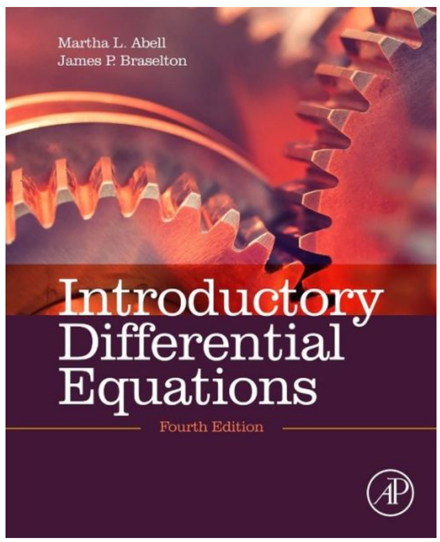
A Solution Manual For

INTRODUCTORY DIFFERENTIAL EQUATIONS. Martha L. Abell, James P. Braselton. Fourth edition 2014. ElScAe. 2014



Nasser M. Abbasi

May 16, 2024

Contents

1 Chapter 1. Introduction to Differential Equations. Exercises 1.1, page 10

1.1 problem 1

Internal problem ID [14045]

Book: INTRODUCTORY DIFFERENTIAL EQUATIONS. Martha L. Abell, James P. Braselton. Fourth edition 2014. ElScAe. 2014
Section: Chapter 1. Introduction to Differential Equations. Exercises 1.1, page 10
Problem number: 1.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _linear, _nonhomogeneous]]`

$$\boxed{y'' + y' - 2y = x^3}$$

✓ Solution by Maple
Time used: 0.0 (sec). Leaf size: 39

```
dsolve(diff(y(x),x$2)+diff(y(x),x)-2*y(x)=x^3,y(x), singsol=all)
```

$$y(x) = -\frac{e^{-2x}\left(\left(x^3 + \frac{3}{2}x^2 + \frac{9}{2}x + \frac{15}{4}\right)e^{2x} - 2c_2 e^{3x} - 2c_1\right)}{2}$$

✓ Solution by Mathematica
Time used: 0.077 (sec). Leaf size: 39

```
DSolve[y''[x]+y'[x]-2*y[x]==x^3,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to \frac{1}{8}\left(-4x^3 - 6x^2 - 18x - 15\right) + c_1 e^{-2x} + c_2 e^x$$

1.2 problem 2

Internal problem ID [14046]

Book: INTRODUCTORY DIFFERENTIAL EQUATIONS. Martha L. Abell, James P. Braselton. Fourth edition 2014. ElScAe. 2014
Section: Chapter 1. Introduction to Differential Equations. Exercises 1.1, page 10
Problem number: 2.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[‘y=_G(x,y’)‘]`

$$\boxed{yy' + y^4 = \sin(x)}$$

✓ Solution by Maple
Time used: 0.015 (sec). Leaf size: 164

```
dsolve(y(x)*diff(y(x),x)+y(x)^4=sin(x),y(x), singsol=all)
```

$$y(x) = -\frac{\sqrt{\left(c_1 \operatorname{MathieuC}\left(0, 8, -\frac{\pi}{4} + \frac{x}{2}\right) + \operatorname{MathieuS}\left(0, 8, -\frac{\pi}{4} + \frac{x}{2}\right)\right) \left(c_1 \operatorname{MathieuCPrime}\left(0, 8, -\frac{\pi}{4} + \frac{x}{2}\right) + \operatorname{Ma}\right.}}{2c_1 \operatorname{MathieuC}\left(0, 8, -\frac{\pi}{4} + \frac{x}{2}\right) + 2 \operatorname{MathieuS}\left(0, 8, -\frac{\pi}{4} + \frac{x}{2}\right)}$$

$$y(x) = \frac{\sqrt{\left(c_1 \operatorname{MathieuC}\left(0, 8, -\frac{\pi}{4} + \frac{x}{2}\right) + \operatorname{MathieuS}\left(0, 8, -\frac{\pi}{4} + \frac{x}{2}\right)\right) \left(c_1 \operatorname{MathieuCPrime}\left(0, 8, -\frac{\pi}{4} + \frac{x}{2}\right) + \operatorname{Math}\right.}}{2c_1 \operatorname{MathieuC}\left(0, 8, -\frac{\pi}{4} + \frac{x}{2}\right) + 2 \operatorname{MathieuS}\left(0, 8, -\frac{\pi}{4} + \frac{x}{2}\right)}$$

✗ Solution by Mathematica
Time used: 0.0 (sec). Leaf size: 0

```
DSolve[y[x]*y'[x]+y[x]^4==Sin[x],y[x],x,IncludeSingularSolutions -> True]
```

Not solved

1.3 problem 4

Internal problem ID [14047]

Book: INTRODUCTORY DIFFERENTIAL EQUATIONS. Martha L. Abell, James P. Braselton. Fourth edition 2014. ElScAe. 2014
Section: Chapter 1. Introduction to Differential Equations. Exercises 1.1, page 10
Problem number: 4.
ODE order: 3.
ODE degree: 1.

CAS Maple gives this as type `[[_3rd_order, _with_linear_symmetries]]`

$$\boxed{y''' - 2y'' + 5y' + y = e^x}$$

✓ Solution by Maple

Time used: 0.015 (sec). Leaf size: 251

```
dsolve(diff(y(x),x$3)-2*diff(y(x),x$2)+5*diff(y(x),x)+y(x)=exp(x),y(x), singsol=all)
```

$$y(x) = \frac{e^x}{5} + c_1 e^{\frac{\left(15\sqrt{69}\left(404+60\sqrt{69}\right)^{\frac{2}{3}} - 101\left(404+60\sqrt{69}\right)^{\frac{2}{3}} - 484\left(404+60\sqrt{69}\right)^{\frac{1}{3}} + 1936\right)x}{2904}}$$
$$+ c_2 e^{-\frac{\left(15\sqrt{69}\left(404+60\sqrt{69}\right)^{\frac{2}{3}} - 101\left(404+60\sqrt{69}\right)^{\frac{2}{3}} - 484\left(404+60\sqrt{69}\right)^{\frac{1}{3}} - 3872\right)x}{5808}} \cos\left(\frac{\left(404 + 60\sqrt{3}\sqrt{23}\right)^{\frac{1}{3}}\sqrt{3}\left(15(404 + 60\sqrt{3}\right.}{}\right.$$
$$+ c_3 e^{-\frac{\left(15\sqrt{69}\left(404+60\sqrt{69}\right)^{\frac{2}{3}} - 101\left(404+60\sqrt{69}\right)^{\frac{2}{3}} - 484\left(404+60\sqrt{69}\right)^{\frac{1}{3}} - 3872\right)x}{5808}} \sin\left(\frac{\left(404 + 60\sqrt{3}\sqrt{23}\right)^{\frac{1}{3}}\sqrt{3}\left(15(404 + 60\sqrt{3}\right.}{}\right.$$

✓ Solution by Mathematica

Time used: 0.389 (sec). Leaf size: 2831

```
DSolve[y'''[x]-2*y''[x]+5*y'[x]+y[x]==Exp[x],y[x],x,IncludeSingularSolutions -> True]
```

Too large to display

1.4 problem 5

Internal problem ID [14048]

Book: INTRODUCTORY DIFFERENTIAL EQUATIONS. Martha L. Abell, James P. Braselton. Fourth edition 2014. ElScAe. 2014
Section: Chapter 1. Introduction to Differential Equations. Exercises 1.1, page 10
Problem number: 5.
ODE order: 1.
ODE degree: 2.

CAS Maple gives this as type `[_quadrature]`

$$\boxed{y'^2 + y = 0}$$

✓ Solution by Maple
Time used: 0.078 (sec). Leaf size: 17

```
dsolve(diff(y(x),x)^2+y(x)=0,y(x), singsol=all)
```

$$y(x) = 0$$
$$y(x) = -\frac{(x - c_1)^2}{4}$$

✓ Solution by Mathematica
Time used: 0.062 (sec). Leaf size: 42

```
DSolve[y'[x]^2+y[x]==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to -\frac{1}{4}(x + ic_1)^2$$
$$y(x) \to -\frac{1}{4}(x - ic_1)^2$$
$$y(x) \to 0$$

1.5 problem 6

Internal problem ID [14049]

Book: INTRODUCTORY DIFFERENTIAL EQUATIONS. Martha L. Abell, James P. Braselton. Fourth edition 2014. ElScAe. 2014
Section: Chapter 1. Introduction to Differential Equations. Exercises 1.1, page 10
Problem number: 6.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_Emden, _Fowler], [_2nd_order, _linear, '_with_symmetry_[0,F`

$$\boxed{t^2y'' + y't + 2y = 0}$$

✓ Solution by Maple
Time used: 0.0 (sec). Leaf size: 23

```
dsolve(t^2*diff(y(t),t$2)+t*diff(y(t),t)+2*y(t)=0,y(t), singsol=all)
```

$$y(t) = c_1 \sin\left(\sqrt{2}\,\ln(t)\right) + c_2 \cos\left(\sqrt{2}\,\ln(t)\right)$$

✓ Solution by Mathematica
Time used: 0.027 (sec). Leaf size: 30

```
DSolve[t^2*y''[t]+t*y'[t]+2*y[t]==0,y[t],t,IncludeSingularSolutions -> True]
```

$$y(t) \to c_1 \cos\left(\sqrt{2}\log(t)\right) + c_2 \sin\left(\sqrt{2}\log(t)\right)$$

1.6 problem 9

Internal problem ID [14050]

Book: INTRODUCTORY DIFFERENTIAL EQUATIONS. Martha L. Abell, James P. Braselton. Fourth edition 2014. ElScAe. 2014
Section: Chapter 1. Introduction to Differential Equations. Exercises 1.1, page 10
Problem number: 9.
ODE order: 2.
ODE degree: 2.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$\boxed{x{y''}^2 + 2y = 2x}$$

✗ Solution by Maple

```
dsolve(x*diff(y(x),x$2)^2+2*y(x)=2*x,y(x), singsol=all)
```

No solution found

✗ Solution by Mathematica
Time used: 0.0 (sec). Leaf size: 0

```
DSolve[x*y''[x]^2+2*y[x]==2*x,y[x],x,IncludeSingularSolutions -> True]
```

Not solved

1.7 problem 10

Internal problem ID [14051]

Book: INTRODUCTORY DIFFERENTIAL EQUATIONS. Martha L. Abell, James P. Braselton. Fourth edition 2014. ElScAe. 2014
Section: Chapter 1. Introduction to Differential Equations. Exercises 1.1, page 10
Problem number: 10.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[NONE]`

$$\boxed{x'' + 2\sin(x) = \sin(2t)}$$

✗ Solution by Maple

```
dsolve(diff(x(t),t$2)+2*sin(x(t))=sin(2*t),x(t), singsol=all)
```

No solution found

✗ Solution by Mathematica
Time used: 0.0 (sec). Leaf size: 0

```
DSolve[x''[t]+2*Sin[x[t]]==Sin[2*t],x[t],t,IncludeSingularSolutions -> True]
```

Not solved

1.8 problem 12

Internal problem ID [14052]

Book: INTRODUCTORY DIFFERENTIAL EQUATIONS. Martha L. Abell, James P. Braselton. Fourth edition 2014. ElScAe. 2014
Section: Chapter 1. Introduction to Differential Equations. Exercises 1.1, page 10
Problem number: 12.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_quadrature]`

$$\boxed{-y' = -2x + 1}$$

✓ Solution by Maple
Time used: 0.0 (sec). Leaf size: 12

```
dsolve((2*x-1)-diff(y(x),x)=0,y(x), singsol=all)
```

$$y(x) = x^2 + c_1 - x$$

✓ Solution by Mathematica
Time used: 0.002 (sec). Leaf size: 7

```
DSolve[(2*x-1)*y'[x]==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to c_1$$

1.9 problem 15

Internal problem ID [14053]

Book: INTRODUCTORY DIFFERENTIAL EQUATIONS. Martha L. Abell, James P. Braselton. Fourth edition 2014. ElScAe. 2014
Section: Chapter 1. Introduction to Differential Equations. Exercises 1.1, page 10
Problem number: 15.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[[_homogeneous, ‘class A‘], _rational, [_Abel, ‘2nd type‘, ‘cl`

$$\boxed{-y - yy' = -2x}$$

✓ Solution by Maple

Time used: 0.891 (sec). Leaf size: 1041

