A Solution Manual For

An elementary treatise on differential equations by Abraham Cohen. DC heath publishers. 1906

AN ELEMENTARY TREATISE

ON

DIFFERENTIAL EQUATIONS

BY

ABRAHAM COHEN, PH.D.
ASSOCIATE PROFESSOR OF MATHEMATICS
JOHNS HOPKINS UNIVERSITY

D. C. HEATH & CO., PUBLISHERS
BOSTON NEW YORK CHICAGO

Nasser M. Abbasi

May 16, 2024

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1 Chapter 2, differential equations of the first order and the first degree. Article 8. Exact differential equations. Page 11

1.1 problem Ex 1

Internal problem ID [11122]

Book: An elementary treatise on differential equations by Abraham Cohen. DC heath publishers. 1906
Section: Chapter 2, differential equations of the first order and the first degree. Article 8. Exact differential equations. Page 11
Problem number: Ex 1.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[[_homogeneous, 'class D'], _exact, _rational, [_Abel, '2nd ty`

$$\boxed{\frac{2yx+1}{y}+\frac{(y-x)\,y'}{y^2}=0}$$

✓ Solution by Maple
Time used: 0.062 (sec). Leaf size: 18

```
dsolve((2*x*y(x)+1)/y(x)+ (y(x)-x)/y(x)^2*diff(y(x),x)=0,y(x), singsol=all)
```

$$y(x)=-\frac{x}{\text{LambertW}\left(-\mathrm{e}^{x^2}c_1x\right)}$$

✓ Solution by Mathematica
Time used: 7.151 (sec). Leaf size: 29

```
DSolve[(2*x*y[x]+1)/y[x]+ (y[x]-x)/y[x]^2*y'[x]==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x)\to -\frac{x}{W\left(x\left(-e^{x^2-c_1}\right)\right)}$$
$$y(x)\to 0$$

1.2 problem Ex 2

Internal problem ID [11123]

Book: An elementary treatise on differential equations by Abraham Cohen. DC heath publishers. 1906
Section: Chapter 2, differential equations of the first order and the first degree. Article 8. Exact differential equations. Page 11
Problem number: Ex 2.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[[_homogeneous, 'class A'], _exact, _rational, _dAlembert]`

$$\boxed{\frac{y^2-2x^2}{y^2x-x^3}+\frac{(2y^2-x^2)\,y'}{y^3-x^2y}=0}$$

✓ Solution by Maple
Time used: 0.266 (sec). Leaf size: 71

```
dsolve((y(x)^2-2*x^2)/(x*y(x)^2-x^3)+ (2*y(x)^2-x^2)/(y(x)^3-x^2*y(x))*diff(y(x),x)=0,y(x),
```

$$y(x)=\frac{\sqrt{\frac{2c_1x^3-2\sqrt{c_1^2x^6+4}}{c_1x^3}}\,x}{2}$$
$$y(x)=\frac{\sqrt{2}\sqrt{\frac{c_1x^3+\sqrt{c_1^2x^6+4}}{c_1x^3}}\,x}{2}$$

✓ Solution by Mathematica

Time used: 15.598 (sec). Leaf size: 277

```
DSolve[(y[x]^2-2*x^2)/(x*y[x]^2-x^3)+ (2*y[x]^2-x^2)/(y[x]^3-x^2*y[x])*y'[x]==0,y[x],x,Inclu
```

$$y(x) \to -\frac{\sqrt{x^2 - \frac{\sqrt{x^6-4e^{2c_1}}}{x}}}{\sqrt{2}}$$

$$y(x) \to \frac{\sqrt{x^2 - \frac{\sqrt{x^6-4e^{2c_1}}}{x}}}{\sqrt{2}}$$

$$y(x) \to -\frac{\sqrt{\frac{x^3+\sqrt{x^6-4e^{2c_1}}}{x}}}{\sqrt{2}}$$

$$y(x) \to \frac{\sqrt{\frac{x^3+\sqrt{x^6-4e^{2c_1}}}{x}}}{\sqrt{2}}$$

$$y(x) \to -\frac{\sqrt{x^2 - \frac{\sqrt{x^6}}{x}}}{\sqrt{2}}$$

$$y(x) \to \frac{\sqrt{x^2 - \frac{\sqrt{x^6}}{x}}}{\sqrt{2}}$$

$$y(x) \to -\frac{\sqrt{\frac{\sqrt{x^6}+x^3}{x}}}{\sqrt{2}}$$

$$y(x) \to \frac{\sqrt{\frac{\sqrt{x^6}+x^3}{x}}}{\sqrt{2}}$$

1.3 problem Ex 3

Internal problem ID [11124]

Book: An elementary treatise on differential equations by Abraham Cohen. DC heath publishers. 1906
Section: Chapter 2, differential equations of the first order and the first degree. Article 8. Exact differential equations. Page 11
Problem number: Ex 3.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[[_homogeneous, 'class A'], _exact, _rational, _dAlembert]`

$$\boxed{\frac{1}{\sqrt{x^2+y^2}}+\left(\frac{1}{y}-\frac{x}{y\sqrt{x^2+y^2}}\right)y'=0}$$

✓ Solution by Maple
Time used: 0.078 (sec). Leaf size: 18

```
dsolve(1/sqrt(x^2+y(x)^2)+ ( 1/y(x)-(x/(y(x)*sqrt(x^2+y(x)^2))))*diff(y(x),x)=0,y(x), singso
```

$$-c_1+\sqrt{y\left(x\right)^2+x^2}+x=0$$

✓ Solution by Mathematica
Time used: 0.893 (sec). Leaf size: 62

```
DSolve[1/Sqrt[x^2+y[x]^2]+ ( 1/y[x]-(x/(y[x]*Sqrt[x^2+y[x]^2])))*y'[x]==0,y[x],x,IncludeSing
```

$$y(x)\to -e^{\frac{c_1}{2}}\sqrt{-2x+e^{c_1}}$$
$$y(x)\to e^{\frac{c_1}{2}}\sqrt{-2x+e^{c_1}}$$
$$y(x)\to 0$$
$$y(x)\to \text{ComplexInfinity}$$

1.4 problem Ex 4

Internal problem ID [11125]

Book: An elementary treatise on differential equations by Abraham Cohen. DC heath publishers. 1906
Section: Chapter 2, differential equations of the first order and the first degree. Article 8. Exact differential equations. Page 11
Problem number: Ex 4.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_linear]`

$$\boxed{y + y'x = -x}$$

✓ Solution by Maple
Time used: 0.0 (sec). Leaf size: 13

```
dsolve((y(x)+x)+ x*diff(y(x),x)=0,y(x), singsol=all)
```

$$y(x) = -\frac{x}{2} + \frac{c_1}{x}$$

✓ Solution by Mathematica
Time used: 0.038 (sec). Leaf size: 17

```
DSolve[(y[x]+x)+ x*y'[x]==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to -\frac{x}{2} + \frac{c_1}{x}$$

1.5 problem Ex 5

Internal problem ID [11126]

Book: An elementary treatise on differential equations by Abraham Cohen. DC heath publishers. 1906
Section: Chapter 2, differential equations of the first order and the first degree. Article 8. Exact differential equations. Page 11
Problem number: Ex 5.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[[_homogeneous, 'class C'], _exact, _rational, [_Abel, '2nd ty`

$$\boxed{-2y + (2y - 2x - 3)\,y' = -6x - 1}$$

✓ Solution by Maple
Time used: 0.469 (sec). Leaf size: 33

```
dsolve((6*x-2*y(x)+1)+(2*y(x)-2*x-3)*diff(y(x),x)=0,y(x), singsol=all)
```

$$y(x) = \frac{-\sqrt{1 - 8\left(x - \frac{1}{2}\right)^2 c_1^2} + (2x + 3)\,c_1}{2c_1}$$

✓ Solution by Mathematica
Time used: 0.208 (sec). Leaf size: 67

```
DSolve[(6*x-2*y[x]+1)+(2*y[x]-2*x-3)*y'[x]==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to -\frac{1}{2}i\sqrt{8x^2 - 8x - 9 - 4c_1} + x + \frac{3}{2}$$
$$y(x) \to \frac{1}{2}i\sqrt{8x^2 - 8x - 9 - 4c_1} + x + \frac{3}{2}$$

2 Chapter 2, differential equations of the first order and the first degree. Article 9. Variables searated or separable. Page 13

2.1 problem Ex 1

Internal problem ID [11127]

Book: An elementary treatise on differential equations by Abraham Cohen. DC heath publishers. 1906
Section: Chapter 2, differential equations of the first order and the first degree. Article 9. Variables searated or separable. Page 13
Problem number: Ex 1.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_separable]`

$$\boxed{\sec(x)\cos(y)^2 - \cos(x)\sin(y)\,y' = 0}$$

✓ Solution by Maple
Time used: 0.016 (sec). Leaf size: 11

```
dsolve((sec(x)*cos(y(x))^2)-(cos(x)*sin(y(x)))*diff(y(x),x)=0,y(x), singsol=all)
```

$$y(x) = \arccos\left(\frac{1}{\tan(x) + c_1}\right)$$

✓ Solution by Mathematica
Time used: 1.366 (sec). Leaf size: 45

```
DSolve[(Sec[x]*Cos[y[x]]^2)-(Cos[x]*Sin[y[x]])*y'[x]==0,y[x],x,IncludeSingularSolutions -> T
```

$$y(x) \to -\sec^{-1}(\tan(x) + 2c_1)$$
$$y(x) \to \sec^{-1}(\tan(x) + 2c_1)$$
$$y(x) \to -\frac{\pi}{2}$$
$$y(x) \to \frac{\pi}{2}$$

2.2 problem Ex 2

Internal problem ID [11128]

Book: An elementary treatise on differential equations by Abraham Cohen. DC heath publishers. 1906
Section: Chapter 2, differential equations of the first order and the first degree. Article 9. Variables searated or separable. Page 13
Problem number: Ex 2.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_separable]`

$$\boxed{(1+x)\,y^2 - y' x^3 = 0}$$

✓ Solution by Maple
Time used: 0.0 (sec). Leaf size: 22

```
dsolve((1+x)*y(x)^2-x^3*diff(y(x),x)=0,y(x), singsol=all)
```

$$y(x) = \frac{2x^2}{2c_1 x^2 + 2x + 1}$$

✓ Solution by Mathematica
Time used: 0.231 (sec). Leaf size: 29

```
DSolve[(1+x)*y[x]^2-x^3*y'[x]==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to \frac{2x^2}{-2c_1 x^2 + 2x + 1}$$
$$y(x) \to 0$$

2.3 problem Ex 3

Internal problem ID [11129]

Book: An elementary treatise on differential equations by Abraham Cohen. DC heath publishers. 1906
Section: Chapter 2, differential equations of the first order and the first degree. Article 9. Variables searated or separable. Page 13
Problem number: Ex 3.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_separable]`

$$\boxed{2\left(1-y^2\right)xy+\left(x^2+1\right)\left(1+y^2\right)y'=0}$$

✓ Solution by Maple
Time used: 0.047 (sec). Leaf size: 61

```
dsolve(2*(1-y(x)^2)*x*y(x)+(1+x^2)*(1+y(x)^2)*diff(y(x),x)=0,y(x), singsol=all)
```

$$y(x)=\frac{c_1x^2}{2}+\frac{c_1}{2}-\frac{\sqrt{4+\left(x^2+1\right)^2c_1^2}}{2}$$
$$y(x)=\frac{c_1x^2}{2}+\frac{c_1}{2}+\frac{\sqrt{4+\left(x^2+1\right)^2c_1^2}}{2}$$

✓ Solution by Mathematica
Time used: 8.437 (sec). Leaf size: 98

```
DSolve[2*(1-y[x]^2)*x*y[x]+(1+x^2)*(1+y[x]^2)*y'[x]==0,y[x],x,IncludeSingularSolutions -> Tr
```

$$y(x)\to\frac{1}{2}\left(-e^{c_1}\left(x^2+1\right)-\sqrt{4+e^{2c_1}\left(x^2+1\right)^2}\right)$$
$$y(x)\to\frac{1}{2}\left(\sqrt{4+e^{2c_1}\left(x^2+1\right)^2}-e^{c_1}\left(x^2+1\right)\right)$$
$$y(x)\to-1$$
$$y(x)\to0$$
$$y(x)\to1$$

2.4 problem Ex 4

Internal problem ID [11130]

Book: An elementary treatise on differential equations by Abraham Cohen. DC heath publishers. 1906
Section: Chapter 2, differential equations of the first order and the first degree. Article 9. Variables searated or separable. Page 13
Problem number: Ex 4.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_separable]`

$$\sin(x)\cos(y)^2+\cos(x)^2y'=0$$

✓ Solution by Maple
Time used: 0.0 (sec). Leaf size: 11

```
dsolve(sin(x)*cos(y(x))^2+cos(x)^2*diff(y(x),x)=0,y(x), singsol=all)
```

$$y(x)=-\arctan(\sec(x)+c_1)$$

✓ Solution by Mathematica
Time used: 2.833 (sec). Leaf size: 31

```
DSolve[Sin[x]*Cos[y[x]]^2+Cos[x]^2*y'[x]==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x)\to \arctan(-\sec(x)+c_1)$$
$$y(x)\to -\frac{\pi}{2}$$
$$y(x)\to \frac{\pi}{2}$$

3 Chapter 2, differential equations of the first order and the first degree. Article 10. Homogeneous equations. Page 15

3.1 problem Ex 1

Internal problem ID [11131]

Book: An elementary treatise on differential equations by Abraham Cohen. DC heath publishers. 1906
Section: Chapter 2, differential equations of the first order and the first degree. Article 10. Homogeneous equations. Page 15
Problem number: Ex 1.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[[_homogeneous, 'class A'], _dAlembert]`

$$\boxed{x\,\mathrm{e}^{\frac{y}{x}} + y - y'x = 0}$$

✓ Solution by Maple
Time used: 0.0 (sec). Leaf size: 15

```
dsolve((x*exp(y(x)/x)+y(x))-x*diff(y(x),x)=0,y(x), singsol=all)
```

$$y(x) = \ln\left(-\frac{1}{\ln(x) + c_1}\right) x$$

✓ Solution by Mathematica
Time used: 0.527 (sec). Leaf size: 18

```
DSolve[(x*Exp[y[x]/x]+y[x])-x*y'[x]==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to -x \log(-\log(x) - c_1)$$

3.2 problem Ex 2

Internal problem ID [11132]

Book: An elementary treatise on differential equations by Abraham Cohen. DC heath publishers. 1906
Section: Chapter 2, differential equations of the first order and the first degree. Article 10. Homogeneous equations. Page 15
Problem number: Ex 2.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[[_homogeneous, 'class A'], _rational, _dAlembert]`

$$\boxed{2x^2y + 3y^3 - \left(x^3 + 2y^2x\right) y' = 0}$$

✓ Solution by Maple
Time used: 0.641 (sec). Leaf size: 89

```
dsolve((2*x^2*y(x)+3*y(x)^3)-(x^3+2*x*y(x)^2)*diff(y(x),x)=0,y(x), singsol=all)
```

$$y(x) = -\frac{\sqrt{-2 - 2\sqrt{4c_1x^2 + 1}}\, x}{2}$$
$$y(x) = \frac{\sqrt{-2 - 2\sqrt{4c_1x^2 + 1}}\, x}{2}$$
$$y(x) = -\frac{\sqrt{-2 + 2\sqrt{4c_1x^2 + 1}}\, x}{2}$$
$$y(x) = \frac{\sqrt{-2 + 2\sqrt{4c_1x^2 + 1}}\, x}{2}$$

✓ Solution by Mathematica

Time used: 47.499 (sec). Leaf size: 277

```
DSolve[(2*x^2*y[x]+3*y[x]^3)-(x^3+2*x*y[x]^2)*y'[x]==0,y[x],x,IncludeSingularSolutions -> Tr
```

$$y(x) \to -\frac{\sqrt{-x^2-\sqrt{x^4+4e^{2c_1}x^6}}}{\sqrt{2}}$$

$$y(x) \to \frac{\sqrt{-x^2-\sqrt{x^4+4e^{2c_1}x^6}}}{\sqrt{2}}$$

$$y(x) \to -\frac{\sqrt{-x^2+\sqrt{x^4+4e^{2c_1}x^6}}}{\sqrt{2}}$$

$$y(x) \to \sqrt{-\frac{x^2}{2}+\frac{1}{2}\sqrt{x^4+4e^{2c_1}x^6}}$$

$$y(x) \to -\frac{\sqrt{-\sqrt{x^4}-x^2}}{\sqrt{2}}$$

$$y(x) \to \frac{\sqrt{-\sqrt{x^4}-x^2}}{\sqrt{2}}$$

$$y(x) \to -\frac{\sqrt{\sqrt{x^4}-x^2}}{\sqrt{2}}$$

$$y(x) \to \frac{\sqrt{\sqrt{x^4}-x^2}}{\sqrt{2}}$$

3.3 problem Ex 3

Internal problem ID [11133]

Book: An elementary treatise on differential equations by Abraham Cohen. DC heath publishers. 1906
Section: Chapter 2, differential equations of the first order and the first degree. Article 10. Homogeneous equations. Page 15
Problem number: Ex 3.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[[_homogeneous, 'class A'], _rational, _Bernoulli]`

$$\boxed{y^2 - yx + y'x^2 = 0}$$

✓ Solution by Maple
Time used: 0.016 (sec). Leaf size: 12

```
dsolve((y(x)^2-x*y(x))+x^2*diff(y(x),x)=0,y(x), singsol=all)
```

$$y(x) = \frac{x}{\ln(x) + c_1}$$

✓ Solution by Mathematica
Time used: 0.15 (sec). Leaf size: 19

```
DSolve[(y[x]^2-x*y[x])+x^2*y'[x]==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to \frac{x}{\log(x) + c_1}$$
$$y(x) \to 0$$

3.4 problem Ex 4

Internal problem ID [11134]

Book: An elementary treatise on differential equations by Abraham Cohen. DC heath publishers. 1906
Section: Chapter 2, differential equations of the first order and the first degree. Article 10. Homogeneous equations. Page 15
Problem number: Ex 4.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[[_homogeneous, 'class A'], _rational, _Bernoulli]`

$$\boxed{2x^2y + y^3 - y'x^3 = 0}$$

✓ Solution by Maple
Time used: 0.0 (sec). Leaf size: 34

```
dsolve(2*x^2*y(x)+y(x)^3-x^3*diff(y(x),x)=0,y(x), singsol=all)
```

$$y(x) = \frac{x^2}{\sqrt{-x^2 + c_1}}$$
$$y(x) = -\frac{x^2}{\sqrt{-x^2 + c_1}}$$

✓ Solution by Mathematica
Time used: 0.181 (sec). Leaf size: 47

```
DSolve[2*x^2*y[x]+y[x]^3-x^3*y'[x]==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to -\frac{x^2}{\sqrt{-x^2 + c_1}}$$
$$y(x) \to \frac{x^2}{\sqrt{-x^2 + c_1}}$$
$$y(x) \to 0$$

3.5 problem Ex 5

Internal problem ID [11135]

Book: An elementary treatise on differential equations by Abraham Cohen. DC heath publishers. 1906
Section: Chapter 2, differential equations of the first order and the first degree. Article 10. Homogeneous equations. Page 15
Problem number: Ex 5.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_separable]`

$$\boxed{y^3 + y' x^3 = 0}$$

✓ Solution by Maple
Time used: 0.0 (sec). Leaf size: 30

```
dsolve(y(x)^3+x^3*diff(y(x),x)=0,y(x), singsol=all)
```

$$y(x) = \frac{x}{\sqrt{c_1 x^2 - 1}}$$
$$y(x) = -\frac{x}{\sqrt{c_1 x^2 - 1}}$$

✓ Solution by Mathematica
Time used: 0.356 (sec). Leaf size: 45

```
DSolve[y[x]^3+x^3*y'[x]==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to -\frac{x}{\sqrt{-1 - 2c_1 x^2}}$$
$$y(x) \to \frac{x}{\sqrt{-1 - 2c_1 x^2}}$$
$$y(x) \to 0$$

3.6 problem Ex 6

Internal problem ID [11136]

Book: An elementary treatise on differential equations by Abraham Cohen. DC heath publishers. 1906
Section: Chapter 2, differential equations of the first order and the first degree. Article 10. Homogeneous equations. Page 15
Problem number: Ex 6.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[[_homogeneous, 'class A'], _dAlembert]`

$$\boxed{y \cos\left(\frac{y}{x}\right) - x \cos\left(\frac{y}{x}\right) y' = -x}$$

✓ Solution by Maple
Time used: 0.0 (sec). Leaf size: 11

```
dsolve((x+y(x)*cos(y(x)/x))-x*cos(y(x)/x)*diff(y(x),x)=0,y(x), singsol=all)
```

$$y(x) = \arcsin\left(\ln\left(x\right) + c_1\right) x$$

✓ Solution by Mathematica
Time used: 0.395 (sec). Leaf size: 13

```
DSolve[(x+y[x]*Cos[y[x]/x])-x*Cos[y[x]/x]*y'[x]==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to x \arcsin(\log(x) + c_1)$$

4 Chapter 2, differential equations of the first order and the first degree. Article 11. Equations in which M and N are linear but not homogeneous. Page 16

4.1 problem Ex 1

Internal problem ID [11137]

Book: An elementary treatise on differential equations by Abraham Cohen. DC heath publishers. 1906
Section: Chapter 2, differential equations of the first order and the first degree. Article 11. Equations in which M and N are linear but not homogeneous. Page 16
Problem number: Ex 1.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[[_homogeneous, 'class C'], _rational, [_Abel, '2nd type', 'cl`

$$\boxed{3y + (x + y + 1)\, y' = -4x - 1}$$

✓ Solution by Maple
Time used: 0.328 (sec). Leaf size: 29

```
dsolve((4*x+3*y(x)+1)+(x+y(x)+1)*diff(y(x),x)=0,y(x), singsol=all)
```

$$y(x) = -3 - \frac{(x-2)\,(2\,\mathrm{LambertW}\,(c_1(x-2)) + 1)}{\mathrm{LambertW}\,(c_1\,(x-2))}$$

✓ Solution by Mathematica
Time used: 1.385 (sec). Leaf size: 159

```
DSolve[(4*x+3*y[x]+1)+(x+y[x]+1)*y'[x]==0,y[x],x,IncludeSingularSolutions -> True]
```

$$\text{Solve}\left[\frac{(-2)^{2/3}\left(-2x\log\left(\frac{3(-2)^{2/3}(y(x)+2x-1)}{y(x)+x+1}\right)+(2x-1)\log\left(-\frac{3(-2)^{2/3}(x-2)}{y(x)+x+1}\right)+\log\left(\frac{3(-2)^{2/3}(y(x)+2x-1)}{y(x)+x+1}\right)+\right.}{9(y(x)+2x-1)}\right.$$

4.2 problem Ex 2

Internal problem ID [11138]

Book: An elementary treatise on differential equations by Abraham Cohen. DC heath publishers. 1906
Section: Chapter 2, differential equations of the first order and the first degree. Article 11. Equations in which M and N are linear but not homogeneous. Page 16
Problem number: Ex 2.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[[_homogeneous, 'class C'], _rational, [_Abel, '2nd type', 'cl`

$$\boxed{-y + (x + y + 3)\, y' = -4x - 2}$$

✓ Solution by Maple
Time used: 0.031 (sec). Leaf size: 36

```
dsolve((4*x-y(x)+2)+(x+y(x)+3)*diff(y(x),x)=0,y(x), singsol=all)
```

$$y(x) = -2 + \tan\left(\text{RootOf}\left(2\ln(2) + \ln\left(\sec\left(\_Z\right)^2\right) - \_Z + 2\ln(1+x) + 2c_1\right)\right)(-2x-2)$$

✓ Solution by Mathematica
Time used: 0.07 (sec). Leaf size: 67

```
DSolve[(4*x-y[x]+2)+(x+y[x]+3)*y'[x]==0,y[x],x,IncludeSingularSolutions -> True]
```

$$\text{Solve}\left[2\arctan\left(\frac{1}{2} - \frac{5(x+1)}{2(y(x)+x+3)}\right) + 2\log\left(\frac{4x^2 + y(x)^2 + 4y(x) + 8x + 8}{5(x+1)^2}\right) + 4\log(x+1) + 5c_1 = 0, y(x)\right]$$

4.3 problem Ex 3

Internal problem ID [11139]

Book: An elementary treatise on differential equations by Abraham Cohen. DC heath publishers. 1906
Section: Chapter 2, differential equations of the first order and the first degree. Article 11. Equations in which M and N are linear but not homogeneous. Page 16
Problem number: Ex 3.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[[_homogeneous, ‘class C‘], _rational, [_Abel, ‘2nd type‘, ‘cl`

$$\boxed{y - (4x + 2y - 1)\, y' = -2x}$$

✓ Solution by Maple
Time used: 0.031 (sec). Leaf size: 23

```
dsolve((2*x+y(x))-(4*x+2*y(x)-1)*diff(y(x),x)=0,y(x), singsol=all)
```

$$y(x) = -\frac{\text{LambertW}\left(-2\,\mathrm{e}^{4-25x+25c_1}\right)}{10} + \frac{2}{5} - 2x$$

✓ Solution by Mathematica
Time used: 4.725 (sec). Leaf size: 39

```
DSolve[(2*x+y[x])-(4*x+2*y[x]-1)*y'[x]==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to -\frac{1}{10} W\left(-e^{-25x-1+c_1}\right) - 2x + \frac{2}{5}$$
$$y(x) \to \frac{2}{5} - 2x$$

5 Chapter 2, differential equations of the first order and the first degree. Article 12. Equations of form $yf_1(xy) + xf_2(xy)y' = 0$. Page 18

5.1 problem Ex 1

Internal problem ID [11140]

Book: An elementary treatise on differential equations by Abraham Cohen. DC heath publishers. 1906
Section: Chapter 2, differential equations of the first order and the first degree. Article 12. Equations of form $yf_1(xy) + xf_2(xy)y' = 0$. Page 18
Problem number: Ex 1.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[[_homogeneous, 'class G'], _rational, _Riccati]`

$$\boxed{y + 2y^2x - y^3x^2 + 2x^2yy' = 0}$$

✓ Solution by Maple
Time used: 0.0 (sec). Leaf size: 21

```
dsolve((y(x)+2*x*y(x)^2-x^2*y(x)^3)+(2*x^2*y(x))*diff(y(x),x)=0,y(x), singsol=all)
```

$$y(x) = 0$$
$$y(x) = \frac{\tanh\left(-\frac{\ln(x)}{2} + \frac{c_1}{2}\right)}{x}$$

✓ Solution by Mathematica
Time used: 1.44 (sec). Leaf size: 71

```
DSolve[(y[x]+2*x*y[x]^2-x^2*y[x]^3)+(2*x^2*y[x])*y'[x]==0,y[x],x,IncludeSingularSolutions ->
```

$$y(x) \to 0$$
$$y(x) \to \frac{i\tan\left(\frac{1}{2}i\log(x) + c_1\right)}{x}$$
$$y(x) \to 0$$
$$y(x) \to \frac{-x + e^{2i\text{Interval}[\{0,\pi\}]}}{x^2 + xe^{2i\text{Interval}[\{0,\pi\}]}}$$

5.2 problem Ex 2

Internal problem ID [11141]

Book: An elementary treatise on differential equations by Abraham Cohen. DC heath publishers. 1906
Section: Chapter 2, differential equations of the first order and the first degree. Article 12. Equations of form $yf_1(xy) + xf_2(xy)y' = 0$. Page 18
Problem number: Ex 2.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[[_homogeneous, 'class G'], _rational, [_Abel, '2nd type', 'cl`

$$\boxed{2y + 3y^2x + \left(x + 2x^2y\right)y' = 0}$$

✓ Solution by Maple
Time used: 0.016 (sec). Leaf size: 45

```
dsolve((2*y(x)+3*x*y(x)^2)+(x+2*x^2*y(x))*diff(y(x),x)=0,y(x), singsol=all)
```

$$y(x) = \frac{-x + \sqrt{x\left(4c_1 + x\right)}}{2x^2}$$
$$y(x) = \frac{-x - \sqrt{x\left(4c_1 + x\right)}}{2x^2}$$

✓ Solution by Mathematica
Time used: 0.888 (sec). Leaf size: 69

```
DSolve[(2*y[x]+3*x*y[x]^2)+(x+2*x^2*y[x])*y'[x]==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to -\frac{x^{3/2} + \sqrt{x^2(x + 4c_1)}}{2x^{5/2}}$$
$$y(x) \to \frac{-x^{3/2} + \sqrt{x^2(x + 4c_1)}}{2x^{5/2}}$$

5.3 problem Ex 3

Internal problem ID [11142]

Book: An elementary treatise on differential equations by Abraham Cohen. DC heath publishers. 1906
Section: Chapter 2, differential equations of the first order and the first degree. Article 12. Equations of form $yf_1(xy) + xf_2(xy)y' = 0$. Page 18
Problem number: Ex 3.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[[_homogeneous, ‘class G‘], _rational, [_Abel, ‘2nd type‘, ‘cl`

$$\boxed{y + y^2x + (x - x^2y)\,y' = 0}$$

✓ Solution by Maple
Time used: 0.031 (sec). Leaf size: 18

```
dsolve((y(x)+x*y(x)^2)+(x-x^2*y(x))*diff(y(x),x)=0,y(x), singsol=all)
```

$$y(x) = -\frac{1}{\text{LambertW}\left(-\frac{c_1}{x^2}\right) x}$$

✓ Solution by Mathematica
Time used: 8.358 (sec). Leaf size: 35

```
DSolve[(y[x]+x*y[x]^2)+(x-x^2*y[x])*y'[x]==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to -\frac{1}{xW\left(\frac{e^{-1+\frac{9c_1}{2^{2/3}}}}{x^2}\right)}$$
$$y(x) \to 0$$

6 Chapter 2, differential equations of the first order and the first degree. Article 13. Linear equations of first order. Page 19

6.1 problem Ex 1

Internal problem ID [11143]

Book: An elementary treatise on differential equations by Abraham Cohen. DC heath publishers. 1906
Section: Chapter 2, differential equations of the first order and the first degree. Article 13. Linear equations of first order. Page 19
Problem number: Ex 1.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_linear]`

$$\boxed{y' + y\cot(x) = \sec(x)}$$

✓ Solution by Maple
Time used: 0.016 (sec). Leaf size: 14

```
dsolve(diff(y(x),x)+y(x)*cot(x)=sec(x),y(x), singsol=all)
```

$$y(x) = \csc(x)\left(-\ln(\cos(x)) + c_1\right)$$

✓ Solution by Mathematica
Time used: 0.081 (sec). Leaf size: 16

```
DSolve[y'[x]+y[x]*Cot[x]==Sec[x],y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to \csc(x)(-\log(\cos(x)) + c_1)$$

6.2 problem Ex 2

Internal problem ID [11144]

Book: An elementary treatise on differential equations by Abraham Cohen. DC heath publishers. 1906
Section: Chapter 2, differential equations of the first order and the first degree. Article 13. Linear equations of first order. Page 19
Problem number: Ex 2.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_linear]`

$$\boxed{y'x + y(1+x) = \mathrm{e}^x}$$

✓ Solution by Maple
Time used: 0.0 (sec). Leaf size: 19

```
dsolve(x*diff(y(x),x)+(1+x)*y(x)=exp(x),y(x), singsol=all)
```

$$y(x) = \frac{\mathrm{e}^x + 2c_1\mathrm{e}^{-x}}{2x}$$

✓ Solution by Mathematica
Time used: 0.078 (sec). Leaf size: 25

```
DSolve[x*y'[x]+(1+x)*y[x]==Exp[x],y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to \frac{e^x + 2c_1e^{-x}}{2x}$$

6.3 problem Ex 3

Internal problem ID [11145]

Book: An elementary treatise on differential equations by Abraham Cohen. DC heath publishers. 1906
Section: Chapter 2, differential equations of the first order and the first degree. Article 13. Linear equations of first order. Page 19
Problem number: Ex 3.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_linear]`

$$\boxed{y' - \frac{2y}{1+x} = (1+x)^3}$$

✓ Solution by Maple
Time used: 0.0 (sec). Leaf size: 18

```
dsolve(diff(y(x),x)-2*y(x)/(1+x)=(x+1)^3,y(x), singsol=all)
```

$$y(x) = \left(x + \frac{1}{2}x^2 + c_1\right)(1+x)^2$$

✓ Solution by Mathematica
Time used: 0.051 (sec). Leaf size: 22

```
DSolve[y'[x]-2*y[x]/(1+x)==(x+1)^3,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to (x+1)^2\left(\frac{x^2}{2} + x + c_1\right)$$

6.4 problem Ex 4

Internal problem ID [11146]

Book: An elementary treatise on differential equations by Abraham Cohen. DC heath publishers. 1906
Section: Chapter 2, differential equations of the first order and the first degree. Article 13. Linear equations of first order. Page 19
Problem number: Ex 4.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_linear]`

$$\boxed{\left(x^3+x\right)y'+4x^2y=2}$$

✓ Solution by Maple
Time used: 0.0 (sec). Leaf size: 21

```
dsolve((x+x^3)*diff(y(x),x)+4*x^2*y(x)=2,y(x), singsol=all)
```

$$y(x)=\frac{x^2+2\ln\left(x\right)+c_1}{\left(x^2+1\right)^2}$$

✓ Solution by Mathematica
Time used: 0.054 (sec). Leaf size: 23

```
DSolve[(x+x^3)*y'[x]+4*x^2*y[x]==2,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x)\to\frac{x^2+2\log(x)+c_1}{\left(x^2+1\right)^2}$$

6.5 problem Ex 5

Internal problem ID [11147]

Book: An elementary treatise on differential equations by Abraham Cohen. DC heath publishers. 1906
Section: Chapter 2, differential equations of the first order and the first degree. Article 13. Linear equations of first order. Page 19
Problem number: Ex 5.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_linear]`

$$\boxed{y'x^2 + (-2x+1)\,y = x^2}$$

✓ Solution by Maple
Time used: 0.0 (sec). Leaf size: 16

```
dsolve(x^2*diff(y(x),x)+(1-2*x)*y(x)=x^2,y(x), singsol=all)
```

$$y(x) = x^2\left(1 + \mathrm{e}^{\frac{1}{x}} c_1\right)$$

✓ Solution by Mathematica
Time used: 0.06 (sec). Leaf size: 19

```
DSolve[x^2*y'[x]+(1-2*x)*y[x]==x^2,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to x^2\left(1 + c_1 e^{\frac{1}{x}}\right)$$

7 Chapter 2, differential equations of the first order and the first degree. Article 14. Equations reducible to linear equations (Bernoulli). Page 21

7.1 problem Ex 1

Internal problem ID [11148]

Book: An elementary treatise on differential equations by Abraham Cohen. DC heath publishers. 1906
Section: Chapter 2, differential equations of the first order and the first degree. Article 14. Equations reducible to linear equations (Bernoulli). Page 21
Problem number: Ex 1.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_rational, _Bernoulli]`

$$\boxed{\left(-x^2+1\right)y' - 2y(1+x) - y^{\frac{5}{2}} = 0}$$

✓ Solution by Maple
Time used: 0.0 (sec). Leaf size: 64

```
dsolve((1-x^2)*diff(y(x),x)-2*(1+x)*y(x)=y(x)^(5/2),y(x), singsol=all)
```

$$-\frac{-1+\left(-\frac{3(-1+x)^3\ln(-1+x)}{32}+\frac{3(-1+x)^3\ln(1+x)}{32}+c_1x^3+\left(-3c_1-\frac{3}{16}\right)x^2+\left(3c_1+\frac{9}{16}\right)x-c_1-\frac{5}{8}\right)y(x)^{\frac{3}{2}}}{y\left(x\right)^{\frac{3}{2}}}$$
$$= 0$$

✓ Solution by Mathematica
Time used: 1.042 (sec). Leaf size: 76

```
DSolve[(1-x^2)*y'[x]-2*(1+x)*y[x]==y[x]^(5/2),y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to \frac{8\sqrt[3]{2}}{\left(32c_1x^3-6x^2-96c_1x^2+18x-3(x-1)^3\log(x-1)+3(x-1)^3\log(x+1)+96c_1x-20-32c_1\right)^{2/3}}$$
$$y(x) \to 0$$

7.2 problem Ex 2

Internal problem ID [11149]

Book: An elementary treatise on differential equations by Abraham Cohen. DC heath publishers. 1906
Section: Chapter 2, differential equations of the first order and the first degree. Article 14. Equations reducible to linear equations (Bernoulli). Page 21
Problem number: Ex 2.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_separable]`

$$\boxed{yy' + y^2 x = x}$$

✓ Solution by Maple
Time used: 0.016 (sec). Leaf size: 33

```
dsolve(y(x)*diff(y(x),x)+x*y(x)^2=x,y(x), singsol=all)
```

$$y(x) = \sqrt{\mathrm{e}^{-x^2} c_1 + 1}$$
$$y(x) = -\sqrt{\mathrm{e}^{-x^2} c_1 + 1}$$

✓ Solution by Mathematica
Time used: 2.1 (sec). Leaf size: 57

```
DSolve[y[x]*y'[x]+x*y[x]^2==x,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to -\sqrt{1 + e^{-x^2+2c_1}}$$
$$y(x) \to \sqrt{1 + e^{-x^2+2c_1}}$$
$$y(x) \to -1$$
$$y(x) \to 1$$

7.3 problem Ex 3

Internal problem ID [11150]

Book: An elementary treatise on differential equations by Abraham Cohen. DC heath publishers. 1906
Section: Chapter 2, differential equations of the first order and the first degree. Article 14. Equations reducible to linear equations (Bernoulli). Page 21
Problem number: Ex 3.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_separable]`

$$\boxed{y' \sin(y) + \sin(x) \cos(y) = \sin(x)}$$

✓ Solution by Maple
Time used: 0.14 (sec). Leaf size: 14

```
dsolve(sin(y(x))*diff(y(x),x)+sin(x)*cos(y(x))=sin(x),y(x), singsol=all)
```

$$y(x) = \arccos\left(\mathrm{e}^{-\cos(x)} c_1 + 1\right)$$

✓ Solution by Mathematica
Time used: 1.53 (sec). Leaf size: 81

```
DSolve[Sin[y[x]]*y'[x]+Sin[x]*Cos[y[x]]==Sin[x],y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to 0$$
$$\text{Solve}\left[2\cos(x)\tan\left(\frac{y(x)}{2}\right) e^{\text{arctanh}(\cos(y(x)))} - \sqrt{\sin^2(y(x))}\csc\left(\frac{y(x)}{2}\right)\sec\left(\frac{y(x)}{2}\right)\left(\log\left(\sec^2\left(\frac{y(x)}{2}\right)\right) - 2\log\left(\tan\left(\frac{y(x)}{2}\right)\right)\right) = c_1, y(x)\right]$$
$$y(x) \to 0$$

7.4 problem Ex 4

Internal problem ID [11151]

Book: An elementary treatise on differential equations by Abraham Cohen. DC heath publishers. 1906
Section: Chapter 2, differential equations of the first order and the first degree. Article 14. Equations reducible to linear equations (Bernoulli). Page 21
Problem number: Ex 4.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_Bernoulli]`

$$\boxed{4y'x + 3y + \mathrm{e}^x x^4 y^5 = 0}$$

✓ Solution by Maple
Time used: 0.031 (sec). Leaf size: 67

