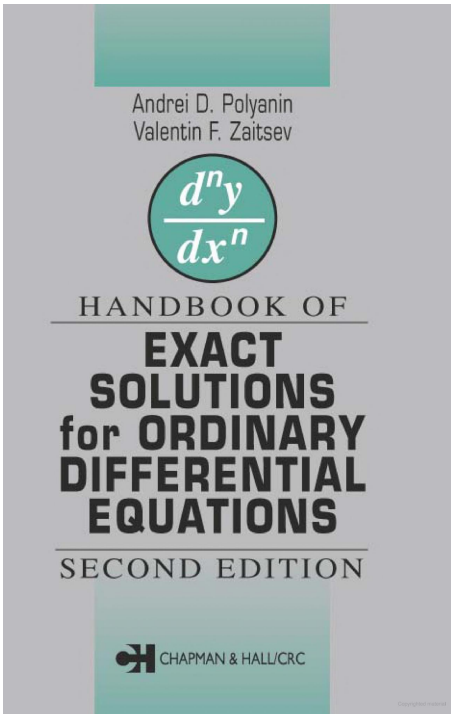
A Solution Manual For

Handbook of exact solutions for ordinary differential equations. By Polyanin and Zaitsev. Second edition



Nasser M. Abbasi

May 16, 2024

Contents

1 Chapter 1, First-Order differential equations **4**

2 Chapter 1, section 1.2. Riccati Equation. 1.2.2. Equations Containing Power Functions **11**

3 Chapter 1, section 1.2. Riccati Equation. subsection 1.2.3. Equations Containing Exponential Functions **120**

4 Chapter 1, section 1.2. Riccati Equation. subsection 1.2.3-2. Equations with power and exponential functions **150**

5 Chapter 1, section 1.2. Riccati Equation. subsection 1.2.4-1. Equations with hyperbolic sine and cosine **171**

6 Chapter 1, section 1.2. Riccati Equation. subsection 1.2.4-2. Equations with hyperbolic tangent and cotangent. **197**

7 Chapter 1, section 1.2. Riccati Equation. subsection 1.2.5-1. Equations Containing Logarithmic Functions **210**

8 Chapter 1, section 1.2. Riccati Equation. subsection 1.2.5-2 **222**

9 Chapter 1, section 1.2. Riccati Equation. subsection 1.2.6-1. Equations with sine **240**

10 Chapter 1, section 1.2. Riccati Equation. subsection 1.2.6-2. Equations with cosine. **255**

11 Chapter 1, section 1.2. Riccati Equation. subsection 1.2.6-3. Equations with tangent. **271**

12 Chapter 1, section 1.2. Riccati Equation. subsection 1.2.6-4. Equations with cotangent. **285**

13 Chapter 1, section 1.2. Riccati Equation. subsection 1.2.6-5. Equations containing combinations of trigonometric functions. **296**

14 Chapter 1, section 1.2. Riccati Equation. subsection 1.2.7-1. Equations containing arcsine. **312**

15 Chapter 1, section 1.2. Riccati Equation. subsection 1.2.7-2. Equations containing arccosine. 323

16 Chapter 1, section 1.2. Riccati Equation. subsection 1.2.7-3. Equations containing arctangent. 333

17 Chapter 1, section 1.2. Riccati Equation. subsection 1.2.7-4. Equations containing arccotangent. 343

18 Chapter 1, section 1.2. Riccati Equation. subsection 1.2.7-3. Equations containing arctangent. 346

19 Chapter 1, section 1.2. Riccati Equation. subsection 1.2.8-1. Equations containing arbitrary functions (but not containing their derivatives). 356

20 Chapter 1, section 1.2. Riccati Equation. subsection 1.2.8-2. Equations containing arbitrary functions and their derivatives. 392

21 Chapter 1, section 1.2. Riccati Equation. subsection 1.2.9. Some Transformations 402

22 Chapter 1, section 1.3. Abel Equations of the Second Kind. Form $yy'-y=f(x)$. subsection 1.3.1-2. Solvable equations and their solutions 417

23 Chapter 1, section 1.3. Abel Equations of the Second Kind. subsection 1.3.2. Equations of the form $yy'=f(x)y+1$ 507

24 Chapter 1, section 1.3. Abel Equations of the Second Kind. subsection 1.3.3-2. Equations of the form $yy'=f_1(x)y+f_0(x)$ 523

25 Chapter 1, section 1.3. Abel Equations of the Second Kind. subsection 1.3.4-2. Equations of the form $(g_1(x)+g_0(x))y'=f_2(x)y^2+f_1(x)y+f_0(x)$ 609

26 Chapter 2, Second-Order Differential Equations. section 2.1.2 Equations Containing Power Functions. page 213 617

27 Chapter 2, Second-Order Differential Equations. section 2.1.2-2 Equation of form $y''+f(x)y'+g(x)y=0$ 628

28 Chapter 2, Second-Order Differential Equations. section 2.1.2-3 Equation of form $(ax+b)y''+f(x)y'+g(x)y=0$ 682

29 Chapter 2, Second-Order Differential Equations. section 2.1.2-4 Equation of form $x^2y''+f(x)y'+g(x)y=0$ 735

30 Chapter 2, Second-Order Differential Equations. section 2.1.2-5 Equation of form $(ax^2+bx+c)y''+f(x)y'+g(x)y=0$ 780

31 Chapter 2, Second-Order Differential Equations. section 2.1.2-6 Equation of form $(a_3x^3+a_2x^2x+a_1x+a_0)y''+f(x)y'+g(x)y=0$ 821

32 Chapter 2, Second-Order Differential Equations. section 2.1.2-7 Equation of form $(a_4x^4+a_3x^3+a_2x^2x+a_1x+a_0)y''+f(x)y'+g(x)y=0$ 856

33 Chapter 2, Second-Order Differential Equations. section 2.1.2-8. Other equations. 891

34 Chapter 2, Second-Order Differential Equations. section 2.1.3-1. Equations with exponential functions 919

1 Chapter 1, First-Order differential equations

1.1 problem 1.1.1 5
1.2 problem 1.1.2 6
1.3 problem 1.1.3 7
1.4 problem 1.1.4 8
1.5 problem 1.1.5 9
1.6 problem 1.1.6 10

1.1 problem 1.1.1

Internal problem ID [10325]

Book: Handbook of exact solutions for ordinary differential equations. By Polyanin and Zaitsev. Second edition
Section: Chapter 1, First-Order differential equations
Problem number: 1.1.1.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_quadrature]`

$$\boxed{y' = f(x)}$$

✓ Solution by Maple
Time used: 0.0 (sec). Leaf size: 10

```
dsolve(diff(y(x),x)=f(x),y(x), singsol=all)
```

$$y(x) = \int f(x)\,dx + c_1$$

✓ Solution by Mathematica
Time used: 0.004 (sec). Leaf size: 18

```
DSolve[y'[x]==f[x],y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to \int_1^x f(K[1])dK[1] + c_1$$

1.2 problem 1.1.2

Internal problem ID [10326]

Book: Handbook of exact solutions for ordinary differential equations. By Polyanin and Zaitsev. Second edition
Section: Chapter 1, First-Order differential equations
Problem number: 1.1.2.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_quadrature]`

$$\boxed{y' - f(y) = 0}$$

✓ Solution by Maple
Time used: 0.0 (sec). Leaf size: 17

```
dsolve(diff(y(x),x)=f(y(x)),y(x), singsol=all)
```

$$x - \left(\int^{y(x)} \frac{1}{f(\_a)} d\_a \right) + c_1 = 0$$

✓ Solution by Mathematica
Time used: 0.442 (sec). Leaf size: 33

```
DSolve[y'[x]==f[y[x]],y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to \text{InverseFunction}\left[\int_1^{\#1} \frac{1}{f(K[1])} dK[1] \& \right][x + c_1]$$
$$y(x) \to f^{(-1)}(0)$$

1.3 problem 1.1.3

Internal problem ID [10327]

Book: Handbook of exact solutions for ordinary differential equations. By Polyanin and Zaitsev. Second edition
Section: Chapter 1, First-Order differential equations
Problem number: 1.1.3.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_separable]`

$$\boxed{y' - f(x)\, g(y) = 0}$$

✓ Solution by Maple
Time used: 0.0 (sec). Leaf size: 20

```
dsolve(diff(y(x),x)=f(x)*g(y(x)),y(x), singsol=all)
```

$$\int f(x)\, dx - \left(\int^{y(x)} \frac{1}{g\left(\_a\right)} d\_a \right) + c_1 = 0$$

✓ Solution by Mathematica
Time used: 0.452 (sec). Leaf size: 42

```
DSolve[y'[x]==f[x]*g[y[x]],y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to \text{InverseFunction}\left[\int_1^{\#1} \frac{1}{g(K[1])} dK[1]\&\right]\left[\int_1^x f(K[2]) dK[2] + c_1\right]$$
$$y(x) \to g^{(-1)}(0)$$

1.4 problem 1.1.4

Internal problem ID [10328]

Book: Handbook of exact solutions for ordinary differential equations. By Polyanin and Zaitsev. Second edition
Section: Chapter 1, First-Order differential equations
Problem number: 1.1.4.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_linear]`

$$\boxed{g(x)\,y' - f_1(x)\,y = f_0(x)}$$

✓ Solution by Maple
Time used: 0.0 (sec). Leaf size: 38

```
dsolve(g(x)*diff(y(x),x)=f__1(x)*y(x)+f__0(x),y(x), singsol=all)
```

$$y(x) = \left(\int \frac{f_0(x)\, \mathrm{e}^{-\left(\int \frac{f_1(x)}{g(x)} dx\right)}}{g\left(x\right)} dx + c_1 \right) \mathrm{e}^{\int \frac{f_1(x)}{g(x)} dx}$$

✓ Solution by Mathematica
Time used: 0.135 (sec). Leaf size: 64

```
DSolve[g[x]*y'[x]==f1[x]*y[x]+f0[x],y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to \exp\left(\int_1^x \frac{\mathrm{f1}(K[1])}{g(K[1])} dK[1] \right) \left(\int_1^x \frac{\exp\left(-\int_1^{K[2]} \frac{\mathrm{f1}(K[1])}{g(K[1])} dK[1] \right) \mathrm{f0}(K[2])}{g(K[2])} dK[2] + c_1 \right)$$

1.5 problem 1.1.5

Internal problem ID [10329]

Book: Handbook of exact solutions for ordinary differential equations. By Polyanin and Zaitsev. Second edition
Section: Chapter 1, First-Order differential equations
Problem number: 1.1.5.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_Bernoulli]`

$$\boxed{g(x)\,y' - f_1(x)\,y - f_n(x)\,y^n = 0}$$

✓ Solution by Maple
Time used: 0.078 (sec). Leaf size: 74

```
dsolve(g(x)*diff(y(x),x)=f__1(x)*y(x)+f__n(x)*y(x)^n,y(x), singsol=all)
```

$$y(x) = e^{\int \frac{f_1(x)}{g(x)}dx}\left(-n\left(\int \frac{f_n(x)\,e^{(n-1)\left(\int \frac{f_1(x)}{g(x)}dx\right)}}{g(x)}dx\right) + c_1 + \int \frac{f_n(x)\,e^{(n-1)\left(\int \frac{f_1(x)}{g(x)}dx\right)}}{g(x)}dx\right)^{-\frac{1}{n-1}}$$

✓ Solution by Mathematica
Time used: 14.019 (sec). Leaf size: 84

```
DSolve[g[x]*y'[x]==f1[x]*y[x]+fn[x]*y[x]^n,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to \left(\exp\left(-\left((n-1)\int_1^x \frac{\text{f1}(K[1])}{g(K[1])}dK[1]\right)\right)\left(-(n-1)\int_1^x \frac{\exp\left((n-1)\int_1^{K[2]} \frac{\text{f1}(K[1])}{g(K[1])}dK[1]\right)\text{fn}(K[2])}{g(K[2])}dK[2] + c_1\right)\right)^{\frac{1}{1-n}}$$

1.6 problem 1.1.6

Internal problem ID [10330]

Book: Handbook of exact solutions for ordinary differential equations. By Polyanin and Zaitsev. Second edition
Section: Chapter 1, First-Order differential equations
Problem number: 1.1.6.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[[_homogeneous, 'class A'], _dAlembert]`

$$\boxed{y' - f\left(\frac{y}{x}\right) = 0}$$

✓ Solution by Maple
Time used: 0.0 (sec). Leaf size: 27

```
dsolve(diff(y(x),x)=f(y(x)/x),y(x), singsol=all)
```

$$y(x) = \text{RootOf}\left(-\left(\int^{\_Z} -\frac{1}{-f\left(\_a\right)+\_a} d\_a\right) + \ln\left(x\right) + c_1\right) x$$

✓ Solution by Mathematica
Time used: 0.106 (sec). Leaf size: 33

```
DSolve[y'[x]==f[y[x]/x],y[x],x,IncludeSingularSolutions -> True]
```

$$\text{Solve}\left[\int_1^{\frac{y(x)}{x}} \frac{1}{K[1] - f(K[1])} dK[1] = -\log(x) + c_1, y(x)\right]$$

2 Chapter 1, section 1.2. Riccati Equation. 1.2.2. Equations Containing Power Functions

2.1 problem 1 14
2.2 problem 2 15
2.3 problem 3 16
2.4 problem 4 18
2.5 problem 5 19
2.6 problem 6 20
2.7 problem 7 22
2.8 problem 8 23
2.9 problem 9 25
2.10 problem 10 26
2.11 problem 11 28
2.12 problem 12 30
2.13 problem 13 32
2.14 problem 14 33
2.15 problem 15 34
2.16 problem 16 36
2.17 problem 17 37
2.18 problem 18 39
2.19 problem 19 40
2.20 problem 20 41
2.21 problem 21 43
2.22 problem 22 45
2.23 problem 23 46
2.24 problem 24 47
2.25 problem 25 49
2.26 problem 26 50
2.27 problem 27 52
2.28 problem 28 53
2.29 problem 29 54
2.30 problem 30 55
2.31 problem 31 57
2.32 problem 32 58
2.33 problem 33 60
2.34 problem 34 61
2.35 problem 35 62
2.36 problem 36 63

2.37 problem 37 64
2.38 problem 38 66
2.39 problem 39 68
2.40 problem 40 69
2.41 problem 41 70
2.42 problem 42 71
2.43 problem 43 72
2.44 problem 44 73
2.45 problem 45 74
2.46 problem 46 75
2.47 problem 47 76
2.48 problem 48 77
2.49 problem 49 78
2.50 problem 50 79
2.51 problem 51 81
2.52 problem 52 82
2.53 problem 53 84
2.54 problem 54 86
2.55 problem 55 87
2.56 problem 56 88
2.57 problem 57 89
2.58 problem 58 91
2.59 problem 59 92
2.60 problem 60 93
2.61 problem 61 94
2.62 problem 62 96
2.63 problem 63 97
2.64 problem 64 98
2.65 problem 65 99
2.66 problem 66 101
2.67 problem 67 103
2.68 problem 68 105
2.69 problem 69 107
2.70 problem 70 108
2.71 problem 71 110
2.72 problem 72 111
2.73 problem 73 112
2.74 problem 74 114
2.75 problem 75 115

2.76 problem 76 116
2.77 problem 77 117
2.78 problem 78 119

2.1 problem 1

Internal problem ID [10331]

Book: Handbook of exact solutions for ordinary differential equations. By Polyanin and Zaitsev. Second edition
Section: Chapter 1, section 1.2. Riccati Equation. 1.2.2. Equations Containing Power Functions
Problem number: 1.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_Riccati]`

$$\boxed{y' - ay^2 = bx + c}$$

✓ Solution by Maple
Time used: 0.015 (sec). Leaf size: 85

```
dsolve(diff(y(x),x)=a*y(x)^2+b*x+c,y(x), singsol=all)
```

$$y(x) = \frac{\left(\frac{b}{\sqrt{a}}\right)^{\frac{1}{3}} \left(\text{AiryAi}\left(1, -\frac{bx+c}{\left(\frac{b}{\sqrt{a}}\right)^{\frac{2}{3}}}\right) c_1 + \text{AiryBi}\left(1, -\frac{bx+c}{\left(\frac{b}{\sqrt{a}}\right)^{\frac{2}{3}}}\right)\right)}{\sqrt{a}\left(c_1 \,\text{AiryAi}\left(-\frac{bx+c}{\left(\frac{b}{\sqrt{a}}\right)^{\frac{2}{3}}}\right) + \text{AiryBi}\left(-\frac{bx+c}{\left(\frac{b}{\sqrt{a}}\right)^{\frac{2}{3}}}\right)\right)}$$

✓ Solution by Mathematica
Time used: 0.325 (sec). Leaf size: 143

```
DSolve[y'[x]==a*y[x]^2+b*x+c,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to \frac{b\left(\text{AiryBiPrime}\left(-\frac{a(c+bx)}{(-ab)^{2/3}}\right) + c_1 \,\text{AiryAiPrime}\left(-\frac{a(c+bx)}{(-ab)^{2/3}}\right)\right)}{(-ab)^{2/3}\left(\text{AiryBi}\left(-\frac{a(c+bx)}{(-ab)^{2/3}}\right) + c_1 \,\text{AiryAi}\left(-\frac{a(c+bx)}{(-ab)^{2/3}}\right)\right)}$$

$$y(x) \to \frac{b \,\text{AiryAiPrime}\left(-\frac{a(c+bx)}{(-ab)^{2/3}}\right)}{(-ab)^{2/3} \,\text{AiryAi}\left(-\frac{a(c+bx)}{(-ab)^{2/3}}\right)}$$

2.2 problem 2

Internal problem ID [10332]

Book: Handbook of exact solutions for ordinary differential equations. By Polyanin and Zaitsev. Second edition
Section: Chapter 1, section 1.2. Riccati Equation. 1.2.2. Equations Containing Power Functions
Problem number: 2.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_Riccati]`

$$\boxed{y' - y^2 = -a^2x^2 + 3a}$$

✓ Solution by Maple
Time used: 0.0 (sec). Leaf size: 82

```
dsolve(diff(y(x),x)=y(x)^2-a^2*x^2+3*a,y(x), singsol=all)
```

$$y(x) = \frac{\mathrm{e}^{a\,x^2} c_1 a x - c_1\sqrt{\pi}\left((-a)^{\frac{3}{2}}\,x^2 + \sqrt{-a}\right)\operatorname{erf}\left(\sqrt{-a}\,x\right) + a\,x^2 - 1}{\sqrt{\pi}\,\sqrt{-a}\,\operatorname{erf}\left(\sqrt{-a}\,x\right) c_1 x + \mathrm{e}^{a\,x^2} c_1 + x}$$

✓ Solution by Mathematica
Time used: 0.79 (sec). Leaf size: 192

```
DSolve[y'[x]==y[x]^2-a^2*x^2+3*a,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x)$$
$$\to \frac{ax\,\text{ParabolicCylinderD}\left(-2, i\sqrt{2}\sqrt{a}x\right) + i\sqrt{2}\sqrt{a}\,\text{ParabolicCylinderD}\left(-1, i\sqrt{2}\sqrt{a}x\right) - ac_1x\,\text{Parabolic}}{\text{ParabolicCylinderD}\left(-2, i\sqrt{2}\sqrt{a}x\right) + c_1\,\text{ParabolicCyli}}$$
$$y(x) \to \frac{\sqrt{2}\sqrt{a}\,\text{ParabolicCylinderD}\left(2, \sqrt{2}\sqrt{a}x\right)}{\text{ParabolicCylinderD}\left(1, \sqrt{2}\sqrt{a}x\right)} - ax$$

2.3 problem 3

Internal problem ID [10333]

Book: Handbook of exact solutions for ordinary differential equations. By Polyanin and Zaitsev. Second edition
Section: Chapter 1, section 1.2. Riccati Equation. 1.2.2. Equations Containing Power Functions
Problem number: 3.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_Riccati]`

$$\boxed{y' - y^2 = a^2x^2 + bx + c}$$

✓ Solution by Maple
Time used: 0.0 (sec). Leaf size: 393

```
dsolve(diff(y(x),x)=y(x)^2+a^2*x^2+b*x+c,y(x), singsol=all)
```

$$y(x) = \frac{-48\left(ia^3 - \frac{1}{3}a^2c + \frac{1}{12}b^2\right)\left(a^2x + \frac{b}{2}\right)^2 c_1 \,\mathrm{hypergeom}\left(\left[\frac{4ia^2c+28a^3-ib^2}{16a^3}\right], \left[\frac{5}{2}\right], \frac{i\left(2a^2x+b\right)^2}{4a^3}\right) + 48c_1a^3\left(ia^4x^2 + ia\right.}{48\left(\left(a\right.\right.}$$

✓ Solution by Mathematica

Time used: 1.582 (sec). Leaf size: 664

```
DSolve[y'[x]==y[x]^2+a^2*x^2+b*x+c,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x)$$

$$\to \frac{2i\sqrt{2}a^2x\,\text{ParabolicCylinderD}\left(\frac{1}{8}\left(-\frac{ib^2}{a^3}+\frac{4ic}{a}-4\right),-\frac{\left(\frac{1}{2}-\frac{i}{2}\right)(2xa^2+b)}{a^{3/2}}\right)+4(-1)^{3/4}a^{3/2}\,\text{ParabolicCylinderD}$$

$$y(x)\to \frac{(1+i)\sqrt{a}\,\text{ParabolicCylinderD}\left(\frac{1}{8}\left(\frac{ib^2}{a^3}-\frac{4ic}{a}+4\right),\frac{\left(\frac{1}{2}+\frac{i}{2}\right)(2xa^2+b)}{a^{3/2}}\right)}{\text{ParabolicCylinderD}\left(\frac{1}{8}\left(\frac{ib^2}{a^3}-\frac{4ic}{a}-4\right),\frac{\left(\frac{1}{2}+\frac{i}{2}\right)(2xa^2+b)}{a^{3/2}}\right)}-\frac{i(2a^2x+b)}{2a}$$

$$y(x)\to \frac{(1+i)\sqrt{a}\,\text{ParabolicCylinderD}\left(\frac{1}{8}\left(\frac{ib^2}{a^3}-\frac{4ic}{a}+4\right),\frac{\left(\frac{1}{2}+\frac{i}{2}\right)(2xa^2+b)}{a^{3/2}}\right)}{\text{ParabolicCylinderD}\left(\frac{1}{8}\left(\frac{ib^2}{a^3}-\frac{4ic}{a}-4\right),\frac{\left(\frac{1}{2}+\frac{i}{2}\right)(2xa^2+b)}{a^{3/2}}\right)}-\frac{i(2a^2x+b)}{2a}$$

2.4 problem 4

Internal problem ID [10334]

Book: Handbook of exact solutions for ordinary differential equations. By Polyanin and Zaitsev. Second edition
Section: Chapter 1, section 1.2. Riccati Equation. 1.2.2. Equations Containing Power Functions
Problem number: 4.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[[_Riccati, _special]]`

$$\boxed{y' - ay^2 = b\,x^n}$$

✓ Solution by Maple
Time used: 0.0 (sec). Leaf size: 207

```
dsolve(diff(y(x),x)=a*y(x)^2+b*x^n,y(x), singsol=all)
```

$$y(x) = \frac{\text{BesselJ}\left(\frac{3+n}{n+2}, \frac{2\sqrt{ab}\,x^{\frac{n}{2}+1}}{n+2}\right)\sqrt{ab}\,x^{\frac{n}{2}+1}c_1 + \text{BesselY}\left(\frac{3+n}{n+2}, \frac{2\sqrt{ab}\,x^{\frac{n}{2}+1}}{n+2}\right)\sqrt{ab}\,x^{\frac{n}{2}+1} - c_1\,\text{BesselJ}\left(\frac{1}{n+2}, \frac{2\sqrt{ab}\,x^{\frac{n}{2}+1}}{n+2}\right.$$
$$xa\left(c_1\,\text{BesselJ}\left(\frac{1}{n+2}, \frac{2\sqrt{ab}\,x^{\frac{n}{2}+1}}{n+2}\right) + \text{BesselY}\left(\frac{1}{n+2}, \frac{2\sqrt{ab}\,x^{\frac{n}{2}+1}}{n+2}\right)\right)$$

✓ Solution by Mathematica
Time used: 0.696 (sec). Leaf size: 605

```
DSolve[y'[x]==a*y[x]^2+b*x^n,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to -\frac{\sqrt{a}\sqrt{b}x^{\frac{n}{2}+1}\,\text{Gamma}\left(1+\frac{1}{n+2}\right)\text{BesselJ}\left(\frac{1}{n+2}-1, \frac{2\sqrt{a}\sqrt{b}x^{\frac{n}{2}+1}}{n+2}\right) - \sqrt{a}\sqrt{b}x^{\frac{n}{2}+1}\,\text{Gamma}\left(1+\frac{1}{n+2}\right)\text{BesselJ}}{}$$

$$y(x) \to \frac{\frac{\sqrt{a}\sqrt{b}x^{n/2}\left(\text{BesselJ}\left(\frac{n+1}{n+2}, \frac{2\sqrt{a}\sqrt{b}x^{\frac{n}{2}+1}}{n+2}\right) - \text{BesselJ}\left(-\frac{n+3}{n+2}, \frac{2\sqrt{a}\sqrt{b}x^{\frac{n}{2}+1}}{n+2}\right)\right)}{\text{BesselJ}\left(-\frac{1}{n+2}, \frac{2\sqrt{a}\sqrt{b}x^{\frac{n}{2}+1}}{n+2}\right)} - \frac{1}{x}}{2a}$$

2.5 problem 5

Internal problem ID [10335]

Book: Handbook of exact solutions for ordinary differential equations. By Polyanin and Zaitsev. Second edition
Section: Chapter 1, section 1.2. Riccati Equation. 1.2.2. Equations Containing Power Functions
Problem number: 5.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_Riccati]`

$$\boxed{y' - y^2 = an\,x^{n-1} - a^2x^{2n}}$$

✓ Solution by Maple
Time used: 0.0 (sec). Leaf size: 388

```
dsolve(diff(y(x),x)=y(x)^2+a*n*x^(n-1)-a^2*x^(2*n),y(x), singsol=all)
```

$$y(x) = \frac{-3(n+2)\,c_1\left(\left(\frac{1}{3}n^2+n+\frac{2}{3}\right)x^{-\frac{3n}{2}} + ax\,x^{-\frac{n}{2}}\left(n+\frac{4}{3}\right)\right)\mathrm{e}^{\frac{ax\,x^n}{n+1}}\,\mathrm{WhittakerM}\left(\frac{n+2}{2n+2},\frac{2n+3}{2n+2},-\frac{2ax\,x^n}{n+1}\right) + 2c_1\mathrm{e}^{\frac{ax\,x}{n+}}}{2\left(-\frac{\mathrm{e}^{\frac{ax\,x^n}{n+1}}x^{-\frac{3n}{2}}c_1(n+2)^2\,\mathrm{WhittakerM}\left(\frac{n+2}{2n+2},\frac{2n+3}{2n+2},\right.}{2}\right.}$$

✓ Solution by Mathematica
Time used: 1.61 (sec). Leaf size: 227

```
DSolve[y'[x]==y[x]^2+a*n*x^(n-1)-a^2*x^(2*n),y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to \frac{2^{\frac{1}{n+1}}(n+1)\left(-\frac{ax^{n+1}}{n+1}\right)^{\frac{1}{n+1}}\left(ax^n - c_1e^{\frac{2ax^{n+1}}{n+1}}\right) - ac_1x^{n+1}\Gamma\left(\frac{1}{n+1},-\frac{2ax^{n+1}}{n+1}\right)}{2^{\frac{1}{n+1}}(n+1)\left(-\frac{ax^{n+1}}{n+1}\right)^{\frac{1}{n+1}} - c_1x\Gamma\left(\frac{1}{n+1},-\frac{2ax^{n+1}}{n+1}\right)}$$

$$y(x) \to \frac{2^{\frac{1}{n+1}}(n+1)e^{\frac{2ax^{n+1}}{n+1}}\left(-\frac{ax^{n+1}}{n+1}\right)^{\frac{1}{n+1}}}{x\Gamma\left(\frac{1}{n+1},-\frac{2ax^{n+1}}{n+1}\right)} + ax^n$$

2.6 problem 6

Internal problem ID [10336]

Book: Handbook of exact solutions for ordinary differential equations. By Polyanin and Zaitsev. Second edition
Section: Chapter 1, section 1.2. Riccati Equation. 1.2.2. Equations Containing Power Functions
Problem number: 6.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_Riccati]`

$$\boxed{y' - ay^2 = b\,x^{2n} + c\,x^{n-1}}$$

✓ Solution by Maple
Time used: 0.0 (sec). Leaf size: 357

```
dsolve(diff(y(x),x)=a*y(x)^2+b*x^(2*n)+c*x^(n-1),y(x), singsol=all)
```

$$y(x) = -\frac{\left(\left(\frac{n}{2}+1\right)\sqrt{b} - \frac{ic\sqrt{a}}{2}\right)\text{WhittakerM}\left(-\frac{(-2n-2)\sqrt{b}+ic\sqrt{a}}{\sqrt{b}\,(2n+2)}, \frac{1}{2n+2}, \frac{2i\sqrt{a}\,\sqrt{b}\,x\,x^n}{n+1}\right) - \sqrt{b}\,c_1(n+1)\,\text{WhittakerW}\left($$
$$\sqrt{b}\,\left(\text{WhittakerW}\left(-\right.\right.$$

✓ Solution by Mathematica

Time used: 1.818 (sec). Leaf size: 982

```
DSolve[y'[x]==a*y[x]^2+b*x^(2*n)+c*x^(n-1),y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to -\frac{x^n\left(\sqrt{b}c_1(n+1)\sqrt{-(n+1)^2}\,\text{HypergeometricU}\left(\frac{1}{2}\left(\frac{\sqrt{a}c}{\sqrt{b}\sqrt{-(n+1)^2}}+\frac{n}{n+1}\right),\frac{n}{n+1},\frac{2\sqrt{a}\sqrt{b}x^{n+1}}{\sqrt{-(n+1)^2}}\right)+c_1\left(\sqrt{a}c(n\right.\right.}{\sqrt{a}(n+1)^2}$$

$$y(x) \to x^n\left(-\frac{\left(\sqrt{a}c(n+1)+\sqrt{b}\sqrt{-(n+1)^2}n\right)\text{HypergeometricU}\left(\frac{1}{2}\left(\frac{\sqrt{a}c}{\sqrt{b}\sqrt{-(n+1)^2}}+\frac{n}{n+1}+2\right),\frac{n}{n+1}+1,\frac{2\sqrt{a}\sqrt{b}x^{n+1}}{\sqrt{-(n+1)^2}}\right)}{\text{HypergeometricU}\left(\frac{1}{2}\left(\frac{\sqrt{a}c}{\sqrt{b}\sqrt{-(n+1)^2}}+\frac{n}{n+1}\right),\frac{n}{n+1},\frac{2\sqrt{a}\sqrt{b}x^{n+1}}{\sqrt{-(n+1)^2}}\right)}-\sqrt{b}\sqrt{-(n+1)^2}(n\right)\Big/\sqrt{a}(n+1)^2$$

$$y(x) \to x^n\left(-\frac{\left(\sqrt{a}c(n+1)+\sqrt{b}\sqrt{-(n+1)^2}n\right)\text{HypergeometricU}\left(\frac{1}{2}\left(\frac{\sqrt{a}c}{\sqrt{b}\sqrt{-(n+1)^2}}+\frac{n}{n+1}+2\right),\frac{n}{n+1}+1,\frac{2\sqrt{a}\sqrt{b}x^{n+1}}{\sqrt{-(n+1)^2}}\right)}{\text{HypergeometricU}\left(\frac{1}{2}\left(\frac{\sqrt{a}c}{\sqrt{b}\sqrt{-(n+1)^2}}+\frac{n}{n+1}\right),\frac{n}{n+1},\frac{2\sqrt{a}\sqrt{b}x^{n+1}}{\sqrt{-(n+1)^2}}\right)}-\sqrt{b}\sqrt{-(n+1)^2}(n\right)\Big/\sqrt{a}(n+1)^2$$

2.7 problem 7

Internal problem ID [10337]

Book: Handbook of exact solutions for ordinary differential equations. By Polyanin and Zaitsev. Second edition
Section: Chapter 1, section 1.2. Riccati Equation. 1.2.2. Equations Containing Power Functions
Problem number: 7.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[[_homogeneous, 'class G'], _Riccati]`

$$\boxed{y' - a\,x^n y^2 = b\,x^{-n-2}}$$

✓ Solution by Maple
Time used: 0.015 (sec). Leaf size: 62

```
dsolve(diff(y(x),x)=a*x^n*y(x)^2+b*x^(-n-2),y(x), singsol=all)
```

$$y(x) = -\frac{x^{-n-1}\left(n+1-\tan\left(\frac{\sqrt{4ab-n^2-2n-1}\,(\ln(x)-c_1)}{2}\right)\sqrt{4ab-n^2-2n-1}\right)}{2a}$$

✓ Solution by Mathematica
Time used: 0.778 (sec). Leaf size: 135

```
DSolve[y'[x]==a*x^n*y[x]^2+b*x^(-n-2),y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to \frac{x^{-n-1}\left(-\left(\sqrt{(n+1)^2-4ab}+n+1\right)x^{\sqrt{(n+1)^2-4ab}}+c_1\left(\sqrt{(n+1)^2-4ab}-n-1\right)\right)}{2a\left(x^{\sqrt{(n+1)^2-4ab}}+c_1\right)}$$

$$y(x) \to \frac{x^{-n-1}\left(\sqrt{(n+1)^2-4ab}-n-1\right)}{2a}$$

2.8 problem 8

Internal problem ID [10338]

Book: Handbook of exact solutions for ordinary differential equations. By Polyanin and Zaitsev. Second edition
Section: Chapter 1, section 1.2. Riccati Equation. 1.2.2. Equations Containing Power Functions
Problem number: 8.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_Riccati]`

$$\boxed{y' - a\,x^n y^2 = b\,x^m}$$

✓ Solution by Maple
Time used: 0.016 (sec). Leaf size: 170

```
dsolve(diff(y(x),x)=a*x^n*y(x)^2+b*x^m,y(x), singsol=all)
```

$$y(x) = \frac{x^{-\frac{n}{2}+\frac{m}{2}}\sqrt{ab}\left(\text{BesselY}\left(\frac{1+m}{n+m+2}, \frac{2\sqrt{ab}\,x^{\frac{n}{2}+\frac{m}{2}+1}}{n+m+2}\right)c_1 + \text{BesselJ}\left(\frac{1+m}{n+m+2}, \frac{2\sqrt{ab}\,x^{\frac{n}{2}+\frac{m}{2}+1}}{n+m+2}\right)\right)}{a\left(\text{BesselY}\left(\frac{-n-1}{n+m+2}, \frac{2\sqrt{ab}\,x^{\frac{n}{2}+\frac{m}{2}+1}}{n+m+2}\right)c_1 + \text{BesselJ}\left(\frac{-n-1}{n+m+2}, \frac{2\sqrt{ab}\,x^{\frac{n}{2}+\frac{m}{2}+1}}{n+m+2}\right)\right)}$$

✓ Solution by Mathematica

Time used: 2.978 (sec). Leaf size: 1805

```
DSolve[y'[x]==a*x^n*y[x]^2+b*x^m,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to -\frac{a^{-\frac{2m+3n+5}{2(m+n+2)}} b^{-\frac{n+1}{2(m+n+2)}} (m+n+1)^{\frac{n+1}{m+n+2}} \left((m+n+1)^2\right)^{\frac{n+1}{m+n+2}-\frac{1}{2}} x^{-n-1} \left(x^{m+n+1}\right)^{-\frac{n+1}{2(m+n+1)}} \left(a^{\frac{n+1}{2(m+n+2)}} b^{\frac{}{2(}}\right.}{}$$

$$y(x) \to \frac{x^{-n-1}\left(\sqrt{a}\sqrt{b}(m+n+1)\left(x^{m+n+1}\right)^{\frac{1}{2}\left(\frac{1}{m+n+1}+1\right)} \text{BesselJ}\left(\frac{m+1}{m+n+2}, \frac{2\sqrt{a}\sqrt{b}(m+n+1)\left(x^{m+n+1}\right)^{\frac{1}{2}\left(1+\frac{1}{m+n+1}\right)}}{\sqrt{(m+n+1)^2}(m+n+2)}\right) - \sqrt{}\right.}{}$$

$$y(x) \to \frac{x^{-n-1}\left(\sqrt{a}\sqrt{b}(m+n+1)\left(x^{m+n+1}\right)^{\frac{1}{2}\left(\frac{1}{m+n+1}+1\right)} \text{BesselJ}\left(\frac{m+1}{m+n+2}, \frac{2\sqrt{a}\sqrt{b}(m+n+1)\left(x^{m+n+1}\right)^{\frac{1}{2}\left(1+\frac{1}{m+n+1}\right)}}{\sqrt{(m+n+1)^2}(m+n+2)}\right) - \sqrt{}\right.}{}$$

2.9 problem 9

Internal problem ID [10339]

Book: Handbook of exact solutions for ordinary differential equations. By Polyanin and Zaitsev. Second edition
Section: Chapter 1, section 1.2. Riccati Equation. 1.2.2. Equations Containing Power Functions
Problem number: 9.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_Riccati]`

$$\boxed{y' - y^2 = k(xa+b)^n (cx+d)^{-n-4}}$$

✗ Solution by Maple

```
dsolve(diff(y(x),x)=y(x)^2+k*(a*x+b)^n*(c*x+d)^(-n-4),y(x), singsol=all)
```

No solution found

✗ Solution by Mathematica
Time used: 0.0 (sec). Leaf size: 0

```
DSolve[y'[x]==y[x]^2+k*(a*x+b)^n*(c*x+d)^(-n-4),y[x],x,IncludeSingularSolutions -> True]
```

Not solved

2.10 problem 10

Internal problem ID [10340]

Book: Handbook of exact solutions for ordinary differential equations. By Polyanin and Zaitsev. Second edition
Section: Chapter 1, section 1.2. Riccati Equation. 1.2.2. Equations Containing Power Functions
Problem number: 10.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_Riccati]`

$$\boxed{y' - a\,x^n y^2 = bm\,x^{m-1} - a\,b^2 x^{n+2m}}$$

✓ Solution by Maple
Time used: 0.0 (sec). Leaf size: 522

```
dsolve(diff(y(x),x)=a*x^n*y(x)^2+b*m*x^(m-1)-a*b^2*x^(n+2*m),y(x), singsol=all)
```

$$y(x) = \frac{x^{-n-1}\left(-\frac{3\left(ab(m+\frac{4n}{3}+\frac{4}{3})x^{n+1-\frac{m}{2}}+\frac{x^{-\frac{3m}{2}}(m+2n+2)(1+m+n)}{3}\right)c_1(m+2n+2)\mathrm{e}^{\frac{ab\,x^{1+m+n}}{1+m+n}}\,\mathrm{WhittakerM}\left(\frac{m+2n+2}{2n+2m+2},\frac{2m+3n+3}{2n+2m+2},-\frac{2ab\,x^{1+m+n}}{1+m+n}\right)}{2}\right.}{\left(-\frac{\mathrm{e}^{\frac{ab\,x^{1+m+n}}{1+m+n}}x^{-\frac{3m}{2}}c_1(m+2n+2)^2\,\mathrm{Whitt}\right.}$$

✓ Solution by Mathematica

Time used: 2.322 (sec). Leaf size: 306

```
DSolve[y'[x]==a*x^n*y[x]^2+b*m*x^(m-1)-a*b^2*x^(n+2*m),y[x],x,IncludeSingularSolutions -> Tr
```

$$y(x)$$

$$\to \frac{2^{\frac{n+1}{m+n+1}}(m+n+1)\left(-\frac{abx^{m+n+1}}{m+n+1}\right)^{\frac{n+1}{m+n+1}}\left(abx^m-c_1e^{\frac{2abx^{m+n+1}}{m+n+1}}\right)-abc_1x^{m+n+1}\Gamma\left(\frac{n+1}{m+n+1},-\frac{2abx^{m+n+1}}{m+n+1}\right)}{a\left(2^{\frac{n+1}{m+n+1}}(m+n+1)\left(-\frac{abx^{m+n+1}}{m+n+1}\right)^{\frac{n+1}{m+n+1}}-c_1x^{n+1}\Gamma\left(\frac{n+1}{m+n+1},-\frac{2abx^{m+n+1}}{m+n+1}\right)\right)}$$

$$y(x)\to bx^m-\frac{b2^{\frac{n+1}{m+n+1}}x^me^{\frac{2abx^{m+n+1}}{m+n+1}}\left(-\frac{abx^{m+n+1}}{m+n+1}\right)^{-\frac{m}{m+n+1}}}{\Gamma\left(\frac{n+1}{m+n+1},-\frac{2abx^{m+n+1}}{m+n+1}\right)}$$

2.11 problem 11

Internal problem ID [10341]

Book: Handbook of exact solutions for ordinary differential equations. By Polyanin and Zaitsev. Second edition
Section: Chapter 1, section 1.2. Riccati Equation. 1.2.2. Equations Containing Power Functions
Problem number: 11.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_Riccati]`

$$\boxed{y' - \left(a\,x^{2n} + b\,x^{n-1}\right) y^2 = c}$$

✓ Solution by Maple
Time used: 0.0 (sec). Leaf size: 1237

```
dsolve(diff(y(x),x)=(a*x^(2*n)+b*x^(n-1))*y(x)^2+c,y(x), singsol=all)
```

Expression too large to display

✓ Solution by Mathematica

Time used: 2.128 (sec). Leaf size: 1384

```
DSolve[y'[x]==(a*x^(2*n)+b*x^(n-1))*y[x]^2+c,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x)$$

$$\to \frac{\sqrt{c}(n+1)^2 x^{-n}\left(\sqrt{a}c_1(n+1)\sqrt{-(n+1)^2}\,\text{HypergeometricU}\left(\frac{1}{2}\left(\frac{\sqrt{c}b}{\sqrt{a}\sqrt{-(n+1)^2}}+\frac{n}{n+1}\right),\frac{n}{n+1},\frac{2\sqrt{a}\sqrt{c}x^{n+1}}{\sqrt{-(n+1)^2}}\right)+c_1\left(\sqrt{a}\sqrt{-(n+1)^2}\,\text{HypergeometricU}\left(\frac{1}{2}\left(\frac{\sqrt{c}b}{\sqrt{a}\sqrt{-(n+1)^2}}+\frac{n}{n+1}\right)+1,\frac{n}{n+1}+1,\frac{2\sqrt{a}\sqrt{c}x^{n+1}}{\sqrt{-(n+1)^2}}\right)+\left(\frac{\sqrt{c}b}{\sqrt{a}\sqrt{-(n+1)^2}}+\frac{n}{n+1}\right)\text{HypergeometricU}\left(\frac{1}{2}\left(\frac{\sqrt{c}b}{\sqrt{a}\sqrt{-(n+1)^2}}+\frac{n}{n+1}\right),\frac{n}{n+1},\frac{2\sqrt{a}\sqrt{c}x^{n+1}}{\sqrt{-(n+1)^2}}\right)\right)\right)}{\sqrt{a}c_1(n+1)\sqrt{-(n+1)^2}\,\text{HypergeometricU}\left(\frac{1}{2}\left(\frac{\sqrt{c}b}{\sqrt{a}\sqrt{-(n+1)^2}}+\frac{n}{n+1}\right),\frac{n}{n+1},\frac{2\sqrt{a}\sqrt{c}x^{n+1}}{\sqrt{-(n+1)^2}}\right)+c_1\left(\sqrt{a}\sqrt{-(n+1)^2}\,\text{HypergeometricU}\left(\frac{1}{2}\left(\frac{\sqrt{c}b}{\sqrt{a}\sqrt{-(n+1)^2}}+\frac{n}{n+1}\right)+1,\frac{n}{n+1}+1,\frac{2\sqrt{a}\sqrt{c}x^{n+1}}{\sqrt{-(n+1)^2}}\right)+\left(\frac{\sqrt{c}b}{\sqrt{a}\sqrt{-(n+1)^2}}+\frac{n}{n+1}\right)\text{HypergeometricU}\left(\frac{1}{2}\left(\frac{\sqrt{c}b}{\sqrt{a}\sqrt{-(n+1)^2}}+\frac{n}{n+1}\right),\frac{n}{n+1},\frac{2\sqrt{a}\sqrt{c}x^{n+1}}{\sqrt{-(n+1)^2}}\right)\right)\right)}$$

$$y(x)$$

$$\to \frac{\sqrt{c}(n+1)^2 x^{-n}\,\text{HypergeometricU}\left(\frac{1}{2}\left(\frac{\sqrt{c}b}{\sqrt{a}\sqrt{-(n+1)^2}}+\frac{n}{n+1}\right),\frac{n}{n+1},\frac{2\sqrt{a}\sqrt{c}x^{n+1}}{\sqrt{-(n+1)^2}}\right)}{\sqrt{a}(n+1)\sqrt{-(n+1)^2}\,\text{HypergeometricU}\left(\frac{1}{2}\left(\frac{\sqrt{c}b}{\sqrt{a}\sqrt{-(n+1)^2}}+\frac{n}{n+1}\right),\frac{n}{n+1},\frac{2\sqrt{a}\sqrt{c}x^{n+1}}{\sqrt{-(n+1)^2}}\right)+\left(\sqrt{a}\sqrt{-(n+1)^2}\,\text{HypergeometricU}\left(\frac{1}{2}\left(\frac{\sqrt{c}b}{\sqrt{a}\sqrt{-(n+1)^2}}+\frac{n}{n+1}\right)+1,\frac{n}{n+1}+1,\frac{2\sqrt{a}\sqrt{c}x^{n+1}}{\sqrt{-(n+1)^2}}\right)+\left(\frac{\sqrt{c}b}{\sqrt{a}\sqrt{-(n+1)^2}}+\frac{n}{n+1}\right)\text{HypergeometricU}\left(\frac{1}{2}\left(\frac{\sqrt{c}b}{\sqrt{a}\sqrt{-(n+1)^2}}+\frac{n}{n+1}\right),\frac{n}{n+1},\frac{2\sqrt{a}\sqrt{c}x^{n+1}}{\sqrt{-(n+1)^2}}\right)\right)\right)}$$

$$y(x)$$

$$\to \frac{\sqrt{c}(n+1)^2 x^{-n}\,\text{HypergeometricU}\left(\frac{1}{2}\left(\frac{\sqrt{c}b}{\sqrt{a}\sqrt{-(n+1)^2}}+\frac{n}{n+1}\right),\frac{n}{n+1},\frac{2\sqrt{a}\sqrt{c}x^{n+1}}{\sqrt{-(n+1)^2}}\right)}{\sqrt{a}(n+1)\sqrt{-(n+1)^2}\,\text{HypergeometricU}\left(\frac{1}{2}\left(\frac{\sqrt{c}b}{\sqrt{a}\sqrt{-(n+1)^2}}+\frac{n}{n+1}\right),\frac{n}{n+1},\frac{2\sqrt{a}\sqrt{c}x^{n+1}}{\sqrt{-(n+1)^2}}\right)+\left(\sqrt{a}\sqrt{-(n+1)^2}\,\text{HypergeometricU}\left(\frac{1}{2}\left(\frac{\sqrt{c}b}{\sqrt{a}\sqrt{-(n+1)^2}}+\frac{n}{n+1}\right)+1,\frac{n}{n+1}+1,\frac{2\sqrt{a}\sqrt{c}x^{n+1}}{\sqrt{-(n+1)^2}}\right)+\left(\frac{\sqrt{c}b}{\sqrt{a}\sqrt{-(n+1)^2}}+\frac{n}{n+1}\right)\text{HypergeometricU}\left(\frac{1}{2}\left(\frac{\sqrt{c}b}{\sqrt{a}\sqrt{-(n+1)^2}}+\frac{n}{n+1}\right),\frac{n}{n+1},\frac{2\sqrt{a}\sqrt{c}x^{n+1}}{\sqrt{-(n+1)^2}}\right)\right)\right)}$$

$$y(x)$$

$$\to \frac{\sqrt{c}(n+1)x^{-n}L^{-\frac{1}{n+1}}_{-\frac{\sqrt{c}b}{2\sqrt{a}\sqrt{-(n+1)^2}}-\frac{n}{2(n+1)}}\left(\frac{2\sqrt{a}\sqrt{c}x^{n+1}}{\sqrt{-(n+1)^2}}\right)}{\sqrt{a}\sqrt{-(n+1)^2}\left(2L^{\frac{n}{n+1}}_{-\frac{\sqrt{c}b}{2\sqrt{a}\sqrt{-(n+1)^2}}-\frac{n}{2(n+1)}-1}\left(\frac{2\sqrt{a}\sqrt{c}x^{n+1}}{\sqrt{-(n+1)^2}}\right)+L^{-\frac{1}{n+1}}_{-\frac{\sqrt{c}b}{2\sqrt{a}\sqrt{-(n+1)^2}}-\frac{n}{2(n+1)}}\left(\frac{2\sqrt{a}\sqrt{c}x^{n+1}}{\sqrt{-(n+1)^2}}\right)\right)}$$

2.12 problem 12

Internal problem ID [10342]

Book: Handbook of exact solutions for ordinary differential equations. By Polyanin and Zaitsev. Second edition
Section: Chapter 1, section 1.2. Riccati Equation. 1.2.2. Equations Containing Power Functions
Problem number: 12.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_rational, _Riccati]`

$$\boxed{(a_2x+b_2)\left(y'+\lambda y^2\right)=-a_0x-b_0}$$

✓ Solution by Maple
Time used: 0.031 (sec). Leaf size: 461

```
dsolve((a__2*x+b__2)*(diff(y(x),x)+lambda*y(x)^2)+a__0*x+b__0=0,y(x), singsol=all)
```

$$y(x)=$$
$$-\frac{\left(\frac{c_1\lambda(a_0b_2-a_2b_0)\,\mathrm{KummerU}\left(-\frac{\sqrt{-a_2\lambda a_0}\,a_0b_2-\sqrt{-a_2\lambda a_0}\,a_2b_0-2a_0a_2^2}{2a_0a_2^2},1,\frac{2(a_2x+b_2)\sqrt{-a_2\lambda a_0}}{a_2^2}\right)}{2}+\sqrt{-a_2\lambda a_0}\,a_2\left(c_1\,\mathrm{KummerU}\left(\right.\right.\right.}{\left(\frac{c_1\sqrt{-a_2\lambda a_0}\,(a_0b_2-a_2b_0)\,\mathrm{KummerU}\left(-\frac{\sqrt{-a_2\lambda a_0}\,a_0b_2-\sqrt{-a_2\lambda a_0}\,a_2b_0-2a_0a_2^2}{2a_0a_2^2},1,\frac{2(a_2x+b_2)\sqrt{-a_2\lambda a_0}}{a_2^2}\right)}{2}+a_0a_2^2\left(\mathrm{KummerM}\left(-\right.\right.\right.}$$

✓ Solution by Mathematica

Time used: 1.895 (sec). Leaf size: 690

```
DSolve[(a2*x+b2)*(y'[x]+\[Lambda]*y[x]^2)+a0*x+b0==0,y[x],x,IncludeSingularSolutions -> True
```

$$y(x)$$

$$\to \frac{c_1\sqrt{\lambda}(\text{a2b0}-\text{a0b2})\,\text{HypergeometricU}\left(\frac{i\sqrt{\lambda}(\text{a2b0}-\text{a0b2})}{2\sqrt{\text{a0}}\text{a2}^{3/2}}+1,1,\frac{2i\sqrt{\text{a0}}(\text{b2}+\text{a2}x)\sqrt{\lambda}}{\text{a2}^{3/2}}\right)-i\sqrt{\text{a0}}\text{a2}^{3/2}\left(c_1\,\text{Hy}\right.}{\text{a2}^2\sqrt{\lambda}\left(c_1\,\text{HypergeometricU}\right.}$$

$$y(x)\to \frac{(\text{a2b0}-\text{a0b2})\,\text{HypergeometricU}\left(\frac{i\sqrt{\lambda}(\text{a2b0}-\text{a0b2})}{2\sqrt{\text{a0}}\text{a2}^{3/2}}+1,1,\frac{2i\sqrt{\text{a0}}(\text{b2}+\text{a2}x)\sqrt{\lambda}}{\text{a2}^{3/2}}\right)}{\text{a2}^2\,\text{HypergeometricU}\left(\frac{i(\text{a2b0}-\text{a0b2})\sqrt{\lambda}}{2\sqrt{\text{a0}}\text{a2}^{3/2}},0,\frac{2i\sqrt{\text{a0}}(\text{b2}+\text{a2}x)\sqrt{\lambda}}{\text{a2}^{3/2}}\right)}$$
$$-\frac{i\sqrt{\text{a0}}}{\sqrt{\text{a2}}\sqrt{\lambda}}$$

$$y(x)\to \frac{(\text{a2b0}-\text{a0b2})\,\text{HypergeometricU}\left(\frac{i\sqrt{\lambda}(\text{a2b0}-\text{a0b2})}{2\sqrt{\text{a0}}\text{a2}^{3/2}}+1,1,\frac{2i\sqrt{\text{a0}}(\text{b2}+\text{a2}x)\sqrt{\lambda}}{\text{a2}^{3/2}}\right)}{\text{a2}^2\,\text{HypergeometricU}\left(\frac{i(\text{a2b0}-\text{a0b2})\sqrt{\lambda}}{2\sqrt{\text{a0}}\text{a2}^{3/2}},0,\frac{2i\sqrt{\text{a0}}(\text{b2}+\text{a2}x)\sqrt{\lambda}}{\text{a2}^{3/2}}\right)}$$
$$-\frac{i\sqrt{\text{a0}}}{\sqrt{\text{a2}}\sqrt{\lambda}}$$

2.13 problem 13

Internal problem ID [10343]

Book: Handbook of exact solutions for ordinary differential equations. By Polyanin and Zaitsev. Second edition
Section: Chapter 1, section 1.2. Riccati Equation. 1.2.2. Equations Containing Power Functions
Problem number: 13.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[[_homogeneous, 'class G'], _rational, [_Riccati, _special]]`

$$\boxed{x^2 y' - a\, x^2 y^2 = b}$$

✓ Solution by Maple
Time used: 0.015 (sec). Leaf size: 40

```
dsolve(x^2*diff(y(x),x)=a*x^2*y(x)^2+b,y(x), singsol=all)
```

$$y(x) = \frac{-1 + \tan\left(\frac{\sqrt{4ab-1}\,(\ln(x)-c_1)}{2}\right)\sqrt{4ab-1}}{2ax}$$

✓ Solution by Mathematica
Time used: 0.292 (sec). Leaf size: 77

```
DSolve[x^2*y'[x]==a*x^2*y[x]^2+b,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to \frac{-1 + \sqrt{1-4ab}\left(-1 + \frac{2c_1}{x^{\sqrt{1-4ab}}+c_1}\right)}{2ax}$$
$$y(x) \to \frac{\sqrt{1-4ab}-1}{2ax}$$

2.14 problem 14

Internal problem ID [10344]

Book: Handbook of exact solutions for ordinary differential equations. By Polyanin and Zaitsev. Second edition
Section: Chapter 1, section 1.2. Riccati Equation. 1.2.2. Equations Containing Power Functions
Problem number: 14.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_rational, _Riccati]`

$$\boxed{x^2 y' - x^2 y^2 = -a^2 x^4 + a(1-2b)\,x^2 - b(b+1)}$$

✓ Solution by Maple
Time used: 0.0 (sec). Leaf size: 84

```
dsolve(x^2*diff(y(x),x)=x^2*y(x)^2-a^2*x^4+a*(1-2*b)*x^2-b*(b+1),y(x), singsol=all)
```

$$y(x) = \frac{-2(-a\,x^2)^{b-\frac{1}{2}}\,\mathrm{e}^{a\,x^2} c_1 a\,x^2 + \left(-c_1\Gamma\left(b+\frac{1}{2}\right) + c_1\Gamma\left(b+\frac{1}{2},-a\,x^2\right) - 1\right)(a\,x^2+b)}{x\left(-c_1\Gamma\left(b+\frac{1}{2}\right) + c_1\Gamma\left(b+\frac{1}{2},-a\,x^2\right) - 1\right)}$$

✓ Solution by Mathematica
Time used: 1.225 (sec). Leaf size: 128

```
DSolve[x^2*y'[x]==x^2*y[x]^2-a^2*x^4+a*(1-2*b)*x^2-b*(b+1),y[x],x,IncludeSingularSolutions -
```

$$y(x) \to \frac{x^{2b+1}(ax^2+b)\,\Gamma\left(b+\frac{1}{2},-ax^2\right) - 2(-ax^2)^{b+\frac{1}{2}}\left(-e^{ax^2}x^{2b+1} + c_1(ax^2+b)\right)}{x^{2b+2}\Gamma\left(b+\frac{1}{2},-ax^2\right) - 2c_1 x\,(-ax^2)^{b+\frac{1}{2}}}$$

$$y(x) \to ax + \frac{b}{x}$$

2.15 problem 15

Internal problem ID [10345]

Book: Handbook of exact solutions for ordinary differential equations. By Polyanin and Zaitsev. Second edition
Section: Chapter 1, section 1.2. Riccati Equation. 1.2.2. Equations Containing Power Functions
Problem number: 15.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_rational, _Riccati]`

$$\boxed{x^2 y' - a\,x^2 y^2 = b\,x^n + c}$$

✓ Solution by Maple
Time used: 0.0 (sec). Leaf size: 220

```
dsolve(x^2*diff(y(x),x)=a*x^2*y(x)^2+b*x^n+c,y(x), singsol=all)
```

$$y(x) = \frac{2\sqrt{ab}\left(\text{BesselY}\left(\frac{\sqrt{-4ac+1}}{n}+1, \frac{2\sqrt{ab}\,x^{\frac{n}{2}}}{n}\right)c_1 + \text{BesselJ}\left(\frac{\sqrt{-4ac+1}}{n}+1, \frac{2\sqrt{ab}\,x^{\frac{n}{2}}}{n}\right)\right)x^{\frac{n}{2}} - \left(\sqrt{-4ac+1}+1\right)\left(\text{B}\right.}{2xa\left(\text{BesselY}\left(\frac{\sqrt{-4ac+1}}{n}, \frac{2\sqrt{ab}\,x^{\frac{n}{2}}}{n}\right)c_1 + \text{BesselJ}\left(\frac{\sqrt{-4ac+1}}{n}\right.\right.}$$

✓ Solution by Mathematica

Time used: 1.898 (sec). Leaf size: 1779

```
DSolve[x^2*y'[x]==a*x^2*y[x]^2+b*x^n+c,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x)$$

$$\to \frac{-a^{\frac{i\sqrt{4ac-1}}{n}+\frac{1}{2}} n^{\frac{2\sqrt{(1-4ac)n^2}}{n^2}+1} (x^n)^{\frac{i\sqrt{4ac-1}}{n}+1} \text{BesselJ}\left(\frac{\sqrt{(1-4ac)n^2}}{n^2}-1, \frac{2\sqrt{a}\sqrt{b}\sqrt{x^n}}{n}\right) \text{Gamma}\left(\frac{n+\sqrt{1-4ac}}{n}\right) b^{\frac{i\sqrt{4ac-1}}{n}}$$

$$y(x)$$

$$\to \frac{\frac{\sqrt{a}\sqrt{b}\sqrt{x^n}\left(\text{BesselJ}\left(1-\frac{\sqrt{(1-4ac)n^2}}{n^2}, \frac{2\sqrt{a}\sqrt{b}\sqrt{x^n}}{n}\right)-\text{BesselJ}\left(-\frac{\sqrt{(1-4ac)n^2}}{n^2}-1, \frac{2\sqrt{a}\sqrt{b}\sqrt{x^n}}{n}\right)\right)}{\text{BesselJ}\left(-\frac{\sqrt{(1-4ac)n^2}}{n^2}, \frac{2\sqrt{a}\sqrt{b}\sqrt{x^n}}{n}\right)} - \frac{\sqrt{n^2(1-4ac)}}{n} + i\sqrt{4ac-1} - 1}{2ax}$$

2.16 problem 16

Internal problem ID [10346]

Book: Handbook of exact solutions for ordinary differential equations. By Polyanin and Zaitsev. Second edition
Section: Chapter 1, section 1.2. Riccati Equation. 1.2.2. Equations Containing Power Functions
Problem number: 16.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_Riccati]`

$$x^2 y' - x^2 y^2 = a\, x^{2m} (b\, x^m + c)^n - \frac{n^2}{4} + \frac{1}{4}$$

✗ Solution by Maple

```
dsolve(x^2*diff(y(x),x)=x^2*y(x)^2+a*x^(2*m)*(b*x^m+c)^n+1/4*(1-n^2),y(x), singsol=all)
```

No solution found

✗ Solution by Mathematica
Time used: 0.0 (sec). Leaf size: 0

```
DSolve[x^2*y'[x]==x^2*y[x]^2+a*x^(2*m)*(b*x^m+c)^n+1/4*(1-n^2),y[x],x,IncludeSingularSolutio
```

Not solved

2.17 problem 17

Internal problem ID [10347]

Book: Handbook of exact solutions for ordinary differential equations. By Polyanin and Zaitsev. Second edition
Section: Chapter 1, section 1.2. Riccati Equation. 1.2.2. Equations Containing Power Functions
Problem number: 17.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_rational, _Riccati]`

$$\boxed{\left(c_2 x^2+b_2 x+a_2\right)\left(y'+\lambda y^2\right)=-a_0}$$

✓ Solution by Maple
Time used: 0.015 (sec). Leaf size: 1127

```
dsolve((c__2*x^2+b__2*x+a__2)*(diff(y(x),x)+lambda*y(x)^2)+a__0=0,y(x), singsol=all)
```

Expression too large to display

✓ Solution by Mathematica

Time used: 5.964 (sec). Leaf size: 1046

```
DSolve[(c2*x^2+b2*x+a2)*(y'[x]+\[Lambda]*y[x]^2)+a0==0,y[x],x,IncludeSingularSolutions -> Tr
```

$$y(x)$$

$$\to \frac{(\text{b2}+2\text{c2}x)\left(8\text{c2}\left(\text{b2}^2-4\text{a2c2}\right)G^{2,0}_{2,2}\left(-\frac{4\text{c2}(\text{a2}+x(\text{b2}+\text{c2}x))}{\text{b2}^2-4\text{a2c2}}\middle|\begin{matrix}\frac{1}{4}-\frac{\sqrt{\text{c2}-4\text{a0}\lambda}}{4\sqrt{\text{c2}}},\frac{1}{4}\left(\frac{\sqrt{\text{c2}-4\text{a0}\lambda}}{\sqrt{\text{c2}}}+1\right)\\ 0,0\end{matrix}\right)+c_1\right.}{2\lambda\left(\text{b2}^2\right.}$$

$$y(x)$$

$$\to \frac{(\text{b2}+2\text{c2}x)\left(2\left(\text{b2}^2-4\text{a2c2}\right)\text{Hypergeometric2F1}\left(\frac{3\text{c2}+\sqrt{\text{c2}(\text{c2}-4\text{a0}\lambda)}}{4\text{c2}},\frac{1}{4}\left(3-\frac{\sqrt{\text{c2}(\text{c2}-4\text{a0}\lambda)}}{\text{c2}}\right),2,-\frac{4\text{c2}\right.\right.}{2\lambda\left(\text{b2}^2-4\text{a2c2}\right)(\text{a2}+x(\text{b2}+\text{c2}x))\text{Hyperg}}$$

$$y(x)$$

$$\to \frac{(\text{b2}+2\text{c2}x)\left(2\left(\text{b2}^2-4\text{a2c2}\right)\text{Hypergeometric2F1}\left(\frac{3\text{c2}+\sqrt{\text{c2}(\text{c2}-4\text{a0}\lambda)}}{4\text{c2}},\frac{1}{4}\left(3-\frac{\sqrt{\text{c2}(\text{c2}-4\text{a0}\lambda)}}{\text{c2}}\right),2,-\frac{4\text{c2}\right.\right.}{2\lambda\left(\text{b2}^2-4\text{a2c2}\right)(\text{a2}+x(\text{b2}+\text{c2}x))\text{Hyperg}}$$

2.18 problem 18

Internal problem ID [10348]

Book: Handbook of exact solutions for ordinary differential equations. By Polyanin and Zaitsev. Second edition
Section: Chapter 1, section 1.2. Riccati Equation. 1.2.2. Equations Containing Power Functions
Problem number: 18.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_rational, [_Riccati, _special]]`

$$\boxed{x^4 y' + x^4 y^2 = -a^2}$$

✓ Solution by Maple
Time used: 0.0 (sec). Leaf size: 24

```
dsolve(x^4*diff(y(x),x)=-x^4*y(x)^2-a^2,y(x), singsol=all)
```

$$y(x) = \frac{-a \tan\left(\frac{a(c_1 x - 1)}{x}\right) + x}{x^2}$$

✓ Solution by Mathematica
Time used: 1.107 (sec). Leaf size: 87

```
DSolve[x^4*y'[x]==-x^4*y[x]^2-a^2,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to \frac{-2ia^2 c_1 e^{\frac{2ia}{x}} + 2ac_1 x e^{\frac{2ia}{x}} + a - ix}{x^2\left(2ac_1 e^{\frac{2ia}{x}} - i\right)}$$
$$y(x) \to \frac{x - ia}{x^2}$$

2.19 problem 19

Internal problem ID [10349]

Book: Handbook of exact solutions for ordinary differential equations. By Polyanin and Zaitsev. Second edition
Section: Chapter 1, section 1.2. Riccati Equation. 1.2.2. Equations Containing Power Functions
Problem number: 19.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_rational, _Riccati]`

$$\boxed{a\,x^2(x-1)^2\left(y'+\lambda y^2\right)=-b\,x^2-cx-s}$$

✓ Solution by Maple
Time used: 0.0 (sec). Leaf size: 1087

```
dsolve(a*x^2*(x-1)^2*(diff(y(x),x)+lambda*y(x)^2)+b*x^2+c*x+s=0,y(x), singsol=all)
```

Expression too large to display

✗ Solution by Mathematica
Time used: 0.0 (sec). Leaf size: 0

```
DSolve[a*x^2*(x-1)^2*(y'[x]+\[Lambda]*y[x]^2)+b*x^2+c*x+s==0,y[x],x,IncludeSingularSolutions
```

Timed out

2.20 problem 20

Internal problem ID [10350]

Book: Handbook of exact solutions for ordinary differential equations. By Polyanin and Zaitsev. Second edition
Section: Chapter 1, section 1.2. Riccati Equation. 1.2.2. Equations Containing Power Functions
Problem number: 20.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_rational, _Riccati]`

$$\boxed{\left(a\,x^2+bx+c\right)^2\left(y'+y^2\right)=-A}$$

✓ Solution by Maple
Time used: 0.0 (sec). Leaf size: 493

```
dsolve((a*x^2+b*x+c)^2*(diff(y(x),x)+y(x)^2)+A=0,y(x), singsol=all)
```

$$y(x)$$

$$=\frac{2\left(c_1\left(i\sqrt{\frac{-4ac+b^2-4A}{a^2}}\,a\sqrt{4ac-b^2}-2\sqrt{-4ac+b^2}\left(\frac{b}{2}+ax\right)\right)\left(\frac{-b+i\sqrt{4ac-b^2}-2ax}{i\sqrt{4ac-b^2}+2ax+b}\right)^{-\frac{a\sqrt{\frac{-4ac+b^2-4A}{a^2}}}{2\sqrt{-4ac+b^2}}}-\left(i\sqrt{-4a\right.\right.}{\sqrt{-4ac+b^2}\left(i\sqrt{4ac-b^2}+2ax+b\right)\left(-b+i\sqrt{4ac-b^2}-2ax\right)\left(c_1\left(\frac{-b+i\sqrt{4ac-}}{i\sqrt{4ac-b^2}+}\right.\right.}$$

✓ Solution by Mathematica

Time used: 5.579 (sec). Leaf size: 743

```
DSolve[(a*x^2+b*x+c)^2*(y'[x]+y[x]^2)+A==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x)$$

$$\to \frac{b^2 c_1 \left(-\exp\left(\frac{2\sqrt{4ac-b^2}\sqrt{1-\frac{4A}{b^2-4ac}}\arctan\left(\frac{2ax+b}{\sqrt{4ac-b^2}}\right)}{\sqrt{b^2-4ac}}\right)\right)+bc_1\sqrt{b^2-4ac}\sqrt{1-\frac{4A}{b^2-4ac}}\exp\left(\frac{2\sqrt{4ac-b^2}\sqrt{1-\frac{4A}{b^2-4ac}}\,a}{\sqrt{b^2-4a}}\right.}{}$$

$$y(x)\to \frac{2ax\sqrt{b^2-4ac}\sqrt{1-\frac{4A}{b^2-4ac}}+b\sqrt{b^2-4ac}\sqrt{1-\frac{4A}{b^2-4ac}}+4ac+4A-b^2}{2\sqrt{b^2-4ac}\sqrt{1-\frac{4A}{b^2-4ac}}(x(ax+b)+c)}$$

2.21 problem 21

Internal problem ID [10351]

Book: Handbook of exact solutions for ordinary differential equations. By Polyanin and Zaitsev. Second edition
Section: Chapter 1, section 1.2. Riccati Equation. 1.2.2. Equations Containing Power Functions
Problem number: 21.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_Riccati]`

$$\boxed{x^{n+1}y' - x^{2n}y^2 a = c\,x^m + d}$$

✓ Solution by Maple
Time used: 0.0 (sec). Leaf size: 235

```
dsolve(x^(n+1)*diff(y(x),x)=a*x^(2*n)*y(x)^2+c*x^m+d,y(x), singsol=all)
```

$$y(x) = -\frac{x^{-n}\left(-2\sqrt{ac}\left(\text{BesselY}\left(\frac{\sqrt{-4ad+n^2}}{m}+1,\frac{2\sqrt{ac}\,x^{\frac{m}{2}}}{m}\right)c_1+\text{BesselJ}\left(\frac{\sqrt{-4ad+n^2}}{m}+1,\frac{2\sqrt{ac}\,x^{\frac{m}{2}}}{m}\right)\right)x^{\frac{m}{2}}+\left(\text{Bessel}\right.\right.}{2a\left(\text{BesselY}\left(\frac{\sqrt{-4ad+n^2}}{m},\frac{2\sqrt{ac}\,x^{\frac{m}{2}}}{m}\right)c_1+\text{Bessel}\right.}$$

✓ Solution by Mathematica

Time used: 2.124 (sec). Leaf size: 1890

```
DSolve[x^(n+1)*y'[x]==a*x^(2*n)*y[x]^2+c*x^m+d,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x)$$

$$\to x^{-n}\left(a^{\frac{\sqrt{n^2-4ad}}{m}} m^{\frac{2\sqrt{m^2\left(n^2-4ad\right)}}{m^2}} \left(\sqrt{m^2\left(n^2-4ad\right)}-m\left(n+\sqrt{n^2-4ad}\right)\right)\left(x^m\right)^{\frac{\sqrt{n^2-4ad}}{m}+\frac{1}{2}} \text{BesselJ}\left(\frac{\sqrt{m^2(n^2-}\right.\right.$$

$$y(x)$$

$$\to x^{-n}\left(\frac{\sqrt{a}\sqrt{c}\sqrt{x^m}\left(\text{BesselJ}\left(1-\frac{\sqrt{m^2\left(n^2-4ad\right)}}{m^2},\frac{2\sqrt{a}\sqrt{c}\sqrt{x^m}}{m}\right)-\text{BesselJ}\left(-\frac{\sqrt{m^2\left(n^2-4ad\right)}}{m^2}-1,\frac{2\sqrt{a}\sqrt{c}\sqrt{x^m}}{m}\right)\right)}{\text{BesselJ}\left(-\frac{\sqrt{m^2\left(n^2-4ad\right)}}{m^2},\frac{2\sqrt{a}\sqrt{c}\sqrt{x^m}}{m}\right)}-\frac{\sqrt{m^2(n^2-4ad)}}{m}+\sqrt{n^2}\right.$$

$$\to \frac{}{2a}$$

2.22 problem 22

Internal problem ID [10352]

Book: Handbook of exact solutions for ordinary differential equations. By Polyanin and Zaitsev. Second edition
Section: Chapter 1, section 1.2. Riccati Equation. 1.2.2. Equations Containing Power Functions
Problem number: 22.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_rational, _Riccati]`

$$\boxed{(x^n a + b)\, y' - b y^2 = a\, x^{-2+n}}$$

✓ Solution by Maple
Time used: 0.0 (sec). Leaf size: 224

```
dsolve((a*x^n+b)*diff(y(x),x)=b*y(x)^2+a*x^(n-2),y(x), singsol=all)
```

$$y(x) = \frac{\left(\frac{a\,x^n+b}{b}\right)^{\frac{2}{n}} \left(anc_1(a^2x^{3n} + 2ab\,x^{2n} + b^2x^n)\,\text{hypergeom}\left(\left[2, \frac{n+1}{n}\right], \left[\frac{2n-1}{n}\right], -\frac{a\,x^n}{b}\right) - (n-1)\,b\left(ac_1(a\,x^{2n} + b\right.\right.}{b^2\,(n-1)\,x\,(a\,x^n+b)\left(x + \text{hypergeom}\left(\left[1, \frac{1}{n}\right], \left[\frac{n-1}{n}\right],\right.\right.}$$

✓ Solution by Mathematica
Time used: 1.899 (sec). Leaf size: 289

```
DSolve[(a*x^n+b)*y'[x]==b*y[x]^2+a*x^(n-2),y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to \frac{-b^2(-1)^{\frac{1}{n}}(n-1)\left(-\frac{ax^n}{b}\right)^{\frac{1}{n}} - abc_1(n-1)x^n\left(\frac{ax^n}{b}+1\right)^{2/n}\text{Hypergeometric2F1}\left(1, \frac{1}{n}, \frac{n-1}{n}, -\frac{ax^n}{b}\right) + ac_1nx}{b^2(n-1)x\left((-1)^{\frac{1}{n}}\left(-\frac{ax^n}{b}\right)^{\frac{1}{n}} + c_1\left(\frac{ax^n}{b}+1\right)^{2/n}\text{Hypergeomet}\right.}$$

$$y(x) \to \frac{ax^{n-1}\left(\frac{n(ax^n+b)\,\text{Hypergeometric2F1}\left(2,1+\frac{1}{n},2-\frac{1}{n},-\frac{ax^n}{b}\right)}{\text{Hypergeometric2F1}\left(1,\frac{1}{n},\frac{n-1}{n},-\frac{ax^n}{b}\right)} + b(-n) + b\right)}{b^2(n-1)}$$

2.23 problem 23

Internal problem ID [10353]

Book: Handbook of exact solutions for ordinary differential equations. By Polyanin and Zaitsev. Second edition
Section: Chapter 1, section 1.2. Riccati Equation. 1.2.2. Equations Containing Power Functions
Problem number: 23.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_rational, _Riccati]`

$$\boxed{(x^n a + b\,x^m + c)\left(y' - y^2\right) = -an(n-1)\,x^{-2+n} - bm(m-1)\,x^{m-2}}$$

✓ Solution by Maple
Time used: 0.0 (sec). Leaf size: 141

```
dsolve((a*x^n+b*x^m+c)*(diff(y(x),x)-y(x)^2)+a*n*(n-1)*x^(n-2)+b*m*(m-1)*x^(m-2)=0,y(x), sin
```

$$y(x) = \frac{-(an\,x^n + bm\,x^m)(a\,x^n + b\,x^m + c)\left(\int \frac{1}{(a\,x^n+b\,x^m+c)^2}dx\right) - x^{2m}c_1 b^2 m - b(a(n+m)\,x^n + cm)\,c_1 x^m - x^2}{(a\,x^n + b\,x^m + c)^2\,x\left(c_1 + \int \frac{1}{(a\,x^n+b\,x^m+c)^2}dx\right)}$$

✓ Solution by Mathematica
Time used: 35.099 (sec). Leaf size: 201

```
DSolve[(a*x^n+b*x^m+c)*(y'[x]-y[x]^2)+a*n*(n-1)*x^(n-2)+b*m*(m-1)*x^(m-2)==0,y[x],x,IncludeS
```

$$y(x) \to -\frac{c_1\left(\frac{(anx^n+bmx^m)\int_1^x \frac{1}{(bK[1]^m+aK[1]^n+c)^2}dK[1]}{x} + \frac{1}{ax^n+bx^m+c}\right) + anx^{n-1} + bmx^{m-1}}{(ax^n+bx^m+c)\left(1 + c_1\int_1^x \frac{1}{(bK[1]^m+aK[1]^n+c)^2}dK[1]\right)}$$

$$y(x) \to -\frac{\frac{1}{\int_1^x \frac{1}{(bK[1]^m+aK[1]^n+c)^2}dK[1]} + \frac{(anx^n+bmx^m)(ax^n+bx^m+c)}{x}}{(ax^n+bx^m+c)^2}$$

2.24 problem 24

Internal problem ID [10354]

Book: Handbook of exact solutions for ordinary differential equations. By Polyanin and Zaitsev. Second edition
Section: Chapter 1, section 1.2. Riccati Equation. 1.2.2. Equations Containing Power Functions
Problem number: 24.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_Riccati]`

$$\boxed{y' - ay^2 - yb = cx + k}$$

✓ Solution by Maple
Time used: 0.016 (sec). Leaf size: 194

```
dsolve(diff(y(x),x)=a*y(x)^2+b*y(x)+c*x+k,y(x), singsol=all)
```

$$y(x) = \frac{2\sqrt{a}\left(\frac{c}{\sqrt{a}}\right)^{\frac{1}{3}}\left(\operatorname{AiryAi}\left(1, -\frac{a(cx+k)-\frac{b^2}{4}}{\left(\frac{c}{\sqrt{a}}\right)^{\frac{2}{3}}a}\right)c_1 + \operatorname{AiryBi}\left(1, -\frac{a(cx+k)-\frac{b^2}{4}}{\left(\frac{c}{\sqrt{a}}\right)^{\frac{2}{3}}a}\right)\right) - b\left(c_1 \operatorname{AiryAi}\left(-\frac{a(cx+k)-\frac{b^2}{4}}{\left(\frac{c}{\sqrt{a}}\right)^{\frac{2}{3}}a}\right)\right.}{2a\left(c_1 \operatorname{AiryAi}\left(-\frac{a(cx+k)-\frac{b^2}{4}}{\left(\frac{c}{\sqrt{a}}\right)^{\frac{2}{3}}a}\right) + \operatorname{AiryBi}\left(-\frac{a(cx+k)-\frac{b^2}{4}}{\left(\frac{c}{\sqrt{a}}\right)^{\frac{2}{3}}a}\right)\right)}$$

✓ Solution by Mathematica

Time used: 0.51 (sec). Leaf size: 359

```
DSolve[y'[x]==a*y[x]^2+b*y[x]+c*x+k,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to -\frac{c\left(-b(-ac)^{2/3}\,\text{AiryBi}\left(\frac{b^2-4a(k+cx)}{4(-ac)^{2/3}}\right)+2ac\,\text{AiryBiPrime}\left(\frac{b^2-4a(k+cx)}{4(-ac)^{2/3}}\right)+c_1\left(2ac\,\text{AiryAiPrime}\left(\frac{b^2-4a(k+\right.\right.}{2(-ac)^{5/3}\left(\text{AiryBi}\left(\frac{b^2-4a(k+cx)}{4(-ac)^{2/3}}\right)+c_1\,\text{AiryAi}\left(\frac{b^2-4a(k+cx)}{4(-ac)^{2/3}}\right)\right)}$$

$$y(x) \to -\frac{\frac{2\sqrt[3]{-ac}\,\text{AiryAiPrime}\left(\frac{b^2-4a(k+cx)}{4(-ac)^{2/3}}\right)}{\text{AiryAi}\left(\frac{b^2-4a(k+cx)}{4(-ac)^{2/3}}\right)}+b}{2a}$$

$$y(x) \to -\frac{\frac{2\sqrt[3]{-ac}\,\text{AiryAiPrime}\left(\frac{b^2-4a(k+cx)}{4(-ac)^{2/3}}\right)}{\text{AiryAi}\left(\frac{b^2-4a(k+cx)}{4(-ac)^{2/3}}\right)}+b}{2a}$$

2.25 problem 25

Internal problem ID [10355]

Book: Handbook of exact solutions for ordinary differential equations. By Polyanin and Zaitsev. Second edition
Section: Chapter 1, section 1.2. Riccati Equation. 1.2.2. Equations Containing Power Functions
Problem number: 25.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_Riccati]`

$$\boxed{y' - y^2 - a\,x^n y = a\,x^{n-1}}$$

✓ Solution by Maple
Time used: 0.0 (sec). Leaf size: 400

```
dsolve(diff(y(x),x)=y(x)^2+a*x^n*y(x)+a*x^(n-1),y(x), singsol=all)
```

$$y(x) = \frac{-\mathrm{e}^{\frac{x^n a x}{2n+2}}\left(-\frac{a x\,x^n}{n+1}\right)^{-\frac{n}{2n+2}}(n+1)^2\left(x^n a x - n\right)\text{WhittakerM}\left(\frac{-n-2}{2n+2},\frac{2n+1}{2n+2},-\frac{a x\,x^n}{n+1}\right) - 2n\left(-\frac{(n+1)n\,\mathrm{e}^{\frac{x^n a x}{2n+2}}\left(-\frac{a x\,x^n}{n+1}\right)}{}\right.}{x\left(\mathrm{e}^{\frac{x^n a x}{2n+2}}\left(-\frac{a x\,x^n}{n+1}\right)^{-\frac{n}{2n+2}}(n+1)^2\left(x^n a x - n\right)\text{WhittakerM}\left(\frac{-n-2}{2n+2},\frac{2n+1}{2n+2},-\frac{a x\,x^n}{n+1}\right) + 2n\left(-\frac{(n+1)n\,\mathrm{e}^{\frac{x^n a x}{2n+2}}\left(-\right.\right.\right.}$$

✓ Solution by Mathematica
Time used: 2.82 (sec). Leaf size: 136

```
DSolve[y'[x]==y[x]^2+a*x^n*y[x]+a*x^(n-1),y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to \frac{\left(-\frac{a x^{n+1}}{n+1}\right)^{\frac{1}{n+1}}\Gamma\left(-\frac{1}{n+1},-\frac{a x^{n+1}}{n+1}\right) - (n+1)\left(e^{\frac{a x^{n+1}}{n+1}} + c_1 x\right)}{x\left(-\left(-\frac{a x^{n+1}}{n+1}\right)^{\frac{1}{n+1}}\Gamma\left(-\frac{1}{n+1},-\frac{a x^{n+1}}{n+1}\right) + c_1(n+1)x\right)}$$

$$y(x) \to -\frac{1}{x}$$

2.26 problem 26

Internal problem ID [10356]

Book: Handbook of exact solutions for ordinary differential equations. By Polyanin and Zaitsev. Second edition
Section: Chapter 1, section 1.2. Riccati Equation. 1.2.2. Equations Containing Power Functions
Problem number: 26.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_Riccati]`

$$\boxed{y' - y^2 - a\,x^n y = b\,x^{n-1}}$$

✓ Solution by Maple
Time used: 0.0 (sec). Leaf size: 267

```
dsolve(diff(y(x),x)=y(x)^2+a*x^n*y(x)+b*x^(n-1),y(x), singsol=all)
```

$$y(x) = \frac{-a(n+1)(a-b)\,\mathrm{KummerM}\left(\frac{a(n+2)-b}{a(n+1)}, \frac{n+2}{n+1}, \frac{a\,x\,x^n}{n+1}\right) + \left((a-b)\,c_1\,\mathrm{KummerU}\left(\frac{2+n-\frac{b}{a}}{n+1}, \frac{n+2}{n+1}, \frac{a\,x\,x^n}{n+1}\right) - a\left(\mathrm{K}\right.\right.}{a^2(n+1)\,x\left(\mathrm{KummerU}\left(\frac{a-b}{a(n+1)}, \frac{n+2}{n+1}, \frac{a\,x\,x^n}{n+1}\right)c_1 + \mathrm{Kum}\right.}$$

✓ Solution by Mathematica

Time used: 0.956 (sec). Leaf size: 453

```
DSolve[y'[x]==y[x]^2+a*x^n*y[x]+b*x^(n-1),y[x],x,IncludeSingularSolutions -> True]
```

$$y(x)$$

$$\to \frac{(x^n)^{\frac{1}{n}}\left(-(-1)^{\frac{1}{n+1}}n(n+2)a^{\frac{1}{n+1}}\,\text{Hypergeometric1F1}\left(\frac{a-b}{na+a},\frac{n+2}{n+1},\frac{a(x^n)^{1+\frac{1}{n}}}{n+1}\right)+x^n\left(-(-1)^{\frac{1}{n+1}}n(a-b)a^{\frac{}{}}\right.\right.}{n(n+2)x\left((-1)^{\frac{1}{n+1}}a^{\frac{1}{n+1}}\,(x^n)^{\frac{1}{n}}\,\text{Hypergeometric1F1}\left(\frac{a}{n}\right.\right.}$$

$$y(x)\to \frac{bx^{n-1}(x^n)^{\frac{1}{n}}\,\text{Hypergeometric1F1}\left(\frac{na+a-b}{na+a},\frac{2n+1}{n+1},\frac{a(x^n)^{1+\frac{1}{n}}}{n+1}\right)}{n\,\text{Hypergeometric1F1}\left(-\frac{b}{na+a},\frac{n}{n+1},\frac{a(x^n)^{1+\frac{1}{n}}}{n+1}\right)}$$

2.27 problem 27

Internal problem ID [10357]

Book: Handbook of exact solutions for ordinary differential equations. By Polyanin and Zaitsev. Second edition
Section: Chapter 1, section 1.2. Riccati Equation. 1.2.2. Equations Containing Power Functions
Problem number: 27.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_Riccati]`

$$\boxed{y' - y^2 - (\alpha x + \beta)\, y = a\, x^2 + b x + c}$$

✓ Solution by Maple
Time used: 0.0 (sec). Leaf size: 974

```
dsolve(diff(y(x),x)=y(x)^2+(alpha*x+beta)*y(x)+a*x^2+b*x+c,y(x), singsol=all)
```

Expression too large to display

✓ Solution by Mathematica
Time used: 4.264 (sec). Leaf size: 1291

```
DSolve[y'[x]==y[x]^2+(\[Alpha]*x+\[Beta])*y[x]+a*x^2+b*x+c,y[x],x,IncludeSingularSolutions -
```

$$y(x) \to$$

$$-\frac{2\left(2b + 4ax + \left(\sqrt{\alpha^2 - 4a} - \alpha\right)(x\alpha + \beta)\right) \text{Hypergeometric1F1}\left(-\frac{2b^2 - 2\alpha\beta b + \alpha^2\left(2c + \alpha - \sqrt{\alpha^2 - 4a}\right) + 2a\left(\beta^2 - 4c - \right.}{4(\alpha^2 - 4a)^{3/2}}\right.$$

$$y(x)$$

$$\to \frac{(4a - \alpha^2)\left(\left(\sqrt{\alpha^2 - 4a} - \alpha\right)(\beta + \alpha x) + 4ax + 2b\right) - \frac{\sqrt{2}\sqrt[4]{\alpha^2 - 4a}\left(2a\left(2\sqrt{\alpha^2 - 4a} - 2\alpha + \beta^2 - 4c\right) + \alpha^2\left(-\sqrt{\alpha^2 - 4a} + \alpha + \right.\right.}{\text{HermiteH}\left(-\frac{-2b^2 + 2\alpha\beta b + \alpha}{2(\alpha^2 - 4a)^{3/2}}\right.}$$

2.28 problem 28

Internal problem ID [10358]

Book: Handbook of exact solutions for ordinary differential equations. By Polyanin and Zaitsev. Second edition
Section: Chapter 1, section 1.2. Riccati Equation. 1.2.2. Equations Containing Power Functions
Problem number: 28.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_Riccati]`

$$\boxed{y' - y^2 - a\,x^n y = -a\,x^n b - b^2}$$

✓ Solution by Maple
Time used: 0.0 (sec). Leaf size: 74

```
dsolve(diff(y(x),x)=y(x)^2+a*x^n*y(x)-a*b*x^n-b^2,y(x), singsol=all)
```

$$\frac{(b-y(x))\left(\int^x \mathrm{e}^{\frac{(\_a^n a+2b(n+1))\_a}{n+1}} d\_a\right) + c_1 b - c_1 y(x) - \mathrm{e}^{\frac{(a\,x^n+2b(n+1))x}{n+1}}}{b-y\left(x\right)} = 0$$

✓ Solution by Mathematica
Time used: 1.948 (sec). Leaf size: 195

```
DSolve[y'[x]==y[x]^2+a*x^n*y[x]-a*b*x^n-b^2,y[x],x,IncludeSingularSolutions -> True]
```

$$\begin{aligned}&\text{Solve}\left[\int_1^{y(x)}\left(\frac{e^{\frac{ax^{n+1}}{n+1}+2bx}}{an(K[2]-b)^2}\right.\right.\\&\left.-\int_1^x\left(\frac{e^{\frac{aK[1]^{n+1}}{n+1}+2bK[1]}(aK[1]^n+b+K[2])}{an(b-K[2])^2}+\frac{e^{\frac{aK[1]^{n+1}}{n+1}+2bK[1]}}{an(b-K[2])}\right)dK[1]\right)dK[2]\\&\left.+\int_1^x\frac{e^{\frac{aK[1]^{n+1}}{n+1}+2bK[1]}(aK[1]^n+b+y(x))}{an(b-y(x))}dK[1]=c_1,y(x)\right]\end{aligned}$$

2.29 problem 29

Internal problem ID [10359]

Book: Handbook of exact solutions for ordinary differential equations. By Polyanin and Zaitsev. Second edition
Section: Chapter 1, section 1.2. Riccati Equation. 1.2.2. Equations Containing Power Functions
Problem number: 29.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_Riccati]`

$$y' + (n+1)\, x^n y^2 = a\, x^{1+m+n} - a\, x^m$$

✗ Solution by Maple

```
dsolve(diff(y(x),x)=-(n+1)*x^n*y(x)^2+a*x^(n+m+1)-a*x^m,y(x), singsol=all)
```

No solution found

✗ Solution by Mathematica
Time used: 0.0 (sec). Leaf size: 0

```
DSolve[y'[x]==-(n+1)*x^n*y[x]^2+a*x^(n+m+1)-a*x^m,y[x],x,IncludeSingularSolutions -> True]
```

Not solved

2.30 problem 30

Internal problem ID [10360]

Book: Handbook of exact solutions for ordinary differential equations. By Polyanin and Zaitsev. Second edition
Section: Chapter 1, section 1.2. Riccati Equation. 1.2.2. Equations Containing Power Functions
Problem number: 30.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_Riccati]`

$$\boxed{y' - a\,x^n y^2 - b\,x^m y = bc\,x^m - a\,c^2 x^n}$$

✓ Solution by Maple
Time used: 0.015 (sec). Leaf size: 98

```
dsolve(diff(y(x),x)=a*x^n*y(x)^2+b*x^m*y(x)+b*c*x^m-a*c^2*x^n,y(x), singsol=all)
```

$$\frac{a(c+y(x))\left(\int^x \mathrm{e}^{-\frac{2\left(-\frac{b(n+1)\_a^m}{2}+a\,\_a^n c(1+m)\right)\_a}{(1+m)(n+1)}}\_a^n d\_a\right)+c_1 y(x)+c_1 c+\mathrm{e}^{-\frac{2\left(-\frac{b(n+1)x^m}{2}+a\,x^n c(1+m)\right)x}{(1+m)(n+1)}}}{c+y\,(x)}$$

$= 0$

✓ Solution by Mathematica

Time used: 3.436 (sec). Leaf size: 286

```
DSolve[y'[x]==a*x^n*y[x]^2+b*x^m*y[x]+b*c*x^m-a*c^2*x^n,y[x],x,IncludeSingularSolutions -> T
```

$$\text{Solve}\left[\int_1^{y(x)}\left(\frac{e^{\frac{bx^{m+1}}{m+1}-\frac{2acx^{n+1}}{n+1}}}{ab(m-n)(c+K[2])^2}\right.\right.$$
$$-\int_1^x\left(-\frac{\exp\left(\frac{bK[1]^{m+1}}{m+1}-\frac{2acK[1]^{n+1}}{n+1}\right)K[1]^n}{b(m-n)(c+K[2])}-\frac{\exp\left(\frac{bK[1]^{m+1}}{m+1}-\frac{2acK[1]^{n+1}}{n+1}\right)(-bK[1]^m+acK[1]^n-aK[2]K[}{ab(m-n)(c+K[2])^2}\right.$$
$$\left.\left.+\int_1^x\frac{\exp\left(\frac{bK[1]^{m+1}}{m+1}-\frac{2acK[1]^{n+1}}{n+1}\right)(-bK[1]^m+acK[1]^n-ay(x)K[1]^n)}{ab(m-n)(c+y(x))}dK[1]=c_1,y(x)\right]\right.$$

2.31 problem 31

Internal problem ID [10361]

Book: Handbook of exact solutions for ordinary differential equations. By Polyanin and Zaitsev. Second edition
Section: Chapter 1, section 1.2. Riccati Equation. 1.2.2. Equations Containing Power Functions
Problem number: 31.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_Riccati]`

$$y' - a\,x^n y^2 + a\,x^n (b\,x^m + c)\,y = bm\,x^{m-1}$$

✗ Solution by Maple

```
dsolve(diff(y(x),x)=a*x^n*y(x)^2-a*x^n*(b*x^m+c)*y(x)+b*m*x^(m-1),y(x), singsol=all)
```

No solution found

✓ Solution by Mathematica
Time used: 59.342 (sec). Leaf size: 353

```
DSolve[y'[x]==a*x^n*y[x]^2-a*x^n*(b*x^m+c)*y[x]+b*m*x^(m-1),y[x],x,IncludeSingularSolutions
```

$$y(x) \to \frac{bm(bx^m+c)^2\left(1+c_1\int_1^x \frac{\exp\left(aK[1]^{n+1}\left(\frac{bK[1]^m}{m+n+1}+\frac{c}{n+1}\right)\right)K[1]^{m-1}}{(bK[1]^m+c)^2}dK[1]\right)}{bc_1 m\,(bx^m+c)\int_1^x \frac{\exp\left(aK[1]^{n+1}\left(\frac{bK[1]^m}{m+n+1}+\frac{c}{n+1}\right)\right)K[1]^{m-1}}{(bK[1]^m+c)^2}dK[1] + c_1 e^{ax^{n+1}\left(\frac{bx^m}{m+n+1}+\frac{c}{n+1}\right)} + b^2 m x^m + bcm}$$

$$y(x) \to \frac{bm(bx^m+c)^2\int_1^x \frac{\exp\left(aK[1]^{n+1}\left(\frac{bK[1]^m}{m+n+1}+\frac{c}{n+1}\right)\right)K[1]^{m-1}}{(bK[1]^m+c)^2}dK[1]}{bm\,(bx^m+c)\int_1^x \frac{\exp\left(aK[1]^{n+1}\left(\frac{bK[1]^m}{m+n+1}+\frac{c}{n+1}\right)\right)K[1]^{m-1}}{(bK[1]^m+c)^2}dK[1] + e^{ax^{n+1}\left(\frac{bx^m}{m+n+1}+\frac{c}{n+1}\right)}}$$

2.32 problem 32

Internal problem ID [10362]

Book: Handbook of exact solutions for ordinary differential equations. By Polyanin and Zaitsev. Second edition
Section: Chapter 1, section 1.2. Riccati Equation. 1.2.2. Equations Containing Power Functions
Problem number: 32.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_Riccati]`

$$\boxed{y' + an\,x^{n-1}y^2 - c\,x^m(x^n a + b)\,y = -c\,x^m}$$

✓ Solution by Maple
Time used: 0.0 (sec). Leaf size: 199

```
dsolve(diff(y(x),x)=-a*n*x^(n-1)*y(x)^2+c*x^m*(a*x^n+b)*y(x)-c*x^m,y(x), singsol=all)
```

$$y(x) = \frac{an(a\,x^n+b)\left(\int \frac{x^{n-1}\mathrm{e}^{\frac{c\left(a(1+m)x^{1+m+n}+b\,x^{1+m}(1+m+n)\right)}{(1+m)(1+m+n)}}}{(a\,x^n+b)^2}dx\right) - x^n c_1 a - c_1 b + \mathrm{e}^{\frac{c\left(a(1+m)x^{1+m+n}+b\,x^{1+m}(1+m+n)\right)}{(1+m)(1+m+n)}}}{\left(a\left(\int \frac{x^{n-1}\mathrm{e}^{\frac{x(a(1+m)x^n+b(1+m+n))c\,x^m}{(1+m)(1+m+n)}}}{(a\,x^n+b)^2}dx\right)n - c_1\right)(a^2x^{2n}+2x^n ab+b^2)}$$

✓ Solution by Mathematica

Time used: 8.659 (sec). Leaf size: 304

```
DSolve[y'[x]==-a*n*x^(n-1)*y[x]^2+c*x^m*(a*x^n+b)*y[x]-c*x^m,y[x],x,IncludeSingularSolutions
```

$$y(x)$$

$$\to \frac{ac_1 n(ax^n+b)\int_1^x \frac{\exp\left(cK[1]^{m+1}\left(\frac{aK[1]^n}{m+n+1}+\frac{b}{m+1}\right)\right)K[1]^{n-1}}{(aK[1]^n+b)^2}dK[1]+a^2nx^n+c_1 e^{cx^{m+1}\left(\frac{ax^n}{m+n+1}+\frac{b}{m+1}\right)}+abn}{an(ax^n+b)^2\left(1+c_1\int_1^x \frac{\exp\left(cK[1]^{m+1}\left(\frac{aK[1]^n}{m+n+1}+\frac{b}{m+1}\right)\right)K[1]^{n-1}}{(aK[1]^n+b)^2}dK[1]\right)}$$

$$y(x)\to \frac{\frac{e^{cx^{m+1}\left(\frac{ax^n}{m+n+1}+\frac{b}{m+1}\right)}}{an\int_1^x \frac{\exp\left(cK[1]^{m+1}\left(\frac{aK[1]^n}{m+n+1}+\frac{b}{m+1}\right)\right)K[1]^{n-1}}{(aK[1]^n+b)^2}dK[1]}+ax^n+b}{(ax^n+b)^2}$$

2.33 problem 33

Internal problem ID [10363]

Book: Handbook of exact solutions for ordinary differential equations. By Polyanin and Zaitsev. Second edition
Section: Chapter 1, section 1.2. Riccati Equation. 1.2.2. Equations Containing Power Functions
Problem number: 33.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_Riccati]`

$$y' - a\,x^n y^2 - b\,x^m y = ck\,x^{k-1} - bc\,x^{k+m} - a\,c^2 x^{n+2k}$$

✗ Solution by Maple

```
dsolve(diff(y(x),x)=a*x^n*y(x)^2+b*x^m*y(x)+c*k*x^(k-1)-b*c*x^(m+k)-a*c^2*x^(n+2*k),y(x), si
```

No solution found

✗ Solution by Mathematica
Time used: 0.0 (sec). Leaf size: 0

```
DSolve[y'[x]==a*x^n*y[x]^2+b*x^m*y[x]+c*k*x^(k-1)-b*c*x^(m+k)-a*c^2*x^(n+2*k),y[x],x,Include
```

Not solved

2.34 problem 34

Internal problem ID [10364]

Book: Handbook of exact solutions for ordinary differential equations. By Polyanin and Zaitsev. Second edition
Section: Chapter 1, section 1.2. Riccati Equation. 1.2.2. Equations Containing Power Functions
Problem number: 34.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_rational, _Riccati]`

$$\boxed{y'x - ay^2 - yb = c\,x^{2b}}$$

✓ Solution by Maple
Time used: 0.016 (sec). Leaf size: 34

```
dsolve(x*diff(y(x),x)=a*y(x)^2+b*y(x)+c*x^(2*b),y(x), singsol=all)
```

$$y(x) = \frac{\tan\left(\frac{x^b\sqrt{a}\sqrt{c}-c_1 b}{b}\right)\sqrt{c}\,x^b}{\sqrt{a}}$$

✓ Solution by Mathematica
Time used: 0.544 (sec). Leaf size: 139

```
DSolve[x*y'[x]==a*y[x]^2+b*y[x]+c*x^(2*b),y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to \frac{\sqrt{c}x^b\left(-\cos\left(\frac{\sqrt{a}\sqrt{c}x^b}{b}\right)+c_1\sin\left(\frac{\sqrt{a}\sqrt{c}x^b}{b}\right)\right)}{\sqrt{a}\left(\sin\left(\frac{\sqrt{a}\sqrt{c}x^b}{b}\right)+c_1\cos\left(\frac{\sqrt{a}\sqrt{c}x^b}{b}\right)\right)}$$
$$y(x) \to \frac{\sqrt{c}x^b\tan\left(\frac{\sqrt{a}\sqrt{c}x^b}{b}\right)}{\sqrt{a}}$$

2.35 problem 35

Internal problem ID [10365]

Book: Handbook of exact solutions for ordinary differential equations. By Polyanin and Zaitsev. Second edition
Section: Chapter 1, section 1.2. Riccati Equation. 1.2.2. Equations Containing Power Functions
Problem number: 35.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_rational, _Riccati]`

$$\boxed{y'x - ay^2 - yb = c\,x^n}$$

✓ Solution by Maple
Time used: 0.0 (sec). Leaf size: 164

```
dsolve(x*diff(y(x),x)=a*y(x)^2+b*y(x)+c*x^n,y(x), singsol=all)
```

$$y(x) = \frac{\sqrt{ac}\left(\text{BesselY}\left(\frac{b+n}{n}, \frac{2\sqrt{ac}\,x^{\frac{n}{2}}}{n}\right)c_1 + \text{BesselJ}\left(\frac{b+n}{n}, \frac{2\sqrt{ac}\,x^{\frac{n}{2}}}{n}\right)\right)x^{\frac{n}{2}} - b\left(\text{BesselY}\left(\frac{b}{n}, \frac{2\sqrt{ac}\,x^{\frac{n}{2}}}{n}\right)c_1 + \text{BesselJ}\left(\frac{b}{n}, \frac{2\sqrt{ac}\,x^{\frac{n}{2}}}{n}\right)\right)}{a\left(\text{BesselY}\left(\frac{b}{n}, \frac{2\sqrt{ac}\,x^{\frac{n}{2}}}{n}\right)c_1 + \text{BesselJ}\left(\frac{b}{n}, \frac{2\sqrt{ac}\,x^{\frac{n}{2}}}{n}\right)\right)}$$

✓ Solution by Mathematica
Time used: 0.513 (sec). Leaf size: 402

```
DSolve[x*y'[x]==a*y[x]^2+b*y[x]+c*x^n,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to \frac{\sqrt{a}\sqrt{c}x^{n/2}\left(-2\,\text{BesselJ}\left(\frac{b}{n}-1, \frac{2\sqrt{a}\sqrt{c}x^{n/2}}{n}\right) + c_1\left(\text{BesselJ}\left(1-\frac{b}{n}, \frac{2\sqrt{a}\sqrt{c}x^{n/2}}{n}\right) - \text{BesselJ}\left(-\frac{b+n}{n}, \frac{2\sqrt{a}\sqrt{c}x^{n/2}}{n}\right)\right)\right.}{2a\left(\text{BesselJ}\left(\frac{b}{n}, \frac{2\sqrt{a}\sqrt{c}x^{n/2}}{n}\right) + c_1\,\text{BesselJ}\left(-\frac{b}{n}, \frac{2\sqrt{a}\sqrt{c}x^{n/2}}{n}\right)\right)}$$

$$y(x) \to -\frac{-\sqrt{a}\sqrt{c}x^{n/2}\,\text{BesselJ}\left(1-\frac{b}{n}, \frac{2\sqrt{a}\sqrt{c}x^{n/2}}{n}\right) + \sqrt{a}\sqrt{c}x^{n/2}\,\text{BesselJ}\left(-\frac{b+n}{n}, \frac{2\sqrt{a}\sqrt{c}x^{n/2}}{n}\right) + b\,\text{BesselJ}\left(-\frac{b}{n}, \frac{2\sqrt{a}\sqrt{c}x^{n/2}}{n}\right)}{2a\,\text{BesselJ}\left(-\frac{b}{n}, \frac{2\sqrt{a}\sqrt{c}x^{n/2}}{n}\right)}$$

2.36 problem 36

Internal problem ID [10366]

Book: Handbook of exact solutions for ordinary differential equations. By Polyanin and Zaitsev. Second edition
Section: Chapter 1, section 1.2. Riccati Equation. 1.2.2. Equations Containing Power Functions
Problem number: 36.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_rational, _Riccati]`

$$\boxed{y'x - ay^2 - (n + b\,x^n)\,y = c\,x^{2n}}$$

✓ Solution by Maple
Time used: 0.015 (sec). Leaf size: 69

```
dsolve(x*diff(y(x),x)=a*y(x)^2+(n+b*x^n)*y(x)+c*x^(2*n),y(x), singsol=all)
```

$$y(x) = -\frac{x^n\left(b^2 - \sqrt{4a\,b^2c - b^4}\,\tan\left(\frac{\sqrt{4a\,b^2c-b^4}\,(b\,x^n+c_1 n)}{2b^2 n}\right)\right)}{2ab}$$

✓ Solution by Mathematica
Time used: 1.07 (sec). Leaf size: 114

```
DSolve[x*y'[x]==a*y[x]^2+(n+b*x^n)*y[x]+c*x^(2*n),y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to \frac{x^n\left(-b + \frac{\sqrt{b^2-4ac}\left(-e^{\frac{x^n\sqrt{b^2-4ac}}{n}}+c_1\right)}{e^{\frac{x^n\sqrt{b^2-4ac}}{n}}+c_1}\right)}{2a}$$
$$y(x) \to \frac{x^n\left(\sqrt{b^2-4ac}-b\right)}{2a}$$

2.37 problem 37

Internal problem ID [10367]

Book: Handbook of exact solutions for ordinary differential equations. By Polyanin and Zaitsev. Second edition
Section: Chapter 1, section 1.2. Riccati Equation. 1.2.2. Equations Containing Power Functions
Problem number: 37.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_rational, _Riccati]`

$$\boxed{y'x - y^2x - ay = b\,x^n}$$

✓ Solution by Maple
Time used: 0.0 (sec). Leaf size: 139

```
dsolve(x*diff(y(x),x)=x*y(x)^2+a*y(x)+b*x^n,y(x), singsol=all)
```

$$y(x) = \frac{x^{\frac{n}{2}-\frac{1}{2}}\sqrt{b}\left(\text{BesselY}\left(\frac{-a+n}{n+1}, \frac{2\sqrt{b}\,x^{\frac{n}{2}+\frac{1}{2}}}{n+1}\right)c_1 + \text{BesselJ}\left(\frac{-a+n}{n+1}, \frac{2\sqrt{b}\,x^{\frac{n}{2}+\frac{1}{2}}}{n+1}\right)\right)}{\text{BesselY}\left(\frac{-a-1}{n+1}, \frac{2\sqrt{b}\,x^{\frac{n}{2}+\frac{1}{2}}}{n+1}\right)c_1 + \text{BesselJ}\left(\frac{-a-1}{n+1}, \frac{2\sqrt{b}\,x^{\frac{n}{2}+\frac{1}{2}}}{n+1}\right)}$$

✓ Solution by Mathematica

Time used: 1.381 (sec). Leaf size: 855

```
DSolve[x*y'[x]==x*y[x]^2+a*y[x]+b*x^n,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to -\frac{\sqrt{b}(x^n)^{\frac{n+1}{2n}} \text{ Gamma}\left(\frac{a+n+2}{n+1}\right) \text{BesselJ}\left(\frac{a-n}{n+1}, \frac{2\sqrt{b}(x^n)^{\frac{n+1}{2n}}}{n+1}\right) - \sqrt{b}(x^n)^{\frac{n+1}{2n}} \text{ Gamma}\left(\frac{a+n+2}{n+1}\right) \text{BesselJ}\left(\frac{a+n+2}{n+1}\right.}{}$$

$$y(x) \to -\frac{\frac{\sqrt{b}(x^n)^{\frac{n+1}{2n}}\left(\text{BesselJ}\left(-\frac{a+n+2}{n+1}, \frac{2\sqrt{b}(x^n)^{\frac{n+1}{2n}}}{n+1}\right) - \text{BesselJ}\left(\frac{n-a}{n+1}, \frac{2\sqrt{b}(x^n)^{\frac{n+1}{2n}}}{n+1}\right)\right)}{\text{BesselJ}\left(-\frac{a+1}{n+1}, \frac{2\sqrt{b}(x^n)^{\frac{n+1}{2n}}}{n+1}\right)} + a + 1}{2x}$$

$$y(x) \to -\frac{\frac{\sqrt{b}(x^n)^{\frac{n+1}{2n}}\left(\text{BesselJ}\left(-\frac{a+n+2}{n+1}, \frac{2\sqrt{b}(x^n)^{\frac{n+1}{2n}}}{n+1}\right) - \text{BesselJ}\left(\frac{n-a}{n+1}, \frac{2\sqrt{b}(x^n)^{\frac{n+1}{2n}}}{n+1}\right)\right)}{\text{BesselJ}\left(-\frac{a+1}{n+1}, \frac{2\sqrt{b}(x^n)^{\frac{n+1}{2n}}}{n+1}\right)} + a + 1}{2x}$$

2.38 problem 38

Internal problem ID [10368]

Book: Handbook of exact solutions for ordinary differential equations. By Polyanin and Zaitsev. Second edition
Section: Chapter 1, section 1.2. Riccati Equation. 1.2.2. Equations Containing Power Functions
Problem number: 38.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_rational, _Riccati]`

$$\boxed{y'x + a_3 x y^2 + a_2 y = -a_1 x - a_0}$$

✓ Solution by Maple
Time used: 0.031 (sec). Leaf size: 403

```
dsolve(x*diff(y(x),x)+a__3*x*y(x)^2+a__2*y(x)+a__1*x+a__0=0,y(x), singsol=all)
```

$$y(x) = -\frac{4a_1\left(a_1^3 a_3\left(a_3 a_0 - a_2\sqrt{-a_1 a_3}\right)\operatorname{KummerM}\left(\frac{\sqrt{-a_1 a_3}\,a_0+a_1(a_2+2)}{2a_1}, a_2+1, 2x\sqrt{-a_1 a_3}\right)\right.}{4a_1^3 a_3^2\left(\sqrt{-a_1 a_3}\,a_0 + a_1 a_2\right)\operatorname{KummerM}\left(\frac{\sqrt{-a_1 a_3}\,a_0+a_1(a_2+2)}{2a_1}, a_2+1, 2x\sqrt{-a_1 a_3}\right) - c_1\sqrt{-a_1 a_3}\,(a_0^2 a_3 + a}$$

✓ Solution by Mathematica

Time used: 0.748 (sec). Leaf size: 541

```
DSolve[x*y'[x]+a3*x*y[x]^2+a2*y[x]+a1*x+a0==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to -\frac{i\left(\sqrt{\text{a1}}c_1 \text{HypergeometricU}\left(\frac{1}{2}\left(\frac{i\sqrt{\text{a3}}\text{a0}}{\sqrt{\text{a1}}}+\text{a2}\right),\text{a2},2i\sqrt{\text{a1}}\sqrt{\text{a3}}x\right)+c_1\left(\sqrt{\text{a1}}\text{a2}+i\text{a0}\sqrt{\text{a3}}\right)\text{Hypergeom}\right.}{\sqrt{\text{a3}}\left(c_1 \text{HypergeometricU}\left(\frac{1}{2}\left(\frac{i\sqrt{\text{a3}}}{\sqrt{\text{a}}}\right.\right.\right.}$$

$$y(x) \to \frac{\left(\text{a0}\sqrt{\text{a3}}-i\sqrt{\text{a1}}\text{a2}\right)\text{HypergeometricU}\left(\frac{1}{2}\left(\frac{i\sqrt{\text{a3}}\text{a0}}{\sqrt{\text{a1}}}+\text{a2}+2\right),\text{a2}+1,2i\sqrt{\text{a1}}\sqrt{\text{a3}}x\right)}{\text{HypergeometricU}\left(\frac{1}{2}\left(\frac{i\sqrt{\text{a3}}\text{a0}}{\sqrt{\text{a1}}}+\text{a2}\right),\text{a2},2i\sqrt{\text{a1}}\sqrt{\text{a3}}x\right)}-i\sqrt{\text{a1}}}{\sqrt{\text{a3}}}$$

$$y(x) \to \frac{\left(\text{a0}\sqrt{\text{a3}}-i\sqrt{\text{a1}}\text{a2}\right)\text{HypergeometricU}\left(\frac{1}{2}\left(\frac{i\sqrt{\text{a3}}\text{a0}}{\sqrt{\text{a1}}}+\text{a2}+2\right),\text{a2}+1,2i\sqrt{\text{a1}}\sqrt{\text{a3}}x\right)}{\text{HypergeometricU}\left(\frac{1}{2}\left(\frac{i\sqrt{\text{a3}}\text{a0}}{\sqrt{\text{a1}}}+\text{a2}\right),\text{a2},2i\sqrt{\text{a1}}\sqrt{\text{a3}}x\right)}-i\sqrt{\text{a1}}}{\sqrt{\text{a3}}}$$

2.39 problem 39

Internal problem ID [10369]

Book: Handbook of exact solutions for ordinary differential equations. By Polyanin and Zaitsev. Second edition
Section: Chapter 1, section 1.2. Riccati Equation. 1.2.2. Equations Containing Power Functions
Problem number: 39.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[[_homogeneous, 'class G'], _rational, _Riccati]`

$$\boxed{y'x - a\,x^n y^2 - yb = c\,x^{-n}}$$

✓ Solution by Maple
Time used: 0.016 (sec). Leaf size: 73

```
dsolve(x*diff(y(x),x)=a*x^n*y(x)^2+b*y(x)+c*x^(-n),y(x), singsol=all)
```

$$y(x) = \frac{x^{-n}\left(\tan\left(\frac{\sqrt{4ac-b^2-2bn-n^2}\,(\ln(x)-c_1)}{2}\right)\sqrt{4ac-b^2-2bn-n^2}-b-n\right)}{2a}$$

✓ Solution by Mathematica
Time used: 0.978 (sec). Leaf size: 138

```
DSolve[x*y'[x]==a*x^n*y[x]^2+b*y[x]+c*x^(-n),y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to \frac{x^{-n}\left(\frac{\sqrt{-4ac+b^2+2bn+n^2}\left(-x^{\sqrt{-4ac+b^2+2bn+n^2}}+c_1\right)}{x^{\sqrt{-4ac+b^2+2bn+n^2}}+c_1}-b-n\right)}{2a}$$
$$y(x) \to \frac{x^{-n}\left(\sqrt{-4ac+b^2+2bn+n^2}-b-n\right)}{2a}$$

2.40 problem 40

Internal problem ID [10370]

Book: Handbook of exact solutions for ordinary differential equations. By Polyanin and Zaitsev. Second edition
Section: Chapter 1, section 1.2. Riccati Equation. 1.2.2. Equations Containing Power Functions
Problem number: 40.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_rational, _Riccati]`

$$\boxed{y'x - a\,x^n y^2 - ym = -a\,b^2 x^{n+2m}}$$

✓ Solution by Maple
Time used: 0.016 (sec). Leaf size: 34

```
dsolve(x*diff(y(x),x)=a*x^n*y(x)^2+m*y(x)-a*b^2*x^(n+2*m),y(x), singsol=all)
```

$$y(x) = i \tan\left(\frac{c_1(n+m) + iab\,x^{n+m}}{n+m}\right) b\,x^m$$

✓ Solution by Mathematica
Time used: 1.736 (sec). Leaf size: 43

```
DSolve[x*y'[x]==a*x^n*y[x]^2+m*y[x]-a*b^2*x^(n+2*m),y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to \sqrt{-b^2}x^m \tan\left(\frac{a\sqrt{-b^2}x^{m+n}}{m+n} + c_1\right)$$

2.41 problem 41

Internal problem ID [10371]

Book: Handbook of exact solutions for ordinary differential equations. By Polyanin and Zaitsev. Second edition
Section: Chapter 1, section 1.2. Riccati Equation. 1.2.2. Equations Containing Power Functions
Problem number: 41.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_rational, _Riccati]`

$$\boxed{y'x - x^{2n}y^2 - (m-n)\,y = x^{2m}}$$

✓ Solution by Maple
Time used: 0.0 (sec). Leaf size: 34

```
dsolve(x*diff(y(x),x)=x^(2*n)*y(x)^2+(m-n)*y(x)+x^(2*m),y(x), singsol=all)
```

$$y(x) = \tan\left(\frac{x^{n+m} + (-n-m)\,c_1}{n+m}\right) x^{-n+m}$$

✓ Solution by Mathematica
Time used: 0.727 (sec). Leaf size: 28

```
DSolve[x*y'[x]==x^(2*n)*y[x]^2+(m-n)*y[x]+x^(2*m),y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to x^{m-n} \tan\left(\frac{x^{m+n}}{m+n} + c_1\right)$$

2.42 problem 42

Internal problem ID [10372]

Book: Handbook of exact solutions for ordinary differential equations. By Polyanin and Zaitsev. Second edition
Section: Chapter 1, section 1.2. Riccati Equation. 1.2.2. Equations Containing Power Functions
Problem number: 42.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_rational, _Riccati]`

$$\boxed{y'x - a\,x^n y^2 - yb = c\,x^m}$$

✓ Solution by Maple
Time used: 0.0 (sec). Leaf size: 166

```
dsolve(x*diff(y(x),x)=a*x^(n)*y(x)^2+b*y(x)+c*x^(m),y(x), singsol=all)
```

$$y(x) = \frac{x^{-\frac{n}{2}+\frac{m}{2}}\sqrt{ac}\left(\text{BesselY}\left(\frac{-b+m}{n+m}, \frac{2\sqrt{ac}\,x^{\frac{m}{2}+\frac{n}{2}}}{n+m}\right)c_1 + \text{BesselJ}\left(\frac{-b+m}{n+m}, \frac{2\sqrt{ac}\,x^{\frac{m}{2}+\frac{n}{2}}}{n+m}\right)\right)}{a\left(\text{BesselY}\left(\frac{-b-n}{n+m}, \frac{2\sqrt{ac}\,x^{\frac{m}{2}+\frac{n}{2}}}{n+m}\right)c_1 + \text{BesselJ}\left(\frac{-b-n}{n+m}, \frac{2\sqrt{ac}\,x^{\frac{m}{2}+\frac{n}{2}}}{n+m}\right)\right)}$$

✓ Solution by Mathematica
Time used: 1.49 (sec). Leaf size: 1321

```
DSolve[x*y'[x]==a*x^(n)*y[x]^2+b*y[x]+c*x^(m),y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to \frac{x^{-n}\left(\sqrt{a}\sqrt{c}(m+n)x^{m+n}\,\text{BesselJ}\left(\frac{m-b}{m+n}, \frac{2\sqrt{a}\sqrt{c}\sqrt{x^{m+n}}}{\sqrt{(m+n)^2}}\right)c_1\,\text{Gamma}\left(\frac{m-b}{m+n}\right)((m+n)^2)^{\frac{b+n}{m+n}} - \sqrt{a}\sqrt{c}mx^{m+n}\right.$$

$$y(x) \to \frac{x^{-n}\left(\sqrt{a}\sqrt{c}(m+n)\sqrt{x^{m+n}}\,\text{BesselJ}\left(\frac{m-b}{m+n}, \frac{2\sqrt{a}\sqrt{c}\sqrt{x^{m+n}}}{\sqrt{(m+n)^2}}\right) - (b+n)\sqrt{(m+n)^2}\,\text{BesselJ}\left(-\frac{b+n}{m+n}, \frac{2\sqrt{a}\sqrt{c}\sqrt{x^{m}}\right.\right.}{2a\sqrt{(m+n)^2}\,\text{BesselJ}\left(-\frac{b+n}{m+n}, \frac{2\sqrt{a}\sqrt{c}\sqrt{x^{m+}}\right.}$$

2.43 problem 43

Internal problem ID [10373]

Book: Handbook of exact solutions for ordinary differential equations. By Polyanin and Zaitsev. Second edition
Section: Chapter 1, section 1.2. Riccati Equation. 1.2.2. Equations Containing Power Functions
Problem number: 43.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_rational, _Riccati]`

$$\boxed{y'x - x^{2n}y^2a - (b\,x^n - n)\,y = c}$$

✓ Solution by Maple
Time used: 0.016 (sec). Leaf size: 72

```
dsolve(x*diff(y(x),x)=a*x^(2*n)*y(x)^2+(b*x^n-n)*y(x)+c,y(x), singsol=all)
```

$$y(x) = \frac{\left(\sqrt{4a\,b^2c - b^4}\,\tan\left(\frac{\sqrt{4a\,b^2c-b^4}\,(b\,x^n+c_1 n)}{2b^2n}\right) - b^2\right)x^{-n}}{2ba}$$

✓ Solution by Mathematica
Time used: 1.071 (sec). Leaf size: 118

```
DSolve[x*y'[x]==a*x^(2*n)*y[x]^2+(b*x^n-n)*y[x]+c,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to \frac{x^{-n}\left(-b + \frac{\sqrt{b^2-4ac}\left(-e^{\frac{x^n\sqrt{b^2-4ac}}{n}}+c_1\right)}{e^{\frac{x^n\sqrt{b^2-4ac}}{n}}+c_1}\right)}{2a}$$
$$y(x) \to \frac{x^{-n}\left(\sqrt{b^2-4ac} - b\right)}{2a}$$

2.44 problem 44

Internal problem ID [10374]

Book: Handbook of exact solutions for ordinary differential equations. By Polyanin and Zaitsev. Second edition
Section: Chapter 1, section 1.2. Riccati Equation. 1.2.2. Equations Containing Power Functions
Problem number: 44.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_rational, _Riccati]`

$$\boxed{y'x - a\,x^{m+2n}y^2 - \left(b\,x^{m+n} - n\right)y = c\,x^m}$$

✓ Solution by Maple
Time used: 0.031 (sec). Leaf size: 78

```
dsolve(x*diff(y(x),x)=a*x^(2*n+m)*y(x)^2+(b*x^(n+m)-n)*y(x)+c*x^m,y(x), singsol=all)
```

$$y(x) = \frac{x^{-n}\left(\sqrt{4a\,b^2c - b^4}\,\tan\left(\frac{\left(x^{n+m}b + c_1(n+m)\right)\sqrt{4a\,b^2c-b^4}}{2b^2(n+m)}\right) - b^2\right)}{2ab}$$

✓ Solution by Mathematica
Time used: 1.566 (sec). Leaf size: 126

```
DSolve[x*y'[x]==a*x^(2*n+m)*y[x]^2+(b*x^(n+m)-n)*y[x]+c*x^m,y[x],x,IncludeSingularSolutions
```

$$y(x) \to \frac{x^{-n}\left(-b + \frac{\sqrt{b^2-4ac}\left(-e^{\frac{\sqrt{b^2-4ac}\,x^{m+n}}{m+n}} + c_1\right)}{e^{\frac{\sqrt{b^2-4ac}\,x^{m+n}}{m+n}} + c_1}\right)}{2a}$$
$$y(x) \to \frac{x^{-n}\left(\sqrt{b^2-4ac} - b\right)}{2a}$$

2.45 problem 45

Internal problem ID [10375]

Book: Handbook of exact solutions for ordinary differential equations. By Polyanin and Zaitsev. Second edition
Section: Chapter 1, section 1.2. Riccati Equation. 1.2.2. Equations Containing Power Functions
Problem number: 45.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_rational, _Riccati]`

$$\boxed{(a_2x+b_2)\left(y'+\lambda y^2\right)+(a_1x+b_1)\,y=-a_0x-b_0}$$

✓ Solution by Maple
Time used: 0.0 (sec). Leaf size: 827

```
dsolve((a__2*x+b__2)*(diff(y(x),x)+lambda*y(x)^2)+(a__1*x+b__1)*y(x)+a__0*x+b__0=0,y(x), sin
```

Expression too large to display

✓ Solution by Mathematica
Time used: 3.165 (sec). Leaf size: 1418

```
DSolve[(a2*x+b2)*(y'[x]+\[Lambda]*y[x]^2)+(a1*x+b1)*y[x]+a0*x+b0==0,y[x],x,IncludeSingularSo
```

Too large to display

2.46 problem 46

Internal problem ID [10376]

Book: Handbook of exact solutions for ordinary differential equations. By Polyanin and Zaitsev. Second edition
Section: Chapter 1, section 1.2. Riccati Equation. 1.2.2. Equations Containing Power Functions
Problem number: 46.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[[_1st_order, _with_linear_symmetries], _rational, _Riccati]`

$$\boxed{(xa+c)\,y' - \alpha(ay+bx)^2 - \beta(ay+bx) = -bx+\gamma}$$

✓ Solution by Maple
Time used: 0.0 (sec). Leaf size: 94

```
dsolve((a*x+c)*diff(y(x),x)=alpha*(a*y(x)+b*x)^2+beta*(a*y(x)+b*x)-b*x+gamma,y(x), singsol=a
```

$$y(x) = \frac{-2a^2\alpha x b - a^2\beta + \sqrt{-\left(\left(-4\gamma\alpha+\beta^2\right)a - 4\alpha bc\right)a^3}\,\tan\left(\frac{-2c_1a^2+\ln(ax+c)\sqrt{-\left(\left(-4\gamma\alpha+\beta^2\right)a-4\alpha bc\right)a^3}}{2a^2}\right)}{2a^3\alpha}$$

✓ Solution by Mathematica
Time used: 60.527 (sec). Leaf size: 98

```
DSolve[(a*x+c)*y'[x]==\[Alpha]*(a*y[x]+b*x)^2+\[Beta]*(a*y[x]+b*x)-b*x+\[Gamma],y[x],x,Inclu
```

$$y(x) \to -\frac{-a\alpha\sqrt{\frac{4a\alpha\gamma-a\beta^2+4\alpha bc}{a^3\alpha^2}}\,\tan\left(\frac{1}{2}a\alpha\log(ax+c)\sqrt{\frac{4a\alpha\gamma-a\beta^2+4\alpha bc}{a^3\alpha^2}}+c_1\right)+2\alpha bx+\beta}{2a\alpha}$$

2.47 problem 47

Internal problem ID [10377]

Book: Handbook of exact solutions for ordinary differential equations. By Polyanin and Zaitsev. Second edition
Section: Chapter 1, section 1.2. Riccati Equation. 1.2.2. Equations Containing Power Functions
Problem number: 47.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_rational, _Riccati]`

$$\boxed{2x^2y' - 2y^2 - xy = -2a^2x}$$

✓ Solution by Maple
Time used: 0.0 (sec). Leaf size: 25

```
dsolve(2*x^2*diff(y(x),x)=2*y(x)^2+x*y(x)-2*a^2*x,y(x), singsol=all)
```

$$y(x) = \tanh\left(\frac{ic_1\sqrt{x} + 2a}{\sqrt{x}}\right)\sqrt{x}\, a$$

✓ Solution by Mathematica
Time used: 0.619 (sec). Leaf size: 43

```
DSolve[2*x^2*y'[x]==2*y[x]^2+x*y[x]-2*a^2*x,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to -\sqrt{-a^2}\sqrt{x}\tan\left(\frac{2\sqrt{-a^2}}{\sqrt{x}} - c_1\right)$$

2.48 problem 48

Internal problem ID [10378]

Book: Handbook of exact solutions for ordinary differential equations. By Polyanin and Zaitsev. Second edition
Section: Chapter 1, section 1.2. Riccati Equation. 1.2.2. Equations Containing Power Functions
Problem number: 48.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_rational, _Riccati]`

$$\boxed{2x^2y' - 2y^2 - 3xy = -2a^2x}$$

✓ Solution by Maple
Time used: 0.016 (sec). Leaf size: 102

```
dsolve(2*x^2*diff(y(x),x)=2*y(x)^2+3*x*y(x)-2*a^2*x,y(x), singsol=all)
```

$$y(x) = \frac{\left(-2xc_1\sqrt{-\frac{a^2}{x}} - x\right)\sin\left(2\sqrt{-\frac{a^2}{x}}\right) - x\left(c_1 - 2\sqrt{-\frac{a^2}{x}}\right)\cos\left(2\sqrt{-\frac{a^2}{x}}\right)}{2\cos\left(2\sqrt{-\frac{a^2}{x}}\right)c_1 + 2\sin\left(2\sqrt{-\frac{a^2}{x}}\right)}$$

✓ Solution by Mathematica
Time used: 0.457 (sec). Leaf size: 94

```
DSolve[2*x^2*y'[x]==2*y[x]^2+3*x*y[x]-2*a^2*x,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to \frac{4a^2c_1\sqrt{x} + 2a\sqrt{x}e^{\frac{4a}{\sqrt{x}}} - xe^{\frac{4a}{\sqrt{x}}} + 2ac_1x}{2e^{\frac{4a}{\sqrt{x}}} - 4ac_1}$$
$$y(x) \to a\left(-\sqrt{x}\right) - \frac{x}{2}$$

2.49 problem 49

Internal problem ID [10379]

Book: Handbook of exact solutions for ordinary differential equations. By Polyanin and Zaitsev. Second edition
Section: Chapter 1, section 1.2. Riccati Equation. 1.2.2. Equations Containing Power Functions
Problem number: 49.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[[_homogeneous, 'class G'], _rational, _Riccati]`

$$\boxed{x^2 y' - a\,x^2 y^2 - ybx = c}$$

✓ Solution by Maple
Time used: 0.0 (sec). Leaf size: 59

```
dsolve(x^2*diff(y(x),x)=a*x^2*y(x)^2+b*x*y(x)+c,y(x), singsol=all)
```

$$y(x) = \frac{-1 - b + \tan\left(\frac{\sqrt{4ac-b^2-2b-1}\,(\ln(x)-c_1)}{2}\right)\sqrt{4ac-b^2-2b-1}}{2ax}$$

✓ Solution by Mathematica
Time used: 0.43 (sec). Leaf size: 99

```
DSolve[x^2*y'[x]==a*x^2*y[x]^2+b*x*y[x]+c,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to -\frac{\sqrt{-4ac+b^2+2b+1}\left(1 - \frac{2c_1}{x^{\sqrt{-4ac+b^2+2b+1}}+c_1}\right) + b + 1}{2ax}$$
$$y(x) \to -\frac{-\sqrt{-4ac+b^2+2b+1} + b + 1}{2ax}$$

2.50 problem 50

Internal problem ID [10380]

Book: Handbook of exact solutions for ordinary differential equations. By Polyanin and Zaitsev. Second edition
Section: Chapter 1, section 1.2. Riccati Equation. 1.2.2. Equations Containing Power Functions
Problem number: 50.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_rational, _Riccati]`

$$\boxed{x^2 y' - y^2 c\, x^2 - \left(a\, x^2 + bx\right) y = \alpha\, x^2 + \beta x + \gamma}$$

✓ Solution by Maple
Time used: 0.016 (sec). Leaf size: 443

```
dsolve(x^2*diff(y(x),x)=c*x^2*y(x)^2+(a*x^2+b*x)*y(x)+alpha*x^2+beta*x+gamma,y(x), singsol=a
```

$$y(x) = -\frac{\left(\sqrt{b^2-4c\gamma+2b+1}\,\sqrt{a^2-4\alpha c}-ab+2\beta c+\sqrt{a^2-4\alpha c}\right)\text{WhittakerM}\left(-\frac{ab-2\beta c-2\sqrt{a^2-4\alpha c}}{2\sqrt{a^2-4\alpha c}},\frac{\sqrt{b^2-4c\gamma+2}}{2}\right.$$

✓ Solution by Mathematica

Time used: 1.712 (sec). Leaf size: 1584

```
DSolve[x^2*y'[x]==c*x^2*y[x]^2+(a*x^2+b*x)*y[x]+\[Alpha]*x^2+\[Beta]*x+\[Gamma],y[x],x,Inclu
```

$$y(x)\to$$

$$-\frac{\left(b+ax-x\sqrt{a^2-4c\alpha}+\sqrt{b^2+2b-4c\gamma+1}+1\right)c_1\,\text{HypergeometricU}\left(\frac{ab-2c\beta+\sqrt{a^2-4c\alpha}\left(\sqrt{b^2+2b-4c\gamma+1}\right.}{2\sqrt{a^2-4c\alpha}}\right.}{}$$

$$y(x)$$

$$\to\frac{\left(\sqrt{a^2-4\alpha c}\left(\sqrt{b^2+2b-4c\gamma+1}+1\right)+ab-2\beta c\right)\text{HypergeometricU}\left(\frac{ab-2c\beta+\sqrt{a^2-4c\alpha}\left(\sqrt{b^2+2b-4c\gamma+1}+3\right)}{2\sqrt{a^2-4c\alpha}},\sqrt{b^2+2b-4c\gamma+1}+2,x\sqrt{a^2-4c\alpha}\right)}{\text{HypergeometricU}\left(\frac{ab-2c\beta+\sqrt{a^2-4c\alpha}\left(\sqrt{b^2+2b-4c\gamma+1}+1\right)}{2\sqrt{a^2-4c\alpha}},\sqrt{b^2+2b-4c\gamma+1}+1,x\sqrt{a^2-4c\alpha}\right)}$$

$$y(x)$$

$$\to\frac{\left(\sqrt{a^2-4\alpha c}\left(\sqrt{b^2+2b-4c\gamma+1}+1\right)+ab-2\beta c\right)\text{HypergeometricU}\left(\frac{ab-2c\beta+\sqrt{a^2-4c\alpha}\left(\sqrt{b^2+2b-4c\gamma+1}+3\right)}{2\sqrt{a^2-4c\alpha}},\sqrt{b^2+2b-4c\gamma+1}+2,x\sqrt{a^2-4c\alpha}\right)}{\text{HypergeometricU}\left(\frac{ab-2c\beta+\sqrt{a^2-4c\alpha}\left(\sqrt{b^2+2b-4c\gamma+1}+1\right)}{2\sqrt{a^2-4c\alpha}},\sqrt{b^2+2b-4c\gamma+1}+1,x\sqrt{a^2-4c\alpha}\right)}$$

2.51 problem 51

Internal problem ID [10381]

Book: Handbook of exact solutions for ordinary differential equations. By Polyanin and Zaitsev. Second edition
Section: Chapter 1, section 1.2. Riccati Equation. 1.2.2. Equations Containing Power Functions
Problem number: 51.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_rational, _Riccati]`

$$\boxed{x^2y' - a\,x^2y^2 - ybx = c\,x^n + s}$$

✓ Solution by Maple
Time used: 0.0 (sec). Leaf size: 263

```
dsolve(x^2*diff(y(x),x)=a*x^2*y(x)^2+b*x*y(x)+c*x^n+s,y(x), singsol=all)
```

$$y(x) = \frac{2\left(\text{BesselY}\left(\frac{\sqrt{-4as+b^2+2b+1}}{n}+1, \frac{2\sqrt{ac}\,x^{\frac{n}{2}}}{n}\right)c_1 + \text{BesselJ}\left(\frac{\sqrt{-4as+b^2+2b+1}}{n}+1, \frac{2\sqrt{ac}\,x^{\frac{n}{2}}}{n}\right)\right)\sqrt{ac}\,x^{\frac{n}{2}} - \left(\text{BesselY}\right.}{2xa\left(\text{BesselY}\left(\frac{\sqrt{-4as+b^2+2b+1}}{n}, \frac{2\sqrt{ac}\,x^{\frac{n}{2}}}{n}\right)c_1 +\right.}$$

✓ Solution by Mathematica
Time used: 2.637 (sec). Leaf size: 2281

```
DSolve[x^2*y'[x]==a*x^2*y[x]^2+b*x*y[x]+c*x^n+s,y[x],x,IncludeSingularSolutions -> True]
```

Too large to display

2.52 problem 52

Internal problem ID [10382]

Book: Handbook of exact solutions for ordinary differential equations. By Polyanin and Zaitsev. Second edition
Section: Chapter 1, section 1.2. Riccati Equation. 1.2.2. Equations Containing Power Functions
Problem number: 52.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_rational, _Riccati]`

$$x^2y' - a\,x^2y^2 - ybx = c\,x^{2n} + s\,x^n$$

✓ Solution by Maple
Time used: 0.0 (sec). Leaf size: 373

```
dsolve(x^2*diff(y(x),x)=a*x^2*y(x)^2+b*x*y(x)+c*x^(2*n)+s*x^n,y(x), singsol=all)
```

$$y(x) = \frac{\text{KummerM}\left(\frac{(b-n+1)\sqrt{c}+i\sqrt{a}\,s}{2\sqrt{c}\,n}, \frac{b+n+1}{n}, \frac{2i\sqrt{a}\sqrt{c}\,x^n}{n}\right)\left((-b-n-1)\sqrt{c}+i\sqrt{a}\,s\right)+2\sqrt{c}\,\text{KummerU}\left(\frac{(b-n+1)\sqrt{c}}{2\sqrt{c}\,n}\right.}{2\sqrt{c}\,xa\left(\text{KummerU}\left(\frac{(b+}$$

✓ Solution by Mathematica

Time used: 1.839 (sec). Leaf size: 819

```
DSolve[x^2*y'[x]==a*x^2*y[x]^2+b*x*y[x]+c*x^(2*n)+s*x^n,y[x],x,IncludeSingularSolutions -> T
```

$$y(x) \to$$

$$-\frac{i\sqrt{a}c_1x^n\left(\sqrt{c}(b+n+1)-i\sqrt{a}s\right)\text{HypergeometricU}\left(\frac{b+3n-\frac{i\sqrt{a}s}{\sqrt{c}}+1}{2n},\frac{b+2n+1}{n},-\frac{2i\sqrt{a}\sqrt{c}x^n}{n}\right)+c_1n\left(i\sqrt{a}\sqrt{c}x^n+b+1\right)\text{HypergeometricU}\left(\frac{b+n-\frac{i\sqrt{a}s}{\sqrt{c}}+1}{2n},\frac{b+n+1}{n},-\frac{2i\sqrt{a}\sqrt{c}x^n}{n}\right)}{anx\left(c_1\text{Hyperge}\right.}$$

$$y(x)$$

$$\to -\frac{\frac{\sqrt{a}x^n\left(\sqrt{a}s+i\sqrt{c}(b+n+1)\right)\text{HypergeometricU}\left(\frac{b+3n-\frac{i\sqrt{a}s}{\sqrt{c}}+1}{2n},\frac{b+2n+1}{n},-\frac{2i\sqrt{a}\sqrt{c}x^n}{n}\right)}{n\,\text{HypergeometricU}\left(\frac{b+n-\frac{i\sqrt{a}s}{\sqrt{c}}+1}{2n},\frac{b+n+1}{n},-\frac{2i\sqrt{a}\sqrt{c}x^n}{n}\right)}+i\sqrt{a}\sqrt{c}x^n+b+1}{ax}$$

$$y(x)$$

$$\to -\frac{\frac{\sqrt{a}x^n\left(\sqrt{a}s+i\sqrt{c}(b+n+1)\right)\text{HypergeometricU}\left(\frac{b+3n-\frac{i\sqrt{a}s}{\sqrt{c}}+1}{2n},\frac{b+2n+1}{n},-\frac{2i\sqrt{a}\sqrt{c}x^n}{n}\right)}{n\,\text{HypergeometricU}\left(\frac{b+n-\frac{i\sqrt{a}s}{\sqrt{c}}+1}{2n},\frac{b+n+1}{n},-\frac{2i\sqrt{a}\sqrt{c}x^n}{n}\right)}+i\sqrt{a}\sqrt{c}x^n+b+1}{ax}$$

2.53 problem 53

Internal problem ID [10383]

Book: Handbook of exact solutions for ordinary differential equations. By Polyanin and Zaitsev. Second edition
Section: Chapter 1, section 1.2. Riccati Equation. 1.2.2. Equations Containing Power Functions
Problem number: 53.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_rational, _Riccati]`

$$x^2y' - c\,x^2y^2 - (a\,x^n + b)\,xy = \gamma + \beta\,x^n + \alpha\,x^{2n}$$

✓ Solution by Maple
Time used: 0.0 (sec). Leaf size: 560

```
dsolve(x^2*diff(y(x),x)=c*x^2*y(x)^2+(a*x^n+b)*x*y(x)+alpha*x^(2*n)+beta*x^n+gamma,y(x), sin
```

$$y(x) = -\frac{\left(\sqrt{b^2-4c\gamma+2b+1}\,\sqrt{a^2-4\alpha c}+\sqrt{a^2-4\alpha c}\,n+(n-b-1)\,a+2\beta c\right)\text{WhittakerM}\left(-\frac{-2\sqrt{a^2-4\alpha c}\,n+a}{2\sqrt{a^2-4}}\right.$$

✓ Solution by Mathematica

Time used: 3.672 (sec). Leaf size: 1837

```
DSolve[x^2*y'[x]==c*x^2*y[x]^2+(a*x^n+b)*x*y[x]+\[Alpha]*x^(2*n)+\[Beta]*x^n+\[Gamma],y[x],x
```

$y(x)$

$$-\left(\left(-\left(\left(n^2+\sqrt{n^2\left(b^2+2b-4c\gamma+1\right)}\right)a^2\right)+n(-b+n-1)\sqrt{a^2-4c\alpha}a+2c\left(2\alpha n^2+\sqrt{a^2-4c\alpha}\beta n\right.\right.\right.$$

$$\to$$

$y(x)$

$$-\frac{x^n\left(2c\left(\beta n\sqrt{a^2-4\alpha c}+2\alpha\sqrt{n^2(b^2+2b-4c\gamma+1)}+2\alpha n^2\right)-\left(a^2\left(\sqrt{n^2(b^2+2b-4c\gamma+1)}+n^2\right)\right)+an(-b+n-1)\sqrt{a^2-4\alpha c}\right)\text{HypergeometricU}\left(\frac{}{}\right.}{\text{HypergeometricU}\left(\frac{\left(n^2+\sqrt{n^2\left(b^2+2b-4c\gamma+1\right)}\right)a^2+(b-n+1)n\sqrt{a^2-4c\alpha}a-2c\left(2\alpha n^2\right.}{2n^2\left(a^2-4c\alpha\right)}\right.}$$

$$\to$$

2.54 problem 54

Internal problem ID [10384]

Book: Handbook of exact solutions for ordinary differential equations. By Polyanin and Zaitsev. Second edition
Section: Chapter 1, section 1.2. Riccati Equation. 1.2.2. Equations Containing Power Functions
Problem number: 54.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_rational, _Riccati]`

$$x^2y' - \left(\alpha\, x^{2n} + \beta\, x^n + \gamma\right) y^2 - \left(x^n a + b\right) xy = c\, x^2$$

✓ Solution by Maple
Time used: 0.031 (sec). Leaf size: 215200

```
dsolve(x^2*diff(y(x),x)=(alpha*x^(2*n)+beta*x^n+gamma)*y(x)^2+(a*x^n+b)*x*y(x)+c*x^2,y(x), s
```

Expression too large to display

✓ Solution by Mathematica
Time used: 4.676 (sec). Leaf size: 2649

```
DSolve[x^2*y'[x]==(\[Alpha]*x^(2*n)+\[Beta]*x^n+\[Gamma])*y[x]^2+(a*x^n+b)*x*y[x]+c*x^2,y[x]
```

Too large to display

2.55 problem 55

Internal problem ID [10385]

Book: Handbook of exact solutions for ordinary differential equations. By Polyanin and Zaitsev. Second edition
Section: Chapter 1, section 1.2. Riccati Equation. 1.2.2. Equations Containing Power Functions
Problem number: 55.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_rational, _Riccati]`

$$\boxed{\left(x^2-1\right)y'+\lambda\left(y^2-2xy+1\right)=0}$$

✓ Solution by Maple
Time used: 0.016 (sec). Leaf size: 280

```
dsolve((x^2-1)*diff(y(x),x)+lambda*(y(x)^2-2*x*y(x)+1)=0,y(x), singsol=all)
```

$$y(x)=$$
$$-\frac{2\left(\left(-\frac{x}{2}-\frac{1}{2}\right)^{-2\lambda}(1+x)(-1+x)^2\,\mathrm{HeunCPrime}\left(0,2\lambda-1,0,0,\lambda^2-\lambda+\frac{1}{2},\frac{2}{1+x}\right)+8(-1+x)^2\,\mathrm{Heun}\right.}{\left(8c_1\,\mathrm{hyp}\right.}$$

✓ Solution by Mathematica
Time used: 0.643 (sec). Leaf size: 47

```
DSolve[(x^2-1)*y'[x]+\[Lambda]*(y[x]^2-2*x*y[x]+1)==0,y[x],x,IncludeSingularSolutions -> Tru
```

$$y(x)\to\frac{\mathrm{LegendreQ}(\lambda,x)+c_1\,\mathrm{LegendreP}(\lambda,x)}{\mathrm{LegendreQ}(\lambda-1,x)+c_1\,\mathrm{LegendreP}(\lambda-1,x)}$$
$$y(x)\to\frac{\mathrm{LegendreP}(\lambda,x)}{\mathrm{LegendreP}(\lambda-1,x)}$$

2.56 problem 56

Internal problem ID [10386]

Book: Handbook of exact solutions for ordinary differential equations. By Polyanin and Zaitsev. Second edition
Section: Chapter 1, section 1.2. Riccati Equation. 1.2.2. Equations Containing Power Functions
Problem number: 56.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_rational, _Riccati]`

$$\boxed{\left(a\,x^2+b\right)y'+\alpha y^2+\beta xy=-\frac{b(a+\beta)}{\alpha}}$$

✓ Solution by Maple
Time used: 0.016 (sec). Leaf size: 517

```
dsolve((a*x^2+b)*diff(y(x),x)+alpha*y(x)^2+beta*x*y(x)+b/alpha*(a+beta)=0,y(x), singsol=all)
```

$$y(x)=$$
$$-\frac{b\,a^2\left(-\frac{\left(-\frac{-ax+\sqrt{-ab}}{2\sqrt{-ab}}\right)^{\frac{\beta}{a}}\left(a\,x^2+b\right)\left(a\,x^2+2\sqrt{-ab}\,x-b\right)\text{HeunCPrime}\left(0,-1-\frac{\beta}{a},1+\frac{\beta}{2a},0,\frac{1}{2}+\frac{\beta}{2a}+\frac{\beta^2}{4a^2},\frac{2\sqrt{-ab}}{-ax+\sqrt{-ab}}\right)}{2}-2c_1b\left((3a\,x^2-\right.}{}$$

✓ Solution by Mathematica
Time used: 1.111 (sec). Leaf size: 27

```
DSolve[(a*x^2+b)*y'[x]+\[Alpha]*y[x]^2+\[Beta]*x*y[x]+b/\[Alpha]*(a+\[Beta])==0,y[x],x,Inclu
```

$$y(x)\to -\frac{x(a+\beta)}{\alpha}$$
$$y(x)\to -\frac{x(a+\beta)}{\alpha}$$

2.57 problem 57

Internal problem ID [10387]

Book: Handbook of exact solutions for ordinary differential equations. By Polyanin and Zaitsev. Second edition
Section: Chapter 1, section 1.2. Riccati Equation. 1.2.2. Equations Containing Power Functions
Problem number: 57.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_rational, _Riccati]`

$$\boxed{\left(a\,x^2+b\right)y'+\alpha y^2+\beta xy=-\gamma}$$

✓ Solution by Maple
Time used: 0.016 (sec). Leaf size: 858

```
dsolve((a*x^2+b)*diff(y(x),x)+alpha*y(x)^2+beta*x*y(x)+gamma=0,y(x), singsol=all)
```

$y(x)=$

$$-\frac{}{2\left(-ab\left(-\sqrt{-ab}\,x+b\right)\left(a\,x^2+b\right)\text{HeunCPrime}\left(0,\frac{-a+\beta}{a},-\frac{\sqrt{4\gamma\alpha ab+\beta^2b^2}}{2ab},0,\frac{2a^2-2\beta a+\beta^2}{4a^2},-\frac{2\sqrt{-ab}}{ax-\sqrt{-ab}}\right)+\right.}$$

✓ Solution by Mathematica

Time used: 1.098 (sec). Leaf size: 598

```
DSolve[(a*x^2+b)*y'[x]+\[Alpha]*y[x]^2+\[Beta]*x*y[x]+\[Gamma]==0,y[x],x,IncludeSingularSolu
```

$$y(x)$$

$$\to \frac{i\left(c_1\left(\sqrt{4a\alpha\gamma+b\beta^2}-2a\sqrt{b}-\sqrt{b}\beta\right)P_{\frac{\beta}{2a}+1}^{\frac{\sqrt{b\beta^2+4a\alpha\gamma}}{2a\sqrt{b}}}\left(\frac{i\sqrt{a}x}{\sqrt{b}}\right)+2i\sqrt{a}x(a+\beta)Q_{\frac{\beta}{2a}}^{\frac{\sqrt{b\beta^2+4a\alpha\gamma}}{2a\sqrt{b}}}\left(\frac{i\sqrt{a}x}{\sqrt{b}}\right)+\left(\sqrt{4a\alpha\gamma+}\right.\right.}{2\sqrt{a}\alpha\left(c_1P_{\frac{\beta}{2a}}^{\frac{\sqrt{b\beta^2+4a\alpha\gamma}}{2a\sqrt{b}}}\left(\frac{i\sqrt{a}x}{\sqrt{b}}\right)+Q_{\frac{\beta}{2a}}^{\frac{\sqrt{b\beta^2+}}{2a}}\right.}$$

$$y(x)\to\frac{-2x(a+\beta)+\frac{i\left(\sqrt{4a\alpha\gamma+b\beta^2}-2a\sqrt{b}-\sqrt{b}\beta\right)P_{\frac{\beta}{2a}+1}^{\frac{\sqrt{b\beta^2+4a\alpha\gamma}}{2a\sqrt{b}}}\left(\frac{i\sqrt{a}x}{\sqrt{b}}\right)}{\sqrt{a}P_{\frac{\beta}{2a}}^{\frac{\sqrt{b\beta^2+4a\alpha\gamma}}{2a\sqrt{b}}}\left(\frac{i\sqrt{a}x}{\sqrt{b}}\right)}}{2\alpha}$$

2.58 problem 58

Internal problem ID [10388]

Book: Handbook of exact solutions for ordinary differential equations. By Polyanin and Zaitsev. Second edition
Section: Chapter 1, section 1.2. Riccati Equation. 1.2.2. Equations Containing Power Functions
Problem number: 58.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_rational, [_1st_order, '_with_symmetry_[F(x),G(x)]'], _Ricca`

$$\boxed{\left(a\,x^2+b\right)y'+y^2-2xy=-(1-a)\,x^2+b}$$

✓ Solution by Maple
Time used: 0.0 (sec). Leaf size: 31

```
dsolve((a*x^2+b)*diff(y(x),x)+y(x)^2-2*x*y(x)+(1-a)*x^2-b=0,y(x), singsol=all)
```

$$y(x)=x+\frac{\sqrt{ab}}{c_1\sqrt{ab}+\arctan\left(\frac{ax}{\sqrt{ab}}\right)}$$

✓ Solution by Mathematica
Time used: 0.562 (sec). Leaf size: 41

```
DSolve[(a*x^2+b)*y'[x]+y[x]^2-2*x*y[x]+(1-a)*x^2-b==0,y[x],x,IncludeSingularSolutions -> Tru
```

$$y(x)\to x+\frac{1}{\frac{\arctan\left(\frac{\sqrt{a}x}{\sqrt{b}}\right)}{\sqrt{a}\sqrt{b}}+c_1}$$
$$y(x)\to x$$

2.59 problem 59

Internal problem ID [10389]

Book: Handbook of exact solutions for ordinary differential equations. By Polyanin and Zaitsev. Second edition
Section: Chapter 1, section 1.2. Riccati Equation. 1.2.2. Equations Containing Power Functions
Problem number: 59.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_rational, [_1st_order, '_with_symmetry_[F(x),G(x)]'], _Ricca`

$$\boxed{(a\,x^2+bx+c)\,y'-y^2-(2\lambda x+b)\,y=\lambda(\lambda-a)\,x^2+\mu}$$

✓ Solution by Maple
Time used: 0.0 (sec). Leaf size: 542

```
dsolve((a*x^2+b*x+c)*diff(y(x),x)=y(x)^2+(2*lambda*x+b)*y(x)+lambda*(lambda-a)*x^2+mu,y(x),
```

$$y(x)=-\frac{8\left(\left(ia\sqrt{4ac-b^2}\,\sqrt{\frac{b^2-4c\lambda-4\mu}{a^2}}-\sqrt{-4ac+b^2}\,(2x\lambda+b)\right)c_1\left(\frac{-b+i\sqrt{4ac-b^2}-2ax}{i\sqrt{4ac-b^2}+2ax+b}\right)^{-\frac{a\sqrt{\frac{b^2-4c\lambda-4\mu}{a^2}}}{2\sqrt{-4ac+b^2}}}-\left(ia\sqrt{4ac}\right.\right.}{\sqrt{-4ac+b^2}\,\left(2ax-\sqrt{-4ac+b^2}+b\right)\left(2ax+\sqrt{-4ac+b^2}+b\right)\left(i\sqrt{4ac-b^2}+2ax+b\right)(-b+}$$

✓ Solution by Mathematica
Time used: 17.168 (sec). Leaf size: 93

```
DSolve[(a*x^2+b*x+c)*y'[x]==y[x]^2+(2*\[Lambda]*x+b)*y[x]+\[Lambda]*(\[Lambda]-a)*x^2+\[Mu],
```

$$y(x)\to\frac{1}{2}\left(\sqrt{4(c\lambda+\mu)-b^2}\tan\left(\frac{\sqrt{-b^2+4c\lambda+4\mu}\arctan\left(\frac{2ax+b}{\sqrt{4ac-b^2}}\right)}{\sqrt{4ac-b^2}}+c_1\right)-b-2\lambda x\right)$$

2.60 problem 60

Internal problem ID [10390]

Book: Handbook of exact solutions for ordinary differential equations. By Polyanin and Zaitsev. Second edition
Section: Chapter 1, section 1.2. Riccati Equation. 1.2.2. Equations Containing Power Functions
Problem number: 60.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_rational, _Riccati]`

$$\left(a\,x^2+bx+c\right)y'-y^2-(ax+\mu)\,y=\lambda c+\lambda(b-\mu)\,x-\lambda^2x^2$$

✓ Solution by Maple
Time used: 0.016 (sec). Leaf size: 6204

```
dsolve((a*x^2+b*x+c)*diff(y(x),x)=y(x)^2+(a*x+mu)*y(x)-lambda^2*x^2+lambda*(b-mu)*x+lambda*c
```

Expression too large to display

✓ Solution by Mathematica
Time used: 23.352 (sec). Leaf size: 433

```
DSolve[(a*x^2+b*x+c)*y'[x]==y[x]^2+(a*x+\[Mu])*y[x]-\[Lambda]^2*x^2+\[Lambda]*(b-\[Mu])*x+\[
```

$y(x)$

$$\to \frac{(x(ax+b)+c)^{\frac{\lambda}{a}-\frac{1}{2}}\exp\left(-\frac{(a(b-2\mu)+2b\lambda)\arctan\left(\frac{2ax+b}{\sqrt{4ac-b^2}}\right)}{a\sqrt{4ac-b^2}}\right)\left(\lambda x(x(ax+b)+c)^{\frac{1}{2}-\frac{\lambda}{a}}\exp\left(\frac{(a(b-2\mu)+2b\lambda)\arctan}{a\sqrt{4ac-b^2}}\right.\right.}{\int_1^x \exp}$$

$y(x)\to \lambda x$

2.61 problem 61

Internal problem ID [10391]

Book: Handbook of exact solutions for ordinary differential equations. By Polyanin and Zaitsev. Second edition
Section: Chapter 1, section 1.2. Riccati Equation. 1.2.2. Equations Containing Power Functions
Problem number: 61.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_rational, _Riccati]`

$$\left(a_2x^2+b_2x+c_2\right)y'-y^2-\left(a_1x+b_1\right)y=\lambda c_2+\lambda(b_2-b_1)\,x-\lambda(\lambda+a_1-a_2)\,x^2$$

✓ Solution by Maple
Time used: 0.031 (sec). Leaf size: 5771

```
dsolve((a__2*x^2+b__2*x+c__2)*diff(y(x),x)=y(x)^2+(a__1*x+b__1)*y(x)-lambda*(lambda+a__1-a__
```

Expression too large to display

✓ Solution by Mathematica

Time used: 34.34 (sec). Leaf size: 284

```
DSolve[(a2*x^2+b2*x+c2)*y'[x]==y[x]^2+(a1*x+b1)*y[x]-\[Lambda]*(\[Lambda]+a1-a2)*x^2+\[Lambd
```

$$y(x)$$

$$\to \frac{\lambda x \int_1^x \exp\left(\frac{(a1-2a2+2\lambda)\log(c2+K[1](b2+a2K[1]))-\frac{2(b2(a1+2\lambda)-2a2b1)\arctan\left(\frac{b2+2a2K[1]}{\sqrt{4a2c2-b2^2}}\right)}{\sqrt{4a2c2-b2^2}}}{2a2}\right) dK[1] + (x(a2x -}{\int_1^x \exp\left(\frac{(a1-2a2+2\lambda)\log(c2+K[1](b2+a2K[1]))-\frac{2(b2(a1+2\lambda)-2a2b}{\sqrt{4a}}}{2a2}\right)}$$

$$y(x) \to \lambda x$$

2.62 problem 62

Internal problem ID [10392]

Book: Handbook of exact solutions for ordinary differential equations. By Polyanin and Zaitsev. Second edition
Section: Chapter 1, section 1.2. Riccati Equation. 1.2.2. Equations Containing Power Functions
Problem number: 62.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_rational, _Riccati]`

$$\boxed{\left(a_2x^2+b_2x+c_2\right)y'-y^2-\left(a_1x+b_1\right)y=a_0x^2+b_0x+c_0}$$

✓ Solution by Maple
Time used: 0.015 (sec). Leaf size: 6462

```
dsolve((a__2*x^2+b__2*x+c__2)*diff(y(x),x)=y(x)^2+(a__1*x+b__1)*y(x)+a__0*x^2+b__0*x+c__0,y(
```

Expression too large to display

✗ Solution by Mathematica
Time used: 0.0 (sec). Leaf size: 0

```
DSolve[(a2*x^2+b2*x+c2)*y'[x]==y[x]^2+(a1*x+b1)*y[x]+a0*x^2+b0*x+c0,y[x],x,IncludeSingularSo
```

Timed out

2.63 problem 63

Internal problem ID [10393]

Book: Handbook of exact solutions for ordinary differential equations. By Polyanin and Zaitsev. Second edition
Section: Chapter 1, section 1.2. Riccati Equation. 1.2.2. Equations Containing Power Functions
Problem number: 63.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_rational, [_1st_order, '_with_symmetry_[F(x),G(x)]'], _Ricca`

$$\boxed{(x-a)(x-b)y' + y^2 + k(y+x-a)(y+x-b) = 0}$$

✓ Solution by Maple
Time used: 0.0 (sec). Leaf size: 54

```
dsolve((x-a)*(x-b)*diff(y(x),x)+y(x)^2+k*(y(x)+x-a)*(y(x)+x-b)=0,y(x), singsol=all)
```

$$y(x) = \frac{k\left((b-x)^{1+k} + c_1(a-x)^k(a-x)\right)}{(1+k)\left(c_1(a-x)^k + (b-x)^k\right)}$$

✓ Solution by Mathematica
Time used: 60.572 (sec). Leaf size: 99

```
DSolve[(x-a)*(x-b)*y'[x]+y[x]^2+k*(y[x]+x-a)*(y[x]+x-b)==0,y[x],x,IncludeSingularSolutions -
```

$$y(x) \to \frac{1}{2}\left(\frac{k(a+b-2x)}{k+1} + \sqrt{-\frac{k^2(a-b)^2}{(k+1)^2}}\tan\left(\frac{(k+1)\sqrt{-\frac{k^2(a-b)^2}{(k+1)^2}}(\log(x-b)-\log(x-a))}{2(a-b)} + c_1\right)\right)$$

2.64 problem 64

Internal problem ID [10394]

Book: Handbook of exact solutions for ordinary differential equations. By Polyanin and Zaitsev. Second edition
Section: Chapter 1, section 1.2. Riccati Equation. 1.2.2. Equations Containing Power Functions
Problem number: 64.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_rational, _Riccati]`

$$\boxed{\left(c_2x^2+b_2x+a_2\right)\left(y'+\lambda y^2\right)+\left(b_1x+a_1\right)y=-a_0}$$

✓ Solution by Maple
Time used: 0.016 (sec). Leaf size: 2160

```
dsolve((c__2*x^2+b__2*x+a__2)*(diff(y(x),x)+lambda*y(x)^2)+(b__1*x+a__1)*y(x)+a__0=0,y(x), s
```

Expression too large to display

✓ Solution by Mathematica
Time used: 14.836 (sec). Leaf size: 1986

```
DSolve[(c2*x^2+b2*x+a2)*(y'[x]+\[Lambda]*y[x]^2)+(b1*x+a1)*y[x]+a0==0,y[x],x,IncludeSingular
```

Too large to display

2.65 problem 65

Internal problem ID [10395]

Book: Handbook of exact solutions for ordinary differential equations. By Polyanin and Zaitsev. Second edition
Section: Chapter 1, section 1.2. Riccati Equation. 1.2.2. Equations Containing Power Functions
Problem number: 65.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_rational, _Riccati]`

$$\boxed{x^3 y' - a\,x^3 y^2 - \left(b\,x^2 + c\right) y = s x}$$

✓ Solution by Maple
Time used: 0.0 (sec). Leaf size: 435

```
dsolve(x^3*diff(y(x),x)=a*x^3*y(x)^2+(b*x^2+c)*y(x)+s*x,y(x), singsol=all)
```

$$y(x) = -\frac{\left(\frac{\left(1-\sqrt{-4as+b^2+2b+1}+b\right)\text{KummerM}\left(\frac{\sqrt{-4as+b^2+2b+1}}{4}\right.}{4}\right.}{2a\left(\frac{((b-1)x^2+c)\left(1-\sqrt{-4as+b^2+2b+1}+b\right)\text{KummerM}\left(\frac{\sqrt{-4as+b^2+2b+1}}{4}+\frac{b}{4}+\frac{1}{4},1+\frac{\sqrt{-4as+b^2+2b+1}}{2},\frac{c}{2x^2}\right)}{2}+\left(\frac{1}{2}+\frac{(b-1)\sqrt{-4as+b}}{2}\right.\right.}$$

✓ Solution by Mathematica

Time used: 3.199 (sec). Leaf size: 907

```
DSolve[x^3*y'[x]==a*x^3*y[x]^2+(b*x^2+c)*y[x]+s*x,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to$$

$$-\frac{-\left(\left(\sqrt{-4as+b^2+2b+1}-b-1\right) c^{\frac{1}{2}\sqrt{-4as+b^2+2b+1}} \left(\frac{1}{x}\right)^{\sqrt{-4as+b^2+2b+1}} \text{Hypergeometric1F1}\left(\frac{1}{4}\left(-b+\sqrt{b\right.}\right.\right.}{}$$

$$y(x)$$

$$\to \frac{c\left(b\left(\sqrt{-4as+b^2+2b+1}+4\right)+3\sqrt{-4as+b^2+2b+1}-4as+b^2+3\right) \text{Hypergeometric1F1}\left(\frac{1}{4}\left(-b-\sqrt{b^2+2b-4as+1}+3\right),2-\frac{1}{2}\sqrt{b^2+2b-4as+1},-\frac{c}{2x^2}\right.}{\text{Hypergeometric1F1}\left(\frac{1}{4}\left(-b-\sqrt{b^2+2b-4as+1}-1\right),1-\frac{1}{2}\sqrt{b^2+2b-4as+1},-\frac{c}{2x^2}\right)}$$

$$2ax^3\left(4as-b^2-2b+3\right)$$

2.66 problem 66

Internal problem ID [10396]

Book: Handbook of exact solutions for ordinary differential equations. By Polyanin and Zaitsev. Second edition
Section: Chapter 1, section 1.2. Riccati Equation. 1.2.2. Equations Containing Power Functions
Problem number: 66.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_rational, _Riccati]`

$$\boxed{x^3 y' - a\,x^3 y^2 - x(bx+c)\,y = \alpha x + \beta}$$

✓ Solution by Maple
Time used: 0.0 (sec). Leaf size: 438

```
dsolve(x^3*diff(y(x),x)=a*x^3*y(x)^2+x*(b*x+c)*y(x)+alpha*x+beta,y(x), singsol=all)
```

$$y(x) = \frac{\left((a\alpha+b+2)\,c^2 - a\beta(b+3)\,c + a^2\beta^2\right) x c_1\,\mathrm{KummerU}\left(\frac{\sqrt{-4a\alpha+b^2+2b+1}\,c+(b+5)c-2\beta a}{2c}, 1+\sqrt{-4a\alpha+b^2+2b}\right.}{}$$

✓ Solution by Mathematica

Time used: 2.395 (sec). Leaf size: 908

```
DSolve[x^3*y'[x]==a*x^3*y[x]^2+x*(b*x+c)*y[x]+\[Alpha]*x+\[Beta],y[x],x,IncludeSingularSolut
```

$$y(x) \to$$

$$-\frac{c^{\sqrt{-4a\alpha+b^2+2b+1}}\left(\frac{1}{x}\right)^{\sqrt{-4a\alpha+b^2+2b+1}+1}\left(c\left(\sqrt{-4a\alpha+b^2+2b+1}-b-1\right)+2a\beta\right)\text{Hypergeometric1F1}\left(\frac{1}{2}\left(-b+\frac{2a\beta}{c}+\sqrt{b^2+2b-4a\alpha+1}+1\right),\sqrt{b^2}\right.}{\sqrt{-4a\alpha+b^2+2b+1}+1}$$

$$y(x)$$

$$\to \frac{\left(c\left(b\left(\sqrt{-4a\alpha+b^2+2b+1}+3\right)+2\left(-2a\alpha+\sqrt{-4a\alpha+b^2+2b+1}+1\right)+b^2\right)-2a\beta\left(\sqrt{-4a\alpha+b^2+2b+1}+1\right)\right)\text{Hypergeometric1F1}\left(\frac{2a\beta-c\left(b+\sqrt{b^2+2b}\right.}{2c}\right.}{\text{Hypergeometric1F1}\left(\frac{2a\beta-c\left(b+\sqrt{b^2+2b-4a\alpha+1}+1\right)}{2c},1-\sqrt{b^2+2b-4a\alpha+1},-\frac{c}{x}\right)}$$

$$2ax^2\left(4a\alpha-b^2-2b\right)$$

2.67 problem 67

Internal problem ID [10397]

Book: Handbook of exact solutions for ordinary differential equations. By Polyanin and Zaitsev. Second edition
Section: Chapter 1, section 1.2. Riccati Equation. 1.2.2. Equations Containing Power Functions
Problem number: 67.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_rational, _Riccati]`

$$\boxed{x\left(x^2+a\right)\left(y'+\lambda y^2\right)+\left(b\,x^2+c\right)y=-sx}$$

✓ Solution by Maple
Time used: 0.0 (sec). Leaf size: 613

```
dsolve(x*(x^2+a)*(diff(y(x),x)+lambda*y(x)^2)+(b*x^2+c)*y(x)+s*x=0,y(x), singsol=all)
```

$$y(x)$$

$$= \frac{\left(a-\frac{c}{3}\right)\left((-\lambda s+b-2)\,a^2+c(b-3)\,a-c^2\right)\left(a\,x^{\frac{a+c}{a}}+x^{\frac{3a+c}{a}}\right)c_1\,\text{hypergeom}\left(\left[-\frac{\sqrt{b^2-4\lambda s-2b+1}\,a+ab-7a-2c}{4a}\right.\right.}{}$$

✓ Solution by Mathematica

Time used: 2.874 (sec). Leaf size: 862

```
DSolve[x*(x^2+a)*(y'[x]+\[Lambda]*y[x]^2)+(b*x^2+c)*y[x]+s*x==0,y[x],x,IncludeSingularSoluti
```

$y(x)$

$$\to \frac{a^{\frac{1}{2}\left(\frac{c}{a}-3\right)}(a-c)x^{-\frac{c}{a}}\,\text{Hypergeometric2F1}\left(\frac{ba-\sqrt{b^2-2b-4s\lambda+1}a+a-2c}{4a},\frac{a\left(b+\sqrt{b^2-2b-4s\lambda+1}+1\right)-2c}{4a},\frac{3}{2}-\frac{c}{2a},-\frac{x^2}{a}\right)+}{\lambda a^{\frac{1}{2}\left(\frac{c}{a}-1\right)}x^{1-\frac{c}{a}}\,\text{Hypergeometric2F1}\left(\frac{ba-\sqrt{b^2-2b-4s\lambda-}}{4a}\right.}$$

$y(x) \to$

$$-\frac{sx\,\text{Hypergeometric2F1}\left(\frac{1}{4}\left(b-\sqrt{b^2-2b-4s\lambda+1}+3\right),\frac{1}{4}\left(b+\sqrt{b^2-2b-4s\lambda+1}+3\right),\frac{1}{2}\left(\frac{c}{a}+3\right),\right.}{(a+c)\,\text{Hypergeometric2F1}\left(\frac{1}{4}\left(b-\sqrt{b^2-2b-4s\lambda+1}-1\right),\frac{1}{4}\left(b+\sqrt{b^2-2b-4s\lambda+1}-1\right),\frac{a+c}{2a},\right.}$$

$y(x) \to$

$$-\frac{sx\,\text{Hypergeometric2F1}\left(\frac{1}{4}\left(b-\sqrt{b^2-2b-4s\lambda+1}+3\right),\frac{1}{4}\left(b+\sqrt{b^2-2b-4s\lambda+1}+3\right),\frac{1}{2}\left(\frac{c}{a}+3\right),\right.}{(a+c)\,\text{Hypergeometric2F1}\left(\frac{1}{4}\left(b-\sqrt{b^2-2b-4s\lambda+1}-1\right),\frac{1}{4}\left(b+\sqrt{b^2-2b-4s\lambda+1}-1\right),\frac{a+c}{2a},\right.}$$

2.68 problem 68

Internal problem ID [10398]

Book: Handbook of exact solutions for ordinary differential equations. By Polyanin and Zaitsev. Second edition
Section: Chapter 1, section 1.2. Riccati Equation. 1.2.2. Equations Containing Power Functions
Problem number: 68.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_rational, _Riccati]`

$$x^2(x+a)\left(y'+\lambda y^2\right)+x(bx+c)\,y=-\alpha x-\beta$$

✓ Solution by Maple
Time used: 0.0 (sec). Leaf size: 1508

```
dsolve(x^2*(x+a)*(diff(y(x),x)+lambda*y(x)^2)+x*(b*x+c)*y(x)+alpha*x+beta=0,y(x), singsol=al
```

Expression too large to display

✓ Solution by Mathematica

Time used: 5.239 (sec). Leaf size: 1770

```
DSolve[x^2*(x+a)*(y'[x]+\[Lambda]*y[x]^2)+x*(b*x+c)*y[x]+\[Alpha]*x+\[Beta]==0,y[x],x,Includ
```

$y(x)$

$$\rightarrow \frac{2a\left(a-c+\sqrt{a^2-2(c+2\beta\lambda)a+c^2}\right)\text{Hypergeometric2F1}\left(\frac{-c+a\left(b-\sqrt{b^2-2b-4\alpha\lambda+1}\right)+\sqrt{a^2-2(c+2\beta\lambda)a+c^2}}{2a},\frac{-c-}{}\right.}{}$$

$y(x)$

$$\rightarrow \frac{a\left(c^2-2a(2\beta\lambda+c)\right)\left(\sqrt{a^2-2a(2\beta\lambda+c)+c^2}-a+c\right)}{x}-\frac{\left(2\alpha a^3\lambda+a^2\left(2a\lambda\sqrt{a^2-2a(2\beta\lambda+c)+c^2}+4b\beta\lambda+bc-2\beta\lambda\right)-a\left(bc\sqrt{a^2-2a(2\beta\lambda+c)+c^2}+2\right.\right.}{\text{Hy}}$$

2.69 problem 69

Internal problem ID [10399]

Book: Handbook of exact solutions for ordinary differential equations. By Polyanin and Zaitsev. Second edition
Section: Chapter 1, section 1.2. Riccati Equation. 1.2.2. Equations Containing Power Functions
Problem number: 69.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[[_homogeneous, 'class D'], _rational, _Riccati]`

$$\boxed{\left(a\,x^2+bx+e\right)\left(y'x-y\right)-y^2=-x^2}$$

✓ Solution by Maple
Time used: 0.0 (sec). Leaf size: 58

```
dsolve((a*x^2+b*x+e)*(x*diff(y(x),x)-y(x))-y(x)^2+x^2=0,y(x), singsol=all)
```

$$y(x)=-\tanh\left(\frac{c_1\sqrt{4ea-b^2}+2\arctan\left(\frac{2ax+b}{\sqrt{4ea-b^2}}\right)}{\sqrt{4ea-b^2}}\right)x$$

✓ Solution by Mathematica
Time used: 1.973 (sec). Leaf size: 116

```
DSolve[(a*x^2+b*x+e)*(x*y'[x]-y[x])-y[x]^2+x^2==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x)\to -\frac{x\left(-1+\exp\left(\frac{4\arctan\left(\frac{2ax+b}{\sqrt{4ae-b^2}}\right)}{\sqrt{4ae-b^2}}+2c_1\right)\right)}{1+\exp\left(\frac{4\arctan\left(\frac{2ax+b}{\sqrt{4ae-b^2}}\right)}{\sqrt{4ae-b^2}}+2c_1\right)}$$
$$y(x)\to -x$$
$$y(x)\to x$$

2.70 problem 70

Internal problem ID [10400]

Book: Handbook of exact solutions for ordinary differential equations. By Polyanin and Zaitsev. Second edition
Section: Chapter 1, section 1.2. Riccati Equation. 1.2.2. Equations Containing Power Functions
Problem number: 70.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_rational, _Riccati]`

$$\boxed{x^2\left(x^2+a\right)\left(y'+\lambda y^2\right)+x\left(b\,x^2+c\right)y=-s}$$

✓ Solution by Maple
Time used: 0.0 (sec). Leaf size: 1329

```
dsolve(x^2*(x^2+a)*(diff(y(x),x)+lambda*y(x)^2)+x*(b*x^2+c)*y(x)+s=0,y(x), singsol=all)
```

Expression too large to display

✓ Solution by Mathematica

Time used: 7.158 (sec). Leaf size: 1470

```
DSolve[x^2*(x^2+a)*(y'[x]+\[Lambda]*y[x]^2)+x*(b*x^2+c)*y[x]+s==0,y[x],x,IncludeSingularSolu
```

$$y(x)$$

$$\to \frac{\left(a-c-\sqrt{a^2-2(c+2s\lambda)a+c^2}\right)c_1\left(\left(-2ba+a+c+\sqrt{a^2-2(c+2s\lambda)a+c^2}\right)\text{Hypergeometric2F1}\left(-\frac{-5a+c+\sqrt{a^2-2(c+2s\lambda)a+c^2}}{4a},-\frac{-a(2b+3)+c+\sqrt{a}\right.\right.}{4}$$

$$y(x)$$

$$\to \frac{x\left(a^3(-b)+a^2\left(b\sqrt{a^2-2a(c+2\lambda s)+c^2}-4(b-1)\lambda s+c\right)+a\left(bc\left(\sqrt{a^2-2a(c+2\lambda s)+c^2}+c\right)-c\right.\right.}{2a^2\lambda\left(3a^2+2a\right.}$$

$$-\frac{\sqrt{a^2-2a(c+2\lambda s)+c^2}-a+c}{2a\lambda x}$$

2.71 problem 71

Internal problem ID [10401]

Book: Handbook of exact solutions for ordinary differential equations. By Polyanin and Zaitsev. Second edition
Section: Chapter 1, section 1.2. Riccati Equation. 1.2.2. Equations Containing Power Functions
Problem number: 71.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_rational, _Riccati]`

$$\boxed{a\left(x^2-1\right)\left(y'+\lambda y^2\right)+bx\left(x^2-1\right)y=-c\,x^2-dx-s}$$

✗ Solution by Maple

```
dsolve(a*(x^2-1)*(diff(y(x),x)+lambda*y(x)^2)+b*x*(x^2-1)*y(x)+c*x^2+d*x+s=0,y(x), singsol=a
```

No solution found

✗ Solution by Mathematica
Time used: 0.0 (sec). Leaf size: 0

```
DSolve[a*(x^2-1)*(y'[x]+\[Lambda]*y[x]^2)+b*x*(x^2-1)*y[x]+c*x^2+d*x+s==0,y[x],x,IncludeSing
```

Not solved

2.72 problem 72

Internal problem ID [10402]

Book: Handbook of exact solutions for ordinary differential equations. By Polyanin and Zaitsev. Second edition
Section: Chapter 1, section 1.2. Riccati Equation. 1.2.2. Equations Containing Power Functions
Problem number: 72.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_Riccati]`

$$\boxed{x^{n+1}y' - x^{2n}y^2a - yx^nb = cx^m + d}$$

✓ Solution by Maple
Time used: 0.0 (sec). Leaf size: 284

```
dsolve(x^(n+1)*diff(y(x),x)=a*x^(2*n)*y(x)^2+b*x^n*y(x)+c*x^m+d,y(x), singsol=all)
```

$$y(x) = \frac{\left(\sqrt{ac}\left(\text{BesselY}\left(\frac{\sqrt{-4ad+b^2+2bn+n^2}}{m}+1, \frac{2\sqrt{ac}\,x^{\frac{m}{2}}}{m}\right)c_1 + \text{BesselJ}\left(\frac{\sqrt{-4ad+b^2+2bn+n^2}}{m}+1, \frac{2\sqrt{ac}\,x^{\frac{m}{2}}}{m}\right)\right)x^{\frac{m}{2}} - \frac{\left(\text{B}\right.}{}}{a\left(\text{BesselY}\left(\frac{\sqrt{-4ad+b^2+2bn+n^2}}{m}, \frac{2\sqrt{ac}\,x^{\frac{m}{2}}}{m}\right)c_1 + \text{Be}\right.}$$

✓ Solution by Mathematica
Time used: 3.153 (sec). Leaf size: 2576

```
DSolve[x^(n+1)*y'[x]==a*x^(2*n)*y[x]^2+b*x^n*y[x]+c*x^m+d,y[x],x,IncludeSingularSolutions ->
```

Too large to display

2.73 problem 73

Internal problem ID [10403]

Book: Handbook of exact solutions for ordinary differential equations. By Polyanin and Zaitsev. Second edition
Section: Chapter 1, section 1.2. Riccati Equation. 1.2.2. Equations Containing Power Functions
Problem number: 73.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_rational, _Riccati]`

$$\boxed{x\left(a\,x^{k}+b\right)y'-\alpha\,x^{n}y^{2}-\left(\beta-an\,x^{k}\right)y=\gamma\,x^{-n}}$$

✓ Solution by Maple
Time used: 0.032 (sec). Leaf size: 138

```
dsolve(x*(a*x^k+b)*diff(y(x),x)=alpha*x^n*y(x)^2+(beta-a*n*x^k)*y(x)+gamma*x^(-n),y(x), sing
```

$$y(x)=-\frac{x^{-n}\left(\tanh\left(\frac{((-bn-\beta)\ln(a\,x^{k}+b)+((bn+\beta)\ln(x)+c_1 b)k)\sqrt{(bn+\beta)^2(n^2b^2+2b\beta n-4\gamma\alpha+\beta^2)}}{2kb(bn+\beta)^2}\right)\sqrt{(bn+\beta)^2\left(n^2b^2+2b\beta n-\right.}}{2\alpha\,(bn+\beta)}$$

✓ Solution by Mathematica

Time used: 4.641 (sec). Leaf size: 663

```
DSolve[x*(a*x^k+b)*y'[x]==\[Alpha]*x^n*y[x]^2+(\[Beta]-a*n*x^k)*y[x]+\[Gamma]*x^(-n),y[x],x,
```

$$y(x)$$

$$\to \frac{x^{-n}\left(b\left(n\left(-\exp\left(-\frac{\left(\log\left(ax^k+b\right)+\log(b)-k\log(x)+\log(k)\right)\left(\sqrt{\alpha}\sqrt{\gamma}\sqrt{\frac{-4\alpha\gamma+b^2n^2+\beta^2+2b\beta n}{\alpha\gamma}}+bn+\beta\right)}{2bk}\right)\right)\right)-c_1 n\exp\left(-\right.\right.}{}$$

$$y(x)\to \frac{x^{-n}\left(\sqrt{\alpha}\sqrt{\gamma}\sqrt{\frac{(bn+\beta)^2}{\alpha\gamma}-4}-bn-\beta\right)}{2\alpha}$$

2.74 problem 74

Internal problem ID [10404]

Book: Handbook of exact solutions for ordinary differential equations. By Polyanin and Zaitsev. Second edition
Section: Chapter 1, section 1.2. Riccati Equation. 1.2.2. Equations Containing Power Functions
Problem number: 74.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_rational, _Riccati]`

$$x^2(x^n a-1)\left(y'+\lambda y^2\right)+\left(p\,x^n+q\right)xy=-r\,x^n-s$$

✓ Solution by Maple
Time used: 0.0 (sec). Leaf size: 1222

