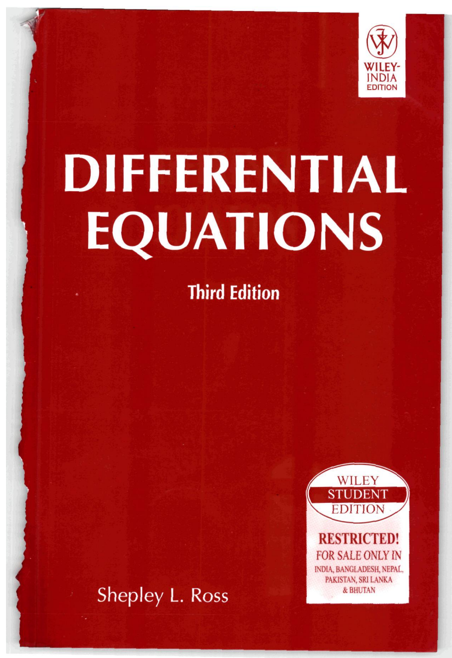
A Solution Manual For

# Differential Equations by Shepley L. Ross. Third edition. John Willey. New Delhi. 2004.

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# Contents

# 1 Chapter 1, Differential equations and their solutions. Exercises page 13

## 1.1 problem 1(a)

Internal problem ID [11570]

**Book**: Differential Equations by Shepley L. Ross. Third edition. John Willey. New Delhi. 2004.
**Section**: Chapter 1, Differential equations and their solutions. Exercises page 13
**Problem number**: 1(a).
**ODE order**: 1.
**ODE degree**: 1.

CAS Maple gives this as type `[[_linear, ‘class A‘]]`

$$\boxed{y' + y = 1 + x}$$

✓ Solution by Maple
Time used: 0.0 (sec). Leaf size: 12

```
dsolve(diff(y(x),x)+y(x)=1+x,y(x), singsol=all)
```

$$y(x) = x + c_1 e^{-x}$$

✓ Solution by Mathematica
Time used: 0.04 (sec). Leaf size: 15

```
DSolve[y'[x]+y[x]==1+x,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to x + c_1 e^{-x}$$

## 1.2 problem 1(b)

Internal problem ID [11571]

**Book**: Differential Equations by Shepley L. Ross. Third edition. John Willey. New Delhi. 2004.
**Section**: Chapter 1, Differential equations and their solutions. Exercises page 13
**Problem number**: 1(b).
**ODE order**: 2.
**ODE degree**: 1.

CAS Maple gives this as type `[[_2nd_order, _missing_x]]`

$$\boxed{y'' - 7y' + 12y = 0}$$

✓ Solution by Maple
Time used: 0.0 (sec). Leaf size: 17

```
dsolve(diff(y(x),x$2)-7*diff(y(x),x)+12*y(x)=0,y(x), singsol=all)
```

$$y(x) = c_1 \mathrm{e}^{4x} + c_2 \mathrm{e}^{3x}$$

✓ Solution by Mathematica
Time used: 0.022 (sec). Leaf size: 20

```
DSolve[y''[x]-7*y'[x]+12*y[x]==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to e^{3x}(c_2 e^x + c_1)$$

## 1.3 problem 1(c)

Internal problem ID [11572]

**Book**: Differential Equations by Shepley L. Ross. Third edition. John Willey. New Delhi. 2004.
**Section**: Chapter 1, Differential equations and their solutions. Exercises page 13
**Problem number**: 1(c).
**ODE order**: 2.
**ODE degree**: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$\boxed{y'' - 3y' + 2y = 4x^2}$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 24

```
dsolve(diff(y(x),x$2)-3*diff(y(x),x)+2*y(x)=4*x^2,y(x), singsol=all)
```

$$y(x) = e^{2x} c_1 + c_2 e^x + 2x^2 + 6x + 7$$

✓ Solution by Mathematica

Time used: 0.03 (sec). Leaf size: 29

```
DSolve[y''[x]-3*y'[x]+2*y[x]==4*x^2,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to 2x^2 + 6x + c_1 e^x + c_2 e^{2x} + 7$$

## 1.4 problem 1(d)

Internal problem ID [11573]

**Book**: Differential Equations by Shepley L. Ross. Third edition. John Willey. New Delhi. 2004.
**Section**: Chapter 1, Differential equations and their solutions. Exercises page 13
**Problem number**: 1(d).
**ODE order**: 2.
**ODE degree**: 1.

CAS Maple gives this as type `[[_2nd_order, _exact, _linear, _homogeneous]]`

$$\boxed{(x^2+1)\,y''+4y'x+2y=0}$$

✓ Solution by Maple
Time used: 0.0 (sec). Leaf size: 17

```
dsolve((1+x^2)*diff(y(x),x$2)+4*x*diff(y(x),x)+2*y(x)=0,y(x), singsol=all)
```

$$y(x)=\frac{c_1x+c_2}{x^2+1}$$

✓ Solution by Mathematica
Time used: 0.039 (sec). Leaf size: 20

```
DSolve[(1+x^2)*y''[x]+4*x*y'[x]+2*y[x]==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x)\to\frac{c_2x+c_1}{x^2+1}$$

## 1.5 problem 2(a)

Internal problem ID [11574]

**Book**: Differential Equations by Shepley L. Ross. Third edition. John Willey. New Delhi. 2004.
**Section**: Chapter 1, Differential equations and their solutions. Exercises page 13
**Problem number**: 2(a).
**ODE order**: 1.
**ODE degree**: 1.

CAS Maple gives this as type `[[_homogeneous, ‘class A‘], _exact, _rational, _Bernoulli]`

$$\boxed{2xyy' + y^2 = -x^2}$$

✓ Solution by Maple
Time used: 0.0 (sec). Leaf size: 47

```
dsolve(2*x*y(x)*diff(y(x),x)+x^2+y(x)^2=0,y(x), singsol=all)
```

$$y(x) = -\frac{\sqrt{3}\,\sqrt{-x\,(x^3 - 3c_1)}}{3x}$$
$$y(x) = \frac{\sqrt{3}\,\sqrt{-x\,(x^3 - 3c_1)}}{3x}$$

✓ Solution by Mathematica
Time used: 0.377 (sec). Leaf size: 60

```
DSolve[2*x*y[x]*y'[x]+x^2+y[x]^2==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to -\frac{\sqrt{-x^3 + 3c_1}}{\sqrt{3}\sqrt{x}}$$
$$y(x) \to \frac{\sqrt{-x^3 + 3c_1}}{\sqrt{3}\sqrt{x}}$$

## 1.6 problem 2(b)

Internal problem ID [11575]

**Book**: Differential Equations by Shepley L. Ross. Third edition. John Willey. New Delhi. 2004.
**Section**: Chapter 1, Differential equations and their solutions. Exercises page 13
**Problem number**: 2(b).
**ODE order**: 1.
**ODE degree**: 1.

CAS Maple gives this as type `[[_homogeneous, ‘class G‘], _rational, _Bernoulli]`

$$\boxed{y'x + y - y^3x^3 = 0}$$

✓ Solution by Maple
Time used: 0.0 (sec). Leaf size: 30

```
dsolve(x*diff(y(x),x)+y(x)=x^3*y(x)^3,y(x), singsol=all)
```

$$y(x) = \frac{1}{\sqrt{-2x + c_1}\, x}$$
$$y(x) = -\frac{1}{\sqrt{-2x + c_1}\, x}$$

✓ Solution by Mathematica
Time used: 0.6 (sec). Leaf size: 44

```
DSolve[x*y'[x]+y[x]==x^3*y[x]^3,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to -\frac{1}{\sqrt{x^2(-2x + c_1)}}$$
$$y(x) \to \frac{1}{\sqrt{x^2(-2x + c_1)}}$$
$$y(x) \to 0$$

## 1.7 problem 3(a)

Internal problem ID [11576]

**Book**: Differential Equations by Shepley L. Ross. Third edition. John Willey. New Delhi. 2004.
**Section**: Chapter 1, Differential equations and their solutions. Exercises page 13
**Problem number**: 3(a).
**ODE order**: 1.
**ODE degree**: 1.

CAS Maple gives this as type `[[_linear, 'class A']]`

$$\boxed{y' + 3y = 3x^2 \mathrm{e}^{-3x}}$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 14

```
dsolve(diff(y(x),x)+3*y(x)=3*x^2*exp(-3*x),y(x), singsol=all)
```

$$y(x) = \left(x^3 + c_1\right) \mathrm{e}^{-3x}$$

✓ Solution by Mathematica

Time used: 0.098 (sec). Leaf size: 17

```
DSolve[y'[x]+3*y[x]==3*x^2*Exp[-3*x],y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to e^{-3x}\left(x^3 + c_1\right)$$

## 1.8 problem 3(b)

Internal problem ID [11577]

**Book**: Differential Equations by Shepley L. Ross. Third edition. John Willey. New Delhi. 2004.
**Section**: Chapter 1, Differential equations and their solutions. Exercises page 13
**Problem number**: 3(b).
**ODE order**: 1.
**ODE degree**: 1.

CAS Maple gives this as type **[_separable]**

$$\boxed{y' + 4yx = 8x}$$

✓ Solution by Maple
Time used: 0.0 (sec). Leaf size: 14

```
dsolve(diff(y(x),x)+4*x*y(x)=8*x,y(x), singsol=all)
```

$$y(x) = 2 + \mathrm{e}^{-2x^2} c_1$$

✓ Solution by Mathematica
Time used: 0.074 (sec). Leaf size: 22

```
DSolve[y'[x]+4*x*y[x]==8*x,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to 2 + c_1 e^{-2x^2}$$
$$y(x) \to 2$$

## 1.9 problem 4(a)

Internal problem ID [11578]

**Book**: Differential Equations by Shepley L. Ross. Third edition. John Willey. New Delhi. 2004.
**Section**: Chapter 1, Differential equations and their solutions. Exercises page 13
**Problem number**: 4(a).
**ODE order**: 2.
**ODE degree**: 1.

CAS Maple gives this as type `[[_2nd_order, _missing_x]]`

$$\boxed{y'' - 2y' - 8y = 0}$$

✓ Solution by Maple
Time used: 0.0 (sec). Leaf size: 17

```
dsolve(diff(y(x),x$2)-2*diff(y(x),x)-8*y(x)=0,y(x), singsol=all)
```

$$y(x) = c_1 e^{4x} + e^{-2x} c_2$$

✓ Solution by Mathematica
Time used: 0.023 (sec). Leaf size: 22

```
DSolve[y''[x]-2*y'[x]-8*y[x]==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to e^{-2x}\left(c_2 e^{6x} + c_1\right)$$

## 1.10 problem 4(b)

Internal problem ID [11579]

**Book**: Differential Equations by Shepley L. Ross. Third edition. John Willey. New Delhi. 2004.
**Section**: Chapter 1, Differential equations and their solutions. Exercises page 13
**Problem number**: 4(b).
**ODE order**: 3.
**ODE degree**: 1.

CAS Maple gives this as type `[[_3rd_order, _missing_x]]`

$$\boxed{y''' - 2y'' - 4y' + 8y = 0}$$

✓ Solution by Maple
Time used: 0.0 (sec). Leaf size: 21

```
dsolve(diff(y(x),x$3)-2*diff(y(x),x$2)-4*diff(y(x),x)+8*y(x)=0,y(x), singsol=all)
```

$$y(x) = (c_3 x + c_2) e^{2x} + e^{-2x} c_1$$

✓ Solution by Mathematica
Time used: 0.005 (sec). Leaf size: 27

```
DSolve[y'''[x]-2*y''[x]-4*y'[x]+8*y[x]==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to e^{-2x} \left(e^{4x} (c_3 x + c_2) + c_1\right)$$

## 1.11 problem 5(a)

Internal problem ID [11580]

**Book**: Differential Equations by Shepley L. Ross. Third edition. John Willey. New Delhi. 2004.
**Section**: Chapter 1, Differential equations and their solutions. Exercises page 13
**Problem number**: 5(a).
**ODE order**: 3.
**ODE degree**: 1.

CAS Maple gives this as type `[[_3rd_order, _missing_x]]`

$$\boxed{y''' - 3y'' - 4y' + 12y = 0}$$

✓ Solution by Maple
Time used: 0.0 (sec). Leaf size: 23

```
dsolve(diff(y(x),x$3)-3*diff(y(x),x$2)-4*diff(y(x),x)+12*y(x)=0,y(x), singsol=all)
```

$$y(x) = \left(c_1 e^{5x} + c_2 e^{4x} + c_3\right) e^{-2x}$$

✓ Solution by Mathematica
Time used: 0.005 (sec). Leaf size: 29

```
DSolve[y'''[x]-3*y''[x]-4*y'[x]+12*y[x]==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to e^{-2x}\left(e^{4x}(c_3 e^x + c_2) + c_1\right)$$

## 1.12 problem 5(b)

Internal problem ID [11581]

**Book**: Differential Equations by Shepley L. Ross. Third edition. John Willey. New Delhi. 2004.
**Section**: Chapter 1, Differential equations and their solutions. Exercises page 13
**Problem number**: 5(b).
**ODE order**: 3.
**ODE degree**: 1.

CAS Maple gives this as type `[[_3rd_order, _fully, _exact, _linear]]`

$$\boxed{x^3y''' + 2x^2y'' - 10y'x - 8y = 0}$$

✓ Solution by Maple

Time used: 0.016 (sec). Leaf size: 18

```
dsolve(x^3*diff(y(x),x$3)+2*x^2*diff(y(x),x$2)-10*x*diff(y(x),x)-8*y(x)=0,y(x), singsol=all)
```

$$y(x) = \frac{c_1x^6 + c_2x + c_3}{x^2}$$

✓ Solution by Mathematica

Time used: 0.006 (sec). Leaf size: 22

```
DSolve[x^3*y'''[x]+2*x^2*y''[x]-10*x*y'[x]-8*y[x]==0,y[x],x,IncludeSingularSolutions -> True
```

$$y(x) \to \frac{c_3x^6 + c_2x + c_1}{x^2}$$

## 1.13 problem 6(a)

Internal problem ID [11582]

**Book**: Differential Equations by Shepley L. Ross. Third edition. John Willey. New Delhi. 2004.
**Section**: Chapter 1, Differential equations and their solutions. Exercises page 13
**Problem number**: 6(a).
**ODE order**: 1.
**ODE degree**: 1.

CAS Maple gives this as type `[[_linear, 'class A']]`

$$\boxed{y' + 2y = 6\,\mathrm{e}^{x} + 4x\,\mathrm{e}^{-2x}}$$

✓ Solution by Maple
Time used: 0.0 (sec). Leaf size: 22

```
dsolve(diff(y(x),x)+2*y(x)=6*exp(x)+4*x*exp(-2*x),y(x), singsol=all)
```

$$y(x) = \left(2x^2 + 2\,\mathrm{e}^{3x} + c_1\right)\mathrm{e}^{-2x}$$

✓ Solution by Mathematica
Time used: 0.129 (sec). Leaf size: 26

```
DSolve[y'[x]+2*y[x]==6*Exp[x]+4*x*Exp[-2*x],y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to e^{-2x}\left(2x^2 + 2e^{3x} + c_1\right)$$

## 1.14 problem 6(b)

Internal problem ID [11583]

**Book**: Differential Equations by Shepley L. Ross. Third edition. John Willey. New Delhi. 2004.
**Section**: Chapter 1, Differential equations and their solutions. Exercises page 13
**Problem number**: 6(b).
**ODE order**: 2.
**ODE degree**: 1.

CAS Maple gives this as type `[[_2nd_order, _linear, _nonhomogeneous]]`

$$\boxed{y'' - 4y' + 4y = -8\sin(2x)}$$

✓ Solution by Maple
Time used: 0.0 (sec). Leaf size: 21

```
dsolve(diff(y(x),x$2)-4*diff(y(x),x)+4*y(x)=-8*sin(2*x),y(x), singsol=all)
```

$$y(x) = (c_1 x + c_2)\,\mathrm{e}^{2x} - \cos(2x)$$

✓ Solution by Mathematica
Time used: 0.039 (sec). Leaf size: 25

```
DSolve[y''[x]-4*y'[x]+4*y[x]==-8*Sin[2*x],y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to -\cos(2x) + e^{2x}(c_2 x + c_1)$$

## 1.15 problem 7(a)

Internal problem ID [11584]

**Book**: Differential Equations by Shepley L. Ross. Third edition. John Willey. New Delhi. 2004.
**Section**: Chapter 1, Differential equations and their solutions. Exercises page 13
**Problem number**: 7(a).
**ODE order**: 1.
**ODE degree**: 2.

CAS Maple gives this as type `[_quadrature]`

$$\boxed{y'^2 - 4y = 0}$$

✓ Solution by Maple
Time used: 0.078 (sec). Leaf size: 15

```
dsolve(diff(y(x),x)^2-4*y(x)=0,y(x), singsol=all)
```

$$y(x) = 0$$
$$y(x) = (x - c_1)^2$$

✓ Solution by Mathematica
Time used: 0.079 (sec). Leaf size: 38

```
DSolve[(y'[x])^2-4*y[x]==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to \frac{1}{4}(-2x + c_1)^2$$
$$y(x) \to \frac{1}{4}(2x + c_1)^2$$
$$y(x) \to 0$$

# 2 Chapter 1, section 1.3. Exercises page 22

## 2.1 problem 1

Internal problem ID [11585]

**Book**: Differential Equations by Shepley L. Ross. Third edition. John Willey. New Delhi. 2004.
**Section**: Chapter 1, section 1.3. Exercises page 22
**Problem number**: 1.
**ODE order**: 2.
**ODE degree**: 1.

CAS Maple gives this as type `[[_2nd_order, _missing_x]]`

$$\boxed{y'' + y' - 6y = 0}$$

With initial conditions

$$[y(0) = 6, y'(0) = 2]$$

✓ Solution by Maple
Time used: 0.016 (sec). Leaf size: 17

```
dsolve([diff(y(x),x$2)+diff(y(x),x)-6*y(x)=0,y(0) = 6, D(y)(0) = 2],y(x), singsol=all)
```

$$y(x) = \left(4\,\mathrm{e}^{5x} + 2\right) \mathrm{e}^{-3x}$$

✓ Solution by Mathematica
Time used: 0.024 (sec). Leaf size: 20

```
DSolve[{y''[x]+y'[x]-6*y[x]==0,{y[0]==6,y'[0]==2}},y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to e^{-3x}\left(4e^{5x} + 2\right)$$

## 2.2 problem 2(a)

Internal problem ID [11586]

**Book**: Differential Equations by Shepley L. Ross. Third edition. John Willey. New Delhi. 2004.
**Section**: Chapter 1, section 1.3. Exercises page 22
**Problem number**: 2(a).
**ODE order**: 1.
**ODE degree**: 1.

CAS Maple gives this as type `[[_linear, 'class A']]`

$$\boxed{y' + y = 2x\,\mathrm{e}^{-x}}$$

With initial conditions

$$[y(0) = 2]$$

✓ Solution by Maple
Time used: 0.016 (sec). Leaf size: 14

```
dsolve([diff(y(x),x)+y(x)=2*x*exp(-x),y(0) = 2],y(x), singsol=all)
```

$$y(x) = \left(x^2 + 2\right)\mathrm{e}^{-x}$$

✓ Solution by Mathematica
Time used: 0.087 (sec). Leaf size: 16

```
DSolve[{y'[x]+y[x]==2*x*Exp[-x],{y[0]==2}},y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to e^{-x}\left(x^2 + 2\right)$$

## 2.3 problem 2(b)

Internal problem ID [11587]

**Book**: Differential Equations by Shepley L. Ross. Third edition. John Willey. New Delhi. 2004.
**Section**: Chapter 1, section 1.3. Exercises page 22
**Problem number**: 2(b).
**ODE order**: 1.
**ODE degree**: 1.

CAS Maple gives this as type `[[_linear, ‘class A‘]]`

$$\boxed{y' + y = 2x\,\mathrm{e}^{-x}}$$

With initial conditions

$$[y(-1) = \mathrm{e} + 3]$$

✓ Solution by Maple
Time used: 0.016 (sec). Leaf size: 17

```
dsolve([diff(y(x),x)+y(x)=2*x*exp(-x),y(-1) = exp(1)+3],y(x), singsol=all)
```

$$y(x) = \left(x^2 + 3\,\mathrm{e}^{-1}\right)\mathrm{e}^{-x}$$

✓ Solution by Mathematica
Time used: 0.086 (sec). Leaf size: 20

```
DSolve[{y'[x]+y[x]==2*x*Exp[-x],{y[-1]==Exp[1]+3}},y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to e^{-x-1}\left(ex^2 + 3\right)$$

## 2.4 problem 3(a)

Internal problem ID [11588]

**Book**: Differential Equations by Shepley L. Ross. Third edition. John Willey. New Delhi. 2004.
**Section**: Chapter 1, section 1.3. Exercises page 22
**Problem number**: 3(a).
**ODE order**: 2.
**ODE degree**: 1.

CAS Maple gives this as type **[[_2nd_order, _missing_x]]**

$$\boxed{y'' - y' - 12y = 0}$$

With initial conditions

$$[y(0) = 5, y'(0) = 6]$$

✓ Solution by Maple
Time used: 0.0 (sec). Leaf size: 17

```
dsolve([diff(y(x),x$2)-diff(y(x),x)-12*y(x)=0,y(0) = 5, D(y)(0) = 6],y(x), singsol=all)
```

$$y(x) = \left(3\,\mathrm{e}^{7x} + 2\right)\mathrm{e}^{-3x}$$

✓ Solution by Mathematica
Time used: 0.022 (sec). Leaf size: 20

```
DSolve[{y''[x]-y'[x]-12*y[x]==0,{y[0]==5,y'[0]==6}},y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to e^{-3x}\left(3e^{7x} + 2\right)$$

## 2.5 problem 4(a)

Internal problem ID [11589]

**Book**: Differential Equations by Shepley L. Ross. Third edition. John Willey. New Delhi. 2004.
**Section**: Chapter 1, section 1.3. Exercises page 22
**Problem number**: 4(a).
**ODE order**: 2.
**ODE degree**: 1.

CAS Maple gives this as type **`[[_2nd_order, _missing_x]]`**

$$\boxed{y'' + y = 0}$$

With initial conditions

$$\left[y(0) = 0, y'\left(\frac{\pi}{2}\right) = 1\right]$$

✗ Solution by Maple

```
dsolve([diff(y(x),x$2)+y(x)=0,y(0) = 0, D(y)(1/2*Pi) = 1],y(x), singsol=all)
```

No solution found

✗ Solution by Mathematica
Time used: 0.0 (sec). Leaf size: 0

```
DSolve[{y''[x]+y[x]==0,{y[0]==0,y'[Pi/2]==1}},y[x],x,IncludeSingularSolutions -> True]
```

{}

## 2.6 problem 4(b)

Internal problem ID [11590]

**Book**: Differential Equations by Shepley L. Ross. Third edition. John Willey. New Delhi. 2004.
**Section**: Chapter 1, section 1.3. Exercises page 22
**Problem number**: 4(b).
**ODE order**: 2.
**ODE degree**: 1.

CAS Maple gives this as type `[[_2nd_order, _missing_x]]`

$$\boxed{y'' + y = 0}$$

With initial conditions

$$\left[y(0) = 0, y'\left(\frac{\pi}{2}\right) = -1\right]$$

✗ Solution by Maple

```
dsolve([diff(y(x),x$2)+y(x)=0,y(0) = 0, D(y)(1/2*Pi) = -1],y(x), singsol=all)
```

No solution found

✗ Solution by Mathematica

Time used: 0.0 (sec). Leaf size: 0

```
DSolve[{y''[x]+y[x]==0,{y[0]==0,y'[Pi/2]==-1}},y[x],x,IncludeSingularSolutions -> True]
```

$\{\}$

## 2.7 problem 4(c)

Internal problem ID [11591]

**Book**: Differential Equations by Shepley L. Ross. Third edition. John Willey. New Delhi. 2004.
**Section**: Chapter 1, section 1.3. Exercises page 22
**Problem number**: 4(c).
**ODE order**: 2.
**ODE degree**: 1.

CAS Maple gives this as type **`[[_2nd_order, _missing_x]]`**

$$\boxed{y'' + y = 0}$$

With initial conditions

$$[y(0) = 0, y'(\pi) = 1]$$

✓ Solution by Maple
Time used: 0.0 (sec). Leaf size: 8

```
dsolve([diff(y(x),x$2)+y(x)=0,y(0) = 0, D(y)(Pi) = 1],y(x), singsol=all)
```

$$y(x) = -\sin(x)$$

✓ Solution by Mathematica
Time used: 0.018 (sec). Leaf size: 9

```
DSolve[{y''[x]+y[x]==0,{y[0]==0,y'[Pi]==1}},y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to -\sin(x)$$

## 2.8 problem 5

Internal problem ID [11592]

**Book**: Differential Equations by Shepley L. Ross. Third edition. John Willey. New Delhi. 2004.
**Section**: Chapter 1, section 1.3. Exercises page 22
**Problem number**: 5.
**ODE order**: 3.
**ODE degree**: 1.

CAS Maple gives this as type `[[_3rd_order, _with_linear_symmetries]]`

$$\boxed{x^3y''' - 3x^2y'' + 6y'x - 6y = 0}$$

With initial conditions

$$[y(2) = 0, y'(2) = 2, y''(2) = 6]$$

✓ Solution by Maple

Time used: 0.016 (sec). Leaf size: 16

```
dsolve([x^3*diff(y(x),x$3)-3*x^2*diff(y(x),x$2)+6*x*diff(y(x),x)-6*y(x)=0,y(2) = 0, D(y)(2)
```

$$y(x) = x^3 - 3x^2 + 2x$$

✓ Solution by Mathematica

Time used: 0.013 (sec). Leaf size: 15

```
DSolve[{x^3*y'''[x]-3*x^2*y''[x]+6*x*y'[x]-6*y[x]==0,{y[2]==0,y'[2]==2,y''[2]==6}},y[x],x,In
```

$$y(x) \to x(x^2 - 3x + 2)$$

## 2.9 problem 6(a)

Internal problem ID [11593]

**Book**: Differential Equations by Shepley L. Ross. Third edition. John Willey. New Delhi. 2004.
**Section**: Chapter 1, section 1.3. Exercises page 22
**Problem number**: 6(a).
**ODE order**: 1.
**ODE degree**: 1.

CAS Maple gives this as type `[_separable]`

$$\boxed{y' - x^2 \sin(y) = 0}$$

With initial conditions

$$[y(1) = -2]$$

✓ Solution by Maple
Time used: 1.844 (sec). Leaf size: 97

```
dsolve([diff(y(x),x)=x^2*sin(y(x)),y(1) = -2],y(x), singsol=all)
```

$$y(x) = \arctan\left(\frac{2\sin(2)\,\mathrm{e}^{\frac{(-1+x)(x^2+x+1)}{3}}}{(-1+\cos(2))\,\mathrm{e}^{\frac{2(-1+x)(x^2+x+1)}{3}} - 1 - \cos(2)}, \frac{(1-\cos(2))\,\mathrm{e}^{\frac{2(-1+x)(x^2+x+1)}{3}} - 1 - \cos(2)}{(-1+\cos(2))\,\mathrm{e}^{\frac{2(-1+x)(x^2+x+1)}{3}} - 1 - \cos(2)}\right)$$

✓ Solution by Mathematica
Time used: 0.68 (sec). Leaf size: 23

```
DSolve[{y'[x]==x^2*Sin[y[x]],{y[1]==-2}},y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to -\arccos\left(\tanh\left(\operatorname{arctanh}(\cos(2)) - \frac{x^3}{3} + \frac{1}{3}\right)\right)$$

## 2.10 problem 6(b)

Internal problem ID [11594]

**Book**: Differential Equations by Shepley L. Ross. Third edition. John Willey. New Delhi. 2004.
**Section**: Chapter 1, section 1.3. Exercises page 22
**Problem number**: 6(b).
**ODE order**: 1.
**ODE degree**: 1.

CAS Maple gives this as type **`[_separable]`**

$$\boxed{y' - \frac{y^2}{x-2} = 0}$$

With initial conditions

$$[y(1) = 0]$$

✓ Solution by Maple
Time used: 0.0 (sec). Leaf size: 5

```
dsolve([diff(y(x),x)=y(x)^2/(x-2),y(1) = 0],y(x), singsol=all)
```

$$y(x) = 0$$

✓ Solution by Mathematica
Time used: 0.002 (sec). Leaf size: 6

```
DSolve[{y'[x]==y[x]^2/(x-2),{y[1]==0}},y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to 0$$

## 2.11 problem 8

Internal problem ID [11595]

**Book**: Differential Equations by Shepley L. Ross. Third edition. John Willey. New Delhi. 2004.
**Section**: Chapter 1, section 1.3. Exercises page 22
**Problem number**: 8.
**ODE order**: 1.
**ODE degree**: 1.

CAS Maple gives this as type `[_quadrature]`

$$\boxed{y' - y^{\frac{1}{3}} = 0}$$

With initial conditions

$$[y(0) = 0]$$

✓ Solution by Maple
Time used: 0.0 (sec). Leaf size: 5

```
dsolve([diff(y(x),x)=y(x)^(1/3),y(0) = 0],y(x), singsol=all)
```

$$y(x) = 0$$

✓ Solution by Mathematica
Time used: 0.007 (sec). Leaf size: 21

```
DSolve[{y'[x]==y[x]^(1/3),{y[0]==0}},y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to \frac{2}{3}\sqrt{\frac{2}{3}}x^{3/2}$$

# 3 Chapter 2, section 2.1 (Exact differential equations and integrating factors). Exercises page 37

## 3.1 problem 1

Internal problem ID [11596]

**Book**: Differential Equations by Shepley L. Ross. Third edition. John Willey. New Delhi. 2004.
**Section**: Chapter 2, section 2.1 (Exact differential equations and integrating factors). Exercises page 37
**Problem number**: 1.
**ODE order**: 1.
**ODE degree**: 1.

CAS Maple gives this as type `[[_homogeneous, ‘class A‘], _exact, _rational, [_Abel, ‘2nd ty`

$$\boxed{2y + (2x + y)\, y' = -3x}$$

✓ Solution by Maple
Time used: 0.063 (sec). Leaf size: 49

```
dsolve((3*x+2*y(x))+(2*x+y(x))*diff(y(x),x)=0,y(x), singsol=all)
```

$$y(x) = \frac{-2c_1x - \sqrt{c_1^2x^2 + 1}}{c_1}$$
$$y(x) = \frac{-2c_1x + \sqrt{c_1^2x^2 + 1}}{c_1}$$

✓ Solution by Mathematica
Time used: 0.781 (sec). Leaf size: 79

```
DSolve[(3*x+2*y[x])+(2*x+y[x])*y'[x]==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to -2x - \sqrt{x^2 + e^{2c_1}}$$
$$y(x) \to -2x + \sqrt{x^2 + e^{2c_1}}$$
$$y(x) \to -\sqrt{x^2} - 2x$$
$$y(x) \to \sqrt{x^2} - 2x$$

## 3.2 problem 2

Internal problem ID [11597]

**Book**: Differential Equations by Shepley L. Ross. Third edition. John Willey. New Delhi. 2004.
**Section**: Chapter 2, section 2.1 (Exact differential equations and integrating factors). Exercises page 37
**Problem number**: 2.
**ODE order**: 1.
**ODE degree**: 1.

CAS Maple gives this as type `[_exact, _rational, [_1st_order, '_with_symmetry_[F(x)*G(y),0]`

$$\boxed{y^2 + (2yx - 4)\, y' = -3}$$

✓ Solution by Maple

Time used: 0.016 (sec). Leaf size: 62

```
dsolve((y(x)^2+3)+(2*x*y(x)-4)*diff(y(x),x)=0,y(x), singsol=all)
```

$$\frac{-ic_1\left(y(x)^2 x + 3x - 4y(x)\right)\sqrt{3} + 12c_1 + i}{\left(-y(x)\sqrt{3}\,x + 4\sqrt{3} - 3ix\right)\left(\sqrt{3} + iy(x)\right)} = 0$$

✓ Solution by Mathematica

Time used: 0.615 (sec). Leaf size: 79

```
DSolve[(y[x]^2+3)+(2*x*y[x]-4)*y'[x]==0,y[x],x,IncludeSingularSolutions -> True]
```

$$\begin{aligned}
y(x) &\to \frac{2 - \sqrt{-3x^2 + c_1 x + 4}}{x} \\
y(x) &\to \frac{2 + \sqrt{-3x^2 + c_1 x + 4}}{x} \\
y(x) &\to -i\sqrt{3} \\
y(x) &\to i\sqrt{3}
\end{aligned}$$

## 3.3 problem 3

Internal problem ID [11598]

**Book**: Differential Equations by Shepley L. Ross. Third edition. John Willey. New Delhi. 2004.
**Section**: Chapter 2, section 2.1 (Exact differential equations and integrating factors). Exercises page 37
**Problem number**: 3.
**ODE order**: 1.
**ODE degree**: 1.

CAS Maple gives this as type `[_exact, _rational, [_1st_order, '_with_symmetry_[F(x),G(x)]']`

$$\boxed{2yx + (x^2 + 4y)\, y' = -1}$$

✓ Solution by Maple
Time used: 0.0 (sec). Leaf size: 47

```
dsolve((2*x*y(x)+1)+(x^2+4*y(x))*diff(y(x),x)=0,y(x), singsol=all)
```

$$y(x) = -\frac{x^2}{4} - \frac{\sqrt{x^4 - 8c_1 - 8x}}{4}$$
$$y(x) = -\frac{x^2}{4} + \frac{\sqrt{x^4 - 8c_1 - 8x}}{4}$$

✓ Solution by Mathematica
Time used: 0.223 (sec). Leaf size: 61

```
DSolve[(2*x*y[x]+1)+(x^2+4*y[x])*y'[x]==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to \frac{1}{4}\left(-x^2 - \sqrt{x^4 - 8x + 16c_1}\right)$$
$$y(x) \to \frac{1}{4}\left(-x^2 + \sqrt{x^4 - 8x + 16c_1}\right)$$

## 3.4 problem 4

Internal problem ID [11599]

**Book**: Differential Equations by Shepley L. Ross. Third edition. John Willey. New Delhi. 2004.
**Section**: Chapter 2, section 2.1 (Exact differential equations and integrating factors). Exercises page 37
**Problem number**: 4.
**ODE order**: 1.
**ODE degree**: 1.

CAS Maple gives this as type `[_rational, [_Abel, '2nd type', 'class A']]`

$$\boxed{3x^2y - \left(x^3 + y\right)y' = -2}$$

✗ Solution by Maple

```
dsolve((3*x^2*y(x)+2)-(x^3+y(x))*diff(y(x),x)=0,y(x), singsol=all)
```

No solution found

✗ Solution by Mathematica
Time used: 0.0 (sec). Leaf size: 0

```
DSolve[(3*x^2+2)-(x^3+y[x])*y'[x]==0,y[x],x,IncludeSingularSolutions -> True]
```

Not solved

## 3.5 problem 5

Internal problem ID [11600]

**Book**: Differential Equations by Shepley L. Ross. Third edition. John Willey. New Delhi. 2004.
**Section**: Chapter 2, section 2.1 (Exact differential equations and integrating factors). Exercises page 37
**Problem number**: 5.
**ODE order**: 1.
**ODE degree**: 1.

CAS Maple gives this as type `[_exact, _rational, [_Abel, '2nd type', 'class B']]`

$$\boxed{6yx + 2y^2 + \left(3x^2 + 4yx - 6\right) y' = 5}$$

✓ Solution by Maple
Time used: 0.016 (sec). Leaf size: 69

```
dsolve((6*x*y(x)+2*y(x)^2-5)+(3*x^2+4*x*y(x)-6)*diff(y(x),x)=0,y(x), singsol=all)
```

$$y(x) = \frac{-3x^2 + 6 + \sqrt{9x^4 - 8c_1x + 4x^2 + 36}}{4x}$$
$$y(x) = \frac{-3x^2 + 6 - \sqrt{9x^4 - 8c_1x + 4x^2 + 36}}{4x}$$

✓ Solution by Mathematica
Time used: 0.709 (sec). Leaf size: 79

```
DSolve[(6*x*y[x]+2*y[x]^2-5)+(3*x^2+4*x*y[x]-6)*y'[x]==0,y[x],x,IncludeSingularSolutions ->
```

$$y(x) \to -\frac{3x^2 + \sqrt{9x^4 + 4x^2 + 16c_1x + 36} - 6}{4x}$$
$$y(x) \to \frac{-3x^2 + \sqrt{9x^4 + 4x^2 + 16c_1x + 36} + 6}{4x}$$

## 3.6 problem 7

Internal problem ID [11601]

**Book**: Differential Equations by Shepley L. Ross. Third edition. John Willey. New Delhi. 2004.
**Section**: Chapter 2, section 2.1 (Exact differential equations and integrating factors). Exercises page 37
**Problem number**: 7.
**ODE order**: 1.
**ODE degree**: 1.

CAS Maple gives this as type `[_exact, [_Abel, '2nd type', 'class A']]`

$$\boxed{y \sec(x)^2 + (\tan(x) + 2y)\, y' = -\sec(x)\tan(x)}$$

✓ Solution by Maple
Time used: 0.047 (sec). Leaf size: 61

```
dsolve((y(x)*sec(x)^2+sec(x)*tan(x))+(tan(x)+2*y(x))*diff(y(x),x)=0,y(x), singsol=all)
```

$$y(x) = -\frac{\tan(x)}{2} + \frac{\sec(x)\sqrt{-4\cos(x)^2 c_1 + \sin(x)^2 - 4\cos(x)}}{2}$$
$$y(x) = -\frac{\tan(x)}{2} - \frac{\sec(x)\sqrt{-4\cos(x)^2 c_1 + \sin(x)^2 - 4\cos(x)}}{2}$$

✓ Solution by Mathematica
Time used: 1.831 (sec). Leaf size: 101

```
DSolve[(y[x]*Sec[x]^2+Sec[x]*Tan[x])+(Tan[x]+2*y[x])*y'[x]==0,y[x],x,IncludeSingularSolution
```

$$y(x) \to \frac{1}{4}\left(-2\tan(x) - \sqrt{2}\sqrt{\sec^2(x)}\sqrt{-8\cos(x) + (-1+4c_1)\cos(2x) + 1 + 4c_1}\right)$$
$$y(x) \to \frac{1}{4}\left(-2\tan(x) + \sqrt{\sec^2(x)}\sqrt{-16\cos(x) + (-2+8c_1)\cos(2x) + 2 + 8c_1}\right)$$

## 3.7 problem 8

Internal problem ID [11602]

**Book**: Differential Equations by Shepley L. Ross. Third edition. John Willey. New Delhi. 2004.
**Section**: Chapter 2, section 2.1 (Exact differential equations and integrating factors). Exercises page 37
**Problem number**: 8.
**ODE order**: 1.
**ODE degree**: 1.

CAS Maple gives this as type `[_rational, [_1st_order, '_with_symmetry_[F(x)*G(y),0]']]`

$$\boxed{\frac{x}{y^2}+\left(\frac{x^2}{y^3}+y\right)y'=-x}$$

✓ Solution by Maple
Time used: 0.062 (sec). Leaf size: 52

```
dsolve((x/y(x)^2+x)+(x^2/y(x)^3+y(x))*diff(y(x),x)=0,y(x), singsol=all)
```

$$\frac{\left(-2y(x)^2-2\right)\ln\left(y(x)^2+1\right)+y(x)^4+\left(x^2+2c_1+1\right)y(x)^2+2c_1-1}{2y\left(x\right)^2+2}=0$$

✓ Solution by Mathematica
Time used: 0.4 (sec). Leaf size: 55

```
DSolve[(x/y[x]^2+x)+(x^2/y[x]^3+y[x])*y'[x]==0,y[x],x,IncludeSingularSolutions -> True]
```

$$\text{Solve}\left[\frac{x^2y(x)^2}{2\left(y(x)^2+1\right)}+\frac{y(x)^2}{2}-\frac{1}{2\left(y(x)^2+1\right)}-\log\left(y(x)^2+1\right)=c_1,y(x)\right]$$

## 3.8 problem 9

Internal problem ID [11603]

**Book**: Differential Equations by Shepley L. Ross. Third edition. John Willey. New Delhi. 2004.
**Section**: Chapter 2, section 2.1 (Exact differential equations and integrating factors). Exercises page 37
**Problem number**: 9.
**ODE order**: 1.
**ODE degree**: 1.

CAS Maple gives this as type `[_separable]`

$$\boxed{\frac{(2s-1)\,s'}{t}+\frac{s-s^2}{t^2}=0}$$

✓ Solution by Maple
Time used: 0.016 (sec). Leaf size: 31

```
dsolve((2*s(t)-1)/t*diff(s(t),t)+(s(t)-s(t)^2)/t^2=0,s(t), singsol=all)
```

$$s(t)=\frac{1}{2}-\frac{\sqrt{4c_1t+1}}{2}$$
$$s(t)=\frac{1}{2}+\frac{\sqrt{4c_1t+1}}{2}$$

✓ Solution by Mathematica
Time used: 0.682 (sec). Leaf size: 59

```
DSolve[(2*s[t]-1)/t*s'[t]+(s[t]-s[t]^2)/t^2==0,s[t],t,IncludeSingularSolutions -> True]
```

$$s(t)\to\frac{1}{2}\left(1-\sqrt{1-4e^{c_1}t}\right)$$
$$s(t)\to\frac{1}{2}\left(1+\sqrt{1-4e^{c_1}t}\right)$$
$$s(t)\to 0$$
$$s(t)\to 1$$

## 3.9 problem 10

Internal problem ID [11604]

**Book**: Differential Equations by Shepley L. Ross. Third edition. John Willey. New Delhi. 2004.
**Section**: Chapter 2, section 2.1 (Exact differential equations and integrating factors). Exercises page 37
**Problem number**: 10.
**ODE order**: 1.
**ODE degree**: 1.

CAS Maple gives this as type `[_rational]`

$$\boxed{\frac{2y^{\frac{3}{2}}+1}{x^{\frac{1}{3}}}+\left(3\sqrt{x}\,\sqrt{y}-1\right)y'=0}$$

✗ Solution by Maple

```
dsolve((2*y(x)^(3/2)+1)/x^(1/3)+(3*x^(1/2)*y(x)^(1/2)-1)*diff(y(x),x)=0,y(x), singsol=all)
```

No solution found

✗ Solution by Mathematica
Time used: 0.0 (sec). Leaf size: 0

```
DSolve[(2*y[x]^(3/2)+1)/x^(1/3)+(3*x^(1/2)*y[x]^(1/2)-1)*y'[x]==0,y[x],x,IncludeSingularSolu
```

Timed out

## 3.10 problem 11

Internal problem ID [11605]

**Book**: Differential Equations by Shepley L. Ross. Third edition. John Willey. New Delhi. 2004.
**Section**: Chapter 2, section 2.1 (Exact differential equations and integrating factors). Exercises page 37
**Problem number**: 11.
**ODE order**: 1.
**ODE degree**: 1.

CAS Maple gives this as type `[_exact, _rational, [_1st_order, '_with_symmetry_[F(x),G(x)]']`

$$\boxed{2yx + (x^2 + 4y)\, y' = 3}$$

With initial conditions

$$[y(1) = 2]$$

✓ Solution by Maple
Time used: 0.047 (sec). Leaf size: 22

```
dsolve([(2*x*y(x)-3)+(x^2+4*y(x))*diff(y(x),x)=0,y(1) = 2],y(x), singsol=all)
```

$$y(x) = -\frac{x^2}{4} + \frac{\sqrt{x^4 + 24x + 56}}{4}$$

✓ Solution by Mathematica
Time used: 0.218 (sec). Leaf size: 27

```
DSolve[{(2*x*y[x]-3)+(x^2+4*y[x])*y'[x]==0,{y[1]==2}},y[x],x,IncludeSingularSolutions -> Tru
```

$$y(x) \to \frac{1}{4}\left(\sqrt{x^4 + 24x + 56} - x^2\right)$$

## 3.11 problem 12

Internal problem ID [11606]

**Book**: Differential Equations by Shepley L. Ross. Third edition. John Willey. New Delhi. 2004.
**Section**: Chapter 2, section 2.1 (Exact differential equations and integrating factors). Exercises page 37
**Problem number**: 12.
**ODE order**: 1.
**ODE degree**: 1.

CAS Maple gives this as type `[_exact, _rational]`

$$\boxed{3x^2y^2 - y^3 + \left(2yx^3 - 3y^2x + 1\right) y' = -2x}$$

With initial conditions

$$[y(-2) = 1]$$

✓ Solution by Maple

Time used: 28.797 (sec). Leaf size: 210

```
dsolve([(3*x^2*y(x)^2-y(x)^3+2*x)+(2*x^3*y(x)-3*x*y(x)^2+1)*diff(y(x),x)=0,y(-2) = 1],y(x),
```

$$y(x) = \frac{-\frac{2^{\frac{2}{3}}\left(1+i\sqrt{3}\right)\left(\left(2x^7+3\sqrt{3}\,\sqrt{\frac{4x^{10}+4x^8+44x^5+72x^3+27x-4}{x}}+36x^2+27\right)x^2\right)^{\frac{2}{3}}}{2} + x\left(2x^2\left(\left(2x^7 + 3\sqrt{3}\,\sqrt{\frac{4x^{10}+4x^8+44x^5+72x^3+2}{x}}\right.\right.\right.}{6\left(\left(2x^7 + 3\sqrt{3}\,\sqrt{\frac{4x^{10}+4x^8+44x^5+72x^3+27x-4}{x}} + 36x^2 + 27\right) x\right.}$$

✓ Solution by Mathematica

Time used: 60.368 (sec). Leaf size: 250

```
DSolve[{(3*x^2*y[x]^2-y[x]^3+2*x)+(2*x^3*y[x]-3*x*y[x]^2+1)*y'[x]==0,{y[-2]==1}},y[x],x,Incl
```

$y(x)$

$$\rightarrow \frac{2\sqrt[3]{2}\left(1-i\sqrt{3}\right)x^6+4\sqrt[3]{-2x^9-36x^4-27x^2+3\sqrt{3}\sqrt{x^3\left(4x^{10}+4x^8+44x^5+72x^3+27x-4\right)}x^3+\left(1}{12x\sqrt[3]{-2x^9-36x^4-27x^2+3\sqrt{3}\sqrt{x^}}$$

## 3.12 problem 13

Internal problem ID [11607]

**Book**: Differential Equations by Shepley L. Ross. Third edition. John Willey. New Delhi. 2004.
**Section**: Chapter 2, section 2.1 (Exact differential equations and integrating factors). Exercises page 37
**Problem number**: 13.
**ODE order**: 1.
**ODE degree**: 1.

CAS Maple gives this as type `[_exact, [_Abel, ‘2nd type‘, ‘class B‘]]`

$$\boxed{2\sin(x)\cos(x)\,y+\sin(x)\,y^2+\left(\sin(x)^2-2y\cos(x)\right)y'=0}$$

With initial conditions

$$[y(0)=3]$$

✓ Solution by Maple
Time used: 11.656 (sec). Leaf size: 24

```
dsolve([(2*y(x)*sin(x)*cos(x)+y(x)^2*sin(x))+(sin(x)^2-2*y(x)*cos(x))*diff(y(x),x)=0,y(0) =
```

$$y(x)=\frac{\sec(x)\left(\sin(x)^2+\sqrt{\sin(x)^4+36\cos(x)}\right)}{2}$$

✓ Solution by Mathematica
Time used: 2.029 (sec). Leaf size: 34

```
DSolve[{(2*y[x]*Sin[x]*Cos[x]+y[x]^2*Sin[x])+(Sin[x]^2-2*y[x]*Cos[x])*y'[x]==0,{y[0]==3}},y[
```

$$y(x)\to\frac{1}{4}\sec(x)\left(-\cos(2x)+2\sqrt{\sin^4(x)+36\cos(x)}+1\right)$$

## 3.13 problem 14

Internal problem ID [11608]

**Book**: Differential Equations by Shepley L. Ross. Third edition. John Willey. New Delhi. 2004.
**Section**: Chapter 2, section 2.1 (Exact differential equations and integrating factors). Exercises page 37
**Problem number**: 14.
**ODE order**: 1.
**ODE degree**: 1.

CAS Maple gives this as type `[_exact, [_Abel, '2nd type', 'class B']]`

$$\boxed{y\,\mathrm{e}^{x} + y^{2} + (\mathrm{e}^{x} + 2yx)\,y' = -2\,\mathrm{e}^{x}}$$

With initial conditions

$$[y(0) = 6]$$

✓ Solution by Maple

Time used: 0.344 (sec). Leaf size: 29

```
dsolve([(y(x)*exp(x)+2*exp(x)+y(x)^2)+(exp(x)+2*x*y(x))*diff(y(x),x)=0,y(0) = 6],y(x), sings
```

$$y(x) = \frac{-\mathrm{e}^{x} + \sqrt{\mathrm{e}^{2x} - 8\,\mathrm{e}^{x} x + 32x}}{2x}$$

✓ Solution by Mathematica

Time used: 32.264 (sec). Leaf size: 37

```
DSolve[{(y[x]*Exp[x]+2*Exp[x]+y[x]^2)+(Exp[x]+2*x*y[x])*y'[x]==0,{y[0]==6}},y[x],x,IncludeSi
```

$$y(x) \to \frac{\sqrt{-8e^{x}x + 32x + e^{2x}} - e^{x}}{2x}$$

## 3.14 problem 15

Internal problem ID [11609]

**Book**: Differential Equations by Shepley L. Ross. Third edition. John Willey. New Delhi. 2004.
**Section**: Chapter 2, section 2.1 (Exact differential equations and integrating factors). Exercises page 37
**Problem number**: 15.
**ODE order**: 1.
**ODE degree**: 1.

CAS Maple gives this as type `[_exact, _rational, [_1st_order, '_with_symmetry_[F(x)*G(y),0]`

$$\boxed{\frac{3-y}{x^2}+\frac{(y^2-2x)\,y'}{y^2x}=0}$$

With initial conditions

$$[y(-1)=2]$$

✓ Solution by Maple
Time used: 0.047 (sec). Leaf size: 21

```
dsolve([(3-y(x))/x^2+((y(x)^2-2*x)/(x*y(x)^2))*diff(y(x),x)=0,y(-1) = 2],y(x), singsol=all)
```

$$y(x)=x+\frac{3}{2}+\frac{\sqrt{4x^2+4x+9}}{2}$$

✓ Solution by Mathematica
Time used: 1.961 (sec). Leaf size: 28

```
DSolve[{(3-y[x])/x^2+( (y[x]^2-2*x)/(x*y[x]^2) )*y'[x]==0,{y[-1]==2}},y[x],x,IncludeSingular
```

$$y(x)\to\frac{1}{2}\left(\sqrt{4x^2+4x+9}+2x+3\right)$$

## 3.15 problem 16

Internal problem ID [11610]

**Book**: Differential Equations by Shepley L. Ross. Third edition. John Willey. New Delhi. 2004.
**Section**: Chapter 2, section 2.1 (Exact differential equations and integrating factors). Exercises page 37
**Problem number**: 16.
**ODE order**: 1.
**ODE degree**: 1.

CAS Maple gives this as type `[[_homogeneous, ‘class G‘], _exact, _rational]`

$$\boxed{\frac{1+8x y^{\frac{2}{3}}}{x^{\frac{2}{3}} y^{\frac{1}{3}}}+\frac{\left(2 x^{\frac{4}{3}} y^{\frac{2}{3}}-x^{\frac{1}{3}}\right) y'}{y^{\frac{4}{3}}}=0}$$

With initial conditions

$$[y(1) = 8]$$

✓ Solution by Maple
Time used: 0.172 (sec). Leaf size: 55

```
dsolve([(1+8*x*y(x)^(2/3))/(x^(2/3)*y(x)^(1/3))+((2*x^(4/3)*y(x)^(2/3)-x^(1/3))/(y(x)^(4/3))
```

$$y(x) = \mathrm{RootOf}\left(64\_Z^{\frac{7}{3}}x^4+96\_Z^{\frac{5}{3}}x^3-729\_Z^{\frac{4}{3}}+48x^2\_Z+8x\_Z^{\frac{1}{3}}\right)$$

✗ Solution by Mathematica
Time used: 0.0 (sec). Leaf size: 0

```
DSolve[{(1+8*x*y[x]^(2/3))/(x^(2/3)*y[x]^(1/3))+((2*x^(4/3)*y[x]^(2/3)-x^(1/3))/(y[x]^(4/3))
```

{}

## 3.16 problem 21

Internal problem ID [11611]

**Book**: Differential Equations by Shepley L. Ross. Third edition. John Willey. New Delhi. 2004.
**Section**: Chapter 2, section 2.1 (Exact differential equations and integrating factors). Exercises page 37
**Problem number**: 21.
**ODE order**: 1.
**ODE degree**: 1.

CAS Maple gives this as type `[[_homogeneous, ‘class G‘], _rational, _Bernoulli]`

$$\boxed{3y^2 + 2xyy' = -4x}$$

✓ Solution by Maple
Time used: 0.0 (sec). Leaf size: 38

```
dsolve((4*x+3*y(x)^2)+(2*x*y(x))*diff(y(x),x)=0,y(x), singsol=all)
```

$$y(x) = \frac{\sqrt{x\left(-x^4 + c_1\right)}}{x^2}$$
$$y(x) = -\frac{\sqrt{x\left(-x^4 + c_1\right)}}{x^2}$$

✓ Solution by Mathematica
Time used: 0.349 (sec). Leaf size: 46

```
DSolve[(4*x+3*y[x]^2)+(2*x*y[x])*y'[x]==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to -\frac{\sqrt{-x^4 + c_1}}{x^{3/2}}$$
$$y(x) \to \frac{\sqrt{-x^4 + c_1}}{x^{3/2}}$$

## 3.17 problem 22

Internal problem ID [11612]

**Book**: Differential Equations by Shepley L. Ross. Third edition. John Willey. New Delhi. 2004.
**Section**: Chapter 2, section 2.1 (Exact differential equations and integrating factors). Exercises page 37
**Problem number**: 22.
**ODE order**: 1.
**ODE degree**: 1.

CAS Maple gives this as type `[[_homogeneous, ‘class A‘], _rational, _Bernoulli]`

$$\boxed{y^2 + 2yx - x^2y' = 0}$$

✓ Solution by Maple
Time used: 0.0 (sec). Leaf size: 15

```
dsolve((y(x)^2+2*x*y(x))-x^2*diff(y(x),x)=0,y(x), singsol=all)
```

$$y(x) = \frac{x^2}{c_1 - x}$$

✓ Solution by Mathematica
Time used: 0.221 (sec). Leaf size: 23

```
DSolve[(y[x]^2+2*x*y[x])-x^2*y'[x]==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to -\frac{x^2}{x - c_1}$$
$$y(x) \to 0$$

## 3.18 problem 24

Internal problem ID [11613]

**Book**: Differential Equations by Shepley L. Ross. Third edition. John Willey. New Delhi. 2004.
**Section**: Chapter 2, section 2.1 (Exact differential equations and integrating factors). Exercises page 37
**Problem number**: 24.
**ODE order**: 1.
**ODE degree**: 1.

CAS Maple gives this as type `[[_1st_order, _with_linear_symmetries], _rational]`

$$\boxed{y + x(x^2+y^2)^2 + \left(y(x^2+y^2)^2 - x\right) y' = 0}$$

✓ Solution by Maple
Time used: 0.156 (sec). Leaf size: 28

```
dsolve((y(x)+x*(x^2+y(x)^2)^2)+(y(x)*(x^2+y(x)^2)^2-x)*diff(y(x),x)=0,y(x), singsol=all)
```

$$y(x) = \cot\left(\operatorname{RootOf}\left(4c_1 \sin(\_Z)^4 - 4\_Z \sin(\_Z)^4 - x^4\right)\right) x$$

✓ Solution by Mathematica
Time used: 0.144 (sec). Leaf size: 40

```
DSolve[(y[x]+x*(x^2+y[x]^2)^2)+(y[x]*(x^2+y[x]^2)^2-x)*y'[x]==0,y[x],x,IncludeSingularSoluti
```

$$\text{Solve}\left[\arctan\left(\frac{x}{y(x)}\right) + \frac{x^4}{4} + \frac{1}{2}x^2 y(x)^2 + \frac{y(x)^4}{4} = c_1, y(x)\right]$$

# 4 Chapter 2, section 2.2 (Separable equations). Exercises page 47

## 4.1 problem 1

Internal problem ID [11614]

**Book**: Differential Equations by Shepley L. Ross. Third edition. John Willey. New Delhi. 2004.
**Section**: Chapter 2, section 2.2 (Separable equations). Exercises page 47
**Problem number**: 1.
**ODE order**: 1.
**ODE degree**: 1.

CAS Maple gives this as type `[_separable]`

$$\boxed{4yx + (x^2 + 1)\, y' = 0}$$

✓ Solution by Maple
Time used: 0.0 (sec). Leaf size: 13

```
dsolve((4*x*y(x))+(x^2+1)*diff(y(x),x)=0,y(x), singsol=all)
```

$$y(x) = \frac{c_1}{(x^2 + 1)^2}$$

✓ Solution by Mathematica
Time used: 0.047 (sec). Leaf size: 20

```
DSolve[(4*x*y[x])+(x^2+1)*y'[x]==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to \frac{c_1}{(x^2 + 1)^2}$$
$$y(x) \to 0$$

## 4.2 problem 2

Internal problem ID [11615]

**Book**: Differential Equations by Shepley L. Ross. Third edition. John Willey. New Delhi. 2004.
**Section**: Chapter 2, section 2.2 (Separable equations). Exercises page 47
**Problem number**: 2.
**ODE order**: 1.
**ODE degree**: 1.

CAS Maple gives this as type `[_separable]`

$$\boxed{yx + y + (x^2 + 2x)\, y' = -2x - 2}$$

✓ Solution by Maple
Time used: 0.0 (sec). Leaf size: 15

```
dsolve((x*y(x)+2*x+y(x)+2)+(x^2+2*x)*diff(y(x),x)=0,y(x), singsol=all)
```

$$y(x) = -2 + \frac{c_1}{\sqrt{x\,(x+2)}}$$

✓ Solution by Mathematica
Time used: 0.149 (sec). Leaf size: 27

```
DSolve[(x*y[x]+2*x+y[x]+2)+(x^2+2*x)*y'[x]==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to -2 + \frac{c_1}{\sqrt{x}\sqrt{x+2}}$$
$$y(x) \to -2$$

## 4.3 problem 3

Internal problem ID [11616]

**Book**: Differential Equations by Shepley L. Ross. Third edition. John Willey. New Delhi. 2004.
**Section**: Chapter 2, section 2.2 (Separable equations). Exercises page 47
**Problem number**: 3.
**ODE order**: 1.
**ODE degree**: 1.

CAS Maple gives this as type `[_separable]`

$$\boxed{2r(s^2+1)+(r^4+1)\,s'=0}$$

✓ Solution by Maple
Time used: 0.0 (sec). Leaf size: 15

```
dsolve(2*r*(s(r)^2+1)+(r^4+1)*diff(s(r),r)=0,s(r), singsol=all)
```

$$s(r) = -\tan\left(\arctan\left(r^2\right)+2c_1\right)$$

✓ Solution by Mathematica
Time used: 0.478 (sec). Leaf size: 31

```
DSolve[2*r*(s[r]^2+1)+(r^4+1)*s'[r]==0,s[r],r,IncludeSingularSolutions -> True]
```

$$\begin{aligned} s(r) &\to -\tan\left(\arctan\left(r^2\right)-c_1\right) \\ s(r) &\to -i \\ s(r) &\to i \end{aligned}$$

## 4.4 problem 4

Internal problem ID [11617]

**Book**: Differential Equations by Shepley L. Ross. Third edition. John Willey. New Delhi. 2004.
**Section**: Chapter 2, section 2.2 (Separable equations). Exercises page 47
**Problem number**: 4.
**ODE order**: 1.
**ODE degree**: 1.

CAS Maple gives this as type `[_separable]`

$$\boxed{\csc(y) + y' \sec(x) = 0}$$

✓ Solution by Maple
Time used: 0.0 (sec). Leaf size: 9

```
dsolve(csc(y(x))+sec(x)*diff(y(x),x)=0,y(x), singsol=all)
```

$$y(x) = \arccos(\sin(x) + c_1)$$

✓ Solution by Mathematica
Time used: 0.696 (sec). Leaf size: 27

```
DSolve[Csc[y[x]]+Sec[x]*y'[x]==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to -\arccos(\sin(x) - c_1)$$
$$y(x) \to \arccos(\sin(x) - c_1)$$

## 4.5 problem 5

Internal problem ID [11618]

**Book**: Differential Equations by Shepley L. Ross. Third edition. John Willey. New Delhi. 2004.
**Section**: Chapter 2, section 2.2 (Separable equations). Exercises page 47
**Problem number**: 5.
**ODE order**: 1.
**ODE degree**: 1.

CAS Maple gives this as type `[_separable]`

$$\boxed{\tan(\theta) + 2r\theta' = 0}$$

✓ Solution by Maple
Time used: 0.313 (sec). Leaf size: 21

```
dsolve(tan(theta(r))+2*r*diff(theta(r),r)=0,theta(r), singsol=all)
```

$$\theta(r) = \arcsin\left(\frac{1}{\sqrt{c_1 r}}\right)$$
$$\theta(r) = -\arcsin\left(\frac{1}{\sqrt{c_1 r}}\right)$$

✓ Solution by Mathematica
Time used: 15.319 (sec). Leaf size: 21

```
DSolve[Tan[theta[r]]+2*r*theta'[r]==0,theta[r],r,IncludeSingularSolutions -> True]
```

$$\theta(r) \to \arcsin\left(\frac{e^{c_1}}{\sqrt{r}}\right)$$
$$\theta(r) \to 0$$

## 4.6 problem 6

Internal problem ID [11619]

**Book**: Differential Equations by Shepley L. Ross. Third edition. John Willey. New Delhi. 2004.
**Section**: Chapter 2, section 2.2 (Separable equations). Exercises page 47
**Problem number**: 6.
**ODE order**: 1.
**ODE degree**: 1.

CAS Maple gives this as type `[_separable]`

$$\boxed{(\mathrm{e}^{v}+1)\cos(u)+\mathrm{e}^{v}(1+\sin(u))\,v'=0}$$

✓ Solution by Maple
Time used: 0.64 (sec). Leaf size: 29

```
dsolve((exp(v(u))+1)*cos(u) + exp(v(u))*(1+sin(u))*diff(v(u),u)=0,v(u), singsol=all)
```

$$v(u)=-\ln\left(\frac{-1-\sin(u)}{-1+(1+\sin(u))\,\mathrm{e}^{c_1}}\right)-c_1$$

✓ Solution by Mathematica
Time used: 5.457 (sec). Leaf size: 37

```
DSolve[(Exp[v[u]]+1)*Cos[u] + Exp[v[u]]*(1+Sin[u])*v'[u]==0,v[u],u,IncludeSingularSolutions
```

$$\begin{aligned} v(u)&\to \log\left(-1+\frac{e^{c_1}}{\left(\sin\left(\frac{u}{2}\right)+\cos\left(\frac{u}{2}\right)\right)^2}\right)\\ v(u)&\to i\pi \end{aligned}$$

## 4.7 problem 7

Internal problem ID [11620]

**Book**: Differential Equations by Shepley L. Ross. Third edition. John Willey. New Delhi. 2004.
**Section**: Chapter 2, section 2.2 (Separable equations). Exercises page 47
**Problem number**: 7.
**ODE order**: 1.
**ODE degree**: 1.

CAS Maple gives this as type `[_separable]`

$$\boxed{(x+4)\left(1+y^2\right)+y\left(x^2+3x+2\right)y'=0}$$

✓ Solution by Maple
Time used: 0.0 (sec). Leaf size: 114

```
dsolve((x+4)*(y(x)^2+1) + y(x)*(x^2+3*x+2)*diff(y(x),x)=0,y(x), singsol=all)
```

$$y(x) = \frac{\sqrt{-x^6-6x^5+x^4c_1+(8c_1+100)\,x^3+(24c_1+345)\,x^2+(32c_1+474)\,x+16c_1+239}}{(1+x)^3}$$

$$y(x) = -\frac{\sqrt{-x^6-6x^5+x^4c_1+(8c_1+100)\,x^3+(24c_1+345)\,x^2+(32c_1+474)\,x+16c_1+239}}{(1+x)^3}$$

✓ Solution by Mathematica

Time used: 5.501 (sec). Leaf size: 126

```
DSolve[(x+4)*(y[x]^2+1) + y[x]*(x^2+3*x+2)*y'[x]==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to -\frac{\sqrt{-(x+1)^6 + e^{2c_1}(x+2)^4}}{(x+1)^3}$$
$$y(x) \to \frac{\sqrt{-(x+1)^6 + e^{2c_1}(x+2)^4}}{(x+1)^3}$$
$$y(x) \to -i$$
$$y(x) \to i$$
$$y(x) \to \frac{(x+1)^3}{\sqrt{-(x+1)^6}}$$
$$y(x) \to \frac{\sqrt{-(x+1)^6}}{(x+1)^3}$$

## 4.8 problem 8

Internal problem ID [11621]

**Book**: Differential Equations by Shepley L. Ross. Third edition. John Willey. New Delhi. 2004.
**Section**: Chapter 2, section 2.2 (Separable equations). Exercises page 47
**Problem number**: 8.
**ODE order**: 1.
**ODE degree**: 1.

CAS Maple gives this as type `[_linear]`

$$\boxed{y - y'x = -x}$$

✓ Solution by Maple
Time used: 0.0 (sec). Leaf size: 10

```
dsolve((x+y(x))- x*diff(y(x),x)=0,y(x), singsol=all)
```

$$y(x) = (\ln(x) + c_1)\, x$$

✓ Solution by Mathematica
Time used: 0.037 (sec). Leaf size: 12

```
DSolve[(x+y[x])- x*y'[x]==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to x(\log(x) + c_1)$$

## 4.9 problem 9

Internal problem ID [11622]

**Book**: Differential Equations by Shepley L. Ross. Third edition. John Willey. New Delhi. 2004.
**Section**: Chapter 2, section 2.2 (Separable equations). Exercises page 47
**Problem number**: 9.
**ODE order**: 1.
**ODE degree**: 1.

CAS Maple gives this as type `[[_homogeneous, ‘class A‘], _rational, [_Abel, ‘2nd type‘, ‘cl`

$$\boxed{2yx + 3y^2 - \left(2yx + x^2\right) y' = 0}$$

✓ Solution by Maple
Time used: 0.016 (sec). Leaf size: 33

```
dsolve((2*x*y(x)+3*y(x)^2)- (2*x*y(x)+x^2)*diff(y(x),x)=0,y(x), singsol=all)
```

$$y(x) = -\frac{\left(1+\sqrt{4c_1x+1}\right)x}{2}$$
$$y(x) = \frac{\left(-1+\sqrt{4c_1x+1}\right)x}{2}$$

✓ Solution by Mathematica
Time used: 0.618 (sec). Leaf size: 61

```
DSolve[(2*x*y[x]+3*y[x]^2)- (2*x*y[x]+x^2)*y'[x]==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to -\frac{1}{2}x\left(1+\sqrt{1+4e^{c_1}x}\right)$$
$$y(x) \to \frac{1}{2}x\left(-1+\sqrt{1+4e^{c_1}x}\right)$$
$$y(x) \to 0$$
$$y(x) \to -x$$

## 4.10 problem 10

Internal problem ID [11623]

**Book**: Differential Equations by Shepley L. Ross. Third edition. John Willey. New Delhi. 2004.
**Section**: Chapter 2, section 2.2 (Separable equations). Exercises page 47
**Problem number**: 10.
**ODE order**: 1.
**ODE degree**: 1.