```
dsolve(4*x*diff(y(x),x)+3*y(x)+exp(x)*x^4*y(x)^5=0,y(x), singsol=all)
```

$$y(x) = \frac{1}{\sqrt{\sqrt{(\mathrm{e}^x + c_1)\, x}\, x}}$$
$$y(x) = \frac{1}{\sqrt{-\sqrt{(\mathrm{e}^x + c_1)\, x}\, x}}$$
$$y(x) = -\frac{1}{\sqrt{\sqrt{(\mathrm{e}^x + c_1)\, x}\, x}}$$
$$y(x) = -\frac{1}{\sqrt{-\sqrt{(\mathrm{e}^x + c_1)\, x}\, x}}$$

✓ Solution by Mathematica

Time used: 14.931 (sec). Leaf size: 88

```
DSolve[4*x*y'[x]+3*y[x]+Exp[x]*x^4*y[x]^5==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to -\frac{1}{\sqrt[4]{x^3\left(e^x+c_1\right)}}$$
$$y(x) \to -\frac{i}{\sqrt[4]{x^3\left(e^x+c_1\right)}}$$
$$y(x) \to \frac{i}{\sqrt[4]{x^3\left(e^x+c_1\right)}}$$
$$y(x) \to \frac{1}{\sqrt[4]{x^3\left(e^x+c_1\right)}}$$
$$y(x) \to 0$$

7.5 problem Ex 5

Internal problem ID [11152]

Book: An elementary treatise on differential equations by Abraham Cohen. DC heath publishers. 1906
Section: Chapter 2, differential equations of the first order and the first degree. Article 14. Equations reducible to linear equations (Bernoulli). Page 21
Problem number: Ex 5.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[[_1st_order, _with_linear_symmetries]]`

$$\boxed{y' - \frac{y+1}{1+x} - \sqrt{y+1} = 0}$$

✓ Solution by Maple
Time used: 0.015 (sec). Leaf size: 81

```
dsolve(diff(y(x),x)- (y(x)+1)/(x+1)=sqrt(1+y(x)),y(x), singsol=all)
```

$$\frac{\left(-c_1 y(x) + 1 + c_1 x^2 + (2c_1+1)\,x\right)\sqrt{y(x)+1} - (1+x)\left(-c_1 y(x) - 1 + c_1 x^2 + (2c_1-1)\,x\right)}{\left(x^2 + 2x - y(x)\right)\left(-\sqrt{y(x)+1} + 1 + x\right)}$$
$$= 0$$

✓ Solution by Mathematica
Time used: 0.418 (sec). Leaf size: 60

```
DSolve[y'[x]- (y[x]+1)/(x+1)==Sqrt[1+y[x]],y[x],x,IncludeSingularSolutions -> True]
```

$$\text{Solve}\left[\frac{2\sqrt{y(x)+1}\arctan\left(\frac{x+1}{\sqrt{-y(x)-1}}\right)}{\sqrt{-y(x)-1}} + \log\left(y(x) - (x+1)^2 + 1\right) - \log(x+1) = c_1, y(x)\right]$$

8 Chapter 2, differential equations of the first order and the first degree. Article 15. Page 22

8.1 problem Ex 1

Internal problem ID [11153]

Book: An elementary treatise on differential equations by Abraham Cohen. DC heath publishers. 1906
Section: Chapter 2, differential equations of the first order and the first degree. Article 15. Page 22
Problem number: Ex 1.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[[_homogeneous, 'class G'], _rational, [_Abel, '2nd type', 'cl`

$$\boxed{x^4 y(3y + 2y'x) + x^2(4y + 3y'x) = 0}$$

✓ Solution by Maple
Time used: 1.547 (sec). Leaf size: 39

```
dsolve(x^4*y(x)*(3*y(x)+2*x*diff(y(x),x))+ x^2*(4*y(x)+3*x*diff(y(x),x))=0,y(x), singsol=all
```

$$y(x) = \frac{\text{RootOf}\left(x^2\_Z^8 - 2\_Z^2 c_1 - c_1\right)^6 x^2 - 2c_1}{x^2 c_1}$$

✓ Solution by Mathematica

Time used: 60.464 (sec). Leaf size: 1769

```
DSolve[x^4*y[x]*(3*y[x]+2*x*y'[x])+ x^2*(4*y[x]+3*x*y'[x])==0,y[x],x,IncludeSingularSolution
```

$$y(x)\to -\frac{1}{2x^2}$$

$$+\frac{\sqrt{\frac{3}{x^4}-\frac{2\ 6^{2/3}e^{-2c_1}}{\sqrt[3]{e^{-6c_1}\left(\sqrt{48e^{6c_1}x^{18}+81e^{8c_1}x^{16}}-9e^{4c_1}x^8\right)}}+\frac{\sqrt[3]{6}\sqrt[3]{e^{-6c_1}\left(\sqrt{48e^{6c_1}x^{18}+81e^{8c_1}x^{16}}-9e^{4c_1}x^8\right)}}{x^6}}}{2\sqrt{3}}$$

$$-\frac{1}{2}\sqrt{\frac{2}{x^4}+\frac{2\ 2^{2/3}e^{-2c_1}}{\sqrt[3]{3}\sqrt[3]{e^{-6c_1}\left(\sqrt{48e^{6c_1}x^{18}+81e^{8c_1}x^{16}}-9e^{4c_1}x^8\right)}}-\frac{\sqrt[3]{2}\sqrt[3]{e^{-6c_1}\left(\sqrt{48e^{6c_1}x^{18}+81e^{8c_1}x^{16}}-9e^{4c_1}x^8\right)}}{3^{2/3}x^6}}$$

$$y(x)\to -\frac{1}{2x^2}$$

$$+\frac{\sqrt{\frac{3}{x^4}-\frac{2\ 6^{2/3}e^{-2c_1}}{\sqrt[3]{e^{-6c_1}\left(\sqrt{48e^{6c_1}x^{18}+81e^{8c_1}x^{16}}-9e^{4c_1}x^8\right)}}+\frac{\sqrt[3]{6}\sqrt[3]{e^{-6c_1}\left(\sqrt{48e^{6c_1}x^{18}+81e^{8c_1}x^{16}}-9e^{4c_1}x^8\right)}}{x^6}}}{2\sqrt{3}}$$

$$+\frac{1}{2}\sqrt{\frac{2}{x^4}+\frac{2\ 2^{2/3}e^{-2c_1}}{\sqrt[3]{3}\sqrt[3]{e^{-6c_1}\left(\sqrt{48e^{6c_1}x^{18}+81e^{8c_1}x^{16}}-9e^{4c_1}x^8\right)}}-\frac{\sqrt[3]{2}\sqrt[3]{e^{-6c_1}\left(\sqrt{48e^{6c_1}x^{18}+81e^{8c_1}x^{16}}-9e^{4c_1}x^8\right)}}{3^{2/3}x^6}}$$

$$y(x)\to -\frac{1}{2x^2}$$

$$-\frac{\sqrt{\frac{3}{x^4}-\frac{2\ 6^{2/3}e^{-2c_1}}{\sqrt[3]{e^{-6c_1}\left(\sqrt{48e^{6c_1}x^{18}+81e^{8c_1}x^{16}}-9e^{4c_1}x^8\right)}}+\frac{\sqrt[3]{6}\sqrt[3]{e^{-6c_1}\left(\sqrt{48e^{6c_1}x^{18}+81e^{8c_1}x^{16}}-9e^{4c_1}x^8\right)}}{x^6}}}{2\sqrt{3}}$$

$$-\frac{1}{2}\sqrt{\frac{2}{x^4}+\frac{2\ 2^{2/3}e^{-2c_1}47}{\sqrt[3]{3}\sqrt[3]{e^{-6c_1}\left(\sqrt{48e^{6c_1}x^{18}+81e^{8c_1}x^{16}}-9e^{4c_1}x^8\right)}}-\frac{\sqrt[3]{2}\sqrt[3]{e^{-6c_1}\left(\sqrt{48e^{6c_1}x^{18}+81e^{8c_1}x^{16}}-9e\right.}}{3^{2/3}x^6}}$$

8.2 problem Ex 2

Internal problem ID [11154]

Book: An elementary treatise on differential equations by Abraham Cohen. DC heath publishers. 1906
Section: Chapter 2, differential equations of the first order and the first degree. Article 15. Page 22
Problem number: Ex 2.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_separable]`

$$\boxed{y^2(3y-6y'x)-x(y-2y'x)=0}$$

✓ Solution by Maple
Time used: 0.0 (sec). Leaf size: 31

```
dsolve(y(x)^2*(3*y(x)-6*x*diff(y(x),x))-x*(y(x)-2*x*diff(y(x),x))=0,y(x), singsol=all)
```

$$y(x)=-\frac{\sqrt{3}\sqrt{x}}{3}$$
$$y(x)=\frac{\sqrt{3}\sqrt{x}}{3}$$
$$y(x)=c_1\sqrt{x}$$

✓ Solution by Mathematica
Time used: 6.194 (sec). Leaf size: 74

```
DSolve[y[x]^2*(3*y[x]-6*x*y'[x])- x*(4*y[x]-2*x*y'[x])==0,y[x],x,IncludeSingularSolutions ->
```

$$y(x)\to -\frac{i\sqrt{x}\sqrt{W\left(-3e^{-3c_1}x^3\right)}}{\sqrt{3}}$$
$$y(x)\to \frac{i\sqrt{x}\sqrt{W\left(-3e^{-3c_1}x^3\right)}}{\sqrt{3}}$$
$$y(x)\to 0$$

8.3 problem Ex 3

Internal problem ID [11155]

Book: An elementary treatise on differential equations by Abraham Cohen. DC heath publishers. 1906
Section: Chapter 2, differential equations of the first order and the first degree. Article 15. Page 22
Problem number: Ex 3.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[[_homogeneous, 'class G'], _rational, [_Abel, '2nd type', 'cl`

$$\boxed{2yx^3 - y^2 - \left(2x^4 + yx\right) y' = 0}$$

✓ Solution by Maple
Time used: 0.594 (sec). Leaf size: 47

```
dsolve((2*x^3*y(x)-y(x)^2)-(2*x^4+x*y(x))*diff(y(x),x)=0,y(x), singsol=all)
```

$$y(x) = \frac{c_1\left(\sqrt{4x^4 + c_1^2} + c_1\right)}{2x}$$
$$y(x) = -\frac{c_1\left(-c_1 + \sqrt{4x^4 + c_1^2}\right)}{2x}$$

✓ Solution by Mathematica
Time used: 1.279 (sec). Leaf size: 76

```
DSolve[(2*x^3*y[x]-y[x]^2)-(2*x^4+x*y[x])*y'[x]==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to \frac{2x^4}{-x + \frac{\sqrt{1+4c_1x^4}}{\sqrt{\frac{1}{x^2}}}}$$
$$y(x) \to -\frac{2x^4}{x + \frac{\sqrt{1+4c_1x^4}}{\sqrt{\frac{1}{x^2}}}}$$
$$y(x) \to 0$$

9 Chapter 2, differential equations of the first order and the first degree. Article 16. Integrating factors by inspection. Page 23

9.1 problem Ex 1

Internal problem ID [11156]

Book: An elementary treatise on differential equations by Abraham Cohen. DC heath publishers. 1906
Section: Chapter 2, differential equations of the first order and the first degree. Article 16. Integrating factors by inspection. Page 23
Problem number: Ex 1.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[[_homogeneous, 'class A'], _rational, _Bernoulli]`

$$\boxed{y^2 - yx + y'x^2 = 0}$$

✓ Solution by Maple
Time used: 0.015 (sec). Leaf size: 12

```
dsolve((y(x)^2-x*y(x))+x^2*diff(y(x),x)=0,y(x), singsol=all)
```

$$y(x) = \frac{x}{\ln(x) + c_1}$$

✓ Solution by Mathematica
Time used: 0.227 (sec). Leaf size: 19

```
DSolve[(y[x]^2-x*y[x])+x^2*y'[x]==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to \frac{x}{\log(x) + c_1}$$
$$y(x) \to 0$$

9.2 problem Ex 2

Internal problem ID [11157]

Book: An elementary treatise on differential equations by Abraham Cohen. DC heath publishers. 1906
Section: Chapter 2, differential equations of the first order and the first degree. Article 16. Integrating factors by inspection. Page 23
Problem number: Ex 2.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `['y=_G(x,y')']`

$$\boxed{\frac{y'x-y}{\sqrt{x^2-y^2}}-y'x=0}$$

✓ Solution by Maple
Time used: 0.032 (sec). Leaf size: 27

```
dsolve((x*diff(y(x),x)-y(x))/sqrt(x^2-y(x)^2)=x*diff(y(x),x),y(x), singsol=all)
```

$$y(x)-\arctan\left(\frac{y(x)}{\sqrt{x^2-y\left(x\right)^2}}\right)-c_1=0$$

✓ Solution by Mathematica
Time used: 0.896 (sec). Leaf size: 29

```
DSolve[(x*y'[x]-y[x])/Sqrt[x^2-y[x]^2]==x*y'[x],y[x],x,IncludeSingularSolutions -> True]
```

$$\text{Solve}\left[\arctan\left(\frac{\sqrt{x^2-y(x)^2}}{y(x)}\right)+y(x)=c_1,y(x)\right]$$

9.3 problem Ex 3

Internal problem ID [11158]

Book: An elementary treatise on differential equations by Abraham Cohen. DC heath publishers. 1906
Section: Chapter 2, differential equations of the first order and the first degree. Article 16. Integrating factors by inspection. Page 23
Problem number: Ex 3.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[[_homogeneous, 'class A'], _rational, [_Abel, '2nd type', 'cl`

$$\boxed{y - (-y + x)\,y' = -x}$$

✓ Solution by Maple
Time used: 0.015 (sec). Leaf size: 24

```
dsolve((x+y(x))-(x-y(x))*diff(y(x),x)=0,y(x), singsol=all)
```

$$y(x) = \tan\left(\text{RootOf}\left(-2\_Z + \ln\left(\sec\left(\_Z\right)^2\right) + 2\ln\left(x\right) + 2c_1\right)\right) x$$

✓ Solution by Mathematica
Time used: 0.056 (sec). Leaf size: 36

```
DSolve[(x+y[x])-(x-y[x])*y'[x]==0,y[x],x,IncludeSingularSolutions -> True]
```

$$\text{Solve}\left[\frac{1}{2}\log\left(\frac{y(x)^2}{x^2}+1\right) - \arctan\left(\frac{y(x)}{x}\right) = -\log(x) + c_1, y(x)\right]$$

9.4 problem Ex 4

Internal problem ID [11159]

Book: An elementary treatise on differential equations by Abraham Cohen. DC heath publishers. 1906
Section: Chapter 2, differential equations of the first order and the first degree. Article 16. Integrating factors by inspection. Page 23
Problem number: Ex 4.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[[_homogeneous, 'class A'], _rational, _Bernoulli]`

$$\boxed{y^2 - 2xyy' = -x^2}$$

✓ Solution by Maple
Time used: 0.0 (sec). Leaf size: 23

```
dsolve((x^2+y(x)^2)-2*x*y(x)*diff(y(x),x)=0,y(x), singsol=all)
```

$$y(x) = \sqrt{(c_1 + x)\, x}$$
$$y(x) = -\sqrt{(c_1 + x)\, x}$$

✓ Solution by Mathematica
Time used: 0.304 (sec). Leaf size: 38

```
DSolve[(x^2+y[x]^2)-2*x*y[x]*y'[x]==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to -\sqrt{x}\sqrt{x + c_1}$$
$$y(x) \to \sqrt{x}\sqrt{x + c_1}$$

9.5 problem Ex 5

Internal problem ID [11160]

Book: An elementary treatise on differential equations by Abraham Cohen. DC heath publishers. 1906
Section: Chapter 2, differential equations of the first order and the first degree. Article 16. Integrating factors by inspection. Page 23
Problem number: Ex 5.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[[_homogeneous, 'class G'], _rational, _Bernoulli]`

$$\boxed{-y^2+2xyy'=-x}$$

✓ Solution by Maple
Time used: 0.016 (sec). Leaf size: 30

```
dsolve((x-y(x)^2)+2*x*y(x)*diff(y(x),x)=0,y(x), singsol=all)
```

$$y(x)=\sqrt{-x\left(\ln\left(x\right)-c_1\right)}$$
$$y(x)=-\sqrt{\left(-\ln\left(x\right)+c_1\right)x}$$

✓ Solution by Mathematica
Time used: 0.298 (sec). Leaf size: 44

```
DSolve[(x-y[x]^2)+2*x*y[x]*y'[x]==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x)\to -\sqrt{x}\sqrt{-\log(x)+c_1}$$
$$y(x)\to \sqrt{x}\sqrt{-\log(x)+c_1}$$

9.6 problem Ex 6

Internal problem ID [11161]

Book: An elementary treatise on differential equations by Abraham Cohen. DC heath publishers. 1906
Section: Chapter 2, differential equations of the first order and the first degree. Article 16. Integrating factors by inspection. Page 23
Problem number: Ex 6.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[[_homogeneous, 'class D'], _rational, _Riccati]`

$$\boxed{y'x - y - y^2 = x^2}$$

✓ Solution by Maple
Time used: 0.0 (sec). Leaf size: 10

```
dsolve(x*diff(y(x),x)-y(x)=x^2+y(x)^2,y(x), singsol=all)
```

$$y(x) = \tan\left(c_1 + x\right) x$$

✓ Solution by Mathematica
Time used: 0.277 (sec). Leaf size: 12

```
DSolve[x*y'[x]-y[x]==x^2+y[x]^2,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to x \tan(x + c_1)$$

10 Chapter 2, differential equations of the first order and the first degree. Article 17. Other forms which Integrating factors can be found. Page 25

10.1 problem Ex 1

Internal problem ID [11162]

Book: An elementary treatise on differential equations by Abraham Cohen. DC heath publishers. 1906
Section: Chapter 2, differential equations of the first order and the first degree. Article 17. Other forms which Integrating factors can be found. Page 25
Problem number: Ex 1.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[[_homogeneous, 'class A'], _rational, [_Abel, '2nd type', 'cl`

$$\boxed{6yx + 3y^2 + \left(2x^2 + 3yx\right) y' = -3x^2}$$

✓ Solution by Maple
Time used: 0.047 (sec). Leaf size: 63

```
dsolve((3*x^2+6*x*y(x)+3*y(x)^2)+(2*x^2+3*x*y(x))*diff(y(x),x)=0,y(x), singsol=all)
```

$$y(x) = \frac{-4c_1x^2 - \sqrt{-2c_1^2x^4 + 6}}{6c_1x}$$
$$y(x) = \frac{-4c_1x^2 + \sqrt{-2c_1^2x^4 + 6}}{6c_1x}$$

✓ Solution by Mathematica
Time used: 2.7 (sec). Leaf size: 135

```
DSolve[(3*x^2+6*x*y[x]+3*y[x]^2)+(2*x^2+3*x*y[x])*y'[x]==0,y[x],x,IncludeSingularSolutions -
```

$$y(x) \to -\frac{4x^2 + \sqrt{-2x^4 + 6e^{4c_1}}}{6x}$$
$$y(x) \to \frac{-4x^2 + \sqrt{-2x^4 + 6e^{4c_1}}}{6x}$$
$$y(x) \to -\frac{\sqrt{2}\sqrt{-x^4} + 4x^2}{6x}$$
$$y(x) \to \frac{\sqrt{2}\sqrt{-x^4} - 4x^2}{6x}$$

10.2 problem Ex 2

Internal problem ID [11163]

Book: An elementary treatise on differential equations by Abraham Cohen. DC heath publishers. 1906
Section: Chapter 2, differential equations of the first order and the first degree. Article 17. Other forms which Integrating factors can be found. Page 25
Problem number: Ex 2.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_rational, [_1st_order, '_with_symmetry_[F(x)*G(y),0]']]`

$$\boxed{\left(x^2+y^2+2y\right)y' = -2x}$$

✓ Solution by Maple
Time used: 0.016 (sec). Leaf size: 17

```
dsolve((2*x)+(x^2+y(x)^2+2*y(x))*diff(y(x),x)=0,y(x), singsol=all)
```

$$\left(y(x)^2+x^2\right)\mathrm{e}^{y(x)} + c_1 = 0$$

✓ Solution by Mathematica
Time used: 0.245 (sec). Leaf size: 24

```
DSolve[(2*x)+(x^2+y[x]^2+2*y[x])*y'[x]==0,y[x],x,IncludeSingularSolutions -> True]
```

$$\text{Solve}\left[x^2 e^{y(x)} + e^{y(x)} y(x)^2 = c_1, y(x)\right]$$

10.3 problem Ex 3

Internal problem ID [11164]

Book: An elementary treatise on differential equations by Abraham Cohen. DC heath publishers. 1906
Section: Chapter 2, differential equations of the first order and the first degree. Article 17. Other forms which Integrating factors can be found. Page 25
Problem number: Ex 3.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_rational, [_1st_order, '_with_symmetry_[F(x)*G(y),0]']]`

$$\boxed{y^4 + 2y + \left(y^3 x + 2y^4 - 4x\right) y' = 0}$$

✓ Solution by Maple
Time used: 0.016 (sec). Leaf size: 27

```
dsolve((y(x)^4+2*y(x))+(x*y(x)^3+2*y(x)^4-4*x)*diff(y(x),x)=0,y(x), singsol=all)
```

$$x - \frac{\left(-y(x)^2 + c_1\right) y(x)^2}{y(x)^3 + 2} = 0$$

✓ Solution by Mathematica

Time used: 60.318 (sec). Leaf size: 2021

```
DSolve[(y[x]^4+2*y[x])+(x*y[x]^3+2*y[x]^4-4*x)*y'[x]==0,y[x],x,IncludeSingularSolutions -> T
```

$$y(x)\to -\frac{1}{2}\sqrt{\frac{\sqrt[3]{54x^3+\sqrt{\left(54x^3+144c_1x-2{c_1}^3\right)^2-4\left(24x+{c_1}^2\right)^3}+144c_1x-2{c_1}^3}{3\sqrt[3]{2}}+\frac{3\sqrt[3]{54x^3+\sqrt{(54x^3+}}}{}}$$
$$-\frac{1}{2}\sqrt{-\frac{\sqrt[3]{54x^3+\sqrt{\left(54x^3+144c_1x-2{c_1}^3\right)^2-4\left(24x+{c_1}^2\right)^3}+144c_1x-2{c_1}^3}{3\sqrt[3]{2}}-\frac{}{3\sqrt[3]{54x^3+\sqrt{(54x^3-}}}}$$
$$-\frac{x}{4}$$

$$y(x)\to -\frac{1}{2}\sqrt{\frac{\sqrt[3]{54x^3+\sqrt{\left(54x^3+144c_1x-2{c_1}^3\right)^2-4\left(24x+{c_1}^2\right)^3}+144c_1x-2{c_1}^3}{3\sqrt[3]{2}}+\frac{3\sqrt[3]{54x^3+\sqrt{(54x^3+}}}{}}$$
$$+\frac{1}{2}\sqrt{-\frac{\sqrt[3]{54x^3+\sqrt{\left(54x^3+144c_1x-2{c_1}^3\right)^2-4\left(24x+{c_1}^2\right)^3}+144c_1x-2{c_1}^3}{3\sqrt[3]{2}}-\frac{}{3\sqrt[3]{54x^3+\sqrt{(54x^3-}}}}$$
$$-\frac{x}{4}$$

$$y(x)$$
$$\to \frac{1}{2}\sqrt{\frac{\sqrt[3]{54x^3+\sqrt{\left(54x^3+144c_1x-2{c_1}^3\right)^2-4\left(24x+{c_1}^2\right)^3}+144c_1x-2{c_1}^3}{3\sqrt[3]{2}}+\frac{3\sqrt[3]{54x^3+\sqrt{(54x^3+14}}}{}}$$
$$-\frac{1}{2}\sqrt{-\frac{\sqrt[3]{54x^3+\sqrt{\left(54x^3+144c_1x-2{c_1}^3\right)^2-4\left(24x+{c_1}^2\right)^3}+144c_1x-2{c_1}^3}{3\sqrt[3]{2}}-\frac{}{3\sqrt[3]{54x^3+\sqrt{(54x^3-}}}}$$
$$-\frac{x}{4}$$

$$y(x)$$
$$\to \frac{1}{2}\sqrt{\frac{\sqrt[3]{54x^3+\sqrt{\left(54x^3+144c_1x-2{c_1}^3\right)^2-4\left(24x+{c_1}^2\right)^3}+144c_1x-2{c_1}^3}{3\sqrt[3]{2}}+\frac{3\sqrt[3]{54x^3+\sqrt{(54x^3+14}}}{}}$$

10.4 problem Ex 4

Internal problem ID [11165]

Book: An elementary treatise on differential equations by Abraham Cohen. DC heath publishers. 1906
Section: Chapter 2, differential equations of the first order and the first degree. Article 17. Other forms which Integrating factors can be found. Page 25
Problem number: Ex 4.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_separable]`

$$\boxed{yx^3 - y^4 + \left(y^3 x - x^4\right) y' = 0}$$

✓ Solution by Maple
Time used: 0.0 (sec). Leaf size: 39

```
dsolve((x^3*y(x)-y(x)^4)+(y(x)^3*x-x^4)*diff(y(x),x)=0,y(x), singsol=all)
```

$$y(x) = -\frac{\left(1 + i\sqrt{3}\right) x}{2}$$
$$y(x) = \frac{\left(i\sqrt{3} - 1\right) x}{2}$$
$$y(x) = x$$
$$y(x) = c_1 x$$

✓ Solution by Mathematica

Time used: 0.043 (sec). Leaf size: 99

```
DSolve[(x^3*y[x]-y[x]^4)+(y[x]^3*x-x^4)*y'[x]==0,y[x],x,IncludeSingularSolutions -> True]
```

$$
\begin{aligned}
y(x) &\to x \\
y(x) &\to -\frac{1}{2}i\left(\sqrt{3}-i\right)x \\
y(x) &\to \frac{1}{2}i\left(\sqrt{3}+i\right)x \\
y(x) &\to c_1 x \\
y(x) &\to x \\
y(x) &\to -\frac{1}{2}i\left(\sqrt{3}-i\right)x \\
y(x) &\to \frac{1}{2}i\left(\sqrt{3}+i\right)x
\end{aligned}
$$

10.5 problem Ex 6

Internal problem ID [11166]

Book: An elementary treatise on differential equations by Abraham Cohen. DC heath publishers. 1906
Section: Chapter 2, differential equations of the first order and the first degree. Article 17. Other forms which Integrating factors can be found. Page 25
Problem number: Ex 6.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[[_homogeneous, 'class A'], _rational, _dAlembert]`

$$\boxed{y^2 + 2ymx + \left(y^2 m - m\,x^2 - 2yx\right) y' = x^2}$$

✓ Solution by Maple
Time used: 0.031 (sec). Leaf size: 59

```
dsolve((y(x)^2-x^2+2*m*x*y(x))+(m*y(x)^2-m*x^2-2*x*y(x))*diff(y(x),x)=0,y(x), singsol=all)
```

$$y(x) = \frac{m - \sqrt{-4c_1^2x^2 - 4c_1x + m^2}}{2c_1}$$
$$y(x) = \frac{m + \sqrt{-4c_1^2x^2 - 4c_1x + m^2}}{2c_1}$$

✓ Solution by Mathematica
Time used: 3.604 (sec). Leaf size: 89

```
DSolve[(y[x]^2-x^2+2*m*x*y[x])+(m*y[x]^2-m*x^2-2*x*y[x])*y'[x]==0,y[x],x,IncludeSingularSolu
```

$$y(x) \to \frac{1}{2}\left(-\sqrt{e^{2c_1}m^2 - 4x^2 + 4e^{c_1}x} - e^{c_1}m\right)$$
$$y(x) \to \frac{1}{2}\left(\sqrt{e^{2c_1}m^2 - 4x^2 + 4e^{c_1}x} - e^{c_1}m\right)$$

11 Chapter 2, differential equations of the first order and the first degree. Article 18. Transformation of variables. Page 26

11.1 problem Ex 1

Internal problem ID [11167]

Book: An elementary treatise on differential equations by Abraham Cohen. DC heath publishers. 1906
Section: Chapter 2, differential equations of the first order and the first degree. Article 18. Transformation of variables. Page 26
Problem number: Ex 1.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_linear]`

$$\boxed{y'x - y + 2x^2y = x^3}$$

✓ Solution by Maple
Time used: 0.0 (sec). Leaf size: 17

```
dsolve(x*diff(y(x),x)-y(x)+2*x^2*y(x)-x^3=0,y(x), singsol=all)
```

$$y(x) = \frac{x}{2} + x\,\mathrm{e}^{-x^2}c_1$$

✓ Solution by Mathematica
Time used: 0.082 (sec). Leaf size: 21

```
DSolve[x*y'[x]-y[x]+2*x^2*y[x]-x^3==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to x\left(\frac{1}{2} + c_1 e^{-x^2}\right)$$

11.2 problem Ex 2

Internal problem ID [11168]

Book: An elementary treatise on differential equations by Abraham Cohen. DC heath publishers. 1906
Section: Chapter 2, differential equations of the first order and the first degree. Article 18. Transformation of variables. Page 26
Problem number: Ex 2.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[[_homogeneous, 'class C'], [_Abel, '2nd type', 'class C'], _d`

$$\boxed{(y+x)\,y'=1}$$

✓ Solution by Maple
Time used: 0.0 (sec). Leaf size: 21

```
dsolve((x+y(x))*diff(y(x),x)-1=0,y(x), singsol=all)
```

$$y(x)=-\operatorname{LambertW}\left(-c_1 \mathrm{e}^{-x-1}\right)-1-x$$

✓ Solution by Mathematica
Time used: 0.04 (sec). Leaf size: 24

```
DSolve[(x+y[x])*y'[x]-1==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x)\to -W\left(c_1\left(-e^{-x-1}\right)\right)-x-1$$

11.3 problem Ex 3

Internal problem ID [11169]

Book: An elementary treatise on differential equations by Abraham Cohen. DC heath publishers. 1906
Section: Chapter 2, differential equations of the first order and the first degree. Article 18. Transformation of variables. Page 26
Problem number: Ex 3.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[[_homogeneous, 'class A'], _rational, [_Abel, '2nd type', 'cl`

$$\boxed{yy' - y'x + y = -x}$$

✓ Solution by Maple
Time used: 0.0 (sec). Leaf size: 24

```
dsolve(x+y(x)*diff(y(x),x)+y(x)-x*diff(y(x),x)=0,y(x), singsol=all)
```

$$y(x) = \tan\left(\text{RootOf}\left(-2\_Z + \ln\left(\sec\left(\_Z\right)^2\right) + 2\ln\left(x\right) + 2c_1\right)\right) x$$

✓ Solution by Mathematica
Time used: 0.052 (sec). Leaf size: 36

```
DSolve[x+y[x]*y'[x]+y[x]-x*y'[x]==0,y[x],x,IncludeSingularSolutions -> True]
```

$$\text{Solve}\left[\frac{1}{2}\log\left(\frac{y(x)^2}{x^2}+1\right) - \arctan\left(\frac{y(x)}{x}\right) = -\log(x) + c_1, y(x)\right]$$

11.4 problem Ex 4

Internal problem ID [11170]

Book: An elementary treatise on differential equations by Abraham Cohen. DC heath publishers. 1906
Section: Chapter 2, differential equations of the first order and the first degree. Article 18. Transformation of variables. Page 26
Problem number: Ex 4.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_rational, _Riccati]`

$$\boxed{y'x - ay + y^2 b = c\,x^{2a}}$$

✓ Solution by Maple
Time used: 0.015 (sec). Leaf size: 35

```
dsolve(x*diff(y(x),x)-a*y(x)+b*y(x)^2=c*x^(2*a),y(x), singsol=all)
```

$$y(x) = \frac{\tanh\left(\frac{x^a\sqrt{b}\sqrt{c}+ic_1 a}{a}\right)\sqrt{c}\,x^a}{\sqrt{b}}$$

✓ Solution by Mathematica
Time used: 0.533 (sec). Leaf size: 153

```
DSolve[x*y'[x]-a*y[x]+b*y[x]^2==c*x^(2*a),y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to \frac{\sqrt{c}x^a\left(-\cos\left(\frac{\sqrt{-b}\sqrt{c}x^a}{a}\right)+c_1\sin\left(\frac{\sqrt{-b}\sqrt{c}x^a}{a}\right)\right)}{\sqrt{-b}\left(\sin\left(\frac{\sqrt{-b}\sqrt{c}x^a}{a}\right)+c_1\cos\left(\frac{\sqrt{-b}\sqrt{c}x^a}{a}\right)\right)}$$
$$y(x) \to \frac{\sqrt{c}x^a\tan\left(\frac{\sqrt{-b}\sqrt{c}x^a}{a}\right)}{\sqrt{-b}}$$

12 Chapter 2, differential equations of the first order and the first degree. Article 19. Summary. Page 29

12.1 problem Ex 1

Internal problem ID [11171]

Book: An elementary treatise on differential equations by Abraham Cohen. DC heath publishers. 1906
Section: Chapter 2, differential equations of the first order and the first degree. Article 19. Summary. Page 29
Problem number: Ex 1.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_separable]`

$$\boxed{x\sqrt{1-y^2}+y\sqrt{-x^2+1}\,y'=0}$$

✓ Solution by Maple
Time used: 0.0 (sec). Leaf size: 40

```
dsolve(x*sqrt(1-y(x)^2)+y(x)*sqrt(1-x^2)*diff(y(x),x)=0,y(x), singsol=all)
```

$$\frac{(-1+x)(1+x)}{\sqrt{-x^2+1}}+\frac{(-1+y(x))(y(x)+1)}{\sqrt{1-y(x)^2}}+c_1=0$$

✓ Solution by Mathematica
Time used: 3.778 (sec). Leaf size: 77

```
DSolve[x*Sqrt[1-y[x]^2]+y[x]*Sqrt[1-x^2]*y'[x]==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x)\to -\sqrt{x^2-c_1\left(2\sqrt{1-x^2}+c_1\right)}$$
$$y(x)\to \sqrt{x^2-c_1\left(2\sqrt{1-x^2}+c_1\right)}$$
$$y(x)\to -1$$
$$y(x)\to 1$$

12.2 problem Ex 2

Internal problem ID [11172]

Book: An elementary treatise on differential equations by Abraham Cohen. DC heath publishers. 1906
Section: Chapter 2, differential equations of the first order and the first degree. Article 19. Summary. Page 29
Problem number: Ex 2.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_separable]`

$$\boxed{\sqrt{1-y^2}+\sqrt{-x^2+1}\,y'=0}$$

✓ Solution by Maple
Time used: 0.0 (sec). Leaf size: 11

```
dsolve(sqrt(1-y(x)^2)+sqrt(1-x^2)*diff(y(x),x)=0,y(x), singsol=all)
```

$$y(x)=-\sin\left(\arcsin\left(x\right)+c_1\right)$$

✓ Solution by Mathematica
Time used: 0.496 (sec). Leaf size: 47

```
DSolve[Sqrt[1-y[x]^2]+Sqrt[1-x^2]*y'[x]==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x)\to\cos\left(2\arctan\left(\frac{\sqrt{1-x^2}}{x+1}\right)+c_1\right)$$
$$y(x)\to -1$$
$$y(x)\to 1$$
$$y(x)\to \text{Interval}[\{-1,1\}]$$

12.3 problem Ex 3

Internal problem ID [11173]

Book: An elementary treatise on differential equations by Abraham Cohen. DC heath publishers. 1906
Section: Chapter 2, differential equations of the first order and the first degree. Article 19. Summary. Page 29
Problem number: Ex 3.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_linear]`

$$\boxed{y' - x^2 y = x^5}$$

✓ Solution by Maple
Time used: 0.0 (sec). Leaf size: 19

```
dsolve(diff(y(x),x)-x^2*y(x)=x^5,y(x), singsol=all)
```

$$y(x) = -x^3 - 3 + \mathrm{e}^{\frac{x^3}{3}} c_1$$

✓ Solution by Mathematica
Time used: 0.111 (sec). Leaf size: 24

```
DSolve[y'[x]-x^2*y[x]==x^5,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to -x^3 + c_1 e^{\frac{x^3}{3}} - 3$$

12.4 problem Ex 4

Internal problem ID [11174]

Book: An elementary treatise on differential equations by Abraham Cohen. DC heath publishers. 1906
Section: Chapter 2, differential equations of the first order and the first degree. Article 19. Summary. Page 29
Problem number: Ex 4.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[[_homogeneous, 'class C'], _dAlembert]`

$$\boxed{(y-x)^2 y' = 1}$$

✓ Solution by Maple
Time used: 0.109 (sec). Leaf size: 29

```
dsolve((y(x)-x)^2*diff(y(x),x)=1,y(x), singsol=all)
```

$$y(x) + \frac{\ln(y(x)-x-1)}{2} - \frac{\ln(y(x)-x+1)}{2} - c_1 = 0$$

✓ Solution by Mathematica
Time used: 0.23 (sec). Leaf size: 33

```
DSolve[(y[x]-x)^2*y'[x]==1,y[x],x,IncludeSingularSolutions -> True]
```

$$\text{Solve}\left[y(x) + \frac{1}{2}\log(-y(x)+x+1) - \frac{1}{2}\log(y(x)-x+1) = c_1, y(x)\right]$$

12.5 problem Ex 5

Internal problem ID [11175]

Book: An elementary treatise on differential equations by Abraham Cohen. DC heath publishers. 1906
Section: Chapter 2, differential equations of the first order and the first degree. Article 19. Summary. Page 29
Problem number: Ex 5.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_Bernoulli]`

$$\boxed{y'x + y + \mathrm{e}^x x^4 y^4 = 0}$$

✓ Solution by Maple
Time used: 0.015 (sec). Leaf size: 64

```
dsolve(x*diff(y(x),x)+y(x)+x^4*y(x)^4*exp(x)=0,y(x), singsol=all)
```

$$y(x) = \frac{1}{(3\,\mathrm{e}^x + c_1)^{\frac{1}{3}}\, x}$$
$$y(x) = -\frac{1 + i\sqrt{3}}{2\,(3\,\mathrm{e}^x + c_1)^{\frac{1}{3}}\, x}$$
$$y(x) = \frac{i\sqrt{3} - 1}{2\,(3\,\mathrm{e}^x + c_1)^{\frac{1}{3}}\, x}$$

✓ Solution by Mathematica

Time used: 11.276 (sec). Leaf size: 79

```
DSolve[x*y'[x]+y[x]+x^4*y[x]^4*Exp[x]==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to \frac{1}{\sqrt[3]{x^3\left(3e^x+c_1\right)}}$$
$$y(x) \to -\frac{\sqrt[3]{-1}}{\sqrt[3]{x^3\left(3e^x+c_1\right)}}$$
$$y(x) \to \frac{(-1)^{2/3}}{\sqrt[3]{x^3\left(3e^x+c_1\right)}}$$
$$y(x) \to 0$$

12.6 problem Ex 6

Internal problem ID [11176]

