A Solution Manual For

Second order enumerated odes

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May 16, 2024

Contents

1 section 1

1.1 problem 1

Internal problem ID [7390]

Book: Second order enumerated odes
Section: section 1
Problem number: 1.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _quadrature]]`

$$\boxed{y'' = 0}$$

✓ Solution by Maple
Time used: 0.0 (sec). Leaf size: 9

```
dsolve(diff(y(x),x$2)=0,y(x), singsol=all)
```

$$y(x) = c_1 x + c_2$$

✓ Solution by Mathematica
Time used: 0.002 (sec). Leaf size: 12

```
DSolve[y''[x]==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to c_2 x + c_1$$

1.2 problem 2

Internal problem ID [7391]

Book: Second order enumerated odes
Section: section 1
Problem number: 2.
ODE order: 2.
ODE degree: 2.

CAS Maple gives this as type `[[_2nd_order, _quadrature]]`

$$\boxed{y''^2 = 0}$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 9

```
dsolve(diff(y(x),x$2)^2=0,y(x), singsol=all)
```

$$y(x) = c_1x + c_2$$

✓ Solution by Mathematica

Time used: 0.002 (sec). Leaf size: 12

```
DSolve[(y''[x])^2==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to c_2x + c_1$$

1.3 problem 3

Internal problem ID [7392]

Book: Second order enumerated odes
Section: section 1
Problem number: 3.
ODE order: 2.
ODE degree: 0.

CAS Maple gives this as type `[[_2nd_order, _quadrature]]`

$$\boxed{y''^n = 0}$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 9

```
dsolve(diff(y(x),x$2)^n=0,y(x), singsol=all)
```

$$y(x) = c_1 x + c_2$$

✓ Solution by Mathematica

Time used: 0.003 (sec). Leaf size: 24

```
DSolve[(y''[x])^n==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to \frac{1}{2} 0^{\frac{1}{n}} x^2 + c_2 x + c_1$$

1.4 problem 4

Internal problem ID [7393]

Book: Second order enumerated odes
Section: section 1
Problem number: 4.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _quadrature]]`

$$\boxed{ay'' = 0}$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 9

```
dsolve(a*diff(y(x),x$2)=0,y(x), singsol=all)
```

$$y(x) = c_1 x + c_2$$

✓ Solution by Mathematica

Time used: 0.002 (sec). Leaf size: 12

```
DSolve[a*y''[x]==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to c_2 x + c_1$$

1.5 problem 5

Internal problem ID [7394]

Book: Second order enumerated odes
Section: section 1
Problem number: 5.
ODE order: 2.
ODE degree: 2.

CAS Maple gives this as type `[[_2nd_order, _quadrature]]`

$$\boxed{ay''^2 = 0}$$

✓ Solution by Maple
Time used: 0.015 (sec). Leaf size: 9

```
dsolve(a*diff(y(x),x$2)^2=0,y(x), singsol=all)
```

$$y(x) = c_1 x + c_2$$

✓ Solution by Mathematica
Time used: 0.002 (sec). Leaf size: 12

```
DSolve[a*(y''[x])^2==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to c_2 x + c_1$$

1.6 problem 6

Internal problem ID [7395]

Book: Second order enumerated odes
Section: section 1
Problem number: 6.
ODE order: 2.
ODE degree: 0.

CAS Maple gives this as type `[[_2nd_order, _quadrature]]`

$$\boxed{a{y''}^{n}=0}$$

✓ Solution by Maple
Time used: 0.0 (sec). Leaf size: 9

```
dsolve(a*diff(y(x),x$2)^n=0,y(x), singsol=all)
```

$$y(x) = c_1x + c_2$$

✓ Solution by Mathematica
Time used: 0.003 (sec). Leaf size: 24

```
DSolve[a*(y''[x])^n==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to \frac{1}{2}0^{\frac{1}{n}}x^2 + c_2x + c_1$$

1.7 problem 7

Internal problem ID [7396]

Book: Second order enumerated odes
Section: section 1
Problem number: 7.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _quadrature]]`

$$\boxed{y'' = 1}$$

✓ Solution by Maple
Time used: 0.016 (sec). Leaf size: 14

```
dsolve(diff(y(x),x$2)=1,y(x), singsol=all)
```

$$y(x) = \frac{1}{2}x^2 + c_1 x + c_2$$

✓ Solution by Mathematica
Time used: 0.002 (sec). Leaf size: 19

```
DSolve[y''[x]==1,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to \frac{x^2}{2} + c_2 x + c_1$$

1.8 problem 8

Internal problem ID [7397]

Book: Second order enumerated odes
Section: section 1
Problem number: 8.
ODE order: 2.
ODE degree: 2.

CAS Maple gives this as type `[[_2nd_order, _quadrature]]`

$$\boxed{y''^2 = 1}$$

✓ Solution by Maple
Time used: 0.015 (sec). Leaf size: 27

```
dsolve(diff(y(x),x$2)^2=1,y(x), singsol=all)
```

$$y(x) = \frac{1}{2}x^2 + c_1x + c_2$$
$$y(x) = -\frac{1}{2}x^2 + c_1x + c_2$$

✓ Solution by Mathematica
Time used: 0.002 (sec). Leaf size: 37

```
DSolve[(y''[x])^2==1,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to -\frac{x^2}{2} + c_2x + c_1$$
$$y(x) \to \frac{x^2}{2} + c_2x + c_1$$

1.9 problem 9

Internal problem ID [7398]

Book: Second order enumerated odes
Section: section 1
Problem number: 9.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _quadrature]]`

$$\boxed{y'' = x}$$

✓ Solution by Maple
Time used: 0.0 (sec). Leaf size: 14

```
dsolve(diff(y(x),x$2)=x,y(x), singsol=all)
```

$$y(x) = \frac{1}{6}x^3 + c_1 x + c_2$$

✓ Solution by Mathematica
Time used: 0.002 (sec). Leaf size: 19

```
DSolve[y''[x]==x,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to \frac{x^3}{6} + c_2 x + c_1$$

1.10 problem 10

Internal problem ID [7399]

Book: Second order enumerated odes
Section: section 1
Problem number: 10.
ODE order: 2.
ODE degree: 2.

CAS Maple gives this as type `[[_2nd_order, _quadrature]]`

$$\boxed{y''^2 = x}$$

✓ Solution by Maple

Time used: 0.016 (sec). Leaf size: 27

```
dsolve(diff(y(x),x$2)^2=x,y(x), singsol=all)
```

$$y(x) = \frac{4x^{\frac{5}{2}}}{15} + c_1 x + c_2$$
$$y(x) = -\frac{4x^{\frac{5}{2}}}{15} + c_1 x + c_2$$

✓ Solution by Mathematica

Time used: 0.004 (sec). Leaf size: 41

```
DSolve[(y''[x])^2==x,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to -\frac{4x^{5/2}}{15} + c_2 x + c_1$$
$$y(x) \to \frac{4x^{5/2}}{15} + c_2 x + c_1$$

1.11 problem 11

Internal problem ID [7400]

Book: Second order enumerated odes
Section: section 1
Problem number: 11.
ODE order: 2.
ODE degree: 3.

CAS Maple gives this as type `[[_2nd_order, _quadrature]]`

$$\boxed{y''^3 = 0}$$

✓ Solution by Maple

Time used: 0.016 (sec). Leaf size: 9

```
dsolve(diff(y(x),x$2)^3=0,y(x), singsol=all)
```

$$y(x) = c_1 x + c_2$$

✓ Solution by Mathematica

Time used: 0.002 (sec). Leaf size: 12

```
DSolve[(y''[x])^3==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to c_2 x + c_1$$

1.12 problem 12

Internal problem ID [7401]

Book: Second order enumerated odes
Section: section 1
Problem number: 12.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _missing_x]]`

$$\boxed{y'' + y' = 0}$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 12

```
dsolve(diff(y(x),x$2)+diff(y(x),x)=0,y(x), singsol=all)
```

$$y(x) = c_1 + \mathrm{e}^{-x} c_2$$

✓ Solution by Mathematica

Time used: 0.01 (sec). Leaf size: 17

```
DSolve[y''[x]+y'[x]==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to c_2 - c_1 e^{-x}$$

1.13 problem 13

Internal problem ID [7402]

Book: Second order enumerated odes
Section: section 1
Problem number: 13.
ODE order: 2.
ODE degree: 2.

CAS Maple gives this as type `[[_2nd_order, _missing_x]]`

$$\boxed{y''^2 + y' = 0}$$

✓ Solution by Maple
Time used: 0.078 (sec). Leaf size: 27

```
dsolve(diff(y(x),x$2)^2+diff(y(x),x)=0,y(x), singsol=all)
```

$$y(x) = c_1$$
$$y(x) = -\frac{1}{12}x^3 + \frac{1}{2}c_1x^2 - x\,c_1^2 + c_2$$

✓ Solution by Mathematica
Time used: 0.022 (sec). Leaf size: 69

```
DSolve[(y''[x])^2+y'[x]==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to -\frac{x^3}{12} - \frac{1}{4}ic_1x^2 + \frac{c_1{}^2x}{4} + c_2$$
$$y(x) \to -\frac{x^3}{12} + \frac{1}{4}ic_1x^2 + \frac{c_1{}^2x}{4} + c_2$$

1.14 problem 14

Internal problem ID [7403]

Book: Second order enumerated odes
Section: section 1
Problem number: 14.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _missing_x], _Liouville, [_2nd_order, _reducible`

$$\boxed{y'' + {y'}^2 = 0}$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 10

```
dsolve(diff(y(x),x$2)+diff(y(x),x)^2=0,y(x), singsol=all)
```

$$y(x) = \ln\left(c_1 x + c_2\right)$$

✓ Solution by Mathematica

Time used: 0.205 (sec). Leaf size: 15

```
DSolve[y''[x]+(y'[x])^2==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to \log(x - c_1) + c_2$$

1.15 problem 15

Internal problem ID [7404]

Book: Second order enumerated odes
Section: section 1
Problem number: 15.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _missing_x]]`

$$\boxed{y'' + y' = 1}$$

✓ Solution by Maple
Time used: 0.0 (sec). Leaf size: 14

```
dsolve(diff(y(x),x$2)+diff(y(x),x)=1,y(x), singsol=all)
```

$$y(x) = -\mathrm{e}^{-x} c_1 + x + c_2$$

✓ Solution by Mathematica
Time used: 0.012 (sec). Leaf size: 18

```
DSolve[y''[x]+y'[x]==1,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to x - c_1 e^{-x} + c_2$$

1.16 problem 16

Internal problem ID [7405]

Book: Second order enumerated odes
Section: section 1
Problem number: 16.
ODE order: 2.
ODE degree: 2.

CAS Maple gives this as type `[[_2nd_order, _missing_x]]`

$$\boxed{y''^2 + y' = 1}$$

✓ Solution by Maple

Time used: 0.078 (sec). Leaf size: 30

```
dsolve(diff(y(x),x$2)^2+diff(y(x),x)=1,y(x), singsol=all)
```

$$y(x) = x + c_1$$
$$y(x) = -\frac{1}{12}x^3 + \frac{1}{2}c_1x^2 - x\,c_1^2 + x + c_2$$

✓ Solution by Mathematica

Time used: 0.027 (sec). Leaf size: 67

```
DSolve[(y''[x])^2+y'[x]==1,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to -\frac{x^3}{12} - \frac{c_1x^2}{4} + x - \frac{{c_1}^2x}{4} + c_2$$
$$y(x) \to -\frac{x^3}{12} + \frac{c_1x^2}{4} + x - \frac{{c_1}^2x}{4} + c_2$$

1.17 problem 17

Internal problem ID [7406]

Book: Second order enumerated odes
Section: section 1
Problem number: 17.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _missing_x], [_2nd_order, _reducible, _mu_xy]]`

$$\boxed{y'' + {y'}^2 = 1}$$

✓ Solution by Maple

Time used: 0.016 (sec). Leaf size: 21

```
dsolve(diff(y(x),x$2)+diff(y(x),x)^2=1,y(x), singsol=all)
```

$$y(x) = x - \ln(2) + \ln\left(\mathrm{e}^{-2x} c_1 - c_2\right)$$

✓ Solution by Mathematica

Time used: 0.333 (sec). Leaf size: 46

```
DSolve[y''[x]+(y'[x])^2==1,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to -\log\left(e^x\right) + \log\left(e^{2x} + e^{2c_1}\right) + c_2$$
$$y(x) \to -\log\left(e^x\right) + \log\left(e^{2x}\right) + c_2$$

1.18 problem 18

Internal problem ID [7407]

Book: Second order enumerated odes
Section: section 1
Problem number: 18.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _missing_y]]`

$$\boxed{y'' + y' = x}$$

✓ Solution by Maple
Time used: 0.0 (sec). Leaf size: 21

```
dsolve(diff(y(x),x$2)+diff(y(x),x)=x,y(x), singsol=all)
```

$$y(x) = \frac{x^2}{2} - \mathrm{e}^{-x} c_1 - x + c_2$$

✓ Solution by Mathematica
Time used: 0.04 (sec). Leaf size: 27

```
DSolve[y''[x]+y'[x]==x,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to \frac{x^2}{2} - x - c_1 e^{-x} + c_2$$

1.19 problem 19

Internal problem ID [7408]

Book: Second order enumerated odes
Section: section 1
Problem number: 19.
ODE order: 2.
ODE degree: 2.

CAS Maple gives this as type `[[_2nd_order, _missing_y]]`

$$y''^2 + y' = x$$

✓ Solution by Maple
Time used: 0.047 (sec). Leaf size: 122

```
dsolve(diff(y(x),x$2)^2+diff(y(x),x)=x,y(x), singsol=all)
```

$$y(x) = \int \left(-e^{2\,\mathrm{RootOf}\left(\_Z - x - 2\,e^{\_Z} + 2 + c_1 - \ln\left(e^{\_Z}\left(e^{\_Z} - 2\right)^2\right)\right)} + 2\,e^{\mathrm{RootOf}\left(\_Z - x - 2\,e^{\_Z} + 2 + c_1 - \ln\left(e^{\_Z}\left(e^{\_Z} - 2\right)^2\right)\right)} + x\right) dx - x + c_2$$

$$y(x) = \frac{2\,\mathrm{LambertW}\left(-c_1 e^{-\frac{x}{2}-1}\right)^3}{3} + 3\,\mathrm{LambertW}\left(-c_1 e^{-\frac{x}{2}-1}\right)^2 + 4\,\mathrm{LambertW}\left(-c_1 e^{-\frac{x}{2}-1}\right) + \frac{x^2}{2} - x + c_2$$

✓ Solution by Mathematica

Time used: 24.995 (sec). Leaf size: 237

```
DSolve[(y''[x])^2+y'[x]==x,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to \frac{2}{3} W\left(e^{-\frac{x}{2}-1-\frac{c_1}{2}}\right)^3 + 3W\left(e^{-\frac{x}{2}-1-\frac{c_1}{2}}\right)^2 + 4W\left(e^{-\frac{x}{2}-1-\frac{c_1}{2}}\right) + \frac{x^2}{2} - x + c_2$$

$$y(x) \to \frac{2}{3} W\left(-e^{\frac{1}{2}(-x-2+c_1)}\right)^3 + 3W\left(-e^{\frac{1}{2}(-x-2+c_1)}\right)^2 + 4W\left(-e^{\frac{1}{2}(-x-2+c_1)}\right) + \frac{x^2}{2} - x + c_2$$

$$y(x) \to \frac{x^2}{2} - x + c_2$$

$$y(x) \to \frac{2}{3} W\left(-e^{-\frac{x}{2}-1}\right)^3 + 3W\left(-e^{-\frac{x}{2}-1}\right)^2 + 4W\left(-e^{-\frac{x}{2}-1}\right) + \frac{x^2}{2} - x + c_2$$

1.20 problem 20

Internal problem ID [7409]

Book: Second order enumerated odes
Section: section 1
Problem number: 20.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _missing_y], [_2nd_order, _reducible, _mu_xy]]`

$$\boxed{y'' + {y'}^2 = x}$$

✓ Solution by Maple
Time used: 0.0 (sec). Leaf size: 18

```
dsolve(diff(y(x),x$2)+diff(y(x),x)^2=x,y(x), singsol=all)
```

$$y(x) = \ln(\pi) + \ln(c_1 \operatorname{AiryAi}(x) - c_2 \operatorname{AiryBi}(x))$$

✓ Solution by Mathematica
Time used: 0.114 (sec). Leaf size: 15

```
DSolve[y''[x]+(y'[x])^2==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to \log(x - c_1) + c_2$$

1.21 problem 21

Internal problem ID [7410]

Book: Second order enumerated odes
Section: section 1
Problem number: 21.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _missing_x]]`

$$\boxed{y'' + y' + y = 0}$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 28

```
dsolve(diff(y(x),x$2)+diff(y(x),x)+y(x)=0,y(x), singsol=all)
```

$$y(x) = \mathrm{e}^{-\frac{x}{2}}\left(c_1 \sin\left(\frac{\sqrt{3}\,x}{2}\right) + c_2 \cos\left(\frac{\sqrt{3}\,x}{2}\right)\right)$$