```
dsolve((2*x-y(x))-y(x)*diff(y(x),x)=0,y(x), singsol=all)
```

$$y(x) = x\left(-1+\frac{\left(8c_1^3x^3+4\sqrt{4c_1^3x^3+1}+4\right)^{\frac{1}{3}}}{2c_1x}+\frac{2c_1x}{\left(8c_1^3x^3+4\sqrt{4c_1^3x^3+1}+4\right)^{\frac{1}{3}}}\right)$$

$$y(x) = \frac{4c_1^2x^2-2c_1x\left(8c_1^3x^3+4\sqrt{4c_1^3x^3+1}+4\right)^{\frac{1}{3}}+\left(8c_1^3x^3+4\sqrt{4c_1^3x^3+1}+4\right)^{\frac{2}{3}}}{2c_1\left(8c_1^3x^3+4\sqrt{4c_1^3x^3+1}+4\right)^{\frac{1}{3}}}$$

$$y(x) = \frac{4c_1^2x^2-2c_1x\left(8c_1^3x^3+4\sqrt{4c_1^3x^3+1}+4\right)^{\frac{1}{3}}+\left(8c_1^3x^3+4\sqrt{4c_1^3x^3+1}+4\right)^{\frac{2}{3}}}{2c_1\left(8c_1^3x^3+4\sqrt{4c_1^3x^3+1}+4\right)^{\frac{1}{3}}}$$

$$y(x) = \frac{4i\sqrt{3}\,c_1^2x^2-i\left(8c_1^3x^3+4\sqrt{4c_1^3x^3+1}+4\right)^{\frac{2}{3}}\sqrt{3}-4c_1^2x^2-4c_1x\left(8c_1^3x^3+4\sqrt{4c_1^3x^3+1}+4\right)^{\frac{1}{3}}-\left(8c_1^3x^3\right.}{4c_1\left(8c_1^3x^3+4\sqrt{4c_1^3x^3+1}+4\right)^{\frac{1}{3}}}$$

$$y(x) = \frac{\left(i\sqrt{3}-1\right)\left(8c_1^3x^3+4\sqrt{4c_1^3x^3+1}+4\right)^{\frac{2}{3}}-4c_1\left(ixc_1\sqrt{3}+c_1x+\left(8c_1^3x^3+4\sqrt{4c_1^3x^3+1}+4\right)^{\frac{1}{3}}\right)x}{4\left(8c_1^3x^3+4\sqrt{4c_1^3x^3+1}+4\right)^{\frac{1}{3}}c_1}$$

$$y(x) = \frac{4i\sqrt{3}\,c_1^2x^2-i\left(8c_1^3x^3+4\sqrt{4c_1^3x^3+1}+4\right)^{\frac{2}{3}}\sqrt{3}-4c_1^2x^2-4c_1x\left(8c_1^3x^3+4\sqrt{4c_1^3x^3+1}+4\right)^{\frac{1}{3}}-\left(8c_1^3x^3\right.}{4c_1\left(8c_1^3x^3+4\sqrt{4c_1^3x^3+1}+4\right)^{\frac{1}{3}}}$$

$$y(x) = \frac{\left(i\sqrt{3}-1\right)\left(8c_1^3x^3+4\sqrt{4c_1^3x^3+1}+4\right)^{\frac{2}{3}}-4c_1\left(ixc_1\sqrt{3}+c_1x+\left(8c_1^3x^3+4\sqrt{4c_1^3x^3+1}+4\right)^{\frac{1}{3}}\right)x}{4\left(8c_1^3x^3+4\sqrt{4c_1^3x^3+1}+4\right)^{\frac{1}{3}}c_1}$$

$$y(x) = \frac{4i\sqrt{3}\,c_1^2x^2-i\left(8c_1^3x^3+4\sqrt{4c_1^3x^3+1}+4\right)^{\frac{2}{3}}\sqrt{3}-4c_1^2x^2-4c_1x\left(8c_1^3x^3+4\sqrt{4c_1^3x^3+1}+4\right)^{\frac{1}{3}}-\left(8c_1^3x^3\right.}{4c_1\left(8c_1^3x^3+4\sqrt{4c_1^3x^3+1}+4\right)^{\frac{1}{3}}}$$

$$y(x) = \frac{\left(i\sqrt{3}-1\right)\left(8c_1^3x^3+4\sqrt{4c_1^3x^3+1}+4\right)^{\frac{2}{3}}-4c_1\left(ixc_1\sqrt{3}+c_1x+\left(8c_1^3x^3+4\sqrt{4c_1^3x^3+1}+4\right)^{\frac{1}{3}}\right)x}{4\left(8c_1^3x^3+4\sqrt{4c_1^3x^3+1}+4\right)^{\frac{1}{3}}c_1}$$

✓ Solution by Mathematica

Time used: 54.579 (sec). Leaf size: 496

```
DSolve[(2*x-y[x])-y[x]*y'[x]==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to \frac{\sqrt[3]{2x^3 + \sqrt{e^{6c_1} - 4e^{3c_1}x^3} - e^{3c_1}}}{\sqrt[3]{2}} + \frac{\sqrt[3]{2}x^2}{\sqrt[3]{2x^3 + \sqrt{e^{6c_1} - 4e^{3c_1}x^3} - e^{3c_1}}} - x$$

$$y(x) \to \frac{i(\sqrt{3}+i)\sqrt[3]{2x^3 + \sqrt{e^{6c_1} - 4e^{3c_1}x^3} - e^{3c_1}}}{2\sqrt[3]{2}} - \frac{(1+i\sqrt{3})\,x^2}{2^{2/3}\sqrt[3]{2x^3 + \sqrt{e^{6c_1} - 4e^{3c_1}x^3} - e^{3c_1}}} - x$$

$$y(x) \to -\frac{(1+i\sqrt{3})\sqrt[3]{2x^3 + \sqrt{e^{6c_1} - 4e^{3c_1}x^3} - e^{3c_1}}}{2\sqrt[3]{2}} + \frac{i(\sqrt{3}+i)\,x^2}{2^{2/3}\sqrt[3]{2x^3 + \sqrt{e^{6c_1} - 4e^{3c_1}x^3} - e^{3c_1}}} - x$$

$$y(x) \to \sqrt[3]{x^3} + \frac{(x^3)^{2/3}}{x} - x$$

$$y(x) \to \frac{1}{2}\left(i\left(\sqrt{3}+i\right)\sqrt[3]{x^3} + \frac{(-1-i\sqrt{3})\,(x^3)^{2/3}}{x} - 2x\right)$$

$$y(x) \to \frac{1}{2}\left(\left(-1-i\sqrt{3}\right)\sqrt[3]{x^3} + \frac{i(\sqrt{3}+i)\,(x^3)^{2/3}}{x} - 2x\right)$$

1.10 problem 17

Internal problem ID [14054]

Book: INTRODUCTORY DIFFERENTIAL EQUATIONS. Martha L. Abell, James P. Braselton. Fourth edition 2014. ElScAe. 2014
Section: Chapter 1. Introduction to Differential Equations. Exercises 1.1, page 10
Problem number: 17.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_quadrature]`

$$\boxed{y' + 2y = 0}$$

✓ Solution by Maple
Time used: 0.0 (sec). Leaf size: 10

```
dsolve(diff(y(x),x)+2*y(x)=0,y(x), singsol=all)
```

$$y(x) = \mathrm{e}^{-2x} c_1$$

✓ Solution by Mathematica
Time used: 0.022 (sec). Leaf size: 18

```
DSolve[y'[x]+2*y[x]==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to c_1 e^{-2x}$$
$$y(x) \to 0$$

1.11 problem 18

Internal problem ID [14055]

Book: INTRODUCTORY DIFFERENTIAL EQUATIONS. Martha L. Abell, James P. Braselton. Fourth edition 2014. ElScAe. 2014
Section: Chapter 1. Introduction to Differential Equations. Exercises 1.1, page 10
Problem number: 18.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_separable]`

$$\boxed{yx + y' = 0}$$

✓ Solution by Maple
Time used: 0.0 (sec). Leaf size: 12

```
dsolve(diff(y(x),x)+x*y(x)=0,y(x), singsol=all)
```

$$y(x) = e^{-\frac{x^2}{2}} c_1$$

✓ Solution by Mathematica
Time used: 0.021 (sec). Leaf size: 22

```
DSolve[y'[x]+x*y[x]==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to c_1 e^{-\frac{x^2}{2}}$$
$$y(x) \to 0$$

1.12 problem 19

Internal problem ID [14056]

Book: INTRODUCTORY DIFFERENTIAL EQUATIONS. Martha L. Abell, James P. Braselton. Fourth edition 2014. ElScAe. 2014
Section: Chapter 1. Introduction to Differential Equations. Exercises 1.1, page 10
Problem number: 19.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[[_linear, 'class A']]`

$$\boxed{y' + y = \sin(x)}$$

✓ Solution by Maple
Time used: 0.0 (sec). Leaf size: 19

```
dsolve(diff(y(x),x)+y(x)=sin(x),y(x), singsol=all)
```

$$y(x) = -\frac{\cos(x)}{2} + \frac{\sin(x)}{2} + c_1 e^{-x}$$

✓ Solution by Mathematica
Time used: 0.056 (sec). Leaf size: 25

```
DSolve[y'[x]+y[x]==Sin[x],y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to \frac{1}{2}(\sin(x) - \cos(x) + 2c_1 e^{-x})$$

1.13 problem 20

Internal problem ID [14057]

Book: INTRODUCTORY DIFFERENTIAL EQUATIONS. Martha L. Abell, James P. Braselton. Fourth edition 2014. ElScAe. 2014
Section: Chapter 1. Introduction to Differential Equations. Exercises 1.1, page 10
Problem number: 20.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _missing_x]]`

$$\boxed{y'' - y' - 12y = 0}$$

✓ Solution by Maple

Time used: 0.015 (sec). Leaf size: 17

```
dsolve(diff(y(t),t$2)-diff(y(t),t)-12*y(t)=0,y(t), singsol=all)
```

$$y(t) = \left(\mathrm{e}^{7t} c_2 + c_1\right) \mathrm{e}^{-3t}$$

✓ Solution by Mathematica

Time used: 0.014 (sec). Leaf size: 22

```
DSolve[y''[t]-y'[t]-12*y[t]==0,y[t],t,IncludeSingularSolutions -> True]
```

$$y(t) \to e^{-3t}\left(c_2 e^{7t} + c_1\right)$$

1.14 problem 21

Internal problem ID [14058]

Book: INTRODUCTORY DIFFERENTIAL EQUATIONS. Martha L. Abell, James P. Braselton. Fourth edition 2014. ElScAe. 2014
Section: Chapter 1. Introduction to Differential Equations. Exercises 1.1, page 10
Problem number: 21.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _missing_x]]`

$$\boxed{y'' + 9y' = 0}$$

✓ Solution by Maple

Time used: 0.016 (sec). Leaf size: 12

```
dsolve(diff(y(t),t$2)+9*diff(y(t),t)=0,y(t), singsol=all)
```

$$y(t) = c_1 + c_2 e^{-9t}$$

✓ Solution by Mathematica

Time used: 0.015 (sec). Leaf size: 19

```
DSolve[y''[t]+9*y'[t]==0,y[t],t,IncludeSingularSolutions -> True]
```

$$y(t) \to c_2 - \frac{1}{9} c_1 e^{-9t}$$

1.15 problem 22

Internal problem ID [14059]

Book: INTRODUCTORY DIFFERENTIAL EQUATIONS. Martha L. Abell, James P. Braselton. Fourth edition 2014. ElScAe. 2014
Section: Chapter 1. Introduction to Differential Equations. Exercises 1.1, page 10
Problem number: 22.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _missing_x]]`

$$\boxed{x'' + 2x' - 10x = 0}$$

✓ Solution by Maple
Time used: 0.0 (sec). Leaf size: 26

```
dsolve(diff(x(t),t$2)+2*diff(x(t),t)-10*x(t)=0,x(t), singsol=all)
```

$$x(t) = c_1 \mathrm{e}^{\left(-1+\sqrt{11}\right)t} + c_2 \mathrm{e}^{-\left(1+\sqrt{11}\right)t}$$

✓ Solution by Mathematica
Time used: 0.023 (sec). Leaf size: 34