Book: An elementary treatise on differential equations by Abraham Cohen. DC heath publishers. 1906
Section: Chapter 2, differential equations of the first order and the first degree. Article 19. Summary. Page 29
Problem number: Ex 6.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_separable]`

$$\boxed{(1-x)\,y+(1-y)\,xy'=0}$$

✓ Solution by Maple
Time used: 0.016 (sec). Leaf size: 15

```
dsolve((1-x)*y(x)+(1-y(x))*x*diff(y(x),x)=0,y(x), singsol=all)
```

$$y(x)=-\operatorname{LambertW}\left(-\frac{c_1 \mathrm{e}^{x}}{x}\right)$$

✓ Solution by Mathematica
Time used: 4.764 (sec). Leaf size: 26

```
DSolve[(1-x)*y[x]+(1-y[x])*x*y'[x]==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x)\to -W\left(-\frac{e^{x-c_1}}{x}\right)$$
$$y(x)\to 0$$

12.7 problem Ex 7

Internal problem ID [11177]

Book: An elementary treatise on differential equations by Abraham Cohen. DC heath publishers. 1906
Section: Chapter 2, differential equations of the first order and the first degree. Article 19. Summary. Page 29
Problem number: Ex 7.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[[_homogeneous, 'class A'], _rational, [_Abel, '2nd type', 'cl`

$$\boxed{(y-x)\,y'+y=0}$$

✓ Solution by Maple
Time used: 0.016 (sec). Leaf size: 17

```
dsolve((y(x)-x)*diff(y(x),x)+y(x)=0,y(x), singsol=all)
```

$$y(x)=-\frac{x}{\text{LambertW}\left(-x\,\mathrm{e}^{-c_1}\right)}$$

✓ Solution by Mathematica
Time used: 5.289 (sec). Leaf size: 25

```
DSolve[(y[x]-x)*y'[x]+y[x]==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x)\to -\frac{x}{W\left(-e^{-c_1}x\right)}$$
$$y(x)\to 0$$

12.8 problem Ex 8

Internal problem ID [11178]

Book: An elementary treatise on differential equations by Abraham Cohen. DC heath publishers. 1906
Section: Chapter 2, differential equations of the first order and the first degree. Article 19. Summary. Page 29
Problem number: Ex 8.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[[_homogeneous, 'class A'], _rational, _dAlembert]`

$$\boxed{y'x - y - \sqrt{x^2 + y^2} = 0}$$

✓ Solution by Maple
Time used: 0.0 (sec). Leaf size: 26

```
dsolve(x*diff(y(x),x)-y(x)=sqrt(x^2+y(x)^2),y(x), singsol=all)
```

$$\frac{-c_1 x^2 + y(x) + \sqrt{y(x)^2 + x^2}}{x^2} = 0$$

✓ Solution by Mathematica
Time used: 0.582 (sec). Leaf size: 27

```
DSolve[x*y'[x]-y[x]==Sqrt[x^2+y[x]^2],y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to \frac{1}{2} e^{-c_1} \left(-1 + e^{2c_1} x^2\right)$$

12.9 problem Ex 10

Internal problem ID [11179]

Book: An elementary treatise on differential equations by Abraham Cohen. DC heath publishers. 1906
Section: Chapter 2, differential equations of the first order and the first degree. Article 19. Summary. Page 29
Problem number: Ex 10.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[[_homogeneous, 'class A'], _rational, _dAlembert]`

$$\boxed{y'x - y - \sqrt{x^2 - y^2} = 0}$$

✓ Solution by Maple
Time used: 0.016 (sec). Leaf size: 27

```
dsolve(x*diff(y(x),x)-y(x)=sqrt(x^2-y(x)^2),y(x), singsol=all)
```

$$-\arctan\left(\frac{y(x)}{\sqrt{x^2 - y(x)^2}}\right) + \ln(x) - c_1 = 0$$

✓ Solution by Mathematica
Time used: 0.395 (sec). Leaf size: 18

```
DSolve[x*y'[x]-y[x]==Sqrt[x^2-y[x]^2],y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to -x \cosh(i \log(x) + c_1)$$

12.10 problem Ex 11

Internal problem ID [11180]

Book: An elementary treatise on differential equations by Abraham Cohen. DC heath publishers. 1906
Section: Chapter 2, differential equations of the first order and the first degree. Article 19. Summary. Page 29
Problem number: Ex 11.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[[_homogeneous, 'class A'], _dAlembert]`

$$\boxed{x \sin\left(\frac{y}{x}\right) - y \cos\left(\frac{y}{x}\right) + x \cos\left(\frac{y}{x}\right) y' = 0}$$

✓ Solution by Maple
Time used: 0.0 (sec). Leaf size: 14

```
dsolve((x*sin(y(x)/x)-y(x)*cos(y(x)/x))+x*cos(y(x)/x)*diff(y(x),x)=0,y(x), singsol=all)
```

$$y(x) = x \arcsin\left(\frac{1}{x c_1}\right)$$

✓ Solution by Mathematica
Time used: 15.438 (sec). Leaf size: 21

```
DSolve[(x*Sin[y[x]/x]-y[x]*Cos[y[x]/x])+x*Cos[y[x]/x]*y'[x]==0,y[x],x,IncludeSingularSolutio
```

$$y(x) \to x \arcsin\left(\frac{e^{c_1}}{x}\right)$$
$$y(x) \to 0$$

12.11 problem Ex 12

Internal problem ID [11181]

Book: An elementary treatise on differential equations by Abraham Cohen. DC heath publishers. 1906
Section: Chapter 2, differential equations of the first order and the first degree. Article 19. Summary. Page 29
Problem number: Ex 12.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[[_homogeneous, 'class C'], _rational, [_Abel, '2nd type', 'cl`

$$\boxed{-2y + (2x - y + 4)\,y' = -x - 5}$$

✓ Solution by Maple
Time used: 0.766 (sec). Leaf size: 117

```
dsolve((x-2*y(x)+5)+(2*x-y(x)+4)*diff(y(x),x)=0,y(x), singsol=all)
```

$$y(x) = -\frac{(i\sqrt{3}-1)\left(3\sqrt{3}\sqrt{27c_1^2(1+x)^2-1}+27c_1(1+x)\right)^{\frac{2}{3}} - 3i\sqrt{3} - 3 + 6\left(3\sqrt{3}\sqrt{27c_1^2(1+x)^2-1}+2\right.}{6\left(3\sqrt{3}\sqrt{27c_1^2(1+x)^2-1}+27c_1(1+x)\right)^{\frac{1}{3}}c_1}$$

✓ Solution by Mathematica
Time used: 60.282 (sec). Leaf size: 1601

```
DSolve[(x-2*y[x]+5)+(2*x-y[x]+4)*y'[x]==0,y[x],x,IncludeSingularSolutions -> True]
```

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12.12 problem Ex 13

Internal problem ID [11182]

Book: An elementary treatise on differential equations by Abraham Cohen. DC heath publishers. 1906
Section: Chapter 2, differential equations of the first order and the first degree. Article 19. Summary. Page 29
Problem number: Ex 13.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_linear]`

$$\boxed{y' + \frac{y}{(-x^2+1)^{\frac{3}{2}}} = \frac{x+\sqrt{-x^2+1}}{(-x^2+1)^2}}$$

✓ Solution by Maple
Time used: 0.0 (sec). Leaf size: 56

```
dsolve(diff(y(x),x)+y(x)/(1-x^2)^(3/2)= (x+(1-x^2)^(1/2))/(1-x^2)^2,y(x), singsol=all)
```

$$y(x) = \left(\int \frac{\mathrm{e}^{\frac{x}{\sqrt{-x^2+1}}} \left(x+\sqrt{-x^2+1}\right)}{(-1+x)^2 (1+x)^2} dx + c_1 \right) \mathrm{e}^{-\frac{x}{\sqrt{-x^2+1}}}$$

✓ Solution by Mathematica
Time used: 0.358 (sec). Leaf size: 38

```
DSolve[y'[x]+y[x]/(1-x^2)^(3/2)== (x+(1-x^2)^(1/2))/(1-x^2)^2,y[x],x,IncludeSingularSolution
```

$$y(x) \to \frac{x}{\sqrt{1-x^2}} + c_1 e^{-\frac{x}{\sqrt{1-x^2}}}$$

12.13 problem Ex 14

Internal problem ID [11183]

Book: An elementary treatise on differential equations by Abraham Cohen. DC heath publishers. 1906
Section: Chapter 2, differential equations of the first order and the first degree. Article 19. Summary. Page 29
Problem number: Ex 14.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_separable]`

$$\boxed{\left(-x^2+1\right)y' - yx - y^2 a x = 0}$$

✓ Solution by Maple
Time used: 0.0 (sec). Leaf size: 22

```
dsolve((1-x^2)*diff(y(x),x)-x*y(x)=a*x*y(x)^2,y(x), singsol=all)
```

$$y(x) = \frac{1}{\sqrt{-1+x}\,\sqrt{1+x}\,c_1 - a}$$

✓ Solution by Mathematica
Time used: 4.13 (sec). Leaf size: 47

```
DSolve[(1-x^2)*y'[x]-x*y[x]==a*x*y[x]^2,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to -\frac{e^{c_1}}{-\sqrt{1-x^2} + a e^{c_1}}$$
$$y(x) \to 0$$
$$y(x) \to -\frac{1}{a}$$

12.14 problem Ex 15

Internal problem ID [11184]

Book: An elementary treatise on differential equations by Abraham Cohen. DC heath publishers. 1906
Section: Chapter 2, differential equations of the first order and the first degree. Article 19. Summary. Page 29
Problem number: Ex 15.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[[_homogeneous, 'class G'], _rational]`

$$\boxed{xy^2(3y+y'x)-2y+y'x=0}$$

✓ Solution by Maple
Time used: 0.125 (sec). Leaf size: 45

```
dsolve((x*y(x)^2)*(3*y(x)+x*diff(y(x),x))-(2*y(x)-x*diff(y(x),x))=0,y(x), singsol=all)
```

$$y(x)=\frac{c_1-\sqrt{4x^5+c_1^2}}{2x^3}$$
$$y(x)=\frac{c_1+\sqrt{4x^5+c_1^2}}{2x^3}$$

✓ Solution by Mathematica
Time used: 1.836 (sec). Leaf size: 75

```
DSolve[(x*y[x]^2)*(3*y[x]+x*y'[x])-(2*y[x]-x*y'[x])==0,y[x],x,IncludeSingularSolutions -> Tr
```

$$y(x)\to -\frac{\sqrt{4x^5+e^{5c_1}}+e^{\frac{5c_1}{2}}}{2x^3}$$
$$y(x)\to \frac{\sqrt{4x^5+e^{5c_1}}-e^{\frac{5c_1}{2}}}{2x^3}$$

12.15 problem Ex 16

Internal problem ID [11185]

Book: An elementary treatise on differential equations by Abraham Cohen. DC heath publishers. 1906
Section: Chapter 2, differential equations of the first order and the first degree. Article 19. Summary. Page 29
Problem number: Ex 16.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_linear]`

$$\boxed{\left(x^2+1\right)y'+y=\arctan\left(x\right)}$$

✓ Solution by Maple
Time used: 0.0 (sec). Leaf size: 15

```
dsolve((1+x^2)*diff(y(x),x)+y(x)=arctan(x),y(x), singsol=all)
```

$$y(x)=\arctan\left(x\right)-1+\mathrm{e}^{-\arctan(x)}c_1$$

✓ Solution by Mathematica
Time used: 0.23 (sec). Leaf size: 18

```
DSolve[(1+x^2)*y'[x]+y[x]==ArcTan[x],y[x],x,IncludeSingularSolutions -> True]
```

$$y(x)\to \arctan(x)+c_1e^{-\arctan(x)}-1$$

12.16 problem Ex 17

Internal problem ID [11186]

Book: An elementary treatise on differential equations by Abraham Cohen. DC heath publishers. 1906
Section: Chapter 2, differential equations of the first order and the first degree. Article 19. Summary. Page 29
Problem number: Ex 17.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[[_homogeneous, 'class G'], _rational]`

$$\boxed{5yx - 3y^3 + \left(3x^2 - 7y^2x\right) y' = 0}$$

✓ Solution by Maple
Time used: 1.453 (sec). Leaf size: 49

```
dsolve((5*x*y(x)-3*y(x)^3)+(3*x^2-7*x*y(x)^2)*diff(y(x),x)=0,y(x), singsol=all)
```

$$y(x) = \text{RootOf}\left(x^{\frac{3}{2}}\_Z^7 - \_Z^3 x^{\frac{5}{2}} - c_1\right)^2$$
$$y(x) = \text{RootOf}\left(x^{\frac{3}{2}}\_Z^7 - \_Z^3 x^{\frac{5}{2}} + c_1\right)^2$$

✓ Solution by Mathematica
Time used: 7.756 (sec). Leaf size: 288

```
DSolve[(5*x*y[x]-3*y[x]^3)+(3*x^2-7*x*y[x]^2)*y'[x]==0,y[x],x,IncludeSingularSolutions -> Tr
```

$$y(x) \to \text{Root}\left[4\#1^7x^3 - 8\#1^5x^4 + 4\#1^3x^5 - {c_1}^2\&, 1\right]$$
$$y(x) \to \text{Root}\left[4\#1^7x^3 - 8\#1^5x^4 + 4\#1^3x^5 - {c_1}^2\&, 2\right]$$
$$y(x) \to \text{Root}\left[4\#1^7x^3 - 8\#1^5x^4 + 4\#1^3x^5 - {c_1}^2\&, 3\right]$$
$$y(x) \to \text{Root}\left[4\#1^7x^3 - 8\#1^5x^4 + 4\#1^3x^5 - {c_1}^2\&, 4\right]$$
$$y(x) \to \text{Root}\left[4\#1^7x^3 - 8\#1^5x^4 + 4\#1^3x^5 - {c_1}^2\&, 5\right]$$
$$y(x) \to \text{Root}\left[4\#1^7x^3 - 8\#1^5x^4 + 4\#1^3x^5 - {c_1}^2\&, 6\right]$$
$$y(x) \to \text{Root}\left[4\#1^7x^3 - 8\#1^5x^4 + 4\#1^3x^5 - {c_1}^2\&, 7\right]$$

12.17 problem Ex 18

Internal problem ID [11187]

Book: An elementary treatise on differential equations by Abraham Cohen. DC heath publishers. 1906
Section: Chapter 2, differential equations of the first order and the first degree. Article 19. Summary. Page 29
Problem number: Ex 18.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_linear]`

$$\boxed{y' + y\cos(x) = \frac{\sin(2x)}{2}}$$

✓ Solution by Maple
Time used: 0.0 (sec). Leaf size: 15

```
dsolve(diff(y(x),x)+y(x)*cos(x)=1/2*sin(2*x),y(x), singsol=all)
```

$$y(x) = \sin(x) - 1 + \mathrm{e}^{-\sin(x)} c_1$$

✓ Solution by Mathematica
Time used: 0.089 (sec). Leaf size: 18

```
DSolve[y'[x]+y[x]*Cos[x]==1/2*Sin[2*x],y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to \sin(x) + c_1 e^{-\sin(x)} - 1$$

12.18 problem Ex 19

Internal problem ID [11188]

Book: An elementary treatise on differential equations by Abraham Cohen. DC heath publishers. 1906
Section: Chapter 2, differential equations of the first order and the first degree. Article 19. Summary. Page 29
Problem number: Ex 19.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[[_homogeneous, 'class D'], _rational, _Bernoulli]`

$$\boxed{y^2x + y - y'x = 0}$$

✓ Solution by Maple
Time used: 0.016 (sec). Leaf size: 16

```
dsolve((x*y(x)^2+y(x))-x*diff(y(x),x)=0,y(x), singsol=all)
```

$$y(x) = -\frac{2x}{x^2 - 2c_1}$$

✓ Solution by Mathematica
Time used: 0.207 (sec). Leaf size: 23

```
DSolve[(x*y[x]^2+y[x])-x*y'[x]==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to -\frac{2x}{x^2 - 2c_1}$$
$$y(x) \to 0$$

12.19 problem Ex 20

Internal problem ID [11189]

Book: An elementary treatise on differential equations by Abraham Cohen. DC heath publishers. 1906
Section: Chapter 2, differential equations of the first order and the first degree. Article 19. Summary. Page 29
Problem number: Ex 20.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_separable]`

$$\boxed{(1-x)\,y-(y+1)\,xy'=0}$$

✓ Solution by Maple
Time used: 0.016 (sec). Leaf size: 14

```
dsolve((1-x)*y(x)-(1+y(x))*x*diff(y(x),x)=0,y(x), singsol=all)
```

$$y(x)=\text{LambertW}\left(\frac{\mathrm{e}^{-x}x}{c_1}\right)$$

✓ Solution by Mathematica
Time used: 5.134 (sec). Leaf size: 21

```
DSolve[(1-x)*y[x]-(1+y[x])*x*y'[x]==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x)\to W\left(xe^{-x+c_1}\right)$$
$$y(x)\to 0$$

12.20 problem Ex 21

Internal problem ID [11190]

Book: An elementary treatise on differential equations by Abraham Cohen. DC heath publishers. 1906
Section: Chapter 2, differential equations of the first order and the first degree. Article 19. Summary. Page 29
Problem number: Ex 21.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_separable]`

$$\boxed{3x^2y + \left(x^3 + y^2x^3\right)y' = 0}$$

✓ Solution by Maple
Time used: 0.016 (sec). Leaf size: 14

```
dsolve(3*x^2*y(x)+(x^3+x^3*y(x)^2)*diff(y(x),x)=0,y(x), singsol=all)
```

$$y(x) = \frac{1}{\sqrt{\frac{1}{\text{LambertW}\left(\frac{c_1}{x^6}\right)}}}$$

✓ Solution by Mathematica
Time used: 6.245 (sec). Leaf size: 46

```
DSolve[3*x^2*y[x]+(x^3+x^3*y[x]^2)*y'[x]==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to -\sqrt{W\left(\frac{e^{2c_1}}{x^6}\right)}$$
$$y(x) \to \sqrt{W\left(\frac{e^{2c_1}}{x^6}\right)}$$
$$y(x) \to 0$$

12.21 problem Ex 22

Internal problem ID [11191]

Book: An elementary treatise on differential equations by Abraham Cohen. DC heath publishers. 1906
Section: Chapter 2, differential equations of the first order and the first degree. Article 19. Summary. Page 29
Problem number: Ex 22.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_rational]`

$$\boxed{\left(x^2+y^2\right)\left(x+yy'\right)-\left(x^2+y^2+x\right)\left(y'x-y\right)=0}$$

✓ Solution by Maple
Time used: 0.015 (sec). Leaf size: 45

```
dsolve((x^2+y(x)^2)*(x+y(x)*diff(y(x),x))=(x^2+y(x)^2+x)*(x*diff(y(x),x)-y(x)),y(x), singsol
```

$$y(x)=-\cot\left(\text{RootOf}\left(-2\_Z+2\ln\left(2\csc\left(\_Z\right)^2x^2+\cot\left(\_Z\right)x+x\right)\right.\right.$$
$$\left.\left.-\ln\left(\csc\left(\_Z\right)^2x^2\right)+2c_1\right)\right)x$$

✓ Solution by Mathematica
Time used: 0.548 (sec). Leaf size: 53

```
DSolve[(x^2+y[x]^2)*(x+y[x]*y'[x])==(x^2+y[x]^2+x)*(x*y'[x]-y[x]),y[x],x,IncludeSingularSolu
```

$$\text{Solve}\left[\frac{1}{2}\arctan\left(\frac{x}{y(x)}\right)-\frac{1}{4}\log\left(x^2+y(x)^2\right)+\frac{1}{2}\log\left(2x^2+2y(x)^2-y(x)+x\right)=c_1,y(x)\right]$$

12.22 problem Ex 23

Internal problem ID [11192]

Book: An elementary treatise on differential equations by Abraham Cohen. DC heath publishers. 1906
Section: Chapter 2, differential equations of the first order and the first degree. Article 19. Summary. Page 29
Problem number: Ex 23.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[[_homogeneous, 'class C'], _rational, [_Abel, '2nd type', 'cl`

$$\boxed{3y + (2x + 3y - 5)\,y' = -2x + 1}$$

✓ Solution by Maple
Time used: 0.015 (sec). Leaf size: 21

```
dsolve((2*x+3*y(x)-1)+(2*x+3*y(x)-5)*diff(y(x),x)=0,y(x), singsol=all)
```

$$y(x) = -\frac{2x}{3} - 4\,\mathrm{LambertW}\left(-\frac{c_1 \mathrm{e}^{\frac{x}{12}-\frac{7}{12}}}{12}\right) - \frac{7}{3}$$

✓ Solution by Mathematica
Time used: 5.457 (sec). Leaf size: 43

```
DSolve[(2*x+3*y[x]-1)+(2*x+3*y[x]-5)*y'[x]==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to -4W\left(-e^{\frac{x}{12}-1+c_1}\right) - \frac{2x}{3} - \frac{7}{3}$$
$$y(x) \to \frac{1}{3}(-2x - 7)$$

12.23 problem Ex 24

Internal problem ID [11193]

Book: An elementary treatise on differential equations by Abraham Cohen. DC heath publishers. 1906
Section: Chapter 2, differential equations of the first order and the first degree. Article 19. Summary. Page 29
Problem number: Ex 24.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[[_homogeneous, 'class A'], _rational, _dAlembert]`

$$\boxed{y^3 - 2x^2y + \left(2y^2x - x^3\right)y' = 0}$$

✓ Solution by Maple
Time used: 0.266 (sec). Leaf size: 71

```
dsolve((y(x)^3-2*x^2*y(x))+(2*x*y(x)^2-x^3)*diff(y(x),x)=0,y(x), singsol=all)
```

$$y(x) = \frac{\sqrt{\frac{2c_1x^3 - 2\sqrt{c_1^2x^6+4}}{c_1x^3}}\,x}{2}$$
$$y(x) = \frac{\sqrt{2}\sqrt{\frac{c_1x^3 + \sqrt{c_1^2x^6+4}}{c_1x^3}}\,x}{2}$$

✓ Solution by Mathematica

Time used: 15.638 (sec). Leaf size: 277

```
DSolve[(y[x]^3-2*x^2*y[x])+(2*x*y[x]^2-x^3)*y'[x]==0,y[x],x,IncludeSingularSolutions -> True
```

$$y(x) \to -\frac{\sqrt{x^2 - \frac{\sqrt{x^6-4e^{2c_1}}}{x}}}{\sqrt{2}}$$

$$y(x) \to \frac{\sqrt{x^2 - \frac{\sqrt{x^6-4e^{2c_1}}}{x}}}{\sqrt{2}}$$

$$y(x) \to -\frac{\sqrt{\frac{x^3+\sqrt{x^6-4e^{2c_1}}}{x}}}{\sqrt{2}}$$

$$y(x) \to \frac{\sqrt{\frac{x^3+\sqrt{x^6-4e^{2c_1}}}{x}}}{\sqrt{2}}$$

$$y(x) \to -\frac{\sqrt{x^2 - \frac{\sqrt{x^6}}{x}}}{\sqrt{2}}$$

$$y(x) \to \frac{\sqrt{x^2 - \frac{\sqrt{x^6}}{x}}}{\sqrt{2}}$$

$$y(x) \to -\frac{\sqrt{\frac{\sqrt{x^6}+x^3}{x}}}{\sqrt{2}}$$

$$y(x) \to \frac{\sqrt{\frac{\sqrt{x^6}+x^3}{x}}}{\sqrt{2}}$$

12.24 problem Ex 25

Internal problem ID [11194]

Book: An elementary treatise on differential equations by Abraham Cohen. DC heath publishers. 1906
Section: Chapter 2, differential equations of the first order and the first degree. Article 19. Summary. Page 29
Problem number: Ex 25.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_rational]`

$$\boxed{2y^2x^3 - y + (2y^3x^2 - x)\,y' = 0}$$

✓ Solution by Maple
Time used: 0.0 (sec). Leaf size: 361

```
dsolve((2*x^3*y(x)^2-y(x))+(2*x^2*y(x)^3-x)*diff(y(x),x)=0,y(x), singsol=all)
```

$$y(x) = -\frac{\left(-\left(\left(-9+\sqrt{12x^8-36c_1x^6+36c_1^2x^4-12c_1^3x^2+81}\right)x^2\right)^{\frac{2}{3}}+x^2 12^{\frac{1}{3}}(x^2-c_1)\right)12^{\frac{1}{3}}}{6\left(\left(-9+\sqrt{12x^8-36c_1x^6+36c_1^2x^4-12c_1^3x^2+81}\right)x^2\right)^{\frac{1}{3}}x}$$

$$y(x) = -\frac{\left((1+i\sqrt{3})\left(\left(-9+\sqrt{12x^8-36c_1x^6+36c_1^2x^4-12c_1^3x^2+81}\right)x^2\right)^{\frac{2}{3}}+\left(i3^{\frac{5}{6}}-3^{\frac{1}{3}}\right)2^{\frac{2}{3}}x^2(x^2-c_1)\right)2}{12\left(\left(-9+\sqrt{12x^8-36c_1x^6+36c_1^2x^4-12c_1^3x^2+81}\right)x^2\right)^{\frac{1}{3}}x}$$

$$y(x) = \frac{2^{\frac{2}{3}}3^{\frac{1}{3}}\left((i\sqrt{3}-1)\left(\left(-9+\sqrt{12x^8-36c_1x^6+36c_1^2x^4-12c_1^3x^2+81}\right)x^2\right)^{\frac{2}{3}}+2^{\frac{2}{3}}x^2(x^2-c_1)\left(i3^{\frac{5}{6}}+3^{\frac{1}{3}}\right)\right.}{12\left(\left(-9+\sqrt{12x^8-36c_1x^6+36c_1^2x^4-12c_1^3x^2+81}\right)x^2\right)^{\frac{1}{3}}x}$$

✓ Solution by Mathematica

Time used: 46.278 (sec). Leaf size: 358

```
DSolve[(2*x^3*y[x]^2-y[x])+(2*x^2*y[x]^3-x)*y'[x]==0,y[x],x,IncludeSingularSolutions -> True
```

$$y(x) \to \frac{\sqrt[3]{2}(-x^3+c_1 x)}{\sqrt[3]{-27x^2+\sqrt{729x^4+108x^3\left(x^3-c_1 x\right)^3}}} + \frac{\sqrt[3]{-27x^2+\sqrt{729x^4+108x^3\left(x^3-c_1 x\right)^3}}}{3\sqrt[3]{2}x}$$

$$y(x) \to \frac{\left(1+i\sqrt{3}\right)\left(x^3-c_1 x\right)}{2^{2/3}\sqrt[3]{-27x^2+\sqrt{729x^4+108x^3\left(x^3-c_1 x\right)^3}}} - \frac{\left(1-i\sqrt{3}\right)\sqrt[3]{-27x^2+\sqrt{729x^4+108x^3\left(x^3-c_1 x\right)^3}}}{6\sqrt[3]{2}x}$$

$$y(x) \to \frac{\left(1-i\sqrt{3}\right)\left(x^3-c_1 x\right)}{2^{2/3}\sqrt[3]{-27x^2+\sqrt{729x^4+108x^3\left(x^3-c_1 x\right)^3}}} - \frac{\left(1+i\sqrt{3}\right)\sqrt[3]{-27x^2+\sqrt{729x^4+108x^3\left(x^3-c_1 x\right)^3}}}{6\sqrt[3]{2}x}$$

12.25 problem Ex 26

Internal problem ID [11195]

Book: An elementary treatise on differential equations by Abraham Cohen. DC heath publishers. 1906
Section: Chapter 2, differential equations of the first order and the first degree. Article 19. Summary. Page 29
Problem number: Ex 26.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[[_1st_order, _with_linear_symmetries]]`

$$\boxed{\left(x^2+y^2\right)\left(x+yy'\right)+\sqrt{1+x^2+y^2}\left(y-y'x\right)=0}$$

✓ Solution by Maple
Time used: 0.125 (sec). Leaf size: 25

```
dsolve((x^2+y(x)^2)*(x+y(x)*diff(y(x),x))+(1+x^2+y(x)^2)^(1/2)*(y(x)-x*diff(y(x),x))=0,y(x),
```

$$\arctan\left(\frac{x}{y\left(x\right)}\right)+\sqrt{1+x^2+y\left(x\right)^2}-c_1=0$$

✓ Solution by Mathematica
Time used: 0.454 (sec). Leaf size: 27

```
DSolve[(x^2+y[x]^2)*(x+y[x]*y'[x])+(1+x^2+y[x]^2)^(1/2)*(y[x]-x*y'[x])==0,y[x],x,IncludeSing
```

$$\text{Solve}\left[\arctan\left(\frac{x}{y(x)}\right)+\sqrt{x^2+y(x)^2+1}=c_1,y(x)\right]$$

12.26 problem Ex 27

Internal problem ID [11196]

Book: An elementary treatise on differential equations by Abraham Cohen. DC heath publishers. 1906
Section: Chapter 2, differential equations of the first order and the first degree. Article 19. Summary. Page 29
Problem number: Ex 27.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[[_homogeneous, 'class A'], _dAlembert]`

$$\boxed{\mathrm{e}^{\frac{y}{x}}+\mathrm{e}^{\frac{x}{y}}\left(1-\frac{x}{y}\right)y'=-1}$$

✓ Solution by Maple
Time used: 0.0 (sec). Leaf size: 44

```
dsolve((1+exp(y(x)/x))+exp(x/y(x))*(1-x/y(x))*diff(y(x),x)=0,y(x), singsol=all)
```

$$y(x)=\text{RootOf}\left(\int^{-Z}\frac{\mathrm{e}^{\frac{1}{\_a}}(\_a-1)}{\_a\left(\_a\mathrm{e}^{\frac{1}{\_a}}+\mathrm{e}^{-\_a}-\mathrm{e}^{\frac{1}{\_a}}+1\right)}d\_a+\ln(x)+c_1\right)x$$

✓ Solution by Mathematica
Time used: 0.429 (sec). Leaf size: 63

```
DSolve[(1+Exp[y[x]/x])+Exp[x/y[x]]*(1-x/y[x])*y'[x]==0,y[x],x,IncludeSingularSolutions -> Tr
```

$$\text{Solve}\left[\int_1^{\frac{y(x)}{x}}\frac{e^{\frac{1}{K[1]}}(K[1]-1)}{K[1]\left(e^{\frac{1}{K[1]}}K[1]+e^{K[1]}-e^{\frac{1}{K[1]}}+1\right)}dK[1]=-\log(x)+c_1,y(x)\right]$$

12.27 problem Ex 28

Internal problem ID [11197]

Book: An elementary treatise on differential equations by Abraham Cohen. DC heath publishers. 1906
Section: Chapter 2, differential equations of the first order and the first degree. Article 19. Summary. Page 29
Problem number: Ex 28.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_Bernoulli]`

$$\boxed{y'x + y - y^2 \ln(x) = 0}$$

✓ Solution by Maple
Time used: 0.0 (sec). Leaf size: 13

```
dsolve(x*diff(y(x),x)+y(x)-y(x)^2*ln(x)=0,y(x), singsol=all)
```

$$y(x) = \frac{1}{1 + c_1 x + \ln(x)}$$

✓ Solution by Mathematica
Time used: 0.233 (sec). Leaf size: 20

```
DSolve[x*y'[x]+y[x]-y[x]^2*Log[x]==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to \frac{1}{\log(x) + c_1 x + 1}$$
$$y(x) \to 0$$

12.28 problem Ex 29

Internal problem ID [11198]

Book: An elementary treatise on differential equations by Abraham Cohen. DC heath publishers. 1906
Section: Chapter 2, differential equations of the first order and the first degree. Article 19. Summary. Page 29
Problem number: Ex 29.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_rational]`

$$\boxed{x^3y^4+y^3x^2+y^2x+y+\left(y^3x^4-y^2x^3-yx^3+x\right)y'=0}$$

✗ Solution by Maple

```
dsolve((x^3*y(x)^4+x^2*y(x)^3+x*y(x)^2+y(x))+(x^4*y(x)^3-x^3*y(x)^2-x^3*y(x)+x)*diff(y(x),x)
```

No solution found

✗ Solution by Mathematica
Time used: 0.0 (sec). Leaf size: 0

```
DSolve[(x^3*y[x]^4+x^2*y[x]^3+x*y[x]^2+y[x])+(x^4*y[x]^3-x^3*y[x]^2-x^3*y[x]+x)*y'[x]==0,y[x
```

Not solved

12.29 problem Ex 30

Internal problem ID [11199]

Book: An elementary treatise on differential equations by Abraham Cohen. DC heath publishers. 1906
Section: Chapter 2, differential equations of the first order and the first degree. Article 19. Summary. Page 29
Problem number: Ex 30.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[[_homogeneous, 'class A'], _dAlembert]`

$$\boxed{(2\sqrt{yx}-x)\,y'+y=0}$$

✓ Solution by Maple
Time used: 0.015 (sec). Leaf size: 18

```
dsolve((2*sqrt(x*y(x))-x)*diff(y(x),x)+y(x)=0,y(x), singsol=all)
```

$$\ln(y(x))+\frac{x}{\sqrt{y(x)\,x}}-c_1=0$$

✓ Solution by Mathematica
Time used: 0.376 (sec). Leaf size: 33

```
DSolve[(2*Sqrt[x*y[x]]-x)*y'[x]+y[x]==0,y[x],x,IncludeSingularSolutions -> True]
```

$$\text{Solve}\left[\frac{2}{\sqrt{\frac{y(x)}{x}}}+2\log\left(\frac{y(x)}{x}\right)=-2\log(x)+c_1,y(x)\right]$$

13 Chapter IV, differential equations of the first order and higher degree than the first. Article 24. Equations solvable for p. Page 49

13.1 problem Ex 1

Internal problem ID [11200]

Book: An elementary treatise on differential equations by Abraham Cohen. DC heath publishers. 1906
Section: Chapter IV, differential equations of the first order and higher degree than the first. Article 24. Equations solvable for p. Page 49
Problem number: Ex 1.
ODE order: 1.
ODE degree: 2.

CAS Maple gives this as type `[_quadrature]`

$$\boxed{y'^2 + (y+x)\,y' + yx = 0}$$

✓ Solution by Maple
Time used: 0.0 (sec). Leaf size: 20

```
dsolve(diff(y(x),x)^2+(x+y(x))*diff(y(x),x)+x*y(x)=0,y(x), singsol=all)
```

$$y(x) = -\frac{x^2}{2} + c_1$$
$$y(x) = c_1 \mathrm{e}^{-x}$$

✓ Solution by Mathematica
Time used: 0.056 (sec). Leaf size: 32

```
DSolve[(y'[x])^2+(x+y[x])*y'[x]+x*y[x]==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to c_1 e^{-x}$$
$$y(x) \to -\frac{x^2}{2} + c_1$$
$$y(x) \to 0$$

13.2 problem Ex 2

Internal problem ID [11201]

Book: An elementary treatise on differential equations by Abraham Cohen. DC heath publishers. 1906
Section: Chapter IV, differential equations of the first order and higher degree than the first. Article 24. Equations solvable for p. Page 49
Problem number: Ex 2.
ODE order: 1.
ODE degree: 2.

CAS Maple gives this as type `[[_homogeneous, 'class A'], _rational, _dAlembert]`

$$\boxed{xy'^2 - 2yy' = x}$$

✓ Solution by Maple
Time used: 0.094 (sec). Leaf size: 32

```
dsolve(x*diff(y(x),x)^2-2*y(x)*diff(y(x),x)-x=0,y(x), singsol=all)
```

$$\begin{aligned} y(x) &= -ix \\ y(x) &= ix \\ y(x) &= \frac{-c_1^2 + x^2}{2c_1} \end{aligned}$$

✓ Solution by Mathematica
Time used: 0.225 (sec). Leaf size: 71

```
DSolve[x*(y'[x])^2-2*y[x]*y'[x]-x==0,y[x],x,IncludeSingularSolutions -> True]
```

$$\begin{aligned} y(x) &\to \frac{1}{2}e^{-c_1}\left(-x^2 + e^{2c_1}\right) \\ y(x) &\to \frac{1}{2}e^{-c_1}\left(-1 + e^{2c_1}x^2\right) \\ y(x) &\to -ix \\ y(x) &\to ix \end{aligned}$$

13.3 problem Ex 3

Internal problem ID [11202]

Book: An elementary treatise on differential equations by Abraham Cohen. DC heath publishers. 1906
Section: Chapter IV, differential equations of the first order and higher degree than the first. Article 24. Equations solvable for p. Page 49
Problem number: Ex 3.
ODE order: 1.
ODE degree: 2.

CAS Maple gives this as type `[_quadrature]`

$$\boxed{y^2 + y'^2 = 1}$$

✓ Solution by Maple
Time used: 0.078 (sec). Leaf size: 29

```
dsolve(y(x)^2+diff(y(x),x)^2=1,y(x), singsol=all)
```

$$\begin{aligned} y(x) &= -1 \\ y(x) &= 1 \\ y(x) &= -\sin(c_1 - x) \\ y(x) &= \sin(c_1 - x) \end{aligned}$$

✓ Solution by Mathematica
Time used: 0.211 (sec). Leaf size: 39

```
DSolve[y[x]^2+(y'[x])^2==1,y[x],x,IncludeSingularSolutions -> True]
```

$$\begin{aligned} y(x) &\to \cos(x + c_1) \\ y(x) &\to \cos(x - c_1) \\ y(x) &\to -1 \\ y(x) &\to 1 \\ y(x) &\to \text{Interval}[\{-1, 1\}] \end{aligned}$$

13.4 problem Ex 4

Internal problem ID [11203]

Book: An elementary treatise on differential equations by Abraham Cohen. DC heath publishers. 1906
Section: Chapter IV, differential equations of the first order and higher degree than the first. Article 24. Equations solvable for p. Page 49
Problem number: Ex 4.
ODE order: 1.
ODE degree: 2.

CAS Maple gives this as type `[_linear]`

$$\boxed{(2y'x - y)^2 = 8x^3}$$

✓ Solution by Maple
Time used: 0.062 (sec). Leaf size: 30

```
dsolve((2*x*diff(y(x),x)-y(x))^2=8*x^3,y(x), singsol=all)
```

$$y(x) = \left(-x\sqrt{2} + c_1\right)\sqrt{x}$$
$$y(x) = \left(x\sqrt{2} + c_1\right)\sqrt{x}$$

✓ Solution by Mathematica
Time used: 0.128 (sec). Leaf size: 42

```
DSolve[(2*x*y'[x]-y[x])^2==8*x^3,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to \sqrt{x}\left(-\sqrt{2}x + c_1\right)$$
$$y(x) \to \sqrt{x}\left(\sqrt{2}x + c_1\right)$$

13.5 problem Ex 5

Internal problem ID [11204]

Book: An elementary treatise on differential equations by Abraham Cohen. DC heath publishers. 1906
Section: Chapter IV, differential equations of the first order and higher degree than the first. Article 24. Equations solvable for p. Page 49
Problem number: Ex 5.
ODE order: 1.
ODE degree: 2.

CAS Maple gives this as type `[_quadrature]`

$$\boxed{\left(x^2+1\right) y'^2 = 1}$$

✓ Solution by Maple
Time used: 0.078 (sec). Leaf size: 17

```
dsolve((1+x^2)*diff(y(x),x)^2=1,y(x), singsol=all)
```

$$y(x) = \operatorname{arcsinh}(x) + c_1$$
$$y(x) = -\operatorname{arcsinh}(x) + c_1$$

✓ Solution by Mathematica
Time used: 0.006 (sec). Leaf size: 45

```
DSolve[(1+x^2)*(y'[x])^2==1,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to -\log\left(\sqrt{x^2+1} - x\right) + c_1$$
$$y(x) \to \log\left(\sqrt{x^2+1} - x\right) + c_1$$

13.6 problem Ex 6

Internal problem ID [11205]

Book: An elementary treatise on differential equations by Abraham Cohen. DC heath publishers. 1906
Section: Chapter IV, differential equations of the first order and higher degree than the first. Article 24. Equations solvable for p. Page 49
Problem number: Ex 6.
ODE order: 1.
ODE degree: 3.