CAS Maple gives this as type `[[_homogeneous, 'class A'], _rational, _dAlembert]`

$$\boxed{v^3 + (u^3 - uv^2)\, v' = 0}$$

✓ Solution by Maple
Time used: 0.031 (sec). Leaf size: 32

```
dsolve(v(u)^3+ (u^3-u*v(u)^2)*diff(v(u),u)=0,v(u), singsol=all)
```

$$v(u) = \frac{e^{-c_1}}{\sqrt{-\frac{e^{-2c_1}}{u^2\,\mathrm{LambertW}\left(-\frac{e^{-2c_1}}{u^2}\right)}}}$$

✓ Solution by Mathematica
Time used: 9.023 (sec). Leaf size: 56

```
DSolve[v[u]^3+ (u^3-u*v[u]^2)*v'[u]==0,v[u],u,IncludeSingularSolutions -> True]
```

$$v(u) \to -iu\sqrt{W\left(-\frac{e^{-2c_1}}{u^2}\right)}$$
$$v(u) \to iu\sqrt{W\left(-\frac{e^{-2c_1}}{u^2}\right)}$$
$$v(u) \to 0$$

## 4.11 problem 11

Internal problem ID [11624]

**Book**: Differential Equations by Shepley L. Ross. Third edition. John Willey. New Delhi. 2004.
**Section**: Chapter 2, section 2.2 (Separable equations). Exercises page 47
**Problem number**: 11.
**ODE order**: 1.
**ODE degree**: 1.

CAS Maple gives this as type `[[_homogeneous, ‘class A‘], _dAlembert]`

$$\boxed{x\tan\left(\frac{y}{x}\right)+y-y'x=0}$$

✓ Solution by Maple
Time used: 0.016 (sec). Leaf size: 10

```
dsolve((x*tan(y(x)/x)+y(x))- x*diff(y(x),x)=0,y(x), singsol=all)
```

$$y(x)=\arcsin\left(c_1x\right)x$$

✓ Solution by Mathematica
Time used: 8.002 (sec). Leaf size: 19

```
DSolve[(x*Tan[y[x]/x]+y[x])- x*y'[x]==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x)\to x\arcsin\left(e^{c_1}x\right)$$
$$y(x)\to 0$$

## 4.12 problem 12

Internal problem ID [11625]

**Book**: Differential Equations by Shepley L. Ross. Third edition. John Willey. New Delhi. 2004.
**Section**: Chapter 2, section 2.2 (Separable equations). Exercises page 47
**Problem number**: 12.
**ODE order**: 1.
**ODE degree**: 1.

CAS Maple gives this as type `[[_homogeneous, ‘class A‘], _exact, _rational, _dAlembert]`

$$\boxed{(2s^2+2st+t^2)\, s'+s^2+2st=t^2}$$

✓ Solution by Maple
Time used: 0.063 (sec). Leaf size: 348

```
dsolve((2*s(t)^2+2*s(t)*t+t^2)*diff(s(t),t)+(s(t)^2+2*s(t)*t-t^2)=0,s(t), singsol=all)
```

$$s(t)=\frac{\left(4t^3c_1^3+2+\sqrt{17c_1^6t^6+16t^3c_1^3+4}\right)^{\frac{1}{3}}-\frac{t^2c_1^2}{\left(4t^3c_1^3+2+\sqrt{17c_1^6t^6+16t^3c_1^3+4}\right)^{\frac{1}{3}}}-c_1t}{2c_1}$$

$$s(t)=-\frac{(1+i\sqrt{3})\left(4t^3c_1^3+2+\sqrt{17c_1^6t^6+16t^3c_1^3+4}\right)^{\frac{2}{3}}+c_1t\left(2\left(4t^3c_1^3+2+\sqrt{17c_1^6t^6+16t^3c_1^3+4}\right)^{\frac{1}{3}}+(i\sqrt{}\right.}{4\left(4t^3c_1^3+2+\sqrt{17c_1^6t^6+16t^3c_1^3+4}\right)^{\frac{1}{3}}c_1}$$

$$s(t)=\frac{(i\sqrt{3}-1)\left(4t^3c_1^3+2+\sqrt{17c_1^6t^6+16t^3c_1^3+4}\right)^{\frac{2}{3}}+\left(-2\left(4t^3c_1^3+2+\sqrt{17c_1^6t^6+16t^3c_1^3+4}\right)^{\frac{1}{3}}+c_1t(1-\right.}{4\left(4t^3c_1^3+2+\sqrt{17c_1^6t^6+16t^3c_1^3+4}\right)^{\frac{1}{3}}c_1}$$

✓ Solution by Mathematica

Time used: 48.03 (sec). Leaf size: 616

```
DSolve[(2*s[t]^2+2*s[t]*t+t^2)*s'[t]+(s[t]^2+2*s[t]*t-t^2)==0,s[t],t,IncludeSingularSolution
```

$$s(t) \to \frac{1}{2}\left(\sqrt[3]{4t^3+\sqrt{17t^6+16e^{3c_1}t^3+4e^{6c_1}}+2e^{3c_1}} - \frac{t^2}{\sqrt[3]{4t^3+\sqrt{17t^6+16e^{3c_1}t^3+4e^{6c_1}}+2e^{3c_1}}} - t\right)$$

$$s(t) \to \frac{1}{8}\left(2i\left(\sqrt{3}+i\right)\sqrt[3]{4t^3+\sqrt{17t^6+16e^{3c_1}t^3+4e^{6c_1}}+2e^{3c_1}} + \frac{2\left(1+i\sqrt{3}\right)t^2}{\sqrt[3]{4t^3+\sqrt{17t^6+16e^{3c_1}t^3+4e^{6c_1}}+2e^{3c_1}}} - 4t\right)$$

$$s(t) \to \frac{1}{8}\left(-2\left(1+i\sqrt{3}\right)\sqrt[3]{4t^3+\sqrt{17t^6+16e^{3c_1}t^3+4e^{6c_1}}+2e^{3c_1}} + \frac{2\left(1-i\sqrt{3}\right)t^2}{\sqrt[3]{4t^3+\sqrt{17t^6+16e^{3c_1}t^3+4e^{6c_1}}+2e^{3c_1}}} - 4t\right)$$

$$s(t) \to \frac{1}{2}\left(\sqrt[3]{\sqrt{17}\sqrt{t^6}+4t^3} - \frac{t^2}{\sqrt[3]{\sqrt{17}\sqrt{t^6}+4t^3}} - t\right)$$

$$s(t) \to \frac{1}{4}\left(\left(-1-i\sqrt{3}\right)\sqrt[3]{\sqrt{17}\sqrt{t^6}+4t^3} + \frac{\left(1-i\sqrt{3}\right)t^2}{\sqrt[3]{\sqrt{17}\sqrt{t^6}+4t^3}} - 2t\right)$$

$$s(t) \to \frac{1}{4}\left(i\left(\sqrt{3}+i\right)\sqrt[3]{\sqrt{17}\sqrt{t^6}+4t^3} + \frac{\left(1+i\sqrt{3}\right)t^2}{\sqrt[3]{\sqrt{17}\sqrt{t^6}+4t^3}} - 2t\right)$$

## 4.13 problem 13

Internal problem ID [11626]

**Book**: Differential Equations by Shepley L. Ross. Third edition. John Willey. New Delhi. 2004.
**Section**: Chapter 2, section 2.2 (Separable equations). Exercises page 47
**Problem number**: 13.
**ODE order**: 1.
**ODE degree**: 1.

CAS Maple gives this as type `[[_homogeneous, ‘class A‘], _dAlembert]`

$$\boxed{y^2\sqrt{x^2+y^2}-xy\sqrt{x^2+y^2}\,y'=-x^3}$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 42

```
dsolve((x^3+y(x)^2*sqrt(x^2+y(x)^2))-x*y(x)*sqrt(x^2+y(x)^2)*diff(y(x),x)=0,y(x), singsol=al
```

$$\frac{\left(-y(x)^2-x^2\right)\sqrt{y\left(x\right)^2+x^2}-x^3(c_1-3\ln\left(x\right))}{x^3}=0$$

✓ Solution by Mathematica

Time used: 28.664 (sec). Leaf size: 265

```
DSolve[(x^3+y[x]^2*Sqrt[x^2+y[x]^2])-x*y[x]*Sqrt[x^2+y[x]^2]*y'[x]==0,y[x],x,IncludeSingular
```

$$y(x) \to -\sqrt{-x^2 - \frac{1}{2}\sqrt[6]{3}\left(\sqrt{3}+3i\right)\sqrt[3]{x^6(\log(x)+c_1)^2}}$$
$$y(x) \to \sqrt{-x^2 - \frac{1}{2}\sqrt[6]{3}\left(\sqrt{3}+3i\right)\sqrt[3]{x^6(\log(x)+c_1)^2}}$$
$$y(x) \to -\sqrt{-x^2 - \frac{1}{2}\sqrt[6]{3}\left(\sqrt{3}-3i\right)\sqrt[3]{x^6(\log(x)+c_1)^2}}$$
$$y(x) \to \sqrt{-x^2 - \frac{1}{2}\sqrt[6]{3}\left(\sqrt{3}-3i\right)\sqrt[3]{x^6(\log(x)+c_1)^2}}$$
$$y(x) \to -\sqrt{-x^2 + 3^{2/3}\sqrt[3]{x^6(\log(x)+c_1)^2}}$$
$$y(x) \to \sqrt{-x^2 + 3^{2/3}\sqrt[3]{x^6(\log(x)+c_1)^2}}$$

## 4.14 problem 14

Internal problem ID [11627]

**Book**: Differential Equations by Shepley L. Ross. Third edition. John Willey. New Delhi. 2004.
**Section**: Chapter 2, section 2.2 (Separable equations). Exercises page 47
**Problem number**: 14.
**ODE order**: 1.
**ODE degree**: 1.

CAS Maple gives this as type `[[_homogeneous, ‘class C‘], _dAlembert]`

$$\boxed{\sqrt{y+x}+\sqrt{-y+x}+\left(\sqrt{-y+x}-\sqrt{y+x}\right)y'=0}$$

✓ Solution by Maple

Time used: 2.796 (sec). Leaf size: 36

```
dsolve((sqrt(x+y(x))+sqrt(x-y(x)))+(sqrt(x-y(x))-sqrt(x+y(x)))*diff(y(x),x)=0,y(x), singsol=
```

$$\ln(x)+\ln\left(\frac{y(x)}{x}\right)-\operatorname{arctanh}\left(\frac{1}{\sqrt{-\frac{-x^2+y(x)^2}{x^2}}}\right)-c_1=0$$

✓ Solution by Mathematica

Time used: 2.828 (sec). Leaf size: 84

```
DSolve[(Sqrt[x+y[x]]+Sqrt[x-y[x]])+(Sqrt[x-y[x]]-Sqrt[x+y[x]])*y'[x]==0,y[x],x,IncludeSingul
```

$$y(x)\to -\frac{1}{4}\left(\cosh\left(\frac{c_1}{2}\right)+\sinh\left(\frac{c_1}{2}\right)\right)\sqrt{-8ix+\cosh(c_1)+\sinh(c_1)}$$
$$y(x)\to \frac{1}{4}\left(\cosh\left(\frac{c_1}{2}\right)+\sinh\left(\frac{c_1}{2}\right)\right)\sqrt{-8ix+\cosh(c_1)+\sinh(c_1)}$$
$$y(x)\to 0$$

## 4.15 problem 15

Internal problem ID [11628]

**Book**: Differential Equations by Shepley L. Ross. Third edition. John Willey. New Delhi. 2004.
**Section**: Chapter 2, section 2.2 (Separable equations). Exercises page 47
**Problem number**: 15.
**ODE order**: 1.
**ODE degree**: 1.

CAS Maple gives this as type **[_separable]**

$$\boxed{y + y(x+4)\,y' = -2}$$

With initial conditions

$$[y(-3) = -1]$$

✓ Solution by Maple
Time used: 0.156 (sec). Leaf size: 18

```
dsolve([(y(x)+2)+(y(x)*(x+4))*diff(y(x),x)=0,y(-3) = -1],y(x), singsol=all)
```

$$y(x) = -2\,\mathrm{LambertW}\left(-\frac{\sqrt{x+4}\,\mathrm{e}^{-\frac{1}{2}}}{2}\right) - 2$$

✓ Solution by Mathematica
Time used: 12.779 (sec). Leaf size: 26

```
DSolve[{(y[x]+2)+(y[x]*(x+4))*y'[x]==0,{y[-3]==-1}},y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to -2\left(W\left(-\frac{\sqrt{x+4}}{2\sqrt{e}}\right)+1\right)$$

## 4.16 problem 16

Internal problem ID [11629]

**Book**: Differential Equations by Shepley L. Ross. Third edition. John Willey. New Delhi. 2004.
**Section**: Chapter 2, section 2.2 (Separable equations). Exercises page 47
**Problem number**: 16.
**ODE order**: 1.
**ODE degree**: 1.

CAS Maple gives this as type `[_separable]`

$$\boxed{8\cos(y)^2 + \csc(x)^2 y' = 0}$$

With initial conditions

$$\left[y\left(\frac{\pi}{12}\right) = \frac{\pi}{4}\right]$$

✓ Solution by Maple
Time used: 0.156 (sec). Leaf size: 20

```
dsolve([(8*cos(y(x))^2)+csc(x)^2*diff(y(x),x)=0,y(1/12*Pi) = 1/4*Pi],y(x), singsol=all)
```

$$y(x) = -\arctan\left(-\frac{\pi}{3} + 4x - 2\sin(2x)\right)$$

✓ Solution by Mathematica
Time used: 1.156 (sec). Leaf size: 21

```
DSolve[{(8*Cos[y[x]]^2)+Csc[x]^2*y'[x]==0,{y[Pi/12]==Pi/4}},y[x],x,IncludeSingularSolutions
```

$$y(x) \to \arctan\left(-4x + 2\sin(2x) + \frac{\pi}{3}\right)$$

## 4.17 problem 17

Internal problem ID [11630]

**Book**: Differential Equations by Shepley L. Ross. Third edition. John Willey. New Delhi. 2004.
**Section**: Chapter 2, section 2.2 (Separable equations). Exercises page 47
**Problem number**: 17.
**ODE order**: 1.
**ODE degree**: 1.

CAS Maple gives this as type `[_separable]`

$$\boxed{(3x+8)\left(y^2+4\right)-4y\left(x^2+5x+6\right)y'=0}$$

With initial conditions

$$[y(1)=2]$$

✓ Solution by Maple
Time used: 0.047 (sec). Leaf size: 21

```
dsolve([(3*x+8)*(y(x)^2+4)-4*y(x)*(x^2+5*x+6)*diff(y(x),x)=0,y(1) = 2],y(x), singsol=all)
```

$$y(x)=\frac{2\sqrt{-9+(3x+6)\sqrt{x+3}}}{3}$$

✓ Solution by Mathematica
Time used: 4.88 (sec). Leaf size: 36

```
DSolve[{(3*x+8)*(y[x]^2+4)-4*y[x]*(x^2+5*x+6)*y'[x]==0,{y[1]==2}},y[x],x,IncludeSingularSolu
```

$$y(x)\to\frac{2\sqrt{\sqrt{x+3}x+2\sqrt{x+3}-3}}{\sqrt{3}}$$

## 4.18 problem 18

Internal problem ID [11631]

**Book**: Differential Equations by Shepley L. Ross. Third edition. John Willey. New Delhi. 2004.
**Section**: Chapter 2, section 2.2 (Separable equations). Exercises page 47
**Problem number**: 18.
**ODE order**: 1.
**ODE degree**: 1.

CAS Maple gives this as type `[[_homogeneous, ‘class A‘], _rational, _Bernoulli]`

$$\boxed{3y^2 - 2xyy' = -x^2}$$

With initial conditions

$$[y(2) = 6]$$

✓ Solution by Maple
Time used: 0.062 (sec). Leaf size: 13

```
dsolve([(x^2+3*y(x)^2)-2*x*y(x)*diff(y(x),x)=0,y(2) = 6],y(x), singsol=all)
```

$$y(x) = \sqrt{5x-1}\,x$$

✓ Solution by Mathematica
Time used: 0.455 (sec). Leaf size: 16

```
DSolve[{(x^2+3*y[x]^2)-2*x*y[x]*y'[x]==0,{y[2]==6}},y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to x\sqrt{5x-1}$$

## 4.19 problem 19

Internal problem ID [11632]

**Book**: Differential Equations by Shepley L. Ross. Third edition. John Willey. New Delhi. 2004.
**Section**: Chapter 2, section 2.2 (Separable equations). Exercises page 47
**Problem number**: 19.
**ODE order**: 1.
**ODE degree**: 1.

CAS Maple gives this as type `[[_homogeneous, ‘class A‘], _rational, [_Abel, ‘2nd type‘, ‘cl`

$$\boxed{-5y + (4x - y)\, y' = -2x}$$

With initial conditions

$$[y(1) = 4]$$

✓ Solution by Maple
Time used: 0.218 (sec). Leaf size: 35

```
dsolve([(2*x-5*y(x))+(4*x-y(x))*diff(y(x),x)=0,y(1) = 4],y(x), singsol=all)
```

$$y(x) = 6 - 2x - 6\sqrt{1-x}$$
$$y(x) = 6 - 2x + 6\sqrt{1-x}$$

✓ Solution by Mathematica
Time used: 2.199 (sec). Leaf size: 41

```
DSolve[{(2*x-5*y[x])+(4*x-y[x])*y'[x]==0,{y[1]==4}},y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to -2x - 6i\sqrt{x-1} + 6$$
$$y(x) \to -2x + 6i\sqrt{x-1} + 6$$

## 4.20 problem 20

Internal problem ID [11633]

**Book**: Differential Equations by Shepley L. Ross. Third edition. John Willey. New Delhi. 2004.
**Section**: Chapter 2, section 2.2 (Separable equations). Exercises page 47
**Problem number**: 20.
**ODE order**: 1.
**ODE degree**: 1.

CAS Maple gives this as type `[[_homogeneous, ‘class A‘], _rational, [_Abel, ‘2nd type‘, ‘cl`

$$\boxed{9yx + 5y^2 - (6x^2 + 4yx)\, y' = -3x^2}$$

With initial conditions

$$[y(2) = -6]$$

✓ Solution by Maple
Time used: 0.265 (sec). Leaf size: 21

```
dsolve([(3*x^2+9*x*y(x)+5*y(x)^2)-(6*x^2+4*x*y(x))*diff(y(x),x)=0,y(2) = -6],y(x), singsol=a
```

$$y(x) = -\frac{\left(3 + \sqrt{-3 + 6\sqrt{2}\,\sqrt{x}}\right) x}{2}$$

✓ Solution by Mathematica
Time used: 37.251 (sec). Leaf size: 30

```
DSolve[{(3*x^2+9*x*y[x]+5*y[x]^2)-(6*x^2+4*x*y[x])*y'[x]==0,{y[2]==-6}},y[x],x,IncludeSingul
```

$$y(x) \to -\frac{1}{2}\left(\sqrt{6\sqrt{2}\sqrt{x} - 3} + 3\right) x$$

## 4.21 problem 22(a)

Internal problem ID [11634]

**Book**: Differential Equations by Shepley L. Ross. Third edition. John Willey. New Delhi. 2004.
**Section**: Chapter 2, section 2.2 (Separable equations). Exercises page 47
**Problem number**: 22(a).
**ODE order**: 1.
**ODE degree**: 1.

CAS Maple gives this as type `[[_homogeneous, ‘class A‘], _exact, _rational, [_Abel, ‘2nd ty`

$$\boxed{2y + (2x - y)\, y' = -x}$$

✓ Solution by Maple
Time used: 0.047 (sec). Leaf size: 51

```
dsolve((x+2*y(x))+(2*x-y(x))*diff(y(x),x)=0,y(x), singsol=all)
```

$$y(x) = \frac{2c_1x - \sqrt{5c_1^2x^2 + 1}}{c_1}$$
$$y(x) = \frac{2c_1x + \sqrt{5c_1^2x^2 + 1}}{c_1}$$

✓ Solution by Mathematica
Time used: 0.777 (sec). Leaf size: 94

```
DSolve[(x+2*y[x])+(2*x-y[x])*y'[x]==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to 2x - \sqrt{5x^2 + e^{2c_1}}$$
$$y(x) \to 2x + \sqrt{5x^2 + e^{2c_1}}$$
$$y(x) \to 2x - \sqrt{5}\sqrt{x^2}$$
$$y(x) \to \sqrt{5}\sqrt{x^2} + 2x$$

## 4.22 problem 22(b)

Internal problem ID [11635]

**Book**: Differential Equations by Shepley L. Ross. Third edition. John Willey. New Delhi. 2004.
**Section**: Chapter 2, section 2.2 (Separable equations). Exercises page 47
**Problem number**: 22(b).
**ODE order**: 1.
**ODE degree**: 1.

CAS Maple gives this as type `[[_homogeneous, 'class A'], _exact, _rational, [_Abel, '2nd ty`

$$\boxed{-y-(y+x)\,y'=-3x}$$

✓ Solution by Maple
Time used: 0.047 (sec). Leaf size: 51

```
dsolve((3*x-y(x))-(x+y(x))*diff(y(x),x)=0,y(x), singsol=all)
```

$$y(x)=\frac{-c_1x-\sqrt{4c_1^2x^2+1}}{c_1}$$
$$y(x)=\frac{-c_1x+\sqrt{4c_1^2x^2+1}}{c_1}$$

✓ Solution by Mathematica
Time used: 0.752 (sec). Leaf size: 85

```
DSolve[(3*x-y[x])-(x+y[x])*y'[x]==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x)\to -x-\sqrt{4x^2+e^{2c_1}}$$
$$y(x)\to -x+\sqrt{4x^2+e^{2c_1}}$$
$$y(x)\to -2\sqrt{x^2}-x$$
$$y(x)\to 2\sqrt{x^2}-x$$

## 4.23 problem 23(a)

Internal problem ID [11636]

**Book**: Differential Equations by Shepley L. Ross. Third edition. John Willey. New Delhi. 2004.
**Section**: Chapter 2, section 2.2 (Separable equations). Exercises page 47
**Problem number**: 23(a).
**ODE order**: 1.
**ODE degree**: 1.

CAS Maple gives this as type `[[_homogeneous, ‘class A‘], _exact, _rational, _dAlembert]`

$$\boxed{2y^2 + (4yx - y^2)\, y' = -x^2}$$

✓ Solution by Maple
Time used: 0.063 (sec). Leaf size: 439

```
dsolve((x^2+2*y(x)^2)+(4*x*y(x)-y(x)^2)*diff(y(x),x)=0,y(x), singsol=all)
```

$$y(x) = \frac{\frac{\left(4+68c_1^3x^3+4\sqrt{33c_1^6x^6+34c_1^3x^3+1}\right)^{\frac{1}{3}}}{2} + \frac{8x^2c_1^2}{\left(4+68c_1^3x^3+4\sqrt{33c_1^6x^6+34c_1^3x^3+1}\right)^{\frac{1}{3}}} + 2c_1x}{c_1}$$

$$y(x) = \frac{-\frac{\left(4+68c_1^3x^3+4\sqrt{33c_1^6x^6+34c_1^3x^3+1}\right)^{\frac{1}{3}}}{4} - \frac{4x^2c_1^2}{\left(4+68c_1^3x^3+4\sqrt{33c_1^6x^6+34c_1^3x^3+1}\right)^{\frac{1}{3}}} + 2c_1x - \frac{i\sqrt{3}\left(-16c_1^2x^2+\left(4+68c_1^3x^3+4\sqrt{33c_1^6x^6}\right.\right.}{4\left(4+68c_1^3x^3+4\sqrt{33c_1^6x^6+34c_1^3}\right.}}{c_1}$$

$$y(x) = -\frac{16i\sqrt{3}\,c_1^2x^2 - i\sqrt{3}\left(4+68c_1^3x^3+4\sqrt{33c_1^6x^6+34c_1^3x^3+1}\right)^{\frac{2}{3}} + 16c_1^2x^2 - 8c_1x\left(4+68c_1^3x^3+4\sqrt{33c_1^6x}\right.}{4\left(4+68c_1^3x^3+4\sqrt{33c_1^6x^6+34c_1^3x^3+1}\right)}$$

✓ Solution by Mathematica

Time used: 33.481 (sec). Leaf size: 731

```
DSolve[(x^2+2*y[x]^2)+(4*x*y[x]-y[x]^2)*y'[x]==0,y[x],x,IncludeSingularSolutions -> True]
```

$$
\begin{aligned}
y(x) \to{} & \frac{\sqrt[3]{17x^3+\sqrt{33x^6+34e^{3c_1}x^3+e^{6c_1}}+e^{3c_1}}}{\sqrt[3]{2}} \\
& + \frac{4\sqrt[3]{2}x^2}{\sqrt[3]{17x^3+\sqrt{33x^6+34e^{3c_1}x^3+e^{6c_1}}+e^{3c_1}}} + 2x \\
y(x) \to{} & -\frac{\left(1-i\sqrt{3}\right)\sqrt[3]{17x^3+\sqrt{33x^6+34e^{3c_1}x^3+e^{6c_1}}+e^{3c_1}}}{2\sqrt[3]{2}} \\
& - \frac{2\sqrt[3]{2}\left(1+i\sqrt{3}\right)x^2}{\sqrt[3]{17x^3+\sqrt{33x^6+34e^{3c_1}x^3+e^{6c_1}}+e^{3c_1}}} + 2x \\
y(x) \to{} & -\frac{\left(1+i\sqrt{3}\right)\sqrt[3]{17x^3+\sqrt{33x^6+34e^{3c_1}x^3+e^{6c_1}}+e^{3c_1}}}{2\sqrt[3]{2}} \\
& - \frac{2\sqrt[3]{2}\left(1-i\sqrt{3}\right)x^2}{\sqrt[3]{17x^3+\sqrt{33x^6+34e^{3c_1}x^3+e^{6c_1}}+e^{3c_1}}} + 2x \\
y(x) \to{} & \frac{8\sqrt[3]{2}x^2+4\sqrt[3]{\sqrt{33}\sqrt{x^6}+17x^3}x+2^{2/3}\left(\sqrt{33}\sqrt{x^6}+17x^3\right)^{2/3}}{2\sqrt[3]{\sqrt{33}\sqrt{x^6}+17x^3}} \\
y(x) & \\
\to{} & \frac{8i\sqrt[3]{2}\sqrt{3}x^2-8\sqrt[3]{2}x^2+8\sqrt[3]{\sqrt{33}\sqrt{x^6}+17x^3}x-i2^{2/3}\sqrt{3}\left(\sqrt{33}\sqrt{x^6}+17x^3\right)^{2/3}-2^{2/3}\left(\sqrt{33}\sqrt{x^6}+17x^3\right)}{4\sqrt[3]{\sqrt{33}\sqrt{x^6}+17x^3}} \\
y(x) \to{} & \frac{\left(\sqrt{33}\sqrt{x^6}+17x^3\right)^{2/3}\mathrm{Root}\left[2\#1^3-1\&,3\right]-4\sqrt[3]{-2}x^2+2\sqrt[3]{\sqrt{33}\sqrt{x^6}+17x^3}x}{\sqrt[3]{\sqrt{33}\sqrt{x^6}+17x^3}}
\end{aligned}
$$

## 4.24 problem 23(b)

Internal problem ID [11637]

**Book**: Differential Equations by Shepley L. Ross. Third edition. John Willey. New Delhi. 2004.
**Section**: Chapter 2, section 2.2 (Separable equations). Exercises page 47
**Problem number**: 23(b).
**ODE order**: 1.
**ODE degree**: 1.

CAS Maple gives this as type `[[_homogeneous, ‘class A‘], _exact, _rational, [_Abel, ‘2nd ty`

$$\boxed{2yx + y^2 + \left(2yx + x^2\right) y' = -2x^2}$$

✓ Solution by Maple
Time used: 0.047 (sec). Leaf size: 80

```
dsolve((2*x^2+2*x*y(x)+y(x)^2)+(x^2+2*x*y(x))*diff(y(x),x)=0,y(x), singsol=all)
```

$$y(x) = \frac{-3c_1^2x^2 + \sqrt{3}\,\sqrt{-5\left(c_1^3x^3 - \frac{4}{5}\right)c_1x}}{6c_1^2x}$$
$$y(x) = \frac{-3c_1^2x^2 - \sqrt{3}\,\sqrt{-5\left(c_1^3x^3 - \frac{4}{5}\right)c_1x}}{6c_1^2x}$$

✓ Solution by Mathematica

Time used: 1.277 (sec). Leaf size: 150

```
DSolve[(2*x^2+2*x*y[x]+y[x]^2)+(x^2+2*x*y[x])*y'[x]==0,y[x],x,IncludeSingularSolutions -> Tr
```

$$
\begin{aligned}
y(x) &\to \frac{1}{6}\left(-3x - \frac{\sqrt{3}\sqrt{-5x^3+4e^{3c_1}}}{\sqrt{x}}\right)\\
y(x) &\to \frac{1}{6}\left(-3x + \frac{\sqrt{3}\sqrt{-5x^3+4e^{3c_1}}}{\sqrt{x}}\right)\\
y(x) &\to \frac{1}{6}x\left(\frac{\sqrt{15}x^{3/2}}{\sqrt{-x^3}} - 3\right)\\
y(x) &\to \frac{\sqrt{\frac{5}{3}}\sqrt{-x^3}}{2\sqrt{x}} - \frac{x}{2}
\end{aligned}
$$

# 5 Chapter 2, section 2.3 (Linear equations). Exercises page 56

## 5.1 problem 1

Internal problem ID [11638]

**Book**: Differential Equations by Shepley L. Ross. Third edition. John Willey. New Delhi. 2004.
**Section**: Chapter 2, section 2.3 (Linear equations). Exercises page 56
**Problem number**: 1.
**ODE order**: 1.
**ODE degree**: 1.

CAS Maple gives this as type `[_linear]`

$$\boxed{y' + \frac{3y}{x} = 6x^2}$$

✓ Solution by Maple
Time used: 0.0 (sec). Leaf size: 13

```
dsolve(diff(y(x),x)+3*y(x)/x=6*x^2,y(x), singsol=all)
```

$$y(x) = \frac{x^6 + c_1}{x^3}$$

✓ Solution by Mathematica
Time used: 0.071 (sec). Leaf size: 15

```
DSolve[y'[x]+3*y[x]/x==6*x^2,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to \frac{x^6 + c_1}{x^3}$$

## 5.2 problem 2

Internal problem ID [11639]

**Book**: Differential Equations by Shepley L. Ross. Third edition. John Willey. New Delhi. 2004.
**Section**: Chapter 2, section 2.3 (Linear equations). Exercises page 56
**Problem number**: 2.
**ODE order**: 1.
**ODE degree**: 1.

CAS Maple gives this as type `[_linear]`

$$\boxed{x^4y' + 2yx^3 = 1}$$

✓ Solution by Maple
Time used: 0.015 (sec). Leaf size: 13

```
dsolve(x^4*diff(y(x),x)+2*x^3*y(x)=1,y(x), singsol=all)
```

$$y(x) = \frac{c_1x - 1}{x^3}$$

✓ Solution by Mathematica
Time used: 0.027 (sec). Leaf size: 15

```
DSolve[x^4*y'[x]+2*x^3*y[x]==1,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to \frac{-1 + c_1x}{x^3}$$

## 5.3 problem 3

Internal problem ID [11640]

**Book**: Differential Equations by Shepley L. Ross. Third edition. John Willey. New Delhi. 2004.
**Section**: Chapter 2, section 2.3 (Linear equations). Exercises page 56
**Problem number**: 3.
**ODE order**: 1.
**ODE degree**: 1.

CAS Maple gives this as type `[[_linear, ‘class A‘]]`

$$\boxed{y' + 3y = 3x^2 e^{-3x}}$$

✓ Solution by Maple
Time used: 0.0 (sec). Leaf size: 14

```
dsolve(diff(y(x),x)+3*y(x)=3*x^2*exp(-3*x),y(x), singsol=all)
```

$$y(x) = \left(x^3 + c_1\right) e^{-3x}$$

✓ Solution by Mathematica
Time used: 0.057 (sec). Leaf size: 17

```
DSolve[y'[x]+3*y[x]==3*x^2*Exp[-3*x],y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to e^{-3x}\left(x^3 + c_1\right)$$

## 5.4 problem 4

Internal problem ID [11641]

**Book**: Differential Equations by Shepley L. Ross. Third edition. John Willey. New Delhi. 2004.
**Section**: Chapter 2, section 2.3 (Linear equations). Exercises page 56
**Problem number**: 4.
**ODE order**: 1.
**ODE degree**: 1.

CAS Maple gives this as type **[_separable]**

$$\boxed{y' + 4yx = 8x}$$

✓ Solution by Maple
Time used: 0.0 (sec). Leaf size: 14

```
dsolve(diff(y(x),x)+4*x*y(x)=8*x,y(x), singsol=all)
```

$$y(x) = 2 + e^{-2x^2} c_1$$

✓ Solution by Mathematica
Time used: 0.041 (sec). Leaf size: 22

```
DSolve[y'[x]+4*x*y[x]==8*x,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to 2 + c_1 e^{-2x^2}$$
$$y(x) \to 2$$

## 5.5 problem 5

Internal problem ID [11642]

**Book**: Differential Equations by Shepley L. Ross. Third edition. John Willey. New Delhi. 2004.
**Section**: Chapter 2, section 2.3 (Linear equations). Exercises page 56
**Problem number**: 5.
**ODE order**: 1.
**ODE degree**: 1.

CAS Maple gives this as type `[_separable]`

$$\boxed{x' + \frac{x}{t^2} = \frac{1}{t^2}}$$

✓ Solution by Maple
Time used: 0.0 (sec). Leaf size: 12

```
dsolve(diff(x(t),t)+x(t)/t^2=1/t^2,x(t), singsol=all)
```

$$x(t) = 1 + e^{\frac{1}{t}} c_1$$

✓ Solution by Mathematica
Time used: 0.029 (sec). Leaf size: 20

```
DSolve[x'[t]+x[t]/t^2==1/t^2,x[t],t,IncludeSingularSolutions -> True]
```

$$x(t) \to 1 + c_1 e^{\frac{1}{t}}$$
$$x(t) \to 1$$

## 5.6 problem 6

Internal problem ID [11643]

**Book**: Differential Equations by Shepley L. Ross. Third edition. John Willey. New Delhi. 2004.
**Section**: Chapter 2, section 2.3 (Linear equations). Exercises page 56
**Problem number**: 6.
**ODE order**: 1.
**ODE degree**: 1.

CAS Maple gives this as type `[_separable]`

$$\boxed{(u^2+1)\,v'+4vu=3u}$$

✓ Solution by Maple
Time used: 0.0 (sec). Leaf size: 15

```
dsolve((u^2+1)*diff(v(u),u)+4*u*v(u)=3*u,v(u), singsol=all)
```

$$v(u)=\frac{3}{4}+\frac{c_1}{(u^2+1)^2}$$

✓ Solution by Mathematica
Time used: 0.032 (sec). Leaf size: 38

```
DSolve[(u^2+1)*v'[u]+4*u*v[u]==3*u,v[u],u,IncludeSingularSolutions -> True]
```

$$v(u)\to\frac{3u^4+6u^2+4c_1}{4\,(u^2+1)^2}$$
$$v(u)\to\frac{3}{4}$$

## 5.7 problem 7

Internal problem ID [11644]

**Book**: Differential Equations by Shepley L. Ross. Third edition. John Willey. New Delhi. 2004.
**Section**: Chapter 2, section 2.3 (Linear equations). Exercises page 56
**Problem number**: 7.
**ODE order**: 1.
**ODE degree**: 1.

CAS Maple gives this as type `[_linear]`

$$\boxed{y'x + \frac{(2x+1)\,y}{1+x} = x-1}$$

✓ Solution by Maple
Time used: 0.0 (sec). Leaf size: 24

```
dsolve(x*diff(y(x),x)+(2*x+1)/(x+1)*y(x)=x-1,y(x), singsol=all)
```

$$y(x) = \frac{x^3+3c_1-3x}{3x\,(1+x)}$$

✓ Solution by Mathematica
Time used: 0.037 (sec). Leaf size: 28

```
DSolve[x*y'[x]+(2*x+1)/(x+1)*y[x]==x-1,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to \frac{x^3-3x+3c_1}{3x(x+1)}$$

## 5.8 problem 8

Internal problem ID [11645]

**Book**: Differential Equations by Shepley L. Ross. Third edition. John Willey. New Delhi. 2004.
**Section**: Chapter 2, section 2.3 (Linear equations). Exercises page 56
**Problem number**: 8.
**ODE order**: 1.
**ODE degree**: 1.

CAS Maple gives this as type `[_linear]`

$$\boxed{(x^2 + x - 2)\, y' + 3y(1 + x) = x - 1}$$

✓ Solution by Maple
Time used: 0.016 (sec). Leaf size: 24

```
dsolve((x^2+x-2)*diff(y(x),x)+3*(x+1)*y(x)=x-1,y(x), singsol=all)
```

$$y(x) = \frac{\frac{(-1+x)^3}{3} + c_1}{(x + 2)\,(-1 + x)^2}$$

✓ Solution by Mathematica
Time used: 0.041 (sec). Leaf size: 34

```
DSolve[(x^2+x-2)*y'[x]+3*(x+1)*y[x]==x-1,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to \frac{x^3 - 3x^2 + 3x + 3c_1}{3x^3 - 9x + 6}$$

## 5.9 problem 9

Internal problem ID [11646]

**Book**: Differential Equations by Shepley L. Ross. Third edition. John Willey. New Delhi. 2004.
**Section**: Chapter 2, section 2.3 (Linear equations). Exercises page 56
**Problem number**: 9.
**ODE order**: 1.
**ODE degree**: 1.

CAS Maple gives this as type `[_linear]`

$$\boxed{y'x + yx + y = 1}$$

✓ Solution by Maple
Time used: 0.0 (sec). Leaf size: 16

```
dsolve(x*diff(y(x),x)+(x*y(x)+y(x)-1)=0,y(x), singsol=all)
```

$$y(x) = \frac{c_1 e^{-x} + 1}{x}$$

✓ Solution by Mathematica
Time used: 0.028 (sec). Leaf size: 19

```
DSolve[x*y'[x]+(x*y[x]+y[x]-1)==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to \frac{1 + c_1 e^{-x}}{x}$$

## 5.10 problem 10

Internal problem ID [11647]

**Book**: Differential Equations by Shepley L. Ross. Third edition. John Willey. New Delhi. 2004.
**Section**: Chapter 2, section 2.3 (Linear equations). Exercises page 56
**Problem number**: 10.
**ODE order**: 1.
**ODE degree**: 1.

CAS Maple gives this as type `[_rational, [_1st_order, '_with_symmetry_[F(x)*G(y),0]']]`

$$\boxed{y + (y^2x + x - y)\, y' = 0}$$

✓ Solution by Maple

Time used: 0.031 (sec). Leaf size: 35

```
dsolve(y(x)+(x*y(x)^2+x-y(x))*diff(y(x),x)=0,y(x), singsol=all)
```

$$y(x) = \frac{e^{\mathrm{RootOf}\left(c_1^2 e^{2\_Z} + 2x^2\_Z + 2c_1 e^{\_Z} + 1\right)} c_1 + 1}{x}$$

✓ Solution by Mathematica

Time used: 0.206 (sec). Leaf size: 27

```
DSolve[y[x]+(x*y[x]^2+x-y[x])*y'[x]==0,y[x],x,IncludeSingularSolutions -> True]
```

$$\mathrm{Solve}\left[x = \frac{1}{y(x)} + \frac{c_1 e^{-\frac{1}{2}y(x)^2}}{y(x)}, y(x)\right]$$

## 5.11 problem 11

Internal problem ID [11648]

**Book**: Differential Equations by Shepley L. Ross. Third edition. John Willey. New Delhi. 2004.
**Section**: Chapter 2, section 2.3 (Linear equations). Exercises page 56
**Problem number**: 11.
**ODE order**: 1.
**ODE degree**: 1.

CAS Maple gives this as type `[_linear]`

$$\boxed{r' + r\tan(t) = \cos(t)}$$

✓ Solution by Maple
Time used: 0.0 (sec). Leaf size: 10

```
dsolve(diff(r(t),t)+r(t)*tan(t)=cos(t),r(t), singsol=all)
```

$$r(t) = (t + c_1)\cos(t)$$

✓ Solution by Mathematica
Time used: 0.06 (sec). Leaf size: 12

```
DSolve[r'[t]+r[t]*Tan[t]==Cos[t],r[t],t,IncludeSingularSolutions -> True]
```

$$r(t) \to (t + c_1)\cos(t)$$

## 5.12 problem 12

Internal problem ID [11649]

**Book**: Differential Equations by Shepley L. Ross. Third edition. John Willey. New Delhi. 2004.
**Section**: Chapter 2, section 2.3 (Linear equations). Exercises page 56
**Problem number**: 12.
**ODE order**: 1.
**ODE degree**: 1.

CAS Maple gives this as type `[_linear]`

$$\boxed{\cos(t)\, r' + r\sin(t) = \cos(t)^4}$$

✓ Solution by Maple
Time used: 0.0 (sec). Leaf size: 19

```
dsolve(cos(t)*diff(r(t),t)+(r(t)*sin(t)-cos(t)^4)=0,r(t), singsol=all)
```

$$r(t) = \frac{(2t + \sin(2t) + 4c_1)\cos(t)}{4}$$

✓ Solution by Mathematica
Time used: 0.052 (sec). Leaf size: 22

```
DSolve[Cos[t]*r'[t]+(r[t]*Sin[t]-Cos[t]^4)==0,r[t],t,IncludeSingularSolutions -> True]
```

$$r(t) \to \frac{1}{2}\cos(t)(t + \sin(t)\cos(t) + 2c_1)$$

## 5.13 problem 13

Internal problem ID [11650]

**Book**: Differential Equations by Shepley L. Ross. Third edition. John Willey. New Delhi. 2004.
**Section**: Chapter 2, section 2.3 (Linear equations). Exercises page 56
**Problem number**: 13.
**ODE order**: 1.
**ODE degree**: 1.

CAS Maple gives this as type `[_linear]`

$$\boxed{-y\cos(x) - (1+\sin(x))\, y' = -\cos(x)^2}$$

✓ Solution by Maple
Time used: 0.0 (sec). Leaf size: 23

```
dsolve((cos(x)^2-y(x)*cos(x))-(1+sin(x))*diff(y(x),x)=0,y(x), singsol=all)
```

$$y(x) = \frac{\sin(x)\cos(x) + 2c_1 + x}{2 + 2\sin(x)}$$

✓ Solution by Mathematica
Time used: 0.314 (sec). Leaf size: 25

```
DSolve[(Cos[x]^2-y[x]*Cos[x])-(1+Sin[x])*y'[x]==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to \frac{x + \sin(x)\cos(x) + 2c_1}{2\sin(x) + 2}$$

## 5.14 problem 14

Internal problem ID [11651]

**Book**: Differential Equations by Shepley L. Ross. Third edition. John Willey. New Delhi. 2004.
**Section**: Chapter 2, section 2.3 (Linear equations). Exercises page 56
**Problem number**: 14.
**ODE order**: 1.
**ODE degree**: 1.

CAS Maple gives this as type `[_linear]`

$$\boxed{y\sin(2x) + \left(1+\sin(x)^2\right)y' = \cos(x)}$$

✓ Solution by Maple
Time used: 0.0 (sec). Leaf size: 21

```
dsolve((y(x)*sin(2*x)-cos(x))+(1+sin(x)^2)*diff(y(x),x)=0,y(x), singsol=all)
```

$$y(x) = \frac{-\sin(x) - c_1}{\cos(x)^2 - 2}$$

✓ Solution by Mathematica
Time used: 0.173 (sec). Leaf size: 21

```
DSolve[(y[x]*Sin[2*x]-Cos[x])+(1+Sin[x]^2)*y'[x]==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to \frac{-2\sin(x) + c_1}{\cos(2x) - 3}$$

## 5.15 problem 15

Internal problem ID [11652]

**Book**: Differential Equations by Shepley L. Ross. Third edition. John Willey. New Delhi. 2004.
**Section**: Chapter 2, section 2.3 (Linear equations). Exercises page 56
**Problem number**: 15.
**ODE order**: 1.
**ODE degree**: 1.

CAS Maple gives this as type `[_separable]`

$$\boxed{y' - \frac{y}{x} + \frac{y^2}{x} = 0}$$

✓ Solution by Maple
Time used: 0.0 (sec). Leaf size: 11

```
dsolve(diff(y(x),x)-y(x)/x=-y(x)^2/x,y(x), singsol=all)
```

$$y(x) = \frac{x}{c_1 + x}$$

✓ Solution by Mathematica
Time used: 0.238 (sec). Leaf size: 25

```
DSolve[y'[x]-y[x]/x==-y[x]^2/x,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to \frac{x}{x + e^{c_1}}$$
$$y(x) \to 0$$
$$y(x) \to 1$$

## 5.16 problem 16

Internal problem ID [11653]

**Book**: Differential Equations by Shepley L. Ross. Third edition. John Willey. New Delhi. 2004.
**Section**: Chapter 2, section 2.3 (Linear equations). Exercises page 56
**Problem number**: 16.
**ODE order**: 1.
**ODE degree**: 1.

CAS Maple gives this as type `[[_homogeneous, ‘class G‘], _rational, _Bernoulli]`

$$\boxed{y'x + y + 2x^6y^4 = 0}$$

✓ Solution by Maple
Time used: 0.015 (sec). Leaf size: 67

```
dsolve(x*diff(y(x),x)+y(x)=-2*x^6*y(x)^4,y(x), singsol=all)
```

$$y(x) = \frac{1}{(2x^3 + c_1)^{\frac{1}{3}}\, x}$$
$$y(x) = -\frac{1 + i\sqrt{3}}{2\,(2x^3 + c_1)^{\frac{1}{3}}\, x}$$
$$y(x) = \frac{i\sqrt{3} - 1}{2\,(2x^3 + c_1)^{\frac{1}{3}}\, x}$$

✓ Solution by Mathematica

Time used: 0.87 (sec). Leaf size: 79

```
DSolve[x*y'[x]+y[x]==-2*x^6*y[x]^4,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to \frac{1}{\sqrt[3]{x^3 (2x^3 + c_1)}}$$
$$y(x) \to -\frac{\sqrt[3]{-1}}{\sqrt[3]{x^3 (2x^3 + c_1)}}$$
$$y(x) \to \frac{(-1)^{2/3}}{\sqrt[3]{x^3 (2x^3 + c_1)}}$$
$$y(x) \to 0$$

## 5.17 problem 17

Internal problem ID [11654]

**Book**: Differential Equations by Shepley L. Ross. Third edition. John Willey. New Delhi. 2004.
**Section**: Chapter 2, section 2.3 (Linear equations). Exercises page 56
**Problem number**: 17.
**ODE order**: 1.
**ODE degree**: 1.

CAS Maple gives this as type `[_separable]`

$$\boxed{y' + \left(4y - \frac{8}{y^3}\right)x = 0}$$

✓ Solution by Maple

Time used: 0.015 (sec). Leaf size: 94

```
dsolve(diff(y(x),x)+(4*y(x)-8/y(x)^3)*x=0,y(x), singsol=all)
```

$$y(x) = \left(2\,\mathrm{e}^{8x^2} + c_1\right)^{\frac{1}{4}} \mathrm{e}^{-2x^2}$$
$$y(x) = -\left(2\,\mathrm{e}^{8x^2} + c_1\right)^{\frac{1}{4}} \mathrm{e}^{-2x^2}$$
$$y(x) = -i\left(2\,\mathrm{e}^{8x^2} + c_1\right)^{\frac{1}{4}} \mathrm{e}^{-2x^2}$$
$$y(x) = i\left(2\,\mathrm{e}^{8x^2} + c_1\right)^{\frac{1}{4}} \mathrm{e}^{-2x^2}$$

✓ Solution by Mathematica

Time used: 1.939 (sec). Leaf size: 145

```
DSolve[y'[x]+(4*y[x]-8/y[x]^3)*x==0,y[x],x,IncludeSingularSolutions -> True]
```

$$
\begin{aligned}
y(x) &\to -\sqrt[4]{2+e^{-8x^2+4c_1}} \\
y(x) &\to -i\sqrt[4]{2+e^{-8x^2+4c_1}} \\
y(x) &\to i\sqrt[4]{2+e^{-8x^2+4c_1}} \\
y(x) &\to \sqrt[4]{2+e^{-8x^2+4c_1}} \\
y(x) &\to -\sqrt[4]{2} \\
y(x) &\to -i\sqrt[4]{2} \\
y(x) &\to i\sqrt[4]{2} \\
y(x) &\to \sqrt[4]{2}
\end{aligned}
$$

## 5.18 problem 18

Internal problem ID [11655]

**Book**: Differential Equations by Shepley L. Ross. Third edition. John Willey. New Delhi. 2004.
**Section**: Chapter 2, section 2.3 (Linear equations). Exercises page 56
**Problem number**: 18.
**ODE order**: 1.
**ODE degree**: 1.

CAS Maple gives this as type `[_separable]`

$$\boxed{x' + \frac{(1+t)\,x}{2t} - \frac{1+t}{xt} = 0}$$

✓ Solution by Maple

Time used: 0.031 (sec). Leaf size: 46

```
dsolve(diff(x(t),t)+(t+1)/(2*t)*x(t)=(t+1)/(x(t)*t),x(t), singsol=all)
```

$$x(t) = \frac{\sqrt{t\,\mathrm{e}^{-t}c_1 + 2t^2}}{t}$$
$$x(t) = -\frac{\sqrt{t\,\mathrm{e}^{-t}c_1 + 2t^2}}{t}$$

✓ Solution by Mathematica

Time used: 3.335 (sec). Leaf size: 78

```
DSolve[x'[t]+(t+1)/(2*t)*x[t]==(t+1)/(x[t]*t),x[t],t,IncludeSingularSolutions -> True]
```

$$x(t) \to -\frac{\sqrt{2t + e^{-t+2c_1}}}{\sqrt{t}}$$
$$x(t) \to \frac{\sqrt{2t + e^{-t+2c_1}}}{\sqrt{t}}$$
$$x(t) \to -\sqrt{2}$$
$$x(t) \to \sqrt{2}$$

## 5.19 problem 19

Internal problem ID [11656]

**Book**: Differential Equations by Shepley L. Ross. Third edition. John Willey. New Delhi. 2004.
**Section**: Chapter 2, section 2.3 (Linear equations). Exercises page 56
**Problem number**: 19.
**ODE order**: 1.
**ODE degree**: 1.

CAS Maple gives this as type `[_linear]`

$$\boxed{y'x - 2y = 2x^4}$$

With initial conditions

$$[y(2) = 8]$$

✓ Solution by Maple
Time used: 0.0 (sec). Leaf size: 13

```
dsolve([x*diff(y(x),x)-2*y(x)=2*x^4,y(2) = 8],y(x), singsol=all)
```

$$y(x) = (x^2 - 2)\, x^2$$

✓ Solution by Mathematica
Time used: 0.043 (sec). Leaf size: 14

```
DSolve[{x*y'[x]-2*y[x]==2*x^4,{y[2]==8}},y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to x^2(x^2 - 2)$$

## 5.20 problem 20

Internal problem ID [11657]

**Book**: Differential Equations by Shepley L. Ross. Third edition. John Willey. New Delhi. 2004.
**Section**: Chapter 2, section 2.3 (Linear equations). Exercises page 56
**Problem number**: 20.
**ODE order**: 1.
**ODE degree**: 1.

CAS Maple gives this as type `[_separable]`

$$\boxed{y' + 3x^2 y = x^2}$$

With initial conditions

$$[y(0) = 2]$$

✓ Solution by Maple
Time used: 0.016 (sec). Leaf size: 14

```
dsolve([diff(y(x),x)+3*x^2*y(x)=x^2,y(0) = 2],y(x), singsol=all)
```

$$y(x) = \frac{1}{3} + \frac{5\,\mathrm{e}^{-x^3}}{3}$$

✓ Solution by Mathematica
Time used: 2.884 (sec). Leaf size: 20

```
DSolve[{y'[x]+3*x^2*y[x]==x^2,{y[0]==2}},y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to \frac{5e^{-x^3}}{3} + \frac{1}{3}$$

## 5.21 problem 21

Internal problem ID [11658]

**Book**: Differential Equations by Shepley L. Ross. Third edition. John Willey. New Delhi. 2004.
**Section**: Chapter 2, section 2.3 (Linear equations). Exercises page 56
**Problem number**: 21.
**ODE order**: 1.
**ODE degree**: 1.

CAS Maple gives this as type `[_linear]`

$$\boxed{e^x\left(y-3(e^x+1)^2\right)+(e^x+1)\,y'=0}$$

With initial conditions

$$[y(0)=4]$$

✓ Solution by Maple
Time used: 0.016 (sec). Leaf size: 14

```
dsolve([exp(x)*(y(x)-3*(exp(x)+1)^2)+(exp(x)+1)*diff(y(x),x)=0,y(0) = 4],y(x), singsol=all)
```

$$y(x)=e^{2x}+2\,e^x+1$$

✓ Solution by Mathematica
Time used: 0.104 (sec). Leaf size: 12

```
DSolve[{Exp[x]*(y[x]-3*(Exp[x]+1)^2)+(Exp[x]+1)*y'[x]==0,{y[0]==4}},y[x],x,IncludeSingularSo
```

$$y(x)\to (e^x+1)^2$$

## 5.22 problem 22

Internal problem ID [11659]

**Book**: Differential Equations by Shepley L. Ross. Third edition. John Willey. New Delhi. 2004.
**Section**: Chapter 2, section 2.3 (Linear equations). Exercises page 56
**Problem number**: 22.
**ODE order**: 1.
**ODE degree**: 1.

CAS Maple gives this as type `[_separable]`

$$\boxed{2(y+1)\,x-\left(x^2+1\right)y'=0}$$

With initial conditions

$$[y(1)=-5]$$

✓ Solution by Maple
Time used: 0.016 (sec). Leaf size: 11

```
dsolve([2*x*(y(x)+1)-(x^2+1)*diff(y(x),x)=0,y(1) = -5],y(x), singsol=all)
```

$$y(x)=-2x^2-3$$

✓ Solution by Mathematica
Time used: 0.032 (sec). Leaf size: 12

```
DSolve[{2*x*(y[x]+1)-(x^2+1)*y'[x]==0,{y[1]==-5}},y[x],x,IncludeSingularSolutions -> True]
```

$$y(x)\to -2x^2-3$$

## 5.23 problem 23

Internal problem ID [11660]

**Book**: Differential Equations by Shepley L. Ross. Third edition. John Willey. New Delhi. 2004.
**Section**: Chapter 2, section 2.3 (Linear equations). Exercises page 56
**Problem number**: 23.
**ODE order**: 1.
**ODE degree**: 1.

CAS Maple gives this as type `[_linear]`

$$\boxed{r' + r\tan(t) = \cos(t)^2}$$

With initial conditions

$$\left[r\left(\frac{\pi}{4}\right) = 1\right]$$

✓ Solution by Maple

Time used: 0.016 (sec). Leaf size: 15

```
dsolve([diff(r(t),t)+r(t)*tan(t)=cos(t)^2,r(1/4*Pi) = 1],r(t), singsol=all)
```

$$r(t) = \frac{(2\sin(t) + \sqrt{2})\cos(t)}{2}$$

✓ Solution by Mathematica

Time used: 0.077 (sec). Leaf size: 16

```
DSolve[{r'[t]+r[t]*Tan[t]==Cos[t]^2,{r[Pi/4]==1}},r[t],t,IncludeSingularSolutions -> True]
```

$$r(t) \to \left(\sin(t) + \frac{1}{\sqrt{2}}\right)\cos(t)$$

## 5.24 problem 24

Internal problem ID [11661]

**Book**: Differential Equations by Shepley L. Ross. Third edition. John Willey. New Delhi. 2004.
**Section**: Chapter 2, section 2.3 (Linear equations). Exercises page 56
**Problem number**: 24.
**ODE order**: 1.
**ODE degree**: 1.

CAS Maple gives this as type `[[_linear, 'class A']]`

$$\boxed{-x + x' = \sin(2t)}$$

With initial conditions

$$[x(0) = 0]$$

✓ Solution by Maple
Time used: 0.016 (sec). Leaf size: 21

```
dsolve([diff(x(t),t)-x(t)=sin(2*t),x(0) = 0],x(t), singsol=all)
```

$$x(t) = -\frac{2\cos(2t)}{5} - \frac{\sin(2t)}{5} + \frac{2\,\mathrm{e}^{t}}{5}$$

✓ Solution by Mathematica
Time used: 0.083 (sec). Leaf size: 27

```
DSolve[{x'[t]-x[t]==Sin[2*t],{x[0]==0}},x[t],t,IncludeSingularSolutions -> True]
```

$$x(t) \to \frac{1}{5}\left(2e^{t} - \sin(2t) - 2\cos(2t)\right)$$

## 5.25 problem 25

Internal problem ID [11662]

**Book**: Differential Equations by Shepley L. Ross. Third edition. John Willey. New Delhi. 2004.
**Section**: Chapter 2, section 2.3 (Linear equations). Exercises page 56
**Problem number**: 25.
**ODE order**: 1.
**ODE degree**: 1.

CAS Maple gives this as type `[[_homogeneous, ‘class G‘], _rational, _Bernoulli]`

$$\boxed{y' + \frac{y}{2x} - \frac{x}{y^3} = 0}$$

With initial conditions

$$[y(1) = 2]$$

✓ Solution by Maple
Time used: 0.078 (sec). Leaf size: 17

```
dsolve([diff(y(x),x)+y(x)/(2*x)=x/y(x)^3,y(1) = 2],y(x), singsol=all)
```

$$y(x) = \sqrt{\frac{\sqrt{x^4 + 15}}{x}}$$

✓ Solution by Mathematica
Time used: 0.277 (sec). Leaf size: 20

```
DSolve[{y'[x]+y[x]/(2*x)==x/y[x]^3,{y[1]==2}},y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to \frac{\sqrt[4]{x^4 + 15}}{\sqrt{x}}$$

## 5.26 problem 26

Internal problem ID [11663]

**Book**: Differential Equations by Shepley L. Ross. Third edition. John Willey. New Delhi. 2004.
**Section**: Chapter 2, section 2.3 (Linear equations). Exercises page 56
**Problem number**: 26.
**ODE order**: 1.
**ODE degree**: 1.

CAS Maple gives this as type `[[_homogeneous, ‘class G‘], _rational]`

$$\boxed{y'x + y - (yx)^{\frac{3}{2}} = 0}$$

With initial conditions

$$[y(1) = 4]$$

✓ Solution by Maple

Time used: 0.203 (sec). Leaf size: 9

```
dsolve([x*diff(y(x),x)+y(x)=(x*y(x))^(3/2),y(1) = 4],y(x), singsol=all)
```

$$y(x) = \frac{4}{x^3}$$

✓ Solution by Mathematica

Time used: 0.258 (sec). Leaf size: 24

```
DSolve[{x*y'[x]+y[x]==(x*y[x])^(3/2),{y[1]==4}},y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to \frac{4}{x^3}$$
$$y(x) \to \frac{4}{(x-2)^2 x}$$

## 5.27 problem 27

Internal problem ID [11664]

**Book**: Differential Equations by Shepley L. Ross. Third edition. John Willey. New Delhi. 2004.
**Section**: Chapter 2, section 2.3 (Linear equations). Exercises page 56
**Problem number**: 27.
**ODE order**: 1.
**ODE degree**: 1.

CAS Maple gives this as type `[[_linear, ‘class A‘]]`

$$\boxed{y' + y = \begin{cases} 2 & 0 \leq x < 1 \\ 0 & 1 \leq x \end{cases}}$$

With initial conditions

$$[y(0) = 0]$$

✓ Solution by Maple
Time used: 0.328 (sec). Leaf size: 38

```
dsolve([diff(y(x),x)+y(x)=piecewise(0<=x and x<1,2,x>=1,0),y(0) = 0],y(x), singsol=all)
```

$$y(x) = \begin{cases} 0 & x < 0 \\ 2 - 2\,\mathrm{e}^{-x} & x < 1 \\ 2\,\mathrm{e}^{1-x} - 2\,\mathrm{e}^{-x} & 1 \leq x \end{cases}$$

✓ Solution by Mathematica

Time used: 0.062 (sec). Leaf size: 38

```
DSolve[{y'[x]+y[x]==Piecewise[{{2,0<=x<1},{0,x>=1}}],{y[0]==0}},y[x],x,IncludeSingularSoluti
```

$$y(x) \to \begin{cases} 0 & x \leq 0 \\ 2 - 2e^{-x} & 0 < x \leq 1 \\ 2(-1+e)e^{-x} & \text{True} \end{cases}$$

## 5.28 problem 28

Internal problem ID [11665]

**Book**: Differential Equations by Shepley L. Ross. Third edition. John Willey. New Delhi. 2004.
**Section**: Chapter 2, section 2.3 (Linear equations). Exercises page 56
**Problem number**: 28.
**ODE order**: 1.
**ODE degree**: 1.

CAS Maple gives this as type `[[_linear, 'class A']]`

$$\boxed{y' + y = \begin{cases} 5 & 0 \leq x < 10 \\ 1 & 10 \leq x \end{cases}}$$

With initial conditions

$$[y(0) = 6]$$

✓ Solution by Maple

Time used: 7.219 (sec). Leaf size: 40

```
dsolve([diff(y(x),x)+y(x)=piecewise(0<=x and x<10,5,x>=10,1),y(0) = 6],y(x), singsol=all)
```

$$y(x) = \begin{cases} 6\,\mathrm{e}^{-x} & x < 0 \\ \mathrm{e}^{-x} + 5 & x < 10 \\ \mathrm{e}^{-x} + 1 + 4\,\mathrm{e}^{10-x} & 10 \leq x \end{cases}$$

✓ Solution by Mathematica

Time used: 0.061 (sec). Leaf size: 45

```
DSolve[{y'[x]+y[x]==Piecewise[{{5,0<=x<10},{1,x>=10}}],{y[0]==6}},y[x],x,IncludeSingularSolu
```

$$y(x) \to \begin{cases} 6e^{-x} & x \leq 0 \\ e^{-x}(1 + 4e^{10} + e^{x}) & x > 10 \\ 5 + e^{-x} & \text{True} \end{cases}$$

## 5.29 problem 29

Internal problem ID [11666]

**Book**: Differential Equations by Shepley L. Ross. Third edition. John Willey. New Delhi. 2004.
**Section**: Chapter 2, section 2.3 (Linear equations). Exercises page 56
**Problem number**: 29.
**ODE order**: 1.
**ODE degree**: 1.

CAS Maple gives this as type `[[_linear, 'class A']]`

$$\boxed{y' + y = \begin{cases} e^{-x} & 0 \leq x < 2 \\ e^{-2} & 2 \leq x \end{cases}}$$

With initial conditions

$$[y(0) = 1]$$

✓ Solution by Maple

Time used: 0.359 (sec). Leaf size: 35

```
dsolve([diff(y(x),x)+y(x)=piecewise(0<=x and x<2,exp(-x),x>=2,exp(-2)),y(0) = 1],y(x), sings
```

$$y(x) = \begin{cases} e^{-x} & x < 0 \\ e^{-x}(1+x) & x < 2 \\ 2\,e^{-x} + e^{-2} & 2 \leq x \end{cases}$$

✓ Solution by Mathematica

Time used: 0.103 (sec). Leaf size: 40

```
DSolve[{y'[x]+y[x]==Piecewise[{{Exp[-x],0<=x<2},{Exp[-2],x>=2}}],{y[0]==1}},y[x],x,IncludeSi
```

$$y(x) \to \begin{cases} e^{-x} & x \leq 0 \\ \frac{1}{e^2} + 2e^{-x} & x > 2 \\ e^{-x}(x+1) & \text{True} \end{cases}$$

## 5.30 problem 30

Internal problem ID [11667]

**Book**: Differential Equations by Shepley L. Ross. Third edition. John Willey. New Delhi. 2004.
**Section**: Chapter 2, section 2.3 (Linear equations). Exercises page 56
**Problem number**: 30.
**ODE order**: 1.
**ODE degree**: 1.

CAS Maple gives this as type `[_linear]`

$$\boxed{(x+2)\,y'+y=\begin{cases} 2x & 0\le x<2 \\ 4 & 2\le x \end{cases}}$$

With initial conditions

$$[y(0)=4]$$

✓ Solution by Maple

Time used: 0.265 (sec). Leaf size: 31

```
dsolve([(x+2)*diff(y(x),x)+y(x)=piecewise(0<=x and x<2,2*x,x>=2,4),y(0) = 4],y(x), singsol=a
```

$$y(x)=\frac{\begin{cases} 8 & x<0 \\ x^2+8 & x<2 \\ 4+4x & 2\le x \end{cases}}{x+2}$$

✓ Solution by Mathematica

Time used: 0.074 (sec). Leaf size: 43

```
DSolve[{(x+2)*y'[x]+y[x]==Piecewise[{{2*x,0<=x<2},{4,x>=2}}],{y[0]==4}},y[x],x,IncludeSingul
```

$$y(x) \to \begin{cases} \frac{8}{x+2} & x \leq 0 \\ \frac{4(x+1)}{x+2} & x > 2 \\ \frac{x^2+8}{x+2} & \text{True} \end{cases}$$

## 5.31 problem 31

Internal problem ID [11668]

**Book**: Differential Equations by Shepley L. Ross. Third edition. John Willey. New Delhi. 2004.
**Section**: Chapter 2, section 2.3 (Linear equations). Exercises page 56
**Problem number**: 31.
**ODE order**: 1.
**ODE degree**: 1.

CAS Maple gives this as type `[[_linear, 'class A']]`

$$\boxed{ay' + yb = k\,\mathrm{e}^{-\lambda x}}$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 49

```
dsolve(a*diff(y(x),x)+b*y(x)=k*exp(-lambda*x),y(x), singsol=all)
```

$$y(x) = \frac{\left(-k\,\mathrm{e}^{-\frac{x(a\lambda-b)}{a}} + c_1(a\lambda - b)\right)\mathrm{e}^{-\frac{bx}{a}}}{a\lambda - b}$$

✓ Solution by Mathematica

Time used: 0.087 (sec). Leaf size: 44

```
DSolve[a*y'[x]+b*y[x]==k*Exp[\[Lambda]*x],y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to \frac{e^{-\frac{bx}{a}}\left(ke^{x\left(\frac{b}{a}+\lambda\right)} + c_1(a\lambda + b)\right)}{a\lambda + b}$$

## 5.32 problem 35 (b)

Internal problem ID [11669]

**Book**: Differential Equations by Shepley L. Ross. Third edition. John Willey. New Delhi. 2004.
**Section**: Chapter 2, section 2.3 (Linear equations). Exercises page 56
**Problem number**: 35 (b).
**ODE order**: 1.
**ODE degree**: 1.

