

**A Solution Manual For**

**Selected problems from homeworks from  
different courses**

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## 1.1 problem HW 1 problem 6(a)

Internal problem ID [7029]

**Book:** Selected problems from homeworks from different courses

**Section:** Math 2520, summer 2021. Differential Equations and Linear Algebra. Normandale college, Bloomington, Minnesota

**Problem number:** HW 1 problem 6(a).

**ODE order:** 1.

**ODE degree:** 1.

CAS Maple gives this as type [\_separable]

$$y' - \frac{y}{x \ln(x)} = 0$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 8

```
dsolve(diff(y(x),x)=y(x)/(x*ln(x)),y(x), singsol=all)
```

$$y(x) = c_1 \ln(x)$$

✓ Solution by Mathematica

Time used: 0.043 (sec). Leaf size: 15

```
DSolve[y'[x]==y[x]/(x*Log[x]),y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \rightarrow c_1 \log(x)$$

$$y(x) \rightarrow 0$$

## 1.2 problem HW 1 problem 6(b)

Internal problem ID [7030]

**Book:** Selected problems from homeworks from different courses

**Section:** Math 2520, summer 2021. Differential Equations and Linear Algebra. Normandale college, Bloomington, Minnesota

**Problem number:** HW 1 problem 6(b).

**ODE order:** 1.

**ODE degree:** 1.

CAS Maple gives this as type [\_separable]

$$(x^2 + 1)y' + y^2 = -1$$

With initial conditions

$$[y(0) = 1]$$

✓ Solution by Maple

Time used: 0.062 (sec). Leaf size: 11

```
dsolve([(x^2+1)*diff(y(x),x)+y(x)^2=-1,y(0) = 1],y(x), singsol=all)
```

$$y(x) = \cot\left(\arctan(x) + \frac{\pi}{4}\right)$$

✓ Solution by Mathematica

Time used: 0.264 (sec). Leaf size: 14

```
DSolve[{(x^2+1)*y'[x]+y[x]^2== -1,{y[0]==1}},y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \rightarrow \cot\left(\arctan(x) + \frac{\pi}{4}\right)$$

### 1.3 problem HW 1 problem 7(a)

Internal problem ID [7031]

**Book:** Selected problems from homeworks from different courses

**Section:** Math 2520, summer 2021. Differential Equations and Linear Algebra. Normandale college, Bloomington, Minnesota

**Problem number:** HW 1 problem 7(a).

**ODE order:** 1.

**ODE degree:** 1.

CAS Maple gives this as type [linear]

$$y' + \frac{2y}{x} = 5x^2$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 13

```
dsolve(diff(y(x),x)+2/x*y(x)=5*x^2,y(x), singsol=all)
```

$$y(x) = \frac{x^5 + c_1}{x^2}$$

✓ Solution by Mathematica

Time used: 0.028 (sec). Leaf size: 15

```
DSolve[y'[x]+2/x*y[x]==5*x^2,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \rightarrow \frac{x^5 + c_1}{x^2}$$

## 1.4 problem HW 1 problem 7(b)

Internal problem ID [7032]

**Book:** Selected problems from homeworks from different courses

**Section:** Math 2520, summer 2021. Differential Equations and Linear Algebra. Normandale college, Bloomington, Minnesota

**Problem number:** HW 1 problem 7(b).

**ODE order:** 1.

**ODE degree:** 1.

CAS Maple gives this as type [linear]

$$tx' + 2x = 4e^t$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 18

```
dsolve(t*diff(x(t),t)+2*x(t)=4*exp(t),x(t), singsol=all)
```

$$x(t) = \frac{(4t - 4)e^t + c_1}{t^2}$$

✓ Solution by Mathematica

Time used: 0.051 (sec). Leaf size: 20

```
DSolve[t*x'[t]+2*x[t]==4*Exp[t],x[t],t,IncludeSingularSolutions -> True]
```

$$x(t) \rightarrow \frac{4e^t(t - 1) + c_1}{t^2}$$

## 1.5 problem HW 1 problem 10

Internal problem ID [7033]

**Book:** Selected problems from homeworks from different courses

**Section:** Math 2520, summer 2021. Differential Equations and Linear Algebra. Normandale college, Bloomington, Minnesota

**Problem number:** HW 1 problem 10.

**ODE order:** 1.

**ODE degree:** 1.

CAS Maple gives this as type `[_homogeneous, 'class A', _rational, [_Abel, '2nd type', 'cl`

$$y' - \frac{2x - y}{x + 4y} = 0$$

With initial conditions

$$[y(1) = 1]$$

✓ Solution by Maple

Time used: 0.157 (sec). Leaf size: 19