CAS Maple gives this as type `[_quadrature]`

$$\boxed{y'^3 - \left(2x + y^2\right) y'^2 + \left(x^2 - y^2 + 2y^2 x\right) y' - \left(x^2 - y^2\right) y^2 = 0}$$

✓ Solution by Maple
Time used: 0.0 (sec). Leaf size: 35

```
dsolve(diff(y(x),x)^3-(2*x+y(x)^2)*diff(y(x),x)^2+(x^2-y(x)^2+2*x*y(x)^2)*diff(y(x),x)-(x^2-
```

$$y(x) = \frac{1}{c_1 - x}$$
$$y(x) = -x - 1 + c_1 \mathrm{e}^{x}$$
$$y(x) = x - 1 + c_1 \mathrm{e}^{-x}$$

✓ Solution by Mathematica
Time used: 0.276 (sec). Leaf size: 48

```
DSolve[(y'[x])^3-(2*x+y[x]^2)*(y'[x])^2+(x^2-y[x]^2+2*x*y[x]^2)*y'[x]-(x^2-y[x]^2)*y[x]^2==0
```

$$y(x) \to -\frac{1}{x + c_1}$$
$$y(x) \to x + c_1 e^{-x} - 1$$
$$y(x) \to -x + c_1 e^{x} - 1$$
$$y(x) \to 0$$

14 Chapter IV, differential equations of the first order and higher degree than the first. Article 25. Equations solvable for y. Page 52

14.1 problem Ex 1

Internal problem ID [11206]

Book: An elementary treatise on differential equations by Abraham Cohen. DC heath publishers. 1906
Section: Chapter IV, differential equations of the first order and higher degree than the first. Article 25. Equations solvable for y. Page 52
Problem number: Ex 1.
ODE order: 1.
ODE degree: 0.

CAS Maple gives this as type `[[_1st_order, _with_linear_symmetries], _dAlembert]`

$$\boxed{2y'x - y + \ln(y') = 0}$$

✓ Solution by Maple
Time used: 0.016 (sec). Leaf size: 69

```
dsolve(2*diff(y(x),x)*x-y(x)+ln(diff(y(x),x))=0,y(x), singsol=all)
```

$$y(x) = -1 + \sqrt{4c_1x + 1} - \ln(2) + \ln\left(\frac{-1 + \sqrt{4c_1x + 1}}{x}\right)$$
$$y(x) = -1 - \sqrt{4c_1x + 1} - \ln(2) + \ln\left(\frac{-1 - \sqrt{4c_1x + 1}}{x}\right)$$

✓ Solution by Mathematica
Time used: 0.157 (sec). Leaf size: 32

```
DSolve[2*y'[x]*x-y[x]+Log[y'[x]]==0,y[x],x,IncludeSingularSolutions -> True]
```

$$\text{Solve}\left[W\left(2xe^{y(x)}\right) - \log\left(W\left(2xe^{y(x)}\right) + 2\right) - y(x) = c_1, y(x)\right]$$

14.2 problem Ex 2

Internal problem ID [11207]

Book: An elementary treatise on differential equations by Abraham Cohen. DC heath publishers. 1906
Section: Chapter IV, differential equations of the first order and higher degree than the first. Article 25. Equations solvable for y. Page 52
Problem number: Ex 2.
ODE order: 1.
ODE degree: 2.

CAS Maple gives this as type `[[_homogeneous, 'class A'], _rational, _dAlembert]`

$$\boxed{4xy'^2 + 2y'x - y = 0}$$

✓ Solution by Maple
Time used: 0.063 (sec). Leaf size: 35

```
dsolve(4*x*diff(y(x),x)^2+2*x*diff(y(x),x)-y(x)=0,y(x), singsol=all)
```

$$y(x) = -\frac{x}{4}$$
$$y(x) = 4c_1 + 2\sqrt{c_1 x}$$
$$y(x) = 4c_1 - 2\sqrt{c_1 x}$$

✓ Solution by Mathematica
Time used: 0.196 (sec). Leaf size: 72

```
DSolve[4*x*(y'[x])^2+2*x*y'[x]-y[x]==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to \frac{1}{4}e^{2c_1}\left(-2\sqrt{x} + e^{2c_1}\right)$$
$$y(x) \to \frac{1}{4}e^{-4c_1}\left(1 + 2e^{2c_1}\sqrt{x}\right)$$
$$y(x) \to 0$$
$$y(x) \to -\frac{x}{4}$$

14.3 problem Ex 3

Internal problem ID [11208]

Book: An elementary treatise on differential equations by Abraham Cohen. DC heath publishers. 1906
Section: Chapter IV, differential equations of the first order and higher degree than the first. Article 25. Equations solvable for y. Page 52
Problem number: Ex 3.
ODE order: 1.
ODE degree: 2.

CAS Maple gives this as type `[[_homogeneous, 'class A'], _rational, _dAlembert]`

$$\boxed{xy'^2 - 2yy' = x}$$

✓ Solution by Maple
Time used: 0.015 (sec). Leaf size: 32

```
dsolve(x*diff(y(x),x)^2-2*y(x)*diff(y(x),x)-x=0,y(x), singsol=all)
```

$$\begin{aligned} y(x) &= -ix \\ y(x) &= ix \\ y(x) &= \frac{-c_1^2 + x^2}{2c_1} \end{aligned}$$

✓ Solution by Mathematica
Time used: 0.186 (sec). Leaf size: 71

```
DSolve[x*(y'[x])^2-2*y[x]*y'[x]-x==0,y[x],x,IncludeSingularSolutions -> True]
```

$$\begin{aligned} y(x) &\to \frac{1}{2}e^{-c_1}\left(-x^2 + e^{2c_1}\right) \\ y(x) &\to \frac{1}{2}e^{-c_1}\left(-1 + e^{2c_1}x^2\right) \\ y(x) &\to -ix \\ y(x) &\to ix \end{aligned}$$

14.4 problem Ex 4

Internal problem ID [11209]

Book: An elementary treatise on differential equations by Abraham Cohen. DC heath publishers. 1906
Section: Chapter IV, differential equations of the first order and higher degree than the first. Article 25. Equations solvable for y. Page 52
Problem number: Ex 4.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[[_homogeneous, 'class C'], _Riccati]`

$$\boxed{y' + 2yx - y^2 = x^2}$$

✓ Solution by Maple
Time used: 0.031 (sec). Leaf size: 29

```
dsolve(diff(y(x),x)+2*x*y(x)=x^2+y(x)^2,y(x), singsol=all)
```

$$y(x) = \frac{c_1(-1+x)\,\mathrm{e}^{2x} - x - 1}{-1 + \mathrm{e}^{2x} c_1}$$

✓ Solution by Mathematica
Time used: 0.208 (sec). Leaf size: 29

```
DSolve[y'[x]+2*x*y[x]==x^2+y[x]^2,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to x + \frac{1}{\frac{1}{2} + c_1 e^{2x}} - 1$$
$$y(x) \to x - 1$$

14.5 problem Ex 5

Internal problem ID [11210]

Book: An elementary treatise on differential equations by Abraham Cohen. DC heath publishers. 1906
Section: Chapter IV, differential equations of the first order and higher degree than the first. Article 25. Equations solvable for y. Page 52
Problem number: Ex 5.
ODE order: 1.
ODE degree: 2.

CAS Maple gives this as type `[[_homogeneous, 'class G'], _rational]`

$$\boxed{y + y'x - x^4 y'^2 = 0}$$

✓ Solution by Maple
Time used: 0.047 (sec). Leaf size: 81

```
dsolve(y(x)=-x*diff(y(x),x)+x^4*diff(y(x),x)^2,y(x), singsol=all)
```

$$y(x) = -\frac{1}{4x^2}$$
$$y(x) = \frac{-c_1 i - x}{c_1^2 x}$$
$$y(x) = \frac{c_1 i - x}{x\, c_1^2}$$
$$y(x) = \frac{c_1 i - x}{x\, c_1^2}$$
$$y(x) = \frac{-c_1 i - x}{c_1^2 x}$$

✓ Solution by Mathematica

Time used: 0.809 (sec). Leaf size: 123

```
DSolve[y[x]==-x*y'[x]+x^4*(y'[x])^2,y[x],x,IncludeSingularSolutions -> True]
```

$$\text{Solve}\left[-\frac{x\sqrt{4x^2y(x)+1}\operatorname{arctanh}\left(\sqrt{4x^2y(x)+1}\right)}{\sqrt{4x^4y(x)+x^2}}-\frac{1}{2}\log(y(x))=c_1,y(x)\right]$$

$$\text{Solve}\left[\frac{x\sqrt{4x^2y(x)+1}\operatorname{arctanh}\left(\sqrt{4x^2y(x)+1}\right)}{\sqrt{4x^4y(x)+x^2}}-\frac{1}{2}\log(y(x))=c_1,y(x)\right]$$

$$y(x)\to 0$$

14.6 problem Ex 6

Internal problem ID [11211]

Book: An elementary treatise on differential equations by Abraham Cohen. DC heath publishers. 1906
Section: Chapter IV, differential equations of the first order and higher degree than the first. Article 25. Equations solvable for y. Page 52
Problem number: Ex 6.
ODE order: 1.
ODE degree: 2.

CAS Maple gives this as type `[[_1st_order, _with_linear_symmetries], _dAlembert]`

$$\boxed{y'^2 + 2y'x - y = 0}$$

✓ Solution by Maple
Time used: 0.078 (sec). Leaf size: 650

```
dsolve(diff(y(x),x)^2+2*x*diff(y(x),x)-y(x)=0,y(x), singsol=all)
```

$$y(x) = \frac{\left(x^2 - x\left(-x^3 + 2\sqrt{3}\,\sqrt{-c_1\left(x^3 - 3c_1\right)} + 6c_1\right)^{\frac{1}{3}} + \left(-x^3 + 2\sqrt{3}\,\sqrt{-c_1\left(x^3 - 3c_1\right)} + 6c_1\right)^{\frac{2}{3}}\right)\left(x^2 + 3x\left(-\right.\right.}{4\left(-x^3 + 2\sqrt{3}\,\sqrt{-c_1\left(x^3 - 3c_1\right)}\right.}$$

$$y(x) = \frac{\left(i\left(-x^3 + 2\sqrt{3}\,\sqrt{-c_1\left(x^3 - 3c_1\right)} + 6c_1\right)^{\frac{2}{3}}\sqrt{3} - i\sqrt{3}\,x^2 + \left(-x^3 + 2\sqrt{3}\,\sqrt{-c_1\left(x^3 - 3c_1\right)} + 6c_1\right)^{\frac{2}{3}} + 2x\left(\right.}{}$$

$$y(x) = \frac{\left(i\sqrt{3}\,x^2 - i\left(-x^3 + 2\sqrt{3}\,\sqrt{-c_1\left(x^3 - 3c_1\right)} + 6c_1\right)^{\frac{2}{3}}\sqrt{3} + x^2 + 2x\left(-x^3 + 2\sqrt{3}\,\sqrt{-c_1\left(x^3 - 3c_1\right)} + 6c_1\right)^{\frac{1}{3}}\right.}{}$$

✓ Solution by Mathematica

Time used: 60.154 (sec). Leaf size: 931

```
DSolve[(y'[x])^2+2*x*y'[x]-y[x]==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x)\to \frac{1}{4}\left(-x^2+\frac{x\left(x^3+8e^{3c_1}\right)}{\sqrt[3]{-x^6+20e^{3c_1}x^3+8\sqrt{e^{3c_1}\left(-x^3+e^{3c_1}\right)^3}+8e^{6c_1}}}+\sqrt[3]{-x^6+20e^{3c_1}x^3+8\sqrt{e^{3c_1}\left(-x^3+e^{3c_1}\right)^3}+8e^{6c_1}}\right)$$

$$y(x)\to \frac{1}{72}\left(-18x^2-\frac{9i\left(\sqrt{3}-i\right)x\left(x^3+8e^{3c_1}\right)}{\sqrt[3]{-x^6+20e^{3c_1}x^3+8\sqrt{e^{3c_1}\left(-x^3+e^{3c_1}\right)^3}+8e^{6c_1}}}+9i\left(\sqrt{3}+i\right)\sqrt[3]{-x^6+20e^{3c_1}x^3+8\sqrt{e^{3c_1}\left(-x^3+e^{3c_1}\right)^3}+8e^{6c_1}}\right)$$

$$y(x)\to \frac{1}{72}\left(-18x^2+\frac{9i\left(\sqrt{3}+i\right)x\left(x^3+8e^{3c_1}\right)}{\sqrt[3]{-x^6+20e^{3c_1}x^3+8\sqrt{e^{3c_1}\left(-x^3+e^{3c_1}\right)^3}+8e^{6c_1}}}-9\left(1+i\sqrt{3}\right)\sqrt[3]{-x^6+20e^{3c_1}x^3+8\sqrt{e^{3c_1}\left(-x^3+e^{3c_1}\right)^3}+8e^{6c_1}}\right)$$

$$y(x)\to \frac{1}{4}\left(-x^2+\frac{x\left(x^3-8e^{3c_1}\right)}{\sqrt[3]{-x^6-20e^{3c_1}x^3+8\sqrt{e^{3c_1}\left(x^3+e^{3c_1}\right)^3}+8e^{6c_1}}}+\sqrt[3]{-x^6-20e^{3c_1}x^3+8\sqrt{e^{3c_1}\left(x^3+e^{3c_1}\right)^3}+8e^{6c_1}}\right)$$

$$y(x)\to \frac{1}{72}\left(-18x^2+\frac{9\left(1+i\sqrt{3}\right)x\left(-x^3+8e^{3c_1}\right)}{\sqrt[3]{-x^6-20e^{3c_1}x^3+8\sqrt{e^{3c_1}\left(x^3+e^{3c_1}\right)^3}+8e^{6c_1}}}+9i\left(\sqrt{3}+i\right)\sqrt[3]{-x^6-20e^{3c_1}x^3+8\sqrt{e^{3c_1}\left(x^3+e^{3c_1}\right)^3}+8e^{6c_1}}\right)$$

$$y(x)\to \frac{1}{72}\left(-18x^2+\frac{9i\left(\sqrt{3}+i\right)x\left(x^3-8e^{3c_1}\right)}{\sqrt[3]{-x^6-20e^{3c_1}x^3+8\sqrt{e^{3c_1}\left(x^3+e^{3c_1}\right)^3}+8e^{6c_1}}}-9\left(1+i\sqrt{3}\right)\sqrt[3]{-x^6-20e^{3c_1}x^3+8\sqrt{e^{3c_1}\left(x^3+e^{3c_1}\right)^3}+8e^{6c_1}}\right)$$

15 Chapter IV, differential equations of the first order and higher degree than the first. Article 26. Equations solvable for x. Page 55

15.1 problem Ex 1

Internal problem ID [11212]

Book: An elementary treatise on differential equations by Abraham Cohen. DC heath publishers. 1906
Section: Chapter IV, differential equations of the first order and higher degree than the first. Article 26. Equations solvable for x. Page 55
Problem number: Ex 1.
ODE order: 1.
ODE degree: 3.

CAS Maple gives this as type `[[_homogeneous, 'class A'], _dAlembert]`

$$\boxed{y' y \left(2 {y'}^2 + 3\right) = -x}$$

✓ Solution by Maple

Time used: 0.14 (sec). Leaf size: 776

```
dsolve(x+diff(y(x),x)*y(x)*(2*diff(y(x),x)^2+3)=0,y(x), singsol=all)
```

$$y(x) = -\frac{i\sqrt{2}\,x}{2}$$

$$y(x) = \frac{i\sqrt{2}\,x}{2}$$

$$y(x) = \text{RootOf}\left(-\ln(x) + \int^{\_Z} -\frac{-2\left(\frac{\left(\_a^2-\sqrt{2\_a^2+1}+1\right)\_a}{(2\_a^2+1)^{\frac{3}{2}}}\right)^{\frac{2}{3}}\_a^2 + 2\left(\frac{\left(\_a^2-\sqrt{2\_a^2+1}+1\right)\_a}{(2\_a^2+1)^{\frac{3}{2}}}\right)^{\frac{1}{3}}\_a^3 - \left(\frac{\left(\_a^2-\sqrt{2\_a^2+1}+1\right)\_a}{(2\_a^2+1)^{\frac{3}{2}}}\right)^{\frac{2}{3}} + \_}{\left(\frac{\left(\_a^2-\sqrt{2\_a^2+1}+1\right)\_a}{(2\_a^2+1)^{\frac{3}{2}}}\right)^{\frac{1}{3}}(2\_a^4+3\_a^2+1)} + c_1\right) x$$

$$y(x) = \text{RootOf}\left(-2\ln(x) + \int^{\_Z} \frac{2i\left(\frac{\left(\_a^2-\sqrt{2\_a^2+1}+1\right)\_a}{(2\_a^2+1)^{\frac{3}{2}}}\right)^{\frac{2}{3}}\sqrt{3}\_a^2 + i\left(\frac{\left(\_a^2-\sqrt{2\_a^2+1}+1\right)\_a}{(2\_a^2+1)^{\frac{3}{2}}}\right)^{\frac{2}{3}}\sqrt{3} - 2\left(\frac{\left(\_a^2-\sqrt{2\_a^2+1}+1\right)\_a}{(2\_a^2+1)^{\frac{3}{2}}}\right.}{\left(\frac{\left(\_a^2-\sqrt{}\right.}{(2\_}} + 2c_1\right) x$$

$$y(x) = \text{RootOf}\left(-2\ln(x) - \left(\int^{\_Z} \frac{2i\left(\frac{\left(\_a^2-\sqrt{2\_a^2+1}+1\right)\_a}{(2\_a^2+1)^{\frac{3}{2}}}\right)^{\frac{2}{3}}\frac{121}{\sqrt{3}}\_a^2 + i\left(\frac{\left(\_a^2-\sqrt{2\_a^2+1}+1\right)\_a}{(2\_a^2+1)^{\frac{3}{2}}}\right)^{\frac{2}{3}}\sqrt{3} + 2\left(\frac{\left(\_a^2-\sqrt{2\_a^2+1}+1\right)\_}{(2\_a^2+1)^{\frac{3}{2}}}\right.}{}\right.$$

✗ Solution by Mathematica
Time used: 0.0 (sec). Leaf size: 0

```
DSolve[x+y'[x]*y[x]*(2*(y'[x])^2+3)==0,y[x],x,IncludeSingularSolutions -> True]
```

Timed out

15.2 problem Ex 2

Internal problem ID [11213]

Book: An elementary treatise on differential equations by Abraham Cohen. DC heath publishers. 1906
Section: Chapter IV, differential equations of the first order and higher degree than the first. Article 26. Equations solvable for x. Page 55
Problem number: Ex 2.
ODE order: 1.
ODE degree: 2.

CAS Maple gives this as type `[[_homogeneous, 'class A'], _rational, _dAlembert]`

$$\boxed{a^2 y y'^2 - 2y'x + y = 0}$$

✓ Solution by Maple
Time used: 0.141 (sec). Leaf size: 51

```
dsolve(a^2*y(x)*diff(y(x),x)^2-2*x*diff(y(x),x)+y(x)=0,y(x), singsol=all)
```

$$y(x) = -\frac{x}{a}$$
$$y(x) = \frac{x}{a}$$
$$y(x) = 0$$
$$y(x) = \mathrm{e}^{\mathrm{RootOf}\left(\mathrm{e}^{2\_Z} \sinh(-\_Z + c_1 - \ln(x))^2 a^2 + 1\right)} x$$

✓ Solution by Mathematica

Time used: 30.099 (sec). Leaf size: 244

```
DSolve[a^2*y[x]*(y'[x])^2-2*x*y'[x]+y[x]==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to -\frac{\left(\cosh\left(\frac{a^2c_1}{2}\right)+\sinh\left(\frac{a^2c_1}{2}\right)\right)\sqrt{\cosh\left(a^2c_1\right)+\sinh\left(a^2c_1\right)-8ix}}{4a}$$

$$y(x) \to \frac{\left(\cosh\left(\frac{a^2c_1}{2}\right)+\sinh\left(\frac{a^2c_1}{2}\right)\right)\sqrt{\cosh\left(a^2c_1\right)+\sinh\left(a^2c_1\right)-8ix}}{4a}$$

$$y(x) \to -\frac{\left(\cosh\left(\frac{a^2c_1}{2}\right)+\sinh\left(\frac{a^2c_1}{2}\right)\right)\sqrt{\cosh\left(a^2c_1\right)+\sinh\left(a^2c_1\right)+8ix}}{4a}$$

$$y(x) \to \frac{\left(\cosh\left(\frac{a^2c_1}{2}\right)+\sinh\left(\frac{a^2c_1}{2}\right)\right)\sqrt{\cosh\left(a^2c_1\right)+\sinh\left(a^2c_1\right)+8ix}}{4a}$$

$$y(x) \to -\frac{x}{a}$$

$$y(x) \to \frac{x}{a}$$

15.3 problem Ex 3

Internal problem ID [11214]

Book: An elementary treatise on differential equations by Abraham Cohen. DC heath publishers. 1906
Section: Chapter IV, differential equations of the first order and higher degree than the first. Article 26. Equations solvable for x. Page 55
Problem number: Ex 3.
ODE order: 1.
ODE degree: 2.

CAS Maple gives this as type `[[_homogeneous, 'class A'], _rational, _dAlembert]`

$$\boxed{xy'^2 - 2yy' = x}$$

✓ Solution by Maple
Time used: 0.015 (sec). Leaf size: 32

```
dsolve(x*diff(y(x),x)^2-2*y(x)*diff(y(x),x)-x=0,y(x), singsol=all)
```

$$\begin{aligned} y(x) &= -ix \\ y(x) &= ix \\ y(x) &= \frac{-c_1^2 + x^2}{2c_1} \end{aligned}$$

✓ Solution by Mathematica
Time used: 0.213 (sec). Leaf size: 71

```
DSolve[x*(y'[x])^2-2*y[x]*y'[x]-x==0,y[x],x,IncludeSingularSolutions -> True]
```

$$\begin{aligned} y(x) &\to \frac{1}{2}e^{-c_1}\left(-x^2 + e^{2c_1}\right) \\ y(x) &\to \frac{1}{2}e^{-c_1}\left(-1 + e^{2c_1}x^2\right) \\ y(x) &\to -ix \\ y(x) &\to ix \end{aligned}$$

15.4 problem Ex 4

Internal problem ID [11215]

Book: An elementary treatise on differential equations by Abraham Cohen. DC heath publishers. 1906
Section: Chapter IV, differential equations of the first order and higher degree than the first. Article 26. Equations solvable for x. Page 55
Problem number: Ex 4.
ODE order: 1.
ODE degree: 3.

CAS Maple gives this as type `[[_1st_order, _with_linear_symmetries]]`

$$\boxed{y'^3 - 4xyy' + 8y^2 = 0}$$

✓ Solution by Maple
Time used: 0.11 (sec). Leaf size: 29

```
dsolve(diff(y(x),x)^3-4*x*y(x)*diff(y(x),x)+8*y(x)^2=0,y(x), singsol=all)
```

$$y(x) = \frac{4x^3}{27}$$
$$y(x) = 0$$
$$y(x) = \frac{(4c_1x - 1)^2}{64c_1^3}$$

✗ Solution by Mathematica
Time used: 0.0 (sec). Leaf size: 0

```
DSolve[(y'[x])^3-4*x*y[x]*y'[x]+8*y[x]^2==0,y[x],x,IncludeSingularSolutions -> True]
```

Timed out

16 Chapter IV, differential equations of the first order and higher degree than the first. Article 27. Clairaut equation. Page 56

16.1 problem Ex 1

Internal problem ID [11216]

Book: An elementary treatise on differential equations by Abraham Cohen. DC heath publishers. 1906
Section: Chapter IV, differential equations of the first order and higher degree than the first. Article 27. Clairaut equation. Page 56
Problem number: Ex 1.
ODE order: 1.
ODE degree: 2.

CAS Maple gives this as type `[[_1st_order, _with_linear_symmetries], _rational, _Clairaut]`

$$\boxed{(y'x-y)^2-y'^2=1}$$

✓ Solution by Maple
Time used: 0.11 (sec). Leaf size: 57

```
dsolve((diff(y(x),x)*x-y(x))^2=diff(y(x),x)^2+1,y(x), singsol=all)
```

$$y(x)=\sqrt{-x^2+1}$$
$$y(x)=-\sqrt{-x^2+1}$$
$$y(x)=c_1x-\sqrt{c_1^2+1}$$
$$y(x)=c_1x+\sqrt{c_1^2+1}$$

✓ Solution by Mathematica
Time used: 0.192 (sec). Leaf size: 73

```
DSolve[(y'[x]*x-y[x])^2==(y'[x])^2+1,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x)\to c_1x-\sqrt{1+{c_1}^2}$$
$$y(x)\to c_1x+\sqrt{1+{c_1}^2}$$
$$y(x)\to -\sqrt{1-x^2}$$
$$y(x)\to \sqrt{1-x^2}$$

16.2 problem Ex 2

Internal problem ID [11217]

Book: An elementary treatise on differential equations by Abraham Cohen. DC heath publishers. 1906
Section: Chapter IV, differential equations of the first order and higher degree than the first. Article 27. Clairaut equation. Page 56
Problem number: Ex 2.
ODE order: 1.
ODE degree: 2.

CAS Maple gives this as type `[[_1st_order, '_with_symmetry_[F(x),G(y)]']]`

$$\boxed{4\,\mathrm{e}^{2y}y'^2+2y'x=1}$$

✓ Solution by Maple
Time used: 0.047 (sec). Leaf size: 23

```
dsolve(4*exp(2*y(x))*diff(y(x),x)^2+2*x*diff(y(x),x)-1=0,y(x), singsol=all)
```

$$y(x)=\frac{\ln(2)}{2}-\frac{\ln\left(\frac{1}{2\,\mathrm{e}^{2c_1}+x}\right)}{2}+c_1$$

✓ Solution by Mathematica
Time used: 12.616 (sec). Leaf size: 119

```
DSolve[4*Exp[2*y[x]]*(y'[x])^2+2*x*y'[x]-1==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x)\to\log\left(-e^{\frac{c_1}{2}}\sqrt{-x+e^{c_1}}\right)$$
$$y(x)\to\log\left(e^{\frac{c_1}{2}}\sqrt{-x+e^{c_1}}\right)$$
$$y(x)\to\log\left(-e^{\frac{c_1}{2}}\sqrt{x+e^{c_1}}\right)$$
$$y(x)\to\log\left(e^{\frac{c_1}{2}}\sqrt{x+e^{c_1}}\right)$$
$$y(x)\to\frac{1}{2}\log\left(-\frac{x^2}{4}\right)$$

16.3 problem Ex 3

Internal problem ID [11218]

Book: An elementary treatise on differential equations by Abraham Cohen. DC heath publishers. 1906
Section: Chapter IV, differential equations of the first order and higher degree than the first. Article 27. Clairaut equation. Page 56
Problem number: Ex 3.
ODE order: 1.
ODE degree: 2.

CAS Maple gives this as type `[[_homogeneous, 'class C'], _dAlembert]`

$$\boxed{4\,\mathrm{e}^{2y}y'^2+2\,\mathrm{e}^{2x}y'=\mathrm{e}^{2x}}$$

✓ Solution by Maple
Time used: 1.171 (sec). Leaf size: 87

```
dsolve(4*exp(2*y(x))*diff(y(x),x)^2+2*exp(2*x)*diff(y(x),x)-exp(2*x)=0,y(x), singsol=all)
```

$$y(x)=\operatorname{arctanh}\left(\operatorname{RootOf}\left(-1+\left(\mathrm{e}^4+4\,\mathrm{e}^{\operatorname{RootOf}\left(-4\,\mathrm{e}^{-Z}\sinh\left(-\frac{Z}{2}+2+c_1-x\right)^2+\mathrm{e}^4\right)}\right)\_Z^2\right)\mathrm{e}^2\right)+c_1$$

$$y(x)=-\operatorname{arctanh}\left(\operatorname{RootOf}\left(-1+\left(\mathrm{e}^4+4\,\mathrm{e}^{\operatorname{RootOf}\left(-4\,\mathrm{e}^{-Z}\sinh\left(-\frac{Z}{2}+2+c_1-x\right)^2+\mathrm{e}^4\right)}\right)\_Z^2\right)\mathrm{e}^2\right)+c_1$$

✓ Solution by Mathematica

Time used: 2.772 (sec). Leaf size: 332

```
DSolve[4*Exp[2*y[x]]*(y'[x])^2+2*Exp[2*x]*y'[x]-Exp[2*x]==0,y[x],x,IncludeSingularSolutions
```

$$\text{Solve}\left[-\frac{2e^{-x}\sqrt{4e^{2(y(x)+x)}+e^{4x}}\text{arctanh}\left(\frac{-\sqrt{4e^{2y(x)}+e^{2x}}+e^{x}+1}{\sqrt{4e^{2y(x)}+e^{2x}}-e^{x}+1}\right)}{\sqrt{4e^{2y(x)}+e^{2x}}}\right.$$
$$\left.-\frac{e^{-x}\sqrt{4e^{2(y(x)+x)}+e^{4x}}y(x)}{\sqrt{4e^{2y(x)}+e^{2x}}}+y(x)=c_1,y(x)\right]$$

$$\text{Solve}\left[\frac{2e^{-x}\sqrt{4e^{2(y(x)+x)}+e^{4x}}\text{arctanh}\left(\frac{-\sqrt{4e^{2y(x)}+e^{2x}}+e^{x}+1}{\sqrt{4e^{2y(x)}+e^{2x}}-e^{x}+1}\right)}{\sqrt{4e^{2y(x)}+e^{2x}}}\right.$$
$$\left.+\frac{e^{-x}\sqrt{4e^{2(y(x)+x)}+e^{4x}}y(x)}{\sqrt{4e^{2y(x)}+e^{2x}}}+y(x)=c_1,y(x)\right]$$

$$y(x)\to\frac{1}{2}\left(\log\left(-\frac{e^{4x}}{4}\right)-2x\right)$$

16.4 problem Ex 4

Internal problem ID [11219]

Book: An elementary treatise on differential equations by Abraham Cohen. DC heath publishers. 1906
Section: Chapter IV, differential equations of the first order and higher degree than the first. Article 27. Clairaut equation. Page 56
Problem number: Ex 4.
ODE order: 1.
ODE degree: 3.

CAS Maple gives this as type `['y=_G(x,y')']`

$$\boxed{e^{2y} y'^3 + \left(e^{2x} + e^{3x}\right) y' = e^{3x}}$$

✓ Solution by Maple
Time used: 1.188 (sec). Leaf size: 24

```
dsolve(exp(2*y(x))*diff(y(x),x)^3+(exp(2*x)+exp(3*x))*diff(y(x),x)-exp(3*x)=0,y(x), singsol=
```

$$y(x) = \frac{\ln\left(-(c_1+1)\left(c_1 e^{-x} - 1\right)^2\right)}{2} + x$$

✗ Solution by Mathematica
Time used: 0.0 (sec). Leaf size: 0

```
DSolve[Exp[2*y[x]]*(y'[x])^3+(Exp[2*x]+Exp[3*x])*y'[x]-Exp[3*x]==0,y[x],x,IncludeSingularSol
```

Timed out

16.5 problem Ex 5

Internal problem ID [11220]

Book: An elementary treatise on differential equations by Abraham Cohen. DC heath publishers. 1906
Section: Chapter IV, differential equations of the first order and higher degree than the first. Article 27. Clairaut equation. Page 56
Problem number: Ex 5.
ODE order: 1.
ODE degree: 2.

CAS Maple gives this as type `[[_homogeneous, 'class G'], _rational]`

$$\boxed{xy^2y'^2 - y^3y' = -x}$$

✓ Solution by Maple
Time used: 0.188 (sec). Leaf size: 140

```
dsolve(x*y(x)^2*diff(y(x),x)^2-y(x)^3*diff(y(x),x)+x=0,y(x), singsol=all)
```

$$y(x) = \sqrt{2}\sqrt{-x}$$
$$y(x) = -\sqrt{2}\sqrt{-x}$$
$$y(x) = \sqrt{2}\sqrt{x}$$
$$y(x) = -\sqrt{2}\sqrt{x}$$
$$y(x) = \frac{\mathrm{e}^{\frac{c_1}{2}+\frac{\mathrm{RootOf}\left(16x\,\mathrm{e}^{2\_Z+2c_1}+\mathrm{e}^{2\_Z}x^3-4\,\mathrm{e}^{3\_Z+2c_1}\right)}{2}}}{\sqrt{x}}$$
$$y(x) = \sqrt{x}\,\mathrm{e}^{-\frac{c_1}{2}+\frac{\mathrm{RootOf}\left(x^2\left(16x^2\mathrm{e}^{2\_Z-2c_1}+\mathrm{e}^{2\_Z}-4\,\mathrm{e}^{3\_Z-2c_1}x\right)\right)}{2}}$$

✓ Solution by Mathematica

Time used: 6.367 (sec). Leaf size: 187

```
DSolve[x*y[x]^2*(y'[x])^2-y[x]^3*y'[x]+x==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to -\sqrt{-2e^{-c_1}x^2 - \frac{e^{c_1}}{2}}$$
$$y(x) \to \sqrt{-2e^{-c_1}x^2 - \frac{e^{c_1}}{2}}$$
$$y(x) \to -\frac{\sqrt{4e^{-c_1}x^2 + e^{c_1}}}{\sqrt{2}}$$
$$y(x) \to \frac{\sqrt{4e^{-c_1}x^2 + e^{c_1}}}{\sqrt{2}}$$
$$y(x) \to -\sqrt{2}\sqrt{x}$$
$$y(x) \to -i\sqrt{2}\sqrt{x}$$
$$y(x) \to i\sqrt{2}\sqrt{x}$$
$$y(x) \to \sqrt{2}\sqrt{x}$$

16.6 problem Ex 6

Internal problem ID [11221]

Book: An elementary treatise on differential equations by Abraham Cohen. DC heath publishers. 1906
Section: Chapter IV, differential equations of the first order and higher degree than the first. Article 27. Clairaut equation. Page 56
Problem number: Ex 6.
ODE order: 1.
ODE degree: 2.

CAS Maple gives this as type `[[_homogeneous, 'class A'], _dAlembert]`

$$\boxed{\left(x^2+y^2\right)\left(1+y'\right)^2-2(y+x)\left(1+y'\right)\left(x+yy'\right)+\left(x+yy'\right)^2=0}$$

✓ Solution by Maple
Time used: 0.156 (sec). Leaf size: 103

```
dsolve((x^2+y(x)^2)*(1+diff(y(x),x))^2-2*(x+y(x))*(1+diff(y(x),x))*(x+y(x)*diff(y(x),x))+(x+
```

$$y(x)=0$$

$$y(x)=\operatorname{RootOf}\left(-2\ln\left(x\right)-\left(\int^{\_Z}\frac{2\_a^2+\sqrt{2}\sqrt{\_a\left(\_a-1\right)^2}}{\_a\left(\_a^2+1\right)}d\_a\right)+2c_1\right)x$$

$$y(x)=\operatorname{RootOf}\left(-2\ln\left(x\right)+\int^{\_Z}\frac{\sqrt{2}\sqrt{\_a\left(\_a-1\right)^2}-2\_a^2}{\_a\left(\_a^2+1\right)}d\_a+2c_1\right)x$$

✓ Solution by Mathematica

Time used: 7.379 (sec). Leaf size: 167

```
DSolve[(x^2+y[x]^2)*(1+y'[x])^2-2*(x+y[x])*(1+y'[x])*(x+y[x]*y'[x])+(x+y[x]*y'[x])^2==0,y[x]
```

$$y(x) \to -\sqrt{-x\left(x+2e^{\frac{c_1}{2}}\right)}-e^{\frac{c_1}{2}}$$

$$y(x) \to \sqrt{-x\left(x+2e^{\frac{c_1}{2}}\right)}-e^{\frac{c_1}{2}}$$

$$y(x) \to e^{\frac{c_1}{2}}-\sqrt{x\left(-x+2e^{\frac{c_1}{2}}\right)}$$

$$y(x) \to \sqrt{x\left(-x+2e^{\frac{c_1}{2}}\right)}+e^{\frac{c_1}{2}}$$

$$y(x) \to -\sqrt{-x^2}$$

$$y(x) \to \sqrt{-x^2}$$

16.7 problem Ex 7

Internal problem ID [11222]

Book: An elementary treatise on differential equations by Abraham Cohen. DC heath publishers. 1906
Section: Chapter IV, differential equations of the first order and higher degree than the first. Article 27. Clairaut equation. Page 56
Problem number: Ex 7.
ODE order: 1.
ODE degree: 3.

CAS Maple gives this as type `[[_1st_order, _with_linear_symmetries]]`

$$\boxed{y - 2y'x - y^2y'^3 = 0}$$

✓ Solution by Maple
Time used: 0.141 (sec). Leaf size: 97

```
dsolve(y(x)=2*diff(y(x),x)*x+y(x)^2*diff(y(x),x)^3,y(x), singsol=all)
```

$$y(x) = -\frac{2(-x^3)^{\frac{1}{4}}\,6^{\frac{1}{4}}}{3}$$
$$y(x) = \frac{2(-x^3)^{\frac{1}{4}}\,6^{\frac{1}{4}}}{3}$$
$$y(x) = -\frac{2i(-x^3)^{\frac{1}{4}}\,6^{\frac{1}{4}}}{3}$$
$$y(x) = \frac{2i(-x^3)^{\frac{1}{4}}\,6^{\frac{1}{4}}}{3}$$
$$y(x) = 0$$
$$y(x) = \sqrt{c_1\,(c_1^2 + 2x)}$$
$$y(x) = -\sqrt{c_1\,(c_1^2 + 2x)}$$

✓ Solution by Mathematica

Time used: 0.183 (sec). Leaf size: 119

```
DSolve[y[x]==2*y'[x]*x+y[x]^2*(y'[x])^3,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to -\sqrt{2c_1x + {c_1}^3}$$
$$y(x) \to \sqrt{2c_1x + {c_1}^3}$$
$$y(x) \to (-1-i)\left(\frac{2}{3}\right)^{3/4} x^{3/4}$$
$$y(x) \to (1-i)\left(\frac{2}{3}\right)^{3/4} x^{3/4}$$
$$y(x) \to (-1+i)\left(\frac{2}{3}\right)^{3/4} x^{3/4}$$
$$y(x) \to (1+i)\left(\frac{2}{3}\right)^{3/4} x^{3/4}$$

16.8 problem Ex 8

Internal problem ID [11223]

Book: An elementary treatise on differential equations by Abraham Cohen. DC heath publishers. 1906
Section: Chapter IV, differential equations of the first order and higher degree than the first. Article 27. Clairaut equation. Page 56
Problem number: Ex 8.
ODE order: 1.
ODE degree: 2.