```
dsolve(x^2*(a*x^n-1)*(diff(y(x),x)+lambda*y(x)^2)+(p*x^n+q)*x*y(x)+r*x^n+s=0,y(x), singsol=a
```

Expression too large to display

✓ Solution by Mathematica
Time used: 7.968 (sec). Leaf size: 2419

```
DSolve[x^2*(a*x^n-1)*(y'[x]+\[Lambda]*y[x]^2)+(p*x^n+q)*x*y[x]+r*x^n+s==0,y[x],x,IncludeSing
```

Too large to display

2.75 problem 75

Internal problem ID [10405]

Book: Handbook of exact solutions for ordinary differential equations. By Polyanin and Zaitsev. Second edition
Section: Chapter 1, section 1.2. Riccati Equation. 1.2.2. Equations Containing Power Functions
Problem number: 75.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_rational, _Riccati]`

$$\boxed{(x^n a + b\,x^m + c)\,y' - cy^2 + b\,x^{m-1}y = a\,x^{-2+n}}$$

✓ Solution by Maple
Time used: 0.0 (sec). Leaf size: 1236

```
dsolve((a*x^n+b*x^m+c)*diff(y(x),x)=c*y(x)^2-b*x^(m-1)*y(x)+a*x^(n-2),y(x), singsol=all)
```

Expression too large to display

✗ Solution by Mathematica
Time used: 0.0 (sec). Leaf size: 0

```
DSolve[(a*x^n+b*x^m+c)*y'[x]==c*y[x]^2-b*x^(m-1)*y[x]+a*x^(n-2),y[x],x,IncludeSingularSoluti
```

Not solved

2.76 problem 76

Internal problem ID [10406]

Book: Handbook of exact solutions for ordinary differential equations. By Polyanin and Zaitsev. Second edition
Section: Chapter 1, section 1.2. Riccati Equation. 1.2.2. Equations Containing Power Functions
Problem number: 76.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_rational, _Riccati]`

$$\boxed{(x^n a + b\,x^m + c)\,y' - a\,x^{-2+n} y^2 - b\,x^{m-1} y = c}$$

✗ Solution by Maple

```
dsolve((a*x^n+b*x^m+c)*diff(y(x),x)=a*x^(n-2)*y(x)^2+b*x^(m-1)*y(x)+c,y(x), singsol=all)
```

No solution found

✗ Solution by Mathematica
Time used: 0.0 (sec). Leaf size: 0

```
DSolve[(a*x^n+b*x^m+c)*y'[x]==a*x^(n-2)*y[x]^2+b*x^(m-1)*y[x]+c,y[x],x,IncludeSingularSoluti
```

Not solved

2.77 problem 77

Internal problem ID [10407]

Book: Handbook of exact solutions for ordinary differential equations. By Polyanin and Zaitsev. Second edition
Section: Chapter 1, section 1.2. Riccati Equation. 1.2.2. Equations Containing Power Functions
Problem number: 77.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_rational, _Riccati]`

$$\boxed{(x^n a + b\,x^m + c)\,y' - \alpha\,x^k y^2 - \beta\,x^s y = -\alpha\,\lambda^2 x^k + \beta\lambda\,x^s}$$

✓ Solution by Maple
Time used: 0.015 (sec). Leaf size: 164

```
dsolve((a*x^n+b*x^m+c)*diff(y(x),x)=alpha*x^k*y(x)^2+beta*x^s*y(x)-alpha*lambda^2*x^k+beta*l
```

$$y(x) = \frac{-\alpha\left(\int \frac{x^k \mathrm{e}^{-\left(\int \frac{2\alpha\,x^k\lambda - x^s\beta}{a\,x^n+b\,x^m+c}dx\right)}}{a\,x^n+b\,x^m+c}dx\right)\lambda - \lambda c_1 - \mathrm{e}^{-\left(\int \frac{2\alpha\,x^k\lambda - x^s\beta}{a\,x^n+b\,x^m+c}dx\right)}}{c_1 + \alpha\left(\int \frac{x^k \mathrm{e}^{-\left(\int \frac{2\alpha\,x^k\lambda - x^s\beta}{a\,x^n+b\,x^m+c}dx\right)}}{a\,x^n+b\,x^m+c}dx\right)}$$

✓ Solution by Mathematica

Time used: 13.649 (sec). Leaf size: 389

```
DSolve[(a*x^n+b*x^m+c)*y'[x]==\[Alpha]*x^k*y[x]^2+\[Beta]*x^s*y[x]-\[Alpha]*\[Lambda]^2*x^k+
```

$$\text{Solve}\left[\int_1^x \frac{\exp\left(-\int_1^{K[2]} -\frac{\beta K[1]^s-2\alpha\lambda K[1]^k}{bK[1]^m+aK[1]^n+c}dK[1]\right)\left(-\alpha\lambda K[2]^k+\alpha y(x)K[2]^k+\beta K[2]^s\right)}{(k-s)\alpha\beta\left(bK[2]^m+aK[2]^n+c\right)(\lambda+y(x))}dK[2]\right.$$

$$+\int_1^{y(x)}\left(-\int_1^x\left(\frac{\exp\left(-\int_1^{K[2]} -\frac{\beta K[1]^s-2\alpha\lambda K[1]^k}{bK[1]^m+aK[1]^n+c}dK[1]\right)K[2]^k}{(k-s)\beta\left(bK[2]^m+aK[2]^n+c\right)(\lambda+K[3])}-\frac{\exp\left(-\int_1^{K[2]} -\frac{\beta K[1]^s-2\alpha\lambda K[1]^k}{bK[1]^m+aK[1]^n+c}dK[1]\right)(-\alpha\right.\right.$$

$$\left.\left.-\frac{\exp\left(-\int_1^x -\frac{\beta K[1]^s-2\alpha\lambda K[1]^k}{bK[1]^m+aK[1]^n+c}dK[1]\right)}{(k-s)\alpha\beta(\lambda+K[3])^2}\right)dK[3]=c_1,y(x)\right]$$

2.78 problem 78

Internal problem ID [10408]

Book: Handbook of exact solutions for ordinary differential equations. By Polyanin and Zaitsev. Second edition
Section: Chapter 1, section 1.2. Riccati Equation. 1.2.2. Equations Containing Power Functions
Problem number: 78.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[[_homogeneous, 'class D'], _rational, _Riccati]`

$$\boxed{(x^n a + b\,x^m + c)\,(y'x - y) + s\,x^k\left(y^2 - \lambda\,x^2\right) = 0}$$

✓ Solution by Maple
Time used: 0.015 (sec). Leaf size: 37

```
dsolve((a*x^n+b*x^m+c)*(x*diff(y(x),x)-y(x))+s*x^k*(y(x)^2-lambda*x^2)=0,y(x), singsol=all)
```

$$y(x) = \tanh\left(s\sqrt{\lambda}\left(\int \frac{x^k}{a\,x^n + b\,x^m + c}dx + c_1\right)\right) x\sqrt{\lambda}$$

✓ Solution by Mathematica
Time used: 22.652 (sec). Leaf size: 53

```
DSolve[(a*x^n+b*x^m+c)*(x*y'[x]-y[x])+s*x^k*(y[x]^2-\[Lambda]*x^2)==0,y[x],x,IncludeSingular
```

$$y(x) \to \sqrt{\lambda}(-x)\tanh\left(\sqrt{\lambda}\left(\int_1^x -\frac{sK[1]^k}{bK[1]^m + aK[1]^n + c}dK[1] + c_1\right)\right)$$

3 Chapter 1, section 1.2. Riccati Equation. subsection 1.2.3. Equations Containing Exponential Functions

3.1 problem 1 121
3.2 problem 2 123
3.3 problem 3 124
3.4 problem 4 126
3.5 problem 5 127
3.6 problem 6 128
3.7 problem 7 129
3.8 problem 8 130
3.9 problem 9 131
3.10 problem 10 133
3.11 problem 11 135
3.12 problem 12 136
3.13 problem 13 137
3.14 problem 14 138
3.15 problem 15 139
3.16 problem 16 141
3.17 problem 17 143
3.18 problem 18 144
3.19 problem 19 145
3.20 problem 20 146
3.21 problem 21 148

3.1 problem 1

Internal problem ID [10409]

Book: Handbook of exact solutions for ordinary differential equations. By Polyanin and Zaitsev. Second edition
Section: Chapter 1, section 1.2. Riccati Equation. subsection 1.2.3. Equations Containing Exponential Functions
Problem number: 1.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_Riccati]`

$$\boxed{y' - ay^2 = b\,\mathrm{e}^{\lambda x}}$$

✓ Solution by Maple
Time used: 0.0 (sec). Leaf size: 96

```
dsolve(diff(y(x),x)=a*y(x)^2+b*exp(lambda*x),y(x), singsol=all)
```

$$y(x) = \frac{\sqrt{b}\,\mathrm{e}^{\frac{x\lambda}{2}}\left(\mathrm{BesselY}\left(1, \frac{2\sqrt{a}\,\sqrt{b}\,\mathrm{e}^{\frac{x\lambda}{2}}}{\lambda}\right) c_1 + \mathrm{BesselJ}\left(1, \frac{2\sqrt{a}\,\sqrt{b}\,\mathrm{e}^{\frac{x\lambda}{2}}}{\lambda}\right)\right)}{\sqrt{a}\left(c_1\,\mathrm{BesselY}\left(0, \frac{2\sqrt{a}\,\sqrt{b}\,\mathrm{e}^{\frac{x\lambda}{2}}}{\lambda}\right) + \mathrm{BesselJ}\left(0, \frac{2\sqrt{a}\,\sqrt{b}\,\mathrm{e}^{\frac{x\lambda}{2}}}{\lambda}\right)\right)}$$

✓ Solution by Mathematica

Time used: 0.551 (sec). Leaf size: 266

```
DSolve[y'[x]==a*y[x]^2+b*Exp[\[Lambda]*x],y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to \frac{\sqrt{be^{\lambda x}}\left(2\,\text{BesselY}\left(1,\frac{2\sqrt{a}\sqrt{be^{x\lambda}}}{\lambda}\right)+c_1\,\text{BesselJ}\left(1,\frac{2\sqrt{a}\sqrt{be^{x\lambda}}}{\lambda}\right)\right)}{\sqrt{a}\left(2\,\text{BesselY}\left(0,\frac{2\sqrt{a}\sqrt{be^{x\lambda}}}{\lambda}\right)+c_1\,\text{BesselJ}\left(0,\frac{2\sqrt{a}\sqrt{be^{x\lambda}}}{\lambda}\right)\right)}$$

$$y(x) \to \frac{\sqrt{be^{\lambda x}}\,\text{BesselJ}\left(1,\frac{2\sqrt{a}\sqrt{be^{x\lambda}}}{\lambda}\right)}{\sqrt{a}\,\text{BesselJ}\left(0,\frac{2\sqrt{a}\sqrt{be^{x\lambda}}}{\lambda}\right)}$$

$$y(x) \to \frac{\sqrt{be^{\lambda x}}\,\text{BesselJ}\left(1,\frac{2\sqrt{a}\sqrt{be^{x\lambda}}}{\lambda}\right)}{\sqrt{a}\,\text{BesselJ}\left(0,\frac{2\sqrt{a}\sqrt{be^{x\lambda}}}{\lambda}\right)}$$

3.2 problem 2

Internal problem ID [10410]

Book: Handbook of exact solutions for ordinary differential equations. By Polyanin and Zaitsev. Second edition
Section: Chapter 1, section 1.2. Riccati Equation. subsection 1.2.3. Equations Containing Exponential Functions
Problem number: 2.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_Riccati]`

$$\boxed{y' - y^2 = a\lambda\,\mathrm{e}^{\lambda x} - a^2\mathrm{e}^{2\lambda x}}$$

✓ Solution by Maple
Time used: 0.0 (sec). Leaf size: 63

```
dsolve(diff(y(x),x)=y(x)^2+a*lambda*exp(lambda*x)-a^2*exp(2*lambda*x),y(x), singsol=all)
```

$$y(x) = \frac{\mathrm{e}^{x\lambda}\,\mathrm{expIntegral}_1\left(-\frac{2a\,\mathrm{e}^{x\lambda}}{\lambda}\right)c_1 a + \mathrm{e}^{\frac{2a\,\mathrm{e}^{x\lambda}}{\lambda}}c_1\lambda + \mathrm{e}^{x\lambda}a}{\mathrm{expIntegral}_1\left(-\frac{2a\,\mathrm{e}^{x\lambda}}{\lambda}\right)c_1 + 1}$$

✓ Solution by Mathematica
Time used: 2.507 (sec). Leaf size: 79

```
DSolve[y'[x]==y[x]^2+a*\[Lambda]*Exp[\[Lambda]*x]-a^2*Exp[2*\[Lambda]*x],y[x],x,IncludeSingu
```

$$y(x) \to \frac{ae^{\lambda x}\,\mathrm{ExpIntegralEi}\left(\frac{2ae^{x\lambda}}{\lambda}\right) + \lambda\left(-e^{\frac{2ae^{\lambda x}}{\lambda}}\right) + ac_1 e^{\lambda x}}{\mathrm{ExpIntegralEi}\left(\frac{2ae^{x\lambda}}{\lambda}\right) + c_1}$$
$$y(x) \to ae^{\lambda x}$$

3.3 problem 3

Internal problem ID [10411]

Book: Handbook of exact solutions for ordinary differential equations. By Polyanin and Zaitsev. Second edition
Section: Chapter 1, section 1.2. Riccati Equation. subsection 1.2.3. Equations Containing Exponential Functions
Problem number: 3.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_Riccati]`

$$\boxed{y' - \sigma y^2 = a + b\,\mathrm{e}^{\lambda x} + c\,\mathrm{e}^{2\lambda x}}$$

✓ Solution by Maple
Time used: 0.0 (sec). Leaf size: 348

```
dsolve(diff(y(x),x)=sigma*y(x)^2+a+b*exp(lambda*x)+c*exp(2*lambda*x),y(x), singsol=all)
```

$$y(x) = \frac{-\text{WhittakerM}\left(-\frac{i\sqrt{\sigma}\,b-2\lambda\sqrt{c}}{2\lambda\sqrt{c}}, \frac{i\sqrt{a}\sqrt{\sigma}}{\lambda}, \frac{2i\sqrt{\sigma}\sqrt{c}\,\mathrm{e}^{x\lambda}}{\lambda}\right)\left(i\left(\sqrt{c}\sqrt{a}-\frac{b}{2}\right)\sqrt{\sigma}+\frac{\lambda\sqrt{c}}{2}\right)+\lambda c_1\,\text{WhittakerW}\left(-\frac{i\sqrt{\sigma}\,b-}{2\lambda}\right.}{\sqrt{c}\,\sigma\left(\text{WhittakerW}\left(-\frac{i\sqrt{}}{2}\right.\right.}$$

✓ Solution by Mathematica

Time used: 3.251 (sec). Leaf size: 1081

```
DSolve[y'[x]==sigma*y[x]^2+a+b*Exp[\[Lambda]*x]+c*Exp[2*\[Lambda]*x],y[x],x,IncludeSingularS
```

$$y(x) \to$$

$$-\frac{i\left(c_1\lambda\left(\sqrt{a}-\sqrt{c}e^{\lambda x}\right)\text{HypergeometricU}\left(\frac{\frac{i\sqrt{\sigma}b}{\sqrt{c}}+\lambda+2i\sqrt{a}\sqrt{\sigma}}{2\lambda},\frac{2i\sqrt{a}\sqrt{\sigma}}{\lambda}+1,\frac{2i\sqrt{c}e^{x\lambda}\sqrt{\sigma}}{\lambda}\right)-ic_1e^{\lambda x}\left(b\sqrt{\sigma}+\sqrt{c}(2\right.\right.}{\lambda\sqrt{\sigma}\left(c_1\text{ Hyperge}\right.}$$

$$y(x)$$

$$\to -\frac{e^{\lambda x}\left(b\sqrt{\sigma}+\sqrt{c}(2\sqrt{a}\sqrt{\sigma}-i\lambda)\right)\text{HypergeometricU}\left(\frac{\frac{i\sqrt{\sigma}b}{\sqrt{c}}+3\lambda+2i\sqrt{a}\sqrt{\sigma}}{2\lambda},\frac{2i\sqrt{a}\sqrt{\sigma}}{\lambda}+2,\frac{2i\sqrt{c}e^{x\lambda}\sqrt{\sigma}}{\lambda}\right)}{\lambda\,\text{HypergeometricU}\left(\frac{\frac{i\sqrt{\sigma}b}{\sqrt{c}}+\lambda+2i\sqrt{a}\sqrt{\sigma}}{2\lambda},\frac{2i\sqrt{a}\sqrt{\sigma}}{\lambda}+1,\frac{2i\sqrt{c}e^{x\lambda}\sqrt{\sigma}}{\lambda}\right)}-i\left(\sqrt{a}-\sqrt{c}e^{\lambda x}\right)}{\sqrt{\sigma}}$$

$$y(x)$$

$$\to -\frac{e^{\lambda x}\left(b\sqrt{\sigma}+\sqrt{c}(2\sqrt{a}\sqrt{\sigma}-i\lambda)\right)\text{HypergeometricU}\left(\frac{\frac{i\sqrt{\sigma}b}{\sqrt{c}}+3\lambda+2i\sqrt{a}\sqrt{\sigma}}{2\lambda},\frac{2i\sqrt{a}\sqrt{\sigma}}{\lambda}+2,\frac{2i\sqrt{c}e^{x\lambda}\sqrt{\sigma}}{\lambda}\right)}{\lambda\,\text{HypergeometricU}\left(\frac{\frac{i\sqrt{\sigma}b}{\sqrt{c}}+\lambda+2i\sqrt{a}\sqrt{\sigma}}{2\lambda},\frac{2i\sqrt{a}\sqrt{\sigma}}{\lambda}+1,\frac{2i\sqrt{c}e^{x\lambda}\sqrt{\sigma}}{\lambda}\right)}-i\left(\sqrt{a}-\sqrt{c}e^{\lambda x}\right)}{\sqrt{\sigma}}$$

3.4 problem 4

Internal problem ID [10412]

Book: Handbook of exact solutions for ordinary differential equations. By Polyanin and Zaitsev. Second edition
Section: Chapter 1, section 1.2. Riccati Equation. subsection 1.2.3. Equations Containing Exponential Functions
Problem number: 4.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_Riccati]`

$$\boxed{y' - \sigma y^2 - ay = b\,\mathrm{e}^x + c}$$

✓ Solution by Maple
Time used: 0.0 (sec). Leaf size: 200

```
dsolve(diff(y(x),x)=sigma*y(x)^2+a*y(x)+b*exp(x)+c,y(x), singsol=all)
```

$$y(x) = -\frac{-2\sqrt{b}\,\mathrm{e}^{\frac{x}{2}}\,\mathrm{BesselJ}\left(\sqrt{a^2-4\sigma c}+1, 2\sqrt{\sigma}\,\sqrt{b}\,\mathrm{e}^{\frac{x}{2}}\right)\sigma - 2\sqrt{b}\,\mathrm{e}^{\frac{x}{2}}\,\mathrm{BesselY}\left(\sqrt{a^2-4\sigma c}+1, 2\sqrt{\sigma}\,\sqrt{b}\,\mathrm{e}^{\frac{x}{2}}\right)c_1\sigma}{\sigma^{\frac{3}{2}}\left(2\,\mathrm{BesselY}\left(\sqrt{a^2-4\sigma c}, 2\sqrt{\sigma}\,\sqrt{b}\,\mathrm{e}^{\frac{x}{2}}\right)c\right.}$$

✓ Solution by Mathematica
Time used: 0.971 (sec). Leaf size: 546

```
DSolve[y'[x]==sigma*y[x]^2+a*y[x]+b*Exp[x]+c,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to -\frac{a\sqrt{b\sigma e^x}\,\Gamma\left(\sqrt{a^2-4c\sigma}+1\right)\mathrm{BesselJ}\left(\sqrt{a^2-4c\sigma}, 2\sqrt{be^x\sigma}\right) + b\sigma e^x\,\Gamma\left(\sqrt{a^2-4c\sigma}+1\right)\mathrm{Be}}{}$$

$$y(x) \to \frac{\frac{\sqrt{b\sigma e^x}\left(\mathrm{BesselJ}\left(1-\sqrt{a^2-4c\sigma}, 2\sqrt{be^x\sigma}\right) - \mathrm{BesselJ}\left(-\sqrt{a^2-4c\sigma}-1, 2\sqrt{be^x\sigma}\right)\right)}{\mathrm{BesselJ}\left(-\sqrt{a^2-4c\sigma}, 2\sqrt{be^x\sigma}\right)} - a}{2\sigma}$$

3.5 problem 5

Internal problem ID [10413]

Book: Handbook of exact solutions for ordinary differential equations. By Polyanin and Zaitsev. Second edition
Section: Chapter 1, section 1.2. Riccati Equation. subsection 1.2.3. Equations Containing Exponential Functions
Problem number: 5.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_Riccati]`

$$y' - y^2 - yb = a(\lambda - b)\,\mathrm{e}^{\lambda x} - a^2\mathrm{e}^{2\lambda x}$$

✓ Solution by Maple
Time used: 0.015 (sec). Leaf size: 80

```
dsolve(diff(y(x),x)=y(x)^2+b*y(x)+a*(lambda-b)*exp(lambda*x)-a^2*exp(2*lambda*x),y(x), sings
```

$$y(x) = \frac{\mathrm{e}^{x\lambda}a\left(\int \mathrm{e}^{\frac{b\lambda x+2\,\mathrm{e}^{x\lambda}a}{\lambda}}dx\right) + \mathrm{e}^{x\lambda}c_1 a - \mathrm{e}^{\frac{b\lambda x+2\,\mathrm{e}^{x\lambda}a}{\lambda}}}{\int \mathrm{e}^{\frac{b\lambda x+2\,\mathrm{e}^{x\lambda}a}{\lambda}}dx + c_1}$$

✓ Solution by Mathematica
Time used: 3.226 (sec). Leaf size: 191

```
DSolve[y'[x]==y[x]^2+b*y[x]+a*(\[Lambda]-b)*Exp[\[Lambda]*x]-a^2*Exp[2*\[Lambda]*x],y[x],x,I
```

$$y(x) \to \frac{-2^{b/\lambda}\left(b - ae^{\lambda x}\right)\left(\frac{ae^{\lambda x}}{\lambda}\right)^{b/\lambda} L^{\frac{b}{\lambda}}_{-\frac{b}{\lambda}}\left(\frac{2ae^{x\lambda}}{\lambda}\right) + ae^{\lambda x}\left(2^{\frac{b+\lambda}{\lambda}}\left(\frac{ae^{\lambda x}}{\lambda}\right)^{b/\lambda} L^{\frac{b+\lambda}{\lambda}}_{-\frac{b+\lambda}{\lambda}}\left(\frac{2ae^{x\lambda}}{\lambda}\right) + c_1\right)}{2^{b/\lambda}\left(\frac{ae^{\lambda x}}{\lambda}\right)^{b/\lambda} L^{\frac{b}{\lambda}}_{-\frac{b}{\lambda}}\left(\frac{2ae^{x\lambda}}{\lambda}\right) + c_1}$$

$$y(x) \to ae^{\lambda x}$$

3.6 problem 6

Internal problem ID [10414]

Book: Handbook of exact solutions for ordinary differential equations. By Polyanin and Zaitsev. Second edition
Section: Chapter 1, section 1.2. Riccati Equation. subsection 1.2.3. Equations Containing Exponential Functions
Problem number: 6.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_Riccati]`

$$\boxed{y' - y^2 - a\,\mathrm{e}^{\lambda x} y = -a\,\mathrm{e}^{\lambda x} b - b^2}$$

✓ Solution by Maple
Time used: 0.0 (sec). Leaf size: 73

```
dsolve(diff(y(x),x)=y(x)^2+a*exp(lambda*x)*y(x)-a*b*exp(lambda*x)-b^2,y(x), singsol=all)
```

$$y(x) = \frac{-b\left(\int \mathrm{e}^{\frac{2b\lambda x+\mathrm{e}^{x\lambda}a}{\lambda}}dx\right) + c_1 b + \mathrm{e}^{\frac{2b\lambda x+\mathrm{e}^{x\lambda}a}{\lambda}}}{-\left(\int \mathrm{e}^{\frac{2b\lambda x+\mathrm{e}^{x\lambda}a}{\lambda}}dx\right) + c_1}$$

✓ Solution by Mathematica
Time used: 0.944 (sec). Leaf size: 115

```
DSolve[y'[x]==y[x]^2+a*Exp[\[Lambda]*x]*y[x]-a*b*Exp[\[Lambda]*x]-b^2,y[x],x,IncludeSingular
```

$$y(x) \to \frac{b\left(-2\lambda e^{\frac{ae^{\lambda x}}{\lambda}}\left(-\frac{ae^{\lambda x}}{\lambda}\right)^{\frac{2b}{\lambda}} + 2b\Gamma\left(\frac{2b}{\lambda}, 0, -\frac{ae^{x\lambda}}{\lambda}\right) + c_1\lambda(-1)^{b/\lambda}\right)}{2b\Gamma\left(\frac{2b}{\lambda}, 0, -\frac{ae^{x\lambda}}{\lambda}\right) + c_1\lambda(-1)^{b/\lambda}}$$

$$y(x) \to b$$

3.7 problem 7

Internal problem ID [10415]

Book: Handbook of exact solutions for ordinary differential equations. By Polyanin and Zaitsev. Second edition
Section: Chapter 1, section 1.2. Riccati Equation. subsection 1.2.3. Equations Containing Exponential Functions
Problem number: 7.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_Riccati]`

$$y' - y^2 = a\,\mathrm{e}^{2\lambda x}\left(\mathrm{e}^{\lambda x} + b\right)^n - \frac{\lambda^2}{4}$$

✓ Solution by Maple
Time used: 0.0 (sec). Leaf size: 1342

```
dsolve(diff(y(x),x)=y(x)^2+a*exp(2*lambda*x)*(exp(lambda*x)+b)^n-1/4*lambda^2,y(x), singsol=
```

Expression too large to display

✗ Solution by Mathematica
Time used: 0.0 (sec). Leaf size: 0

```
DSolve[y'[x]==y[x]^2+a*Exp[2*\[Lambda]*x]*(Exp[\[Lambda]*x]+b)^n-1/4*\[Lambda]^2,y[x],x,Incl
```

Not solved

3.8 problem 8

Internal problem ID [10416]

Book: Handbook of exact solutions for ordinary differential equations. By Polyanin and Zaitsev. Second edition
Section: Chapter 1, section 1.2. Riccati Equation. subsection 1.2.3. Equations Containing Exponential Functions
Problem number: 8.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_Riccati]`

$$y' - y^2 = a\,\mathrm{e}^{8\lambda x} + b\,\mathrm{e}^{6\lambda x} + c\,\mathrm{e}^{4\lambda x} - \lambda^2$$

✓ Solution by Maple
Time used: 0.0 (sec). Leaf size: 1078

```
dsolve(diff(y(x),x)=y(x)^2+a*exp(8*lambda*x)+b*exp(6*lambda*x)+c*exp(4*lambda*x)-lambda^2,y(
```

Expression too large to display

✓ Solution by Mathematica
Time used: 4.991 (sec). Leaf size: 1282

```
DSolve[y'[x]==y[x]^2+a*Exp[8*\[Lambda]*x]+b*Exp[6*\[Lambda]*x]+c*Exp[4*\[Lambda]*x]-\[Lambda
```

$$y(x) \to \frac{-e^{2x\lambda}\,\text{Hypergeometric1F1}\left(\frac{-ib^2+4iac+40a^{3/2}\lambda}{32a^{3/2}\lambda}, \frac{3}{2}, \frac{i\left(2e^{2x\lambda}a+b\right)^2}{8a^{3/2}\lambda}\right) b^3 - 2ae^{4x\lambda}\,\text{Hypergeometric1F1}\left(\frac{-ib^2+4iac+}{32a^{3/}\right.$$

$$y(x) \to \frac{\left(\frac{1}{8}+\frac{i}{8}\right)e^{2\lambda x}\left(8a^{3/2}\lambda+4iac-ib^2\right)\text{HermiteH}\left(\frac{i\left(b^2-4ac+24ia^{3/2}\lambda\right)}{16a^{3/2}\lambda}, \frac{\left(\frac{1}{4}+\frac{i}{4}\right)\left(2e^{2x\lambda}a+b\right)}{a^{3/4}\sqrt{\lambda}}\right)}{a^{5/4}\sqrt{\lambda}\,\text{HermiteH}\left(\frac{i\left(b^2-4ac+8ia^{3/2}\lambda\right)}{16a^{3/2}\lambda}, \frac{\left(\frac{1}{4}+\frac{i}{4}\right)\left(2e^{2x\lambda}a+b\right)}{a^{3/4}\sqrt{\lambda}}\right)}$$
$$+\frac{ibe^{2\lambda x}}{2\sqrt{a}} + i\sqrt{a}e^{4\lambda x} + \lambda$$

3.9 problem 9

Internal problem ID [10417]

Book: Handbook of exact solutions for ordinary differential equations. By Polyanin and Zaitsev. Second edition
Section: Chapter 1, section 1.2. Riccati Equation. subsection 1.2.3. Equations Containing Exponential Functions
Problem number: 9.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_Riccati]`

$$\boxed{y' - a\,\mathrm{e}^{kx} y^2 = b\,\mathrm{e}^{sx}}$$

✓ Solution by Maple
Time used: 0.0 (sec). Leaf size: 228

```
dsolve(diff(y(x),x)=a*exp(k*x)*y(x)^2+b*exp(s*x),y(x), singsol=all)
```

$$y(x) = -\frac{b\,\mathrm{e}^{sx}\left(\mathrm{BesselY}\left(\frac{s}{s+k}, \frac{2\sqrt{a}\,\sqrt{b}\,\mathrm{e}^{\frac{x(s+k)}{2}}}{s+k}\right) c_1 + \mathrm{BesselJ}\left(\frac{s}{s+k}, \right.\right.}{\mathrm{BesselJ}\left(\frac{2s+k}{s+k}, \frac{2\sqrt{a}\,\sqrt{b}\,\mathrm{e}^{\frac{x(s+k)}{2}}}{s+k}\right)\sqrt{a}\,\sqrt{b}\,\mathrm{e}^{\frac{x(s+k)}{2}} + \sqrt{a}\,\sqrt{b}\,\mathrm{e}^{\frac{x(s+k)}{2}}\,\mathrm{BesselY}\left(\frac{2s+k}{s+k}, \frac{2\sqrt{a}\,\sqrt{b}\,\mathrm{e}^{\frac{x(s+k)}{2}}}{s+k}\right) c_1 - s\left(\mathrm{Be}$$

✓ Solution by Mathematica

Time used: 6.491 (sec). Leaf size: 1097

```
DSolve[y'[x]==a*Exp[k*x]*y[x]^2+b*Exp[s*x],y[x],x,IncludeSingularSolutions -> True]
```

$$y(x)$$

$$\to \frac{e^{-kx}\left(-kK_{\frac{k\log\left(e^{k+s}\right)}{(k+s)^2}}\left(2\sqrt{-\frac{ab\left(\left(e^{k+s}\right)^x\right)^{\frac{k+s}{\log\left(e^{k+s}\right)}}\log^2\left(e^{k+s}\right)}{(k+s)^4}}\right)-c_1k(-1)^{\frac{k\log\left(e^{k+s}\right)}{(k+s)^2}}\text{BesselI}\left(\frac{k\log\left(e^{k+s}\right)}{(k+s)^2},2\sqrt{-\frac{ab}{}}\right.\right.}{}$$

$$y(x)$$

$$\to \frac{e^{-kx}\left(-\frac{(k+s)\sqrt{-\frac{ab\log^2\left(e^{k+s}\right)\left(\left(e^{k+s}\right)^x\right)^{\frac{k+s}{\log\left(e^{k+s}\right)}}}{(k+s)^4}}\left(\text{BesselI}\left(\frac{k\log\left(e^{k+s}\right)}{(k+s)^2}-1,2\sqrt{-\frac{ab\left(\left(e^{k+s}\right)^x\right)^{\frac{k+s}{\log\left(e^{k+s}\right)}}\log^2\left(e^{k+s}\right)}{(k+s)^4}}\right)+\text{BesselI}\left(\frac{k\log}{(k}\right.\right.\right.}{\text{BesselI}\left(\frac{k\log\left(e^{k+s}\right)}{(k+s)^2},2\sqrt{-\frac{ab\left(\left(e^{k+s}\right)^x\right)^{\frac{k+s}{\log\left(e^{k+s}\right)}}\log^2\left(e^{k+s}\right)}{(k+s)^4}}\right)}$$

$$2a$$

$$y(x)$$

$$\to \frac{e^{-kx}\left(-\frac{(k+s)\sqrt{-\frac{ab\log^2\left(e^{k+s}\right)\left(\left(e^{k+s}\right)^x\right)^{\frac{k+s}{\log\left(e^{k+s}\right)}}}{(k+s)^4}}\left(\text{BesselI}\left(\frac{k\log\left(e^{k+s}\right)}{(k+s)^2}-1,2\sqrt{-\frac{ab\left(\left(e^{k+s}\right)^x\right)^{\frac{k+s}{\log\left(e^{k+s}\right)}}\log^2\left(e^{k+s}\right)}{(k+s)^4}}\right)+\text{BesselI}\left(\frac{k\log}{(k}\right.\right.\right.}{\text{BesselI}\left(\frac{k\log\left(e^{k+s}\right)}{(k+s)^2},2\sqrt{-\frac{ab\left(\left(e^{k+s}\right)^x\right)^{\frac{k+s}{\log\left(e^{k+s}\right)}}\log^2\left(e^{k+s}\right)}{(k+s)^4}}\right)}$$

$$2a$$

3.10 problem 10

Internal problem ID [10418]

Book: Handbook of exact solutions for ordinary differential equations. By Polyanin and Zaitsev. Second edition
Section: Chapter 1, section 1.2. Riccati Equation. subsection 1.2.3. Equations Containing Exponential Functions
Problem number: 10.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_Riccati]`

$$y' - b\,\mathrm{e}^{x\mu} y^2 = a\lambda\,\mathrm{e}^{\lambda x} - a^2 b\,\mathrm{e}^{(\mu+2\lambda)x}$$

✗ Solution by Maple

```
dsolve(diff(y(x),x)=b*exp(mu*x)*y(x)^2+a*lambda*exp(lambda*x)-a^2*b*exp((mu+2*lambda)*x),y(x
```

No solution found

✓ Solution by Mathematica

Time used: 8.808 (sec). Leaf size: 844

```
DSolve[y'[x]==b*Exp[\[Mu]*x]*y[x]^2+a*\[Lambda]*Exp[\[Lambda]*x]-a^2*b*Exp[(\[Mu]+2*\[Lambda]
```

$$y(x)$$

$$\to \frac{e^{\mu(-x)}\left(-2ab\log\left(e^{\lambda+\mu}\right)\left(\left(e^{\lambda+\mu}\right)^{x}\right)^{\frac{\lambda+\mu}{\log\left(e^{\lambda+\mu}\right)}}\left(2(\lambda+\mu)L^{\frac{\mu\log\left(e^{\lambda+\mu}\right)}{(\lambda+\mu)^2}+1}_{-\frac{\log\left(e^{\lambda+\mu}\right)}{2(\lambda+\mu)}-\frac{3}{2}}\left(-\frac{2ab\left(\left(e^{\lambda+\mu}\right)^{x}\right)^{\frac{\lambda+\mu}{\log\left(e^{\lambda+\mu}\right)}}\log\left(e^{\lambda+\mu}\right)}{(\lambda+\mu)^2}\right)+c\right.\right.}{}$$

$$y(x) \to$$

$$-\frac{ae^{\mu(-x)}\log\left(e^{\lambda+\mu}\right)\left(\log\left(e^{\lambda+\mu}\right)+\lambda+\mu\right)\left(\left(e^{\lambda+\mu}\right)^{x}\right)^{\frac{\lambda+\mu}{\log\left(e^{\lambda+\mu}\right)}}\text{HypergeometricU}\left(\frac{1}{2}\left(\frac{\log\left(e^{\lambda+\mu}\right)}{\lambda+\mu}+3\right),\frac{2(\lambda+\mu}{}\right.}{(\lambda+\mu)^2\,\text{HypergeometricU}\left(\frac{\lambda+\mu+\log\left(e^{\lambda+\mu}\right)}{2(\lambda+\mu)},\frac{\mu\log\left(e^{\lambda+\mu}\right)}{(\lambda+\mu)^2}+1,-\frac{2ab\left(\left(e^{\lambda+\mu}\right)^{x}\right)^{\frac{\log(}{(\lambda+}}\right.}$$

$$-\frac{e^{\mu(-x)}\left(\log\left(e^{\lambda+\mu}\right)\left(2ab\left(\left(e^{\lambda+\mu}\right)^{x}\right)^{\frac{\lambda+\mu}{\log\left(e^{\lambda+\mu}\right)}}+\mu\right)+\mu(\lambda+\mu)\right)}{2b(\lambda+\mu)}$$

3.11 problem 11

Internal problem ID [10419]

Book: Handbook of exact solutions for ordinary differential equations. By Polyanin and Zaitsev. Second edition
Section: Chapter 1, section 1.2. Riccati Equation. subsection 1.2.3. Equations Containing Exponential Functions
Problem number: 11.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[[_1st_order, _with_linear_symmetries], _Riccati]`

$$\boxed{y' - a\,\mathrm{e}^{\lambda x} y^2 - yb = c\,\mathrm{e}^{-\lambda x}}$$

✓ Solution by Maple
Time used: 0.015 (sec). Leaf size: 96

```
dsolve(diff(y(x),x)=a*exp(lambda*x)*y(x)^2+b*y(x)+c*exp(-lambda*x),y(x), singsol=all)
```

$$y(x) = -\frac{\left(-\sqrt{(b+\lambda)^2\left(4ac-b^2-2\lambda b-\lambda^2\right)}\,\tan\left(\frac{((b+\lambda)x+c_1)\sqrt{(b+\lambda)^2(4ac-b^2-2\lambda b-\lambda^2)}}{2(b+\lambda)^2}\right)+(b+\lambda)^2\right)\mathrm{e}^{-x\lambda}}{2a\,(b+\lambda)}$$

✓ Solution by Mathematica
Time used: 0.927 (sec). Leaf size: 188

```
DSolve[y'[x]==a*Exp[\[Lambda]*x]*y[x]^2+b*y[x]+c*Exp[-\[Lambda]*x],y[x],x,IncludeSingularSol
```

$$y(x) \to \frac{e^{\lambda(-x)}\left(-\sqrt{-4ac+b^2+2b\lambda+\lambda^2}+\frac{2}{\frac{1}{\sqrt{-4ac+b^2+2b\lambda+\lambda^2}}+c_1 e^{x\sqrt{-4ac+b^2+2b\lambda+\lambda^2}}}-b-\lambda\right)}{2a}$$

$$y(x) \to -\frac{e^{\lambda(-x)}\left(b\left(\sqrt{-4ac+b^2+2b\lambda+\lambda^2}+2\lambda\right)+\lambda\left(\sqrt{-4ac+b^2+2b\lambda+\lambda^2}+\lambda\right)-4ac+b^2\right)}{2a\sqrt{-4ac+b^2+2b\lambda+\lambda^2}}$$

3.12 problem 12

Internal problem ID [10420]

Book: Handbook of exact solutions for ordinary differential equations. By Polyanin and Zaitsev. Second edition
Section: Chapter 1, section 1.2. Riccati Equation. subsection 1.2.3. Equations Containing Exponential Functions
Problem number: 12.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_Riccati]`

$$y' - a\,\mathrm{e}^{x\mu} y^2 - \lambda y = -a\,b^2 \mathrm{e}^{(\mu+2\lambda)x}$$

✓ Solution by Maple
Time used: 0.0 (sec). Leaf size: 79

```
dsolve(diff(y(x),x)=a*exp(mu*x)*y(x)^2+lambda*y(x)-a*b^2*exp((mu+2*lambda)*x),y(x), singsol=
```

$$y(x) = -\frac{b\left(c_1 \sinh\left(\frac{ab\,\mathrm{e}^{x(\lambda+\mu)}}{\lambda+\mu}\right) + \cosh\left(\frac{ab\,\mathrm{e}^{x(\lambda+\mu)}}{\lambda+\mu}\right)\right)\mathrm{e}^{x\lambda}}{c_1 \cosh\left(\frac{ab\,\mathrm{e}^{x(\lambda+\mu)}}{\lambda+\mu}\right) + \sinh\left(\frac{ab\,\mathrm{e}^{x(\lambda+\mu)}}{\lambda+\mu}\right)}$$

✓ Solution by Mathematica
Time used: 2.706 (sec). Leaf size: 286

```
DSolve[y'[x]==a*Exp[\[Mu]*x]*y[x]^2+\[Lambda]*y[x]-a*b^2*Exp[(\[Mu]+2*\[Lambda])*x],y[x],x,I
```

$$y(x) \to \frac{-\tan\left(\frac{ab^2 e^{x(2\lambda+\mu)}\sqrt{-\frac{e^{-2x\lambda}}{b^2}}}{\lambda+\mu} - c_1\right)}{\sqrt{-\frac{e^{-2x\lambda}}{b^2}}} \quad \text{if condition}$$

3.13 problem 13

Internal problem ID [10421]

Book: Handbook of exact solutions for ordinary differential equations. By Polyanin and Zaitsev. Second edition
Section: Chapter 1, section 1.2. Riccati Equation. subsection 1.2.3. Equations Containing Exponential Functions
Problem number: 13.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_Riccati]`

$$y' - \mathrm{e}^{\lambda x} y^2 - a\,\mathrm{e}^{x\mu} y = a\lambda\,\mathrm{e}^{(\mu-\lambda)x}$$

✓ Solution by Maple
Time used: 0.016 (sec). Leaf size: 97

```
dsolve(diff(y(x),x)=exp(lambda*x)*y(x)^2+a*exp(mu*x)*y(x)+a*lambda*exp((mu-lambda)*x),y(x),
```

$$y(x) = \frac{\lambda\left(a c_1 \mathrm{e}^{(\mu-\lambda)x}\,\mathrm{hypergeom}\left(\left[\frac{\mu-\lambda}{\mu}\right], \left[\frac{-\lambda+2\mu}{\mu}\right], \frac{a\,\mathrm{e}^{x\mu}}{\mu}\right) + \lambda - \mu\right)}{(\mu-\lambda)\left(c_1\,\mathrm{hypergeom}\left(\left[-\frac{\lambda}{\mu}\right], \left[\frac{\mu-\lambda}{\mu}\right], \frac{a\,\mathrm{e}^{x\mu}}{\mu}\right) + \mathrm{e}^{x\lambda}\right)}$$

✓ Solution by Mathematica
Time used: 4.392 (sec). Leaf size: 148

```
DSolve[y'[x]==Exp[\[Lambda]*x]*y[x]^2+a*Exp[\[Mu]*x]*y[x]+a*\[Lambda]*Exp[(\[Mu]-\[Lambda])*
```

$$y(x) \to -\frac{e^{\lambda(-x)}\left(-\lambda\left(-\frac{ae^{\mu x}}{\mu}\right)^{\lambda/\mu}\Gamma\left(-\frac{\lambda}{\mu}, -\frac{ae^{x\mu}}{\mu}\right) + \mu e^{\frac{ae^{\mu x}}{\mu}} + c_1\lambda(e^{\mu x})^{\lambda/\mu}\right)}{-\left(-\frac{ae^{\mu x}}{\mu}\right)^{\lambda/\mu}\Gamma\left(-\frac{\lambda}{\mu}, -\frac{ae^{x\mu}}{\mu}\right) + c_1\,(e^{\mu x})^{\lambda/\mu}}$$
$$y(x) \to \lambda\left(-e^{\lambda(-x)}\right)$$

3.14 problem 14

Internal problem ID [10422]

Book: Handbook of exact solutions for ordinary differential equations. By Polyanin and Zaitsev. Second edition
Section: Chapter 1, section 1.2. Riccati Equation. subsection 1.2.3. Equations Containing Exponential Functions
Problem number: 14.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_Riccati]`

$$y' + \mathrm{e}^{\lambda x} y^2 \lambda - a\,\mathrm{e}^{x\mu} y = -a\,\mathrm{e}^{(\mu-\lambda)x}$$

✓ Solution by Maple
Time used: 0.0 (sec). Leaf size: 96

```
dsolve(diff(y(x),x)=-lambda*exp(lambda*x)*y(x)^2+a*exp(mu*x)*y(x)-a*exp((mu-lambda)*x),y(x),
```

$$y(x) = \frac{a c_1 \mathrm{e}^{(\mu-\lambda)x}\,\mathrm{hypergeom}\left(\left[\frac{\mu-\lambda}{\mu}\right], \left[\frac{-\lambda+2\mu}{\mu}\right], \frac{a\,\mathrm{e}^{x\mu}}{\mu}\right) + \lambda - \mu}{(\lambda-\mu)\left(c_1\,\mathrm{hypergeom}\left(\left[-\frac{\lambda}{\mu}\right], \left[\frac{\mu-\lambda}{\mu}\right], \frac{a\,\mathrm{e}^{x\mu}}{\mu}\right) + \mathrm{e}^{x\lambda}\right)}$$

✓ Solution by Mathematica
Time used: 4.358 (sec). Leaf size: 147

```
DSolve[y'[x]==-\[Lambda]*Exp[\[Lambda]*x]*y[x]^2+a*Exp[\[Mu]*x]*y[x]-a*Exp[(\[Mu]-\[Lambda])
```

$$y(x) \to \frac{e^{\lambda(-x)}\left(-\lambda\left(-\frac{ae^{\mu x}}{\mu}\right)^{\lambda/\mu}\Gamma\left(-\frac{\lambda}{\mu}, -\frac{ae^{x\mu}}{\mu}\right) + \mu e^{\frac{ae^{\mu x}}{\mu}} + c_1\lambda(e^{\mu x})^{\lambda/\mu}\right)}{\lambda\left(-\left(-\frac{ae^{\mu x}}{\mu}\right)^{\lambda/\mu}\Gamma\left(-\frac{\lambda}{\mu}, -\frac{ae^{x\mu}}{\mu}\right) + c_1\,(e^{\mu x})^{\lambda/\mu}\right)}$$

$$y(x) \to e^{\lambda(-x)}$$

3.15 problem 15

Internal problem ID [10423]

Book: Handbook of exact solutions for ordinary differential equations. By Polyanin and Zaitsev. Second edition
Section: Chapter 1, section 1.2. Riccati Equation. subsection 1.2.3. Equations Containing Exponential Functions
Problem number: 15.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_Riccati]`

$$y' - a\,\mathrm{e}^{x\mu} y^2 - ab\,\mathrm{e}^{x(\lambda+\mu)} y = -b\lambda\,\mathrm{e}^{\lambda x}$$

✓ Solution by Maple
Time used: 0.0 (sec). Leaf size: 629

```
dsolve(diff(y(x),x)=a*exp(mu*x)*y(x)^2+a*b*exp((lambda+mu)*x)*y(x)-b*lambda*exp(lambda*x),y(
```

$$y(x) = \frac{-6\left(-\frac{2(\lambda+\mu)\left(\mu+\frac{\lambda}{2}\right)\mathrm{e}^{\frac{ab\,\mathrm{e}^{x(\lambda+\mu)}-4(\lambda+\mu)x\left(\mu+\frac{3\lambda}{4}\right)}{2\lambda+2\mu}}}{3} + a\,\mathrm{e}^{\frac{ab\,\mathrm{e}^{x(\lambda+\mu)}-2(\lambda+\mu)x\left(\mu+\frac{\lambda}{2}\right)}{2\lambda+2\mu}} b\left(\frac{2\lambda}{3}+\mu\right)\right) c_1\left(\mu+\frac{\lambda}{2}\right) \text{WhittakerM}\left(\frac{}{}\right.}{\left(4\,\mathrm{e}^{\frac{ab\,\mathrm{e}^{x(\lambda+\mu)}-2(\lambda+\mu)x\left(\frac{3\lambda}{2}+\mu\right.}{2\lambda+2\mu}}\right.}$$

✓ Solution by Mathematica

Time used: 12.587 (sec). Leaf size: 902

```
DSolve[y'[x]==a*Exp[\[Mu]*x]*y[x]^2+a*b*Exp[(\[Lambda]+\[Mu])*x]*y[x]-b*\[Lambda]*Exp[\[Lamb
```

$$y(x)$$

$$\to \frac{e^{\mu(-x)}\left(ab\log\left(e^{\lambda+\mu}\right)\left(\left(e^{\lambda+\mu}\right)^{x}\right)^{\frac{\lambda+\mu}{\log\left(e^{\lambda+\mu}\right)}}\left(2(\lambda+\mu)L^{\frac{\mu\log\left(e^{\lambda+\mu}\right)}{(\lambda+\mu)^2}+1}_{-\frac{\log\left(e^{\lambda+\mu}\right)}{2(\lambda+\mu)}-\frac{3}{2}}\left(\frac{ab\left(\left(e^{\lambda+\mu}\right)^{x}\right)^{\frac{\lambda+\mu}{\log\left(e^{\lambda+\mu}\right)}}\log\left(e^{\lambda+\mu}\right)}{(\lambda+\mu)^2}\right)+c_1\left(\log\right.\right.\right.}$$

$$y(x)$$

$$\to \frac{be^{\mu(-x)}\log\left(e^{\lambda+\mu}\right)\left(\log\left(e^{\lambda+\mu}\right)+\lambda+\mu\right)\left(\left(e^{\lambda+\mu}\right)^{x}\right)^{\frac{\lambda+\mu}{\log\left(e^{\lambda+\mu}\right)}}\text{HypergeometricU}\left(\frac{1}{2}\left(\frac{\log\left(e^{\lambda+\mu}\right)}{\lambda+\mu}+3\right),\frac{\mu\log\left(e^{\lambda+\mu}\right)}{(\lambda+\mu)^2}\right.}{2(\lambda+\mu)^2\,\text{HypergeometricU}\left(\frac{\lambda+\mu+\log\left(e^{\lambda+\mu}\right)}{2(\lambda+\mu)},\frac{\mu\log\left(e^{\lambda+\mu}\right)}{(\lambda+\mu)^2}+1,\frac{ab\left(\left(e^{\lambda+\mu}\right)^{x}\right)^{\frac{\lambda+\mu}{\log\left(e^{\lambda+\mu}\right)}}}{(\lambda+\mu)^2}\right.}$$

$$-\frac{e^{\mu(-x)}\left((\lambda+\mu)\left(ab\left(\left(e^{\lambda+\mu}\right)^{x}\right)^{\frac{\lambda+\mu}{\log\left(e^{\lambda+\mu}\right)}}+\mu\right)+\log\left(e^{\lambda+\mu}\right)\left(\mu-ab\left(\left(e^{\lambda+\mu}\right)^{x}\right)^{\frac{\lambda+\mu}{\log\left(e^{\lambda+\mu}\right)}}\right)\right)}{2a(\lambda+\mu)}$$

3.16 problem 16

Internal problem ID [10424]

Book: Handbook of exact solutions for ordinary differential equations. By Polyanin and Zaitsev. Second edition
Section: Chapter 1, section 1.2. Riccati Equation. subsection 1.2.3. Equations Containing Exponential Functions
Problem number: 16.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_Riccati]`

$$\boxed{y' - a\,\mathrm{e}^{kx} y^2 - yb = c\,\mathrm{e}^{sx} + d\,\mathrm{e}^{-kx}}$$

✓ Solution by Maple
Time used: 0.0 (sec). Leaf size: 332

```
dsolve(diff(y(x),x)=a*exp(k*x)*y(x)^2+b*y(x)+c*exp(s*x)+d*exp(-k*x),y(x), singsol=all)
```

$$y(x) = \frac{\left(\sqrt{c}\,a\left(\mathrm{BesselY}\left(\frac{\sqrt{-4ad+b^2+2kb+k^2}+s+k}{s+k}, \frac{2\sqrt{c}\sqrt{a}\,\mathrm{e}^{\frac{x(s+k)}{2}}}{s+k}\right) c_1 + \mathrm{BesselJ}\left(\frac{\sqrt{-4ad+b^2+2kb+k^2}+s+k}{s+k}, \frac{2\sqrt{c}\sqrt{a}\,\mathrm{e}^{\frac{x(s+k)}{2}}}{s+k}\right)\right.\right.}{a^{\frac{3}{2}}\left(\mathrm{BesselY}\left(\frac{\sqrt{-4ad+b^2+2kb+k^2}}{s+k}, \frac{2\sqrt{c}\sqrt{a}\,\mathrm{e}^{\frac{}{}}}{s+k}\right.\right.}$$

✓ Solution by Mathematica

Time used: 18.386 (sec). Leaf size: 1636

```
DSolve[y'[x]==a*Exp[k*x]*y[x]^2+b*y[x]+c*Exp[s*x]+d*Exp[-k*x],y[x],x,IncludeSingularSolution
```

$$y(x)$$

$$\to \frac{e^{-kx}\left(-\left((b+k)K_{\frac{\sqrt{\left(b^2+2kb+k^2-4ad\right)(k+s)^4\log^2\left(e^{k+s}\right)}}{(k+s)^4}}\left(2\sqrt{-\frac{ac\left(\left(e^{k+s}\right)^x\right)^{\frac{k+s}{\log\left(e^{k+s}\right)}}\log^2\left(e^{k+s}\right)}{(k+s)^4}}\right)\right)+(-1)^{\frac{k^4+4sk^3+6s^2k^2}{}}\right.}{}$$

$$y(x)$$

$$\to \frac{e^{-kx}\left(-(b+k)(k+s)^3\sqrt{-\frac{ac\log^2\left(e^{k+s}\right)\left(\left(e^{k+s}\right)^x\right)^{\frac{k+s}{\log\left(e^{k+s}\right)}}}{(k+s)^4}}\,\text{BesselI}\left(\frac{\sqrt{(b^2+2kb+k^2-4ad)(k+s)^4\log^2\left(e^{k+s}\right)}}{(k+s)^4},2\sqrt{-\frac{ac(}{}}\right.\right.}{}$$

3.17 problem 17

Internal problem ID [10425]

Book: Handbook of exact solutions for ordinary differential equations. By Polyanin and Zaitsev. Second edition
Section: Chapter 1, section 1.2. Riccati Equation. subsection 1.2.3. Equations Containing Exponential Functions
Problem number: 17.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_Riccati]`

$$\boxed{y' - a\,\mathrm{e}^{(\mu+2\lambda)x} y^2 - \left(\mathrm{e}^{x(\lambda+\mu)} b - \lambda\right) y = c\,\mathrm{e}^{x\mu}}$$

✓ Solution by Maple
Time used: 0.016 (sec). Leaf size: 79

```
dsolve(diff(y(x),x)=a*exp((2*lambda+mu)*x)*y(x)^2+(b*exp((lambda+mu)*x)-lambda)*y(x)+c*exp(m
```

$$y(x) = \frac{\mathrm{e}^{-x\lambda}\left(\sqrt{4a\,b^2c - b^4}\,\tan\left(\frac{\left(\mathrm{e}^{x(\lambda+\mu)}b+(\lambda+\mu)c_1\right)\sqrt{4a\,b^2c-b^4}}{2b^2(\lambda+\mu)}\right) - b^2\right)}{2ab}$$

✓ Solution by Mathematica
Time used: 6.375 (sec). Leaf size: 349

```
DSolve[y'[x]==a*Exp[(2*\[Lambda]+\[Mu])*x]*y[x]^2+(b*Exp[(\[Lambda]+\[Mu])*x]-\[Lambda])*y[x
```

$$y(x)$$

$$\to \frac{e^{\lambda(-x)}\left(b^2e^{x(\lambda+\mu)}\left(\pi + ic_1\left(e^{\sqrt{\frac{\left(b^2-4ac\right)e^{2x(\lambda+\mu)}}{(\lambda+\mu)^2}}} - 1\right)\right) - b(\lambda+\mu)\sqrt{\frac{(b^2-4ac)e^{2x(\lambda+\mu)}}{(\lambda+\mu)^2}}\left(\pi - ic_1\left(e^{\sqrt{\frac{\left(b^2-4ac\right)e^{2x}}{(\lambda+\mu)}}}\right.\right.\right.}{2a(\lambda+\mu)\sqrt{\frac{(b^2-4ac)e^{2x(\lambda+\mu)}}{(\lambda+\mu)^2}}\left(\pi - ic_1\left(e^{\sqrt{\frac{\left(b^2-4ac\right)e^{2x}}{(\lambda+\mu)^2}}}\right.\right.}$$

$$y(x) \to \frac{e^{\lambda(-x)}\left(-(\lambda+\mu)e^{-x(\lambda+\mu)}\sqrt{\frac{(b^2-4ac)e^{2x(\lambda+\mu)}}{(\lambda+\mu)^2}}\tanh\left(\frac{1}{2}\sqrt{\frac{(b^2-4ac)e^{2x(\lambda+\mu)}}{(\lambda+\mu)^2}}\right) - b\right)}{2a}$$

3.18 problem 18

Internal problem ID [10426]

Book: Handbook of exact solutions for ordinary differential equations. By Polyanin and Zaitsev. Second edition
Section: Chapter 1, section 1.2. Riccati Equation. subsection 1.2.3. Equations Containing Exponential Functions
Problem number: 18.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_Riccati]`

$$\boxed{y' - a\,\mathrm{e}^{kx} y^2 - yb = c\,\mathrm{e}^{knx} + d\,\mathrm{e}^{k(1+2n)x}}$$

✗ Solution by Maple

```
dsolve(diff(y(x),x)=a*exp(k*x)*y(x)^2+b*y(x)+c*exp(k*n*x)+d*exp(k*(2*n+1)*x),y(x), singsol=a
```

No solution found

✓ Solution by Mathematica
Time used: 27.598 (sec). Leaf size: 2503

```
DSolve[y'[x]==a*Exp[k*x]*y[x]^2+b*y[x]+c*Exp[k*n*x]+d*Exp[k*(2*n+1)*x],y[x],x,IncludeSingula
```

Too large to display

3.19 problem 19

Internal problem ID [10427]

Book: Handbook of exact solutions for ordinary differential equations. By Polyanin and Zaitsev. Second edition
Section: Chapter 1, section 1.2. Riccati Equation. subsection 1.2.3. Equations Containing Exponential Functions
Problem number: 19.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[[_1st_order, '_with_symmetry_[F(x),G(x)]'], _Riccati]`

$$\boxed{y' - \mathrm{e}^{x\mu}\left(y - b\,\mathrm{e}^{\lambda x}\right)^2 = b\lambda\,\mathrm{e}^{\lambda x}}$$

✓ Solution by Maple
Time used: 0.0 (sec). Leaf size: 44

```
dsolve(diff(y(x),x)=exp(mu*x)*(y(x)-b*exp(lambda*x))^2+b*lambda*exp(lambda*x),y(x), singsol=
```

$$y(x) = \frac{\left(\mathrm{e}^{x(\lambda+\mu)}c_1 b\mu + b\,\mathrm{e}^{x\lambda} - c_1\mu^2\right)\mathrm{e}^{-x\mu}}{c_1\mu + \mathrm{e}^{-x\mu}}$$

✓ Solution by Mathematica
Time used: 1.524 (sec). Leaf size: 40

```
DSolve[y'[x]==Exp[\[Mu]*x]*(y[x]-b*Exp[\[Lambda]*x])^2+b*\[Lambda]*Exp[\[Lambda]*x],y[x],x,I
```

$$y(x) \to be^{\lambda x} + \frac{\mu}{-e^{\mu x} + c_1\mu}$$
$$y(x) \to be^{\lambda x}$$

3.20 problem 20

Internal problem ID [10428]

Book: Handbook of exact solutions for ordinary differential equations. By Polyanin and Zaitsev. Second edition
Section: Chapter 1, section 1.2. Riccati Equation. subsection 1.2.3. Equations Containing Exponential Functions
Problem number: 20.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_Riccati]`

$$\boxed{\left(\mathrm{e}^{\lambda x} a + b\,\mathrm{e}^{x\mu} + c\right) y' - y^2 - k\,\mathrm{e}^{\nu x} y = -m^2 + k m\,\mathrm{e}^{\nu x}}$$

✓ Solution by Maple
Time used: 0.0 (sec). Leaf size: 202

```
dsolve((a*exp(lambda*x)+b*exp(mu*x)+c)*diff(y(x),x)=y(x)^2+k*exp(nu*x)*y(x)-m^2+k*m*exp(nu*x)
```

$$y(x) = \frac{-m\left(\int \frac{\mathrm{e}^{k\left(\int \frac{\mathrm{e}^{\nu x}}{\mathrm{e}^{x\lambda} a + b\,\mathrm{e}^{x\mu} + c} dx\right) - 2m\left(\int \frac{1}{\mathrm{e}^{x\lambda} a + b\,\mathrm{e}^{x\mu} + c} dx\right)}}{\mathrm{e}^{x\lambda} a + b\,\mathrm{e}^{x\mu} + c} dx\right) + c_1 m - \mathrm{e}^{k\left(\int \frac{\mathrm{e}^{\nu x}}{\mathrm{e}^{x\lambda} a + b\,\mathrm{e}^{x\mu} + c} dx\right) - 2m\left(\int \frac{1}{\mathrm{e}^{x\lambda} a + b\,\mathrm{e}^{x\mu} + c} dx\right)}}{\int \frac{\mathrm{e}^{k\left(\int \frac{\mathrm{e}^{\nu x}}{\mathrm{e}^{x\lambda} a + b\,\mathrm{e}^{x\mu} + c} dx\right) - 2m\left(\int \frac{1}{\mathrm{e}^{x\lambda} a + b\,\mathrm{e}^{x\mu} + c} dx\right)}}{\mathrm{e}^{x\lambda} a + b\,\mathrm{e}^{x\mu} + c} dx - c_1}$$

✓ Solution by Mathematica

Time used: 16.545 (sec). Leaf size: 358

```
DSolve[(a*Exp[\[Lambda]*x]+b*Exp[\[Mu]*x]+c)*y'[x]==y[x]^2+k*Exp[\[Nu]*x]*y[x]-m^2+k*m*Exp[\
```

$$\text{Solve}\left[\int_1^x -\frac{\exp\left(-\int_1^{K[2]} -\frac{e^{\nu K[1]}k-2m}{e^{\lambda K[1]}a+be^{\mu K[1]}+c}dK[1]\right)\left(e^{\nu K[2]}k-m+y(x)\right)}{\left(e^{\lambda K[2]}a+be^{\mu K[2]}+c\right)k\nu(m+y(x))}dK[2]\right.$$

$$+\int_1^{y(x)}\left(\frac{\exp\left(-\int_1^x -\frac{e^{\nu K[1]}k-2m}{e^{\lambda K[1]}a+be^{\mu K[1]}+c}dK[1]\right)}{k\nu(m+K[3])^2}\right.$$

$$\left.\left.-\int_1^x\left(\frac{\exp\left(-\int_1^{K[2]} -\frac{e^{\nu K[1]}k-2m}{e^{\lambda K[1]}a+be^{\mu K[1]}+c}dK[1]\right)\left(e^{\nu K[2]}k-m+K[3]\right)}{\left(e^{\lambda K[2]}a+be^{\mu K[2]}+c\right)k\nu(m+K[3])^2}-\frac{\exp\left(-\int_1^{K[2]} -\frac{e^{\nu K[1]}k-2m}{e^{\lambda K[1]}a+be^{\mu K[1]}+c}dK[1]\right.\right.\right.\right.$$

3.21 problem 21

Internal problem ID [10429]

Book: Handbook of exact solutions for ordinary differential equations. By Polyanin and Zaitsev. Second edition
Section: Chapter 1, section 1.2. Riccati Equation. subsection 1.2.3. Equations Containing Exponential Functions
Problem number: 21.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_Riccati]`

$$\boxed{\left(\mathrm{e}^{\lambda x} a + b\,\mathrm{e}^{x\mu} + c\right)\left(y' - y^2\right) = -a\,\lambda^2 \mathrm{e}^{\lambda x} - b\,\mu^2 \mathrm{e}^{x\mu}}$$

✓ Solution by Maple
Time used: 0.015 (sec). Leaf size: 176

```
dsolve((a*exp(lambda*x)+b*exp(mu*x)+c)*(diff(y(x),x)-y(x)^2)+a*lambda^2*exp(lambda*x)+b*mu^2
```

$$y(x) = \frac{\left(-ab(\lambda+\mu)\,\mathrm{e}^{x(\lambda+\mu)} - a^2\lambda\,\mathrm{e}^{2x\lambda} - \mathrm{e}^{2x\mu}b^2\mu - c\left(a\lambda\,\mathrm{e}^{x\lambda} + b\mu\,\mathrm{e}^{x\mu}\right)\right)\left(\int \frac{1}{\left(\mathrm{e}^{x\lambda}a + b\,\mathrm{e}^{x\mu}+c\right)^2}dx\right) - abc_1(\lambda+\mu)\,\mathrm{e}^{x(\lambda}}{\left(\mathrm{e}^{x\lambda}a + b\,\mathrm{e}^{x\mu}+c\right)^2\left(c_1 + \int \frac{1}{\left(\mathrm{e}^{x\lambda}a + b\,\mathrm{e}^{x\mu}+c\right)^2}dx\right)}$$

✓ Solution by Mathematica

Time used: 24.922 (sec). Leaf size: 393

```
DSolve[(a*Exp[\[Lambda]*x]+b*Exp[\[Mu]*x]+c)*(y'[x]-y[x]^2)+a*\[Lambda]^2*Exp[\[Lambda]*x]+b
```

$$\text{Solve}\left[\int_1^x -\frac{-ae^{\lambda K[1]}\lambda^2-be^{\mu K[1]}\mu^2+ae^{\lambda K[1]}y(x)^2+be^{\mu K[1]}y(x)^2+cy(x)^2}{\left(e^{\lambda K[1]}a+be^{\mu K[1]}+c\right)\left(ae^{\lambda K[1]}\lambda+be^{\mu K[1]}\mu+ae^{\lambda K[1]}y(x)+be^{\mu K[1]}y(x)+cy(x)\right)^2}dK[1]\right.$$

$$+\int_1^{y(x)}\left(\frac{1}{\left(ae^{x\lambda}\lambda+be^{x\mu}\mu+ae^{x\lambda}K[2]+be^{x\mu}K[2]+cK[2]\right)^2}\right.$$

$$-\int_1^x\left(\frac{2\left(-ae^{\lambda K[1]}\lambda^2-be^{\mu K[1]}\mu^2+ae^{\lambda K[1]}K[2]^2+be^{\mu K[1]}K[2]^2+cK[2]^2\right)}{\left(ae^{\lambda K[1]}\lambda+be^{\mu K[1]}\mu+ae^{\lambda K[1]}K[2]+be^{\mu K[1]}K[2]+cK[2]\right)^3}-\frac{2ae^{\lambda K[1]}}{\left(e^{\lambda K[1]}a+be^{\mu K[1]}+c\right)\left(ae^{\lambda K[1]}\right.}\right.$$

4 Chapter 1, section 1.2. Riccati Equation. subsection 1.2.3-2. Equations with power and exponential functions

4.1 problem 22 151
4.2 problem 23 152
4.3 problem 24 153
4.4 problem 25 154
4.5 problem 26 155
4.6 problem 27 156
4.7 problem 28 157
4.8 problem 29 158
4.9 problem 30 159
4.10 problem 31 160
4.11 problem 32 161
4.12 problem 33 162
4.13 problem 34 163
4.14 problem 35 164
4.15 problem 36 165
4.16 problem 37 166
4.17 problem 38 167
4.18 problem 39 168
4.19 problem 40 169

4.1 problem 22

Internal problem ID [10430]

Book: Handbook of exact solutions for ordinary differential equations. By Polyanin and Zaitsev. Second edition
Section: Chapter 1, section 1.2. Riccati Equation. subsection 1.2.3-2. Equations with power and exponential functions
Problem number: 22.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_Riccati]`

$$\boxed{y' - y^2 - ax\,\mathrm{e}^{\lambda x} y = \mathrm{e}^{\lambda x} a}$$

✓ Solution by Maple
Time used: 0.016 (sec). Leaf size: 85

```
dsolve(diff(y(x),x)=y(x)^2+a*x*exp(lambda*x)*y(x)+a*exp(lambda*x),y(x), singsol=all)
```

$$y(x) = \frac{-c_1\lambda^2 x + \left(\int \frac{\mathrm{e}^{\frac{\mathrm{e}^{x\lambda} a(x\lambda-1)}{\lambda^2}}}{x^2} dx\right) x + \mathrm{e}^{\frac{\mathrm{e}^{x\lambda} a(x\lambda-1)}{\lambda^2}}}{x^2\left(c_1\lambda^2 - \left(\int \frac{\mathrm{e}^{\frac{\mathrm{e}^{x\lambda} a(x\lambda-1)}{\lambda^2}}}{x^2} dx\right)\right)}$$

✓ Solution by Mathematica
Time used: 2.132 (sec). Leaf size: 110

```
DSolve[y'[x]==y[x]^2+a*x*Exp[\[Lambda]*x]*y[x]+a*Exp[\[Lambda]*x],y[x],x,IncludeSingularSolu
```

$$y(x) \to -\frac{x\int_1^x \frac{e^{\frac{ae^{\lambda K[1]}(\lambda K[1]-1)}{\lambda^2}}}{K[1]^2} dK[1] + e^{\frac{ae^{\lambda x}(\lambda x-1)}{\lambda^2}} + c_1 x}{x^2\left(\int_1^x \frac{e^{\frac{ae^{\lambda K[1]}(\lambda K[1]-1)}{\lambda^2}}}{K[1]^2} dK[1] + c_1\right)}$$

$$y(x) \to -\frac{1}{x}$$

4.2 problem 23

Internal problem ID [10431]

Book: Handbook of exact solutions for ordinary differential equations. By Polyanin and Zaitsev. Second edition
Section: Chapter 1, section 1.2. Riccati Equation. subsection 1.2.3-2. Equations with power and exponential functions
Problem number: 23.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[[_1st_order, _with_linear_symmetries], _Riccati]`

$$\boxed{y' - a\,\mathrm{e}^{\lambda x} y^2 = b\,\mathrm{e}^{-\lambda x}}$$

✓ Solution by Maple
Time used: 0.0 (sec). Leaf size: 64

```
dsolve(diff(y(x),x)=a*exp(lambda*x)*y(x)^2+b*exp(-lambda*x),y(x), singsol=all)
```

$$y(x) = -\frac{\left(\lambda^2 - \tan\left(\frac{\sqrt{4ab\,\lambda^2-\lambda^4}\,(x\lambda+c_1)}{2\lambda^2}\right)\sqrt{4ab\,\lambda^2-\lambda^4}\right)\mathrm{e}^{-x\lambda}}{2a\lambda}$$

✓ Solution by Mathematica
Time used: 0.624 (sec). Leaf size: 123

```
DSolve[y'[x]==a*Exp[\[Lambda]*x]*y[x]^2+b*Exp[-\[Lambda]*x],y[x],x,IncludeSingularSolutions
```

$$y(x) \to \frac{e^{\lambda(-x)}\left(-\sqrt{\lambda^2-4ab} + \frac{2}{\frac{1}{\sqrt{\lambda^2-4ab}}+c_1 e^{x\sqrt{\lambda^2-4ab}}} - \lambda\right)}{2a}$$
$$y(x) \to \frac{e^{\lambda(-x)}\left(4ab - \lambda\left(\sqrt{\lambda^2-4ab}+\lambda\right)\right)}{2a\sqrt{\lambda^2-4ab}}$$

4.3 problem 24

Internal problem ID [10432]

Book: Handbook of exact solutions for ordinary differential equations. By Polyanin and Zaitsev. Second edition
Section: Chapter 1, section 1.2. Riccati Equation. subsection 1.2.3-2. Equations with power and exponential functions
Problem number: 24.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_Riccati]`

$$y' - a\,\mathrm{e}^{\lambda x} y^2 = b\,x^{n-1} n - a\,b^2 \mathrm{e}^{\lambda x} x^{2n}$$

✗ Solution by Maple

```
dsolve(diff(y(x),x)=a*exp(lambda*x)*y(x)^2+b*n*x^(n-1)-a*b^2*exp(lambda*x)*x^(2*n),y(x), sin
```

No solution found

✗ Solution by Mathematica
Time used: 0.0 (sec). Leaf size: 0

```
DSolve[y'[x]==a*Exp[\[Lambda]*x]*y[x]^2+b*n*x^(n-1)-a*b^2*Exp[\[Lambda]*x]*x^(2*n),y[x],x,In
```

Not solved

4.4 problem 25

Internal problem ID [10433]

Book: Handbook of exact solutions for ordinary differential equations. By Polyanin and Zaitsev. Second edition
Section: Chapter 1, section 1.2. Riccati Equation. subsection 1.2.3-2. Equations with power and exponential functions
Problem number: 25.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_Riccati]`

$$\boxed{y' - \mathrm{e}^{\lambda x} y^2 - a\, x^n y = a\lambda\, x^n \mathrm{e}^{-\lambda x}}$$

✓ Solution by Maple
Time used: 0.016 (sec). Leaf size: 89

```
dsolve(diff(y(x),x)=exp(lambda*x)*y(x)^2+a*x^(n)*y(x)+a*lambda*x^n*exp(-lambda*x),y(x), sing
```

$$y(x) = -\frac{\mathrm{e}^{-x\lambda}\left(\left(\int \mathrm{e}^{\frac{x^{n+1}a - x\lambda(n+1)}{n+1}} dx\right)\lambda + \lambda c_1 + \mathrm{e}^{\frac{x^{n+1}a - x\lambda(n+1)}{n+1}}\right)}{c_1 + \int \mathrm{e}^{\frac{x(a\,x^n - (n+1)\lambda)}{n+1}} dx}$$

✓ Solution by Mathematica
Time used: 1.93 (sec). Leaf size: 254

```
DSolve[y'[x]==Exp[\[Lambda]*x]*y[x]^2+a*x^(n)*y[x]+a*\[Lambda]*x^n*Exp[-\[Lambda]*x],y[x],x,
```

$$\text{Solve}\left[\int_1^{y(x)} \left(\frac{e^{\frac{ax^{n+1}}{n+1}}}{\left(\lambda + e^{x\lambda} K[2]\right)^2}\right.\right.$$
$$\left. - \int_1^x \left(\frac{2e^{\frac{aK[1]^{n+1}}{n+1}}\left(a\lambda K[1]^n + ae^{\lambda K[1]} K[2] K[1]^n + e^{2\lambda K[1]} K[2]^2\right)}{\left(\lambda + e^{\lambda K[1]} K[2]\right)^3} - \frac{e^{\frac{aK[1]^{n+1}}{n+1} - \lambda K[1]}\left(ae^{\lambda K[1]} K[1]^n + 2e^{2\lambda K[1]} K\right.\right.\right.$$
$$\left.\left.\left. + \int_1^x - \frac{e^{\frac{aK[1]^{n+1}}{n+1} - \lambda K[1]}\left(a\lambda K[1]^n + ae^{\lambda K[1]} y(x) K[1]^n + e^{2\lambda K[1]} y(x)^2\right)}{\left(\lambda + e^{\lambda K[1]} y(x)\right)^2} dK[1] = c_1, y(x)\right]\right.\right.$$

4.5 problem 26

Internal problem ID [10434]

Book: Handbook of exact solutions for ordinary differential equations. By Polyanin and Zaitsev. Second edition
Section: Chapter 1, section 1.2. Riccati Equation. subsection 1.2.3-2. Equations with power and exponential functions
Problem number: 26.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_Riccati]`

$$\boxed{y' + \mathrm{e}^{\lambda x} y^2 \lambda - a\, x^n y\, \mathrm{e}^{\lambda x} = -x^n a}$$

✓ Solution by Maple
Time used: 0.0 (sec). Leaf size: 92

```
dsolve(diff(y(x),x)=-lambda*exp(lambda*x)*y(x)^2+a*x^(n)*exp(lambda*x)*y(x)-a*x^n,y(x), sing
```

$$y(x) = \frac{\mathrm{e}^{-x\lambda}\left(\int \mathrm{e}^{-x\lambda + a\left(\int \mathrm{e}^{x\lambda} x^n dx\right)} dx\right) c_1 \lambda + \lambda^2 \mathrm{e}^{-x\lambda} + c_1 \mathrm{e}^{-2x\lambda + a\left(\int \mathrm{e}^{x\lambda} x^n dx\right)}}{\lambda\left(\left(\int \mathrm{e}^{-x\lambda + a\left(\int \mathrm{e}^{x\lambda} x^n dx\right)} dx\right) c_1 + \lambda\right)}$$

✓ Solution by Mathematica
Time used: 6.627 (sec). Leaf size: 185

```
DSolve[y'[x]==-\[Lambda]*Exp[\[Lambda]*x]*y[x]^2+a*x^(n)*Exp[\[Lambda]*x]*y[x]-a*x^n,y[x],x,
```

$$y(x) \to \frac{e^{-2\lambda x}\left(e^{\lambda x}\int_1^{e^{x\lambda}} \frac{\exp\left(\frac{a\Gamma(n+1,-\log(K[1]))(-\log(K[1]))^{-n}\left(\frac{\log(K[1])}{\lambda}\right)^n}{\lambda}\right)}{K[1]^2} dK[1] + \exp\left(\frac{a(-\log(e^{\lambda x}))^{-n}\left(\frac{\log(e^{\lambda x})}{\lambda}\right)^n \Gamma(n+1,-\log}{\lambda}\right.\right.}{\int_1^{e^{x\lambda}} \frac{\exp\left(\frac{a\Gamma(n+1,-\log(K[1]))(-\log(K[1]))^{-n}\left(\frac{\log(K[1])}{\lambda}\right)^n}{\lambda}\right)}{K[1]^2} dK[1] + c_1}$$

$$y(x) \to e^{\lambda(-x)}$$

4.6 problem 27

Internal problem ID [10435]

Book: Handbook of exact solutions for ordinary differential equations. By Polyanin and Zaitsev. Second edition
Section: Chapter 1, section 1.2. Riccati Equation. subsection 1.2.3-2. Equations with power and exponential functions
Problem number: 27.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_Riccati]`

$$y' - a\,\mathrm{e}^{\lambda x} y^2 + ab\,x^n \mathrm{e}^{\lambda x} y = b\,x^{n-1} n$$

✓ Solution by Maple
Time used: 0.0 (sec). Leaf size: 88

```
dsolve(diff(y(x),x)=a*exp(lambda*x)*y(x)^2-a*b*x^(n)*exp(lambda*x)*y(x)+b*n*x^(n-1),y(x), si
```

$$y(x) = \frac{x^n \lambda \left(\int \mathrm{e}^{x\lambda + ab\left(\int \mathrm{e}^{x\lambda} x^n dx\right)} dx\right) c_1 ab + x^n ab - c_1 \lambda\, \mathrm{e}^{ab\left(\int \mathrm{e}^{x\lambda} x^n dx\right)}}{a\left(\lambda \left(\int \mathrm{e}^{x\lambda + ab\left(\int \mathrm{e}^{x\lambda} x^n dx\right)} dx\right) c_1 + 1\right)}$$

✓ Solution by Mathematica
Time used: 63.132 (sec). Leaf size: 188

```
DSolve[y'[x]==a*Exp[\[Lambda]*x]*y[x]^2-a*b*x^(n)*Exp[\[Lambda]*x]*y[x]+b*n*x^(n-1),y[x],x,I
```

$$y(x)$$

$$\to \frac{abc_1 \left(\frac{\log\left(e^{\lambda x}\right)}{\lambda}\right)^n \int_1^{e^{x\lambda}} \exp\left(\frac{ab\Gamma(n+1,-\log(K[1]))(-\log(K[1]))^{-n}\left(\frac{\log(K[1])}{\lambda}\right)^n}{\lambda}\right) dK[1] - c_1 \lambda \exp\left(\frac{ab(-\log\left(e^{\lambda x}\right))^{-n}\left(\frac{\log(}{}\right.\right.}{a + ac_1 \int_1^{e^{x\lambda}} \exp\left(\frac{ab\Gamma(n+1,-\log(K[1]))(-\log(K[1]))^{-n}\left(\frac{\log(K[1])}{\lambda}\right)^n}{\lambda}\right) dK[1]}$$

4.7 problem 28

Internal problem ID [10436]

Book: Handbook of exact solutions for ordinary differential equations. By Polyanin and Zaitsev. Second edition
Section: Chapter 1, section 1.2. Riccati Equation. subsection 1.2.3-2. Equations with power and exponential functions
Problem number: 28.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_Riccati]`

$$y' - a\,x^n y^2 = b\lambda\,\mathrm{e}^{\lambda x} - a\,b^2 x^n \mathrm{e}^{2\lambda x}$$

✗ Solution by Maple

```
dsolve(diff(y(x),x)=a*x^n*y(x)^2+b*lambda*exp(lambda*x)-a*b^2*x^n*exp(2*lambda*x),y(x), sing
```

No solution found

✗ Solution by Mathematica
Time used: 0.0 (sec). Leaf size: 0

```
DSolve[y'[x]==a*x^n*y[x]^2+b*\[Lambda]*Exp[\[Lambda]*x]-a*b^2*x^n*Exp[2*\[Lambda]*x],y[x],x,
```

Not solved

4.8 problem 29

Internal problem ID [10437]

Book: Handbook of exact solutions for ordinary differential equations. By Polyanin and Zaitsev. Second edition
Section: Chapter 1, section 1.2. Riccati Equation. subsection 1.2.3-2. Equations with power and exponential functions
Problem number: 29.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_Riccati]`

$$y' - a\,x^n y^2 - \lambda y = -a\,b^2 x^n \mathrm{e}^{2\lambda x}$$

✓ Solution by Maple
Time used: 0.0 (sec). Leaf size: 62

```
dsolve(diff(y(x),x)=a*x^n*y(x)^2+lambda*y(x)-a*b^2*x^n*exp(2*lambda*x),y(x), singsol=all)
```

$$y(x) = \tanh\left(\frac{-ab\,x^n(n\Gamma(n,-x\lambda)-\Gamma(n+1))\,(-x\lambda)^{-n} - ba\,\mathrm{e}^{x\lambda}x^n + i\lambda c_1}{\lambda}\right) b\,\mathrm{e}^{x\lambda}$$

✓ Solution by Mathematica
Time used: 1.69 (sec). Leaf size: 57

```
DSolve[y'[x]==a*x^n*y[x]^2+\[Lambda]*y[x]-a*b^2*x^n*Exp[2*\[Lambda]*x],y[x],x,IncludeSingula
```

$$y(x) \to \sqrt{-b^2}e^{\lambda x}\tan\left(\frac{a\sqrt{-b^2}x^n(\lambda(-x))^{-n}\Gamma(n+1,-x\lambda)}{\lambda} + c_1\right)$$

4.9 problem 30

Internal problem ID [10438]

Book: Handbook of exact solutions for ordinary differential equations. By Polyanin and Zaitsev. Second edition
Section: Chapter 1, section 1.2. Riccati Equation. subsection 1.2.3-2. Equations with power and exponential functions
Problem number: 30.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_Riccati]`

$$y' - a\,x^n y^2 + ab\,x^n \mathrm{e}^{\lambda x} y = b\lambda\,\mathrm{e}^{\lambda x}$$

✗ Solution by Maple

```
dsolve(diff(y(x),x)=a*x^n*y(x)^2-a*b*x^n*exp(lambda*x)*y(x)+b*lambda*exp(lambda*x),y(x), sin
```

No solution found

✓ Solution by Mathematica
Time used: 53.05 (sec). Leaf size: 190

```
DSolve[y'[x]==a*x^n*y[x]^2-a*b*x^n*Exp[\[Lambda]*x]*y[x]+b*\[Lambda]*Exp[\[Lambda]*x],y[x],x
```

$y(x)$

$$\to \frac{be^{2\lambda x}\left(\int_1^{e^{x\lambda}} \frac{\exp\left(\frac{ab\Gamma(n+1,-\log(K[1]))(-\log(K[1]))^{-n}\left(\frac{\log(K[1])}{\lambda}\right)^n}{\lambda}\right)}{K[1]^2} dK[1] + c_1\right)}{e^{\lambda x}\int_1^{e^{x\lambda}} \frac{\exp\left(\frac{ab\Gamma(n+1,-\log(K[1]))(-\log(K[1]))^{-n}\left(\frac{\log(K[1])}{\lambda}\right)^n}{\lambda}\right)}{K[1]^2} dK[1] + \exp\left(\frac{ab(-\log(e^{\lambda x}))^{-n}\left(\frac{\log\left(e^{\lambda x}\right)}{\lambda}\right)^n \Gamma(n+1,-\log(e^{x\lambda}))}{\lambda}\right.}$$

$y(x) \to be^{\lambda x}$

4.10 problem 31

Internal problem ID [10439]

Book: Handbook of exact solutions for ordinary differential equations. By Polyanin and Zaitsev. Second edition
Section: Chapter 1, section 1.2. Riccati Equation. subsection 1.2.3-2. Equations with power and exponential functions
Problem number: 31.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_Riccati]`

$$\boxed{y' + (k+1)\,x^k y^2 - a\,x^{k+1} \mathrm{e}^{\lambda x} y = -\mathrm{e}^{\lambda x} a}$$

✓ Solution by Maple
Time used: 0.032 (sec). Leaf size: 184

```
dsolve(diff(y(x),x)=-(k+1)*x^k*y(x)^2+a*x^(k+1)*exp(lambda*x)*y(x)-a*exp(lambda*x),y(x), sin
```

$$y(x) = \frac{x^{-1-k}\left(x^{1+k}\mathrm{e}^{\int \frac{x^{1+k}\mathrm{e}^{x\lambda}ax-2k-2}{x}dx} + \left(\int x^k \mathrm{e}^{a\left(\int x^{1+k}\mathrm{e}^{x\lambda}dx\right)-2\left(\int \frac{1}{x}dx\right)(1+k)}dx\right)k + \int x^k \mathrm{e}^{a\left(\int x^{1+k}\mathrm{e}^{x\lambda}dx\right)-2\left(\int \frac{1}{x}dx\right)(1+k)}\right.}{\left(\int x^k \mathrm{e}^{a\left(\int x^{1+k}\mathrm{e}^{x\lambda}dx\right)-2\left(\int \frac{1}{x}dx\right)(1+k)}dx\right)k + \int x^k \mathrm{e}^{a\left(\int x^{1+k}\mathrm{e}^{x\lambda}dx\right)-2\left(\int \frac{1}{x}dx\right)(1+k)}dx - c_1}$$

✓ Solution by Mathematica
Time used: 86.249 (sec). Leaf size: 280

```
DSolve[y'[x]==-(k+1)*x^k*y[x]^2+a*x^(k+1)*Exp[\[Lambda]*x]*y[x]-a*Exp[\[Lambda]*x],y[x],x,In
```

$$y(x) \to \frac{a\lambda \exp\left(\frac{a(-\log(e^{\lambda x}))^{-k}\left(\frac{\log(e^{\lambda x})}{\lambda}\right)^k \Gamma(k+2,-\log(e^{x\lambda}))}{\lambda^2}\right)\left(1 + c_1 \int_1^{e^{x\lambda}} \mathrm{ex}\right.}{ac_1\lambda\left(\frac{\log(e^{\lambda x})}{\lambda}\right)^{k+1}\exp\left(\frac{a(-\log(e^{\lambda x}))^{-k}\left(\frac{\log(e^{\lambda x})}{\lambda}\right)^k \Gamma(k+2,-\log(e^{x\lambda}))}{\lambda^2}\right)\int_1^{e^{x\lambda}} \exp\left(-\frac{a\Gamma(k+2,-\log(K[1]))(-\log(K[1]))^{-}}{\lambda^2}\right.}$$

4.11 problem 32

Internal problem ID [10440]

Book: Handbook of exact solutions for ordinary differential equations. By Polyanin and Zaitsev. Second edition
Section: Chapter 1, section 1.2. Riccati Equation. subsection 1.2.3-2. Equations with power and exponential functions
Problem number: 32.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_Riccati]`

$$y' - a\,x^n y^2 + a\,x^n\left(b\,\mathrm{e}^{\lambda x} + c\right) y = c\,x^n$$

✗ Solution by Maple

```
dsolve(diff(y(x),x)=a*x^n*y(x)^2-a*x^n*(b*exp(lambda*x)+c)*y(x)+c*x^n,y(x), singsol=all)
```

No solution found

✗ Solution by Mathematica
Time used: 0.0 (sec). Leaf size: 0

```
DSolve[y'[x]==a*x^n*y[x]^2-a*x^n*(b*Exp[\[Lambda]*x]+c)*y[x]+c*x^n,y[x],x,IncludeSingularSol
```

Not solved

4.12 problem 33

Internal problem ID [10441]

Book: Handbook of exact solutions for ordinary differential equations. By Polyanin and Zaitsev. Second edition
Section: Chapter 1, section 1.2. Riccati Equation. subsection 1.2.3-2. Equations with power and exponential functions
Problem number: 33.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_Riccati]`

$$y' - a\,x^n \mathrm{e}^{2\lambda x} y^2 - \left(b\,x^n \mathrm{e}^{\lambda x} - \lambda\right) y = c\,x^n$$

✓ Solution by Maple
Time used: 0.032 (sec). Leaf size: 114

```
dsolve(diff(y(x),x)=a*x^n*exp(2*lambda*x)*y(x)^2+(b*x^n*exp(lambda*x)-lambda)*y(x)+c*x^n,y(x
```

$$y(x) = \frac{\left(\tan\left(\frac{\sqrt{4a\,b^2c-b^4}\left(x^n(-x\lambda)^{-n}\Gamma(n,-x\lambda)bn-x^n(-x\lambda)^{-n}\Gamma(n+1)b+\mathrm{e}^{x\lambda}x^n b+\lambda c_1\right)}{2b^2\lambda}\right)\sqrt{4a\,b^2c-b^4}-b^2\right)\mathrm{e}^{-x\lambda}}{2ab}$$

✓ Solution by Mathematica
Time used: 3.112 (sec). Leaf size: 102

```
DSolve[y'[x]==a*x^n*Exp[2*\[Lambda]*x]*y[x]^2+(b*x^n*Exp[\[Lambda]*x]-\[Lambda])*y[x]+c*x^n,
```

$$\text{Solve}\left[\int_1^{\sqrt{\frac{ae^{2x\lambda}}{c}}y(x)} \frac{1}{K[1]^2-\sqrt{\frac{b^2}{ac}}K[1]+1}dK[1] = \frac{cx^n e^{\lambda(-x)}(\lambda(-x))^{-n}\sqrt{\frac{ae^{2\lambda x}}{c}}\Gamma(n+1,-x\lambda)}{\lambda} + c_1, y(x)\right]$$

4.13 problem 34

Internal problem ID [10442]

Book: Handbook of exact solutions for ordinary differential equations. By Polyanin and Zaitsev. Second edition
Section: Chapter 1, section 1.2. Riccati Equation. subsection 1.2.3-2. Equations with power and exponential functions
Problem number: 34.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[[_1st_order, '_with_symmetry_[F(x),G(x)]'], _Riccati]`

$$\boxed{y' - a\,\mathrm{e}^{\lambda x}(y - b\,x^n - c)^2 = b\,x^{n-1}n}$$

✓ Solution by Maple
Time used: 0.0 (sec). Leaf size: 50

```
dsolve(diff(y(x),x)=a*exp(lambda*x)*(y(x)-b*x^n-c)^2+b*n*x^(n-1),y(x), singsol=all)
```

$$y(x) = \frac{ac_1\lambda(b\,x^n + c)\,\mathrm{e}^{x\lambda} + x^n ab - c_1\lambda^2 + ac}{(\lambda c_1 \mathrm{e}^{x\lambda} + 1)\,a}$$

✓ Solution by Mathematica
Time used: 1.563 (sec). Leaf size: 40

```
DSolve[y'[x]==a*Exp[\[Lambda]*x]*(y[x]-b*x^n-c)^2+b*n*x^(n-1),y[x],x,IncludeSingularSolution
```

$$y(x) \to \frac{\lambda}{-ae^{\lambda x} + c_1\lambda} + bx^n + c$$
$$y(x) \to bx^n + c$$

4.14 problem 35

Internal problem ID [10443]

Book: Handbook of exact solutions for ordinary differential equations. By Polyanin and Zaitsev. Second edition
Section: Chapter 1, section 1.2. Riccati Equation. subsection 1.2.3-2. Equations with power and exponential functions
Problem number: 35.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_Riccati]`

$$y'x - a\,\mathrm{e}^{\lambda x}y^2 - ky = a\,b^2x^{2k}\mathrm{e}^{\lambda x}$$

✓ Solution by Maple
Time used: 0.015 (sec). Leaf size: 38

```
dsolve(x*diff(y(x),x)=a*exp(lambda*x)*y(x)^2+k*y(x)+a*b^2*x^(2*k)*exp(lambda*x),y(x), singso
```

$$y(x) = -\tan\left(ab\,x^k(\Gamma(k,-x\lambda)-\Gamma(k))\,(-x\lambda)^{-k}+c_1\right)b\,x^k$$

✓ Solution by Mathematica
Time used: 1.593 (sec). Leaf size: 47

```
DSolve[x*y'[x]==a*Exp[\[Lambda]*x]*y[x]^2+k*y[x]+a*b^2*x^(2*k)*Exp[\[Lambda]*x],y[x],x,Inclu
```

$$y(x) \to \sqrt{b^2}x^k\tan\left(-a\sqrt{b^2}x^k(\lambda(-x))^{-k}\Gamma(k,-x\lambda)+c_1\right)$$

4.15 problem 36

Internal problem ID [10444]

Book: Handbook of exact solutions for ordinary differential equations. By Polyanin and Zaitsev. Second edition
Section: Chapter 1, section 1.2. Riccati Equation. subsection 1.2.3-2. Equations with power and exponential functions
Problem number: 36.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_Riccati]`

$$\boxed{y'x - a\,x^{2n}\mathrm{e}^{\lambda x}y^2 - \left(b\,x^n\mathrm{e}^{\lambda x} - n\right)y = \mathrm{e}^{\lambda x}c}$$

✓ Solution by Maple
Time used: 0.015 (sec). Leaf size: 86

```
dsolve(x*diff(y(x),x)=a*x^(2*n)*exp(lambda*x)*y(x)^2+(b*x^n*exp(lambda*x)-n)*y(x)+c*exp(lamb
```

$$y(x) = -\frac{\left(\tan\left(\frac{\left(x^n b(\Gamma(n,-x\lambda)-\Gamma(n))(-x\lambda)^{-n}-c_1\right)\sqrt{4a\,b^2c-b^4}}{2b^2}\right)\sqrt{4a\,b^2c-b^4}+b^2\right)x^{-n}}{2ab}$$

✓ Solution by Mathematica
Time used: 3.62 (sec). Leaf size: 87

```
DSolve[x*y'[x]==a*x^(2*n)*Exp[\[Lambda]*x]*y[x]^2+(b*x^n*Exp[\[Lambda]*x]-n)*y[x]+c*Exp[\[La
```

$$\text{Solve}\left[\int_1^{\sqrt{\frac{ax^{2n}}{c}}y(x)}\frac{1}{K[1]^2-\sqrt{\frac{b^2}{ac}}K[1]+1}dK[1] = -c(\lambda(-x))^{-n}\sqrt{\frac{ax^{2n}}{c}}\Gamma(n,-x\lambda)+c_1, y(x)\right]$$

4.16 problem 37

Internal problem ID [10445]

Book: Handbook of exact solutions for ordinary differential equations. By Polyanin and Zaitsev. Second edition
Section: Chapter 1, section 1.2. Riccati Equation. subsection 1.2.3-2. Equations with power and exponential functions
Problem number: 37.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_Riccati]`

$$y' - y^2 = 2a\lambda x\, \mathrm{e}^{\lambda x^2} - a^2 \mathrm{e}^{2\lambda x^2}$$

✗ Solution by Maple

```
dsolve(diff(y(x),x)=y(x)^2+2*a*lambda*x*exp(lambda*x^2)-a^2*exp(2*lambda*x^2),y(x), singsol=
```

No solution found

✗ Solution by Mathematica
Time used: 0.0 (sec). Leaf size: 0

```
DSolve[y'[x]==y[x]^2+2*a*\[Lambda]*x*Exp[\[Lambda]*x^2]-a^2*Exp[2*\[Lambda]*x^2],y[x],x,Incl
```

Not solved

4.17 problem 38

Internal problem ID [10446]

Book: Handbook of exact solutions for ordinary differential equations. By Polyanin and Zaitsev. Second edition
Section: Chapter 1, section 1.2. Riccati Equation. subsection 1.2.3-2. Equations with power and exponential functions
Problem number: 38.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_Riccati]`

$$\boxed{y' - a\,\mathrm{e}^{-\lambda x^2} y^2 - y\lambda x = a\,b^2}$$

✓ Solution by Maple
Time used: 0.016 (sec). Leaf size: 45

```
dsolve(diff(y(x),x)=a*exp(-lambda*x^2)*y(x)^2+lambda*x*y(x)+a*b^2,y(x), singsol=all)
```

$$y(x) = \tan\left(\frac{ab\sqrt{\pi}\,\sqrt{2}\,\operatorname{erf}\left(\frac{\sqrt{2}\sqrt{\lambda}\,x}{2}\right) - 2c_1\sqrt{\lambda}}{2\sqrt{\lambda}}\right) b\,\mathrm{e}^{\frac{x^2\lambda}{2}}$$

✓ Solution by Mathematica
Time used: 2.252 (sec). Leaf size: 63

```
DSolve[y'[x]==a*Exp[-\[Lambda]*x^2]*y[x]^2+\[Lambda]*x*y[x]+a*b^2,y[x],x,IncludeSingularSolu
```

$$y(x) \to \sqrt{b^2} e^{\frac{\lambda x^2}{2}} \tan\left(\frac{\sqrt{\frac{\pi}{2}} a \sqrt{b^2} \operatorname{erf}\left(\frac{\sqrt{\lambda} x}{\sqrt{2}}\right)}{\sqrt{\lambda}} + c_1\right)$$

4.18 problem 39

Internal problem ID [10447]

Book: Handbook of exact solutions for ordinary differential equations. By Polyanin and Zaitsev. Second edition
Section: Chapter 1, section 1.2. Riccati Equation. subsection 1.2.3-2. Equations with power and exponential functions
Problem number: 39.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_Riccati]`

$$y' - a\,x^n y^2 - y\lambda x = a\,b^2 x^n \mathrm{e}^{\lambda x^2}$$

✓ Solution by Maple
Time used: 0.016 (sec). Leaf size: 90

```
dsolve(diff(y(x),x)=a*x^n*y(x)^2+lambda*x*y(x)+a*b^2*x^n*exp(lambda*x^2),y(x), singsol=all)
```

$$y(x) = -\tan\left(-ab2^{\frac{n}{2}-\frac{1}{2}}x^{n+1}\Gamma\left(\frac{n}{2}+\frac{1}{2}\right)\left(-x^2\lambda\right)^{-\frac{n}{2}-\frac{1}{2}} + ab2^{\frac{n}{2}-\frac{1}{2}}x^{n+1}\left(-x^2\lambda\right)^{-\frac{n}{2}-\frac{1}{2}}\Gamma\left(\frac{n}{2}+\frac{1}{2},-\frac{x^2\lambda}{2}\right)+c_1\right)b\,\mathrm{e}^{\frac{x^2\lambda}{2}}$$

✓ Solution by Mathematica
Time used: 2.366 (sec). Leaf size: 83

```
DSolve[y'[x]==a*x^n*y[x]^2+\[Lambda]*x*y[x]+a*b^2*x^n*Exp[\[Lambda]*x^2],y[x],x,IncludeSingu
```

$$y(x) \to \sqrt{b^2}e^{\frac{\lambda x^2}{2}}\tan\left(a\sqrt{b^2}\lambda 2^{\frac{n-1}{2}}x^{n+3}\left(\lambda\left(-x^2\right)\right)^{-\frac{n}{2}-\frac{3}{2}}\Gamma\left(\frac{n+1}{2},-\frac{x^2\lambda}{2}\right)+c_1\right)$$

4.19 problem 40

Internal problem ID [10448]

Book: Handbook of exact solutions for ordinary differential equations. By Polyanin and Zaitsev. Second edition
Section: Chapter 1, section 1.2. Riccati Equation. subsection 1.2.3-2. Equations with power and exponential functions
Problem number: 40.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_Riccati]`

$$\boxed{x^4\left(y'-y^2\right)=a+b\,\mathrm{e}^{\frac{k}{x}}+c\,\mathrm{e}^{\frac{2k}{x}}}$$

✓ Solution by Maple
Time used: 0.0 (sec). Leaf size: 302

```
dsolve(x^4*(diff(y(x),x)-y(x)^2)=a+b*exp(k/x)+c*exp(2*k/x),y(x), singsol=all)
```

$$y(x)$$
$$=\frac{\left(\left(i\sqrt{a}+\frac{k}{2}\right)\sqrt{c}-\frac{ib}{2}\right)\operatorname{WhittakerM}\left(-\frac{ib-2k\sqrt{c}}{2k\sqrt{c}},\frac{i\sqrt{a}}{k},\frac{2i\sqrt{c}\,\mathrm{e}^{\frac{k}{x}}}{k}\right)-c_1 k\operatorname{WhittakerW}\left(-\frac{ib-2k\sqrt{c}}{2k\sqrt{c}},\frac{i\sqrt{a}}{k},\frac{2i\sqrt{c}\,\mathrm{e}^{\frac{k}{x}}}{k}\right)}{\sqrt{c}\,x^2\left(\operatorname{WhittakerW}\left(-\frac{ib}{2k\sqrt{c}},\frac{i\sqrt{a}}{k},\frac{2i\sqrt{}}{}\right.\right.}$$

✓ Solution by Mathematica

Time used: 4.039 (sec). Leaf size: 940

```
DSolve[x^4*(y'[x]-y[x]^2)==a+b*Exp[k/x]+c*Exp[2*k/x],y[x],x,IncludeSingularSolutions -> True
```

$$y(x)$$

$$\to \frac{e^{k/x}\log\left(e^{k/x}\right)\left(c_1\left(b+\sqrt{c}\left(2\sqrt{a}-ik\right)\right)\text{HypergeometricU}\left(\frac{\frac{ib}{\sqrt{c}}+3k+2i\sqrt{a}}{2k},\frac{2i\sqrt{a}}{k}+2,\frac{2i\sqrt{c}e^{k/x}}{k}\right)-2i\sqrt{c}kL^{\frac{2i}{-}}\right.}{kx^2\log\left(e^{k/x}\right)}$$

$$y(x)$$

$$\to \frac{\frac{e^{k/x}\left(b+\sqrt{c}\left(2\sqrt{a}-ik\right)\right)\text{HypergeometricU}\left(\frac{\frac{ib}{\sqrt{c}}+3k+2i\sqrt{a}}{2k},\frac{2i\sqrt{a}}{k}+2,\frac{2i\sqrt{c}e^{k/x}}{k}\right)}{k\,\text{HypergeometricU}\left(\frac{\frac{ib}{\sqrt{c}}+k+2i\sqrt{a}}{2k},\frac{2i\sqrt{a}}{k}+1,\frac{2i\sqrt{c}e^{k/x}}{k}\right)}+i\left(\sqrt{a}-\sqrt{c}e^{k/x}\right)-\frac{k}{\log\left(e^{k/x}\right)}}{x^2}$$

$$y(x)$$

$$\to \frac{\frac{e^{k/x}\left(b+\sqrt{c}\left(2\sqrt{a}-ik\right)\right)\text{HypergeometricU}\left(\frac{\frac{ib}{\sqrt{c}}+3k+2i\sqrt{a}}{2k},\frac{2i\sqrt{a}}{k}+2,\frac{2i\sqrt{c}e^{k/x}}{k}\right)}{k\,\text{HypergeometricU}\left(\frac{\frac{ib}{\sqrt{c}}+k+2i\sqrt{a}}{2k},\frac{2i\sqrt{a}}{k}+1,\frac{2i\sqrt{c}e^{k/x}}{k}\right)}+i\left(\sqrt{a}-\sqrt{c}e^{k/x}\right)-\frac{k}{\log\left(e^{k/x}\right)}}{x^2}$$

5 Chapter 1, section 1.2. Riccati Equation. subsection 1.2.4-1. Equations with hyperbolic sine and cosine

5.1 problem 1 172
5.2 problem 2 174
5.3 problem 3 175
5.4 problem 4 176
5.5 problem 5 177
5.6 problem 6 179
5.7 problem 7 181
5.8 problem 8 183
5.9 problem 9 184
5.10 problem 10 185
5.11 problem 11 186
5.12 problem 12 188
5.13 problem 13 190
5.14 problem 14 191
5.15 problem 15 193
5.16 problem 16 194
5.17 problem 17 196

5.1 problem 1

Internal problem ID [10449]

Book: Handbook of exact solutions for ordinary differential equations. By Polyanin and Zaitsev. Second edition
Section: Chapter 1, section 1.2. Riccati Equation. subsection 1.2.4-1. Equations with hyperbolic sine and cosine
Problem number: 1.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_Riccati]`

$$\boxed{y' - y^2 = -a^2 + a\lambda \sinh(\lambda x) - a^2 \sinh(\lambda x)^2}$$

✓ Solution by Maple
Time used: 0.0 (sec). Leaf size: 318

```
dsolve(diff(y(x),x)=y(x)^2-a^2+a*lambda*sinh(lambda*x)-a^2*sinh(lambda*x)^2,y(x), singsol=al
```

$$y(x) = \frac{\left(-2\sinh\left(\frac{i\pi}{4}+\frac{x\lambda}{2}\right)\cosh(x\lambda)\,c_1 a - \cosh\left(\frac{i\pi}{4}+\frac{x\lambda}{2}\right)c_1\lambda\right)\mathrm{HeunC}\left(\frac{4ia}{\lambda},\frac{1}{2},-\frac{1}{2},\frac{2ia}{\lambda},-\frac{8ia-3\lambda}{8\lambda},-\frac{i\sinh(x\lambda)}{2}+\frac{1}{2}\right)+}{2\sinh\left(\frac{i}{4}\right.}$$

✓ Solution by Mathematica

Time used: 11.807 (sec). Leaf size: 162

```
DSolve[y'[x]==y[x]^2-a^2+a*\[Lambda]*Sinh[\[Lambda]*x]-a^2*Sinh[\[Lambda]*x]^2,y[x],x,Includ
```

$$y(x)$$

$$\to \frac{e^{\lambda(-x)}\left(a\left(e^{2\lambda x}+1\right)\int_1^{e^{x\lambda}} \frac{e^{\frac{a\left(K[1]^2-1\right)}{\lambda K[1]}}}{K[1]}dK[1]-2\lambda e^{\frac{ae^{\lambda(-x)}\left(e^{2\lambda x}-1\right)}{\lambda}+\lambda x}+ac_1e^{2\lambda x}+ac_1\right)}{2\left(\int_1^{e^{x\lambda}} \frac{e^{\frac{a\left(K[1]^2-1\right)}{\lambda K[1]}}}{K[1]}dK[1]+c_1\right)}$$

$$y(x)\to \frac{1}{2}ae^{\lambda(-x)}\left(e^{2\lambda x}+1\right)$$

5.2 problem 2

Internal problem ID [10450]

Book: Handbook of exact solutions for ordinary differential equations. By Polyanin and Zaitsev. Second edition
Section: Chapter 1, section 1.2. Riccati Equation. subsection 1.2.4-1. Equations with hyperbolic sine and cosine
Problem number: 2.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_Riccati]`

$$\boxed{y' - y^2 - a\sinh(\beta x)\, y = ab\sinh(\beta x) - b^2}$$

✓ Solution by Maple
Time used: 0.0 (sec). Leaf size: 73

```
dsolve(diff(y(x),x)=y(x)^2+a*sinh(beta*x)*y(x)+a*b*sinh(beta*x)-b^2,y(x), singsol=all)
```

$$y(x) = \frac{b\left(\int \mathrm{e}^{\frac{-2b\beta x + a\cosh(x\beta)}{\beta}} dx\right) - c_1 b + \mathrm{e}^{\frac{-2b\beta x + a\cosh(x\beta)}{\beta}}}{-\left(\int \mathrm{e}^{\frac{-2b\beta x + a\cosh(x\beta)}{\beta}} dx\right) + c_1}$$

✓ Solution by Mathematica
Time used: 9.168 (sec). Leaf size: 183

```
DSolve[y'[x]==y[x]^2+a*Sinh[\[Beta]*x]*y[x]+a*b*Sinh[\[Beta]*x]-b^2,y[x],x,IncludeSingularSo
```

$$\text{Solve}\left[\int_1^x -\frac{e^{\frac{a\cosh(\beta K[1])}{\beta} - 2bK[1]}(-b + a\sinh(\beta K[1]) + y(x))}{a\beta(b + y(x))} dK[1] + \int_1^{y(x)} \left(\frac{e^{\frac{a\cosh(x\beta)}{\beta} - 2bx}}{a\beta(b + K[2])^2} - \int_1^x \left(\frac{e^{\frac{a\cosh(\beta K[1])}{\beta} - 2bK[1]}(-b + K[2] + a\sinh(\beta K[1]))}{a\beta(b + K[2])^2} - \frac{e^{\frac{a\cosh(\beta K[1])}{\beta} - 2bK[1]}}{a\beta(b + K[2])} \right) dK[1] \right) dK[2] = c_1, y(x)\right]$$

5.3 problem 3

Internal problem ID [10451]

Book: Handbook of exact solutions for ordinary differential equations. By Polyanin and Zaitsev. Second edition
Section: Chapter 1, section 1.2. Riccati Equation. subsection 1.2.4-1. Equations with hyperbolic sine and cosine
Problem number: 3.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_Riccati]`

$$\boxed{y' - y^2 - ax\sinh(bx)^m y = a\sinh(bx)^m}$$

✓ Solution by Maple
Time used: 0.0 (sec). Leaf size: 85

```
dsolve(diff(y(x),x)=y(x)^2+a*x*sinh(b*x)^m*y(x)+a*sinh(b*x)^m,y(x), singsol=all)
```

$$y(x) = \frac{-e^{\int \frac{\sinh(bx)^m x^2 a - 2}{x} dx} x - \left(\int e^{\int \frac{\sinh(bx)^m x^2 a - 2}{x} dx} dx\right) + c_1}{\left(-c_1 + \int e^{\int \frac{\sinh(bx)^m x^2 a - 2}{x} dx} dx\right) x}$$

✓ Solution by Mathematica
Time used: 7.437 (sec). Leaf size: 379

```
DSolve[y'[x]==y[x]^2+a*x*Sinh[b*x]^m*y[x]+a*Sinh[b*x]^m,y[x],x,IncludeSingularSolutions -> T
```

$$y(x) \to -\frac{\int_1^x \frac{\exp\left(-\frac{a\left(-e^{-bK[1]}+e^{bK[1]}\right)^m \left(2-2e^{2bK[1]}\right)^{-m}\left({}_3F_2\left(-m,-\frac{m}{2},-\frac{m}{2};1-\frac{m}{2},1-\frac{m}{2};e^{2bK[1]}\right)+bm\,\text{Hypergeometric2F1}\left(-m,-\frac{m}{2},1-\frac{m}{2},e^{2bK[1]}\right)K[1}{b^2m^2}\right.}{K[1]^2}}{x\left(\int_1^x \frac{\exp\left(-\frac{a\left(-e^{-bK[1]}+e^{bK[1]}\right)^m \left(2-2e^{2bK[1]}\right)^{-m}\left({}_3F_2\left(-m,-\frac{m}{2},-\frac{m}{2};1-\frac{m}{2}\right.\right.}{b^2}\right.}{K[1]}\right.}$$

$$y(x) \to -\frac{1}{x}$$

5.4 problem 4

Internal problem ID [10452]

Book: Handbook of exact solutions for ordinary differential equations. By Polyanin and Zaitsev. Second edition
Section: Chapter 1, section 1.2. Riccati Equation. subsection 1.2.4-1. Equations with hyperbolic sine and cosine
Problem number: 4.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_Riccati]`

$$\boxed{y' - \lambda \sinh(\lambda x)\, y^2 = -\lambda \sinh(\lambda x)^3}$$

✓ Solution by Maple
Time used: 0.0 (sec). Leaf size: 51

```
dsolve(diff(y(x),x)=lambda*sinh(lambda*x)*y(x)^2-lambda*sinh(lambda*x)^3,y(x), singsol=all)
```

$$y(x) = -\frac{2\left(\mathrm{e}^{\frac{\cosh(2x\lambda)}{2}+\frac{1}{2}}c_1 - \frac{\cosh(x\lambda)\sqrt{\pi}\,(\operatorname{erfi}(\cosh(x\lambda))c_1+1)}{2}\right)}{\sqrt{\pi}\,(\operatorname{erfi}(\cosh(x\lambda))\,c_1+1)}$$

✗ Solution by Mathematica
Time used: 0.0 (sec). Leaf size: 0

```
DSolve[y'[x]==\[Lambda]*Sinh[\[Lambda]*x]*y[x]^2-\[Lambda]*Sinh[\[Lambda]*x]^3,y[x],x,Includ
```

Not solved

5.5 problem 5

Internal problem ID [10453]

Book: Handbook of exact solutions for ordinary differential equations. By Polyanin and Zaitsev. Second edition
Section: Chapter 1, section 1.2. Riccati Equation. subsection 1.2.4-1. Equations with hyperbolic sine and cosine
Problem number: 5.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_Riccati]`

$$\boxed{y' - \left(a \sinh\left(\lambda x\right)^2 - \lambda\right) y^2 = -a \sinh\left(\lambda x\right)^2 + \lambda - a}$$

✓ Solution by Maple
Time used: 0.015 (sec). Leaf size: 104

```
dsolve(diff(y(x),x)=(a*sinh(lambda*x)^2-lambda)*y(x)^2-a*sinh(lambda*x)^2+lambda-a,y(x), sin
```

$$y(x) = \frac{2 \coth\left(x\lambda\right) \lambda \left(\int -\mathrm{e}^{\frac{a \cosh(2x\lambda)}{2\lambda}} \left(a - \operatorname{csch}\left(x\lambda\right)^2 \lambda\right) dx\right) c_1 + 2 \operatorname{csch}\left(x\lambda\right)^2 \mathrm{e}^{\frac{a \cosh(2x\lambda)}{2\lambda}} c_1 \lambda - \coth\left(x\lambda\right)}{2\lambda \left(\int -\mathrm{e}^{\frac{a \cosh(2x\lambda)}{2\lambda}} \left(a - \operatorname{csch}\left(x\lambda\right)^2 \lambda\right) dx\right) c_1 - 1}$$

✓ Solution by Mathematica

Time used: 50.151 (sec). Leaf size: 211

```
DSolve[y'[x]==(a*Sinh[\[Lambda]*x]^2-\[Lambda])*y[x]^2-a*Sinh[\[Lambda]*x]^2+\[Lambda]-a,y[x
```

$$y(x)$$

$$\to \frac{\operatorname{csch}^2(\lambda x)\left(c_1 \sinh(2\lambda x)\int_1^x e^{\frac{a\sinh^2(\lambda K[1])}{\lambda}}\operatorname{csch}^2(\lambda K[1])\left(\lambda - a\sinh^2(\lambda K[1])\right)dK[1] + 2c_1 e^{\frac{a\sinh^2(\lambda x)}{\lambda}} + \sinh($$

$$2 + 2c_1\int_1^x e^{\frac{a\sinh^2(\lambda K[1])}{\lambda}}\operatorname{csch}^2(\lambda K[1])\left(\lambda - a\sinh^2(\lambda K[1])\right)dK[1]$$

$$y(x) \to \frac{1}{2}\operatorname{csch}^2(\lambda x)\left(\frac{2e^{\frac{a\sinh^2(\lambda x)}{\lambda}}}{\int_1^x e^{\frac{a\sinh^2(\lambda K[1])}{\lambda}}\operatorname{csch}^2(\lambda K[1])\left(\lambda - a\sinh^2(\lambda K[1])\right)dK[1]} + \sinh(2\lambda x)\right)$$

5.6 problem 6

Internal problem ID [10454]

Book: Handbook of exact solutions for ordinary differential equations. By Polyanin and Zaitsev. Second edition
Section: Chapter 1, section 1.2. Riccati Equation. subsection 1.2.4-1. Equations with hyperbolic sine and cosine
Problem number: 6.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_Riccati]`

$$\boxed{(a\sinh(\lambda x)+b)\,y'-y^2-c\sinh(x\mu)\,y=-d^2+cd\sinh(x\mu)}$$

✓ Solution by Maple
Time used: 0.0 (sec). Leaf size: 253

```
dsolve((a*sinh(lambda*x)+b)*diff(y(x),x)=y(x)^2+c*sinh(mu*x)*y(x)-d^2+c*d*sinh(mu*x),y(x), s
```

$$y(x)$$

$$= \frac{-d\left(\int \frac{\mathrm{e}^{\frac{c\left(\int \frac{\sinh(x\mu)}{\sinh(x\lambda)a+b}dx\right)\lambda\sqrt{a^2+b^2}+4d\ \operatorname{arctanh}\left(\frac{-\tanh\left(\frac{x\lambda}{2}\right)b+a}{\sqrt{a^2+b^2}}\right)}{\lambda\sqrt{a^2+b^2}}}{\sinh(x\lambda)a+b}dx\right)+dc_1-\mathrm{e}^{\frac{c\left(\int \frac{\sinh(x\mu)}{\sinh(x\lambda)a+b}dx\right)\lambda\sqrt{a^2+b^2}+4d\ \operatorname{arctanh}\left(\frac{-\tanh\left(\frac{x\lambda}{2}\right)b+a}{\sqrt{a^2+b^2}}\right)}{\lambda\sqrt{a^2+b^2}}}}{\int \frac{\mathrm{e}^{\frac{c\left(\int \frac{\sinh(x\mu)}{\sinh(x\lambda)a+b}dx\right)\lambda\sqrt{a^2+b^2}+4d\ \operatorname{arctanh}\left(\frac{-\tanh\left(\frac{x\lambda}{2}\right)b+a}{\sqrt{a^2+b^2}}\right)}{\lambda\sqrt{a^2+b^2}}}}{\sinh(x\lambda)a+b}dx-c_1}$$

✓ Solution by Mathematica

Time used: 28.506 (sec). Leaf size: 289

```
DSolve[(a*Sinh[\[Lambda]*x]+b)*y'[x]==y[x]^2+c*Sinh[\[Mu]*x]*y[x]-d^2+c*d*Sinh[\[Mu]*x],y[x]
```

$$\text{Solve}\left[\int_1^x -\frac{\exp\left(-\int_1^{K[2]} \frac{2d-c\sinh(\mu K[1])}{b+a\sinh(\lambda K[1])}dK[1]\right)(-d+c\sinh(\mu K[2])+y(x))}{c\mu(b+a\sinh(\lambda K[2]))(d+y(x))}dK[2]\right.$$

$$+\int_1^{y(x)}\left(\frac{\exp\left(-\int_1^x \frac{2d-c\sinh(\mu K[1])}{b+a\sinh(\lambda K[1])}dK[1]\right)}{c\mu(d+K[3])^2}\right.$$

$$-\int_1^x\left(\frac{\exp\left(-\int_1^{K[2]} \frac{2d-c\sinh(\mu K[1])}{b+a\sinh(\lambda K[1])}dK[1]\right)(-d+K[3]+c\sinh(\mu K[2]))}{c\mu(d+K[3])^2(b+a\sinh(\lambda K[2]))}-\frac{\exp\left(-\int_1^{K[2]} \frac{2d-c\sinh(\mu K[1])}{b+a\sinh(\lambda K[1])}dK[1\right.}{c\mu(d+K[3])(b+a\sinh(\lambda K[2])}$$

5.7 problem 7

Internal problem ID [10455]

Book: Handbook of exact solutions for ordinary differential equations. By Polyanin and Zaitsev. Second edition
Section: Chapter 1, section 1.2. Riccati Equation. subsection 1.2.4-1. Equations with hyperbolic sine and cosine
Problem number: 7.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_Riccati]`

$$\boxed{(a\sinh(\lambda x)+b)\left(y'-y^2\right)=-a\,\lambda^2\sinh(\lambda x)}$$

✓ Solution by Maple
Time used: 0.016 (sec). Leaf size: 250

```
dsolve((a*sinh(lambda*x)+b)*(diff(y(x),x)-y(x)^2)+a*lambda^2*sinh(lambda*x)=0,y(x), singsol=
```

$$y(x)=$$
$$-\frac{4\left(\left(\operatorname{arctanh}\left(\frac{-\tanh\left(\frac{x\lambda}{2}\right)b+a}{\sqrt{a^2+b^2}}\right)a^2b^2+\operatorname{arctanh}\left(\frac{-\tanh\left(\frac{x\lambda}{2}\right)b+a}{\sqrt{a^2+b^2}}\right)b^4-c_1\right)a\left(\cosh\left(\frac{x\lambda}{2}\right)^2-\frac{1}{2}\right)\sqrt{a^2}}{\sqrt{a^2+b^2}\left(2a\cosh\left(\frac{x\lambda}{2}\right)\left(a^2+b^2\right)^{\frac{3}{2}}\left(a\sinh\left(\frac{x\lambda}{2}\right)+b\cosh\left(\frac{x\lambda}{2}\right)\right)+4\left(\operatorname{arctanh}\left(\frac{-\tanh\left(\frac{x\lambda}{2}\right)b+a}{\sqrt{a^2+b^2}}\right)a^2b^2+a\right.\right.}$$

✓ Solution by Mathematica

Time used: 24.532 (sec). Leaf size: 202

```
DSolve[(a*Sinh[\[Lambda]*x]+b)*(y'[x]-y[x]^2)+a*\[Lambda]^2*Sinh[\[Lambda]*x]==0,y[x],x,Incl
```

$$y(x) \to$$

$$-\frac{\lambda\left(\sqrt{-a^2-b^2}(b-a\sinh(\lambda x))+a\cosh(\lambda x)\left(2b\arctan\left(\frac{a-b\tanh\left(\frac{\lambda x}{2}\right)}{\sqrt{-a^2-b^2}}\right)-c_1\lambda(-a^2-b^2)^{3/2}\right)\right)}{-a\sqrt{-a^2-b^2}\cosh(\lambda x)+(a\sinh(\lambda x)+b)\left(2b\arctan\left(\frac{a-b\tanh\left(\frac{\lambda x}{2}\right)}{\sqrt{-a^2-b^2}}\right)-c_1\lambda\left(-a^2-b^2\right)^{3/2}\right)}$$

$$y(x) \to -\frac{a\lambda\cosh(\lambda x)}{a\sinh(\lambda x)+b}$$

5.8 problem 8

Internal problem ID [10456]

Book: Handbook of exact solutions for ordinary differential equations. By Polyanin and Zaitsev. Second edition
Section: Chapter 1, section 1.2. Riccati Equation. subsection 1.2.4-1. Equations with hyperbolic sine and cosine
Problem number: 8.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_Riccati]`

$$\boxed{y' - \alpha y^2 = \beta + \gamma \cosh(x)}$$

✓ Solution by Maple
Time used: 0.0 (sec). Leaf size: 70

```
dsolve(diff(y(x),x)=alpha*y(x)^2+beta+gamma*cosh(x),y(x), singsol=all)
```

$$y(x) = -\frac{i\left(c_1 \operatorname{MathieuSPrime}\left(-4\alpha\beta, 2\gamma\alpha, \frac{ix}{2}\right) + \operatorname{MathieuCPrime}\left(-4\alpha\beta, 2\gamma\alpha, \frac{ix}{2}\right)\right)}{2\alpha\left(c_1 \operatorname{MathieuS}\left(-4\alpha\beta, 2\gamma\alpha, \frac{ix}{2}\right) + \operatorname{MathieuC}\left(-4\alpha\beta, 2\gamma\alpha, \frac{ix}{2}\right)\right)}$$

✓ Solution by Mathematica
Time used: 0.543 (sec). Leaf size: 140

```
DSolve[y'[x]==\[Alpha]*y[x]^2+\[Beta]+\[Gamma]*Cosh[x],y[x],x,IncludeSingularSolutions -> Tr
```

$$y(x) \to -\frac{ic_1 \operatorname{MathieuCPrime}\left[-4\alpha\beta, 2\alpha\gamma, \frac{ix}{2}\right] - i\operatorname{MathieuSPrime}\left[-4\alpha\beta, 2\alpha\gamma, \frac{ix}{2}\right]}{2\alpha c_1 \operatorname{MathieuC}\left[-4\alpha\beta, 2\alpha\gamma, \frac{ix}{2}\right] - 2\alpha \operatorname{MathieuS}\left[-4\alpha\beta, 2\alpha\gamma, \frac{ix}{2}\right]}$$
$$y(x) \to -\frac{i\operatorname{MathieuCPrime}\left[-4\alpha\beta, 2\alpha\gamma, \frac{ix}{2}\right]}{2\alpha \operatorname{MathieuC}\left[-4\alpha\beta, 2\alpha\gamma, \frac{ix}{2}\right]}$$

5.9 problem 9

Internal problem ID [10457]

Book: Handbook of exact solutions for ordinary differential equations. By Polyanin and Zaitsev. Second edition
Section: Chapter 1, section 1.2. Riccati Equation. subsection 1.2.4-1. Equations with hyperbolic sine and cosine
Problem number: 9.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_Riccati]`

$$y' - y^2 - a\cosh(\beta x)\, y = ab\cosh(\beta x) - b^2$$

✓ Solution by Maple
Time used: 0.016 (sec). Leaf size: 73

```
dsolve(diff(y(x),x)=y(x)^2+a*cosh(beta*x)*y(x)+a*b*cosh(beta*x)-b^2,y(x), singsol=all)
```

$$y(x) = \frac{b\left(\int \mathrm{e}^{\frac{-2b\beta x+\sinh(x\beta)a}{\beta}} dx\right) - c_1 b + \mathrm{e}^{\frac{-2b\beta x+\sinh(x\beta)a}{\beta}}}{-\left(\int \mathrm{e}^{\frac{-2b\beta x+\sinh(x\beta)a}{\beta}} dx\right) + c_1}$$

✓ Solution by Mathematica
Time used: 9.815 (sec). Leaf size: 242

```
DSolve[y'[x]==y[x]^2+a*Cosh[\[Beta]*x]*y[x]+a*b*Cosh[\[Beta]*x]-b^2,y[x],x,IncludeSingularSo
```

$$y(x) \to -\frac{b\int_1^{e^{x\beta}} e^{\frac{a\left(K[1]^2-1\right)}{2\beta K[1]}} K[1]^{-\frac{2b}{\beta}-1} dK[1] + \beta e^{\frac{ae^{\beta(-x)}\left(e^{2\beta x}-1\right)}{2\beta}} \left(e^{\beta x}\right)^{-\frac{2b}{\beta}} + bc_1}{\int_1^{e^{x\beta}} e^{\frac{a\left(K[1]^2-1\right)}{2\beta K[1]}} K[1]^{-\frac{2b}{\beta}-1} dK[1] + c_1}$$

$$y(x) \to -b$$

$$y(x) \to -\frac{\beta e^{\frac{ae^{\beta(-x)}\left(e^{2\beta x}-1\right)}{2\beta}} \left(e^{\beta x}\right)^{-\frac{2b}{\beta}}}{\int_1^{e^{x\beta}} e^{\frac{a\left(K[1]^2-1\right)}{2\beta K[1]}} K[1]^{-\frac{2b}{\beta}-1} dK[1]} - b$$

5.10 problem 10

Internal problem ID [10458]

Book: Handbook of exact solutions for ordinary differential equations. By Polyanin and Zaitsev. Second edition
Section: Chapter 1, section 1.2. Riccati Equation. subsection 1.2.4-1. Equations with hyperbolic sine and cosine
Problem number: 10.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_Riccati]`

$$\boxed{y' - y^2 - ax\cosh(bx)^m y = a\cosh(bx)^m}$$

✓ Solution by Maple
Time used: 0.016 (sec). Leaf size: 85

```
dsolve(diff(y(x),x)=y(x)^2+a*x*cosh(b*x)^m*y(x)+a*cosh(b*x)^m,y(x), singsol=all)
```

$$y(x) = \frac{-e^{\int \frac{\cosh(bx)^m x^2 a-2}{x}dx} x - \left(\int e^{\int \frac{\cosh(bx)^m x^2 a-2}{x}dx}dx\right) + c_1}{\left(-c_1 + \int e^{\int \frac{\cosh(bx)^m x^2 a-2}{x}dx}dx\right) x}$$

✓ Solution by Mathematica
Time used: 7.557 (sec). Leaf size: 394

```
DSolve[y'[x]==y[x]^2+a*x*Cosh[b*x]^m*y[x]+a*Cosh[b*x]^m,y[x],x,IncludeSingularSolutions -> T
```

$$y(x) \to -\frac{\int_1^x \frac{\exp\left(-\frac{2^{-m}a\left(e^{-bK[1]}+e^{bK[1]}\right)^m\left(1+e^{2bK[1]}\right)^{-m}\left({}_3F_2\left(-m,-\frac{m}{2},-\frac{m}{2};1-\frac{m}{2},1-\frac{m}{2};-e^{2bK[1]}\right)+bm\,\text{Hypergeometric2F1}\left(-m,-\frac{m}{2},1-\frac{m}{2},-e^{2bK[1]}\right.\right.\right.}{b^2m^2}\right.}{K[1]^2}}{x\left(\int_1^x \frac{\exp\left(-\frac{2^{-m}a\left(e^{-bK[1]}+e^{bK[1]}\right)^m\left(1+e^{2bK[1]}\right)^{-m}\left({}_3F_2\left(-m,-\frac{m}{2},-\frac{m}{2};\right.\right.\right.\right.\right.}\right.}$$

$$y(x) \to -\frac{1}{x}$$

5.11 problem 11

Internal problem ID [10459]

Book: Handbook of exact solutions for ordinary differential equations. By Polyanin and Zaitsev. Second edition
Section: Chapter 1, section 1.2. Riccati Equation. subsection 1.2.4-1. Equations with hyperbolic sine and cosine
Problem number: 11.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_Riccati]`

$$\boxed{y' - \left(a \cosh\left(\lambda x\right)^2 - \lambda\right) y^2 = -a \cosh\left(\lambda x\right)^2 + a + \lambda}$$

✓ Solution by Maple
Time used: 0.016 (sec). Leaf size: 104

```
dsolve(diff(y(x),x)=(a*cosh(lambda*x)^2-lambda)*y(x)^2+a+lambda-a*cosh(lambda*x)^2,y(x), sin
```

$$y(x) = \frac{2 \tanh\left(x\lambda\right) \lambda \left(\int -\mathrm{e}^{\frac{a \cosh(2x\lambda)}{2\lambda}} \left(a - \operatorname{sech}\left(x\lambda\right)^2 \lambda\right) dx\right) c_1 + 2 \operatorname{sech}\left(x\lambda\right)^2 \mathrm{e}^{\frac{a \cosh(2x\lambda)}{2\lambda}} c_1 \lambda - \tanh\left(x\lambda\right)}{2\lambda \left(\int -\mathrm{e}^{\frac{a \cosh(2x\lambda)}{2\lambda}} \left(a - \operatorname{sech}\left(x\lambda\right)^2 \lambda\right) dx\right) c_1 - 1}$$

✓ Solution by Mathematica

Time used: 49.81 (sec). Leaf size: 211

```
DSolve[y'[x]==(a*Cosh[\[Lambda]*x]^2-\[Lambda])*y[x]^2+a+\[Lambda]-a*Cosh[\[Lambda]*x]^2,y[x
```

$$y(x)$$

$$\to \frac{\operatorname{sech}^2(\lambda x)\left(c_1 \sinh(2\lambda x)\int_1^x e^{\frac{a\cosh^2(\lambda K[1])}{\lambda}}\left(\lambda - a\cosh^2(\lambda K[1])\right)\operatorname{sech}^2(\lambda K[1])dK[1] + 2c_1 e^{\frac{a\cosh^2(\lambda x)}{\lambda}} + \sinh(\right.}{2 + 2c_1\int_1^x e^{\frac{a\cosh^2(\lambda K[1])}{\lambda}}\left(\lambda - a\cosh^2(\lambda K[1])\right)\operatorname{sech}^2(\lambda K[1])dK[1]}$$

$$y(x) \to \frac{1}{2}\operatorname{sech}^2(\lambda x)\left(\frac{2e^{\frac{a\cosh^2(\lambda x)}{\lambda}}}{\int_1^x e^{\frac{a\cosh^2(\lambda K[1])}{\lambda}}\left(\lambda - a\cosh^2(\lambda K[1])\right)\operatorname{sech}^2(\lambda K[1])dK[1]} + \sinh(2\lambda x)\right)$$

5.12 problem 12

Internal problem ID [10460]

Book: Handbook of exact solutions for ordinary differential equations. By Polyanin and Zaitsev. Second edition
Section: Chapter 1, section 1.2. Riccati Equation. subsection 1.2.4-1. Equations with hyperbolic sine and cosine
Problem number: 12.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_Riccati]`

$$\boxed{2y' - (a - \lambda + a\cosh(\lambda x))\, y^2 = a + \lambda - a\cosh(\lambda x)}$$

✓ Solution by Maple
Time used: 0.0 (sec). Leaf size: 101

```
dsolve(2*diff(y(x),x)=(a-lambda+a*cosh(lambda*x))*y(x)^2+a+lambda-a*cosh(lambda*x),y(x), sin
```

$$y(x) = \frac{\tanh\left(\frac{x\lambda}{2}\right)\lambda\left(\int \mathrm{e}^{\frac{a\cosh(x\lambda)}{\lambda}}\left(-2a + \operatorname{sech}\left(\frac{x\lambda}{2}\right)^2\lambda\right)dx\right)c_1 + 2\operatorname{sech}\left(\frac{x\lambda}{2}\right)^2\mathrm{e}^{\frac{a\cosh(x\lambda)}{\lambda}}c_1\lambda - 2\tanh\left(\frac{x\lambda}{2}\right)}{\lambda\left(\int \mathrm{e}^{\frac{a\cosh(x\lambda)}{\lambda}}\left(-2a + \operatorname{sech}\left(\frac{x\lambda}{2}\right)^2\lambda\right)dx\right)c_1 - 2}$$

✓ Solution by Mathematica

Time used: 59.899 (sec). Leaf size: 338

```
DSolve[2*y'[x]==(a-\[Lambda]+a*Cosh[\[Lambda]*x])*y[x]^2+a+\[Lambda]-a*Cosh[\[Lambda]*x],y[x
```

$$y(x)$$

$$\to \frac{\operatorname{sech}^2\left(\frac{\lambda x}{2}\right)\left(c_1 \sinh(\lambda x)\int_1^x -e^{\frac{2a\cosh^2\left(\frac{1}{2}\lambda K[1]\right)}{\lambda}}(\cosh(\lambda K[1])a+a-\lambda)\operatorname{sech}^2\left(\frac{1}{2}\lambda K[1]\right)dK[1]+4c_1 e^{\frac{2a\cosh^2\left(\frac{\lambda x}{2}\right.}{\lambda}}\right.}{2+2c_1\int_1^x -e^{\frac{2a\cosh^2\left(\frac{1}{2}\lambda K[1]\right)}{\lambda}}(\cosh(\lambda K[1])a+a-\lambda)\operatorname{sech}^2\left(\frac{1}{2}\lambda K[1]\right)dK[1]}$$

$$y(x)\to \frac{1}{2}\operatorname{sech}^2\left(\frac{\lambda x}{2}\right)\left(\frac{4e^{\frac{2a\cosh^2\left(\frac{\lambda x}{2}\right)}{\lambda}}}{\int_1^x -e^{\frac{2a\cosh^2\left(\frac{1}{2}\lambda K[1]\right)}{\lambda}}(\cosh(\lambda K[1])a+a-\lambda)\operatorname{sech}^2\left(\frac{1}{2}\lambda K[1]\right)dK[1]}+\sinh(\lambda x)\right)$$

$$y(x)\to \frac{1}{2}\operatorname{sech}^2\left(\frac{\lambda x}{2}\right)\left(\frac{4e^{\frac{2a\cosh^2\left(\frac{\lambda x}{2}\right)}{\lambda}}}{\int_1^x -e^{\frac{2a\cosh^2\left(\frac{1}{2}\lambda K[1]\right)}{\lambda}}(\cosh(\lambda K[1])a+a-\lambda)\operatorname{sech}^2\left(\frac{1}{2}\lambda K[1]\right)dK[1]}+\sinh(\lambda x)\right)$$

5.13 problem 13

Internal problem ID [10461]

Book: Handbook of exact solutions for ordinary differential equations. By Polyanin and Zaitsev. Second edition
Section: Chapter 1, section 1.2. Riccati Equation. subsection 1.2.4-1. Equations with hyperbolic sine and cosine
Problem number: 13.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_Riccati]`

$$y' - y^2 = -\lambda^2 + a \cosh(\lambda x)^n \sinh(\lambda x)^{-n-4}$$

✗ Solution by Maple

```
dsolve(diff(y(x),x)=y(x)^2-lambda^2+a*cosh(lambda*x)^n*sinh(lambda*x)^(-n-4),y(x), singsol=a
```

No solution found

✗ Solution by Mathematica
Time used: 0.0 (sec). Leaf size: 0

```
DSolve[y'[x]==y[x]^2-\[Lambda]^2+a*Cosh[\[Lambda]*x]^n*Sinh[\[Lambda]*x]^(-n-4),y[x],x,Inclu
```

Not solved

5.14 problem 14

Internal problem ID [10462]

Book: Handbook of exact solutions for ordinary differential equations. By Polyanin and Zaitsev. Second edition
Section: Chapter 1, section 1.2. Riccati Equation. subsection 1.2.4-1. Equations with hyperbolic sine and cosine
Problem number: 14.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_Riccati]`

$$y' - \sinh(\lambda x)\, y^2 a = b \sinh(\lambda x) \cosh(\lambda x)^n$$

✓ Solution by Maple
Time used: 0.0 (sec). Leaf size: 245

```
dsolve(diff(y(x),x)=a*sinh(lambda*x)*y(x)^2+b*sinh(lambda*x)*cosh(lambda*x)^n,y(x), singsol=
```

$$y(x) = \frac{\operatorname{sech}(x\lambda)\left(-\lambda\sqrt{a}\left(\operatorname{BesselY}\left(\frac{1}{n+2}, \frac{2\sqrt{a}\sqrt{b}\cosh(x\lambda)^{\frac{n}{2}+1}}{\lambda(n+2)}\right)c_1 + \operatorname{BesselJ}\left(\frac{1}{n+2}, \frac{2\sqrt{a}\sqrt{b}\cosh(x\lambda)^{\frac{n}{2}+1}}{\lambda(n+2)}\right)\right) + \left(\operatorname{BesselY}\right.\right.}{a^{\frac{3}{2}}\left(\operatorname{BesselY}\left(\frac{1}{n+2}, \frac{2\sqrt{a}\sqrt{b}\cosh(x\lambda)^{\frac{n}{2}+1}}{\lambda(n+2)}\right)c_1 + \operatorname{Besse}\right.}$$

✓ Solution by Mathematica

Time used: 1.376 (sec). Leaf size: 667

```
DSolve[y'[x]==a*Sinh[\[Lambda]*x]*y[x]^2+b*Sinh[\[Lambda]*x]*Cosh[\[Lambda]*x]^n,y[x],x,Incl
```

$$y(x)$$

$$\to \frac{\sqrt{a}\sqrt{b}c_1 \text{ Gamma}\left(\frac{n+1}{n+2}\right) \cosh^{\frac{n}{2}}(\lambda x) \text{ BesselJ}\left(\frac{n+1}{n+2}, \frac{2\sqrt{a}\sqrt{b}\cosh^{\frac{n}{2}+1}(x\lambda)}{n\lambda+2\lambda}\right) - \text{sech}(\lambda x)\left(\text{Gamma}\left(1+\frac{1}{n+2}\right)\left(\sqrt{a}\right.\right.}{}$$

$$y(x)$$

$$\to \frac{\frac{\sqrt{a}\sqrt{b}\cosh^{\frac{n}{2}}(\lambda x)\left(\text{BesselJ}\left(\frac{n+1}{n+2}, \frac{2\sqrt{a}\sqrt{b}\cosh^{\frac{n}{2}+1}(x\lambda)}{n\lambda+2\lambda}\right) - \text{BesselJ}\left(-\frac{n+3}{n+2}, \frac{2\sqrt{a}\sqrt{b}\cosh^{\frac{n}{2}+1}(x\lambda)}{n\lambda+2\lambda}\right)\right)}{\text{BesselJ}\left(-\frac{1}{n+2}, \frac{2\sqrt{a}\sqrt{b}\cosh^{\frac{n}{2}+1}(x\lambda)}{n\lambda+2\lambda}\right)} - \lambda\text{sech}(\lambda x)}{2a}$$

5.15 problem 15

Internal problem ID [10463]

Book: Handbook of exact solutions for ordinary differential equations. By Polyanin and Zaitsev. Second edition
Section: Chapter 1, section 1.2. Riccati Equation. subsection 1.2.4-1. Equations with hyperbolic sine and cosine
Problem number: 15.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_Riccati]`

$$y' - y^2 \cosh(\lambda x)\, a = b \cosh(\lambda x) \sinh(\lambda x)^n$$

✓ Solution by Maple
Time used: 0.0 (sec). Leaf size: 951

```
dsolve(diff(y(x),x)=a*cosh(lambda*x)*y(x)^2+b*cosh(lambda*x)*sinh(lambda*x)^n,y(x), singsol=
```

Expression too large to display

✓ Solution by Mathematica
Time used: 1.277 (sec). Leaf size: 633

```
DSolve[y'[x]==a*Cosh[\[Lambda]*x]*y[x]^2+b*Cosh[\[Lambda]*x]*Sinh[\[Lambda]*x]^n,y[x],x,Incl
```

$$y(x) \to \frac{\operatorname{csch}(\lambda x)\left(-\lambda \operatorname{Gamma}\left(1+\frac{1}{n+2}\right) \operatorname{BesselJ}\left(\frac{1}{n+2}, \frac{2\sqrt{a}\sqrt{b}\sinh^{\frac{n}{2}+1}(x\lambda)}{n\lambda+2\lambda}\right) + \sqrt{a}\sqrt{b}\sinh^{\frac{n}{2}+1}(\lambda x)\left(\operatorname{Gamma}\left(1+\right.\right.\right.}{}$$

$$y(x) \to \frac{\frac{\sqrt{a}\sqrt{b}\sinh^{\frac{n}{2}}(\lambda x)\left(\operatorname{BesselJ}\left(\frac{n+1}{n+2}, \frac{2\sqrt{a}\sqrt{b}\sinh^{\frac{n}{2}+1}(x\lambda)}{n\lambda+2\lambda}\right) - \operatorname{BesselJ}\left(-\frac{n+3}{n+2}, \frac{2\sqrt{a}\sqrt{b}\sinh^{\frac{n}{2}+1}(x\lambda)}{n\lambda+2\lambda}\right)\right)}{\operatorname{BesselJ}\left(-\frac{1}{n+2}, \frac{2\sqrt{a}\sqrt{b}\sinh^{\frac{n}{2}+1}(x\lambda)}{n\lambda+2\lambda}\right)} - \lambda \operatorname{csch}(\lambda x)}{2a}$$

5.16 problem 16

Internal problem ID [10464]

Book: Handbook of exact solutions for ordinary differential equations. By Polyanin and Zaitsev. Second edition
Section: Chapter 1, section 1.2. Riccati Equation. subsection 1.2.4-1. Equations with hyperbolic sine and cosine
Problem number: 16.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_Riccati]`

$$(a\cosh(\lambda x)+b)\,y'-y^2-c\cosh(x\mu)\,y=cd\cosh(x\mu)-d^2$$

✓ Solution by Maple
Time used: 0.015 (sec). Leaf size: 268

```
dsolve((a*cosh(lambda*x)+b)*diff(y(x),x)=y(x)^2+c*cosh(mu*x)*y(x)-d^2+c*d*cosh(mu*x),y(x), s
```

$$y(x)$$

$$= \frac{-d\left(\int \frac{\mathrm{e}^{\frac{c\left(\int \frac{\cosh(x\mu)}{a\cosh(x\lambda)+b}dx\right)\sqrt{a^2-b^2}\,\lambda-4d\arctan\left(\frac{(a-b)\tanh\left(\frac{x\lambda}{2}\right)}{\sqrt{a^2-b^2}}\right)}{\sqrt{a^2-b^2}\,\lambda}}{a\cosh(x\lambda)+b}dx\right)+dc_1-\mathrm{e}^{\frac{c\left(\int \frac{\cosh(x\mu)}{a\cosh(x\lambda)+b}dx\right)\sqrt{a^2-b^2}\,\lambda-4d\arctan\left(\frac{(a-b)\tanh}{\sqrt{a^2-b}}\right.}}}{\int \frac{\mathrm{e}^{\frac{c\left(\int \frac{\cosh(x\mu)}{a\cosh(x\lambda)+b}dx\right)\sqrt{a^2-b^2}\,\lambda-4d\arctan\left(\frac{(a-b)\tanh\left(\frac{x\lambda}{2}\right)}{\sqrt{a^2-b^2}}\right)}{\sqrt{a^2-b^2}\,\lambda}}{a\cosh(x\lambda)+b}dx-c_1}$$

✓ Solution by Mathematica

Time used: 24.309 (sec). Leaf size: 289

```
DSolve[(a*Cosh[\[Lambda]*x]+b)*y'[x]==y[x]^2+c*Cosh[\[Mu]*x]*y[x]-d^2+c*d*Cosh[\[Mu]*x],y[x]
```

$$\text{Solve}\left[\int_1^x -\frac{\exp\left(-\int_1^{K[2]} \frac{2d-c\cosh(\mu K[1])}{b+a\cosh(\lambda K[1])}dK[1]\right)(-d+c\cosh(\mu K[2])+y(x))}{c\mu(b+a\cosh(\lambda K[2]))(d+y(x))}dK[2]\right.$$

$$+\int_1^{y(x)} \left(\frac{\exp\left(-\int_1^x \frac{2d-c\cosh(\mu K[1])}{b+a\cosh(\lambda K[1])}dK[1]\right)}{c\mu(d+K[3])^2}\right.$$

$$\left.-\int_1^x \left(\frac{\exp\left(-\int_1^{K[2]} \frac{2d-c\cosh(\mu K[1])}{b+a\cosh(\lambda K[1])}dK[1]\right)(-d+c\cosh(\mu K[2])+K[3])}{c\mu(b+a\cosh(\lambda K[2]))(d+K[3])^2}-\frac{\exp\left(-\int_1^{K[2]} \frac{2d-c\cosh(\mu K[1])}{b+a\cosh(\lambda K[1])}dK[1]\right.\right.\right.$$

5.17 problem 17

Internal problem ID [10465]

Book: Handbook of exact solutions for ordinary differential equations. By Polyanin and Zaitsev. Second edition
Section: Chapter 1, section 1.2. Riccati Equation. subsection 1.2.4-1. Equations with hyperbolic sine and cosine
Problem number: 17.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_Riccati]`

$$\boxed{(a\cosh(\lambda x)+b)\left(y'-y^2\right)=-a\,\lambda^2\cosh(\lambda x)}$$

✓ Solution by Maple
Time used: 0.0 (sec). Leaf size: 204

```
dsolve((a*cosh(lambda*x)+b)*(diff(y(x),x)-y(x)^2)+a*lambda^2*cosh(lambda*x)=0,y(x), singsol=
```

$$y(x)$$
$$=\frac{\lambda\left(-2\arctan\left(\frac{(a-b)\tanh\left(\frac{x\lambda}{2}\right)}{\sqrt{a^2-b^2}}\right)\sqrt{a^2-b^2}\,ab\cosh\left(\frac{x\lambda}{2}\right)\sinh\left(\frac{x\lambda}{2}\right)+2\sqrt{a^2-b^2}\,c_1a\cosh\left(\frac{x\lambda}{2}\right)\sinh\left(\frac{x\lambda}{2}\right)+(a\right.}{\sqrt{a^2-b^2}\left(2\left(\cosh\left(\frac{x\lambda}{2}\right)^2a-\frac{a}{2}+\frac{b}{2}\right)b\arctan\left(\frac{(a-b)\tanh\left(\frac{x\lambda}{2}\right)}{\sqrt{a^2-b^2}}\right)-\sqrt{a^2-b^2}\,a\cosh\left(\frac{x\lambda}{2}\right)\sinh\left(\frac{x\lambda}{2}\right)-\right.}$$

✓ Solution by Mathematica
Time used: 7.749 (sec). Leaf size: 246

```
DSolve[(a*Cosh[\[Lambda]*x]+b)*(y'[x]-y[x]^2)+a*\[Lambda]^2*Cosh[\[Lambda]*x]==0,y[x],x,Incl
```

$$y(x)\to$$
$$-\frac{\lambda\left(a\sinh(\lambda x)\left(2b\arctan\left(\frac{(b-a)\tanh\left(\frac{\lambda x}{2}\right)}{\sqrt{a^2-b^2}}\right)+c_1\lambda(a^2-b^2)^{3/2}\right)+a\sqrt{a^2-b^2}\cosh(\lambda x)+\right.}{b\left(2b\arctan\left(\frac{(b-a)\tanh\left(\frac{\lambda x}{2}\right)}{\sqrt{a^2-b^2}}\right)+c_1\lambda\left(a^2-b^2\right)^{3/2}\right)+a\cosh(\lambda x)\left(2b\arctan\left(\frac{(b-a)\tanh\left(\frac{\lambda x}{2}\right)}{\sqrt{a^2-b^2}}\right)+c_1\lambda\left(a^2\right.}$$
$$y(x)\to-\frac{a\lambda\sinh(\lambda x)}{a\cosh(\lambda x)+b}$$

6 Chapter 1, section 1.2. Riccati Equation. subsection 1.2.4-2. Equations with hyperbolic tangent and cotangent.

6.1 problem 18 198
6.2 problem 19 199
6.3 problem 20 200
6.4 problem 21 201
6.5 problem 22 203
6.6 problem 23 204
6.7 problem 24 205
6.8 problem 25 206
6.9 problem 26 208
6.10 problem 27 209

6.1 problem 18

Internal problem ID [10466]

Book: Handbook of exact solutions for ordinary differential equations. By Polyanin and Zaitsev. Second edition
Section: Chapter 1, section 1.2. Riccati Equation. subsection 1.2.4-2. Equations with hyperbolic tangent and cotangent.
Problem number: 18.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_Riccati]`

$$y' - y^2 = \lambda a - a(a+\lambda)\tanh(\lambda x)^2$$

✓ Solution by Maple
Time used: 0.0 (sec). Leaf size: 122

```
dsolve(diff(y(x),x)=y(x)^2+a*lambda-a*(a+lambda)*tanh(lambda*x)^2,y(x), singsol=all)
```

$$y(x) = \frac{\text{LegendreP}\left(\frac{a+\lambda}{\lambda}, \frac{a}{\lambda}, \tanh(x\lambda)\right)\lambda + \text{LegendreQ}\left(\frac{a+\lambda}{\lambda}, \frac{a}{\lambda}, \tanh(x\lambda)\right)c_1\lambda - \tanh(x\lambda)\left(c_1 \text{LegendreQ}\left(\frac{a}{\lambda}, \frac{a}{\lambda},\right.\right.}{c_1 \text{LegendreQ}\left(\frac{a}{\lambda}, \frac{a}{\lambda}, \tanh(x\lambda)\right) + \text{LegendreP}\left(\frac{a}{\lambda}, \frac{a}{\lambda}, \tanh($$

✓ Solution by Mathematica
Time used: 8.574 (sec). Leaf size: 177

```
DSolve[y'[x]==y[x]^2+a*\[Lambda]-a*(a+\[Lambda])*Tanh[\[Lambda]*x]^2,y[x],x,IncludeSingularS
```

$$y(x) \to \frac{a\left(-\lambda\left(e^{2\lambda x}-1\right)\text{Hypergeometric2F1}\left(-\frac{2a}{\lambda}, -\frac{a}{\lambda}, 1-\frac{a}{\lambda}, -e^{2x\lambda}\right) - 2\lambda\left(e^{2\lambda x}+1\right)^{\frac{2a}{\lambda}+1} + ac_1\left(e^{2\lambda x}-1\right)\left(e^{2\lambda}\right.\right.}{\left(e^{2\lambda x}+1\right)\left(-\lambda\,\text{Hypergeometric2F1}\left(-\frac{2a}{\lambda}, -\frac{a}{\lambda}, 1-\frac{a}{\lambda}, -e^{2x\lambda}\right) + ac_1\left(e^{2\lambda x}\right)^{a/\lambda}\right)}$$

$$y(x) \to \frac{a\left(e^{2\lambda x}-1\right)}{e^{2\lambda x}+1}$$

6.2 problem 19

Internal problem ID [10467]

Book: Handbook of exact solutions for ordinary differential equations. By Polyanin and Zaitsev. Second edition
Section: Chapter 1, section 1.2. Riccati Equation. subsection 1.2.4-2. Equations with hyperbolic tangent and cotangent.
Problem number: 19.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_Riccati]`

$$y' - y^2 = 3\lambda a - \lambda^2 - a(a+\lambda)\tanh(\lambda x)^2$$

✓ Solution by Maple
Time used: 0.0 (sec). Leaf size: 148

```
dsolve(diff(y(x),x)=y(x)^2+3*a*lambda-lambda^2-a*(a+lambda)*tanh(lambda*x)^2,y(x), singsol=a
```

$$y(x) = \frac{2\,\text{LegendreP}\left(\frac{a+\lambda}{\lambda}, \frac{a-\lambda}{\lambda}, \tanh(x\lambda)\right)\lambda + 2\,\text{LegendreQ}\left(\frac{a+\lambda}{\lambda}, \frac{a-\lambda}{\lambda}, \tanh(x\lambda)\right)c_1\lambda - \tanh(x\lambda)\left(c_1\,\text{LegendreQ}\right.}{c_1\,\text{LegendreQ}\left(\frac{a}{\lambda}, \frac{a-\lambda}{\lambda}, \tanh(x\lambda)\right) + \text{LegendreP}\left(\frac{a}{\lambda}, \frac{a-}{\lambda}\right.}$$

✓ Solution by Mathematica
Time used: 12.804 (sec). Leaf size: 631

```
DSolve[y'[x]==y[x]^2+3*a*\[Lambda]-\[Lambda]^2-a*(a+\[Lambda])*Tanh[\[Lambda]*x]^2,y[x],x,In
```

$$y(x) \to \frac{-\lambda(a-2\lambda)\left(e^{2\lambda x}-1\right)\left(e^{2\lambda x}+1\right)^{\frac{2a}{\lambda}}\left(\frac{1}{e^{2\lambda x}-1}+1\right)^{a/\lambda}\left(a\left(4e^{2\lambda x}+e^{4\lambda x}-1\right)+\lambda-\lambda e^{4\lambda x}\right)\text{AppellF1}\left(1-\right.}{}$$

$$y(x) \to \frac{a\left(e^{2\lambda x}-1\right)^2 - \lambda\left(e^{2\lambda x}+1\right)^2}{e^{4\lambda x}-1}$$

6.3 problem 20

Internal problem ID [10468]

Book: Handbook of exact solutions for ordinary differential equations. By Polyanin and Zaitsev. Second edition
Section: Chapter 1, section 1.2. Riccati Equation. subsection 1.2.4-2. Equations with hyperbolic tangent and cotangent.
Problem number: 20.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_Riccati]`

$$\boxed{y' - y^2 - ax \tanh(bx)^m y = a \tanh(bx)^m}$$

✓ Solution by Maple
Time used: 0.015 (sec). Leaf size: 85

```
dsolve(diff(y(x),x)=y(x)^2+a*x*tanh(b*x)^m*y(x)+a*tanh(b*x)^m,y(x), singsol=all)
```

$$y(x) = \frac{-e^{\int \frac{a \tanh(bx)^m x^2 - 2}{x} dx} x - \left(\int e^{\int \frac{a \tanh(bx)^m x^2 - 2}{x} dx} dx \right) + c_1}{\left(-c_1 + \int e^{\int \frac{a \tanh(bx)^m x^2 - 2}{x} dx} dx \right) x}$$

✓ Solution by Mathematica
Time used: 12.331 (sec). Leaf size: 126

```
DSolve[y'[x]==y[x]^2+a*x*Tanh[b*x]^m*y[x]+a*Tanh[b*x]^m,y[x],x,IncludeSingularSolutions -> T
```

$$y(x) \to -\frac{\exp\left(-\int_1^x -aK[1] \tanh^m(bK[1]) dK[1]\right) + x \int_1^x \frac{\exp\left(-\int_1^{K[2]} -aK[1] \tanh^m(bK[1]) dK[1]\right)}{K[2]^2} dK[2] + c_1 x}{x^2 \left(\int_1^x \frac{\exp\left(-\int_1^{K[2]} -aK[1] \tanh^m(bK[1]) dK[1]\right)}{K[2]^2} dK[2] + c_1 \right)}$$

$$y(x) \to -\frac{1}{x}$$

6.4 problem 21

Internal problem ID [10469]

Book: Handbook of exact solutions for ordinary differential equations. By Polyanin and Zaitsev. Second edition
Section: Chapter 1, section 1.2. Riccati Equation. subsection 1.2.4-2. Equations with hyperbolic tangent and cotangent.
Problem number: 21.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_Riccati]`

$$\boxed{(a\tanh(\lambda x)+b)\,y'-y^2-c\tanh(x\mu)\,y=cd\tanh(x\mu)-d^2}$$

✓ Solution by Maple
Time used: 0.015 (sec). Leaf size: 302

```
dsolve((a*tanh(lambda*x)+b)*diff(y(x),x)=y(x)^2+c*tanh(mu*x)*y(x)-d^2+c*d*tanh(mu*x),y(x), s
```

$$y(x)$$

$$=\frac{-e^{c\left(\int \frac{\tanh(x\mu)}{a\tanh(x\lambda)+b}dx\right)}(\tanh(x\lambda)+1)^{\frac{d}{\lambda(a-b)}}(\tanh(x\lambda)-1)^{\frac{d}{\lambda(a+b)}}(a\tanh(x\lambda)+b)^{-\frac{2ad}{\lambda\left(a^2-b^2\right)}}-d\left(\int(a\tanh($$

$$\int(a\tanh(x\lambda)+b)^{\frac{\left(-a^2+b^2\right)\lambda-2ad}{\lambda\left(a^2-b^2\right)}}(\tanh(x\lambda)-1)^{\frac{d}{\lambda(a+b)}}(\tan$$

✓ Solution by Mathematica

Time used: 163.692 (sec). Leaf size: 800

```
DSolve[(a*Tanh[\[Lambda]*x]+b)*y'[x]==y[x]^2+c*Tanh[\[Mu]*x]*y[x]-d^2+c*d*Tanh[\[Mu]*x],y[x]
```

$$\text{Solve}\left[\int_1^x \frac{e^{-\int_1^{K[2]} \frac{\text{sech}(\mu K[1])(2d\cosh(\lambda K[1]-\mu K[1])+2d\cosh(\lambda K[1]+\mu K[1])+c\sinh(\lambda K[1]-\mu K[1])-c\sinh(\lambda K[1]+\mu K[1]))}{2(b\cosh(\lambda K[1])+a\sinh(\lambda K[1]))}dK[1]}(d\cosh(\lambda K[2]}{c\mu(b\cosh(\lambda K[2]-\mu K[2])+b\,\text{co}}\right.$$

$$+\int_1^{y(x)} \left(\frac{e^{-\int_1^x \frac{\text{sech}(\mu K[1])(2d\cosh(\lambda K[1]-\mu K[1])+2d\cosh(\lambda K[1]+\mu K[1])+c\sinh(\lambda K[1]-\mu K[1])-c\sinh(\lambda K[1]+\mu K[1]))}{2(b\cosh(\lambda K[1])+a\sinh(\lambda K[1]))}dK[1]}}{c\mu(d+K[3])^2}\right.$$

$$-\int_1^x \left(\frac{e^{-\int_1^{K[2]} \frac{\text{sech}(\mu K[1])(2d\cosh(\lambda K[1]-\mu K[1])+2d\cosh(\lambda K[1]+\mu K[1])+c\sinh(\lambda K[1]-\mu K[1])-c\sinh(\lambda K[1]+\mu K[1]))}{2(b\cosh(\lambda K[1])+a\sinh(\lambda K[1]))}dK[1]}(-\cosh(\lambda K[2]-}{c\mu(d+K[3])(b\cosh(\lambda K[2]-\mu K[2])+b\cosh(\lambda K[2]+\mu K[2])+a\sinh(\lambda K[2]-\mu K[2])+}\right.$$

6.5 problem 22

Internal problem ID [10470]

Book: Handbook of exact solutions for ordinary differential equations. By Polyanin and Zaitsev. Second edition
Section: Chapter 1, section 1.2. Riccati Equation. subsection 1.2.4-2. Equations with hyperbolic tangent and cotangent.
Problem number: 22.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_Riccati]`

$$\boxed{y' - y^2 = \lambda a - a(a+\lambda)\coth(\lambda x)^2}$$

✓ Solution by Maple
Time used: 0.0 (sec). Leaf size: 122

```
dsolve(diff(y(x),x)=y(x)^2+a*lambda-a*(a+lambda)*coth(lambda*x)^2,y(x), singsol=all)
```

$$y(x) = \frac{\text{LegendreP}\left(\frac{a+\lambda}{\lambda}, \frac{a}{\lambda}, \coth(x\lambda)\right)\lambda + \text{LegendreQ}\left(\frac{a+\lambda}{\lambda}, \frac{a}{\lambda}, \coth(x\lambda)\right)c_1\lambda - \coth(x\lambda)\left(c_1 \text{LegendreQ}\left(\frac{a}{\lambda}, \frac{a}{\lambda}, \right.\right.}{c_1 \text{LegendreQ}\left(\frac{a}{\lambda}, \frac{a}{\lambda}, \coth(x\lambda)\right) + \text{LegendreP}\left(\frac{a}{\lambda}, \frac{a}{\lambda}, \coth(x\lambda)\right)}$$

✓ Solution by Mathematica
Time used: 8.402 (sec). Leaf size: 175

```
DSolve[y'[x]==y[x]^2+a*\[Lambda]-a*(a+\[Lambda])*Coth[\[Lambda]*x]^2,y[x],x,IncludeSingularS
```

$$y(x) \to \frac{a\left(-\lambda\left(e^{2\lambda x}+1\right)\text{Hypergeometric2F1}\left(-\frac{2a}{\lambda}, -\frac{a}{\lambda}, 1-\frac{a}{\lambda}, e^{2x\lambda}\right) + 2\lambda\left(1-e^{2\lambda x}\right)^{\frac{2a}{\lambda}+1} + ac_1\left(e^{2\lambda x}+1\right)\left(e^{2\lambda x}\right)\right.}{\left(e^{2\lambda x}-1\right)\left(-\lambda\,\text{Hypergeometric2F1}\left(-\frac{2a}{\lambda}, -\frac{a}{\lambda}, 1-\frac{a}{\lambda}, e^{2x\lambda}\right) + ac_1\left(e^{2\lambda x}\right)^{a/\lambda}\right)}$$

$$y(x) \to \frac{a\left(e^{2\lambda x}+1\right)}{e^{2\lambda x}-1}$$

6.6 problem 23

Internal problem ID [10471]

Book: Handbook of exact solutions for ordinary differential equations. By Polyanin and Zaitsev. Second edition
Section: Chapter 1, section 1.2. Riccati Equation. subsection 1.2.4-2. Equations with hyperbolic tangent and cotangent.
Problem number: 23.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_Riccati]`

$$y' - y^2 = 3\lambda a - \lambda^2 - a(a+\lambda)\coth(\lambda x)^2$$

✓ Solution by Maple
Time used: 0.0 (sec). Leaf size: 148

```
dsolve(diff(y(x),x)=y(x)^2-lambda^2+3*a*lambda-a*(a+lambda)*coth(lambda*x)^2,y(x), singsol=a
```

$$y(x) = \frac{2\,\text{LegendreP}\left(\frac{a+\lambda}{\lambda}, \frac{a-\lambda}{\lambda}, \coth(x\lambda)\right)\lambda + 2\,\text{LegendreQ}\left(\frac{a+\lambda}{\lambda}, \frac{a-\lambda}{\lambda}, \coth(x\lambda)\right)c_1\lambda - \coth(x\lambda)\left(c_1\,\text{LegendreQ}\right.}{c_1\,\text{LegendreQ}\left(\frac{a}{\lambda}, \frac{a-\lambda}{\lambda}, \coth(x\lambda)\right) + \text{LegendreP}\left(\frac{a}{\lambda}, \frac{a-\lambda}{\lambda}\right.}$$

✓ Solution by Mathematica
Time used: 14.312 (sec). Leaf size: 659

```
DSolve[y'[x]==y[x]^2-\[Lambda]^2+3*a*\[Lambda]-a*(a+\[Lambda])*Coth[\[Lambda]*x]^2,y[x],x,In
```

$$y(x) \to \frac{-\lambda(a-2\lambda)\left(e^{2\lambda x}+1\right)\left(1-e^{2\lambda x}\right)^{\frac{2a}{\lambda}}\left(\frac{e^{2\lambda x}}{e^{2\lambda x}+1}\right)^{a/\lambda}\left(a\left(-4e^{2\lambda x}+e^{4\lambda x}-1\right)+\lambda-\lambda e^{4\lambda x}\right)\text{AppellF1}\left(1-\frac{a}{\lambda},\right.}{}$$

$$y(x) \to \frac{a\left(e^{2\lambda x}+1\right)^2 - \lambda\left(e^{2\lambda x}-1\right)^2}{e^{4\lambda x}-1}$$

6.7 problem 24

Internal problem ID [10472]

Book: Handbook of exact solutions for ordinary differential equations. By Polyanin and Zaitsev. Second edition
Section: Chapter 1, section 1.2. Riccati Equation. subsection 1.2.4-2. Equations with hyperbolic tangent and cotangent.
Problem number: 24.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_Riccati]`

$$\boxed{y' - y^2 - ax \coth(bx)^m y = a \coth(bx)^m}$$

✓ Solution by Maple
Time used: 0.016 (sec). Leaf size: 85

```
dsolve(diff(y(x),x)=y(x)^2+a*x*coth(b*x)^m*y(x)+a*coth(b*x)^m,y(x), singsol=all)
```

$$y(x) = \frac{-e^{\int \frac{a \coth(bx)^m x^2 - 2}{x} dx} x - \left(\int e^{\int \frac{a \coth(bx)^m x^2 - 2}{x} dx} dx \right) + c_1}{\left(-c_1 + \int e^{\int \frac{a \coth(bx)^m x^2 - 2}{x} dx} dx \right) x}$$

✓ Solution by Mathematica
Time used: 11.817 (sec). Leaf size: 126

```
DSolve[y'[x]==y[x]^2+a*x*Coth[b*x]^m*y[x]+a*Coth[b*x]^m,y[x],x,IncludeSingularSolutions -> T
```

$$y(x) \to -\frac{\exp\left(-\int_1^x -a \coth^m(bK[1]) K[1] dK[1]\right) + x \int_1^x \frac{\exp\left(-\int_1^{K[2]} -a \coth^m(bK[1]) K[1] dK[1]\right)}{K[2]^2} dK[2] + c_1 x}{x^2 \left(\int_1^x \frac{\exp\left(-\int_1^{K[2]} -a \coth^m(bK[1]) K[1] dK[1]\right)}{K[2]^2} dK[2] + c_1 \right)}$$

$$y(x) \to -\frac{1}{x}$$

6.8 problem 25

Internal problem ID [10473]

Book: Handbook of exact solutions for ordinary differential equations. By Polyanin and Zaitsev. Second edition
Section: Chapter 1, section 1.2. Riccati Equation. subsection 1.2.4-2. Equations with hyperbolic tangent and cotangent.
Problem number: 25.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_Riccati]`

$$(a\coth(\lambda x)+b)\,y'-y^2-c\coth(x\mu)\,y=cd\coth(x\mu)-d^2$$

✓ Solution by Maple
Time used: 0.031 (sec). Leaf size: 302

```
dsolve((a*coth(lambda*x)+b)*diff(y(x),x)=y(x)^2+c*coth(mu*x)*y(x)-d^2+c*d*coth(mu*x),y(x), s
```

$$y(x)$$

$$=\frac{-e^{c\left(\int \frac{\coth(x\mu)}{a\coth(x\lambda)+b}dx\right)}(\coth(x\lambda)+1)^{\frac{d}{\lambda(a-b)}}(\coth(x\lambda)-1)^{\frac{d}{\lambda(a+b)}}(a\coth(x\lambda)+b)^{-\frac{2ad}{\lambda\left(a^2-b^2\right)}}-d\left(\int(a\coth(x\lambda)+b)^{\frac{\left(-a^2+b^2\right)\lambda-2ad}{\lambda\left(a^2-b^2\right)}}(\coth(x\lambda)-1)^{\frac{d}{\lambda(a+b)}}(\coth(x\lambda)+1)^{\frac{d}{\lambda(a-b)}}e^{c\left(\int \frac{\coth(x\mu)}{a\coth(x\lambda)+b}dx\right)}dx+c_1\right)}{\int(a\coth(x\lambda)+b)^{\frac{\left(-a^2+b^2\right)\lambda-2ad}{\lambda\left(a^2-b^2\right)}}(\coth(x\lambda)-1)^{\frac{d}{\lambda(a+b)}}(\coth(x\lambda)+1)^{\frac{d}{\lambda(a-b)}}e^{c\left(\int \frac{\coth(x\mu)}{a\coth(x\lambda)+b}dx\right)}dx+c_1}$$

✓ Solution by Mathematica

Time used: 153.106 (sec). Leaf size: 808

```
DSolve[(a*Coth[\[Lambda]*x]+b)*y'[x]==y[x]^2+c*Coth[\[Mu]*x]*y[x]-d^2+c*d*Coth[\[Mu]*x],y[x]
```

$$\text{Solve}\left[\int_1^x -\frac{e^{-\int_1^{K[2]} \frac{\text{csch}(\mu K[1])(-2d\cosh(\lambda K[1]-\mu K[1])+2d\cosh(\lambda K[1]+\mu K[1])-c\sinh(\lambda K[1]-\mu K[1])-c\sinh(\lambda K[1]+\mu K[1]))}{2(a\cosh(\lambda K[1])+b\sinh(\lambda K[1]))}dK[1]}(d\cosh(\lambda K[2]-\mu K[2]}{c\mu(b\cosh(\lambda K[2]-\mu K[2])-b\cosh(\lambda K[}+\int_1^{y(x)}\left(-\int_1^x\left(\frac{e^{-\int_1^{K[2]} \frac{\text{csch}(\mu K[1])(-2d\cosh(\lambda K[1]-\mu K[1])+2d\cosh(\lambda K[1]+\mu K[1])-c\sinh(\lambda K[1]-\mu K[1])-c\sinh(\lambda K[1]+\mu K[1]))}{2(a\cosh(\lambda K[1])+b\sinh(\lambda K[1]))}dK[1]}(d\cosh}{c\mu(d+K[3])^2(b\cosh(\lambda K}-\frac{e^{-\int_1^x \frac{\text{csch}(\mu K[1])(-2d\cosh(\lambda K[1]-\mu K[1])+2d\cosh(\lambda K[1]+\mu K[1])-c\sinh(\lambda K[1]-\mu K[1])-c\sinh(\lambda K[1]+\mu K[1]))}{2(a\cosh(\lambda K[1])+b\sinh(\lambda K[1]))}dK[1]}}{c\mu(d+K[3])^2}\right)dK[3]=c_1,y(x)\right]$$

6.9 problem 26

Internal problem ID [10474]

Book: Handbook of exact solutions for ordinary differential equations. By Polyanin and Zaitsev. Second edition
Section: Chapter 1, section 1.2. Riccati Equation. subsection 1.2.4-2. Equations with hyperbolic tangent and cotangent.
Problem number: 26.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_Riccati]`

$$y' - y^2 = -2\tanh(\lambda x)^2 \lambda^2 - 2\lambda^2 \coth(\lambda x)^2$$

✓ Solution by Maple
Time used: 0.0 (sec). Leaf size: 143

```
dsolve(diff(y(x),x)=y(x)^2-2*lambda^2*tanh(lambda*x)^2-2*lambda^2*coth(lambda*x)^2,y(x), sin
```

$$y(x) = \frac{2\left(-\frac{1}{2} + c_1\left(-\cosh(x\lambda)^2 + \frac{1}{2}\right)\ln(\coth(x\lambda) - 1) + c_1\left(\cosh(x\lambda)^2 - \frac{1}{2}\right)\ln(\coth(x\lambda) + 1) + 4\cosh(x\lambda)^5 c\right.}{-4\cosh(x\lambda)^3 c_1 \sinh(x\lambda) + 2\sinh(x\lambda)\cosh(x\lambda) c_1 +}$$

✓ Solution by Mathematica
Time used: 7.989 (sec). Leaf size: 132

```
DSolve[y'[x]==y[x]^2-2*\[Lambda]^2*Tanh[\[Lambda]*x]^2-2*\[Lambda]^2*Coth[\[Lambda]*x]^2,y[x
```

$$y(x) \to -\frac{2\lambda\left(e^{12\lambda x} + 2e^{4\lambda x}\left(e^{4\lambda x} + 1\right)\log\left(e^{4\lambda x}\right) + (-7 + c_1)\left(-e^{4\lambda x}\right) - (7 + c_1)e^{8\lambda x} - 1\right)}{\left(e^{4\lambda x} - 1\right)\left(e^{8\lambda x} - 2e^{4\lambda x}\log\left(e^{4\lambda x}\right) + c_1 e^{4\lambda x} - 1\right)}$$

$$y(x) \to \frac{2\lambda\left(e^{4\lambda x} + 1\right)}{e^{4\lambda x} - 1}$$

6.10 problem 27

Internal problem ID [10475]

Book: Handbook of exact solutions for ordinary differential equations. By Polyanin and Zaitsev. Second edition
Section: Chapter 1, section 1.2. Riccati Equation. subsection 1.2.4-2. Equations with hyperbolic tangent and cotangent.
Problem number: 27.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_Riccati]`

$$y' - y^2 = -2ab + \lambda a + b\lambda - a(a+\lambda)\tanh(\lambda x)^2 - b(b+\lambda)\coth(\lambda x)^2$$

✓ Solution by Maple
Time used: 0.0 (sec). Leaf size: 289

```
dsolve(diff(y(x),x)=y(x)^2+a*lambda+b*lambda-2*a*b-a*(a+lambda)*tanh(lambda*x)^2-b*(b+lambda
```

$$y(x) = \frac{-4c_1\lambda\left(b-\frac{\lambda}{2}\right)\coth(x\lambda)^{\frac{2a+2\lambda}{\lambda}}\operatorname{csch}(x\lambda)^2\operatorname{hypergeom}\left(\left[2,-\frac{2b-3\lambda}{2\lambda}\right],\left[\frac{2a+5\lambda}{2\lambda}\right],\coth(x\lambda)^2\right) - 2c_1\left(\left(\frac{3a}{2}+\frac{3b}{2}\right)\right.}{\left(\text{hyp}\right.}$$

✓ Solution by Mathematica
Time used: 40.238 (sec). Leaf size: 493

```
DSolve[y'[x]==y[x]^2+a*\[Lambda]+b*\[Lambda]-2*a*b-a*(a+\[Lambda])*Tanh[\[Lambda]*x]^2-b*(b+
```

$$y(x) \to -\frac{(a+b)\left(e^{2\lambda x}\right)^{\frac{a+b}{\lambda}}\left(\frac{2\lambda\left(a\left(e^{2\lambda x}-1\right)^2+b\left(e^{2\lambda x}+1\right)^2\right)\left(e^{2\lambda x}\right)^{-\frac{a+b}{\lambda}}\operatorname{AppellF1}\left(-\frac{a+b}{\lambda},-\frac{2b}{\lambda},-\frac{2a}{\lambda},-\frac{a+b-\lambda}{\lambda},e^{2x\lambda},-e^{2x\lambda}\right)}{(a+b)\left(e^{2\lambda x}-1\right)\left(e^{2\lambda x}+1\right)} + 4\lambda\left(e^{2\lambda x}\right)^{-}\right.}{}$$

$$y(x) \to \frac{a\left(e^{2\lambda x}-1\right)^2+b\left(e^{2\lambda x}+1\right)^2}{e^{4\lambda x}-1}$$

7 Chapter 1, section 1.2. Riccati Equation. subsection 1.2.5-1. Equations Containing Logarithmic Functions

7.1 problem 1 211
7.2 problem 2 212
7.3 problem 3 213
7.4 problem 4 215
7.5 problem 5 216
7.6 problem 6 217
7.7 problem 7 218
7.8 problem 8 220
7.9 problem 9 221

7.1 problem 1

Internal problem ID [10476]

Book: Handbook of exact solutions for ordinary differential equations. By Polyanin and Zaitsev. Second edition
Section: Chapter 1, section 1.2. Riccati Equation. subsection 1.2.5-1. Equations Containing Logarithmic Functions
Problem number: 1.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_Riccati]`

$$y' - a \ln(x)^n y^2 = bm x^{m-1} - a b^2 x^{2m} \ln(x)^n$$

✗ Solution by Maple

```
dsolve(diff(y(x),x)=a*(ln(x))^n*y(x)^2+b*m*x^(m-1)-a*b^2*x^(2*m)*(ln(x))^n,y(x), singsol=all
```

No solution found

✗ Solution by Mathematica
Time used: 0.0 (sec). Leaf size: 0

```
DSolve[y'[x]==a*(Log[x])^n*y[x]^2+b*m*x^(m-1)-a*b^2*x^(2*m)*(Log[x])^n,y[x],x,IncludeSingula
```

Not solved

7.2 problem 2

Internal problem ID [10477]

Book: Handbook of exact solutions for ordinary differential equations. By Polyanin and Zaitsev. Second edition
Section: Chapter 1, section 1.2. Riccati Equation. subsection 1.2.5-1. Equations Containing Logarithmic Functions
Problem number: 2.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_Riccati]`

$$\boxed{y'x - ay^2 = b\ln(x) + c}$$

✓ Solution by Maple
Time used: 0.0 (sec). Leaf size: 91

```
dsolve(x*diff(y(x),x)=a*y(x)^2+b*ln(x)+c,y(x), singsol=all)
```

$$y(x) = \frac{(ab)^{\frac{1}{3}}\left(\text{AiryBi}\left(1, -\frac{(ab)^{\frac{1}{3}}(b\ln(x)+c)}{b}\right)c_1 + \text{AiryAi}\left(1, -\frac{(ab)^{\frac{1}{3}}(b\ln(x)+c)}{b}\right)\right)}{a\left(c_1\,\text{AiryBi}\left(-\frac{(ab)^{\frac{1}{3}}(b\ln(x)+c)}{b}\right) + \text{AiryAi}\left(-\frac{(ab)^{\frac{1}{3}}(b\ln(x)+c)}{b}\right)\right)}$$

✓ Solution by Mathematica
Time used: 1.682 (sec). Leaf size: 149

```
DSolve[x*y'[x]==a*y[x]^2+b*Log[x]+c,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to \frac{b\left(\text{AiryBiPrime}\left(-\frac{a(c+b\log(x))}{(-ab)^{2/3}}\right) + c_1\,\text{AiryAiPrime}\left(-\frac{a(c+b\log(x))}{(-ab)^{2/3}}\right)\right)}{(-ab)^{2/3}\left(\text{AiryBi}\left(-\frac{a(c+b\log(x))}{(-ab)^{2/3}}\right) + c_1\,\text{AiryAi}\left(-\frac{a(c+b\log(x))}{(-ab)^{2/3}}\right)\right)}$$

$$y(x) \to \frac{b\,\text{AiryAiPrime}\left(-\frac{a(c+b\log(x))}{(-ab)^{2/3}}\right)}{(-ab)^{2/3}\,\text{AiryAi}\left(-\frac{a(c+b\log(x))}{(-ab)^{2/3}}\right)}$$

7.3 problem 3

Internal problem ID [10478]

