^{-x/2} \sin\left(\frac{\sqrt{15}x}{2}\right) + \frac{1}{4}$$

1.43 problem 43

Internal problem ID [7087]

Book: Own collection of miscellaneous problems
Section: section 1.0
Problem number: 43.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _linear, _nonhomogeneous]]`

$$\boxed{y'' + y' + 4y = \sin(x)}$$

✓ Solution by Maple

Time used: 0.015 (sec). Leaf size: 39

```
dsolve(diff(y(x),x$2)+diff(y(x),x)+4*y(x)=sin(x),y(x), singsol=all)
```

$$y(x) = \mathrm{e}^{-\frac{x}{2}} \sin\left(\frac{\sqrt{15}\, x}{2}\right) c_2 + \mathrm{e}^{-\frac{x}{2}} \cos\left(\frac{\sqrt{15}\, x}{2}\right) c_1 + \frac{3\sin(x)}{10} - \frac{\cos(x)}{10}$$

✓ Solution by Mathematica

Time used: 1.949 (sec). Leaf size: 60

```
DSolve[y''[x]+y'[x]+4*y[x]==Sin[x],y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to \frac{3\sin(x)}{10} - \frac{\cos(x)}{10} + c_2 e^{-x/2} \cos\left(\frac{\sqrt{15}x}{2}\right) + c_1 e^{-x/2} \sin\left(\frac{\sqrt{15}x}{2}\right)$$

1.44 problem 44

Internal problem ID [7088]

Book: Own collection of miscellaneous problems
Section: section 1.0
Problem number: 44.
ODE order: 1.
ODE degree: 2.

CAS Maple gives this as type `[[_homogeneous, ‘class A‘], _rational, _dAlembert]`

$$\boxed{y - x{y'}^2 = 0}$$

✓ Solution by Maple
Time used: 0.047 (sec). Leaf size: 39

```
dsolve(y(x)=x*(diff(y(x),x))^2,y(x), singsol=all)
```

$$y(x) = 0$$
$$y(x) = \frac{\left(x + \sqrt{c_1 x}\right)^2}{x}$$
$$y(x) = \frac{\left(-x + \sqrt{c_1 x}\right)^2}{x}$$

✓ Solution by Mathematica
Time used: 0.053 (sec). Leaf size: 46

```
DSolve[y[x]==x*(y'[x])^2,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to \frac{1}{4}\left(-2\sqrt{x} + c_1\right)^2$$
$$y(x) \to \frac{1}{4}\left(2\sqrt{x} + c_1\right)^2$$
$$y(x) \to 0$$

1.45 problem 45

Internal problem ID [7089]

Book: Own collection of miscellaneous problems
Section: section 1.0
Problem number: 45.
ODE order: 1.
ODE degree: 3.

CAS Maple gives this as type `[_dAlembert]`

$$\boxed{y'y + xy'^3 = 1}$$

✓ Solution by Maple

Time used: 0.047 (sec). Leaf size: 1817

```
dsolve(diff(y(x),x)*y(x)=1-x*(diff(y(x),x))^3,y(x), singsol=all)
```

$$\frac{12\left(-2\left(\left(\sqrt{3}\sqrt{\frac{4y(x)^3+27x}{x}}+9\right)x^2\right)^{\frac{2}{3}}y(x)+\left(\frac{3^{\frac{2}{3}}2^{\frac{1}{3}}\left(\sqrt{3}\sqrt{\frac{4y(x)^3+27x}{x}}+9\right)\left(\left(\sqrt{3}\sqrt{\frac{4y(x)^3+27x}{x}}+9\right)x^2\right)^{\frac{1}{3}}}{6}+2^{\frac{2}{3}}3^{\frac{1}{3}}y(x)^2\right.}{\left(2^{\frac{2}{3}}3^{\frac{1}{3}}\left(\left(\sqrt{3}\sqrt{\frac{4y(x)^3+27x}{x}}+9\right)^2x^4\right)^{\frac{1}{3}}-2x\left(y(x)3^{\frac{2}{3}}2^{\frac{1}{3}}-3\left(\left(\sqrt{3}\sqrt{\frac{4y(x)^3+27x}{x}}+9\right)x^2\right)^{\frac{1}{3}}\right)\right)^2\left(y(x)2^{\frac{2}{3}}\right.}$$

$$+x$$

$$-\frac{18x^4\left(\sqrt{\frac{4y(x)^3+27x}{x}}\,2^{\frac{2}{3}}3^{\frac{5}{6}}x-2\left(\left(\sqrt{3}\sqrt{\frac{4y(x)^3+27x}{x}}+9\right)x^2\right)^{\frac{1}{3}}3^{\frac{2}{3}}2^{\frac{1}{3}}y(x)+9\,3^{\frac{1}{3}}2^{\frac{2}{3}}x+3\left(\left(\sqrt{3}\sqrt{\frac{4y(x)^3+27x}{x}}\right.\right.\right.}{\left(-2y(x)3^{\frac{2}{3}}2^{\frac{1}{3}}x+2^{\frac{2}{3}}3^{\frac{1}{3}}\left(\left(\sqrt{3}\sqrt{\frac{4y(x)^3+27x}{x}}+9\right)^2x^4\right)^{\frac{1}{3}}+6x\left(\left(\sqrt{3}\sqrt{\frac{4y(x)^3+27x}{x}}+9\right)x^2\right)^{\frac{1}{3}}\right)^2\left(-y\right.}$$

$$=0$$

$$-\frac{\left(\left(\sqrt{3}\sqrt{\frac{4y(x)^3+27x}{x}}+9\right)x^2\right)^{\frac{2}{3}}\left(-8\left(\left(\sqrt{3}\sqrt{\frac{4y(x)^3+27x}{x}}+9\right)x^2\right)^{\frac{2}{3}}y(x)+x\left(\left(\sqrt{3}\sqrt{\frac{4y(x)^3+27x}{x}}+9\right)2^{\frac{1}{3}}\right.\right.}{6\left(\frac{\left(i3^{\frac{5}{6}}+3^{\frac{1}{3}}\right)2^{\frac{2}{3}}\left(\left(\sqrt{3}\sqrt{\frac{4y(x)^3+27x}{x}}+9\right)^2x^4\right)^{\frac{1}{3}}}{6}+\left(-2\left(\left(\sqrt{3}\sqrt{\frac{4y(x)^3+27x}{x}}+9\right)x^2\right)^{\frac{1}{3}}+y(x)2^{\frac{1}{3}}\left(i3^{\frac{1}{6}}-\frac{3^{\frac{2}{3}}}{3}\right)\right)x\right.}$$

$$+x$$

$$+\frac{24\,2^{\frac{2}{3}}\left(\left(\sqrt{3}\sqrt{\frac{4y(x)^3+27x}{x}}+9\right)^2x^4\right)^{\frac{1}{3}}x^4 3^{\frac{1}{3}}\left(-\left(\left(\sqrt{3}\sqrt{\frac{4y(x)^3+27x}{x}}+9\right)x^2\right)^{\frac{2}{3}}+y(x)\left(i3^{\frac{1}{6}}-\frac{3^{\frac{2}{3}}}{3}\right)2^{\frac{1}{3}}\left(\left(\right.\right.\right.}{\left(\left(-i\sqrt{3}-1\right)\left(\left(\sqrt{3}\sqrt{\frac{4y(x)^3+27x}{x}}+9\right)x^2\right)^{\frac{2}{3}}+\left(-i3^{\frac{5}{6}}+3^{\frac{1}{3}}\right)2^{\frac{2}{3}}y(x)x\right)^2\left(\frac{\left(i3^{\frac{5}{6}}+3^{\frac{1}{3}}\right)2^{\frac{2}{3}}\left(\left(\sqrt{3}\sqrt{\frac{4y(x)^3+27x}{x}}+9\right.\right.}{6}\right.}$$

$$=0$$

$$\frac{\left(\left(\sqrt{3}\sqrt{\frac{4y(x)^3+27x}{x}}+9\right)x^2\right)^{\frac{2}{3}}c_1x^3\left(8\left(\left(\sqrt{3}\sqrt{\frac{4y(x)^3+27x}{x}}+9\right)x^2\right)^{\frac{2}{3}}y(x)+\left(\left(\sqrt{3}\sqrt{\frac{4y(x)^3+27x}{x}}+9\right)2^{\frac{1}{3}}\left(i\right.\right.\right.}{6\left(\left(i\sqrt{3}-1\right)\left(\left(\sqrt{3}\sqrt{\frac{4y(x)^3+27x}{x}}+9\right)x^2\right)^{\frac{2}{3}}+y(x)\left(i3^{\frac{5}{6}}+3^{\frac{1}{3}}\right)x2^{\frac{2}{3}}\right)^2\left(\frac{\left(-3^{\frac{1}{3}}+i3^{\frac{5}{6}}\right)2^{\frac{2}{3}}\left(\left(\sqrt{3}\sqrt{\frac{4y(x)^3+27x}{x}}+9\right)^2\right.}{6}\right.}$$

$$+x$$

$$+\frac{24\,2^{\frac{2}{3}}\left(\left(\sqrt{3}\sqrt{\frac{4y(x)^3+27x}{x}}+9\right)^2x^4\right)^{\frac{1}{3}}x^4 3^{\frac{1}{3}}\left(\left(\left(\sqrt{3}\sqrt{\frac{4y(x)^3+27x}{x}}+9\right)x^2\right)^{\frac{2}{3}}+y(x)\left(i3^{\frac{1}{6}}+\frac{3^{\frac{2}{3}}}{3}\right)2^{\frac{1}{3}}\left(\left(\sqrt{3}\right.\right.\right.}{}$$

✓ Solution by Mathematica

Time used: 89.497 (sec). Leaf size: 20717

```
DSolve[y'[x]*y[x]==1-x*(y'[x])^3,y[x],x,IncludeSingularSolutions -> True]
```

Too large to display

1.46 problem 46

Internal problem ID [7090]

Book: Own collection of miscellaneous problems
Section: section 1.0
Problem number: 46.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_quadrature]`

$$\boxed{f' - \frac{1}{f} = 0}$$

✓ Solution by Maple
Time used: 0.0 (sec). Leaf size: 23

```
dsolve(diff(f(x),x)=f(x)^(-1),f(x), singsol=all)
```

$$f(x) = \sqrt{c_1 + 2x}$$
$$f(x) = -\sqrt{c_1 + 2x}$$

✓ Solution by Mathematica
Time used: 0.078 (sec). Leaf size: 38

```
DSolve[f'[x]==f[x]^(-1),f[x],x,IncludeSingularSolutions -> True]
```

$$f(x) \to -\sqrt{2}\sqrt{x + c_1}$$
$$f(x) \to \sqrt{2}\sqrt{x + c_1}$$

1.47 problem 47

Internal problem ID [7091]

Book: Own collection of miscellaneous problems
Section: section 1.0
Problem number: 47.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _missing_y]]`

$$\boxed{ty'' + 4y' = t^2}$$

✓ Solution by Maple
Time used: 0.0 (sec). Leaf size: 17

```
dsolve(t*diff(y(t),t$2)+4*diff(y(t),t)=t^2,y(t), singsol=all)
```

$$y(t) = \frac{t^3}{18} - \frac{c_1}{3t^3} + c_2$$

✓ Solution by Mathematica
Time used: 0.034 (sec). Leaf size: 24

```
DSolve[t*y''[t]+4*y'[t]==t^2,y[t],t,IncludeSingularSolutions -> True]
```

$$y(t) \to \frac{t^3}{18} - \frac{c_1}{3t^3} + c_2$$

1.48 problem 48

Internal problem ID [7092]

Book: Own collection of miscellaneous problems
Section: section 1.0
Problem number: 48.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _missing_y]]`

$$\boxed{(t^2+9)\,y''+2ty'=0}$$

With initial conditions

$$\left[y(3)=2\pi, y'(3)=\frac{2}{3}\right]$$

✓ Solution by Maple

Time used: 0.062 (sec). Leaf size: 12

```
dsolve([(t^2+9)*diff(y(t),t$2)+2*t*diff(y(t),t)=0,y(3) = 2*Pi, D(y)(3) = 2/3],y(t), singsol=
```

$$y(t)=\pi+4\arctan\left(\frac{t}{3}\right)$$

✓ Solution by Mathematica

Time used: 0.023 (sec). Leaf size: 15

```
DSolve[{(t^2+9)*y''[t]+2*t*y'[t]==0,{y[3]==2*Pi,y'[3]==2/3}},y[t],t,IncludeSingularSolutions
```

$$y(t)\to 4\arctan\left(\frac{t}{3}\right)+\pi$$

1.49 problem 49

Internal problem ID [7093]

Book: Own collection of miscellaneous problems
Section: section 1.0
Problem number: 49.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_Emden, _Fowler]]`

$$\boxed{t^2y'' - 3ty' + 5y = 0}$$

✓ Solution by Maple
Time used: 0.0 (sec). Leaf size: 19

```
dsolve(t^2*diff(y(t),t$2)-3*t*diff(y(t),t)+5*y(t)=0,y(t), singsol=all)
```

$$y(t) = t^2(c_1 \sin(\ln(t)) + c_2 \cos(\ln(t)))$$

✓ Solution by Mathematica
Time used: 0.026 (sec). Leaf size: 22

```
DSolve[t^2*y''[t]-3*t*y'[t]+5*y[t]==0,y[t],t,IncludeSingularSolutions -> True]
```

$$y(t) \to t^2(c_2 \cos(\log(t)) + c_1 \sin(\log(t)))$$

1.50 problem 50

Internal problem ID [7094]

Book: Own collection of miscellaneous problems
Section: section 1.0
Problem number: 50.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _missing_y]]`

$$\boxed{ty'' + y' = 0}$$

✓ Solution by Maple
Time used: 0.0 (sec). Leaf size: 10

```
dsolve(t*diff(y(t),t$2)+diff(y(t),t)=0,y(t), singsol=all)
```

$$y(t) = c_2 \ln(t) + c_1$$

✓ Solution by Mathematica
Time used: 0.01 (sec). Leaf size: 13

```
DSolve[t*y''[t]+y'[t]==0,y[t],t,IncludeSingularSolutions -> True]
```

$$y(t) \to c_1 \log(t) + c_2$$

1.51 problem 51

Internal problem ID [7095]

Book: Own collection of miscellaneous problems
Section: section 1.0
Problem number: 51.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _missing_y]]`

$$\boxed{t^2y'' - 2y' = 0}$$

✓ Solution by Maple

Time used: 0.016 (sec). Leaf size: 25

```
dsolve(t^2*diff(y(t),t$2)-2*diff(y(t),t)=0,y(t), singsol=all)
```

$$y(t) = \mathrm{e}^{-\frac{2}{t}} c_2 t - 2\ \mathrm{expIntegral}_1\left(\frac{2}{t}\right) c_2 + c_1$$

✓ Solution by Mathematica

Time used: 0.02 (sec). Leaf size: 29

```
DSolve[t^2*y''[t]-2*y'[t]==0,y[t],t,IncludeSingularSolutions -> True]
```

$$y(t) \to 2c_1 \operatorname{ExpIntegralEi}\left(-\frac{2}{t}\right) + c_1 e^{-2/t} t + c_2$$

1.52 problem 52

Internal problem ID [7096]

Book: Own collection of miscellaneous problems
Section: section 1.0
Problem number: 52.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$y'' + \frac{(t^2-1)\, y'}{t} + \frac{t^2 y}{\left(1+\mathrm{e}^{\frac{t^2}{2}}\right)^2} = 0$$

✓ Solution by Maple

Time used: 0.032 (sec). Leaf size: 84

```
dsolve(diff(y(t),t$2)+(t^2-1)/t*diff(y(t),t)+t^2/(1 + exp(t^2/2))^2*y(t)=0,y(t), singsol=all
```

$$y(t) = \frac{\left(c_1\left(1+\mathrm{e}^{\frac{t^2}{2}}\right)^{-\frac{i\sqrt{3}}{2}}\left(\mathrm{e}^{\frac{t^2}{2}}\right)^{\frac{i\sqrt{3}}{2}} + c_2\left(1+\mathrm{e}^{\frac{t^2}{2}}\right)^{\frac{i\sqrt{3}}{2}}\left(\mathrm{e}^{\frac{t^2}{2}}\right)^{-\frac{i\sqrt{3}}{2}}\right)\sqrt{1+\mathrm{e}^{\frac{t^2}{2}}}}{\sqrt{\mathrm{e}^{\frac{t^2}{2}}}}$$

✓ Solution by Mathematica

Time used: 0.116 (sec). Leaf size: 72

```
DSolve[y''[t]+(t^2-1)/t*y'[t]+t^2/(1 + Exp[t^2/2])^2*y[t]==0,y[t],t,IncludeSingularSolutions
```

$$y(t) \to e^{\operatorname{arctanh}\left(2e^{\frac{t^2}{2}}+1\right)}\left(c_2 \cos\left(\sqrt{3}\operatorname{arctanh}\left(2e^{\frac{t^2}{2}}+1\right)\right) - c_1 \sin\left(\sqrt{3}\operatorname{arctanh}\left(2e^{\frac{t^2}{2}}+1\right)\right)\right)$$

1.53 problem 53

Internal problem ID [7097]

Book: Own collection of miscellaneous problems
Section: section 1.0
Problem number: 53.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_Emden, _Fowler], [_2nd_order, _linear, '_with_symmetry_[0,F`

$$\boxed{ty'' - y' + 4t^3 y = 0}$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 17

```
dsolve(t*diff(y(t),t$2)-diff(y(t),t)+4*t^3*y(t)=0,y(t), singsol=all)
```

$$y(t) = c_1 \sin(t^2) + c_2 \cos(t^2)$$

✓ Solution by Mathematica

Time used: 0.019 (sec). Leaf size: 20

```
DSolve[t*y''[t]-y'[t]+4*t^3*y[t]==0,y[t],t,IncludeSingularSolutions -> True]
```

$$y(t) \to c_1 \cos(t^2) + c_2 \sin(t^2)$$

1.54 problem 54

Internal problem ID [7098]

Book: Own collection of miscellaneous problems
Section: section 1.0
Problem number: 54.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _quadrature]]`

$$\boxed{y'' = 0}$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 9

```
dsolve(diff(y(t),t$2)=0,y(t), singsol=all)
```

$$y(t) = c_1 t + c_2$$

✓ Solution by Mathematica

Time used: 0.002 (sec). Leaf size: 12

```
DSolve[y''[t]==0,y[t],t,IncludeSingularSolutions -> True]
```

$$y(t) \to c_2 t + c_1$$

1.55 problem 55

Internal problem ID [7099]

Book: Own collection of miscellaneous problems
Section: section 1.0
Problem number: 55.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _quadrature]]`

$$\boxed{y'' = 1}$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 14

```
dsolve(diff(y(t),t$2)=1,y(t), singsol=all)
```

$$y(t) = \frac{1}{2}t^2 + c_1 t + c_2$$

✓ Solution by Mathematica

Time used: 0.002 (sec). Leaf size: 19

```
DSolve[y''[t]==1,y[t],t,IncludeSingularSolutions -> True]
```

$$y(t) \to \frac{t^2}{2} + c_2 t + c_1$$

1.56 problem 56

Internal problem ID [7100]

Book: Own collection of miscellaneous problems
Section: section 1.0
Problem number: 56.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _quadrature]]`

$$\boxed{y'' = f(t)}$$

✓ Solution by Maple
Time used: 0.0 (sec). Leaf size: 15

```
dsolve(diff(y(t),t$2)=f(t),y(t), singsol=all)
```

$$y(t) = \int \int f(t)\, dt dt + c_1 t + c_2$$

✓ Solution by Mathematica
Time used: 0.01 (sec). Leaf size: 30

```
DSolve[y''[t]==f[t],y[t],t,IncludeSingularSolutions -> True]
```

$$y(t) \to \int_1^t \int_1^{K[2]} f(K[1]) dK[1] dK[2] + c_2 t + c_1$$

1.57 problem 57

Internal problem ID [7101]

Book: Own collection of miscellaneous problems
Section: section 1.0
Problem number: 57.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _quadrature]]`

$$\boxed{y'' = k}$$

✓ Solution by Maple
Time used: 0.016 (sec). Leaf size: 15

```
dsolve(diff(y(t),t$2)=k,y(t), singsol=all)
```

$$y(t) = \frac{1}{2}k\,t^2 + c_1 t + c_2$$

✓ Solution by Mathematica
Time used: 0.002 (sec). Leaf size: 20

```
DSolve[y''[t]==k,y[t],t,IncludeSingularSolutions -> True]
```

$$y(t) \to \frac{kt^2}{2} + c_2 t + c_1$$

1.58 problem 58

Internal problem ID [7102]

Book: Own collection of miscellaneous problems
Section: section 1.0
Problem number: 58.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[[_homogeneous, ‘class C‘], _dAlembert]`

$$\boxed{y' + 4\sin(x - y) = -4}$$

✓ Solution by Maple
Time used: 0.047 (sec). Leaf size: 21

```
dsolve(diff(y(x),x)=4*sin(y(x)-x)-4,y(x), singsol=all)
```

$$y(x) = x + 2\arctan\left(\frac{3\tan\left(-\frac{3x}{2} + \frac{3c_1}{2}\right)}{5} + \frac{4}{5}\right)$$

✗ Solution by Mathematica
Time used: 0.0 (sec). Leaf size: 0

```
DSolve[y'[x]==4*Sin[y[x]-x]-4,y[x],x,IncludeSingularSolutions -> True]
```

Timed out

1.59 problem 59

Internal problem ID [7103]

Book: Own collection of miscellaneous problems
Section: section 1.0
Problem number: 59.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[[_homogeneous, ‘class C‘], _dAlembert]`

$$\boxed{y' + \sin(x - y) = 0}$$

✓ Solution by Maple
Time used: 0.032 (sec). Leaf size: 23

```
dsolve(diff(y(x),x)-sin(y(x)-x)=0,y(x), singsol=all)
```

$$y(x) = x + 2\arctan\left(\frac{c_1 - x - 2}{-x + c_1}\right)$$

✓ Solution by Mathematica

Time used: 37.233 (sec). Leaf size: 553

```
DSolve[y'[x]-Sin[y[x]-x]==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to -2\arccos\left(\frac{(-x+2+c_1)\cos\left(\frac{x}{2}\right)+(x-c_1)\sin\left(\frac{x}{2}\right)}{\sqrt{2}\sqrt{x^2-2(1+c_1)x+2+c_1{}^2+2c_1}}\right)$$
$$y(x) \to 2\arccos\left(\frac{(-x+2+c_1)\cos\left(\frac{x}{2}\right)+(x-c_1)\sin\left(\frac{x}{2}\right)}{\sqrt{2}\sqrt{x^2-2(1+c_1)x+2+c_1{}^2+2c_1}}\right)$$
$$y(x) \to -2\arccos\left(\frac{(x-2-c_1)\cos\left(\frac{x}{2}\right)+(-x+c_1)\sin\left(\frac{x}{2}\right)}{\sqrt{2}\sqrt{x^2-2(1+c_1)x+2+c_1{}^2+2c_1}}\right)$$
$$y(x) \to 2\arccos\left(\frac{(x-2-c_1)\cos\left(\frac{x}{2}\right)+(-x+c_1)\sin\left(\frac{x}{2}\right)}{\sqrt{2}\sqrt{x^2-2(1+c_1)x+2+c_1{}^2+2c_1}}\right)$$
$$y(x) \to -2\arccos\left(\frac{\cos\left(\frac{x}{2}\right)-\sin\left(\frac{x}{2}\right)}{\sqrt{2}}\right)$$
$$y(x) \to 2\arccos\left(\frac{\cos\left(\frac{x}{2}\right)-\sin\left(\frac{x}{2}\right)}{\sqrt{2}}\right)$$
$$y(x) \to -2\arccos\left(\frac{\sin\left(\frac{x}{2}\right)-\cos\left(\frac{x}{2}\right)}{\sqrt{2}}\right)$$
$$y(x) \to 2\arccos\left(\frac{\sin\left(\frac{x}{2}\right)-\cos\left(\frac{x}{2}\right)}{\sqrt{2}}\right)$$
$$y(x) \to -2\arccos\left(\frac{(x-2)\cos\left(\frac{x}{2}\right)-x\sin\left(\frac{x}{2}\right)}{\sqrt{2}\sqrt{x^2-2x+2}}\right)$$
$$y(x) \to 2\arccos\left(\frac{(x-2)\cos\left(\frac{x}{2}\right)-x\sin\left(\frac{x}{2}\right)}{\sqrt{2}\sqrt{x^2-2x+2}}\right)$$
$$y(x) \to -2\arccos\left(\frac{x\sin\left(\frac{x}{2}\right)-(x-2)\cos\left(\frac{x}{2}\right)}{\sqrt{2}\sqrt{x^2-2x+2}}\right)$$
$$y(x) \to 2\arccos\left(\frac{x\sin\left(\frac{x}{2}\right)-(x-2)\cos\left(\frac{x}{2}\right)}{\sqrt{2}\sqrt{x^2-2x+2}}\right)$$

1.60 problem 60

Internal problem ID [7104]

Book: Own collection of miscellaneous problems
Section: section 1.0
Problem number: 60.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _quadrature]]`

$$\boxed{y'' = 4\sin(x) - 4}$$

✓ Solution by Maple
Time used: 0.0 (sec). Leaf size: 18

```
dsolve(diff(y(x),x$2)=4*sin(x)-4,y(x), singsol=all)
```

$$y(x) = -2x^2 - 4\sin(x) + c_1x + c_2$$

✓ Solution by Mathematica
Time used: 0.031 (sec). Leaf size: 21

```
DSolve[y''[x]==4*Sin[x]-4,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to -2x^2 - 4\sin(x) + c_2x + c_1$$

1.61 problem 61

Internal problem ID [7105]

Book: Own collection of miscellaneous problems
Section: section 1.0
Problem number: 61.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _quadrature]]`

$$\boxed{yy'' = 0}$$

✓ Solution by Maple
Time used: 0.0 (sec). Leaf size: 13

```
dsolve(y(x)*diff(y(x),x$2)=0,y(x), singsol=all)
```

$$y(x) = 0$$
$$y(x) = c_1x + c_2$$

✓ Solution by Mathematica
Time used: 0.003 (sec). Leaf size: 17

```
DSolve[y[x]*y''[x]==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to 0$$
$$y(x) \to c_2x + c_1$$

1.62 problem 62

Internal problem ID [7106]

Book: Own collection of miscellaneous problems
Section: section 1.0
Problem number: 62.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _missing_x], [_2nd_order, _reducible, _mu_x_y1]]`

$$\boxed{yy'' = 1}$$

✓ Solution by Maple

Time used: 0.063 (sec). Leaf size: 51

```
dsolve(y(x)*diff(y(x),x$2)=1,y(x), singsol=all)
```

$$\int^{y(x)} \frac{1}{\sqrt{2\ln(\_a) - c_1}} d\_a - x - c_2 = 0$$
$$-\left(\int^{y(x)} \frac{1}{\sqrt{2\ln(\_a) - c_1}} d\_a\right) - x - c_2 = 0$$

✓ Solution by Mathematica

Time used: 60.072 (sec). Leaf size: 93

```
DSolve[y[x]*y''[x]==1,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to \exp\left(-\text{erf}^{-1}\left(-i\sqrt{\frac{2}{\pi}}\sqrt{e^{c_1}(x + c_2)^2}\right)^2 - \frac{c_1}{2}\right)$$
$$y(x) \to \exp\left(-\text{erf}^{-1}\left(i\sqrt{\frac{2}{\pi}}\sqrt{e^{c_1}(x + c_2)^2}\right)^2 - \frac{c_1}{2}\right)$$

1.63 problem 63

Internal problem ID [7107]

Book: Own collection of miscellaneous problems
Section: section 1.0
Problem number: 63.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_Emden, _Fowler], [_2nd_order, _with_linear_symmetries]]`

$$\boxed{yy'' = x}$$

✗ Solution by Maple

```
dsolve(y(x)*diff(y(x),x$2)=x,y(x), singsol=all)
```

No solution found

✗ Solution by Mathematica

Time used: 0.0 (sec). Leaf size: 0

```
DSolve[y[x]*y''[x]==x,y[x],x,IncludeSingularSolutions -> True]
```

Not solved

1.64 problem 64

Internal problem ID [7108]

Book: Own collection of miscellaneous problems
Section: section 1.0
Problem number: 64.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_Emden, _Fowler], [_2nd_order, _with_linear_symmetries]]`

$$\boxed{y^2 y'' = x}$$

✓ Solution by Maple

Time used: 0.078 (sec). Leaf size: 106

```
dsolve(y(x)^2*diff(y(x),x$2)=x,y(x), singsol=all)
```

$$y(x) = \text{RootOf}\left(\ln(x) + 2^{\frac{1}{3}}\left(\int^{\_Z} \frac{1}{2^{\frac{1}{3}}\_f + 2\,\text{RootOf}\left(\text{AiryBi}\left(\frac{2\_Z^2\_f + 2^{\frac{2}{3}}}{2\_f}\right) c_1\_Z + \_Z\,\text{AiryAi}\left(\frac{2\_Z^2\_f + 2^{\frac{2}{3}}}{2\_f}\right) + \text{AiryBi}\left(1, \right.\right.}\right.\right.$$
$$\left.\left. - c_2\right) x$$

✗ Solution by Mathematica

Time used: 0.0 (sec). Leaf size: 0

```
DSolve[y[x]^2*y''[x]==x,y[x],x,IncludeSingularSolutions -> True]
```

Not solved

1.65 problem 65

Internal problem ID [7109]

Book: Own collection of miscellaneous problems
Section: section 1.0
Problem number: 65.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _quadrature]]`

$$\boxed{y^2 y'' = 0}$$

✓ Solution by Maple
Time used: 0.0 (sec). Leaf size: 13

```
dsolve(y(x)^2*diff(y(x),x$2)=0,y(x), singsol=all)
```

$$y(x) = 0$$
$$y(x) = c_1 x + c_2$$

✓ Solution by Mathematica
Time used: 0.003 (sec). Leaf size: 17

```
DSolve[y[x]^2*y''[x]==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to 0$$
$$y(x) \to c_2 x + c_1$$

1.66 problem 66

Internal problem ID [7110]

Book: Own collection of miscellaneous problems
Section: section 1.0
Problem number: 66.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[NONE]`

$$\boxed{3yy'' = \sin(x)}$$

✗ Solution by Maple

```
dsolve(3*y(x)*diff(y(x),x$2)=sin(x),y(x), singsol=all)
```

No solution found

✗ Solution by Mathematica
Time used: 0.0 (sec). Leaf size: 0

```
DSolve[3*y[x]*y''[x]==Sin[x],y[x],x,IncludeSingularSolutions -> True]
```

Not solved

1.67 problem 67

Internal problem ID [7111]

Book: Own collection of miscellaneous problems
Section: section 1.0
Problem number: 67.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _missing_x], [_2nd_order, _reducible, _mu_x_y1]]`

$$\boxed{3yy'' + y = 5}$$

✓ Solution by Maple

Time used: 0.063 (sec). Leaf size: 59

```
dsolve(3*y(x)*diff(y(x),x$2)+y(x)=5,y(x), singsol=all)
```

$$-3\left(\int^{y(x)} \frac{1}{\sqrt{30\ln(\_a) + 9c_1 - 6\_a}} d\_a\right) - x - c_2 = 0$$
$$3\left(\int^{y(x)} \frac{1}{\sqrt{30\ln(\_a) + 9c_1 - 6\_a}} d\_a\right) - x - c_2 = 0$$

✓ Solution by Mathematica

Time used: 0.333 (sec). Leaf size: 41

```
DSolve[3*y[x]*y''[x]+y[x]==5,y[x],x,IncludeSingularSolutions -> True]
```

$$\text{Solve}\left[\int_1^{y(x)} \frac{1}{\sqrt{c_1 + \frac{2}{3}(5\log(K[1]) - K[1])}} dK[1]^2 = (x + c_2)^2, y(x)\right]$$

1.68 problem 68

Internal problem ID [7112]

Book: Own collection of miscellaneous problems
Section: section 1.0
Problem number: 68.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _missing_x], [_2nd_order, _reducible, _mu_x_y1]]`

$$\boxed{ayy'' + by = c}$$

✓ Solution by Maple
Time used: 0.062 (sec). Leaf size: 68

```
dsolve(a*y(x)*diff(y(x),x$2)+b*y(x)=c,y(x), singsol=all)
```

$$a\left(\int^{y(x)} \frac{1}{\sqrt{a\left(2c\ln\left(\_a\right)+c_1a-2\_ab\right)}}d\_a\right)-x-c_2=0$$
$$-a\left(\int^{y(x)} \frac{1}{\sqrt{a\left(2c\ln\left(\_a\right)+c_1a-2\_ab\right)}}d\_a\right)-x-c_2=0$$

✓ Solution by Mathematica
Time used: 0.43 (sec). Leaf size: 43

```
DSolve[a*y[x]*y''[x]+b*y[x]==c,y[x],x,IncludeSingularSolutions -> True]
```

$$\text{Solve}\left[\int_1^{y(x)} \frac{1}{\sqrt{c_1+\frac{2(c\log(K[1])-bK[1])}{a}}}dK[1]^2=(x+c_2)^2,y(x)\right]$$

1.69 problem 69

Internal problem ID [7113]

Book: Own collection of miscellaneous problems
Section: section 1.0
Problem number: 69.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _missing_x], [_2nd_order, _reducible, _mu_x_y1]]`

$$\boxed{a y^2 y'' + b y^2 = c}$$

✓ Solution by Maple
Time used: 0.062 (sec). Leaf size: 76

```
dsolve(a*y(x)^2*diff(y(x),x$2)+b*y(x)^2=c,y(x), singsol=all)
```

$$a\left(\int^{y(x)} \frac{\_a}{\sqrt{\_aa\left(-2b\_a^2+\_aac_1-2c\right)}}d\_a\right)-x-c_2=0$$
$$-a\left(\int^{y(x)} \frac{\_a}{\sqrt{\_aa\left(-2b\_a^2+\_aac_1-2c\right)}}d\_a\right)-x-c_2=0$$

✓ Solution by Mathematica
Time used: 0.801 (sec). Leaf size: 346

```
DSolve[a*y[x]^2*y''[x]+b*y[x]^2==c,y[x],x,IncludeSingularSolutions -> True]
```

$$\text{Solve}\left[-\frac{\left(\sqrt{-16bc+a^2{c_1}^2}-ac_1\right)\left(\sqrt{-16bc+a^2{c_1}^2}+ac_1\right)^2\left(1+\frac{4by(x)}{\sqrt{-16bc+a^2{c_1}^2}-ac_1}\right)\left(1-\frac{4by(x)}{\sqrt{-16bc+a^2{c_1}^2}+ac_1}\right)}{}\right.$$

1.70 problem 70

Internal problem ID [7114]

Book: Own collection of miscellaneous problems
Section: section 1.0
Problem number: 70.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _quadrature]]`

$$\boxed{ayy'' + by = 0}$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 22

```
dsolve(a*y(x)*diff(y(x),x$2)+b*y(x)=0,y(x), singsol=all)
```

$$y(x) = 0$$
$$y(x) = -\frac{b\,x^2}{2a} + c_1x + c_2$$

✓ Solution by Mathematica

Time used: 0.002 (sec). Leaf size: 28

```
DSolve[a*y[x]*y''[x]+b*y[x]==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to 0$$
$$y(x) \to -\frac{bx^2}{2a} + c_2x + c_1$$

1.71 problem 71

Internal problem ID [7115]

Book: Own collection of miscellaneous problems
Section: section 1.0
Problem number: 71.
ODE order: 1.
ODE degree: 1.

Solve

$$
\begin{aligned}
x'(t) &= 9x(t) + 4y(t) \\
y'(t) &= -6x(t) - y(t) \\
z'(t) &= 6x(t) + 4y(t) + 3z(t)
\end{aligned}
$$

✓ Solution by Maple

Time used: 0.031 (sec). Leaf size: 58

```
dsolve([diff(x(t),t)=9*x(t)+4*y(t),diff(y(t),t)=-6*x(t)-y(t),diff(z(t),t)=6*x(t)+4*y(t)+3*z(
```

$$
\begin{aligned}
x(t) &= c_2 e^{3t} + c_3 e^{5t} \\
y(t) &= -\frac{3c_2 e^{3t}}{2} - c_3 e^{5t} \\
z(t) &= c_2 e^{3t} + c_3 e^{5t} + c_1 e^{3t}
\end{aligned}
$$

✓ Solution by Mathematica

Time used: 0.005 (sec). Leaf size: 103

```
DSolve[{x'[t]==9*x[t]+4*y[t],y'[t]==-6*x[t]-y[t],z'[t]==6*x[t]+4*y[t]+3*z[t]},{x[t],y[t],z[t
```

$$
\begin{aligned}
x(t) &\to e^{3t}\left(c_1\left(3e^{2t} - 2\right) + 2c_2\left(e^{2t} - 1\right)\right) \\
y(t) &\to -e^{3t}\left(3c_1\left(e^{2t} - 1\right) + c_2\left(2e^{2t} - 3\right)\right) \\
z(t) &\to e^{3t}\left(3c_1\left(e^{2t} - 1\right) + 2c_2\left(e^{2t} - 1\right) + c_3\right)
\end{aligned}
$$

1.72 problem 72

Internal problem ID [7116]

Book: Own collection of miscellaneous problems
Section: section 1.0
Problem number: 72.
ODE order: 1.
ODE degree: 1.

Solve

$$x'(t) = x(t) - 3y(t)$$
$$y'(t) = 3x(t) + 7y(t)$$

✓ Solution by Maple

Time used: 0.015 (sec). Leaf size: 33

```
dsolve([diff(x(t),t)=x(t)-3*y(t),diff(y(t),t)=3*x(t)+7*y(t)],singsol=all)
```

$$x(t) = e^{4t}(c_2 t + c_1)$$
$$y(t) = -\frac{e^{4t}(3c_2 t + 3c_1 + c_2)}{3}$$

✓ Solution by Mathematica

Time used: 0.002 (sec). Leaf size: 46

```
DSolve[{x'[t]==x[t]-3*y[t],y'[t]==3*x[t]+7*y[t]},{x[t],y[t]},t,IncludeSingularSolutions -> T
```

$$x(t) \to -e^{4t}(c_1(3t - 1) + 3c_2 t)$$
$$y(t) \to e^{4t}(3(c_1 + c_2)t + c_2)$$

1.73 problem 73

Internal problem ID [7117]

Book: Own collection of miscellaneous problems
Section: section 1.0
Problem number: 73.
ODE order: 1.
ODE degree: 1.

Solve

$$x'(t) = x(t) - 2y(t)$$
$$y'(t) = 2x(t) + 5y(t)$$

✓ Solution by Maple

Time used: 0.016 (sec). Leaf size: 33

```
dsolve([diff(x(t),t) = x(t)-2*y(t), diff(y(t),t) = 2*x(t)+5*y(t)],singsol=all)
```

$$x(t) = e^{3t}(c_2 t + c_1)$$
$$y(t) = -\frac{e^{3t}(2c_2 t + 2c_1 + c_2)}{2}$$

✓ Solution by Mathematica

Time used: 0.004 (sec). Leaf size: 46

```
DSolve[{x'[t]== x[t]-2*y[t],y'[t] == 2*x[t]+5*y[t]},{x[t],y[t]},t,IncludeSingularSolutions -
```

$$x(t) \to -e^{3t}(c_1(2t-1) + 2c_2 t)$$
$$y(t) \to e^{3t}(2(c_1 + c_2)t + c_2)$$

1.74 problem 74

Internal problem ID [7118]

Book: Own collection of miscellaneous problems
Section: section 1.0
Problem number: 74.
ODE order: 1.
ODE degree: 1.

Solve

$$x'(t) = 7x(t) + y(t)$$
$$y'(t) = -4x(t) + 3y(t)$$

✓ Solution by Maple
Time used: 0.0 (sec). Leaf size: 35

```
dsolve([diff(x(t),t) = 7*x(t)+y(t), diff(y(t),t) = -4*x(t)+3*y(t)],singsol=all)
```

$$x(t) = e^{5t}(c_2 t + c_1)$$
$$y(t) = -e^{5t}(2c_2 t + 2c_1 - c_2)$$

✓ Solution by Mathematica
Time used: 0.003 (sec). Leaf size: 45

```
DSolve[{x'[t]== 7*x[t]+y[t],y'[t] == -4*x[t]+3*y[t]},{x[t],y[t]},t,IncludeSingularSolutions
```

$$x(t) \to e^{5t}(2c_1 t + c_2 t + c_1)$$
$$y(t) \to e^{5t}(c_2 - 2(2c_1 + c_2)t)$$

1.75 problem 75

Internal problem ID [7119]

Book: Own collection of miscellaneous problems
Section: section 1.0
Problem number: 75.
ODE order: 1.
ODE degree: 1.

Solve

$$x'(t) = x(t) + y(t)$$
$$y'(t) = y(t)$$
$$z'(t) = z(t)$$

✓ Solution by Maple

Time used: 0.016 (sec). Leaf size: 27

```
dsolve([diff(x(t),t)=x(t)+y(t),diff(y(t),t)=y(t),diff(z(t),t)=z(t)],singsol=all)
```

$$x(t) = \mathrm{e}^{t}(c_2 t + c_1)$$
$$y(t) = c_2 \mathrm{e}^{t}$$
$$z(t) = c_3 \mathrm{e}^{t}$$

✓ Solution by Mathematica

Time used: 0.023 (sec). Leaf size: 62

```
DSolve[{x'[t]== x[t]+y[t],y'[t] == y[t],z'[t]==z[t]},{x[t],y[t],z[t]},t,IncludeSingularSolut
```

$$x(t) \to e^t(c_2 t + c_1)$$
$$y(t) \to c_2 e^t$$
$$z(t) \to c_3 e^t$$
$$x(t) \to e^t(c_2 t + c_1)$$
$$y(t) \to c_2 e^t$$
$$z(t) \to 0$$

1.76 problem 76

Internal problem ID [7120]

Book: Own collection of miscellaneous problems
Section: section 1.0
Problem number: 76.
ODE order: 1.
ODE degree: 1.