CAS Maple gives this as type `[[_homogeneous, 'class A'], _rational, _dAlembert]`

$$\boxed{a^2 y y'^2 - 2y'x + y = 0}$$

✓ Solution by Maple
Time used: 0.125 (sec). Leaf size: 51

```
dsolve(a^2*y(x)*diff(y(x),x)^2-2*x*diff(y(x),x)+y(x)=0,y(x), singsol=all)
```

$$y(x) = -\frac{x}{a}$$
$$y(x) = \frac{x}{a}$$
$$y(x) = 0$$
$$y(x) = \mathrm{e}^{\mathrm{RootOf}\left(\mathrm{e}^{2\_Z} \sinh(-\_Z + c_1 - \ln(x))^2 a^2 + 1\right)} x$$

✓ Solution by Mathematica

Time used: 31.661 (sec). Leaf size: 244

```
DSolve[a^2*y[x]*(y'[x])^2-2*x*y'[x]+y[x]==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to -\frac{\left(\cosh\left(\frac{a^2c_1}{2}\right)+\sinh\left(\frac{a^2c_1}{2}\right)\right)\sqrt{\cosh\left(a^2c_1\right)+\sinh\left(a^2c_1\right)-8ix}}{4a}$$

$$y(x) \to \frac{\left(\cosh\left(\frac{a^2c_1}{2}\right)+\sinh\left(\frac{a^2c_1}{2}\right)\right)\sqrt{\cosh\left(a^2c_1\right)+\sinh\left(a^2c_1\right)-8ix}}{4a}$$

$$y(x) \to -\frac{\left(\cosh\left(\frac{a^2c_1}{2}\right)+\sinh\left(\frac{a^2c_1}{2}\right)\right)\sqrt{\cosh\left(a^2c_1\right)+\sinh\left(a^2c_1\right)+8ix}}{4a}$$

$$y(x) \to \frac{\left(\cosh\left(\frac{a^2c_1}{2}\right)+\sinh\left(\frac{a^2c_1}{2}\right)\right)\sqrt{\cosh\left(a^2c_1\right)+\sinh\left(a^2c_1\right)+8ix}}{4a}$$

$$y(x) \to -\frac{x}{a}$$

$$y(x) \to \frac{x}{a}$$

16.9 problem Ex 9

Internal problem ID [11224]

Book: An elementary treatise on differential equations by Abraham Cohen. DC heath publishers. 1906
Section: Chapter IV, differential equations of the first order and higher degree than the first. Article 27. Clairaut equation. Page 56
Problem number: Ex 9.
ODE order: 1.
ODE degree: 2.

CAS Maple gives this as type `['y=_G(x,y')']`

$$\boxed{(x-y'-y)^2-x^2\left(2yx-x^2y'\right)=0}$$

✗ Solution by Maple

```
dsolve((x-diff(y(x),x)-y(x))^2=x^2*(2*x*y(x)-x^2*diff(y(x),x)),y(x), singsol=all)
```

No solution found

✗ Solution by Mathematica
Time used: 0.0 (sec). Leaf size: 0

```
DSolve[(x-y'[x]-y[x])^2==x^2*(2*x*y[x]-x^2*y'[x]),y[x],x,IncludeSingularSolutions -> True]
```

Not solved

17 Chapter IV, differential equations of the first order and higher degree than the first. Article 28. Summary. Page 59

17.1 problem Ex 1

Internal problem ID [11225]

Book: An elementary treatise on differential equations by Abraham Cohen. DC heath publishers. 1906
Section: Chapter IV, differential equations of the first order and higher degree than the first. Article 28. Summary. Page 59
Problem number: Ex 1.
ODE order: 1.
ODE degree: 2.

CAS Maple gives this as type `[_quadrature]`

$$\boxed{y^2\left(y'^2+1\right)=a^2}$$

✓ Solution by Maple
Time used: 0.141 (sec). Leaf size: 54

```
dsolve(y(x)^2*(1+diff(y(x),x)^2)=a^2,y(x), singsol=all)
```

$$\begin{aligned}y(x) &= -a\\ y(x) &= a\\ y(x) &= \sqrt{a^2-c_1^2+2c_1x-x^2}\\ y(x) &= -\sqrt{(a+x-c_1)\,(c_1+a-x)}\end{aligned}$$

✓ Solution by Mathematica
Time used: 0.344 (sec). Leaf size: 101

```
DSolve[y[x]^2*(1+(y'[x])^2)==a^2,y[x],x,IncludeSingularSolutions -> True]
```

$$\begin{aligned}y(x) &\to -\sqrt{a^2-(x+c_1)^2}\\ y(x) &\to \sqrt{a^2-(x+c_1)^2}\\ y(x) &\to -\sqrt{a^2-(x-c_1)^2}\\ y(x) &\to \sqrt{a^2-(x-c_1)^2}\\ y(x) &\to -a\\ y(x) &\to a\end{aligned}$$

17.2 problem Ex 2

Internal problem ID [11226]

Book: An elementary treatise on differential equations by Abraham Cohen. DC heath publishers. 1906
Section: Chapter IV, differential equations of the first order and higher degree than the first. Article 28. Summary. Page 59
Problem number: Ex 2.
ODE order: 1.
ODE degree: 2.

CAS Maple gives this as type `[[_1st_order, _with_linear_symmetries], _Clairaut]`

$$\boxed{yy' - (x-b)\,y'^2 = a}$$

✓ Solution by Maple
Time used: 0.125 (sec). Leaf size: 49

```
dsolve(y(x)*diff(y(x),x)=(x-b)*diff(y(x),x)^2+a,y(x), singsol=all)
```

$$y(x) = -2\sqrt{-a\,(b-x)}$$
$$y(x) = 2\sqrt{-a\,(b-x)}$$
$$y(x) = \frac{(-b+x)\,c_1^2 + a}{c_1}$$

✓ Solution by Mathematica
Time used: 0.1 (sec). Leaf size: 59

```
DSolve[y[x]*y'[x]==(x-b)*(y'[x])^2+a,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to \frac{a}{c_1} + c_1(x-b)$$
$$y(x) \to \text{Indeterminate}$$
$$y(x) \to -2\sqrt{a(x-b)}$$
$$y(x) \to 2\sqrt{a(x-b)}$$

17.3 problem Ex 3

Internal problem ID [11227]

Book: An elementary treatise on differential equations by Abraham Cohen. DC heath publishers. 1906
Section: Chapter IV, differential equations of the first order and higher degree than the first. Article 28. Summary. Page 59
Problem number: Ex 3.
ODE order: 1.
ODE degree: 2.

CAS Maple gives this as type `[[_homogeneous, 'class G'], _rational]`

$$\boxed{x^3 y'^2 + x^2 y y' = -1}$$

✓ Solution by Maple
Time used: 0.281 (sec). Leaf size: 53

```
dsolve(x^3*diff(y(x),x)^2+x^2*y(x)*diff(y(x),x)+1=0,y(x), singsol=all)
```

$$y(x) = -\frac{2}{\sqrt{x}}$$
$$y(x) = \frac{2}{\sqrt{x}}$$
$$y(x) = \frac{c_1^2 x + 4}{2 c_1 x}$$
$$y(x) = \frac{c_1^2 + 4x}{2 c_1 x}$$

✓ Solution by Mathematica

Time used: 0.934 (sec). Leaf size: 77

```
DSolve[x^3*(y'[x])^2+x^2*y[x]*y'[x]+1==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to -\frac{e^{-\frac{c_1}{2}}\left(x+16e^{c_1}\right)}{4x}$$
$$y(x) \to \frac{e^{-\frac{c_1}{2}}\left(x+16e^{c_1}\right)}{4x}$$
$$y(x) \to -\frac{2}{\sqrt{x}}$$
$$y(x) \to \frac{2}{\sqrt{x}}$$

17.4 problem Ex 4

Internal problem ID [11228]

Book: An elementary treatise on differential equations by Abraham Cohen. DC heath publishers. 1906
Section: Chapter IV, differential equations of the first order and higher degree than the first. Article 28. Summary. Page 59
Problem number: Ex 4.
ODE order: 1.
ODE degree: 2.

CAS Maple gives this as type `[[_homogeneous, 'class A'], _rational, _dAlembert]`

$$\boxed{3xy'^2 - 6yy' + 2y = -x}$$

✓ Solution by Maple
Time used: 0.062 (sec). Leaf size: 32

```
dsolve(3*x*diff(y(x),x)^2-6*y(x)*diff(y(x),x)+x+2*y(x)=0,y(x), singsol=all)
```

$$\begin{aligned} y(x) &= x \\ y(x) &= -\frac{x}{3} \\ y(x) &= \frac{4c_1^2 + 2c_1 x + x^2}{6c_1} \end{aligned}$$

✓ Solution by Mathematica
Time used: 0.505 (sec). Leaf size: 67

```
DSolve[3*x*(y'[x])^2-6*y[x]*y'[x]+x+2*y[x]==0,y[x],x,IncludeSingularSolutions -> True]
```

$$\begin{aligned} y(x) &\to -\frac{1}{3}x\left(-1 + 2\cosh\left(-\log(x) + \sqrt{3}c_1\right)\right) \\ y(x) &\to -\frac{1}{3}x\left(-1 + 2\cosh\left(\log(x) + \sqrt{3}c_1\right)\right) \\ y(x) &\to -\frac{x}{3} \\ y(x) &\to x \end{aligned}$$

17.5 problem Ex 5

Internal problem ID [11229]

Book: An elementary treatise on differential equations by Abraham Cohen. DC heath publishers. 1906
Section: Chapter IV, differential equations of the first order and higher degree than the first. Article 28. Summary. Page 59
Problem number: Ex 5.
ODE order: 1.
ODE degree: 2.

CAS Maple gives this as type `[[_homogeneous, 'class C'], _rational, _dAlembert]`

$$\boxed{y - y'^2(1+x) = 0}$$

✓ Solution by Maple
Time used: 0.078 (sec). Leaf size: 53

```
dsolve(y(x)=diff(y(x),x)^2*(x+1),y(x), singsol=all)
```

$$y(x) = 0$$
$$y(x) = \frac{\left(x+1+\sqrt{(1+x)(c_1+1)}\right)^2}{1+x}$$
$$y(x) = \frac{\left(-x-1+\sqrt{(1+x)(c_1+1)}\right)^2}{1+x}$$

✓ Solution by Mathematica
Time used: 0.1 (sec). Leaf size: 57

```
DSolve[y[x]==(y'[x])^2*(x+1),y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to x - c_1\sqrt{x+1} + 1 + \frac{{c_1}^2}{4}$$
$$y(x) \to x + c_1\sqrt{x+1} + 1 + \frac{{c_1}^2}{4}$$
$$y(x) \to 0$$

17.6 problem Ex 6

Internal problem ID [11230]

Book: An elementary treatise on differential equations by Abraham Cohen. DC heath publishers. 1906
Section: Chapter IV, differential equations of the first order and higher degree than the first. Article 28. Summary. Page 59
Problem number: Ex 6.
ODE order: 1.
ODE degree: 2.

CAS Maple gives this as type `[_rational]`

$$\boxed{(y'x - y)(x + yy') - a^2 y' = 0}$$

✗ Solution by Maple

```
dsolve((diff(y(x),x)*x-y(x))*(diff(y(x),x)*y(x)+x)=a^2*diff(y(x),x),y(x), singsol=all)
```

No solution found

✓ Solution by Mathematica
Time used: 0.6 (sec). Leaf size: 75

```
DSolve[(y'[x]*x-y[x])*(y'[x]*y[x]+x)==a^2*y'[x],y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to \sqrt{c_1 \left(x^2 - \frac{a^2}{1+c_1}\right)}$$
$$y(x) \to -i(a - x)$$
$$y(x) \to i(a - x)$$
$$y(x) \to -i(a + x)$$
$$y(x) \to i(a + x)$$

17.7 problem Ex 7

Internal problem ID [11231]

Book: An elementary treatise on differential equations by Abraham Cohen. DC heath publishers. 1906
Section: Chapter IV, differential equations of the first order and higher degree than the first. Article 28. Summary. Page 59
Problem number: Ex 7.
ODE order: 1.
ODE degree: 2.

CAS Maple gives this as type `[_separable]`

$$\boxed{y'^2 + 2y'y\cot(x) - y^2 = 0}$$

✓ Solution by Maple
Time used: 0.172 (sec). Leaf size: 39

```
dsolve(diff(y(x),x)^2+2*diff(y(x),x)*y(x)*cot(x)=y(x)^2,y(x), singsol=all)
```

$$y(x) = 0$$
$$y(x) = \frac{\operatorname{csgn}(\sin(x))\, c_1}{\cos(x) + \operatorname{csgn}(\sec(x))}$$
$$y(x) = \csc(x)^2 (\cos(x) + \operatorname{csgn}(\sec(x))) \operatorname{csgn}(\sin(x))\, c_1$$

✓ Solution by Mathematica
Time used: 0.241 (sec). Leaf size: 36

```
DSolve[(y'[x])^2+2*y'[x]*y[x]*Cot[x]==y[x]^2,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to c_1 \csc^2\left(\frac{x}{2}\right)$$
$$y(x) \to c_1 \sec^2\left(\frac{x}{2}\right)$$
$$y(x) \to 0$$

17.8 problem Ex 8

Internal problem ID [11232]

Book: An elementary treatise on differential equations by Abraham Cohen. DC heath publishers. 1906
Section: Chapter IV, differential equations of the first order and higher degree than the first. Article 28. Summary. Page 59
Problem number: Ex 8.
ODE order: 1.
ODE degree: 2.

CAS Maple gives this as type `[[_1st_order, _with_linear_symmetries], _rational, _Clairaut]`

$$\boxed{\left(x^2+1\right) y'^2 - 2xyy' + y^2 = 1}$$

✓ Solution by Maple
Time used: 0.156 (sec). Leaf size: 57

```
dsolve((1+x^2)*diff(y(x),x)^2-2*x*y(x)*diff(y(x),x)+y(x)^2-1=0,y(x), singsol=all)
```

$$\begin{aligned} y(x) &= \sqrt{x^2+1} \\ y(x) &= -\sqrt{x^2+1} \\ y(x) &= c_1 x - \sqrt{-c_1^2+1} \\ y(x) &= c_1 x + \sqrt{-c_1^2+1} \end{aligned}$$

✓ Solution by Mathematica
Time used: 0.168 (sec). Leaf size: 73

```
DSolve[(1+x^2)*(y'[x])^2-2*x*y[x]*y'[x]+y[x]^2-1==0,y[x],x,IncludeSingularSolutions -> True]
```

$$\begin{aligned} y(x) &\to c_1 x - \sqrt{1-{c_1}^2} \\ y(x) &\to c_1 x + \sqrt{1-{c_1}^2} \\ y(x) &\to -\sqrt{x^2+1} \\ y(x) &\to \sqrt{x^2+1} \end{aligned}$$

17.9 problem Ex 9

Internal problem ID [11233]

Book: An elementary treatise on differential equations by Abraham Cohen. DC heath publishers. 1906
Section: Chapter IV, differential equations of the first order and higher degree than the first. Article 28. Summary. Page 59
Problem number: Ex 9.
ODE order: 1.
ODE degree: 2.

CAS Maple gives this as type `[_separable]`

$$\boxed{x^2 y'^2 - 2(yx + 2y')\, y' + y^2 = 0}$$

✓ Solution by Maple
Time used: 0.0 (sec). Leaf size: 17

```
dsolve(x^2*diff(y(x),x)^2-2*(x*y(x)+2*diff(y(x),x))*diff(y(x),x)+y(x)^2=0,y(x), singsol=all)
```

$$y(x) = c_1(x-2)$$
$$y(x) = c_1(x+2)$$

✓ Solution by Mathematica
Time used: 0.074 (sec). Leaf size: 26

```
DSolve[x^2*(y'[x])^2-2*(x*y[x]+2*y'[x])*y'[x]+y[x]^2==0,y[x],x,IncludeSingularSolutions -> T
```

$$y(x) \to c_1(x-2)$$
$$y(x) \to c_1(x+2)$$
$$y(x) \to 0$$

17.10 problem Ex 10

Internal problem ID [11234]

Book: An elementary treatise on differential equations by Abraham Cohen. DC heath publishers. 1906
Section: Chapter IV, differential equations of the first order and higher degree than the first. Article 28. Summary. Page 59
Problem number: Ex 10.
ODE order: 1.
ODE degree: 2.

CAS Maple gives this as type `[[_1st_order, _with_linear_symmetries]]`

$$\boxed{y - y'x - \frac{y{y'}^2}{x^2} = 0}$$

✓ Solution by Maple
Time used: 0.344 (sec). Leaf size: 89

```
dsolve(y(x)=x*diff(y(x),x)+y(x)*diff(y(x),x)^2/x^2,y(x), singsol=all)
```

$$y(x) = -\frac{ix^2}{2}$$
$$y(x) = \frac{ix^2}{2}$$
$$y(x) = 0$$
$$y(x) = -\frac{\sqrt{c_1\left(-4x^2+c_1\right)}}{4}$$
$$y(x) = \frac{\sqrt{c_1\left(-4x^2+c_1\right)}}{4}$$
$$y(x) = -\frac{2\sqrt{c_1x^2+4}}{c_1}$$
$$y(x) = \frac{2\sqrt{c_1x^2+4}}{c_1}$$

✓ Solution by Mathematica

Time used: 0.986 (sec). Leaf size: 244

```
DSolve[y[x]==x*y'[x]+y[x]*(y'[x])^2/x^2,y[x],x,IncludeSingularSolutions -> True]
```

$$\text{Solve}\left[\frac{\sqrt{x^6+4x^2y(x)^2}\log\left(\sqrt{x^4+4y(x)^2}+x^2\right)}{2x\sqrt{x^4+4y(x)^2}}+\frac{1}{2}\left(1-\frac{\sqrt{x^6+4x^2y(x)^2}}{x\sqrt{x^4+4y(x)^2}}\right)\log(y(x))=c_1,y(x)\right]$$

$$\text{Solve}\left[\frac{1}{2}\left(\frac{\sqrt{x^6+4x^2y(x)^2}}{x\sqrt{x^4+4y(x)^2}}+1\right)\log(y(x))-\frac{\sqrt{x^6+4x^2y(x)^2}\log\left(\sqrt{x^4+4y(x)^2}+x^2\right)}{2x\sqrt{x^4+4y(x)^2}}=c_1,y(x)\right]$$

$$y(x)\to -\frac{ix^2}{2}$$

$$y(x)\to \frac{ix^2}{2}$$

17.11 problem Ex 11

Internal problem ID [11235]

Book: An elementary treatise on differential equations by Abraham Cohen. DC heath publishers. 1906
Section: Chapter IV, differential equations of the first order and higher degree than the first. Article 28. Summary. Page 59
Problem number: Ex 11.
ODE order: 1.
ODE degree: 2.

CAS Maple gives this as type `[_rational, [_1st_order, '_with_symmetry_[F(x),G(x)*y+H(x)]']]`

$$\boxed{x^2y'^2 - 2xyy' + y^2 - x^2y^2 = x^4}$$

✓ Solution by Maple
Time used: 0.578 (sec). Leaf size: 58

```
dsolve(x^2*diff(y(x),x)^2-2*x*y(x)*diff(y(x),x)+y(x)^2=x^2*y(x)^2+x^4,y(x), singsol=all)
```

$$\begin{aligned} y(x) &= -ix \\ y(x) &= ix \\ y(x) &= -\frac{x(\mathrm{e}^{x} - c_1^2\mathrm{e}^{-x})}{2c_1} \\ y(x) &= \frac{x(c_1^2\mathrm{e}^{x} - \mathrm{e}^{-x})}{2c_1} \end{aligned}$$

✓ Solution by Mathematica
Time used: 0.366 (sec). Leaf size: 60

```
DSolve[x^2*(y'[x])^2-2*x*y[x]*y'[x]+y[x]^2==x^2*y[x]^2+x^4,y[x],x,IncludeSingularSolutions -
```

$$\begin{aligned} y(x) &\to \frac{1}{2}xe^{-x-c_1}\left(-1 + e^{2(x+c_1)}\right) \\ y(x) &\to \frac{1}{2}\left(xe^{-x+c_1} - xe^{x-c_1}\right) \end{aligned}$$

18 Chapter V, Singular solutions. Article 30. Page 63

18.1 problem Ex 1

Internal problem ID [11236]

Book: An elementary treatise on differential equations by Abraham Cohen. DC heath publishers. 1906
Section: Chapter V, Singular solutions. Article 30. Page 63
Problem number: Ex 1.
ODE order: 1.
ODE degree: 2.

CAS Maple gives this as type `[[_homogeneous, 'class G'], _rational, _Clairaut]`

$$\boxed{y - y'x - \frac{1}{y'} = 0}$$

✓ Solution by Maple
Time used: 0.047 (sec). Leaf size: 27

```
dsolve(y(x)=diff(y(x),x)*x+1/diff(y(x),x),y(x), singsol=all)
```

$$y(x) = -2\sqrt{x}$$
$$y(x) = 2\sqrt{x}$$
$$y(x) = c_1 x + \frac{1}{c_1}$$

✓ Solution by Mathematica
Time used: 0.016 (sec). Leaf size: 41

```
DSolve[y[x]==y'[x]*x+1/y'[x],y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to c_1 x + \frac{1}{c_1}$$
$$y(x) \to \text{Indeterminate}$$
$$y(x) \to -2\sqrt{x}$$
$$y(x) \to 2\sqrt{x}$$

18.2 problem Ex 2

Internal problem ID [11237]

Book: An elementary treatise on differential equations by Abraham Cohen. DC heath publishers. 1906
Section: Chapter V, Singular solutions. Article 30. Page 63
Problem number: Ex 2.
ODE order: 1.
ODE degree: 2.

CAS Maple gives this as type `[[_homogeneous, 'class A'], _rational, _dAlembert]`

$$\boxed{x{y'}^2 - 2yy' = x}$$

✓ Solution by Maple
Time used: 0.062 (sec). Leaf size: 32

```
dsolve(x*diff(y(x),x)^2-2*y(x)*diff(y(x),x)-x=0,y(x), singsol=all)
```

$$\begin{aligned} y(x) &= -ix \\ y(x) &= ix \\ y(x) &= \frac{-c_1^2 + x^2}{2c_1} \end{aligned}$$

✓ Solution by Mathematica
Time used: 0.213 (sec). Leaf size: 71

```
DSolve[x*(y'[x])^2-2*y[x]*y'[x]-x==0,y[x],x,IncludeSingularSolutions -> True]
```

$$\begin{aligned} y(x) &\to \frac{1}{2}e^{-c_1}\left(-x^2 + e^{2c_1}\right) \\ y(x) &\to \frac{1}{2}e^{-c_1}\left(-1 + e^{2c_1}x^2\right) \\ y(x) &\to -ix \\ y(x) &\to ix \end{aligned}$$

19 Chapter V, Singular solutions. Article 32. Page 69

19.1 problem Ex 5

Internal problem ID [11238]

Book: An elementary treatise on differential equations by Abraham Cohen. DC heath publishers. 1906
Section: Chapter V, Singular solutions. Article 32. Page 69
Problem number: Ex 5.
ODE order: 1.
ODE degree: 2.

CAS Maple gives this as type `[[_homogeneous, 'class G'], _Clairaut]`

$$\boxed{x^2 y'^2 - 2(yx-2)\,y' + y^2 = 0}$$

✓ Solution by Maple
Time used: 0.063 (sec). Leaf size: 35

```
dsolve(x^2*diff(y(x),x)^2-2*(x*y(x)-2)*diff(y(x),x)+y(x)^2=0,y(x), singsol=all)
```

$$\begin{aligned} y(x) &= \frac{1}{x} \\ y(x) &= c_1 x - 2\sqrt{-c_1} \\ y(x) &= c_1 x + 2\sqrt{-c_1} \end{aligned}$$

✓ Solution by Mathematica
Time used: 0.416 (sec). Leaf size: 43

```
DSolve[x^2*(y'[x])^2-2*(x*y[x]-2)*y'[x]+y[x]^2==0,y[x],x,IncludeSingularSolutions -> True]
```

$$\begin{aligned} y(x) &\to \frac{4(-x+c_1)}{{c_1}^2} \\ y(x) &\to -\frac{4(x+c_1)}{{c_1}^2} \\ y(x) &\to 0 \\ y(x) &\to \frac{1}{x} \end{aligned}$$

20 Chapter V, Singular solutions. Article 33. Page 73

20.1 problem Ex 1

Internal problem ID [11239]

Book: An elementary treatise on differential equations by Abraham Cohen. DC heath publishers. 1906
Section: Chapter V, Singular solutions. Article 33. Page 73
Problem number: Ex 1.
ODE order: 1.
ODE degree: 2.

CAS Maple gives this as type `[_quadrature]`

$$\boxed{x^2 y'^2 = (x-1)^2}$$

✓ Solution by Maple
Time used: 0.0 (sec). Leaf size: 21

```
dsolve(x^2*diff(y(x),x)^2-(x-1)^2=0,y(x), singsol=all)
```

$$y(x) = x - \ln(x) + c_1$$
$$y(x) = -x + \ln(x) + c_1$$

✓ Solution by Mathematica
Time used: 0.006 (sec). Leaf size: 25

```
DSolve[x^2*(y'[x])^2-(x-1)^2==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to x - \log(x) + c_1$$
$$y(x) \to -x + \log(x) + c_1$$

20.2 problem Ex 2

Internal problem ID [11240]

Book: An elementary treatise on differential equations by Abraham Cohen. DC heath publishers. 1906
Section: Chapter V, Singular solutions. Article 33. Page 73
Problem number: Ex 2.
ODE order: 1.
ODE degree: 3.

CAS Maple gives this as type `[[_homogeneous, 'class C'], _dAlembert]`

$$\boxed{8(1+y')^3 - 27(x+y)(1-y')^3 = 0}$$

✓ Solution by Maple
Time used: 0.344 (sec). Leaf size: 140

```
dsolve(8*(1+diff(y(x),x))^3=27*(x+y(x))*(1-diff(y(x),x))^3,y(x), singsol=all)
```

$$y(x) = -x$$

$$\frac{x}{2} - \frac{4\ln(27y(x)+27x+8)}{27} + \frac{4\ln\left(2+3(x+y(x))^{\frac{1}{3}}\right)}{27} + \frac{4\ln\left(9(x+y(x))^{\frac{2}{3}} - 6(x+y(x))^{\frac{1}{3}} + 4\right)}{27} - \frac{y(x)}{2} - \frac{(x+y(x))^{\frac{2}{3}}}{2} - c_1 = 0$$

$$\frac{x}{2} - \frac{y(x)}{2} - \frac{i\sqrt{3}\,(x+y(x))^{\frac{2}{3}}}{4} + \frac{(x+y(x))^{\frac{2}{3}}}{4} - c_1 = 0$$

$$\frac{x}{2} - \frac{y(x)}{2} + \frac{i\sqrt{3}\,(x+y(x))^{\frac{2}{3}}}{4} + \frac{(x+y(x))^{\frac{2}{3}}}{4} - c_1 = 0$$

✗ Solution by Mathematica
Time used: 0.0 (sec). Leaf size: 0

```
DSolve[8*(1+y'[x])^3==27*(x+y[x])*(1-y'[x])^3,y[x],x,IncludeSingularSolutions -> True]
```

Timed out

20.3 problem Ex 3

Internal problem ID [11241]

Book: An elementary treatise on differential equations by Abraham Cohen. DC heath publishers. 1906
Section: Chapter V, Singular solutions. Article 33. Page 73
Problem number: Ex 3.
ODE order: 1.
ODE degree: 2.

CAS Maple gives this as type `[_quadrature]`

$$\boxed{4{y'}^2 = 9x}$$

✓ Solution by Maple
Time used: 0.063 (sec). Leaf size: 19

```
dsolve(4*diff(y(x),x)^2=9*x,y(x), singsol=all)
```

$$y(x) = -x^{\frac{3}{2}} + c_1$$
$$y(x) = x^{\frac{3}{2}} + c_1$$

✓ Solution by Mathematica
Time used: 0.006 (sec). Leaf size: 27

```
DSolve[4*y'[x]^2==9*x,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to -x^{3/2} + c_1$$
$$y(x) \to x^{3/2} + c_1$$

20.4 problem Ex 4

Internal problem ID [11242]

Book: An elementary treatise on differential equations by Abraham Cohen. DC heath publishers. 1906
Section: Chapter V, Singular solutions. Article 33. Page 73
Problem number: Ex 4.
ODE order: 1.
ODE degree: 2.

CAS Maple gives this as type `[_quadrature]`

$$\boxed{y(3-4y)^2 y'^2 + 4y = 4}$$

✓ Solution by Maple
Time used: 0.078 (sec). Leaf size: 58

```
dsolve(y(x)*(3-4*y(x))^2*diff(y(x),x)^2=4*(1-y(x)),y(x), singsol=all)
```

$$y(x) = 1$$
$$x + \frac{y(x)^2 (-1+y(x))}{\sqrt{-y(x)(-1+y(x))}} - c_1 = 0$$
$$x - \frac{y(x)^2 (-1+y(x))}{\sqrt{-y(x)(-1+y(x))}} - c_1 = 0$$

✓ Solution by Mathematica
Time used: 60.436 (sec). Leaf size: 3751

```
DSolve[y[x]*(3-4*y[x])^2*y'[x]^2==4*(1-y[x]),y[x],x,IncludeSingularSolutions -> True]
```

Too large to display

21 Chapter VII, Linear differential equations with constant coefficients. Article 43. Page 92

21.1 problem Ex 1

Internal problem ID [11243]

Book: An elementary treatise on differential equations by Abraham Cohen. DC heath publishers. 1906
Section: Chapter VII, Linear differential equations with constant coefficients. Article 43. Page 92
Problem number: Ex 1.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _missing_x]]`

$$\boxed{y'' - 3y' + 2y = 0}$$

✓ Solution by Maple
Time used: 0.0 (sec). Leaf size: 15

```
dsolve(diff(y(x),x$2)-3*diff(y(x),x)+2*y(x)=0,y(x), singsol=all)
```

$$y(x) = c_1 \mathrm{e}^{x} + c_2 \mathrm{e}^{2x}$$

✓ Solution by Mathematica
Time used: 0.019 (sec). Leaf size: 18

```
DSolve[y''[x]-3*y'[x]+2*y[x]==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to e^{x}(c_2 e^{x} + c_1)$$

21.2 problem Ex 2

Internal problem ID [11244]

Book: An elementary treatise on differential equations by Abraham Cohen. DC heath publishers. 1906
Section: Chapter VII, Linear differential equations with constant coefficients. Article 43. Page 92
Problem number: Ex 2.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _missing_x]]`

$$\boxed{y'' - 6y' + 25y = 0}$$

✓ Solution by Maple
Time used: 0.0 (sec). Leaf size: 22

```
dsolve(diff(y(x),x$2)-6*diff(y(x),x)+25*y(x)=0,y(x), singsol=all)
```

$$y(x) = \mathrm{e}^{3x}(c_1 \sin(4x) + c_2 \cos(4x))$$

✓ Solution by Mathematica
Time used: 0.032 (sec). Leaf size: 26

```
DSolve[y''[x]-6*y'[x]+25*y[x]==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to e^{3x}(c_2 \cos(4x) + c_1 \sin(4x))$$

21.3 problem Ex 3

Internal problem ID [11245]

Book: An elementary treatise on differential equations by Abraham Cohen. DC heath publishers. 1906
Section: Chapter VII, Linear differential equations with constant coefficients. Article 43. Page 92
Problem number: Ex 3.
ODE order: 3.
ODE degree: 1.

CAS Maple gives this as type `[[_3rd_order, _missing_x]]`

$$\boxed{y''' - y' = 0}$$

✓ Solution by Maple
Time used: 0.015 (sec). Leaf size: 16

```
dsolve(diff(y(x),x$3)-diff(y(x),x)=0,y(x), singsol=all)
```

$$y(x) = c_1 + c_2 \mathrm{e}^{-x} + c_3 \mathrm{e}^{x}$$

✓ Solution by Mathematica
Time used: 0.034 (sec). Leaf size: 23

```
DSolve[y'''[x]-y'[x]==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to c_1 e^{x} - c_2 e^{-x} + c_3$$

21.4 problem Ex 4

Internal problem ID [11246]

Book: An elementary treatise on differential equations by Abraham Cohen. DC heath publishers. 1906
Section: Chapter VII, Linear differential equations with constant coefficients. Article 43. Page 92
Problem number: Ex 4.
ODE order: 3.
ODE degree: 1.

CAS Maple gives this as type `[[_3rd_order, _missing_x]]`

$$\boxed{y''' - 2y'' - y' + 2y = 0}$$

✓ Solution by Maple
Time used: 0.0 (sec). Leaf size: 21

```
dsolve(diff(y(x),x$3)-2*diff(y(x),x$2)-diff(y(x),x)+2*y(x)=0,y(x), singsol=all)
```

$$y(x) = c_1 \mathrm{e}^{-x} + c_2 \mathrm{e}^{x} + c_3 \mathrm{e}^{2x}$$

✓ Solution by Mathematica
Time used: 0.005 (sec). Leaf size: 28

```
DSolve[y'''[x]-2*y''[x]-y'[x]+2*y[x]==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to c_1 e^{-x} + c_2 e^{x} + c_3 e^{2x}$$

22 Chapter VII, Linear differential equations with constant coefficients. Article 44. Roots of auxiliary equation repeated. Page 94

22.1 problem Ex 1

Internal problem ID [11247]

Book: An elementary treatise on differential equations by Abraham Cohen. DC heath publishers. 1906
Section: Chapter VII, Linear differential equations with constant coefficients. Article 44. Roots of auxiliary equation repeated. Page 94
Problem number: Ex 1.
ODE order: 3.
ODE degree: 1.

CAS Maple gives this as type `[[_3rd_order, _missing_x]]`

$$\boxed{4y''' - 3y' + y = 0}$$

✓ Solution by Maple
Time used: 0.0 (sec). Leaf size: 21

```
dsolve(4*diff(y(x),x$3)-3*diff(y(x),x)+y(x)=0,y(x), singsol=all)
```

$$y(x) = \mathrm{e}^{-x}\left(\left(c_3 x + c_2\right)\mathrm{e}^{\frac{3x}{2}} + c_1\right)$$

✓ Solution by Mathematica
Time used: 0.005 (sec). Leaf size: 29

```
DSolve[4*y'''[x]-3*y'[x]+y[x]==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to e^{-x}\left(e^{3x/2}(c_2 x + c_1) + c_3\right)$$

22.2 problem Ex 2

Internal problem ID [11248]

Book: An elementary treatise on differential equations by Abraham Cohen. DC heath publishers. 1906
Section: Chapter VII, Linear differential equations with constant coefficients. Article 44. Roots of auxiliary equation repeated. Page 94
Problem number: Ex 2.
ODE order: 3.
ODE degree: 1.

CAS Maple gives this as type `[[_3rd_order, _missing_x]]`

$$\boxed{y''' - y'' - y' + y = 0}$$

✓ Solution by Maple
Time used: 0.0 (sec). Leaf size: 19

```
dsolve(diff(y(x),x$3)-diff(y(x),x$2)-diff(y(x),x)+y(x)=0,y(x), singsol=all)
```

$$y(x) = c_1 \mathrm{e}^{-x} + (c_3 x + c_2)\, \mathrm{e}^{x}$$

✓ Solution by Mathematica
Time used: 0.004 (sec). Leaf size: 25

```
DSolve[y'''[x]-y''[x]-y'[x]+y[x]==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to c_1 e^{-x} + e^{x}(c_3 x + c_2)$$

22.3 problem Ex 3

Internal problem ID [11249]

Book: An elementary treatise on differential equations by Abraham Cohen. DC heath publishers. 1906
Section: Chapter VII, Linear differential equations with constant coefficients. Article 44. Roots of auxiliary equation repeated. Page 94
Problem number: Ex 3.
ODE order: 4.
ODE degree: 1.

CAS Maple gives this as type `[[_high_order, _missing_x]]`

$$\boxed{y'''' + 2y''' - 2y' - y = 0}$$

✓ Solution by Maple
Time used: 0.0 (sec). Leaf size: 24

```
dsolve(diff(y(x),x$4)+2*diff(y(x),x$3)-2*diff(y(x),x)-y(x)=0,y(x), singsol=all)
```

$$y(x) = \left(c_4 x^2 + c_3 x + c_2\right) \mathrm{e}^{-x} + c_1 \mathrm{e}^{x}$$

✓ Solution by Mathematica
Time used: 0.005 (sec). Leaf size: 32

```
DSolve[y''''[x]+2*y'''[x]-2*y'[x]-y[x]==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to e^{-x}\left(c_3 x^2 + c_2 x + c_4 e^{2x} + c_1\right)$$

22.4 problem Ex 4

Internal problem ID [11250]

Book: An elementary treatise on differential equations by Abraham Cohen. DC heath publishers. 1906
Section: Chapter VII, Linear differential equations with constant coefficients. Article 44. Roots of auxiliary equation repeated. Page 94
Problem number: Ex 4.
ODE order: 3.
ODE degree: 1.

CAS Maple gives this as type `[[_3rd_order, _missing_x]]`

$$\boxed{y''' - 6y'' + 9y' = 0}$$

✓ Solution by Maple
Time used: 0.0 (sec). Leaf size: 16

```
dsolve(diff(y(x),x$3)-6*diff(y(x),x$2)+9*diff(y(x),x)=0,y(x), singsol=all)
```

$$y(x) = (c_3 x + c_2)\,\mathrm{e}^{3x} + c_1$$

✓ Solution by Mathematica
Time used: 0.078 (sec). Leaf size: 30

```
DSolve[y'''[x]-6*y''[x]+9*y'[x]==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to \frac{1}{9} e^{3x} (c_2(3x-1) + 3c_1) + c_3$$

23 Chapter VII, Linear differential equations with constant coefficients. Article 45. Roots of auxiliary equation complex. Page 95

23.1 problem Ex 2

Internal problem ID [11251]

Book: An elementary treatise on differential equations by Abraham Cohen. DC heath publishers. 1906
Section: Chapter VII, Linear differential equations with constant coefficients. Article 45. Roots of auxiliary equation complex. Page 95
Problem number: Ex 2.
ODE order: 4.
ODE degree: 1.

CAS Maple gives this as type `[[_high_order, _missing_x]]`

$$\boxed{y'''' + 2y'' + y = 0}$$

✓ Solution by Maple
Time used: 0.0 (sec). Leaf size: 21

```
dsolve(diff(y(x),x$4)+2*diff(y(x),x$2)+y(x)=0,y(x), singsol=all)
```

$$y(x) = (c_4 x + c_2)\cos(x) + \sin(x)(c_3 x + c_1)$$

✓ Solution by Mathematica
Time used: 0.004 (sec). Leaf size: 26

```
DSolve[y''''[x]+2*y''[x]+y[x]==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to (c_2 x + c_1)\cos(x) + (c_4 x + c_3)\sin(x)$$

23.2 problem Ex 3

Internal problem ID [11252]

Book: An elementary treatise on differential equations by Abraham Cohen. DC heath publishers. 1906
Section: Chapter VII, Linear differential equations with constant coefficients. Article 45. Roots of auxiliary equation complex. Page 95
Problem number: Ex 3.
ODE order: 3.
ODE degree: 1.