CAS Maple gives this as type `[[_linear, 'class A']]`

$$\boxed{y' + y = 2\sin(x) + 5\sin(2x)}$$

✓ Solution by Maple
Time used: 0.0 (sec). Leaf size: 27

```
dsolve(diff(y(x),x)+y(x)=2*sin(x)+5*sin(2*x),y(x), singsol=all)
```

$$y(x) = -\cos(x) - 2\cos(2x) + \sin(x) + \sin(2x) + c_1 e^{-x}$$

✓ Solution by Mathematica
Time used: 0.131 (sec). Leaf size: 30

```
DSolve[y'[x]+y[x]==2*Sin[x]+5*Sin[2*x],y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to \sin(x) + \sin(2x) - \cos(x) - 2\cos(2x) + c_1 e^{-x}$$

## 5.33 problem 37 (a)

Internal problem ID [11670]

**Book**: Differential Equations by Shepley L. Ross. Third edition. John Willey. New Delhi. 2004.
**Section**: Chapter 2, section 2.3 (Linear equations). Exercises page 56
**Problem number**: 37 (a).
**ODE order**: 1.
**ODE degree**: 1.

CAS Maple gives this as type `[‘y=_G(x,y’)‘]`

$$\boxed{\cos(y)\, y' + \frac{\sin(y)}{x} = 1}$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 21

```
dsolve(cos(y(x))*diff(y(x),x)+1/x*sin(y(x))=1,y(x), singsol=all)
```

$$y(x) = -\arcsin\left(\frac{-x^2+2c_1}{2x}\right)$$

✓ Solution by Mathematica

Time used: 8.67 (sec). Leaf size: 18

```
DSolve[Cos[y[x]]*y'[x]+1/x*Sin[y[x]]==1,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to \arcsin\left(\frac{x}{2}+\frac{c_1}{x}\right)$$

## 5.34 problem 37 (b)

Internal problem ID [11671]

**Book**: Differential Equations by Shepley L. Ross. Third edition. John Willey. New Delhi. 2004.
**Section**: Chapter 2, section 2.3 (Linear equations). Exercises page 56
**Problem number**: 37 (b).
**ODE order**: 1.
**ODE degree**: 1.

CAS Maple gives this as type `[_separable]`

$$\boxed{(y+1)\,y'+x\left(y^2+2y\right)=x}$$

✓ Solution by Maple
Time used: 0.015 (sec). Leaf size: 37

```
dsolve((y(x)+1)*diff(y(x),x)+x*(y(x)^2+2*y(x))=x,y(x), singsol=all)
```

$$y(x)=-1-\sqrt{2+\mathrm{e}^{-x^2}c_1}$$
$$y(x)=-1+\sqrt{2+\mathrm{e}^{-x^2}c_1}$$

✓ Solution by Mathematica
Time used: 29.843 (sec). Leaf size: 163

```
DSolve[(y[x]+1)*y'[x]+x*(y[x]^2+2*y[x])==x,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x)\to -1-e^{-x^2}\sqrt{e^{x^2}\left(2e^{x^2}+e^{2c_1}\right)}$$
$$y(x)\to -1+e^{-x^2}\sqrt{e^{x^2}\left(2e^{x^2}+e^{2c_1}\right)}$$
$$y(x)\to -1-\sqrt{2}$$
$$y(x)\to \sqrt{2}-1$$
$$y(x)\to \sqrt{2}e^{-x^2}\sqrt{e^{2x^2}}-1$$
$$y(x)\to -\sqrt{2}e^{-x^2}\sqrt{e^{2x^2}}-1$$

## 5.35 problem 39

Internal problem ID [11672]

**Book**: Differential Equations by Shepley L. Ross. Third edition. John Willey. New Delhi. 2004.
**Section**: Chapter 2, section 2.3 (Linear equations). Exercises page 56
**Problem number**: 39.
**ODE order**: 1.
**ODE degree**: 1.

CAS Maple gives this as type `[_Riccati]`

$$\boxed{y' - (1-x)\,y^2 - (2x-1)\,y = -x}$$

✓ Solution by Maple
Time used: 0.0 (sec). Leaf size: 31

```
dsolve(diff(y(x),x)=(1-x)*y(x)^2+(2*x-1)*y(x)-x,y(x), singsol=all)
```

$$y(x) = \frac{(2x-2)\,\mathrm{e}^{x} - c_1}{(2x-4)\,\mathrm{e}^{x} - c_1}$$

✓ Solution by Mathematica
Time used: 0.197 (sec). Leaf size: 28

```
DSolve[y'[x]==(1-x)*y[x]^2+(2*x-1)*y[x]-x,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to 1 + \frac{e^x}{e^x(x-2) + c_1}$$
$$y(x) \to 1$$

## 5.36 problem 40

Internal problem ID [11673]

**Book**: Differential Equations by Shepley L. Ross. Third edition. John Willey. New Delhi. 2004.
**Section**: Chapter 2, section 2.3 (Linear equations). Exercises page 56
**Problem number**: 40.
**ODE order**: 1.
**ODE degree**: 1.

CAS Maple gives this as type `[_Riccati]`

$$\boxed{y' + y^2 - yx = 1}$$

✓ Solution by Maple
Time used: 0.031 (sec). Leaf size: 53

```
dsolve(diff(y(x),x)=-y(x)^2+x*y(x)+1,y(x), singsol=all)
```

$$y(x) = \frac{\sqrt{\pi}\,\sqrt{2}\,\mathrm{erf}\left(\frac{x\sqrt{2}}{2}\right) x + 2c_1 x + 2\,\mathrm{e}^{-\frac{x^2}{2}}}{\sqrt{\pi}\,\sqrt{2}\,\mathrm{erf}\left(\frac{x\sqrt{2}}{2}\right) + 2c_1}$$

✓ Solution by Mathematica
Time used: 0.154 (sec). Leaf size: 45

```
DSolve[y'[x]==-y[x]^2+x*y[x]+1,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to x + \frac{e^{-\frac{x^2}{2}}}{\sqrt{\frac{\pi}{2}}\mathrm{erf}\left(\frac{x}{\sqrt{2}}\right) + c_1}$$
$$y(x) \to x$$

## 5.37 problem 41

Internal problem ID [11674]

**Book**: Differential Equations by Shepley L. Ross. Third edition. John Willey. New Delhi. 2004.
**Section**: Chapter 2, section 2.3 (Linear equations). Exercises page 56
**Problem number**: 41.
**ODE order**: 1.
**ODE degree**: 1.

CAS Maple gives this as type `[[_1st_order, `_with_symmetry_[F(x),G(x)]`], _Riccati]`

$$\boxed{y' + 8y^2x - 4x(4x+1)\,y = -8x^3 - 4x^2 + 1}$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 60

```
dsolve(diff(y(x),x)=-8*x*y(x)^2+4*x*(4*x+1)*y(x)-(8*x^3+4*x^2-1),y(x), singsol=all)
```

$$y(x) = \frac{c_1(2x+1)\,\mathrm{e}^{\frac{8}{3}x^3+2x^2} + 2\,\mathrm{e}^{\frac{8x^3}{3}}x}{2c_1\mathrm{e}^{\frac{8}{3}x^3+2x^2} + 2\,\mathrm{e}^{\frac{8x^3}{3}}}$$

✓ Solution by Mathematica

Time used: 0.196 (sec). Leaf size: 30

```
DSolve[y'[x]==-8*x*y[x]^2+4*x*(4*x+1)*y[x]-(8*x^3+4*x^2-1),y[x],x,IncludeSingularSolutions -
```

$$y(x) \to \frac{1}{4}\left(\tanh\left(x^2 + ic_1\right) + 4x + 1\right)$$
$$y(x) \to \text{Indeterminate}$$

# 6 Chapter 2, Miscellaneous Review. Exercises page 60

## 6.1 problem 1

Internal problem ID [11675]

**Book**: Differential Equations by Shepley L. Ross. Third edition. John Willey. New Delhi. 2004.
**Section**: Chapter 2, Miscellaneous Review. Exercises page 60
**Problem number**: 1.
**ODE order**: 1.
**ODE degree**: 1.

CAS Maple gives this as type `[_separable]`

$$\boxed{6x^2y - (x^3+1)\,y' = 0}$$

✓ Solution by Maple
Time used: 0.0 (sec). Leaf size: 13

```
dsolve(6*x^2*y(x)-(x^3+1)*diff(y(x),x)=0,y(x), singsol=all)
```

$$y(x) = c_1(x^3+1)^2$$

✓ Solution by Mathematica
Time used: 0.031 (sec). Leaf size: 20

```
DSolve[6*x^2*y[x]-(x^3+1)*y'[x]==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to c_1(x^3+1)^2$$
$$y(x) \to 0$$

## 6.2 problem 2

Internal problem ID [11676]

**Book**: Differential Equations by Shepley L. Ross. Third edition. John Willey. New Delhi. 2004.
**Section**: Chapter 2, Miscellaneous Review. Exercises page 60
**Problem number**: 2.
**ODE order**: 1.
**ODE degree**: 1.

CAS Maple gives this as type `[[_homogeneous, ‘class G‘], _exact, _rational]`

$$\boxed{(3x^2y^2 - x)\, y' + 2y^3x - y = 0}$$

✓ Solution by Maple

Time used: 0.078 (sec). Leaf size: 1127

```
dsolve((3*x^2*y(x)^2-x)*diff(y(x),x)+(2*x*y(x)^3-y(x))=0,y(x), singsol=all)
```

$$y(x) = -\frac{\sqrt{3}\,\sqrt{2}\,\sqrt{\left(\left(3\sqrt{3}\,\sqrt{27c_1^2-4c_1x}+27c_1-2x\right)x^2\right)^{\frac{1}{3}}\,(2x^2 2^{\frac{1}{3}}+2^{\frac{2}{3}}((3\sqrt{3}\,\sqrt{27c_1^2-4c_1x}+27c_1-2x)}}{6\left(\left(3\sqrt{3}\,\sqrt{27c_1^2-4c_1x}+27c_1-2x\right)x^2\right)^{\frac{1}{3}}x}$$

$$y(x) = \frac{\sqrt{3}\,\sqrt{2}\,\sqrt{\left(\left(3\sqrt{3}\,\sqrt{27c_1^2-4c_1x}+27c_1-2x\right)x^2\right)^{\frac{1}{3}}\,(2x^2 2^{\frac{1}{3}}+2^{\frac{2}{3}}((3\sqrt{3}\,\sqrt{27c_1^2-4c_1x}+27c_1-2x)^2\,x}}{6\left(\left(3\sqrt{3}\,\sqrt{27c_1^2-4c_1x}+27c_1-2x\right)x^2\right)^{\frac{1}{3}}x}$$

$$y(x) = -\frac{\sqrt{3}\,\sqrt{\left(i\left(-2x^2 2^{\frac{1}{3}}+2^{\frac{2}{3}}\left(\left(3\sqrt{3}\,\sqrt{27c_1^2-4c_1x}+27c_1-2x\right)^2x^4\right)^{\frac{1}{3}}\right)\sqrt{3}-2x^2 2^{\frac{1}{3}}+8x((3\sqrt{3}\,\sqrt{27c}}}{6((3\sqrt{3}\,\sqrt{2}}$$

$$y(x) = \frac{\sqrt{3}\,\sqrt{\left(i\left(-2x^2 2^{\frac{1}{3}}+2^{\frac{2}{3}}\left(\left(3\sqrt{3}\,\sqrt{27c_1^2-4c_1x}+27c_1-2x\right)^2x^4\right)^{\frac{1}{3}}\right)\sqrt{3}-2x^2 2^{\frac{1}{3}}+8x((3\sqrt{3}\,\sqrt{27c_1^2-}}}{6((3\sqrt{3}\,\sqrt{27c}}$$

$$y(x) = -\frac{\sqrt{3}\,\sqrt{\left(-i\left(-2x^2 2^{\frac{1}{3}}+2^{\frac{2}{3}}\left(\left(3\sqrt{3}\,\sqrt{27c_1^2-4c_1x}+27c_1-2x\right)^2x^4\right)^{\frac{1}{3}}\right)\sqrt{3}-2x^2 2^{\frac{1}{3}}+8x((3\sqrt{3}\,\sqrt{27}}}{6((3\sqrt{3}\,\sqrt{}}$$

$$y(x) = \frac{\sqrt{3}\,\sqrt{\left(-i\left(-2x^2 2^{\frac{1}{3}}+2^{\frac{2}{3}}\left(\left(3\sqrt{3}\,\sqrt{27c_1^2-4c_1x}+27c_1-2x\right)^2x^4\right)^{\frac{1}{3}}\right)\sqrt{3}-2x^2 2^{\frac{1}{3}}+8x((3\sqrt{3}\,\sqrt{27c_1^2}}}{6((3\sqrt{3}\,\sqrt{27c}}$$

✓ Solution by Mathematica

Time used: 30.566 (sec). Leaf size: 356

```
DSolve[(3*x^2*y[x]^2-x)*y'[x]+(2*x*y[x]^3-y[x])==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to \frac{2\sqrt[3]{3}x^3 + \sqrt[3]{2}\left(\sqrt{3}\sqrt{x^8\left(-4x+27c_1{}^2\right)}+9c_1x^4\right)^{2/3}}{6^{2/3}x^2\sqrt[3]{\sqrt{3}\sqrt{x^8\left(-4x+27c_1{}^2\right)}+9c_1x^4}}$$

$$y(x) \to \frac{i\sqrt[3]{3}\left(\sqrt{3}+i\right)\left(2\sqrt{3}\sqrt{x^8\left(-4x+27c_1{}^2\right)}+18c_1x^4\right)^{2/3} - 2\sqrt[3]{2}\sqrt[6]{3}\left(\sqrt{3}+3i\right)x^3}{12x^2\sqrt[3]{\sqrt{3}\sqrt{x^8\left(-4x+27c_1{}^2\right)}+9c_1x^4}}$$

$$y(x) \to \frac{\sqrt[3]{3}\left(-1-i\sqrt{3}\right)\left(2\sqrt{3}\sqrt{x^8\left(-4x+27c_1{}^2\right)}+18c_1x^4\right)^{2/3} - 2\sqrt[3]{2}\sqrt[6]{3}\left(\sqrt{3}-3i\right)x^3}{12x^2\sqrt[3]{\sqrt{3}\sqrt{x^8\left(-4x+27c_1{}^2\right)}+9c_1x^4}}$$

## 6.3 problem 3

Internal problem ID [11677]

**Book**: Differential Equations by Shepley L. Ross. Third edition. John Willey. New Delhi. 2004.
**Section**: Chapter 2, Miscellaneous Review. Exercises page 60
**Problem number**: 3.
**ODE order**: 1.
**ODE degree**: 1.

CAS Maple gives this as type `[_separable]`

$$\boxed{y + x(1 + x)\, y' = 1}$$

✓ Solution by Maple
Time used: 0.015 (sec). Leaf size: 14

```
dsolve((y(x)-1)+(x*(x+1))*diff(y(x),x)=0,y(x), singsol=all)
```

$$y(x) = \frac{c_1 x + c_1 - 1}{x}$$

✓ Solution by Mathematica
Time used: 0.03 (sec). Leaf size: 22

```
DSolve[(y[x]-1)+(x*(x+1))*y'[x]==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to \frac{-1 + c_1(x + 1)}{x}$$
$$y(x) \to 1$$

## 6.4 problem 4

Internal problem ID [11678]

**Book**: Differential Equations by Shepley L. Ross. Third edition. John Willey. New Delhi. 2004.
**Section**: Chapter 2, Miscellaneous Review. Exercises page 60
**Problem number**: 4.
**ODE order**: 1.
**ODE degree**: 1.

CAS Maple gives this as type `[_linear]`

$$\boxed{-2y + y'x = -x^2}$$

✓ Solution by Maple
Time used: 0.0 (sec). Leaf size: 14

```
dsolve((x^2-2*y(x))+(x)*diff(y(x),x)=0,y(x), singsol=all)
```

$$y(x) = (-\ln(x) + c_1)\, x^2$$

✓ Solution by Mathematica
Time used: 0.025 (sec). Leaf size: 16

```
DSolve[(x^2-2*y[x])+(x)*y'[x]==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to x^2(-\log(x) + c_1)$$

## 6.5 problem 5

Internal problem ID [11679]

**Book**: Differential Equations by Shepley L. Ross. Third edition. John Willey. New Delhi. 2004.
**Section**: Chapter 2, Miscellaneous Review. Exercises page 60
**Problem number**: 5.
**ODE order**: 1.
**ODE degree**: 1.

CAS Maple gives this as type `[[_homogeneous, ‘class A‘], _rational, [_Abel, ‘2nd type‘, ‘cl`

$$\boxed{-5y + (y + x)\, y' = -3x}$$

✓ Solution by Maple
Time used: 0.031 (sec). Leaf size: 47

```
dsolve((3*x-5*y(x))+(x+y(x))*diff(y(x),x)=0,y(x), singsol=all)
```

$$y(x) = \frac{6c_1x - \sqrt{8c_1x + 1} + 1}{2c_1}$$
$$y(x) = \frac{6c_1x + 1 + \sqrt{8c_1x + 1}}{2c_1}$$

✓ Solution by Mathematica
Time used: 1.033 (sec). Leaf size: 80

```
DSolve[(3*x-5*y[x])+(x+y[x])*y'[x]==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to \frac{1}{2}\left(6x - e^{\frac{c_1}{2}}\sqrt{-8x + e^{c_1}} - e^{c_1}\right)$$
$$y(x) \to \frac{1}{2}\left(6x + e^{\frac{c_1}{2}}\sqrt{-8x + e^{c_1}} - e^{c_1}\right)$$

## 6.6 problem 6

Internal problem ID [11680]

**Book**: Differential Equations by Shepley L. Ross. Third edition. John Willey. New Delhi. 2004.
**Section**: Chapter 2, Miscellaneous Review. Exercises page 60
**Problem number**: 6.
**ODE order**: 1.
**ODE degree**: 1.

CAS Maple gives this as type `[_separable]`

$$\boxed{e^{2x}y^2 + \left(y\, e^{2x} - 2y\right) y' = 0}$$

✓ Solution by Maple
Time used: 0.0 (sec). Leaf size: 58

```
dsolve((exp(2*x)*y(x)^2)+(exp(2*x)*y(x)-2*y(x))*diff(y(x),x)=0,y(x), singsol=all)
```

$$y(x) = 0$$
$$y(x) = \frac{\sqrt{2}\,\sqrt{-\left(e^{2x}-2\right)c_1}}{e^{2x}-2}$$
$$y(x) = -\frac{\sqrt{2}\,\sqrt{-\left(e^{2x}-2\right)c_1}}{e^{2x}-2}$$

✓ Solution by Mathematica
Time used: 0.071 (sec). Leaf size: 29

```
DSolve[(Exp[2*x]*y[x]^2)+(Exp[2*x]*y[x]-2*y[x])*y'[x]==0,y[x],x,IncludeSingularSolutions ->
```

$$y(x) \to 0$$
$$y(x) \to \frac{c_1}{\sqrt{e^{2x}-2}}$$
$$y(x) \to 0$$

## 6.7 problem 7

Internal problem ID [11681]

**Book**: Differential Equations by Shepley L. Ross. Third edition. John Willey. New Delhi. 2004.
**Section**: Chapter 2, Miscellaneous Review. Exercises page 60
**Problem number**: 7.
**ODE order**: 1.
**ODE degree**: 1.

CAS Maple gives this as type `[_separable]`

$$\boxed{8yx^3 + (x^4 + 1)\, y' = 12x^3}$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 15

```
dsolve((8*x^3*y(x)-12*x^3)+(1+x^4)*diff(y(x),x)=0,y(x), singsol=all)
```

$$y(x) = \frac{3}{2} + \frac{c_1}{(x^4+1)^2}$$

✓ Solution by Mathematica

Time used: 0.036 (sec). Leaf size: 38

```
DSolve[(8*x^3*y[x]-12*x^3)+(1+x^4)*y'[x]==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to \frac{3x^8 + 6x^4 + 2c_1}{2(x^4+1)^2}$$
$$y(x) \to \frac{3}{2}$$

## 6.8 problem 8

Internal problem ID [11682]

**Book**: Differential Equations by Shepley L. Ross. Third edition. John Willey. New Delhi. 2004.
**Section**: Chapter 2, Miscellaneous Review. Exercises page 60
**Problem number**: 8.
**ODE order**: 1.
**ODE degree**: 1.

CAS Maple gives this as type `[[_homogeneous, ‘class A‘], _rational, _Riccati]`

$$\boxed{yx + y^2 + 2x^2 y' = -2x^2}$$

✓ Solution by Maple
Time used: 0.016 (sec). Leaf size: 49

```
dsolve((2*x^2+x*y(x)+y(x)^2)+(2*x^2)*diff(y(x),x)=0,y(x), singsol=all)
```

$$y(x) = -\frac{\left(c_1 x - \sqrt{c_1 x} - 2\right) x}{c_1 x - 1}$$
$$y(x) = -\frac{\left(c_1 x + \sqrt{c_1 x} - 2\right) x}{c_1 x - 1}$$

✓ Solution by Mathematica
Time used: 2.203 (sec). Leaf size: 47

```
DSolve[(2*x^2+x*y[x]+y[x]^2)+(2*x^2)*y'[x]==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to \frac{x\left(\sqrt{x} - 2e^{c_1}\right)}{-\sqrt{x} + e^{c_1}}$$
$$y(x) \to -2x$$
$$y(x) \to -x$$

## 6.9 problem 9

Internal problem ID [11683]

**Book**: Differential Equations by Shepley L. Ross. Third edition. John Willey. New Delhi. 2004.
**Section**: Chapter 2, Miscellaneous Review. Exercises page 60
**Problem number**: 9.
**ODE order**: 1.
**ODE degree**: 1.

CAS Maple gives this as type `[[_homogeneous, ‘class G‘], _rational, [_Abel, ‘2nd type‘, ‘cl`

$$\boxed{y' - \frac{4y^2x^3 - 3x^2y}{x^3 - 2yx^4} = 0}$$

✓ Solution by Maple
Time used: 0.078 (sec). Leaf size: 41

```
dsolve(diff(y(x),x)=(4*x^3*y(x)^2-3*x^2*y(x))/(x^3-2*x^4*y(x)),y(x), singsol=all)
```

$$y(x) = \frac{x - \sqrt{x^2 + 4c_1}}{2x^2}$$
$$y(x) = \frac{x + \sqrt{x^2 + 4c_1}}{2x^2}$$

✓ Solution by Mathematica
Time used: 0.575 (sec). Leaf size: 78

```
DSolve[y'[x]==(4*x^3*y[x]^2-3*x^2*y[x])/(x^3-2*x^4*y[x]),y[x],x,IncludeSingularSolutions ->
```

$$y(x) \to \frac{x^3 - \sqrt{x^2}\sqrt{x^4 + 4c_1x^2}}{2x^4}$$
$$y(x) \to \frac{x^3 + \sqrt{x^2}\sqrt{x^4 + 4c_1x^2}}{2x^4}$$

## 6.10 problem 10

Internal problem ID [11684]

**Book**: Differential Equations by Shepley L. Ross. Third edition. John Willey. New Delhi. 2004.
**Section**: Chapter 2, Miscellaneous Review. Exercises page 60
**Problem number**: 10.
**ODE order**: 1.
**ODE degree**: 1.

CAS Maple gives this as type `[_linear]`

$$\boxed{(1+x)\,y'+yx=\mathrm{e}^{-x}}$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 15

```
dsolve((x+1)*diff(y(x),x)+x*y(x)=exp(-x),y(x), singsol=all)
```

$$y(x)=\mathrm{e}^{-x}(c_1x+c_1-1)$$

✓ Solution by Mathematica

Time used: 0.062 (sec). Leaf size: 19

```
DSolve[(x+1)*y'[x]+x*y[x]==Exp[-x],y[x],x,IncludeSingularSolutions -> True]
```

$$y(x)\to e^{-x}(-1+c_1(x+1))$$

## 6.11 problem 11

Internal problem ID [11685]

**Book**: Differential Equations by Shepley L. Ross. Third edition. John Willey. New Delhi. 2004.
**Section**: Chapter 2, Miscellaneous Review. Exercises page 60
**Problem number**: 11.
**ODE order**: 1.
**ODE degree**: 1.

CAS Maple gives this as type `[[_homogeneous, 'class A'], _rational, [_Abel, '2nd type', 'cl`

$$\boxed{y' - \frac{2x - 7y}{3y - 8x} = 0}$$

✓ Solution by Maple
Time used: 0.032 (sec). Leaf size: 47

```
dsolve(diff(y(x),x)=(2*x-7*y(x))/(3*y(x)-8*x),y(x), singsol=all)
```

$$y(x) = \frac{-12c_1x - \sqrt{-60c_1x + 1} + 1}{18c_1}$$
$$y(x) = \frac{-12c_1x + 1 + \sqrt{-60c_1x + 1}}{18c_1}$$

✓ Solution by Mathematica
Time used: 0.969 (sec). Leaf size: 80

```
DSolve[y'[x]==(2*x-7*y[x])/(3*y[x]-8*x),y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to \frac{1}{18}\left(-12x - e^{\frac{c_1}{2}}\sqrt{60x + e^{c_1}} - e^{c_1}\right)$$
$$y(x) \to \frac{1}{18}\left(-12x + e^{\frac{c_1}{2}}\sqrt{60x + e^{c_1}} - e^{c_1}\right)$$

## 6.12 problem 12

Internal problem ID [11686]

**Book**: Differential Equations by Shepley L. Ross. Third edition. John Willey. New Delhi. 2004.
**Section**: Chapter 2, Miscellaneous Review. Exercises page 60
**Problem number**: 12.
**ODE order**: 1.
**ODE degree**: 1.

CAS Maple gives this as type `[_separable]`

$$\boxed{x^2y' + yx - y^3x = 0}$$

✓ Solution by Maple
Time used: 0.016 (sec). Leaf size: 27

```
dsolve(x^2*diff(y(x),x)+x*y(x)=x*y(x)^3,y(x), singsol=all)
```

$$y(x) = \frac{1}{\sqrt{c_1x^2+1}}$$
$$y(x) = -\frac{1}{\sqrt{c_1x^2+1}}$$

✓ Solution by Mathematica
Time used: 0.242 (sec). Leaf size: 58

```
DSolve[x^2*y'[x]+x*y[x]==x*y[x]^3,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to -\frac{1}{\sqrt{1+e^{2c_1}x^2}}$$
$$y(x) \to \frac{1}{\sqrt{1+e^{2c_1}x^2}}$$
$$y(x) \to -1$$
$$y(x) \to 0$$
$$y(x) \to 1$$

## 6.13 problem 13

Internal problem ID [11687]

**Book**: Differential Equations by Shepley L. Ross. Third edition. John Willey. New Delhi. 2004.
**Section**: Chapter 2, Miscellaneous Review. Exercises page 60
**Problem number**: 13.
**ODE order**: 1.
**ODE degree**: 1.

CAS Maple gives this as type **[_separable]**

$$\boxed{(x^3+1)\,y'+6x^2y=6x^2}$$

✓ Solution by Maple
Time used: 0.0 (sec). Leaf size: 22

```
dsolve((x^3+1)*diff(y(x),x)+6*x^2*y(x)=6*x^2,y(x), singsol=all)
```

$$y(x)=\frac{x^6+2x^3+c_1}{(x^3+1)^2}$$

✓ Solution by Mathematica
Time used: 0.033 (sec). Leaf size: 29

```
DSolve[(x^3+1)*y'[x]+6*x^2*y[x]==6*x^2,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x)\to\frac{x^6+2x^3+c_1}{(x^3+1)^2}$$
$$y(x)\to 1$$

## 6.14 problem 14

Internal problem ID [11688]

**Book**: Differential Equations by Shepley L. Ross. Third edition. John Willey. New Delhi. 2004.
**Section**: Chapter 2, Miscellaneous Review. Exercises page 60
**Problem number**: 14.
**ODE order**: 1.
**ODE degree**: 1.

CAS Maple gives this as type `[[_homogeneous, ‘class A‘], _rational, [_Abel, ‘2nd type‘, ‘cl`

$$\boxed{y' - \frac{2x^2 + y^2}{2yx - x^2} = 0}$$

✓ Solution by Maple
Time used: 0.031 (sec). Leaf size: 57

```
dsolve(diff(y(x),x)=(2*x^2+y(x)^2)/(2*x*y(x)-x^2),y(x), singsol=all)
```

$$y(x) = \frac{c_1 x - \sqrt{9c_1^2x^2 + 4c_1 x}}{2c_1}$$
$$y(x) = \frac{c_1 x + \sqrt{9c_1^2x^2 + 4c_1 x}}{2c_1}$$

✓ Solution by Mathematica
Time used: 2.748 (sec). Leaf size: 93

```
DSolve[y'[x]==(2*x^2+y[x]^2)/(2*x*y[x]-x^2),y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to \frac{1}{2}\left(x - \sqrt{x\,(9x - 4e^{c_1})}\right)$$
$$y(x) \to \frac{1}{2}\left(x + \sqrt{x\,(9x - 4e^{c_1})}\right)$$
$$y(x) \to \frac{1}{2}\left(x - 3\sqrt{x^2}\right)$$
$$y(x) \to \frac{1}{2}\left(3\sqrt{x^2} + x\right)$$

## 6.15 problem 15

Internal problem ID [11689]

**Book**: Differential Equations by Shepley L. Ross. Third edition. John Willey. New Delhi. 2004.
**Section**: Chapter 2, Miscellaneous Review. Exercises page 60
**Problem number**: 15.
**ODE order**: 1.
**ODE degree**: 1.

CAS Maple gives this as type `[[_homogeneous, ‘class A‘], _rational, _Bernoulli]`

$$\boxed{y^2 - 2xyy' = -x^2}$$

With initial conditions

$$[y(1) = 2]$$

✓ Solution by Maple
Time used: 0.047 (sec). Leaf size: 13

```
dsolve([(x^2+y(x)^2)-2*x*y(x)*diff(y(x),x)=0,y(1) = 2],y(x), singsol=all)
```

$$y(x) = \sqrt{(x+3)\,x}$$

✓ Solution by Mathematica
Time used: 0.175 (sec). Leaf size: 18

```
DSolve[{(x^2+y[x]^2)-2*x*y[x]*y'[x]==0,{y[1]==2}},y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to \sqrt{x}\sqrt{x+3}$$

## 6.16 problem 16

Internal problem ID [11690]

**Book**: Differential Equations by Shepley L. Ross. Third edition. John Willey. New Delhi. 2004.
**Section**: Chapter 2, Miscellaneous Review. Exercises page 60
**Problem number**: 16.
**ODE order**: 1.
**ODE degree**: 1.

CAS Maple gives this as type `[_separable]`

$$\boxed{2y^2 + \left(-x^2 + 1\right) y y' = -8}$$

With initial conditions

$$[y(3) = 0]$$

✓ Solution by Maple
Time used: 0.046 (sec). Leaf size: 45

```
dsolve([2*(y(x)^2+4)+(1-x^2)*y(x)*diff(y(x),x)=0,y(3) = 0],y(x), singsol=all)
```

$$y(x) = -\frac{2\sqrt{3x^2 - 10x + 3}}{1 + x}$$
$$y(x) = \frac{2\sqrt{3x^2 - 10x + 3}}{1 + x}$$

✓ Solution by Mathematica
Time used: 0.886 (sec). Leaf size: 51

```
DSolve[{2*(y[x]^2+4)+(1-x^2)*y[x]*y'[x]==0,{y[3]==0}},y[x],x,IncludeSingularSolutions -> Tru
```

$$y(x) \to -\frac{2\sqrt{3x^2 - 10x + 3}}{x + 1}$$
$$y(x) \to \frac{2\sqrt{3x^2 - 10x + 3}}{x + 1}$$

## 6.17 problem 17

Internal problem ID [11691]

**Book**: Differential Equations by Shepley L. Ross. Third edition. John Willey. New Delhi. 2004.
**Section**: Chapter 2, Miscellaneous Review. Exercises page 60
**Problem number**: 17.
**ODE order**: 1.
**ODE degree**: 1.

CAS Maple gives this as type `[_exact, _Bernoulli]`

$$\boxed{e^{2x}y^2 + e^{2x}yy' = 2x}$$

With initial conditions

$$[y(0) = 2]$$

✓ Solution by Maple
Time used: 0.141 (sec). Leaf size: 24

```
dsolve([(exp(2*x)*y(x)^2-2*x)+(exp(2*x)*y(x))*diff(y(x),x)=0,y(0) = 2],y(x), singsol=all)
```

$$y(x) = e^{-2x}\sqrt{2}\,\sqrt{e^{2x}\,(x^2+2)}$$

✓ Solution by Mathematica
Time used: 0.337 (sec). Leaf size: 25

```
DSolve[{(Exp[2*x]*y[x]^2-2*x)+(Exp[2*x]*y[x])*y'[x]==0,{y[0]==2}},y[x],x,IncludeSingularSolu
```

$$y(x) \to \sqrt{2}e^{-x}\sqrt{x^2+2}$$

## 6.18 problem 18

Internal problem ID [11692]

**Book**: Differential Equations by Shepley L. Ross. Third edition. John Willey. New Delhi. 2004.
**Section**: Chapter 2, Miscellaneous Review. Exercises page 60
**Problem number**: 18.
**ODE order**: 1.
**ODE degree**: 1.

CAS Maple gives this as type `[_exact, _rational]`

$$\boxed{2y^2x + (2x^2y + 6y^2)\,y' = -3x^2}$$

With initial conditions

$$[y(1) = 2]$$

✓ Solution by Maple
Time used: 0.156 (sec). Leaf size: 87

```
dsolve([(3*x^2+2*x*y(x)^2)+(2*x^2*y(x)+6*y(x)^2)*diff(y(x),x)=0,y(1) = 2],y(x), singsol=all)
```

$$y(x) = \frac{\left(1134 - 54x^3 - x^6 + 6\sqrt{3x^9 + 18x^6 - 3402x^3 + 35721}\right)^{\frac{1}{3}}}{6} + \frac{x^4}{6\left(1134 - 54x^3 - x^6 + 6\sqrt{3x^9 + 18x^6 - 3402x^3 + 35721}\right)^{\frac{1}{3}}} - \frac{x^2}{6}$$

✓ Solution by Mathematica

Time used: 4.797 (sec). Leaf size: 103

```
DSolve[{(3*x^2+2*x*y[x]^2)+(2*x^2*y[x]+6*y[x]^2)*y'[x]==0,{y[1]==2}},y[x],x,IncludeSingularS
```

$$y(x) \to \frac{1}{6}\left(-x^2 + \sqrt[3]{-x^6 - 54x^3 + 6\sqrt{3}\sqrt{x^9 + 6x^6 - 1134x^3 + 11907} + 1134} + \frac{x^4}{\sqrt[3]{-x^6 - 54x^3 + 6\sqrt{3}\sqrt{x^9 + 6x^6 - 1134x^3 + 11907} + 1134}}\right)$$

## 6.19 problem 19

Internal problem ID [11693]

**Book**: Differential Equations by Shepley L. Ross. Third edition. John Willey. New Delhi. 2004.
**Section**: Chapter 2, Miscellaneous Review. Exercises page 60
**Problem number**: 19.
**ODE order**: 1.
**ODE degree**: 1.

CAS Maple gives this as type `[_separable]`

$$\boxed{4xyy' - y^2 = 1}$$

With initial conditions

$$[y(2) = 1]$$

✓ Solution by Maple
Time used: 0.062 (sec). Leaf size: 15

```
dsolve([4*x*y(x)*diff(y(x),x)=y(x)^2+1,y(2) = 1],y(x), singsol=all)
```

$$y(x) = \sqrt{\sqrt{2}\,\sqrt{x} - 1}$$

✓ Solution by Mathematica
Time used: 3.741 (sec). Leaf size: 22

```
DSolve[{4*x*y[x]*y'[x]==y[x]^2+1,{y[2]==1}},y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to \sqrt{\sqrt{2}\sqrt{x} - 1}$$

## 6.20 problem 20

Internal problem ID [11694]

**Book**: Differential Equations by Shepley L. Ross. Third edition. John Willey. New Delhi. 2004.
**Section**: Chapter 2, Miscellaneous Review. Exercises page 60
**Problem number**: 20.
**ODE order**: 1.
**ODE degree**: 1.

CAS Maple gives this as type `[[_homogeneous, 'class A'], _rational, [_Abel, '2nd type', 'cl`

$$\boxed{y' - \frac{2x+7y}{-2y+2x} = 0}$$

With initial conditions

$$[y(1) = 2]$$

✓ Solution by Maple
Time used: 0.203 (sec). Leaf size: 18

```
dsolve([diff(y(x),x)=(2*x+7*y(x))/(2*x-2*y(x)),y(1) = 2],y(x), singsol=all)
```

$$y(x) = \frac{4\sqrt{16-15x}}{5} - 2x + \frac{16}{5}$$

✓ Solution by Mathematica
Time used: 1.383 (sec). Leaf size: 25

```
DSolve[{y'[x]==(2*x+7*y[x])/(2*x-2*y[x]),{y[1]==2}},y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to \frac{2}{5}\left(-5x + 2\sqrt{16-15x} + 8\right)$$

## 6.21 problem 21

Internal problem ID [11695]

**Book**: Differential Equations by Shepley L. Ross. Third edition. John Willey. New Delhi. 2004.
**Section**: Chapter 2, Miscellaneous Review. Exercises page 60
**Problem number**: 21.
**ODE order**: 1.
**ODE degree**: 1.

CAS Maple gives this as type `[_separable]`

$$\boxed{y' - \frac{xy}{x^2+1} = 0}$$

With initial conditions

$$\left[y\left(\sqrt{15}\right) = 2\right]$$

✓ Solution by Maple
Time used: 0.015 (sec). Leaf size: 13

```
dsolve([diff(y(x),x)=(x*y(x))/(x^2+1),y(15^(1/2)) = 2],y(x), singsol=all)
```

$$y(x) = \frac{\sqrt{x^2+1}}{2}$$

✓ Solution by Mathematica
Time used: 0.028 (sec). Leaf size: 18

```
DSolve[{y'[x]==(x*y[x])/(x^2+1),{y[Sqrt[15]]==2}},y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to \frac{\sqrt{x^2+1}}{2}$$

## 6.22 problem 22

Internal problem ID [11696]

**Book**: Differential Equations by Shepley L. Ross. Third edition. John Willey. New Delhi. 2004.
**Section**: Chapter 2, Miscellaneous Review. Exercises page 60
**Problem number**: 22.
**ODE order**: 1.
**ODE degree**: 1.

CAS Maple gives this as type `[[_linear, 'class A']]`

$$\boxed{y' + y = \begin{cases} 1 & 0 \leq x < 2 \\ 0 & 0 < x \end{cases}}$$

With initial conditions

$$[y(0) = 0]$$

✓ Solution by Maple

Time used: 0.313 (sec). Leaf size: 36

```
dsolve([diff(y(x),x)+y(x)=piecewise(0<=x and x<2,1,x>0,0),y(0) = 0],y(x), singsol=all)
```

$$y(x) = \begin{cases} 0 & x < 0 \\ 1 - e^{-x} & x < 2 \\ e^{2-x} - e^{-x} & 2 \leq x \end{cases}$$

✓ Solution by Mathematica

Time used: 0.059 (sec). Leaf size: 39

```
DSolve[{y'[x]+y[x]==Piecewise[{{1,0<=x<2},{0,x>2}}],{y[0]==0}},y[x],x,IncludeSingularSolutio
```

$$y(x) \to \begin{cases} 0 & x \leq 0 \\ 1 - e^{-x} & 0 < x \leq 2 \\ e^{-x}(-1+e^2) & \text{True} \end{cases}$$

## 6.23 problem 23

Internal problem ID [11697]

**Book**: Differential Equations by Shepley L. Ross. Third edition. John Willey. New Delhi. 2004.
**Section**: Chapter 2, Miscellaneous Review. Exercises page 60
**Problem number**: 23.
**ODE order**: 1.
**ODE degree**: 1.

CAS Maple gives this as type `[_linear]`

$$\boxed{(x+2)\,y'+y=\begin{cases} 2x & 0\le x\le 2\\ 4 & 2<x\end{cases}}$$

With initial conditions

$$[y(0)=4]$$

✓ Solution by Maple
Time used: 0.063 (sec). Leaf size: 31

```
dsolve([(x+2)*diff(y(x),x)+y(x)=piecewise(0<=x and x<=2,2*x,x>2,4),y(0) = 4],y(x), singsol=a
```

$$y(x)=\frac{\begin{cases} 8 & x<0\\ x^2+8 & x<2\\ 4+4x & 2\le x\end{cases}}{x+2}$$

✓ Solution by Mathematica

Time used: 0.248 (sec). Leaf size: 43

```
DSolve[{(x+2)*y'[x]+y[x]==Piecewise[{{2*x,0<=x<=2},{4,x>2}}],{y[0]==4}},y[x],x,IncludeSingul
```

$$y(x) \to \begin{cases} \frac{8}{x+2} & x \leq 0 \\ \frac{4(x+1)}{x+2} & x > 2 \\ \frac{x^2+8}{x+2} & \text{True} \end{cases}$$

## 6.24 problem 24

Internal problem ID [11698]

**Book**: Differential Equations by Shepley L. Ross. Third edition. John Willey. New Delhi. 2004.
**Section**: Chapter 2, Miscellaneous Review. Exercises page 60
**Problem number**: 24.
**ODE order**: 1.
**ODE degree**: 1.

CAS Maple gives this as type `[[_homogeneous, ‘class A‘], _rational, _Bernoulli]`

$$\boxed{x^2y' + yx - \frac{y^3}{x} = 0}$$

With initial conditions

$$[y(1) = 1]$$

✓ Solution by Maple
Time used: 0.063 (sec). Leaf size: 16

```
dsolve([x^2*diff(y(x),x)+x*y(x)=y(x)^3/x,y(1) = 1],y(x), singsol=all)
```

$$y(x) = \frac{2x}{\sqrt{2x^4 + 2}}$$

✓ Solution by Mathematica
Time used: 0.355 (sec). Leaf size: 21

```
DSolve[{x^2*y'[x]+x*y[x]==y[x]^3/x,{y[1]==1}},y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to \frac{\sqrt{2}x}{\sqrt{x^4 + 1}}$$

# 7 Chapter 2, Section 2.4. Special integrating factors and transformations. Exercises page 67

## 7.1 problem 1

Internal problem ID [11699]

**Book**: Differential Equations by Shepley L. Ross. Third edition. John Willey. New Delhi. 2004.
**Section**: Chapter 2, Section 2.4. Special integrating factors and transformations. Exercises page 67
**Problem number**: 1.
**ODE order**: 1.
**ODE degree**: 1.

CAS Maple gives this as type `[_rational, [_Abel, '2nd type', 'class B']]`

$$\boxed{5yx + 4y^2 + (2yx + x^2)\, y' = -1}$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 59

```
dsolve((5*x*y(x)+4*y(x)^2+1)+(x^2+2*x*y(x))*diff(y(x),x)=0,y(x), singsol=all)
```

$$y(x) = \frac{-x^3 - \sqrt{x^6 - x^4 - 4c_1}}{2x^2}$$
$$y(x) = \frac{-x^3 + \sqrt{x^6 - x^4 - 4c_1}}{2x^2}$$

✓ Solution by Mathematica

Time used: 0.558 (sec). Leaf size: 84

```
DSolve[(5*x*y[x]+4*y[x]^2+1)+(x^2+2*x*y[x])*y'[x]==0,y[x],x,IncludeSingularSolutions -> True
```

$$y(x) \to -\frac{x^5 + \sqrt{x^3}\sqrt{x^7 - x^5 + 4c_1 x}}{2x^4}$$
$$y(x) \to -\frac{x}{2} + \frac{\sqrt{x^3}\sqrt{x^7 - x^5 + 4c_1 x}}{2x^4}$$

## 7.2 problem 2

Internal problem ID [11700]

**Book**: Differential Equations by Shepley L. Ross. Third edition. John Willey. New Delhi. 2004.
**Section**: Chapter 2, Section 2.4. Special integrating factors and transformations. Exercises page 67
**Problem number**: 2.
**ODE order**: 1.
**ODE degree**: 1.

CAS Maple gives this as type `[[_1st_order, '_with_symmetry_[F(x),G(x)]']]`

$$\boxed{\tan(y) + \left(x - x^2 \tan(y)\right) y' = -2x}$$

✓ Solution by Maple
Time used: 0.047 (sec). Leaf size: 134

```
dsolve((2*x+tan(y(x)))+(x-x^2*tan(y(x)))*diff(y(x),x)=0,y(x), singsol=all)
```

$$y(x) = \arctan\left(\frac{-\sqrt{x^4 - c_1^2 + x^2}\, x - c_1}{(x^2+1)\, x}, \frac{-c_1 x + \sqrt{x^4 - c_1^2 + x^2}}{(x^2+1)\, x}\right)$$
$$y(x) = \arctan\left(\frac{\sqrt{x^4 - c_1^2 + x^2}\, x - c_1}{(x^2+1)\, x}, \frac{-c_1 x - \sqrt{x^4 - c_1^2 + x^2}}{(x^2+1)\, x}\right)$$

✓ Solution by Mathematica

Time used: 38.283 (sec). Leaf size: 177

```
DSolve[(2*x+Tan[y[x]])+(x-x^2*Tan[y[x]])*y'[x]==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to -\arccos\left(-\frac{c_1x^2 + \sqrt{x^6 + x^4 - c_1{}^2x^2}}{x^4 + x^2}\right)$$
$$y(x) \to \arccos\left(-\frac{c_1x^2 + \sqrt{x^6 + x^4 - c_1{}^2x^2}}{x^4 + x^2}\right)$$
$$y(x) \to -\arccos\left(\frac{\sqrt{x^6 + x^4 - c_1{}^2x^2} - c_1x^2}{x^4 + x^2}\right)$$
$$y(x) \to \arccos\left(\frac{\sqrt{x^6 + x^4 - c_1{}^2x^2} - c_1x^2}{x^4 + x^2}\right)$$

## 7.3 problem 3

Internal problem ID [11701]

**Book**: Differential Equations by Shepley L. Ross. Third edition. John Willey. New Delhi. 2004.
**Section**: Chapter 2, Section 2.4. Special integrating factors and transformations. Exercises page 67
**Problem number**: 3.
**ODE order**: 1.
**ODE degree**: 1.

CAS Maple gives this as type `[_rational, [_Abel, '2nd type', 'class B']]`

$$\boxed{(1+x)\,y^2+y+(2yx+1)\,y'=0}$$

✓ Solution by Maple
Time used: 0.0 (sec). Leaf size: 56

```
dsolve((y(x)^2*(x+1)+y(x))+(2*x*y(x)+1)*diff(y(x),x)=0,y(x), singsol=all)
```

$$y(x)=\frac{-1+\sqrt{\mathrm{e}^{x}\left(-4c_1x+\mathrm{e}^{x}\right)}\,\mathrm{e}^{-x}}{2x}$$
$$y(x)=\frac{-\sqrt{\mathrm{e}^{x}\left(-4c_1x+\mathrm{e}^{x}\right)}\,\mathrm{e}^{-x}-1}{2x}$$

✓ Solution by Mathematica
Time used: 2.638 (sec). Leaf size: 69

```
DSolve[(y[x]^2*(x+1)+y[x])+(2*x*y[x]+1)*y'[x]==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x)\to-\frac{1+\frac{\sqrt{e^x+4c_1x}}{\sqrt{e^x}}}{2x}$$
$$y(x)\to\frac{-1+\frac{\sqrt{e^x+4c_1x}}{\sqrt{e^x}}}{2x}$$

## 7.4 problem 4

Internal problem ID [11702]

**Book**: Differential Equations by Shepley L. Ross. Third edition. John Willey. New Delhi. 2004.
**Section**: Chapter 2, Section 2.4. Special integrating factors and transformations. Exercises page 67
**Problem number**: 4.
**ODE order**: 1.
**ODE degree**: 1.

CAS Maple gives this as type `[_rational]`

$$\boxed{2y^2x + y + (2y^3 - x)\, y' = 0}$$

✓ Solution by Maple
Time used: 0.0 (sec). Leaf size: 301

```
dsolve((2*x*y(x)^2+y(x))+(2*y(x)^3-x)*diff(y(x),x)=0,y(x), singsol=all)
```

$$y(x) = \frac{-12x^2 - 12c_1 + \left(-108x + 12\sqrt{12x^6 + 36x^4c_1 + (36c_1^2 + 81)\, x^2 + 12c_1^3}\right)^{\frac{2}{3}}}{6\left(-108x + 12\sqrt{12x^6 + 36x^4c_1 + (36c_1^2 + 81)\, x^2 + 12c_1^3}\right)^{\frac{1}{3}}}$$

$$y(x) = -\frac{\left(\frac{i\sqrt{3}}{12} + \frac{1}{12}\right)\left(-108x + 12\sqrt{12x^6 + 36x^4c_1 + (36c_1^2 + 81)\, x^2 + 12c_1^3}\right)^{\frac{2}{3}} + (x^2 + c_1)\,(i\sqrt{3} - 1)}{\left(-108x + 12\sqrt{12x^6 + 36x^4c_1 + (36c_1^2 + 81)\, x^2 + 12c_1^3}\right)^{\frac{1}{3}}}$$

$$y(x) = \frac{\frac{(i\sqrt{3}-1)\left(-108x+12\sqrt{12x^6+36x^4c_1+(36c_1^2+81)x^2+12c_1^3}\right)^{\frac{2}{3}}}{12} + (x^2 + c_1)\,(1 + i\sqrt{3})}{\left(-108x + 12\sqrt{12x^6 + 36x^4c_1 + (36c_1^2 + 81)\, x^2 + 12c_1^3}\right)^{\frac{1}{3}}}$$

✓ Solution by Mathematica

Time used: 6.163 (sec). Leaf size: 316

```
DSolve[(2*x*y[x]^2+y[x])+(2*y[x]^3-x)*y'[x]==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to \frac{2^{2/3}\left(-27x+\sqrt{729x^2+108\left(x^2-c_1\right)^3}\right)^{2/3}-6\sqrt[3]{2}(x^2-c_1)}{6\sqrt[3]{-27x+\sqrt{729x^2+108\left(x^2-c_1\right)^3}}}$$

$$y(x) \to \frac{\left(1-i\sqrt{3}\right)\left(x^2-c_1\right)}{2^{2/3}\sqrt[3]{-27x+\sqrt{729x^2+108\left(x^2-c_1\right)^3}}} - \frac{\left(1+i\sqrt{3}\right)\sqrt[3]{-27x+\sqrt{729x^2+108\left(x^2-c_1\right)^3}}}{6\sqrt[3]{2}}$$

$$y(x) \to \frac{\left(1+i\sqrt{3}\right)\left(x^2-c_1\right)}{2^{2/3}\sqrt[3]{-27x+\sqrt{729x^2+108\left(x^2-c_1\right)^3}}} + \frac{\left(-1+i\sqrt{3}\right)\sqrt[3]{-27x+\sqrt{729x^2+108\left(x^2-c_1\right)^3}}}{6\sqrt[3]{2}}$$

$$y(x) \to 0$$

## 7.5 problem 5

Internal problem ID [11703]

**Book**: Differential Equations by Shepley L. Ross. Third edition. John Willey. New Delhi. 2004.
**Section**: Chapter 2, Section 2.4. Special integrating factors and transformations. Exercises page 67
**Problem number**: 5.
**ODE order**: 1.
**ODE degree**: 1.

CAS Maple gives this as type `[[_homogeneous, ‘class G‘], _rational, [_Abel, ‘2nd type‘, ‘cl`

$$\boxed{4y^2x + 6y + (5x^2y + 8x)\, y' = 0}$$

✓ Solution by Maple

Time used: 0.078 (sec). Leaf size: 23

```
dsolve((4*x*y(x)^2+6*y(x))+(5*x^2*y(x)+8*x)*diff(y(x),x)=0,y(x), singsol=all)
```

$$y(x) = \frac{\operatorname{RootOf}(-\ln(x) + c_1 + \ln(\_Z + 2) + 4\ln(\_Z))}{x}$$

✓ Solution by Mathematica

Time used: 1.767 (sec). Leaf size: 156

```
DSolve[(4*x*y[x]^2+6*y[x])+(5*x^2*y[x]+8*x)*y'[x]==0,y[x],x,IncludeSingularSolutions -> True
```

$$y(x) \to \operatorname{Root}\left[-\#1^5 - \frac{2\#1^4}{x} + \frac{e^{c_1}}{x^4}\&, 1\right]$$
$$y(x) \to \operatorname{Root}\left[-\#1^5 - \frac{2\#1^4}{x} + \frac{e^{c_1}}{x^4}\&, 2\right]$$
$$y(x) \to \operatorname{Root}\left[-\#1^5 - \frac{2\#1^4}{x} + \frac{e^{c_1}}{x^4}\&, 3\right]$$
$$y(x) \to \operatorname{Root}\left[-\#1^5 - \frac{2\#1^4}{x} + \frac{e^{c_1}}{x^4}\&, 4\right]$$
$$y(x) \to \operatorname{Root}\left[-\#1^5 - \frac{2\#1^4}{x} + \frac{e^{c_1}}{x^4}\&, 5\right]$$

## 7.6 problem 6

Internal problem ID [11704]

**Book**: Differential Equations by Shepley L. Ross. Third edition. John Willey. New Delhi. 2004.
**Section**: Chapter 2, Section 2.4. Special integrating factors and transformations. Exercises page 67
**Problem number**: 6.
**ODE order**: 1.
**ODE degree**: 1.

CAS Maple gives this as type `[[_homogeneous, ‘class G‘], _rational, [_Abel, ‘2nd type‘, ‘cl`

$$\boxed{8y^3x^2 - 2y^4 + \left(5y^2x^3 - 8y^3x\right) y' = 0}$$

✓ Solution by Maple
Time used: 0.703 (sec). Leaf size: 34

```
dsolve((8*x^2*y(x)^3-2*y(x)^4)+(5*x^3*y(x)^2-8*x*y(x)^3)*diff(y(x),x)=0,y(x), singsol=all)
```

$$y(x) = 0$$
$$y(x) = \operatorname{RootOf}\left(x^6\_Z^{48} - x^6\_Z^{30} - c_1\right)^{18} x^2$$

✓ Solution by Mathematica

Time used: 3.924 (sec). Leaf size: 411

```
DSolve[(8*x^2*y[x]^3-2*y[x]^4)+(5*x^3*y[x]^2-8*x*y[x]^3)*y'[x]==0,y[x],x,IncludeSingularSolu
```

$$y(x) \to 0$$
$$y(x) \to \text{Root}\left[-\#1^8 + 3\#1^7x^2 - 3\#1^6x^4 + \#1^5x^6 + \frac{e^{18c_1}}{x^2}\&, 1\right]$$
$$y(x) \to \text{Root}\left[-\#1^8 + 3\#1^7x^2 - 3\#1^6x^4 + \#1^5x^6 + \frac{e^{18c_1}}{x^2}\&, 2\right]$$
$$y(x) \to \text{Root}\left[-\#1^8 + 3\#1^7x^2 - 3\#1^6x^4 + \#1^5x^6 + \frac{e^{18c_1}}{x^2}\&, 3\right]$$
$$y(x) \to \text{Root}\left[-\#1^8 + 3\#1^7x^2 - 3\#1^6x^4 + \#1^5x^6 + \frac{e^{18c_1}}{x^2}\&, 4\right]$$
$$y(x) \to \text{Root}\left[-\#1^8 + 3\#1^7x^2 - 3\#1^6x^4 + \#1^5x^6 + \frac{e^{18c_1}}{x^2}\&, 5\right]$$
$$y(x) \to \text{Root}\left[-\#1^8 + 3\#1^7x^2 - 3\#1^6x^4 + \#1^5x^6 + \frac{e^{18c_1}}{x^2}\&, 6\right]$$
$$y(x) \to \text{Root}\left[-\#1^8 + 3\#1^7x^2 - 3\#1^6x^4 + \#1^5x^6 + \frac{e^{18c_1}}{x^2}\&, 7\right]$$
$$y(x) \to \text{Root}\left[-\#1^8 + 3\#1^7x^2 - 3\#1^6x^4 + \#1^5x^6 + \frac{e^{18c_1}}{x^2}\&, 8\right]$$
$$y(x) \to 0$$

## 7.7 problem 7

Internal problem ID [11705]

**Book**: Differential Equations by Shepley L. Ross. Third edition. John Willey. New Delhi. 2004.
**Section**: Chapter 2, Section 2.4. Special integrating factors and transformations. Exercises page 67
**Problem number**: 7.
**ODE order**: 1.
**ODE degree**: 1.

CAS Maple gives this as type `[[_homogeneous, ‘class C‘], _exact, _rational, [_Abel, ‘2nd ty`

$$\boxed{2y + (2x + y + 1)\, y' = -5x - 1}$$

✓ Solution by Maple
Time used: 0.562 (sec). Leaf size: 32

```
dsolve((5*x+2*y(x)+1)+(2*x+y(x)+1)*diff(y(x),x)=0,y(x), singsol=all)
```

$$y(x) = \frac{-\sqrt{-\left(-1+x\right)^{2} c_1^{2}+1}+\left(-2x-1\right) c_1}{c_1}$$

✓ Solution by Mathematica
Time used: 0.134 (sec). Leaf size: 53

```
DSolve[(5*x+2*y[x]+1)+(2*x+y[x]+1)*y'[x]==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to -\sqrt{-x^2+2x+1+c_1} - 2x - 1$$
$$y(x) \to \sqrt{-x^2+2x+1+c_1} - 2x - 1$$

## 7.8 problem 8

Internal problem ID [11706]

**Book**: Differential Equations by Shepley L. Ross. Third edition. John Willey. New Delhi. 2004.
**Section**: Chapter 2, Section 2.4. Special integrating factors and transformations. Exercises page 67
**Problem number**: 8.
**ODE order**: 1.
**ODE degree**: 1.

CAS Maple gives this as type `[[_homogeneous, ‘class C‘], _rational, [_Abel, ‘2nd type‘, ‘cl`

$$\boxed{-y-(6x-2y-3)\,y'=-3x-1}$$

✓ Solution by Maple
Time used: 0.031 (sec). Leaf size: 23

```
dsolve((3*x-y(x)+1)-(6*x-2*y(x)-3)*diff(y(x),x)=0,y(x), singsol=all)
```

$$y(x)=-\frac{\mathrm{LambertW}\left(-2\,\mathrm{e}^{5x-4-5c_1}\right)}{2}+3x-2$$

✓ Solution by Mathematica
Time used: 3.097 (sec). Leaf size: 35

```
DSolve[(3*x-y[x]+1)-(6*x-2*y[x]-3)*y'[x]==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x)\to -\frac{1}{2}W\left(-e^{5x-1+c_1}\right)+3x-2$$
$$y(x)\to 3x-2$$

## 7.9 problem 9

Internal problem ID [11707]

**Book**: Differential Equations by Shepley L. Ross. Third edition. John Willey. New Delhi. 2004.
**Section**: Chapter 2, Section 2.4. Special integrating factors and transformations. Exercises page 67
**Problem number**: 9.
**ODE order**: 1.
**ODE degree**: 1.

CAS Maple gives this as type `[[_homogeneous, ‘class C‘], _rational, [_Abel, ‘2nd type‘, ‘cl`

$$\boxed{-2y + (2x + y - 1)\, y' = -x + 3}$$

✓ Solution by Maple

Time used: 0.031 (sec). Leaf size: 31

```
dsolve((x-2*y(x)-3)+(2*x+y(x)-1)*diff(y(x),x)=0,y(x), singsol=all)
```

$$y(x) = -1 - \tan\left(\operatorname{RootOf}\left(-4\_Z + \ln\left(\sec\left(\_Z\right)^2\right) + 2\ln\left(-1 + x\right) + 2c_1\right)\right)\left(-1 + x\right)$$

✓ Solution by Mathematica

Time used: 0.061 (sec). Leaf size: 66

```
DSolve[(x-2*y[x]-3)+(2*x+y[x]-1)*y'[x]==0,y[x],x,IncludeSingularSolutions -> True]
```

$$\text{Solve}\left[32\arctan\left(\frac{2y(x) - x + 3}{y(x) + 2x - 1}\right) + 8\log\left(\frac{x^2 + y(x)^2 + 2y(x) - 2x + 2}{5(x-1)^2}\right) + 16\log(x-1) + 5c_1 = 0, y(x)\right]$$

## 7.10 problem 10

Internal problem ID [11708]

**Book**: Differential Equations by Shepley L. Ross. Third edition. John Willey. New Delhi. 2004.
**Section**: Chapter 2, Section 2.4. Special integrating factors and transformations. Exercises page 67
**Problem number**: 10.
**ODE order**: 1.
**ODE degree**: 1.

CAS Maple gives this as type `[[_homogeneous, ‘class C‘], _rational, [_Abel, ‘2nd type‘, ‘cl`

$$\boxed{-4y-(x+5y+3)\,y'=-10x-12}$$

✓ Solution by Maple
Time used: 0.859 (sec). Leaf size: 129

```
dsolve((10*x-4*y(x)+12)-(x+5*y(x)+3)*diff(y(x),x)=0,y(x), singsol=all)
```

$$y(x) = \frac{(-3x-4)\operatorname{RootOf}\left(-1+\left(243c_1x^5+1620x^4c_1+4320c_1x^3+5760c_1x^2+3840c_1x+1024c_1\right)\_Z^{25}+(14\right.}{}$$

$$-2x-3$$

✓ Solution by Mathematica
Time used: 60.443 (sec). Leaf size: 3061

```
DSolve[(10*x-4*y[x]+12)-(x+5*y[x]+3)*y'[x]==0,y[x],x,IncludeSingularSolutions -> True]
```

Too large to display

## 7.11 problem 11

Internal problem ID [11709]

**Book**: Differential Equations by Shepley L. Ross. Third edition. John Willey. New Delhi. 2004.
**Section**: Chapter 2, Section 2.4. Special integrating factors and transformations. Exercises page 67
**Problem number**: 11.
**ODE order**: 1.
**ODE degree**: 1.

CAS Maple gives this as type `[[_homogeneous, ‘class C‘], _exact, _rational, [_Abel, ‘2nd ty`

$$\boxed{4y + (4x + 2y + 2)\, y' = -6x - 1}$$

With initial conditions

$$\left[y\left(\frac{1}{2}\right) = 3\right]$$

✓ Solution by Maple
Time used: 0.297 (sec). Leaf size: 23

```
dsolve([(6*x+4*y(x)+1)+(4*x+2*y(x)+2)*diff(y(x),x)=0,y(1/2) = 3],y(x), singsol=all)
```

$$y(x) = -2x - 1 + \frac{\sqrt{4x^2 + 12x + 93}}{2}$$

✓ Solution by Mathematica
Time used: 0.143 (sec). Leaf size: 28

```
DSolve[{(6*x+4*y[x]+1)+(4*x+2*y[x]+2)*y'[x]==0,{y[1/2]==3}},y[x],x,IncludeSingularSolutions
```

$$y(x) \to \frac{1}{2}\left(\sqrt{4x^2 + 12x + 93} - 4x - 2\right)$$

## 7.12 problem 12

Internal problem ID [11710]

**Book**: Differential Equations by Shepley L. Ross. Third edition. John Willey. New Delhi. 2004.
**Section**: Chapter 2, Section 2.4. Special integrating factors and transformations. Exercises page 67
**Problem number**: 12.
**ODE order**: 1.
**ODE degree**: 1.

CAS Maple gives this as type `[[_homogeneous, ‘class C‘], _rational, [_Abel, ‘2nd type‘, ‘cl`

$$\boxed{-y + (x + y + 2)\, y' = -3x + 6}$$

With initial conditions

$$[y(2) = -2]$$

✓ Solution by Maple

Time used: 9.453 (sec). Leaf size: 120

```
dsolve([(3*x-y(x)-6)+(x+y(x)+2)*diff(y(x),x)=0,y(2) = -2],y(x), singsol=all)
```

$$y(x) = -3 - \sqrt{3}\, \tan\left(\operatorname{RootOf}\left(-3\sqrt{3}\, \ln(3) + 6\sqrt{3}\, \ln(2) - 3\sqrt{3}\, \ln\left(\sec(\_Z)^2 \left(-1+x\right)^2\right) + \pi + 6\_Z\right)\right)(-1+x)$$

✓ Solution by Mathematica

Time used: 0.141 (sec). Leaf size: 90

```
DSolve[{(3*x-y[x]-6)+(x+y[x]+2)*y'[x]==0,{y[2]==-2}},y[x],x,IncludeSingularSolutions -> True
```

$$\text{Solve}\left[\frac{\arctan\left(\frac{-y(x)+3x-6}{\sqrt{3}(y(x)+x+2)}\right)}{\sqrt{3}} + \log(2) = \frac{1}{2}\log\left(\frac{3x^2 + y(x)^2 + 6y(x) - 6x + 12}{(x-1)^2}\right) + \log(x-1) + \frac{1}{18}\left(\sqrt{3}\pi + 18\log(2) - 9\log(4)\right), y(x)\right]$$

## 7.13 problem 13

Internal problem ID [11711]

**Book**: Differential Equations by Shepley L. Ross. Third edition. John Willey. New Delhi. 2004.
**Section**: Chapter 2, Section 2.4. Special integrating factors and transformations. Exercises page 67
**Problem number**: 13.
**ODE order**: 1.
**ODE degree**: 1.

CAS Maple gives this as type `[[_homogeneous, ‘class C‘], _rational, [_Abel, ‘2nd type‘, ‘cl`

$$\boxed{3y + (4x + 6y + 1)\, y' = -2x - 1}$$

With initial conditions

$$[y(-2) = 2]$$

✓ Solution by Maple

Time used: 0.156 (sec). Leaf size: 20

```
dsolve([(2*x+3*y(x)+1)+(4*x+6*y(x)+1)*diff(y(x),x)=0,y(-2) = 2],y(x), singsol=all)
```

$$y(x) = \frac{1}{3} - \frac{2x}{3} + \frac{\operatorname{LambertW}\left(\frac{2\,\mathrm{e}^{\frac{4}{3}+\frac{x}{3}}}{3}\right)}{2}$$

✓ Solution by Mathematica

Time used: 4.146 (sec). Leaf size: 30

```
DSolve[{(2*x+3*y[x]+1)+(4*x+6*y[x]+1)*y'[x]==0,{y[-2]==2}},y[x],x,IncludeSingularSolutions -
```

$$y(x) \to \frac{1}{6}\left(3W\left(\frac{2}{3}e^{\frac{x+4}{3}}\right) - 4x + 2\right)$$

## 7.14 problem 14

Internal problem ID [11712]

**Book**: Differential Equations by Shepley L. Ross. Third edition. John Willey. New Delhi. 2004.
**Section**: Chapter 2, Section 2.4. Special integrating factors and transformations. Exercises page 67
**Problem number**: 14.
**ODE order**: 1.
**ODE degree**: 1.