Solve

$$
\begin{aligned}
x'(t) &= 2x(t) + y(t) - z(t) \\
y'(t) &= -x(t) + 2z(t) \\
z'(t) &= -x(t) - 2y(t) + 4z(t)
\end{aligned}
$$

✓ Solution by Maple
Time used: 0.015 (sec). Leaf size: 59

```
dsolve([diff(x(t),t)=2*x(t)+y(t)-z(t),diff(y(t),t)=-x(t)+2*z(t),diff(z(t),t)=-x(t)-2*y(t)+4*
```

$$
\begin{aligned}
x(t) &= -e^{2t}(2c_3t + c_2 - 4c_3) \\
y(t) &= e^{2t}\left(c_3t^2 + c_2t + c_1\right) \\
z(t) &= e^{2t}\left(c_3t^2 + c_2t + c_1 + 2c_3\right)
\end{aligned}
$$

✓ Solution by Mathematica
Time used: 0.003 (sec). Leaf size: 107

```
DSolve[{x'[t]== 2*x[t]+y[t]-z[t],y'[t] == -x[t]+2*z[t],z'[t]==-x[t]-2*y[t]+4*z[t]},{x[t],y[t
```

$$
\begin{aligned}
x(t) &\to e^{2t}((c_2 - c_3)t + c_1) \\
y(t) &\to -\frac{1}{2}e^{2t}\left((c_2 - c_3)t^2 + 2(c_1 + 2c_2 - 2c_3)t - 2c_2\right) \\
z(t) &\to -\frac{1}{2}e^{2t}\left((c_2 - c_3)t^2 + 2(c_1 + 2c_2 - 2c_3)t - 2c_3\right)
\end{aligned}
$$

1.77 problem 77

Internal problem ID [7121]

Book: Own collection of miscellaneous problems
Section: section 1.0
Problem number: 77.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_quadrature]`

$$\boxed{x' - 4Ak\left(\frac{x}{A}\right)^{\frac{3}{4}} + 3kx = 0}$$

✓ Solution by Maple
Time used: 0.0 (sec). Leaf size: 85

```
dsolve(diff(x(t),t)=4*A*k*(x(t)/A)^(3/4)-3*k*x(t),x(t), singsol=all)
```

$$\frac{\ln\left(9\sqrt{\frac{x(t)}{A}} - 16\right) - \ln\left(9\sqrt{\frac{x(t)}{A}} + 16\right) + 2\ln\left(3\left(\frac{x(t)}{A}\right)^{\frac{1}{4}} - 4\right) - 2\ln\left(3\left(\frac{x(t)}{A}\right)^{\frac{1}{4}} + 4\right) + \ln\left(256A - 81x(t\right.}{3k}$$
$$= 0$$

✓ Solution by Mathematica
Time used: 0.409 (sec). Leaf size: 51

```
DSolve[x'[t]==4*A*k*(x[t]/A)^(3/4)-3*k*x[t],x[t],t,IncludeSingularSolutions -> True]
```

$$x(t) \to \frac{1}{81}Ae^{-3kt}\left(4e^{\frac{3kt}{4}} + e^{\frac{3c_1}{4}}\right)^4$$
$$x(t) \to 0$$
$$x(t) \to \frac{256A}{81}$$

1.78 problem 78

Internal problem ID [7122]

Book: Own collection of miscellaneous problems
Section: section 1.0
Problem number: 78.
ODE order: 1.
ODE degree: 2.

CAS Maple gives this as type `[[_homogeneous, ‘class A‘], _dAlembert]`

$$\boxed{\frac{y'y}{1+\frac{\sqrt{1+y'^2}}{2}} = -x}$$

✓ Solution by Maple
Time used: 1.891 (sec). Leaf size: 187

```
dsolve(diff(y(x),x)*y(x)/(1+1/2*sqrt(1+diff(y(x),x)^2))=-x,y(x), singsol=all)
```

$$y(x) = -\frac{\sqrt{-x^2+c_1}\left(2+\sqrt{\frac{c_1}{-x^2+c_1}}\right)}{2}$$
$$y(x) = \frac{\sqrt{-x^2+c_1}\left(2+\sqrt{\frac{c_1}{-x^2+c_1}}\right)}{2}$$
$$y(x) = -\frac{\sqrt{-9x^2+15c_1-6\sqrt{-3c_1x^2+4c_1^2}}}{3}$$
$$y(x) = \frac{\sqrt{-9x^2+15c_1-6\sqrt{-3c_1x^2+4c_1^2}}}{3}$$
$$y(x) = -\frac{\sqrt{-9x^2+15c_1+6\sqrt{-3c_1x^2+4c_1^2}}}{3}$$
$$y(x) = \frac{\sqrt{-9x^2+15c_1+6\sqrt{-3c_1x^2+4c_1^2}}}{3}$$

✓ Solution by Mathematica

Time used: 2.255 (sec). Leaf size: 153

```
DSolve[y'[x]*y[x]/(1+1/2*Sqrt[1+(y'[x])^2])==-x,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x)\to \frac{1}{3}\left(e^{c_1}-\sqrt{-9x^2+4e^{2c_1}}\right)$$
$$y(x)\to \frac{1}{3}\left(\sqrt{-9x^2+4e^{2c_1}}+e^{c_1}\right)$$
$$y(x)\to -\sqrt{-x^2+4e^{2c_1}}-e^{c_1}$$
$$y(x)\to \sqrt{-x^2+4e^{2c_1}}-e^{c_1}$$
$$y(x)\to -\sqrt{-x^2}$$
$$y(x)\to \sqrt{-x^2}$$

1.79 problem 78

Internal problem ID [7123]

Book: Own collection of miscellaneous problems
Section: section 1.0
Problem number: 78.
ODE order: 1.
ODE degree: 2.

CAS Maple gives this as type `[[_homogeneous, 'class A'], _dAlembert]`

$$\boxed{\frac{y'y}{1+\frac{\sqrt{1+y'^2}}{2}} = -x}$$

With initial conditions

$$[y(0) = 3]$$

✓ Solution by Maple

Time used: 5.672 (sec). Leaf size: 33

```
dsolve([diff(y(x),x)*y(x)/(1+1/2*sqrt(1+diff(y(x),x)^2))=-x,y(0) = 3],y(x), singsol=all)
```

$$y(x) = -3 + \sqrt{-x^2 + 36}$$
$$y(x) = 1 + \sqrt{-x^2 + 4}$$

✓ Solution by Mathematica

Time used: 0.55 (sec). Leaf size: 35

```
DSolve[{y'[x]*y[x]/(1+1/2*Sqrt[1+(y'[x])^2])==-x,y[0]==3},y[x],x,IncludeSingularSolutions ->
```

$$y(x) \to \sqrt{4 - x^2} + 1$$
$$y(x) \to \sqrt{36 - x^2} - 3$$

1.80 problem 79

Internal problem ID [7124]

Book: Own collection of miscellaneous problems
Section: section 1.0
Problem number: 79.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_separable]`

$$y' - \frac{y\left(1 + \frac{a^2 x}{\sqrt{a^2(x^2+1)}}\right)}{\sqrt{a^2 (x^2 + 1)}} = 0$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 36

```
dsolve(diff(y(x),x) = y(x)*(1+ a^2*x/sqrt(a^2*(x^2+1)))/sqrt(a^2*(x^2+1)),y(x), singsol=all)
```

$$y(x) = c_1 \left(ax \operatorname{csgn}(a) + \sqrt{a^2 (x^2 + 1)}\right)^{\frac{1}{\sqrt{a^2}}} \sqrt{x^2 + 1}$$

✓ Solution by Mathematica

Time used: 0.365 (sec). Leaf size: 116

```
DSolve[y'[x]== y[x]*(1+ a^2*x/Sqrt[a^2*(x^2+1)])/Sqrt[a^2*(x^2+1)],y[x],x,IncludeSingularSol
```

$$y(x) \to c_1 \left(a\left(-\sqrt{a^2 (x^2 + 1)} + \sqrt{a^2} + ax\right)\right)^{-\frac{a+1}{a}} \left(a\left(\sqrt{a^2 (x^2 + 1)} - \sqrt{a^2} + ax\right)\right)^{\frac{1}{a}-1} \left(\sqrt{a^2}\sqrt{a^2 (x^2 + 1)} - a^2 (x^2 + 1)\right)$$

$$y(x) \to 0$$

1.81 problem 80

Internal problem ID [7125]

Book: Own collection of miscellaneous problems
Section: section 1.0
Problem number: 80.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[[_Riccati, _special]]`

$$\boxed{y' - y^2 = x^2}$$

✓ Solution by Maple
Time used: 0.0 (sec). Leaf size: 43

```
dsolve(diff(y(x),x)=x^2+y(x)^2,y(x), singsol=all)
```

$$y(x) = -\frac{x\left(\text{BesselJ}\left(-\frac{3}{4}, \frac{x^2}{2}\right) c_1 + \text{BesselY}\left(-\frac{3}{4}, \frac{x^2}{2}\right)\right)}{c_1 \,\text{BesselJ}\left(\frac{1}{4}, \frac{x^2}{2}\right) + \text{BesselY}\left(\frac{1}{4}, \frac{x^2}{2}\right)}$$

✓ Solution by Mathematica
Time used: 0.127 (sec). Leaf size: 169

```
DSolve[y'[x]==x^2+y[x]^2,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to \frac{x^2\left(-2\,\text{BesselJ}\left(-\frac{3}{4}, \frac{x^2}{2}\right) + c_1\left(\text{BesselJ}\left(\frac{3}{4}, \frac{x^2}{2}\right) - \text{BesselJ}\left(-\frac{5}{4}, \frac{x^2}{2}\right)\right)\right) - c_1\,\text{BesselJ}\left(-\frac{1}{4}, \frac{x^2}{2}\right)}{2x\left(\text{BesselJ}\left(\frac{1}{4}, \frac{x^2}{2}\right) + c_1\,\text{BesselJ}\left(-\frac{1}{4}, \frac{x^2}{2}\right)\right)}$$

$$y(x) \to -\frac{x^2\,\text{BesselJ}\left(-\frac{5}{4}, \frac{x^2}{2}\right) - x^2\,\text{BesselJ}\left(\frac{3}{4}, \frac{x^2}{2}\right) + \text{BesselJ}\left(-\frac{1}{4}, \frac{x^2}{2}\right)}{2x\,\text{BesselJ}\left(-\frac{1}{4}, \frac{x^2}{2}\right)}$$

1.82 problem 81

Internal problem ID [7126]

Book: Own collection of miscellaneous problems
Section: section 1.0
Problem number: 81.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_quadrature]`

$$\boxed{y' - 2\sqrt{y} = 0}$$

With initial conditions

$$[y(0) = 0]$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 5

```
dsolve([diff(y(x),x) = 2*sqrt(y(x)),y(0) = 0],y(x), singsol=all)
```

$$y(x) = 0$$

✓ Solution by Mathematica

Time used: 0.003 (sec). Leaf size: 8

```
DSolve[{y'[x]==2*Sqrt[y[x]],{y[0]==0}},y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to x^2$$

1.83 problem 82

Internal problem ID [7127]

Book: Own collection of miscellaneous problems
Section: section 1.0
Problem number: 82.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _linear, _nonhomogeneous]]`

$$\boxed{z'' + 3z' + 2z = 24\,\mathrm{e}^{-3t} - 24\,\mathrm{e}^{-4t}}$$

✓ Solution by Maple
Time used: 0.015 (sec). Leaf size: 30

```
dsolve(diff(z(t),t$2)+3*diff(z(t),t)+2*z(t)=24*(exp(-3*t)-exp(-4*t)),z(t), singsol=all)
```

$$z(t) = \left(-\mathrm{e}^{-t} c_1 - 4\,\mathrm{e}^{-3t} + 12\,\mathrm{e}^{-2t} + c_2\right)\mathrm{e}^{-t}$$

✓ Solution by Mathematica
Time used: 0.084 (sec). Leaf size: 34

```
DSolve[z''[t]+3*z'[t]+2*z[t]==24*(Exp[-3*t]-Exp[-4*t]),z[t],t,IncludeSingularSolutions -> Tr
```

$$z(t) \to e^{-4t}\left(12e^t + c_1 e^{2t} + c_2 e^{3t} - 4\right)$$

1.84 problem 83

Internal problem ID [7128]

Book: Own collection of miscellaneous problems
Section: section 1.0
Problem number: 83.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_quadrature]`

$$\boxed{y' - \sqrt{1 - y^2} = 0}$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 8

```
dsolve(diff(y(x),x)=sqrt(1-y(x)^2),y(x), singsol=all)
```

$$y(x) = \sin(x + c_1)$$

✓ Solution by Mathematica

Time used: 0.228 (sec). Leaf size: 28

```
DSolve[y'[x]==Sqrt[1-y[x]^2],y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to \cos(x + c_1)$$
$$y(x) \to -1$$
$$y(x) \to 1$$
$$y(x) \to \text{Interval}[\{-1, 1\}]$$

1.85 problem 84

Internal problem ID [7129]

Book: Own collection of miscellaneous problems
Section: section 1.0
Problem number: 84.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_Riccati]`

$$\boxed{y' - y^2 = x^2 - 1}$$

✓ Solution by Maple
Time used: 0.0 (sec). Leaf size: 107

```
dsolve(diff(y(x),x)=x^2+y(x)^2-1,y(x), singsol=all)
```

$$y(x) = \frac{(-3 - i)\,\text{WhittakerM}\left(1 + \frac{i}{4}, \frac{1}{4}, ix^2\right) + 4\,\text{WhittakerW}\left(1 + \frac{i}{4}, \frac{1}{4}, ix^2\right) c_1 + (-2ix^2 + i + 1)\,\text{WhittakerM}\,(}{2x\left(c_1\,\text{WhittakerW}\left(\frac{i}{4}, \frac{1}{4}, ix^2\right) + \text{WhittakerM}\left(\frac{i}{4}, \frac{1}{4}, i\right.\right.}$$

✓ Solution by Mathematica
Time used: 0.236 (sec). Leaf size: 153

```
DSolve[y'[x]==x^2+y[x]^2-1,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \rightarrow \frac{i\left(x\,\text{ParabolicCylinderD}\left(-\frac{1}{2} - \frac{i}{2}, (-1 + i)x\right) + (1 + i)\,\text{ParabolicCylinderD}\left(\frac{1}{2} - \frac{i}{2}, (-1 + i)x\right) - c_1 x\,\text{P}\right.}{\text{ParabolicCylinderD}\left(-\frac{1}{2} - \frac{i}{2}, (-1 + i)x\right) + c_1\,\text{Par}}$$

$$y(x) \rightarrow \frac{(1 + i)\,\text{ParabolicCylinderD}\left(\frac{1}{2} + \frac{i}{2}, (1 + i)x\right)}{\text{ParabolicCylinderD}\left(-\frac{1}{2} + \frac{i}{2}, (1 + i)x\right)} - ix$$

1.86 problem 85

Internal problem ID [7130]

Book: Own collection of miscellaneous problems
Section: section 1.0
Problem number: 85.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_Bernoulli]`

$$\boxed{y' - 2y(x\sqrt{y} - 1) = 0}$$

With initial conditions

$$[y(0) = 1]$$

✓ Solution by Maple

Time used: 0.046 (sec). Leaf size: 9

```
dsolve([diff(y(x),x)= 2*y(x)*(x*sqrt(y(x)) - 1),y(0) = 1],y(x), singsol=all)
```

$$y(x) = \frac{1}{(x+1)^2}$$

✓ Solution by Mathematica

Time used: 0.684 (sec). Leaf size: 20

```
DSolve[{y'[x]==2*y[x]*(x*Sqrt[y[x]-1]),{y[0]==1}},y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to 1$$
$$y(x) \to \sec^2\left(\frac{x^2}{2}\right)$$

1.87 problem 86

Internal problem ID [7131]

Book: Own collection of miscellaneous problems
Section: section 1.0
Problem number: 86.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _exact, _nonlinear], [_2nd_order, _with_linear_s`

$$\boxed{y'' - \frac{1}{y} + \frac{xy'}{y^2} = 0}$$

✓ Solution by Maple

Time used: 0.031 (sec). Leaf size: 56

```
dsolve(diff(y(x),x$2)=1/y(x)-x/y(x)^2*diff(y(x),x),y(x), singsol=all)
```

$$y(x) = \mathrm{RootOf}\left(\_Z^2 - \mathrm{e}^{\mathrm{RootOf}\left(x^2\left(4\,\mathrm{e}^{-\_Z}\cosh\left(\frac{\sqrt{c_1^2+4}\,(2c_2+\_Z+2\ln(x))}{2c_1}\right)^2+c_1^2+4\right)\right)} - 1 + \_Zc_1\right)x$$

✓ Solution by Mathematica

Time used: 0.199 (sec). Leaf size: 77

```
DSolve[y''[x]==1/y[x]-x/y[x]^2*y'[x],y[x],x,IncludeSingularSolutions -> True]
```

$$\mathrm{Solve}\left[\frac{1}{2}\log\left(-\frac{y(x)^2}{x^2} - \frac{c_1 y(x)}{x} + 1\right) - \frac{c_1\arctan\left(\frac{\frac{2y(x)}{x}+c_1}{\sqrt{-4-c_1{}^2}}\right)}{\sqrt{-4-c_1{}^2}} = -\log(x) + c_2, y(x)\right]$$

1.88 problem 87

Internal problem ID [7132]

Book: Own collection of miscellaneous problems
Section: section 1.0
Problem number: 87.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _missing_x]]`

$$\boxed{y'' + y' + y = 0}$$

With initial conditions

$$[y(0) = 0]$$

✓ Solution by Maple
Time used: 0.016 (sec). Leaf size: 17

```
dsolve([diff(y(x),x$2)+diff(y(x),x)+y(x)=0,y(0) = 0],y(x), singsol=all)
```

$$y(x) = c_1 \mathrm{e}^{-\frac{x}{2}} \sin\left(\frac{\sqrt{3}\, x}{2}\right)$$

✓ Solution by Mathematica
Time used: 0.025 (sec). Leaf size: 26

```
DSolve[{y''[x]+y'[x]+y[x]==0,{y[0]==0}},y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to c_1 e^{-x/2} \sin\left(\frac{\sqrt{3}x}{2}\right)$$

1.89 problem 88

Internal problem ID [7133]

Book: Own collection of miscellaneous problems
Section: section 1.0
Problem number: 88.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _missing_x]]`

$$\boxed{y'' + y' + y = 0}$$

With initial conditions

$$[y'(0) = 0]$$

✓ Solution by Maple
Time used: 0.016 (sec). Leaf size: 29

```
dsolve([diff(y(x),x$2)+diff(y(x),x)+y(x)=0,D(y)(0) = 0],y(x), singsol=all)
```

$$y(x) = c_1 e^{-\frac{x}{2}} \left(\sqrt{3}\,\cos\left(\frac{\sqrt{3}\,x}{2}\right) + \sin\left(\frac{\sqrt{3}\,x}{2}\right)\right)$$

✓ Solution by Mathematica
Time used: 0.022 (sec). Leaf size: 44

```
DSolve[{y''[x]+y'[x]+y[x]==0,{y'[0]==0}},y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to c_1 e^{-x/2} \left(\sin\left(\frac{\sqrt{3}x}{2}\right) + \sqrt{3}\cos\left(\frac{\sqrt{3}x}{2}\right)\right)$$

1.90 problem 88

Internal problem ID [7134]

Book: Own collection of miscellaneous problems
Section: section 1.0
Problem number: 88.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _missing_x]]`

$$\boxed{y'' + y' + y = 0}$$

With initial conditions

$$[y'(0) = 0, y(0) = 1]$$

✓ Solution by Maple
Time used: 0.031 (sec). Leaf size: 31

```
dsolve([diff(y(x),x$2)+diff(y(x),x)+y(x)=0,D(y)(0) = 0, y(0) = 1],y(x), singsol=all)
```

$$y(x) = \frac{e^{-\frac{x}{2}}\left(\sqrt{3}\,\sin\left(\frac{\sqrt{3}\,x}{2}\right) + 3\cos\left(\frac{\sqrt{3}\,x}{2}\right)\right)}{3}$$

✓ Solution by Mathematica
Time used: 0.025 (sec). Leaf size: 47

```
DSolve[{y''[x]+y'[x]+y[x]==0,{y'[0]==0,y[0]==1}},y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to \frac{1}{3}e^{-x/2}\left(\sqrt{3}\sin\left(\frac{\sqrt{3}x}{2}\right) + 3\cos\left(\frac{\sqrt{3}x}{2}\right)\right)$$

1.91 problem 89

Internal problem ID [7135]

Book: Own collection of miscellaneous problems
Section: section 1.0
Problem number: 89.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _exact, _nonlinear], [_2nd_order, _reducible, _m`

$$\boxed{y'' - yy' = 2x}$$

✓ Solution by Maple
Time used: 0.016 (sec). Leaf size: 161

```
dsolve(diff(y(x),x$2)-diff(y(x),x)*y(x)=2*x,y(x), singsol=all)
```

$$y(x) = \frac{-\operatorname{WhittakerM}\left(\frac{ic_1\sqrt{2}}{8}+1, \frac{1}{4}, \frac{i\sqrt{2}\,x^2}{2}\right)\left(6+ic_1\sqrt{2}\right)+8c_2\operatorname{WhittakerW}\left(\frac{ic_1\sqrt{2}}{8}+1, \frac{1}{4}, \frac{i\sqrt{2}\,x^2}{2}\right)+2\left(1-i\left(x^2\right.\right.}{2x\left(c_2\operatorname{WhittakerW}\left(\frac{ic_1\sqrt{2}}{8}, \frac{1}{4}, \frac{i\sqrt{2}\,x^2}{2}\right)+\operatorname{Whittak}\right.}$$

✓ Solution by Mathematica

Time used: 42.411 (sec). Leaf size: 318

```
DSolve[y''[x]+y'[x]*y[x]==2*x,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to -\frac{\sqrt[4]{2}\left(\sqrt[4]{2}x\,\text{ParabolicCylinderD}\left(\frac{1}{4}\left(-\sqrt{2}c_1-2\right), i\sqrt[4]{2}x\right) + 2i\,\text{ParabolicCylinderD}\left(\frac{1}{4}\left(2-\sqrt{2}c_1\right), i\sqrt[4]{2}\right.\right.}{\text{ParabolicCylinderD}\left(\frac{1}{4}\left(-\sqrt{2}c_1-2\right), i\sqrt[4]{2}x\right.}$$

$$y(x) \to \sqrt{2}x - \frac{2\sqrt[4]{2}\,\text{ParabolicCylinderD}\left(\frac{1}{4}\left(\sqrt{2}c_1+2\right), \sqrt[4]{2}x\right)}{\text{ParabolicCylinderD}\left(\frac{1}{4}\left(\sqrt{2}c_1-2\right), \sqrt[4]{2}x\right)}$$

$$y(x) \to \sqrt{2}x - \frac{2\sqrt[4]{2}\,\text{ParabolicCylinderD}\left(\frac{1}{4}\left(\sqrt{2}c_1+2\right), \sqrt[4]{2}x\right)}{\text{ParabolicCylinderD}\left(\frac{1}{4}\left(\sqrt{2}c_1-2\right), \sqrt[4]{2}x\right)}$$

1.92 problem 90

Internal problem ID [7136]

Book: Own collection of miscellaneous problems
Section: section 1.0
Problem number: 90.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_Riccati]`

$$y' - y^2 = x^2 + x$$

✓ Solution by Maple
Time used: 0.0 (sec). Leaf size: 155

```
dsolve(diff(y(x),x)-y(x)^2-x-x^2=0,y(x), singsol=all)
```

$$y(x) = \frac{2\left(ix^2 + ix - 1 + \frac{1}{4}i\right) c_1 \operatorname{hypergeom}\left(\left[\frac{3}{4} - \frac{i}{16}\right], \left[\frac{3}{2}\right], \frac{i(2x+1)^2}{4}\right) + 2\left(\left(-\frac{1}{12} - i\right) c_1 \left(x + \frac{1}{2}\right) \operatorname{hypergeom}\left(\left[\frac{7}{4} -\right.\right.\right.}{(2x+1)\, c_1 \operatorname{hypergeom}\left(\left[\frac{3}{4} - \frac{i}{16}\right], \left[\frac{3}{2}\right], \frac{i(}{}\right.}$$

✓ Solution by Mathematica
Time used: 0.306 (sec). Leaf size: 298

```
DSolve[y'[x]-y[x]^2-x-x^2==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to \frac{i\left((2x+1) \operatorname{ParabolicCylinderD}\left(-\frac{1}{2} - \frac{i}{8}, \left(-\frac{1}{2} + \frac{i}{2}\right)(2x+1)\right) - c_1(2x+1) \operatorname{ParabolicCylinderD}\left(-\frac{1}{2} + \right.\right.}{2\left(\operatorname{ParabolicCylinderD}\left(-\frac{1}{2} - \frac{i}{8}, \left(-\right.\right.\right.}$$

$$y(x) \to \frac{(1+i) \operatorname{ParabolicCylinderD}\left(\frac{1}{2} + \frac{i}{8}, (1+i)x + \left(\frac{1}{2} + \frac{i}{2}\right)\right)}{\operatorname{ParabolicCylinderD}\left(-\frac{1}{2} + \frac{i}{8}, (1+i)x + \left(\frac{1}{2} + \frac{i}{2}\right)\right)} - \frac{1}{2}i(2x+1)$$

$$y(x) \to \frac{(1+i) \operatorname{ParabolicCylinderD}\left(\frac{1}{2} + \frac{i}{8}, (1+i)x + \left(\frac{1}{2} + \frac{i}{2}\right)\right)}{\operatorname{ParabolicCylinderD}\left(-\frac{1}{2} + \frac{i}{8}, (1+i)x + \left(\frac{1}{2} + \frac{i}{2}\right)\right)} - \frac{1}{2}i(2x+1)$$

2 section 2.0

2.1 problem 1

Internal problem ID [7137]

Book: Own collection of miscellaneous problems
Section: section 2.0
Problem number: 1.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$\boxed{y'' - xy' - yx = x}$$

✓ Solution by Maple
Time used: 0.0 (sec). Leaf size: 55

```
dsolve(diff(y(x),x$2)-x*diff(y(x),x)-x*y(x)-x=0,y(x), singsol=all)
```

$$y(x) = -\pi\, \mathrm{e}^{-2-x} c_1 (x+2) \operatorname{erf}\left(\frac{i\sqrt{2}\,(x+2)}{2}\right) + i\sqrt{\pi}\,\sqrt{2}\,\mathrm{e}^{\frac{x(x+2)}{2}} c_1 - 1 + \mathrm{e}^{-x}(x+2)\, c_2$$

✓ Solution by Mathematica
Time used: 4.759 (sec). Leaf size: 216

```
DSolve[y''[x]-x*y'[x]-x*y[x]-x==0,y[x],x,IncludeSingularSolutions -> True]
```

$$\begin{aligned} y(x) \to \frac{1}{2} e^{-\frac{1}{2}(x+2)^2} \Bigg(& 2\sqrt{2} e^{\frac{x^2}{2}+x+2} (x+2) \int_1^x \Bigg(\frac{e^{K[1]} K[1]}{\sqrt{2}} \\ & - \frac{1}{2} e^{-\frac{1}{2}K[1]^2 - K[1] - 2} \sqrt{\pi} \operatorname{erfi}\left(\frac{\sqrt{(K[1]+2)^2}}{\sqrt{2}} \right) K[1] \sqrt{(K[1]+2)^2} \Bigg) dK[1] \\ & - \sqrt{2\pi} \sqrt{(x+2)^2} \left(c_2 e^{\frac{x^2}{2}+x+2} + x + 1 \right) \operatorname{erfi}\left(\frac{\sqrt{(x+2)^2}}{\sqrt{2}} \right) \\ & + 2e^{\frac{x^2}{2}+x+2} \left(e^x (x+1) + \sqrt{2} c_1 (x+2) + c_2 e^{\frac{1}{2}(x+2)^2} \right) \Bigg) \end{aligned}$$

2.2 problem 2

Internal problem ID [7138]

Book: Own collection of miscellaneous problems
Section: section 2.0
Problem number: 2.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$y'' - xy' - yx = 2x$$

✓ Solution by Maple

Time used: 0.015 (sec). Leaf size: 54

```
dsolve(diff(y(x),x$2)-x*diff(y(x),x)-x*y(x)-2*x=0,y(x), singsol=all)
```

$$y(x) = \pi\, \mathrm{e}^{-2-x} c_1 (x+2) \operatorname{erf}\left(\frac{i\sqrt{2}\,(x+2)}{2}\right) - i\sqrt{\pi}\,\sqrt{2}\,\mathrm{e}^{\frac{x(x+2)}{2}} c_1 - 2 + \mathrm{e}^{-x}(x+2)\, c_2$$

✓ Solution by Mathematica

Time used: 1.745 (sec). Leaf size: 217

```
DSolve[y''[x]-x*y'[x]-x*y[x]-2*x==0,y[x],x,IncludeSingularSolutions -> True]
```

$$\begin{aligned} y(x) \to \frac{1}{2} e^{-\frac{1}{2}(x+2)^2} \Bigg(& 2\sqrt{2} e^{\frac{x^2}{2}+x+2}(x+2) \int_1^x \Bigg(\sqrt{2} e^{K[1]} K[1] \\ & - e^{-\frac{1}{2}K[1]^2-K[1]-2}\sqrt{\pi}\operatorname{erfi}\left(\frac{\sqrt{(K[1]+2)^2}}{\sqrt{2}}\right) K[1]\sqrt{(K[1]+2)^2}\Bigg) dK[1] \\ & - \sqrt{2\pi}\sqrt{(x+2)^2}\left(c_2 e^{\frac{x^2}{2}+x+2} + 2x + 2\right) \operatorname{erfi}\left(\frac{\sqrt{(x+2)^2}}{\sqrt{2}}\right) \\ & + 2e^{\frac{x^2}{2}+x+2}\left(2e^x(x+1) + \sqrt{2}c_1(x+2) + c_2 e^{\frac{1}{2}(x+2)^2}\right)\Bigg) \end{aligned}$$

2.3 problem 3

Internal problem ID [7139]

Book: Own collection of miscellaneous problems
Section: section 2.0
Problem number: 3.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$\boxed{y'' - xy' - yx = 3x}$$

✓ Solution by Maple
Time used: 0.016 (sec). Leaf size: 54

```
dsolve(diff(y(x),x$2)-x*diff(y(x),x)-x*y(x)-3*x=0,y(x), singsol=all)
```

$$y(x) = \pi \, \mathrm{e}^{-2-x} c_1 (x+2) \operatorname{erf}\left(\frac{i\sqrt{2}\,(x+2)}{2}\right) - i\sqrt{\pi}\,\sqrt{2}\,\mathrm{e}^{\frac{x(x+2)}{2}} c_1 - 3 + \mathrm{e}^{-x}(x+2)\, c_2$$

✓ Solution by Mathematica
Time used: 2.238 (sec). Leaf size: 220

```
DSolve[y''[x]-x*y'[x]-x*y[x]-3*x==0,y[x],x,IncludeSingularSolutions -> True]
```

$$\begin{aligned} y(x) \to \frac{1}{2} e^{-\frac{1}{2}(x+2)^2} \Bigg(& 2\sqrt{2} e^{\frac{x^2}{2}+x+2} (x+2) \int_1^x \Bigg(\frac{3e^{K[1]} K[1]}{\sqrt{2}} \\ & - \frac{3}{2} e^{-\frac{1}{2}K[1]^2 - K[1] - 2} \sqrt{\pi} \operatorname{erfi}\left(\frac{\sqrt{(K[1]+2)^2}}{\sqrt{2}}\right) K[1] \sqrt{(K[1]+2)^2} \Bigg) dK[1] \\ & - \sqrt{2\pi} \sqrt{(x+2)^2} \left(c_2 e^{\frac{x^2}{2}+x+2} + 3x + 3 \right) \operatorname{erfi}\left(\frac{\sqrt{(x+2)^2}}{\sqrt{2}} \right) \\ & + 2e^{\frac{x^2}{2}+x+2} \left(3e^x (x+1) + \sqrt{2} c_1 (x+2) + c_2 e^{\frac{1}{2}(x+2)^2} \right) \Bigg) \end{aligned}$$

2.4 problem 4

Internal problem ID [7140]

Book: Own collection of miscellaneous problems
Section: section 2.0
Problem number: 4.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$\boxed{y'' - xy' - yx = x^2 + x}$$

✓ Solution by Maple
Time used: 0.0 (sec). Leaf size: 56

```
dsolve(diff(y(x),x$2)-x*diff(y(x),x)-x*y(x)-x^2-x=0,y(x), singsol=all)
```

$$y(x) = \pi \,\mathrm{e}^{-2-x} c_1 (x+2) \operatorname{erf}\left(\frac{i\sqrt{2}\,(x+2)}{2}\right) - i\sqrt{\pi}\,\sqrt{2}\,\mathrm{e}^{\frac{x(x+2)}{2}} c_1 + \mathrm{e}^{-x}(x+2)\,c_2 - x$$

✓ Solution by Mathematica
Time used: 2.153 (sec). Leaf size: 84

```
DSolve[y''[x]-x*y'[x]-x*y[x]-x^2-x==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to \frac{1}{2} e^{-x}\left(-\sqrt{2\pi} c_2 \sqrt{(x+2)^2} \operatorname{erfi}\left(\frac{\sqrt{(x+2)^2}}{\sqrt{2}}\right) - 2e^x x + 2\sqrt{2} c_1 (x+2) + 2c_2 e^{\frac{1}{2}(x+2)^2}\right)$$

2.5 problem 5

Internal problem ID [7141]

Book: Own collection of miscellaneous problems
Section: section 2.0
Problem number: 5.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _linear, _nonhomogeneous]]`

$$\boxed{y'' - xy' - yx = x^3 - 2}$$

✓ Solution by Maple

Time used: 0.016 (sec). Leaf size: 62

```
dsolve(diff(y(x),x$2)-x*diff(y(x),x)-x*y(x)-x^3+2=0,y(x), singsol=all)
```

$$y(x) = \pi\,\mathrm{e}^{-2-x} c_1 (x+2)\,\mathrm{erf}\left(\frac{i\sqrt{2}\,(x+2)}{2}\right) - i\sqrt{\pi}\,\sqrt{2}\,\mathrm{e}^{\frac{x(x+2)}{2}} c_1 + \mathrm{e}^{-x}(x+2)\,c_2 - x^2 + 2x - 2$$

✓ Solution by Mathematica

Time used: 5.186 (sec). Leaf size: 91

```
DSolve[y''[x]-x*y'[x]-x*y[x]-x^3+2==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to \frac{1}{2}e^{-x}\left(-\sqrt{2\pi}c_2\sqrt{(x+2)^2}\mathrm{erfi}\left(\frac{\sqrt{(x+2)^2}}{\sqrt{2}}\right) - 2e^x\left(x^2 - 2x + 2\right) + 2\sqrt{2}c_1(x+2) + 2c_2e^{\frac{1}{2}(x+2)^2}\right)$$

2.6 problem 6

Internal problem ID [7142]

Book: Own collection of miscellaneous problems
Section: section 2.0
Problem number: 6.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _linear, _nonhomogeneous]]`

$$\boxed{y'' - xy' - yx = x^4 + 6}$$

✓ Solution by Maple

Time used: 0.016 (sec). Leaf size: 66

```
dsolve(diff(y(x),x$2)-x*diff(y(x),x)-x*y(x)-x^4-6=0,y(x), singsol=all)
```

$$y(x) = \pi \,\mathrm{e}^{-2-x} c_1 (x+2) \,\mathrm{erf}\left(\frac{i\sqrt{2}\,(x+2)}{2}\right) - i\sqrt{\pi}\,\sqrt{2}\,\mathrm{e}^{\frac{x(x+2)}{2}} c_1 + \mathrm{e}^{-x}(x+2)\, c_2 - x^3 + 3x^2 - 6x$$

✓ Solution by Mathematica

Time used: 7.359 (sec). Leaf size: 92

```
DSolve[y''[x]-x*y'[x]-x*y[x]-x^4-6==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to \frac{1}{2} e^{-x} \left(-\sqrt{2\pi} c_2 \sqrt{(x+2)^2} \mathrm{erfi}\left(\frac{\sqrt{(x+2)^2}}{\sqrt{2}}\right) - 2e^x x \left(x^2 - 3x + 6\right) + 2\sqrt{2} c_1 (x+2) + 2c_2 e^{\frac{1}{2}(x+2)^2}\right)$$

2.7 problem 7

Internal problem ID [7143]

Book: Own collection of miscellaneous problems
Section: section 2.0
Problem number: 7.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _linear, _nonhomogeneous]]`

$$y'' - xy' - yx = x^5 - 24$$

✓ Solution by Maple
Time used: 0.0 (sec). Leaf size: 72

```
dsolve(diff(y(x),x$2)-x*diff(y(x),x)-x*y(x)-x^5+24=0,y(x), singsol=all)
```

$$y(x) = \pi\,\mathrm{e}^{-2-x} c_1 (x+2)\,\mathrm{erf}\left(\frac{i\sqrt{2}\,(x+2)}{2}\right) - i\sqrt{\pi}\,\sqrt{2}\,\mathrm{e}^{\frac{x(x+2)}{2}} c_1 + \mathrm{e}^{-x}(x+2)\,c_2 - x^4 + 4x^3 - 12x^2 + 12x + 12$$

✓ Solution by Mathematica
Time used: 3.222 (sec). Leaf size: 102

```
DSolve[y''[x]-x*y'[x]-x*y[x]-x^5+24==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to \frac{1}{2}e^{-x}\left(-\sqrt{2\pi}c_2\sqrt{(x+2)^2}\mathrm{erfi}\left(\frac{\sqrt{(x+2)^2}}{\sqrt{2}}\right) + e^x\left(-2x^4 + 8x^3 - 24x^2 + 24x + 24\right) + 2\sqrt{2}c_1(x+2) + 2c_2e^{\frac{1}{2}(x+2)^2}\right)$$

2.8 problem 8

Internal problem ID [7144]

Book: Own collection of miscellaneous problems
Section: section 2.0
Problem number: 8.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$\boxed{y'' - xy' - yx = x}$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 54

```
dsolve(diff(y(x),x$2)-x*diff(y(x),x)-x*y(x)-x=0,y(x), singsol=all)
```

$$y(x) = \pi \, \mathrm{e}^{-2-x} c_1 (x+2) \, \mathrm{erf}\left(\frac{i\sqrt{2}\,(x+2)}{2}\right) - i\sqrt{\pi}\,\sqrt{2}\,\mathrm{e}^{\frac{x(x+2)}{2}} c_1 - 1 + \mathrm{e}^{-x}(x+2)\,c_2$$

✓ Solution by Mathematica

Time used: 0.689 (sec). Leaf size: 216

```
DSolve[y''[x]-x*y'[x]-x*y[x]-x==0,y[x],x,IncludeSingularSolutions -> True]
```

$$\begin{aligned} y(x) \to \frac{1}{2} e^{-\frac{1}{2}(x+2)^2} \Bigg(& 2\sqrt{2} e^{\frac{x^2}{2}+x+2}(x+2) \int_1^x \Bigg(\frac{e^{K[1]} K[1]}{\sqrt{2}} \\ & - \frac{1}{2} e^{-\frac{1}{2}K[1]^2 - K[1] - 2} \sqrt{\pi} \mathrm{erfi}\left(\frac{\sqrt{(K[1]+2)^2}}{\sqrt{2}} \right) K[1] \sqrt{(K[1]+2)^2} \Bigg) dK[1] \\ & - \sqrt{2\pi} \sqrt{(x+2)^2} \left(c_2 e^{\frac{x^2}{2}+x+2} + x + 1 \right) \mathrm{erfi}\left(\frac{\sqrt{(x+2)^2}}{\sqrt{2}} \right) \\ & + 2 e^{\frac{x^2}{2}+x+2} \left(e^x (x+1) + \sqrt{2} c_1 (x+2) + c_2 e^{\frac{1}{2}(x+2)^2} \right) \Bigg) \end{aligned}$$

2.9 problem 9

Internal problem ID [7145]

Book: Own collection of miscellaneous problems
Section: section 2.0
Problem number: 9.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$\boxed{y'' - xy' - yx = x^2}$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 57

```
dsolve(diff(y(x),x$2)-x*diff(y(x),x)-x*y(x)-x^2=0,y(x), singsol=all)
```

$$y(x) = \pi \, \mathrm{e}^{-2-x} c_1 (x+2) \,\mathrm{erf}\left(\frac{i\sqrt{2}\,(x+2)}{2}\right) - i\sqrt{\pi}\,\sqrt{2}\,\mathrm{e}^{\frac{x(x+2)}{2}} c_1 + \mathrm{e}^{-x}(x+2)\,c_2 - x + 1$$

✓ Solution by Mathematica

Time used: 4.289 (sec). Leaf size: 226

```
DSolve[y''[x]-x*y'[x]-x*y[x]-x^2==0,y[x],x,IncludeSingularSolutions -> True]
```

$$\begin{aligned} y(x) \to \frac{1}{2} e^{-\frac{1}{2}(x+2)^2} \Bigg(& 2\sqrt{2} e^{\frac{x^2}{2}+x+2} (x+2) \int_1^x \Bigg(\frac{e^{K[1]} K[1]^2}{\sqrt{2}} \\ & - \frac{1}{2} e^{-\frac{1}{2}K[1]^2 - K[1] - 2} \sqrt{\pi} \mathrm{erfi}\left(\frac{\sqrt{(K[1]+2)^2}}{\sqrt{2}} \right) K[1]^2 \sqrt{(K[1]+2)^2} \Bigg) dK[1] \\ & - \sqrt{2\pi} \sqrt{(x+2)^2} \left(x^2 + c_2 e^{\frac{x^2}{2}+x+2} + x + 1 \right) \mathrm{erfi}\left(\frac{\sqrt{(x+2)^2}}{\sqrt{2}} \right) \\ & + 2e^{\frac{x^2}{2}+x+2} \left(e^x \left(x^2+x+1 \right) + \sqrt{2} c_1 (x+2) + c_2 e^{\frac{1}{2}(x+2)^2} \right) \Bigg) \end{aligned}$$

2.10 problem 10

Internal problem ID [7146]

Book: Own collection of miscellaneous problems
Section: section 2.0
Problem number: 10.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _linear, _nonhomogeneous]]`

$$y'' - xy' - yx = x^3$$

✓ Solution by Maple
Time used: 0.141 (sec). Leaf size: 211

```
dsolve(diff(y(x),x$2)-x*diff(y(x),x)-x*y(x)-x^3=0,y(x), singsol=all)
```

$$y(x) = \frac{\sqrt{2}\,\mathrm{e}^{-x}(x+2)\left(\int x^3\mathrm{e}^{-\frac{x(x+2)}{2}}\left(i\pi\,\mathrm{e}^{-2}(x+2)\,\mathrm{erf}\left(\frac{i\sqrt{2}\,(x+2)}{2}\right)+\sqrt{2}\,\sqrt{\pi}\,\mathrm{e}^{\frac{x(x+4)}{2}}\right)dx\right)+i\sqrt{2}\,(x+2)\,x\ \mathrm{erf}\left(\frac{i\sqrt{}}{}\right.}{}$$

✓ Solution by Mathematica

Time used: 6.619 (sec). Leaf size: 453