```
DSolve[x''[t]+2*x'[t]-10*x[t]==0,x[t],t,IncludeSingularSolutions -> True]
```

$$x(t) \to e^{-\left(\left(1+\sqrt{11}\right)t\right)}\left(c_2 e^{2\sqrt{11}t} + c_1\right)$$

1.16 problem 23

Internal problem ID [14060]

Book: INTRODUCTORY DIFFERENTIAL EQUATIONS. Martha L. Abell, James P. Braselton. Fourth edition 2014. ElScAe. 2014
Section: Chapter 1. Introduction to Differential Equations. Exercises 1.1, page 10
Problem number: 23.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _linear, _nonhomogeneous]]`

$$\boxed{x'' + x = \cos(t)\, t - \cos(t)}$$

✓ Solution by Maple
Time used: 0.0 (sec). Leaf size: 30

```
dsolve(diff(x(t),t$2)+x(t)=t*cos(t)-cos(t),x(t), singsol=all)
```

$$x(t) = \frac{(t^2 + 4c_2 - 2t - 1)\sin(t)}{4} + \frac{\cos(t)\,(t + 4c_1 - 2)}{4}$$

✓ Solution by Mathematica
Time used: 0.089 (sec). Leaf size: 38

```
DSolve[x''[t]+x[t]==t*Cos[t]-Cos[t],x[t],t,IncludeSingularSolutions -> True]
```

$$x(t) \to \frac{1}{8}\left((2t^2 - 4t - 1 + 8c_2)\sin(t) + 2(t - 2 + 4c_1)\cos(t)\right)$$

1.17 problem 24

Internal problem ID [14061]

Book: INTRODUCTORY DIFFERENTIAL EQUATIONS. Martha L. Abell, James P. Braselton. Fourth edition 2014. ElScAe. 2014
Section: Chapter 1. Introduction to Differential Equations. Exercises 1.1, page 10
Problem number: 24.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _missing_x]]`

$$\boxed{y'' - 12y' + 40y = 0}$$

✓ Solution by Maple
Time used: 0.0 (sec). Leaf size: 22

```
dsolve(diff(y(x),x$2)-12*diff(y(x),x)+40*y(x)=0,y(x), singsol=all)
```

$$y(x) = \mathrm{e}^{6x}(c_1 \sin (2x) + c_2 \cos (2x))$$

✓ Solution by Mathematica
Time used: 0.017 (sec). Leaf size: 26

```
DSolve[y''[x]-12*y'[x]+40*y[x]==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \rightarrow e^{6x}(c_2 \cos(2x) + c_1 \sin(2x))$$

1.18 problem 25

Internal problem ID [14062]

Book: INTRODUCTORY DIFFERENTIAL EQUATIONS. Martha L. Abell, James P. Braselton. Fourth edition 2014. ElScAe. 2014
Section: Chapter 1. Introduction to Differential Equations. Exercises 1.1, page 10
Problem number: 25.
ODE order: 3.
ODE degree: 1.

CAS Maple gives this as type `[[_3rd_order, _missing_x]]`

$$\boxed{y''' - 4y'' = 0}$$

✓ Solution by Maple
Time used: 0.0 (sec). Leaf size: 15

```
dsolve(diff(y(x),x$3)-4*diff(y(x),x$2)=0,y(x), singsol=all)
```

$$y(x) = c_1 + c_2 x + c_3 e^{4x}$$

✓ Solution by Mathematica
Time used: 0.035 (sec). Leaf size: 23

```
DSolve[y'''[x]-4*y''[x]==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to \frac{1}{16} c_1 e^{4x} + c_3 x + c_2$$

1.19 problem 26

Internal problem ID [14063]

Book: INTRODUCTORY DIFFERENTIAL EQUATIONS. Martha L. Abell, James P. Braselton. Fourth edition 2014. ElScAe. 2014
Section: Chapter 1. Introduction to Differential Equations. Exercises 1.1, page 10
Problem number: 26.
ODE order: 3.
ODE degree: 1.

CAS Maple gives this as type `[[_3rd_order, _missing_x]]`

$$\boxed{y''' - 2y'' = 0}$$

✓ Solution by Maple
Time used: 0.0 (sec). Leaf size: 15

```
dsolve(diff(y(x),x$3)-2*diff(y(x),x$2)=0,y(x), singsol=all)
```

$$y(x) = c_1 + c_2x + c_3e^{2x}$$

✓ Solution by Mathematica
Time used: 0.035 (sec). Leaf size: 23

```
DSolve[y'''[x]-2*y''[x]==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to \frac{1}{4}c_1e^{2x} + c_3x + c_2$$

1.20 problem 27

Internal problem ID [14064]

Book: INTRODUCTORY DIFFERENTIAL EQUATIONS. Martha L. Abell, James P. Braselton. Fourth edition 2014. ElScAe. 2014
Section: Chapter 1. Introduction to Differential Equations. Exercises 1.1, page 10
Problem number: 27.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_Emden, _Fowler]]`

$$\boxed{x^2y'' - 12y'x + 42y = 0}$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 13

```
dsolve(x^2*diff(y(x),x$2)-12*x*diff(y(x),x)+42*y(x)=0,y(x), singsol=all)
```

$$y(x) = x^6(c_2x + c_1)$$

✓ Solution by Mathematica

Time used: 0.011 (sec). Leaf size: 16

```
DSolve[x^2*y''[x]-12*x*y'[x]+42*y[x]==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to x^6(c_2x + c_1)$$

1.21 problem 28

Internal problem ID [14065]

Book: INTRODUCTORY DIFFERENTIAL EQUATIONS. Martha L. Abell, James P. Braselton. Fourth edition 2014. ElScAe. 2014
Section: Chapter 1. Introduction to Differential Equations. Exercises 1.1, page 10
Problem number: 28.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_Emden, _Fowler]]`

$$\boxed{t^2 y'' + 3y't + 5y = 0}$$

✓ Solution by Maple
Time used: 0.0 (sec). Leaf size: 23

```
dsolve(t^2*diff(y(t),t$2)+3*t*diff(y(t),t)+5*y(t)=0,y(t), singsol=all)
```

$$y(t) = \frac{c_1 \sin (2 \ln (t)) + c_2 \cos (2 \ln (t))}{t}$$

✓ Solution by Mathematica
Time used: 0.028 (sec). Leaf size: 26

```
DSolve[t^2*y''[t]+3*t*y'[t]+5*y[t]==0,y[t],t,IncludeSingularSolutions -> True]
```

$$y(t) \to \frac{c_2 \cos(2 \log(t)) + c_1 \sin(2 \log(t))}{t}$$

1.22 problem 29

Internal problem ID [14066]

Book: INTRODUCTORY DIFFERENTIAL EQUATIONS. Martha L. Abell, James P. Braselton. Fourth edition 2014. ElScAe. 2014
Section: Chapter 1. Introduction to Differential Equations. Exercises 1.1, page 10
Problem number: 29.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_separable]`

$$\boxed{y' + \frac{x}{y} = 0}$$

✓ Solution by Maple
Time used: 0.0 (sec). Leaf size: 27

```
dsolve(diff(y(x),x)=-x/y(x),y(x), singsol=all)
```

$$y(x) = \sqrt{-x^2 + c_1}$$
$$y(x) = -\sqrt{-x^2 + c_1}$$

✓ Solution by Mathematica
Time used: 0.201 (sec). Leaf size: 39

```
DSolve[y'[x]==-x/y[x],y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to -\sqrt{-x^2 + 2c_1}$$
$$y(x) \to \sqrt{-x^2 + 2c_1}$$

1.23 problem 30

Internal problem ID [14067]

Book: INTRODUCTORY DIFFERENTIAL EQUATIONS. Martha L. Abell, James P. Braselton. Fourth edition 2014. ElScAe. 2014
Section: Chapter 1. Introduction to Differential Equations. Exercises 1.1, page 10
Problem number: 30.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[[_homogeneous, 'class G'], _exact, _rational, [_Abel, '2nd ty`

$$\boxed{3y(t^2+y)+t(t^2+6y)\,y'=0}$$

✓ Solution by Maple
Time used: 0.094 (sec). Leaf size: 53

```
dsolve(3*y(t)*(t^2+y(t))+t*(t^2+6*y(t))*diff(y(t),t)=0,y(t), singsol=all)
```

$$y(t)=\frac{-t^3+\sqrt{t\,(t^5+12c_1)}}{6t}$$
$$y(t)=\frac{-t^3-\sqrt{t\,(t^5+12c_1)}}{6t}$$

✓ Solution by Mathematica
Time used: 0.003 (sec). Leaf size: 114

```
DSolve[3*y[t]*(t^2+y[t])+t*(t^2+6*y[t])*y'[x]==0,y[t],t,IncludeSingularSolutions -> True]
```

$$y(t)\to\frac{1}{6}\left(-\sqrt{3}\sqrt{t^2\,(3t^2+8ty'(x)+12y'(x)^2)}-3t^2-6ty'(x)\right)$$
$$y(t)\to\frac{1}{6}\left(\sqrt{3}\sqrt{t^2\,(3t^2+8ty'(x)+12y'(x)^2)}-3t^2-6ty'(x)\right)$$

1.24 problem 31

Internal problem ID [14068]

Book: INTRODUCTORY DIFFERENTIAL EQUATIONS. Martha L. Abell, James P. Braselton. Fourth edition 2014. ElScAe. 2014
Section: Chapter 1. Introduction to Differential Equations. Exercises 1.1, page 10
Problem number: 31.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_linear]`

$$\boxed{y' + \frac{2y}{x} = -3}$$

✓ Solution by Maple
Time used: 0.0 (sec). Leaf size: 13

```
dsolve(diff(y(x),x)=-2*y(x)/x-3,y(x), singsol=all)
```

$$y(x) = -x + \frac{c_1}{x^2}$$

✓ Solution by Mathematica
Time used: 0.022 (sec). Leaf size: 15

```
DSolve[y'[x]==-2*y[x]/x-3,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to -x + \frac{c_1}{x^2}$$

1.25 problem 32

Internal problem ID [14069]

Book: INTRODUCTORY DIFFERENTIAL EQUATIONS. Martha L. Abell, James P. Braselton. Fourth edition 2014. ElScAe. 2014
Section: Chapter 1. Introduction to Differential Equations. Exercises 1.1, page 10
Problem number: 32.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_exact, [_1st_order, '_with_symmetry_[F(x),G(y)]'], [_Abel, '`

$$\boxed{\cos(t)\, y + (2y + \sin(t))\, y' = 0}$$

✓ Solution by Maple
Time used: 0.0 (sec). Leaf size: 41

```
dsolve(y(t)*cos(t)+(2*y(t)+sin(t))*diff(y(t),t)=0,y(t), singsol=all)
```

$$y(t) = -\frac{\sin(t)}{2} - \frac{\sqrt{\sin(t)^2 - 4c_1}}{2}$$
$$y(t) = -\frac{\sin(t)}{2} + \frac{\sqrt{\sin(t)^2 - 4c_1}}{2}$$

✓ Solution by Mathematica
Time used: 0.151 (sec). Leaf size: 60

```
DSolve[y[t]*Cos[t]+(2*y[t]+Sin[t])*y'[t]==0,y[t],t,IncludeSingularSolutions -> True]
```

$$y(t) \to \frac{1}{2}\left(-\sin(t) - \sqrt{\sin^2(t) + 4c_1}\right)$$
$$y(t) \to \frac{1}{2}\left(-\sin(t) + \sqrt{\sin^2(t) + 4c_1}\right)$$
$$y(t) \to 0$$

1.26 problem 33

Internal problem ID [14070]

Book: INTRODUCTORY DIFFERENTIAL EQUATIONS. Martha L. Abell, James P. Braselton. Fourth edition 2014. ElScAe. 2014
Section: Chapter 1. Introduction to Differential Equations. Exercises 1.1, page 10
Problem number: 33.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_exact]`

$$\boxed{\frac{y}{x}+\cos(y)+(\ln(x)-\sin(y)\,x)\,y'=0}$$

✓ Solution by Maple
Time used: 0.015 (sec). Leaf size: 15