CAS Maple gives this as type `[[_homogeneous, ‘class C‘], _rational, [_Abel, ‘2nd type‘, ‘cl`

$$\boxed{3y + (x + y + 1)\, y' = -4x - 1}$$

With initial conditions

$$[y(3) = -4]$$

✓ Solution by Maple
Time used: 0.265 (sec). Leaf size: 39

```
dsolve([(4*x+3*y(x)+1)+(x+y(x)+1)*diff(y(x),x)=0,y(3) = -4],y(x), singsol=all)
```

$$y(x) = \frac{-2x \operatorname{LambertW}\left(-(x-2)\,\mathrm{e}^{-1}\right) + \operatorname{LambertW}\left(-(x-2)\,\mathrm{e}^{-1}\right) - x + 2}{\operatorname{LambertW}\left(-(x-2)\,\mathrm{e}^{-1}\right)}$$

✓ Solution by Mathematica
Time used: 65.902 (sec). Leaf size: 197

```
DSolve[{(4*x+3*y[x]+1)+(x+y[x]+1)*y'[x]==0,{y[-2]==2}},y[x],x,IncludeSingularSolutions -> Tr
```

$$\text{Solve}\left[\frac{(-2)^{2/3}\left(-2x\log\left(\frac{3(-2)^{2/3}(y(x)+2x-1)}{y(x)+x+1}\right) + (2x-1)\log\left(-\frac{3(-2)^{2/3}(x-2)}{y(x)+x+1}\right) + \log\left(\frac{3(-2)^{2/3}(y(x)+2x-1)}{y(x)+x+1}\right) + \cdots\right.}{9(y(x)+2x-1)}\right.$$

# 8 Chapter 4, Section 4.1. Basic theory of linear differential equations. Exercises page 113

## 8.1 problem 1 (a)

Internal problem ID [11713]

**Book**: Differential Equations by Shepley L. Ross. Third edition. John Willey. New Delhi. 2004.
**Section**: Chapter 4, Section 4.1. Basic theory of linear differential equations. Exercises page 113
**Problem number**: 1 (a).
**ODE order**: 2.
**ODE degree**: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$\boxed{y'' + 5y' + 6y = \mathrm{e}^{x}}$$

With initial conditions

$$[y(0) = 5, y'(0) = 7]$$

✓ Solution by Maple
Time used: 0.015 (sec). Leaf size: 20

```
dsolve([diff(y(x),x$2)+5*diff(y(x),x)+6*y(x)=exp(x),y(0) = 5, D(y)(0) = 7],y(x), singsol=all
```

$$y(x) = \frac{(\mathrm{e}^{4x} + 260\,\mathrm{e}^{x} - 201)\,\mathrm{e}^{-3x}}{12}$$

✓ Solution by Mathematica
Time used: 0.057 (sec). Leaf size: 26

```
DSolve[{y''[x]+5*y'[x]+6*y[x]==Exp[x],{y[0]==5,y'[0]==7}},y[x],x,IncludeSingularSolutions ->
```

$$y(x) \to \frac{1}{12}e^{-3x}\left(260e^{x} + e^{4x} - 201\right)$$

## 8.2 problem 1 (b)

Internal problem ID [11714]

**Book**: Differential Equations by Shepley L. Ross. Third edition. John Willey. New Delhi. 2004.
**Section**: Chapter 4, Section 4.1. Basic theory of linear differential equations. Exercises page 113
**Problem number**: 1 (b).
**ODE order**: 2.
**ODE degree**: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$\boxed{y'' + 5y' + 6y = \mathrm{e}^{x}}$$

With initial conditions

$$[y(0) = 5, y'(1) = 7]$$

✓ Solution by Maple

Time used: 0.125 (sec). Leaf size: 55

```
dsolve([diff(y(x),x$2)+5*diff(y(x),x)+6*y(x)=exp(x),y(0) = 5, D(y)(1) = 7],y(x), singsol=all
```

$$y(x) = \frac{(-\mathrm{e}^{4-x} + 84\,\mathrm{e}^{3-x} + \mathrm{e}^{4} + 2\,\mathrm{e}^{3x+1} - 84\,\mathrm{e}^{3} + 118\,\mathrm{e}^{1-x} - 3\,\mathrm{e}^{3x} - 177)\,\mathrm{e}^{-2x}}{24\,\mathrm{e} - 36}$$

✓ Solution by Mathematica

Time used: 0.022 (sec). Leaf size: 68

```
DSolve[{y''[x]+5*y'[x]+6*y[x]==Exp[x],{y[0]==5,y'[1]==7}},y[x],x,IncludeSingularSolutions ->
```

$$y(x) \to \frac{e^{-3x}(-177e^{x} - 3e^{4x} - 84e^{x+3} + e^{x+4} + 2e^{4x+1} + 118e + 84e^{3} - e^{4})}{12(2e - 3)}$$

## 8.3 problem 2

Internal problem ID [11715]

**Book**: Differential Equations by Shepley L. Ross. Third edition. John Willey. New Delhi. 2004.
**Section**: Chapter 4, Section 4.1. Basic theory of linear differential equations. Exercises page 113
**Problem number**: 2.
**ODE order**: 2.
**ODE degree**: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$\boxed{y'' + y'x + x^2 y = 0}$$

With initial conditions

$$[y(1) = 0, y'(1) = 0]$$

✓ Solution by Maple
Time used: 0.641 (sec). Leaf size: 5

```
dsolve([diff(y(x),x$2)+x*diff(y(x),x)+x^2*y(x)=0,y(1) = 0, D(y)(1) = 0],y(x), singsol=all)
```

$$y(x) = 0$$

✓ Solution by Mathematica
Time used: 0.101 (sec). Leaf size: 6

```
DSolve[{y''[x]+x*y'[x]+x^2*y[x]==0,{y[1]==0,y'[1]==0}},y[x],x,IncludeSingularSolutions -> Tr
```

$$y(x) \to 0$$

## 8.4 problem 4 (a)

Internal problem ID [11716]

**Book**: Differential Equations by Shepley L. Ross. Third edition. John Willey. New Delhi. 2004.
**Section**: Chapter 4, Section 4.1. Basic theory of linear differential equations. Exercises page 113
**Problem number**: 4 (a).
**ODE order**: 2.
**ODE degree**: 1.

CAS Maple gives this as type `[[_2nd_order, _missing_x]]`

$$\boxed{y'' - 4y' + 3y = 0}$$

✓ Solution by Maple
Time used: 0.016 (sec). Leaf size: 15

```
dsolve(diff(y(x),x$2)-4*diff(y(x),x)+3*y(x)=0,y(x), singsol=all)
```

$$y(x) = c_1 e^{x} + c_2 e^{3x}$$

✓ Solution by Mathematica
Time used: 0.011 (sec). Leaf size: 20

```
DSolve[y''[x]-4*y'[x]+3*y[x]==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to e^{x}\left(c_2 e^{2x} + c_1\right)$$

## 8.5 problem 8

Internal problem ID [11717]

**Book**: Differential Equations by Shepley L. Ross. Third edition. John Willey. New Delhi. 2004.
**Section**: Chapter 4, Section 4.1. Basic theory of linear differential equations. Exercises page 113
**Problem number**: 8.
**ODE order**: 2.
**ODE degree**: 1.

CAS Maple gives this as type `[[_2nd_order, _missing_x]]`

$$\boxed{y'' - 2y' + y = 0}$$

With initial conditions

$$[y(0) = 1, y'(0) = 4]$$

✓ Solution by Maple
Time used: 0.015 (sec). Leaf size: 12

```
dsolve([diff(y(x),x$2)-2*diff(y(x),x)+y(x)=0,y(0) = 1, D(y)(0) = 4],y(x), singsol=all)
```

$$y(x) = e^x(3x + 1)$$

✓ Solution by Mathematica
Time used: 0.011 (sec). Leaf size: 14

```
DSolve[{y''[x]-2*y'[x]+y[x]==0,{y[0]==1,y'[0]==4}},y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to e^x(3x + 1)$$

## 8.6 problem 9

Internal problem ID [11718]

**Book**: Differential Equations by Shepley L. Ross. Third edition. John Willey. New Delhi. 2004.
**Section**: Chapter 4, Section 4.1. Basic theory of linear differential equations. Exercises page 113
**Problem number**: 9.
**ODE order**: 2.
**ODE degree**: 1.

CAS Maple gives this as type `[[_Emden, _Fowler], [_2nd_order, _linear, '_with_symmetry_[0,F`

$$\boxed{x^2y'' - 2y'x + 2y = 0}$$

With initial conditions

$$[y(1) = 3, y'(1) = 2]$$

✓ Solution by Maple

Time used: 0.016 (sec). Leaf size: 13

```
dsolve([x^2*diff(y(x),x$2)-2*x*diff(y(x),x)+2*y(x)=0,y(1) = 3, D(y)(1) = 2],y(x), singsol=al
```

$$y(x) = -x^2 + 4x$$

✓ Solution by Mathematica

Time used: 0.01 (sec). Leaf size: 11

```
DSolve[{x^2*y''[x]-2*x*y'[x]+2*y[x]==0,{y[1]==3,y'[1]==2}},y[x],x,IncludeSingularSolutions -
```

$$y(x) \to -((x - 4)x)$$

## 8.7 problem 10

Internal problem ID [11719]

**Book**: Differential Equations by Shepley L. Ross. Third edition. John Willey. New Delhi. 2004.
**Section**: Chapter 4, Section 4.1. Basic theory of linear differential equations. Exercises page 113
**Problem number**: 10.
**ODE order**: 2.
**ODE degree**: 1.

CAS Maple gives this as type `[[_Emden, _Fowler], [_2nd_order, _linear, '_with_symmetry_[0,F`

$$\boxed{x^2y'' + y'x - 4y = 0}$$

With initial conditions

$$[y(2) = 3, y'(2) = -1]$$

✓ Solution by Maple
Time used: 0.0 (sec). Leaf size: 14

```
dsolve([x^2*diff(y(x),x$2)+x*diff(y(x),x)-4*y(x)=0,y(2) = 3, D(y)(2) = -1],y(x), singsol=all
```

$$y(x) = \frac{x^4 + 32}{4x^2}$$

✓ Solution by Mathematica
Time used: 0.011 (sec). Leaf size: 17

```
DSolve[{x^2*y''[x]+x*y'[x]-4*y[x]==0,{y[2]==3,y'[2]==-1}},y[x],x,IncludeSingularSolutions ->
```

$$y(x) \to \frac{x^4 + 32}{4x^2}$$

## 8.8 problem 11

Internal problem ID [11720]

**Book**: Differential Equations by Shepley L. Ross. Third edition. John Willey. New Delhi. 2004.
**Section**: Chapter 4, Section 4.1. Basic theory of linear differential equations. Exercises page 113
**Problem number**: 11.
**ODE order**: 2.
**ODE degree**: 1.

CAS Maple gives this as type `[[_2nd_order, _missing_x]]`

$$\boxed{y'' - 5y' + 4y = 0}$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 15

```
dsolve(diff(y(x),x$2)-5*diff(y(x),x)+4*y(x)=0,y(x), singsol=all)
```

$$y(x) = c_1 \mathrm{e}^{4x} + c_2 \mathrm{e}^{x}$$

✓ Solution by Mathematica

Time used: 0.011 (sec). Leaf size: 20

```
DSolve[y''[x]-5*y'[x]+4*y[x]==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to e^x \left(c_2 e^{3x} + c_1\right)$$

## 8.9 problem 12

Internal problem ID [11721]

**Book**: Differential Equations by Shepley L. Ross. Third edition. John Willey. New Delhi. 2004.
**Section**: Chapter 4, Section 4.1. Basic theory of linear differential equations. Exercises page 113
**Problem number**: 12.
**ODE order**: 3.
**ODE degree**: 1.

CAS Maple gives this as type `[[_3rd_order, _missing_x]]`

$$\boxed{y''' - 6y'' + 5y' + 12y = 0}$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 23

```
dsolve(diff(y(x),x$3)-6*diff(y(x),x$2)+5*diff(y(x),x)+12*y(x)=0,y(x), singsol=all)
```

$$y(x) = c_1 e^{4x} + c_2 e^{-x} + c_3 e^{3x}$$

✓ Solution by Mathematica

Time used: 0.009 (sec). Leaf size: 29

```
DSolve[y'''[x]-6*y''[x]+5*y'[x]+12*y[x]==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to e^{-x}\left(e^{4x}(c_3 e^x + c_2) + c_1\right)$$

## 8.10 problem 13

Internal problem ID [11722]

**Book**: Differential Equations by Shepley L. Ross. Third edition. John Willey. New Delhi. 2004.
**Section**: Chapter 4, Section 4.1. Basic theory of linear differential equations. Exercises page 113
**Problem number**: 13.
**ODE order**: 3.
**ODE degree**: 1.

CAS Maple gives this as type `[[_3rd_order, _with_linear_symmetries]]`

$$\boxed{x^3y''' - 4x^2y'' + 8y'x - 8y = 0}$$

✓ Solution by Maple
Time used: 0.015 (sec). Leaf size: 16

```
dsolve(x^3*diff(y(x),x$3)-4*x^2*diff(y(x),x$2)+8*x*diff(y(x),x)-8*y(x)=0,y(x), singsol=all)
```

$$y(x) = x\left(c_1x^3 + c_3x + c_2\right)$$

✓ Solution by Mathematica
Time used: 0.004 (sec). Leaf size: 20

```
DSolve[x^3*y'''[x]-4*x^2*y''[x]+8*x*y'[x]-8*y[x]==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to x\left(c_3x^3 + c_2x + c_1\right)$$

# 9 Chapter 4, Section 4.1. Basic theory of linear differential equations. Exercises page 124

## 9.1 problem 1

Internal problem ID [11723]

**Book**: Differential Equations by Shepley L. Ross. Third edition. John Willey. New Delhi. 2004.
**Section**: Chapter 4, Section 4.1. Basic theory of linear differential equations. Exercises page 124
**Problem number**: 1.
**ODE order**: 2.
**ODE degree**: 1.

CAS Maple gives this as type `[[_Emden, _Fowler]]`

$$\boxed{x^2y'' - 4y'x + 4y = 0}$$

Given that one solution of the ode is

$$y_1 = x$$

✓ Solution by Maple
Time used: 0.0 (sec). Leaf size: 13

```
dsolve([x^2*diff(y(x),x$2)-4*x*diff(y(x),x)+4*y(x)=0,x],singsol=all)
```

$$y(x) = x\left(c_1x^3 + c_2\right)$$

✓ Solution by Mathematica
Time used: 0.01 (sec). Leaf size: 16

```
DSolve[x^2*y''[x]-4*x*y'[x]+4*y[x]==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to x\left(c_2x^3 + c_1\right)$$

## 9.2 problem 2

Internal problem ID [11724]

**Book**: Differential Equations by Shepley L. Ross. Third edition. John Willey. New Delhi. 2004.
**Section**: Chapter 4, Section 4.1. Basic theory of linear differential equations. Exercises page 124
**Problem number**: 2.
**ODE order**: 2.
**ODE degree**: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$\boxed{(1+x)^2 y'' - 3(1+x) y' + 3y = 0}$$

Given that one solution of the ode is

$$y_1 = 1 + x$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 17

```
dsolve([(x+1)^2*diff(y(x),x$2)-3*(x+1)*diff(y(x),x)+3*y(x)=0,x+1],singsol=all)
```

$$y(x) = (1+x)\left(c_1 + c_2(1+x)^2\right)$$

✓ Solution by Mathematica

Time used: 0.026 (sec). Leaf size: 20

```
DSolve[(x+1)^2*y''[x]-3*(x+1)*y'[x]+3*y[x]==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to c_2(x+1)^3 + c_1(x+1)$$

## 9.3 problem 3

Internal problem ID [11725]

**Book**: Differential Equations by Shepley L. Ross. Third edition. John Willey. New Delhi. 2004.
**Section**: Chapter 4, Section 4.1. Basic theory of linear differential equations. Exercises page 124
**Problem number**: 3.
**ODE order**: 2.
**ODE degree**: 1.

CAS Maple gives this as type `[_Gegenbauer]`

$$\boxed{(x^2-1)\,y''-2y'x+2y=0}$$

Given that one solution of the ode is

$$y_1=x$$

✓ Solution by Maple
Time used: 0.0 (sec). Leaf size: 14

```
dsolve([(x^2-1)*diff(y(x),x$2)-2*x*diff(y(x),x)+2*y(x)=0,x],singsol=all)
```

$$y(x)=c_2x^2+c_1x+c_2$$

✓ Solution by Mathematica
Time used: 0.086 (sec). Leaf size: 39

```
DSolve[(x^2-1)*y''[x]-2*x*y'[x]+2*y[x]==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x)\to\frac{\sqrt{x^2-1}(c_1(x-1)^2+c_2x)}{\sqrt{1-x^2}}$$

## 9.4 problem 4

Internal problem ID [11726]

**Book**: Differential Equations by Shepley L. Ross. Third edition. John Willey. New Delhi. 2004.
**Section**: Chapter 4, Section 4.1. Basic theory of linear differential equations. Exercises page 124
**Problem number**: 4.
**ODE order**: 2.
**ODE degree**: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$\boxed{(x^2 - x + 1)\, y'' - (x^2 + x)\, y' + y(1 + x) = 0}$$

Given that one solution of the ode is

$$y_1 = x$$

✓ Solution by Maple
Time used: 0.0 (sec). Leaf size: 15

```
dsolve([(x^2-x+1)*diff(y(x),x$2)-(x^2+x)*diff(y(x),x)+(x+1)*y(x)=0,x],singsol=all)
```

$$y(x) = c_1 x + c_2 \mathrm{e}^{x}(-1 + x)$$

✓ Solution by Mathematica
Time used: 0.093 (sec). Leaf size: 19

```
DSolve[(x^2-x+1)*y''[x]-(x^2+x)*y'[x]+(x+1)*y[x]==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to c_1 x + c_2 e^{x}(x - 1)$$

## 9.5 problem 5

Internal problem ID [11727]

**Book**: Differential Equations by Shepley L. Ross. Third edition. John Willey. New Delhi. 2004.
**Section**: Chapter 4, Section 4.1. Basic theory of linear differential equations. Exercises page 124
**Problem number**: 5.
**ODE order**: 2.
**ODE degree**: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$\boxed{(2x+1)\, y'' - 4(1+x)\, y' + 4y = 0}$$

Given that one solution of the ode is

$$y_1 = e^{2x}$$

✓ Solution by Maple
Time used: 0.0 (sec). Leaf size: 15

```
dsolve([(2*x+1)*diff(y(x),x$2)-4*(x+1)*diff(y(x),x)+4*y(x)=0,exp(2*x)],singsol=all)
```

$$y(x) = c_2 e^{2x} + c_1 x + c_1$$

✓ Solution by Mathematica
Time used: 0.125 (sec). Leaf size: 23

```
DSolve[(2*x+1)*y''[x]-4*(x+1)*y'[x]+4*y[x]==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to c_1 e^{2x+1} - c_2(x+1)$$

## 9.6 problem 6

Internal problem ID [11728]

**Book**: Differential Equations by Shepley L. Ross. Third edition. John Willey. New Delhi. 2004.
**Section**: Chapter 4, Section 4.1. Basic theory of linear differential equations. Exercises page 124
**Problem number**: 6.
**ODE order**: 2.
**ODE degree**: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$\boxed{(x^3 - x^2)\, y'' - (x^3 + 2x^2 - 2x)\, y' + (2x^2 + 2x - 2)\, y = 0}$$

Given that one solution of the ode is

$$y_1 = x^2$$

✓ Solution by Maple
Time used: 0.0 (sec). Leaf size: 14

```
dsolve([(x^3-x^2)*diff(y(x),x$2)-(x^3+2*x^2-2*x)*diff(y(x),x)+(2*x^2+2*x-2)*y(x)=0,x^2],sing
```

$$y(x) = x(c_2 e^x + c_1 x)$$

✓ Solution by Mathematica
Time used: 0.101 (sec). Leaf size: 36

```
DSolve[(x^3-x^2)*y''[x]-(x^3+2*x^2-2*x)*y'[x]+(2*x^2+2*x-2)*y[x]==0,y[x],x,IncludeSingularSo
```

$$y(x) \to -\frac{\sqrt{1-x}x(c_2 x - c_1 e^x)}{\sqrt{x-1}}$$

## 9.7 problem 8

Internal problem ID [11729]

**Book**: Differential Equations by Shepley L. Ross. Third edition. John Willey. New Delhi. 2004.
**Section**: Chapter 4, Section 4.1. Basic theory of linear differential equations. Exercises page 124
**Problem number**: 8.
**ODE order**: 2.
**ODE degree**: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$\boxed{y'' - 3y' + 2y = 4x^2}$$

✓ Solution by Maple
Time used: 0.0 (sec). Leaf size: 24

```
dsolve(diff(y(x),x$2)-3*diff(y(x),x)+2*y(x)=4*x^2,y(x), singsol=all)
```

$$y(x) = e^{2x}c_1 + c_2e^x + 2x^2 + 6x + 7$$

✓ Solution by Mathematica
Time used: 0.014 (sec). Leaf size: 29

```
DSolve[y''[x]-3*y'[x]+2*y[x]==4*x^2,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to 2x^2 + 6x + c_1e^x + c_2e^{2x} + 7$$

## 9.8 problem 9

Internal problem ID [11730]

**Book**: Differential Equations by Shepley L. Ross. Third edition. John Willey. New Delhi. 2004.
**Section**: Chapter 4, Section 4.1. Basic theory of linear differential equations. Exercises page 124
**Problem number**: 9.
**ODE order**: 2.
**ODE degree**: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$\boxed{y'' - 5y' + 6y = 2 - 12x + 6\,e^{x}}$$

✓ Solution by Maple

Time used: 0.016 (sec). Leaf size: 25

```
dsolve(diff(y(x),x$2)-5*diff(y(x),x)+6*y(x)=2-12*x+6*exp(x),y(x), singsol=all)
```

$$y(x) = c_2 e^{2x} + c_1 e^{3x} + 3\,e^{x} - 2x - \frac{4}{3}$$

✓ Solution by Mathematica

Time used: 0.15 (sec). Leaf size: 33

```
DSolve[y''[x]-5*y'[x]+6*y[x]==2-12*x+6*Exp[x],y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to -2x + 3e^{x} + c_1 e^{2x} + c_2 e^{3x} - \frac{4}{3}$$

# 10 Chapter 4, Section 4.2. The homogeneous linear equation with constant coefficients. Exercises page 135

## 10.1 problem 1

Internal problem ID [11731]

**Book**: Differential Equations by Shepley L. Ross. Third edition. John Willey. New Delhi. 2004.
**Section**: Chapter 4, Section 4.2. The homogeneous linear equation with constant coefficients. Exercises page 135
**Problem number**: 1.
**ODE order**: 2.
**ODE degree**: 1.

CAS Maple gives this as type `[[_2nd_order, _missing_x]]`

$$\boxed{y'' - 5y' + 6y = 0}$$

✓ Solution by Maple
Time used: 0.0 (sec). Leaf size: 17

```
dsolve(diff(y(x),x$2)-5*diff(y(x),x)+6*y(x)=0,y(x), singsol=all)
```

$$y(x) = \mathrm{e}^{2x}c_1 + c_2\mathrm{e}^{3x}$$

✓ Solution by Mathematica
Time used: 0.013 (sec). Leaf size: 20

```
DSolve[y''[x]-5*y'[x]+6*y[x]==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to e^{2x}(c_2e^x + c_1)$$

## 10.2 problem 2

Internal problem ID [11732]

**Book**: Differential Equations by Shepley L. Ross. Third edition. John Willey. New Delhi. 2004.
**Section**: Chapter 4, Section 4.2. The homogeneous linear equation with constant coefficients. Exercises page 135
**Problem number**: 2.
**ODE order**: 2.
**ODE degree**: 1.

CAS Maple gives this as type `[[_2nd_order, _missing_x]]`

$$\boxed{y'' - 2y' - 3y = 0}$$

✓ Solution by Maple
Time used: 0.0 (sec). Leaf size: 17

```
dsolve(diff(y(x),x$2)-2*diff(y(x),x)-3*y(x)=0,y(x), singsol=all)
```

$$y(x) = c_1 e^{-x} + c_2 e^{3x}$$

✓ Solution by Mathematica
Time used: 0.012 (sec). Leaf size: 22

```
DSolve[y''[x]-2*y'[x]-3*y[x]==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to e^{-x}(c_2 e^{4x} + c_1)$$

## 10.3 problem 3

Internal problem ID [11733]

**Book**: Differential Equations by Shepley L. Ross. Third edition. John Willey. New Delhi. 2004.
**Section**: Chapter 4, Section 4.2. The homogeneous linear equation with constant coefficients. Exercises page 135
**Problem number**: 3.
**ODE order**: 2.
**ODE degree**: 1.

CAS Maple gives this as type `[[_2nd_order, _missing_x]]`

$$\boxed{4y'' - 12y' + 5y = 0}$$

✓ Solution by Maple
Time used: 0.0 (sec). Leaf size: 17

```
dsolve(4*diff(y(x),x$2)-12*diff(y(x),x)+5*y(x)=0,y(x), singsol=all)
```

$$y(x) = c_1 e^{\frac{x}{2}} + c_2 e^{\frac{5x}{2}}$$

✓ Solution by Mathematica
Time used: 0.013 (sec). Leaf size: 24

```
DSolve[4*y''[x]-12*y'[x]+5*y[x]==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to e^{x/2}(c_2 e^{2x} + c_1)$$

## 10.4 problem 4

Internal problem ID [11734]

**Book**: Differential Equations by Shepley L. Ross. Third edition. John Willey. New Delhi. 2004.
**Section**: Chapter 4, Section 4.2. The homogeneous linear equation with constant coefficients. Exercises page 135
**Problem number**: 4.
**ODE order**: 2.
**ODE degree**: 1.

CAS Maple gives this as type `[[_2nd_order, _missing_x]]`

$$\boxed{3y'' - 14y' - 5y = 0}$$

✓ Solution by Maple
Time used: 0.015 (sec). Leaf size: 17

```
dsolve(3*diff(y(x),x$2)-14*diff(y(x),x)-5*y(x)=0,y(x), singsol=all)
```

$$y(x) = c_1 e^{5x} + c_2 e^{-\frac{x}{3}}$$

✓ Solution by Mathematica
Time used: 0.013 (sec). Leaf size: 24

```
DSolve[3*y''[x]-14*y'[x]-5*y[x]==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to c_1 e^{-x/3} + c_2 e^{5x}$$

## 10.5 problem 5

Internal problem ID [11735]

**Book**: Differential Equations by Shepley L. Ross. Third edition. John Willey. New Delhi. 2004.
**Section**: Chapter 4, Section 4.2. The homogeneous linear equation with constant coefficients. Exercises page 135
**Problem number**: 5.
**ODE order**: 3.
**ODE degree**: 1.

CAS Maple gives this as type `[[_3rd_order, _missing_x]]`

$$\boxed{y''' - 3y'' - y' + 3y = 0}$$

✓ Solution by Maple
Time used: 0.0 (sec). Leaf size: 21

```
dsolve(diff(y(x),x$3)-3*diff(y(x),x$2)-diff(y(x),x)+3*y(x)=0,y(x), singsol=all)
```

$$y(x) = c_1 e^{-x} + c_2 e^{x} + c_3 e^{3x}$$

✓ Solution by Mathematica
Time used: 0.003 (sec). Leaf size: 28

```
DSolve[y'''[x]-3*y''[x]-y'[x]+3*y[x]==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to c_1 e^{-x} + c_2 e^{x} + c_3 e^{3x}$$

## 10.6 problem 6

Internal problem ID [11736]

**Book**: Differential Equations by Shepley L. Ross. Third edition. John Willey. New Delhi. 2004.
**Section**: Chapter 4, Section 4.2. The homogeneous linear equation with constant coefficients. Exercises page 135
**Problem number**: 6.
**ODE order**: 3.
**ODE degree**: 1.

CAS Maple gives this as type `[[_3rd_order, _missing_x]]`

$$\boxed{y''' - 6y'' + 5y' + 12y = 0}$$

✓ Solution by Maple
Time used: 0.0 (sec). Leaf size: 23

```
dsolve(diff(y(x),x$3)-6*diff(y(x),x$2)+5*diff(y(x),x)+12*y(x)=0,y(x), singsol=all)
```

$$y(x) = c_1 e^{4x} + c_2 e^{-x} + c_3 e^{3x}$$

✓ Solution by Mathematica
Time used: 0.003 (sec). Leaf size: 29

```
DSolve[y'''[x]-6*y''[x]+5*y'[x]+12*y[x]==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to e^{-x}\left(e^{4x}(c_3 e^x + c_2) + c_1\right)$$

## 10.7 problem 7

Internal problem ID [11737]

**Book**: Differential Equations by Shepley L. Ross. Third edition. John Willey. New Delhi. 2004.
**Section**: Chapter 4, Section 4.2. The homogeneous linear equation with constant coefficients. Exercises page 135
**Problem number**: 7.
**ODE order**: 2.
**ODE degree**: 1.

CAS Maple gives this as type `[[_2nd_order, _missing_x]]`

$$\boxed{y'' - 8y' + 16y = 0}$$

✓ Solution by Maple
Time used: 0.016 (sec). Leaf size: 14

```
dsolve(diff(y(x),x$2)-8*diff(y(x),x)+16*y(x)=0,y(x), singsol=all)
```

$$y(x) = \mathrm{e}^{4x}(c_2 x + c_1)$$

✓ Solution by Mathematica
Time used: 0.013 (sec). Leaf size: 18

```
DSolve[y''[x]-8*y'[x]+16*y[x]==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to e^{4x}(c_2 x + c_1)$$

## 10.8 problem 8

Internal problem ID [11738]

**Book**: Differential Equations by Shepley L. Ross. Third edition. John Willey. New Delhi. 2004.
**Section**: Chapter 4, Section 4.2. The homogeneous linear equation with constant coefficients. Exercises page 135
**Problem number**: 8.
**ODE order**: 2.
**ODE degree**: 1.

CAS Maple gives this as type `[[_2nd_order, _missing_x]]`

$$\boxed{4y'' + 4y' + y = 0}$$

✓ Solution by Maple
Time used: 0.0 (sec). Leaf size: 14

```
dsolve(4*diff(y(x),x$2)+4*diff(y(x),x)+y(x)=0,y(x), singsol=all)
```

$$y(x) = \mathrm{e}^{-\frac{x}{2}}(c_2 x + c_1)$$

✓ Solution by Mathematica
Time used: 0.015 (sec). Leaf size: 20

```
DSolve[4*y''[x]+4*y'[x]+y[x]==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to e^{-x/2}(c_2 x + c_1)$$

## 10.9 problem 9

Internal problem ID [11739]

**Book**: Differential Equations by Shepley L. Ross. Third edition. John Willey. New Delhi. 2004.
**Section**: Chapter 4, Section 4.2. The homogeneous linear equation with constant coefficients. Exercises page 135
**Problem number**: 9.
**ODE order**: 2.
**ODE degree**: 1.

CAS Maple gives this as type `[[_2nd_order, _missing_x]]`

$$\boxed{y'' - 4y' + 13y = 0}$$

✓ Solution by Maple

Time used: 0.016 (sec). Leaf size: 22

```
dsolve(diff(y(x),x$2)-4*diff(y(x),x)+13*y(x)=0,y(x), singsol=all)
```

$$y(x) = \mathrm{e}^{2x}(c_1 \sin(3x) + c_2 \cos(3x))$$

✓ Solution by Mathematica

Time used: 0.018 (sec). Leaf size: 26

```
DSolve[y''[x]-4*y'[x]+13*y[x]==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to e^{2x}(c_2 \cos(3x) + c_1 \sin(3x))$$

## 10.10 problem 10

Internal problem ID [11740]

**Book**: Differential Equations by Shepley L. Ross. Third edition. John Willey. New Delhi. 2004.
**Section**: Chapter 4, Section 4.2. The homogeneous linear equation with constant coefficients. Exercises page 135
**Problem number**: 10.
**ODE order**: 2.
**ODE degree**: 1.

CAS Maple gives this as type `[[_2nd_order, _missing_x]]`

$$\boxed{y'' + 6y' + 25y = 0}$$

✓ Solution by Maple
Time used: 0.0 (sec). Leaf size: 22

```
dsolve(diff(y(x),x$2)+6*diff(y(x),x)+25*y(x)=0,y(x), singsol=all)
```

$$y(x) = \mathrm{e}^{-3x}(c_1 \sin(4x) + c_2 \cos(4x))$$

✓ Solution by Mathematica
Time used: 0.019 (sec). Leaf size: 26

```
DSolve[y''[x]+6*y'[x]+25*y[x]==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to e^{-3x}(c_2 \cos(4x) + c_1 \sin(4x))$$

## 10.11 problem 11

Internal problem ID [11741]

**Book**: Differential Equations by Shepley L. Ross. Third edition. John Willey. New Delhi. 2004.
**Section**: Chapter 4, Section 4.2. The homogeneous linear equation with constant coefficients. Exercises page 135
**Problem number**: 11.
**ODE order**: 2.
**ODE degree**: 1.

CAS Maple gives this as type `[[_2nd_order, _missing_x]]`

$$\boxed{y'' + 9y = 0}$$

✓ Solution by Maple
Time used: 0.0 (sec). Leaf size: 17

```
dsolve(diff(y(x),x$2)+9*y(x)=0,y(x), singsol=all)
```

$$y(x) = c_1 \sin(3x) + c_2 \cos(3x)$$

✓ Solution by Mathematica
Time used: 0.012 (sec). Leaf size: 20

```
DSolve[y''[x]+9*y[x]==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to c_1 \cos(3x) + c_2 \sin(3x)$$

## 10.12 problem 12

Internal problem ID [11742]

**Book**: Differential Equations by Shepley L. Ross. Third edition. John Willey. New Delhi. 2004.
**Section**: Chapter 4, Section 4.2. The homogeneous linear equation with constant coefficients. Exercises page 135
**Problem number**: 12.
**ODE order**: 2.
**ODE degree**: 1.

CAS Maple gives this as type `[[_2nd_order, _missing_x]]`

$$\boxed{4y'' + y = 0}$$

✓ Solution by Maple
Time used: 0.0 (sec). Leaf size: 17

```
dsolve(4*diff(y(x),x$2)+y(x)=0,y(x), singsol=all)
```

$$y(x) = c_1 \sin\left(\frac{x}{2}\right) + c_2 \cos\left(\frac{x}{2}\right)$$

✓ Solution by Mathematica
Time used: 0.013 (sec). Leaf size: 24

```
DSolve[4*y''[x]+y[x]==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to c_1 \cos\left(\frac{x}{2}\right) + c_2 \sin\left(\frac{x}{2}\right)$$

## 10.13 problem 13

Internal problem ID [11743]

**Book**: Differential Equations by Shepley L. Ross. Third edition. John Willey. New Delhi. 2004.
**Section**: Chapter 4, Section 4.2. The homogeneous linear equation with constant coefficients. Exercises page 135
**Problem number**: 13.
**ODE order**: 3.
**ODE degree**: 1.

CAS Maple gives this as type `[[_3rd_order, _missing_x]]`

$$\boxed{y''' - 5y'' + 7y' - 3y = 0}$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 19

```
dsolve(diff(y(x),x$3)-5*diff(y(x),x$2)+7*diff(y(x),x)-3*y(x)=0,y(x), singsol=all)
```

$$y(x) = c_1 \mathrm{e}^{3x} + \mathrm{e}^{x}(c_3 x + c_2)$$

✓ Solution by Mathematica

Time used: 0.003 (sec). Leaf size: 24

```
DSolve[y'''[x]-5*y''[x]+7*y'[x]-3*y[x]==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to e^{x}\left(c_2 x + c_3 e^{2x} + c_1\right)$$

## 10.14 problem 14

Internal problem ID [11744]

**Book**: Differential Equations by Shepley L. Ross. Third edition. John Willey. New Delhi. 2004.
**Section**: Chapter 4, Section 4.2. The homogeneous linear equation with constant coefficients. Exercises page 135
**Problem number**: 14.
**ODE order**: 3.
**ODE degree**: 1.

CAS Maple gives this as type `[[_3rd_order, _missing_x]]`

$$\boxed{4y''' + 4y'' - 7y' + 2y = 0}$$

✓ Solution by Maple

Time used: 0.016 (sec). Leaf size: 21

```
dsolve(4*diff(y(x),x$3)+4*diff(y(x),x$2)-7*diff(y(x),x)+2*y(x)=0,y(x), singsol=all)
```

$$y(x) = \left((c_3 x + c_2)\,e^{\frac{5x}{2}} + c_1\right) e^{-2x}$$

✓ Solution by Mathematica

Time used: 0.003 (sec). Leaf size: 93

```
DSolve[4*y'''[x]+4*y''[x]+7*y'[x]+2*y[x]==0,y[x],x,IncludeSingularSolutions -> True]
```

$$\begin{aligned} y(x) \to\ & c_1 \exp\left(x\text{Root}\left[4\#1^3 + 4\#1^2 + 7\#1 + 2\&, 1\right]\right) \\ & + c_2 \exp\left(x\text{Root}\left[4\#1^3 + 4\#1^2 + 7\#1 + 2\&, 2\right]\right) \\ & + c_3 \exp\left(x\text{Root}\left[4\#1^3 + 4\#1^2 + 7\#1 + 2\&, 3\right]\right) \end{aligned}$$

## 10.15 problem 15

Internal problem ID [11745]

**Book**: Differential Equations by Shepley L. Ross. Third edition. John Willey. New Delhi. 2004.
**Section**: Chapter 4, Section 4.2. The homogeneous linear equation with constant coefficients. Exercises page 135
**Problem number**: 15.
**ODE order**: 3.
**ODE degree**: 1.

CAS Maple gives this as type `[[_3rd_order, _missing_x]]`

$$\boxed{y''' - 6y'' + 12y' - 8y = 0}$$

✓ Solution by Maple
Time used: 0.0 (sec). Leaf size: 19

```
dsolve(diff(y(x),x$3)-6*diff(y(x),x$2)+12*diff(y(x),x)-8*y(x)=0,y(x), singsol=all)
```

$$y(x) = e^{2x}(c_3x^2 + c_2x + c_1)$$

✓ Solution by Mathematica
Time used: 0.003 (sec). Leaf size: 23

```
DSolve[y'''[x]-6*y''[x]+12*y'[x]-8*y[x]==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to e^{2x}(x(c_3x + c_2) + c_1)$$

## 10.16 problem 16

Internal problem ID [11746]

**Book**: Differential Equations by Shepley L. Ross. Third edition. John Willey. New Delhi. 2004.
**Section**: Chapter 4, Section 4.2. The homogeneous linear equation with constant coefficients. Exercises page 135
**Problem number**: 16.
**ODE order**: 3.
**ODE degree**: 1.

CAS Maple gives this as type `[[_3rd_order, _missing_x]]`

$$\boxed{y''' + 4y'' + 5y' + 6y = 0}$$

✓ Solution by Maple
Time used: 0.0 (sec). Leaf size: 37

```
dsolve(diff(y(x),x$3)+4*diff(y(x),x$2)+5*diff(y(x),x)+6*y(x)=0,y(x), singsol=all)
```

$$y(x) = c_1 e^{-3x} + c_2 e^{-\frac{x}{2}} \sin\left(\frac{\sqrt{7}\,x}{2}\right) + c_3 e^{-\frac{x}{2}} \cos\left(\frac{\sqrt{7}\,x}{2}\right)$$

✓ Solution by Mathematica
Time used: 0.004 (sec). Leaf size: 56

```
DSolve[y'''[x]+4*y''[x]+5*y'[x]+6*y[x]==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to e^{-3x}\left(c_2 e^{5x/2} \cos\left(\frac{\sqrt{7}x}{2}\right) + c_1 e^{5x/2} \sin\left(\frac{\sqrt{7}x}{2}\right) + c_3\right)$$

## 10.17 problem 17

Internal problem ID [11747]

**Book**: Differential Equations by Shepley L. Ross. Third edition. John Willey. New Delhi. 2004.
**Section**: Chapter 4, Section 4.2. The homogeneous linear equation with constant coefficients. Exercises page 135
**Problem number**: 17.
**ODE order**: 3.
**ODE degree**: 1.

CAS Maple gives this as type `[[_3rd_order, _missing_x]]`

$$\boxed{y''' - y'' + y' - y = 0}$$

✓ Solution by Maple
Time used: 0.0 (sec). Leaf size: 17

```
dsolve(diff(y(x),x$3)-diff(y(x),x$2)+diff(y(x),x)-y(x)=0,y(x), singsol=all)
```

$$y(x) = c_1 e^x + \sin(x) c_2 + c_3 \cos(x)$$

✓ Solution by Mathematica
Time used: 0.003 (sec). Leaf size: 22

```
DSolve[y'''[x]-y''[x]+y'[x]-y[x]==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to c_3 e^x + c_1 \cos(x) + c_2 \sin(x)$$

## 10.18 problem 18

Internal problem ID [11748]

**Book**: Differential Equations by Shepley L. Ross. Third edition. John Willey. New Delhi. 2004.
**Section**: Chapter 4, Section 4.2. The homogeneous linear equation with constant coefficients. Exercises page 135
**Problem number**: 18.
**ODE order**: 4.
**ODE degree**: 1.

CAS Maple gives this as type `[[_high_order, _missing_x]]`

$$\boxed{y'''' + 8y'' + 16y = 0}$$

✓ Solution by Maple
Time used: 0.0 (sec). Leaf size: 25

```
dsolve(diff(y(x),x$4)+8*diff(y(x),x$2)+16*y(x)=0,y(x), singsol=all)
```

$$y(x) = (c_4x + c_2)\cos(2x) + \sin(2x)(c_3x + c_1)$$

✓ Solution by Mathematica
Time used: 0.003 (sec). Leaf size: 30

```
DSolve[y''''[x]+8*y''[x]+16*y[x]==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to (c_2x + c_1)\cos(2x) + (c_4x + c_3)\sin(2x)$$

## 10.19 problem 19

Internal problem ID [11749]

**Book**: Differential Equations by Shepley L. Ross. Third edition. John Willey. New Delhi. 2004.
**Section**: Chapter 4, Section 4.2. The homogeneous linear equation with constant coefficients. Exercises page 135
**Problem number**: 19.
**ODE order**: 5.
**ODE degree**: 1.

CAS Maple gives this as type `[[_high_order, _missing_x]]`

$$\boxed{y^{(5)} - 2y'''' + y''' = 0}$$

✓ Solution by Maple
Time used: 0.015 (sec). Leaf size: 22

```
dsolve(diff(y(x),x$5)-2*diff(y(x),x$4)+diff(y(x),x$3)=0,y(x), singsol=all)
```

$$y(x) = (c_5 x + c_4)\, \mathrm{e}^{x} + c_3 x^2 + c_2 x + c_1$$

✓ Solution by Mathematica
Time used: 0.096 (sec). Leaf size: 30

```
DSolve[y'''''[x]-2*y''''[x]+y'''[x]==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to e^x (c_2 (x-3) + c_1) + x (c_5 x + c_4) + c_3$$

## 10.20 problem 20

Internal problem ID [11750]

**Book**: Differential Equations by Shepley L. Ross. Third edition. John Willey. New Delhi. 2004.
**Section**: Chapter 4, Section 4.2. The homogeneous linear equation with constant coefficients. Exercises page 135
**Problem number**: 20.
**ODE order**: 4.
**ODE degree**: 1.

CAS Maple gives this as type `[[_high_order, _missing_x]]`

$$\boxed{y'''' - y''' - 3y'' + y' + 2y = 0}$$

✓ Solution by Maple
Time used: 0.016 (sec). Leaf size: 25

```
dsolve(diff(y(x),x$4)-diff(y(x),x$3)-3*diff(y(x),x$2)+diff(y(x),x)+2*y(x)=0,y(x), singsol=al
```

$$y(x) = (c_4x + c_3)\,\mathrm{e}^{-x} + c_1\mathrm{e}^{x} + c_2\mathrm{e}^{2x}$$

✓ Solution by Mathematica
Time used: 0.003 (sec). Leaf size: 33

```
DSolve[y''''[x]-y'''[x]-3*y''[x]+y'[x]+2*y[x]==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to e^{-x}\left(c_2x + e^{2x}(c_4e^x + c_3) + c_1\right)$$

## 10.21 problem 21

Internal problem ID [11751]

**Book**: Differential Equations by Shepley L. Ross. Third edition. John Willey. New Delhi. 2004.
**Section**: Chapter 4, Section 4.2. The homogeneous linear equation with constant coefficients. Exercises page 135
**Problem number**: 21.
**ODE order**: 4.
**ODE degree**: 1.

CAS Maple gives this as type `[[_high_order, _missing_x]]`

$$\boxed{y'''' - 3y''' - 2y'' + 2y' + 12y = 0}$$

✓ Solution by Maple
Time used: 0.0 (sec). Leaf size: 33

```
dsolve(diff(y(x),x$4)-3*diff(y(x),x$3)-2*diff(y(x),x$2)+2*diff(y(x),x)+12*y(x)=0,y(x), sings
```

$$y(x) = \mathrm{e}^{2x} c_1 + c_2 \mathrm{e}^{3x} + c_3 \mathrm{e}^{-x} \sin(x) + c_4 \mathrm{e}^{-x} \cos(x)$$

✓ Solution by Mathematica
Time used: 0.003 (sec). Leaf size: 37

```
DSolve[y''''[x]-3*y'''[x]-2*y''[x]+2*y'[x]+12*y[x]==0,y[x],x,IncludeSingularSolutions -> Tru
```

$$y(x) \to e^{-x} \left(e^{3x} (c_4 e^x + c_3) + c_2 \cos(x) + c_1 \sin(x)\right)$$

## 10.22 problem 22

Internal problem ID [11752]

**Book**: Differential Equations by Shepley L. Ross. Third edition. John Willey. New Delhi. 2004.
**Section**: Chapter 4, Section 4.2. The homogeneous linear equation with constant coefficients. Exercises page 135
**Problem number**: 22.
**ODE order**: 4.
**ODE degree**: 1.

CAS Maple gives this as type `[[_high_order, _missing_x]]`

$$\boxed{y'''' + 6y''' + 15y'' + 20y' + 12y = 0}$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 39

```
dsolve(diff(y(x),x$4)+6*diff(y(x),x$3)+15*diff(y(x),x$2)+20*diff(y(x),x)+12*y(x)=0,y(x), sin
```

$$y(x) = c_4 \mathrm{e}^{-x} \cos\left(x\sqrt{2}\right) + c_3 \mathrm{e}^{-x} \sin\left(x\sqrt{2}\right) + \mathrm{e}^{-2x}(c_2 x + c_1)$$

✓ Solution by Mathematica

Time used: 0.003 (sec). Leaf size: 46

```
DSolve[y''''[x]+6*y'''[x]+15*y''[x]+20*y'[x]+12*y[x]==0,y[x],x,IncludeSingularSolutions -> T
```

$$y(x) \to e^{-2x}\left(c_4 x + c_2 e^x \cos\left(\sqrt{2}x\right) + c_1 e^x \sin\left(\sqrt{2}x\right) + c_3\right)$$

## 10.23 problem 23

Internal problem ID [11753]

**Book**: Differential Equations by Shepley L. Ross. Third edition. John Willey. New Delhi. 2004.
**Section**: Chapter 4, Section 4.2. The homogeneous linear equation with constant coefficients. Exercises page 135
**Problem number**: 23.
**ODE order**: 4.
**ODE degree**: 1.

CAS Maple gives this as type `[[_high_order, _missing_x]]`

$$\boxed{y'''' + y = 0}$$

✓ Solution by Maple
Time used: 0.0 (sec). Leaf size: 61

```
dsolve(diff(y(x),x$4)+y(x)=0,y(x), singsol=all)
```

$$y(x) = \left(-c_1 e^{-\frac{x\sqrt{2}}{2}} - c_2 e^{\frac{x\sqrt{2}}{2}}\right)\sin\left(\frac{x\sqrt{2}}{2}\right) + \left(c_3 e^{-\frac{x\sqrt{2}}{2}} + c_4 e^{\frac{x\sqrt{2}}{2}}\right)\cos\left(\frac{x\sqrt{2}}{2}\right)$$

✓ Solution by Mathematica
Time used: 0.003 (sec). Leaf size: 65

```
DSolve[y''''[x]+y[x]==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to e^{-\frac{x}{\sqrt{2}}}\left(\left(c_1 e^{\sqrt{2}x} + c_2\right)\cos\left(\frac{x}{\sqrt{2}}\right) + \left(c_4 e^{\sqrt{2}x} + c_3\right)\sin\left(\frac{x}{\sqrt{2}}\right)\right)$$

## 10.24 problem 24

Internal problem ID [11754]

**Book**: Differential Equations by Shepley L. Ross. Third edition. John Willey. New Delhi. 2004.
**Section**: Chapter 4, Section 4.2. The homogeneous linear equation with constant coefficients. Exercises page 135
**Problem number**: 24.
**ODE order**: 5.
**ODE degree**: 1.

CAS Maple gives this as type `[[_high_order, _quadrature]]`

$$\boxed{y^{(5)} = 0}$$

✓ Solution by Maple
Time used: 0.0 (sec). Leaf size: 27

```
dsolve(diff(y(x),x$5)=0,y(x), singsol=all)
```

$$y(x) = \frac{1}{24}x^4c_1 + \frac{1}{6}c_2x^3 + \frac{1}{2}c_3x^2 + c_4x + c_5$$

✓ Solution by Mathematica
Time used: 0.002 (sec). Leaf size: 27

```
DSolve[y'''''[x]==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to x(x(x(c_5x + c_4) + c_3) + c_2) + c_1$$

## 10.25 problem 25

Internal problem ID [11755]

**Book**: Differential Equations by Shepley L. Ross. Third edition. John Willey. New Delhi. 2004.
**Section**: Chapter 4, Section 4.2. The homogeneous linear equation with constant coefficients. Exercises page 135
**Problem number**: 25.
**ODE order**: 2.
**ODE degree**: 1.

CAS Maple gives this as type `[[_2nd_order, _missing_x]]`

$$\boxed{y'' - y' - 12y = 0}$$

With initial conditions

$$[y(0) = 3, y'(0) = 5]$$

✓ Solution by Maple

Time used: 0.016 (sec). Leaf size: 15

```
dsolve([diff(y(x),x$2)-diff(y(x),x)-12*y(x)=0,y(0) = 3, D(y)(0) = 5],y(x), singsol=all)
```

$$y(x) = \left(2\,\mathrm{e}^{7x} + 1\right)\mathrm{e}^{-3x}$$

✓ Solution by Mathematica

Time used: 0.012 (sec). Leaf size: 18

```
DSolve[{y''[x]-y'[x]-12*y[x]==0,{y[0]==3,y'[0]==5}},y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to e^{-3x} + 2e^{4x}$$

## 10.26 problem 26

Internal problem ID [11756]

**Book**: Differential Equations by Shepley L. Ross. Third edition. John Willey. New Delhi. 2004.
**Section**: Chapter 4, Section 4.2. The homogeneous linear equation with constant coefficients. Exercises page 135
**Problem number**: 26.
**ODE order**: 2.
**ODE degree**: 1.

CAS Maple gives this as type `[[_2nd_order, _missing_x]]`

$$\boxed{y'' + 7y' + 10y = 0}$$

With initial conditions

$$[y(0) = -4, y'(0) = 2]$$

✓ Solution by Maple

Time used: 0.015 (sec). Leaf size: 17

```
dsolve([diff(y(x),x$2)+7*diff(y(x),x)+10*y(x)=0,y(0) = -4, D(y)(0) = 2],y(x), singsol=all)
```

$$y(x) = 2\,\mathrm{e}^{-5x} - 6\,\mathrm{e}^{-2x}$$

✓ Solution by Mathematica

Time used: 0.013 (sec). Leaf size: 20

```
DSolve[{y''[x]+7*y'[x]+10*y[x]==0,{y[0]==-4,y'[0]==2}},y[x],x,IncludeSingularSolutions -> Tr
```

$$y(x) \to e^{-5x}(2 - 6e^{3x})$$

## 10.27 problem 27

Internal problem ID [11757]

**Book**: Differential Equations by Shepley L. Ross. Third edition. John Willey. New Delhi. 2004.
**Section**: Chapter 4, Section 4.2. The homogeneous linear equation with constant coefficients. Exercises page 135
**Problem number**: 27.
**ODE order**: 2.
**ODE degree**: 1.

CAS Maple gives this as type `[[_2nd_order, _missing_x]]`

$$\boxed{y'' - 6y' + 8y = 0}$$

With initial conditions

$$[y(0) = 1, y'(0) = 6]$$

✓ Solution by Maple
Time used: 0.016 (sec). Leaf size: 17

```
dsolve([diff(y(x),x$2)-6*diff(y(x),x)+8*y(x)=0,y(0) = 1, D(y)(0) = 6],y(x), singsol=all)
```

$$y(x) = 2\,\mathrm{e}^{4x} - \mathrm{e}^{2x}$$

✓ Solution by Mathematica
Time used: 0.012 (sec). Leaf size: 20

```
DSolve[{y''[x]-6*y'[x]+8*y[x]==0,{y[0]==1,y'[0]==6}},y[x],x,IncludeSingularSolutions -> True
```

$$y(x) \to e^{2x}\left(2e^{2x} - 1\right)$$

## 10.28 problem 28

Internal problem ID [11758]

**Book**: Differential Equations by Shepley L. Ross. Third edition. John Willey. New Delhi. 2004.
**Section**: Chapter 4, Section 4.2. The homogeneous linear equation with constant coefficients. Exercises page 135
**Problem number**: 28.
**ODE order**: 2.
**ODE degree**: 1.

CAS Maple gives this as type `[[_2nd_order, _missing_x]]`

$$\boxed{3y'' + 4y' - 4y = 0}$$

With initial conditions

$$[y(0) = 2, y'(0) = -4]$$

✓ Solution by Maple
Time used: 0.0 (sec). Leaf size: 10

```
dsolve([3*diff(y(x),x$2)+4*diff(y(x),x)-4*y(x)=0,y(0) = 2, D(y)(0) = -4],y(x), singsol=all)
```

$$y(x) = 2\,\mathrm{e}^{-2x}$$

✓ Solution by Mathematica
Time used: 0.013 (sec). Leaf size: 12

```
DSolve[{3*y''[x]+4*y'[x]-4*y[x]==0,{y[0]==2,y'[0]==-4}},y[x],x,IncludeSingularSolutions -> T
```

$$y(x) \to 2e^{-2x}$$

## 10.29 problem 29

Internal problem ID [11759]

**Book**: Differential Equations by Shepley L. Ross. Third edition. John Willey. New Delhi. 2004.
**Section**: Chapter 4, Section 4.2. The homogeneous linear equation with constant coefficients. Exercises page 135
**Problem number**: 29.
**ODE order**: 2.
**ODE degree**: 1.

CAS Maple gives this as type `[[_2nd_order, _missing_x]]`

$$\boxed{y'' + 6y' + 9y = 0}$$

With initial conditions

$$[y(0) = 2, y'(0) = -3]$$

✓ Solution by Maple
Time used: 0.015 (sec). Leaf size: 14

```
dsolve([diff(y(x),x$2)+6*diff(y(x),x)+9*y(x)=0,y(0) = 2, D(y)(0) = -3],y(x), singsol=all)
```

$$y(x) = \mathrm{e}^{-3x}(3x + 2)$$

✓ Solution by Mathematica
Time used: 0.013 (sec). Leaf size: 16

```
DSolve[{y''[x]+6*y'[x]+9*y[x]==0,{y[0]==2,y'[0]==-3}},y[x],x,IncludeSingularSolutions -> Tru
```

$$y(x) \to e^{-3x}(3x + 2)$$

## 10.30 problem 30

Internal problem ID [11760]

**Book**: Differential Equations by Shepley L. Ross. Third edition. John Willey. New Delhi. 2004.
**Section**: Chapter 4, Section 4.2. The homogeneous linear equation with constant coefficients. Exercises page 135
**Problem number**: 30.
**ODE order**: 2.
**ODE degree**: 1.

CAS Maple gives this as type `[[_2nd_order, _missing_x]]`

$$\boxed{4y'' - 12y' + 9y = 0}$$

With initial conditions

$$[y(0) = 4, y'(0) = 9]$$

✓ Solution by Maple

Time used: 0.016 (sec). Leaf size: 14

```
dsolve([4*diff(y(x),x$2)-12*diff(y(x),x)+9*y(x)=0,y(0) = 4, D(y)(0) = 9],y(x), singsol=all)
```

$$y(x) = e^{\frac{3x}{2}}(3x + 4)$$

✓ Solution by Mathematica

Time used: 0.014 (sec). Leaf size: 18

```
DSolve[{4*y''[x]-12*y'[x]+9*y[x]==0,{y[0]==4,y'[0]==9}},y[x],x,IncludeSingularSolutions -> T
```

$$y(x) \to e^{3x/2}(3x + 4)$$

## 10.31 problem 31

Internal problem ID [11761]

**Book**: Differential Equations by Shepley L. Ross. Third edition. John Willey. New Delhi. 2004.
**Section**: Chapter 4, Section 4.2. The homogeneous linear equation with constant coefficients. Exercises page 135
**Problem number**: 31.
**ODE order**: 2.
**ODE degree**: 1.

CAS Maple gives this as type `[[_2nd_order, _missing_x]]`

$$\boxed{y'' + 4y' + 4y = 0}$$

With initial conditions

$$[y(0) = 3, y'(0) = 7]$$

✓ Solution by Maple
Time used: 0.0 (sec). Leaf size: 14

```
dsolve([diff(y(x),x$2)+4*diff(y(x),x)+4*y(x)=0,y(0) = 3, D(y)(0) = 7],y(x), singsol=all)
```

$$y(x) = \mathrm{e}^{-2x}(3 + 13x)$$

✓ Solution by Mathematica
Time used: 0.013 (sec). Leaf size: 16

```
DSolve[{y''[x]+4*y'[x]+4*y[x]==0,{y[0]==3,y'[0]==7}},y[x],x,IncludeSingularSolutions -> True
```

$$y(x) \to e^{-2x}(13x + 3)$$

## 10.32 problem 32

Internal problem ID [11762]

**Book**: Differential Equations by Shepley L. Ross. Third edition. John Willey. New Delhi. 2004.
**Section**: Chapter 4, Section 4.2. The homogeneous linear equation with constant coefficients. Exercises page 135
**Problem number**: 32.
**ODE order**: 2.
**ODE degree**: 1.

CAS Maple gives this as type `[[_2nd_order, _missing_x]]`

$$\boxed{9y'' - 6y' + y = 0}$$

With initial conditions

$$[y(0) = 3, y'(0) = -1]$$

✓ Solution by Maple

Time used: 0.015 (sec). Leaf size: 14

```
dsolve([9*diff(y(x),x$2)-6*diff(y(x),x)+y(x)=0,y(0) = 3, D(y)(0) = -1],y(x), singsol=all)
```

$$y(x) = e^{\frac{x}{3}}(-2x + 3)$$

✓ Solution by Mathematica

Time used: 0.014 (sec). Leaf size: 18

```
DSolve[{9*y''[x]-6*y'[x]+y[x]==0,{y[0]==3,y'[0]==-1}},y[x],x,IncludeSingularSolutions -> Tru
```

$$y(x) \to e^{x/3}(3 - 2x)$$

## 10.33 problem 33

Internal problem ID [11763]

**Book**: Differential Equations by Shepley L. Ross. Third edition. John Willey. New Delhi. 2004.
**Section**: Chapter 4, Section 4.2. The homogeneous linear equation with constant coefficients. Exercises page 135
**Problem number**: 33.
**ODE order**: 2.
**ODE degree**: 1.

CAS Maple gives this as type `[[_2nd_order, _missing_x]]`

$$\boxed{y'' - 4y' + 29y = 0}$$

With initial conditions

$$[y(0) = 0, y'(0) = 5]$$

✓ Solution by Maple
Time used: 0.0 (sec). Leaf size: 13

```
dsolve([diff(y(x),x$2)-4*diff(y(x),x)+29*y(x)=0,y(0) = 0, D(y)(0) = 5],y(x), singsol=all)
```

$$y(x) = e^{2x} \sin(5x)$$

✓ Solution by Mathematica
Time used: 0.018 (sec). Leaf size: 15

```
DSolve[{y''[x]-4*y'[x]+29*y[x]==0,{y[0]==0,y'[0]==5}},y[x],x,IncludeSingularSolutions -> Tru
```

$$y(x) \to e^{2x} \sin(5x)$$

## 10.34 problem 34

Internal problem ID [11764]

**Book**: Differential Equations by Shepley L. Ross. Third edition. John Willey. New Delhi. 2004.
**Section**: Chapter 4, Section 4.2. The homogeneous linear equation with constant coefficients. Exercises page 135
**Problem number**: 34.
**ODE order**: 2.
**ODE degree**: 1.

CAS Maple gives this as type `[[_2nd_order, _missing_x]]`

$$\boxed{y'' + 6y' + 58y = 0}$$

With initial conditions

$$[y(0) = -1, y'(0) = 5]$$

✓ Solution by Maple
Time used: 0.031 (sec). Leaf size: 23

```
dsolve([diff(y(x),x$2)+6*diff(y(x),x)+58*y(x)=0,y(0) = -1, D(y)(0) = 5],y(x), singsol=all)
```

$$y(x) = \frac{e^{-3x}(2\sin(7x) - 7\cos(7x))}{7}$$

✓ Solution by Mathematica
Time used: 0.018 (sec). Leaf size: 27

```
DSolve[{y''[x]+6*y'[x]+58*y[x]==0,{y[0]==-1,y'[0]==5}},y[x],x,IncludeSingularSolutions -> Tr
```

$$y(x) \to \frac{1}{7}e^{-3x}(2\sin(7x) - 7\cos(7x))$$

## 10.35 problem 35

Internal problem ID [11765]

**Book**: Differential Equations by Shepley L. Ross. Third edition. John Willey. New Delhi. 2004.
**Section**: Chapter 4, Section 4.2. The homogeneous linear equation with constant coefficients. Exercises page 135
**Problem number**: 35.
**ODE order**: 2.
**ODE degree**: 1.

CAS Maple gives this as type `[[_2nd_order, _missing_x]]`

$$\boxed{y'' + 6y' + 13y = 0}$$

With initial conditions

$$[y(0) = 3, y'(0) = -1]$$

✓ Solution by Maple
Time used: 0.016 (sec). Leaf size: 22

```
dsolve([diff(y(x),x$2)+6*diff(y(x),x)+13*y(x)=0,y(0) = 3, D(y)(0) = -1],y(x), singsol=all)
```

$$y(x) = e^{-3x}(4\sin(2x) + 3\cos(2x))$$

✓ Solution by Mathematica
Time used: 0.018 (sec). Leaf size: 24

```
DSolve[{y''[x]+6*y'[x]+13*y[x]==0,{y[0]==3,y'[0]==-1}},y[x],x,IncludeSingularSolutions -> Tr
```

$$y(x) \to e^{-3x}(4\sin(2x) + 3\cos(2x))$$

## 10.36 problem 36

Internal problem ID [11766]

**Book**: Differential Equations by Shepley L. Ross. Third edition. John Willey. New Delhi. 2004.
**Section**: Chapter 4, Section 4.2. The homogeneous linear equation with constant coefficients. Exercises page 135
**Problem number**: 36.
**ODE order**: 2.
**ODE degree**: 1.

CAS Maple gives this as type `[[_2nd_order, _missing_x]]`

$$\boxed{y'' + 2y' + 5y = 0}$$

With initial conditions

$$[y(0) = 2, y'(0) = 6]$$

✓ Solution by Maple
Time used: 0.016 (sec). Leaf size: 21

```
dsolve([diff(y(x),x$2)+2*diff(y(x),x)+5*y(x)=0,y(0) = 2, D(y)(0) = 6],y(x), singsol=all)
```

$$y(x) = 2\,\mathrm{e}^{-x}(2\sin(2x) + \cos(2x))$$

✓ Solution by Mathematica
Time used: 0.017 (sec). Leaf size: 23

```
DSolve[{y''[x]+2*y'[x]+5*y[x]==0,{y[0]==2,y'[0]==6}},y[x],x,IncludeSingularSolutions -> True
```

$$y(x) \to 2e^{-x}(2\sin(2x) + \cos(2x))$$

## 10.37 problem 37

Internal problem ID [11767]

**Book**: Differential Equations by Shepley L. Ross. Third edition. John Willey. New Delhi. 2004.
**Section**: Chapter 4, Section 4.2. The homogeneous linear equation with constant coefficients. Exercises page 135
**Problem number**: 37.
**ODE order**: 2.
**ODE degree**: 1.

CAS Maple gives this as type `[[_2nd_order, _missing_x]]`

$$\boxed{9y'' + 6y' + 5y = 0}$$

With initial conditions

$$[y(0) = 6, y'(0) = 0]$$

✓ Solution by Maple
Time used: 0.016 (sec). Leaf size: 21

```
dsolve([9*diff(y(x),x$2)+6*diff(y(x),x)+5*y(x)=0,y(0) = 6, D(y)(0) = 0],y(x), singsol=all)
```

$$y(x) = 3\,\mathrm{e}^{-\frac{x}{3}}\left(\sin\left(\frac{2x}{3}\right) + 2\cos\left(\frac{2x}{3}\right)\right)$$

✓ Solution by Mathematica
Time used: 0.019 (sec). Leaf size: 29

```
DSolve[{9*y''[x]+6*y'[x]+5*y[x]==0,{y[0]==6,y'[0]==0}},y[x],x,IncludeSingularSolutions -> Tr
```

$$y(x) \to 3e^{-x/3}\left(\sin\left(\frac{2x}{3}\right) + 2\cos\left(\frac{2x}{3}\right)\right)$$

## 10.38 problem 38

Internal problem ID [11768]

**Book**: Differential Equations by Shepley L. Ross. Third edition. John Willey. New Delhi. 2004.
**Section**: Chapter 4, Section 4.2. The homogeneous linear equation with constant coefficients. Exercises page 135
**Problem number**: 38.
**ODE order**: 2.
**ODE degree**: 1.

CAS Maple gives this as type `[[_2nd_order, _missing_x]]`

$$\boxed{4y'' + 4y' + 37y = 0}$$

With initial conditions

$$[y(0) = 2, y'(0) = -4]$$

✓ Solution by Maple
Time used: 0.015 (sec). Leaf size: 22

```
dsolve([4*diff(y(x),x$2)+4*diff(y(x),x)+37*y(x)=0,y(0) = 2, D(y)(0) = -4],y(x), singsol=all)
```

$$y(x) = \mathrm{e}^{-\frac{x}{2}}(-\sin(3x) + 2\cos(3x))$$

✓ Solution by Mathematica
Time used: 0.017 (sec). Leaf size: 26

```
DSolve[{4*y''[x]+4*y'[x]+37*y[x]==0,{y[0]==2,y'[0]==-4}},y[x],x,IncludeSingularSolutions ->
```

$$y(x) \to e^{-x/2}(2\cos(3x) - \sin(3x))$$

## 10.39 problem 39

Internal problem ID [11769]

**Book**: Differential Equations by Shepley L. Ross. Third edition. John Willey. New Delhi. 2004.
**Section**: Chapter 4, Section 4.2. The homogeneous linear equation with constant coefficients. Exercises page 135
**Problem number**: 39.
**ODE order**: 3.
**ODE degree**: 1.