```
DSolve[y''[x]-x*y'[x]-x*y[x]-x^3==0,y[x],x,IncludeSingularSolutions -> True]
```

$$
\begin{aligned}
y(x) \to \frac{1}{2}e^{-\frac{1}{2}(x+2)^2}\Bigg(2\sqrt{2}e^{\frac{x^2}{2}+x+2}(x+2)\int_1^x\Bigg(\frac{e^{K[1]}K[1]^3}{\sqrt{2}} \\
-\frac{1}{2}e^{-\frac{1}{2}K[1]^2-K[1]-2}\sqrt{\pi}\,\mathrm{erfi}\left(\frac{\sqrt{(K[1]+2)^2}}{\sqrt{2}}\right)K[1]^3\sqrt{(K[1]+2)^2}\Bigg)dK[1] \\
-2\,\mathrm{erf}\left(\frac{x+1}{\sqrt{2}}\right)\left(\sqrt{2\pi}e^{x^2+3x+\frac{5}{2}}-\pi e^{\frac{1}{2}(x+1)^2}\sqrt{(x+2)^2}\,\mathrm{erfi}\left(\frac{\sqrt{(x+2)^2}}{\sqrt{2}}\right)\right) \\
-\sqrt{2\pi}\sqrt{(x+2)^2}x^3\,\mathrm{erfi}\left(\frac{\sqrt{(x+2)^2}}{\sqrt{2}}\right)-\sqrt{2\pi}\sqrt{(x+2)^2}x^2\,\mathrm{erfi}\left(\frac{\sqrt{(x+2)^2}}{\sqrt{2}}\right) \\
-\sqrt{2\pi}c_2e^{\frac{x^2}{2}+x+2}\sqrt{(x+2)^2}\,\mathrm{erfi}\left(\frac{\sqrt{(x+2)^2}}{\sqrt{2}}\right) \\
-2\sqrt{2\pi}\sqrt{(x+2)^2}x\,\mathrm{erfi}\left(\frac{\sqrt{(x+2)^2}}{\sqrt{2}}\right)+2e^{\frac{1}{2}(x+2)^2}x^3+2e^{\frac{1}{2}(x+2)^2}x^2 \\
+2\sqrt{2}c_1e^{\frac{x^2}{2}+x+2}x+4\sqrt{2}c_1e^{\frac{x^2}{2}+x+2}+2c_2e^{x^2+3x+4}+4e^{\frac{1}{2}(x+2)^2}x\Bigg)
\end{aligned}
$$

2.11 problem 11

Internal problem ID [7147]

Book: Own collection of miscellaneous problems
Section: section 2.0
Problem number: 11.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$y'' - axy' - bxy = cx$$

✓ Solution by Maple
Time used: 0.015 (sec). Leaf size: 86

```
dsolve(diff(y(x),x$2)-a*x*diff(y(x),x)-b*x*y(x)-c*x=0,y(x), singsol=all)
```

$$y(x) = \frac{e^{-\frac{bx}{a}} \operatorname{KummerU}\left(-\frac{b^2}{2a^3}, \frac{1}{2}, \frac{(a^2x+2b)^2}{2a^3}\right) c_1 b + e^{-\frac{bx}{a}} \operatorname{KummerM}\left(-\frac{b^2}{2a^3}, \frac{1}{2}, \frac{(a^2x+2b)^2}{2a^3}\right) c_2 b - c}{b}$$

✓ Solution by Mathematica
Time used: 5.384 (sec). Leaf size: 565

```
DSolve[y''[x]-a*x*y'[x]-b*x*y[x]-c*x==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to e^{-\frac{bx}{a}} \left(\operatorname{HermiteH}\left(\frac{b^2}{a^3}, \frac{xa^2+2b}{\sqrt{2}a^{3/2}}\right) \int_1^x \frac{a^4 c e^{\frac{bK[}{a}}}{b^2 \left(\sqrt{2} \operatorname{HermiteH}\left(\frac{b^2}{a^3} - 1, \frac{K[1]a^2+2b}{\sqrt{2}a^{3/2}}\right) \operatorname{Hypergeometric1F1}\left(-\frac{b^2}{2a^3},\right.\right.}\right.$$

2.12 problem 12

Internal problem ID [7148]

Book: Own collection of miscellaneous problems
Section: section 2.0
Problem number: 12.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$\boxed{y'' - axy' - bxy = c\,x^2}$$

✓ Solution by Maple

Time used: 0.031 (sec). Leaf size: 95

```
dsolve(diff(y(x),x$2)-a*x*diff(y(x),x)-b*x*y(x)-c*x^2=0,y(x), singsol=all)
```

$$y(x) = \frac{\mathrm{e}^{-\frac{bx}{a}}\,\mathrm{KummerM}\left(-\frac{b^2}{2a^3}, \frac{1}{2}, \frac{(a^2x+2b)^2}{2a^3}\right) c_2 b^2 + \mathrm{e}^{-\frac{bx}{a}}\,\mathrm{KummerU}\left(-\frac{b^2}{2a^3}, \frac{1}{2}, \frac{(a^2x+2b)^2}{2a^3}\right) c_1 b^2 + c(-bx+a)}{b^2}$$

✓ Solution by Mathematica

Time used: 2.978 (sec). Leaf size: 569

```
DSolve[y''[x]-a*x*y'[x]-b*x*y[x]-c*x^2==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to e^{-\frac{bx}{a}}\left(\mathrm{HermiteH}\left(\frac{b^2}{a^3}, \frac{xa^2+2b}{\sqrt{2}a^{3/2}}\right)\int_1^x \frac{a^4ce^{\frac{bK[1}{a}}}{b^2\left(\sqrt{2}\,\mathrm{HermiteH}\left(\frac{b^2}{a^3}-1, \frac{K[1]a^2+2b}{\sqrt{2}a^{3/2}}\right)\mathrm{Hypergeometric1F1}\left(-\frac{b^2}{2a^3},\right.\right.}\right.$$

2.13 problem 13

Internal problem ID [7149]

Book: Own collection of miscellaneous problems
Section: section 2.0
Problem number: 13.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _linear, _nonhomogeneous]]`

$$\boxed{y'' - axy' - bxy = c\,x^3}$$

✓ Solution by Maple
Time used: 0.0 (sec). Leaf size: 517

```
dsolve(diff(y(x),x$2)-a*x*diff(y(x),x)-b*x*y(x)-c*x^3=0,y(x), singsol=all)
```

$$y(x) = \frac{e^{-\frac{bx}{a}}\left(2\,\mathrm{KummerU}\left(-\frac{b^2}{2a^3},\frac{1}{2},\frac{(a^2x+2b)^2}{2a^3}\right)\left(\int -\frac{(a^2x+2b)x^3\,\mathrm{KummerM}\left(-\right.}{\mathrm{KummerM}\left(-\frac{b^2}{2a^3},\frac{1}{2},\frac{(a^2x+2b)^2}{2a^3}\right)(a^3-b^2)\,\mathrm{KummerU}\left(\frac{2a^3-b^2}{2a^3},\frac{1}{2},\frac{(a^2x+2b)^2}{2a^3}\right)}\right.\right.}{}$$

✓ Solution by Mathematica
Time used: 3.085 (sec). Leaf size: 569

```
DSolve[y''[x]-a*x*y'[x]-b*x*y[x]-c*x^3==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to e^{-\frac{bx}{a}}\left(\mathrm{HermiteH}\left(\frac{b^2}{a^3},\frac{xa^2+2b}{\sqrt{2}a^{3/2}}\right)\int_1^x \frac{a^4ce^{\frac{bK[1}{a}}}{b^2\left(\sqrt{2}\,\mathrm{HermiteH}\left(\frac{b^2}{a^3}-1,\frac{K[1]a^2+2b}{\sqrt{2}a^{3/2}}\right)\mathrm{Hypergeometric1F1}\left(-\frac{b^2}{2a^3},\right.\right.}\right.$$

2.14 problem 14

Internal problem ID [7150]

Book: Own collection of miscellaneous problems
Section: section 2.0
Problem number: 14.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$\boxed{y'' - y' - yx = x}$$

✓ Solution by Maple
Time used: 0.015 (sec). Leaf size: 26

```
dsolve(diff(y(x),x$2)-diff(y(x),x)-x*y(x)-x=0,y(x), singsol=all)
```

$$y(x) = \mathrm{e}^{\frac{x}{2}} \operatorname{AiryAi}\left(\frac{1}{4} + x\right) c_2 + \mathrm{e}^{\frac{x}{2}} \operatorname{AiryBi}\left(\frac{1}{4} + x\right) c_1 - 1$$

✓ Solution by Mathematica
Time used: 13.6 (sec). Leaf size: 99

```
DSolve[y''[x]-y'[x]-x*y[x]-x==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to e^{x/2}\left(\operatorname{AiryAi}\left(x+\frac{1}{4}\right)\int_1^x -e^{-\frac{K[1]}{2}}\pi \operatorname{AiryBi}\left(K[1]+\frac{1}{4}\right)K[1]dK[1] + \operatorname{AiryBi}\left(x+\frac{1}{4}\right)\int_1^x e^{-\frac{K[2]}{2}}\pi \operatorname{AiryAi}\left(K[2]+\frac{1}{4}\right)K[2]dK[2] + c_1 \operatorname{AiryAi}\left(x+\frac{1}{4}\right) + c_2 \operatorname{AiryBi}\left(x+\frac{1}{4}\right)\right)$$

2.15 problem 15

Internal problem ID [7151]

Book: Own collection of miscellaneous problems
Section: section 2.0
Problem number: 15.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _linear, _nonhomogeneous]]`

$$\boxed{y'' - y' - yx = x^2}$$

✓ Solution by Maple
Time used: 0.015 (sec). Leaf size: 63

```
dsolve(diff(y(x),x$2)-diff(y(x),x)-x*y(x)-x^2=0,y(x), singsol=all)
```

$$y(x) = e^{\frac{x}{2}}\left(\text{AiryBi}\left(\frac{1}{4}+x\right)\pi\left(\int x^2 \text{AiryAi}\left(\frac{1}{4}+x\right)e^{-\frac{x}{2}}dx\right) - \text{AiryAi}\left(\frac{1}{4}+x\right)\pi\left(\int x^2 \text{AiryBi}\left(\frac{1}{4}+x\right)e^{-\frac{x}{2}}dx\right) + c_1 \text{AiryBi}\left(\frac{1}{4}+x\right) + c_2 \text{AiryAi}\left(\frac{1}{4}+x\right)\right)$$

✓ Solution by Mathematica
Time used: 9.743 (sec). Leaf size: 103

```
DSolve[y''[x]-y'[x]-x*y[x]-x^2==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to e^{x/2}\left(\text{AiryAi}\left(x+\frac{1}{4}\right)\int_1^x -e^{-\frac{K[1]}{2}}\pi \text{AiryBi}\left(K[1]+\frac{1}{4}\right)K[1]^2 dK[1] + \text{AiryBi}\left(x+\frac{1}{4}\right)\int_1^x e^{-\frac{K[2]}{2}}\pi \text{AiryAi}\left(K[2]+\frac{1}{4}\right)K[2]^2 dK[2] + c_1 \text{AiryAi}\left(x+\frac{1}{4}\right) + c_2 \text{AiryBi}\left(x+\frac{1}{4}\right)\right)$$

2.16 problem 16

Internal problem ID [7152]

Book: Own collection of miscellaneous problems
Section: section 2.0
Problem number: 16.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$\boxed{y'' - y' - yx = x^2 + 1}$$

✓ Solution by Maple

Time used: 0.015 (sec). Leaf size: 28

```
dsolve(diff(y(x),x$2)-diff(y(x),x)-x*y(x)-x^2-1=0,y(x), singsol=all)
```

$$y(x) = \mathrm{e}^{\frac{x}{2}} \operatorname{AiryAi}\left(\frac{1}{4} + x\right) c_2 + \mathrm{e}^{\frac{x}{2}} \operatorname{AiryBi}\left(\frac{1}{4} + x\right) c_1 - x$$

✓ Solution by Mathematica

Time used: 4.468 (sec). Leaf size: 107

```
DSolve[y''[x]-y'[x]-x*y[x]-x^2-1==0,y[x],x,IncludeSingularSolutions -> True]
```

$$\begin{aligned} y(x) \rightarrow e^{x/2} \Bigg(& \operatorname{AiryAi}\left(x + \frac{1}{4}\right) \int_1^x -e^{-\frac{K[1]}{2}} \pi \operatorname{AiryBi}\left(K[1] + \frac{1}{4}\right) \left(K[1]^2 + 1\right) dK[1] \\ & + \operatorname{AiryBi}\left(x + \frac{1}{4}\right) \int_1^x e^{-\frac{K[2]}{2}} \pi \operatorname{AiryAi}\left(K[2] + \frac{1}{4}\right) \left(K[2]^2 + 1\right) dK[2] \\ & + c_1 \operatorname{AiryAi}\left(x + \frac{1}{4}\right) + c_2 \operatorname{AiryBi}\left(x + \frac{1}{4}\right) \Bigg) \end{aligned}$$

2.17 problem 16

Internal problem ID [7153]

Book: Own collection of miscellaneous problems
Section: section 2.0
Problem number: 16.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$y'' - y' - yx = x^2 + 1$$

✓ Solution by Maple
Time used: 0.016 (sec). Leaf size: 28

```
dsolve(diff(y(x),x$2)-diff(y(x),x)-x*y(x)-x^2-1=0,y(x), singsol=all)
```

$$y(x) = \mathrm{e}^{\frac{x}{2}} \operatorname{AiryAi}\left(\frac{1}{4}+x\right) c_2 + \mathrm{e}^{\frac{x}{2}} \operatorname{AiryBi}\left(\frac{1}{4}+x\right) c_1 - x$$

✓ Solution by Mathematica
Time used: 1.289 (sec). Leaf size: 107

```
DSolve[y''[x]-y'[x]-x*y[x]-x^2-1==0,y[x],x,IncludeSingularSolutions -> True]
```

$$\begin{aligned} y(x) \to e^{x/2}\Bigg(&\operatorname{AiryAi}\left(x+\frac{1}{4}\right)\int_1^x -e^{-\frac{K[1]}{2}}\pi \operatorname{AiryBi}\left(K[1]+\frac{1}{4}\right)\left(K[1]^2+1\right)dK[1] \\ &+ \operatorname{AiryBi}\left(x+\frac{1}{4}\right)\int_1^x e^{-\frac{K[2]}{2}}\pi \operatorname{AiryAi}\left(K[2]+\frac{1}{4}\right)\left(K[2]^2+1\right)dK[2] \\ &+ c_1 \operatorname{AiryAi}\left(x+\frac{1}{4}\right) + c_2 \operatorname{AiryBi}\left(x+\frac{1}{4}\right)\Bigg) \end{aligned}$$

2.18 problem 17

Internal problem ID [7154]

Book: Own collection of miscellaneous problems
Section: section 2.0
Problem number: 17.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$\boxed{y'' - 2y' - yx = x^2 + 2}$$

✓ Solution by Maple
Time used: 0.0 (sec). Leaf size: 24

```
dsolve(diff(y(x),x$2)-2*diff(y(x),x)-x*y(x)-x^2-2=0,y(x), singsol=all)
```

$$y(x) = e^x \operatorname{AiryAi}(x+1) c_2 + e^x \operatorname{AiryBi}(x+1) c_1 - x$$

✓ Solution by Mathematica
Time used: 5.71 (sec). Leaf size: 87

```
DSolve[y''[x]-2*y'[x]-x*y[x]-x^2-2==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to e^x \left(\operatorname{AiryAi}(x+1) \int_1^x -e^{-K[1]} \pi \operatorname{AiryBi}(K[1]+1) \left(K[1]^2+2\right) dK[1] + \operatorname{AiryBi}(x+1) \int_1^x e^{-K[2]} \pi \operatorname{AiryAi}(K[2]+1) \left(K[2]^2+2\right) dK[2] + c_1 \operatorname{AiryAi}(x+1) + c_2 \operatorname{AiryBi}(x+1) \right)$$

2.19 problem 18

Internal problem ID [7155]

Book: Own collection of miscellaneous problems
Section: section 2.0
Problem number: 18.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$\boxed{y'' - 4y' - yx = x^2 + 4}$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 28

```
dsolve(diff(y(x),x$2)-4*diff(y(x),x)-x*y(x)-x^2-4=0,y(x), singsol=all)
```

$$y(x) = e^{2x}\operatorname{AiryAi}(x+4)\, c_2 + e^{2x}\operatorname{AiryBi}(x+4)\, c_1 - x$$

✓ Solution by Mathematica

Time used: 6.139 (sec). Leaf size: 89

```
DSolve[y''[x]-4*y'[x]-x*y[x]-x^2-4==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to e^{2x}\left(\operatorname{AiryAi}(x+4)\int_1^x -e^{-2K[1]}\pi \operatorname{AiryBi}(K[1]+4)\left(K[1]^2+4\right) dK[1] + \operatorname{AiryBi}(x+4)\int_1^x e^{-2K[2]}\pi \operatorname{AiryAi}(K[2]+4)\left(K[2]^2+4\right) dK[2] + c_1 \operatorname{AiryAi}(x+4) + c_2 \operatorname{AiryBi}(x+4)\right)$$

2.20 problem 19

Internal problem ID [7156]

Book: Own collection of miscellaneous problems
Section: section 2.0
Problem number: 19.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _linear, _nonhomogeneous]]`

$$\boxed{y'' - y' - yx = x^3 - 1}$$

✓ Solution by Maple

Time used: 0.015 (sec). Leaf size: 67

```
dsolve(diff(y(x),x$2)-diff(y(x),x)-x*y(x)-x^3+1=0,y(x), singsol=all)
```

$$y(x) = e^{\frac{x}{2}} \left(-\operatorname{AiryAi}\left(\frac{1}{4}+x\right) \pi \left(\int (x^3-1) \operatorname{AiryBi}\left(\frac{1}{4}+x\right) e^{-\frac{x}{2}} dx \right) + \operatorname{AiryBi}\left(\frac{1}{4}+x\right) \pi \left(\int (x^3-1) \operatorname{AiryAi}\left(\frac{1}{4}+x\right) e^{-\frac{x}{2}} dx \right) + c_2 \operatorname{AiryAi}\left(\frac{1}{4}+x\right) + c_1 \operatorname{AiryBi}\left(\frac{1}{4}+x\right) \right)$$

✓ Solution by Mathematica

Time used: 3.972 (sec). Leaf size: 107

```
DSolve[y''[x]-y'[x]-x*y[x]-x^3+1==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to e^{x/2} \left(\operatorname{AiryAi}\left(x+\frac{1}{4}\right) \int_1^x -e^{-\frac{K[1]}{2}} \pi \operatorname{AiryBi}\left(K[1]+\frac{1}{4}\right) \left(K[1]^3-1\right) dK[1] + \operatorname{AiryBi}\left(x+\frac{1}{4}\right) \int_1^x e^{-\frac{K[2]}{2}} \pi \operatorname{AiryAi}\left(K[2]+\frac{1}{4}\right) \left(K[2]^3-1\right) dK[2] + c_1 \operatorname{AiryAi}\left(x+\frac{1}{4}\right) + c_2 \operatorname{AiryBi}\left(x+\frac{1}{4}\right) \right)$$

2.21 problem 20

Internal problem ID [7157]

Book: Own collection of miscellaneous problems
Section: section 2.0
Problem number: 20.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _linear, _nonhomogeneous]]`

$$y'' - 2y' - yx = x^3 + x^2$$

✓ Solution by Maple
Time used: 0.0 (sec). Leaf size: 30

```
dsolve(diff(y(x),x$2)-2*diff(y(x),x)-x*y(x)-x^3-x^2=0,y(x), singsol=all)
```

$$y(x) = e^x \operatorname{AiryAi}(x+1)\, c_2 + e^x \operatorname{AiryBi}(x+1)\, c_1 - x^2 - x + 4$$

✓ Solution by Mathematica
Time used: 8.466 (sec). Leaf size: 91

```
DSolve[y''[x]-2*y'[x]-x*y[x]-x^3-x^2==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to e^x \left(\operatorname{AiryAi}(x+1) \int_1^x -e^{-K[1]} \pi \operatorname{AiryBi}(K[1]+1) K[1]^2 (K[1]+1) dK[1] + \operatorname{AiryBi}(x+1) \int_1^x e^{-K[2]} \pi \operatorname{AiryAi}(K[2]+1) K[2]^2 (K[2]+1) dK[2] + c_1 \operatorname{AiryAi}(x+1) + c_2 \operatorname{AiryBi}(x+1) \right)$$

2.22 problem 21

Internal problem ID [7158]

Book: Own collection of miscellaneous problems
Section: section 2.0
Problem number: 21.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _linear, _nonhomogeneous]]`

$$\boxed{y'' - y' - yx = x^3 - 2}$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 31

```
dsolve(diff(y(x),x$2)-diff(y(x),x)-x*y(x)-x^3+2=0,y(x), singsol=all)
```

$$y(x) = e^{\frac{x}{2}} \operatorname{AiryAi}\left(\frac{1}{4} + x\right) c_2 + e^{\frac{x}{2}} \operatorname{AiryBi}\left(\frac{1}{4} + x\right) c_1 - x^2 + 2$$

✓ Solution by Mathematica

Time used: 3.963 (sec). Leaf size: 107

```
DSolve[y''[x]-y'[x]-x*y[x]-x^3+2==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to e^{x/2}\left(\operatorname{AiryAi}\left(x+\frac{1}{4}\right) \int_1^x -e^{-\frac{K[1]}{2}} \pi \operatorname{AiryBi}\left(K[1]+\frac{1}{4}\right) \left(K[1]^3-2\right) dK[1] + \operatorname{AiryBi}\left(x+\frac{1}{4}\right) \int_1^x e^{-\frac{K[2]}{2}} \pi \operatorname{AiryAi}\left(K[2]+\frac{1}{4}\right) \left(K[2]^3-2\right) dK[2] + c_1 \operatorname{AiryAi}\left(x+\frac{1}{4}\right) + c_2 \operatorname{AiryBi}\left(x+\frac{1}{4}\right)\right)$$

2.23 problem 22

Internal problem ID [7159]

Book: Own collection of miscellaneous problems
Section: section 2.0
Problem number: 22.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _linear, _nonhomogeneous]]`

$$\boxed{y'' - 2y' - yx = x^3 - 2}$$

✓ Solution by Maple
Time used: 0.0 (sec). Leaf size: 27

```
dsolve(diff(y(x),x$2)-2*diff(y(x),x)-x*y(x)-x^3+2=0,y(x), singsol=all)
```

$$y(x) = \mathrm{e}^{x} \operatorname{AiryAi}(x+1) c_2 + \mathrm{e}^{x} \operatorname{AiryBi}(x+1) c_1 - x^2 + 4$$

✓ Solution by Mathematica
Time used: 2.673 (sec). Leaf size: 87

```
DSolve[y''[x]-2*y'[x]-x*y[x]-x^3+2==0,y[x],x,IncludeSingularSolutions -> True]
```

$$\begin{aligned} y(x) \to e^x \bigg(&\operatorname{AiryAi}(x+1) \int_1^x -e^{-K[1]} \pi \operatorname{AiryBi}(K[1]+1) \left(K[1]^3 - 2\right) dK[1] \\ &+ \operatorname{AiryBi}(x+1) \int_1^x e^{-K[2]} \pi \operatorname{AiryAi}(K[2]+1) \left(K[2]^3 - 2\right) dK[2] \\ &+ c_1 \operatorname{AiryAi}(x+1) + c_2 \operatorname{AiryBi}(x+1) \bigg) \end{aligned}$$

2.24 problem 23

Internal problem ID [7160]

Book: Own collection of miscellaneous problems
Section: section 2.0
Problem number: 23.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _linear, _nonhomogeneous]]`

$$\boxed{y'' - 4y' - yx = x^3 - 2}$$

✓ Solution by Maple
Time used: 0.0 (sec). Leaf size: 31

```
dsolve(diff(y(x),x$2)-4*diff(y(x),x)-x*y(x)-x^3+2=0,y(x), singsol=all)
```

$$y(x) = \mathrm{e}^{2x} \operatorname{AiryAi}(x+4) c_2 + \mathrm{e}^{2x} \operatorname{AiryBi}(x+4) c_1 - x^2 + 8$$

✓ Solution by Mathematica
Time used: 2.795 (sec). Leaf size: 89

```
DSolve[y''[x]-4*y'[x]-x*y[x]-x^3+2==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to e^{2x} \left(\operatorname{AiryAi}(x+4) \int_1^x -e^{-2K[1]} \pi \operatorname{AiryBi}(K[1]+4) \left(K[1]^3 - 2\right) dK[1] + \operatorname{AiryBi}(x+4) \int_1^x e^{-2K[2]} \pi \operatorname{AiryAi}(K[2]+4) \left(K[2]^3 - 2\right) dK[2] + c_1 \operatorname{AiryAi}(x+4) + c_2 \operatorname{AiryBi}(x+4) \right)$$

2.25 problem 24

Internal problem ID [7161]

Book: Own collection of miscellaneous problems
Section: section 2.0
Problem number: 24.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _linear, _nonhomogeneous]]`

$$\boxed{y'' - 6y' - yx = x^3 - 2}$$

✓ Solution by Maple
Time used: 0.016 (sec). Leaf size: 31

```
dsolve(diff(y(x),x$2)-6*diff(y(x),x)-x*y(x)-x^3+2=0,y(x), singsol=all)
```

$$y(x) = e^{3x} \operatorname{AiryAi}(9 + x) c_2 + e^{3x} \operatorname{AiryBi}(9 + x) c_1 - x^2 + 12$$

✓ Solution by Mathematica
Time used: 6.656 (sec). Leaf size: 89

```
DSolve[y''[x]-6*y'[x]-x*y[x]-x^3+2==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to e^{3x} \left(\operatorname{AiryAi}(x + 9) \int_1^x -e^{-3K[1]} \pi \operatorname{AiryBi}(K[1] + 9) \left(K[1]^3 - 2\right) dK[1] + \operatorname{AiryBi}(x + 9) \int_1^x e^{-3K[2]} \pi \operatorname{AiryAi}(K[2] + 9) \left(K[2]^3 - 2\right) dK[2] + c_1 \operatorname{AiryAi}(x + 9) + c_2 \operatorname{AiryBi}(x + 9) \right)$$

2.26 problem 25

Internal problem ID [7162]

Book: Own collection of miscellaneous problems
Section: section 2.0
Problem number: 25.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _linear, _nonhomogeneous]]`

$$\boxed{y'' - 8y' - yx = x^3 - 2}$$

✓ Solution by Maple
Time used: 0.0 (sec). Leaf size: 31

```
dsolve(diff(y(x),x$2)-8*diff(y(x),x)-x*y(x)-x^3+2=0,y(x), singsol=all)
```

$$y(x) = e^{4x} \operatorname{AiryAi}(16 + x) c_2 + e^{4x} \operatorname{AiryBi}(16 + x) c_1 - x^2 + 16$$

✓ Solution by Mathematica
Time used: 6.555 (sec). Leaf size: 89

```
DSolve[y''[x]-8*y'[x]-x*y[x]-x^3+2==0,y[x],x,IncludeSingularSolutions -> True]
```

$$\begin{aligned} y(x) \to e^{4x} \bigg(& \operatorname{AiryAi}(x+16) \int_1^x -e^{-4K[1]} \pi \operatorname{AiryBi}(K[1]+16) \left(K[1]^3 - 2\right) dK[1] \\ & + \operatorname{AiryBi}(x+16) \int_1^x e^{-4K[2]} \pi \operatorname{AiryAi}(K[2]+16) \left(K[2]^3 - 2\right) dK[2] \\ & + c_1 \operatorname{AiryAi}(x+16) + c_2 \operatorname{AiryBi}(x+16) \bigg) \end{aligned}$$

2.27 problem 26

Internal problem ID [7163]

Book: Own collection of miscellaneous problems
Section: section 2.0
Problem number: 26.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _linear, _nonhomogeneous]]`

$$\boxed{y'' - y' - yx = x^4 - 3}$$

✓ Solution by Maple
Time used: 0.016 (sec). Leaf size: 34

```
dsolve(diff(y(x),x$2)-diff(y(x),x)-x*y(x)-x^4+3=0,y(x), singsol=all)
```

$$y(x) = e^{\frac{x}{2}} \operatorname{AiryAi}\left(\frac{1}{4} + x\right) c_2 + e^{\frac{x}{2}} \operatorname{AiryBi}\left(\frac{1}{4} + x\right) c_1 - x^3 + 3x - 6$$

✓ Solution by Mathematica
Time used: 4.059 (sec). Leaf size: 107

```
DSolve[y''[x]-y'[x]-x*y[x]-x^4+3==0,y[x],x,IncludeSingularSolutions -> True]
```

$$\begin{aligned} y(x) \to e^{x/2}\Bigg(&\operatorname{AiryAi}\left(x + \frac{1}{4}\right) \int_1^x -e^{-\frac{K[1]}{2}} \pi \operatorname{AiryBi}\left(K[1] + \frac{1}{4}\right) \left(K[1]^4 - 3\right) dK[1] \\ &+ \operatorname{AiryBi}\left(x + \frac{1}{4}\right) \int_1^x e^{-\frac{K[2]}{2}} \pi \operatorname{AiryAi}\left(K[2] + \frac{1}{4}\right) \left(K[2]^4 - 3\right) dK[2] \\ &+ c_1 \operatorname{AiryAi}\left(x + \frac{1}{4}\right) + c_2 \operatorname{AiryBi}\left(x + \frac{1}{4}\right)\Bigg) \end{aligned}$$

2.28 problem 27

Internal problem ID [7164]

Book: Own collection of miscellaneous problems
Section: section 2.0
Problem number: 27.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _linear, _nonhomogeneous]]`

$$y'' - y' - yx = x^3$$

✓ Solution by Maple

Time used: 0.016 (sec). Leaf size: 63

```
dsolve(diff(y(x),x$2)-diff(y(x),x)-x*y(x)-x^3=0,y(x), singsol=all)
```

$$y(x) = e^{\frac{x}{2}}\left(-\operatorname{AiryAi}\left(\frac{1}{4}+x\right)\pi\left(\int x^3 \operatorname{AiryBi}\left(\frac{1}{4}+x\right)e^{-\frac{x}{2}}dx\right) + \operatorname{AiryBi}\left(\frac{1}{4}+x\right)\pi\left(\int x^3 \operatorname{AiryAi}\left(\frac{1}{4}+x\right)e^{-\frac{x}{2}}dx\right) + c_2 \operatorname{AiryAi}\left(\frac{1}{4}+x\right) + c_1 \operatorname{AiryBi}\left(\frac{1}{4}+x\right)\right)$$

✓ Solution by Mathematica

Time used: 10.277 (sec). Leaf size: 103

```
DSolve[y''[x]-y'[x]-x*y[x]-x^3==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to e^{x/2}\left(\operatorname{AiryAi}\left(x+\frac{1}{4}\right)\int_1^x -e^{-\frac{K[1]}{2}}\pi \operatorname{AiryBi}\left(K[1]+\frac{1}{4}\right)K[1]^3 dK[1] + \operatorname{AiryBi}\left(x+\frac{1}{4}\right)\int_1^x e^{-\frac{K[2]}{2}}\pi \operatorname{AiryAi}\left(K[2]+\frac{1}{4}\right)K[2]^3 dK[2] + c_1 \operatorname{AiryAi}\left(x+\frac{1}{4}\right) + c_2 \operatorname{AiryBi}\left(x+\frac{1}{4}\right)\right)$$

2.29 problem 28

Internal problem ID [7165]

Book: Own collection of miscellaneous problems
Section: section 2.0
Problem number: 28.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _linear, _nonhomogeneous]]`

$$\boxed{y'' - yx = x^3 - 2}$$

✓ Solution by Maple
Time used: 0.016 (sec). Leaf size: 18

```
dsolve(diff(y(x),x$2)-x*y(x)-x^3+2=0,y(x), singsol=all)
```

$$y(x) = \text{AiryAi}(x)\, c_2 + \text{AiryBi}(x)\, c_1 - x^2$$

✓ Solution by Mathematica
Time used: 0.458 (sec). Leaf size: 290

```
DSolve[y''[x]-x*y[x]-x^3+2==0,y[x],x,IncludeSingularSolutions -> True]
```

$y(x)$

$$\rightarrow \frac{6\sqrt[3]{3}\pi x \,\text{Gamma}\left(\frac{1}{3}\right) \text{Gamma}\left(\frac{5}{3}\right) \text{Gamma}\left(\frac{7}{3}\right) \text{Gamma}\left(\frac{8}{3}\right) \left(\sqrt{3}\,\text{AiryAi}(x) - \text{AiryBi}(x)\right) {}_1F_2\left(\frac{1}{3}; \frac{2}{3}, \frac{4}{3}; \frac{x^3}{9}\right)}{}$$

2.30 problem 29

Internal problem ID [7166]

Book: Own collection of miscellaneous problems
Section: section 2.0
Problem number: 29.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _linear, _nonhomogeneous]]`

$$\boxed{y'' - yx = x^6 - 64}$$

✓ Solution by Maple
Time used: 0.015 (sec). Leaf size: 149

```
dsolve(diff(y(x),x$2)-x*y(x)-x^6+64=0,y(x), singsol=all)
```

$$y(x) = \frac{16x^7\pi\left(\mathrm{AiryBi}\left(x\right)3^{\frac{1}{3}} - 3^{\frac{5}{6}}\,\mathrm{AiryAi}\left(x\right)\right)\mathrm{hypergeom}\left(\left[\frac{7}{3}\right],\left[\frac{2}{3},\frac{10}{3}\right],\frac{x^3}{9}\right) - 21x^8\Gamma\left(\frac{2}{3}\right)^2\left(3^{\frac{1}{6}}\,\mathrm{AiryBi}\left(x\right) + 3^{\frac{2}{3}}\right.}{}$$

✓ Solution by Mathematica
Time used: 0.493 (sec). Leaf size: 256

```
DSolve[y''[x]-x*y[x]-x^6+64==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to \frac{192\sqrt[3]{3}\pi x\,\mathrm{Gamma}\left(\frac{1}{3}\right)\left(\sqrt{3}\,\mathrm{AiryAi}(x) - \mathrm{AiryBi}(x)\right)\,{}_1F_2\left(\frac{1}{3};\frac{2}{3},\frac{4}{3};\frac{x^3}{9}\right) - \frac{\sqrt[6]{3}\pi x^8\,\mathrm{Gamma}\left(\frac{2}{3}\right)\mathrm{Gamma}\left(\frac{8}{3}\right)\left(3\,\mathrm{AiryAi}\right.}{\mathrm{Gamma}(}}{}$$

2.31 problem 30

Internal problem ID [7167]

Book: Own collection of miscellaneous problems
Section: section 2.0
Problem number: 30.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$\boxed{y'' - yx = x}$$

✓ Solution by Maple
Time used: 0.0 (sec). Leaf size: 14

```
dsolve(diff(y(x),x$2)-x*y(x)-x=0,y(x), singsol=all)
```

$$y(x) = \text{AiryAi}\,(x)\,c_2 + \text{AiryBi}\,(x)\,c_1 - 1$$

✓ Solution by Mathematica
Time used: 0.036 (sec). Leaf size: 28

```
DSolve[y''[x]-x*y[x]-x==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to \pi\,\text{AiryAiPrime}(x)\,\text{AiryBi}(x) + c_2\,\text{AiryBi}(x) + \text{AiryAi}(x)(-\pi\,\text{AiryBiPrime}(x) + c_1)$$

2.32 problem 31

Internal problem ID [7168]

Book: Own collection of miscellaneous problems
Section: section 2.0
Problem number: 31.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$\boxed{y'' - yx = x^2}$$

✓ Solution by Maple
Time used: 0.0 (sec). Leaf size: 16

```
dsolve(diff(y(x),x$2)-x*y(x)-x^2=0,y(x), singsol=all)
```

$$y(x) = \text{AiryAi}\,(x)\, c_2 + \text{AiryBi}\,(x)\, c_1 - x$$

✓ Solution by Mathematica
Time used: 0.041 (sec). Leaf size: 30

```
DSolve[y''[x]-x*y[x]-x^2==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to \pi x \,\text{AiryAiPrime}(x)\,\text{AiryBi}(x) + c_2\,\text{AiryBi}(x) + \text{AiryAi}(x)(-\pi x\,\text{AiryBiPrime}(x) + c_1)$$

2.33 problem 32

Internal problem ID [7169]

Book: Own collection of miscellaneous problems
Section: section 2.0
Problem number: 32.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _linear, _nonhomogeneous]]`

$$\boxed{y'' - yx = x^3}$$

✓ Solution by Maple
Time used: 0.0 (sec). Leaf size: 87

```
dsolve(diff(y(x),x$2)-x*y(x)-x^3=0,y(x), singsol=all)
```

$$y(x) = \frac{5x^4\pi \,\text{hypergeom}\left(\left[\frac{4}{3}\right], \left[\frac{2}{3}, \frac{7}{3}\right], \frac{x^3}{9}\right)\left(\text{AiryBi}\,(x)\, 3^{\frac{1}{3}} - 3^{\frac{5}{6}}\,\text{AiryAi}\,(x)\right) - 6\Gamma\left(\frac{2}{3}\right)\left(x^5\,\text{hypergeom}\left(\left[\frac{5}{3}\right], \left[\frac{4}{3}, \frac{8}{3}\right.\right.\right.}{60\Gamma\left(\frac{2}{3}\right)}$$

✓ Solution by Mathematica
Time used: 0.093 (sec). Leaf size: 137

```
DSolve[y''[x]-x*y[x]-x^3==0,y[x],x,IncludeSingularSolutions -> True]
```

$$\begin{aligned} y(x) \to & -\frac{\pi x^5 \,\text{Gamma}\left(\frac{5}{3}\right)\left(3\,\text{AiryAi}(x) + \sqrt{3}\,\text{AiryBi}(x)\right)\,{}_1F_2\left(\frac{5}{3}; \frac{4}{3}, \frac{8}{3}; \frac{x^3}{9}\right)}{9\ 3^{5/6}\,\text{Gamma}\left(\frac{4}{3}\right)\text{Gamma}\left(\frac{8}{3}\right)} \\ & + \frac{\pi x^4 \,\text{Gamma}\left(\frac{4}{3}\right)\left(\text{AiryBi}(x) - \sqrt{3}\,\text{AiryAi}(x)\right)\,{}_1F_2\left(\frac{4}{3}; \frac{2}{3}, \frac{7}{3}; \frac{x^3}{9}\right)}{3\ 3^{2/3}\,\text{Gamma}\left(\frac{2}{3}\right)\text{Gamma}\left(\frac{7}{3}\right)} \\ & + c_1\,\text{AiryAi}(x) + c_2\,\text{AiryBi}(x) \end{aligned}$$

2.34 problem 33

Internal problem ID [7170]

Book: Own collection of miscellaneous problems
Section: section 2.0
Problem number: 33.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _linear, _nonhomogeneous]]`

$$\boxed{y'' - yx = x^6 + x^3 - 42}$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 23

```
dsolve(diff(y(x),x$2)-x*y(x)-x^6-x^3+42=0,y(x), singsol=all)
```

$$y(x) = \text{AiryAi}\,(x)\, c_2 + \text{AiryBi}\,(x)\, c_1 - x^5 - 21x^2$$

✓ Solution by Mathematica

Time used: 1.142 (sec). Leaf size: 367

```
DSolve[y''[x]-x*y[x]-x^6-x^3+42==0,y[x],x,IncludeSingularSolutions -> True]
```

$y(x) \rightarrow$

$$-\frac{-126\sqrt[3]{3}\pi x\,\text{Gamma}\left(\frac{1}{3}\right)\left(\sqrt{3}\,\text{AiryAi}(x) - \text{AiryBi}(x)\right)\,{}_1F_2\left(\frac{1}{3};\frac{2}{3},\frac{4}{3};\frac{x^3}{9}\right) + \frac{\sqrt[6]{3}\pi x^8\,\text{Gamma}\left(\frac{2}{3}\right)\text{Gamma}\left(\frac{8}{3}\right)\left(3\,\text{Ai}\right.}{\text{Gam}}}{}$$

2.35 problem 34

Internal problem ID [7171]

Book: Own collection of miscellaneous problems
Section: section 2.0
Problem number: 34.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$\boxed{y'' - x^2 y = x^2}$$

✓ Solution by Maple

Time used: 0.015 (sec). Leaf size: 30

```
dsolve(diff(y(x),x$2)-x^2*y(x)-x^2=0,y(x), singsol=all)
```

$$y(x) = \sqrt{x}\,\mathrm{BesselI}\left(\frac{1}{4}, \frac{x^2}{2}\right) c_2 + \sqrt{x}\,\mathrm{BesselK}\left(\frac{1}{4}, \frac{x^2}{2}\right) c_1 - 1$$

✓ Solution by Mathematica

Time used: 6.053 (sec). Leaf size: 213

```
DSolve[y''[x]-x^2*y[x]-x^2==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to \mathrm{ParabolicCylinderD}\left(-\frac{1}{2}, \sqrt{2}x\right)\left(\int_1^x \frac{K[1]^2\,\mathrm{ParabolicCyl}}{\sqrt{2}\left(\mathrm{HermiteH}\left(-\frac{1}{2}, K[1]\right)\left(i\,\mathrm{HermiteH}\left(\frac{1}{2}, iK[1]\right) + 2\,\mathrm{HermiteH}\right.\right.} + c_1\right)$$

$$+\mathrm{ParabolicCylinderD}\left(-\frac{1}{2}, i\sqrt{2}x\right)\left(\int_1^x \frac{K[2]^2\,\mathrm{Parabolic}}{\sqrt{2}\left(\mathrm{HermiteH}\left(-\frac{1}{2}, iK[2]\right)\mathrm{HermiteH}\left(\frac{1}{2}, K[2]\right) + \mathrm{HermiteH}\right.} + c_2\right)$$

2.36 problem 35

Internal problem ID [7172]

Book: Own collection of miscellaneous problems
Section: section 2.0
Problem number: 35.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$\boxed{y'' - x^2 y = x^3}$$

✓ Solution by Maple
Time used: 0.0 (sec). Leaf size: 32

```
dsolve(diff(y(x),x$2)-x^2*y(x)-x^3=0,y(x), singsol=all)
```

$$y(x) = \sqrt{x}\,\text{BesselI}\left(\frac{1}{4}, \frac{x^2}{2}\right) c_2 + \sqrt{x}\,\text{BesselK}\left(\frac{1}{4}, \frac{x^2}{2}\right) c_1 - x$$

✓ Solution by Mathematica
Time used: 4.871 (sec). Leaf size: 213

```
DSolve[y''[x]-x^2*y[x]-x^3==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to \text{ParabolicCylinderD}\left(-\frac{1}{2}, \sqrt{2}x\right) \left(\int_1^x \frac{K[1]^3\,\text{ParabolicCyl}}{\sqrt{2}\left(\text{HermiteH}\left(-\frac{1}{2}, K[1]\right)\left(i\,\text{HermiteH}\left(\frac{1}{2}, iK[1]\right) + 2\,\text{HermiteH}\right.\right.} + c_1\right) + \text{ParabolicCylinderD}\left(-\frac{1}{2}, i\sqrt{2}x\right) \left(\int_1^x \frac{K[2]^3\,\text{Parabolic}}{\sqrt{2}\left(\text{HermiteH}\left(-\frac{1}{2}, iK[2]\right)\text{HermiteH}\left(\frac{1}{2}, K[2]\right) + \text{HermiteH}\right.} + c_2\right)$$

2.37 problem 36

Internal problem ID [7173]

Book: Own collection of miscellaneous problems
Section: section 2.0
Problem number: 36.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _linear, _nonhomogeneous]]`

$$y'' - x^2 y = x^4$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 124