```
dsolve((y(x)/x+cos(y(x)))+(ln(x)-x*sin(y(x)))*diff(y(x),x)=0,y(x), singsol=all)
```

$$\cos(y(x))\,x+y(x)\ln(x)+c_1=0$$

✓ Solution by Mathematica
Time used: 0.221 (sec). Leaf size: 19

```
DSolve[(y[x]/x+Cos[y[x]])+(Log[x]-x*Sin[y[x]])*y'[x]==0,y[x],x,IncludeSingularSolutions -> T
```

$$\text{Solve}[-y(x)\log(x)-x\cos(y(x))=c_1,y(x)]$$

1.27 problem 34

Internal problem ID [14071]

Book: INTRODUCTORY DIFFERENTIAL EQUATIONS. Martha L. Abell, James P. Braselton. Fourth edition 2014. ElScAe. 2014
Section: Chapter 1. Introduction to Differential Equations. Exercises 1.1, page 10
Problem number: 34.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_quadrature]`

$$\boxed{y' = (x^2 - 1)(x^3 - 3x)^3}$$

✓ Solution by Maple
Time used: 0.0 (sec). Leaf size: 31

```
dsolve(diff(y(x),x)=(x^2-1)*(x^3-3*x)^3,y(x), singsol=all)
```

$$y(x) = \frac{1}{12}x^{12} - x^{10} + \frac{9}{2}x^8 - 9x^6 + \frac{27}{4}x^4 + c_1$$

✓ Solution by Mathematica
Time used: 0.002 (sec). Leaf size: 39

```
DSolve[y'[x]==(x^2-1)*(x^3-3*x)^3,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to \frac{x^{12}}{12} - x^{10} + \frac{9x^8}{2} - 9x^6 + \frac{27x^4}{4} + c_1$$

1.28 problem 35

Internal problem ID [14072]

Book: INTRODUCTORY DIFFERENTIAL EQUATIONS. Martha L. Abell, James P. Braselton. Fourth edition 2014. ElScAe. 2014
Section: Chapter 1. Introduction to Differential Equations. Exercises 1.1, page 10
Problem number: 35.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_quadrature]`

$$\boxed{y' = \sin(x^2)\, x}$$

✓ Solution by Maple
Time used: 0.0 (sec). Leaf size: 12

```
dsolve(diff(y(x),x)=x*sin(x^2),y(x), singsol=all)
```

$$y(x) = -\frac{\cos(x^2)}{2} + c_1$$

✓ Solution by Mathematica
Time used: 0.014 (sec). Leaf size: 16

```
DSolve[y'[x]==x*Sin[x^2],y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to -\frac{\cos(x^2)}{2} + c_1$$

1.29 problem 36

Internal problem ID [14073]

Book: INTRODUCTORY DIFFERENTIAL EQUATIONS. Martha L. Abell, James P. Braselton. Fourth edition 2014. ElScAe. 2014
Section: Chapter 1. Introduction to Differential Equations. Exercises 1.1, page 10
Problem number: 36.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_quadrature]`

$$\boxed{y' = \frac{x}{\sqrt{x^2 - 16}}}$$

✓ Solution by Maple
Time used: 0.0 (sec). Leaf size: 20

```
dsolve(diff(y(x),x)=x/sqrt(x^2-16),y(x), singsol=all)
```

$$y(x) = \frac{(x-4)(x+4)}{\sqrt{x^2-16}} + c_1$$

✓ Solution by Mathematica
Time used: 0.004 (sec). Leaf size: 17

```
DSolve[y'[x]==x/Sqrt[x^2-16],y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to \sqrt{x^2-16} + c_1$$

1.30 problem 37

Internal problem ID [14074]

Book: INTRODUCTORY DIFFERENTIAL EQUATIONS. Martha L. Abell, James P. Braselton. Fourth edition 2014. ElScAe. 2014
Section: Chapter 1. Introduction to Differential Equations. Exercises 1.1, page 10
Problem number: 37.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_quadrature]`

$$\boxed{y' = \frac{1}{x \ln (x)}}$$

✓ Solution by Maple
Time used: 0.0 (sec). Leaf size: 9

```
dsolve(diff(y(x),x)=1/(x*ln(x)),y(x), singsol=all)
```

$$y(x) = \ln (\ln (x)) + c_1$$

✓ Solution by Mathematica
Time used: 0.006 (sec). Leaf size: 11

```
DSolve[y'[x]==1/(x*Log[x]),y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to \log(\log(x)) + c_1$$

1.31 problem 38

Internal problem ID [14075]

Book: INTRODUCTORY DIFFERENTIAL EQUATIONS. Martha L. Abell, James P. Braselton. Fourth edition 2014. ElScAe. 2014
Section: Chapter 1. Introduction to Differential Equations. Exercises 1.1, page 10
Problem number: 38.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_quadrature]`

$$\boxed{y' = x \ln(x)}$$

✓ Solution by Maple
Time used: 0.0 (sec). Leaf size: 18

```
dsolve(diff(y(x),x)=x*ln(x),y(x), singsol=all)
```

$$y(x) = \frac{\ln(x)\, x^2}{2} - \frac{x^2}{4} + c_1$$

✓ Solution by Mathematica
Time used: 0.003 (sec). Leaf size: 24

```
DSolve[y'[x]==x*Log[x],y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to -\frac{x^2}{4} + \frac{1}{2}x^2 \log(x) + c_1$$

1.32 problem 39

Internal problem ID [14076]

Book: INTRODUCTORY DIFFERENTIAL EQUATIONS. Martha L. Abell, James P. Braselton. Fourth edition 2014. ElScAe. 2014
Section: Chapter 1. Introduction to Differential Equations. Exercises 1.1, page 10
Problem number: 39.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_quadrature]`

$$\boxed{y' = x\,\mathrm{e}^{-x}}$$

✓ Solution by Maple
Time used: 0.0 (sec). Leaf size: 16

```
dsolve(diff(y(x),x)=x*exp(-x),y(x), singsol=all)
```

$$y(x) = (-x-1)\,\mathrm{e}^{-x} + c_1$$

✓ Solution by Mathematica
Time used: 0.012 (sec). Leaf size: 18

```
DSolve[y'[x]==x*Exp[-x],y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to -e^{-x}(x+1) + c_1$$

1.33 problem 40

Internal problem ID [14077]

Book: INTRODUCTORY DIFFERENTIAL EQUATIONS. Martha L. Abell, James P. Braselton. Fourth edition 2014. ElScAe. 2014
Section: Chapter 1. Introduction to Differential Equations. Exercises 1.1, page 10
Problem number: 40.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_quadrature]`

$$\boxed{y' = \frac{-2x - 10}{(x+2)(-4+x)}}$$

✓ Solution by Maple
Time used: 0.0 (sec). Leaf size: 16

```
dsolve(diff(y(x),x)=(-2*(x+5))/((x+2)*(x-4)),y(x), singsol=all)
```

$$y(x) = -3\ln(x-4) + \ln(x+2) + c_1$$

✓ Solution by Mathematica
Time used: 0.005 (sec). Leaf size: 20

```
DSolve[y'[x]==(-2*(x+5))/((x+2)*(x-4)),y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to -3\log(4-x) + \log(x+2) + c_1$$

1.34 problem 41

Internal problem ID [14078]

Book: INTRODUCTORY DIFFERENTIAL EQUATIONS. Martha L. Abell, James P. Braselton. Fourth edition 2014. ElScAe. 2014
Section: Chapter 1. Introduction to Differential Equations. Exercises 1.1, page 10
Problem number: 41.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_quadrature]`

$$\boxed{y' = \frac{-x^2 + x}{(x+1)(x^2+1)}}$$

✓ Solution by Maple
Time used: 0.0 (sec). Leaf size: 14

```
dsolve(diff(y(x),x)=(x-x^2)/((x+1)*(x^2+1)),y(x), singsol=all)
```

$$y(x) = -\ln(1+x) + \arctan(x) + c_1$$

✓ Solution by Mathematica
Time used: 0.009 (sec). Leaf size: 16

```
DSolve[y'[x]==(x-x^2)/((x+1)*(x^2+1)),y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to \arctan(x) - \log(x+1) + c_1$$

1.35 problem 42

Internal problem ID [14079]

Book: INTRODUCTORY DIFFERENTIAL EQUATIONS. Martha L. Abell, James P. Braselton. Fourth edition 2014. ElScAe. 2014
Section: Chapter 1. Introduction to Differential Equations. Exercises 1.1, page 10
Problem number: 42.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_quadrature]`

$$\boxed{y' = \frac{\sqrt{x^2-16}}{x}}$$

✓ Solution by Maple
Time used: 0.0 (sec). Leaf size: 25

```
dsolve(diff(y(x),x)=sqrt(x^2-16)/x,y(x), singsol=all)
```

$$y(x) = \sqrt{x^2-16} + 4 \arctan\left(\frac{4}{\sqrt{x^2-16}}\right) + c_1$$

✓ Solution by Mathematica
Time used: 0.016 (sec). Leaf size: 33

```
DSolve[y'[x]==Sqrt[x^2-16]/x,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to -4 \arctan\left(\frac{\sqrt{x^2-16}}{4}\right) + \sqrt{x^2-16} + c_1$$

1.36 problem 43

Internal problem ID [14080]

Book: INTRODUCTORY DIFFERENTIAL EQUATIONS. Martha L. Abell, James P. Braselton. Fourth edition 2014. ElScAe. 2014
Section: Chapter 1. Introduction to Differential Equations. Exercises 1.1, page 10
Problem number: 43.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_quadrature]`

$$\boxed{y' = (-x^2+4)^{\frac{3}{2}}}$$

✓ Solution by Maple
Time used: 0.015 (sec). Leaf size: 32

```
dsolve(diff(y(x),x)=(4-x^2)^(3/2),y(x), singsol=all)
```

$$y(x) = \frac{(-x^3+10x)\sqrt{-x^2+4}}{4} + c_1 + 6\arcsin\left(\frac{x}{2}\right)$$

✓ Solution by Mathematica
Time used: 0.062 (sec). Leaf size: 49

```
DSolve[y'[x]==(4-x^2)^(3/2),y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to -12\arctan\left(\frac{\sqrt{4-x^2}}{x+2}\right) - \frac{1}{4}x\sqrt{4-x^2}(x^2-10) + c_1$$

1.37 problem 44

Internal problem ID [14081]

Book: INTRODUCTORY DIFFERENTIAL EQUATIONS. Martha L. Abell, James P. Braselton. Fourth edition 2014. ElScAe. 2014
Section: Chapter 1. Introduction to Differential Equations. Exercises 1.1, page 10
Problem number: 44.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_quadrature]`

$$\boxed{y' = \frac{1}{x^2 - 16}}$$

✓ Solution by Maple
Time used: 0.0 (sec). Leaf size: 18

```
dsolve(diff(y(x),x)=1/(x^2-16),y(x), singsol=all)
```

$$y(x) = -\frac{\ln(x+4)}{8} + \frac{\ln(x-4)}{8} + c_1$$

✓ Solution by Mathematica
Time used: 0.004 (sec). Leaf size: 26

```
DSolve[y'[x]==1/(x^2-16),y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to \frac{1}{8}(\log(4-x) - \log(x+4) + 8c_1)$$

1.38 problem 45

Internal problem ID [14082]

Book: INTRODUCTORY DIFFERENTIAL EQUATIONS. Martha L. Abell, James P. Braselton. Fourth edition 2014. ElScAe. 2014
Section: Chapter 1. Introduction to Differential Equations. Exercises 1.1, page 10
Problem number: 45.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_quadrature]`

$$\boxed{y' = \cos(x)\cot(x)}$$

✓ Solution by Maple
Time used: 0.0 (sec). Leaf size: 16

```
dsolve(diff(y(x),x)=cos(x)*cot(x),y(x), singsol=all)
```

$$y(x) = \cos(x) + \ln(\csc(x) - \cot(x)) + c_1$$

✓ Solution by Mathematica
Time used: 0.021 (sec). Leaf size: 26

```
DSolve[y'[x]==Cos[x]*Cot[x],y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to \cos(x) + \log\left(\sin\left(\frac{x}{2}\right)\right) - \log\left(\cos\left(\frac{x}{2}\right)\right) + c_1$$

1.39 problem 46

Internal problem ID [14083]

Book: INTRODUCTORY DIFFERENTIAL EQUATIONS. Martha L. Abell, James P. Braselton. Fourth edition 2014. ElScAe. 2014
Section: Chapter 1. Introduction to Differential Equations. Exercises 1.1, page 10
Problem number: 46.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_quadrature]`

$$\boxed{y' = \sin(x)^3 \tan(x)}$$

✓ Solution by Maple
Time used: 0.0 (sec). Leaf size: 22