CAS Maple gives this as type `[[_3rd_order, _missing_x]]`

$$\boxed{y''' - 6y'' + 11y' - 6y = 0}$$

With initial conditions

$$[y(0) = 0, y'(0) = 0, y''(0) = 2]$$

✓ Solution by Maple

Time used: 0.016 (sec). Leaf size: 17

```
dsolve([diff(y(x),x$3)-6*diff(y(x),x$2)+11*diff(y(x),x)-6*y(x)=0,y(0) = 0, D(y)(0) = 0, (D@@
```

$$y(x) = e^{x} - 2\,e^{2x} + e^{3x}$$

✓ Solution by Mathematica

Time used: 0.003 (sec). Leaf size: 16

```
DSolve[{y'''[x]-6*y''[x]+11*y'[x]-6*y[x]==0,{y[0]==0,y'[0]==0,y''[0]==2}},y[x],x,IncludeSing
```

$$y(x) \to e^{x}(e^{x} - 1)^{2}$$

## 10.40 problem 40

Internal problem ID [11770]

**Book**: Differential Equations by Shepley L. Ross. Third edition. John Willey. New Delhi. 2004.
**Section**: Chapter 4, Section 4.2. The homogeneous linear equation with constant coefficients. Exercises page 135
**Problem number**: 40.
**ODE order**: 3.
**ODE degree**: 1.

CAS Maple gives this as type `[[_3rd_order, _missing_x]]`

$$\boxed{y''' - 2y'' + 4y' - 8y = 0}$$

With initial conditions

$$[y(0) = 2, y'(0) = 0, y''(0) = 0]$$

✓ Solution by Maple
Time used: 0.015 (sec). Leaf size: 19

```
dsolve([diff(y(x),x$3)-2*diff(y(x),x$2)+4*diff(y(x),x)-8*y(x)=0,y(0) = 2, D(y)(0) = 0, (D@@2
```

$$y(x) = e^{2x} - \sin(2x) + \cos(2x)$$

✓ Solution by Mathematica
Time used: 0.003 (sec). Leaf size: 21

```
DSolve[{y'''[x]-2*y''[x]+4*y'[x]-8*y[x]==0,{y[0]==2,y'[0]==0,y''[0]==0}},y[x],x,IncludeSingu
```

$$y(x) \to e^{2x} - \sin(2x) + \cos(2x)$$

## 10.41 problem 41

Internal problem ID [11771]

**Book**: Differential Equations by Shepley L. Ross. Third edition. John Willey. New Delhi. 2004.
**Section**: Chapter 4, Section 4.2. The homogeneous linear equation with constant coefficients. Exercises page 135
**Problem number**: 41.
**ODE order**: 3.
**ODE degree**: 1.

CAS Maple gives this as type `[[_3rd_order, _missing_x]]`

$$\boxed{y''' - 3y'' + 4y = 0}$$

With initial conditions

$$[y(0) = 1, y'(0) = -8, y''(0) = -4]$$

✓ Solution by Maple

Time used: 0.016 (sec). Leaf size: 22

```
dsolve([diff(y(x),x$3)-3*diff(y(x),x$2)+4*y(x)=0,y(0) = 1, D(y)(0) = -8, (D@@2)(y)(0) = -4],
```

$$y(x) = \frac{(6x - 23)\,\mathrm{e}^{2x}}{9} + \frac{32\,\mathrm{e}^{-x}}{9}$$

✓ Solution by Mathematica

Time used: 0.003 (sec). Leaf size: 27

```
DSolve[{y'''[x]-3*y''[x]+4*y[x]==0,{y[0]==1,y'[0]==-8,y''[0]==-4}},y[x],x,IncludeSingularSol
```

$$y(x) \to \frac{1}{9}e^{-x}\left(e^{3x}(6x - 23) + 32\right)$$

## 10.42 problem 42

Internal problem ID [11772]

**Book**: Differential Equations by Shepley L. Ross. Third edition. John Willey. New Delhi. 2004.
**Section**: Chapter 4, Section 4.2. The homogeneous linear equation with constant coefficients. Exercises page 135
**Problem number**: 42.
**ODE order**: 3.
**ODE degree**: 1.

CAS Maple gives this as type `[[_3rd_order, _missing_x]]`

$$\boxed{y''' - 5y'' + 9y' - 5y = 0}$$

With initial conditions

$$[y(0) = 0, y'(0) = 1, y''(0) = 6]$$

✓ Solution by Maple
Time used: 0.016 (sec). Leaf size: 23

```
dsolve([diff(y(x),x$3)-5*diff(y(x),x$2)+9*diff(y(x),x)-5*y(x)=0,y(0) = 0, D(y)(0) = 1, (D@@2
```

$$y(x) = e^x + (2\sin(x) - \cos(x))\, e^{2x}$$

✓ Solution by Mathematica
Time used: 0.003 (sec). Leaf size: 25

```
DSolve[{y'''[x]-5*y''[x]+9*y'[x]-5*y[x]==0,{y[0]==0,y'[0]==1,y''[0]==6}},y[x],x,IncludeSingu
```

$$y(x) \to e^x(2e^x \sin(x) - e^x \cos(x) + 1)$$

## 10.43 problem 45

Internal problem ID [11773]

**Book**: Differential Equations by Shepley L. Ross. Third edition. John Willey. New Delhi. 2004.
**Section**: Chapter 4, Section 4.2. The homogeneous linear equation with constant coefficients. Exercises page 135
**Problem number**: 45.
**ODE order**: 4.
**ODE degree**: 1.

CAS Maple gives this as type `[[_high_order, _missing_x]]`

$$\boxed{y'''' + 2y''' + 6y'' + 2y' + 5y = 0}$$

✓ Solution by Maple
Time used: 0.0 (sec). Leaf size: 36

```
dsolve(diff(y(x),x$4)+2*diff(y(x),x$3)+6*diff(y(x),x$2)+2*diff(y(x),x)+5*y(x)=0,y(x), singsol
```

$$y(x) = \left(2c_3 \cos(x) \sin(x) + 2c_4 \cos(x)^2 - c_4\right) e^{-x} + c_1 \sin(x) + c_2 \cos(x)$$

✓ Solution by Mathematica
Time used: 0.003 (sec). Leaf size: 40

```
DSolve[y''''[x]+2*y'''[x]+6*y''[x]+2*y'[x]+5*y[x]==0,y[x],x,IncludeSingularSolutions -> True
```

$$y(x) \to c_3 \cos(x) + e^{-x}(c_2 \cos(2x) + c_4 e^x \sin(x) + c_1 \sin(2x))$$

## 10.44 problem 46

Internal problem ID [11774]

**Book**: Differential Equations by Shepley L. Ross. Third edition. John Willey. New Delhi. 2004.
**Section**: Chapter 4, Section 4.2. The homogeneous linear equation with constant coefficients. Exercises page 135
**Problem number**: 46.
**ODE order**: 4.
**ODE degree**: 1.

CAS Maple gives this as type `[[_high_order, _missing_x]]`

$$\boxed{y'''' + 3y''' + y'' + 13y' + 30y = 0}$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 35

```
dsolve(diff(y(x),x$4)+3*diff(y(x),x$3)+diff(y(x),x$2)+13*diff(y(x),x)+30*y(x)=0,y(x), singso
```

$$y(x) = \left(c_3 \mathrm{e}^{4x} \sin(2x) + c_4 \mathrm{e}^{4x} \cos(2x) + c_2 \mathrm{e}^{x} + c_1\right) \mathrm{e}^{-3x}$$

✓ Solution by Mathematica

Time used: 0.003 (sec). Leaf size: 43

```
DSolve[y''''[x]+3*y'''[x]+y''[x]+13*y'[x]+30*y[x]==0,y[x],x,IncludeSingularSolutions -> True
```

$$y(x) \to c_2 e^{x} \cos(2x) + e^{-3x}\left(c_4 e^{x} + c_1 e^{4x} \sin(2x) + c_3\right)$$

# 11 Chapter 4, Section 4.3. The method of undetermined coefficients. Exercises page 151

## 11.1 problem 1

Internal problem ID [11775]

**Book**: Differential Equations by Shepley L. Ross. Third edition. John Willey. New Delhi. 2004.
**Section**: Chapter 4, Section 4.3. The method of undetermined coefficients. Exercises page 151
**Problem number**: 1.
**ODE order**: 2.
**ODE degree**: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$\boxed{y'' - 3y' + 8y = 4x^2}$$

✓ Solution by Maple
Time used: 0.0 (sec). Leaf size: 40

```
dsolve(diff(y(x),x$2)-3*diff(y(x),x)+8*y(x)=4*x^2,y(x), singsol=all)
```

$$y(x) = \mathrm{e}^{\frac{3x}{2}} \sin\left(\frac{\sqrt{23}\, x}{2}\right) c_2 + \mathrm{e}^{\frac{3x}{2}} \cos\left(\frac{\sqrt{23}\, x}{2}\right) c_1 + \frac{x^2}{2} + \frac{3x}{8} + \frac{1}{64}$$

✓ Solution by Mathematica
Time used: 0.026 (sec). Leaf size: 63

```
DSolve[y''[x]-3*y'[x]+8*y[x]==4*x^2,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to \frac{x^2}{2} + \frac{3x}{8} + c_2 e^{3x/2} \cos\left(\frac{\sqrt{23}x}{2}\right) + c_1 e^{3x/2} \sin\left(\frac{\sqrt{23}x}{2}\right) + \frac{1}{64}$$

## 11.2 problem 2

Internal problem ID [11776]

**Book**: Differential Equations by Shepley L. Ross. Third edition. John Willey. New Delhi. 2004.
**Section**: Chapter 4, Section 4.3. The method of undetermined coefficients. Exercises page 151
**Problem number**: 2.
**ODE order**: 2.
**ODE degree**: 1.

CAS Maple gives this as type `[[_2nd_order, _linear, _nonhomogeneous]]`

$$\boxed{y'' - 2y' - 8y = 4\,e^{2x} - 21\,e^{-3x}}$$

✓ Solution by Maple
Time used: 0.016 (sec). Leaf size: 30

```
dsolve(diff(y(x),x$2)-2*diff(y(x),x)-8*y(x)=4*exp(2*x)-21*exp(-3*x),y(x), singsol=all)
```

$$y(x) = \frac{(2c_2e^{7x} - e^{5x} + 2c_1e^{x} - 6)\,e^{-3x}}{2}$$

✓ Solution by Mathematica
Time used: 0.098 (sec). Leaf size: 38

```
DSolve[y''[x]-2*y'[x]-8*y[x]==4*Exp[2*x]-21*Exp[-3*x],y[x],x,IncludeSingularSolutions -> Tru
```

$$y(x) \to -\frac{1}{2}e^{-3x}\left(e^{5x} + 6\right) + c_1e^{-2x} + c_2e^{4x}$$

## 11.3 problem 3

Internal problem ID [11777]

**Book**: Differential Equations by Shepley L. Ross. Third edition. John Willey. New Delhi. 2004.
**Section**: Chapter 4, Section 4.3. The method of undetermined coefficients. Exercises page 151
**Problem number**: 3.
**ODE order**: 2.
**ODE degree**: 1.

CAS Maple gives this as type `[[_2nd_order, _linear, _nonhomogeneous]]`

$$\boxed{y'' + 2y' + 5y = 6\sin(2x) + 7\cos(2x)}$$

✓ Solution by Maple
Time used: 0.0 (sec). Leaf size: 35

```
dsolve(diff(y(x),x$2)+2*diff(y(x),x)+5*y(x)=6*sin(2*x)+7*cos(2*x),y(x), singsol=all)
```

$$y(x) = (\cos(2x)\, c_1 + c_2 \sin(2x))\, e^{-x} - \cos(2x) + 2\sin(2x)$$

✓ Solution by Mathematica
Time used: 0.026 (sec). Leaf size: 38

```
DSolve[y''[x]+2*y'[x]+5*y[x]==6*Sin[2*x]+7*Cos[2*x],y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to e^{-x}((-e^x + c_2)\cos(2x) + (2e^x + c_1)\sin(2x))$$

## 11.4 problem 4

Internal problem ID [11778]

**Book**: Differential Equations by Shepley L. Ross. Third edition. John Willey. New Delhi. 2004.
**Section**: Chapter 4, Section 4.3. The method of undetermined coefficients. Exercises page 151
**Problem number**: 4.
**ODE order**: 2.
**ODE degree**: 1.

CAS Maple gives this as type `[[_2nd_order, _linear, _nonhomogeneous]]`

$$\boxed{y'' + 2y' + 2y = 10\sin(4x)}$$

✓ Solution by Maple
Time used: 0.0 (sec). Leaf size: 33

```
dsolve(diff(y(x),x$2)+2*diff(y(x),x)+2*y(x)=10*sin(4*x),y(x), singsol=all)
```

$$y(x) = e^{-x}\sin(x)\,c_2 + e^{-x}\cos(x)\,c_1 - \frac{7\sin(4x)}{13} - \frac{4\cos(4x)}{13}$$

✓ Solution by Mathematica
Time used: 0.026 (sec). Leaf size: 42

```
DSolve[y''[x]+2*y'[x]+2*y[x]==10*Sin[4*x],y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to -\frac{7}{13}\sin(4x) - \frac{4}{13}\cos(4x) + c_2 e^{-x}\cos(x) + c_1 e^{-x}\sin(x)$$

## 11.5 problem 5

Internal problem ID [11779]

**Book**: Differential Equations by Shepley L. Ross. Third edition. John Willey. New Delhi. 2004.
**Section**: Chapter 4, Section 4.3. The method of undetermined coefficients. Exercises page 151
**Problem number**: 5.
**ODE order**: 2.
**ODE degree**: 1.

CAS Maple gives this as type `[[_2nd_order, _linear, _nonhomogeneous]]`

$$\boxed{y'' + 2y' + 4y = \cos(4x)}$$

✓ Solution by Maple

Time used: 0.016 (sec). Leaf size: 41

```
dsolve(diff(y(x),x$2)+2*diff(y(x),x)+4*y(x)=cos(4*x),y(x), singsol=all)
```

$$y(x) = \mathrm{e}^{-x}\sin\left(\sqrt{3}\,x\right)c_2 + \mathrm{e}^{-x}\cos\left(\sqrt{3}\,x\right)c_1 + \frac{\sin(4x)}{26} - \frac{3\cos(4x)}{52}$$

✓ Solution by Mathematica

Time used: 1.15 (sec). Leaf size: 54

```
DSolve[y''[x]+2*y'[x]+4*y[x]==Cos[4*x],y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to \frac{1}{26}\sin(4x) - \frac{3}{52}\cos(4x) + c_2 e^{-x}\cos\left(\sqrt{3}x\right) + c_1 e^{-x}\sin\left(\sqrt{3}x\right)$$

## 11.6 problem 6

Internal problem ID [11780]

**Book**: Differential Equations by Shepley L. Ross. Third edition. John Willey. New Delhi. 2004.
**Section**: Chapter 4, Section 4.3. The method of undetermined coefficients. Exercises page 151
**Problem number**: 6.
**ODE order**: 2.
**ODE degree**: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$\boxed{y'' - 3y' - 4y = 16x - 12\,\mathrm{e}^{2x}}$$

✓ Solution by Maple
Time used: 0.0 (sec). Leaf size: 27

```
dsolve(diff(y(x),x$2)-3*diff(y(x),x)-4*y(x)=16*x-12*exp(2*x),y(x), singsol=all)
```

$$y(x) = c_2 \mathrm{e}^{4x} + c_1 \mathrm{e}^{-x} + 2\,\mathrm{e}^{2x} - 4x + 3$$

✓ Solution by Mathematica
Time used: 0.136 (sec). Leaf size: 33

```
DSolve[y''[x]-3*y'[x]-4*y[x]==16*x-12*Exp[2*x],y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to -4x + 2e^{2x} + c_1 e^{-x} + c_2 e^{4x} + 3$$

## 11.7 problem 7

Internal problem ID [11781]

**Book**: Differential Equations by Shepley L. Ross. Third edition. John Willey. New Delhi. 2004.
**Section**: Chapter 4, Section 4.3. The method of undetermined coefficients. Exercises page 151
**Problem number**: 7.
**ODE order**: 2.
**ODE degree**: 1.

CAS Maple gives this as type `[[_2nd_order, _linear, _nonhomogeneous]]`

$$\boxed{y'' + 6y' + 5y = 2\,e^{x} + 10\,e^{5x}}$$

✓ Solution by Maple
Time used: 0.016 (sec). Leaf size: 29

```
dsolve(diff(y(x),x$2)+6*diff(y(x),x)+5*y(x)=2*exp(x)+10*exp(5*x),y(x), singsol=all)
```

$$y(x) = \frac{\left(e^{10x} + e^{6x} + 6c_1 e^{4x} + 6c_2\right) e^{-5x}}{6}$$

✓ Solution by Mathematica
Time used: 0.089 (sec). Leaf size: 36

```
DSolve[y''[x]+6*y'[x]+5*y[x]==2*Exp[x]+10*Exp[5*x],y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to \frac{1}{6} e^{x} \left(e^{4x} + 1\right) + c_1 e^{-5x} + c_2 e^{-x}$$

## 11.8 problem 8

Internal problem ID [11782]

**Book**: Differential Equations by Shepley L. Ross. Third edition. John Willey. New Delhi. 2004.
**Section**: Chapter 4, Section 4.3. The method of undetermined coefficients. Exercises page 151
**Problem number**: 8.
**ODE order**: 2.
**ODE degree**: 1.

CAS Maple gives this as type `[[_2nd_order, _linear, _nonhomogeneous]]`

$$\boxed{y'' + 2y' + 10y = 5x\,\mathrm{e}^{-2x}}$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 37

```
dsolve(diff(y(x),x$2)+2*diff(y(x),x)+10*y(x)=5*x*exp(-2*x),y(x), singsol=all)
```

$$y(x) = \frac{(10c_1 \cos(3x) + 10c_2 \sin(3x))\,\mathrm{e}^{-x}}{10} + \frac{(5x+1)\,\mathrm{e}^{-2x}}{10}$$

✓ Solution by Mathematica

Time used: 0.026 (sec). Leaf size: 41

```
DSolve[y''[x]+2*y'[x]+10*y[x]==5*x*Exp[-2*x],y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to \frac{1}{10}e^{-2x}(5x + 10c_2e^x \cos(3x) + 10c_1e^x \sin(3x) + 1)$$

## 11.9 problem 9

Internal problem ID [11783]

**Book**: Differential Equations by Shepley L. Ross. Third edition. John Willey. New Delhi. 2004.
**Section**: Chapter 4, Section 4.3. The method of undetermined coefficients. Exercises page 151
**Problem number**: 9.
**ODE order**: 3.
**ODE degree**: 1.

CAS Maple gives this as type `[[_3rd_order, _with_linear_symmetries]]`

$$\boxed{y''' + 4y'' + y' - 6y = -18x^2 + 1}$$

✓ Solution by Maple
Time used: 0.0 (sec). Leaf size: 34

```
dsolve(diff(y(x),x$3)+4*diff(y(x),x$2)+diff(y(x),x)-6*y(x)=-18*x^2+1,y(x), singsol=all)
```

$$y(x) = \mathrm{e}^{-3x}\left(\left(3x^2 + x + 4\right)\mathrm{e}^{3x} + c_1\mathrm{e}^{4x} + c_3\mathrm{e}^{x} + c_2\right)$$

✓ Solution by Mathematica
Time used: 0.004 (sec). Leaf size: 35

```
DSolve[y'''[x]+4*y''[x]+y'[x]-6*y[x]==-18*x^2+1,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to 3x^2 + x + c_1e^{-3x} + c_2e^{-2x} + c_3e^{x} + 4$$

## 11.10 problem 10

Internal problem ID [11784]

**Book**: Differential Equations by Shepley L. Ross. Third edition. John Willey. New Delhi. 2004.
**Section**: Chapter 4, Section 4.3. The method of undetermined coefficients. Exercises page 151
**Problem number**: 10.
**ODE order**: 3.
**ODE degree**: 1.

CAS Maple gives this as type `[[_3rd_order, _linear, _nonhomogeneous]]`

$$\boxed{y''' + 2y'' - 3y' - 10y = 8x\,\mathrm{e}^{-2x}}$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 32

```
dsolve(diff(y(x),x$3)+2*diff(y(x),x$2)-3*diff(y(x),x)-10*y(x)=8*x*exp(-2*x),y(x), singsol=al
```

$$y(x) = \frac{(2c_2\cos(x) + 2c_3\sin(x) - 4x - 1)\,\mathrm{e}^{-2x}}{2} + \mathrm{e}^{2x}c_1$$

✓ Solution by Mathematica

Time used: 0.007 (sec). Leaf size: 40

```
DSolve[y'''[x]+2*y''[x]-3*y'[x]-10*y[x]==8*x*Exp[-2*x],y[x],x,IncludeSingularSolutions -> Tr
```

$$y(x) \to \frac{1}{2}e^{-2x}\left(-4x + 2c_3e^{4x} + 2c_2\cos(x) + 2c_1\sin(x) - 1\right)$$

## 11.11 problem 11

Internal problem ID [11785]

**Book**: Differential Equations by Shepley L. Ross. Third edition. John Willey. New Delhi. 2004.
**Section**: Chapter 4, Section 4.3. The method of undetermined coefficients. Exercises page 151
**Problem number**: 11.
**ODE order**: 3.
**ODE degree**: 1.

CAS Maple gives this as type `[[_3rd_order, _linear, _nonhomogeneous]]`

$$\boxed{y''' + y'' + 3y' - 5y = 5\sin(2x) + 10x^2 + 3x + 7}$$

✓ Solution by Maple

Time used: 0.016 (sec). Leaf size: 49

```
dsolve(diff(y(x),x$3)+diff(y(x),x$2)+3*diff(y(x),x)-5*y(x)=5*sin(2*x)+10*x^2+3*x+7,y(x), sin
```

$$y(x) = \frac{(17c_3 e^{-x} - 9)\sin(2x)}{17} + c_2 e^{-x}\cos(2x) - 2x^2 + c_1 e^{x} - 3x + \frac{2\cos(2x)}{17} - 4$$

✓ Solution by Mathematica

Time used: 0.419 (sec). Leaf size: 55

```
DSolve[y'''[x]+y''[x]+3*y'[x]-5*y[x]==5*Sin[2*x]+10*x^2+3*x+7,y[x],x,IncludeSingularSolution
```

$$y(x) \to -2x^2 - 3x + c_3 e^{x} + \left(\frac{2}{17} + c_2 e^{-x}\right)\cos(2x) + \left(-\frac{9}{17} + c_1 e^{-x}\right)\sin(2x) - 4$$

## 11.12 problem 12

Internal problem ID [11786]

**Book**: Differential Equations by Shepley L. Ross. Third edition. John Willey. New Delhi. 2004.
**Section**: Chapter 4, Section 4.3. The method of undetermined coefficients. Exercises page 151
**Problem number**: 12.
**ODE order**: 3.
**ODE degree**: 1.

CAS Maple gives this as type `[[_3rd_order, _linear, _nonhomogeneous]]`

$$\boxed{4y''' - 4y'' - 5y' + 3y = 3x^3 - 8x}$$

✓ Solution by Maple
Time used: 0.0 (sec). Leaf size: 39

```
dsolve(4*diff(y(x),x$3)-4*diff(y(x),x$2)-5*diff(y(x),x)+3*y(x)=3*x^3-8*x,y(x), singsol=all)
```

$$y(x) = \left(c_2 \mathrm{e}^{\frac{3x}{2}} + c_3 \mathrm{e}^{\frac{5x}{2}} + \left(x^3 + 5x^2 + 22x + 42\right) \mathrm{e}^{x} + c_1\right) \mathrm{e}^{-x}$$

✓ Solution by Mathematica
Time used: 0.005 (sec). Leaf size: 46

```
DSolve[4*y'''[x]-4*y''[x]-5*y'[x]+3*y[x]==3*x^3-8*x,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to x^3 + 5x^2 + 22x + c_1 e^{x/2} + c_2 e^{3x/2} + c_3 e^{-x} + 42$$

## 11.13 problem 13

Internal problem ID [11787]

**Book**: Differential Equations by Shepley L. Ross. Third edition. John Willey. New Delhi. 2004.
**Section**: Chapter 4, Section 4.3. The method of undetermined coefficients. Exercises page 151
**Problem number**: 13.
**ODE order**: 2.
**ODE degree**: 1.

CAS Maple gives this as type `[[_2nd_order, _linear, _nonhomogeneous]]`

$$\boxed{y'' + y' - 6y = 10\,\mathrm{e}^{2x} - 18\,\mathrm{e}^{3x} - 6x - 11}$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 36

```
dsolve(diff(y(x),x$2)+diff(y(x),x)-6*y(x)=10*exp(2*x)-18*exp(3*x)-6*x-11,y(x), singsol=all)
```

$$y(x) = \mathrm{e}^{-3x}\left(\left(2x + c_1 - \frac{2}{5}\right)\mathrm{e}^{5x} + (x+2)\,\mathrm{e}^{3x} + c_2 - 3\,\mathrm{e}^{6x}\right)$$

✓ Solution by Mathematica

Time used: 0.299 (sec). Leaf size: 38

```
DSolve[y''[x]+y'[x]-6*y[x]==10*Exp[2*x]-18*Exp[3*x]-6*x-11,y[x],x,IncludeSingularSolutions -
```

$$y(x) \to x - 3e^{3x} + c_1 e^{-3x} + e^{2x}\left(2x - \frac{2}{5} + c_2\right) + 2$$

## 11.14 problem 14

Internal problem ID [11788]

**Book**: Differential Equations by Shepley L. Ross. Third edition. John Willey. New Delhi. 2004.
**Section**: Chapter 4, Section 4.3. The method of undetermined coefficients. Exercises page 151
**Problem number**: 14.
**ODE order**: 2.
**ODE degree**: 1.

CAS Maple gives this as type `[[_2nd_order, _linear, _nonhomogeneous]]`

$$\boxed{y'' + y' - 2y = 6\,\mathrm{e}^{-2x} + 3\,\mathrm{e}^{x} - 4x^2}$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 39

```
dsolve(diff(y(x),x$2)+diff(y(x),x)-2*y(x)=6*exp(-2*x)+3*exp(x)-4*x^2,y(x), singsol=all)
```

$$y(x) = \mathrm{e}^{-2x}\left(\left(2x^2 + 2x + 3\right)\mathrm{e}^{2x} + \left(c_2 + x - \frac{1}{3}\right)\mathrm{e}^{3x} - 2x + c_1 - \frac{2}{3}\right)$$

✓ Solution by Mathematica

Time used: 0.163 (sec). Leaf size: 54

```
DSolve[y''[x]+y'[x]-2*y[x]==6*Exp[-2*x]+3*Exp[x]-4*x^2,y[x],x,IncludeSingularSolutions -> Tr
```

$$y(x) \to \frac{1}{3}e^{-2x}\left(e^{2x}\left(6x^2 + 6x + 9\right) - 6x + e^{3x}(3x - 1 + 3c_2) - 2 + 3c_1\right)$$

## 11.15 problem 15

Internal problem ID [11789]

**Book**: Differential Equations by Shepley L. Ross. Third edition. John Willey. New Delhi. 2004.
**Section**: Chapter 4, Section 4.3. The method of undetermined coefficients. Exercises page 151
**Problem number**: 15.
**ODE order**: 3.
**ODE degree**: 1.

CAS Maple gives this as type `[[_3rd_order, _linear, _nonhomogeneous]]`

$$\boxed{y''' - 3y'' + 4y = 4\,\mathrm{e}^{x} - 18\,\mathrm{e}^{-x}}$$

✓ Solution by Maple
Time used: 0.0 (sec). Leaf size: 33

```
dsolve(diff(y(x),x$3)-3*diff(y(x),x$2)+4*y(x)=4*exp(x)-18*exp(-x),y(x), singsol=all)
```

$$y(x) = \frac{(-6x + 3c_1 - 4)\,\mathrm{e}^{-x}}{3} + (c_3 x + c_2)\,\mathrm{e}^{2x} + 2\,\mathrm{e}^{x}$$

✓ Solution by Mathematica
Time used: 0.078 (sec). Leaf size: 44

```
DSolve[y'''[x]-3*y''[x]+4*y[x]==4*Exp[x]-18*Exp[-x],y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to \frac{1}{3}e^{-x}\left(-6x + 6e^{2x} + 3e^{3x}(c_3 x + c_2) - 4 + 3c_1\right)$$

## 11.16 problem 16

Internal problem ID [11790]

**Book**: Differential Equations by Shepley L. Ross. Third edition. John Willey. New Delhi. 2004.
**Section**: Chapter 4, Section 4.3. The method of undetermined coefficients. Exercises page 151
**Problem number**: 16.
**ODE order**: 3.
**ODE degree**: 1.

CAS Maple gives this as type `[[_3rd_order, _linear, _nonhomogeneous]]`

$$\boxed{y''' - 2y'' - y' + 2y = 9\,\mathrm{e}^{2x} - 8\,\mathrm{e}^{3x}}$$

✓ Solution by Maple

Time used: 0.015 (sec). Leaf size: 32

```
dsolve(diff(y(x),x$3)-2*diff(y(x),x$2)-diff(y(x),x)+2*y(x)=9*exp(2*x)-8*exp(3*x),y(x), sings
```

$$y(x) = (3x + c_3 - 4)\,\mathrm{e}^{2x} + c_1\mathrm{e}^{x} + c_2\mathrm{e}^{-x} - \mathrm{e}^{3x}$$

✓ Solution by Mathematica

Time used: 0.033 (sec). Leaf size: 44

```
DSolve[y'''[x]-2*y''[x]-y'[x]+2*y[x]==9*Exp[2*x]-8*Exp[3*x],y[x],x,IncludeSingularSolutions
```

$$y(x) \to -e^{3x} + c_1e^{-x} + \left(\frac{81}{32} + c_2\right)e^{x} + e^{2x}(3x - 4 + c_3)$$

## 11.17 problem 17

Internal problem ID [11791]

**Book**: Differential Equations by Shepley L. Ross. Third edition. John Willey. New Delhi. 2004.
**Section**: Chapter 4, Section 4.3. The method of undetermined coefficients. Exercises page 151
**Problem number**: 17.
**ODE order**: 3.
**ODE degree**: 1.

CAS Maple gives this as type `[[_3rd_order, _missing_y]]`

$$\boxed{y''' + y' = 2x^2 + 4\sin(x)}$$

✓ Solution by Maple
Time used: 0.016 (sec). Leaf size: 30

```
dsolve(diff(y(x),x$3)+diff(y(x),x)=2*x^2+4*sin(x),y(x), singsol=all)
```

$$y(x) = (-2 - c_2)\cos(x) + (-2x + c_1)\sin(x) + \frac{2x^3}{3} - 4x + c_3$$

✓ Solution by Mathematica
Time used: 0.233 (sec). Leaf size: 35

```
DSolve[y'''[x]+y'[x]==2*x^2+4*Sin[x],y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to \frac{2x^3}{3} - 4x - (2 + c_2)\cos(x) + (-2x + c_1)\sin(x) + c_3$$

## 11.18 problem 18

Internal problem ID [11792]

**Book**: Differential Equations by Shepley L. Ross. Third edition. John Willey. New Delhi. 2004.
**Section**: Chapter 4, Section 4.3. The method of undetermined coefficients. Exercises page 151
**Problem number**: 18.
**ODE order**: 4.
**ODE degree**: 1.

CAS Maple gives this as type `[[_high_order, _missing_y]]`

$$\boxed{y'''' - 3y''' + 2y'' = 3\,\mathrm{e}^{-x} + 6\,\mathrm{e}^{2x} - 6x}$$

✓ Solution by Maple

Time used: 0.016 (sec). Leaf size: 41

```
dsolve(diff(y(x),x$4)-3*diff(y(x),x$3)+2*diff(y(x),x$2)=3*exp(-x)+6*exp(2*x)-6*x,y(x), sings
```

$$y(x) = \frac{(6x + c_1 - 12)\,\mathrm{e}^{2x}}{4} - \frac{x^3}{2} - \frac{9x^2}{4} + c_3 x + c_2 \mathrm{e}^{x} + c_4 + \frac{\mathrm{e}^{-x}}{2}$$

✓ Solution by Mathematica

Time used: 0.372 (sec). Leaf size: 54

```
DSolve[y''''[x]-3*y'''[x]+2*y''[x]==3*Exp[-x]+6*Exp[2*x]-6*x,y[x],x,IncludeSingularSolutions
```

$$y(x) \to \frac{1}{4}\left(-\left((2x+9)x^2\right) + 2e^{-x} + 4c_1 e^{x} + e^{2x}(6x - 12 + c_2)\right) + c_4 x + c_3$$

## 11.19 problem 19

Internal problem ID [11793]

**Book**: Differential Equations by Shepley L. Ross. Third edition. John Willey. New Delhi. 2004.
**Section**: Chapter 4, Section 4.3. The method of undetermined coefficients. Exercises page 151
**Problem number**: 19.
**ODE order**: 3.
**ODE degree**: 1.

CAS Maple gives this as type `[[_3rd_order, _linear, _nonhomogeneous]]`

$$\boxed{y''' - 6y'' + 11y' - 6y = x\,\mathrm{e}^{x} - 4\,\mathrm{e}^{2x} + 6\,\mathrm{e}^{4x}}$$

✓ Solution by Maple
Time used: 0.015 (sec). Leaf size: 42

```
dsolve(diff(y(x),x$3)-6*diff(y(x),x$2)+11*diff(y(x),x)-6*y(x)=x*exp(x)-4*exp(2*x)+6*exp(4*x)
```

$$y(x) = (4x + c_2)\,\mathrm{e}^{2x} + c_3\mathrm{e}^{3x} + \mathrm{e}^{4x} + \frac{(2x^2 + 8c_1 + 6x + 7)\,\mathrm{e}^{x}}{8}$$

✓ Solution by Mathematica
Time used: 0.074 (sec). Leaf size: 53

```
DSolve[y'''[x]-6*y''[x]+11*y'[x]-6*y[x]==x*Exp[x]-4*Exp[2*x]+6*Exp[4*x],y[x],x,IncludeSingul
```

$$y(x) \to \frac{1}{8}e^{x}\left(2x^2 + 6x + 8e^{3x} + 8e^{x}(4x + c_2) + 8c_3e^{2x} + 7 + 8c_1\right)$$

## 11.20 problem 20

Internal problem ID [11794]

**Book**: Differential Equations by Shepley L. Ross. Third edition. John Willey. New Delhi. 2004.
**Section**: Chapter 4, Section 4.3. The method of undetermined coefficients. Exercises page 151
**Problem number**: 20.
**ODE order**: 3.
**ODE degree**: 1.

CAS Maple gives this as type `[[_3rd_order, _linear, _nonhomogeneous]]`

$$\boxed{y''' - 4y'' + 5y' - 2y = 3x^2 \mathrm{e}^x - 7\,\mathrm{e}^x}$$

✓ Solution by Maple

Time used: 0.016 (sec). Leaf size: 34

```
dsolve(diff(y(x),x$3)-4*diff(y(x),x$2)+5*diff(y(x),x)-2*y(x)=3*x^2*exp(x)-7*exp(x),y(x), sin
```

$$y(x) = -\frac{\mathrm{e}^x (x^4 + 4x^3 - 4c_2 \mathrm{e}^x - 4c_3 x - 2x^2 - 4c_1)}{4}$$

✓ Solution by Mathematica

Time used: 0.038 (sec). Leaf size: 47

```
DSolve[y'''[x]-4*y''[x]+5*y'[x]-2*y[x]==3*x^2*Exp[x]-7*Exp[x],y[x],x,IncludeSingularSolution
```

$$y(x) \to \frac{1}{4} e^x \left(-x^4 - 4x^3 + 2x^2 + 4(1 + c_2)x + 4(c_3 e^x + 1 + c_1)\right)$$

## 11.21 problem 21

Internal problem ID [11795]

**Book**: Differential Equations by Shepley L. Ross. Third edition. John Willey. New Delhi. 2004.
**Section**: Chapter 4, Section 4.3. The method of undetermined coefficients. Exercises page 151
**Problem number**: 21.
**ODE order**: 2.
**ODE degree**: 1.

CAS Maple gives this as type `[[_2nd_order, _linear, _nonhomogeneous]]`

$$\boxed{y'' + y = x \sin(x)}$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 27

```
dsolve(diff(y(x),x$2)+y(x)=x*sin(x),y(x), singsol=all)
```

$$y(x) = \frac{(-x^2 + 4c_1)\cos(x)}{4} + \frac{\sin(x)(4c_2 + x)}{4}$$

✓ Solution by Mathematica

Time used: 0.046 (sec). Leaf size: 34

```
DSolve[y''[x]+y[x]==x*Sin[x],y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to \frac{1}{8}\left((-2x^2 + 1 + 8c_1)\cos(x) + 2(x + 4c_2)\sin(x)\right)$$

## 11.22 problem 22

Internal problem ID [11796]

**Book**: Differential Equations by Shepley L. Ross. Third edition. John Willey. New Delhi. 2004.
**Section**: Chapter 4, Section 4.3. The method of undetermined coefficients. Exercises page 151
**Problem number**: 22.
**ODE order**: 2.
**ODE degree**: 1.

CAS Maple gives this as type `[[_2nd_order, _linear, _nonhomogeneous]]`

$$\boxed{y'' + 4y = 12x^2 - 16x\cos(2x)}$$

✓ Solution by Maple
Time used: 0.0 (sec). Leaf size: 37

```
dsolve(diff(y(x),x$2)+4*y(x)=12*x^2-16*x*cos(2*x),y(x), singsol=all)
```

$$y(x) = -\frac{3}{2} + \frac{(-8x^2 + 4c_2 + 1)\sin(2x)}{4} + (c_1 - x)\cos(2x) + 3x^2$$

✓ Solution by Mathematica
Time used: 0.251 (sec). Leaf size: 44

```
DSolve[y''[x]+4*y[x]==12*x^2-16*x*Cos[2*x],y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to 3x^2 + \frac{1}{4}(-8x^2 + 1 + 4c_2)\sin(2x) + (-x + c_1)\cos(2x) - \frac{3}{2}$$

## 11.23 problem 23

Internal problem ID [11797]

**Book**: Differential Equations by Shepley L. Ross. Third edition. John Willey. New Delhi. 2004.
**Section**: Chapter 4, Section 4.3. The method of undetermined coefficients. Exercises page 151
**Problem number**: 23.
**ODE order**: 4.
**ODE degree**: 1.

CAS Maple gives this as type `[[_high_order, _missing_y]]`

$$\boxed{y'''' + 2y''' - 3y'' = 18x^2 + 16x\,\mathrm{e}^x + 4\,\mathrm{e}^{3x} - 9}$$

✓ Solution by Maple
Time used: 0.0 (sec). Leaf size: 64

```
dsolve(diff(y(x),x$4)+2*diff(y(x),x$3)-3*diff(y(x),x$2)=18*x^2+16*x*exp(x)+4*exp(3*x)-9,y(x)
```

$$y(x) = -\frac{\left(\left(x^4 + \frac{8}{3}x^3 + \frac{19}{3}x^2 - 2c_3x - 2c_4\right)\mathrm{e}^{3x} + \left(-4x^2 + 18x - 2c_2 - \frac{57}{2}\right)\mathrm{e}^{4x} - \frac{2c_1}{9} - \frac{2\,\mathrm{e}^{6x}}{27}\right)\mathrm{e}^{-3x}}{2}$$

✓ Solution by Mathematica
Time used: 1.232 (sec). Leaf size: 70

```
DSolve[y''''[x]+2*y'''[x]-3*y''[x]==18*x^2+16*x*Exp[x]+4*Exp[3*x]-9,y[x],x,IncludeSingularSo
```

$$y(x) \to -\frac{1}{6}\left(3x^2 + 8x + 19\right)x^2 + \frac{1}{4}e^x\left(8x^2 - 36x + 57 + 4c_2\right) + \frac{e^{3x}}{27} + c_4x + \frac{1}{9}c_1e^{-3x} + c_3$$

## 11.24 problem 24

Internal problem ID [11798]

**Book**: Differential Equations by Shepley L. Ross. Third edition. John Willey. New Delhi. 2004.
**Section**: Chapter 4, Section 4.3. The method of undetermined coefficients. Exercises page 151
**Problem number**: 24.
**ODE order**: 4.
**ODE degree**: 1.

CAS Maple gives this as type `[[_high_order, _linear, _nonhomogeneous]]`

$$\boxed{y'''' - 5y''' + 7y'' - 5y' + 6y = 5\sin(x) - 12\sin(2x)}$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 51

```
dsolve(diff(y(x),x$4)-5*diff(y(x),x$3)+7*diff(y(x),x$2)-5*diff(y(x),x)+6*y(x)=5*sin(x)-12*si
```

$$y(x) = \frac{5\cos(2x)}{13} + c_3 e^{2x} + c_4 e^{3x} + \frac{\sin(2x)}{13} + \frac{(-2 - 5x + 20c_1)\cos(x)}{20} + \frac{(1 + x + 4c_2)\sin(x)}{4}$$

✓ Solution by Mathematica

Time used: 0.232 (sec). Leaf size: 71

```
DSolve[y''''[x]-5*y'''[x]+7*y''[x]-5*y'[x]+6*y[x]==5*Sin[x]-12*Sin[2*x],y[x],x,IncludeSingul
```

$$y(x) \to -\frac{5\sin^2(x)}{13} + \frac{5\cos^2(x)}{13} + e^{2x}(c_4 e^x + c_3) + \left(\frac{x}{4} + \frac{3}{8} + c_2\right)\sin(x) + \cos(x)\left(-\frac{x}{4} + \frac{2\sin(x)}{13} - \frac{1}{10} + c_1\right)$$

## 11.25 problem 25

Internal problem ID [11799]

**Book**: Differential Equations by Shepley L. Ross. Third edition. John Willey. New Delhi. 2004.
**Section**: Chapter 4, Section 4.3. The method of undetermined coefficients. Exercises page 151
**Problem number**: 25.
**ODE order**: 2.
**ODE degree**: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$\boxed{y'' - 4y' + 3y = 9x^2 + 4}$$

With initial conditions

$$[y(0) = 6, y'(0) = 8]$$

✓ Solution by Maple
Time used: 0.016 (sec). Leaf size: 24

```
dsolve([diff(y(x),x$2)-4*diff(y(x),x)+3*y(x)=9*x^2+4,y(0) = 6, D(y)(0) = 8],y(x), singsol=al
```

$$y(x) = -6\,\mathrm{e}^{x} + 2\,\mathrm{e}^{3x} + 3x^2 + 8x + 10$$

✓ Solution by Mathematica
Time used: 0.015 (sec). Leaf size: 27

```
DSolve[{y''[x]-4*y'[x]+3*y[x]==9*x^2+4,{y[0]==6,y'[0]==8}},y[x],x,IncludeSingularSolutions -
```

$$y(x) \to 3x^2 + 8x - 6e^x + 2e^{3x} + 10$$

## 11.26 problem 26

Internal problem ID [11800]

**Book**: Differential Equations by Shepley L. Ross. Third edition. John Willey. New Delhi. 2004.
**Section**: Chapter 4, Section 4.3. The method of undetermined coefficients. Exercises page 151
**Problem number**: 26.
**ODE order**: 2.
**ODE degree**: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$\boxed{y'' + 5y' + 4y = 16x + 20\,\mathrm{e}^{x}}$$

With initial conditions

$$[y(0) = 0, y'(0) = 3]$$

✓ Solution by Maple
Time used: 0.016 (sec). Leaf size: 19

```
dsolve([diff(y(x),x$2)+5*diff(y(x),x)+4*y(x)=16*x+20*exp(x),y(0) = 0, D(y)(0) = 3],y(x), sin
```

$$y(x) = 3\,\mathrm{e}^{-x} - 5 + 2\,\mathrm{e}^{x} + 4x$$

✓ Solution by Mathematica
Time used: 0.125 (sec). Leaf size: 22

```
DSolve[{y''[x]+5*y'[x]+4*y[x]==16*x+20*Exp[x],{y[0]==0,y'[0]==3}},y[x],x,IncludeSingularSolu
```

$$y(x) \to 4x + 3e^{-x} + 2e^{x} - 5$$

## 11.27 problem 27

Internal problem ID [11801]

**Book**: Differential Equations by Shepley L. Ross. Third edition. John Willey. New Delhi. 2004.
**Section**: Chapter 4, Section 4.3. The method of undetermined coefficients. Exercises page 151
**Problem number**: 27.
**ODE order**: 2.
**ODE degree**: 1.

CAS Maple gives this as type `[[_2nd_order, _linear, _nonhomogeneous]]`

$$\boxed{y'' - 8y' + 15y = 9\,\mathrm{e}^{2x}x}$$

With initial conditions

$$[y(0) = 5, y'(0) = 10]$$

✓ Solution by Maple
Time used: 0.015 (sec). Leaf size: 27

```
dsolve([diff(y(x),x$2)-8*diff(y(x),x)+15*y(x)=9*x*exp(2*x),y(0) = 5, D(y)(0) = 10],y(x), sin
```

$$y(x) = -2\,\mathrm{e}^{5x} + 3\,\mathrm{e}^{3x} + (3x+4)\,\mathrm{e}^{2x}$$

✓ Solution by Mathematica
Time used: 0.019 (sec). Leaf size: 28

```
DSolve[{y''[x]-8*y'[x]+15*y[x]==9*x*Exp[2*x],{y[0]==5,y'[0]==10}},y[x],x,IncludeSingularSolu
```

$$y(x) \to e^{2x}\left(3x + 3e^{x} - 2e^{3x} + 4\right)$$

## 11.28 problem 28

Internal problem ID [11802]

**Book**: Differential Equations by Shepley L. Ross. Third edition. John Willey. New Delhi. 2004.
**Section**: Chapter 4, Section 4.3. The method of undetermined coefficients. Exercises page 151
**Problem number**: 28.
**ODE order**: 2.
**ODE degree**: 1.

CAS Maple gives this as type `[[_2nd_order, _linear, _nonhomogeneous]]`

$$\boxed{y'' + 7y' + 10y = 4x\,\mathrm{e}^{-3x}}$$

With initial conditions

$$[y(0) = 0, y'(0) = -1]$$

✓ Solution by Maple
Time used: 0.016 (sec). Leaf size: 19

```
dsolve([diff(y(x),x$2)+7*diff(y(x),x)+10*y(x)=4*x*exp(-3*x),y(0) = 0, D(y)(0) = -1],y(x), si
```

$$y(x) = \mathrm{e}^{-2x} + (-2x - 1)\,\mathrm{e}^{-3x}$$

✓ Solution by Mathematica
Time used: 0.022 (sec). Leaf size: 19

```
DSolve[{y''[x]+7*y'[x]+10*y[x]==4*x*Exp[-3*x],{y[0]==0,y'[0]==-1}},y[x],x,IncludeSingularSol
```

$$y(x) \to e^{-3x}(-2x + e^x - 1)$$

## 11.29 problem 29

Internal problem ID [11803]

**Book**: Differential Equations by Shepley L. Ross. Third edition. John Willey. New Delhi. 2004.
**Section**: Chapter 4, Section 4.3. The method of undetermined coefficients. Exercises page 151
**Problem number**: 29.
**ODE order**: 2.
**ODE degree**: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$\boxed{y'' + 8y' + 16y = 8\,e^{-2x}}$$

With initial conditions

$$[y(0) = 2, y'(0) = 0]$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 18

```
dsolve([diff(y(x),x$2)+8*diff(y(x),x)+16*y(x)=8*exp(-2*x),y(0) = 2, D(y)(0) = 0],y(x), sings
```

$$y(x) = 4\,e^{-4x}x + 2\,e^{-2x}$$

✓ Solution by Mathematica

Time used: 0.02 (sec). Leaf size: 21

```
DSolve[{y''[x]+8*y'[x]+16*y[x]==8*Exp[-2*x],{y[0]==2,y'[0]==0}},y[x],x,IncludeSingularSoluti
```

$$y(x) \to 2e^{-4x}\left(2x + e^{2x}\right)$$

## 11.30 problem 30

Internal problem ID [11804]

**Book**: Differential Equations by Shepley L. Ross. Third edition. John Willey. New Delhi. 2004.
**Section**: Chapter 4, Section 4.3. The method of undetermined coefficients. Exercises page 151
**Problem number**: 30.
**ODE order**: 2.
**ODE degree**: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$\boxed{y'' + 6y' + 9y = 27\,\mathrm{e}^{-6x}}$$

With initial conditions

$$[y(0) = -2, y'(0) = 0]$$

✓ Solution by Maple

Time used: 0.016 (sec). Leaf size: 21

```
dsolve([diff(y(x),x$2)+6*diff(y(x),x)+9*y(x)=27*exp(-6*x),y(0) = -2, D(y)(0) = 0],y(x), sing
```

$$y(x) = (3x - 5)\,\mathrm{e}^{-3x} + 3\,\mathrm{e}^{-6x}$$

✓ Solution by Mathematica

Time used: 0.022 (sec). Leaf size: 24

```
DSolve[{y''[x]+6*y'[x]+9*y[x]==27*Exp[-6*x],{y[0]==-2,y'[0]==0}},y[x],x,IncludeSingularSolut
```

$$y(x) \to e^{-6x}\left(e^{3x}(3x - 5) + 3\right)$$

## 11.31 problem 31

Internal problem ID [11805]

**Book**: Differential Equations by Shepley L. Ross. Third edition. John Willey. New Delhi. 2004.
**Section**: Chapter 4, Section 4.3. The method of undetermined coefficients. Exercises page 151
**Problem number**: 31.
**ODE order**: 2.
**ODE degree**: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$\boxed{y'' + 4y' + 13y = 18\,\mathrm{e}^{-2x}}$$

With initial conditions

$$[y(0) = 0, y'(0) = 4]$$

✓ Solution by Maple

Time used: 0.015 (sec). Leaf size: 24

```
dsolve([diff(y(x),x$2)+4*diff(y(x),x)+13*y(x)=18*exp(-2*x),y(0) = 0, D(y)(0) = 4],y(x), sing
```

$$y(x) = \frac{2\,\mathrm{e}^{-2x}(2\sin(3x) - 3\cos(3x) + 3)}{3}$$

✓ Solution by Mathematica

Time used: 0.024 (sec). Leaf size: 28

```
DSolve[{y''[x]+4*y'[x]+13*y[x]==18*Exp[-2*x],{y[0]==0,y'[0]==4}},y[x],x,IncludeSingularSolut
```

$$y(x) \to \frac{1}{3}e^{-2x}(4\sin(3x) - 6\cos(3x) + 6)$$

## 11.32 problem 32

Internal problem ID [11806]

**Book**: Differential Equations by Shepley L. Ross. Third edition. John Willey. New Delhi. 2004.
**Section**: Chapter 4, Section 4.3. The method of undetermined coefficients. Exercises page 151
**Problem number**: 32.
**ODE order**: 2.
**ODE degree**: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$\boxed{y'' - 10y' + 29y = 8\,\mathrm{e}^{5x}}$$

With initial conditions

$$[y(0) = 0, y'(0) = 8]$$

✓ Solution by Maple

Time used: 0.016 (sec). Leaf size: 22

```
dsolve([diff(y(x),x$2)-10*diff(y(x),x)+29*y(x)=8*exp(5*x),y(0) = 0, D(y)(0) = 8],y(x), sings
```

$$y(x) = -2\,\mathrm{e}^{5x}(-1 - 2\sin(2x) + \cos(2x))$$

✓ Solution by Mathematica

Time used: 0.021 (sec). Leaf size: 24

```
DSolve[{y''[x]-10*y'[x]+29*y[x]==8*Exp[5*x],{y[0]==0,y'[0]==8}},y[x],x,IncludeSingularSoluti
```

$$y(x) \to -2e^{5x}(-2\sin(2x) + \cos(2x) - 1)$$

## 11.33 problem 33

Internal problem ID [11807]

**Book**: Differential Equations by Shepley L. Ross. Third edition. John Willey. New Delhi. 2004.
**Section**: Chapter 4, Section 4.3. The method of undetermined coefficients. Exercises page 151
**Problem number**: 33.
**ODE order**: 2.
**ODE degree**: 1.

CAS Maple gives this as type `[[_2nd_order, _linear, _nonhomogeneous]]`

$$\boxed{y'' - 4y' + 13y = 8\sin(3x)}$$

With initial conditions

$$[y(0) = 1, y'(0) = 2]$$

✓ Solution by Maple
Time used: 0.032 (sec). Leaf size: 31

```
dsolve([diff(y(x),x$2)-4*diff(y(x),x)+13*y(x)=8*sin(3*x),y(0) = 1, D(y)(0) = 2],y(x), singso
```

$$y(x) = \frac{(2\,e^{2x} + 3)\cos(3x)}{5} + \frac{\sin(3x)\,(e^{2x} + 1)}{5}$$

✓ Solution by Mathematica
Time used: 0.026 (sec). Leaf size: 36

```
DSolve[{y''[x]-4*y'[x]+13*y[x]==8*Sin[3*x],{y[0]==1,y'[0]==2}},y[x],x,IncludeSingularSolutio
```

$$y(x) \to \frac{1}{5}\left(\left(e^{2x} + 1\right)\sin(3x) + \left(2e^{2x} + 3\right)\cos(3x)\right)$$

## 11.34 problem 34

Internal problem ID [11808]

**Book**: Differential Equations by Shepley L. Ross. Third edition. John Willey. New Delhi. 2004.
**Section**: Chapter 4, Section 4.3. The method of undetermined coefficients. Exercises page 151
**Problem number**: 34.
**ODE order**: 2.
**ODE degree**: 1.

CAS Maple gives this as type `[[_2nd_order, _linear, _nonhomogeneous]]`

$$\boxed{y'' - y' - 6y = 8\,e^{2x} - 5\,e^{3x}}$$

With initial conditions

$$[y(0) = 3, y'(0) = 5]$$

✓ Solution by Maple

Time used: 0.015 (sec). Leaf size: 28

```
dsolve([diff(y(x),x$2)-diff(y(x),x)-6*y(x)=8*exp(2*x)-5*exp(3*x),y(0) = 3, D(y)(0) = 5],y(x)
```

$$y(x) = -\left((-4 + x)\,e^{5x} + 2\,e^{4x} - 1\right) e^{-2x}$$

✓ Solution by Mathematica

Time used: 0.053 (sec). Leaf size: 28

```
DSolve[{y''[x]-y'[x]-6*y[x]==8*Exp[2*x]-5*Exp[3*x],{y[0]==3,y'[0]==5}},y[x],x,IncludeSingula
```

$$y(x) \to -e^{3x}(x - 4) + e^{-2x} - 2e^{2x}$$

## 11.35 problem 35

Internal problem ID [11809]

**Book**: Differential Equations by Shepley L. Ross. Third edition. John Willey. New Delhi. 2004.
**Section**: Chapter 4, Section 4.3. The method of undetermined coefficients. Exercises page 151
**Problem number**: 35.
**ODE order**: 2.
**ODE degree**: 1.

CAS Maple gives this as type `[[_2nd_order, _linear, _nonhomogeneous]]`

$$\boxed{y'' - 2y' + y = 2\,e^{2x}x + 6\,e^{x}}$$

With initial conditions

$$[y(0) = 1, y'(0) = 0]$$

✓ Solution by Maple

Time used: 0.016 (sec). Leaf size: 30

```
dsolve([diff(y(x),x$2)-2*diff(y(x),x)+y(x)=2*x*exp(2*x)+6*exp(x),y(0) = 1, D(y)(0) = 0],y(x)
```

$$y(x) = (2x - 4)\,e^{2x} + e^{x}\left(3x^2 + x + 5\right)$$

✓ Solution by Mathematica

Time used: 0.104 (sec). Leaf size: 25

```
DSolve[{y''[x]-2*y'[x]+y[x]==2*x*Exp[2*x]+6*Exp[x],{y[0]==1,y'[0]==0}},y[x],x,IncludeSingula
```

$$y(x) \to e^{x}\left(3x^2 + x + 2e^{x}(x - 2) + 5\right)$$

## 11.36 problem 36

Internal problem ID [11810]

**Book**: Differential Equations by Shepley L. Ross. Third edition. John Willey. New Delhi. 2004.
**Section**: Chapter 4, Section 4.3. The method of undetermined coefficients. Exercises page 151
**Problem number**: 36.
**ODE order**: 2.
**ODE degree**: 1.

CAS Maple gives this as type `[[_2nd_order, _linear, _nonhomogeneous]]`

$$\boxed{y'' - y = 3x^2 \mathrm{e}^x}$$

With initial conditions

$$[y(0) = 1, y'(0) = 2]$$

✓ Solution by Maple
Time used: 0.016 (sec). Leaf size: 30

```
dsolve([diff(y(x),x$2)-y(x)=3*x^2*exp(x),y(0) = 1, D(y)(0) = 2],y(x), singsol=all)
```

$$y(x) = -\frac{\mathrm{e}^{-x}}{8} + \frac{(4x^3 - 6x^2 + 6x + 9)\,\mathrm{e}^x}{8}$$

✓ Solution by Mathematica
Time used: 0.03 (sec). Leaf size: 37

```
DSolve[{y''[x]-y[x]==3*x^2*Exp[x],{y[0]==1,y'[0]==2}},y[x],x,IncludeSingularSolutions -> Tru
```

$$y(x) \to \frac{1}{8}e^{-x}\left(e^{2x}\left(4x^3 - 6x^2 + 6x + 9\right) - 1\right)$$

## 11.37 problem 37

Internal problem ID [11811]

**Book**: Differential Equations by Shepley L. Ross. Third edition. John Willey. New Delhi. 2004.
**Section**: Chapter 4, Section 4.3. The method of undetermined coefficients. Exercises page 151
**Problem number**: 37.
**ODE order**: 2.
**ODE degree**: 1.

CAS Maple gives this as type `[[_2nd_order, _linear, _nonhomogeneous]]`

$$\boxed{y'' + y = 3x^2 - 4\sin(x)}$$

With initial conditions

$$[y(0) = 0, y'(0) = 1]$$

✓ Solution by Maple
Time used: 0.016 (sec). Leaf size: 24

```
dsolve([diff(y(x),x$2)+y(x)=3*x^2-4*sin(x),y(0) = 0, D(y)(0) = 1],y(x), singsol=all)
```

$$y(x) = (2x + 6)\cos(x) + 3x^2 - \sin(x) - 6$$

✓ Solution by Mathematica
Time used: 0.158 (sec). Leaf size: 23

```
DSolve[{y''[x]+y[x]==3*x^2-4*Sin[x],{y[0]==0,y'[0]==1}},y[x],x,IncludeSingularSolutions -> T
```

$$y(x) \to 3x^2 - \sin(x) + 2(x+3)\cos(x) - 6$$

## 11.38 problem 38

Internal problem ID [11812]

**Book**: Differential Equations by Shepley L. Ross. Third edition. John Willey. New Delhi. 2004.
**Section**: Chapter 4, Section 4.3. The method of undetermined coefficients. Exercises page 151
**Problem number**: 38.
**ODE order**: 2.
**ODE degree**: 1.

CAS Maple gives this as type `[[_2nd_order, _linear, _nonhomogeneous]]`

$$\boxed{y'' + 4y = 8\sin(2x)}$$

With initial conditions

$$[y(0) = 6, y'(0) = 8]$$

✓ Solution by Maple
Time used: 0.016 (sec). Leaf size: 21

```
dsolve([diff(y(x),x$2)+4*y(x)=8*sin(2*x),y(0) = 6, D(y)(0) = 8],y(x), singsol=all)
```

$$y(x) = (-2x + 6)\cos(2x) + 5\sin(2x)$$

✓ Solution by Mathematica
Time used: 0.108 (sec). Leaf size: 19

```
DSolve[{y''[x]+4*y[x]==8*Sin[2*x],{y[0]==0,y'[0]==1}},y[x],x,IncludeSingularSolutions -> Tru
```

$$y(x) \to 3\sin(x)\cos(x) - 2x\cos(2x)$$

## 11.39 problem 39

Internal problem ID [11813]

**Book**: Differential Equations by Shepley L. Ross. Third edition. John Willey. New Delhi. 2004.
**Section**: Chapter 4, Section 4.3. The method of undetermined coefficients. Exercises page 151
**Problem number**: 39.
**ODE order**: 3.
**ODE degree**: 1.

CAS Maple gives this as type `[[_3rd_order, _linear, _nonhomogeneous]]`

$$\boxed{y''' - 4y'' + y' + 6y = 3x\,\mathrm{e}^{x} + 2\,\mathrm{e}^{x} - \sin(x)}$$

With initial conditions

$$\left[y(0) = \frac{33}{40}, y'(0) = 0, y''(0) = 0\right]$$

✓ Solution by Maple
Time used: 0.031 (sec). Leaf size: 30

```
dsolve([diff(y(x),x$3)-4*diff(y(x),x$2)+diff(y(x),x)+6*y(x)=3*x*exp(x)+2*exp(x)-sin(x),y(0)
```

$$y(x) = \frac{7\,\mathrm{e}^{-x}}{20} - \frac{31\,\mathrm{e}^{2x}}{40} + \frac{(3x+5)\,\mathrm{e}^{x}}{4} - \frac{\sin(x)}{10}$$

✓ Solution by Mathematica
Time used: 0.285 (sec). Leaf size: 38

```
DSolve[{y'''[x]-4*y''[x]+y'[x]+6*y[x]==3*x*Exp[x]+2*Exp[x]-Sin[x],{y[0]==33/40,y'[0]==0,y''[
```

$$y(x) \to \frac{1}{40}\left(10e^{x}(3x+5) + 14e^{-x} - 31e^{2x} - 4\sin(x)\right)$$

## 11.40 problem 40

Internal problem ID [11814]

**Book**: Differential Equations by Shepley L. Ross. Third edition. John Willey. New Delhi. 2004.
**Section**: Chapter 4, Section 4.3. The method of undetermined coefficients. Exercises page 151
**Problem number**: 40.
**ODE order**: 3.
**ODE degree**: 1.

CAS Maple gives this as type `[[_3rd_order, _linear, _nonhomogeneous]]`

$$\boxed{y''' - 6y'' + 9y' - 4y = 8x^2 + 3 - 6\,\mathrm{e}^{2x}}$$

With initial conditions

$$[y(0) = 1, y'(0) = 7, y''(0) = 0]$$

✓ Solution by Maple
Time used: 0.015 (sec). Leaf size: 35

```
dsolve([diff(y(x),x$3)-6*diff(y(x),x$2)+9*diff(y(x),x)-4*y(x)=8*x^2+3-6*exp(2*x),y(0) = 1, D
```

$$y(x) = -2x^2 - 9x + 3\,\mathrm{e}^{2x} - 15 + \frac{44\,\mathrm{e}^{x}}{3} - \frac{5\,\mathrm{e}^{4x}}{3} + 2\,\mathrm{e}^{x} x$$

✓ Solution by Mathematica
Time used: 0.21 (sec). Leaf size: 42

```
DSolve[{y'''[x]-6*y''[x]+9*y'[x]-4*y[x]==8*x^2+3-6*Exp[2*x],{y[0]==1,y'[0]==7,y''[0]==0}},y[
```

$$y(x) \to -2x^2 - 9x + 3e^{2x} - \frac{5e^{4x}}{3} + e^{x}\left(2x + \frac{44}{3}\right) - 15$$

## 11.41 problem 41

Internal problem ID [11815]

**Book**: Differential Equations by Shepley L. Ross. Third edition. John Willey. New Delhi. 2004.
**Section**: Chapter 4, Section 4.3. The method of undetermined coefficients. Exercises page 151
**Problem number**: 41.
**ODE order**: 2.
**ODE degree**: 1.

CAS Maple gives this as type `[[_2nd_order, _linear, _nonhomogeneous]]`

$$\boxed{y'' - 6y' + 8y = x^3 + x + e^{-2x}}$$

✓ Solution by Maple
Time used: 0.0 (sec). Leaf size: 38

```
dsolve(diff(y(x),x$2)-6*diff(y(x),x)+8*y(x)=x^3+x+exp(-2*x),y(x), singsol=all)
```

$$y(x) = \frac{c_1 e^{4x}}{2} + \frac{e^{-2x}}{24} + \frac{69}{256} + \frac{29x}{64} + \frac{9x^2}{32} + \frac{x^3}{8} + c_2 e^{2x}$$

✓ Solution by Mathematica
Time used: 0.699 (sec). Leaf size: 50

```
DSolve[y''[x]-6*y'[x]+8*y[x]==x^3+x+Exp[-2*x],y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to \frac{1}{256}(32x^3 + 72x^2 + 116x + 69) + \frac{e^{-2x}}{24} + c_1 e^{2x} + c_2 e^{4x}$$

## 11.42 problem 42

Internal problem ID [11816]

**Book**: Differential Equations by Shepley L. Ross. Third edition. John Willey. New Delhi. 2004.
**Section**: Chapter 4, Section 4.3. The method of undetermined coefficients. Exercises page 151
**Problem number**: 42.
**ODE order**: 2.
**ODE degree**: 1.

CAS Maple gives this as type `[[_2nd_order, _linear, _nonhomogeneous]]`

$$\boxed{y'' + 9y = e^{3x} + e^{-3x} + e^{3x} \sin(3x)}$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 43

```
dsolve(diff(y(x),x$2)+9*y(x)=exp(3*x)+exp(-3*x)+exp(3*x)*sin(3*x),y(x), singsol=all)
```

$$y(x) = c_2 \sin(3x) + c_1 \cos(3x) + \frac{(2\sin(3x) - 4\cos(3x) + 5)\,e^{3x}}{90} + \frac{e^{-3x}}{18}$$

✓ Solution by Mathematica

Time used: 0.997 (sec). Leaf size: 57

```
DSolve[y''[x]+9*y[x]==Exp[3*x]+Exp[-3*x]+Exp[3*x]*Sin[3*x],y[x],x,IncludeSingularSolutions -
```

$$y(x) \to \frac{1}{90}\left(5e^{-3x}\left(e^{6x}+1\right) + \left(-4e^{3x} + 90c_1\right)\cos(3x) + 2\left(e^{3x} + 45c_2\right)\sin(3x)\right)$$

## 11.43 problem 43

Internal problem ID [11817]

**Book**: Differential Equations by Shepley L. Ross. Third edition. John Willey. New Delhi. 2004.
**Section**: Chapter 4, Section 4.3. The method of undetermined coefficients. Exercises page 151
**Problem number**: 43.
**ODE order**: 2.
**ODE degree**: 1.