✓ Solution by Mathematica

Time used: 0.026 (sec). Leaf size: 42

```
DSolve[y''[x]+y'[x]+y[x]==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to e^{-x/2}\left(c_2 \cos\left(\frac{\sqrt{3}x}{2}\right) + c_1 \sin\left(\frac{\sqrt{3}x}{2}\right)\right)$$

1.22 problem 22

Internal problem ID [7411]

Book: Second order enumerated odes
Section: section 1
Problem number: 22.
ODE order: 2.
ODE degree: 2.

CAS Maple gives this as type `[[_2nd_order, _missing_x]]`

$$\boxed{y''^2 + y' + y = 0}$$

✗ Solution by Maple

```
dsolve(diff(y(x),x$2)^2+diff(y(x),x)+y(x)=0,y(x), singsol=all)
```

No solution found

✗ Solution by Mathematica

Time used: 0.0 (sec). Leaf size: 0

```
DSolve[(y''[x])^2+y'[x]+y[x]==0,y[x],x,IncludeSingularSolutions -> True]
```

Not solved

1.23 problem 23

Internal problem ID [7412]

Book: Second order enumerated odes
Section: section 1
Problem number: 23.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _missing_x]]`

$$\boxed{y'' + y'^2 + y = 0}$$

✓ Solution by Maple

Time used: 0.016 (sec). Leaf size: 61

```
dsolve(diff(y(x),x$2)+diff(y(x),x)^2+y(x)=0,y(x), singsol=all)
```

$$-2\left(\int^{y(x)} \frac{1}{\sqrt{2 + 4\,\mathrm{e}^{-2\_a} c_1 - 4\_a}} d\_a\right) - x - c_2 = 0$$
$$2\left(\int^{y(x)} \frac{1}{\sqrt{2 + 4\,\mathrm{e}^{-2\_a} c_1 - 4\_a}} d\_a\right) - x - c_2 = 0$$

✓ Solution by Mathematica

Time used: 0.786 (sec). Leaf size: 272

```
DSolve[y''[x]+(y'[x])^2+y[x]==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x)\to \text{InverseFunction}\left[\int_1^{\#1}-\frac{\sqrt{2}}{\sqrt{2e^{-2K[1]}c_1-2K[1]+1}}dK[1]\&\right][x+c_2]$$

$$y(x)\to \text{InverseFunction}\left[\int_1^{\#1}\frac{\sqrt{2}}{\sqrt{2e^{-2K[2]}c_1-2K[2]+1}}dK[2]\&\right][x+c_2]$$

$$y(x)\to \text{InverseFunction}\left[\int_1^{\#1}-\frac{\sqrt{2}}{\sqrt{2e^{-2K[1]}(-c_1)-2K[1]+1}}dK[1]\&\right][x+c_2]$$

$$y(x)\to \text{InverseFunction}\left[\int_1^{\#1}-\frac{\sqrt{2}}{\sqrt{2e^{-2K[1]}c_1-2K[1]+1}}dK[1]\&\right][x+c_2]$$

$$y(x)\to \text{InverseFunction}\left[\int_1^{\#1}\frac{\sqrt{2}}{\sqrt{2e^{-2K[2]}(-c_1)-2K[2]+1}}dK[2]\&\right][x+c_2]$$

$$y(x)\to \text{InverseFunction}\left[\int_1^{\#1}\frac{\sqrt{2}}{\sqrt{2e^{-2K[2]}c_1-2K[2]+1}}dK[2]\&\right][x+c_2]$$

1.24 problem 24

Internal problem ID [7413]

Book: Second order enumerated odes
Section: section 1
Problem number: 24.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _missing_x]]`

$$\boxed{y'' + y' + y = 1}$$

✓ Solution by Maple
Time used: 0.0 (sec). Leaf size: 32

```
dsolve(diff(y(x),x$2)+diff(y(x),x)+y(x)=1,y(x), singsol=all)
```

$$y(x) = \mathrm{e}^{-\frac{x}{2}} \sin\left(\frac{\sqrt{3}\,x}{2}\right) c_2 + \mathrm{e}^{-\frac{x}{2}} \cos\left(\frac{\sqrt{3}\,x}{2}\right) c_1 + 1$$

✓ Solution by Mathematica
Time used: 0.02 (sec). Leaf size: 49

```
DSolve[y''[x]+y'[x]+y[x]==1,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to e^{-x/2}\left(e^{x/2} + c_2 \cos\left(\frac{\sqrt{3}x}{2}\right) + c_1 \sin\left(\frac{\sqrt{3}x}{2}\right)\right)$$

1.25 problem 25

Internal problem ID [7414]

Book: Second order enumerated odes
Section: section 1
Problem number: 25.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$\boxed{y'' + y' + y = x}$$

✓ Solution by Maple
Time used: 0.0 (sec). Leaf size: 33

```
dsolve(diff(y(x),x$2)+diff(y(x),x)+y(x)=x,y(x), singsol=all)
```

$$y(x) = \mathrm{e}^{-\frac{x}{2}} \sin\left(\frac{\sqrt{3}\, x}{2}\right) c_2 + \mathrm{e}^{-\frac{x}{2}} \cos\left(\frac{\sqrt{3}\, x}{2}\right) c_1 + x - 1$$

✓ Solution by Mathematica
Time used: 0.02 (sec). Leaf size: 50

```
DSolve[y''[x]+y'[x]+y[x]==x,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to x + c_2 e^{-x/2} \cos\left(\frac{\sqrt{3}x}{2}\right) + c_1 e^{-x/2} \sin\left(\frac{\sqrt{3}x}{2}\right) - 1$$

1.26 problem 26

Internal problem ID [7415]

Book: Second order enumerated odes
Section: section 1
Problem number: 26.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$\boxed{y'' + y' + y = 1 + x}$$

✓ Solution by Maple
Time used: 0.0 (sec). Leaf size: 32

```
dsolve(diff(y(x),x$2)+diff(y(x),x)+y(x)=1+x,y(x), singsol=all)
```

$$y(x) = \mathrm{e}^{-\frac{x}{2}} \sin\left(\frac{\sqrt{3}\, x}{2}\right) c_2 + \mathrm{e}^{-\frac{x}{2}} \cos\left(\frac{\sqrt{3}\, x}{2}\right) c_1 + x$$

✓ Solution by Mathematica
Time used: 0.019 (sec). Leaf size: 49

```
DSolve[y''[x]+y'[x]+y[x]==1+x,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to x + c_2 e^{-x/2} \cos\left(\frac{\sqrt{3}x}{2}\right) + c_1 e^{-x/2} \sin\left(\frac{\sqrt{3}x}{2}\right)$$

1.27 problem 27

Internal problem ID [7416]

Book: Second order enumerated odes
Section: section 1
Problem number: 27.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$y'' + y' + y = x^2 + x + 1$$

✓ Solution by Maple
Time used: 0.0 (sec). Leaf size: 37

```
dsolve(diff(y(x),x$2)+diff(y(x),x)+y(x)=1+x+x^2,y(x), singsol=all)
```

$$y(x) = \mathrm{e}^{-\frac{x}{2}} \sin\left(\frac{\sqrt{3}\,x}{2}\right) c_2 + \mathrm{e}^{-\frac{x}{2}} \cos\left(\frac{\sqrt{3}\,x}{2}\right) c_1 + x^2 - x$$

✓ Solution by Mathematica
Time used: 0.021 (sec). Leaf size: 54

```
DSolve[y''[x]+y'[x]+y[x]==1+x+x^2,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to e^{-x/2}\left(e^{x/2}(x-1)x + c_2 \cos\left(\frac{\sqrt{3}x}{2}\right) + c_1 \sin\left(\frac{\sqrt{3}x}{2}\right)\right)$$

1.28 problem 28

Internal problem ID [7417]

Book: Second order enumerated odes
Section: section 1
Problem number: 28.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _linear, _nonhomogeneous]]`

$$\boxed{y'' + y' + y = x^3 + x^2 + x + 1}$$

✓ Solution by Maple
Time used: 0.0 (sec). Leaf size: 43

```
dsolve(diff(y(x),x$2)+diff(y(x),x)+y(x)=1+x+x^2+x^3,y(x), singsol=all)
```

$$y(x) = e^{-\frac{x}{2}} \sin\left(\frac{\sqrt{3}\, x}{2}\right) c_2 + e^{-\frac{x}{2}} \cos\left(\frac{\sqrt{3}\, x}{2}\right) c_1 + x^3 - 2x^2 - x + 6$$

✓ Solution by Mathematica
Time used: 0.02 (sec). Leaf size: 60

```
DSolve[y''[x]+y'[x]+y[x]==1+x+x^2+x^3,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to x^3 - 2x^2 - x + c_2 e^{-x/2} \cos\left(\frac{\sqrt{3}x}{2}\right) + c_1 e^{-x/2} \sin\left(\frac{\sqrt{3}x}{2}\right) + 6$$

1.29 problem 29

Internal problem ID [7418]

Book: Second order enumerated odes
Section: section 1
Problem number: 29.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _linear, _nonhomogeneous]]`

$$\boxed{y'' + y' + y = \sin(x)}$$

✓ Solution by Maple
Time used: 0.016 (sec). Leaf size: 35

```
dsolve(diff(y(x),x$2)+diff(y(x),x)+y(x)=sin(x),y(x), singsol=all)
```

$$y(x) = \mathrm{e}^{-\frac{x}{2}} \sin\left(\frac{\sqrt{3}\,x}{2}\right) c_2 + \mathrm{e}^{-\frac{x}{2}} \cos\left(\frac{\sqrt{3}\,x}{2}\right) c_1 - \cos(x)$$

✓ Solution by Mathematica
Time used: 0.309 (sec). Leaf size: 53

```
DSolve[y''[x]+y'[x]+y[x]==Sin[x],y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to e^{-x/2}\left(-e^{x/2}\cos(x) + c_2\cos\left(\frac{\sqrt{3}x}{2}\right) + c_1\sin\left(\frac{\sqrt{3}x}{2}\right)\right)$$

1.30 problem 30

Internal problem ID [7419]

Book: Second order enumerated odes
Section: section 1
Problem number: 30.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _linear, _nonhomogeneous]]`

$$\boxed{y'' + y' + y = \cos(x)}$$

✓ Solution by Maple
Time used: 0.0 (sec). Leaf size: 33

```
dsolve(diff(y(x),x$2)+diff(y(x),x)+y(x)=cos(x),y(x), singsol=all)
```

$$y(x) = \mathrm{e}^{-\frac{x}{2}} \sin\left(\frac{\sqrt{3}\, x}{2}\right) c_2 + \mathrm{e}^{-\frac{x}{2}} \cos\left(\frac{\sqrt{3}\, x}{2}\right) c_1 + \sin(x)$$

✓ Solution by Mathematica
Time used: 0.63 (sec). Leaf size: 50

```
DSolve[y''[x]+y'[x]+y[x]==Cos[x],y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to \sin(x) + c_2 e^{-x/2} \cos\left(\frac{\sqrt{3}x}{2}\right) + c_1 e^{-x/2} \sin\left(\frac{\sqrt{3}x}{2}\right)$$

1.31 problem 31

Internal problem ID [7420]

Book: Second order enumerated odes
Section: section 1
Problem number: 31.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _missing_x]]`

$$\boxed{y'' + y' = 1}$$

✓ Solution by Maple
Time used: 0.0 (sec). Leaf size: 14

```
dsolve(diff(y(x),x$2)+diff(y(x),x)=1,y(x), singsol=all)
```

$$y(x) = -\mathrm{e}^{-x} c_1 + x + c_2$$

✓ Solution by Mathematica
Time used: 0.012 (sec). Leaf size: 18

```
DSolve[y''[x]+y'[x]==1,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to x - c_1 e^{-x} + c_2$$

1.32 problem 32

Internal problem ID [7421]

Book: Second order enumerated odes
Section: section 1
Problem number: 32.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _missing_y]]`

$$\boxed{y'' + y' = x}$$

✓ Solution by Maple

Time used: 0.015 (sec). Leaf size: 21

```
dsolve(diff(y(x),x$2)+diff(y(x),x)=x,y(x), singsol=all)
```

$$y(x) = \frac{x^2}{2} - \mathrm{e}^{-x} c_1 - x + c_2$$

✓ Solution by Mathematica

Time used: 0.036 (sec). Leaf size: 27

```
DSolve[y''[x]+y'[x]==x,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \rightarrow \frac{x^2}{2} - x - c_1 e^{-x} + c_2$$

1.33 problem 33

Internal problem ID [7422]

Book: Second order enumerated odes
Section: section 1
Problem number: 33.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _missing_y]]`

$$\boxed{y'' + y' = 1 + x}$$

✓ Solution by Maple
Time used: 0.016 (sec). Leaf size: 18

```
dsolve(diff(y(x),x$2)+diff(y(x),x)=1+x,y(x), singsol=all)
```

$$y(x) = \frac{x^2}{2} - \mathrm{e}^{-x}c_1 + c_2$$

✓ Solution by Mathematica
Time used: 0.031 (sec). Leaf size: 24

```
DSolve[y''[x]+y'[x]==1+x,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to \frac{x^2}{2} - c_1e^{-x} + c_2$$

1.34 problem 34

Internal problem ID [7423]

Book: Second order enumerated odes
Section: section 1
Problem number: 34.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _missing_y]]`

$$\boxed{y'' + y' = x^2 + x + 1}$$

✓ Solution by Maple
Time used: 0.0 (sec). Leaf size: 26

```
dsolve(diff(y(x),x$2)+diff(y(x),x)=1+x+x^2,y(x), singsol=all)
```

$$y(x) = \frac{x^3}{3} - e^{-x}c_1 - \frac{x^2}{2} + 2x + c_2$$

✓ Solution by Mathematica
Time used: 0.083 (sec). Leaf size: 34

```
DSolve[y''[x]+y'[x]==1+x+x^2,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to \frac{x^3}{3} - \frac{x^2}{2} + 2x - c_1e^{-x} + c_2$$

1.35 problem 35

Internal problem ID [7424]

Book: Second order enumerated odes
Section: section 1
Problem number: 35.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _missing_y]]`

$$\boxed{y'' + y' = x^3 + x^2 + x + 1}$$

✓ Solution by Maple
Time used: 0.0 (sec). Leaf size: 31

```
dsolve(diff(y(x),x$2)+diff(y(x),x)=1+x+x^2+x^3,y(x), singsol=all)
```

$$y(x) = \frac{x^4}{4} - e^{-x} c_1 + \frac{5x^2}{2} - \frac{2x^3}{3} - 4x + c_2$$

✓ Solution by Mathematica
Time used: 0.131 (sec). Leaf size: 41

```
DSolve[y''[x]+y'[x]==1+x+x^2+x^3,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to \frac{x^4}{4} - \frac{2x^3}{3} + \frac{5x^2}{2} - 4x - c_1 e^{-x} + c_2$$

1.36 problem 36

Internal problem ID [7425]

Book: Second order enumerated odes
Section: section 1
Problem number: 36.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _missing_y]]`

$$\boxed{y'' + y' = \sin(x)}$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 21

```
dsolve(diff(y(x),x$2)+diff(y(x),x)=sin(x),y(x), singsol=all)
```

$$y(x) = -e^{-x}c_1 - \frac{\sin(x)}{2} - \frac{\cos(x)}{2} + c_2$$

✓ Solution by Mathematica

Time used: 0.112 (sec). Leaf size: 29

```
DSolve[y''[x]+y'[x]==Sin[x],y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to -\frac{\sin(x)}{2} - \frac{\cos(x)}{2} + c_1(-e^{-x}) + c_2$$

1.37 problem 37

Internal problem ID [7426]

Book: Second order enumerated odes
Section: section 1
Problem number: 37.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _missing_y]]`

$$\boxed{y'' + y' = \cos(x)}$$

✓ Solution by Maple
Time used: 0.0 (sec). Leaf size: 21

```
dsolve(diff(y(x),x$2)+diff(y(x),x)=cos(x),y(x), singsol=all)
```

$$y(x) = -e^{-x}c_1 + \frac{\sin(x)}{2} - \frac{\cos(x)}{2} + c_2$$

✓ Solution by Mathematica
Time used: 0.08 (sec). Leaf size: 28

```
DSolve[y''[x]+y'[x]==Cos[x],y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to \frac{1}{2}\left(\sin(x) - \cos(x) - 2c_1e^{-x}\right) + c_2$$

1.38 problem 38

Internal problem ID [7427]

Book: Second order enumerated odes
Section: section 1
Problem number: 38.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _missing_x]]`

$$\boxed{y'' + y = 1}$$

✓ Solution by Maple
Time used: 0.0 (sec). Leaf size: 14

```
dsolve(diff(y(x),x$2)+y(x)=1,y(x), singsol=all)
```

$$y(x) = \sin(x)\, c_2 + \cos(x)\, c_1 + 1$$

✓ Solution by Mathematica
Time used: 0.01 (sec). Leaf size: 17

```
DSolve[y''[x]+y[x]==1,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to c_1 \cos(x) + c_2 \sin(x) + 1$$

1.39 problem 39

Internal problem ID [7428]

Book: Second order enumerated odes
Section: section 1
Problem number: 39.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$\boxed{y'' + y = x}$$

✓ Solution by Maple
Time used: 0.0 (sec). Leaf size: 14