```
dsolve(diff(y(x),x$2)-x^2*y(x)-x^4=0,y(x), singsol=all)
```

$$y(x) = -\frac{\left(-\frac{6x^5\pi^2 \operatorname{csgn}(x) \operatorname{hypergeom}\left(\left[\frac{5}{4}\right],\left[\frac{3}{4},\frac{5}{2}\right],\frac{x^4}{16}\right) \operatorname{BesselI}\left(\frac{1}{4},\frac{x^2}{2}\right)}{5} + \Gamma\left(\frac{3}{4}\right)\left(2x^6\Gamma\left(\frac{3}{4}\right) \operatorname{BesselK}\left(\frac{1}{4},\frac{x^2}{2}\right) \operatorname{hypergeom}\left(\left[\frac{3}{2}\right],\left[\frac{5}{4}\right.\right.\right.\right.}{}$$

✓ Solution by Mathematica

Time used: 3.699 (sec). Leaf size: 213

```
DSolve[y''[x]-x^2*y[x]-x^4==0,y[x],x,IncludeSingularSolutions -> True]
```

$y(x)$

$$\to \text{ParabolicCylinderD}\left(-\frac{1}{2},\sqrt{2}x\right)\left(\int_1^x \frac{K[1]^4\,\text{ParabolicCyl}}{\sqrt{2}\left(\text{HermiteH}\left(-\frac{1}{2},K[1]\right)\left(i\,\text{HermiteH}\left(\frac{1}{2},iK[1]\right)+2\,\text{HermiteH}\right.\right.} + c_1\right)$$

$$+\text{ParabolicCylinderD}\left(-\frac{1}{2},i\sqrt{2}x\right)\left(\int_1^x \frac{K[2]^4\,\text{Parabolic}}{\sqrt{2}\left(\text{HermiteH}\left(-\frac{1}{2},iK[2]\right)\text{HermiteH}\left(\frac{1}{2},K[2]\right)+\text{HermiteH}\right.} + c_2\right)$$

2.38 problem 37

Internal problem ID [7174]

Book: Own collection of miscellaneous problems
Section: section 2.0
Problem number: 37.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _linear, _nonhomogeneous]]`

$$\boxed{y'' - x^2 y = x^4 - 2}$$

✓ Solution by Maple
Time used: 0.015 (sec). Leaf size: 34

```
dsolve(diff(y(x),x$2)-x^2*y(x)-x^4+2=0,y(x), singsol=all)
```

$$y(x) = \sqrt{x}\ \text{BesselI}\left(\frac{1}{4}, \frac{x^2}{2}\right) c_2 + \sqrt{x}\ \text{BesselK}\left(\frac{1}{4}, \frac{x^2}{2}\right) c_1 - x^2$$

✓ Solution by Mathematica
Time used: 4.998 (sec). Leaf size: 217

```
DSolve[y''[x]-x^2*y[x]-x^4+2==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to \text{ParabolicCylinderD}\left(-\frac{1}{2}, \sqrt{2}x\right)\left(\int_1^x -\frac{(K[1]^4-2)\,\text{ParabolicCylinderD}\left(-\frac{1}{2}, i\sqrt{2}K[1]\right)}{\sqrt{2}\left(\text{HermiteH}\left(-\frac{1}{2}, iK[1]\right)\text{HermiteH}\left(\frac{1}{2}, K[1]\right) + \text{HermiteH}\left(-\frac{1}{2}, K[1]\right)\left(-i\,\text{HermiteH}\left(\frac{1}{2}, iK[1]\right) - 2\right.\right.} + c_1\right)$$
$$+\text{ParabolicCylinderD}\left(-\frac{1}{2}, i\sqrt{2}x\right)\left(\int_1^x \frac{(K[2]^4-2)\,\text{Parabo}}{\sqrt{2}\left(\text{HermiteH}\left(-\frac{1}{2}, iK[2]\right)\text{HermiteH}\left(\frac{1}{2}, K[2]\right) + \text{HermiteH}\right.} + c_2\right)$$

2.39 problem 38

Internal problem ID [7175]

Book: Own collection of miscellaneous problems
Section: section 2.0
Problem number: 38.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _linear, _nonhomogeneous]]`

$$\boxed{y'' - 2x^2 y = x^4 - 1}$$

✓ Solution by Maple
Time used: 0.016 (sec). Leaf size: 40

```
dsolve(diff(y(x),x$2)-2*x^2*y(x)-x^4+1=0,y(x), singsol=all)
```

$$y(x) = \sqrt{x}\,\text{BesselI}\left(\frac{1}{4}, \frac{\sqrt{2}\,x^2}{2}\right) c_2 + \sqrt{x}\,\text{BesselK}\left(\frac{1}{4}, \frac{\sqrt{2}\,x^2}{2}\right) c_1 - \frac{x^2}{2}$$

✓ Solution by Mathematica
Time used: 3.94 (sec). Leaf size: 288

```
DSolve[y''[x]-2*x^2*y[x]-x^4+1==0,y[x],x,IncludeSingularSolutions -> True]
```

$y(x)$

$$\rightarrow \text{ParabolicCylinderD}\left(-\frac{1}{2}, 2^{3/4}x\right)\left(\left(\int_1^x \frac{}{i2^{3/4}\,\text{HermiteH}\left(-\frac{1}{2}, \sqrt[4]{2}K[1]\right)\text{HermiteH}\left(\frac{1}{2}, i\sqrt[4]{2}K[1]\right) + \text{Para}}\right.\right.$$

2.40 problem 39

Internal problem ID [7176]

Book: Own collection of miscellaneous problems
Section: section 2.0
Problem number: 39.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$\boxed{y'' - yx^3 = x^3}$$

✓ Solution by Maple

Time used: 0.016 (sec). Leaf size: 30

```
dsolve(diff(y(x),x$2)-x^3*y(x)-x^3=0,y(x), singsol=all)
```

$$y(x) = \sqrt{x}\,\text{BesselI}\left(\frac{1}{5}, \frac{2x^{\frac{5}{2}}}{5}\right)c_2 + \sqrt{x}\,\text{BesselK}\left(\frac{1}{5}, \frac{2x^{\frac{5}{2}}}{5}\right)c_1 - 1$$

✓ Solution by Mathematica

Time used: 0.275 (sec). Leaf size: 217

```
DSolve[y''[x]-x^3*y[x]-x^3==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to \frac{\sqrt[5]{-1}\,\text{Gamma}\left(\frac{4}{5}\right)\left(5^{4/5}x^5\,\text{Gamma}\left(\frac{6}{5}\right)\text{Hypergeometric0F1Regularized}\left(\frac{9}{5}, \frac{x^5}{25}\right)\text{BesselI}\left(\frac{1}{5}, \frac{2x^{5/2}}{5}\right) + 5\,5^{\cdots}\right.}{25\sqrt[5]{x^{5/2}}\text{Root}\left[25\cdots\right.} + \frac{c_1\sqrt{x}\,\text{Gamma}\left(\frac{4}{5}\right)\text{BesselI}\left(-\frac{1}{5}, \frac{2x^{5/2}}{5}\right)}{\sqrt[5]{5}} + \sqrt[5]{-\frac{1}{5}}c_2\sqrt{x}\,\text{Gamma}\left(\frac{6}{5}\right)\text{BesselI}\left(\frac{1}{5}, \frac{2x^{5/2}}{5}\right)$$

2.41 problem 40

Internal problem ID [7177]

Book: Own collection of miscellaneous problems
Section: section 2.0
Problem number: 40.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$\boxed{y'' - yx^3 = x^4}$$

✓ Solution by Maple
Time used: 0.0 (sec). Leaf size: 32

```
dsolve(diff(y(x),x$2)-x^3*y(x)-x^4=0,y(x), singsol=all)
```

$$y(x) = \sqrt{x}\,\mathrm{BesselI}\left(\frac{1}{5}, \frac{2x^{\frac{5}{2}}}{5}\right) c_2 + \sqrt{x}\,\mathrm{BesselK}\left(\frac{1}{5}, \frac{2x^{\frac{5}{2}}}{5}\right) c_1 - x$$

✓ Solution by Mathematica
Time used: 0.182 (sec). Leaf size: 219

```
DSolve[y''[x]-x^3*y[x]-x^4==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to \frac{\sqrt[5]{-1}\,\mathrm{Gamma}\left(\frac{6}{5}\right)\left(-5^{2/5}\sqrt[5]{x^{5/2}}x^{15/2}\,\mathrm{Gamma}\left(\frac{4}{5}\right)\mathrm{Hypergeometric0F1Regularized}\left(\frac{11}{5}, \frac{x^5}{25}\right)\mathrm{BesselI}\left(-\frac{1}{5}\right.\right.}{25x^{3/2}\mathrm{Root}\left[\right.}$$
$$+ \frac{c_1\sqrt{x}\,\mathrm{Gamma}\left(\frac{4}{5}\right)\mathrm{BesselI}\left(-\frac{1}{5}, \frac{2x^{5/2}}{5}\right)}{\sqrt[5]{5}}$$
$$+ \sqrt[5]{-\frac{1}{5}}c_2\sqrt{x}\,\mathrm{Gamma}\left(\frac{6}{5}\right)\mathrm{BesselI}\left(\frac{1}{5}, \frac{2x^{5/2}}{5}\right)$$

2.42 problem 41

Internal problem ID [7178]

Book: Own collection of miscellaneous problems
Section: section 2.0
Problem number: 41.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$\boxed{y'' - x^2 y' - x^2 y = x^2}$$

✓ Solution by Maple

Time used: 0.031 (sec). Leaf size: 55

```
dsolve(diff(y(x),x$2)-x^2*diff(y(x),x)-x^2*y(x)-x^2=0,y(x), singsol=all)
```

$$y(x) = \operatorname{HeunT}\left(3^{\frac{2}{3}}, 3, 2\,3^{\frac{1}{3}}, \frac{3^{\frac{2}{3}}x}{3}\right) \mathrm{e}^{-x} c_2 + \operatorname{HeunT}\left(3^{\frac{2}{3}}, -3, 2\,3^{\frac{1}{3}}, -\frac{3^{\frac{2}{3}}x}{3}\right) \mathrm{e}^{\frac{x(x^2+3)}{3}} c_1 - 1$$

✗ Solution by Mathematica

Time used: 0.0 (sec). Leaf size: 0

```
DSolve[y''[x]-x^2*y'[x]-x^2*y[x]-x^2==0,y[x],x,IncludeSingularSolutions -> True]
```

Not solved

2.43 problem 42

Internal problem ID [7179]

Book: Own collection of miscellaneous problems
Section: section 2.0
Problem number: 42.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$\boxed{y'' - y'x^3 - yx^3 = x^3}$$

✗ Solution by Maple

```
dsolve(diff(y(x),x$2)-x^3*diff(y(x),x)-x^3*y(x)-x^3=0,y(x), singsol=all)
```

No solution found

✗ Solution by Mathematica
Time used: 0.0 (sec). Leaf size: 0

```
DSolve[y''[x]-x^3*y'[x]-x^3*y[x]-x^3==0,y[x],x,IncludeSingularSolutions -> True]
```

Not solved

2.44 problem 43

Internal problem ID [7180]

Book: Own collection of miscellaneous problems
Section: section 2.0
Problem number: 43.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$\boxed{y'' - xy' - yx = x}$$

✓ Solution by Maple
Time used: 0.016 (sec). Leaf size: 54

```
dsolve(diff(y(x),x$2)-x*diff(y(x),x)-x*y(x)-x=0,y(x), singsol=all)
```

$$y(x) = \pi \,\mathrm{e}^{-2-x} c_1 (x+2) \operatorname{erf}\left(\frac{i\sqrt{2}\,(x+2)}{2}\right) - i\sqrt{\pi}\,\sqrt{2}\,\mathrm{e}^{\frac{x(x+2)}{2}} c_1 - 1 + \mathrm{e}^{-x}(x+2)\, c_2$$

✓ Solution by Mathematica
Time used: 0.809 (sec). Leaf size: 216

```
DSolve[y''[x]-x*y'[x]-x*y[x]-x==0,y[x],x,IncludeSingularSolutions -> True]
```

$$\begin{aligned} y(x) \to \frac{1}{2}e^{-\frac{1}{2}(x+2)^2}\Bigg(&2\sqrt{2}e^{\frac{x^2}{2}+x+2}(x+2)\int_1^x \Bigg(\frac{e^{K[1]}K[1]}{\sqrt{2}} \\ &- \frac{1}{2}e^{-\frac{1}{2}K[1]^2-K[1]-2}\sqrt{\pi}\operatorname{erfi}\left(\frac{\sqrt{(K[1]+2)^2}}{\sqrt{2}}\right)K[1]\sqrt{(K[1]+2)^2}\Bigg)dK[1] \\ &- \sqrt{2\pi}\sqrt{(x+2)^2}\left(c_2e^{\frac{x^2}{2}+x+2}+x+1\right)\operatorname{erfi}\left(\frac{\sqrt{(x+2)^2}}{\sqrt{2}}\right) \\ &+ 2e^{\frac{x^2}{2}+x+2}\left(e^x(x+1)+\sqrt{2}c_1(x+2)+c_2e^{\frac{1}{2}(x+2)^2}\right)\Bigg) \end{aligned}$$

2.45 problem 44

Internal problem ID [7181]

Book: Own collection of miscellaneous problems
Section: section 2.0
Problem number: 44.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$y'' - x^2 y' - yx = x^2$$

✓ Solution by Maple
Time used: 0.0 (sec). Leaf size: 44

```
dsolve(diff(y(x),x$2)-x^2*diff(y(x),x)-x*y(x)-x^2=0,y(x), singsol=all)
```

$$y(x) = \mathrm{e}^{\frac{x^3}{6}} \sqrt{x}\, \mathrm{BesselI}\left(\frac{1}{6}, \frac{x^3}{6}\right) c_2 + \mathrm{e}^{\frac{x^3}{6}} \sqrt{x}\, \mathrm{BesselK}\left(\frac{1}{6}, \frac{x^3}{6}\right) c_1 - \frac{x}{2}$$

✓ Solution by Mathematica
Time used: 0.344 (sec). Leaf size: 224

```
DSolve[y''[x]-x^2*y'[x]-x*y[x]-x^2==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to -\frac{e^{\frac{x^3}{6}} \left(12(x^3)^{5/6} \operatorname{Gamma}\left(\frac{1}{6}\right) \operatorname{Gamma}\left(\frac{7}{6}\right) \operatorname{BesselI}\left(\frac{1}{6}, \frac{x^3}{6}\right) {}_1F_1\left(-\frac{2}{3}; -\frac{1}{3}; -\frac{x^3}{3}\right) + \sqrt[3]{2}3^{2/3} \sqrt[6]{x^3} x^6 \operatorname{Gamma}\left(\right.\right.}{}$$

2.46 problem 45

Internal problem ID [7182]

Book: Own collection of miscellaneous problems
Section: section 2.0
Problem number: 45.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$y'' - x^2 y' - x^2 y = x^3 + x^2$$

✓ Solution by Maple
Time used: 0.047 (sec). Leaf size: 57

```
dsolve(diff(y(x),x$2)-x^2*diff(y(x),x)-x^2*y(x)-x^3-x^2=0,y(x), singsol=all)
```

$$y(x) = \operatorname{HeunT}\left(3^{\frac{2}{3}}, 3, 2\,3^{\frac{1}{3}}, \frac{3^{\frac{2}{3}}x}{3}\right)\mathrm{e}^{-x} c_2 + \operatorname{HeunT}\left(3^{\frac{2}{3}}, -3, 2\,3^{\frac{1}{3}}, -\frac{3^{\frac{2}{3}}x}{3}\right)\mathrm{e}^{\frac{x\left(x^2+3\right)}{3}} c_1 - x$$

✗ Solution by Mathematica
Time used: 0.0 (sec). Leaf size: 0

```
DSolve[y''[x]-x^2*y'[x]-x^2*y[x]-x^3-x^2==0,y[x],x,IncludeSingularSolutions -> True]
```

Not solved

2.47 problem 46

Internal problem ID [7183]

Book: Own collection of miscellaneous problems
Section: section 2.0
Problem number: 46.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$\boxed{y'' - x^2 y' - y x^3 = x^4 + x^2}$$

✓ Solution by Maple
Time used: 0.031 (sec). Leaf size: 74

```
dsolve(diff(y(x),x$2)-x^2*diff(y(x),x)-x^3*y(x)-x^4-x^2=0,y(x), singsol=all)
```

$$y(x) = \mathrm{e}^{-\frac{x(x-2)}{2}} \operatorname{HeunT}\left(2\,3^{\frac{2}{3}}, -3, -3\,3^{\frac{1}{3}}, \frac{3^{\frac{2}{3}}(x+1)}{3}\right) c_2 + \mathrm{e}^{\frac{1}{3}x^3+\frac{1}{2}x^2-x} \operatorname{HeunT}\left(2\,3^{\frac{2}{3}}, 3, -3\,3^{\frac{1}{3}}, -\frac{3^{\frac{2}{3}}(x+1)}{3}\right) c_1 - x$$

✗ Solution by Mathematica
Time used: 0.0 (sec). Leaf size: 0

```
DSolve[y''[x]-x^2*y'[x]-x^3*y[x]-x^4-x^2==0,y[x],x,IncludeSingularSolutions -> True]
```

Not solved

2.48 problem 47

Internal problem ID [7184]

Book: Own collection of miscellaneous problems
Section: section 2.0
Problem number: 47.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$
\boxed{y'' - \frac{y'}{x} - yx = x^2 + \frac{1}{x}}
$$

✓ Solution by Maple

Time used: 0.016 (sec). Leaf size: 26

```
dsolve(diff(y(x),x$2)-1/x*diff(y(x),x)-x*y(x)-x^2-1/x=0,y(x), singsol=all)
```

$$
y(x) = x\left(-1 + \operatorname{BesselI}\left(\frac{2}{3}, \frac{2x^{\frac{3}{2}}}{3}\right)c_2 + \operatorname{BesselK}\left(\frac{2}{3}, \frac{2x^{\frac{3}{2}}}{3}\right)c_1\right)
$$

✓ Solution by Mathematica

Time used: 0.487 (sec). Leaf size: 253

```
DSolve[y''[x]-1/x*y'[x]-x*y[x]-x^2-1/x==0,y[x],x,IncludeSingularSolutions -> True]
```

$$
y(x) \to \frac{\frac{3\sqrt[6]{3}\pi \operatorname{Gamma}\left(-\frac{1}{3}\right)\left(3\operatorname{AiryAiPrime}(x)+\sqrt{3}\operatorname{AiryBiPrime}(x)\right){}_1F_2\left(-\frac{1}{3};\frac{1}{3},\frac{2}{3};\frac{x^3}{9}\right)}{x\operatorname{Gamma}\left(\frac{2}{3}\right)} + \frac{\sqrt[3]{3}\pi x \operatorname{Gamma}\left(\frac{1}{3}\right)^2\left(\sqrt{3}\operatorname{AiryAiPrime}(x)-\operatorname{AiryBiPrime}(x)\right)}{\operatorname{Gamma}\left(\frac{4}{3}\right)}}{}
$$

2.49 problem 48

Internal problem ID [7185]

Book: Own collection of miscellaneous problems
Section: section 2.0
Problem number: 48.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$y'' - \frac{y'}{x} - x^2 y = x^3 + \frac{1}{x}$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 24

```
dsolve(diff(y(x),x$2)-1/x*diff(y(x),x)-x^2*y(x)-x^3-1/x=0,y(x), singsol=all)
```

$$y(x) = \sinh\left(\frac{x^2}{2}\right) c_2 + \cosh\left(\frac{x^2}{2}\right) c_1 - x$$

✓ Solution by Mathematica

Time used: 0.094 (sec). Leaf size: 34

```
DSolve[y''[x]-1/x*y'[x]-x^2*y[x]-x^3-1/x==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to c_1 \cosh\left(\frac{x^2}{2}\right) + ic_2 \sinh\left(\frac{x^2}{2}\right) - x$$

2.50 problem 49

Internal problem ID [7186]

Book: Own collection of miscellaneous problems
Section: section 2.0
Problem number: 49.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$y'' - \frac{y'}{x} - yx^3 = x^4 + \frac{1}{x}$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 26

```
dsolve(diff(y(x),x$2)-1/x*diff(y(x),x)-x^3*y(x)-x^4-1/x=0,y(x), singsol=all)
```

$$y(x) = x\left(-1 + \text{BesselI}\left(\frac{2}{5}, \frac{2x^{\frac{5}{2}}}{5}\right)c_2 + \text{BesselK}\left(\frac{2}{5}, \frac{2x^{\frac{5}{2}}}{5}\right)c_1\right)$$

✓ Solution by Mathematica

Time used: 0.364 (sec). Leaf size: 316

```
DSolve[y''[x]-1/x*y'[x]-x^3*y[x]-x^4-1/x==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to \frac{\frac{5(x^{5/2})^{13/5}\,\text{Gamma}\left(\frac{4}{5}\right)\text{Gamma}\left(\frac{7}{5}\right)\text{BesselI}\left(\frac{2}{5},\frac{2x^{5/2}}{5}\right){}_1F_2\left(\frac{4}{5};\frac{3}{5},\frac{9}{5};\frac{x^5}{25}\right)}{\text{Gamma}\left(\frac{9}{5}\right)} - \frac{\sqrt[5]{5}(x^{5/2})^{7/5}\,\text{Gamma}\left(\frac{1}{5}\right)\text{Gamma}\left(\frac{3}{5}\right)\text{BesselI}\left(-\frac{2}{5},\frac{2x^{5/2}}{5}\right){}_1F}{\text{Gamma}\left(\frac{6}{5}\right)}}{}$$

2.51 problem 50

Internal problem ID [7187]

Book: Own collection of miscellaneous problems
Section: section 2.0
Problem number: 50.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$y'' - y'x^3 - yx = x^3 + x^2$$

✗ Solution by Maple

```
dsolve(diff(y(x),x$2)-x^3*diff(y(x),x)-x*y(x)-x^3-x^2=0,y(x), singsol=all)
```

No solution found

✗ Solution by Mathematica
Time used: 0.0 (sec). Leaf size: 0

```
DSolve[y''[x]-x^3*y'[x]-x*y[x]-x^3-x^2==0,y[x],x,IncludeSingularSolutions -> True]
```

Not solved

2.52 problem 51

Internal problem ID [7188]

Book: Own collection of miscellaneous problems
Section: section 2.0
Problem number: 51.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$y'' - y'x^3 - x^2y = x^3$$

✓ Solution by Maple

Time used: 0.047 (sec). Leaf size: 28

```
dsolve(diff(y(x),x$2)-x^3*diff(y(x),x)-x^2*y(x)-x^3=0,y(x), singsol=all)
```

$$y(x) = x\left(\text{KummerU}\left(\frac{1}{2}, \frac{5}{4}, \frac{x^4}{4}\right)c_1 + \text{KummerM}\left(\frac{1}{2}, \frac{5}{4}, \frac{x^4}{4}\right)c_2 - \frac{1}{2}\right)$$

✓ Solution by Mathematica

Time used: 1.216 (sec). Leaf size: 337

```
DSolve[y''[x]-x^3*y'[x]-x^2*y[x]-x^3==0,y[x],x,IncludeSingularSolutions -> True]
```

$y(x)$

$$\to \text{Hypergeometric1F1}\left(\frac{1}{4}, \frac{3}{4}, \frac{x^4}{4}\right)\int_1^x \frac{}{5\,\text{Hypergeometric1F1}\left(\frac{1}{2}, \frac{5}{4}, \frac{K[1]^4}{4}\right)\text{Hypergeometric1F1}\left(\frac{5}{4}, \frac{7}{4}, \frac{K[1]^4}{4}\right.}$$

$$+ \frac{\sqrt[4]{-1}x\,\text{Hypergeometric1F1}\left(\frac{1}{2}, \frac{5}{4}, \frac{x^4}{4}\right)\int_1^x \frac{(15-15i)\,\text{H}}{3\,\text{Hypergeometric1F1}\left(\frac{1}{4}, \frac{3}{4}, \frac{K[2]^4}{4}\right)\left(2\,\text{Hypergeometric1F1}\left(\frac{3}{2}, \frac{9}{4}, \frac{K[2]^4}{4}\right)K[2]^4 + 5\,\text{Hy}\right.}}{\sqrt{2}}$$

$$+ c_1\,\text{Hypergeometric1F1}\left(\frac{1}{4}, \frac{3}{4}, \frac{x^4}{4}\right) + \left(\frac{1}{2} + \frac{i}{2}\right)c_2x\,\text{Hypergeometric1F1}\left(\frac{1}{2}, \frac{5}{4}, \frac{x^4}{4}\right)$$

2.53 problem 52

Internal problem ID [7189]

Book: Own collection of miscellaneous problems
Section: section 2.0
Problem number: 52.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$\boxed{y'' - y'x^3 - yx^3 = x^4 + x^3}$$

✗ Solution by Maple

```
dsolve(diff(y(x),x$2)-x^3*diff(y(x),x)-x^3*y(x)-x^4-x^3=0,y(x), singsol=all)
```

No solution found

✗ Solution by Mathematica

Time used: 0.0 (sec). Leaf size: 0

```
DSolve[y''[x]-x^3*y'[x]-x^3*y[x]-x^4-x^3==0,y[x],x,IncludeSingularSolutions -> True]
```

Not solved

2.54 problem 50

Internal problem ID [7190]

Book: Own collection of miscellaneous problems
Section: section 2.0
Problem number: 50.
ODE order: 3.
ODE degree: 1.

CAS Maple gives this as type `[[_3rd_order, _with_linear_symmetries]]`

$$y''' - y'x^3 - x^2y = x^3$$

✓ Solution by Maple
Time used: 0.0 (sec). Leaf size: 51

```
dsolve(diff(y(x),x$3)-x^3*diff(y(x),x)-x^2*y(x)-x^3=0,y(x), singsol=all)
```

$$y(x) = -\frac{x}{2} + c_1 \,\mathrm{hypergeom}\left(\left[\frac{1}{5}\right], \left[\frac{3}{5}, \frac{4}{5}\right], \frac{x^5}{25}\right) + c_2 x \,\mathrm{hypergeom}\left(\left[\frac{2}{5}\right], \left[\frac{4}{5}, \frac{6}{5}\right], \frac{x^5}{25}\right) + c_3 x^2 \,\mathrm{hypergeom}\left(\left[\frac{3}{5}\right], \left[\frac{6}{5}, \frac{7}{5}\right], \frac{x^5}{25}\right)$$

✓ Solution by Mathematica
Time used: 12.206 (sec). Leaf size: 2548

```
DSolve[y'''[x]-x^3*y'[x]-x^2*y[x]-x^3==0,y[x],x,IncludeSingularSolutions -> True]
```

Too large to display

3 section 3.0

3.1 problem 1

Internal problem ID [7191]

Book: Own collection of miscellaneous problems
Section: section 3.0
Problem number: 1.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _missing_x]]`

$$y'' + y'c + ky = 0$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 41

```
dsolve(diff(y(x),x$2)+c*diff(y(x),x)+k*y(x)=0,y(x), singsol=all)
```

$$y(x) = c_1 e^{\frac{\left(-c+\sqrt{c^2-4k}\right)x}{2}} + c_2 e^{-\frac{\left(c+\sqrt{c^2-4k}\right)x}{2}}$$

✓ Solution by Mathematica

Time used: 8.987 (sec). Leaf size: 2548

```
DSolve[y'''[x]-x^3*y'[x]-x^2*y[x]-x^3==0,y[x],x,IncludeSingularSolutions -> True]
```

Too large to display

3.2 problem 2

Internal problem ID [7192]

Book: Own collection of miscellaneous problems
Section: section 3.0
Problem number: 2.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_quadrature]`

$$\boxed{w' + \frac{\sqrt{1-12w}}{2} = -\frac{1}{2}}$$

With initial conditions

$$[w(1) = -1]$$

✓ Solution by Maple
Time used: 0.64 (sec). Leaf size: 66

```
dsolve([diff(w(z),z) = -1/2 - sqrt(1/4 - 3*w(z)),w(1) = -1],w(z), singsol=all)
```

$$w(z) = \text{RootOf}\left(-i\pi + 2\sqrt{13} - 2\sqrt{1-12\_Z} + \ln(\_Z) - \ln\left(-1+\sqrt{1-12\_Z}\right) + \ln\left(1+\sqrt{1-12\_Z}\right) - \ln\left(1+\sqrt{13}\right) + \ln\left(-1+\sqrt{13}\right) + 6z - 6\right)$$

✓ Solution by Mathematica
Time used: 14.307 (sec). Leaf size: 105

```
DSolve[{w'[z] == -1/2 - Sqrt[1/4 - 3*w[z]],{w[1] == -1}},w[z],z,IncludeSingularSolutions ->
```

$$w(z) \to -\frac{1}{12}W\left(\left(\sqrt{13}-1\right)e^{-3z+\sqrt{13}+2}\right)\left(W\left(\left(\sqrt{13}-1\right)e^{-3z+\sqrt{13}+2}\right)+2\right)$$
$$w(z) \to -\frac{1}{12}W\left(\left(\sqrt{13}-1\right)e^{-3z+\sqrt{13}+2}\right)\left(W\left(\left(\sqrt{13}-1\right)e^{-3z+\sqrt{13}+2}\right)+2\right)$$

3.3 problem 3

Internal problem ID [7193]

Book: Own collection of miscellaneous problems
Section: section 3.0
Problem number: 3.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _linear, _nonhomogeneous]]`

$$\boxed{y'' + y = \sin(x)}$$

With initial conditions

$$[y(0) = 1]$$

✓ Solution by Maple

Time used: 0.016 (sec). Leaf size: 20

```
dsolve([diff(y(x),x$2)+y(x)=sin(x),y(0) = 1],y(x), singsol=all)
```

$$y(x) = \frac{\sin(x)(2c_2 + 1)}{2} - \frac{\cos(x)(x - 2)}{2}$$

✓ Solution by Mathematica

Time used: 0.025 (sec). Leaf size: 20

```
DSolve[{y''[x]+y[x]==Sin[x],{y[0] == 1}},y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to -\frac{1}{2}x\cos(x) + \cos(x) + c_2\sin(x)$$

3.4 problem 4

Internal problem ID [7194]

Book: Own collection of miscellaneous problems
Section: section 3.0
Problem number: 4.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _linear, _nonhomogeneous]]`

$$\boxed{y'' + y = \sin(x)}$$

With initial conditions

$$[y'(0) = 1]$$

✓ Solution by Maple

Time used: 0.016 (sec). Leaf size: 18

```
dsolve([diff(y(x),x$2)+y(x)=sin(x),D(y)(0) = 1],y(x), singsol=all)
```

$$y(x) = \frac{(-x + 2c_1)\cos(x)}{2} + \frac{3\sin(x)}{2}$$

✓ Solution by Mathematica

Time used: 0.023 (sec). Leaf size: 23

```
DSolve[{y''[x]+y[x]==Sin[x],{y'[0] == 1}},y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to \frac{3\sin(x)}{2} + \left(-\frac{x}{2} + c_1\right)\cos(x)$$

3.5 problem 5

Internal problem ID [7195]

Book: Own collection of miscellaneous problems
Section: section 3.0
Problem number: 5.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _linear, _nonhomogeneous]]`

$$\boxed{y'' + y = \sin(x)}$$

With initial conditions

$$[y'(0) = 1, y(0) = 0]$$

✓ Solution by Maple
Time used: 0.015 (sec). Leaf size: 14

```
dsolve([diff(y(x),x$2)+y(x)=sin(x),D(y)(0) = 1, y(0) = 0],y(x), singsol=all)
```

$$y(x) = \frac{3\sin(x)}{2} - \frac{\cos(x)\, x}{2}$$

✓ Solution by Mathematica
Time used: 0.024 (sec). Leaf size: 19

```
DSolve[{y''[x]+y[x]==Sin[x],{y'[0] == 1,y[0]==0}},y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to \frac{1}{2}(3\sin(x) - x\cos(x))$$

3.6 problem 6

Internal problem ID [7196]

Book: Own collection of miscellaneous problems
Section: section 3.0
Problem number: 6.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _linear, _nonhomogeneous]]`

$$\boxed{y'' + y = \sin(x)}$$

With initial conditions

$$[y(1) = 0]$$

✓ Solution by Maple
Time used: 0.063 (sec). Leaf size: 32

```
dsolve([diff(y(x),x$2)+y(x)=sin(x),y(1) = 0],y(x), singsol=all)
```

$$y(x) = \frac{((-2c_2 - 1)\tan(1) - x + 1)\cos(x)}{2} + \frac{\sin(x)(2c_2 + 1)}{2}$$

✓ Solution by Mathematica
Time used: 0.027 (sec). Leaf size: 18

```
DSolve[{y''[x]+y[x]==Sin[x],{y[0] == 0}},y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to -\frac{1}{2}x\cos(x) + c_2\sin(x)$$

3.7 problem 7

Internal problem ID [7197]

Book: Own collection of miscellaneous problems
Section: section 3.0
Problem number: 7.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _linear, _nonhomogeneous]]`

$$\boxed{y'' + y = \sin(x)}$$

With initial conditions

$$[y'(1) = 0]$$

✓ Solution by Maple

Time used: 0.046 (sec). Leaf size: 27

```
dsolve([diff(y(x),x$2)+y(x)=sin(x),D(y)(1) = 0],y(x), singsol=all)
```

$$y(x) = \frac{(2\cot(1)\, c_2 - x + 1)\cos(x)}{2} + \frac{\sin(x)\,(2c_2 + 1)}{2}$$

✓ Solution by Mathematica

Time used: 0.022 (sec). Leaf size: 35

```
DSolve[{y''[x]+y[x]==Sin[x],{y'[1] == 0}},y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to \frac{1}{2}((1 - \tan(1) + 2c_1\tan(1))\sin(x) - (x - 2c_1)\cos(x))$$

3.8 problem 8

Internal problem ID [7198]

Book: Own collection of miscellaneous problems
Section: section 3.0
Problem number: 8.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _linear, _nonhomogeneous]]`

$$\boxed{y'' + y = \sin(x)}$$

With initial conditions

$$[y'(1) = 0, y(0) = 0]$$

✓ Solution by Maple
Time used: 0.032 (sec). Leaf size: 20

```
dsolve([diff(y(x),x$2)+y(x)=sin(x),D(y)(1) = 0, y(0) = 0],y(x), singsol=all)
```

$$y(x) = \frac{(-\tan(1) + 1)\sin(x)}{2} - \frac{\cos(x)\, x}{2}$$

✓ Solution by Mathematica
Time used: 0.029 (sec). Leaf size: 23

```
DSolve[{y''[x]+y[x]==Sin[x],{y'[1] == 0,y[0]==0}},y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to \frac{1}{2}(\sin(x) - x\cos(x) - \tan(1)\sin(x))$$

3.9 problem 9

Internal problem ID [7199]

Book: Own collection of miscellaneous problems
Section: section 3.0
Problem number: 9.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _linear, _nonhomogeneous]]`

$$\boxed{y'' + y = \sin(x)}$$

With initial conditions

$$[y'(1) = 0, y(2) = 0]$$

✓ Solution by Maple
Time used: 0.11 (sec). Leaf size: 33

```
dsolve([diff(y(x),x$2)+y(x)=sin(x),D(y)(1) = 0, y(2) = 0],y(x), singsol=all)
```

$$y(x) = \frac{(-x + \cos(2) - \sin(2) + 1)\cos(x)}{2} + \frac{\sin(x)(\sin(2) - \tan(1) + \cos(2))}{2}$$

✓ Solution by Mathematica
Time used: 0.026 (sec). Leaf size: 39

```
DSolve[{y''[x]+y[x]==Sin[x],{y'[1] == 0,y[2]==0}},y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to \frac{1}{4}(\sec(1)\sin(x)(-\sin(1) + \sin(3) + \cos(1) + \cos(3)) - 2\cos(x)(x - 1 + \sin(2) - \cos(2)))$$

3.10 problem 10

Internal problem ID [7200]

Book: Own collection of miscellaneous problems
Section: section 3.0
Problem number: 10.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _linear, _nonhomogeneous]]`

$$\boxed{y'' + y = \sin(x)}$$

With initial conditions

$$[y'(1) = 0, y(0) = 0]$$

✓ Solution by Maple
Time used: 0.016 (sec). Leaf size: 20

```
dsolve([diff(y(x),x$2)+y(x)=sin(x),D(y)(1) = 0, y(0) = 0],y(x), singsol=all)
```

$$y(x) = \frac{(-\tan(1) + 1)\sin(x)}{2} - \frac{\cos(x)\,x}{2}$$

✓ Solution by Mathematica
Time used: 0.031 (sec). Leaf size: 23

```
DSolve[{y''[x]+y[x]==Sin[x],{y'[1] == 0,y[0]==0}},y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to \frac{1}{2}(\sin(x) - x\cos(x) - \tan(1)\sin(x))$$

3.11 problem 11

Internal problem ID [7201]

Book: Own collection of miscellaneous problems
Section: section 3.0
Problem number: 11.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _linear, _nonhomogeneous]]`

$$\boxed{y'' + y' + y = \sin(x)}$$

With initial conditions

$$[y'(1) = 0, y(2) = 0]$$

✓ Solution by Maple

Time used: 0.469 (sec). Leaf size: 144

```
dsolve([diff(y(x),x$2)+diff(y(x),x)+y(x)=sin(x),D(y)(1) = 0, y(2) = 0],y(x), singsol=all)
```

$$y(x) = \frac{2\sin(1)\left(\cos\left(\frac{\sqrt{3}\,x}{2}\right)\sin\left(\sqrt{3}\right) - \sin\left(\frac{\sqrt{3}\,x}{2}\right)\cos\left(\sqrt{3}\right)\right)e^{\frac{1}{2}-\frac{x}{2}} - \cos(2)\left(\left(-\sqrt{3}\cos\left(\frac{\sqrt{3}}{2}\right) + \sin\left(\frac{\sqrt{3}}{2}\right)\right)\cos}{\sqrt{3}\cos\left(\frac{\sqrt{3}}{2}\right) + \mathrm{si}}$$

✓ Solution by Mathematica

Time used: 1.065 (sec). Leaf size: 12765

```
DSolve[{y'''[x]+y'[x]+y[x]==Sin[x],{y'[1] == 0,y[2]==0}},y[x],x,IncludeSingularSolutions ->
```

Too large to display

3.12 problem 12

Internal problem ID [7202]

Book: Own collection of miscellaneous problems
Section: section 3.0
Problem number: 12.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _linear, _nonhomogeneous]]`

$$\boxed{y'' + y' + y = \sin(x)}$$

With initial conditions

$$[y'(1) = 0]$$

✓ Solution by Maple

Time used: 0.297 (sec). Leaf size: 80

```
dsolve([diff(y(x),x$2)+diff(y(x),x)+y(x)=sin(x),D(y)(1) = 0],y(x), singsol=all)
```

$$y(x) = \frac{2\sin(1)\,e^{\frac{1}{2}-\frac{x}{2}}\cos\left(\frac{\sqrt{3}\,x}{2}\right) + c_2 e^{-\frac{x}{2}}\left(\sqrt{3}\cos\left(\frac{\sqrt{3}}{2}\right) - \sin\left(\frac{\sqrt{3}}{2}\right)\right)\cos\left(\frac{\sqrt{3}\,x}{2}\right) + \left(\sqrt{3}\sin\left(\frac{\sqrt{3}}{2}\right) + \cos\left(\frac{\sqrt{3}}{2}\right)\right)(}{\sqrt{3}\sin\left(\frac{\sqrt{3}}{2}\right) + \cos\left(\frac{\sqrt{3}}{2}\right)}$$

✓ Solution by Mathematica

Time used: 0.346 (sec). Leaf size: 4176

```
DSolve[{y''[x]+y'[x]+y[x]==Sin[x],{y'[1] == 0}},y[x],x,IncludeSingularSolutions -> True]
```

Too large to display

3.13 problem 13

Internal problem ID [7203]

Book: Own collection of miscellaneous problems
Section: section 3.0
Problem number: 13.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _linear, _nonhomogeneous]]`

$$\boxed{y'' + y' + y = \sin(x)}$$

With initial conditions

$$[y'(1) = 0, y(2) = 0]$$

✓ Solution by Maple

Time used: 0.062 (sec). Leaf size: 144

```
dsolve([diff(y(x),x$2)+diff(y(x),x)+y(x)=sin(x),D(y)(1) = 0, y(2) = 0],y(x), singsol=all)
```

$$y(x) = \frac{2\sin(1)\left(\cos\left(\frac{\sqrt{3}\,x}{2}\right)\sin\left(\sqrt{3}\right) - \sin\left(\frac{\sqrt{3}\,x}{2}\right)\cos\left(\sqrt{3}\right)\right)\mathrm{e}^{\frac{1}{2}-\frac{x}{2}} - \cos(2)\left(\left(-\sqrt{3}\cos\left(\frac{\sqrt{3}}{2}\right) + \sin\left(\frac{\sqrt{3}}{2}\right)\right)\cos\right.}{\sqrt{3}\cos\left(\frac{\sqrt{3}}{2}\right) + \mathrm{si}}$$

✓ Solution by Mathematica

Time used: 0.786 (sec). Leaf size: 12765

```
DSolve[{y'''[x]+y'[x]+y[x]==Sin[x],{y'[1] == 0,y[2]==0}},y[x],x,IncludeSingularSolutions ->
```

Too large to display

3.14 problem 14

Internal problem ID [7204]

Book: Own collection of miscellaneous problems
Section: section 3.0
Problem number: 14.
ODE order: 3.
ODE degree: 1.

CAS Maple gives this as type `[[_3rd_order, _with_linear_symmetries]]`

$$\boxed{y''' + y' + y = x}$$

With initial conditions

$$[y'(0) = 0, y(0) = 0, y''(0) = 1]$$

✓ Solution by Maple
Time used: 0.609 (sec). Leaf size: 359

```
dsolve([diff(y(x),x$3)+diff(y(x),x)+y(x)=x,D(y)(0) = 0, y(0) = 0, (D@@2)(y)(0) = 1],y(x), si
```

$$y(x) = \frac{\frac{10\,\mathrm{e}^{-\frac{x\left(108+12\sqrt{93}\right)^{\frac{1}{3}}\left(-12+\left(\sqrt{93}-9\right)\left(108+12\sqrt{93}\right)^{\frac{1}{3}}\right)}{144}}\left(\left(108+12\sqrt{3}\,\sqrt{31}\right)^{\frac{1}{3}}\sqrt{3}\,\sqrt{31}+\frac{3\sqrt{3}\left(108+12\sqrt{3}\,\sqrt{31}\right)^{\frac{2}{3}}\sqrt{31}}{5}-\frac{6\sqrt{3}\,\sqrt{31}}{5}-\frac{39\left(108+12\sqrt{3}\,\sqrt{31}\right)^{\frac{1}{3}}}{5}\right.}{3}}{}$$

✓ Solution by Mathematica
Time used: 0.024 (sec). Leaf size: 1546

```
DSolve[{y'''[x]+y'[x]+y[x]==x,{y'[1] == 0,y[0]==0,y''[0]==1}},y[x],x,IncludeSingularSolution
```

Too large to display

3.15 problem 15

Internal problem ID [7205]

Book: Own collection of miscellaneous problems
Section: section 3.0
Problem number: 15.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$\boxed{x^4y'' + y'x^3 - 4x^2y = 1}$$

✓ Solution by Maple
Time used: 0.0 (sec). Leaf size: 24

```
dsolve(x^4*diff(y(x),x$2)+x^3*diff(y(x),x)-4*x^2*y(x)=1,y(x), singsol=all)
```

$$y(x) = \frac{16c_2x^4 - 4\ln(x) + 16c_1 - 1}{16x^2}$$

✓ Solution by Mathematica
Time used: 0.019 (sec). Leaf size: 29

```
DSolve[x^4*y''[x]+x^3*y'[x]-4*x^2*y[x]==1,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to \frac{16c_2x^4 - 4\log(x) - 1 + 16c_1}{16x^2}$$

3.16 problem 16

Internal problem ID [7206]

Book: Own collection of miscellaneous problems
Section: section 3.0
Problem number: 16.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$\boxed{x^4y'' + y'x^3 - 4x^2y = x}$$

✓ Solution by Maple
Time used: 0.0 (sec). Leaf size: 22

```
dsolve(x^4*diff(y(x),x$2)+x^3*diff(y(x),x)-4*x^2*y(x)=x,y(x), singsol=all)
```

$$y(x) = \frac{3c_2x^4 + 3c_1 - x}{3x^2}$$

✓ Solution by Mathematica
Time used: 0.015 (sec). Leaf size: 25

```
DSolve[x^4*y''[x]+x^3*y'[x]-4*x^2*y[x]==x,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to c_2x^2 + \frac{c_1}{x^2} - \frac{1}{3x}$$

3.17 problem 17

Internal problem ID [7207]

Book: Own collection of miscellaneous problems
Section: section 3.0
Problem number: 17.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$\boxed{x^2y'' + xy' - 4y = x}$$

✓ Solution by Maple

Time used: 0.016 (sec). Leaf size: 18

```
dsolve(x^2*diff(diff(y(x),x),x)+x*diff(y(x),x)-4*y(x) = x,y(x), singsol=all)
```

$$y(x) = c_2x^2 + \frac{c_1}{x^2} - \frac{x}{3}$$

✓ Solution by Mathematica

Time used: 0.022 (sec). Leaf size: 23

```
DSolve[x^2*y''[x]+x*y'[x]-4*y[x] == x,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to c_2x^2 + \frac{c_1}{x^2} - \frac{x}{3}$$

3.18 problem 18

Internal problem ID [7208]

Book: Own collection of miscellaneous problems
Section: section 3.0
Problem number: 18.
ODE order: 3.
ODE degree: 1.