```
dsolve(diff(y(x),x)=sin(x)^3*tan(x),y(x), singsol=all)
```

$$y(x) = -\frac{\sin(x)^3}{3} - \sin(x) + \ln(\sec(x) + \tan(x)) + c_1$$

✓ Solution by Mathematica
Time used: 0.014 (sec). Leaf size: 23

```
DSolve[y'[x]==Sin[x]^3*Tan[x],y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to \operatorname{arctanh}(\sin(x)) - \frac{1}{3}\sin^3(x) - \sin(x) + c_1$$

1.40 problem 47

Internal problem ID [14084]

Book: INTRODUCTORY DIFFERENTIAL EQUATIONS. Martha L. Abell, James P. Braselton. Fourth edition 2014. ElScAe. 2014
Section: Chapter 1. Introduction to Differential Equations. Exercises 1.1, page 10
Problem number: 47.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_quadrature]`

$$\boxed{y' + 2y = 0}$$

With initial conditions

$$[y(0) = 2]$$

✓ Solution by Maple
Time used: 0.0 (sec). Leaf size: 10

```
dsolve([diff(y(x),x)+2*y(x)=0,y(0) = 2],y(x), singsol=all)
```

$$y(x) = 2\,\mathrm{e}^{-2x}$$

✓ Solution by Mathematica
Time used: 0.072 (sec). Leaf size: 12

```
DSolve[{y'[x]+2*y[x]==0,{y[0]==2}},y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to 2e^{-2x}$$

1.41 problem 48

Internal problem ID [14085]

Book: INTRODUCTORY DIFFERENTIAL EQUATIONS. Martha L. Abell, James P. Braselton. Fourth edition 2014. ElScAe. 2014
Section: Chapter 1. Introduction to Differential Equations. Exercises 1.1, page 10
Problem number: 48.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[[_linear, 'class A']]`

$$\boxed{y' + y = \sin(t)}$$

With initial conditions

$$[y(0) = -1]$$

✓ Solution by Maple
Time used: 0.015 (sec). Leaf size: 19

```
dsolve([diff(y(t),t)+y(t)=sin(t),y(0) = -1],y(t), singsol=all)
```

$$y(t) = -\frac{\cos(t)}{2} + \frac{\sin(t)}{2} - \frac{e^{-t}}{2}$$

✓ Solution by Mathematica
Time used: 0.057 (sec). Leaf size: 23

```
DSolve[{y'[t]+y[t]==Sin[t],{y[0]==-1}},y[t],t,IncludeSingularSolutions -> True]
```

$$y(t) \to \frac{1}{2}\left(-e^{-t} + \sin(t) - \cos(t)\right)$$

1.42 problem 49

Internal problem ID [14086]

Book: INTRODUCTORY DIFFERENTIAL EQUATIONS. Martha L. Abell, James P. Braselton. Fourth edition 2014. ElScAe. 2014
Section: Chapter 1. Introduction to Differential Equations. Exercises 1.1, page 10
Problem number: 49.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _missing_x]]`

$$\boxed{y'' - y' - 12y = 0}$$

With initial conditions

$$[y(0) = 1, y'(0) = -1]$$

✓ Solution by Maple
Time used: 0.016 (sec). Leaf size: 18

```
dsolve([diff(y(x),x$2)-diff(y(x),x)-12*y(x)=0,y(0) = 1, D(y)(0) = -1],y(x), singsol=all)
```

$$y(x) = \frac{(2\,e^{7x} + 5)\,e^{-3x}}{7}$$

✓ Solution by Mathematica
Time used: 0.024 (sec). Leaf size: 23

```
DSolve[{y''[x]-y'[x]-12*y[x]==0,{y[0]==1,y'[0]==-1}},y[x],x,IncludeSingularSolutions -> True
```

$$y(x) \to \frac{1}{7}e^{-3x}(2e^{7x} + 5)$$

1.43 problem 50

Internal problem ID [14087]

Book: INTRODUCTORY DIFFERENTIAL EQUATIONS. Martha L. Abell, James P. Braselton. Fourth edition 2014. ElScAe. 2014
Section: Chapter 1. Introduction to Differential Equations. Exercises 1.1, page 10
Problem number: 50.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _missing_x]]`

$$\boxed{y'' + 9y' = 0}$$

With initial conditions

$$[y(0) = 2, y'(0) = -1]$$

✓ Solution by Maple
Time used: 0.016 (sec). Leaf size: 12

```
dsolve([diff(y(x),x$2)+9*diff(y(x),x)=0,y(0) = 2, D(y)(0) = -1],y(x), singsol=all)
```

$$y(x) = \frac{17}{9} + \frac{e^{-9x}}{9}$$

✓ Solution by Mathematica
Time used: 0.014 (sec). Leaf size: 16

```
DSolve[{y''[x]+9*y'[x]==0,{y[0]==2,y'[0]==-1}},y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to \frac{1}{9}\left(e^{-9x} + 17\right)$$

1.44 problem 51

Internal problem ID [14088]

Book: INTRODUCTORY DIFFERENTIAL EQUATIONS. Martha L. Abell, James P. Braselton. Fourth edition 2014. ElScAe. 2014
Section: Chapter 1. Introduction to Differential Equations. Exercises 1.1, page 10
Problem number: 51.
ODE order: 3.
ODE degree: 1.

CAS Maple gives this as type `[[_3rd_order, _missing_x]]`

$$\boxed{y''' - 2y'' = 0}$$

With initial conditions

$$[y(0) = 0, y'(0) = 1, y''(0) = 3]$$

✓ Solution by Maple
Time used: 0.015 (sec). Leaf size: 15

```
dsolve([diff(y(x),x$3)-2*diff(y(x),x$2)=0,y(0) = 0, D(y)(0) = 1, (D@@2)(y)(0) = 3],y(x), sin
```

$$y(x) = -\frac{3}{4} - \frac{x}{2} + \frac{3\,e^{2x}}{4}$$

✓ Solution by Mathematica
Time used: 0.033 (sec). Leaf size: 21

```
DSolve[{y'''[x]-2*y''[x]==0,{y[0]==0,y'[0]==1,y''[0]==3}},y[x],x,IncludeSingularSolutions ->
```

$$y(x) \to \frac{1}{4}\left(-2x + 3e^{2x} - 3\right)$$

1.45 problem 52

Internal problem ID [14089]

Book: INTRODUCTORY DIFFERENTIAL EQUATIONS. Martha L. Abell, James P. Braselton. Fourth edition 2014. ElScAe. 2014
Section: Chapter 1. Introduction to Differential Equations. Exercises 1.1, page 10
Problem number: 52.
ODE order: 3.
ODE degree: 1.

CAS Maple gives this as type `[[_3rd_order, _missing_x]]`

$$\boxed{y''' - 4y' = 0}$$

With initial conditions

$$[y(0) = 1, y'(0) = -1, y''(0) = 0]$$

✓ Solution by Maple
Time used: 0.015 (sec). Leaf size: 18

```
dsolve([diff(y(x),x$3)-4*diff(y(x),x)=0,y(0) = 1, D(y)(0) = -1, (D@@2)(y)(0) = 0],y(x), sing
```

$$y(x) = 1 - \frac{e^{2x}}{4} + \frac{e^{-2x}}{4}$$

✓ Solution by Mathematica
Time used: 0.026 (sec). Leaf size: 23

```
DSolve[{y'''[x]-4*y'[x]==0,{y[0]==1,y'[0]==-1,y''[0]==0}},y[x],x,IncludeSingularSolutions ->
```

$$y(x) \to \frac{1}{4}\left(e^{-2x} - e^{2x} + 4\right)$$

1.46 problem 53

Internal problem ID [14090]

Book: INTRODUCTORY DIFFERENTIAL EQUATIONS. Martha L. Abell, James P. Braselton. Fourth edition 2014. ElScAe. 2014
Section: Chapter 1. Introduction to Differential Equations. Exercises 1.1, page 10
Problem number: 53.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_Emden, _Fowler]]`

$$\boxed{t^2y'' - 12y't + 42y = 0}$$

With initial conditions

$$[y(1) = 0, y'(1) = -1]$$

✓ Solution by Maple
Time used: 0.016 (sec). Leaf size: 13

```
dsolve([t^2*diff(y(t),t$2)-12*t*diff(y(t),t)+42*y(t)=0,y(1) = 0, D(y)(1) = -1],y(t), singsol
```

$$y(t) = -t^7 + t^6$$

✓ Solution by Mathematica
Time used: 0.01 (sec). Leaf size: 13

```
DSolve[{t^2*y''[t]-12*t*y'[t]+42*y[t]==0,{y[1]==0,y'[1]==-1}},y[t],t,IncludeSingularSolution
```

$$y(t) \to -\left((t-1)t^6\right)$$

1.47 problem 54

Internal problem ID [14091]

Book: INTRODUCTORY DIFFERENTIAL EQUATIONS. Martha L. Abell, James P. Braselton. Fourth edition 2014. ElScAe. 2014
Section: Chapter 1. Introduction to Differential Equations. Exercises 1.1, page 10
Problem number: 54.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_Emden, _Fowler]]`

$$\boxed{x^2y'' + 3y'x + 5y = 0}$$

With initial conditions

$$[y(1) = 0, y'(1) = 1]$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 14

```
dsolve([x^2*diff(y(x),x$2)+3*x*diff(y(x),x)+5*y(x)=0,y(1) = 0, D(y)(1) = 1],y(x), singsol=al
```

$$y(x) = \frac{\sin(2\ln(x))}{2x}$$

✓ Solution by Mathematica

Time used: 0.032 (sec). Leaf size: 15

```
DSolve[{x^2*y''[x]+3*x*y'[x]+5*y[x]==0,{y[1]==0,y'[1]==1}},y[x],x,IncludeSingularSolutions -
```

$$y(x) \to \frac{\sin(\log(x))\cos(\log(x))}{x}$$

1.48 problem 55

Internal problem ID [14092]

Book: INTRODUCTORY DIFFERENTIAL EQUATIONS. Martha L. Abell, James P. Braselton. Fourth edition 2014. ElScAe. 2014
Section: Chapter 1. Introduction to Differential Equations. Exercises 1.1, page 10
Problem number: 55.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_quadrature]`

$$\boxed{y' = 4x^3 - x + 2}$$

With initial conditions

$$[y(0) = 1]$$

✓ Solution by Maple
Time used: 0.0 (sec). Leaf size: 17

```
dsolve([diff(y(x),x)=4*x^3-x+2,y(0) = 1],y(x), singsol=all)
```

$$y(x) = x^4 - \frac{1}{2}x^2 + 1 + 2x$$

✓ Solution by Mathematica
Time used: 0.003 (sec). Leaf size: 20

```
DSolve[{y'[x]==4*x^3-x+2,{y[0]==1}},y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to x^4 - \frac{x^2}{2} + 2x + 1$$

1.49 problem 56

Internal problem ID [14093]

Book: INTRODUCTORY DIFFERENTIAL EQUATIONS. Martha L. Abell, James P. Braselton. Fourth edition 2014. ElScAe. 2014
Section: Chapter 1. Introduction to Differential Equations. Exercises 1.1, page 10
Problem number: 56.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_quadrature]`

$$\boxed{y' = -\cos(2t) + \sin(2t)}$$

With initial conditions

$$[y(0) = 0]$$

✓ Solution by Maple
Time used: 0.016 (sec). Leaf size: 18

```
dsolve([diff(y(t),t)=sin(2*t)-cos(2*t),y(0) = 0],y(t), singsol=all)
```

$$y(t) = -\frac{\sin(2t)}{2} - \frac{\cos(2t)}{2} + \frac{1}{2}$$

✓ Solution by Mathematica
Time used: 0.044 (sec). Leaf size: 23