```
dsolve(diff(y(x),x$2)+y(x)=x,y(x), singsol=all)
```

$$y(x) = \sin(x) c_2 + \cos(x) c_1 + x$$

✓ Solution by Mathematica
Time used: 0.011 (sec). Leaf size: 17

```
DSolve[y''[x]+y[x]==x,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to x + c_1 \cos(x) + c_2 \sin(x)$$

1.40 problem 40

Internal problem ID [7429]

Book: Second order enumerated odes
Section: section 1
Problem number: 40.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$\boxed{y'' + y = 1 + x}$$

✓ Solution by Maple
Time used: 0.0 (sec). Leaf size: 15

```
dsolve(diff(y(x),x$2)+y(x)=1+x,y(x), singsol=all)
```

$$y(x) = \sin(x) c_2 + \cos(x) c_1 + x + 1$$

✓ Solution by Mathematica
Time used: 0.012 (sec). Leaf size: 18

```
DSolve[y''[x]+y[x]==1+x,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to x + c_1 \cos(x) + c_2 \sin(x) + 1$$

1.41 problem 41

Internal problem ID [7430]

Book: Second order enumerated odes
Section: section 1
Problem number: 41.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$\boxed{y'' + y = x^2 + x + 1}$$

✓ Solution by Maple
Time used: 0.0 (sec). Leaf size: 18

```
dsolve(diff(y(x),x$2)+y(x)=1+x+x^2,y(x), singsol=all)
```

$$y(x) = \sin(x) c_2 + \cos(x) c_1 + x^2 + x - 1$$

✓ Solution by Mathematica
Time used: 0.012 (sec). Leaf size: 21

```
DSolve[y''[x]+y[x]==1+x+x^2,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to x^2 + x + c_1 \cos(x) + c_2 \sin(x) - 1$$

1.42 problem 42

Internal problem ID [7431]

Book: Second order enumerated odes
Section: section 1
Problem number: 42.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _linear, _nonhomogeneous]]`

$$\boxed{y'' + y = x^3 + x^2 + x + 1}$$

✓ Solution by Maple
Time used: 0.0 (sec). Leaf size: 23

```
dsolve(diff(y(x),x$2)+y(x)=1+x+x^2+x^3,y(x), singsol=all)
```

$$y(x) = \sin(x) c_2 + \cos(x) c_1 + x^3 + x^2 - 5x - 1$$

✓ Solution by Mathematica
Time used: 0.012 (sec). Leaf size: 26

```
DSolve[y''[x]+y[x]==1+x+x^2+x^3,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to x^3 + x^2 - 5x + c_1 \cos(x) + c_2 \sin(x) - 1$$

1.43 problem 43

Internal problem ID [7432]

Book: Second order enumerated odes
Section: section 1
Problem number: 43.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _linear, _nonhomogeneous]]`

$$\boxed{y'' + y = \sin(x)}$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 25

```
dsolve(diff(y(x),x$2)+y(x)=sin(x),y(x), singsol=all)
```

$$y(x) = \frac{(-x + 2c_1)\cos(x)}{2} + \frac{\sin(x)(2c_2 + 1)}{2}$$

✓ Solution by Mathematica

Time used: 0.025 (sec). Leaf size: 22

```
DSolve[y''[x]+y[x]==Sin[x],y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to \left(-\frac{x}{2} + c_1\right)\cos(x) + c_2\sin(x)$$

1.44 problem 44

Internal problem ID [7433]

Book: Second order enumerated odes
Section: section 1
Problem number: 44.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _linear, _nonhomogeneous]]`

$$\boxed{y'' + y = \cos(x)}$$

✓ Solution by Maple
Time used: 0.0 (sec). Leaf size: 18

```
dsolve(diff(y(x),x$2)+y(x)=cos(x),y(x), singsol=all)
```

$$y(x) = \frac{(2c_2 + x)\sin(x)}{2} + \cos(x)\, c_1$$

✓ Solution by Mathematica
Time used: 0.025 (sec). Leaf size: 28

```
DSolve[y''[x]+y[x]==Cos[x],y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to \frac{1}{2}(x\sin(x) + \cos(x) + 2c_1\cos(x) + 2c_2\sin(x))$$

1.45 problem 45

Internal problem ID [7434]

Book: Second order enumerated odes
Section: section 1
Problem number: 45.
ODE order: 2.
ODE degree: 2.

CAS Maple gives this as type `[[_2nd_order, _missing_x]]`

$$\boxed{yy''^2 + y' = 0}$$

✓ Solution by Maple

Time used: 0.078 (sec). Leaf size: 271

```
dsolve(y(x)*diff(y(x),x$2)^2+diff(y(x),x)=0,y(x), singsol=all)
```

$$y(x) = c_1$$
$$y(x) = 0$$
$$-\left(\int^{y(x)} \frac{\_a}{\left(\_a^{\frac{3}{2}}\left(c_1 - 3\sqrt{\_a}\right)\right)^{\frac{2}{3}}} d\_a\right) - x - c_2 = 0$$
$$-\left(\int^{y(x)} \frac{\_a}{\left(\_a^{\frac{3}{2}}\left(c_1 + 3\sqrt{\_a}\right)\right)^{\frac{2}{3}}} d\_a\right) - x - c_2 = 0$$
$$\frac{-4\left(\int^{y(x)} \frac{\_a}{\left(\_a^{\frac{3}{2}}(c_1 - 3\sqrt{\_a})\right)^{\frac{2}{3}}} d\_a\right) + 2i(-x - c_2)\sqrt{3} + 2x + 2c_2}{\left(-i\sqrt{3} - 1\right)^2} = 0$$
$$\frac{-4\left(\int^{y(x)} \frac{\_a}{\left(\_a^{\frac{3}{2}}(c_1 - 3\sqrt{\_a})\right)^{\frac{2}{3}}} d\_a\right) + 2i(x + c_2)\sqrt{3} + 2x + 2c_2}{\left(1 - i\sqrt{3}\right)^2} = 0$$
$$\frac{-4\left(\int^{y(x)} \frac{\_a}{\left(\_a^{\frac{3}{2}}(c_1 + 3\sqrt{\_a})\right)^{\frac{2}{3}}} d\_a\right) + 2i(-x - c_2)\sqrt{3} + 2x + 2c_2}{\left(-i\sqrt{3} - 1\right)^2} = 0$$
$$\frac{-4\left(\int^{y(x)} \frac{\_a}{\left(\_a^{\frac{3}{2}}(c_1 + 3\sqrt{\_a})\right)^{\frac{2}{3}}} d\_a\right) + 2i(x + c_2)\sqrt{3} + 2x + 2c_2}{\left(1 - i\sqrt{3}\right)^2} = 0$$

✓ Solution by Mathematica

Time used: 61.116 (sec). Leaf size: 23861

```
DSolve[y[x]*y''[x]^2+y'[x]==0,y[x],x,IncludeSingularSolutions -> True]
```

Too large to display

1.46 problem 46

Internal problem ID [7435]

Book: Second order enumerated odes
Section: section 1
Problem number: 46.
ODE order: 2.
ODE degree: 2.

CAS Maple gives this as type `[[_2nd_order, _missing_x]]`

$$y{y''}^2 + {y'}^3 = 0$$

✓ Solution by Maple

Time used: 0.093 (sec). Leaf size: 166

```
dsolve(y(x)*diff(y(x),x$2)^2+diff(y(x),x)^3=0,y(x), singsol=all)
```

$$y(x) = c_1$$
$$y(x) = 0$$
$$y(x) = \frac{c_2 \left(\operatorname{LambertW}\left(c_1 \mathrm{e}^{-1+\frac{x}{2}}\right)+1\right)^2}{\operatorname{LambertW}\left(c_1 \mathrm{e}^{-1+\frac{x}{2}}\right)^2}$$
$$y(x) = \frac{c_2 \left(\operatorname{LambertW}\left(-c_1 \mathrm{e}^{-1+\frac{x}{2}}\right)+1\right)^2}{\operatorname{LambertW}\left(-c_1 \mathrm{e}^{-1+\frac{x}{2}}\right)^2}$$
$$y(x) = \mathrm{e}^{-\left(\int \mathrm{e}^{2 \operatorname{RootOf}\left(\mathrm{e}^{\_Z} \ln\left(\left(\mathrm{e}^{\_Z}+1\right)^2\right)+c_1 \mathrm{e}^{\_Z}-2 \mathrm{e}^{\_Z}\_Z+x \mathrm{e}^{\_Z}+\ln\left(\left(\mathrm{e}^{\_Z}+1\right)^2\right)+c_1-2\_Z+x-2\right)} dx\right)-2\left(\int \mathrm{e}^{\operatorname{RootOf}\left(\mathrm{e}^{\_Z} \ln\left(\left(\mathrm{e}^{\_Z}+1\right)^2\right)+c_1 \mathrm{e}^{\_Z}-2 \mathrm{e}^{\_Z}\right.}\right.}$$

✓ Solution by Mathematica

Time used: 2.165 (sec). Leaf size: 361

```
DSolve[y[x]*y''[x]^2+y'[x]^3==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to \text{InverseFunction}\left[-4\left(\frac{1}{2}\log\left(2\sqrt{\#1}-ic_1\right)-\frac{ic_1}{2\left(2\sqrt{\#1}-ic_1\right)}\right)\&\right][x+c_2]$$
$$y(x) \to \text{InverseFunction}\left[-4\left(\frac{ic_1}{2\left(2\sqrt{\#1}+ic_1\right)}+\frac{1}{2}\log\left(2\sqrt{\#1}+ic_1\right)\right)\&\right][x+c_2]$$
$$y(x) \to \text{InverseFunction}\left[-4\left(\frac{1}{2}\log\left(2\sqrt{\#1}-i(-c_1)\right)-\frac{i(-c_1)}{2\left(2\sqrt{\#1}-i(-c_1)\right)}\right)\&\right][x+c_2]$$
$$y(x) \to \text{InverseFunction}\left[-4\left(\frac{i(-c_1)}{2\left(2\sqrt{\#1}+i(-1)c_1\right)}+\frac{1}{2}\log\left(2\sqrt{\#1}+i(-1)c_1\right)\right)\&\right][x+c_2]$$
$$y(x) \to \text{InverseFunction}\left[-4\left(\frac{1}{2}\log\left(2\sqrt{\#1}-ic_1\right)-\frac{ic_1}{2\left(2\sqrt{\#1}-ic_1\right)}\right)\&\right][x+c_2]$$
$$y(x) \to \text{InverseFunction}\left[-4\left(\frac{ic_1}{2\left(2\sqrt{\#1}+ic_1\right)}+\frac{1}{2}\log\left(2\sqrt{\#1}+ic_1\right)\right)\&\right][x+c_2]$$

1.47 problem 47

Internal problem ID [7436]

Book: Second order enumerated odes
Section: section 1
Problem number: 47.
ODE order: 2.
ODE degree: 2.

CAS Maple gives this as type `[[_2nd_order, _missing_x], [_2nd_order, _reducible, _mu_x_y1]]`

$$\boxed{y^2 y''^2 + y' = 0}$$

✓ Solution by Maple

Time used: 0.063 (sec). Leaf size: 241

```
dsolve(y(x)^2*diff(y(x),x$2)^2+diff(y(x),x)=0,y(x), singsol=all)
```

$$y(x) = c_1$$
$$y(x) = 0$$
$$-4\left(\int^{y(x)} \frac{1}{(-12\ln(\_a) + 8c_1)^{\frac{2}{3}}} d\_a\right) - x - c_2 = 0$$
$$-4\left(\int^{y(x)} \frac{1}{(12\ln(\_a) - 8c_1)^{\frac{2}{3}}} d\_a\right) - x - c_2 = 0$$
$$\frac{-16\left(\int^{y(x)} \frac{1}{(-12\ln(\_a)+8c_1)^{\frac{2}{3}}} d\_a\right) + 2i(-x - c_2)\sqrt{3} + 2x + 2c_2}{\left(-i\sqrt{3} - 1\right)^2} = 0$$
$$\frac{-16\left(\int^{y(x)} \frac{1}{(-12\ln(\_a)+8c_1)^{\frac{2}{3}}} d\_a\right) + 2i(x + c_2)\sqrt{3} + 2x + 2c_2}{\left(1 - i\sqrt{3}\right)^2} = 0$$
$$\frac{-16\left(\int^{y(x)} \frac{1}{(12\ln(\_a)-8c_1)^{\frac{2}{3}}} d\_a\right) + 2i(-x - c_2)\sqrt{3} + 2x + 2c_2}{\left(-i\sqrt{3} - 1\right)^2} = 0$$
$$\frac{-16\left(\int^{y(x)} \frac{1}{(12\ln(\_a)-8c_1)^{\frac{2}{3}}} d\_a\right) + 2i(x + c_2)\sqrt{3} + 2x + 2c_2}{\left(1 - i\sqrt{3}\right)^2} = 0$$

✓ Solution by Mathematica

Time used: 2.57 (sec). Leaf size: 449

```
DSolve[y[x]^2*y''[x]^2+y'[x]==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to \text{InverseFunction}\left[\frac{\left(\frac{2}{3}\right)^{2/3} e^{-ic_1}(-\log(\#1)-ic_1)^{2/3}\Gamma\left(\frac{1}{3},-ic_1-\log(\#1)\right)}{(c_1-i\log(\#1))^{2/3}}\&\right][x+c_2]$$

$$y(x) \to \text{InverseFunction}\left[\frac{\left(\frac{2}{3}\right)^{2/3} e^{ic_1}(-\log(\#1)+ic_1)^{2/3}\Gamma\left(\frac{1}{3},ic_1-\log(\#1)\right)}{(i\log(\#1)+c_1)^{2/3}}\&\right][x+c_2]$$

$$y(x) \to \text{InverseFunction}\left[\frac{\left(\frac{2}{3}\right)^{2/3} e^{-i(-c_1)}(-\log(\#1)-i(-1)c_1)^{2/3}\Gamma\left(\frac{1}{3},-i(-1)c_1-\log(\#1)\right)}{(-i\log(\#1)-c_1)^{2/3}}\&\right][x+c_2]$$

$$y(x) \to \text{InverseFunction}\left[\frac{\left(\frac{2}{3}\right)^{2/3} e^{-ic_1}(-\log(\#1)-ic_1)^{2/3}\Gamma\left(\frac{1}{3},-ic_1-\log(\#1)\right)}{(c_1-i\log(\#1))^{2/3}}\&\right][x+c_2]$$

$$y(x) \to \text{InverseFunction}\left[\frac{\left(\frac{2}{3}\right)^{2/3} e^{i(-c_1)}(-\log(\#1)+i(-c_1))^{2/3}\Gamma\left(\frac{1}{3},i(-c_1)-\log(\#1)\right)}{(i\log(\#1)-c_1)^{2/3}}\&\right][x+c_2]$$

$$y(x) \to \text{InverseFunction}\left[\frac{\left(\frac{2}{3}\right)^{2/3} e^{ic_1}(-\log(\#1)+ic_1)^{2/3}\Gamma\left(\frac{1}{3},ic_1-\log(\#1)\right)}{(i\log(\#1)+c_1)^{2/3}}\&\right][x+c_2]$$

1.48 problem 48

Internal problem ID [7437]

Book: Second order enumerated odes
Section: section 1
Problem number: 48.
ODE order: 2.
ODE degree: 4.

CAS Maple gives this as type `[[_2nd_order, _missing_x]]`

$$\boxed{yy''^4 + y'^2 = 0}$$

✓ Solution by Maple

Time used: 0.157 (sec). Leaf size: 2926