CAS Maple gives this as type `[[_3rd_order, _with_linear_symmetries]]`

$$\boxed{x^4y''' + x^3y'' + x^2y' + yx = 0}$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 184

```
dsolve(x^4*diff(y(x),x$3)+x^3*diff(y(x),x$2)+x^2*diff(y(x),x)+x*y(x)= 0,y(x), singsol=all)
```

$$
\begin{aligned}
y(x) = {} & c_1 x^{-\frac{\left(188+12\sqrt{249}\right)^{\frac{2}{3}}-4\left(188+12\sqrt{249}\right)^{\frac{1}{3}}-8}{6\left(188+12\sqrt{249}\right)^{\frac{1}{3}}}} \\
& + c_2 x^{\frac{-8+\left(188+12\sqrt{249}\right)^{\frac{2}{3}}+8\left(188+12\sqrt{249}\right)^{\frac{1}{3}}}{12\left(188+12\sqrt{249}\right)^{\frac{1}{3}}}} \sin\left(\frac{\sqrt{3}\left(\left(188+12\sqrt{3}\,\sqrt{83}\right)^{\frac{2}{3}}+8\right)\ln\left(x\right)}{12\left(188+12\sqrt{3}\,\sqrt{83}\right)^{\frac{1}{3}}}\right) \\
& + c_3 x^{\frac{-8+\left(188+12\sqrt{249}\right)^{\frac{2}{3}}+8\left(188+12\sqrt{249}\right)^{\frac{1}{3}}}{12\left(188+12\sqrt{249}\right)^{\frac{1}{3}}}} \cos\left(\frac{\sqrt{3}\left(\left(188+12\sqrt{3}\,\sqrt{83}\right)^{\frac{2}{3}}+8\right)\ln\left(x\right)}{12\left(188+12\sqrt{3}\,\sqrt{83}\right)^{\frac{1}{3}}}\right)
\end{aligned}
$$

✓ Solution by Mathematica

Time used: 0.004 (sec). Leaf size: 81

```
DSolve[x^4*y'''[x]+x^3*y''[x]+x^2*y'[x]+x*y[x]== 0,y[x],x,IncludeSingularSolutions -> True]
```

$$
\begin{aligned}
y(x) \to {} & c_1 x^{\text{Root}[\#1^3-2\#1^2+2\#1+1\&,1]} + c_3 x^{\text{Root}[\#1^3-2\#1^2+2\#1+1\&,3]} \\
& + c_2 x^{\text{Root}[\#1^3-2\#1^2+2\#1+1\&,2]}
\end{aligned}
$$

3.19 problem 19

Internal problem ID [7209]

Book: Own collection of miscellaneous problems
Section: section 3.0
Problem number: 19.
ODE order: 3.
ODE degree: 1.

CAS Maple gives this as type `[[_3rd_order, _with_linear_symmetries]]`

$$\boxed{x^4 y''' + x^3 y'' + x^2 y' + yx = x}$$

✓ Solution by Maple
Time used: 0.015 (sec). Leaf size: 223

```
dsolve(x^4*diff(y(x),x$3)+x^3*diff(y(x),x$2)+x^2*diff(y(x),x)+x*y(x)= x,y(x), singsol=all)
```

$$y(x) = c_2 x^{\frac{(47-3\sqrt{249})(188+12\sqrt{249})^{\frac{2}{3}}}{192}+\frac{(188+12\sqrt{249})^{\frac{1}{3}}}{12}+\frac{2}{3}} \cos\left(\frac{(188+12\sqrt{3}\sqrt{83})^{\frac{1}{3}}\sqrt{3}\left(3(188+12\sqrt{3}\sqrt{83})^{\frac{1}{3}}\sqrt{3}\sqrt{83}-\right.}{192}\right.$$
$$+ c_3 x^{\frac{(47-3\sqrt{249})(188+12\sqrt{249})^{\frac{2}{3}}}{192}+\frac{(188+12\sqrt{249})^{\frac{1}{3}}}{12}+\frac{2}{3}} \sin\left(\frac{(188+12\sqrt{3}\sqrt{83})^{\frac{1}{3}}\sqrt{3}\left(3(188+12\sqrt{3}\sqrt{83})^{\frac{1}{3}}\sqrt{3}\sqrt{83}\right.}{192}\right.$$
$$+ x^{\frac{(188+12\sqrt{249})^{\frac{2}{3}}(-47+3\sqrt{249})}{96}-\frac{(188+12\sqrt{249})^{\frac{1}{3}}}{6}+\frac{2}{3}} c_1 + 1$$

✓ Solution by Mathematica
Time used: 0.004 (sec). Leaf size: 82

```
DSolve[x^4*y'''[x]+x^3*y''[x]+x^2*y'[x]+x*y[x]== x,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to c_1 x^{\text{Root}[\#1^3-2\#1^2+2\#1+1\&,1]} + c_3 x^{\text{Root}[\#1^3-2\#1^2+2\#1+1\&,3]} + c_2 x^{\text{Root}[\#1^3-2\#1^2+2\#1+1\&,2]} + 1$$

3.20 problem 20

Internal problem ID [7210]

Book: Own collection of miscellaneous problems
Section: section 3.0
Problem number: 20.
ODE order: 4.
ODE degree: 1.

CAS Maple gives this as type `[[_high_order, _with_linear_symmetries]]`

$$5x^5y'''' + 4x^4y''' + x^2y' + xy = 0$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 38

```
dsolve(5*x^5*diff(y(x),x$4)+4*x^4*diff(y(x),x$3)+x^2*diff(y(x),x)+x*y(x)= 0,y(x), singsol=al
```

$$y(x) = \sum_{\_a=1}^{4} x^{\text{RootOf}(5\_Z^4-26\_Z^3+43\_Z^2-21\_Z+1,\text{index}=\_a)}\_C\_a$$

✓ Solution by Mathematica

Time used: 1.114 (sec). Leaf size: 1931

```
DSolve[5*x^5*y''''[x]+4*x^4*y'''[x]+x^2*y'[x]+x*y[x]== Sin[x],y[x],x,IncludeSingularSolution
```

Too large to display

3.21 problem 21

Internal problem ID [7211]

Book: Own collection of miscellaneous problems
Section: section 3.0
Problem number: 21.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _missing_y], [_2nd_order, _reducible, _mu_y_y1]]`

$$\boxed{(x^2+1)\, y'' + y'^2 = -1}$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 33

```
dsolve((1+x^2)*diff(y(x),x$2)+1+diff(y(x),x)^2=0,y(x), singsol=all)
```

$$y(x) = \frac{\ln(c_1x - 1)\, c_1^2 + c_2c_1^2 + c_1x + \ln(c_1x - 1)}{c_1^2}$$

✓ Solution by Mathematica

Time used: 8.017 (sec). Leaf size: 33

```
DSolve[(1+x^2)*y''[x]+1+(y'[x])^2==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to -x\cot(c_1) + \csc^2(c_1)\log(-x\sin(c_1) - \cos(c_1)) + c_2$$

3.22 problem 22

Internal problem ID [7212]

Book: Own collection of miscellaneous problems
Section: section 3.0
Problem number: 22.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _missing_y]]`

$$\boxed{\left(x^{2}+1\right) y''+{y'}^{2}=x-1}$$

✓ Solution by Maple

Time used: 0.016 (sec). Leaf size: 460

```
dsolve((1+x^2)*diff(y(x),x$2)+1+diff(y(x),x)^2=x,y(x), singsol=all)
```

$$y(x) = -\left(\int \frac{-\left(\frac{1}{2}-\frac{ix}{2}\right)^{\frac{i\sqrt{-2+2\sqrt{2}}}{2}} (x+i)\left(\frac{1}{2}+\frac{ix}{2}\right)^{i\sqrt{-1+i}} \sqrt{-1+i}\ \mathrm{hypergeom}\left(\left[\frac{i\sqrt{-2+2\sqrt{2}}}{2}, \frac{i\sqrt{-1+i}}{2}+\frac{\sqrt{1+i}}{2}+1\right], \right.}{\left(4\left(\frac{1}{2}-\frac{ix}{2}\right)^{\frac{\sqrt{2+2\sqrt{2}}}{2}} c_1 \right.} \right. + c_2$$

✗ Solution by Mathematica

Time used: 0.0 (sec). Leaf size: 0

```
DSolve[(1+x^2)*y''[x]+1+(y'[x])^2==x,y[x],x,IncludeSingularSolutions -> True]
```

Not solved

3.23 problem 23

Internal problem ID [7213]

Book: Own collection of miscellaneous problems
Section: section 3.0
Problem number: 23.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _missing_y], [_2nd_order, _reducible, _mu_y_y1]]`

$$(x^2+1)\, y'' + x{y'}^2 = 0$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 22

```
dsolve((1+x^2)*diff(y(x),x$2)+1+x*diff(y(x),x)^2=1,y(x), singsol=all)
```

$$y(x) = 2\left(\int \frac{1}{\ln(x^2+1)+2c_1}dx\right)+c_2$$

✓ Solution by Mathematica

Time used: 60.288 (sec). Leaf size: 33

```
DSolve[(1+x^2)*y''[x]+1+x*(y'[x])^2==1,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to \int_1^x -\frac{2}{2c_1 - \log(K[1]^2+1)}dK[1]+c_2$$

3.24 problem 24

Internal problem ID [7214]

Book: Own collection of miscellaneous problems
Section: section 3.0
Problem number: 24.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[NONE]`

$$(x^2+1)\,y''+yy'^2=0$$

✗ Solution by Maple

```
dsolve((1+x^2)*diff(y(x),x$2)+y(x)*diff(y(x),x)^2=0,y(x), singsol=all)
```

No solution found

✗ Solution by Mathematica
Time used: 0.0 (sec). Leaf size: 0

```
DSolve[(1+x^2)*y''[x]+y[x]*(y'[x])^2==0,y[x],x,IncludeSingularSolutions -> True]
```

Not solved

3.25 problem 25

Internal problem ID [7215]

Book: Own collection of miscellaneous problems
Section: section 3.0
Problem number: 25.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _missing_y], [_2nd_order, _reducible, _mu_y_y1]]`

$$\boxed{(x^2+1)\, y''+{y'}^2=0}$$

✓ Solution by Maple

Time used: 0.015 (sec). Leaf size: 14

```
dsolve((1+x^2)*diff(y(x),x$2)+diff(y(x),x)^2=0,y(x), singsol=all)
```

$$y(x)=\int \frac{1}{\arctan\left(x\right)+c_1}dx+c_2$$

✓ Solution by Mathematica

Time used: 60.278 (sec). Leaf size: 25

```
DSolve[(1+x^2)*y''[x]+(y'[x])^2==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x)\to \int_1^x \frac{1}{\arctan(K[1])-c_1}dK[1]+c_2$$

3.26 problem 26

Internal problem ID [7216]

Book: Own collection of miscellaneous problems
Section: section 3.0
Problem number: 26.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _missing_x], _Liouville, [_2nd_order, _reducible`

$$\boxed{y'' + \sin(y)\, y'^2 = 0}$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 21

```
dsolve(diff(y(x),x$2)+sin(y(x))*diff(y(x),x)^2=0,y(x), singsol=all)
```

$$\int^{y(x)} \mathrm{e}^{-\cos(\_a)} d\_a - c_1 x - c_2 = 0$$

✓ Solution by Mathematica

Time used: 1.584 (sec). Leaf size: 111

```
DSolve[y''[x]+y[x]*Sin[y[x]](y'[x])^2==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to \text{InverseFunction}\left[\int_1^{\#1} \frac{e^{\sin(K[1]) - \cos(K[1])K[1]}}{c_1} dK[1]\&\right][x + c_2]$$
$$y(x) \to \text{InverseFunction}\left[\int_1^{\#1} -\frac{e^{\sin(K[1]) - \cos(K[1])K[1]}}{c_1} dK[1]\&\right][x + c_2]$$
$$y(x) \to \text{InverseFunction}\left[\int_1^{\#1} \frac{e^{\sin(K[1]) - \cos(K[1])K[1]}}{c_1} dK[1]\&\right][x + c_2]$$

3.27 problem 27

Internal problem ID [7217]

Book: Own collection of miscellaneous problems
Section: section 3.0
Problem number: 27.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _missing_y], [_2nd_order, _reducible, _mu_y_y1]]`

$$(x^2+1)\,y''+y'^3=0$$

✓ Solution by Maple
Time used: 0.016 (sec). Leaf size: 33

```
dsolve((1+x^2)*diff(y(x),x$2)+diff(y(x),x)^3=0,y(x), singsol=all)
```

$$y(x)=\int \frac{1}{\sqrt{c_1+2\arctan(x)}}dx+c_2$$
$$y(x)=-\left(\int \frac{1}{\sqrt{c_1+2\arctan(x)}}dx\right)+c_2$$

✓ Solution by Mathematica
Time used: 62.161 (sec). Leaf size: 59

```
DSolve[(1+x^2)*y''[x]+y'[x]^3==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x)\to \int_1^x -\frac{1}{\sqrt{2\arctan(K[1])-2c_1}}dK[1]+c_2$$
$$y(x)\to \int_1^x \frac{1}{\sqrt{2\arctan(K[2])-2c_1}}dK[2]+c_2$$

3.28 problem 28

Internal problem ID [7218]

Book: Own collection of miscellaneous problems
Section: section 3.0
Problem number: 28.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[[_homogeneous, 'class A'], _dAlembert]`

$$\boxed{y' - e^{-\frac{y}{x}} = 0}$$

✓ Solution by Maple

Time used: 0.015 (sec). Leaf size: 29

```
dsolve(diff(y(x),x)=exp(-y(x)/x),y(x), singsol=all)
```

$$y(x) = \text{RootOf}\left(-\left(\int^{\_Z} -\frac{1}{-e^{-\_a} + \_a} d\_a\right) + \ln(x) + c_1\right) x$$

✓ Solution by Mathematica

Time used: 0.166 (sec). Leaf size: 39

```
DSolve[y'[x]==Exp[-y[x]/x],y[x],x,IncludeSingularSolutions -> True]
```

$$\text{Solve}\left[\int_1^{\frac{y(x)}{x}} \frac{e^{K[1]}}{e^{K[1]}K[1] - 1} dK[1] = -\log(x) + c_1, y(x)\right]$$

3.29 problem 29

Internal problem ID [7219]

Book: Own collection of miscellaneous problems
Section: section 3.0
Problem number: 29.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[[_homogeneous, 'class D']]`

$$\boxed{y' - 2x^2 \sin\left(\frac{y}{x}\right)^2 - \frac{y}{x} = 0}$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 18

```
dsolve(diff(y(x),x)= 2*x^2 * sin(y(x)/x)^2 + y(x)/x,y(x), singsol=all)
```

$$y(x) = \left(\frac{\pi}{2} + \arctan\left(x^2 + 2c_1\right)\right) x$$

✓ Solution by Mathematica

Time used: 0.341 (sec). Leaf size: 22

```
DSolve[y'[x]== 2*x^2 * Sin[y[x]/x]^2 + y[x]/x,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to -x \cot^{-1}\left(x^2 - 2c_1\right)$$
$$y(x) \to 0$$

3.30 problem 30

Internal problem ID [7220]

Book: Own collection of miscellaneous problems
Section: section 3.0
Problem number: 30.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _linear, _nonhomogeneous]]`

$$\boxed{4x^2y'' + y = 8\sqrt{x}\,(1 + \ln(x))}$$

✓ Solution by Maple

Time used: 0.016 (sec). Leaf size: 24

```
dsolve(4*x^2*diff(y(x),x$2)+ y(x) = 8*sqrt(x)*(1+ln(x)),y(x), singsol=all)
```

$$y(x) = \left(c_2 + \ln(x)\, c_1 + \frac{\ln(x)^3}{3} + \ln(x)^2\right)\sqrt{x}$$

✓ Solution by Mathematica

Time used: 0.037 (sec). Leaf size: 37

```
DSolve[4*x^2*y''[x]+y[x] == 8*Sqrt[x]*(1+Log[x]),y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to \frac{1}{6}\sqrt{x}\left(2\log^3(x) + 6\log^2(x) + 3c_2\log(x) + 6c_1\right)$$

3.31 problem 31

Internal problem ID [7221]

Book: Own collection of miscellaneous problems
Section: section 3.0
Problem number: 31.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_rational, _Bernoulli]`

$$v v' - \frac{2v^2}{r^3} = \frac{\lambda r}{3}$$

✓ Solution by Maple

Time used: 0.015 (sec). Leaf size: 97

```
dsolve(v(r)*diff(v(r),r)=2*v(r)^2/r^3+1/3*lambda*r,v(r), singsol=all)
```

$$v(r) = -\frac{\sqrt{3}\,\sqrt{\mathrm{e}^{\frac{2}{r^2}} \left(\lambda\, \mathrm{e}^{\frac{2}{r^2}} r^2 + 2\lambda \operatorname{expIntegral}_1\left(-\frac{2}{r^2}\right) + 3c_1\right) \mathrm{e}^{-\frac{2}{r^2}}}}{3}$$
$$v(r) = \frac{\sqrt{3}\,\sqrt{\mathrm{e}^{\frac{2}{r^2}} \left(\lambda\, \mathrm{e}^{\frac{2}{r^2}} r^2 + 2\lambda \operatorname{expIntegral}_1\left(-\frac{2}{r^2}\right) + 3c_1\right) \mathrm{e}^{-\frac{2}{r^2}}}}{3}$$

✓ Solution by Mathematica

Time used: 10.758 (sec). Leaf size: 98

```
DSolve[v[r]*v'[r]==2*v[r]^2/r^3+1/3*\[Lambda]*r,v[r],r,IncludeSingularSolutions -> True]
```

$$v(r) \to -\frac{\sqrt{e^{-\frac{2}{r^2}} \left(-2\lambda \operatorname{ExpIntegralEi}\left(\frac{2}{r^2}\right) + \lambda e^{\frac{2}{r^2}} r^2 + 3c_1\right)}}{\sqrt{3}}$$
$$v(r) \to \frac{\sqrt{e^{-\frac{2}{r^2}} \left(-2\lambda \operatorname{ExpIntegralEi}\left(\frac{2}{r^2}\right) + \lambda e^{\frac{2}{r^2}} r^2 + 3c_1\right)}}{\sqrt{3}}$$

4 section 4.0

4.1 problem 1

Internal problem ID [7222]

Book: Own collection of miscellaneous problems
Section: section 4.0
Problem number: 1.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$\boxed{2x^2y'' - xy' + (1 - x^2)\, y = 0}$$

With the expansion point for the power series method at $x = 0$.

✓ Solution by Maple
Time used: 0.015 (sec). Leaf size: 33

```
Order:=6;
dsolve(2*x^2*diff(y(x), x$2) - x*diff(y(x), x) + (1-x^2 )*y(x) = 0,y(x),type='series',x=0);
```

$$y(x) = c_1\sqrt{x}\left(1 + \frac{1}{6}x^2 + \frac{1}{168}x^4 + \mathrm{O}\left(x^6\right)\right) + c_2x\left(1 + \frac{1}{10}x^2 + \frac{1}{360}x^4 + \mathrm{O}\left(x^6\right)\right)$$

✓ Solution by Mathematica
Time used: 0.003 (sec). Leaf size: 48

```
AsymptoticDSolveValue[2*x^2*y''[x] - x*y'[x] + (1-x^2 )*y[x] ==0,y[x],{x,0,5}]
```

$$y(x) \to c_1x\left(\frac{x^4}{360} + \frac{x^2}{10} + 1\right) + c_2\sqrt{x}\left(\frac{x^4}{168} + \frac{x^2}{6} + 1\right)$$

4.2 problem 2

Internal problem ID [7223]

Book: Own collection of miscellaneous problems
Section: section 4.0
Problem number: 2.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _linear, _nonhomogeneous]]`

$$\boxed{2x^2y'' - xy' + (1 - x^2)\, y = 1}$$

With the expansion point for the power series method at $x = 0$.

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 43

```
Order:=6;
dsolve(2*x^2*diff(y(x), x$2) - x*diff(y(x), x) + (1-x^2 )*y(x) = 1,y(x),type='series',x=0);
```

$$y(x) = c_1\sqrt{x}\left(1 + \frac{1}{6}x^2 + \frac{1}{168}x^4 + \mathrm{O}\left(x^6\right)\right) + c_2x\left(1 + \frac{1}{10}x^2 + \frac{1}{360}x^4 + \mathrm{O}\left(x^6\right)\right) + \left(1 + \frac{1}{3}x^2 + \frac{1}{63}x^4 + \mathrm{O}\left(x^6\right)\right)$$

✓ Solution by Mathematica

Time used: 0.024 (sec). Leaf size: 176

```
AsymptoticDSolveValue[2*x^2*y''[x] - x*y'[x] + (1-x^2 )*y[x] ==1,y[x],{x,0,5}]
```

$$y(x) \to c_2x\left(\frac{x^6}{28080} + \frac{x^4}{360} + \frac{x^2}{10} + 1\right) + c_1\sqrt{x}\left(\frac{x^6}{11088} + \frac{x^4}{168} + \frac{x^2}{6} + 1\right) + \sqrt{x}\left(-\frac{x^{11/2}}{154440} - \frac{x^{7/2}}{1260} - \frac{x^{3/2}}{15} + \frac{2}{\sqrt{x}}\right)\left(\frac{x^6}{11088} + \frac{x^4}{168} + \frac{x^2}{6} + 1\right) + x\left(\frac{x^5}{55440} + \frac{x^3}{504} + \frac{x}{6} - \frac{1}{x}\right)\left(\frac{x^6}{28080} + \frac{x^4}{360} + \frac{x^2}{10} + 1\right)$$

4.3 problem 3

Internal problem ID [7224]

Book: Own collection of miscellaneous problems
Section: section 4.0
Problem number: 3.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _linear, _nonhomogeneous]]`

$$\boxed{2x^2y'' - xy' + (1 - x^2)\, y = 1 + x}$$

With the expansion point for the power series method at $x = 0$.

✗ Solution by Maple

```
Order:=6;
dsolve(2*x^2*diff(y(x), x$2) - x*diff(y(x), x) + (1-x^2 )*y(x) = 1+x,y(x),type='series',x=0)
```

No solution found

✓ Solution by Mathematica

Time used: 0.033 (sec). Leaf size: 224

```
AsymptoticDSolveValue[2*x^2*y''[x] - x*y'[x] + (1-x^2 )*y[x] ==1+x,y[x],{x,0,5}]
```

$$y(x) \to c_1\sqrt{x}\left(\frac{x^6}{11088} + \frac{x^4}{168} + \frac{x^2}{6} + 1\right) + c_2x\left(\frac{x^6}{28080} + \frac{x^4}{360} + \frac{x^2}{10} + 1\right)$$
$$+ \sqrt{x}\left(-\frac{x^{11/2}}{154440} - \frac{x^{9/2}}{1620} - \frac{x^{7/2}}{1260} - \frac{x^{5/2}}{25} - \frac{x^{3/2}}{15} - 2\sqrt{x}\right.$$
$$\left.+ \frac{2}{\sqrt{x}}\right)\left(\frac{x^6}{11088} + \frac{x^4}{168} + \frac{x^2}{6} + 1\right) + x\left(\frac{x^6}{28080} + \frac{x^4}{360} + \frac{x^2}{10} + 1\right)\left(\frac{x^6}{66528} + \frac{x^5}{55440} + \frac{x^4}{672} + \frac{x^3}{504} + \frac{x^2}{12} + \frac{x}{6} - \frac{1}{x} +\right.$$

4.4 problem 4

Internal problem ID [7225]

Book: Own collection of miscellaneous problems
Section: section 4.0
Problem number: 4.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _linear, _nonhomogeneous]]`

$$\boxed{2x^2y'' - xy' + (1 - x^2)\, y = x}$$

With the expansion point for the power series method at $x = 0$.

✗ Solution by Maple

```
Order:=6;
dsolve(2*x^2*diff(y(x), x$2) - x*diff(y(x), x) + (1-x^2 )*y(x) = x,y(x),type='series',x=0);
```

No solution found

✓ Solution by Mathematica

Time used: 0.039 (sec). Leaf size: 166

```
AsymptoticDSolveValue[2*x^2*y''[x] - x*y'[x] + (1-x^2 )*y[x] ==x,y[x],{x,0,5}]
```

$$\begin{aligned} y(x) \to\ & x\left(\frac{x^6}{28080} + \frac{x^4}{360} + \frac{x^2}{10} + 1\right)\left(\frac{x^6}{66528} + \frac{x^4}{672} + \frac{x^2}{12} + \log(x)\right) \\ & + c_1\sqrt{x}\left(\frac{x^6}{11088} + \frac{x^4}{168} + \frac{x^2}{6} + 1\right) + c_2x\left(\frac{x^6}{28080} + \frac{x^4}{360} + \frac{x^2}{10} + 1\right) \\ & + \sqrt{x}\left(-\frac{x^{9/2}}{1620} - \frac{x^{5/2}}{25} - 2\sqrt{x}\right)\left(\frac{x^6}{11088} + \frac{x^4}{168} + \frac{x^2}{6} + 1\right) \end{aligned}$$

4.5 problem 5

Internal problem ID [7226]

Book: Own collection of miscellaneous problems
Section: section 4.0
Problem number: 5.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _linear, _nonhomogeneous]]`

$$2x^2y'' - xy' + (1 - x^2)\, y = x^2 + x + 1$$

With the expansion point for the power series method at $x = 0$.

✗ Solution by Maple

```
Order:=6;
dsolve(2*x^2*diff(y(x), x$2) - x*diff(y(x), x) + (1-x^2 )*y(x) = 1+x+x^2,y(x),type='series',
```

No solution found

✓ Solution by Mathematica

Time used: 0.036 (sec). Leaf size: 224

```
AsymptoticDSolveValue[2*x^2*y''[x] - x*y'[x] + (1-x^2 )*y[x] ==1+x+x^2,y[x],{x,0,5}]
```

$$y(x) \to c_1\sqrt{x}\left(\frac{x^6}{11088} + \frac{x^4}{168} + \frac{x^2}{6} + 1\right) + c_2x\left(\frac{x^6}{28080} + \frac{x^4}{360} + \frac{x^2}{10} + 1\right)$$
$$+ \sqrt{x}\left(-\frac{79x^{11/2}}{154440} - \frac{x^{9/2}}{1620} - \frac{37x^{7/2}}{1260} - \frac{x^{5/2}}{25} - \frac{11x^{3/2}}{15} - 2\sqrt{x}\right.$$
$$\left.+ \frac{2}{\sqrt{x}}\right)\left(\frac{x^6}{11088} + \frac{x^4}{168} + \frac{x^2}{6} + 1\right) + x\left(\frac{x^6}{28080} + \frac{x^4}{360} + \frac{x^2}{10} + 1\right)\left(\frac{x^6}{66528} + \frac{67x^5}{55440} + \frac{x^4}{672} + \frac{29x^3}{504} + \frac{x^2}{12} + \frac{7x}{6} - \right.$$

4.6 problem 6

Internal problem ID [7227]

Book: Own collection of miscellaneous problems
Section: section 4.0
Problem number: 6.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _linear, _nonhomogeneous]]`

$$2x^2y'' - xy' + (1 - x^2)\, y = x^2$$

With the expansion point for the power series method at $x = 0$.

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 45

```
Order:=6;
dsolve(2*x^2*diff(y(x), x$2) - x*diff(y(x), x) + (1-x^2 )*y(x) = x^2,y(x),type='series',x=0)
```

$$y(x) = c_1\sqrt{x}\left(1 + \frac{1}{6}x^2 + \frac{1}{168}x^4 + \mathrm{O}\left(x^6\right)\right) + c_2x\left(1 + \frac{1}{10}x^2 + \frac{1}{360}x^4 + \mathrm{O}\left(x^6\right)\right) + x^2\left(\frac{1}{3} + \frac{1}{63}x^2 + \mathrm{O}\left(x^4\right)\right)$$

✓ Solution by Mathematica

Time used: 0.023 (sec). Leaf size: 160

```
AsymptoticDSolveValue[2*x^2*y''[x] - x*y'[x] + (1-x^2 )*y[x] ==x^2,y[x],{x,0,5}]
```

$$y(x) \to c_2x\left(\frac{x^6}{28080} + \frac{x^4}{360} + \frac{x^2}{10} + 1\right) + c_1\sqrt{x}\left(\frac{x^6}{11088} + \frac{x^4}{168} + \frac{x^2}{6} + 1\right) + \sqrt{x}\left(-\frac{x^{11/2}}{1980} - \frac{x^{7/2}}{35} - \frac{2x^{3/2}}{3}\right)\left(\frac{x^6}{11088} + \frac{x^4}{168} + \frac{x^2}{6} + 1\right) + x\left(\frac{x^5}{840} + \frac{x^3}{18} + x\right)\left(\frac{x^6}{28080} + \frac{x^4}{360} + \frac{x^2}{10} + 1\right)$$

4.7 problem 7

Internal problem ID [7228]

Book: Own collection of miscellaneous problems
Section: section 4.0
Problem number: 7.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _linear, _nonhomogeneous]]`

$$\boxed{2x^2y'' - xy' + (1 - x^2)\, y = x^2 + 1}$$

With the expansion point for the power series method at $x = 0$.

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 43

```
Order:=6;
dsolve(2*x^2*diff(y(x), x$2) - x*diff(y(x), x) + (1-x^2 )*y(x) = 1+x^2,y(x),type='series',x=
```

$$y(x) = c_1\sqrt{x}\left(1 + \frac{1}{6}x^2 + \frac{1}{168}x^4 + \mathrm{O}\left(x^6\right)\right) + c_2x\left(1 + \frac{1}{10}x^2 + \frac{1}{360}x^4 + \mathrm{O}\left(x^6\right)\right) + \left(1 + \frac{2}{3}x^2 + \frac{2}{63}x^4 + \mathrm{O}\left(x^6\right)\right)$$

✓ Solution by Mathematica

Time used: 0.032 (sec). Leaf size: 176

```
AsymptoticDSolveValue[2*x^2*y''[x] - x*y'[x] + (1-x^2 )*y[x] ==1+x^2,y[x],{x,0,5}]
```

$$y(x) \to c_2x\left(\frac{x^6}{28080} + \frac{x^4}{360} + \frac{x^2}{10} + 1\right) + c_1\sqrt{x}\left(\frac{x^6}{11088} + \frac{x^4}{168} + \frac{x^2}{6} + 1\right) + \sqrt{x}\left(-\frac{79x^{11/2}}{154440} - \frac{37x^{7/2}}{1260} - \frac{11x^{3/2}}{15} + \frac{2}{\sqrt{x}}\right)\left(\frac{x^6}{11088} + \frac{x^4}{168} + \frac{x^2}{6} + 1\right) + x\left(\frac{67x^5}{55440} + \frac{29x^3}{504} + \frac{7x}{6} - \frac{1}{x}\right)\left(\frac{x^6}{28080} + \frac{x^4}{360} + \frac{x^2}{10} + 1\right)$$

4.8 problem 8

Internal problem ID [7229]

Book: Own collection of miscellaneous problems
Section: section 4.0
Problem number: 8.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _linear, _nonhomogeneous]]`

$$2x^2y'' - xy' + (1 - x^2)\, y = x^4$$

With the expansion point for the power series method at $x = 0$.

✓ Solution by Maple

Time used: 0.015 (sec). Leaf size: 43

```
Order:=6;
dsolve(2*x^2*diff(y(x), x$2) - x*diff(y(x), x) + (1-x^2 )*y(x) = x^4,y(x),type='series',x=0)
```

$$y(x) = c_1\sqrt{x}\left(1 + \frac{1}{6}x^2 + \frac{1}{168}x^4 + \mathrm{O}\left(x^6\right)\right) + c_2x\left(1 + \frac{1}{10}x^2 + \frac{1}{360}x^4 + \mathrm{O}\left(x^6\right)\right) + x^4\left(\frac{1}{21} + \mathrm{O}\left(x^2\right)\right)$$

✓ Solution by Mathematica

Time used: 0.028 (sec). Leaf size: 150

```
AsymptoticDSolveValue[2*x^2*y''[x] - x*y'[x] + (1-x^2 )*y[x] ==x^4,y[x],{x,0,5}]
```

$$y(x) \to c_2x\left(\frac{x^6}{28080} + \frac{x^4}{360} + \frac{x^2}{10} + 1\right) + c_1\sqrt{x}\left(\frac{x^6}{11088} + \frac{x^4}{168} + \frac{x^2}{6} + 1\right) + \sqrt{x}\left(-\frac{x^{11/2}}{55} - \frac{2x^{7/2}}{7}\right)\left(\frac{x^6}{11088} + \frac{x^4}{168} + \frac{x^2}{6} + 1\right) + x\left(\frac{x^5}{30} + \frac{x^3}{3}\right)\left(\frac{x^6}{28080} + \frac{x^4}{360} + \frac{x^2}{10} + 1\right)$$

4.9 problem 9

Internal problem ID [7230]

Book: Own collection of miscellaneous problems
Section: section 4.0
Problem number: 9.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _linear, _nonhomogeneous]]`

$$\boxed{2x^2y'' - xy' + (1 - x^2)\, y = \sin(x)}$$

With the expansion point for the power series method at $x = 0$.

✗ Solution by Maple

```
Order:=6;
dsolve(2*x^2*diff(y(x), x$2) - x*diff(y(x), x) + (1-x^2 )*y(x) = sin(x),y(x),type='series',x
```

No solution found

✓ Solution by Mathematica
Time used: 0.04 (sec). Leaf size: 159

```
AsymptoticDSolveValue[2*x^2*y''[x] - x*y'[x] + (1-x^2 )*y[x] ==Sin[x],y[x],{x,0,5}]
```

$$\begin{aligned} y(x) \to & \, x\left(\frac{x^6}{28080} + \frac{x^4}{360} + \frac{x^2}{10} + 1\right)\left(\frac{x^6}{20790} - \frac{17x^4}{5040} + \log(x)\right) \\ & + c_1\sqrt{x}\left(\frac{x^6}{11088} + \frac{x^4}{168} + \frac{x^2}{6} + 1\right) + c_2x\left(\frac{x^6}{28080} + \frac{x^4}{360} + \frac{x^2}{10} + 1\right) \\ & + \sqrt{x}\left(\frac{x^{9/2}}{810} + \frac{2x^{5/2}}{75} - 2\sqrt{x}\right)\left(\frac{x^6}{11088} + \frac{x^4}{168} + \frac{x^2}{6} + 1\right) \end{aligned}$$

4.10 problem 10

Internal problem ID [7231]

Book: Own collection of miscellaneous problems
Section: section 4.0
Problem number: 10.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _linear, _nonhomogeneous]]`

$$\boxed{2x^2y'' - xy' + (1 - x^2)\, y = 1 + \sin(x)}$$

With the expansion point for the power series method at $x = 0$.

✗ Solution by Maple

```
Order:=6;
dsolve(2*x^2*diff(y(x), x$2) - x*diff(y(x), x) + (1-x^2 )*y(x) = 1+sin(x),y(x),type='series'
```

No solution found

✓ Solution by Mathematica

Time used: 0.142 (sec). Leaf size: 217

```
AsymptoticDSolveValue[2*x^2*y''[x] - x*y'[x] + (1-x^2 )*y[x] ==1+Sin[x],y[x],{x,0,5}]
```

$$y(x) \to c_1\sqrt{x}\left(\frac{x^6}{11088} + \frac{x^4}{168} + \frac{x^2}{6} + 1\right) + c_2x\left(\frac{x^6}{28080} + \frac{x^4}{360} + \frac{x^2}{10} + 1\right) + \sqrt{x}\left(-\frac{x^{11/2}}{154440} + \frac{x^{9/2}}{810} - \frac{x^{7/2}}{1260} + \frac{2x^{5/2}}{75} - \frac{x^{3/2}}{15} - 2\sqrt{x} + \frac{2}{\sqrt{x}}\right)\left(\frac{x^6}{11088} + \frac{x^4}{168} + \frac{x^2}{6} + 1\right) + x\left(\frac{x^6}{28080} + \frac{x^4}{360} + \frac{x^2}{10} + 1\right)\left(\frac{x^6}{20790} + \frac{x^5}{55440} - \frac{17x^4}{5040} + \frac{x^3}{504} + \frac{x}{6} - \frac{1}{x} + \log\right.$$

4.11 problem 11

Internal problem ID [7232]

Book: Own collection of miscellaneous problems
Section: section 4.0
Problem number: 11.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _linear, _nonhomogeneous]]`

$$2x^2y'' - xy' + (1 - x^2)\, y = x \sin(x)$$

With the expansion point for the power series method at $x = 0$.

✓ Solution by Maple

Time used: 0.016 (sec). Leaf size: 45

```
Order:=6;
dsolve(2*x^2*diff(y(x), x$2) - x*diff(y(x), x) + (1-x^2 )*y(x) = x*sin(x),y(x),type='series'
```

$$y(x) = c_1\sqrt{x}\left(1 + \frac{1}{6}x^2 + \frac{1}{168}x^4 + \mathrm{O}\left(x^6\right)\right) + c_2x\left(1 + \frac{1}{10}x^2 + \frac{1}{360}x^4 + \mathrm{O}\left(x^6\right)\right) + x^2\left(\frac{1}{3} + \frac{1}{126}x^2 + \mathrm{O}\left(x^4\right)\right)$$

✓ Solution by Mathematica

Time used: 0.023 (sec). Leaf size: 167

```
AsymptoticDSolveValue[2*x^2*y''[x] - x*y'[x] + (1-x^2 )*y[x] ==x*sin(x),y[x],{x,0,5}]
```

$$y(x) \to c_2x\left(\frac{x^6}{28080} + \frac{x^4}{360} + \frac{x^2}{10} + 1\right) + c_1\sqrt{x}\left(\frac{x^6}{11088} + \frac{x^4}{168} + \frac{x^2}{6} + 1\right) + \sqrt{x}\left(\frac{x^6}{11088} + \frac{x^4}{168} + \frac{x^2}{6} + 1\right)\left(-\frac{x^{11/2}\sin}{1980} - \frac{1}{35}x^{7/2}\sin - \frac{2}{3}x^{3/2}\sin\right) + x\left(\frac{x^6}{28080} + \frac{x^4}{360} + \frac{x^2}{10} + 1\right)\left(\frac{x^5\sin}{840} + \frac{x^3\sin}{18} + x\sin\right)$$

4.12 problem 12

Internal problem ID [7233]

Book: Own collection of miscellaneous problems
Section: section 4.0
Problem number: 12.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _linear, _nonhomogeneous]]`

$$2x^2y'' - xy' + (1 - x^2)\, y = \cos(x) + \sin(x)$$

With the expansion point for the power series method at $x = 0$.

✗ Solution by Maple

```
Order:=6;
dsolve(2*x^2*diff(y(x), x$2) - x*diff(y(x), x) + (1-x^2 )*y(x) = sin(x)+cos(x),y(x),type='se
```

No solution found

✓ Solution by Mathematica

Time used: 0.048 (sec). Leaf size: 217

```
AsymptoticDSolveValue[2*x^2*y''[x] - x*y'[x] + (1-x^2 )*y[x] ==Sin[x]+Cos[x],y[x],{x,0,5}]
```

$$y(x) \to c_1\sqrt{x}\left(\frac{x^6}{11088} + \frac{x^4}{168} + \frac{x^2}{6} + 1\right) + c_2x\left(\frac{x^6}{28080} + \frac{x^4}{360} + \frac{x^2}{10} + 1\right) + \sqrt{x}\left(-\frac{x^{11/2}}{3861} + \frac{x^{9/2}}{810} + \frac{x^{7/2}}{630} + \frac{2x^{5/2}}{75} + \frac{4x^{3/2}}{15} - 2\sqrt{x} + \frac{2}{\sqrt{x}}\right)\left(\frac{x^6}{11088}+\frac{x^4}{168}+\frac{x^2}{6}+1\right)+x\left(\frac{x^6}{28080}+\frac{x^4}{360}+\frac{x^2}{10}+1\right)\left(\frac{x^6}{20790}+\frac{37x^5}{69300}-\frac{17x^4}{5040}-\frac{x^3}{84}-\frac{x}{3}-\frac{1}{x}+\log(

4.13 problem 13

Internal problem ID [7234]

Book: Own collection of miscellaneous problems
Section: section 4.0
Problem number: 13.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$x^2y'' + (\cos(x) - 1)\,y' + e^x y = 0$$

With the expansion point for the power series method at $x = 0$.