CAS Maple gives this as type `[[_2nd_order, _linear, _nonhomogeneous]]`

$$\boxed{y'' + 4y' + 5y = \mathrm{e}^{-2x}(\cos(x) + 1)}$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 28

```
dsolve(diff(y(x),x$2)+4*diff(y(x),x)+5*y(x)=exp(-2*x)*(1+cos(x)),y(x), singsol=all)
```

$$y(x) = \frac{((2c_1 + 1)\cos(x) + 2 + (2c_2 + x)\sin(x))\,\mathrm{e}^{-2x}}{2}$$

✓ Solution by Mathematica

Time used: 0.122 (sec). Leaf size: 35

```
DSolve[y''[x]+4*y'[x]+5*y[x]==Exp[-2*x]*(1+Cos[x]),y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to \frac{1}{4}e^{-2x}((1 + 4c_2)\cos(x) + 2(x + 2c_1)\sin(x) + 4)$$

## 11.44 problem 44

Internal problem ID [11818]

**Book**: Differential Equations by Shepley L. Ross. Third edition. John Willey. New Delhi. 2004.
**Section**: Chapter 4, Section 4.3. The method of undetermined coefficients. Exercises page 151
**Problem number**: 44.
**ODE order**: 2.
**ODE degree**: 1.

CAS Maple gives this as type `[[_2nd_order, _linear, _nonhomogeneous]]`

$$\boxed{y'' - 6y' + 9y = e^{x} x^{4} + x^{3} e^{2x} + e^{3x} x^{2}}$$

✓ Solution by Maple
Time used: 0.0 (sec). Leaf size: 62

```
dsolve(diff(y(x),x$2)-6*diff(y(x),x)+9*y(x)=x^4*exp(x)+x^3*exp(2*x)+x^2*exp(3*x),y(x), sings
```

$$y(x) = \left(x^{3}+6x^{2}+18x+24\right) e^{2x} + \frac{\left(x^{4}+12c_{1}x+12c_{2}\right) e^{3x}}{12} + \frac{\left(x^{4}+4x^{3}+9x^{2}+12x+\frac{15}{2}\right) e^{x}}{4}$$

✓ Solution by Mathematica
Time used: 1.391 (sec). Leaf size: 70

```
DSolve[y''[x]-6*y'[x]+9*y[x]==x^4*Exp[x]+x^3*Exp[2*x]+x^2*Exp[3*x],y[x],x,IncludeSingularSol
```

$$y(x) \to e^{x}\left(\frac{x^{4}}{4} + e^{2x}\left(\frac{x^{4}}{12} + c_{2}x + c_{1}\right) + x^{3} + \frac{9x^{2}}{4} + e^{x}\left(x^{3}+6x^{2}+18x+24\right) + 3x + \frac{15}{8}\right)$$

## 11.45 problem 45

Internal problem ID [11819]

**Book**: Differential Equations by Shepley L. Ross. Third edition. John Willey. New Delhi. 2004.
**Section**: Chapter 4, Section 4.3. The method of undetermined coefficients. Exercises page 151
**Problem number**: 45.
**ODE order**: 2.
**ODE degree**: 1.

CAS Maple gives this as type `[[_2nd_order, _linear, _nonhomogeneous]]`

$$\boxed{y''+6y'+13y = x\,\mathrm{e}^{-3x}\sin(2x)+x^2\mathrm{e}^{-2x}\sin(3x)}$$

✓ Solution by Maple
Time used: 0.015 (sec). Leaf size: 68

```
dsolve(diff(y(x),x$2)+6*diff(y(x),x)+13*y(x)=x*exp(-3*x)*sin(2*x)+x^2*exp(-2*x)*sin(3*x),y(x
```

$$y(x) = -\frac{3\left(\left(\frac{13x^2}{12}-\frac{26c_1}{3}-\frac{39}{16}\right)\cos(2x)+\mathrm{e}^{x}\left(x^2-\frac{2}{13}x-\frac{180}{169}\right)\cos(3x)+\frac{2\,\mathrm{e}^{x}\left(x^2-\frac{41}{13}x+\frac{563}{338}\right)\sin(3x)}{3}-\frac{13\sin(2x)(x+16c_2)}{24}\right.}{26}$$

✓ Solution by Mathematica
Time used: 1.921 (sec). Leaf size: 82

```
DSolve[y''[x]+6*y'[x]+13*y[x]==x*Exp[-3*x]*Sin[2*x]+x^2*Exp[-2*x]*Sin[3*x],y[x],x,IncludeSin
```

$$y(x) \to \frac{e^{-3x}(-32e^{x}(338x^2-1066x+563)\sin(3x)-96e^{x}(169x^2-26x-180)\cos(3x)-2197(8x^2-1-64c_2)}{140608}$$

## 11.46 problem 46

Internal problem ID [11820]

**Book**: Differential Equations by Shepley L. Ross. Third edition. John Willey. New Delhi. 2004.
**Section**: Chapter 4, Section 4.3. The method of undetermined coefficients. Exercises page 151
**Problem number**: 46.
**ODE order**: 3.
**ODE degree**: 1.

CAS Maple gives this as type `[[_3rd_order, _missing_y]]`

$$\boxed{y''' - 3y'' + 2y' = x^2 e^x + 3 e^{2x} x + 5x^2}$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 55

```
dsolve(diff(y(x),x$3)-3*diff(y(x),x$2)+2*diff(y(x),x)=x^2*exp(x)+3*x*exp(2*x)+5*x^2,y(x), si
```

$$y(x) = \frac{(6x^2 + 4c_1 - 18x + 21) e^{2x}}{8} + \frac{(-x^3 + 3c_2 - 6x + 6) e^x}{3} + \frac{5x^3}{6} + \frac{15x^2}{4} + \frac{35x}{4} + c_3$$

✓ Solution by Mathematica

Time used: 0.885 (sec). Leaf size: 67

```
DSolve[y'''[x]-3*y''[x]+2*y'[x]==x^2*Exp[x]+3*x*Exp[2*x]+5*x^2,y[x],x,IncludeSingularSolutio
```

$$y(x) \to \frac{5x^3}{6} + e^x\left(-\frac{x^3}{3} - 2x + c_1\right) + \frac{15x^2}{4} + \frac{1}{8}e^{2x}(6x^2 - 18x + 21 + 4c_2) + \frac{35x}{4} + c_3$$

## 11.47 problem 47

Internal problem ID [11821]

**Book**: Differential Equations by Shepley L. Ross. Third edition. John Willey. New Delhi. 2004.
**Section**: Chapter 4, Section 4.3. The method of undetermined coefficients. Exercises page 151
**Problem number**: 47.
**ODE order**: 3.
**ODE degree**: 1.

CAS Maple gives this as type `[[_3rd_order, _linear, _nonhomogeneous]]`

$$\boxed{y''' - 6y'' + 12y' - 8y = \mathrm{e}^{2x} x + \mathrm{e}^{3x} x^2}$$

✓ Solution by Maple
Time used: 0.0 (sec). Leaf size: 41

```
dsolve(diff(y(x),x$3)-6*diff(y(x),x$2)+12*diff(y(x),x)-8*y(x)=x*exp(2*x)+x^2*exp(3*x),y(x),
```

$$y(x) = \frac{(x^4 + 24c_3x^2 + 24c_2x + 24c_1)\,\mathrm{e}^{2x}}{24} + \mathrm{e}^{3x}\left(x^2 - 6x + 12\right)$$

✓ Solution by Mathematica
Time used: 0.084 (sec). Leaf size: 47

```
DSolve[y'''[x]-6*y''[x]+12*y'[x]-8*y[x]==x*Exp[2*x]+x^2*Exp[3*x],y[x],x,IncludeSingularSolut
```

$$y(x) \to \frac{1}{24}e^{2x}\left(x^4 + 24e^x\left(x^2 - 6x + 12\right) + 24c_3x^2 + 24c_2x + 24c_1\right)$$

## 11.48 problem 48

Internal problem ID [11822]

**Book**: Differential Equations by Shepley L. Ross. Third edition. John Willey. New Delhi. 2004.
**Section**: Chapter 4, Section 4.3. The method of undetermined coefficients. Exercises page 151
**Problem number**: 48.
**ODE order**: 4.
**ODE degree**: 1.

CAS Maple gives this as type `[[_high_order, _linear, _nonhomogeneous]]`

$$\boxed{y'''' + 3y''' + 4y'' + 3y' + y = x^2 \mathrm{e}^{-x} + 3\,\mathrm{e}^{-\frac{x}{2}} \cos\left(\frac{\sqrt{3}\,x}{2}\right)}$$

✓ Solution by Maple

Time used: 0.016 (sec). Leaf size: 73

```
dsolve(diff(y(x),x$4)+3*diff(y(x),x$3)+4*diff(y(x),x$2)+3*diff(y(x),x)+y(x)=x^2*exp(-x)+3*ex
```

$$y(x) = -\frac{3\,\mathrm{e}^{-\frac{x}{2}}\left(x - \frac{2c_3}{3} + \frac{1}{3}\right)\cos\left(\frac{\sqrt{3}\,x}{2}\right)}{2} - \frac{\mathrm{e}^{-\frac{x}{2}}\left((x-5)\sqrt{3} - 2c_4\right)\sin\left(\frac{\sqrt{3}\,x}{2}\right)}{2} + \frac{\left(-24 + x^4 + 4x^3 + 12(-2+c_2)\,x + 12c_1\right)\mathrm{e}^{-x}}{12}$$

✓ Solution by Mathematica

Time used: 2.054 (sec). Leaf size: 104

```
DSolve[y''''[x]+3*y'''[x]+4*y''[x]+3*y'[x]+y[x]==x^2*Exp[-x]+3*Exp[-x/2]*Cos[Sqrt[3]/2*x],y[
```

$$y(x) \to \frac{1}{12}e^{-x}\left(x^4 + 4x^3 - 24x + 12c_4x - 6e^{x/2}(3x + 1 - 2c_2)\cos\left(\frac{\sqrt{3}x}{2}\right) - 6e^{x/2}\left(\sqrt{3}x - 5\sqrt{3} - 2c_1\right)\sin\left(\frac{\sqrt{3}x}{2}\right) - 24 + 12c_3\right)$$

## 11.49 problem 49

Internal problem ID [11823]

**Book**: Differential Equations by Shepley L. Ross. Third edition. John Willey. New Delhi. 2004.
**Section**: Chapter 4, Section 4.3. The method of undetermined coefficients. Exercises page 151
**Problem number**: 49.
**ODE order**: 4.
**ODE degree**: 1.

CAS Maple gives this as type `[[_high_order, _linear, _nonhomogeneous]]`

$$\boxed{y'''' - 16y = x^2 \sin(2x) + e^{2x}x^4}$$

✓ Solution by Maple
Time used: 0.0 (sec). Leaf size: 79

```
dsolve(diff(y(x),x$4)-16*y(x)=x^2*sin(2*x)+x^4*exp(2*x),y(x), singsol=all)
```

$$y(x) = \frac{(128x^5 - 480x^4 + 800x^3 - 600x^2 + 20480c_3 + 60x + 105)\,e^{2x}}{20480} + \frac{(8x^3 + 768c_1 - 15x)\cos(2x)}{768} + \frac{(-6x^2 + 256c_4 - 11)\sin(2x)}{256} + e^{-2x}c_2$$

✓ Solution by Mathematica
Time used: 0.562 (sec). Leaf size: 92

```
DSolve[y''''[x]-16*y[x]==x^2*Sin[2*x]+x^4*Exp[2*x],y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to \frac{1}{768}(8x^3 - 15x + 768c_2)\cos(2x) - \frac{1}{512}(24x^2 - 5 - 1024c_4)\sin(x)\cos(x) + \frac{e^{2x}(128x^5 - 480x^4 + 800x^3 - 600x^2 + 60x + 105 + 20480c_1)}{20480} + c_3e^{-2x}$$

## 11.50 problem 50

Internal problem ID [11824]

**Book**: Differential Equations by Shepley L. Ross. Third edition. John Willey. New Delhi. 2004.
**Section**: Chapter 4, Section 4.3. The method of undetermined coefficients. Exercises page 151
**Problem number**: 50.
**ODE order**: 6.
**ODE degree**: 1.

CAS Maple gives this as type `[[_high_order, _missing_y]]`

$$\boxed{y^{(6)} + 2y^{(5)} + 5y'''' = x^3 + x^2 e^{-x} + e^{-x} \sin(2x)}$$

✓ Solution by Maple

Time used: 0.031 (sec). Leaf size: 91

```
dsolve(diff(y(x),x$6)+2*diff(y(x),x$5)+5*diff(y(x),x$4)=x^3+x^2*exp(-x)+exp(-x)*sin(2*x),y(x
```

$$y(x) = c_5 x + c_6 + \frac{\left(\int \left(\left(\left(-330x + 1320c_1 + 240c_2 + 69\right)\cos(2x) + \left(60x - 240c_1 + 1320c_2 + 567\right)\sin(2x) - 3750x^2 - 22\right.\right.\right.}{15000}$$

✓ Solution by Mathematica

Time used: 11.809 (sec). Leaf size: 119

```
DSolve[y''''''[x]+2*y'''''[x]+5*y''''[x]==x^3+x^2*Exp[-x]+Exp[-x]*Sin[2*x],y[x],x,IncludeSin
```

$$y(x) \to c_6 x^3 + c_5 x^2 + \frac{e^{-x}\left(10\left(25e^x x^7 - 70e^x x^6 - 42e^x x^5 + 504e^x x^4 + 26250x^2 + 210000x + 511875\right) + 84\left(35x - 2\left(97 + 24\right.\right.\right.}{1050000} + c_4 x + c_3$$

## 11.51 problem 51

Internal problem ID [11825]

**Book**: Differential Equations by Shepley L. Ross. Third edition. John Willey. New Delhi. 2004.
**Section**: Chapter 4, Section 4.3. The method of undetermined coefficients. Exercises page 151
**Problem number**: 51.
**ODE order**: 4.
**ODE degree**: 1.

CAS Maple gives this as type `[[_high_order, _linear, _nonhomogeneous]]`

$$\boxed{y'''' + 2y'' + y = \cos(x)\, x^2}$$

✓ Solution by Maple
Time used: 0.016 (sec). Leaf size: 46

```
dsolve(diff(y(x),x$4)+2*diff(y(x),x$2)+y(x)=x^2*cos(x),y(x), singsol=all)
```

$$y(x) = \frac{(-4x^4 + 192c_4x + 36x^2 + 192c_1 - 21)\cos(x)}{192} + \frac{(x^3 + (12c_3 - 3)\, x + 12c_2)\sin(x)}{12}$$

✓ Solution by Mathematica
Time used: 0.138 (sec). Leaf size: 56

```
DSolve[y''''[x]+2*y''[x]+y[x]==x^2*Cos[x],y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to \frac{1}{12}(x^3 + 3(-1 + 4c_4)x + 12c_3)\sin(x) + \left(-\frac{x^4}{48} + \frac{3x^2}{16} + c_2x - \frac{5}{32} + c_1\right)\cos(x)$$

## 11.52 problem 52

Internal problem ID [11826]

**Book**: Differential Equations by Shepley L. Ross. Third edition. John Willey. New Delhi. 2004.
**Section**: Chapter 4, Section 4.3. The method of undetermined coefficients. Exercises page 151
**Problem number**: 52.
**ODE order**: 4.
**ODE degree**: 1.

CAS Maple gives this as type `[[_high_order, _linear, _nonhomogeneous]]`

$$\boxed{y'''' + 16y = x\,\mathrm{e}^{\sqrt{2}\,x}\sin\left(\sqrt{2}\,x\right) + \mathrm{e}^{-\sqrt{2}\,x}\cos\left(\sqrt{2}\,x\right)}$$

✓ Solution by Maple

Time used: 0.015 (sec). Leaf size: 108

```
dsolve(diff(y(x),x$4)+16*y(x)=x*exp(sqrt(2)*x)*sin(sqrt(2)*x)+exp(-sqrt(2)*x)*cos(sqrt(2)*x)
```

$$y(x) = \frac{\left(\left(2x\sqrt{2}+128c_3+3\right)\cos\left(x\sqrt{2}\right)+2\sin\left(x\sqrt{2}\right)\left(x\sqrt{2}+64c_4\right)\right)\mathrm{e}^{-x\sqrt{2}}}{128} - \frac{\left(\left(x^2\sqrt{2}-128c_1-\frac{5\sqrt{2}}{8}\right)\cos\left(x\sqrt{2}\right)+\sin\left(x\sqrt{2}\right)\left(x^2\sqrt{2}-3x-128c_2+\frac{5\sqrt{2}}{8}\right)\right)\mathrm{e}^{x\sqrt{2}}}{128}$$

✓ Solution by Mathematica

Time used: 2.857 (sec). Leaf size: 140

```
DSolve[y''''[x]+16*y[x]==x*Exp[Sqrt[2]*x]*Sin[Sqrt[2]*x]+Exp[-Sqrt[2]*x]*Cos[Sqrt[2]*x],y[x]
```

$$y(x) \to \frac{e^{-\sqrt{2}x}\left(\left(e^{2\sqrt{2}x}\left(-8\sqrt{2}x^2+5\sqrt{2}+1024c_1\right)+8\left(2\sqrt{2}x+3+128c_2\right)\right)\cos\left(\sqrt{2}x\right)-\left(e^{2\sqrt{2}x}\left(8\sqrt{2}x^2-24x\right.\right.\right.}{1024}$$

## 11.53 problem 53

Internal problem ID [11827]

**Book**: Differential Equations by Shepley L. Ross. Third edition. John Willey. New Delhi. 2004.
**Section**: Chapter 4, Section 4.3. The method of undetermined coefficients. Exercises page 151
**Problem number**: 53.
**ODE order**: 4.
**ODE degree**: 1.

CAS Maple gives this as type `[[_high_order, _linear, _nonhomogeneous]]`

$$\boxed{y'''' + 3y'' - 4y = \cos(x)^2 - \cosh(x)}$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 56

```
dsolve(diff(y(x),x$4)+3*diff(y(x),x$2)-4*y(x)=cos(x)^2-cosh(x),y(x), singsol=all)
```

$$y(x) = -\frac{1}{8} + \frac{(10x + 200c_3 + 9)\,\mathrm{e}^{-x}}{200} + \frac{(200c_2 - 9)\cos(2x)}{200} + \frac{(-x + 40c_4)\sin(2x)}{40} + \frac{(-10x + 200c_1 + 9)\,\mathrm{e}^{x}}{200}$$

✓ Solution by Mathematica

Time used: 0.21 (sec). Leaf size: 75

```
DSolve[y''''[x]+3*y''[x]-4*y[x]==Cos[x]^2-Cosh[x],y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to \frac{1}{400}e^{-x}\big((-13 + 400c_1)e^{x}\cos(2x) + 2\big(10x - 25e^{x} + e^{2x}(-10x + 9 + 200c_4) - 5e^{x}(x - 40c_2)\sin(2x) + 9 + 200c_3\big)\big)$$

## 11.54 problem 54

Internal problem ID [11828]

**Book**: Differential Equations by Shepley L. Ross. Third edition. John Willey. New Delhi. 2004.
**Section**: Chapter 4, Section 4.3. The method of undetermined coefficients. Exercises page 151
**Problem number**: 54.
**ODE order**: 4.
**ODE degree**: 1.

CAS Maple gives this as type `[[_high_order, _linear, _nonhomogeneous]]`

$$\boxed{y'''' + 10y'' + 9y = \sin(x)\sin(2x)}$$

✓ Solution by Maple
Time used: 0.016 (sec). Leaf size: 45

```
dsolve(diff(y(x),x$4)+10*diff(y(x),x$2)+9*y(x)=sin(x)*sin(2*x),y(x), singsol=all)
```

$$y(x) = \frac{(11 + 1152c_3)\cos(3x)}{1152} + \frac{(x + 96c_4)\sin(3x)}{96} + \frac{(-1 + 64c_1)\cos(x)}{64} + \frac{\sin(x)(x + 32c_2)}{32}$$

✓ Solution by Mathematica
Time used: 0.079 (sec). Leaf size: 54

```
DSolve[y''''[x]+10*y''[x]+9*y[x]==Sin[x]*Sin[2*x],y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to \frac{1}{32}x\sin(x) + \frac{1}{96}x\sin(3x) + \left(-\frac{1}{64} + c_3\right)\cos(x) + \left(\frac{13}{576} + c_1\right)\cos(3x) + c_4\sin(x) + c_2\sin(3x)$$

# 12 Chapter 4, Section 4.4. Variation of parameters. Exercises page 162

## 12.1 problem 1

Internal problem ID [11829]

**Book**: Differential Equations by Shepley L. Ross. Third edition. John Willey. New Delhi. 2004.
**Section**: Chapter 4, Section 4.4. Variation of parameters. Exercises page 162
**Problem number**: 1.
**ODE order**: 2.
**ODE degree**: 1.

CAS Maple gives this as type `[[_2nd_order, _linear, _nonhomogeneous]]`

$$\boxed{y'' + y = \cot(x)}$$

✓ Solution by Maple
Time used: 0.0 (sec). Leaf size: 24

```
dsolve(diff(y(x),x$2)+y(x)=cot(x),y(x), singsol=all)
```

$$y(x) = \sin(x) c_2 + c_1 \cos(x) + \sin(x) \ln(\csc(x) - \cot(x))$$

✓ Solution by Mathematica
Time used: 0.052 (sec). Leaf size: 33

```
DSolve[y''[x]+y[x]==Cot[x],y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to c_1 \cos(x) + \sin(x) \left(\log\left(\sin\left(\frac{x}{2}\right)\right) - \log\left(\cos\left(\frac{x}{2}\right)\right) + c_2\right)$$

## 12.2 problem 2

Internal problem ID [11830]

**Book**: Differential Equations by Shepley L. Ross. Third edition. John Willey. New Delhi. 2004.
**Section**: Chapter 4, Section 4.4. Variation of parameters. Exercises page 162
**Problem number**: 2.
**ODE order**: 2.
**ODE degree**: 1.

CAS Maple gives this as type `[[_2nd_order, _linear, _nonhomogeneous]]`

$$\boxed{y'' + y = \tan(x)^2}$$

✓ Solution by Maple
Time used: 0.0 (sec). Leaf size: 23

```
dsolve(diff(y(x),x$2)+y(x)=tan(x)^2,y(x), singsol=all)
```

$$y(x) = \sin(x) c_2 + c_1 \cos(x) - 2 + \sin(x) \ln(\sec(x) + \tan(x))$$

✓ Solution by Mathematica
Time used: 0.124 (sec). Leaf size: 23

```
DSolve[y''[x]+y[x]==Tan[x]^2,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to \sin(x)\text{arctanh}(\sin(x)) + c_1 \cos(x) + c_2 \sin(x) - 2$$

## 12.3 problem 3

Internal problem ID [11831]

**Book**: Differential Equations by Shepley L. Ross. Third edition. John Willey. New Delhi. 2004.
**Section**: Chapter 4, Section 4.4. Variation of parameters. Exercises page 162
**Problem number**: 3.
**ODE order**: 2.
**ODE degree**: 1.

CAS Maple gives this as type `[[_2nd_order, _linear, _nonhomogeneous]]`

$$\boxed{y'' + y = \sec(x)}$$

✓ Solution by Maple
Time used: 0.0 (sec). Leaf size: 22

```
dsolve(diff(y(x),x$2)+y(x)=sec(x),y(x), singsol=all)
```

$$y(x) = -\ln(\sec(x))\cos(x) + c_1 \cos(x) + \sin(x)(c_2 + x)$$

✓ Solution by Mathematica
Time used: 0.029 (sec). Leaf size: 22

```
DSolve[y''[x]+y[x]==Sec[x],y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to (x + c_2)\sin(x) + \cos(x)(\log(\cos(x)) + c_1)$$

## 12.4 problem 4

Internal problem ID [11832]

**Book**: Differential Equations by Shepley L. Ross. Third edition. John Willey. New Delhi. 2004.
**Section**: Chapter 4, Section 4.4. Variation of parameters. Exercises page 162
**Problem number**: 4.
**ODE order**: 2.
**ODE degree**: 1.

CAS Maple gives this as type `[[_2nd_order, _linear, _nonhomogeneous]]`

$$\boxed{y'' + y = \sec(x)^3}$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 19

```
dsolve(diff(y(x),x$2)+y(x)=sec(x)^3,y(x), singsol=all)
```

$$y(x) = (-1 + c_1)\cos(x) + \sin(x)\, c_2 + \frac{\sec(x)}{2}$$

✓ Solution by Mathematica

Time used: 0.065 (sec). Leaf size: 25

```
DSolve[y''[x]+y[x]==Sec[x]^3,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to -\frac{\sec(x)}{2} + c_1 \cos(x) + \sin(x)(\tan(x) + c_2)$$

## 12.5 problem 5

Internal problem ID [11833]

**Book**: Differential Equations by Shepley L. Ross. Third edition. John Willey. New Delhi. 2004.
**Section**: Chapter 4, Section 4.4. Variation of parameters. Exercises page 162
**Problem number**: 5.
**ODE order**: 2.
**ODE degree**: 1.

CAS Maple gives this as type `[[_2nd_order, _linear, _nonhomogeneous]]`

$$\boxed{y'' + 4y = \sec(x)^2}$$

✓ Solution by Maple
Time used: 0.0 (sec). Leaf size: 42

```
dsolve(diff(y(x),x$2)+4*y(x)=sec(x)^2,y(x), singsol=all)
```

$$y(x) = \left(-2\cos(x)^2 + 1\right)\ln(\sec(x)) + 2\cos(x)^2 c_1 + 2\sin(x)(c_2 + x)\cos(x) - \sin(x)^2 - c_1$$

✓ Solution by Mathematica
Time used: 0.098 (sec). Leaf size: 33

```
DSolve[y''[x]+4*y[x]==Sec[x]^2,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to \cos(2x)(\log(\cos(x)) + c_1) + \sin(x)(-\sin(x) + 2(x + c_2)\cos(x))$$

## 12.6 problem 6

Internal problem ID [11834]

**Book**: Differential Equations by Shepley L. Ross. Third edition. John Willey. New Delhi. 2004.
**Section**: Chapter 4, Section 4.4. Variation of parameters. Exercises page 162
**Problem number**: 6.
**ODE order**: 2.
**ODE degree**: 1.

CAS Maple gives this as type `[[_2nd_order, _linear, _nonhomogeneous]]`

$$\boxed{y'' + y = \sec(x)\tan(x)}$$

✓ Solution by Maple
Time used: 0.0 (sec). Leaf size: 23

```
dsolve(diff(y(x),x$2)+y(x)=tan(x)*sec(x),y(x), singsol=all)
```

$$y(x) = \ln(\sec(x))\sin(x) + (c_2 - 1)\sin(x) + \cos(x)(c_1 + x)$$

✓ Solution by Mathematica
Time used: 0.039 (sec). Leaf size: 29

```
DSolve[y''[x]+y[x]==Tan[x]*Sec[x],y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to \cos(x)\arctan(\tan(x)) + c_1\cos(x) + \sin(x)(-\log(\cos(x)) - 1 + c_2)$$

## 12.7 problem 7

Internal problem ID [11835]

**Book**: Differential Equations by Shepley L. Ross. Third edition. John Willey. New Delhi. 2004.
**Section**: Chapter 4, Section 4.4. Variation of parameters. Exercises page 162
**Problem number**: 7.
**ODE order**: 2.
**ODE degree**: 1.

CAS Maple gives this as type `[[_2nd_order, _linear, _nonhomogeneous]]`

$$\boxed{y'' + 4y' + 5y = e^{-2x} \sec(x)}$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 27

```
dsolve(diff(y(x),x$2)+4*diff(y(x),x)+5*y(x)=exp(-2*x)*sec(x),y(x), singsol=all)
```

$$y(x) = e^{-2x}(-\ln(\sec(x))\cos(x) + c_1\cos(x) + \sin(x)(c_2 + x))$$

✓ Solution by Mathematica

Time used: 0.048 (sec). Leaf size: 28

```
DSolve[y''[x]+4*y'[x]+5*y[x]==Exp[-2*x]*Sec[x],y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to e^{-2x}((x + c_1)\sin(x) + \cos(x)(\log(\cos(x)) + c_2))$$

## 12.8 problem 8

Internal problem ID [11836]

**Book**: Differential Equations by Shepley L. Ross. Third edition. John Willey. New Delhi. 2004.
**Section**: Chapter 4, Section 4.4. Variation of parameters. Exercises page 162
**Problem number**: 8.
**ODE order**: 2.
**ODE degree**: 1.

CAS Maple gives this as type `[[_2nd_order, _linear, _nonhomogeneous]]`

$$\boxed{y'' - 2y' + 5y = e^{x} \tan (2x)}$$

✓ Solution by Maple
Time used: 0.0 (sec). Leaf size: 39

```
dsolve(diff(y(x),x$2)-2*diff(y(x),x)+5*y(x)=exp(x)*tan(2*x),y(x), singsol=all)
```

$$y(x) = \frac{e^{x}(4c_2 \sin (2x) - \ln (\sec (2x) + \tan (2x)) \cos (2x) + 4 \cos (2x) c_1)}{4}$$

✓ Solution by Mathematica
Time used: 0.069 (sec). Leaf size: 42

```
DSolve[y''[x]-2*y'[x]+5*y[x]==Exp[x]*Tan[2*x],y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to -\frac{1}{4}e^{x}(\cos(2x)\text{arctanh}(\sin(2x)) - 4c_2 \cos(2x) + (1 - 4c_1) \sin(2x))$$

## 12.9 problem 9

Internal problem ID [11837]

**Book**: Differential Equations by Shepley L. Ross. Third edition. John Willey. New Delhi. 2004.
**Section**: Chapter 4, Section 4.4. Variation of parameters. Exercises page 162
**Problem number**: 9.
**ODE order**: 2.
**ODE degree**: 1.

CAS Maple gives this as type `[[_2nd_order, _linear, _nonhomogeneous]]`

$$\boxed{y'' + 6y' + 9y = \frac{e^{-3x}}{x^3}}$$

✓ Solution by Maple
Time used: 0.0 (sec). Leaf size: 25

```
dsolve(diff(y(x),x$2)+6*diff(y(x),x)+9*y(x)=exp(-3*x)/x^3,y(x), singsol=all)
```

$$y(x) = \frac{e^{-3x}(2c_1x^2 + 2c_2x + 1)}{2x}$$

✓ Solution by Mathematica
Time used: 0.03 (sec). Leaf size: 31

```
DSolve[y''[x]+6*y'[x]+9*y[x]==Exp[-3*x]/x^3,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to \frac{e^{-3x}(2c_2x^2 + 2c_1x + 1)}{2x}$$

## 12.10 problem 10

Internal problem ID [11838]

**Book**: Differential Equations by Shepley L. Ross. Third edition. John Willey. New Delhi. 2004.
**Section**: Chapter 4, Section 4.4. Variation of parameters. Exercises page 162
**Problem number**: 10.
**ODE order**: 2.
**ODE degree**: 1.

CAS Maple gives this as type `[[_2nd_order, _linear, _nonhomogeneous]]`

$$\boxed{y'' - 2y' + y = x\,\mathrm{e}^{x} \ln(x)}$$

✓ Solution by Maple
Time used: 0.0 (sec). Leaf size: 27

```
dsolve(diff(y(x),x$2)-2*diff(y(x),x)+y(x)=x*exp(x)*ln(x),y(x), singsol=all)
```

$$y(x) = \frac{\left(\ln(x)\,x^{3} - \frac{5x^{3}}{6} + 6c_{1}x + 6c_{2}\right)\mathrm{e}^{x}}{6}$$

✓ Solution by Mathematica
Time used: 0.028 (sec). Leaf size: 32

```
DSolve[y''[x]-2*y'[x]+y[x]==x*Exp[x]*Log[x],y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to \frac{1}{36}e^{x}\left(x^{3}(6\log(x) - 5) + 36c_{2}x + 36c_{1}\right)$$

## 12.11 problem 11

Internal problem ID [11839]

**Book**: Differential Equations by Shepley L. Ross. Third edition. John Willey. New Delhi. 2004.
**Section**: Chapter 4, Section 4.4. Variation of parameters. Exercises page 162
**Problem number**: 11.
**ODE order**: 2.
**ODE degree**: 1.

CAS Maple gives this as type `[[_2nd_order, _linear, _nonhomogeneous]]`

$$\boxed{y'' + y = \sec(x)\csc(x)}$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 34

```
dsolve(diff(y(x),x$2)+y(x)=sec(x)*csc(x),y(x), singsol=all)
```

$$y(x) = \sin(x) c_2 + c_1 \cos(x) + \sin(x) \ln(\csc(x) - \cot(x)) - \cos(x) \ln(\sec(x) + \tan(x))$$

✓ Solution by Mathematica

Time used: 0.127 (sec). Leaf size: 30

```
DSolve[y''[x]+y[x]==Sec[x]*Csc[x],y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to -\sin(x)\text{arctanh}(\cos(x)) + c_1 \cos(x) + c_2 \sin(x) + \cos(x)\left(-\coth^{-1}(\sin(x))\right)$$

## 12.12 problem 12

Internal problem ID [11840]

**Book**: Differential Equations by Shepley L. Ross. Third edition. John Willey. New Delhi. 2004.
**Section**: Chapter 4, Section 4.4. Variation of parameters. Exercises page 162
**Problem number**: 12.
**ODE order**: 2.
**ODE degree**: 1.

CAS Maple gives this as type `[[_2nd_order, _linear, _nonhomogeneous]]`

$$\boxed{y'' + y = \tan(x)^3}$$

✓ Solution by Maple
Time used: 0.0 (sec). Leaf size: 27

```
dsolve(diff(y(x),x$2)+y(x)=tan(x)^3,y(x), singsol=all)
```

$$y(x) = \sin(x) c_2 + c_1 \cos(x) + \frac{\tan(x)}{2} + \frac{3\cos(x)\ln(\sec(x) + \tan(x))}{2}$$

✓ Solution by Mathematica
Time used: 0.078 (sec). Leaf size: 39

```
DSolve[y''[x]+y[x]==Tan[x]^3,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to \frac{1}{2}\sec(x)\left(3\cos^2(x)\mathrm{arctanh}(\sin(x)) + \sin(x) + c_1\cos(2x) + c_2\sin(2x) + c_1\right)$$

## 12.13 problem 13

Internal problem ID [11841]

**Book**: Differential Equations by Shepley L. Ross. Third edition. John Willey. New Delhi. 2004.
**Section**: Chapter 4, Section 4.4. Variation of parameters. Exercises page 162
**Problem number**: 13.
**ODE order**: 2.
**ODE degree**: 1.

CAS Maple gives this as type `[[_2nd_order, _linear, _nonhomogeneous]]`

$$\boxed{y'' + 3y' + 2y = \frac{1}{e^x + 1}}$$

✓ Solution by Maple
Time used: 0.0 (sec). Leaf size: 36

```
dsolve(diff(y(x),x$2)+3*diff(y(x),x)+2*y(x)=1/(1+exp(x)),y(x), singsol=all)
```

$$y(x) = e^{-2x}(\ln(e^x + 1)(e^x + 1) - \ln(e^x)e^x + (c_2 + x)e^x - c_1)$$

✓ Solution by Mathematica
Time used: 0.074 (sec). Leaf size: 34

```
DSolve[y''[x]+3*y'[x]+2*y[x]==1/(1+Exp[x]),y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to e^{-2x}((e^x + 1)\log(e^x + 1) + (-1 + c_2)e^x + c_1)$$

## 12.14 problem 14

Internal problem ID [11842]

**Book**: Differential Equations by Shepley L. Ross. Third edition. John Willey. New Delhi. 2004.
**Section**: Chapter 4, Section 4.4. Variation of parameters. Exercises page 162
**Problem number**: 14.
**ODE order**: 2.
**ODE degree**: 1.

CAS Maple gives this as type `[[_2nd_order, _linear, _nonhomogeneous]]`

$$\boxed{y'' + 3y' + 2y = \frac{1}{e^{2x} + 1}}$$

✓ Solution by Maple
Time used: 0.015 (sec). Leaf size: 38

```
dsolve(diff(y(x),x$2)+3*diff(y(x),x)+2*y(x)=1/(1+exp(2*x)),y(x), singsol=all)
```

$$y(x) = -\frac{(\ln(e^{2x} + 1) e^{-x} + 2c_1 e^{-x} - 2\arctan(e^{x}) - 2c_2) e^{-x}}{2}$$

✓ Solution by Mathematica
Time used: 0.074 (sec). Leaf size: 45

```
DSolve[y''[x]+3*y'[x]+2*y[x]==1/(1+Exp[2*x]),y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to \frac{1}{2} e^{-2x} \left(2e^{x} \arctan(e^{x}) - \log\left(e^{2x} + 1\right) + 2(c_2 e^{x} + c_1)\right)$$

## 12.15 problem 15

Internal problem ID [11843]

**Book**: Differential Equations by Shepley L. Ross. Third edition. John Willey. New Delhi. 2004.
**Section**: Chapter 4, Section 4.4. Variation of parameters. Exercises page 162
**Problem number**: 15.
**ODE order**: 2.
**ODE degree**: 1.

CAS Maple gives this as type `[[_2nd_order, _linear, _nonhomogeneous]]`

$$\boxed{y'' + y = \frac{1}{1 + \sin(x)}}$$

✓ Solution by Maple
Time used: 0.0 (sec). Leaf size: 29

```
dsolve(diff(y(x),x$2)+y(x)=1/(1+sin(x)),y(x), singsol=all)
```

$$y(x) = \ln(1 + \sin(x))\sin(x) + (-x + c_1 - 1)\cos(x) - 1 + (c_2 + 1)\sin(x)$$

✓ Solution by Mathematica
Time used: 0.188 (sec). Leaf size: 40

```
DSolve[y''[x]+y[x]==1/(1+Sin[x]),y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to (-x + 1 + c_1)\cos(x) + \sin(x)\left(2\log\left(\sin\left(\frac{x}{2}\right) + \cos\left(\frac{x}{2}\right)\right) + 1 + c_2\right) - 1$$

## 12.16 problem 16

Internal problem ID [11844]

**Book**: Differential Equations by Shepley L. Ross. Third edition. John Willey. New Delhi. 2004.
**Section**: Chapter 4, Section 4.4. Variation of parameters. Exercises page 162
**Problem number**: 16.
**ODE order**: 2.
**ODE degree**: 1.

CAS Maple gives this as type `[[_2nd_order, _linear, _nonhomogeneous]]`

$$\boxed{y'' - 2y' + y = e^x \arcsin(x)}$$

✓ Solution by Maple
Time used: 0.016 (sec). Leaf size: 37

```
dsolve(diff(y(x),x$2)-2*diff(y(x),x)+y(x)=exp(x)*arcsin(x),y(x), singsol=all)
```

$$y(x) = \frac{e^x \left(2x^2 \arcsin(x) + 3x\sqrt{-x^2+1} + 4c_1 x + \arcsin(x) + 4c_2\right)}{4}$$

✓ Solution by Mathematica
Time used: 0.043 (sec). Leaf size: 45

```
DSolve[y''[x]-2*y'[x]+y[x]==Exp[x]*ArcSin[x],y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to \frac{1}{4} e^x \left(2x^2 \arcsin(x) + \arcsin(x) + 3\sqrt{1-x^2}x + 4c_2 x + 4c_1\right)$$

## 12.17 problem 17

Internal problem ID [11845]

**Book**: Differential Equations by Shepley L. Ross. Third edition. John Willey. New Delhi. 2004.
**Section**: Chapter 4, Section 4.4. Variation of parameters. Exercises page 162
**Problem number**: 17.
**ODE order**: 2.
**ODE degree**: 1.

CAS Maple gives this as type `[[_2nd_order, _linear, _nonhomogeneous]]`

$$\boxed{y'' + 3y' + 2y = \frac{e^{-x}}{x}}$$

✓ Solution by Maple
Time used: 0.0 (sec). Leaf size: 29

```
dsolve(diff(y(x),x$2)+3*diff(y(x),x)+2*y(x)=exp(-x)/x,y(x), singsol=all)
```

$$y(x) = \left(-\left(\int e^{-x}(\text{expIntegral}_1(-x) - c_1)\, dx\right) + c_2\right) e^{-x}$$

✓ Solution by Mathematica
Time used: 0.041 (sec). Leaf size: 30

```
DSolve[y''[x]+3*y'[x]+2*y[x]==Exp[-x]/x,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to e^{-2x}(-\text{ExpIntegralEi}(x) + e^x \log(x) + c_2 e^x + c_1)$$

## 12.18 problem 18

Internal problem ID [11846]

**Book**: Differential Equations by Shepley L. Ross. Third edition. John Willey. New Delhi. 2004.
**Section**: Chapter 4, Section 4.4. Variation of parameters. Exercises page 162
**Problem number**: 18.
**ODE order**: 2.
**ODE degree**: 1.

CAS Maple gives this as type `[[_2nd_order, _linear, _nonhomogeneous]]`

$$\boxed{y'' - 2y' + y = x \ln(x)}$$

✓ Solution by Maple
Time used: 0.0 (sec). Leaf size: 30

```
dsolve(diff(y(x),x$2)-2*diff(y(x),x)+y(x)=x*ln(x),y(x), singsol=all)
```

$$y(x) = -(x-2)\,\mathrm{e}^{x}\,\mathrm{expIntegral}_1(x) + (c_1x + c_2)\,\mathrm{e}^{x} + 3 + (x+2)\ln(x)$$

✓ Solution by Mathematica
Time used: 0.059 (sec). Leaf size: 37

```
DSolve[y''[x]-2*y'[x]+y[x]==x*Log[x],y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to e^x(x-2)\,\text{ExpIntegralEi}(-x) + (x+2)\log(x) + c_1e^x + c_2e^x x + 3$$

## 12.19 problem 19

Internal problem ID [11847]

**Book**: Differential Equations by Shepley L. Ross. Third edition. John Willey. New Delhi. 2004.
**Section**: Chapter 4, Section 4.4. Variation of parameters. Exercises page 162
**Problem number**: 19.
**ODE order**: 2.
**ODE degree**: 1.

CAS Maple gives this as type `[[_2nd_order, _linear, _nonhomogeneous]]`

$$\boxed{x^2y'' - 6y'x + 10y = 3x^4 + 6x^3}$$

✓ Solution by Maple
Time used: 0.0 (sec). Leaf size: 26

```
dsolve(x^2*diff(y(x),x$2)-6*x*diff(y(x),x)+10*y(x)=3*x^4+6*x^3,y(x), singsol=all)
```

$$y(x) = -\frac{3}{2}x^4 - 3x^3 + \frac{1}{3}c_1x^5 + c_2x^2$$

✓ Solution by Mathematica
Time used: 0.016 (sec). Leaf size: 28

```
DSolve[x^2*y''[x]-6*x*y'[x]+10*y[x]==3*x^4+6*x^3,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to c_2x^5 - \frac{3}{2}(x+2)x^3 + c_1x^2$$

## 12.20 problem 20

Internal problem ID [11848]

**Book**: Differential Equations by Shepley L. Ross. Third edition. John Willey. New Delhi. 2004.
**Section**: Chapter 4, Section 4.4. Variation of parameters. Exercises page 162
**Problem number**: 20.
**ODE order**: 2.
**ODE degree**: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$\boxed{(1+x)^2 y'' - 2(1+x) y' + 2y = 1}$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 17

```
dsolve((x+1)^2*diff(y(x),x$2)-2*(x+1)*diff(y(x),x)+2*y(x)=1,y(x), singsol=all)
```

$$y(x) = (1+x)^2 c_1 + c_2 x + c_2 + \frac{1}{2}$$

✓ Solution by Mathematica

Time used: 0.032 (sec). Leaf size: 23

```
DSolve[(x+1)^2*y''[x]-2*(x+1)*y'[x]+2*y[x]==1,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to c_2(x+1)^2 + c_1(x+1) + \frac{1}{2}$$

## 12.21 problem 21

Internal problem ID [11849]

**Book**: Differential Equations by Shepley L. Ross. Third edition. John Willey. New Delhi. 2004.
**Section**: Chapter 4, Section 4.4. Variation of parameters. Exercises page 162
**Problem number**: 21.
**ODE order**: 2.
**ODE degree**: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$\boxed{(x^2+2x)\,y''-2(x+1)\,y'+2y=(x+2)^2}$$

✓ Solution by Maple
Time used: 0.016 (sec). Leaf size: 24

```
dsolve((x^2+2*x)*diff(y(x),x$2)-2*(x+1)*diff(y(x),x)+2*y(x)=(x+2)^2,y(x), singsol=all)
```

$$y(x)=\ln(x)\,x^2+(c_2-1)\,x^2+(-2+c_1)\,x+c_1$$

✓ Solution by Mathematica
Time used: 0.056 (sec). Leaf size: 31

```
DSolve[(x^2+2*x)*y''[x]-2*(x+1)*y'[x]+2*y[x]==(x+2)^2,y[x],x,IncludeSingularSolutions -> Tru
```

$$y(x)\to x^2\log(x)+(-1+c_1)x^2-(2+c_2)x-c_2$$

## 12.22 problem 22

Internal problem ID [11850]

**Book**: Differential Equations by Shepley L. Ross. Third edition. John Willey. New Delhi. 2004.
**Section**: Chapter 4, Section 4.4. Variation of parameters. Exercises page 162
**Problem number**: 22.
**ODE order**: 2.
**ODE degree**: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$\boxed{x^2y'' - x(x+2)\,y' + (x+2)\,y = x^3}$$

✓ Solution by Maple

Time used: 0.015 (sec). Leaf size: 15

```
dsolve(x^2*diff(y(x),x$2)-x*(x+2)*diff(y(x),x)+(x+2)*y(x)=x^3,y(x), singsol=all)
```

$$y(x) = x(-x + c_1 e^x + c_2)$$

✓ Solution by Mathematica

Time used: 0.035 (sec). Leaf size: 22

```
DSolve[x^2*y''[x]-x*(x+2)*y'[x]+(x+2)*y[x]==x^3,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to -x(x - c_2 e^x + 1 - c_1)$$

## 12.23 problem 23

Internal problem ID [11851]

**Book**: Differential Equations by Shepley L. Ross. Third edition. John Willey. New Delhi. 2004.
**Section**: Chapter 4, Section 4.4. Variation of parameters. Exercises page 162
**Problem number**: 23.
**ODE order**: 2.
**ODE degree**: 1.

CAS Maple gives this as type `[[_2nd_order, _linear, _nonhomogeneous]]`

$$\boxed{x(x-2)\,y''-\left(x^2-2\right)y'+2y(x-1)=3x^2(x-2)^2\,\mathrm{e}^x}$$

✓ Solution by Maple
Time used: 0.0 (sec). Leaf size: 23

```
dsolve(x*(x-2)*diff(y(x),x$2)-(x^2-2)*diff(y(x),x)+2*(x-1)*y(x)=3*x^2*(x-2)^2*exp(x),y(x), s
```

$$y(x)=\left(x^3-3x^2+c_1\right)\mathrm{e}^x+c_2x^2$$

✓ Solution by Mathematica
Time used: 0.1 (sec). Leaf size: 27

```
DSolve[x*(x-2)*y''[x]-(x^2-2)*y'[x]+2*(x-1)*y[x]==3*x^2*(x-2)^2*Exp[x],y[x],x,IncludeSingula
```

$$y(x)\to c_2x^2+e^x\left(x^3-3x^2+c_1\right)$$

## 12.24 problem 24

Internal problem ID [11852]

**Book**: Differential Equations by Shepley L. Ross. Third edition. John Willey. New Delhi. 2004.
**Section**: Chapter 4, Section 4.4. Variation of parameters. Exercises page 162
**Problem number**: 24.
**ODE order**: 2.
**ODE degree**: 1.

CAS Maple gives this as type `[[_2nd_order, _exact, _linear, _nonhomogeneous]]`

$$\boxed{(2x+1)(x+1)y''+2y'x-2y=(2x+1)^2}$$

✓ Solution by Maple
Time used: 0.0 (sec). Leaf size: 43

```
dsolve((2*x+1)*(x+1)*diff(y(x),x$2)+2*x*diff(y(x),x)-2*y(x)=(2*x+1)^2,y(x), singsol=all)
```

$$y(x)=\frac{4x^3+(6c_1+24c_2+4)x^2+(6c_1+24c_2+1)x+6c_2}{6x+6}$$

✓ Solution by Mathematica
Time used: 1.049 (sec). Leaf size: 72

```
DSolve[(2*x+1)*(x+1)*y''[x]+2*x*y'[x]-2*y[x]==(2*x+1)^2,y[x],x,IncludeSingularSolutions -> T
```

$$y(x)\to\frac{\sqrt{-2x-1}(4x+3)x^2-6c_2(x+1)\sqrt{2x+1}x+6c_1\sqrt{2x+1}}{6\sqrt{-2x-1}(x+1)}$$

## 12.25 problem 25

Internal problem ID [11853]

**Book**: Differential Equations by Shepley L. Ross. Third edition. John Willey. New Delhi. 2004.
**Section**: Chapter 4, Section 4.4. Variation of parameters. Exercises page 162
**Problem number**: 25.
**ODE order**: 2.
**ODE degree**: 1.

CAS Maple gives this as type `[[_2nd_order, _linear, _nonhomogeneous]]`

$$\boxed{\sin(x)^2 y'' - 2\sin(x)\cos(x)\, y' + \left(\cos(x)^2 + 1\right) y = \sin(x)^3}$$

✓ Solution by Maple
Time used: 0.0 (sec). Leaf size: 17

```
dsolve(sin(x)^2*diff(y(x),x$2)-2*sin(x)*cos(x)*diff(y(x),x)+(cos(x)^2+1)*y(x)=sin(x)^3,y(x),
```

$$y(x) = \sin(x)\left(c_2 + c_1 x + \frac{1}{2}x^2\right)$$

✓ Solution by Mathematica
Time used: 0.092 (sec). Leaf size: 24

```
DSolve[Sin[x]^2*y''[x]-2*Sin[x]*Cos[x]*y'[x]+(Cos[x]^2+1)*y[x]==Sin[x]^3,y[x],x,IncludeSingu
```

$$y(x) \rightarrow \frac{1}{2}\left(x^2 + 2c_2 x + 2c_1\right)\sin(x)$$

## 12.26 problem 26

Internal problem ID [11854]

**Book**: Differential Equations by Shepley L. Ross. Third edition. John Willey. New Delhi. 2004.
**Section**: Chapter 4, Section 4.4. Variation of parameters. Exercises page 162
**Problem number**: 26.
**ODE order**: 3.
**ODE degree**: 1.

CAS Maple gives this as type `[[_3rd_order, _linear, _nonhomogeneous]]`

$$\boxed{y''' - 3y'' - y' + 3y = x^2 e^x}$$

✓ Solution by Maple
Time used: 0.0 (sec). Leaf size: 31

```
dsolve(diff(y(x),x$3)-3*diff(y(x),x$2)-diff(y(x),x)+3*y(x)=x^2*exp(x),y(x), singsol=all)
```

$$y(x) = c_2 e^{-x} + c_3 e^{3x} - \frac{\left(x^3 + \frac{3}{2}x - 12c_1\right) e^x}{12}$$

✓ Solution by Mathematica
Time used: 0.038 (sec). Leaf size: 41

```
DSolve[y'''[x]-3*y''[x]-y'[x]+3*y[x]==x^2*Exp[x],y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to e^x \left(-\frac{x^3}{12} - \frac{x}{8} + c_2\right) + c_1 e^{-x} + c_3 e^{3x}$$

# 13 Chapter 4, Section 4.5. The Cauchy-Euler Equation. Exercises page 169

## 13.1 problem 1

Internal problem ID [11855]

**Book**: Differential Equations by Shepley L. Ross. Third edition. John Willey. New Delhi. 2004.
**Section**: Chapter 4, Section 4.5. The Cauchy-Euler Equation. Exercises page 169
**Problem number**: 1.
**ODE order**: 2.
**ODE degree**: 1.

CAS Maple gives this as type `[[_Emden, _Fowler]]`

$$\boxed{x^2y'' - 3y'x + 3y = 0}$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 13

```
dsolve(x^2*diff(y(x),x$2)-3*x*diff(y(x),x)+3*y(x)=0,y(x), singsol=all)
```

$$y(x) = x(c_2x^2 + c_1)$$

✓ Solution by Mathematica

Time used: 0.011 (sec). Leaf size: 16

```
DSolve[x^2*y''[x]-3*x*y'[x]+3*y[x]==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to x(c_2x^2 + c_1)$$

## 13.2 problem 2

Internal problem ID [11856]

**Book**: Differential Equations by Shepley L. Ross. Third edition. John Willey. New Delhi. 2004.
**Section**: Chapter 4, Section 4.5. The Cauchy-Euler Equation. Exercises page 169
**Problem number**: 2.
**ODE order**: 2.
**ODE degree**: 1.

CAS Maple gives this as type `[[_Emden, _Fowler], [_2nd_order, _linear, '_with_symmetry_[0,F`

$$\boxed{x^2y'' + y'x - 4y = 0}$$

✓ Solution by Maple

Time used: 0.016 (sec). Leaf size: 15

```
dsolve(x^2*diff(y(x),x$2)+x*diff(y(x),x)-4*y(x)=0,y(x), singsol=all)
```

$$y(x) = \frac{x^4c_1 + c_2}{x^2}$$

✓ Solution by Mathematica

Time used: 0.01 (sec). Leaf size: 18

```
DSolve[x^2*y''[x]+x*y'[x]-4*y[x]==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to \frac{c_2x^4 + c_1}{x^2}$$

## 13.3 problem 3

Internal problem ID [11857]

**Book**: Differential Equations by Shepley L. Ross. Third edition. John Willey. New Delhi. 2004.
**Section**: Chapter 4, Section 4.5. The Cauchy-Euler Equation. Exercises page 169
**Problem number**: 3.
**ODE order**: 2.
**ODE degree**: 1.

CAS Maple gives this as type `[[_Emden, _Fowler], [_2nd_order, _linear, '_with_symmetry_[0,F`

$$\boxed{4x^2y'' - 4y'x + 3y = 0}$$

✓ Solution by Maple

Time used: 0.016 (sec). Leaf size: 13

```
dsolve(4*x^2*diff(y(x),x$2)-4*x*diff(y(x),x)+3*y(x)=0,y(x), singsol=all)
```

$$y(x) = \sqrt{x}\,(c_2x + c_1)$$

✓ Solution by Mathematica

Time used: 0.012 (sec). Leaf size: 18

```
DSolve[4*x^2*y''[x]-4*x*y'[x]+3*y[x]==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to \sqrt{x}(c_2x + c_1)$$

## 13.4 problem 4

Internal problem ID [11858]

**Book**: Differential Equations by Shepley L. Ross. Third edition. John Willey. New Delhi. 2004.
**Section**: Chapter 4, Section 4.5. The Cauchy-Euler Equation. Exercises page 169
**Problem number**: 4.
**ODE order**: 2.
**ODE degree**: 1.

CAS Maple gives this as type `[[_Emden, _Fowler], [_2nd_order, _linear, '_with_symmetry_[0,F`

$$\boxed{x^2y'' - 3y'x + 4y = 0}$$

✓ Solution by Maple

Time used: 0.016 (sec). Leaf size: 14

```
dsolve(x^2*diff(y(x),x$2)-3*x*diff(y(x),x)+4*y(x)=0,y(x), singsol=all)
```

$$y(x) = x^2(c_1 + c_2 \ln(x))$$

✓ Solution by Mathematica

Time used: 0.018 (sec). Leaf size: 18

```
DSolve[x^2*y''[x]-3*x*y'[x]+4*y[x]==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to x^2(2c_2 \log(x) + c_1)$$

## 13.5 problem 5

Internal problem ID [11859]

**Book**: Differential Equations by Shepley L. Ross. Third edition. John Willey. New Delhi. 2004.
**Section**: Chapter 4, Section 4.5. The Cauchy-Euler Equation. Exercises page 169
**Problem number**: 5.
**ODE order**: 2.
**ODE degree**: 1.

CAS Maple gives this as type `[[_Emden, _Fowler], [_2nd_order, _linear, '_with_symmetry_[0,F`

$$\boxed{x^2y'' + y'x + 4y = 0}$$

✓ Solution by Maple
Time used: 0.0 (sec). Leaf size: 19

```
dsolve(x^2*diff(y(x),x$2)+x*diff(y(x),x)+4*y(x)=0,y(x), singsol=all)
```

$$y(x) = c_1 \sin(2\ln(x)) + c_2 \cos(2\ln(x))$$

✓ Solution by Mathematica
Time used: 0.021 (sec). Leaf size: 22

```
DSolve[x^2*y''[x]+x*y'[x]+4*y[x]==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to c_1 \cos(2\log(x)) + c_2 \sin(2\log(x))$$

## 13.6 problem 6

Internal problem ID [11860]

**Book**: Differential Equations by Shepley L. Ross. Third edition. John Willey. New Delhi. 2004.
**Section**: Chapter 4, Section 4.5. The Cauchy-Euler Equation. Exercises page 169
**Problem number**: 6.
**ODE order**: 2.
**ODE degree**: 1.

CAS Maple gives this as type `[[_Emden, _Fowler]]`

$$\boxed{x^2y'' - 3y'x + 13y = 0}$$

✓ Solution by Maple
Time used: 0.0 (sec). Leaf size: 23

```
dsolve(x^2*diff(y(x),x$2)-3*x*diff(y(x),x)+13*y(x)=0,y(x), singsol=all)
```

$$y(x) = x^2(c_1 \sin(3\ln(x)) + c_2 \cos(3\ln(x)))$$

✓ Solution by Mathematica
Time used: 0.028 (sec). Leaf size: 26

```
DSolve[x^2*y''[x]-3*x*y'[x]+13*y[x]==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to x^2(c_2 \cos(3\log(x)) + c_1 \sin(3\log(x)))$$

## 13.7 problem 7

Internal problem ID [11861]

**Book**: Differential Equations by Shepley L. Ross. Third edition. John Willey. New Delhi. 2004.
**Section**: Chapter 4, Section 4.5. The Cauchy-Euler Equation. Exercises page 169
**Problem number**: 7.
**ODE order**: 2.
**ODE degree**: 1.

CAS Maple gives this as type `[[_Emden, _Fowler], [_2nd_order, _linear, '_with_symmetry_[0,F`

$$\boxed{3x^2y'' - 4y'x + 2y = 0}$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 15

```
dsolve(3*x^2*diff(y(x),x$2)-4*x*diff(y(x),x)+2*y(x)=0,y(x), singsol=all)
```

$$y(x) = c_1x^2 + c_2x^{\frac{1}{3}}$$

✓ Solution by Mathematica

Time used: 0.015 (sec). Leaf size: 20

```
DSolve[3*x^2*y''[x]-4*x*y'[x]+2*y[x]==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to c_2x^2 + c_1\sqrt[3]{x}$$

## 13.8 problem 8

Internal problem ID [11862]

**Book**: Differential Equations by Shepley L. Ross. Third edition. John Willey. New Delhi. 2004.
**Section**: Chapter 4, Section 4.5. The Cauchy-Euler Equation. Exercises page 169
**Problem number**: 8.
**ODE order**: 2.
**ODE degree**: 1.

CAS Maple gives this as type `[[_Emden, _Fowler], [_2nd_order, _linear, '_with_symmetry_[0,F`

$$\boxed{x^2y'' + y'x + 9y = 0}$$

✓ Solution by Maple
Time used: 0.0 (sec). Leaf size: 19

```
dsolve(x^2*diff(y(x),x$2)+x*diff(y(x),x)+9*y(x)=0,y(x), singsol=all)
```

$$y(x) = c_1 \sin(3\ln(x)) + c_2 \cos(3\ln(x))$$

✓ Solution by Mathematica
Time used: 0.021 (sec). Leaf size: 22

```
DSolve[x^2*y''[x]+x*y'[x]+9*y[x]==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to c_1 \cos(3\log(x)) + c_2 \sin(3\log(x))$$

## 13.9 problem 9

Internal problem ID [11863]

**Book**: Differential Equations by Shepley L. Ross. Third edition. John Willey. New Delhi. 2004.
**Section**: Chapter 4, Section 4.5. The Cauchy-Euler Equation. Exercises page 169
**Problem number**: 9.
**ODE order**: 2.
**ODE degree**: 1.

CAS Maple gives this as type `[[_Emden, _Fowler], [_2nd_order, _linear, ‘_with_symmetry_[0,F`

$$\boxed{9x^2y'' + 3y'x + y = 0}$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 14

```
dsolve(9*x^2*diff(y(x),x$2)+3*x*diff(y(x),x)+y(x)=0,y(x), singsol=all)
```

$$y(x) = (c_1 + c_2 \ln(x))\, x^{\frac{1}{3}}$$

✓ Solution by Mathematica

Time used: 0.017 (sec). Leaf size: 24

```
DSolve[9*x^2*y''[x]+3*x*y'[x]+y[x]==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to \frac{1}{3}\sqrt[3]{x}(c_2 \log(x) + 3c_1)$$

## 13.10 problem 10

Internal problem ID [11864]

**Book**: Differential Equations by Shepley L. Ross. Third edition. John Willey. New Delhi. 2004.
**Section**: Chapter 4, Section 4.5. The Cauchy-Euler Equation. Exercises page 169
**Problem number**: 10.
**ODE order**: 2.
**ODE degree**: 1.

CAS Maple gives this as type `[[_Emden, _Fowler]]`

$$\boxed{x^2y'' - 5y'x + 10y = 0}$$

✓ Solution by Maple
Time used: 0.0 (sec). Leaf size: 19

```
dsolve(x^2*diff(y(x),x$2)-5*x*diff(y(x),x)+10*y(x)=0,y(x), singsol=all)
```

$$y(x) = x^3(c_1 \sin(\ln(x)) + \cos(\ln(x)) c_2)$$

✓ Solution by Mathematica
Time used: 0.024 (sec). Leaf size: 22

```
DSolve[x^2*y''[x]-5*x*y'[x]+10*y[x]==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \rightarrow x^3(c_2 \cos(\log(x)) + c_1 \sin(\log(x)))$$

## 13.11 problem 11

Internal problem ID [11865]

**Book**: Differential Equations by Shepley L. Ross. Third edition. John Willey. New Delhi. 2004.
**Section**: Chapter 4, Section 4.5. The Cauchy-Euler Equation. Exercises page 169
**Problem number**: 11.
**ODE order**: 3.
**ODE degree**: 1.

CAS Maple gives this as type `[[_3rd_order, _with_linear_symmetries]]`

$$\boxed{x^3y''' - 3x^2y'' + 6y'x - 6y = 0}$$

✓ Solution by Maple
Time used: 0.0 (sec). Leaf size: 16

```
dsolve(x^3*diff(y(x),x$3)-3*x^2*diff(y(x),x$2)+6*x*diff(y(x),x)-6*y(x)=0,y(x), singsol=all)
```

$$y(x) = x\left(c_3x^2 + c_2x + c_1\right)$$

✓ Solution by Mathematica
Time used: 0.004 (sec). Leaf size: 19

```
DSolve[x^3*y'''[x]-3*x^2*y''[x]+6*x*y'[x]-6*y[x]==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to x(x(c_3x + c_2) + c_1)$$

## 13.12 problem 12

Internal problem ID [11866]

**Book**: Differential Equations by Shepley L. Ross. Third edition. John Willey. New Delhi. 2004.
**Section**: Chapter 4, Section 4.5. The Cauchy-Euler Equation. Exercises page 169
**Problem number**: 12.
**ODE order**: 3.
**ODE degree**: 1.

CAS Maple gives this as type `[[_3rd_order, _fully, _exact, _linear]]`

$$\boxed{x^3y''' + 2x^2y'' - 10y'x - 8y = 0}$$

✓ Solution by Maple
Time used: 0.0 (sec). Leaf size: 18

```
dsolve(x^3*diff(y(x),x$3)+2*x^2*diff(y(x),x$2)-10*x*diff(y(x),x)-8*y(x)=0,y(x), singsol=all)
```

$$y(x) = \frac{c_1x^6 + c_2x + c_3}{x^2}$$

✓ Solution by Mathematica
Time used: 0.004 (sec). Leaf size: 22

```
DSolve[x^3*y'''[x]+2*x^2*y''[x]-10*x*y'[x]-8*y[x]==0,y[x],x,IncludeSingularSolutions -> True
```

$$y(x) \to \frac{c_3x^6 + c_2x + c_1}{x^2}$$

## 13.13 problem 13

Internal problem ID [11867]

**Book**: Differential Equations by Shepley L. Ross. Third edition. John Willey. New Delhi. 2004.
**Section**: Chapter 4, Section 4.5. The Cauchy-Euler Equation. Exercises page 169
**Problem number**: 13.
**ODE order**: 3.
**ODE degree**: 1.