```
dsolve([diff(y(x),x)=(2*x-y(x))/(x+4*y(x)),y(1) = 1],y(x), singsol=all)
```

$$y(x) = -\frac{x}{4} + \frac{\sqrt{9x^2 + 16}}{4}$$

✓ Solution by Mathematica

Time used: 0.482 (sec). Leaf size: 24

```
DSolve[{y'[x]==(2*x-y[x])/(x+4*y[x]),{y[1]==1}},y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \rightarrow \frac{1}{4} \left( \sqrt{9x^2 + 16} - x \right)$$

## 1.6 problem HW 1 problem 11

Internal problem ID [7034]

**Book:** Selected problems from homeworks from different courses

**Section:** Math 2520, summer 2021. Differential Equations and Linear Algebra. Normandale college, Bloomington, Minnesota

**Problem number:** HW 1 problem 11.

**ODE order:** 1.

**ODE degree:** 1.

CAS Maple gives this as type `[_homogeneous, 'class G', _rational, _Bernoulli]`

$$y' + \frac{2y}{x} - 6x^4y^2 = 0$$

### ✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 17

```
dsolve(diff(y(x),x)+2*y(x)/x=6*y(x)^2*x^4,y(x), singsol=all)
```

$$y(x) = \frac{1}{(-2x^3 + c_1)x^2}$$

### ✓ Solution by Mathematica

Time used: 0.153 (sec). Leaf size: 24

```
DSolve[y'[x]+2*y[x]/x==6*y[x]^2*x^4,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \rightarrow \frac{1}{-2x^5 + c_1x^2}$$
$$y(x) \rightarrow 0$$

## 1.7 problem HW 1 problem 13

Internal problem ID [7035]

**Book:** Selected problems from homeworks from different courses

**Section:** Math 2520, summer 2021. Differential Equations and Linear Algebra. Normandale college, Bloomington, Minnesota

**Problem number:** HW 1 problem 13.

**ODE order:** 1.

**ODE degree:** 1.

CAS Maple gives this as type [exact]

$$y^2 + (2yx + \sin(y))y' = -\cos(x)$$

✓ Solution by Maple

Time used: 0.031 (sec). Leaf size: 18

```
dsolve((y(x)^2+cos(x))+(2*x*y(x)+sin(y(x)))*diff(y(x),x)=0,y(x), singsol=all)
```

$$xy(x)^2 + \sin(x) - \cos(y(x)) + c_1 = 0$$

✓ Solution by Mathematica

Time used: 0.229 (sec). Leaf size: 20

```
DSolve[(y[x]^2+Cos[x])+(2*x*y[x]+Sin[y[x]])*y'[x]==0,y[x],x,IncludeSingularSolutions -> True
```

$$\text{Solve}[xy(x)^2 - \cos(y(x)) + \sin(x) = c_1, y(x)]$$

## 1.8 problem HW 1 problem 14

Internal problem ID [7036]

**Book:** Selected problems from homeworks from different courses

**Section:** Math 2520, summer 2021. Differential Equations and Linear Algebra. Normandale college, Bloomington, Minnesota

**Problem number:** HW 1 problem 14.

**ODE order:** 1.

**ODE degree:** 1.

CAS Maple gives this as type [linear]

$$yx + x^2y' = 1$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 12

```
dsolve((x*y(x)-1)+x^2*diff(y(x),x)=0,y(x), singsol=all)
```

$$y(x) = \frac{\ln(x) + c_1}{x}$$

✓ Solution by Mathematica

Time used: 0.025 (sec). Leaf size: 14

```
DSolve[(x*y[x]-1)+x^2*y'[x]==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \rightarrow \frac{\log(x) + c_1}{x}$$

## 1.9 problem HW 5 problem 1(a)

Internal problem ID [7037]

**Book:** Selected problems from homeworks from different courses

**Section:** Math 2520, summer 2021. Differential Equations and Linear Algebra. Normandale college, Bloomington, Minnesota

**Problem number:** HW 5 problem 1(a).

**ODE order:** 2.

**ODE degree:** 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$y'' - y' - 2y = 5e^{2x}$$

✓ Solution by Maple

Time used: 0.016 (sec). Leaf size: 24

```
dsolve(diff(y(x),x$2)-diff(y(x),x)-2*y(x)=5*exp(2*x),y(x), singsol=all)
```

$$y(x) = \frac{(5x + 3c_2)e^{2x}}{3} + c_1e^{-x}$$

✓ Solution by Mathematica

Time used: 0.029 (sec). Leaf size: 31

```
DSolve[y''[x]-y'[x]-2*y[x]==5*Exp[2*x],y[x],x,IncludeSingularSolutions->True]
```

$$y(x) \rightarrow c_1e^{-x} + e^{2x} \left( \frac{5x}{3} - \frac{5}{9} + c_2 \right)$$