✓ Solution by Maple

Time used: 0.125 (sec). Leaf size: 1171

```
Order:=6;
dsolve(x^2*diff(y(x), x$2) + (cos(x)-1)*diff(y(x), x) + exp(x)*y(x) = 0,y(x),type='series',x
```

$$y(x) = \sqrt{x}\left(c_2 x^{\frac{i\sqrt{3}}{2}}\left(1 + \frac{1}{4}i\sqrt{3}x + \frac{-i\sqrt{3}-11}{32i\sqrt{3}+64}x^2 + \frac{\frac{55\sqrt{3}}{288}+\frac{55i}{96}}{(i-\sqrt{3})(i\sqrt{3}+2)(i\sqrt{3}+3)}x^3 + \frac{1}{384}\frac{112i\sqrt{3}+199}{(-\sqrt{3}+2i)(-i+\sqrt{3})(i\sqrt{3}+3)(i\sqrt{3}+4)}x^4 + \frac{\frac{18491\sqrt{3}}{38400}+\frac{4387i}{12800}}{(-i+\sqrt{3})(i\sqrt{3}+2)(i\sqrt{3}+3)(i\sqrt{3}+4)(i\sqrt{3}+5)}x^5 + \mathrm{O}\left(x^6\right)\right) + c_1 x^{-\frac{i\sqrt{3}}{2}}\left(1 - \frac{1}{4}i\sqrt{3}x + \frac{-\sqrt{3}-11i}{32\sqrt{3}+64i}x^2 + \frac{55\sqrt{3}-165i}{3456i-2304\sqrt{3}}x^3 + \frac{199i+112\sqrt{3}}{-27648i+7680\sqrt{3}}x^4 + \frac{\frac{18491\sqrt{3}}{38400}-\frac{4387i}{12800}}{(\sqrt{3}+i)(\sqrt{3}+2i)(\sqrt{3}+3i)(\sqrt{3}+4i)(\sqrt{3}+5i)}x^5 + \mathrm{O}\left(x^6\right)\right)\right)$$

✓ Solution by Mathematica

Time used: 0.005 (sec). Leaf size: 2502

```
AsymptoticDSolveValue[x^2*y''[x] + (Cos[x]-1)*y'[x] + Exp[x]*y[x] ==0,y[x],{x,0,5}]
```

Too large to display

4.14 problem 14

Internal problem ID [7235]

Book: Own collection of miscellaneous problems
Section: section 4.0
Problem number: 14.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$(x-2)\,y'' + \frac{y'}{x} + (1+x)\,y = 0$$

With the expansion point for the power series method at $x = 0$.

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 42

```
Order:=6;
dsolve((x-2)*diff(y(x), x$2) + 1/x*diff(y(x), x) + (x+1)*y(x) = 0,y(x),type='series',x=0);
```

$$y(x) = c_1 x^{\frac{3}{2}} \left(1 + \frac{3}{20}x + \frac{25}{224}x^2 + \frac{1361}{17280}x^3 + \frac{80753}{2365440}x^4 + \frac{616517}{38707200}x^5 + \mathrm{O}\left(x^6\right)\right) + c_2 \left(1 + \frac{1}{2}x^2 + \frac{2}{9}x^3 + \frac{11}{120}x^4 + \frac{82}{1575}x^5 + \mathrm{O}\left(x^6\right)\right)$$

✓ Solution by Mathematica

Time used: 0.004 (sec). Leaf size: 80

```
AsymptoticDSolveValue[(x-2)*y''[x] + 1/x*y'[x] + (x+1)*y[x] ==0,y[x],{x,0,5}]
```

$$y(x) \to c_2 \left(\frac{82x^5}{1575} + \frac{11x^4}{120} + \frac{2x^3}{9} + \frac{x^2}{2} + 1\right) + c_1 \left(\frac{616517x^5}{38707200} + \frac{80753x^4}{2365440} + \frac{1361x^3}{17280} + \frac{25x^2}{224} + \frac{3x}{20} + 1\right) x^{3/2}$$

4.15 problem 15

Internal problem ID [7236]

Book: Own collection of miscellaneous problems
Section: section 4.0
Problem number: 15.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$(x-2)\,y''+\frac{y'}{x}+(1+x)\,y=0$$

With the expansion point for the power series method at $x=2$.

✓ Solution by Maple

Time used: 0.015 (sec). Leaf size: 46

```
Order:=6;
dsolve((x-2)*diff(y(x), x$2) + 1/x*diff(y(x), x) + (x+1)*y(x) = 0,y(x),type='series',x=2);
```

$$y(x)=c_1\sqrt{x-2}\left(1-\frac{23}{12}(x-2)+\frac{127}{160}(x-2)^2+\frac{1621}{40320}(x-2)^3-\frac{426599}{5806080}(x-2)^4+\frac{4670443}{425779200}(x-2)^5+\mathrm{O}\left((x-2)^6\right)\right)+c_2\left(1-6(x-2)+\frac{31}{6}(x-2)^2-\frac{37}{45}(x-2)^3-\frac{299}{840}(x-2)^4+\frac{6743}{56700}(x-2)^5+\mathrm{O}\left((x-2)^6\right)\right)$$

✓ <u>Solution by Mathematica</u>

Time used: 0.006 (sec). Leaf size: 105

```
AsymptoticDSolveValue[(x-2)*y''[x] + 1/x*y'[x] + (x+1)*y[x] ==0,y[x],{x,2,5}]
```

$$y(x) \to c_1\left(\frac{4670443(x-2)^5}{425779200} - \frac{426599(x-2)^4}{5806080} + \frac{1621(x-2)^3}{40320} + \frac{127}{160}(x-2)^2 - \frac{23(x-2)}{12} + 1\right)\sqrt{x-2} + c_2\left(\frac{6743(x-2)^5}{56700} - \frac{299}{840}(x-2)^4 - \frac{37}{45}(x-2)^3 + \frac{31}{6}(x-2)^2 - 6(x-2) + 1\right)$$

4.16 problem 16

Internal problem ID [7237]

Book: Own collection of miscellaneous problems
Section: section 4.0
Problem number: 16.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$(1+x)(3x-1)y''+y'\cos(x)-3yx=0$$

With the expansion point for the power series method at $x=0$.

✓ Solution by Maple
Time used: 0.0 (sec). Leaf size: 49

```
Order:=6;
dsolve((x+1)*(3*x-1)*diff(y(x),x$2)+cos(x)*diff(y(x),x)-3*x*y(x)=0,y(x),type='series',x=0);
```

$$y(x)=\left(1-\frac{1}{2}x^3-\frac{5}{8}x^4-\frac{53}{40}x^5\right)y(0)+\left(x+\frac{1}{2}x^2+\frac{1}{2}x^3+\frac{7}{12}x^4+\frac{7}{6}x^5\right)D(y)(0)+O\left(x^6\right)$$

✓ Solution by Mathematica
Time used: 0.001 (sec). Leaf size: 63

```
AsymptoticDSolveValue[(x+1)*(3*x-1)*y''[x]+Cos[x]*y'[x]-3*x*y[x]==0,y[x],{x,0,5}]
```

$$y(x)\to c_1\left(-\frac{53x^5}{40}-\frac{5x^4}{8}-\frac{x^3}{2}+1\right)+c_2\left(\frac{7x^5}{6}+\frac{7x^4}{12}+\frac{x^3}{2}+\frac{x^2}{2}+x\right)$$

4.17 problem 17

Internal problem ID [7238]

Book: Own collection of miscellaneous problems
Section: section 4.0
Problem number: 17.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[_Lienard]`

$$\boxed{xy'' + 2y' + yx = 0}$$

With initial conditions

$$[y(0) = 1, y'(0) = 0]$$

With the expansion point for the power series method at $x = 0$.

✓ Solution by Maple
Time used: 0.0 (sec). Leaf size: 14

```
Order:=6;
dsolve([x*diff(y(x),x$2)+2*diff(y(x),x)+x*y(x)=0,y(0) = 1, D(y)(0) = 0],y(x),type='series',x
```

$$y(x) = 1 - \frac{1}{6}x^2 + \frac{1}{120}x^4 + \mathrm{O}\left(x^6\right)$$

✓ Solution by Mathematica
Time used: 0.005 (sec). Leaf size: 19

```
AsymptoticDSolveValue[{x*y''[x]+2*y'[x]+x*y[x]==0,{y[0]==1,y'[0]==0}},y[x],{x,0,5}]
```

$$y(x) \to \frac{x^4}{120} - \frac{x^2}{6} + 1$$

4.18 problem 18

Internal problem ID [7239]

Book: Own collection of miscellaneous problems
Section: section 4.0
Problem number: 18.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$2x^2y'' + 3xy' - yx = x^2 + 2x$$

With the expansion point for the power series method at $x = 0$.

✓ Solution by Maple

Time used: 0.031 (sec). Leaf size: 60

```
Order:=6;
dsolve(2*x^2*diff(y(x),x$2)+3*x*diff(y(x),x)-x*y(x)=x^2+2*x,y(x),type='series',x=0);
```

$$\begin{aligned} y(x) = &\frac{c_1\left(1 + x + \frac{1}{6}x^2 + \frac{1}{90}x^3 + \frac{1}{2520}x^4 + \frac{1}{113400}x^5 + \mathrm{O}\left(x^6\right)\right)}{\sqrt{x}} \\ &+ c_2\left(1 + \frac{1}{3}x + \frac{1}{30}x^2 + \frac{1}{630}x^3 + \frac{1}{22680}x^4 + \frac{1}{1247400}x^5 + \mathrm{O}\left(x^6\right)\right) \\ &+ x\left(\frac{2}{3} + \frac{1}{6}x + \frac{1}{126}x^2 + \frac{1}{4536}x^3 + \frac{1}{249480}x^4 + \mathrm{O}\left(x^5\right)\right) \end{aligned}$$

✓ Solution by Mathematica

Time used: 0.046 (sec). Leaf size: 239

```
AsymptoticDSolveValue[2*x^2*y''[x]+3*x*y'[x]-x*y[x]==x^2+2*x,y[x],{x,0,5}]
```

$$
\begin{aligned}
y(x) &\to c_1\left(\frac{x^5}{1247400}+\frac{x^4}{22680}+\frac{x^3}{630}+\frac{x^2}{30}+\frac{x}{3}+1\right)+\frac{c_2\left(\frac{x^5}{113400}+\frac{x^4}{2520}+\frac{x^3}{90}+\frac{x^2}{6}+x+1\right)}{\sqrt{x}} \\
&+\frac{\left(\frac{x^5}{113400}+\frac{x^4}{2520}+\frac{x^3}{90}+\frac{x^2}{6}+x+1\right)\left(-\frac{19x^{11/2}}{62370}-\frac{23x^{9/2}}{2835}-\frac{4x^{7/2}}{35}-\frac{2x^{5/2}}{3}-\frac{4x^{3/2}}{3}\right)}{\sqrt{x}} \\
&+\left(\frac{x^5}{1247400}+\frac{x^4}{22680}+\frac{x^3}{630}+\frac{x^2}{30}+\frac{x}{3}+1\right)\left(\frac{47x^6}{680400}+\frac{x^5}{420}+\frac{17x^4}{360}+\frac{4x^3}{9}+\frac{3x^2}{2}+2x\right)
\end{aligned}
$$

4.19 problem 19

Internal problem ID [7240]

Book: Own collection of miscellaneous problems
Section: section 4.0
Problem number: 19.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _linear, _nonhomogeneous]]`

$$\boxed{2x^2y'' - xy' + (1 - x^2)\, y = 1}$$

With the expansion point for the power series method at $x = 0$.

✓ Solution by Maple

Time used: 0.015 (sec). Leaf size: 43

```
Order:=6;
dsolve(2*x^2*diff(y(x), x$2) - x*diff(y(x), x) + (-x^2 + 1)*y(x) = 1,y(x),type='series',x=0)
```

$$y(x) = c_1\sqrt{x}\left(1 + \frac{1}{6}x^2 + \frac{1}{168}x^4 + \mathrm{O}\left(x^6\right)\right) + c_2x\left(1 + \frac{1}{10}x^2 + \frac{1}{360}x^4 + \mathrm{O}\left(x^6\right)\right) + \left(1 + \frac{1}{3}x^2 + \frac{1}{63}x^4 + \mathrm{O}\left(x^6\right)\right)$$

✓ Solution by Mathematica

Time used: 0.02 (sec). Leaf size: 176

```
AsymptoticDSolveValue[2*x^2*y''[x]-x*y'[x]+(1-x^2)*y[x]==1,y[x],{x,0,5}]
```

$$y(x) \to c_2x\left(\frac{x^6}{28080} + \frac{x^4}{360} + \frac{x^2}{10} + 1\right) + c_1\sqrt{x}\left(\frac{x^6}{11088} + \frac{x^4}{168} + \frac{x^2}{6} + 1\right) + \sqrt{x}\left(-\frac{x^{11/2}}{154440} - \frac{x^{7/2}}{1260} - \frac{x^{3/2}}{15} + \frac{2}{\sqrt{x}}\right)\left(\frac{x^6}{11088} + \frac{x^4}{168} + \frac{x^2}{6} + 1\right) + x\left(\frac{x^5}{55440} + \frac{x^3}{504} + \frac{x}{6} - \frac{1}{x}\right)\left(\frac{x^6}{28080} + \frac{x^4}{360} + \frac{x^2}{10} + 1\right)$$

4.20 problem 20

Internal problem ID [7241]

Book: Own collection of miscellaneous problems
Section: section 4.0
Problem number: 20.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _linear, _nonhomogeneous]]`

$$\boxed{2x^2y'' + 2xy' - yx = 1}$$

With the expansion point for the power series method at $x = 0$.

✗ Solution by Maple

```
Order:=6;
dsolve(2*x^2*diff(y(x), x, x) + 2*x*diff(y(x), x) - x*y(x) = 1,y(x),type='series',x=0);
```

No solution found

✓ Solution by Mathematica

Time used: 0.148 (sec). Leaf size: 360

```
AsymptoticDSolveValue[2*x^2*y''[x]+2*x*y'[x]-x*y[x]==1,y[x],{x,0,5}]
```

$$
\begin{aligned}
y(x) \to\ & c_2\left(\frac{x^5}{460800}+\frac{x^4}{9216}+\frac{x^3}{288}+\frac{x^2}{16}+\frac{x}{2}+1\right) \\
&+c_1\left(x^5\left(\frac{\log(x)}{460800}-\frac{107}{13824000}\right)+x^4\left(\frac{\log(x)}{9216}-\frac{19}{55296}\right)+x^3\left(\frac{\log(x)}{288}-\frac{1}{108}\right)\right. \\
&\left.+x^2\left(\frac{\log(x)}{16}-\frac{1}{8}\right)+x\left(\frac{\log(x)}{2}-\frac{1}{2}\right)+\log(x)+1\right) \\
&+\left(-\frac{137x^6}{1990656000}+\frac{x^5}{4608000}+\frac{x^4}{73728}+\frac{x^3}{1728}+\frac{x^2}{64}+\frac{x}{4}\right. \\
&\left.+\frac{\log(x)}{2}\right)\left(x^5\left(\frac{\log(x)}{460800}-\frac{107}{13824000}\right)+x^4\left(\frac{\log(x)}{9216}-\frac{19}{55296}\right)\right. \\
&\left.+x^3\left(\frac{\log(x)}{288}-\frac{1}{108}\right)+x^2\left(\frac{\log(x)}{16}-\frac{1}{8}\right)+x\left(\frac{\log(x)}{2}-\frac{1}{2}\right)+\log(x)+1\right) \\
&+\left(\frac{x^5}{460800}+\frac{x^4}{9216}+\frac{x^3}{288}+\frac{x^2}{16}+\frac{x}{2}+1\right)\left(\frac{137x^6(6\log(x)+5)}{11943936000}\right. \\
&+\frac{x^5(113-30\log(x))}{138240000}+\frac{x^4(41-12\log(x))}{884736}+\frac{x^3(3-\log(x))}{1728} \\
&\left.+\frac{1}{128}x^2(5-2\log(x))+\frac{1}{4}x(2-\log(x))-\frac{1}{4}\log(x)(\log(x)+2)\right)
\end{aligned}
$$

4.21 problem 21

Internal problem ID [7242]

Book: Own collection of miscellaneous problems
Section: section 4.0
Problem number: 21.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$\boxed{y'' + (x - 6)\, y = 0}$$

With the expansion point for the power series method at $x = 0$.

✓ Solution by Maple
Time used: 0.0 (sec). Leaf size: 47

```
Order:=6;
dsolve(diff(y(x), x, x) + (x-6)*y(x) = 0,y(x),type='series',x=0);
```

$$y(x) = \left(1 + 3x^2 - \frac{1}{6}x^3 + \frac{3}{2}x^4 - \frac{1}{5}x^5\right) y(0) + \left(x + x^3 - \frac{1}{12}x^4 + \frac{3}{10}x^5\right) D(y)(0) + O(x^6)$$

✓ Solution by Mathematica
Time used: 0.001 (sec). Leaf size: 57

```
AsymptoticDSolveValue[y''[x]+(x-6)*y[x]==0,y[x],{x,0,5}]
```

$$y(x) \to c_2\left(\frac{3x^5}{10} - \frac{x^4}{12} + x^3 + x\right) + c_1\left(-\frac{x^5}{5} + \frac{3x^4}{2} - \frac{x^3}{6} + 3x^2 + 1\right)$$

4.22 problem 22

Internal problem ID [7243]

Book: Own collection of miscellaneous problems
Section: section 4.0
Problem number: 22.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$\boxed{x^2y'' + (3x^2 + 2x)\, y' - 2y = 0}$$

With the expansion point for the power series method at $x = 0$.

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 45

```
Order:=6;
dsolve(x^2*diff(y(x), x, x) + (2*x+3*x^2)*diff(y(x),x)-2*y(x) = 0,y(x),type='series',x=0);
```

$$y(x) = c_1 x\left(1 - \frac{3}{4}x + \frac{9}{20}x^2 - \frac{9}{40}x^3 + \frac{27}{280}x^4 - \frac{81}{2240}x^5 + \mathrm{O}\left(x^6\right)\right) + \frac{c_2\left(12 - 36x + 54x^2 - 54x^3 + \frac{81}{2}x^4 - \frac{243}{10}x^5 + \mathrm{O}\left(x^6\right)\right)}{x^2}$$

✓ Solution by Mathematica

Time used: 0.024 (sec). Leaf size: 64

```
AsymptoticDSolveValue[x^2*y''[x]+(2*x+3*x^2)*y'[x]-2*y[x]==0,y[x],{x,0,5}]
```

$$y(x) \to c_1\left(\frac{27x^2}{8} + \frac{1}{x^2} - \frac{9x}{2} - \frac{3}{x} + \frac{9}{2}\right) + c_2\left(\frac{27x^5}{280} - \frac{9x^4}{40} + \frac{9x^3}{20} - \frac{3x^2}{4} + x\right)$$

4.23 problem 23

Internal problem ID [7244]

Book: Own collection of miscellaneous problems
Section: section 4.0
Problem number: 23.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _linear, _nonhomogeneous]]`

$$2x^2y'' - xy' + (1 - x^2)\, y = x^2 + \cos(x)$$

With the expansion point for the power series method at $x = 0$.

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 43

```
Order:=6;
dsolve(2*x^2*diff(y(x), x, x) - x*diff(y(x), x) + (-x^2 + 1)*y(x) = x^2+cos(x),y(x),type='se
```

$$y(x) = c_1\sqrt{x}\left(1 + \frac{1}{6}x^2 + \frac{1}{168}x^4 + \mathrm{O}\left(x^6\right)\right) + c_2 x\left(1 + \frac{1}{10}x^2 + \frac{1}{360}x^4 + \mathrm{O}\left(x^6\right)\right) + \left(1 + \frac{1}{2}x^2 + \frac{13}{504}x^4 + \mathrm{O}\left(x^6\right)\right)$$

✓ Solution by Mathematica

Time used: 0.107 (sec). Leaf size: 176

```
AsymptoticDSolveValue[2*x^2*y''[x]-x*y'[x]+(1-x^2)*y[x]==x^2+Cos[x],y[x],{x,0,5}]
```

$$y(x) \to c_2 x\left(\frac{x^6}{28080} + \frac{x^4}{360} + \frac{x^2}{10} + 1\right) + c_1\sqrt{x}\left(\frac{x^6}{11088} + \frac{x^4}{168} + \frac{x^2}{6} + 1\right) + \sqrt{x}\left(-\frac{59x^{11/2}}{77220} - \frac{17x^{7/2}}{630} - \frac{2x^{3/2}}{5} + \frac{2}{\sqrt{x}}\right)\left(\frac{x^6}{11088} + \frac{x^4}{168} + \frac{x^2}{6} + 1\right) + x\left(\frac{239x^5}{138600} + \frac{11x^3}{252} + \frac{2x}{3} - \frac{1}{x}\right)\left(\frac{x^6}{28080} + \frac{x^4}{360} + \frac{x^2}{10} + 1\right)$$

4.24 problem 24

Internal problem ID [7245]

Book: Own collection of miscellaneous problems
Section: section 4.0
Problem number: 24.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _linear, _nonhomogeneous]]`

$$2x^2y'' - xy' + (1 - x^2)\, y = \cos(x)$$

With the expansion point for the power series method at $x = 0$.

✓ Solution by Maple

Time used: 0.015 (sec). Leaf size: 43

```
Order:=6;
dsolve(2*x^2*diff(y(x), x, x) - x*diff(y(x), x) + (-x^2 + 1)*y(x) = cos(x),y(x),type='series
```

$$y(x) = c_1\sqrt{x}\left(1 + \frac{1}{6}x^2 + \frac{1}{168}x^4 + \mathrm{O}\left(x^6\right)\right) + c_2x\left(1 + \frac{1}{10}x^2 + \frac{1}{360}x^4 + \mathrm{O}\left(x^6\right)\right) + \left(1 + \frac{1}{6}x^2 + \frac{5}{504}x^4 + \mathrm{O}\left(x^6\right)\right)$$

✓ Solution by Mathematica

Time used: 0.038 (sec). Leaf size: 176

```
AsymptoticDSolveValue[2*x^2*y''[x]-x*y'[x]+(1-x^2)*y[x]==Cos[x],y[x],{x,0,5}]
```

$$y(x) \to c_2x\left(\frac{x^6}{28080} + \frac{x^4}{360} + \frac{x^2}{10} + 1\right) + c_1\sqrt{x}\left(\frac{x^6}{11088} + \frac{x^4}{168} + \frac{x^2}{6} + 1\right) + \sqrt{x}\left(-\frac{x^{11/2}}{3861} + \frac{x^{7/2}}{630} + \frac{4x^{3/2}}{15} + \frac{2}{\sqrt{x}}\right)\left(\frac{x^6}{11088} + \frac{x^4}{168} + \frac{x^2}{6} + 1\right) + x\left(\frac{37x^5}{69300} - \frac{x^3}{84} - \frac{x}{3} - \frac{1}{x}\right)\left(\frac{x^6}{28080} + \frac{x^4}{360} + \frac{x^2}{10} + 1\right)$$

4.25 problem 24

Internal problem ID [7246]

Book: Own collection of miscellaneous problems
Section: section 4.0
Problem number: 24.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _linear, _nonhomogeneous]]`

$$2x^2y'' - xy' + (1 - x^2)\, y = x^3 + \cos(x)$$

With the expansion point for the power series method at $x = 0$.

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 47

```
Order:=6;
dsolve(2*x^2*diff(y(x), x, x) - x*diff(y(x), x) + (-x^2 + 1)*y(x) = x^3+cos(x),y(x),type='se
```

$$y(x) = c_1\sqrt{x}\left(1 + \frac{1}{6}x^2 + \frac{1}{168}x^4 + \mathrm{O}\left(x^6\right)\right) + c_2x\left(1 + \frac{1}{10}x^2 + \frac{1}{360}x^4 + \mathrm{O}\left(x^6\right)\right) + \left(1 + \frac{1}{6}x^2 + \frac{1}{10}x^3 + \frac{5}{504}x^4 + \frac{1}{360}x^5 + \mathrm{O}\left(x^6\right)\right)$$

✓ Solution by Mathematica

Time used: 0.038 (sec). Leaf size: 176

```
AsymptoticDSolveValue[2*x^2*y''[x]-x*y'[x]+(1-x^2)*y[x]==Cos[x],y[x],{x,0,5}]
```

$$y(x) \to c_2x\left(\frac{x^6}{28080} + \frac{x^4}{360} + \frac{x^2}{10} + 1\right) + c_1\sqrt{x}\left(\frac{x^6}{11088} + \frac{x^4}{168} + \frac{x^2}{6} + 1\right) + \sqrt{x}\left(-\frac{x^{11/2}}{3861} + \frac{x^{7/2}}{630} + \frac{4x^{3/2}}{15} + \frac{2}{\sqrt{x}}\right)\left(\frac{x^6}{11088} + \frac{x^4}{168} + \frac{x^2}{6} + 1\right) + x\left(\frac{37x^5}{69300} - \frac{x^3}{84} - \frac{x}{3} - \frac{1}{x}\right)\left(\frac{x^6}{28080} + \frac{x^4}{360} + \frac{x^2}{10} + 1\right)$$

4.26 problem 24

Internal problem ID [7247]

Book: Own collection of miscellaneous problems
Section: section 4.0
Problem number: 24.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _linear, _nonhomogeneous]]`

$$2x^2y'' - xy' + (1 - x^2)\, y = \cos(x)\, x^3$$

With the expansion point for the power series method at $x = 0$.

✓ Solution by Maple

Time used: 0.016 (sec). Leaf size: 45

```
Order:=6;
dsolve(2*x^2*diff(y(x), x, x) - x*diff(y(x), x) + (-x^2 + 1)*y(x) = x^3*cos(x),y(x),type='se
```

$$y(x) = c_1\sqrt{x}\left(1 + \frac{1}{6}x^2 + \frac{1}{168}x^4 + \mathrm{O}(x^6)\right) + c_2 x\left(1 + \frac{1}{10}x^2 + \frac{1}{360}x^4 + \mathrm{O}(x^6)\right) + x^3\left(\frac{1}{10} - \frac{1}{90}x^2 + \mathrm{O}(x^4)\right)$$

✓ Solution by Mathematica

Time used: 0.138 (sec). Leaf size: 215

```
AsymptoticDSolveValue[2*x^2*y''[x]-x*y'[x]+(1-x^2)*y[x]==x^3+Cos[x],y[x],{x,0,5}]
```

$$y(x) \to c_1\sqrt{x}\left(\frac{x^6}{11088} + \frac{x^4}{168} + \frac{x^2}{6} + 1\right) + c_2 x\left(\frac{x^6}{28080} + \frac{x^4}{360} + \frac{x^2}{10} + 1\right) + \sqrt{x}\left(-\frac{x^{11/2}}{3861} - \frac{x^{9/2}}{45} + \frac{x^{7/2}}{630} - \frac{2x^{5/2}}{5} + \frac{4x^{3/2}}{15} + \frac{2}{\sqrt{x}}\right)\left(\frac{x^6}{11088} + \frac{x^4}{168} + \frac{x^2}{6} + 1\right) + x\left(\frac{x^6}{28080} + \frac{x^4}{360} + \frac{x^2}{10} + 1\right)\left(\frac{x^6}{1008} + \frac{37x^5}{69300} + \frac{x^4}{24} - \frac{x^3}{84} + \frac{x^2}{2} - \frac{x}{3} - \frac{1}{x}\right)$$

4.27 problem 24

Internal problem ID [7248]

Book: Own collection of miscellaneous problems
Section: section 4.0
Problem number: 24.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _linear, _nonhomogeneous]]`

$$2x^2y'' - xy' + (1 - x^2)\, y = \cos(x)\, x^3 + \sin(x)^2$$

With the expansion point for the power series method at $x = 0$.

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 47

```
Order:=6;
dsolve(2*x^2*diff(y(x), x, x) - x*diff(y(x), x) + (-x^2 + 1)*y(x) = x^3*cos(x)+sin(x)^2,y(x)
```

$$y(x) = c_1\sqrt{x}\left(1 + \frac{1}{6}x^2 + \frac{1}{168}x^4 + \mathrm{O}\left(x^6\right)\right) + c_2 x\left(1 + \frac{1}{10}x^2 + \frac{1}{360}x^4 + \mathrm{O}\left(x^6\right)\right) + x^2\left(\frac{1}{3} + \frac{1}{10}x - \frac{1}{90}x^3 + \mathrm{O}\left(x^4\right)\right)$$

✓ Solution by Mathematica

Time used: 0.514 (sec). Leaf size: 199

```
AsymptoticDSolveValue[2*x^2*y''[x]-x*y'[x]+(1-x^2)*y[x]==x^3*Cos[x]+Sin[x]^2,y[x],{x,0,5}]
```

$$y(x) \to c_2 x\left(\frac{x^6}{28080} + \frac{x^4}{360} + \frac{x^2}{10} + 1\right) + c_1\sqrt{x}\left(\frac{x^6}{11088} + \frac{x^4}{168} + \frac{x^2}{6} + 1\right) + \sqrt{x}\left(-\frac{x^{11/2}}{396} + \frac{4x^{9/2}}{45} + \frac{x^{7/2}}{15} - \frac{2x^{5/2}}{5} - \frac{2x^{3/2}}{3}\right)\left(\frac{x^6}{11088} + \frac{x^4}{168} + \frac{x^2}{6} + 1\right) + x\left(-\frac{x^6}{168} - \frac{13x^5}{12600} - \frac{x^4}{12} - \frac{x^3}{18} + \frac{x^2}{2} + x\right)\left(\frac{x^6}{28080} + \frac{x^4}{360} + \frac{x^2}{10} + 1\right)$$

4.28 problem 24

Internal problem ID [7249]

Book: Own collection of miscellaneous problems
Section: section 4.0
Problem number: 24.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _linear, _nonhomogeneous]]`

$$2x^2y'' - xy' + (1 - x^2)\, y = \ln(x)$$

With the expansion point for the power series method at $x = 1$.

✓ Solution by Maple
Time used: 0.0 (sec). Leaf size: 52

```
Order:=6;
dsolve(2*x^2*diff(y(x), x, x) - x*diff(y(x), x) + (-x^2 + 1)*y(x) = ln(x),y(x),type='series'
```

$$\begin{aligned} y(x) = {} & \left(1 + \frac{(x-1)^3}{6} - \frac{5(x-1)^4}{48} + \frac{37(x-1)^5}{480}\right) y(1) \\ & + \left(x - 1 + \frac{(x-1)^2}{4} - \frac{(x-1)^3}{24} + \frac{19(x-1)^4}{192} - \frac{119(x-1)^5}{1920}\right) D(y)(1) \\ & + \frac{(x-1)^3}{12} - \frac{3(x-1)^4}{32} + \frac{89(x-1)^5}{960} + O(x^6) \end{aligned}$$

✓ Solution by Mathematica

Time used: 0.023 (sec). Leaf size: 105

```
AsymptoticDSolveValue[2*x^2*y''[x]-x*y'[x]+(1-x^2)*y[x]==Log[x],y[x],{x,1,5}]
```

$$
\begin{aligned}
y(x) \rightarrow & \frac{89}{960}(x-1)^5-\frac{3}{32}(x-1)^4+\frac{1}{12}(x-1)^3 \\
& +c_1\left(\frac{37}{480}(x-1)^5-\frac{5}{48}(x-1)^4+\frac{1}{6}(x-1)^3+1\right) \\
& +c_2\left(-\frac{119(x-1)^5}{1920}+\frac{19}{192}(x-1)^4-\frac{1}{24}(x-1)^3+\frac{1}{4}(x-1)^2+x-1\right)
\end{aligned}
$$

4.29 problem 25

Internal problem ID [7250]

Book: Own collection of miscellaneous problems
Section: section 4.0
Problem number: 25.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$\boxed{2x^2(x^2+x+1)\,y''+x(11x^2+11x+9)\,y'+(7x^2+10x+6)\,y=0}$$

With the expansion point for the power series method at $x=0$.

✓ Solution by Maple

Time used: 0.015 (sec). Leaf size: 45

```
Order:=6;
dsolve(2*x^2*(1+x+x^2)*diff(y(x), x$2) + x*(9+11*x+11*x^2)*diff(y(x), x) + (6+10*x+7*x^2)*y(
```

$$y(x)=\frac{c_1\left(1+\frac{1}{2}x^2-\frac{1}{3}x^3+\frac{1}{8}x^4+\frac{1}{30}x^5+\mathrm{O}\left(x^6\right)\right)}{x^2}+\frac{c_2\left(1-\frac{1}{3}x+\frac{2}{5}x^2-\frac{5}{21}x^3+\frac{7}{135}x^4+\frac{76}{1155}x^5+\mathrm{O}\left(x^6\right)\right)}{x^{\frac{3}{2}}}$$

✓ Solution by Mathematica

Time used: 0.008 (sec). Leaf size: 83

```
AsymptoticDSolveValue[2*x^2*(1+x+x^2)*y''[x] + x*(9+11*x+11*x^2)*y'[x] + (6+10*x+7*x^2)*y[x]
```

$$y(x)\to\frac{c_2\left(\frac{x^5}{30}+\frac{x^4}{8}-\frac{x^3}{3}+\frac{x^2}{2}+1\right)}{x^2}+\frac{c_1\left(\frac{76x^5}{1155}+\frac{7x^4}{135}-\frac{5x^3}{21}+\frac{2x^2}{5}-\frac{x}{3}+1\right)}{x^{3/2}}$$

4.30 problem 26

Internal problem ID [7251]

Book: Own collection of miscellaneous problems
Section: section 4.0
Problem number: 26.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _exact, _linear, _homogeneous]]`

$$x^2(x+3)\,y''+5x(1+x)\,y'-(1-4x)\,y=0$$

With the expansion point for the power series method at $x=0$.

✓ Solution by Maple

Time used: 0.047 (sec). Leaf size: 47

```
Order:=6;
dsolve(x^2*(3+x)*diff(y(x), x$2) + 5*x*(1+x)*diff(y(x), x) - (1-4*x)*y(x) = 0,y(x),type='ser
```

$$y(x) = \frac{c_2 x^{\frac{4}{3}}\left(1-\frac{7}{9}x+\frac{35}{81}x^2-\frac{455}{2187}x^3+\frac{1820}{19683}x^4-\frac{6916}{177147}x^5+\mathrm{O}\left(x^6\right)\right)+c_1\left(1+x-x^2+\frac{3}{5}x^3-\frac{3}{10}x^4+\frac{3}{22}x^5+\mathrm{O}\right.}{x}$$

✓ Solution by Mathematica

Time used: 0.006 (sec). Leaf size: 82

```
AsymptoticDSolveValue[x^2*(3+x)*y''[x] + 5*x*(1+x)*y'[x] - (1-4*x)*y[x] == 0,y[x],{x,0,5}]
```

$$y(x)\to c_1\sqrt[3]{x}\left(-\frac{6916x^5}{177147}+\frac{1820x^4}{19683}-\frac{455x^3}{2187}+\frac{35x^2}{81}-\frac{7x}{9}+1\right)+\frac{c_2\left(\frac{3x^5}{22}-\frac{3x^4}{10}+\frac{3x^3}{5}-x^2+x+1\right)}{x}$$

4.31 problem 27

Internal problem ID [7252]

Book: Own collection of miscellaneous problems
Section: section 4.0
Problem number: 27.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$x^2(-x^2+2)\,y'' - x(4x^2+3)\,y' + (-2x^2+2)\,y = 0$$

With the expansion point for the power series method at $x = 0$.

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 35

```
Order:=6;
dsolve(x^2*(2-x^2)*diff(y(x), x$2) - x*(3+4*x^2)*diff(y(x), x) + (2-2*x^2)*y(x) = 0,y(x),typ
```

$$y(x) = c_1\sqrt{x}\left(1+\frac{15}{8}x^2+\frac{189}{128}x^4+\mathrm{O}\left(x^6\right)\right)+c_2x^2\left(1+\frac{6}{7}x^2+\frac{45}{77}x^4+\mathrm{O}\left(x^6\right)\right)$$

✓ Solution by Mathematica

Time used: 0.006 (sec). Leaf size: 50

```
AsymptoticDSolveValue[x^2*(2-x^2)*y''[x] - x*(3+4*x^2)*y'[x] + (2-2*x^2)*y[x] == 0,y[x],{x,0
```

$$y(x) \to c_1\left(\frac{45x^4}{77}+\frac{6x^2}{7}+1\right)x^2+c_2\left(\frac{189x^4}{128}+\frac{15x^2}{8}+1\right)\sqrt{x}$$

4.32 problem 28

Internal problem ID [7253]

Book: Own collection of miscellaneous problems
Section: section 4.0
Problem number: 28.
ODE order: 1.
ODE degree: 2.

CAS Maple gives this as type `[‘y=_G(x,y’)‘]`

$$\boxed{y'^2 + y^2 = \sec(x)^4}$$

✗ Solution by Maple

```
dsolve(diff(y(x),x)^2+y(x)^2=sec(x)^4,y(x), singsol=all)
```

No solution found

✗ Solution by Mathematica

Time used: 0.0 (sec). Leaf size: 0

```
DSolve[y'[x]^2+y[x]^2==Sec[x]^4,y[x],x,IncludeSingularSolutions -> True]
```

Not solved

4.33 problem 29

Internal problem ID [7254]

Book: Own collection of miscellaneous problems
Section: section 4.0
Problem number: 29.
ODE order: 1.
ODE degree: 3.

CAS Maple gives this as type `[[_1st_order, _with_linear_symmetries], _dAlembert]`

$$\boxed{(y - 2xy')^2 - y'^3 = 0}$$

✓ Solution by Maple

Time used: 0.094 (sec). Leaf size: 73

```
dsolve((y(x)-2*x*diff(y(x),x))^2= diff(y(x),x)^3,y(x), singsol=all)
```

$$y(x) = 0$$
$$\left[x(\_T) = \frac{3\_T^{\frac{5}{2}} + 5c_1}{5\_T^2}, y(\_T) = \frac{\_T^{\frac{5}{2}} + 10c_1}{5\_T}\right]$$
$$\left[x(\_T) = \frac{-3\_T^{\frac{5}{2}} + 5c_1}{5\_T^2}, y(\_T) = \frac{-\_T^{\frac{5}{2}} + 10c_1}{5\_T}\right]$$

✗ Solution by Mathematica

Time used: 0.0 (sec). Leaf size: 0

```
DSolve[(y[x]-2*x*y'[x])^2== y'[x]^3,y[x],x,IncludeSingularSolutions -> True]
```

Timed out

4.34 problem 31

Internal problem ID [7255]

Book: Own collection of miscellaneous problems
Section: section 4.0
Problem number: 31.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_Emden, _Fowler]]`

$$\boxed{x^2y'' + y = 0}$$

With the expansion point for the power series method at $x = 0$.

✓ Solution by Maple
Time used: 0.0 (sec). Leaf size: 41

```
Order:=6;
dsolve(x^2*diff(y(x), x$2) +y(x) = 0,y(x),type='series',x=0);
```

$$y(x) = \sqrt{x}\left(c_1 x^{-\frac{i\sqrt{3}}{2}} + c_2 x^{\frac{i\sqrt{3}}{2}}\right) + O(x^6)$$

✓ Solution by Mathematica
Time used: 0.003 (sec). Leaf size: 26

```
AsymptoticDSolveValue[x^2*y''[x] +y[x] == 0,y[x],{x,0,5}]
```

$$y(x) \to c_1 x^{-(-1)^{2/3}} + c_2 x^{\sqrt[3]{-1}}$$

4.35 problem 32

Internal problem ID [7256]

Book: Own collection of miscellaneous problems
Section: section 4.0
Problem number: 32.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_Emden, _Fowler]]`

$$xy'' + y' - y = 0$$

With the expansion point for the power series method at $x = 0$.

✓ Solution by Maple
Time used: 0.016 (sec). Leaf size: 59

```
Order:=6;
dsolve(x*diff(y(x), x$2) +diff(y(x),x)-y(x) = 0,y(x),type='series',x=0);
```

$$y(x) = (c_2 \ln(x) + c_1)\left(1 + x + \frac{1}{4}x^2 + \frac{1}{36}x^3 + \frac{1}{576}x^4 + \frac{1}{14400}x^5 + \mathrm{O}\left(x^6\right)\right) + \left((-2)\,x - \frac{3}{4}x^2 - \frac{11}{108}x^3 - \frac{25}{3456}x^4 - \frac{137}{432000}x^5 + \mathrm{O}\left(x^6\right)\right) c_2$$

✓ Solution by Mathematica
Time used: 0.003 (sec). Leaf size: 107

```
AsymptoticDSolveValue[x*y''[x] +y'[x]-y[x] == 0,y[x],{x,0,5}]
```

$$y(x) \to c_1\left(\frac{x^5}{14400} + \frac{x^4}{576} + \frac{x^3}{36} + \frac{x^2}{4} + x + 1\right) + c_2\left(-\frac{137x^5}{432000} - \frac{25x^4}{3456} - \frac{11x^3}{108} - \frac{3x^2}{4} + \left(\frac{x^5}{14400} + \frac{x^4}{576} + \frac{x^3}{36} + \frac{x^2}{4} + x + 1\right)\log(x) - 2x\right)$$

4.36 problem 33

Internal problem ID [7257]

Book: Own collection of miscellaneous problems
Section: section 4.0
Problem number: 33.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_Emden, _Fowler], [_2nd_order, _linear, '_with_symmetry_[0,F`

$$\boxed{4xy'' + 2y' + y = 0}$$

With the expansion point for the power series method at $x = 0$.

✓ Solution by Maple

Time used: 0.015 (sec). Leaf size: 44

```
Order:=6;
dsolve(4*x*diff(y(x), x$2) +2*diff(y(x),x)+y(x) = 0,y(x),type='series',x=0);
```

$$y(x) = c_1\sqrt{x}\left(1 - \frac{1}{6}x + \frac{1}{120}x^2 - \frac{1}{5040}x^3 + \frac{1}{362880}x^4 - \frac{1}{39916800}x^5 + \mathrm{O}\left(x^6\right)\right) + c_2\left(1 - \frac{1}{2}x + \frac{1}{24}x^2 - \frac{1}{720}x^3 + \frac{1}{40320}x^4 - \frac{1}{3628800}x^5 + \mathrm{O}\left(x^6\right)\right)$$

✓ Solution by Mathematica

Time used: 0.003 (sec). Leaf size: 85

```
AsymptoticDSolveValue[4*x*y''[x] +2*y'[x]+y[x] == 0,y[x],{x,0,5}]
```

$$y(x) \to c_1\sqrt{x}\left(-\frac{x^5}{39916800} + \frac{x^4}{362880} - \frac{x^3}{5040} + \frac{x^2}{120} - \frac{x}{6} + 1\right) + c_2\left(-\frac{x^5}{3628800} + \frac{x^4}{40320} - \frac{x^3}{720} + \frac{x^2}{24} - \frac{x}{2} + 1\right)$$

4.37 problem 34

Internal problem ID [7258]

Book: Own collection of miscellaneous problems
Section: section 4.0
Problem number: 34.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_Emden, _Fowler]]`

$$\boxed{xy'' + y' - y = 0}$$

With the expansion point for the power series method at $x = 0$.