CAS Maple gives this as type `[[_3rd_order, _missing_x]]`

$$\boxed{y''' - y'' + y' = 0}$$

✓ Solution by Maple
Time used: 0.0 (sec). Leaf size: 32

```
dsolve(diff(y(x),x$3)-diff(y(x),x$2)+diff(y(x),x)=0,y(x), singsol=all)
```

$$y(x) = c_1 + c_2 \mathrm{e}^{\frac{x}{2}} \sin\left(\frac{\sqrt{3}\,x}{2}\right) + c_3 \mathrm{e}^{\frac{x}{2}} \cos\left(\frac{\sqrt{3}\,x}{2}\right)$$

✓ Solution by Mathematica
Time used: 0.799 (sec). Leaf size: 75

```
DSolve[y'''[x]-y''[x]+y'[x]==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to \frac{1}{2}\left(c_1 - \sqrt{3}c_2\right) e^{x/2} \cos\left(\frac{\sqrt{3}x}{2}\right) + \frac{1}{2}\left(\sqrt{3}c_1 + c_2\right) e^{x/2} \sin\left(\frac{\sqrt{3}x}{2}\right) + c_3$$

24 Chapter VII, Linear differential equations with constant coefficients. Article 47. Particular integral. Page 100

24.1 problem Ex 1

Internal problem ID [11253]

Book: An elementary treatise on differential equations by Abraham Cohen. DC heath publishers. 1906
Section: Chapter VII, Linear differential equations with constant coefficients. Article 47. Particular integral. Page 100
Problem number: Ex 1.
ODE order: 3.
ODE degree: 1.

CAS Maple gives this as type `[[_3rd_order, _missing_y]]`

$$\boxed{y''' - y'' - 2y' = \mathrm{e}^{-x}}$$

✓ Solution by Maple
Time used: 0.0 (sec). Leaf size: 27

```
dsolve(diff(y(x),x$3)-diff(y(x),x$2)-2*diff(y(x),x)=exp(-x),y(x), singsol=all)
```

$$y(x) = \frac{(2x - 6c_2 + 2)\,\mathrm{e}^{-x}}{6} + \frac{\mathrm{e}^{2x} c_1}{2} + c_3$$

✓ Solution by Mathematica
Time used: 0.166 (sec). Leaf size: 37

```
DSolve[y'''[x]-y''[x]-2*y'[x]==Exp[-x],y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to \frac{1}{9} e^{-x}(3x + 4 - 9c_1) + \frac{1}{2} c_2 e^{2x} + c_3$$

24.2 problem Ex 2

Internal problem ID [11254]

Book: An elementary treatise on differential equations by Abraham Cohen. DC heath publishers. 1906
Section: Chapter VII, Linear differential equations with constant coefficients. Article 47. Particular integral. Page 100
Problem number: Ex 2.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _linear, _nonhomogeneous]]`

$$y'' + 3y' + 2y = \mathrm{e}^{\mathrm{e}^x}$$

✓ Solution by Maple
Time used: 0.0 (sec). Leaf size: 20

```
dsolve(diff(y(x),x$2)+3*diff(y(x),x)+2*y(x)=exp(exp(x)),y(x), singsol=all)
```

$$y(x) = \left(\mathrm{e}^{\mathrm{e}^x} + c_2 \mathrm{e}^x - c_1\right) \mathrm{e}^{-2x}$$

✓ Solution by Mathematica
Time used: 0.082 (sec). Leaf size: 25

```
DSolve[y''[x]+3*y'[x]+2*y[x]==Exp[Exp[x]],y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to e^{-2x}\left(e^{e^x} + c_2 e^x + c_1\right)$$

24.3 problem Ex 3

Internal problem ID [11255]

Book: An elementary treatise on differential equations by Abraham Cohen. DC heath publishers. 1906
Section: Chapter VII, Linear differential equations with constant coefficients. Article 47. Particular integral. Page 100
Problem number: Ex 3.
ODE order: 3.
ODE degree: 1.

CAS Maple gives this as type `[[_3rd_order, _linear, _nonhomogeneous]]`

$$y''' + 3y'' + 3y' + y = 2\,\mathrm{e}^{-x} - x^2\mathrm{e}^{-x}$$

✓ Solution by Maple
Time used: 0.0 (sec). Leaf size: 32

```
dsolve(diff(y(x),x$3)+3*diff(y(x),x$2)+3*diff(y(x),x)+y(x)=2*exp(-x)-x^2*exp(-x),y(x), sings
```

$$y(x) = -\frac{\mathrm{e}^{-x}(x^5 - 60c_2x^2 - 20x^3 - 60c_3x - 60c_1)}{60}$$

✓ Solution by Mathematica
Time used: 0.017 (sec). Leaf size: 41

```
DSolve[y'''[x]+3*y''[x]+3*y'[x]+y[x]==2*Exp[-x]-x^2*Exp[-x],y[x],x,IncludeSingularSolutions
```

$$y(x) \to \frac{1}{60}e^{-x}\left(-x^5 + 20x^3 + 60c_3x^2 + 60c_2x + 60c_1\right)$$

24.4 problem Ex 4

Internal problem ID [11256]

Book: An elementary treatise on differential equations by Abraham Cohen. DC heath publishers. 1906
Section: Chapter VII, Linear differential equations with constant coefficients. Article 47. Particular integral. Page 100
Problem number: Ex 4.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _linear, _nonhomogeneous]]`

$$\boxed{y'' - 2y' + y = \frac{\mathrm{e}^x}{(1-x)^2}}$$

✓ Solution by Maple
Time used: 0.0 (sec). Leaf size: 19

```
dsolve(diff(y(x),x$2)-2*diff(y(x),x)+y(x)=exp(x)/(1-x)^2,y(x), singsol=all)
```

$$y(x) = \mathrm{e}^x(-1 + c_1 x - \ln(-1+x) + c_2)$$

✓ Solution by Mathematica
Time used: 0.052 (sec). Leaf size: 23

```
DSolve[y''[x]-2*y'[x]+y[x]==Exp[x]/(1-x)^2,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to e^x(-\log(x-1) + c_2 x - 1 + c_1)$$

25 Chapter VII, Linear differential equations with constant coefficients. Article 48. Page 103

25.1 problem Ex 1

Internal problem ID [11257]

Book: An elementary treatise on differential equations by Abraham Cohen. DC heath publishers. 1906
Section: Chapter VII, Linear differential equations with constant coefficients. Article 48. Page 103
Problem number: Ex 1.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$\boxed{y'' - 3y' + 2y = \mathrm{e}^{x}}$$

✓ Solution by Maple
Time used: 0.0 (sec). Leaf size: 16

```
dsolve(diff(y(x),x$2)-3*diff(y(x),x)+2*y(x)=exp(x),y(x), singsol=all)
```

$$y(x) = (-x + c_1 \mathrm{e}^{x} + c_2)\, \mathrm{e}^{x}$$

✓ Solution by Mathematica
Time used: 0.034 (sec). Leaf size: 22

```
DSolve[y''[x]-3*y'[x]+2*y[x]==Exp[x],y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to e^{x}(-x + c_2 e^{x} - 1 + c_1)$$

25.2 problem Ex 2

Internal problem ID [11258]

Book: An elementary treatise on differential equations by Abraham Cohen. DC heath publishers. 1906
Section: Chapter VII, Linear differential equations with constant coefficients. Article 48. Page 103
Problem number: Ex 2.
ODE order: 3.
ODE degree: 1.

CAS Maple gives this as type `[[_3rd_order, _with_linear_symmetries]]`

$$y''' - 3y'' - y' + 3y = x^2$$

✓ Solution by Maple
Time used: 0.0 (sec). Leaf size: 30

```
dsolve(diff(y(x),x$3)-3*diff(y(x),x$2)-diff(y(x),x)+3*y(x)=x^2,y(x), singsol=all)
```

$$y(x) = \frac{x^2}{3} + \frac{2x}{9} + \frac{20}{27} + c_1 \mathrm{e}^{x} + c_2 \mathrm{e}^{-x} + c_3 \mathrm{e}^{3x}$$

✓ Solution by Mathematica
Time used: 0.013 (sec). Leaf size: 42

```
DSolve[y'''[x]-3*y''[x]-y'[x]+3*y[x]==x^2,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to \frac{1}{27}\left(9x^2 + 6x + 20\right) + c_1 e^{-x} + c_2 e^{x} + c_3 e^{3x}$$

25.3 problem Ex 3

Internal problem ID [11259]

Book: An elementary treatise on differential equations by Abraham Cohen. DC heath publishers. 1906
Section: Chapter VII, Linear differential equations with constant coefficients. Article 48. Page 103
Problem number: Ex 3.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _linear, _nonhomogeneous]]`

$$\boxed{y'' + y = \sec(x)}$$

✓ Solution by Maple
Time used: 0.0 (sec). Leaf size: 22

```
dsolve(diff(y(x),x$2)+y(x)=sec(x),y(x), singsol=all)
```

$$y(x) = -\ln(\sec(x))\cos(x) + c_1\cos(x) + \sin(x)(c_2 + x)$$

✓ Solution by Mathematica
Time used: 0.037 (sec). Leaf size: 22

```
DSolve[y''[x]+y[x]==Sec[x],y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to (x + c_2)\sin(x) + \cos(x)(\log(\cos(x)) + c_1)$$

25.4 problem Ex 4

Internal problem ID [11260]

Book: An elementary treatise on differential equations by Abraham Cohen. DC heath publishers. 1906
Section: Chapter VII, Linear differential equations with constant coefficients. Article 48. Page 103
Problem number: Ex 4.
ODE order: 3.
ODE degree: 1.

CAS Maple gives this as type `[[_3rd_order, _with_linear_symmetries]]`

$$y''' - 4y'' + 5y' - 2y = x$$

✓ Solution by Maple
Time used: 0.015 (sec). Leaf size: 23

```
dsolve(diff(y(x),x$3)-4*diff(y(x),x$2)+5*diff(y(x),x)-2*y(x)=x,y(x), singsol=all)
```

$$y(x) = -\frac{5}{4} + c_2 \mathrm{e}^{2x} + (c_3 x + c_1)\,\mathrm{e}^{x} - \frac{x}{2}$$

✓ Solution by Mathematica
Time used: 0.006 (sec). Leaf size: 35

```
DSolve[y'''[x]-4*y''[x]+5*y'[x]-2*y[x]==x,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to c_1 e^{x} + x\left(-\frac{1}{2} + c_2 e^{x}\right) + c_3 e^{2x} - \frac{5}{4}$$

26 Chapter VII, Linear differential equations with constant coefficients. Article 49. Variation of parameters. Page 106

26.1 problem Ex 1

Internal problem ID [11261]

Book: An elementary treatise on differential equations by Abraham Cohen. DC heath publishers. 1906
Section: Chapter VII, Linear differential equations with constant coefficients. Article 49. Variation of parameters. Page 106
Problem number: Ex 1.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _linear, _nonhomogeneous]]`

$$\boxed{y'' + y = \sec(x)}$$

✓ Solution by Maple
Time used: 0.0 (sec). Leaf size: 22

```
dsolve(diff(y(x),x$2)+y(x)=sec(x),y(x), singsol=all)
```

$$y(x) = -\ln(\sec(x))\cos(x) + c_1\cos(x) + \sin(x)(c_2 + x)$$

✓ Solution by Mathematica
Time used: 0.03 (sec). Leaf size: 22

```
DSolve[y''[x]+y[x]==Sec[x],y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to (x + c_2)\sin(x) + \cos(x)(\log(\cos(x)) + c_1)$$

26.2 problem Ex 2

Internal problem ID [11262]

Book: An elementary treatise on differential equations by Abraham Cohen. DC heath publishers. 1906
Section: Chapter VII, Linear differential equations with constant coefficients. Article 49. Variation of parameters. Page 106
Problem number: Ex 2.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _linear, _nonhomogeneous]]`

$$\boxed{y'' + y = \tan(x)}$$

✓ Solution by Maple
Time used: 0.0 (sec). Leaf size: 23

```
dsolve(diff(y(x),x$2)+y(x)=tan(x),y(x), singsol=all)
```

$$y(x) = \sin(x)\, c_2 + c_1 \cos(x) - \cos(x) \ln(\sec(x) + \tan(x))$$

✓ Solution by Mathematica
Time used: 0.046 (sec). Leaf size: 23

```
DSolve[y''[x]+y[x]==Tan[x],y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to \cos(x)(-\operatorname{arctanh}(\sin(x))) + c_1 \cos(x) + c_2 \sin(x)$$

27 Chapter VII, Linear differential equations with constant coefficients. Article 50. Method of undetermined coefficients. Page 107

27.1 problem Ex 1

Internal problem ID [11263]

Book: An elementary treatise on differential equations by Abraham Cohen. DC heath publishers. 1906
Section: Chapter VII, Linear differential equations with constant coefficients. Article 50. Method of undetermined coefficients. Page 107
Problem number: Ex 1.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _linear, _nonhomogeneous]]`

$$\boxed{y'' + 4y = x^2 + \cos(x)}$$

✓ Solution by Maple
Time used: 0.015 (sec). Leaf size: 27

```
dsolve(diff(y(x),x$2)+4*y(x)=x^2+cos(x),y(x), singsol=all)
```

$$y(x) = \sin(2x)\, c_2 + c_1 \cos(2x) + \frac{x^2}{4} - \frac{1}{8} + \frac{\cos(x)}{3}$$

✓ Solution by Mathematica
Time used: 0.321 (sec). Leaf size: 36

```
DSolve[y''[x]+4*y[x]==x^2+Cos[x],y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to \frac{x^2}{4} + \frac{\cos(x)}{3} + c_1 \cos(2x) + c_2 \sin(2x) - \frac{1}{8}$$

27.2 problem Ex 2

Internal problem ID [11264]

Book: An elementary treatise on differential equations by Abraham Cohen. DC heath publishers. 1906
Section: Chapter VII, Linear differential equations with constant coefficients. Article 50. Method of undetermined coefficients. Page 107
Problem number: Ex 2.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _linear, _nonhomogeneous]]`

$$y'' - 2y' + y = 2\,\mathrm{e}^{2x}x - \sin(x)^2$$

✓ Solution by Maple
Time used: 0.0 (sec). Leaf size: 35

```
dsolve(diff(y(x),x$2)-2*diff(y(x),x)+y(x)=2*x*exp(2*x)-sin(x)^2,y(x), singsol=all)
```

$$y(x) = -\frac{1}{2} + 2(x-2)\,\mathrm{e}^{2x} - \frac{3\cos(2x)}{50} - \frac{2\sin(2x)}{25} + (c_1 x + c_2)\,\mathrm{e}^{x}$$

✓ Solution by Mathematica
Time used: 1.17 (sec). Leaf size: 53

```
DSolve[y''[x]-2*y'[x]+y[x]==2*x*Exp[2*x]-Sin[x]^2,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to 2e^{2x}x - 4e^{2x} - \frac{2}{25}\sin(2x) - \frac{3}{50}\cos(2x) + c_2 e^{x}x + c_1 e^{x} - \frac{1}{2}$$

27.3 problem Ex 3

Internal problem ID [11265]

Book: An elementary treatise on differential equations by Abraham Cohen. DC heath publishers. 1906
Section: Chapter VII, Linear differential equations with constant coefficients. Article 50. Method of undetermined coefficients. Page 107
Problem number: Ex 3.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _linear, _nonhomogeneous]]`

$$\boxed{y'' + y = 2\,\mathrm{e}^{x} + x^{3} - x}$$

✓ Solution by Maple
Time used: 0.0 (sec). Leaf size: 21

```
dsolve(diff(y(x),x$2)+y(x)=2*exp(x)+x^3-x,y(x), singsol=all)
```

$$y(x) = \sin(x)\, c_2 + c_1 \cos(x) + x^3 + \mathrm{e}^{x} - 7x$$

✓ Solution by Mathematica
Time used: 0.234 (sec). Leaf size: 25

```
DSolve[y''[x]+y[x]==2*Exp[x]+x^3-x,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to x^3 - 7x + e^x + c_1 \cos(x) + c_2 \sin(x)$$

27.4 problem Ex 4

Internal problem ID [11266]

Book: An elementary treatise on differential equations by Abraham Cohen. DC heath publishers. 1906
Section: Chapter VII, Linear differential equations with constant coefficients. Article 50. Method of undetermined coefficients. Page 107
Problem number: Ex 4.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _linear, _nonhomogeneous]]`

$$\boxed{y'' + 2y' + y = 3\,\mathrm{e}^{2x} - \cos(x)}$$

✓ Solution by Maple
Time used: 0.0 (sec). Leaf size: 25

```
dsolve(diff(y(x),x$2)+2*diff(y(x),x)+y(x)=3*exp(2*x)-cos(x),y(x), singsol=all)
```

$$y(x) = (c_1 x + c_2)\,\mathrm{e}^{-x} - \frac{\sin(x)}{2} + \frac{\mathrm{e}^{2x}}{3}$$

✓ Solution by Mathematica
Time used: 0.454 (sec). Leaf size: 38

```
DSolve[y''[x]+2*y'[x]+y[x]==3*Exp[2*x]-Cos[x],y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to \frac{1}{6}e^{-x}\left(2e^{3x} - 3e^{x}\sin(x) + 6c_2 x + 6c_1\right)$$

27.5 problem Ex 5

Internal problem ID [11267]

Book: An elementary treatise on differential equations by Abraham Cohen. DC heath publishers. 1906
Section: Chapter VII, Linear differential equations with constant coefficients. Article 50. Method of undetermined coefficients. Page 107
Problem number: Ex 5.
ODE order: 3.
ODE degree: 1.

CAS Maple gives this as type `[[_3rd_order, _with_linear_symmetries]]`

$$\boxed{y''' - y = x^2}$$

✓ Solution by Maple
Time used: 0.0 (sec). Leaf size: 40

```
dsolve(diff(y(x),x$3)-y(x)=x^2,y(x), singsol=all)
```

$$y(x) = -x^2 + c_1 \mathrm{e}^{x} + c_2 \mathrm{e}^{-\frac{x}{2}} \cos\left(\frac{\sqrt{3}\,x}{2}\right) + c_3 \mathrm{e}^{-\frac{x}{2}} \sin\left(\frac{\sqrt{3}\,x}{2}\right)$$

✓ Solution by Mathematica
Time used: 0.006 (sec). Leaf size: 59

```
DSolve[y'''[x]-y[x]==x^2,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to -x^2 + c_1 e^{x} + c_2 e^{-x/2} \cos\left(\frac{\sqrt{3}x}{2}\right) + c_3 e^{-x/2} \sin\left(\frac{\sqrt{3}x}{2}\right)$$

27.6 problem Ex 6

Internal problem ID [11268]

Book: An elementary treatise on differential equations by Abraham Cohen. DC heath publishers. 1906
Section: Chapter VII, Linear differential equations with constant coefficients. Article 50. Method of undetermined coefficients. Page 107
Problem number: Ex 6.
ODE order: 3.
ODE degree: 1.

CAS Maple gives this as type `[[_3rd_order, _missing_y]]`

$$\boxed{y''' - 2y'' - 3y' = 3x^2 + \sin(x)}$$

✓ Solution by Maple
Time used: 0.0 (sec). Leaf size: 41

```
dsolve(diff(y(x),x$3)-2*diff(y(x),x$2)-3*diff(y(x),x)=3*x^2+sin(x),y(x), singsol=all)
```

$$y(x) = -\frac{x^3}{3} + \frac{2x^2}{3} - c_1 e^{-x} + \frac{c_2 e^{3x}}{3} + \frac{\sin(x)}{10} + \frac{\cos(x)}{5} - \frac{14x}{9} + c_3$$

✓ Solution by Mathematica
Time used: 0.68 (sec). Leaf size: 58

```
DSolve[y'''[x]-2*y''[x]-3*y'[x]==3*x^2+Sin[x],y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to \frac{1}{9}\left(-3x^3 + 6x^2 - 14x - 9c_1 e^{-x} + 3c_2 e^{3x} + 9c_3\right) + \frac{\sin(x)}{10} + \frac{\cos(x)}{5}$$

27.7 problem Ex 7

Internal problem ID [11269]

Book: An elementary treatise on differential equations by Abraham Cohen. DC heath publishers. 1906
Section: Chapter VII, Linear differential equations with constant coefficients. Article 50. Method of undetermined coefficients. Page 107
Problem number: Ex 7.
ODE order: 4.
ODE degree: 1.

CAS Maple gives this as type `[[_high_order, _with_linear_symmetries]]`

$$\boxed{y'''' - 2y'' + y = \mathrm{e}^{x} + 4}$$

✓ Solution by Maple
Time used: 0.0 (sec). Leaf size: 38

```
dsolve(diff(y(x),x$4)-2*diff(y(x),x$2)+y(x)=exp(x)+4,y(x), singsol=all)
```

$$y(x) = 4 + (c_4 x + c_2)\,\mathrm{e}^{-x} + \frac{(3 + 2x^2 + 4(-1 + 4c_3)\,x + 16c_1)\,\mathrm{e}^{x}}{16}$$

✓ Solution by Mathematica
Time used: 0.174 (sec). Leaf size: 47

```
DSolve[y''''[x]-2*y''[x]+y[x]==Exp[x]+4,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to e^{x}\left(\frac{x^2}{8} + \left(-\frac{1}{4} + c_4\right)x + \frac{3}{16} + c_3\right) + e^{-x}((2 + c_2)x + c_1) + 4$$

27.8 problem Ex 8

Internal problem ID [11270]

Book: An elementary treatise on differential equations by Abraham Cohen. DC heath publishers. 1906
Section: Chapter VII, Linear differential equations with constant coefficients. Article 50. Method of undetermined coefficients. Page 107
Problem number: Ex 8.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _missing_y]]`

$$\boxed{y'' - 2y' = \mathrm{e}^{2x} + 1}$$

✓ Solution by Maple
Time used: 0.0 (sec). Leaf size: 23

```
dsolve(diff(y(x),x$2)-2*diff(y(x),x)=exp(2*x)+1,y(x), singsol=all)
```

$$y(x) = \frac{(2x + 2c_1 - 1)\,\mathrm{e}^{2x}}{4} - \frac{x}{2} + c_2$$

✓ Solution by Mathematica
Time used: 0.126 (sec). Leaf size: 31

```
DSolve[y''[x]-2*y'[x]==Exp[2*x]+1,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to -\frac{x}{2} + \frac{1}{4}e^{2x}(2x - 1 + 2c_1) + c_2$$

27.9 problem Ex 9

Internal problem ID [11271]

Book: An elementary treatise on differential equations by Abraham Cohen. DC heath publishers. 1906
Section: Chapter VII, Linear differential equations with constant coefficients. Article 50. Method of undetermined coefficients. Page 107
Problem number: Ex 9.
ODE order: 4.
ODE degree: 1.

CAS Maple gives this as type `[[_high_order, _linear, _nonhomogeneous]]`

$$\boxed{y'''' + 2y'' + y = \cos(x)}$$

✓ Solution by Maple
Time used: 0.0 (sec). Leaf size: 33

```
dsolve(diff(y(x),x$4)+2*diff(y(x),x$2)+y(x)=cos(x),y(x), singsol=all)
```

$$y(x) = \frac{(8c_4x - x^2 + 8c_1 + 2)\cos(x)}{8} + \left(\left(c_3 + \frac{1}{8}\right)x + c_2\right)\sin(x)$$

✓ Solution by Mathematica
Time used: 0.065 (sec). Leaf size: 43

```
DSolve[y''''[x]+2*y''[x]+y[x]==Cos[x],y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to \left(-\frac{x^2}{8} + c_2x + \frac{5}{16} + c_1\right)\cos(x) + \frac{1}{4}(x + 4c_4x + 4c_3)\sin(x)$$

28 Chapter VII, Linear differential equations with constant coefficients. Article 51. Cauchy linear equation. Page 114

28.1 problem Ex 1

Internal problem ID [11272]

Book: An elementary treatise on differential equations by Abraham Cohen. DC heath publishers. 1906
Section: Chapter VII, Linear differential equations with constant coefficients. Article 51. Cauchy linear equation. Page 114
Problem number: Ex 1.
ODE order: 3.
ODE degree: 1.

CAS Maple gives this as type `[[_3rd_order, _with_linear_symmetries]]`

$$\boxed{x^3 y''' + y' x - y = x \ln(x)}$$

✓ Solution by Maple
Time used: 0.0 (sec). Leaf size: 24

```
dsolve(x^3*diff(y(x),x$3)+x*diff(y(x),x)-y(x)=x*ln(x),y(x), singsol=all)
```

$$y(x) = x\left(\frac{\ln(x)^4}{24} + c_1 + c_2 \ln(x) + c_3 \ln(x)^2\right)$$

✓ Solution by Mathematica
Time used: 0.015 (sec). Leaf size: 33

```
DSolve[x^3*y'''[x]+x*y'[x]-y[x]==x*Log[x],y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to \frac{1}{24} x \log^4(x) + c_1 x + c_3 x \log^2(x) + c_2 x \log(x)$$

28.2 problem Ex 2

Internal problem ID [11273]

Book: An elementary treatise on differential equations by Abraham Cohen. DC heath publishers. 1906
Section: Chapter VII, Linear differential equations with constant coefficients. Article 51. Cauchy linear equation. Page 114
Problem number: Ex 2.
ODE order: 3.
ODE degree: 1.

CAS Maple gives this as type `[[_3rd_order, _exact, _linear, _nonhomogeneous]]`

$$\boxed{x^3 y''' + 2x^2 y'' + 2y = 10x + \frac{10}{x}}$$

✓ Solution by Maple
Time used: 0.015 (sec). Leaf size: 120

```
dsolve(x^3*diff(y(x),x$3)+2*x^2*diff(y(x),x$2)+2*y(x)=10*(x+1/x),y(x), singsol=all)
```

$$\begin{aligned} y(x) = & \frac{((20-10i)\ln(x)+6-8i+(2-i)c_1)(i\cos(\ln(x))-\sin(\ln(x)))x^{-1-i}}{10} \\ & + \frac{((20+10i)\ln(x)+6+8i+(2+i)c_1)(-\sin(\ln(x))-i\cos(\ln(x)))x^{-1+i}}{10} \\ & + \frac{5x^{1-i}(i\sin(\ln(x))+\cos(\ln(x)))}{2} \\ & + \frac{5(-i\sin(\ln(x))+\cos(\ln(x)))x^{1+i}}{2} + x(\cos(\ln(x))c_2+\sin(\ln(x))c_3) \end{aligned}$$

✓ Solution by Mathematica
Time used: 0.187 (sec). Leaf size: 42

```
DSolve[x^3*y'''[x]+2*x^2*y''[x]+2*y[x]==10*(x+1/x),y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to \frac{25x^2+10\log(x)+8+5c_3}{5x} + c_2 x\cos(\log(x)) + c_1 x\sin(\log(x))$$

28.3 problem Ex 3

Internal problem ID [11274]

Book: An elementary treatise on differential equations by Abraham Cohen. DC heath publishers. 1906
Section: Chapter VII, Linear differential equations with constant coefficients. Article 51. Cauchy linear equation. Page 114
Problem number: Ex 3.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _exact, _linear, _nonhomogeneous]]`

$$\boxed{x^2y'' + 3y'x + y = \frac{1}{(1-x)^2}}$$

✓ Solution by Maple
Time used: 0.0 (sec). Leaf size: 22

```
dsolve(x^2*diff(y(x),x$2)+3*x*diff(y(x),x)+y(x)=1/(1-x)^2,y(x), singsol=all)
```

$$y(x) = \frac{c_1 \ln(x) - \ln(-1+x) + \ln(x) + c_2}{x}$$

✓ Solution by Mathematica
Time used: 0.061 (sec). Leaf size: 27

```
DSolve[x^2*y''[x]+3*x*y'[x]+y[x]==1/(1-x)^2,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to \frac{-\log(1-x) + \log(x) + c_2 \log(x) + c_1}{x}$$

28.4 problem Ex 4

Internal problem ID [11275]

Book: An elementary treatise on differential equations by Abraham Cohen. DC heath publishers. 1906
Section: Chapter VII, Linear differential equations with constant coefficients. Article 51. Cauchy linear equation. Page 114
Problem number: Ex 4.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$\boxed{(1+x)^2 y'' - (1+x) y' + 6y = x}$$

✓ Solution by Maple
Time used: 0.016 (sec). Leaf size: 37

```
dsolve((x+1)^2*diff(y(x),x$2)-(x+1)*diff(y(x),x)+6*y(x)=x,y(x), singsol=all)
```

$$y(x) = (1+x)\sin\left(\sqrt{5}\,\ln(1+x)\right)c_2 + (1+x)\cos\left(\sqrt{5}\,\ln(1+x)\right)c_1 + \frac{x}{5} + \frac{1}{30}$$

✓ Solution by Mathematica
Time used: 0.508 (sec). Leaf size: 49

```
DSolve[(x+1)^2*y''[x]-(x+1)*y'[x]+6*y[x]==x,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to \frac{1}{30}(6x+1) + c_2(x+1)\cos\left(\sqrt{5}\log(x+1)\right) + c_1(x+1)\sin\left(\sqrt{5}\log(x+1)\right)$$

29 Chapter VII, Linear differential equations with constant coefficients. Article 52. Summary. Page 117

29.1 problem Ex 1

Internal problem ID [11276]

Book: An elementary treatise on differential equations by Abraham Cohen. DC heath publishers. 1906
Section: Chapter VII, Linear differential equations with constant coefficients. Article 52. Summary. Page 117
Problem number: Ex 1.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _linear, _nonhomogeneous]]`

$$y'' - 5y' + 6y = \cos(x) - \mathrm{e}^{2x}$$

✓ Solution by Maple
Time used: 0.0 (sec). Leaf size: 28

```
dsolve(diff(y(x),x$2)-5*diff(y(x),x)+6*y(x)=cos(x)-exp(2*x),y(x), singsol=all)
```

$$y(x) = (1 + x + c_2)\,\mathrm{e}^{2x} + c_1\mathrm{e}^{3x} + \frac{\cos(x)}{10} - \frac{\sin(x)}{10}$$

✓ Solution by Mathematica
Time used: 0.345 (sec). Leaf size: 34

```
DSolve[y''[x]-5*y'[x]+6*y[x]==Cos[x]-Exp[2*x],y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to \frac{1}{10}\left(-\sin(x) + \cos(x) + 10e^{2x}(x + c_2e^x + 1 + c_1)\right)$$

29.2 problem Ex 2

Internal problem ID [11277]

Book: An elementary treatise on differential equations by Abraham Cohen. DC heath publishers. 1906
Section: Chapter VII, Linear differential equations with constant coefficients. Article 52. Summary. Page 117
Problem number: Ex 2.
ODE order: 4.
ODE degree: 1.

CAS Maple gives this as type `[[_high_order, _linear, _nonhomogeneous]]`

$$\boxed{y'''' - y = \mathrm{e}^{x} \cos(x)}$$

✓ Solution by Maple
Time used: 0.016 (sec). Leaf size: 31

```
dsolve(diff(y(x),x$4)-y(x)=exp(x)*cos(x),y(x), singsol=all)
```

$$y(x) = c_4 \mathrm{e}^{-x} + \frac{(5c_1 - \mathrm{e}^{x}) \cos(x)}{5} + c_2 \mathrm{e}^{x} + c_3 \sin(x)$$

✓ Solution by Mathematica
Time used: 0.071 (sec). Leaf size: 38

```
DSolve[y''''[x]-y[x]==Exp[x]*Cos[x],y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to c_1 e^{x} + c_3 e^{-x} + \left(-\frac{e^{x}}{5} + c_2\right) \cos(x) + c_4 \sin(x)$$

29.3 problem Ex 3

Internal problem ID [11278]

Book: An elementary treatise on differential equations by Abraham Cohen. DC heath publishers. 1906
Section: Chapter VII, Linear differential equations with constant coefficients. Article 52. Summary. Page 117
Problem number: Ex 3.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _linear, _nonhomogeneous]]`

$$\boxed{y'' + 2y' + y = 2x^3 - x\,\mathrm{e}^{3x}}$$

✓ Solution by Maple
Time used: 0.0 (sec). Leaf size: 40

```
dsolve(diff(y(x),x$2)+2*diff(y(x),x)+y(x)=2*x^3-x*exp(3*x),y(x), singsol=all)
```

$$y(x) = -48 + (c_1 x + c_2)\,\mathrm{e}^{-x} + \frac{(1-2x)\,\mathrm{e}^{3x}}{32} + 2x^3 - 12x^2 + 36x$$

✓ Solution by Mathematica
Time used: 0.335 (sec). Leaf size: 48

```
DSolve[y''[x]+2*y'[x]+y[x]==2*x^3-x*Exp[3*x],y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to 2\left(x^3 - 6x^2 + 18x - 24\right) + \frac{1}{32}e^{3x}(1-2x) + e^{-x}(c_2 x + c_1)$$

29.4 problem Ex 5

Internal problem ID [11279]

Book: An elementary treatise on differential equations by Abraham Cohen. DC heath publishers. 1906
Section: Chapter VII, Linear differential equations with constant coefficients. Article 52. Summary. Page 117
Problem number: Ex 5.
ODE order: 3.
ODE degree: 1.

CAS Maple gives this as type `[[_3rd_order, _missing_y]]`

$$\boxed{y''' - 4y' = x^2 - 3\,\mathrm{e}^{2x}}$$

✓ Solution by Maple
Time used: 0.016 (sec). Leaf size: 35

```
dsolve(diff(y(x),x$3)-4*diff(y(x),x)=x^2-3*exp(2*x),y(x), singsol=all)
```

$$y(x) = \frac{(9 - 12x + 16c_1)\,\mathrm{e}^{2x}}{32} - \frac{x^3}{12} - \frac{\mathrm{e}^{-2x}c_2}{2} - \frac{x}{8} + c_3$$

✓ Solution by Mathematica
Time used: 0.311 (sec). Leaf size: 49

```
DSolve[y'''[x]-4*y'[x]==x^2-3*Exp[2*x],y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to -\frac{x^3}{12} - \frac{x}{8} + \frac{1}{32}e^{2x}(-12x + 9 + 16c_1) - \frac{1}{2}c_2e^{-2x} + c_3$$

29.5 problem Ex 6

Internal problem ID [11280]

Book: An elementary treatise on differential equations by Abraham Cohen. DC heath publishers. 1906
Section: Chapter VII, Linear differential equations with constant coefficients. Article 52. Summary. Page 117
Problem number: Ex 6.
ODE order: 4.
ODE degree: 1.

CAS Maple gives this as type `[[_high_order, _linear, _nonhomogeneous]]`

$$\boxed{y'''' - 2y'' + y = \cos(x)}$$

✓ Solution by Maple
Time used: 0.0 (sec). Leaf size: 27

```
dsolve(diff(y(x),x$4)-2*diff(y(x),x$2)+y(x)=cos(x),y(x), singsol=all)
```

$$y(x) = (c_4 x + c_2)\,\mathrm{e}^{-x} + (c_3 x + c_1)\,\mathrm{e}^{x} + \frac{\cos(x)}{4}$$

✓ Solution by Mathematica
Time used: 0.131 (sec). Leaf size: 42

```
DSolve[y''''[x]-2*y''[x]+y[x]==Cos[x],y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to \frac{\cos(x)}{4} + e^{-x}\left(c_2 x + c_3 e^{2x} + c_4 e^{2x} x + c_1\right)$$

29.6 problem Ex 7

Internal problem ID [11281]

Book: An elementary treatise on differential equations by Abraham Cohen. DC heath publishers. 1906
Section: Chapter VII, Linear differential equations with constant coefficients. Article 52. Summary. Page 117
Problem number: Ex 7.
ODE order: 4.
ODE degree: 1.

CAS Maple gives this as type `[[_high_order, _linear, _nonhomogeneous]]`

$$x^4 y'''' + 6x^3 y''' + 9x^2 y'' + 3y'x + y = (\ln(x) + 1)^2$$

✓ Solution by Maple
Time used: 0.016 (sec). Leaf size: 34

```
dsolve(x^4*diff(y(x),x$4)+6*x^3*diff(y(x),x$3)+9*x^2*diff(y(x),x$2)+3*x*diff(y(x),x)+y(x)=(1
```

$$y(x) = (c_3 \ln(x) + c_1)\cos(\ln(x)) + (c_4 \ln(x) + c_2)\sin(\ln(x)) + \ln(x)^2 + 2\ln(x) - 3$$

✓ Solution by Mathematica
Time used: 0.27 (sec). Leaf size: 39

```
DSolve[x^4*y''''[x]+6*x^3*y'''[x]+9*x^2*y''[x]+3*x*y'[x]+y[x]==(1+Log[x])^2,y[x],x,IncludeSi
```

$$y(x) \to \log^2(x) + 2\log(x) + (c_2 \log(x) + c_1)\cos(\log(x)) + (c_4 \log(x) + c_3)\sin(\log(x)) - 3$$

29.7 problem Ex 8

Internal problem ID [11282]

Book: An elementary treatise on differential equations by Abraham Cohen. DC heath publishers. 1906
Section: Chapter VII, Linear differential equations with constant coefficients. Article 52. Summary. Page 117
Problem number: Ex 8.
ODE order: 3.
ODE degree: 1.