Book: Handbook of exact solutions for ordinary differential equations. By Polyanin and Zaitsev. Second edition
Section: Chapter 1, section 1.2. Riccati Equation. subsection 1.2.5-1. Equations Containing Logarithmic Functions
Problem number: 3.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_Riccati]`

$$\boxed{y'x - ay^2 = b\ln(x)^k + c\ln(x)^{2k+2}}$$

✓ Solution by Maple
Time used: 0.0 (sec). Leaf size: 480

```
dsolve(x*diff(y(x),x)=a*y(x)^2+b*(ln(x))^k+c*(ln(x))^(2*k+2),y(x), singsol=all)
```

$$y(x) = \frac{-\ln(x)^{1+k}(k+3)\left(i\sqrt{c}(1+k)\sqrt{a}-ab\right)\text{hypergeom}\left(\left[\frac{(3k+5)\sqrt{c}+i\sqrt{a}\,b}{\sqrt{c}(2k+4)}\right],\left[\frac{2k+3}{k+2}\right],\frac{2i\sqrt{a}\sqrt{c}\ln(x)^{k+2}}{k+2}\right)+\left(-(i$$

✓ Solution by Mathematica

Time used: 3.775 (sec). Leaf size: 807

```
DSolve[x*y'[x]==a*y[x]^2+b*(Log[x])^k+c*(Log[x])^(2*k+2),y[x],x,IncludeSingularSolutions ->
```

$$y(x) \to$$

$$-\frac{\log^{k+1}(x)\left(\sqrt{c}c_1(k+2)\sqrt{-(k+2)^2}\,\text{HypergeometricU}\left(\frac{1}{2}\left(\frac{\sqrt{a}b}{\sqrt{c}\sqrt{-(k+2)^2}}+\frac{k+1}{k+2}\right),\frac{k+1}{k+2},\frac{2\sqrt{a}\sqrt{c}\log^{k+2}(x)}{\sqrt{-(k+2)^2}}\right)+\right.}{\sqrt{a}}$$

$$y(x)$$

$$\to \frac{\log^{k+1}(x)\left(-\frac{\left(\sqrt{a}b(k+2)+\sqrt{c}\sqrt{-(k+2)^2}(k+1)\right)\text{HypergeometricU}\left(\frac{1}{2}\left(\frac{\sqrt{a}b}{\sqrt{c}\sqrt{-(k+2)^2}}+\frac{3k+5}{k+2}\right),\frac{2k+3}{k+2},\frac{2\sqrt{a}\sqrt{c}\log^{k+2}(x)}{\sqrt{-(k+2)^2}}\right)}{\text{HypergeometricU}\left(\frac{1}{2}\left(\frac{\sqrt{a}b}{\sqrt{c}\sqrt{-(k+2)^2}}+\frac{k+1}{k+2}\right),\frac{k+1}{k+2},\frac{2\sqrt{a}\sqrt{c}\log^{k+2}(x)}{\sqrt{-(k+2)^2}}\right)}-\sqrt{c}\sqrt{-(}\right.}{\sqrt{a}(k+2)^2}$$

7.4 problem 4

Internal problem ID [10479]

Book: Handbook of exact solutions for ordinary differential equations. By Polyanin and Zaitsev. Second edition
Section: Chapter 1, section 1.2. Riccati Equation. subsection 1.2.5-1. Equations Containing Logarithmic Functions
Problem number: 4.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_Riccati]`

$$\boxed{y'x - y^2x = -a^2x\ln(\beta x)^2 + a}$$

✗ Solution by Maple

```
dsolve(x*diff(y(x),x)=x*y(x)^2-a^2*x*(ln(beta*x))^2+a,y(x), singsol=all)
```

No solution found

✗ Solution by Mathematica
Time used: 0.0 (sec). Leaf size: 0

```
DSolve[x*y'[x]==x*y[x]^2-a^2*x*(Log[\[Beta]*x])^2+a,y[x],x,IncludeSingularSolutions -> True]
```

Not solved

7.5 problem 5

Internal problem ID [10480]

Book: Handbook of exact solutions for ordinary differential equations. By Polyanin and Zaitsev. Second edition
Section: Chapter 1, section 1.2. Riccati Equation. subsection 1.2.5-1. Equations Containing Logarithmic Functions
Problem number: 5.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_Riccati]`

$$\boxed{y'x - y^2x = -a^2x\ln(\beta x)^{2k} + ak\ln(\beta x)^{k-1}}$$

✗ Solution by Maple

```
dsolve(x*diff(y(x),x)=x*y(x)^2-a^2*x*(ln(beta*x))^(2*k)+a*k*(ln(beta*x))^(k-1),y(x), singsol
```

No solution found

✗ Solution by Mathematica
Time used: 0.0 (sec). Leaf size: 0

```
DSolve[x*y'[x]==x*y[x]^2-a^2*x*(Log[\[Beta]*x])^(2*k)+a*k*(Log[\[Beta]*x])^(k-1),y[x],x,Incl
```

Not solved

7.6 problem 6

Internal problem ID [10481]

Book: Handbook of exact solutions for ordinary differential equations. By Polyanin and Zaitsev. Second edition
Section: Chapter 1, section 1.2. Riccati Equation. subsection 1.2.5-1. Equations Containing Logarithmic Functions
Problem number: 6.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_Riccati]`

$$y'x - a\,x^n y^2 = b - a\,b^2 x^n \ln(x)^2$$

✗ Solution by Maple

```
dsolve(x*diff(y(x),x)=a*x^n*y(x)^2+b-a*b^2*x^n*(ln(x))^2,y(x), singsol=all)
```

No solution found

✗ Solution by Mathematica
Time used: 0.0 (sec). Leaf size: 0

```
DSolve[x*y'[x]==a*x^n*y[x]^2+b-a*b^2*x^n*(Log[x])^2,y[x],x,IncludeSingularSolutions -> True]
```

Not solved

7.7 problem 7

Internal problem ID [10482]

Book: Handbook of exact solutions for ordinary differential equations. By Polyanin and Zaitsev. Second edition
Section: Chapter 1, section 1.2. Riccati Equation. subsection 1.2.5-1. Equations Containing Logarithmic Functions
Problem number: 7.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_Riccati]`

$$\boxed{x^2y' - x^2y^2 = a\ln(x)^2 + b\ln(x) + c}$$

✓ Solution by Maple
Time used: 0.0 (sec). Leaf size: 477

```
dsolve(x^2*diff(y(x),x)=x^2*y(x)^2+a*(ln(x))^2+b*ln(x)+c,y(x), singsol=all)
```

$y(x)$

$$= \frac{\left(-\frac{b\ln(x)(b\ln(x)-4c+1)a^{\frac{5}{2}}}{12} - \frac{(b\ln(x)-c+\frac{1}{4})b^2a^{\frac{3}{2}}}{12} + \frac{(c-\frac{1}{4})\ln(x)^2a^{\frac{7}{2}}}{3} - \frac{\sqrt{a}\,b^4}{48} - i\left(a\ln(x)+\frac{b}{2}\right)^2a^2\right)c_1\,\text{hypergeom}\left(\left[\right.\right.}{}$$

✓ Solution by Mathematica

Time used: 1.151 (sec). Leaf size: 868

```
DSolve[x^2*y'[x]==x^2*y[x]^2+a*(Log[x])^2+b*Log[x]+c,y[x],x,IncludeSingularSolutions -> True
```

$$y(x)$$

$$\to \frac{ib\,\text{ParabolicCylinderD}\left(\frac{-ib^2-4a^{3/2}+ia(4c-1)}{8a^{3/2}},-\frac{\left(\frac{1}{2}-\frac{i}{2}\right)(b+2a\log(x))}{a^{3/4}}\right)+2ia\log(x)\,\text{ParabolicCylinderD}\left(\frac{-ib^2-4}{}\right.}{}$$

$$y(x)\to -\frac{2\sqrt[4]{-1}\sqrt{2}\sqrt[4]{a}\,\text{ParabolicCylinderD}\left(\frac{ib^2+4a^{3/2}-ia(4c-1)}{8a^{3/2}},\frac{\left(\frac{1}{2}+\frac{i}{2}\right)(b+2a\log(x))}{a^{3/4}}\right)}{\text{ParabolicCylinderD}\left(\frac{ib^2-4a^{3/2}-ia(4c-1)}{8a^{3/2}},\frac{\left(\frac{1}{2}+\frac{i}{2}\right)(b+2a\log(x))}{a^{3/4}}\right)}+\frac{ib}{\sqrt{a}}+2i\sqrt{a}\log(x)+1}{2x}$$

$$y(x)\to -\frac{2\sqrt[4]{-1}\sqrt{2}\sqrt[4]{a}\,\text{ParabolicCylinderD}\left(\frac{ib^2+4a^{3/2}-ia(4c-1)}{8a^{3/2}},\frac{\left(\frac{1}{2}+\frac{i}{2}\right)(b+2a\log(x))}{a^{3/4}}\right)}{\text{ParabolicCylinderD}\left(\frac{ib^2-4a^{3/2}-ia(4c-1)}{8a^{3/2}},\frac{\left(\frac{1}{2}+\frac{i}{2}\right)(b+2a\log(x))}{a^{3/4}}\right)}+\frac{ib}{\sqrt{a}}+2i\sqrt{a}\log(x)+1}{2x}$$

7.8 problem 8

Internal problem ID [10483]

Book: Handbook of exact solutions for ordinary differential equations. By Polyanin and Zaitsev. Second edition
Section: Chapter 1, section 1.2. Riccati Equation. subsection 1.2.5-1. Equations Containing Logarithmic Functions
Problem number: 8.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_Riccati]`

$$\boxed{x^2y' - x^2y^2 = a(b\ln(x) + c)^n + \frac{1}{4}}$$

✗ Solution by Maple

```
dsolve(x^2*diff(y(x),x)=x^2*y(x)^2+a*(b*ln(x)+c)^n+1/4,y(x), singsol=all)
```

No solution found

✗ Solution by Mathematica
Time used: 0.0 (sec). Leaf size: 0

```
DSolve[x^2*y'[x]==x^2*y[x]^2+a*(b*Log[x]+c)^n+1/4,y[x],x,IncludeSingularSolutions -> True]
```

Not solved

7.9 problem 9

Internal problem ID [10484]

Book: Handbook of exact solutions for ordinary differential equations. By Polyanin and Zaitsev. Second edition
Section: Chapter 1, section 1.2. Riccati Equation. subsection 1.2.5-1. Equations Containing Logarithmic Functions
Problem number: 9.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_Riccati]`

$$\boxed{x^2 \ln(xa)\left(y' - y^2\right) = 1}$$

✓ Solution by Maple
Time used: 0.015 (sec). Leaf size: 45

```
dsolve(x^2*ln(a*x)*(diff(y(x),x)-y(x)^2)=1,y(x), singsol=all)
```

$$y(x) = \frac{-c_1 \operatorname{expIntegral}_1(-\ln(ax)) + 1}{x\left(\left(c_1 \operatorname{expIntegral}_1(-\ln(ax)) - 1\right)\ln(ax) + c_1 ax\right)}$$

✓ Solution by Mathematica
Time used: 0.616 (sec). Leaf size: 74

```
DSolve[x^2*Log[a*x]*(y'[x]-y[x]^2)==1,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to \frac{a + c_1 \operatorname{LogIntegral}(ax)}{-c_1 x \operatorname{LogIntegral}(ax)\log(ax) + ac_1 x^2 - ax\log(ax)}$$
$$y(x) \to \frac{\operatorname{LogIntegral}(ax)}{ax^2 - x\operatorname{LogIntegral}(ax)\log(ax)}$$

8 Chapter 1, section 1.2. Riccati Equation. subsection 1.2.5-2

8.1 problem 10 223
8.2 problem 11 224
8.3 problem 12 225
8.4 problem 13 226
8.5 problem 14 227
8.6 problem 15 228
8.7 problem 16 229
8.8 problem 17 230
8.9 problem 18 231
8.10 problem 19 232
8.11 problem 20 233
8.12 problem 21 234
8.13 problem 22 236
8.14 problem 23 238

8.1 problem 10

Internal problem ID [10485]

Book: Handbook of exact solutions for ordinary differential equations. By Polyanin and Zaitsev. Second edition
Section: Chapter 1, section 1.2. Riccati Equation. subsection 1.2.5-2
Problem number: 10.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_Riccati]`

$$\boxed{y' - y^2 - a \ln(\beta x)\, y = -ab \ln(\beta x) - b^2}$$

✓ Solution by Maple
Time used: 0.0 (sec). Leaf size: 74

```
dsolve(diff(y(x),x)=y(x)^2+a*ln(beta*x)*y(x)-a*b*ln(beta*x)-b^2,y(x), singsol=all)
```

$$y(x) = \frac{\left(\int (x\beta)^{ax} \mathrm{e}^{-(a-2b)x} dx - c_1\right) b - (x\beta)^{ax} \mathrm{e}^{-(a-2b)x}}{\int (x\beta)^{ax} \mathrm{e}^{-(a-2b)x} dx - c_1}$$

✓ Solution by Mathematica
Time used: 1.584 (sec). Leaf size: 187

```
DSolve[y'[x]==y[x]^2+a*Log[\[Beta]*x]*y[x]-a*b*Log[\[Beta]*x]-b^2,y[x],x,IncludeSingularSolu
```

$$\text{Solve}\left[\int_1^x \frac{e^{2bK[1]-aK[1]}(\beta K[1])^{aK[1]}(b + a\log(\beta K[1]) + y(x))}{a(b - y(x))} dK[1]\right.$$
$$+ \int_1^{y(x)} \left(\frac{e^{2bx-ax}(x\beta)^{ax}}{a(K[2]-b)^2}\right.$$
$$\left.\left. - \int_1^x \left(\frac{e^{2bK[1]-aK[1]}(b + K[2] + a\log(\beta K[1]))(\beta K[1])^{aK[1]}}{a(b-K[2])^2} + \frac{e^{2bK[1]-aK[1]}(\beta K[1])^{aK[1]}}{a(b-K[2])}\right) dK[1]\right) dK[2] = c_1,$$

8.2 problem 11

Internal problem ID [10486]

Book: Handbook of exact solutions for ordinary differential equations. By Polyanin and Zaitsev. Second edition
Section: Chapter 1, section 1.2. Riccati Equation. subsection 1.2.5-2
Problem number: 11.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_Riccati]`

$$\boxed{y' - y^2 - ax \ln(bx)^m y = a \ln(bx)^m}$$

✓ Solution by Maple
Time used: 0.0 (sec). Leaf size: 85

```
dsolve(diff(y(x),x)=y(x)^2+a*x*(ln(b*x))^m*y(x)+a*(ln(b*x))^m,y(x), singsol=all)
```

$$y(x) = \frac{-e^{\int \frac{a \ln(bx)^m x^2 - 2}{x} dx} x - \left(\int e^{\int \frac{a \ln(bx)^m x^2 - 2}{x} dx} dx \right) + c_1}{\left(-c_1 + \int e^{\int \frac{a \ln(bx)^m x^2 - 2}{x} dx} dx \right) x}$$

✓ Solution by Mathematica
Time used: 3.589 (sec). Leaf size: 181

```
DSolve[y'[x]==y[x]^2+a*x*(Log[b*x])^m*y[x]+a*(Log[b*x])^m,y[x],x,IncludeSingularSolutions ->
```

$$y(x) \to -\frac{x \int_1^x \frac{\exp\left(\frac{2^{-m-1} a \Gamma(m+1,-2\log(bK[1]))(-\log(bK[1]))^{-m} \log^m(bK[1])}{b^2}\right)}{K[1]^2} dK[1] + \exp\left(\frac{a 2^{-m-1}(-\log(bx))^{-m} \log^m(bx)\Gamma(m+1,-2\log(}{b^2}\right.}{x^2 \left(\int_1^x \frac{\exp\left(\frac{2^{-m-1} a \Gamma(m+1,-2\log(bK[1]))(-\log(bK[1]))^{-m} \log^m(bK[1])}{b^2}\right)}{K[1]^2} dK[1] + c_1 \right)}$$

$$y(x) \to -\frac{1}{x}$$

8.3 problem 12

Internal problem ID [10487]

Book: Handbook of exact solutions for ordinary differential equations. By Polyanin and Zaitsev. Second edition
Section: Chapter 1, section 1.2. Riccati Equation. subsection 1.2.5-2
Problem number: 12.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_Riccati]`

$$\boxed{y' - a\,x^n y^2 + ab\,x^{n+1}\ln(x)\,y = b\ln(x) + b}$$

✗ Solution by Maple

```
dsolve(diff(y(x),x)=a*x^n*y(x)^2-a*b*x^(n+1)*ln(x)*y(x)+b*ln(x)+b,y(x), singsol=all)
```

No solution found

✗ Solution by Mathematica
Time used: 0.0 (sec). Leaf size: 0

```
DSolve[y'[x]==a*x^n*y[x]^2-a*b*x^(n+1)*Log[x]*y[x]+b*Log[x]+b,y[x],x,IncludeSingularSolution
```

Not solved

8.4 problem 13

Internal problem ID [10488]

Book: Handbook of exact solutions for ordinary differential equations. By Polyanin and Zaitsev. Second edition
Section: Chapter 1, section 1.2. Riccati Equation. subsection 1.2.5-2
Problem number: 13.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_Riccati]`

$$\boxed{y' + (n+1)\,x^n y^2 - a\,x^{n+1}\ln(x)^m\,y = -a\ln(x)^m}$$

✓ Solution by Maple
Time used: 0.0 (sec). Leaf size: 184

```
dsolve(diff(y(x),x)=-(n+1)*x^n*y(x)^2+a*x^(n+1)*(ln(x))^m*y(x)-a*(ln(x))^m,y(x), singsol=all
```

$$y(x) = \frac{x^{-n-1}\left(x^{n+1}\mathrm{e}^{\int \frac{a\,x^{n+1}\ln(x)^m x-2n-2}{x}dx} + \left(\int x^n \mathrm{e}^{a\left(\int x^{n+1}\ln(x)^m dx\right)-2\left(\int \frac{1}{x}dx\right)(n+1)}dx\right) n + \int x^n \mathrm{e}^{a\left(\int x^{n+1}\ln(x)^m dx\right)-2\left(\int \frac{1}{x}dx\right)(n+1)}dx\right.}{\left(\int x^n \mathrm{e}^{a\left(\int x^{n+1}\ln(x)^m dx\right)-2\left(\int \frac{1}{x}dx\right)(n+1)}dx\right) n + \int x^n \mathrm{e}^{a\left(\int x^{n+1}\ln(x)^m dx\right)-2\left(\int \frac{1}{x}dx\right)(n+1)}dx - c_1}$$

✓ Solution by Mathematica
Time used: 5.364 (sec). Leaf size: 311

```
DSolve[y'[x]==-(n+1)*x^n*y[x]^2+a*x^(n+1)*(Log[x])^m*y[x]-a*(Log[x])^m,y[x],x,IncludeSingula
```

$$y(x) \to \frac{x^{-2(n+1)}\left(c_1(n+1)x^{n+1}\int_1^x \exp\left(\frac{a\Gamma(m+1,-((n+2)\log(K[1])))\log^m(K[1])(-((n+2)\log(K[1])))^{-m}}{n+2} - (n+2)\log(K[1])\right)\right.}{(n+1)\left(1 + c_1\int_1^x \exp\left(\frac{a\Gamma(m+1,-((n+2)\log(K[1])))\log^m(K[1])(-((n+2)\log}{n+2}\right.\right.}$$

$$y(x) \to \frac{x^{-2(n+1)}\left(\frac{\exp\left(\frac{a\log^m(x)(-((n+2)\log(x)))^{-m}\Gamma(m+1,-((n+2)\log(x)))}{n+2}\right)}{\int_1^x \exp\left(\frac{a\Gamma(m+1,-((n+2)\log(K[1])))\log^m(K[1])(-((n+2)\log(K[1])))^{-m}}{n+2} - (n+2)\log(K[1])\right)dK[1]} + (n+1)x^{n+1}\right)}{n+1}$$

8.5 problem 14

Internal problem ID [10489]

Book: Handbook of exact solutions for ordinary differential equations. By Polyanin and Zaitsev. Second edition
Section: Chapter 1, section 1.2. Riccati Equation. subsection 1.2.5-2
Problem number: 14.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_linear]`

$$\boxed{y' - a\ln(x)^n y + abx\ln(x)^{n+1} y = b\ln(x) + b}$$

✓ Solution by Maple
Time used: 0.0 (sec). Leaf size: 51

```
dsolve(diff(y(x),x)=a*(ln(x))^n*y(x)-a*b*x*(ln(x))^(n+1)*y(x)+b*ln(x)+b,y(x), singsol=all)
```

$$y(x) = \left(b\left(\int e^{a\left(\int \ln(x)^n(-1+\ln(x)bx)dx\right)}(\ln(x)+1)\,dx\right) + c_1\right) e^{-a\left(\int \ln(x)^n(-1+\ln(x)bx)dx\right)}$$

✓ Solution by Mathematica
Time used: 0.86 (sec). Leaf size: 124

```
DSolve[y'[x]==a*(Log[x])^n*y[x]-a*b*x*(Log[x])^(n+1)*y[x]+b*Log[x]+b,y[x],x,IncludeSingularS
```

$$y(x) \to \exp\left(a2^{-n-2}(-\log(x))^{-n}\log^n(x)\left(b\Gamma(n+2,-2\log(x))\right.\right.$$
$$\left.\left.+2^{n+2}\Gamma(n+1,-\log(x))\right)\right)\left(\int_1^x b\exp\left(-2^{-n-2}a\left(2^{n+2}\Gamma(n+1,-\log(K[1]))+b\Gamma(n+2,-2\log(K[1]))\right)(-\right.\right.$$
$$\left.\left.+1)dK[1]+c_1\right)$$

8.6 problem 15

Internal problem ID [10490]

Book: Handbook of exact solutions for ordinary differential equations. By Polyanin and Zaitsev. Second edition
Section: Chapter 1, section 1.2. Riccati Equation. subsection 1.2.5-2
Problem number: 15.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[[_1st_order, '_with_symmetry_[F(x),G(x)]'], _Riccati]`

$$\boxed{y' - a \ln(x)^k (y - b x^n - c)^2 = b x^{n-1} n}$$

✓ Solution by Maple
Time used: 0.016 (sec). Leaf size: 24

```
dsolve(diff(y(x),x)=a*(ln(x))^k*(y(x)-b*x^n-c)^2+b*n*x^(n-1),y(x), singsol=all)
```

$$y(x) = b x^n + c + \frac{1}{c_1 - a \left(\int \ln(x)^k \, dx \right)}$$

✓ Solution by Mathematica
Time used: 1.572 (sec). Leaf size: 51

```
DSolve[y'[x]==a*(Log[x])^k*(y[x]-b*x^n-c)^2+b*n*x^(n-1),y[x],x,IncludeSingularSolutions -> T
```

$$y(x) \to \frac{1}{-a(-\log(x))^{-k} \log^k(x) \Gamma(k+1, -\log(x)) + c_1} + b x^n + c$$
$$y(x) \to b x^n + c$$

8.7 problem 16

Internal problem ID [10491]

Book: Handbook of exact solutions for ordinary differential equations. By Polyanin and Zaitsev. Second edition
Section: Chapter 1, section 1.2. Riccati Equation. subsection 1.2.5-2
Problem number: 16.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_Riccati]`

$$\boxed{y' - a\ln(x)^n y^2 - b\ln(x)^m y = bc\ln(x)^m - a\,c^2\ln(x)^n}$$

✓ Solution by Maple
Time used: 0.015 (sec). Leaf size: 99

```
dsolve(diff(y(x),x)=a*(ln(x))^n*y(x)^2+b*(ln(x))^m*y(x)+b*c*(ln(x))^m-a*c^2*(ln(x))^n,y(x),
```

$$y(x) = \frac{-ca\left(\int \ln(x)^n \mathrm{e}^{-\left(\int (2\ln(x)^n ac - \ln(x)^m b)dx\right)} dx\right) - c_1 c - \mathrm{e}^{-\left(\int (2\ln(x)^n ac - \ln(x)^m b)dx\right)}}{c_1 + a\left(\int \ln(x)^n \mathrm{e}^{-\left(\int (2\ln(x)^n ac - \ln(x)^m b)dx\right)} dx\right)}$$

✓ Solution by Mathematica
Time used: 3.927 (sec). Leaf size: 385

```
DSolve[y'[x]==a*(Log[x])^n*y[x]^2+b*(Log[x])^m*y[x]+b*c*(Log[x])^m-a*c^2*(Log[x])^n,y[x],x,I
```

$$\text{Solve}\left[\int_1^x \frac{\exp\left(b\Gamma(m+1,-\log(K[1]))(-\log(K[1]))^{-m}\log^m(K[1]) - 2ac\Gamma(n+1,-\log(K[1]))(-\log(K[1]\right.}{ab(m-n)(c+y(x))}$$
$$+\int_1^{y(x)} \left(\frac{\exp\left(b\Gamma(m+1,-\log(x))(-\log(x))^{-m}\log^m(x) - 2ac\Gamma(n+1,-\log(x))(-\log(x))^{-n}\log^n(x)\right)}{ab(m-n)(c+K[2])^2}\right.$$
$$-\int_1^x \left(-\frac{\exp\left(b\Gamma(m+1,-\log(K[1]))(-\log(K[1]))^{-m}\log^m(K[1]) - 2ac\Gamma(n+1,-\log(K[1]))(-\log(K[1]\right.}{b(m-n)(c+K[2])}$$

8.8 problem 17

Internal problem ID [10492]

Book: Handbook of exact solutions for ordinary differential equations. By Polyanin and Zaitsev. Second edition
Section: Chapter 1, section 1.2. Riccati Equation. subsection 1.2.5-2
Problem number: 17.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[[_1st_order, _with_linear_symmetries], _Riccati]`

$$\boxed{y'x - (ay + b\ln(x))^2 = 0}$$

✓ Solution by Maple
Time used: 0.016 (sec). Leaf size: 32

```
dsolve(x*diff(y(x),x)=(a*y(x)+b*ln(x))^2,y(x), singsol=all)
```

$$y(x) = \frac{-\ln(x)\,ab + \tan\left(\left(\ln(x) + c_1\right)\sqrt{ab}\right)\sqrt{ab}}{a^2}$$

✓ Solution by Mathematica
Time used: 6.524 (sec). Leaf size: 43

```
DSolve[x*y'[x]==(a*y[x]+b*Log[x])^2,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to -\frac{b\log(x)}{a} + \sqrt{\frac{b}{a^3}}\tan\left(a^2\sqrt{\frac{b}{a^3}}\log(x) + c_1\right)$$

8.9 problem 18

Internal problem ID [10493]

Book: Handbook of exact solutions for ordinary differential equations. By Polyanin and Zaitsev. Second edition
Section: Chapter 1, section 1.2. Riccati Equation. subsection 1.2.5-2
Problem number: 18.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_Riccati]`

$$y'x - a\ln(\lambda x)^m y^2 - ky = a\,b^2 x^{2k}\ln(\lambda x)^m$$

✓ Solution by Maple
Time used: 0.047 (sec). Leaf size: 31

```
dsolve(x*diff(y(x),x)=a*(ln(lambda*x))^m*y(x)^2+k*y(x)+a*b^2*x^(2*k)*(ln(lambda*x))^m,y(x),
```

$$y(x) = -\tan\left(-ab\left(\int x^{-1+k}\ln(x\lambda)^m\,dx\right)+c_1\right)b\,x^k$$

✓ Solution by Mathematica
Time used: 2.161 (sec). Leaf size: 70

```
DSolve[x*y'[x]==a*(Log[\[Lambda]*x])^m*y[x]^2+k*y[x]+a*b^2*x^(2*k)*(Log[\[Lambda]*x])^m,y[x]
```

$$y(x)\to\sqrt{b^2}x^k\tan\left(\frac{a\sqrt{b^2}x^k(\lambda x)^{-k}\log^m(\lambda x)(-k\log(\lambda x))^{-m}\Gamma(m+1,-k\log(x\lambda))}{k}+c_1\right)$$

8.10 problem 19

Internal problem ID [10494]

Book: Handbook of exact solutions for ordinary differential equations. By Polyanin and Zaitsev. Second edition
Section: Chapter 1, section 1.2. Riccati Equation. subsection 1.2.5-2
Problem number: 19.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[[_1st_order, '_with_symmetry_[F(x),G(x)]'], _Riccati]`

$$\boxed{y'x - a\,x^n(y + b\ln(x))^2 = -b}$$

✓ Solution by Maple
Time used: 0.0 (sec). Leaf size: 24

```
dsolve(x*diff(y(x),x)=a*x^n*(y(x)+b*ln(x))^2-b,y(x), singsol=all)
```

$$y(x) = -b\ln(x) + \frac{n}{c_1 n - a\,x^n}$$

✓ Solution by Mathematica
Time used: 0.649 (sec). Leaf size: 35

```
DSolve[x*y'[x]==a*x^n*(y[x]+b*Log[x])^2-b,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to -b\log(x) + \frac{n}{-ax^n + c_1 n}$$
$$y(x) \to -b\log(x)$$

8.11 problem 20

Internal problem ID [10495]

Book: Handbook of exact solutions for ordinary differential equations. By Polyanin and Zaitsev. Second edition
Section: Chapter 1, section 1.2. Riccati Equation. subsection 1.2.5-2
Problem number: 20.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_Riccati]`

$$\boxed{y' x - a\, x^{2n} \ln(x)\, y^2 - (b\, x^n \ln(x) - n)\, y = c \ln(x)}$$

✓ Solution by Maple
Time used: 0.032 (sec). Leaf size: 80

```
dsolve(x*diff(y(x),x)=a*x^(2*n)*ln(x)*y(x)^2+(b*x^n*ln(x)-n)*y(x)+c*ln(x),y(x), singsol=all)
```

$$y(x) = \frac{\left(\tan\left(\frac{(b(n\ln(x)-1)x^n+c_1 n^2)\sqrt{4a\,b^2c-b^4}}{2b^2n^2}\right)\sqrt{4a\,b^2c-b^4}-b^2\right)x^{-n}}{2ab}$$

✓ Solution by Mathematica
Time used: 1.872 (sec). Leaf size: 130

```
DSolve[x*y'[x]==a*x^(2*n)*Log[x]*y[x]^2+(b*x^n*Log[x]-n)*y[x]+c*Log[x],y[x],x,IncludeSingula
```

$$y(x) \to \frac{x^{-n}\left(-b+\frac{\sqrt{b^2-4ac}\left(-e^{\frac{x^n\sqrt{b^2-4ac}(n\log(x)-1)}{n^2}}+c_1\right)}{e^{\frac{x^n\sqrt{b^2-4ac}(n\log(x)-1)}{n^2}}+c_1}\right)}{2a}$$

$$y(x) \to \frac{x^{-n}\left(\sqrt{b^2-4ac}-b\right)}{2a}$$

8.12 problem 21

Internal problem ID [10496]

Book: Handbook of exact solutions for ordinary differential equations. By Polyanin and Zaitsev. Second edition
Section: Chapter 1, section 1.2. Riccati Equation. subsection 1.2.5-2
Problem number: 21.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_Riccati]`

$$\boxed{x^2y' - y^2a^2x^2 + xy = b^2 \ln(x)^n}$$

✓ Solution by Maple
Time used: 0.0 (sec). Leaf size: 251

```
dsolve(x^2*diff(y(x),x)=a^2*x^2*y(x)^2-x*y(x)+b^2*(ln(x))^n,y(x), singsol=all)
```

$$y(x) = \frac{\ln(x)^{\frac{n}{2}+1}\,\text{BesselY}\left(\frac{3+n}{n+2}, \frac{2\sqrt{a^2b^2}\,\ln(x)^{\frac{n}{2}+1}}{n+2}\right)\sqrt{a^2b^2}\,c_1 + \text{BesselJ}\left(\frac{3+n}{n+2}, \frac{2\sqrt{a^2b^2}\,\ln(x)^{\frac{n}{2}+1}}{n+2}\right)\sqrt{a^2b^2}\,\ln(x)^{\frac{n}{2}+1} - \text{Bes}}{\left(\text{BesselY}\left(\frac{1}{n+2}, \frac{2\sqrt{a^2b^2}\,\ln(x)^{\frac{n}{2}+1}}{n+2}\right)c_1 + \text{BesselJ}\left(\frac{1}{n+2}, \frac{2\sqrt{a^2b^2}\,\ln(x)^{\frac{n}{2}+1}}{n+2}\right)\right.}$$

✓ Solution by Mathematica

Time used: 45.846 (sec). Leaf size: 1769

```
DSolve[x^2*y'[x]==a^2*x^2*y[x]^2-x*y[x]+b^2*(Log[x])^n,y[x],x,IncludeSingularSolutions -> Tr
```

$y(x)$

$$\to \frac{}{x\left(2a(n+2)^{\frac{2(n+1)}{n+2}}\,\text{BesselJ}\left(-\frac{1}{n+2},\frac{2a\sqrt{\left(b^2\log^{n+1}(x)\right)^{1+\frac{1}{n+1}}}}{\sqrt{b^{\frac{2}{n+1}}(n+2)^2}}\right)\text{Gamma}\left(\frac{2n+3}{n+2}\right)\left(b^2\log^{n+1}(x)\right)^{1+\frac{1}{n+1}}b^{\frac{2}{n+2}}+an\right.$$

$y(x)$

$$\to \frac{2b^2\sqrt{(n+2)^2b^{\frac{2}{n+1}}}\log^{n+1}(x)\sqrt{(b}}{x\left(-a(n+2)\left(b^2\log^{n+1}(x)\right)^{\frac{1}{n+1}+1}\text{BesselJ}\left(\frac{1}{n+2},\frac{2a\sqrt{\left(b^2\log^{n+1}(x)\right)^{1+\frac{1}{n+1}}}}{\sqrt{b^{\frac{2}{n+1}}(n+2)^2}}\right)+a(n+2)\left(b^2\log^{n+1}(x)\right)^{\frac{1}{n+1}+1}\right.}$$

8.13 problem 22

Internal problem ID [10497]

Book: Handbook of exact solutions for ordinary differential equations. By Polyanin and Zaitsev. Second edition
Section: Chapter 1, section 1.2. Riccati Equation. subsection 1.2.5-2
Problem number: 22.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_Riccati]`

$$\boxed{(a\ln(x)+b)\,y' - y^2 - c\ln(x)^n\,y = -\lambda^2 + \lambda c\ln(x)^n}$$

✓ Solution by Maple
Time used: 0.0 (sec). Leaf size: 107

```
dsolve((a*ln(x)+b)*diff(y(x),x)=y(x)^2+c*(ln(x))^n*y(x)-lambda^2+lambda*c*(ln(x))^n,y(x), si
```

$$y(x) = \frac{-\left(\int \frac{e^{\int \frac{\ln(x)^n c-2\lambda}{a\ln(x)+b}dx}}{a\ln(x)+b}dx\right)\lambda - \lambda c_1 - e^{\int \frac{\ln(x)^n c-2\lambda}{a\ln(x)+b}dx}}{c_1 + \int \frac{e^{\int \frac{\ln(x)^n c-2\lambda}{a\ln(x)+b}dx}}{a\ln(x)+b}dx}$$

✓ Solution by Mathematica

Time used: 5.348 (sec). Leaf size: 275

```
DSolve[(a*Log[x]+b)*y'[x]==y[x]^2+c*(Log[x])^n*y[x]-\[Lambda]^2+\[Lambda]*c*(Log[x])^n,y[x],
```

$$\text{Solve}\left[\int_1^x -\frac{\exp\left(-\int_1^{K[2]} \frac{2\lambda - c\log^n(K[1])}{b+a\log(K[1])} dK[1]\right)\left(c\log^n(K[2]) - \lambda + y(x)\right)}{cn(b+a\log(K[2]))(\lambda+y(x))} dK[2]\right.$$

$$+\int_1^{y(x)} \left(\frac{\exp\left(-\int_1^x \frac{2\lambda - c\log^n(K[1])}{b+a\log(K[1])} dK[1]\right)}{cn(\lambda+K[3])^2}\right.$$

$$\left.\left.-\int_1^x \left(\frac{\exp\left(-\int_1^{K[2]} \frac{2\lambda - c\log^n(K[1])}{b+a\log(K[1])} dK[1]\right)\left(c\log^n(K[2]) - \lambda + K[3]\right)}{cn(\lambda+K[3])^2(b+a\log(K[2]))} - \frac{\exp\left(-\int_1^{K[2]} \frac{2\lambda - c\log^n(K[1])}{b+a\log(K[1])} dK[1]\right)}{cn(\lambda+K[3])(b+a\log(K[2]))}\right)\right.\right.$$

8.14 problem 23

Internal problem ID [10498]

Book: Handbook of exact solutions for ordinary differential equations. By Polyanin and Zaitsev. Second edition
Section: Chapter 1, section 1.2. Riccati Equation. subsection 1.2.5-2
Problem number: 23.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_Riccati]`

$$\boxed{(a\ln(x)+b)\,y' - \ln(x)^n\,y^2 - yc = -\lambda^2\ln(x)^n + c\lambda}$$

✓ Solution by Maple
Time used: 0.0 (sec). Leaf size: 124

```
dsolve((a*ln(x)+b)*diff(y(x),x)=(ln(x))^n*y(x)^2+c*y(x)-lambda^2*(ln(x))^n+c*lambda,y(x), si
```

$$y(x) = \frac{-\lambda c_1 - \left(\int \frac{\ln(x)^n \mathrm{e}^{-\left(\int \frac{2\ln(x)^n\lambda - c}{a\ln(x)+b}dx\right)}}{a\ln(x)+b}dx\right)\lambda - \mathrm{e}^{-\left(\int \frac{2\ln(x)^n\lambda - c}{a\ln(x)+b}dx\right)}}{c_1 + \int \frac{\ln(x)^n \mathrm{e}^{-\left(\int \frac{2\ln(x)^n\lambda - c}{a\ln(x)+b}dx\right)}}{a\ln(x)+b}dx}$$

✓ Solution by Mathematica

Time used: 5.137 (sec). Leaf size: 286

```
DSolve[(a*Log[x]+b)*y'[x]==(Log[x])^n*y[x]^2+c*y[x]-\[Lambda]^2*(Log[x])^n+c*\[Lambda],y[x],
```

$$\text{Solve}\left[\int_1^x \frac{\exp\left(-\int_1^{K[2]} -\frac{c-2\lambda \log^n(K[1])}{b+a\log(K[1])}dK[1]\right)\left(-\lambda \log^n(K[2])+y(x)\log^n(K[2])+c\right)}{cn(b+a\log(K[2]))(\lambda+y(x))}dK[2]\right.$$

$$+\int_1^{y(x)}\left(-\int_1^x\left(\frac{\exp\left(-\int_1^{K[2]} -\frac{c-2\lambda \log^n(K[1])}{b+a\log(K[1])}dK[1]\right)\log^n(K[2])}{cn(\lambda+K[3])(b+a\log(K[2]))}-\frac{\exp\left(-\int_1^{K[2]} -\frac{c-2\lambda \log^n(K[1])}{b+a\log(K[1])}dK[1]\right)(-\lambda}{cn(\lambda+K[3])^2(b+}\right.\right.$$

$$\left.\left.-\frac{\exp\left(-\int_1^{x} -\frac{c-2\lambda \log^n(K[1])}{b+a\log(K[1])}dK[1]\right)}{cn(\lambda+K[3])^2}\right)dK[3]=c_1,y(x)\right]$$

9 Chapter 1, section 1.2. Riccati Equation. subsection 1.2.6-1. Equations with sine

9.1 problem 1 241
9.2 problem 2 242
9.3 problem 3 243
9.4 problem 4 244
9.5 problem 5 245
9.6 problem 6 246
9.7 problem 7 247
9.8 problem 8 248
9.9 problem 9 249
9.10 problem 10 250
9.11 problem 11 251
9.12 problem 12 252
9.13 problem 13 254

9.1 problem 1

Internal problem ID [10499]

Book: Handbook of exact solutions for ordinary differential equations. By Polyanin and Zaitsev. Second edition
Section: Chapter 1, section 1.2. Riccati Equation. subsection 1.2.6-1. Equations with sine
Problem number: 1.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_Riccati]`

$$\boxed{y' - \alpha y^2 = \beta + \gamma \sin(\lambda x)}$$

✓ Solution by Maple
Time used: 0.0 (sec). Leaf size: 110

```
dsolve(diff(y(x),x)=alpha*y(x)^2+beta+gamma*sin(lambda*x),y(x), singsol=all)
```

$$y(x) = -\frac{\lambda\left(c_1 \operatorname{MathieuSPrime}\left(\frac{4\alpha\beta}{\lambda^2}, -\frac{2\gamma\alpha}{\lambda^2}, -\frac{\pi}{4}+\frac{x\lambda}{2}\right) + \operatorname{MathieuCPrime}\left(\frac{4\alpha\beta}{\lambda^2}, -\frac{2\gamma\alpha}{\lambda^2}, -\frac{\pi}{4}+\frac{x\lambda}{2}\right)\right)}{2\alpha\left(c_1 \operatorname{MathieuS}\left(\frac{4\alpha\beta}{\lambda^2}, -\frac{2\gamma\alpha}{\lambda^2}, -\frac{\pi}{4}+\frac{x\lambda}{2}\right) + \operatorname{MathieuC}\left(\frac{4\alpha\beta}{\lambda^2}, -\frac{2\gamma\alpha}{\lambda^2}, -\frac{\pi}{4}+\frac{x\lambda}{2}\right)\right)}$$

✓ Solution by Mathematica
Time used: 0.612 (sec). Leaf size: 191

```
DSolve[y'[x]==\[Alpha]*y[x]^2+\[Beta]+\[Gamma]*Sin[\[Lambda]*x],y[x],x,IncludeSingularSoluti
```

$$y(x) \to -\frac{\lambda\left(\operatorname{MathieuSPrime}\left[\frac{4\alpha\beta}{\lambda^2}, -\frac{2\alpha\gamma}{\lambda^2}, \frac{1}{4}(\pi - 2\lambda x)\right] + c_1 \operatorname{MathieuCPrime}\left[\frac{4\alpha\beta}{\lambda^2}, -\frac{2\alpha\gamma}{\lambda^2}, \frac{1}{4}(2\lambda x - \pi)\right]\right)}{2\alpha\left(\operatorname{MathieuS}\left[\frac{4\alpha\beta}{\lambda^2}, -\frac{2\alpha\gamma}{\lambda^2}, \frac{1}{4}(2\lambda x - \pi)\right] + c_1 \operatorname{MathieuC}\left[\frac{4\alpha\beta}{\lambda^2}, -\frac{2\alpha\gamma}{\lambda^2}, \frac{1}{4}(\pi - 2\lambda x)\right]\right)}$$

$$y(x) \to \frac{\lambda \operatorname{MathieuCPrime}\left[\frac{4\alpha\beta}{\lambda^2}, -\frac{2\alpha\gamma}{\lambda^2}, \frac{1}{4}(\pi - 2\lambda x)\right]}{2\alpha \operatorname{MathieuC}\left[\frac{4\alpha\beta}{\lambda^2}, -\frac{2\alpha\gamma}{\lambda^2}, \frac{1}{4}(\pi - 2\lambda x)\right]}$$

9.2 problem 2

Internal problem ID [10500]

Book: Handbook of exact solutions for ordinary differential equations. By Polyanin and Zaitsev. Second edition
Section: Chapter 1, section 1.2. Riccati Equation. subsection 1.2.6-1. Equations with sine
Problem number: 2.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_Riccati]`

$$\boxed{y' - y^2 = -a^2 + a\lambda \sin(\lambda x) + a^2 \sin(\lambda x)^2}$$

✓ Solution by Maple
Time used: 0.015 (sec). Leaf size: 289

```
dsolve(diff(y(x),x)=y(x)^2-a^2+a*lambda*sin(lambda*x)+a^2*sin(lambda*x)^2,y(x), singsol=all)
```

$$y(x) = \frac{\left(-2ac_1 \cos(x\lambda) \sin\left(\frac{\pi}{4} + \frac{x\lambda}{2}\right) - c_1\lambda \cos\left(\frac{\pi}{4} + \frac{x\lambda}{2}\right)\right) \text{HeunC}\left(\frac{4a}{\lambda}, \frac{1}{2}, -\frac{1}{2}, -\frac{2a}{\lambda}, \frac{8a+3\lambda}{8\lambda}, \frac{\sin(x\lambda)}{2} + \frac{1}{2}\right) - 2\left(a \text{ Heu}\right.}{2\,\text{HeunC}\left(\frac{4a}{\lambda}, \frac{1}{2}, -\frac{1}{2}, -\frac{2a}{\lambda}, \frac{8a+3\lambda}{8\lambda}, \frac{\sin}{}\right.}$$

✓ Solution by Mathematica
Time used: 4.337 (sec). Leaf size: 132

```
DSolve[y'[x]==y[x]^2-a^2+a*\[Lambda]*Sin[\[Lambda]*x]+a^2*Sin[\[Lambda]*x]^2,y[x],x,IncludeS
```

$$y(x) \to -\frac{ac_1 \cos(\lambda x) \int_1^x e^{-\frac{2a \sin(\lambda K[1])}{\lambda}} dK[1] + a\cos(\lambda x) + c_1 e^{-\frac{2a \sin(\lambda x)}{\lambda}}}{1 + c_1 \int_1^x e^{-\frac{2a \sin(\lambda K[1])}{\lambda}} dK[1]}$$

$$y(x) \to -\frac{e^{-\frac{2a \sin(\lambda x)}{\lambda}}}{\int_1^x e^{-\frac{2a \sin(\lambda K[1])}{\lambda}} dK[1]} - a\cos(\lambda x)$$

9.3 problem 3

Internal problem ID [10501]

Book: Handbook of exact solutions for ordinary differential equations. By Polyanin and Zaitsev. Second edition
Section: Chapter 1, section 1.2. Riccati Equation. subsection 1.2.6-1. Equations with sine
Problem number: 3.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_Riccati]`

$$\boxed{y' - y^2 = c \sin(\lambda x + a)^n \sin(\lambda x + b)^{-n-4} + \lambda^2}$$

✗ Solution by Maple

```
dsolve(diff(y(x),x)=y(x)^2+lambda^2+c*sin(lambda*x+a)^n*sin(lambda*x+b)^(-n-4),y(x), singsol
```

No solution found

✗ Solution by Mathematica
Time used: 0.0 (sec). Leaf size: 0

```
DSolve[y'[x]==y[x]^2+\[Lambda]^2+c*Sin[\[Lambda]*x+a]^n*Sin[\[Lambda]*x+b]^(-n-4),y[x],x,Inc
```

Not solved

9.4 problem 4

Internal problem ID [10502]

Book: Handbook of exact solutions for ordinary differential equations. By Polyanin and Zaitsev. Second edition
Section: Chapter 1, section 1.2. Riccati Equation. subsection 1.2.6-1. Equations with sine
Problem number: 4.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_Riccati]`

$$\boxed{y' - y^2 - a\sin(\beta x)\, y = ab\sin(\beta x) - b^2}$$

✓ Solution by Maple
Time used: 0.0 (sec). Leaf size: 76

```
dsolve(diff(y(x),x)=y(x)^2+a*sin(beta*x)*y(x)+a*b*sin(beta*x)-b^2,y(x), singsol=all)
```

$$y(x) = \frac{b\left(\int \mathrm{e}^{\frac{-2b\beta x - a\cos(x\beta)}{\beta}} dx\right) - c_1 b + \mathrm{e}^{\frac{-2b\beta x - a\cos(x\beta)}{\beta}}}{-\left(\int \mathrm{e}^{\frac{-2b\beta x - a\cos(x\beta)}{\beta}} dx\right) + c_1}$$

✓ Solution by Mathematica
Time used: 9.066 (sec). Leaf size: 187

```
DSolve[y'[x]==y[x]^2+a*Sin[\[Beta]*x]*y[x]+a*b*Sin[\[Beta]*x]-b^2,y[x],x,IncludeSingularSolu
```

$$\text{Solve}\left[\int_1^x -\frac{e^{-\frac{a\cos(\beta K[1])}{\beta}-2bK[1]}(-b+a\sin(\beta K[1])+y(x))}{a\beta(b+y(x))}dK[1] + \int_1^{y(x)}\left(\frac{e^{-2bx-\frac{a\cos(x\beta)}{\beta}}}{a\beta(b+K[2])^2}\right.\right.$$
$$\left.\left. -\int_1^x \left(\frac{e^{-\frac{a\cos(\beta K[1])}{\beta}-2bK[1]}(-b+K[2]+a\sin(\beta K[1]))}{a\beta(b+K[2])^2} - \frac{e^{-\frac{a\cos(\beta K[1])}{\beta}-2bK[1]}}{a\beta(b+K[2])}\right) dK[1]\right) dK[2] = c_1, y(x)\right]$$

9.5 problem 5

Internal problem ID [10503]

Book: Handbook of exact solutions for ordinary differential equations. By Polyanin and Zaitsev. Second edition
Section: Chapter 1, section 1.2. Riccati Equation. subsection 1.2.6-1. Equations with sine
Problem number: 5.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_Riccati]`

$$\boxed{y' - y^2 - a \sin(bx)^m y = a \sin(bx)^m}$$

✗ Solution by Maple

```
dsolve(diff(y(x),x)=y(x)^2+a*sin(b*x)^m*y(x)+a*sin(b*x)^m,y(x), singsol=all)
```

No solution found

✗ Solution by Mathematica
Time used: 0.0 (sec). Leaf size: 0

```
DSolve[y'[x]==y[x]^2+a*Sin[b*x]^m*y[x]+a*Sin[b*x]^m,y[x],x,IncludeSingularSolutions -> True]
```

Not solved

9.6 problem 6

Internal problem ID [10504]

Book: Handbook of exact solutions for ordinary differential equations. By Polyanin and Zaitsev. Second edition
Section: Chapter 1, section 1.2. Riccati Equation. subsection 1.2.6-1. Equations with sine
Problem number: 6.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_Riccati]`

$$\boxed{y' - \lambda \sin(\lambda x)\, y^2 = \lambda \sin(\lambda x)^3}$$

✓ Solution by Maple
Time used: 0.016 (sec). Leaf size: 51

```
dsolve(diff(y(x),x)=lambda*sin(lambda*x)*y(x)^2+lambda*sin(lambda*x)^3,y(x), singsol=all)
```

$$y(x) = \frac{2\,\mathrm{e}^{\frac{\cos(2x\lambda)}{2}+\frac{1}{2}} c_1 - \cos(x\lambda)\sqrt{\pi}\,(\operatorname{erfi}(\cos(x\lambda))\,c_1 + 1)}{\sqrt{\pi}\,(\operatorname{erfi}(\cos(x\lambda))\,c_1 + 1)}$$

✗ Solution by Mathematica
Time used: 0.0 (sec). Leaf size: 0

```
DSolve[y'[x]==\[Lambda]*Sin[\[Lambda]*x]*y[x]^2+\[Lambda]*Sin[\[Lambda]*x]^3,y[x],x,IncludeS
```

Not solved

9.7 problem 7

Internal problem ID [10505]

Book: Handbook of exact solutions for ordinary differential equations. By Polyanin and Zaitsev. Second edition
Section: Chapter 1, section 1.2. Riccati Equation. subsection 1.2.6-1. Equations with sine
Problem number: 7.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_Riccati]`

$$2y' - (\lambda + a - a\sin(\lambda x))\, y^2 = -a\sin(\lambda x) - a + \lambda$$

✓ Solution by Maple
Time used: 0.0 (sec). Leaf size: 220

```
dsolve(2*diff(y(x),x)=(lambda+a-a*sin(lambda*x))*y(x)^2+lambda-a-a*sin(lambda*x),y(x), sings
```

$$y(x) = -\frac{\left(\left(\left(\int^{\sin(x\lambda)} \frac{(a(\_a-1)-\lambda)\mathrm{e}^{\frac{a\_a}{\lambda}}}{(\_a-1)^{\frac{3}{2}}\sqrt{\_a+1}} d\_a\right) c_1 + 1\right)\sqrt{-\cos\left(\frac{\pi}{4}+\frac{x\lambda}{2}\right)^2}\left(a\cos(x\lambda)+\tan\left(\frac{\pi}{4}+\frac{x\lambda}{2}\right)\lambda\right)\operatorname{csgn}\left(\sin\right.\right.}{\sqrt{-\cos\left(\frac{\pi}{4}+\frac{x\lambda}{2}\right)^2}\left(\left(\int^{\sin(x\lambda)} \frac{(a(\_a-1)-\lambda)\mathrm{e}^{\frac{a\_a}{\lambda}}}{(\_a-1)^{\frac{3}{2}}\sqrt{\_a+1}} d\_\right.\right.}$$

✗ Solution by Mathematica
Time used: 0.0 (sec). Leaf size: 0

```
DSolve[2*y'[x]==(\[Lambda]+a-a*Sin[\[Lambda]*x])*y[x]^2+\[Lambda]-a-a*Sin[\[Lambda]*x],y[x],
```

Not solved

9.8 problem 8

Internal problem ID [10506]

Book: Handbook of exact solutions for ordinary differential equations. By Polyanin and Zaitsev. Second edition
Section: Chapter 1, section 1.2. Riccati Equation. subsection 1.2.6-1. Equations with sine
Problem number: 8.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_Riccati]`

$$\boxed{y' - \left(\lambda + \sin\left(\lambda x\right)^2 a\right) y^2 = -a + \lambda + \sin\left(\lambda x\right)^2 a}$$

✓ Solution by Maple
Time used: 0.0 (sec). Leaf size: 102

```
dsolve(diff(y(x),x)=(lambda+a*sin(lambda*x)^2)*y(x)^2+lambda-a+a*sin(lambda*x)^2,y(x), sings
```

$$y(x) = \frac{2\cot\left(x\lambda\right)\lambda\left(\int \mathrm{e}^{\frac{a\cos(2x\lambda)}{2\lambda}}\left(\csc\left(x\lambda\right)^2\lambda + a\right)dx\right)c_1 + 2\csc\left(x\lambda\right)^2 \mathrm{e}^{\frac{a\cos(2x\lambda)}{2\lambda}} c_1\lambda - i\cot\left(x\lambda\right)}{-2\lambda\left(\int \mathrm{e}^{\frac{a\cos(2x\lambda)}{2\lambda}}\left(\csc\left(x\lambda\right)^2\lambda + a\right)dx\right)c_1 + i}$$

✓ Solution by Mathematica
Time used: 41.676 (sec). Leaf size: 187

```
DSolve[y'[x]==(\[Lambda]+a*Sin[\[Lambda]*x]^2)*y[x]^2+\[Lambda]-a+a*Sin[\[Lambda]*x]^2,y[x],
```

$$y(x) \to -\frac{2\left(c_1\cot(\lambda x)\int_1^x e^{-\frac{a\sin^2(\lambda K[1])}{\lambda}}\left(\lambda\csc^2(\lambda K[1]) + a\right)dK[1] + c_1\csc^2(\lambda x)e^{-\frac{a\sin^2(\lambda x)}{\lambda}} + \cot(\lambda x)\right)}{2 + 2c_1\int_1^x e^{-\frac{a\sin^2(\lambda K[1])}{\lambda}}\left(\lambda\csc^2(\lambda K[1]) + a\right)dK[1]}$$

$$y(x) \to -\frac{\csc^2(\lambda x)e^{-\frac{a\sin^2(\lambda x)}{\lambda}}}{\int_1^x e^{-\frac{a\sin^2(\lambda K[1])}{\lambda}}\left(\lambda\csc^2(\lambda K[1]) + a\right)dK[1]} - \cot(\lambda x)$$

9.9 problem 9

Internal problem ID [10507]

Book: Handbook of exact solutions for ordinary differential equations. By Polyanin and Zaitsev. Second edition
Section: Chapter 1, section 1.2. Riccati Equation. subsection 1.2.6-1. Equations with sine
Problem number: 9.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_Riccati]`

$$\boxed{y' + (k+1)\, x^k y^2 - a\, x^{k+1} \sin(x)^m\, y = -a \sin(x)^m}$$

✓ Solution by Maple
Time used: 0.016 (sec). Leaf size: 174

```
dsolve(diff(y(x),x)=-(k+1)*x^k*y(x)^2+a*x^(k+1)*sin(x)^m*y(x)-a*sin(x)^m,y(x), singsol=all)
```

$$y(x) = \frac{x^{-1-k}\left(x^{1+k} e^{\int \frac{x^{1+k}\sin(x)^m a x-2k-2}{x}dx} + \left(\int x^k e^{\int \frac{x^{1+k}\sin(x)^m a x-2k-2}{x}dx}dx\right) k + \int x^k e^{\int \frac{x^{1+k}\sin(x)^m a x-2k-2}{x}dx}dx - c_1\right.}{\left(\int x^k e^{\int \frac{a\, x^{k+2}\sin(x)^m-2k-2}{x}dx}dx\right) k + \int x^k e^{\int \frac{a\, x^{k+2}\sin(x)^m-2k-2}{x}dx}dx - c_1}$$

✓ Solution by Mathematica
Time used: 16.483 (sec). Leaf size: 248

```
DSolve[y'[x]==-(k+1)*x^k*y[x]^2+a*x^(k+1)*Sin[x]^m*y[x]-a*Sin[x]^m,y[x],x,IncludeSingularSol
```

$$y(x) \to \frac{x^{-k-1}\left(c_1 x \exp\left(\int_1^x -\frac{-a\sin^m(K[1])K[1]^{k+2}+k+2}{K[1]}dK[1]\right) + c_1(k+1)\int_1^x \exp\left(\int_1^{K[2]} -\frac{-a\sin^m(K[1])K[1]^{k+2}+k+2}{K[1]}dK[1]\right)dK[2]\right)}{(k+1)\left(1 + c_1 \int_1^x \exp\left(\int_1^{K[2]} -\frac{-a\sin^m(K[1])K[1]^{k+2}+k+2}{K[1]}dK[1]\right)dK[2]\right)}$$

$$y(x) \to \frac{x^{-k}\left(\frac{\exp\left(\int_1^x -\frac{-a\sin^m(K[1])K[1]^{k+2}+k+2}{K[1]}dK[1]\right)}{\int_1^x \exp\left(\int_1^{K[2]} -\frac{-a\sin^m(K[1])K[1]^{k+2}+k+2}{K[1]}dK[1]\right)dK[2]} + \frac{k+1}{x}\right)}{k+1}$$

9.10 problem 10

Internal problem ID [10508]

Book: Handbook of exact solutions for ordinary differential equations. By Polyanin and Zaitsev. Second edition
Section: Chapter 1, section 1.2. Riccati Equation. subsection 1.2.6-1. Equations with sine
Problem number: 10.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[[_1st_order, '_with_symmetry_[F(x),G(x)]'], _Riccati]`

$$\boxed{y' - a \sin(\lambda x + \mu)^k (y - b x^n - c)^2 = b x^{n-1} n}$$

✓ Solution by Maple
Time used: 0.0 (sec). Leaf size: 37

```
dsolve(diff(y(x),x)=a*sin(lambda*x+mu)^k*(y(x)-b*x^n-c)^2+b*n*x^(n-1),y(x), singsol=all)
```

$$y(x) = b x^n + c + \frac{1}{c_1 - a \left(\int (\sin(x\lambda)\cos(\mu) + \cos(x\lambda)\sin(\mu))^k \, dx \right)}$$

✓ Solution by Mathematica
Time used: 5.928 (sec). Leaf size: 93

```
DSolve[y'[x]==a*Sin[\[Lambda]*x+\[Mu]]^k*(y[x]-b*x^n-c)^2+b*n*x^(n-1),y[x],x,IncludeSingular
```

$$y(x) \to \frac{1}{-\frac{a\sqrt{\cos^2(\mu+\lambda x)}\sec(\mu+\lambda x)\sin^{k+1}(\mu+\lambda x)\,\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{k+1}{2},\frac{k+3}{2},\sin^2(x\lambda+\mu)\right)}{(k+1)\lambda} + c_1} + bx^n + c$$
$$y(x) \to bx^n + c$$

9.11 problem 11

Internal problem ID [10509]

Book: Handbook of exact solutions for ordinary differential equations. By Polyanin and Zaitsev. Second edition
Section: Chapter 1, section 1.2. Riccati Equation. subsection 1.2.6-1. Equations with sine
Problem number: 11.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_Riccati]`

$$y'x - a\sin(\lambda x)^m y^2 - ky = a\,b^2 x^{2k}\sin(\lambda x)^m$$

✓ Solution by Maple
Time used: 0.031 (sec). Leaf size: 31

```
dsolve(x*diff(y(x),x)=a*sin(lambda*x)^m*y(x)^2+k*y(x)+a*b^2*x^(2*k)*sin(lambda*x)^m,y(x), si
```

$$y(x) = -\tan\left(-ab\left(\int x^{-1+k}\sin(x\lambda)^m\,dx\right) + c_1\right) b\,x^k$$

✓ Solution by Mathematica
Time used: 1.774 (sec). Leaf size: 50

```
DSolve[x*y'[x]==a*Sin[\[Lambda]*x]^m*y[x]^2+k*y[x]+a*b^2*x^(2*k)*Sin[\[Lambda]*x]^m,y[x],x,I
```

$$y(x) \to \sqrt{b^2}x^k \tan\left(\sqrt{b^2}\int_1^x aK[1]^{k-1}\sin^m(\lambda K[1])dK[1] + c_1\right)$$

9.12 problem 12

Internal problem ID [10510]

Book: Handbook of exact solutions for ordinary differential equations. By Polyanin and Zaitsev. Second edition
Section: Chapter 1, section 1.2. Riccati Equation. subsection 1.2.6-1. Equations with sine
Problem number: 12.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_Riccati]`

$$\boxed{(a\sin(\lambda x)+b)\,y'-y^2-c\sin(x\mu)\,y=-d^2+cd\sin(x\mu)}$$

✓ Solution by Maple
Time used: 0.015 (sec). Leaf size: 265

```
dsolve((a*sin(lambda*x)+b)*diff(y(x),x)=y(x)^2+c*sin(mu*x)*y(x)-d^2+c*d*sin(mu*x),y(x), sing
```

$$y(x)$$

$$= \frac{-d\left(\int \frac{\mathrm{e}^{\frac{c\left(\int \frac{\sin(x\mu)}{a\sin(x\lambda)+b}dx\right)\sqrt{-a^2+b^2}\,\lambda-4d\arctan\left(\frac{\tan\left(\frac{x\lambda}{2}\right)b+a}{\sqrt{-a^2+b^2}}\right)}{\sqrt{-a^2+b^2}\,\lambda}}{a\sin(x\lambda)+b}dx\right)+dc_1-\mathrm{e}^{\frac{c\left(\int \frac{\sin(x\mu)}{a\sin(x\lambda)+b}dx\right)\sqrt{-a^2+b^2}\,\lambda-4d\arctan\left(\frac{\tan\left(\frac{x\lambda}{2}\right)b+a}{\sqrt{-a^2+b^2}}\right)}{\sqrt{-a^2+b^2}\,\lambda}}}{\int \frac{\mathrm{e}^{\frac{c\left(\int \frac{\sin(x\mu)}{a\sin(x\lambda)+b}dx\right)\sqrt{-a^2+b^2}\,\lambda-4d\arctan\left(\frac{\tan\left(\frac{x\lambda}{2}\right)b+a}{\sqrt{-a^2+b^2}}\right)}{\sqrt{-a^2+b^2}\,\lambda}}}{a\sin(x\lambda)+b}dx-c_1}$$

✓ Solution by Mathematica

Time used: 15.846 (sec). Leaf size: 289

```
DSolve[(a*Sin[\[Lambda]*x]+b)*y'[x]==y[x]^2+c*Sin[\[Mu]*x]*y[x]-d^2+c*d*Sin[\[Mu]*x],y[x],x,
```

$$\text{Solve}\left[\int_1^x -\frac{\exp\left(-\int_1^{K[2]} \frac{2d-c\sin(\mu K[1])}{b+a\sin(\lambda K[1])}dK[1]\right)(-d+c\sin(\mu K[2])+y(x))}{c\mu(b+a\sin(\lambda K[2]))(d+y(x))}dK[2]\right.$$

$$+\int_1^{y(x)}\left(\frac{\exp\left(-\int_1^x \frac{2d-c\sin(\mu K[1])}{b+a\sin(\lambda K[1])}dK[1]\right)}{c\mu(d+K[3])^2}\right.$$

$$\left.\left.-\int_1^x\left(\frac{\exp\left(-\int_1^{K[2]} \frac{2d-c\sin(\mu K[1])}{b+a\sin(\lambda K[1])}dK[1]\right)(-d+K[3]+c\sin(\mu K[2]))}{c\mu(d+K[3])^2(b+a\sin(\lambda K[2]))}-\frac{\exp\left(-\int_1^{K[2]} \frac{2d-c\sin(\mu K[1])}{b+a\sin(\lambda K[1])}dK[1]\right)}{c\mu(d+K[3])(b+a\sin(\lambda K[2]))}\right)\right.\right.$$

9.13 problem 13

Internal problem ID [10511]

Book: Handbook of exact solutions for ordinary differential equations. By Polyanin and Zaitsev. Second edition
Section: Chapter 1, section 1.2. Riccati Equation. subsection 1.2.6-1. Equations with sine
Problem number: 13.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_Riccati]`

$$\boxed{(a\sin(\lambda x)+b)\left(y'-y^2\right)=a\,\lambda^2\sin(\lambda x)}$$

✓ Solution by Maple
Time used: 0.015 (sec). Leaf size: 261

```
dsolve((a*sin(lambda*x)+b)*(diff(y(x),x)-y(x)^2)-a*lambda^2*sin(lambda*x)=0,y(x), singsol=al
```

$$y(x)$$
$$=\frac{\left(-2(a-b)\,b^2(a+b)\left(\cos\left(\frac{x\lambda}{2}\right)^2-\frac{1}{2}\right)a\sqrt{-a^2+b^2}\,\arctan\left(\frac{\tan\left(\frac{x\lambda}{2}\right)b+a}{\sqrt{-a^2+b^2}}\right)+2c_1\left(\cos\left(\frac{x\lambda}{2}\right)^2-\frac{1}{2}\right)a\sqrt{-a^2+\right.}}{\sqrt{-a^2+b^2}\left(2\,(a-b)\,b^2\,(a+b)\left(\sin\left(\frac{x\lambda}{2}\right)a\cos\left(\frac{x\lambda}{2}\right)+\frac{b}{2}\right)\arctan\left(\frac{\tan\left(\frac{x\lambda}{2}\right)b+a}{\sqrt{-a^2+b^2}}\right)+a\cos\left(\frac{x\lambda}{2}\right)(a-b)\,(a+\right.}$$

✓ Solution by Mathematica
Time used: 24.795 (sec). Leaf size: 189

```
DSolve[(a*Sin[\[Lambda]*x]+b)*(y'[x]-y[x]^2)-a*\[Lambda]^2*Sin[\[Lambda]*x]==0,y[x],x,Includ
```

$$y(x)$$
$$\to\frac{\lambda\left(2ab\cos(\lambda x)\arctan\left(\frac{a+b\tan\left(\frac{\lambda x}{2}\right)}{\sqrt{b^2-a^2}}\right)+\sqrt{b^2-a^2}(-ac_1\lambda(a^2-b^2)\cos(\lambda x)-a\sin(\lambda x)+b)\right)}{-2b(a\sin(\lambda x)+b)\arctan\left(\frac{a+b\tan\left(\frac{\lambda x}{2}\right)}{\sqrt{b^2-a^2}}\right)+\sqrt{b^2-a^2}\,(-a\cos(\lambda x)+c_1\lambda\,(a^2-b^2)\,(a\sin(\lambda x)+b))}$$
$$y(x)\to-\frac{a\lambda\cos(\lambda x)}{a\sin(\lambda x)+b}$$

10 Chapter 1, section 1.2. Riccati Equation. subsection 1.2.6-2. Equations with cosine.

10.1 problem 14 256
10.2 problem 15 257
10.3 problem 16 258
10.4 problem 17 259
10.5 problem 18 260
10.6 problem 19 261
10.7 problem 20 262
10.8 problem 21 263
10.9 problem 22 265
10.10 problem 23 266
10.11 problem 24 267
10.12 problem 25 268
10.13 problem 26 270

10.1 problem 14

Internal problem ID [10512]

Book: Handbook of exact solutions for ordinary differential equations. By Polyanin and Zaitsev. Second edition
Section: Chapter 1, section 1.2. Riccati Equation. subsection 1.2.6-2. Equations with cosine.
Problem number: 14.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_Riccati]`

$$\boxed{y' - \alpha y^2 = \beta + \gamma \cos(\lambda x)}$$

✓ Solution by Maple
Time used: 0.0 (sec). Leaf size: 94

```
dsolve(diff(y(x),x)=alpha*y(x)^2+beta+gamma*cos(lambda*x),y(x), singsol=all)
```

$$y(x) = -\frac{\lambda\left(c_1 \operatorname{MathieuSPrime}\left(\frac{4\alpha\beta}{\lambda^2}, -\frac{2\gamma\alpha}{\lambda^2}, \frac{x\lambda}{2}\right) + \operatorname{MathieuCPrime}\left(\frac{4\alpha\beta}{\lambda^2}, -\frac{2\gamma\alpha}{\lambda^2}, \frac{x\lambda}{2}\right)\right)}{2\alpha\left(c_1 \operatorname{MathieuS}\left(\frac{4\alpha\beta}{\lambda^2}, -\frac{2\gamma\alpha}{\lambda^2}, \frac{x\lambda}{2}\right) + \operatorname{MathieuC}\left(\frac{4\alpha\beta}{\lambda^2}, -\frac{2\gamma\alpha}{\lambda^2}, \frac{x\lambda}{2}\right)\right)}$$

✓ Solution by Mathematica
Time used: 0.577 (sec). Leaf size: 163

```
DSolve[y'[x]==\[Alpha]*y[x]^2+\[Beta]+\[Gamma]*Cos[\[Lambda]*x],y[x],x,IncludeSingularSoluti
```

$$y(x) \to -\frac{\lambda\left(\operatorname{MathieuSPrime}\left[\frac{4\alpha\beta}{\lambda^2}, -\frac{2\alpha\gamma}{\lambda^2}, \frac{\lambda x}{2}\right] + c_1 \operatorname{MathieuCPrime}\left[\frac{4\alpha\beta}{\lambda^2}, -\frac{2\alpha\gamma}{\lambda^2}, \frac{\lambda x}{2}\right]\right)}{2\alpha\left(\operatorname{MathieuS}\left[\frac{4\alpha\beta}{\lambda^2}, -\frac{2\alpha\gamma}{\lambda^2}, \frac{\lambda x}{2}\right] + c_1 \operatorname{MathieuC}\left[\frac{4\alpha\beta}{\lambda^2}, -\frac{2\alpha\gamma}{\lambda^2}, \frac{\lambda x}{2}\right]\right)}$$

$$y(x) \to -\frac{\lambda \operatorname{MathieuCPrime}\left[\frac{4\alpha\beta}{\lambda^2}, -\frac{2\alpha\gamma}{\lambda^2}, \frac{\lambda x}{2}\right]}{2\alpha \operatorname{MathieuC}\left[\frac{4\alpha\beta}{\lambda^2}, -\frac{2\alpha\gamma}{\lambda^2}, \frac{\lambda x}{2}\right]}$$

10.2 problem 15

Internal problem ID [10513]

Book: Handbook of exact solutions for ordinary differential equations. By Polyanin and Zaitsev. Second edition
Section: Chapter 1, section 1.2. Riccati Equation. subsection 1.2.6-2. Equations with cosine.
Problem number: 15.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_Riccati]`

$$y' - y^2 = -a^2 + a\lambda \cos(\lambda x) + a^2 \cos(\lambda x)^2$$

✓ Solution by Maple
Time used: 0.0 (sec). Leaf size: 272

```
dsolve(diff(y(x),x)=y(x)^2-a^2+a*lambda*cos(lambda*x)+a^2*cos(lambda*x)^2,y(x), singsol=all)
```

$$y(x) = \frac{\left(2ac_1 \sin(x\lambda) \cos\left(\frac{x\lambda}{2}\right) + c_1 \lambda \sin\left(\frac{x\lambda}{2}\right)\right) \text{HeunC}\left(\frac{4a}{\lambda}, \frac{1}{2}, -\frac{1}{2}, -\frac{2a}{\lambda}, \frac{8a+3\lambda}{8\lambda}, \frac{\cos(x\lambda)}{2} + \frac{1}{2}\right) + 2\sin(x\lambda)\left(a\,\text{HeunC}\right.}{2\cos\left(\frac{x\lambda}{2}\right) \text{HeunC}\left(\frac{4a}{\lambda}, \frac{1}{2}, -\frac{1}{2}, -\frac{2a}{\lambda}, \frac{8a+3\lambda}{8\lambda}\right.}$$

✓ Solution by Mathematica
Time used: 3.942 (sec). Leaf size: 131

```
DSolve[y'[x]==y[x]^2-a^2+a*\[Lambda]*Cos[\[Lambda]*x]+a^2*Cos[\[Lambda]*x]^2,y[x],x,IncludeS
```

$$y(x) \to \frac{ac_1 \sin(\lambda x) \int_1^x e^{-\frac{2a\cos(\lambda K[1])}{\lambda}} dK[1] + a\sin(\lambda x) + c_1\left(-e^{-\frac{2a\cos(\lambda x)}{\lambda}}\right)}{1 + c_1 \int_1^x e^{-\frac{2a\cos(\lambda K[1])}{\lambda}} dK[1]}$$

$$y(x) \to a\sin(\lambda x) - \frac{e^{-\frac{2a\cos(\lambda x)}{\lambda}}}{\int_1^x e^{-\frac{2a\cos(\lambda K[1])}{\lambda}} dK[1]}$$

10.3 problem 16

Internal problem ID [10514]

Book: Handbook of exact solutions for ordinary differential equations. By Polyanin and Zaitsev. Second edition
Section: Chapter 1, section 1.2. Riccati Equation. subsection 1.2.6-2. Equations with cosine.
Problem number: 16.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_Riccati]`

$$y' - y^2 = c\cos(\lambda x + a)^n \cos(\lambda x + b)^{-n-4} + \lambda^2$$

✗ Solution by Maple

```
dsolve(diff(y(x),x)=y(x)^2+lambda^2+c*cos(lambda*x+a)^n*cos(lambda*x+b)^(-n-4),y(x), singsol
```

No solution found

✗ Solution by Mathematica
Time used: 0.0 (sec). Leaf size: 0

```
DSolve[y'[x]==y[x]^2+\[Lambda]^2+c*Cos[\[Lambda]*x+a]^n*Cos[\[Lambda]*x+b]^(-n-4),y[x],x,Inc
```

Not solved

10.4 problem 17

Internal problem ID [10515]

Book: Handbook of exact solutions for ordinary differential equations. By Polyanin and Zaitsev. Second edition
Section: Chapter 1, section 1.2. Riccati Equation. subsection 1.2.6-2. Equations with cosine.
Problem number: 17.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_Riccati]`

$$\boxed{y' - y^2 - a\cos(\beta x)\, y = ab\cos(\beta x) - b^2}$$

✓ Solution by Maple
Time used: 0.0 (sec). Leaf size: 73

```
dsolve(diff(y(x),x)=y(x)^2+a*cos(beta*x)*y(x)+a*b*cos(beta*x)-b^2,y(x), singsol=all)
```

$$y(x) = \frac{b\left(\int \mathrm{e}^{\frac{-2b\beta x+\sin(x\beta)a}{\beta}} dx\right) - c_1 b + \mathrm{e}^{\frac{-2b\beta x+\sin(x\beta)a}{\beta}}}{-\left(\int \mathrm{e}^{\frac{-2b\beta x+\sin(x\beta)a}{\beta}} dx\right) + c_1}$$

✓ Solution by Mathematica
Time used: 9.071 (sec). Leaf size: 183

```
DSolve[y'[x]==y[x]^2+a*Cos[\[Beta]*x]*y[x]+a*b*Cos[\[Beta]*x]-b^2,y[x],x,IncludeSingularSolu
```

$$\text{Solve}\left[\int_1^x \frac{e^{\frac{a\sin(\beta K[1])}{\beta}-2bK[1]}(-b+a\cos(\beta K[1])+y(x))}{a\beta(b+y(x))}dK[1]\right.$$
$$+\int_1^{y(x)}\left(-\int_1^x\left(\frac{e^{\frac{a\sin(\beta K[1])}{\beta}-2bK[1]}}{a\beta(b+K[2])}-\frac{e^{\frac{a\sin(\beta K[1])}{\beta}-2bK[1]}(-b+a\cos(\beta K[1])+K[2])}{a\beta(b+K[2])^2}\right)dK[1]\right.$$
$$\left.\left.-\frac{e^{\frac{a\sin(x\beta)}{\beta}-2bx}}{a\beta(b+K[2])^2}\right)dK[2]=c_1, y(x)\right]$$

10.5 problem 18

Internal problem ID [10516]

Book: Handbook of exact solutions for ordinary differential equations. By Polyanin and Zaitsev. Second edition
Section: Chapter 1, section 1.2. Riccati Equation. subsection 1.2.6-2. Equations with cosine.
Problem number: 18.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_Riccati]`

$$y' - y^2 - a\cos(bx)^m y = a\cos(bx)^m$$

✗ Solution by Maple

```
dsolve(diff(y(x),x)=y(x)^2+a*cos(b*x)^m*y(x)+a*cos(b*x)^m,y(x), singsol=all)
```

No solution found

✗ Solution by Mathematica
Time used: 0.0 (sec). Leaf size: 0

```
DSolve[y'[x]==y[x]^2+a*Cos[b*x]^m*y[x]+a*Cos[b*x]^m,y[x],x,IncludeSingularSolutions -> True]
```

Not solved

10.6 problem 19

Internal problem ID [10517]

Book: Handbook of exact solutions for ordinary differential equations. By Polyanin and Zaitsev. Second edition
Section: Chapter 1, section 1.2. Riccati Equation. subsection 1.2.6-2. Equations with cosine.
Problem number: 19.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_Riccati]`

$$y' - \cos(\lambda x)\, y^2 \lambda = \lambda \cos(\lambda x)^3$$

✓ Solution by Maple
Time used: 0.016 (sec). Leaf size: 95

```
dsolve(diff(y(x),x)=lambda*cos(lambda*x)*y(x)^2+lambda*cos(lambda*x)^3,y(x), singsol=all)
```

$$y(x) = -\frac{4\csc(x\lambda)\left(-\frac{\sqrt{\pi}\,\sin(x\lambda)^2\left(c_1-\frac{1}{2}\right)\operatorname{erf}\left(\sqrt{-\sin(x\lambda)^2}\right)}{2} + \left(c_1-\frac{1}{2}\right)\mathrm{e}^{\sin(x\lambda)^2}\sqrt{-\sin(x\lambda)^2} + \frac{\sin(x\lambda)^2\sqrt{\pi}\,c_1}{2}\right)}{\sqrt{\pi}\left(\operatorname{erf}\left(\sqrt{-\sin(x\lambda)^2}\right)(2c_1-1) - 2c_1\right)}$$

✗ Solution by Mathematica
Time used: 0.0 (sec). Leaf size: 0

```
DSolve[y'[x]==\[Lambda]*Cos[\[Lambda]*x]*y[x]^2+\[Lambda]*Cos[\[Lambda]*x]^3,y[x],x,IncludeS
```

Not solved

10.7 problem 20

Internal problem ID [10518]

Book: Handbook of exact solutions for ordinary differential equations. By Polyanin and Zaitsev. Second edition
Section: Chapter 1, section 1.2. Riccati Equation. subsection 1.2.6-2. Equations with cosine.
Problem number: 20.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_Riccati]`

$$\boxed{2y' - (\lambda + a - \cos(\lambda x)\, a)\, y^2 = -a + \lambda - \cos(\lambda x)\, a}$$

✓ Solution by Maple
Time used: 0.016 (sec). Leaf size: 122

```
dsolve(2*diff(y(x),x)=(lambda+a-a*cos(lambda*x))*y(x)^2+lambda-a-a*cos(lambda*x),y(x), sings
```

$$y(x) = \frac{-\cot\left(\frac{x\lambda}{2}\right)\lambda\left(\int \mathrm{e}^{\frac{a\cos(x\lambda)}{\lambda}}\operatorname{csgn}\left(\sin\left(\frac{x\lambda}{2}\right)\right)\left(\csc\left(\frac{x\lambda}{2}\right)^2\lambda + 2a\right)dx\right)c_1 - 2\csc\left(\frac{x\lambda}{2}\right)^2\operatorname{csgn}\left(\sin\left(\frac{x\lambda}{2}\right)\right)\mathrm{e}^{\frac{a\cos(x\lambda)}{\lambda}}c_1}{\lambda\left(\int \mathrm{e}^{\frac{a\cos(x\lambda)}{\lambda}}\operatorname{csgn}\left(\sin\left(\frac{x\lambda}{2}\right)\right)\left(\csc\left(\frac{x\lambda}{2}\right)^2\lambda + 2a\right)dx\right)c_1 - 2i}$$

✓ Solution by Mathematica
Time used: 34.139 (sec). Leaf size: 234

```
DSolve[2*y'[x]==(\[Lambda]+a-a*Cos[\[Lambda]*x])*y[x]^2+\[Lambda]-a-a*Cos[\[Lambda]*x],y[x],
```

$$y(x) \to -\frac{2\left(c_1\cot\left(\frac{\lambda x}{2}\right)\int_1^x e^{-\frac{2a\sin^2\left(\frac{1}{2}\lambda K[1]\right)}{\lambda}}\left(\lambda\csc^2\left(\frac{1}{2}\lambda K[1]\right)+2a\right)dK[1] + 2c_1\csc^2\left(\frac{\lambda x}{2}\right)e^{-\frac{2a\sin^2\left(\frac{\lambda x}{2}\right)}{\lambda}} + \cot\left(\frac{\lambda x}{2}\right)\right)}{2 + 2c_1\int_1^x e^{-\frac{2a\sin^2\left(\frac{1}{2}\lambda K[1]\right)}{\lambda}}\left(\lambda\csc^2\left(\frac{1}{2}\lambda K[1]\right)+2a\right)dK[1]}$$

$$y(x) \to \frac{1}{2}\csc^2\left(\frac{\lambda x}{2}\right)\left(-\frac{4e^{-\frac{2a\sin^2\left(\frac{\lambda x}{2}\right)}{\lambda}}}{\int_1^x e^{-\frac{2a\sin^2\left(\frac{1}{2}\lambda K[1]\right)}{\lambda}}\left(\lambda\csc^2\left(\frac{1}{2}\lambda K[1]\right)+2a\right)dK[1]} - \sin(\lambda x)\right)$$

10.8 problem 21

Internal problem ID [10519]

Book: Handbook of exact solutions for ordinary differential equations. By Polyanin and Zaitsev. Second edition
Section: Chapter 1, section 1.2. Riccati Equation. subsection 1.2.6-2. Equations with cosine.
Problem number: 21.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_Riccati]`

$$\boxed{y' - \left(\lambda + a\cos\left(\lambda x\right)^2\right) y^2 = -a + \lambda + a\cos\left(\lambda x\right)^2}$$

✓ Solution by Maple
Time used: 0.0 (sec). Leaf size: 102

```
dsolve(diff(y(x),x)=(lambda+a*cos(lambda*x)^2)*y(x)^2+lambda-a+a*cos(lambda*x)^2,y(x), sings
```

$$y(x) = \frac{2\sec\left(x\lambda\right)^2 \mathrm{e}^{-\frac{a\cos(2x\lambda)}{2\lambda}} c_1\lambda - 2\tan\left(x\lambda\right)\lambda\left(\int \mathrm{e}^{-\frac{a\cos(2x\lambda)}{2\lambda}}\left(\sec\left(x\lambda\right)^2\lambda + a\right)dx\right)c_1 + i\tan\left(x\lambda\right)}{-2\lambda\left(\int \mathrm{e}^{-\frac{a\cos(2x\lambda)}{2\lambda}}\left(\sec\left(x\lambda\right)^2\lambda + a\right)dx\right)c_1 + i}$$

✓ Solution by Mathematica

Time used: 36.333 (sec). Leaf size: 263

```
DSolve[y'[x]==(\[Lambda]+a*Cos[\[Lambda]*x]^2)*y[x]^2+\[Lambda]-a+a*Cos[\[Lambda]*x]^2,y[x],
```

$$y(x)$$

$$\to \frac{2\left(c_1 \tan(\lambda x) \int_1^x e^{-\frac{a\cos^2(\lambda K[1])}{\lambda}} (\lambda \sec^2(\lambda K[1]) + a)\, dK[1] + c_1 \sec^2(\lambda x) \left(-e^{-\frac{a\cos^2(\lambda x)}{\lambda}}\right) + \tan(\lambda x)\right)}{2 + 2c_1 \int_1^x e^{-\frac{a\cos^2(\lambda K[1])}{\lambda}} (\lambda \sec^2(\lambda K[1]) + a)\, dK[1]}$$

$$y(x) \to \frac{1}{2}\sec^2(\lambda x)\left(\sin(2\lambda x) - \frac{2e^{-\frac{a\cos^2(\lambda x)}{\lambda}}}{\int_1^x e^{-\frac{a\cos^2(\lambda K[1])}{\lambda}} (\lambda \sec^2(\lambda K[1]) + a)\, dK[1]}\right)$$

$$y(x) \to \frac{1}{2}\sec^2(\lambda x)\left(\sin(2\lambda x) - \frac{2e^{-\frac{a\cos^2(\lambda x)}{\lambda}}}{\int_1^x e^{-\frac{a\cos^2(\lambda K[1])}{\lambda}} (\lambda \sec^2(\lambda K[1]) + a)\, dK[1]}\right)$$

10.9 problem 22

Internal problem ID [10520]

Book: Handbook of exact solutions for ordinary differential equations. By Polyanin and Zaitsev. Second edition
Section: Chapter 1, section 1.2. Riccati Equation. subsection 1.2.6-2. Equations with cosine.
Problem number: 22.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_Riccati]`

$$\boxed{y' + (k+1)\, x^k y^2 - a\, x^{k+1} \cos(x)^m\, y = -a \cos(x)^m}$$

✓ Solution by Maple
Time used: 0.015 (sec). Leaf size: 174

```
dsolve(diff(y(x),x)=-(k+1)*x^k*y(x)^2+a*x^(k+1)*cos(x)^m*y(x)-a*cos(x)^m,y(x), singsol=all)
```

$$y(x) = \frac{x^{-1-k}\left(x^{1+k} e^{\int \frac{\cos(x)^m x^{1+k} a x - 2k - 2}{x} dx} + \left(\int x^k e^{\int \frac{\cos(x)^m x^{1+k} a x - 2k - 2}{x} dx} dx\right) k + \int x^k e^{\int \frac{\cos(x)^m x^{1+k} a x - 2k - 2}{x} dx} dx - c_1\right)}{\left(\int x^k e^{\int \frac{a\, x^{k+2} \cos(x)^m - 2k - 2}{x} dx} dx\right) k + \int x^k e^{\int \frac{a\, x^{k+2} \cos(x)^m - 2k - 2}{x} dx} dx - c_1}$$

✓ Solution by Mathematica
Time used: 20.002 (sec). Leaf size: 248

```
DSolve[y'[x]==-(k+1)*x^k*y[x]^2+a*x^(k+1)*Cos[x]^m*y[x]-a*Cos[x]^m,y[x],x,IncludeSingularSol
```

$$y(x) \to \frac{x^{-k-1}\left(c_1 x \exp\left(\int_1^x -\frac{-a\cos^m(K[1]) K[1]^{k+2}+k+2}{K[1]} dK[1]\right) + c_1 (k+1) \int_1^x \exp\left(\int_1^{K[2]} -\frac{-a\cos^m(K[1]) K[1]^{k+2}+k+2}{K[1]} dK[1]\right) dK[2] + (k+1) \int_1^x \exp\left(\int_1^{K[2]} -\frac{-a\cos^m(K[1]) K[1]^{k+2}+k+2}{K[1]} dK[1]\right) dK[2]\right)}{(k+1)\left(1 + c_1 \int_1^x \exp\left(\int_1^{K[2]} -\frac{-a\cos^m(K[1]) K[1]^{k+2}+k+2}{K[1]} dK[1]\right) dK[2]\right)}$$

$$y(x) \to \frac{x^{-k}\left(\frac{\exp\left(\int_1^x -\frac{-a\cos^m(K[1]) K[1]^{k+2}+k+2}{K[1]} dK[1]\right)}{\int_1^x \exp\left(\int_1^{K[2]} -\frac{-a\cos^m(K[1]) K[1]^{k+2}+k+2}{K[1]} dK[1]\right) dK[2]} + \frac{k+1}{x}\right)}{k+1}$$

10.10 problem 23

Internal problem ID [10521]

Book: Handbook of exact solutions for ordinary differential equations. By Polyanin and Zaitsev. Second edition
Section: Chapter 1, section 1.2. Riccati Equation. subsection 1.2.6-2. Equations with cosine.
Problem number: 23.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[[_1st_order, '_with_symmetry_[F(x),G(x)]'], _Riccati]`

$$\boxed{y' - a\cos\left(\lambda x + \mu\right)^k \left(y - b\,x^n - c\right)^2 = b\,x^{n-1} n}$$

✓ Solution by Maple
Time used: 0.0 (sec). Leaf size: 38

```
dsolve(diff(y(x),x)=a*cos(lambda*x+mu)^k*(y(x)-b*x^n-c)^2+b*n*x^(n-1),y(x), singsol=all)
```

$$y(x) = b\,x^n + c + \frac{1}{c_1 - a\left(\int \left(\cos\left(x\lambda\right)\cos\left(\mu\right) - \sin\left(x\lambda\right)\sin\left(\mu\right)\right)^k dx\right)}$$

✓ Solution by Mathematica
Time used: 6.016 (sec). Leaf size: 92

```
DSolve[y'[x]==a*Cos[\[Lambda]*x+\[Mu]]^k*(y[x]-b*x^n-c)^2+b*n*x^(n-1),y[x],x,IncludeSingular
```

$$y(x) \to \frac{1}{\frac{a\sqrt{\sin^2(\mu+\lambda x)}\csc(\mu+\lambda x)\cos^{k+1}(\mu+\lambda x)\,\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{k+1}{2},\frac{k+3}{2},\cos^2(x\lambda+\mu)\right)}{(k+1)\lambda} + c_1} + bx^n + c$$

$$y(x) \to bx^n + c$$

10.11 problem 24

Internal problem ID [10522]

Book: Handbook of exact solutions for ordinary differential equations. By Polyanin and Zaitsev. Second edition
Section: Chapter 1, section 1.2. Riccati Equation. subsection 1.2.6-2. Equations with cosine.
Problem number: 24.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_Riccati]`

$$y'x - a\cos(\lambda x)^m y^2 - ky = a\,b^2 x^{2k}\cos(\lambda x)^m$$

✓ Solution by Maple
Time used: 0.032 (sec). Leaf size: 31

```
dsolve(x*diff(y(x),x)=a*cos(lambda*x)^m*y(x)^2+k*y(x)+a*b^2*x^(2*k)*cos(lambda*x)^m,y(x), si
```

$$y(x) = -\tan\left(-ab\left(\int x^{-1+k}\cos(x\lambda)^m\,dx\right) + c_1\right) b\,x^k$$

✓ Solution by Mathematica
Time used: 1.628 (sec). Leaf size: 50

```
DSolve[x*y'[x]==a*Cos[\[Lambda]*x]^m*y[x]^2+k*y[x]+a*b^2*x^(2*k)*Cos[\[Lambda]*x]^m,y[x],x,I
```

$$y(x) \to \sqrt{b^2}x^k \tan\left(\sqrt{b^2}\int_1^x a\cos^m(\lambda K[1])K[1]^{k-1}dK[1] + c_1\right)$$

10.12 problem 25

Internal problem ID [10523]

Book: Handbook of exact solutions for ordinary differential equations. By Polyanin and Zaitsev. Second edition
Section: Chapter 1, section 1.2. Riccati Equation. subsection 1.2.6-2. Equations with cosine.
Problem number: 25.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_Riccati]`

$$\boxed{(\cos(\lambda x)\, a + b)\, y' - y^2 - c\cos(x\mu)\, y = -d^2 + cd\cos(x\mu)}$$

✓ Solution by Maple
Time used: 0.016 (sec). Leaf size: 268

```
dsolve((a*cos(lambda*x)+b)*diff(y(x),x)=y(x)^2+c*cos(mu*x)*y(x)-d^2+c*d*cos(mu*x),y(x), sing
```

$$y(x) = \frac{-d\left(\int \frac{\mathrm{e}^{\frac{c\left(\int \frac{\cos(x\mu)}{a\cos(x\lambda)+b}dx\right)\sqrt{a^2-b^2}\,\lambda - 4d\,\operatorname{arctanh}\left(\frac{(a-b)\tan\left(\frac{x\lambda}{2}\right)}{\sqrt{a^2-b^2}}\right)}{\sqrt{a^2-b^2}\,\lambda}}}{a\cos(x\lambda)+b}dx\right) + dc_1 - \mathrm{e}^{\frac{c\left(\int \frac{\cos(x\mu)}{a\cos(x\lambda)+b}dx\right)\sqrt{a^2-b^2}\,\lambda - 4d\,\operatorname{arctanh}\left(\frac{(a-b)\tan\left(\frac{x}{}\right.\right.}{\sqrt{a^2-b^2}\,\lambda}}}{\int \frac{\mathrm{e}^{\frac{c\left(\int \frac{\cos(x\mu)}{a\cos(x\lambda)+b}dx\right)\sqrt{a^2-b^2}\,\lambda - 4d\,\operatorname{arctanh}\left(\frac{(a-b)\tan\left(\frac{x\lambda}{2}\right)}{\sqrt{a^2-b^2}}\right)}{\sqrt{a^2-b^2}\,\lambda}}}{a\cos(x\lambda)+b}dx - c_1}$$

✓ Solution by Mathematica

Time used: 12.31 (sec). Leaf size: 289

```
DSolve[(a*Cos[\[Lambda]*x]+b)*y'[x]==y[x]^2+c*Cos[\[Mu]*x]*y[x]-d^2+c*d*Cos[\[Mu]*x],y[x],x,
```

$$\text{Solve}\left[\int_1^x \frac{\exp\left(-\int_1^{K[2]} \frac{2d-c\cos(\mu K[1])}{b+a\cos(\lambda K[1])}dK[1]\right)(-d+c\cos(\mu K[2])+y(x))}{c\mu(b+a\cos(\lambda K[2]))(d+y(x))}dK[2]\right.$$

$$+\int_1^{y(x)}\left(-\int_1^x\left(\frac{\exp\left(-\int_1^{K[2]} \frac{2d-c\cos(\mu K[1])}{b+a\cos(\lambda K[1])}dK[1]\right)}{c\mu(b+a\cos(\lambda K[2]))(d+K[3])}-\frac{\exp\left(-\int_1^{K[2]} \frac{2d-c\cos(\mu K[1])}{b+a\cos(\lambda K[1])}dK[1]\right)(-d+c\cos(\mu K[2]}{c\mu(b+a\cos(\lambda K[2]))(d+K[3])^2}\right.\right.$$

$$\left.\left.-\frac{\exp\left(-\int_1^x \frac{2d-c\cos(\mu K[1])}{b+a\cos(\lambda K[1])}dK[1]\right)}{c\mu(d+K[3])^2}\right)dK[3]=c_1,y(x)\right]$$

10.13 problem 26

Internal problem ID [10524]

Book: Handbook of exact solutions for ordinary differential equations. By Polyanin and Zaitsev. Second edition
Section: Chapter 1, section 1.2. Riccati Equation. subsection 1.2.6-2. Equations with cosine.
Problem number: 26.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_Riccati]`

$$\boxed{(\cos(\lambda x)\, a + b)\left(y' - y^2\right) = a\,\lambda^2 \cos(\lambda x)}$$

✓ Solution by Maple
Time used: 0.0 (sec). Leaf size: 204

```
dsolve((a*cos(lambda*x)+b)*(diff(y(x),x)-y(x)^2)-a*lambda^2*cos(lambda*x)=0,y(x), singsol=al
```

$$y(x) = \frac{\left(2\ \operatorname{arctanh}\left(\frac{(a-b)\tan\left(\frac{x\lambda}{2}\right)}{\sqrt{a^2-b^2}}\right)\sqrt{a^2-b^2}\, ab\cos\left(\frac{x\lambda}{2}\right)\sin\left(\frac{x\lambda}{2}\right) - 2\sqrt{a^2-b^2}\, c_1 a\cos\left(\frac{x\lambda}{2}\right)\sin\left(\frac{x\lambda}{2}\right) + \left(a\cos\left(\frac{x\lambda}{2}\right)\right.\right.}{\sqrt{a^2-b^2}\left(2\left(a\cos\left(\frac{x\lambda}{2}\right)^2 - \frac{a}{2} + \frac{b}{2}\right) b\ \operatorname{arctanh}\left(\frac{(a-b)\tan\left(\frac{x\lambda}{2}\right)}{\sqrt{a^2-b^2}}\right) - \sqrt{a^2-b^2}\, a\cos\left(\frac{x\lambda}{2}\right)\sin\left(\frac{x\lambda}{2}\right) - 2c_1\right.}$$

✓ Solution by Mathematica
Time used: 7.903 (sec). Leaf size: 202

```
DSolve[(a*Cos[\[Lambda]*x]+b)*(y'[x]-y[x]^2)-a*\[Lambda]^2*Cos[\[Lambda]*x]==0,y[x],x,Includ
```

$$y(x) \to -\frac{\lambda\left(-2ab\sin(\lambda x)\operatorname{arctanh}\left(\frac{(b-a)\tan\left(\frac{\lambda x}{2}\right)}{\sqrt{a^2-b^2}}\right) + \sqrt{a^2-b^2}\left(-ac_1\lambda(a^2-b^2)\sin(\lambda x) + a\cos(\lambda x) - b\right)\right.}{2b(a\cos(\lambda x)+b)\operatorname{arctanh}\left(\frac{(b-a)\tan\left(\frac{\lambda x}{2}\right)}{\sqrt{a^2-b^2}}\right) + \sqrt{a^2-b^2}\left(bc_1\lambda\left(a^2-b^2\right) + ac_1\lambda\left(a^2-b^2\right)\cos(\lambda x) + a\sin(\right.}$$

$$y(x) \to \frac{a\lambda\sin(\lambda x)}{a\cos(\lambda x)+b}$$

11 Chapter 1, section 1.2. Riccati Equation. subsection 1.2.6-3. Equations with tangent.

11.1 problem 27 272
11.2 problem 28 273
11.3 problem 29 274
11.4 problem 30 276
11.5 problem 31 277
11.6 problem 32 278
11.7 problem 33 279
11.8 problem 34 280
11.9 problem 35 281
11.10 problem 36 282
11.11 problem 37 283

11.1 problem 27

Internal problem ID [10525]

Book: Handbook of exact solutions for ordinary differential equations. By Polyanin and Zaitsev. Second edition
Section: Chapter 1, section 1.2. Riccati Equation. subsection 1.2.6-3. Equations with tangent.
Problem number: 27.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_Riccati]`

$$\boxed{y' - y^2 = \lambda a + a(\lambda - a)\tan(\lambda x)^2}$$

✓ Solution by Maple
Time used: 0.0 (sec). Leaf size: 205

```
dsolve(diff(y(x),x)=y(x)^2+a*lambda+a*(lambda-a)*tan(lambda*x)^2,y(x), singsol=all)
```

$$y(x) = \_\frac{\left(\sin(x\lambda)\,\text{LegendreP}\left(\frac{2a-\lambda}{2\lambda},\frac{2a-\lambda}{2\lambda},\sin(x\lambda)\right)a + \text{LegendreQ}\left(\frac{2a-\lambda}{2\lambda},\frac{2a-\lambda}{2\lambda},\sin(x\lambda)\right)c_1 a\sin(x\lambda) - \lambda(\text{Lege}\right.}{\text{LegendreQ}\left(\frac{2a-\lambda}{2\lambda},\frac{2a-\lambda}{2\lambda},\sin(x\lambda)\right)c_1 + \text{Legendre}}$$

✓ Solution by Mathematica
Time used: 2.982 (sec). Leaf size: 259

```
DSolve[y'[x]==y[x]^2+a*\[Lambda]+a*(\[Lambda]-a)*Tan[\[Lambda]*x]^2,y[x],x,IncludeSingularSo
```

$$y(x) \to \frac{2\left(ac_1\sin^2(\lambda x)\,\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{1}{2}-\frac{a}{\lambda},\frac{3}{2}-\frac{a}{\lambda},\cos^2(x\lambda)\right) + (2a-\lambda)\sqrt{\sin^2(\lambda x)}\left(a\sin(\lambda x)\cos^{\frac{2a}{\lambda}-1}\right.\right.}{2(2a-\lambda)\sqrt{\sin^2(\lambda x)}\cos^{\frac{2a}{\lambda}}(\lambda x) + c_1\sin(2\lambda x)\,\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{1}{2}-\frac{a}{\lambda},\frac{3}{2}-\frac{a}{\lambda},\cos^2(x\lambda)\right.}$$

$$y(x) \to \frac{\tan(\lambda x)\left(a\sqrt{\sin^2(\lambda x)}\,\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{1}{2}-\frac{a}{\lambda},\frac{3}{2}-\frac{a}{\lambda},\cos^2(x\lambda)\right) - 2a + \lambda\right)}{\sqrt{\sin^2(\lambda x)}\,\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{1}{2}-\frac{a}{\lambda},\frac{3}{2}-\frac{a}{\lambda},\cos^2(x\lambda)\right)}$$

11.2 problem 28

Internal problem ID [10526]

Book: Handbook of exact solutions for ordinary differential equations. By Polyanin and Zaitsev. Second edition
Section: Chapter 1, section 1.2. Riccati Equation. subsection 1.2.6-3. Equations with tangent.
Problem number: 28.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_Riccati]`

$$\boxed{y' - y^2 = 3\lambda a + \lambda^2 + a(\lambda - a)\tan(\lambda x)^2}$$

✓ Solution by Maple
Time used: 0.0 (sec). Leaf size: 200

```
dsolve(diff(y(x),x)=y(x)^2+lambda^2+3*a*lambda+a*(lambda-a)*tan(lambda*x)^2,y(x), singsol=al
```

$$y(x) = -\frac{\left(-2\,\mathrm{LegendreP}\left(\frac{2a+3\lambda}{2\lambda}, \frac{2a-\lambda}{2\lambda}, \sin(x\lambda)\right)\lambda - 2\,\mathrm{LegendreQ}\left(\frac{2a+3\lambda}{2\lambda}, \frac{2a-\lambda}{2\lambda}, \sin(x\lambda)\right) c_1\lambda + \sin(x\lambda)\left(\mathrm{Legend}\right.\right.}{\mathrm{LegendreQ}\left(\frac{2a+\lambda}{2\lambda}, \frac{2a-\lambda}{2\lambda}, \sin(x\lambda)\right) c_1 + \mathrm{Legend}}$$

✓ Solution by Mathematica
Time used: 76.241 (sec). Leaf size: 319

```
DSolve[y'[x]==y[x]^2+\[Lambda]^2+3*a*\[Lambda]+a*(\[Lambda]-a)*Tan[\[Lambda]*x]^2,y[x],x,Inc
```

$$y(x) \to -\frac{\sin^{-\frac{a+\lambda}{\lambda}}(2\lambda x) e^{-\frac{a\,\mathrm{arctanh}(\cos(2\lambda x))}{\lambda}}\left(c_1 \sin^{\frac{a}{\lambda}}(2\lambda x)((a+\lambda)\cos(2\lambda x) - a + \lambda) e^{\frac{a\,\mathrm{arctanh}(\cos(2\lambda x))}{\lambda}} \int_1^x e^{-\frac{(a-\lambda)\mathrm{arctanh}}{\lambda}}\right.}{1 + c_1 \int_1^x e^{-\frac{(a-\lambda)\mathrm{arctanh}(\cos}{\lambda}}}$$

$$y(x) \to \csc(2\lambda x)\left(-\frac{\sin^{-\frac{a}{\lambda}}(2\lambda x) e^{-\frac{(a-\lambda)\mathrm{arctanh}(\cos(2\lambda x))}{\lambda}}}{\int_1^x e^{-\frac{(a-\lambda)\mathrm{arctanh}(\cos(2\lambda K[1]))}{\lambda}} \sin^{-\frac{a+\lambda}{\lambda}}(2\lambda K[1]) dK[1]} - (a+\lambda)\cos(2\lambda x) + a - \lambda\right)$$

11.3 problem 29

Internal problem ID [10527]

Book: Handbook of exact solutions for ordinary differential equations. By Polyanin and Zaitsev. Second edition
Section: Chapter 1, section 1.2. Riccati Equation. subsection 1.2.6-3. Equations with tangent.
Problem number: 29.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_Riccati]`

$$\boxed{y' - ay^2 - b\tan(x)\, y = c}$$

✓ Solution by Maple
Time used: 0.0 (sec). Leaf size: 187

```
dsolve(diff(y(x),x)=a*y(x)^2+b*tan(x)*y(x)+c,y(x), singsol=all)
```

$$y(x) = \frac{\sec(x)\left(-\left(\text{LegendreQ}\left(\frac{\sqrt{4ac+b^2}}{2}-\frac{1}{2},-\frac{1}{2}+\frac{b}{2},\sin(x)\right)c_1+\text{LegendreP}\left(\frac{\sqrt{4ac+b^2}}{2}-\frac{1}{2},-\frac{1}{2}+\frac{b}{2},\sin(x)\right)\right)(\right.}{2\left(\text{LegendreQ}\left(\frac{\sqrt{4ac+b^2}}{2}-\frac{1}{2},-\right.\right.}$$

✓ Solution by Mathematica

Time used: 2.211 (sec). Leaf size: 608

```
DSolve[y'[x]==a*y[x]^2+b*Tan[x]*y[x]+c,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x)$$

$$\to \frac{\sin(x)\left((-b^3+3b^2+b-3)\,\text{Hypergeometric2F1}\left(\frac{1}{4}\left(-b-\sqrt{b^2+4ac}+2\right),\frac{1}{4}\left(-b+\sqrt{b^2+4ac}+2\right),\frac{3-b}{2}\right.\right.}{a(b-3)(b+1)\left(\cos(x)\,\text{Hyperg}\right.}$$

$$y(x)\to -\frac{c\sin(x)\cos(x)\,\text{Hypergeometric2F1}\left(\frac{1}{4}\left(b-\sqrt{b^2+4ac}+4\right),\frac{1}{4}\left(b+\sqrt{b^2+4ac}+4\right),\frac{b+3}{2},\cos^2(x)\right)}{(b+1)\,\text{Hypergeometric2F1}\left(\frac{1}{4}\left(b-\sqrt{b^2+4ac}\right),\frac{1}{4}\left(b+\sqrt{b^2+4ac}\right),\frac{b+1}{2},\cos^2(x)\right)}$$

$$y(x)\to -\frac{c\sin(x)\cos(x)\,\text{Hypergeometric2F1}\left(\frac{1}{4}\left(b-\sqrt{b^2+4ac}+4\right),\frac{1}{4}\left(b+\sqrt{b^2+4ac}+4\right),\frac{b+3}{2},\cos^2(x)\right)}{(b+1)\,\text{Hypergeometric2F1}\left(\frac{1}{4}\left(b-\sqrt{b^2+4ac}\right),\frac{1}{4}\left(b+\sqrt{b^2+4ac}\right),\frac{b+1}{2},\cos^2(x)\right)}$$

11.4 problem 30

Internal problem ID [10528]

Book: Handbook of exact solutions for ordinary differential equations. By Polyanin and Zaitsev. Second edition
Section: Chapter 1, section 1.2. Riccati Equation. subsection 1.2.6-3. Equations with tangent.
Problem number: 30.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[[_1st_order, '_with_symmetry_[F(x),G(x)]'], _Riccati]`

$$\boxed{y' - ay^2 - 2ab\tan(x)\,y = b(ab-1)\tan(x)^2}$$

✓ Solution by Maple
Time used: 0.0 (sec). Leaf size: 87

```
dsolve(diff(y(x),x)=a*y(x)^2+2*a*b*tan(x)*y(x)+b*(a*b-1)*tan(x)^2,y(x), singsol=all)
```

$$y(x) = \frac{2c_1ab - 2i\tan(x)\,a^{\frac{3}{2}}b^{\frac{3}{2}}c_1 + i\sqrt{a}\,\sqrt{b}\,\mathrm{e}^{-2i\sqrt{a}\,\sqrt{b}\,x} - \tan(x)\,\mathrm{e}^{-2i\sqrt{a}\,\sqrt{b}\,x}ab}{a\left(2ic_1\sqrt{a}\,\sqrt{b} + \mathrm{e}^{-2i\sqrt{a}\,\sqrt{b}\,x}\right)}$$

✓ Solution by Mathematica
Time used: 12.833 (sec). Leaf size: 37

```
DSolve[y'[x]==a*y[x]^2+2*a*b*Tan[x]*y[x]+b*(a*b-1)*Tan[x]^2,y[x],x,IncludeSingularSolutions
```

$$y(x) \to -b\tan(x) + \sqrt{\frac{b}{a}}\tan\left(ax\sqrt{\frac{b}{a}} + c_1\right)$$

11.5 problem 31

Internal problem ID [10529]

Book: Handbook of exact solutions for ordinary differential equations. By Polyanin and Zaitsev. Second edition
Section: Chapter 1, section 1.2. Riccati Equation. subsection 1.2.6-3. Equations with tangent.
Problem number: 31.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_Riccati]`

$$y' - y^2 - a\tan(\beta x)\, y = ab\tan(\beta x) - b^2$$

✓ Solution by Maple
Time used: 0.0 (sec). Leaf size: 81

```
dsolve(diff(y(x),x)=y(x)^2+a*tan(beta*x)*y(x)+a*b*tan(beta*x)-b^2,y(x), singsol=all)
```

$$y(x) = \frac{-\left(\sec(x\beta)^2\right)^{\frac{a}{2\beta}} \mathrm{e}^{-2bx} - b\left(\int \left(\sec(x\beta)^2\right)^{\frac{a}{2\beta}} \mathrm{e}^{-2bx} dx - c_1\right)}{\int \left(\sec(x\beta)^2\right)^{\frac{a}{2\beta}} \mathrm{e}^{-2bx} dx - c_1}$$

✓ Solution by Mathematica
Time used: 25.611 (sec). Leaf size: 408

```
DSolve[y'[x]==y[x]^2+a*Tan[\[Beta]*x]*y[x]+a*b*Tan[\[Beta]*x]-b^2,y[x],x,IncludeSingularSolu
```

$$y(x)$$
$$\to \frac{2^{-\frac{a}{\beta}} \cos^{-\frac{a}{\beta}}(\beta x)\left(ib(a+2ib+2\beta)\,\text{Hypergeometric2F1}\left(1, -\frac{a-2ib}{2\beta}, \frac{a+2ib+2\beta}{2\beta}, -e^{2ix\beta}\right)(\sin(2\beta x)\csc(\beta x))^{a}\right.}{(a+2ib)\left(a\beta c_1 e^{2bx}(a+2ib+2\beta)\cos^{\frac{a}{\beta}}(\beta x) - ie^{2i\beta x}\,\text{Hyper}\right.}$$
$$y(x) \to -b$$

11.6 problem 32

Internal problem ID [10530]

Book: Handbook of exact solutions for ordinary differential equations. By Polyanin and Zaitsev. Second edition
Section: Chapter 1, section 1.2. Riccati Equation. subsection 1.2.6-3. Equations with tangent.
Problem number: 32.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_Riccati]`

$$\boxed{y' - y^2 - ax\tan(bx)^m y = a\tan(bx)^m}$$

✓ Solution by Maple
Time used: 0.0 (sec). Leaf size: 85

```
dsolve(diff(y(x),x)=y(x)^2+a*x*tan(b*x)^m*y(x)+a*tan(b*x)^m,y(x), singsol=all)
```

$$y(x) = \frac{-e^{\int \frac{a\tan(bx)^m x^2-2}{x}dx}x - \left(\int e^{\int \frac{a\tan(bx)^m x^2-2}{x}dx}dx\right) + c_1}{\left(-c_1 + \int e^{\int \frac{a\tan(bx)^m x^2-2}{x}dx}dx\right)x}$$

✓ Solution by Mathematica
Time used: 8.199 (sec). Leaf size: 126

```
DSolve[y'[x]==y[x]^2+a*x*Tan[b*x]^m*y[x]+a*Tan[b*x]^m,y[x],x,IncludeSingularSolutions -> Tru
```

$$y(x) \to -\frac{\exp\left(-\int_1^x -aK[1]\tan^m(bK[1])dK[1]\right) + x\int_1^x \frac{\exp\left(-\int_1^{K[2]} -aK[1]\tan^m(bK[1])dK[1]\right)}{K[2]^2}dK[2] + c_1 x}{x^2\left(\int_1^x \frac{\exp\left(-\int_1^{K[2]} -aK[1]\tan^m(bK[1])dK[1]\right)}{K[2]^2}dK[2] + c_1\right)}$$

$$y(x) \to -\frac{1}{x}$$

11.7 problem 33

Internal problem ID [10531]

Book: Handbook of exact solutions for ordinary differential equations. By Polyanin and Zaitsev. Second edition
Section: Chapter 1, section 1.2. Riccati Equation. subsection 1.2.6-3. Equations with tangent.
Problem number: 33.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_Riccati]`

$$\boxed{y' + (k+1)\, x^k y^2 - a\, x^{k+1} \tan(x)^m\, y = -a \tan(x)^m}$$

✓ Solution by Maple
Time used: 0.016 (sec). Leaf size: 174

```
dsolve(diff(y(x),x)=-(k+1)*x^k*y(x)^2+a*x^(k+1)*tan(x)^m*y(x)-a*tan(x)^m,y(x), singsol=all)
```

$$y(x) = \frac{x^{-1-k}\left(x^{1+k} e^{\int \frac{x^{1+k}\tan(x)^m a x - 2k - 2}{x} dx} + \left(\int x^k e^{\int \frac{x^{1+k}\tan(x)^m a x - 2k - 2}{x} dx} dx\right) k + \int x^k e^{\int \frac{x^{1+k}\tan(x)^m a x - 2k - 2}{x} dx} dx - c_1\right.}{\left(\int x^k e^{\int \frac{a\, x^{k+2}\tan(x)^m - 2k - 2}{x} dx} dx\right) k + \int x^k e^{\int \frac{a\, x^{k+2}\tan(x)^m - 2k - 2}{x} dx} dx - c_1}$$

✓ Solution by Mathematica
Time used: 19.083 (sec). Leaf size: 248

```
DSolve[y'[x]==-(k+1)*x^k*y[x]^2+a*x^(k+1)*Tan[x]^m*y[x]-a*Tan[x]^m,y[x],x,IncludeSingularSol
```

$$y(x) \to \frac{x^{-k-1}\left(c_1 x \exp\left(\int_1^x -\frac{-a\tan^m(K[1])K[1]^{k+2}+k+2}{K[1]} dK[1]\right) + c_1 (k+1) \int_1^x \exp\left(\int_1^{K[2]} -\frac{-a\tan^m(K[1])K[1]^{k+2}+k+2}{K[1]} d\right.\right.}{(k+1)\left(1 + c_1 \int_1^x \exp\left(\int_1^{K[2]} -\frac{-a\tan^m(K[1])K[1]^{k+2}+k+2}{K[1]} dK[1]\right) dK[2]\right)}$$

$$y(x) \to \frac{x^{-k}\left(\frac{\exp\left(\int_1^x -\frac{-a\tan^m(K[1])K[1]^{k+2}+k+2}{K[1]} dK[1]\right)}{\int_1^x \exp\left(\int_1^{K[2]} -\frac{-a\tan^m(K[1])K[1]^{k+2}+k+2}{K[1]} dK[1]\right) dK[2]} + \frac{k+1}{x}\right)}{k+1}$$

11.8 problem 34

Internal problem ID [10532]

Book: Handbook of exact solutions for ordinary differential equations. By Polyanin and Zaitsev. Second edition
Section: Chapter 1, section 1.2. Riccati Equation. subsection 1.2.6-3. Equations with tangent.
Problem number: 34.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_Riccati]`

$$\boxed{y' - a\tan(\lambda x)^n y^2 = -a\,b^2\tan(\lambda x)^{2+n} + b\lambda\tan(\lambda x)^2 + b\lambda}$$

✗ Solution by Maple

```
dsolve(diff(y(x),x)=a*tan(lambda*x)^n*y(x)^2-a*b^2*tan(lambda*x)^(n+2)+b*lambda*tan(lambda*x
```

No solution found

✗ Solution by Mathematica
Time used: 0.0 (sec). Leaf size: 0

```
DSolve[y'[x]==a*Tan[\[Lambda]*x]^n*y[x]^2-a*b^2*Tan[\[Lambda]*x]^(n+2)+b*\[Lambda]*Tan[\[Lam
```

Not solved

11.9 problem 35

Internal problem ID [10533]

Book: Handbook of exact solutions for ordinary differential equations. By Polyanin and Zaitsev. Second edition
Section: Chapter 1, section 1.2. Riccati Equation. subsection 1.2.6-3. Equations with tangent.
Problem number: 35.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[[_1st_order, '_with_symmetry_[F(x),G(x)]'], _Riccati]`

$$\boxed{y' - a\tan\left(\lambda x+\mu\right)^k\left(y - b\,x^n - c\right)^2 = b\,x^{n-1}n}$$

✓ Solution by Maple
Time used: 0.016 (sec). Leaf size: 42

```
dsolve(diff(y(x),x)=a*tan(lambda*x+mu)^k*(y(x)-b*x^n-c)^2+b*n*x^(n-1),y(x), singsol=all)
```

$$y(x) = b\,x^n + c + \frac{1}{c_1 - a\left(\int\left(-\frac{\tan(\mu)+\tan(x\lambda)}{\tan(\mu)\tan(x\lambda)-1}\right)^k dx\right)}$$

✓ Solution by Mathematica
Time used: 6.024 (sec). Leaf size: 75

```
DSolve[y'[x]==a*Tan[\[Lambda]*x+mu]^k*(y[x]-b*x^n-c)^2+b*n*x^(n-1),y[x],x,IncludeSingularSol
```

$$y(x) \to \frac{1}{-\frac{a\tan^{k+1}(\mu+\lambda x)\,\text{Hypergeometric2F1}\left(1,\frac{k+1}{2},\frac{k+3}{2},-\tan^2(\mu+x\lambda)\right)}{(k+1)\lambda} + c_1} + bx^n + c$$
$$y(x) \to bx^n + c$$

11.10 problem 36

Internal problem ID [10534]

Book: Handbook of exact solutions for ordinary differential equations. By Polyanin and Zaitsev. Second edition
Section: Chapter 1, section 1.2. Riccati Equation. subsection 1.2.6-3. Equations with tangent.
Problem number: 36.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_Riccati]`

$$y'x - a\tan(\lambda x)^m y^2 - ky = a\,b^2 x^{2k}\tan(\lambda x)^m$$

✓ Solution by Maple
Time used: 0.031 (sec). Leaf size: 31

```
dsolve(x*diff(y(x),x)=a*tan(lambda*x)^m*y(x)^2+k*y(x)+a*b^2*x^(2*k)*tan(lambda*x)^m,y(x), si
```

$$y(x) = -\tan\left(-ab\left(\int x^{-1+k}\tan(x\lambda)^m\,dx\right) + c_1\right) b\,x^k$$

✓ Solution by Mathematica
Time used: 1.817 (sec). Leaf size: 50

```
DSolve[x*y'[x]==a*Tan[\[Lambda]*x]^m*y[x]^2+k*y[x]+a*b^2*x^(2*k)*Tan[\[Lambda]*x]^m,y[x],x,I
```

$$y(x) \to \sqrt{b^2}x^k \tan\left(\sqrt{b^2}\int_1^x aK[1]^{k-1}\tan^m(\lambda K[1])dK[1] + c_1\right)$$

11.11 problem 37

Internal problem ID [10535]

Book: Handbook of exact solutions for ordinary differential equations. By Polyanin and Zaitsev. Second edition
Section: Chapter 1, section 1.2. Riccati Equation. subsection 1.2.6-3. Equations with tangent.
Problem number: 37.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_Riccati]`

$$\boxed{(a\tan(\lambda x)+b)\,y'-y^2-k\tan(x\mu)\,y=-d^2+kd\tan(x\mu)}$$

✓ Solution by Maple
Time used: 0.015 (sec). Leaf size: 351

```
dsolve((a*tan(lambda*x)+b)*diff(y(x),x)=y(x)^2+k*tan(mu*x)*y(x)-d^2+k*d*tan(mu*x),y(x), sing
```

$$y(x)$$

$$= \frac{-\left(\sec(x\lambda)^2\right)^{\frac{ad}{\lambda\left(a^2+b^2\right)}}(a\tan(x\lambda)+b)^{-\frac{2ad}{\lambda\left(a^2+b^2\right)}}\mathrm{e}^{\frac{\lambda k\left(a^2+b^2\right)\left(\int\frac{\tan(x\mu)}{a\tan(x\lambda)+b}dx\right)-2\arctan(\tan(x\lambda))bd}{\lambda\left(a^2+b^2\right)}}-d\left(\int(a\tan(x\lambda)+\right.}{\int(a\tan(x\lambda)+b)^{\frac{\left(-a^2-b^2\right)\lambda-2ad}{\lambda\left(a^2+b^2\right)}}\left(\sec(x\lambda)^2\right)^{\frac{ad}{\lambda\left(a^2+b^2\right)}}\mathrm{e}^{\frac{\lambda k\left(a^2+b^2\right)}}$$

✓ Solution by Mathematica

Time used: 130.719 (sec). Leaf size: 800

```
DSolve[(a*Tan[\[Lambda]*x]+b)*y'[x]==y[x]^2+k*Tan[\[Mu]*x]*y[x]-d^2+k*d*Tan[\[Mu]*x],y[x],x,
```

$$\text{Solve}\left[\int_1^x \frac{e^{-\int_1^{K[2]} \frac{\sec(\mu K[1])(2d\cos(\lambda K[1]-\mu K[1])+2d\cos(\lambda K[1]+\mu K[1])+k\sin(\lambda K[1]-\mu K[1])-k\sin(\lambda K[1]+\mu K[1]))}{2(b\cos(\lambda K[1])+a\sin(\lambda K[1]))}dK[1]}(d\cos(\lambda K[2]-\mu K[2])+b\cos(\lambda K[2]-\mu K[2])+a\sin(\lambda K[2]-\mu K[2])+k\mu)}{k\mu(b\cos(\lambda K[2]-\mu K[2])+b\cos(\lambda K[2]+\mu K[2])+a\sin(\lambda K[2]-\mu K[2])+a\sin(\lambda K[2]+\mu K[2])+2k\mu)}dK[2]\right.$$

$$+\int_1^{y(x)}\left(\frac{e^{-\int_1^x \frac{\sec(\mu K[1])(2d\cos(\lambda K[1]-\mu K[1])+2d\cos(\lambda K[1]+\mu K[1])+k\sin(\lambda K[1]-\mu K[1])-k\sin(\lambda K[1]+\mu K[1]))}{2(b\cos(\lambda K[1])+a\sin(\lambda K[1]))}dK[1]}}{k\mu(d+K[3])^2}\right.$$

$$\left.\left.-\int_1^x\left(\frac{e^{-\int_1^{K[2]} \frac{\sec(\mu K[1])(2d\cos(\lambda K[1]-\mu K[1])+2d\cos(\lambda K[1]+\mu K[1])+k\sin(\lambda K[1]-\mu K[1])-k\sin(\lambda K[1]+\mu K[1]))}{2(b\cos(\lambda K[1])+a\sin(\lambda K[1]))}dK[1]}(-\cos(\lambda K[2]-\mu K[2])+b\cos(\lambda K[2]+\mu K[2])+a\sin(\lambda K[2]-\mu K[2])+a\sin(\lambda K[2]+\mu K[2])+2k\mu)}{k\mu(d+K[3])(b\cos(\lambda K[2]-\mu K[2])+b\cos(\lambda K[2]+\mu K[2])+a\sin(\lambda K[2]-\mu K[2])+a\sin(\lambda K[2]+\mu K[2])+2k\mu)}dK[2]\right)\right.$$

12 Chapter 1, section 1.2. Riccati Equation. subsection 1.2.6-4. Equations with cotangent.

12.1 problem 38 286
12.2 problem 39 287
12.3 problem 40 288
12.4 problem 41 289
12.5 problem 42 290
12.6 problem 43 291
12.7 problem 44 292
12.8 problem 45 293
12.9 problem 46 294

12.1 problem 38

Internal problem ID [10536]

Book: Handbook of exact solutions for ordinary differential equations. By Polyanin and Zaitsev. Second edition
Section: Chapter 1, section 1.2. Riccati Equation. subsection 1.2.6-4. Equations with cotangent.
Problem number: 38.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_Riccati]`

$$y' - y^2 = \lambda a + a(\lambda - a)\cot(\lambda x)^2$$

✓ Solution by Maple
Time used: 0.0 (sec). Leaf size: 204

```
dsolve(diff(y(x),x)=y(x)^2+a*lambda+a*(lambda-a)*cot(lambda*x)^2,y(x), singsol=all)
```

$$y(x) = \frac{(\cos(x\lambda)\,\text{LegendreP}\left(\frac{2a-\lambda}{2\lambda},\frac{2a-\lambda}{2\lambda},\cos(x\lambda)\right)a + \text{LegendreQ}\left(\frac{2a-\lambda}{2\lambda},\frac{2a-\lambda}{2\lambda},\cos(x\lambda)\right)\cos(x\lambda)\,c_1 a - \lambda(\text{Legen}}{\text{LegendreQ}\left(\frac{2a-\lambda}{2\lambda},\frac{2a-\lambda}{2\lambda},\cos(x\lambda)\right)c_1 + \text{LegendreP}}$$

✗ Solution by Mathematica
Time used: 0.0 (sec). Leaf size: 0

```
DSolve[y'[x]==y[x]^2+a*\[Lambda]+a*(\[Lambda]-a)*Cot[\[Lambda]*x]^2,y[x],x,IncludeSingularSo
```

Not solved

12.2 problem 39

Internal problem ID [10537]

Book: Handbook of exact solutions for ordinary differential equations. By Polyanin and Zaitsev. Second edition
Section: Chapter 1, section 1.2. Riccati Equation. subsection 1.2.6-4. Equations with cotangent.
Problem number: 39.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_Riccati]`

$$y' - y^2 = 3\lambda a + \lambda^2 + a(\lambda - a)\cot(\lambda x)^2$$

✓ Solution by Maple
Time used: 0.016 (sec). Leaf size: 199

```
dsolve(diff(y(x),x)=y(x)^2+lambda^2+3*a*lambda+a*(lambda-a)*cot(lambda*x)^2,y(x), singsol=al
```

$$y(x) = \frac{\csc(x\lambda)\left(-2\,\text{LegendreP}\left(\frac{2a+3\lambda}{2\lambda}, \frac{2a-\lambda}{2\lambda}, \cos(x\lambda)\right)\lambda - 2\,\text{LegendreQ}\left(\frac{2a+3\lambda}{2\lambda}, \frac{2a-\lambda}{2\lambda}, \cos(x\lambda)\right)c_1\lambda + \cos(x\lambda)\right.}{\text{LegendreQ}\left(\frac{2a+\lambda}{2\lambda}, \frac{2a-\lambda}{2\lambda}, \cos(x\lambda)\right)c_1 + \text{Legendr}}$$

✓ Solution by Mathematica
Time used: 67.099 (sec). Leaf size: 306

```
DSolve[y'[x]==y[x]^2+\[Lambda]^2+3*a*\[Lambda]+a*(\[Lambda]-a)*Cot[\[Lambda]*x]^2,y[x],x,Inc
```

$$y(x) \to -\frac{\sin^{-\frac{a+\lambda}{\lambda}}(2\lambda x)e^{-\text{arctanh}(\cos(2\lambda x))}\left(c_1 \sin^{\frac{a}{\lambda}}(2\lambda x)((a+\lambda)\cos(2\lambda x)+a-\lambda)e^{\text{arctanh}(\cos(2\lambda x))}\int_1^x e^{\frac{(a-\lambda)\text{arctanh}(\cos}{\lambda}}\right.}{1 + c_1\int_1^x e^{\frac{(a-\lambda)\text{arctanh}(\cos(}{\lambda}}$$

$$y(x) \to \csc(2\lambda x)\left(-\frac{\sin^{-\frac{a}{\lambda}}(2\lambda x)e^{\frac{(a-\lambda)\text{arctanh}(\cos(2\lambda x))}{\lambda}}}{\int_1^x e^{\frac{(a-\lambda)\text{arctanh}(\cos(2\lambda K[1]))}{\lambda}}\sin^{-\frac{a+\lambda}{\lambda}}(2\lambda K[1])dK[1]} - (a+\lambda)\cos(2\lambda x)\right.$$
$$\left. - a + \lambda\right)$$

12.3 problem 40

Internal problem ID [10538]

Book: Handbook of exact solutions for ordinary differential equations. By Polyanin and Zaitsev. Second edition
Section: Chapter 1, section 1.2. Riccati Equation. subsection 1.2.6-4. Equations with cotangent.
Problem number: 40.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_Riccati]`

$$\boxed{y' - y^2 + 2ab\cot(xa)\,y = -a^2 + b^2}$$

✓ Solution by Maple
Time used: 0.016 (sec). Leaf size: 291

```
dsolve(diff(y(x),x)=y(x)^2-2*a*b*cot(a*x)*y(x)+b^2-a^2,y(x), singsol=all)
```

$$y(x) = \frac{\left(\cos(ax)\left(ab+\sqrt{(b^2-1)a^2+b^2}\right)\text{LegendreP}\left(\frac{-a+2\sqrt{(b^2-1)a^2+b^2}}{2a}, b-\frac{1}{2}, \cos(ax)\right) + c_1\cos(ax)\left(ab+\sqrt{}\right.\right.}{}$$

✗ Solution by Mathematica
Time used: 0.0 (sec). Leaf size: 0

```
DSolve[y'[x]==y[x]^2-2*a*b*Cot[a*x]*y[x]+b^2-a^2,y[x],x,IncludeSingularSolutions -> True]
```

Not solved

12.4 problem 41

Internal problem ID [10539]

Book: Handbook of exact solutions for ordinary differential equations. By Polyanin and Zaitsev. Second edition
Section: Chapter 1, section 1.2. Riccati Equation. subsection 1.2.6-4. Equations with cotangent.
Problem number: 41.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_Riccati]`

$$\boxed{y' - y^2 - a\cot(\beta x)\, y = ab\cot(\beta x) - b^2}$$

✓ Solution by Maple
Time used: 0.0 (sec). Leaf size: 81

```
dsolve(diff(y(x),x)=y(x)^2+a*cot(beta*x)*y(x)+a*b*cot(beta*x)-b^2,y(x), singsol=all)
```

$$y(x) = \frac{-\left(\csc(x\beta)^2\right)^{-\frac{a}{2\beta}} \mathrm{e}^{-2bx} - b\left(\int \left(\csc(x\beta)^2\right)^{-\frac{a}{2\beta}} \mathrm{e}^{-2bx} dx - c_1\right)}{\int \left(\csc(x\beta)^2\right)^{-\frac{a}{2\beta}} \mathrm{e}^{-2bx} dx - c_1}$$

✓ Solution by Mathematica
Time used: 26.26 (sec). Leaf size: 462

```
DSolve[y'[x]==y[x]^2+a*Cot[\[Beta]*x]*y[x]+a*b*Cot[\[Beta]*x]-b^2,y[x],x,IncludeSingularSolu
```

$$y(x)$$
$$\to \frac{b(ia+2b-2i\beta)\left(-ie^{-i\beta x}\left(-1+e^{2i\beta x}\right)\right)^{a/\beta} \text{Hypergeometric2F1}\left(1, \frac{a+2ib}{2\beta}, -\frac{a-2ib-2\beta}{2\beta}, e^{2ix\beta}\right) + (a-2ib)}{i(-a+2ib+2\beta)\left(-ie^{-i\beta x}\left(-1+e^{2i\beta x}\right)\right)^{a/\beta} \text{Hypergeometric2F1}\left(1, \frac{a+2ib}{2\beta}, -\frac{a-2ib-2\beta}{2\beta}, e^{2}\right.}$$
$$y(x) \to -b$$

12.5 problem 42

Internal problem ID [10540]

Book: Handbook of exact solutions for ordinary differential equations. By Polyanin and Zaitsev. Second edition
Section: Chapter 1, section 1.2. Riccati Equation. subsection 1.2.6-4. Equations with cotangent.
Problem number: 42.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_Riccati]`

$$\boxed{y' - y^2 - ax\cot(bx)^m y = a\cot(bx)^m}$$

✓ Solution by Maple
Time used: 0.0 (sec). Leaf size: 85

```
dsolve(diff(y(x),x)=y(x)^2+a*x*cot(b*x)^m*y(x)+a*cot(b*x)^m,y(x), singsol=all)
```

$$y(x) = \frac{-e^{\int \frac{a\cot(bx)^m x^2-2}{x}dx}x - \left(\int e^{\int \frac{a\cot(bx)^m x^2-2}{x}dx}dx\right) + c_1}{\left(-c_1 + \int e^{\int \frac{a\cot(bx)^m x^2-2}{x}dx}dx\right)x}$$

✓ Solution by Mathematica
Time used: 8.36 (sec). Leaf size: 126

```
DSolve[y'[x]==y[x]^2+a*x*Cot[b*x]^m*y[x]+a*Cot[b*x]^m,y[x],x,IncludeSingularSolutions -> Tru
```

$$y(x) \to -\frac{\exp\left(-\int_1^x -a\cot^m(bK[1])K[1]dK[1]\right) + x\int_1^x \frac{\exp\left(-\int_1^{K[2]} -a\cot^m(bK[1])K[1]dK[1]\right)}{K[2]^2}dK[2] + c_1 x}{x^2\left(\int_1^x \frac{\exp\left(-\int_1^{K[2]} -a\cot^m(bK[1])K[1]dK[1]\right)}{K[2]^2}dK[2] + c_1\right)}$$

$$y(x) \to -\frac{1}{x}$$

12.6 problem 43

Internal problem ID [10541]

Book: Handbook of exact solutions for ordinary differential equations. By Polyanin and Zaitsev. Second edition
Section: Chapter 1, section 1.2. Riccati Equation. subsection 1.2.6-4. Equations with cotangent.
Problem number: 43.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_Riccati]`

$$y' + (k+1)\, x^k y^2 - a\, x^{k+1} \cot(x)^m\, y = -a \cot(x)^m$$

✓ Solution by Maple
Time used: 0.062 (sec). Leaf size: 170

```
dsolve(diff(y(x),x)=-(k+1)*x^k*y(x)^2+a*x^(k+1)*cot(x)^m*y(x)-a*cot(x)^m,y(x), singsol=all)
```

$$y(x) = \frac{x^{-1-k}\left(x^{1+k} e^{\int \frac{x^{1+k}\cot(x)^m ax-2k-2}{x}dx} + \left(\int x^k e^{\int \frac{x^{1+k}\cot(x)^m ax-2k-2}{x}dx}dx\right) k + \int x^k e^{\int \frac{x^{1+k}\cot(x)^m ax-2k-2}{x}dx}dx + c_1\right.}{\left(\int x^k e^{\int \frac{a\,x^{k+2}\cot(x)^m-2k-2}{x}dx}dx\right) k + c_1 + \int x^k e^{\int \frac{a\,x^{k+2}\cot(x)^m-2k-2}{x}dx}dx}$$

✗ Solution by Mathematica
Time used: 0.0 (sec). Leaf size: 0

```
DSolve[y'[x]==-(k+1)*x^k*y[x]^2+a*x^(k+1)*Cot[x]^m*y[x]-a*Cot[x]^m,y[x],x,IncludeSingularSol
```

Not solved

12.7 problem 44

Internal problem ID [10542]

Book: Handbook of exact solutions for ordinary differential equations. By Polyanin and Zaitsev. Second edition
Section: Chapter 1, section 1.2. Riccati Equation. subsection 1.2.6-4. Equations with cotangent.
Problem number: 44.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[[_1st_order, '_with_symmetry_[F(x),G(x)]'], _Riccati]`

$$\boxed{y' - a\cot\left(\lambda x + \mu\right)^k \left(y - b\,x^n - c\right)^2 = b\,x^{n-1} n}$$

✓ Solution by Maple
Time used: 0.0 (sec). Leaf size: 41

```
dsolve(diff(y(x),x)=a*cot(lambda*x+mu)^k*(y(x)-b*x^n-c)^2+b*n*x^(n-1),y(x), singsol=all)
```

$$y(x) = b\,x^n + c + \frac{1}{c_1 - a\left(\int \left(\frac{-1+\cot(\mu)\cot(x\lambda)}{\cot(\mu)+\cot(x\lambda)}\right)^k dx\right)}$$

✓ Solution by Mathematica
Time used: 5.758 (sec). Leaf size: 74

```
DSolve[y'[x]==a*Cot[\[Lambda]*x+mu]^k*(y[x]-b*x^n-c)^2+b*n*x^(n-1),y[x],x,IncludeSingularSol
```

$$y(x) \to \frac{1}{\frac{a\cot^{k+1}(\mu+\lambda x)\,\text{Hypergeometric2F1}\left(1,\frac{k+1}{2},\frac{k+3}{2},-\cot^2(\mu+x\lambda)\right)}{(k+1)\lambda} + c_1} + bx^n + c$$
$$y(x) \to bx^n + c$$

12.8 problem 45

Internal problem ID [10543]

Book: Handbook of exact solutions for ordinary differential equations. By Polyanin and Zaitsev. Second edition
Section: Chapter 1, section 1.2. Riccati Equation. subsection 1.2.6-4. Equations with cotangent.
Problem number: 45.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_Riccati]`

$$\boxed{y'x - a\cot(\lambda x)^m y^2 - ky = a\,b^2 x^{2k}\cot(\lambda x)^m}$$

✓ Solution by Maple
Time used: 0.031 (sec). Leaf size: 31

```
dsolve(x*diff(y(x),x)=a*cot(lambda*x)^m*y(x)^2+k*y(x)+a*b^2*x^(2*k)*cot(lambda*x)^m,y(x), si
```

$$y(x) = -\tan\left(-ab\left(\int x^{-1+k}\cot(x\lambda)^m\,dx\right) + c_1\right) b\,x^k$$

✓ Solution by Mathematica
Time used: 1.805 (sec). Leaf size: 50

```
DSolve[x*y'[x]==a*Cot[\[Lambda]*x]^m*y[x]^2+k*y[x]+a*b^2*x^(2*k)*Cot[\[Lambda]*x]^m,y[x],x,I
```

$$y(x) \to \sqrt{b^2}x^k \tan\left(\sqrt{b^2}\int_1^x a\cot^m(\lambda K[1])K[1]^{k-1}dK[1] + c_1\right)$$

12.9 problem 46

Internal problem ID [10544]

Book: Handbook of exact solutions for ordinary differential equations. By Polyanin and Zaitsev. Second edition
Section: Chapter 1, section 1.2. Riccati Equation. subsection 1.2.6-4. Equations with cotangent.
Problem number: 46.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_Riccati]`

$$(a\cot(\lambda x)+b)\,y'-y^2-c\cot(x\mu)\,y=cd\cot(x\mu)-d^2$$

✓ Solution by Maple
Time used: 0.015 (sec). Leaf size: 366

```
dsolve((a*cot(lambda*x)+b)*diff(y(x),x)=y(x)^2+c*cot(mu*x)*y(x)-d^2+c*d*cot(mu*x),y(x), sing
```

$$y(x)$$

$$=\frac{-\mathrm{e}^{\frac{\lambda c\left(a^2+b^2\right)\left(\int \frac{\cot(x\mu)}{a\cot(x\lambda)+b}dx\right)-2d\left(\operatorname{arccot}(\cot(x\lambda))-\frac{\pi}{2}\right)b}{\lambda\left(a^2+b^2\right)}}\left(\csc(x\lambda)^2\right)^{-\frac{ad}{\lambda\left(a^2+b^2\right)}}(a\cot(x\lambda)+b)^{\frac{2ad}{\lambda\left(a^2+b^2\right)}}-d\left(\int(a\cot(x\lambda)\right.}{\int(a\cot(x\lambda)+b)^{\frac{\left(-a^2-b^2\right)\lambda+2ad}{\lambda\left(a^2+b^2\right)}}\mathrm{e}^{\frac{\lambda c\left(a^2+b^2\right)\left(\int \frac{\cot(x\mu)}{a\cot(x\lambda)+b}dx\right)-2d\left(\mathrm{arc}\right.}{\lambda\left(a^2+b^2\right)}}}$$

✓ Solution by Mathematica

Time used: 87.594 (sec). Leaf size: 799

```
DSolve[(a*Cot[\[Lambda]*x]+b)*y'[x]==y[x]^2+c*Cot[\[Mu]*x]*y[x]-d^2+c*d*Cot[\[Mu]*x],y[x],x,
```

$$\text{Solve}\left[\int_1^x \frac{e^{-\int_1^{K[2]} -\frac{\csc(\mu K[1])(-2d\cos(\lambda K[1]-\mu K[1])+2d\cos(\lambda K[1]+\mu K[1])+c\sin(\lambda K[1]-\mu K[1])+c\sin(\lambda K[1]+\mu K[1]))}{2(a\cos(\lambda K[1])+b\sin(\lambda K[1]))}dK[1]}(-d\cos(\lambda K[2]-}{c\mu(b\cos(\lambda K[2]-\mu K[2])-b\cos(}\right.$$

$$+\int_1^{y(x)}\left(-\int_1^x\left(\frac{e^{-\int_1^{K[2]} -\frac{\csc(\mu K[1])(-2d\cos(\lambda K[1]-\mu K[1])+2d\cos(\lambda K[1]+\mu K[1])+c\sin(\lambda K[1]-\mu K[1])+c\sin(\lambda K[1]+\mu K[1]))}{2(a\cos(\lambda K[1])+b\sin(\lambda K[1]))}dK[1]}(\cos(\lambda K[2}{c\mu(d+K[3])(b\cos(\lambda K[2]-\mu K[2])-b\cos(\lambda K[2]+\mu K[2])-a\sin(\lambda K[2]-\mu K[2}\right.\right.\right.$$

$$\left.\left.\left.-\frac{e^{-\int_1^x -\frac{\csc(\mu K[1])(-2d\cos(\lambda K[1]-\mu K[1])+2d\cos(\lambda K[1]+\mu K[1])+c\sin(\lambda K[1]-\mu K[1])+c\sin(\lambda K[1]+\mu K[1]))}{2(a\cos(\lambda K[1])+b\sin(\lambda K[1]))}dK[1]}}{c\mu(d+K[3])^2}\right)dK[3]=c_1,y(x)\right]$$

13 Chapter 1, section 1.2. Riccati Equation. subsection 1.2.6-5. Equations containing combinations of trigonometric functions.

13.1 problem 47 297
13.2 problem 48 298
13.3 problem 49 300
13.4 problem 50 302
13.5 problem 51 303
13.6 problem 52 304
13.7 problem 53 305
13.8 problem 54 306
13.9 problem 55 307
13.10 problem 56 308
13.11 problem 57 309
13.12 problem 58 310
13.13 problem 59 311

13.1 problem 47

Internal problem ID [10545]

Book: Handbook of exact solutions for ordinary differential equations. By Polyanin and Zaitsev. Second edition
Section: Chapter 1, section 1.2. Riccati Equation. subsection 1.2.6-5. Equations containing combinations of trigonometric functions.
Problem number: 47.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_Riccati]`

$$y' - y^2 = \lambda^2 + c \sin(\lambda x)^n \cos(\lambda x)^{-n-4}$$

✗ Solution by Maple

```
dsolve(diff(y(x),x)=y(x)^2+lambda^2+c*sin(lambda*x)^n*cos(lambda*x)^(-n-4),y(x), singsol=all
```

No solution found

✗ Solution by Mathematica
Time used: 0.0 (sec). Leaf size: 0

```
DSolve[y'[x]==y[x]^2+\[Lambda]^2+c*Sin[\[Lambda]*x]^n*Cos[\[Lambda]*x]^(-n-4),y[x],x,Include
```

Not solved

13.2 problem 48

Internal problem ID [10546]

Book: Handbook of exact solutions for ordinary differential equations. By Polyanin and Zaitsev. Second edition
Section: Chapter 1, section 1.2. Riccati Equation. subsection 1.2.6-5. Equations containing combinations of trigonometric functions.
Problem number: 48.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_Riccati]`

$$\boxed{y' - y^2 \sin(\lambda x)\, a = b \sin(\lambda x) \cos(\lambda x)^n}$$

✓ Solution by Maple
Time used: 0.0 (sec). Leaf size: 256

```
dsolve(diff(y(x),x)=a*sin(lambda*x)*y(x)^2+b*sin(lambda*x)*cos(lambda*x)^n,y(x), singsol=all
```

$$y(x) = \frac{\left(-\sqrt{\frac{ab}{\lambda^2}}\ \mathrm{BesselY}\left(\frac{3+n}{n+2}, \frac{2\sqrt{\frac{ab}{\lambda^2}}\ \cos(x\lambda)^{\frac{n}{2}+1}}{n+2}\right) \cos(x\lambda)^{\frac{n}{2}+1} c_1 - \mathrm{BesselJ}\left(\frac{3+n}{n+2}, \frac{2\sqrt{\frac{ab}{\lambda^2}}\ \cos(x\lambda)^{\frac{n}{2}+1}}{n+2}\right) \sqrt{\frac{ab}{\lambda^2}}\ \cos(x\lambda)\right.}{\left(\mathrm{BesselY}\left(\frac{1}{n+2}, \frac{2\sqrt{\frac{ab}{\lambda^2}}\ \cos(x\lambda)^{\frac{n}{2}+1}}{n+2}\right) c_1 + \mathrm{Bessel}\right.}$$

✓ Solution by Mathematica

Time used: 1.409 (sec). Leaf size: 695

```
DSolve[y'[x]==a*Sin[\[Lambda]*x]*y[x]^2+b*Sin[\[Lambda]*x]*Cos[\[Lambda]*x]^n,y[x],x,Include
```

$$y(x)$$

$$\to \frac{\sqrt{a}\sqrt{b}\,\text{Gamma}\left(1+\frac{1}{n+2}\right)\cos^{\frac{n}{2}}(\lambda x)\,\text{BesselJ}\left(\frac{1}{n+2}-1,\frac{2\sqrt{a}\sqrt{b}\cos^{\frac{n}{2}+1}(x\lambda)}{n\lambda+2\lambda}\right)-\sqrt{a}\sqrt{b}\,\text{Gamma}\left(1+\frac{1}{n+2}\right)\cos^{\frac{n}{2}}$$

$$y(x)\to \frac{\frac{\sqrt{a}\sqrt{b}\cos^{\frac{n}{2}}(\lambda x)\left(\text{BesselJ}\left(-\frac{n+3}{n+2},\frac{2\sqrt{a}\sqrt{b}\cos^{\frac{n}{2}+1}(x\lambda)}{n\lambda+2\lambda}\right)-\text{BesselJ}\left(\frac{n+1}{n+2},\frac{2\sqrt{a}\sqrt{b}\cos^{\frac{n}{2}+1}(x\lambda)}{n\lambda+2\lambda}\right)\right)}{\text{BesselJ}\left(-\frac{1}{n+2},\frac{2\sqrt{a}\sqrt{b}\cos^{\frac{n}{2}+1}(x\lambda)}{n\lambda+2\lambda}\right)}+\lambda\sec(\lambda x)}{2a}$$

13.3 problem 49

Internal problem ID [10547]

Book: Handbook of exact solutions for ordinary differential equations. By Polyanin and Zaitsev. Second edition
Section: Chapter 1, section 1.2. Riccati Equation. subsection 1.2.6-5. Equations containing combinations of trigonometric functions.
Problem number: 49.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_Riccati]`

$$y' - \lambda \sin(\lambda x)\, y^2 - a \cos(\lambda x)^n\, y = -a \cos(\lambda x)^{n-1}$$

✗ Solution by Maple

```
dsolve(diff(y(x),x)=lambda*sin(lambda*x)*y(x)^2+a*cos(lambda*x)^n*y(x)-a*cos(lambda*x)^(n-1)
```

No solution found

✓ Solution by Mathematica

Time used: 150.623 (sec). Leaf size: 467

```
DSolve[y'[x]==\[Lambda]*Sin[\[Lambda]*x]*y[x]^2+a*Cos[\[Lambda]*x]^n*y[x]-a*Cos[\[Lambda]*x]
```

$$\text{Solve}\left[\int_1^x -\frac{\exp\left(-\frac{a\cos^{n+1}(\lambda K[1])\csc(\lambda K[1])\,\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{n+1}{2},\frac{n+3}{2},\cos^2(\lambda K[1])\right)\sqrt{\sin^2(\lambda K[1])}}{(n+1)\lambda}\right)\tan(\lambda K[1])\,(-a\csc(\lambda K[1])\,\text{co}}{(\cos(\lambda K[1])y(x)-1)^2}\right.$$

$$+\int_1^{y(x)}\left(\frac{\exp\left(-\frac{a\cos^{n+1}(x\lambda)\csc(x\lambda)\,\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{n+1}{2},\frac{n+3}{2},\cos^2(x\lambda)\right)\sqrt{\sin^2(x\lambda)}}{(n+1)\lambda}\right)}{(\cos(x\lambda)K[2]-1)^2}\right.$$

$$-\int_1^x\left(\frac{2\exp\left(-\frac{a\cos^{n+1}(\lambda K[1])\csc(\lambda K[1])\,\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{n+1}{2},\frac{n+3}{2},\cos^2(\lambda K[1])\right)\sqrt{\sin^2(\lambda K[1])}}{(n+1)\lambda}\right)(-a\csc(\lambda K[1])\cos^n(\lambda}{(\cos(\lambda K[1])K[2]-1)^3}\right.$$

13.4 problem 50

Internal problem ID [10548]

Book: Handbook of exact solutions for ordinary differential equations. By Polyanin and Zaitsev. Second edition
Section: Chapter 1, section 1.2. Riccati Equation. subsection 1.2.6-5. Equations containing combinations of trigonometric functions.
Problem number: 50.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_Riccati]`

$$y' - y^2 \cos(\lambda x)\, a = b \cos(\lambda x) \sin(\lambda x)^n$$

✓ Solution by Maple
Time used: 0.0 (sec). Leaf size: 949

```
dsolve(diff(y(x),x)=a*cos(lambda*x)*y(x)^2+b*cos(lambda*x)*sin(lambda*x)^n,y(x), singsol=all
```

Expression too large to display

✓ Solution by Mathematica
Time used: 1.376 (sec). Leaf size: 633

```
DSolve[y'[x]==a*Cos[\[Lambda]*x]*y[x]^2+b*Cos[\[Lambda]*x]*Sin[\[Lambda]*x]^n,y[x],x,Include
```

$$y(x) \to \frac{\csc(\lambda x)\left(-\lambda \,\text{Gamma}\left(1+\frac{1}{n+2}\right)\text{BesselJ}\left(\frac{1}{n+2},\frac{2\sqrt{a}\sqrt{b}\sin^{\frac{n}{2}+1}(x\lambda)}{n\lambda+2\lambda}\right)+\sqrt{a}\sqrt{b}\sin^{\frac{n}{2}+1}(\lambda x)\left(\text{Gamma}\left(1+\frac{1}{n+2}\right)\right.}{}$$

$$y(x) \to \frac{\frac{\sqrt{a}\sqrt{b}\sin^{\frac{n}{2}}(\lambda x)\left(\text{BesselJ}\left(\frac{n+1}{n+2},\frac{2\sqrt{a}\sqrt{b}\sin^{\frac{n}{2}+1}(x\lambda)}{n\lambda+2\lambda}\right)-\text{BesselJ}\left(-\frac{n+3}{n+2},\frac{2\sqrt{a}\sqrt{b}\sin^{\frac{n}{2}+1}(x\lambda)}{n\lambda+2\lambda}\right)\right)}{\text{BesselJ}\left(-\frac{1}{n+2},\frac{2\sqrt{a}\sqrt{b}\sin^{\frac{n}{2}+1}(x\lambda)}{n\lambda+2\lambda}\right)}-\lambda\csc(\lambda x)}{2a}$$

13.5 problem 51

Internal problem ID [10549]

Book: Handbook of exact solutions for ordinary differential equations. By Polyanin and Zaitsev. Second edition
Section: Chapter 1, section 1.2. Riccati Equation. subsection 1.2.6-5. Equations containing combinations of trigonometric functions.
Problem number: 51.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_Riccati]`

$$y' - \lambda \sin(\lambda x)\, y^2 - a\, x^n \cos(\lambda x)\, y = -x^n a$$

✓ Solution by Maple
Time used: 0.015 (sec). Leaf size: 103

```
dsolve(diff(y(x),x)=lambda*sin(lambda*x)*y(x)^2+a*x^n*cos(lambda*x)*y(x)-a*x^n,y(x), singsol
```

$$y(x) = \frac{-c_1 \mathrm{e}^{\int (x^n \cos(x\lambda) a + 2\tan(x\lambda)\lambda)dx} + \sec(x\lambda)\,\lambda \left(\int \mathrm{e}^{\int (x^n \cos(x\lambda) a + 2\tan(x\lambda)\lambda)dx} \sin(x\lambda)\, dx\right) c_1 - \sec(x\lambda)}{\lambda \left(\int \mathrm{e}^{\int (x^n \cos(x\lambda) a + 2\tan(x\lambda)\lambda)dx} \sin(x\lambda)\, dx\right) c_1 - 1}$$

✗ Solution by Mathematica
Time used: 0.0 (sec). Leaf size: 0

```
DSolve[y'[x]==\[Lambda]*Sin[\[Lambda]*x]*y[x]^2+a*x^n*Cos[\[Lambda]*x]*y[x]-a*x^n,y[x],x,Inc
```

Not solved

13.6 problem 52

Internal problem ID [10550]

Book: Handbook of exact solutions for ordinary differential equations. By Polyanin and Zaitsev. Second edition
Section: Chapter 1, section 1.2. Riccati Equation. subsection 1.2.6-5. Equations containing combinations of trigonometric functions.
Problem number: 52.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_Riccati]`

$$\boxed{\sin(2x)^{n+1} y' - a y^2 \sin(x)^{2n} = b \cos(x)^{2n}}$$

✓ Solution by Maple
Time used: 0.172 (sec). Leaf size: 232

```
dsolve(sin(2*x)^(n+1)*diff(y(x),x)=a*y(x)^2*sin(x)^(2*n)+b*cos(x)^(2*n),y(x), singsol=all)
```

$$y(x) = -\frac{\csc(x)\sin(2x)^n \left(\sin(x)^{\frac{\sqrt{n^2-4^{-n}ab}}{2}} \left(n+\sqrt{n^2-4^{-n}ab}\right)\cos(x)^{-\frac{\sqrt{n^2-4^{-n}ab}}{2}} - \cos(x)^{\frac{\sqrt{n^2-4^{-n}ab}}{2}} \sin(x)^{-\frac{\sqrt{n}}{}}\right.}{a\left(\cos(x)^{\frac{\sqrt{n^2-4^{-n}ab}}{2}} \sin(x)^{-\frac{\sqrt{n^2-4^{-n}ab}}{2}} c_1 + \cos(x)^{-\frac{\sqrt{n^2-4^{-n}ab}}{2}} \sin(x)\right.}$$

✓ Solution by Mathematica
Time used: 33.745 (sec). Leaf size: 132

```
DSolve[Sin[2*x]^(n+1)*y'[x]==a*y[x]^2*Sin[x]^(2*n)+b*Cos[x]^(2*n),y[x],x,IncludeSingularSolu
```

$$\text{Solve}\left[\int_1^{\sqrt{\frac{a\cos^{-2n}(x)\sin^{2n}(x)}{b}}y(x)} \frac{1}{K[1]^2 - \sqrt{\frac{2^{2n+2}n^2}{ab}}K[1]+1}dK[1] = \frac{1}{2}b\sin^{-n}(2x)\cos^{2n}(x)\left(\log\left(\tan\left(\frac{x}{2}\right)\right)\right.\right.$$
$$\left.\left.-\log\left(\cos(x)\sec^2\left(\frac{x}{2}\right)\right)\right)\sqrt{\frac{a\sin^{2n}(x)\cos^{-2n}(x)}{b}} + c_1, y(x)\right]$$

13.7 problem 53

Internal problem ID [10551]

Book: Handbook of exact solutions for ordinary differential equations. By Polyanin and Zaitsev. Second edition
Section: Chapter 1, section 1.2. Riccati Equation. subsection 1.2.6-5. Equations containing combinations of trigonometric functions.
Problem number: 53.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_Riccati]`

$$\boxed{y' - y^2 + \tan(x)\, y = a(1-a)\cot(x)^2}$$

✓ Solution by Maple
Time used: 0.0 (sec). Leaf size: 37

```
dsolve(diff(y(x),x)=y(x)^2-y(x)*tan(x)+a*(1-a)*cot(x)^2,y(x), singsol=all)
```

$$y(x) = \frac{-\cot(x)\sin(x)^{2a}\, a + c_1 \cos(x)\,(a-1)}{c_1 \sin(x) + \sin(x)^{2a}}$$

✓ Solution by Mathematica
Time used: 7.444 (sec). Leaf size: 230

```
DSolve[y'[x]==y[x]^2-y[x]*Tan[x]+a*(1-a)*Cot[x]^2,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to -\frac{i\cot(x)\left(\left(\sqrt{a-1}\sqrt{a}\sqrt{-\frac{(2a-1)^2}{(a-1)a}} - i\right)(-\sin^2(x))^{\frac{1}{2}i\sqrt{a-1}\sqrt{a}\sqrt{-\frac{(2a-1)^2}{(a-1)a}}} - \left(\sqrt{a-1}\sqrt{a}\sqrt{-\frac{(2a-1)^2}{(a-1)a}} + i\right)c_1\right.}{2\left((-\sin^2(x))^{\frac{1}{2}i\sqrt{a-1}\sqrt{a}\sqrt{\frac{1}{a-a^2}-4}} + c_1\right)}$$

$$y(x) \to \frac{1}{2}i\left(\sqrt{a-1}\sqrt{a}\sqrt{\frac{1}{a-a^2}-4} + i\right)\cot(x)$$

13.8 problem 54

Internal problem ID [10552]

Book: Handbook of exact solutions for ordinary differential equations. By Polyanin and Zaitsev. Second edition
Section: Chapter 1, section 1.2. Riccati Equation. subsection 1.2.6-5. Equations containing combinations of trigonometric functions.
Problem number: 54.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_Riccati]`

$$\boxed{y' - y^2 + my\tan(x) = b^2\cos(x)^{2m}}$$

✓ Solution by Maple
Time used: 0.0 (sec). Leaf size: 222

```
dsolve(diff(y(x),x)=y(x)^2-m*y(x)*tan(x)+b^2*cos(x)^(2*m),y(x), singsol=all)
```

$$y(x) = \frac{\left(-\frac{\sin(x)^2(m-1)\,\text{hypergeom}\left(\left[\frac{3}{2},-\frac{m}{2}+\frac{3}{2}\right],\left[\frac{5}{2}\right],\sin(x)^2\right)}{3} + \text{hypergeom}\left(\left[\frac{1}{2},-\frac{m}{2}+\frac{1}{2}\right],\left[\frac{3}{2}\right],\sin(x)^2\right)\right) b\cos(x)^{-m+1}\left(s\right.}{c_1\cos\left(b\sqrt{\cos(x)^{2m-2}}\,\cos(x)^{-m+}\right.}$$

✓ Solution by Mathematica
Time used: 4.179 (sec). Leaf size: 73

```
DSolve[y'[x]==y[x]^2-m*y[x]*Tan[x]+b^2*Cos[x]^(2*m),y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to \sqrt{b^2}\cos^m(x)\tan\left(-\frac{\sqrt{b^2}\sqrt{\sin^2(x)}\csc(x)\cos^{m+1}(x)\,\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{m+1}{2},\frac{m+3}{2},\cos^2(x)\right)}{m+1} + c_1\right)$$

13.9 problem 55

Internal problem ID [10553]

Book: Handbook of exact solutions for ordinary differential equations. By Polyanin and Zaitsev. Second edition
Section: Chapter 1, section 1.2. Riccati Equation. subsection 1.2.6-5. Equations containing combinations of trigonometric functions.
Problem number: 55.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_Riccati]`

$$\boxed{y' - y^2 - my\cot(x) = b^2 \sin(x)^{2m}}$$

✓ Solution by Maple
Time used: 0.016 (sec). Leaf size: 281

```
dsolve(diff(y(x),x)=y(x)^2+m*y(x)*cot(x)+b^2*sin(x)^(2*m),y(x), singsol=all)
```

$$y(x) = \frac{\sqrt{\left(\csc(x)^2\right)^{-m}\sin(x)^4}\left(-c_1 \sin\left(\sqrt{\left(\csc(x)^2\right)^{-m}\sin(x)^4}\,\csc(x)^2\left(\csc(x)^2\right)^{\frac{m}{2}}\cot(x)\,\text{hypergeom}\left(\left[\frac{1}{2}, 1 -\right.\right.\right.}{c_1 \cos\left(\sqrt{\left(\csc(x)^2\right)^{-m}\sin(x)^4}\right.}$$

✓ Solution by Mathematica
Time used: 5.352 (sec). Leaf size: 72

```
DSolve[y'[x]==y[x]^2+m*y[x]*Cot[x]+b^2*Sin[x]^(2*m),y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to \sqrt{b^2}\sin^m(x)\tan\left(\frac{\sqrt{b^2}\sqrt{\cos^2(x)}\sec(x)\sin^{m+1}(x)\,\text{Hypergeometric2F1}\left(\frac{1}{2}, \frac{m+1}{2}, \frac{m+3}{2}, \sin^2(x)\right)}{m+1} + c_1\right)$$

13.10 problem 56

Internal problem ID [10554]

Book: Handbook of exact solutions for ordinary differential equations. By Polyanin and Zaitsev. Second edition
Section: Chapter 1, section 1.2. Riccati Equation. subsection 1.2.6-5. Equations containing combinations of trigonometric functions.
Problem number: 56.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_Riccati]`

$$y' - y^2 = -2\lambda^2 \cot(\lambda x)^2 - 2\lambda^2 \tan(x)^2$$

✗ Solution by Maple

```
dsolve(diff(y(x),x)=y(x)^2-2*lambda^2*tan(x)^2-2*lambda^2*cot(lambda*x)^2,y(x), singsol=all)
```

No solution found

✗ Solution by Mathematica
Time used: 0.0 (sec). Leaf size: 0

```
DSolve[y'[x]==y[x]^2-2*\[Lambda]^2*Tan[x]^2-2*\[Lambda]^2*Cot[\[Lambda]*x]^2,y[x],x,IncludeS
```

Not solved

13.11 problem 57

Internal problem ID [10555]

Book: Handbook of exact solutions for ordinary differential equations. By Polyanin and Zaitsev. Second edition
Section: Chapter 1, section 1.2. Riccati Equation. subsection 1.2.6-5. Equations containing combinations of trigonometric functions.
Problem number: 57.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_Riccati]`

$$y' - y^2 = 2ab + \lambda a + b\lambda + a(\lambda - a)\tan(\lambda x)^2 + b(\lambda - b)\cot(\lambda x)^2$$

✓ Solution by Maple
Time used: 0.0 (sec). Leaf size: 268

```
dsolve(diff(y(x),x)=y(x)^2+lambda*a+lambda*b+2*a*b+a*(lambda-a)*tan(lambda*x)^2+b*(lambda-b)
```

$$y(x) = \frac{4c_1\lambda\cos(x\lambda)^2\sin(x\lambda)^2(b-\lambda+a)\,\text{hypergeom}\left(\left[2,\frac{2\lambda-b-a}{\lambda}\right],\left[-\frac{2a-5\lambda}{2\lambda}\right],\cos(x\lambda)^2\right) - 2c_1\left(\left(-3\lambda^2+\left(\frac{7a}{2}+\right.\right.\right.}{(2a-3\lambda)\left(c_1\cos(x\lambda)\sin\right.}$$

✗ Solution by Mathematica
Time used: 0.0 (sec). Leaf size: 0

```
DSolve[y'[x]==y[x]^2+\[Lambda]*a+\[Lambda]*b+2*a*b+a*(\[Lambda]-a)*Tan[\[Lambda]*x]^2+b*(\[L
```

Not solved

13.12 problem 58

Internal problem ID [10556]

Book: Handbook of exact solutions for ordinary differential equations. By Polyanin and Zaitsev. Second edition
Section: Chapter 1, section 1.2. Riccati Equation. subsection 1.2.6-5. Equations containing combinations of trigonometric functions.
Problem number: 58.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_Riccati]`

$$y' - y^2 = -\frac{\lambda^2}{2} - \frac{3\lambda^2 \tan(\lambda x)^2}{4} + a\cos(\lambda x)^2 \sin(\lambda x)^n$$

✓ Solution by Maple
Time used: 0.0 (sec). Leaf size: 1194

```
dsolve(diff(y(x),x)=y(x)^2-1/2*lambda^2-3/4*lambda^2*tan(lambda*x)^2+a*cos(lambda*x)^2*sin(l
```

Expression too large to display

✗ Solution by Mathematica
Time used: 0.0 (sec). Leaf size: 0

```
DSolve[y'[x]==y[x]^2-1/2*\[Lambda]^2-3/4*\[Lambda]^2*Tan[\[Lambda]*x]^2+a*Cos[\[Lambda]*x]^2
```

Not solved

13.13 problem 59

Internal problem ID [10557]

Book: Handbook of exact solutions for ordinary differential equations. By Polyanin and Zaitsev. Second edition
Section: Chapter 1, section 1.2. Riccati Equation. subsection 1.2.6-5. Equations containing combinations of trigonometric functions.
Problem number: 59.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_Riccati]`

$$y' - \lambda \sin(\lambda x)\, y^2 - a \sin(\lambda x)\, y = -a \tan(\lambda x)$$

✓ Solution by Maple
Time used: 0.0 (sec). Leaf size: 61

```
dsolve(diff(y(x),x)=lambda*sin(lambda*x)*y(x)^2+a*sin(lambda*x)*y(x)-a*tan(lambda*x),y(x), s
```

$$y(x) = \frac{\text{expIntegral}_1\left(\frac{a\cos(x\lambda)}{\lambda}\right) c_1 a + 1}{\cos(x\lambda)\,\text{expIntegral}_1\left(\frac{a\cos(x\lambda)}{\lambda}\right) c_1 a - \mathrm{e}^{-\frac{a\cos(x\lambda)}{\lambda}} c_1 \lambda + \cos(x\lambda)}$$

✗ Solution by Mathematica
Time used: 0.0 (sec). Leaf size: 0

```
DSolve[y'[x]==\[Lambda]*Sin[\[Lambda]*x]*y[x]^2+a*Sin[\[Lambda]*x]*y[x]-a*Tan[\[Lambda]*x],y
```

Not solved

14 Chapter 1, section 1.2. Riccati Equation. subsection 1.2.7-1. Equations containing arcsine.

14.1 problem 1 313
14.2 problem 2 315
14.3 problem 3 316
14.4 problem 4 317
14.5 problem 5 318
14.6 problem 6 319
14.7 problem 7 320
14.8 problem 8 321
14.9 problem 9 322

14.1 problem 1

Internal problem ID [10558]

Book: Handbook of exact solutions for ordinary differential equations. By Polyanin and Zaitsev. Second edition
Section: Chapter 1, section 1.2. Riccati Equation. subsection 1.2.7-1. Equations containing arcsine.
Problem number: 1.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_Riccati]`

$$\boxed{y' - y^2 - \lambda \arcsin(x)^n y = -a^2 + a\lambda \arcsin(x)^n}$$

✓ Solution by Maple
Time used: 0.0 (sec). Leaf size: 71

```
dsolve(diff(y(x),x)=y(x)^2+lambda*arcsin(x)^n*y(x)-a^2+a*lambda*arcsin(x)^n,y(x), singsol=al
```

$$y(x) = \frac{-c_1 a - a\left(\int \mathrm{e}^{-\left(\int (-\lambda \arcsin(x)^n + 2a)dx\right)} dx\right) - \mathrm{e}^{-\left(\int (-\lambda \arcsin(x)^n + 2a)dx\right)}}{c_1 + \int \mathrm{e}^{-\left(\int (-\lambda \arcsin(x)^n + 2a)dx\right)} dx}$$

✓ Solution by Mathematica

Time used: 6.556 (sec). Leaf size: 398

```
DSolve[y'[x]==y[x]^2+\[Lambda]*ArcSin[x]^n*y[x]-a^2+a*\[Lambda]*ArcSin[x]^n,y[x],x,IncludeSi
```

$$\text{Solve}\left[\int_1^x -\frac{\exp\left(\frac{1}{2}i\lambda \arcsin(K[1])^n \left(\arcsin(K[1])^2\right)^{-n}\left((-i\arcsin(K[1]))^n\Gamma(n+1,i\arcsin(K[1]))-(i\arcsin(K[1]))^n\right.\right.\right.}{n\lambda(a+y(x))}\right.$$

$$+\int_1^{y(x)}\left(\frac{\exp\left(\frac{1}{2}i\lambda \arcsin(x)^n \left(\arcsin(x)^2\right)^{-n}\left((-i\arcsin(x))^n\Gamma(n+1,i\arcsin(x))-(i\arcsin(x))^n\Gamma(n+1,\right.\right.\right.}{n\lambda(a+K[2])^2}\right.$$

$$\left.-\int_1^x\left(\frac{\exp\left(\frac{1}{2}i\lambda \arcsin(K[1])^n \left(\arcsin(K[1])^2\right)^{-n}\left((-i\arcsin(K[1]))^n\Gamma(n+1,i\arcsin(K[1]))-(i\arcsin(K[1]))^n\right.\right.\right.}{n\lambda(a+K[2])^2}\right.$$

14.2 problem 2

Internal problem ID [10559]

Book: Handbook of exact solutions for ordinary differential equations. By Polyanin and Zaitsev. Second edition
Section: Chapter 1, section 1.2. Riccati Equation. subsection 1.2.7-1. Equations containing arcsine.
Problem number: 2.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_Riccati]`

$$\boxed{y' - y^2 - \lambda x \arcsin(x)^n y = \arcsin(x)^n \lambda}$$

✓ Solution by Maple
Time used: 0.016 (sec). Leaf size: 78

```
dsolve(diff(y(x),x)=y(x)^2+lambda*x*arcsin(x)^n*y(x)+lambda*arcsin(x)^n,y(x), singsol=all)
```

$$y(x) = \frac{e^{\int \frac{\lambda \arcsin(x)^n x^2 - 2}{x} dx} x + \int e^{\int \frac{\lambda \arcsin(x)^n x^2 - 2}{x} dx} dx - c_1}{\left(c_1 - \left(\int e^{\int \frac{\lambda \arcsin(x)^n x^2 - 2}{x} dx} dx\right)\right) x}$$

✓ Solution by Mathematica
Time used: 4.147 (sec). Leaf size: 256

```
DSolve[y'[x]==y[x]^2+\[Lambda]*x*ArcSin[x]^n*y[x]+\[Lambda]*ArcSin[x]^n,y[x],x,IncludeSingul
```

$$y(x) \to -\frac{\int_1^x \frac{\exp\left(-2^{-n-3}\lambda \arcsin(K[1])^n (\arcsin(K[1])^2)^{-n} (\Gamma(n+1,2i\arcsin(K[1]))(-i\arcsin(K[1]))^n + (i\arcsin(K[1]))^n \Gamma(n+1,-2i\arcsin(K[1]))\right.}{K[1]^2}}{x\left(\int_1^x \frac{\exp\left(-2^{-n-3}\lambda \arcsin(K[1])^n (\arcsin(K[1])^2)^{-n} (\Gamma(n+1,2i\arcsin(K[1]))\right.}{K[1]^2}}$$

$$y(x) \to -\frac{1}{x}$$

14.3 problem 3

Internal problem ID [10560]

Book: Handbook of exact solutions for ordinary differential equations. By Polyanin and Zaitsev. Second edition
Section: Chapter 1, section 1.2. Riccati Equation. subsection 1.2.7-1. Equations containing arcsine.
Problem number: 3.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_Riccati]`

$$\boxed{y' + (k+1)\, x^k y^2 - \lambda \arcsin(x)^n \left(x^{k+1} y - 1\right) = 0}$$

✓ Solution by Maple
Time used: 0.016 (sec). Leaf size: 180

```
dsolve(diff(y(x),x)=-(k+1)*x^k*y(x)^2+lambda*arcsin(x)^n*(x^(k+1)*y(x)-1),y(x), singsol=all)
```

$$y(x) = \frac{x^{-1-k}\left(x^{1+k} e^{\int \frac{\arcsin(x)^n x^{1+k} \lambda x - 2k - 2}{x} dx} + \left(\int x^k e^{\lambda\left(\int \arcsin(x)^n x^{1+k} dx\right) - 2\left(\int \frac{1}{x} dx\right)(1+k)} dx\right) k + \int x^k e^{\lambda\left(\int \arcsin(x)^n x^{1+k} d\right.}\right.}{\left(\int x^k e^{\lambda\left(\int \arcsin(x)^n x^{1+k} dx\right) - 2\left(\int \frac{1}{x} dx\right)(1+k)} dx\right) k + \int x^k e^{\lambda\left(\int \arcsin(x)^n x^{1+k} dx\right) - 2\left(\int \frac{1}{x} dx\right)(1+k)} dx}$$

✗ Solution by Mathematica
Time used: 0.0 (sec). Leaf size: 0

```
DSolve[y'[x]==-(k+1)*x^k*y[x]^2+\[Lambda]*ArcSin[x]^n*(x^(k+1)*y[x]-1),y[x],x,IncludeSingula
```

Not solved

14.4 problem 4

Internal problem ID [10561]

Book: Handbook of exact solutions for ordinary differential equations. By Polyanin and Zaitsev. Second edition
Section: Chapter 1, section 1.2. Riccati Equation. subsection 1.2.7-1. Equations containing arcsine.
Problem number: 4.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_Riccati]`

$$\boxed{y' - \lambda \arcsin(x)^n y^2 - ay = ab - b^2 \lambda \arcsin(x)^n}$$

✓ Solution by Maple
Time used: 0.0 (sec). Leaf size: 87

```
dsolve(diff(y(x),x)=lambda*arcsin(x)^n*y(x)^2+a*y(x)+a*b-b^2*lambda*arcsin(x)^n,y(x), singso
```

$$y(x) = \frac{-b\lambda\left(\int \arcsin(x)^n \mathrm{e}^{-\left(\int (2\arcsin(x)^n \lambda b - a)dx\right)} dx\right) - c_1 b - \mathrm{e}^{-\left(\int (2\arcsin(x)^n \lambda b - a)dx\right)}}{c_1 + \lambda\left(\int \arcsin(x)^n \mathrm{e}^{-\left(\int (2\arcsin(x)^n \lambda b - a)dx\right)} dx\right)}$$

✓ Solution by Mathematica
Time used: 7.093 (sec). Leaf size: 428

```
DSolve[y'[x]==\[Lambda]*ArcSin[x]^n*y[x]^2+a*y[x]+a*b-b^2*\[Lambda]*ArcSin[x]^n,y[x],x,Inclu
```

$$\text{Solve}\left[\int_1^x \frac{i \exp\left(aK[1] - ib\lambda \arcsin(K[1])^n \left(\arcsin(K[1])^2\right)^{-n} \left((-i\arcsin(K[1]))^n \Gamma(n+1, i\arcsin(K[1]))\right.\right.}{an\lambda(b-}$$

$$+\int_1^{y(x)} \left(-\int_1^x \left(\frac{i \exp\left(aK[1] - ib\lambda \arcsin(K[1])^n \left(\arcsin(K[1])^2\right)^{-n} \left((-i\arcsin(K[1]))^n \Gamma(n+1, i\arcsin(\right.\right.\right.}{an(b+K[2])}$$

$$\left.\left.\left. - \frac{i \exp\left(ax - ib\lambda \arcsin(x)^n \left(\arcsin(x)^2\right)^{-n} \left((-i\arcsin(x))^n \Gamma(n+1, i\arcsin(x)) - (i\arcsin(x))^n \Gamma(n+1,\right.\right.\right.\right.}{an\lambda(b+K[2])^2}$$

14.5 problem 5

Internal problem ID [10562]

Book: Handbook of exact solutions for ordinary differential equations. By Polyanin and Zaitsev. Second edition
Section: Chapter 1, section 1.2. Riccati Equation. subsection 1.2.7-1. Equations containing arcsine.
Problem number: 5.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_Riccati]`

$$y' - \lambda \arcsin(x)^n y^2 + b\lambda x^m \arcsin(x)^n y = bm x^{m-1}$$

✗ Solution by Maple

```
dsolve(diff(y(x),x)=lambda*arcsin(x)^n*y(x)^2-b*lambda*x^m*arcsin(x)^n*y(x)+b*m*x^(m-1),y(x)
```

No solution found

✗ Solution by Mathematica
Time used: 0.0 (sec). Leaf size: 0

```
DSolve[y'[x]==\[Lambda]*ArcSin[x]^n*y[x]^2-b*\[Lambda]*x^m*ArcSin[x]^n*y[x]+b*m*x^(m-1),y[x]
```

Not solved

14.6 problem 6

Internal problem ID [10563]

Book: Handbook of exact solutions for ordinary differential equations. By Polyanin and Zaitsev. Second edition
Section: Chapter 1, section 1.2. Riccati Equation. subsection 1.2.7-1. Equations containing arcsine.
Problem number: 6.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_Riccati]`

$$y' - \lambda \arcsin(x)^n y^2 = \beta m x^{m-1} - \lambda \beta^2 x^{2m} \arcsin(x)^n$$

✗ Solution by Maple

```
dsolve(diff(y(x),x)=lambda*arcsin(x)^n*y(x)^2+beta*m*x^(m-1)-lambda*beta^2*x^(2*m)*arcsin(x)
```

No solution found

✗ Solution by Mathematica
Time used: 0.0 (sec). Leaf size: 0

```
DSolve[y'[x]==\[Lambda]*ArcSin[x]^n*y[x]^2+\[Beta]*m*x^(m-1)-\[Lambda]*\[Beta]^2*x^(2*m)*Arc
```

Not solved

14.7 problem 7

Internal problem ID [10564]

Book: Handbook of exact solutions for ordinary differential equations. By Polyanin and Zaitsev. Second edition
Section: Chapter 1, section 1.2. Riccati Equation. subsection 1.2.7-1. Equations containing arcsine.
Problem number: 7.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[[_1st_order, '_with_symmetry_[F(x),G(x)]'], _Riccati]`

$$\boxed{y' - \lambda \arcsin(x)^n (y - a x^m - b)^2 = a m x^{m-1}}$$

✓ Solution by Maple
Time used: 0.0 (sec). Leaf size: 24

```
dsolve(diff(y(x),x)=lambda*arcsin(x)^n*(y(x)-a*x^m-b)^2+a*m*x^(m-1),y(x), singsol=all)
```

$$y(x) = a x^m + b + \frac{1}{c_1 - \lambda \left(\int \arcsin(x)^n \, dx \right)}$$

✓ Solution by Mathematica
Time used: 4.054 (sec). Leaf size: 87

```
DSolve[y'[x]==\[Lambda]*ArcSin[x]^n*(y[x]-a*x^m-b)^2+a*m*x^(m-1),y[x],x,IncludeSingularSolut
```

$$y(x) \to a x^m + \frac{1}{\frac{1}{2} i \lambda \arcsin(x)^n (\arcsin(x)^2)^{-n} ((i \arcsin(x))^n \Gamma(n+1, -i \arcsin(x)) - (-i \arcsin(x))^n \Gamma(n+1, i \arcsin(} + b$$

$$y(x) \to a x^m + b$$

14.8 problem 8

Internal problem ID [10565]

Book: Handbook of exact solutions for ordinary differential equations. By Polyanin and Zaitsev. Second edition
Section: Chapter 1, section 1.2. Riccati Equation. subsection 1.2.7-1. Equations containing arcsine.
Problem number: 8.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_Riccati]`

$$y'x - \lambda \arcsin(x)^n y^2 - ky = \lambda b^2 x^{2k} \arcsin(x)^n$$

✓ Solution by Maple
Time used: 0.031 (sec). Leaf size: 29

```
dsolve(x*diff(y(x),x)=lambda*arcsin(x)^n*y(x)^2+k*y(x)+lambda*b^2*x^(2*k)*arcsin(x)^n,y(x),
```

$$y(x) = -\tan\left(-\lambda b\left(\int x^{-1+k} \arcsin(x)^n \, dx\right) + c_1\right) b\, x^k$$

✓ Solution by Mathematica
Time used: 1.716 (sec). Leaf size: 48