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 59

```
Order:=6;
dsolve(x*diff(y(x), x$2) +diff(y(x),x)-y(x) = 0,y(x),type='series',x=0);
```

$$y(x) = (c_2 \ln(x) + c_1)\left(1 + x + \frac{1}{4}x^2 + \frac{1}{36}x^3 + \frac{1}{576}x^4 + \frac{1}{14400}x^5 + \mathrm{O}\left(x^6\right)\right) + \left((-2)\,x - \frac{3}{4}x^2 - \frac{11}{108}x^3 - \frac{25}{3456}x^4 - \frac{137}{432000}x^5 + \mathrm{O}\left(x^6\right)\right) c_2$$

✓ Solution by Mathematica

Time used: 0.003 (sec). Leaf size: 107

```
AsymptoticDSolveValue[x*y''[x] +y'[x]-y[x] == 0,y[x],{x,0,5}]
```

$$y(x) \to c_1\left(\frac{x^5}{14400} + \frac{x^4}{576} + \frac{x^3}{36} + \frac{x^2}{4} + x + 1\right) + c_2\left(-\frac{137x^5}{432000} - \frac{25x^4}{3456} - \frac{11x^3}{108} - \frac{3x^2}{4} + \left(\frac{x^5}{14400} + \frac{x^4}{576} + \frac{x^3}{36} + \frac{x^2}{4} + x + 1\right)\log(x) - 2x\right)$$

4.38 problem 35

Internal problem ID [7259]

Book: Own collection of miscellaneous problems
Section: section 4.0
Problem number: 35.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$\boxed{xy'' + (1+x)\,y' + 2y = 0}$$

With the expansion point for the power series method at $x = 0$.

✓ Solution by Maple

Time used: 0.016 (sec). Leaf size: 59

```
Order:=6;
dsolve(x*diff(y(x), x$2) +(1+x)*diff(y(x),x)+2*y(x) = 0,y(x),type='series',x=0);
```

$$y(x) = (c_2 \ln(x) + c_1)\left(1 - 2x + \frac{3}{2}x^2 - \frac{2}{3}x^3 + \frac{5}{24}x^4 - \frac{1}{20}x^5 + \mathrm{O}\left(x^6\right)\right) + \left(3x - \frac{13}{4}x^2 + \frac{31}{18}x^3 - \frac{173}{288}x^4 + \frac{187}{1200}x^5 + \mathrm{O}\left(x^6\right)\right)c_2$$

✓ Solution by Mathematica

Time used: 0.004 (sec). Leaf size: 111

```
AsymptoticDSolveValue[x*y''[x] +(1+x)*y'[x]+2*y[x] == 0,y[x],{x,0,5}]
```

$$y(x) \to c_1\left(-\frac{x^5}{20} + \frac{5x^4}{24} - \frac{2x^3}{3} + \frac{3x^2}{2} - 2x + 1\right) + c_2\left(\frac{187x^5}{1200} - \frac{173x^4}{288} + \frac{31x^3}{18} - \frac{13x^2}{4} + \left(-\frac{x^5}{20} + \frac{5x^4}{24} - \frac{2x^3}{3} + \frac{3x^2}{2} - 2x + 1\right)\log(x) + 3x\right)$$

4.39 problem 36

Internal problem ID [7260]

Book: Own collection of miscellaneous problems
Section: section 4.0
Problem number: 36.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _exact, _linear, _homogeneous]]`

$$\boxed{x(x-1)\,y''+3xy'+y=0}$$

With the expansion point for the power series method at $x=0$.

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 60

```
Order:=6;
dsolve(x*(x-1)*diff(y(x), x$2) +3*x*diff(y(x),x)+y(x) = 0,y(x),type='series',x=0);
```

$$\begin{aligned}y(x)=&\ c_1x\left(1+2x+3x^2+4x^3+5x^4+6x^5+\mathrm{O}\left(x^6\right)\right)\\&+\left(x+2x^2+3x^3+4x^4+5x^5+\mathrm{O}\left(x^6\right)\right)\ln\left(x\right)c_2\\&+\left(1+3x+5x^2+7x^3+9x^4+11x^5+\mathrm{O}\left(x^6\right)\right)c_2\end{aligned}$$

✓ Solution by Mathematica

Time used: 0.039 (sec). Leaf size: 63

```
AsymptoticDSolveValue[x*(x-1)*y''[x] +3*x*y'[x]+y[x] == 0,y[x],{x,0,5}]
```

$$\begin{aligned}y(x)\to&\ c_1\left(x^4+x^3+x^2+\left(4x^3+3x^2+2x+1\right)x\log(x)+x+1\right)\\&+c_2\left(5x^5+4x^4+3x^3+2x^2+x\right)\end{aligned}$$

4.40 problem 37

Internal problem ID [7261]

Book: Own collection of miscellaneous problems
Section: section 4.0
Problem number: 37.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$x^2(x^2-2x+1)\,y''-x(x+3)\,y'+(4+x)\,y=0$$

With the expansion point for the power series method at $x=0$.

✓ Solution by Maple

Time used: 0.016 (sec). Leaf size: 69

```
Order:=6;
dsolve(x^2*(1-2*x+x^2)*diff(y(x), x$2) -x*(3+x)*diff(y(x),x)+(4+x)*y(x) = 0,y(x),type='serie
```

$$y(x)=\left((c_2\ln(x)+c_1)\left(1+5x+17x^2+\frac{143}{3}x^3+\frac{355}{3}x^4+\frac{4043}{15}x^5+\mathrm{O}\left(x^6\right)\right)+\left((-3)\,x-\frac{29}{2}x^2-\frac{859}{18}x^3-\frac{4693}{36}x^4-\frac{285181}{900}x^5+\mathrm{O}\left(x^6\right)\right)c_2\right)x^2$$

✓ Solution by Mathematica

Time used: 0.009 (sec). Leaf size: 118

```
AsymptoticDSolveValue[x^2*(1-2*x+x^2)*y''[x] -x*(3+x)*y'[x]+(4+x)*y[x] == 0,y[x],{x,0,5}]
```

$$y(x)\to c_1\left(\frac{4043x^5}{15}+\frac{355x^4}{3}+\frac{143x^3}{3}+17x^2+5x+1\right)x^2+c_2\left(\left(-\frac{285181x^5}{900}-\frac{4693x^4}{36}-\frac{859x^3}{18}-\frac{29x^2}{2}-3x\right)x^2+\left(\frac{4043x^5}{15}+\frac{355x^4}{3}+\frac{143x^3}{3}+17x^2+5x+1\right)x^2\log(x)\right)$$

4.41 problem 38

Internal problem ID [7262]

Book: Own collection of miscellaneous problems
Section: section 4.0
Problem number: 38.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$2x^2(x+2)\,y''+5x^2y'+(1+x)\,y=0$$

With the expansion point for the power series method at $x=0$.

✓ Solution by Maple
Time used: 0.016 (sec). Leaf size: 69

```
Order:=6;
dsolve(2*x^2*(2+x)*diff(y(x), x$2) +5*x^2*diff(y(x),x)+(1+x)*y(x) = 0,y(x),type='series',x=0
```

$$y(x)=\left((c_2\ln(x)+c_1)\left(1-\frac{3}{4}x+\frac{15}{32}x^2-\frac{35}{128}x^3+\frac{315}{2048}x^4-\frac{693}{8192}x^5+\mathrm{O}\left(x^6\right)\right)+\left(\frac{1}{4}x-\frac{13}{64}x^2+\frac{101}{768}x^3-\frac{641}{8192}x^4+\frac{7303}{163840}x^5+\mathrm{O}\left(x^6\right)\right)c_2\right)\sqrt{x}$$

✓ Solution by Mathematica
Time used: 0.015 (sec). Leaf size: 134

```
AsymptoticDSolveValue[2*x^2*(2+x)*y''[x] +5*x^2*y'[x]+(1+x)*y[x] == 0,y[x],{x,0,5}]
```

$$y(x)\to c_1\sqrt{x}\left(-\frac{693x^5}{8192}+\frac{315x^4}{2048}-\frac{35x^3}{128}+\frac{15x^2}{32}-\frac{3x}{4}+1\right)+c_2\left(\sqrt{x}\left(\frac{7303x^5}{163840}-\frac{641x^4}{8192}+\frac{101x^3}{768}-\frac{13x^2}{64}+\frac{x}{4}\right)+\sqrt{x}\left(-\frac{693x^5}{8192}+\frac{315x^4}{2048}-\frac{35x^3}{128}+\frac{15x^2}{32}-\frac{3x}{4}+1\right)\log(x)\right)$$

4.42 problem 39

Internal problem ID [7263]

Book: Own collection of miscellaneous problems
Section: section 4.0
Problem number: 39.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$2x^2y'' + xy' + (x-5)\,y = 0$$

With the expansion point for the power series method at $x = 0$.

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 665

```
Order:=6;
dsolve(2*x^2*diff(y(x), x, x) +x*diff(y(x), x) +(x-5)*y(x) = 0,y(x),type='series',x=0);
```

$$y(x) = x^{\frac{1}{4}}\left(c_1 x^{-\frac{\sqrt{41}}{4}}\left(1 + \frac{1}{-2+\sqrt{41}}x + \frac{1}{2}\frac{1}{\left(-2+\sqrt{41}\right)\left(-4+\sqrt{41}\right)}x^2 + \frac{1}{6}\frac{1}{\left(-2+\sqrt{41}\right)\left(-4+\sqrt{41}\right)\left(-6+\sqrt{41}\right)}x^3 + \frac{1}{24}\frac{1}{\left(-2+\sqrt{41}\right)\left(-4+\sqrt{41}\right)\left(-6+\sqrt{41}\right)\left(-8+\sqrt{41}\right)}x^4 + \frac{1}{120}\frac{1}{\left(-2+\sqrt{41}\right)\left(-4+\sqrt{41}\right)\left(-6+\sqrt{41}\right)\left(-8+\sqrt{41}\right)\left(-10+\sqrt{41}\right)}x^5 + \mathrm{O}\left(x^6\right)\right) + c_2 x^{\frac{\sqrt{41}}{4}}\left(1 + \frac{1}{-2-\sqrt{41}}x + \frac{1}{2}\frac{1}{\left(2+\sqrt{41}\right)\left(4+\sqrt{41}\right)}x^2 - \frac{1}{6}\frac{1}{\left(2+\sqrt{41}\right)\left(4+\sqrt{41}\right)\left(6+\sqrt{41}\right)}x^3 + \frac{1}{24}\frac{1}{\left(2+\sqrt{41}\right)\left(4+\sqrt{41}\right)\left(6+\sqrt{41}\right)\left(8+\sqrt{41}\right)}x^4 - \frac{1}{120}\frac{1}{\left(2+\sqrt{41}\right)\left(4+\sqrt{41}\right)\left(6+\sqrt{41}\right)\left(8+\sqrt{41}\right)\left(10+\sqrt{41}\right)}x^5 + \mathrm{O}\left(x^6\right)\right)\right)$$

✓ Solution by Mathematica

Time used: 0.004 (sec). Leaf size: 1668

```
AsymptoticDSolveValue[2*x^2*y''[x]+x*y'[x]+(x-5)*y[x]==0,y[x],{x,0,5}]
```

Too large to display

4.43 problem 40

Internal problem ID [7264]

Book: Own collection of miscellaneous problems
Section: section 4.0
Problem number: 40.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _linear, _nonhomogeneous]]`

$$2x^2y'' + 2xy' - yx = \sin(x)$$

With the expansion point for the power series method at $x = 0$.

✓ Solution by Maple

Time used: 0.015 (sec). Leaf size: 77

```
Order:=6;
dsolve(2*x^2*diff(y(x), x, x) + 2*x*diff(y(x), x) - x*y(x) = sin(x),y(x),type='series',x=0);
```

$$y(x) = (c_2 \ln(x) + c_1)\left(1 + \frac{1}{2}x + \frac{1}{16}x^2 + \frac{1}{288}x^3 + \frac{1}{9216}x^4 + \frac{1}{460800}x^5 + \mathrm{O}\left(x^6\right)\right) + x\left(\frac{1}{2} + \frac{1}{16}x - \frac{5}{864}x^2 - \frac{5}{27648}x^3 + \frac{1127}{6912000}x^4 + \frac{1127}{497664000}x^5 + \mathrm{O}\left(x^6\right)\right) + \left(-x - \frac{3}{16}x^2 - \frac{11}{864}x^3 - \frac{25}{55296}x^4 - \frac{137}{13824000}x^5 + \mathrm{O}\left(x^6\right)\right)c_2$$

✓ Solution by Mathematica

Time used: 0.156 (sec). Leaf size: 340

```
AsymptoticDSolveValue[2*x^2*y''[x]+2*x*y'[x]-x*y[x]==Sin[x],y[x],{x,0,5}]
```

$$
\begin{aligned}
y(x) \to\; & c_2\left(\frac{x^5}{460800}+\frac{x^4}{9216}+\frac{x^3}{288}+\frac{x^2}{16}+\frac{x}{2}+1\right) \\
& + c_1\left(x^5\left(\frac{\log(x)}{460800}-\frac{107}{13824000}\right)+x^4\left(\frac{\log(x)}{9216}-\frac{19}{55296}\right)+x^3\left(\frac{\log(x)}{288}-\frac{1}{108}\right)\right. \\
& \left.+x^2\left(\frac{\log(x)}{16}-\frac{1}{8}\right)+x\left(\frac{\log(x)}{2}-\frac{1}{2}\right)+\log(x)+1\right)+\left(\frac{4963x^6}{16588800}-\frac{91x^5}{460800}\right. \\
& \left.-\frac{23x^4}{2304}-\frac{5x^3}{288}+\frac{x^2}{8}+\frac{x}{2}\right)\left(x^5\left(\frac{\log(x)}{460800}-\frac{107}{13824000}\right)+x^4\left(\frac{\log(x)}{9216}-\frac{19}{55296}\right)\right. \\
& \left.+x^3\left(\frac{\log(x)}{288}-\frac{1}{108}\right)+x^2\left(\frac{\log(x)}{16}-\frac{1}{8}\right)+x\left(\frac{\log(x)}{2}-\frac{1}{2}\right)+\log(x)+1\right) \\
& +\left(\frac{x^5}{460800}+\frac{x^4}{9216}+\frac{x^3}{288}+\frac{x^2}{16}+\frac{x}{2}+1\right)\left(\frac{x^6(66968-74445\log(x))}{248832000}\right. \\
& +\frac{13x^5(210\log(x)-3107)}{13824000}+\frac{x^4(276\log(x)-325)}{27648}+\frac{1}{864}x^3(15\log(x)+37) \\
& \left.+\frac{1}{16}x^2(3-2\log(x))-\frac{1}{2}x\log(x)\right)
\end{aligned}
$$

4.44 problem 41

Internal problem ID [7265]

Book: Own collection of miscellaneous problems
Section: section 4.0
Problem number: 41.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _linear, _nonhomogeneous]]`

$$2x^2y'' + 2xy' - yx = x \sin(x)$$

With the expansion point for the power series method at $x = 0$.

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 75

```
Order:=6;
dsolve(2*x^2*diff(y(x), x, x) + 2*x*diff(y(x), x) - x*y(x) = x*sin(x),y(x),type='series',x=0
```

$$y(x) = (c_2 \ln(x) + c_1)\left(1 + \frac{1}{2}x + \frac{1}{16}x^2 + \frac{1}{288}x^3 + \frac{1}{9216}x^4 + \frac{1}{460800}x^5 + \mathrm{O}\left(x^6\right)\right) + x^2\left(\frac{1}{8} + \frac{1}{144}x - \frac{23}{4608}x^2 - \frac{23}{230400}x^3 + \mathrm{O}\left(x^4\right)\right) + \left(-x - \frac{3}{16}x^2 - \frac{11}{864}x^3 - \frac{25}{55296}x^4 - \frac{137}{13824000}x^5 + \mathrm{O}\left(x^6\right)\right)c_2$$

✓ Solution by Mathematica

Time used: 0.186 (sec). Leaf size: 328

```
AsymptoticDSolveValue[2*x^2*y''[x]+2*x*y'[x]-x*y[x]==x*Sin[x],y[x],{x,0,5}]
```

$$y(x) \to c_2\left(\frac{x^5}{460800}+\frac{x^4}{9216}+\frac{x^3}{288}+\frac{x^2}{16}+\frac{x}{2}+1\right) + c_1\left(x^5\left(\frac{\log(x)}{460800}-\frac{107}{13824000}\right)+x^4\left(\frac{\log(x)}{9216}-\frac{19}{55296}\right)+x^3\left(\frac{\log(x)}{288}-\frac{1}{108}\right) + x^2\left(\frac{\log(x)}{16}-\frac{1}{8}\right)+x\left(\frac{\log(x)}{2}-\frac{1}{2}\right)+\log(x)+1\right)+\left(-\frac{91x^6}{552960}-\frac{23x^5}{2880} - \frac{5x^4}{384}+\frac{x^3}{12}+\frac{x^2}{4}\right)\left(x^5\left(\frac{\log(x)}{460800}-\frac{107}{13824000}\right)+x^4\left(\frac{\log(x)}{9216}-\frac{19}{55296}\right) + x^3\left(\frac{\log(x)}{288}-\frac{1}{108}\right)+x^2\left(\frac{\log(x)}{16}-\frac{1}{8}\right)+x\left(\frac{\log(x)}{2}-\frac{1}{2}\right)+\log(x)+1\right) + \left(\frac{x^5}{460800}+\frac{x^4}{9216}+\frac{x^3}{288}+\frac{x^2}{16}+\frac{x}{2}+1\right)\left(\frac{13x^6(21\log(x)-310)}{1658880} + \frac{x^5(345\log(x)-389)}{43200}+\frac{x^4(20\log(x)+51)}{1536}+\frac{1}{36}x^3(4-3\log(x)) + \frac{1}{8}x^2(-2\log(x)-1)\right)$$

4.45 problem 42

Internal problem ID [7266]

Book: Own collection of miscellaneous problems
Section: section 4.0
Problem number: 42.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _linear, _nonhomogeneous]]`

$$\boxed{2x^2y'' + 2xy' - yx = \sin(x)\cos(x)}$$

With the expansion point for the power series method at $x = 0$.

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 77

```
Order:=6;
dsolve(2*x^2*diff(y(x), x, x) + 2*x*diff(y(x), x) - x*y(x) = cos(x)*sin(x),y(x),type='series
```

$$\begin{aligned} y(x) = &\left(c_2 \ln(x) + c_1\right)\left(1 + \frac{1}{2}x + \frac{1}{16}x^2 + \frac{1}{288}x^3 + \frac{1}{9216}x^4 + \frac{1}{460800}x^5 + \mathrm{O}\left(x^6\right)\right) \\ &+ x\left(\frac{1}{2} + \frac{1}{16}x - \frac{29}{864}x^2 - \frac{29}{27648}x^3 + \frac{18287}{6912000}x^4 + \frac{18287}{497664000}x^5 + \mathrm{O}\left(x^6\right)\right) \\ &+ \left(-x - \frac{3}{16}x^2 - \frac{11}{864}x^3 - \frac{25}{55296}x^4 - \frac{137}{13824000}x^5 + \mathrm{O}\left(x^6\right)\right)c_2 \end{aligned}$$

✓ Solution by Mathematica

Time used: 0.157 (sec). Leaf size: 340

```
AsymptoticDSolveValue[2*x^2*y''[x]+2*x*y'[x]-x*y[x]==Cos[x]*Sin[x],y[x],{x,0,5}]
```

$$
\begin{aligned}
y(x) \to\ & c_2\left(\frac{x^5}{460800}+\frac{x^4}{9216}+\frac{x^3}{288}+\frac{x^2}{16}+\frac{x}{2}+1\right) \\
&+c_1\left(x^5\left(\frac{\log(x)}{460800}-\frac{107}{13824000}\right)+x^4\left(\frac{\log(x)}{9216}-\frac{19}{55296}\right)+x^3\left(\frac{\log(x)}{288}-\frac{1}{108}\right)\right. \\
&\left.+x^2\left(\frac{\log(x)}{16}-\frac{1}{8}\right)+x\left(\frac{\log(x)}{2}-\frac{1}{2}\right)+\log(x)+1\right)+\left(\frac{88963x^6}{16588800}+\frac{4229x^5}{460800}\right. \\
&\left.-\frac{95x^4}{2304}-\frac{29x^3}{288}+\frac{x^2}{8}+\frac{x}{2}\right)\left(x^5\left(\frac{\log(x)}{460800}-\frac{107}{13824000}\right)+x^4\left(\frac{\log(x)}{9216}-\frac{19}{55296}\right)\right. \\
&\left.+x^3\left(\frac{\log(x)}{288}-\frac{1}{108}\right)+x^2\left(\frac{\log(x)}{16}-\frac{1}{8}\right)+x\left(\frac{\log(x)}{2}-\frac{1}{2}\right)+\log(x)+1\right) \\
&+\left(\frac{x^5}{460800}+\frac{x^4}{9216}+\frac{x^3}{288}+\frac{x^2}{16}+\frac{x}{2}+1\right)\left(\frac{x^6(1476968-1334445\log(x))}{248832000}\right. \\
&+\frac{x^5(-126870\log(x)-273671)}{13824000}+\frac{5x^4(228\log(x)-281)}{27648} \\
&\left.+\frac{1}{864}x^3(87\log(x)+85)+\frac{1}{16}x^2(3-2\log(x))-\frac{1}{2}x\log(x)\right)
\end{aligned}
$$

4.46 problem 43

Internal problem ID [7267]

Book: Own collection of miscellaneous problems
Section: section 4.0
Problem number: 43.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _linear, _nonhomogeneous]]`

$$\boxed{2x^2y'' + 2xy' - yx = x^3 + x\sin(x)}$$

With the expansion point for the power series method at $x = 0$.

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 75

```
Order:=6;
dsolve(2*x^2*diff(y(x), x, x) + 2*x*diff(y(x), x) - x*y(x) = x^3+x*sin(x),y(x),type='series'
```

$$\begin{aligned} y(x) = &(c_2 \ln(x) + c_1)\left(1 + \frac{1}{2}x + \frac{1}{16}x^2 + \frac{1}{288}x^3 + \frac{1}{9216}x^4 + \frac{1}{460800}x^5 + \mathrm{O}\left(x^6\right)\right) \\ &+ x^2\left(\frac{1}{8} + \frac{1}{16}x - \frac{5}{1536}x^2 - \frac{1}{15360}x^3 + \mathrm{O}\left(x^4\right)\right) \\ &+ \left(-x - \frac{3}{16}x^2 - \frac{11}{864}x^3 - \frac{25}{55296}x^4 - \frac{137}{13824000}x^5 + \mathrm{O}\left(x^6\right)\right)c_2 \end{aligned}$$

✓ Solution by Mathematica

Time used: 0.296 (sec). Leaf size: 268

```
AsymptoticDSolveValue[2*x^2*y''[x]+2*x*y'[x]-x*y[x]==x^3*x*Sin[x],y[x],{x,0,5}]
```

$$
\begin{aligned}
y(x) \to\ & c_2\left(\frac{x^5}{460800}+\frac{x^4}{9216}+\frac{x^3}{288}+\frac{x^2}{16}+\frac{x}{2}+1\right) \\
& + c_1\left(x^5\left(\frac{\log(x)}{460800}-\frac{107}{13824000}\right)+x^4\left(\frac{\log(x)}{9216}-\frac{19}{55296}\right)+x^3\left(\frac{\log(x)}{288}-\frac{1}{108}\right)\right. \\
& \left.+x^2\left(\frac{\log(x)}{16}-\frac{1}{8}\right)+x\left(\frac{\log(x)}{2}-\frac{1}{2}\right)+\log(x)+1\right)+\left(\frac{x^5}{460800}+\frac{x^4}{9216}+\frac{x^3}{288}\right. \\
& \left.+\frac{x^2}{16}+\frac{x}{2}+1\right)\left(\frac{1}{144}x^6(7-6\log(x))+\frac{1}{50}x^5(-5\log(x)-4)\right) \\
& +\left(\frac{x^6}{24}+\frac{x^5}{10}\right)\left(x^5\left(\frac{\log(x)}{460800}-\frac{107}{13824000}\right)+x^4\left(\frac{\log(x)}{9216}-\frac{19}{55296}\right)\right. \\
& \left.+x^3\left(\frac{\log(x)}{288}-\frac{1}{108}\right)+x^2\left(\frac{\log(x)}{16}-\frac{1}{8}\right)+x\left(\frac{\log(x)}{2}-\frac{1}{2}\right)+\log(x)+1\right)
\end{aligned}
$$

4.47 problem 44

Internal problem ID [7268]

Book: Own collection of miscellaneous problems
Section: section 4.0
Problem number: 44.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$\boxed{y'' \cos(x) + 2xy' - yx = 0}$$

With the expansion point for the power series method at $x = 0$.

✓ Solution by Maple
Time used: 0.015 (sec). Leaf size: 39

```
Order:=6;
dsolve(cos(x)*diff(y(x), x, x) + 2*x*diff(y(x), x) - x*y(x) = 0,y(x),type='series',x=0);
```

$$y(x) = \left(1 + \frac{1}{6}x^3 - \frac{1}{40}x^5\right) y(0) + \left(x - \frac{1}{3}x^3 + \frac{1}{12}x^4 + \frac{1}{20}x^5\right) D(y)(0) + O(x^6)$$

✓ Solution by Mathematica
Time used: 0.002 (sec). Leaf size: 49

```
AsymptoticDSolveValue[Cos[x]*y''[x]+2*x*y'[x]-x*y[x]==0,y[x],{x,0,5}]
```

$$y(x) \to c_1\left(-\frac{x^5}{40} + \frac{x^3}{6} + 1\right) + c_2\left(\frac{x^5}{20} + \frac{x^4}{12} - \frac{x^3}{3} + x\right)$$

4.48 problem 45

Internal problem ID [7269]

Book: Own collection of miscellaneous problems
Section: section 4.0
Problem number: 45.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$\boxed{x^2y'' + 4xy' + (x^2 + 2)\, y = 0}$$

With the expansion point for the power series method at $x = 0$.

✓ Solution by Maple

Time used: 0.015 (sec). Leaf size: 35

```
Order:=6;
dsolve(x^2*diff(y(x), x, x) + 4*x*diff(y(x), x) + (x^2+2)*y(x) = 0,y(x),type='series',x=0);
```

$$y(x) = \frac{c_1\left(1 - \frac{1}{6}x^2 + \frac{1}{120}x^4 + \mathrm{O}\left(x^6\right)\right)x + c_2\left(1 - \frac{1}{2}x^2 + \frac{1}{24}x^4 + \mathrm{O}\left(x^6\right)\right)}{x^2}$$

✓ Solution by Mathematica

Time used: 0.011 (sec). Leaf size: 40

```
AsymptoticDSolveValue[x^2*y''[x]+4*x*y'[x]+(x^2+2)*y[x]==0,y[x],{x,0,5}]
```

$$y(x) \to c_2\left(\frac{x^3}{120} - \frac{x}{6} + \frac{1}{x}\right) + c_1\left(\frac{x^2}{24} + \frac{1}{x^2} - \frac{1}{2}\right)$$

4.49 problem 46

Internal problem ID [7270]

Book: Own collection of miscellaneous problems
Section: section 4.0
Problem number: 46.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_Emden, _Fowler]]`

$$\boxed{x^2y'' + xy' - yx = 0}$$

With the expansion point for the power series method at $x = 0$.

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 59

```
Order:=6;
dsolve(x^2*diff(y(x), x, x) + x*diff(y(x), x) - x*y(x) = 0,y(x),type='series',x=0);
```

$$y(x) = (c_2 \ln(x) + c_1)\left(1 + x + \frac{1}{4}x^2 + \frac{1}{36}x^3 + \frac{1}{576}x^4 + \frac{1}{14400}x^5 + \mathrm{O}\left(x^6\right)\right) + \left((-2)\,x - \frac{3}{4}x^2 - \frac{11}{108}x^3 - \frac{25}{3456}x^4 - \frac{137}{432000}x^5 + \mathrm{O}\left(x^6\right)\right)c_2$$

✓ Solution by Mathematica

Time used: 0.003 (sec). Leaf size: 107

```
AsymptoticDSolveValue[x^2*y''[x]+x*y'[x]-x*y[x]==0,y[x],{x,0,5}]
```

$$y(x) \to c_1\left(\frac{x^5}{14400} + \frac{x^4}{576} + \frac{x^3}{36} + \frac{x^2}{4} + x + 1\right) + c_2\left(-\frac{137x^5}{432000} - \frac{25x^4}{3456} - \frac{11x^3}{108} - \frac{3x^2}{4} + \left(\frac{x^5}{14400} + \frac{x^4}{576} + \frac{x^3}{36} + \frac{x^2}{4} + x + 1\right)\log(x) - 2x\right)$$

4.50 problem 47

Internal problem ID [7271]

Book: Own collection of miscellaneous problems
Section: section 4.0
Problem number: 47.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$x^2y'' + xy' + \left(x^2 - \frac{1}{4}\right)y = 0$$

With the expansion point for the power series method at $x = 0$.

✓ Solution by Maple

Time used: 0.015 (sec). Leaf size: 35

```
Order:=6;
dsolve(x^2*diff(y(x),x$2)+x*diff(y(x),x)+(x^2-1/4)*y(x) = 0,y(x),type='series',x=0);
```

$$y(x) = \frac{c_1\left(1 - \frac{1}{6}x^2 + \frac{1}{120}x^4 + \mathrm{O}\left(x^6\right)\right)x + c_2\left(1 - \frac{1}{2}x^2 + \frac{1}{24}x^4 + \mathrm{O}\left(x^6\right)\right)}{\sqrt{x}}$$

✓ Solution by Mathematica

Time used: 0.01 (sec). Leaf size: 58

```
AsymptoticDSolveValue[x^2*y''[x]+x*y'[x]+(x^2-1/4)*y[x]==0,y[x],{x,0,5}]
```

$$y(x) \to c_1\left(\frac{x^{7/2}}{24} - \frac{x^{3/2}}{2} + \frac{1}{\sqrt{x}}\right) + c_2\left(\frac{x^{9/2}}{120} - \frac{x^{5/2}}{6} + \sqrt{x}\right)$$

4.51 problem 48

Internal problem ID [7272]

Book: Own collection of miscellaneous problems
Section: section 4.0
Problem number: 48.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$\boxed{(x^2 - x)\, y'' - x y' + y = 0}$$

With the expansion point for the power series method at $x = 0$.

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 34

```
Order:=6;
dsolve((x^2-x)*diff(y(x), x$2)-x*diff(y(x), x)+y(x) = 0,y(x),type='series',x=0);
```

$$y(x) = \ln(x) \left(x + \mathrm{O}\left(x^6\right)\right) c_2 + c_1 x \left(1 + \mathrm{O}\left(x^6\right)\right) + \left(1 - x + \mathrm{O}\left(x^6\right)\right) c_2$$

✓ Solution by Mathematica

Time used: 0.047 (sec). Leaf size: 20

```
AsymptoticDSolveValue[(x^2-x)*y''[x]-x*y'[x]+y[x]==0,y[x],{x,0,5}]
```

$$y(x) \to c_2 x + c_1(-3x + x\log(x) + 1)$$

4.52 problem 49

Internal problem ID [7273]

Book: Own collection of miscellaneous problems
Section: section 4.0
Problem number: 49.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries], [_2nd_order, _linear,`

$$x^2y'' + (x^2 + 6x)\, y' + yx = 0$$

With the expansion point for the power series method at $x = 0$.

✓ Solution by Maple
Time used: 0.0 (sec). Leaf size: 44

```
Order:=6;
dsolve(x^2*diff(y(x), x$2)+(6*x+x^2)*diff(y(x), x)+x*y(x) = 0,y(x),type='series',x=0);
```

$$y(x) = c_1\left(1 - \frac{1}{6}x + \frac{1}{42}x^2 - \frac{1}{336}x^3 + \frac{1}{3024}x^4 - \frac{1}{30240}x^5 + \mathrm{O}\left(x^6\right)\right) + \frac{c_2(2880 - 2880x + 1440x^2 - 480x^3 + 120x^4 - 24x^5 + \mathrm{O}\left(x^6\right))}{x^5}$$

✓ Solution by Mathematica
Time used: 0.034 (sec). Leaf size: 68

```
AsymptoticDSolveValue[x^2*y''[x]+(6*x+x^2)*y'[x]+x*y[x]==0,y[x],{x,0,5}]
```

$$y(x) \to c_2\left(\frac{x^4}{3024} - \frac{x^3}{336} + \frac{x^2}{42} - \frac{x}{6} + 1\right) + c_1\left(\frac{1}{x^5} - \frac{1}{x^4} + \frac{1}{2x^3} - \frac{1}{6x^2} + \frac{1}{24x}\right)$$

4.53 problem 50

Internal problem ID [7274]

Book: Own collection of miscellaneous problems
Section: section 4.0
Problem number: 50.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$\boxed{x^2y'' - xy' + (x^2 - 8)\, y = 0}$$

With the expansion point for the power series method at $x = 0$.

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 35

```
Order:=6;
dsolve(x^2*diff(y(x), x$2)-x*diff(y(x), x)+(x^2-8)*y(x) = 0,y(x),type='series',x=0);
```

$$y(x) = c_1x^4\left(1 - \frac{1}{16}x^2 + \frac{1}{640}x^4 + \mathrm{O}\left(x^6\right)\right) + \frac{c_2(-86400 - 10800x^2 - 1350x^4 + \mathrm{O}\left(x^6\right))}{x^2}$$

✓ Solution by Mathematica

Time used: 0.01 (sec). Leaf size: 42

```
AsymptoticDSolveValue[x^2*y''[x]-x*y'[x]+(x^2-8)*y[x]==0,y[x],{x,0,5}]
```

$$y(x) \to c_1\left(\frac{x^2}{64} + \frac{1}{x^2} + \frac{1}{8}\right) + c_2\left(\frac{x^8}{640} - \frac{x^6}{16} + x^4\right)$$

4.54 problem 51

Internal problem ID [7275]

Book: Own collection of miscellaneous problems
Section: section 4.0
Problem number: 51.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_Emden, _Fowler]]`

$$\boxed{x^2y'' - 9xy' + 25y = 0}$$

With the expansion point for the power series method at $x = 0$.

✓ Solution by Maple
Time used: 0.0 (sec). Leaf size: 29

```
Order:=6;
dsolve(x^2*diff(y(x), x$2)-9*x*diff(y(x), x)+25*y(x) = 0,y(x),type='series',x=0);
```

$$y(x) = x^5(c_2 \ln(x) + c_1) + O(x^6)$$

✓ Solution by Mathematica
Time used: 0.002 (sec). Leaf size: 18

```
AsymptoticDSolveValue[x^2*y''[x]-9*x*y'[x]+25*y[x]==0,y[x],{x,0,5}]
```

$$y(x) \to c_1x^5 + c_2x^5 \log(x)$$

4.55 problem 52

Internal problem ID [7276]

Book: Own collection of miscellaneous problems
Section: section 4.0
Problem number: 52.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$\boxed{x^2y'' - xy' - \left(x^2 + \frac{5}{4}\right)y = 0}$$

With the expansion point for the power series method at $x = 0$.

✓ Solution by Maple

Time used: 0.016 (sec). Leaf size: 35

```
Order:=6;
dsolve(x^2*diff(y(x),x$2)-x*diff(y(x),x)-(x^2+5/4)*y(x) = 0,y(x),type='series',x=0);
```

$$y(x) = \frac{c_1 x^3 \left(1 + \frac{1}{10}x^2 + \frac{1}{280}x^4 + \mathrm{O}\left(x^6\right)\right) + c_2\left(12 - 6x^2 - \frac{3}{2}x^4 + \mathrm{O}\left(x^6\right)\right)}{\sqrt{x}}$$

✓ Solution by Mathematica

Time used: 0.011 (sec). Leaf size: 58

```
AsymptoticDSolveValue[x^2*y''[x]-x*y'[x]-(x^2+5/4)*y[x]==0,y[x],{x,0,5}]
```

$$y(x) \to c_1\left(-\frac{x^{7/2}}{8} - \frac{x^{3/2}}{2} + \frac{1}{\sqrt{x}}\right) + c_2\left(\frac{x^{13/2}}{280} + \frac{x^{9/2}}{10} + x^{5/2}\right)$$

4.56 problem 53

Internal problem ID [7277]

Book: Own collection of miscellaneous problems
Section: section 4.0
Problem number: 53.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$\boxed{x^2y'' + xy' + \left(x^2 - \frac{1}{4}\right)y = 0}$$

With the expansion point for the power series method at $x = 0$.

✓ Solution by Maple

Time used: 0.016 (sec). Leaf size: 35

```
Order:=6;
dsolve(x^2*diff(y(x),x$2)+x*diff(y(x),x)+(x^2-1/4)*y(x) = 0,y(x),type='series',x=0);
```

$$y(x) = \frac{c_1\left(1 - \frac{1}{6}x^2 + \frac{1}{120}x^4 + \mathrm{O}\left(x^6\right)\right)x + c_2\left(1 - \frac{1}{2}x^2 + \frac{1}{24}x^4 + \mathrm{O}\left(x^6\right)\right)}{\sqrt{x}}$$

✓ Solution by Mathematica

Time used: 0.011 (sec). Leaf size: 58

```
AsymptoticDSolveValue[x^2*y''[x]+x*y'[x]+(x^2-1/4)*y[x]==0,y[x],{x,0,5}]
```

$$y(x) \to c_1\left(\frac{x^{7/2}}{24} - \frac{x^{3/2}}{2} + \frac{1}{\sqrt{x}}\right) + c_2\left(\frac{x^{9/2}}{120} - \frac{x^{5/2}}{6} + \sqrt{x}\right)$$

4.57 problem 54

Internal problem ID [7278]

Book: Own collection of miscellaneous problems
Section: section 4.0
Problem number: 54.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _exact, _linear, _homogeneous]]`

$$\boxed{xy'' + (-x+2)\,y' - y = 0}$$

With the expansion point for the power series method at $x = 0$.

✓ Solution by Maple
Time used: 0.016 (sec). Leaf size: 44

```
Order:=6;
dsolve(x*diff(y(x),x$2)+(2-x)*diff(y(x),x)-y(x) = 0,y(x),type='series',x=0);
```

$$y(x) = c_1\left(1 + \frac{1}{2}x + \frac{1}{6}x^2 + \frac{1}{24}x^3 + \frac{1}{120}x^4 + \frac{1}{720}x^5 + \mathrm{O}\left(x^6\right)\right) + \frac{c_2\left(1 + x + \frac{1}{2}x^2 + \frac{1}{6}x^3 + \frac{1}{24}x^4 + \frac{1}{120}x^5 + \mathrm{O}\left(x^6\right)\right)}{x}$$

✓ Solution by Mathematica
Time used: 0.022 (sec). Leaf size: 62

```
AsymptoticDSolveValue[x*y''[x]+(2-x)*y'[x]-y[x]==0,y[x],{x,0,5}]
```

$$y(x) \to c_1\left(\frac{x^3}{24} + \frac{x^2}{6} + \frac{x}{2} + \frac{1}{x} + 1\right) + c_2\left(\frac{x^4}{120} + \frac{x^3}{24} + \frac{x^2}{6} + \frac{x}{2} + 1\right)$$

4.58 problem 55

Internal problem ID [7279]

Book: Own collection of miscellaneous problems
Section: section 4.0
Problem number: 55.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _exact, _linear, _homogeneous]]`

$$\boxed{2x^2y'' + 3xy' - y = 0}$$

With the expansion point for the power series method at $x = 0$.

✓ Solution by Maple
Time used: 0.015 (sec). Leaf size: 27

```
Order:=6;
dsolve(2*x^2*diff(y(x),x$2)+3*x*diff(y(x),x)-y(x) = 0,y(x),type='series',x=0);
```

$$y(x) = \frac{x^{\frac{3}{2}}c_2 + c_1}{x} + O(x^6)$$

✓ Solution by Mathematica
Time used: 0.003 (sec). Leaf size: 18

```
AsymptoticDSolveValue[2*x^2*y''[x]+3*x*y'[x]-y[x]==0,y[x],{x,0,5}]
```

$$y(x) \to c_1\sqrt{x} + \frac{c_2}{x}$$

4.59 problem 56

Internal problem ID [7280]

Book: Own collection of miscellaneous problems
Section: section 4.0
Problem number: 56.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_Emden, _Fowler]]`

$$2x^2y'' + 5xy' + 4y = 0$$

With the expansion point for the power series method at $x = 0$.

✓ Solution by Maple
Time used: 0.0 (sec). Leaf size: 41

```
Order:=6;
dsolve(2*x^2*diff(y(x),x$2)+5*x*diff(y(x),x)+4*y(x) = 0,y(x),type='series',x=0);
```

$$y(x) = \frac{x^{-\frac{i\sqrt{23}}{4}}c_1 + x^{\frac{i\sqrt{23}}{4}}c_2}{x^{\frac{3}{4}}} + O(x^6)$$

✓ Solution by Mathematica
Time used: 0.003 (sec). Leaf size: 44

```
AsymptoticDSolveValue[2*x^2*y''[x]+5*x*y'[x]+4*y[x]==0,y[x],{x,0,5}]
```

$$y(x) \to c_1x^{\frac{1}{4}\left(-3+i\sqrt{23}\right)} + c_2x^{\frac{1}{4}\left(-3-i\sqrt{23}\right)}$$

4.60 problem 57

Internal problem ID [7281]

Book: Own collection of miscellaneous problems
Section: section 4.0
Problem number: 57.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_Emden, _Fowler]]`

$$\boxed{x^2y'' + 3xy' + 4yx^4 = 0}$$

With the expansion point for the power series method at $x = 0$.

✓ Solution by Maple
Time used: 0.0 (sec). Leaf size: 28

```
Order:=6;
dsolve(x^2*diff(y(x),x$2)+3*x*diff(y(x),x)+4*x^4*y(x) = 0,y(x),type='series',x=0);
```

$$y(x) = c_1 \left(1 - \frac{1}{6}x^4 + \mathrm{O}\left(x^6\right)\right) + \frac{c_2(-2 + x^4 + \mathrm{O}\left(x^6\right))}{x^2}$$

✓ Solution by Mathematica
Time used: 0.008 (sec). Leaf size: 30

```
AsymptoticDSolveValue[x^2*y''[x]+3*x*y'[x]+4*x^4*y[x]==0,y[x],{x,0,5}]
```

$$y(x) \to c_2 \left(1 - \frac{x^4}{6}\right) + c_1 \left(\frac{1}{x^2} - \frac{x^2}{2}\right)$$

4.61 problem 58

Internal problem ID [7282]

Book: Own collection of miscellaneous problems
Section: section 4.0
Problem number: 58.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_Emden, _Fowler]]`

$$\boxed{x^2y'' - yx = 0}$$

With the expansion point for the power series method at $x = 0$.