CAS Maple gives this as type `[[_3rd_order, _missing_y]]`

$$\boxed{y''' + 2y'' + y' = x^2 - x}$$

✓ Solution by Maple
Time used: 0.0 (sec). Leaf size: 35

```
dsolve(diff(y(x),x$3)+2*diff(y(x),x$2)+diff(y(x),x)=x^2-x,y(x), singsol=all)
```

$$y(x) = (-c_1 x - c_1 - c_2)\,\mathrm{e}^{-x} + \frac{x^3}{3} - \frac{5x^2}{2} + 8x + c_3$$

✓ Solution by Mathematica
Time used: 0.243 (sec). Leaf size: 39

```
DSolve[y'''[x]+2*y''[x]+y'[x]==x^2-x,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to \frac{1}{6}x\left(2x^2 - 15x + 48\right) - e^{-x}(c_2(x+1) + c_1) + c_3$$

29.8 problem Ex 9

Internal problem ID [11283]

Book: An elementary treatise on differential equations by Abraham Cohen. DC heath publishers. 1906
Section: Chapter VII, Linear differential equations with constant coefficients. Article 52. Summary. Page 117
Problem number: Ex 9.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _linear, _nonhomogeneous]]`

$$\boxed{y'' + 4y = \sin(x)^2}$$

✓ Solution by Maple
Time used: 0.0 (sec). Leaf size: 30

```
dsolve(diff(y(x),x$2)+4*y(x)=sin(x)^2,y(x), singsol=all)
```

$$y(x) = \frac{(8c_1 - 1)\cos(2x)}{8} + \frac{1}{8} + \frac{(8c_2 - x)\sin(2x)}{8}$$

✓ Solution by Mathematica
Time used: 0.16 (sec). Leaf size: 34

```
DSolve[y''[x]+4*y[x]==Sin[x]^2,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to \frac{1}{8}((-1 + 8c_1)\cos(2x) - (x - 8c_2)\sin(2x) + 1)$$

29.9 problem Ex 10

Internal problem ID [11284]

Book: An elementary treatise on differential equations by Abraham Cohen. DC heath publishers. 1906
Section: Chapter VII, Linear differential equations with constant coefficients. Article 52. Summary. Page 117
Problem number: Ex 10.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _linear, _nonhomogeneous]]`

$$y'' + 4y = \sec(x)^2$$

✓ Solution by Maple
Time used: 0.0 (sec). Leaf size: 42

```
dsolve(diff(y(x),x$2)+4*y(x)=sec(x)^2,y(x), singsol=all)
```

$$y(x) = \left(-2\cos(x)^2+1\right)\ln(\sec(x)) + 2\cos(x)^2 c_1 + 2\sin(x)(c_2+x)\cos(x) - \sin(x)^2 - c_1$$

✓ Solution by Mathematica
Time used: 0.168 (sec). Leaf size: 33

```
DSolve[y''[x]+4*y[x]==Sec[x]^2,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to \cos(2x)(\log(\cos(x)) + c_1) + \sin(x)(-\sin(x) + 2(x + c_2)\cos(x))$$

29.10 problem Ex 12

Internal problem ID [11285]

Book: An elementary treatise on differential equations by Abraham Cohen. DC heath publishers. 1906
Section: Chapter VII, Linear differential equations with constant coefficients. Article 52. Summary. Page 117
Problem number: Ex 12.
ODE order: 4.
ODE degree: 1.

CAS Maple gives this as type `[[_high_order, _with_linear_symmetries]]`

$$y'''' - y''' - 3y'' + 5y' - 2y = \mathrm{e}^{3x}$$

✓ Solution by Maple
Time used: 0.0 (sec). Leaf size: 32

```
dsolve(diff(y(x),x$4)-diff(y(x),x$3)-3*diff(y(x),x$2)+5*diff(y(x),x)-2*y(x)=exp(3*x),y(x), s
```

$$y(x) = \left(\left(c_3 x^2 + c_4 x + c_1\right) \mathrm{e}^{3x} + c_2 + \frac{\mathrm{e}^{5x}}{40}\right) \mathrm{e}^{-2x}$$

✓ Solution by Mathematica
Time used: 0.091 (sec). Leaf size: 39

```
DSolve[y''''[x]-y'''[x]-3*y''[x]+5*y'[x]-2*y[x]==Exp[3*x],y[x],x,IncludeSingularSolutions ->
```

$$y(x) \to \frac{e^{3x}}{40} + c_1 e^{-2x} + e^x(x(c_4 x + c_3) + c_2)$$

29.11 problem Ex 13

Internal problem ID [11286]

Book: An elementary treatise on differential equations by Abraham Cohen. DC heath publishers. 1906
Section: Chapter VII, Linear differential equations with constant coefficients. Article 52. Summary. Page 117
Problem number: Ex 13.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _linear, _nonhomogeneous]]`

$$\boxed{y'' + y = \cos(x)\, x}$$

✓ Solution by Maple
Time used: 0.0 (sec). Leaf size: 26

```
dsolve(diff(y(x),x$2)+y(x)=x*cos(x),y(x), singsol=all)
```

$$y(x) = \frac{(x^2 + 4c_2 - 1)\sin(x)}{4} + \frac{\cos(x)\,(4c_1 + x)}{4}$$

✓ Solution by Mathematica
Time used: 0.05 (sec). Leaf size: 34

```
DSolve[y''[x]+y[x]==x*Cos[x],y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to \frac{1}{8}\left(\left(2x^2 - 1 + 8c_2\right)\sin(x) + 2(x + 4c_1)\cos(x)\right)$$

29.12 problem Ex 14

Internal problem ID [11287]

Book: An elementary treatise on differential equations by Abraham Cohen. DC heath publishers. 1906
Section: Chapter VII, Linear differential equations with constant coefficients. Article 52. Summary. Page 117
Problem number: Ex 14.
ODE order: 3.
ODE degree: 1.

CAS Maple gives this as type `[[_3rd_order, _exact, _linear, _nonhomogeneous]]`

$$\boxed{x^3 y''' + 2x^2 y'' - y' x + y = \frac{1}{x}}$$

✓ Solution by Maple
Time used: 0.0 (sec). Leaf size: 28

```
dsolve(x^3*diff(y(x),x$3)+2*x^2*diff(y(x),x$2)-x*diff(y(x),x)+y(x)=1/x,y(x), singsol=all)
```

$$y(x) = \frac{4\ln(x)\, c_2 x^2 + 4c_3 x^2 + \ln(x) + c_1 + 1}{4x}$$

✓ Solution by Mathematica
Time used: 0.013 (sec). Leaf size: 33

```
DSolve[x^3*y'''[x]+2*x^2*y''[x]-x*y'[x]+y[x]==1/x,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to \frac{\log(x) + 1}{4x} + \frac{c_1}{x} + c_2 x + c_3 x \log(x)$$

29.13 problem Ex 15

Internal problem ID [11288]

Book: An elementary treatise on differential equations by Abraham Cohen. DC heath publishers. 1906
Section: Chapter VII, Linear differential equations with constant coefficients. Article 52. Summary. Page 117
Problem number: Ex 15.
ODE order: 3.
ODE degree: 1.

CAS Maple gives this as type `[[_3rd_order, _linear, _nonhomogeneous]]`

$$\boxed{y''' - y = x\,\mathrm{e}^{x} + \cos(x)^2}$$

✓ Solution by Maple
Time used: 0.016 (sec). Leaf size: 61

```
dsolve(diff(y(x),x$3)-y(x)=x*exp(x)+cos(x)^2,y(x), singsol=all)
```

$$y(x) = -\frac{1}{2} + c_2 \mathrm{e}^{-\frac{x}{2}} \cos\left(\frac{\sqrt{3}\,x}{2}\right) + c_3 \mathrm{e}^{-\frac{x}{2}} \sin\left(\frac{\sqrt{3}\,x}{2}\right) - \frac{\cos(2x)}{130} - \frac{4\sin(2x)}{65} + \frac{(3x^2 + 18c_1 - 6x + 4)\,\mathrm{e}^{x}}{18}$$

✓ Solution by Mathematica
Time used: 7.274 (sec). Leaf size: 98

```
DSolve[y'''[x]-y[x]==x*Exp[x]+Cos[x]^2,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to \frac{e^x x^2}{6} - \frac{e^x x}{3} + \frac{2e^x}{9} - \frac{4}{65}\sin(2x) - \frac{1}{130}\cos(2x) + c_1 e^x + c_2 e^{-x/2} \cos\left(\frac{\sqrt{3}x}{2}\right) + c_3 e^{-x/2} \sin\left(\frac{\sqrt{3}x}{2}\right) - \frac{1}{2}$$

30 Chapter VIII, Linear differential equations of the second order. Article 53. Change of dependent variable. Page 125

30.1 problem Ex 1

Internal problem ID [11289]

Book: An elementary treatise on differential equations by Abraham Cohen. DC heath publishers. 1906
Section: Chapter VIII, Linear differential equations of the second order. Article 53. Change of dependent variable. Page 125
Problem number: Ex 1.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$\boxed{y'' - x^2 y' + yx = x}$$

✓ Solution by Maple
Time used: 0.016 (sec). Leaf size: 57

```
dsolve(diff(y(x),x$2)-x^2*diff(y(x),x)+x*y(x)=x,y(x), singsol=all)
```

$$y(x) = -\frac{\left(-3^{\frac{1}{3}} \mathrm{e}^{\frac{x^3}{3}} c_1 - c_2 x - 1\right) (-x^3)^{\frac{2}{3}} + x^3 c_1 \left(\Gamma\left(\frac{2}{3}\right) - \Gamma\left(\frac{2}{3}, -\frac{x^3}{3}\right)\right)}{(-x^3)^{\frac{2}{3}}}$$

✓ Solution by Mathematica
Time used: 0.286 (sec). Leaf size: 42

```
DSolve[y''[x]-x^2*y'[x]+x*y[x]==x,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to -\frac{c_2 \sqrt[3]{-x^3} \Gamma\left(-\frac{1}{3}, -\frac{x^3}{3}\right)}{3\sqrt[3]{3}} + c_1 x + 1$$

30.2 problem Ex 2

Internal problem ID [11290]

Book: An elementary treatise on differential equations by Abraham Cohen. DC heath publishers. 1906
Section: Chapter VIII, Linear differential equations of the second order. Article 53. Change of dependent variable. Page 125
Problem number: Ex 2.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$\boxed{xy'' - (2x+1)\,y' + y(1+x) = x^2 - x - 1}$$

✓ Solution by Maple
Time used: 0.0 (sec). Leaf size: 16

```
dsolve(x*diff(y(x),x$2)-(2*x+1)*diff(y(x),x)+(x+1)*y(x)=x^2-x-1,y(x), singsol=all)
```

$$y(x) = \left(c_1 x^2 + c_2\right) \mathrm{e}^x + x$$

✓ Solution by Mathematica
Time used: 0.275 (sec). Leaf size: 25

```
DSolve[x*y''[x]-(2*x+1)*y'[x]+(x+1)*y[x]==x^2-x-1,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to \frac{1}{2} c_2 e^x x^2 + x + c_1 e^x$$

30.3 problem Ex 3

Internal problem ID [11291]

Book: An elementary treatise on differential equations by Abraham Cohen. DC heath publishers. 1906
Section: Chapter VIII, Linear differential equations of the second order. Article 53. Change of dependent variable. Page 125
Problem number: Ex 3.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$\boxed{\left(x^2+1\right) y'' + 2y'x - 2y = 0}$$

✓ Solution by Maple
Time used: 0.016 (sec). Leaf size: 14

```
dsolve((1+x^2)*diff(y(x),x$2)+2*x*diff(y(x),x)-2*y(x)=0,y(x), singsol=all)
```

$$y(x) = c_1 x + \arctan\left(x\right) x c_2 + c_2$$

✓ Solution by Mathematica
Time used: 0.034 (sec). Leaf size: 48

```
DSolve[(1+x^2)*y''[x]+2*x*y'[x]-2*y[x]==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to \frac{1}{2} i (2c_1 x - c_2 x \log(1 - ix) + c_2 x \log(1 + ix) + 2ic_2)$$

30.4 problem Ex 4

Internal problem ID [11292]

Book: An elementary treatise on differential equations by Abraham Cohen. DC heath publishers. 1906
Section: Chapter VIII, Linear differential equations of the second order. Article 53. Change of dependent variable. Page 125
Problem number: Ex 4.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$\boxed{(1-x)\,y'' + y'x - y = (1-x)^2}$$

✓ Solution by Maple
Time used: 0.0 (sec). Leaf size: 16

```
dsolve((1-x)*diff(y(x),x$2)+x*diff(y(x),x)-y(x)=(1-x)^2,y(x), singsol=all)
```

$$y(x) = c_2 x + c_1 \mathrm{e}^x + x^2 + 1$$

✓ Solution by Mathematica
Time used: 0.067 (sec). Leaf size: 22

```
DSolve[(1-x)*y''[x]+x*y'[x]-y[x]==(1-x)^2,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to x^2 + x - c_2 x + c_1 e^x + 1$$

30.5 problem Ex 5

Internal problem ID [11293]

Book: An elementary treatise on differential equations by Abraham Cohen. DC heath publishers. 1906
Section: Chapter VIII, Linear differential equations of the second order. Article 53. Change of dependent variable. Page 125
Problem number: Ex 5.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _linear, _nonhomogeneous]]`

$$\boxed{\sin(x)\,y'' + 2\cos(x)\,y' + 3\sin(x)\,y = \mathrm{e}^{x}}$$

✓ Solution by Maple
Time used: 0.015 (sec). Leaf size: 28

```
dsolve(sin(x)*diff(y(x),x$2)+2*cos(x)*diff(y(x),x)+3*sin(x)*y(x)=exp(x),y(x), singsol=all)
```

$$y(x) = \frac{\csc(x)\left(10\cos(x)^2 c_1 + 10\sin(x)\cos(x)\,c_2 + \mathrm{e}^{x} - 5c_1\right)}{5}$$

✓ Solution by Mathematica
Time used: 0.229 (sec). Leaf size: 56

```
DSolve[Sin[x]*y''[x]+2*Cos[x]*y'[x]+3*Sin[x]*y[x]==Exp[x],y[x],x,IncludeSingularSolutions ->
```

$$y(x) \to \frac{e^{-ix}\left(4ie^{(1+2i)x} + 5c_2 e^{4ix} + 20ic_1\right)}{10\left(-1 + e^{2ix}\right)}$$

30.6 problem Ex 6

Internal problem ID [11294]

Book: An elementary treatise on differential equations by Abraham Cohen. DC heath publishers. 1906
Section: Chapter VIII, Linear differential equations of the second order. Article 53. Change of dependent variable. Page 125
Problem number: Ex 6.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$\boxed{y'' - 2y' \tan(x) - \left(a^2 + 1\right) y = 0}$$

✓ Solution by Maple
Time used: 0.0 (sec). Leaf size: 20

```
dsolve(diff(y(x),x$2)-2*tan(x)*diff(y(x),x)-(a^2+1)*y(x)=0,y(x), singsol=all)
```

$$y(x) = \sec(x) \left(c_1 \sinh(ax) + c_2 \cosh(ax)\right)$$

✓ Solution by Mathematica
Time used: 0.117 (sec). Leaf size: 32

```
DSolve[y''[x]-2*Tan[x]*y'[x]-(a^2+1)*y[x]==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to \sec(x) \left(c_1 e^{-ax} + \frac{c_2 e^{ax}}{2a}\right)$$

30.7 problem Ex 7

Internal problem ID [11295]

Book: An elementary treatise on differential equations by Abraham Cohen. DC heath publishers. 1906
Section: Chapter VIII, Linear differential equations of the second order. Article 53. Change of dependent variable. Page 125
Problem number: Ex 7.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$\boxed{4x^2y'' + 4y'x^3 + \left(x^2+1\right)y = 0}$$

✓ Solution by Maple
Time used: 0.156 (sec). Leaf size: 35

```
dsolve(4*x^2*diff(y(x),x$2)+4*x^3*diff(y(x),x)+(x^2+1)*y(x)=0,y(x), singsol=all)
```

$$y(x) = \frac{\mathrm{e}^{-\frac{x^2}{4}}\left(\text{WhittakerM}\left(-\frac{1}{8},0,\frac{x^2}{2}\right)c_1 + \text{WhittakerW}\left(-\frac{1}{8},0,\frac{x^2}{2}\right)c_2\right)}{\sqrt{x}}$$

✓ Solution by Mathematica
Time used: 0.239 (sec). Leaf size: 60

```
DSolve[4*x^2*y''[x]+4*x^3*y'[x]+(x^2+1)*y[x]==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to c_2 G^{2,0}_{1,2}\left(\frac{x^2}{16}\middle|\begin{matrix}\frac{7}{8}\\ \frac{1}{4},\frac{1}{4}\end{matrix}\right) + \frac{1}{2}\sqrt[4]{-1}c_1\sqrt{x}\,\text{Hypergeometric1F1}\left(\frac{3}{8},1,-\frac{x^2}{16}\right)$$

30.8 problem Ex 8

Internal problem ID [11296]

Book: An elementary treatise on differential equations by Abraham Cohen. DC heath publishers. 1906
Section: Chapter VIII, Linear differential equations of the second order. Article 53. Change of dependent variable. Page 125
Problem number: Ex 8.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _linear, _nonhomogeneous]]`

$$\boxed{xy'' + 2y' - yx = 2\,\mathrm{e}^{x}}$$

✓ Solution by Maple
Time used: 0.0 (sec). Leaf size: 21

```
dsolve(x*diff(y(x),x$2)+2*diff(y(x),x)-x*y(x)=2*exp(x),y(x), singsol=all)
```

$$y(x) = \frac{\mathrm{e}^{x} x + \sinh(x)\, c_2 + \cosh(x)\, c_1}{x}$$

✓ Solution by Mathematica
Time used: 0.051 (sec). Leaf size: 35

```
DSolve[x*y''[x]+2*y'[x]-x*y[x]==2*Exp[x],y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to \frac{e^{-x}(e^{2x}(2x - 1 + c_2) + 2c_1)}{2x}$$

31 Chapter VIII, Linear differential equations of the second order. Article 54. Change of independent variable. Page 127

31.1 problem Ex 1

Internal problem ID [11297]

Book: An elementary treatise on differential equations by Abraham Cohen. DC heath publishers. 1906
Section: Chapter VIII, Linear differential equations of the second order. Article 54. Change of independent variable. Page 127
Problem number: Ex 1.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _linear, _nonhomogeneous]]`

$$\boxed{y'' + (2\,\mathrm{e}^{x} - 1)\,y' + y\,\mathrm{e}^{2x} = \mathrm{e}^{4x}}$$

✓ Solution by Maple
Time used: 0.016 (sec). Leaf size: 44

```
dsolve(diff(y(x),x$2)+(2*exp(x)-1)*diff(y(x),x)+exp(2*x)*y(x)=exp(4*x),y(x), singsol=all)
```

$$y(x) = \mathrm{e}^{\frac{x}{2}-\mathrm{e}^{x}} \sinh\left(\frac{x}{2}\right) c_2 + \mathrm{e}^{\frac{x}{2}-\mathrm{e}^{x}} \cosh\left(\frac{x}{2}\right) c_1 + \mathrm{e}^{2x} - 4\,\mathrm{e}^{x} + 6$$

✓ Solution by Mathematica
Time used: 0.121 (sec). Leaf size: 39

```
DSolve[y''[x]+(2*Exp[x]-1)*y'[x]+Exp[2*x]*y[x]==Exp[4*x],y[x],x,IncludeSingularSolutions ->
```

$$y(x) \to -4e^{x} + e^{2x} + c_1 e^{-e^{x}} + c_2 e^{x-e^{x}} + 6$$

31.2 problem Ex 2

Internal problem ID [11298]

Book: An elementary treatise on differential equations by Abraham Cohen. DC heath publishers. 1906
Section: Chapter VIII, Linear differential equations of the second order. Article 54. Change of independent variable. Page 127
Problem number: Ex 2.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[_Gegenbauer, [_2nd_order, _linear, '_with_symmetry_[0,F(x)]']]`

$$\boxed{\left(-x^2+1\right) y'' - y' x + 4y = 0}$$

✓ Solution by Maple
Time used: 0.0 (sec). Leaf size: 50

```
dsolve((1-x^2)*diff(y(x),x$2)-x*diff(y(x),x)+4*y(x)=0,y(x), singsol=all)
```

$$y(x) = \frac{\left(8x^3 - 4x\right) c_2 \sqrt{x^2-1} + \left(8x^4 - 8x^2 + 1\right) c_2 + c_1}{\left(x + \sqrt{x^2-1}\right)^2}$$

✓ Solution by Mathematica
Time used: 0.316 (sec). Leaf size: 97

```
DSolve[(1-x^2)*y''[x]-x*y'[x]+4*y[x]==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to c_1 \cosh\left(\frac{4\sqrt{1-x^2}\arctan\left(\frac{\sqrt{1-x^2}}{x+1}\right)}{\sqrt{x^2-1}}\right) - ic_2 \sinh\left(\frac{4\sqrt{1-x^2}\arctan\left(\frac{\sqrt{1-x^2}}{x+1}\right)}{\sqrt{x^2-1}}\right)$$

31.3 problem Ex 3

Internal problem ID [11299]

Book: An elementary treatise on differential equations by Abraham Cohen. DC heath publishers. 1906
Section: Chapter VIII, Linear differential equations of the second order. Article 54. Change of independent variable. Page 127
Problem number: Ex 3.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$\boxed{y'' + y' \tan(x) + \cos(x)^2 y = 0}$$

✓ Solution by Maple
Time used: 0.093 (sec). Leaf size: 15

```
dsolve(diff(y(x),x$2)+tan(x)*diff(y(x),x)+cos(x)^2*y(x)=0,y(x), singsol=all)
```

$$y(x) = c_1 \sin(\sin(x)) + c_2 \cos(\sin(x))$$

✓ Solution by Mathematica
Time used: 0.078 (sec). Leaf size: 18

```
DSolve[y''[x]+Tan[x]*y'[x]+Cos[x]^2*y[x]==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to c_2 \sin(\sin(x)) + c_1 \cos(\sin(x))$$

31.4 problem Ex 4

Internal problem ID [11300]

Book: An elementary treatise on differential equations by Abraham Cohen. DC heath publishers. 1906
Section: Chapter VIII, Linear differential equations of the second order. Article 54. Change of independent variable. Page 127
Problem number: Ex 4.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _linear, _nonhomogeneous]]`

$$\boxed{x^6 y'' + 3x^5 y' + y = \frac{1}{x^2}}$$

✓ Solution by Maple
Time used: 0.0 (sec). Leaf size: 24

```
dsolve(x^6*diff(y(x),x$2)+3*x^5*diff(y(x),x)+y(x)=1/x^2,y(x), singsol=all)
```

$$y(x) = \sin\left(\frac{1}{2x^2}\right) c_2 + \cos\left(\frac{1}{2x^2}\right) c_1 + \frac{1}{x^2}$$

✓ Solution by Mathematica
Time used: 0.099 (sec). Leaf size: 32

```
DSolve[x^6*y''[x]+3*x^5*y'[x]+y[x]==1/x^2,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to \frac{1}{x^2} + c_1 \cos\left(\frac{1}{2x^2}\right) - c_2 \sin\left(\frac{1}{2x^2}\right)$$

31.5 problem Ex 5

Internal problem ID [11301]

Book: An elementary treatise on differential equations by Abraham Cohen. DC heath publishers. 1906
Section: Chapter VIII, Linear differential equations of the second order. Article 54. Change of independent variable. Page 127
Problem number: Ex 5.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _linear, _nonhomogeneous]]`

$$\boxed{xy'' - \left(2x^2+1\right)y' - 8yx^3 = 4x^3 \mathrm{e}^{-x^2}}$$

✓ Solution by Maple
Time used: 0.016 (sec). Leaf size: 30

```
dsolve(x*diff(y(x),x$2)-(2*x^2+1)*diff(y(x),x)-8*x^3*y(x)=4*x^3*exp(-x^2),y(x), singsol=all)
```

$$y(x) = \frac{\left(-x^2+3c_1\right)\mathrm{e}^{-x^2}}{3} + \mathrm{e}^{2x^2} c_2$$

✓ Solution by Mathematica
Time used: 0.105 (sec). Leaf size: 38

```
DSolve[x*y''[x]-(2*x^2+1)*y'[x]-8*x^3*y[x]==4*x^3*Exp[-x^2],y[x],x,IncludeSingularSolutions
```

$$y(x) \to \frac{1}{9}e^{-x^2}\left(-3x^2+9c_1e^{3x^2}-1+9c_2\right)$$

32 Chapter VIII, Linear differential equations of the second order. Article 55. Summary. Page 129

32.1 problem Ex 1

Internal problem ID [11302]

Book: An elementary treatise on differential equations by Abraham Cohen. DC heath publishers. 1906
Section: Chapter VIII, Linear differential equations of the second order. Article 55. Summary. Page 129
Problem number: Ex 1.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[_Laguerre]`

$$\boxed{xy'' - (x+3)\,y' + 3y = 0}$$

✓ Solution by Maple
Time used: 0.0 (sec). Leaf size: 24

```
dsolve(x*diff(y(x),x$2)-(x+3)*diff(y(x),x)+3*y(x)=0,y(x), singsol=all)
```

$$y(x) = c_1 \mathrm{e}^{x} + c_2\left(x^3 + 3x^2 + 6x + 6\right)$$

✓ Solution by Mathematica
Time used: 0.087 (sec). Leaf size: 29

```
DSolve[x*y''[x]-(x+3)*y'[x]+3*y[x]==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to c_1 e^{x} - c_2\left(x^3 + 3x^2 + 6x + 6\right)$$

32.2 problem Ex 2

Internal problem ID [11303]

Book: An elementary treatise on differential equations by Abraham Cohen. DC heath publishers. 1906
Section: Chapter VIII, Linear differential equations of the second order. Article 55. Summary. Page 129
Problem number: Ex 2.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$\boxed{(x-3)\,y'' - (4x-9)\,y' + (3x-6)\,y = 0}$$

✓ Solution by Maple
Time used: 0.016 (sec). Leaf size: 29

```
dsolve((x-3)*diff(y(x),x$2)-(4*x-9)*diff(y(x),x)+(3*x-6)*y(x)=0,y(x), singsol=all)
```

$$y(x) = 4c_2\left(x^3 - \frac{21}{2}x^2 + \frac{75}{2}x - \frac{183}{4}\right)\mathrm{e}^{3x} + c_1\mathrm{e}^{x}$$

✓ Solution by Mathematica
Time used: 0.091 (sec). Leaf size: 42

```
DSolve[(x-3)*y''[x]-(4*x-9)*y'[x]+(3*x-6)*y[x]==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to \frac{1}{8}c_2 e^{3x-9}\left(4x^3 - 42x^2 + 150x - 183\right) + c_1 e^{x-3}$$

32.3 problem Ex 3

Internal problem ID [11304]

Book: An elementary treatise on differential equations by Abraham Cohen. DC heath publishers. 1906
Section: Chapter VIII, Linear differential equations of the second order. Article 55. Summary. Page 129
Problem number: Ex 3.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$\boxed{y''x^2 + 4y'x + \left(-x^2 + 2\right) y = 0}$$

✓ Solution by Maple
Time used: 0.0 (sec). Leaf size: 17

```
dsolve(x^2*diff(y(x),x$2)+4*x*diff(y(x),x)+(2-x^2)*y(x)=0,y(x), singsol=all)
```

$$y(x) = \frac{c_1 \sinh(x) + c_2 \cosh(x)}{x^2}$$

✓ Solution by Mathematica
Time used: 0.045 (sec). Leaf size: 28

```
DSolve[x^2*y''[x]+4*x*y'[x]+(2-x^2)*y[x]==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to \frac{2c_1 e^{-x} + c_2 e^{x}}{2x^2}$$

32.4 problem Ex 4

Internal problem ID [11305]

Book: An elementary treatise on differential equations by Abraham Cohen. DC heath publishers. 1906
Section: Chapter VIII, Linear differential equations of the second order. Article 55. Summary. Page 129
Problem number: Ex 4.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$\boxed{\left(x^2+1\right) y'' - 2y'x + 2y = 0}$$

✓ Solution by Maple
Time used: 0.0 (sec). Leaf size: 16

```
dsolve((x^2+1)*diff(y(x),x$2)-2*x*diff(y(x),x)+2*y(x)=0,y(x), singsol=all)
```

$$y(x) = c_2 x^2 + c_1 x - c_2$$

✓ Solution by Mathematica
Time used: 0.07 (sec). Leaf size: 21

```
DSolve[(x^2+1)*y''[x]-2*x*y'[x]+2*y[x]==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to c_2 x - c_1 (x-i)^2$$

32.5 problem Ex 5

Internal problem ID [11306]

Book: An elementary treatise on differential equations by Abraham Cohen. DC heath publishers. 1906
Section: Chapter VIII, Linear differential equations of the second order. Article 55. Summary. Page 129
Problem number: Ex 5.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$\boxed{xy'' - (2x-1)\,y' + y(x-1) = 0}$$

✓ Solution by Maple
Time used: 0.0 (sec). Leaf size: 13

```
dsolve(x*diff(y(x),x$2)-(2*x-1)*diff(y(x),x)+(x-1)*y(x)=0,y(x), singsol=all)
```

$$y(x) = \mathrm{e}^{x}(c_1 + c_2 \ln(x))$$

✓ Solution by Mathematica
Time used: 0.038 (sec). Leaf size: 17

```
DSolve[x*y''[x]-(2*x-1)*y'[x]+(x-1)*y[x]==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to e^{x}(c_2 \log(x) + c_1)$$

32.6 problem Ex 6

Internal problem ID [11307]

Book: An elementary treatise on differential equations by Abraham Cohen. DC heath publishers. 1906
Section: Chapter VIII, Linear differential equations of the second order. Article 55. Summary. Page 129
Problem number: Ex 6.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$\boxed{y''x^2 - 4y'x + \left(x^2 + 6\right) y = 0}$$

✓ Solution by Maple
Time used: 0.016 (sec). Leaf size: 17

```
dsolve(x^2*diff(y(x),x$2)-4*x*diff(y(x),x)+(6+x^2)*y(x)=0,y(x), singsol=all)
```

$$y(x) = x^2(c_1 \sin(x) + c_2 \cos(x))$$

✓ Solution by Mathematica
Time used: 0.05 (sec). Leaf size: 37

```
DSolve[x^2*y''[x]-4*x*y'[x]+(6+x^2)*y[x]==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to \frac{1}{2} e^{-ix} x^2 \left(2c_1 - ic_2 e^{2ix}\right)$$

32.7 problem Ex 7

Internal problem ID [11308]

Book: An elementary treatise on differential equations by Abraham Cohen. DC heath publishers. 1906
Section: Chapter VIII, Linear differential equations of the second order. Article 55. Summary. Page 129
Problem number: Ex 7.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$\boxed{\left(2x^3-1\right)y''-6x^2y'+6yx=0}$$

✓ Solution by Maple
Time used: 0.0 (sec). Leaf size: 14

```
dsolve((2*x^3-1)*diff(y(x),x$2)-6*x^2*diff(y(x),x)+6*x*y(x)=0,y(x), singsol=all)
```

$$y(x)=c_2x^3+c_1x+c_2$$

✓ Solution by Mathematica
Time used: 2.452 (sec). Leaf size: 19

```
DSolve[(2*x^3-1)*y''[x]-6*x^2*y'[x]+6*x*y[x]==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x)\to c_1x-c_2\left(x^3+1\right)$$

32.8 problem Ex 8

Internal problem ID [11309]

Book: An elementary treatise on differential equations by Abraham Cohen. DC heath publishers. 1906
Section: Chapter VIII, Linear differential equations of the second order. Article 55. Summary. Page 129
Problem number: Ex 8.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$\boxed{y''x^2 - 2x(1+x)\,y' + 2y(1+x) = x^3}$$

✓ Solution by Maple
Time used: 0.0 (sec). Leaf size: 19

```
dsolve(x^2*diff(y(x),x$2)-2*x*(1+x)*diff(y(x),x)+2*(1+x)*y(x)=x^3,y(x), singsol=all)
```

$$y(x) = -\frac{x(-2\,\mathrm{e}^{2x}c_1 - 2c_2 + x)}{2}$$

✓ Solution by Mathematica
Time used: 0.051 (sec). Leaf size: 28

```
DSolve[x^2*y''[x]-2*x*(1+x)*y'[x]+2*(1+x)*y[x]==x^3,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to -\frac{1}{4}x\left(2x - 2c_2 e^{2x} + 1 - 4c_1\right)$$

32.9 problem Ex 9

Internal problem ID [11310]

Book: An elementary treatise on differential equations by Abraham Cohen. DC heath publishers. 1906
Section: Chapter VIII, Linear differential equations of the second order. Article 55. Summary. Page 129
Problem number: Ex 9.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$\boxed{y''x^2 - 2nx(1+x)\,y' + \left(a^2x^2 + n^2 + n\right)y = 0}$$

✓ Solution by Maple
Time used: 0.047 (sec). Leaf size: 89

```
dsolve(x^2*diff(y(x),x$2)-2*n*x*(1+x)*diff(y(x),x)+(n^2+n+a^2*x^2)*y(x)=0,y(x), singsol=all)
```

$$y(x) = \mathrm{e}^{xn}x^n\left(\mathrm{WhittakerM}\left(\frac{in^2}{\sqrt{a-n}\,\sqrt{a+n}}, \frac{1}{2}, 2i\sqrt{a-n}\,\sqrt{a+n}\,x\right)c_1 + \mathrm{WhittakerW}\left(\frac{in^2}{\sqrt{a-n}\,\sqrt{a+n}}, \frac{1}{2}, 2i\sqrt{a-n}\,\sqrt{a+n}\,x\right)c_2\right)$$

✗ Solution by Mathematica
Time used: 0.0 (sec). Leaf size: 0

```
DSolve[x^2*y''[x]-2*n*x*(1+x)*y'[x]+(n^2+n+a^2*x^2)*y[x]==0,y[x],x,IncludeSingularSolutions
```

Not solved

32.10 problem Ex 10

Internal problem ID [11311]

Book: An elementary treatise on differential equations by Abraham Cohen. DC heath publishers. 1906
Section: Chapter VIII, Linear differential equations of the second order. Article 55. Summary. Page 129
Problem number: Ex 10.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$x^4 y'' + 2x^3(1+x)\,y' + yn^2 = 0$$

✓ Solution by Maple
Time used: 0.422 (sec). Leaf size: 297

```
dsolve(x^4*diff(y(x),x$2)+2*x^3*(1+x)*diff(y(x),x)+n^2*y(x)=0,y(x), singsol=all)
```

$$y(x) = \frac{c_1 \,\mathrm{HeunD}\left(8(-n^2)^{\frac{1}{4}}, \frac{-8i(-n^2)^{\frac{3}{4}}-n+8\sqrt{-n^2}\,n}{n}, -\frac{16i(-n^2)^{\frac{3}{4}}}{n}, \frac{n-8i(-n^2)^{\frac{3}{4}}-8\sqrt{-n^2}\,n}{n}, \frac{(-n^2)^{\frac{1}{4}}x-in}{(-n^2)^{\frac{1}{4}}x+in}\right) e^{\frac{i\sqrt{-n^2}\,x^2+in^2-n\,x^2}{xn}}}{}$$

✗ Solution by Mathematica
Time used: 0.0 (sec). Leaf size: 0

```
DSolve[x^4*y''[x]+2*x^3*(1+x)*y'[x]+n^2*y[x]==0,y[x],x,IncludeSingularSolutions -> True]
```

Not solved

33 Chapter IX, Miscellaneous methods for solving equations of higher order than first. Article 57. Dependent variable absent. Page 132

33.1 problem Ex 1

Internal problem ID [11312]

Book: An elementary treatise on differential equations by Abraham Cohen. DC heath publishers. 1906
Section: Chapter IX, Miscellaneous methods for solving equations of higher order than first. Article 57. Dependent variable absent. Page 132
Problem number: Ex 1.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _missing_y], [_2nd_order, _reducible, _mu_y_y1]]`

$$\boxed{\left(x^2+1\right) y'' + y'^2 = -1}$$

✓ Solution by Maple
Time used: 0.032 (sec). Leaf size: 33

```
dsolve((1+x^2)*diff(y(x),x$2)+1+diff(y(x),x)^2=0,y(x), singsol=all)
```

$$y(x) = \frac{\ln\left(c_1 x - 1\right) c_1^2 + c_2 c_1^2 + c_1 x + \ln\left(c_1 x - 1\right)}{c_1^2}$$

✓ Solution by Mathematica
Time used: 12.052 (sec). Leaf size: 33

```
DSolve[(1+x^2)*y''[x]+1+(y'[x])^2==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to -x\cot(c_1) + \csc^2(c_1)\log(-x\sin(c_1) - \cos(c_1)) + c_2$$

33.2 problem Ex 2

Internal problem ID [11313]

Book: An elementary treatise on differential equations by Abraham Cohen. DC heath publishers. 1906
Section: Chapter IX, Miscellaneous methods for solving equations of higher order than first. Article 57. Dependent variable absent. Page 132
Problem number: Ex 2.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_3rd_order, _missing_y], [_3rd_order, _with_linear_symmetrie`

✓ Solution by Maple
Time used: 0.093 (sec). Leaf size: 94

```
dsolve((x*diff(y(x),x$3)-diff(y(x),x$2))^2=diff(y(x),x$3)^2+1,y(x), singsol=all)
```

$$y(x) = \frac{(x^2+2)\sqrt{-x^2+1}}{6} + c_1 x + \frac{x \arcsin(x)}{2} + c_2$$
$$y(x) = -\frac{x^2\sqrt{-x^2+1}}{6} - \frac{\sqrt{-x^2+1}}{3} - \frac{x \arcsin(x)}{2} + c_1 x + c_2$$
$$y(x) = \frac{\sqrt{c_1^2-1}\, x^3}{6} + \frac{c_1 x^2}{2} + c_2 x + c_3$$

✓ Solution by Mathematica
Time used: 0.241 (sec). Leaf size: 75

```
DSolve[(x*y'''[x]-y''[x])^2==(y'''[x])^2+1,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to \frac{c_1 x^3}{6} - \frac{1}{2}\sqrt{1+{c_1}^2}\, x^2 + c_3 x + c_2$$
$$y(x) \to \frac{c_1 x^3}{6} + \frac{1}{2}\sqrt{1+{c_1}^2}\, x^2 + c_3 x + c_2$$

33.3 problem Ex 3

Internal problem ID [11314]

Book: An elementary treatise on differential equations by Abraham Cohen. DC heath publishers. 1906
Section: Chapter IX, Miscellaneous methods for solving equations of higher order than first. Article 57. Dependent variable absent. Page 132
Problem number: Ex 3.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _missing_y]]`

$$\boxed{y'' + y'x = x}$$

✓ Solution by Maple
Time used: 0.0 (sec). Leaf size: 23

```
dsolve(diff(y(x),x$2)+x*diff(y(x),x)=x,y(x), singsol=all)
```

$$y(x) = \frac{c_1\sqrt{\pi}\,\sqrt{2}\,\operatorname{erf}\left(\frac{x\sqrt{2}}{2}\right)}{2} + x + c_2$$

✓ Solution by Mathematica
Time used: 0.137 (sec). Leaf size: 29

```
DSolve[y''[x]+x*y'[x]==x,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to \sqrt{\frac{\pi}{2}}c_1 \operatorname{erf}\left(\frac{x}{\sqrt{2}}\right) + x + c_2$$

33.4 problem Ex 4

Internal problem ID [11315]

Book: An elementary treatise on differential equations by Abraham Cohen. DC heath publishers. 1906
Section: Chapter IX, Miscellaneous methods for solving equations of higher order than first. Article 57. Dependent variable absent. Page 132
Problem number: Ex 4.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _quadrature]]`

$$\boxed{y'' = x\,\mathrm{e}^{x}}$$

✓ Solution by Maple
Time used: 0.0 (sec). Leaf size: 15

```
dsolve(diff(y(x),x$2)=x*exp(x),y(x), singsol=all)
```

$$y(x) = (x-2)\,\mathrm{e}^{x} + c_1 x + c_2$$

✓ Solution by Mathematica
Time used: 0.033 (sec). Leaf size: 19

```
DSolve[y''[x]==x*Exp[x],y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to e^{x}(x-2) + c_2 x + c_1$$

33.5 problem Ex 5

Internal problem ID [11316]

Book: An elementary treatise on differential equations by Abraham Cohen. DC heath publishers. 1906
Section: Chapter IX, Miscellaneous methods for solving equations of higher order than first. Article 57. Dependent variable absent. Page 132
Problem number: Ex 5.
ODE order: 2.
ODE degree: 2.