```
DSolve[x*y'[x]==\[Lambda]*ArcSin[x]^n*y[x]^2+k*y[x]+\[Lambda]*b^2*x^(2*k)*ArcSin[x]^n,y[x],x
```

$$y(x) \to \sqrt{b^2} x^k \tan\left(\sqrt{b^2} \int_1^x \lambda \arcsin(K[1])^n K[1]^{k-1} dK[1] + c_1\right)$$

14.9 problem 9

Internal problem ID [10566]

Book: Handbook of exact solutions for ordinary differential equations. By Polyanin and Zaitsev. Second edition
Section: Chapter 1, section 1.2. Riccati Equation. subsection 1.2.7-1. Equations containing arcsine.
Problem number: 9.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_Riccati]`

$$y'x - \left(a\,x^{2m}y^2 + yx^n b + c\right)\arcsin\left(x\right)^m + yn = 0$$

✗ Solution by Maple

```
dsolve(x*diff(y(x),x)=(a*x^(2*m)*y(x)^2+b*x^n*y(x)+c)*arcsin(x)^m-n*y(x),y(x), singsol=all)
```

No solution found

✗ Solution by Mathematica
Time used: 0.0 (sec). Leaf size: 0

```
DSolve[x*y'[x]==(a*x^(2*m)*y[x]^2+b*x^n*y[x]+c)*ArcSin[x]^m-n*y[x],y[x],x,IncludeSingularSol
```

Not solved

15 Chapter 1, section 1.2. Riccati Equation. subsection 1.2.7-2. Equations containing arccosine.

15.1 problem 10 324
15.2 problem 11 325
15.3 problem 12 326
15.4 problem 13 327
15.5 problem 14 328
15.6 problem 15 329
15.7 problem 16 330
15.8 problem 17 331
15.9 problem 18 332

15.1 problem 10

Internal problem ID [10567]

Book: Handbook of exact solutions for ordinary differential equations. By Polyanin and Zaitsev. Second edition
Section: Chapter 1, section 1.2. Riccati Equation. subsection 1.2.7-2. Equations containing arccosine.
Problem number: 10.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_Riccati]`

$$y' - y^2 - \lambda \arccos(x)^n y = -a^2 + a\lambda \arccos(x)^n$$

✓ Solution by Maple
Time used: 0.0 (sec). Leaf size: 386

```
dsolve(diff(y(x),x)=y(x)^2+lambda*arccos(x)^n*y(x)-a^2+a*lambda*arccos(x)^n,y(x), singsol=al
```

Expression too large to display

✓ Solution by Mathematica
Time used: 8.046 (sec). Leaf size: 404

```
DSolve[y'[x]==y[x]^2+\[Lambda]*ArcCos[x]^n*y[x]-a^2+a*\[Lambda]*ArcCos[x]^n,y[x],x,IncludeSi
```

$$\text{Solve}\left[\int_1^x \frac{i \exp\left(\frac{1}{2}\lambda \arccos(K[1])^n \Gamma(n+1,-i\arccos(K[1]))(-i\arccos(K[1]))^{-n} + \frac{1}{2}\lambda(i\arccos(K[1]))^{-n} a}{n\lambda(a+y(x))}\right.$$
$$+\int_1^{y(x)} \left(-\int_1^x \left(\frac{i \exp\left(\frac{1}{2}\lambda \arccos(K[1])^n \Gamma(n+1,-i\arccos(K[1]))(-i\arccos(K[1]))^{-n} + \frac{1}{2}\lambda(i\arccos(K[1]\right.\right.\right.$$
$$\left.\left.\left.-\frac{i \exp\left(\frac{1}{2}\lambda \arccos(x)^n \Gamma(n+1,-i\arccos(x))(-i\arccos(x))^{-n} - 2ax + \frac{1}{2}\lambda(i\arccos(x))^{-n} \arccos(x)^n \Gamma(n+\right.}{n\lambda(a+K[2])^2}\right.\right.\right.$$

15.2 problem 11

Internal problem ID [10568]

Book: Handbook of exact solutions for ordinary differential equations. By Polyanin and Zaitsev. Second edition
Section: Chapter 1, section 1.2. Riccati Equation. subsection 1.2.7-2. Equations containing arccosine.
Problem number: 11.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_Riccati]`

$$\boxed{y' - y^2 - \lambda x \arccos(x)^n y = \arccos(x)^n \lambda}$$

✓ Solution by Maple
Time used: 0.0 (sec). Leaf size: 78

```
dsolve(diff(y(x),x)=y(x)^2+lambda*x*arccos(x)^n*y(x)+lambda*arccos(x)^n,y(x), singsol=all)
```

$$y(x) = \frac{e^{\int \frac{\arccos(x)^n \lambda x^2 - 2}{x} dx} x + \int e^{\int \frac{\arccos(x)^n \lambda x^2 - 2}{x} dx} dx - c_1}{\left(c_1 - \left(\int e^{\int \frac{\arccos(x)^n \lambda x^2 - 2}{x} dx} dx\right)\right) x}$$

✓ Solution by Mathematica
Time used: 5.617 (sec). Leaf size: 253

```
DSolve[y'[x]==y[x]^2+\[Lambda]*x*ArcCos[x]^n*y[x]+\[Lambda]*ArcCos[x]^n,y[x],x,IncludeSingul
```

$$y(x) \to -\frac{x \int_1^x \frac{\exp\left(2^{-n-3}\lambda \arccos(K[1])^n (\arccos(K[1])^2)^{-n} (\Gamma(n+1,2i\arccos(K[1]))(-i\arccos(K[1]))^n + (i\arccos(K[1]))^n \Gamma(n+1,-2i\arccos(K[1]\right.}{K[1]^2}}{x^2 \left(\int_1^x \frac{\exp\left(2^{-n-3}\lambda \arccos(K[1])^n (\arccos(K[1])^2)^{-n}\right.}\right.}$$

$$y(x) \to -\frac{1}{x}$$

15.3 problem 12

Internal problem ID [10569]

Book: Handbook of exact solutions for ordinary differential equations. By Polyanin and Zaitsev. Second edition
Section: Chapter 1, section 1.2. Riccati Equation. subsection 1.2.7-2. Equations containing arccosine.
Problem number: 12.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_Riccati]`

$$\boxed{y' + (k+1)\, x^k y^2 - \lambda \arccos(x)^n \left(x^{k+1} y - 1\right) = 0}$$

✓ Solution by Maple
Time used: 0.0 (sec). Leaf size: 180

```
dsolve(diff(y(x),x)=-(k+1)*x^k*y(x)^2+lambda*arccos(x)^n*(x^(k+1)*y(x)-1),y(x), singsol=all)
```

$$y(x) = \frac{x^{-1-k}\left(x^{1+k} e^{\int \frac{\arccos(x)^n x^{1+k} \lambda x - 2k - 2}{x} dx} + \left(\int x^k e^{\lambda\left(\int \arccos(x)^n x^{1+k} dx\right) - 2\left(\int \frac{1}{x} dx\right)(1+k)} dx\right) k + \int x^k e^{\lambda\left(\int \arccos(x)^n x^{1+k} dx\right) - 2\left(\int \frac{1}{x} dx\right)(1+k)} dx\right)}{\left(\int x^k e^{\lambda\left(\int \arccos(x)^n x^{1+k} dx\right) - 2\left(\int \frac{1}{x} dx\right)(1+k)} dx\right) k + \int x^k e^{\lambda\left(\int \arccos(x)^n x^{1+k} dx\right) - 2\left(\int \frac{1}{x} dx\right)(1+k)} dx}$$

✗ Solution by Mathematica
Time used: 0.0 (sec). Leaf size: 0

```
DSolve[y'[x]==-(k+1)*x^k*y[x]^2+\[Lambda]*ArcCos[x]^n*(x^(k+1)*y[x]-1),y[x],x,IncludeSingula
```

Not solved

15.4 problem 13

Internal problem ID [10570]

Book: Handbook of exact solutions for ordinary differential equations. By Polyanin and Zaitsev. Second edition
Section: Chapter 1, section 1.2. Riccati Equation. subsection 1.2.7-2. Equations containing arccosine.
Problem number: 13.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_Riccati]`

$$y' - \lambda \arccos(x)^n y^2 - ay = ab - b^2 \lambda \arccos(x)^n$$

✓ Solution by Maple
Time used: 0.016 (sec). Leaf size: 382

```
dsolve(diff(y(x),x)=lambda*arccos(x)^n*y(x)^2+a*y(x)+a*b-b^2*lambda*arccos(x)^n,y(x), singso
```

Expression too large to display

✓ Solution by Mathematica
Time used: 9.288 (sec). Leaf size: 420

```
DSolve[y'[x]==\[Lambda]*ArcCos[x]^n*y[x]^2+a*y[x]+a*b-b^2*\[Lambda]*ArcCos[x]^n,y[x],x,Inclu
```

$$\text{Solve}\left[\int_1^x \right.$$
$$-\frac{i \exp\left(-b\lambda \arccos(K[1])^n \Gamma(n+1,-i\arccos(K[1]))(-i\arccos(K[1]))^{-n} - b\lambda(i\arccos(K[1]))^{-n}\arccos(K\right.}{an\lambda(b+y(x))}$$
$$+\int_1^{y(x)} \left(\frac{i \exp\left(-b\lambda \arccos(x)^n \Gamma(n+1,-i\arccos(x))(-i\arccos(x))^{-n} + ax - b\lambda(i\arccos(x))^{-n}\arccos(x)\right.}{an\lambda(b+K[2])^2}\right.$$
$$-\int_1^x \left(\frac{i \exp\left(-b\lambda \arccos(K[1])^n \Gamma(n+1,-i\arccos(K[1]))(-i\arccos(K[1]))^{-n} - b\lambda(i\arccos(K[1]))^{-n}\arc\right.}{an\lambda(b+K\right.}$$

15.5 problem 14

Internal problem ID [10571]

Book: Handbook of exact solutions for ordinary differential equations. By Polyanin and Zaitsev. Second edition
Section: Chapter 1, section 1.2. Riccati Equation. subsection 1.2.7-2. Equations containing arccosine.
Problem number: 14.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_Riccati]`

$$y' - \lambda \arccos(x)^n y^2 + b\lambda x^m \arccos(x)^n y = bm x^{m-1}$$

✗ Solution by Maple

```
dsolve(diff(y(x),x)=lambda*arccos(x)^n*y(x)^2-b*lambda*x^m*arccos(x)^n*y(x)+b*m*x^(m-1),y(x)
```

No solution found

✗ Solution by Mathematica
Time used: 0.0 (sec). Leaf size: 0

```
DSolve[y'[x]==\[Lambda]*ArcCos[x]^n*y[x]^2-b*\[Lambda]*x^m*ArcCos[x]^n*y[x]+b*m*x^(m-1),y[x]
```

Not solved

15.6 problem 15

Internal problem ID [10572]

Book: Handbook of exact solutions for ordinary differential equations. By Polyanin and Zaitsev. Second edition
Section: Chapter 1, section 1.2. Riccati Equation. subsection 1.2.7-2. Equations containing arccosine.
Problem number: 15.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_Riccati]`

$$y' - \lambda \arccos(x)^n y^2 = \beta m x^{m-1} - \lambda \beta^2 x^{2m} \arccos(x)^n$$

✗ Solution by Maple

```
dsolve(diff(y(x),x)=lambda*arccos(x)^n*y(x)^2+beta*m*x^(m-1)-lambda*beta^2*x^(2*m)*arccos(x)
```

No solution found

✗ Solution by Mathematica
Time used: 0.0 (sec). Leaf size: 0

```
DSolve[y'[x]==\[Lambda]*ArcCos[x]^n*y[x]^2+\[Beta]*m*x^(m-1)-\[Lambda]*\[Beta]^2*x^(2*m)*Arc
```

Not solved

15.7 problem 16

Internal problem ID [10573]

Book: Handbook of exact solutions for ordinary differential equations. By Polyanin and Zaitsev. Second edition
Section: Chapter 1, section 1.2. Riccati Equation. subsection 1.2.7-2. Equations containing arccosine.
Problem number: 16.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[[_1st_order, '_with_symmetry_[F(x),G(x)]'], _Riccati]`

$$y' - \lambda \arccos(x)^n \left(y - a\,x^m - b\right)^2 = am\,x^{m-1}$$

✓ Solution by Maple
Time used: 0.0 (sec). Leaf size: 165

```
dsolve(diff(y(x),x)=lambda*arccos(x)^n*(y(x)-a*x^m-b)^2+a*m*x^(m-1),y(x), singsol=all)
```

$$y(x) = \frac{\lambda(a\,x^m+b)\left((n+2)\,\text{LommelS1}\left(n+\frac{1}{2},\frac{1}{2},\arccos(x)\right) - \text{LommelS1}\left(n+\frac{3}{2},\frac{3}{2},\arccos(x)\right)\arccos(x) + a\right.}{\lambda\left((n+2)\,\text{LommelS1}\left(n+\frac{1}{2},\frac{1}{2},\arccos(x)\right) - \text{LommelS1}\left(n+\frac{3}{2},\frac{3}{2},\arccos(x)\right)\arccos\right.}$$

✓ Solution by Mathematica
Time used: 4.776 (sec). Leaf size: 86

```
DSolve[y'[x]==\[Lambda]*ArcCos[x]^n*(y[x]-a*x^m-b)^2+a*m*x^(m-1),y[x],x,IncludeSingularSolut
```

$$y(x) \to ax^m + \frac{1}{-\frac{1}{2}\lambda \arccos(x)^n(-i\arccos(x))^{-n}\Gamma(n+1,-i\arccos(x)) - \frac{1}{2}\lambda(i\arccos(x))^{-n}\arccos(x)^n\Gamma(n+1,i\arccos(x))} + b$$

$$y(x) \to ax^m + b$$

15.8 problem 17

Internal problem ID [10574]

Book: Handbook of exact solutions for ordinary differential equations. By Polyanin and Zaitsev. Second edition
Section: Chapter 1, section 1.2. Riccati Equation. subsection 1.2.7-2. Equations containing arccosine.
Problem number: 17.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_Riccati]`

$$y'x - \lambda \arccos(x)^n y^2 - ky = \lambda b^2 x^{2k} \arccos(x)^n$$

✓ Solution by Maple
Time used: 0.047 (sec). Leaf size: 29

```
dsolve(x*diff(y(x),x)=lambda*arccos(x)^n*y(x)^2+k*y(x)+lambda*b^2*x^(2*k)*arccos(x)^n,y(x),
```

$$y(x) = -\tan\left(-\lambda b\left(\int x^{-1+k} \arccos(x)^n \, dx\right) + c_1\right) b x^k$$

✓ Solution by Mathematica
Time used: 2.128 (sec). Leaf size: 48

```
DSolve[x*y'[x]==\[Lambda]*ArcCos[x]^n*y[x]^2+k*y[x]+\[Lambda]*b^2*x^(2*k)*ArcCos[x]^n,y[x],x
```

$$y(x) \to \sqrt{b^2} x^k \tan\left(\sqrt{b^2} \int_1^x \lambda \arccos(K[1])^n K[1]^{k-1} dK[1] + c_1\right)$$

15.9 problem 18

Internal problem ID [10575]

Book: Handbook of exact solutions for ordinary differential equations. By Polyanin and Zaitsev. Second edition
Section: Chapter 1, section 1.2. Riccati Equation. subsection 1.2.7-2. Equations containing arccosine.
Problem number: 18.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_Riccati]`

$$\boxed{y'x - \left(a\,x^{2m}y^2 + yx^n b + c\right)\arccos\left(x\right)^m + yn = 0}$$

✗ Solution by Maple

```
dsolve(x*diff(y(x),x)=(a*x^(2*m)*y(x)^2+b*x^n*y(x)+c)*arccos(x)^m-n*y(x),y(x), singsol=all)
```

No solution found

✗ Solution by Mathematica
Time used: 0.0 (sec). Leaf size: 0

```
DSolve[x*y'[x]==(a*x^(2*m)*y[x]^2+b*x^n*y[x]+c)*ArcCos[x]^m-n*y[x],y[x],x,IncludeSingularSol
```

Not solved

16 Chapter 1, section 1.2. Riccati Equation. subsection 1.2.7-3. Equations containing arctangent.

16.1 problem 19 334
16.2 problem 20 335
16.3 problem 21 336
16.4 problem 22 337
16.5 problem 23 338
16.6 problem 24 339
16.7 problem 25 340
16.8 problem 26 341
16.9 problem 27 342

16.1 problem 19

Internal problem ID [10576]

Book: Handbook of exact solutions for ordinary differential equations. By Polyanin and Zaitsev. Second edition
Section: Chapter 1, section 1.2. Riccati Equation. subsection 1.2.7-3. Equations containing arctangent.
Problem number: 19.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_Riccati]`

$$y' - y^2 - \lambda \arctan(x)^n y = -a^2 + a\lambda \arctan(x)^n$$

✓ Solution by Maple
Time used: 0.015 (sec). Leaf size: 71

```
dsolve(diff(y(x),x)=y(x)^2+lambda*arctan(x)^n*y(x)-a^2+a*lambda*arctan(x)^n,y(x), singsol=al
```

$$y(x) = \frac{-c_1 a - a\left(\int \mathrm{e}^{-\left(\int (-\arctan(x)^n \lambda + 2a)dx\right)} dx\right) - \mathrm{e}^{-\left(\int (-\arctan(x)^n \lambda + 2a)dx\right)}}{c_1 + \int \mathrm{e}^{-\left(\int (-\arctan(x)^n \lambda + 2a)dx\right)} dx}$$

✓ Solution by Mathematica
Time used: 7.862 (sec). Leaf size: 210

```
DSolve[y'[x]==y[x]^2+\[Lambda]*ArcTan[x]^n*y[x]-a^2+a*\[Lambda]*ArcTan[x]^n,y[x],x,IncludeSi
```

$$\text{Solve}\left[\int_1^x \frac{\exp\left(-\int_1^{K[2]} (2a - \lambda \arctan(K[1])^n)\, dK[1]\right)(-\lambda \arctan(K[2])^n + a - y(x))}{n\lambda(a + y(x))} dK[2]\right.$$
$$+ \int_1^{y(x)} \left(\frac{\exp\left(-\int_1^x (2a - \lambda \arctan(K[1])^n)\, dK[1]\right)}{n\lambda(a + K[3])^2}\right.$$
$$\left. - \int_1^x \left(-\frac{\exp\left(-\int_1^{K[2]} (2a - \lambda \arctan(K[1])^n)\, dK[1]\right)(-\lambda \arctan(K[2])^n + a - K[3])}{n\lambda(a + K[3])^2} - \frac{\exp\left(-\int_1^{K[2]} (2a\right.\right.\right.$$
$$\left.\left.\left. n\right.\right.\right.$$

16.2 problem 20

Internal problem ID [10577]

Book: Handbook of exact solutions for ordinary differential equations. By Polyanin and Zaitsev. Second edition
Section: Chapter 1, section 1.2. Riccati Equation. subsection 1.2.7-3. Equations containing arctangent.
Problem number: 20.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_Riccati]`

$$\boxed{y' - y^2 - \lambda x \arctan(x)^n y = \arctan(x)^n \lambda}$$

✓ Solution by Maple
Time used: 0.0 (sec). Leaf size: 78

```
dsolve(diff(y(x),x)=y(x)^2+lambda*x*arctan(x)^n*y(x)+lambda*arctan(x)^n,y(x), singsol=all)
```

$$y(x) = \frac{e^{\int \frac{\arctan(x)^n \lambda x^2 - 2}{x} dx} x + \int e^{\int \frac{\arctan(x)^n \lambda x^2 - 2}{x} dx} dx - c_1}{\left(c_1 - \left(\int e^{\int \frac{\arctan(x)^n \lambda x^2 - 2}{x} dx} dx\right)\right) x}$$

✓ Solution by Mathematica
Time used: 7.063 (sec). Leaf size: 120

```
DSolve[y'[x]==y[x]^2+\[Lambda]*x*ArcTan[x]^n*y[x]+\[Lambda]*ArcTan[x]^n,y[x],x,IncludeSingul
```

$$y(x) \to -\frac{\exp\left(-\int_1^x -\lambda \arctan(K[1])^n K[1] dK[1]\right) + x \int_1^x \frac{\exp\left(-\int_1^{K[2]} -\lambda \arctan(K[1])^n K[1] dK[1]\right)}{K[2]^2} dK[2] + c_1 x}{x^2 \left(\int_1^x \frac{\exp\left(-\int_1^{K[2]} -\lambda \arctan(K[1])^n K[1] dK[1]\right)}{K[2]^2} dK[2] + c_1\right)}$$

$$y(x) \to -\frac{1}{x}$$

16.3 problem 21

Internal problem ID [10578]

Book: Handbook of exact solutions for ordinary differential equations. By Polyanin and Zaitsev. Second edition
Section: Chapter 1, section 1.2. Riccati Equation. subsection 1.2.7-3. Equations containing arctangent.
Problem number: 21.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_Riccati]`

$$\boxed{y' + (k+1)\, x^k y^2 - \lambda \arctan(x)^n \left(x^{k+1} y - 1\right) = 0}$$

✓ Solution by Maple
Time used: 0.016 (sec). Leaf size: 180

```
dsolve(diff(y(x),x)=-(k+1)*x^k*y(x)^2+lambda*arctan(x)^n*(x^(k+1)*y(x)-1),y(x), singsol=all)
```

$$y(x) = \frac{x^{-1-k}\left(\left(\int x^k \mathrm{e}^{\lambda\left(\int \arctan(x)^n x^{1+k} dx\right)-2\left(\int \frac{1}{x} dx\right)(1+k)} dx\right) k + x^{1+k} \mathrm{e}^{\int \frac{x^{1+k} \arctan(x)^n x \lambda - 2k - 2}{x} dx} + \int x^k \mathrm{e}^{\lambda\left(\int \arctan(x)^n x^{1+k}\right.}\right.}{\left(\int x^k \mathrm{e}^{\lambda\left(\int \arctan(x)^n x^{1+k} dx\right)-2\left(\int \frac{1}{x} dx\right)(1+k)} dx\right) k + \int x^k \mathrm{e}^{\lambda\left(\int \arctan(x)^n x^{1+k} dx\right)-2\left(\int \frac{1}{x} dx\right)(1+k)} dx}$$

✗ Solution by Mathematica
Time used: 0.0 (sec). Leaf size: 0

```
DSolve[y'[x]==-(k+1)*x^k*y[x]^2+\[Lambda]*ArcTan[x]^n*(x^(k+1)*y[x]-1),y[x],x,IncludeSingula
```

Not solved

16.4 problem 22

Internal problem ID [10579]

Book: Handbook of exact solutions for ordinary differential equations. By Polyanin and Zaitsev. Second edition
Section: Chapter 1, section 1.2. Riccati Equation. subsection 1.2.7-3. Equations containing arctangent.
Problem number: 22.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_Riccati]`

$$y' - \lambda \arctan(x)^n y^2 - ay = ab - b^2 \lambda \arctan(x)^n$$

✓ Solution by Maple
Time used: 0.0 (sec). Leaf size: 87

```
dsolve(diff(y(x),x)=lambda*arctan(x)^n*y(x)^2+a*y(x)+a*b-b^2*lambda*arctan(x)^n,y(x), singso
```

$$y(x) = \frac{-b\lambda\left(\int \arctan(x)^n \mathrm{e}^{-\left(\int (2\arctan(x)^n \lambda b - a)dx\right)} dx\right) - c_1 b - \mathrm{e}^{-\left(\int (2\arctan(x)^n \lambda b - a)dx\right)}}{c_1 + \lambda\left(\int \arctan(x)^n \mathrm{e}^{-\left(\int (2\arctan(x)^n \lambda b - a)dx\right)} dx\right)}$$

✓ Solution by Mathematica
Time used: 10.998 (sec). Leaf size: 240

```
DSolve[y'[x]==\[Lambda]*ArcTan[x]^n*y[x]^2+a*y[x]+a*b-b^2*\[Lambda]*ArcTan[x]^n,y[x],x,Inclu
```

$$\text{Solve}\left[\int_1^x \frac{\exp\left(-\int_1^{K[2]} (2b\lambda \arctan(K[1])^n - a)\, dK[1]\right)(-b\lambda \arctan(K[2])^n + \lambda y(x) \arctan(K[2])^n + a)}{an\lambda(b + y(x))} a \right.$$
$$+ \int_1^{y(x)} \left(- \int_1^x \left(\frac{\exp\left(-\int_1^{K[2]} (2b\lambda \arctan(K[1])^n - a)\, dK[1]\right) \arctan(K[2])^n}{an(b + K[3])} - \frac{\exp\left(-\int_1^{K[2]} (2b\lambda \arctan(\right.}{}\right.$$
$$\left.\left. - \frac{\exp\left(-\int_1^x (2b\lambda \arctan(K[1])^n - a)\, dK[1]\right)}{an\lambda(b + K[3])^2} \right) dK[3] = c_1, y(x) \right]$$

16.5 problem 23

Internal problem ID [10580]

Book: Handbook of exact solutions for ordinary differential equations. By Polyanin and Zaitsev. Second edition
Section: Chapter 1, section 1.2. Riccati Equation. subsection 1.2.7-3. Equations containing arctangent.
Problem number: 23.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_Riccati]`

$$y' - \lambda \arctan(x)^n y^2 + b\lambda x^m \arctan(x)^n y = bm x^{m-1}$$

✗ Solution by Maple

```
dsolve(diff(y(x),x)=lambda*arctan(x)^n*y(x)^2-b*lambda*x^m*arctan(x)^n*y(x)+b*m*x^(m-1),y(x)
```

No solution found

✗ Solution by Mathematica
Time used: 0.0 (sec). Leaf size: 0

```
DSolve[y'[x]==\[Lambda]*ArcTan[x]^n*y[x]^2-b*\[Lambda]*x^m*ArcTan[x]^n*y[x]+b*m*x^(m-1),y[x]
```

Not solved

16.6 problem 24

Internal problem ID [10581]

Book: Handbook of exact solutions for ordinary differential equations. By Polyanin and Zaitsev. Second edition
Section: Chapter 1, section 1.2. Riccati Equation. subsection 1.2.7-3. Equations containing arctangent.
Problem number: 24.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_Riccati]`

$$\boxed{y' - \lambda \arctan(x)^n y^2 = \beta m x^{m-1} - \lambda \beta^2 x^{2m} \arctan(x)^n}$$

✗ Solution by Maple

```
dsolve(diff(y(x),x)=lambda*arctan(x)^n*y(x)^2+beta*m*x^(m-1)-lambda*beta^2*x^(2*m)*arctan(x)
```

No solution found

✗ Solution by Mathematica
Time used: 0.0 (sec). Leaf size: 0

```
DSolve[y'[x]==\[Lambda]*ArcTan[x]^n*y[x]^2+\[Beta]*m*x^(m-1)-\[Lambda]*\[Beta]^2*x^(2*m)*Arc
```

Not solved

16.7 problem 25

Internal problem ID [10582]

Book: Handbook of exact solutions for ordinary differential equations. By Polyanin and Zaitsev. Second edition
Section: Chapter 1, section 1.2. Riccati Equation. subsection 1.2.7-3. Equations containing arctangent.
Problem number: 25.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[[_1st_order, '_with_symmetry_[F(x),G(x)]'], _Riccati]`

$$\boxed{y' - \lambda \arctan(x)^n (y - a x^m - b)^2 = a m x^{m-1}}$$

✓ Solution by Maple
Time used: 0.015 (sec). Leaf size: 24

```
dsolve(diff(y(x),x)=lambda*arctan(x)^n*(y(x)-a*x^m-b)^2+a*m*x^(m-1),y(x), singsol=all)
```

$$y(x) = a x^m + b + \frac{1}{c_1 - \lambda \left(\int \arctan(x)^n \, dx \right)}$$

✓ Solution by Mathematica
Time used: 2.089 (sec). Leaf size: 44

```
DSolve[y'[x]==\[Lambda]*ArcTan[x]^n*(y[x]-a*x^m-b)^2+a*m*x^(m-1),y[x],x,IncludeSingularSolut
```

$$y(x) \to \frac{1}{-\int_1^x \lambda \arctan(K[2])^n dK[2] + c_1} + a x^m + b$$
$$y(x) \to a x^m + b$$

16.8 problem 26

Internal problem ID [10583]

Book: Handbook of exact solutions for ordinary differential equations. By Polyanin and Zaitsev. Second edition
Section: Chapter 1, section 1.2. Riccati Equation. subsection 1.2.7-3. Equations containing arctangent.
Problem number: 26.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_Riccati]`

$$y'x - \lambda \arctan(x)^n y^2 - ky = \lambda b^2 x^{2k} \arctan(x)^n$$

✓ Solution by Maple
Time used: 0.031 (sec). Leaf size: 29

```
dsolve(x*diff(y(x),x)=lambda*arctan(x)^n*y(x)^2+k*y(x)+lambda*b^2*x^(2*k)*arctan(x)^n,y(x),
```

$$y(x) = -\tan\left(-\lambda b\left(\int x^{-1+k} \arctan(x)^n \, dx\right) + c_1\right) b\, x^k$$

✓ Solution by Mathematica
Time used: 1.992 (sec). Leaf size: 48

```
DSolve[x*y'[x]==\[Lambda]*ArcTan[x]^n*y[x]^2+k*y[x]+\[Lambda]*b^2*x^(2*k)*ArcTan[x]^n,y[x],x
```

$$y(x) \to \sqrt{b^2} x^k \tan\left(\sqrt{b^2} \int_1^x \lambda \arctan(K[1])^n K[1]^{k-1} dK[1] + c_1\right)$$

16.9 problem 27

Internal problem ID [10584]

Book: Handbook of exact solutions for ordinary differential equations. By Polyanin and Zaitsev. Second edition
Section: Chapter 1, section 1.2. Riccati Equation. subsection 1.2.7-3. Equations containing arctangent.
Problem number: 27.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_Riccati]`

$$\boxed{y'x - \left(a\,x^{2m}y^2 + yx^n b + c\right)\arctan\left(x\right)^m + yn = 0}$$

✗ Solution by Maple

```
dsolve(x*diff(y(x),x)=(a*x^(2*m)*y(x)^2+b*x^n*y(x)+c)*arctan(x)^m-n*y(x),y(x), singsol=all)
```

No solution found

✗ Solution by Mathematica
Time used: 0.0 (sec). Leaf size: 0

```
DSolve[x*y'[x]==(a*x^(2*m)*y[x]^2+b*x^n*y[x]+c)*ArcTan[x]^m-n*y[x],y[x],x,IncludeSingularSol
```

Not solved

17 Chapter 1, section 1.2. Riccati Equation. subsection 1.2.7-4. Equations containing arccotangent.

17.1 problem 28 344

17.1 problem 28

Internal problem ID [10585]

Book: Handbook of exact solutions for ordinary differential equations. By Polyanin and Zaitsev. Second edition
Section: Chapter 1, section 1.2. Riccati Equation. subsection 1.2.7-4. Equations containing arccotangent.
Problem number: 28.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_Riccati]`

$$\boxed{y' - y^2 - \lambda \operatorname{arccot}(x)^n y = -a^2 + a\lambda \operatorname{arccot}(x)^n}$$

✓ Solution by Maple
Time used: 0.0 (sec). Leaf size: 71

```
dsolve(diff(y(x),x)=y(x)^2+lambda*arccot(x)^n*y(x)-a^2+a*lambda*arccot(x)^n,y(x), singsol=al
```

$$y(x) = \frac{-c_1 a - a\left(\int \mathrm{e}^{-\left(\int (-\operatorname{arccot}(x)^n \lambda + 2a)dx\right)} dx\right) - \mathrm{e}^{-\left(\int (-\operatorname{arccot}(x)^n \lambda + 2a)dx\right)}}{c_1 + \int \mathrm{e}^{-\left(\int (-\operatorname{arccot}(x)^n \lambda + 2a)dx\right)} dx}$$

✓ Solution by Mathematica

Time used: 8.548 (sec). Leaf size: 210

```
DSolve[y'[x]==y[x]^2+\[Lambda]*ArcCot[x]^n*y[x]-a^2+a*\[Lambda]*ArcCot[x]^n,y[x],x,IncludeSi
```

$$\text{Solve}\left[\int_1^x -\frac{\exp\left(-\int_1^{K[2]}\left(2a-\lambda\cot^{-1}(K[1])^n\right)dK[1]\right)\left(-\lambda\cot^{-1}(K[2])^n+a-y(x)\right)}{n\lambda(a+y(x))}dK[2]\right.$$
$$+\int_1^{y(x)}\left(-\int_1^x\left(\frac{\exp\left(-\int_1^{K[2]}\left(2a-\lambda\cot^{-1}(K[1])^n\right)dK[1]\right)\left(-\lambda\cot^{-1}(K[2])^n+a-K[3]\right)}{n\lambda(a+K[3])^2}+\frac{\exp\left(-\int_1^{K[}\right.\right.\right.$$
$$\left.\left.\left.-\frac{\exp\left(-\int_1^x\left(2a-\lambda\cot^{-1}(K[1])^n\right)dK[1]\right)}{n\lambda(a+K[3])^2}\right)dK[3]=c_1,y(x)\right]$$

18 Chapter 1, section 1.2. Riccati Equation. subsection 1.2.7-3. Equations containing arctangent.

18.1 problem 29 347
18.2 problem 30 348
18.3 problem 31 349
18.4 problem 32 351
18.5 problem 33 352
18.6 problem 34 353
18.7 problem 35 354
18.8 problem 36 355

18.1 problem 29

Internal problem ID [10586]

Book: Handbook of exact solutions for ordinary differential equations. By Polyanin and Zaitsev. Second edition
Section: Chapter 1, section 1.2. Riccati Equation. subsection 1.2.7-3. Equations containing arctangent.
Problem number: 29.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_Riccati]`

$$\boxed{y' - y^2 - \lambda x \operatorname{arccot}(x)^n y = \operatorname{arccot}(x)^n \lambda}$$

✓ Solution by Maple
Time used: 0.0 (sec). Leaf size: 78

```
dsolve(diff(y(x),x)=y(x)^2+lambda*x*arccot(x)^n*y(x)+lambda*arccot(x)^n,y(x), singsol=all)
```

$$y(x) = \frac{e^{\int \frac{x^2 \operatorname{arccot}(x)^n \lambda - 2}{x} dx} x + \int e^{\int \frac{x^2 \operatorname{arccot}(x)^n \lambda - 2}{x} dx} dx - c_1}{\left(c_1 - \left(\int e^{\int \frac{x^2 \operatorname{arccot}(x)^n \lambda - 2}{x} dx} dx\right)\right) x}$$

✓ Solution by Mathematica
Time used: 7.258 (sec). Leaf size: 120

```
DSolve[y'[x]==y[x]^2+\[Lambda]*x*ArcCot[x]^n*y[x]+\[Lambda]*ArcCot[x]^n,y[x],x,IncludeSingul
```

$$y(x) \to -\frac{\exp\left(-\int_1^x -\lambda \cot^{-1}(K[1])^n K[1] dK[1]\right) + x \int_1^x \frac{\exp\left(-\int_1^{K[2]} -\lambda \cot^{-1}(K[1])^n K[1] dK[1]\right)}{K[2]^2} dK[2] + c_1 x}{x^2 \left(\int_1^x \frac{\exp\left(-\int_1^{K[2]} -\lambda \cot^{-1}(K[1])^n K[1] dK[1]\right)}{K[2]^2} dK[2] + c_1\right)}$$

$$y(x) \to -\frac{1}{x}$$

18.2 problem 30

Internal problem ID [10587]

Book: Handbook of exact solutions for ordinary differential equations. By Polyanin and Zaitsev. Second edition
Section: Chapter 1, section 1.2. Riccati Equation. subsection 1.2.7-3. Equations containing arctangent.
Problem number: 30.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_Riccati]`

$$y' + (k+1)\, x^k y^2 - \lambda \operatorname{arccot}(x)^n \left(x^{k+1} y - 1\right) = 0$$

✓ Solution by Maple
Time used: 0.016 (sec). Leaf size: 184

```
dsolve(diff(y(x),x)=-(k+1)*x^k*y(x)^2+lambda*arccot(x)^n*(x^(k+1)*y(x)-1),y(x), singsol=all)
```

$$y(x) = \frac{x^{-1-k}\left(\left(\int x^k \mathrm{e}^{\lambda\left(\int x^{1+k}\operatorname{arccot}(x)^n dx\right)-2\left(\int \frac{1}{x}dx\right)(1+k)}dx\right)k + x^{1+k}\mathrm{e}^{\int \frac{x^{1+k}\operatorname{arccot}(x)^n x\lambda-2k-2}{x}dx} + \int x^k \mathrm{e}^{\lambda\left(\int x^{1+k}\operatorname{arccot}(x)^n\right.}\right.}{\left(\int x^k \mathrm{e}^{\lambda\left(\int x^{1+k}\operatorname{arccot}(x)^n dx\right)-2\left(\int \frac{1}{x}dx\right)(1+k)}dx\right)k + \int x^k \mathrm{e}^{\lambda\left(\int x^{1+k}\operatorname{arccot}(x)^n dx\right)-2\left(\int \frac{1}{x}dx\right)(1+k)}dx}$$

✗ Solution by Mathematica
Time used: 0.0 (sec). Leaf size: 0

```
DSolve[y'[x]==-(k+1)*x^k*y[x]^2+\[Lambda]*ArcCot[x]^n*(x^(k+1)*y[x]-1),y[x],x,IncludeSingula
```

Not solved

18.3 problem 31

Internal problem ID [10588]

Book: Handbook of exact solutions for ordinary differential equations. By Polyanin and Zaitsev. Second edition
Section: Chapter 1, section 1.2. Riccati Equation. subsection 1.2.7-3. Equations containing arctangent.
Problem number: 31.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_Riccati]`

$$\boxed{y' - \lambda \operatorname{arccot}(x)^n y^2 - ay = ab - b^2 \lambda \operatorname{arccot}(x)^n}$$

✓ Solution by Maple
Time used: 0.0 (sec). Leaf size: 87

```
dsolve(diff(y(x),x)=lambda*arccot(x)^n*y(x)^2+a*y(x)+a*b-b^2*lambda*arccot(x)^n,y(x), singso
```

$$y(x) = \frac{-b\lambda\left(\int \operatorname{arccot}(x)^n \mathrm{e}^{-\left(\int (2\operatorname{arccot}(x)^n \lambda b - a)dx\right)} dx\right) - c_1 b - \mathrm{e}^{-\left(\int (2\operatorname{arccot}(x)^n \lambda b - a)dx\right)}}{c_1 + \lambda\left(\int \operatorname{arccot}(x)^n \mathrm{e}^{-\left(\int (2\operatorname{arccot}(x)^n \lambda b - a)dx\right)} dx\right)}$$

✓ Solution by Mathematica

Time used: 11.807 (sec). Leaf size: 240

```
DSolve[y'[x]==\[Lambda]*ArcCot[x]^n*y[x]^2+a*y[x]+a*b-b^2*\[Lambda]*ArcCot[x]^n,y[x],x,Inclu
```

$$\text{Solve}\left[\int_1^x -\frac{\exp\left(-\int_1^{K[2]}\left(2b\lambda\cot^{-1}(K[1])^n-a\right)dK[1]\right)\left(-b\lambda\cot^{-1}(K[2])^n+\lambda y(x)\cot^{-1}(K[2])^n+a\right)}{an\lambda(b+y(x))}dK[2]\right.$$

$$+\int_1^{y(x)}\left(\frac{\exp\left(-\int_1^x\left(2b\lambda\cot^{-1}(K[1])^n-a\right)dK[1]\right)}{an\lambda(b+K[3])^2}\right.$$

$$\left.-\int_1^x\left(\frac{\exp\left(-\int_1^{K[2]}\left(2b\lambda\cot^{-1}(K[1])^n-a\right)dK[1]\right)\left(-b\lambda\cot^{-1}(K[2])^n+\lambda K[3]\cot^{-1}(K[2])^n+a\right)}{an\lambda(b+K[3])^2}-\frac{\exp\right.\right.$$

18.4 problem 32

Internal problem ID [10589]

Book: Handbook of exact solutions for ordinary differential equations. By Polyanin and Zaitsev. Second edition
Section: Chapter 1, section 1.2. Riccati Equation. subsection 1.2.7-3. Equations containing arctangent.
Problem number: 32.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_Riccati]`

$$y' - \lambda \operatorname{arccot}(x)^n y^2 + b\lambda x^m \operatorname{arccot}(x)^n y = bm x^{m-1}$$

✗ Solution by Maple

```
dsolve(diff(y(x),x)=lambda*arccot(x)^n*y(x)^2-b*lambda*x^m*arccot(x)^n*y(x)+b*m*x^(m-1),y(x)
```

No solution found

✗ Solution by Mathematica
Time used: 0.0 (sec). Leaf size: 0

```
DSolve[y'[x]==\[Lambda]*ArcCot[x]^n*y[x]^2-b*\[Lambda]*x^m*ArcCot[x]^n*y[x]+b*m*x^(m-1),y[x]
```

Not solved

18.5 problem 33

Internal problem ID [10590]

Book: Handbook of exact solutions for ordinary differential equations. By Polyanin and Zaitsev. Second edition
Section: Chapter 1, section 1.2. Riccati Equation. subsection 1.2.7-3. Equations containing arctangent.
Problem number: 33.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_Riccati]`

$$y' - \lambda \operatorname{arccot}(x)^n y^2 = \beta m x^{m-1} - \lambda \beta^2 x^{2m} \operatorname{arccot}(x)^n$$

✗ Solution by Maple

```
dsolve(diff(y(x),x)=lambda*arccot(x)^n*y(x)^2+beta*m*x^(m-1)-lambda*beta^2*x^(2*m)*arccot(x)
```

No solution found

✗ Solution by Mathematica
Time used: 0.0 (sec). Leaf size: 0

```
DSolve[y'[x]==\[Lambda]*ArcCot[x]^n*y[x]^2+\[Beta]*m*x^(m-1)-\[Lambda]*\[Beta]^2*x^(2*m)*Arc
```

Not solved

18.6 problem 34

Internal problem ID [10591]

Book: Handbook of exact solutions for ordinary differential equations. By Polyanin and Zaitsev. Second edition
Section: Chapter 1, section 1.2. Riccati Equation. subsection 1.2.7-3. Equations containing arctangent.
Problem number: 34.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[[_1st_order, '_with_symmetry_[F(x),G(x)]'], _Riccati]`

$$\boxed{y' - \lambda \operatorname{arccot}(x)^n \left(y - a\,x^m - b\right)^2 = am\,x^{m-1}}$$

✓ Solution by Maple
Time used: 0.0 (sec). Leaf size: 24

```
dsolve(diff(y(x),x)=lambda*arccot(x)^n*(y(x)-a*x^m-b)^2+a*m*x^(m-1),y(x), singsol=all)
```

$$y(x) = a\,x^m + b + \frac{1}{c_1 - \lambda \left(\int \operatorname{arccot}(x)^n\,dx\right)}$$

✓ Solution by Mathematica
Time used: 2.259 (sec). Leaf size: 44

```
DSolve[y'[x]==\[Lambda]*ArcCot[x]^n*(y[x]-a*x^m-b)^2+a*m*x^(m-1),y[x],x,IncludeSingularSolut
```

$$y(x) \to \frac{1}{-\int_1^x \lambda \cot^{-1}(K[2])^n dK[2] + c_1} + ax^m + b$$
$$y(x) \to ax^m + b$$

18.7 problem 35

Internal problem ID [10592]

Book: Handbook of exact solutions for ordinary differential equations. By Polyanin and Zaitsev. Second edition
Section: Chapter 1, section 1.2. Riccati Equation. subsection 1.2.7-3. Equations containing arctangent.
Problem number: 35.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_Riccati]`

$$y'x - \lambda \operatorname{arccot}(x)^n y^2 - ky = \lambda b^2 x^{2k} \operatorname{arccot}(x)^n$$

✓ Solution by Maple
Time used: 0.031 (sec). Leaf size: 29

```
dsolve(x*diff(y(x),x)=lambda*arccot(x)^n*y(x)^2+k*y(x)+lambda*b^2*x^(2*k)*arccot(x)^n,y(x),
```

$$y(x) = -\tan\left(-\lambda b\left(\int x^{-1+k} \operatorname{arccot}(x)^n \, dx\right) + c_1\right) b x^k$$

✓ Solution by Mathematica
Time used: 2.591 (sec). Leaf size: 48

```
DSolve[x*y'[x]==\[Lambda]*ArcCot[x]^n*y[x]^2+k*y[x]+\[Lambda]*b^2*x^(2*k)*ArcCot[x]^n,y[x],x
```

$$y(x) \to \sqrt{b^2} x^k \tan\left(\sqrt{b^2} \int_1^x \lambda \cot^{-1}(K[1])^n K[1]^{k-1} dK[1] + c_1\right)$$

18.8 problem 36

Internal problem ID [10593]

Book: Handbook of exact solutions for ordinary differential equations. By Polyanin and Zaitsev. Second edition
Section: Chapter 1, section 1.2. Riccati Equation. subsection 1.2.7-3. Equations containing arctangent.
Problem number: 36.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_Riccati]`

$$\boxed{y'x - \left(a\,x^{2m}y^2 + yx^n b + c\right)\operatorname{arccot}(x)^m + yn = 0}$$

✗ Solution by Maple

```
dsolve(x*diff(y(x),x)=(a*x^(2*m)*y(x)^2+b*x^n*y(x)+c)*arccot(x)^m-n*y(x),y(x), singsol=all)
```

No solution found

✗ Solution by Mathematica
Time used: 0.0 (sec). Leaf size: 0

```
DSolve[x*y'[x]==(a*x^(2*m)*y[x]^2+b*x^n*y[x]+c)*ArcCot[x]^m-n*y[x],y[x],x,IncludeSingularSol
```

Not solved

19 Chapter 1, section 1.2. Riccati Equation. subsection 1.2.8-1. Equations containing arbitrary functions (but not containing their derivatives).

19.1 problem 1 357
19.2 problem 2 358
19.3 problem 3 359
19.4 problem 4 360
19.5 problem 5 361
19.6 problem 6 362
19.7 problem 7 363
19.8 problem 8 364
19.9 problem 9 365
19.10problem 10 367
19.11problem 11 368
19.12problem 12 369
19.13problem 13 371
19.14problem 14 372
19.15problem 15 373
19.16problem 16 374
19.17problem 17 375
19.18problem 18 376
19.19problem 19 377
19.20problem 20 378
19.21problem 21 379
19.22problem 22 380
19.23problem 23 381
19.24problem 24 382
19.25problem 25 383
19.26problem 26 384
19.27problem 27 385
19.28problem 28 386
19.29problem 29 387
19.30problem 30 388
19.31problem 31 389
19.32problem 32 390
19.33problem 33 391

19.1 problem 1

Internal problem ID [10594]

Book: Handbook of exact solutions for ordinary differential equations. By Polyanin and Zaitsev. Second edition
Section: Chapter 1, section 1.2. Riccati Equation. subsection 1.2.8-1. Equations containing arbitrary functions (but not containing their derivatives).
Problem number: 1.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_Riccati]`

$$\boxed{y' - y^2 - f(x)\,y = -a^2 - f(x)\,a}$$

✓ Solution by Maple
Time used: 0.0 (sec). Leaf size: 52

```
dsolve(diff(y(x),x)=y(x)^2+f(x)*y(x)-a^2-a*f(x),y(x), singsol=all)
```

$$y(x) = \frac{-a\left(\int \mathrm{e}^{\int f(x)dx+2ax}dx\right) + c_1 a + \mathrm{e}^{\int f(x)dx+2ax}}{-\left(\int \mathrm{e}^{\int f(x)dx+2ax}dx\right) + c_1}$$

✓ Solution by Mathematica
Time used: 0.719 (sec). Leaf size: 166

```
DSolve[y'[x]==y[x]^2+f[x]*y[x]-a^2-a*f[x],y[x],x,IncludeSingularSolutions -> True]
```

$$\text{Solve}\left[\int_1^x \frac{\exp\left(-\int_1^{K[2]}(-2a - f(K[1]))dK[1]\right)(a + f(K[2]) + y(x))}{a - y(x)}dK[2]\right.$$
$$+\int_1^{y(x)}\left(\frac{\exp\left(-\int_1^x(-2a - f(K[1]))dK[1]\right)}{(K[3]-a)^2}\right.$$
$$\left.-\int_1^x\left(\frac{\exp\left(-\int_1^{K[2]}(-2a - f(K[1]))dK[1]\right)(a + f(K[2]) + K[3])}{(a-K[3])^2}+\frac{\exp\left(-\int_1^{K[2]}(-2a - f(K[1]))dK[1]\right.}{a-K[3]}\right.$$

19.2 problem 2

Internal problem ID [10595]

Book: Handbook of exact solutions for ordinary differential equations. By Polyanin and Zaitsev. Second edition
Section: Chapter 1, section 1.2. Riccati Equation. subsection 1.2.8-1. Equations containing arbitrary functions (but not containing their derivatives).
Problem number: 2.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_Riccati]`

$$\boxed{y' - y^2 f(x) + ay = -ab - b^2 f(x)}$$

✓ Solution by Maple
Time used: 0.0 (sec). Leaf size: 65

```
dsolve(diff(y(x),x)=f(x)*y(x)^2-a*y(x)-a*b-b^2*f(x),y(x), singsol=all)
```

$$y(x) = \frac{-c_1 b - b\left(\int f(x)\, \mathrm{e}^{-\left(\int (2f(x)b+a)dx\right)} dx\right) - \mathrm{e}^{-\left(\int (2f(x)b+a)dx\right)}}{c_1 + \int f(x)\, \mathrm{e}^{-\left(\int (2f(x)b+a)dx\right)} dx}$$

✓ Solution by Mathematica
Time used: 0.955 (sec). Leaf size: 185

```
DSolve[y'[x]==f[x]*y[x]^2-a*y[x]-a*b-b^2*f[x],y[x],x,IncludeSingularSolutions -> True]
```

$$\text{Solve}\left[\int_1^x \frac{\exp\left(-\int_1^{K[2]} (a + 2bf(K[1]))dK[1]\right)(a + bf(K[2]) - f(K[2])y(x))}{a(b + y(x))} dK[2]\right.$$
$$+ \int_1^{y(x)} \left(\frac{\exp\left(-\int_1^x (a + 2bf(K[1]))dK[1]\right)}{a(b + K[3])^2}\right.$$
$$\left.\left. - \int_1^x \left(-\frac{\exp\left(-\int_1^{K[2]} (a + 2bf(K[1]))dK[1]\right) f(K[2])}{a(b + K[3])} - \frac{\exp\left(-\int_1^{K[2]} (a + 2bf(K[1]))dK[1]\right)(a + bf(K[2]}{a(b + K[3])^2}\right.\right.$$

19.3 problem 3

Internal problem ID [10596]

Book: Handbook of exact solutions for ordinary differential equations. By Polyanin and Zaitsev. Second edition
Section: Chapter 1, section 1.2. Riccati Equation. subsection 1.2.8-1. Equations containing arbitrary functions (but not containing their derivatives).
Problem number: 3.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_Riccati]`

$$\boxed{y' - y^2 - xf(x)\,y = f(x)}$$

✓ Solution by Maple
Time used: 0.0 (sec). Leaf size: 69

```
dsolve(diff(y(x),x)=y(x)^2+x*f(x)*y(x)+f(x),y(x), singsol=all)
```

$$y(x) = \frac{e^{\int \frac{f(x)x^2-2}{x}dx}x + \int e^{\int \frac{f(x)x^2-2}{x}dx}dx - c_1}{\left(c_1 - \left(\int e^{\int \frac{f(x)x^2-2}{x}dx}dx\right)\right)x}$$

✓ Solution by Mathematica
Time used: 1.074 (sec). Leaf size: 111

```
DSolve[y'[x]==y[x]^2+x*f[x]*y[x]+f[x],y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to -\frac{\exp\left(-\int_1^x -f(K[1])K[1]dK[1]\right) + x\int_1^x \frac{\exp\left(-\int_1^{K[2]} -f(K[1])K[1]dK[1]\right)}{K[2]^2}dK[2] + c_1 x}{x^2\left(\int_1^x \frac{\exp\left(-\int_1^{K[2]} -f(K[1])K[1]dK[1]\right)}{K[2]^2}dK[2] + c_1\right)}$$

$$y(x) \to -\frac{1}{x}$$

19.4 problem 4

Internal problem ID [10597]

Book: Handbook of exact solutions for ordinary differential equations. By Polyanin and Zaitsev. Second edition
Section: Chapter 1, section 1.2. Riccati Equation. subsection 1.2.8-1. Equations containing arbitrary functions (but not containing their derivatives).
Problem number: 4.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_Riccati]`

$$y' - y^2 f(x) + a\, x^n f(x)\, y = an\, x^{n-1}$$

✗ Solution by Maple

```
dsolve(diff(y(x),x)=f(x)*y(x)^2-a*x^n*f(x)*y(x)+a*n*x^(n-1),y(x), singsol=all)
```

No solution found

✗ Solution by Mathematica
Time used: 0.0 (sec). Leaf size: 0

```
DSolve[y'[x]==f[x]*y[x]^2-a*x^n*f[x]*y[x]+a*n*x^(n-1),y[x],x,IncludeSingularSolutions -> Tru
```

Not solved

19.5 problem 5

Internal problem ID [10598]

Book: Handbook of exact solutions for ordinary differential equations. By Polyanin and Zaitsev. Second edition
Section: Chapter 1, section 1.2. Riccati Equation. subsection 1.2.8-1. Equations containing arbitrary functions (but not containing their derivatives).
Problem number: 5.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_Riccati]`

$$y' - y^2 f(x) = an\, x^{n-1} - a^2 x^{2n} f(x)$$

✗ Solution by Maple

```
dsolve(diff(y(x),x)=f(x)*y(x)^2+a*n*x^(n-1)-a^2*x^(2*n)*f(x),y(x), singsol=all)
```

No solution found

✗ Solution by Mathematica
Time used: 0.0 (sec). Leaf size: 0

```
DSolve[y'[x]==f[x]*y[x]^2+a*n*x^(n-1)-a^2*x^(2*n)*f[x],y[x],x,IncludeSingularSolutions -> Tr
```

Not solved

19.6 problem 6

Internal problem ID [10599]

Book: Handbook of exact solutions for ordinary differential equations. By Polyanin and Zaitsev. Second edition
Section: Chapter 1, section 1.2. Riccati Equation. subsection 1.2.8-1. Equations containing arbitrary functions (but not containing their derivatives).
Problem number: 6.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_Riccati]`

$$\boxed{y' + (n+1)\, x^n y^2 - x^{n+1} f(x)\, y = -f(x)}$$

✓ Solution by Maple
Time used: 0.015 (sec). Leaf size: 169

```
dsolve(diff(y(x),x)=-(n+1)*x^n*y(x)^2+x^(n+1)*f(x)*y(x)-f(x),y(x), singsol=all)
```

$$y(x) = \frac{x^{-n-1}\left(x^{n+1} e^{\int \frac{x^{n+1} f(x) x - 2n - 2}{x} dx} + \left(\int x^n e^{\int x^{n+1} f(x) dx + (-2n-2)\left(\int \frac{1}{x} dx\right)} dx\right) n + \int x^n e^{\int x^{n+1} f(x) dx + (-2n-2)\left(\int \frac{1}{x} dx\right)} d\right.}{\left(\int x^n e^{\int x^{n+1} f(x) dx + (-2n-2)\left(\int \frac{1}{x} dx\right)} dx\right) n + \int x^n e^{\int x^{n+1} f(x) dx + (-2n-2)\left(\int \frac{1}{x} dx\right)} dx - c_1}$$

✗ Solution by Mathematica
Time used: 0.0 (sec). Leaf size: 0

```
DSolve[y'[x]==-(n+1)*x^n*y[x]^2+x^(n+1)*f[x]*y[x]-f[x],y[x],x,IncludeSingularSolutions -> Tr
```

Not solved

19.7 problem 7

Internal problem ID [10600]

Book: Handbook of exact solutions for ordinary differential equations. By Polyanin and Zaitsev. Second edition
Section: Chapter 1, section 1.2. Riccati Equation. subsection 1.2.8-1. Equations containing arbitrary functions (but not containing their derivatives).
Problem number: 7.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_Riccati]`

$$\boxed{y'x - y^2 f(x) - yn = f(x)\, x^{2n} a}$$

✓ Solution by Maple
Time used: 0.031 (sec). Leaf size: 30

```
dsolve(x*diff(y(x),x)=f(x)*y(x)^2+n*y(x)+a*x^(2*n)*f(x),y(x), singsol=all)
```

$$y(x) = -\tan\left(-\sqrt{a}\left(\int f(x)\, x^{n-1} dx\right) + c_1\right)\sqrt{a}\, x^n$$

✓ Solution by Mathematica
Time used: 0.577 (sec). Leaf size: 41

```
DSolve[x*y'[x]==f[x]*y[x]^2+n*y[x]+a*x^(2*n)*f[x],y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to \sqrt{a} x^n \tan\left(\sqrt{a}\int_1^x f(K[1]) K[1]^{n-1} dK[1] + c_1\right)$$

19.8 problem 8

Internal problem ID [10601]

Book: Handbook of exact solutions for ordinary differential equations. By Polyanin and Zaitsev. Second edition
Section: Chapter 1, section 1.2. Riccati Equation. subsection 1.2.8-1. Equations containing arbitrary functions (but not containing their derivatives).
Problem number: 8.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_Riccati]`

$$y'x - x^{2n} f(x)\, y^2 - (f(x)\, a\, x^n - n)\, y = f(x)\, b$$

✓ Solution by Maple
Time used: 0.031 (sec). Leaf size: 65

```
dsolve(x*diff(y(x),x)=x^(2*n)*f(x)*y(x)^2+(a*x^n*f(x)-n)*y(x)+b*f(x),y(x), singsol=all)
```

$$y(x) = -\frac{\left(a^2 + \tanh\left(\frac{\sqrt{a^2(a^2-4b)}\,(a(\int f(x)x^{n-1}dx)+c_1)}{2a^2}\right)\sqrt{a^2\,(a^2-4b)}\right)x^{-n}}{2a}$$

✓ Solution by Mathematica
Time used: 2.272 (sec). Leaf size: 82

```
DSolve[x*y'[x]==x^(2*n)*f[x]*y[x]^2+(a*x^n*f[x]-n)*y[x]+b*f[x],y[x],x,IncludeSingularSolutio
```

$$\text{Solve}\left[\int_1^{\sqrt{\frac{x^{2n}}{b}}y(x)} \frac{1}{K[1]^2 - \sqrt{\frac{a^2}{b}}K[1] + 1}dK[1] = \int_1^x \frac{bf(K[2])\sqrt{\frac{K[2]^{2n}}{b}}}{K[2]}dK[2] + c_1, y(x)\right]$$

19.9 problem 9

Internal problem ID [10602]

Book: Handbook of exact solutions for ordinary differential equations. By Polyanin and Zaitsev. Second edition
Section: Chapter 1, section 1.2. Riccati Equation. subsection 1.2.8-1. Equations containing arbitrary functions (but not containing their derivatives).
Problem number: 9.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_Riccati]`

$$\boxed{y' - y^2 f(x) - g(x)\, y = -f(x)\, a^2 - a g(x)}$$

✓ Solution by Maple
Time used: 0.0 (sec). Leaf size: 67

```
dsolve(diff(y(x),x)=f(x)*y(x)^2+g(x)*y(x)-a^2*f(x)-a*g(x),y(x), singsol=all)
```

$$y(x) = \frac{-a\left(\int \mathrm{e}^{\int g(x)dx+2a\left(\int f(x)dx\right)} f(x)\, dx\right) + c_1 a + \mathrm{e}^{\int g(x)dx+2a\left(\int f(x)dx\right)}}{-\left(\int \mathrm{e}^{\int g(x)dx+2a\left(\int f(x)dx\right)} f\left(x\right) dx\right) + c_1}$$

✓ Solution by Mathematica

Time used: 1.122 (sec). Leaf size: 201

```
DSolve[y'[x]==f[x]*y[x]^2+g[x]*y[x]-a^2*f[x]-a*g[x],y[x],x,IncludeSingularSolutions -> True]
```

$$\text{Solve}\left[\int_1^x -\frac{\exp\left(-\int_1^{K[2]}(-2af(K[1])-g(K[1]))dK[1]\right)(af(K[2])+y(x)f(K[2])+g(K[2]))}{a-y(x)}dK[2]\right.$$
$$+\int_1^{y(x)}\left(-\int_1^x\left(-\frac{\exp\left(-\int_1^{K[2]}(-2af(K[1])-g(K[1]))dK[1]\right)f(K[2])}{a-K[3]}-\frac{\exp\left(-\int_1^{K[2]}(-2af(K[1])-g\right.\right.$$
$$\left.\left.-\frac{\exp\left(-\int_1^x(-2af(K[1])-g(K[1]))dK[1]\right)}{(K[3]-a)^2}\right)dK[3]=c_1,y(x)\right]$$

19.10 problem 10

Internal problem ID [10603]

Book: Handbook of exact solutions for ordinary differential equations. By Polyanin and Zaitsev. Second edition
Section: Chapter 1, section 1.2. Riccati Equation. subsection 1.2.8-1. Equations containing arbitrary functions (but not containing their derivatives).
Problem number: 10.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_Riccati]`

$$y' - y^2 f(x) - g(x)\, y = an\, x^{n-1} - a\, x^n g(x) - a^2 x^{2n} f(x)$$

✗ Solution by Maple

```
dsolve(diff(y(x),x)=f(x)*y(x)^2+g(x)*y(x)+a*n*x^(n-1)-a*x^n*g(x)-a^2*f(x)*x^(2*n),y(x), sing
```

No solution found

✗ Solution by Mathematica
Time used: 0.0 (sec). Leaf size: 0

```
DSolve[y'[x]==f[x]*y[x]^2+g[x]*y[x]+a*n*x^(n-1)-a*x^n*g[x]-a^2*f[x]*x^(2*n),y[x],x,IncludeSi
```

Not solved

19.11 problem 11

Internal problem ID [10604]

Book: Handbook of exact solutions for ordinary differential equations. By Polyanin and Zaitsev. Second edition
Section: Chapter 1, section 1.2. Riccati Equation. subsection 1.2.8-1. Equations containing arbitrary functions (but not containing their derivatives).
Problem number: 11.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_Riccati]`

$$y' - y^2 f(x) + a\,x^n g(x)\,y = an\,x^{n-1} + a^2 x^{2n}(g(x) - f(x))$$

✗ Solution by Maple

```
dsolve(diff(y(x),x)=f(x)*y(x)^2-a*x^n*g(x)*y(x)+a*n*x^(n-1)+a^2*x^(2*n)*(g(x)-f(x)),y(x), si
```

No solution found

✗ Solution by Mathematica
Time used: 0.0 (sec). Leaf size: 0

```
DSolve[y'[x]==f[x]*y[x]^2-a*x^n*g[x]*y[x]+a*n*x^(n-1)+a^2*x^(2*n)*(g[x]-f[x]),y[x],x,Include
```

Not solved

19.12 problem 12

Internal problem ID [10605]

Book: Handbook of exact solutions for ordinary differential equations. By Polyanin and Zaitsev. Second edition
Section: Chapter 1, section 1.2. Riccati Equation. subsection 1.2.8-1. Equations containing arbitrary functions (but not containing their derivatives).
Problem number: 12.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_Riccati]`

$$\boxed{y' - a\,\mathrm{e}^{\lambda x} y^2 - a\,\mathrm{e}^{\lambda x} f(x)\,y = \lambda f(x)}$$

✓ Solution by Maple
Time used: 0.0 (sec). Leaf size: 92

```
dsolve(diff(y(x),x)=a*exp(lambda*x)*y(x)^2+a*exp(lambda*x)*f(x)*y(x)+lambda*f(x),y(x), sings
```

$$y(x) = \frac{-c_1 \mathrm{e}^{-2x\lambda + a\left(\int \mathrm{e}^{x\lambda} f(x) dx\right)} - \mathrm{e}^{-x\lambda}\left(\int \mathrm{e}^{-x\lambda + a\left(\int \mathrm{e}^{x\lambda} f(x) dx\right)} dx\right) c_1 \lambda - \lambda^2 \mathrm{e}^{-x\lambda}}{a\left(\left(\int \mathrm{e}^{-x\lambda + a\left(\int \mathrm{e}^{x\lambda} f(x) dx\right)} dx\right) c_1 + \lambda\right)}$$

✓ Solution by Mathematica

Time used: 4.45 (sec). Leaf size: 166

```
DSolve[y'[x]==a*Exp[\[Lambda]*x]*y[x]^2+a*Exp[\[Lambda]*x]*f[x]*y[x]+\[Lambda]*f[x],y[x],x,I
```

$$y(x)\to \frac{\lambda e^{-2\lambda x}\left(\exp\left(-\int_1^{e^{x\lambda}}-\frac{af\left(\frac{\log(K[1])}{\lambda}\right)}{\lambda}dK[1]\right)+e^{\lambda x}\int_1^{e^{x\lambda}}\frac{\exp\left(-\int_1^{K[2]}-\frac{af\left(\frac{\log(K[1])}{\lambda}\right)}{\lambda}dK[1]\right)}{K[2]^2}dK[2]+c_1e^{\lambda x}\right)}{a\left(\int_1^{e^{x\lambda}}\frac{\exp\left(-\int_1^{K[2]}-\frac{af\left(\frac{\log(K[1])}{\lambda}\right)}{\lambda}dK[1]\right)}{K[2]^2}dK[2]+c_1\right)}$$

$$y(x)\to -\frac{\lambda e^{\lambda(-x)}}{a}$$

19.13 problem 13

Internal problem ID [10606]

Book: Handbook of exact solutions for ordinary differential equations. By Polyanin and Zaitsev. Second edition
Section: Chapter 1, section 1.2. Riccati Equation. subsection 1.2.8-1. Equations containing arbitrary functions (but not containing their derivatives).
Problem number: 13.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_Riccati]`

$$y' - y^2 f(x) + a\,\mathrm{e}^{\lambda x} f(x)\, y = a\lambda\,\mathrm{e}^{\lambda x}$$

✗ Solution by Maple

```
dsolve(diff(y(x),x)=f(x)*y(x)^2-a*exp(lambda*x)*f(x)*y(x)+a*lambda*exp(lambda*x),y(x), sings
```

No solution found

✓ Solution by Mathematica
Time used: 48.456 (sec). Leaf size: 207

```
DSolve[y'[x]==f[x]*y[x]^2-a*Exp[\[Lambda]*x]*f[x]*y[x]+a*\[Lambda]*Exp[\[Lambda]*x],y[x],x,I
```

$$y(x)$$

$$\to \frac{a \exp\left(\int_1^{e^{x\lambda}} -\frac{af\left(\frac{\log(K[1])}{\lambda}\right)}{\lambda} dK[1] + 2\lambda x\right) \left(\int_1^{e^{x\lambda}} \frac{\exp\left(-\int_1^{K[2]} -\frac{af\left(\frac{\log(K[1])}{\lambda}\right)}{\lambda} dK[1]\right)}{K[2]^2} dK[2] + c_1\right.}{\exp\left(\int_1^{e^{x\lambda}} -\frac{af\left(\frac{\log(K[1])}{\lambda}\right)}{\lambda} dK[1] + \lambda x\right) \int_1^{e^{x\lambda}} \frac{\exp\left(-\int_1^{K[2]} -\frac{af\left(\frac{\log(K[1])}{\lambda}\right)}{\lambda} dK[1]\right)}{K[2]^2} dK[2] + c_1 \exp\left(\int_1^{e^{x\lambda}} -\frac{af\left(\frac{\log(K[1])}{\lambda}\right)}{\lambda}\right.}$$

$$y(x) \to ae^{\lambda x}$$

19.14 problem 14

Internal problem ID [10607]

Book: Handbook of exact solutions for ordinary differential equations. By Polyanin and Zaitsev. Second edition
Section: Chapter 1, section 1.2. Riccati Equation. subsection 1.2.8-1. Equations containing arbitrary functions (but not containing their derivatives).
Problem number: 14.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_Riccati]`

$$y' - y^2 f(x) = a\lambda \, \mathrm{e}^{\lambda x} - a^2 \mathrm{e}^{2\lambda x} f(x)$$

✗ Solution by Maple

```
dsolve(diff(y(x),x)=f(x)*y(x)^2+a*lambda*exp(lambda*x)-a^2*exp(2*lambda*x)*f(x),y(x), singso
```

No solution found

✗ Solution by Mathematica
Time used: 0.0 (sec). Leaf size: 0

```
DSolve[y'[x]==f[x]*y[x]^2+a*\[Lambda]*Exp[\[Lambda]*x]-a^2*Exp[2*\[Lambda]*x]*f[x],y[x],x,In
```

Not solved

19.15 problem 15

Internal problem ID [10608]

Book: Handbook of exact solutions for ordinary differential equations. By Polyanin and Zaitsev. Second edition
Section: Chapter 1, section 1.2. Riccati Equation. subsection 1.2.8-1. Equations containing arbitrary functions (but not containing their derivatives).
Problem number: 15.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_Riccati]`

$$\boxed{y' - y^2 f(x) - \lambda y = a^2 \mathrm{e}^{2\lambda x} f(x)}$$

✓ Solution by Maple
Time used: 0.032 (sec). Leaf size: 26

```
dsolve(diff(y(x),x)=f(x)*y(x)^2+lambda*y(x)+a^2*exp(2*lambda*x)*f(x),y(x), singsol=all)
```

$$y(x) = -\tan\left(-a\left(\int \mathrm{e}^{x\lambda} f(x)\,dx\right) + c_1\right) a\,\mathrm{e}^{x\lambda}$$

✓ Solution by Mathematica
Time used: 0.615 (sec). Leaf size: 47

```
DSolve[y'[x]==f[x]*y[x]^2+\[Lambda]*y[x]+a^2*Exp[2*\[Lambda]*x]*f[x],y[x],x,IncludeSingularS
```

$$y(x) \to \sqrt{a^2} e^{\lambda x} \tan\left(\sqrt{a^2} \int_1^x e^{\lambda K[1]} f(K[1]) dK[1] + c_1\right)$$

19.16 problem 16

Internal problem ID [10609]

Book: Handbook of exact solutions for ordinary differential equations. By Polyanin and Zaitsev. Second edition
Section: Chapter 1, section 1.2. Riccati Equation. subsection 1.2.8-1. Equations containing arbitrary functions (but not containing their derivatives).
Problem number: 16.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_Riccati]`

$$y' - y^2 f(x) + f(x)\left(\mathrm{e}^{\lambda x} a + b\right) y = a\lambda\,\mathrm{e}^{\lambda x}$$

✗ Solution by Maple

```
dsolve(diff(y(x),x)=f(x)*y(x)^2-f(x)*(a*exp(lambda*x)+b)*y(x)+a*lambda*exp(lambda*x),y(x), s
```

No solution found

✗ Solution by Mathematica
Time used: 0.0 (sec). Leaf size: 0

```
DSolve[y'[x]==f[x]*y[x]^2-f[x]*(a*Exp[\[Lambda]*x]+b)*y[x]+a*\[Lambda]*Exp[\[Lambda]*x],y[x]
```

Not solved

19.17 problem 17

Internal problem ID [10610]

Book: Handbook of exact solutions for ordinary differential equations. By Polyanin and Zaitsev. Second edition
Section: Chapter 1, section 1.2. Riccati Equation. subsection 1.2.8-1. Equations containing arbitrary functions (but not containing their derivatives).
Problem number: 17.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_Riccati]`

$$\boxed{y' - \mathrm{e}^{\lambda x} f(x)\, y^2 - (f(x)\, a - \lambda)\, y = b\, \mathrm{e}^{-\lambda x} f(x)}$$

✓ Solution by Maple
Time used: 0.031 (sec). Leaf size: 59

```
dsolve(diff(y(x),x)=exp(lambda*x)*f(x)*y(x)^2+(a*f(x)-lambda)*y(x)+b*exp(-lambda*x)*f(x),y(x
```

$$y(x) = -\frac{\left(a^2 + \tanh\left(\frac{\sqrt{a^2(a^2-4b)}\,(a(\int f(x)dx)+c_1)}{2a^2}\right)\sqrt{a^2\,(a^2-4b)}\right)\mathrm{e}^{-x\lambda}}{2a}$$

✓ Solution by Mathematica
Time used: 1.392 (sec). Leaf size: 87

```
DSolve[y'[x]==Exp[\[Lambda]*x]*f[x]*y[x]^2+(a*f[x]-\[Lambda])*y[x]+b*Exp[-\[Lambda]*x]*f[x],
```

$$\text{Solve}\left[\int_1^{\sqrt{\frac{e^{2x\lambda}}{b}}y(x)} \frac{1}{K[1]^2 - \sqrt{\frac{a^2}{b}}K[1] + 1}dK[1] = \int_1^x be^{-\lambda K[2]}\sqrt{\frac{e^{2\lambda K[2]}}{b}}f(K[2])dK[2] + c_1, y(x)\right]$$

19.18 problem 18

Internal problem ID [10611]

Book: Handbook of exact solutions for ordinary differential equations. By Polyanin and Zaitsev. Second edition
Section: Chapter 1, section 1.2. Riccati Equation. subsection 1.2.8-1. Equations containing arbitrary functions (but not containing their derivatives).
Problem number: 18.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_Riccati]`

$$y' - y^2 f(x) - g(x)\, y = a\lambda\, \mathrm{e}^{\lambda x} - a\, \mathrm{e}^{\lambda x} g(x) - a^2 \mathrm{e}^{2\lambda x} f(x)$$

✗ Solution by Maple

```
dsolve(diff(y(x),x)=f(x)*y(x)^2+g(x)*y(x)+a*lambda*exp(lambda*x)-a*exp(lambda*x)*g(x)-a^2*ex
```

No solution found

✗ Solution by Mathematica
Time used: 0.0 (sec). Leaf size: 0

```
DSolve[y'[x]==f[x]*y[x]^2+g[x]*y[x]+a*\[Lambda]*Exp[\[Lambda]*x]-a*Exp[\[Lambda]*x]*g[x]-a^2
```

Not solved

19.19 problem 19

Internal problem ID [10612]

Book: Handbook of exact solutions for ordinary differential equations. By Polyanin and Zaitsev. Second edition
Section: Chapter 1, section 1.2. Riccati Equation. subsection 1.2.8-1. Equations containing arbitrary functions (but not containing their derivatives).
Problem number: 19.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_Riccati]`

$$y' - y^2 f(x) + a\,\mathrm{e}^{\lambda x} g(x)\, y = a\lambda\,\mathrm{e}^{\lambda x} + a^2 \mathrm{e}^{2\lambda x}(g(x) - f(x))$$

✗ Solution by Maple

```
dsolve(diff(y(x),x)=f(x)*y(x)^2-a*exp(lambda*x)*g(x)*y(x)+a*lambda*exp(lambda*x)+a^2*exp(2*l
```

No solution found

✗ Solution by Mathematica
Time used: 0.0 (sec). Leaf size: 0

```
DSolve[y'[x]==f[x]*y[x]^2-a*Exp[\[Lambda]*x]*g[x]*y[x]+a*\[Lambda]*Exp[\[Lambda]*x]+a^2*Exp[
```

Not solved

19.20 problem 20

Internal problem ID [10613]

Book: Handbook of exact solutions for ordinary differential equations. By Polyanin and Zaitsev. Second edition
Section: Chapter 1, section 1.2. Riccati Equation. subsection 1.2.8-1. Equations containing arbitrary functions (but not containing their derivatives).
Problem number: 20.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_Riccati]`

$$y' - y^2 f(x) = 2a\lambda x\, \mathrm{e}^{\lambda x^2} - a^2 f(x)\, \mathrm{e}^{2\lambda x^2}$$

✗ Solution by Maple

```
dsolve(diff(y(x),x)=f(x)*y(x)^2+2*a*lambda*x*exp(lambda*x^2)-a^2*f(x)*exp(2*lambda*x^2),y(x)
```

No solution found

✗ Solution by Mathematica
Time used: 0.0 (sec). Leaf size: 0

```
DSolve[y'[x]==f[x]*y[x]^2+2*a*\[Lambda]*x*Exp[\[Lambda]*x^2]-a^2*f[x]*Exp[2*\[Lambda]*x^2],y
```

Not solved

19.21 problem 21

Internal problem ID [10614]

Book: Handbook of exact solutions for ordinary differential equations. By Polyanin and Zaitsev. Second edition
Section: Chapter 1, section 1.2. Riccati Equation. subsection 1.2.8-1. Equations containing arbitrary functions (but not containing their derivatives).
Problem number: 21.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_Riccati]`

$$y' - y^2 f(x) - y\lambda x = a\,\mathrm{e}^{\lambda x} f(x)$$

✗ Solution by Maple

```
dsolve(diff(y(x),x)=f(x)*y(x)^2+lambda*x*y(x)+a*f(x)*exp(lambda*x),y(x), singsol=all)
```

No solution found

✗ Solution by Mathematica
Time used: 0.0 (sec). Leaf size: 0

```
DSolve[y'[x]==f[x]*y[x]^2+\[Lambda]*x*y[x]+a*f[x]*Exp[\[Lambda]*x],y[x],x,IncludeSingularSol
```

Not solved

19.22 problem 22

Internal problem ID [10615]

Book: Handbook of exact solutions for ordinary differential equations. By Polyanin and Zaitsev. Second edition
Section: Chapter 1, section 1.2. Riccati Equation. subsection 1.2.8-1. Equations containing arbitrary functions (but not containing their derivatives).
Problem number: 22.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_Riccati]`

$$y' - f(x)\,y^2 = a\lambda - a\tanh(\lambda x)^2\,(af(x) + \lambda)$$

✗ Solution by Maple

```
dsolve(diff(y(x),x)=f(x)*y(x)^2-a*tanh(lambda*x)^2*(a*f(x)+lambda)+a*lambda,y(x), singsol=al
```

No solution found

✗ Solution by Mathematica
Time used: 0.0 (sec). Leaf size: 0

```
DSolve[y'[x]==f[x]*y[x]^2-a*Tanh[\[Lambda]*x]^2*(a*f[x]+\[Lambda])+a*\[Lambda],y[x],x,Includ
```

Not solved

19.23 problem 23

Internal problem ID [10616]

Book: Handbook of exact solutions for ordinary differential equations. By Polyanin and Zaitsev. Second edition
Section: Chapter 1, section 1.2. Riccati Equation. subsection 1.2.8-1. Equations containing arbitrary functions (but not containing their derivatives).
Problem number: 23.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_Riccati]`

$$y' - f(x)\,y^2 = a\lambda - a\coth(\lambda x)^2\,(af(x) + \lambda)$$

✗ Solution by Maple

```
dsolve(diff(y(x),x)=f(x)*y(x)^2-a*coth(lambda*x)^2*(a*f(x)+lambda)+a*lambda,y(x), singsol=al
```

No solution found

✗ Solution by Mathematica
Time used: 0.0 (sec). Leaf size: 0

```
DSolve[y'[x]==f[x]*y[x]^2-a*Coth[\[Lambda]*x]^2*(a*f[x]+\[Lambda])+a*\[Lambda],y[x],x,Includ
```

Not solved

19.24 problem 24

Internal problem ID [10617]

Book: Handbook of exact solutions for ordinary differential equations. By Polyanin and Zaitsev. Second edition
Section: Chapter 1, section 1.2. Riccati Equation. subsection 1.2.8-1. Equations containing arbitrary functions (but not containing their derivatives).
Problem number: 24.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_Riccati]`

$$y' - y^2 f(x) = -f(x)\, a^2 + a\lambda \sinh(\lambda x) - \sinh(\lambda x)^2 f(x)\, a^2$$

✗ Solution by Maple

```
dsolve(diff(y(x),x)=f(x)*y(x)^2-a^2*f(x)+a*lambda*sinh(lambda*x)-a^2*f(x)*sinh(lambda*x)^2,y
```

No solution found

✗ Solution by Mathematica
Time used: 0.0 (sec). Leaf size: 0

```
DSolve[y'[x]==f[x]*y[x]^2-a^2*f[x]+a*\[Lambda]*Sinh[\[Lambda]*x]-a^2*f[x]*Sinh[\[Lambda]*x]^
```

Not solved

19.25 problem 25

Internal problem ID [10618]

Book: Handbook of exact solutions for ordinary differential equations. By Polyanin and Zaitsev. Second edition
Section: Chapter 1, section 1.2. Riccati Equation. subsection 1.2.8-1. Equations containing arbitrary functions (but not containing their derivatives).
Problem number: 25.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_Riccati]`

$$y'x - y^2 f(x) = a - a^2 f(x) \ln(x)^2$$

✗ Solution by Maple

```
dsolve(x*diff(y(x),x)=f(x)*y(x)^2+a-a^2*f(x)*(ln(x))^2,y(x), singsol=all)
```

No solution found

✗ Solution by Mathematica
Time used: 0.0 (sec). Leaf size: 0

```
DSolve[x*y'[x]==f[x]*y[x]^2+a-a^2*f[x]*(Log[x])^2,y[x],x,IncludeSingularSolutions -> True]
```

Not solved

19.26 problem 26

Internal problem ID [10619]

Book: Handbook of exact solutions for ordinary differential equations. By Polyanin and Zaitsev. Second edition
Section: Chapter 1, section 1.2. Riccati Equation. subsection 1.2.8-1. Equations containing arbitrary functions (but not containing their derivatives).
Problem number: 26.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[[_1st_order, '_with_symmetry_[F(x),G(x)]'], _Riccati]`

$$\boxed{y'x - f(x)\left(y + a\ln(x)\right)^2 = -a}$$

✓ Solution by Maple
Time used: 0.0 (sec). Leaf size: 24

```
dsolve(x*diff(y(x),x)=f(x)*(y(x)+a*ln(x))^2-a,y(x), singsol=all)
```

$$y(x) = -a\ln(x) + \frac{1}{c_1 - \left(\int \frac{f(x)}{x} dx\right)}$$

✓ Solution by Mathematica
Time used: 0.48 (sec). Leaf size: 42

```
DSolve[x*y'[x]==f[x]*(y[x]+a*Log[x])^2-a,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to -a\log(x) + \frac{1}{-\int_1^x \frac{f(K[2])}{K[2]} dK[2] + c_1}$$
$$y(x) \to -a\log(x)$$

19.27 problem 27

Internal problem ID [10620]

Book: Handbook of exact solutions for ordinary differential equations. By Polyanin and Zaitsev. Second edition
Section: Chapter 1, section 1.2. Riccati Equation. subsection 1.2.8-1. Equations containing arbitrary functions (but not containing their derivatives).
Problem number: 27.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_Riccati]`

$$y' - y^2 f(x) + ax \ln(x) f(x) y = a \ln(x) + a$$

✗ Solution by Maple

```
dsolve(diff(y(x),x)=f(x)*y(x)^2-a*x*ln(x)*f(x)*y(x)+a*ln(x)+a,y(x), singsol=all)
```

No solution found

✗ Solution by Mathematica
Time used: 0.0 (sec). Leaf size: 0

```
DSolve[y'[x]==f[x]*y[x]^2-a*x*Log[x]*f[x]*y[x]+a*Log[x]+a,y[x],x,IncludeSingularSolutions ->
```

Not solved

19.28 problem 28

Internal problem ID [10621]

Book: Handbook of exact solutions for ordinary differential equations. By Polyanin and Zaitsev. Second edition
Section: Chapter 1, section 1.2. Riccati Equation. subsection 1.2.8-1. Equations containing arbitrary functions (but not containing their derivatives).
Problem number: 28.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_Riccati]`

$$y' + a\ln(x)\,y^2 - af(x)\,(\ln(x)\,x - x)\,y = -f(x)$$

✓ Solution by Maple
Time used: 0.016 (sec). Leaf size: 227

```
dsolve(diff(y(x),x)=-a*ln(x)*y(x)^2+a*f(x)*(x*ln(x)-x)*y(x)-f(x),y(x), singsol=all)
```

$$y(x) = \frac{-x(\ln(x)-1)\,\mathrm{e}^{\int \frac{f(x)\ln(x)^2 a\,x^2+\left(-2x^2af(x)-2\right)\ln(x)+x^2af(x)}{x(\ln(x)-1)}dx} + c_1 a - \left(\int \ln(x)\,\mathrm{e}^{a\left(\int \frac{xf(x)\ln(x)^2}{\ln(x)-1}dx\right)-2a\left(\int \frac{xf(x)\ln(x)}{\ln(x)-1}dx\right)+}}{ax\,(\ln(x)-1)\left(c_1 a - \left(\int \ln(x)\,\mathrm{e}^{a\left(\int \frac{xf(x)\ln(x)^2}{\ln(x)-1}dx\right)-2a\left(\int \frac{xf(x)\ln(x)}{\ln(x)-1}dx\right)+a\left(\int \frac{xf(x)}{\ln(x)-1}dx\right)-2\left(\int \frac{1}{x(\ln}\right.\right.\right.$$

✗ Solution by Mathematica
Time used: 0.0 (sec). Leaf size: 0

```
DSolve[y'[x]==-a*Log[x]*y[x]^2+a*f[x]*(x*Log[x]-x)*y[x]-f[x],y[x],x,IncludeSingularSolutions
```

Not solved

19.29 problem 29

Internal problem ID [10622]

Book: Handbook of exact solutions for ordinary differential equations. By Polyanin and Zaitsev. Second edition
Section: Chapter 1, section 1.2. Riccati Equation. subsection 1.2.8-1. Equations containing arbitrary functions (but not containing their derivatives).
Problem number: 29.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_Riccati]`

$$\boxed{y' - \lambda \sin(\lambda x)\, y^2 - f(x) \cos(\lambda x)\, y = -f(x)}$$

✓ Solution by Maple
Time used: 0.016 (sec). Leaf size: 97

```
dsolve(diff(y(x),x)=lambda*sin(lambda*x)*y(x)^2+f(x)*cos(lambda*x)*y(x)-f(x),y(x), singsol=a
```

$$y(x) = \frac{\sec(x\lambda)\,\lambda\left(\int e^{\int (f(x)\cos(x\lambda)+2\tan(x\lambda)\lambda)dx}\sin(x\lambda)\,dx\right)c_1 - c_1 e^{\int (f(x)\cos(x\lambda)+2\tan(x\lambda)\lambda)dx} - \sec(x\lambda)}{\lambda\left(\int e^{\int (f(x)\cos(x\lambda)+2\tan(x\lambda)\lambda)dx}\sin(x\lambda)\,dx\right)c_1 - 1}$$

✗ Solution by Mathematica
Time used: 0.0 (sec). Leaf size: 0

```
DSolve[y'[x]==\[Lambda]*Sin[\[Lambda]*x]*y[x]^2+f[x]*Cos[\[Lambda]*x]*y[x]-f[x],y[x],x,Inclu
```

Not solved

19.30 problem 30

Internal problem ID [10623]

Book: Handbook of exact solutions for ordinary differential equations. By Polyanin and Zaitsev. Second edition
Section: Chapter 1, section 1.2. Riccati Equation. subsection 1.2.8-1. Equations containing arbitrary functions (but not containing their derivatives).
Problem number: 30.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_Riccati]`

$$y' - y^2 f(x) = -f(x)\, a^2 + a\lambda \sin(\lambda x) + a^2 f(x) \sin(\lambda x)^2$$

✗ Solution by Maple

```
dsolve(diff(y(x),x)=f(x)*y(x)^2-a^2*f(x)+a*lambda*sin(lambda*x)+a^2*f(x)*sin(lambda*x)^2,y(x
```

No solution found

✗ Solution by Mathematica
Time used: 0.0 (sec). Leaf size: 0

```
DSolve[y'[x]==f[x]*y[x]^2-a^2*f[x]+a*\[Lambda]*Sin[\[Lambda]*x]+a^2*f[x]*Sin[\[Lambda]*x]^2,
```

Not solved

19.31 problem 31

Internal problem ID [10624]

Book: Handbook of exact solutions for ordinary differential equations. By Polyanin and Zaitsev. Second edition
Section: Chapter 1, section 1.2. Riccati Equation. subsection 1.2.8-1. Equations containing arbitrary functions (but not containing their derivatives).
Problem number: 31.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_Riccati]`

$$y' - f(x)\,y^2 = -a^2 f(x) + a\lambda\cos(\lambda x) + a^2 f(x)\cos(\lambda x)^2$$

✗ Solution by Maple

```
dsolve(diff(y(x),x)=f(x)*y(x)^2-a^2*f(x)+a*lambda*cos(lambda*x)+a^2*f(x)*cos(lambda*x)^2,y(x
```

No solution found

✗ Solution by Mathematica
Time used: 0.0 (sec). Leaf size: 0

```
DSolve[y'[x]==f[x]*y[x]^2-a^2*f[x]+a*\[Lambda]*cos[\[Lambda]*x]+a^2*f[x]*Cos[\[Lambda]*x]^2,
```

Not solved

19.32 problem 32

Internal problem ID [10625]

Book: Handbook of exact solutions for ordinary differential equations. By Polyanin and Zaitsev. Second edition
Section: Chapter 1, section 1.2. Riccati Equation. subsection 1.2.8-1. Equations containing arbitrary functions (but not containing their derivatives).
Problem number: 32.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_Riccati]`

$$y' - f(x)\,y^2 = a\lambda - a\tan(\lambda x)^2\,(af(x) - \lambda)$$

✗ Solution by Maple

```
dsolve(diff(y(x),x)=f(x)*y(x)^2-a*tan(lambda*x)^2*(a*f(x)-lambda)+a*lambda,y(x), singsol=all
```

No solution found

✗ Solution by Mathematica
Time used: 0.0 (sec). Leaf size: 0

```
DSolve[y'[x]==f[x]*y[x]^2-a*Tan[\[Lambda]*x]^2*(a*f[x]-\[Lambda])+a*\[Lambda],y[x],x,Include
```

Not solved

19.33 problem 33

Internal problem ID [10626]

Book: Handbook of exact solutions for ordinary differential equations. By Polyanin and Zaitsev. Second edition
Section: Chapter 1, section 1.2. Riccati Equation. subsection 1.2.8-1. Equations containing arbitrary functions (but not containing their derivatives).
Problem number: 33.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_Riccati]`

$$y' - f(x)\,y^2 = a\lambda - a\cot(\lambda x)^2\,(af(x) - \lambda)$$

✗ Solution by Maple

```
dsolve(diff(y(x),x)=f(x)*y(x)^2-a*cot(lambda*x)^2*(a*f(x)-lambda)+a*lambda,y(x), singsol=all
```

No solution found

✗ Solution by Mathematica
Time used: 0.0 (sec). Leaf size: 0

```
DSolve[y'[x]==f[x]*y[x]^2-a*Cot[\[Lambda]*x]^2*(a*f[x]-\[Lambda])+a*\[Lambda],y[x],x,Include
```

Not solved

20 Chapter 1, section 1.2. Riccati Equation. subsection 1.2.8-2. Equations containing arbitrary functions and their derivatives.

20.1 problem 34 393
20.2 problem 35 394
20.3 problem 36 395
20.4 problem 37 396
20.5 problem 38 397
20.6 problem 39 398
20.7 problem 40 399
20.8 problem 41 400
20.9 problem 42 401

20.1 problem 34

Internal problem ID [10627]

Book: Handbook of exact solutions for ordinary differential equations. By Polyanin and Zaitsev. Second edition
Section: Chapter 1, section 1.2. Riccati Equation. subsection 1.2.8-2. Equations containing arbitrary functions and their derivatives.
Problem number: 34.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_Riccati]`

$$\boxed{y' - y^2 = -f(x)^2 + f'(x)}$$

✓ Solution by Maple
Time used: 0.0 (sec). Leaf size: 45

```
dsolve(diff(y(x),x)=y(x)^2-f(x)^2+diff(f(x),x),y(x), singsol=all)
```

$$y(x) = \frac{-f(x)\left(\int \mathrm{e}^{2\left(\int f(x)dx\right)}dx\right) + f(x)\,c_1 + \mathrm{e}^{2\left(\int f(x)dx\right)}}{c_1 - \left(\int \mathrm{e}^{2\left(\int f(x)dx\right)}dx\right)}$$

✗ Solution by Mathematica
Time used: 0.0 (sec). Leaf size: 0

```
DSolve[y'[x]==y[x]^2-f[x]^2+f'[x],y[x],x,IncludeSingularSolutions -> True]
```

Not solved

20.2 problem 35

Internal problem ID [10628]

Book: Handbook of exact solutions for ordinary differential equations. By Polyanin and Zaitsev. Second edition
Section: Chapter 1, section 1.2. Riccati Equation. subsection 1.2.8-2. Equations containing arbitrary functions and their derivatives.
Problem number: 35.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_Riccati]`

$$\boxed{y' - f(x)\,y^2 + f(x)\,g(x)\,y = g'(x)}$$

✗ Solution by Maple

```
dsolve(diff(y(x),x)=f(x)*y(x)^2-f(x)*g(x)*y(x)+diff(g(x),x),y(x), singsol=all)
```

No solution found

✗ Solution by Mathematica
Time used: 0.0 (sec). Leaf size: 0

```
DSolve[y'[x]==f[x]*y[x]^2-f[x]*g[x]*y[x]+g'[x],y[x],x,IncludeSingularSolutions -> True]
```

Not solved

20.3 problem 36

Internal problem ID [10629]

Book: Handbook of exact solutions for ordinary differential equations. By Polyanin and Zaitsev. Second edition
Section: Chapter 1, section 1.2. Riccati Equation. subsection 1.2.8-2. Equations containing arbitrary functions and their derivatives.
Problem number: 36.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_Riccati]`

$$\boxed{y' + f'(x)\,y^2 - f(x)\,g(x)\,y = -g(x)}$$

✓ Solution by Maple
Time used: 0.0 (sec). Leaf size: 102

```
dsolve(diff(y(x),x)=-diff(f(x),x)*y(x)^2+f(x)*g(x)*y(x)-g(x),y(x), singsol=all)
```

$$y(x) = \frac{f(x)\,\mathrm{e}^{\int \frac{g(x)f(x)^2 - 2\frac{d}{dx}f(x)}{f(x)}dx} + \int \left(\frac{d}{dx}f(x)\right)\mathrm{e}^{\int g(x)f(x)dx - 2\left(\int \frac{\frac{d}{dx}f(x)}{f(x)}dx\right)}dx - c_1}{f(x)\left(\int \left(\frac{d}{dx}f(x)\right)\mathrm{e}^{\int g(x)f(x)dx - 2\left(\int \frac{\frac{d}{dx}f(x)}{f(x)}dx\right)}dx - c_1\right)}$$

✗ Solution by Mathematica
Time used: 0.0 (sec). Leaf size: 0

```
DSolve[y'[x]==-f'[x]*y[x]^2+f[x]*g[x]*y[x]-g[x],y[x],x,IncludeSingularSolutions -> True]
```

Not solved

20.4 problem 37

Internal problem ID [10630]

Book: Handbook of exact solutions for ordinary differential equations. By Polyanin and Zaitsev. Second edition
Section: Chapter 1, section 1.2. Riccati Equation. subsection 1.2.8-2. Equations containing arbitrary functions and their derivatives.
Problem number: 37.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[[_1st_order, '_with_symmetry_[F(x),G(x)]'], _Riccati]`

$$\boxed{y' - g(x)\left(y - f(x)\right)^2 = f'(x)}$$

✓ Solution by Maple
Time used: 0.0 (sec). Leaf size: 17

```
dsolve(diff(y(x),x)=g(x)*(y(x)-f(x))^2+diff(f(x),x),y(x), singsol=all)
```

$$y(x) = f(x) + \frac{1}{c_1 - \left(\int g\left(x\right) dx\right)}$$

✓ Solution by Mathematica
Time used: 0.35 (sec). Leaf size: 31

```
DSolve[y'[x]==g[x]*(y[x]-f[x])^2+f'[x],y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to f(x) + \frac{1}{-\int_1^x g(K[2])dK[2] + c_1}$$
$$y(x) \to f(x)$$

20.5 problem 38

Internal problem ID [10631]

Book: Handbook of exact solutions for ordinary differential equations. By Polyanin and Zaitsev. Second edition
Section: Chapter 1, section 1.2. Riccati Equation. subsection 1.2.8-2. Equations containing arbitrary functions and their derivatives.
Problem number: 38.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_Riccati]`

$$\boxed{y' - \frac{f'(x)\,y^2}{g\,(x)} = -\frac{g'(x)}{f\,(x)}}$$

✓ Solution by Maple
Time used: 0.0 (sec). Leaf size: 58

```
dsolve(diff(y(x),x)=diff(f(x),x)/g(x)*y(x)^2-diff(g(x),x)/f(x),y(x), singsol=all)
```

$$y(x) = \frac{-\left(\int \frac{\frac{d}{dx}f(x)}{g(x)f(x)^2}dx\right) g(x)\,f(x) - c_1 f(x)\,g(x) - 1}{f\,(x)^2\left(\int \frac{\frac{d}{dx}f(x)}{g(x)f(x)^2}dx + c_1\right)}$$

✓ Solution by Mathematica
Time used: 0.347 (sec). Leaf size: 160

```
DSolve[y'[x]==f'[x]/g[x]*y[x]^2-g'[x]/f[x],y[x],x,IncludeSingularSolutions -> True]
```

$$\text{Solve}\left[\int_1^{y(x)} \left(\frac{1}{(g(x)+f(x)K[2])^2} - \int_1^x \left(\frac{2(f(K[1])K[2]^2 f'(K[1]) - g(K[1])g'(K[1]))}{g(K[1])(g(K[1])+f(K[1])K[2])^3} - \frac{2K[2]f'(K[1])}{g(K[1])(g(K[1])+f(K[1])K[2])^2}\right) dK[1]\right) dK[2] + \int_1^x -\frac{f(K[1])y(x)^2 f'(K[1]) - g(K[1])g'(K[1])}{f(K[1])g(K[1])(g(K[1])+f(K[1])y(x))^2} dK[1] = c_1, y(x)\right]$$

20.6 problem 39

Internal problem ID [10632]

Book: Handbook of exact solutions for ordinary differential equations. By Polyanin and Zaitsev. Second edition
Section: Chapter 1, section 1.2. Riccati Equation. subsection 1.2.8-2. Equations containing arbitrary functions and their derivatives.
Problem number: 39.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_Riccati]`

$$\boxed{f(x)^2 y' - f'(x) y^2 + g(x) (y - f(x)) = 0}$$

✗ Solution by Maple

```
dsolve(f(x)^2*diff(y(x),x)-diff(f(x),x)*y(x)^2+g(x)*(y(x)-f(x))=0,y(x), singsol=all)
```

No solution found

✗ Solution by Mathematica
Time used: 0.0 (sec). Leaf size: 0

```
DSolve[f[x]^2*y'[x]-f'[x]*y[x]^2+g[x]*(y[x]-f[x])==0,y[x],x,IncludeSingularSolutions -> True
```

Not solved

20.7 problem 40

Internal problem ID [10633]

Book: Handbook of exact solutions for ordinary differential equations. By Polyanin and Zaitsev. Second edition
Section: Chapter 1, section 1.2. Riccati Equation. subsection 1.2.8-2. Equations containing arbitrary functions and their derivatives.
Problem number: 40.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_Riccati]`

$$y' - f'(x)\, y^2 - a\, \mathrm{e}^{\lambda x} f(x)\, y = a\, \mathrm{e}^{\lambda x}$$

✓ Solution by Maple
Time used: 0.0 (sec). Leaf size: 114

```
dsolve(diff(y(x),x)=diff(f(x),x)*y(x)^2+a*exp(lambda*x)*f(x)*y(x)+a*exp(lambda*x),y(x), sing
```

$$y(x) = \frac{-f(x)\, \mathrm{e}^{\int \frac{\mathrm{e}^{x\lambda} f(x)^2 a - 2\frac{d}{dx} f(x)}{f(x)} dx} - \left(\int \left(\frac{d}{dx} f(x) \right) \mathrm{e}^{a\left(\int \mathrm{e}^{x\lambda} f(x) dx \right) - 2\left(\int \frac{\frac{d}{dx} f(x)}{f(x)} dx \right)} dx \right) - c_1}{f(x) \left(c_1 + \int \left(\frac{d}{dx} f(x) \right) \mathrm{e}^{a\left(\int \mathrm{e}^{x\lambda} f(x) dx \right) - 2\left(\int \frac{\frac{d}{dx} f(x)}{f(x)} dx \right)} dx \right)}$$

✓ Solution by Mathematica
Time used: 84.356 (sec). Leaf size: 167

```
DSolve[y'[x]==f'[x]*y[x]^2+a*Exp[\[Lambda]*x]*f[x]*y[x]+a*Exp[\[Lambda]*x],y[x],x,IncludeSin
```

$$y(x) \to -\frac{a \exp\left(\int_1^{e^{x\lambda}} -\frac{af\left(\frac{\log(K[1])}{\lambda}\right)}{\lambda} dK[1] \right) \left(1 + c_1 \int_1^{e^{x\lambda}} \exp\left(-\int_1^{K[2]} -\frac{af\left(\frac{\log(K[1])}{\lambda}\right)}{\lambda} dK[1] \right) dK[2] \right)}{af\left(\frac{\log(e^{\lambda x})}{\lambda}\right) \exp\left(\int_1^{e^{x\lambda}} -\frac{af\left(\frac{\log(K[1])}{\lambda}\right)}{\lambda} dK[1] \right) \left(1 + c_1 \int_1^{e^{x\lambda}} \exp\left(-\int_1^{K[2]} -\frac{af\left(\frac{\log(K[1])}{\lambda}\right)}{\lambda} dK[1] \right) dK[2] \right) -$$

20.8 problem 41

Internal problem ID [10634]

Book: Handbook of exact solutions for ordinary differential equations. By Polyanin and Zaitsev. Second edition
Section: Chapter 1, section 1.2. Riccati Equation. subsection 1.2.8-2. Equations containing arbitrary functions and their derivatives.
Problem number: 41.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_Riccati]`

$$\boxed{y' - f(x)\,y^2 - g'(x)\,y = a f(x)\,\mathrm{e}^{2g(x)}}$$

✓ Solution by Maple
Time used: 0.031 (sec). Leaf size: 28

```
dsolve(diff(y(x),x)=f(x)*y(x)^2+diff(g(x),x)*y(x)+a*f(x)*exp(2*g(x)),y(x), singsol=all)
```

$$y(x) = -\tan\left(-\sqrt{a}\left(\int f(x)\,\mathrm{e}^{g(x)}dx\right) + c_1\right)\sqrt{a}\,\mathrm{e}^{g(x)}$$

✓ Solution by Mathematica
Time used: 0.635 (sec). Leaf size: 41

```
DSolve[y'[x]==f[x]*y[x]^2+g'[x]*y[x]+a*f[x]*Exp[2*g[x]],y[x],x,IncludeSingularSolutions -> T
```

$$y(x) \to \sqrt{a}e^{g(x)}\tan\left(\sqrt{a}\int_1^x e^{g(K[1])}f(K[1])dK[1] + c_1\right)$$

20.9 problem 42

Internal problem ID [10635]

Book: Handbook of exact solutions for ordinary differential equations. By Polyanin and Zaitsev. Second edition
Section: Chapter 1, section 1.2. Riccati Equation. subsection 1.2.8-2. Equations containing arbitrary functions and their derivatives.
Problem number: 42.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[_Riccati]`

$$\boxed{y' - y^2 = -\frac{f''(x)}{f(x)}}$$

✓ Solution by Maple
Time used: 0.0 (sec). Leaf size: 44

```
dsolve(diff(y(x),x)=y(x)^2-diff(f(x),x$2)/f(x),y(x), singsol=all)
```

$$y(x) = \frac{-\left(\int \frac{1}{f(x)^2} dx\right) \left(\frac{d}{dx} f(x)\right) f(x) - \left(\frac{d}{dx} f(x)\right) c_1 f(x) - 1}{\left(\int \frac{1}{f(x)^2} dx + c_1\right) f(x)^2}$$

✓ Solution by Mathematica
Time used: 0.365 (sec). Leaf size: 132

```
DSolve[y'[x]==y[x]^2-f''[x]/f[x],y[x],x,IncludeSingularSolutions -> True]
```

$$\text{Solve}\left[\int_1^{y(x)} \left(\frac{1}{(f(x)K[2] + f'(x))^2} - \int_1^x \left(\frac{2(f(K[1])K[2]^2 - f''(K[1]))}{(f(K[1])K[2] + f'(K[1]))^3} - \frac{2K[2]}{(f(K[1])K[2] + f'(K[1]))^2}\right) dK[1]\right) dK[2] + \int_1^x -\frac{f(K[1])y(x)^2 - f''(K[1])}{f(K[1])\,(f(K[1])y(x) + f'(K[1]))^2} dK[1] = c_1, y(x)\right]$$

21 Chapter 1, section 1.2. Riccati Equation. subsection 1.2.9. Some Transformations

21.1 problem 1 403
21.2 problem 2 404
21.3 problem 3 405
21.4 problem 4 406
21.5 problem 5 407
21.6 problem 6 408
21.7 problem 7 409
21.8 problem 8 410
21.9 problem 9 411
21.10 problem 10 412
21.11 problem 11 413
21.12 problem 12 414
21.13 problem 13 415
21.14 problem 14 416

21.1 problem 1

Internal problem ID [10636]

Book: Handbook of exact solutions for ordinary differential equations. By Polyanin and Zaitsev. Second edition
Section: Chapter 1, section 1.2. Riccati Equation. subsection 1.2.9. Some Transformations
Problem number: 1.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_Riccati]`

$$\boxed{y' - y^2 = a^2 f(ax+b)}$$

✗ Solution by Maple

```
dsolve(diff(y(x),x)=y(x)^2+a^2*f(a*x+b),y(x), singsol=all)
```

No solution found

✗ Solution by Mathematica
Time used: 0.0 (sec). Leaf size: 0

```
DSolve[y'[x]==y[x]^2+a^2*f[a*x+b],y[x],x,IncludeSingularSolutions -> True]
```

Not solved

21.2 problem 2

Internal problem ID [10637]

Book: Handbook of exact solutions for ordinary differential equations. By Polyanin and Zaitsev. Second edition
Section: Chapter 1, section 1.2. Riccati Equation. subsection 1.2.9. Some Transformations
Problem number: 2.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_Riccati]`

$$\boxed{y' - y^2 = \frac{f\left(\frac{1}{x}\right)}{x^4}}$$

✗ Solution by Maple

```
dsolve(diff(y(x),x)=y(x)^2+1/x^4*f(1/x),y(x), singsol=all)
```

No solution found

✗ Solution by Mathematica
Time used: 0.0 (sec). Leaf size: 0

```
DSolve[y'[x]==y[x]^2+1/x^4*f[1/x],y[x],x,IncludeSingularSolutions -> True]
```

Not solved

21.3 problem 3

Internal problem ID [10638]

Book: Handbook of exact solutions for ordinary differential equations. By Polyanin and Zaitsev. Second edition
Section: Chapter 1, section 1.2. Riccati Equation. subsection 1.2.9. Some Transformations
Problem number: 3.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_Riccati]`

$$\boxed{y' - y^2 = \frac{f\left(\frac{ax+b}{cx+d}\right)}{(cx+d)^4}}$$

✗ Solution by Maple

```
dsolve(diff(y(x),x)=y(x)^2+1/(c*x+d)^4*f((a*x+b)/(c*x+d)),y(x), singsol=all)
```

No solution found

✗ Solution by Mathematica
Time used: 0.0 (sec). Leaf size: 0

```
DSolve[y'[x]==y[x]^2+1/(c*x+d)^4*f[(a*x+b)/(c*x+d)],y[x],x,IncludeSingularSolutions -> True]
```

Not solved

21.4 problem 4

Internal problem ID [10639]

Book: Handbook of exact solutions for ordinary differential equations. By Polyanin and Zaitsev. Second edition
Section: Chapter 1, section 1.2. Riccati Equation. subsection 1.2.9. Some Transformations
Problem number: 4.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_Riccati]`

$$\boxed{x^2 y' - x^4 f(x) y^2 = 1}$$

✗ Solution by Maple

```
dsolve(x^2*diff(y(x),x)=x^4*f(x)*y(x)^2+1,y(x), singsol=all)
```

No solution found

✗ Solution by Mathematica
Time used: 0.0 (sec). Leaf size: 0

```
DSolve[x^2*y'[x]==x^4*f[x]*y[x]^2+1,y[x],x,IncludeSingularSolutions -> True]
```

Not solved

21.5 problem 5

Internal problem ID [10640]

Book: Handbook of exact solutions for ordinary differential equations. By Polyanin and Zaitsev. Second edition
Section: Chapter 1, section 1.2. Riccati Equation. subsection 1.2.9. Some Transformations
Problem number: 5.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_Riccati]`

$$\boxed{x^2 y' - y^2 x^4 = x^{2n} f(a\,x^n + b) - \frac{n^2}{4} + \frac{1}{4}}$$

✗ Solution by Maple

```
dsolve(x^2*diff(y(x),x)=x^4*y(x)^2+x^(2*n)*f(a*x^n+b)+1/4*(1-n^2),y(x), singsol=all)
```

No solution found

✗ Solution by Mathematica
Time used: 0.0 (sec). Leaf size: 0

```
DSolve[x^2*y'[x]==x^4*y[x]^2+x^(2*n)*f[a*x^n+b]+1/4*(1-n^2),y[x],x,IncludeSingularSolutions
```

Not solved

21.6 problem 6

Internal problem ID [10641]

Book: Handbook of exact solutions for ordinary differential equations. By Polyanin and Zaitsev. Second edition
Section: Chapter 1, section 1.2. Riccati Equation. subsection 1.2.9. Some Transformations
Problem number: 6.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_Riccati]`

$$\boxed{y' - f(x)\,y^2 - g(x)\,y = h(x)}$$

✗ Solution by Maple

```
dsolve(diff(y(x),x)=f(x)*y(x)^2+g(x)*y(x)+h(x),y(x), singsol=all)
```

No solution found

✗ Solution by Mathematica
Time used: 0.0 (sec). Leaf size: 0

```
DSolve[y'[x]==f[x]*y[x]^2+g[x]*y[x]+h[x],y[x],x,IncludeSingularSolutions -> True]
```

Not solved

21.7 problem 7

Internal problem ID [10642]

Book: Handbook of exact solutions for ordinary differential equations. By Polyanin and Zaitsev. Second edition
Section: Chapter 1, section 1.2. Riccati Equation. subsection 1.2.9. Some Transformations
Problem number: 7.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_Riccati]`

$$\boxed{y' - y^2 = \mathrm{e}^{2\lambda x} f\left(\mathrm{e}^{\lambda x}\right) - \frac{\lambda^2}{4}}$$

✗ Solution by Maple

```
dsolve(diff(y(x),x)=y(x)^2+exp(2*lambda*x)*f(exp(lambda*x))-1/4*lambda^2,y(x), singsol=all)
```

No solution found

✗ Solution by Mathematica
Time used: 0.0 (sec). Leaf size: 0

```
DSolve[y'[x]==y[x]^2+Exp[2*\[Lambda]*x]*f[Exp[\[Lambda]*x]]-1/4*\[Lambda]^2,y[x],x,IncludeSi
```

Not solved

21.8 problem 8

Internal problem ID [10643]

Book: Handbook of exact solutions for ordinary differential equations. By Polyanin and Zaitsev. Second edition
Section: Chapter 1, section 1.2. Riccati Equation. subsection 1.2.9. Some Transformations
Problem number: 8.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_Riccati]`

$$y' - y^2 = -\frac{\lambda^2}{4} + \frac{\mathrm{e}^{2\lambda x} f\left(\frac{a\,\mathrm{e}^{\lambda x}+b}{c\,\mathrm{e}^{\lambda x}+d}\right)}{\left(c\,\mathrm{e}^{\lambda x}+d\right)^4}$$

✗ Solution by Maple

```
dsolve(diff(y(x),x)=y(x)^2-lambda^2/4+exp(2*lambda*x)/(c*exp(lambda*x)+d)^4*f((a*exp(lambda*
```

No solution found

✗ Solution by Mathematica
Time used: 0.0 (sec). Leaf size: 0

```
DSolve[y'[x]==y[x]^2-\[Lambda]^2/4+Exp[2*\[Lambda]*x]/(c*Exp[\[Lambda]*x]+d)^4*f[(a*Exp[\[La
```

Not solved

21.9 problem 9

Internal problem ID [10644]

Book: Handbook of exact solutions for ordinary differential equations. By Polyanin and Zaitsev. Second edition
Section: Chapter 1, section 1.2. Riccati Equation. subsection 1.2.9. Some Transformations
Problem number: 9.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_Riccati]`

$$\boxed{y' - y^2 = -\lambda^2 + \frac{f(\coth(\lambda x))}{\sinh(\lambda x)^4}}$$

✗ Solution by Maple

```
dsolve(diff(y(x),x)=y(x)^2-lambda^2+sinh(lambda*x)^(-4)*f(coth(lambda*x)),y(x), singsol=all)
```

No solution found

✗ Solution by Mathematica
Time used: 0.0 (sec). Leaf size: 0

```
DSolve[y'[x]==y[x]^2-\[Lambda]^2+Sinh[\[Lambda]*x]^(-4)*f[Coth[\[Lambda]*x]],y[x],x,IncludeS
```

Not solved

21.10 problem 10

Internal problem ID [10645]

Book: Handbook of exact solutions for ordinary differential equations. By Polyanin and Zaitsev. Second edition
Section: Chapter 1, section 1.2. Riccati Equation. subsection 1.2.9. Some Transformations
Problem number: 10.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_Riccati]`

$$\boxed{y' - y^2 = -\lambda^2 + \frac{f(\tanh(\lambda x))}{\cosh(\lambda x)^4}}$$

✗ Solution by Maple

```
dsolve(diff(y(x),x)=y(x)^2-lambda^2+cosh(lambda*x)^(-4)*f(tanh(lambda*x)),y(x), singsol=all)
```

No solution found

✗ Solution by Mathematica
Time used: 0.0 (sec). Leaf size: 0

```
DSolve[y'[x]==y[x]^2-\[Lambda]^2+Cosh[\[Lambda]*x]^(-4)*f[Tanh[\[Lambda]*x]],y[x],x,IncludeS
```

Not solved

21.11 problem 11

Internal problem ID [10646]

Book: Handbook of exact solutions for ordinary differential equations. By Polyanin and Zaitsev. Second edition
Section: Chapter 1, section 1.2. Riccati Equation. subsection 1.2.9. Some Transformations
Problem number: 11.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_Riccati]`

$$\boxed{x^2 y' - x^2 y^2 = f(a \ln(x) + b) + \frac{1}{4}}$$

✗ Solution by Maple

```
dsolve(x^2*diff(y(x),x)=x^2*y(x)^2+f(a*ln(x)+b)+1/4,y(x), singsol=all)
```

No solution found

✗ Solution by Mathematica
Time used: 0.0 (sec). Leaf size: 0

```
DSolve[x^2*y'[x]==x^2*y[x]^2+f[a*Log[x]+b]+1/4,y[x],x,IncludeSingularSolutions -> True]
```

Not solved

21.12 problem 12

Internal problem ID [10647]

Book: Handbook of exact solutions for ordinary differential equations. By Polyanin and Zaitsev. Second edition
Section: Chapter 1, section 1.2. Riccati Equation. subsection 1.2.9. Some Transformations
Problem number: 12.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_Riccati]`

$$y' - y^2 = \lambda^2 + \frac{f(\cot(\lambda x))}{\sin(\lambda x)^4}$$

✗ Solution by Maple

```
dsolve(diff(y(x),x)=y(x)^2+lambda^2+sin(lambda*x)^(-4)*f(cot(lambda*x)),y(x), singsol=all)
```

No solution found

✗ Solution by Mathematica
Time used: 0.0 (sec). Leaf size: 0

```
DSolve[y'[x]==y[x]^2+\[Lambda]^2+Sin[\[Lambda]*x]^(-4)*f[Cot[\[Lambda]*x]],y[x],x,IncludeSin
```

Not solved

21.13 problem 13

Internal problem ID [10648]

Book: Handbook of exact solutions for ordinary differential equations. By Polyanin and Zaitsev. Second edition
Section: Chapter 1, section 1.2. Riccati Equation. subsection 1.2.9. Some Transformations
Problem number: 13.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_Riccati]`

$$y' - y^2 = \lambda^2 + \frac{f(\tan(\lambda x))}{\cos(\lambda x)^4}$$

✗ Solution by Maple

```
dsolve(diff(y(x),x)=y(x)^2+lambda^2+cos(lambda*x)^(-4)*f(tan(lambda*x)),y(x), singsol=all)
```

No solution found

✗ Solution by Mathematica
Time used: 0.0 (sec). Leaf size: 0

```
DSolve[y'[x]==y[x]^2+\[Lambda]^2+Cos[\[Lambda]*x]^(-4)*f[Tan[\[Lambda]*x]],y[x],x,IncludeSin
```

Not solved

21.14 problem 14

Internal problem ID [10649]

Book: Handbook of exact solutions for ordinary differential equations. By Polyanin and Zaitsev. Second edition
Section: Chapter 1, section 1.2. Riccati Equation. subsection 1.2.9. Some Transformations
Problem number: 14.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_Riccati]`

$$y' - y^2 = \lambda^2 + \frac{f\left(\frac{\sin(\lambda x+a)}{\sin(\lambda x+b)}\right)}{\sin\left(\lambda x+b\right)^4}$$

✗ Solution by Maple

```
dsolve(diff(y(x),x)=y(x)^2+lambda^2+sin(lambda*x+b)^(-4)*f(sin(lambda*x+a)/sin(lambda*x+b)),
```

No solution found

✗ Solution by Mathematica
Time used: 0.0 (sec). Leaf size: 0

```
DSolve[y'[x]==y[x]^2+\[Lambda]^2+Sin[\[Lambda]*x+b]^(-4)*f[Sin[\[Lambda]*x+a]/Sin[\[Lambda]*
```

Not solved

22 Chapter 1, section 1.3. Abel Equations of the Second Kind. Form $yy' - y = f(x)$. subsection 1.3.1-2. Solvable equations and their solutions

22.1 problem 1 420
22.2 problem 2 421
22.3 problem 3 422
22.4 problem 4 424
22.5 problem 5 425
22.6 problem 6 426
22.7 problem 7 427
22.8 problem 8 428
22.9 problem 9 429
22.10problem 10 430
22.11problem 11 431
22.12problem 12 433
22.13problem 13 434
22.14problem 14 435
22.15problem 15 437
22.16problem 16 438
22.17problem 17 439
22.18problem 18 440
22.19problem 19 441
22.20problem 20 443
22.21problem 21 444
22.22problem 22 445
22.23problem 23 446
22.24problem 24 447
22.25problem 25 448
22.26problem 26 449
22.27problem 27 451
22.28problem 28 452
22.29problem 29 453
22.30problem 30 454
22.31problem 31 455
22.32problem 32 456
22.33problem 33 458
22.34problem 34 460

22.35problem 35 461
22.36problem 36 462
22.37problem 37 463
22.38problem 38 464
22.39problem 39 465
22.40problem 40 466
22.41problem 41 467
22.42problem 42 468
22.43problem 43 469
22.44problem 44 470
22.45problem 45 472
22.46problem 46 474
22.47problem 47 476
22.48problem 48 477
22.49problem 49 478
22.50problem 50 479
22.51problem 51 480
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22.53problem 53 482
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22.56problem 56 486
22.57problem 57 487
22.58problem 58 488
22.59problem 59 489
22.60problem 60 490
22.61problem 61 491
22.62problem 62 492
22.63problem 63 493
22.64problem 64 494
22.65problem 65 495
22.66problem 66 496
22.67problem 67 497
22.68problem 68 498
22.69problem 69 499
22.70problem 70 500
22.71problem 71 501
22.72problem 72 502
22.73problem 73 503

22.74problem 74 504
22.75problem 75 505
22.76problem 76 506

22.1 problem 1

Internal problem ID [10650]

Book: Handbook of exact solutions for ordinary differential equations. By Polyanin and Zaitsev. Second edition
Section: Chapter 1, section 1.3. Abel Equations of the Second Kind. Form $yy' - y = f(x)$. subsection 1.3.1-2. Solvable equations and their solutions
Problem number: 1.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_quadrature]`

$$\boxed{yy' - y = A}$$

✓ Solution by Maple
Time used: 0.015 (sec). Leaf size: 30

```
dsolve(y(x)*diff(y(x),x)-y(x)=A,y(x), singsol=all)
```

$$y(x) = -A\left(\text{LambertW}\left(-\frac{\mathrm{e}^{\frac{-A-c_1-x}{A}}}{A}\right)+1\right)$$

✓ Solution by Mathematica
Time used: 60.032 (sec). Leaf size: 28

```
DSolve[y[x]*y'[x]-y[x]==A,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to -A\left(1+W\left(-\frac{e^{-\frac{A+x+c_1}{A}}}{A}\right)\right)$$

22.2 problem 2

Internal problem ID [10651]

Book: Handbook of exact solutions for ordinary differential equations. By Polyanin and Zaitsev. Second edition
Section: Chapter 1, section 1.3. Abel Equations of the Second Kind. Form $yy' - y = f(x)$. subsection 1.3.1-2. Solvable equations and their solutions
Problem number: 2.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[[_homogeneous, 'class C'], _rational, [_Abel, '2nd type', 'cl`

$$\boxed{yy' - y = Ax + B}$$

✓ Solution by Maple
Time used: 4.266 (sec). Leaf size: 68

```
dsolve(y(x)*diff(y(x),x)-y(x)=A*x+B,y(x), singsol=all)
```

$$y(x) = -\frac{(xA+B)\,\mathrm{RootOf}\left(\_Z^2 - A + \_Z + \mathrm{e}^{\mathrm{RootOf}\left((xA+B)^2\left(-2\,\mathrm{e}^{-\_Z}\cosh\left((\_Z+2\ln(xA+B)+2c_1)\sqrt{4A+1}\right)+4A-2\,\mathrm{e}^{-\_Z}+1\right)\right)}\right)}{A}$$

✓ Solution by Mathematica
Time used: 0.184 (sec). Leaf size: 88

```
DSolve[y[x]*y'[x]-y[x]==A*x+B,y[x],x,IncludeSingularSolutions -> True]
```

$$\text{Solve}\left[-\frac{\frac{2\arctan\left(\frac{\frac{2Ay(x)}{Ax+B}-1}{\sqrt{-4A-1}}\right)}{\sqrt{-4A-1}} + \log\left(-\frac{Ay(x)^2}{(Ax+B)^2} + \frac{y(x)}{Ax+B} + 1\right)}{2A} = \frac{\log(Ax+B)}{A} + c_1, y(x)\right]$$

22.3 problem 3

Internal problem ID [10652]

Book: Handbook of exact solutions for ordinary differential equations. By Polyanin and Zaitsev. Second edition
Section: Chapter 1, section 1.3. Abel Equations of the Second Kind. Form $yy' - y = f(x)$. subsection 1.3.1-2. Solvable equations and their solutions
Problem number: 3.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_rational, [_Abel, '2nd type', 'class B']]`

$$\boxed{yy' - y = -\frac{2x}{9} + A + \frac{B}{\sqrt{x}}}$$

✓ Solution by Maple
Time used: 0.0 (sec). Leaf size: 90

```
dsolve(y(x)*diff(y(x),x)-y(x)=-2/9*x+A+B*x^(-1/2),y(x), singsol=all)
```

$$y(x) = -\frac{9\left(A\sqrt{x} + B - \frac{2x^{\frac{3}{2}}}{9}\right)A}{3A\sqrt{x} + 3\,\mathrm{RootOf}\left(18A^3\left(\int^{-Z} \frac{1}{-2\_a^3B^2+9\_aA^3-9A^3}d\_a\right) - 9A\left(\int \frac{1}{9xA-2x^2+9B\sqrt{x}}dx\right) + 2c_1\right)B}$$

✓ Solution by Mathematica

Time used: 8.154 (sec). Leaf size: 415

```
DSolve[y[x]*y'[x]-y[x]==-2/9*x+A+B*x^(-1/2),y[x],x,IncludeSingularSolutions -> True]
```

$$\text{Solve}\left[6\text{RootSum}\left[8\#1^6-72\#1^4A-36\#1^4y(x)-72\#1^3B+162\#1^2A^2\right.\right.$$
$$\left.+162\#1^2Ay(x)+54\#1^2y(x)^2+324\#1AB+162\#1By(x)-81Ay(x)^2+162B^2$$
$$\left.-27y(x)^3\&,\frac{-2\#1^3\log\left(\sqrt{x}-\#1\right)+9\#1A\log\left(\sqrt{x}-\#1\right)+9B\log\left(\sqrt{x}-\#1\right)+9\#1y(x)\log\left(\sqrt{x}-\right.\right.\right.$$
$$\left.\left.\left.8\#1^5-48\#1^3A-24\#1^3y(x)-36\#1^2B+54\#1A^2+54\#1Ay(x)+18\#1y(x)^2+54AB+\right.\right.\right.$$
$$\left.\left.+\int_1^{y(x)}\left(\frac{162K[1]}{8x^3-72Ax^2-36K[1]x^2-72Bx^{3/2}+162A^2x+54K[1]^2x+162AK[1]x+324AB\sqrt{x}+162BK\right.\right.\right.$$
$$\left.\left.\left.+\frac{162K[1]}{-8x^3+72Ax^2+36K[1]x^2+72Bx^{3/2}-162A^2x-54K[1]^2x-162AK[1]x-324AB\sqrt{x}-162BK[1]\sqrt{}\right.\right.\right.$$

22.4 problem 4

Internal problem ID [10653]

Book: Handbook of exact solutions for ordinary differential equations. By Polyanin and Zaitsev. Second edition
Section: Chapter 1, section 1.3. Abel Equations of the Second Kind. Form $yy' - y = f(x)$. subsection 1.3.1-2. Solvable equations and their solutions
Problem number: 4.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_rational, [_Abel, '2nd type', 'class B']]`

$$\boxed{yy' - y = 2A\left(\sqrt{x} + 4A + \frac{3A^2}{\sqrt{x}}\right)}$$

✓ Solution by Maple
Time used: 0.015 (sec). Leaf size: 120

```
dsolve(y(x)*diff(y(x),x)-y(x)=2*A*(x^(1/2)+4*A+3*A^2*x^(-1/2)),y(x), singsol=all)
```

$$\frac{-\sqrt{\frac{-6A^2-8A\sqrt{x}-2x+2y(x)}{y(x)}}\sqrt{2}+4\sqrt{-\frac{A^2}{y(x)}}\operatorname{arctanh}\left(\frac{\sqrt{-\frac{A^2}{y(x)}}\left(3A+\sqrt{x}\right)}{\sqrt{\frac{-3A^2-4A\sqrt{x}-x+y(x)}{y(x)}}A}\right)+c_1\sqrt{-\frac{A^2}{y(x)}}}{\sqrt{-\frac{A^2}{y(x)}}}$$
$$= 0$$

✗ Solution by Mathematica
Time used: 0.0 (sec). Leaf size: 0

```
DSolve[y[x]*y'[x]-y[x]==2*A*(x^(1/2)+4*A+3*A^2*x^(-1/2)),y[x],x,IncludeSingularSolutions ->
```

Not solved

22.5 problem 5

Internal problem ID [10654]

Book: Handbook of exact solutions for ordinary differential equations. By Polyanin and Zaitsev. Second edition
Section: Chapter 1, section 1.3. Abel Equations of the Second Kind. Form $yy' - y = f(x)$. subsection 1.3.1-2. Solvable equations and their solutions
Problem number: 5.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_rational, [_Abel, '2nd type', 'class B']]`

$$\boxed{yy' - y = Ax + \frac{B}{x} - \frac{B^2}{x^3}}$$

✓ Solution by Maple
Time used: 0.0 (sec). Leaf size: 179

```
dsolve(y(x)*diff(y(x),x)-y(x)=A*x+B/x-B^2*x^(-3),y(x), singsol=all)
```

$$\frac{(-y(x)\,x^2B - B^2x)\left(\int^{-\frac{x^2}{2y(x)x+2B}} \frac{\mathrm{e}^{\frac{2\,\operatorname{arctanh}\left(\frac{4A\_a-1}{\sqrt{4A+1}}\right)}{\sqrt{4A+1}}}(4A\_a^2-2\_a-1)}{\_a^2}d\_a\right) + 2y(x)\left(-y(x)^2x^2 + (x^3-2Bx)\right.}{x\,(y\,(x)\,x+B)}$$

$$= 0$$

✗ Solution by Mathematica
Time used: 0.0 (sec). Leaf size: 0

```
DSolve[y[x]*y'[x]-y[x]==A*x+B/x-B^2*x^(-3),y[x],x,IncludeSingularSolutions -> True]
```

Not solved

22.6 problem 6

Internal problem ID [10655]

Book: Handbook of exact solutions for ordinary differential equations. By Polyanin and Zaitsev. Second edition
Section: Chapter 1, section 1.3. Abel Equations of the Second Kind. Form $yy' - y = f(x)$. subsection 1.3.1-2. Solvable equations and their solutions
Problem number: 6.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_rational, [_Abel, '2nd type', 'class A']]`

$$\boxed{yy' - y = A\,x^{k-1} - kB\,x^k + k\,B^2 x^{2k-1}}$$

✗ Solution by Maple

```
dsolve(y(x)*diff(y(x),x)-y(x)=A*x^(k-1)-k*B*x^k+k*B^2*x^(2*k-1),y(x), singsol=all)
```

No solution found

✗ Solution by Mathematica
Time used: 0.0 (sec). Leaf size: 0

```
DSolve[y[x]*y'[x]-y[x]==A*x^(k-1)-k*B*x^k+k*B^2*x^(2*k-1),y[x],x,IncludeSingularSolutions ->
```

Not solved

22.7 problem 7

Internal problem ID [10656]

Book: Handbook of exact solutions for ordinary differential equations. By Polyanin and Zaitsev. Second edition
Section: Chapter 1, section 1.3. Abel Equations of the Second Kind. Form $yy' - y = f(x)$. subsection 1.3.1-2. Solvable equations and their solutions
Problem number: 7.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_rational, [_Abel, '2nd type', 'class B']]`

$$\boxed{yy' - y = \frac{A}{x} - \frac{A^2}{x^3}}$$

✓ Solution by Maple
Time used: 0.0 (sec). Leaf size: 118

```
dsolve(y(x)*diff(y(x),x)-y(x)=A*x^(-1)-A^2*x^(-3),y(x), singsol=all)
```

$$y(x) = -\frac{\left(-c_1 x^2 + A\,\mathrm{e}^{\mathrm{RootOf}\left(2\_ZA\,\mathrm{e}^{2\_Z} - x^2\mathrm{e}^{2\_Z} + 2c_1x^2\mathrm{e}^{\_Z} - c_1^2x^2 - 2A\,\mathrm{e}^{2\_Z} + 2Ac_1\mathrm{e}^{\_Z}\right)}\right)\mathrm{e}^{-\mathrm{RootOf}\left(2\_ZA\,\mathrm{e}^{2\_Z} - x^2\mathrm{e}^{2\_Z} + 2c_1x^2\mathrm{e}^{\_Z} - c_1^2}}{x}$$

✓ Solution by Mathematica
Time used: 0.569 (sec). Leaf size: 63

```
DSolve[y[x]*y'[x]-y[x]==A*x^(-1)-A^2*x^(-3),y[x],x,IncludeSingularSolutions -> True]
```

$$\text{Solve}\left[x^2\left(-\frac{1}{A} + \frac{2x^2\log\left(\frac{x^2}{A+xy(x)}\right) + 2A - c_1x^2 + 2xy(x)}{(A - x^2 + xy(x))^2}\right) = 0, y(x)\right]$$

22.8 problem 8

Internal problem ID [10657]

Book: Handbook of exact solutions for ordinary differential equations. By Polyanin and Zaitsev. Second edition
Section: Chapter 1, section 1.3. Abel Equations of the Second Kind. Form $yy' - y = f(x)$. subsection 1.3.1-2. Solvable equations and their solutions
Problem number: 8.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[[_Abel, '2nd type', 'class A']]`

$$\boxed{yy' - y = A + B\,\mathrm{e}^{-\frac{2x}{A}}}$$

✓ Solution by Maple
Time used: 0.0 (sec). Leaf size: 76

```
dsolve(y(x)*diff(y(x),x)-y(x)=A+B*exp(-2*x/A),y(x), singsol=all)
```

$$c_1 - 2\arctan\left(\frac{y(x)+A}{y(x)\sqrt{\frac{-AB\,\mathrm{e}^{-\frac{2x}{A}}-(y(x)+A)^2}{y(x)^2}}}\right) A - 2\sqrt{\frac{-AB\,\mathrm{e}^{-\frac{2x}{A}}-(y(x)+A)^2}{y(x)^2}}\,y(x) = 0$$

✗ Solution by Mathematica
Time used: 0.0 (sec). Leaf size: 0

```
DSolve[y[x]*y'[x]-y[x]==A+B*Exp[-2*x/A],y[x],x,IncludeSingularSolutions -> True]
```

Not solved

22.9 problem 9

Internal problem ID [10658]

Book: Handbook of exact solutions for ordinary differential equations. By Polyanin and Zaitsev. Second edition
Section: Chapter 1, section 1.3. Abel Equations of the Second Kind. Form $yy' - y = f(x)$. subsection 1.3.1-2. Solvable equations and their solutions
Problem number: 9.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[[_Abel, '2nd type', 'class A']]`

$$\boxed{yy' - y = A\left(\mathrm{e}^{\frac{2x}{A}} - 1\right)}$$

✓ Solution by Maple
Time used: 0.0 (sec). Leaf size: 82

```
dsolve(y(x)*diff(y(x),x)-y(x)=A*(exp(2*x/A)-1),y(x), singsol=all)
```

$$c_1 + 2A \arctan\left(\frac{A - y(x)}{y(x)\sqrt{\frac{\mathrm{e}^{\frac{2x}{A}}A^2 - (A-y(x))^2}{y(x)^2}}}\right) + 2y(x)\sqrt{\frac{\mathrm{e}^{\frac{2x}{A}}A^2 - (A-y(x))^2}{y(x)^2}} = 0$$

✗ Solution by Mathematica
Time used: 0.0 (sec). Leaf size: 0

```
DSolve[y[x]*y'[x]-y[x]==A*(Exp[2*x/A]-1),y[x],x,IncludeSingularSolutions -> True]
```

{}

22.10 problem 10

Internal problem ID [10659]

Book: Handbook of exact solutions for ordinary differential equations. By Polyanin and Zaitsev. Second edition
Section: Chapter 1, section 1.3. Abel Equations of the Second Kind. Form $yy' - y = f(x)$. subsection 1.3.1-2. Solvable equations and their solutions
Problem number: 10.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_rational, [_Abel, '2nd type', 'class A']]`

$$\boxed{yy' - y = -\frac{2(m+1)}{(m+3)^2} + A\,x^m}$$

✗ Solution by Maple

```
dsolve(y(x)*diff(y(x),x)-y(x)=-(2*(m+1))/(m+3)^2+A*x^m,y(x), singsol=all)
```

No solution found

✗ Solution by Mathematica
Time used: 0.0 (sec). Leaf size: 0

```
DSolve[y[x]*y'[x]-y[x]==-(2*(m+1))/(m+3)^2+A*x^m,y[x],x,IncludeSingularSolutions -> True]
```

Not solved

22.11 problem 11

Internal problem ID [10660]

Book: Handbook of exact solutions for ordinary differential equations. By Polyanin and Zaitsev. Second edition
Section: Chapter 1, section 1.3. Abel Equations of the Second Kind. Form $yy' - y = f(x)$. subsection 1.3.1-2. Solvable equations and their solutions
Problem number: 11.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_rational, [_Abel, '2nd type', 'class B']]`

$$\boxed{yy' - y = -\frac{2x}{9} + 6A^2\left(1 + \frac{2A}{\sqrt{x}}\right)}$$

✓ Solution by Maple
Time used: 0.0 (sec). Leaf size: 354

```
dsolve(y(x)*diff(y(x),x)-y(x)=-2/9*x+6*A^2*(1+2*A*x^(-1/2)),y(x), singsol=all)
```

$y(x)$

$$= \frac{}{3\,\mathrm{e}^{\mathrm{RootOf}\left(36A^2\mathrm{e}^{-Z}\ln(2)+18A^2\mathrm{e}^{-Z}\ln\left(\frac{(3A-\sqrt{x})(6A-\sqrt{x})\left(36A^2-x\right)}{\left(9A^2-x\right)(6A+\sqrt{x})(3A+\sqrt{x})\left(\mathrm{e}^{-Z}+9\right)^2}\right)+108A^2c_1\mathrm{e}^{-Z}+36A^2\mathrm{e}^{-Z}\_Z+6A\sqrt{x}\,\mathrm{e}^{-Z}\ln(2)+3A\sqrt{x}\,\mathrm{e}^{-Z}\ln\right.}$$

✓ Solution by Mathematica

Time used: 12.331 (sec). Leaf size: 488

```
DSolve[y[x]*y'[x]-y[x]==-2/9*x+6*A^2*(1+2*A*x^(-1/2)),y[x],x,IncludeSingularSolutions -> Tru
```

$$\text{Solve}\left[\frac{2^{2/3}\left(\frac{-\frac{6(6A-\sqrt{x})(3A+\sqrt{x})^2}{y(x)}-9\sqrt{x}}{\sqrt[3]{A^3}}+54\right)\left(\frac{6(6A-\sqrt{x})(3A+\sqrt{x})^2+9\sqrt{x}y(x)}{\sqrt[3]{A^3}y(x)}+27\right)\left(-\frac{\left(3\left(3\sqrt[3]{A^3}+\sqrt{x}\right)y(x)+2(6A-\sqrt{x}\right.\right.}{6561\left(\frac{\left(2(6A-\right.\right.}\right.\right.\right.

22.12 problem 12

Internal problem ID [10661]

Book: Handbook of exact solutions for ordinary differential equations. By Polyanin and Zaitsev. Second edition
Section: Chapter 1, section 1.3. Abel Equations of the Second Kind. Form $yy' - y = f(x)$. subsection 1.3.1-2. Solvable equations and their solutions
Problem number: 12.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_rational, [_Abel, '2nd type', 'class B']]`

$$yy' - y = \frac{2m-2}{(m-3)^2} + \frac{2A\left(m(m+3)\sqrt{x} + (4m^2+3m+9)A + \frac{3m(m+3)A^2}{\sqrt{x}}\right)}{(m-3)^2}$$

✗ Solution by Maple

```
dsolve(y(x)*diff(y(x),x)-y(x)=2*(m-1)/(m-3)^2+2*A/(m-3)^2*(m*(m+3)*x^(1/2)+(4*m^2+3*m+9)*A+3
```

No solution found

✗ Solution by Mathematica
Time used: 0.0 (sec). Leaf size: 0

```
DSolve[y[x]*y'[x]-y[x]==2*(m-1)/(m-3)^2+2*A/(m-3)^2*(m*(m+3)*x^(1/2)+(4*m^2+3*m+9)*A+3*m*(m+
```

Not solved

22.13 problem 13

Internal problem ID [10662]

Book: Handbook of exact solutions for ordinary differential equations. By Polyanin and Zaitsev. Second edition
Section: Chapter 1, section 1.3. Abel Equations of the Second Kind. Form $yy' - y = f(x)$. subsection 1.3.1-2. Solvable equations and their solutions
Problem number: 13.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_rational, [_Abel, '2nd type', 'class B']]`

$$\boxed{yy' - y = \frac{(2m+1)\,x}{4m^2} + \frac{A}{x} - \frac{A^2}{x^3}}$$

✓ Solution by Maple
Time used: 0.0 (sec). Leaf size: 166

```
dsolve(y(x)*diff(y(x),x)-y(x)=(2*m+1)/(4*m^2)*x+A*1/x-A^2*1/(x^3),y(x), singsol=all)
```

$$\frac{2^{-\frac{m}{1+m}}\,y(x)\left(\frac{-2y(x)mx-2Am-x^2}{2y(x)x+2A}\right)^{\frac{1}{1+m}}\left(y(x)\,x+A\right)\left(\frac{(-1-2m)x^2+2y(x)mx+2Am}{y(x)x+A}\right)^{\frac{1+2m}{1+m}} - x\left(A\left(\int^{-\frac{x^2}{2y(x)x+2A}}\frac{(-m+\_$$

$$= 0$$

✗ Solution by Mathematica
Time used: 0.0 (sec). Leaf size: 0

```
DSolve[y[x]*y'[x]-y[x]==(2*m+1)/(4*m^2)*x+A*1/x-A^2*1/(x^3),y[x],x,IncludeSingularSolutions
```

Not solved

22.14 problem 14

Internal problem ID [10663]

Book: Handbook of exact solutions for ordinary differential equations. By Polyanin and Zaitsev. Second edition
Section: Chapter 1, section 1.3. Abel Equations of the Second Kind. Form $yy' - y = f(x)$. subsection 1.3.1-2. Solvable equations and their solutions
Problem number: 14.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_rational, [_Abel, ‘2nd type‘, ‘class A‘]]`

$$\boxed{yy' - y = \frac{4}{9}x + 2A\,x^2 + 2A^2x^3}$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 177

```
dsolve(y(x)*diff(y(x),x)-y(x)=4/9*x+2*A*x^2+2*A^2*x^3,y(x), singsol=all)
```

$$-\frac{27\left(Ay(x)\,3^{\frac{1}{4}}\left(xA+\frac{1}{3}\right)\left(\frac{A\left(9A^2x^2-9y(x)A+9xA+2\right)\left(3A\,x^2+3y(x)+x\right)}{\left(-9y(x)A+3xA+1\right)^2}\right)^{\frac{1}{4}}-\frac{\left(\int^{\frac{(3xA+1)^2}{-9y(x)A+3xA+1}}\frac{\left(\_a^2-1\right)^{\frac{1}{4}}}{\sqrt{\_a}}d\_a+c_1\right)\left(\frac{1}{3}-3y(\right.}{9}\right.}{\sqrt{\frac{(3xA+1)^2}{-9y(x)A+3xA+1}}\left(-9y\left(x\right)A+3xA+1\right)}$$

$= 0$

✓ Solution by Mathematica

Time used: 3.439 (sec). Leaf size: 170

```
DSolve[y[x]*y'[x]-y[x]==4/9*x+2*A*x^2+2*A^2*x^3,y[x],x,IncludeSingularSolutions -> True]
```

$$\text{Solve}\left[\sqrt[4]{\frac{(-9Ay(x)+3Ax+1)^2}{(3Ax+1)^4}}-1\left(\frac{(-9Ay(x)+3Ax+1)\,\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{3}{4},\frac{3}{2},\frac{(3Ax}{}\right.\right.\right.$$

$$\left.\left.\left.\frac{}{2\sqrt[4]{3}(3Ax+1)\sqrt{(3Ax+1)^2}\sqrt[4]{\frac{A\,(6(3Ax+1)y(x)-27Ay(x)^2+x}{(3Ax+1)^4}}}\right.\right.\right.$$

$$\left.\left.+\sqrt{(3Ax+1)^2}\right)+c_1=0,y(x)\right]$$

22.15 problem 15

Internal problem ID [10664]

Book: Handbook of exact solutions for ordinary differential equations. By Polyanin and Zaitsev. Second edition
Section: Chapter 1, section 1.3. Abel Equations of the Second Kind. Form $yy' - y = f(x)$. subsection 1.3.1-2. Solvable equations and their solutions
Problem number: 15.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_rational, [_Abel, '2nd type', 'class B']]`

$$\boxed{yy' - y = -\frac{3x}{16} + \frac{5A}{x^{\frac{1}{3}}} - \frac{12A^2}{x^{\frac{5}{3}}}}$$

✗ Solution by Maple

```
dsolve(y(x)*diff(y(x),x)-y(x)=-3/16*x+5*A*x^(-1/3)-12*A^2*x^(-5/3),y(x), singsol=all)
```

No solution found

✗ Solution by Mathematica
Time used: 0.0 (sec). Leaf size: 0

```
DSolve[y[x]*y'[x]-y[x]==-3/16*x+5*A*x^(-1/3)-12*A^2*x^(-5/3),y[x],x,IncludeSingularSolutions
```

Not solved

22.16 problem 16

Internal problem ID [10665]

Book: Handbook of exact solutions for ordinary differential equations. By Polyanin and Zaitsev. Second edition
Section: Chapter 1, section 1.3. Abel Equations of the Second Kind. Form $yy' - y = f(x)$. subsection 1.3.1-2. Solvable equations and their solutions
Problem number: 16.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_rational, [_Abel, '2nd type', 'class B']]`

$$\boxed{yy' - y = \frac{A}{x}}$$

✓ Solution by Maple
Time used: 0.0 (sec). Leaf size: 57

```
dsolve(y(x)*diff(y(x),x)-y(x)=A*1/x,y(x), singsol=all)
```

$$\frac{\operatorname{erf}\left(\frac{(y(x)-x)\sqrt{2}}{2\sqrt{-A}}\right)\sqrt{2}\sqrt{\pi}\,x - 2\,\mathrm{e}^{\frac{(y(x)-x)^2}{2A}}\sqrt{-A} + c_1 x}{x} = 0$$

✓ Solution by Mathematica
Time used: 0.827 (sec). Leaf size: 64

```
DSolve[y[x]*y'[x]-y[x]==A*1/x,y[x],x,IncludeSingularSolutions -> True]
```

$$\text{Solve}\left[-\frac{x}{\sqrt{A}} = \frac{2e^{\frac{(x-y(x))^2}{2A}}}{\sqrt{2\pi}\operatorname{erfi}\left(\frac{y(x)-x}{\sqrt{2}\sqrt{A}}\right) + 2c_1}, y(x)\right]$$

22.17 problem 17

Internal problem ID [10666]

Book: Handbook of exact solutions for ordinary differential equations. By Polyanin and Zaitsev. Second edition
Section: Chapter 1, section 1.3. Abel Equations of the Second Kind. Form $yy' - y = f(x)$. subsection 1.3.1-2. Solvable equations and their solutions
Problem number: 17.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_rational, [_Abel, '2nd type', 'class B']]`

$$\boxed{yy' - y = -\frac{x}{4} + \frac{A\left(\sqrt{x} + 5A + \frac{3A^2}{\sqrt{x}}\right)}{4}}$$

✓ Solution by Maple
Time used: 0.0 (sec). Leaf size: 257

```
dsolve(y(x)*diff(y(x),x)-y(x)=-1/4*x+1/4*A*(x^(1/2)+5*A+3*A^2*x^(-1/2)),y(x), singsol=all)
```

$$3\left(-4\left(\left(\int \frac{6A\sqrt{x}-2x+3y(x)}{12A^2-4A\sqrt{x}+2y(x)} \frac{e^{-\frac{2}{2\_a+1}}\sqrt{2\_a+1}}{\sqrt{2\_a-3}} d\_a\right) A + \frac{c_1}{2}\right)\left(A^2 - \frac{A\sqrt{x}}{3} + \frac{y(x)}{6}\right)\sqrt{-\frac{(3A-\sqrt{x})^2}{6A^2-2A\sqrt{x}+y(x)}} + \frac{\sqrt{\frac{3A^2+2A}{6A^2-}}}{\sqrt{-\frac{(3A-\sqrt{x})^2}{6A^2-2A\sqrt{x}+y(x)}}\left(6A^2 - 2A\sqrt{x} + y(x)\right)}\right)$$

$= 0$

✗ Solution by Mathematica
Time used: 0.0 (sec). Leaf size: 0

```
DSolve[y[x]*y'[x]-y[x]==-1/4*x+1/4*A*(x^(1/2)+5*A+3*A^2*x^(-1/2)),y[x],x,IncludeSingularSolu
```

Not solved

22.18 problem 18

Internal problem ID [10667]

Book: Handbook of exact solutions for ordinary differential equations. By Polyanin and Zaitsev. Second edition
Section: Chapter 1, section 1.3. Abel Equations of the Second Kind. Form $yy' - y = f(x)$. subsection 1.3.1-2. Solvable equations and their solutions
Problem number: 18.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[[_Abel, '2nd type', 'class B']]`

$$\boxed{yy' - y = \frac{2a^2}{\sqrt{8a^2 + x^2}}}$$

✓ Solution by Maple
Time used: 0.016 (sec). Leaf size: 715

```
dsolve(y(x)*diff(y(x),x)-y(x)=2*a^2/sqrt(x^2+8*a^2),y(x), singsol=all)
```

$$512\left(-\frac{33(a^4+\frac{23}{66}a^2x^2+\frac{1}{66}x^4)x\sqrt{8a^2+x^2}}{64}+a^6+\frac{75a^4x^2}{64}+\frac{27a^2x^4}{128}+\frac{x^6}{128}\right)\mathrm{e}^{-\frac{(-y(x)+x)^2\left(-64\sqrt{8a^2+x^2}\,a^6-108\sqrt{8a^2+x^2}\,a^4x^2-25\sqrt{8a^2+}\right.}{2\left(128a^6+150a^4x^2-66\sqrt{8a^2+x^2}\,a^4x+27a^2x^4-23\sqrt{8a^2+x^2}\right.}}$$

$$= 0$$

✗ Solution by Mathematica
Time used: 0.0 (sec). Leaf size: 0

```
DSolve[y[x]*y'[x]-y[x]==2*a^2/Sqrt[x^2+8*a^2],y[x],x,IncludeSingularSolutions -> True]
```

Not solved

22.19 problem 19

Internal problem ID [10668]

Book: Handbook of exact solutions for ordinary differential equations. By Polyanin and Zaitsev. Second edition
Section: Chapter 1, section 1.3. Abel Equations of the Second Kind. Form $yy' - y = f(x)$. subsection 1.3.1-2. Solvable equations and their solutions
Problem number: 19.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_rational, [_Abel, '2nd type', 'class B']]`

$$\boxed{yy' - y = 2x + \frac{A}{x^2}}$$

✓ Solution by Maple
Time used: 0.0 (sec). Leaf size: 170

```
dsolve(y(x)*diff(y(x),x)-y(x)=2*x+A*x^(-2),y(x), singsol=all)
```

$$\frac{6\left(\sqrt{3}\,\operatorname{arctanh}\left(\frac{\sqrt{\frac{x\left(A^2\right)^{\frac{1}{3}}}{A}}\left(-2x+y(x)\right)}{\sqrt{\frac{\left(4x^3-4y(x)x^2+y(x)^2x+2A\right)\left(A^2\right)^{\frac{1}{3}}}{y(x)^2A}}\,y(x)}\right)A+\frac{c_1}{6}\right)x\sqrt{\frac{x(A^2)^{\frac{1}{3}}}{A}}+2\sqrt{3}\,y(x)\left(-x^3-\frac{y(x)x^2}{2}+\frac{y(x)^2x}{2}-\right.}{\sqrt{\frac{x(A^2)^{\frac{1}{3}}}{A}}\,x}$$

$$= 0$$

✓ Solution by Mathematica

Time used: 2.08 (sec). Leaf size: 233

```
DSolve[y[x]*y'[x]-y[x]==2*x+A*x^(-2),y[x],x,IncludeSingularSolutions -> True]
```

$$\text{Solve}\left[c_1 = -\frac{i\sqrt{-\frac{2A+4x^3-4x^2y(x)+xy(x)^2}{A}}\left(-6\sqrt{A}x^{3/2}\text{arcsinh}\left(\frac{\sqrt{x}(2x-y(x))}{\sqrt{2}\sqrt{A}}\right)+x^2(-y(x))\sqrt{\frac{2A+4x^3-4x^2y(x)+xy(x)^2}{A}}+xy(x)^2\right.}{4\sqrt{A}x^{3/2}\sqrt{\frac{2A+4x^3-4x^2y(x)+xy(x)^2}{A}}}\right.$$

22.20 problem 20

Internal problem ID [10669]

Book: Handbook of exact solutions for ordinary differential equations. By Polyanin and Zaitsev. Second edition
Section: Chapter 1, section 1.3. Abel Equations of the Second Kind. Form $yy' - y = f(x)$. subsection 1.3.1-2. Solvable equations and their solutions
Problem number: 20.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_rational, [_Abel, '2nd type', 'class B']]`

$$\boxed{yy' - y = -\frac{6X}{25} + \frac{2A\left(2\sqrt{x} + 19A + \frac{6A^2}{\sqrt{x}}\right)}{25}}$$

✗ Solution by Maple

```
dsolve(y(x)*diff(y(x),x)-y(x)=-6/25*X+2/25*A*(2*x^(1/2)+19*A+6*A^2*x^(-1/2)),y(x), singsol=a
```

No solution found

✗ Solution by Mathematica
Time used: 0.0 (sec). Leaf size: 0

```
DSolve[y[x]*y'[x]-y[x]==-6/25*X+2/25*A*(2*x^(1/2)+19*A+6*A^2*x^(-1/2)),y[x],x,IncludeSingula
```

Not solved

22.21 problem 21

Internal problem ID [10670]

Book: Handbook of exact solutions for ordinary differential equations. By Polyanin and Zaitsev. Second edition
Section: Chapter 1, section 1.3. Abel Equations of the Second Kind. Form $yy' - y = f(x)$. subsection 1.3.1-2. Solvable equations and their solutions
Problem number: 21.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[[_Abel, '2nd type', 'class B']]`

$$\boxed{yy' - y = \frac{3x}{8} + \frac{3\sqrt{a^2+x^2}}{8} - \frac{a^2}{16\sqrt{a^2+x^2}}}$$

✗ Solution by Maple

```
dsolve(y(x)*diff(y(x),x)-y(x)=3/8*x+3/8*sqrt(x^2+a^2)-a^2/(16*sqrt(x^2+a^2)),y(x), singsol=a
```

No solution found

✗ Solution by Mathematica
Time used: 0.0 (sec). Leaf size: 0

```
DSolve[y[x]*y'[x]-y[x]==3/8*x+3/8*Sqrt[x^2+a^2]-a^2/(16*Sqrt[x^2+a^2]),y[x],x,IncludeSingula
```

Not solved

22.22 problem 22

Internal problem ID [10671]

Book: Handbook of exact solutions for ordinary differential equations. By Polyanin and Zaitsev. Second edition
Section: Chapter 1, section 1.3. Abel Equations of the Second Kind. Form $yy' - y = f(x)$. subsection 1.3.1-2. Solvable equations and their solutions
Problem number: 22.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_rational, [_Abel, '2nd type', 'class B']]`

$$\boxed{yy' - y = -\frac{4x}{25} + \frac{A}{\sqrt{x}}}$$

✓ Solution by Maple
Time used: 0.0 (sec). Leaf size: 270

```
dsolve(y(x)*diff(y(x),x)-y(x)=-4/25*x+A*x^(-1/2),y(x), singsol=all)
```

$$\frac{625\sqrt{A\,x^{\frac{3}{2}}}\,c_1\left(-\frac{Ay(x)^2\sqrt{x}}{2}+\frac{16x^4}{625}-\frac{16x^3y(x)}{125}+\frac{6y(x)^2x^2}{25}-\frac{xy(x)^3}{5}+\frac{y(x)^4}{16}+A^2x+\frac{4Ay(x)x^{\frac{3}{2}}}{5}-\frac{8A\,x^{\frac{5}{2}}}{25}\right)\sqrt{\frac{A\sqrt{x}-\frac{4($$

$= 0$

✗ Solution by Mathematica
Time used: 0.0 (sec). Leaf size: 0

```
DSolve[y[x]*y'[x]-y[x]==-4/25*x+A*x^(-1/2),y[x],x,IncludeSingularSolutions -> True]
```

Not solved

22.23 problem 23

Internal problem ID [10672]

Book: Handbook of exact solutions for ordinary differential equations. By Polyanin and Zaitsev. Second edition
Section: Chapter 1, section 1.3. Abel Equations of the Second Kind. Form $yy' - y = f(x)$. subsection 1.3.1-2. Solvable equations and their solutions
Problem number: 23.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_rational, [_Abel, '2nd type', 'class B']]`

$$\boxed{yy' - y = -\frac{9x}{100} + \frac{A}{x^{\frac{5}{3}}}}$$

✓ Solution by Maple
Time used: 0.0 (sec). Leaf size: 581

```
dsolve(y(x)*diff(y(x),x)-y(x)=-9/100*x+A*x^(-5/3),y(x), singsol=all)
```

Expression too large to display

✓ Solution by Mathematica
Time used: 60.566 (sec). Leaf size: 7909

```
DSolve[y[x]*y'[x]-y[x]==-9/100*x+A*x^(-5/3),y[x],x,IncludeSingularSolutions -> True]
```

Too large to display

22.24 problem 24

Internal problem ID [10673]

Book: Handbook of exact solutions for ordinary differential equations. By Polyanin and Zaitsev. Second edition
Section: Chapter 1, section 1.3. Abel Equations of the Second Kind. Form $yy' - y = f(x)$. subsection 1.3.1-2. Solvable equations and their solutions
Problem number: 24.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_rational, [_Abel, '2nd type', 'class B']]`

$$\boxed{yy' - y = -\frac{12x}{49} + \frac{2A\left(5\sqrt{x} + 34A + \frac{15A^2}{\sqrt{x}}\right)}{49}}$$

✓ Solution by Maple
Time used: 0.0 (sec). Leaf size: 270

```
dsolve(y(x)*diff(y(x),x)-y(x)=-12/49*x+2/49*A*(5*x^(1/2)+34*A+15*A^2*x^(-1/2)),y(x), singsol
```

$$\frac{\left(3A-\sqrt{x}\right)\left(36A^4+120A^3\sqrt{x}-80Ax^{\frac{3}{2}}+52A^2x+84A^2y(x)+140A\sqrt{x}\,y(x)+16x^2-56y(x)\,x+49y(\right.}{8\sqrt{-\frac{\left(3A-\sqrt{x}\right)^2}{6A^2-2A\sqrt{x}+y(x)}}\left(\frac{15A^2+4A\sqrt{x}-3x+7y(x)}{6A^2-2A\sqrt{x}+y(x)}\right)^{\frac{3}{2}}\left(6A^2-2A\sqrt{x}+y\left(x\right)\right)^3A}$$
$$+\frac{\left(-54A^2-6A\sqrt{x}+8x-21y(x)\right)\sqrt{-\frac{\left(3A-\sqrt{x}\right)^2}{6A^2-2A\sqrt{x}+y(x)}}}{\sqrt{\frac{15A^2+4A\sqrt{x}-3x+7y(x)}{6A^2-2A\sqrt{x}+y(x)}}\left(36A^2-12A\sqrt{x}+6y\left(x\right)\right)}+c_1=0$$

✗ Solution by Mathematica
Time used: 0.0 (sec). Leaf size: 0

```
DSolve[y[x]*y'[x]-y[x]==-12/49*x+2/49*A*(5*x^(1/2)+34*A+15*A^2*x^(-1/2)),y[x],x,IncludeSingu
```

Not solved

22.25 problem 25

Internal problem ID [10674]

Book: Handbook of exact solutions for ordinary differential equations. By Polyanin and Zaitsev. Second edition
Section: Chapter 1, section 1.3. Abel Equations of the Second Kind. Form $yy' - y = f(x)$. subsection 1.3.1-2. Solvable equations and their solutions
Problem number: 25.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_rational, [_Abel, '2nd type', 'class B']]`

$$\boxed{yy' - y = -\frac{12x}{49} + \frac{A\left(25\sqrt{x} + 41A + \frac{10A^2}{\sqrt{x}}\right)}{98}}$$

✓ Solution by Maple
Time used: 0.0 (sec). Leaf size: 1531

```
dsolve(y(x)*diff(y(x),x)-y(x)=-12/49*x+1/98*A*(25*x^(1/2)+41*A+10*A^2*x^(-1/2)),y(x), singso
```

Expression too large to display

✗ Solution by Mathematica
Time used: 0.0 (sec). Leaf size: 0

```
DSolve[y[x]*y'[x]-y[x]==-12/49*x+1/98*A*(25*x^(1/2)+41*A+10*A^2*x^(-1/2)),y[x],x,IncludeSing
```

Not solved

22.26 problem 26

Internal problem ID [10675]

Book: Handbook of exact solutions for ordinary differential equations. By Polyanin and Zaitsev. Second edition
Section: Chapter 1, section 1.3. Abel Equations of the Second Kind. Form $yy' - y = f(x)$. subsection 1.3.1-2. Solvable equations and their solutions
Problem number: 26.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_rational, [_Abel, '2nd type', 'class B']]`

$$\boxed{yy' - y = -\frac{2x}{9} + \frac{A}{\sqrt{x}}}$$

✓ Solution by Maple
Time used: 0.0 (sec). Leaf size: 134

```
dsolve(y(x)*diff(y(x),x)-y(x)=-2/9*x+A*x^(-1/2),y(x), singsol=all)
```

$$y(x) = \frac{2\,3^{\frac{5}{6}}2^{\frac{1}{3}}\left(-2x^{\frac{3}{2}}}{\sqrt{x}\left(\left(27\tan\left(\text{RootOf}\left(18\,3^{\frac{5}{6}}2^{\frac{1}{3}}\left(\int \frac{\left(\frac{A}{x^{\frac{3}{2}}}\right)^{\frac{2}{3}}\sqrt{x}}{-2x^{\frac{3}{2}}+9A}dx\right) + \ln\left(-8\sqrt{3}\,\sin(\_Z)\cos(\_Z)^3 - 8\cos(\_Z)^4 - 4\right.\right.\right.\right.$$

✓ Solution by Mathematica

Time used: 1.355 (sec). Leaf size: 282

```
DSolve[y[x]*y'[x]-y[x]==-2/9*x+A*x^(-1/2),y[x],x,IncludeSingularSolutions -> True]
```

$$\text{Solve}\left[\log\left(9A^{2/3}+3\sqrt[3]{6}\sqrt[3]{A}\sqrt{x}\right.\right.$$

$$\left.\left.+6^{2/3}x\right)+2\sqrt{3}\arctan\left(\frac{-\frac{6\sqrt[3]{6}(9A-2x^{3/2}+3\sqrt{x}y(x))}{\sqrt[3]{A}y(x)}-27}{27\sqrt{3}}\right)+2\sqrt{3}\arctan\left(\frac{\frac{2\sqrt[3]{6}\sqrt{x}}{\sqrt[3]{A}}+3}{3\sqrt{3}}\right)+2\log\left(\frac{1}{27}\left(27-\frac{3\sqrt[3]{}\right.\right.\right.$$

22.27 problem 27

Internal problem ID [10676]

Book: Handbook of exact solutions for ordinary differential equations. By Polyanin and Zaitsev. Second edition
Section: Chapter 1, section 1.3. Abel Equations of the Second Kind. Form $yy' - y = f(x)$. subsection 1.3.1-2. Solvable equations and their solutions
Problem number: 27.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_rational, [_Abel, '2nd type', 'class B']]`

$$\boxed{yy' - y = -\frac{5x}{36} + \frac{A}{x^{\frac{7}{5}}}}$$

✗ Solution by Maple

```
dsolve(y(x)*diff(y(x),x)-y(x)=-5/36*x+A*x^(-7/5),y(x), singsol=all)
```

No solution found

✗ Solution by Mathematica
Time used: 0.0 (sec). Leaf size: 0

```
DSolve[y[x]*y'[x]-y[x]==-5/36*x+A*x^(-7/5),y[x],x,IncludeSingularSolutions -> True]
```

Not solved

22.28 problem 28

Internal problem ID [10677]

Book: Handbook of exact solutions for ordinary differential equations. By Polyanin and Zaitsev. Second edition
Section: Chapter 1, section 1.3. Abel Equations of the Second Kind. Form $yy' - y = f(x)$. subsection 1.3.1-2. Solvable equations and their solutions
Problem number: 28.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_rational, [_Abel, '2nd type', 'class B']]`

$$\boxed{yy' - y = -\frac{12x}{49} + \frac{6A\left(-3\sqrt{x} + 23A + \frac{12A^2}{\sqrt{x}}\right)}{49}}$$

✗ Solution by Maple

```
dsolve(y(x)*diff(y(x),x)-y(x)=-12/49*x+6/49*A*(-3*x^(1/2)+23*A+12*A^2*x^(-1/2)),y(x), singso
```

No solution found

✗ Solution by Mathematica
Time used: 0.0 (sec). Leaf size: 0

```
DSolve[y[x]*y'[x]-y[x]==-12/49*x+6/49*A*(-3*x^(1/2)+23*A+12*A^2*x^(-1/2)),y[x],x,IncludeSing
```

Not solved

22.29 problem 29

Internal problem ID [10678]

Book: Handbook of exact solutions for ordinary differential equations. By Polyanin and Zaitsev. Second edition
Section: Chapter 1, section 1.3. Abel Equations of the Second Kind. Form $yy' - y = f(x)$. subsection 1.3.1-2. Solvable equations and their solutions
Problem number: 29.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_rational, [_Abel, '2nd type', 'class B']]`

$$\boxed{yy' - y = -\frac{30x}{121} + \frac{3A\left(21\sqrt{x} + 35A + \frac{6A^2}{\sqrt{x}}\right)}{242}}$$

✗ Solution by Maple

```
dsolve(y(x)*diff(y(x),x)-y(x)=-30/121*x+3/242*A*(21*x^(1/2)+35*A+6*A^2*x^(-1/2)),y(x), sings
```

No solution found

✗ Solution by Mathematica
Time used: 0.0 (sec). Leaf size: 0

```
DSolve[y[x]*y'[x]-y[x]==-30/121*x+3/242*A*(21*x^(1/2)+35*A+6*A^2*x^(-1/2)),y[x],x,IncludeSin
```

Not solved

22.30 problem 30

Internal problem ID [10679]

Book: Handbook of exact solutions for ordinary differential equations. By Polyanin and Zaitsev. Second edition
Section: Chapter 1, section 1.3. Abel Equations of the Second Kind. Form $yy' - y = f(x)$. subsection 1.3.1-2. Solvable equations and their solutions
Problem number: 30.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_rational, [_Abel, '2nd type', 'class B']]`

$$\boxed{yy' - y = -\frac{3x}{16} + \frac{A}{x^{\frac{5}{3}}}}$$

✗ Solution by Maple

```
dsolve(y(x)*diff(y(x),x)-y(x)=-3/16*x+A*x^(-5/3),y(x), singsol=all)
```

No solution found

✗ Solution by Mathematica
Time used: 0.0 (sec). Leaf size: 0

```
DSolve[y[x]*y'[x]-y[x]==-3/16*x+A*x^(-5/3),y[x],x,IncludeSingularSolutions -> True]
```

Not solved

22.31 problem 31

Internal problem ID [10680]

Book: Handbook of exact solutions for ordinary differential equations. By Polyanin and Zaitsev. Second edition
Section: Chapter 1, section 1.3. Abel Equations of the Second Kind. Form $yy' - y = f(x)$. subsection 1.3.1-2. Solvable equations and their solutions
Problem number: 31.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_rational, [_Abel, '2nd type', 'class B']]`

$$\boxed{yy' - y = -\frac{12x}{49} + \frac{4A\left(-10\sqrt{x} + 27A + \frac{10A^2}{\sqrt{x}}\right)}{49}}$$

✗ Solution by Maple

```
dsolve(y(x)*diff(y(x),x)-y(x)=-12/49*x+4/49*A*(-10*x^(1/2)+27*A+10*A^2*x^(-1/2)),y(x), sings
```

No solution found

✗ Solution by Mathematica
Time used: 0.0 (sec). Leaf size: 0

```
DSolve[y[x]*y'[x]-y[x]==-12/49*x+4/49*A*(-10*x^(1/2)+27*A+10*A^2*x^(-1/2)),y[x],x,IncludeSin
```

Not solved

22.32 problem 32

Internal problem ID [10681]

Book: Handbook of exact solutions for ordinary differential equations. By Polyanin and Zaitsev. Second edition
Section: Chapter 1, section 1.3. Abel Equations of the Second Kind. Form $yy' - y = f(x)$. subsection 1.3.1-2. Solvable equations and their solutions
Problem number: 32.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_rational, [_Abel, '2nd type', 'class B']]`

$$\boxed{yy' - y = \frac{A}{\sqrt{x}}}$$

✓ Solution by Maple
Time used: 0.015 (sec). Leaf size: 222

```
dsolve(y(x)*diff(y(x),x)-y(x)=A*x^(-1/2),y(x), singsol=all)
```

$$\frac{\left(\operatorname{AiryBi}\left(-\frac{2^{\frac{1}{3}}\left(-A^2x^{\frac{3}{2}}\right)^{\frac{2}{3}}(y(x)-x)}{2A^2x}\right)c_1 - \operatorname{AiryAi}\left(-\frac{2^{\frac{1}{3}}\left(-A^2x^{\frac{3}{2}}\right)^{\frac{2}{3}}(y(x)-x)}{2A^2x}\right)\right)2^{\frac{2}{3}}\left(-A^2x^{\frac{3}{2}}\right)^{\frac{1}{3}} - 2A\left(-\operatorname{AiryBi}\left(\right.\right.}{2^{\frac{2}{3}}\left(-A^2x^{\frac{3}{2}}\right)^{\frac{1}{3}}\operatorname{AiryBi}\left(-\frac{2^{\frac{1}{3}}\left(-A^2x^{\frac{3}{2}}\right)^{\frac{2}{3}}(y(x)-x)}{2A^2x}\right) + 2\operatorname{AiryBi}\left(1,-\right.\right.}$$

$$= 0$$

✓ Solution by Mathematica

Time used: 0.566 (sec). Leaf size: 139

```
DSolve[y[x]*y'[x]-y[x]==A*x^(-1/2),y[x],x,IncludeSingularSolutions -> True]
```

$$\text{Solve}\left[\frac{\sqrt[3]{-1}2^{2/3}\sqrt{x}\,\text{AiryAi}\left(\frac{\left(-\frac{1}{2}\right)^{2/3}(x-y(x))}{A^{2/3}}\right)+2\sqrt[3]{A}\,\text{AiryAiPrime}\left(\frac{\left(-\frac{1}{2}\right)^{2/3}(x-y(x))}{A^{2/3}}\right)}{\sqrt[3]{-1}2^{2/3}\sqrt{x}\,\text{AiryBi}\left(\frac{\left(-\frac{1}{2}\right)^{2/3}(x-y(x))}{A^{2/3}}\right)+2\sqrt[3]{A}\,\text{AiryBiPrime}\left(\frac{\left(-\frac{1}{2}\right)^{2/3}(x-y(x))}{A^{2/3}}\right)}+c_1=0,y(x)\right]$$

22.33 problem 33

Internal problem ID [10682]

Book: Handbook of exact solutions for ordinary differential equations. By Polyanin and Zaitsev. Second edition
Section: Chapter 1, section 1.3. Abel Equations of the Second Kind. Form $yy' - y = f(x)$. subsection 1.3.1-2. Solvable equations and their solutions
Problem number: 33.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_rational, [_Abel, '2nd type', 'class B']]`

$$\boxed{yy' - y = \frac{A}{x^2}}$$

✓ Solution by Maple
Time used: 0.0 (sec). Leaf size: 279

```
dsolve(y(x)*diff(y(x),x)-y(x)=A*x^(-2),y(x), singsol=all)
```

$$\frac{-\left(\text{AiryBi}\left(-\frac{\left(x^3-2y(x)x^2+y(x)^2x+2A\right)2^{\frac{2}{3}}}{4(-A^2)^{\frac{1}{3}}x}\right)c_1-\text{AiryAi}\left(-\frac{\left(x^3-2y(x)x^2+y(x)^2x+2A\right)2^{\frac{2}{3}}}{4(-A^2)^{\frac{1}{3}}x}\right)\right)A(-y(x)+x)\,2^{\frac{1}{3}}+2(}{-A2^{\frac{1}{3}}\left(-y\left(x\right)+x\right)\text{AiryBi}\left(-\frac{\left(x^3-2y(x)x^2+y(x)^2x+2A\right)2^{\frac{2}{3}}}{4(-A^2)^{\frac{1}{3}}x}\right)+2}$$

$$= 0$$

✓ Solution by Mathematica

Time used: 1.053 (sec). Leaf size: 201

```
DSolve[y[x]*y'[x]-y[x]==A*x^(-2),y[x],x,IncludeSingularSolutions -> True]
```

$$\text{Solve}\left[\frac{\text{AiryAiPrime}\left(\frac{x^3-2y(x)x^2+y(x)^2x+2A}{2\sqrt[3]{2}A^{2/3}x}\right)-\frac{(x-y(x))\,\text{AiryAi}\left(\frac{x^3-2y(x)x^2+y(x)^2x+2A}{2\sqrt[3]{2}A^{2/3}x}\right)}{2^{2/3}\sqrt[3]{A}}}{\text{AiryBiPrime}\left(\frac{x^3-2y(x)x^2+y(x)^2x+2A}{2\sqrt[3]{2}A^{2/3}x}\right)-\frac{(x-y(x))\,\text{AiryBi}\left(\frac{x^3-2y(x)x^2+y(x)^2x+2A}{2\sqrt[3]{2}A^{2/3}x}\right)}{2^{2/3}\sqrt[3]{A}}}+c_1=0,y(x)\right]$$

22.34 problem 34

Internal problem ID [10683]

Book: Handbook of exact solutions for ordinary differential equations. By Polyanin and Zaitsev. Second edition
Section: Chapter 1, section 1.3. Abel Equations of the Second Kind. Form $yy' - y = f(x)$. subsection 1.3.1-2. Solvable equations and their solutions
Problem number: 34.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_rational, [_Abel, '2nd type', 'class B']]`

$$\boxed{yy' - y = A(2+n)\left(\sqrt{x} + 2(2+n)\,A + \frac{(1+n)\,(n+3)\,A^2}{\sqrt{x}}\right)}$$

✓ Solution by Maple
Time used: 0.016 (sec). Leaf size: 410

```
dsolve(y(x)*diff(y(x),x)-y(x)=A*(n+2)*(x^(1/2)+2*(n+2)*A+(n+1)*(n+3)*A^2*x^(-1/2)),y(x), sin
```

$$\frac{(n+2)\left(\text{BesselI}\left(\frac{3+n}{n+2}, -\sqrt{\frac{2(n+2)A\sqrt{x}+A^2(n^2+4n+3)+x-y(x)}{(n+2)^2A^2}}\right)c_1 + \text{BesselK}\left(\frac{3+n}{n+2}, -\sqrt{\frac{2(n+2)A\sqrt{x}+A^2(n^2+4n+3)+x-}{(n+2)^2A^2}}\right.\right.}{A\sqrt{\frac{2(n+2)A\sqrt{x}+A^2(n^2+4n+3)+x-y(x)}{(n+2)^2A^2}}\,(n+2)\,\text{BesselI}}$$

$= 0$

✗ Solution by Mathematica
Time used: 0.0 (sec). Leaf size: 0

```
DSolve[y[x]*y'[x]-y[x]==A*(n+2)*(x^(1/2)+2*(n+2)*A+(n+1)*(n+3)*A^2*x^(-1/2)),y[x],x,IncludeS
```

Not solved

22.35 problem 35

Internal problem ID [10684]

Book: Handbook of exact solutions for ordinary differential equations. By Polyanin and Zaitsev. Second edition
Section: Chapter 1, section 1.3. Abel Equations of the Second Kind. Form $yy' - y = f(x)$. subsection 1.3.1-2. Solvable equations and their solutions
Problem number: 35.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_rational, [_Abel, '2nd type', 'class B']]`

$$\boxed{yy' - y = A(2+n)\left(\sqrt{x} + 2(2+n)A + \frac{(3+2n)A^2}{\sqrt{x}}\right)}$$

✓ Solution by Maple
Time used: 0.0 (sec). Leaf size: 474

```
dsolve(y(x)*diff(y(x),x)-y(x)=A*(n+2)*(x^(1/2)+2*(n+2)*A+(2*n+3)*A^2*x^(-1/2)),y(x), singsol
```

$$\frac{-(n+2)\left(\text{BesselI}\left(\sqrt{\frac{(n+1)^2}{(n+2)^2}}+1,-\sqrt{\frac{2(n+2)A\sqrt{x}+(2n+3)A^2+x-y(x)}{(n+2)^2A^2}}\right)c_1+\text{BesselK}\left(\sqrt{\frac{(n+1)^2}{(n+2)^2}}+1,-\sqrt{\frac{2(n+2)A\sqrt{}}{}}\right.\right.}{-A\sqrt{\frac{2(n+2)A\sqrt{x}+(2n+3)A^2+x-y(x)}{(n+2)^2A^2}}\,(n+2)\,\text{Bessel}}$$

$$= 0$$

✗ Solution by Mathematica
Time used: 0.0 (sec). Leaf size: 0

```
DSolve[y[x]*y'[x]-y[x]==A*(n+2)*(x^(1/2)+2*(n+2)*A+(2*n+3)*A^2*x^(-1/2)),y[x],x,IncludeSingu
```

Not solved

22.36 problem 36

Internal problem ID [10685]

Book: Handbook of exact solutions for ordinary differential equations. By Polyanin and Zaitsev. Second edition
Section: Chapter 1, section 1.3. Abel Equations of the Second Kind. Form $yy' - y = f(x)$. subsection 1.3.1-2. Solvable equations and their solutions
Problem number: 36.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_rational, [_Abel, '2nd type', 'class B']]`

$$\boxed{yy' - y = A\sqrt{x} + 2A^2 + \frac{B}{\sqrt{x}}}$$

✓ Solution by Maple
Time used: 0.0 (sec). Leaf size: 407

```
dsolve(y(x)*diff(y(x),x)-y(x)=A*x^(1/2)+2*A^2+B*x^(-1/2),y(x), singsol=all)
```

$$\frac{-c_1\left(\sqrt{\frac{A^3-B}{A^3}}\,A - A - \sqrt{x}\right)\text{BesselI}\left(\sqrt{\frac{A^3-B}{A^3}}, -\sqrt{\frac{2A^2\sqrt{x}-y(x)A+xA+B}{A^3}}\right) + A\sqrt{\frac{2A^2\sqrt{x}-y(x)A+xA+B}{A^3}}\,\text{BesselI}\left(\right.}{A\sqrt{\frac{2A^2\sqrt{x}-y(x)A+xA+B}{A^3}}\,\text{BesselI}\left(\right.} = 0$$

✗ Solution by Mathematica
Time used: 0.0 (sec). Leaf size: 0

```
DSolve[y[x]*y'[x]-y[x]==A*x^(1/2)+2*A^2+B*x^(-1/2),y[x],x,IncludeSingularSolutions -> True]
```

Not solved

22.37 problem 37

Internal problem ID [10686]

Book: Handbook of exact solutions for ordinary differential equations. By Polyanin and Zaitsev. Second edition
Section: Chapter 1, section 1.3. Abel Equations of the Second Kind. Form $yy' - y = f(x)$. subsection 1.3.1-2. Solvable equations and their solutions
Problem number: 37.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_rational, [_Abel, '2nd type', 'class A']]`

$$\boxed{yy' - y = 2A^2 - A\sqrt{x}}$$

✓ Solution by Maple
Time used: 0.0 (sec). Leaf size: 228

```
dsolve(y(x)*diff(y(x),x)-y(x)=2*A^2-A*x^(1/2),y(x), singsol=all)
```

$$\frac{\left(-2A+\sqrt{x}\right)\operatorname{BesselK}\left(1,-\sqrt{-\frac{2A\sqrt{x}-x+y(x)}{A^2}}\right)+\operatorname{BesselK}\left(0,-\sqrt{-\frac{2A\sqrt{x}-x+y(x)}{A^2}}\right)\sqrt{-\frac{2A\sqrt{x}-x+y(x)}{A^2}}\,A+c_1\left(\right.}{A\operatorname{BesselI}\left(0,\sqrt{-\frac{2A\sqrt{x}-x+y(x)}{A^2}}\right)\sqrt{-\frac{2A\sqrt{x}-x+y(x)}{A^2}}+\left(\right.}$$
$$= 0$$

✗ Solution by Mathematica
Time used: 0.0 (sec). Leaf size: 0

```
DSolve[y[x]*y'[x]-y[x]==2*A^2-A*x^(1/2),y[x],x,IncludeSingularSolutions -> True]
```

Not solved

22.38 problem 38

Internal problem ID [10687]

Book: Handbook of exact solutions for ordinary differential equations. By Polyanin and Zaitsev. Second edition
Section: Chapter 1, section 1.3. Abel Equations of the Second Kind. Form $yy' - y = f(x)$. subsection 1.3.1-2. Solvable equations and their solutions
Problem number: 38.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_rational, [_Abel, '2nd type', 'class B']]`

$$\boxed{yy' - y = -\frac{x}{4} + \frac{6A\left(\sqrt{x} + 8A + \frac{5A^2}{\sqrt{x}}\right)}{49}}$$

✗ Solution by Maple

```
dsolve(y(x)*diff(y(x),x)-y(x)=-12/48*x+6/49*A*(x^(1/2)+8*A+5*A^2*x^(-1/2)),y(x), singsol=all
```