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 58

```
Order:=6;
dsolve(x^2*diff(y(x),x$2)-x*y(x) = 0,y(x),type='series',x=0);
```

$$y(x) = c_1 x \left(1 + \frac{1}{2}x + \frac{1}{12}x^2 + \frac{1}{144}x^3 + \frac{1}{2880}x^4 + \frac{1}{86400}x^5 + \mathrm{O}\left(x^6\right)\right) + c_2 \left(\ln(x)\left(x + \frac{1}{2}x^2 + \frac{1}{12}x^3 + \frac{1}{144}x^4 + \frac{1}{2880}x^5 + \mathrm{O}\left(x^6\right)\right) + \left(1 - \frac{3}{4}x^2 - \frac{7}{36}x^3 - \frac{35}{1728}x^4 - \frac{101}{86400}x^5 + \mathrm{O}\left(x^6\right)\right)\right)$$

✓ Solution by Mathematica

Time used: 0.018 (sec). Leaf size: 85

```
AsymptoticDSolveValue[x^2*y''[x]-x*y[x]==0,y[x],{x,0,5}]
```

$$y(x) \to c_1 \left(\frac{1}{144}x\left(x^3 + 12x^2 + 72x + 144\right)\log(x) + \frac{-47x^4 - 480x^3 - 2160x^2 - 1728x + 1728}{1728}\right) + c_2\left(\frac{x^5}{2880} + \frac{x^4}{144} + \frac{x^3}{12} + \frac{x^2}{2} + x\right)$$

4.62 problem 59

Internal problem ID [7283]

Book: Own collection of miscellaneous problems
Section: section 4.0
Problem number: 59.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _linear, _nonhomogeneous]]`

$$\boxed{(1-x^2)\, y''+y'+y=x\,\mathrm{e}^{x}}$$

With the expansion point for the power series method at $x=0$.

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 53

```
Order:=6;
dsolve((1-x^2)*diff(y(x),x$2)+diff(y(x),x)+y(x)=x*exp(x),y(x),type='series',x=0);
```

$$y(x)=\left(1-\frac{1}{2}x^2+\frac{1}{6}x^3-\frac{1}{12}x^4+\frac{7}{120}x^5\right)y(0)+\left(x-\frac{1}{2}x^2-\frac{1}{24}x^4+\frac{1}{120}x^5\right)D(y)(0)+\frac{x^3}{6}+\frac{x^4}{24}+\frac{7x^5}{120}+O(x^6)$$

✓ Solution by Mathematica

Time used: 0.018 (sec). Leaf size: 63

```
AsymptoticDSolveValue[(1-x^2)*y''[x]+y'[x]+y[x]==x*Exp[x],y[x],{x,0,5}]
```

$$y(x)\to c_2\left(\frac{x^5}{120}-\frac{x^4}{24}-\frac{x^2}{2}+x\right)+c_1\left(\frac{7x^5}{120}-\frac{x^4}{12}+\frac{x^3}{6}-\frac{x^2}{2}+1\right)$$

4.63 problem 60

Internal problem ID [7284]

Book: Own collection of miscellaneous problems
Section: section 4.0
Problem number: 60.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_quadrature]`

$$\boxed{y' - y(1 - y^2) = 0}$$

✓ Solution by Maple
Time used: 0.0 (sec). Leaf size: 29

```
dsolve(diff(y(x),x)=y(x)*(1-y(x)^2),y(x), singsol=all)
```

$$y(x) = \frac{1}{\sqrt{e^{-2x}c_1 + 1}}$$
$$y(x) = -\frac{1}{\sqrt{e^{-2x}c_1 + 1}}$$

✓ Solution by Mathematica

Time used: 0.787 (sec). Leaf size: 100

```
DSolve[y'[x]==y[x]*(1-y[x]^2),y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to -\frac{e^x}{\sqrt{e^{2x} + e^{2c_1}}}$$
$$y(x) \to \frac{e^x}{\sqrt{e^{2x} + e^{2c_1}}}$$
$$y(x) \to -1$$
$$y(x) \to 0$$
$$y(x) \to 1$$
$$y(x) \to -\frac{e^x}{\sqrt{e^{2x}}}$$
$$y(x) \to \frac{e^x}{\sqrt{e^{2x}}}$$

4.64 problem 61

Internal problem ID [7285]

Book: Own collection of miscellaneous problems
Section: section 4.0
Problem number: 61.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _linear, _nonhomogeneous]]`

$$\boxed{\frac{xy''}{1-x}+y=\frac{1}{1-x}}$$

✓ Solution by Maple
Time used: 0.016 (sec). Leaf size: 167

```
dsolve(x/(1-x)*diff(y(x),x$2)+y(x)=1/(1-x),y(x), singsol=all)
```

$$y(x) = -x\Bigg((\mathrm{BesselK}(0,-x)$$
$$-\mathrm{BesselK}(1,-x))\left(\int\frac{-\mathrm{BesselI}(0,-x)-\mathrm{BesselI}(1,-x)}{x(\mathrm{BesselI}(0,x)(x+1)\mathrm{BesselK}(1,-x)+1-(x+1)\mathrm{BesselK}(0,-x)\mathrm{BesselI}(}\right.$$
$$+(-\mathrm{BesselI}(0,-x)$$
$$-\mathrm{BesselI}(1,-x))\left(\int\frac{-\mathrm{BesselK}(0,-x)+\mathrm{BesselK}(1,-x)}{(\mathrm{BesselI}(0,x)(x+1)\mathrm{BesselK}(1,-x)+1-(x+1)\mathrm{BesselK}(0,-x)\mathrm{BesselI}(1,x}\right.$$
$$-\mathrm{BesselK}(0,-x)c_1+\mathrm{BesselK}(1,-x)c_1-\mathrm{BesselI}(0,-x)c_2-\mathrm{BesselI}(1,-x)c_2\Bigg)$$

✓ Solution by Mathematica

Time used: 0.266 (sec). Leaf size: 136

```
DSolve[x/(1-x)*y''[x]+y[x]==1/(1-x),y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to e^{-x}x\Bigg(e^x(\text{BesselI}(0,x) - \text{BesselI}(1,x))\int_1^x 2e^{-K[1]}\sqrt{\pi}\,\text{HypergeometricU}\left(\frac{1}{2},2,2K[1]\right)dK[1] - 2\sqrt{\pi}x\,\text{HypergeometricU}\left(\frac{1}{2},2,2x\right){}_1F_2\left(\frac{1}{2};1,\frac{3}{2};\frac{x^2}{4}\right) + 2\sqrt{\pi}\,\text{HypergeometricU}\left(\frac{1}{2},2,2x\right)\text{BesselI}(0,x) + c_1\,\text{HypergeometricU}\left(\frac{1}{2},2,2x\right) + c_2e^x\,\text{BesselI}(0,x) - c_2e^x\,\text{BesselI}(1,x)\Bigg)$$

4.65 problem 62

Internal problem ID [7286]

Book: Own collection of miscellaneous problems
Section: section 4.0
Problem number: 62.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$\boxed{\frac{xy''}{1-x} + yx = 0}$$

✓ Solution by Maple
Time used: 0.0 (sec). Leaf size: 17

```
dsolve(x/(1-x)*diff(y(x),x$2)+x*y(x)=0,y(x), singsol=all)
```

$$y(x) = c_1 \,\mathrm{AiryAi}\,(x-1) + c_2 \,\mathrm{AiryBi}\,(x-1)$$

✓ Solution by Mathematica
Time used: 0.02 (sec). Leaf size: 20

```
DSolve[x/(1-x)*y''[x]+x*y[x]==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to c_1 \,\mathrm{AiryAi}(x-1) + c_2 \,\mathrm{AiryBi}(x-1)$$

4.66 problem 63

Internal problem ID [7287]

Book: Own collection of miscellaneous problems
Section: section 4.0
Problem number: 63.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _linear, _nonhomogeneous]]`

$$\boxed{\frac{xy''}{1-x}+y=\cos(x)}$$

✓ Solution by Maple

Time used: 0.078 (sec). Leaf size: 169

```
dsolve(x/(1-x)*diff(y(x),x$2)+y(x)=cos(x),y(x), singsol=all)
```

$$y(x) = -\left((\operatorname{BesselI}(0,-x)+\operatorname{BesselI}(1,-x))\left(\int -\frac{\cos(x)\,(\operatorname{BesselK}(0,-x)-\operatorname{BesselK}(1,-x))\,(x-1)}{x\,(\operatorname{BesselI}(0,x)\,(x+1)\operatorname{BesselK}(1,-x)+1-(x+1)\operatorname{BesselK}(0,-x)\operatorname{BesselI}(1,x))}dx\right)\right.$$
$$+(-\operatorname{BesselK}(0,-x)+\operatorname{BesselK}(1,-x))\left(\int -\frac{\cos(x)\,(\operatorname{BesselI}(0,x)-\operatorname{BesselI}(1,x))\,(x-1)}{x\,(\operatorname{BesselI}(0,x)\,(x+1)\operatorname{BesselK}(1,-x)+1-(x+1)\operatorname{BesselK}(0,-x)\operatorname{BesselI}(1,x))}dx\right)$$
$$\left.+\operatorname{BesselK}(1,-x)\,c_1-\operatorname{BesselK}(0,-x)\,c_1-\operatorname{BesselI}(0,-x)\,c_2-\operatorname{BesselI}(1,-x)\,c_2\right)x$$

✓ Solution by Mathematica

Time used: 8.805 (sec). Leaf size: 133

```
DSolve[x/(1-x)*y''[x]+y[x]==Cos[x],y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to e^{-x}x\Bigg(\text{HypergeometricU}\left(\frac{1}{2},2,2x\right)\int_1^x 2\sqrt{\pi}(\text{BesselI}(0,K[1]) - \text{BesselI}(1,K[1]))\cos(K[1])(K[1]-1)dK[1] + e^x(\text{BesselI}(0,x)-\text{BesselI}(1,x))\int_1^x -2e^{-K[2]}\sqrt{\pi}\cos(K[2])\,\text{HypergeometricU}\left(\frac{1}{2},2,2K[2]\right)(K[2]-1)dK[2] + c_1\,\text{HypergeometricU}\left(\frac{1}{2},2,2x\right) + c_2e^x\,\text{BesselI}(0,x) - c_2e^x\,\text{BesselI}(1,x)\Bigg)$$

4.67 problem 64

Internal problem ID [7288]

Book: Own collection of miscellaneous problems
Section: section 4.0
Problem number: 64.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$\frac{xy''}{1-x^2} + y = 0$$

✗ Solution by Maple

```
dsolve(x/(1-x^2)*diff(y(x),x$2)+y(x)=0,y(x), singsol=all)
```

No solution found

✗ Solution by Mathematica

Time used: 0.0 (sec). Leaf size: 0

```
DSolve[x/(1-x^2)*y''[x]+y[x]==0,y[x],x,IncludeSingularSolutions -> True]
```

Not solved

4.68 problem 65

Internal problem ID [7289]

Book: Own collection of miscellaneous problems
Section: section 4.0
Problem number: 65.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$\boxed{y'' - (x^2 + 3)\, y = 0}$$

✓ Solution by Maple
Time used: 0.015 (sec). Leaf size: 30

```
dsolve(diff(y(x),x$2)=(x^2+3)*y(x),y(x), singsol=all)
```

$$y(x) = x\left(c_2\sqrt{\pi}\ \mathrm{erf}\left(x\right) + c_1\right) \mathrm{e}^{\frac{x^2}{2}} + \mathrm{e}^{-\frac{x^2}{2}} c_2$$

✓ Solution by Mathematica
Time used: 0.125 (sec). Leaf size: 46

```
DSolve[y''[x]==(x^2+3)*y[x],y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to e^{-\frac{x^2}{2}} \left(-\sqrt{\pi} c_2 e^{x^2} x \mathrm{erf}(x) + c_1 e^{x^2} x - c_2\right)$$

4.69 problem 66

Internal problem ID [7290]

Book: Own collection of miscellaneous problems
Section: section 4.0
Problem number: 66.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$\boxed{y'' + (x-1)\,y = 0}$$

With the expansion point for the power series method at $x = 0$.

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 49

```
Order:=6;
dsolve(diff(y(x),x$2)+(x-1)*y(x)=0,y(x),type='series',x=0);
```

$$y(x) = \left(1+\frac{1}{2}x^2-\frac{1}{6}x^3+\frac{1}{24}x^4-\frac{1}{30}x^5\right)y(0) + \left(x+\frac{1}{6}x^3-\frac{1}{12}x^4+\frac{1}{120}x^5\right)D(y)(0)+O(x^6)$$

✓ Solution by Mathematica

Time used: 0.001 (sec). Leaf size: 63

```
AsymptoticDSolveValue[y''[x]+(x-1)*y[x]==0,y[x],{x,0,5}]
```

$$y(x) \to c_2\left(\frac{x^5}{120}-\frac{x^4}{12}+\frac{x^3}{6}+x\right)+c_1\left(-\frac{x^5}{30}+\frac{x^4}{24}-\frac{x^3}{6}+\frac{x^2}{2}+1\right)$$

4.70 problem 67

Internal problem ID [7291]

Book: Own collection of miscellaneous problems
Section: section 4.0
Problem number: 67.
ODE order: 1.
ODE degree: 1.

Solve

$$
\begin{aligned}
x'(t) &= x(t) + 2y(t) + 2t + 1 \\
y'(t) &= 5x(t) + y(t) + 3t - 1
\end{aligned}
$$

✓ Solution by Maple
Time used: 0.016 (sec). Leaf size: 68

```
dsolve([diff(x(t),t)=x(t)+2*y(t)+2*t+1,diff(y(t),t)=5*x(t)+y(t)+3*t-1],singsol=all)
```

$$
\begin{aligned}
x(t) &= \mathrm{e}^{\left(1+\sqrt{10}\right)t} c_2 + \mathrm{e}^{-\left(-1+\sqrt{10}\right)t} c_1 - \frac{4t}{9} + \frac{17}{81} \\
y(t) &= \frac{\mathrm{e}^{\left(1+\sqrt{10}\right)t} c_2 \sqrt{10}}{2} - \frac{\mathrm{e}^{-\left(-1+\sqrt{10}\right)t} c_1 \sqrt{10}}{2} - \frac{7t}{9} - \frac{67}{81}
\end{aligned}
$$

✓ Solution by Mathematica
Time used: 10.731 (sec). Leaf size: 158

```
DSolve[{x'[t]==x[t]+2*y[t]+2*t+1,y'[t]==5*x[t]+y[t]+3*t-1},{x[t],y[t]},t,IncludeSingularSolu
```

$$
\begin{aligned}
x(t) &\to \frac{1}{810} e^{t-\sqrt{10}t} \left(e^{\left(\sqrt{10}-1\right)t} (170 - 360t) + 81 \left(5c_1 + \sqrt{10} c_2 \right) e^{2\sqrt{10}t} + 81 \left(5c_1 - \sqrt{10} c_2 \right) \right) \\
y(t) &\to \frac{1}{324} e^{t-\sqrt{10}t} \left(-4e^{\left(\sqrt{10}-1\right)t} (63t + 67) + 81 \left(\sqrt{10} c_1 + 2c_2 \right) e^{2\sqrt{10}t} - 81 \left(\sqrt{10} c_1 - 2c_2 \right) \right)
\end{aligned}
$$

4.71 problem 68

Internal problem ID [7292]

Book: Own collection of miscellaneous problems
Section: section 4.0
Problem number: 68.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _linear, _nonhomogeneous]]`

$$y'' + 20y' + 500y = 100000\cos(100x)$$

✓ Solution by Maple

Time used: 0.031 (sec). Leaf size: 37

```
dsolve(diff(diff(y(x),x),x)+20*diff(y(x),x)+500*y(x) = 100000*cos(100*x),y(x), singsol=all)
```

$$y(x) = e^{-10x}\sin(20x)c_2 + e^{-10x}\cos(20x)c_1 - \frac{3800\cos(100x)}{377} + \frac{800\sin(100x)}{377}$$

✓ Solution by Mathematica

Time used: 0.033 (sec). Leaf size: 47

```
DSolve[y''[x]+20*y'[x]+500*y[x] == 100000*Cos[100*x],y[x],x,IncludeSingularSolutions -> True
```

$$y(x) \to -\frac{200}{377}(19\cos(100x) - 4\sin(100x)) + c_2e^{-10x}\cos(20x) + c_1e^{-10x}\sin(20x)$$

4.72 problem 69

Internal problem ID [7293]

Book: Own collection of miscellaneous problems
Section: section 4.0
Problem number: 69.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries], [_2nd_order, _linear,`

$$\boxed{y'' \sin(2x)^2 + y' \sin(4x) - 4y = 0}$$

✓ Solution by Maple
Time used: 0.063 (sec). Leaf size: 17

```
dsolve(diff(y(x),x$2)*sin(2*x)^2+diff(y(x),x)*sin(4*x)-4*y(x)=0,y(x), singsol=all)
```

$$y(x) = c_1 \csc(2x) + \cot(2x) c_2$$

✓ Solution by Mathematica
Time used: 0.063 (sec). Leaf size: 29

```
DSolve[y''[x]*Sin[2*x]^2+y'[x]*Sin[4*x]-4*y[x]==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to \frac{c_1 - ic_2 \cos(2x)}{\sqrt{\sin^2(2x)}}$$

5 section 5.0

5.1 problem 1

Internal problem ID [7294]

Book: Own collection of miscellaneous problems
Section: section 5.0
Problem number: 1.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _missing_x], [_2nd_order, _reducible, _mu_x_y1]]`

$$\boxed{y'' - Ay^{\frac{2}{3}} = 0}$$

✓ Solution by Maple

Time used: 0.031 (sec). Leaf size: 61

```
dsolve(diff(y(x),x$2)=A*y(x)^(2/3),y(x), singsol=all)
```

$$y(x) = 0$$
$$-5\left(\int^{y(x)} \frac{1}{\sqrt{30\_a^{\frac{5}{3}}A - 5c_1}}d\_a\right) - x - c_2 = 0$$
$$5\left(\int^{y(x)} \frac{1}{\sqrt{30\_a^{\frac{5}{3}}A - 5c_1}}d\_a\right) - x - c_2 = 0$$

✓ Solution by Mathematica

Time used: 0.108 (sec). Leaf size: 75

```
DSolve[y''[x]==A*y[x]^(2/3),y[x],x,IncludeSingularSolutions -> True]
```

$$\text{Solve}\left[\frac{y(x)^2\left(1+\frac{6Ay(x)^{5/3}}{5c_1}\right)\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{3}{5},\frac{8}{5},-\frac{6Ay(x)^{5/3}}{5c_1}\right)^2}{\frac{6}{5}Ay(x)^{5/3}+c_1} = (x+c_2)^2, y(x)\right]$$

5.2 problem 2

Internal problem ID [7295]

Book: Own collection of miscellaneous problems
Section: section 5.0
Problem number: 2.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$\boxed{y'' + 2xy' + (x^2 + 1)\, y = 0}$$

✓ Solution by Maple
Time used: 0.0 (sec). Leaf size: 16

```
dsolve(diff(y(x),x$2)+2*x*diff(y(x),x)+(x^2+1)*y(x)=0,y(x), singsol=all)
```

$$y(x) = \mathrm{e}^{-\frac{x^2}{2}} (c_2 x + c_1)$$

✓ Solution by Mathematica
Time used: 0.026 (sec). Leaf size: 22

```
DSolve[y''[x]+2*x*y'[x]+(x^2+1)*y[x]==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to e^{-\frac{x^2}{2}} (c_2 x + c_1)$$

5.3 problem 3

Internal problem ID [7296]

Book: Own collection of miscellaneous problems
Section: section 5.0
Problem number: 3.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$\boxed{y'' + 2\cot(x)\, y' - y = 0}$$

✓ Solution by Maple
Time used: 0.0 (sec). Leaf size: 12

```
dsolve(diff(y(x),x$2)+2*cot(x)*diff(y(x),x)-y(x)=0,y(x), singsol=all)
```

$$y(x) = \csc(x)\,(c_2 x + c_1)$$

✓ Solution by Mathematica
Time used: 0.058 (sec). Leaf size: 15

```
DSolve[y''[x]+2*Cot[x]*y'[x]-y[x]==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to (c_2 x + c_1)\csc(x)$$

5.4 problem 4

Internal problem ID [7297]

Book: Own collection of miscellaneous problems
Section: section 5.0
Problem number: 4.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$x^2y'' + xy' + \left(x^2 - \frac{1}{4}\right)y = 0$$

✓ Solution by Maple
Time used: 0.016 (sec). Leaf size: 17

```
dsolve(x^2*diff(diff(y(x),x),x)+x*diff(y(x),x)+(x^2-1/4)*y(x) = 0,y(x), singsol=all)
```

$$y(x) = \frac{c_1 \sin(x) + c_2 \cos(x)}{\sqrt{x}}$$

✓ Solution by Mathematica
Time used: 0.042 (sec). Leaf size: 39

```
DSolve[x^2*y''[x]+x*y'[x]+(x^2-1/4)*y[x] == 0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to \frac{e^{-ix}(2c_1 - ic_2e^{2ix})}{2\sqrt{x}}$$

5.5 problem 5

Internal problem ID [7298]

Book: Own collection of miscellaneous problems
Section: section 5.0
Problem number: 5.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _linear, _nonhomogeneous]]`

$$\boxed{4x^2y'' + (-8x^2 + 4x)\, y' + (4x^2 - 4x - 1)\, y = 4\sqrt{x}\, e^x}$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 21

```
dsolve(4*x^2*diff(diff(y(x),x),x)+(-8*x^2+4*x)*diff(y(x),x)+(4*x^2-4*x-1)*y(x) = 4*x^(1/2)*e
```

$$y(x) = \frac{(x\ln(x) + (-1 + c_1)\, x + c_2)\, e^x}{\sqrt{x}}$$

✓ Solution by Mathematica

Time used: 0.055 (sec). Leaf size: 27

```
DSolve[4*x^2*y''[x]+(-8*x^2+4*x)*y'[x]+(4*x^2-4*x-1)*y[x] == 4*x^(1/2)*Exp[x],y[x],x,Include
```

$$y(x) \to \frac{e^x(x\log(x) + (-1 + c_2)x + c_1)}{\sqrt{x}}$$

5.6 problem 6

Internal problem ID [7299]

Book: Own collection of miscellaneous problems
Section: section 5.0
Problem number: 6.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _linear, _nonhomogeneous]]`

$$\boxed{xy'' - (2x+2)\,y' + (x+2)\,y = 6x^3\mathrm{e}^x}$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 19

```
dsolve(x*diff(diff(y(x),x),x)-(2*x+2)*diff(y(x),x)+(2+x)*y(x) = 6*x^3*exp(x),y(x), singsol=a
```

$$y(x) = \mathrm{e}^x\left(c_2 + c_1x^3 + \frac{3}{2}x^4\right)$$

✓ Solution by Mathematica

Time used: 0.04 (sec). Leaf size: 29

```
DSolve[x*y''[x]-(2*x+2)*y'[x]+(2+x)*y[x] == 6*x^3*Exp[x],y[x],x,IncludeSingularSolutions ->
```

$$y(x) \to \frac{1}{6}e^x\left(9x^4 + 2c_2x^3 + 6c_1\right)$$

5.7 problem 7

Internal problem ID [7300]

Book: Own collection of miscellaneous problems
Section: section 5.0
Problem number: 7.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[[_linear, ‘class A‘]]`

$$\boxed{y' + y = \frac{1}{x}}$$

With the expansion point for the power series method at $x = 0$.

✗ Solution by Maple

```
Order:=6;
dsolve(diff(y(x),x)+y(x)=1/x,y(x),type='series',x=0);
```

No solution found

✓ Solution by Mathematica
Time used: 0.032 (sec). Leaf size: 113

```
AsymptoticDSolveValue[y'[x]+y[x]==1/x,y[x],{x,0,5}]
```

$$y(x) \rightarrow \left(-\frac{x^5}{120} + \frac{x^4}{24} - \frac{x^3}{6} + \frac{x^2}{2} - x + 1\right)\left(\frac{x^6}{2160} + \frac{x^5}{600} + \frac{x^4}{96} + \frac{x^3}{18} + \frac{x^2}{4} + x + \log(x)\right) + c_1\left(-\frac{x^5}{120} + \frac{x^4}{24} - \frac{x^3}{6} + \frac{x^2}{2} - x + 1\right)$$

5.8 problem 8

Internal problem ID [7301]

Book: Own collection of miscellaneous problems
Section: section 5.0
Problem number: 8.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[[_linear, 'class A']]`

$$\boxed{y' + y = \frac{1}{x^2}}$$

With the expansion point for the power series method at $x = 0$.

✗ Solution by Maple

```
Order:=6;
dsolve(diff(y(x),x)+y(x)=1/x^2,y(x),type='series',x=0);
```

No solution found

✓ Solution by Mathematica

Time used: 0.01 (sec). Leaf size: 122

```
AsymptoticDSolveValue[y'[x]+y[x]==1/x^2,y[x],{x,0,5}]
```

$$y(x) \to \left(-\frac{x^5}{120}+\frac{x^4}{24}-\frac{x^3}{6}+\frac{x^2}{2}-x+1\right)\left(\frac{x^6}{2160}+\frac{x^5}{1800}+\frac{x^4}{480}+\frac{x^3}{72}+\frac{x^2}{12}+\frac{x}{2}-\frac{1}{x}+\log(x)\right) + c_1\left(-\frac{x^5}{120}+\frac{x^4}{24}-\frac{x^3}{6}+\frac{x^2}{2}-x+1\right)$$

5.9 problem 9

Internal problem ID [7302]

Book: Own collection of miscellaneous problems
Section: section 5.0
Problem number: 9.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_separable]`

$$\boxed{xy' + y = 0}$$

With the expansion point for the power series method at $x = 0$.

✓ Solution by Maple
Time used: 0.0 (sec). Leaf size: 15

```
Order:=6;
dsolve(x*diff(y(x),x)+y(x)=0,y(x),type='series',x=0);
```

$$y(x) = \frac{c_1}{x} + O(x^6)$$

✓ Solution by Mathematica
Time used: 0.001 (sec). Leaf size: 9

```
AsymptoticDSolveValue[x*y'[x]+y[x]==0,y[x],{x,0,5}]
```

$$y(x) \to \frac{c_1}{x}$$

5.10 problem 10

Internal problem ID [7303]

Book: Own collection of miscellaneous problems
Section: section 5.0
Problem number: 10.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_quadrature]`

$$\boxed{y' = \frac{1}{x}}$$

With the expansion point for the power series method at $x = 0$.

✗ Solution by Maple

```
Order:=6;
dsolve(diff(y(x),x)=1/x,y(x),type='series',x=0);
```

No solution found

✓ Solution by Mathematica

Time used: 0.007 (sec). Leaf size: 8

```
AsymptoticDSolveValue[y'[x]==1/x,y[x],{x,0,5}]
```

$$y(x) \to \log(x) + c_1$$

5.11 problem 11

Internal problem ID [7304]

Book: Own collection of miscellaneous problems
Section: section 5.0
Problem number: 11.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _quadrature]]`

$$\boxed{y'' = \frac{1}{x}}$$

With the expansion point for the power series method at $x = 0$.

✗ Solution by Maple

```
Order:=6;
dsolve(diff(y(x),x$2)=1/x,y(x),type='series',x=0);
```

No solution found

✓ Solution by Mathematica
Time used: 0.026 (sec). Leaf size: 17

```
AsymptoticDSolveValue[y''[x]==1/x,y[x],{x,0,5}]
```

$$y(x) \rightarrow -x + x\log(x) + c_2x + c_1$$

5.12 problem 12

Internal problem ID [7305]

Book: Own collection of miscellaneous problems
Section: section 5.0
Problem number: 12.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _missing_y]]`

$$y'' + y' = \frac{1}{x}$$

With the expansion point for the power series method at $x = 0$.

✗ Solution by Maple

```
Order:=6;
dsolve(diff(y(x),x$2)+diff(y(x),x)=1/x,y(x),type='series',x=0);
```

No solution found

✓ Solution by Mathematica

Time used: 0.02 (sec). Leaf size: 159

```
AsymptoticDSolveValue[y''[x]+y'[x]==1/x,y[x],{x,0,5}]
```

$$y(x) \to -\frac{x^6}{4320} - \frac{x^5}{600} - \frac{x^4}{96} - \frac{x^3}{18} - \frac{x^2}{4} + c_2\left(-\frac{x^5}{720} + \frac{x^4}{120} - \frac{x^3}{24} + \frac{x^2}{6} - \frac{x}{2} + 1\right)x + \left(-\frac{x^5}{720} + \frac{x^4}{120} - \frac{x^3}{24} + \frac{x^2}{6} - \frac{x}{2} + 1\right)x\left(\frac{x^6}{2160} + \frac{x^5}{600} + \frac{x^4}{96} + \frac{x^3}{18} + \frac{x^2}{4} + x + \log(x)\right) - x + c_1$$

5.13 problem 13

Internal problem ID [7306]

Book: Own collection of miscellaneous problems
Section: section 5.0
Problem number: 13.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _linear, _nonhomogeneous]]`

$$\boxed{y'' + y = \frac{1}{x}}$$

With the expansion point for the power series method at $x = 0$.

✗ Solution by Maple

```
Order:=6;
dsolve(diff(y(x),x$2)+y(x)=1/x,y(x),type='series',x=0);
```

No solution found

✓ Solution by Mathematica
Time used: 0.015 (sec). Leaf size: 148

```
AsymptoticDSolveValue[y''[x]+y[x]==1/x,y[x],{x,0,5}]
```

$$\begin{aligned}
y(x) \to\ & x\left(-\frac{x^6}{5040} + \frac{x^4}{120} - \frac{x^2}{6} + 1\right)\left(-\frac{x^6}{4320} + \frac{x^4}{96} - \frac{x^2}{4} + \log(x)\right) \\
& + c_1\left(-\frac{x^6}{720} + \frac{x^4}{24} - \frac{x^2}{2} + 1\right) + c_2 x\left(-\frac{x^6}{5040} + \frac{x^4}{120} - \frac{x^2}{6} + 1\right) \\
& + \left(-\frac{x^5}{600} + \frac{x^3}{18} - x\right)\left(-\frac{x^6}{720} + \frac{x^4}{24} - \frac{x^2}{2} + 1\right)
\end{aligned}$$

5.14 problem 14

Internal problem ID [7307]

Book: Own collection of miscellaneous problems
Section: section 5.0
Problem number: 14.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _linear, _nonhomogeneous]]`

$$y'' + y' + y = \frac{1}{x}$$

With the expansion point for the power series method at $x = 0$.

✗ Solution by Maple

```
Order:=6;
dsolve(diff(y(x),x$2)+diff(y(x),x)+y(x)=1/x,y(x),type='series',x=0);
```

No solution found

✓ Solution by Mathematica

Time used: 0.055 (sec). Leaf size: 152

```
AsymptoticDSolveValue[y''[x]+y'[x]+y[x]==1/x,y[x],{x,0,5}]
```

$$\begin{aligned} y(x) \to\ & c_2 x\left(-\frac{x^4}{120}+\frac{x^3}{24}-\frac{x}{2}+1\right)+c_1\left(\frac{x^3}{6}-\frac{x^2}{2}+1\right) \\ & +x\left(-\frac{x^4}{120}+\frac{x^3}{24}-\frac{x}{2}+1\right)\left(\frac{41x^6}{4320}+\frac{x^5}{120}-\frac{x^4}{96}-\frac{x^3}{18}+x+\log(x)\right) \\ & +\left(\frac{x^3}{6}-\frac{x^2}{2}+1\right)\left(-\frac{x^6}{180}+\frac{x^5}{600}+\frac{x^4}{96}-\frac{x^2}{4}-x\right) \end{aligned}$$

5.15 problem 15

Internal problem ID [7308]

Book: Own collection of miscellaneous problems
Section: section 5.0
Problem number: 15.
ODE order: 1.
ODE degree: 2.

CAS Maple gives this as type `[_quadrature]`

$$\boxed{h^2 + \frac{2ah}{\sqrt{1 + h'^2}} = b^2}$$

✓ Solution by Maple
Time used: 0.438 (sec). Leaf size: 103

```
dsolve(h(u)^2 + 2*a*h(u)/sqrt(1 + diff(h(u), u)^2) = b^2,h(u), singsol=all)
```

$$u - \left(\int^{h(u)} \frac{\_a^2 - b^2}{\sqrt{-\_a^4 + (4a^2 + 2b^2)\_a^2 - b^4}} d\_a\right) - c_1 = 0$$
$$u + \int^{h(u)} \frac{\_a^2 - b^2}{\sqrt{-\_a^4 + (4a^2 + 2b^2)\_a^2 - b^4}} d\_a - c_1 = 0$$

✓ Solution by Mathematica

Time used: 24.41 (sec). Leaf size: 913

```
DSolve[h[u]^2 + 2*a*h[u]/Sqrt[1 + (h'[u])^2] == b^2,h[u],u,IncludeSingularSolutions -> True]
```

$$h(u) \to \text{InverseFunction}\left[-\frac{i\sqrt{\left(b^2-\#1^2\right)^2}\sqrt{1-\frac{\#1^2}{-2\sqrt{a^2(a^2+b^2)}+2a^2+b^2}}\sqrt{1-\frac{\#1^2}{2\sqrt{a^2(a^2+b^2)}+2a^2+b^2}}\left(\left(2\sqrt{a^2\left(a^2+b^2\right)}\right.\right.}{} + c_1\right]$$

$$h(u) \to \text{InverseFunction}\left[-\frac{i\sqrt{\left(b^2-\#1^2\right)^2}\sqrt{1-\frac{\#1^2}{-2\sqrt{a^2(a^2+b^2)}+2a^2+b^2}}\sqrt{1-\frac{\#1^2}{2\sqrt{a^2(a^2+b^2)}+2a^2+b^2}}\left(\left(2\sqrt{a^2\left(a^2+b^2\right)}\right.\right.}{} + c_1\right]$$

$$h(u) \to -\sqrt{a^2+b^2}-a$$

$$h(u) \to \sqrt{a^2+b^2}-a$$

5.16 problem 16

Internal problem ID [7309]

Book: Own collection of miscellaneous problems
Section: section 5.0
Problem number: 16.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _linear, _nonhomogeneous]]`

$$\boxed{y'' + 2y' - 24y = 16 - (x + 2)\,e^{4x}}$$

✓ Solution by Maple
Time used: 0.015 (sec). Leaf size: 36

```
dsolve(diff(y(x),x$2)+2*diff(y(x),x)-24*y(x)=16-(x+2)*exp(4*x),y(x), singsol=all)
```

$$y(x) = -\frac{\left(\left(x^2 + \frac{19}{5}x - 20c_2 - \frac{19}{50}\right)e^{10x} - 20c_1 + \frac{40\,e^{6x}}{3}\right)e^{-6x}}{20}$$

✓ Solution by Mathematica
Time used: 0.186 (sec). Leaf size: 41

```
DSolve[y''[x]+2*y'[x]-24*y[x]==16-(x+2)*Exp[4*x],y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to e^{4x}\left(-\frac{x^2}{20} - \frac{19x}{100} + \frac{19}{1000} + c_2\right) + c_1 e^{-6x} - \frac{2}{3}$$

5.17 problem 17

Internal problem ID [7310]

Book: Own collection of miscellaneous problems
Section: section 5.0
Problem number: 17.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$\boxed{y'' + 3y' - 4y = 6\,\mathrm{e}^{2t-2}}$$

With initial conditions

$$[y(1) = 4, y'(1) = 5]$$

✓ Solution by Maple
Time used: 0.297 (sec). Leaf size: 17

```
dsolve([diff(y(t),t$2)+3*diff(y(t),t)-4*y(t)=6*exp(2*t-2),y(1) = 4, D(y)(1) = 5],y(t), sings
```

$$y(t) = \mathrm{e}^{2t-2} + 3\,\mathrm{e}^{t-1}$$

✓ Solution by Mathematica
Time used: 0.078 (sec). Leaf size: 18

```
DSolve[{y''[t]+3*y'[t]-4*y[t]==6*Exp[2*t-2],{y[1]==4,y'[1]==5}},y[t],t,IncludeSingularSoluti
```

$$y(t) \to e^{t-2}(e^t + 3e)$$

5.18 problem 18

Internal problem ID [7311]

Book: Own collection of miscellaneous problems
Section: section 5.0
Problem number: 18.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _linear, _nonhomogeneous]]`

$$\boxed{y'' + y = \mathrm{e}^{a\cos(x)}}$$

With the expansion point for the power series method at $x = 0$.

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 76

```
Order:=8;
dsolve(diff(y(x),x$2)+y(x)=exp(a*cos(x)),y(x),type='series',x=0);
```

$$y(x) = \left(1 - \frac{1}{2}x^2 + \frac{1}{24}x^4 - \frac{1}{720}x^6\right) y(0) + \left(x - \frac{1}{6}x^3 + \frac{1}{120}x^5 - \frac{1}{5040}x^7\right) D(y)(0) + \frac{\mathrm{e}^a x^2}{2} + \frac{(-a-1)\,\mathrm{e}^a x^4}{24} + \frac{(3a^2+2a+1)\,\mathrm{e}^a x^6}{720} + O(x^8)$$

✓ Solution by Mathematica

Time used: 0.036 (sec). Leaf size: 239

```
AsymptoticDSolveValue[y''[x]+y[x]==Exp[a*Cos[x]],y[x],{x,0,7}]
```

$$y(x) \to \left(-\frac{x^7}{5040}+\frac{x^5}{120}-\frac{x^3}{6}+x\right)\left(\frac{1}{120}\left(3a^2+7a+1\right)e^a x^5 - \frac{\left(15a^3+60a^2+31a+1\right)e^a x^7}{5040} - \frac{1}{6}(a+1)e^a x^3 + e^a x\right) + \left(-\frac{x^6}{720}+\frac{x^4}{24}-\frac{x^2}{2}+1\right)\left(-\frac{1}{720}\left(15a^2+15a+1\right)e^a x^6 + \frac{\left(105a^3+210a^2+63a+1\right)e^a x^8}{40320} + \frac{1}{24}(3a+1)e^a x^4 - \frac{e^a x^2}{2}\right) + c_2\left(-\frac{x^7}{5040}+\frac{x^5}{120}-\frac{x^3}{6}+x\right) + c_1\left(-\frac{x^6}{720}+\frac{x^4}{24}-\frac{x^2}{2}+1\right)$$

5.19 problem 19

Internal problem ID [7312]

Book: Own collection of miscellaneous problems
Section: section 5.0
Problem number: 19.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[[_1st_order, _with_linear_symmetries]]`

$$\boxed{y' - \frac{y}{2y \ln(y) + y - x} = 0}$$

✓ Solution by Maple

Time used: 0.016 (sec). Leaf size: 19

```
dsolve(diff(y(x),x)=y(x)/(2*y(x)*ln(y(x))+y(x)-x),y(x), singsol=all)
```

$$y(x) = e^{\mathrm{RootOf}\left(\_Z\,e^{2\_Z} - x\,e^{\_Z} + c_1\right)}$$

✓ Solution by Mathematica

Time used: 0.198 (sec). Leaf size: 19

```
DSolve[y'[x]==y[x]/(2*y[x]*Log[y[x]]+y[x]-x),y[x],x,IncludeSingularSolutions -> True]
```

$$\text{Solve}\left[x = y(x)\log(y(x)) + \frac{c_1}{y(x)}, y(x)\right]$$

5.20 problem 20

Internal problem ID [7313]

Book: Own collection of miscellaneous problems
Section: section 5.0
Problem number: 20.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$\boxed{xy'' - (2x+1)\,y' + (1+x)\,y = 0}$$

✓ Solution by Maple
Time used: 0.0 (sec). Leaf size: 14

```
dsolve(x*diff(y(x),x$2)-(2*x+1)*diff(y(x),x)+(x+1)*y(x)=0,y(x), singsol=all)
```

$$y(x) = \mathrm{e}^{x}\left(c_2x^2 + c_1\right)$$

✓ Solution by Mathematica
Time used: 0.023 (sec). Leaf size: 23

```
DSolve[x*y''[x]-(2*x+1)*y'[x]+(x+1)*y[x]==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to \frac{1}{2}e^{x}\left(c_2x^2 + 2c_1\right)$$

5.21 problem 21

Internal problem ID [7314]

Book: Own collection of miscellaneous problems
Section: section 5.0
Problem number: 21.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_separable]`

$$\boxed{x^2y' + \mathrm{e}^{-y} = 0}$$

✓ Solution by Maple
Time used: 0.016 (sec). Leaf size: 15

```
dsolve(x^2*diff(y(x),x)+exp(-y(x))=0,y(x), singsol=all)
```

$$y(x) = \ln\left(\frac{-c_1x + 1}{x}\right)$$

✓ Solution by Mathematica
Time used: 0.441 (sec). Leaf size: 12

```
DSolve[x^2*y'[x]+Exp[-y[x]]==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to \log\left(\frac{1}{x} + c_1\right)$$

5.22 problem 22

Internal problem ID [7315]

Book: Own collection of miscellaneous problems
Section: section 5.0
Problem number: 22.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _missing_x], [_2nd_order, _reducible, _mu_x_y1]]`

$$\boxed{y'' + e^y = 0}$$

✓ Solution by Maple

Time used: 0.046 (sec). Leaf size: 25

```
dsolve(diff(y(x),x$2)+exp(y(x))=0,y(x), singsol=all)
```

$$y(x) = -\ln(2) + \ln\left(\frac{\operatorname{sech}\left(\frac{x+c_2}{2c_1}\right)^2}{c_1^2}\right)$$

✓ Solution by Mathematica

Time used: 29.642 (sec). Leaf size: 60

```
DSolve[y''[x]+Exp[y[x]]==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to \log\left(\frac{1}{2}c_1\operatorname{sech}^2\left(\frac{1}{2}\sqrt{c_1(x+c_2)^2}\right)\right)$$
$$y(x) \to \log\left(\frac{1}{2}c_1\operatorname{sech}^2\left(\frac{\sqrt{c_1x^2}}{2}\right)\right)$$

5.23 problem 23

Internal problem ID [7316]

Book: Own collection of miscellaneous problems
Section: section 5.0
Problem number: 23.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_rational, [_Abel, '2nd type', 'class B']]`

$$y' - \frac{yx + 3x - 2y + 6}{yx - 3x - 2y + 6} = 0$$

✗ Solution by Maple

```
dsolve(diff(y(x),x)=(x*y(x)+3*x-2*y(x)+6)/(x*y(x)-3*x-2*y(x)+6),y(x), singsol=all)
```

No solution found

✗ Solution by Mathematica

Time used: 0.0 (sec). Leaf size: 0

```
DSolve[y'[x]==(x*y[x]+3*x-2*y[x]+6)/(x*y[x]-3*x-2*y[x]+6),y[x],x,IncludeSingularSolutions ->
```

Not solved