```
DSolve[{y'[t]==Sin[2*t]-Cos[2*t],{y[0]==0}},y[t],t,IncludeSingularSolutions -> True]
```

$$y(t) \to \frac{1}{2}(-\sin(2t) - \cos(2t) + 1)$$

1.50 problem 57

Internal problem ID [14094]

Book: INTRODUCTORY DIFFERENTIAL EQUATIONS. Martha L. Abell, James P. Braselton. Fourth edition 2014. ElScAe. 2014
Section: Chapter 1. Introduction to Differential Equations. Exercises 1.1, page 10
Problem number: 57.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type **[_quadrature]**

$$\boxed{y' = \frac{\cos\left(\frac{1}{x}\right)}{x^2}}$$

With initial conditions

$$\left[y\left(\frac{2}{\pi}\right) = 1\right]$$

✓ Solution by Maple
Time used: 0.0 (sec). Leaf size: 12

```
dsolve([diff(y(x),x)=cos(1/x)/x^2,y(2/Pi) = 1],y(x), singsol=all)
```

$$y(x) = -\sin\left(\frac{1}{x}\right) + 2$$

✓ Solution by Mathematica
Time used: 0.008 (sec). Leaf size: 13

```
DSolve[{y'[x]==Cos[1/x]/x^2,{y[2/Pi]==1}},y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to 2 - \sin\left(\frac{1}{x}\right)$$

1.51 problem 58

Internal problem ID [14095]

Book: INTRODUCTORY DIFFERENTIAL EQUATIONS. Martha L. Abell, James P. Braselton. Fourth edition 2014. ElScAe. 2014
Section: Chapter 1. Introduction to Differential Equations. Exercises 1.1, page 10
Problem number: 58.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_quadrature]`

$$\boxed{y' = \frac{\ln(x)}{x}}$$

With initial conditions

$$[y(1) = 0]$$

✓ Solution by Maple
Time used: 0.0 (sec). Leaf size: 10

```
dsolve([diff(y(x),x)=ln(x)/x,y(1) = 0],y(x), singsol=all)
```

$$y(x) = \frac{\ln(x)^2}{2}$$

✓ Solution by Mathematica
Time used: 0.004 (sec). Leaf size: 13

```
DSolve[{y'[x]==Log[x]/x,{y[1]==0}},y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to \frac{\log^2(x)}{2}$$

1.52 problem 72

Internal problem ID [14096]

Book: INTRODUCTORY DIFFERENTIAL EQUATIONS. Martha L. Abell, James P. Braselton. Fourth edition 2014. ElScAe. 2014
Section: Chapter 1. Introduction to Differential Equations. Exercises 1.1, page 10
Problem number: 72.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_separable]`

$$\boxed{y' - \frac{(-4+x)\,y^3}{x^3\,(y-2)} = 0}$$

✓ Solution by Maple
Time used: 0.0 (sec). Leaf size: 76

```
dsolve(diff(y(x),x)=(x-4)*y(x)^3/(x^3*(y(x)-2)),y(x), singsol=all)
```

$$y(x) = -\frac{\left(x+\sqrt{8+(4c_1+1)\,x^2-4x}\right)x}{2c_1x^2-2x+4}$$
$$y(x) = \frac{x\left(-x+\sqrt{8+(4c_1+1)\,x^2-4x}\right)}{2c_1x^2-2x+4}$$

✓ Solution by Mathematica
Time used: 0.66 (sec). Leaf size: 91

```
DSolve[y'[x]==(x-4)*y[x]^3/(x^3*(y[x]-2)),y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to \frac{x\left(-x+\sqrt{x^2+4c_1x^2-4x+8}\right)}{2c_1x^2-2x+4}$$
$$y(x) \to -\frac{x\left(x+\sqrt{x^2+4c_1x^2-4x+8}\right)}{2c_1x^2-2x+4}$$
$$y(x) \to 0$$

1.53 problem 73

Internal problem ID [14097]

Book: INTRODUCTORY DIFFERENTIAL EQUATIONS. Martha L. Abell, James P. Braselton. Fourth edition 2014. ElScAe. 2014
Section: Chapter 1. Introduction to Differential Equations. Exercises 1.1, page 10
Problem number: 73.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[[_homogeneous, ‘class A‘], _rational, _Bernoulli]`

$$\boxed{y' - \frac{y^2 + 2yx}{x^2} = 0}$$

✓ Solution by Maple
Time used: 0.015 (sec). Leaf size: 15

```
dsolve(diff(y(x),x)=(y(x)^2+2*x*y(x))/x^2,y(x), singsol=all)
```

$$y(x) = \frac{x^2}{c_1 - x}$$

✓ Solution by Mathematica
Time used: 0.124 (sec). Leaf size: 23

```
DSolve[y'[x]==(y[x]^2+2*x*y[x])/x^2,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to -\frac{x^2}{x - c_1}$$
$$y(x) \to 0$$

1.54 problem 75

Internal problem ID [14098]

Book: INTRODUCTORY DIFFERENTIAL EQUATIONS. Martha L. Abell, James P. Braselton. Fourth edition 2014. ElScAe. 2014
Section: Chapter 1. Introduction to Differential Equations. Exercises 1.1, page 10
Problem number: 75.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_linear]`

$$\boxed{y'x + y = \cos(x)}$$

✓ Solution by Maple
Time used: 0.0 (sec). Leaf size: 12

```
dsolve(x*diff(y(x),x)+y(x)=cos(x),y(x), singsol=all)
```

$$y(x) = \frac{\sin(x) + c_1}{x}$$

✓ Solution by Mathematica
Time used: 0.035 (sec). Leaf size: 14

```
DSolve[x*y'[x]+y[x]==Cos[x],y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to \frac{\sin(x) + c_1}{x}$$

1.55 problem 76

Internal problem ID [14099]

Book: INTRODUCTORY DIFFERENTIAL EQUATIONS. Martha L. Abell, James P. Braselton. Fourth edition 2014. ElScAe. 2014
Section: Chapter 1. Introduction to Differential Equations. Exercises 1.1, page 10
Problem number: 76.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _missing_x]]`

$$\boxed{16y'' + 24y' + 153y = 0}$$

✓ Solution by Maple
Time used: 0.0 (sec). Leaf size: 22

```
dsolve(16*diff(y(x),x$2)+24*diff(y(x),x)+153*y(x)=0,y(x), singsol=all)
```

$$y(x) = \mathrm{e}^{-\frac{3x}{4}}(c_1 \sin(3x) + c_2 \cos(3x))$$

✓ Solution by Mathematica
Time used: 0.018 (sec). Leaf size: 28

```
DSolve[16*y''[x]+24*y'[x]+153*y[x]==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to e^{-3x/4}(c_2 \cos(3x) + c_1 \sin(3x))$$

1.56 problem 77

Internal problem ID [14100]

Book: INTRODUCTORY DIFFERENTIAL EQUATIONS. Martha L. Abell, James P. Braselton. Fourth edition 2014. ElScAe. 2014
Section: Chapter 1. Introduction to Differential Equations. Exercises 1.1, page 10
Problem number: 77.
ODE order: 1.
ODE degree: 1.

Solve

$$x'(t) = 4y(t)$$
$$y'(t) = -x(t) - 2y(t)$$

✓ Solution by Maple
Time used: 0.031 (sec). Leaf size: 76

```
dsolve([diff(x(t),t)=4*y(t),diff(y(t),t)=-x(t)-2*y(t)],singsol=all)
```

$$x(t) = \mathrm{e}^{-t}\left(\sin\left(\sqrt{3}\,t\right) c_1 + \cos\left(\sqrt{3}\,t\right) c_2\right)$$
$$y(t) = -\frac{\mathrm{e}^{-t}\left(\sqrt{3}\,\sin\left(\sqrt{3}\,t\right) c_2 - \sqrt{3}\,\cos\left(\sqrt{3}\,t\right) c_1 + \sin\left(\sqrt{3}\,t\right) c_1 + \cos\left(\sqrt{3}\,t\right) c_2\right)}{4}$$

✓ Solution by Mathematica
Time used: 0.034 (sec). Leaf size: 93

```
DSolve[{x'[t]==4*y[t],y'[t]==-x[t]-2*y[t]},{x[t],y[t]},t,IncludeSingularSolutions -> True]
```

$$x(t) \to \frac{1}{3}e^{-t}\left(3c_1\cos\left(\sqrt{3}t\right) + \sqrt{3}(c_1 + 4c_2)\sin\left(\sqrt{3}t\right)\right)$$
$$y(t) \to \frac{1}{3}e^{-t}\left(3c_2\cos\left(\sqrt{3}t\right) - \sqrt{3}(c_1 + c_2)\sin\left(\sqrt{3}t\right)\right)$$

1.57 problem 79

Internal problem ID [14101]

Book: INTRODUCTORY DIFFERENTIAL EQUATIONS. Martha L. Abell, James P. Braselton. Fourth edition 2014. ElScAe. 2014
Section: Chapter 1. Introduction to Differential Equations. Exercises 1.1, page 10
Problem number: 79.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_rational]`

$$\boxed{4x(x^2+y^2)-5y+4y(x^2+y^2-5x)\,y'=0}$$

✗ Solution by Maple

```
dsolve((4*x*(x^2+y(x)^2)-5*y(x))+(4*y(x)*(x^2+y(x)^2-5*x))*diff(y(x),x)=0,y(x), singsol=all)
```

No solution found

✗ Solution by Mathematica
Time used: 0.0 (sec). Leaf size: 0

```
DSolve[(4*x*(x^2+y[x]^2)-5*y[x])+(4*y[x]*(x^2+y[x]^2-5*x))*y'[x]==0,y[x],x,IncludeSingularSo
```

Not solved

1.58 problem 80

Internal problem ID [14102]

Book: INTRODUCTORY DIFFERENTIAL EQUATIONS. Martha L. Abell, James P. Braselton. Fourth edition 2014. ElScAe. 2014
Section: Chapter 1. Introduction to Differential Equations. Exercises 1.1, page 10
Problem number: 80.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_quadrature]`

$$\boxed{y' = \sin(x)^4}$$

With initial conditions

$$[y(0) = 0]$$

✓ Solution by Maple
Time used: 0.047 (sec). Leaf size: 20

```
dsolve([diff(y(x),x)=sin(x)^4,y(0) = 0],y(x), singsol=all)
```

$$y(x) = \frac{3x}{8} + \frac{\sin(4x)}{32} - \frac{\sin(2x)}{4}$$

✓ Solution by Mathematica
Time used: 0.012 (sec). Leaf size: 23

```
DSolve[{y'[x]==Sin[x]^4,{y[0]==0}},y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to \frac{1}{32}(12x - 8\sin(2x) + \sin(4x))$$

1.59 problem 81

Internal problem ID [14103]

Book: INTRODUCTORY DIFFERENTIAL EQUATIONS. Martha L. Abell, James P. Braselton. Fourth edition 2014. ElScAe. 2014
Section: Chapter 1. Introduction to Differential Equations. Exercises 1.1, page 10
Problem number: 81.
ODE order: 4.
ODE degree: 1.

CAS Maple gives this as type `[[_high_order, _missing_x]]`

$$\boxed{y'''' + \frac{25y''}{2} - 5y' + \frac{629y}{16} = 0}$$

With initial conditions

$$[y(0) = 0, y'(0) = 1, y''(0) = -1, y'''(0) = 1]$$

✓ Solution by Maple
Time used: 0.062 (sec). Leaf size: 41

```
dsolve([diff(y(x),x$4)+25/2*diff(y(x),x$2)-5*diff(y(x),x)+629/16*y(x)=0,y(0) = 0, D(y)(0) =
```

$$y(x) = \frac{(74\cos(3x) + 20\sin(3x))\, e^{-\frac{x}{2}}}{208} - \frac{37\left(\cos(2x) - \frac{3\sin(2x)}{2}\right) e^{\frac{x}{2}}}{104}$$

✓ Solution by Mathematica
Time used: 0.01 (sec). Leaf size: 47

```
DSolve[{y''''[x]+25/2*y''[x]-5*y'[x]+629/16*y[x]==0,{y[0]==0,y'[0]==1,y''[0]==-1,y'''[0]==1}
```

$$y(x) \to \frac{1}{208} e^{-x/2}(111e^x \sin(2x) + 20\sin(3x) - 74e^x \cos(2x) + 74\cos(3x))$$

1.60 problem 82

Internal problem ID [14104]

Book: INTRODUCTORY DIFFERENTIAL EQUATIONS. Martha L. Abell, James P. Braselton. Fourth edition 2014. ElScAe. 2014
Section: Chapter 1. Introduction to Differential Equations. Exercises 1.1, page 10
Problem number: 82.
ODE order: 1.
ODE degree: 1.