No solution found

✗ Solution by Mathematica
Time used: 0.0 (sec). Leaf size: 0

```
DSolve[y[x]*y'[x]-y[x]==-12/48*x+6/49*A*(x^(1/2)+8*A+5*A^2*x^(-1/2)),y[x],x,IncludeSingularS
```

Not solved

22.39 problem 39

Internal problem ID [10688]

Book: Handbook of exact solutions for ordinary differential equations. By Polyanin and Zaitsev. Second edition
Section: Chapter 1, section 1.3. Abel Equations of the Second Kind. Form $yy' - y = f(x)$. subsection 1.3.1-2. Solvable equations and their solutions
Problem number: 39.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_rational, [_Abel, '2nd type', 'class B']]`

$$\boxed{yy' - y = -\frac{6x}{25} + \frac{6A\left(2\sqrt{x} + 7A + \frac{4A^2}{\sqrt{x}}\right)}{25}}$$

✗ Solution by Maple

```
dsolve(y(x)*diff(y(x),x)-y(x)=-6/25*x+6/25*A*(2*x^(1/2)+7*A+4*A^2*x^(-1/2)),y(x), singsol=al
```

No solution found

✗ Solution by Mathematica
Time used: 0.0 (sec). Leaf size: 0

```
DSolve[y[x]*y'[x]-y[x]==-6/25*x+6/25*A*(2*x^(1/2)+7*A+4*A^2*x^(-1/2)),y[x],x,IncludeSingular
```

Not solved

22.40 problem 40

Internal problem ID [10689]

Book: Handbook of exact solutions for ordinary differential equations. By Polyanin and Zaitsev. Second edition
Section: Chapter 1, section 1.3. Abel Equations of the Second Kind. Form $yy' - y = f(x)$. subsection 1.3.1-2. Solvable equations and their solutions
Problem number: 40.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_rational, [_Abel, '2nd type', 'class B']]`

$$\boxed{yy' - y = -\frac{3x}{16} + \frac{3A}{x^{\frac{1}{3}}} - \frac{12A^2}{x^{\frac{5}{3}}}}$$

✓ Solution by Maple
Time used: 0.0 (sec). Leaf size: 1488

```
dsolve(y(x)*diff(y(x),x)-y(x)=-3/16*x+3*A*x^(-1/3)-12*A^2*x^(-5/3),y(x), singsol=all)
```

Expression too large to display

✗ Solution by Mathematica
Time used: 0.0 (sec). Leaf size: 0

```
DSolve[y[x]*y'[x]-y[x]==-3/16*x+3*A*x^(-1/3)-12*A^2*x^(-5/3),y[x],x,IncludeSingularSolutions
```

Not solved

22.41 problem 41

Internal problem ID [10690]

Book: Handbook of exact solutions for ordinary differential equations. By Polyanin and Zaitsev. Second edition
Section: Chapter 1, section 1.3. Abel Equations of the Second Kind. Form $yy' - y = f(x)$. subsection 1.3.1-2. Solvable equations and their solutions
Problem number: 41.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[[_Abel, '2nd type', 'class B']]`

$$\boxed{yy' - y = \frac{3x}{8} + \frac{3\sqrt{b^2+x^2}}{8} + \frac{3b^2}{16\sqrt{b^2+x^2}}}$$

✗ Solution by Maple

```
dsolve(y(x)*diff(y(x),x)-y(x)=3/8*x+3/8*sqrt(x^2+b^2)+3*b^2/(16*sqrt(x^2+b^2)),y(x), singsol
```

No solution found

✗ Solution by Mathematica
Time used: 0.0 (sec). Leaf size: 0

```
DSolve[y[x]*y'[x]-y[x]==3/8*x+3/8*Sqrt[x^2+b^2]+3*b^2/(16*Sqrt[x^2+b^2]),y[x],x,IncludeSingu
```

Not solved

22.42 problem 42

Internal problem ID [10691]

Book: Handbook of exact solutions for ordinary differential equations. By Polyanin and Zaitsev. Second edition
Section: Chapter 1, section 1.3. Abel Equations of the Second Kind. Form $yy' - y = f(x)$. subsection 1.3.1-2. Solvable equations and their solutions
Problem number: 42.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[[_Abel, '2nd type', 'class B']]`

$$\boxed{yy' - y = \frac{9x}{32} + \frac{15\sqrt{b^2+x^2}}{32} + \frac{3b^2}{64\sqrt{b^2+x^2}}}$$

✗ Solution by Maple

```
dsolve(y(x)*diff(y(x),x)-y(x)=9/32*x+15/32*sqrt(x^2+b^2)+3*b^2/(64*sqrt(x^2+b^2)),y(x), sing
```

No solution found

✗ Solution by Mathematica
Time used: 0.0 (sec). Leaf size: 0

```
DSolve[y[x]*y'[x]-y[x]==9/32*x+15/32*Sqrt[x^2+b^2]+3*b^2/(64*Sqrt[x^2+b^2]),y[x],x,IncludeSi
```

Not solved

22.43 problem 43

Internal problem ID [10692]

Book: Handbook of exact solutions for ordinary differential equations. By Polyanin and Zaitsev. Second edition
Section: Chapter 1, section 1.3. Abel Equations of the Second Kind. Form $yy' - y = f(x)$. subsection 1.3.1-2. Solvable equations and their solutions
Problem number: 43.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[[_Abel, '2nd type', 'class B']]`

$$\boxed{yy' - y = -\frac{3x}{32} - \frac{3\sqrt{a^2+x^2}}{32} + \frac{15a^2}{64\sqrt{a^2+x^2}}}$$

✗ Solution by Maple

```
dsolve(y(x)*diff(y(x),x)-y(x)=-3/32*x-3/32*sqrt(x^2+a^2)+15*a^2/(64*sqrt(x^2+a^2)),y(x), sin
```

No solution found

✗ Solution by Mathematica
Time used: 0.0 (sec). Leaf size: 0

```
DSolve[y[x]*y'[x]-y[x]==-3/32*x-3/32*Sqrt[x^2+a^2]+15*a^2/(64*Sqrt[x^2+a^2]),y[x],x,IncludeS
```

Not solved

22.44 problem 44

Internal problem ID [10693]

Book: Handbook of exact solutions for ordinary differential equations. By Polyanin and Zaitsev. Second edition
Section: Chapter 1, section 1.3. Abel Equations of the Second Kind. Form $yy' - y = f(x)$. subsection 1.3.1-2. Solvable equations and their solutions
Problem number: 44.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_rational, [_Abel, '2nd type', 'class A']]`

$$\boxed{yy' - y = A\,x^2 - \frac{9}{625A}}$$

✓ Solution by Maple
Time used: 0.0 (sec). Leaf size: 186

```
dsolve(y(x)*diff(y(x),x)-y(x)=A*x^2-9/625*A^(-1),y(x), singsol=all)
```

$$-\frac{125\,2^{\frac{5}{6}}\left(\frac{-46875A^2y(x)^2+\left(37500A^2x+4500A\right)y(x)+31250\left(xA-\frac{3}{25}\right)\left(xA+\frac{3}{25}\right)^2}{(50xA-125y(x)A+6)^2}\right)^{\frac{1}{6}}Ay(x)\sqrt{25xA+3}}{2}+50\left(xA-\frac{5y(x)A}{2}+\frac{3}{25}\right)\left(\frac{(25xA$$

$$\left(\frac{(25xA+3)^{\frac{3}{2}}}{50xA-125y(x)A+6}\right)^{\frac{1}{3}}(50xA-125y\,(x)\,A+6)$$

$$=0$$

✓ Solution by Mathematica

Time used: 2.438 (sec). Leaf size: 198

```
DSolve[y[x]*y'[x]-y[x]==A*x^2-9/625*A^(-1),y[x],x,IncludeSingularSolutions -> True]
```

$$\text{Solve}\left[\frac{\sqrt[6]{\frac{46875A^2y(x)^2-1500A(25Ax+3)y(x)-2(25Ax-3)(25Ax+3)^2}{(25Ax+3)^3}}\left(\frac{(-125Ay(x)+}{\sqrt[3]{2}\sqrt{3}(25Ax+3)^{3/2}\sqrt[6]{\frac{-46875}{}}}\right.}{\sqrt[6]{2}}\right.$$

$$\left.\left.+c_1=0,y(x)\right]\right]$$

22.45 problem 45

Internal problem ID [10694]

Book: Handbook of exact solutions for ordinary differential equations. By Polyanin and Zaitsev. Second edition
Section: Chapter 1, section 1.3. Abel Equations of the Second Kind. Form $yy' - y = f(x)$. subsection 1.3.1-2. Solvable equations and their solutions
Problem number: 45.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_rational, [_Abel, '2nd type', 'class A']]`

$$\boxed{yy' - y = -\frac{6}{25}x - A\,x^2}$$

✓ Solution by Maple
Time used: 0.0 (sec). Leaf size: 132

```
dsolve(y(x)*diff(y(x),x)-y(x)=-6/25*x-A*x^2,y(x), singsol=all)
```

$$c_1 + \frac{(2x-5y(x))\left(\int^{-\frac{10\sqrt{-xA}\,x}{-2x+5y(x)}} \frac{(\_a^2-6)^{\frac{1}{6}}}{\_a^{\frac{1}{3}}}d\_a\right) - \frac{5\,2^{\frac{5}{6}}\left(\frac{-50A\,x^3-12x^2+60y(x)x-75y(x)^2}{(-2x+5y(x))^2}\right)^{\frac{1}{6}} 5^{\frac{2}{3}}\sqrt{-xA}\,y(x)}{2\left(-\frac{\sqrt{-xA}\,x}{-2x+5y(x)}\right)^{\frac{1}{3}}}}{2x-5y(x)} = 0$$

✓ Solution by Mathematica

Time used: 2.139 (sec). Leaf size: 162

```
DSolve[y[x]*y'[x]-y[x]==-6/25*x-A*x^2,y[x],x,IncludeSingularSolutions -> True]
```

$$\text{Solve}\left[c_1 = \frac{i\sqrt[6]{\frac{-2x^2(25Ax+6)+60xy(x)-75y(x)^2}{Ax^3}}\left(25Ax^2-\frac{\sqrt[6]{2}\sqrt[3]{5}(2x-5y(x))\,\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{5}{6},\frac{3}{2},-\frac{3(2}{\sqrt[6]{\frac{2x^2(25Ax+6)-60xy(x)+75y(}{Ax^3}}}\right.\right.}{5\ 2^{2/3}\sqrt{3}\sqrt[3]{5}\sqrt{A}x^{3/2}}\right.$$

22.46 problem 46

Internal problem ID [10695]

Book: Handbook of exact solutions for ordinary differential equations. By Polyanin and Zaitsev. Second edition
Section: Chapter 1, section 1.3. Abel Equations of the Second Kind. Form $yy' - y = f(x)$. subsection 1.3.1-2. Solvable equations and their solutions
Problem number: 46.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_rational, [_Abel, '2nd type', 'class A']]`

$$\boxed{yy' - y = \frac{6}{25}x - A\,x^2}$$

✓ Solution by Maple
Time used: 0.0 (sec). Leaf size: 180

```
dsolve(y(x)*diff(y(x),x)-y(x)=6/25*x-A*x^2,y(x), singsol=all)
```

$$125\left(5^{\frac{1}{3}}2^{\frac{5}{6}}\left(-\frac{1250\left(\frac{3y(x)^2A}{2}+\left(-\frac{6xA}{5}+\frac{36}{125}\right)y(x)+\left(xA-\frac{6}{25}\right)^2x\right)A}{(50xA-125y(x)A-12)^2}\right)^{\frac{1}{6}}Ay(x)\sqrt{-25xA+6}-\frac{4\left(xA-\frac{5y(x)A}{2}-\frac{6}{25}\right)\left(\int^{\frac{2(-25}{-50xA+1}\right.\right.$$

$$-\frac{}{\left(\frac{(-25xA+6)^{\frac{3}{2}}}{-50xA+125y(x)A+12}\right)^{\frac{1}{3}}(100xA-250y\,(x)\,A-24)}$$

$$=0$$

✓ Solution by Mathematica

Time used: 3.324 (sec). Leaf size: 189

```
DSolve[y[x]*y'[x]-y[x]==6/25*x-A*x^2,y[x],x,IncludeSingularSolutions -> True]
```

$$\text{Solve}\left[\frac{\sqrt[3]{5}\sqrt[6]{-\frac{A\left(1875Ay(x)^2-60(25Ax-6)y(x)+2x(6-25Ax)^2\right)}{(25Ax-6)^3}}\left(\frac{(-125Ay(x)+50Ax-12)\,\text{Hyperg}}{\sqrt[3]{10}\sqrt{18-75Ax}(25Ax-6)\sqrt[6]{\frac{A\left(1875A}{}}}\right.}{\sqrt[6]{2}}\right.$$

$$\left.+c_1=0,y(x)\right]$$

22.47 problem 47

Internal problem ID [10696]

Book: Handbook of exact solutions for ordinary differential equations. By Polyanin and Zaitsev. Second edition
Section: Chapter 1, section 1.3. Abel Equations of the Second Kind. Form $yy' - y = f(x)$. subsection 1.3.1-2. Solvable equations and their solutions
Problem number: 47.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_rational, [_Abel, '2nd type', 'class B']]`

$$\boxed{yy' - y = 12x + \frac{A}{x^{\frac{5}{2}}}}$$

✓ Solution by Maple
Time used: 0.0 (sec). Leaf size: 110

```
dsolve(y(x)*diff(y(x),x)-y(x)=12*x+A*x^(-5/2),y(x), singsol=all)
```

$$c_1 + \frac{12\sqrt{3}\left(2^{\frac{2}{3}}\left(\frac{3y(x)^2x^{\frac{3}{2}}}{4} - 6y(x)\,x^{\frac{5}{2}} + A + 12x^{\frac{7}{2}}\right)\left(\frac{48x^{\frac{7}{2}} - 24y(x)x^{\frac{5}{2}} + 3y(x)^2x^{\frac{3}{2}} + 4A}{A}\right)^{\frac{1}{6}} - 56\,\text{hypergeom}\left(\left[-\frac{1}{6}, \frac{1}{2}\right],\right.\right.}{\sqrt{-A\,x^{\frac{7}{2}}}}$$

$$= 0$$

✗ Solution by Mathematica
Time used: 0.0 (sec). Leaf size: 0

```
DSolve[y[x]*y'[x]-y[x]==12*x+A*x^(-5/2),y[x],x,IncludeSingularSolutions -> True]
```

Not solved

22.48 problem 48

Internal problem ID [10697]

Book: Handbook of exact solutions for ordinary differential equations. By Polyanin and Zaitsev. Second edition
Section: Chapter 1, section 1.3. Abel Equations of the Second Kind. Form $yy' - y = f(x)$. subsection 1.3.1-2. Solvable equations and their solutions
Problem number: 48.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_rational, [_Abel, '2nd type', 'class B']]`

$$\boxed{yy' - y = \frac{63x}{4} + \frac{A}{x^{\frac{5}{3}}}}$$

✗ Solution by Maple

```
dsolve(y(x)*diff(y(x),x)-y(x)=63/4*x+A*x^(-5/3),y(x), singsol=all)
```

No solution found

✗ Solution by Mathematica
Time used: 0.0 (sec). Leaf size: 0

```
DSolve[y[x]*y'[x]-y[x]==63/4*x+A*x^(-5/3),y[x],x,IncludeSingularSolutions -> True]
```

Not solved

22.49 problem 49

Internal problem ID [10698]

Book: Handbook of exact solutions for ordinary differential equations. By Polyanin and Zaitsev. Second edition
Section: Chapter 1, section 1.3. Abel Equations of the Second Kind. Form $yy' - y = f(x)$. subsection 1.3.1-2. Solvable equations and their solutions
Problem number: 49.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_rational, [_Abel, '2nd type', 'class B']]`

$$\boxed{yy' - y = 2x + 2A\left(10\sqrt{x} + 31A + \frac{30A^2}{\sqrt{x}}\right)}$$

✓ Solution by Maple
Time used: 0.016 (sec). Leaf size: 196

```
dsolve(y(x)*diff(y(x),x)-y(x)=2*x+2*A*(10*x^(1/2)+31*A+30*A^2*x^(-1/2)),y(x), singsol=all)
```

$$c_1 - \frac{\left(3A+\sqrt{x}\right) 2^{\frac{1}{3}} \left(\frac{12A^2+10A\sqrt{x}+2x-y(x)}{6A^2+2A\sqrt{x}+y(x)}\right)^{\frac{1}{3}} \left(\frac{15A^2+8A\sqrt{x}+x+y(x)}{6A^2+2A\sqrt{x}+y(x)}\right)^{\frac{1}{6}} y(x)}{4\sqrt{\frac{\left(3A+\sqrt{x}\right)^2}{6A^2+2A\sqrt{x}+y(x)}}\left(6A^2+2A\sqrt{x}+y\left(x\right)\right)A} - \left(\int^{\frac{6A\sqrt{x}+2x-3y(x)}{12A^2+4A\sqrt{x}+2y(x)}} \frac{\left(\_a+1\right)^{\frac{1}{3}}\left(2\_a+5\right)^{\frac{1}{6}}}{\sqrt{2\_a+3}} d\_a\right) = 0$$

✗ Solution by Mathematica
Time used: 0.0 (sec). Leaf size: 0

```
DSolve[y[x]*y'[x]-y[x]==2*x+2*A*(10*x^(1/2)+31*A+30*A^2*x^(-1/2)),y[x],x,IncludeSingularSolu
```

Not solved

22.50 problem 50

Internal problem ID [10699]

Book: Handbook of exact solutions for ordinary differential equations. By Polyanin and Zaitsev. Second edition
Section: Chapter 1, section 1.3. Abel Equations of the Second Kind. Form $yy' - y = f(x)$. subsection 1.3.1-2. Solvable equations and their solutions
Problem number: 50.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_rational, [_Abel, '2nd type', 'class B']]`

$$\boxed{yy' - y = 2x + 2A\left(-10\sqrt{x} + 19A + \frac{30A^2}{\sqrt{x}}\right)}$$

✗ Solution by Maple

```
dsolve(y(x)*diff(y(x),x)-y(x)=2*x+2*A*(-10*x^(1/2)+19*A+30*A^2*x^(-1/2)),y(x), singsol=all)
```

No solution found

✗ Solution by Mathematica
Time used: 0.0 (sec). Leaf size: 0

```
DSolve[y[x]*y'[x]-y[x]==2*x+2*A*(-10*x^(1/2)+19*A+30*A^2*x^(-1/2)),y[x],x,IncludeSingularSol
```

Not solved

22.51 problem 51

Internal problem ID [10700]

Book: Handbook of exact solutions for ordinary differential equations. By Polyanin and Zaitsev. Second edition
Section: Chapter 1, section 1.3. Abel Equations of the Second Kind. Form $yy' - y = f(x)$. subsection 1.3.1-2. Solvable equations and their solutions
Problem number: 51.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_rational, [_Abel, '2nd type', 'class B']]`

$$\boxed{yy' - y = -\frac{28x}{121} + \frac{2A\left(5\sqrt{x} + 106A + \frac{65A^2}{\sqrt{x}}\right)}{121}}$$

✗ Solution by Maple

```
dsolve(y(x)*diff(y(x),x)-y(x)=-28/121*x+2/121*A*(5*x^(1/2)+106*A+65*A^2*x^(-1/2)),y(x), sing
```

No solution found

✗ Solution by Mathematica
Time used: 0.0 (sec). Leaf size: 0

```
DSolve[y[x]*y'[x]-y[x]==-28/121*x+2/121*A*(5*x^(1/2)+106*A+65*A^2*x^(-1/2)),y[x],x,IncludeSi
```

Not solved

22.52 problem 52

Internal problem ID [10701]

Book: Handbook of exact solutions for ordinary differential equations. By Polyanin and Zaitsev. Second edition
Section: Chapter 1, section 1.3. Abel Equations of the Second Kind. Form $yy' - y = f(x)$. subsection 1.3.1-2. Solvable equations and their solutions
Problem number: 52.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_rational, [_Abel, '2nd type', 'class B']]`

$$\boxed{yy' - y = -\frac{12x}{49} + \frac{A\left(5\sqrt{x} + 262A + \frac{65A^2}{\sqrt{x}}\right)}{49}}$$

✓ Solution by Maple
Time used: 0.0 (sec). Leaf size: 4129

```
dsolve(y(x)*diff(y(x),x)-y(x)=-12/49*x+1/49*A*(5*x^(1/2)+262*A+65*A^2*x^(-1/2)),y(x), singso
```

Expression too large to display

✗ Solution by Mathematica
Time used: 0.0 (sec). Leaf size: 0

```
DSolve[y[x]*y'[x]-y[x]==-28/121*x+2/121*A*(5*x^(1/2)+262*A+65*A^2*x^(-1/2)),y[x],x,IncludeSi
```

Not solved

22.53 problem 53

Internal problem ID [10702]

Book: Handbook of exact solutions for ordinary differential equations. By Polyanin and Zaitsev. Second edition
Section: Chapter 1, section 1.3. Abel Equations of the Second Kind. Form $yy' - y = f(x)$. subsection 1.3.1-2. Solvable equations and their solutions
Problem number: 53.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_rational, [_Abel, '2nd type', 'class A']]`

$$\boxed{yy' - y = -\frac{12x}{49} + A\sqrt{x}}$$

✓ Solution by Maple
Time used: 0.0 (sec). Leaf size: 133

```
dsolve(y(x)*diff(y(x),x)-y(x)=-12/49*x+A*x^(1/2),y(x), singsol=all)
```

$$\frac{196^{\frac{5}{6}}\left(\left(\frac{4\left(x-\frac{7y(x)}{4}\right)\sqrt{3}\ \text{hypergeom}\left(\left[\frac{1}{2},\frac{7}{6}\right],\left[\frac{3}{2}\right],\frac{3(-4x+7y(x))^2}{196x^{\frac{3}{2}}A}\right)}{7}+\sqrt{x}\,\sqrt{A\sqrt{x}}\,c_1\right)196^{\frac{1}{6}}\left(\frac{Ax^{\frac{3}{2}}-\frac{12\left(x-\frac{7y(x)}{4}\right)^2}{49}}{x^{\frac{3}{2}}A}\right)^{\frac{1}{6}}-7\,14^{\frac{1}{3}}A}{196\left(\frac{Ax^{\frac{3}{2}}-\frac{12\left(x-\frac{7y(x)}{4}\right)^2}{49}}{x^{\frac{3}{2}}A}\right)^{\frac{1}{6}}\sqrt{A\sqrt{x}}\,\sqrt{x}}$$

$= 0$

✗ Solution by Mathematica
Time used: 0.0 (sec). Leaf size: 0

```
DSolve[y[x]*y'[x]-y[x]==-12/49*x+A*x^(1/2),y[x],x,IncludeSingularSolutions -> True]
```

Not solved

22.54 problem 54

Internal problem ID [10703]

Book: Handbook of exact solutions for ordinary differential equations. By Polyanin and Zaitsev. Second edition
Section: Chapter 1, section 1.3. Abel Equations of the Second Kind. Form $yy' - y = f(x)$. subsection 1.3.1-2. Solvable equations and their solutions
Problem number: 54.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_rational, [_Abel, '2nd type', 'class B']]`

$$\boxed{yy' - y = 6x + \frac{A}{x^4}}$$

✓ Solution by Maple
Time used: 0.016 (sec). Leaf size: 217

```
dsolve(y(x)*diff(y(x),x)-y(x)=6*x+A*x^(-4),y(x), singsol=all)
```

$$c_1 \left(x + \frac{y(x)}{2} \right) 5\,5^{\frac{2}{3}} \left(xA3^{\frac{5}{6}} \text{hypergeom}\left(\left[\frac{1}{6}, \frac{2}{3} \right], \left[\frac{5}{3} \right], -\frac{2A}{3x^3(2x+y(x))^2} \right) \left(\frac{12x^5+12y(x)x^4+3y(x)^2x^3+2A}{x^9(2x+y(x))^6} \right)^{\frac{1}{6}} + 24 \left(\frac{\left(-\frac{1}{x^{\frac{3}{2}}(2} \right.}{} \right. \right.$$

$$\left. \left. \left. + \frac{}{2\left(-\frac{1}{x^{\frac{3}{2}}(2x+y(x))} \right)^{\frac{7}{3}} x^{\frac{11}{2}} \left(\frac{12x^5+12y(x)x^4+3y(x)^2x^3+2A}{x^3(2x+y(x))^2} \right)^{\frac{1}{6}} (2} \right) \right) \right)$$

$$= 0$$

✓ Solution by Mathematica

Time used: 2.079 (sec). Leaf size: 213

```
DSolve[y[x]*y'[x]-y[x]==6*x+A*x^(-4),y[x],x,IncludeSingularSolutions -> True]
```

$$\text{Solve}\left[c_1 = \frac{i\left(-\frac{2A+12x^5+12x^4y(x)+3x^3y(x)^2}{A}\right)^{5/6}\left(-10\ 2^{5/6}x^5\ \text{Hypergeometric2F1}\left(\frac{1}{6},\frac{1}{2},\frac{3}{2},-\frac{3x^3(2x+y(x))^2}{2A}\right)-5\right.}{2\sqrt[3]{2}\sqrt{3}\sqrt{A}x^{5/2}\left(\frac{2A+12x^5}{}\right.}\right.$$

22.55 problem 55

Internal problem ID [10704]

Book: Handbook of exact solutions for ordinary differential equations. By Polyanin and Zaitsev. Second edition
Section: Chapter 1, section 1.3. Abel Equations of the Second Kind. Form $yy' - y = f(x)$. subsection 1.3.1-2. Solvable equations and their solutions
Problem number: 55.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_rational, [_Abel, '2nd type', 'class B']]`

$$\boxed{yy' - y = 20x + \frac{A}{\sqrt{x}}}$$

✗ Solution by Maple

```
dsolve(y(x)*diff(y(x),x)-y(x)=20*x+A*x^(-1/2),y(x), singsol=all)
```

No solution found

✗ Solution by Mathematica
Time used: 0.0 (sec). Leaf size: 0

```
DSolve[y[x]*y'[x]-y[x]==20*x+A*x^(-1/2),y[x],x,IncludeSingularSolutions -> True]
```

Not solved

22.56 problem 56

Internal problem ID [10705]

Book: Handbook of exact solutions for ordinary differential equations. By Polyanin and Zaitsev. Second edition
Section: Chapter 1, section 1.3. Abel Equations of the Second Kind. Form $yy' - y = f(x)$. subsection 1.3.1-2. Solvable equations and their solutions
Problem number: 56.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_rational, [_Abel, '2nd type', 'class B']]`

$$\boxed{yy' - y = \frac{15x}{4} + \frac{A}{x^7}}$$

✗ Solution by Maple

```
dsolve(y(x)*diff(y(x),x)-y(x)=15/4*x+A*x^(-7),y(x), singsol=all)
```

No solution found

✗ Solution by Mathematica
Time used: 0.0 (sec). Leaf size: 0

```
DSolve[y[x]*y'[x]-y[x]==15/4*x+A*x^(-7),y[x],x,IncludeSingularSolutions -> True]
```

Not solved

22.57 problem 57

Internal problem ID [10706]

Book: Handbook of exact solutions for ordinary differential equations. By Polyanin and Zaitsev. Second edition
Section: Chapter 1, section 1.3. Abel Equations of the Second Kind. Form $yy' - y = f(x)$. subsection 1.3.1-2. Solvable equations and their solutions
Problem number: 57.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_rational, [_Abel, '2nd type', 'class B']]`

$$\boxed{yy' - y = -\frac{10x}{49} + \frac{2A\left(4\sqrt{x} + 61A + \frac{12A^2}{\sqrt{x}}\right)}{49}}$$

✓ Solution by Maple
Time used: 0.0 (sec). Leaf size: 200

```
dsolve(y(x)*diff(y(x),x)-y(x)=-10/49*x+2/49*A*(4*x^(1/2)+61*A+12*A^2*x^(-1/2)),y(x), singsol
```

$$c_1 - \frac{\left(3A+\sqrt{x}\right) 2^{\frac{2}{3}} \left(\frac{3A^2+16A\sqrt{x}+5x-7y(x)}{6A^2+2A\sqrt{x}+y(x)}\right)^{\frac{5}{6}} y(x)}{2\sqrt{\frac{\left(3A+\sqrt{x}\right)^2}{6A^2+2A\sqrt{x}+y(x)}} \left(\frac{-24A^2-2A\sqrt{x}+2x-7y(x)}{6A^2+2A\sqrt{x}+y(x)}\right)^{\frac{1}{3}} \left(6A^2+2A\sqrt{x}+y\left(x\right)\right) A} - \left(\int^{\frac{6A\sqrt{x}+2x-3y(x)}{12A^2+4A\sqrt{x}+2y(x)}} \frac{\left(10\_a+1\right)^{\frac{5}{6}}}{\sqrt{2\_a+3}\,\left(\_a-2\right)^{\frac{1}{3}}} d\_a\right) = 0$$

✗ Solution by Mathematica
Time used: 0.0 (sec). Leaf size: 0

```
DSolve[y[x]*y'[x]-y[x]==-10/49*x+2/49*A*(4*x^(1/2)+61*A+12*A^2*x^(-1/2)),y[x],x,IncludeSingu
```

Not solved

22.58 problem 58

Internal problem ID [10707]

Book: Handbook of exact solutions for ordinary differential equations. By Polyanin and Zaitsev. Second edition
Section: Chapter 1, section 1.3. Abel Equations of the Second Kind. Form $yy' - y = f(x)$. subsection 1.3.1-2. Solvable equations and their solutions
Problem number: 58.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_rational, [_Abel, '2nd type', 'class B']]`

$$\boxed{yy' - y = -\frac{12x}{49} + \frac{2A\left(\sqrt{x} + 166A + \frac{55A^2}{\sqrt{x}}\right)}{49}}$$

✓ Solution by Maple
Time used: 0.0 (sec). Leaf size: 686

```
dsolve(y(x)*diff(y(x),x)-y(x)=-12/49*x+2/49*A*(x^(1/2)+166*A+55*A^2*x^(-1/2)),y(x), singsol=
```

$$c_1 + \frac{3\sqrt{6}\,4^{\frac{2}{3}}\left(\left(\sqrt{-35A^2-7A\sqrt{x}}\left(A\left(3+\right.\right.\right.\right.}{4\left(\left(\left(2\left(-18i-7\sqrt{6}\right)A\sqrt{x}-70A^2\sqrt{6}+120iA^2-12ix+21iy\left(x\right)\right)\sqrt{-35A^2-7A\sqrt{x}}+350\left(-\frac{\sqrt{6}x}{25}+\right.\right.\right.\right.}$$
$$= 0$$

✗ Solution by Mathematica
Time used: 0.0 (sec). Leaf size: 0

```
DSolve[y[x]*y'[x]-y[x]==-12/49*x+2/49*A*(x^(1/2)+166*A+55*A^2*x^(-1/2)),y[x],x,IncludeSingul
```

Not solved

22.59 problem 59

Internal problem ID [10708]

Book: Handbook of exact solutions for ordinary differential equations. By Polyanin and Zaitsev. Second edition
Section: Chapter 1, section 1.3. Abel Equations of the Second Kind. Form $yy' - y = f(x)$. subsection 1.3.1-2. Solvable equations and their solutions
Problem number: 59.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_rational, [_Abel, '2nd type', 'class B']]`

$$\boxed{yy' - y = -\frac{4x}{25} + \frac{A\left(7\sqrt{x} + 49A + \frac{6A^2}{\sqrt{x}}\right)}{50}}$$

✗ Solution by Maple

```
dsolve(y(x)*diff(y(x),x)-y(x)=-4/25*x+1/50*A*(7*x^(1/2)+49*A+6*A^2*x^(-1/2)),y(x), singsol=a
```

No solution found

✗ Solution by Mathematica
Time used: 0.0 (sec). Leaf size: 0

```
DSolve[y[x]*y'[x]-y[x]==-4/25*x+1/50*A*(7*x^(1/2)+49*A+6*A^2*x^(-1/2)),y[x],x,IncludeSingula
```

Not solved

22.60 problem 60

Internal problem ID [10709]

Book: Handbook of exact solutions for ordinary differential equations. By Polyanin and Zaitsev. Second edition
Section: Chapter 1, section 1.3. Abel Equations of the Second Kind. Form $yy' - y = f(x)$. subsection 1.3.1-2. Solvable equations and their solutions
Problem number: 60.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_rational, [_Abel, '2nd type', 'class B']]`

$$\boxed{yy' - y = \frac{15x}{4} + \frac{6A}{x^{\frac{1}{3}}} - \frac{3A^2}{x^{\frac{5}{3}}}}$$

✗ Solution by Maple

```
dsolve(y(x)*diff(y(x),x)-y(x)=15/4*x+6*A*x^(-1/3)-3*A^2*x^(-5/3),y(x), singsol=all)
```

No solution found

✗ Solution by Mathematica
Time used: 0.0 (sec). Leaf size: 0

```
DSolve[y[x]*y'[x]-y[x]==15/4*x+6*A*x^(-1/3)-3*A^2*x^(-5/3),y[x],x,IncludeSingularSolutions -
```

Not solved

22.61 problem 61

Internal problem ID [10710]

Book: Handbook of exact solutions for ordinary differential equations. By Polyanin and Zaitsev. Second edition
Section: Chapter 1, section 1.3. Abel Equations of the Second Kind. Form $yy' - y = f(x)$. subsection 1.3.1-2. Solvable equations and their solutions
Problem number: 61.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_rational, [_Abel, '2nd type', 'class B']]`

$$\boxed{yy' - y = -\frac{3x}{16} + \frac{A}{x^{\frac{1}{3}}} + \frac{B}{x^{\frac{5}{3}}}}$$

✗ Solution by Maple

```
dsolve(y(x)*diff(y(x),x)-y(x)=-3/16*x+A*x^(-1/3)+B*x^(-5/3),y(x), singsol=all)
```

No solution found

✗ Solution by Mathematica
Time used: 0.0 (sec). Leaf size: 0

```
DSolve[y[x]*y'[x]-y[x]==-3/16*x+A*x^(-1/3)+B*x^(-5/3),y[x],x,IncludeSingularSolutions -> Tru
```

Not solved

22.62 problem 62

Internal problem ID [10711]

Book: Handbook of exact solutions for ordinary differential equations. By Polyanin and Zaitsev. Second edition
Section: Chapter 1, section 1.3. Abel Equations of the Second Kind. Form $yy' - y = f(x)$. subsection 1.3.1-2. Solvable equations and their solutions
Problem number: 62.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_rational, [_Abel, '2nd type', 'class B']]`

$$\boxed{yy' - y = -\frac{5x}{36} + \frac{A}{x^{\frac{3}{5}}} - \frac{B}{x^{\frac{7}{5}}}}$$

✗ Solution by Maple

```
dsolve(y(x)*diff(y(x),x)-y(x)=-5/36*x+A*x^(-3/5)-B*x^(-7/5),y(x), singsol=all)
```

No solution found

✗ Solution by Mathematica
Time used: 0.0 (sec). Leaf size: 0

```
DSolve[y[x]*y'[x]-y[x]==-5/36*x+A*x^(-3/5)-B*x^(-7/5),y[x],x,IncludeSingularSolutions -> Tru
```

Timed out

22.63 problem 63

Internal problem ID [10712]

Book: Handbook of exact solutions for ordinary differential equations. By Polyanin and Zaitsev. Second edition
Section: Chapter 1, section 1.3. Abel Equations of the Second Kind. Form $yy' - y = f(x)$. subsection 1.3.1-2. Solvable equations and their solutions
Problem number: 63.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[[_Abel, '2nd type', 'class B']]`

$$\boxed{yy' - y = \frac{k}{\sqrt{A\,x^2 + Bx + c}}}$$

✗ Solution by Maple

```
dsolve(y(x)*diff(y(x),x)-y(x)=k*(A*x^2+B*x+c)^(-1/2),y(x), singsol=all)
```

No solution found

✗ Solution by Mathematica
Time used: 0.0 (sec). Leaf size: 0

```
DSolve[y[x]*y'[x]-y[x]==k*(A*x^2+B*x+c)^(-1/2),y[x],x,IncludeSingularSolutions -> True]
```

Timed out

22.64 problem 64

Internal problem ID [10713]

Book: Handbook of exact solutions for ordinary differential equations. By Polyanin and Zaitsev. Second edition
Section: Chapter 1, section 1.3. Abel Equations of the Second Kind. Form $yy' - y = f(x)$. subsection 1.3.1-2. Solvable equations and their solutions
Problem number: 64.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_rational, [_Abel, '2nd type', 'class B']]`

$$yy' - y = -\frac{12x}{49} + 3A\left(\frac{1}{49} + B\right)\sqrt{x} + 3A^2\left(\frac{4}{49} - \frac{5B}{2}\right) + \frac{15A^3\left(\frac{1}{49} - \frac{5B}{4}\right)}{4\sqrt{x}}$$

✗ Solution by Maple

```
dsolve(y(x)*diff(y(x),x)-y(x)=-12/49*x+3*A*(1/49+B)*x^(1/2)+3*A^2*(4/49-5/2*B)+15/4*A^3*(1/4
```

No solution found

✗ Solution by Mathematica
Time used: 0.0 (sec). Leaf size: 0

```
DSolve[y[x]*y'[x]-y[x]==-12/49*x+3*A*(1/49+B)*x^(1/2)+3*A^2*(4/49-5/2*B)+15/4*A^3*(1/49-5/4*
```

Not solved

22.65 problem 65

Internal problem ID [10714]

Book: Handbook of exact solutions for ordinary differential equations. By Polyanin and Zaitsev. Second edition
Section: Chapter 1, section 1.3. Abel Equations of the Second Kind. Form $yy' - y = f(x)$. subsection 1.3.1-2. Solvable equations and their solutions
Problem number: 65.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_rational, [_Abel, '2nd type', 'class B']]`

$$\boxed{yy' - y = -\frac{6x}{25} + \frac{4B^2\left((2-A)\,x^{\frac{1}{3}} - \frac{3B(2A+1)}{2} + \frac{B^2(1-3A)}{x^{\frac{1}{3}}} - \frac{AB^3}{x^{\frac{2}{3}}}\right)}{75}}$$

✓ Solution by Maple
Time used: 0.0 (sec). Leaf size: 3187

```
dsolve(y(x)*diff(y(x),x)-y(x)=-6/25*x+4/75*B^2*((2-A)*x^(1/3)-3/2*B*(2*A+1)+B^2*(1-3*A)*x^(-
```

Expression too large to display

✗ Solution by Mathematica
Time used: 0.0 (sec). Leaf size: 0

```
DSolve[y[x]*y'[x]-y[x]==-6/25*x+4/75*B^2*((2-A)*x^(1/3)-3/2*B*(2*A+1)+B^2*(1-3*A)*x^(-1/3)-A
```

Not solved

22.66 problem 66

Internal problem ID [10715]

Book: Handbook of exact solutions for ordinary differential equations. By Polyanin and Zaitsev. Second edition
Section: Chapter 1, section 1.3. Abel Equations of the Second Kind. Form $yy' - y = f(x)$. subsection 1.3.1-2. Solvable equations and their solutions
Problem number: 66.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_rational, [_Abel, '2nd type', 'class B']]`

$$\boxed{yy' - y = \frac{3x}{4} - \frac{3A\,x^{\frac{1}{3}}}{2} + \frac{3A^2}{4x^{\frac{1}{3}}} - \frac{27A^4}{625x^{\frac{5}{3}}}}$$

✗ Solution by Maple

```
dsolve(y(x)*diff(y(x),x)-y(x)=3/4*x-3/2*A*x^(1/3)+3/4*A^2*x^(-1/3)-27/625*A^4*x^(-5/3),y(x),
```

No solution found

✗ Solution by Mathematica
Time used: 0.0 (sec). Leaf size: 0

```
DSolve[y[x]*y'[x]-y[x]==3/4*x-3/2*A*x^(1/3)+3/4*A^2*x^(-1/3)-27/625*A^4*x^(-5/3),y[x],x,Incl
```

Not solved

22.67 problem 67

Internal problem ID [10716]

Book: Handbook of exact solutions for ordinary differential equations. By Polyanin and Zaitsev. Second edition
Section: Chapter 1, section 1.3. Abel Equations of the Second Kind. Form $yy' - y = f(x)$. subsection 1.3.1-2. Solvable equations and their solutions
Problem number: 67.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_rational, [_Abel, '2nd type', 'class B']]`

$$yy' - y = -\frac{6x}{25} + \frac{7A\,x^{\frac{1}{3}}}{5} + \frac{31A^2}{3x^{\frac{1}{3}}} - \frac{100A^4}{3x^{\frac{5}{3}}}$$

✗ Solution by Maple

```
dsolve(y(x)*diff(y(x),x)-y(x)=-6/25*x+7/5*A*x^(1/3)+31/3*A^2*x^(-1/3)-100/3*A^4*x^(-5/3),y(x
```

No solution found

✗ Solution by Mathematica
Time used: 0.0 (sec). Leaf size: 0

```
DSolve[y[x]*y'[x]-y[x]==-6/25*x+7/5*A*x^(1/3)+31/3*A^2*x^(-1/3)-100/3*A^4*x^(-5/3),y[x],x,In
```

Not solved

22.68 problem 68

Internal problem ID [10717]

Book: Handbook of exact solutions for ordinary differential equations. By Polyanin and Zaitsev. Second edition
Section: Chapter 1, section 1.3. Abel Equations of the Second Kind. Form $yy' - y = f(x)$. subsection 1.3.1-2. Solvable equations and their solutions
Problem number: 68.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_rational, [_Abel, '2nd type', 'class B']]`

$$\boxed{yy' - y = -\frac{10x}{49} + \frac{13A^2}{5x^{\frac{1}{5}}} - \frac{7A^3}{20x^{\frac{4}{5}}}}$$

✗ Solution by Maple

```
dsolve(y(x)*diff(y(x),x)-y(x)=-10/49*x+13/5*A^2*x^(-1/5)-7/20*A^3*x^(-4/5),y(x), singsol=all
```

No solution found

✗ Solution by Mathematica
Time used: 0.0 (sec). Leaf size: 0

```
DSolve[y[x]*y'[x]-y[x]==-10/49*x+13/5*A^2*x^(-1/5)-7/20*A^3*x^(-4/5),y[x],x,IncludeSingularS
```

Not solved

22.69 problem 69

Internal problem ID [10718]

Book: Handbook of exact solutions for ordinary differential equations. By Polyanin and Zaitsev. Second edition
Section: Chapter 1, section 1.3. Abel Equations of the Second Kind. Form $yy' - y = f(x)$. subsection 1.3.1-2. Solvable equations and their solutions
Problem number: 69.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_rational, [_Abel, '2nd type', 'class B']]`

$$\boxed{yy' - y = -\frac{33x}{169} + \frac{286A^2}{3x^{\frac{5}{11}}} - \frac{770A^3}{9x^{\frac{13}{11}}}}$$

✗ Solution by Maple

```
dsolve(y(x)*diff(y(x),x)-y(x)=-33/169*x+286/3*A^2*x^(-5/11)-770/9*A^3*x^(-13/11),y(x), sings
```

No solution found

✗ Solution by Mathematica
Time used: 0.0 (sec). Leaf size: 0

```
DSolve[y[x]*y'[x]-y[x]==-33/169*x+286/3*A^2*x^(-5/11)-770/9*A^3*x^(-13/11),y[x],x,IncludeSin
```

Timed out

22.70 problem 70

Internal problem ID [10719]

Book: Handbook of exact solutions for ordinary differential equations. By Polyanin and Zaitsev. Second edition
Section: Chapter 1, section 1.3. Abel Equations of the Second Kind. Form $yy' - y = f(x)$. subsection 1.3.1-2. Solvable equations and their solutions
Problem number: 70.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_rational, [_Abel, '2nd type', 'class B']]`

$$\boxed{yy' - y = -\frac{21x}{100} + \frac{7A^2\left(\frac{123}{x^{\frac{1}{7}}} + \frac{280A}{x^{\frac{5}{7}}} - \frac{400A^2}{x^{\frac{9}{7}}}\right)}{9}}$$

✗ Solution by Maple

```
dsolve(y(x)*diff(y(x),x)-y(x)=-21/100*x+7/9*A^2*(123*x^(-1/7)+280*A*x^(-5/7)-400*A^2*x^(-9/7
```

No solution found

✗ Solution by Mathematica
Time used: 0.0 (sec). Leaf size: 0

```
DSolve[y[x]*y'[x]-y[x]==-21/100*x+7/9*A^2*(123*x^(-1/7)+280*A*x^(-5/7)-400*A^2*x^(-9/7)),y[x
```

Not solved

22.71 problem 71

Internal problem ID [10720]

Book: Handbook of exact solutions for ordinary differential equations. By Polyanin and Zaitsev. Second edition
Section: Chapter 1, section 1.3. Abel Equations of the Second Kind. Form $yy' - y = f(x)$. subsection 1.3.1-2. Solvable equations and their solutions
Problem number: 71.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_rational, [_Abel, '2nd type', 'class A']]`

$$\boxed{yy' - y = ax + b\,x^m}$$

✗ Solution by Maple

```
dsolve(y(x)*diff(y(x),x)-y(x)=a*x+b*x^m,y(x), singsol=all)
```

No solution found

✗ Solution by Mathematica
Time used: 0.0 (sec). Leaf size: 0

```
DSolve[y[x]*y'[x]-y[x]==a*x+b*x^m,y[x],x,IncludeSingularSolutions -> True]
```

Not solved

22.72 problem 72

Internal problem ID [10721]

Book: Handbook of exact solutions for ordinary differential equations. By Polyanin and Zaitsev. Second edition
Section: Chapter 1, section 1.3. Abel Equations of the Second Kind. Form $yy' - y = f(x)$. subsection 1.3.1-2. Solvable equations and their solutions
Problem number: 72.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_rational, [_Abel, '2nd type', 'class A']]`

$$\boxed{yy' - y = -\frac{(m+1)\,x}{(m+2)^2} + A\,x^{2m+1} + B\,x^{3m+1}}$$

✗ Solution by Maple

```
dsolve(y(x)*diff(y(x),x)-y(x)=-(m+1)/(m+2)^2*x+A*x^(2*m+1)+B*x^(3*m+1),y(x), singsol=all)
```

No solution found

✗ Solution by Mathematica
Time used: 0.0 (sec). Leaf size: 0

```
DSolve[y[x]*y'[x]-y[x]==-(m+1)/(m+2)^2*x+A*x^(2*m+1)+B*x^(3*m+1),y[x],x,IncludeSingularSolut
```

Not solved

22.73 problem 73

Internal problem ID [10722]

Book: Handbook of exact solutions for ordinary differential equations. By Polyanin and Zaitsev. Second edition
Section: Chapter 1, section 1.3. Abel Equations of the Second Kind. Form $yy' - y = f(x)$. subsection 1.3.1-2. Solvable equations and their solutions
Problem number: 73.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[[_Abel, '2nd type', 'class A']]`

$$\boxed{yy' - y = a^2\lambda\,\mathrm{e}^{2\lambda x} - a(b\lambda + 1)\,\mathrm{e}^{\lambda x} + b}$$

✗ Solution by Maple

```
dsolve(y(x)*diff(y(x),x)-y(x)=a^2*lambda*exp(2*lambda*x)-a*(b*lambda+1)*exp(lambda*x)+b,y(x)
```

No solution found

✗ Solution by Mathematica
Time used: 0.0 (sec). Leaf size: 0

```
DSolve[y[x]*y'[x]-y[x]==a^2*\[Lambda]*Exp[2*\[Lambda]*x]-a*(b*\[Lambda]+1)*Exp[\[Lambda]*x]+
```

Not solved

22.74 problem 74

Internal problem ID [10723]

Book: Handbook of exact solutions for ordinary differential equations. By Polyanin and Zaitsev. Second edition
Section: Chapter 1, section 1.3. Abel Equations of the Second Kind. Form $yy' - y = f(x)$. subsection 1.3.1-2. Solvable equations and their solutions
Problem number: 74.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[[_Abel, '2nd type', 'class A']]`

$$\boxed{yy' - y = a^2\lambda\,\mathrm{e}^{2\lambda x} + a\lambda x\,\mathrm{e}^{\lambda x} + b\,\mathrm{e}^{\lambda x}}$$

✗ Solution by Maple

```
dsolve(y(x)*diff(y(x),x)-y(x)=a^2*lambda*exp(2*lambda*x)+a*lambda*x*exp(lambda*x)+b*exp(lamb
```

No solution found

✗ Solution by Mathematica
Time used: 0.0 (sec). Leaf size: 0

```
DSolve[y[x]*y'[x]-y[x]==a^2*\[Lambda]*Exp[2*\[Lambda]*x]+a*\[Lambda]*x*Exp[\[Lambda]*x]+b*Ex
```

Not solved

22.75 problem 75

Internal problem ID [10724]

Book: Handbook of exact solutions for ordinary differential equations. By Polyanin and Zaitsev. Second edition
Section: Chapter 1, section 1.3. Abel Equations of the Second Kind. Form $yy' - y = f(x)$. subsection 1.3.1-2. Solvable equations and their solutions
Problem number: 75.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[[_Abel, '2nd type', 'class A']]`

$$\boxed{yy' - y = 2a^2\lambda \sin(2\lambda x) + 2a \sin(\lambda x)}$$

✗ Solution by Maple

```
dsolve(y(x)*diff(y(x),x)-y(x)=2*a^2*lambda*sin(2*lambda*x)+2*a*sin(lambda*x),y(x), singsol=a
```

No solution found

✗ Solution by Mathematica
Time used: 0.0 (sec). Leaf size: 0

```
DSolve[y[x]*y'[x]-y[x]==2*a^2*\[Lambda]*Sin[2*\[Lambda]*x]+2*a*Sin[\[Lambda]*x],y[x],x,Inclu
```

Not solved

22.76 problem 76

Internal problem ID [10725]

Book: Handbook of exact solutions for ordinary differential equations. By Polyanin and Zaitsev. Second edition
Section: Chapter 1, section 1.3. Abel Equations of the Second Kind. Form $yy' - y = f(x)$. subsection 1.3.1-2. Solvable equations and their solutions
Problem number: 76.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_Abel, '2nd type', 'class B']]`

$$\boxed{yy' - y = a^2 f'(x) f''(x) - \frac{(f(x)+b)^2 f''(x)}{f'(x)^3}}$$

✗ Solution by Maple

```
dsolve(y(x)*diff(y(x),x)-y(x)=a^2*diff(f(x),x)*diff(f(x),x$2)-(f(x)+b)^2/( diff(f(x),x)^3)*d
```

No solution found

✗ Solution by Mathematica
Time used: 0.0 (sec). Leaf size: 0

```
DSolve[y[x]*y'[x]-y[x]==a^2*f'[x]*f''[x]-(f[x]+b)^2/( (f'[x])^3)*f''[x],y[x],x,IncludeSingul
```

Timed out

23 Chapter 1, section 1.3. Abel Equations of the Second Kind. subsection 1.3.2. Equations of the form $yy' = f(x)y + 1$

23.1 problem 1 508
23.2 problem 2 510
23.3 problem 3 512
23.4 problem 4 513
23.5 problem 5 515
23.6 problem 6 516
23.7 problem 7 517
23.8 problem 8 518
23.9 problem 9 519
23.10 problem 10 520
23.11 problem 11 521
23.12 problem 12 522

23.1 problem 1

Internal problem ID [10726]

Book: Handbook of exact solutions for ordinary differential equations. By Polyanin and Zaitsev. Second edition
Section: Chapter 1, section 1.3. Abel Equations of the Second Kind. subsection 1.3.2. Equations of the form $yy' = f(x)y + 1$
Problem number: 1.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_rational, [_Abel, '2nd type', 'class A']]`

$$\boxed{yy' - (ax+b)\,y = 1}$$

✓ Solution by Maple
Time used: 0.0 (sec). Leaf size: 215

```
dsolve(y(x)*diff(y(x),x)=(a*x+b)*y(x)+1,y(x), singsol=all)
```

$$\frac{(ax+b)\left(\operatorname{AiryBi}\left(-\frac{\left(-2ay(x)+(ax+b)^2\right)2^{\frac{2}{3}}}{4(-a^2)^{\frac{1}{3}}}\right)c_1 - \operatorname{AiryAi}\left(-\frac{\left(-2ay(x)+(ax+b)^2\right)2^{\frac{2}{3}}}{4(-a^2)^{\frac{1}{3}}}\right)\right)(-a^2)^{\frac{1}{3}}\,2^{\frac{1}{3}} + 2\left(\operatorname{AiryBi}\left(\right.\right.}{2^{\frac{1}{3}}\,(-a^2)^{\frac{1}{3}}\,(ax+b)\operatorname{AiryBi}\left(-\frac{\left(-2ay(x)+(ax+b)^2\right)2^{\frac{2}{3}}}{4(-a^2)^{\frac{1}{3}}}\right) + 2\operatorname{AiryBi}\left(1\right.} = 0$$

✓ Solution by Mathematica

Time used: 0.901 (sec). Leaf size: 161

```
DSolve[y[x]*y'[x]==(a*x+b)*y[x]+1,y[x],x,IncludeSingularSolutions -> True]
```

$$\text{Solve}\left[\frac{\sqrt[3]{2}(ax+b)\,\text{AiryAi}\left(\frac{(b+ax)^2-2ay(x)}{2\sqrt[3]{2}a^{2/3}}\right)-2\sqrt[3]{a}\,\text{AiryAiPrime}\left(\frac{(b+ax)^2-2ay(x)}{2\sqrt[3]{2}a^{2/3}}\right)}{\sqrt[3]{2}(ax+b)\,\text{AiryBi}\left(\frac{(b+ax)^2-2ay(x)}{2\sqrt[3]{2}a^{2/3}}\right)-2\sqrt[3]{a}\,\text{AiryBiPrime}\left(\frac{(b+ax)^2-2ay(x)}{2\sqrt[3]{2}a^{2/3}}\right)}+c_1=0,y(x)\right]$$

23.2 problem 2

Internal problem ID [10727]

Book: Handbook of exact solutions for ordinary differential equations. By Polyanin and Zaitsev. Second edition
Section: Chapter 1, section 1.3. Abel Equations of the Second Kind. subsection 1.3.2. Equations of the form $yy' = f(x)y + 1$
Problem number: 2.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_rational, [_Abel, '2nd type', 'class B']]`

$$\boxed{yy' - \frac{y}{(ax+b)^2} = 1}$$

✓ Solution by Maple
Time used: 0.0 (sec). Leaf size: 557

```
dsolve(y(x)*diff(y(x),x)=(a*x+b)^(-2)*y(x)+1,y(x), singsol=all)
```

$$\frac{-a\left(-\operatorname{AiryBi}\left(-\frac{2^{\frac{2}{3}}\left(-\frac{a^2(ax+b)^2y(x)^2}{2}+(-a^2x-ab)y(x)+a^4x^3+3a^3bx^2+3a^2b^2x+ab^3-\frac{1}{2}\right)}{2(a^2)^{\frac{1}{3}}(ax+b)^2}\right)c_1+\operatorname{AiryAi}\left(-\frac{2^{\frac{2}{3}}\left(-\frac{a^2(ax+b)^2y(}{2}\right.\right.\right.}{a2^{\frac{1}{3}}\left(1+a\left(ax+b\right)y\left(x\right)\right.}$$

$$= 0$$

✓ Solution by Mathematica

Time used: 2.233 (sec). Leaf size: 561

```
DSolve[y[x]*y'[x]==(a*x+b)^(-2)*y[x]+1,y[x],x,IncludeSingularSolutions -> True]
```

$$\text{Solve}\left[\frac{ay(x)(ax+b)\,\text{AiryAi}\left(\frac{-2x^3a^4-6bx^2a^3+(b+ax)^2y(x)^2a^2-6b^2xa^2-2b^3a+2(b+ax)y(x)a+1}{2\sqrt[3]{2}(a(b+ax)^3)^{2/3}}\right)+\text{AiryAi}\left(\frac{-2x^3a^4-6bx^2}{}\right.}{ay(x)(ax+b)\,\text{AiryBi}\left(\frac{-2x^3a^4-6bx^2a^3+(b+ax)^2y(x)^2a^2-6b^2xa^2-2b^3a+2(b+ax)y(x)a+1}{2\sqrt[3]{2}(a(b+ax)^3)^{2/3}}\right)+\text{AiryBi}\left(\frac{-2x^3a^4-6bx^2}{}\right.}+c_1=0,y(x)\right]$$

23.3 problem 3

Internal problem ID [10728]

Book: Handbook of exact solutions for ordinary differential equations. By Polyanin and Zaitsev. Second edition
Section: Chapter 1, section 1.3. Abel Equations of the Second Kind. subsection 1.3.2. Equations of the form $yy' = f(x)y + 1$
Problem number: 3.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_rational, [_Abel, '2nd type', 'class B']]`

$$\boxed{yy' - \left(a - \frac{1}{ax}\right) y = 1}$$

✓ Solution by Maple
Time used: 0.0 (sec). Leaf size: 39

```
dsolve(y(x)*diff(y(x),x)=(a-1/(a*x))*y(x)+1,y(x), singsol=all)
```

$$y(x) = \frac{a^2 x - \text{RootOf}\left(-\mathrm{e}^{-Z} - \text{expIntegral}_1\left(-\_Z\right) a^2 x + c_1 a^2 x\right)}{a}$$

✓ Solution by Mathematica
Time used: 0.268 (sec). Leaf size: 37

```
DSolve[y[x]*y'[x]==(a-1/(a*x))*y[x]+1,y[x],x,IncludeSingularSolutions -> True]
```

$$\text{Solve}\left[\text{ExpIntegralEi}(a(ax - y(x))) + c_1 = \frac{e^{a(ax-y(x))}}{a^2 x}, y(x)\right]$$

23.4 problem 4

Internal problem ID [10729]

Book: Handbook of exact solutions for ordinary differential equations. By Polyanin and Zaitsev. Second edition
Section: Chapter 1, section 1.3. Abel Equations of the Second Kind. subsection 1.3.2. Equations of the form $yy' = f(x)y + 1$
Problem number: 4.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[[_1st_order, _with_linear_symmetries], [_Abel, '2nd type', 'c`

$$\boxed{yy' - \frac{y}{\sqrt{ax+b}} = 1}$$

✓ Solution by Maple
Time used: 0.203 (sec). Leaf size: 153

```
dsolve(y(x)*diff(y(x),x)=(a*x+b)^(-1/2)*y(x)+1,y(x), singsol=all)
```

$$\frac{2\,\operatorname{arctanh}\left(\frac{-\sqrt{ax+b}\,y(x)a+ax+b}{\sqrt{(1+2a)(ax+b)^2}}\right)ax}{\sqrt{(1+2a)\,(ax+b)^2}}$$
$$+\ln\left(\left(a(y(x))^2-2ax-2b\right)\sqrt{ax+b}-2(ax+b)\,y(x)\right)$$
$$+\frac{2\,\operatorname{arctanh}\left(\frac{-\sqrt{ax+b}\,y(x)a+ax+b}{\sqrt{(1+2a)(ax+b)^2}}\right)b}{\sqrt{(1+2a)\,(ax+b)^2}}-\frac{\ln(ax+b)}{2}-c_1=0$$

✓ Solution by Mathematica

Time used: 0.256 (sec). Leaf size: 90

```
DSolve[y[x]*y'[x]==(a*x+b)^(-1/2)*y[x]+1,y[x],x,IncludeSingularSolutions -> True]
```

$$\text{Solve}\left[-\frac{\frac{2\arctan\left(\frac{\frac{ay(x)}{\sqrt{ax+b}}-1}{\sqrt{-2a-1}}\right)}{\sqrt{-2a-1}}+\log\left(-\frac{ay(x)^2}{ax+b}+\frac{2y(x)}{\sqrt{ax+b}}+2\right)}{a}=\frac{\log(ax+b)}{a}+c_1,y(x)\right]$$

23.5 problem 5

Internal problem ID [10730]

Book: Handbook of exact solutions for ordinary differential equations. By Polyanin and Zaitsev. Second edition
Section: Chapter 1, section 1.3. Abel Equations of the Second Kind. subsection 1.3.2. Equations of the form $yy' = f(x)y + 1$
Problem number: 5.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[[_Abel, '2nd type', 'class B']]`

$$\boxed{yy' - \frac{3y}{\sqrt{a\,x^{\frac{3}{2}} + 8x}} = 1}$$

✓ Solution by Maple
Time used: 0.0 (sec). Leaf size: 293

```
dsolve(y(x)*diff(y(x),x)=3*(a*x^(3/2)+8*x)^(-1/2)*y(x)+1,y(x), singsol=all)
```

$$\left(-\frac{a\sqrt{x}\left(-2a\,x^{\frac{3}{2}}+\sqrt{x}\,a y(x)^2-8\sqrt{x(8+\sqrt{x}\,a)}\,y(x)-16x\right)}{\left(\sqrt{x}\,a y(x)-4\sqrt{x(8+\sqrt{x}\,a)}\right)^2}\right)^{\frac{1}{4}}\sqrt{2\sqrt{x}\,a+16}\,a\sqrt{x}\,y(x)+4\sqrt{-\frac{\sqrt{2\sqrt{x}\,a+16}\,\sqrt{x(8+\sqrt{x}\,a)}}{\sqrt{x}\,a y(x)-4\sqrt{x(8+\sqrt{x}\,a)}}}\left(\sqrt{x}\,a$$

$$\sqrt{-\frac{\sqrt{2\sqrt{x}\,a+16}\,\sqrt{x(8+\sqrt{x}\,a)}}{\sqrt{x}\,a y(x)-4\sqrt{x(8+\sqrt{x}\,a)}}}\left(\sqrt{x}\,a y(x)-4\sqrt{x(8$$

$= 0$

✗ Solution by Mathematica
Time used: 0.0 (sec). Leaf size: 0

```
DSolve[y[x]*y'[x]==3*(a*x^(3/2)+8*x)^(-1/2)*y[x]+1,y[x],x,IncludeSingularSolutions -> True]
```

Not solved

23.6 problem 6

Internal problem ID [10731]

Book: Handbook of exact solutions for ordinary differential equations. By Polyanin and Zaitsev. Second edition
Section: Chapter 1, section 1.3. Abel Equations of the Second Kind. subsection 1.3.2. Equations of the form $yy' = f(x)y + 1$
Problem number: 6.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_rational, [_Abel, '2nd type', 'class B']]`

$$\boxed{yy' - \left(\frac{a}{x^{\frac{2}{3}}} - \frac{2}{3a\,x^{\frac{1}{3}}}\right) y = 1}$$

✓ Solution by Maple
Time used: 0.0 (sec). Leaf size: 187

```
dsolve(y(x)*diff(y(x),x)=(a*x^(-2/3)-2/3*a^(-1)*x^(-1/3))*y(x)+1,y(x), singsol=all)
```

$$\frac{-\sqrt{\frac{x^{\frac{2}{3}}+ay(x)}{a^4}}\ \mathrm{BesselI}\left(1, \frac{2\sqrt{\frac{x^{\frac{2}{3}}+ay(x)}{a^4}}}{3}\right) c_1 a^2 + \mathrm{BesselK}\left(1, -\frac{2\sqrt{\frac{x^{\frac{2}{3}}+ay(x)}{a^4}}}{3}\right) \sqrt{\frac{x^{\frac{2}{3}}+ay(x)}{a^4}}\ a^2 + x^{\frac{1}{3}}\ \mathrm{BesselI}\left(0, \frac{2\sqrt{}}{}\right.}{-\mathrm{BesselI}\left(1, \frac{2\sqrt{\frac{x^{\frac{2}{3}}+ay(x)}{a^4}}}{3}\right) \sqrt{\frac{x^{\frac{2}{3}}+ay(x)}{a^4}}\ a^2 + x^{\frac{1}{3}}\ \mathrm{BesselI}\left(0, \frac{2\sqrt{\frac{x^{\frac{2}{3}}+a}{}}}{3}\right.} = 0$$

✗ Solution by Mathematica
Time used: 0.0 (sec). Leaf size: 0

```
DSolve[y[x]*y'[x]==(a*x^(-2/3)-2/3*a^(-1)*x^(-1/3))*y[x]+1,y[x],x,IncludeSingularSolutions -
```

Not solved

23.7 problem 7

Internal problem ID [10732]

Book: Handbook of exact solutions for ordinary differential equations. By Polyanin and Zaitsev. Second edition
Section: Chapter 1, section 1.3. Abel Equations of the Second Kind. subsection 1.3.2. Equations of the form $yy' = f(x)y + 1$
Problem number: 7.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[[_Abel, '2nd type', 'class A']]`

$$\boxed{yy' - \mathrm{e}^{\lambda x} y a = 1}$$

✓ Solution by Maple
Time used: 0.0 (sec). Leaf size: 83

```
dsolve(y(x)*diff(y(x),x)=a*exp(lambda*x)*y(x)+1,y(x), singsol=all)
```

$$c_1 + a \operatorname{erf}\left(\frac{\left(\lambda y(x) - \mathrm{e}^{x\lambda} a\right)\sqrt{2}}{2\sqrt{-\lambda}}\right)\sqrt{2}\sqrt{\pi} - 2\sqrt{-\lambda}\,\mathrm{e}^{\frac{y(x)^2\lambda^2 - 2y(x)\mathrm{e}^{x\lambda}a\lambda + a^2\mathrm{e}^{2x\lambda} - 2x\lambda^2}{2\lambda}} = 0$$

✓ Solution by Mathematica
Time used: 1.687 (sec). Leaf size: 83

```
DSolve[y[x]*y'[x]==a*Exp[\[Lambda]*x]*y[x]+1,y[x],x,IncludeSingularSolutions -> True]
```

$$\text{Solve}\left[-\frac{ae^{\lambda x}}{\sqrt{\lambda}} = \frac{2e^{\frac{\left(ae^{\lambda x} - \lambda y(x)\right)^2}{2\lambda}}}{\sqrt{2\pi}\operatorname{erfi}\left(\frac{\lambda y(x) - ae^{\lambda x}}{\sqrt{2}\sqrt{\lambda}}\right) + 2c_1}, y(x)\right]$$

23.8 problem 8

Internal problem ID [10733]

Book: Handbook of exact solutions for ordinary differential equations. By Polyanin and Zaitsev. Second edition
Section: Chapter 1, section 1.3. Abel Equations of the Second Kind. subsection 1.3.2. Equations of the form $yy' = f(x)y + 1$
Problem number: 8.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[[_Abel, '2nd type', 'class A']]`

$$\boxed{yy' - \left(a\,\mathrm{e}^{\lambda x} + b\,\mathrm{e}^{-\lambda x}\right) y = 1}$$

✗ Solution by Maple

```
dsolve(y(x)*diff(y(x),x)=(a*exp(lambda*x)+b*exp(-lambda*x))*y(x)+1,y(x), singsol=all)
```

No solution found

✗ Solution by Mathematica
Time used: 0.0 (sec). Leaf size: 0

```
DSolve[y[x]*y'[x]==(a*Exp[\[Lambda]*x]+b*Exp[-\[Lambda]*x])*y[x]+1,y[x],x,IncludeSingularSol
```

Not solved

23.9 problem 9

Internal problem ID [10734]

Book: Handbook of exact solutions for ordinary differential equations. By Polyanin and Zaitsev. Second edition
Section: Chapter 1, section 1.3. Abel Equations of the Second Kind. subsection 1.3.2. Equations of the form $yy' = f(x)y + 1$
Problem number: 9.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[[_Abel, '2nd type', 'class A']]`

$$\boxed{yy' - ay\cosh(x) = 1}$$

✗ Solution by Maple

```
dsolve(y(x)*diff(y(x),x)=a*y(x)*cosh(x)+1,y(x), singsol=all)
```

No solution found

✗ Solution by Mathematica
Time used: 0.0 (sec). Leaf size: 0

```
DSolve[y[x]*y'[x]==a*y[x]*Cosh[x]+1,y[x],x,IncludeSingularSolutions -> True]
```

Not solved

23.10 problem 10

Internal problem ID [10735]

Book: Handbook of exact solutions for ordinary differential equations. By Polyanin and Zaitsev. Second edition
Section: Chapter 1, section 1.3. Abel Equations of the Second Kind. subsection 1.3.2. Equations of the form $yy' = f(x)y + 1$
Problem number: 10.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[[_Abel, '2nd type', 'class A']]`

$$\boxed{yy' - ay\sinh(x) = 1}$$

✗ Solution by Maple

```
dsolve(y(x)*diff(y(x),x)=a*y(x)*sinh(x)+1,y(x), singsol=all)
```

No solution found

✗ Solution by Mathematica
Time used: 0.0 (sec). Leaf size: 0

```
DSolve[y[x]*y'[x]==a*y[x]*Sinh[x]+1,y[x],x,IncludeSingularSolutions -> True]
```

Not solved

23.11 problem 11

Internal problem ID [10736]

Book: Handbook of exact solutions for ordinary differential equations. By Polyanin and Zaitsev. Second edition
Section: Chapter 1, section 1.3. Abel Equations of the Second Kind. subsection 1.3.2. Equations of the form $yy' = f(x)y + 1$
Problem number: 11.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[[_Abel, '2nd type', 'class A']]`

$$\boxed{yy' - a\cos(\lambda x)\, y = 1}$$

✗ Solution by Maple

```
dsolve(y(x)*diff(y(x),x)=a*cos(lambda*x)*y(x)+1,y(x), singsol=all)
```

No solution found

✗ Solution by Mathematica
Time used: 0.0 (sec). Leaf size: 0

```
DSolve[y[x]*y'[x]==a*Cos[\[Lambda]*x]*y[x]+1,y[x],x,IncludeSingularSolutions -> True]
```

Not solved

23.12 problem 12

Internal problem ID [10737]

Book: Handbook of exact solutions for ordinary differential equations. By Polyanin and Zaitsev. Second edition
Section: Chapter 1, section 1.3. Abel Equations of the Second Kind. subsection 1.3.2. Equations of the form $yy' = f(x)y + 1$
Problem number: 12.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[[_Abel, '2nd type', 'class A']]`

$$\boxed{yy' - \sin(\lambda x)\, ya = 1}$$

✗ Solution by Maple

```
dsolve(y(x)*diff(y(x),x)=a*sin(lambda*x)*y(x)+1,y(x), singsol=all)
```

No solution found

✗ Solution by Mathematica
Time used: 0.0 (sec). Leaf size: 0

```
DSolve[y[x]*y'[x]==a*Sin[\[Lambda]*x]*y[x]+1,y[x],x,IncludeSingularSolutions -> True]
```

Not solved

24 Chapter 1, section 1.3. Abel Equations of the Second Kind. subsection 1.3.3-2. Equations of the form $yy' = f_1(x)y + f_0(x)$

24.1 problem 1 526
24.2 problem 2 527
24.3 problem 3 528
24.4 problem 4 529
24.5 problem 5 530
24.6 problem 6 531
24.7 problem 7 532
24.8 problem 8 533
24.9 problem 9 534
24.10problem 10 536
24.11problem 11 537
24.12problem 12 538
24.13problem 13 539
24.14problem 14 540
24.15problem 15 541
24.16problem 16 542
24.17problem 17 544
24.18problem 18 545
24.19problem 19 546
24.20problem 20 547
24.21problem 21 548
24.22problem 22 549
24.23problem 23 550
24.24problem 24 551
24.25problem 25 552
24.26problem 26 553
24.27problem 27 554
24.28problem 28 555
24.29problem 29 556
24.30problem 30 557
24.31problem 31 558
24.32problem 32 559
24.33problem 33 560
24.34problem 34 561

24.35problem 35 562
24.36problem 36 563
24.37problem 37 564
24.38problem 38 565
24.39problem 39 566
24.40problem 40 567
24.41problem 41 568
24.42problem 42 569
24.43problem 43 570
24.44problem 44 571
24.45problem 45 572
24.46problem 46 573
24.47problem 47 574
24.48problem 48 575
24.49problem 49 576
24.50problem 50 577
24.51problem 51 578
24.52problem 52 579
24.53problem 53 580
24.54problem 54 581
24.55problem 55 582
24.56problem 56 583
24.57problem 57 584
24.58problem 58 585
24.59problem 59 586
24.60problem 60 587
24.61problem 61 588
24.62problem 62 589
24.63problem 63 590
24.64problem 64 591
24.65problem 65 592
24.66problem 66 593
24.67problem 67 594
24.68problem 68 595
24.69problem 69 596
24.70problem 70 598
24.71problem 71 599
24.72problem 72 600
24.73problem 73 601

24.74problem 74 602
24.75problem 75 603
24.76problem 76 604
24.77problem 77 605
24.78problem 78 606
24.79problem 79 607
24.80problem 80 608

24.1 problem 1

Internal problem ID [10738]

Book: Handbook of exact solutions for ordinary differential equations. By Polyanin and Zaitsev. Second edition
Section: Chapter 1, section 1.3. Abel Equations of the Second Kind. subsection 1.3.3-2. Equations of the form $yy' = f_1(x)y + f_0(x)$
Problem number: 1.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_rational, [_Abel, '2nd type', 'class A']]`

$$\boxed{yy' - (ax + 3b)\, y = -ab\, x^2 + c\, x^3 - 2b^2 x}$$

✓ Solution by Maple
Time used: 0.0 (sec). Leaf size: 224

```
dsolve(y(x)*diff(y(x),x)=(a*x+3*b)*y(x)+c*x^3-a*b*x^2-2*b^2*x,y(x), singsol=all)
```

$$x\left(\frac{-2y(x)^2+x(ax+4b)y(x)-ab\,x^3+c\,x^4-2b^2x^2}{(bx-y(x))^2}\right)^{\frac{1}{4}} \mathrm{e}^{-\frac{a\ \operatorname{arctanh}\left(\frac{-abx+2c\,x^2+ay(x)}{(-bx+y(x))\sqrt{a^2+8c}}\right)}{2\sqrt{a^2+8c}}} y(x) + \sqrt{\frac{x^2}{-bx+y(x)}}\,(bx-y(x)) \left(\left(\int \frac{x^2}{-bx+y(} \right.\right.$$

$$\sqrt{\frac{x^2}{-bx+y(x)}}\,(bx-y(x))$$

$= 0$

✗ Solution by Mathematica
Time used: 0.0 (sec). Leaf size: 0

```
DSolve[y[x]*y'[x]==(a*x+3*b)*y[x]+c*x^3-a*b*x^2-2*b^2*x,y[x],x,IncludeSingularSolutions -> T
```

Not solved

24.2 problem 2

Internal problem ID [10739]

Book: Handbook of exact solutions for ordinary differential equations. By Polyanin and Zaitsev. Second edition
Section: Chapter 1, section 1.3. Abel Equations of the Second Kind. subsection 1.3.3-2. Equations of the form $yy' = f_1(x)y + f_0(x)$
Problem number: 2.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_rational, [_Abel, '2nd type', 'class A']]`

$$\boxed{yy' - (3ax + b)\, y = -x^3 a^2 - ab\, x^2 + cx}$$

✓ Solution by Maple
Time used: 0.0 (sec). Leaf size: 826

```
dsolve(y(x)*diff(y(x),x)=(3*a*x+b)*y(x)-a^2*x^3-a*b*x^2+c*x,y(x), singsol=all)
```

$$y(x) = \frac{}{-b\,\mathrm{e}^{\mathrm{RootOf}\left(2a\,b^8 \operatorname{arctanh}\left(\frac{b^2\left(9b^2+27c+2\,\mathrm{e}^{\_Z}\right)}{9\sqrt{b^2\left(b^2+4c\right)\left(b^2+3c\right)^2}}\right)+20a\,b^6c \operatorname{arctanh}\left(\frac{b^2\left(9b^2+27c+2\,\mathrm{e}^{\_Z}\right)}{9\sqrt{b^2\left(b^2+4c\right)\left(b^2+3c\right)^2}}\right)+66a\,b^4c^2 \operatorname{arctanh}\left(\frac{b^2\left(9b^2+27c+2\,\mathrm{e}^{\_Z}\right)}{9\sqrt{b^2\left(b^2+4c\right)\left(b^2+3c\right)^2}}\right)\right.}$$

✓ Solution by Mathematica
Time used: 6.592 (sec). Leaf size: 194

```
DSolve[y[x]*y'[x]==(3*a*x+b)*y[x]-a^2*x^3-a*b*x^2+c*x,y[x],x,IncludeSingularSolutions -> Tru
```

$$\text{Solve}\left[\frac{2ab\left(\text{RootSum}\left[\#1^4a^2+\#1^3ab-2\#1^2ay(x)-\#1^2c-\#1by(x)+y(x)^2\&,\ \frac{-2\#1^3a^2\log(x-\#1)-\#1^2}{}\right.\right.}{c(3a+b-}\right.$$

24.3 problem 3

Internal problem ID [10740]

Book: Handbook of exact solutions for ordinary differential equations. By Polyanin and Zaitsev. Second edition
Section: Chapter 1, section 1.3. Abel Equations of the Second Kind. subsection 1.3.3-2. Equations of the form $yy' = f_1(x)y + f_0(x)$
Problem number: 3.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_rational, [_Abel, '2nd type', 'class A']]`

$$2yy' - (7ax + 5b)\, y = -3x^3a^2 - 3b^2x - 2c\,x^2$$

✓ Solution by Maple
Time used: 0.0 (sec). Leaf size: 4589

```
dsolve(2*y(x)*diff(y(x),x)=(7*a*x+5*b)*y(x)-3*a^2*x^3-2*c*x^2-3*b^2*x,y(x), singsol=all)
```

Expression too large to display

✗ Solution by Mathematica
Time used: 0.0 (sec). Leaf size: 0

```
DSolve[2*y[x]*y'[x]==(7*a*x+5*b)*y[x]-3*a^2*x^3-2*c*x^2-3*b^2*x,y[x],x,IncludeSingularSoluti
```

Not solved

24.4 problem 4

Internal problem ID [10741]

Book: Handbook of exact solutions for ordinary differential equations. By Polyanin and Zaitsev. Second edition
Section: Chapter 1, section 1.3. Abel Equations of the Second Kind. subsection 1.3.3-2. Equations of the form $yy' = f_1(x)y + f_0(x)$
Problem number: 4.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_rational, [_Abel, '2nd type', 'class A']]`

$$\boxed{yy' - ((3-m)\,x - 1)\,y = -(m-1)\,ax}$$

✗ Solution by Maple

```
dsolve(y(x)*diff(y(x),x)=((3-m)*x-1)*y(x)+(m-1)*(x^2-x^2-a*x),y(x), singsol=all)
```

No solution found

✗ Solution by Mathematica
Time used: 0.0 (sec). Leaf size: 0

```
DSolve[y[x]*y'[x]==((3-m)*x-1)*y[x]+(m-1)*(x^2-x^2-a*x),y[x],x,IncludeSingularSolutions -> T
```

Not solved

24.5 problem 5

Internal problem ID [10742]

Book: Handbook of exact solutions for ordinary differential equations. By Polyanin and Zaitsev. Second edition
Section: Chapter 1, section 1.3. Abel Equations of the Second Kind. subsection 1.3.3-2. Equations of the form $yy' = f_1(x)y + f_0(x)$
Problem number: 5.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_rational, [_Abel, '2nd type', 'class A']]`

$$\boxed{yy' + x\left(a\,x^2 + b\right)y = -x}$$

✓ Solution by Maple
Time used: 0.0 (sec). Leaf size: 179

```
dsolve(y(x)*diff(y(x),x)+x*(a*x^2+b)*y(x)+x=0,y(x), singsol=all)
```

$$\frac{2\,\mathrm{AiryBi}\left(1, \frac{4ay(x)+\left(a\,x^2+b\right)^2}{4a^{\frac{2}{3}}}\right) a^{\frac{1}{3}} c_1 + c_1\left(a\,x^2+b\right)\mathrm{AiryBi}\left(\frac{4ay(x)+\left(a\,x^2+b\right)^2}{4a^{\frac{2}{3}}}\right) - 2\,\mathrm{AiryAi}\left(1, \frac{4ay(x)+\left(a\,x^2+b\right)^2}{4a^{\frac{2}{3}}}\right) a}{2\,\mathrm{AiryBi}\left(1, \frac{4ay(x)+\left(a\,x^2+b\right)^2}{4a^{\frac{2}{3}}}\right) a^{\frac{1}{3}} + \mathrm{AiryBi}\left(\frac{4ay(x)+\left(a\,x^2+b\right)^2}{4a^{\frac{2}{3}}}\right)\left(a\,x^2+b\right)} = 0$$

✓ Solution by Mathematica
Time used: 0.492 (sec). Leaf size: 143

```
DSolve[y[x]*y'[x]+x*(a*x^2+b)*y[x]+x==0,y[x],x,IncludeSingularSolutions -> True]
```

$$\mathrm{Solve}\left[\frac{\left(ax^2+b\right)\mathrm{AiryAi}\left(\frac{\left(ax^2+b\right)^2+4ay(x)}{4a^{2/3}}\right) + 2\sqrt[3]{a}\,\mathrm{AiryAiPrime}\left(\frac{\left(ax^2+b\right)^2+4ay(x)}{4a^{2/3}}\right)}{\left(ax^2+b\right)\mathrm{AiryBi}\left(\frac{\left(ax^2+b\right)^2+4ay(x)}{4a^{2/3}}\right) + 2\sqrt[3]{a}\,\mathrm{AiryBiPrime}\left(\frac{\left(ax^2+b\right)^2+4ay(x)}{4a^{2/3}}\right)} + c_1 = 0, y(x)\right]$$

24.6 problem 6

Internal problem ID [10743]

Book: Handbook of exact solutions for ordinary differential equations. By Polyanin and Zaitsev. Second edition
Section: Chapter 1, section 1.3. Abel Equations of the Second Kind. subsection 1.3.3-2. Equations of the form $yy' = f_1(x)y + f_0(x)$
Problem number: 6.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_rational, [_Abel, '2nd type', 'class B']]`

$$\boxed{yy' + a\left(1 - \frac{1}{x}\right)y = a^2}$$

✓ Solution by Maple
Time used: 0.0 (sec). Leaf size: 27

```
dsolve(y(x)*diff(y(x),x)+a*(1-x^(-1))*y(x)=a^2,y(x), singsol=all)
```

$$y(x) = a\left(-x + \text{RootOf}\left(-\mathrm{e}^{-Z} - \text{expIntegral}_1\left(-\_Z\right)x + c_1 x\right)\right)$$

✓ Solution by Mathematica
Time used: 0.208 (sec). Leaf size: 30

```
DSolve[y[x]*y'[x]+a*(1-x^(-1))*y[x]==a^2,y[x],x,IncludeSingularSolutions -> True]
```

$$\text{Solve}\left[\text{ExpIntegralEi}\left(x + \frac{y(x)}{a}\right) + c_1 = \frac{e^{\frac{y(x)}{a}+x}}{x}, y(x)\right]$$

24.7 problem 7

Internal problem ID [10744]

Book: Handbook of exact solutions for ordinary differential equations. By Polyanin and Zaitsev. Second edition
Section: Chapter 1, section 1.3. Abel Equations of the Second Kind. subsection 1.3.3-2. Equations of the form $yy' = f_1(x)y + f_0(x)$
Problem number: 7.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_rational, [_Abel, '2nd type', 'class B']]`

$$\boxed{yy' - a\left(1 - \frac{b}{x}\right)y = a^2 b}$$

✓ Solution by Maple
Time used: 0.0 (sec). Leaf size: 29

```
dsolve(y(x)*diff(y(x),x)-a*(1-b*x^(-1))*y(x)=a^2*b,y(x), singsol=all)
```

$$y(x) = a\left(-\operatorname{RootOf}\left(-\mathrm{e}^{-Z}b - \operatorname{expIntegral}_1\left(-\_Z\right)x + c_1 x\right)b + x\right)$$

✓ Solution by Mathematica
Time used: 0.293 (sec). Leaf size: 45

```
DSolve[y[x]*y'[x]-a*(1-b*x^(-1))*y[x]==a^2*b,y[x],x,IncludeSingularSolutions -> True]
```

$$\operatorname{Solve}\left[\operatorname{ExpIntegralEi}\left(\frac{ax - y(x)}{ab}\right) + c_1 = \frac{be^{\frac{ax-y(x)}{ab}}}{x}, y(x)\right]$$

24.8 problem 8

Internal problem ID [10745]

Book: Handbook of exact solutions for ordinary differential equations. By Polyanin and Zaitsev. Second edition
Section: Chapter 1, section 1.3. Abel Equations of the Second Kind. subsection 1.3.3-2. Equations of the form $yy' = f_1(x)y + f_0(x)$
Problem number: 8.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_rational, [_Abel, '2nd type', 'class A']]`

$$\boxed{yy' - x^{n-1}((2n+1)x + an)y = -n x^{2n}(x+a)}$$

✓ Solution by Maple
Time used: 0.0 (sec). Leaf size: 153

```
dsolve(y(x)*diff(y(x),x)=x^(n-1)*((1+2*n)*x+a*n)*y(x)-n*x^(2*n)*(x+a),y(x), singsol=all)
```

$$y(x) = \frac{2\left(-\frac{\sqrt{-n^2}\,x\tan\left(\frac{\text{RootOf}\left(-2an\,\mathrm{e}_{\_}Z+\_a-nx\,\mathrm{e}_{\_}Z+\_a-\tan\left(\frac{\_a\sqrt{-n^2}}{2}\right)\_Z\sqrt{-n^2}\,x+2c_1x\,\mathrm{e}_{\_}a\right)\sqrt{-n^2}}{2}\right)}{2}+n\left(a+\frac{x}{2}\right)\right)x^n}{\tan\left(\frac{\text{RootOf}\left(-2an\,\mathrm{e}_{\_}Z+\_a-nx\,\mathrm{e}_{\_}Z+\_a-\tan\left(\frac{\_a\sqrt{-n^2}}{2}\right)\_Z\sqrt{-n^2}\,x+2c_1x\,\mathrm{e}_{\_}a\right)\sqrt{-n^2}}{2}\right)\sqrt{-n^2}-n}$$

✗ Solution by Mathematica
Time used: 0.0 (sec). Leaf size: 0

```
DSolve[y[x]*y'[x]==x^(n-1)*((1+2*n)*x+a*n)*y[x]-n*x^(2*n)*(x+a),y[x],x,IncludeSingularSoluti
```

Not solved

24.9 problem 9

Internal problem ID [10746]

Book: Handbook of exact solutions for ordinary differential equations. By Polyanin and Zaitsev. Second edition
Section: Chapter 1, section 1.3. Abel Equations of the Second Kind. subsection 1.3.3-2. Equations of the form $yy' = f_1(x)y + f_0(x)$
Problem number: 9.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_rational, [_Abel, '2nd type', 'class A']]`

$$\boxed{yy' - a(-nb + x)\, x^{n-1} y = c\left(x^2 - (2n+1)\, bx + n(1+n)\, b^2\right) x^{-1+2n}}$$

✓ Solution by Maple
Time used: 0.0 (sec). Leaf size: 1972

```
dsolve(y(x)*diff(y(x),x)=a*(x-n*b)*x^(n-1)*y(x)+c*(x^2-(2*n+1)*b*x+n*(n+1)*b^2)*x^(2*n-1),y(
```

Expression too large to display

✓ Solution by Mathematica

Time used: 0.744 (sec). Leaf size: 200

```
DSolve[y[x]*y'[x]==a*(x-n*b)*x^(n-1)*y[x]+c*(x^2-(2*n+1)*b*x+n*(n+1)*b^2)*x^(2*n-1),y[x],x,I
```

$$\text{Solve}\left[\frac{a^2\left(-\frac{2a\arctan\left(\frac{a^2-\frac{2ac(n+1)y(x)}{-bcx^n-bcnx^n+cx^{n+1}}}{a\sqrt{a^2+4c(n+1)}}\right)}{\sqrt{a^2+4c(n+1)}}-\log\left(a^2\left(\frac{ay(x)}{-bcx^n-bcnx^n+cx^{n+1}}+1\right)-\frac{a^2c(n+1)y(x)^2}{(-bcx^n-bcnx^n+cx^{n+1})^2}\right)\right)}{2c(n+1)}\right.$$

$$\left.+c_1,y(x)\right]$$

24.10 problem 10

Internal problem ID [10747]

Book: Handbook of exact solutions for ordinary differential equations. By Polyanin and Zaitsev. Second edition
Section: Chapter 1, section 1.3. Abel Equations of the Second Kind. subsection 1.3.3-2. Equations of the form $yy' = f_1(x)y + f_0(x)$
Problem number: 10.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_rational, [_Abel, '2nd type', 'class A']]`

$$\boxed{yy' - \left(a(2n+k)\,x^k + b\right)x^{n-1}y = \left(-a^2 n\,x^{2k} - ab\,x^k + c\right)x^{-1+2n}}$$

✗ Solution by Maple

```
dsolve(y(x)*diff(y(x),x)=(a*(2*n+k)*x^k+b)*x^(n-1)*y(x)+(-a^2*n*x^(2*k)-a*b*x^k+c)*x^(2*n-1)
```

No solution found

✗ Solution by Mathematica
Time used: 0.0 (sec). Leaf size: 0

```
DSolve[y[x]*y'[x]==(a*(2*n+k)*x^k+b)*x^(n-1)*y[x]+(-a^2*n*x^(2*k)-a*b*x^k+c)*x^(2*n-1),y[x],
```

Not solved

24.11 problem 11

Internal problem ID [10748]

Book: Handbook of exact solutions for ordinary differential equations. By Polyanin and Zaitsev. Second edition
Section: Chapter 1, section 1.3. Abel Equations of the Second Kind. subsection 1.3.3-2. Equations of the form $yy' = f_1(x)y + f_0(x)$
Problem number: 11.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_rational, [_Abel, '2nd type', 'class B']]`

$$\boxed{yy' - \left(a(2n+k)\,x^{2k} + b(2m-k)\right)x^{m-k-1}y = -\frac{a^2 m\,x^{4k} + c\,x^{2k} + b^2 m}{x}}$$

✗ Solution by Maple

```
dsolve(y(x)*diff(y(x),x)=(a*(2*n+k)*x^(2*k)+b*(2*m-k))*x^(m-k-1)*y(x)-(a^2*m*x^(4*k)+c*x^(2*
```

No solution found

✗ Solution by Mathematica
Time used: 0.0 (sec). Leaf size: 0

```
DSolve[y[x]*y'[x]==(a*(2*n+k)*x^(2*k)+b*(2*m-k))*x^(m-k-1)*y[x]-(a^2*m*x^(4*k)+c*x^(2*k)+b^2
```

Timed out

24.12 problem 12

Internal problem ID [10749]

Book: Handbook of exact solutions for ordinary differential equations. By Polyanin and Zaitsev. Second edition
Section: Chapter 1, section 1.3. Abel Equations of the Second Kind. subsection 1.3.3-2. Equations of the form $yy' = f_1(x)y + f_0(x)$
Problem number: 12.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_rational, [_Abel, '2nd type', 'class B']]`

$$y y' - \frac{\left((m+2L-3)\,x+n-2L+3\right)y}{x} = \left((m-L-1)\,x^2+(n-m-2L+3)\,x-n+L-2\right)x^{1-2L}$$

✗ Solution by Maple

```
dsolve(y(x)*diff(y(x),x)=((m+2*L-3)*x+n-2*L+3)*1/x*y(x)+((m-L-1)*x^2+(n-m-2*L+3)*x-n+L-2)*x^
```

No solution found

✗ Solution by Mathematica
Time used: 0.0 (sec). Leaf size: 0

```
DSolve[y[x]*y'[x]==((m+2*L-3)*x+n-2*L+3)*1/x*y[x]+((m-L-1)*x^2+(n-m-2*L+3)*x-n+L-2)*x^(1-2*L
```

Timed out

24.13 problem 13

Internal problem ID [10750]

Book: Handbook of exact solutions for ordinary differential equations. By Polyanin and Zaitsev. Second edition
Section: Chapter 1, section 1.3. Abel Equations of the Second Kind. subsection 1.3.3-2. Equations of the form $yy' = f_1(x)y + f_0(x)$
Problem number: 13.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_rational, [_Abel, '2nd type', 'class A']]`

$$yy' - \left(a(2n+1)\,x^2 + cx + b(-1+2n)\right)x^{n-2}y = -\left(n\,a^2x^4 + ac\,x^3 + n\,b^2 + bcx + d\,x^2\right)x^{2n-3}$$

✗ Solution by Maple

```
dsolve(y(x)*diff(y(x),x)=(a*(2*n+1)*x^2+c*x+b*(2*n-1))*x^(n-2)*y(x)-(n*a^2*x^4+a*c*x^3+d*x^2
```

No solution found

✗ Solution by Mathematica
Time used: 0.0 (sec). Leaf size: 0

```
DSolve[y[x]*y'[x]==(a*(2*n+1)*x^2+c*x+b*(2*n-1))*x^(n-2)*y[x]-(n*a^2*x^4+a*c*x^3+d*x^2+b*c*x
```

Timed out

24.14 problem 14

Internal problem ID [10751]

Book: Handbook of exact solutions for ordinary differential equations. By Polyanin and Zaitsev. Second edition
Section: Chapter 1, section 1.3. Abel Equations of the Second Kind. subsection 1.3.3-2. Equations of the form $yy' = f_1(x)y + f_0(x)$
Problem number: 14.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[[_Abel, '2nd type', 'class A']]`

$$yy' - (a(n-1)\,x + b(2\lambda+n))\,x^{\lambda-1}(ax+b)^{-\lambda-2}\,y = -(anx + b(\lambda+n))\,x^{2\lambda-1}(ax+b)^{-2\lambda-3}$$

✗ Solution by Maple

```
dsolve(y(x)*diff(y(x),x)=(a*(n-1)*x+b*(2*lambda+n))*x^(lambda-1)*(a*x+b)^(-lambda-2)*y(x)-(a
```

No solution found

✗ Solution by Mathematica
Time used: 0.0 (sec). Leaf size: 0

```
DSolve[y[x]*y'[x]==(a*(n-1)*x+b*(2*\[Lambda]+n))*x^(\[Lambda]-1)*(a*x+b)^(-\[Lambda]-2)*y[x]
```

Not solved

24.15 problem 15

Internal problem ID [10752]

Book: Handbook of exact solutions for ordinary differential equations. By Polyanin and Zaitsev. Second edition
Section: Chapter 1, section 1.3. Abel Equations of the Second Kind. subsection 1.3.3-2. Equations of the form $yy' = f_1(x)y + f_0(x)$
Problem number: 15.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_rational, [_Abel, '2nd type', 'class B']]`

$$\boxed{yy' - \frac{a((m-1)x+1)y}{x} = \frac{a^2(mx+1)(x-1)}{x}}$$

✓ Solution by Maple
Time used: 0.0 (sec). Leaf size: 279

```
dsolve(y(x)*diff(y(x),x)-a*((m-1)*x+1)*1/x*y(x)=a^2*1/x*(m*x+1)*(x-1),y(x), singsol=all)
```

$$-\frac{27(m-1)\left(-54m^4xa(m+2)\left(m+\frac{1}{2}\right)\left(\int^{\frac{9m\left((m-1)y(x)+3\left(\frac{1}{3}+\left(x-\frac{1}{3}\right)m\right)a\right)}{(m-1)(1+2m)(m+2)(-y(x)+a)}} \frac{\_a((m^2+m-2)\_a-9m)^{\frac{1}{1+m}}((2m^2-m}{8((m^2-\frac{1}{2}m-\frac{1}{2})\_a+\frac{9m}{2})((m^2+m-2)\_a-9m}\right.\right.}{m(2}$$

$= 0$

✗ Solution by Mathematica
Time used: 0.0 (sec). Leaf size: 0

```
DSolve[y[x]*y'[x]-a*((m-1)*x+1)*1/x*y[x]==a^2*1/x*(m*x+1)*(x-1),y[x],x,IncludeSingularSoluti
```

Not solved

24.16 problem 16

Internal problem ID [10753]

Book: Handbook of exact solutions for ordinary differential equations. By Polyanin and Zaitsev. Second edition
Section: Chapter 1, section 1.3. Abel Equations of the Second Kind. subsection 1.3.3-2. Equations of the form $yy' = f_1(x)y + f_0(x)$
Problem number: 16.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_rational, [_Abel, '2nd type', 'class B']]`

$$\boxed{yy' - a\left(1 - \frac{b}{\sqrt{x}}\right)y = \frac{a^2 b}{\sqrt{x}}}$$

✓ Solution by Maple
Time used: 0.0 (sec). Leaf size: 269

```
dsolve(y(x)*diff(y(x),x)-a*(1-b*x^(-1/2))*y(x)=a^2*b*x^(-1/2),y(x), singsol=all)
```

$$\frac{(b^2)^{\frac{1}{3}} c_1 2^{\frac{2}{3}} \left(-\sqrt{x}+b\right) \operatorname{AiryBi}\left(\frac{2^{\frac{1}{3}}\left(-2\sqrt{x}\,ab+(b^2+x)a-y(x)\right)}{2(b^2)^{\frac{1}{3}}a}\right) + 2\operatorname{AiryBi}\left(1, \frac{2^{\frac{1}{3}}\left(-2\sqrt{x}\,ab+(b^2+x)a-y(x)\right)}{2(b^2)^{\frac{1}{3}}a}\right) c_1 b - 2\,\mathrm{A}}{(b^2)^{\frac{1}{3}} 2^{\frac{2}{3}} \left(-\sqrt{x}+b\right) \operatorname{AiryBi}\left(\frac{2^{\frac{1}{3}}\left(-2\sqrt{x}\,ab+(b^2+x)a-y(x)\right)}{2(b^2)^{\frac{1}{3}}a}\right) -}$$

$$= 0$$

✓ Solution by Mathematica

Time used: 1.905 (sec). Leaf size: 323

```
DSolve[y[x]*y'[x]-a*(1-b*x^(-1/2))*y[x]==a^2*b*x^(-1/2),y[x],x,IncludeSingularSolutions -> T
```

$$\text{Solve}\left[\frac{\sqrt[3]{-1}2^{2/3}\sqrt[3]{\left(b-\sqrt{x}\right)^3}\,\text{AiryAi}\left(\frac{\left(-\frac{1}{2}\right)^{2/3}\left(\left(b-\sqrt{x}\right)^3\right)^{2/3}\left(a\left(b-\sqrt{x}\right)^2-y(x)\right)}{ab^{2/3}\left(b-\sqrt{x}\right)^2}\right)-2\sqrt[3]{b}\,\text{AiryAiPrime}\left(\frac{\left(-\frac{1}{2}\right)^{2/3}\left(\left(b-\right.\right.\right.}{\sqrt[3]{-1}2^{2/3}\sqrt[3]{\left(b-\sqrt{x}\right)^3}\,\text{AiryBi}\left(\frac{\left(-\frac{1}{2}\right)^{2/3}\left(\left(b-\sqrt{x}\right)^3\right)^{2/3}\left(a\left(b-\sqrt{x}\right)^2-y(x)\right)}{ab^{2/3}\left(b-\sqrt{x}\right)^2}\right)-2\sqrt[3]{b}\,\text{AiryBiPrime}\left(\frac{\left(-\frac{1}{2}\right)^{2/3}\left(\left(b-\right.\right.\right.}+c_1=0,y(x)\right]$$

24.17 problem 17

Internal problem ID [10754]

Book: Handbook of exact solutions for ordinary differential equations. By Polyanin and Zaitsev. Second edition
Section: Chapter 1, section 1.3. Abel Equations of the Second Kind. subsection 1.3.3-2. Equations of the form $yy' = f_1(x)y + f_0(x)$
Problem number: 17.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[[_Abel, '2nd type', 'class B']]`

$$\boxed{yy' - \frac{3y}{(ax+b)^{\frac{1}{3}} x^{\frac{5}{3}}} = \frac{3}{(ax+b)^{\frac{2}{3}} x^{\frac{7}{3}}}}$$

✓ Solution by Maple
Time used: 0.0 (sec). Leaf size: 143

```
dsolve(y(x)*diff(y(x),x)=3*(a*x+b)^(-1/3)*x^(-5/3)*y(x)+3*(a*x+b)^(-2/3)*x^(-7/3),y(x), sing
```

$$y(x) = -\frac{6\sqrt{3}}{(ax+b)^{\frac{1}{3}} x^{\frac{2}{3}} \left(\left((ax+b)^{\frac{1}{3}} x^{\frac{5}{3}} \left(\frac{a}{(ax+b)^2 x^4} \right)^{\frac{1}{3}} + 2 \right) \sqrt{3} + 3x^{\frac{5}{3}} (ax+b)^{\frac{1}{3}} \left(\frac{a}{(ax+b)^2 x^4} \right)^{\frac{1}{3}} \tan\left(\text{RootOf}\left(\sqrt{3}\right.\right.\right.\right.$$

✓ Solution by Mathematica
Time used: 1.769 (sec). Leaf size: 312

```
DSolve[y[x]*y'[x]==3*(a*x+b)^(-1/3)*x^(-5/3)*y[x]+3*(a*x+b)^(-2/3)*x^(-7/3),y[x],x,IncludeSi
```

$$\text{Solve}\left[\frac{1}{6} \left(2\sqrt{3} \arctan\left(\frac{-\frac{2\left(x^{2/3} y(x) \sqrt[3]{ax+b} + 3 \right)}{\sqrt[3]{ax^3} y(x)} - 1}{\sqrt{3}} \right) + 2\log\left(\frac{-x^{2/3} y(x) \sqrt[3]{ax+b} - 3}{\sqrt[3]{ax^3} y(x)} + 1 \right) - \log\left(\frac{\left(x^{2/3}\right.\right.\right.\right.$$

24.18 problem 18

Internal problem ID [10755]

Book: Handbook of exact solutions for ordinary differential equations. By Polyanin and Zaitsev. Second edition
Section: Chapter 1, section 1.3. Abel Equations of the Second Kind. subsection 1.3.3-2. Equations of the form $yy' = f_1(x)y + f_0(x)$
Problem number: 18.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_rational, [_Abel, '2nd type', 'class B']]`

$$3yy' - \frac{(-7\lambda s(3s+4\lambda)\,x + 6s - 2\lambda)\,y}{x^{\frac{1}{3}}} = \frac{6\lambda s x - 6}{x^{\frac{2}{3}}} + 2(\lambda s(3s+4\lambda)\,x + 5\lambda)\,(-\lambda s(3s+4\lambda)\,x + 3s + 4\lambda$$

✗ Solution by Maple

```
dsolve(3*y(x)*diff(y(x),x)=(-7*lambda*s*(3*s+4*lambda)*x+6*s-2*lambda)*x^(-1/3)*y(x)+6*(lamb
```

No solution found

✗ Solution by Mathematica
Time used: 0.0 (sec). Leaf size: 0

```
DSolve[3*y[x]*y'[x]==(-7*\[Lambda]*s*(3*s+4*\[Lambda])*x+6*s-2*\[Lambda])*x^(-1/3)*y[x]+6*(\
```

Timed out

24.19 problem 19

Internal problem ID [10756]

Book: Handbook of exact solutions for ordinary differential equations. By Polyanin and Zaitsev. Second edition
Section: Chapter 1, section 1.3. Abel Equations of the Second Kind. subsection 1.3.3-2. Equations of the form $yy' = f_1(x)y + f_0(x)$
Problem number: 19.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_rational, [_Abel, '2nd type', 'class B']]`

$$\boxed{yy' + \frac{a(6x-1)\,y}{2x} = -\frac{a^2(x-1)\,(4x-1)}{2x}}$$

✓ Solution by Maple
Time used: 0.015 (sec). Leaf size: 364

```
dsolve(y(x)*diff(y(x),x)+1/2*a*(6*x-1)*1/x*y(x)=-1/2*a^2*(x-1)*(4*x-1)*1/x,y(x), singsol=all
```

$$c_1 + \frac{\sqrt{2}\left(\frac{i\left(i\sqrt{-x}\,a+2ax+y(x)-a\right)\sqrt{-x}}{xa}\right)^{\frac{3}{2}}\left(-\frac{i\left(i\sqrt{-x}\,a+2ax+y(x)-a\right)\sqrt{-x}\ \text{hypergeom}\left(\left[\frac{1}{2},\frac{3}{2}\right],\left[\frac{7}{2}\right],\frac{i\left(i\sqrt{-x}\,a+2ax+y(x)-a\right)\sqrt{-x}}{2xa}\right)}{8xa}+\frac{5\left(4i\sqrt{2}\right.}{}\right.}{2\left(\frac{3}{2}-\frac{4i\sqrt{2}\,x-i\sqrt{2}-6i\sqrt{-x}+4x+2}{2\left(4i\sqrt{2}\,x+i\sqrt{2}-2i\sqrt{-x}-4\sqrt{2}\,\sqrt{-x}+4x-2\right)}\right)\text{hypergeom}\left([-2,-1],\left[-\frac{1}{2}\right],\frac{i\left(i\sqrt{-x}\,a+2ax\right.}{2x}\right.}$$
$$= 0$$

✗ Solution by Mathematica
Time used: 0.0 (sec). Leaf size: 0

```
DSolve[y[x]*y'[x]+1/2*a*(6*x-1)*1/x*y[x]==-1/2*a^2*(x-1)*(4*x-1)*1/x,y[x],x,IncludeSingularS
```

Not solved

24.20 problem 20

Internal problem ID [10757]

Book: Handbook of exact solutions for ordinary differential equations. By Polyanin and Zaitsev. Second edition
Section: Chapter 1, section 1.3. Abel Equations of the Second Kind. subsection 1.3.3-2. Equations of the form $yy' = f_1(x)y + f_0(x)$
Problem number: 20.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_rational, [_Abel, '2nd type', 'class B']]`

$$\boxed{yy' - \frac{a\left(1+\frac{2b}{x^2}\right)y}{2} = \frac{a^2\left(3x+\frac{4b}{x}\right)}{16}}$$

✗ Solution by Maple

```
dsolve(y(x)*diff(y(x),x)-1/2*a*(1+2*b*x^(-2))*y(x)=1/16*a^2*(3*x+4*b/x),y(x), singsol=all)
```

No solution found

✗ Solution by Mathematica
Time used: 0.0 (sec). Leaf size: 0

```
DSolve[y[x]*y'[x]-1/2*a*(1+2*b*x^(-2))*y[x]==1/16*a^2*(3*x+4*b/x),y[x],x,IncludeSingularSolu
```

Not solved

24.21 problem 21

Internal problem ID [10758]

Book: Handbook of exact solutions for ordinary differential equations. By Polyanin and Zaitsev. Second edition
Section: Chapter 1, section 1.3. Abel Equations of the Second Kind. subsection 1.3.3-2. Equations of the form $yy' = f_1(x)y + f_0(x)$
Problem number: 21.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_rational, [_Abel, '2nd type', 'class B']]`

$$\boxed{yy' + \frac{a(13x-20)\,y}{14x^{\frac{9}{7}}} = -\frac{3a^2(x-1)\,(x-8)}{14x^{\frac{11}{17}}}}$$

✗ Solution by Maple

```
dsolve(y(x)*diff(y(x),x)+1/14*a*(13*x-20)*x^(-9/7)*y(x)=-3/14*a^2*(x-1)*(x-8)*x^(-11/17),y(x
```

No solution found

✗ Solution by Mathematica
Time used: 0.0 (sec). Leaf size: 0

```
DSolve[y[x]*y'[x]+1/14*a*(13*x-20)*x^(-9/7)*y[x]==-3/14*a^2*(x-1)*(x-8)*x^(-11/17),y[x],x,In
```

Timed out

24.22 problem 22

Internal problem ID [10759]

Book: Handbook of exact solutions for ordinary differential equations. By Polyanin and Zaitsev. Second edition
Section: Chapter 1, section 1.3. Abel Equations of the Second Kind. subsection 1.3.3-2. Equations of the form $yy' = f_1(x)y + f_0(x)$
Problem number: 22.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_rational, [_Abel, '2nd type', 'class B']]`

$$\boxed{yy' + \frac{5a(23x-16)\,y}{56x^{\frac{9}{7}}} = -\frac{3a^2(x-1)\,(25x-32)}{56x^{\frac{11}{17}}}}$$

✗ Solution by Maple

```
dsolve(y(x)*diff(y(x),x)+5/56*a*(23*x-16)*x^(-9/7)*y(x)=-3/56*a^2*(x-1)*(25*x-32)*x^(-11/17)
```

No solution found

✗ Solution by Mathematica
Time used: 0.0 (sec). Leaf size: 0

```
DSolve[y[x]*y'[x]+5/56*a*(23*x-16)*x^(-9/7)*y[x]==-3/56*a^2*(x-1)*(25*x-32)*x^(-11/17),y[x],
```

Timed out

24.23 problem 23

Internal problem ID [10760]

Book: Handbook of exact solutions for ordinary differential equations. By Polyanin and Zaitsev. Second edition
Section: Chapter 1, section 1.3. Abel Equations of the Second Kind. subsection 1.3.3-2. Equations of the form $yy' = f_1(x)y + f_0(x)$
Problem number: 23.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_rational, [_Abel, '2nd type', 'class B']]`

$$\boxed{yy' + \frac{a(19x+85)\,y}{26x^{\frac{18}{13}}} = -\frac{3a^2(x-1)\,(x+25)}{26x^{\frac{23}{13}}}}$$

✗ Solution by Maple

```
dsolve(y(x)*diff(y(x),x)+1/26*a*(19*x+85)*x^(-18/13)*y(x)=-3/26*a^2*(x-1)*(x+25)*x^(-23/13),
```

No solution found

✗ Solution by Mathematica
Time used: 0.0 (sec). Leaf size: 0

```
DSolve[y[x]*y'[x]+1/26*a*(19*x+85)*x^(-18/13)*y[x]==-3/26*a^2*(x-1)*(x+25)*x^(-23/13),y[x],x
```

Timed out

24.24 problem 24

Internal problem ID [10761]

Book: Handbook of exact solutions for ordinary differential equations. By Polyanin and Zaitsev. Second edition
Section: Chapter 1, section 1.3. Abel Equations of the Second Kind. subsection 1.3.3-2. Equations of the form $yy' = f_1(x)y + f_0(x)$
Problem number: 24.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_rational, [_Abel, '2nd type', 'class B']]`

$$\boxed{yy' + \frac{a(13x-18)\,y}{15x^{\frac{7}{5}}} = -\frac{4a^2(x-1)\,(x-6)}{15x^{\frac{9}{5}}}}$$

✗ Solution by Maple

```
dsolve(y(x)*diff(y(x),x)+1/15*a*(13*x-18)*x^(-7/5)*y(x)=-4/15*a^2*(x-1)*(x-6)*x^(-9/5),y(x),
```

No solution found

✗ Solution by Mathematica
Time used: 0.0 (sec). Leaf size: 0

```
DSolve[y[x]*y'[x]+1/15*a*(13*x-18)*x^(-7/5)*y[x]==-4/15*a^2*(x-1)*(x-6)*x^(-9/5),y[x],x,Incl
```

Timed out

24.25 problem 25

Internal problem ID [10762]

Book: Handbook of exact solutions for ordinary differential equations. By Polyanin and Zaitsev. Second edition
Section: Chapter 1, section 1.3. Abel Equations of the Second Kind. subsection 1.3.3-2. Equations of the form $yy' = f_1(x)y + f_0(x)$
Problem number: 25.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_rational, [_Abel, '2nd type', 'class B']]`

$$\boxed{yy' + \frac{a(5x+1)\,y}{2\sqrt{x}} = a^2\left(-x^2+1\right)}$$

✗ Solution by Maple

```
dsolve(y(x)*diff(y(x),x)+1/2*a*(5*x+1)*x^(-1/2)*y(x)=a^2*(1-x^2),y(x), singsol=all)
```

No solution found

✗ Solution by Mathematica
Time used: 0.0 (sec). Leaf size: 0

```
DSolve[y[x]*y'[x]+1/2*a*(5*x+1)*x^(-1/2)*y[x]==a^2*(1-x^2),y[x],x,IncludeSingularSolutions -
```

Not solved

24.26 problem 26

Internal problem ID [10763]

Book: Handbook of exact solutions for ordinary differential equations. By Polyanin and Zaitsev. Second edition
Section: Chapter 1, section 1.3. Abel Equations of the Second Kind. subsection 1.3.3-2. Equations of the form $yy' = f_1(x)y + f_0(x)$
Problem number: 26.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_rational, [_Abel, '2nd type', 'class A']]`

$$\boxed{yy' + \frac{3a(19x-14)\,x^{\frac{7}{5}}y}{35} = -\frac{4a^2(x-1)\,(9x-14)\,x^{\frac{9}{5}}}{35}}$$

✗ Solution by Maple

```
dsolve(y(x)*diff(y(x),x)+3/35*a*(19*x-14)*x^(7/5)*y(x)=-4/35*a^2*(x-1)*(9*x-14)*x^(9/5),y(x)
```

No solution found

✗ Solution by Mathematica
Time used: 0.0 (sec). Leaf size: 0

```
DSolve[y[x]*y'[x]+3/35*a*(19*x-14)*x^(7/5)*y[x]==-4/35*a^2*(x-1)*(9*x-14)*x^(9/5),y[x],x,Inc
```

Timed out

24.27 problem 27

Internal problem ID [10764]

Book: Handbook of exact solutions for ordinary differential equations. By Polyanin and Zaitsev. Second edition
Section: Chapter 1, section 1.3. Abel Equations of the Second Kind. subsection 1.3.3-2. Equations of the form $yy' = f_1(x)y + f_0(x)$
Problem number: 27.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_rational, [_Abel, '2nd type', 'class B']]`

$$\boxed{yy' + \frac{3a(3x+7)\,y}{10x^{\frac{13}{10}}} = -\frac{a^2(x-1)\,(x+9)}{5x^{\frac{8}{5}}}}$$

✗ Solution by Maple

```
dsolve(y(x)*diff(y(x),x)+3/10*a*(3*x+7)*x^(-13/10)*y(x)=-1/5*a^2*(x-1)*(x+9)*x^(-8/5),y(x),
```

No solution found

✗ Solution by Mathematica
Time used: 0.0 (sec). Leaf size: 0

```
DSolve[y[x]*y'[x]+3/10*a*(3*x+7)*x^(-13/10)*y[x]==-1/5*a^2*(x-1)*(x+9)*x^(-8/5),y[x],x,Inclu
```

Timed out

24.28 problem 28

Internal problem ID [10765]

Book: Handbook of exact solutions for ordinary differential equations. By Polyanin and Zaitsev. Second edition
Section: Chapter 1, section 1.3. Abel Equations of the Second Kind. subsection 1.3.3-2. Equations of the form $yy' = f_1(x)y + f_0(x)$
Problem number: 28.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_rational, [_Abel, '2nd type', 'class B']]`

$$\boxed{yy' + \frac{a(7x-12)\,y}{10x^{\frac{7}{5}}} = -\frac{a^2(x-1)\,(x-16)}{10x^{\frac{9}{5}}}}$$

✓ Solution by Maple
Time used: 0.0 (sec). Leaf size: 758

```
dsolve(y(x)*diff(y(x),x)+1/10*a*(7*x-12)*x^(-7/5)*y(x)=-1/10*a^2*(x-1)*(x-16)*x^(-9/5),y(x),
```

Expression too large to display

✗ Solution by Mathematica
Time used: 0.0 (sec). Leaf size: 0

```
DSolve[y[x]*y'[x]+1/10*a*(7*x-12)*x^(-7/5)*y[x]==-1/10*a^2*(x-1)*(x-16)*x^(-9/5),y[x],x,Incl
```

Timed out

24.29 problem 29

Internal problem ID [10766]

Book: Handbook of exact solutions for ordinary differential equations. By Polyanin and Zaitsev. Second edition
Section: Chapter 1, section 1.3. Abel Equations of the Second Kind. subsection 1.3.3-2. Equations of the form $yy' = f_1(x)y + f_0(x)$
Problem number: 29.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_rational, [_Abel, '2nd type', 'class B']]`

$$\boxed{yy' + \frac{3a(13x-8)\,y}{20x^{\frac{7}{5}}} = -\frac{a^2(x-1)\,(27x-32)}{20x^{\frac{9}{5}}}}$$

✗ Solution by Maple

```
dsolve(y(x)*diff(y(x),x)+3/20*a*(13*x-8)*x^(-7/5)*y(x)=-1/20*a^2*(x-1)*(27*x-32)*x^(-9/5),y(
```

No solution found

✗ Solution by Mathematica
Time used: 0.0 (sec). Leaf size: 0

```
DSolve[y[x]*y'[x]+3/20*a*(13*x-8)*x^(-7/5)*y[x]==-1/20*a^2*(x-1)*(27*x-32)*x^(-9/5),y[x],x,I
```

Timed out

24.30 problem 30

Internal problem ID [10767]

Book: Handbook of exact solutions for ordinary differential equations. By Polyanin and Zaitsev. Second edition
Section: Chapter 1, section 1.3. Abel Equations of the Second Kind. subsection 1.3.3-2. Equations of the form $yy' = f_1(x)y + f_0(x)$
Problem number: 30.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_rational, [_Abel, '2nd type', 'class B']]`

$$\boxed{yy' + \frac{3a(3x+11)\,y}{14x^{\frac{10}{7}}} = -\frac{a^2(x-1)\,(x-27)}{14x^{\frac{13}{7}}}}$$

✗ Solution by Maple

```
dsolve(y(x)*diff(y(x),x)+3/14*a*(3*x+11)*x^(-10/7)*y(x)=-1/14*a^2*(x-1)*(x-27)*x^(-13/7),y(x
```

No solution found

✗ Solution by Mathematica
Time used: 0.0 (sec). Leaf size: 0

```
DSolve[y[x]*y'[x]+3/14*a*(3*x+11)*x^(-10/7)*y[x]==-1/14*a^2*(x-1)*(x-27)*x^(-13/7),y[x],x,In
```

Timed out

24.31 problem 31

Internal problem ID [10768]

Book: Handbook of exact solutions for ordinary differential equations. By Polyanin and Zaitsev. Second edition
Section: Chapter 1, section 1.3. Abel Equations of the Second Kind. subsection 1.3.3-2. Equations of the form $yy' = f_1(x)y + f_0(x)$
Problem number: 31.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_rational, [_Abel, '2nd type', 'class B']]`

$$\boxed{yy' - \frac{a(1+x)\,y}{2x^{\frac{7}{4}}} = \frac{a^2(x-1)\,(3x+5)}{4x^{\frac{5}{2}}}}$$

✓ Solution by Maple
Time used: 0.0 (sec). Leaf size: 187

```
dsolve(y(x)*diff(y(x),x)-1/2*a*(x+1)*x^(-7/4)*y(x)=1/4*a^2*(x-1)*(3*x+5)*x^(-5/2),y(x), sing
```

$$\frac{36\sqrt{-\frac{(-1+x)a+x^{\frac{3}{4}}y(x)}{x^{\frac{3}{4}}\left(y(x)+x^{\frac{1}{4}}a\right)}}\sqrt{13}\,55^{\frac{1}{6}}\left(x-\frac{15}{2}\right)\left(\frac{(3x+5)a+3x^{\frac{3}{4}}y(x)}{x^{\frac{3}{4}}\left(y(x)+x^{\frac{1}{4}}a\right)}\right)^{\frac{5}{6}}}{20449}+1458000\left(\int^{-\frac{90\left(2x^{\frac{3}{4}}y(x)+2ax-15a\right)}{143\left(x^{\frac{3}{4}}y(x)+ax\right)}}\frac{\_a\sqrt{11}\,\_a-90\,(13\_}{(143\_a+180)^{\frac{4}{3}}\left(20449\_a^3-119\right.}\right.$$

$$\left.\left(\frac{a}{x^{\frac{3}{4}}\left(y(x)+x^{\frac{1}{4}}a\right)}\right)^{\frac{4}{3}}x\right)$$

$$= 0$$

✗ Solution by Mathematica
Time used: 0.0 (sec). Leaf size: 0

```
DSolve[y[x]*y'[x]-1/2*a*(x+1)*x^(-7/4)*y[x]==1/4*a^2*(x-1)*(3*x+5)*x^(-5/2),y[x],x,IncludeSi
```

Timed out

24.32 problem 32

Internal problem ID [10769]

Book: Handbook of exact solutions for ordinary differential equations. By Polyanin and Zaitsev. Second edition
Section: Chapter 1, section 1.3. Abel Equations of the Second Kind. subsection 1.3.3-2. Equations of the form $yy' = f_1(x)y + f_0(x)$
Problem number: 32.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_rational, [_Abel, '2nd type', 'class B']]`

$$\boxed{yy' - \frac{a(1+x)\,y}{2x^{\frac{7}{4}}} = \frac{a^2(x-1)\,(x+5)}{4x^{\frac{5}{2}}}}$$

✗ Solution by Maple

```
dsolve(y(x)*diff(y(x),x)-1/2*a*(x+1)*x^(-7/4)*y(x)=1/4*a^2*(x-1)*(x+5)*x^(-5/2),y(x), singso
```

No solution found

✗ Solution by Mathematica
Time used: 0.0 (sec). Leaf size: 0

```
DSolve[y[x]*y'[x]-1/2*a*(x+1)*x^(-7/4)*y[x]==1/4*a^2*(x-1)*(x+5)*x^(-5/2),y[x],x,IncludeSing
```

Timed out

24.33 problem 33

Internal problem ID [10770]

Book: Handbook of exact solutions for ordinary differential equations. By Polyanin and Zaitsev. Second edition
Section: Chapter 1, section 1.3. Abel Equations of the Second Kind. subsection 1.3.3-2. Equations of the form $yy' = f_1(x)y + f_0(x)$
Problem number: 33.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_rational, [_Abel, '2nd type', 'class B']]`

$$\boxed{yy' - \frac{a(4x+3)\,y}{14x^{\frac{8}{7}}} = -\frac{a^2(x-1)\,(16x+5)}{14x^{\frac{9}{7}}}}$$

✗ Solution by Maple

```
dsolve(y(x)*diff(y(x),x)-1/14*a*(4*x+3)*x^(-8/7)*y(x)=-1/14*a^2*(x-1)*(16*x+5)*x^(-9/7),y(x)
```

No solution found

✗ Solution by Mathematica
Time used: 0.0 (sec). Leaf size: 0

```
DSolve[y[x]*y'[x]-1/14*a*(4*x+3)*x^(-8/7)*y[x]==-1/14*a^2*(x-1)*(16*x+5)*x^(-9/7),y[x],x,Inc
```

Timed out

24.34 problem 34

Internal problem ID [10771]

Book: Handbook of exact solutions for ordinary differential equations. By Polyanin and Zaitsev. Second edition
Section: Chapter 1, section 1.3. Abel Equations of the Second Kind. subsection 1.3.3-2. Equations of the form $yy' = f_1(x)y + f_0(x)$
Problem number: 34.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_rational, [_Abel, '2nd type', 'class B']]`

$$\boxed{yy' + \frac{a(13x-3)\,y}{6x^{\frac{2}{3}}} = -\frac{a^2(x-1)\,(5x-1)}{6x^{\frac{1}{3}}}}$$

✗ Solution by Maple

```
dsolve(y(x)*diff(y(x),x)+1/6*a*(13*x-3)*x^(-2/3)*y(x)=-1/6*a^2*(x-1)*(5*x-1)*x^(-1/3),y(x),
```

No solution found

✗ Solution by Mathematica
Time used: 0.0 (sec). Leaf size: 0

```
DSolve[y[x]*y'[x]+1/6*a*(13*x-3)*x^(-2/3)*y[x]==-1/6*a^2*(x-1)*(5*x-1)*x^(-1/3),y[x],x,Inclu
```

Not solved

24.35 problem 35

Internal problem ID [10772]

Book: Handbook of exact solutions for ordinary differential equations. By Polyanin and Zaitsev. Second edition
Section: Chapter 1, section 1.3. Abel Equations of the Second Kind. subsection 1.3.3-2. Equations of the form $yy' = f_1(x)y + f_0(x)$
Problem number: 35.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_rational, [_Abel, '2nd type', 'class B']]`

$$\boxed{yy' - \frac{a(8x-1)\,y}{28x^{\frac{8}{7}}} = \frac{a^2(x-1)\,(32x+3)}{28x^{\frac{9}{7}}}}$$

✗ Solution by Maple

```
dsolve(y(x)*diff(y(x),x)-1/28*a*(8*x-1)*x^(-8/7)*y(x)=1/28*a^2*(x-1)*(32*x+3)*x^(-9/7),y(x),
```

No solution found

✗ Solution by Mathematica
Time used: 0.0 (sec). Leaf size: 0

```
DSolve[y[x]*y'[x]-1/28*a*(8*x-1)*x^(-8/7)*y[x]==1/28*a^2*(x-1)*(32*x+3)*x^(-9/7),y[x],x,Incl
```

Timed out

24.36 problem 36

Internal problem ID [10773]

Book: Handbook of exact solutions for ordinary differential equations. By Polyanin and Zaitsev. Second edition
Section: Chapter 1, section 1.3. Abel Equations of the Second Kind. subsection 1.3.3-2. Equations of the form $yy' = f_1(x)y + f_0(x)$
Problem number: 36.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_rational, [_Abel, '2nd type', 'class B']]`

$$\boxed{yy' - \frac{a(5x-4)\,y}{x^4} = \frac{a^2(x-1)\,(3x-1)}{x^7}}$$

✓ Solution by Maple
Time used: 0.0 (sec). Leaf size: 167

```
dsolve(y(x)*diff(y(x),x)-a*(5*x-4)*x^(-4)*y(x)=a^2*(x-1)*(3*x-1)*x^(-7),y(x), singsol=all)
```

$$c_1 - \frac{9\,2^{\frac{2}{3}}\sqrt{\frac{x^3y(x)+ax-a}{(y(x)x^2+a)x}}\left(x-\frac{3}{4}\right)5^{\frac{1}{6}}}{5x\left(-\frac{a}{(y(x)x^2+a)x}\right)^{\frac{1}{3}}\left(\frac{3x^3y(x)+3ax-a}{(y(x)x^2+a)x}\right)^{\frac{1}{6}}} - 729\left(\int^{\frac{\frac{9x^3y(x)}{5}+\frac{9ax}{5}-\frac{27a}{20}}{\left(y(x)x^2+a\right)x}} \frac{\_a\sqrt{20\_a-9}}{(5\_a-9)^{\frac{1}{3}}\,(9+4\_a)^{\frac{1}{6}}\,(400\_a^3-1701\_a+729)}d\_a\right) = 0$$

✗ Solution by Mathematica
Time used: 0.0 (sec). Leaf size: 0

```
DSolve[y[x]*y'[x]-a*(5*x-4)*x^(-4)*y[x]==a^2*(x-1)*(3*x-1)*x^(-7),y[x],x,IncludeSingularSolu
```

Not solved

24.37 problem 37

Internal problem ID [10774]

Book: Handbook of exact solutions for ordinary differential equations. By Polyanin and Zaitsev. Second edition
Section: Chapter 1, section 1.3. Abel Equations of the Second Kind. subsection 1.3.3-2. Equations of the form $yy' = f_1(x)y + f_0(x)$
Problem number: 37.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_rational, [_Abel, '2nd type', 'class B']]`

$$\boxed{yy' - \frac{2a(3x-10)\,y}{5x^4} = \frac{a^2(x-1)\,(8x-5)}{5x^7}}$$

✗ Solution by Maple

```
dsolve(y(x)*diff(y(x),x)-2/5*a*(3*x-10)*x^(-4)*y(x)=1/5*a^2*(x-1)*(8*x-5)*x^(-7),y(x), sings
```

No solution found

✗ Solution by Mathematica
Time used: 0.0 (sec). Leaf size: 0

```
DSolve[y[x]*y'[x]-2/5*a*(3*x-10)*x^(-4)*y[x]==1/5*a^2*(x-1)*(8*x-5)*x^(-7),y[x],x,IncludeSin
```

Not solved

24.38 problem 38

Internal problem ID [10775]

Book: Handbook of exact solutions for ordinary differential equations. By Polyanin and Zaitsev. Second edition
Section: Chapter 1, section 1.3. Abel Equations of the Second Kind. subsection 1.3.3-2. Equations of the form $yy' = f_1(x)y + f_0(x)$
Problem number: 38.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_rational, [_Abel, '2nd type', 'class B']]`

$$\boxed{yy' + \frac{a(39x-4)\,y}{42x^{\frac{9}{7}}} = -\frac{a^2(x-1)\,(9x-1)}{42x^{\frac{11}{7}}}}$$

✗ Solution by Maple

```
dsolve(y(x)*diff(y(x),x)+1/42*a*(39*x-4)*x^(-9/7)*y(x)=-1/42*a^2*(x-1)*(9*x-1)*x^(-11/7),y(x
```

No solution found

✗ Solution by Mathematica
Time used: 0.0 (sec). Leaf size: 0

```
DSolve[y[x]*y'[x]+1/42*a*(39*x-4)*x^(-9/7)*y[x]==-1/42*a^2*(x-1)*(9*x-1)*x^(-11/7),y[x],x,In
```

Timed out

24.39 problem 39

Internal problem ID [10776]

Book: Handbook of exact solutions for ordinary differential equations. By Polyanin and Zaitsev. Second edition
Section: Chapter 1, section 1.3. Abel Equations of the Second Kind. subsection 1.3.3-2. Equations of the form $yy' = f_1(x)y + f_0(x)$
Problem number: 39.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_rational, [_Abel, '2nd type', 'class B']]`

$$\boxed{yy' + \frac{a(x-2)\,y}{x} = \frac{2a^2(x-1)}{x}}$$

✓ Solution by Maple
Time used: 0.0 (sec). Leaf size: 116

```
dsolve(y(x)*diff(y(x),x)+a*(x-2)*x^(-1)*y(x)=2*a^2*(x-1)*x^(-1),y(x), singsol=all)
```

$$\frac{\sqrt{\frac{(1-x)a-y(x)}{ax+y(x)}}\,\mathrm{e}^{\frac{ax+y(x)}{2a}}\,y(x) + x\left(\int \frac{a}{ax+y(x)}\,\frac{\sqrt{\_a-1}\,\mathrm{e}^{\frac{1}{2\_a}}}{\sqrt{\_a}}d\_a + c_1\right)\sqrt{\frac{a}{ax+y(x)}}\,(ax+y(x))}{\sqrt{\frac{a}{ax+y(x)}}\,x\,(ax+y\,(x))} = 0$$

✗ Solution by Mathematica
Time used: 0.0 (sec). Leaf size: 0

```
DSolve[y[x]*y'[x]+a*(x-2)*x^(-1)*y[x]==2*a^2*(x-1)*x^(-1),y[x],x,IncludeSingularSolutions ->
```

Not solved

24.40 problem 40

Internal problem ID [10777]

Book: Handbook of exact solutions for ordinary differential equations. By Polyanin and Zaitsev. Second edition
Section: Chapter 1, section 1.3. Abel Equations of the Second Kind. subsection 1.3.3-2. Equations of the form $yy' = f_1(x)y + f_0(x)$
Problem number: 40.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_rational, [_Abel, '2nd type', 'class B']]`

$$\boxed{yy' + \frac{a(3x-2)\,y}{x} = -\frac{2a^2(x-1)^2}{x}}$$

✗ Solution by Maple

```
dsolve(y(x)*diff(y(x),x)+a*(3*x-2)*x^(-1)*y(x)=-2*a^2*(x-1)^2*x^(-1),y(x), singsol=all)
```

No solution found

✗ Solution by Mathematica
Time used: 0.0 (sec). Leaf size: 0

```
DSolve[y[x]*y'[x]+a*(3*x-2)*x^(-1)*y[x]==-2*a^2*(x-1)^2*x^(-1),y[x],x,IncludeSingularSolutio
```

Not solved

24.41 problem 41

Internal problem ID [10778]

Book: Handbook of exact solutions for ordinary differential equations. By Polyanin and Zaitsev. Second edition
Section: Chapter 1, section 1.3. Abel Equations of the Second Kind. subsection 1.3.3-2. Equations of the form $yy' = f_1(x)y + f_0(x)$
Problem number: 41.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_rational, [_Abel, '2nd type', 'class B']]`

$$\boxed{yy' + \frac{a\left(1 - \frac{b}{x^2}\right) y}{x} = \frac{a^2 b}{x}}$$

✗ Solution by Maple

```
dsolve(y(x)*diff(y(x),x)+a*(1-b*x^(-2))*x^(-1)*y(x)=a^2*b*x^(-1),y(x), singsol=all)
```

No solution found

✗ Solution by Mathematica
Time used: 0.0 (sec). Leaf size: 0

```
DSolve[y[x]*y'[x]+a*(1-b*x^(-2))*x^(-1)*y[x]==a^2*b*x^(-1),y[x],x,IncludeSingularSolutions -
```

Not solved

24.42 problem 42

Internal problem ID [10779]

Book: Handbook of exact solutions for ordinary differential equations. By Polyanin and Zaitsev. Second edition
Section: Chapter 1, section 1.3. Abel Equations of the Second Kind. subsection 1.3.3-2. Equations of the form $yy' = f_1(x)y + f_0(x)$
Problem number: 42.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_rational, [_Abel, '2nd type', 'class B']]`

$$\boxed{yy' - \frac{a(3x-4)\,y}{4x^{\frac{5}{2}}} = \frac{a^2(x-1)\,(x+2)}{4x^4}}$$

✗ Solution by Maple

```
dsolve(y(x)*diff(y(x),x)-1/4*a*(3*x-4)*x^(-5/2)*y(x)=1/4*a^2*(x-1)*(x+2)*x^(-4),y(x), singso
```

No solution found

✗ Solution by Mathematica
Time used: 0.0 (sec). Leaf size: 0

```
DSolve[y[x]*y'[x]-1/4*a*(3*x-4)*x^(-5/2)*y[x]==1/4*a^2*(x-1)*(x+2)*x^(-4),y[x],x,IncludeSing
```

Not solved

24.43 problem 43

Internal problem ID [10780]

Book: Handbook of exact solutions for ordinary differential equations. By Polyanin and Zaitsev. Second edition
Section: Chapter 1, section 1.3. Abel Equations of the Second Kind. subsection 1.3.3-2. Equations of the form $yy' = f_1(x)y + f_0(x)$
Problem number: 43.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_rational, [_Abel, '2nd type', 'class B']]`

$$\boxed{yy' + \frac{a(33x+2)\,y}{30x^{\frac{6}{5}}} = -\frac{a^2(x-1)\,(9x-4)}{30x^{\frac{7}{5}}}}$$

✓ Solution by Maple
Time used: 0.0 (sec). Leaf size: 4330

```
dsolve(y(x)*diff(y(x),x)+1/30*a*(33*x+2)*x^(-6/5)*y(x)=-1/30*a^2*(x-1)*(9*x-4)*x^(-7/5),y(x)
```

Expression too large to display

✗ Solution by Mathematica
Time used: 0.0 (sec). Leaf size: 0

```
DSolve[y[x]*y'[x]+1/30*a*(33*x+2)*x^(-6/5)*y[x]==-1/30*a^2*(x-1)*(9*x-4)*x^(-7/5),y[x],x,Inc
```

Timed out

24.44 problem 44

Internal problem ID [10781]

Book: Handbook of exact solutions for ordinary differential equations. By Polyanin and Zaitsev. Second edition
Section: Chapter 1, section 1.3. Abel Equations of the Second Kind. subsection 1.3.3-2. Equations of the form $yy' = f_1(x)y + f_0(x)$
Problem number: 44.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_rational, [_Abel, '2nd type', 'class B']]`

$$\boxed{yy' - \frac{a(x-8)\,y}{8x^{\frac{5}{2}}} = -\frac{a^2(x-1)\,(3x-4)}{8x^4}}$$

✗ Solution by Maple

```
dsolve(y(x)*diff(y(x),x)-1/8*a*(x-8)*x^(-5/2)*y(x)=-1/8*a^2*(x-1)*(3*x-4)*x^(-4),y(x), sings
```

No solution found

✗ Solution by Mathematica
Time used: 0.0 (sec). Leaf size: 0

```
DSolve[y[x]*y'[x]-1/8*a*(x-8)*x^(-5/2)*y[x]==-1/8*a^2*(x-1)*(3*x-4)*x^(-4),y[x],x,IncludeSin
```

Not solved

24.45 problem 45

Internal problem ID [10782]

Book: Handbook of exact solutions for ordinary differential equations. By Polyanin and Zaitsev. Second edition
Section: Chapter 1, section 1.3. Abel Equations of the Second Kind. subsection 1.3.3-2. Equations of the form $yy' = f_1(x)y + f_0(x)$
Problem number: 45.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_rational, [_Abel, '2nd type', 'class B']]`

$$\boxed{yy' + \frac{a(17x+18)\,y}{30x^{\frac{22}{15}}} = -\frac{a^2(x-1)\,(x+4)}{30x^{\frac{29}{15}}}}$$

✗ Solution by Maple

```
dsolve(y(x)*diff(y(x),x)+1/30*a*(17*x+18)*x^(-22/15)*y(x)=-1/30*a^2*(x-1)*(x+4)*x^(-29/15),y
```

No solution found

✗ Solution by Mathematica
Time used: 0.0 (sec). Leaf size: 0

```
DSolve[y[x]*y'[x]+1/30*a*(17*x+18)*x^(-22/15)*y[x]==-1/30*a^2*(x-1)*(x+4)*x^(-29/15),y[x],x,
```

Timed out

24.46 problem 46

Internal problem ID [10783]

Book: Handbook of exact solutions for ordinary differential equations. By Polyanin and Zaitsev. Second edition
Section: Chapter 1, section 1.3. Abel Equations of the Second Kind. subsection 1.3.3-2. Equations of the form $yy' = f_1(x)y + f_0(x)$
Problem number: 46.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_rational, [_Abel, '2nd type', 'class B']]`

$$\boxed{yy' - \frac{a(6x-13)\,y}{13x^{\frac{5}{2}}} = -\frac{a^2(x-1)\,(x-13)}{26x^4}}$$

✗ Solution by Maple

```
dsolve(y(x)*diff(y(x),x)-1/13*a*(6*x-13)*x^(-5/2)*y(x)=-1/26*a^2*(x-1)*(x-13)*x^(-4),y(x), s
```

No solution found

✗ Solution by Mathematica
Time used: 0.0 (sec). Leaf size: 0

```
DSolve[y[x]*y'[x]-1/13*a*(6*x-13)*x^(-5/2)*y[x]==-1/26*a^2*(x-1)*(x-13)*x^(-4),y[x],x,Includ
```

Not solved

24.47 problem 47

Internal problem ID [10784]

Book: Handbook of exact solutions for ordinary differential equations. By Polyanin and Zaitsev. Second edition
Section: Chapter 1, section 1.3. Abel Equations of the Second Kind. subsection 1.3.3-2. Equations of the form $yy' = f_1(x)y + f_0(x)$
Problem number: 47.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_rational, [_Abel, '2nd type', 'class B']]`

$$\boxed{yy' + \frac{a(24x+11)\,x^{\frac{27}{20}}y}{30} = -\frac{a^2(x-1)\,(9x+1)}{60x^{\frac{17}{10}}}}$$

✗ Solution by Maple

```
dsolve(y(x)*diff(y(x),x)+1/30*a*(24*x+11)*x^(27/20)*y(x)=-1/60*a^2*(x-1)*(9*x+1)*x^(-17/10),
```

No solution found

✗ Solution by Mathematica
Time used: 0.0 (sec). Leaf size: 0

```
DSolve[y[x]*y'[x]+1/30*a*(24*x+11)*x^(27/20)*y[x]==-1/60*a^2*(x-1)*(9*x+1)*x^(-17/10),y[x],x
```

Timed out

24.48 problem 48

Internal problem ID [10785]

Book: Handbook of exact solutions for ordinary differential equations. By Polyanin and Zaitsev. Second edition
Section: Chapter 1, section 1.3. Abel Equations of the Second Kind. subsection 1.3.3-2. Equations of the form $yy' = f_1(x)y + f_0(x)$
Problem number: 48.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_rational, [_Abel, '2nd type', 'class B']]`

$$\boxed{yy' - \frac{2a(3x+2)\,y}{5x^{\frac{8}{5}}} = \frac{a^2(x-1)\,(8x+1)}{5x^{\frac{11}{5}}}}$$

✗ Solution by Maple

```
dsolve(y(x)*diff(y(x),x)-2/5*a*(3*x+2)*x^(-8/5)*y(x)=1/5*a^2*(x-1)*(8*x+1)*x^(-11/5),y(x), s
```

No solution found

✗ Solution by Mathematica
Time used: 0.0 (sec). Leaf size: 0

```
DSolve[y[x]*y'[x]-2/5*a*(3*x+2)*x^(-8/5)*y[x]==1/5*a^2*(x-1)*(8*x+1)*x^(-11/5),y[x],x,Includ
```

Timed out

24.49 problem 49

Internal problem ID [10786]

Book: Handbook of exact solutions for ordinary differential equations. By Polyanin and Zaitsev. Second edition
Section: Chapter 1, section 1.3. Abel Equations of the Second Kind. subsection 1.3.3-2. Equations of the form $yy' = f_1(x)y + f_0(x)$
Problem number: 49.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_rational, [_Abel, '2nd type', 'class B']]`

$$\boxed{yy' - \frac{6a(4x+1)\,y}{5x^{\frac{7}{5}}} = \frac{a^2(x-1)\,(27x+8)}{5x^{\frac{9}{5}}}}$$

✗ Solution by Maple

```
dsolve(y(x)*diff(y(x),x)-6/5*a*(4*x+1)*x^(-7/5)*y(x)=1/5*a^2*(x-1)*(27*x+8)*x^(-9/5),y(x), s
```

No solution found

✗ Solution by Mathematica
Time used: 0.0 (sec). Leaf size: 0

```
DSolve[y[x]*y'[x]-6/5*a*(4*x+1)*x^(-7/5)*y[x]==1/5*a^2*(x-1)*(27*x+8)*x^(-9/5),y[x],x,Includ
```

Timed out

24.50 problem 50

Internal problem ID [10787]

Book: Handbook of exact solutions for ordinary differential equations. By Polyanin and Zaitsev. Second edition
Section: Chapter 1, section 1.3. Abel Equations of the Second Kind. subsection 1.3.3-2. Equations of the form $yy' = f_1(x)y + f_0(x)$
Problem number: 50.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_rational, [_Abel, '2nd type', 'class B']]`

$$\boxed{yy' - \frac{a(x+4)\,y}{5x^{\frac{8}{5}}} = \frac{a^2(x-1)\,(3x+7)}{5x^{\frac{3}{5}}}}$$

✗ Solution by Maple

```
dsolve(y(x)*diff(y(x),x)-1/5*a*(x+4)*x^(-8/5)*y(x)=1/5*a^2*(x-1)*(3*x+7)*x^(-3/5),y(x), sing
```

No solution found

✗ Solution by Mathematica
Time used: 0.0 (sec). Leaf size: 0

```
DSolve[y[x]*y'[x]-1/5*a*(x+4)*x^(-8/5)*y[x]==1/5*a^2*(x-1)*(3*x+7)*x^(-3/5),y[x],x,IncludeSi
```

Not solved

24.51 problem 51

Internal problem ID [10788]

Book: Handbook of exact solutions for ordinary differential equations. By Polyanin and Zaitsev. Second edition
Section: Chapter 1, section 1.3. Abel Equations of the Second Kind. subsection 1.3.3-2. Equations of the form $yy' = f_1(x)y + f_0(x)$
Problem number: 51.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_rational, [_Abel, '2nd type', 'class B']]`

$$\boxed{yy' - \frac{a(x+4)\,y}{5x^{\frac{8}{5}}} = \frac{a^2(x-1)\,(3x+7)}{5x^{\frac{11}{5}}}}$$

✓ Solution by Maple
Time used: 0.015 (sec). Leaf size: 190

```
dsolve(y(x)*diff(y(x),x)-1/5*a*(x+4)*x^(-8/5)*y(x)=1/5*a^2*(x-1)*(3*x+7)*x^(-11/5),y(x), sin
```

$$\frac{360\,2^{\frac{1}{3}}\sqrt{17}\sqrt{-\frac{(-1+x)a+y(x)x^{\frac{3}{5}}}{x^{\frac{3}{5}}\left(y(x)+a\,x^{\frac{2}{5}}\right)}}\,91^{\frac{5}{6}}\left(x-\frac{21}{4}\right)\left(\frac{(3x+7)a+3y(x)x^{\frac{3}{5}}}{x^{\frac{3}{5}}\left(y(x)+a\,x^{\frac{2}{5}}\right)}\right)^{\frac{7}{6}}}{4444531} + 31255875x\left(\int^{-\frac{315\left(4y(x)x^{\frac{3}{5}}+4ax-21a\right)}{884\left(y(x)x^{\frac{3}{5}}+ax\right)}} \frac{\sqrt{52\_\_a-315}\,(68}{(11492\_\_a^2-53235\_\_a-}\right)$$

$$\Big/\; x\left(\frac{a}{x^{\frac{3}{5}}\left(y(x)+a\,x^{\frac{2}{5}}\right)}\right)^{\frac{5}{3}}$$

$$= 0$$

✗ Solution by Mathematica
Time used: 0.0 (sec). Leaf size: 0

```
DSolve[y[x]*y'[x]-1/5*a*(x+4)*x^(-8/5)*y[x]==1/5*a^2*(x-1)*(3*x+7)*x^(-11/5),y[x],x,IncludeS
```

Timed out

24.52 problem 52

Internal problem ID [10789]

Book: Handbook of exact solutions for ordinary differential equations. By Polyanin and Zaitsev. Second edition
Section: Chapter 1, section 1.3. Abel Equations of the Second Kind. subsection 1.3.3-2. Equations of the form $yy' = f_1(x)y + f_0(x)$
Problem number: 52.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_rational, [_Abel, '2nd type', 'class B']]`

$$\boxed{yy' - \frac{a(2x-1)\,y}{x^{\frac{5}{2}}} = \frac{a^2(x-1)\,(3x+1)}{2x^4}}$$

✓ Solution by Maple
Time used: 0.0 (sec). Leaf size: 189

```
dsolve(y(x)*diff(y(x),x)-a*(2*x-1)*x^(-5/2)*y(x)=1/2*a^2*(x-1)*(3*x+1)*x^(-4),y(x), singsol=
```

$$\frac{18\sqrt{\frac{(-1+x)a+y(x)x^{\frac{3}{2}}}{x(y(x)\sqrt{x}+a)}}\,\sqrt{5}\,7^{\frac{5}{6}}\left(x+\frac{3}{2}\right)\left(\frac{(-3x-1)a-3y(x)x^{\frac{3}{2}}}{x(y(x)\sqrt{x}+a)}\right)^{\frac{1}{6}}}{1225} + 1458\left(\int^{-\frac{18y(x)x^{\frac{3}{2}}}{35}+\frac{9(-2x-3)a}{35}} \frac{\_a(5\_a-9)^{\frac{1}{6}}\sqrt{7\_a+9}}{(35\_a+18)^{\frac{2}{3}}\,(1225\_a^3-3159\_a-1458)}\right)$$
$$\Big/\; x\left(-\frac{a}{x(y(x)\sqrt{x}+a)}\right)^{\frac{2}{3}}$$

$= 0$

✗ Solution by Mathematica
Time used: 0.0 (sec). Leaf size: 0

```
DSolve[y[x]*y'[x]-a*(2*x-1)*x^(-5/2)*y[x]==1/2*a^2*(x-1)*(3*x+1)*x^(-4),y[x],x,IncludeSingul
```

Not solved

24.53 problem 53

Internal problem ID [10790]

Book: Handbook of exact solutions for ordinary differential equations. By Polyanin and Zaitsev. Second edition
Section: Chapter 1, section 1.3. Abel Equations of the Second Kind. subsection 1.3.3-2. Equations of the form $yy' = f_1(x)y + f_0(x)$
Problem number: 53.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_rational, [_Abel, '2nd type', 'class B']]`

$$\boxed{yy' + \frac{a(x-6)\,y}{5x^{\frac{7}{5}}} = \frac{2a^2(x-1)\,(x+4)}{5x^{\frac{9}{5}}}}$$

✓ Solution by Maple
Time used: 0.0 (sec). Leaf size: 156

```
dsolve(y(x)*diff(y(x),x)+1/5*a*(x-6)*x^(-7/5)*y(x)=2/5*a^2*(x-1)*(x+4)*x^(-9/5),y(x), singso
```

$$c_1 - \frac{80\sqrt{3}\left(ay(x)\,x^{\frac{2}{5}} + \frac{x^{\frac{4}{5}}y(x)^2}{8} + \frac{\left(y(x)x^{\frac{7}{5}} - 2a(x+24)(-1+x)\right)a}{24}\right)\left(ay(x)\,x^{\frac{2}{5}} + \frac{x^{\frac{4}{5}}y(x)^2}{8} + \frac{a\left(y(x)x^{\frac{7}{5}} + \frac{a(x+4)^2}{2}\right)}{4}\right)\sqrt{\frac{-y(x)}{x^{\frac{2}{5}}\left(y}}}{9\left(\frac{a}{x^{\frac{2}{5}}\left(y(x)+x^{\frac{3}{5}}a\right)}\right)^{\frac{5}{2}}\left(a\,(x+4)+y\,(x)\,x^{\frac{2}{5}}\right)^2\left(y\,(x)\,x^{\frac{2}{5}}+ax\right)^2 x} = 0$$

✗ Solution by Mathematica
Time used: 0.0 (sec). Leaf size: 0

```
DSolve[y[x]*y'[x]+1/5*a*(x-6)*x^(-7/5)*y[x]==2/5*a^2*(x-1)*(x+4)*x^(-9/5),y[x],x,IncludeSing
```

Timed out

24.54 problem 54

Internal problem ID [10791]

Book: Handbook of exact solutions for ordinary differential equations. By Polyanin and Zaitsev. Second edition
Section: Chapter 1, section 1.3. Abel Equations of the Second Kind. subsection 1.3.3-2. Equations of the form $yy' = f_1(x)y + f_0(x)$
Problem number: 54.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_rational, [_Abel, '2nd type', 'class B']]`

$$\boxed{yy' + \frac{a(21x+19)\,y}{5x^{\frac{7}{5}}} = -\frac{2a^2(x-1)\,(9x-4)}{5x^{\frac{9}{5}}}}$$

✗ Solution by Maple

```
dsolve(y(x)*diff(y(x),x)+1/5*a*(21*x+19)*x^(-7/5)*y(x)=-2/5*a^2*(x-1)*(9*x-4)*x^(-9/5),y(x),
```

No solution found

✗ Solution by Mathematica
Time used: 0.0 (sec). Leaf size: 0

```
DSolve[y[x]*y'[x]+1/5*a*(21*x+19)*x^(-7/5)*y[x]==-2/5*a^2*(x-1)*(9*x-4)*x^(-9/5),y[x],x,Incl
```

Timed out

24.55 problem 55

Internal problem ID [10792]

Book: Handbook of exact solutions for ordinary differential equations. By Polyanin and Zaitsev. Second edition
Section: Chapter 1, section 1.3. Abel Equations of the Second Kind. subsection 1.3.3-2. Equations of the form $yy' = f_1(x)y + f_0(x)$
Problem number: 55.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_rational, [_Abel, '2nd type', 'class B']]`

$$\boxed{yy' - \frac{3ay}{x^{\frac{7}{4}}} = \frac{a^2(x-1)(x-9)}{4x^{\frac{5}{2}}}}$$

✗ Solution by Maple

```
dsolve(y(x)*diff(y(x),x)-3*a*x^(-7/4)*y(x)=1/4*a^2*(x-1)*(x-9)*x^(-5/2),y(x), singsol=all)
```

No solution found

✗ Solution by Mathematica
Time used: 0.0 (sec). Leaf size: 0

```
DSolve[y[x]*y'[x]-3*a*x^(-7/4)*y[x]==1/4*a^2*(x-1)*(x-9)*x^(-5/2),y[x],x,IncludeSingularSolu
```

Not solved

24.56 problem 56

Internal problem ID [10793]

Book: Handbook of exact solutions for ordinary differential equations. By Polyanin and Zaitsev. Second edition
Section: Chapter 1, section 1.3. Abel Equations of the Second Kind. subsection 1.3.3-2. Equations of the form $yy' = f_1(x)y + f_0(x)$
Problem number: 56.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_rational, [_Abel, '2nd type', 'class B']]`

$$\boxed{yy' - \frac{a((1+k)\,x-1)\,y}{x^2} = \frac{a^2(1+k)\,(x-1)}{x^2}}$$

✓ Solution by Maple
Time used: 0.016 (sec). Leaf size: 143

```
dsolve(y(x)*diff(y(x),x)-a*((k+1)*x-1)*x^(-2)*y(x)=a^2*(k+1)*(x-1)*x^(-2),y(x), singsol=all)
```

$$\frac{\left(\frac{ax}{-y(x)x+a}\right)^{-\frac{1}{1+k}} x^2 \left(\frac{(-1+x)a+y(x)x}{-y(x)x+a}\right)^{\frac{1}{1+k}} \mathrm{e}^{\frac{-y(x)x+a}{a(1+k)x}} y(x) - \left(\int \frac{ax}{-y(x)x+a} (\_a-1)^{\frac{1}{1+k}} \mathrm{e}^{\frac{1}{(1+k)\_a}} \_a^{-\frac{1}{1+k}} d\_a - c_1\right)(-}{-y\,(x)\,x+a}$$
$$= 0$$

✗ Solution by Mathematica
Time used: 0.0 (sec). Leaf size: 0

```
DSolve[y[x]*y'[x]-a*((k+1)*x-1)*x^(-2)*y[x]==a^2*(k+1)*(x-1)*x^(-2),y[x],x,IncludeSingularSo
```

Not solved

24.57 problem 57

Internal problem ID [10794]

Book: Handbook of exact solutions for ordinary differential equations. By Polyanin and Zaitsev. Second edition
Section: Chapter 1, section 1.3. Abel Equations of the Second Kind. subsection 1.3.3-2. Equations of the form $yy' = f_1(x)y + f_0(x)$
Problem number: 57.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_rational, [_Abel, '2nd type', 'class A']]`

$$\boxed{yy' - a((k-2)\,x + 2k - 3)\,x^{-k}y = a^2(k-2)\,(x-1)^2\,x^{1-2k}}$$

✗ Solution by Maple

```
dsolve(y(x)*diff(y(x),x)-a*( (k-2)*x + 2*k - 3)*x^(-k)*y(x)=a^2*(k-2)*(x-1)^2*x^(1-2*k),y(x)
```

No solution found

✗ Solution by Mathematica
Time used: 0.0 (sec). Leaf size: 0

```
DSolve[y[x]*y'[x]-a*( (k-2)*x + 2*k - 3)*x^(-k)*y[x]==a^2*(k-2)*(x-1)^2*x^(1-2*k),y[x],x,Inc
```

Not solved

24.58 problem 58

Internal problem ID [10795]

Book: Handbook of exact solutions for ordinary differential equations. By Polyanin and Zaitsev. Second edition
Section: Chapter 1, section 1.3. Abel Equations of the Second Kind. subsection 1.3.3-2. Equations of the form $yy' = f_1(x)y + f_0(x)$
Problem number: 58.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_rational, [_Abel, '2nd type', 'class A']]`

$$\boxed{yy' - \frac{a((4k-7)\,x - 4k + 5)\,x^{-k}y}{2} = \frac{a^2(-3+2k)\,(x-1)^2\,x^{1-2k}}{2}}$$

✗ Solution by Maple

```
dsolve(y(x)*diff(y(x),x)-1/2*a*( (4*k-7)*x - 4*k + 5)*x^(-k)*y(x)=1/2*a^2*(2*k-3)*(x-1)^2*x^
```

No solution found

✗ Solution by Mathematica
Time used: 0.0 (sec). Leaf size: 0

```
DSolve[y[x]*y'[x]-1/2*a*( (4*k-7)*x - 4*k + 5)*x^(-k)*y[x]==1/2*a^2*(2*k-3)*(x-1)^2*x^(1-2*k
```

Not solved

24.59 problem 59

Internal problem ID [10796]

Book: Handbook of exact solutions for ordinary differential equations. By Polyanin and Zaitsev. Second edition
Section: Chapter 1, section 1.3. Abel Equations of the Second Kind. subsection 1.3.3-2. Equations of the form $yy' = f_1(x)y + f_0(x)$
Problem number: 59.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_rational, [_Abel, '2nd type', 'class A']]`

$$\boxed{yy' - ((-1+2n)x - an)x^{-n-1}y = n(x-a)x^{-2n}}$$

✓ Solution by Maple
Time used: 0.0 (sec). Leaf size: 151

```
dsolve(y(x)*diff(y(x),x)-((2*n-1)*x-a*n)*x^(-n-1)*y(x)=n*(x-a)*x^(-2*n),y(x), singsol=all)
```

$$y(x) = \frac{2\left(-\frac{\sqrt{-n^2}\,x\tan\left(\frac{\operatorname{RootOf}\left(-2an\,\mathrm{e}^{\_Z+\_a+nx\,\mathrm{e}^{\_Z+\_a}-\tan\left(\frac{\_a\sqrt{-n^2}}{2}\right)\_Z\sqrt{-n^2}\,x+2c_1x\,\mathrm{e}^{\_a}}\right)\sqrt{-n^2}}{2}\right)}{2}+\left(a-\frac{x}{2}\right)n\right)x^{-n}}{\tan\left(\frac{\operatorname{RootOf}\left(-2an\,\mathrm{e}^{\_Z+\_a+nx\,\mathrm{e}^{\_Z+\_a}-\tan\left(\frac{\_a\sqrt{-n^2}}{2}\right)\_Z\sqrt{-n^2}\,x+2c_1x\,\mathrm{e}^{\_a}}\right)\sqrt{-n^2}}{2}\right)\sqrt{-n^2}+n}$$

✗ Solution by Mathematica
Time used: 0.0 (sec). Leaf size: 0

```
DSolve[y[x]*y'[x]-((2*n-1)*x-a*n)*x^(-n-1)*y[x]==n*(x-a)*x^(-2*n),y[x],x,IncludeSingularSolu
```

Not solved

24.60 problem 60

Internal problem ID [10797]

Book: Handbook of exact solutions for ordinary differential equations. By Polyanin and Zaitsev. Second edition
Section: Chapter 1, section 1.3. Abel Equations of the Second Kind. subsection 1.3.3-2. Equations of the form $yy' = f_1(x)y + f_0(x)$
Problem number: 60.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[[_Abel, '2nd type', 'class A']]`

$$\boxed{yy' - ((1+n)\,x - an)\,x^{n-1}(x-a)^{-2-n}\,y = n\,x^{2n}(x-a)^{-2n-3}}$$

✗ Solution by Maple

```
dsolve(y(x)*diff(y(x),x)-((n+1)*x-a*n)*x^(n-1)*(x-a)^(-n-2)*y(x)=n*x^(2*n)*(x-a)^(-2*n-3),y(
```

No solution found

✗ Solution by Mathematica
Time used: 0.0 (sec). Leaf size: 0

```
DSolve[y[x]*y'[x]-((n+1)*x-a*n)*x^(n-1)*(x-a)^(-n-2)*y[x]==n*x^(2*n)*(x-a)^(-2*n-3),y[x],x,I
```

Not solved

24.61 problem 61

Internal problem ID [10798]

Book: Handbook of exact solutions for ordinary differential equations. By Polyanin and Zaitsev. Second edition
Section: Chapter 1, section 1.3. Abel Equations of the Second Kind. subsection 1.3.3-2. Equations of the form $yy' = f_1(x)y + f_0(x)$
Problem number: 61.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_rational, [_Abel, '2nd type', 'class A']]`

$$\boxed{yy' - a((-3+2k)x+1)x^{-k}y = a^2(k-2)((k-1)x+1)x^{2-2k}}$$

✗ Solution by Maple

```
dsolve(y(x)*diff(y(x),x)-a*((2*k-3)*x+1)*x^(-k)*y(x)=a^2*(k-2)*((k-1)*x+1)*x^(2*(1-k)),y(x),
```

No solution found

✗ Solution by Mathematica
Time used: 0.0 (sec). Leaf size: 0

```
DSolve[y[x]*y'[x]-a*((2*k-3)*x+1)*x^(-k)*y[x]==a^2*(k-2)*((k-1)*x+1)*x^(2*(1-k)),y[x],x,Incl
```

Not solved

24.62 problem 62

Internal problem ID [10799]

Book: Handbook of exact solutions for ordinary differential equations. By Polyanin and Zaitsev. Second edition
Section: Chapter 1, section 1.3. Abel Equations of the Second Kind. subsection 1.3.3-2. Equations of the form $yy' = f_1(x)y + f_0(x)$
Problem number: 62.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_rational, [_Abel, '2nd type', 'class A']]`

$$yy' - a((n+2k-3)\,x + 3 - 2k)\,x^{-k}y = a^2\big((k+n-1)\,x^2 - (n+2k-3)\,x + k - 2\big)\,x^{1-2k}$$

✗ Solution by Maple

```
dsolve(y(x)*diff(y(x),x)-a*((n+2*k-3)*x+3-2*k)*x^(-k)*y(x)=a^2*((n+k-1)*x^2-(n+2*k-3)*x+k-2)
```

No solution found

✗ Solution by Mathematica
Time used: 0.0 (sec). Leaf size: 0

```
DSolve[y[x]*y'[x]-a*((n+2*k-3)*x+3-2*k)*x^(-k)*y[x]==a^2*((n+k-1)*x^2-(n+2*k-3)*x+k-2)*x^(1-
```

Timed out

24.63 problem 63

Internal problem ID [10800]

Book: Handbook of exact solutions for ordinary differential equations. By Polyanin and Zaitsev. Second edition
Section: Chapter 1, section 1.3. Abel Equations of the Second Kind. subsection 1.3.3-2. Equations of the form $yy' = f_1(x)y + f_0(x)$
Problem number: 63.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_rational, [_Abel, '2nd type', 'class A']]`

$$\boxed{yy' - \frac{a((2+n)x-2)x^{-\frac{2n+1}{n}}y}{n} = \frac{a^2((1+n)x^2-2x-n+1)x^{-\frac{2+3n}{n}}}{n}}$$

✗ Solution by Maple

```
dsolve(y(x)*diff(y(x),x)-a/n*((n+2)*x-2)*x^(-(2*n+1)/n)*y(x)=a^2/n*((n+1)*x^2-2*x-n+1)*x^(-(
```

No solution found

✗ Solution by Mathematica
Time used: 0.0 (sec). Leaf size: 0

```
DSolve[y[x]*y'[x]-a/n*((n+2)*x-2)*x^(-(2*n+1)/n)*y[x]==a^2/n*((n+1)*x^2-2*x-n+1)*x^(-(3*n+2)
```

Not solved

24.64 problem 64

Internal problem ID [10801]

Book: Handbook of exact solutions for ordinary differential equations. By Polyanin and Zaitsev. Second edition
Section: Chapter 1, section 1.3. Abel Equations of the Second Kind. subsection 1.3.3-2. Equations of the form $yy' = f_1(x)y + f_0(x)$
Problem number: 64.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_rational, [_Abel, '2nd type', 'class A']]`

$$y y' - \frac{a\left(\frac{(n+4)x}{2+n} - 2\right) x^{-\frac{2n+1}{n}} y}{n} = \frac{a^2(2x^2 + (n^2+n-4)\,x - (n-1)\,(2+n))\,x^{-\frac{2+3n}{n}}}{n\,(2+n)}$$

✗ Solution by Maple

```
dsolve(y(x)*diff(y(x),x)-a/n*((n+4)/(n+2)*x-2)*x^(-(2*n+1)/n)*y(x)=a^2/(n*(n+2))*(2*x^2+(n^2
```

No solution found

✗ Solution by Mathematica
Time used: 0.0 (sec). Leaf size: 0

```
DSolve[y[x]*y'[x]-a/n*((n+4)/(n+2)*x-2)*x^(-(2*n+1)/n)*y[x]==a^2/(n*(n+2))*(2*x^2+(n^2+n-4)*
```

Not solved

24.65 problem 65

Internal problem ID [10802]

Book: Handbook of exact solutions for ordinary differential equations. By Polyanin and Zaitsev. Second edition
Section: Chapter 1, section 1.3. Abel Equations of the Second Kind. subsection 1.3.3-2. Equations of the form $yy' = f_1(x)y + f_0(x)$
Problem number: 65.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_rational, [_Abel, '2nd type', 'class A']]`

$$y y' + \frac{a\left(\frac{(3n+5)x}{2} + \frac{n-1}{1+n}\right) x^{-\frac{n+4}{n+3}} y}{n+3} = -\frac{a^2\left((1+n)\,x^2 - \frac{(n^2+2n+5)x}{1+n} + \frac{4}{1+n}\right) x^{-\frac{n+5}{n+3}}}{2n+6}$$

✗ Solution by Maple

```
dsolve(y(x)*diff(y(x),x)+a/(n+3)*((3*n+5)/(2)*x+(n-1)/(n+1))*x^(-(n+4)/(n+3))*y(x)=-a^2/(2*(
```

No solution found

✗ Solution by Mathematica
Time used: 0.0 (sec). Leaf size: 0

```
DSolve[y[x]*y'[x]+a/(n+3)*((3*n+5)/(2)*x+(n-1)/(n+1))*x^(-(n+4)/(n+3))*y[x]==-a^2/(2*(n+3))*
```

Timed out

24.66 problem 66

Internal problem ID [10803]

Book: Handbook of exact solutions for ordinary differential equations. By Polyanin and Zaitsev. Second edition
Section: Chapter 1, section 1.3. Abel Equations of the Second Kind. subsection 1.3.3-2. Equations of the form $yy' = f_1(x)y + f_0(x)$
Problem number: 66.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_rational, [_Abel, '2nd type', 'class A']]`

$$\boxed{yy' - a\left(\frac{2+n}{n} + b\,x^n\right)y = -\frac{a^2 x\left(\frac{1+n}{n} + b\,x^n\right)}{n}}$$

✓ Solution by Maple
Time used: 0.015 (sec). Leaf size: 192

```
dsolve(y(x)*diff(y(x),x)-a*((n+2)/n+b*x^n)*y(x)=-a^2/n*x*((n+1)/n+b*x^n),y(x), singsol=all)
```

$$-n\sqrt{-\frac{(n+1)^2}{n^2}}\left(\int^{\frac{2\arctan\left(\frac{2x^{n+1}abn+(n+1)(ax-y(x)n)}{\sqrt{-\frac{(n+1)^2}{n^2}}\,n(ax-y(x)n)}\right)}{\sqrt{-\frac{(n+1)^2}{n^2}}}} \tan\left(\frac{-a\sqrt{-\frac{(n+1)^2}{n^2}}}{2}\right)\mathrm{e}^{-\_a}d\_a\right)$$
$$+\left(-2bn\,x^n - n - 1\right)\mathrm{e}^{-\frac{2\arctan\left(\frac{2x^{n+1}abn+(n+1)(ax-y(x)n)}{\sqrt{-\frac{(n+1)^2}{n^2}}\,n(ax-y(x)n)}\right)}{\sqrt{-\frac{(n+1)^2}{n^2}}}} + c_1 = 0$$

✗ Solution by Mathematica
Time used: 0.0 (sec). Leaf size: 0

```
DSolve[y[x]*y'[x]-a*((n+2)/n+b*x^n)*y[x]==-a^2/n*x*((n+1)/n+b*x^n),y[x],x,IncludeSingularSol
```

Not solved

24.67 problem 67

Internal problem ID [10804]

Book: Handbook of exact solutions for ordinary differential equations. By Polyanin and Zaitsev. Second edition
Section: Chapter 1, section 1.3. Abel Equations of the Second Kind. subsection 1.3.3-2. Equations of the form $yy' = f_1(x)y + f_0(x)$
Problem number: 67.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[[_Abel, '2nd type', 'class A']]`

$$\boxed{yy' - (a\,\mathrm{e}^x + b)\,y = c\,\mathrm{e}^{2x} - ab\,\mathrm{e}^x - b^2}$$

✓ Solution by Maple
Time used: 0.0 (sec). Leaf size: 153

```
dsolve(y(x)*diff(y(x),x)=(a*exp(x)+b)*y(x)+c*exp(2*x)-a*b*exp(x)-b^2,y(x), singsol=all)
```

$$\sqrt{\frac{c\,\mathrm{e}^{2x} - (b - y(x))\,(a\,\mathrm{e}^x + b - y(x))}{(b - y(x))^2}}\,y(x)\,\mathrm{e}^{-\frac{a\,\operatorname{arctanh}\left(\frac{(b-y(x))a - 2\,\mathrm{e}^x c}{\sqrt{a^2+4c}\,(b-y(x))}\right)}{\sqrt{a^2+4c}}}$$
$$- b\left(\int^{\frac{\mathrm{e}^x}{-b+y(x)}} \frac{\sqrt{\_a^2 c + a\_a - 1}\,\mathrm{e}^{-\frac{a\,\operatorname{arctanh}\left(\frac{2c\_a + a}{\sqrt{a^2+4c}}\right)}{\sqrt{a^2+4c}}}}{\_a}\,d\_a\right) + c_1 = 0$$

✗ Solution by Mathematica
Time used: 0.0 (sec). Leaf size: 0

```
DSolve[y[x]*y'[x]==(a*Exp[x]+b)*y[x]+c*Exp[2*x]-a*b*Exp[x]-b^2,y[x],x,IncludeSingularSolutio
```

Not solved

24.68 problem 68

Internal problem ID [10805]

Book: Handbook of exact solutions for ordinary differential equations. By Polyanin and Zaitsev. Second edition
Section: Chapter 1, section 1.3. Abel Equations of the Second Kind. subsection 1.3.3-2. Equations of the form $yy' = f_1(x)y + f_0(x)$
Problem number: 68.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[[_Abel, '2nd type', 'class A']]`

$$\boxed{yy' - \left(a(2\mu+\lambda)\,\mathrm{e}^{\lambda x} + b\right)\mathrm{e}^{\mu x} y = \left(-a^2\mu\,\mathrm{e}^{2\lambda x} - ab\,\mathrm{e}^{\lambda x} + c\right)\mathrm{e}^{2\mu x}}$$

✗ Solution by Maple

```
dsolve(y(x)*diff(y(x),x)=(a*(2*mu+lambda)*exp(lambda*x)+b)*exp(mu*x)*y(x)+(-a^2*mu*exp(2*lam
```

No solution found

✗ Solution by Mathematica
Time used: 0.0 (sec). Leaf size: 0

```
DSolve[y[x]*y'[x]==(a*(2*\[Mu]+\[Lambda])*Exp[\[Lambda]*x]+b)*Exp[\[Mu]*x]*y[x]+(-a^2*\[Mu]*
```

Not solved

24.69 problem 69

Internal problem ID [10806]

Book: Handbook of exact solutions for ordinary differential equations. By Polyanin and Zaitsev. Second edition
Section: Chapter 1, section 1.3. Abel Equations of the Second Kind. subsection 1.3.3-2. Equations of the form $yy' = f_1(x)y + f_0(x)$
Problem number: 69.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[[_Abel, '2nd type', 'class A']]`

$$\boxed{yy' - \left(a\,\mathrm{e}^{\lambda x} + b\right) y = c\left(a^2\mathrm{e}^{2\lambda x} + ab(\lambda x + 1)\,\mathrm{e}^{\lambda x} + b^2\lambda x\right)}$$

✓ Solution by Maple
Time used: 0.0 (sec). Leaf size: 257

```
dsolve(y(x)*diff(y(x),x)=(a*exp(lambda*x)+b)*y(x)+c*(a^2*exp(2*lambda*x)+a*b*(lambda*x+1)*ex
```

$$\frac{\sqrt{(4c\lambda+1)\,(3c\lambda+1)^2}\,\left(\frac{c\lambda}{2}+\frac{1}{6}\right)\ln\left(\frac{(3c\lambda+1)^2\left(b^2c\,\lambda^2x^2+2\,\mathrm{e}^{x\lambda}abc\lambda x+\mathrm{e}^{2x\lambda}a^2c+b\lambda xy(x)+a\,\mathrm{e}^{x\lambda}y(x)-\lambda y(x)^2\right)c}{(9c\lambda+2)y(x)^2}\right)-3\left(c\lambda+\frac{1}{3}\right)}{}$$

$$= 0$$

✓ Solution by Mathematica

Time used: 0.494 (sec). Leaf size: 134

```
DSolve[y[x]*y'[x]==(a*Exp[\[Lambda]*x]+b)*y[x]+c*(a^2*Exp[2*\[Lambda]*x]+a*b*(\[Lambda]*x+1)
```

$$\text{Solve}\left[-\frac{2\arctan\left(\frac{\frac{2c\lambda y(x)}{ace^{\lambda x}+bc\lambda x}-1}{\sqrt{-4c\lambda-1}}\right)}{\sqrt{-4c\lambda-1}}+\log\left(-\frac{c\lambda y(x)^2}{\left(ace^{\lambda x}+bc\lambda x\right)^2}+\frac{y(x)}{ace^{\lambda x}+bc\lambda x}+1\right)}{2c\lambda}=\frac{\log\left(ace^{\lambda x}+bc\lambda x\right)}{c\lambda}+c_1,y(x)\right]$$

24.70 problem 70

Internal problem ID [10807]

Book: Handbook of exact solutions for ordinary differential equations. By Polyanin and Zaitsev. Second edition
Section: Chapter 1, section 1.3. Abel Equations of the Second Kind. subsection 1.3.3-2. Equations of the form $yy' = f_1(x)y + f_0(x)$
Problem number: 70.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[[_Abel, '2nd type', 'class A']]`

$$\boxed{yy' - \mathrm{e}^{\lambda x}(2a\lambda x + a + b)\, y = -\mathrm{e}^{2\lambda x}\left(a^2\lambda\, x^2 + abx + c\right)}$$

✓ Solution by Maple
Time used: 0.0 (sec). Leaf size: 118

```
dsolve(y(x)*diff(y(x),x)=exp(lambda*x)*(2*a*lambda*x+a+b)*y(x)-exp(2*lambda*x)*(a^2*lambda*x
```

$$y(x) = \frac{\left(\tan\left(\frac{\mathrm{RootOf}\left(2ax\lambda\,\mathrm{e}^{-\_Z+\_a-\sqrt{-\frac{b^2-4c\lambda}{a^2}}}\tan\left(\frac{\_a\sqrt{-\frac{b^2-4c\lambda}{a^2}}}{2}\right)-\_Z a+b\,\mathrm{e}^{-\_Z+\_a+2c_1 a\,\mathrm{e}^{-a}}\right)\sqrt{\frac{-b^2+4c\lambda}{a^2}}}{2}\right) a\sqrt{\frac{-b^2+4c\lambda}{a^2}} + 2x\alpha\right.}{2\lambda}$$

✗ Solution by Mathematica
Time used: 0.0 (sec). Leaf size: 0

```
DSolve[y[x]*y'[x]==Exp[\[Lambda]*x]*(2*a*\[Lambda]*x+a+b)*y[x]-Exp[2*\[Lambda]*x]*(a^2*\[Lam
```

Not solved

24.71 problem 71

Internal problem ID [10808]

Book: Handbook of exact solutions for ordinary differential equations. By Polyanin and Zaitsev. Second edition
Section: Chapter 1, section 1.3. Abel Equations of the Second Kind. subsection 1.3.3-2. Equations of the form $yy' = f_1(x)y + f_0(x)$
Problem number: 71.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[[_Abel, '2nd type', 'class A']]`

$$\boxed{yy' - \mathrm{e}^{ax}\left(2a\,x^2 + b + 2x\right)y = \mathrm{e}^{2ax}\left(-a\,x^4 - b\,x^2 + c\right)}$$

✗ Solution by Maple

```
dsolve(y(x)*diff(y(x),x)=exp(a*x)*(2*a*x^2+2*x+b)*y(x)+exp(2*a*x)*(-a*x^4-b*x^2+c),y(x), sin
```

No solution found

✗ Solution by Mathematica
Time used: 0.0 (sec). Leaf size: 0

```
DSolve[y[x]*y'[x]==Exp[a*x]*(2*a*x^2+2*x+b)*y[x]+Exp[2*a*x]*(-a*x^4-b*x^2+c),y[x],x,IncludeS
```

Not solved

24.72 problem 72

Internal problem ID [10809]

Book: Handbook of exact solutions for ordinary differential equations. By Polyanin and Zaitsev. Second edition
Section: Chapter 1, section 1.3. Abel Equations of the Second Kind. subsection 1.3.3-2. Equations of the form $yy' = f_1(x)y + f_0(x)$
Problem number: 72.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[[_Abel, '2nd type', 'class A']]`

$$\boxed{yy' + a(2bx+1)\,\mathrm{e}^{bx}y = -a^2 b\,x^2\mathrm{e}^{2bx}}$$

✓ Solution by Maple
Time used: 0.0 (sec). Leaf size: 53

```
dsolve(y(x)*diff(y(x),x)+a*(1+2*b*x)*exp(b*x)*y(x)=-a^2*b*x^2*exp(2*b*x),y(x), singsol=all)
```

$$y(x) = -\frac{\mathrm{e}^{bx}a\left(bx\,\mathrm{RootOf}\left(-\mathrm{e}^{-Z}bx - \mathrm{expIntegral}_1\left(-\_Z\right) + c_1\right) - 1\right)}{\mathrm{RootOf}\left(-\mathrm{e}^{-Z}bx - \mathrm{expIntegral}_1\left(-\_Z\right) + c_1\right)b}$$

✓ Solution by Mathematica
Time used: 0.736 (sec). Leaf size: 59

```
DSolve[y[x]*y'[x]+a*(1+2*b*x)*Exp[b*x]*y[x]==-a^2*b*x^2*Exp[2*b*x],y[x],x,IncludeSingularSol
```

$$\text{Solve}\left[bxe^{\frac{ae^{bx}}{abxe^{bx}+by(x)}} = \text{ExpIntegralEi}\left(\frac{ae^{bx}}{abe^{bx}x + by(x)}\right) + c_1, y(x)\right]$$

24.73 problem 73

Internal problem ID [10810]

Book: Handbook of exact solutions for ordinary differential equations. By Polyanin and Zaitsev. Second edition
Section: Chapter 1, section 1.3. Abel Equations of the Second Kind. subsection 1.3.3-2. Equations of the form $yy' = f_1(x)y + f_0(x)$
Problem number: 73.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[[_Abel, '2nd type', 'class A']]`

$$yy' - a(1 + 2n + 2n(1+n)\,x)\,\mathrm{e}^{(1+n)x}y = -a^2n(1+n)\,(xn+1)\,x\,\mathrm{e}^{2(1+n)x}$$

✓ Solution by Maple
Time used: 0.016 (sec). Leaf size: 130

```
dsolve(y(x)*diff(y(x),x)-a*(1+2*n+2*n*(n+1)*x)*exp((n+1)*x)*y(x)=-a^2*n*(n+1)*(1+n*x)*x*exp(
```

$$y(x) = \frac{\mathrm{e}^{(n+1)x}a\left(1 + 2x\,n^2 + \left(\tan\left(\frac{\mathrm{RootOf}\left(2x\,n^2\mathrm{e}^{\_Z+\_a}-\tan\left(\frac{\_a\sqrt{-\frac{(n+1)^2}{n^2}}}{2}\right)\_Z\sqrt{-\frac{(n+1)^2}{n^2}}\,n+2nx\,\mathrm{e}^{\_Z+\_a}+n\,\mathrm{e}^{\_Z+\_a}+2c_1n\right.\right.\right.\right.}{2}\right.\right.\right.\right.}{2n+2}$$

✗ Solution by Mathematica
Time used: 0.0 (sec). Leaf size: 0

```
DSolve[y[x]*y'[x]-a*(1+2*n+2*n*(n+1)*x)*Exp[(n+1)*x]*y[x]==-a^2*n*(n+1)*(1+n*x)*x*Exp[2*(n+1
```

Not solved

24.74 problem 74

Internal problem ID [10811]

Book: Handbook of exact solutions for ordinary differential equations. By Polyanin and Zaitsev. Second edition
Section: Chapter 1, section 1.3. Abel Equations of the Second Kind. subsection 1.3.3-2. Equations of the form $yy' = f_1(x)y + f_0(x)$
Problem number: 74.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[[_Abel, '2nd type', 'class A']]`

$$\boxed{yy' + a\left(1 + 2b\sqrt{x}\right) \mathrm{e}^{2b\sqrt{x}} y = -a^2 b\, x^{\frac{3}{2}} \mathrm{e}^{4b\sqrt{x}}}$$