```
dsolve(y(x)*diff(y(x),x$2)^4+diff(y(x),x)^2=0,y(x), singsol=all)
```

$$y(x) = c_1$$
$$y(x) = 0$$
$$\int^{y(x)} \frac{\_a^2}{\sqrt{\_a^3 \left(2\_a - (c_1\_a)^{\frac{1}{4}}\right) \left(-2\_a^3 + \_a^2 (c_1\_a)^{\frac{1}{4}}\right)^{\frac{1}{3}}}} d\_a - x - c_2 = 0$$
$$\int^{y(x)} \frac{\_a^2}{\sqrt{-\_a^3 \left(i (c_1\_a)^{\frac{1}{4}} - 2\_a\right) \left(\left(i (c_1\_a)^{\frac{1}{4}} - 2\_a\right) \_a^2\right)^{\frac{1}{3}}}} d\_a - x - c_2 = 0$$
$$\int^{y(x)} \frac{\_a^2}{\sqrt{\_a^3 \left(2\_a + (c_1\_a)^{\frac{1}{4}}\right) \left(-2\_a^3 - \_a^2 (c_1\_a)^{\frac{1}{4}}\right)^{\frac{1}{3}}}} d\_a - x - c_2 = 0$$
$$\int^{y(x)} \frac{\_a^2}{\sqrt{\_a^3 \left(i (c_1\_a)^{\frac{1}{4}} + 2\_a\right) \left(-\left(i (c_1\_a)^{\frac{1}{4}} + 2\_a\right) \_a^2\right)^{\frac{1}{3}}}} d\_a - x - c_2 = 0$$
$$\sqrt{2} \left(\int^{y(x)} \frac{\_a^2}{\sqrt{\left(-2\_a + (c_1\_a)^{\frac{1}{4}}\right) (1 + i\sqrt{3}) \_a^3 \left(-2\_a^3 + \_a^2 (c_1\_a)^{\frac{1}{4}}\right)^{\frac{1}{3}}}} d\_a\right) - x - c_2 = 0$$
$$\sqrt{2} \left(\int^{y(x)} \frac{\_a^2}{\sqrt{(i - \sqrt{3}) \_a^3 \left(\left(i (c_1\_a)^{\frac{1}{4}} - 2\_a\right) \_a^2\right)^{\frac{1}{3}} \left((c_1\_a)^{\frac{1}{4}} + 2i\_a\right)}} d\_a\right) - x - c_2 = 0$$
$$\sqrt{2} \left(\int^{y(x)} \frac{\_a^2}{\sqrt{-2 (1 + i\sqrt{3}) \left(-2\_a^3 - \_a^2 (c_1\_a)^{\frac{1}{4}}\right)^{\frac{1}{3}} \left(\_a + \frac{(c_1\_a)^{\frac{1}{4}}}{2}\right) \_a^3}} d\_a\right) - x - c_2 = 0$$
$$\sqrt{2} \left(\int^{y(x)} \frac{\_a^2}{\sqrt{-\_a^3 \left(i (c_1\_a)^{\frac{1}{4}} + 2\_a\right) \left(-\left(i (c_1\_a)^{\frac{1}{4}} + 2\_a\right) \_a^2\right)^{\frac{1}{3}} (1 + i\sqrt{3})}} d\_a\right) - x - c_2 = 0$$
$$-\left(\int^{y(x)} \frac{\_a^2}{\sqrt{\_a^3 \left(2\_a - (c_1\_a)^{\frac{1}{4}}\right) \left(-2\_a^3 + \_a^2 (c_1\_a)^{\frac{1}{4}}\right)^{\frac{1}{3}}}} d\_a\right) - x - c_2 = 0$$

✓ Solution by Mathematica

Time used: 4.322 (sec). Leaf size: 1237

```
DSolve[y[x]*y''[x]^4+y'[x]^2==0,y[x],x,IncludeSingularSolutions -> True]
```

Too large to display

1.49 problem 49

Internal problem ID [7438]

Book: Second order enumerated odes
Section: section 1
Problem number: 49.
ODE order: 2.
ODE degree: 2.

CAS Maple gives this as type `[[_2nd_order, _missing_x], [_2nd_order, _reducible, _mu_x_y1]]`

$$\boxed{y^3 y''^2 + y y' = 0}$$

✓ Solution by Maple

Time used: 0.047 (sec). Leaf size: 241

```
dsolve(y(x)^3*diff(y(x),x$2)^2+y(x)*diff(y(x),x)=0,y(x), singsol=all)
```

$$y(x) = c_1$$
$$y(x) = 0$$
$$-4\left(\int^{y(x)} \frac{1}{\left(-12\ln\left(\_a\right) + 8c_1\right)^{\frac{2}{3}}} d\_a\right) - x - c_2 = 0$$
$$-4\left(\int^{y(x)} \frac{1}{\left(12\ln\left(\_a\right) - 8c_1\right)^{\frac{2}{3}}} d\_a\right) - x - c_2 = 0$$
$$\frac{-16\left(\int^{y(x)} \frac{1}{(-12\ln(\_a)+8c_1)^{\frac{2}{3}}} d\_a\right) + 2i(-x - c_2)\sqrt{3} + 2x + 2c_2}{\left(-i\sqrt{3} - 1\right)^2} = 0$$
$$\frac{-16\left(\int^{y(x)} \frac{1}{(-12\ln(\_a)+8c_1)^{\frac{2}{3}}} d\_a\right) + 2i(x + c_2)\sqrt{3} + 2x + 2c_2}{\left(1 - i\sqrt{3}\right)^2} = 0$$
$$\frac{-16\left(\int^{y(x)} \frac{1}{(12\ln(\_a)-8c_1)^{\frac{2}{3}}} d\_a\right) + 2i(-x - c_2)\sqrt{3} + 2x + 2c_2}{\left(-i\sqrt{3} - 1\right)^2} = 0$$
$$\frac{-16\left(\int^{y(x)} \frac{1}{(12\ln(\_a)-8c_1)^{\frac{2}{3}}} d\_a\right) + 2i(x + c_2)\sqrt{3} + 2x + 2c_2}{\left(1 - i\sqrt{3}\right)^2} = 0$$

✓ Solution by Mathematica

Time used: 2.526 (sec). Leaf size: 459

```
DSolve[y[x]^3*y''[x]^2+y[x]*y'[x]==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to 0$$

$$y(x) \to \text{InverseFunction}\left[\frac{\left(\frac{2}{3}\right)^{2/3} e^{-ic_1}(-\log(\#1)-ic_1)^{2/3}\Gamma\left(\frac{1}{3},-ic_1-\log(\#1)\right)}{(c_1-i\log(\#1))^{2/3}}\&\right][x+c_2]$$

$$y(x) \to \text{InverseFunction}\left[\frac{\left(\frac{2}{3}\right)^{2/3} e^{ic_1}(-\log(\#1)+ic_1)^{2/3}\Gamma\left(\frac{1}{3},ic_1-\log(\#1)\right)}{(i\log(\#1)+c_1)^{2/3}}\&\right][x+c_2]$$

$$y(x) \to 0$$

$$y(x) \to \text{InverseFunction}\left[\frac{\left(\frac{2}{3}\right)^{2/3} e^{-i(-c_1)}(-\log(\#1)-i(-1)c_1)^{2/3}\Gamma\left(\frac{1}{3},-i(-1)c_1-\log(\#1)\right)}{(-i\log(\#1)-c_1)^{2/3}}\&\right][x+c_2]$$

$$y(x) \to \text{InverseFunction}\left[\frac{\left(\frac{2}{3}\right)^{2/3} e^{-ic_1}(-\log(\#1)-ic_1)^{2/3}\Gamma\left(\frac{1}{3},-ic_1-\log(\#1)\right)}{(c_1-i\log(\#1))^{2/3}}\&\right][x+c_2]$$

$$y(x) \to \text{InverseFunction}\left[\frac{\left(\frac{2}{3}\right)^{2/3} e^{i(-c_1)}(-\log(\#1)+i(-c_1))^{2/3}\Gamma\left(\frac{1}{3},i(-c_1)-\log(\#1)\right)}{(i\log(\#1)-c_1)^{2/3}}\&\right][x+c_2]$$

$$y(x) \to \text{InverseFunction}\left[\frac{\left(\frac{2}{3}\right)^{2/3} e^{ic_1}(-\log(\#1)+ic_1)^{2/3}\Gamma\left(\frac{1}{3},ic_1-\log(\#1)\right)}{(i\log(\#1)+c_1)^{2/3}}\&\right][x+c_2]$$

1.50 problem 50

Internal problem ID [7439]

Book: Second order enumerated odes
Section: section 1
Problem number: 50.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _missing_x], [_2nd_order, _reducible, _mu_x_y1],`

$$\boxed{yy'' + y'^3 = 0}$$

✓ Solution by Maple

Time used: 0.032 (sec). Leaf size: 27

```
dsolve(y(x)*diff(y(x),x$2)+diff(y(x),x)^3=0,y(x), singsol=all)
```

$$y(x) = 0$$
$$y(x) = c_1$$
$$y(x) = \frac{x + c_2}{\operatorname{LambertW}\left((x + c_2)\, \mathrm{e}^{c_1 - 1}\right)}$$

✓ Solution by Mathematica

Time used: 60.106 (sec). Leaf size: 26

```
DSolve[y[x]*y''[x]+y'[x]^3==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to \frac{x + c_2}{W\left(e^{-1-c_1}(x + c_2)\right)}$$

1.51 problem 51

Internal problem ID [7440]

Book: Second order enumerated odes
Section: section 1
Problem number: 51.
ODE order: 2.
ODE degree: 3.

CAS Maple gives this as type `[[_2nd_order, _missing_x]]`

$$\boxed{y{y''}^3 + y^3 y' = 0}$$

✓ Solution by Maple

Time used: 0.125 (sec). Leaf size: 128

```
dsolve(y(x)*diff(y(x),x$2)^3+y(x)^3*diff(y(x),x)=0,y(x), singsol=all)
```

$$
\begin{aligned}
y(x) &= 0 \\
y(x) &= c_1 \\
y(x) &= \mathrm{e}^{\int \mathrm{RootOf}\left(x-\left(\int^{\_Z} -\frac{1}{\_f^2-(-\_f)^{\frac{1}{3}}}d\_f\right)+c_1\right)dx+c_2} \\
y(x) &= \mathrm{e}^{\int \mathrm{RootOf}\left(x+2\left(\int^{\_Z} \frac{1}{i\sqrt{3}\,(-\_f)^{\frac{1}{3}}+2\_f^2+(-\_f)^{\frac{1}{3}}}d\_f\right)+c_1\right)dx+c_2} \\
y(x) &= \mathrm{e}^{\int \mathrm{RootOf}\left(x-2\left(\int^{\_Z} \frac{1}{i\sqrt{3}\,(-\_f)^{\frac{1}{3}}-2\_f^2-(-\_f)^{\frac{1}{3}}}d\_f\right)+c_1\right)dx+c_2}
\end{aligned}
$$

✓ Solution by Mathematica

Time used: 3.023 (sec). Leaf size: 800

```
DSolve[y[x]*y''[x]^3+y[x]^3*y'[x]==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x)\to 0$$

$$y(x)\to \text{InverseFunction}\left[\frac{\#1\left(1-\frac{3\#1^{5/3}}{5c_1}\right)^{3/5}\text{Hypergeometric2F1}\left(\frac{3}{5},\frac{3}{5},\frac{8}{5},\frac{3\#1^{5/3}}{5c_1}\right)}{\left(-\#1^{5/3}+\frac{5c_1}{3}\right)^{3/5}}\&\right][x+c_2]$$

$$y(x)\to \text{InverseFunction}\left[\frac{\#1\left(1+\frac{3\sqrt[3]{-1}\#1^{5/3}}{5c_1}\right)^{3/5}\text{Hypergeometric2F1}\left(\frac{3}{5},\frac{3}{5},\frac{8}{5},-\frac{3\sqrt[3]{-1}\#1^{5/3}}{5c_1}\right)}{\left(\sqrt[3]{-1}\#1^{5/3}+\frac{5c_1}{3}\right)^{3/5}}\&\right][x+c_2]$$

$$y(x)\to \text{InverseFunction}\left[\frac{\#1\left(1-\frac{3(-1)^{2/3}\#1^{5/3}}{5c_1}\right)^{3/5}\text{Hypergeometric2F1}\left(\frac{3}{5},\frac{3}{5},\frac{8}{5},\frac{3(-1)^{2/3}\#1^{5/3}}{5c_1}\right)}{\left(-(-1)^{2/3}\#1^{5/3}+\frac{5c_1}{3}\right)^{3/5}}\&\right][x+c_2]$$

$$y(x)\to 0$$

$$y(x)\to \text{InverseFunction}\left[\frac{\#1\left(1-\frac{3\#1^{5/3}}{5(-c_1)}\right)^{3/5}\text{Hypergeometric2F1}\left(\frac{3}{5},\frac{3}{5},\frac{8}{5},\frac{3\#1^{5/3}}{5(-c_1)}\right)}{\left(-\#1^{5/3}+\frac{5(-c_1)}{3}\right)^{3/5}}\&\right][x+c_2]$$

$$y(x)\to \text{InverseFunction}\left[\frac{\#1\left(1+\frac{3\sqrt[3]{-1}\#1^{5/3}}{5(-c_1)}\right)^{3/5}\text{Hypergeometric2F1}\left(\frac{3}{5},\frac{3}{5},\frac{8}{5},-\frac{3\sqrt[3]{-1}\#1^{5/3}}{5(-c_1)}\right)}{\left(\sqrt[3]{-1}\#1^{5/3}+\frac{5}{3}(-1)c_1\right)^{3/5}}\&\right][x+c_2]$$

$$y(x)\to \text{InverseFunction}\left[\frac{\#1\left(1-\frac{3(-1)^{2/3}\#1^{5/3}}{5(-c_1)}\right)^{3/5}\text{Hypergeometric2F1}\left(\frac{3}{5},\frac{3}{5},\frac{8}{5},\frac{3(-1)^{2/3}\#1^{5/3}}{5(-c_1)}\right)}{\left(-(-1)^{2/3}\#1^{5/3}+\frac{5(-c_1)}{3}\right)^{3/5}}\&\right][x+c_2]$$

$$y(x)\to \text{InverseFunction}\left[\frac{\#1\left(1-\frac{3\#1^{5/3}}{5c_1}\right)^{3/5}\text{Hypergeometric2F1}\left(\frac{3}{5},\frac{3}{5},\frac{8}{5},\frac{3\#1^{5/3}}{5c_1}\right)}{\left(-\#1^{5/3}+\frac{5c_1}{3}\right)^{3/5}}\&\right][x+c_2]$$

$$y(x)$$

$$\left[\#1\left(1+\frac{3\sqrt[3]{-1}\#1^{5/3}}{5c_1}\right)^{3/5}\text{Hypergeometric2F1}\left(\frac{3}{5},\frac{3}{5},\frac{8}{5},-\frac{3\sqrt[3]{-1}\#1^{5/3}}{5c_1}\right)\right.$$

1.52 problem 52

Internal problem ID [7441]

Book: Second order enumerated odes
Section: section 1
Problem number: 52.
ODE order: 2.
ODE degree: 3.

CAS Maple gives this as type `[[_2nd_order, _missing_x]]`

$$
y{y''}^3+y^3{y'}^5=0
$$

✓ Solution by Maple

Time used: 0.125 (sec). Leaf size: 214

```
dsolve(y(x)*diff(y(x),x$2)^3+y(x)^3*diff(y(x),x)^5=0,y(x), singsol=all)
```

$$
y(x)=0
$$
$$
y(x)=c_1
$$
$$
\int^{y(x)}\frac{1}{\operatorname{RootOf}\left(5\left(\int_{\_g}^{\_Z}\frac{1}{\_a\left(-\_f^2\_a^2\right)^{\frac{1}{3}}-5\_f}d\_f\right)-\ln\left(\_a^5+125\right)+5c_1\right)}d\_a
$$
$$
-x-c_2=0
$$
$$
\int^{y(x)}\frac{1}{\operatorname{RootOf}\left(-i\ln\left(\_a^5+125\right)+\sqrt{3}\ln\left(\_a^5+125\right)+20\left(\int_{\_g}^{\_Z}\frac{1}{2i\_a\left(-\_f^2\_a^2\right)^{\frac{1}{3}}+5i\_f+5\sqrt{3}\_f}d\_f\right)-2
$$
$$
-x-c_2=0
$$
$$
\int^{y(x)}\frac{1}{\operatorname{RootOf}\left(20\left(\int_{\_g}^{\_Z}\frac{1}{-2i\_a\left(-\_f^2\_a^2\right)^{\frac{1}{3}}-5i\_f+5\sqrt{3}\_f}d\_f\right)+i\ln\left(\_a^5+125\right)+\sqrt{3}\ln\left(\_a^5+125\right)+2
$$
$$
-x-c_2=0
$$

✓ Solution by Mathematica

Time used: 24.581 (sec). Leaf size: 449

```
DSolve[y[x]*y''[x]^3+y[x]^3*y'[x]^5==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to 0$$

$$y(x) \to \text{InverseFunction}\left[\frac{27\#1\,\text{Hypergeometric2F1}\left(\frac{3}{5},3,\frac{8}{5},\frac{3\#1^{5/3}}{5c_1}\right)}{c_1{}^3}\&\right][x+c_2]$$

$$y(x) \to \text{InverseFunction}\left[\frac{27\#1\,\text{Hypergeometric2F1}\left(\frac{3}{5},3,\frac{8}{5},-\frac{3i\left(-i+\sqrt{3}\right)\#1^{5/3}}{10c_1}\right)}{c_1{}^3}\&\right][x+c_2]$$

$$y(x) \to \text{InverseFunction}\left[\frac{27\#1\,\text{Hypergeometric2F1}\left(\frac{3}{5},3,\frac{8}{5},\frac{3i\left(i+\sqrt{3}\right)\#1^{5/3}}{10c_1}\right)}{c_1{}^3}\&\right][x+c_2]$$

$$y(x) \to 0$$

$$y(x) \to \text{InverseFunction}\left[\frac{27\#1\,\text{Hypergeometric2F1}\left(\frac{3}{5},3,\frac{8}{5},\frac{3\#1^{5/3}}{5(-c_1)}\right)}{(-c_1)^3}\&\right][x+c_2]$$

$$y(x) \to \text{InverseFunction}\left[\frac{27\#1\,\text{Hypergeometric2F1}\left(\frac{3}{5},3,\frac{8}{5},-\frac{3i\left(-i+\sqrt{3}\right)\#1^{5/3}}{10(-c_1)}\right)}{(-c_1)^3}\&\right][x+c_2]$$

$$y(x) \to \text{InverseFunction}\left[\frac{27\#1\,\text{Hypergeometric2F1}\left(\frac{3}{5},3,\frac{8}{5},\frac{3i\left(i+\sqrt{3}\right)\#1^{5/3}}{10(-c_1)}\right)}{(-c_1)^3}\&\right][x+c_2]$$

$$y(x) \to \text{InverseFunction}\left[\frac{27\#1\,\text{Hypergeometric2F1}\left(\frac{3}{5},3,\frac{8}{5},\frac{3\#1^{5/3}}{5c_1}\right)}{c_1{}^3}\&\right][x+c_2]$$

$$y(x) \to \text{InverseFunction}\left[\frac{27\#1\,\text{Hypergeometric2F1}\left(\frac{3}{5},3,\frac{8}{5},-\frac{3i\left(-i+\sqrt{3}\right)\#1^{5/3}}{10c_1}\right)}{c_1{}^3}\&\right][x$$

2 section 2

2.1 problem 1

Internal problem ID [7442]

Book: Second order enumerated odes
Section: section 2
Problem number: 1.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[_Liouville, [_2nd_order, _reducible, _mu_xy]]`

$$\boxed{y'' + xy' + y{y'}^2 = 0}$$

✓ Solution by Maple
Time used: 0.015 (sec). Leaf size: 40