Solve

$$x'(t) = 4y(t)$$
$$y'(t) = -4x(t)$$

With initial conditions

$$[x(0) = 4, y(0) = 0]$$

✓ Solution by Maple
Time used: 0.016 (sec). Leaf size: 20

```
dsolve([diff(x(t),t) = 4*y(t), diff(y(t),t) = -4*x(t), x(0) = 4, y(0) = 0], singsol=all)
```

$$x(t) = 4\cos(4t)$$
$$y(t) = -4\sin(4t)$$

✓ Solution by Mathematica
Time used: 0.002 (sec). Leaf size: 20

```
DSolve[{x'[t]==4*y[t],y'[t]==-4*x[t]},{x[0]==4,y[0]==0},{x[t],y[t]},t,IncludeSingularSolutio
```

$$x(t) \to 4\cos(4t)$$
$$y(t) \to -4\sin(4t)$$

1.61 problem 83

Internal problem ID [14105]

Book: INTRODUCTORY DIFFERENTIAL EQUATIONS. Martha L. Abell, James P. Braselton. Fourth edition 2014. ElScAe. 2014
Section: Chapter 1. Introduction to Differential Equations. Exercises 1.1, page 10
Problem number: 83.
ODE order: 1.
ODE degree: 1.

Solve

$$
\begin{aligned}
x'(t) &= -5x(t) + 4y(t) \\
y'(t) &= 2x(t) + 2y(t)
\end{aligned}
$$

With initial conditions

$$
[x(0) = 4, y(0) = 0]
$$

✓ Solution by Maple

Time used: 0.031 (sec). Leaf size: 34

```
dsolve([diff(x(t),t) = -5*x(t)+4*y(t), diff(y(t),t) = 2*x(t)+2*y(t), x(0) = 4, y(0) = 0], si
```

$$
\begin{aligned}
x(t) &= \frac{4\,\mathrm{e}^{3t}}{9} + \frac{32\,\mathrm{e}^{-6t}}{9} \\
y(t) &= \frac{8\,\mathrm{e}^{3t}}{9} - \frac{8\,\mathrm{e}^{-6t}}{9}
\end{aligned}
$$

✓ Solution by Mathematica

Time used: 0.008 (sec). Leaf size: 40

```
DSolve[{x'[t]==-5*x[t]+4*y[t],y'[t]==2*x[t]+2*y[t]},{x[0]==4,y[0]==0},{x[t],y[t]},t,IncludeS
```

$$
\begin{aligned}
x(t) &\to \frac{4}{9}e^{-6t}\left(e^{9t} + 8\right) \\
y(t) &\to \frac{8}{9}e^{-6t}\left(e^{9t} - 1\right)
\end{aligned}
$$

2 Chapter 1. Introduction to Differential Equations. Review exercises, page 23

2.1 problem 6

Internal problem ID [14106]

Book: INTRODUCTORY DIFFERENTIAL EQUATIONS. Martha L. Abell, James P. Braselton. Fourth edition 2014. ElScAe. 2014
Section: Chapter 1. Introduction to Differential Equations. Review exercises, page 23
Problem number: 6.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_separable]`

$$\boxed{y' + y\cos(x) = 0}$$

✓ Solution by Maple
Time used: 0.0 (sec). Leaf size: 11

```
dsolve(diff(y(x),x)+y(x)*cos(x)=0,y(x), singsol=all)
```

$$y(x) = \mathrm{e}^{-\sin(x)} c_1$$

✓ Solution by Mathematica
Time used: 0.028 (sec). Leaf size: 19

```
DSolve[y'[x]+y[x]*Cos[x]==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to c_1 e^{-\sin(x)}$$
$$y(x) \to 0$$

2.2 problem 7

Internal problem ID [14107]

Book: INTRODUCTORY DIFFERENTIAL EQUATIONS. Martha L. Abell, James P. Braselton. Fourth edition 2014. ElScAe. 2014
Section: Chapter 1. Introduction to Differential Equations. Review exercises, page 23
Problem number: 7.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[[_linear, 'class A']]`

$$\boxed{y' - y = \sin(x)}$$

✓ Solution by Maple
Time used: 0.0 (sec). Leaf size: 17

```
dsolve(diff(y(x),x)-y(x)=sin(x),y(x), singsol=all)
```

$$y(x) = -\frac{\cos(x)}{2} - \frac{\sin(x)}{2} + c_1 e^x$$

✓ Solution by Mathematica
Time used: 0.046 (sec). Leaf size: 24

```
DSolve[y'[x]-y[x]==Sin[x],y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to -\frac{\sin(x)}{2} - \frac{\cos(x)}{2} + c_1 e^x$$

2.3 problem 8

Internal problem ID [14108]

Book: INTRODUCTORY DIFFERENTIAL EQUATIONS. Martha L. Abell, James P. Braselton. Fourth edition 2014. ElScAe. 2014
Section: Chapter 1. Introduction to Differential Equations. Review exercises, page 23
Problem number: 8.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _missing_x]]`

$$\boxed{y'' + 4y' - 5y = 0}$$

✓ Solution by Maple
Time used: 0.0 (sec). Leaf size: 17

```
dsolve(diff(y(x),x$2)+4*diff(y(x),x)-5*y(x)=0,y(x), singsol=all)
```

$$y(x) = \left(e^{6x}c_2 + c_1\right) e^{-5x}$$

✓ Solution by Mathematica
Time used: 0.011 (sec). Leaf size: 20

```
DSolve[y''[x]+4*y'[x]-5*y[x]==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to c_1 e^{-5x} + c_2 e^x$$

2.4 problem 9

Internal problem ID [14109]

Book: INTRODUCTORY DIFFERENTIAL EQUATIONS. Martha L. Abell, James P. Braselton. Fourth edition 2014. ElScAe. 2014
Section: Chapter 1. Introduction to Differential Equations. Review exercises, page 23
Problem number: 9.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _missing_x]]`

$$\boxed{y'' - 6y' + 45y = 0}$$

✓ Solution by Maple
Time used: 0.0 (sec). Leaf size: 22

```
dsolve(diff(y(x),x$2)-6*diff(y(x),x)+45*y(x)=0,y(x), singsol=all)
```

$$y(x) = e^{3x}(c_1 \sin(6x) + c_2 \cos(6x))$$

✓ Solution by Mathematica
Time used: 0.022 (sec). Leaf size: 26

```
DSolve[y''[x]-6*y'[x]+45*y[x]==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to e^{3x}(c_2 \cos(6x) + c_1 \sin(6x))$$

2.5 problem 10

Internal problem ID [14110]

Book: INTRODUCTORY DIFFERENTIAL EQUATIONS. Martha L. Abell, James P. Braselton. Fourth edition 2014. ElScAe. 2014
Section: Chapter 1. Introduction to Differential Equations. Review exercises, page 23
Problem number: 10.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_Emden, _Fowler]]`

$$\boxed{x^2y'' - y'x - 16y = 0}$$

✓ Solution by Maple
Time used: 0.0 (sec). Leaf size: 23

```
dsolve(x^2*diff(y(x),x$2)-x*diff(y(x),x)-16*y(x)=0,y(x), singsol=all)
```

$$y(x) = x\left(x^{\sqrt{17}}c_1 + x^{-\sqrt{17}}c_2\right)$$

✓ Solution by Mathematica
Time used: 0.014 (sec). Leaf size: 32

```
DSolve[x^2*y''[x]-x*y'[x]-16*y[x]==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to x^{1-\sqrt{17}}\left(c_2x^{2\sqrt{17}} + c_1\right)$$

2.6 problem 11

Internal problem ID [14111]

Book: INTRODUCTORY DIFFERENTIAL EQUATIONS. Martha L. Abell, James P. Braselton. Fourth edition 2014. ElScAe. 2014
Section: Chapter 1. Introduction to Differential Equations. Review exercises, page 23
Problem number: 11.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_Emden, _Fowler]]`

$$\boxed{x^2y'' + 3y'x + 2y = 0}$$

✓ Solution by Maple
Time used: 0.0 (sec). Leaf size: 19

```
dsolve(x^2*diff(y(x),x$2)+3*x*diff(y(x),x)+2*y(x)=0,y(x), singsol=all)
```

$$y(x) = \frac{c_1 \sin(\ln(x)) + c_2 \cos(\ln(x))}{x}$$

✓ Solution by Mathematica
Time used: 0.024 (sec). Leaf size: 22

```
DSolve[x^2*y''[x]+3*x*y'[x]+2*y[x]==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to \frac{c_2 \cos(\log(x)) + c_1 \sin(\log(x))}{x}$$

2.7 problem 12

Internal problem ID [14112]

Book: INTRODUCTORY DIFFERENTIAL EQUATIONS. Martha L. Abell, James P. Braselton. Fourth edition 2014. ElScAe. 2014
Section: Chapter 1. Introduction to Differential Equations. Review exercises, page 23
Problem number: 12.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$\boxed{y'' + 2y' + 2y = x}$$

✓ Solution by Maple
Time used: 0.015 (sec). Leaf size: 25

```
dsolve(diff(y(x),x$2)+2*diff(y(x),x)+2*y(x)=x,y(x), singsol=all)
```

$$y(x) = \mathrm{e}^{-x} \sin(x) c_2 + \mathrm{e}^{-x} \cos(x) c_1 - \frac{1}{2} + \frac{x}{2}$$

✓ Solution by Mathematica
Time used: 0.015 (sec). Leaf size: 33

```
DSolve[y''[x]+2*y'[x]+2*y[x]==x,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to \frac{x-1}{2} + c_2 e^{-x} \cos(x) + c_1 e^{-x} \sin(x)$$

2.8 problem 13

Internal problem ID [14113]

Book: INTRODUCTORY DIFFERENTIAL EQUATIONS. Martha L. Abell, James P. Braselton. Fourth edition 2014. ElScAe. 2014
Section: Chapter 1. Introduction to Differential Equations. Review exercises, page 23
Problem number: 13.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _missing_x]]`

$$\boxed{y'' - 7y' + 12y = 2}$$

✓ Solution by Maple
Time used: 0.0 (sec). Leaf size: 18

```
dsolve(diff(y(x),x$2)-7*diff(y(x),x)+12*y(x)=2,y(x), singsol=all)
```

$$y(x) = c_2 \mathrm{e}^{4x} + c_1 \mathrm{e}^{3x} + \frac{1}{6}$$

✓ Solution by Mathematica
Time used: 0.013 (sec). Leaf size: 25

```
DSolve[y''[x]-7*y'[x]+12*y[x]==2,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to c_1 e^{3x} + c_2 e^{4x} + \frac{1}{6}$$

2.9 problem 14

Internal problem ID [14114]

Book: INTRODUCTORY DIFFERENTIAL EQUATIONS. Martha L. Abell, James P. Braselton. Fourth edition 2014. ElScAe. 2014
Section: Chapter 1. Introduction to Differential Equations. Review exercises, page 23
Problem number: 14.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[[_homogeneous, ‘class A‘], _exact, _rational, [_Abel, ‘2nd ty`

$$\boxed{-3y + (2y - 3x)\, y' = -2x}$$

✓ Solution by Maple
Time used: 0.078 (sec). Leaf size: 53

```
dsolve((2*x-3*y(x))+(2*y(x)-3*x)*diff(y(x),x)=0,y(x), singsol=all)
```

$$y(x) = \frac{3c_1x - \sqrt{5c_1^2x^2 + 4}}{2c_1}$$
$$y(x) = \frac{3c_1x + \sqrt{5c_1^2x^2 + 4}}{2c_1}$$

✓ Solution by Mathematica
Time used: 0.48 (sec). Leaf size: 110

```
DSolve[(2*x-3*y[x])+(2*y[x]-3*x)*y'[x]==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to \frac{1}{2}\left(3x - \sqrt{5x^2 + 4e^{c_1}}\right)$$
$$y(x) \to \frac{1}{2}\left(3x + \sqrt{5x^2 + 4e^{c_1}}\right)$$
$$y(x) \to \frac{1}{2}\left(3x - \sqrt{5}\sqrt{x^2}\right)$$
$$y(x) \to \frac{1}{2}\left(\sqrt{5}\sqrt{x^2} + 3x\right)$$

2.10 problem 15

Internal problem ID [14115]

Book: INTRODUCTORY DIFFERENTIAL EQUATIONS. Martha L. Abell, James P. Braselton. Fourth edition 2014. ElScAe. 2014
Section: Chapter 1. Introduction to Differential Equations. Review exercises, page 23
Problem number: 15.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_exact, [_1st_order, '_with_symmetry_[F(x),G(x)*y+H(x)]']]`

$$\boxed{y\cos(yx) + x\cos(yx)\, y' = -\sin(x)}$$

✓ Solution by Maple
Time used: 0.0 (sec). Leaf size: 16