✓ Solution by Maple
Time used: 0.0 (sec). Leaf size: 307

```
dsolve(y(x)*diff(y(x),x)+a*(1+2*b*x^(1/2))*exp(2*b*x^(1/2))*y(x)=-a^2*b*x^(3/2)*exp(4*b*x^(1
```

$$\frac{\sqrt{\frac{a\,\mathrm{e}^{2b\sqrt{x}}}{b^2\left(\mathrm{e}^{2b\sqrt{x}}ax+y(x)\right)}}\,\sqrt{x}\,\mathrm{BesselI}\left(1,\sqrt{\frac{a\,\mathrm{e}^{2b\sqrt{x}}}{b^2\left(\mathrm{e}^{2b\sqrt{x}}ax+y(x)\right)}}\right)c_1 b - \mathrm{BesselK}\left(1,-\sqrt{\frac{a\,\mathrm{e}^{2b\sqrt{x}}}{b^2\left(\mathrm{e}^{2b\sqrt{x}}ax+y(x)\right)}}\right)\sqrt{\frac{a\,\mathrm{e}^{2b\sqrt{x}}}{b^2\left(\mathrm{e}^{2b\sqrt{x}}ax+y(x)\right)}}}{\mathrm{BesselI}\left(1,\sqrt{\frac{a\,\mathrm{e}^{2b\sqrt{x}}}{b^2\left(\mathrm{e}^{2b\sqrt{x}}ax+y(x)\right)}}\right)\sqrt{\frac{a\,\mathrm{e}^{2b\sqrt{x}}}{b^2\left(\mathrm{e}^{2b\sqrt{x}}ax+y(x)\right)}}\,b\sqrt{x} - \mathrm{Be}}$$

$= 0$

✗ Solution by Mathematica
Time used: 0.0 (sec). Leaf size: 0

```
DSolve[y[x]*y'[x]+a*(1+2*b*x^(1/2))*Exp[2*b*x^(1/2)]*y[x]==-a^2*b*x^(3/2)*exp(4*b*x^(1/2)),y
```

Not solved

24.75 problem 75

Internal problem ID [10812]

Book: Handbook of exact solutions for ordinary differential equations. By Polyanin and Zaitsev. Second edition
Section: Chapter 1, section 1.3. Abel Equations of the Second Kind. subsection 1.3.3-2. Equations of the form $yy' = f_1(x)y + f_0(x)$
Problem number: 75.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[[_Abel, '2nd type', 'class A']]`

$$\boxed{yy' - (a\cosh(x) + b)\, y = -ab\sinh(x) + c}$$

✗ Solution by Maple

```
dsolve(y(x)*diff(y(x),x)=(a*cosh(x)+b)*y(x)-a*b*sinh(x)+c,y(x), singsol=all)
```

No solution found

✗ Solution by Mathematica
Time used: 0.0 (sec). Leaf size: 0

```
DSolve[y[x]*y'[x]==(a*Cosh[x]+b)*y[x]-a*b*Sinh[x]+c,y[x],x,IncludeSingularSolutions -> True]
```

Not solved

24.76 problem 76

Internal problem ID [10813]

Book: Handbook of exact solutions for ordinary differential equations. By Polyanin and Zaitsev. Second edition
Section: Chapter 1, section 1.3. Abel Equations of the Second Kind. subsection 1.3.3-2. Equations of the form $yy' = f_1(x)y + f_0(x)$
Problem number: 76.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[[_Abel, '2nd type', 'class A']]`

$$\boxed{yy' - (a\sinh(x) + b)\, y = -ab\cosh(x) + c}$$

✗ Solution by Maple

```
dsolve(y(x)*diff(y(x),x)=(a*sinh(x)+b)*y(x)-a*b*cosh(x)+c,y(x), singsol=all)
```

No solution found

✗ Solution by Mathematica
Time used: 0.0 (sec). Leaf size: 0

```
DSolve[y[x]*y'[x]==(a*Sinh[x]+b)*y[x]-a*b*Cosh[x]+c,y[x],x,IncludeSingularSolutions -> True]
```

Not solved

24.77 problem 77

Internal problem ID [10814]

Book: Handbook of exact solutions for ordinary differential equations. By Polyanin and Zaitsev. Second edition
Section: Chapter 1, section 1.3. Abel Equations of the Second Kind. subsection 1.3.3-2. Equations of the form $yy' = f_1(x)y + f_0(x)$
Problem number: 77.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[[_Abel, '2nd type', 'class A']]`

$$\boxed{yy' - (2\ln(x) + a + 1)\, y = x\left(-\ln(x)^2 - a\ln(x) + b\right)}$$

✓ Solution by Maple
Time used: 0.0 (sec). Leaf size: 163

```
dsolve(y(x)*diff(y(x),x)=(2*ln(x)+a+1)*y(x)+x*( -(ln(x))^2-a*ln(x)+b),y(x), singsol=all)
```

$$y(x) = x\left(-\tanh\left(\frac{\text{RootOf}\left(-\sqrt{a^2+4b}\,\tanh\left(\frac{\_Z\sqrt{a^2+4b}}{2}\right)\mathrm{e}^{\_Z} + \mathrm{e}^{-\frac{2\,\operatorname{arctanh}\left(\frac{2\_a-a}{\sqrt{a^2+4b}}\right)}{\sqrt{a^2+4b}}}\tanh\left(\frac{\_Z\sqrt{a^2+4b}}{2}\right)\sqrt{a^2+4b} + 2\ln(x)\mathrm{e}^{\_Z} + \mathrm{e}^{\_Z}a - \mathrm{e}\right.\right.}{2}\right.\right.$$

✗ Solution by Mathematica
Time used: 0.0 (sec). Leaf size: 0

```
DSolve[y[x]*y'[x]==(2*Log[x]+a+1)*y[x]+x*( -(Log[x])^2-a*Log[x]+b),y[x],x,IncludeSingularSol
```

Not solved

24.78 problem 78

Internal problem ID [10815]

Book: Handbook of exact solutions for ordinary differential equations. By Polyanin and Zaitsev. Second edition
Section: Chapter 1, section 1.3. Abel Equations of the Second Kind. subsection 1.3.3-2. Equations of the form $yy' = f_1(x)y + f_0(x)$
Problem number: 78.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[[_Abel, '2nd type', 'class A']]`

$$\boxed{yy' - \left(2\ln(x)^2 + 2\ln(x) + a\right)y = x\left(-\ln(x)^4 - \ln(x)^2 a + b\right)}$$

✗ Solution by Maple

```
dsolve(y(x)*diff(y(x),x)=(2*(ln(x))^2+2*ln(x)+a)*y(x)+x*(- (ln(x))^4-a*(ln(x))^2+b),y(x), si
```

No solution found

✗ Solution by Mathematica
Time used: 0.0 (sec). Leaf size: 0

```
DSolve[y[x]*y'[x]==(2*(Log[x])^2+2*Log[x]+a)*y[x]+x*(- (Log[x])^4-a*(Log[x])^2+b),y[x],x,Inc
```

Not solved

24.79 problem 79

Internal problem ID [10816]

Book: Handbook of exact solutions for ordinary differential equations. By Polyanin and Zaitsev. Second edition
Section: Chapter 1, section 1.3. Abel Equations of the Second Kind. subsection 1.3.3-2. Equations of the form $yy' = f_1(x)y + f_0(x)$
Problem number: 79.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[[_Abel, '2nd type', 'class A']]`

$$\boxed{yy' - ax\cos\left(\lambda x^2\right) y = x}$$

✗ Solution by Maple

```
dsolve(y(x)*diff(y(x),x)=a*x*cos(lambda*x^2)*y(x)+x,y(x), singsol=all)
```

No solution found

✗ Solution by Mathematica
Time used: 0.0 (sec). Leaf size: 0

```
DSolve[y[x]*y'[x]==a*x*Cos[\[Lambda]*x^2]*y[x]+x,y[x],x,IncludeSingularSolutions -> True]
```

Not solved

24.80 problem 80

Internal problem ID [10817]

Book: Handbook of exact solutions for ordinary differential equations. By Polyanin and Zaitsev. Second edition
Section: Chapter 1, section 1.3. Abel Equations of the Second Kind. subsection 1.3.3-2. Equations of the form $yy' = f_1(x)y + f_0(x)$
Problem number: 80.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[[_Abel, '2nd type', 'class A']]`

$$\boxed{yy' - ax\sin\left(\lambda x^2\right)y = x}$$

✗ Solution by Maple

```
dsolve(y(x)*diff(y(x),x)=a*x*sin(lambda*x^2)*y(x)+x,y(x), singsol=all)
```

No solution found

✗ Solution by Mathematica
Time used: 0.0 (sec). Leaf size: 0

```
DSolve[y[x]*y'[x]==a*x*Sin[\[Lambda]*x^2]*y[x]+x,y[x],x,IncludeSingularSolutions -> True]
```

Not solved

25 Chapter 1, section 1.3. Abel Equations of the Second Kind. subsection 1.3.4-2. Equations of the form $(g_1(x)+g_0(x))y'=f_2(x)y^2+f_1(x)y+f_0(x)$

25.1 problem 1 610
25.2 problem 2 611
25.3 problem 3 612
25.4 problem 4 613
25.5 problem 5 614
25.6 problem 6 615
25.7 problem 7 616

25.1 problem 1

Internal problem ID [10818]

Book: Handbook of exact solutions for ordinary differential equations. By Polyanin and Zaitsev. Second edition
Section: Chapter 1, section 1.3. Abel Equations of the Second Kind. subsection 1.3.4-2. Equations of the form $(g_1(x)+g_0(x))y'=f_2(x)y^2+f_1(x)y+f_0(x)$
Problem number: 1.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[[_homogeneous, 'class C'], _exact, _rational, [_Abel, '2nd ty`

$$\boxed{(Ay+Bx+a)\,y'+By=-kx-b}$$

✓ Solution by Maple
Time used: 0.344 (sec). Leaf size: 87

```
dsolve((A*y(x)+B*x+a)*diff(y(x),x)+B*y(x)+k*x+b=0,y(x), singsol=all)
```

$$y(x)$$
$$=\frac{-\sqrt{-\left(Ak-B^2\right)\left((kx+b)\,A-B^2x-Ba\right)^2c_1^2+A}+\left(k(-Bx-a)\,A+x\,B^3+a\,B^2\right)c_1}{Ac_1\left(Ak-B^2\right)}$$

✓ Solution by Mathematica
Time used: 18.19 (sec). Leaf size: 106

```
DSolve[(A*y[x]+B*x+a)*y'[x]+B*y[x]+k*x+b==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x)\to -\frac{\frac{\sqrt{\frac{(a+Bx)^2}{A}+Ac_1-x(2b+kx)}}{\sqrt{\frac{1}{A}}}+a+Bx}{A}$$
$$y(x)\to -\frac{a+Bx}{A}+\sqrt{\frac{1}{A}}\sqrt{\frac{(a+Bx)^2}{A}+Ac_1-x(2b+kx)}$$

25.2 problem 2

Internal problem ID [10819]

Book: Handbook of exact solutions for ordinary differential equations. By Polyanin and Zaitsev. Second edition
Section: Chapter 1, section 1.3. Abel Equations of the Second Kind. subsection 1.3.4-2. Equations of the form $(g_1(x)+g_0(x))y' = f_2(x)y^2 + f_1(x)y + f_0(x)$
Problem number: 2.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[[_homogeneous, 'class C'], _rational, [_Abel, '2nd type', 'cl`

$$\boxed{(y+ax+b)\,y' - \alpha y = \beta x + \gamma}$$

✓ Solution by Maple
Time used: 0.219 (sec). Leaf size: 211

```
dsolve((y(x)+a*x+b)*diff(y(x),x)=alpha*y(x)+beta*x+gamma,y(x), singsol=all)
```

$$y(x) = \frac{((ax+b)\,\alpha - x\beta - \gamma)\,\sqrt{-a^2+2a\alpha-\alpha^2-4\beta}\,\tan\left(\text{RootOf}\left(-2\sqrt{-a^2+2a\alpha-\alpha^2-4\beta}\,\ln(2)+\sqrt{-a^2}\right.\right.$$

✗ Solution by Mathematica
Time used: 0.0 (sec). Leaf size: 0

```
DSolve[(y[x]*a*x+b)*y'[x]==\[Alpha]*y[x]+\[Beta]*x+\[Gamma],y[x],x,IncludeSingularSolutions
```

Not solved

25.3 problem 3

Internal problem ID [10820]

Book: Handbook of exact solutions for ordinary differential equations. By Polyanin and Zaitsev. Second edition
Section: Chapter 1, section 1.3. Abel Equations of the Second Kind. subsection 1.3.4-2. Equations of the form $(g_1(x)+g_0(x))y'=f_2(x)y^2+f_1(x)y+f_0(x)$
Problem number: 3.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_rational, [_Abel, '2nd type', 'class A']]`

$$\boxed{\left(y+ak\,x^2+bx+c\right)y'+y^2a-2yakx-ym=k(k+b-m)\,x+s}$$

✗ Solution by Maple

```
dsolve((y(x)+a*k*x^2+b*x+c)*diff(y(x),x)=-a*y(x)^2+2*a*k*x*y(x)+m*y(x)+k*(k+b-m)*x+s,y(x), s
```

No solution found

✗ Solution by Mathematica
Time used: 0.0 (sec). Leaf size: 0

```
DSolve[(y[x]+a*k*x^2+b*x+c)*y'[x]==-a*y[x]^2+2*a*k*x*y[x]+m*y[x]+k*(k+b-m)*x+s,y[x],x,Includ
```

Timed out

25.4 problem 4

Internal problem ID [10821]

Book: Handbook of exact solutions for ordinary differential equations. By Polyanin and Zaitsev. Second edition
Section: Chapter 1, section 1.3. Abel Equations of the Second Kind. subsection 1.3.4-2. Equations of the form $(g_1(x)+g_0(x))y'=f_2(x)y^2+f_1(x)y+f_0(x)$
Problem number: 4.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_exact, _rational, [_1st_order, '_with_symmetry_[F(x),G(x)]']`

$$\boxed{(y+A\,x^n+a)\,y'+nA\,x^{n-1}y=-k\,x^m-b}$$

✓ Solution by Maple
Time used: 0.016 (sec). Leaf size: 149

```
dsolve((y(x)+A*x^n+a)*diff(y(x),x)+n*A*x^(n-1)*y(x)+k*x^m+b=0,y(x), singsol=all)
```

$$y(x)=\frac{\sqrt{(-2x^{1+m}k+(1+m)\left(x^{2n}A^2+2A\,x^n a+a^2-2bx-2c_1\right))(1+m)}+A(-m-1)\,x^n-am-a}{1+m}$$
$$y(x)=\frac{-\sqrt{(-2x^{1+m}k+(1+m)\left(x^{2n}A^2+2A\,x^n a+a^2-2bx-2c_1\right))(1+m)}+A(-m-1)\,x^n-am-a}{1+m}$$

✓ Solution by Mathematica
Time used: 21.171 (sec). Leaf size: 118

```
DSolve[(y[x]+A*x^n+a)*y'[x]+n*A*x^(n-1)*y[x]+k*x^m+b==0,y[x],x,IncludeSingularSolutions -> T
```

$$y(x)\to -\sqrt{\frac{1}{x}}\sqrt{x\left(\left(a+Ax^n\right)^2-\frac{2x\left(bm+b+kx^m\right)}{m+1}+c_1\right)}-a-Ax^n$$
$$y(x)\to \sqrt{\frac{1}{x}}\sqrt{x\left(\left(a+Ax^n\right)^2-\frac{2x\left(bm+b+kx^m\right)}{m+1}+c_1\right)}-a-Ax^n$$

25.5 problem 5

Internal problem ID [10822]

Book: Handbook of exact solutions for ordinary differential equations. By Polyanin and Zaitsev. Second edition
Section: Chapter 1, section 1.3. Abel Equations of the Second Kind. subsection 1.3.4-2. Equations of the form $(g_1(x)+g_0(x))y'=f_2(x)y^2+f_1(x)y+f_0(x)$
Problem number: 5.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_rational, [_Abel, '2nd type', 'class A']]`

$$\left(y+a\,x^{1+n}+b\,x^{n}\right)y'-\left(x^{n}na+c\,x^{n-1}\right)y=0$$

✗ Solution by Maple

```
dsolve((y(x)+a*x^(n+1)+b*x^n)*diff(y(x),x)=(a*n*x^n+c*x^(n-1))*y(x),y(x), singsol=all)
```

No solution found

✗ Solution by Mathematica
Time used: 0.0 (sec). Leaf size: 0

```
DSolve[(y[x]+a*x^(n+1)+b*x^n)*y'[x]==(a*n*x^n+c*x^(n-1))*y[x],y[x],x,IncludeSingularSolution
```

Not solved

25.6 problem 6

Internal problem ID [10823]

Book: Handbook of exact solutions for ordinary differential equations. By Polyanin and Zaitsev. Second edition
Section: Chapter 1, section 1.3. Abel Equations of the Second Kind. subsection 1.3.4-2. Equations of the form $(g_1(x)+g_0(x))y'=f_2(x)y^2+f_1(x)y+f_0(x)$
Problem number: 6.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_rational, [_Abel, '2nd type', 'class B']]`

$$\boxed{xyy'-y^2a-yb=x^nc+s}$$

✗ Solution by Maple

```
dsolve(x*y(x)*diff(y(x),x)=a*y(x)^2+b*y(x)+c*x^n+s,y(x), singsol=all)
```

No solution found

✗ Solution by Mathematica
Time used: 0.0 (sec). Leaf size: 0

```
DSolve[x*y[x]*y'[x]==a*y[x]^2+b*y[x]+c*x^n+s,y[x],x,IncludeSingularSolutions -> True]
```

Not solved

25.7 problem 7

Internal problem ID [10824]

Book: Handbook of exact solutions for ordinary differential equations. By Polyanin and Zaitsev. Second edition
Section: Chapter 1, section 1.3. Abel Equations of the Second Kind. subsection 1.3.4-2. Equations of the form $(g_1(x)+g_0(x))y' = f_2(x)y^2 + f_1(x)y + f_0(x)$
Problem number: 7.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_rational, [_Abel, '2nd type', 'class B']]`

$$\boxed{xyy' + y^2 n - a(2n+1)\,xy - yb = -a^2 n\,x^2 - abx + c}$$

✓ Solution by Maple
Time used: 0.0 (sec). Leaf size: 224

```
dsolve(x*y(x)*diff(y(x),x)=-n*y(x)^2+a*(2*n+1)*x*y(x)+b*y(x)-a^2*n*x^2-a*b*x+c,y(x), singsol
```

$$\frac{\left(\frac{-ny(x)^2+(2axn+b)y(x)-a^2n\,x^2-abx+c}{(ax-y(x))^2}\right)^{-\frac{1}{2n}}\left(\frac{1}{ax-y(x)}\right)^{\frac{1}{n}} y(x)\,\mathrm{e}^{\frac{b\,\operatorname{arctanh}\left(\frac{-abx+by(x)+2c}{\sqrt{b^2+4cn}\,(-ax+y(x))}\right)}{\sqrt{b^2+4cn}\,n}} - \left(\left(\int \frac{1}{ax-y(x)}\,(\_a^2c - \_a\right.\right.}{x\,(ax-y\,(x))}$$

$= 0$

✗ Solution by Mathematica
Time used: 0.0 (sec). Leaf size: 0

```
DSolve[x*y[x]*y'[x]==-n*y[x]^2+a*(2*n+1)*x*y[x]+b*y[x]-a^2*n*x^2-a*b*x+c,y[x],x,IncludeSingu
```

Not solved

26 Chapter 2, Second-Order Differential Equations. section 2.1.2 Equations Containing Power Functions. page 213

26.1 problem 1 618
26.2 problem 2 619
26.3 problem 3 620
26.4 problem 4 621
26.5 problem 5 622
26.6 problem 6 623
26.7 problem 7 624
26.8 problem 8 625
26.9 problem 9 626
26.10 problem 10 627

26.1 problem 1

Internal problem ID [10825]

Book: Handbook of exact solutions for ordinary differential equations. By Polyanin and Zaitsev. Second edition
Section: Chapter 2, Second-Order Differential Equations. section 2.1.2 Equations Containing Power Functions. page 213
Problem number: 1.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _missing_x]]`

$$\boxed{y'' + ay = 0}$$

✓ Solution by Maple
Time used: 0.0 (sec). Leaf size: 21

```
dsolve(diff(y(x),x$2)+a*y(x)=0,y(x), singsol=all)
```

$$y(x) = c_1 \sin\left(\sqrt{a}\, x\right) + c_2 \cos\left(\sqrt{a}\, x\right)$$

✓ Solution by Mathematica
Time used: 0.028 (sec). Leaf size: 28

```
DSolve[y''[x]+a*y[x]==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to c_1 \cos\left(\sqrt{a}x\right) + c_2 \sin\left(\sqrt{a}x\right)$$

26.2 problem 2

Internal problem ID [10826]

Book: Handbook of exact solutions for ordinary differential equations. By Polyanin and Zaitsev. Second edition
Section: Chapter 2, Second-Order Differential Equations. section 2.1.2 Equations Containing Power Functions. page 213
Problem number: 2.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$\boxed{y'' - (ax+b)\,y = 0}$$

✓ Solution by Maple
Time used: 0.016 (sec). Leaf size: 33

```
dsolve(diff(y(x),x$2)-(a*x+b)*y(x)=0,y(x), singsol=all)
```

$$y(x) = c_1 \operatorname{AiryAi}\left(\frac{ax+b}{(-a)^{\frac{2}{3}}}\right) + c_2 \operatorname{AiryBi}\left(\frac{ax+b}{(-a)^{\frac{2}{3}}}\right)$$

✓ Solution by Mathematica
Time used: 0.029 (sec). Leaf size: 36

```
DSolve[y''[x]-(a*x+b)*y[x]==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to c_1 \operatorname{AiryAi}\left(\frac{b+ax}{a^{2/3}}\right) + c_2 \operatorname{AiryBi}\left(\frac{b+ax}{a^{2/3}}\right)$$

26.3 problem 3

Internal problem ID [10827]

Book: Handbook of exact solutions for ordinary differential equations. By Polyanin and Zaitsev. Second edition
Section: Chapter 2, Second-Order Differential Equations. section 2.1.2 Equations Containing Power Functions. page 213
Problem number: 3.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$\boxed{y'' - \left(a^2 x^2 + a\right) y = 0}$$

✓ Solution by Maple
Time used: 0.0 (sec). Leaf size: 22

```
dsolve(diff(y(x),x$2)-(a^2*x^2+a)*y(x)=0,y(x), singsol=all)
```

$$y(x) = \mathrm{e}^{\frac{a\,x^2}{2}} \left(c_1 + \operatorname{erf}\left(\sqrt{a}\,x\right) c_2\right)$$

✓ Solution by Mathematica
Time used: 0.041 (sec). Leaf size: 43

```
DSolve[y''[x]-(a^2*x^2+a)*y[x]==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to c_1 \operatorname{ParabolicCylinderD}\left(-1, \sqrt{2}\sqrt{a}x\right) + c_2 \operatorname{ParabolicCylinderD}\left(0, i\sqrt{2}\sqrt{a}x\right)$$

26.4 problem 4

Internal problem ID [10828]

Book: Handbook of exact solutions for ordinary differential equations. By Polyanin and Zaitsev. Second edition
Section: Chapter 2, Second-Order Differential Equations. section 2.1.2 Equations Containing Power Functions. page 213
Problem number: 4.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$\boxed{y'' - (a\,x^2 + b)\,y = 0}$$

✓ Solution by Maple
Time used: 0.094 (sec). Leaf size: 43

```
dsolve(diff(y(x),x$2)-(a*x^2+b)*y(x)=0,y(x), singsol=all)
```

$$y(x) = \frac{c_1 \,\text{WhittakerM}\left(-\frac{b}{4\sqrt{a}}, \frac{1}{4}, \sqrt{a}\,x^2\right) + c_2 \,\text{WhittakerW}\left(-\frac{b}{4\sqrt{a}}, \frac{1}{4}, \sqrt{a}\,x^2\right)}{\sqrt{x}}$$

✓ Solution by Mathematica
Time used: 0.047 (sec). Leaf size: 68

```
DSolve[y''[x]-(a*x^2+b)*y[x]==0,y[x],x,IncludeSingularSolutions -> True]
```

$$\begin{aligned} y(x) \to\ & c_1 \,\text{ParabolicCylinderD}\left(-\frac{b}{2\sqrt{a}} - \frac{1}{2}, \sqrt{2}\sqrt[4]{a}x\right) \\ & + c_2 \,\text{ParabolicCylinderD}\left(\frac{1}{2}\left(\frac{b}{\sqrt{a}} - 1\right), i\sqrt{2}\sqrt[4]{a}x\right) \end{aligned}$$

26.5 problem 5

Internal problem ID [10829]

Book: Handbook of exact solutions for ordinary differential equations. By Polyanin and Zaitsev. Second edition
Section: Chapter 2, Second-Order Differential Equations. section 2.1.2 Equations Containing Power Functions. page 213
Problem number: 5.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$\boxed{y'' + a^3 x(-ax+2)\, y = 0}$$

✓ Solution by Maple
Time used: 0.0 (sec). Leaf size: 28

```
dsolve(diff(y(x),x$2)+a^3*x*(2-a*x)*y(x)=0,y(x), singsol=all)
```

$$y(x) = \mathrm{e}^{-\frac{ax(ax-2)}{2}} \left(c_1 + \operatorname{erf}\left(iax - i\right) c_2\right)$$

✓ Solution by Mathematica
Time used: 0.28 (sec). Leaf size: 50

```
DSolve[y''[x]+a^3*x*(2-a*x)*y[x]==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to \frac{e^{-\frac{1}{2}ax(ax-2)-1}\left(2eac_1 - \sqrt{\pi}c_2 \operatorname{erfi}(1-ax)\right)}{2a}$$

26.6 problem 6

Internal problem ID [10830]

Book: Handbook of exact solutions for ordinary differential equations. By Polyanin and Zaitsev. Second edition
Section: Chapter 2, Second-Order Differential Equations. section 2.1.2 Equations Containing Power Functions. page 213
Problem number: 6.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$\boxed{y'' - (a\,x^2 + bcx)\,y = 0}$$

✓ Solution by Maple
Time used: 0.046 (sec). Leaf size: 142

```
dsolve(diff(y(x),x$2)-(a*x^2+b*x*c)*y(x)=0,y(x), singsol=all)
```

$$y(x) = \mathrm{e}^{-\frac{x(ax+bc)}{2\sqrt{a}}}\left(2xac_2\,\mathrm{hypergeom}\left(\left[-\frac{b^2c^2-12a^{\frac{3}{2}}}{16a^{\frac{3}{2}}}\right],\left[\frac{3}{2}\right],\frac{(2ax+bc)^2}{4a^{\frac{3}{2}}}\right)\right.$$
$$+ cbc_2\,\mathrm{hypergeom}\left(\left[-\frac{b^2c^2-12a^{\frac{3}{2}}}{16a^{\frac{3}{2}}}\right],\left[\frac{3}{2}\right],\frac{(2ax+bc)^2}{4a^{\frac{3}{2}}}\right)$$
$$\left.+\,\mathrm{hypergeom}\left(\left[-\frac{b^2c^2-4a^{\frac{3}{2}}}{16a^{\frac{3}{2}}}\right],\left[\frac{1}{2}\right],\frac{(2ax+bc)^2}{4a^{\frac{3}{2}}}\right)c_1\right)$$

✓ Solution by Mathematica
Time used: 0.08 (sec). Leaf size: 92

```
DSolve[y''[x]-(a*x^2+b*x*c)*y[x]==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to c_2\,\mathrm{ParabolicCylinderD}\left(-\frac{b^2c^2}{8a^{3/2}}\right.$$
$$\left.-\frac{1}{2},\frac{i(bc+2ax)}{\sqrt{2}a^{3/4}}\right)+c_1\,\mathrm{ParabolicCylinderD}\left(\frac{1}{8}\left(\frac{b^2c^2}{a^{3/2}}-4\right),\frac{bc+2ax}{\sqrt{2}a^{3/4}}\right)$$

26.7 problem 7

Internal problem ID [10831]

Book: Handbook of exact solutions for ordinary differential equations. By Polyanin and Zaitsev. Second edition
Section: Chapter 2, Second-Order Differential Equations. section 2.1.2 Equations Containing Power Functions. page 213
Problem number: 7.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_Emden, _Fowler]]`

$$\boxed{y'' - a\,x^n y = 0}$$

✓ Solution by Maple
Time used: 0.281 (sec). Leaf size: 63

```
dsolve(diff(y(x),x$2)-a*x^n*y(x)=0,y(x), singsol=all)
```

$$y(x) = \sqrt{x}\left(\text{BesselY}\left(\frac{1}{n+2}, \frac{2\sqrt{-a}\,x^{\frac{n}{2}+1}}{n+2}\right)c_2 + \text{BesselJ}\left(\frac{1}{n+2}, \frac{2\sqrt{-a}\,x^{\frac{n}{2}+1}}{n+2}\right)c_1\right)$$

✓ Solution by Mathematica
Time used: 0.125 (sec). Leaf size: 119

```
DSolve[y''[x]-a*x^n*y[x]==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to (n+2)^{-\frac{1}{n+2}}\sqrt{x}\,a^{\frac{1}{2n+4}}\left(c_1 \text{Gamma}\left(\frac{n+1}{n+2}\right)\text{BesselI}\left(-\frac{1}{n+2}, \frac{2\sqrt{a}x^{\frac{n}{2}+1}}{n+2}\right)\right.$$
$$\left. + c_2(-1)^{\frac{1}{n+2}}\,\text{Gamma}\left(1+\frac{1}{n+2}\right)\text{BesselI}\left(\frac{1}{n+2}, \frac{2\sqrt{a}x^{\frac{n}{2}+1}}{n+2}\right)\right)$$

26.8 problem 8

Internal problem ID [10832]

Book: Handbook of exact solutions for ordinary differential equations. By Polyanin and Zaitsev. Second edition
Section: Chapter 2, Second-Order Differential Equations. section 2.1.2 Equations Containing Power Functions. page 213
Problem number: 8.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$\boxed{y'' - a\left(a\,x^{2n} + n\,x^{n-1}\right) y = 0}$$

✓ Solution by Maple
Time used: 0.203 (sec). Leaf size: 136

```
dsolve(diff(y(x),x$2)-a*(a*x^(2*n)+n*x^(n-1))*y(x)=0,y(x), singsol=all)
```

$$y(x) = \frac{c_2 x^{-\frac{3n}{2}-1}(n+2)^2\,\text{WhittakerM}\left(\frac{n+2}{2n+2}, \frac{2n+3}{2n+2}, \frac{2a\,x^{n+1}}{n+1}\right)}{2} + \left(\left(\frac{n}{2}+1\right)x^{-\frac{3n}{2}-1} + a\,x^{-\frac{n}{2}}\right)(n+1)\,c_2\,\text{WhittakerM}\left(-\frac{n}{2n+2}, \frac{2n+3}{2n+2}, \frac{2a\,x^{n+1}}{n+1}\right) + c_1 \mathrm{e}^{\frac{a\,x^{n+1}}{n+1}}$$

✓ Solution by Mathematica
Time used: 0.596 (sec). Leaf size: 81

```
DSolve[y''[x]-a*(a*x^(2*n)+n*x^(n-1))*y[x]==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to e^{\frac{a x^{n+1}}{n+1}}\left(c_2 - \frac{c_1 2^{-\frac{1}{n+1}} x \left(\frac{a x^{n+1}}{n+1}\right)^{-\frac{1}{n+1}} \Gamma\left(\frac{1}{n+1}, \frac{2a x^{n+1}}{n+1}\right)}{n+1}\right)$$

26.9 problem 9

Internal problem ID [10833]

Book: Handbook of exact solutions for ordinary differential equations. By Polyanin and Zaitsev. Second edition
Section: Chapter 2, Second-Order Differential Equations. section 2.1.2 Equations Containing Power Functions. page 213
Problem number: 9.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$\boxed{y'' - a\,x^{n-2}(a\,x^n + n + 1)\,y = 0}$$

✓ Solution by Maple
Time used: 0.187 (sec). Leaf size: 113

```
dsolve(diff(y(x),x$2)-a*x^(n-2)*(a*x^n+n+1)*y(x)=0,y(x), singsol=all)
```

$$y(x) = \frac{c_2 x^{-\frac{3n}{2}+\frac{1}{2}}(n-1)^2\,\text{WhittakerM}\left(\frac{n-1}{2n}, \frac{2n-1}{2n}, \frac{2a\,x^n}{n}\right)}{2} + \left(\frac{(n-1)\,x^{-\frac{3n}{2}+\frac{1}{2}}}{2} + x^{-\frac{n}{2}+\frac{1}{2}}a\right) n c_2\,\text{WhittakerM}\left(-\frac{n+1}{2n}, \frac{2n-1}{2n}, \frac{2a\,x^n}{n}\right) + c_1 x\,\mathrm{e}^{\frac{a\,x^n}{n}}$$

✗ Solution by Mathematica
Time used: 0.0 (sec). Leaf size: 0

```
DSolve[y''[x]-a*x^(n-2)*(a*x^n+n+1)*y[x]==0,y[x],x,IncludeSingularSolutions -> True]
```

Not solved

26.10 problem 10

Internal problem ID [10834]

Book: Handbook of exact solutions for ordinary differential equations. By Polyanin and Zaitsev. Second edition
Section: Chapter 2, Second-Order Differential Equations. section 2.1.2 Equations Containing Power Functions. page 213
Problem number: 10.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$\boxed{y'' + \left(a\,x^{2n} + b\,x^{n-1}\right) y = 0}$$

✓ Solution by Maple
Time used: 0.375 (sec). Leaf size: 89

```
dsolve(diff(y(x),x$2)+(a*x^(2*n)+b*x^(n-1))*y(x)=0,y(x), singsol=all)
```

$$y(x) = x^{-\frac{n}{2}}\left(c_1\,\text{WhittakerM}\left(-\frac{ib}{\sqrt{a}\,(2n+2)}, \frac{1}{2n+2}, \frac{2i\sqrt{a}\,x\,x^n}{n+1}\right) + c_2\,\text{WhittakerW}\left(-\frac{ib}{\sqrt{a}\,(2n+2)}, \frac{1}{2n+2}, \frac{2i\sqrt{a}\,x\,x^n}{n+1}\right)\right)$$

✓ Solution by Mathematica
Time used: 0.399 (sec). Leaf size: 225

```
DSolve[y''[x]+(a*x^(2*n)+b*x^(n-1))*y[x]==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x)$$

$$\to 2^{\frac{n}{2n+2}} x^{-n/2} \left(x^{n+1}\right)^{\frac{n}{2n+2}} e^{-\frac{\sqrt{a}x^{n+1}}{\sqrt{-(n+1)^2}}} \left(c_1\,\text{HypergeometricU}\left(-\frac{(n+1)\left(nb+b+\sqrt{a}n\sqrt{-(n+1)^2}\right)}{2\sqrt{a}\left(-(n+1)^2\right)^{3/2}}, \frac{n}{n+1}\right.\right.$$

27 Chapter 2, Second-Order Differential Equations. section 2.1.2-2 Equation of form $y'' + f(x)y' + g(x)y = 0$

27.1 problem 11 630
27.2 problem 12 631
27.3 problem 13 632
27.4 problem 14 633
27.5 problem 15 634
27.6 problem 16 635
27.7 problem 17 636
27.8 problem 18 637
27.9 problem 19 638
27.10 problem 20 639
27.11 problem 21 640
27.12 problem 22 641
27.13 problem 23 642
27.14 problem 24 643
27.15 problem 25 644
27.16 problem 26 645
27.17 problem 27 646
27.18 problem 28 647
27.19 problem 29 648
27.20 problem 30 649
27.21 problem 31 650
27.22 problem 32 651
27.23 problem 33 652
27.24 problem 34 653
27.25 problem 35 654
27.26 problem 36 655
27.27 problem 37 656
27.28 problem 38 657
27.29 problem 39 659
27.30 problem 40 660
27.31 problem 41 661
27.32 problem 42 662
27.33 problem 43 663
27.34 problem 44 664

27.35problem 45 665
27.36problem 46 666
27.37problem 47 667
27.38problem 48 669
27.39problem 49 670
27.40problem 50 671
27.41problem 51 672
27.42problem 52 673
27.43problem 53 674
27.44problem 54 675
27.45problem 55 676
27.46problem 56 677
27.47problem 57 678
27.48problem 58 679
27.49problem 59 680
27.50problem 60 681

27.1 problem 11

Internal problem ID [10835]

Book: Handbook of exact solutions for ordinary differential equations. By Polyanin and Zaitsev. Second edition
Section: Chapter 2, Second-Order Differential Equations. section 2.1.2-2 Equation of form $y'' + f(x)y' + g(x)y = 0$
Problem number: 11.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _missing_x]]`

$$\boxed{y'' + ay' + yb = 0}$$

✓ Solution by Maple
Time used: 0.0 (sec). Leaf size: 41

```
dsolve(diff(y(x),x$2)+a*diff(y(x),x)+b*y(x)=0,y(x), singsol=all)
```

$$y(x) = c_1 \mathrm{e}^{\frac{\left(-a+\sqrt{a^2-4b}\right)x}{2}} + c_2 \mathrm{e}^{-\frac{\left(a+\sqrt{a^2-4b}\right)x}{2}}$$

✓ Solution by Mathematica
Time used: 0.048 (sec). Leaf size: 47

```
DSolve[y''[x]+a*y'[x]+b*y[x]==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to e^{-\frac{1}{2}x\left(\sqrt{a^2-4b}+a\right)}\left(c_2 e^{x\sqrt{a^2-4b}} + c_1\right)$$

27.2 problem 12

Internal problem ID [10836]

Book: Handbook of exact solutions for ordinary differential equations. By Polyanin and Zaitsev. Second edition
Section: Chapter 2, Second-Order Differential Equations. section 2.1.2-2 Equation of form $y'' + f(x)y' + g(x)y = 0$
Problem number: 12.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$\boxed{y'' + ay' + (bx + c)\, y = 0}$$

✓ Solution by Maple
Time used: 0.016 (sec). Leaf size: 49

```
dsolve(diff(y(x),x$2)+a*diff(y(x),x)+(b*x+c)*y(x)=0,y(x), singsol=all)
```

$$y(x) = \mathrm{e}^{-\frac{ax}{2}} \left(\mathrm{AiryAi}\left(\frac{a^2 - 4bx - 4c}{4b^{\frac{2}{3}}} \right) c_1 + \mathrm{AiryBi}\left(\frac{a^2 - 4bx - 4c}{4b^{\frac{2}{3}}} \right) c_2 \right)$$

✓ Solution by Mathematica
Time used: 0.046 (sec). Leaf size: 67

```
DSolve[y''[x]+a*y'[x]+(b*x+c)*y[x]==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to e^{-\frac{ax}{2}} \left(c_1 \,\mathrm{AiryAi}\left(\frac{a^2 - 4(c + bx)}{4(-b)^{2/3}} \right) + c_2 \,\mathrm{AiryBi}\left(\frac{a^2 - 4(c + bx)}{4(-b)^{2/3}} \right) \right)$$

27.3 problem 13

Internal problem ID [10837]

Book: Handbook of exact solutions for ordinary differential equations. By Polyanin and Zaitsev. Second edition
Section: Chapter 2, Second-Order Differential Equations. section 2.1.2-2 Equation of form $y'' + f(x)y' + g(x)y = 0$
Problem number: 13.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$\boxed{y'' + ay' - \left(b\,x^2 + c\right) y = 0}$$

✓ Solution by Maple
Time used: 0.079 (sec). Leaf size: 74

```
dsolve(diff(y(x),x$2)+a*diff(y(x),x)-(b*x^2+c)*y(x)=0,y(x), singsol=all)
```

$$y(x) = \mathrm{e}^{-\frac{x\left(\sqrt{b}\,x+a\right)}{2}} x \left(\mathrm{KummerM}\left(\frac{a^2 + 12\sqrt{b} + 4c}{16\sqrt{b}}, \frac{3}{2}, \sqrt{b}\,x^2 \right) c_1 + \mathrm{KummerU}\left(\frac{a^2 + 12\sqrt{b} + 4c}{16\sqrt{b}}, \frac{3}{2}, \sqrt{b}\,x^2 \right) c_2 \right)$$

✓ Solution by Mathematica
Time used: 0.088 (sec). Leaf size: 96

```
DSolve[y''[x]+a*y'[x]-(b*x^2+c)*y[x]==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to e^{-\frac{1}{2}x\left(a+\sqrt{b}x\right)} \left(c_1 \,\mathrm{HermiteH}\left(\frac{-a^2 - 4\left(c+\sqrt{b}\right)}{8\sqrt{b}}, \sqrt[4]{b}x \right) + c_2 \,\mathrm{Hypergeometric1F1}\left(\frac{a^2 + 4\left(c+\sqrt{b}\right)}{16\sqrt{b}}, \frac{1}{2}, \sqrt{b}x^2 \right) \right)$$

27.4 problem 14

Internal problem ID [10838]

Book: Handbook of exact solutions for ordinary differential equations. By Polyanin and Zaitsev. Second edition
Section: Chapter 2, Second-Order Differential Equations. section 2.1.2-2 Equation of form $y'' + f(x)y' + g(x)y = 0$
Problem number: 14.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries], [_2nd_order, _linear,`

$$\boxed{y'' + ay' + b\left(-b\,x^2 + ax + 1\right) y = 0}$$

✓ Solution by Maple
Time used: 0.0 (sec). Leaf size: 30

```
dsolve(diff(y(x),x$2)+a*diff(y(x),x)+b*(-b*x^2+a*x+1)*y(x)=0,y(x), singsol=all)
```

$$y(x) = \mathrm{e}^{-\frac{x^2 b}{2}} \left(c_1 \operatorname{erf}\left(\frac{-2bx + a}{2\sqrt{-b}} \right) + c_2 \right)$$

✓ Solution by Mathematica
Time used: 0.28 (sec). Leaf size: 67

```
DSolve[y''[x]+a*y'[x]+b*(-b*x^2+a*x+1)*y[x]==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to \frac{1}{2} e^{-\frac{bx^2}{2}} \left(\frac{\sqrt{\pi} c_2 e^{-\frac{a^2}{4b}} \operatorname{erfi}\left(\frac{2bx - a}{2\sqrt{b}} \right)}{\sqrt{b}} + 2c_1 \right)$$

27.5 problem 15

Internal problem ID [10839]

Book: Handbook of exact solutions for ordinary differential equations. By Polyanin and Zaitsev. Second edition
Section: Chapter 2, Second-Order Differential Equations. section 2.1.2-2 Equation of form $y'' + f(x)y' + g(x)y = 0$
Problem number: 15.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries], [_2nd_order, _linear,`

$$\boxed{y'' + ay' + bx\left(-x^3 b + ax + 2\right) y = 0}$$

✓ Solution by Maple
Time used: 0.0 (sec). Leaf size: 30

```
dsolve(diff(y(x),x$2)+a*diff(y(x),x)+b*x*(-b*x^3+a*x+2)*y(x)=0,y(x), singsol=all)
```

$$y(x) = \left(\left(\int \mathrm{e}^{-ax+\frac{2}{3}x^3 b} dx\right) c_1 + c_2\right) \mathrm{e}^{-\frac{x^3 b}{3}}$$

✓ Solution by Mathematica
Time used: 0.913 (sec). Leaf size: 46

```
DSolve[y''[x]+a*y'[x]+b*x*(-b*x^3+a*x+2)*y[x]==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to e^{-\frac{bx^3}{3}} \left(c_2 \int_1^x e^{\frac{2}{3}bK[1]^3 - aK[1]} dK[1] + c_1\right)$$

27.6 problem 16

Internal problem ID [10840]

Book: Handbook of exact solutions for ordinary differential equations. By Polyanin and Zaitsev. Second edition
Section: Chapter 2, Second-Order Differential Equations. section 2.1.2-2 Equation of form $y'' + f(x)y' + g(x)y = 0$
Problem number: 16.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries], [_2nd_order, _linear,`

$$\boxed{y'' + ay' + b\left(-b\,x^{2n} + a\,x^{n} + n\,x^{n-1}\right)y = 0}$$

✓ Solution by Maple
Time used: 0.0 (sec). Leaf size: 48

```
dsolve(diff(y(x),x$2)+a*diff(y(x),x)+b*(-b*x^(2*n)+a*x^n+n*x^(n-1))*y(x)=0,y(x), singsol=all
```

$$y(x) = \left(\left(\int \mathrm{e}^{\frac{2b\,x^{n+1}-xa(n+1)}{n+1}}dx\right)c_1 + c_2\right)\mathrm{e}^{-\frac{b\,x^{n+1}}{n+1}}$$

✗ Solution by Mathematica
Time used: 0.0 (sec). Leaf size: 0

```
DSolve[y''[x]+a*y'[x]+b*(-b*x^(2*n)+a*x^n+n*x^(n-1))*y[x]==0,y[x],x,IncludeSingularSolutions
```

Not solved

27.7 problem 17

Internal problem ID [10841]

Book: Handbook of exact solutions for ordinary differential equations. By Polyanin and Zaitsev. Second edition
Section: Chapter 2, Second-Order Differential Equations. section 2.1.2-2 Equation of form $y'' + f(x)y' + g(x)y = 0$
Problem number: 17.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$\boxed{y'' + ay' + b\left(-b\,x^{2n} - a\,x^{n} + n\,x^{n-1}\right)y = 0}$$

✓ Solution by Maple
Time used: 0.187 (sec). Leaf size: 51

```
dsolve(diff(y(x),x$2)+a*diff(y(x),x)+b*(-b*x^(2*n)-a*x^n+n*x^(n-1))*y(x)=0,y(x), singsol=all
```

$$y(x) = \mathrm{e}^{-\frac{x\left(b\,x^{n}+a(n+1)\right)}{n+1}}\left(c_1 + \left(\int \mathrm{e}^{\frac{x\left(2b\,x^{n}+a(n+1)\right)}{n+1}}dx\right)c_2\right)$$

✗ Solution by Mathematica
Time used: 0.0 (sec). Leaf size: 0

```
DSolve[y''[x]+a*y'[x]+b*(-b*x^(2*n)-a*x^n+n*x^(n-1))*y[x]==0,y[x],x,IncludeSingularSolutions
```

Not solved

27.8 problem 18

Internal problem ID [10842]

Book: Handbook of exact solutions for ordinary differential equations. By Polyanin and Zaitsev. Second edition
Section: Chapter 2, Second-Order Differential Equations. section 2.1.2-2 Equation of form $y'' + f(x)y' + g(x)y = 0$
Problem number: 18.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$\boxed{y'' + y'x + (n-1)\,y = 0}$$

✓ Solution by Maple
Time used: 0.062 (sec). Leaf size: 104

```
dsolve(diff(y(x),x$2)+x*diff(y(x),x)+(n-1)*y(x)=0,y(x), singsol=all)
```

$$y(x) = -\frac{\left(-2\left(-\frac{x^2}{2}+n+\frac{1}{2}\right)c_1 n\,\text{KummerM}\left(-\frac{n}{2}+\frac{1}{2},\frac{3}{2},\frac{x^2}{2}\right)+2(-x^2+2n+1)\,c_2\,\text{KummerU}\left(-\frac{n}{2}+\frac{1}{2},\frac{3}{2},\frac{x^2}{2}\right)\right.}{n\,(n-1)}$$

✓ Solution by Mathematica
Time used: 0.048 (sec). Leaf size: 51

```
DSolve[y''[x]+x*y'[x]+(n-1)*y[x]==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to e^{-\frac{x^2}{2}}\left(c_1\,\text{HermiteH}\left(n-2,\frac{x}{\sqrt{2}}\right)+c_2\,\text{Hypergeometric1F1}\left(1-\frac{n}{2},\frac{1}{2},\frac{x^2}{2}\right)\right)$$

27.9 problem 19

Internal problem ID [10843]

Book: Handbook of exact solutions for ordinary differential equations. By Polyanin and Zaitsev. Second edition
Section: Chapter 2, Second-Order Differential Equations. section 2.1.2-2 Equation of form $y'' + f(x)y' + g(x)y = 0$
Problem number: 19.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$\boxed{y'' - 2y'x + 2yn = 0}$$

✓ Solution by Maple
Time used: 0.047 (sec). Leaf size: 31

```
dsolve(diff(y(x),x$2)-2*x*diff(y(x),x)+2*n*y(x)=0,y(x), singsol=all)
```

$$y(x) = x\left(\text{KummerU}\left(-\frac{n}{2}+\frac{1}{2},\frac{3}{2},x^2\right)c_2 + \text{KummerM}\left(-\frac{n}{2}+\frac{1}{2},\frac{3}{2},x^2\right)c_1\right)$$

✓ Solution by Mathematica
Time used: 0.032 (sec). Leaf size: 27

```
DSolve[y''[x]-2*x*y'[x]+2*n*y[x]==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to c_1 \,\text{HermiteH}(n,x) + c_2 \,\text{Hypergeometric1F1}\left(-\frac{n}{2},\frac{1}{2},x^2\right)$$

27.10 problem 20

Internal problem ID [10844]

Book: Handbook of exact solutions for ordinary differential equations. By Polyanin and Zaitsev. Second edition
Section: Chapter 2, Second-Order Differential Equations. section 2.1.2-2 Equation of form $y'' + f(x)y' + g(x)y = 0$
Problem number: 20.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$\boxed{y'' + axy' + yb = 0}$$

✓ Solution by Maple
Time used: 0.062 (sec). Leaf size: 58

```
dsolve(diff(y(x),x$2)+a*x*diff(y(x),x)+b*y(x)=0,y(x), singsol=all)
```

$$y(x) = \mathrm{e}^{-\frac{a\,x^2}{2}} x \left(\mathrm{KummerM}\left(\frac{-b+2a}{2a}, \frac{3}{2}, \frac{a\,x^2}{2} \right) c_1 + \mathrm{KummerU}\left(\frac{-b+2a}{2a}, \frac{3}{2}, \frac{a\,x^2}{2} \right) c_2 \right)$$

✓ Solution by Mathematica
Time used: 0.061 (sec). Leaf size: 67

```
DSolve[y''[x]+a*x*y'[x]+b*y[x]==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to e^{-\frac{ax^2}{2}} \left(c_1 \,\mathrm{HermiteH}\left(\frac{b}{a} - 1, \frac{\sqrt{a}x}{\sqrt{2}} \right) + c_2 \,\mathrm{Hypergeometric1F1}\left(\frac{a-b}{2a}, \frac{1}{2}, \frac{ax^2}{2} \right) \right)$$

27.11 problem 21

Internal problem ID [10845]

Book: Handbook of exact solutions for ordinary differential equations. By Polyanin and Zaitsev. Second edition
Section: Chapter 2, Second-Order Differential Equations. section 2.1.2-2 Equation of form $y'' + f(x)y' + g(x)y = 0$
Problem number: 21.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$\boxed{y'' + axy' + bxy = 0}$$

✓ Solution by Maple
Time used: 0.031 (sec). Leaf size: 70

```
dsolve(diff(y(x),x$2)+a*x*diff(y(x),x)+b*x*y(x)=0,y(x), singsol=all)
```

$$y(x) = \mathrm{e}^{-\frac{bx}{a}} \left(\mathrm{KummerM}\left(\frac{b^2}{2a^3}, \frac{1}{2}, -\frac{(a^2x - 2b)^2}{2a^3} \right) c_1 + \mathrm{KummerU}\left(\frac{b^2}{2a^3}, \frac{1}{2}, -\frac{(a^2x - 2b)^2}{2a^3} \right) c_2 \right)$$

✓ Solution by Mathematica
Time used: 0.106 (sec). Leaf size: 96

```
DSolve[y''[x]+a*x*y'[x]+b*x*y[x]==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to e^{\frac{bx}{a} - \frac{ax^2}{2}} \left(c_2 \,\mathrm{Hypergeometric1F1}\left(\frac{1}{2} - \frac{b^2}{2a^3}, \frac{1}{2}, \frac{(a^2x - 2b)^2}{2a^3} \right) + c_1 \,\mathrm{HermiteH}\left(\frac{b^2}{a^3} - 1, \frac{a^2x - 2b}{\sqrt{2}a^{3/2}} \right) \right)$$

27.12 problem 22

Internal problem ID [10846]

Book: Handbook of exact solutions for ordinary differential equations. By Polyanin and Zaitsev. Second edition
Section: Chapter 2, Second-Order Differential Equations. section 2.1.2-2 Equation of form $y'' + f(x)y' + g(x)y = 0$
Problem number: 22.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$\boxed{y'' + axy' + (bx + c)\, y = 0}$$

✓ Solution by Maple
Time used: 0.031 (sec). Leaf size: 82

```
dsolve(diff(y(x),x$2)+a*x*diff(y(x),x)+(b*x+c)*y(x)=0,y(x), singsol=all)
```

$$y(x) = \mathrm{e}^{-\frac{bx}{a}} \left(\mathrm{KummerM}\left(\frac{a^2 c + b^2}{2a^3}, \frac{1}{2}, -\frac{(a^2 x - 2b)^2}{2a^3} \right) c_1 + \mathrm{KummerU}\left(\frac{a^2 c + b^2}{2a^3}, \frac{1}{2}, -\frac{(a^2 x - 2b)^2}{2a^3} \right) c_2 \right)$$

✓ Solution by Mathematica
Time used: 0.085 (sec). Leaf size: 108

```
DSolve[y''[x]+a*x*y'[x]+(b*x+c)*y[x]==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to e^{\frac{bx}{a} - \frac{ax^2}{2}} \left(c_2 \,\mathrm{Hypergeometric1F1}\left(-\frac{-a^3 + ca^2 + b^2}{2a^3}, \frac{1}{2}, \frac{(a^2 x - 2b)^2}{2a^3} \right) + c_1 \,\mathrm{HermiteH}\left(\frac{b^2}{a^3} + \frac{c}{a} - 1, \frac{a^2 x - 2b}{\sqrt{2} a^{3/2}} \right) \right)$$

27.13 problem 23

Internal problem ID [10847]

Book: Handbook of exact solutions for ordinary differential equations. By Polyanin and Zaitsev. Second edition
Section: Chapter 2, Second-Order Differential Equations. section 2.1.2-2 Equation of form $y'' + f(x)y' + g(x)y = 0$
Problem number: 23.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$\boxed{y'' + 2axy' + \left(b\,x^4 + a^2x^2 + cx + a\right)y = 0}$$

✓ Solution by Maple
Time used: 0.14 (sec). Leaf size: 80

```
dsolve(diff(y(x),x$2)+2*a*x*diff(y(x),x)+(b*x^4+a^2*x^2+c*x+a)*y(x)=0,y(x), singsol=all)
```

$$y(x) = \mathrm{e}^{-\frac{\left(i\sqrt{b}\,x+\frac{3a}{2}\right)x^2}{3}} x \left(\mathrm{KummerM}\left(\frac{ic+4\sqrt{b}}{6\sqrt{b}}, \frac{4}{3}, \frac{2i\sqrt{b}\,x^3}{3}\right) c_1 + \mathrm{KummerU}\left(\frac{ic+4\sqrt{b}}{6\sqrt{b}}, \frac{4}{3}, \frac{2i\sqrt{b}\,x^3}{3}\right) c_2 \right)$$

✓ Solution by Mathematica
Time used: 0.322 (sec). Leaf size: 121

```
DSolve[y''[x]+2*a*x*y'[x]+(b*x^4+a^2*x^2+c*x+a)*y[x]==0,y[x],x,IncludeSingularSolutions -> T
```

$$y(x) \to \frac{\sqrt[3]{2}\sqrt[3]{x^3}\,e^{\frac{1}{6}ix^2\left(2\sqrt{b}x+3ia\right)}\left(c_1\,\mathrm{HypergeometricU}\left(\frac{1}{3}-\frac{ic}{6\sqrt{b}}, \frac{2}{3}, -\frac{2}{3}i\sqrt{b}x^3\right) + c_2 L^{-\frac{1}{3}}_{\frac{ic}{6\sqrt{b}}-\frac{1}{3}}\left(-\frac{2}{3}i\sqrt{b}x^3\right)\right)}{x}$$

27.14 problem 24

Internal problem ID [10848]

Book: Handbook of exact solutions for ordinary differential equations. By Polyanin and Zaitsev. Second edition
Section: Chapter 2, Second-Order Differential Equations. section 2.1.2-2 Equation of form $y'' + f(x)y' + g(x)y = 0$
Problem number: 24.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$\boxed{y'' + (ax+b)\,y' - ay = 0}$$

✓ Solution by Maple
Time used: 0.0 (sec). Leaf size: 65

```
dsolve(diff(y(x),x$2)+(a*x+b)*diff(y(x),x)-a*y(x)=0,y(x), singsol=all)
```

$$y(x) = \mathrm{e}^{\frac{b^2}{2a}} \pi c_2 (ax+b) \operatorname{erf}\left(\frac{\sqrt{2}\,(ax+b)}{2\sqrt{a}}\right) + \sqrt{\pi}\,\sqrt{2}\,\sqrt{a}\,\mathrm{e}^{-\frac{x(ax+2b)}{2}} c_2 + c_1 (ax+b)$$

✓ Solution by Mathematica
Time used: 0.959 (sec). Leaf size: 82

```
DSolve[y''[x]+(a*x+b)*y'[x]-a*y[x]==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to \frac{(ax+b)\left(-\frac{\sqrt{\frac{\pi}{2}}c_2 \operatorname{erf}\left(\frac{ax+b}{\sqrt{2}\sqrt{a}}\right)}{a^{3/2}} - \frac{c_2 e^{-\frac{(ax+b)^2}{2a}}}{a(ax+b)} + c_1\right)}{b}$$

27.15 problem 25

Internal problem ID [10849]

Book: Handbook of exact solutions for ordinary differential equations. By Polyanin and Zaitsev. Second edition
Section: Chapter 2, Second-Order Differential Equations. section 2.1.2-2 Equation of form $y'' + f(x)y' + g(x)y = 0$
Problem number: 25.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _exact, _linear, _homogeneous]]`

$$\boxed{y'' + (ax+b)\,y' + ay = 0}$$

✓ Solution by Maple
Time used: 0.0 (sec). Leaf size: 36

```
dsolve(diff(y(x),x$2)+(a*x+b)*diff(y(x),x)+a*y(x)=0,y(x), singsol=all)
```

$$y(x) = \left(\operatorname{erf}\left(\frac{(ax+b)\sqrt{2}}{2\sqrt{-a}} \right) c_1 + c_2 \right) \mathrm{e}^{-\frac{x(ax+2b)}{2}}$$

✓ Solution by Mathematica
Time used: 0.067 (sec). Leaf size: 79

```
DSolve[y''[x]+(a*x+b)*y'[x]+a*y[x]==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to \frac{e^{-\frac{(ax+b)^2}{2a}} \left(2\sqrt{a} c_2 e^{\frac{b^2}{2a}} + \sqrt{2\pi} c_1 \operatorname{erfi}\left(\frac{ax+b}{\sqrt{2}\sqrt{a}} \right) \right)}{2\sqrt{a}}$$

27.16 problem 26

Internal problem ID [10850]

Book: Handbook of exact solutions for ordinary differential equations. By Polyanin and Zaitsev. Second edition
Section: Chapter 2, Second-Order Differential Equations. section 2.1.2-2 Equation of form $y'' + f(x)y' + g(x)y = 0$
Problem number: 26.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$\boxed{y'' + (ax+b)\,y' + c(ax+b-c)\,y = 0}$$

✓ Solution by Maple
Time used: 0.0 (sec). Leaf size: 31

```
dsolve(diff(y(x),x$2)+(a*x+b)*diff(y(x),x)+c*(a*x+b-c)*y(x)=0,y(x), singsol=all)
```

$$y(x) = \mathrm{e}^{-cx}\left(c_1 + \operatorname{erf}\left(\frac{\sqrt{2}\,(ax+b-2c)}{2\sqrt{a}}\right)c_2\right)$$

✓ Solution by Mathematica
Time used: 0.091 (sec). Leaf size: 70

```
DSolve[y''[x]+(a*x+b)*y'[x]+c*(a*x+b-c)*y[x]==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to e^{-\frac{1}{2}x(ax+2b-2c)}\left(c_1\,\text{HermiteH}\left(-1, \frac{b-2c+ax}{\sqrt{2}\sqrt{a}}\right) + c_2 e^{\frac{(ax+b-2c)^2}{2a}}\right)$$

27.17 problem 27

Internal problem ID [10851]

Book: Handbook of exact solutions for ordinary differential equations. By Polyanin and Zaitsev. Second edition
Section: Chapter 2, Second-Order Differential Equations. section 2.1.2-2 Equation of form $y'' + f(x)y' + g(x)y = 0$
Problem number: 27.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$\boxed{y'' + (ax + 2b)\, y' + \left(abx + b^2 - a\right) y = 0}$$

✓ Solution by Maple
Time used: 0.0 (sec). Leaf size: 49

```
dsolve(diff(y(x),x$2)+(a*x+2*b)*diff(y(x),x)+(a*b*x-a+b^2)*y(x)=0,y(x), singsol=all)
```

$$y(x) = 2\,\mathrm{e}^{-\frac{x(ax+2b)}{2}} c_2 + \mathrm{e}^{-bx} x \left(c_2 \sqrt{a}\, \sqrt{\pi}\, \sqrt{2}\, \operatorname{erf}\left(\frac{\sqrt{2}\, \sqrt{a}\, x}{2} \right) + c_1 \right)$$

✓ Solution by Mathematica
Time used: 0.405 (sec). Leaf size: 64

```
DSolve[y''[x]+(a*x+2*b)*y'[x]+(a*b*x-a+b^2)*y[x]==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to xe^{-bx} \left(-\sqrt{\frac{\pi}{2}} \sqrt{a} c_2 \operatorname{erf}\left(\frac{\sqrt{a}x}{\sqrt{2}} \right) - \frac{c_2 e^{-\frac{ax^2}{2}}}{x} + c_1 \right)$$

27.18 problem 28

Internal problem ID [10852]

Book: Handbook of exact solutions for ordinary differential equations. By Polyanin and Zaitsev. Second edition
Section: Chapter 2, Second-Order Differential Equations. section 2.1.2-2 Equation of form $y'' + f(x)y' + g(x)y = 0$
Problem number: 28.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$\boxed{y'' + (ax+b)\,y' + (cx+d)\,y = 0}$$

✓ Solution by Maple
Time used: 0.031 (sec). Leaf size: 98

```
dsolve(diff(y(x),x$2)+(a*x+b)*diff(y(x),x)+(c*x+d)*y(x)=0,y(x), singsol=all)
```

$$y(x) = \mathrm{e}^{-\frac{cx}{a}} \left(\mathrm{KummerM}\left(\frac{d\,a^2 - abc + c^2}{2a^3}, \frac{1}{2}, -\frac{\left(a^2 x + ab - 2c\right)^2}{2a^3} \right) c_1 + \mathrm{KummerU}\left(\frac{d\,a^2 - abc + c^2}{2a^3}, \frac{1}{2}, -\frac{\left(a^2 x + ab - 2c\right)^2}{2a^3} \right) c_2 \right)$$

✓ Solution by Mathematica
Time used: 0.116 (sec). Leaf size: 132

```
DSolve[y''[x]+(a*x+b)*y'[x]+(c*x+d)*y[x]==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to e^{\frac{cx}{a} - \frac{ax^2}{2} - bx} \left(c_2\, \mathrm{Hypergeometric1F1}\left(\frac{a^3 - da^2 + bca - c^2}{2a^3}, \frac{1}{2}, \frac{\left(xa^2 + ba - 2c\right)^2}{2a^3} \right) + c_1\, \mathrm{HermiteH}\left(\frac{-a^3 + da^2 - bca + c^2}{a^3}, \frac{xa^2 + ba - 2c}{\sqrt{2}a^{3/2}} \right) \right)$$

27.19 problem 29

Internal problem ID [10853]

Book: Handbook of exact solutions for ordinary differential equations. By Polyanin and Zaitsev. Second edition
Section: Chapter 2, Second-Order Differential Equations. section 2.1.2-2 Equation of form $y'' + f(x)y' + g(x)y = 0$
Problem number: 29.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$\boxed{y'' + (ax+b)\,y' + c\big((a-c)\,x^2 + bx + 1\big)\,y = 0}$$

✓ Solution by Maple
Time used: 0.0 (sec). Leaf size: 36

```
dsolve(diff(y(x),x$2)+(a*x+b)*diff(y(x),x)+c*((a-c)*x^2+b*x+1)*y(x)=0,y(x), singsol=all)
```

$$y(x) = \mathrm{e}^{-\frac{c\,x^2}{2}}\left(c_1 + \operatorname{erf}\left(\frac{(-2c+a)\,x+b}{\sqrt{2a-4c}}\right)c_2\right)$$

✓ Solution by Mathematica
Time used: 0.135 (sec). Leaf size: 81

```
DSolve[y''[x]+(a*x+b)*y'[x]+c*((a-c)*x^2+b*x+1)*y[x]==0,y[x],x,IncludeSingularSolutions -> T
```

$$y(x) \to e^{-\frac{1}{2}x(x(a-c)+2b)}\left(c_1\,\text{HermiteH}\left(-1,\frac{b+(a-2c)x}{\sqrt{2}\sqrt{a-2c}}\right) + c_2 e^{\frac{(x(a-2c)+b)^2}{2(a-2c)}}\right)$$

27.20 problem 30

Internal problem ID [10854]

Book: Handbook of exact solutions for ordinary differential equations. By Polyanin and Zaitsev. Second edition
Section: Chapter 2, Second-Order Differential Equations. section 2.1.2-2 Equation of form $y'' + f(x)y' + g(x)y = 0$
Problem number: 30.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$\boxed{y'' + 2(ax+b)\,y' + \left(a^2x^2 + 2abx + c\right)y = 0}$$

✓ Solution by Maple
Time used: 0.0 (sec). Leaf size: 55

```
dsolve(diff(y(x),x$2)+2*(a*x+b)*diff(y(x),x)+(a^2*x^2+2*a*b*x+c)*y(x)=0,y(x), singsol=all)
```

$$y(x) = c_1 \mathrm{e}^{-\frac{x\left(ax-2\sqrt{b^2+a-c}+2b\right)}{2}} + c_2 \mathrm{e}^{-\frac{x\left(ax+2\sqrt{b^2+a-c}+2b\right)}{2}}$$

✓ Solution by Mathematica
Time used: 0.228 (sec). Leaf size: 86

```
DSolve[y''[x]+2*(a*x+b)*y'[x]+(a^2*x^2+2*a*b*x+c)*y[x]==0,y[x],x,IncludeSingularSolutions ->
```

$$y(x) \to \frac{e^{-\frac{1}{2}x\left(2\sqrt{a+b^2-c}+ax+2b\right)}\left(c_2 e^{2x\sqrt{a+b^2-c}} + 2c_1\sqrt{a+b^2-c}\right)}{2\sqrt{a+b^2-c}}$$

27.21 problem 31

Internal problem ID [10855]

Book: Handbook of exact solutions for ordinary differential equations. By Polyanin and Zaitsev. Second edition
Section: Chapter 2, Second-Order Differential Equations. section 2.1.2-2 Equation of form $y'' + f(x)y' + g(x)y = 0$
Problem number: 31.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$\boxed{y'' + (ax+b)\,y' + \left(\alpha\,x^2 + \beta x + \gamma\right) y = 0}$$

✓ Solution by Maple
Time used: 0.046 (sec). Leaf size: 254

```
dsolve(diff(y(x),x$2)+(a*x+b)*diff(y(x),x)+(alpha*x^2+beta*x+gamma)*y(x)=0,y(x), singsol=all
```

$$y(x) = \mathrm{e}^{-\frac{x\left((ax+2b)\sqrt{a^2-4\alpha}+x\left(a^2-4\alpha\right)+2ab-4\beta\right)}{4\sqrt{a^2-4\alpha}}}\left(c_2\left(a^2x+ab-4\alpha x\right.\right.$$
$$\left.-2\beta\right)\,\mathrm{hypergeom}\left(\left[\frac{3(a^2-4\alpha)^{\frac{3}{2}}+a^3-2a^2\gamma+2(b\beta-2\alpha)\,a+2(-b^2+4\gamma)\,\alpha-2\beta^2}{4\,(a^2-4\alpha)^{\frac{3}{2}}}\right],\left[\frac{3}{2}\right],\frac{(a^2x+a}{2\,(}\right.$$
$$\left.+\mathrm{hypergeom}\left(\left[\frac{(a^2-4\alpha)^{\frac{3}{2}}+a^3-2a^2\gamma+(2b\beta-4\alpha)\,a+(-2b^2+8\gamma)\,\alpha-2\beta^2}{4\,(a^2-4\alpha)^{\frac{3}{2}}}\right],\left[\frac{1}{2}\right],\frac{(a^2x+ab-4}{2\,(a^2-4}\right.\right.$$

✓ Solution by Mathematica
Time used: 0.467 (sec). Leaf size: 307

```
DSolve[y''[x]+(a*x+b)*y'[x]+(\[Alpha]*x^2+\[Beta]*x+\[Gamma])*y[x]==0,y[x],x,IncludeSingular
```

$$y(x)$$
$$\to \exp\left(-\frac{x\left(2b\sqrt{a^2-4\alpha}+a\left(x\sqrt{a^2-4\alpha}+2b\right)+a^2x-4(\beta+\alpha x)\right)}{4\sqrt{a^2-4\alpha}}\right)\left(c_1\,\mathrm{HermiteH}\left(\frac{-a^3-\left(\sqrt{a^2-4\alpha}\right.\right.\right.$$

27.22 problem 32

Internal problem ID [10856]

Book: Handbook of exact solutions for ordinary differential equations. By Polyanin and Zaitsev. Second edition
Section: Chapter 2, Second-Order Differential Equations. section 2.1.2-2 Equation of form $y'' + f(x)y' + g(x)y = 0$
Problem number: 32.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$y'' + (ax+b)\,y' + c\left(-c\,x^{2n} + a\,x^{1+n} + b\,x^{n} + n\,x^{n-1}\right) y = 0$$

✗ Solution by Maple

```
dsolve(diff(y(x),x$2)+(a*x+b)*diff(y(x),x)+c*(-c*x^(2*n)+a*x^(n+1)+b*x^n+n*x^(n-1) )*y(x)=0,
```

No solution found

✗ Solution by Mathematica
Time used: 0.0 (sec). Leaf size: 0

```
DSolve[y''[x]+(a*x+b)*y'[x]+c*(-c*x^(2*n)+a*x^(n+1)+b*x^n+n*x^(n-1) )*y[x]==0,y[x],x,Include
```

Not solved

27.23 problem 33

Internal problem ID [10857]

Book: Handbook of exact solutions for ordinary differential equations. By Polyanin and Zaitsev. Second edition
Section: Chapter 2, Second-Order Differential Equations. section 2.1.2-2 Equation of form $y'' + f(x)y' + g(x)y = 0$
Problem number: 33.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$\boxed{y'' + a\left(-b^2 + x^2\right) y' - a(x+b)\, y = 0}$$

✓ Solution by Maple
Time used: 0.297 (sec). Leaf size: 141

```
dsolve(diff(y(x),x$2)+a*(x^2-b^2)*diff(y(x),x)-a*(x+b)*y(x)=0,y(x), singsol=all)
```

$$y(x) = c_1 \operatorname{HeunT}\left(-\frac{a3^{\frac{2}{3}}b}{(a^2)^{\frac{1}{3}}}, -6\ \operatorname{csgn}(a), -\frac{a^2b^2 3^{\frac{1}{3}}}{(a^2)^{\frac{2}{3}}}, \frac{3^{\frac{2}{3}}(a^2)^{\frac{1}{6}}\, x}{3}\right) \mathrm{e}^{\frac{x\left(3b^2-x^2\right)\operatorname{csgn}(a)a(\operatorname{csgn}(a)+1)}{6}}$$
$$+ c_2 \operatorname{HeunT}\left(-\frac{a3^{\frac{2}{3}}b}{(a^2)^{\frac{1}{3}}}, 6\ \operatorname{csgn}(a), -\frac{a^2b^2 3^{\frac{1}{3}}}{(a^2)^{\frac{2}{3}}}, -\frac{3^{\frac{2}{3}}(a^2)^{\frac{1}{6}}\, x}{3}\right) \mathrm{e}^{\frac{x\left(3b^2-x^2\right)\operatorname{csgn}(a)a(\operatorname{csgn}(a)-1)}{6}}$$

✓ Solution by Mathematica
Time used: 3.893 (sec). Leaf size: 55

```
DSolve[y''[x]+a*(x^2-b^2)*y'[x]-a*(x+b)*y[x]==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to \frac{(b-x)\left(c_2 \int_1^x \frac{e^{ab^2 K[1] - \frac{1}{3}aK[1]^3}}{(b-K[1])^2} dK[1] + c_1\right)}{b}$$

27.24 problem 34

Internal problem ID [10858]

Book: Handbook of exact solutions for ordinary differential equations. By Polyanin and Zaitsev. Second edition
Section: Chapter 2, Second-Order Differential Equations. section 2.1.2-2 Equation of form $y'' + f(x)y' + g(x)y = 0$
Problem number: 34.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$\boxed{y'' + \left(a\,x^2 + b\right) y' + c\left(a\,x^2 + b - c\right) y = 0}$$

✓ Solution by Maple
Time used: 0.203 (sec). Leaf size: 134

```
dsolve(diff(y(x),x$2)+(a*x^2+b)*diff(y(x),x)+c*(a*x^2+b-c)*y(x)=0,y(x), singsol=all)
```

$$y(x) = c_1 \,\mathrm{HeunT}\left(0, \right.$$
$$\left. -3\ \mathrm{csgn}\left(a\right), \frac{a(b-2c)\,3^{\frac{1}{3}}}{\left(a^2\right)^{\frac{2}{3}}}, \frac{3^{\frac{2}{3}}\left(a^2\right)^{\frac{1}{6}} x}{3}\right) \mathrm{e}^{-\frac{x\left(\left(a\,x^2+3b\right)\mathrm{csgn}(a)+a\,x^2+3b-6c\right)\mathrm{csgn}(a)}{6}}$$
$$+ c_2 \,\mathrm{HeunT}\left(0, 3\ \mathrm{csgn}\left(a\right), \frac{a(b-2c)\,3^{\frac{1}{3}}}{\left(a^2\right)^{\frac{2}{3}}}, \right.$$
$$\left. -\frac{3^{\frac{2}{3}}\left(a^2\right)^{\frac{1}{6}} x}{3}\right) \mathrm{e}^{-\frac{x\left(\left(a\,x^2+3b\right)\mathrm{csgn}(a)-a\,x^2-3b+6c\right)\mathrm{csgn}(a)}{6}}$$

✓ Solution by Mathematica
Time used: 0.915 (sec). Leaf size: 46

```
DSolve[y''[x]+(a*x^2+b)*y'[x]+c*(a*x^2+b-c)*y[x]==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to e^{-cx}\left(c_2 \int_1^x e^{-\frac{1}{3}K[1]\left(aK[1]^2+3b-6c\right)} dK[1] + c_1\right)$$

27.25 problem 35

Internal problem ID [10859]

Book: Handbook of exact solutions for ordinary differential equations. By Polyanin and Zaitsev. Second edition
Section: Chapter 2, Second-Order Differential Equations. section 2.1.2-2 Equation of form $y'' + f(x)y' + g(x)y = 0$
Problem number: 35.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$\boxed{y'' + \left(a\,x^2 + 2b\right) y' + \left(ab\,x^2 - ax + b^2\right) y = 0}$$

✓ Solution by Maple
Time used: 0.0 (sec). Leaf size: 91

```
dsolve(diff(y(x),x$2)+(a*x^2+2*b)*diff(y(x),x)+(a*b*x^2-a*x+b^2)*y(x)=0,y(x), singsol=all)
```

$$y(x) = \frac{5\left(3^{\frac{2}{3}} c_2 a (a\,x^3)^{\frac{1}{3}} \left(a\,x^3+2\right) \mathrm{e}^{-\frac{x\left(a\,x^2+6b\right)}{6}} + \frac{9x^2\left(c_2 a^2 x\,\mathrm{e}^{-bx}\,\mathrm{WhittakerM}\left(\frac{1}{3},\frac{5}{6},\frac{a\,x^3}{3}\right) + c_1 \mathrm{e}^{\frac{x\left(a\,x^2-6b\right)}{6}}\right)}{5}\right) \mathrm{e}^{-\frac{a\,x^3}{6}}}{9x}$$

✓ Solution by Mathematica
Time used: 0.407 (sec). Leaf size: 51

```
DSolve[y''[x]+(a*x^2+2*b)*y'[x]+(a*b*x^2-a*x+b^2)*y[x]==0,y[x],x,IncludeSingularSolutions ->
```

$$y(x) \to \frac{1}{9} e^{-bx} \left(9c_1 x - 3^{2/3} c_2 \sqrt[3]{a x^3}\,\Gamma\left(-\frac{1}{3}, \frac{a x^3}{3}\right)\right)$$

27.26 problem 36

Internal problem ID [10860]

Book: Handbook of exact solutions for ordinary differential equations. By Polyanin and Zaitsev. Second edition
Section: Chapter 2, Second-Order Differential Equations. section 2.1.2-2 Equation of form $y'' + f(x)y' + g(x)y = 0$
Problem number: 36.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$\boxed{y'' + \left(2x^2 + a\right) y' + \left(x^4 + a\,x^2 + b + 2x\right) y = 0}$$

✓ Solution by Maple
Time used: 0.0 (sec). Leaf size: 57

```
dsolve(diff(y(x),x$2)+(2*x^2+a)*diff(y(x),x)+(x^4+a*x^2+2*x+b)*y(x)=0,y(x), singsol=all)
```

$$y(x) = c_1 \mathrm{e}^{\frac{x\left(-2x^2+3\sqrt{a^2-4b}-3a\right)}{6}} + c_2 \mathrm{e}^{-\frac{x\left(2x^2+3\sqrt{a^2-4b}+3a\right)}{6}}$$

✓ Solution by Mathematica
Time used: 0.217 (sec). Leaf size: 79

```
DSolve[y''[x]+(2*x^2+a)*y'[x]+(x^4+a*x^2+2*x+b)*y[x]==0,y[x],x,IncludeSingularSolutions -> T
```

$$y(x) \to \frac{e^{-\frac{1}{6}x\left(3\sqrt{a^2-4b}+3a+2x^2\right)}\left(c_2 e^{x\sqrt{a^2-4b}} + c_1\sqrt{a^2-4b}\right)}{\sqrt{a^2-4b}}$$

27.27 problem 37

Internal problem ID [10861]

Book: Handbook of exact solutions for ordinary differential equations. By Polyanin and Zaitsev. Second edition
Section: Chapter 2, Second-Order Differential Equations. section 2.1.2-2 Equation of form $y'' + f(x)y' + g(x)y = 0$
Problem number: 37.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$\boxed{y'' + \left(a\,x^2 + bx\right) y' + \left(\alpha\,x^2 + \beta x + \gamma\right) y = 0}$$

✓ Solution by Maple
Time used: 0.265 (sec). Leaf size: 271

```
dsolve(diff(y(x),x$2)+(a*x^2+b*x)*diff(y(x),x)+(alpha*x^2+beta*x+gamma)*y(x)=0,y(x), singsol
```

$$y(x) = c_1 \mathrm{e}^{-\frac{\operatorname{csgn}(a) x \left(2a^2x^2 \operatorname{csgn}(a)+3abx\ \operatorname{csgn}(a)+2a^2x^2+3abx-12\alpha\right)}{12a}} \operatorname{HeunT}\left(\frac{3^{\frac{2}{3}}(2a^2\gamma - ab\beta + \alpha\,b^2 + 2\alpha^2)}{2a^2\,(a^2)^{\frac{1}{3}}}, -\frac{3(a^2 - \beta a + b\alpha)\operatorname{csgn}(a)}{a^2}, -\frac{3^{\frac{1}{3}}(b^2+8\alpha)}{4\,(a^2)^{\frac{2}{3}}}, \frac{3^{\frac{2}{3}}a(2ax+b)}{6\,(a^2)^{\frac{5}{6}}}\right) + c_2 \mathrm{e}^{-\frac{\operatorname{csgn}(a) x \left(2a^2x^2 \operatorname{csgn}(a)+3abx\ \operatorname{csgn}(a)-2a^2x^2-3abx+12\alpha\right)}{12a}} \operatorname{HeunT}\left(\frac{3^{\frac{2}{3}}(2a^2\gamma - ab\beta + \alpha\,b^2 + 2\alpha^2)}{2a^2\,(a^2)^{\frac{1}{3}}}, \frac{3(a^2 - \beta a + b\alpha)}{a^2}, -\frac{3^{\frac{1}{3}}(b^2+8\alpha)}{4\,(a^2)^{\frac{2}{3}}}, -\frac{3^{\frac{2}{3}}a(2ax+b)}{6\,(a^2)^{\frac{5}{6}}}\right)$$

✗ Solution by Mathematica
Time used: 0.0 (sec). Leaf size: 0

```
DSolve[y''[x]+(a*x^2+b*x)*y'[x]+(\[Alpha]*x^2+\[Beta]*x+\[Gamma])*y[x]==0,y[x],x,IncludeSing
```

Not solved

27.28 problem 38

Internal problem ID [10862]

Book: Handbook of exact solutions for ordinary differential equations. By Polyanin and Zaitsev. Second edition
Section: Chapter 2, Second-Order Differential Equations. section 2.1.2-2 Equation of form $y'' + f(x)y' + g(x)y = 0$
Problem number: 38.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$\boxed{y'' + \left(ab\,x^2 + bx + 2a\right)y' + a^2\left(b\,x^2 + 1\right)y = 0}$$

✓ Solution by Maple
Time used: 0.36 (sec). Leaf size: 294

```
dsolve(diff(y(x),x$2)+(a*b*x^2+b*x+2*a)*diff(y(x),x)+a^2*(b*x^2+1)*y(x)=0,y(x), singsol=all)
```

$$y(x) = c_1 \mathrm{e}^{-\frac{x\left(2a^2b^2x^2+2ab\,x^2\sqrt{a^2b^2}+3a\,b^2x+3bx\sqrt{a^2b^2}+12a\sqrt{a^2b^2}\right)}{12\sqrt{a^2b^2}}} \operatorname{HeunT}\left(\frac{b3^{\frac{2}{3}}}{2\left(a^2b^2\right)^{\frac{1}{3}}}, -\frac{6ab}{\sqrt{a^2b^2}}, -\frac{b^2 3^{\frac{1}{3}}}{4\left(a^2b^2\right)^{\frac{2}{3}}}, \frac{3^{\frac{2}{3}}a\,b^2(2ax+1)}{6\left(a^2b^2\right)^{\frac{5}{6}}}\right)$$
$$+ c_2 \mathrm{e}^{-\frac{\left(-2a^2b^2x^2+2ab\,x^2\sqrt{a^2b^2}-3a\,b^2x+3bx\sqrt{a^2b^2}+12a\sqrt{a^2b^2}\right)x}{12\sqrt{a^2b^2}}} \operatorname{HeunT}\left(\frac{b3^{\frac{2}{3}}}{2\left(a^2b^2\right)^{\frac{1}{3}}}, \frac{6ab}{\sqrt{a^2b^2}}, -\frac{b^2 3^{\frac{1}{3}}}{4\left(a^2b^2\right)^{\frac{2}{3}}}, -\frac{\left(ax+\frac{1}{2}\right)3^{\frac{2}{3}}b^2a}{3\left(a^2b^2\right)^{\frac{5}{6}}}\right)$$

✓ Solution by Mathematica

Time used: 2.136 (sec). Leaf size: 57

```
DSolve[y''[x]+(a*b*x^2+b*x+2*a)*y'[x]+a^2*(b*x^2+1)*y[x]==0,y[x],x,IncludeSingularSolutions
```

$$y(x) \to e^{-ax}(ax+1)\left(c_2 \int_1^x \frac{e^{-\frac{1}{6}bK[1]^2(2aK[1]+3)}}{(aK[1]+1)^2} dK[1] + c_1\right)$$

27.29 problem 39

Internal problem ID [10863]

Book: Handbook of exact solutions for ordinary differential equations. By Polyanin and Zaitsev. Second edition
Section: Chapter 2, Second-Order Differential Equations. section 2.1.2-2 Equation of form $y'' + f(x)y' + g(x)y = 0$
Problem number: 39.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$\boxed{y'' + \left(a\,x^2 + bx + c\right) y' + x\left(ab\,x^2 + bc + 2a\right) y = 0}$$

✓ Solution by Maple
Time used: 0.344 (sec). Leaf size: 169

```
dsolve(diff(y(x),x$2)+(a*x^2+b*x+c)*diff(y(x),x)+x*(a*b*x^2+b*c+2*a)*y(x)=0,y(x), singsol=al
```

$$y(x) = c_1 \mathrm{e}^{-\frac{x\ \mathrm{csgn}(a)\left(\left(a\,x^2+\frac{3}{2}bx+3c\right)\mathrm{csgn}(a)+a\,x^2-\frac{3bx}{2}+3c\right)}{6}} \mathrm{HeunT}\left(0, 3\ \mathrm{csgn}\left(a\right), \frac{3^{\frac{1}{3}}(4ac-b^2)}{4\left(a^2\right)^{\frac{2}{3}}}, \frac{3^{\frac{2}{3}}a(2ax-b)}{6\left(a^2\right)^{\frac{5}{6}}}\right)$$
$$+ c_2 \mathrm{e}^{-\frac{x\left(\left(a\,x^2+\frac{3}{2}bx+3c\right)\mathrm{csgn}(a)-a\,x^2+\frac{3bx}{2}-3c\right)\mathrm{csgn}(a)}{6}} \mathrm{HeunT}\left(0, -3\ \mathrm{csgn}\left(a\right), \frac{3^{\frac{1}{3}}(4ac-b^2)}{4\left(a^2\right)^{\frac{2}{3}}}, -\frac{3^{\frac{2}{3}}\left(ax-\frac{b}{2}\right)a}{3\left(a^2\right)^{\frac{5}{6}}}\right)$$

✓ Solution by Mathematica
Time used: 1.085 (sec). Leaf size: 59

```
DSolve[y''[x]+(a*x^2+b*x+c)*y'[x]+x*(a*b*x^2+b*c+2*a)*y[x]==0,y[x],x,IncludeSingularSolution
```

$$y(x) \to e^{-\frac{1}{3}x(ax^2+3c)}\left(c_2 \int_1^x \exp\left(\frac{1}{6}K[1](6c + K[1](2aK[1] - 3b))\right) dK[1] + c_1\right)$$

27.30 problem 40

Internal problem ID [10864]

Book: Handbook of exact solutions for ordinary differential equations. By Polyanin and Zaitsev. Second edition
Section: Chapter 2, Second-Order Differential Equations. section 2.1.2-2 Equation of form $y'' + f(x)y' + g(x)y = 0$
Problem number: 40.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$\boxed{y'' + \left(a\,x^2 + bx + c\right) y' + \left(ab\,x^3 + ac\,x^2 + b\right) y = 0}$$

✓ Solution by Maple
Time used: 0.344 (sec). Leaf size: 165

```
dsolve(diff(y(x),x$2)+(a*x^2+b*x+c)*diff(y(x),x)+(a*b*x^3+a*c*x^2+b)*y(x)=0,y(x), singsol=al
```

$$y(x) = c_1 \mathrm{e}^{-\frac{x\ \mathrm{csgn}(a)\left(\left(a\,x^2+\frac{3}{2}bx+3c\right)\mathrm{csgn}(a)+a\,x^2-\frac{3bx}{2}-3c\right)}{6}} \mathrm{HeunT}\left(0, -3\ \mathrm{csgn}\left(a\right), -\frac{3^{\frac{1}{3}}(4ac+b^2)}{4\left(a^2\right)^{\frac{2}{3}}}, \frac{3^{\frac{2}{3}}a(2ax-b)}{6\left(a^2\right)^{\frac{5}{6}}}\right)$$
$$+ c_2 \mathrm{e}^{-\frac{x\ \mathrm{csgn}(a)\left(\left(a\,x^2+\frac{3}{2}bx+3c\right)\mathrm{csgn}(a)-a\,x^2+\frac{3bx}{2}+3c\right)}{6}} \mathrm{HeunT}\left(0, 3\ \mathrm{csgn}\left(a\right), -\frac{3^{\frac{1}{3}}(4ac+b^2)}{4\left(a^2\right)^{\frac{2}{3}}}, -\frac{3^{\frac{2}{3}}\left(ax-\frac{b}{2}\right)a}{3\left(a^2\right)^{\frac{5}{6}}}\right)$$

✓ Solution by Mathematica
Time used: 1.096 (sec). Leaf size: 57

```
DSolve[y''[x]+(a*x^2+b*x+c)*y'[x]+(a*b*x^3+a*c*x^2+b)*y[x]==0,y[x],x,IncludeSingularSolution
```

$$y(x) \to e^{-\frac{1}{2}x(bx+2c)}\left(c_2 \int_1^x \exp\left(\frac{1}{6}K[1](6c + K[1](3b - 2aK[1]))\right) dK[1] + c_1\right)$$

27.31 problem 41

Internal problem ID [10865]

Book: Handbook of exact solutions for ordinary differential equations. By Polyanin and Zaitsev. Second edition
Section: Chapter 2, Second-Order Differential Equations. section 2.1.2-2 Equation of form $y'' + f(x)y' + g(x)y = 0$
Problem number: 41.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$\boxed{y'' + \left(a\,x^3 + 2b\right)y' + \left(ab\,x^3 - a\,x^2 + b^2\right)y = 0}$$

✓ Solution by Maple
Time used: 0.016 (sec). Leaf size: 85

```
dsolve(diff(y(x),x$2)+(a*x^3+2*b)*diff(y(x),x)+(a*b*x^3-a*x^2+b^2)*y(x)=0,y(x), singsol=all)
```

$$y(x) = \frac{\frac{7\,2^{\frac{1}{4}} c_2 a (x^4 a)^{\frac{3}{8}} (x^4 a+3) \mathrm{e}^{-\frac{x\left(a\,x^3+4b\right)}{4}}}{8} + \mathrm{e}^{-\frac{x\left(a\,x^3+8b\right)}{8}} \text{WhittakerM}\left(\frac{3}{8}, \frac{7}{8}, \frac{x^4 a}{4}\right) c_2 a^2 x^4 + \mathrm{e}^{-bx} c_1 x^{\frac{5}{2}}}{x^{\frac{3}{2}}}$$

✓ Solution by Mathematica
Time used: 0.431 (sec). Leaf size: 51

```
DSolve[y''[x]+(a*x^3+2*b)*y'[x]+(a*b*x^3-a*x^2+b^2)*y[x]==0,y[x],x,IncludeSingularSolutions
```

$$y(x) \to \frac{1}{8} e^{-bx} \left(8 c_1 x - \sqrt{2} c_2 \sqrt[4]{a x^4}\, \Gamma\left(-\frac{1}{4}, \frac{a x^4}{4}\right)\right)$$

27.32 problem 42

Internal problem ID [10866]

Book: Handbook of exact solutions for ordinary differential equations. By Polyanin and Zaitsev. Second edition
Section: Chapter 2, Second-Order Differential Equations. section 2.1.2-2 Equation of form $y'' + f(x)y' + g(x)y = 0$
Problem number: 42.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$\boxed{y'' + \left(a\,x^3 + bx\right)y' + 2\left(2a\,x^2 + b\right)y = 0}$$

✓ Solution by Maple
Time used: 0.406 (sec). Leaf size: 70

```
dsolve(diff(y(x),x$2)+(a*x^3+b*x)*diff(y(x),x)+2*(2*a*x^2+b)*y(x)=0,y(x), singsol=all)
```

$$y(x) = \mathrm{e}^{-\frac{\left(a\,x^2+2b\right)x^2}{4}}\left(\mathrm{HeunB}\left(\frac{1}{2}, \frac{b}{\sqrt{a}}, \frac{5}{2}, -\frac{3b}{2\sqrt{a}}, \frac{\sqrt{a}\,x^2}{2}\right)c_1 x + \mathrm{HeunB}\left(-\frac{1}{2}, \frac{b}{\sqrt{a}}, \frac{5}{2}, -\frac{3b}{2\sqrt{a}}, \frac{\sqrt{a}\,x^2}{2}\right)c_2\right)$$

✓ Solution by Mathematica
Time used: 2.589 (sec). Leaf size: 63

```
DSolve[y''[x]+(a*x^3+b*x)*y'[x]+2*(2*a*x^2+b)*y[x]==0,y[x],x,IncludeSingularSolutions -> Tru
```

$$y(x) \to x e^{-\frac{1}{4}x^2(ax^2+2b)}\left(c_2 \int_1^x \frac{e^{\frac{1}{4}(aK[1]^4+2bK[1]^2)}}{K[1]^2} dK[1] + c_1\right)$$

27.33 problem 43

Internal problem ID [10867]

Book: Handbook of exact solutions for ordinary differential equations. By Polyanin and Zaitsev. Second edition
Section: Chapter 2, Second-Order Differential Equations. section 2.1.2-2 Equation of form $y'' + f(x)y' + g(x)y = 0$
Problem number: 43.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$\boxed{y'' + \left(ab\,x^3 + b\,x^2 + 2a\right)y' + a^2\left(x^3 b + 1\right)y = 0}$$

✓ Solution by Maple
Time used: 0.437 (sec). Leaf size: 41

```
dsolve(diff(y(x),x$2)+(a*b*x^3+b*x^2+2*a)*diff(y(x),x)+a^2*(b*x^3+1)*y(x)=0,y(x), singsol=al
```

$$y(x) = \mathrm{e}^{-ax}\left(c_2\left(\int \frac{\mathrm{e}^{-\frac{b\,x^3\left(ax+\frac{4}{3}\right)}{4}}}{(ax+1)^2}dx\right) + c_1\right)(ax+1)$$

✓ Solution by Mathematica
Time used: 3.356 (sec). Leaf size: 57

```
DSolve[y''[x]+(a*b*x^3+b*x^2+2*a)*y'[x]+a^2*(b*x^3+1)*y[x]==0,y[x],x,IncludeSingularSolution
```

$$y(x) \to e^{-ax}(ax+1)\left(c_2\int_1^x \frac{e^{-\frac{1}{12}bK[1]^3(3aK[1]+4)}}{(aK[1]+1)^2}dK[1] + c_1\right)$$

27.34 problem 44

Internal problem ID [10868]

Book: Handbook of exact solutions for ordinary differential equations. By Polyanin and Zaitsev. Second edition
Section: Chapter 2, Second-Order Differential Equations. section 2.1.2-2 Equation of form $y'' + f(x)y' + g(x)y = 0$
Problem number: 44.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _missing_y]]`

$$\boxed{y'' + a\,x^n y' = 0}$$

✓ Solution by Maple
Time used: 0.0 (sec). Leaf size: 244

```
dsolve(diff(y(x),x$2)+a*x^n*diff(y(x),x)=0,y(x), singsol=all)
```

$$y(x) = \frac{x^{-n}\left(\left(\frac{a\,x^n}{n+1}\right)^{\frac{-n-2}{2n+2}} c_2 \left(\frac{a}{n+1}\right)^{\frac{1}{n+1}} \mathrm{e}^{-\frac{x^n a x}{2n+2}} (n+2)^2 (n+1)^2 \,\mathrm{WhittakerM}\left(\frac{n+2}{2n+2}, \frac{2n+3}{2n+2}, \frac{a\,x^n}{n+1}\right) + \left(\frac{a\,x^n}{n+1}\right)^{\frac{-n-2}{2n+2}} c_2 \left(\frac{a}{n+1}\right.\right.}{(n+2)\,(2n}$$

✓ Solution by Mathematica
Time used: 0.054 (sec). Leaf size: 56

```
DSolve[y''[x]+a*x^n*y'[x]==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to c_2 - \frac{c_1 x \left(\frac{a x^{n+1}}{n+1}\right)^{-\frac{1}{n+1}} \Gamma\left(\frac{1}{n+1}, \frac{a x^{n+1}}{n+1}\right)}{n+1}$$

27.35 problem 45

Internal problem ID [10869]

Book: Handbook of exact solutions for ordinary differential equations. By Polyanin and Zaitsev. Second edition
Section: Chapter 2, Second-Order Differential Equations. section 2.1.2-2 Equation of form $y'' + f(x)y' + g(x)y = 0$
Problem number: 45.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$\boxed{y'' + a\,x^n y' + y x^{n-1} b = 0}$$

✓ Solution by Maple
Time used: 0.407 (sec). Leaf size: 96

```
dsolve(diff(y(x),x$2)+a*x^n*diff(y(x),x)+b*x^(n-1)*y(x)=0,y(x), singsol=all)
```

$$y(x) = x\left(\text{KummerU}\left(\frac{1+n-\frac{b}{a}}{n+1}, \frac{n+2}{n+1}, \frac{a x\, x^n}{n+1}\right) c_2 + \text{KummerM}\left(\frac{1+n-\frac{b}{a}}{n+1}, \frac{n+2}{n+1}, \frac{a x\, x^n}{n+1}\right) c_1\right) \mathrm{e}^{-\frac{a x\, x^n}{n+1}}$$

✓ Solution by Mathematica
Time used: 0.143 (sec). Leaf size: 120

```
DSolve[y''[x]+a*x^n*y'[x]+b*x^(n-1)*y[x]==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to c_2\left(\frac{1}{n}+1\right)^{-\frac{1}{n+1}} n^{-\frac{1}{n+1}} a^{\frac{1}{n+1}} (x^n)^{\frac{1}{n}}\, \text{Hypergeometric1F1}\left(\frac{a+b}{na+a}, \frac{n+2}{n+1}, -\frac{a(x^n)^{1+\frac{1}{n}}}{n+1}\right) + c_1\, \text{Hypergeometric1F1}\left(\frac{b}{na+a}, \frac{n}{n+1}, -\frac{a(x^n)^{1+\frac{1}{n}}}{n+1}\right)$$

27.36 problem 46

Internal problem ID [10870]

Book: Handbook of exact solutions for ordinary differential equations. By Polyanin and Zaitsev. Second edition
Section: Chapter 2, Second-Order Differential Equations. section 2.1.2-2 Equation of form $y'' + f(x)y' + g(x)y = 0$
Problem number: 46.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$\boxed{y'' + 2a\,x^n y' + a\left(a\,x^{2n} + n\,x^{n-1}\right) y = 0}$$

✓ Solution by Maple
Time used: 0.0 (sec). Leaf size: 24

```
dsolve(diff(y(x),x$2)+2*a*x^n*diff(y(x),x)+a*(a*x^(2*n)+n*x^(n-1))*y(x)=0,y(x), singsol=all)
```

$$y(x) = \mathrm{e}^{-\frac{a\,x^{n+1}}{n+1}} (c_2 x + c_1)$$

✓ Solution by Mathematica
Time used: 0.112 (sec). Leaf size: 28

```
DSolve[y''[x]+2*a*x^n*y'[x]+a*(a*x^(2*n)+n*x^(n-1))*y[x]==0,y[x],x,IncludeSingularSolutions
```

$$y(x) \to (c_2 x + c_1) e^{-\frac{a x^{n+1}}{n+1}}$$

27.37 problem 47

Internal problem ID [10871]

Book: Handbook of exact solutions for ordinary differential equations. By Polyanin and Zaitsev. Second edition
Section: Chapter 2, Second-Order Differential Equations. section 2.1.2-2 Equation of form $y'' + f(x)y' + g(x)y = 0$
Problem number: 47.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$\boxed{y'' + a\,x^n y' + \left(b\,x^{2n} + c\,x^{n-1}\right) y = 0}$$

✓ Solution by Maple
Time used: 0.375 (sec). Leaf size: 171

```
dsolve(diff(y(x),x$2)+a*x^n*diff(y(x),x)+(b*x^(2*n)+c*x^(n-1))*y(x)=0,y(x), singsol=all)
```

$$y(x) = \mathrm{e}^{-\frac{x^{n+1}\left(a+\sqrt{a^2-4b}\right)}{2n+2}} x \left(\mathrm{KummerM}\left(\frac{(n+2)\sqrt{a^2-4b}+an-2c}{\sqrt{a^2-4b}\,(2n+2)}, \frac{n+2}{n+1}, \frac{\sqrt{a^2-4b}\,x^{n+1}}{n+1} \right) c_1 + \mathrm{KummerU}\left(\frac{(n+2)\sqrt{a^2-4b}+an-2c}{\sqrt{a^2-4b}\,(2n+2)}, \frac{n+2}{n+1}, \frac{\sqrt{a^2-4b}\,x^{n+1}}{n+1} \right) c_2 \right)$$

✓ Solution by Mathematica

Time used: 0.492 (sec). Leaf size: 333

```
DSolve[y''[x]+a*x^n*y'[x]+(b*x^(2*n)+c*x^(n-1))*y[x]==0,y[x],x,IncludeSingularSolutions -> T
```

$$y(x)$$

$$\to 2^{\frac{n}{2n+2}} x^{-n/2} \left(x^{n+1}\right)^{\frac{n}{2n+2}} \exp\left(-\frac{1}{2}x^{n+1}\left(\frac{\sqrt{a^2-4b}}{\sqrt{(n+1)^2}}+\frac{a}{n+1}\right)\right)\left(c_1 \text{HypergeometricU}\left(\frac{n\left(\sqrt{(n+1)^2}a^2\right.}{}\right.\right.$$

$$\left.\left.+\, c_2 L^{-\frac{1}{n+1}}_{\frac{2\sqrt{a^2-4b}c(n+1)-n\left(\sqrt{(n+1)^2}a^2+\sqrt{a^2-4b}(n+1)a-4b\sqrt{(n+1)^2}\right)}{2\left(a^2-4b\right)(n+1)\sqrt{(n+1)^2}}}\left(\frac{\sqrt{a^2-4b}x^{n+1}}{\sqrt{(n+1)^2}}\right)\right)$$

27.38 problem 48

Internal problem ID [10872]

Book: Handbook of exact solutions for ordinary differential equations. By Polyanin and Zaitsev. Second edition
Section: Chapter 2, Second-Order Differential Equations. section 2.1.2-2 Equation of form $y'' + f(x)y' + g(x)y = 0$
Problem number: 48.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$\boxed{y'' + a\,x^n y' - b\left(a\,x^{m+n} + b\,x^{2m} + m\,x^{m-1}\right) y = 0}$$

✗ Solution by Maple

```
dsolve(diff(y(x),x$2)+a*x^n*diff(y(x),x)-b*(a*x^(n+m)+b*x^(2*m)+m*x^(m-1))*y(x)=0,y(x), sing
```

No solution found

✗ Solution by Mathematica
Time used: 0.0 (sec). Leaf size: 0

```
DSolve[y''[x]+a*x^n*y'[x]-b*(a*x^(n+m)+b*x^(2*m)+m*x^(m-1))*y[x]==0,y[x],x,IncludeSingularSo
```

Not solved

27.39 problem 49

Internal problem ID [10873]

Book: Handbook of exact solutions for ordinary differential equations. By Polyanin and Zaitsev. Second edition
Section: Chapter 2, Second-Order Differential Equations. section 2.1.2-2 Equation of form $y'' + f(x)y' + g(x)y = 0$
Problem number: 49.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$\boxed{y'' + 2a\,x^n y' + \left(x^{2n}a^2 + b\,x^{2m} + an\,x^{n-1} + c\,x^{m-1}\right) y = 0}$$

✓ Solution by Maple
Time used: 0.406 (sec). Leaf size: 147

```
dsolve(diff(y(x),x$2)+2*a*x^n*diff(y(x),x)+(a^2*x^(2*n)+b*x^(2*m)+a*n*x^(n-1)+c*x^(m-1))*y(x
```

$$y(x) = x\left(\text{KummerM}\left(\frac{(m+2)\sqrt{b}+ic}{\sqrt{b}\,(2m+2)}, \frac{m+2}{1+m}, \frac{2i\sqrt{b}\,x^{1+m}}{1+m}\right)c_1 + \text{KummerU}\left(\frac{(m+2)\sqrt{b}+ic}{\sqrt{b}\,(2m+2)}, \frac{m+2}{1+m}, \frac{2i\sqrt{b}\,x^{1+m}}{1+m}\right)c_2\right) \mathrm{e}^{\frac{-i(n+1)\sqrt{b}\,x^{1+m}-x^{n+1}a(1+m)}{(n+1)(1+m)}}$$

✓ Solution by Mathematica
Time used: 0.376 (sec). Leaf size: 236

```
DSolve[y''[x]+2*a*x^n*y'[x]+(a^2*x^(2*n)+b*x^(2*m)+a*n*x^(n-1)+c*x^(m-1))*y[x]==0,y[x],x,Inc
```

$$y(x) \to 2^{\frac{m}{2m+2}} x^{-m/2}\left(x^{m+1}\right)^{\frac{m}{2m+2}} \exp\left(-x\left(\frac{ax^n}{n+1}+\frac{\sqrt{b}x^m}{\sqrt{-(m+1)^2}}\right)\right)\left(c_1 \text{ HypergeometricU}\left(-\frac{(m+1)\left(mc+\right.\right.\right.$$

27.40 problem 50

Internal problem ID [10874]

Book: Handbook of exact solutions for ordinary differential equations. By Polyanin and Zaitsev. Second edition
Section: Chapter 2, Second-Order Differential Equations. section 2.1.2-2 Equation of form $y'' + f(x)y' + g(x)y = 0$
Problem number: 50.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$y'' + (a\,x^n + b)\,y' + c(a\,x^n + b - c)\,y = 0$$

✗ Solution by Maple

```
dsolve(diff(y(x),x$2)+(a*x^n+b)*diff(y(x),x)+c*(a*x^n+b-c)*y(x)=0,y(x), singsol=all)
```

No solution found

✗ Solution by Mathematica
Time used: 0.0 (sec). Leaf size: 0

```
DSolve[y''[x]+(a*x^n+b)*y'[x]+c*(a*x^n+b-c)*y[x]==0,y[x],x,IncludeSingularSolutions -> True]
```

Not solved

27.41 problem 51

Internal problem ID [10875]

Book: Handbook of exact solutions for ordinary differential equations. By Polyanin and Zaitsev. Second edition
Section: Chapter 2, Second-Order Differential Equations. section 2.1.2-2 Equation of form $y'' + f(x)y' + g(x)y = 0$
Problem number: 51.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$\boxed{y'' + (a\,x^n + 2b)\,y' + \left(ab\,x^n - a\,x^{n-1} + b^2\right)y = 0}$$

✓ Solution by Maple
Time used: 0.203 (sec). Leaf size: 167

```
dsolve(diff(y(x),x$2)+(a*x^n+2*b)*diff(y(x),x)+(a*b*x^n-a*x^(n-1)+b^2)*y(x)=0,y(x), singsol=
```

$$y(x) = \mathrm{e}^{-\frac{(a\,x^n+2(n+1)b)x}{2n+2}} c_2(n+1)\left(a\,x^{-\frac{n}{2}} + x^{-\frac{3n}{2}-1}n\right)\text{WhittakerM}\left(\frac{-n-2}{2n+2}, \frac{2n+1}{2n+2}, \frac{a\,x^{n+1}}{n+1}\right) + c_2 n^2 x^{-\frac{3n}{2}-1}\mathrm{e}^{-\frac{(a\,x^n+2(n+1)b)x}{2n+2}}\text{WhittakerM}\left(\frac{n}{2n+2}, \frac{2n+1}{2n+2}, \frac{a\,x^{n+1}}{n+1}\right) + c_1\mathrm{e}^{-bx}x$$

✗ Solution by Mathematica
Time used: 0.0 (sec). Leaf size: 0

```
DSolve[y''[x]+(a*x^n+2*b)*y'[x]+(a*b*x^n-a*x^(n-1)+b^2)*y[x]==0,y[x],x,IncludeSingularSoluti
```

Not solved

27.42 problem 52

Internal problem ID [10876]

Book: Handbook of exact solutions for ordinary differential equations. By Polyanin and Zaitsev. Second edition
Section: Chapter 2, Second-Order Differential Equations. section 2.1.2-2 Equation of form $y'' + f(x)y' + g(x)y = 0$
Problem number: 52.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$y'' + \left(ab\,x^n + b\,x^{n-1} + 2a\right) y' + a^2(b\,x^n + 1)\,y = 0$$

✗ Solution by Maple

```
dsolve(diff(y(x),x$2)+(a*b*x^n+b*x^(n-1)+2*a)*diff(y(x),x)+a^2*(b*x^n+1)*y(x)=0,y(x), singso
```

No solution found

✗ Solution by Mathematica
Time used: 0.0 (sec). Leaf size: 0

```
DSolve[y''[x]+(a*b*x^n+b*x^(n-1)+2*a)*y'[x]+a^2*(b*x^n+1)*y[x]==0,y[x],x,IncludeSingularSolu
```

Not solved

27.43 problem 53

Internal problem ID [10877]

Book: Handbook of exact solutions for ordinary differential equations. By Polyanin and Zaitsev. Second edition
Section: Chapter 2, Second-Order Differential Equations. section 2.1.2-2 Equation of form $y'' + f(x)y' + g(x)y = 0$
Problem number: 53.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$\boxed{y'' + \left(ab\,x^n + 2b\,x^{n-1} - a^2x\right)y' + a\left(ab\,x^n + b\,x^{n-1} - a^2x\right)y = 0}$$

✗ Solution by Maple

```
dsolve(diff(y(x),x$2)+(a*b*x^n+2*b*x^(n-1)-a^2*x)*diff(y(x),x)+a*(a*b*x^n+b*x^(n-1)-a^2*x)*y
```

No solution found

✗ Solution by Mathematica
Time used: 0.0 (sec). Leaf size: 0

```
DSolve[y''[x]+(a*b*x^n+2*b*x^(n-1)-a^2*x)*y'[x]+a*(a*b*x^n+b*x^(n-1)-a^2*x)*y[x]==0,y[x],x,I
```

Not solved

27.44 problem 54

Internal problem ID [10878]

Book: Handbook of exact solutions for ordinary differential equations. By Polyanin and Zaitsev. Second edition
Section: Chapter 2, Second-Order Differential Equations. section 2.1.2-2 Equation of form $y'' + f(x)y' + g(x)y = 0$
Problem number: 54.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$\boxed{y'' + x^n\left(a\,x^2 + (ac+b)\,x + bc\right)y' - x^n(ax+b)\,y = 0}$$

✓ Solution by Maple
Time used: 0.484 (sec). Leaf size: 78

```
dsolve(diff(y(x),x$2)+x^n*(a*x^2+(a*c+b)*x+b*c)*diff(y(x),x)-x^n*(a*x+b)*y(x)=0,y(x), singso
```

$$y(x) = -(x+c)\left(\left(\int \frac{e^{-\frac{\left(a\,x^2(n+2)(n+1)+(ac+b)x(3+n)(n+1)+bc(3+n)(n+2)\right)x^{n+1}}{(3+n)(n+1)(n+2)}}}{(x+c)^2}dx\right)c_1 + c_2\right)$$

✗ Solution by Mathematica
Time used: 0.0 (sec). Leaf size: 0

```
DSolve[y''[x]+x^n*(a*x^2+(a*c+b)*x+b*c)*y'[x]-x^n*(a*x+b)*y[x]==0,y[x],x,IncludeSingularSolu
```

Not solved

27.45 problem 55

Internal problem ID [10879]

Book: Handbook of exact solutions for ordinary differential equations. By Polyanin and Zaitsev. Second edition
Section: Chapter 2, Second-Order Differential Equations. section 2.1.2-2 Equation of form $y'' + f(x)y' + g(x)y = 0$
Problem number: 55.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$\boxed{y'' + \left(a\,x^n + b\,x^m\right)y' - \left(a\,x^{n-1} + b\,x^{m-1}\right)y = 0}$$

✓ Solution by Maple
Time used: 0.328 (sec). Leaf size: 47

```
dsolve(diff(y(x),x$2)+(a*x^n+b*x^m)*diff(y(x),x)-(a*x^(n-1)+b*x^(m-1))*y(x)=0,y(x), singsol=
```

$$y(x) = x\left(c_1 + c_2\left(\int \frac{\mathrm{e}^{-\frac{(b(n+1)x^m + a(1+m)x^n)x}{(1+m)(n+1)}}}{x^2}dx\right)\right)$$

✓ Solution by Mathematica
Time used: 1.216 (sec). Leaf size: 55

```
DSolve[y''[x]+(a*x^n+b*x^m)*y'[x]-(a*x^(n-1)+b*x^(m-1))*y[x]==0,y[x],x,IncludeSingularSoluti
```

$$y(x) \to x\left(c_2 \int_1^x \frac{\exp\left(K[1]\left(-\frac{bK[1]^m}{m+1} - \frac{aK[1]^n}{n+1}\right)\right)}{K[1]^2}dK[1] + c_1\right)$$

27.46 problem 56

Internal problem ID [10880]

Book: Handbook of exact solutions for ordinary differential equations. By Polyanin and Zaitsev. Second edition
Section: Chapter 2, Second-Order Differential Equations. section 2.1.2-2 Equation of form $y'' + f(x)y' + g(x)y = 0$
Problem number: 56.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries], [_2nd_order, _linear,`

$$\boxed{y'' + \left(a\,x^n + b\,x^m\right)y' + \left(an\,x^{n-1} + x^{m-1}bm\right)y = 0}$$

✓ Solution by Maple
Time used: 0.063 (sec). Leaf size: 72

```
dsolve(diff(y(x),x$2)+(a*x^n+b*x^m)*diff(y(x),x)+(a*n*x^(n-1)+b*m*x^(m-1))*y(x)=0,y(x), sing
```

$$y(x) = \left(c_1\left(\int \mathrm{e}^{\frac{(b(n+1)x^m+a(1+m)x^n)x}{(1+m)(n+1)}}dx\right) + c_2\right)\mathrm{e}^{-\frac{(b(n+1)x^m+a(1+m)x^n)x}{(1+m)(n+1)}}$$

✓ Solution by Mathematica
Time used: 0.139 (sec). Leaf size: 74

```
DSolve[y''[x]+(a*x^n+b*x^m)*y'[x]+(a*n*x^(n-1)+b*m*x^(m-1))*y[x]==0,y[x],x,IncludeSingularSo
```

$$y(x) \to e^{x\left(-\frac{ax^n}{n+1}-\frac{bx^m}{m+1}\right)}\left(\int_1^x \exp\left(K[1]\left(\frac{bK[1]^m}{m+1}+\frac{aK[1]^n}{n+1}\right)\right)c_1 dK[1] + c_2\right)$$

27.47 problem 57

Internal problem ID [10881]

Book: Handbook of exact solutions for ordinary differential equations. By Polyanin and Zaitsev. Second edition
Section: Chapter 2, Second-Order Differential Equations. section 2.1.2-2 Equation of form $y'' + f(x)y' + g(x)y = 0$
Problem number: 57.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$\boxed{y'' + \left(a\,x^n + b\,x^m\right)y' + \left(a(n+1)\,x^{n-1} + b(m+1)\,x^{m-1}\right)y = 0}$$

✓ Solution by Maple
Time used: 0.313 (sec). Leaf size: 77

```
dsolve(diff(y(x),x$2)+(a*x^n+b*x^m)*diff(y(x),x)+(a*(n+1)*x^(n-1)+b*(m+1)*x^(m-1))*y(x)=0,y(
```

$$y(x) = x\left(c_1 + \left(\int \frac{\mathrm{e}^{\frac{(b(n+1)x^m + a(1+m)x^n)x}{(1+m)(n+1)}}}{x^2}dx\right)c_2\right)\mathrm{e}^{-\frac{(b(n+1)x^m + a(1+m)x^n)x}{(1+m)(n+1)}}$$

✗ Solution by Mathematica
Time used: 0.0 (sec). Leaf size: 0

```
DSolve[y''[x]+(a*x^n+b*x^m)*y'[x]+(a*(n+1)*x^(n-1)+b*(m+1)*x^(m-1))*y[x]==0,y[x],x,IncludeSi
```

Not solved

27.48 problem 58

Internal problem ID [10882]

Book: Handbook of exact solutions for ordinary differential equations. By Polyanin and Zaitsev. Second edition
Section: Chapter 2, Second-Order Differential Equations. section 2.1.2-2 Equation of form $y'' + f(x)y' + g(x)y = 0$
Problem number: 58.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$\boxed{y'' + (a\,x^n + b\,x^m)\,y' + c(a\,x^n + b\,x^m - c)\,y = 0}$$

✗ Solution by Maple

```
dsolve(diff(y(x),x$2)+(a*x^n+b*x^m)*diff(y(x),x)+c*(a*x^n+b*x^m-c)*y(x)=0,y(x), singsol=all)
```

No solution found

✗ Solution by Mathematica
Time used: 0.0 (sec). Leaf size: 0

```
DSolve[y''[x]+(a*x^n+b*x^m)*y'[x]+c*(a*x^n+b*x^m-c)*y[x]==0,y[x],x,IncludeSingularSolutions
```

Not solved

27.49 problem 59

Internal problem ID [10883]

Book: Handbook of exact solutions for ordinary differential equations. By Polyanin and Zaitsev. Second edition
Section: Chapter 2, Second-Order Differential Equations. section 2.1.2-2 Equation of form $y'' + f(x)y' + g(x)y = 0$
Problem number: 59.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$y'' + \left(a\,x^n + b\,x^m\right) y' + \left(x^{m+n} ab + b(m+1)\,x^{m-1} - a\,x^{n-1}\right) y = 0$$

✗ Solution by Maple

```
dsolve(diff(y(x),x$2)+(a*x^n+b*x^m)*diff(y(x),x)+(a*b*x^(n+m)+b*(m+1)*x^(m-1)-a*x^(n-1))*y(x
```

No solution found

✗ Solution by Mathematica
Time used: 0.0 (sec). Leaf size: 0

```
DSolve[y''[x]+(a*x^n+b*x^m)*y'[x]+(a*b*x^(n+m)+b*(m+1)*x^(m-1)-a*x^(n-1))*y[x]==0,y[x],x,Inc
```

Not solved

27.50 problem 60

Internal problem ID [10884]

Book: Handbook of exact solutions for ordinary differential equations. By Polyanin and Zaitsev. Second edition
Section: Chapter 2, Second-Order Differential Equations. section 2.1.2-2 Equation of form $y'' + f(x)y' + g(x)y = 0$
Problem number: 60.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$\boxed{y'' + (a\,x^n + b\,x^m + c)\,y' + \left(x^{m+n}ab + x^m bc + an\,x^{n-1}\right)y = 0}$$

✗ Solution by Maple

```
dsolve(diff(y(x),x$2)+(a*x^n+b*x^m+c)*diff(y(x),x)+(a*b*x^(n+m)+b*c*x^m+a*n*x^(n-1))*y(x)=0,
```

No solution found

✗ Solution by Mathematica
Time used: 0.0 (sec). Leaf size: 0

```
DSolve[y''[x]+(a*x^n+b*x^m+c)*y'[x]+(a*b*x^(n+m)+b*c*x^m+a*n*x^(n-1))*y[x]==0,y[x],x,Include
```

Not solved

28 Chapter 2, Second-Order Differential Equations. section 2.1.2-3 Equation of form $(ax+b)y''+f(x)y'+g(x)y=0$

28.1 problem 61 684
28.2 problem 62 685
28.3 problem 63 686
28.4 problem 64 687
28.5 problem 65 688
28.6 problem 66 689
28.7 problem 67 690
28.8 problem 68 691
28.9 problem 69 692
28.10 problem 70 693
28.11 problem 71 694
28.12 problem 72 695
28.13 problem 73 696
28.14 problem 74 697
28.15 problem 75 698
28.16 problem 76 699
28.17 problem 77 700
28.18 problem 78 701
28.19 problem 79 702
28.20 problem 80 703
28.21 problem 81 704
28.22 problem 82 705
28.23 problem 83 706
28.24 problem 84 707
28.25 problem 85 708
28.26 problem 86 709
28.27 problem 87 710
28.28 problem 88 711
28.29 problem 89 712
28.30 problem 90 713
28.31 problem 91 714
28.32 problem 92 715
28.33 problem 93 716
28.34 problem 94 717

28.35problem 95 718
28.36problem 96 719
28.37problem 97 720
28.38problem 98 721
28.39problem 99 722
28.40problem 100 724
28.41problem 101 725
28.42problem 102 726
28.43problem 103 727
28.44problem 104 728
28.45problem 105 729
28.46problem 106 730
28.47problem 107 731
28.48problem 108 732
28.49problem 109 734

28.1 problem 61

Internal problem ID [10885]

Book: Handbook of exact solutions for ordinary differential equations. By Polyanin and Zaitsev. Second edition
Section: Chapter 2, Second-Order Differential Equations. section 2.1.2-3 Equation of form $(ax+b)y''+f(x)y'+g(x)y=0$
Problem number: 61.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_Emden, _Fowler], [_2nd_order, _linear, '_with_symmetry_[0,F`

$$\boxed{xy''+\frac{y'}{2}+ay=0}$$

✓ Solution by Maple
Time used: 0.0 (sec). Leaf size: 27

```
dsolve(x*diff(y(x),x$2)+1/2*diff(y(x),x)+a*y(x)=0,y(x), singsol=all)
```

$$y(x)=c_1\sin\left(2\sqrt{x}\,\sqrt{a}\right)+c_2\cos\left(2\sqrt{x}\,\sqrt{a}\right)$$

✓ Solution by Mathematica
Time used: 0.035 (sec). Leaf size: 38

```
DSolve[x*y''[x]+1/2*y'[x]+a*y[x]==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x)\to c_1\cos\left(2\sqrt{a}\sqrt{x}\right)+c_2\sin\left(2\sqrt{a}\sqrt{x}\right)$$

28.2 problem 62

Internal problem ID [10886]

Book: Handbook of exact solutions for ordinary differential equations. By Polyanin and Zaitsev. Second edition
Section: Chapter 2, Second-Order Differential Equations. section 2.1.2-3 Equation of form $(ax+b)y''+f(x)y'+g(x)y=0$
Problem number: 62.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_Emden, _Fowler]]`

$$\boxed{xy''+ay'+yb=0}$$

✓ Solution by Maple
Time used: 0.0 (sec). Leaf size: 83

```
dsolve(x*diff(y(x),x$2)+a*diff(y(x),x)+b*y(x)=0,y(x), singsol=all)
```

$$y(x) = \frac{\left(-\sqrt{x}\ \text{BesselJ}\left(a+1,2\sqrt{b}\,\sqrt{x}\right)\sqrt{b}\,c_1-\sqrt{x}\ \text{BesselY}\left(a+1,2\sqrt{b}\,\sqrt{x}\right)\sqrt{b}\,c_2+a\left(\text{BesselJ}\left(a,2\sqrt{b}\,\sqrt{x}\right)\right.\right.}{\sqrt{b}}$$

✓ Solution by Mathematica
Time used: 0.111 (sec). Leaf size: 77

```
DSolve[x*y''[x]+a*y'[x]+b*y[x]==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x)\to b^{\frac{1}{2}-\frac{a}{2}}x^{\frac{1}{2}-\frac{a}{2}}\left(c_2\,\text{Gamma}(2-a)\,\text{BesselJ}\left(1-a,2\sqrt{b}\sqrt{x}\right)\right.$$
$$\left.+c_1\,\text{Gamma}(a)\,\text{BesselJ}\left(a-1,2\sqrt{b}\sqrt{x}\right)\right)$$

28.3 problem 63

Internal problem ID [10887]

Book: Handbook of exact solutions for ordinary differential equations. By Polyanin and Zaitsev. Second edition
Section: Chapter 2, Second-Order Differential Equations. section 2.1.2-3 Equation of form $(ax+b)y''+f(x)y'+g(x)y=0$
Problem number: 63.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$\boxed{xy''+ay'+bxy=0}$$

✓ Solution by Maple
Time used: 0.0 (sec). Leaf size: 39

```
dsolve(x*diff(y(x),x$2)+a*diff(y(x),x)+b*x*y(x)=0,y(x), singsol=all)
```

$$y(x)=\left(\text{BesselJ}\left(\frac{a}{2}-\frac{1}{2},\sqrt{b}\,x\right)c_1+\text{BesselY}\left(\frac{a}{2}-\frac{1}{2},\sqrt{b}\,x\right)c_2\right)x^{-\frac{a}{2}+\frac{1}{2}}$$

✓ Solution by Mathematica
Time used: 0.057 (sec). Leaf size: 54

```
DSolve[x*y''[x]+a*y'[x]+b*x*y[x]==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x)\to x^{\frac{1}{2}-\frac{a}{2}}\left(c_1\,\text{BesselJ}\left(\frac{a-1}{2},\sqrt{b}x\right)+c_2\,\text{BesselY}\left(\frac{a-1}{2},\sqrt{b}x\right)\right)$$

28.4 problem 64

Internal problem ID [10888]

Book: Handbook of exact solutions for ordinary differential equations. By Polyanin and Zaitsev. Second edition
Section: Chapter 2, Second-Order Differential Equations. section 2.1.2-3 Equation of form $(ax+b)y''+f(x)y'+g(x)y=0$
Problem number: 64.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$\boxed{xy''+ay'+(bx+c)\,y=0}$$

✓ Solution by Maple
Time used: 0.078 (sec). Leaf size: 66

```
dsolve(x*diff(y(x),x$2)+a*diff(y(x),x)+(b*x+c)*y(x)=0,y(x), singsol=all)
```

$$y(x)=\mathrm{e}^{-i\sqrt{b}\,x}\left(\mathrm{KummerU}\left(\frac{ic+a\sqrt{b}}{2\sqrt{b}},a,2i\sqrt{b}\,x\right)c_2 + \mathrm{KummerM}\left(\frac{ic+a\sqrt{b}}{2\sqrt{b}},a,2i\sqrt{b}\,x\right)c_1\right)$$

✓ Solution by Mathematica
Time used: 0.099 (sec). Leaf size: 85

```
DSolve[x*y''[x]+a*y'[x]+(b*x+c)*y[x]==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x)\to e^{-i\sqrt{b}x}\left(c_1\,\mathrm{HypergeometricU}\left(\frac{1}{2}\left(a+\frac{ic}{\sqrt{b}}\right),a,2i\sqrt{b}x\right)+c_2L^{a-1}_{-\frac{a}{2}-\frac{ic}{2\sqrt{b}}}\left(2i\sqrt{b}x\right)\right)$$

28.5 problem 65

Internal problem ID [10889]

Book: Handbook of exact solutions for ordinary differential equations. By Polyanin and Zaitsev. Second edition
Section: Chapter 2, Second-Order Differential Equations. section 2.1.2-3 Equation of form $(ax+b)y''+f(x)y'+g(x)y=0$
Problem number: 65.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_Emden, _Fowler], [_2nd_order, _linear, '_with_symmetry_[0,F`

$$\boxed{xy''+ny'+b\,x^{1-2n}y=0}$$

✓ Solution by Maple
Time used: 0.016 (sec). Leaf size: 43

```
dsolve(x*diff(y(x),x$2)+n*diff(y(x),x)+b*x^(1-2*n)*y(x)=0,y(x), singsol=all)
```

$$y(x)=c_1\sin\left(\frac{x^{-n+1}\sqrt{b}}{n-1}\right)+c_2\cos\left(\frac{x^{-n+1}\sqrt{b}}{n-1}\right)$$

✓ Solution by Mathematica
Time used: 0.072 (sec). Leaf size: 52

```
DSolve[x*y''[x]+n*y'[x]+b*x^(1-2*n)*y[x]==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x)\to c_1\cos\left(\frac{\sqrt{b}x^{1-n}}{n-1}\right)+c_2\sin\left(\frac{\sqrt{b}x^{1-n}}{1-n}\right)$$

28.6 problem 66

Internal problem ID [10890]

Book: Handbook of exact solutions for ordinary differential equations. By Polyanin and Zaitsev. Second edition
Section: Chapter 2, Second-Order Differential Equations. section 2.1.2-3 Equation of form $(ax+b)y''+f(x)y'+g(x)y=0$
Problem number: 66.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_Emden, _Fowler]]`

$$\boxed{xy''+(1-3n)y'-a^2n^2x^{-1+2n}y=0}$$

✓ Solution by Maple
Time used: 0.094 (sec). Leaf size: 62

```
dsolve(x*diff(y(x),x$2)+(1-3*n)*diff(y(x),x)-a^2*n^2*x^(2*n-1)*y(x)=0,y(x), singsol=all)
```

$$y(x)=c_2\mathrm{e}^{-a\,x^n}\left(a\,x^n+x^{-n}\sqrt{x^{2n}}\right)-\mathrm{e}^{a\,x^n}c_1\left(a\,x^n-x^{-n}\sqrt{x^{2n}}\right)$$

✓ Solution by Mathematica
Time used: 0.196 (sec). Leaf size: 77

```
DSolve[x*y''[x]+(1-3*n)*y'[x]-a^2*n^2*x^(2*n-1)*y[x]==0,y[x],x,IncludeSingularSolutions -> T
```

$$y(x)\to\left(c_1-\frac{3}{8}iac_2\sqrt{x^{2n}}\right)\cosh\left(a\sqrt{x^{2n}}\right)+\frac{1}{8}\left(3ic_2-8ac_1\sqrt{x^{2n}}\right)\sinh\left(a\sqrt{x^{2n}}\right)$$

28.7 problem 67

Internal problem ID [10891]

Book: Handbook of exact solutions for ordinary differential equations. By Polyanin and Zaitsev. Second edition
Section: Chapter 2, Second-Order Differential Equations. section 2.1.2-3 Equation of form $(ax+b)y''+f(x)y'+g(x)y=0$
Problem number: 67.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_Emden, _Fowler]]`

$$\boxed{xy''+ay'+bx^n y=0}$$

✓ Solution by Maple
Time used: 0.218 (sec). Leaf size: 71

```
dsolve(x*diff(y(x),x$2)+a*diff(y(x),x)+b*x^n*y(x)=0,y(x), singsol=all)
```

$$y(x)=\left(\text{BesselY}\left(\frac{a-1}{n+1},\frac{2\sqrt{b}\,x^{\frac{n}{2}+\frac{1}{2}}}{n+1}\right)c_2+\text{BesselJ}\left(\frac{a-1}{n+1},\frac{2\sqrt{b}\,x^{\frac{n}{2}+\frac{1}{2}}}{n+1}\right)c_1\right)x^{-\frac{a}{2}+\frac{1}{2}}$$

✓ Solution by Mathematica
Time used: 0.245 (sec). Leaf size: 165

```
DSolve[x*y''[x]+a*y'[x]+b*x^n*y[x]==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x)\to\left(\frac{1}{n}+1\right)^{\frac{a-1}{n+1}}n^{\frac{a-1}{n+1}}b^{\frac{1-a}{2n+2}}(x^n)^{-\frac{a-1}{2n}}\left(c_2\,\text{Gamma}\left(\frac{-a+n+2}{n+1}\right)\text{BesselJ}\left(\frac{1-a}{n+1},\frac{2\sqrt{b}(x^n)^{\frac{n+1}{2n}}}{n+1}\right)+c_1\,\text{Gamma}\left(\frac{a+n}{n+1}\right)\text{BesselJ}\left(\frac{a-1}{n+1},\frac{2\sqrt{b}(x^n)^{\frac{n+1}{2n}}}{n+1}\right)\right)$$

28.8 problem 68

Internal problem ID [10892]

Book: Handbook of exact solutions for ordinary differential equations. By Polyanin and Zaitsev. Second edition
Section: Chapter 2, Second-Order Differential Equations. section 2.1.2-3 Equation of form $(ax+b)y''+f(x)y'+g(x)y=0$
Problem number: 68.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$\boxed{xy''+ay'+b\,x^n\left(-b\,x^{n+1}+a+n\right)y=0}$$

✓ Solution by Maple
Time used: 0.594 (sec). Leaf size: 166

```
dsolve(x*diff(y(x),x$2)+a*diff(y(x),x)+b*x^n*(-b*x^(n+1)+a+n)*y(x)=0,y(x), singsol=all)
```

$$\begin{aligned}y(x)=&-\left(\left(a-n-2\right)x^{-\frac{3n}{2}-\frac{a}{2}-1}+2b\,x^{-\frac{n}{2}-\frac{a}{2}}\right)c_2(n\\&+1)\,\mathrm{WhittakerM}\left(\frac{-a-n}{2n+2},\frac{-a+2n+3}{2n+2},-\frac{2b\,x^{n+1}}{n+1}\right)\\&+x^{-\frac{3n}{2}-\frac{a}{2}-1}c_2(a-n-2)^2\,\mathrm{WhittakerM}\left(\frac{n+2-a}{2n+2},\frac{-a+2n+3}{2n+2},-\frac{2b\,x^{n+1}}{n+1}\right)\\&+c_1\mathrm{e}^{-\frac{b\,x^{n+1}}{n+1}}\end{aligned}$$

✗ Solution by Mathematica
Time used: 0.0 (sec). Leaf size: 0

```
DSolve[x*y''[x]+a*y'[x]+b*x^n*(-b*x^(n+1)+a+n)*y[x]==0,y[x],x,IncludeSingularSolutions -> Tr
```

Not solved

28.9 problem 69

Internal problem ID [10893]

Book: Handbook of exact solutions for ordinary differential equations. By Polyanin and Zaitsev. Second edition
Section: Chapter 2, Second-Order Differential Equations. section 2.1.2-3 Equation of form $(ax+b)y''+f(x)y'+g(x)y=0$
Problem number: 69.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _exact, _linear, _homogeneous]]`

$$\boxed{xy''+axy'+ay=0}$$

✓ Solution by Maple
Time used: 0.016 (sec). Leaf size: 29

```
dsolve(x*diff(y(x),x$2)+a*x*diff(y(x),x)+a*y(x)=0,y(x), singsol=all)
```

$$y(x)=\operatorname{expIntegral}_1(-ax)\,c_1 ax\,\mathrm{e}^{-ax}+c_1+c_2 x\,\mathrm{e}^{-ax}$$

✓ Solution by Mathematica
Time used: 0.177 (sec). Leaf size: 35

```
DSolve[x*y''[x]+a*x*y'[x]+a*y[x]==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x)\to e^{-ax}(ac_2 x\,\operatorname{ExpIntegralEi}(ax)-c_2 e^{ax}+c_1 x)$$

28.10 problem 70

Internal problem ID [10894]

Book: Handbook of exact solutions for ordinary differential equations. By Polyanin and Zaitsev. Second edition
Section: Chapter 2, Second-Order Differential Equations. section 2.1.2-3 Equation of form $(ax+b)y''+f(x)y'+g(x)y=0$
Problem number: 70.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[_Laguerre]`

$$\boxed{xy''+(-x+b)y'-ay=0}$$

✓ Solution by Maple
Time used: 0.047 (sec). Leaf size: 17

```
dsolve(x*diff(y(x),x$2)+(b-x)*diff(y(x),x)-a*y(x)=0,y(x), singsol=all)
```

$$y(x)=c_1\,\mathrm{KummerM}\left(a,b,x\right)+c_2\,\mathrm{KummerU}\left(a,b,x\right)$$

✓ Solution by Mathematica
Time used: 0.049 (sec). Leaf size: 24

```
DSolve[x*y''[x]+(b-x)*y'[x]-a*y[x]==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x)\to c_1\,\mathrm{HypergeometricU}(a,b,x)+c_2L_{-a}^{b-1}(x)$$

28.11 problem 71

Internal problem ID [10895]

Book: Handbook of exact solutions for ordinary differential equations. By Polyanin and Zaitsev. Second edition
Section: Chapter 2, Second-Order Differential Equations. section 2.1.2-3 Equation of form $(ax+b)y''+f(x)y'+g(x)y=0$
Problem number: 71.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$\boxed{xy''+(ax+b)\,y'+c((a-c)\,x+b)\,y=0}$$

✓ Solution by Maple
Time used: 0.0 (sec). Leaf size: 40

```
dsolve(x*diff(y(x),x$2)+(a*x+b)*diff(y(x),x)+c*((a-c)*x+b)*y(x)=0,y(x), singsol=all)
```

$$y(x)=c_1\mathrm{e}^{-cx}+c_2x^{-\frac{b}{2}}\,\mathrm{WhittakerM}\left(-\frac{b}{2},\frac{1}{2}-\frac{b}{2},(-2c+a)\,x\right)\mathrm{e}^{-\frac{ax}{2}}$$

✓ Solution by Mathematica
Time used: 0.333 (sec). Leaf size: 50

```
DSolve[x*y''[x]+(a*x+b)*y'[x]+c*((a-c)*x+b)*y[x]==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x)\to e^{-cx}\left(c_1-c_2x^{1-b}(x(a-2c))^{b-1}\Gamma(1-b,(a-2c)x)\right)$$

28.12 problem 72

Internal problem ID [10896]

Book: Handbook of exact solutions for ordinary differential equations. By Polyanin and Zaitsev. Second edition
Section: Chapter 2, Second-Order Differential Equations. section 2.1.2-3 Equation of form $(ax+b)y''+f(x)y'+g(x)y=0$
Problem number: 72.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$\boxed{xy''+(2ax+b)\,y'+a(ax+b)\,y=0}$$

✓ Solution by Maple
Time used: 0.016 (sec). Leaf size: 21

```
dsolve(x*diff(y(x),x$2)+(2*a*x+b)*diff(y(x),x)+a*(a*x+b)*y(x)=0,y(x), singsol=all)
```

$$y(x)=\mathrm{e}^{-ax}\left(c_1+x^{-b+1}c_2\right)$$

✓ Solution by Mathematica
Time used: 0.233 (sec). Leaf size: 70

```
DSolve[x*y''[x]+(2*a*x+b)*y'[x]+a*(a*x+b)*y[x]==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x)\to \frac{e^{-ax}x^{\frac{1}{2}\left(-b-\sqrt{(b-1)^2}+1\right)}\left(c_2x^{\sqrt{(b-1)^2}}+\sqrt{(b-1)^2}c_1\right)}{\sqrt{(b-1)^2}}$$

28.13 problem 73

Internal problem ID [10897]

Book: Handbook of exact solutions for ordinary differential equations. By Polyanin and Zaitsev. Second edition
Section: Chapter 2, Second-Order Differential Equations. section 2.1.2-3 Equation of form $(ax+b)y''+f(x)y'+g(x)y=0$
Problem number: 73.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$\boxed{xy''+((a+b)x+n+m)y'+(abx+an+bm)y=0}$$

✓ Solution by Maple
Time used: 0.062 (sec). Leaf size: 39

```
dsolve(x*diff(y(x),x$2)+((a+b)*x+n+m)*diff(y(x),x)+(a*b*x+a*n+b*m)*y(x)=0,y(x), singsol=all)
```

$$y(x)=\mathrm{e}^{-ax}(\mathrm{KummerU}\left(m,n+m,(a-b)x\right)c_2+\mathrm{KummerM}\left(m,n+m,(a-b)x\right)c_1)$$

✓ Solution by Mathematica
Time used: 0.131 (sec). Leaf size: 46

```
DSolve[x*y''[x]+((a+b)*x+n+m)*y'[x]+(a*b*x+a*n+b*m)*y[x]==0,y[x],x,IncludeSingularSolutions
```

$$y(x)\to e^{-ax}(c_1\,\mathrm{HypergeometricU}(m,m+n,(a-b)x)+c_2L_{-m}^{m+n-1}((a-b)x))$$

28.14 problem 74

Internal problem ID [10898]

Book: Handbook of exact solutions for ordinary differential equations. By Polyanin and Zaitsev. Second edition
Section: Chapter 2, Second-Order Differential Equations. section 2.1.2-3 Equation of form $(ax+b)y''+f(x)y'+g(x)y=0$
Problem number: 74.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$\boxed{xy''+(ax+b)y'+(cx+d)y=0}$$

✓ Solution by Maple
Time used: 0.093 (sec). Leaf size: 109

```
dsolve(x*diff(y(x),x$2)+(a*x+b)*diff(y(x),x)+(c*x+d)*y(x)=0,y(x), singsol=all)
```

$$y(x)=\mathrm{e}^{-\frac{x\left(\sqrt{a^2-4c}+a\right)}{2}}\left(\mathrm{KummerM}\left(\frac{b\sqrt{a^2-4c}+ab-2d}{2\sqrt{a^2-4c}},b,\sqrt{a^2-4c}\,x\right)c_1+\mathrm{KummerU}\left(\frac{b\sqrt{a^2-4c}+ab-2d}{2\sqrt{a^2-4c}},b,\sqrt{a^2-4c}\,x\right)c_2\right)$$

✓ Solution by Mathematica
Time used: 0.135 (sec). Leaf size: 135

```
DSolve[x*y''[x]+(a*x+b)*y'[x]+(c*x+d)*y[x]==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x)\to e^{-\frac{1}{2}x\left(\sqrt{a^2-4c}+a\right)}\left(c_1\,\mathrm{HypergeometricU}\left(\frac{ab+\sqrt{a^2-4c}b-2d}{2\sqrt{a^2-4c}},b,\sqrt{a^2-4c}x\right)+c_2L^{b-1}_{-\frac{ab+\sqrt{a^2-4c}b-2d}{2\sqrt{a^2-4c}}}\left(\sqrt{a^2-4c}x\right)\right)$$

28.15 problem 75

Internal problem ID [10899]

Book: Handbook of exact solutions for ordinary differential equations. By Polyanin and Zaitsev. Second edition
Section: Chapter 2, Second-Order Differential Equations. section 2.1.2-3 Equation of form $(ax+b)y''+f(x)y'+g(x)y=0$
Problem number: 75.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$\boxed{xy''-(ax+1)y'-bx^2(bx+a)y=0}$$

✓ Solution by Maple
Time used: 0.0 (sec). Leaf size: 73

```
dsolve(x*diff(y(x),x$2)-(a*x+1)*diff(y(x),x)-b*x^2*(b*x+a)*y(x)=0,y(x), singsol=all)
```

$$y(x)=-\mathrm{e}^{-\frac{2b^2x^2+a^2}{4b}}\sqrt{\pi}\sqrt{-b}\,\operatorname{erf}\left(\frac{2bx+a}{2\sqrt{-b}}\right)c_2a+2\,\mathrm{e}^{\frac{1}{2}x^2b+ax}c_2b+c_1\mathrm{e}^{-\frac{x^2b}{2}}$$

✓ Solution by Mathematica
Time used: 0.444 (sec). Leaf size: 88

```
DSolve[x*y''[x]-(a*x+1)*y'[x]-b*x^2*(b*x+a)*y[x]==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x)\to\frac{e^{-\frac{bx^2}{2}}\left(2\sqrt{b}\left(c_2e^{x(a+bx)}+2bc_1\right)-\sqrt{\pi}ac_2e^{-\frac{a^2}{4b}}\operatorname{erfi}\left(\frac{a+2bx}{2\sqrt{b}}\right)\right)}{4b^{3/2}}$$

28.16 problem 76

Internal problem ID [10900]

Book: Handbook of exact solutions for ordinary differential equations. By Polyanin and Zaitsev. Second edition
Section: Chapter 2, Second-Order Differential Equations. section 2.1.2-3 Equation of form $(ax+b)y''+f(x)y'+g(x)y=0$
Problem number: 76.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$\boxed{xy''-(2ax+1)y'+\left(x^3b+a^2x+a\right)y=0}$$

✓ Solution by Maple
Time used: 0.016 (sec). Leaf size: 39

```
dsolve(x*diff(y(x),x$2)-(2*a*x+1)*diff(y(x),x)+(b*x^3+a^2*x+a)*y(x)=0,y(x), singsol=all)
```

$$y(x)=c_1\mathrm{e}^{ax+\frac{x^2\sqrt{-b}}{2}}+c_2\mathrm{e}^{ax-\frac{x^2\sqrt{-b}}{2}}$$

✓ Solution by Mathematica
Time used: 0.276 (sec). Leaf size: 59

```
DSolve[x*y''[x]-(2*a*x+1)*y'[x]+(b*x^3+a^2*x+a)*y[x]==0,y[x],x,IncludeSingularSolutions -> T
```

$$y(x)\to\frac{1}{2}e^{ax-\frac{1}{2}i\sqrt{b}x^2}\left(2c_1-\frac{ic_2e^{i\sqrt{b}x^2}}{\sqrt{b}}\right)$$

28.17 problem 77

Internal problem ID [10901]

Book: Handbook of exact solutions for ordinary differential equations. By Polyanin and Zaitsev. Second edition
Section: Chapter 2, Second-Order Differential Equations. section 2.1.2-3 Equation of form $(ax+b)y''+f(x)y'+g(x)y=0$
Problem number: 77.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _missing_y]]`

$$\boxed{xy''+(ax+b)y'=-cx\left(-cx^2+ax+b+1\right)}$$

✓ Solution by Maple
Time used: 0.016 (sec). Leaf size: 115

```
dsolve(x*diff(y(x),x$2)+(a*x+b)*diff(y(x),x)+c*x*(-c*x^2+a*x+b+1)=0,y(x), singsol=all)
```

$$y(x) = \frac{c_2a^3-\left(\int\left(-c^2((b^3+3b^2+2b)\,\Gamma(b,-ax)-\Gamma(b+3))\,\mathrm{e}^{-ax}(-ax)^{-b}-\mathrm{e}^{-ax}x^{-b}c_1a^3+((-b^2+(ax-3)\,b\right.\right.}{a^3}$$

✓ Solution by Mathematica
Time used: 61.322 (sec). Leaf size: 92

```
DSolve[x*y''[x]+(a*x+b)*y'[x]+c*x*(-c*x^2+a*x+b+1)==0,y[x],x,IncludeSingularSolutions -> Tru
```

$$y(x) \to \int_1^x e^{-aK[1]}K[1]^{-b}\left(\frac{c(-((b+1)\Gamma(b+1,-aK[1])a^2)+\Gamma(b+2,-aK[1])a^2+c\Gamma(b+3,-aK[1]))\,K[1]^b(}{a^3}\right.$$
$$\left.+c_1\right)dK[1]+c_2$$

28.18 problem 78

Internal problem ID [10902]

Book: Handbook of exact solutions for ordinary differential equations. By Polyanin and Zaitsev. Second edition
Section: Chapter 2, Second-Order Differential Equations. section 2.1.2-3 Equation of form $(ax+b)y''+f(x)y'+g(x)y=0$
Problem number: 78.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$\boxed{xy''-(2ax+1)y'+ybx^3=0}$$

✓ Solution by Maple
Time used: 0.188 (sec). Leaf size: 106

```
dsolve(x*diff(y(x),x$2)-(2*a*x+1)*diff(y(x),x)+b*x^3*y(x)=0,y(x), singsol=all)
```

$$y(x)=x^2\,\mathrm{HeunB}\left(2,0,\frac{a^2}{\sqrt{-b}},-\frac{2ia}{(-b)^{\frac{1}{4}}},i(-b)^{\frac{1}{4}}x\right)\mathrm{e}^{ax+\frac{x^2\sqrt{-b}}{2}}\left(c_1+c_2\left(\int\frac{\mathrm{e}^{-x^2\sqrt{-b}}}{\mathrm{HeunB}\left(2,0,\frac{a^2}{\sqrt{-b}},-\frac{2ia}{(-b)^{\frac{1}{4}}},i\left(-b\right)^{\frac{1}{4}}x\right)^2x^3}dx\right)\right)$$

✗ Solution by Mathematica
Time used: 0.0 (sec). Leaf size: 0

```
DSolve[x*y''[x]-(2*a*x+1)*y'[x]+b*x^3*y[x]==0,y[x],x,IncludeSingularSolutions -> True]
```

Not solved

28.19 problem 79

Internal problem ID [10903]

Book: Handbook of exact solutions for ordinary differential equations. By Polyanin and Zaitsev. Second edition
Section: Chapter 2, Second-Order Differential Equations. section 2.1.2-3 Equation of form $(ax+b)y''+f(x)y'+g(x)y=0$
Problem number: 79.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$\boxed{xy''+\left(ab\,x^2+b-5\right)y'+2a^2(-2+b)\,x^3y=0}$$

✓ Solution by Maple
Time used: 0.187 (sec). Leaf size: 92

```
dsolve(x*diff(y(x),x$2)+(a*b*x^2+b-5)*diff(y(x),x)+2*a^2*(b-2)*x^3*y(x)=0,y(x), singsol=all)
```

$$y(x) = \frac{\left(-3\,\mathrm{KummerU}\left(\frac{b}{2}+1,-2+\frac{b}{2},\frac{a(b-4)x^2}{2}\right)c_2 b+\left(a(b-4)\,x^2+b+4\right)c_2\,\mathrm{KummerU}\left(\frac{b}{2},-2+\frac{b}{2},\frac{a(b-4)x^2}{2}\right)\right.}{2}$$

✓ Solution by Mathematica
Time used: 3.578 (sec). Leaf size: 67

```
DSolve[x*y''[x]+(a*b*x^2+b-5)*y'[x]+2*a^2*(b-2)*x^3*y[x]==0,y[x],x,IncludeSingularSolutions
```

$$y(x)\to e^{-ax^2}\left(ax^2+1\right)\left(c_2\int_1^x\frac{e^{-\frac{1}{2}a(b-4)K[1]^2}K[1]^{5-b}}{\left(aK[1]^2+1\right)^2}dK[1]+c_1\right)$$

28.20 problem 80

Internal problem ID [10904]

Book: Handbook of exact solutions for ordinary differential equations. By Polyanin and Zaitsev. Second edition
Section: Chapter 2, Second-Order Differential Equations. section 2.1.2-3 Equation of form $(ax+b)y''+f(x)y'+g(x)y=0$
Problem number: 80.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$\boxed{xy'' + \left(a\,x^2 + bx\right)y' - \left(ac\,x^2 + \left(bc + c^2 + a\right)x + b + 2c\right)y = 0}$$

✓ Solution by Maple
Time used: 8.546 (sec). Leaf size: 34

```
dsolve(x*diff(y(x),x$2)+(a*x^2+b*x)*diff(y(x),x)-(a*c*x^2+(a+b*c+c^2)*x+b+2*c)*y(x)=0,y(x),
```

$$y(x) = \mathrm{e}^{cx} x \left(c_1 + c_2 \left(\int \frac{\mathrm{e}^{-\frac{x(ax+2b+4c)}{2}}}{x^2} dx \right) \right)$$

✓ Solution by Mathematica
Time used: 3.129 (sec). Leaf size: 49

```
DSolve[x*y''[x]+(a*x^2+b*x)*y'[x]-(a*c*x^2+(a+b*c+c^2)*x+b+2*c)*y[x]==0,y[x],x,IncludeSingul
```

$$y(x) \to x e^{cx} \left(c_2 \int_1^x \frac{e^{-\frac{1}{2}K[1](2b+4c+aK[1])}}{K[1]^2} dK[1] + c_1 \right)$$

28.21 problem 81

Internal problem ID [10905]

Book: Handbook of exact solutions for ordinary differential equations. By Polyanin and Zaitsev. Second edition
Section: Chapter 2, Second-Order Differential Equations. section 2.1.2-3 Equation of form $(ax+b)y''+f(x)y'+g(x)y=0$
Problem number: 81.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$\boxed{xy'' + \left(a\,x^2 + bx + 2\right)y' + yb = 0}$$

✓ Solution by Maple
Time used: 0.016 (sec). Leaf size: 69

```
dsolve(x*diff(y(x),x$2)+(a*x^2+b*x+2)*diff(y(x),x)+b*y(x)=0,y(x), singsol=all)
```

$$y(x) = \frac{\mathrm{e}^{\frac{b^2}{2a}}\pi c_2(ax+b)\,\mathrm{erf}\left(\frac{\sqrt{2}\,(ax+b)}{2\sqrt{a}}\right) + \sqrt{\pi}\,\sqrt{2}\,\sqrt{a}\,\mathrm{e}^{-\frac{x(ax+2b)}{2}}c_2 + c_1(ax+b)}{x}$$

✓ Solution by Mathematica
Time used: 0.535 (sec). Leaf size: 85

```
DSolve[x*y''[x]+(a*x^2+b*x+2)*y'[x]+b*y[x]==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to \frac{(ax+b)\left(-\frac{\sqrt{\frac{\pi}{2}}c_2\mathrm{erf}\left(\frac{ax+b}{\sqrt{2}\sqrt{a}}\right)}{a^{3/2}} - \frac{c_2 e^{-\frac{(ax+b)^2}{2a}}}{a(ax+b)} + c_1\right)}{bx}$$

28.22 problem 82

Internal problem ID [10906]

Book: Handbook of exact solutions for ordinary differential equations. By Polyanin and Zaitsev. Second edition
Section: Chapter 2, Second-Order Differential Equations. section 2.1.2-3 Equation of form $(ax+b)y''+f(x)y'+g(x)y=0$
Problem number: 82.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _exact, _linear, _homogeneous]]`

$$\boxed{xy'' + \left(a\,x^2 + bx + c\right)y' + (2ax + b)\,y = 0}$$

✓ Solution by Maple
Time used: 0.109 (sec). Leaf size: 46

```
dsolve(x*diff(y(x),x$2)+(a*x^2+b*x+c)*diff(y(x),x)+(2*a*x+b)*y(x)=0,y(x), singsol=all)
```

$$y(x) = x^{-c+1}\mathrm{e}^{-\frac{x(ax+2b)}{2}}\left(c_1\left(\int x^{c-2}\mathrm{e}^{\frac{1}{2}a\,x^2+bx}dx\right) + c_2\right)$$

✓ Solution by Mathematica
Time used: 1.772 (sec). Leaf size: 63

```
DSolve[x*y''[x]+(a*x^2+b*x+c)*y'[x]+(2*a*x+b)*y[x]==0,y[x],x,IncludeSingularSolutions -> Tru
```

$$y(x) \to x^{1-c}e^{-\frac{1}{2}x(ax+2b)}\left(c_2\int_1^x e^{\frac{1}{2}aK[1]^2+bK[1]}K[1]^{c-2}dK[1] + c_1\right)$$

28.23 problem 83

Internal problem ID [10907]

Book: Handbook of exact solutions for ordinary differential equations. By Polyanin and Zaitsev. Second edition
Section: Chapter 2, Second-Order Differential Equations. section 2.1.2-3 Equation of form $(ax+b)y''+f(x)y'+g(x)y=0$
Problem number: 83.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$\boxed{xy''+\left(a\,x^2+bx+c\right)y'+(c-1)\,(ax+b)\,y=0}$$

✓ Solution by Maple
Time used: 0.297 (sec). Leaf size: 102

```
dsolve(x*diff(y(x),x$2)+(a*x^2+b*x+c)*diff(y(x),x)+(c-1)*(a*x+b)*y(x)=0,y(x), singsol=all)
```

$$y(x)=\mathrm{e}^{-\frac{x(ax+2b)}{2}}\left(\mathrm{HeunB}\left(c-1,\frac{b\sqrt{2}}{\sqrt{a}},c-3,-\frac{\sqrt{2}\,b(c-2)}{\sqrt{a}},\frac{\sqrt{2}\,\sqrt{a}\,x}{2}\right)c_1+\mathrm{HeunB}\left(-c+1,\frac{b\sqrt{2}}{\sqrt{a}},c-3,-\frac{\sqrt{2}\,b(c-2)}{\sqrt{a}},\frac{\sqrt{2}\,\sqrt{a}\,x}{2}\right)x^{-c+1}c_2\right)$$

✓ Solution by Mathematica
Time used: 1.579 (sec). Leaf size: 49

```
DSolve[x*y''[x]+(a*x^2+b*x+c)*y'[x]+(c-1)*(a*x+b)*y[x]==0,y[x],x,IncludeSingularSolutions ->
```

$$y(x)\to x^{1-c}\left(c_2\int_1^x e^{-\frac{1}{2}K[1](2b+aK[1])}K[1]^{c-2}dK[1]+c_1\right)$$

28.24 problem 84

Internal problem ID [10908]

Book: Handbook of exact solutions for ordinary differential equations. By Polyanin and Zaitsev. Second edition
Section: Chapter 2, Second-Order Differential Equations. section 2.1.2-3 Equation of form $(ax+b)y''+f(x)y'+g(x)y=0$
Problem number: 84.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$\boxed{xy''+\left(a\,x^2+bx+c\right)y'+\left(A\,x^2+Bx+\mathrm{C0}\right)y=0}$$

✓ Solution by Maple
Time used: 0.281 (sec). Leaf size: 186

```
dsolve(x*diff(y(x),x$2)+(a*x^2+b*x+c)*diff(y(x),x)+(A*x^2+B*x+C0)*y(x)=0,y(x), singsol=all)
```

$$y(x)=\mathrm{e}^{\frac{x\left(-a^2x-2ab+2A\right)}{2a}}\left(x^{-c+1}\,\mathrm{HeunB}\left(-c+1,\right.\right.$$
$$\left.-\frac{\sqrt{2}\,(-ab+2A)}{a^{\frac{3}{2}}},\frac{(-c-1)\,a^3+2B\,a^2-2Aab+2A^2}{a^3},\frac{(bc-2\,\mathrm{C0})\sqrt{2}}{\sqrt{a}},\frac{\sqrt{2}\sqrt{a}\,x}{2}\right)c_2$$
$$\left.+\mathrm{HeunB}\left(c-1,\right.\right.$$
$$\left.\left.-\frac{\sqrt{2}\,(-ab+2A)}{a^{\frac{3}{2}}},\frac{(-c-1)\,a^3+2B\,a^2-2Aab+2A^2}{a^3},\frac{(bc-2\,\mathrm{C0})\sqrt{2}}{\sqrt{a}},\frac{\sqrt{2}\sqrt{a}\,x}{2}\right)c_1\right)$$

✗ Solution by Mathematica
Time used: 0.0 (sec). Leaf size: 0

```
DSolve[x*y''[x]+(a*x^2+b*x+c)*y'[x]+(A*x^2+B*x+C0)*y[x]==0,y[x],x,IncludeSingularSolutions -
```

Not solved

28.25 problem 85

Internal problem ID [10909]

Book: Handbook of exact solutions for ordinary differential equations. By Polyanin and Zaitsev. Second edition
Section: Chapter 2, Second-Order Differential Equations. section 2.1.2-3 Equation of form $(ax+b)y''+f(x)y'+g(x)y=0$
Problem number: 85.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$\boxed{xy''+\left(a\,x^2+bx+2\right)y'+\left(c\,x^2+dx+b\right)y=0}$$

✓ Solution by Maple
Time used: 0.046 (sec). Leaf size: 143

```
dsolve(x*diff(y(x),x$2)+(a*x^2+b*x+2)*diff(y(x),x)+(c*x^2+d*x+b)*y(x)=0,y(x), singsol=all)
```

$$y(x) = \frac{e^{-\frac{x\left(a^2x+2ab-2c\right)}{2a}}\left(\text{hypergeom}\left(\left[\frac{3a^3-da^2+abc-c^2}{2a^3}\right],\left[\frac{3}{2}\right],\frac{\left(a^2x+ab-2c\right)^2}{2a^3}\right)\left(a^2x+ab-2c\right)c_2+c_1\,\text{hypergeom}\left(\left[\frac{2a}{}\right.\right.\right.}{x}$$

✓ Solution by Mathematica
Time used: 0.155 (sec). Leaf size: 134

```
DSolve[x*y''[x]+(a*x^2+b*x+2)*y'[x]+(c*x^2+d*x+b)*y[x]==0,y[x],x,IncludeSingularSolutions ->
```

$$y(x) \to \frac{e^{-\frac{1}{2}x\left(-\frac{2c}{a}+ax+2b\right)}\left(c_2\,\text{Hypergeometric1F1}\left(-\frac{-2a^3+da^2-bca+c^2}{2a^3},\frac{1}{2},\frac{\left(xa^2+ba-2c\right)^2}{2a^3}\right)+c_1\,\text{HermiteH}\left(\frac{-2a^3+da^2-bc}{a^3}\right.\right.}{x}$$

28.26 problem 86

Internal problem ID [10910]

Book: Handbook of exact solutions for ordinary differential equations. By Polyanin and Zaitsev. Second edition
Section: Chapter 2, Second-Order Differential Equations. section 2.1.2-3 Equation of form $(ax+b)y''+f(x)y'+g(x)y=0$
Problem number: 86.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$\boxed{xy''+\left(a\,x^3+b\right)y'+a(b-1)\,x^2y=0}$$

✓ Solution by Maple
Time used: 0.0 (sec). Leaf size: 107

```
dsolve(x*diff(y(x),x$2)+(a*x^3+b)*diff(y(x),x)+a*(b-1)*x^2*y(x)=0,y(x), singsol=all)
```

$$y(x)$$
$$=\frac{9c_2a^2x^{-\frac{b}{2}+3}\mathrm{e}^{-\frac{a\,x^3}{6}}\,\mathrm{WhittakerM}\left(\frac{1}{3}+\frac{b}{6},\frac{b}{6}+\frac{5}{6},\frac{a\,x^3}{3}\right)+\left(a\,x^{-\frac{b}{2}+3}+x^{-\frac{b}{2}}(b+2)\right)c_2\mathrm{e}^{-\frac{a\,x^3}{3}}a3^{-\frac{b}{6}+\frac{2}{3}}(b+5)\,(a}{9x}$$

✓ Solution by Mathematica
Time used: 0.424 (sec). Leaf size: 60

```
DSolve[x*y''[x]+(a*x^3+b)*y'[x]+a*(b-1)*x^2*y[x]==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x)\to c_1x^{1-b}-3^{\frac{b-4}{3}}c_2\left(ax^3\right)^{\frac{1}{3}-\frac{b}{3}}\Gamma\left(\frac{b-1}{3},\frac{ax^3}{3}\right)$$

28.27 problem 87

Internal problem ID [10911]

Book: Handbook of exact solutions for ordinary differential equations. By Polyanin and Zaitsev. Second edition
Section: Chapter 2, Second-Order Differential Equations. section 2.1.2-3 Equation of form $(ax+b)y''+f(x)y'+g(x)y=0$
Problem number: 87.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _exact, _linear, _homogeneous]]`

$$\boxed{xy''+x\left(a\,x^2+b\right)y'+\left(3a\,x^2+b\right)y=0}$$

✓ Solution by Maple
Time used: 0.141 (sec). Leaf size: 42

```
dsolve(x*diff(y(x),x$2)+x*(a*x^2+b)*diff(y(x),x)+(3*a*x^2+b)*y(x)=0,y(x), singsol=all)
```

$$y(x)=x\,\mathrm{e}^{-\frac{x\left(a\,x^2+3b\right)}{3}}\left(c_1\left(\int\frac{\mathrm{e}^{\frac{x\left(a\,x^2+3b\right)}{3}}}{x^2}dx\right)+c_2\right)$$

✓ Solution by Mathematica
Time used: 2.43 (sec). Leaf size: 56

```
DSolve[x*y''[x]+x*(a*x^2+b)*y'[x]+(3*a*x^2+b)*y[x]==0,y[x],x,IncludeSingularSolutions -> Tru
```

$$y(x)\to xe^{-\frac{ax^3}{3}-bx}\left(c_2\int_1^x\frac{e^{\frac{1}{3}aK[1]^3+bK[1]}}{K[1]^2}dK[1]+c_1\right)$$

28.28 problem 88

Internal problem ID [10912]

Book: Handbook of exact solutions for ordinary differential equations. By Polyanin and Zaitsev. Second edition
Section: Chapter 2, Second-Order Differential Equations. section 2.1.2-3 Equation of form $(ax+b)y''+f(x)y'+g(x)y=0$
Problem number: 88.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$\boxed{xy''+\left(a\,x^3+b\,x^2+2\right)y'+bxy=0}$$

✓ Solution by Maple
Time used: 0.281 (sec). Leaf size: 143

```
dsolve(x*diff(y(x),x$2)+(a*x^3+b*x^2+2)*diff(y(x),x)+b*x*y(x)=0,y(x), singsol=all)
```

$$y(x) = \frac{c_1 \mathrm{e}^{-\frac{\operatorname{csgn}(a) x^2 (\operatorname{csgn}(a)+1)\left(ax+\frac{3b}{2}\right)}{6}} \operatorname{HeunT}\left(\frac{3^{\frac{2}{3}} b}{2(a^2)^{\frac{1}{3}}}, -6\operatorname{csgn}(a), -\frac{b^2 3^{\frac{1}{3}}}{4(a^2)^{\frac{2}{3}}}, \frac{3^{\frac{2}{3}} a(2ax+b)}{6(a^2)^{\frac{5}{6}}}\right) + c_2 \mathrm{e}^{-\frac{\operatorname{csgn}(a) x^2 (\operatorname{csgn}(a)-1)\left(ax+\frac{3b}{2}\right)}{6}}}{x}$$

✓ Solution by Mathematica
Time used: 1.962 (sec). Leaf size: 58

```
DSolve[x*y''[x]+(a*x^3+b*x^2+2)*y'[x]+b*x*y[x]==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to \frac{(ax+b)\left(c_2 \int_1^x \frac{e^{-\frac{1}{6}K[1]^2(3b+2aK[1])}}{(b+aK[1])^2} dK[1] + c_1\right)}{bx}$$

28.29 problem 89

Internal problem ID [10913]

Book: Handbook of exact solutions for ordinary differential equations. By Polyanin and Zaitsev. Second edition
Section: Chapter 2, Second-Order Differential Equations. section 2.1.2-3 Equation of form $(ax+b)y''+f(x)y'+g(x)y=0$
Problem number: 89.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$\boxed{xy'' + \left(ab\,x^3 + b\,x^2 + ax - 1\right)y' + a^2 b\,x^3 y = 0}$$

✓ Solution by Maple
Time used: 0.313 (sec). Leaf size: 48

```
dsolve(x*diff(y(x),x$2)+(a*b*x^3+b*x^2+a*x-1)*diff(y(x),x)+a^2*b*x^3*y(x)=0,y(x), singsol=al
```

$$y(x) = \mathrm{e}^{-ax}\left(c_2\left(\int \frac{x\,\mathrm{e}^{-\frac{\left(ab\,x^2+\frac{3}{2}bx-3a\right)x}{3}}}{(ax+1)^2}dx\right)+c_1\right)(ax+1)$$

✓ Solution by Mathematica
Time used: 4.606 (sec). Leaf size: 72

```
DSolve[x*y''[x]+(a*b*x^3+b*x^2+a*x-1)*y'[x]+a^2*b*x^3*y[x]==0,y[x],x,IncludeSingularSolution
```

$$y(x) \to \frac{e^{-ax}(ax+1)\left(c_2\int_1^x \frac{a^2\exp\left(-\frac{1}{6}K[1]\left(3bK[1]+2a\left(bK[1]^2-3\right)\right)\right)K[1]}{(aK[1]+1)^2}dK[1]+c_1\right)}{a}$$

28.30 problem 90

Internal problem ID [10914]

Book: Handbook of exact solutions for ordinary differential equations. By Polyanin and Zaitsev. Second edition
Section: Chapter 2, Second-Order Differential Equations. section 2.1.2-3 Equation of form $(ax+b)y''+f(x)y'+g(x)y=0$
Problem number: 90.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$\boxed{xy''+\left(a\,x^3+b\,x^2+cx+d\right)y'+(d-1)\left(a\,x^2+bx+c\right)y=0}$$

✓ Solution by Maple
Time used: 0.359 (sec). Leaf size: 42

```
dsolve(x*diff(y(x),x$2)+(a*x^3+b*x^2+c*x+d)*diff(y(x),x)+(d-1)*(a*x^2+b*x+c)*y(x)=0,y(x), si
```

$$y(x)=x^{-d+1}\left(\left(\int x^{d-2}\mathrm{e}^{-\frac{1}{3}a\,x^3-\frac{1}{2}x^2b-cx}dx\right)c_2+c_1\right)$$

✓ Solution by Mathematica
Time used: 1.839 (sec). Leaf size: 57

```
DSolve[x*y''[x]+(a*x^3+b*x^2+c*x+d)*y'[x]+(d-1)*(a*x^2+b*x+c)*y[x]==0,y[x],x,IncludeSingular
```

$$y(x)\to x^{1-d}\left(c_2\int_1^x\exp\left(-\frac{1}{6}K[1](6c+K[1](3b+2aK[1]))\right)K[1]^{d-2}dK[1]+c_1\right)$$

28.31 problem 91

Internal problem ID [10915]

Book: Handbook of exact solutions for ordinary differential equations. By Polyanin and Zaitsev. Second edition
Section: Chapter 2, Second-Order Differential Equations. section 2.1.2-3 Equation of form $(ax+b)y''+f(x)y'+g(x)y=0$
Problem number: 91.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$\boxed{xy''+a\,x^n y'+\left(ab\,x^n-a\,x^{n-1}-b^2x+2b\right)y=0}$$

✗ Solution by Maple

```
dsolve(x*diff(y(x),x$2)+a*x^n*diff(y(x),x)+(a*b*x^n-a*x^(n-1)-b^2*x+2*b)*y(x)=0,y(x), singso
```

No solution found

✗ Solution by Mathematica
Time used: 0.0 (sec). Leaf size: 0

```
DSolve[x*y''[x]+a*x^n*y'[x]+(a*b*x^n-a*x^(n-1)-b^2*x+2*b)*y[x]==0,y[x],x,IncludeSingularSolu
```

Not solved

28.32 problem 92

Internal problem ID [10916]

Book: Handbook of exact solutions for ordinary differential equations. By Polyanin and Zaitsev. Second edition
Section: Chapter 2, Second-Order Differential Equations. section 2.1.2-3 Equation of form $(ax+b)y''+f(x)y'+g(x)y=0$
Problem number: 92.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$\boxed{xy''+(a\,x^n+2)\,y'+x^{n-1}ay=0}$$

✓ Solution by Maple
Time used: 0.203 (sec). Leaf size: 122

```
dsolve(x*diff(y(x),x$2)+(a*x^n+2)*diff(y(x),x)+a*x^(n-1)*y(x)=0,y(x), singsol=all)
```

$$y(x) = \frac{nc_2 \mathrm{e}^{-\frac{a\,x^n}{2n}}\left((n+1)\,x^{-\frac{3n}{2}+\frac{1}{2}}+x^{-\frac{n}{2}+\frac{1}{2}}a\right)\text{WhittakerM}\left(-\frac{n-1}{2n},\frac{2n+1}{2n},\frac{a\,x^n}{n}\right)+c_2x^{-\frac{3n}{2}+\frac{1}{2}}\mathrm{e}^{-\frac{a\,x^n}{2n}}(n+1)^2\,\text{Whitta}}{x}$$

✓ Solution by Mathematica
Time used: 0.099 (sec). Leaf size: 62

```
DSolve[x*y''[x]+(a*x^n+2)*y'[x]+a*x^(n-1)*y[x]==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x)\to(-1)^{-1/n}n^{\frac{1}{n}-1}a^{-1/n}(x^n)^{-1/n}\left(c_1(-1)^{\frac{1}{n}}\Gamma\left(\frac{1}{n},0,\frac{ax^n}{n}\right)+c_2n\right)$$

28.33 problem 93

Internal problem ID [10917]

Book: Handbook of exact solutions for ordinary differential equations. By Polyanin and Zaitsev. Second edition
Section: Chapter 2, Second-Order Differential Equations. section 2.1.2-3 Equation of form $(ax+b)y''+f(x)y'+g(x)y=0$
Problem number: 93.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$\boxed{xy''+\left(x^n+1-n\right)y'+b\,x^{-1+2n}y=0}$$

✓ Solution by Maple
Time used: 0.046 (sec). Leaf size: 53

```
dsolve(x*diff(y(x),x$2)+(x^n+1-n)*diff(y(x),x)+b*x^(2*n-1)*y(x)=0,y(x), singsol=all)
```

$$y(x)=\mathrm{e}^{-\frac{x^n}{2n}}\left(c_1\sinh\left(\frac{x^n\sqrt{\frac{-4b+1}{n^2}}}{2}\right)+c_2\cosh\left(\frac{x^n\sqrt{\frac{-4b+1}{n^2}}}{2}\right)\right)$$

✓ Solution by Mathematica
Time used: 0.094 (sec). Leaf size: 53

```
DSolve[x*y''[x]+(x^n+1-n)*y'[x]+b*x^(2*n-1)*y[x]==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x)\to e^{-\frac{\left(\sqrt{1-4b}+1\right)x^n}{2n}}\left(c_2e^{\frac{\sqrt{1-4b}x^n}{n}}+c_1\right)$$

28.34 problem 94

Internal problem ID [10918]

Book: Handbook of exact solutions for ordinary differential equations. By Polyanin and Zaitsev. Second edition
Section: Chapter 2, Second-Order Differential Equations. section 2.1.2-3 Equation of form $(ax+b)y''+f(x)y'+g(x)y=0$
Problem number: 94.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries], [_2nd_order, _linear,`

$$\boxed{xy''+(a\,x^n+b)\,y'+yx^{n-1}an=0}$$

✓ Solution by Maple
Time used: 0.094 (sec). Leaf size: 53

```
dsolve(x*diff(y(x),x$2)+(a*x^n+b)*diff(y(x),x)+a*n*x^(n-1)*y(x)=0,y(x), singsol=all)
```

$$y(x)=\mathrm{e}^{-\frac{a\,x^n}{n}}\left(\text{hypergeom}\left(\left[\frac{b-1}{n}\right],\left[\frac{b+n-1}{n}\right],\frac{a\,x^n}{n}\right)c_1+x^{-b+1}c_2\right)$$

✓ Solution by Mathematica
Time used: 0.244 (sec). Leaf size: 121

```
DSolve[x*y''[x]+(a*x^n+b)*y'[x]+a*n*x^(n-1)*y[x]==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x)\to(-1)^{-\frac{b}{n}}n^{\frac{b-n-1}{n}}a^{\frac{1-b}{n}}e^{-\frac{ax^n}{n}}(x^n)^{\frac{1-b}{n}}\left(-(b-1)c_1(-1)^{\frac{1}{n}}\Gamma\left(\frac{b-1}{n},-\frac{ax^n}{n}\right)\right.$$
$$\left.+c_2n(-1)^{b/n}+(b-1)c_1(-1)^{\frac{1}{n}}\,\text{Gamma}\left(\frac{b-1}{n}\right)\right)$$

28.35 problem 95

Internal problem ID [10919]

Book: Handbook of exact solutions for ordinary differential equations. By Polyanin and Zaitsev. Second edition
Section: Chapter 2, Second-Order Differential Equations. section 2.1.2-3 Equation of form $(ax+b)y''+f(x)y'+g(x)y=0$
Problem number: 95.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$\boxed{xy''+(a\,x^n+b)\,y'+a(b-1)\,x^{n-1}y=0}$$

✓ Solution by Maple
Time used: 0.094 (sec). Leaf size: 143

```
dsolve(x*diff(y(x),x$2)+(a*x^n+b)*diff(y(x),x)+a*(b-1)*x^(n-1)*y(x)=0,y(x), singsol=all)
```

$$\begin{aligned}y(x)=\mathrm{e}^{-\frac{a\,x^n}{2n}}nc_2\Big(&(b+n-1)\,x^{-\frac{3n}{2}+\frac{1}{2}-\frac{b}{2}}\\&+a\,x^{\frac{1}{2}-\frac{b}{2}-\frac{n}{2}}\Big)\,\mathrm{WhittakerM}\left(\frac{b-n-1}{2n},\frac{b+2n-1}{2n},\frac{a\,x^n}{n}\right)\\&+x^{-\frac{3n}{2}+\frac{1}{2}-\frac{b}{2}}\mathrm{e}^{-\frac{a\,x^n}{2n}}c_2(b+n-1)^2\,\mathrm{WhittakerM}\left(\frac{b+n-1}{2n},\frac{b+2n-1}{2n},\frac{a\,x^n}{n}\right)\\&+c_1x^{-b+1}\end{aligned}$$

✓ Solution by Mathematica
Time used: 0.182 (sec). Leaf size: 90

```
DSolve[x*y''[x]+(a*x^n+b)*y'[x]+a*(b-1)*x^(n-1)*y[x]==0,y[x],x,IncludeSingularSolutions -> T
```

$$y(x)\to(-1)^{-\frac{b}{n}}n^{\frac{b-n-1}{n}}a^{\frac{1-b}{n}}(x^n)^{\frac{1-b}{n}}\left((b-1)c_1(-1)^{b/n}\Gamma\left(\frac{b-1}{n},0,\frac{ax^n}{n}\right)+c_2(-1)^{\frac{1}{n}}n\right)$$

28.36 problem 96

Internal problem ID [10920]

Book: Handbook of exact solutions for ordinary differential equations. By Polyanin and Zaitsev. Second edition
Section: Chapter 2, Second-Order Differential Equations. section 2.1.2-3 Equation of form $(ax+b)y''+f(x)y'+g(x)y=0$
Problem number: 96.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$\boxed{xy''+(a\,x^n+b)\,y'+a(b+n-1)\,x^{n-1}y=0}$$

✓ Solution by Maple
Time used: 0.11 (sec). Leaf size: 56

```
dsolve(x*diff(y(x),x$2)+(a*x^n+b)*diff(y(x),x)+a*(b+n-1)*x^(n-1)*y(x)=0,y(x), singsol=all)
```

$$y(x)=\mathrm{e}^{-\frac{a\,x^n}{n}}\left(c_1+x^{-b+1}c_2\,\mathrm{hypergeom}\left(\left[\frac{-b+1}{n}\right],\left[\frac{-b+n+1}{n}\right],\frac{a\,x^n}{n}\right)\right)$$

✓ Solution by Mathematica
Time used: 0.178 (sec). Leaf size: 93

```
DSolve[x*y''[x]+(a*x^n+b)*y'[x]+a*(b+n-1)*x^(n-1)*y[x]==0,y[x],x,IncludeSingularSolutions ->
```

$$y(x)\\ \to \frac{(-1)^{-1/n}e^{-\frac{ax^n}{n}}\left((b-1)c_2(-1)^{b/n}\Gamma\left(\frac{1-b}{n},-\frac{ax^n}{n}\right)-(b-1)c_2(-1)^{b/n}\,\mathrm{Gamma}\left(\frac{1-b}{n}\right)+c_1(-1)^{\frac{1}{n}}n\right)}{n}$$

28.37 problem 97

Internal problem ID [10921]

Book: Handbook of exact solutions for ordinary differential equations. By Polyanin and Zaitsev. Second edition
Section: Chapter 2, Second-Order Differential Equations. section 2.1.2-3 Equation of form $(ax+b)y''+f(x)y'+g(x)y=0$
Problem number: 97.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$\boxed{x y'' + (a\,x^n + b)\,y' + c(a\,x^n - cx + b)\,y = 0}$$

✗ Solution by Maple

```
dsolve(x*diff(y(x),x$2)+(a*x^n+b)*diff(y(x),x)+c*(a*x^n-c*x+b)*y(x)=0,y(x), singsol=all)
```

No solution found

✗ Solution by Mathematica
Time used: 0.0 (sec). Leaf size: 0

```
DSolve[x*y''[x]+(a*x^n+b)*y'[x]+c*(a*x^n-c*x+b)*y[x]==0,y[x],x,IncludeSingularSolutions -> T
```

Not solved

28.38 problem 98

Internal problem ID [10922]

Book: Handbook of exact solutions for ordinary differential equations. By Polyanin and Zaitsev. Second edition
Section: Chapter 2, Second-Order Differential Equations. section 2.1.2-3 Equation of form $(ax+b)y''+f(x)y'+g(x)y=0$
Problem number: 98.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$\boxed{xy''+\left(ab\,x^{n}+b-3n+1\right)y'+a^{2}n(-n+b)\,x^{-1+2n}y=0}$$

✗ Solution by Maple

```
dsolve(x*diff(y(x),x$2)+(a*b*x^n+b-3*n+1)*diff(y(x),x)+a^2*n*(b-n)*x^(2*n-1)*y(x)=0,y(x), si
```

No solution found

✗ Solution by Mathematica
Time used: 0.0 (sec). Leaf size: 0

```
DSolve[x*y''[x]+(a*b*x^n+b-3*n+1)*y'[x]+a^2*n*(b-n)*x^(2*n-1)*y[x]==0,y[x],x,IncludeSingular
```

Not solved

28.39 problem 99

Internal problem ID [10923]

Book: Handbook of exact solutions for ordinary differential equations. By Polyanin and Zaitsev. Second edition
Section: Chapter 2, Second-Order Differential Equations. section 2.1.2-3 Equation of form $(ax+b)y''+f(x)y'+g(x)y=0$
Problem number: 99.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$\boxed{x y'' + (a\,x^n + b)\,y' + \left(x^{-1+2n} c + d\,x^{n-1}\right) y = 0}$$

✓ Solution by Maple
Time used: 0.172 (sec). Leaf size: 156

```
dsolve(x*diff(y(x),x$2)+(a*x^n+b)*diff(y(x),x)+(c*x^(2*n-1)+d*x^(n-1))*y(x)=0,y(x), singsol=
```

$$y(x) = \mathrm{e}^{-\frac{x^n\left(\sqrt{a^2-4c}+a\right)}{2n}}\left(\mathrm{KummerU}\left(\frac{(b+n-1)\sqrt{a^2-4c}+a(b+n-1)-2d}{2\sqrt{a^2-4c}\,n}, \frac{b+n-1}{n}, \frac{\sqrt{a^2-4c}\,x^n}{n}\right)c_2 + \mathrm{KummerM}\left(\frac{(b+n-1)\sqrt{a^2-4c}+a(b+n-1)-2d}{2\sqrt{a^2-4c}\,n}, \frac{b+n-1}{n}, \frac{\sqrt{a^2-4c}\,x^n}{n}\right)c_1\right)$$