CAS Maple gives this as type `[[_3rd_order, _with_linear_symmetries]]`

$$\boxed{x^3 y''' - x^2 y'' - 6y'x + 18y = 0}$$

✓ Solution by Maple
Time used: 0.0 (sec). Leaf size: 22

```
dsolve(x^3*diff(y(x),x$3)-x^2*diff(y(x),x$2)-6*x*diff(y(x),x)+18*y(x)=0,y(x), singsol=all)
```

$$y(x) = \frac{c_3 x^5 \ln(x) + c_2 x^5 + c_1}{x^2}$$

✓ Solution by Mathematica
Time used: 0.004 (sec). Leaf size: 26

```
DSolve[x^3*y'''[x]-x^2*y''[x]-6*x*y'[x]+18*y[x]==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to \frac{c_2 x^5 + c_3 x^5 \log(x) + c_1}{x^2}$$

## 13.14 problem 14

Internal problem ID [11868]

**Book**: Differential Equations by Shepley L. Ross. Third edition. John Willey. New Delhi. 2004.
**Section**: Chapter 4, Section 4.5. The Cauchy-Euler Equation. Exercises page 169
**Problem number**: 14.
**ODE order**: 2.
**ODE degree**: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$\boxed{x^2y'' - 4y'x + 6y = 4x - 6}$$

✓ Solution by Maple
Time used: 0.015 (sec). Leaf size: 19

```
dsolve(x^2*diff(y(x),x$2)-4*x*diff(y(x),x)+6*y(x)=4*x-6,y(x), singsol=all)
```

$$y(x) = c_1x^3 + c_2x^2 + 2x - 1$$

✓ Solution by Mathematica
Time used: 0.02 (sec). Leaf size: 22

```
DSolve[x^2*y''[x]-4*x*y'[x]+6*y[x]==4*x-6,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to c_2x^3 + c_1x^2 + 2x - 1$$

## 13.15 problem 15

Internal problem ID [11869]

**Book**: Differential Equations by Shepley L. Ross. Third edition. John Willey. New Delhi. 2004.
**Section**: Chapter 4, Section 4.5. The Cauchy-Euler Equation. Exercises page 169
**Problem number**: 15.
**ODE order**: 2.
**ODE degree**: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$\boxed{x^2y'' - 5y'x + 8y = 2x^3}$$

✓ Solution by Maple
Time used: 0.015 (sec). Leaf size: 18

```
dsolve(x^2*diff(y(x),x$2)-5*x*diff(y(x),x)+8*y(x)=2*x^3,y(x), singsol=all)
```

$$y(x) = x^2(c_2x^2 + c_1 - 2x)$$

✓ Solution by Mathematica
Time used: 0.015 (sec). Leaf size: 21

```
DSolve[x^2*y''[x]-5*x*y'[x]+8*y[x]==2*x^3,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to x^2(c_2x^2 - 2x + c_1)$$

## 13.16 problem 16

Internal problem ID [11870]

**Book**: Differential Equations by Shepley L. Ross. Third edition. John Willey. New Delhi. 2004.
**Section**: Chapter 4, Section 4.5. The Cauchy-Euler Equation. Exercises page 169
**Problem number**: 16.
**ODE order**: 2.
**ODE degree**: 1.

CAS Maple gives this as type `[[_2nd_order, _exact, _linear, _nonhomogeneous]]`

$$\boxed{x^2y'' + 4y'x + 2y = 4\ln(x)}$$

✓ Solution by Maple
Time used: 0.0 (sec). Leaf size: 20

```
dsolve(x^2*diff(y(x),x$2)+4*x*diff(y(x),x)+2*y(x)=4*ln(x),y(x), singsol=all)
```

$$y(x) = 2\ln(x) + \frac{c_1}{x} - 3 + \frac{c_2}{x^2}$$

✓ Solution by Mathematica
Time used: 0.022 (sec). Leaf size: 23

```
DSolve[x^2*y''[x]+4*x*y'[x]+2*y[x]==4*Log[x],y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to \frac{c_1}{x^2} + 2\log(x) + \frac{c_2}{x} - 3$$

## 13.17 problem 17

Internal problem ID [11871]

**Book**: Differential Equations by Shepley L. Ross. Third edition. John Willey. New Delhi. 2004.
**Section**: Chapter 4, Section 4.5. The Cauchy-Euler Equation. Exercises page 169
**Problem number**: 17.
**ODE order**: 2.
**ODE degree**: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$\boxed{x^2y'' + y'x + 4y = 2x\ln(x)}$$

✓ Solution by Maple
Time used: 0.016 (sec). Leaf size: 27

```
dsolve(x^2*diff(y(x),x$2)+x*diff(y(x),x)+4*y(x)=2*x*ln(x),y(x), singsol=all)
```

$$y(x) = \sin(2\ln(x))\, c_2 + \cos(2\ln(x))\, c_1 + \frac{2\ln(x)\, x}{5} - \frac{4x}{25}$$

✓ Solution by Mathematica
Time used: 0.11 (sec). Leaf size: 33

```
DSolve[x^2*y''[x]+x*y'[x]+4*y[x]==2*x*Log[x],y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to \frac{2}{25}x(5\log(x) - 2) + c_1\cos(2\log(x)) + c_2\sin(2\log(x))$$

## 13.18 problem 18

Internal problem ID [11872]

**Book**: Differential Equations by Shepley L. Ross. Third edition. John Willey. New Delhi. 2004.
**Section**: Chapter 4, Section 4.5. The Cauchy-Euler Equation. Exercises page 169
**Problem number**: 18.
**ODE order**: 2.
**ODE degree**: 1.

CAS Maple gives this as type `[[_2nd_order, _linear, _nonhomogeneous]]`

$$\boxed{x^2y'' + y'x + 4y = 4\sin(\ln(x))}$$

✓ Solution by Maple
Time used: 0.0 (sec). Leaf size: 24

```
dsolve(x^2*diff(y(x),x$2)+x*diff(y(x),x)+4*y(x)=4*sin(ln(x)),y(x), singsol=all)
```

$$y(x) = \sin(2\ln(x))\, c_2 + \cos(2\ln(x))\, c_1 + \frac{4\sin(\ln(x))}{3}$$

✓ Solution by Mathematica
Time used: 0.176 (sec). Leaf size: 29

```
DSolve[x^2*y''[x]+x*y'[x]+4*y[x]==4*Sin[Log[x]],y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to \frac{4}{3}\sin(\log(x)) + c_1\cos(2\log(x)) + c_2\sin(2\log(x))$$

## 13.19 problem 19

Internal problem ID [11873]

**Book**: Differential Equations by Shepley L. Ross. Third edition. John Willey. New Delhi. 2004.
**Section**: Chapter 4, Section 4.5. The Cauchy-Euler Equation. Exercises page 169
**Problem number**: 19.
**ODE order**: 3.
**ODE degree**: 1.

CAS Maple gives this as type `[[_3rd_order, _with_linear_symmetries]]`

$$\boxed{x^3 y''' - x^2 y'' + 2y'x - 2y = x^3}$$

✓ Solution by Maple
Time used: 0.016 (sec). Leaf size: 23

```
dsolve(x^3*diff(y(x),x$3)-x^2*diff(y(x),x$2)+2*x*diff(y(x),x)-2*y(x)=x^3,y(x), singsol=all)
```

$$y(x) = \frac{x(4c_3 \ln(x) + 4c_2 x + x^2 + 4c_1)}{4}$$

✓ Solution by Mathematica
Time used: 0.005 (sec). Leaf size: 29

```
DSolve[x^3*y'''[x]-x^2*y''[x]+2*x*y'[x]-2*y[x]==x^3,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to \frac{1}{4} x(x^2 + 4c_3 x + 4c_2 \log(x) + 4c_1)$$

## 13.20 problem 20

Internal problem ID [11874]

**Book**: Differential Equations by Shepley L. Ross. Third edition. John Willey. New Delhi. 2004.
**Section**: Chapter 4, Section 4.5. The Cauchy-Euler Equation. Exercises page 169
**Problem number**: 20.
**ODE order**: 2.
**ODE degree**: 1.

CAS Maple gives this as type `[[_Emden, _Fowler], [_2nd_order, _linear, '_with_symmetry_[0,F`

$$\boxed{x^2y'' - 2y'x - 10y = 0}$$

With initial conditions

$$[y(1) = 5, y'(1) = 4]$$

✓ Solution by Maple

Time used: 0.016 (sec). Leaf size: 15

```
dsolve([x^2*diff(y(x),x$2)-2*x*diff(y(x),x)-10*y(x)=0,y(1) = 5, D(y)(1) = 4],y(x), singsol=a
```

$$y(x) = 2x^5 + \frac{3}{x^2}$$

✓ Solution by Mathematica

Time used: 0.02 (sec). Leaf size: 16

```
DSolve[{x^2*y''[x]-2*x*y'[x]-10*y[x]==0,{y[1]==5,y'[1]==4}},y[x],x,IncludeSingularSolutions
```

$$y(x) \to \frac{2x^7 + 3}{x^2}$$

## 13.21 problem 21

Internal problem ID [11875]

**Book**: Differential Equations by Shepley L. Ross. Third edition. John Willey. New Delhi. 2004.
**Section**: Chapter 4, Section 4.5. The Cauchy-Euler Equation. Exercises page 169
**Problem number**: 21.
**ODE order**: 2.
**ODE degree**: 1.

CAS Maple gives this as type `[[_Emden, _Fowler], [_2nd_order, _linear, '_with_symmetry_[0,F`

$$\boxed{x^2y'' - 4y'x + 6y = 0}$$

With initial conditions

$$[y(2) = 0, y'(2) = 4]$$

✓ Solution by Maple

Time used: 0.015 (sec). Leaf size: 11

```
dsolve([x^2*diff(y(x),x$2)-4*x*diff(y(x),x)+6*y(x)=0,y(2) = 0, D(y)(2) = 4],y(x), singsol=al
```

$$y(x) = x^2(x - 2)$$

✓ Solution by Mathematica

Time used: 0.011 (sec). Leaf size: 12

```
DSolve[{x^2*y''[x]-4*x*y'[x]+6*y[x]==0,{y[2]==0,y'[2]==4}},y[x],x,IncludeSingularSolutions -
```

$$y(x) \to (x - 2)x^2$$

## 13.22 problem 22

Internal problem ID [11876]

**Book**: Differential Equations by Shepley L. Ross. Third edition. John Willey. New Delhi. 2004.
**Section**: Chapter 4, Section 4.5. The Cauchy-Euler Equation. Exercises page 169
**Problem number**: 22.
**ODE order**: 2.
**ODE degree**: 1.

CAS Maple gives this as type `[[_2nd_order, _exact, _linear, _homogeneous]]`

$$\boxed{x^2 y'' + 5y'x + 3y = 0}$$

With initial conditions

$$[y(1) = 1, y'(1) = -5]$$

✓ Solution by Maple
Time used: 0.016 (sec). Leaf size: 15

```
dsolve([x^2*diff(y(x),x$2)+5*x*diff(y(x),x)+3*y(x)=0,y(1) = 1, D(y)(1) = -5],y(x), singsol=a
```

$$y(x) = \frac{-x^2 + 2}{x^3}$$

✓ Solution by Mathematica
Time used: 0.011 (sec). Leaf size: 16

```
DSolve[{x^2*y''[x]+5*x*y'[x]+3*y[x]==0,{y[1]==1,y'[1]==-5}},y[x],x,IncludeSingularSolutions
```

$$y(x) \to \frac{2 - x^2}{x^3}$$

## 13.23 problem 23

Internal problem ID [11877]

**Book**: Differential Equations by Shepley L. Ross. Third edition. John Willey. New Delhi. 2004.
**Section**: Chapter 4, Section 4.5. The Cauchy-Euler Equation. Exercises page 169
**Problem number**: 23.
**ODE order**: 2.
**ODE degree**: 1.

CAS Maple gives this as type `[[_2nd_order, _exact, _linear, _nonhomogeneous]]`

$$\boxed{x^2 y'' - 2y = 4x - 8}$$

With initial conditions

$$[y(1) = 4, y'(1) = -1]$$

✓ Solution by Maple
Time used: 0.0 (sec). Leaf size: 15

```
dsolve([x^2*diff(y(x),x$2)-2*y(x)=4*x-8,y(1) = 4, D(y)(1) = -1],y(x), singsol=all)
```

$$y(x) = x^2 + 4 - 2x + \frac{1}{x}$$

✓ Solution by Mathematica
Time used: 0.016 (sec). Leaf size: 16

```
DSolve[{x^2*y''[x]-2*y[x]==4*x-8,{y[1]==4,y'[1]==-1}},y[x],x,IncludeSingularSolutions -> Tru
```

$$y(x) \to x^2 - 2x + \frac{1}{x} + 4$$

## 13.24 problem 24

Internal problem ID [11878]

**Book**: Differential Equations by Shepley L. Ross. Third edition. John Willey. New Delhi. 2004.
**Section**: Chapter 4, Section 4.5. The Cauchy-Euler Equation. Exercises page 169
**Problem number**: 24.
**ODE order**: 2.
**ODE degree**: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$\boxed{x^2y'' - 4y'x + 4y = -6x^3 + 4x^2}$$

With initial conditions

$$[y(2) = 4, y'(2) = -1]$$

✓ Solution by Maple
Time used: 0.016 (sec). Leaf size: 23

```
dsolve([x^2*diff(y(x),x$2)-4*x*diff(y(x),x)+4*y(x)=4*x^2-6*x^3,y(2) = 4, D(y)(2) = -1],y(x),
```

$$y(x) = -\frac{23}{24}x^4 + 3x^3 - 2x^2 + \frac{5}{3}x$$

✓ Solution by Mathematica
Time used: 0.02 (sec). Leaf size: 28

```
DSolve[{x^2*y''[x]-4*x*y'[x]+4*y[x]==4*x^2-6*x^3,{y[2]==4,y'[2]==-1}},y[x],x,IncludeSingular
```

$$y(x) \to -\frac{23x^4}{24} + 3x^3 - 2x^2 + \frac{5x}{3}$$

## 13.25 problem 25

Internal problem ID [11879]

**Book**: Differential Equations by Shepley L. Ross. Third edition. John Willey. New Delhi. 2004.
**Section**: Chapter 4, Section 4.5. The Cauchy-Euler Equation. Exercises page 169
**Problem number**: 25.
**ODE order**: 2.
**ODE degree**: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$\boxed{x^2y'' + 2y'x - 6y = 10x^2}$$

With initial conditions

$$[y(1) = 1, y'(1) = -6]$$

✓ Solution by Maple

Time used: 0.015 (sec). Leaf size: 22

```
dsolve([x^2*diff(y(x),x$2)+2*x*diff(y(x),x)-6*y(x)=10*x^2,y(1) = 1, D(y)(1) = -6],y(x), sing
```

$$y(x) = \frac{2x^5 \ln(x) - x^5 + 2}{x^3}$$

✓ Solution by Mathematica

Time used: 0.018 (sec). Leaf size: 23

```
DSolve[{x^2*y''[x]+2*x*y'[x]-6*y[x]==10*x^2,{y[1]==1,y'[1]==-6}},y[x],x,IncludeSingularSolut
```

$$y(x) \to \frac{-x^5 + 2x^5 \log(x) + 2}{x^3}$$

## 13.26 problem 26

Internal problem ID [11880]

**Book**: Differential Equations by Shepley L. Ross. Third edition. John Willey. New Delhi. 2004.
**Section**: Chapter 4, Section 4.5. The Cauchy-Euler Equation. Exercises page 169
**Problem number**: 26.
**ODE order**: 2.
**ODE degree**: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$\boxed{x^2y'' - 5y'x + 8y = 2x^3}$$

With initial conditions

$$[y(2) = 0, y'(2) = -8]$$

✓ Solution by Maple
Time used: 0.016 (sec). Leaf size: 15

```
dsolve([x^2*diff(y(x),x$2)-5*x*diff(y(x),x)+8*y(x)=2*x^3,y(2) = 0, D(y)(2) = -8],y(x), sings
```

$$y(x) = -2x^3 + 4x^2$$

✓ Solution by Mathematica
Time used: 0.013 (sec). Leaf size: 13

```
DSolve[{x^2*y''[x]-5*x*y'[x]+8*y[x]==2*x^3,{y[2]==0,y'[2]==-8}},y[x],x,IncludeSingularSoluti
```

$$y(x) \to -2(x-2)x^2$$

## 13.27 problem 27

Internal problem ID [11881]

**Book**: Differential Equations by Shepley L. Ross. Third edition. John Willey. New Delhi. 2004.
**Section**: Chapter 4, Section 4.5. The Cauchy-Euler Equation. Exercises page 169
**Problem number**: 27.
**ODE order**: 2.
**ODE degree**: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$\boxed{x^2y'' - 6y = \ln(x)}$$

With initial conditions

$$\left[y(1) = \frac{1}{6}, y'(1) = -\frac{1}{6}\right]$$

✓ Solution by Maple

Time used: 0.016 (sec). Leaf size: 20

```
dsolve([x^2*diff(y(x),x$2)-6*y(x)=ln(x),y(1) = 1/6, D(y)(1) = -1/6],y(x), singsol=all)
```

$$y(x) = \frac{1}{12x^2} + \frac{x^3}{18} - \frac{\ln(x)}{6} + \frac{1}{36}$$

✓ Solution by Mathematica

Time used: 0.019 (sec). Leaf size: 29

```
DSolve[{x^2*y''[x]-6*y[x]==Log[x],{y[1]==1/6,y'[1]==-1/6}},y[x],x,IncludeSingularSolutions -
```

$$y(x) \to \frac{2x^5 + x^2 - 6x^2\log(x) + 3}{36x^2}$$

## 13.28 problem 28

Internal problem ID [11882]

**Book**: Differential Equations by Shepley L. Ross. Third edition. John Willey. New Delhi. 2004.
**Section**: Chapter 4, Section 4.5. The Cauchy-Euler Equation. Exercises page 169
**Problem number**: 28.
**ODE order**: 2.
**ODE degree**: 1.

CAS Maple gives this as type `[[_2nd_order, _exact, _linear, _homogeneous]]`

$$\boxed{(x+2)^2 y'' - (x+2) y' - 3y = 0}$$

✓ Solution by Maple
Time used: 0.015 (sec). Leaf size: 19

```
dsolve((x+2)^2*diff(y(x),x$2)-(x+2)*diff(y(x),x)-3*y(x)=0,y(x), singsol=all)
```

$$y(x) = \frac{c_1 + c_2(x+2)^4}{x+2}$$

✓ Solution by Mathematica
Time used: 0.031 (sec). Leaf size: 22

```
DSolve[(x+2)^2*y''[x]-(x+2)*y'[x]-3*y[x]==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to c_1(x+2)^3 + \frac{c_2}{x+2}$$

## 13.29 problem 29

Internal problem ID [11883]

**Book**: Differential Equations by Shepley L. Ross. Third edition. John Willey. New Delhi. 2004.
**Section**: Chapter 4, Section 4.5. The Cauchy-Euler Equation. Exercises page 169
**Problem number**: 29.
**ODE order**: 2.
**ODE degree**: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$\boxed{(2x-3)^2 y'' - 6(2x-3) y' + 12y = 0}$$

✓ Solution by Maple
Time used: 0.0 (sec). Leaf size: 17

```
dsolve((2*x-3)^2*diff(y(x),x$2)-6*(2*x-3)*diff(y(x),x)+12*y(x)=0,y(x), singsol=all)
```

$$y(x) = \left(x - \frac{3}{2}\right)\left(c_1 + c_2\left(x - \frac{3}{2}\right)^2\right)$$

✓ Solution by Mathematica
Time used: 0.034 (sec). Leaf size: 24

```
DSolve[(2*x-3)^2*y''[x]-6*(2*x-3)*y'[x]+12*y[x]==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to c_2(3-2x)^3 + c_1(3-2x)$$

# 14 Chapter 6, Series solutions of linear differential equations. Section 6.1. Exercises page 232

## 14.1 problem 1

Internal problem ID [11884]

**Book**: Differential Equations by Shepley L. Ross. Third edition. John Willey. New Delhi. 2004.
**Section**: Chapter 6, Series solutions of linear differential equations. Section 6.1. Exercises page 232
**Problem number**: 1.
**ODE order**: 2.
**ODE degree**: 1.

CAS Maple gives this as type `[[_2nd_order, _exact, _linear, _homogeneous]]`

$$\boxed{y'' + y'x + y = 0}$$

With the expansion point for the power series method at $x = 0$.

✓ Solution by Maple
Time used: 0.016 (sec). Leaf size: 34

```
Order:=6;
dsolve(diff(y(x),x$2)+x*diff(y(x),x)+y(x)=0,y(x),type='series',x=0);
```

$$y(x) = \left(1 - \frac{1}{2}x^2 + \frac{1}{8}x^4\right) y(0) + \left(x - \frac{1}{3}x^3 + \frac{1}{15}x^5\right) D(y)(0) + O(x^6)$$

✓ Solution by Mathematica
Time used: 0.007 (sec). Leaf size: 42

```
AsymptoticDSolveValue[y''[x]+x*y'[x]+y[x]==0,y[x],{x,0,5}]
```

$$y(x) \to c_2\left(\frac{x^5}{15} - \frac{x^3}{3} + x\right) + c_1\left(\frac{x^4}{8} - \frac{x^2}{2} + 1\right)$$

## 14.2 problem 2

Internal problem ID [11885]

**Book**: Differential Equations by Shepley L. Ross. Third edition. John Willey. New Delhi. 2004.
**Section**: Chapter 6, Series solutions of linear differential equations. Section 6.1. Exercises page 232
**Problem number**: 2.
**ODE order**: 2.
**ODE degree**: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$\boxed{y'' + 8y'x - 4y = 0}$$

With the expansion point for the power series method at $x = 0$.

✓ Solution by Maple
Time used: 0.0 (sec). Leaf size: 34

```
Order:=6;
dsolve(diff(y(x),x$2)+8*x*diff(y(x),x)-4*y(x)=0,y(x),type='series',x=0);
```

$$y(x) = \left(-2x^4 + 2x^2 + 1\right) y(0) + \left(x - \frac{2}{3}x^3 + \frac{2}{3}x^5\right) D(y)(0) + O\left(x^6\right)$$

✓ Solution by Mathematica
Time used: 0.001 (sec). Leaf size: 38

```
AsymptoticDSolveValue[y''[x]+8*x*y'[x]-4*y[x]==0,y[x],{x,0,5}]
```

$$y(x) \to c_2\left(\frac{2x^5}{3} - \frac{2x^3}{3} + x\right) + c_1\left(-2x^4 + 2x^2 + 1\right)$$

## 14.3 problem 3

Internal problem ID [11886]

**Book**: Differential Equations by Shepley L. Ross. Third edition. John Willey. New Delhi. 2004.
**Section**: Chapter 6, Series solutions of linear differential equations. Section 6.1. Exercises page 232
**Problem number**: 3.
**ODE order**: 2.
**ODE degree**: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$\boxed{y'' + y'x + (2x^2 + 1)\, y = 0}$$

With the expansion point for the power series method at $x = 0$.

✓ Solution by Maple
Time used: 0.0 (sec). Leaf size: 34

```
Order:=6;
dsolve(diff(y(x),x$2)+x*diff(y(x),x)+(2*x^2+1)*y(x)=0,y(x),type='series',x=0);
```

$$y(x) = \left(1 - \frac{1}{2}x^2 - \frac{1}{24}x^4\right) y(0) + \left(x - \frac{1}{3}x^3 - \frac{1}{30}x^5\right) D(y)(0) + O(x^6)$$

✓ Solution by Mathematica
Time used: 0.001 (sec). Leaf size: 42

```
AsymptoticDSolveValue[y''[x]+x*y'[x]+(2*x^2+1)*y[x]==0,y[x],{x,0,5}]
```

$$y(x) \to c_2\left(-\frac{x^5}{30} - \frac{x^3}{3} + x\right) + c_1\left(-\frac{x^4}{24} - \frac{x^2}{2} + 1\right)$$

## 14.4 problem 4

Internal problem ID [11887]

**Book**: Differential Equations by Shepley L. Ross. Third edition. John Willey. New Delhi. 2004.
**Section**: Chapter 6, Series solutions of linear differential equations. Section 6.1. Exercises page 232
**Problem number**: 4.
**ODE order**: 2.
**ODE degree**: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$\boxed{y'' + y'x + (x^2 - 4)\, y = 0}$$

With the expansion point for the power series method at $x = 0$.

✓ Solution by Maple
Time used: 0.0 (sec). Leaf size: 34

```
Order:=6;
dsolve(diff(y(x),x$2)+x*diff(y(x),x)+(x^2-4)*y(x)=0,y(x),type='series',x=0);
```

$$y(x) = \left(1 + 2x^2 + \frac{1}{4}x^4\right) y(0) + \left(x + \frac{1}{2}x^3 - \frac{1}{40}x^5\right) D(y)\,(0) + O\left(x^6\right)$$

✓ Solution by Mathematica
Time used: 0.001 (sec). Leaf size: 40

```
AsymptoticDSolveValue[y''[x]+x*y'[x]+(x^2-4)*y[x]==0,y[x],{x,0,5}]
```

$$y(x) \to c_2\left(-\frac{x^5}{40} + \frac{x^3}{2} + x\right) + c_1\left(\frac{x^4}{4} + 2x^2 + 1\right)$$

## 14.5 problem 5

Internal problem ID [11888]

**Book**: Differential Equations by Shepley L. Ross. Third edition. John Willey. New Delhi. 2004.
**Section**: Chapter 6, Series solutions of linear differential equations. Section 6.1. Exercises page 232
**Problem number**: 5.
**ODE order**: 2.
**ODE degree**: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$\boxed{y'' + y'x + (3x + 2)\, y = 0}$$

With the expansion point for the power series method at $x = 0$.

✓ Solution by Maple
Time used: 0.0 (sec). Leaf size: 49

```
Order:=6;
dsolve(diff(y(x),x$2)+x*diff(y(x),x)+(3*x+2)*y(x)=0,y(x),type='series',x=0);
```

$$y(x) = \left(1 - x^2 - \frac{1}{2}x^3 + \frac{1}{3}x^4 + \frac{11}{40}x^5\right) y(0) + \left(x - \frac{1}{2}x^3 - \frac{1}{4}x^4 + \frac{1}{8}x^5\right) D(y)\,(0) + O(x^6)$$

✓ Solution by Mathematica
Time used: 0.001 (sec). Leaf size: 61

```
AsymptoticDSolveValue[y''[x]+x*y'[x]+(3*x+2)*y[x]==0,y[x],{x,0,5}]
```

$$y(x) \to c_2\left(\frac{x^5}{8} - \frac{x^4}{4} - \frac{x^3}{2} + x\right) + c_1\left(\frac{11x^5}{40} + \frac{x^4}{3} - \frac{x^3}{2} - x^2 + 1\right)$$

## 14.6 problem 6

Internal problem ID [11889]

**Book**: Differential Equations by Shepley L. Ross. Third edition. John Willey. New Delhi. 2004.
**Section**: Chapter 6, Series solutions of linear differential equations. Section 6.1. Exercises page 232
**Problem number**: 6.
**ODE order**: 2.
**ODE degree**: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$\boxed{y'' - y'x + (3x - 2)\,y = 0}$$

With the expansion point for the power series method at $x = 0$.

✓ Solution by Maple
Time used: 0.0 (sec). Leaf size: 47

```
Order:=6;
dsolve(diff(y(x),x$2)-x*diff(y(x),x)+(3*x-2)*y(x)=0,y(x),type='series',x=0);
```

$$y(x) = \left(1 + x^2 - \frac{1}{2}x^3 + \frac{1}{3}x^4 - \frac{11}{40}x^5\right) y(0) + \left(x + \frac{1}{2}x^3 - \frac{1}{4}x^4 + \frac{1}{8}x^5\right) D(y)\,(0) + O(x^6)$$

✓ Solution by Mathematica
Time used: 0.001 (sec). Leaf size: 59

```
AsymptoticDSolveValue[y''[x]-x*y'[x]+(3*x-2)*y[x]==0,y[x],{x,0,5}]
```

$$y(x) \to c_2\left(\frac{x^5}{8} - \frac{x^4}{4} + \frac{x^3}{2} + x\right) + c_1\left(-\frac{11x^5}{40} + \frac{x^4}{3} - \frac{x^3}{2} + x^2 + 1\right)$$

## 14.7 problem 7

Internal problem ID [11890]

**Book**: Differential Equations by Shepley L. Ross. Third edition. John Willey. New Delhi. 2004.
**Section**: Chapter 6, Series solutions of linear differential equations. Section 6.1. Exercises page 232
**Problem number**: 7.
**ODE order**: 2.
**ODE degree**: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$\boxed{(x^2+1)\,y''+y'x+yx=0}$$

With the expansion point for the power series method at $x=0$.

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 39

```
Order:=6;
dsolve((x^2+1)*diff(y(x),x$2)+x*diff(y(x),x)+x*y(x)=0,y(x),type='series',x=0);
```

$$y(x)=\left(1-\frac{1}{6}x^3+\frac{3}{40}x^5\right)y(0)+\left(x-\frac{1}{6}x^3-\frac{1}{12}x^4+\frac{3}{40}x^5\right)D(y)(0)+O(x^6)$$

✓ Solution by Mathematica

Time used: 0.001 (sec). Leaf size: 49

```
AsymptoticDSolveValue[(x^2+1)*y''[x]+x*y'[x]+x*y[x]==0,y[x],{x,0,5}]
```

$$y(x)\to c_1\left(\frac{3x^5}{40}-\frac{x^3}{6}+1\right)+c_2\left(\frac{3x^5}{40}-\frac{x^4}{12}-\frac{x^3}{6}+x\right)$$

## 14.8 problem 8

Internal problem ID [11891]

**Book**: Differential Equations by Shepley L. Ross. Third edition. John Willey. New Delhi. 2004.
**Section**: Chapter 6, Series solutions of linear differential equations. Section 6.1. Exercises page 232
**Problem number**: 8.
**ODE order**: 2.
**ODE degree**: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$\boxed{(x-1)\,y''-(3x-2)\,y'+2yx=0}$$

With the expansion point for the power series method at $x=0$.

✓ Solution by Maple
Time used: 0.016 (sec). Leaf size: 47

```
Order:=6;
dsolve((x-1)*diff(y(x),x$2)-(3*x-2)*diff(y(x),x)+2*x*y(x)=0,y(x),type='series',x=0);
```

$$y(x)=\left(1+\frac{1}{3}x^3+\frac{1}{3}x^4+\frac{11}{60}x^5\right)y(0)+\left(x+x^2+\frac{1}{2}x^3+\frac{1}{6}x^4+\frac{1}{24}x^5\right)D(y)(0)+O\left(x^6\right)$$

✓ Solution by Mathematica
Time used: 0.001 (sec). Leaf size: 59

```
AsymptoticDSolveValue[(x-1)*y''[x]-(3*x-2)*y'[x]+2*x*y[x]==0,y[x],{x,0,5}]
```

$$y(x)\to c_1\left(\frac{11x^5}{60}+\frac{x^4}{3}+\frac{x^3}{3}+1\right)+c_2\left(\frac{x^5}{24}+\frac{x^4}{6}+\frac{x^3}{2}+x^2+x\right)$$

## 14.9 problem 9

Internal problem ID [11892]

**Book**: Differential Equations by Shepley L. Ross. Third edition. John Willey. New Delhi. 2004.
**Section**: Chapter 6, Series solutions of linear differential equations. Section 6.1. Exercises page 232
**Problem number**: 9.
**ODE order**: 2.
**ODE degree**: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$\boxed{(x^3-1)\,y''+x^2y'+yx=0}$$

With the expansion point for the power series method at $x=0$.

✓ Solution by Maple
Time used: 0.015 (sec). Leaf size: 24

```
Order:=6;
dsolve((x^3-1)*diff(y(x),x$2)+x^2*diff(y(x),x)+x*y(x)=0,y(x),type='series',x=0);
```

$$y(x)=\left(1+\frac{x^3}{6}\right)y(0)+\left(x+\frac{1}{6}x^4\right)D(y)\,(0)+O(x^6)$$

✓ Solution by Mathematica
Time used: 0.001 (sec). Leaf size: 28

```
AsymptoticDSolveValue[(x^3-1)*y''[x]+x^2*y'[x]+x*y[x]==0,y[x],{x,0,5}]
```

$$y(x)\to c_2\left(\frac{x^4}{6}+x\right)+c_1\left(\frac{x^3}{6}+1\right)$$

## 14.10 problem 10

Internal problem ID [11893]

**Book**: Differential Equations by Shepley L. Ross. Third edition. John Willey. New Delhi. 2004.
**Section**: Chapter 6, Series solutions of linear differential equations. Section 6.1. Exercises page 232
**Problem number**: 10.
**ODE order**: 2.
**ODE degree**: 1.

CAS Maple gives this as type `[[_2nd_order, _exact, _linear, _homogeneous]]`

$$\boxed{(x+3)\,y''+(x+2)\,y'+y=0}$$

With the expansion point for the power series method at $x=0$.

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 49

```
Order:=6;
dsolve((x+3)*diff(y(x),x$2)+(x+2)*diff(y(x),x)+y(x)=0,y(x),type='series',x=0);
```

$$y(x)=\left(1-\frac{1}{6}x^2+\frac{1}{18}x^3-\frac{1}{216}x^4-\frac{7}{3240}x^5\right)y(0)+\left(x-\frac{1}{3}x^2+\frac{1}{36}x^4-\frac{1}{108}x^5\right)D(y)(0)+O\left(x^6\right)$$

✓ Solution by Mathematica

Time used: 0.001 (sec). Leaf size: 63

```
AsymptoticDSolveValue[(x+3)*y''[x]+(x+2)*y'[x]+y[x]==0,y[x],{x,0,5}]
```

$$y(x)\to c_2\left(-\frac{x^5}{108}+\frac{x^4}{36}-\frac{x^2}{3}+x\right)+c_1\left(-\frac{7x^5}{3240}-\frac{x^4}{216}+\frac{x^3}{18}-\frac{x^2}{6}+1\right)$$

## 14.11 problem 11

Internal problem ID [11894]

**Book**: Differential Equations by Shepley L. Ross. Third edition. John Willey. New Delhi. 2004.
**Section**: Chapter 6, Series solutions of linear differential equations. Section 6.1. Exercises page 232
**Problem number**: 11.
**ODE order**: 2.
**ODE degree**: 1.

CAS Maple gives this as type `[[_2nd_order, _exact, _linear, _homogeneous]]`

$$\boxed{y'' - y'x - y = 0}$$

With initial conditions

$$[y(0) = 1, y'(0) = 0]$$

With the expansion point for the power series method at $x = 0$.

✓ Solution by Maple
Time used: 0.0 (sec). Leaf size: 14

```
Order:=6;
dsolve([diff(y(x),x$2)-x*diff(y(x),x)-y(x)=0,y(0) = 1, D(y)(0) = 0],y(x),type='series',x=0);
```

$$y(x) = 1 + \frac{1}{2}x^2 + \frac{1}{8}x^4 + \mathrm{O}\left(x^6\right)$$

✓ Solution by Mathematica
Time used: 0.001 (sec). Leaf size: 19

```
AsymptoticDSolveValue[{y''[x]-x*y'[x]-y[x]==0,{y[0]==1,y'[0]==0}},y[x],{x,0,5}]
```

$$y(x) \to \frac{x^4}{8} + \frac{x^2}{2} + 1$$

## 14.12 problem 12

Internal problem ID [11895]

**Book**: Differential Equations by Shepley L. Ross. Third edition. John Willey. New Delhi. 2004.
**Section**: Chapter 6, Series solutions of linear differential equations. Section 6.1. Exercises page 232
**Problem number**: 12.
**ODE order**: 2.
**ODE degree**: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$\boxed{y'' + y'x - 2y = 0}$$

With initial conditions

$$[y(0) = 0, y'(0) = 1]$$

With the expansion point for the power series method at $x = 0$.

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 14

```
Order:=6;
dsolve([diff(y(x),x$2)+x*diff(y(x),x)-2*y(x)=0,y(0) = 0, D(y)(0) = 1],y(x),type='series',x=0
```

$$y(x) = x + \frac{1}{6}x^3 - \frac{1}{120}x^5 + \mathrm{O}\left(x^6\right)$$

✓ Solution by Mathematica

Time used: 0.001 (sec). Leaf size: 19

```
AsymptoticDSolveValue[{y''[x]+x*y'[x]-2*y[x]==0,{y[0]==0,y'[0]==1}},y[x],{x,0,5}]
```

$$y(x) \to -\frac{x^5}{120} + \frac{x^3}{6} + x$$

## 14.13 problem 13

Internal problem ID [11896]

**Book**: Differential Equations by Shepley L. Ross. Third edition. John Willey. New Delhi. 2004.
**Section**: Chapter 6, Series solutions of linear differential equations. Section 6.1. Exercises page 232
**Problem number**: 13.
**ODE order**: 2.
**ODE degree**: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$\boxed{(x^2+1)\, y''+y'x+2yx=0}$$

With initial conditions

$$[y(0)=2, y'(0)=3]$$

With the expansion point for the power series method at $x=0$.

✓ Solution by Maple
Time used: 0.0 (sec). Leaf size: 18

```
Order:=6;
dsolve([(x^2+1)*diff(y(x),x$2)+x*diff(y(x),x)+2*x*y(x)=0,y(0) = 2, D(y)(0) = 3],y(x),type='s
```

$$y(x)=2+3x-\frac{7}{6}x^3-\frac{1}{2}x^4+\frac{21}{40}x^5+\mathrm{O}\left(x^6\right)$$

✓ Solution by Mathematica
Time used: 0.002 (sec). Leaf size: 29

```
AsymptoticDSolveValue[{(x^2+1)*y''[x]+x*y'[x]+2*x*y[x]==0,{y[0]==2,y'[0]==3}},y[x],{x,0,5}]
```

$$y(x)\to \frac{21x^5}{40}-\frac{x^4}{2}-\frac{7x^3}{6}+3x+2$$

## 14.14 problem 14

Internal problem ID [11897]

**Book**: Differential Equations by Shepley L. Ross. Third edition. John Willey. New Delhi. 2004.
**Section**: Chapter 6, Series solutions of linear differential equations. Section 6.1. Exercises page 232
**Problem number**: 14.
**ODE order**: 2.
**ODE degree**: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$\boxed{(2x^2-3)\,y''-2y'x+y=0}$$

With initial conditions

$$[y(0)=-1, y'(0)=5]$$

With the expansion point for the power series method at $x=0$.

✓ Solution by Maple
Time used: 0.016 (sec). Leaf size: 20

```
Order:=6;
dsolve([(2*x^2-3)*diff(y(x),x$2)-2*x*diff(y(x),x)+y(x)=0,y(0) = -1, D(y)(0) = 5],y(x),type='
```

$$y(x)=-1+5x-\frac{1}{6}x^2-\frac{5}{18}x^3-\frac{1}{216}x^4-\frac{7}{216}x^5+\mathrm{O}\left(x^6\right)$$

✓ Solution by Mathematica
Time used: 0.001 (sec). Leaf size: 36

```
AsymptoticDSolveValue[{(2*x^2-3)*y''[x]-2*x*y'[x]+y[x]==0,{y[0]==-1,y'[0]==5}},y[x],{x,0,5}]
```

$$y(x)\to -\frac{7x^5}{216}-\frac{x^4}{216}-\frac{5x^3}{18}-\frac{x^2}{6}+5x-1$$

## 14.15 problem 15

Internal problem ID [11898]

**Book**: Differential Equations by Shepley L. Ross. Third edition. John Willey. New Delhi. 2004.
**Section**: Chapter 6, Series solutions of linear differential equations. Section 6.1. Exercises page 232
**Problem number**: 15.
**ODE order**: 2.
**ODE degree**: 1.

CAS Maple gives this as type `[[_Emden, _Fowler], [_2nd_order, _linear, '_with_symmetry_[0,F`

$$\boxed{x^2y'' + y'x + y = 0}$$

With the expansion point for the power series method at $x = 1$.

✓ Solution by Maple
Time used: 0.015 (sec). Leaf size: 49

```
Order:=6;
dsolve(x^2*diff(y(x),x$2)+x*diff(y(x),x)+y(x)=0,y(x),type='series',x=1);
```

$$y(x) = \left(1 - \frac{(-1+x)^2}{2} + \frac{(-1+x)^3}{2} - \frac{5(-1+x)^4}{12} + \frac{(-1+x)^5}{3}\right) y(1) + \left(-1 + x - \frac{(-1+x)^2}{2} + \frac{(-1+x)^3}{6} - \frac{(-1+x)^5}{12}\right) D(y)(1) + O(x^6)$$

✓ Solution by Mathematica
Time used: 0.001 (sec). Leaf size: 78

```
AsymptoticDSolveValue[x^2*y''[x]+x*y'[x]+y[x]==0,y[x],{x,1,5}]
```

$$y(x) \to c_1\left(\frac{1}{3}(x-1)^5 - \frac{5}{12}(x-1)^4 + \frac{1}{2}(x-1)^3 - \frac{1}{2}(x-1)^2 + 1\right) + c_2\left(-\frac{1}{12}(x-1)^5 + \frac{1}{6}(x-1)^3 - \frac{1}{2}(x-1)^2 + x - 1\right)$$

## 14.16 problem 16

Internal problem ID [11899]

**Book**: Differential Equations by Shepley L. Ross. Third edition. John Willey. New Delhi. 2004.
**Section**: Chapter 6, Series solutions of linear differential equations. Section 6.1. Exercises page 232
**Problem number**: 16.
**ODE order**: 2.
**ODE degree**: 1.

CAS Maple gives this as type `[[_Emden, _Fowler]]`

$$\boxed{x^2y'' + 3y'x - y = 0}$$

With the expansion point for the power series method at $x = 1$.

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 54

```
Order:=6;
dsolve(x^2*diff(y(x),x$2)+3*x*diff(y(x),x)-y(x)=0,y(x),type='series',x=1);
```

$$y(x) = \left(1 + \frac{(-1+x)^2}{2} - \frac{5(-1+x)^3}{6} + \frac{7(-1+x)^4}{6} - \frac{91(-1+x)^5}{60}\right) y(1) + \left(-1+x - \frac{3(-1+x)^2}{2} + \frac{13(-1+x)^3}{6} - \frac{35(-1+x)^4}{12} + \frac{56(-1+x)^5}{15}\right) D(y)(1) + O(x^6)$$

✓ Solution by Mathematica

Time used: 0.001 (sec). Leaf size: 87

```
AsymptoticDSolveValue[x^2*y''[x]+3*x*y'[x]-y[x]==0,y[x],{x,1,5}]
```

$$y(x) \to c_1\left(-\frac{91}{60}(x-1)^5 + \frac{7}{6}(x-1)^4 - \frac{5}{6}(x-1)^3 + \frac{1}{2}(x-1)^2 + 1\right) + c_2\left(\frac{56}{15}(x-1)^5 - \frac{35}{12}(x-1)^4 + \frac{13}{6}(x-1)^3 - \frac{3}{2}(x-1)^2 + x - 1\right)$$

## 14.17 problem 17

Internal problem ID [11900]

**Book**: Differential Equations by Shepley L. Ross. Third edition. John Willey. New Delhi. 2004.
**Section**: Chapter 6, Series solutions of linear differential equations. Section 6.1. Exercises page 232
**Problem number**: 17.
**ODE order**: 2.
**ODE degree**: 1.

CAS Maple gives this as type `[[_Emden, _Fowler]]`

$$\boxed{xy'' + y' + 2y = 0}$$

With initial conditions

$$[y(1) = 2, y'(1) = 4]$$

With the expansion point for the power series method at $x = 1$.

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 20

```
Order:=6;
dsolve([x*diff(y(x),x$2)+diff(y(x),x)+2*y(x)=0,y(1) = 2, D(y)(1) = 4],y(x),type='series',x=1
```

$$y(x) = 2 + 4(-1 + x) - 4(-1 + x)^2 + \frac{4}{3}(-1 + x)^3 - \frac{1}{3}(-1 + x)^4 + \frac{2}{15}(-1 + x)^5 + \mathrm{O}\left((-1 + x)^6\right)$$

✓ Solution by Mathematica

Time used: 0.001 (sec). Leaf size: 44

```
AsymptoticDSolveValue[{x*y''[x]+y'[x]+2*y[x]==0,{y[1]==2,y'[1]==4}},y[x],{x,1,5}]
```

$$y(x) \to \frac{2}{15}(x - 1)^5 - \frac{1}{3}(x - 1)^4 + \frac{4}{3}(x - 1)^3 - 4(x - 1)^2 + 4(x - 1) + 2$$

## 14.18 problem 18

Internal problem ID [11901]

**Book**: Differential Equations by Shepley L. Ross. Third edition. John Willey. New Delhi. 2004.
**Section**: Chapter 6, Series solutions of linear differential equations. Section 6.1. Exercises page 232
**Problem number**: 18.
**ODE order**: 2.
**ODE degree**: 1.

CAS Maple gives this as type `[_Gegenbauer]`

$$\boxed{(-x^2+1)\,y''-2y'x+n(n+1)\,y=0}$$

With the expansion point for the power series method at $x=0$.

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 101

```
Order:=6;
dsolve((1-x^2)*diff(y(x),x$2)-2*x*diff(y(x),x)+n*(n+1)*y(x)=0,y(x),type='series',x=0);
```

$$y(x)=\left(1-\frac{n(n+1)\,x^2}{2}+\frac{n(n^3+2n^2-5n-6)\,x^4}{24}\right)y(0)+\left(x-\frac{(n^2+n-2)\,x^3}{6}+\frac{(n^4+2n^3-13n^2-14n+24)\,x^5}{120}\right)D(y)\,(0)+O(x^6)$$

✓ Solution by Mathematica

Time used: 0.001 (sec). Leaf size: 120

```
AsymptoticDSolveValue[(1-x^2)*y''[x]-2*x*y'[x]+n*(n+1)*y[x]==0,y[x],{x,0,5}]
```

$$y(x)\to c_2\left(\frac{1}{120}\left(n^2+n\right)^2x^5+\frac{7}{60}\left(-n^2-n\right)x^5+\frac{1}{6}\left(-n^2-n\right)x^3+\frac{x^5}{5}+\frac{x^3}{3}+x\right)+c_1\left(\frac{1}{24}\left(n^2+n\right)^2x^4+\frac{1}{4}\left(-n^2-n\right)x^4+\frac{1}{2}\left(-n^2-n\right)x^2+1\right)$$

# 15 Chapter 6, Series solutions of linear differential equations. Section 6.2 (Frobenius). Exercises page 251

## 15.1 problem 1

Internal problem ID [11902]

**Book**: Differential Equations by Shepley L. Ross. Third edition. John Willey. New Delhi. 2004.
**Section**: Chapter 6, Series solutions of linear differential equations. Section 6.2 (Frobenius). Exercises page 251
**Problem number**: 1.
**ODE order**: 2.
**ODE degree**: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$\boxed{(x^2-3x)\,y''+(x+2)\,y'+y=0}$$

With the expansion point for the power series method at $x=0$.

✓ Solution by Maple

Time used: 0.032 (sec). Leaf size: 44

```
Order:=6;
dsolve((x^2-3*x)*diff(y(x),x$2)+(x+2)*diff(y(x),x)+y(x)=0,y(x),type='series',x=0);
```

$$y(x)=c_1x^{\frac{5}{3}}\left(1+\frac{17}{36}x+\frac{1241}{7128}x^2+\frac{80665}{1347192}x^3+\frac{972725}{48498912}x^4+\frac{5797441}{872980416}x^5+\mathrm{O}\left(x^6\right)\right)+c_2\left(1-\frac{1}{2}x-\frac{1}{2}x^2-\frac{5}{24}x^3-\frac{25}{336}x^4-\frac{17}{672}x^5+\mathrm{O}\left(x^6\right)\right)$$

✓ Solution by Mathematica

Time used: 0.011 (sec). Leaf size: 85

```
AsymptoticDSolveValue[(x^2-3*x)*y''[x]+(x+2)*y'[x]+y[x]==0,y[x],{x,0,5}]
```

$$y(x)\to c_2\left(-\frac{17x^5}{672}-\frac{25x^4}{336}-\frac{5x^3}{24}-\frac{x^2}{2}-\frac{x}{2}+1\right)+c_1\left(\frac{5797441x^5}{872980416}+\frac{972725x^4}{48498912}+\frac{80665x^3}{1347192}+\frac{1241x^2}{7128}+\frac{17x}{36}+1\right)x^{5/3}$$

## 15.2 problem 2

Internal problem ID [11903]

**Book**: Differential Equations by Shepley L. Ross. Third edition. John Willey. New Delhi. 2004.
**Section**: Chapter 6, Series solutions of linear differential equations. Section 6.2 (Frobenius). Exercises page 251
**Problem number**: 2.
**ODE order**: 2.
**ODE degree**: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$\boxed{(x^3 + x^2)\, y'' + (x^2 - 2x)\, y' + 4y = 0}$$

With the expansion point for the power series method at $x = 0$.

✓ Solution by Maple

Time used: 0.031 (sec). Leaf size: 1227

```
Order:=6;
dsolve((x^3+x^2)*diff(y(x),x$2)+(x^2-2*x)*diff(y(x),x)+4*y(x)=0,y(x),type='series',x=0);
```

$$
\begin{aligned}
y(x) = x^{\frac{3}{2}} \Bigg( c_2 x^{\frac{i\sqrt{7}}{2}} \Bigg( 1 &+ \frac{3\sqrt{7}-i}{-2\sqrt{7}+2i} x + \frac{-4\sqrt{7}-12i}{\left(-\sqrt{7}+i\right)\left(i\sqrt{7}+2\right)} x^2 \\
&+ \frac{224}{3} \frac{1}{\left(\sqrt{7}-2i\right)\left(-\sqrt{7}+i\right)\left(3+i\sqrt{7}\right)} x^3 \\
&+ \frac{84\sqrt{7}-\frac{1036i}{3}}{\left(-\sqrt{7}+i\right)\left(i\sqrt{7}+2\right)\left(3+i\sqrt{7}\right)\left(4+i\sqrt{7}\right)} x^4 \\
&+ \frac{\frac{2576i\sqrt{7}}{3}+\frac{6608}{5}}{\left(-4i+\sqrt{7}\right)\left(-\sqrt{7}+i\right)\left(i\sqrt{7}+2\right)\left(3+i\sqrt{7}\right)\left(i\sqrt{7}+5\right)} x^5 + \mathrm{O}\left(x^6\right) \Bigg) \\
&+ c_1 x^{-\frac{i\sqrt{7}}{2}} \Bigg( 1 + \frac{-3\sqrt{7}-i}{2\sqrt{7}+2i} x + \frac{12+4i\sqrt{7}}{5+3i\sqrt{7}} x^2 \\
&+ \frac{224}{3} \frac{1}{\left(i\sqrt{7}-2\right)\left(\sqrt{7}+3i\right)\left(\sqrt{7}+i\right)} x^3 + \frac{63i\sqrt{7}-259}{15i\sqrt{7}-129} x^4 \\
&+ \frac{-1239i-805\sqrt{7}}{675i+255\sqrt{7}} x^5 + \mathrm{O}\left(x^6\right) \Bigg) \Bigg)
\end{aligned}
$$

✓ Solution by Mathematica

Time used: 0.006 (sec). Leaf size: 5834

```
AsymptoticDSolveValue[(x^3+x^2)*y''[x]+(x^2-2*x)*y'[x]+4*y[x]==0,y[x],{x,0,5}]
```

Too large to display

## 15.3 problem 3

Internal problem ID [11904]

**Book**: Differential Equations by Shepley L. Ross. Third edition. John Willey. New Delhi. 2004.
**Section**: Chapter 6, Series solutions of linear differential equations. Section 6.2 (Frobenius). Exercises page 251
**Problem number**: 3.
**ODE order**: 2.
**ODE degree**: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$\boxed{(x^4-2x^3+x^2)\,y''+2(x-1)\,y'+x^2y=0}$$

With the expansion point for the power series method at $x=0$.

✗ Solution by Maple

```
Order:=6;
dsolve((x^4-2*x^3+x^2)*diff(y(x),x$2)+2*(x-1)*diff(y(x),x)+x^2*y(x)=0,y(x),type='series',x=0
```

No solution found

✓ Solution by Mathematica
Time used: 0.047 (sec). Leaf size: 71

```
AsymptoticDSolveValue[(x^4-2*x^3+x^2)*y''[x]+2*(x-1)*y'[x]+x^2*y[x]==0,y[x],{x,0,5}]
```

$$y(x)\to c_1\left(\frac{3x^5}{10}+\frac{x^4}{4}+\frac{x^3}{6}+1\right)+c_2e^{-2/x}\left(-\frac{429x^5}{5}+\frac{91x^4}{4}-\frac{31x^3}{6}+3x^2+1\right)x^4$$

## 15.4 problem 4

Internal problem ID [11905]

**Book**: Differential Equations by Shepley L. Ross. Third edition. John Willey. New Delhi. 2004.
**Section**: Chapter 6, Series solutions of linear differential equations. Section 6.2 (Frobenius). Exercises page 251
**Problem number**: 4.
**ODE order**: 2.
**ODE degree**: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$\boxed{(x^5 + x^4 - 6x^3)\, y'' + x^2 y' + y(x-2) = 0}$$

With the expansion point for the power series method at $x = 0$.

✗ Solution by Maple

```
Order:=6;
dsolve((x^5+x^4-6*x^3)*diff(y(x),x$2)+x^2*diff(y(x),x)+(x-2)*y(x)=0,y(x),type='series',x=0);
```

No solution found

✓ Solution by Mathematica
Time used: 0.226 (sec). Leaf size: 282

```
AsymptoticDSolveValue[(x^5+x^4-6*x^3)*y''[x]+x^2*y'[x]+(x-2)*y[x]==0,y[x],{x,0,5}]
```

$$y(x) \to c_1 e^{-\frac{2i}{\sqrt{3}\sqrt{x}}} x^{5/6} \left( -\frac{70670717962217 i x^{9/2}}{8463329722368\sqrt{3}} + \frac{454703707 i x^{7/2}}{544195584\sqrt{3}} - \frac{287057 i x^{5/2}}{1679616\sqrt{3}} + \frac{22 i x^{3/2}}{243\sqrt{3}} + \frac{28128149072197063 x^5}{1523399350026240} - \frac{222818846149 x^4}{156728328192} + \frac{35197783 x^3}{181398528} - \frac{14123 x^2}{279936} + \frac{17x}{216} - \frac{7 i\sqrt{x}}{6\sqrt{3}} + 1 \right) + c_2 e^{\frac{2i}{\sqrt{3}\sqrt{x}}} x^{5/6} \left( \frac{70670717962217 i x^{9/2}}{8463329722368\sqrt{3}} - \frac{454703707 i x^{7/2}}{544195584\sqrt{3}} + \frac{287057 i x^{5/2}}{1679616\sqrt{3}} - \frac{22 i x^{3/2}}{243\sqrt{3}} + \frac{28128149072197}{152339935002}\right.$$

## 15.5 problem 5

Internal problem ID [11906]

**Book**: Differential Equations by Shepley L. Ross. Third edition. John Willey. New Delhi. 2004.
**Section**: Chapter 6, Series solutions of linear differential equations. Section 6.2 (Frobenius). Exercises page 251
**Problem number**: 5.
**ODE order**: 2.
**ODE degree**: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$\boxed{2x^2y'' + y'x + y(x^2-1) = 0}$$

With the expansion point for the power series method at $x = 0$.

✓ Solution by Maple
Time used: 0.016 (sec). Leaf size: 33

```
Order:=6;
dsolve(2*x^2*diff(y(x),x$2)+x*diff(y(x),x)+(x^2-1)*y(x)=0,y(x),type='series',x=0);
```

$$y(x) = \frac{c_1\left(1-\frac{1}{2}x^2+\frac{1}{40}x^4+\mathrm{O}\left(x^6\right)\right)}{\sqrt{x}} + c_2x\left(1-\frac{1}{14}x^2+\frac{1}{616}x^4+\mathrm{O}\left(x^6\right)\right)$$

✓ Solution by Mathematica
Time used: 0.002 (sec). Leaf size: 48

```
AsymptoticDSolveValue[2*x^2*y''[x]+x*y'[x]+(x^2-1)*y[x]==0,y[x],{x,0,5}]
```

$$y(x) \to c_1x\left(\frac{x^4}{616}-\frac{x^2}{14}+1\right) + \frac{c_2\left(\frac{x^4}{40}-\frac{x^2}{2}+1\right)}{\sqrt{x}}$$

## 15.6 problem 6

Internal problem ID [11907]

**Book**: Differential Equations by Shepley L. Ross. Third edition. John Willey. New Delhi. 2004.
**Section**: Chapter 6, Series solutions of linear differential equations. Section 6.2 (Frobenius). Exercises page 251
**Problem number**: 6.
**ODE order**: 2.
**ODE degree**: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$\boxed{2x^2y'' + y'x + (2x^2 - 3)\, y = 0}$$

With the expansion point for the power series method at $x = 0$.

✓ Solution by Maple

Time used: 0.015 (sec). Leaf size: 35

```
Order:=6;
dsolve(2*x^2*diff(y(x),x$2)+x*diff(y(x),x)+(2*x^2-3)*y(x)=0,y(x),type='series',x=0);
```

$$y(x) = \frac{c_2 x^{\frac{5}{2}} \left(1 - \frac{1}{9}x^2 + \frac{1}{234}x^4 + \mathrm{O}\left(x^6\right)\right) + c_1\left(1 + x^2 - \frac{1}{6}x^4 + \mathrm{O}\left(x^6\right)\right)}{x}$$

✓ Solution by Mathematica

Time used: 0.003 (sec). Leaf size: 46

```
AsymptoticDSolveValue[2*x^2*y''[x]+x*y'[x]+(2*x^2-3)*y[x]==0,y[x],{x,0,5}]
```

$$y(x) \to \frac{c_2\left(-\frac{x^4}{6} + x^2 + 1\right)}{x} + c_1\left(\frac{x^4}{234} - \frac{x^2}{9} + 1\right) x^{3/2}$$

## 15.7 problem 7

Internal problem ID [11908]

**Book**: Differential Equations by Shepley L. Ross. Third edition. John Willey. New Delhi. 2004.
**Section**: Chapter 6, Series solutions of linear differential equations. Section 6.2 (Frobenius). Exercises page 251
**Problem number**: 7.
**ODE order**: 2.
**ODE degree**: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$\boxed{x^2y'' - y'x + \left(x^2 + \frac{8}{9}\right)y = 0}$$

With the expansion point for the power series method at $x = 0$.

✓ Solution by Maple

Time used: 0.031 (sec). Leaf size: 35

```
Order:=6;
dsolve(x^2*diff(y(x),x$2)-x*diff(y(x),x)+(x^2+8/9)*y(x)=0,y(x),type='series',x=0);
```

$$y(x) = c_1 x^{\frac{2}{3}}\left(1 - \frac{3}{8}x^2 + \frac{9}{320}x^4 + \mathrm{O}\left(x^6\right)\right) + c_2 x^{\frac{4}{3}}\left(1 - \frac{3}{16}x^2 + \frac{9}{896}x^4 + \mathrm{O}\left(x^6\right)\right)$$

✓ Solution by Mathematica

Time used: 0.002 (sec). Leaf size: 52

```
AsymptoticDSolveValue[x^2*y''[x]-x*y'[x]+(x^2+8/9)*y[x]==0,y[x],{x,0,5}]
```

$$y(x) \to c_1\left(\frac{9x^4}{896} - \frac{3x^2}{16} + 1\right)x^{4/3} + c_2\left(\frac{9x^4}{320} - \frac{3x^2}{8} + 1\right)x^{2/3}$$

## 15.8 problem 8

Internal problem ID [11909]

**Book**: Differential Equations by Shepley L. Ross. Third edition. John Willey. New Delhi. 2004.
**Section**: Chapter 6, Series solutions of linear differential equations. Section 6.2 (Frobenius). Exercises page 251
**Problem number**: 8.
**ODE order**: 2.
**ODE degree**: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$\boxed{x^2y'' - y'x + \left(2x^2 + \frac{5}{9}\right)y = 0}$$

With the expansion point for the power series method at $x = 0$.

✓ Solution by Maple

Time used: 0.031 (sec). Leaf size: 35

```
Order:=6;
dsolve(x^2*diff(y(x),x$2)-x*diff(y(x),x)+(2*x^2+5/9)*y(x)=0,y(x),type='series',x=0);
```

$$y(x) = c_1 x^{\frac{1}{3}}\left(1 - \frac{3}{2}x^2 + \frac{9}{32}x^4 + \mathrm{O}\left(x^6\right)\right) + c_2 x^{\frac{5}{3}}\left(1 - \frac{3}{10}x^2 + \frac{9}{320}x^4 + \mathrm{O}\left(x^6\right)\right)$$

✓ Solution by Mathematica

Time used: 0.003 (sec). Leaf size: 52

```
AsymptoticDSolveValue[x^2*y''[x]-x*y'[x]+(2*x^2+5/9)*y[x]==0,y[x],{x,0,5}]
```

$$y(x) \to c_2\left(\frac{9x^4}{32} - \frac{3x^2}{2} + 1\right)\sqrt[3]{x} + c_1\left(\frac{9x^4}{320} - \frac{3x^2}{10} + 1\right)x^{5/3}$$

## 15.9 problem 9

Internal problem ID [11910]

**Book**: Differential Equations by Shepley L. Ross. Third edition. John Willey. New Delhi. 2004.
**Section**: Chapter 6, Series solutions of linear differential equations. Section 6.2 (Frobenius). Exercises page 251
**Problem number**: 9.
**ODE order**: 2.
**ODE degree**: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$\boxed{x^2y'' + y'x + \left(x^2 - \frac{1}{9}\right)y = 0}$$

With the expansion point for the power series method at $x = 0$.

✓ Solution by Maple

Time used: 0.016 (sec). Leaf size: 35

```
Order:=6;
dsolve(x^2*diff(y(x),x$2)+x*diff(y(x),x)+(x^2-1/9)*y(x)=0,y(x),type='series',x=0);
```

$$y(x) = \frac{x^{\frac{2}{3}}\left(1 - \frac{3}{16}x^2 + \frac{9}{896}x^4 + \mathrm{O}\left(x^6\right)\right)c_2 + \left(1 - \frac{3}{8}x^2 + \frac{9}{320}x^4 + \mathrm{O}\left(x^6\right)\right)c_1}{x^{\frac{1}{3}}}$$

✓ Solution by Mathematica

Time used: 0.002 (sec). Leaf size: 52

```
AsymptoticDSolveValue[x^2*y''[x]+x*y'[x]+(x^2-1/9)*y[x]==0,y[x],{x,0,5}]
```

$$y(x) \to c_1\sqrt[3]{x}\left(\frac{9x^4}{896} - \frac{3x^2}{16} + 1\right) + \frac{c_2\left(\frac{9x^4}{320} - \frac{3x^2}{8} + 1\right)}{\sqrt[3]{x}}$$

## 15.10 problem 10

Internal problem ID [11911]

**Book**: Differential Equations by Shepley L. Ross. Third edition. John Willey. New Delhi. 2004.
**Section**: Chapter 6, Series solutions of linear differential equations. Section 6.2 (Frobenius). Exercises page 251
**Problem number**: 10.
**ODE order**: 2.
**ODE degree**: 1.

CAS Maple gives this as type `[[_Emden, _Fowler], [_2nd_order, _linear, '_with_symmetry_[0,F`

$$\boxed{2xy'' + y' + 2y = 0}$$

With the expansion point for the power series method at $x = 0$.

✓ Solution by Maple

Time used: 0.032 (sec). Leaf size: 44

```
Order:=6;
dsolve(2*x*diff(y(x),x$2)+diff(y(x),x)+2*y(x)=0,y(x),type='series',x=0);
```

$$y(x) = c_1\sqrt{x}\left(1 - \frac{2}{3}x + \frac{2}{15}x^2 - \frac{4}{315}x^3 + \frac{2}{2835}x^4 - \frac{4}{155925}x^5 + \mathrm{O}\left(x^6\right)\right) + c_2\left(1 - 2x + \frac{2}{3}x^2 - \frac{4}{45}x^3 + \frac{2}{315}x^4 - \frac{4}{14175}x^5 + \mathrm{O}\left(x^6\right)\right)$$

✓ Solution by Mathematica

Time used: 0.002 (sec). Leaf size: 83

```
AsymptoticDSolveValue[2*x*y''[x]+y'[x]+2*y[x]==0,y[x],{x,0,5}]
```

$$y(x) \to c_1\sqrt{x}\left(-\frac{4x^5}{155925} + \frac{2x^4}{2835} - \frac{4x^3}{315} + \frac{2x^2}{15} - \frac{2x}{3} + 1\right) + c_2\left(-\frac{4x^5}{14175} + \frac{2x^4}{315} - \frac{4x^3}{45} + \frac{2x^2}{3} - 2x + 1\right)$$

## 15.11 problem 11

Internal problem ID [11912]

**Book**: Differential Equations by Shepley L. Ross. Third edition. John Willey. New Delhi. 2004.
**Section**: Chapter 6, Series solutions of linear differential equations. Section 6.2 (Frobenius). Exercises page 251
**Problem number**: 11.
**ODE order**: 2.
**ODE degree**: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$\boxed{3xy'' - (x-2)\,y' - 2y = 0}$$

With the expansion point for the power series method at $x = 0$.

✓ Solution by Maple
Time used: 0.031 (sec). Leaf size: 44

```
Order:=6;
dsolve(3*x*diff(y(x),x$2)-(x-2)*diff(y(x),x)-2*y(x)=0,y(x),type='series',x=0);
```

$$y(x) = c_1 x^{\frac{1}{3}}\left(1 + \frac{7}{12}x + \frac{5}{36}x^2 + \frac{13}{648}x^3 + \frac{1}{486}x^4 + \frac{19}{116640}x^5 + \mathrm{O}\left(x^6\right)\right) + c_2\left(1 + x + \frac{3}{10}x^2 + \frac{1}{20}x^3 + \frac{1}{176}x^4 + \frac{3}{6160}x^5 + \mathrm{O}\left(x^6\right)\right)$$

✓ Solution by Mathematica
Time used: 0.002 (sec). Leaf size: 81

```
AsymptoticDSolveValue[3*x*y''[x]-(x-2)*y'[x]-2*y[x]==0,y[x],{x,0,5}]
```

$$y(x) \to c_1 \sqrt[3]{x}\left(\frac{19x^5}{116640} + \frac{x^4}{486} + \frac{13x^3}{648} + \frac{5x^2}{36} + \frac{7x}{12} + 1\right) + c_2\left(\frac{3x^5}{6160} + \frac{x^4}{176} + \frac{x^3}{20} + \frac{3x^2}{10} + x + 1\right)$$

## 15.12 problem 12

Internal problem ID [11913]

**Book**: Differential Equations by Shepley L. Ross. Third edition. John Willey. New Delhi. 2004.
**Section**: Chapter 6, Series solutions of linear differential equations. Section 6.2 (Frobenius). Exercises page 251
**Problem number**: 12.
**ODE order**: 2.
**ODE degree**: 1.

CAS Maple gives this as type `[_Lienard]`

$$\boxed{xy'' + 2y' + yx = 0}$$

With the expansion point for the power series method at $x = 0$.

✓ Solution by Maple

Time used: 0.031 (sec). Leaf size: 32

```
Order:=6;
dsolve(x*diff(y(x),x$2)+2*diff(y(x),x)+x*y(x)=0,y(x),type='series',x=0);
```

$$y(x) = c_1\left(1 - \frac{1}{6}x^2 + \frac{1}{120}x^4 + \mathrm{O}\left(x^6\right)\right) + \frac{c_2\left(1 - \frac{1}{2}x^2 + \frac{1}{24}x^4 + \mathrm{O}\left(x^6\right)\right)}{x}$$

✓ Solution by Mathematica

Time used: 0.008 (sec). Leaf size: 42

```
AsymptoticDSolveValue[x*y''[x]+2*y'[x]+x*y[x]==0,y[x],{x,0,5}]
```

$$y(x) \to c_1\left(\frac{x^3}{24} - \frac{x}{2} + \frac{1}{x}\right) + c_2\left(\frac{x^4}{120} - \frac{x^2}{6} + 1\right)$$

## 15.13 problem 13

Internal problem ID [11914]

**Book**: Differential Equations by Shepley L. Ross. Third edition. John Willey. New Delhi. 2004.
**Section**: Chapter 6, Series solutions of linear differential equations. Section 6.2 (Frobenius). Exercises page 251
**Problem number**: 13.
**ODE order**: 2.
**ODE degree**: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$\boxed{x^2y'' + y'x + \left(x^2 - \frac{1}{4}\right)y = 0}$$

With the expansion point for the power series method at $x = 0$.

✓ Solution by Maple

Time used: 0.047 (sec). Leaf size: 35

```
Order:=6;
dsolve(x^2*diff(y(x),x$2)+x*diff(y(x),x)+(x^2-1/4)*y(x)=0,y(x),type='series',x=0);
```

$$y(x) = \frac{c_1\left(1 - \frac{1}{6}x^2 + \frac{1}{120}x^4 + \mathrm{O}\left(x^6\right)\right)x + c_2\left(1 - \frac{1}{2}x^2 + \frac{1}{24}x^4 + \mathrm{O}\left(x^6\right)\right)}{\sqrt{x}}$$

✓ Solution by Mathematica

Time used: 0.01 (sec). Leaf size: 58

```
AsymptoticDSolveValue[x^2*y''[x]+x*y'[x]+(x^2-1/4)*y[x]==0,y[x],{x,0,5}]
```

$$y(x) \to c_1\left(\frac{x^{7/2}}{24} - \frac{x^{3/2}}{2} + \frac{1}{\sqrt{x}}\right) + c_2\left(\frac{x^{9/2}}{120} - \frac{x^{5/2}}{6} + \sqrt{x}\right)$$

## 15.14 problem 14

Internal problem ID [11915]

**Book**: Differential Equations by Shepley L. Ross. Third edition. John Willey. New Delhi. 2004.
**Section**: Chapter 6, Series solutions of linear differential equations. Section 6.2 (Frobenius). Exercises page 251
**Problem number**: 14.
**ODE order**: 2.
**ODE degree**: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$\boxed{x^2y'' + (x^4 + x)\, y' - y = 0}$$

With the expansion point for the power series method at $x = 0$.

✓ Solution by Maple

Time used: 0.031 (sec). Leaf size: 29