```
dsolve(diff(y(x),x$2)+x*diff(y(x),x)+y(x)*diff(y(x),x)^2=0,y(x), singsol=all)
```

$$y(x) = -i \operatorname{RootOf}\left(i\sqrt{\pi}\,\operatorname{erf}\left(\frac{\sqrt{2}\,x}{2}\right)c_1 + i\sqrt{2}\,c_2 - \operatorname{erf}(\_Z)\sqrt{\pi}\right)\sqrt{2}$$

✓ Solution by Mathematica
Time used: 0.125 (sec). Leaf size: 44

```
DSolve[y''[x]+x*y'[x]+y[x]*(y'[x])^2==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to -i\sqrt{2}\operatorname{erf}^{-1}\left(i\left(\sqrt{\frac{2}{\pi}}c_2 - c_1\operatorname{erf}\left(\frac{x}{\sqrt{2}}\right)\right)\right)$$

2.2 problem 2

Internal problem ID [7443]

Book: Second order enumerated odes
Section: section 2
Problem number: 2.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[_Liouville, [_2nd_order, _reducible, _mu_xy]]`

$$y'' + y' \sin(x) + y{y'}^2 = 0$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 38

```
dsolve(diff(y(x),x$2)+sin(x)*diff(y(x),x)+y(x)*diff(y(x),x)^2=0,y(x), singsol=all)
```

$$y(x) = -i \operatorname{RootOf}\left(i\sqrt{2}\, c_1 \left(\int \mathrm{e}^{\cos(x)} dx\right) + i\sqrt{2}\, c_2 - \operatorname{erf}(\_Z)\sqrt{\pi}\right)\sqrt{2}$$

✓ Solution by Mathematica

Time used: 0.329 (sec). Leaf size: 47

```
DSolve[y''[x]+Sin[x]*y'[x]+y[x]*(y'[x])^2==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to -i\sqrt{2}\operatorname{erf}^{-1}\left(i\sqrt{\frac{2}{\pi}}\left(\int_1^x -e^{\cos(K[1])}c_1 dK[1] + c_2\right)\right)$$

2.3 problem 3

Internal problem ID [7444]

Book: Second order enumerated odes
Section: section 2
Problem number: 3.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[_Liouville, [_2nd_order, _reducible, _mu_x_y1], [_2nd_order,`

$$\boxed{y'' + (1-x)\,y' + y^2 y'^2 = 0}$$

✓ Solution by Maple

Time used: 0.016 (sec). Leaf size: 62

```
dsolve(diff(y(x),x$2)+(1-x)*diff(y(x),x)+y(x)^2*diff(y(x),x)^2=0,y(x), singsol=all)
```

$$c_1 \operatorname{erf}\left(\frac{i\sqrt{2}\,(x-1)}{2}\right) - c_2 + \frac{2\,3^{\frac{5}{6}} y(x)\,\pi}{9\Gamma\left(\frac{2}{3}\right)\left(-y\left(x\right)^3\right)^{\frac{1}{3}}} - \frac{y(x)\,\Gamma\left(\frac{1}{3}, -\frac{y(x)^3}{3}\right) 3^{\frac{1}{3}}}{3\left(-y\left(x\right)^3\right)^{\frac{1}{3}}} = 0$$

✓ Solution by Mathematica

Time used: 0.374 (sec). Leaf size: 67

```
DSolve[y''[x]+(1-x)*y'[x]+y[x]^2*(y'[x])^2==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to \text{InverseFunction}\left[-\frac{\#1\Gamma\left(\frac{1}{3}, -\frac{\#1^3}{3}\right)}{3^{2/3}\sqrt[3]{-\#1^3}}\&\right]\left[c_2 - \sqrt{\frac{\pi}{2e}}c_1\text{erfi}\left(\frac{x-1}{\sqrt{2}}\right)\right]$$

2.4 problem 4

Internal problem ID [7445]

Book: Second order enumerated odes
Section: section 2
Problem number: 4.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[_Liouville, [_2nd_order, _reducible, _mu_x_y1], [_2nd_order,`

$$y'' + (\sin(x) + 2x) y' + \cos(y) y y'^2 = 0$$

✓ Solution by Maple

Time used: 0.016 (sec). Leaf size: 34

```
dsolve(diff(y(x),x$2)+(sin(x)+2*x)*diff(y(x),x)+cos(y(x))*y(x)*diff(y(x),x)^2=0,y(x), singso
```

$$\int^{y(x)} \mathrm{e}^{\cos(\_a)+\sin(\_a)\_a} d\_a - c_1 \left(\int \mathrm{e}^{-x^2+\cos(x)} dx \right) - c_2 = 0$$

✓ Solution by Mathematica

Time used: 1.16 (sec). Leaf size: 53

```
DSolve[y''[x]+(Sin[x]+2*x)*y'[x]+Cos[y[x]]*y[x]*(y'[x])^2==0,y[x],x,IncludeSingularSolutions
```

$$y(x) \to \text{InverseFunction}\left[\int_1^{\#1} e^{\cos(K[1])+K[1]\sin(K[1])} dK[1]\&\right]\left[\int_1^x -e^{\cos(K[2])-K[2]^2} c_1 dK[2] + c_2\right]$$

2.5 problem 5

Internal problem ID [7446]

Book: Second order enumerated odes
Section: section 2
Problem number: 5.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _missing_x], [_2nd_order, _reducible, _mu_x_y1]]`

$$y''y' + y^2 = 0$$

✓ Solution by Maple

Time used: 0.078 (sec). Leaf size: 61

```
dsolve(diff(y(x),x$2)*diff(y(x),x)+y(x)^2=0,y(x), singsol=all)
```

$$y(x) = 0$$
$$y(x) = e^{\frac{\sqrt{3}\left(\int \tan\left(\mathrm{RootOf}\left(-\sqrt{3}\,\ln\left(\cos\left(\_Z\right)^2\right)-2\sqrt{3}\,\ln\left(\tan\left(\_Z\right)+\sqrt{3}\right)+6\sqrt{3}\,c_1+6\sqrt{3}\,x+6\_Z\right)\right)dx\right)}{2}+c_2+\frac{x}{2}}$$

✓ Solution by Mathematica

Time used: 1.356 (sec). Leaf size: 180

```
DSolve[y''[x]*y'[x]+y[x]^2==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \rightarrow \frac{}{\sqrt[3]{1+\mathrm{InverseFunction}\left[\frac{1}{6}\log\left(\#1^2-\#1+1\right)+\frac{\arctan\left(\frac{2\#1-1}{\sqrt{3}}\right)}{\sqrt{3}}-\frac{1}{3}\log(\#1+1)\&\right][-x+c_1]}\sqrt[3]{\mathrm{Inv}}}$$

2.6 problem 6

Internal problem ID [7447]

Book: Second order enumerated odes
Section: section 2
Problem number: 6.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _missing_x], [_2nd_order, _reducible, _mu_x_y1]]`

$$\boxed{y''y' + y^n = 0}$$

✓ Solution by Maple

Time used: 0.031 (sec). Leaf size: 174

```
dsolve(diff(y(x),x$2)*diff(y(x),x)+y(x)^n=0,y(x), singsol=all)
```

$$\frac{(-2-2n)\left(\int^{y(x)} \frac{1}{\left(-(3\_a^{1+n}-c_1)(1+n)^2\right)^{\frac{1}{3}}}d\_a\right) - \left(1+i\sqrt{3}\right)(x+c_2)}{1+i\sqrt{3}} = 0$$

$$-\frac{2i(1+n)\left(\int^{y(x)} \frac{1}{\left(-(3\_a^{1+n}-c_1)(1+n)^2\right)^{\frac{1}{3}}}d\_a\right) + (x+c_2)\left(\sqrt{3}+i\right)}{\sqrt{3}+i} = 0$$

$$\left(\int^{y(x)} \frac{1}{\left(-\left(3\_a^{1+n}-c_1\right)(1+n)^2\right)^{\frac{1}{3}}}d\_a\right) n + \int^{y(x)} \frac{1}{\left(-\left(3\_a^{1+n}-c_1\right)(1+n)^2\right)^{\frac{1}{3}}}d\_a - c_2 - x = 0$$

✓ Solution by Mathematica

Time used: 2.4 (sec). Leaf size: 910

```
DSolve[y''[x]*y'[x]+y[x]^n==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to \text{InverseFunction}\left[\frac{\#1\sqrt[3]{n+1}\sqrt[3]{1-\frac{\#1^{n+1}}{c_1(n+1)}}\,\text{Hypergeometric2F1}\left(\frac{1}{3},\frac{1}{n+1},1+\frac{1}{n+1},\frac{\#1^{n+1}}{(n+1)c_1}\right)}{\sqrt[3]{-3\#1^{n+1}+3c_1(n+1)}}\&\right][x+c_2]$$

$$y(x) \to \text{InverseFunction}\left[\frac{(-1)^{2/3}\#1\sqrt[3]{n+1}\sqrt[3]{1-\frac{\#1^{n+1}}{c_1(n+1)}}\,\text{Hypergeometric2F1}\left(\frac{1}{3},\frac{1}{n+1},1+\frac{1}{n+1},\frac{\#1^{n+1}}{(n+1)c_1}\right)}{\sqrt[3]{-3\#1^{n+1}+3c_1(n+1)}}\&\right] + c_2]$$

$$y(x) \to \text{InverseFunction}\left[-\frac{\sqrt[3]{-\frac{1}{3}}\#1\sqrt[3]{n+1}\sqrt[3]{1-\frac{\#1^{n+1}}{c_1(n+1)}}\,\text{Hypergeometric2F1}\left(\frac{1}{3},\frac{1}{n+1},1+\frac{1}{n+1},\frac{\#1^{n+1}}{(n+1)c_1}\right)}{\sqrt[3]{-\#1^{n+1}+c_1(n+1)}}\&\right] + c_2]$$

$$y(x) \to \text{InverseFunction}\left[\frac{\#1\sqrt[3]{n+1}\sqrt[3]{1-\frac{\#1^{n+1}}{(-c_1)(n+1)}}\,\text{Hypergeometric2F1}\left(\frac{1}{3},\frac{1}{n+1},1+\frac{1}{n+1},\frac{\#1^{n+1}}{(n+1)(-c_1)}\right)}{\sqrt[3]{-3\#1^{n+1}+3(-c_1)(n+1)}}\&\right] + c_2]$$

$$y(x) \to \text{InverseFunction}\left[\frac{(-1)^{2/3}\#1\sqrt[3]{n+1}\sqrt[3]{1-\frac{\#1^{n+1}}{(-c_1)(n+1)}}\,\text{Hypergeometric2F1}\left(\frac{1}{3},\frac{1}{n+1},1+\frac{1}{n+1},\frac{\#1^{n+1}}{(n+1)(-c_1)}\right)}{\sqrt[3]{-3\#1^{n+1}+3(-c_1)(n+1)}} + c_2]$$

$$y(x) \to \text{InverseFunction}\left[-\frac{\sqrt[3]{-\frac{1}{3}}\#1\sqrt[3]{n+1}\sqrt[3]{174\,\frac{\#1^{n+1}}{(-c_1)(n+1)}}\,\text{Hypergeometric2F1}\left(\frac{1}{3},\frac{1}{n+1},1+\frac{1}{n+1},\frac{\#1^{n+1}}{(n+1)(-c_1)}\right)}{\sqrt[3]{-\#1^{n+1}+(-c_1)(n+1)}}\right.$$

2.7 problem 8

Internal problem ID [7448]

Book: Second order enumerated odes
Section: section 2
Problem number: 8.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[[_homogeneous, 'class C'], _dAlembert]`

$$\boxed{y' - (x+y)^4 = 0}$$

✓ Solution by Maple

Time used: 0.391 (sec). Leaf size: 882

```
dsolve(diff(y(x), x) = (x + y(x))^4,y(x), singsol=all)
```

Expression too large to display

✓ Solution by Mathematica

Time used: 0.115 (sec). Leaf size: 88

```
DSolve[y'[x] == (x + y[x])^4,y[x],x,IncludeSingularSolutions -> True]
```

$$\text{Solve}\left[\frac{1}{4}\text{RootSum}\left[\#1^4 + 4\#1^3 y(x) + 6\#1^2 y(x)^2 + 4\#1 y(x)^3 + y(x)^4 + 1\&, \frac{\log(x-\#1)}{\#1^3 + 3\#1^2 y(x) + 3\#1 y(x)^2 + y(x)^3}\&\right] - x = c_1, y(x)\right]$$

2.8 problem 9

Internal problem ID [7449]

Book: Second order enumerated odes
Section: section 2
Problem number: 9.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[_Liouville, [_2nd_order, _reducible, _mu_x_y1], [_2nd_order,`

$$y'' + (x+3)\, y' + \left(3 + y^2\right) y'^2 = 0$$

✓ Solution by Maple

Time used: 0.016 (sec). Leaf size: 32

```
dsolve(diff(y(x),x$2)+(3+x)*diff(y(x),x)+(3+y(x)^2)*(diff(y(x),x))^2=0,y(x), singsol=all)
```

$$c_1 \operatorname{erf}\left(\frac{\sqrt{2}\,(x+3)}{2}\right) - c_2 + \int^{y(x)} \mathrm{e}^{\frac{\_a\left(\_a^2+9\right)}{3}} d\_a = 0$$

✓ Solution by Mathematica

Time used: 0.359 (sec). Leaf size: 61

```
DSolve[y''[x]+(3+x)*y'[x]+(3+y[x]^2)*(y'[x])^2==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to \text{InverseFunction}\left[\int_1^{\#1} e^{\frac{K[1]^3}{3}+3K[1]} dK[1]\&\right]\left[c_2 - e^{9/2}\sqrt{\frac{\pi}{2}} c_1 \operatorname{erf}\left(\frac{x+3}{\sqrt{2}}\right)\right]$$

2.9 problem 10

Internal problem ID [7450]

Book: Second order enumerated odes
Section: section 2
Problem number: 10.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[_Liouville, [_2nd_order, _reducible, _mu_xy]]`

$$\boxed{y'' + xy' + yy'^2 = 0}$$

✓ Solution by Maple
Time used: 0.0 (sec). Leaf size: 40

```
dsolve(diff(y(x),x$2)+x*diff(y(x),x)+y(x)*(diff(y(x),x))^2=0,y(x), singsol=all)
```

$$y(x) = -i\,\mathrm{RootOf}\left(i\sqrt{\pi}\,\mathrm{erf}\left(\frac{\sqrt{2}\,x}{2}\right)c_1 + i\sqrt{2}\,c_2 - \mathrm{erf}\left(\_Z\right)\sqrt{\pi}\right)\sqrt{2}$$

✓ Solution by Mathematica
Time used: 0.088 (sec). Leaf size: 44

```
DSolve[y''[x]+x*y'[x]+y[x]*(y'[x])^2==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to -i\sqrt{2}\mathrm{erf}^{-1}\left(i\left(\sqrt{\frac{2}{\pi}}c_2 - c_1\mathrm{erf}\left(\frac{x}{\sqrt{2}}\right)\right)\right)$$

2.10 problem 11

Internal problem ID [7451]

Book: Second order enumerated odes
Section: section 2
Problem number: 11.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _missing_y], _Liouville, [_2nd_order, _reducible`

$$y'' + y' \sin(x) + y'^2 = 0$$

✓ Solution by Maple
Time used: 0.0 (sec). Leaf size: 14

```
dsolve(diff(y(x),x$2)+sin(x)*diff(y(x),x)+(diff(y(x),x))^2=0,y(x), singsol=all)
```

$$y(x) = \ln\left(c_1\left(\int \mathrm{e}^{\cos(x)}dx\right) + c_2\right)$$

✓ Solution by Mathematica
Time used: 60.089 (sec). Leaf size: 43

```
DSolve[y''[x]+Sin[x]*y'[x]+(y'[x])^2==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to \int_1^x \frac{e^{\cos(K[2])}}{c_1 - \int_1^{K[2]} -e^{\cos(K[1])}dK[1]}dK[2] + c_2$$

2.11 problem 12

Internal problem ID [7452]

Book: Second order enumerated odes
Section: section 2
Problem number: 12.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[_Liouville, [_2nd_order, _reducible, _mu_x_y1], [_2nd_order,`

$$\boxed{3y'' + y' \cos(x) + \sin(y)\, {y'}^2 = 0}$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 27