✓ Solution by Mathematica

Time used: 0.38 (sec). Leaf size: 255

```
DSolve[x*y''[x]+(a*x^n+b)*y'[x]+(c*x^(2*n-1)+d*x^(n-1))*y[x]==0,y[x],x,IncludeSingularSoluti
```

$$y(x)$$

$$\to 2^{\frac{b+n-1}{2n}} x^{\frac{1}{2}-\frac{n}{2}} \left(x^n\right)^{\frac{n-1}{2n}} e^{-\frac{\left(\sqrt{a^2-4c}+a\right)x^n}{2n}} \left(c_1 \text{HypergeometricU}\left(\frac{(b+n-1)a^2+\sqrt{a^2-4c}(b+n-1)a-2\sqrt{}}{2\left(a^2-4c\right)n} \right.\right.$$

$$\left.\left. + c_2 L^{\frac{b-1}{n}}_{-\frac{(b+n-1)a^2+\sqrt{a^2-4c}(b+n-1)a-2\sqrt{a^2-4c}cd-4c(b+n-1)}{2\left(a^2-4c\right)n}}\left(\frac{\sqrt{a^2-4c}x^n}{n}\right)\right)\right)$$

28.40 problem 100

Internal problem ID [10924]

Book: Handbook of exact solutions for ordinary differential equations. By Polyanin and Zaitsev. Second edition
Section: Chapter 2, Second-Order Differential Equations. section 2.1.2-3 Equation of form $(ax+b)y''+f(x)y'+g(x)y=0$
Problem number: 100.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$\boxed{xy''+\left(a\,x^{n}+b\,x^{n-1}+2\right)y'+yx^{n-2}b=0}$$

✓ Solution by Maple
Time used: 0.344 (sec). Leaf size: 53

```
dsolve(x*diff(y(x),x$2)+(a*x^n+b*x^(n-1)+2)*diff(y(x),x)+(b*x^(n-2))*y(x)=0,y(x), singsol=al
```

$$y(x)=\frac{(ax+b)\left(c_2\left(\int \frac{\mathrm{e}^{-\frac{(ax(n-1)+bn)x^{n-1}}{n(n-1)}}}{(ax+b)^2}dx\right)+c_1\right)}{x}$$

✗ Solution by Mathematica
Time used: 0.0 (sec). Leaf size: 0

```
DSolve[x*y''[x]+(a*x^n+b*x^(n-1)+2)*y'[x]+(b*x^(n-2))*y[x]==0,y[x],x,IncludeSingularSolution
```

Not solved

28.41 problem 101

Internal problem ID [10925]

Book: Handbook of exact solutions for ordinary differential equations. By Polyanin and Zaitsev. Second edition
Section: Chapter 2, Second-Order Differential Equations. section 2.1.2-3 Equation of form $(ax+b)y''+f(x)y'+g(x)y=0$
Problem number: 101.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$\boxed{xy''+\left(a\,x^{n}+bx\right)y'+\left(ab\,x^{n}+an\,x^{n-1}-b\right)y=0}$$

✗ Solution by Maple

```
dsolve(x*diff(y(x),x$2)+(a*x^n+b*x)*diff(y(x),x)+(a*b*x^n+a*n*x^(n-1)-b)*y(x)=0,y(x), singso
```

No solution found

✗ Solution by Mathematica
Time used: 0.0 (sec). Leaf size: 0

```
DSolve[x*y''[x]+(a*x^n+b*x)*y'[x]+(a*b*x^n+a*n*x^(n-1)-b)*y[x]==0,y[x],x,IncludeSingularSolu
```

Not solved

28.42 problem 102

Internal problem ID [10926]

Book: Handbook of exact solutions for ordinary differential equations. By Polyanin and Zaitsev. Second edition
Section: Chapter 2, Second-Order Differential Equations. section 2.1.2-3 Equation of form $(ax+b)y''+f(x)y'+g(x)y=0$
Problem number: 102.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$\boxed{xy''+\left(ab\,x^n+b\,x^{n-1}+ax-1\right)y'+a^2b\,x^n y=0}$$

✗ Solution by Maple

```
dsolve(x*diff(y(x),x$2)+(a*b*x^n+b*x^(n-1)+a*x-1)*diff(y(x),x)+(a^2*b*x^n)*y(x)=0,y(x), sing
```

No solution found

✗ Solution by Mathematica
Time used: 0.0 (sec). Leaf size: 0

```
DSolve[x*y''[x]+(a*b*x^n+b*x^(n-1)+a*x-1)*y'[x]+(a^2*b*x^n)*y[x]==0,y[x],x,IncludeSingularSo
```

Not solved

28.43 problem 103

Internal problem ID [10927]

Book: Handbook of exact solutions for ordinary differential equations. By Polyanin and Zaitsev. Second edition
Section: Chapter 2, Second-Order Differential Equations. section 2.1.2-3 Equation of form $(ax+b)y''+f(x)y'+g(x)y=0$
Problem number: 103.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$\boxed{x y'' + \left(a\,x^n + b\,x^m + c\right) y' + (c-1)\left(a\,x^{n-1} + b\,x^{m-1}\right) y = 0}$$

✗ Solution by Maple

```
dsolve(x*diff(y(x),x$2)+(a*x^n+b*x^m+c)*diff(y(x),x)+(c-1)*(a*x^(n-1)+b*x^(m-1))*y(x)=0,y(x)
```

No solution found

✗ Solution by Mathematica
Time used: 0.0 (sec). Leaf size: 0

```
DSolve[x*y''[x]+(a*x^n+b*x^m+c)*y'[x]+(c-1)*(a*x^(n-1)+b*x^(m-1))*y[x]==0,y[x],x,IncludeSing
```

Not solved

28.44 problem 104

Internal problem ID [10928]

Book: Handbook of exact solutions for ordinary differential equations. By Polyanin and Zaitsev. Second edition
Section: Chapter 2, Second-Order Differential Equations. section 2.1.2-3 Equation of form $(ax+b)y''+f(x)y'+g(x)y=0$
Problem number: 104.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$\boxed{xy''+\left(x^{m+n}ab+x^n na+b\,x^m+1-2n\right)y'+a^2bn\,x^{2n+m-1}y=0}$$

✗ Solution by Maple

```
dsolve(x*diff(y(x),x$2)+(a*b*x^(n+m)+a*n*x^n+b*x^m+1-2*n)*diff(y(x),x)+a^2*b*n*x^(2*n+m-1)*y
```

No solution found

✗ Solution by Mathematica
Time used: 0.0 (sec). Leaf size: 0

```
DSolve[x*y''[x]+(a*b*x^(n+m)+a*n*x^n+b*x^m+1-2*n)*y'[x]+a^2*b*n*x^(2*n+m-1)*y[x]==0,y[x],x,I
```

Not solved

28.45 problem 105

Internal problem ID [10929]

Book: Handbook of exact solutions for ordinary differential equations. By Polyanin and Zaitsev. Second edition
Section: Chapter 2, Second-Order Differential Equations. section 2.1.2-3 Equation of form $(ax+b)y''+f(x)y'+g(x)y=0$
Problem number: 105.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _exact, _linear, _homogeneous]]`

$$\boxed{(x+a)\,y''+(bx+c)\,y'+yb=0}$$

✓ Solution by Maple
Time used: 0.031 (sec). Leaf size: 78

```
dsolve((x+a)*diff(y(x),x$2)+(b*x+c)*diff(y(x),x)+b*y(x)=0,y(x), singsol=all)
```

$$y(x)=-\left(-(a+x)^{ab-c+1}\,c_1+\left(\Gamma(-ab+c)+\Gamma(-ab+c-1,-b(a+x))\,(ab-c+1)\right)b(a+x)\,c_2(-b(a+x))^{ab-c}\right)\mathrm{e}^{-bx}$$

✓ Solution by Mathematica
Time used: 0.559 (sec). Leaf size: 90

```
DSolve[(x+a)*y''[x]+(b*x+c)*y'[x]+b*y[x]==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x)\to e^{-b(a+x)}(a+x)^{1-c}(-b(a+x))^{-c}\left(c_1e^{ab}(a+x)^{ab}(-b(a+x))^{c}+bc_2(-b(a+x))^{ab}(a+x)^{c}\Gamma(-ab+c-1,-b(a+x))\right)$$

28.46 problem 106

Internal problem ID [10930]

Book: Handbook of exact solutions for ordinary differential equations. By Polyanin and Zaitsev. Second edition
Section: Chapter 2, Second-Order Differential Equations. section 2.1.2-3 Equation of form $(ax+b)y''+f(x)y'+g(x)y=0$
Problem number: 106.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$\boxed{(a_1x+a_0)\,y''+(b_1x+b_0)\,y'-mb_1y=0}$$

✓ Solution by Maple
Time used: 0.063 (sec). Leaf size: 101

```
dsolve((a__1*x+a__0)*diff(y(x),x$2)+(b__1*x+b__0)*diff(y(x),x)-m*b__1*y(x)=0,y(x), singsol=a
```

$$\begin{aligned}y(x)=(a_1x&+a_0)^{\frac{a_0b_1+a_1^2-a_1b_0}{a_1^2}}\,\mathrm{e}^{-\frac{b_1x}{a_1}}\left(\operatorname{KummerM}\left(1+m,\frac{a_0b_1+2a_1^2-a_1b_0}{a_1^2},\frac{b_1(a_1x+a_0)}{a_1^2}\right)c_1\right.\\&\left.+\operatorname{KummerU}\left(1+m,\frac{a_0b_1+2a_1^2-a_1b_0}{a_1^2},\frac{b_1(a_1x+a_0)}{a_1^2}\right)c_2\right)\end{aligned}$$

✓ Solution by Mathematica
Time used: 0.278 (sec). Leaf size: 102

```
DSolve[(a1*x+a0)*y''[x]+(b1*x+b0)*y'[x]-m*b1*y[x]==0,y[x],x,IncludeSingularSolutions -> True
```

$$\begin{aligned}y(x)\to e^{-\frac{\mathrm{b1}x}{\mathrm{a1}}}(\mathrm{a0}+\mathrm{a1}x)^{\frac{\mathrm{a0b1}+\mathrm{a1}^2-\mathrm{a1b0}}{\mathrm{a1}^2}}&\left(c_1\operatorname{HypergeometricU}\left(m+1,-\frac{\mathrm{b0}}{\mathrm{a1}}+\frac{\mathrm{a0b1}}{\mathrm{a1}^2}\right.\right.\\&\left.\left.+2,\frac{\mathrm{b1}(\mathrm{a0}+\mathrm{a1}x)}{\mathrm{a1}^2}\right)+c_2L_{-m-1}^{\frac{\mathrm{a1}^2-\mathrm{b0a1}+\mathrm{a0b1}}{\mathrm{a1}^2}}\left(\frac{\mathrm{b1}(\mathrm{a0}+\mathrm{a1}x)}{\mathrm{a1}^2}\right)\right)\end{aligned}$$

28.47 problem 107

Internal problem ID [10931]

Book: Handbook of exact solutions for ordinary differential equations. By Polyanin and Zaitsev. Second edition
Section: Chapter 2, Second-Order Differential Equations. section 2.1.2-3 Equation of form $(ax+b)y''+f(x)y'+g(x)y=0$
Problem number: 107.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$\boxed{(ax+b)\,y''+s(cx+d)\,y'-s^2((a+c)\,x+b+d)\,y=0}$$

✓ Solution by Maple
Time used: 0.031 (sec). Leaf size: 166

```
dsolve((a*x+b)*diff(y(x),x$2)+s*(c*x+d)*diff(y(x),x)-s^2*((a+c)*x+b+d)*y(x)=0,y(x), singsol=
```

$$y(x)$$
$$= \frac{\left(((-c_1+c_2)\,a^2+adsc_1-bcsc_1)\,\Gamma\left(\frac{-dsa+bcs+a^2}{a^2},\frac{s(2a+c)(ax+b)}{a^2}\right)+\Gamma\left(\frac{-dsa+bcs+2a^2}{a^2}\right)c_1a^2\right)(ax+b)^{\frac{-dsa+bcs+}{a^2}}}{a^2}$$

✓ Solution by Mathematica
Time used: 1.269 (sec). Leaf size: 122

```
DSolve[(a*x+b)*y''[x]+s*(c*x+d)*y'[x]-s^2*((a+c)*x+b+d)*y[x]==0,y[x],x,IncludeSingularSoluti
```

$$y(x)$$
$$\to c_1e^{sx}$$
$$-\frac{c_2e^{s\left(\frac{b(2a+c)}{a^2}+x\right)}(ax+b)^{\frac{s(bc-ad)}{a^2}+1}\left(\frac{s(2a+c)(ax+b)}{a^2}\right)^{\frac{s(ad-bc)}{a^2}-1}\Gamma\left(\frac{a^2-dsa+bcs}{a^2},\frac{(2a+c)s(b+ax)}{a^2}\right)}{a}$$

28.48 problem 108

Internal problem ID [10932]

Book: Handbook of exact solutions for ordinary differential equations. By Polyanin and Zaitsev. Second edition
Section: Chapter 2, Second-Order Differential Equations. section 2.1.2-3 Equation of form $(ax+b)y''+f(x)y'+g(x)y=0$
Problem number: 108.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$\boxed{(a_2x+b_2)\,y''+(a_1x+b_1)\,y'+(a_0x+b_0)\,y=0}$$

✓ Solution by Maple
Time used: 0.047 (sec). Leaf size: 248

```
dsolve((a__2*x+b__2)*diff(y(x),x$2)+(a__1*x+b__1)*diff(y(x),x)+(a__0*x+b__0)*y(x)=0,y(x), si
```

$$y(x)=(a_2x+b_2)^{\frac{a_1b_2+a_2^2-a_2b_1}{a_2^2}}\,\mathrm{e}^{-\frac{\left(\sqrt{-4a_0a_2+a_1^2}+a_1\right)x}{2a_2}}\left(\mathrm{KummerM}\left(\frac{(a_1b_2+2a_2^2-a_2b_1)\sqrt{-4a_0a_2+a_1^2}-2a_2^2b_0+(2a_0b_2-\right.\right.$$

$$\left.\left.+\mathrm{KummerU}\left(\frac{(a_1b_2+2a_2^2-a_2b_1)\sqrt{-4a_0a_2+a_1^2}-2a_2^2b_0+(2a_0b_2+b_1a_1)a_2-a_1^2b_2}{2\sqrt{-4a_0a_2+a_1^2}\,a_2^2},\frac{a_1b_2+2a_2^2-a_2b}{a_2^2}\right.\right.\right.$$

✓ Solution by Mathematica

Time used: 0.526 (sec). Leaf size: 301

```
DSolve[(a2*x+b2)*y''[x]+(a1*x+b1)*y'[x]+(a0*x+b0)*y[x]==0,y[x],x,IncludeSingularSolutions ->
```

$$y(x) \to e^{-\frac{x\left(\sqrt{\text{a1}^2-4\text{a0a2}}+\text{a1}\right)}{2\text{a2}}}(\text{a2}x+\text{b2})^{\frac{\text{a1b2}+\text{a2}^2-\text{a2b1}}{\text{a2}^2}}\left(c_1\,\text{HypergeometricU}\left(\frac{2\left(\sqrt{\text{a1}^2-4\text{a0a2}}-\text{b0}\right)\text{a2}^2+\left(\text{a1b1}-\sqrt{\text{a1}^2-4\text{a0a2}}\text{b1}+2\right.}{2\text{a2}^2\sqrt{\text{a1}^2-4\text{a0a2}}}\right.\right.$$

$$\left.\left.-\frac{\text{b1}}{\text{a2}}+\frac{\text{a1b2}}{\text{a2}^2}+2,\frac{\sqrt{\text{a1}^2-4\text{a0a2}}(\text{b2}+\text{a2}x)}{\text{a2}^2}\right)\right.$$

$$\left.+c_2L_{\frac{\frac{\text{a2}^2-\text{b1a2}+\text{a1b2}}{\text{a2}^2}}{\frac{-2\left(\sqrt{\text{a1}^2-4\text{a0a2}}-\text{b0}\right)\text{a2}^2+\left(-\text{a1b1}+\sqrt{\text{a1}^2-4\text{a0a2}}\text{b1}-2\text{a0b2}\right)\text{a2}+\text{a1}\left(\text{a1}-\sqrt{\text{a1}^2-4\text{a0a2}}\right)\text{b2}}{2\text{a2}^2\sqrt{\text{a1}^2-4\text{a0a2}}}}}\left(\frac{\sqrt{\text{a1}^2-4\text{a0a2}}(\text{b2}+\text{a2}x)}{\text{a2}^2}\right)\right)$$

28.49 problem 109

Internal problem ID [10933]

Book: Handbook of exact solutions for ordinary differential equations. By Polyanin and Zaitsev. Second edition
Section: Chapter 2, Second-Order Differential Equations. section 2.1.2-3 Equation of form $(ax+b)y''+f(x)y'+g(x)y=0$
Problem number: 109.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries], [_2nd_order, _linear,`

$$\boxed{(x+\gamma)\,y''+(a\,x^n+b\,x^m+c)\,y'+\left(an\,x^{n-1}+x^{m-1}bm\right)y=0}$$

✓ Solution by Maple
Time used: 0.0 (sec). Leaf size: 63

```
dsolve((x+gamma)*diff(y(x),x$2)+(a*x^n+b*x^m+c)*diff(y(x),x)+(a*n*x^(n-1)+b*m*x^(m-1))*y(x)=
```

$$y(x)=\left(c_1\left(\int\frac{\mathrm{e}^{\int\frac{a\,x^n+b\,x^m+c-1}{x+\gamma}dx}}{x+\gamma}dx\right)+c_2\right)\mathrm{e}^{-\left(\int\frac{a\,x^n+b\,x^m+c-1}{x+\gamma}dx\right)}$$

✗ Solution by Mathematica
Time used: 0.0 (sec). Leaf size: 0

```
DSolve[(x+\[Gamma])*y''[x]+(a*x^n+b*x^m+c)*y'[x]+(a*n*x^(n-1)+b*m*x^(m-1))*y[x]==0,y[x],x,In
```

Not solved

29 Chapter 2, Second-Order Differential Equations. section 2.1.2-4 Equation of form $x^2y'' + f(x)y' + g(x)y = 0$

29.1 problem 110 737
29.2 problem 111 738
29.3 problem 112 739
29.4 problem 113 740
29.5 problem 114 741
29.6 problem 115 742
29.7 problem 116 743
29.8 problem 117 744
29.9 problem 118 745
29.10problem 119 746
29.11problem 120 747
29.12problem 121 748
29.13problem 122 749
29.14problem 123 750
29.15problem 124 751
29.16problem 125 752
29.17problem 126 753
29.18problem 127 754
29.19problem 128 755
29.20problem 129 756
29.21problem 130 757
29.22problem 131 758
29.23problem 132 759
29.24problem 133 761
29.25problem 134 762
29.26problem 135 764
29.27problem 136 765
29.28problem 137 766
29.29problem 138 767
29.30problem 139 768
29.31problem 140 770
29.32problem 141 771
29.33problem 142 772
29.34problem 143 773

29.35problem 144 774
29.36problem 145 775
29.37problem 146 776
29.38problem 147 778
29.39problem 148 779

29.1 problem 110

Internal problem ID [10934]

Book: Handbook of exact solutions for ordinary differential equations. By Polyanin and Zaitsev. Second edition
Section: Chapter 2, Second-Order Differential Equations. section 2.1.2-4 Equation of form $x^2y'' + f(x)y' + g(x)y = 0$
Problem number: 110.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_Emden, _Fowler]]`

$$\boxed{x^2y'' + ay = 0}$$

✓ Solution by Maple
Time used: 0.0 (sec). Leaf size: 35

```
dsolve(x^2*diff(y(x),x$2)+a*y(x)=0,y(x), singsol=all)
```

$$y(x) = \sqrt{x}\left(c_1 x^{\frac{\sqrt{-4a+1}}{2}} + c_2 x^{-\frac{\sqrt{-4a+1}}{2}}\right)$$

✓ Solution by Mathematica
Time used: 0.057 (sec). Leaf size: 42

```
DSolve[x^2*y''[x]+a*y[x]==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to x^{\frac{1}{2}-\frac{1}{2}\sqrt{1-4a}}\left(c_2 x^{\sqrt{1-4a}} + c_1\right)$$

29.2 problem 111

Internal problem ID [10935]

Book: Handbook of exact solutions for ordinary differential equations. By Polyanin and Zaitsev. Second edition
Section: Chapter 2, Second-Order Differential Equations. section 2.1.2-4 Equation of form $x^2y'' + f(x)y' + g(x)y = 0$
Problem number: 111.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$\boxed{x^2y'' + (ax+b)\,y = 0}$$

✓ Solution by Maple
Time used: 0.0 (sec). Leaf size: 45

```
dsolve(x^2*diff(y(x),x$2)+(a*x+b)*y(x)=0,y(x), singsol=all)
```

$$y(x) = \left(\text{BesselJ}\left(\sqrt{-4b+1}, 2\sqrt{x}\,\sqrt{a}\right) c_1 + \text{BesselY}\left(\sqrt{-4b+1}, 2\sqrt{x}\,\sqrt{a}\right) c_2\right)\sqrt{x}$$

✓ Solution by Mathematica
Time used: 0.147 (sec). Leaf size: 95

```
DSolve[x^2*y''[x]+(a*x+b)*y[x]==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to \sqrt{a}\sqrt{x}\left(c_1 \,\text{Gamma}\left(1-\sqrt{1-4b}\right)\text{BesselJ}\left(-\sqrt{1-4b}, 2\sqrt{a}\sqrt{x}\right) + c_2 \,\text{Gamma}\left(\sqrt{1-4b}+1\right)\text{BesselJ}\left(\sqrt{1-4b}, 2\sqrt{a}\sqrt{x}\right)\right)$$

29.3 problem 112

Internal problem ID [10936]

Book: Handbook of exact solutions for ordinary differential equations. By Polyanin and Zaitsev. Second edition
Section: Chapter 2, Second-Order Differential Equations. section 2.1.2-4 Equation of form $x^2y'' + f(x)y' + g(x)y = 0$
Problem number: 112.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$\boxed{x^2y'' + \left(a^2x^2 - n(n+1)\right)y = 0}$$

✓ Solution by Maple
Time used: 0.0 (sec). Leaf size: 27

```
dsolve(x^2*diff(y(x),x$2)+(a^2*x^2-n*(n+1))*y(x)=0,y(x), singsol=all)
```

$$y(x) = \left(\text{BesselJ}\left(n + \frac{1}{2}, ax\right)c_1 + \text{BesselY}\left(n + \frac{1}{2}, ax\right)c_2\right)\sqrt{x}$$

✓ Solution by Mathematica
Time used: 0.056 (sec). Leaf size: 36

```
DSolve[x^2*y''[x]+(a^2*x^2-n*(n+1))*y[x]==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to \sqrt{x}\left(c_1 \text{BesselJ}\left(n + \frac{1}{2}, ax\right) + c_2 \text{BesselY}\left(n + \frac{1}{2}, ax\right)\right)$$

29.4 problem 113

Internal problem ID [10937]

Book: Handbook of exact solutions for ordinary differential equations. By Polyanin and Zaitsev. Second edition
Section: Chapter 2, Second-Order Differential Equations. section 2.1.2-4 Equation of form $x^2y'' + f(x)y' + g(x)y = 0$
Problem number: 113.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$\boxed{x^2y'' - \left(a^2x^2 + n(n+1)\right)y = 0}$$

✓ Solution by Maple
Time used: 0.0 (sec). Leaf size: 39

```
dsolve(x^2*diff(y(x),x$2)-(a^2*x^2+n*(n+1))*y(x)=0,y(x), singsol=all)
```

$$y(x) = \sqrt{x}\left(\text{BesselJ}\left(n+\frac{1}{2}, \sqrt{-a^2}\,x\right)c_1 + \text{BesselY}\left(n+\frac{1}{2}, \sqrt{-a^2}\,x\right)c_2\right)$$

✓ Solution by Mathematica
Time used: 0.054 (sec). Leaf size: 42

```
DSolve[x^2*y''[x]-(a^2*x^2+n*(n+1))*y[x]==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to \sqrt{x}\left(c_1 \text{BesselJ}\left(n+\frac{1}{2}, -iax\right) + c_2 \text{BesselY}\left(n+\frac{1}{2}, -iax\right)\right)$$

29.5 problem 114

Internal problem ID [10938]

Book: Handbook of exact solutions for ordinary differential equations. By Polyanin and Zaitsev. Second edition
Section: Chapter 2, Second-Order Differential Equations. section 2.1.2-4 Equation of form $x^2y'' + f(x)y' + g(x)y = 0$
Problem number: 114.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$\boxed{x^2y'' - \left(a^2x^2 + 2abx + b^2 - b\right)y = 0}$$

✓ Solution by Maple
Time used: 0.016 (sec). Leaf size: 29

```
dsolve(x^2*diff(y(x),x$2)-(a^2*x^2+2*a*b*x+b^2-b)*y(x)=0,y(x), singsol=all)
```

$$y(x) = c_1 x^b \mathrm{e}^{ax} + c_2 \,\mathrm{WhittakerM}\left(-b, \frac{1}{2} - b, 2ax\right)$$

✓ Solution by Mathematica
Time used: 0.055 (sec). Leaf size: 38

```
DSolve[x^2*y''[x]-(a^2*x^2+2*a*b*x+b^2-b)*y[x]==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to c_1 M_{-b,b-\frac{1}{2}}(2ax) + c_2 W_{-b,b-\frac{1}{2}}(2ax)$$

29.6 problem 115

Internal problem ID [10939]

Book: Handbook of exact solutions for ordinary differential equations. By Polyanin and Zaitsev. Second edition
Section: Chapter 2, Second-Order Differential Equations. section 2.1.2-4 Equation of form $x^2y'' + f(x)y' + g(x)y = 0$
Problem number: 115.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$\boxed{x^2y'' + \left(a\,x^2 + bx + c\right)y = 0}$$

✓ Solution by Maple
Time used: 0.078 (sec). Leaf size: 57

```
dsolve(x^2*diff(y(x),x$2)+(a*x^2+b*x+c)*y(x)=0,y(x), singsol=all)
```

$$y(x) = c_1\,\text{WhittakerM}\left(-\frac{ib}{2\sqrt{a}}, \frac{\sqrt{-4c+1}}{2}, 2i\sqrt{a}\,x\right) + c_2\,\text{WhittakerW}\left(-\frac{ib}{2\sqrt{a}}, \frac{\sqrt{-4c+1}}{2}, 2i\sqrt{a}\,x\right)$$

✓ Solution by Mathematica
Time used: 0.067 (sec). Leaf size: 88

```
DSolve[x^2*y''[x]+(a*x^2+b*x+c)*y[x]==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to c_1 M_{-\frac{ib}{2\sqrt{a}},-\frac{1}{2}i\sqrt{4c-1}}\left(2i\sqrt{a}x\right) + c_2 W_{-\frac{ib}{2\sqrt{a}},-\frac{1}{2}i\sqrt{4c-1}}\left(2i\sqrt{a}x\right)$$

29.7 problem 116

Internal problem ID [10940]

Book: Handbook of exact solutions for ordinary differential equations. By Polyanin and Zaitsev. Second edition
Section: Chapter 2, Second-Order Differential Equations. section 2.1.2-4 Equation of form $x^2y'' + f(x)y' + g(x)y = 0$
Problem number: 116.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$\boxed{x^2y'' - \left(a\,x^3 + \frac{5}{16}\right)y = 0}$$

✓ Solution by Maple
Time used: 0.032 (sec). Leaf size: 31

```
dsolve(x^2*diff(y(x),x$2)-(a*x^3+5/16)*y(x)=0,y(x), singsol=all)
```

$$y(x) = \frac{c_1 \sinh\left(\frac{2x^{\frac{3}{2}}\sqrt{a}}{3}\right) + c_2 \cosh\left(\frac{2x^{\frac{3}{2}}\sqrt{a}}{3}\right)}{x^{\frac{1}{4}}}$$

✓ Solution by Mathematica
Time used: 0.102 (sec). Leaf size: 60

```
DSolve[x^2*y''[x]-(a*x^3+5/16)*y[x]==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to \frac{e^{-\frac{2}{3}\sqrt{a}x^{3/2}}\left(2c_1 e^{\frac{4}{3}\sqrt{a}x^{3/2}} - \frac{c_2}{\sqrt{a}}\right)}{2\sqrt[4]{x}}$$

29.8 problem 117

Internal problem ID [10941]

Book: Handbook of exact solutions for ordinary differential equations. By Polyanin and Zaitsev. Second edition
Section: Chapter 2, Second-Order Differential Equations. section 2.1.2-4 Equation of form $x^2y'' + f(x)y' + g(x)y = 0$
Problem number: 117.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$\boxed{x^2y'' - \left(a^2x^4 + a(-1+2b)\,x^2 + b(1+b)\right)y = 0}$$

✓ Solution by Maple
Time used: 0.0 (sec). Leaf size: 38

```
dsolve(x^2*diff(y(x),x$2)-(a^2*x^4+a*(2*b-1)*x^2+b*(b+1))*y(x)=0,y(x), singsol=all)
```

$$y(x) = x^{-b}\mathrm{e}^{-\frac{a\,x^2}{2}}\left(c_2\Gamma\left(b+\frac{1}{2}\right) - c_2\Gamma\left(b+\frac{1}{2},-a\,x^2\right) + c_1\right)$$

✓ Solution by Mathematica
Time used: 0.45 (sec). Leaf size: 66

```
DSolve[x^2*y''[x]-(a^2*x^4+a*(2*b-1)*x^2+b*(b+1))*y[x]==0,y[x],x,IncludeSingularSolutions ->
```

$$y(x) \to \frac{1}{2}e^{-\frac{ax^2}{2}}x^{-b}\left(ac_2x^{2b+3}\left(-ax^2\right)^{-b-\frac{3}{2}}\Gamma\left(b+\frac{1}{2},-ax^2\right) + 2c_1\right)$$

29.9 problem 118

Internal problem ID [10942]

Book: Handbook of exact solutions for ordinary differential equations. By Polyanin and Zaitsev. Second edition
Section: Chapter 2, Second-Order Differential Equations. section 2.1.2-4 Equation of form $x^2y'' + f(x)y' + g(x)y = 0$
Problem number: 118.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$\boxed{x^2y'' + (a\,x^n + b)\,y = 0}$$

✓ Solution by Maple
Time used: 0.047 (sec). Leaf size: 63

```
dsolve(x^2*diff(y(x),x$2)+(a*x^n+b)*y(x)=0,y(x), singsol=all)
```

$$y(x) = \left(\text{BesselJ}\left(\frac{\sqrt{-4b+1}}{n}, \frac{2\sqrt{a}\,x^{\frac{n}{2}}}{n}\right)c_1 + \text{BesselY}\left(\frac{\sqrt{-4b+1}}{n}, \frac{2\sqrt{a}\,x^{\frac{n}{2}}}{n}\right)c_2\right)\sqrt{x}$$

✓ Solution by Mathematica
Time used: 0.343 (sec). Leaf size: 351

```
DSolve[x^2*y''[x]+(a*x^n+b)*y[x]==0,y[x],x,IncludeSingularSolutions -> True]
```

$$\begin{aligned} &y(x) \\ &\to n^{-\frac{\sqrt{(1-4b)n^2}+i\sqrt{4b-1}n+n}{n^2}} a^{\frac{-\sqrt{(1-4b)n^2}-i\sqrt{4b-1}n+n}{2n^2}} (x^n)^{\frac{-\sqrt{(1-4b)n^2}-i\sqrt{4b-1}n+n}{2n^2}} \left(c_2 n^{\frac{2\sqrt{(1-4b)n^2}}{n^2}} a^{\frac{i\sqrt{4b-1}}{n}} (x^n)^{\frac{i\sqrt{4b-1}}{n}} \text{Gamma}\right. \\ &\qquad + c_1 n^{\frac{2i\sqrt{4b-1}}{n}} a^{\frac{\sqrt{(1-4b)n^2}}{n^2}} (x^n)^{\frac{\sqrt{(1-4b)n^2}}{n^2}} \text{Gamma}\left(1\right. \\ &\qquad \left.\left. - \frac{\sqrt{1-4b}}{n}\right) \text{BesselJ}\left(-\frac{\sqrt{(1-4b)n^2}}{n^2}, \frac{2\sqrt{a}\sqrt{x^n}}{n}\right)\right) \end{aligned}$$

29.10 problem 119

Internal problem ID [10943]

Book: Handbook of exact solutions for ordinary differential equations. By Polyanin and Zaitsev. Second edition
Section: Chapter 2, Second-Order Differential Equations. section 2.1.2-4 Equation of form $x^2y'' + f(x)y' + g(x)y = 0$
Problem number: 119.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$\boxed{x^2y'' - \left(x^{2n}a^2 + a(2b+n-1)\,x^n + b(b-1)\right)y = 0}$$

✓ Solution by Maple
Time used: 0.094 (sec). Leaf size: 134

```
dsolve(x^2*diff(y(x),x$2)-(a^2*x^(2*n)+a*(2*b+n-1)*x^n+b*(b-1))*y(x)=0,y(x), singsol=all)
```

$$y(x) = 2\left(b - \frac{1}{2} - \frac{n}{2}\right)^2 c_2 x^{-\frac{3n}{2}+\frac{1}{2}}\,\text{WhittakerM}\left(\frac{n-2b+1}{2n}, -\frac{2b-2n-1}{2n}, \frac{2a\,x^n}{n}\right) + n\left(\left(-b+\frac{1}{2}+\frac{n}{2}\right)x^{-\frac{3n}{2}+\frac{1}{2}} + x^{-\frac{n}{2}+\frac{1}{2}}a\right)c_2\,\text{WhittakerM}\left(-\frac{2b+n-1}{2n}, -\frac{2b-2n-1}{2n}, \frac{2a\,x^n}{n}\right) + c_1 x^b \mathrm{e}^{\frac{a\,x^n}{n}}$$

✗ Solution by Mathematica
Time used: 0.0 (sec). Leaf size: 0

```
DSolve[x^2*y''[x]-(a^2*x^(2*n)+a*(2*b+n-1)*x^n+b*(b-1))*y[x]==0,y[x],x,IncludeSingularSoluti
```

Not solved

29.11 problem 120

Internal problem ID [10944]

Book: Handbook of exact solutions for ordinary differential equations. By Polyanin and Zaitsev. Second edition
Section: Chapter 2, Second-Order Differential Equations. section 2.1.2-4 Equation of form $x^2y'' + f(x)y' + g(x)y = 0$
Problem number: 120.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$\boxed{x^2y'' + \left(a\,x^{2n} + b\,x^n + c\right)y = 0}$$

✓ Solution by Maple
Time used: 0.172 (sec). Leaf size: 90

```
dsolve(x^2*diff(y(x),x$2)+(a*x^(2*n)+b*x^n+c)*y(x)=0,y(x), singsol=all)
```

$$y(x) = x^{-\frac{n}{2}}\sqrt{x}\left(\text{WhittakerW}\left(-\frac{ib}{2\sqrt{a}\,n}, \frac{i\sqrt{4c-1}}{2n}, \frac{2i\sqrt{a}\,x^n}{n}\right)c_2 + \text{WhittakerM}\left(-\frac{ib}{2\sqrt{a}\,n}, \frac{i\sqrt{4c-1}}{2n}, \frac{2i\sqrt{a}\,x^n}{n}\right)c_1\right)$$

✓ Solution by Mathematica
Time used: 0.313 (sec). Leaf size: 236

```
DSolve[x^2*y''[x]+(a*x^(2*n)+b*x^n+c)*y[x]==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to 2^{\frac{\sqrt{(1-4c)n^2}+n^2}{2n^2}} x^{\frac{1}{2}-\frac{n}{2}} e^{\frac{i\sqrt{a}x^n}{n}} (x^n)^{\frac{\sqrt{(1-4c)n^2}+n^2}{2n^2}} \left(c_1 \text{HypergeometricU}\left(\frac{1}{2}\left(-\frac{ib}{\sqrt{a}n} + \frac{\sqrt{(1-4c)n^2}}{n^2} + 1\right), \frac{\sqrt{(1-4c)n^2}}{n^2} + 1, -\frac{2i\sqrt{a}x^n}{n}\right) + c_2 L^{\frac{\sqrt{(1-4c)n^2}}{n^2}}_{\frac{1}{2}\left(\frac{ib}{\sqrt{a}n} - \frac{\sqrt{(1-4c)n^2}}{n^2} - 1\right)}\left(-\frac{2i\sqrt{a}x^n}{n}\right)\right)$$

29.12 problem 121

Internal problem ID [10945]

Book: Handbook of exact solutions for ordinary differential equations. By Polyanin and Zaitsev. Second edition
Section: Chapter 2, Second-Order Differential Equations. section 2.1.2-4 Equation of form $x^2y'' + f(x)y' + g(x)y = 0$
Problem number: 121.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$\boxed{x^2y'' + \left(a\,x^{3n} + b\,x^{2n} + \frac{1}{4} - \frac{n^2}{4}\right)y = 0}$$

✓ Solution by Maple
Time used: 0.297 (sec). Leaf size: 177

```
dsolve(x^2*diff(y(x),x$2)+(a*x^(3*n)+b*x^(2*n)+1/4-1/4*n^2)*y(x)=0,y(x), singsol=all)
```

$$y(x) = \frac{\left(\frac{2\,3^{\frac{5}{6}}\pi c_2(a\,x^n+b)\,\mathrm{BesselI}\left(\frac{1}{3},\frac{2\sqrt{\frac{-x^{3n}a^3-3x^{2n}a^2b-3x^n a\,b^2-b^3}{n^2a^2}}}{3}\right)}{3} + c_1\,\mathrm{BesselI}\left(-\frac{1}{3},\frac{2\sqrt{\frac{-x^{3n}a^3-3x^{2n}a^2b-3x^n a\,b^2-b^3}{n^2a^2}}}{3}\right)\Gamma\left(\frac{2}{3}\right)^2 3^{\frac{1}{3}}\right)}{3\left(-\frac{(a\,x^n+b)^3}{a^2n^2}\right)^{\frac{1}{6}}\Gamma\left(\frac{2}{3}\right)}$$

✗ Solution by Mathematica
Time used: 0.0 (sec). Leaf size: 0

```
DSolve[x^2*y''[x]+(a*x^(3*n)+b*x^(2*n)+1/4-1/4*n^2)*y[x]==0,y[x],x,IncludeSingularSolutions
```

Not solved

29.13 problem 122

Internal problem ID [10946]

Book: Handbook of exact solutions for ordinary differential equations. By Polyanin and Zaitsev. Second edition
Section: Chapter 2, Second-Order Differential Equations. section 2.1.2-4 Equation of form $x^2y'' + f(x)y' + g(x)y = 0$
Problem number: 122.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$\boxed{x^2y'' + \left(a\,x^{2n}(b\,x^n + c)^m + \frac{1}{4} - \frac{n^2}{4}\right)y = 0}$$

✗ Solution by Maple

```
dsolve(x^2*diff(y(x),x$2)+(a*x^(2*n)*(b*x^n+c)^m+1/4-1/4*n^2)*y(x)=0,y(x), singsol=all)
```

No solution found

✗ Solution by Mathematica
Time used: 0.0 (sec). Leaf size: 0

```
DSolve[x^2*y''[x]+(a*x^(2*n)*(b*x^n+c)^m+1/4-1/4*n^2)*y[x]==0,y[x],x,IncludeSingularSolution
```

Not solved

29.14 problem 123

Internal problem ID [10947]

Book: Handbook of exact solutions for ordinary differential equations. By Polyanin and Zaitsev. Second edition
Section: Chapter 2, Second-Order Differential Equations. section 2.1.2-4 Equation of form $x^2y'' + f(x)y' + g(x)y = 0$
Problem number: 123.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_Emden, _Fowler]]`

$$\boxed{x^2y'' + axy' + yb = 0}$$

✓ Solution by Maple
Time used: 0.016 (sec). Leaf size: 52

```
dsolve(x^2*diff(y(x),x$2)+a*x*diff(y(x),x)+b*y(x)=0,y(x), singsol=all)
```

$$y(x) = x^{-\frac{a}{2}}\sqrt{x}\left(x^{\frac{\sqrt{a^2-2a-4b+1}}{2}}c_1 + x^{-\frac{\sqrt{a^2-2a-4b+1}}{2}}c_2\right)$$

✓ Solution by Mathematica
Time used: 0.069 (sec). Leaf size: 57

```
DSolve[x^2*y''[x]+a*x*y'[x]+b*y[x]==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to x^{\frac{1}{2}\left(-\sqrt{a^2-2a-4b+1}-a+1\right)}\left(c_2 x^{\sqrt{a^2-2a-4b+1}} + c_1\right)$$

29.15 problem 124

Internal problem ID [10948]

Book: Handbook of exact solutions for ordinary differential equations. By Polyanin and Zaitsev. Second edition
Section: Chapter 2, Second-Order Differential Equations. section 2.1.2-4 Equation of form $x^2y'' + f(x)y' + g(x)y = 0$
Problem number: 124.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$\boxed{x^2y'' + y'x + \left(x^2 - \left(n + \frac{1}{2}\right)^2\right)y = 0}$$

✓ Solution by Maple
Time used: 0.0 (sec). Leaf size: 19

```
dsolve(x^2*diff(y(x),x$2)+x*diff(y(x),x)+(x^2-(n+1/2)^2)*y(x)=0,y(x), singsol=all)
```

$$y(x) = c_1 \operatorname{BesselJ}\left(n + \frac{1}{2}, x\right) + c_2 \operatorname{BesselY}\left(n + \frac{1}{2}, x\right)$$

✓ Solution by Mathematica
Time used: 0.387 (sec). Leaf size: 26

```
DSolve[x^2*y''[x]+x*y'[x]+(x^2-(n+1/2)^2)*y[x]==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to c_1 \operatorname{BesselJ}\left(n + \frac{1}{2}, x\right) + c_2 \operatorname{BesselY}\left(n + \frac{1}{2}, x\right)$$

29.16 problem 125

Internal problem ID [10949]

Book: Handbook of exact solutions for ordinary differential equations. By Polyanin and Zaitsev. Second edition
Section: Chapter 2, Second-Order Differential Equations. section 2.1.2-4 Equation of form $x^2y'' + f(x)y' + g(x)y = 0$
Problem number: 125.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$\boxed{x^2y'' + y'x - \left(x^2 + \left(n + \frac{1}{2}\right)^2\right)y = 0}$$

✓ Solution by Maple
Time used: 0.0 (sec). Leaf size: 19

```
dsolve(x^2*diff(y(x),x$2)+x*diff(y(x),x)-(x^2+(n+1/2)^2)*y(x)=0,y(x), singsol=all)
```

$$y(x) = c_1 \operatorname{BesselI}\left(n + \frac{1}{2}, x\right) + c_2 \operatorname{BesselK}\left(n + \frac{1}{2}, x\right)$$

✓ Solution by Mathematica
Time used: 0.056 (sec). Leaf size: 34

```
DSolve[x^2*y''[x]+x*y'[x]-(x^2+(n+1/2)^2)*y[x]==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to c_1 \operatorname{BesselJ}\left(n + \frac{1}{2}, -ix\right) + c_2 \operatorname{BesselY}\left(n + \frac{1}{2}, -ix\right)$$

29.17 problem 126

Internal problem ID [10950]

Book: Handbook of exact solutions for ordinary differential equations. By Polyanin and Zaitsev. Second edition
Section: Chapter 2, Second-Order Differential Equations. section 2.1.2-4 Equation of form $x^2y'' + f(x)y' + g(x)y = 0$
Problem number: 126.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[_Bessel]`

$$\boxed{x^2y'' + y'x + \left(-\nu^2 + x^2\right) y = 0}$$

✓ Solution by Maple
Time used: 0.0 (sec). Leaf size: 15

```
dsolve(x^2*diff(y(x),x$2)+x*diff(y(x),x)+(x^2-nu^2)*y(x)=0,y(x), singsol=all)
```

$$y(x) = c_1 \operatorname{BesselJ}(\nu, x) + c_2 \operatorname{BesselY}(\nu, x)$$

✓ Solution by Mathematica
Time used: 0.102 (sec). Leaf size: 26

```
DSolve[x^2*y''[x]+x*y'[x]+(x^2-\[Nu])*y[x]==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to c_1 \operatorname{BesselJ}\left(\sqrt{\nu}, x\right) + c_2 \operatorname{BesselY}\left(\sqrt{\nu}, x\right)$$

29.18 problem 127

Internal problem ID [10951]

Book: Handbook of exact solutions for ordinary differential equations. By Polyanin and Zaitsev. Second edition
Section: Chapter 2, Second-Order Differential Equations. section 2.1.2-4 Equation of form $x^2y'' + f(x)y' + g(x)y = 0$
Problem number: 127.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_Bessel, _modified]]`

$$\boxed{x^2y'' + y'x - \left(\nu^2 + x^2\right)y = 0}$$

✓ Solution by Maple
Time used: 0.0 (sec). Leaf size: 15

```
dsolve(x^2*diff(y(x),x$2)+x*diff(y(x),x)-(x^2+nu^2)*y(x)=0,y(x), singsol=all)
```

$$y(x) = c_1 \operatorname{BesselI}(\nu, x) + c_2 \operatorname{BesselK}(\nu, x)$$

✓ Solution by Mathematica
Time used: 0.038 (sec). Leaf size: 34

```
DSolve[x^2*y''[x]+x*y'[x]-(x^2+\[Nu])*y[x]==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to c_1 \operatorname{BesselJ}\left(\sqrt{\nu}, -ix\right) + c_2 \operatorname{BesselY}\left(\sqrt{\nu}, -ix\right)$$

29.19 problem 128

Internal problem ID [10952]

Book: Handbook of exact solutions for ordinary differential equations. By Polyanin and Zaitsev. Second edition
Section: Chapter 2, Second-Order Differential Equations. section 2.1.2-4 Equation of form $x^2y'' + f(x)y' + g(x)y = 0$
Problem number: 128.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$\boxed{x^2y'' + 2y'x - \left(a^2x^2 + 2\right)y = 0}$$

✓ Solution by Maple
Time used: 0.016 (sec). Leaf size: 32

```
dsolve(x^2*diff(y(x),x$2)+2*x*diff(y(x),x)-(a^2*x^2+2)*y(x)=0,y(x), singsol=all)
```

$$y(x) = \frac{c_2 \mathrm{e}^{-ax}(ax+1) + c_1 \mathrm{e}^{ax}(ax-1)}{x^2}$$

✓ Solution by Mathematica
Time used: 0.042 (sec). Leaf size: 29

```
DSolve[x^2*y''[x]+2*x*y'[x]-(a^2*x^2+2)*y[x]==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to c_1 j_{-2}(iax) - c_2 y_{-2}(iax)$$

29.20 problem 129

Internal problem ID [10953]

Book: Handbook of exact solutions for ordinary differential equations. By Polyanin and Zaitsev. Second edition
Section: Chapter 2, Second-Order Differential Equations. section 2.1.2-4 Equation of form $x^2y'' + f(x)y' + g(x)y = 0$
Problem number: 129.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$\boxed{x^2y'' - 2axy' + \left(b^2x^2 + a(a+1)\right)y = 0}$$

✓ Solution by Maple
Time used: 0.016 (sec). Leaf size: 21

```
dsolve(x^2*diff(y(x),x$2)-2*a*x*diff(y(x),x)+(b^2*x^2+a*(a+1))*y(x)=0,y(x), singsol=all)
```

$$y(x) = x^a(c_1 \sin(bx) + c_2 \cos(bx))$$

✓ Solution by Mathematica
Time used: 0.066 (sec). Leaf size: 42

```
DSolve[x^2*y''[x]-2*a*x*y'[x]+(b^2*x^2+a*(a+1))*y[x]==0,y[x],x,IncludeSingularSolutions -> T
```

$$y(x) \to c_1 x^a e^{-ibx} - \frac{ic_2 x^a e^{ibx}}{2b}$$

29.21 problem 130

Internal problem ID [10954]

Book: Handbook of exact solutions for ordinary differential equations. By Polyanin and Zaitsev. Second edition
Section: Chapter 2, Second-Order Differential Equations. section 2.1.2-4 Equation of form $x^2y'' + f(x)y' + g(x)y = 0$
Problem number: 130.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$\boxed{x^2y'' - 2axy' + \left(-b^2x^2 + a(a+1)\right)y = 0}$$

✓ Solution by Maple
Time used: 0.0 (sec). Leaf size: 21

```
dsolve(x^2*diff(y(x),x$2)-2*a*x*diff(y(x),x)+(-b^2*x^2+a*(a+1))*y(x)=0,y(x), singsol=all)
```

$$y(x) = x^a(c_1 \sinh(bx) + c_2 \cosh(bx))$$

✓ Solution by Mathematica
Time used: 0.08 (sec). Leaf size: 35

```
DSolve[x^2*y''[x]-2*a*x*y'[x]+(-b^2*x^2+a*(a+1))*y[x]==0,y[x],x,IncludeSingularSolutions ->
```

$$y(x) \to c_1 x^a e^{-bx} + \frac{c_2 x^a e^{bx}}{2b}$$

29.22 problem 131

Internal problem ID [10955]

Book: Handbook of exact solutions for ordinary differential equations. By Polyanin and Zaitsev. Second edition
Section: Chapter 2, Second-Order Differential Equations. section 2.1.2-4 Equation of form $x^2y'' + f(x)y' + g(x)y = 0$
Problem number: 131.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$\boxed{x^2y'' + \lambda xy' + \left(a\,x^2 + bx + c\right)y = 0}$$

✓ Solution by Maple
Time used: 0.11 (sec). Leaf size: 75

```
dsolve(x^2*diff(y(x),x$2)+lambda*x*diff(y(x),x)+(a*x^2+b*x+c)*y(x)=0,y(x), singsol=all)
```

$$y(x) = x^{-\frac{\lambda}{2}}\left(\text{WhittakerW}\left(-\frac{ib}{2\sqrt{a}}, \frac{\sqrt{\lambda^2-4c-2\lambda+1}}{2}, 2i\sqrt{a}\,x\right)c_2 + \text{WhittakerM}\left(-\frac{ib}{2\sqrt{a}}, \frac{\sqrt{\lambda^2-4c-2\lambda+1}}{2}, 2i\sqrt{a}\,x\right)c_1\right)$$

✓ Solution by Mathematica
Time used: 0.224 (sec). Leaf size: 159

```
DSolve[x^2*y''[x]+\[Lambda]*x*y'[x]+(a*x^2+b*x+c)*y[x]==0,y[x],x,IncludeSingularSolutions ->
```

$$y(x) \to e^{-i\sqrt{a}x}x^{\frac{1}{2}\left(\sqrt{(\lambda-1)^2-4c}-\lambda+1\right)}\left(c_1\,\text{HypergeometricU}\left(\frac{1}{2}\left(\frac{ib}{\sqrt{a}}+\sqrt{(\lambda-1)^2-4c}+1\right), \sqrt{(\lambda-1)^2-4c}+1, 2i\sqrt{a}x\right) + c_2 L^{\sqrt{(\lambda-1)^2-4c}}_{\frac{1}{2}\left(-\frac{ib}{\sqrt{a}}-\sqrt{(\lambda-1)^2-4c}-1\right)}\left(2i\sqrt{a}x\right)\right)$$

29.23 problem 132

Internal problem ID [10956]

Book: Handbook of exact solutions for ordinary differential equations. By Polyanin and Zaitsev. Second edition
Section: Chapter 2, Second-Order Differential Equations. section 2.1.2-4 Equation of form $x^2y'' + f(x)y' + g(x)y = 0$
Problem number: 132.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$\boxed{x^2y'' + axy' + (b\,x^n + c)\,y = 0}$$

✓ Solution by Maple
Time used: 0.047 (sec). Leaf size: 80

```
dsolve(x^2*diff(y(x),x$2)+a*x*diff(y(x),x)+(b*x^n+c)*y(x)=0,y(x), singsol=all)
```

$$y(x) = x^{-\frac{a}{2}}\sqrt{x}\left(\text{BesselY}\left(\frac{\sqrt{a^2-2a-4c+1}}{n}, \frac{2\sqrt{b}\,x^{\frac{n}{2}}}{n}\right)c_2 + \text{BesselJ}\left(\frac{\sqrt{a^2-2a-4c+1}}{n}, \frac{2\sqrt{b}\,x^{\frac{n}{2}}}{n}\right)c_1\right)$$

✓ Solution by Mathematica

Time used: 0.287 (sec). Leaf size: 168

```
DSolve[x^2*y''[x]+a*x*y'[x]+(b*x^n+c)*y[x]==0,y[x],x,IncludeSingularSolutions -> True]
```

$$
\begin{aligned}
&y(x) \\
&\to n^{\frac{a-1}{n}} b^{-\frac{a-1}{2n}} (x^n)^{-\frac{a-1}{2n}} \left(c_1 \text{Gamma}\left(1 \right.\right. \\
&\qquad \left. - \frac{\sqrt{a^2-2a-4c+1}}{n}\right) \text{BesselJ}\left(-\frac{\sqrt{a^2-2a-4c+1}}{n}, \frac{2\sqrt{b}\sqrt{x^n}}{n}\right) \\
&\quad + c_2 \text{Gamma}\left(\frac{n+\sqrt{a^2-2a-4c+1}}{n}\right) \text{BesselJ}\left(\frac{\sqrt{a^2-2a-4c+1}}{n}, \frac{2\sqrt{b}\sqrt{x^n}}{n}\right)\Bigg)
\end{aligned}
$$

29.24 problem 133

Internal problem ID [10957]

Book: Handbook of exact solutions for ordinary differential equations. By Polyanin and Zaitsev. Second edition
Section: Chapter 2, Second-Order Differential Equations. section 2.1.2-4 Equation of form $x^2y'' + f(x)y' + g(x)y = 0$
Problem number: 133.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$\boxed{x^2y'' + axy' + x^n(b\,x^n + c)\,y = 0}$$

✓ Solution by Maple
Time used: 0.218 (sec). Leaf size: 82

```
dsolve(x^2*diff(y(x),x$2)+a*x*diff(y(x),x)+x^n*(b*x^n+c)*y(x)=0,y(x), singsol=all)
```

$$y(x) = \left(\text{WhittakerM}\left(-\frac{ic}{2n\sqrt{b}}, \frac{a-1}{2n}, \frac{2i\sqrt{b}\,x^n}{n}\right) c_1 + \text{WhittakerW}\left(-\frac{ic}{2n\sqrt{b}}, \frac{a-1}{2n}, \frac{2i\sqrt{b}\,x^n}{n}\right) c_2\right) x^{-\frac{a}{2}-\frac{n}{2}+\frac{1}{2}}$$

✓ Solution by Mathematica
Time used: 0.269 (sec). Leaf size: 165

```
DSolve[x^2*y''[x]+a*x*y'[x]+x^n*(b*x^n+c)*y[x]==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to 2^{\frac{a+n-1}{2n}} x^{\frac{1}{2}(-a-n+1)} \left(x^n\right)^{\frac{a+n-1}{2n}} e^{\frac{i\sqrt{b}x^n}{n}} \left(c_1 \,\text{HypergeometricU}\left(-\frac{-a+\frac{ic}{\sqrt{b}}-n+1}{2n}, \frac{a+n-1}{n}, -\frac{2i\sqrt{b}x^n}{n}\right) + c_2 L^{\frac{a-1}{n}}_{-\frac{a-\frac{ic}{\sqrt{b}}+n-1}{2n}}\left(-\frac{2i\sqrt{b}x^n}{n}\right)\right)$$

29.25 problem 134

Internal problem ID [10958]

Book: Handbook of exact solutions for ordinary differential equations. By Polyanin and Zaitsev. Second edition
Section: Chapter 2, Second-Order Differential Equations. section 2.1.2-4 Equation of form $x^2y'' + f(x)y' + g(x)y = 0$
Problem number: 134.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$\boxed{x^2y'' + (ax+b)\,y' + yc = 0}$$

✓ Solution by Maple
Time used: 0.093 (sec). Leaf size: 114

```
dsolve(x^2*diff(y(x),x$2)+(a*x+b)*diff(y(x),x)+c*y(x)=0,y(x), singsol=all)
```

$$y(x) = x^{-\frac{\sqrt{a^2-2a-4c+1}}{2}-\frac{a}{2}+\frac{1}{2}}\left(\text{KummerU}\left(-\frac{1}{2}+\frac{\sqrt{a^2-2a-4c+1}}{2}+\frac{a}{2},1+\sqrt{a^2-2a-4c+1},\frac{b}{x}\right)c_2 + \text{KummerM}\left(-\frac{1}{2}+\frac{\sqrt{a^2-2a-4c+1}}{2}+\frac{a}{2},1+\sqrt{a^2-2a-4c+1},\frac{b}{x}\right)c_1\right)$$

✓ Solution by Mathematica

Time used: 0.574 (sec). Leaf size: 243

```
DSolve[x^2*y''[x]+(a*x+b)*y'[x]+c*y[x]==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to -i^{-\sqrt{a^2-2a-4c+1}+a+1} b^{\frac{1}{2}\left(-\sqrt{a^2-2a-4c+1}+a-1\right)} \left(\frac{1}{x}\right)^{\frac{1}{2}\left(-\sqrt{a^2-2a-4c+1}+a-1\right)} \left(c_2 i^{2\sqrt{a^2-2a-4c+1}} b^{\sqrt{a^2-2a-4c+1}} \left(\frac{1}{x}\right)^{\sqrt{a^2-2a-4c+1}} \text{Hypergeometric1F1}\left(\frac{1}{2}\left(a-\sqrt{a^2-2a-4c+1}-1\right), 1-\sqrt{a^2-2a-4c+1}, \frac{b}{x}\right) + c_1 \text{Hypergeometric1F1}\left(\frac{1}{2}\left(a-\sqrt{a^2-2a-4c+1}-1\right), 1, \frac{b}{x}\right)\right)$$

29.26 problem 135

Internal problem ID [10959]

Book: Handbook of exact solutions for ordinary differential equations. By Polyanin and Zaitsev. Second edition
Section: Chapter 2, Second-Order Differential Equations. section 2.1.2-4 Equation of form $x^2y'' + f(x)y' + g(x)y = 0$
Problem number: 135.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$\boxed{x^2y'' + a\,x^2y' + \left(b\,x^2 + cx + d\right)y = 0}$$

✓ Solution by Maple
Time used: 0.078 (sec). Leaf size: 79

```
dsolve(x^2*diff(y(x),x$2)+a*x^2*diff(y(x),x)+(b*x^2+c*x+d)*y(x)=0,y(x), singsol=all)
```

$$y(x) = \mathrm{e}^{-\frac{ax}{2}}\left(\text{WhittakerM}\left(\frac{c}{\sqrt{a^2-4b}}, \frac{\sqrt{1-4d}}{2}, \sqrt{a^2-4b}\,x\right)c_1 + \text{WhittakerW}\left(\frac{c}{\sqrt{a^2-4b}}, \frac{\sqrt{1-4d}}{2}, \sqrt{a^2-4b}\,x\right)c_2\right)$$

✓ Solution by Mathematica
Time used: 0.228 (sec). Leaf size: 157

```
DSolve[x^2*y''[x]+a*x^2*y'[x]+(b*x^2+c*x+d)*y[x]==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to x^{\frac{1}{2}\left(\sqrt{1-4d}+1\right)}e^{-\frac{1}{2}x\left(\sqrt{a^2-4b}+a\right)}\left(c_1\,\text{HypergeometricU}\left(\frac{1}{2}\left(-\frac{2c}{\sqrt{a^2-4b}}+\sqrt{1-4d}+1\right), \sqrt{1-4d}+1, \sqrt{a^2-4b}x\right) + c_2 L^{\sqrt{1-4d}}_{\frac{c}{\sqrt{a^2-4b}}-\frac{1}{2}\sqrt{1-4d}-\frac{1}{2}}\left(\sqrt{a^2-4b}x\right)\right)$$

29.27 problem 136

Internal problem ID [10960]

Book: Handbook of exact solutions for ordinary differential equations. By Polyanin and Zaitsev. Second edition
Section: Chapter 2, Second-Order Differential Equations. section 2.1.2-4 Equation of form $x^2y'' + f(x)y' + g(x)y = 0$
Problem number: 136.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$\boxed{x^2y'' + \left(a\,x^2 + b\right)y' + c\left(\left(a - c\right)x^2 + b\right)y = 0}$$

✓ Solution by Maple
Time used: 0.485 (sec). Leaf size: 243

```
dsolve(x^2*diff(y(x),x$2)+(a*x^2+b)*diff(y(x),x)+c*((a-c)*x^2+b)*y(x)=0,y(x), singsol=all)
```

$$y(x) = \sqrt{x}\left(\text{HeunD}\left(-4\sqrt{-b\left(-2c+a\right)}, -4\sqrt{-b\left(-2c+a\right)} - 1 + \left(-4a+8c\right)b, 8\sqrt{-b\left(-2c+a\right)}, -4\sqrt{-b\left(-2c+a\right)} + 1 + \left(-8c+4a\right)b, \frac{\sqrt{-b\left(-2c+a\right)}\,x - b}{\sqrt{-b\left(-2c+a\right)}\,x + b}\right)\mathrm{e}^{-x(a-c)}c_2 + \text{HeunD}\left(4\sqrt{-b\left(-2c+a\right)}, -4\sqrt{-b\left(-2c+a\right)} - 1 + \left(-4a+8c\right)b, 8\sqrt{-b\left(-2c+a\right)}, -4\sqrt{-b\left(-2c+a\right)} + 1 + \left(-8c+4a\right)b, \frac{\sqrt{-b\left(-2c+a\right)}\,x - b}{\sqrt{-b\left(-2c+a\right)}\,x + b}\right)\mathrm{e}^{\frac{-c\,x^2+b}{x}}c_1\right)$$

✓ Solution by Mathematica
Time used: 1.026 (sec). Leaf size: 44

```
DSolve[x^2*y''[x]+(a*x^2+b)*y'[x]+c*((a-c)*x^2+b)*y[x]==0,y[x],x,IncludeSingularSolutions ->
```

$$y(x) \to e^{-cx}\left(c_2\int_1^x e^{\frac{b}{K[1]} - aK[1] + 2cK[1]}dK[1] + c_1\right)$$

29.28 problem 137

Internal problem ID [10961]

Book: Handbook of exact solutions for ordinary differential equations. By Polyanin and Zaitsev. Second edition
Section: Chapter 2, Second-Order Differential Equations. section 2.1.2-4 Equation of form $x^2y'' + f(x)y' + g(x)y = 0$
Problem number: 137.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$\boxed{x^2y'' + \left(a\,x^2 + bx\right)y' - yb = 0}$$

✓ Solution by Maple
Time used: 0.016 (sec). Leaf size: 51

```
dsolve(x^2*diff(y(x),x$2)+(a*x^2+b*x)*diff(y(x),x)-b*y(x)=0,y(x), singsol=all)
```

$$y(x) = -\mathrm{e}^{-ax}c_2(\Gamma(b,-ax)\,b - \Gamma(b+1))\,(-ax)^{-b} + c_1x^{-b}\mathrm{e}^{-ax} - c_2$$

✓ Solution by Mathematica
Time used: 0.04 (sec). Leaf size: 43

```
DSolve[x^2*y''[x]+(a*x^2+b*x)*y'[x]-b*y[x]==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to e^{-ax}\left(\frac{c_1(-ax)^{-b}\Gamma(b+1,-ax)}{a} + c_2x^{-b}\right)$$

29.29 problem 138

Internal problem ID [10962]

Book: Handbook of exact solutions for ordinary differential equations. By Polyanin and Zaitsev. Second edition
Section: Chapter 2, Second-Order Differential Equations. section 2.1.2-4 Equation of form $x^2y'' + f(x)y' + g(x)y = 0$
Problem number: 138.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$x^2y'' + \left(a\,x^2 + bx\right)y' + \left(k(a-k)\,x^2 + (an + bk - 2kn)\,x + n(-n+b-1)\right)y = 0$$

✓ Solution by Maple
Time used: 0.016 (sec). Leaf size: 48

```
dsolve(x^2*diff(y(x),x$2)+(a*x^2+b*x)*diff(y(x),x)+(k*(a-k)*x^2+(a*n+b*k-2*k*n)*x+n*(b-n-1))
```

$$y(x) = c_1 x^{-n} \mathrm{e}^{-kx} + c_2 x^{-\frac{b}{2}}\,\mathrm{WhittakerM}\left(-\frac{b}{2}+n, -\frac{b}{2}+n+\frac{1}{2}, (-2k+a)\,x\right)\mathrm{e}^{-\frac{ax}{2}}$$

✓ Solution by Mathematica
Time used: 0.504 (sec). Leaf size: 64

```
DSolve[x^2*y''[x]+(a*x^2+b*x)*y'[x]+(k*(a-k)*x^2+(a*n+b*k-2*k*n)*x+n*(b-n-1))*y[x]==0,y[x],x
```

$$y(x) \to e^{-kx}x^{-n}\left(c_1 - c_2 x^{-b+2n+1}(x(a-2k))^{b-2n-1}\Gamma(-b+2n+1,(a-2k)x)\right)$$

29.30 problem 139

Internal problem ID [10963]

Book: Handbook of exact solutions for ordinary differential equations. By Polyanin and Zaitsev. Second edition
Section: Chapter 2, Second-Order Differential Equations. section 2.1.2-4 Equation of form $x^2y'' + f(x)y' + g(x)y = 0$
Problem number: 139.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$\boxed{a_2x^2y'' + \left(a_1x^2 + b_1x\right)y' + \left(x^2a_0 + b_0x + c_0\right)y = 0}$$

✓ Solution by Maple
Time used: 0.11 (sec). Leaf size: 150

```
dsolve(a__2*x^2*diff(y(x),x$2)+(a__1*x^2+b__1*x)*diff(y(x),x)+(a__0*x^2+b__0*x+c__0)*y(x)=0,
```

$$y(x) = \mathrm{e}^{-\frac{a_1x}{2a_2}} x^{-\frac{b_1}{2a_2}} \left(c_1 \operatorname{WhittakerM}\left(-\frac{b_1a_1 - 2a_2b_0}{2a_2\sqrt{-4a_0a_2 + a_1^2}}, \frac{\sqrt{a_2^2 + (-2b_1 - 4c_0)a_2 + b_1^2}}{2a_2}, \frac{\sqrt{-4a_0a_2 + a_1^2}\,x}{a_2} \right) + \operatorname{WhittakerW}\left(-\frac{b_1a_1 - 2a_2b_0}{2a_2\sqrt{-4a_0a_2 + a_1^2}}, \frac{\sqrt{a_2^2 + (-2b_1 - 4c_0)a_2 + b_1^2}}{2a_2}, \frac{\sqrt{-4a_0a_2 + a_1^2}\,x}{a_2} \right) c_2 \right)$$

✓ Solution by Mathematica

Time used: 0.538 (sec). Leaf size: 272

```
DSolve[a2*x^2*y''[x]+(a1*x^2+b1*x)*y'[x]+(a0*x^2+b0*x+c0)*y[x]==0,y[x],x,IncludeSingularSolu
```

$$y(x)$$

$$\to e^{-\frac{x\left(\sqrt{a1^2-4a0a2}+a1\right)}{2a2}} x^{\frac{\sqrt{a2^2-2a2(b1+2c0)+b1^2}+a2-b1}{2a2}} \left(c_1 \text{HypergeometricU}\left(\frac{-\frac{2b0a2}{\sqrt{a1^2-4a0a2}}+a2+\frac{a1b1}{\sqrt{a1^2-4a0a2}}}{2a2} \right.\right.$$

$$\left.\left. + c_2 L^{\frac{\sqrt{a2^2-2(b1+2c0)a2+b1^2}}{a2}}_{-\frac{-\frac{2b0a2}{\sqrt{a1^2-4a0a2}}+a2+\frac{a1b1}{\sqrt{a1^2-4a0a2}}+\sqrt{a2^2-2(b1+2c0)a2+b1^2}}{2a2}}\left(\frac{\sqrt{a1^2-4a0a2}x}{a2}\right)\right)\right)$$

29.31 problem 140

Internal problem ID [10964]

Book: Handbook of exact solutions for ordinary differential equations. By Polyanin and Zaitsev. Second edition
Section: Chapter 2, Second-Order Differential Equations. section 2.1.2-4 Equation of form $x^2y'' + f(x)y' + g(x)y = 0$
Problem number: 140.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$\boxed{x^2y'' + \left(a\,x^2 + (ab-1)\,x + b\right)y' + a^2bxy = 0}$$

✓ Solution by Maple
Time used: 0.312 (sec). Leaf size: 199

```
dsolve(x^2*diff(y(x),x$2)+(a*x^2+(a*b-1)*x+b)*diff(y(x),x)+a^2*b*x*y(x)=0,y(x), singsol=all)
```

$$y(x) = \left(\text{HeunD}\left(4\sqrt{ab}, -a^2b^2 + 4ab - 8\sqrt{ab} - 4, -8\sqrt{ab}\,(ab-1), a^2b^2 - 4ab - 8\sqrt{ab} + 4, \frac{\sqrt{ab}\,x - b}{\sqrt{ab}\,x + b}\right) \mathrm{e}^{\frac{-a\,x^2+b}{x}} c_1 + \text{HeunD}\left(-4\sqrt{ab}, -a^2b^2 + 4ab - 8\sqrt{ab} - 4, -8\sqrt{ab}\,(ab-1), a^2b^2 - 4ab - 8\sqrt{ab} + 4, \frac{\sqrt{ab}\,x - b}{\sqrt{ab}\,x + b}\right) c_2\right) x^{1-\frac{ab}{2}}$$

✓ Solution by Mathematica
Time used: 4.002 (sec). Leaf size: 67

```
DSolve[x^2*y''[x]+(a*x^2+(a*b-1)*x+b)*y'[x]+a^2*b*x*y[x]==0,y[x],x,IncludeSingularSolutions
```

$$y(x) \to \frac{e^{-ax}(ax+1)\left(c_2 \int_1^x \frac{a^2 e^{\frac{b}{K[1]} + aK[1]} K[1]^{1-ab}}{(aK[1]+1)^2} dK[1] + c_1\right)}{a}$$

29.32 problem 141

Internal problem ID [10965]

Book: Handbook of exact solutions for ordinary differential equations. By Polyanin and Zaitsev. Second edition
Section: Chapter 2, Second-Order Differential Equations. section 2.1.2-4 Equation of form $x^2y'' + f(x)y' + g(x)y = 0$
Problem number: 141.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$\boxed{x^2y'' - 2x\left(x^2 - a\right)y' + \left(2n\,x^2 + \left((-1)^n - 1\right)a\right)y = 0}$$

✓ Solution by Maple
Time used: 0.125 (sec). Leaf size: 81

```
dsolve(x^2*diff(y(x),x$2)-2*x*(x^2-a)*diff(y(x),x)+(2*n*x^2+( (-1)^n-1)*a )*y(x)=0,y(x), sin
```

$$y(x) = x^{-a-\frac{1}{2}}\mathrm{e}^{\frac{x^2}{2}}\left(\mathrm{WhittakerM}\left(\frac{a}{2}+\frac{n}{2}+\frac{1}{4},\frac{\sqrt{1-4a\left(-1\right)^n+4a^2}}{4},x^2\right)c_1 + \mathrm{WhittakerW}\left(\frac{a}{2}+\frac{n}{2}+\frac{1}{4},\frac{\sqrt{1-4a\left(-1\right)^n+4a^2}}{4},x^2\right)c_2\right)$$

✓ Solution by Mathematica
Time used: 0.646 (sec). Leaf size: 231

```
DSolve[x^2*y''[x]-2*x*(x^2-a)*y'[x]+(2*n*x^2+( (-1)^n-1)*a )*y[x]==0,y[x],x,IncludeSingularS
```

$$y(x)$$
$$\to i^{-a}(-1)^{\frac{1}{4}\left(1-\sqrt{4a^2-4a(-1)^n+1}\right)}x^{\frac{1}{2}\left(-\sqrt{4a^2-4a(-1)^n+1}-2a+1\right)}\left(c_1\,\mathrm{Hypergeometric1F1}\left(\frac{1}{4}\left(-2a-2n-\sqrt{4a^2-4\right.\right.\right.$$
$$\left.\left.\left.-\frac{1}{2}\sqrt{4a^2-4(-1)^na+1},x^2\right)\right.\right.$$
$$+c_2i^{\sqrt{4a^2-4a(-1)^n+1}}x^{\sqrt{4a^2-4a(-1)^n+1}}\,\mathrm{Hypergeometric1F1}\left(\frac{1}{4}\left(-2a-2n+\sqrt{4a^2-4(-1)^na+1}+1\right),\frac{1}{2}\left(\right.\right.$$

29.33 problem 142

Internal problem ID [10966]

Book: Handbook of exact solutions for ordinary differential equations. By Polyanin and Zaitsev. Second edition
Section: Chapter 2, Second-Order Differential Equations. section 2.1.2-4 Equation of form $x^2y'' + f(x)y' + g(x)y = 0$
Problem number: 142.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$\boxed{x^2y'' + x\left(a\,x^2 + bx + c\right)y' + \left(A\,x^3 + B\,x^2 + Cx + d\right)y = 0}$$

✓ Solution by Maple
Time used: 0.328 (sec). Leaf size: 232

```
dsolve(x^2*diff(y(x),x$2)+x*(a*x^2+b*x+c)*diff(y(x),x)+(A*x^3+B*x^2+C*x+d)*y(x)=0,y(x), sing
```

$$\begin{aligned} y(x) &= x^{-\frac{c}{2}+\frac{1}{2}} \mathrm{e}^{\frac{x\left(-a^2x-2ab+2A\right)}{2a}} \left(c_1 x^{\frac{\sqrt{c^2-2c-4d+1}}{2}} \operatorname{HeunB}\left(\sqrt{c^2-2c-4d+1}, \frac{\sqrt{2}\,(-ab+2A)}{a^{\frac{3}{2}}}, \right.\right. \\ &\qquad \left. -c - \frac{2Ab}{a^2} + \frac{2B}{a} - 1 + \frac{2A^2}{a^3}, \frac{\sqrt{2}\,(-bc+2C)}{\sqrt{a}}, -\frac{\sqrt{2}\,\sqrt{a}\,x}{2} \right) \\ &\quad + c_2 x^{-\frac{\sqrt{c^2-2c-4d+1}}{2}} \operatorname{HeunB}\left(-\sqrt{c^2-2c-4d+1}, \frac{\sqrt{2}\,(-ab+2A)}{a^{\frac{3}{2}}}, -c - \frac{2Ab}{a^2} + \frac{2B}{a} \right. \\ &\qquad \left.\left. -1 + \frac{2A^2}{a^3}, \frac{\sqrt{2}\,(-bc+2C)}{\sqrt{a}}, -\frac{\sqrt{2}\,\sqrt{a}\,x}{2} \right) \right) \end{aligned}$$

✗ Solution by Mathematica
Time used: 0.0 (sec). Leaf size: 0

```
DSolve[x^2*y''[x]+x*(a*x^2+b*x+c)*y'[x]+(A*x^3+B*x^2+C0*x+d)*y[x]==0,y[x],x,IncludeSingularS
```

Not solved

29.34 problem 143

Internal problem ID [10967]

Book: Handbook of exact solutions for ordinary differential equations. By Polyanin and Zaitsev. Second edition
Section: Chapter 2, Second-Order Differential Equations. section 2.1.2-4 Equation of form $x^2y'' + f(x)y' + g(x)y = 0$
Problem number: 143.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$x^2y'' + a\,x^n y' - \left(ab\,x^n + ac\,x^{n-1} + b^2x^2 + 2bcx + c^2 - c\right)y = 0$$

✗ Solution by Maple

```
dsolve(x^2*diff(y(x),x$2)+a*x^n*diff(y(x),x)-(a*b*x^n+a*c*x^(n-1)+b^2*x^2+2*b*c*x+c^2-c)*y(x
```

No solution found

✗ Solution by Mathematica
Time used: 0.0 (sec). Leaf size: 0

```
DSolve[x^2*y''[x]+a*x^n*y'[x]-(a*b*x^n+a*c*x^(n-1)+b^2*x^2+2*b*c*x+c^2-c)*y[x]==0,y[x],x,Inc
```

Not solved

29.35 problem 144

Internal problem ID [10968]

Book: Handbook of exact solutions for ordinary differential equations. By Polyanin and Zaitsev. Second edition
Section: Chapter 2, Second-Order Differential Equations. section 2.1.2-4 Equation of form $x^2y'' + f(x)y' + g(x)y = 0$
Problem number: 144.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$x^2y'' + a\,x^n y' + \left(ab\,x^{n+2m} - b^2x^{4m+2} + am\,x^{n-1} - m^2 - m\right)y = 0$$

✗ Solution by Maple

```
dsolve(x^2*diff(y(x),x$2)+a*x^n*diff(y(x),x)+(a*b*x^(n+2*m)-b^2*x^(4*m+2)+a*m*x^(n-1)-m^2-m)
```

No solution found

✗ Solution by Mathematica
Time used: 0.0 (sec). Leaf size: 0

```
DSolve[x^2*y''[x]+a*x^n*y'[x]+(a*b*x^(n+2*m)-b^2*x^(4*m+2)+a*m*x^(n-1)-m^2-m)*y[x]==0,y[x],x
```

Not solved

29.36 problem 145

Internal problem ID [10969]

Book: Handbook of exact solutions for ordinary differential equations. By Polyanin and Zaitsev. Second edition
Section: Chapter 2, Second-Order Differential Equations. section 2.1.2-4 Equation of form $x^2y'' + f(x)y' + g(x)y = 0$
Problem number: 145.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$x^2y'' + x(a\,x^n + b)\,y' + b(a\,x^n - 1)\,y = 0$$

✓ Solution by Maple
Time used: 0.11 (sec). Leaf size: 141

```
dsolve(x^2*diff(y(x),x$2)+x*(a*x^n+b)*diff(y(x),x)+b*(a*x^n-1)*y(x)=0,y(x), singsol=all)
```

$$\begin{aligned} y(x) = \mathrm{e}^{-\frac{a\,x^n}{2n}} \Big(& (b+n+1)\,x^{-\frac{3n}{2}+\frac{1}{2}-\frac{b}{2}} \\ & + a\,x^{\frac{1}{2}-\frac{b}{2}-\frac{n}{2}} \Big) nc_2\,\mathrm{WhittakerM}\left(\frac{b-n+1}{2n}, \frac{b+2n+1}{2n}, \frac{a\,x^n}{n}\right) \\ & + x^{-\frac{3n}{2}+\frac{1}{2}-\frac{b}{2}} \mathrm{e}^{-\frac{a\,x^n}{2n}} c_2 (b+n+1)^2\,\mathrm{WhittakerM}\left(\frac{b+n+1}{2n}, \frac{b+2n+1}{2n}, \frac{a\,x^n}{n}\right) \\ & + c_1 x^{-b} \end{aligned}$$

✓ Solution by Mathematica
Time used: 0.146 (sec). Leaf size: 76

```
DSolve[x^2*y''[x]+x*(a*x^n+b)*y'[x]+b*(a*x^n-1)*y[x]==0,y[x],x,IncludeSingularSolutions -> T
```

$$y(x) \to (-1)^{-\frac{b}{n}} n^{\frac{b}{n}-1} a^{-\frac{b}{n}} (x^n)^{-\frac{b}{n}} \left((b+1)c_1 (-1)^{b/n} \Gamma\left(\frac{b+1}{n}, 0, \frac{a x^n}{n}\right) + c_2 n \right)$$

29.37 problem 146

Internal problem ID [10970]

Book: Handbook of exact solutions for ordinary differential equations. By Polyanin and Zaitsev. Second edition
Section: Chapter 2, Second-Order Differential Equations. section 2.1.2-4 Equation of form $x^2y'' + f(x)y' + g(x)y = 0$
Problem number: 146.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$\boxed{x^2y'' + x(a\,x^n + b)\,y' + \left(\alpha\,x^{2n} + \beta\,x^n + \gamma\right)y = 0}$$

✓ Solution by Maple
Time used: 0.187 (sec). Leaf size: 148

```
dsolve(x^2*diff(y(x),x$2)+x*(a*x^n+b)*diff(y(x),x)+(alpha*x^(2*n)+beta*x^n+gamma)*y(x)=0,y(x
```

$$y(x) = x^{\frac{1}{2}-\frac{b}{2}-\frac{n}{2}} \mathrm{e}^{-\frac{a\,x^n}{2n}} \left(c_1 \,\mathrm{WhittakerM}\left(-\frac{a(b+n-1)-2\beta}{2\sqrt{a^2-4\alpha}\,n}, \frac{\sqrt{b^2-2b-4\gamma+1}}{2n}, \frac{\sqrt{a^2-4\alpha}\,x^n}{n} \right) + c_2 \,\mathrm{WhittakerW}\left(-\frac{a(b+n-1)-2\beta}{2\sqrt{a^2-4\alpha}\,n}, \frac{\sqrt{b^2-2b-4\gamma+1}}{2n}, \frac{\sqrt{a^2-4\alpha}\,x^n}{n} \right) \right)$$

✓ Solution by Mathematica

Time used: 0.485 (sec). Leaf size: 420

```
DSolve[x^2*y''[x]+x*(a*x^n+b)*y'[x]+(\[Alpha]*x^(2*n)+\[Beta]*x^n+\[Gamma])*y[x]==0,y[x],x,I
```

$$y(x)$$

$$\to x^{\frac{1}{2}-\frac{n}{2}} 2^{\frac{1}{2}\left(\frac{\sqrt{n^2\left(b^2-2b-4\gamma+1\right)}}{n^2}+1\right)} e^{-\frac{\left(\sqrt{a^2-4\alpha}+a\right)x^n}{2n}} (x^n)^{\frac{\sqrt{n^2\left(b^2-2b-4\gamma+1\right)}-bn+n^2}{2n^2}} \left(c_1 \text{HypergeometricU}\left(\frac{\left(n^2+\sqrt{n^2}\right.\right.\right.$$

$$\left.\left.\left.+c_2 L_{\frac{-\left(\left(n^2+\sqrt{n^2\left(b^2-2b-4\gamma+1\right)}\right)a^2\right)-n(b+n-1)\sqrt{a^2-4\alpha}a+4n^2\alpha+2n\sqrt{a^2-4\alpha}\beta+4\alpha\sqrt{n^2\left(b^2-2b-4\gamma+1\right)}}{2n^2\left(a^2-4\alpha\right)}}^{\frac{\sqrt{n^2\left(b^2-2b-4\gamma+1\right)}}{n^2}}\left(\frac{x^n\sqrt{a^2-4\alpha}}{n}\right)\right)\right.$$

29.38 problem 147

Internal problem ID [10971]

Book: Handbook of exact solutions for ordinary differential equations. By Polyanin and Zaitsev. Second edition
Section: Chapter 2, Second-Order Differential Equations. section 2.1.2-4 Equation of form $x^2y'' + f(x)y' + g(x)y = 0$
Problem number: 147.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$x^2y'' + x(2a\,x^n + b)\,y' + \left(x^{2n}a^2 + a(b+n-1)\,x^n + \alpha\,x^{2m} + \beta\,x^m + \gamma\right)y = 0$$

✓ Solution by Maple
Time used: 0.172 (sec). Leaf size: 115

```
dsolve(x^2*diff(y(x),x$2)+x*(2*a*x^n+b)*diff(y(x),x)+(a^2*x^(2*n)+a*(b+n-1)*x^n+alpha*x^(2*m
```

$$y(x) = x^{-\frac{b}{2}}x^{-\frac{m}{2}}\sqrt{x}\,\mathrm{e}^{-\frac{a\,x^n}{n}}\left(c_1\,\mathrm{WhittakerM}\left(-\frac{i\beta}{2m\sqrt{\alpha}},\frac{\sqrt{b^2-2b-4\gamma+1}}{2m},\frac{2i\sqrt{\alpha}\,x^m}{m}\right)\right.$$
$$\left. + c_2\,\mathrm{WhittakerW}\left(-\frac{i\beta}{2m\sqrt{\alpha}},\frac{\sqrt{b^2-2b-4\gamma+1}}{2m},\frac{2i\sqrt{\alpha}\,x^m}{m}\right)\right)$$

✓ Solution by Mathematica
Time used: 0.47 (sec). Leaf size: 291

```
DSolve[x^2*y''[x]+x*(2*a*x^n+b)*y'[x]+(a^2*x^(2*n)+a*(b+n-1)*x^n+\[Alpha]*x^(2*m)+\[Beta]*x^
```

$$y(x)$$
$$\to x^{\frac{1}{2}-\frac{m}{2}}2^{\frac{1}{2}\left(\frac{\sqrt{m^2\left(b^2-2b-4\gamma+1\right)}}{m^2}+1\right)}(x^n)^{-\frac{b}{2n}}(x^m)^{\frac{1}{2}\left(\frac{\sqrt{m^2\left(b^2-2b-4\gamma+1\right)}}{m^2}+1\right)}e^{-\frac{ax^n}{n}+\frac{i\sqrt{\alpha}x^m}{m}}\left(c_1\,\mathrm{HypergeometricU}\left(\frac{m^2}{}\right.\right.$$
$$\left.\left. -\frac{2ix^m\sqrt{\alpha}}{m}\right) + c_2 L_{-\frac{m^2-\frac{i\beta m}{\sqrt{\alpha}}+\sqrt{m^2\left(b^2-2b-4\gamma+1\right)}}{2m^2}}^{\frac{\sqrt{m^2\left(b^2-2b-4\gamma+1\right)}}{m^2}}\left(-\frac{2ix^m\sqrt{\alpha}}{m}\right)\right)$$

29.39 problem 148

Internal problem ID [10972]

Book: Handbook of exact solutions for ordinary differential equations. By Polyanin and Zaitsev. Second edition
Section: Chapter 2, Second-Order Differential Equations. section 2.1.2-4 Equation of form $x^2y'' + f(x)y' + g(x)y = 0$
Problem number: 148.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$\boxed{x^2y'' + \left(a\,x^{2+n} + b\,x^2 + c\right)y' + \left(an\,x^{n+1} + x^n ac + bc\right)y = 0}$$

✗ Solution by Maple

```
dsolve(x^2*diff(y(x),x$2)+(a*x^(n+2)+b*x^2+c)*diff(y(x),x)+(a*n*x^(n+1)+a*c*x^n+b*c)*y(x)=0,
```

No solution found

✗ Solution by Mathematica
Time used: 0.0 (sec). Leaf size: 0

```
DSolve[x^2*y''[x]+(a*x^(n+2)+b*x^2+c)*y'[x]+(a*n*x^(n+1)+a*c*x^n+b*c)*y[x]==0,y[x],x,Include
```

Not solved

30 Chapter 2, Second-Order Differential Equations. section 2.1.2-5 Equation of form $(ax^2+bx+c)y''+f(x)y'+g(x)y=0$

30.1 problem 149 781
30.2 problem 150 782
30.3 problem 151 784
30.4 problem 152 785
30.5 problem 153 786
30.6 problem 154 787
30.7 problem 155 788
30.8 problem 156 789
30.9 problem 157 790
30.10problem 158 791
30.11problem 159 793
30.12problem 160 794
30.13problem 161 795
30.14problem 162 796
30.15problem 163 797
30.16problem 164 798
30.17problem 165 799
30.18problem 166 800
30.19problem 167 801
30.20problem 168 802
30.21problem 169 804
30.22problem 170 806
30.23problem 171 807
30.24problem 172 808
30.25problem 173 810
30.26problem 174 811
30.27problem 175 812
30.28problem 176 813
30.29problem 177 814
30.30problem 178 815
30.31problem 179 816
30.32problem 180 818
30.33problem 181 819

30.1 problem 149

Internal problem ID [10973]

Book: Handbook of exact solutions for ordinary differential equations. By Polyanin and Zaitsev. Second edition
Section: Chapter 2, Second-Order Differential Equations. section 2.1.2-5 Equation of form $(ax^2+bx+c)y''+f(x)y'+g(x)y=0$
Problem number: 149.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[_Gegenbauer]`

$$\boxed{\left(-x^2+1\right)y''+n(n-1)y=0}$$

✓ Solution by Maple
Time used: 0.078 (sec). Leaf size: 52

```
dsolve((1-x^2)*diff(y(x),x$2)+n*(n-1)*y(x)=0,y(x), singsol=all)
```

$$y(x)=-(-1+x)(1+x)\left(\text{hypergeom}\left(\left[\frac{n}{2}+1,\frac{3}{2}-\frac{n}{2}\right],\left[\frac{3}{2}\right],x^2\right)c_2x+c_1\,\text{hypergeom}\left(\left[-\frac{n}{2}+1,\frac{n}{2}+\frac{1}{2}\right],\left[\frac{1}{2}\right],x^2\right)\right)$$

✓ Solution by Mathematica
Time used: 0.149 (sec). Leaf size: 56

```
DSolve[(1-x^2)*y''[x]+n*(n-1)*y[x]==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x)\to ic_2x\,\text{Hypergeometric2F1}\left(\frac{1}{2}-\frac{n}{2},\frac{n}{2},\frac{3}{2},x^2\right)+c_1\,\text{Hypergeometric2F1}\left(\frac{n-1}{2},-\frac{n}{2},\frac{1}{2},x^2\right)$$

30.2 problem 150

Internal problem ID [10974]

Book: Handbook of exact solutions for ordinary differential equations. By Polyanin and Zaitsev. Second edition
Section: Chapter 2, Second-Order Differential Equations. section 2.1.2-5 Equation of form $(ax^2+bx+c)y''+f(x)y'+g(x)y=0$
Problem number: 150.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$\boxed{\left(-a^2+x^2\right)y''+y'b-6y=0}$$

✓ Solution by Maple
Time used: 0.015 (sec). Leaf size: 76

```
dsolve((x^2-a^2)*diff(y(x),x$2)+b*diff(y(x),x)-6*y(x)=0,y(x), singsol=all)
```

$$y(x)=-\frac{c_1(10a^2b+24a^2x-b^3-6b^2x-18x^2b-24x^3)}{24}+c_2(a+x)(a-x)(b-4x)\left(\frac{a+x}{a-x}\right)^{\frac{b}{2a}}$$

✓ Solution by Mathematica

Time used: 13.059 (sec). Leaf size: 1171

```
DSolve[(x^2-a^2)*y''[x]+b*y'[x]-6*y[x]==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x)$$

$$\to e^{\frac{b\operatorname{arctanh}\left(\frac{x}{a}\right)}{2a}+\frac{\left(b^5-20a^2b^3+64a^4b+\sqrt{b^2\left(64a^4-20b^2a^2+b^4\right)^2}\right)\text{RootSum}\left[-b^3-6\#1b^2+10a^2b-18\#1^2b-24\#1^3+24a^2\#1\&,\log(x-\#1)\&\right]}{2\left(b^5-20a^2b^3+64a^4b\right)}}(a+x)^{\frac{1}{2}-\frac{\sqrt{b^2\left(64a^4-20b^2a^2+b^4\right)^2}}{4ab\left(32a^3-16ba^2-2b^2a+b^3\right)}}(4x-b)^{\frac{b^5}{2\left(b^5-20a^2b^3+64a^4b\right)}-\frac{10a^2b^3}{b^5-20a^2b^3+64a^4b}+\frac{32a^4b}{b^5-20a^2b^3+64a^4b}-\frac{\sqrt{b^2\left(64a^4-20b^2a^2+b^4\right)^2}}{2\left(b^5-20a^2b^3+64a^4b\right)}}c_2\int_1^x -e^{-\frac{\left(b^5-20a^2b^3+64a^4b+\sqrt{b^2\left(64a^4-20b^2a^2+b^4\right)^2}\right)\text{RootSum}\left[-b^3-6\#1b^2+10a^2b-18\#1^2b-24\#1^3+24a^2\#1\&,\log(K[1]-\#1)\&\right]}{b^5-20a^2b^3+64a^4b}}(K[1]-a)^{\frac{\sqrt{b^2}}{2a}}$$

$$-a)^{\frac{1}{2}-\frac{\sqrt{b^2\left(64a^4-20b^2a^2+b^4\right)^2}}{4a(4a-b)b(2a+b)(4a+b)}}+e^{\frac{1}{2}\left(\frac{b\operatorname{arctanh}\left(\frac{x}{a}\right)}{a}+\frac{\left(b^5-20a^2b^3+64a^4b+\sqrt{\left(b^5-20a^2b^3+64a^4b\right)^2}\right)\text{RootSum}\left[-b^3-6\#1b^2+10a^2b-18\#1^2b-24\#1^3+24a^2\#1\&,\log(x-\#1)\&\right]}{b^5-20a^2b^3+64a^4b}\right)}(a+x)^{\frac{1}{4}\left(2-\frac{\sqrt{\left(b^5-20a^2b^3+64a^4b\right)^2}}{ab\left(32a^3-16ba^2-2b^2a+b^3\right)}\right)}(4x-b)^{\frac{b^5-20a^2b^3+64a^4b-\sqrt{\left(b^5-20a^2b^3+64a^4b\right)^2}}{2\left(b^5-20a^2b^3+64a^4b\right)}}c_1(x-a)^{\frac{1}{2}-\frac{\sqrt{\left(b^5-20a^2b^3+64a^4b\right)^2}}{4a(4a-b)b(2a+b)(4a+b)}}$$

30.3 problem 151

Internal problem ID [10975]

Book: Handbook of exact solutions for ordinary differential equations. By Polyanin and Zaitsev. Second edition
Section: Chapter 2, Second-Order Differential Equations. section 2.1.2-5 Equation of form $(ax^2+bx+c)y''+f(x)y'+g(x)y=0$
Problem number: 151.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[_Gegenbauer, [_2nd_order, _linear, '_with_symmetry_[0,F(x)]']]`

$$\boxed{\left(x^2-1\right)y''+y'x+ay=0}$$

✓ Solution by Maple
Time used: 0.0 (sec). Leaf size: 41

```
dsolve((x^2-1)*diff(y(x),x$2)+x*diff(y(x),x)+a*y(x)=0,y(x), singsol=all)
```

$$y(x)=c_1\left(x+\sqrt{x^2-1}\right)^{i\sqrt{a}}+c_2\left(x+\sqrt{x^2-1}\right)^{-i\sqrt{a}}$$

✓ Solution by Mathematica
Time used: 0.144 (sec). Leaf size: 97

```
DSolve[(x^2-1)*y''[x]+x*y'[x]+a*y[x]==0,y[x],x,IncludeSingularSolutions -> True]
```

$$\begin{aligned}y(x)\to c_1\cos\left(\frac{1}{2}\sqrt{a}\left(\log\left(1-\frac{x}{\sqrt{x^2-1}}\right)-\log\left(\frac{x}{\sqrt{x^2-1}}+1\right)\right)\right)\\-c_2\sin\left(\frac{1}{2}\sqrt{a}\left(\log\left(1-\frac{x}{\sqrt{x^2-1}}\right)-\log\left(\frac{x}{\sqrt{x^2-1}}+1\right)\right)\right)\end{aligned}$$

30.4 problem 152

Internal problem ID [10976]

Book: Handbook of exact solutions for ordinary differential equations. By Polyanin and Zaitsev. Second edition
Section: Chapter 2, Second-Order Differential Equations. section 2.1.2-5 Equation of form $(ax^2+bx+c)y''+f(x)y'+g(x)y=0$
Problem number: 152.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[_Gegenbauer, [_2nd_order, _linear, '_with_symmetry_[0,F(x)]']]`

$$\boxed{\left(-x^2+1\right)y''-y'x+yn^2=0}$$

✓ Solution by Maple
Time used: 0.0 (sec). Leaf size: 33

```
dsolve((1-x^2)*diff(y(x),x$2)-x*diff(y(x),x)+n^2*y(x)=0,y(x), singsol=all)
```

$$y(x)=c_1\left(x+\sqrt{x^2-1}\right)^{-n}+c_2\left(x+\sqrt{x^2-1}\right)^{n}$$

✓ Solution by Mathematica
Time used: 0.139 (sec). Leaf size: 91

```
DSolve[(1-x^2)*y''[x]-x*y'[x]+n^2*y[x]==0,y[x],x,IncludeSingularSolutions -> True]
```

$$\begin{aligned}y(x)\to c_1\cosh\left(\frac{1}{2}n\left(\log\left(1-\frac{x}{\sqrt{x^2-1}}\right)-\log\left(\frac{x}{\sqrt{x^2-1}}+1\right)\right)\right)\\-ic_2\sinh\left(\frac{1}{2}n\left(\log\left(1-\frac{x}{\sqrt{x^2-1}}\right)-\log\left(\frac{x}{\sqrt{x^2-1}}+1\right)\right)\right)\end{aligned}$$

30.5 problem 153

Internal problem ID [10977]

Book: Handbook of exact solutions for ordinary differential equations. By Polyanin and Zaitsev. Second edition
Section: Chapter 2, Second-Order Differential Equations. section 2.1.2-5 Equation of form $(ax^2+bx+c)y''+f(x)y'+g(x)y=0$
Problem number: 153.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[_Gegenbauer]`

$$\boxed{\left(-x^2+1\right)y''-2y'x+n(n+1)y=0}$$

✓ Solution by Maple
Time used: 0.047 (sec). Leaf size: 15

```
dsolve((1-x^2)*diff(y(x),x$2)-2*x*diff(y(x),x)+n*(n+1)*y(x)=0,y(x), singsol=all)
```

$$y(x)=c_1\,\mathrm{LegendreP}\left(n,x\right)+c_2\,\mathrm{LegendreQ}\left(n,x\right)$$

✓ Solution by Mathematica
Time used: 0.042 (sec). Leaf size: 18

```
DSolve[(1-x^2)*y''[x]-2*x*y'[x]+n*(n+1)*y[x]==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x)\to c_1\,\mathrm{LegendreP}(n,x)+c_2\,\mathrm{LegendreQ}(n,x)$$

30.6 problem 154

Internal problem ID [10978]

Book: Handbook of exact solutions for ordinary differential equations. By Polyanin and Zaitsev. Second edition
Section: Chapter 2, Second-Order Differential Equations. section 2.1.2-5 Equation of form $(ax^2+bx+c)y''+f(x)y'+g(x)y=0$
Problem number: 154.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[_Gegenbauer]`

$$\boxed{\left(-x^2+1\right)y''-2y'x+\nu(\nu+1)y=0}$$

✓ Solution by Maple
Time used: 0.062 (sec). Leaf size: 15

```
dsolve((1-x^2)*diff(y(x),x$2)-2*x*diff(y(x),x)+nu*(nu+1)*y(x)=0,y(x), singsol=all)
```

$$y(x)=c_1\,\mathrm{LegendreP}\left(\nu,x\right)+c_2\,\mathrm{LegendreQ}\left(\nu,x\right)$$

✓ Solution by Mathematica
Time used: 0.042 (sec). Leaf size: 18

```
DSolve[(1-x^2)*y''[x]-2*x*y'[x]+\[Nu]*(\[Nu]+1)*y[x]==0,y[x],x,IncludeSingularSolutions -> T
```

$$y(x)\to c_1\,\mathrm{LegendreP}(\nu,x)+c_2\,\mathrm{LegendreQ}(\nu,x)$$

30.7 problem 155

Internal problem ID [10979]

Book: Handbook of exact solutions for ordinary differential equations. By Polyanin and Zaitsev. Second edition
Section: Chapter 2, Second-Order Differential Equations. section 2.1.2-5 Equation of form $(ax^2+bx+c)y''+f(x)y'+g(x)y=0$
Problem number: 155.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[_Gegenbauer]`

$$\boxed{\left(-x^2+1\right)y''-3y'x+ny(2+n)=0}$$

✓ Solution by Maple
Time used: 0.063 (sec). Leaf size: 68

```
dsolve((1-x^2)*diff(y(x),x$2)-3*x*diff(y(x),x)+n*(n+2)*y(x)=0,y(x), singsol=all)
```

$$y(x)=\frac{c_1\left(-\sqrt{x^2-1}+x\right)\left(x+\sqrt{x^2-1}\right)^{-n-1}-c_2\left(x+\sqrt{x^2-1}\right)^n}{\sqrt{x^2-1}\left(-\sqrt{x^2-1}+x\right)}$$

✓ Solution by Mathematica
Time used: 0.061 (sec). Leaf size: 42

```
DSolve[(1-x^2)*y''[x]-3*x*y'[x]+n*(n+2)*y[x]==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x)\to\frac{c_1P^{\frac{1}{2}}_{n+\frac{1}{2}}(x)+c_2Q^{\frac{1}{2}}_{n+\frac{1}{2}}(x)}{\sqrt[4]{x^2-1}}$$

30.8 problem 156

Internal problem ID [10980]

Book: Handbook of exact solutions for ordinary differential equations. By Polyanin and Zaitsev. Second edition
Section: Chapter 2, Second-Order Differential Equations. section 2.1.2-5 Equation of form $(ax^2+bx+c)y''+f(x)y'+g(x)y=0$
Problem number: 156.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[_Gegenbauer]`

$$\boxed{\left(x^2-1\right)y''+2(n+1)\,xy'-(\nu+n+1)\,(\nu-n)\,y=0}$$

✓ Solution by Maple
Time used: 0.062 (sec). Leaf size: 27

```
dsolve((x^2-1)*diff(y(x),x$2)+2*(n+1)*x*diff(y(x),x)-(nu+n+1)*(nu-n)*y(x)=0,y(x), singsol=al
```

$$y(x)=\left(\text{LegendreP}\left(\nu,n,x\right)c_1+\text{LegendreQ}\left(\nu,n,x\right)c_2\right)\left(x^2-1\right)^{-\frac{n}{2}}$$

✓ Solution by Mathematica
Time used: 0.069 (sec). Leaf size: 32

```
DSolve[(x^2-1)*y''[x]+2*(n+1)*x*y'[x]-(\[Nu]+n+1)*(\[Nu]-n)*y[x]==0,y[x],x,IncludeSingularSo
```

$$y(x)\to\left(x^2-1\right)^{-n/2}\left(c_1P_\nu^n(x)+c_2Q_\nu^n(x)\right)$$

30.9 problem 157

Internal problem ID [10981]

Book: Handbook of exact solutions for ordinary differential equations. By Polyanin and Zaitsev. Second edition
Section: Chapter 2, Second-Order Differential Equations. section 2.1.2-5 Equation of form $(ax^2+bx+c)y''+f(x)y'+g(x)y=0$
Problem number: 157.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[_Gegenbauer]`

$$\boxed{\left(x^2-1\right)y''-2(n-1)\,xy'-(\nu-n+1)\,(\nu+n)\,y=0}$$

✓ Solution by Maple
Time used: 0.062 (sec). Leaf size: 27

```
dsolve((x^2-1)*diff(y(x),x$2)-2*(n-1)*x*diff(y(x),x)-(nu-n+1)*(nu+n)*y(x)=0,y(x), singsol=al
```

$$y(x)=\left(\text{LegendreP}\left(\nu,n,x\right)c_1+\text{LegendreQ}\left(\nu,n,x\right)c_2\right)\left(x^2-1\right)^{\frac{n}{2}}$$

✓ Solution by Mathematica
Time used: 0.064 (sec). Leaf size: 32

```
DSolve[(x^2-1)*y''[x]-2*(n-1)*x*y'[x]-(\[Nu]-n+1)*(\[Nu]+n)*y[x]==0,y[x],x,IncludeSingularSo
```

$$y(x)\to\left(x^2-1\right)^{n/2}\left(c_1P_\nu^n(x)+c_2Q_\nu^n(x)\right)$$

30.10 problem 158

Internal problem ID [10982]

Book: Handbook of exact solutions for ordinary differential equations. By Polyanin and Zaitsev. Second edition
Section: Chapter 2, Second-Order Differential Equations. section 2.1.2-5 Equation of form $(ax^2+bx+c)y''+f(x)y'+g(x)y=0$
Problem number: 158.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$\boxed{\left(x^2-1\right)y''+(2a+1)y'-b(2a+b)y=0}$$

✓ Solution by Maple
Time used: 0.078 (sec). Leaf size: 112

```
dsolve((x^2-1)*diff(y(x),x$2)+(2*a+1)*diff(y(x),x)-b*(2*a+b)*y(x)=0,y(x), singsol=all)
```

$$y(x)=c_1\,\text{hypergeom}\left(\left[-\frac{1}{2}-\frac{\sqrt{8ab+4b^2+1}}{2},\frac{\sqrt{8ab+4b^2+1}}{2}-\frac{1}{2}\right],\left[-a-\frac{1}{2}\right],\frac{1}{2}+\frac{x}{2}\right)+c_2\left(\frac{1}{2}+\frac{x}{2}\right)^{a+\frac{3}{2}}\text{hypergeom}\left(\left[1-\frac{\sqrt{8ab+4b^2+1}}{2}+a,\frac{\sqrt{8ab+4b^2+1}}{2}+1+a\right],\left[\frac{5}{2}+a\right],\frac{1}{2}+\frac{x}{2}\right)$$

✓ Solution by Mathematica

Time used: 0.304 (sec). Leaf size: 152

```
DSolve[(x^2-1)*y''[x]+(2*a+1)*y'[x]-b*(2*a+b)*y[x]==0,y[x],x,IncludeSingularSolutions -> Tru
```

$$y(x) \to 2^{a-\frac{1}{2}} c_2 (x-1)^{\frac{1}{2}-a} \text{Hypergeometric2F1}\left(-a - \frac{1}{2}\sqrt{4b^2+8ab+1}, \frac{1}{2}\sqrt{4b^2+8ab+1} - a, \frac{3}{2} - a, \frac{1}{2} - \frac{x}{2}\right) + c_1 \text{Hypergeometric2F1}\left(\frac{1}{2}\left(-\sqrt{4b^2+8ab+1} - 1\right), \frac{1}{2}\left(\sqrt{4b^2+8ab+1} - 1\right), a + \frac{1}{2}, \frac{1-x}{2}\right)$$

30.11 problem 159

Internal problem ID [10983]

Book: Handbook of exact solutions for ordinary differential equations. By Polyanin and Zaitsev. Second edition
Section: Chapter 2, Second-Order Differential Equations. section 2.1.2-5 Equation of form $(ax^2+bx+c)y''+f(x)y'+g(x)y=0$
Problem number: 159.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[_Gegenbauer]`

$$\boxed{\left(-x^2+1\right)y''+(2a-3)xy'+(n+1)(n+2a-1)y=0}$$

✓ Solution by Maple
Time used: 0.062 (sec). Leaf size: 39

```
dsolve((1-x^2)*diff(y(x),x$2)+(2*a-3)*x*diff(y(x),x)+(n+1)*(n+2*a-1)*y(x)=0,y(x), singsol=al
```

$$y(x)=\left(\text{LegendreP}\left(a+n-\frac{1}{2},a-\frac{1}{2},x\right)c_1+\text{LegendreQ}\left(a+n-\frac{1}{2},a-\frac{1}{2},x\right)c_2\right)\left(x^2-1\right)^{\frac{a}{2}-\frac{1}{4}}$$

✓ Solution by Mathematica
Time used: 0.325 (sec). Leaf size: 158

```
DSolve[(1-x^2)*y''[x]+(2*a-3)*y'[x]+(n+1)*(n+2*a-1)*y[x]==0,y[x],x,IncludeSingularSolutions
```

$$y(x)\to 2^{\frac{1}{2}-a}c_2(x-1)^{a-\frac{1}{2}}\,\text{Hypergeometric2F1}\left(a-\frac{1}{2}\sqrt{4n^2+8a(n+1)-3}-1,a+\frac{1}{2}\sqrt{4n^2+8a(n+1)-3}-1,a+\frac{1}{2},\frac{1-x}{2}\right)+c_1\,\text{Hypergeometric2F1}\left(\frac{1}{2}\left(-\sqrt{4n^2+8a(n+1)-3}-1\right),\frac{1}{2}\left(\sqrt{4n^2+8a(n+1)-3}-1\right),\frac{3}{2}-a,\frac{1-x}{2}\right)$$

30.12 problem 160

Internal problem ID [10984]

Book: Handbook of exact solutions for ordinary differential equations. By Polyanin and Zaitsev. Second edition
Section: Chapter 2, Second-Order Differential Equations. section 2.1.2-5 Equation of form $(ax^2+bx+c)y''+f(x)y'+g(x)y=0$
Problem number: 160.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$\boxed{\left(-x^2+1\right)y''+\left(\beta-\alpha-\left(\alpha+\beta+2\right)x\right)y'+n(n+\alpha+\beta+1)\,y=0}$$

✓ Solution by Maple
Time used: 0.094 (sec). Leaf size: 61

```
dsolve((1-x^2)*diff(y(x),x$2)+(beta-alpha-(alpha+beta+2)*x)*diff(y(x),x)+n*(n+alpha+beta+1)*
```

$$y(x)=c_1\,\text{hypergeom}\left(\left[-n,n+\alpha+\beta+1\right],\left[\beta+1\right],\frac{1}{2}+\frac{x}{2}\right)+c_2\left(\frac{1}{2}+\frac{x}{2}\right)^{-\beta}\text{hypergeom}\left(\left[-n-\beta,n+\alpha+1\right],\left[1-\beta\right],\frac{1}{2}+\frac{x}{2}\right)$$

✓ Solution by Mathematica
Time used: 0.28 (sec). Leaf size: 69

```
DSolve[(1-x^2)*y''[x]+(\[Beta]-\[Alpha]-(\[Alpha]+\[Beta]+2)*x)*y'[x]+n*(n+\[Alpha]+\[Beta]+
```

$$y(x)\to 2^{\alpha}c_2(x-1)^{-\alpha}\,\text{Hypergeometric2F1}\left(-n-\alpha,n+\beta+1,1-\alpha,\frac{1-x}{2}\right)+c_1\,\text{Hypergeometric2F1}\left(-n,n+\alpha+\beta+1,\alpha+1,\frac{1-x}{2}\right)$$

30.13 problem 161

Internal problem ID [10985]

Book: Handbook of exact solutions for ordinary differential equations. By Polyanin and Zaitsev. Second edition
Section: Chapter 2, Second-Order Differential Equations. section 2.1.2-5 Equation of form $(ax^2+bx+c)y''+f(x)y'+g(x)y=0$
Problem number: 161.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$\left(-x^2+1\right)y''+\left(\alpha-\beta+\left(\alpha+\beta-2\right)x\right)y'+\left(n+1\right)\left(n+\alpha+\beta\right)y=0$$

✓ Solution by Maple
Time used: 0.094 (sec). Leaf size: 64

```
dsolve((1-x^2)*diff(y(x),x$2)+(alpha-beta+(alpha+beta-2)*x)*diff(y(x),x)+(n+1)*(n+alpha+beta
```

$$y(x)=c_1\,\text{hypergeom}\left(\left[n+1,-n-\beta-\alpha\right],\left[1-\beta\right],\frac{1}{2}+\frac{x}{2}\right)+c_2\left(\frac{1}{2}+\frac{x}{2}\right)^{\beta}\text{hypergeom}\left(\left[-n-\alpha,n+\beta+1\right],\left[\beta+1\right],\frac{1}{2}+\frac{x}{2}\right)$$

✓ Solution by Mathematica
Time used: 0.225 (sec). Leaf size: 74

```
DSolve[(1-x^2)*y''[x]+(\[Alpha]-\[Beta]+(\[Alpha]+\[Beta]-2)*x)*y'[x]+(n+1)*(n+\[Alpha]+\[Be
```

$$y(x)\to 2^{-\alpha}c_2(x-1)^{\alpha}\,\text{Hypergeometric2F1}\left(n+\alpha+1,-n-\beta,\alpha+1,\frac{1-x}{2}\right)+c_1\,\text{Hypergeometric2F1}\left(n+1,-n-\alpha-\beta,1-\alpha,\frac{1-x}{2}\right)$$

30.14 problem 162

Internal problem ID [10986]

Book: Handbook of exact solutions for ordinary differential equations. By Polyanin and Zaitsev. Second edition
Section: Chapter 2, Second-Order Differential Equations. section 2.1.2-5 Equation of form $(ax^2+bx+c)y''+f(x)y'+g(x)y=0$
Problem number: 162.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries], [_2nd_order, _linear,`

$$\boxed{\left(a\,x^2+b\right)y''+axy'+cy=0}$$

✓ Solution by Maple
Time used: 0.0 (sec). Leaf size: 59

```
dsolve((a*x^2+b)*diff(y(x),x$2)+a*x*diff(y(x),x)+c*y(x)=0,y(x), singsol=all)
```

$$y(x)=c_1\left(\sqrt{a}\,x+\sqrt{a\,x^2+b}\right)^{\frac{i\sqrt{c}}{\sqrt{a}}}+c_2\left(\sqrt{a}\,x+\sqrt{a\,x^2+b}\right)^{-\frac{i\sqrt{c}}{\sqrt{a}}}$$

✓ Solution by Mathematica
Time used: 0.196 (sec). Leaf size: 74

```
DSolve[(a*x^2+b)*y''[x]+a*x*y'[x]+c*y[x]==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x)\to c_1\cos\left(\frac{\sqrt{c}\operatorname{arctanh}\left(\frac{\sqrt{a}x}{\sqrt{ax^2+b}}\right)}{\sqrt{a}}\right)+c_2\sin\left(\frac{\sqrt{c}\operatorname{arctanh}\left(\frac{\sqrt{a}x}{\sqrt{ax^2+b}}\right)}{\sqrt{a}}\right)$$

30.15 problem 163

Internal problem ID [10987]

Book: Handbook of exact solutions for ordinary differential equations. By Polyanin and Zaitsev. Second edition
Section: Chapter 2, Second-Order Differential Equations. section 2.1.2-5 Equation of form $(ax^2+bx+c)y''+f(x)y'+g(x)y=0$
Problem number: 163.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _exact, _linear, _homogeneous]]`

$$\boxed{\left(x^2+a\right)y''+2bxy'+2(b-1)y=0}$$

✓ Solution by Maple
Time used: 0.047 (sec). Leaf size: 41

```
dsolve((x^2+a)*diff(y(x),x$2)+2*b*x*diff(y(x),x)+2*(b-1)*y(x)=0,y(x), singsol=all)
```

$$y(x)=c_1\left(\frac{x^2+a}{a}\right)^{-b+1}+c_2x\,\text{hypergeom}\left(\left[1,b-\frac{1}{2}\right],\left[\frac{3}{2}\right],-\frac{x^2}{a}\right)$$

✓ Solution by Mathematica
Time used: 0.426 (sec). Leaf size: 64

```
DSolve[(x^2+a)*y''[x]+2*b*x*y'[x]+2*(b-1)*y[x]==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x)\to\left(a+x^2\right)\left(\frac{c_2x\left(\frac{a+x^2}{a}\right)^{-b}\text{Hypergeometric2F1}\left(\frac{1}{2},2-b,\frac{3}{2},-\frac{x^2}{a}\right)}{a^2}+c_1\left(a+x^2\right)^{-b}\right)$$

30.16 problem 164

Internal problem ID [10988]

Book: Handbook of exact solutions for ordinary differential equations. By Polyanin and Zaitsev. Second edition
Section: Chapter 2, Second-Order Differential Equations. section 2.1.2-5 Equation of form $(ax^2+bx+c)y''+f(x)y'+g(x)y=0$
Problem number: 164.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$\boxed{\left(-a^2+x^2\right)y''+2bxy'+b(b-1)y=0}$$

✓ Solution by Maple
Time used: 0.0 (sec). Leaf size: 29

```
dsolve((x^2-a^2)*diff(y(x),x$2)+2*b*x*diff(y(x),x)+b*(b-1)*y(x)=0,y(x), singsol=all)
```

$$y(x)=c_1(a+x)^{-b+1}+c_2(a-x)^{-b+1}$$

✓ Solution by Mathematica
Time used: 0.727 (sec). Leaf size: 127

```
DSolve[(x^2-a^2)*y''[x]+2*b*x*y'[x]+b*(b-1)*y[x]==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x)$$
$$\to \frac{(x-a)^{\frac{1}{2}-\frac{1}{2}\sqrt{(b-1)^2}}(a+x)^{\frac{1}{2}-\frac{1}{2}\sqrt{(b-1)^2}}\left(x^2-a^2\right)^{-b/2}\left(2a\sqrt{(b-1)^2}c_1(x-a)^{\sqrt{(b-1)^2}}-c_2(a+x)^{\sqrt{(b-1)^2}}\right)}{2a\sqrt{(b-1)^2}}$$

30.17 problem 165

Internal problem ID [10989]

Book: Handbook of exact solutions for ordinary differential equations. By Polyanin and Zaitsev. Second edition
Section: Chapter 2, Second-Order Differential Equations. section 2.1.2-5 Equation of form $(ax^2+bx+c)y''+f(x)y'+g(x)y=0$
Problem number: 165.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$\boxed{\left(a^2+x^2\right)y''+2bxy'+b(b-1)y=0}$$

✓ Solution by Maple
Time used: 0.14 (sec). Leaf size: 33

```
dsolve((x^2+a^2)*diff(y(x),x$2)+2*b*x*diff(y(x),x)+b*(b-1)*y(x)=0,y(x), singsol=all)
```

$$y(x)=c_1(-ix+a)^{-b+1}+c_2(ix+a)^{-b+1}$$

✓ Solution by Mathematica
Time used: 0.813 (sec). Leaf size: 101

```
DSolve[(x^2+a^2)*y''[x]+2*b*x*y'[x]+b*(b-1)*y[x]==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x)\to \frac{\left(a^2+x^2\right)^{\frac{1}{2}-\frac{b}{2}}e^{-i\sqrt{(b-1)^2}\arctan\left(\frac{a}{x}\right)}\left(ic_2e^{2i\sqrt{(b-1)^2}\arctan\left(\frac{a}{x}\right)}+2a\sqrt{(b-1)^2}c_1\right)}{2a\sqrt{(b-1)^2}}$$

30.18 problem 166

Internal problem ID [10990]

Book: Handbook of exact solutions for ordinary differential equations. By Polyanin and Zaitsev. Second edition
Section: Chapter 2, Second-Order Differential Equations. section 2.1.2-5 Equation of form $(ax^2+bx+c)y''+f(x)y'+g(x)y=0$
Problem number: 166.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$\boxed{\left(a\,x^2+b\right)y''+\left(1+2n\right)axy'+cy=0}$$

✓ Solution by Maple
Time used: 0.078 (sec). Leaf size: 93

```
dsolve((a*x^2+b)*diff(y(x),x$2)+(2*n+1)*a*x*diff(y(x),x)+c*y(x)=0,y(x), singsol=all)
```

$$y(x)=\left(a\,x^2+b\right)^{-\frac{n}{2}+\frac{1}{4}}\left(c_1\,\mathrm{LegendreP}\left(-\frac{-2\sqrt{a\,n^2-c}+\sqrt{a}}{2\sqrt{a}},n-\frac{1}{2},\frac{ax}{\sqrt{-ab}}\right)+c_2\,\mathrm{LegendreQ}\left(-\frac{-2\sqrt{a\,n^2-c}+\sqrt{a}}{2\sqrt{a}},n-\frac{1}{2},\frac{ax}{\sqrt{-ab}}\right)\right)$$

✓ Solution by Mathematica
Time used: 0.149 (sec). Leaf size: 118

```
DSolve[(a*x^2+b)*y''[x]+(2*n+1)*a*x*y'[x]+c*y[x]==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x)\to\left(ax^2+b\right)^{\frac{1}{4}-\frac{n}{2}}\left(c_1P^{n-\frac{1}{2}}_{\frac{\sqrt{an^2-c}}{\sqrt{a}}-\frac{1}{2}}\left(\frac{i\sqrt{a}x}{\sqrt{b}}\right)+c_2Q^{n-\frac{1}{2}}_{\frac{\sqrt{an^2-c}}{\sqrt{a}}-\frac{1}{2}}\left(\frac{i\sqrt{a}x}{\sqrt{b}}\right)\right)$$

30.19 problem 167

Internal problem ID [10991]

Book: Handbook of exact solutions for ordinary differential equations. By Polyanin and Zaitsev. Second edition
Section: Chapter 2, Second-Order Differential Equations. section 2.1.2-5 Equation of form $(ax^2+bx+c)y''+f(x)y'+g(x)y=0$
Problem number: 167.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$\boxed{\left(-x^2+1\right)y''-y'x+\left(2a\,x^2+b\right)y=0}$$

✓ Solution by Maple
Time used: 0.359 (sec). Leaf size: 27

```
dsolve((1-x^2)*diff(y(x),x$2)-x*diff(y(x),x)+(2*a*x^2+b)*y(x)=0,y(x), singsol=all)
```

$$y(x)=c_1\,\text{MathieuC}\left(a+b,-\frac{a}{2},\arccos\left(x\right)\right)+c_2\,\text{MathieuS}\left(a+b,-\frac{a}{2},\arccos\left(x\right)\right)$$

✓ Solution by Mathematica
Time used: 0.051 (sec). Leaf size: 34

```
DSolve[(1-x^2)*y''[x]-x*y'[x]+(2*a*x^2+b)*y[x]==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x)\to c_1\text{MathieuC}\left[a+b,-\frac{a}{2},\arccos(x)\right]+c_2\text{MathieuS}\left[a+b,-\frac{a}{2},\arccos(x)\right]$$

30.20 problem 168

Internal problem ID [10992]

Book: Handbook of exact solutions for ordinary differential equations. By Polyanin and Zaitsev. Second edition
Section: Chapter 2, Second-Order Differential Equations. section 2.1.2-5 Equation of form $(ax^2+bx+c)y''+f(x)y'+g(x)y=0$
Problem number: 168.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$\boxed{\left(-x^2+1\right)y''+\left(ax+b\right)y'+yc=0}$$

✓ Solution by Maple
Time used: 0.079 (sec). Leaf size: 134

```
dsolve((1-x^2)*diff(y(x),x$2)+(a*x+b)*diff(y(x),x)+c*y(x)=0,y(x), singsol=all)
```

$$y(x)=c_1\,\text{hypergeom}\left(\left[-\frac{1}{2}-\frac{a}{2}-\frac{\sqrt{a^2+2a+4c+1}}{2},-\frac{1}{2}-\frac{a}{2}+\frac{\sqrt{a^2+2a+4c+1}}{2}\right],\left[-\frac{a}{2}+\frac{b}{2}\right],\frac{1}{2}+\frac{x}{2}\right)+c_2\left(\frac{1}{2}+\frac{x}{2}\right)^{1+\frac{a}{2}-\frac{b}{2}}\text{hypergeom}\left(\left[\frac{1}{2}-\frac{\sqrt{a^2+2a+4c+1}}{2}-\frac{b}{2},\frac{1}{2}+\frac{\sqrt{a^2+2a+4c+1}}{2}-\frac{b}{2}\right],\left[2+\frac{a}{2}-\frac{b}{2}\right],\frac{1}{2}+\frac{x}{2}\right)$$

✓ Solution by Mathematica

Time used: 0.317 (sec). Leaf size: 184

```
DSolve[(1-x^2)*y''[x]+(a*x+b)*y'[x]+c*y[x]==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to 2^{\frac{1}{2}(-a-b-2)}\left(c_2(x\right.$$
$$-1)^{\frac{1}{2}(a+b+2)}\,\text{Hypergeometric2F1}\left(\frac{1}{2}\left(b-\sqrt{a^2+2a+4c+1}+1\right),\frac{1}{2}\left(b+\sqrt{a^2+2a+4c+1}+1\right),\frac{1}{2}(a+\right.$$
$$+c_1 2^{\frac{1}{2}(a+b+2)}\,\text{Hypergeometric2F1}\left(\frac{1}{2}\left(-a-\sqrt{a^2+2a+4c+1}-1\right),\frac{1}{2}\left(-a+\sqrt{a^2+2a+4c+1}-1\right),\frac{1}{2}\right.$$

30.21 problem 169

Internal problem ID [10993]

Book: Handbook of exact solutions for ordinary differential equations. By Polyanin and Zaitsev. Second edition
Section: Chapter 2, Second-Order Differential Equations. section 2.1.2-5 Equation of form $(ax^2+bx+c)y''+f(x)y'+g(x)y=0$
Problem number: 169.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$\boxed{\left(a\,x^2+b\right)y''+\left(c\,x^2+d\right)y'+\lambda\left(\left(-a\lambda+c\right)x^2+d-b\lambda\right)y=0}$$

✓ Solution by Maple

Time used: 0.281 (sec). Leaf size: 939

```
dsolve((a*x^2+b)*diff(y(x),x$2)+(c*x^2+d)*diff(y(x),x)+lambda*((c-a*lambda)*x^2+d-b*lambda)*
```

$$y(x) = \left(-ax+\sqrt{-ab}\right)^{\frac{2a^2b+\sqrt{4a^2b(ad-bc)\sqrt{-ab}+4a^4b^2-a^3bd^2+2db^2ca^2-b^3c^2a}}{4a^2b}} \left(c_2 \left(ax \right.\right.$$
$$\left. + \sqrt{-ab}\right)^{-\frac{-2a^2b+\sqrt{-ab\left(4\sqrt{-ab}\,a^2d-4\sqrt{-ab}\,abc-4a^3b+a^2d^2-2abcd+b^2c^2\right)}}{4a^2b}} \operatorname{HeunC}\left(\frac{(4a\lambda-2c)\sqrt{-\frac{b}{a}}}{a}, \right.$$
$$\left. -\frac{\sqrt{-ab\left(4\sqrt{-ab}\,a^2d-4\sqrt{-ab}\,abc-4a^3b+a^2d^2-2abcd+b^2c^2\right)}}{2a^2b}, \frac{\sqrt{4a^2b(ad-bc)\sqrt{-ab}+4a^4b^2-a}}{2a^2b} \right.$$
$$\left. - \frac{bc\lambda}{a^2} + \frac{1}{2} - \frac{d^2}{8ab} - \frac{cd}{4a^2} + \frac{3bc^2}{8a^3}, \frac{ax}{2\sqrt{-ab}} \right.$$
$$\left. + \frac{1}{2} \right) \mathrm{e}^{\frac{-i\pi\sqrt{4a^2b(ad-bc)\sqrt{-ab}+4a^4b^2-a^3bd^2+2db^2ca^2-b^3c^2a}+i\pi\sqrt{-ab\left(4\sqrt{-ab}\,a^2d-4\sqrt{-ab}\,abc-4a^3b+a^2d^2-2abcd+b^2c^2\right)}-4b\left(a^2\left(\frac{d}{\sqrt{b}\sqrt{a}}-\frac{\sqrt{b}c}{a^{\frac{3}{2}}}\right.\right.}{8a^2b}}$$
$$+ c_1 \left(ax \right.$$
$$\left. + \sqrt{-ab}\right)^{\frac{2a^2b+\sqrt{-ab\left(4\sqrt{-ab}\,a^2d-4\sqrt{-ab}\,abc-4a^3b+a^2d^2-2abcd+b^2c^2\right)}}{4a^2b}} \mathrm{e}^{x\lambda+\frac{\sqrt{-ab}\,\lambda}{a}-\frac{cx}{a}-\frac{\sqrt{-ab}\,c}{2a^2}-\frac{\arctan\left(\frac{\sqrt{a}\,x}{\sqrt{b}}\right)d}{2\sqrt{a}\sqrt{b}}+\frac{\sqrt{b}\,\arctan\left(\frac{\sqrt{a}\,x}{\sqrt{b}}\right)c}{2a^{\frac{3}{2}}}} \mathrm{He}$$
$$\left. \left. - \frac{bc\lambda}{a^2} + \frac{1}{2} - \frac{d^2}{8ab} - \frac{cd}{4a^2} + \frac{3bc^2}{8a^3}, \frac{ax}{2\sqrt{-ab}} + \frac{1}{2} \right) \right)$$

✓ Solution by Mathematica

Time used: 2.859 (sec). Leaf size: 74

```
DSolve[(a*x^2+b)*y''[x]+(c*x^2+d)*y'[x]+\[Lambda]*((c-a*\[Lambda])*x^2+d-b*\[Lambda])*y[x]==
```

$$y(x) \to e^{\lambda(-x)} \left(c_2 \int_1^x \exp\left(\frac{(bc-ad)\arctan\left(\frac{\sqrt{a}K[1]}{\sqrt{b}}\right)}{a^{3/2}\sqrt{b}} + \left(2\lambda - \frac{c}{a}\right) K[1] \right) dK[1] + c_1 \right)$$

30.22 problem 170

Internal problem ID [10994]

Book: Handbook of exact solutions for ordinary differential equations. By Polyanin and Zaitsev. Second edition
Section: Chapter 2, Second-Order Differential Equations. section 2.1.2-5 Equation of form $(ax^2+bx+c)y''+f(x)y'+g(x)y=0$
Problem number: 170.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$\left(a\,x^2+b\right)y''+\left(\lambda(a+c)\,x^2+(c-a)\,x+2b\lambda\right)y'+\lambda^2\left(c\,x^2+b\right)y=0$$

✓ Solution by Maple
Time used: 0.422 (sec). Leaf size: 1381

```
dsolve((a*x^2+b)*diff(y(x),x$2)+(lambda*(c+a)*x^2+(c-a)*x+2*b*lambda)*diff(y(x),x)+lambda^2*
```

Expression too large to display

✓ Solution by Mathematica
Time used: 5.408 (sec). Leaf size: 104

```
DSolve[(a*x^2+b)*y''[x]+(\[Lambda]*(c+a)*x^2+(c-a)*x+2*b*\[Lambda])*y'[x]+\[Lambda]^2*(c*x^2
```

$$y(x)\to e^{\lambda(-x)}(\lambda x \\ +1)\left(c_2\int_1^x\frac{\exp\left(\frac{(a-c)\lambda\left(\sqrt{a}K[1]-\sqrt{b}\arctan\left(\frac{\sqrt{a}K[1]}{\sqrt{b}}\right)\right)}{a^{3/2}}\right)\left(aK[1]^2+b\right)^{\frac{a-c}{2a}}}{(\lambda K[1]+1)^2}dK[1]+c_1\right)$$

30.23 problem 171

Internal problem ID [10995]

Book: Handbook of exact solutions for ordinary differential equations. By Polyanin and Zaitsev. Second edition
Section: Chapter 2, Second-Order Differential Equations. section 2.1.2-5 Equation of form $(ax^2+bx+c)y''+f(x)y'+g(x)y=0$
Problem number: 171.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[_Jacobi]`

$$x(x-1)\,y''+((\alpha+\beta+1)\,x-\gamma)\,y'+\alpha\beta y=0$$

✓ Solution by Maple
Time used: 0.125 (sec). Leaf size: 44

```
dsolve(x*(x-1)*diff(y(x),x$2)+((alpha+beta+1)*x-gamma)*diff(y(x),x)+alpha*beta*y(x)=0,y(x),
```

$$y(x)=c_1\,\text{hypergeom}\left([\alpha,\beta],[\gamma],x\right)+c_2x^{1-\gamma}\,\text{hypergeom}\left([\beta+1-\gamma,\alpha+1-\gamma],[2-\gamma],x\right)$$

✓ Solution by Mathematica
Time used: 0.281 (sec). Leaf size: 49

```
DSolve[x*(x-1)*y''[x]+((\[Alpha]+\[Beta]+1)*x-\[Gamma])*y'[x]+\[Alpha]*\[Beta]*y[x]==0,y[x],
```

$$\begin{aligned}y(x)\to{}& c_1\,\text{Hypergeometric2F1}(\alpha,\beta,\gamma,x)\\&-(-1)^{-\gamma}c_2x^{1-\gamma}\,\text{Hypergeometric2F1}(\alpha-\gamma+1,\beta-\gamma+1,2-\gamma,x)\end{aligned}$$

30.24 problem 172

Internal problem ID [10996]

Book: Handbook of exact solutions for ordinary differential equations. By Polyanin and Zaitsev. Second edition
Section: Chapter 2, Second-Order Differential Equations. section 2.1.2-5 Equation of form $(ax^2+bx+c)y''+f(x)y'+g(x)y=0$
Problem number: 172.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$\boxed{x(x+a)\,y''+(bx+c)\,y'+yd=0}$$

✓ Solution by Maple
Time used: 0.109 (sec). Leaf size: 230

```
dsolve(x*(x+a)*diff(y(x),x$2)+(b*x+c)*diff(y(x),x)+d*y(x)=0,y(x), singsol=all)
```

$$y(x)=c_2(\operatorname{csgn}(a)\,a+a+2x)^{-\frac{((b-2)\operatorname{csgn}(a)a+ab-2c)\operatorname{csgn}(a)}{2a}}\operatorname{hypergeom}\left(\left[\frac{\operatorname{csgn}(a)\left(\operatorname{csgn}(a)\,a+\sqrt{b^2-2b-4d+1}\,\operatorname{csgn}(a)\,a-ab+2\right.\right.\right.$$
$$\left.\left.\left.-\frac{\operatorname{csgn}(a)\left(\sqrt{b^2-2b-4d+1}\,\operatorname{csgn}(a)\,a-\operatorname{csgn}(a)\,a+ab-2c\right)}{2a}\right],\left[-\frac{\operatorname{csgn}(a)\left((b-4)\operatorname{csgn}(a)\,a+ab\right.\right.\right.$$
$$+c_1\operatorname{hypergeom}\left(\left[-\frac{1}{2}+\frac{b}{2}-\frac{\sqrt{b^2-2b-4d+1}}{2},-\frac{1}{2}+\frac{b}{2}\right.\right.$$
$$\left.\left.+\frac{\sqrt{b^2-2b-4d+1}}{2}\right],\left[\frac{(b\operatorname{csgn}(a)\,a+ab-2c)\operatorname{csgn}(a)}{2a}\right],\frac{\operatorname{csgn}(a)\left(\operatorname{csgn}(a)\,a+a+2x\right)}{2a}\right)$$

✓ Solution by Mathematica

Time used: 0.423 (sec). Leaf size: 165

```
DSolve[x*(x+a)*y''[x]+(b*x+c)*y'[x]+d*y[x]==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to c_2 a^{\frac{c}{a}-1} x^{1-\frac{c}{a}} \text{Hypergeometric2F1}\left(\frac{1}{2}\left(b-\frac{2c}{a}+\sqrt{b^2-2b-4d+1}+1\right), \frac{ba-\sqrt{b^2-2b-4d+1}a+a-2c}{2a}, 2-\frac{c}{a}, -\frac{x}{a}\right) + c_1 \text{Hypergeometric2F1}\left(\frac{1}{2}\left(b-\sqrt{b^2-2b-4d+1}-1\right), \frac{1}{2}\left(b+\sqrt{b^2-2b-4d+1}-1\right), \frac{c}{a}, -\frac{x}{a}\right)$$

30.25 problem 173

Internal problem ID [10997]

Book: Handbook of exact solutions for ordinary differential equations. By Polyanin and Zaitsev. Second edition
Section: Chapter 2, Second-Order Differential Equations. section 2.1.2-5 Equation of form $(ax^2+bx+c)y''+f(x)y'+g(x)y=0$
Problem number: 173.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[_Jacobi]`

$$\boxed{2x(x-1)\,y''+(2x-1)\,y'+(ax+b)\,y=0}$$

✓ Solution by Maple
Time used: 0.297 (sec). Leaf size: 39

```
dsolve(2*x*(x-1)*diff(y(x),x$2)+(2*x-1)*diff(y(x),x)+(a*x+b)*y(x)=0,y(x), singsol=all)
```

$$y(x)=c_1\,\text{MathieuC}\left(-a-2b,\frac{a}{2},\arccos\left(\sqrt{x}\right)\right)+c_2\,\text{MathieuS}\left(-a-2b,\frac{a}{2},\arccos\left(\sqrt{x}\right)\right)$$

✓ Solution by Mathematica
Time used: 0.258 (sec). Leaf size: 50

```
DSolve[2*x*(x-1)*y''[x]+(2*x-1)*y'[x]+(a*x+b)*y[x]==0,y[x],x,IncludeSingularSolutions -> Tru
```

$$y(x)\to c_1\text{MathieuC}\left[-a-2b,\frac{a}{2},\arccos\left(\sqrt{x}\right)\right]+c_2\text{MathieuS}\left[-a-2b,\frac{a}{2},\arccos\left(\sqrt{x}\right)\right]$$

30.26 problem 174

Internal problem ID [10998]

Book: Handbook of exact solutions for ordinary differential equations. By Polyanin and Zaitsev. Second edition
Section: Chapter 2, Second-Order Differential Equations. section 2.1.2-5 Equation of form $(ax^2+bx+c)y''+f(x)y'+g(x)y=0$
Problem number: 174.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries], [_2nd_order, _linear,`

$$\boxed{\left(2ax+x^2+b\right)y''+\left(x+a\right)y'-ym^2=0}$$

✓ Solution by Maple
Time used: 0.0 (sec). Leaf size: 43

```
dsolve((x^2+2*a*x+b)*diff(y(x),x$2)+(x+a)*diff(y(x),x)-m^2*y(x)=0,y(x), singsol=all)
```

$$y(x)=c_1\left(a+x+\sqrt{2ax+x^2+b}\right)^{-m}+c_2\left(a+x+\sqrt{2ax+x^2+b}\right)^{m}$$

✓ Solution by Mathematica
Time used: 0.301 (sec). Leaf size: 63

```
DSolve[(x^2+2*a*x+b)*y''[x]+(x+a)*y'[x]-m^2*y[x]==0,y[x],x,IncludeSingularSolutions -> True]
```

$$\begin{aligned}y(x)\to c_1\cosh\left(m\log\left(\sqrt{2ax+b+x^2}-a-x\right)\right)\\-ic_2\sinh\left(m\log\left(\sqrt{2ax+b+x^2}-a-x\right)\right)\end{aligned}$$

30.27 problem 175

Internal problem ID [10999]

Book: Handbook of exact solutions for ordinary differential equations. By Polyanin and Zaitsev. Second edition
Section: Chapter 2, Second-Order Differential Equations. section 2.1.2-5 Equation of form $(ax^2+bx+c)y''+f(x)y'+g(x)y=0$
Problem number: 175.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _exact, _linear, _homogeneous]]`

$$\boxed{\left(a\,x^2+bx+c\right)y''+\left(dx+k\right)y'+\left(-2a+d\right)y=0}$$

✓ Solution by Maple
Time used: 0.047 (sec). Leaf size: 1412

```
dsolve((a*x^2+b*x+c)*diff(y(x),x$2)+(d*x+k)*diff(y(x),x)+(d-2*a)*y(x)=0,y(x), singsol=all)
```

Expression too large to display

✓ Solution by Mathematica
Time used: 15.225 (sec). Leaf size: 164

```
DSolve[(a*x^2+b*x+c)*y''[x]+(d*x+k)*y'[x]+(d-2*a)*y[x]==0,y[x],x,IncludeSingularSolutions ->
```

$$y(x)\to (x(ax+b)$$
$$+c)^{1-\frac{d}{2a}}\exp\left(\frac{(bd-2ak)\arctan\left(\frac{2ax+b}{\sqrt{4ac-b^2}}\right)}{a\sqrt{4ac-b^2}}\right)\left(c_2\int_1^x\exp\left(\frac{(d-4a)\log(c+K[1](b+aK[1]))-\frac{2(bd-2}{2a}\right.\right.$$
$$\left.\left.+c_1\right)\right.$$

30.28 problem 176

Internal problem ID [11000]

Book: Handbook of exact solutions for ordinary differential equations. By Polyanin and Zaitsev. Second edition
Section: Chapter 2, Second-Order Differential Equations. section 2.1.2-5 Equation of form $(ax^2+bx+c)y''+f(x)y'+g(x)y=0$
Problem number: 176.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$\boxed{\left(a\,x^2+bx+c\right)y''+\left(kx+d\right)y'-yk=0}$$

✓ Solution by Maple
Time used: 0.094 (sec). Leaf size: 315

```
dsolve((a*x^2+b*x+c)*diff(y(x),x$2)+(k*x+d)*diff(y(x),x)-k*y(x)=0,y(x), singsol=all)
```

$$y(x)=c_1(kx+d)$$
$$+c_2\left(2\sqrt{\frac{-4ac+b^2}{a^2}}\,x\,a^2+\sqrt{\frac{-4ac+b^2}{a^2}}\,ba-4ac+b^2\right)^{\frac{a\left(a-\frac{k}{2}\right)\sqrt{\frac{-4ac+b^2}{a^2}}+ad-\frac{kb}{2}}{\sqrt{\frac{-4ac+b^2}{a^2}}\,a^2}}\text{hypergeom}\left(\left[-\frac{k\sqrt{\frac{-4ac+b}{a^2}}}{2a^2}\right.\right.$$

✓ Solution by Mathematica
Time used: 4.256 (sec). Leaf size: 107

```
DSolve[(a*x^2+b*x+c)*y''[x]+(k*x+d)*y'[x]-k*y[x]==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x)\to\frac{(d+kx)\left(c_2\int_1^x\frac{\exp\left(\frac{(bk-2ad)\arctan\left(\frac{b+2aK[1]}{\sqrt{4ac-b^2}}\right)}{a\sqrt{4ac-b^2}}\right)(c+K[1](b+aK[1]))^{-\frac{k}{2a}}}{(d+kK[1])^2}dK[1]+c_1\right)}{d}$$

30.29 problem 177

Internal problem ID [11001]

Book: Handbook of exact solutions for ordinary differential equations. By Polyanin and Zaitsev. Second edition
Section: Chapter 2, Second-Order Differential Equations. section 2.1.2-5 Equation of form $(ax^2+bx+c)y''+f(x)y'+g(x)y=0$
Problem number: 177.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries], [_2nd_order, _linear,`

$$\boxed{\left(a\,x^2+2bx+c\right)y''+\left(ax+b\right)y'+yd=0}$$

✓ Solution by Maple
Time used: 0.0 (sec). Leaf size: 81

```
dsolve((a*x^2+2*b*x+c)*diff(y(x),x$2)+(a*x+b)*diff(y(x),x)+d*y(x)=0,y(x), singsol=all)
```

$$y(x)=c_1\left(\frac{\sqrt{a\,x^2+2bx+c}\,\sqrt{a}+ax+b}{\sqrt{a}}\right)^{\frac{i\sqrt{d}}{\sqrt{a}}}+c_2\left(\frac{\sqrt{a\,x^2+2bx+c}\,\sqrt{a}+ax+b}{\sqrt{a}}\right)^{-\frac{i\sqrt{d}}{\sqrt{a}}}$$

✓ Solution by Mathematica
Time used: 0.404 (sec). Leaf size: 93

```
DSolve[(a*x^2+2*b*x+c)*y''[x]+(a*x+b)*y'[x]+d*y[x]==0,y[x],x,IncludeSingularSolutions -> Tru
```

$$y(x)\to c_1\cos\left(\frac{\sqrt{d}\log\left(-\sqrt{a}\sqrt{ax^2+2bx+c}+ax+b\right)}{\sqrt{a}}\right)-c_2\sin\left(\frac{\sqrt{d}\log\left(-\sqrt{a}\sqrt{ax^2+2bx+c}+ax+b\right)}{\sqrt{a}}\right)$$

30.30 problem 178

Internal problem ID [11002]

Book: Handbook of exact solutions for ordinary differential equations. By Polyanin and Zaitsev. Second edition
Section: Chapter 2, Second-Order Differential Equations. section 2.1.2-5 Equation of form $(ax^2+bx+c)y''+f(x)y'+g(x)y=0$
Problem number: 178.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$\boxed{\left(a\,x^2+2bx+c\right)y''+3(ax+b)\,y'+yd=0}$$

✓ Solution by Maple
Time used: 0.219 (sec). Leaf size: 88

```
dsolve((a*x^2+2*b*x+c)*diff(y(x),x$2)+3*(a*x+b)*diff(y(x),x)+d*y(x)=0,y(x), singsol=all)
```

$$y(x) = \frac{c_2\left(\sqrt{a\left(a\,x^2+2bx+c\right)}+ax+b\right)^{-\frac{\sqrt{-d+a}}{\sqrt{a}}}+c_1\left(\sqrt{a\left(a\,x^2+2bx+c\right)}+ax+b\right)^{\frac{\sqrt{-d+a}}{\sqrt{a}}}}{\sqrt{a\,x^2+2bx+c}}$$

✓ Solution by Mathematica
Time used: 0.171 (sec). Leaf size: 152

```
DSolve[(a*x^2+2*b*x+c)*y''[x]+3*(a*x+b)*y'[x]+d*y[x]==0,y[x],x,IncludeSingularSolutions -> T
```

$$y(x)\to \frac{c_1 P^{\frac{1}{2}}_{\frac{\sqrt{a-d}}{\sqrt{a}}-\frac{1}{2}}\left(\frac{\sqrt{-b^2-ac}(b+ax)}{a\sqrt{c^2-\frac{b^4}{a^2}}}\right)+c_2 Q^{\frac{1}{2}}_{\frac{\sqrt{a-d}}{\sqrt{a}}-\frac{1}{2}}\left(\frac{\sqrt{-b^2-ac}(b+ax)}{a\sqrt{c^2-\frac{b^4}{a^2}}}\right)}{\sqrt[4]{x(ax+2b)+c}}$$

30.31 problem 179

Internal problem ID [11003]

Book: Handbook of exact solutions for ordinary differential equations. By Polyanin and Zaitsev. Second edition
Section: Chapter 2, Second-Order Differential Equations. section 2.1.2-5 Equation of form $(ax^2+bx+c)y''+f(x)y'+g(x)y=0$
Problem number: 179.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$\boxed{\left(a_2x^2+b_2x+c_2\right)y''+\left(b_1x+c_1\right)y'+c_0y=0}$$

✓ Solution by Maple
Time used: 0.14 (sec). Leaf size: 482

```
dsolve((a__2*x^2+b__2*x+c__2)*diff(y(x),x$2)+(b__1*x+c__1)*diff(y(x),x)+c__0*y(x)=0,y(x), si
```

$$y(x)=c_3\,\text{hypergeom}\left(\left[\frac{-a_2+b_1+\sqrt{a_2^2+\left(-2b_1-4c_0\right)a_2+b_1^2}}{2a_2},\right.\right.$$
$$\left.-\frac{a_2-b_1+\sqrt{a_2^2+\left(-2b_1-4c_0\right)a_2+b_1^2}}{2a_2}\right],\left[\frac{b_1\sqrt{\frac{-4c_2a_2+b_2^2}{a_2^2}}\,a_2-2a_2c_1+b_1b_2}{2a_2^2\sqrt{\frac{-4c_2a_2+b_2^2}{a_2^2}}}\right],\frac{\left(-2x\,a_2^2-b_2a_2\right)\sqrt{\frac{-4c_2}{a}}}{8c_2a_2-}$$
$$+c_4\left(2\sqrt{\frac{-4c_2a_2+b_2^2}{a_2^2}}\,x\,a_2^2+\sqrt{\frac{-4c_2a_2+b_2^2}{a_2^2}}\,b_2a_2-4c_2a_2+b_2^2\right)\frac{a_2\left(a_2-\frac{b_1}{2}\right)\sqrt{\frac{-4c_2a_2+b_2^2}{a_2^2}}+a_2c_1-\frac{b_1b_2}{2}}{\sqrt{\frac{-4c_2a_2+b_2^2}{a_2^2}}\,a_2^2}\ \text{hypergeom}$$

✓ Solution by Mathematica

Time used: 6.771 (sec). Leaf size: 498

```
DSolve[(a2*x^2+b2*x+c2)*y''[x]+(b1*x+c1)*y'[x]+c0*y[x]==0,y[x],x,IncludeSingularSolutions ->
```

$$y(x)$$

$$\to c_1 \text{Hypergeometric2F1}\left(-\frac{\text{a2}-\text{b1}+\sqrt{(\text{a2}-\text{b1})^2-4\text{a2c0}}}{2\text{a2}},\frac{-\text{a2}+\text{b1}+\sqrt{(\text{a2}-\text{b1})^2-4\text{a2c0}}}{2\text{a2}},\frac{\text{b1}\left(\text{b2}\right.\right.$$

$$-c_2 2^{\frac{\frac{\text{b1b2}}{\sqrt{\text{b2}^2-4\text{a2c2}}}+\text{b1}}{2\text{a2}}-\frac{\text{c1}}{\sqrt{\text{b2}^2-4\text{a2c2}}}-1}\exp\left(-\frac{i\pi\left(\text{b1}\left(\sqrt{\text{b2}^2-4\text{a2c2}}+\text{b2}\right)-2\text{a2c1}\right)}{2\text{a2}\sqrt{\text{b2}^2-4\text{a2c2}}}\right)\left(\frac{\sqrt{\text{b2}^2-4\text{a2c2}}+2\text{a}}{\sqrt{\text{b2}^2-4\text{a2}}}\right.$$

$$\left.\left.-\frac{\frac{\text{b2b1}}{\sqrt{\text{b2}^2-4\text{a2c2}}}+\text{b1}+\text{a2}\left(-\frac{2\text{c1}}{\sqrt{\text{b2}^2-4\text{a2c2}}}-4\right)}{2\text{a2}},\frac{\text{b2}+2\text{a2}x+\sqrt{\text{b2}^2-4\text{a2c2}}}{2\sqrt{\text{b2}^2-4\text{a2c2}}}\right)\right)$$

30.32 problem 180

Internal problem ID [11004]

Book: Handbook of exact solutions for ordinary differential equations. By Polyanin and Zaitsev. Second edition
Section: Chapter 2, Second-Order Differential Equations. section 2.1.2-5 Equation of form $(ax^2+bx+c)y''+f(x)y'+g(x)y=0$
Problem number: 180.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$\boxed{\left(a\,x^2+bx+c\right)y''-\left(-k^2+x^2\right)y'+(x+k)\,y=0}$$

✓ Solution by Maple
Time used: 0.328 (sec). Leaf size: 1535

```
dsolve((a*x^2+b*x+c)*diff(y(x),x$2)-(x^2-k^2)*diff(y(x),x)+(x+k)*y(x)=0,y(x), singsol=all)
```

Expression too large to display

✓ Solution by Mathematica
Time used: 2.442 (sec). Leaf size: 119

```
DSolve[(a*x^2+b*x+c)*y''[x]-(x^2-k^2)*y'[x]+(x+k)*y[x]==0,y[x],x,IncludeSingularSolutions ->
```

$$y(x)\to \frac{(k-x)\left(c_2\int_1^x \frac{\exp\left(\frac{\left(b^2-2a\left(ak^2+c\right)\right)\arctan\left(\frac{b+2aK[1]}{\sqrt{4ac-b^2}}\right)}{\frac{\sqrt{4ac-b^2}}{a^2}}+aK[1]\right)(c+K[1](b+aK[1]))^{-\frac{b}{2a^2}}}{(k-K[1])^2}dK[1]+c_1\right)}{k}$$

30.33 problem 181

Internal problem ID [11005]

Book: Handbook of exact solutions for ordinary differential equations. By Polyanin and Zaitsev. Second edition
Section: Chapter 2, Second-Order Differential Equations. section 2.1.2-5 Equation of form $(ax^2+bx+c)y''+f(x)y'+g(x)y=0$
Problem number: 181.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$\left(a\,x^2+bx+c\right)y''+\left(k^3+x^3\right)y'-\left(k^2-kx+x^2\right)y=0$$

✓ Solution by Maple
Time used: 0.437 (sec). Leaf size: 246

```
dsolve((a*x^2+b*x+c)*diff(y(x),x$2)+(x^3+k^3)*diff(y(x),x)-(x^2-k*x+k^2)*y(x)=0,y(x), singso
```

$$y(x)=(x+k)\left(\left(\int\frac{\left(2ax+b-\sqrt{-4ac+b^2}\right)^{-\frac{k^3}{\sqrt{-4ac+b^2}}}\left(\frac{-2ax-b+\sqrt{-4ac+b^2}}{2ax+\sqrt{-4ac+b^2}+b}\right)^{-\frac{3bc}{2a^2\sqrt{-4ac+b^2}}}\left(\frac{2ax+\sqrt{-4ac+b^2}+b}{-2ax-b+\sqrt{-4ac+b^2}}\right)^{-\frac{b^3}{2a^3\sqrt{-4}}}}{(x+k)^2}\right.\right.$$

$$\left.\left.+c_1\right)\right)$$

✓ Solution by Mathematica

Time used: 3.224 (sec). Leaf size: 137

```
DSolve[(a*x^2+b*x+c)*y''[x]+(x^3+k^3)*y'[x]-(x^2-k*x+k^2)*y[x]==0,y[x],x,IncludeSingularSolu
```

$$y(x)$$

$$\to \frac{(k+x)\left(c_2 \int_1^x \frac{\exp\left(\frac{\left(b^3-3acb-2a^3k^3\right)\arctan\left(\frac{b+2aK[1]}{\sqrt{4ac-b^2}}\right)}{a^3\sqrt{4ac-b^2}}-\frac{K[1](aK[1]-2b)}{2a^2}\right)(c+K[1](b+aK[1]))^{-\frac{b^2-ac}{2a^3}}}{(k+K[1])^2}dK[1]+c_1\right)}{k}$$

31 Chapter 2, Second-Order Differential Equations. section 2.1.2-6 Equation of form $(a_3x^3+a_2x^2x+a_1x+a_0)y''+f(x)y'+g(x)y=0$

31.1 problem 182 822
31.2 problem 183 823
31.3 problem 184 825
31.4 problem 185 826
31.5 problem 186 827
31.6 problem 187 829
31.7 problem 188 830
31.8 problem 189 831
31.9 problem 190 832
31.10problem 191 834
31.11problem 192 835
31.12problem 193 836
31.13problem 194 837
31.14problem 195 839
31.15problem 196 840
31.16problem 197 841
31.17problem 198 842
31.18problem 199 843
31.19problem 200 844
31.20problem 201 845
31.21problem 202 846
31.22problem 203 847
31.23problem 204 848
31.24problem 205 849
31.25problem 206 850
31.26problem 207 851
31.27problem 208 852
31.28problem 209 853
31.29problem 210 854

31.1 problem 182

Internal problem ID [11006]

Book: Handbook of exact solutions for ordinary differential equations. By Polyanin and Zaitsev. Second edition
Section: Chapter 2, Second-Order Differential Equations. section 2.1.2-6 Equation of form $(a_3x^3+a_2x^2x+a_1x+a_0)y''+f(x)y'+g(x)y=0$
Problem number: 182.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$\boxed{x^3y''+(ax+b)\,y=0}$$

✓ Solution by Maple
Time used: 0.031 (sec). Leaf size: 49

```
dsolve(x^3*diff(y(x),x$2)+(a*x+b)*y(x)=0,y(x), singsol=all)
```

$$y(x)=\left(\text{BesselJ}\left(-\sqrt{-4a+1},\frac{2\sqrt{b}}{\sqrt{x}}\right)c_1+\text{BesselY}\left(-\sqrt{-4a+1},\frac{2\sqrt{b}}{\sqrt{x}}\right)c_2\right)\sqrt{x}$$

✓ Solution by Mathematica
Time used: 0.128 (sec). Leaf size: 101

```
DSolve[x^3*y''[x]+(a*x+b)*y[x]==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x)$$
$$\to \frac{c_1\,\text{Gamma}\left(1-\sqrt{1-4a}\right)\text{BesselJ}\left(-\sqrt{1-4a},2\sqrt{b}\sqrt{\frac{1}{x}}\right)+c_2\,\text{Gamma}\left(\sqrt{1-4a}+1\right)\text{BesselJ}\left(\sqrt{1-4a}\right.}{\sqrt{b}\sqrt{\frac{1}{x}}}$$

31.2 problem 183

Internal problem ID [11007]

Book: Handbook of exact solutions for ordinary differential equations. By Polyanin and Zaitsev. Second edition
Section: Chapter 2, Second-Order Differential Equations. section 2.1.2-6 Equation of form $(a_3x^3+a_2x^2x+a_1x+a_0)y''+f(x)y'+g(x)y=0$
Problem number: 183.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$\boxed{x^3y''+\left(a\,x^2+b\right)y'+ycx=0}$$

✓ Solution by Maple
Time used: 0.156 (sec). Leaf size: 120

```
dsolve(x^3*diff(y(x),x$2)+(a*x^2+b)*diff(y(x),x)+c*x*y(x)=0,y(x), singsol=all)
```

$$y(x)=x^{-\frac{\sqrt{a^2-2a-4c+1}}{2}-\frac{a}{2}+\frac{1}{2}}\left(\text{KummerM}\left(-\frac{1}{4}+\frac{\sqrt{a^2-2a-4c+1}}{4}+\frac{a}{4},1+\frac{\sqrt{a^2-2a-4c+1}}{2},\frac{b}{2x^2}\right)c_1+\text{KummerU}\left(-\frac{1}{4}+\frac{\sqrt{a^2-2a-4c+1}}{4}+\frac{a}{4},1+\frac{\sqrt{a^2-2a-4c+1}}{2},\frac{b}{2x^2}\right)c_2\right)$$

✓ Solution by Mathematica

Time used: 0.646 (sec). Leaf size: 308

```
DSolve[x^3*y''[x]+(a*x^2+b)*y'[x]+c*x*y[x]==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to$$

$$-(-1)^{\frac{1}{4}\left(-\sqrt{a^2-2a-4c+1}+a+3\right)} 2^{\frac{1}{4}\left(-\sqrt{a^2-2a-4c+1}-a+1\right)} b^{\frac{1}{4}\left(-\sqrt{a^2-2a-4c+1}+a-1\right)} \left(\frac{1}{x}\right)^{\frac{1}{2}\left(-\sqrt{a^2-2a-4c+1}+a-1\right)} \left(c_2 i^{\sqrt{a}}\right.$$

$$+ c_1 2^{\frac{1}{2}\sqrt{a^2-2a-4c+1}} \text{Hypergeometric1F1}\left(\frac{1}{4}\left(a-\sqrt{a^2-2a-4c+1}-1\right), 1\right.$$

$$\left.\left. -\frac{1}{2}\sqrt{a^2-2a-4c+1}, \frac{b}{2x^2}\right)\right)$$

31.3 problem 184

Internal problem ID [11008]

Book: Handbook of exact solutions for ordinary differential equations. By Polyanin and Zaitsev. Second edition
Section: Chapter 2, Second-Order Differential Equations. section 2.1.2-6 Equation of form $(a_3x^3+a_2x^2x+a_1x+a_0)y''+f(x)y'+g(x)y=0$
Problem number: 184.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$\boxed{x^3y''+\left(a\,x^2+bx\right)y'+yb=0}$$

✓ Solution by Maple
Time used: 0.047 (sec). Leaf size: 108

```
dsolve(x^3*diff(y(x),x$2)+(a*x^2+b*x)*diff(y(x),x)+b*y(x)=0,y(x), singsol=all)
```

$$y(x)$$
$$=\frac{c_1x\left(\Gamma\left(a,-\frac{b}{x}\right)-\Gamma(a)\right)(-1)^{-a}(a-2)\,b^{-a+1}+c_1\left(\Gamma\left(a,-\frac{b}{x}\right)-\Gamma(a)\right)(-1)^{-a}\,b^{-a+2}+b\,x^{-a+1}c_1\mathrm{e}^{\frac{b}{x}}+c_2(a-}{x}$$

✓ Solution by Mathematica
Time used: 2.653 (sec). Leaf size: 62

```
DSolve[x^3*y''[x]+(a*x^2+b*x)*y'[x]+b*y[x]==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x)\to\frac{((a-2)x+b)\left(c_2\int_1^x\frac{e^{\frac{b}{K[1]}}K[1]^{2-a}}{(b+(a-2)K[1])^2}dK[1]+c_1\right)}{x(a+b-2)}$$

31.4 problem 185

Internal problem ID [11009]

Book: Handbook of exact solutions for ordinary differential equations. By Polyanin and Zaitsev. Second edition
Section: Chapter 2, Second-Order Differential Equations. section 2.1.2-6 Equation of form $(a_3x^3+a_2x^2x+a_1x+a_0)y''+f(x)y'+g(x)y=0$
Problem number: 185.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$\boxed{x^3y''+\left(a\,x^2+bx\right)y'+yc=0}$$

✓ Solution by Maple
Time used: 0.094 (sec). Leaf size: 146

```
dsolve(x^3*diff(y(x),x$2)+(a*x^2+b*x)*diff(y(x),x)+c*y(x)=0,y(x), singsol=all)
```

$$y(x)$$
$$=\frac{x^{-a}\left(-cxc_2(ab-c)\,(b-c)\,\text{KummerU}\left(\frac{(a+1)b-c}{b},a,\frac{b}{x}\right)+\left(c_1xb(ab-c)\,\text{KummerM}\left(\frac{(a+1)b-c}{b},a,\frac{b}{x}\right)-(bc\right.}{b^2c}$$

✓ Solution by Mathematica
Time used: 0.435 (sec). Leaf size: 62

```
DSolve[x^3*y''[x]+(a*x^2+b*x)*y'[x]+c*y[x]==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x)\to c_1\,\text{Hypergeometric1F1}\left(-\frac{c}{b},2-a,\frac{b}{x}\right)$$
$$-(-1)^ac_2b^{a-1}\left(\frac{1}{x}\right)^{a-1}\text{Hypergeometric1F1}\left(a-\frac{b+c}{b},a,\frac{b}{x}\right)$$

31.5 problem 186

Internal problem ID [11010]

Book: Handbook of exact solutions for ordinary differential equations. By Polyanin and Zaitsev. Second edition
Section: Chapter 2, Second-Order Differential Equations. section 2.1.2-6 Equation of form $(a_3x^3+a_2x^2x+a_1x+a_0)y''+f(x)y'+g(x)y=0$
Problem number: 186.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$\boxed{x^3y''+\left(a\,x^2+bx\right)y'+\left(cx+d\right)y=0}$$

✓ Solution by Maple
Time used: 0.125 (sec). Leaf size: 132

```
dsolve(x^3*diff(y(x),x$2)+(a*x^2+b*x)*diff(y(x),x)+(c*x+d)*y(x)=0,y(x), singsol=all)
```

$$y(x)=x^{-\frac{\sqrt{a^2-2a-4c+1}}{2}-\frac{a}{2}+\frac{1}{2}}\left(\text{KummerM}\left(\frac{\sqrt{a^2-2a-4c+1}\,b+b(a-1)-2d}{2b},1+\sqrt{a^2-2a-4c+1},\frac{b}{x}\right)c_1+\text{KummerU}\left(\frac{\sqrt{a^2-2a-4c+1}\,b+b(a-1)-2d}{2b},1+\sqrt{a^2-2a-4c+1},\frac{b}{x}\right)c_2\right)$$

✓ Solution by Mathematica

Time used: 0.641 (sec). Leaf size: 255

```
DSolve[x^3*y''[x]+(a*x^2+b*x)*y'[x]+(c*x+d)*y[x]==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to -i^{-\sqrt{a^2-2a-4c+1}+a+1} b^{\frac{1}{2}\left(-\sqrt{a^2-2a-4c+1}+a-1\right)} \left(\frac{1}{x}\right)^{\frac{1}{2}\left(-\sqrt{a^2-2a-4c+1}+a-1\right)} \left(c_2 i^{2\sqrt{a^2-2a-4c+1}} b^{\sqrt{a^2-2a-4c+1}} \left(\frac{1}{x}\right)^{\sqrt{a^2-2a-4c+1}} \text{Hypergeometric1F1}\left(\frac{1}{2}\left(a-\frac{2d}{b}-\sqrt{a^2-2a-4c+1}-1\right), 1-\sqrt{a^2-2a-4c+1}, \frac{b}{x}\right)\right) + c_1 \text{Hypergeometric1F1}\left(\frac{1}{2}\left(a-\frac{2d}{b}-\sqrt{a^2-2a-4c+1}-1\right), 1-\sqrt{a^2-2a-4c+1}, \frac{b}{x}\right)$$

31.6 problem 187

Internal problem ID [11011]

Book: Handbook of exact solutions for ordinary differential equations. By Polyanin and Zaitsev. Second edition
Section: Chapter 2, Second-Order Differential Equations. section 2.1.2-6 Equation of form $(a_3x^3+a_2x^2x+a_1x+a_0)y''+f(x)y'+g(x)y=0$
Problem number: 187.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$\boxed{x^3y''+\left(a\,x^3+abx-x^2+b\right)y'+a^2bxy=0}$$

✓ Solution by Maple
Time used: 0.454 (sec). Leaf size: 49

```
dsolve(x^3*diff(y(x),x$2)+(a*x^3-x^2+a*b*x+b)*diff(y(x),x)+a^2*b*x*y(x)=0,y(x), singsol=all)
```

$$y(x)=\mathrm{e}^{-ax}\left(c_2\left(\int\frac{x\,\mathrm{e}^{\frac{2a\,x^3+2abx+b}{2x^2}}}{(ax+1)^2}dx\right)+c_1\right)(ax+1)$$

✓ Solution by Mathematica
Time used: 1.347 (sec). Leaf size: 70

```
DSolve[x^3*y''[x]+(a*x^3-x^2+a*b*x+b)*y'[x]+a^2*b*x*y[x]==0,y[x],x,IncludeSingularSolutions
```

$$y(x)\to\frac{e^{-ax}(ax+1)\left(c_2\int_1^x\frac{a^2e^{aK[1]+\frac{2aK[1]b+b}{2K[1]^2}}K[1]}{(aK[1]+1)^2}dK[1]+c_1\right)}{a}$$

31.7 problem 188

Internal problem ID [11012]

Book: Handbook of exact solutions for ordinary differential equations. By Polyanin and Zaitsev. Second edition
Section: Chapter 2, Second-Order Differential Equations. section 2.1.2-6 Equation of form $(a_3x^3+a_2x^2x+a_1x+a_0)y''+f(x)y'+g(x)y=0$
Problem number: 188.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$\boxed{x^3y''+x(a\,x^n+b)\,y'-\left(a\,x^n-x^{n-1}ab+b\right)y=0}$$

✗ Solution by Maple

```
dsolve(x^3*diff(y(x),x$2)+x*(a*x^n+b)*diff(y(x),x)-(a*x^n-a*b*x^(n-1)+b)*y(x)=0,y(x), singso
```

No solution found

✗ Solution by Mathematica
Time used: 0.0 (sec). Leaf size: 0

```
DSolve[x^3*y''[x]+x*(a*x^n+b)*y'[x]-(a*x^n-a*b*x^(n-1)+b)*y[x]==0,y[x],x,IncludeSingularSolu
```

Not solved

31.8 problem 189

Internal problem ID [11013]

Book: Handbook of exact solutions for ordinary differential equations. By Polyanin and Zaitsev. Second edition
Section: Chapter 2, Second-Order Differential Equations. section 2.1.2-6 Equation of form $(a_3x^3+a_2x^2x+a_1x+a_0)y''+f(x)y'+g(x)y=0$
Problem number: 189.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _exact, _linear, _homogeneous]]`

$$\boxed{x\left(a\,x^2+b\right)y''+2\left(a\,x^2+b\right)y'-2yax=0}$$

✓ Solution by Maple
Time used: 0.0 (sec). Leaf size: 46

```
dsolve(x*(a*x^2+b)*diff(y(x),x$2)+2*(a*x^2+b)*diff(y(x),x)-2*a*x*y(x)=0,y(x), singsol=all)
```

$$y(x)=\frac{\left(a\,x^2+b\right)c_2\arctan\left(\frac{\sqrt{ab}\,x}{b}\right)+\sqrt{ab}\,c_2x+c_1\left(a\,x^2+b\right)}{x}$$

✓ Solution by Mathematica
Time used: 0.16 (sec). Leaf size: 78

```
DSolve[x*(a*x^2+b)*y''[x]+2*(a*x^2+b)*y'[x]-2*a*x*y[x]==0,y[x],x,IncludeSingularSolutions ->
```

$$y(x)\to\frac{c_2(ax^2+b)\arctan\left(\frac{\sqrt{a}x}{\sqrt{b}}\right)+\sqrt{a}\sqrt{b}(2abc_1x^2+2b^2c_1+c_2x)}{2\sqrt{a}b^{3/2}x}$$

31.9 problem 190

Internal problem ID [11014]

Book: Handbook of exact solutions for ordinary differential equations. By Polyanin and Zaitsev. Second edition
Section: Chapter 2, Second-Order Differential Equations. section 2.1.2-6 Equation of form $(a_3x^3+a_2x^2x+a_1x+a_0)y''+f(x)y'+g(x)y=0$
Problem number: 190.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$\boxed{x\left(x^2+a\right)y''+\left(bx^2+c\right)y'+sxy=0}$$

✓ Solution by Maple
Time used: 0.157 (sec). Leaf size: 175

```
dsolve(x*(x^2+a)*diff(y(x),x$2)+(b*x^2+c)*diff(y(x),x)+s*x*y(x)=0,y(x), singsol=all)
```

$$y(x)=\left(x^2+a\right)^{\frac{(-b+2)a+c}{2a}}\left(x^{\frac{a-c}{a}}\,\text{hypergeom}\left(\left[-\frac{b}{4}+\frac{5}{4}-\frac{\sqrt{b^2-2b-4s+1}}{4},-\frac{b}{4}+\frac{5}{4}+\frac{\sqrt{b^2-2b-4s+1}}{4}\right],\left[\frac{3a-c}{2a}\right.\right.\right.$$
$$\left.\left.\left.-\frac{x^2}{a}\right)c_1+\text{hypergeom}\left(\left[-\frac{b}{4}+\frac{3}{4}+\frac{c}{2a}+\frac{\sqrt{b^2-2b-4s+1}}{4},-\frac{\sqrt{b^2-2b-4s+1}}{4}-\frac{b}{4}+\frac{3}{4}+\frac{c}{2a}\right],\left[\frac{1}{2}+\frac{c}{2a}\right],\right.\right.\right.$$
$$\left.\left.\left.-\frac{x^2}{a}\right)c_2\right)$$

✓ Solution by Mathematica

Time used: 0.967 (sec). Leaf size: 185

```
DSolve[x*(x^2+a)*y''[x]+(b*x^2+c)*y'[x]+s*x*y[x]==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x)$$

$$\to c_2 a^{\frac{1}{2}\left(\frac{c}{a}-1\right)} x^{1-\frac{c}{a}} \text{Hypergeometric2F1}\left(\frac{a\left(b+\sqrt{b^2-2b-4s+1}+1\right)-2c}{4a}, \frac{ba-\sqrt{b^2-2b-4s+1}a+a}{4a}\right.$$

$$\left. -\frac{c}{2a}, -\frac{x^2}{a}\right) + c_1 \text{Hypergeometric2F1}\left(\frac{1}{4}\left(b-\sqrt{b^2-2b-4s+1}-1\right), \frac{1}{4}\left(b\right.\right.$$

$$\left.\left. +\sqrt{b^2-2b-4s+1}-1\right), \frac{a+c}{2a}, -\frac{x^2}{a}\right)$$

31.10 problem 191

Internal problem ID [11015]

Book: Handbook of exact solutions for ordinary differential equations. By Polyanin and Zaitsev. Second edition
Section: Chapter 2, Second-Order Differential Equations. section 2.1.2-6 Equation of form $(a_3x^3+a_2x^2x+a_1x+a_0)y''+f(x)y'+g(x)y=0$
Problem number: 191.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$\boxed{x^2(ax+b)\,y''+\left(c\,x^2+(a\lambda+2b)\,x+b\lambda\right)y'+\lambda(-2a+c)\,y=0}$$

✓ Solution by Maple
Time used: 0.469 (sec). Leaf size: 169

```
dsolve(x^2*(a*x+b)*diff(y(x),x$2)+(c*x^2+(2*b+a*lambda)*x+b*lambda)*diff(y(x),x)+lambda*(c-2
```

$$y(x)=\frac{(ax+b)^{\frac{3a-c}{a}}\left(c_1x^{\frac{-3a+c}{a}}\,\mathrm{HeunC}\left(\frac{\lambda a}{b},1-\frac{c}{a},3-\frac{c}{a},0,-\frac{\lambda a}{b}+\frac{c\lambda}{2b}+\frac{5}{2}-\frac{2c}{a}+\frac{c^2}{2a^2},-\frac{b}{ax}\right)x^2+c_2\,\mathrm{HeunC}\left(\frac{\lambda a}{b},\frac{c}{a}\right.\right.}{x^2}$$

✓ Solution by Mathematica
Time used: 1.48 (sec). Leaf size: 55

```
DSolve[x^2*(a*x+b)*y''[x]+(c*x^2+(2*b+a*\[Lambda])*x+b*\[Lambda])*y'[x]+\[Lambda]*(c-2*a)*y[
```

$$y(x)\to e^{\frac{\lambda}{x}}\left(c_2\int_1^x\frac{e^{-\frac{\lambda}{K[1]}}(b+aK[1])^{2-\frac{c}{a}}}{K[1]^2}dK[1]+c_1\right)$$

31.11 problem 192

Internal problem ID [11016]

Book: Handbook of exact solutions for ordinary differential equations. By Polyanin and Zaitsev. Second edition
Section: Chapter 2, Second-Order Differential Equations. section 2.1.2-6 Equation of form $(a_3x^3+a_2x^2x+a_1x+a_0)y''+f(x)y'+g(x)y=0$
Problem number: 192.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$\boxed{x^2(ax+b)\,y''-2x(ax+2b)\,y'+2(ax+3b)\,y=0}$$

✓ Solution by Maple
Time used: 0.016 (sec). Leaf size: 20

```
dsolve(x^2*(a*x+b)*diff(y(x),x$2)-2*x*(a*x+2*b)*diff(y(x),x)+2*(a*x+3*b)*y(x)=0,y(x), singso
```

$$y(x)=\frac{x^2(c_2x+c_1)}{ax+b}$$

✓ Solution by Mathematica
Time used: 0.068 (sec). Leaf size: 23

```
DSolve[x^2*(a*x+b)*y''[x]-2*x*(a*x+2*b)*y'[x]+2*(a*x+3*b)*y[x]==0,y[x],x,IncludeSingularSolu
```

$$y(x)\to\frac{x^2(c_2x+c_1)}{ax+b}$$

31.12 problem 193

Internal problem ID [11017]

Book: Handbook of exact solutions for ordinary differential equations. By Polyanin and Zaitsev. Second edition
Section: Chapter 2, Second-Order Differential Equations. section 2.1.2-6 Equation of form $(a_3x^3+a_2x^2x+a_1x+a_0)y''+f(x)y'+g(x)y=0$
Problem number: 193.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$x^2(ax+b)\,y''+\left(a(2-m-n)\,x^2-b(m+n)\,x\right)y'+(am(n-1)\,x+bn(m+1))\,y=0$$

✓ Solution by Maple
Time used: 0.047 (sec). Leaf size: 25

```
dsolve(x^2*(a*x+b)*diff(y(x),x$2)+(a*(2-n-m)*x^2-b*(n+m)*x)*diff(y(x),x)+(a*m*(n-1)*x+b*n*(m
```

$$y(x)=\frac{c_1x^n+c_2x^{1+m}}{ax+b}$$

✓ Solution by Mathematica
Time used: 0.403 (sec). Leaf size: 82

```
DSolve[x^2*(a*x+b)*y''[x]+(a*(2-n-m)*x^2-b*(n+m)*x)*y'[x]+(a*m*(n-1)*x+b*n*(m+1))*y[x]==0,y[
```

$$y(x)\to\frac{x^{\frac{1}{2}\left(-\sqrt{(m-n+1)^2}+m+n+1\right)}\left(c_2x^{\sqrt{(m-n+1)^2}}+c_1\sqrt{(m-n+1)^2}\right)}{\sqrt{(m-n+1)^2}(ax+b)}$$

31.13 problem 194

Internal problem ID [11018]

Book: Handbook of exact solutions for ordinary differential equations. By Polyanin and Zaitsev. Second edition
Section: Chapter 2, Second-Order Differential Equations. section 2.1.2-6 Equation of form $(a_3x^3+a_2x^2x+a_1x+a_0)y''+f(x)y'+g(x)y=0$
Problem number: 194.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$x^2(x+a_2)\,y''+x(b_1x+a_1)\,y'+(b_0x+a_0)\,y=0$$

✓ Solution by Maple
Time used: 0.125 (sec). Leaf size: 317

```
dsolve(x^2*(x+a__2)*diff(y(x),x$2)+x*(b__1*x+a__1)*diff(y(x),x)+(b__0*x+a__0)*y(x)=0,y(x), s
```

$$y(x)$$
$$=c_1x^{\frac{a_2-a_1+\sqrt{a_2^2+(-4a_0-2a_1)a_2+a_1^2}}{2a_2}}\,\text{hypergeom}\left(\left[\frac{a_2b_1-a_1+\sqrt{a_2^2+(-4a_0-2a_1)\,a_2+a_1^2}-\sqrt{b_1^2-4b_0-2b_1+}}{2a_2}\right.\right.$$
$$\left.\left.-\frac{x}{a_2}\right)\right.$$
$$+c_2x^{-\frac{-a_2+a_1+\sqrt{a_2^2+(-4a_0-2a_1)a_2+a_1^2}}{2a_2}}\,\text{hypergeom}\left(\left[-\frac{-\sqrt{b_1^2-4b_0-2b_1+1}\,a_2-a_2b_1+\sqrt{a_2^2+(-4a_0-2a_1}}{2a_2}\right.\right.$$
$$\left.\left.-\frac{\sqrt{b_1^2-4b_0-2b_1+1}\,a_2-a_2b_1+\sqrt{a_2^2+(-4a_0-2a_1)\,a_2+a_1^2}+a_1}{2a_2}\right],\left[\frac{a_2-\sqrt{a_2^2+(-4a_0-2a_1)\,a_2}}{a_2}\right.\right.$$
$$\left.\left.-\frac{x}{a_2}\right)\right.$$

✓ Solution by Mathematica

Time used: 1.236 (sec). Leaf size: 384

```
DSolve[x^2*(x+a2)*y''[x]+x*(b1*x+a1)*y'[x]+(b0*x+a0)*y[x]==0,y[x],x,IncludeSingularSolutions
```

$$y(x)$$

$$\to \text{a2}^{-\frac{\sqrt{-4\text{a0a2}+\text{a1}^2-2\text{a1a2}+\text{a2}^2}-\text{a1}+\text{a2}}{2\text{a2}}} x^{-\frac{\sqrt{-4\text{a0a2}+\text{a1}^2-2\text{a1a2}+\text{a2}^2}+\text{a1}-\text{a2}}{2\text{a2}}} \left(c_2 x^{\frac{\sqrt{-4\text{a0a2}+\text{a1}^2-2\text{a1a2}+\text{a2}^2}}{\text{a2}}} \text{Hypergeometric2F1}\left(\right.\right.$$

$$\left. -\frac{x}{\text{a2}} \right)$$

$$+c_1 \text{a2}^{\frac{\sqrt{-4\text{a0a2}+\text{a1}^2-2\text{a1a2}+\text{a2}^2}}{\text{a2}}} \text{Hypergeometric2F1}\left(-\frac{\text{a1}-\text{a2b1}+\sqrt{\text{a1}^2-2\text{a2a1}+\text{a2}(\text{a2}-4\text{a0})}+\text{a2}\sqrt{(}}{2\text{a2}} \right.$$

$$-\frac{\text{a1}-\text{a2}\left(\text{b1}+\sqrt{(\text{b1}-1)^2-4\text{b0}}\right)+\sqrt{\text{a1}^2-2\text{a2a1}+\text{a2}(\text{a2}-4\text{a0})}}{2\text{a2}}, 1$$

$$\left.\left. -\frac{\sqrt{\text{a1}^2-2\text{a2a1}+\text{a2}^2-4\text{a0a2}}}{\text{a2}}, -\frac{x}{\text{a2}} \right) \right)$$

31.14 problem 195

Internal problem ID [11019]

Book: Handbook of exact solutions for ordinary differential equations. By Polyanin and Zaitsev. Second edition
Section: Chapter 2, Second-Order Differential Equations. section 2.1.2-6 Equation of form $(a_3x^3+a_2x^2x+a_1x+a_0)y''+f(x)y'+g(x)y=0$
Problem number: 195.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$\boxed{\left(a\,x^3+b\,x^2+cx\right)y''+\left(\alpha\,x^2+\beta x+2c\right)y'+\left(\beta-2b\right)y=0}$$

✓ Solution by Maple
Time used: 9.625 (sec). Leaf size: 1505

```
dsolve((a*x^3+b*x^2+c*x)*diff(y(x),x$2)+(alpha*x^2+beta*x+2*c)*diff(y(x),x)+(beta-2*b)*y(x)=
```

Expression too large to display

✓ Solution by Mathematica
Time used: 4.64 (sec). Leaf size: 139

```
DSolve[(a*x^3+b*x^2+c*x)*y''[x]+(\[Alpha]*x^2+\[Beta]*x+2*c)*y'[x]+(\[Beta]-2*b)*y[x]==0,y[x
```

$$y(x)$$

$$\to \frac{(2ax+2b-\beta-\alpha x)\left(c_2\int_1^x \frac{\exp\left(\frac{(b\alpha+2a(b-\beta))\arctan\left(\frac{b+2aK[1]}{\sqrt{4ac-b^2}}\right)}{a\sqrt{4ac-b^2}}\right)(c+K[1](b+aK[1]))^{1-\frac{\alpha}{2a}}}{(-2b+\beta+(\alpha-2a)K[1])^2}dK[1]+c_1\right)}{x(2b-\beta)}$$