CAS Maple gives this as type `[[_2nd_order, _missing_y]]`

$$\boxed{(y' - xy'')^2 - y''^2 = 1}$$

✓ Solution by Maple
Time used: 0.094 (sec). Leaf size: 63

```
dsolve((diff(y(x),x)-x*diff(y(x),x$2))^2=1+diff(y(x),x$2)^2,y(x), singsol=all)
```

$$y(x) = \frac{x\sqrt{-x^2+1}}{2} + \frac{\arcsin(x)}{2} + c_1$$
$$y(x) = -\frac{x\sqrt{-x^2+1}}{2} - \frac{\arcsin(x)}{2} + c_1$$
$$y(x) = \frac{\sqrt{c_1^2-1}\,x^2}{2} + c_1 x + c_2$$

✓ Solution by Mathematica
Time used: 0.215 (sec). Leaf size: 58

```
DSolve[(y'[x]-x*y''[x])^2==1+(y''[x])^2,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to \frac{c_1 x^2}{2} - \sqrt{1+{c_1}^2}\,x + c_2$$
$$y(x) \to \frac{c_1 x^2}{2} + \sqrt{1+{c_1}^2}\,x + c_2$$

34 Chapter IX, Miscellaneous methods for solving equations of higher order than first. Article 58. Independent variable absent. Page 135

34.1 problem Ex 1

Internal problem ID [11317]

Book: An elementary treatise on differential equations by Abraham Cohen. DC heath publishers. 1906
Section: Chapter IX, Miscellaneous methods for solving equations of higher order than first. Article 58. Independent variable absent. Page 135
Problem number: Ex 1.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _missing_x], [_2nd_order, _with_potential_symmet`

$$\boxed{yy'' - y'^2 - y^2 y' = 0}$$

✓ Solution by Maple
Time used: 0.094 (sec). Leaf size: 27

```
dsolve(y(x)*diff(y(x),x$2)-diff(y(x),x)^2-y(x)^2*diff(y(x),x)=0,y(x), singsol=all)
```

$$y(x) = 0$$
$$y(x) = -\frac{c_1 \mathrm{e}^{c_1(c_2+x)}}{-1+\mathrm{e}^{c_1(c_2+x)}}$$

✓ Solution by Mathematica
Time used: 2.444 (sec). Leaf size: 43

```
DSolve[y[x]*y''[x]-y'[x]^2-y[x]^2*y'[x]==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to -\frac{c_1 e^{c_1(x+c_2)}}{-1+e^{c_1(x+c_2)}}$$
$$y(x) \to -\frac{1}{x+c_2}$$

34.2 problem Ex 2

Internal problem ID [11318]

Book: An elementary treatise on differential equations by Abraham Cohen. DC heath publishers. 1906
Section: Chapter IX, Miscellaneous methods for solving equations of higher order than first. Article 58. Independent variable absent. Page 135
Problem number: Ex 2.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _missing_x], [_2nd_order, _reducible, _mu_x_y1]]`

$$\boxed{yy'' - y'^2 = -1}$$

✓ Solution by Maple
Time used: 0.109 (sec). Leaf size: 59

```
dsolve(y(x)*diff(y(x),x$2)-diff(y(x),x)^2+1=0,y(x), singsol=all)
```

$$y(x) = \frac{c_1\left(-\mathrm{e}^{\frac{c_2+x}{c_1}} + \mathrm{e}^{\frac{-c_2-x}{c_1}}\right)}{2}$$
$$y(x) = -\frac{c_1\left(-\mathrm{e}^{\frac{c_2+x}{c_1}} + \mathrm{e}^{\frac{-c_2-x}{c_1}}\right)}{2}$$

✓ Solution by Mathematica
Time used: 60.222 (sec). Leaf size: 85

```
DSolve[y[x]*y''[x]-y'[x]^2+1==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to -\frac{ie^{-c_1}\tanh\left(e^{c_1}(x+c_2)\right)}{\sqrt{-\operatorname{sech}^2\left(e^{c_1}(x+c_2)\right)}}$$
$$y(x) \to \frac{ie^{-c_1}\tanh\left(e^{c_1}(x+c_2)\right)}{\sqrt{-\operatorname{sech}^2\left(e^{c_1}(x+c_2)\right)}}$$

34.3 problem Ex 3

Internal problem ID [11319]

Book: An elementary treatise on differential equations by Abraham Cohen. DC heath publishers. 1906
Section: Chapter IX, Miscellaneous methods for solving equations of higher order than first. Article 58. Independent variable absent. Page 135
Problem number: Ex 3.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _missing_x], [_2nd_order, _reducible, _mu_x_y1]]`

$$\boxed{2y'' - \mathrm{e}^{y} = 0}$$

✓ Solution by Maple
Time used: 0.469 (sec). Leaf size: 20

```
dsolve(2*diff(y(x),x$2)=exp(y(x)),y(x), singsol=all)
```

$$y(x) = \ln\left(\frac{\sec\left(\frac{c_2+x}{2c_1}\right)^2}{c_1^2}\right)$$

✓ Solution by Mathematica
Time used: 60.049 (sec). Leaf size: 30

```
DSolve[2*y''[x]==Exp[y[x]],y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to \log\left(-c_1 \operatorname{sech}^2\left(\frac{1}{2}\sqrt{c_1(x+c_2)^2}\right)\right)$$

34.4 problem Ex 4

Internal problem ID [11320]

Book: An elementary treatise on differential equations by Abraham Cohen. DC heath publishers. 1906
Section: Chapter IX, Miscellaneous methods for solving equations of higher order than first. Article 58. Independent variable absent. Page 135
Problem number: Ex 4.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _missing_x], [_2nd_order, _reducible, _mu_x_y1],`

$$\boxed{yy'' + 2y' - y'^2 = 0}$$

✓ Solution by Maple
Time used: 0.109 (sec). Leaf size: 20

```
dsolve(y(x)*diff(y(x),x$2)+2*diff(y(x),x)-diff(y(x),x)^2=0,y(x), singsol=all)
```

$$y(x) = 0$$
$$y(x) = \frac{\mathrm{e}^{c_1(c_2+x)} - 2}{c_1}$$

✓ Solution by Mathematica
Time used: 2.726 (sec). Leaf size: 26

```
DSolve[y[x]*y''[x]+2*y'[x]-y'[x]^2==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to \frac{-2 + e^{c_1(x+c_2)}}{c_1}$$
$$y(x) \to \text{Indeterminate}$$

35 Chapter IX, Miscellaneous methods for solving equations of higher order than first. Article 59. Linear equations with particular integral known. Page 136

35.1 problem Ex 1

Internal problem ID [11321]

Book: An elementary treatise on differential equations by Abraham Cohen. DC heath publishers. 1906
Section: Chapter IX, Miscellaneous methods for solving equations of higher order than first. Article 59. Linear equations with particular integral known. Page 136
Problem number: Ex 1.
ODE order: 3.
ODE degree: 1.

CAS Maple gives this as type `[[_3rd_order, _with_linear_symmetries]]`

$$\boxed{\left(x^2-2x+2\right)y'''-y''x^2+2y'x-2y=0}$$

✓ Solution by Maple
Time used: 0.016 (sec). Leaf size: 17

```
dsolve((x^2-2*x+2)*diff(y(x),x$3)-x^2*diff(y(x),x$2)+2*x*diff(y(x),x)-2*y(x)=0,y(x), singsol
```

$$y(x)=c_1x+c_2x^2+c_3\mathrm{e}^x$$

✓ Solution by Mathematica
Time used: 0.124 (sec). Leaf size: 27

```
DSolve[(x^2-2*x+2)*y'''[x]-x^2*y''[x]+2*x*y'[x]-2*y[x]==0,y[x],x,IncludeSingularSolutions ->
```

$$y(x)\to\frac{1}{2}\left(c_2x^2+2c_1x+c_3e^x\right)$$

35.2 problem Ex 2

Internal problem ID [11322]

Book: An elementary treatise on differential equations by Abraham Cohen. DC heath publishers. 1906
Section: Chapter IX, Miscellaneous methods for solving equations of higher order than first. Article 59. Linear equations with particular integral known. Page 136
Problem number: Ex 2.
ODE order: 3.
ODE degree: 1.

CAS Maple gives this as type `[[_3rd_order, _with_linear_symmetries]]`

$$\boxed{xy''' - y'' - y'x + y = -x^2 + 1}$$

✓ Solution by Maple
Time used: 0.016 (sec). Leaf size: 22

```
dsolve(x*diff(y(x),x$3)-diff(y(x),x$2)-x*diff(y(x),x)+y(x)=1-x^2,y(x), singsol=all)
```

$$y(x) = x^2 + 3 + c_1 x + c_2 e^x + c_3 e^{-x}$$

✓ Solution by Mathematica
Time used: 0.242 (sec). Leaf size: 28

```
DSolve[x*y'''[x]-y''[x]-x*y'[x]+y[x]==1-x^2,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to x^2 + c_1 x - c_2 \cosh(x) + i c_3 \sinh(x) + 3$$

36 Chapter IX, Miscellaneous methods for solving equations of higher order than first. Article 60. Exact equation. Integrating factor. Page 139

36.1 problem Ex 1

Internal problem ID [11323]

Book: An elementary treatise on differential equations by Abraham Cohen. DC heath publishers. 1906
Section: Chapter IX, Miscellaneous methods for solving equations of higher order than first. Article 60. Exact equation. Integrating factor. Page 139
Problem number: Ex 1.
ODE order: 3.
ODE degree: 1.

CAS Maple gives this as type `[[_3rd_order, _missing_y]]`

$$\boxed{(x+2)^2 y''' + (x+2) y'' + y' = 1}$$

✓ Solution by Maple
Time used: 0.016 (sec). Leaf size: 35

```
dsolve((x+2)^2*diff(y(x),x$3)+(x+2)*diff(y(x),x$2)+diff(y(x),x)=1,y(x), singsol=all)
```

$$y(x) = \frac{(c_1 - c_2)(x+2)\cos(\ln(x+2))}{2} + \frac{(c_2 + c_1)(x+2)\sin(\ln(x+2))}{2} + x + c_3$$

✓ Solution by Mathematica
Time used: 0.202 (sec). Leaf size: 45

```
DSolve[(x+2)^2*y'''[x]+(x+2)*y''[x]+y'[x]==1,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to x + \frac{1}{2}(c_1 - c_2)(x+2)\cos(\log(x+2)) + \frac{1}{2}(c_1 + c_2)(x+2)\sin(\log(x+2)) + c_3$$

36.2 problem Ex 2

Internal problem ID [11324]

Book: An elementary treatise on differential equations by Abraham Cohen. DC heath publishers. 1906
Section: Chapter IX, Miscellaneous methods for solving equations of higher order than first. Article 60. Exact equation. Integrating factor. Page 139
Problem number: Ex 2.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _exact, _linear, _nonhomogeneous]]`

$$\boxed{y''x^2 + 3y'x + y = x}$$

✓ Solution by Maple
Time used: 0.016 (sec). Leaf size: 20

```
dsolve(x^2*diff(y(x),x$2)+3*x*diff(y(x),x)+y(x)=x,y(x), singsol=all)
```

$$y(x) = \frac{c_2}{x} + \frac{x}{4} + \frac{\ln(x)\, c_1}{x}$$

✓ Solution by Mathematica
Time used: 0.035 (sec). Leaf size: 26

```
DSolve[x^2*y''[x]+3*x*y'[x]+y[x]==x,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to \frac{x^2 + 4c_2 \log(x) + 4c_1}{4x}$$

36.3 problem Ex 3

Internal problem ID [11325]

Book: An elementary treatise on differential equations by Abraham Cohen. DC heath publishers. 1906
Section: Chapter IX, Miscellaneous methods for solving equations of higher order than first. Article 60. Exact equation. Integrating factor. Page 139
Problem number: Ex 3.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _exact, _linear, _nonhomogeneous]]`

$$\boxed{(x-1)^2 y'' + 4(x-1) y' + 2y = \cos(x)}$$

✓ Solution by Maple
Time used: 0.0 (sec). Leaf size: 19

```
dsolve((x-1)^2*diff(y(x),x$2)+4*(x-1)*diff(y(x),x)+2*y(x)=cos(x),y(x), singsol=all)
```

$$y(x) = \frac{c_2 + c_1 x - \cos(x)}{(-1+x)^2}$$

✓ Solution by Mathematica
Time used: 0.134 (sec). Leaf size: 24

```
DSolve[(x-1)^2*y''[x]+4*(x-1)*y'[x]+2*y[x]==Cos[x],y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to \frac{-\cos(x) + c_1(x-1) + c_2}{(x-1)^2}$$

36.4 problem Ex 4

Internal problem ID [11326]

Book: An elementary treatise on differential equations by Abraham Cohen. DC heath publishers. 1906
Section: Chapter IX, Miscellaneous methods for solving equations of higher order than first. Article 60. Exact equation. Integrating factor. Page 139
Problem number: Ex 4.
ODE order: 3.
ODE degree: 1.

CAS Maple gives this as type `[[_3rd_order, _fully, _exact, _linear]]`

$$\boxed{\left(x^3-x\right)y'''+\left(8x^2-3\right)y''+14y'x+4y=0}$$

✓ Solution by Maple
Time used: 0.015 (sec). Leaf size: 41

```
dsolve((x^3-x)*diff(y(x),x$3)+(8*x^2-3)*diff(y(x),x$2)+14*x*diff(y(x),x)+4*y(x)=0,y(x), sing
```

$$y(x)=\frac{\frac{c_3}{\sqrt{1+x}\sqrt{-1+x}}+c_1+\frac{c_2\ln\left(x+\sqrt{x^2-1}\right)}{\sqrt{x^2-1}}}{x}$$

✓ Solution by Mathematica
Time used: 0.135 (sec). Leaf size: 51

```
DSolve[(x^3-x)*y'''[x]+(8*x^2-3)*y''[x]+14*x*y'[x]+4*y[x]==0,y[x],x,IncludeSingularSolutions
```

$$y(x)\to\frac{-\frac{c_2}{\sqrt{x^2-1}}+\frac{c_3\log\left(\sqrt{x^2-1}-x\right)}{\sqrt{x^2-1}}+c_1}{x}$$

36.5 problem Ex 5

Internal problem ID [11327]

Book: An elementary treatise on differential equations by Abraham Cohen. DC heath publishers. 1906
Section: Chapter IX, Miscellaneous methods for solving equations of higher order than first. Article 60. Exact equation. Integrating factor. Page 139
Problem number: Ex 5.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_3rd_order, _exact, _nonlinear], [_3rd_order, _with_linear_s`

✓ Solution by Maple
Time used: 0.094 (sec). Leaf size: 56

```
dsolve(2*x^3*y(x)*diff(y(x),x$3)+6*x^3*diff(y(x),x)*diff(y(x),x$2)+18*x^2*y(x)*diff(y(x),x$2
```

$$y(x) = 0$$
$$y(x) = \frac{\sqrt{-x\left(c_1 x^2 + 2c_2 x - 2c_3\right)}}{x^2}$$
$$y(x) = -\frac{\sqrt{-x\left(c_1 x^2 + 2c_2 x - 2c_3\right)}}{x^2}$$

✓ Solution by Mathematica
Time used: 0.389 (sec). Leaf size: 60

```
DSolve[2*x^3*y[x]*y'''[x]+6*x^3*y'[x]*y''[x]+18*x^2*y[x]*y''[x]+18*x^2*y'[x]^2+36*x*y[x]*y'[
```

$$y(x) \to -\frac{\sqrt{c_1 x^2 + c_3 x + 2c_2}}{x^{3/2}}$$
$$y(x) \to \frac{\sqrt{c_1 x^2 + c_3 x + 2c_2}}{x^{3/2}}$$

36.6 problem Ex 6

Internal problem ID [11328]

Book: An elementary treatise on differential equations by Abraham Cohen. DC heath publishers. 1906
Section: Chapter IX, Miscellaneous methods for solving equations of higher order than first. Article 60. Exact equation. Integrating factor. Page 139
Problem number: Ex 6.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$\boxed{x^5 y'' + \left(2x^4 - x\right) y' - \left(2x^3 - 1\right) y = 0}$$

✓ Solution by Maple
Time used: 0.0 (sec). Leaf size: 16

```
dsolve(x^5*diff(y(x),x$2)+(2*x^4-x)*diff(y(x),x)-(2*x^3-1)*y(x)=0,y(x), singsol=all)
```

$$y(x) = x\left(c_1 + c_2 \mathrm{e}^{-\frac{1}{3x^3}}\right)$$

✓ Solution by Mathematica
Time used: 0.152 (sec). Leaf size: 22

```
DSolve[x^5*y''[x]+(2*x^4-x)*y'[x]-(2*x^3-1)*y[x]==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to x\left(c_2 e^{-\frac{1}{3x^3}} + c_1\right)$$

36.7 problem Ex 7

Internal problem ID [11329]

Book: An elementary treatise on differential equations by Abraham Cohen. DC heath publishers. 1906
Section: Chapter IX, Miscellaneous methods for solving equations of higher order than first. Article 60. Exact equation. Integrating factor. Page 139
Problem number: Ex 7.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$x^2\left(-x^3+1\right)y''-y'x^3-2y=0$$

✗ Solution by Maple

```
dsolve(x^2*(1-x^3)*diff(y(x),x$2)-x^3*diff(y(x),x)-2*y(x)=0,y(x), singsol=all)
```

No solution found

✗ Solution by Mathematica
Time used: 0.0 (sec). Leaf size: 0

```
DSolve[x^2*(1-x^3)*y''[x]-x^3*y'[x]-2*y[x]==0,y[x],x,IncludeSingularSolutions -> True]
```

Not solved

36.8 problem Ex 8

Internal problem ID [11330]

Book: An elementary treatise on differential equations by Abraham Cohen. DC heath publishers. 1906
Section: Chapter IX, Miscellaneous methods for solving equations of higher order than first. Article 60. Exact equation. Integrating factor. Page 139
Problem number: Ex 8.
ODE order: 3.
ODE degree: 1.

CAS Maple gives this as type `[[_3rd_order, _with_linear_symmetries]]`

$$\boxed{x^2y''' - 5xy'' + \left(4x^4 + 5\right)y' - 8yx^3 = 0}$$

✓ Solution by Maple
Time used: 0.015 (sec). Leaf size: 22

```
dsolve(x^2*diff(y(x),x$3)-5*x*diff(y(x),x$2)+(4*x^4+5)*diff(y(x),x)-8*x^3*y(x)=0,y(x), sings
```

$$y(x) = c_1x^2 + c_2\cos\left(x^2\right) + c_3\sin\left(x^2\right)$$

✓ Solution by Mathematica
Time used: 0.507 (sec). Leaf size: 44

```
DSolve[x^2*y'''[x]-5*x*y''[x]+(4*x^4+5)*y'[x]-8*x^3*y[x]==0,y[x],x,IncludeSingularSolutions
```

$$y(x) \to c_1x^2 + \frac{1}{2}ic_2e^{-ix^2} - \frac{1}{8}c_3e^{ix^2}$$

36.9 problem Ex 10

Internal problem ID [11331]

Book: An elementary treatise on differential equations by Abraham Cohen. DC heath publishers. 1906
Section: Chapter IX, Miscellaneous methods for solving equations of higher order than first. Article 60. Exact equation. Integrating factor. Page 139
Problem number: Ex 10.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _missing_y]]`

$$y'' + 2\cot(x) y' + 2\tan(x) y'^2 = 0$$

✓ Solution by Maple
Time used: 0.0 (sec). Leaf size: 21

```
dsolve(diff(y(x),x$2)+2*cot(x)*diff(y(x),x)+2*tan(x)*diff(y(x),x)^2=0,y(x), singsol=all)
```

$$y(x) = -\frac{e^{\frac{c_1}{2}} \operatorname{expIntegral}_1\left(\ln(\tan(x)) + \frac{c_1}{2}\right)}{2} + c_2$$

✗ Solution by Mathematica
Time used: 0.0 (sec). Leaf size: 0

```
DSolve[y''[x]+2*Cot[x]*y'[x]+2*Tan[x]*y'[x]^2==0,y[x],x,IncludeSingularSolutions -> True]
```

Not solved

37 Chapter IX, Miscellaneous methods for solving equations of higher order than first. Article 61. Transformation of variables. Page 143

37.1 problem Ex 1

Internal problem ID [11332]

Book: An elementary treatise on differential equations by Abraham Cohen. DC heath publishers. 1906
Section: Chapter IX, Miscellaneous methods for solving equations of higher order than first. Article 61. Transformation of variables. Page 143
Problem number: Ex 1.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries], [_2nd_order, _reducibl`

$$\boxed{x^2 y y'' + (y' x - y)^2 = 0}$$

✓ Solution by Maple
Time used: 0.047 (sec). Leaf size: 44

```
dsolve(x^2*y(x)*diff(y(x),x$2)+(x*diff(y(x),x)-y(x))^2=0,y(x), singsol=all)
```

$$y(x) = 0$$
$$y(x) = \sqrt{2}\sqrt{-x(c_1 x - c_2)}$$
$$y(x) = -\sqrt{2}\sqrt{-x(c_1 x - c_2)}$$

✓ Solution by Mathematica
Time used: 0.388 (sec). Leaf size: 23

```
DSolve[x^2*y[x]*y''[x]+(x*y'[x]-y[x])^2==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to c_2 \sqrt{x} \sqrt{2x + c_1}$$

37.2 problem Ex 2

Internal problem ID [11333]

Book: An elementary treatise on differential equations by Abraham Cohen. DC heath publishers. 1906
Section: Chapter IX, Miscellaneous methods for solving equations of higher order than first. Article 61. Transformation of variables. Page 143
Problem number: Ex 2.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries], [_2nd_order, _reducibl`

$$\boxed{x^3 y'' - (y' x - y)^2 = 0}$$

✓ Solution by Maple
Time used: 0.031 (sec). Leaf size: 19

```
dsolve(x^3*diff(y(x),x$2)-(x*diff(y(x),x)-y(x))^2=0,y(x), singsol=all)
```

$$y(x) = -x \ln\left(\frac{c_1 x - c_2}{x}\right)$$

✓ Solution by Mathematica
Time used: 1.65 (sec). Leaf size: 21

```
DSolve[x^3*y''[x]-(x*y'[x]-y[x])^2==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to -x \log\left(-\frac{c_2 x + c_1}{x}\right)$$

37.3 problem Ex 3

Internal problem ID [11334]

Book: An elementary treatise on differential equations by Abraham Cohen. DC heath publishers. 1906
Section: Chapter IX, Miscellaneous methods for solving equations of higher order than first. Article 61. Transformation of variables. Page 143
Problem number: Ex 3.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _reducible, _mu_xy]]`

$$\boxed{yy'' - y'^2 - y^2 \ln(y) + x^2 y^2 = 0}$$

✓ Solution by Maple
Time used: 0.046 (sec). Leaf size: 22

```
dsolve(y(x)*diff(y(x),x$2)-diff(y(x),x)^2=y(x)^2*ln(y(x))-x^2*y(x)^2,y(x), singsol=all)
```

$$y(x) = \mathrm{e}^{x^2+2-\frac{c_2 \mathrm{e}^x}{2}+\frac{c_1 \mathrm{e}^{-x}}{2}}$$

✓ Solution by Mathematica
Time used: 1.156 (sec). Leaf size: 30

```
DSolve[y[x]*y''[x]-y'[x]^2==y[x]^2*Log[y[x]]-x^2*y[x]^2,y[x],x,IncludeSingularSolutions -> T
```

$$y(x) \to e^{x^2-\frac{c_1 e^x}{2}-c_2 e^{-x}+2}$$

37.4 problem Ex 4

Internal problem ID [11335]

Book: An elementary treatise on differential equations by Abraham Cohen. DC heath publishers. 1906
Section: Chapter IX, Miscellaneous methods for solving equations of higher order than first. Article 61. Transformation of variables. Page 143
Problem number: Ex 4.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$\boxed{\sin(x)^2 y'' - 2y = 0}$$

✓ Solution by Maple
Time used: 0.344 (sec). Leaf size: 31

```
dsolve(sin(x)^2*diff(y(x),x$2)-2*y(x)=0,y(x), singsol=all)
```

$$y(x) = -i\cot(x)\ln(\cos(2x) + i\sin(2x))c_2 - 2c_2 + c_1\cot(x)$$

✓ Solution by Mathematica
Time used: 0.339 (sec). Leaf size: 46

```
DSolve[Sin[x]^2*y''[x]-2*y[x]==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to \frac{\cos(x)\left(c_1 - c_2\log\left(\sqrt{-\sin^2(x)} - \cos(x)\right)\right)}{\sqrt{-\sin^2(x)}} - c_2$$

38 Chapter IX, Miscellaneous methods for solving equations of higher order than first. Article 62. Summary. Page 144

38.1 problem Ex 1

Internal problem ID [11336]

Book: An elementary treatise on differential equations by Abraham Cohen. DC heath publishers. 1906
Section: Chapter IX, Miscellaneous methods for solving equations of higher order than first. Article 62. Summary. Page 144
Problem number: Ex 1.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _missing_x], [_2nd_order, _reducible, _mu_xy]]`

$$\boxed{y'' - y'^2 = 1}$$

✓ Solution by Maple
Time used: 0.031 (sec). Leaf size: 17

```
dsolve(diff(y(x),x$2)=diff(y(x),x)^2+1,y(x), singsol=all)
```

$$y(x) = -\ln\left(c_1 \sin\left(x\right) - c_2 \cos\left(x\right)\right)$$

✓ Solution by Mathematica
Time used: 3.079 (sec). Leaf size: 16

```
DSolve[y''[x]==y'[x]^2+1,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to c_2 - \log(\cos(x + c_1))$$

38.2 problem Ex 2

Internal problem ID [11337]

Book: An elementary treatise on differential equations by Abraham Cohen. DC heath publishers. 1906
Section: Chapter IX, Miscellaneous methods for solving equations of higher order than first. Article 62. Summary. Page 144
Problem number: Ex 2.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _missing_y]]`

$$\boxed{\left(-x^2+1\right)y''-y'x=2}$$

✓ Solution by Maple
Time used: 0.235 (sec). Leaf size: 59

```
dsolve((1-x^2)*diff(y(x),x$2)-x*diff(y(x),x)=2,y(x), singsol=all)
```

$$y(x)=-\left(\int -\frac{-2\sqrt{x^2-1}\,\ln\left(x+\sqrt{x^2-1}\right)\sqrt{-1+x}\,\sqrt{1+x}+c_1(x^2-1)}{(-1+x)^{\frac{3}{2}}\,(1+x)^{\frac{3}{2}}}dx\right)+c_2$$

✓ Solution by Mathematica
Time used: 0.05 (sec). Leaf size: 48

```
DSolve[(1-x^2)*y''[x]-x*y'[x]==2,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x)\to c_2-\frac{1}{4}\left(\log\left(1-\frac{x}{\sqrt{x^2-1}}\right)-\log\left(\frac{x}{\sqrt{x^2-1}}+1\right)+c_1\right)^2$$

38.3 problem Ex 3

Internal problem ID [11338]

Book: An elementary treatise on differential equations by Abraham Cohen. DC heath publishers. 1906
Section: Chapter IX, Miscellaneous methods for solving equations of higher order than first. Article 62. Summary. Page 144
Problem number: Ex 3.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _missing_x], [_2nd_order, _exact, _nonlinear], _`

$$\boxed{y'' + yy' = 0}$$

✓ Solution by Maple
Time used: 0.094 (sec). Leaf size: 23

```
dsolve(diff(y(x),x$2)+y(x)*diff(y(x),x)=0,y(x), singsol=all)
```

$$y(x) = \frac{\tanh\left(\frac{(c_2+x)\sqrt{2}}{2c_1}\right)\sqrt{2}}{c_1}$$

✓ Solution by Mathematica
Time used: 20.03 (sec). Leaf size: 34

```
DSolve[y''[x]+y[x]*y'[x]==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to \sqrt{2}\sqrt{c_1}\tanh\left(\frac{\sqrt{c_1}(x+c_2)}{\sqrt{2}}\right)$$

38.4 problem Ex 4

Internal problem ID [11339]

Book: An elementary treatise on differential equations by Abraham Cohen. DC heath publishers. 1906
Section: Chapter IX, Miscellaneous methods for solving equations of higher order than first. Article 62. Summary. Page 144
Problem number: Ex 4.
ODE order: 3.
ODE degree: 1.

CAS Maple gives this as type `[[_3rd_order, _fully, _exact, _linear]]`

$$\boxed{\left(x^3+1\right)y'''+9y''x^2+18y'x+6y=0}$$

✓ Solution by Maple
Time used: 0.0 (sec). Leaf size: 30

```
dsolve((1+x^3)*diff(y(x),x$3)+9*x^2*diff(y(x),x$2)+18*x*diff(y(x),x)+6*y(x)=0,y(x), singsol=
```

$$y(x)=\frac{c_1x^2+c_2x+c_3}{(1+x)\left(x^2-x+1\right)}$$

✓ Solution by Mathematica
Time used: 0.015 (sec). Leaf size: 31

```
DSolve[(1+x^3)*y'''[x]+9*x^2*y''[x]+18*x*y'[x]+6*y[x]==0,y[x],x,IncludeSingularSolutions ->
```

$$y(x)\to\frac{c_3x^2+2c_2x+2c_1}{2x^3+2}$$

38.5 problem Ex 5

Internal problem ID [11340]

Book: An elementary treatise on differential equations by Abraham Cohen. DC heath publishers. 1906
Section: Chapter IX, Miscellaneous methods for solving equations of higher order than first. Article 62. Summary. Page 144
Problem number: Ex 5.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _exact, _linear, _homogeneous]]`

$$\boxed{\left(x^2-x\right)y''+\left(4x+2\right)y'+2y=0}$$

✓ Solution by Maple
Time used: 0.0 (sec). Leaf size: 41

```
dsolve((x^2-x)*diff(y(x),x$2)+(4*x+2)*diff(y(x),x)+2*y(x)=0,y(x), singsol=all)
```

$$y(x)=\frac{12\ln\left(x\right)c_1x^3+\left(-3x^4+18x^2-6x+1\right)c_1+c_2x^3}{\left(-1+x\right)^5}$$

✓ Solution by Mathematica
Time used: 0.086 (sec). Leaf size: 52

```
DSolve[(x^2-x)*y''[x]+(4*x+2)*y'[x]+2*y[x]==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x)\to\frac{-3c_2x^4-3c_1x^3+12c_2x^3\log(x)+18c_2x^2-6c_2x+c_2}{3(x-1)^5}$$

38.6 problem Ex 6

Internal problem ID [11341]

Book: An elementary treatise on differential equations by Abraham Cohen. DC heath publishers. 1906
Section: Chapter IX, Miscellaneous methods for solving equations of higher order than first. Article 62. Summary. Page 144
Problem number: Ex 6.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _missing_x], _Liouville, [_2nd_order, _reducible`

$$\boxed{y(1-\ln(y))\,y''+(1+\ln(y))\,y'^2=0}$$

✓ Solution by Maple
Time used: 0.016 (sec). Leaf size: 19

```
dsolve(y(x)*(1-ln(y(x)))*diff(y(x),x$2)+(1+ln(y(x)))*diff(y(x),x)^2=0,y(x), singsol=all)
```

$$y(x)=\mathrm{e}^{\frac{c_1x+c_2-1}{c_1x+c_2}}$$

✓ Solution by Mathematica
Time used: 1.021 (sec). Leaf size: 34

```
DSolve[y[x]*(1-Log[y[x]])*y''[x]+(1+Log[y[x]])*y'[x]^2==0,y[x],x,IncludeSingularSolutions ->
```

$$y(x)\to e^{\frac{c_1x-1+c_2c_1}{c_1(x+c_2)}}$$
$$y(x)\to e$$

38.7 problem Ex 7

Internal problem ID [11342]

Book: An elementary treatise on differential equations by Abraham Cohen. DC heath publishers. 1906
Section: Chapter IX, Miscellaneous methods for solving equations of higher order than first. Article 62. Summary. Page 144
Problem number: Ex 7.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _missing_y]]`

$$\boxed{y'' + \frac{y'}{x} = 0}$$

✓ Solution by Maple
Time used: 0.0 (sec). Leaf size: 10

```
dsolve(diff(y(x),x$2)+1/x*diff(y(x),x)=0,y(x), singsol=all)
```

$$y(x) = c_1 + c_2 \ln(x)$$

✓ Solution by Mathematica
Time used: 0.018 (sec). Leaf size: 13

```
DSolve[y''[x]+1/x*y'[x]==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to c_1 \log(x) + c_2$$

38.8 problem Ex 8

Internal problem ID [11343]

Book: An elementary treatise on differential equations by Abraham Cohen. DC heath publishers. 1906
Section: Chapter IX, Miscellaneous methods for solving equations of higher order than first. Article 62. Summary. Page 144
Problem number: Ex 8.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _exact, _nonlinear], [_2nd_order, _reducible, _m`

$$\boxed{x(x+2y)\,y''+2xy'^2+4(y+x)\,y'+2y=-x^2}$$

✓ Solution by Maple
Time used: 0.046 (sec). Leaf size: 80

```
dsolve(x*(x+2*y(x))*diff(y(x),x$2)+2*x*(diff(y(x),x))^2+4*(x+y(x))*diff(y(x),x)+2*y(x)+x^2=0
```

$$y(x)=\frac{-3x^2+\sqrt{3}\,\sqrt{-x\left(x^4-3x^3+12c_2x-12c_1\right)}}{6x}$$
$$y(x)=\frac{-3x^2-\sqrt{3}\,\sqrt{-x\left(x^4-3x^3+12c_2x-12c_1\right)}}{6x}$$

✓ Solution by Mathematica
Time used: 2.35 (sec). Leaf size: 104

```
DSolve[x*(x+2*y[x])*y''[x]+2*x*(y'[x])^2+4*(x+y[x])*y'[x]+2*y[x]+x^2==0,y[x],x,IncludeSingul
```

$$y(x)\to\frac{1}{6}\left(-3x-\sqrt{3}\sqrt{\frac{1}{x^2}}\sqrt{x\left(-x^4+3x^3+12c_2x+12c_1\right)}\right)$$
$$y(x)\to\frac{1}{6}\left(-3x+\sqrt{3}\sqrt{\frac{1}{x^2}}\sqrt{x\left(-x^4+3x^3+12c_2x+12c_1\right)}\right)$$

38.9 problem Ex 9

Internal problem ID [11344]

Book: An elementary treatise on differential equations by Abraham Cohen. DC heath publishers. 1906
Section: Chapter IX, Miscellaneous methods for solving equations of higher order than first. Article 62. Summary. Page 144
Problem number: Ex 9.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _missing_x], [_2nd_order, _reducible, _mu_xy]]`

$$\boxed{y'' + y'^2 = -1}$$

✓ Solution by Maple
Time used: 0.016 (sec). Leaf size: 15

```
dsolve(diff(y(x),x$2)+diff(y(x),x)^2+1=0,y(x), singsol=all)
```

$$y(x) = \ln\left(-c_1 \sin(x) + c_2 \cos(x)\right)$$

✓ Solution by Mathematica
Time used: 3.113 (sec). Leaf size: 16

```
DSolve[y''[x]+y'[x]^2+1==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to \log(\cos(x - c_1)) + c_2$$

38.10 problem Ex 10

Internal problem ID [11345]

Book: An elementary treatise on differential equations by Abraham Cohen. DC heath publishers. 1906
Section: Chapter IX, Miscellaneous methods for solving equations of higher order than first. Article 62. Summary. Page 144
Problem number: Ex 10.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _missing_y]]`

$$\boxed{\left(-x^2+1\right) y'' - \frac{y'}{x} = -x^2}$$

✓ Solution by Maple
Time used: 0.0 (sec). Leaf size: 23

```
dsolve((1-x^2)*diff(y(x),x$2)-1/x*diff(y(x),x)+x^2=0,y(x), singsol=all)
```

$$y(x) = \frac{x^2}{2} + \sqrt{-1+x}\,\sqrt{1+x}\,c_1 + c_2$$

✓ Solution by Mathematica
Time used: 0.084 (sec). Leaf size: 30

```
DSolve[(1-x^2)*y''[x]-1/x*y'[x]+x^2==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to \frac{x^2}{2} - c_1\sqrt{1-x^2} + c_2$$

38.11 problem Ex 11

Internal problem ID [11346]

Book: An elementary treatise on differential equations by Abraham Cohen. DC heath publishers. 1906
Section: Chapter IX, Miscellaneous methods for solving equations of higher order than first. Article 62. Summary. Page 144
Problem number: Ex 11.
ODE order: 3.
ODE degree: 1.

CAS Maple gives this as type `[[_3rd_order, _missing_y]]`

$$\boxed{4x^2y''' + 8xy'' + y' = 0}$$

✓ Solution by Maple
Time used: 0.0 (sec). Leaf size: 16

```
dsolve(4*x^2*diff(y(x),x$3)+8*x*diff(y(x),x$2)+diff(y(x),x)=0,y(x), singsol=all)
```

$$y(x) = (c_3 \ln(x) + c_2)\sqrt{x} + c_1$$

✓ Solution by Mathematica
Time used: 0.05 (sec). Leaf size: 28

```
DSolve[4*x^2*y'''[x]+8*x*y''[x]+y'[x]==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to \sqrt{x}(c_2 \log(x) + 2c_1 - 2c_2) + c_3$$

38.12 problem Ex 12

Internal problem ID [11347]

Book: An elementary treatise on differential equations by Abraham Cohen. DC heath publishers. 1906
Section: Chapter IX, Miscellaneous methods for solving equations of higher order than first. Article 62. Summary. Page 144
Problem number: Ex 12.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _exact, _linear, _homogeneous]]`

$$\sin(x) y'' - \cos(x) y' + 2\sin(x) y = 0$$

✓ Solution by Maple
Time used: 0.14 (sec). Leaf size: 39

```
dsolve(sin(x)*diff(y(x),x$2)-cos(x)*diff(y(x),x)+2*sin(x)*y(x)=0,y(x), singsol=all)
```

$$y(x) = \ln(\cos(x) - 1) c_2 \sin(x)^2 - \ln(\cos(x) + 1) c_2 \sin(x)^2 + c_1 \sin(x)^2 - 2c_2 \cos(x)$$

✓ Solution by Mathematica
Time used: 0.235 (sec). Leaf size: 45

```
DSolve[Sin[x]*y''[x]-Cos[x]*y'[x]+2*Sin[x]*y[x]==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to -c_1 \sin^2(x) - \frac{1}{4} c_2 \left(2\cos(x) + \sin^2(x)(\log(\cos(x)+1) - \log(1-\cos(x)))\right)$$

39 Chapter X, System of simulataneous equations. Article 64. Systems of linear equations with constant coefficients. Page 150

39.1 problem Ex 1

Internal problem ID [11348]

Book: An elementary treatise on differential equations by Abraham Cohen. DC heath publishers. 1906
Section: Chapter X, System of simulataneous equations. Article 64. Systems of linear equations with constant coefficients. Page 150
Problem number: Ex 1.
ODE order: 1.
ODE degree: 1.

Solve

$$x'(t) = -x(t) - \frac{2y(t)}{3} + \frac{\mathrm{e}^t}{3}$$
$$y'(t) = \frac{4x(t)}{3} + y(t) - t$$

✓ Solution by Maple
Time used: 0.062 (sec). Leaf size: 47

```
dsolve([3*diff(x(t),t)+3*x(t)+2*y(t)=exp(t),4*x(t)-3*diff(y(t),t)+3*y(t)=3*t],singsol=all)
```

$$x(t) = -\frac{\mathrm{e}^{\frac{t}{3}} c_2}{2} - \mathrm{e}^{-\frac{t}{3}} c_1 - 6t$$
$$y(t) = \mathrm{e}^{\frac{t}{3}} c_2 + \mathrm{e}^{-\frac{t}{3}} c_1 + 9t + 9 + \frac{\mathrm{e}^t}{2}$$

✓ Solution by Mathematica
Time used: 1.125 (sec). Leaf size: 90

```
DSolve[{3*x'[t]+3*x[t]+2*y[t]==Exp[t],4*x[t]-3*y'[t]+3*y[t]==3*t},{x[t],y[t]},t,IncludeSingu
```

$$x(t) \to e^{-t/3}\left(-6e^{t/3}t - (c_1 + c_2)e^{2t/3} + 2c_1 + c_2\right)$$
$$y(t) \to 9(t+1) + \frac{e^t}{2} + 2(c_1 + c_2)e^{t/3} - (2c_1 + c_2)e^{-t/3}$$