```
dsolve(3*diff(y(x),x$2)+cos(x)*diff(y(x),x)+sin(y(x))*(diff(y(x),x))^2=0,y(x), singsol=all)
```

$$\int^{y(x)} e^{-\frac{\cos(\_a)}{3}} d\_a - c_1\left(\int e^{-\frac{\sin(x)}{3}} dx\right) - c_2 = 0$$

✓ Solution by Mathematica

Time used: 0.601 (sec). Leaf size: 47

```
DSolve[3*y''[x]+Cos[x]*y'[x]+Sin[y[x]]*(y'[x])^2==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to \text{InverseFunction}\left[\int_1^{\#1} e^{-\frac{1}{3}\cos(K[1])} dK[1]\&\right]\left[\int_1^x -e^{-\frac{1}{3}\sin(K[2])} c_1 dK[2] + c_2\right]$$

2.12 problem 13

Internal problem ID [7453]

Book: Second order enumerated odes
Section: section 2
Problem number: 13.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[_Liouville, [_2nd_order, _with_linear_symmetries], [_2nd_orde`

$$10y'' + x^2 y' + \frac{3{y'}^2}{y} = 0$$

✓ Solution by Maple

Time used: 0.016 (sec). Leaf size: 55

```
dsolve(10*diff(y(x),x$2)+x^2*diff(y(x),x)+3/y(x)*(diff(y(x),x))^2=0,y(x), singsol=all)
```

$$-\frac{3\left(c_1 x \,\mathrm{WhittakerM}\left(\frac{1}{6}, \frac{2}{3}, \frac{x^3}{30}\right) \mathrm{e}^{-\frac{x^3}{60}} 30^{\frac{1}{6}} + \frac{4\left(x^3\right)^{\frac{1}{6}}\left(c_1 x\,\mathrm{e}^{-\frac{x^3}{30}} + c_2 - \frac{10 y(x)^{\frac{13}{10}}}{13}\right)}{3}\right)}{4\left(x^3\right)^{\frac{1}{6}}} = 0$$

✓ Solution by Mathematica

Time used: 66.444 (sec). Leaf size: 73

```
DSolve[10*y''[x]+x^2*y'[x]+3/y[x]*(y'[x])^2==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to c_2 \exp\left(\int_1^x \frac{30 e^{-\frac{1}{30}K[1]^3} \sqrt[3]{K[1]^3}}{30 c_1 \sqrt[3]{K[1]^3} - 13\sqrt[3]{30}\,\Gamma\left(\frac{1}{3}, \frac{K[1]^3}{30}\right) K[1]} dK[1]\right)$$

2.13 problem 14

Internal problem ID [7454]

Book: Second order enumerated odes
Section: section 2
Problem number: 14.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[_Liouville, [_2nd_order, _reducible, _mu_x_y1], [_2nd_order,`

$$10y'' + (e^x + 3x)\, y' + \frac{3\, e^y y'^2}{\sin(y)} = 0$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 38

```
dsolve(10*diff(y(x),x$2)+(exp(x)+3*x)*diff(y(x),x)+3/sin(y(x))*exp(y(x))*(diff(y(x),x))^2=0,
```

$$\int^{y(x)} e^{\frac{3\left(\int \csc(\_b)e^{\_b}d\_b\right)}{10}} d\_b - c_1 \left(\int e^{-\frac{3x^2}{20}-\frac{e^x}{10}} dx\right) - c_2 = 0$$

✓ Solution by Mathematica

Time used: 0.234 (sec). Leaf size: 90

```
DSolve[10*y''[x]+(Exp[x]+3*x)*y'[x]+3/Sin[y[x]]*Exp[y[x]]*(y'[x])^2==0,y[x],x,IncludeSingula
```

$$y(x) \to \text{InverseFunction}\left[\int_1^{\#1} \exp\left(\left(-\frac{3}{10}-\frac{3i}{10}\right) e^{(1+i)K[1]} \text{Hypergeometric2F1}\left(\frac{1}{2}-\frac{i}{2}, 1, \frac{3}{2}-\frac{i}{2}, e^{2iK[1]}\right)\right) dK[\right.$$
$$\left. -e^{\frac{1}{20}\left(-3K[2]^2-2e^{K[2]}\right)} c_1 dK[2] + c_2\right]$$

2.14 problem 15

Internal problem ID [7455]

Book: Second order enumerated odes
Section: section 2
Problem number: 15.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _linear, _nonhomogeneous]]`

$$\boxed{y'' - \frac{2y}{x^2} = x\,\mathrm{e}^{-\sqrt{x}}}$$

✓ Solution by Maple
Time used: 0.015 (sec). Leaf size: 51

```
dsolve(diff(diff(y(x),x),x)-2/x^2*y(x) = x*exp(-x^(1/2)),y(x), singsol=all)
```

$$y(x) = \frac{4\,\mathrm{e}^{-\sqrt{x}}\left(7x^{\frac{5}{2}} + 140x^{\frac{3}{2}} + x^3 + 35x^2 + 840\sqrt{x} + 420x + 840\right) + c_1x^3 + c_2}{x}$$

✓ Solution by Mathematica
Time used: 0.05 (sec). Leaf size: 54

```
DSolve[y''[x]-2/x^2*y[x] == x*Exp[-x^(1/2)],y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to \frac{-2e^{-\sqrt{x}}\left(\sqrt{x}+1\right)x^3 + 3(c_2x^3 + c_1) + 2\Gamma\left(8, \sqrt{x}\right)}{3x}$$

2.15 problem 16

Internal problem ID [7456]

Book: Second order enumerated odes
Section: section 2
Problem number: 16.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _linear, _nonhomogeneous]]`

$$y'' - \frac{y'}{\sqrt{x}} + \frac{\left(x + \sqrt{x} - 8\right) y}{4x^2} = x$$

✓ Solution by Maple

Time used: 0.016 (sec). Leaf size: 50

```
dsolve(diff(y(x),x$2)-1/sqrt(x)*diff(y(x),x)+1/(4*x^2)*(x+sqrt(x)-8)*y(x)=x,y(x), singsol=al
```

$$y(x) = \frac{560x^{\frac{3}{2}} + 28x^{\frac{5}{2}} + (c_1 x^3 + c_2)\,e^{\sqrt{x}} + 4x^3 + 140x^2 + 1680x + 3360\sqrt{x} + 3360}{x}$$

✓ Solution by Mathematica

Time used: 0.053 (sec). Leaf size: 63

```
DSolve[y''[x]-1/Sqrt[x]*y'[x]+1/(4*x^2)*(x+Sqrt[x]-8)*y[x]==x,y[x],x,IncludeSingularSolution
```

$$y(x) \to \frac{-2x^{7/2} + x^3\left(-2 + c_2 e^{\sqrt{x}}\right) + 2e^{\sqrt{x}}\Gamma\left(8, \sqrt{x}\right) + 3c_1 e^{\sqrt{x}}}{3x}$$

2.16 problem 17

Internal problem ID [7457]

Book: Second order enumerated odes
Section: section 2
Problem number: 17.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_Emden, _Fowler], [_2nd_order, _linear, '_with_symmetry_[0,F`

$$\boxed{y'' + \frac{2y'}{x} + \frac{a^2 y}{x^4} = 0}$$

✓ Solution by Maple

Time used: 0.015 (sec). Leaf size: 21

```
dsolve(diff(y(x),x$2)+2/x*diff(y(x),x)+a^2/x^4*y(x)=0,y(x), singsol=all)
```

$$y(x) = c_1 \sin\left(\frac{a}{x}\right) + c_2 \cos\left(\frac{a}{x}\right)$$

✓ Solution by Mathematica

Time used: 0.022 (sec). Leaf size: 25

```
DSolve[y''[x]+2/x*y'[x]+a^2/x^4*y[x]==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to c_1 \cos\left(\frac{a}{x}\right) - c_2 \sin\left(\frac{a}{x}\right)$$

2.17 problem 18

Internal problem ID [7458]

Book: Second order enumerated odes
Section: section 2
Problem number: 18.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[_Gegenbauer, [_2nd_order, _linear, '_with_symmetry_[0,F(x)]']]`

$$\boxed{(1 - x^2)\, y'' - xy' - c^2 y = 0}$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 37

```
dsolve((1-x^2)*diff(y(x),x$2)-x*diff(y(x),x)-c^2*y(x)=0,y(x), singsol=all)
```

$$y(x) = c_1 \left(x + \sqrt{x^2 - 1}\right)^{ic} + c_2 \left(x + \sqrt{x^2 - 1}\right)^{-ic}$$

✓ Solution by Mathematica

Time used: 0.09 (sec). Leaf size: 89

```
DSolve[(1-x^2)*y''[x]-x*y'[x]-c^2*y[x]==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to c_1 \cos\left(\frac{1}{2}c\left(\log\left(1 - \frac{x}{\sqrt{x^2 - 1}}\right) - \log\left(\frac{x}{\sqrt{x^2 - 1}} + 1\right)\right)\right) - c_2 \sin\left(\frac{1}{2}c\left(\log\left(1 - \frac{x}{\sqrt{x^2 - 1}}\right) - \log\left(\frac{x}{\sqrt{x^2 - 1}} + 1\right)\right)\right)$$

2.18 problem 19

Internal problem ID [7459]

Book: Second order enumerated odes
Section: section 2
Problem number: 19.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _linear, _nonhomogeneous]]`

$$x^6y'' + 3y'x^5 + a^2y = \frac{1}{x^2}$$

✓ Solution by Maple
Time used: 0.0 (sec). Leaf size: 30

```
dsolve(x^6*diff(y(x),x$2)+3*x^5*diff(y(x),x)+a^2*y(x)=1/x^2,y(x), singsol=all)
```

$$y(x) = \sin\left(\frac{a}{2x^2}\right)c_2 + \cos\left(\frac{a}{2x^2}\right)c_1 + \frac{1}{a^2x^2}$$

✓ Solution by Mathematica
Time used: 0.073 (sec). Leaf size: 38

```
DSolve[x^6*y''[x]+3*x^5*y'[x]+a^2*y[x]==1/x^2,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to \frac{1}{a^2x^2} + c_1\cos\left(\frac{a}{2x^2}\right) - c_2\sin\left(\frac{a}{2x^2}\right)$$

2.19 problem 20

Internal problem ID [7460]

Book: Second order enumerated odes
Section: section 2
Problem number: 20.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$\boxed{x^2 y'' - 3xy' + 3y = 2x^3 - x^2}$$

✓ Solution by Maple
Time used: 0.016 (sec). Leaf size: 28

```
dsolve(x^2*diff(y(x),x$2)-3*x*diff(y(x),x)+3*y(x)=2*x^3-x^2,y(x), singsol=all)
```

$$y(x) = \frac{x(2x^2 \ln(x) + (c_1 - 1) x^2 + 2x + 2c_2)}{2}$$

✓ Solution by Mathematica
Time used: 0.018 (sec). Leaf size: 27

```
DSolve[x^2*y''[x]-3*x*y'[x]+3*y[x]==2*x^3-x^2,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to x\left(x^2 \log(x) + \left(-\frac{1}{2} + c_2\right) x^2 + x + c_1\right)$$

2.20 problem 21

Internal problem ID [7461]

Book: Second order enumerated odes
Section: section 2
Problem number: 21.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries], [_2nd_order, _linear,`

$$y'' + \cot(x)\, y' + 4y\csc(x)^2 = 0$$

✓ Solution by Maple
Time used: 0.0 (sec). Leaf size: 25

```
dsolve(diff(y(x),x$2)+cot(x)*diff(y(x),x)+4*y(x)*csc(x)^2=0,y(x), singsol=all)
```

$$y(x) = c_1(\csc(x) + \cot(x))^{-2i} + c_2(\csc(x) + \cot(x))^{2i}$$

✓ Solution by Mathematica
Time used: 0.06 (sec). Leaf size: 25

```
DSolve[y''[x]+Cot[x]*y'[x]+4*y[x]*Csc[x]^2==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to c_1 \cos(2\text{arctanh}(\cos(x))) - c_2 \sin(2\text{arctanh}(\cos(x)))$$

2.21 problem 22

Internal problem ID [7462]

Book: Second order enumerated odes
Section: section 2
Problem number: 22.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _linear, _nonhomogeneous]]`

$$(x^2+1)\,y''+(1+x)\,y'+y=4\cos\left(\ln\left(1+x\right)\right)$$

✓ Solution by Maple

Time used: 0.031 (sec). Leaf size: 408

```
dsolve((1+x^2)*diff(y(x),x$2)+(1+x)*diff(y(x),x)+y(x)=4*cos(ln(1+x)),y(x), singsol=all)
```

$$y(x)=\mathrm{hypergeom}\left(\left[i,-i\right],\left[\frac{1}{2}+\frac{i}{2}\right],\frac{1}{2}-\frac{ix}{2}\right)c_2$$
$$+(x+i)^{\frac{1}{2}-\frac{i}{2}}\,\mathrm{hypergeom}\left(\left[\frac{1}{2}+\frac{i}{2},\frac{1}{2}-\frac{3i}{2}\right],\left[\frac{3}{2}-\frac{i}{2}\right],\frac{1}{2}-\frac{ix}{2}\right)c_1$$
$$+80\left(\int\frac{}{(x^2+1)\left(10\,\mathrm{hypergeom}\left(\left[\frac{1}{2}+\frac{i}{2},\frac{1}{2}-\frac{3i}{2}\right],\left[\frac{3}{2}-\frac{i}{2}\right],\frac{1}{2}-\frac{ix}{2}\right)\left((-1-i+(-1+i)\,x)\,\mathrm{hypergeom}\,(\right.\right.}\right.$$
$$\left.-i\right],\left[\frac{1}{2}+\frac{i}{2}\right],\frac{1}{2}-\frac{ix}{2}\right)$$
$$-80\left(\int\frac{\cos(\ln(x+1}{7\left(\frac{10((1-i+(-1-i)x)\,\mathrm{hypergeom}([1-i,1+i],[\frac{3}{2}+\frac{i}{2}],\frac{1}{2}-\frac{ix}{2})+(-1+i)\,\mathrm{hypergeom}([i,-i],[\frac{1}{2}+\frac{i}{2}],\frac{1}{2}-\frac{ix}{2}))\,\mathrm{hypergeom}([\frac{1}{2}+\frac{i}{2},\frac{1}{2}-}{7}\right.}\right.$$
$$\left.+i\right)^{\frac{1}{2}-\frac{i}{2}}\,\mathrm{hypergeom}\left(\left[\frac{1}{2}+\frac{i}{2},\frac{1}{2}-\frac{3i}{2}\right],\left[\frac{3}{2}-\frac{i}{2}\right],\frac{1}{2}-\frac{ix}{2}\right)$$

✗ Solution by Mathematica

Time used: 0.0 (sec). Leaf size: 0

```
DSolve[(1+x^2)*y''[x]+(1+x)*y'[x]+y[x]==4*Cos[Log[1+x]],y[x],x,IncludeSingularSolutions -> T
```

Not solved

2.22 problem 23

Internal problem ID [7463]

Book: Second order enumerated odes
Section: section 2
Problem number: 23.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$\boxed{y'' + \tan(x)\, y' + y \cos(x)^2 = 0}$$

✓ Solution by Maple

Time used: 0.031 (sec). Leaf size: 15

```
dsolve(diff(y(x),x$2)+tan(x)*diff(y(x),x)+cos(x)^2*y(x)=0,y(x), singsol=all)
```

$$y(x) = c_1 \sin(\sin(x)) + c_2 \cos(\sin(x))$$

✓ Solution by Mathematica

Time used: 0.085 (sec). Leaf size: 18

```
DSolve[y''[x]+Tan[x]*y'[x]+Cos[x]^2*y[x]==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to c_2 \sin(\sin(x)) + c_1 \cos(\sin(x))$$

2.23 problem 24

Internal problem ID [7464]

Book: Second order enumerated odes
Section: section 2
Problem number: 24.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _linear, _nonhomogeneous]]`

$$x y'' - y' + 4 y x^3 = 8 x^3 \sin(x)^2$$

✓ Solution by Maple
Time used: 0.0 (sec). Leaf size: 124

```
dsolve(x*diff(y(x),x$2)-diff(y(x),x)+4*x^3*y(x)=8*x^3*sin(x)^2,y(x), singsol=all)
```

$$y(x) = \sin(x^2) c_2 + \cos(x^2) c_1 + 1 - \cos(2x) - \frac{\text{FresnelC}\left(\frac{\sqrt{2}(x-1)}{\sqrt{\pi}}\right)\sqrt{\pi}\sqrt{2}\sin(x^2+1)}{2} + \frac{\text{FresnelS}\left(\frac{\sqrt{2}(x-1)}{\sqrt{\pi}}\right)\sqrt{\pi}\sqrt{2}\cos(x^2+1)}{2} + \frac{\text{FresnelC}\left(\frac{\sqrt{2}(x+1)}{\sqrt{\pi}}\right)\sqrt{\pi}\sqrt{2}\sin(x^2+1)}{2} - \frac{\text{FresnelS}\left(\frac{\sqrt{2}(x+1)}{\sqrt{\pi}}\right)\sqrt{\pi}\sqrt{2}\cos(x^2+1)}{2}$$