```
Order:=6;
dsolve(x^2*diff(y(x),x$2)+(x^4+x)*diff(y(x),x)-y(x)=0,y(x),type='series',x=0);
```

$$y(x) = c_1x\left(1 - \frac{1}{15}x^3 + \mathrm{O}\left(x^6\right)\right) + \frac{c_2\left(-2 - \frac{2}{3}x^3 + \mathrm{O}\left(x^6\right)\right)}{x}$$

✓ Solution by Mathematica

Time used: 0.007 (sec). Leaf size: 30

```
AsymptoticDSolveValue[x^2*y''[x]+(x^4+x)*y'[x]-y[x]==0,y[x],{x,0,5}]
```

$$y(x) \to c_2\left(x - \frac{x^4}{15}\right) + c_1\left(\frac{x^2}{3} + \frac{1}{x}\right)$$

## 15.15 problem 15

Internal problem ID [11916]

**Book**: Differential Equations by Shepley L. Ross. Third edition. John Willey. New Delhi. 2004.
**Section**: Chapter 6, Series solutions of linear differential equations. Section 6.2 (Frobenius). Exercises page 251
**Problem number**: 15.
**ODE order**: 2.
**ODE degree**: 1.

CAS Maple gives this as type `[_Lienard]`

$$\boxed{xy'' - (x^2+2)\,y' + yx = 0}$$

With the expansion point for the power series method at $x = 0$.

✓ Solution by Maple
Time used: 0.032 (sec). Leaf size: 32

```
Order:=6;
dsolve(x*diff(y(x),x$2)-(x^2+2)*diff(y(x),x)+x*y(x)=0,y(x),type='series',x=0);
```

$$y(x) = c_1 x^3 \left(1 + \frac{1}{5}x^2 + \frac{1}{35}x^4 + \mathrm{O}\left(x^6\right)\right) + c_2\left(12 + 6x^2 + \frac{3}{2}x^4 + \mathrm{O}\left(x^6\right)\right)$$

✓ Solution by Mathematica
Time used: 0.009 (sec). Leaf size: 44

```
AsymptoticDSolveValue[x*y''[x]-(x^2+2)*y'[x]+x*y[x]==0,y[x],{x,0,5}]
```

$$y(x) \to c_1\left(\frac{x^4}{8} + \frac{x^2}{2} + 1\right) + c_2\left(\frac{x^7}{35} + \frac{x^5}{5} + x^3\right)$$

## 15.16 problem 16

Internal problem ID [11917]

**Book**: Differential Equations by Shepley L. Ross. Third edition. John Willey. New Delhi. 2004.
**Section**: Chapter 6, Series solutions of linear differential equations. Section 6.2 (Frobenius). Exercises page 251
**Problem number**: 16.
**ODE order**: 2.
**ODE degree**: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$\boxed{x^2y'' + x^2y' - 2y = 0}$$

With the expansion point for the power series method at $x = 0$.

✓ Solution by Maple
Time used: 0.031 (sec). Leaf size: 45

```
Order:=6;
dsolve(x^2*diff(y(x),x$2)+x^2*diff(y(x),x)-2*y(x)=0,y(x),type='series',x=0);
```

$$y(x) = c_1x^2\left(1 - \frac{1}{2}x + \frac{3}{20}x^2 - \frac{1}{30}x^3 + \frac{1}{168}x^4 - \frac{1}{1120}x^5 + \mathrm{O}\left(x^6\right)\right) + \frac{c_2\left(12 - 6x + x^3 - \frac{1}{2}x^4 + \frac{3}{20}x^5 + \mathrm{O}\left(x^6\right)\right)}{x}$$

✓ Solution by Mathematica
Time used: 0.019 (sec). Leaf size: 63

```
AsymptoticDSolveValue[x^2*y''[x]+x^2*y'[x]-2*y[x]==0,y[x],{x,0,5}]
```

$$y(x) \to c_1\left(-\frac{x^3}{24} + \frac{x^2}{12} + \frac{1}{x} - \frac{1}{2}\right) + c_2\left(\frac{x^6}{168} - \frac{x^5}{30} + \frac{3x^4}{20} - \frac{x^3}{2} + x^2\right)$$

## 15.17 problem 17

Internal problem ID [11918]

**Book**: Differential Equations by Shepley L. Ross. Third edition. John Willey. New Delhi. 2004.
**Section**: Chapter 6, Series solutions of linear differential equations. Section 6.2 (Frobenius). Exercises page 251
**Problem number**: 17.
**ODE order**: 2.
**ODE degree**: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$\boxed{(2x^2 - x)\, y'' + (2x - 2)\, y' + (-2x^2 + 3x - 2)\, y = 0}$$

With the expansion point for the power series method at $x = 0$.

✓ Solution by Maple
Time used: 0.047 (sec). Leaf size: 44

```
Order:=6;
dsolve((2*x^2-x)*diff(y(x),x$2)+(2*x-2)*diff(y(x),x)+(-2*x^2+3*x-2)*y(x)=0,y(x),type='series
```

$$y(x) = c_1\left(1 - x + \frac{1}{2}x^2 - \frac{1}{6}x^3 + \frac{1}{24}x^4 - \frac{1}{120}x^5 + \mathrm{O}\left(x^6\right)\right) + \frac{c_2\left(1 - 2x + \frac{7}{2}x^2 - \frac{4}{3}x^3 + \frac{13}{24}x^4 - \frac{7}{60}x^5 + \mathrm{O}\left(x^6\right)\right)}{x}$$

✓ Solution by Mathematica
Time used: 0.033 (sec). Leaf size: 60

```
AsymptoticDSolveValue[(2*x^2-x)*y''[x]+(2*x-2)*y'[x]+(-2*x^2+3*x-2)*y[x]==0,y[x],{x,0,5}]
```

$$y(x) \to c_1\left(\frac{7x^3}{8} - \frac{7x^2}{3} + \frac{11x}{2} + \frac{1}{x} - 4\right) + c_2\left(\frac{x^4}{24} - \frac{x^3}{6} + \frac{x^2}{2} - x + 1\right)$$

## 15.18 problem 18

Internal problem ID [11919]

**Book**: Differential Equations by Shepley L. Ross. Third edition. John Willey. New Delhi. 2004.
**Section**: Chapter 6, Series solutions of linear differential equations. Section 6.2 (Frobenius). Exercises page 251
**Problem number**: 18.
**ODE order**: 2.
**ODE degree**: 1.

CAS Maple gives this as type `[[_Emden, _Fowler], [_2nd_order, _linear, '_with_symmetry_[0,F`

$$\boxed{x^2y'' - y'x + \frac{3y}{4} = 0}$$

With the expansion point for the power series method at $x = 0$.

✓ Solution by Maple
Time used: 0.032 (sec). Leaf size: 27

```
Order:=6;
dsolve(x^2*diff(y(x),x$2)-x*diff(y(x),x)+3/4*y(x)=0,y(x),type='series',x=0);
```

$$y(x) = \sqrt{x}\,(c_1x + c_2) + O(x^6)$$

✓ Solution by Mathematica
Time used: 0.004 (sec). Leaf size: 20

```
AsymptoticDSolveValue[x^2*y''[x]-x*y'[x]+3/4*y[x]==0,y[x],{x,0,5}]
```

$$y(x) \to c_2x^{3/2} + c_1\sqrt{x}$$

## 15.19 problem 19

Internal problem ID [11920]

**Book**: Differential Equations by Shepley L. Ross. Third edition. John Willey. New Delhi. 2004.
**Section**: Chapter 6, Series solutions of linear differential equations. Section 6.2 (Frobenius). Exercises page 251
**Problem number**: 19.
**ODE order**: 2.
**ODE degree**: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$\boxed{x^2y'' + y'x + y(x-1) = 0}$$

With the expansion point for the power series method at $x = 0$.

✓ Solution by Maple

Time used: 0.015 (sec). Leaf size: 63

```
Order:=6;
dsolve(x^2*diff(y(x),x$2)+x*diff(y(x),x)+(x-1)*y(x)=0,y(x),type='series',x=0);
```

$$y(x) = \frac{c_1 x^2 \left(1 - \frac{1}{3}x + \frac{1}{24}x^2 - \frac{1}{360}x^3 + \frac{1}{8640}x^4 - \frac{1}{302400}x^5 + \mathrm{O}\left(x^6\right)\right) + c_2 \left(\ln\left(x\right)\left(x^2 - \frac{1}{3}x^3 + \frac{1}{24}x^4 - \frac{1}{360}x^5 + \mathrm{O}\left(x^6 \right.\right.\right.}{x}$$

✓ Solution by Mathematica

Time used: 0.017 (sec). Leaf size: 83

```
AsymptoticDSolveValue[x^2*y''[x]+x*y'[x]+(x-1)*y[x]==0,y[x],{x,0,5}]
```

$$y(x) \to c_1 \left(\frac{31x^4 - 176x^3 + 144x^2 + 576x + 576}{576x} - \frac{1}{48}x\left(x^2 - 8x + 24\right)\log(x)\right) + c_2\left(\frac{x^5}{8640} - \frac{x^4}{360} + \frac{x^3}{24} - \frac{x^2}{3} + x\right)$$

## 15.20 problem 20

Internal problem ID [11921]

**Book**: Differential Equations by Shepley L. Ross. Third edition. John Willey. New Delhi. 2004.
**Section**: Chapter 6, Series solutions of linear differential equations. Section 6.2 (Frobenius). Exercises page 251
**Problem number**: 20.
**ODE order**: 2.
**ODE degree**: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$\boxed{x^2y'' + (x^3 - x)\, y' - 3y = 0}$$

With the expansion point for the power series method at $x = 0$.

✓ Solution by Maple

Time used: 0.015 (sec). Leaf size: 47

```
Order:=6;
dsolve(x^2*diff(y(x),x$2)+(x^3-x)*diff(y(x),x)-3*y(x)=0,y(x),type='series',x=0);
```

$$y(x) = \frac{c_1 x^4 \left(1 - \frac{1}{4}x^2 + \frac{5}{128}x^4 + \mathrm{O}\left(x^6\right)\right) + c_2 \left(\ln\left(x\right)\left(\left(-9\right)x^4 + \mathrm{O}\left(x^6\right)\right) + \left(-144 + 36x^2 + \mathrm{O}\left(x^6\right)\right)\right)}{x}$$

✓ Solution by Mathematica

Time used: 0.01 (sec). Leaf size: 55

```
AsymptoticDSolveValue[x^2*y''[x]+(x^3-x)*y'[x]-3*y[x]==0,y[x],{x,0,5}]
```

$$y(x) \to c_2 \left(\frac{5x^7}{128} - \frac{x^5}{4} + x^3\right) + c_1 \left(\frac{1}{16}x^3 \log(x) - \frac{x^4 + 16x^2 - 64}{64x}\right)$$

## 15.21 problem 21

Internal problem ID [11922]

**Book**: Differential Equations by Shepley L. Ross. Third edition. John Willey. New Delhi. 2004.
**Section**: Chapter 6, Series solutions of linear differential equations. Section 6.2 (Frobenius). Exercises page 251
**Problem number**: 21.
**ODE order**: 2.
**ODE degree**: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$\boxed{x^2y'' - y'x + 8y(x^2-1) = 0}$$

With the expansion point for the power series method at $x = 0$.

✓ Solution by Maple

Time used: 0.031 (sec). Leaf size: 35

```
Order:=6;
dsolve(x^2*diff(y(x),x$2)-x*diff(y(x),x)+8*(x^2-1)*y(x)=0,y(x),type='series',x=0);
```

$$y(x) = c_1x^4\left(1 - \frac{1}{2}x^2 + \frac{1}{10}x^4 + \mathrm{O}\left(x^6\right)\right) + \frac{c_2(-86400 - 86400x^2 - 86400x^4 + \mathrm{O}\left(x^6\right))}{x^2}$$

✓ Solution by Mathematica

Time used: 0.009 (sec). Leaf size: 36

```
AsymptoticDSolveValue[x^2*y''[x]-x*y'[x]+8*(x^2-1)*y[x]==0,y[x],{x,0,5}]
```

$$y(x) \to c_1\left(x^2 + \frac{1}{x^2} + 1\right) + c_2\left(\frac{x^8}{10} - \frac{x^6}{2} + x^4\right)$$

## 15.22 problem 22

Internal problem ID [11923]

**Book**: Differential Equations by Shepley L. Ross. Third edition. John Willey. New Delhi. 2004.
**Section**: Chapter 6, Series solutions of linear differential equations. Section 6.2 (Frobenius). Exercises page 251
**Problem number**: 22.
**ODE order**: 2.
**ODE degree**: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$\boxed{x^2y'' + x^2y' - \frac{3y}{4} = 0}$$

With the expansion point for the power series method at $x = 0$.

✓ Solution by Maple

Time used: 0.031 (sec). Leaf size: 65

```
Order:=6;
dsolve(x^2*diff(y(x),x$2)+x^2*diff(y(x),x)-3/4*y(x)=0,y(x),type='series',x=0);
```

$$y(x) = \frac{c_1x^2\left(1 - \frac{1}{2}x + \frac{5}{32}x^2 - \frac{7}{192}x^3 + \frac{7}{1024}x^4 - \frac{11}{10240}x^5 + \mathrm{O}\left(x^6\right)\right) + c_2\left(\ln\left(x\right)\left(-\frac{1}{4}x^2 + \frac{1}{8}x^3 - \frac{5}{128}x^4 + \frac{7}{768}x^5 + \mathrm{O}\right.\right.}{\sqrt{x}}$$

✓ Solution by Mathematica

Time used: 0.026 (sec). Leaf size: 103

```
AsymptoticDSolveValue[x^2*y''[x]+x^2*y'[x]-3/4*y[x]==0,y[x],{x,0,5}]
```

$$y(x) \to c_2\left(\frac{7x^{11/2}}{1024} - \frac{7x^{9/2}}{192} + \frac{5x^{7/2}}{32} - \frac{x^{5/2}}{2} + x^{3/2}\right) + c_1\left(\frac{1}{256}x^{3/2}\left(5x^2 - 16x + 32\right)\log(x) - \frac{91x^4 - 224x^3 + 192x^2 + 1536x - 3072}{3072\sqrt{x}}\right)$$

## 15.23 problem 23

Internal problem ID [11924]

**Book**: Differential Equations by Shepley L. Ross. Third edition. John Willey. New Delhi. 2004.
**Section**: Chapter 6, Series solutions of linear differential equations. Section 6.2 (Frobenius). Exercises page 251
**Problem number**: 23.
**ODE order**: 2.
**ODE degree**: 1.

CAS Maple gives this as type `[[_Emden, _Fowler]]`

$$\boxed{xy'' + y' + 2y = 0}$$

With the expansion point for the power series method at $x = 0$.

✓ Solution by Maple
Time used: 0.032 (sec). Leaf size: 59

```
Order:=6;
dsolve(x*diff(y(x),x$2)+diff(y(x),x)+2*y(x)=0,y(x),type='series',x=0);
```

$$y(x) = (c_1 + c_2 \ln(x)) \left(1 - 2x + x^2 - \frac{2}{9}x^3 + \frac{1}{36}x^4 - \frac{1}{450}x^5 + \mathrm{O}(x^6)\right) + \left(4x - 3x^2 + \frac{22}{27}x^3 - \frac{25}{216}x^4 + \frac{137}{13500}x^5 + \mathrm{O}(x^6)\right) c_2$$

✓ Solution by Mathematica
Time used: 0.003 (sec). Leaf size: 101

```
AsymptoticDSolveValue[x*y''[x]+y'[x]+2*y[x]==0,y[x],{x,0,5}]
```

$$y(x) \to c_1\left(-\frac{x^5}{450} + \frac{x^4}{36} - \frac{2x^3}{9} + x^2 - 2x + 1\right) + c_2\left(\frac{137x^5}{13500} - \frac{25x^4}{216} + \frac{22x^3}{27} - 3x^2 + \left(-\frac{x^5}{450} + \frac{x^4}{36} - \frac{2x^3}{9} + x^2 - 2x + 1\right)\log(x) + 4x\right)$$

## 15.24 problem 24

Internal problem ID [11925]

**Book**: Differential Equations by Shepley L. Ross. Third edition. John Willey. New Delhi. 2004.
**Section**: Chapter 6, Series solutions of linear differential equations. Section 6.2 (Frobenius). Exercises page 251
**Problem number**: 24.
**ODE order**: 2.
**ODE degree**: 1.

CAS Maple gives this as type `[[_Emden, _Fowler]]`

$$\boxed{2xy'' + 6y' + y = 0}$$

With the expansion point for the power series method at $x = 0$.

✓ Solution by Maple
Time used: 0.031 (sec). Leaf size: 62

```
Order:=6;
dsolve(2*x*diff(y(x),x$2)+6*diff(y(x),x)+y(x)=0,y(x),type='series',x=0);
```

$$y(x) = \frac{c_1\left(1 - \frac{1}{6}x + \frac{1}{96}x^2 - \frac{1}{2880}x^3 + \frac{1}{138240}x^4 - \frac{1}{9676800}x^5 + \mathrm{O}\left(x^6\right)\right)x^2 + c_2\left(\ln\left(x\right)\left(\frac{1}{4}x^2 - \frac{1}{24}x^3 + \frac{1}{384}x^4 - \frac{1}{11520}x\right.\right.}{x^2}$$

✓ Solution by Mathematica
Time used: 0.018 (sec). Leaf size: 80

```
AsymptoticDSolveValue[2*x*y''[x]+6*y'[x]+y[x]==0,y[x],{x,0,5}]
```

$$y(x) \to c_2\left(\frac{x^4}{138240} - \frac{x^3}{2880} + \frac{x^2}{96} - \frac{x}{6} + 1\right) + c_1\left(\frac{31x^4 - 352x^3 + 576x^2 + 4608x + 9216}{9216x^2} - \frac{1}{768}\left(x^2 - 16x + 96\right)\log(x)\right)$$

## 15.25 problem 25

Internal problem ID [11926]

**Book**: Differential Equations by Shepley L. Ross. Third edition. John Willey. New Delhi. 2004.
**Section**: Chapter 6, Series solutions of linear differential equations. Section 6.2 (Frobenius). Exercises page 251
**Problem number**: 25.
**ODE order**: 2.
**ODE degree**: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$\boxed{x^2y'' - y'x + (x^2+1)\,y = 0}$$

With the expansion point for the power series method at $x = 0$.

✓ Solution by Maple

Time used: 0.015 (sec). Leaf size: 45

```
Order:=6;
dsolve(x^2*diff(y(x),x$2)-x*diff(y(x),x)+(x^2+1)*y(x)=0,y(x),type='series',x=0);
```

$$y(x) = \left((c_1 + c_2\ln(x))\left(1 - \frac{1}{4}x^2 + \frac{1}{64}x^4 + \mathrm{O}\left(x^6\right)\right) + \left(\frac{1}{4}x^2 - \frac{3}{128}x^4 + \mathrm{O}\left(x^6\right)\right)c_2\right)x$$

✓ Solution by Mathematica

Time used: 0.002 (sec). Leaf size: 65

```
AsymptoticDSolveValue[x^2*y''[x]-x*y'[x]+(x^2+1)*y[x]==0,y[x],{x,0,5}]
```

$$y(x) \to c_1x\left(\frac{x^4}{64} - \frac{x^2}{4} + 1\right) + c_2\left(x\left(\frac{x^2}{4} - \frac{3x^4}{128}\right) + x\left(\frac{x^4}{64} - \frac{x^2}{4} + 1\right)\log(x)\right)$$

## 15.26 problem 26

Internal problem ID [11927]

**Book**: Differential Equations by Shepley L. Ross. Third edition. John Willey. New Delhi. 2004.
**Section**: Chapter 6, Series solutions of linear differential equations. Section 6.2 (Frobenius). Exercises page 251
**Problem number**: 26.
**ODE order**: 2.
**ODE degree**: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$\boxed{x^2y'' - y'x + (x^2 - 3)\, y = 0}$$

With the expansion point for the power series method at $x = 0$.

✓ Solution by Maple
Time used: 0.031 (sec). Leaf size: 47

```
Order:=6;
dsolve(x^2*diff(y(x),x$2)-x*diff(y(x),x)+(x^2-3)*y(x)=0,y(x),type='series',x=0);
```

$$y(x) = \frac{c_1 x^4 \left(1 - \frac{1}{12}x^2 + \frac{1}{384}x^4 + \mathrm{O}(x^6)\right) + c_2 (\ln(x)(9x^4 + \mathrm{O}(x^6)) + (-144 - 36x^2 + \mathrm{O}(x^6)))}{x}$$

✓ Solution by Mathematica
Time used: 0.008 (sec). Leaf size: 52

```
AsymptoticDSolveValue[x^2*y''[x]-x*y'[x]+(x^2-3)*y[x]==0,y[x],{x,0,5}]
```

$$y(x) \to c_1 \left(\frac{(x^2+8)^2}{64x} - \frac{1}{16}x^3 \log(x)\right) + c_2 \left(\frac{x^7}{384} - \frac{x^5}{12} + x^3\right)$$

# 16 Chapter 7, Systems of linear differential equations. Section 7.1. Exercises page 277

## 16.1 problem 1

Internal problem ID [11928]

**Book**: Differential Equations by Shepley L. Ross. Third edition. John Willey. New Delhi. 2004.
**Section**: Chapter 7, Systems of linear differential equations. Section 7.1. Exercises page 277
**Problem number**: 1.
**ODE order**: 1.
**ODE degree**: 1.

Solve

$$
\begin{aligned}
x'(t) + y'(t) &= 2x(t) + 4y(t) + \mathrm{e}^{t} \\
x'(t) + y'(t) &= y(t) + \mathrm{e}^{4t}
\end{aligned}
$$

✓ Solution by Maple

Time used: 0.015 (sec). Leaf size: 32

```
dsolve([diff(x(t),t)+diff(y(t),t)-2*x(t)-4*y(t)=exp(t),diff(x(t),t)+diff(y(t),t)-y(t)=exp(4*
```

$$
\begin{aligned}
x(t) &= c_1 \mathrm{e}^{-2t} \\
y(t) &= \frac{\mathrm{e}^{4t}}{3} - \frac{\mathrm{e}^{t}}{3} - \frac{2c_1 \mathrm{e}^{-2t}}{3}
\end{aligned}
$$

✓ Solution by Mathematica

Time used: 0.055 (sec). Leaf size: 52

```
DSolve[{x'[t]+y'[t]-2*x[t]-4*y[t]==Exp[t],x'[t]+y'[t]-y[t]==Exp[4*t]},{x[t],y[t]},t,IncludeS
```

$$
\begin{aligned}
x(t) &\to \frac{1}{12}(3+4c_1)e^{-2t} \\
y(t) &\to \frac{1}{18}e^{-2t}\left(-6e^{3t}+6e^{6t}-3-4c_1\right)
\end{aligned}
$$

## 16.2 problem 2

Internal problem ID [11929]

**Book**: Differential Equations by Shepley L. Ross. Third edition. John Willey. New Delhi. 2004.
**Section**: Chapter 7, Systems of linear differential equations. Section 7.1. Exercises page 277
**Problem number**: 2.
**ODE order**: 1.
**ODE degree**: 1.

Solve

$$
\begin{aligned}
x'(t) + y'(t) &= x(t) - 2t \\
x'(t) + y'(t) &= t^2 + 3x(t) + y(t)
\end{aligned}
$$

✓ Solution by Maple

Time used: 0.016 (sec). Leaf size: 33

```
dsolve([diff(x(t),t)+diff(y(t),t)-x(t)=-2*t,diff(x(t),t)+diff(y(t),t)-3*x(t)-y(t)=t^2],sings
```

$$
\begin{aligned}
x(t) &= -2 + \mathrm{e}^{-t} c_1 \\
y(t) &= -t^2 + 4 - 2\,\mathrm{e}^{-t} c_1 - 2t
\end{aligned}
$$

✓ Solution by Mathematica

Time used: 0.056 (sec). Leaf size: 42

```
DSolve[{x'[t]+y'[t]-x[t]==-2*t,x'[t]+y'[t]-3*x[t]-y[t]==t^2},{x[t],y[t]},t,IncludeSingularSo
```

$$
\begin{aligned}
x(t) &\to -2 - \frac{1}{4} c_1 e^{-t} \\
y(t) &\to -t^2 - 2t + \frac{c_1 e^{-t}}{2} + 4
\end{aligned}
$$

## 16.3 problem 3

Internal problem ID [11930]

**Book**: Differential Equations by Shepley L. Ross. Third edition. John Willey. New Delhi. 2004.
**Section**: Chapter 7, Systems of linear differential equations. Section 7.1. Exercises page 277
**Problem number**: 3.
**ODE order**: 1.
**ODE degree**: 1.

Solve

$$
\begin{aligned}
x'(t) + y'(t) &= x(t) + 3y(t) + \mathrm{e}^{t} \\
x'(t) + y'(t) &= -x(t) + \mathrm{e}^{3t}
\end{aligned}
$$

✓ Solution by Maple

Time used: 0.032 (sec). Leaf size: 37

```
dsolve([diff(x(t),t)+diff(y(t),t)-x(t)-3*y(t)=exp(t),diff(x(t),t)+diff(y(t),t)+x(t)=exp(3*t)
```

$$
\begin{aligned}
x(t) &= \frac{\mathrm{e}^{t}}{4} + c_1 \mathrm{e}^{-3t} \\
y(t) &= \frac{\mathrm{e}^{3t}}{3} - \frac{\mathrm{e}^{t}}{2} - \frac{2c_1 \mathrm{e}^{-3t}}{3}
\end{aligned}
$$

✓ Solution by Mathematica

Time used: 0.043 (sec). Leaf size: 55

```
DSolve[{x'[t]+y'[t]-x[t]-3*y[t]==Exp[t],x'[t]+y'[t]+x[t]==Exp[3*t]},{x[t],y[t]},t,IncludeSin
```

$$
\begin{aligned}
x(t) &\to \frac{e^t}{4} + \frac{3}{16} c_1 e^{-3t} \\
y(t) &\to -\frac{e^t}{2} + \frac{e^{3t}}{3} - \frac{1}{8} c_1 e^{-3t}
\end{aligned}
$$

## 16.4 problem 4

Internal problem ID [11931]

**Book**: Differential Equations by Shepley L. Ross. Third edition. John Willey. New Delhi. 2004.
**Section**: Chapter 7, Systems of linear differential equations. Section 7.1. Exercises page 277
**Problem number**: 4.
**ODE order**: 1.
**ODE degree**: 1.

Solve

$$
\begin{aligned}
x'(t) + y'(t) &= x(t) + 2y(t) + 2\,\mathrm{e}^{t} \\
x'(t) + y'(t) &= 3x(t) + 4y(t) + \mathrm{e}^{2t}
\end{aligned}
$$

✓ Solution by Maple
Time used: 0.015 (sec). Leaf size: 23

```
dsolve([diff(x(t),t)+diff(y(t),t)-x(t)-2*y(t)=2*exp(t),diff(x(t),t)+diff(y(t),t)-3*x(t)-4*y(
```

$$
\begin{aligned}
x(t) &= 3\,\mathrm{e}^{t} \\
y(t) &= -\frac{\mathrm{e}^{2t}}{2} - 2\,\mathrm{e}^{t}
\end{aligned}
$$

✓ Solution by Mathematica
Time used: 0.013 (sec). Leaf size: 25

```
DSolve[{x'[t]+y'[t]-x[t]-2*y[t]==2*Exp[t],x'[t]+y'[t]-3*x[t]-4*y[t]==Exp[2*t]},{x[t],y[t]},t
```

$$
\begin{aligned}
x(t) &\to 3e^{t} \\
y(t) &\to -\frac{1}{2}e^{t}\left(e^{t}+4\right)
\end{aligned}
$$

## 16.5 problem 5

Internal problem ID [11932]

**Book**: Differential Equations by Shepley L. Ross. Third edition. John Willey. New Delhi. 2004.
**Section**: Chapter 7, Systems of linear differential equations. Section 7.1. Exercises page 277
**Problem number**: 5.
**ODE order**: 1.
**ODE degree**: 1.

Solve

$$
\begin{aligned}
x'(t) &= 3x(t) + 2y(t) - e^{t} + e^{-t} \\
y'(t) &= -5x(t) - 3y(t) + 2\,e^{t} - e^{-t}
\end{aligned}
$$

✓ Solution by Maple

Time used: 0.031 (sec). Leaf size: 48

```
dsolve([2*diff(x(t),t)+diff(y(t),t)-x(t)-y(t)=exp(-t),diff(x(t),t)+diff(y(t),t)+2*x(t)+y(t)=
```

$$
\begin{aligned}
x(t) &= c_1 \sin(t) + c_2 \cos(t) \\
y(t) &= \frac{c_1 \cos(t)}{2} - \frac{3c_2 \cos(t)}{2} - \frac{3c_1 \sin(t)}{2} - \frac{c_2 \sin(t)}{2} + \frac{e^{t}}{2} - \frac{e^{-t}}{2}
\end{aligned}
$$

✓ Solution by Mathematica

Time used: 0.229 (sec). Leaf size: 60

```
DSolve[{2*x'[t]+y'[t]-x[t]-y[t]==Exp[-t],x'[t]+y'[t]+2*x[t]+y[t]==Exp[t]},{x[t],y[t]},t,Incl
```

$$
\begin{aligned}
x(t) &\to c_1 \cos(t) + (3c_1 + 2c_2) \sin(t) \\
y(t) &\to \frac{1}{2}\left(-e^{-t} + e^{t} + 2c_2 \cos(t) - 2(5c_1 + 3c_2) \sin(t)\right)
\end{aligned}
$$

## 16.6 problem 6

Internal problem ID [11933]

**Book**: Differential Equations by Shepley L. Ross. Third edition. John Willey. New Delhi. 2004.
**Section**: Chapter 7, Systems of linear differential equations. Section 7.1. Exercises page 277
**Problem number**: 6.
**ODE order**: 1.
**ODE degree**: 1.

Solve

$$
\begin{aligned}
x'(t) &= -x(t) + t - e^t \\
y'(t) &= 5x(t) + y(t) - t + 2\,e^t
\end{aligned}
$$

✓ Solution by Maple

Time used: 0.031 (sec). Leaf size: 42

```
dsolve([2*diff(x(t),t)+diff(y(t),t)-3*x(t)-y(t)=t,diff(x(t),t)+diff(y(t),t)-4*x(t)-y(t)=exp(
```

$$
\begin{aligned}
x(t) &= t - 1 - \frac{e^t}{2} + c_2 e^{-t} \\
y(t) &= -\frac{5c_2 e^{-t}}{2} - 4t + 1 + c_1 e^t - \frac{e^t t}{2}
\end{aligned}
$$

✓ Solution by Mathematica

Time used: 0.665 (sec). Leaf size: 72

```
DSolve[{2*x'[t]+y'[t]-3*x[t]-y[t]==t,x'[t]+y'[t]+4*x[t]-y[t]==Exp[t]},{x[t],y[t]},t,IncludeS
```

$$
\begin{aligned}
x(t) &\to -\frac{t}{7} + \frac{e^t}{6} + c_1 e^{7t} - \frac{1}{49} \\
y(t) &\to -\frac{4t}{7} - \frac{11}{6} c_1 e^{7t} + \frac{1}{36} e^t (6t - 11 + 66c_1 + 36c_2) - \frac{39}{49}
\end{aligned}
$$

## 16.7 problem 7

Internal problem ID [11934]

**Book**: Differential Equations by Shepley L. Ross. Third edition. John Willey. New Delhi. 2004.
**Section**: Chapter 7, Systems of linear differential equations. Section 7.1. Exercises page 277
**Problem number**: 7.
**ODE order**: 1.
**ODE degree**: 1.

Solve

$$x'(t) = -t + 2\,e^{3t} + 6y(t)$$
$$y'(t) = x(t) + t - e^{3t}$$

✓ Solution by Maple

Time used: 0.031 (sec). Leaf size: 66

```
dsolve([diff(x(t),t)+diff(y(t),t)-x(t)-6*y(t)=exp(3*t),diff(x(t),t)+2*diff(y(t),t)-2*x(t)-6*
```

$$x(t) = e^{\sqrt{6}\,t} c_2 + e^{-\sqrt{6}\,t} c_1 - t + \frac{1}{6}$$
$$y(t) = \frac{\sqrt{6}\,e^{\sqrt{6}\,t} c_2}{6} - \frac{\sqrt{6}\,e^{-\sqrt{6}\,t} c_1}{6} - \frac{1}{6} + \frac{t}{6} - \frac{e^{3t}}{3}$$

✓ Solution by Mathematica

Time used: 8.119 (sec). Leaf size: 142

```
DSolve[{x'[t]+y'[t]-x[t]-6*y[t]==Exp[3*t],x'[t]+2*y'[t]-2*x[t]-6*y[t]==t},{x[t],y[t]},t,Incl
```

$$x(t) \to \frac{1}{6}\left(-6t + 3\left(c_1 - \sqrt{6}c_2\right) e^{-\sqrt{6}t} + 3\left(c_1 + \sqrt{6}c_2\right) e^{\sqrt{6}t} + 1\right)$$
$$y(t) \to \frac{1}{12} e^{-\sqrt{6}t}\left(2e^{\sqrt{6}t}(t-1) - 4e^{\left(3+\sqrt{6}\right)t} + \left(\sqrt{6}c_1 + 6c_2\right) e^{2\sqrt{6}t} - \sqrt{6}c_1 + 6c_2\right)$$

## 16.8 problem 8

Internal problem ID [11935]

**Book**: Differential Equations by Shepley L. Ross. Third edition. John Willey. New Delhi. 2004.
**Section**: Chapter 7, Systems of linear differential equations. Section 7.1. Exercises page 277
**Problem number**: 8.
**ODE order**: 1.
**ODE degree**: 1.

Solve

$$x'(t) = 6t - 1 + 3y(t)$$
$$y'(t) = x(t) - 3t + 1$$

✓ Solution by Maple

Time used: 0.032 (sec). Leaf size: 60

```
dsolve([diff(x(t),t)+diff(y(t),t)-x(t)-3*y(t)=3*t,diff(x(t),t)+2*diff(y(t),t)-2*x(t)-3*y(t)=
```

$$x(t) = e^{\sqrt{3}\,t}c_2 + e^{-\sqrt{3}\,t}c_1 + 3t - 3$$
$$y(t) = \frac{\sqrt{3}\,e^{\sqrt{3}\,t}c_2}{3} - \frac{\sqrt{3}\,e^{-\sqrt{3}\,t}c_1}{3} + \frac{4}{3} - 2t$$

✓ Solution by Mathematica

Time used: 6.866 (sec). Leaf size: 137

```
DSolve[{x'[t]+y'[t]-x[t]-3*y[t]==3*t,x'[t]+2*y'[t]-2*x[t]-3*y[t]==1},{x[t],y[t]},t,IncludeSi
```

$$x(t) \to \frac{1}{2}e^{-\sqrt{3}t}\left(6e^{\sqrt{3}t}(t-1) + \left(c_1 + \sqrt{3}c_2\right)e^{2\sqrt{3}t} + c_1 - \sqrt{3}c_2\right)$$
$$y(t) \to \frac{1}{6}e^{-\sqrt{3}t}\left(e^{\sqrt{3}t}(8-12t) + \left(\sqrt{3}c_1 + 3c_2\right)e^{2\sqrt{3}t} - \sqrt{3}c_1 + 3c_2\right)$$

## 16.9 problem 9

Internal problem ID [11936]

**Book**: Differential Equations by Shepley L. Ross. Third edition. John Willey. New Delhi. 2004.
**Section**: Chapter 7, Systems of linear differential equations. Section 7.1. Exercises page 277
**Problem number**: 9.
**ODE order**: 1.
**ODE degree**: 1.

Solve

$$
\begin{aligned}
x'(t) + y'(t) &= -2y(t) + \sin(t) \\
x'(t) + y'(t) &= x(t) + y(t)
\end{aligned}
$$

✓ Solution by Maple
Time used: 0.062 (sec). Leaf size: 27

```
dsolve([diff(x(t),t)+diff(y(t),t)+2*y(t)=sin(t),diff(x(t),t)+diff(y(t),t)-x(t)-y(t)=0],sings
```

$$
\begin{aligned}
x(t) &= c_1 e^t - \frac{\sin(t)}{2} \\
y(t) &= -\frac{c_1 e^t}{3} + \frac{\sin(t)}{2}
\end{aligned}
$$

✓ Solution by Mathematica
Time used: 0.02 (sec). Leaf size: 38

```
DSolve[{x'[t]+y'[t]+2*y[t]==Sin[t],x'[t]+y'[t]-x[t]-y[t]==0},{x[t],y[t]},t,IncludeSingularSo
```

$$
\begin{aligned}
x(t) &\to \frac{1}{2}\left(-\sin(t) + 3c_1 e^t\right) \\
y(t) &\to \frac{1}{2}\left(\sin(t) - c_1 e^t\right)
\end{aligned}
$$

## 16.10 problem 10

Internal problem ID [11937]

**Book**: Differential Equations by Shepley L. Ross. Third edition. John Willey. New Delhi. 2004.
**Section**: Chapter 7, Systems of linear differential equations. Section 7.1. Exercises page 277
**Problem number**: 10.
**ODE order**: 1.
**ODE degree**: 1.

Solve

$$
\begin{aligned}
x'(t) &= \frac{3x(t)}{2} - \frac{3y(t)}{2} + \frac{t}{2} + \frac{1}{2} \\
y'(t) &= -\frac{x(t)}{2} + \frac{5y(t)}{2} - \frac{t}{2} + \frac{1}{2}
\end{aligned}
$$

✓ Solution by Maple

Time used: 0.031 (sec). Leaf size: 40

```
dsolve([diff(x(t),t)-diff(y(t),t)-2*x(t)+4*y(t)=t,diff(x(t),t)+diff(y(t),t)-x(t)-y(t)=1],sin
```

$$
\begin{aligned}
x(t) &= c_2 e^t + c_1 e^{3t} - \frac{t}{6} - \frac{13}{18} \\
y(t) &= \frac{c_2 e^t}{3} - c_1 e^{3t} - \frac{5}{18} + \frac{t}{6}
\end{aligned}
$$

✓ Solution by Mathematica

Time used: 0.083 (sec). Leaf size: 74

```
DSolve[{x'[t]-y'[t]-2*x[t]+4*y[t]==t,x'[t]+y'[t]-x[t]-y[t]==1},{x[t],y[t]},t,IncludeSingular
```

$$
\begin{aligned}
x(t) &\to \frac{1}{36}\left(-6t + 9(c_1 - 3c_2)e^{3t} + 27(c_1 + c_2)e^t - 26\right) \\
y(t) &\to \frac{1}{36}\left(6t - 9(c_1 - 3c_2)e^{3t} + 9(c_1 + c_2)e^t - 10\right)
\end{aligned}
$$

## 16.11 problem 11

Internal problem ID [11938]

**Book**: Differential Equations by Shepley L. Ross. Third edition. John Willey. New Delhi. 2004.
**Section**: Chapter 7, Systems of linear differential equations. Section 7.1. Exercises page 277
**Problem number**: 11.
**ODE order**: 1.
**ODE degree**: 1.

Solve

$$
\begin{aligned}
x'(t) &= -2 + x(t) - 3y(t) + 4t \\
y'(t) &= 4 - 3x(t) + y(t) - 4t
\end{aligned}
$$

✓ Solution by Maple
Time used: 0.015 (sec). Leaf size: 40

```
dsolve([2*diff(x(t),t)+diff(y(t),t)+x(t)+5*y(t)=4*t,diff(x(t),t)+diff(y(t),t)+2*x(t)+2*y(t)=
```

$$
\begin{aligned}
x(t) &= c_2 e^{4t} + c_1 e^{-2t} - t + 1 \\
y(t) &= -c_2 e^{4t} + c_1 e^{-2t} + t
\end{aligned}
$$

✓ Solution by Mathematica
Time used: 0.078 (sec). Leaf size: 80

```
DSolve[{2*x'[t]+y'[t]+x[t]+5*y[t]==4*t,x'[t]+y'[t]+2*x[t]+2*y[t]==2},{x[t],y[t]},t,IncludeSi
```

$$
\begin{aligned}
x(t) &\to \frac{1}{2} e^{-2t} \left( -2e^{2t}(t-1) + (c_1 - c_2)e^{6t} + c_1 + c_2 \right) \\
y(t) &\to \frac{1}{2} e^{-2t} \left( 2e^{2t} t + (c_2 - c_1)e^{6t} + c_1 + c_2 \right)
\end{aligned}
$$

## 16.12 problem 12

Internal problem ID [11939]

**Book**: Differential Equations by Shepley L. Ross. Third edition. John Willey. New Delhi. 2004.
**Section**: Chapter 7, Systems of linear differential equations. Section 7.1. Exercises page 277
**Problem number**: 12.
**ODE order**: 1.
**ODE degree**: 1.

Solve

$$x'(t) = 2t^2 - 6y(t) - 2t - 1$$
$$y'(t) = -t^2 + x(t) + y(t) + 2t + 1$$

✓ Solution by Maple

Time used: 0.031 (sec). Leaf size: 116

```
dsolve([diff(x(t),t)+diff(y(t),t)-x(t)+5*y(t)=t^2,diff(x(t),t)+2*diff(y(t),t)-2*x(t)+4*y(t)=
```

$$x(t) = e^{\frac{t}{2}} \sin\left(\frac{\sqrt{23}\,t}{2}\right) c_2 + e^{\frac{t}{2}} \cos\left(\frac{\sqrt{23}\,t}{2}\right) c_1 + \frac{2t^2}{3} - \frac{7t}{9} - \frac{41}{27}$$
$$y(t) = \frac{t^2}{3} - \frac{e^{\frac{t}{2}} \sin\left(\frac{\sqrt{23}\,t}{2}\right) c_2}{12} - \frac{e^{\frac{t}{2}} \sqrt{23} \cos\left(\frac{\sqrt{23}\,t}{2}\right) c_2}{12} - \frac{e^{\frac{t}{2}} \cos\left(\frac{\sqrt{23}\,t}{2}\right) c_1}{12} + \frac{e^{\frac{t}{2}} \sqrt{23} \sin\left(\frac{\sqrt{23}\,t}{2}\right) c_1}{12} - \frac{5t}{9} - \frac{1}{27}$$

✓ Solution by Mathematica

Time used: 11.178 (sec). Leaf size: 143

```
DSolve[{x'[t]+y'[t]-x[t]+5*y[t]==t^2,x'[t]+2*y'[t]-2*x[t]+4*y[t]==2*t+1},{x[t],y[t]},t,Inclu
```

$$x(t) \to \frac{1}{27}\left(18t^2 - 21t - 41\right) + c_1 e^{t/2} \cos\left(\frac{\sqrt{23}t}{2}\right) - \frac{(c_1 + 12c_2)e^{t/2} \sin\left(\frac{\sqrt{23}t}{2}\right)}{\sqrt{23}}$$
$$y(t) \to \frac{1}{27}\left(9t^2 - 15t - 1\right) + c_2 e^{t/2} \cos\left(\frac{\sqrt{23}t}{2}\right) + \frac{(2c_1 + c_2)e^{t/2} \sin\left(\frac{\sqrt{23}t}{2}\right)}{\sqrt{23}}$$

## 16.13 problem 13

Internal problem ID [11940]

**Book**: Differential Equations by Shepley L. Ross. Third edition. John Willey. New Delhi. 2004.
**Section**: Chapter 7, Systems of linear differential equations. Section 7.1. Exercises page 277
**Problem number**: 13.
**ODE order**: 1.
**ODE degree**: 1.

Solve

$$x'(t) = -t^2 + x(t) + y(t) + 6t$$
$$y'(t) = 3t^2 - 3x(t) - 3y(t) - 8t$$

✓ Solution by Maple
Time used: 0.016 (sec). Leaf size: 43

```
dsolve([2*diff(x(t),t)+diff(y(t),t)+x(t)+y(t)=t^2+4*t,diff(x(t),t)+diff(y(t),t)+2*x(t)+2*y(t
```

$$x(t) = -\frac{c_1 e^{-2t}}{2} + 2t^2 + t + c_2$$
$$y(t) = -t^2 + \frac{3c_1 e^{-2t}}{2} - 3t + 1 - c_2$$

✓ Solution by Mathematica
Time used: 0.101 (sec). Leaf size: 82

```
DSolve[{2*x'[t]+y'[t]+x[t]+y[t]==t^2+4*t,x'[t]+y'[t]+2*x[t]+2*y[t]==2*t^2-2*t},{x[t],y[t]},t
```

$$x(t) \to \frac{1}{2}e^{-2t}\left(e^{2t}\left(4t^2 + 2t - 1 + 3c_1 + c_2\right) - c_1 - c_2\right)$$
$$y(t) \to \frac{1}{2}\left(-2t^2 - 6t + 3(c_1 + c_2)e^{-2t} + 3 - 3c_1 - c_2\right)$$

## 16.14 problem 14

Internal problem ID [11941]

**Book**: Differential Equations by Shepley L. Ross. Third edition. John Willey. New Delhi. 2004.
**Section**: Chapter 7, Systems of linear differential equations. Section 7.1. Exercises page 277
**Problem number**: 14.
**ODE order**: 1.
**ODE degree**: 1.

Solve

$$
\begin{aligned}
x'(t) &= -5 - x(t) - t - y(t)\\
y'(t) &= 7 + 2x(t) + 2t + y(t)
\end{aligned}
$$

✓ Solution by Maple

Time used: 0.015 (sec). Leaf size: 42

```
dsolve([3*diff(x(t),t)+2*diff(y(t),t)-x(t)+y(t)=t-1,diff(x(t),t)+diff(y(t),t)-x(t)=t+2],sing
```

$$
\begin{aligned}
x(t) &= c_2 \sin(t) + c_1 \cos(t) - 3 - t\\
y(t) &= -c_2 \cos(t) + c_1 \sin(t) - 1 - c_2 \sin(t) - c_1 \cos(t)
\end{aligned}
$$

✓ Solution by Mathematica

Time used: 0.106 (sec). Leaf size: 44

```
DSolve[{3*x'[t]+2*y'[t]-x[t]+y[t]==t-1,x'[t]+y'[t]-x[t]==t+2},{x[t],y[t]},t,IncludeSingularS
```

$$
\begin{aligned}
x(t) &\to -t + c_1 \cos(t) - (c_1 + c_2) \sin(t) - 3\\
y(t) &\to c_2 \cos(t) + (2c_1 + c_2) \sin(t) - 1
\end{aligned}
$$

## 16.15 problem 15

Internal problem ID [11942]

**Book**: Differential Equations by Shepley L. Ross. Third edition. John Willey. New Delhi. 2004.
**Section**: Chapter 7, Systems of linear differential equations. Section 7.1. Exercises page 277
**Problem number**: 15.
**ODE order**: 1.
**ODE degree**: 1.

Solve

$$
\begin{aligned}
x'(t) &= -\frac{7x(t)}{2} - \frac{9y(t)}{2} + \frac{e^t}{2} \\
y'(t) &= \frac{3x(t)}{2} + \frac{5y(t)}{2} + \frac{e^t}{2}
\end{aligned}
$$

✓ Solution by Maple
Time used: 0.032 (sec). Leaf size: 45

```
dsolve([2*diff(x(t),t)+4*diff(y(t),t)+x(t)-y(t)=3*exp(t),diff(x(t),t)+diff(y(t),t)+2*x(t)+2*
```

$$
\begin{aligned}
x(t) &= c_2 e^t + c_1 e^{-2t} - e^t t \\
y(t) &= -c_2 e^t - \frac{c_1 e^{-2t}}{3} + e^t t + \frac{e^t}{3}
\end{aligned}
$$

✓ Solution by Mathematica
Time used: 0.014 (sec). Leaf size: 76

```
DSolve[{2*x'[t]+4*y'[t]+x[t]-y[t]==3*Exp[t],x'[t]+y'[t]+2*x[t]+2*y[t]==Exp[t]},{x[t],y[t]},t
```

$$
\begin{aligned}
x(t) &\to \frac{3}{2}(c_1 + c_2)e^{-2t} - \frac{1}{2}e^t(2t - 1 + c_1 + 3c_2) \\
y(t) &\to \frac{1}{6}e^t(6t - 1 + 3c_1 + 9c_2) - \frac{1}{2}(c_1 + c_2)e^{-2t}
\end{aligned}
$$

## 16.16 problem 16

Internal problem ID [11943]

**Book**: Differential Equations by Shepley L. Ross. Third edition. John Willey. New Delhi. 2004.
**Section**: Chapter 7, Systems of linear differential equations. Section 7.1. Exercises page 277
**Problem number**: 16.
**ODE order**: 1.
**ODE degree**: 1.

Solve

$$x'(t) = 2x(t) - 2t - t^2$$
$$y'(t) = -3x(t) + y(t) + 2t + 2t^2$$

✓ Solution by Maple

Time used: 0.031 (sec). Leaf size: 45

```
dsolve([2*diff(x(t),t)+diff(y(t),t)-x(t)-y(t)=-2*t,diff(x(t),t)+diff(y(t),t)+x(t)-y(t)=t^2],
```

$$x(t) = \frac{t^2}{2} + \frac{3t}{2} + \frac{3}{4} + c_2 e^{2t}$$
$$y(t) = \frac{15}{4} - 3c_2 e^{2t} + \frac{3t}{2} - \frac{t^2}{2} + c_1 e^t$$

✓ Solution by Mathematica

Time used: 0.215 (sec). Leaf size: 67

```
DSolve[{2*x'[t]+y'[t]-x[t]-y[t]==-2*t,x'[t]+y'[t]+x[t]-y[t]==t^2},{x[t],y[t]},t,IncludeSingu
```

$$x(t) \to \frac{1}{4}\left(2t^2 + 6t + 4c_1 e^{2t} + 3\right)$$
$$y(t) \to -\frac{t^2}{2} + \frac{3t}{2} - 3c_1 e^{2t} + (3c_1 + c_2)e^t + \frac{15}{4}$$

## 16.17 problem 17

Internal problem ID [11944]

**Book**: Differential Equations by Shepley L. Ross. Third edition. John Willey. New Delhi. 2004.
**Section**: Chapter 7, Systems of linear differential equations. Section 7.1. Exercises page 277
**Problem number**: 17.
**ODE order**: 1.
**ODE degree**: 1.

Solve

$$x'(t) = 3x(t) - t + 1$$
$$y'(t) = -5x(t) + y(t) + 2t - 1$$

✓ Solution by Maple

Time used: 0.016 (sec). Leaf size: 35

```
dsolve([2*diff(x(t),t)+diff(y(t),t)-x(t)-y(t)=1,diff(x(t),t)+diff(y(t),t)+2*x(t)-y(t)=t],sin
```

$$x(t) = \frac{t}{3} - \frac{2}{9} + c_2 e^{3t}$$
$$y(t) = -\frac{4}{9} - \frac{5c_2 e^{3t}}{2} - \frac{t}{3} + c_1 e^{t}$$

✓ Solution by Mathematica

Time used: 0.147 (sec). Leaf size: 58

```
DSolve[{2*x'[t]+y'[t]-x[t]-y[t]==1,x'[t]+y'[t]+2*x[t]-y[t]==t},{x[t],y[t]},t,IncludeSingular
```

$$x(t) \to \frac{t}{3} + c_1 e^{3t} - \frac{2}{9}$$
$$y(t) \to -\frac{t}{3} - \frac{5}{2} c_1 e^{3t} + \left(\frac{5c_1}{2} + c_2\right) e^{t} - \frac{4}{9}$$

# 17 Chapter 7, Systems of linear differential equations. Section 7.3. Exercises page 299

## 17.1 problem 1

Internal problem ID [11945]

**Book**: Differential Equations by Shepley L. Ross. Third edition. John Willey. New Delhi. 2004.
**Section**: Chapter 7, Systems of linear differential equations. Section 7.3. Exercises page 299
**Problem number**: 1.
**ODE order**: 1.
**ODE degree**: 1.

Solve

$$x'(t) = 3x(t) + 4y(t)$$
$$y'(t) = 2x(t) + y(t)$$

With initial conditions

$$[x(0) = 1, y(0) = 2]$$

✓ Solution by Maple
Time used: 0.032 (sec). Leaf size: 30

```
dsolve([diff(x(t),t) = 3*x(t)+4*y(t), diff(y(t),t) = 2*x(t)+y(t), x(0) = 1, y(0) = 2], sings
```

$$x(t) = 2\,\mathrm{e}^{5t} - \mathrm{e}^{-t}$$
$$y(t) = \mathrm{e}^{5t} + \mathrm{e}^{-t}$$

✓ Solution by Mathematica
Time used: 0.008 (sec). Leaf size: 34

```
DSolve[{x'[t]==3*x[t]+4*y[t],y'[t]==2*x[t]+y[t]},{x[0]==1,y[0]==2},{x[t],y[t]},t,IncludeSing
```

$$x(t) \to e^{-t}\left(2e^{6t} - 1\right)$$
$$y(t) \to e^{-t} + e^{5t}$$

## 17.2 problem 2

Internal problem ID [11946]

**Book**: Differential Equations by Shepley L. Ross. Third edition. John Willey. New Delhi. 2004.
**Section**: Chapter 7, Systems of linear differential equations. Section 7.3. Exercises page 299
**Problem number**: 2.
**ODE order**: 1.
**ODE degree**: 1.

Solve

$$x'(t) = 5x(t) + 3y(t)$$
$$y'(t) = 4x(t) + y(t)$$

With initial conditions

$$[x(0) = 0, y(0) = 8]$$

✓ Solution by Maple

Time used: 0.016 (sec). Leaf size: 34

```
dsolve([diff(x(t),t) = 5*x(t)+3*y(t), diff(y(t),t) = 4*x(t)+y(t), x(0) = 0, y(0) = 8], sings
```

$$x(t) = 3\,\mathrm{e}^{7t} - 3\,\mathrm{e}^{-t}$$
$$y(t) = 2\,\mathrm{e}^{7t} + 6\,\mathrm{e}^{-t}$$

✓ Solution by Mathematica

Time used: 0.005 (sec). Leaf size: 36

```
DSolve[{x'[t]==5*x[t]+3*y[t],y'[t]==4*x[t]+y[t]},{x[0]==0,y[0]==8},{x[t],y[t]},t,IncludeSing
```

$$x(t) \to 3e^{-t}\left(e^{8t} - 1\right)$$
$$y(t) \to 2e^{-t}\left(e^{8t} + 3\right)$$

## 17.3 problem 3

Internal problem ID [11947]

**Book**: Differential Equations by Shepley L. Ross. Third edition. John Willey. New Delhi. 2004.
**Section**: Chapter 7, Systems of linear differential equations. Section 7.3. Exercises page 299
**Problem number**: 3.
**ODE order**: 1.
**ODE degree**: 1.

Solve

$$x'(t) = 5x(t) + 2y(t) + 5t$$
$$y'(t) = 3x(t) + 4y(t) + 17t$$

✓ Solution by Maple
Time used: 0.031 (sec). Leaf size: 41

```
dsolve([diff(x(t),t)=5*x(t)+2*y(t)+5*t,diff(y(t),t)=3*x(t)+4*y(t)+17*t],singsol=all)
```

$$x(t) = c_2 e^{7t} + c_1 e^{2t} + t + 1$$
$$y(t) = c_2 e^{7t} - \frac{3c_1 e^{2t}}{2} - 2 - 5t$$

✓ Solution by Mathematica
Time used: 0.086 (sec). Leaf size: 84

```
DSolve[{x'[t]==5*x[t]+2*y[t]+5*t,y'[t]==3*x[t]+4*y[t]+17*t},{x[t],y[t]},t,IncludeSingularSol
```

$$x(t) \to t + \frac{1}{5}\left(2(c_1 - c_2)e^{2t} + (3c_1 + 2c_2)e^{7t} + 5\right)$$
$$y(t) \to -5t - \frac{3}{5}(c_1 - c_2)e^{2t} + \frac{1}{5}(3c_1 + 2c_2)e^{7t} - 2$$

# 18 Chapter 7, Systems of linear differential equations. Section 7.4. Exercises page 309

## 18.1 problem 1

Internal problem ID [11948]

**Book**: Differential Equations by Shepley L. Ross. Third edition. John Willey. New Delhi. 2004.
**Section**: Chapter 7, Systems of linear differential equations. Section 7.4. Exercises page 309
**Problem number**: 1.
**ODE order**: 1.
**ODE degree**: 1.

Solve

$$
\begin{aligned}
x'(t) &= 5x(t) - 2y(t) \\
y'(t) &= 4x(t) - y(t)
\end{aligned}
$$

✓ Solution by Maple

Time used: 0.015 (sec). Leaf size: 31

```
dsolve([diff(x(t),t)=5*x(t)-2*y(t),diff(y(t),t)=4*x(t)-y(t)],singsol=all)
```

$$
\begin{aligned}
x(t) &= c_1 \mathrm{e}^t + c_2 \mathrm{e}^{3t} \\
y(t) &= 2c_1 \mathrm{e}^t + c_2 \mathrm{e}^{3t}
\end{aligned}
$$

✓ Solution by Mathematica

Time used: 0.003 (sec). Leaf size: 63

```
DSolve[{x'[t]==5*x[t]-2*y[t],y'[t]==4*x[t]-y[t]},{x[t],y[t]},t,IncludeSingularSolutions -> T
```

$$
\begin{aligned}
x(t) &\to e^t \left(c_1 \left(2e^{2t} - 1\right) - c_2 \left(e^{2t} - 1\right)\right) \\
y(t) &\to e^t \left(2c_1 \left(e^{2t} - 1\right) - c_2 \left(e^{2t} - 2\right)\right)
\end{aligned}
$$

## 18.2 problem 2

Internal problem ID [11949]

**Book**: Differential Equations by Shepley L. Ross. Third edition. John Willey. New Delhi. 2004.
**Section**: Chapter 7, Systems of linear differential equations. Section 7.4. Exercises page 309
**Problem number**: 2.
**ODE order**: 1.
**ODE degree**: 1.

Solve

$$
\begin{aligned}
x'(t) &= 5x(t) - y(t) \\
y'(t) &= 3x(t) + y(t)
\end{aligned}
$$

✓ Solution by Maple
Time used: 0.016 (sec). Leaf size: 35

```
dsolve([diff(x(t),t)=5*x(t)-y(t),diff(y(t),t)=3*x(t)+y(t)],singsol=all)
```

$$
\begin{aligned}
x(t) &= c_1 \mathrm{e}^{4t} + c_2 \mathrm{e}^{2t} \\
y(t) &= c_1 \mathrm{e}^{4t} + 3c_2 \mathrm{e}^{2t}
\end{aligned}
$$

✓ Solution by Mathematica
Time used: 0.003 (sec). Leaf size: 73

```
DSolve[{x'[t]==5*x[t]-y[t],y'[t]==3*x[t]+y[t]},{x[t],y[t]},t,IncludeSingularSolutions -> Tru
```

$$
\begin{aligned}
x(t) &\to \frac{1}{2} e^{2t} \left(c_1 \left(3e^{2t} - 1\right) - c_2 \left(e^{2t} - 1\right)\right) \\
y(t) &\to \frac{1}{2} e^{2t} \left(3c_1 \left(e^{2t} - 1\right) - c_2 \left(e^{2t} - 3\right)\right)
\end{aligned}
$$

## 18.3 problem 23

Internal problem ID [11950]

**Book**: Differential Equations by Shepley L. Ross. Third edition. John Willey. New Delhi. 2004.
**Section**: Chapter 7, Systems of linear differential equations. Section 7.4. Exercises page 309
**Problem number**: 23.
**ODE order**: 1.
**ODE degree**: 1.

Solve

$$x'(t) = -2x(t) + 7y(t)$$
$$y'(t) = 3x(t) + 2y(t)$$

With initial conditions

$$[x(0) = 9, y(0) = -1]$$

✓ Solution by Maple
Time used: 0.015 (sec). Leaf size: 34

```
dsolve([diff(x(t),t) = -2*x(t)+7*y(t), diff(y(t),t) = 3*x(t)+2*y(t), x(0) = 9, y(0) = -1], s
```

$$x(t) = 2\,\mathrm{e}^{5t} + 7\,\mathrm{e}^{-5t}$$
$$y(t) = 2\,\mathrm{e}^{5t} - 3\,\mathrm{e}^{-5t}$$

✓ Solution by Mathematica
Time used: 0.004 (sec). Leaf size: 38

```
DSolve[{x'[t]==-2*x[t]+7*y[t],y'[t]==3*x[t]+2*y[t]},{x[0]==9,y[0]==-1},{x[t],y[t]},t,Include
```

$$x(t) \to 7e^{-5t} + 2e^{5t}$$
$$y(t) \to 2e^{5t} - 3e^{-5t}$$

## 18.4 problem 24

Internal problem ID [11951]

**Book**: Differential Equations by Shepley L. Ross. Third edition. John Willey. New Delhi. 2004.
**Section**: Chapter 7, Systems of linear differential equations. Section 7.4. Exercises page 309
**Problem number**: 24.
**ODE order**: 1.
**ODE degree**: 1.

Solve

$$x'(t) = -2x(t) + y(t)$$
$$y'(t) = 7x(t) + 4y(t)$$

With initial conditions

$$[x(0) = 6, y(0) = 2]$$

✓ Solution by Maple

Time used: 0.015 (sec). Leaf size: 32

```
dsolve([diff(x(t),t) = -2*x(t)+y(t), diff(y(t),t) = 7*x(t)+4*y(t), x(0) = 6, y(0) = 2], sing
```

$$x(t) = e^{5t} + 5\,e^{-3t}$$
$$y(t) = 7\,e^{5t} - 5\,e^{-3t}$$

✓ Solution by Mathematica

Time used: 0.004 (sec). Leaf size: 36

```
DSolve[{x'[t]==-2*x[t]+y[t],y'[t]==7*x[t]+4*y[t]},{x[0]==6,y[0]==2},{x[t],y[t]},t,IncludeSin
```

$$x(t) \to e^{-3t}\left(e^{8t} + 5\right)$$
$$y(t) \to e^{-3t}\left(7e^{8t} - 5\right)$$

# 19 Chapter 7, Systems of linear differential equations. Section 7.7. Exercises page 375

## 19.1 problem 1

Internal problem ID [11966]

**Book**: Differential Equations by Shepley L. Ross. Third edition. John Willey. New Delhi. 2004.
**Section**: Chapter 7, Systems of linear differential equations. Section 7.7. Exercises page 375
**Problem number**: 1.
**ODE order**: 1.
**ODE degree**: 1.

Solve

$$
\begin{aligned}
x'(t) &= x(t) + y(t) - z(t) \\
y'(t) &= 2x(t) + 3y(t) - 4z(t) \\
z'(t) &= 4x(t) + y(t) - 4z(t)
\end{aligned}
$$

✓ Solution by Maple

Time used: 0.031 (sec). Leaf size: 65

```
dsolve([diff(x(t),t)=x(t)+y(t)-z(t),diff(y(t),t)=2*x(t)+3*y(t)-4*z(t),diff(z(t),t)=4*x(t)+y(
```

$$
\begin{aligned}
x(t) &= c_1 e^t + c_2 e^{2t} + c_3 e^{-3t} \\
y(t) &= c_1 e^t + 2c_2 e^{2t} + 7c_3 e^{-3t} \\
z(t) &= c_1 e^t + c_2 e^{2t} + 11c_3 e^{-3t}
\end{aligned}
$$

✓ Solution by Mathematica

Time used: 0.012 (sec). Leaf size: 198

```
DSolve[{x'[t]==x[t]+y[t]-z[t],y'[t]==2*x[t]+3*y[t]-4*z[t],z'[t]==4*x[t]+y[t]-4*z[t]},{x[t],y
```

$$
\begin{aligned}
x(t) &\to \frac{1}{10}e^{-3t}\left(c_1\left(15e^{4t} - 4e^{5t} - 1\right) + 2(5c_2 - 3c_3)e^{5t} + 5(c_3 - 2c_2)e^{4t} + c_3\right) \\
y(t) &\to \frac{1}{10}e^{-3t}\left(c_1\left(15e^{4t} - 8e^{5t} - 7\right) + 4(5c_2 - 3c_3)e^{5t} + 5(c_3 - 2c_2)e^{4t} + 7c_3\right) \\
z(t) &\to \frac{1}{10}e^{-3t}\left(c_1\left(15e^{4t} - 4e^{5t} - 11\right) + 2(5c_2 - 3c_3)e^{5t} + 5(c_3 - 2c_2)e^{4t} + 11c_3\right)
\end{aligned}
$$

## 19.2 problem 2

Internal problem ID [11967]

**Book**: Differential Equations by Shepley L. Ross. Third edition. John Willey. New Delhi. 2004.
**Section**: Chapter 7, Systems of linear differential equations. Section 7.7. Exercises page 375
**Problem number**: 2.
**ODE order**: 1.
**ODE degree**: 1.

Solve

$$
\begin{aligned}
x'(t) &= x(t) - y(t) - z(t) \\
y'(t) &= x(t) + 3y(t) + z(t) \\
z'(t) &= -3x(t) - 6y(t) + 6z(t)
\end{aligned}
$$

✓ Solution by Maple
Time used: 0.031 (sec). Leaf size: 74

```
dsolve([diff(x(t),t)=x(t)-y(t)-z(t),diff(y(t),t)=x(t)+3*y(t)+z(t),diff(z(t),t)=-3*x(t)-6*y(t
```

$$
\begin{aligned}
x(t) &= c_1 e^{3t} + c_2 e^{2t} + c_3 e^{5t} \\
y(t) &= -c_1 e^{3t} - \frac{7c_2 e^{2t}}{10} - c_3 e^{5t} \\
z(t) &= -c_1 e^{3t} - \frac{3c_2 e^{2t}}{10} - 3c_3 e^{5t}
\end{aligned}
$$

✓ Solution by Mathematica
Time used: 0.04 (sec). Leaf size: 217

```
DSolve[{x'[t]==x[t]-y[t]-z[t],y'[t]==x[t]+3*y[t]+z[t],z'[t]==3*x[t]-6*y[t]+6*z[t]},{x[t],y[t
```

$$
\begin{aligned}
x(t) &\to -\frac{1}{45} e^{2t} \left( 5(c_1 + 10c_2) e^{2t} \cos\left(\sqrt{5}t\right) + \sqrt{5}(7c_1 - 11c_2 + 9c_3) e^{2t} \sin\left(\sqrt{5}t\right) \right. \\
&\qquad \left. - 50(c_1 + c_2) \right) \\
y(t) &\to \frac{1}{45} e^{2t} \left( 5(c_1 + 10c_2) e^{2t} \cos\left(\sqrt{5}t\right) + \sqrt{5}(7c_1 - 11c_2 + 9c_3) e^{2t} \sin\left(\sqrt{5}t\right) - 5(c_1 + c_2) \right) \\
z(t) &\to (c_1 + c_2) \left(-e^{2t}\right) + (c_1 + c_2 + c_3) e^{4t} \cos\left(\sqrt{5}t\right) + \frac{(c_1 - 8c_2 + 2c_3) e^{4t} \sin\left(\sqrt{5}t\right)}{\sqrt{5}}
\end{aligned}
$$