## 1.10 problem HW 5 problem 1(b)

Internal problem ID [7038]

**Book:** Selected problems from homeworks from different courses

**Section:** Math 2520, summer 2021. Differential Equations and Linear Algebra. Normandale college, Bloomington, Minnesota

**Problem number:** HW 5 problem 1(b).

**ODE order:** 2.

**ODE degree:** 1.

CAS Maple gives this as type `[[_2nd_order, _linear, _nonhomogeneous]]`

$$y'' + 16y = 4 \cos(x)$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 21

```
dsolve(diff(y(x),x$2)+16*y(x)=4*cos(x),y(x), singsol=all)
```

$$y(x) = \sin(4x) c_2 + \cos(4x) c_1 + \frac{4 \cos(x)}{15}$$

✓ Solution by Mathematica

Time used: 0.022 (sec). Leaf size: 26

```
DSolve[y''[x]+16*y[x]==4*Cos[x],y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \rightarrow \frac{4 \cos(x)}{15} + c_1 \cos(4x) + c_2 \sin(4x)$$

## 1.11 problem HW 5 problem 1(c)

Internal problem ID [7039]

**Book:** Selected problems from homeworks from different courses

**Section:** Math 2520, summer 2021. Differential Equations and Linear Algebra. Normandale college, Bloomington, Minnesota

**Problem number:** HW 5 problem 1(c).

**ODE order:** 2.

**ODE degree:** 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$y'' - 4y' + 3y = 9x^2 + 4$$

With initial conditions

$$[y(0) = 6, y'(0) = 8]$$

✓ Solution by Maple

Time used: 0.015 (sec). Leaf size: 24

```
dsolve([diff(y(x),x$2)-4*diff(y(x),x)+3*y(x)=9*x^2+4,y(0) = 6, D(y)(0) = 8],y(x), singsol=al
```

$$y(x) = 2e^{3x} - 6e^x + 3x^2 + 8x + 10$$

✓ Solution by Mathematica

Time used: 0.015 (sec). Leaf size: 27

```
DSolve[{y'[x]-4*y'[x]+3*y[x]==9*x^2+4,{y[0]==6,y'[0]==8}},y[x],x,IncludeSingularSolutions -
```

$$y(x) \rightarrow 3x^2 + 8x - 6e^x + 2e^{3x} + 10$$

## 1.12 problem HW 5 problem 2

Internal problem ID [7040]

**Book:** Selected problems from homeworks from different courses

**Section:** Math 2520, summer 2021. Differential Equations and Linear Algebra. Normandale college, Bloomington, Minnesota

**Problem number:** HW 5 problem 2.

**ODE order:** 2.

**ODE degree:** 1.

CAS Maple gives this as type `[[_2nd_order, _linear, _nonhomogeneous]]`

$$y'' + y = \tan(x)^2$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 23

```
dsolve(diff(y(x),x$2)+y(x)=tan(x)^2,y(x), singsol=all)
```

$$y(x) = \sin(x) c_2 + \cos(x) c_1 - 2 + \sin(x) \ln(\sec(x) + \tan(x))$$

✓ Solution by Mathematica

Time used: 0.129 (sec). Leaf size: 23

```
DSolve[y''[x]+y[x]==Tan[x]^2,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \rightarrow \sin(x) \operatorname{arctanh}(\sin(x)) + c_1 \cos(x) + c_2 \sin(x) - 2$$

## 1.13 problem HW 5 problem 5

Internal problem ID [7041]

**Book:** Selected problems from homeworks from different courses

**Section:** Math 2520, summer 2021. Differential Equations and Linear Algebra. Normandale college, Bloomington, Minnesota

**Problem number:** HW 5 problem 5.

**ODE order:** 1.

**ODE degree:** 1.

Solve

$$x'(t) = -2x(t) + 3y(t)$$

$$y'(t) = -2x(t) + 5y(t)$$

With initial conditions

$$[x(0) = -2, y(0) = 1]$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 32

```
dsolve([diff(x(t),t) = -2*x(t)+3*y(t), diff(y(t),t) = -2*x(t)+5*y(t), x(0) = -2, y(0) = 1],
```

$$x(t) = -3e^{-t} + e^{4t}$$

$$y(t) = -e^{-t} + 2e^{4t}$$

✓ Solution by Mathematica

Time used: 0.007 (sec). Leaf size: 36

```
DSolve[{x'[t]==-2*x[t]+3*y[t],y'[t]==-2*x[t]+5*y[t]},{x[0]==-2,y[0]==1},{x[t],y[t]},t,Includ
```

$$x(t) \rightarrow e^{-t}(e^{5t} - 3)$$

$$y(t) \rightarrow e^{-t}(2e^{5t} - 1)$$

## 1.14 problem HW 5 problem 6

Internal problem ID [7042]