✓ Solution by Mathematica

Time used: 1.041 (sec). Leaf size: 147

```
DSolve[x*y''[x]-y'[x]+4*x^3*y[x]==8*x^3*Sin[x]^2,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to \frac{1}{2}\left(-\sqrt{2\pi}\,\text{FresnelC}\left(\sqrt{\frac{2}{\pi}}(x-1)\right)\sin\left(x^2+1\right) + \sqrt{2\pi}\,\text{FresnelC}\left(\sqrt{\frac{2}{\pi}}(x+1)\right)\sin\left(x^2+1\right) + \sqrt{2\pi}\,\text{FresnelS}\left(\sqrt{\frac{2}{\pi}}(x-1)\right)\cos\left(x^2+1\right) - \sqrt{2\pi}\,\text{FresnelS}\left(\sqrt{\frac{2}{\pi}}(x+1)\right)\cos\left(x^2+1\right) + 2c_1\cos\left(x^2\right) + 2c_2\sin\left(x^2\right) - 2\cos(2x) + 2\right)$$

2.24 problem 25

Internal problem ID [7465]

Book: Second order enumerated odes
Section: section 2
Problem number: 25.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _linear, _nonhomogeneous]]`

$$\boxed{xy'' - y' + 4yx^3 = x^5}$$

✓ Solution by Maple
Time used: 0.016 (sec). Leaf size: 22

```
dsolve(x*diff(y(x),x$2)-diff(y(x),x)+4*x^3*y(x)=x^5,y(x), singsol=all)
```

$$y(x) = \sin(x^2)\, c_2 + \cos(x^2)\, c_1 + \frac{x^2}{4}$$

✓ Solution by Mathematica
Time used: 0.058 (sec). Leaf size: 27

```
DSolve[x*y''[x]-y'[x]+4*x^3*y[x]==x^5,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to \frac{x^2}{4} + c_1 \cos(x^2) + c_2 \sin(x^2)$$

2.25 problem 25

Internal problem ID [7466]

Book: Second order enumerated odes
Section: section 2
Problem number: 25.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _linear, _nonhomogeneous]]`

$$\boxed{\cos(x)\, y'' + y' \sin(x) - 2y\cos(x)^3 = 2\cos(x)^5}$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 30

```
dsolve(cos(x)*diff(y(x),x$2)+sin(x)*diff(y(x),x)-2*y(x)*cos(x)^3=2*cos(x)^5,y(x), singsol=al
```

$$y(x) = \sinh\left(\sin(x)\sqrt{2}\right) c_2 + \cosh\left(\sin(x)\sqrt{2}\right) c_1 + \frac{1}{2} - \frac{\cos(2x)}{2}$$

✓ Solution by Mathematica

Time used: 17.301 (sec). Leaf size: 167

```
DSolve[Cos[x]*y''[x]+Sin[x]*y'[x]-2*y[x]*Cos[x]^3==2*Cos[x]^5,y[x],x,IncludeSingularSolution
```

$$\begin{aligned} y(x) &\\ \to \cos&\left(\sqrt{-\cos(2x)-1}\tan(x)\right)\int_1^x \cos^2(K[1])\sqrt{-\cos(2K[1])-1}\sin\left(\sqrt{-\cos(2K[1])-1}\tan(K[1])\right) dK \\ &+ \sin\left(\sqrt{-\cos(2x)-1}\tan(x)\right)\int_1^x \\ &- \cos^2(K[2])\sqrt{-\cos(2K[2])-1}\cos\left(\sqrt{-\cos(2K[2])-1}\tan(K[2])\right) dK[2] \\ &+ c_1\cos\left(\sqrt{-\cos(2x)-1}\tan(x)\right) + c_2\sin\left(\sqrt{-\cos(2x)-1}\tan(x)\right) \end{aligned}$$

2.26 problem 26

Internal problem ID [7467]

Book: Second order enumerated odes
Section: section 2
Problem number: 26.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _linear, _nonhomogeneous]]`

$$y'' + \left(1 - \frac{1}{x}\right) y' + 4x^2 y\, e^{-2x} = 4\left(x^3 + x^2\right) e^{-3x}$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 39

```
dsolve(diff(y(x),x$2)+(1-1/x)*diff(y(x),x)+4*x^2*y(x)*exp(-2*x)=4*(x^2+x^3)*exp(-3*x),y(x),
```

$$y(x) = \sin\left(2(x+1)\, e^{-x}\right) c_2 + \cos\left(2(x+1)\, e^{-x}\right) c_1 + e^{-x} x + e^{-x}$$

✓ Solution by Mathematica

Time used: 0.604 (sec). Leaf size: 47

```
DSolve[y''[x]+(1-1/x)*y'[x]+4*x^2*y[x]*Exp[-2*x]==4*(x^2+x^3)*Exp[-3*x],y[x],x,IncludeSingul
```

$$y(x) \to c_1 \cos\left(2e^{-x}(x+1)\right) + e^{-x}\left(x - c_2 e^x \sin\left(2e^{-x}(x+1)\right) + 1\right)$$

2.27 problem 27

Internal problem ID [7468]

Book: Second order enumerated odes
Section: section 2
Problem number: 27.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$\boxed{y'' - x^2 y' + yx = x^{m+1}}$$

✓ Solution by Maple

Time used: 0.032 (sec). Leaf size: 201

```
dsolve(diff(y(x),x$2)-x^2*diff(y(x),x)+x*y(x)=x^(m+1),y(x), singsol=all)
```

$$y(x) = \frac{\left(-3\, 3^{\frac{m}{6}} \mathrm{e}^{\frac{x^3}{6}} \left(x^3\right)^{-\frac{m}{6}} \operatorname{WhittakerM}\left(\frac{m}{6}, \frac{m}{6}+\frac{1}{2}, \frac{x^3}{3}\right) x^m + (m+3) \left(3^{\frac{1}{3}} \mathrm{e}^{\frac{x^3}{3}} c_1 - \frac{\left(\int \frac{\left(-3\left(-x^3\right)^{\frac{2}{3}} + x^3 3^{\frac{2}{3}} \mathrm{e}^{-\frac{x^3}{3}} \left(\Gamma\left(\frac{2}{3}\right) - \Gamma \cdots\right.\right.}{\left(-x^3\right)^{\frac{2}{3}}} \cdots\right.}{3} \cdots\right.\right.}{\left(-x^3\right) \cdots}$$

✓ Solution by Mathematica

Time used: 0.453 (sec). Leaf size: 144

```
DSolve[y''[x]-x^2*y'[x]+x*y[x]==x^(m+1),y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to x \int_1^x \frac{e^{-\frac{1}{3}K[1]^3} \Gamma\left(-\frac{1}{3}, -\frac{1}{3}K[1]^3\right) K[1]^{m+1} \sqrt[3]{-K[1]^3}}{3\sqrt[3]{3}} dK[1] - \frac{\sqrt[3]{-x^3} \left(x^3\right)^{-m/3} \Gamma\left(-\frac{1}{3}, -\frac{x^3}{3}\right) \left(-3^{m/3} x^m \Gamma\left(\frac{m+3}{3}, \frac{x^3}{3}\right) + c_2 \left(x^3\right)^{m/3}\right)}{3\sqrt[3]{3}} + c_1 x$$

2.28 problem 28

Internal problem ID [7469]

Book: Second order enumerated odes
Section: section 2
Problem number: 28.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$y'' - \frac{y'}{\sqrt{x}} + \frac{\left(x + \sqrt{x} - 8\right) y}{4x^2} = 0$$

✓ Solution by Maple
Time used: 0.0 (sec). Leaf size: 19

```
dsolve(diff(y(x),x$2)-1/x^(1/2)*diff(y(x),x)+y(x)/(4*x^2)*(-8+x^(1/2)+x)=0,y(x), singsol=all
```

$$y(x) = \frac{e^{\sqrt{x}}(c_2 x^3 + c_1)}{x}$$

✓ Solution by Mathematica
Time used: 0.041 (sec). Leaf size: 30

```
DSolve[y''[x]-1/x^(1/2)*y'[x]+y[x]/(4*x^2)*(-8+x^(1/2)+x)==0,y[x],x,IncludeSingularSolutions
```

$$y(x) \to \frac{e^{\sqrt{x}}(c_2 x^3 + 3c_1)}{3x}$$

2.29 problem 29

Internal problem ID [7470]

Book: Second order enumerated odes
Section: section 2
Problem number: 29.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[_Lienard]`

$$\boxed{\cos(x)^2 y'' - 2\cos(x)\sin(x) y' + y\cos(x)^2 = 0}$$

✓ Solution by Maple
Time used: 0.016 (sec). Leaf size: 24

```
dsolve(cos(x)^2*diff(y(x),x$2)-2*cos(x)*sin(x)*diff(y(x),x)+y(x)*cos(x)^2=0,y(x), singsol=al
```

$$y(x) = \sec(x)\left(c_1 \sin\left(\sqrt{2}\,x\right) + c_2 \cos\left(\sqrt{2}\,x\right)\right)$$

✓ Solution by Mathematica
Time used: 0.098 (sec). Leaf size: 51

```
DSolve[Cos[x]^2*y''[x]-2*Cos[x]*Sin[x]*y'[x]+y[x]*Cos[x]^2==0,y[x],x,IncludeSingularSolution
```

$$y(x) \to \frac{1}{4} e^{-i\sqrt{2}x}\left(4c_1 - i\sqrt{2}c_2 e^{2i\sqrt{2}x}\right)\sec(x)$$

2.30 problem 30

Internal problem ID [7471]

Book: Second order enumerated odes
Section: section 2
Problem number: 30.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _linear, _nonhomogeneous]]`

$$y'' - 4xy' + (4x^2 - 1)\, y = -3\,\mathrm{e}^{x^2} \sin(x)$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 29

```
dsolve(diff(y(x),x$2)-4*x*diff(y(x),x)+(4*x^2-1)*y(x)=-3*exp(x^2)*sin(x),y(x), singsol=all)
```

$$y(x) = \frac{((2c_2 + 3x)\cos(x) + \sin(x)\,(2c_1 - 3))\,\mathrm{e}^{x^2}}{2}$$

✓ Solution by Mathematica

Time used: 0.094 (sec). Leaf size: 50

```
DSolve[y''[x]-4*x*y'[x]+(4*x^2-1)*y[x]==-3*Exp[x^2]*Sin[x],y[x],x,IncludeSingularSolutions -
```

$$y(x) \to \frac{1}{8} e^{x(x-i)} \left(6x + e^{2ix}(6x + 3i - 4ic_2) - 3i + 8c_1\right)$$

2.31 problem 31

Internal problem ID [7472]

Book: Second order enumerated odes
Section: section 2
Problem number: 31.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _linear, _nonhomogeneous]]`

$$\boxed{y'' - 2bxy' + b^2x^2y = x}$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 137

```
dsolve(diff(y(x),x$2)-2*b*x*diff(y(x),x)+b^2*x^2*y(x)=x,y(x), singsol=all)
```

$$y(x) = \frac{4\,e^{\frac{x(bx+2\sqrt{-b})}{2}}c_2 b^{\frac{3}{2}} + 4\,e^{\frac{x(bx-2\sqrt{-b})}{2}}c_1 b^{\frac{3}{2}} - \operatorname{erf}\left(\frac{\sqrt{2}\,(bx+\sqrt{-b})}{2\sqrt{b}}\right)\sqrt{2}\,\sqrt{\pi}\,e^{\frac{b\,x^2}{2}+x\sqrt{-b}-\frac{1}{2}} + \sqrt{2}\,e^{\frac{b\,x^2}{2}-x\sqrt{-b}-\frac{1}{2}}\sqrt{\pi}\,\operatorname{erf}\Big(}{4b^{\frac{3}{2}}}$$

✓ Solution by Mathematica

Time used: 0.427 (sec). Leaf size: 139

```
DSolve[y''[x]-2*b*x*y'[x]+b^2*x^2*y[x]==x,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to \frac{e^{\frac{1}{2}\left(\sqrt{b}x-i\right)^2}\left(-\sqrt{2\pi}e^{2i\sqrt{b}x}\operatorname{erf}\left(\frac{\sqrt{b}x+i}{\sqrt{2}}\right) + i\sqrt{2\pi}\operatorname{erfi}\left(\frac{1+i\sqrt{b}x}{\sqrt{2}}\right) + 2\sqrt{e}b\left(2\sqrt{b}c_1 - ic_2e^{2i\sqrt{b}x}\right)\right)}{4b^{3/2}}$$

2.32 problem 32

Internal problem ID [7473]

Book: Second order enumerated odes
Section: section 2
Problem number: 32.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _linear, _nonhomogeneous]]`

$$\boxed{y'' - 4xy' + (4x^2 - 3)\, y = e^{x^2}}$$

✓ Solution by Maple
Time used: 0.0 (sec). Leaf size: 27

```
dsolve(diff(y(x),x$2)-4*x*diff(y(x),x)+(4*x^2-3)*y(x)=exp(x^2),y(x), singsol=all)
```

$$y(x) = e^{x(x+1)} c_2 + e^{x(x-1)} c_1 - e^{x^2}$$

✓ Solution by Mathematica
Time used: 0.041 (sec). Leaf size: 34

```
DSolve[y''[x]-4*x*y'[x]+(4*x^2-3)*y[x]==Exp[x^2],y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to \frac{1}{2} e^{(x-1)x} \left(-2e^x + c_2 e^{2x} + 2c_1\right)$$

2.33 problem 33

Internal problem ID [7474]

Book: Second order enumerated odes
Section: section 2
Problem number: 33.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _linear, _nonhomogeneous]]`

$$y'' - 2\tan(x)\, y' + 5y = e^{x^2}\sec(x)$$

✓ Solution by Maple

Time used: 0.015 (sec). Leaf size: 101

```
dsolve(diff(y(x),x$2)-2*tan(x)*diff(y(x),x)+5*y(x)=exp(x^2)*sec(x),y(x), singsol=all)
```

$$y(x) = -\frac{\left(\sqrt{6}\,e^{\frac{3}{2}}\sqrt{\pi}\left(i\sin\left(\sqrt{6}\,x\right)-\cos\left(\sqrt{6}\,x\right)\right)\operatorname{erf}\left(ix-\frac{\sqrt{6}}{2}\right)+\sqrt{6}\,e^{\frac{3}{2}}\left(i\sin\left(\sqrt{6}\,x\right)+\cos\left(\sqrt{6}\,x\right)\right)\sqrt{\pi}\operatorname{erf}\left(ix\right.\right.}{24}$$

✓ Solution by Mathematica

Time used: 0.249 (sec). Leaf size: 118

```
DSolve[y''[x]-2*Tan[x]*y'[x]+5*y[x]==Exp[x^2]*Sec[x],y[x],x,IncludeSingularSolutions -> True
```

$$y(x) \to \frac{1}{24}e^{-i\sqrt{6}x}\sec(x)\left(-e^{3/2}\sqrt{6\pi}\operatorname{erf}\left(\sqrt{\frac{3}{2}}-ix\right)-\sqrt{6\pi}e^{\frac{3}{2}+2i\sqrt{6}x}\operatorname{erf}\left(\sqrt{\frac{3}{2}}+ix\right)\right.$$
$$\left.-2i\sqrt{6}c_2e^{2i\sqrt{6}x}+24c_1\right)$$

2.34 problem 34

Internal problem ID [7475]

Book: Second order enumerated odes
Section: section 2
Problem number: 34.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$x^2y'' - 2xy' + 2(x^2 + 1)\,y = 0$$

✓ Solution by Maple

Time used: 0.016 (sec). Leaf size: 23

```
dsolve(x^2*diff(y(x),x$2)-2*x*diff(y(x),x)+2*(1+x^2)*y(x)=0,y(x), singsol=all)
```

$$y(x) = x\left(c_1 \sin\left(\sqrt{2}\,x\right) + c_2 \cos\left(\sqrt{2}\,x\right)\right)$$

✓ Solution by Mathematica

Time used: 0.042 (sec). Leaf size: 48

```
DSolve[x^2*y''[x]-2*x*y'[x]+2*(1+x^2)*y[x]==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to c_1 e^{-i\sqrt{2}x} x - \frac{ic_2 e^{i\sqrt{2}x} x}{2\sqrt{2}}$$

2.35 problem 35

Internal problem ID [7476]

Book: Second order enumerated odes
Section: section 2
Problem number: 35.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$4x^2y'' + 4y'x^5 + (x^8 + 6x^4 + 4)\,y = 0$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 35

```
dsolve(4*x^2*diff(y(x),x$2)+4*x^5*diff(y(x),x)+(x^8+6*x^4+4)*y(x)=0,y(x), singsol=all)
```

$$y(x) = \sqrt{x}\,\mathrm{e}^{-\frac{x^4}{8}}\left(c_1 x^{\frac{i\sqrt{3}}{2}} + c_2 x^{-\frac{i\sqrt{3}}{2}}\right)$$