31.15 problem 196

Internal problem ID [11020]

Book: Handbook of exact solutions for ordinary differential equations. By Polyanin and Zaitsev. Second edition
Section: Chapter 2, Second-Order Differential Equations. section 2.1.2-6 Equation of form $(a_3x^3+a_2x^2x+a_1x+a_0)y''+f(x)y'+g(x)y=0$
Problem number: 196.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$\boxed{\left(a\,x^3+b\,x^2+cx\right)y''+\left(\alpha\,x^2+\beta x+2c\right)y'-\left(x\alpha+2b-\beta\right)y=0}$$

✓ Solution by Maple
Time used: 1.062 (sec). Leaf size: 1505

```
dsolve((a*x^3+b*x^2+c*x)*diff(y(x),x$2)+(alpha*x^2+beta*x+2*c)*diff(y(x),x)-(alpha*x+2*b-bet
```

Expression too large to display

✓ Solution by Mathematica
Time used: 6.092 (sec). Leaf size: 224

```
DSolve[(a*x^3+b*x^2+c*x)*y''[x]+(\[Alpha]*x^2+\[Beta]*x+2*c)*y'[x]-(\[Alpha]*x+2*b-\[Beta])*
```

$$y(x)$$

$$\to \frac{(b(2ax-3\beta-2\alpha x)-a\alpha x^2-2a\beta x+2b^2+\beta^2+\alpha c+\alpha^2x^2+2\alpha\beta x)\left(c_2\int_1^x \frac{\exp\left(\frac{(b\alpha+2a(b-\beta))\arctan\left(\frac{b}{\sqrt{}}\right.}{a\sqrt{4ac-b^2}}\right.}{(2b^2-3\beta b+2(a-\alpha)K[1]b+\beta^2+\alpha^2}\right.}{x\left(2b^2+\beta^2-3\beta b+\alpha c\right)}$$

31.16 problem 197

Internal problem ID [11021]

Book: Handbook of exact solutions for ordinary differential equations. By Polyanin and Zaitsev. Second edition
Section: Chapter 2, Second-Order Differential Equations. section 2.1.2-6 Equation of form $(a_3x^3+a_2x^2x+a_1x+a_0)y''+f(x)y'+g(x)y=0$
Problem number: 197.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$\boxed{\left(a\,x^3+b\,x^2+cx\right)y''+\left(-2a\,x^2-(1+b)\,x+k\right)y'+2(ax+1)\,y=0}$$

✓ Solution by Maple
Time used: 0.437 (sec). Leaf size: 2278

```
dsolve((a*x^3+b*x^2+c*x)*diff(y(x),x$2)+(-2*a*x^2-(b+1)*x+k)*diff(y(x),x)+2*(a*x+1)*y(x)=0,y
```

Expression too large to display

✓ Solution by Mathematica
Time used: 10.126 (sec). Leaf size: 186

```
DSolve[(a*x^3+b*x^2+c*x)*y''[x]+(-2*a*x^2-(b+1)*x+k)*y'[x]+2*(a*x+1)*y[x]==0,y[x],x,IncludeS
```

$$y(x)\to -\frac{(-kx(ax+2)-(b-1)x^2+c(k-2x)+k^2)\left(c_2\int_1^x \frac{\exp\left(\frac{(2c+bk)\arctan\left(\frac{b+2aK[1]}{\sqrt{4ac-b^2}}\right)}{c\sqrt{4ac-b^2}}\right)K[1]^{-\frac{k}{c}}(c+K[1](b+aK[1]))^{\frac{k}{2c}+1}}{(k^2-K[1](aK[1]+2)k-(b-1)K[1]^2+c(k-2K[1]))^2}\right.}{ak+b-c(k-2)-k^2+2k-1}$$

31.17 problem 198

Internal problem ID [11022]

Book: Handbook of exact solutions for ordinary differential equations. By Polyanin and Zaitsev. Second edition
Section: Chapter 2, Second-Order Differential Equations. section 2.1.2-6 Equation of form $(a_3x^3+a_2x^2x+a_1x+a_0)y''+f(x)y'+g(x)y=0$
Problem number: 198.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$\left(a\,x^3+b\,x^2+cx\right)y''+\left(n\,x^2+mx+k\right)y'+(k-1)\left((-ak+n)\,x+m-bk\right)y=0$$

✗ Solution by Maple

```
dsolve((a*x^3+b*x^2+c*x)*diff(y(x),x$2)+(n*x^2+m*x+k)*diff(y(x),x)+(k-1)*((n-a*k)*x+m-b*k)*y
```

No solution found

✓ Solution by Mathematica
Time used: 148.451 (sec). Leaf size: 570

```
DSolve[(a*x^3+b*x^2+c*x)*y''[x]+(n*x^2+m*x+k)*y'[x]+(k-1)*((n-a*k)*x+m-b*k)*y[x]==0,y[x],x,I
```

$$y(x)\to -\frac{2^{-\frac{k}{c}}\left(-\frac{ax}{\sqrt{b^2-4ac}+b}\right)^{1-\frac{k}{c}}\left(ac_1x\left(-2^{\frac{k}{c}}\right)\left(-\frac{ax}{\sqrt{b^2-4ac}+b}\right)^{\frac{k}{c}-1}\text{HeunG}\left[\frac{b-\sqrt{b^2-4ac}}{\sqrt{b^2-4ac}+b},-\frac{2(k-1)(bk-m)}{\sqrt{b^2-4ac}+b},\frac{1}{2}\left(-\sqrt{\frac{(-2ak+}{a}\right.\right.\right.$$

31.18 problem 199

Internal problem ID [11023]

Book: Handbook of exact solutions for ordinary differential equations. By Polyanin and Zaitsev. Second edition
Section: Chapter 2, Second-Order Differential Equations. section 2.1.2-6 Equation of form $(a_3x^3+a_2x^2x+a_1x+a_0)y''+f(x)y'+g(x)y=0$
Problem number: 199.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$\left(a\,x^3+b\,x^2+cx\right)y''+\left(\left(m-a\right)x^2+\left(2cm-1\right)x-c\right)y'+\left(-2mx+1\right)y=0$$

✗ Solution by Maple

```
dsolve((a*x^3+b*x^2+c*x)*diff(y(x),x$2)+((m-a)*x^2+(2*c*m-1)*x-c)*diff(y(x),x)+(-2*m*x+1)*y(
```

No solution found

✓ Solution by Mathematica
Time used: 17.694 (sec). Leaf size: 192

```
DSolve[(a*x^3+b*x^2+c*x)*y''[x]+((m-a)*x^2+(2*c*m-1)*x-c)*y'[x]+(-2*m*x+1)*y[x]==0,y[x],x,In
```

$$y(x)$$

$$\to \frac{\left(x(ax+2b+mx-1)+c(2b+4mx-1)+4c^2m\right)\left(c_2\int_1^x \frac{\exp\left(\frac{(bm-2a(b+2cm-1))\arctan\left(\frac{b+2aK[1]}{\sqrt{4ac-b^2}}\right)}{a\sqrt{4ac-b^2}}\right)K[1](c+K[1](b-}{(4mc^2+(2b+4mK[1]-1)c+K[1](2b+aK[1]+mK[1]-}\right)}{a+2b(c+1)+4c^2m+4cm-c+m-1}$$

31.19 problem 200

Internal problem ID [11024]

Book: Handbook of exact solutions for ordinary differential equations. By Polyanin and Zaitsev. Second edition
Section: Chapter 2, Second-Order Differential Equations. section 2.1.2-6 Equation of form $(a_3x^3+a_2x^2x+a_1x+a_0)y''+f(x)y'+g(x)y=0$
Problem number: 200.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$\left(a\,x^3+b\,x^2+cx\right)y''+\left(n\,x^2+mx+k\right)y'+\left(-2(a+n)\,x+1\right)y=0$$

✗ Solution by Maple

```
dsolve((a*x^3+b*x^2+c*x)*diff(y(x),x$2)+(n*x^2+m*x+k)*diff(y(x),x)+(-2*(a+n)*x+1)*y(x)=0,y(x
```

No solution found

✓ Solution by Mathematica
Time used: 121.038 (sec). Leaf size: 552

```
DSolve[(a*x^3+b*x^2+c*x)*y''[x]+(n*x^2+m*x+k)*y'[x]+(-2*(a+n)*x+1)*y[x]==0,y[x],x,IncludeSin
```

$$y(x)\to -\frac{2^{-\frac{k}{c}}\left(-\frac{ax}{\sqrt{b^2-4ac}+b}\right)^{1-\frac{k}{c}}\left(ac_1x\left(-2^{\frac{k}{c}}\right)\left(-\frac{ax}{\sqrt{b^2-4ac}+b}\right)^{\frac{k}{c}-1}\text{HeunG}\left[\frac{b-\sqrt{b^2-4ac}}{\sqrt{b^2-4ac}+b},\frac{2}{\sqrt{b^2-4ac}+b},\frac{1}{2}\left(-\sqrt{\frac{(3a+n)^2}{a^2}}+\right.\right.\right.$$

31.20 problem 201

Internal problem ID [11025]

Book: Handbook of exact solutions for ordinary differential equations. By Polyanin and Zaitsev. Second edition
Section: Chapter 2, Second-Order Differential Equations. section 2.1.2-6 Equation of form $(a_3x^3+a_2x^2x+a_1x+a_0)y''+f(x)y'+g(x)y=0$
Problem number: 201.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$\boxed{\left(a\,x^3+x^2+b\right)y''+a^2x\left(x^2-b\right)y'-a^3bxy=0}$$

✓ Solution by Maple
Time used: 0.562 (sec). Leaf size: 79

```
dsolve((a*x^3+x^2+b)*diff(y(x),x$2)+a^2*x*(x^2-b)*diff(y(x),x)-a^3*b*x*y(x)=0,y(x), singsol=
```

$$y(x)=\mathrm{e}^{-ax}\left(c_2\left(\int \mathrm{e}^{a\left(\int \frac{a^2x^4+2a\,x^3+\left(a^2b+2\right)x^2+4abx+2b}{\left(a\,x^3+x^2+b\right)(ax+2)}dx\right)}dx\right)+c_1\right)(ax+2)$$

✗ Solution by Mathematica
Time used: 0.0 (sec). Leaf size: 0

```
DSolve[(a*x^3+x^2+b)*y''[x]+a^2*x*(x^2-b)*y'[x]-a^3*b*x*y[x]==0,y[x],x,IncludeSingularSoluti
```

Timed out

31.21 problem 202

Internal problem ID [11026]

Book: Handbook of exact solutions for ordinary differential equations. By Polyanin and Zaitsev. Second edition
Section: Chapter 2, Second-Order Differential Equations. section 2.1.2-6 Equation of form $(a_3x^3+a_2x^2x+a_1x+a_0)y''+f(x)y'+g(x)y=0$
Problem number: 202.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries], [_2nd_order, _linear,`

$$\boxed{2y''x\left(a\,x^2+bx+c\right)+\left(a\,x^2-c\right)y'+\lambda\,x^2y=0}$$

✓ Solution by Maple
Time used: 7.125 (sec). Leaf size: 65

```
dsolve(2*x*(a*x^2+b*x+c)*diff(y(x),x$2)+(a*x^2-c)*diff(y(x),x)+lambda*x^2*y(x)=0,y(x), sings
```

$$y(x)=c_1\mathrm{e}^{\frac{i\sqrt{2}\,\sqrt{\lambda}\left(\int\frac{\sqrt{x}}{\sqrt{a\,x^2+bx+c}}dx\right)}{2}}+c_2\mathrm{e}^{-\frac{i\sqrt{2}\,\sqrt{\lambda}\left(\int\frac{\sqrt{x}}{\sqrt{a\,x^2+bx+c}}dx\right)}{2}}$$

✓ Solution by Mathematica
Time used: 144.69 (sec). Leaf size: 501

```
DSolve[2*x*(a*x^2+b*x+c)*y''[x]+(a*x^2-c)*y'[x]+\[Lambda]*x^2*y[x]==0,y[x],x,IncludeSingular
```

$$y(x)$$
$$\to c_1\cosh\left(\frac{\sqrt{\lambda}\left(\sqrt{b^2-4ac}-b\right)\sqrt{\sqrt{b^2-4ac}+2ax+b}\sqrt{\frac{2ax}{b-\sqrt{b^2-4ac}}+1}\left(E\left(i\operatorname{arcsinh}\left(\frac{\sqrt{2}\sqrt{a}\sqrt{x}}{\sqrt{b+\sqrt{b^2-4ac}}}\right)\right|\frac{b+\sqrt{b^2-}\right.\right.}{2a^{3/2}\sqrt{x(ax+b)+c}}\right.$$
$$\left.+ic_2\sinh\left(\frac{\sqrt{\lambda}\left(\sqrt{b^2-4ac}-b\right)\sqrt{\sqrt{b^2-4ac}+2ax+b}\sqrt{\frac{2ax}{b-\sqrt{b^2-4ac}}+1}\left(E\left(i\operatorname{arcsinh}\left(\frac{\sqrt{2}\sqrt{a}\sqrt{x}}{\sqrt{b+\sqrt{b^2-4ac}}}\right)\right|\frac{b+\sqrt{}\right.\right.}{2a^{3/2}\sqrt{x(ax+b)+c}}\right.$$

31.22 problem 203

Internal problem ID [11027]

Book: Handbook of exact solutions for ordinary differential equations. By Polyanin and Zaitsev. Second edition
Section: Chapter 2, Second-Order Differential Equations. section 2.1.2-6 Equation of form $(a_3x^3+a_2x^2x+a_1x+a_0)y''+f(x)y'+g(x)y=0$
Problem number: 203.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$\boxed{x\left(a\,x^2+bx+1\right)y''+\left(\alpha\,x^2+\beta x+\gamma\right)y'+(xn+m)\,y=0}$$

✗ Solution by Maple

```
dsolve(x*(a*x^2+b*x+1)*diff(y(x),x$2)+(alpha*x^2+beta*x+gamma)*diff(y(x),x)+(n*x+m)*y(x)=0,y
```

No solution found

✓ Solution by Mathematica
Time used: 108.623 (sec). Leaf size: 524

```
DSolve[x*(a*x^2+b*x+1)*y''[x]+(\[Alpha]*x^2+\[Beta]*x+\[Gamma])*y'[x]+(n*x+m)*y[x]==0,y[x],x
```

$$y(x)\to -\frac{2^{-\gamma}\left(-\frac{ax}{\sqrt{b^2-4a}+b}\right)^{1-\gamma}\left(a(-2^{\gamma})\,c_1x\left(-\frac{ax}{\sqrt{b^2-4a}+b}\right)^{\gamma-1}\text{HeunG}\left[\frac{b-\sqrt{b^2-4a}}{\sqrt{b^2-4a}+b},\frac{2m}{\sqrt{b^2-4a}+b},\frac{1}{2}\left(-\sqrt{\frac{a^2+\alpha^2-2a(\alpha+2n)}{a^2}}\right.\right.\right.$$

31.23 problem 204

Internal problem ID [11028]

Book: Handbook of exact solutions for ordinary differential equations. By Polyanin and Zaitsev. Second edition
Section: Chapter 2, Second-Order Differential Equations. section 2.1.2-6 Equation of form $(a_3x^3+a_2x^2x+a_1x+a_0)y''+f(x)y'+g(x)y=0$
Problem number: 204.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$x(x-1)(x-a)y''+\left((\alpha+\beta+1)x^2-(\alpha+\beta+1+a(\gamma+d)-a)x+a\gamma\right)y'+(\alpha\beta x-q)y=0$$

✓ Solution by Maple
Time used: 0.578 (sec). Leaf size: 82

```
dsolve(x*(x-1)*(x-a)*diff(y(x),x$2)+((alpha+beta+1)*x^2-(alpha+beta+1+a*(gamma+d)-a)*x+a*gam
```

$$y(x)=c_1\,\mathrm{HeunG}\left(a,q,\alpha,\beta,\gamma,\frac{a(d-1)}{a-1},x\right)+c_2x^{1-\gamma}\,\mathrm{HeunG}\left(a,q-(-1+\gamma)\left(a(d-1)+\alpha+\beta-\gamma+1\right),\beta+1-\gamma,\alpha+1-\gamma,2-\gamma,\frac{a(d-1)}{a-1},x\right)$$

✓ Solution by Mathematica
Time used: 2.215 (sec). Leaf size: 85

```
DSolve[x*(x-1)*(x-a)*y''[x]+((\[Alpha]+\[Beta]+1)*x^2-(\[Alpha]+\[Beta]+1+a*(\[Gamma]+d)-a)*
```

$$y(x)\to c_2x^{1-\gamma}\mathrm{HeunG}\left[a,q-(\gamma-1)(a(d-1)+\alpha+\beta-\gamma+1),\alpha-\gamma+1,\beta-\gamma+1,2-\gamma,\frac{a(d-1)}{a-1},x\right]+c_1\mathrm{HeunG}\left[a,q,\alpha,\beta,\gamma,\frac{a(d-1)}{a-1},x\right]$$

31.24 problem 205

Internal problem ID [11029]

Book: Handbook of exact solutions for ordinary differential equations. By Polyanin and Zaitsev. Second edition
Section: Chapter 2, Second-Order Differential Equations. section 2.1.2-6 Equation of form $(a_3x^3+a_2x^2x+a_1x+a_0)y''+f(x)y'+g(x)y=0$
Problem number: 205.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$\boxed{\left(a\,x^3+b\,x^2+cx+d\right)y''-\left(-\lambda^2+x^2\right)y'+\left(x+\lambda\right)y=0}$$

✓ Solution by Maple
Time used: 0.5 (sec). Leaf size: 84

```
dsolve((a*x^3+b*x^2+c*x+d)*diff(y(x),x$2)-(x^2-lambda^2)*diff(y(x),x)+(x+lambda)*y(x)=0,y(x)
```

$$y(x)=(\lambda-x)\left(\left(\int \mathrm{e}^{\int \frac{(1-2a)x^3+(-2b-\lambda)x^2+\left(-\lambda^2-2c\right)x+\lambda^3-2d}{\left(a\,x^3+x^2b+cx+d\right)(-\lambda+x)}dx}dx\right)c_2-c_1\right)$$

✗ Solution by Mathematica
Time used: 0.0 (sec). Leaf size: 0

```
DSolve[(a*x^3+b*x^2+c*x+d)*y''[x]-(x^2-\[Lambda]^2)*y'[x]+(x+\[Lambda])*y[x]==0,y[x],x,Inclu
```

Timed out

31.25 problem 206

Internal problem ID [11030]

Book: Handbook of exact solutions for ordinary differential equations. By Polyanin and Zaitsev. Second edition
Section: Chapter 2, Second-Order Differential Equations. section 2.1.2-6 Equation of form $(a_3x^3+a_2x^2x+a_1x+a_0)y''+f(x)y'+g(x)y=0$
Problem number: 206.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries], [_2nd_order, _linear,`

$$\boxed{2\left(a\,x^3+b\,x^2+cx+d\right)y''+\left(3a\,x^2+2bx+c\right)y'+y\lambda=0}$$

✓ Solution by Maple
Time used: 0.453 (sec). Leaf size: 67

```
dsolve(2*(a*x^3+b*x^2+c*x+d)*diff(y(x),x$2)+(3*a*x^2+2*b*x+c)*diff(y(x),x)+lambda*y(x)=0,y(x
```

$$y(x)=c_1\mathrm{e}^{\frac{i\sqrt{2}\,\sqrt{\lambda}\left(\int\frac{1}{\sqrt{a\,x^3+x^2b+cx+d}}dx\right)}{2}}+c_2\mathrm{e}^{-\frac{i\sqrt{2}\,\sqrt{\lambda}\left(\int\frac{1}{\sqrt{a\,x^3+x^2b+cx+d}}dx\right)}{2}}$$

✗ Solution by Mathematica
Time used: 0.0 (sec). Leaf size: 0

```
DSolve[2*(a*x^3+b*x^2+c*x+d)*y''[x]+(3*a*x^2+2*b*x+c)*y'[x]+lambda*y[x]==0,y[x],x,IncludeSin
```

Timed out

31.26 problem 207

Internal problem ID [11031]

Book: Handbook of exact solutions for ordinary differential equations. By Polyanin and Zaitsev. Second edition
Section: Chapter 2, Second-Order Differential Equations. section 2.1.2-6 Equation of form $(a_3x^3+a_2x^2x+a_1x+a_0)y''+f(x)y'+g(x)y=0$
Problem number: 207.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$2\left(a\,x^3+b\,x^2+cx+d\right)y''+3\left(3a\,x^2+2bx+c\right)y'+\left(6ax+2b+\lambda\right)y=0$$

✓ Solution by Maple
Time used: 0.531 (sec). Leaf size: 101

```
dsolve(2*(a*x^3+b*x^2+c*x+d)*diff(y(x),x$2)+3*(3*a*x^2+2*b*x+c)*diff(y(x),x)+(6*a*x+2*b+lamb
```

$$y(x)=\frac{c_1\mathrm{e}^{\frac{\sqrt{2}\sqrt{-\frac{\lambda}{a}}\left(\int\frac{1}{\sqrt{\frac{a\,x^3+x^2b+cx+d}{a}}}dx\right)}{2}}+c_2\mathrm{e}^{-\frac{\sqrt{2}\sqrt{-\frac{\lambda}{a}}\left(\int\frac{1}{\sqrt{\frac{a\,x^3+x^2b+cx+d}{a}}}dx\right)}{2}}}{\sqrt{a\,x^3+x^2b+cx+d}}$$

✓ Solution by Mathematica
Time used: 135.727 (sec). Leaf size: 3202

```
DSolve[2*(a*x^3+b*x^2+c*x+d)*y''[x]+3*(3*a*x^2+2*b*x+c)*y'[x]+(6*a*x+2*b+\[Lambda])*y[x]==0,
```

Too large to display

31.27 problem 208

Internal problem ID [11032]

Book: Handbook of exact solutions for ordinary differential equations. By Polyanin and Zaitsev. Second edition
Section: Chapter 2, Second-Order Differential Equations. section 2.1.2-6 Equation of form $(a_3x^3+a_2x^2x+a_1x+a_0)y''+f(x)y'+g(x)y=0$
Problem number: 208.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$\left(a\,x^3+b\,x^2+cx+d\right)y''+\left(\alpha\,x^2+(\alpha\gamma+\beta)\,x+\beta\lambda\right)y'-(\alpha x+\beta)\,y=0$$

✗ Solution by Maple

```
dsolve((a*x^3+b*x^2+c*x+d)*diff(y(x),x$2)+(alpha*x^2+(alpha*gamma+beta)*x+beta*lambda)*diff(
```

No solution found

✗ Solution by Mathematica
Time used: 0.0 (sec). Leaf size: 0

```
DSolve[(a*x^3+b*x^2+c*x+d)*y''[x]+(\[Alpha]*x^2+(\[Alpha]*\[Gamma]+\[Beta])*x+\[Beta]*\[Lamb
```

Timed out

31.28 problem 209

Internal problem ID [11033]

Book: Handbook of exact solutions for ordinary differential equations. By Polyanin and Zaitsev. Second edition
Section: Chapter 2, Second-Order Differential Equations. section 2.1.2-6 Equation of form $(a_3x^3+a_2x^2x+a_1x+a_0)y''+f(x)y'+g(x)y=0$
Problem number: 209.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$\boxed{\left(a\,x^3+b\,x^2+cx+d\right)y''+\left(\lambda^3+x^3\right)y'-\left(\lambda^2-x\lambda+x^2\right)y=0}$$

✓ Solution by Maple
Time used: 0.532 (sec). Leaf size: 76

```
dsolve((a*x^3+b*x^2+c*x+d)*diff(y(x),x$2)+(x^3+lambda^3)*diff(y(x),x)-(x^2-lambda*x+lambda^2
```

$$y(x)=(x+\lambda)\left(\left(\int \mathrm{e}^{-\left(\int \frac{x^4+(2a+\lambda)x^3+2x^2b+\left(\lambda^3+2c\right)x+\lambda^4+2d}{\left(a\,x^3+x^2b+cx+d\right)(x+\lambda)}dx\right)}dx\right)c_2+c_1\right)$$

✓ Solution by Mathematica
Time used: 1.343 (sec). Leaf size: 240

```
DSolve[(a*x^3+b*x^2+c*x+d)*y''[x]+(x^3+\[Lambda]^3)*y'[x]-(x^2-\[Lambda]*x+\[Lambda]^2)*y[x]
```

$$y(x)$$

$$\to \frac{c_2(\lambda+x)\int_1^x \exp\left(-\frac{\lambda+K[1]+2a\log(\lambda+K[1])+\text{RootSum}\left[-a\lambda^3+b\lambda^2+3a\#1\lambda^2-3a\#1^2\lambda-c\lambda-2b\#1\lambda+a\#1^3+b\#1^2+d+c\right.}{}\right.}{}$$

$$+\frac{c_1(\lambda+x)}{\lambda}$$

31.29 problem 210

Internal problem ID [11034]

Book: Handbook of exact solutions for ordinary differential equations. By Polyanin and Zaitsev. Second edition
Section: Chapter 2, Second-Order Differential Equations. section 2.1.2-6 Equation of form $(a_3x^3+a_2x^2x+a_1x+a_0)y''+f(x)y'+g(x)y=0$
Problem number: 210.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$2x\left(a\,x^2+bx+c\right)y''+\left(a(2-k)\,x^2+b(-k+1)\,x-ck\right)y'+\lambda\,x^{1+k}y=0$$

✗ Solution by Maple

```
dsolve(2*x(a*x^2+b*x+c)*diff(y(x),x$2)+(a*(2-k)*x^2+b*(1-k)*x-c*k)*diff(y(x),x)+(lambda*x^(k
```

No solution found

✓ Solution by Mathematica

Time used: 157.344 (sec). Leaf size: 790

```
DSolve[2*x(a*x^2+b*x+c)*y''[x]+(a*(2-k)*x^2+b*(1-k)*x-c*k)*y'[x]+(\[Lambda]*x^(k+1))*y[x]==0
```

$$y(x)$$

$$\to \frac{\sqrt{2}\sqrt{c_1}\tan\left(\frac{\sqrt{2}x\sqrt{\frac{-\sqrt{b^2-4ac}+2ax+b}{b-\sqrt{b^2-4ac}}}\sqrt{\frac{\sqrt{b^2-4ac}+2ax+b}{\sqrt{b^2-4ac}+b}}\,\text{AppellF1}\left(\frac{k+2}{2},\frac{1}{2},\frac{1}{2},\frac{k+4}{2},-\frac{2ax}{b+\sqrt{b^2-4ac}},\frac{2ax}{\sqrt{b^2-4ac}-b}\right)}{(k+2)\sqrt{\frac{x^{-k}(x(ax+b)+c)}{\lambda}}}-c_2\right)}{\sqrt{-1-\tan^2\left(\frac{\sqrt{2}x\sqrt{\frac{-\sqrt{b^2-4ac}+2ax+b}{b-\sqrt{b^2-4ac}}}\sqrt{\frac{\sqrt{b^2-4ac}+2ax+b}{\sqrt{b^2-4ac}+b}}\,\text{AppellF1}\left(\frac{k+2}{2},\frac{1}{2},\frac{1}{2},\frac{k+4}{2},-\frac{2ax}{b+\sqrt{b^2-4ac}},\frac{2ax}{\sqrt{b^2-4ac}-b}\right)}{(k+2)\sqrt{\frac{x^{-k}(x(ax+b)+c)}{\lambda}}}-c_2\right)}}$$

$$y(x)\to$$

$$-\frac{\sqrt{2}\sqrt{c_1}\tan\left(\frac{\sqrt{2}x\sqrt{\frac{-\sqrt{b^2-4ac}+2ax+b}{b-\sqrt{b^2-4ac}}}\sqrt{\frac{\sqrt{b^2-4ac}+2ax+b}{\sqrt{b^2-4ac}+b}}\,\text{AppellF1}\left(\frac{k+2}{2},\frac{1}{2},\frac{1}{2},\frac{k+4}{2},-\frac{2ax}{b+\sqrt{b^2-4ac}},\frac{2ax}{\sqrt{b^2-4ac}-b}\right)}{(k+2)\sqrt{\frac{x^{-k}(x(ax+b)+c)}{\lambda}}}+c_2\right)}{\sqrt{-1-\tan^2\left(\frac{\sqrt{2}x\sqrt{\frac{-\sqrt{b^2-4ac}+2ax+b}{b-\sqrt{b^2-4ac}}}\sqrt{\frac{\sqrt{b^2-4ac}+2ax+b}{\sqrt{b^2-4ac}+b}}\,\text{AppellF1}\left(\frac{k+2}{2},\frac{1}{2},\frac{1}{2},\frac{k+4}{2},-\frac{2ax}{b+\sqrt{b^2-4ac}},\frac{2ax}{\sqrt{b^2-4ac}-b}\right)}{(k+2)\sqrt{\frac{x^{-k}(x(ax+b)+c)}{\lambda}}}+c_2\right)}}$$

32 Chapter 2, Second-Order Differential Equations. section 2.1.2-7 Equation of form $(a_4x^4+a_3x^3+a_2x^2x+a_1x+a_0)y''+f(x)y'+g(x)y=0$

32.1 problem 211 857
32.2 problem 212 858
32.3 problem 213 859
32.4 problem 214 860
32.5 problem 215 861
32.6 problem 216 862
32.7 problem 217 863
32.8 problem 218 864
32.9 problem 219 865
32.10 problem 220 867
32.11 problem 221 868
32.12 problem 222 A 869
32.13 problem 222 B 870
32.14 problem 223 871
32.15 problem 224 872
32.16 problem 225 873
32.17 problem 226 874
32.18 problem 227 875
32.19 problem 228 876
32.20 problem 229 877
32.21 problem 230 879
32.22 problem 231 880
32.23 problem 232 881
32.24 problem 233 882
32.25 problem 234 884
32.26 problem 235 886
32.27 problem 236 888
32.28 problem 237 889

32.1 problem 211

Internal problem ID [11035]

Book: Handbook of exact solutions for ordinary differential equations. By Polyanin and Zaitsev. Second edition
Section: Chapter 2, Second-Order Differential Equations. section 2.1.2-7 Equation of form $(a_4x^4+a_3x^3+a_2x^2x+a_1x+a_0)y''+f(x)y'+g(x)y=0$
Problem number: 211.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_Emden, _Fowler]]`

$$\boxed{x^4y''+ay=0}$$

✓ Solution by Maple
Time used: 0.0 (sec). Leaf size: 31

```
dsolve(x^4*diff(y(x),x$2)+a*y(x)=0,y(x), singsol=all)
```

$$y(x)=x\left(c_1\sinh\left(\frac{\sqrt{-a}}{x}\right)+c_2\cosh\left(\frac{\sqrt{-a}}{x}\right)\right)$$

✓ Solution by Mathematica
Time used: 0.199 (sec). Leaf size: 52

```
DSolve[x^4*y''[x]+a*y[x]==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x)\to c_1xe^{\frac{i\sqrt{a}}{x}}-\frac{ic_2xe^{-\frac{i\sqrt{a}}{x}}}{2\sqrt{a}}$$

32.2 problem 212

Internal problem ID [11036]

Book: Handbook of exact solutions for ordinary differential equations. By Polyanin and Zaitsev. Second edition
Section: Chapter 2, Second-Order Differential Equations. section 2.1.2-7 Equation of form $(a_4x^4+a_3x^3+a_2x^2x+a_1x+a_0)y''+f(x)y'+g(x)y=0$
Problem number: 212.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$\boxed{x^4y''+\left(a\,x^2+bx+c\right)y=0}$$

✓ Solution by Maple
Time used: 0.125 (sec). Leaf size: 63

```
dsolve(x^4*diff(y(x),x$2)+(a*x^2+b*x+c)*y(x)=0,y(x), singsol=all)
```

$$y(x)=x\left(c_1\,\text{WhittakerM}\left(-\frac{ib}{2\sqrt{c}},\frac{\sqrt{-4a+1}}{2},\frac{2i\sqrt{c}}{x}\right)+c_2\,\text{WhittakerW}\left(-\frac{ib}{2\sqrt{c}},\frac{\sqrt{-4a+1}}{2},\frac{2i\sqrt{c}}{x}\right)\right)$$

✗ Solution by Mathematica
Time used: 0.0 (sec). Leaf size: 0

```
DSolve[x^4*y''[x]+(a*x^2+b*x+c)*y[x]==0,y[x],x,IncludeSingularSolutions -> True]
```

Not solved

32.3 problem 213

Internal problem ID [11037]

Book: Handbook of exact solutions for ordinary differential equations. By Polyanin and Zaitsev. Second edition
Section: Chapter 2, Second-Order Differential Equations. section 2.1.2-7 Equation of form $(a_4x^4+a_3x^3+a_2x^2x+a_1x+a_0)y''+f(x)y'+g(x)y=0$
Problem number: 213.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$\boxed{x^4y''-(a+b)\,x^2y'+((a+b)\,x+ab)\,y=0}$$

✓ Solution by Maple
Time used: 0.0 (sec). Leaf size: 25

```
dsolve(x^4*diff(y(x),x$2)-(a+b)*x^2*diff(y(x),x)+((a+b)*x+a*b)*y(x)=0,y(x), singsol=all)
```

$$y(x)=x\left(\mathrm{e}^{-\frac{a}{x}}c_1+\mathrm{e}^{-\frac{b}{x}}c_2\right)$$

✓ Solution by Mathematica
Time used: 0.214 (sec). Leaf size: 37

```
DSolve[x^4*y''[x]-(a+b)*x^2*y'[x]+((a+b)*x+a*b)*y[x]==0,y[x],x,IncludeSingularSolutions -> T
```

$$y(x)\to \frac{c_2xe^{-\frac{a}{x}}}{a-b}+c_1xe^{-\frac{b}{x}}$$

32.4 problem 214

Internal problem ID [11038]

Book: Handbook of exact solutions for ordinary differential equations. By Polyanin and Zaitsev. Second edition
Section: Chapter 2, Second-Order Differential Equations. section 2.1.2-7 Equation of form $(a_4x^4+a_3x^3+a_2x^2x+a_1x+a_0)y''+f(x)y'+g(x)y=0$
Problem number: 214.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$\boxed{x^4y''+2x^2(x+a)\,y'+yb=0}$$

✓ Solution by Maple
Time used: 0.0 (sec). Leaf size: 43

```
dsolve(x^4*diff(y(x),x$2)+2*x^2*(x+a)*diff(y(x),x)+b*y(x)=0,y(x), singsol=all)
```

$$y(x)=c_1\mathrm{e}^{\frac{a-\sqrt{a^2-b}}{x}}+c_2\mathrm{e}^{\frac{a+\sqrt{a^2-b}}{x}}$$

✓ Solution by Mathematica
Time used: 0.06 (sec). Leaf size: 51

```
DSolve[x^4*y''[x]+2*x^2*(x+a)*y'[x]+b*y[x]==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x)\to e^{\frac{a-\sqrt{a^2-b}}{x}}\left(c_1e^{\frac{2\sqrt{a^2-b}}{x}}+c_2\right)$$

32.5 problem 215

Internal problem ID [11039]

Book: Handbook of exact solutions for ordinary differential equations. By Polyanin and Zaitsev. Second edition
Section: Chapter 2, Second-Order Differential Equations. section 2.1.2-7 Equation of form $(a_4x^4+a_3x^3+a_2x^2x+a_1x+a_0)y''+f(x)y'+g(x)y=0$
Problem number: 215.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$\boxed{x^4y''+a\,x^ny'-\left(a\,x^{n-1}+ab\,x^{n-2}+b^2\right)y=0}$$

✗ Solution by Maple

```
dsolve(x^4*diff(y(x),x$2)+a*x^n*diff(y(x),x)-(a*x^(n-1)+a*b*x^(n-2)+b^2)*y(x)=0,y(x), singso
```

No solution found

✗ Solution by Mathematica
Time used: 0.0 (sec). Leaf size: 0

```
DSolve[x^4*y''[x]+a*x^n*y'[x]-(a*x^(n-1)+a*b*x^(n-2)+b^2)*y[x]==0,y[x],x,IncludeSingularSolu
```

Not solved

32.6 problem 216

Internal problem ID [11040]

Book: Handbook of exact solutions for ordinary differential equations. By Polyanin and Zaitsev. Second edition
Section: Chapter 2, Second-Order Differential Equations. section 2.1.2-7 Equation of form $(a_4x^4+a_3x^3+a_2x^2x+a_1x+a_0)y''+f(x)y'+g(x)y=0$
Problem number: 216.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$\boxed{x^2(x-a)^2y''+yb=0}$$

✓ Solution by Maple
Time used: 0.015 (sec). Leaf size: 67

```
dsolve(x^2*(x-a)^2*diff(y(x),x$2)+b*y(x)=0,y(x), singsol=all)
```

$$y(x)=\sqrt{x\left(a-x\right)}\left(\left(\frac{x}{a-x}\right)^{\frac{\sqrt{a^2-4b}}{2a}}c_2+\left(\frac{a-x}{x}\right)^{\frac{\sqrt{a^2-4b}}{2a}}c_1\right)$$

✓ Solution by Mathematica
Time used: 0.487 (sec). Leaf size: 121

```
DSolve[x^2*(x-a)^2*y''[x]+b*y[x]==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x)\to\frac{x^{\frac{1}{2}-\frac{1}{2}\sqrt{1-\frac{4b}{a^2}}}(x-a)^{\frac{1}{2}-\frac{1}{2}\sqrt{1-\frac{4b}{a^2}}}\left(ac_1\sqrt{1-\frac{4b}{a^2}}x^{\sqrt{1-\frac{4b}{a^2}}}+c_2(x-a)^{\sqrt{1-\frac{4b}{a^2}}}\right)}{a\sqrt{1-\frac{4b}{a^2}}}$$

32.7 problem 217

Internal problem ID [11041]

Book: Handbook of exact solutions for ordinary differential equations. By Polyanin and Zaitsev. Second edition
Section: Chapter 2, Second-Order Differential Equations. section 2.1.2-7 Equation of form $(a_4x^4+a_3x^3+a_2x^2x+a_1x+a_0)y''+f(x)y'+g(x)y=0$
Problem number: 217.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _linear, _nonhomogeneous]]`

$$\boxed{x^2(x-a)^2y''+yb=cx^2(x-a)^2}$$

✓ Solution by Maple
Time used: 0.015 (sec). Leaf size: 219

```
dsolve(x^2*(x-a)^2*diff(y(x),x$2)+b*y(x)=c*x^2*(x-a)^2,y(x), singsol=all)
```

$$y(x)$$
$$=\frac{\sqrt{x\left(a-x\right)}\left(\left(\frac{x}{a-x}\right)^{\frac{\sqrt{a^2-4b}}{2a}}c_1\sqrt{a^2-4b}+\left(\frac{a-x}{x}\right)^{\frac{\sqrt{a^2-4b}}{2a}}c_2\sqrt{a^2-4b}+\left(\frac{x}{a-x}\right)^{\frac{\sqrt{a^2-4b}}{2a}}\left(\int\sqrt{x\left(a-x\right)}\left(\frac{x}{a-x}\right)^{-\sqrt{}}\right.\right.}{\sqrt{a^2-4b}}$$

✓ Solution by Mathematica
Time used: 0.958 (sec). Leaf size: 371

```
DSolve[x^2*(x-a)^2*y''[x]+b*y[x]==c*x^2*(x-a)^2,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x)$$
$$\to\frac{acx^2(a-x)\left(1-\frac{x}{a}\right)^{-\frac{1}{2}\sqrt{1-\frac{4b}{a^2}}-\frac{1}{2}}\left(\left(\sqrt{1-\frac{4b}{a^2}}-3\right)\left(1-\frac{x}{a}\right)^{\sqrt{1-\frac{4b}{a^2}}}\text{Hypergeometric2F1}\left(\frac{1}{2}\sqrt{1-\frac{4b}{a^2}}-\frac{1}{2},\frac{1}{2}\right.\right.}{}$$

$$+c_1x^{\frac{1}{2}\sqrt{1-\frac{4b}{a^2}}+\frac{1}{2}}(x-a)^{\frac{1}{2}-\frac{1}{2}\sqrt{1-\frac{4b}{a^2}}}+\frac{c_2x^{\frac{1}{2}-\frac{1}{2}\sqrt{1-\frac{4b}{a^2}}}(x-a)^{\frac{1}{2}\sqrt{1-\frac{4b}{a^2}}+\frac{1}{2}}}{a\sqrt{1-\frac{4b}{a^2}}}$$

32.8 problem 218

Internal problem ID [11042]

Book: Handbook of exact solutions for ordinary differential equations. By Polyanin and Zaitsev. Second edition
Section: Chapter 2, Second-Order Differential Equations. section 2.1.2-7 Equation of form $(a_4x^4+a_3x^3+a_2x^2x+a_1x+a_0)y''+f(x)y'+g(x)y=0$
Problem number: 218.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$\boxed{a\,x^2(x-1)^2\,y''+\left(b\,x^2+cx+d\right)y=0}$$

✓ Solution by Maple
Time used: 0.125 (sec). Leaf size: 267

```
dsolve(a*x^2*(x-1)^2*diff(y(x),x$2)+(b*x^2+c*x+d)*y(x)=0,y(x), singsol=all)
```

$$y(x)=(-1+x)^{-\frac{\sqrt{a-4b-4c-4d}-\sqrt{a}}{2\sqrt{a}}}\left(c_1x^{\frac{\sqrt{a}+\sqrt{a-4d}}{2\sqrt{a}}}\,\text{hypergeom}\left(\left[\frac{-\sqrt{a-4b-4c-4d}+\sqrt{a}+\sqrt{a-4d}+\sqrt{a-4b}}{2\sqrt{a}},-\sqrt{}\right.\right.\right.$$
$$+c_2x^{-\frac{-\sqrt{a}+\sqrt{a-4d}}{2\sqrt{a}}}\,\text{hypergeom}\left(\left[\frac{-\sqrt{a-4b-4c-4d}+\sqrt{a}-\sqrt{a-4d}+\sqrt{a-4b}}{2\sqrt{a}},-\frac{\sqrt{a-4b-4c-4d}}{}\right.\right.$$

✓ Solution by Mathematica
Time used: 135.53 (sec). Leaf size: 413606

```
DSolve[a*x^2*(x-1)^2*y''[x]+(b*x^2+c*x+d)*y[x]==0,y[x],x,IncludeSingularSolutions -> True]
```

Too large to display

32.9 problem 219

Internal problem ID [11043]

Book: Handbook of exact solutions for ordinary differential equations. By Polyanin and Zaitsev. Second edition
Section: Chapter 2, Second-Order Differential Equations. section 2.1.2-7 Equation of form $(a_4x^4+a_3x^3+a_2x^2x+a_1x+a_0)y''+f(x)y'+g(x)y=0$
Problem number: 219.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$\boxed{x^2\left(x^2+a\right)y''+\left(bx^2+c\right)xy'+yd=0}$$

✓ Solution by Maple
Time used: 0.156 (sec). Leaf size: 286

```
dsolve(x^2*(x^2+a)*diff(y(x),x$2)+(b*x^2+c)*x*diff(y(x),x)+d*y(x)=0,y(x), singsol=all)
```

$$y(x)=\left(x^2+a\right)^{\frac{(-b+2)a+c}{2a}}\left(c_2x^{-\frac{-a+c+\sqrt{a^2+(-2c-4d)a+c^2}}{2a}}\,\text{hypergeom}\left(\left[-\frac{-3a-c+\sqrt{a^2+(-2c-4d)a+c^2}}{4a},\frac{-\sqrt{a^2+}\right.\right.\right.$$

$$\left.\left.\left.-\frac{x^2}{a}\right)\right.\right.$$

$$+c_1x^{\frac{a-c+\sqrt{a^2+(-2c-4d)a+c^2}}{2a}}\,\text{hypergeom}\left(\left[\frac{3a+c+\sqrt{a^2+(-2c-4d)a+c^2}}{4a},\frac{\sqrt{a^2+(-2c-4d)a+c^2}+}{4a}\right.\right.$$

$$\left.\left.-\frac{x^2}{a}\right)\right)$$

✓ Solution by Mathematica

Time used: 2.385 (sec). Leaf size: 336

```
DSolve[x^2*(x^2+a)*y''[x]+(b*x^2+c)*x*y'[x]+d*y[x]==0,y[x],x,IncludeSingularSolutions -> Tru
```

$$y(x)$$

$$\to a^{-\frac{\sqrt{a^2-2a(c+2d)+c^2}+a-c}{4a}} x^{-\frac{\sqrt{a^2-2a(c+2d)+c^2}-a+c}{2a}} \left(c_2 x^{\frac{\sqrt{a^2-2a(c+2d)+c^2}}{a}} \text{Hypergeometric2F1}\left(-\frac{-2ba+a+c-\sqrt{a}}{4} \right.\right.$$

$$\left. + 1, -\frac{x^2}{a} \right)$$

$$+ c_1 a^{\frac{\sqrt{a^2-2a(c+2d)+c^2}}{2a}} \text{Hypergeometric2F1}\left(-\frac{-a+c+\sqrt{a^2-2(c+2d)a+c^2}}{4a}, \right.$$

$$\left. \left. -\frac{-2ba+a+c+\sqrt{a^2-2(c+2d)a+c^2}}{4a}, 1-\frac{\sqrt{a^2-2(c+2d)a+c^2}}{2a}, -\frac{x^2}{a} \right) \right)$$

32.10 problem 220

Internal problem ID [11044]

Book: Handbook of exact solutions for ordinary differential equations. By Polyanin and Zaitsev. Second edition
Section: Chapter 2, Second-Order Differential Equations. section 2.1.2-7 Equation of form $(a_4x^4+a_3x^3+a_2x^2x+a_1x+a_0)y''+f(x)y'+g(x)y=0$
Problem number: 220.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[_Halm]`

$$\boxed{\left(x^2+1\right)^2 y''+ay=0}$$

✓ Solution by Maple
Time used: 0.0 (sec). Leaf size: 59

```
dsolve((x^2+1)^2*diff(y(x),x$2)+a*y(x)=0,y(x), singsol=all)
```

$$y(x)=\left(\left(\frac{x+i}{-x+i}\right)^{-\frac{\sqrt{a+1}}{2}}c_2+\left(\frac{x+i}{-x+i}\right)^{\frac{\sqrt{a+1}}{2}}c_1\right)\sqrt{x^2+1}$$

✓ Solution by Mathematica
Time used: 0.215 (sec). Leaf size: 83

```
DSolve[(x^2+1)^2*y''[x]+a*y[x]==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x)\to \frac{1}{2}\sqrt{x^2+1}e^{i\sqrt{a+1}\arctan(x)}\left(\frac{ic_2(1-ix)^{\sqrt{a+1}}(1+ix)^{-\sqrt{a+1}}}{\sqrt{a+1}}+2c_1\right)$$

32.11 problem 221

Internal problem ID [11045]

Book: Handbook of exact solutions for ordinary differential equations. By Polyanin and Zaitsev. Second edition
Section: Chapter 2, Second-Order Differential Equations. section 2.1.2-7 Equation of form $(a_4x^4+a_3x^3+a_2x^2x+a_1x+a_0)y''+f(x)y'+g(x)y=0$
Problem number: 221.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$\boxed{\left(x^2-1\right)^2 y''+ay=0}$$

✓ Solution by Maple
Time used: 0.0 (sec). Leaf size: 55

```
dsolve((x^2-1)^2*diff(y(x),x$2)+a*y(x)=0,y(x), singsol=all)
```

$$y(x)=\sqrt{x^2-1}\left(\left(\frac{-1+x}{1+x}\right)^{\frac{\sqrt{-a+1}}{2}}c_1+\left(\frac{-1+x}{1+x}\right)^{-\frac{\sqrt{-a+1}}{2}}c_2\right)$$

✓ Solution by Mathematica
Time used: 0.249 (sec). Leaf size: 88

```
DSolve[(x^2-1)^2*y''[x]+a*y[x]==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x)\to \frac{(1-x^2)^{\frac{1}{2}-\frac{\sqrt{1-a}}{2}}\left(2\sqrt{1-a}c_1(1-x)^{\sqrt{1-a}}+c_2(x+1)^{\sqrt{1-a}}\right)}{2\sqrt{1-a}}$$

32.12 problem 222 A

Internal problem ID [11046]

Book: Handbook of exact solutions for ordinary differential equations. By Polyanin and Zaitsev. Second edition
Section: Chapter 2, Second-Order Differential Equations. section 2.1.2-7 Equation of form $(a_4x^4+a_3x^3+a_2x^2x+a_1x+a_0)y''+f(x)y'+g(x)y=0$
Problem number: 222 A.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_Emden, _Fowler]]`

$$\boxed{\left(a^2+x^2\right)^2 y''+yb^2=0}$$

✓ Solution by Maple
Time used: 0.016 (sec). Leaf size: 83

```
dsolve((x^2+a^2)^2*diff(y(x),x$2)+b^2*y(x)=0,y(x), singsol=all)
```

$$y(x)=\left(\left(\frac{ix-a}{ix+a}\right)^{\frac{\sqrt{a^2+b^2}}{2a}}c_1+\left(\frac{ix-a}{ix+a}\right)^{-\frac{\sqrt{a^2+b^2}}{2a}}c_2\right)\sqrt{a^2+x^2}$$

✓ Solution by Mathematica
Time used: 0.489 (sec). Leaf size: 97

```
DSolve[(x^2+a^2)^2*y''[x]+b^2*y[x]==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x)\to\frac{1}{2}\sqrt{a^2+x^2}e^{-i\sqrt{\frac{b^2}{a^2}+1}\arctan\left(\frac{a}{x}\right)}\left(\frac{ic_2e^{2i\sqrt{\frac{b^2}{a^2}+1}\arctan\left(\frac{a}{x}\right)}}{a\sqrt{\frac{b^2}{a^2}+1}}+2c_1\right)$$

32.13 problem 222 B

Internal problem ID [11047]

Book: Handbook of exact solutions for ordinary differential equations. By Polyanin and Zaitsev. Second edition
Section: Chapter 2, Second-Order Differential Equations. section 2.1.2-7 Equation of form $(a_4x^4+a_3x^3+a_2x^2x+a_1x+a_0)y''+f(x)y'+g(x)y=0$
Problem number: 222 B.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$\boxed{\left(-a^2+x^2\right)^2 y''+yb^2=0}$$

✓ Solution by Maple
Time used: 0.016 (sec). Leaf size: 77

```
dsolve((x^2-a^2)^2*diff(y(x),x$2)+b^2*y(x)=0,y(x), singsol=all)
```

$$y(x)=\sqrt{a^2-x^2}\left(\left(\frac{a-x}{a+x}\right)^{-\frac{\sqrt{a^2-b^2}}{2a}}c_2+\left(\frac{a-x}{a+x}\right)^{\frac{\sqrt{a^2-b^2}}{2a}}c_1\right)$$

✓ Solution by Mathematica
Time used: 0.529 (sec). Leaf size: 142

```
DSolve[(x^2-a^2)^2*y''[x]+b^2*y[x]==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x)\rightarrow\frac{(x-a)^{\frac{1}{2}-\frac{1}{2}\sqrt{1-\frac{b^2}{a^2}}}(a+x)^{\frac{1}{2}-\frac{1}{2}\sqrt{1-\frac{b^2}{a^2}}}\left(2ac_1\sqrt{1-\frac{b^2}{a^2}}(x-a)^{\sqrt{1-\frac{b^2}{a^2}}}-c_2(a+x)^{\sqrt{1-\frac{b^2}{a^2}}}\right)}{2a\sqrt{1-\frac{b^2}{a^2}}}$$

32.14 problem 223

Internal problem ID [11048]

Book: Handbook of exact solutions for ordinary differential equations. By Polyanin and Zaitsev. Second edition
Section: Chapter 2, Second-Order Differential Equations. section 2.1.2-7 Equation of form $(a_4x^4+a_3x^3+a_2x^2x+a_1x+a_0)y''+f(x)y'+g(x)y=0$
Problem number: 223.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[_Halm]`

$$\boxed{4\left(x^2+1\right)^2 y''+\left(a\,x^2+a-3\right) y=0}$$

✓ Solution by Maple
Time used: 0.0 (sec). Leaf size: 55

```
dsolve(4*(x^2+1)^2*diff(y(x),x$2)+(a*x^2+a-3)*y(x)=0,y(x), singsol=all)
```

$$y(x)=\left(x^2+1\right)^{\frac{1}{4}}\left(\left(x+\sqrt{x^2+1}\right)^{-\frac{\sqrt{-a+1}}{2}}c_2+\left(x+\sqrt{x^2+1}\right)^{\frac{\sqrt{-a+1}}{2}}c_1\right)$$

✓ Solution by Mathematica
Time used: 0.067 (sec). Leaf size: 70

```
DSolve[4*(x^2+1)^2*y''[x]+(a*x^2+a-3)*y[x]==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x)\to\sqrt{x^2+1}\left(c_1P^{\frac{1}{2}}_{\frac{1}{2}\left(\sqrt{1-a}-1\right)}(ix)+c_2Q^{\frac{1}{2}}_{\frac{1}{2}\left(\sqrt{1-a}-1\right)}(ix)\right)$$

32.15 problem 224

Internal problem ID [11049]

Book: Handbook of exact solutions for ordinary differential equations. By Polyanin and Zaitsev. Second edition
Section: Chapter 2, Second-Order Differential Equations. section 2.1.2-7 Equation of form $(a_4x^4+a_3x^3+a_2x^2x+a_1x+a_0)y''+f(x)y'+g(x)y=0$
Problem number: 224.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries], [_2nd_order, _linear,`

$$\boxed{\left(a\,x^2+b\right)^2 y''+2ax\left(a\,x^2+b\right)y'+yc=0}$$

✓ Solution by Maple
Time used: 0.016 (sec). Leaf size: 47

```
dsolve((a*x^2+b)^2*diff(y(x),x$2)+2*a*x*(a*x^2+b)*diff(y(x),x)+c*y(x)=0,y(x), singsol=all)
```

$$y(x)=c_1\sin\left(\frac{\sqrt{c}\,\arctan\left(\frac{ax}{\sqrt{ab}}\right)}{\sqrt{ab}}\right)+c_2\cos\left(\frac{\sqrt{c}\,\arctan\left(\frac{ax}{\sqrt{ab}}\right)}{\sqrt{ab}}\right)$$

✓ Solution by Mathematica
Time used: 2.133 (sec). Leaf size: 72

```
DSolve[(a*x^2+b)^2*y''[x]+2*a*x*(a*x^2+b)*y'[x]+c*y[x]==0,y[x],x,IncludeSingularSolutions ->
```

$$y(x)\to c_1\cos\left(\frac{\sqrt{c}\arctan\left(\frac{\sqrt{a}x}{\sqrt{b}}\right)}{\sqrt{a}\sqrt{b}}\right)+c_2\sin\left(\frac{\sqrt{c}\arctan\left(\frac{\sqrt{a}x}{\sqrt{b}}\right)}{\sqrt{a}\sqrt{b}}\right)$$

32.16 problem 225

Internal problem ID [11050]

Book: Handbook of exact solutions for ordinary differential equations. By Polyanin and Zaitsev. Second edition
Section: Chapter 2, Second-Order Differential Equations. section 2.1.2-7 Equation of form $(a_4x^4+a_3x^3+a_2x^2x+a_1x+a_0)y''+f(x)y'+g(x)y=0$
Problem number: 225.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$\boxed{\left(x^2-1\right)^2 y''+2x\left(x^2-1\right) y'-\left(\nu(\nu+1)\left(x^2-1\right)+n^2\right) y=0}$$

✓ Solution by Maple
Time used: 0.063 (sec). Leaf size: 17

```
dsolve((x^2-1)^2*diff(y(x),x$2)+2*x*(x^2-1)*diff(y(x),x)-(nu*(nu+1)*(x^2-1)+n^2)*y(x)=0,y(x)
```

$$y(x)=c_1\,\mathrm{LegendreP}\left(\nu,n,x\right)+c_2\,\mathrm{LegendreQ}\left(\nu,n,x\right)$$

✓ Solution by Mathematica
Time used: 0.051 (sec). Leaf size: 20

```
DSolve[(x^2-1)^2*y''[x]+2*x*(x^2-1)*y'[x]-(\[Nu]*(\[Nu]+1)*(x^2-1)+n^2)*y[x]==0,y[x],x,Inclu
```

$$y(x)\to c_1P_\nu^n(x)+c_2Q_\nu^n(x)$$

32.17 problem 226

Internal problem ID [11051]

Book: Handbook of exact solutions for ordinary differential equations. By Polyanin and Zaitsev. Second edition
Section: Chapter 2, Second-Order Differential Equations. section 2.1.2-7 Equation of form $(a_4x^4+a_3x^3+a_2x^2x+a_1x+a_0)y''+f(x)y'+g(x)y=0$
Problem number: 226.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$\boxed{\left(-x^2+1\right)^2 y''-2x\left(-x^2+1\right) y'+\left(\nu(\nu+1)\left(-x^2+1\right)-\mu^2\right) y=0}$$

✓ Solution by Maple
Time used: 0.063 (sec). Leaf size: 17

```
dsolve((1-x^2)^2*diff(y(x),x$2)-2*x*(1-x^2)*diff(y(x),x)+(nu*(nu+1)*(1-x^2)-mu^2)*y(x)=0,y(x
```

$$y(x)=c_1\,\mathrm{LegendreP}\left(\nu,\mu,x\right)+c_2\,\mathrm{LegendreQ}\left(\nu,\mu,x\right)$$

✓ Solution by Mathematica
Time used: 0.046 (sec). Leaf size: 20

```
DSolve[(1-x^2)^2*y''[x]-2*x*(1-x^2)*y'[x]+(\[Nu]*(\[Nu]+1)*(1-x^2)-\[Mu]^2)*y[x]==0,y[x],x,I
```

$$y(x)\to c_1 P_\nu^\mu(x)+c_2 Q_\nu^\mu(x)$$

32.18 problem 227

Internal problem ID [11052]

Book: Handbook of exact solutions for ordinary differential equations. By Polyanin and Zaitsev. Second edition
Section: Chapter 2, Second-Order Differential Equations. section 2.1.2-7 Equation of form $(a_4x^4+a_3x^3+a_2x^2x+a_1x+a_0)y''+f(x)y'+g(x)y=0$
Problem number: 227.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$\boxed{a\left(x^2-1\right)^2y''+bx\left(x^2-1\right)y'+\left(cx^2+dx+e\right)y=0}$$

✓ Solution by Maple
Time used: 0.141 (sec). Leaf size: 562

```
dsolve(a*(x^2-1)^2*diff(y(x),x$2)+b*x*(x^2-1)*diff(y(x),x)+(c*x^2+d*x+e)*y(x)=0,y(x), singso
```

$$y(x)=\frac{(x^2-1)^{-\frac{b}{4a}}\sqrt{2+2x}\left(c_1\,\text{hypergeom}\left(\left[-\frac{-\sqrt{4a^2+(-4b-4c-4d-4e)a+b^2}+2\sqrt{a^2+(-2b-4c)a+b^2}+\sqrt{4a^2+(-4b-4c+4d-4e)}}{4a}\right.\right.\right.$$

✗ Solution by Mathematica
Time used: 0.0 (sec). Leaf size: 0

```
DSolve[a*(x^2-1)^2*y''[x]+b*x*(x^2-1)*y'[x]+(c*x^2+d*x+e)*y[x]==0,y[x],x,IncludeSingularSolu
```

Timed out

32.19 problem 228

Internal problem ID [11053]

Book: Handbook of exact solutions for ordinary differential equations. By Polyanin and Zaitsev. Second edition
Section: Chapter 2, Second-Order Differential Equations. section 2.1.2-7 Equation of form $(a_4x^4+a_3x^3+a_2x^2x+a_1x+a_0)y''+f(x)y'+g(x)y=0$
Problem number: 228.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$\boxed{\left(a\,x^2+b\right)^2 y''+(2ax+c)\left(a\,x^2+b\right)y'+yk=0}$$

✓ Solution by Maple
Time used: 0.016 (sec). Leaf size: 162

```
dsolve((a*x^2+b)^2*diff(y(x),x$2)+(2*a*x+c)*(a*x^2+b)*diff(y(x),x)+k*y(x)=0,y(x), singsol=al
```

$$y(x)=c_1\left(\frac{-i\sqrt{ab}+ax}{i\sqrt{ab}+ax}\right)^{\frac{i\sqrt{ab}\,c\sqrt{-ab}+a^2\sqrt{\frac{c^2-4k}{a^2}}\,b}{4ab\sqrt{-ab}}}+c_2\left(\frac{-i\sqrt{ab}+ax}{i\sqrt{ab}+ax}\right)^{\frac{i\sqrt{ab}\,c\sqrt{-ab}-a^2\sqrt{\frac{c^2-4k}{a^2}}\,b}{4ab\sqrt{-ab}}}$$

✓ Solution by Mathematica
Time used: 2.188 (sec). Leaf size: 91

```
DSolve[(a*x^2+b)^2*y''[x]+(2*a*x+c)*(a*x^2+b)*y'[x]+k*y[x]==0,y[x],x,IncludeSingularSolution
```

$$y(x)\to e^{-\frac{\left(\sqrt{c^2-4k}+c\right)\arctan\left(\frac{\sqrt{a}x}{\sqrt{b}}\right)}{2\sqrt{a}\sqrt{b}}}\left(c_2e^{\frac{\sqrt{c^2-4k}\arctan\left(\frac{\sqrt{a}x}{\sqrt{b}}\right)}{\sqrt{a}\sqrt{b}}}+c_1\right)$$

32.20 problem 229

Internal problem ID [11054]

Book: Handbook of exact solutions for ordinary differential equations. By Polyanin and Zaitsev. Second edition
Section: Chapter 2, Second-Order Differential Equations. section 2.1.2-7 Equation of form $(a_4x^4+a_3x^3+a_2x^2x+a_1x+a_0)y''+f(x)y'+g(x)y=0$
Problem number: 229.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$\boxed{\left(a\,x^2+b\right)^2 y''+\left(a\,x^2+b\right)\left(c\,x^2+d\right) y'+2(-ad+bc)\,xy=0}$$

✓ Solution by Maple

Time used: 0.312 (sec). Leaf size: 866

```
dsolve((a*x^2+b)^2*diff(y(x),x$2)+(a*x^2+b)*(c*x^2+d)*diff(y(x),x)+2*(b*c-a*d)*x*y(x)=0,y(x)
```

$$y(x) = \left(-ax+\sqrt{-ab}\right)^{\frac{2a^2b+\sqrt{-ab\left(4\sqrt{-ab}\,a^2d-4\sqrt{-ab}\,abc-4a^3b+a^2d^2-2abcd+b^2c^2\right)}}{4a^2b}} \left(c_1 \left(ax \right.\right.$$

$$\left.\left. +\sqrt{-ab}\right)^{\frac{2a^2b+\sqrt{4a^2b(ad-bc)\sqrt{-ab}+4a^4b^2-a^3bd^2+2db^2ca^2-b^3c^2a}}{4a^2b}} \mathrm{e}^{\frac{\sqrt{-ab}\,c}{2a^2}-\frac{\arctan\left(\frac{\sqrt{a}\,x}{\sqrt{b}}\right)d}{2\sqrt{a}\,\sqrt{b}}+\frac{\sqrt{b}\,\arctan\left(\frac{\sqrt{a}\,x}{\sqrt{b}}\right)c}{2a^{\frac{3}{2}}}} \mathrm{HeunC}\left(\frac{2\sqrt{-\frac{b}{a}}\,c}{a}, \right.\right.$$

$$\left.\left. -\frac{d^2}{8ab}-\frac{cd}{4a^2}+\frac{3bc^2}{8a^3}, \frac{ax}{2\sqrt{-ab}}+\frac{1}{2} \right) \right.$$

$$\left. + c_2\left(ax+\sqrt{-ab}\right)^{-\frac{-2a^2b+\sqrt{4a^2b(ad-bc)\sqrt{-ab}+4a^4b^2-a^3bd^2+2db^2ca^2-b^3c^2a}}{4a^2b}} \mathrm{HeunC}\left(\frac{2\sqrt{-\frac{b}{a}}\,c}{a}, \right.\right.$$

$$\left.\left. -\frac{\sqrt{4a^2b\,(ad-bc)\sqrt{-ab}+4a^4b^2-a^3bd^2+2db^2ca^2-b^3c^2a}}{2a^2b}, \frac{\sqrt{-ab\left(4\sqrt{-ab}\,a^2d-4\sqrt{-ab}\,abc-4a^3b\right.}}{2a^2b} \right.\right.$$

$$\left.\left. -\frac{d^2}{8ab}-\frac{cd}{4a^2}+\frac{3bc^2}{8a^3}, \frac{ax}{2\sqrt{-ab}} \right) \right.$$

$$\left. +\frac{1}{2}\right) \mathrm{e}^{\frac{i\pi\sqrt{4a^2b(ad-bc)\sqrt{-ab}+4a^4b^2-a^3bd^2+2db^2ca^2-b^3c^2a}-i\pi\sqrt{-ab\left(4\sqrt{-ab}\,a^2d-4\sqrt{-ab}\,abc-4a^3b+a^2d^2-2abcd+b^2c^2\right)}-4\left(a^2\left(\frac{d}{\sqrt{b}\,\sqrt{a}}-\frac{\sqrt{b}\,c}{a^{\frac{3}{2}}}\right)a\right.}{8a^2b}}$$

✓ Solution by Mathematica

Time used: 0.165 (sec). Leaf size: 104

```
DSolve[(a*x^2+b)^2*y''[x]+(a*x^2+b)*(c*x^2+d)*y'[x]+2*(b*c-a*d)*x*y[x]==0,y[x],x,IncludeSing
```

$$y(x) \to \exp\left(\frac{\arctan\left(\frac{\sqrt{a}x}{\sqrt{b}}\right)(bc-ad)}{a^{3/2}\sqrt{b}} \right.$$

$$\left. -\frac{cx}{a} \right) \left(\int_1^x \exp\left(\frac{(ad-bc)\arctan\left(\frac{\sqrt{a}K[1]}{\sqrt{b}}\right)}{a^{3/2}\sqrt{b}} + \frac{cK[1]}{a} \right) c_1 dK[1] + c_2 \right)$$

32.21 problem 230

Internal problem ID [11055]

Book: Handbook of exact solutions for ordinary differential equations. By Polyanin and Zaitsev. Second edition
Section: Chapter 2, Second-Order Differential Equations. section 2.1.2-7 Equation of form $(a_4x^4+a_3x^3+a_2x^2x+a_1x+a_0)y''+f(x)y'+g(x)y=0$
Problem number: 230.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$\boxed{\left(x^2+a\right)^2 y''+b\,x^n\left(x^2+a\right)y'-\left(b\,x^{n+1}+a\right)y=0}$$

✗ Solution by Maple

```
dsolve((x^2+a)^2*diff(y(x),x$2)+b*x^n*(x^2+a)*diff(y(x),x)-(b*x^(n+1)+a)*y(x)=0,y(x), singso
```

No solution found

✗ Solution by Mathematica
Time used: 0.0 (sec). Leaf size: 0

```
DSolve[(x^2+a)^2*y''[x]+b*x^n*(x^2+a)*y'[x]-(b*x^(n+1)+a)*y[x]==0,y[x],x,IncludeSingularSolu
```

Not solved

32.22 problem 231

Internal problem ID [11056]

Book: Handbook of exact solutions for ordinary differential equations. By Polyanin and Zaitsev. Second edition
Section: Chapter 2, Second-Order Differential Equations. section 2.1.2-7 Equation of form $(a_4x^4+a_3x^3+a_2x^2x+a_1x+a_0)y''+f(x)y'+g(x)y=0$
Problem number: 231.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$\boxed{\left(x^2+a\right)^2 y''+b\,x^n\left(x^2+a\right)y'-m\left(b\,x^{n+1}+(m-1)\,x^2+a\right)y=0}$$

✗ Solution by Maple

```
dsolve((x^2+a)^2*diff(y(x),x$2)+b*x^n*(x^2+a)*diff(y(x),x)-m*(b*x^(n+1)+(m-1)*x^2+a)*y(x)=0,
```

No solution found

✗ Solution by Mathematica
Time used: 0.0 (sec). Leaf size: 0

```
DSolve[(x^2+a)^2*y''[x]+b*x^n*(x^2+a)*y'[x]-m*(b*x^(n+1)+(m-1)*x^2+a)*y[x]==0,y[x],x,Include
```

Not solved

32.23 problem 232

Internal problem ID [11057]

Book: Handbook of exact solutions for ordinary differential equations. By Polyanin and Zaitsev. Second edition
Section: Chapter 2, Second-Order Differential Equations. section 2.1.2-7 Equation of form $(a_4x^4+a_3x^3+a_2x^2x+a_1x+a_0)y''+f(x)y'+g(x)y=0$
Problem number: 232.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$\boxed{(x-a)^2(x-b)^2y''-yc=0}$$

✓ Solution by Maple
Time used: 0.015 (sec). Leaf size: 104

```
dsolve((x-a)^2*(x-b)^2*diff(y(x),x$2)-c*y(x)=0,y(x), singsol=all)
```

$$y(x)=\sqrt{(a-x)(b-x)}\left(\left(\frac{a-x}{b-x}\right)^{\frac{\sqrt{a^2-2ab+b^2+4c}}{2a-2b}}c_1+\left(\frac{a-x}{b-x}\right)^{-\frac{\sqrt{a^2-2ab+b^2+4c}}{2a-2b}}c_2\right)$$

✓ Solution by Mathematica
Time used: 1.085 (sec). Leaf size: 141

```
DSolve[(x-a)^2*(x-b)^2*y''[x]-c*y[x]==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x)\to(x-a)^{\frac{1}{2}\left(1-\sqrt{\frac{4c}{(a-b)^2}+1}\right)}(x-b)^{\frac{1}{2}\left(1-\sqrt{\frac{4c}{(a-b)^2}+1}\right)}\left(c_1(x-a)^{\sqrt{\frac{4c}{(a-b)^2}+1}}-\frac{c_2(x-b)^{\sqrt{\frac{4c}{(a-b)^2}+1}}}{(a-b)\sqrt{\frac{4c}{(a-b)^2}+1}}\right)$$

32.24 problem 233

Internal problem ID [11058]

Book: Handbook of exact solutions for ordinary differential equations. By Polyanin and Zaitsev. Second edition
Section: Chapter 2, Second-Order Differential Equations. section 2.1.2-7 Equation of form $(a_4x^4+a_3x^3+a_2x^2x+a_1x+a_0)y''+f(x)y'+g(x)y=0$
Problem number: 233.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$\boxed{(x-a)^2(x-b)^2y''+(x-a)(x-b)(2x+\lambda)y'+\mu y=0}$$

✓ Solution by Maple
Time used: 0.015 (sec). Leaf size: 143

```
dsolve((x-a)^2*(x-b)^2*diff(y(x),x$2)+(x-a)*(x-b)*(2*x+lambda)*diff(y(x),x)+mu*y(x)=0,y(x),
```

$$y(x)=\left(\left(\frac{a-x}{b-x}\right)^{-\frac{\sqrt{\lambda^2+(2a+2b)\lambda+a^2+2ab+b^2-4\mu}}{2a-2b}}c_2+\left(\frac{a-x}{b-x}\right)^{\frac{\sqrt{\lambda^2+(2a+2b)\lambda+a^2+2ab+b^2-4\mu}}{2a-2b}}c_1\right)\left(\frac{b-x}{a-x}\right)^{\frac{a+b+\lambda}{2a-2b}}$$

✓ Solution by Mathematica

Time used: 2.299 (sec). Leaf size: 152

```
DSolve[(x-a)^2*(x-b)^2*y''[x]+(x-a)*(x-b)*(2*x+\[Lambda])*y'[x]+mu*y[x]==0,y[x],x,IncludeSin
```

$$y(x)$$

$$\to e^{-\frac{(a+b+\lambda)(\log(x-a)-\log(x-b))}{a-b}}\left(c_1 \exp\left(\frac{\left(\sqrt{\mu}\sqrt{\frac{(a+b+\lambda)^2}{\mu}-4}+a+b+\lambda\right)(\log(x-a)-\log(x-b))}{2(a-b)}\right)\right.$$
$$\left.+\,c_2 \exp\left(\frac{\left(-\sqrt{\mu}\sqrt{\frac{(a+b+\lambda)^2}{\mu}-4}+a+b+\lambda\right)(\log(x-a)-\log(x-b))}{2(a-b)}\right)\right)$$

32.25 problem 234

Internal problem ID [11059]

Book: Handbook of exact solutions for ordinary differential equations. By Polyanin and Zaitsev. Second edition
Section: Chapter 2, Second-Order Differential Equations. section 2.1.2-7 Equation of form $(a_4x^4+a_3x^3+a_2x^2x+a_1x+a_0)y''+f(x)y'+g(x)y=0$
Problem number: 234.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_Emden, _Fowler]]`

$$\boxed{\left(a\,x^2+bx+c\right)^2 y''+Ay=0}$$

✓ Solution by Maple
Time used: 0.016 (sec). Leaf size: 178

```
dsolve((a*x^2+b*x+c)^2*diff(y(x),x$2)+A*y(x)=0,y(x), singsol=all)
```

$$y(x)=\left(\left(\frac{-b+i\sqrt{4ac-b^2}-2ax}{i\sqrt{4ac-b^2}+2ax+b}\right)^{-\frac{a\sqrt{\frac{-4ac+b^2-4A}{a^2}}}{2\sqrt{-4ac+b^2}}}c_2+\left(\frac{-b+i\sqrt{4ac-b^2}-2ax}{i\sqrt{4ac-b^2}+2ax+b}\right)^{\frac{a\sqrt{\frac{-4ac+b^2-4A}{a^2}}}{2\sqrt{-4ac+b^2}}}c_1\right)\sqrt{a\,x^2+bx+c}$$

✓ Solution by Mathematica

Time used: 2.154 (sec). Leaf size: 199

```
DSolve[(a*x^2+b*x+c)^2*y''[x]+A*y[x]==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x)$$

$$\to \sqrt{x(ax+b)+c}\exp\left(-\frac{\sqrt{4ac-b^2}\sqrt{1-\frac{4A}{b^2-4ac}}\arctan\left(\frac{2ax+b}{\sqrt{4ac-b^2}}\right)}{\sqrt{b^2-4ac}}\right)\left(c_1\exp\left(\frac{2\sqrt{4ac-b^2}\sqrt{1-\frac{4A}{b^2-4ac}}\,a}{\sqrt{b^2-4a}}\right.\right.$$

$$\left.\left.+\frac{c_2}{\sqrt{b^2-4ac}\sqrt{1-\frac{4A}{b^2-4ac}}}\right)\right.$$

32.26 problem 235

Internal problem ID [11060]

Book: Handbook of exact solutions for ordinary differential equations. By Polyanin and Zaitsev. Second edition
Section: Chapter 2, Second-Order Differential Equations. section 2.1.2-7 Equation of form $(a_4x^4+a_3x^3+a_2x^2x+a_1x+a_0)y''+f(x)y'+g(x)y=0$
Problem number: 235.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$\boxed{\left(x^2-1\right)^2 y''+2x\left(x^2-1\right) y'+\left(\left(x^2-1\right)\left(a^2x^2-\lambda\right)-m^2\right) y=0}$$

✓ Solution by Maple
Time used: 0.5 (sec). Leaf size: 64

```
dsolve((x^2-1)^2*diff(y(x),x$2)+2*x*(x^2-1)*diff(y(x),x)+( (x^2-1)*(a^2*x^2-lambda)-m^2)*y(x
```

$$y(x)=\left(\text{HeunC}\left(0,\frac{1}{2},m,\frac{a^2}{4},\frac{1}{4}+\frac{m^2}{4}-\frac{\lambda}{4},x^2\right)c_2x+\text{HeunC}\left(0,-\frac{1}{2},m,\frac{a^2}{4},\frac{1}{4}+\frac{m^2}{4}-\frac{\lambda}{4},x^2\right)c_1\right)\left(x^2-1\right)^{\frac{m}{2}}$$

✓ Solution by Mathematica

Time used: 0.602 (sec). Leaf size: 234

```
DSolve[(x^2-1)^2*y''[x]+2*x*(x^2-1)*y'[x]+( (x^2-1)*(a^2*x^2-\[Lambda])-m^2)*y[x]==0,y[x],x,
```

$$y(x) \to e^{i\sqrt{a^2}x}\left(\frac{x+1}{x-1}\right)^{\frac{\sqrt{m^2}}{2}}\left(c_2(x-1)^{\sqrt{m^2}}\text{HeunC}\left[-\left(\sqrt{m^2}+1\right)\left(\sqrt{m^2}+2i\sqrt{a^2}\right)-a^2\right.\right.$$
$$\left.+\lambda,-4i\sqrt{a^2}\left(\sqrt{m^2}+1\right),\sqrt{m^2}+1,\sqrt{m^2}+1,-4i\sqrt{a^2},\frac{1-x}{2}\right]$$
$$+c_1\text{HeunC}\left[2i\sqrt{a^2}\left(\sqrt{m^2}-1\right)-a^2+\lambda,-4i\sqrt{a^2},1-\sqrt{m^2},\sqrt{m^2}+1,\right.$$
$$\left.\left.-4i\sqrt{a^2},\frac{1-x}{2}\right]\right)$$

32.27 problem 236

Internal problem ID [11061]

Book: Handbook of exact solutions for ordinary differential equations. By Polyanin and Zaitsev. Second edition
Section: Chapter 2, Second-Order Differential Equations. section 2.1.2-7 Equation of form $(a_4x^4+a_3x^3+a_2x^2x+a_1x+a_0)y''+f(x)y'+g(x)y=0$
Problem number: 236.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$\boxed{\left(x^2+1\right)^2 y''+2x\left(x^2+1\right) y'+\left(\left(x^2+1\right)\left(a^2x^2-\lambda\right)+m^2\right) y=0}$$

✓ Solution by Maple
Time used: 0.5 (sec). Leaf size: 68

```
dsolve((x^2+1)^2*diff(y(x),x$2)+2*x*(x^2+1)*diff(y(x),x)+( (x^2+1)*(a^2*x^2-lambda)+m^2)*y(x
```

$$y(x)=\left(\text{HeunC}\left(0,\frac{1}{2},m,-\frac{a^2}{4},\frac{1}{4}+\frac{m^2}{4}-\frac{\lambda}{4},-x^2\right)c_2x+\text{HeunC}\left(0,-\frac{1}{2},m,-\frac{a^2}{4},\frac{1}{4}+\frac{m^2}{4}-\frac{\lambda}{4},-x^2\right)c_1\right)\left(x^2+1\right)^{\frac{m}{2}}$$

✓ Solution by Mathematica
Time used: 0.605 (sec). Leaf size: 124

```
DSolve[(x^2+1)^2*y''[x]+2*x*(x^2+1)*y'[x]+( (x^2+1)*(a^2*x^2-\[Lambda])+m^2)*y[x]==0,y[x],x,
```

$$y(x)\to\left(x^2+1\right)^{\frac{\sqrt{m^2}}{2}}\left(c_2x\text{HeunC}\left[\frac{1}{4}\left(\lambda-m^2-3\sqrt{m^2}-2\right),-\frac{a^2}{4},\frac{3}{2},\sqrt{m^2}+1,0,-x^2\right]+c_1\text{HeunC}\left[\frac{1}{4}\left(\lambda-m^2-\sqrt{m^2}\right),-\frac{a^2}{4},\frac{1}{2},\sqrt{m^2}+1,0,-x^2\right]\right)$$

32.28 problem 237

Internal problem ID [11062]

Book: Handbook of exact solutions for ordinary differential equations. By Polyanin and Zaitsev. Second edition
Section: Chapter 2, Second-Order Differential Equations. section 2.1.2-7 Equation of form $(a_4x^4+a_3x^3+a_2x^2x+a_1x+a_0)y''+f(x)y'+g(x)y=0$
Problem number: 237.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$\boxed{\left(a\,x^2+bx+c\right)^2 y''+\left(2ax+k\right)\left(a\,x^2+bx+c\right) y'+ym=0}$$

✓ Solution by Maple
Time used: 0.031 (sec). Leaf size: 274

```
dsolve((a*x^2+b*x+c)^2*diff(y(x),x$2)+(2*a*x+k)*(a*x^2+b*x+c)*diff(y(x),x)+m*y(x)=0,y(x), si
```

$$y(x)$$

$$=\left(\frac{-2ax-b+\sqrt{-4ac+b^2}}{2ax+\sqrt{-4ac+b^2}+b}\right)^{-\frac{k}{2\sqrt{-4ac+b^2}}}\left(\frac{2ax+\sqrt{-4ac+b^2}+b}{-2ax-b+\sqrt{-4ac+b^2}}\right)^{-\frac{b}{2\sqrt{-4ac+b^2}}}\left(c_1\left(\frac{-b+i\sqrt{4ac-b^2}-}{i\sqrt{4ac-b^2}+2ax}\right.\right.$$

$$\left.\left.+c_2\left(\frac{-b+i\sqrt{4ac-b^2}-2ax}{i\sqrt{4ac-b^2}+2ax+b}\right)^{-\frac{a\sqrt{\frac{b^2-2kb+k^2-4m}{a^2}}}{2\sqrt{-4ac+b^2}}}\right)\right.$$

✓ Solution by Mathematica

Time used: 2.382 (sec). Leaf size: 157

```
DSolve[(a*x^2+b*x+c)^2*y''[x]+(2*a*x+k)*(a*x^2+b*x+c)*y'[x]+m*y[x]==0,y[x],x,IncludeSingular
```

$$y(x) \to c_1 \exp\left(\frac{\left(-\sqrt{m}\sqrt{\frac{b^2-2bk+k^2-4m}{m}}+b-k\right)\arctan\left(\frac{2ax+b}{\sqrt{4ac-b^2}}\right)}{\sqrt{4ac-b^2}}\right)$$
$$+ c_2 \exp\left(\frac{\left(\sqrt{m}\sqrt{\frac{b^2-2bk+k^2-4m}{m}}+b-k\right)\arctan\left(\frac{2ax+b}{\sqrt{4ac-b^2}}\right)}{\sqrt{4ac-b^2}}\right)$$

33 Chapter 2, Second-Order Differential Equations. section 2.1.2-8. Other equations.

33.1 problem 238 892
33.2 problem 239 893
33.3 problem 241 894
33.4 problem 242 895
33.5 problem 243 896
33.6 problem 244 897
33.7 problem 245 898
33.8 problem 246 899
33.9 problem 247 900
33.10problem 248 901
33.11problem 249 902
33.12problem 250 903
33.13problem 251 904
33.14problem 252 905
33.15problem 253 906
33.16problem 254 907
33.17problem 255 908
33.18problem 256 909
33.19problem 257 910
33.20problem 258 911
33.21problem 259 912
33.22problem 260 914
33.23problem 261 915
33.24problem 262 916
33.25problem 263 917
33.26problem 264 918

33.1 problem 238

Internal problem ID [11063]

Book: Handbook of exact solutions for ordinary differential equations. By Polyanin and Zaitsev. Second edition
Section: Chapter 2, Second-Order Differential Equations. section 2.1.2-8. Other equations.
Problem number: 238.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_Emden, _Fowler]]`

$$\boxed{x^6 y'' - x^5 y' + a y = 0}$$

✓ Solution by Maple
Time used: 0.0 (sec). Leaf size: 35

```
dsolve(x^6*diff(y(x),x$2)-x^5*diff(y(x),x)+a*y(x)=0,y(x), singsol=all)
```

$$y(x) = x^2 \left(c_1 \sinh\left(\frac{\sqrt{-a}}{2x^2} \right) + c_2 \cosh\left(\frac{\sqrt{-a}}{2x^2} \right) \right)$$

✓ Solution by Mathematica
Time used: 0.239 (sec). Leaf size: 58

```
DSolve[x^6*y''[x]-x^5*y'[x]+a*y[x]==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to \frac{1}{2} x^2 e^{-\frac{i\sqrt{a}}{2x^2}} \left(2c_1 e^{\frac{i\sqrt{a}}{x^2}} - \frac{i c_2}{\sqrt{a}} \right)$$

33.2 problem 239

Internal problem ID [11064]

Book: Handbook of exact solutions for ordinary differential equations. By Polyanin and Zaitsev. Second edition
Section: Chapter 2, Second-Order Differential Equations. section 2.1.2-8. Other equations.
Problem number: 239.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$\boxed{x^6 y'' + \left(3x^2 + a\right) x^3 y' + yb = 0}$$

✓ Solution by Maple
Time used: 0.015 (sec). Leaf size: 45

```
dsolve(x^6*diff(y(x),x$2)+(3*x^2+a)*x^3*diff(y(x),x)+b*y(x)=0,y(x), singsol=all)
```

$$y(x) = c_1 \mathrm{e}^{-\frac{-a+\sqrt{a^2-4b}}{4x^2}} + c_2 \mathrm{e}^{\frac{a+\sqrt{a^2-4b}}{4x^2}}$$

✓ Solution by Mathematica
Time used: 0.066 (sec). Leaf size: 56

```
DSolve[x^6*y''[x]+(3*x^2+a)*x^3*y'[x]+b*y[x]==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to e^{\frac{a-\sqrt{a^2-4b}}{4x^2}} \left(c_1 e^{\frac{\sqrt{a^2-4b}}{2x^2}} + c_2 \right)$$

33.3 problem 241

Internal problem ID [11065]

Book: Handbook of exact solutions for ordinary differential equations. By Polyanin and Zaitsev. Second edition
Section: Chapter 2, Second-Order Differential Equations. section 2.1.2-8. Other equations.
Problem number: 241.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$\boxed{x^n y'' + c(ax+b)^{n-4} y = 0}$$

✗ Solution by Maple

```
dsolve(x^n*diff(y(x),x$2)+c*(a*x+b)^(n-4)*y(x)=0,y(x), singsol=all)
```

No solution found

✗ Solution by Mathematica
Time used: 0.0 (sec). Leaf size: 0

```
DSolve[x^n*y''[x]+c*(a*x+b)^(n-4)*y[x]==0,y[x],x,IncludeSingularSolutions -> True]
```

Not solved

33.4 problem 242

Internal problem ID [11066]

Book: Handbook of exact solutions for ordinary differential equations. By Polyanin and Zaitsev. Second edition
Section: Chapter 2, Second-Order Differential Equations. section 2.1.2-8. Other equations.
Problem number: 242.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$\boxed{x^n y'' + axy' - \left(b^2 x^n + 2b\, x^{n-1} + abx + a\right) y = 0}$$

✗ Solution by Maple

```
dsolve(x^n*diff(y(x),x$2)+a*x*diff(y(x),x)-(b^2*x^n+2*b*x^(n-1)+a*b*x+a)*y(x)=0,y(x), singso
```

No solution found

✗ Solution by Mathematica
Time used: 0.0 (sec). Leaf size: 0

```
DSolve[x^n*y''[x]+a*x*y'[x]-(b^2*x^n+2*b*x^(n-1)+a*b*x+a)*y[x]==0,y[x],x,IncludeSingularSolu
```

Not solved

33.5 problem 243

Internal problem ID [11067]

Book: Handbook of exact solutions for ordinary differential equations. By Polyanin and Zaitsev. Second edition
Section: Chapter 2, Second-Order Differential Equations. section 2.1.2-8. Other equations.
Problem number: 243.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$\boxed{x^n y'' + (ax+b)\,y' - ay = 0}$$

✓ Solution by Maple
Time used: 0.375 (sec). Leaf size: 56

```
dsolve(x^n*diff(y(x),x$2)+(a*x+b)*diff(y(x),x)-a*y(x)=0,y(x), singsol=all)
```

$$y(x) = -\left(c_1\left(\int \frac{\mathrm{e}^{\frac{x^{-n+1}(ax(n-1)+b(n-2))}{(n-2)(n-1)}}}{(ax+b)^2}dx\right) + c_2\right)(ax+b)$$

✗ Solution by Mathematica
Time used: 0.0 (sec). Leaf size: 0

```
DSolve[x^n*y''[x]+(a*x+b)*y'[x]-a*y[x]==0,y[x],x,IncludeSingularSolutions -> True]
```

Not solved

33.6 problem 244

Internal problem ID [11068]

Book: Handbook of exact solutions for ordinary differential equations. By Polyanin and Zaitsev. Second edition
Section: Chapter 2, Second-Order Differential Equations. section 2.1.2-8. Other equations.
Problem number: 244.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$\boxed{x^n y'' + \left(a\,x^{n-1} + bx\right) y' + (a-1)\,y = 0}$$

✓ Solution by Maple
Time used: 0.782 (sec). Leaf size: 137

```
dsolve(x^n*diff(y(x),x$2)+(a*x^(n-1)+b*x)*diff(y(x),x)+(a-1)*y(x)=0,y(x), singsol=all)
```

$$y(x) = x^{-\frac{a}{2}-\frac{1}{2}+\frac{n}{2}} \mathrm{e}^{\frac{b\,x^{2-n}}{-4+2n}} \left(\mathrm{WhittakerM}\left(\frac{(-b+2)\,a-2+b(n-1)}{2b\,(n-2)}, \frac{a-1}{-4+2n}, \frac{b\,x^{2-n}}{n-2} \right) c_1 + \mathrm{WhittakerW}\left(\frac{(-b+2)\,a-2+b(n-1)}{2b\,(n-2)}, \frac{a-1}{-4+2n}, \frac{b\,x^{2-n}}{n-2} \right) c_2 \right)$$

✗ Solution by Mathematica
Time used: 0.0 (sec). Leaf size: 0

```
DSolve[x^n*y''[x]+(a*x^(n-1)+b*x)*y'[x]+(a-1)*y[x]==0,y[x],x,IncludeSingularSolutions -> Tru
```

Not solved

33.7 problem 245

Internal problem ID [11069]

Book: Handbook of exact solutions for ordinary differential equations. By Polyanin and Zaitsev. Second edition
Section: Chapter 2, Second-Order Differential Equations. section 2.1.2-8. Other equations.
Problem number: 245.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$\boxed{x^n y'' + \left(2x^{n-1} + a\,x^2 + bx\right) y' + yb = 0}$$

✓ Solution by Maple
Time used: 0.454 (sec). Leaf size: 76

```
dsolve(x^n*diff(y(x),x$2)+(2*x^(n-1)+a*x^2+b*x)*diff(y(x),x)+b*y(x)=0,y(x), singsol=all)
```

$$y(x) = \frac{(ax+b)\left(c_2\left(\int \frac{e^{\frac{b(n-3)x^{2-n}+(n-2)\left(a\,x^{3-n}-2(n-3)\ln(x)\right)}{(n-3)(n-2)}}}{(ax+b)^2}x^2\,dx\right)+c_1\right)}{x}$$

✗ Solution by Mathematica
Time used: 0.0 (sec). Leaf size: 0

```
DSolve[x^n*y''[x]+(2*x^(n-1)+a*x^2+b*x)*y'[x]+b*y[x]==0,y[x],x,IncludeSingularSolutions -> T
```

Not solved

33.8 problem 246

Internal problem ID [11070]

Book: Handbook of exact solutions for ordinary differential equations. By Polyanin and Zaitsev. Second edition
Section: Chapter 2, Second-Order Differential Equations. section 2.1.2-8. Other equations.
Problem number: 246.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$x^n y'' + (a\,x^n + b)\,y' + c((a - c)\,x^n + b)\,y = 0$$

✗ Solution by Maple

```
dsolve(x^n*diff(y(x),x$2)+(a*x^n+b)*diff(y(x),x)+c*((a-c)*x^n+b)*y(x)=0,y(x), singsol=all)
```

No solution found

✗ Solution by Mathematica
Time used: 0.0 (sec). Leaf size: 0

```
DSolve[x^n*y''[x]+(a*x^n+b)*y'[x]+c*((a-c)*x^n+b)*y[x]==0,y[x],x,IncludeSingularSolutions ->
```

Not solved

33.9 problem 247

Internal problem ID [11071]

Book: Handbook of exact solutions for ordinary differential equations. By Polyanin and Zaitsev. Second edition
Section: Chapter 2, Second-Order Differential Equations. section 2.1.2-8. Other equations.
Problem number: 247.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$x^n y'' + \left(a\,x^n - x^{n-1} + abx + b\right) y' + a^2 bxy = 0$$

✗ Solution by Maple

```
dsolve(x^n*diff(y(x),x$2)+(a*x^n-x^(n-1)+a*b*x+b)*diff(y(x),x)+a^2*b*x*y(x)=0,y(x), singsol=
```

No solution found

✗ Solution by Mathematica
Time used: 0.0 (sec). Leaf size: 0

```
DSolve[x^n*y''[x]+(a*x^n-x^(n-1)+a*b*x+b)*y'[x]+a^2*b*x*y[x]==0,y[x],x,IncludeSingularSoluti
```

Not solved

33.10 problem 248

Internal problem ID [11072]

Book: Handbook of exact solutions for ordinary differential equations. By Polyanin and Zaitsev. Second edition
Section: Chapter 2, Second-Order Differential Equations. section 2.1.2-8. Other equations.
Problem number: 248.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$\boxed{x^n y'' + \left(a\, x^{m+n} + 1\right) y' + a\, x^m \left(1 + x^{n-1} m\right) y = 0}$$

✗ Solution by Maple

```
dsolve(x^n*diff(y(x),x$2)+(a*x^(n+m)+1)*diff(y(x),x)+a*x^m*(1+m*x^(n-1))*y(x)=0,y(x), singso
```

No solution found

✗ Solution by Mathematica
Time used: 0.0 (sec). Leaf size: 0

```
DSolve[x^n*y''[x]+(a*x^(n+m)+1)*y'[x]+a*x^m*(1+m*x^(n-1))*y[x]==0,y[x],x,IncludeSingularSolu
```

Not solved

33.11 problem 249

Internal problem ID [11073]

Book: Handbook of exact solutions for ordinary differential equations. By Polyanin and Zaitsev. Second edition
Section: Chapter 2, Second-Order Differential Equations. section 2.1.2-8. Other equations.
Problem number: 249.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$\boxed{(a\,x^n+b)\,y''+(x^n c+d)\,y'+\lambda((-a\lambda+c)\,x^n+d-b\lambda)\,y=0}$$

✗ Solution by Maple

```
dsolve((a*x^n+b)*diff(y(x),x$2)+(c*x^n+d)*diff(y(x),x)+lambda*((c-a*lambda)*x^n+d-b*lambda)*
```

No solution found

✗ Solution by Mathematica
Time used: 0.0 (sec). Leaf size: 0

```
DSolve[(a*x^n+b)*y''[x]+(c*x^n+d)*y'[x]+\[Lambda]*((c-a*\[Lambda])*x^n+d-b*\[Lambda])*y[x]==
```

Not solved

33.12 problem 250

Internal problem ID [11074]

Book: Handbook of exact solutions for ordinary differential equations. By Polyanin and Zaitsev. Second edition
Section: Chapter 2, Second-Order Differential Equations. section 2.1.2-8. Other equations.
Problem number: 250.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries], [_2nd_order, _linear,`

$$\boxed{(a\,x^n+bx+c)\,y''-an(n-1)\,x^{n-2}y=0}$$

✓ Solution by Maple
Time used: 0.0 (sec). Leaf size: 33

```
dsolve((a*x^n+b*x+c)*diff(y(x),x$2)=a*n*(n-1)*x^(n-2)*y(x),y(x), singsol=all)
```

$$y(x)=\left(\left(\int\frac{1}{(a\,x^n+bx+c)^2}dx\right)c_1+c_2\right)(a\,x^n+bx+c)$$

✗ Solution by Mathematica
Time used: 0.0 (sec). Leaf size: 0

```
DSolve[(a*x^n+b*x+c)*y''[x]==a*n*(n-1)*x^(n-2)*y[x],y[x],x,IncludeSingularSolutions -> True]
```

Not solved

33.13 problem 251

Internal problem ID [11075]

Book: Handbook of exact solutions for ordinary differential equations. By Polyanin and Zaitsev. Second edition
Section: Chapter 2, Second-Order Differential Equations. section 2.1.2-8. Other equations.
Problem number: 251.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$\boxed{x(x^n+1)\,y''+((a-b)\,x^n+a-n)\,y'+b(-a+1)\,x^{n-1}y=0}$$

✓ Solution by Maple
Time used: 0.172 (sec). Leaf size: 65

```
dsolve(x*(x^n+1)*diff(y(x),x$2)+((a-b)*x^n+a-n)*diff(y(x),x)+b*(1-a)*x^(n-1)*y(x)=0,y(x), si
```

$$y(x)=\left(x^{-a+n+1}c_2\,\text{hypergeom}\left(\left[\frac{b+n}{n},\frac{-a+n+1}{n}\right],\left[\frac{2n-a+1}{n}\right],-x^n\right)+c_1\right)(x^n+1)^{\frac{b}{n}}$$

✓ Solution by Mathematica
Time used: 0.164 (sec). Leaf size: 69

```
DSolve[x*(x^n+1)*y''[x]+((a-b)*x^n+a-n)*y'[x]+b*(1-a)*x^(n-1)*y[x]==0,y[x],x,IncludeSingular
```

$$y(x)\to c_2(x^n)^{\frac{-a+n+1}{n}}\,\text{Hypergeometric2F1}\left(1,\frac{-a-b+n+1}{n},\frac{-a+2n+1}{n},-x^n\right)+c_1(x^n+1)^{b/n}$$

33.14 problem 252

Internal problem ID [11076]

Book: Handbook of exact solutions for ordinary differential equations. By Polyanin and Zaitsev. Second edition
Section: Chapter 2, Second-Order Differential Equations. section 2.1.2-8. Other equations.
Problem number: 252.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries], [_2nd_order, _linear,`

$$\boxed{x\left(x^{2n}+a\right)y''+\left(x^{2n}+a-an\right)y'-b^2x^{-1+2n}y=0}$$

✓ Solution by Maple
Time used: 0.0 (sec). Leaf size: 61

```
dsolve(x*(x^(2*n)+a)*diff(y(x),x$2)+(x^(2*n)+a-a*n)*diff(y(x),x)-b^2*x^(2*n-1)*y(x)=0,y(x),
```

$$y(x)=c_1\mathrm{e}^{ib\left(\int x^{n-1}\sqrt{-\frac{1}{x^{2n}+a}}dx\right)}+c_2\mathrm{e}^{-ib\left(\int x^{n-1}\sqrt{-\frac{1}{x^{2n}+a}}dx\right)}$$

✓ Solution by Mathematica
Time used: 0.458 (sec). Leaf size: 47

```
DSolve[x*(x^(2*n)+a)*y''[x]+(x^(2*n)+a-a*n)*y'[x]-b^2*x^(2*n-1)*y[x]==0,y[x],x,IncludeSingul
```

$$y(x)\to c_1\cosh\left(\frac{b\operatorname{arcsinh}\left(\frac{x^n}{\sqrt{a}}\right)}{n}\right)+ic_2\sinh\left(\frac{b\operatorname{arcsinh}\left(\frac{x^n}{\sqrt{a}}\right)}{n}\right)$$

33.15 problem 253

Internal problem ID [11077]

Book: Handbook of exact solutions for ordinary differential equations. By Polyanin and Zaitsev. Second edition
Section: Chapter 2, Second-Order Differential Equations. section 2.1.2-8. Other equations.
Problem number: 253.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$\boxed{x^2\left(x^{2n}a^2-1\right)y''+x\left(a^2(n+1)\,x^{2n}+n-1\right)y'-\nu(\nu+1)\,a^2n^2x^{2n}y=0}$$

✓ Solution by Maple
Time used: 0.157 (sec). Leaf size: 23

```
dsolve(x^2*(a^2*x^(2*n)-1)*diff(y(x),x$2)+x*(a^2*(n+1)*x^(2*n)+n-1)*diff(y(x),x)-nu*(nu+1)*a
```

$$y(x)=c_1\,\mathrm{LegendreP}\left(\nu,a\,x^n\right)+c_2\,\mathrm{LegendreQ}\left(\nu,a\,x^n\right)$$

✓ Solution by Mathematica
Time used: 0.21 (sec). Leaf size: 79

```
DSolve[x^2*(a^2*x^(2*n)-1)*y''[x]+x*(a^2*(n+1)*x^(2*n)+n-1)*y'[x]-\[Nu]*(\[Nu]+1)*a^2*n^2*x^
```

$$\begin{aligned}y(x)\to{}&iac_2\sqrt{x^{2n}}\,\mathrm{Hypergeometric2F1}\left(\frac{1}{2}-\frac{\nu}{2},\frac{\nu}{2}+1,\frac{3}{2},a^2x^{2n}\right)\\&+c_1\,\mathrm{Hypergeometric2F1}\left(-\frac{\nu}{2},\frac{\nu+1}{2},\frac{1}{2},a^2x^{2n}\right)\end{aligned}$$

33.16 problem 254

Internal problem ID [11078]

Book: Handbook of exact solutions for ordinary differential equations. By Polyanin and Zaitsev. Second edition
Section: Chapter 2, Second-Order Differential Equations. section 2.1.2-8. Other equations.
Problem number: 254.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$\boxed{x^2\left(x^{2n}a^2-1\right)y''+x(apx^n+q)y'+(arx^n+s)y=0}$$

✓ Solution by Maple
Time used: 1.734 (sec). Leaf size: 273

```
dsolve(x^2*(a^2*x^(2*n)-1)*diff(y(x),x$2)+x*(a*p*x^n+q)*diff(y(x),x)+(a*r*x^n+s)*y(x)=0,y(x)
```

$$\begin{aligned}y(x)&=x^{\frac{q}{2}+\frac{1}{2}}\left(c_1x^{\frac{\sqrt{q^2+2q+4s+1}}{2}}\operatorname{HeunG}\left(-1,\frac{pq+\sqrt{q^2+2q+4s+1}\,p+p+2r}{2n^2},\frac{\sqrt{q^2+2q+4s+1}+q-1}{2n},\frac{\sqrt{q^2}\right.\right.\\&\qquad\left.-\frac{p-q}{2n},-a\,x^n\right)\\&\quad+c_2x^{-\frac{\sqrt{q^2+2q+4s+1}}{2}}\operatorname{HeunG}\left(-1,\frac{-\sqrt{q^2+2q+4s+1}\,p+(q+1)\,p+2r}{2n^2},\right.\\&\quad-\frac{\sqrt{q^2+2q+4s+1}-q-1}{2n},-\frac{\sqrt{q^2+2q+4s+1}-q+1}{2n},\frac{n-\sqrt{q^2+2q+4s+1}}{n},\\&\qquad\left.\left.-\frac{p-q}{2n},-a\,x^n\right)\right)\end{aligned}$$

✗ Solution by Mathematica
Time used: 0.0 (sec). Leaf size: 0

```
DSolve[x^2*(a^2*x^(2*n)-1)*y''[x]+x*(a*p*x^n+q)*y'[x]+(a*r*x^n+s)*y[x]==0,y[x],x,IncludeSing
```

Not solved

33.17 problem 255

Internal problem ID [11079]

Book: Handbook of exact solutions for ordinary differential equations. By Polyanin and Zaitsev. Second edition
Section: Chapter 2, Second-Order Differential Equations. section 2.1.2-8. Other equations.
Problem number: 255.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$\boxed{(x^n+a)^2 y'' - b\,x^{n-2}((b-1)\,x^n+(n-1)\,a)\,y = 0}$$

✓ Solution by Maple
Time used: 0.391 (sec). Leaf size: 75

```
dsolve((x^n+a)^2*diff(y(x),x$2)-b*x^(n-2)*( (b-1)*x^n+a*(n-1))*y(x)=0,y(x), singsol=all)
```

$$y(x) = \left(c_2\left(ax+x^{n+1}\right)\text{hypergeom}\left(\left[1,\frac{n-2b+1}{n}\right],\left[1+\frac{1}{n}\right],-\frac{x^n}{a}\right) + \left(\frac{x^n+a}{a}\right)^{\frac{2b}{n}} a c_1\right)\left(x^n+a\right)^{-\frac{b}{n}}$$

✗ Solution by Mathematica
Time used: 0.0 (sec). Leaf size: 0

```
DSolve[(x^n+a)^2*y''[x]-b*x^(n-2)*( (b-1)*x^n+a*(n-1))*y[x]==0,y[x],x,IncludeSingularSolutio
```

Not solved

33.18 problem 256

Internal problem ID [11080]

Book: Handbook of exact solutions for ordinary differential equations. By Polyanin and Zaitsev. Second edition
Section: Chapter 2, Second-Order Differential Equations. section 2.1.2-8. Other equations.
Problem number: 256.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$\boxed{(a\,x^n+b)^2\,y''+(a\,x^n+b)\,(x^n c+d)\,y'+n(-ad+bc)\,x^{n-1}y=0}$$

✓ Solution by Maple
Time used: 0.406 (sec). Leaf size: 53

```
dsolve((a*x^n+b)^2*diff(y(x),x$2)+(a*x^n+b)*(c*x^n+d)*diff(y(x),x)+n*(b*c-a*d)*x^(n-1)*y(x)=
```

$$y(x)=\mathrm{e}^{-\left(\int \frac{c\,x^n+d}{a\,x^n+b}dx\right)}\left(c_1+\left(\int \mathrm{e}^{\int \frac{c\,x^n+d}{a\,x^n+b}dx}dx\right)c_2\right)$$

✓ Solution by Mathematica
Time used: 0.923 (sec). Leaf size: 106

```
DSolve[(a*x^n+b)^2*y''[x]+(a*x^n+b)*(c*x^n+d)*y'[x]+n*(b*c-a*d)*x^(n-1)*y[x]==0,y[x],x,Inclu
```

$y(x)$

$$\to \exp\left(-\frac{x\left((ad-bc)\,\text{Hypergeometric2F1}\left(1,\tfrac{1}{n},1+\tfrac{1}{n},-\tfrac{ax^n}{b}\right)+bc\right)}{ab}\right)\left(\int_1^x \exp\left(\frac{\left(bc+(ad-bc)\,\text{Hyper}\right.}{}\right.\right.$$

$$\left.\left.+c_2\right)\right)$$

33.19 problem 257

Internal problem ID [11081]

Book: Handbook of exact solutions for ordinary differential equations. By Polyanin and Zaitsev. Second edition
Section: Chapter 2, Second-Order Differential Equations. section 2.1.2-8. Other equations.
Problem number: 257.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$\boxed{(x^n+a)^2 y'' + b\,x^m(x^n+a)\,y' - x^{n-2}\left(b\,x^{m+1} + an - a\right) y = 0}$$

✗ Solution by Maple

```
dsolve((x^n+a)^2*diff(y(x),x$2)+b*x^m*(x^n+a)*diff(y(x),x)-x^(n-2)*(b*x^(m+1)+a*n-a)*y(x)=0,
```

No solution found

✗ Solution by Mathematica
Time used: 0.0 (sec). Leaf size: 0

```
DSolve[(x^n+a)^2*y''[x]+b*x^m*(x^n+a)*y'[x]-x^(n-2)*(b*x^(m+1)+a*n-a)*y[x]==0,y[x],x,Include
```

Not solved

33.20 problem 258

Internal problem ID [11082]

Book: Handbook of exact solutions for ordinary differential equations. By Polyanin and Zaitsev. Second edition
Section: Chapter 2, Second-Order Differential Equations. section 2.1.2-8. Other equations.
Problem number: 258.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$\boxed{(a\,x^n+b)^2\,y''+c\,x^m(a\,x^n+b)\,y'+\left(c\,x^m-an\,x^{n-1}-1\right)y=0}$$

✗ Solution by Maple

```
dsolve((a*x^n+b)^2*diff(y(x),x$2)+c*x^m*(a*x^n+b)*diff(y(x),x)+(c*x^m-a*n*x^(n-1)-1)*y(x)=0,
```

No solution found

✗ Solution by Mathematica
Time used: 0.0 (sec). Leaf size: 0

```
DSolve[(a*x^n+b)^2*y''[x]+c*x^m*(a*x^n+b)*y'[x]+(c*x^m-a*n*x^(n-1)-1)*y[x]==0,y[x],x,Include
```

Not solved

33.21 problem 259

Internal problem ID [11083]

Book: Handbook of exact solutions for ordinary differential equations. By Polyanin and Zaitsev. Second edition
Section: Chapter 2, Second-Order Differential Equations. section 2.1.2-8. Other equations.
Problem number: 259.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$\boxed{x^2(a\,x^n+b)^2\,y''+(n+1)\,x\left(x^{2n}a^2-b^2\right)y'+yc=0}$$

✓ Solution by Maple
Time used: 0.172 (sec). Leaf size: 127

```
dsolve(x^2*(a*x^n+b)^2*diff(y(x),x$2)+(n+1)*x*(a^2*x^(2*n)-b^2)*diff(y(x),x)+c*y(x)=0,y(x),
```

$$y(x)=\sqrt{a\,x^{2n}+b\,x^n}\,(a\,x^n+b)^{\frac{-n-1}{n}}\,x\left(\left(\frac{x^n}{a\,x^n+b}\right)^{-\frac{\sqrt{\frac{(n+2)^2b^2-4c}{n^2a^2}}\,a}{2b}}c_2+\left(\frac{x^n}{a\,x^n+b}\right)^{\frac{\sqrt{\frac{(n+2)^2b^2-4c}{n^2a^2}}\,a}{2b}}c_1\right)$$

✓ Solution by Mathematica

Time used: 0.407 (sec). Leaf size: 149

```
DSolve[x^2*(a*x^n+b)^2*y''[x]+(n+1)*x*(a^2*x^(2*n)-b^2)*y'[x]+c*y[x]==0,y[x],x,IncludeSingul
```

$$y(x)$$

$$\to c_1 \exp\left(\frac{\left(b(n+2)-\sqrt{c}\sqrt{\frac{b^2(n+2)^2-4c}{c}}\right)\left(-\log\left(ax^n+b\right)-\log(b)+n\log(x)-\log(n)\right)}{2bn}\right)$$

$$+c_2 \exp\left(\frac{\left(\sqrt{c}\sqrt{\frac{b^2(n+2)^2-4c}{c}}+b(n+2)\right)\left(-\log\left(ax^n+b\right)-\log(b)+n\log(x)-\log(n)\right)}{2bn}\right)$$

33.22 problem 260

Internal problem ID [11084]

Book: Handbook of exact solutions for ordinary differential equations. By Polyanin and Zaitsev. Second edition
Section: Chapter 2, Second-Order Differential Equations. section 2.1.2-8. Other equations.
Problem number: 260.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$\left(a\,x^{n+1}+b\,x^{n}+c\right)^{2}y''+\left(\alpha\,x^{n}+\beta\,x^{n-1}+\gamma\right)y'+\left(n(-an-a+\alpha)\,x^{n-1}+(n-1)\left(-nb+\beta\right)x^{n-2}\right)y=$$

✗ Solution by Maple

```
dsolve((a*x^(n+1)+b*x^n+c)^2*diff(y(x),x$2)+(alpha*x^n+beta*x^(n-1)+gamma)*diff(y(x),x)+(n*(
```

No solution found

✗ Solution by Mathematica
Time used: 0.0 (sec). Leaf size: 0

```
DSolve[(a*x^(n+1)+b*x^n+c)^2*y''[x]+(\[Alpha]*x^n+\[Beta]*x^(n-1)+\[Gamma])*y'[x]+(n*(\[Alph
```

Not solved

33.23 problem 261

Internal problem ID [11085]

Book: Handbook of exact solutions for ordinary differential equations. By Polyanin and Zaitsev. Second edition
Section: Chapter 2, Second-Order Differential Equations. section 2.1.2-8. Other equations.
Problem number: 261.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$\boxed{(a\,x^n + b\,x^m + c)\,y'' + (\lambda - x)\,y' + y = 0}$$

✓ Solution by Maple
Time used: 0.781 (sec). Leaf size: 68

```
dsolve((a*x^n+b*x^m+c)*diff(y(x),x$2)+(lambda-x)*diff(y(x),x)+y(x)=0,y(x), singsol=all)
```

$$y(x) = -\left(\left(\int \mathrm{e}^{\int \frac{-2a\,x^n - 2b\,x^m - 2c + x^2 - 2x\lambda + \lambda^2}{(a\,x^n + b\,x^m + c)(-\lambda + x)}\,dx}\,dx\right) c_1 + c_2\right)(\lambda - x)$$

✗ Solution by Mathematica
Time used: 0.0 (sec). Leaf size: 0

```
DSolve[(a*x^n+b*x^m+c)*y''[x]+(\[Lambda]-x)*y'[x]+y[x]==0,y[x],x,IncludeSingularSolutions ->
```

Not solved

33.24 problem 262

Internal problem ID [11086]

Book: Handbook of exact solutions for ordinary differential equations. By Polyanin and Zaitsev. Second edition
Section: Chapter 2, Second-Order Differential Equations. section 2.1.2-8. Other equations.
Problem number: 262.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$\boxed{(a\,x^n + b\,x^m + c)\,y'' + \left(\lambda^2 - x^2\right)y' + (\lambda + x)\,y = 0}$$

✓ Solution by Maple
Time used: 1.0 (sec). Leaf size: 76

```
dsolve((a*x^n+b*x^m+c)*diff(y(x),x$2)+(lambda^2-x^2)*diff(y(x),x)+(x+lambda)*y(x)=0,y(x), si
```

$$y(x) = -\left(\left(\int \mathrm{e}^{\int \frac{\lambda^3 - x\,\lambda^2 - x^2\lambda + x^3 - 2a\,x^n - 2b\,x^m - 2c}{(a\,x^n + b\,x^m + c)(-\lambda + x)}dx}dx\right)c_1 + c_2\right)(\lambda - x)$$

✗ Solution by Mathematica
Time used: 0.0 (sec). Leaf size: 0

```
DSolve[(a*x^n+b*x^m+c)*y''[x]+(\[Lambda]^2-x^2)*y'[x]+(x+\[Lambda])*y[x]==0,y[x],x,IncludeSi
```

Not solved

33.25 problem 263

Internal problem ID [11087]

Book: Handbook of exact solutions for ordinary differential equations. By Polyanin and Zaitsev. Second edition
Section: Chapter 2, Second-Order Differential Equations. section 2.1.2-8. Other equations.
Problem number: 263.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$\boxed{2(a\,x^n+b\,x^m+c)\,y''+an\,x^{n-1}bm\,x^{m-1}y'+yd=0}$$

✗ Solution by Maple

```
dsolve(2*(a*x^n+b*x^m+c)*diff(y(x),x$2)+(a*n*x^(n-1)*b*m*x^(m-1))*diff(y(x),x)+d*y(x)=0,y(x)
```

No solution found

✗ Solution by Mathematica
Time used: 0.0 (sec). Leaf size: 0

```
DSolve[2*(a*x^n+b*x^m+c)*y''[x]+(a*n*x^(n-1)*b*m*x^(m-1))*y'[x]+d*y[x]==0,y[x],x,IncludeSing
```

Not solved

33.26 problem 264

Internal problem ID [11088]

Book: Handbook of exact solutions for ordinary differential equations. By Polyanin and Zaitsev. Second edition
Section: Chapter 2, Second-Order Differential Equations. section 2.1.2-8. Other equations.
Problem number: 264.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries], [_2nd_order, _linear,`

$$\boxed{(a\,x^n+b)^{m+1}\,y''+(a\,x^n+b)\,y'-anm\,x^{n-1}y=0}$$

✓ Solution by Maple
Time used: 0.031 (sec). Leaf size: 41

```
dsolve((a*x^n+b)^(m+1)*diff(y(x),x$2)+(a*x^n+b)*diff(y(x),x)-a*n*m*x^(n-1)*y(x)=0,y(x), sing
```

$$y(x)=\mathrm{e}^{-\left(\int (a\,x^n+b)^{-m}dx\right)}\left(c_1+\left(\int \mathrm{e}^{\int (a\,x^n+b)^{-m}dx}dx\right)c_2\right)$$

✓ Solution by Mathematica
Time used: 0.249 (sec). Leaf size: 116

```
DSolve[(a*x^n+b)^(m+1)*y''[x]+(a*x^n+b)*y'[x]-a*n*m*x^(n-1)*y[x]==0,y[x],x,IncludeSingularSo
```

$$y(x)\to \exp\left(-x(ax^n+b)^{-m}\left(\frac{ax^n}{b}+1\right)^m \text{Hypergeometric2F1}\left(m,\frac{1}{n},1+\frac{1}{n},-\frac{ax^n}{b}\right)\right)\left(\int_1^x \exp\left(\text{Hypergeometric2F1}\left(m,\frac{1}{n},1+\frac{1}{n},-\frac{aK[1]^n}{b}\right)K[1]\,(aK[1]^n+b)^{-m}\left(\frac{aK[1]^n}{b}+1\right)^m + c_2\right)$$

34 Chapter 2, Second-Order Differential Equations. section 2.1.3-1. Equations with exponential functions

34.1 problem 1 920
34.2 problem 2 921
34.3 problem 3 922
34.4 problem 4 923
34.5 problem 5 924
34.6 problem 6 925
34.7 problem 7 926
34.8 problem 8 927
34.9 problem 9 928
34.10problem 10 929
34.11problem 11 930
34.12problem 12 931
34.13problem 13 932
34.14problem 14 933
34.15problem 15 934
34.16problem 16 935
34.17problem 17 936
34.18problem 18 937
34.19problem 19 938
34.20problem 20 939
34.21problem 21 940
34.22problem 22 941
34.23problem 23 943
34.24problem 24 944
34.25problem 25 945
34.26problem 26 946
34.27problem 27 947
34.28problem 28 948
34.29problem 29 949
34.30problem 30 950
34.31problem 31 951
34.32problem 32 952
34.33problem 33 953

34.1 problem 1

Internal problem ID [11089]

Book: Handbook of exact solutions for ordinary differential equations. By Polyanin and Zaitsev. Second edition
Section: Chapter 2, Second-Order Differential Equations. section 2.1.3-1. Equations with exponential functions
Problem number: 1.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$\boxed{y'' + \mathrm{e}^{\lambda x} y a = 0}$$

✓ Solution by Maple
Time used: 0.172 (sec). Leaf size: 39

```
dsolve(diff(y(x),x$2)+a*exp(lambda*x)*y(x)=0,y(x), singsol=all)
```

$$y(x) = c_1 \operatorname{BesselJ}\left(0, \frac{2\sqrt{a}\,\mathrm{e}^{\frac{x\lambda}{2}}}{\lambda}\right) + c_2 \operatorname{BesselY}\left(0, \frac{2\sqrt{a}\,\mathrm{e}^{\frac{x\lambda}{2}}}{\lambda}\right)$$

✓ Solution by Mathematica
Time used: 0.062 (sec). Leaf size: 55

```
DSolve[y''[x]+a*Exp[\[Lambda]*x]*y[x]==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to c_1 \operatorname{BesselJ}\left(0, \frac{2\sqrt{a}\sqrt{e^{x\lambda}}}{\lambda}\right) + 2c_2 \operatorname{BesselY}\left(0, \frac{2\sqrt{a}\sqrt{e^{x\lambda}}}{\lambda}\right)$$

34.2 problem 2

Internal problem ID [11090]

Book: Handbook of exact solutions for ordinary differential equations. By Polyanin and Zaitsev. Second edition
Section: Chapter 2, Second-Order Differential Equations. section 2.1.3-1. Equations with exponential functions
Problem number: 2.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$\boxed{y'' + (a\,\mathrm{e}^{x} - b)\,y = 0}$$

✓ Solution by Maple
Time used: 0.109 (sec). Leaf size: 39

```
dsolve(diff(y(x),x$2)+(a*exp(x)-b)*y(x)=0,y(x), singsol=all)
```

$$y(x) = c_1\,\mathrm{BesselJ}\left(2\sqrt{b}, 2\sqrt{a}\,\mathrm{e}^{\frac{x}{2}}\right) + c_2\,\mathrm{BesselY}\left(2\sqrt{b}, 2\sqrt{a}\,\mathrm{e}^{\frac{x}{2}}\right)$$

✓ Solution by Mathematica
Time used: 0.081 (sec). Leaf size: 76

```
DSolve[y''[x]+(a*Exp[x]-b)*y[x]==0,y[x],x,IncludeSingularSolutions -> True]
```

$$\begin{aligned} y(x) \to\ & c_1\,\mathrm{Gamma}\left(1 - 2\sqrt{b}\right)\mathrm{BesselJ}\left(-2\sqrt{b}, 2\sqrt{a}\sqrt{e^{x}}\right) \\ & + c_2\,\mathrm{Gamma}\left(2\sqrt{b} + 1\right)\mathrm{BesselJ}\left(2\sqrt{b}, 2\sqrt{a}\sqrt{e^{x}}\right) \end{aligned}$$

34.3 problem 3

Internal problem ID [11091]

Book: Handbook of exact solutions for ordinary differential equations. By Polyanin and Zaitsev. Second edition
Section: Chapter 2, Second-Order Differential Equations. section 2.1.3-1. Equations with exponential functions
Problem number: 3.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$\boxed{y'' + a\left(\lambda\, \mathrm{e}^{\lambda x} - a\, \mathrm{e}^{2\lambda x}\right) y = 0}$$

✓ Solution by Maple
Time used: 0.125 (sec). Leaf size: 32

```
dsolve(diff(y(x),x$2)+a*(lambda*exp(lambda*x)-a*exp(2*lambda*x))*y(x)=0,y(x), singsol=all)
```

$$y(x) = \mathrm{e}^{-\frac{a\, \mathrm{e}^{x\lambda}}{\lambda}} \left(c_1 + \mathrm{expIntegral}_1\left(-\frac{2a\, \mathrm{e}^{x\lambda}}{\lambda} \right) c_2 \right)$$

✓ Solution by Mathematica
Time used: 1.32 (sec). Leaf size: 37

```
DSolve[y''[x]+a*(\[Lambda]*Exp[\[Lambda]*x]-a*Exp[2*\[Lambda]*x])*y[x]==0,y[x],x,IncludeSing
```

$$y(x) \to e^{-\frac{ae^{\lambda x}}{\lambda}} \left(c_2\, \mathrm{ExpIntegralEi}\left(\frac{2ae^{x\lambda}}{\lambda} \right) + c_1 \right)$$

34.4 problem 4

Internal problem ID [11092]

Book: Handbook of exact solutions for ordinary differential equations. By Polyanin and Zaitsev. Second edition
Section: Chapter 2, Second-Order Differential Equations. section 2.1.3-1. Equations with exponential functions
Problem number: 4.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$\boxed{y'' - \left(a^2 \mathrm{e}^{2x} + a(1+2b)\,\mathrm{e}^{x} + b^2\right) y = 0}$$

✓ Solution by Maple
Time used: 0.297 (sec). Leaf size: 76

```
dsolve(diff(y(x),x$2)-(a^2*exp(2*x)+a*(2*b+1)*exp(x)+b^2)*y(x)=0,y(x), singsol=all)
```

$$\begin{aligned} y(x) = &-c_2 a^{-2b}\,\mathrm{WhittakerM}\left(-b, \frac{1}{2} - b, 2a\,\mathrm{e}^{x}\right) \\ &+ c_1 \mathrm{e}^{bx + a\,\mathrm{e}^{x}} + (a\,\mathrm{e}^{x})^{-b}\,\mathrm{e}^{-a\,\mathrm{e}^{x}} c_2 a^{-2b}\left(b 2^{-b+1} - 2^{-b}\right) \end{aligned}$$

✓ Solution by Mathematica
Time used: 1.8 (sec). Leaf size: 57

```
DSolve[y''[x]-(a^2*Exp[2*x]+a*(2*b+1)*Exp[x]+b^2)*y[x]==0,y[x],x,IncludeSingularSolutions ->
```

$$y(x) \to e^{ae^x} (e^x)^{-b} \left(c_1 (e^x)^{2b} - 4^b c_2 (ae^x)^{2b}\, \Gamma(-2b, 2ae^x)\right)$$

34.5 problem 5

Internal problem ID [11093]

Book: Handbook of exact solutions for ordinary differential equations. By Polyanin and Zaitsev. Second edition
Section: Chapter 2, Second-Order Differential Equations. section 2.1.3-1. Equations with exponential functions
Problem number: 5.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$\boxed{y'' - \left(a\,\mathrm{e}^{2\lambda x} + b\,\mathrm{e}^{\lambda x} + c\right) y = 0}$$

✓ Solution by Maple
Time used: 0.343 (sec). Leaf size: 73

```
dsolve(diff(y(x),x$2)-(a*exp(2*lambda*x)+b*exp(lambda*x)+c)*y(x)=0,y(x), singsol=all)
```

$$y(x) = \mathrm{e}^{-\frac{x\lambda}{2}}\left(\mathrm{WhittakerM}\left(-\frac{b}{2\lambda\sqrt{a}}, \frac{\sqrt{c}}{\lambda}, \frac{2\sqrt{a}\,\mathrm{e}^{x\lambda}}{\lambda}\right) c_1 + \mathrm{WhittakerW}\left(-\frac{b}{2\lambda\sqrt{a}}, \frac{\sqrt{c}}{\lambda}, \frac{2\sqrt{a}\,\mathrm{e}^{x\lambda}}{\lambda}\right) c_2\right)$$

✓ Solution by Mathematica
Time used: 1.158 (sec). Leaf size: 145

```
DSolve[y''[x]-(a*Exp[2*\[Lambda]*x]+b*Exp[\[Lambda]*x]+c)*y[x]==0,y[x],x,IncludeSingularSolu
```

$$y(x) \to e^{-\frac{\sqrt{a}e^{\lambda x}}{\lambda}}\left(e^{\lambda x}\right)^{\frac{\sqrt{c}}{\lambda}}\left(c_1\,\mathrm{HypergeometricU}\left(\frac{\frac{b}{\sqrt{a}}+\lambda+2\sqrt{c}}{2\lambda}, \frac{2\sqrt{c}}{\lambda}+1, \frac{2\sqrt{a}e^{x\lambda}}{\lambda}\right) + c_2 L^{\frac{2\sqrt{c}}{\lambda}}_{-\frac{\frac{b}{\sqrt{a}}+\lambda+2\sqrt{c}}{2\lambda}}\left(\frac{2\sqrt{a}e^{x\lambda}}{\lambda}\right)\right)$$

34.6 problem 6

Internal problem ID [11094]

Book: Handbook of exact solutions for ordinary differential equations. By Polyanin and Zaitsev. Second edition
Section: Chapter 2, Second-Order Differential Equations. section 2.1.3-1. Equations with exponential functions
Problem number: 6.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$\boxed{y'' + \left(a\,\mathrm{e}^{4\lambda x} + b\,\mathrm{e}^{3\lambda x} + \mathrm{e}^{2\lambda x} c - \frac{\lambda^2}{4}\right) y = 0}$$

✓ Solution by Maple
Time used: 0.36 (sec). Leaf size: 221

```
dsolve(diff(y(x),x$2)+(a*exp(4*lambda*x)+b*exp(3*lambda*x)+c*exp(2*lambda*x)-1/4*lambda^2)*y
```

$$y(x) = c_1 \,\mathrm{hypergeom}\left(\left[\frac{4\lambda\, a^{\frac{3}{2}} + 4iac - ib^2}{16\lambda\, a^{\frac{3}{2}}}\right], \left[\frac{1}{2}\right], \frac{i\left(2\,\mathrm{e}^{x\lambda} a + b\right)^2}{4\lambda\, a^{\frac{3}{2}}}\right) \mathrm{e}^{-\frac{i\mathrm{e}^{2x\lambda} a + \lambda^2 x\sqrt{a} + ib\,\mathrm{e}^{x\lambda}}{2\lambda\sqrt{a}}}$$
$$+ c_2 \,\mathrm{hypergeom}\left(\left[\frac{12\lambda\, a^{\frac{3}{2}} + 4iac - ib^2}{16\lambda\, a^{\frac{3}{2}}}\right], \left[\frac{3}{2}\right], \frac{i\left(2\,\mathrm{e}^{x\lambda} a + b\right)^2}{4\lambda\, a^{\frac{3}{2}}}\right) \left(2a\,\mathrm{e}^{-\frac{i\mathrm{e}^{2x\lambda} a - \lambda^2 x\sqrt{a} + ib\,\mathrm{e}^{x\lambda}}{2\lambda\sqrt{a}}}\right.$$
$$\left. + b\,\mathrm{e}^{-\frac{i\mathrm{e}^{2x\lambda} a + \lambda^2 x\sqrt{a} + ib\,\mathrm{e}^{x\lambda}}{2\lambda\sqrt{a}}}\right)$$

✓ Solution by Mathematica
Time used: 1.895 (sec). Leaf size: 178

```
DSolve[y''[x]+(a*Exp[4*\[Lambda]*x]+b*Exp[3*\[Lambda]*x]+c*Exp[2*\[Lambda]*x]-1/4*\[Lambda]^
```

$$y(x)$$
$$\to \frac{e^{-\frac{ie^{\lambda x}\left(ae^{\lambda x}+b\right)}{2\sqrt{a}\lambda}}\left(c_1 \,\mathrm{HermiteH}\left(\frac{i(b^2-4ac+4ia^{3/2}\lambda)}{8a^{3/2}\lambda}, \frac{\sqrt[4]{-1}(2e^{x\lambda}a+b)}{2a^{3/4}\sqrt{\lambda}}\right) + c_2 \,\mathrm{Hypergeometric1F1}\left(\frac{-ib^2+4iac+4a^{3/2}\lambda}{16a^{3/2}\lambda}\right.\right.}{\sqrt{e^{\lambda x}}}$$

34.7 problem 7

Internal problem ID [11095]

Book: Handbook of exact solutions for ordinary differential equations. By Polyanin and Zaitsev. Second edition
Section: Chapter 2, Second-Order Differential Equations. section 2.1.3-1. Equations with exponential functions
Problem number: 7.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$y'' + \left(a\,\mathrm{e}^{2\lambda x}\left(b\,\mathrm{e}^{\lambda x}+c\right)^n - \frac{\lambda^2}{4}\right) y = 0$$

✓ Solution by Maple
Time used: 0.844 (sec). Leaf size: 218

```
dsolve(diff(y(x),x$2)+(a*exp(2*lambda*x)*(b*exp(lambda*x)+c)^n-1/4*lambda^2)*y(x)=0,y(x), si
```

$$y(x) = \frac{\mathrm{e}^{-\frac{x\lambda}{2}}\Gamma\left(\frac{n+1}{n+2}\right)^2\left(-\frac{a\left(b\,\mathrm{e}^{x\lambda}+c\right)^{n+2}}{\lambda^2 b^2 (n+2)^2}\right)^{\frac{1}{2n+4}} c_1 (n+2)\,\mathrm{BesselI}\left(-\frac{1}{n+2}, 2\sqrt{-\frac{a\left(b\,\mathrm{e}^{x\lambda}+c\right)^{n+2}}{\lambda^2 b^2 (n+2)^2}}\right) + \csc\left(\frac{\pi(n+1)}{n+2}\right)\left(-\frac{a\left(b\,\mathrm{e}^{x\lambda}+c\right)^{n+2}}{\lambda^2 b^2 (n+2)^2}\right)^{\frac{1}{2n+4}} c_2 (n+2)\,\mathrm{BesselI}\left(\frac{1}{n+2}, 2\sqrt{-\frac{a\left(b\,\mathrm{e}^{x\lambda}+c\right)^{n+2}}{\lambda^2 b^2 (n+2)^2}}\right)}{(n+2)\,\Gamma\left(\frac{n+1}{n+2}\right)}$$

✗ Solution by Mathematica
Time used: 0.0 (sec). Leaf size: 0

```
DSolve[y''[x]+(a*Exp[2*\[Lambda]*x]*(b*Exp[\[Lambda]*x]+c)^n-1/4*\[Lambda]^2)*y[x]==0,y[x],x
```

Not solved

34.8 problem 8

Internal problem ID [11096]

Book: Handbook of exact solutions for ordinary differential equations. By Polyanin and Zaitsev. Second edition
Section: Chapter 2, Second-Order Differential Equations. section 2.1.3-1. Equations with exponential functions
Problem number: 8.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$\boxed{y'' + ay' + b\,\mathrm{e}^{2ax}y = 0}$$

✓ Solution by Maple
Time used: 0.156 (sec). Leaf size: 39

```
dsolve(diff(y(x),x$2)+a*diff(y(x),x)+b*exp(2*a*x)*y(x)=0,y(x), singsol=all)
```

$$y(x) = \mathrm{e}^{-ax}\left(c_1 \sin\left(\frac{\sqrt{b}\,\mathrm{e}^{ax}}{a}\right) + c_2 \cos\left(\frac{\sqrt{b}\,\mathrm{e}^{ax}}{a}\right)\right)$$

✓ Solution by Mathematica
Time used: 0.133 (sec). Leaf size: 78

```
DSolve[y''[x]+a*y'[x]+b*Exp[2*a*x]*y[x]==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to \frac{\sqrt{a}e^{-\frac{ax}{2}}\left(2c_1 \cos\left(\frac{\sqrt{be^{2ax}}}{a}\right) + c_2 \sin\left(\frac{\sqrt{be^{2ax}}}{a}\right)\right)}{\sqrt{2}\sqrt[4]{be^{2ax}}}$$

34.9 problem 9

Internal problem ID [11097]

Book: Handbook of exact solutions for ordinary differential equations. By Polyanin and Zaitsev. Second edition
Section: Chapter 2, Second-Order Differential Equations. section 2.1.3-1. Equations with exponential functions
Problem number: 9.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries], [_2nd_order, _linear,`

$$\boxed{y'' - ay' + b\,\mathrm{e}^{2ax} y = 0}$$

✓ Solution by Maple
Time used: 0.016 (sec). Leaf size: 33

```
dsolve(diff(y(x),x$2)-a*diff(y(x),x)+b*exp(2*a*x)*y(x)=0,y(x), singsol=all)
```

$$y(x) = c_1 \sin\left(\frac{\sqrt{b}\,\mathrm{e}^{ax}}{a}\right) + c_2 \cos\left(\frac{\sqrt{b}\,\mathrm{e}^{ax}}{a}\right)$$

✓ Solution by Mathematica
Time used: 0.061 (sec). Leaf size: 42

```
DSolve[y''[x]-a*y'[x]+b*Exp[2*a*x]*y[x]==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to c_1 \cos\left(\frac{\sqrt{b}e^{ax}}{a}\right) + c_2 \sin\left(\frac{\sqrt{b}e^{ax}}{a}\right)$$

34.10 problem 10

Internal problem ID [11098]

Book: Handbook of exact solutions for ordinary differential equations. By Polyanin and Zaitsev. Second edition
Section: Chapter 2, Second-Order Differential Equations. section 2.1.3-1. Equations with exponential functions
Problem number: 10.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$\boxed{y'' + ay' + \left(b\,\mathrm{e}^{\lambda x} + c\right) y = 0}$$

✓ Solution by Maple
Time used: 0.344 (sec). Leaf size: 69

```
dsolve(diff(y(x),x$2)+a*diff(y(x),x)+(b*exp(lambda*x)+c)*y(x)=0,y(x), singsol=all)
```

$$y(x) = \mathrm{e}^{-\frac{ax}{2}} \left(\mathrm{BesselJ}\left(\frac{\sqrt{a^2-4c}}{\lambda}, \frac{2\sqrt{b}\,\mathrm{e}^{\frac{x\lambda}{2}}}{\lambda} \right) c_1 + \mathrm{BesselY}\left(\frac{\sqrt{a^2-4c}}{\lambda}, \frac{2\sqrt{b}\,\mathrm{e}^{\frac{x\lambda}{2}}}{\lambda} \right) c_2 \right)$$

✓ Solution by Mathematica
Time used: 0.185 (sec). Leaf size: 123

```
DSolve[y''[x]+a*y'[x]+(b*Exp[\[Lambda]*x]+c)*y[x]==0,y[x],x,IncludeSingularSolutions -> True
```

$$\begin{aligned} y(x) \to e^{-\frac{ax}{2}} \Bigg(& c_1 \,\mathrm{Gamma}\left(1 - \frac{\sqrt{a^2-4c}}{\lambda}\right) \mathrm{BesselJ}\left(-\frac{\sqrt{a^2-4c}}{\lambda}, \frac{2\sqrt{be^{x\lambda}}}{\lambda} \right) \\ & + c_2 \,\mathrm{Gamma}\left(\frac{\lambda + \sqrt{a^2-4c}}{\lambda} \right) \mathrm{BesselJ}\left(\frac{\sqrt{a^2-4c}}{\lambda}, \frac{2\sqrt{be^{x\lambda}}}{\lambda} \right) \Bigg) \end{aligned}$$

34.11 problem 11

Internal problem ID [11099]

Book: Handbook of exact solutions for ordinary differential equations. By Polyanin and Zaitsev. Second edition
Section: Chapter 2, Second-Order Differential Equations. section 2.1.3-1. Equations with exponential functions
Problem number: 11.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$y'' - y' + \left(\mathrm{e}^{3\lambda x} a + b\,\mathrm{e}^{2\lambda x} + \frac{1}{4} - \frac{\lambda^2}{4}\right) y = 0$$

✓ Solution by Maple
Time used: 0.234 (sec). Leaf size: 51

```
dsolve(diff(y(x),x$2)-diff(y(x),x)+(a*exp(3*lambda*x)+b*exp(2*lambda*x)+1/4-1/4*lambda^2  )*
```

$$y(x) = \mathrm{e}^{-\frac{x(\lambda-1)}{2}} \left(\mathrm{AiryAi}\left(-\frac{\mathrm{e}^{x\lambda} a + b}{\lambda^{\frac{2}{3}} a^{\frac{2}{3}}}\right) c_1 + \mathrm{AiryBi}\left(-\frac{\mathrm{e}^{x\lambda} a + b}{\lambda^{\frac{2}{3}} a^{\frac{2}{3}}}\right) c_2\right)$$

✓ Solution by Mathematica
Time used: 1.332 (sec). Leaf size: 77

```
DSolve[y''[x]-y'[x]+(a*Exp[3*\[Lambda]*x]+b*Exp[2*\[Lambda]*x]+1/4-1/4*\[Lambda]^2  )*y[x]==
```

$$y(x) \to \frac{e^{x/2}\left(c_1 \,\mathrm{AiryAi}\left(\frac{\left(e^{x\lambda}a+b\right)\sqrt[3]{-\frac{a}{\lambda^2}}}{a}\right) + c_2 \,\mathrm{AiryBi}\left(\frac{\left(e^{x\lambda}a+b\right)\sqrt[3]{-\frac{a}{\lambda^2}}}{a}\right)\right)}{\sqrt{e^{\lambda x}}}$$

34.12 problem 12

Internal problem ID [11100]

Book: Handbook of exact solutions for ordinary differential equations. By Polyanin and Zaitsev. Second edition
Section: Chapter 2, Second-Order Differential Equations. section 2.1.3-1. Equations with exponential functions
Problem number: 12.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$y'' - y' + \left(a\,\mathrm{e}^{2\lambda x}\left(b\,\mathrm{e}^{\lambda x} + c\right)^n + \frac{1}{4} - \frac{\lambda^2}{4}\right) y = 0$$

✓ Solution by Maple
Time used: 0.796 (sec). Leaf size: 224

```
dsolve(diff(y(x),x$2)-diff(y(x),x)+(a*exp(2*lambda*x)*(b*exp(lambda*x)+c)^n+1/4-1/4*lambda^2
```

$$y(x) = \frac{\Gamma\left(\frac{n+1}{n+2}\right)^2 \mathrm{e}^{-\frac{x(\lambda-1)}{2}} \left(-\frac{a\left(b\,\mathrm{e}^{x\lambda}+c\right)^{n+2}}{\lambda^2 b^2 (n+2)^2}\right)^{\frac{1}{2n+4}} c_1 (n+2)\,\mathrm{BesselI}\left(-\frac{1}{n+2}, 2\sqrt{-\frac{a\left(b\,\mathrm{e}^{x\lambda}+c\right)^{n+2}}{\lambda^2 b^2 (n+2)^2}}\right) + \csc\left(\frac{\pi(n+1)}{n+2}\right)\left(-\frac{a(b}{\lambda^2}\right.}{(n+2)\,\Gamma\left(\frac{n+1}{n+2}\right)}$$

✗ Solution by Mathematica
Time used: 0.0 (sec). Leaf size: 0

```
DSolve[y''[x]-y'[x]+(a*Exp[2*\[Lambda]*x]*(b*Exp[\[Lambda]*x]+c)^n+1/4-1/4*\[Lambda]^2  )*y[
```

Not solved

34.13 problem 13

Internal problem ID [11101]

Book: Handbook of exact solutions for ordinary differential equations. By Polyanin and Zaitsev. Second edition
Section: Chapter 2, Second-Order Differential Equations. section 2.1.3-1. Equations with exponential functions
Problem number: 13.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$\boxed{y'' + 2a\,\mathrm{e}^{\lambda x} y' + a\,\mathrm{e}^{\lambda x}\left(a\,\mathrm{e}^{\lambda x} + \lambda\right) y = 0}$$

✓ Solution by Maple
Time used: 0.0 (sec). Leaf size: 21

```
dsolve(diff(y(x),x$2)+2*a*exp(lambda*x)*diff(y(x),x)+a*exp(lambda*x)*(a*exp(lambda*x)+lambda
```

$$y(x) = \mathrm{e}^{-\frac{a\,\mathrm{e}^{x\lambda}}{\lambda}}(c_2 x + c_1)$$

✓ Solution by Mathematica
Time used: 0.109 (sec). Leaf size: 26

```
DSolve[y''[x]+2*a*Exp[\[Lambda]*x]*y'[x]+a*Exp[\[Lambda]*x]*(a*Exp[\[Lambda]*x]+\[Lambda])*y
```

$$y(x) \to (c_2 x + c_1) e^{-\frac{a e^{\lambda x}}{\lambda}}$$

34.14 problem 14

Internal problem ID [11102]

Book: Handbook of exact solutions for ordinary differential equations. By Polyanin and Zaitsev. Second edition
Section: Chapter 2, Second-Order Differential Equations. section 2.1.3-1. Equations with exponential functions
Problem number: 14.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$\boxed{y'' + (a+b)\,\mathrm{e}^{\lambda x} y' + a\,\mathrm{e}^{\lambda x}\left(b\,\mathrm{e}^{\lambda x} + \lambda\right) y = 0}$$

✓ Solution by Maple
Time used: 0.125 (sec). Leaf size: 36

```
dsolve(diff(y(x),x$2)+(a+b)*exp(lambda*x)*diff(y(x),x)+a*exp(lambda*x)*(b*exp(lambda*x)+lamb
```

$$y(x) = \mathrm{e}^{-\frac{a\,\mathrm{e}^{x\lambda}}{\lambda}}\left(c_1 + \operatorname{expIntegral}_1\left(-\frac{\mathrm{e}^{x\lambda}(a-b)}{\lambda}\right) c_2\right)$$

✓ Solution by Mathematica
Time used: 2.377 (sec). Leaf size: 40

```
DSolve[y''[x]+(a+b)*Exp[\[Lambda]*x]*y'[x]+a*Exp[\[Lambda]*x]*(b*Exp[\[Lambda]*x]+\[Lambda])
```

$$y(x) \to e^{-\frac{a e^{\lambda x}}{\lambda}}\left(c_2 \operatorname{ExpIntegralEi}\left(\frac{(a-b)e^{x\lambda}}{\lambda}\right) + c_1\right)$$

34.15 problem 15

Internal problem ID [11103]

Book: Handbook of exact solutions for ordinary differential equations. By Polyanin and Zaitsev. Second edition
Section: Chapter 2, Second-Order Differential Equations. section 2.1.3-1. Equations with exponential functions
Problem number: 15.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$\boxed{y'' + a\,\mathrm{e}^{\lambda x} y' - b\,\mathrm{e}^{\mu x}\left(a\,\mathrm{e}^{\lambda x} + \mathrm{e}^{\mu x} b + \mu\right) y = 0}$$

✓ Solution by Maple
Time used: 0.0 (sec). Leaf size: 46

```
dsolve(diff(y(x),x$2)+a*exp(lambda*x)*diff(y(x),x)-b*exp(mu*x)*(a*exp(lambda*x)+b*exp(mu*x)+
```

$$y(x) = \left(\left(\int \mathrm{e}^{\frac{-2b\,\mathrm{e}^{x\mu}\lambda - \mathrm{e}^{x\lambda} a\mu}{\mu\lambda}}\,dx\right) c_1 + c_2\right) \mathrm{e}^{\frac{b\,\mathrm{e}^{x\mu}}{\mu}}$$

✗ Solution by Mathematica
Time used: 0.0 (sec). Leaf size: 0

```
DSolve[y''[x]+a*Exp[\[Lambda]*x]*y'[x]-b*Exp[\[Mu]*x]*(a*Exp[\[Lambda]*x]+b*Exp[\[Mu]*x]+\[M
```

Not solved

34.16 problem 16

Internal problem ID [11104]

Book: Handbook of exact solutions for ordinary differential equations. By Polyanin and Zaitsev. Second edition
Section: Chapter 2, Second-Order Differential Equations. section 2.1.3-1. Equations with exponential functions
Problem number: 16.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$y'' + 2k\,\mathrm{e}^{\mu x} y' + \left(a\,\mathrm{e}^{2\lambda x} + b\,\mathrm{e}^{\lambda x} + k^2 \mathrm{e}^{2\mu x} + k\mu\,\mathrm{e}^{\mu x} + c\right) y = 0$$

✗ Solution by Maple

```
dsolve(diff(y(x),x$2)+2*k*exp(mu*x)*diff(y(x),x)+(a*exp(2*lambda*x)+b*exp(lambda*x)+k^2*exp(
```

No solution found

✓ Solution by Mathematica
Time used: 2.531 (sec). Leaf size: 232

```
DSolve[y''[x]+2*k*Exp[\[Mu]*x]*y'[x]+(a*Exp[2*\[Lambda]*x]+b*Exp[\[Lambda]*x]+k^2*Exp[2*\[Mu
```

$$y(x)$$
$$\to 2^{\frac{1}{2}-\frac{i\sqrt{c}}{\lambda}} (e^x)^{\frac{1}{2}-\frac{\lambda}{2}} \left((e^x)^{\mu}\right)^{-\frac{1}{2}/\mu} \left((e^x)^{\lambda}\right)^{\frac{1}{2}-\frac{i\sqrt{c}}{\lambda}} e^{-\frac{k(e^x)^{\mu}}{\mu}+\frac{i\sqrt{a}(e^x)^{\lambda}}{\lambda}} \left(c_1 \text{HypergeometricU}\left(-\frac{\frac{ib}{\sqrt{a}}-\lambda+2i\sqrt{c}}{2\lambda}, 1 - \frac{2i\sqrt{c}}{\lambda}, -\frac{2i\sqrt{a}(e^x)^{\lambda}}{\lambda}\right) + c_2 L^{-\frac{2i\sqrt{c}}{\lambda}}_{\frac{\frac{ib}{\sqrt{a}}-\lambda+2i\sqrt{c}}{2\lambda}}\left(-\frac{2i\sqrt{a}(e^x)^{\lambda}}{\lambda}\right)\right)$$

34.17 problem 17

Internal problem ID [11105]

Book: Handbook of exact solutions for ordinary differential equations. By Polyanin and Zaitsev. Second edition
Section: Chapter 2, Second-Order Differential Equations. section 2.1.3-1. Equations with exponential functions
Problem number: 17.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$y'' - (a + 2\,\mathrm{e}^{ax} b)\, y' + b^2 \mathrm{e}^{2ax} y = 0$$

✓ Solution by Maple
Time used: 0.0 (sec). Leaf size: 39

```
dsolve(diff(y(x),x$2)-(a+2*b*exp(a*x))*diff(y(x),x)+b^2*exp(2*a*x)*y(x)=0,y(x), singsol=all)
```

$$y(x) = \mathrm{e}^{\frac{a^2 x + 2\,\mathrm{e}^{ax} b}{2a}} \left(c_1 \sinh\left(\frac{ax}{2}\right) + c_2 \cosh\left(\frac{ax}{2}\right) \right)$$

✓ Solution by Mathematica
Time used: 0.079 (sec). Leaf size: 35

```
DSolve[y''[x]-(a+2*b*Exp[a*x])*y'[x]+b^2*Exp[2*a*x]*y[x]==0,y[x],x,IncludeSingularSolutions
```

$$y(x) \to \frac{e^{\frac{be^{ax}}{a}} (bc_2 e^{ax} + ac_1)}{a}$$

34.18 problem 18

Internal problem ID [11106]

Book: Handbook of exact solutions for ordinary differential equations. By Polyanin and Zaitsev. Second edition
Section: Chapter 2, Second-Order Differential Equations. section 2.1.3-1. Equations with exponential functions
Problem number: 18.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$\boxed{y'' + \left(a\,\mathrm{e}^{2\lambda x} + \lambda\right) y' - a\lambda\,\mathrm{e}^{2\lambda x} y = 0}$$

✓ Solution by Maple
Time used: 0.593 (sec). Leaf size: 79

```
dsolve(diff(y(x),x$2)+(a*exp(2*lambda*x)+lambda)*diff(y(x),x)-a*lambda*exp(2*lambda*x)*y(x)=
```

$$y(x) = c_2\sqrt{\pi}\left(\mathrm{e}^{x\lambda}a + \mathrm{e}^{-x\lambda}\lambda\right)\operatorname{erf}\left(\frac{\sqrt{2}\,\mathrm{e}^{x\lambda}\sqrt{a}}{2\sqrt{\lambda}}\right) + \sqrt{a}\,\sqrt{\lambda}\,\mathrm{e}^{-\frac{a\,\mathrm{e}^{2x\lambda}}{2\lambda}}\sqrt{2}\,c_2 + c_1\left(\mathrm{e}^{x\lambda}a + \mathrm{e}^{-x\lambda}\lambda\right)$$

✓ Solution by Mathematica
Time used: 0.283 (sec). Leaf size: 129

```
DSolve[y''[x]+(a*Exp[2*\[Lambda]*x]+\[Lambda])*y'[x]-a*\[Lambda]*Exp[2*\[Lambda]*x]*y[x]==0,
```

$$y(x) \to \frac{\sqrt{2\pi}c_2\left(ae^{2\lambda x} + \lambda\right)\operatorname{erf}\left(\frac{\sqrt{a\lambda e^{2\lambda x}}}{\sqrt{2}\lambda}\right) - 4i\sqrt{2}ac_1e^{2\lambda x} + 2c_2e^{-\frac{ae^{2\lambda x}}{2\lambda}}\sqrt{a\lambda e^{2\lambda x}} - 4i\sqrt{2}c_1\lambda}{4\sqrt{a\lambda e^{2\lambda x}}}$$

34.19 problem 19

Internal problem ID [11107]

Book: Handbook of exact solutions for ordinary differential equations. By Polyanin and Zaitsev. Second edition
Section: Chapter 2, Second-Order Differential Equations. section 2.1.3-1. Equations with exponential functions
Problem number: 19.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$\boxed{y'' + \left(a\,\mathrm{e}^{\lambda x} - \lambda\right) y' + yb\,\mathrm{e}^{2\lambda x} = 0}$$

✓ Solution by Maple
Time used: 0.172 (sec). Leaf size: 53

```
dsolve(diff(y(x),x$2)+(a*exp(lambda*x)-lambda)*diff(y(x),x)+b*exp(2*lambda*x)*y(x)=0,y(x), s
```

$$y(x) = c_1 \mathrm{e}^{\frac{\left(-a+\sqrt{a^2-4b}\right)\mathrm{e}^{x\lambda}}{2\lambda}} + c_2 \mathrm{e}^{-\frac{\left(a+\sqrt{a^2-4b}\right)\mathrm{e}^{x\lambda}}{2\lambda}}$$

✓ Solution by Mathematica
Time used: 0.133 (sec). Leaf size: 61

```
DSolve[y''[x]+(a*Exp[\[Lambda]*x]-\[Lambda])*y'[x]+b*Exp[2*\[Lambda]*x]*y[x]==0,y[x],x,Inclu
```

$$y(x) \to e^{-\frac{\left(\sqrt{a^2-4b}+a\right)e^{\lambda x}}{2\lambda}} \left(c_2 e^{\frac{\sqrt{a^2-4b}e^{\lambda x}}{\lambda}} + c_1\right)$$

34.20 problem 20

Internal problem ID [11108]

Book: Handbook of exact solutions for ordinary differential equations. By Polyanin and Zaitsev. Second edition
Section: Chapter 2, Second-Order Differential Equations. section 2.1.3-1. Equations with exponential functions
Problem number: 20.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$\boxed{y'' + \left(a\,\mathrm{e}^{\lambda x} + b\right) y' + c\left(a\,\mathrm{e}^{\lambda x} + b - c\right) y = 0}$$

✓ Solution by Maple
Time used: 0.297 (sec). Leaf size: 176

```
dsolve(diff(y(x),x$2)+(a*exp(lambda*x)+b)*diff(y(x),x)+c*(a*exp(lambda*x)+b-c)*y(x)=0,y(x),
```

$$y(x) = \mathrm{e}^{\frac{-\mathrm{e}^{x\lambda}a-(b+3\lambda)x\lambda}{2\lambda}} c_2(-\lambda-2c+b)^2\,\mathrm{WhittakerM}\left(-\frac{-\lambda-2c+b}{2\lambda}, -\frac{-2\lambda-2c+b}{2\lambda}, \frac{a\,\mathrm{e}^{x\lambda}}{\lambda}\right) + \left((\lambda+2c-b)\,\mathrm{e}^{\frac{-\mathrm{e}^{x\lambda}a-(b+3\lambda)x\lambda}{2\lambda}} + a\,\mathrm{e}^{\frac{-\mathrm{e}^{x\lambda}a-x\lambda(b+\lambda)}{2\lambda}}\right) c_2\lambda\,\mathrm{WhittakerM}\left(-\frac{b-2c+\lambda}{2\lambda}, -\frac{-2\lambda-2c+b}{2\lambda}, \frac{a\,\mathrm{e}^{x\lambda}}{\lambda}\right) + c_1\mathrm{e}^{-cx}$$

✓ Solution by Mathematica
Time used: 0.244 (sec). Leaf size: 96

```
DSolve[y''[x]+(a*Exp[\[Lambda]*x]+b)*y'[x]+c*(a*Exp[\[Lambda]*x]+b-c)*y[x]==0,y[x],x,Include
```

$$y(x) \to (-1)^{-\frac{c}{\lambda}} c^{c/\lambda} \lambda^{\frac{c}{\lambda}-1} a^{-\frac{c}{\lambda}} \left(ce^{\lambda x}\right)^{-\frac{c}{\lambda}} \left(c_2(2c-b)(-1)^{c/\lambda}\Gamma\left(-\frac{b-2c}{\lambda}, 0, \frac{ae^{x\lambda}}{\lambda}\right) + c_1\lambda\right)$$

34.21 problem 21

Internal problem ID [11109]

Book: Handbook of exact solutions for ordinary differential equations. By Polyanin and Zaitsev. Second edition
Section: Chapter 2, Second-Order Differential Equations. section 2.1.3-1. Equations with exponential functions
Problem number: 21.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$y'' + \left(a + b\,\mathrm{e}^{2\lambda x}\right) y' + \lambda\left(a - \lambda - b\,\mathrm{e}^{2\lambda x}\right) y = 0$$

✗ Solution by Maple

```
dsolve(diff(y(x),x$2)+(a+b*exp(2*lambda*x))*diff(y(x),x)+lambda*(a-lambda-b*exp(2*lambda*x))
```

No solution found

✓ Solution by Mathematica
Time used: 0.513 (sec). Leaf size: 248

```
DSolve[y''[x]+(a+b*Exp[2*\[Lambda]*x])*y'[x]+\[Lambda]*(a-\[Lambda]-b*Exp[2*\[Lambda]*x])*y[
```

$$y(x) \to \frac{-\frac{1}{2}c_2(a-2\lambda)e^{-\frac{be^{2\lambda x}}{2\lambda}}\left(b\lambda e^{2\lambda x}\right)^{-\frac{a}{2\lambda}}\left(b2^{\frac{a}{2\lambda}}\lambda^{a/\lambda}e^{2\lambda x} + \text{Gamma}\left(1-\frac{a}{2\lambda}\right)e^{\frac{be^{2\lambda x}}{2\lambda}}\left(a+be^{2\lambda x}\right)\left(b\lambda e^{2\lambda x}\right)^{\frac{a}{2\lambda}} - e^{\frac{be^{2\lambda}}{2\lambda}}\right.}{\sqrt{2}\lambda\sqrt{b\lambda e^{2\lambda x}}}$$

34.22 problem 22

Internal problem ID [11110]

Book: Handbook of exact solutions for ordinary differential equations. By Polyanin and Zaitsev. Second edition
Section: Chapter 2, Second-Order Differential Equations. section 2.1.3-1. Equations with exponential functions
Problem number: 22.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$y'' + \left(a + b\,\mathrm{e}^{\lambda x} + b - 3\lambda\right) y' + a^2\lambda(-\lambda + b)\,\mathrm{e}^{2\lambda x} y = 0$$

✓ Solution by Maple
Time used: 0.406 (sec). Leaf size: 205

```
dsolve(diff(y(x),x$2)+(a+b*exp(lambda*x)+b-3*lambda)*diff(y(x),x)+a^2*lambda*(b-lambda)*exp(
```

$$y(x) = \mathrm{e}^{-\frac{\mathrm{e}^{x\lambda}\left(b+\sqrt{-4\lambda(b-\lambda)a^2+b^2}\right)}{2\lambda}} \left(\mathrm{KummerU}\left(\frac{\left(b+\sqrt{-4\lambda\,(b-\lambda)\,a^2+b^2}\right)(-2\lambda+b+a)}{2\sqrt{-4\lambda\,(b-\lambda)\,a^2+b^2}\,\lambda}, \frac{-2\lambda+b+a}{\lambda}, \frac{\sqrt{-4\lambda(} \right.\right.$$

$$\left.\left. + \mathrm{KummerM}\left(\frac{\left(b+\sqrt{-4\lambda\,(b-\lambda)\,a^2+b^2}\right)(-2\lambda+b+a)}{2\sqrt{-4\lambda\,(b-\lambda)\,a^2+b^2}\,\lambda}, \frac{-2\lambda+b+a}{\lambda}, \frac{\sqrt{-4\lambda\,(b-\lambda)\,a^2+b^2}\,\mathrm{e}^{x\lambda}}{\lambda} \right) \right.$$

✓ Solution by Mathematica

Time used: 3.799 (sec). Leaf size: 260

```
DSolve[y''[x]+(a+b*Exp[\[Lambda]*x]+b-3*\[Lambda])*y'[x]+a^2*\[Lambda]*(b-\[Lambda])*Exp[2*\
```

$$y(x)$$

$$\to \exp\left(-\frac{e^{\lambda x}\left(\sqrt{-4a^2b\lambda+4a^2\lambda^2+b^2}+b\right)}{2\lambda}\right)\left(c_1 \text{HypergeometricU}\left(\frac{(a+b-2\lambda)\left(b+\sqrt{4\lambda^2a^2-4b\lambda a^2}\right.\right.\right.$$

$$\left.\left.\left.+\, c_2 L^{\frac{a+b-3\lambda}{\lambda}}_{-\frac{(a+b-2\lambda)\left(b+\sqrt{4\lambda^2a^2-4b\lambda a^2+b^2}\right)}{2\lambda\sqrt{4\lambda^2a^2-4b\lambda a^2+b^2}}}\left(\frac{e^{x\lambda}\sqrt{4\lambda^2a^2-4b\lambda a^2+b^2}}{\lambda}\right)\right)\right.$$

34.23 problem 23

Internal problem ID [11111]

Book: Handbook of exact solutions for ordinary differential equations. By Polyanin and Zaitsev. Second edition
Section: Chapter 2, Second-Order Differential Equations. section 2.1.3-1. Equations with exponential functions
Problem number: 23.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$y'' + \left(2a\,\mathrm{e}^{\lambda x} - \lambda\right) y' + \left(a^2 \mathrm{e}^{2\lambda x} + \mathrm{e}^{\mu x} c\right) y = 0$$

✗ Solution by Maple

```
dsolve(diff(y(x),x$2)+(2*a*exp(lambda*x)-lambda)*diff(y(x),x)+(a^2*exp(2*lambda*x)+c*exp(mu*
```

No solution found

✓ Solution by Mathematica
Time used: 1.858 (sec). Leaf size: 164

```
DSolve[y''[x]+(2*a*Exp[\[Lambda]*x]-\[Lambda])*y'[x]+(a^2*Exp[2*\[Lambda]*x]+c*Exp[\[Mu]*x])
```

$y(x)$

$$\to (-1)^{-\frac{\lambda}{\mu}} 2^{\frac{\lambda+\mu}{2\mu}} \left((e^x)^\lambda\right)^{\frac{\lambda-1}{2\lambda}} (e^x)^{\frac{1}{2}-\frac{\mu}{2}} e^{-\frac{a(e^x)^\lambda}{\lambda}} ((e^x)^\mu)^{\frac{\lambda+\mu}{2\mu}} \left(-\frac{c(e^x)^\mu}{\mu^2}\right)^{-\frac{\lambda}{2\mu}} \left(c_1(-1)^{\lambda/\mu}\,\text{BesselI}\left(\frac{\lambda}{\mu}, 2\sqrt{-\frac{c\,(e^x)^\mu}{\mu^2}}\right)\right.$$
$$\left. + c_2 K_{\frac{\lambda}{\mu}}\left(2\sqrt{-\frac{c\,(e^x)^\mu}{\mu^2}}\right)\right)$$

34.24 problem 24

Internal problem ID [11112]

Book: Handbook of exact solutions for ordinary differential equations. By Polyanin and Zaitsev. Second edition
Section: Chapter 2, Second-Order Differential Equations. section 2.1.3-1. Equations with exponential functions
Problem number: 24.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$y'' + \left(2a\,\mathrm{e}^{\lambda x} + b\right) y' + \left(a^2\mathrm{e}^{2\lambda x} + a(b+\lambda)\,\mathrm{e}^{\lambda x} + c\right) y = 0$$

✓ Solution by Maple
Time used: 0.016 (sec). Leaf size: 54

```
dsolve(diff(y(x),x$2)+(2*a*exp(lambda*x)+b)*diff(y(x),x)+(a^2*exp(2*lambda*x)+a*(b+lambda)*e
```

$$y(x) = \mathrm{e}^{-\frac{b\lambda x + 2\,\mathrm{e}^{x\lambda}a}{2\lambda}} \left(c_1 \sinh\left(\frac{\sqrt{b^2-4c}\,x}{2}\right) + c_2 \cosh\left(\frac{\sqrt{b^2-4c}\,x}{2}\right)\right)$$

✓ Solution by Mathematica
Time used: 0.589 (sec). Leaf size: 82

```
DSolve[y''[x]+(2*a*Exp[\[Lambda]*x]+b)*y'[x]+(a^2*Exp[2*\[Lambda]*x]+a*(b+\[Lambda])*Exp[\[L
```

$$y(x) \to \frac{\left(c_2 e^{x\sqrt{b^2-4c}} + c_1\sqrt{b^2-4c}\right) e^{-\frac{ae^{\lambda x}}{\lambda} - \frac{1}{2}x\left(\sqrt{b^2-4c}+b\right)}}{\sqrt{b^2-4c}}$$

34.25 problem 25

Internal problem ID [11113]

Book: Handbook of exact solutions for ordinary differential equations. By Polyanin and Zaitsev. Second edition
Section: Chapter 2, Second-Order Differential Equations. section 2.1.3-1. Equations with exponential functions
Problem number: 25.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$\boxed{y'' + \left(a\,\mathrm{e}^{\lambda x} + 2b - \lambda\right) y' + \left(\mathrm{e}^{2\lambda x} c + ab\,\mathrm{e}^{\lambda x} + b^2 - b\lambda\right) y = 0}$$

✓ Solution by Maple
Time used: 0.484 (sec). Leaf size: 74

```
dsolve(diff(y(x),x$2)+(a*exp(lambda*x)+2*b-lambda)*diff(y(x),x)+(c*exp(2*lambda*x)+a*b*exp(l
```

$$y(x) = c_1 \mathrm{e}^{\frac{-2b\lambda x + \mathrm{e}^{x\lambda}\sqrt{a^2-4c} - \mathrm{e}^{x\lambda} a}{2\lambda}} + c_2 \mathrm{e}^{-\frac{2b\lambda x + \mathrm{e}^{x\lambda}\sqrt{a^2-4c} + \mathrm{e}^{x\lambda} a}{2\lambda}}$$

✓ Solution by Mathematica
Time used: 2.119 (sec). Leaf size: 97

```
DSolve[y''[x]+(a*Exp[\[Lambda]*x]+2*b-\[Lambda])*y'[x]+(c*Exp[2*\[Lambda]*x]+a*b*Exp[\[Lambd
```

$$y(x) \to \frac{\left(e^{\lambda x}\right)^{-\frac{b}{\lambda}} e^{-\frac{\left(\sqrt{a^2-4c}+a\right)e^{\lambda x}}{2\lambda}} \left(c_2 \lambda e^{\frac{\sqrt{a^2-4c}e^{\lambda x}}{\lambda}} + c_1 \sqrt{a^2-4c}\right)}{\sqrt{a^2-4c}}$$

34.26 problem 26

Internal problem ID [11114]

Book: Handbook of exact solutions for ordinary differential equations. By Polyanin and Zaitsev. Second edition
Section: Chapter 2, Second-Order Differential Equations. section 2.1.3-1. Equations with exponential functions
Problem number: 26.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$y'' + (a\,\mathrm{e}^{x} + b)\,y' + \left(c(a-c)\,\mathrm{e}^{2x} + (ak + bc - 2ck + c)\,\mathrm{e}^{x} + k(b-k)\right) y = 0$$

✓ Solution by Maple
Time used: 0.391 (sec). Leaf size: 114

```
dsolve(diff(y(x),x$2)+(a*exp(x)+b)*diff(y(x),x)+( c*(a-c)*exp(2*x)+ (a*k+b*c+c-2*c*k)*exp(x)
```

$$y(x) = -\operatorname{WhittakerM}\left(-\frac{b}{2}+k, -\frac{b}{2}+k+\frac{1}{2}, (-2c+a)\,\mathrm{e}^{x}\right) \mathrm{e}^{-\frac{a\,\mathrm{e}^{x}}{2}-\frac{bx}{2}} (-2c+a)^{-b+2k}\, c_2 + \left((-2c+a)\,\mathrm{e}^{x}\right)^{-\frac{b}{2}+k} c_2 (-2c+a)^{-b+2k} (-1+b-2k)\,\mathrm{e}^{(-a+c)\mathrm{e}^{x}-\frac{bx}{2}} + c_1 \mathrm{e}^{-kx-\mathrm{e}^{x}c}$$

✓ Solution by Mathematica
Time used: 3.806 (sec). Leaf size: 71

```
DSolve[y''[x]+(a*Exp[x]+b)*y'[x]+( c*(a-c)*Exp[2*x]+ (a*k+b*c+c-2*c*k)*Exp[x] + k*(b-k) )*y[
```

$$y(x) \to e^{-ce^{x}} (e^{x})^{-k} \left(c_1 - c_2 (e^{x})^{2k-b} \left(e^{x}(a-2c)\right)^{b-2k} \Gamma(2k-b, (a-2c)e^{x})\right)$$

34.27 problem 27

Internal problem ID [11115]

Book: Handbook of exact solutions for ordinary differential equations. By Polyanin and Zaitsev. Second edition
Section: Chapter 2, Second-Order Differential Equations. section 2.1.3-1. Equations with exponential functions
Problem number: 27.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$\boxed{y'' + \left(a\,\mathrm{e}^{\lambda x} + b\right) y' + \left(\alpha\,\mathrm{e}^{2\lambda x} + \beta\,\mathrm{e}^{\lambda x} + \gamma\right) y = 0}$$

✓ Solution by Maple
Time used: 0.422 (sec). Leaf size: 141

```
dsolve(diff(y(x),x$2)+(a*exp(lambda*x)+b)*diff(y(x),x)+( alpha*exp(2*lambda*x)+ beta*exp(lam
```

$$y(x) = \mathrm{e}^{\frac{-\mathrm{e}^{x\lambda}a - x\lambda(b+\lambda)}{2\lambda}} \left(\mathrm{WhittakerM}\left(-\frac{a(b+\lambda)-2\beta}{2\sqrt{a^2-4\alpha}\,\lambda}, \frac{\sqrt{b^2-4\gamma}}{2\lambda}, \frac{\sqrt{a^2-4\alpha}\,\mathrm{e}^{x\lambda}}{\lambda} \right) c_1 + \mathrm{WhittakerW}\left(-\frac{a(b+\lambda)-2\beta}{2\sqrt{a^2-4\alpha}\,\lambda}, \frac{\sqrt{b^2-4\gamma}}{2\lambda}, \frac{\sqrt{a^2-4\alpha}\,\mathrm{e}^{x\lambda}}{\lambda} \right) c_2 \right)$$

✓ Solution by Mathematica
Time used: 2.375 (sec). Leaf size: 248

```
DSolve[y''[x]+(a*Exp[\[Lambda]*x]+b)*y'[x]+( alpha*Exp[2*\[Lambda]*x]+ \[Beta]*Exp[\[Lambda]
```

$$y(x) \to e^{-\frac{\left(\sqrt{a^2-4\alpha}+a\right)e^{\lambda x}}{2\lambda}} \left(e^{\lambda x}\right)^{\frac{\sqrt{b^2-4\gamma}-b}{2\lambda}} \left(c_1 \,\mathrm{HypergeometricU}\left(\frac{-2\beta + a(b+\lambda) + \sqrt{a^2-4\alpha}\left(\lambda+\sqrt{b^2-4\gamma}\right)}{2\sqrt{a^2-4\alpha}\lambda}, \frac{\lambda+}{} \right. \right.$$
$$\left. \left. + c_2 L^{\frac{\sqrt{b^2-4\gamma}}{\lambda}}_{\frac{2\beta-a(b+\lambda)-\sqrt{a^2-4\alpha}\left(\lambda+\sqrt{b^2-4\gamma}\right)}{2\sqrt{a^2-4\alpha}\lambda}} \left(\frac{\sqrt{a^2-4\alpha}e^{x\lambda}}{\lambda} \right) \right)$$

34.28 problem 28

Internal problem ID [11116]

Book: Handbook of exact solutions for ordinary differential equations. By Polyanin and Zaitsev. Second edition
Section: Chapter 2, Second-Order Differential Equations. section 2.1.3-1. Equations with exponential functions
Problem number: 28.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$y'' + \left(2a\,\mathrm{e}^{\lambda x} - \lambda\right) y' + \left(a^2 \mathrm{e}^{2\lambda x} + \mathrm{e}^{2\mu x} b + \mathrm{e}^{\mu x} c + k\right) y = 0$$

✗ Solution by Maple

```
dsolve(diff(y(x),x$2)+(2*a*exp(lambda*x)-lambda)*diff(y(x),x)+( a^2*exp(2*lambda*x) + b*exp(
```

No solution found

✓ Solution by Mathematica
Time used: 2.29 (sec). Leaf size: 290

```
DSolve[y''[x]+(2*a*Exp[\[Lambda]*x]-\[Lambda])*y'[x]+( a^2*Exp[2*\[Lambda]*x] + b*Exp[2*\[Mu
```

$$y(x)$$

$$\to \left((e^x)^{\lambda}\right)^{\frac{\lambda-1}{2\lambda}} (e^x)^{\frac{1}{2}-\frac{\mu}{2}} 2^{\frac{\sqrt{\mu^2\left(\lambda^2-4k\right)}+\mu^2}{2\mu^2}} \left((e^x)^{\mu}\right)^{\frac{\sqrt{\mu^2\left(\lambda^2-4k\right)}+\mu^2}{2\mu^2}} e^{-\frac{a(e^x)^{\lambda}}{\lambda}+\frac{i\sqrt{b}(e^x)^{\mu}}{\mu}} \left(c_1 \text{HypergeometricU}\left(\frac{\mu^2-\frac{ic\mu}{\sqrt{b}}}{2\mu^2}-\frac{\sqrt{\left(\lambda^2-4k\right)\mu^2}}{2\mu^2},\frac{\sqrt{\left(\lambda^2-4k\right)\mu^2}}{\mu^2},-\frac{2i\sqrt{b}(e^x)^{\mu}}{\mu}\right)+c_2 L^{\frac{\sqrt{\left(\lambda^2-4k\right)\mu^2}}{\mu^2}}_{\frac{ic}{2\sqrt{b}\mu}-\frac{\mu^2+\sqrt{\left(\lambda^2-4k\right)\mu^2}}{2\mu^2}}\left(-\frac{2i\sqrt{b}(e^x)^{\mu}}{\mu}\right)\right)$$

34.29 problem 29

Internal problem ID [11117]

Book: Handbook of exact solutions for ordinary differential equations. By Polyanin and Zaitsev. Second edition
Section: Chapter 2, Second-Order Differential Equations. section 2.1.3-1. Equations with exponential functions
Problem number: 29.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$y'' + \left(2a\,\mathrm{e}^{\lambda x} + b - \lambda\right) y' + \left(a^2 \mathrm{e}^{2\lambda x} + ab\,\mathrm{e}^{\lambda x} + \mathrm{e}^{2\mu x} c + d\,\mathrm{e}^{\mu x} + k\right) y = 0$$

✗ Solution by Maple

```
dsolve(diff(y(x),x$2)+(2*a*exp(lambda*x)+b-lambda)*diff(y(x),x)+( a^2*exp(2*lambda*x) + a*b*
```

No solution found

✓ Solution by Mathematica
Time used: 2.625 (sec). Leaf size: 332

```
DSolve[y''[x]+(2*a*Exp[\[Lambda]*x]+b-\[Lambda])*y'[x]+( a^2*Exp[2*\[Lambda]*x] + a*b*Exp[\[
```

$y(x)$

$$\to (e^x)^{\frac{1}{2}-\frac{\mu}{2}} \left((e^x)^{\lambda}\right)^{-\frac{b-\lambda+1}{2\lambda}} 2^{\frac{1}{2}\left(\frac{\sqrt{\mu^2\left(b^2-2b\lambda+\lambda^2-4k\right)}}{\mu^2}+1\right)} e^{-\frac{a(e^x)^{\lambda}}{\lambda}+\frac{i\sqrt{c}(e^x)^{\mu}}{\mu}} ((e^x)^{\mu})^{\frac{1}{2}\left(\frac{\sqrt{\mu^2\left(b^2-2b\lambda+\lambda^2-4k\right)}}{\mu^2}+1\right)} \left(c_1 \text{Hyper}\right.$$

$$\left. -\frac{2i\sqrt{c}(e^x)^{\mu}}{\mu}\right) + c_2 L^{\frac{\sqrt{\left(b^2-2\lambda b+\lambda^2-4k\right)\mu^2}}{\mu^2}}_{-\frac{\mu^2-\frac{id\mu}{\sqrt{c}}+\sqrt{\left(b^2-2\lambda b+\lambda^2-4k\right)\mu^2}}{2\mu^2}}\left(-\frac{2i\sqrt{c}(e^x)^{\mu}}{\mu}\right)\Bigg)$$

34.30 problem 30

Internal problem ID [11118]

Book: Handbook of exact solutions for ordinary differential equations. By Polyanin and Zaitsev. Second edition
Section: Chapter 2, Second-Order Differential Equations. section 2.1.3-1. Equations with exponential functions
Problem number: 30.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$y'' + \left(a\,\mathrm{e}^{\lambda x} + \mathrm{e}^{\mu x} b\right) y' + a\,\mathrm{e}^{\lambda x}\left(\mathrm{e}^{\mu x} b + \lambda\right) y = 0$$

✗ Solution by Maple

```
dsolve(diff(y(x),x$2)+(a*exp(lambda*x)+b*exp(mu*x))*diff(y(x),x)+a*exp(lambda*x)*(b*exp(mu*x
```

No solution found

✗ Solution by Mathematica
Time used: 0.0 (sec). Leaf size: 0

```
DSolve[y''[x]+(a*Exp[\[Lambda]*x]+b*Exp[\[Mu]*x])*y'[x]+a*Exp[\[Lambda]*x]*(b*Exp[\[Mu]*x]+\
```

Not solved

34.31 problem 31

Internal problem ID [11119]

Book: Handbook of exact solutions for ordinary differential equations. By Polyanin and Zaitsev. Second edition
Section: Chapter 2, Second-Order Differential Equations. section 2.1.3-1. Equations with exponential functions
Problem number: 31.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$\boxed{y'' + \mathrm{e}^{\lambda x}\left(a\,\mathrm{e}^{2\mu x} + b\right) y' + \mu\left(\mathrm{e}^{\lambda x}\left(b - a\,\mathrm{e}^{2\mu x}\right) - \mu\right) y = 0}$$

✓ Solution by Maple
Time used: 0.016 (sec). Leaf size: 81

```
dsolve(diff(y(x),x$2)+exp(lambda*x)*(a*exp(2*mu*x)+b)*diff(y(x),x)+mu*(exp(lambda*x)*(b-a*ex
```

$$y(x) = \left(\left(\int \frac{\mathrm{e}^{\frac{-\mathrm{e}^{x(\lambda+2\mu)}a\lambda - 2\left(\frac{\lambda}{2}+\mu\right)\left(-2\lambda\mu x + b\,\mathrm{e}^{x\lambda}\right)}{\lambda(\lambda+2\mu)}}}{\left(\mathrm{e}^{2x\mu}a + b\right)^2} dx\right) c_2 + c_1\right)\left(a\,\mathrm{e}^{x\mu} + \mathrm{e}^{-x\mu}b\right)$$

✗ Solution by Mathematica
Time used: 0.0 (sec). Leaf size: 0

```
DSolve[y''[x]+Exp[\[Lambda]*x]*(a*Exp[2*\[Mu]*x]+b)*y'[x]+\[Mu]*(Exp[\[Lambda]*x]*(b-a*Exp[2
```

Not solved

34.32 problem 32

Internal problem ID [11120]

Book: Handbook of exact solutions for ordinary differential equations. By Polyanin and Zaitsev. Second edition
Section: Chapter 2, Second-Order Differential Equations. section 2.1.3-1. Equations with exponential functions
Problem number: 32.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _exact, _linear, _homogeneous]]`

$$\boxed{y'' + \left(a\,\mathrm{e}^{\lambda x} + \mathrm{e}^{\mu x} b + c\right) y' + \left(a\lambda\,\mathrm{e}^{\lambda x} + \mu\,\mathrm{e}^{\mu x} b\right) y = 0}$$

✓ Solution by Maple
Time used: 0.015 (sec). Leaf size: 70

```
dsolve(diff(y(x),x$2)+(a*exp(lambda*x)+b*exp(mu*x)+c)*diff(y(x),x)+(a*lambda*exp(lambda*x)+b
```

$$y(x) = \left(c_1 \left(\int \mathrm{e}^{\frac{cx\mu\lambda + \mathrm{e}^{x\lambda} a\mu + b\,\mathrm{e}^{x\mu}\lambda}{\mu\lambda}}\,dx\right) + c_2\right) \mathrm{e}^{\frac{-\mathrm{e}^{x\lambda} a\mu - \lambda\left(cx\mu + b\,\mathrm{e}^{x\mu}\right)}{\mu\lambda}}$$

✓ Solution by Mathematica
Time used: 0.146 (sec). Leaf size: 77

```
DSolve[y''[x]+(a*Exp[\[Lambda]*x]+b*Exp[\[Mu]*x]+c)*y'[x]+(a*\[Lambda]*Exp[\[Lambda]*x]+b*\[
```

$$y(x) \to e^{-\frac{ae^{\lambda x}}{\lambda} - \frac{be^{\mu x}}{\mu} - cx} \left(\int_1^x e^{\frac{e^{\lambda K[1]} a}{\lambda} + cK[1] + \frac{be^{\mu K[1]}}{\mu}} c_1 dK[1] + c_2\right)$$

34.33 problem 33

Internal problem ID [11121]

Book: Handbook of exact solutions for ordinary differential equations. By Polyanin and Zaitsev. Second edition
Section: Chapter 2, Second-Order Differential Equations. section 2.1.3-1. Equations with exponential functions
Problem number: 33.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$y'' + \left(a\,\mathrm{e}^{\lambda x} + \mathrm{e}^{\mu x} b + c\right) y' + \left(ab\,\mathrm{e}^{x(\lambda+\mu)} + \mathrm{e}^{\lambda x} ac + \mu\,\mathrm{e}^{\mu x} b\right) y = 0$$

✗ Solution by Maple

```
dsolve(diff(y(x),x$2)+(a*exp(lambda*x)+b*exp(mu*x)+c)*diff(y(x),x)+(a*b*exp((lambda+mu)*x)+a
```

No solution found

✗ Solution by Mathematica
Time used: 0.0 (sec). Leaf size: 0

```
DSolve[y''[x]+(a*Exp[\[Lambda]*x]+b*Exp[\[Mu]*x]+c)*y'[x]+(a*b*Exp[(\[Lambda]+\[Mu])*x]+a*c*
```

Not solved