**Book:** Selected problems from homeworks from different courses

**Section:** Math 2520, summer 2021. Differential Equations and Linear Algebra. Normandale college, Bloomington, Minnesota

**Problem number:** HW 5 problem 6.

**ODE order:** 1.

**ODE degree:** 1.

Solve

$$x'(t) = -x(t) + 4y(t)$$

$$y'(t) = 2x(t) - 3y(t)$$

With initial conditions

$$[x(0) = 3, y(0) = 0]$$

✓ Solution by Maple

Time used: 0.016 (sec). Leaf size: 26

```
dsolve([diff(x(t),t) = -x(t)+4*y(t), diff(y(t),t) = 2*x(t)-3*y(t), x(0) = 3, y(0) = 0], sing
```

$$x(t) = 2e^t + e^{-5t}$$

$$y(t) = e^t - e^{-5t}$$

✓ Solution by Mathematica

Time used: 0.004 (sec). Leaf size: 30

```
DSolve[{x'[t]==-x[t]+4*y[t],y'[t]==2*x[t]-3*y[t]},{x[0]==3,y[0]==0},{x[t],y[t]},t,IncludeSin
```

$$x(t) \rightarrow e^{-5t} + 2e^t$$

$$y(t) \rightarrow e^t - e^{-5t}$$

## 1.15 problem HW 5 problem 7

Internal problem ID [7043]

**Book:** Selected problems from homeworks from different courses

**Section:** Math 2520, summer 2021. Differential Equations and Linear Algebra. Normandale college, Bloomington, Minnesota

**Problem number:** HW 5 problem 7.

**ODE order:** 1.

**ODE degree:** 1.

Solve

$$\begin{aligned}x'(t) &= 2x(t) - y(t) \\y'(t) &= -x(t) + 2y(t) + 4e^t\end{aligned}$$

✓ Solution by Maple

Time used: 0.031 (sec). Leaf size: 45

```
dsolve([diff(x(t),t)=2*x(t)-y(t),diff(y(t),t)=-x(t)+2*y(t)+4*exp(t)],singsol=all)
```

$$\begin{aligned}x(t) &= c_2e^t + c_1e^{3t} + 2e^tt \\y(t) &= c_2e^t - c_1e^{3t} + 2e^tt - 2e^t\end{aligned}$$

✓ Solution by Mathematica

Time used: 0.011 (sec). Leaf size: 74

```
DSolve[{x'[t]==2*x[t]-y[t],y'[t]==-x[t]+2*y[t]+4*Exp[t]},{x[t],y[t]},t,IncludeSingularSoluti
```

$$\begin{aligned}x(t) &\rightarrow \frac{1}{2}e^t(4t + c_1(e^{2t} + 1) - c_2e^{2t} + 2 + c_2) \\y(t) &\rightarrow \frac{1}{2}e^t(4t - c_1e^{2t} + c_2e^{2t} - 2 + c_1 + c_2)\end{aligned}$$

## 1.16 problem Example 8.3.4 from Handout chapter 8.2

Internal problem ID [7044]

**Book:** Selected problems from homeworks from different courses

**Section:** Math 2520, summer 2021. Differential Equations and Linear Algebra. Normandale college, Bloomington, Minnesota

**Problem number:** Example 8.3.4 from Handout chapter 8.2.

**ODE order:** 1.

**ODE degree:** 1.

Solve

$$x'(t) = 6x(t) - 7y(t) + 10$$

$$y'(t) = x(t) - 2y(t) - 2e^t$$

✓ Solution by Maple

Time used: 0.031 (sec). Leaf size: 45

```
dsolve([diff(x(t),t)=6*x(t)-7*y(t)+10,diff(y(t),t)=x(t)-2*y(t)-2*exp(t)],singsol=all)
```

$$x(t) = c_2 e^{5t} + e^{-t} c_1 - \frac{7e^t}{4} - 4$$

$$y(t) = \frac{c_2 e^{5t}}{7} + e^{-t} c_1 - \frac{5e^t}{4} - 2$$

✓ Solution by Mathematica

Time used: 0.053 (sec). Leaf size: 90

```
DSolve[{x'[t]==6*x[t]-7*y[t]+10,y'[t]==x[t]-2*y[t]-2*Exp[t]},{x[t],y[t]},t,IncludeSingularSo
```

$$x(t) \rightarrow -\frac{7e^t}{4} - \frac{1}{6}(c_1 - 7c_2)e^{-t} + \frac{7}{6}(c_1 - c_2)e^{5t} - 4$$

$$y(t) \rightarrow -\frac{5e^t}{4} - \frac{1}{6}(c_1 - 7c_2)e^{-t} + \frac{1}{6}(c_1 - c_2)e^{5t} - 2$$