✓ Solution by Mathematica

Time used: 0.076 (sec). Leaf size: 62

```
DSolve[4*x^2*y''[x]+4*x^5*y'[x]+(x^8+6*x^4+4)*y[x]==0,y[x],x,IncludeSingularSolutions -> Tru
```

$$y(x) \to \frac{1}{3}e^{-\frac{x^4}{8}}x^{\frac{1}{2}-\frac{i\sqrt{3}}{2}}\left(3c_1 - i\sqrt{3}c_2 x^{i\sqrt{3}}\right)$$

2.36 problem 36

Internal problem ID [7477]

Book: Second order enumerated odes
Section: section 2
Problem number: 36.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$\boxed{x^2 y'' + (xy' - y)^2 = 0}$$

✓ Solution by Maple

Time used: 0.032 (sec). Leaf size: 22

```
dsolve(x^2*diff(y(x),x$2)+(x*diff(y(x),x)-y(x))^2=0,y(x), singsol=all)
```

$$y(x) = \left(-e^{c_1} \operatorname{expIntegral}_1\left(-\ln\left(\frac{1}{x}\right) + c_1\right) + c_2\right) x$$

✓ Solution by Mathematica

Time used: 46.789 (sec). Leaf size: 33

```
DSolve[x^2*y''[x]+(x*y'[x]-y[x])^2==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to x(e^{c_1} \operatorname{ExpIntegralEi}(-c_1 - \log(x)) + c_2)$$
$$y(x) \to c_2 x$$

2.37 problem 37

Internal problem ID [7478]

Book: Second order enumerated odes
Section: section 2
Problem number: 37.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$\boxed{xy'' + 2y' - yx = 0}$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 17

```
dsolve(x*diff(y(x),x$2)+2*diff(y(x),x)-x*y(x)=0,y(x), singsol=all)
```

$$y(x) = \frac{c_1 \sinh(x) + c_2 \cosh(x)}{x}$$

✓ Solution by Mathematica

Time used: 0.032 (sec). Leaf size: 28

```
DSolve[x*y''[x]+2*y'[x]-x*y[x]==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to \frac{2c_1 e^{-x} + c_2 e^{x}}{2x}$$

2.38 problem 38

Internal problem ID [7479]

Book: Second order enumerated odes
Section: section 2
Problem number: 38.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[_Lienard]`

$$\boxed{xy'' + 2y' + yx = 0}$$

✓ Solution by Maple
Time used: 0.016 (sec). Leaf size: 17

```
dsolve(x*diff(y(x),x$2)+2*diff(y(x),x)+x*y(x)=0,y(x), singsol=all)
```

$$y(x) = \frac{c_1 \sin(x) + c_2 \cos(x)}{x}$$

✓ Solution by Mathematica
Time used: 0.03 (sec). Leaf size: 37

```
DSolve[x*y''[x]+2*y'[x]+x*y[x]==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to \frac{2c_1e^{-ix} - ic_2e^{ix}}{2x}$$

2.39 problem 39

Internal problem ID [7480]

Book: Second order enumerated odes
Section: section 2
Problem number: 39.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_linear]`

$$\boxed{y' + y\cot(x) = 2\cos(x)}$$

✓ Solution by Maple
Time used: 0.0 (sec). Leaf size: 16

```
dsolve(diff(y(x),x)+y(x)*cot(x)=2*cos(x),y(x), singsol=all)
```

$$y(x) = \csc(x)\left(-\cos(x)^2 + c_1 + \frac{1}{2}\right)$$

✓ Solution by Mathematica
Time used: 0.082 (sec). Leaf size: 20

```
DSolve[y'[x]+y[x]*Cot[x]==2*Cos[x],y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to -\frac{1}{2}\csc(x)(\cos(2x) - 2c_1)$$

2.40 problem 40

Internal problem ID [7481]

Book: Second order enumerated odes
Section: section 2
Problem number: 40.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_rational]`

$$\boxed{2xy^2 - y + (y^2 + x + y)\, y' = 0}$$

✓ Solution by Maple
Time used: 0.032 (sec). Leaf size: 28

```
dsolve((2*x*y(x)^2-y(x))+(y(x)^2+x+y(x))*diff(y(x),x)=0,y(x), singsol=all)
```

$$y(x) = e^{\mathrm{RootOf}\left(x^2 e^{\_Z} + e^{2\_Z} + c_1 e^{\_Z} + e^{\_Z}\_Z - x\right)}$$

✓ Solution by Mathematica
Time used: 0.191 (sec). Leaf size: 22

```
DSolve[(2*x*y[x]^2-y[x])+(y[x]^2+x+y[x])*y'[x]==0,y[x],x,IncludeSingularSolutions -> True]
```

$$\text{Solve}\left[x^2 - \frac{x}{y(x)} + y(x) + \log(y(x)) = c_1, y(x)\right]$$

2.41 problem 41

Internal problem ID [7482]

Book: Second order enumerated odes
Section: section 2
Problem number: 41.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[[_Riccati, _special]]`

$$\boxed{y' + y^2 = x}$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 23

```
dsolve(diff(y(x),x)=x-y(x)^2,y(x), singsol=all)
```

$$y(x) = \frac{c_1 \operatorname{AiryAi}(1, x) + \operatorname{AiryBi}(1, x)}{c_1 \operatorname{AiryAi}(x) + \operatorname{AiryBi}(x)}$$

✓ Solution by Mathematica

Time used: 0.118 (sec). Leaf size: 223

```
DSolve[y'[x]==x-y[x]^2,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to -\frac{-ix^{3/2}\big(2\operatorname{BesselJ}\big(-\frac{2}{3}, \frac{2}{3}ix^{3/2}\big) + c_1\big(\operatorname{BesselJ}\big(-\frac{4}{3}, \frac{2}{3}ix^{3/2}\big) - \operatorname{BesselJ}\big(\frac{2}{3}, \frac{2}{3}ix^{3/2}\big)\big)\big) - c_1 \operatorname{BesselJ}\big(-\frac{1}{3}, \frac{2}{3}i}{2x\left(\operatorname{BesselJ}\left(\frac{1}{3}, \frac{2}{3}ix^{3/2}\right) + c_1 \operatorname{BesselJ}\left(-\frac{1}{3}, \frac{2}{3}ix^{3/2}\right)\right)}$$

$$y(x) \to \frac{ix^{3/2} \operatorname{BesselJ}\left(-\frac{4}{3}, \frac{2}{3}ix^{3/2}\right) - ix^{3/2} \operatorname{BesselJ}\left(\frac{2}{3}, \frac{2}{3}ix^{3/2}\right) + \operatorname{BesselJ}\left(-\frac{1}{3}, \frac{2}{3}ix^{3/2}\right)}{2x \operatorname{BesselJ}\left(-\frac{1}{3}, \frac{2}{3}ix^{3/2}\right)}$$

2.42 problem 42

Internal problem ID [7483]

Book: Second order enumerated odes
Section: section 2
Problem number: 42.
ODE order: 4.
ODE degree: 1.

CAS Maple gives this as type `[[_high_order, _linear, _nonhomogeneous]]`

$$y'''' - y''' - 3y'' + 5y' - 2y = x\,\mathrm{e}^{x} + 3\,\mathrm{e}^{-2x}$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 52

```
dsolve(diff(y(x),x$4)-diff(y(x),x$3)-3*diff(y(x),x$2)+5*diff(y(x),x)-2*y(x)=x*exp(x)+3*exp(-
```

$$y(x) = \frac{\left(\left(x^4 - \frac{4x^3}{3} + \left(72c_4 + \frac{4}{3}\right)x^2 + \left(72c_3 - \frac{8}{9}\right)x + 72c_1 + \frac{8}{27}\right)\mathrm{e}^{3x} - 8x + 72c_2 - 8\right)\mathrm{e}^{-2x}}{72}$$

✓ Solution by Mathematica

Time used: 0.234 (sec). Leaf size: 64

```
DSolve[y''''[x]-y'''[x]-3*y''[x]+5*y'[x]-2*y[x]==x*Exp[x]+3*Exp[-2*x],y[x],x,IncludeSingular
```

$$y(x) \to e^x\left(\frac{x^4}{72} - \frac{x^3}{54} + \left(\frac{1}{54} + c_4\right)x^2 + \left(-\frac{1}{81} + c_3\right)x + \frac{1}{243} + c_2\right) - \frac{1}{9}e^{-2x}(x + 1 - 9c_1)$$

2.43 problem 43

Internal problem ID [7484]

Book: Second order enumerated odes
Section: section 2
Problem number: 43.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$\boxed{x^2y'' - x(x+6)\,y' + 10y = 0}$$

With the expansion point for the power series method at $x = 0$.

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 65

```
Order:=6;
dsolve(x^2*diff(y(x),x$2)-x*(x+6)*diff(y(x),x)+10*y(x)=0,y(x),type='series',x=0);
```

$$y(x) = x^2\left(c_1x^3\left(1+\frac{5}{4}x+\frac{3}{4}x^2+\frac{7}{24}x^3+\frac{1}{12}x^4+\frac{3}{160}x^5+\mathrm{O}\left(x^6\right)\right)\right.$$
$$+c_2\left(\ln\left(x\right)\left(24x^3+30x^4+18x^5+\mathrm{O}\left(x^6\right)\right)\right.$$
$$\left.\left.+\left(12-12x+18x^2+26x^3+x^4-9x^5+\mathrm{O}\left(x^6\right)\right)\right)\right)$$

✓ Solution by Mathematica

Time used: 0.03 (sec). Leaf size: 84

```
AsymptoticDSolveValue[x^2*y''[x]-x*(x+6)*y'[x]+10*y[x]==0,y[x],{x,0,5}]
```

$$y(x) \to c_1\left(\frac{1}{2}x^5(5x+4)\log(x)-\frac{1}{4}x^2\left(3x^4-6x^3-6x^2+4x-4\right)\right)$$
$$+c_2\left(\frac{x^9}{12}+\frac{7x^8}{24}+\frac{3x^7}{4}+\frac{5x^6}{4}+x^5\right)$$

2.44 problem 44

Internal problem ID [7485]

Book: Second order enumerated odes
Section: section 2
Problem number: 44.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[_Bessel]`

$$x^2y'' + xy' + (x^2 - 5)\, y = 0$$

With the expansion point for the power series method at $x = 0$.

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 97

```
Order:=6;
dsolve(x^2*diff(y(x),x$2)+x*diff(y(x),x)+(x^2-5)*y(x)=0,y(x),type='series',x=0);
```

$$y(x) = c_1 x^{-\sqrt{5}} \left(1 + \frac{1}{-4 + 4\sqrt{5}} x^2 + \frac{1}{32} \frac{1}{\left(-2 + \sqrt{5}\right) \left(\sqrt{5} - 1\right)} x^4 + \mathrm{O}\left(x^6\right)\right) + c_2 x^{\sqrt{5}} \left(1 - \frac{1}{4 + 4\sqrt{5}} x^2 + \frac{1}{32} \frac{1}{\left(\sqrt{5} + 2\right) \left(\sqrt{5} + 1\right)} x^4 + \mathrm{O}\left(x^6\right)\right)$$

✓ Solution by Mathematica

Time used: 0.003 (sec). Leaf size: 210

```
AsymptoticDSolveValue[x^2*y''[x]+x*y'[x]+(x^2-5)*y[x]==0,y[x],{x,0,5}]
```

$$y(x) \to c_2\left(\frac{x^4}{\left(-3-\sqrt{5}+\left(1-\sqrt{5}\right)\left(2-\sqrt{5}\right)\right)\left(-1-\sqrt{5}+\left(3-\sqrt{5}\right)\left(4-\sqrt{5}\right)\right)} - \frac{x^2}{-3-\sqrt{5}+\left(1-\sqrt{5}\right)\left(2-\sqrt{5}\right)}+1\right)x^{-\sqrt{5}} + c_1\left(\frac{x^4}{\left(-3+\sqrt{5}+\left(1+\sqrt{5}\right)\left(2+\sqrt{5}\right)\right)\left(-1+\sqrt{5}+\left(3+\sqrt{5}\right)\left(4+\sqrt{5}\right)\right)} - \frac{x^2}{-3+\sqrt{5}+\left(1+\sqrt{5}\right)\left(2+\sqrt{5}\right)}+1\right)x^{\sqrt{5}}$$

2.45 problem 45

Internal problem ID [7486]

Book: Second order enumerated odes
Section: section 2
Problem number: 45.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[_Bessel]`

$$x^2y'' + xy' + (x^2 - 5)\,y = 0$$

✓ Solution by Maple
Time used: 0.0 (sec). Leaf size: 19

```
dsolve(x^2*diff(y(x),x$2)+x*diff(y(x),x)+(x^2-5)*y(x)=0,y(x), singsol=all)
```

$$y(x) = c_1 \operatorname{BesselJ}\left(\sqrt{5}, x\right) + c_2 \operatorname{BesselY}\left(\sqrt{5}, x\right)$$

✓ Solution by Mathematica
Time used: 0.08 (sec). Leaf size: 26

```
DSolve[x^2*y''[x]+x*y'[x]+(x^2-5)*y[x]==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to c_1 \operatorname{BesselJ}\left(\sqrt{5}, x\right) + c_2 \operatorname{BesselY}\left(\sqrt{5}, x\right)$$

2.46 problem 46

Internal problem ID [7487]

Book: Second order enumerated odes
Section: section 2
Problem number: 46.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_Emden, _Fowler], [_2nd_order, _linear, '_with_symmetry_[0,F`

$$\boxed{x^2y'' - 4xy' + 6y = 0}$$

✓ Solution by Maple
Time used: 0.0 (sec). Leaf size: 13

```
dsolve(x^2*diff(y(x),x$2)-4*x*diff(y(x),x)+6*y(x)=0,y(x), singsol=all)
```

$$y(x) = x^2(c_1x + c_2)$$

✓ Solution by Mathematica
Time used: 0.012 (sec). Leaf size: 16

```
DSolve[x^2*y''[x]-4*x*y'[x]+6*y[x]==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to x^2(c_2x + c_1)$$

2.47 problem 47

Internal problem ID [7488]

Book: Second order enumerated odes
Section: section 2
Problem number: 47.
ODE order: 3.
ODE degree: 1.

CAS Maple gives this as type `[[_3rd_order, _with_linear_symmetries]]`

$$\boxed{y''' - yx = 0}$$

✓ Solution by Maple
Time used: 0.0 (sec). Leaf size: 45

```
dsolve(diff(y(x),x$3)-x*y(x)=0,y(x), singsol=all)
```

$$y(x) = c_1 \operatorname{hypergeom}\left([\,], \left[\frac{1}{2}, \frac{3}{4}\right], \frac{x^4}{64}\right) + c_2 x \operatorname{hypergeom}\left([\,], \left[\frac{3}{4}, \frac{5}{4}\right], \frac{x^4}{64}\right) + c_3 x^2 \operatorname{hypergeom}\left([\,], \left[\frac{5}{4}, \frac{3}{2}\right], \frac{x^4}{64}\right)$$

✓ Solution by Mathematica
Time used: 0.009 (sec). Leaf size: 76

```
DSolve[y'''[x]-x*y[x]==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to c_1\, {}_0F_2\left(;\frac{1}{2}, \frac{3}{4}; \frac{x^4}{64}\right) + \frac{1}{8} x \left((2+2i) c_2\, {}_0F_2\left(;\frac{3}{4}, \frac{5}{4}; \frac{x^4}{64}\right) + i c_3 x\, {}_0F_2\left(;\frac{5}{4}, \frac{3}{2}; \frac{x^4}{64}\right)\right)$$

2.48 problem 48

Internal problem ID [7489]

Book: Second order enumerated odes
Section: section 2
Problem number: 48.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_quadrature]`

$$\boxed{y' - y^{\frac{1}{3}} = 0}$$

With initial conditions

$$[y(0) = 0]$$

✓ Solution by Maple
Time used: 0.0 (sec). Leaf size: 5

```
dsolve([diff(y(x),x)=y(x)^(1/3),y(0) = 0],y(x), singsol=all)
```

$$y(x) = 0$$

✓ Solution by Mathematica
Time used: 0.005 (sec). Leaf size: 21

```
DSolve[{y'[x]==y[x]^(1/3),{y[0]==0}},y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \rightarrow \frac{2}{3}\sqrt{\frac{2}{3}}x^{3/2}$$

2.49 problem 49

Internal problem ID [7490]

Book: Second order enumerated odes
Section: section 2
Problem number: 49.
ODE order: 1.
ODE degree: 1.

Solve

$$x'(t) = 3x(t) + y(t)$$
$$y'(t) = -x(t) + y(t)$$

✓ Solution by Maple

Time used: 0.015 (sec). Leaf size: 32

```
dsolve([diff(x(t),t)=3*x(t)+y(t),diff(y(t),t)=-x(t)+y(t)],singsol=all)
```

$$x(t) = e^{2t}(c_2 t + c_1)$$
$$y(t) = -e^{2t}(c_2 t + c_1 - c_2)$$

✓ Solution by Mathematica

Time used: 0.003 (sec). Leaf size: 42

```
DSolve[{x'[t]==3*x[t]+y[t],y'[t]==-x[t]+y[t]},{x[t],y[t]},t,IncludeSingularSolutions -> True
```

$$x(t) \to e^{2t}(c_1(t+1) + c_2 t)$$
$$y(t) \to e^{2t}(c_2 - (c_1 + c_2)t)$$