```
dsolve((y(x)*cos(x*y(x))+sin(x))+(x*cos(x*y(x)))*diff(y(x),x)=0,y(x), singsol=all)
```

$$y(x) = -\frac{\arcsin(-\cos(x) + c_1)}{x}$$

✓ Solution by Mathematica
Time used: 1.144 (sec). Leaf size: 15

```
DSolve[(y[x]*Cos[x*y[x]]+Sin[x])+(x*Cos[x*y[x]])*y'[x]==0,y[x],x,IncludeSingularSolutions ->
```

$$y(x) \to \frac{\arcsin(\cos(x) + c_1)}{x}$$

2.11 problem 16

Internal problem ID [14116]

Book: INTRODUCTORY DIFFERENTIAL EQUATIONS. Martha L. Abell, James P. Braselton. Fourth edition 2014. ElScAe. 2014
Section: Chapter 1. Introduction to Differential Equations. Review exercises, page 23
Problem number: 16.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_quadrature]`

$$\boxed{y' = x\,\mathrm{e}^{-x^2}}$$

✓ Solution by Maple
Time used: 0.0 (sec). Leaf size: 14

```
dsolve(diff(y(x),x)=x*exp(-x^2),y(x), singsol=all)
```

$$y(x) = -\frac{\mathrm{e}^{-x^2}}{2} + c_1$$

✓ Solution by Mathematica
Time used: 0.013 (sec). Leaf size: 19

```
DSolve[y'[x]==x*Exp[-x^2],y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to -\frac{e^{-x^2}}{2} + c_1$$

2.12 problem 17

Internal problem ID [14117]

Book: INTRODUCTORY DIFFERENTIAL EQUATIONS. Martha L. Abell, James P. Braselton. Fourth edition 2014. ElScAe. 2014
Section: Chapter 1. Introduction to Differential Equations. Review exercises, page 23
Problem number: 17.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_quadrature]`

$$\boxed{y' = \sin(x) x^2}$$

✓ Solution by Maple
Time used: 0.0 (sec). Leaf size: 22

```
dsolve(diff(y(x),x)=x^2*sin(x),y(x), singsol=all)
```

$$y(x) = -x^2 \cos(x) + 2\cos(x) + 2x \sin(x) + c_1$$

✓ Solution by Mathematica
Time used: 0.018 (sec). Leaf size: 22

```
DSolve[y'[x]==x^2*Sin[x],y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to -(x^2 - 2)\cos(x) + 2x\sin(x) + c_1$$

2.13 problem 18

Internal problem ID [14118]

Book: INTRODUCTORY DIFFERENTIAL EQUATIONS. Martha L. Abell, James P. Braselton. Fourth edition 2014. ElScAe. 2014
Section: Chapter 1. Introduction to Differential Equations. Review exercises, page 23
Problem number: 18.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_quadrature]`

$$\boxed{y' = \frac{2x^2 - x + 1}{(x-1)(x^2+1)}}$$

✓ Solution by Maple
Time used: 0.0 (sec). Leaf size: 18

```
dsolve(diff(y(x),x)=(2*x^2-x+1)/( (x-1)*(x^2+1)),y(x), singsol=all)
```

$$y(x) = \frac{\ln(x^2+1)}{2} + \ln(-1+x) + c_1$$

✓ Solution by Mathematica
Time used: 0.007 (sec). Leaf size: 24

```
DSolve[y'[x]==(2*x^2-x+1)/( (x-1)*(x^2+1)),y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to \frac{1}{2}\log(x^2+1) + \log(1-x) + c_1$$

2.14 problem 19

Internal problem ID [14119]

Book: INTRODUCTORY DIFFERENTIAL EQUATIONS. Martha L. Abell, James P. Braselton. Fourth edition 2014. ElScAe. 2014
Section: Chapter 1. Introduction to Differential Equations. Review exercises, page 23
Problem number: 19.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_quadrature]`

$$\boxed{y' = \frac{x^2}{\sqrt{x^2-1}}}$$

✓ Solution by Maple
Time used: 0.0 (sec). Leaf size: 28

```
dsolve(diff(y(x),x)=x^2/sqrt(x^2-1),y(x), singsol=all)
```

$$y(x) = \frac{x\sqrt{x^2-1}}{2} + \frac{\ln\left(x+\sqrt{x^2-1}\right)}{2} + c_1$$

✓ Solution by Mathematica
Time used: 0.028 (sec). Leaf size: 40

```
DSolve[y'[x]==x^2/Sqrt[x^2-1],y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to \frac{1}{2}\sqrt{x^2-1}x - \frac{1}{2}\log\left(\sqrt{x^2-1}-x\right) + c_1$$

2.15 problem 20

Internal problem ID [14120]

Book: INTRODUCTORY DIFFERENTIAL EQUATIONS. Martha L. Abell, James P. Braselton. Fourth edition 2014. ElScAe. 2014
Section: Chapter 1. Introduction to Differential Equations. Review exercises, page 23
Problem number: 20.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[[_linear, 'class A']]`

$$\boxed{y' + 2y = x^2}$$

With initial conditions

$$[y(0) = 1]$$

✓ Solution by Maple
Time used: 0.0 (sec). Leaf size: 20

```
dsolve([diff(y(x),x)+2*y(x)=x^2,y(0) = 1],y(x), singsol=all)
```

$$y(x) = \frac{x^2}{2} - \frac{x}{2} + \frac{1}{4} + \frac{3\,e^{-2x}}{4}$$

✓ Solution by Mathematica
Time used: 0.048 (sec). Leaf size: 26

```
DSolve[{y'[x]+2*y[x]==x^2,{y[0]==1}},y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to \frac{1}{4}\left(2x^2 - 2x + 3e^{-2x} + 1\right)$$

2.16 problem 21

Internal problem ID [14121]

Book: INTRODUCTORY DIFFERENTIAL EQUATIONS. Martha L. Abell, James P. Braselton. Fourth edition 2014. ElScAe. 2014
Section: Chapter 1. Introduction to Differential Equations. Review exercises, page 23
Problem number: 21.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$\boxed{y'' + 4y = t}$$

With initial conditions

$$\left[y(0) = 1, y'\left(\frac{\pi}{4}\right) = \frac{\pi}{16}\right]$$

✗ Solution by Maple

```
dsolve([diff(y(t),t$2)+4*y(t)=t,y(0) = 1, D(y)(1/4*Pi) = 1/16*Pi],y(t), singsol=all)
```

No solution found

✗ Solution by Mathematica
Time used: 0.0 (sec). Leaf size: 0

```
DSolve[{y''[t]+4*y[t]==t,{y[0]==1,y'[Pi/4]==Pi/16}},y[t],t,IncludeSingularSolutions -> True]
```

{}

2.17 problem 22

Internal problem ID [14122]

Book: INTRODUCTORY DIFFERENTIAL EQUATIONS. Martha L. Abell, James P. Braselton. Fourth edition 2014. ElScAe. 2014
Section: Chapter 1. Introduction to Differential Equations. Review exercises, page 23
Problem number: 22.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_Emden, _Fowler]]`

$$\boxed{x^2y'' + 5y'x + 4y = 0}$$

With initial conditions

$$[y(1) = 1, y'(1) = 0]$$

✓ Solution by Maple
Time used: 0.016 (sec). Leaf size: 14

```
dsolve([x^2*diff(y(x),x$2)+5*x*diff(y(x),x)+4*y(x)=0,y(1) = 1, D(y)(1) = 0],y(x), singsol=al
```

$$y(x) = \frac{1 + 2\ln(x)}{x^2}$$

✓ Solution by Mathematica
Time used: 0.016 (sec). Leaf size: 15

```
DSolve[{x^2*y''[x]+5*x*y'[x]+4*y[x]==0,{y[1]==1,y'[1]==0}},y[x],x,IncludeSingularSolutions -
```

$$y(x) \to \frac{2\log(x) + 1}{x^2}$$

2.18 problem 23

Internal problem ID [14123]

Book: INTRODUCTORY DIFFERENTIAL EQUATIONS. Martha L. Abell, James P. Braselton. Fourth edition 2014. ElScAe. 2014
Section: Chapter 1. Introduction to Differential Equations. Review exercises, page 23
Problem number: 23.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_quadrature]`

$$\boxed{y' = \cos(x)^2 \sin(x)}$$

With initial conditions

$$[y(0) = 0]$$

✓ Solution by Maple
Time used: 0.016 (sec). Leaf size: 12

```
dsolve([diff(y(x),x)=cos(x)^2*sin(x),y(0) = 0],y(x), singsol=all)
```

$$y(x) = -\frac{\cos(x)^3}{3} + \frac{1}{3}$$

✓ Solution by Mathematica
Time used: 0.006 (sec). Leaf size: 17

```
DSolve[{y'[x]==Cos[x]^2*Sin[x],{y[0]==0}},y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to \frac{1}{3}(1 - \cos^3(x))$$

2.19 problem 24

Internal problem ID [14124]

Book: INTRODUCTORY DIFFERENTIAL EQUATIONS. Martha L. Abell, James P. Braselton. Fourth edition 2014. ElScAe. 2014
Section: Chapter 1. Introduction to Differential Equations. Review exercises, page 23
Problem number: 24.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_quadrature]`

$$\boxed{y' = \frac{4x - 9}{3 (x - 3)^{\frac{2}{3}}}}$$

With initial conditions

$$[y(0) = 0]$$

✓ Solution by Maple
Time used: 0.0 (sec). Leaf size: 11

```
dsolve([diff(y(x),x)=1/3*(4*x-9)*(x-3)^(-2/3),y(0) = 0],y(x), singsol=all)
```

$$y(x) = x(-3 + x)^{\frac{1}{3}}$$

✓ Solution by Mathematica
Time used: 0.04 (sec). Leaf size: 14

```
DSolve[{y'[x]==1/3*(4*x-9)*(x-3)^(-2/3),{y[0]==0}},y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to \sqrt[3]{x - 3}x$$

3 Chapter 2. First Order Equations. Exercises 2.1, page 32

3.1 problem 1 (a)

Internal problem ID [14125]

Book: INTRODUCTORY DIFFERENTIAL EQUATIONS. Martha L. Abell, James P. Braselton. Fourth edition 2014. ElScAe. 2014
Section: Chapter 2. First Order Equations. Exercises 2.1, page 32
Problem number: 1 (a).
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_Riccati]`

$$\boxed{y' - y^2 = -t^2}$$

With initial conditions

$$[y(0) = 0]$$

✓ Solution by Maple
Time used: 0.859 (sec). Leaf size: 59

```
dsolve([diff(y(t),t)+t^2=y(t)^2,y(0) = 0],y(t), singsol=all)
```

$$y(t) = -\left(\begin{cases} 0 & t = 0 \\ \frac{\left(\operatorname{BesselI}\left(-\frac{3}{4}, \frac{t^2}{2}\right)\pi\sqrt{2} - 2\operatorname{BesselK}\left(\frac{3}{4}, \frac{t^2}{2}\right)\right)t}{\pi\sqrt{2}\operatorname{BesselI}\left(\frac{1}{4}, \frac{t^2}{2}\right) + 2\operatorname{BesselK}\left(\frac{1}{4}, \frac{t^2}{2}\right)} & \text{otherwise} \end{cases}\right)$$

✓ Solution by Mathematica
Time used: 0.663 (sec). Leaf size: 81

```
DSolve[{y'[t]+t^2==y[t]^2,{y[0]==0}},y[t],t,IncludeSingularSolutions -> True]
```

$$y(t) \rightarrow -\frac{it^2 \operatorname{BesselJ}\left(-\frac{5}{4}, \frac{it^2}{2}\right) - it^2 \operatorname{BesselJ}\left(\frac{3}{4}, \frac{it^2}{2}\right) + \operatorname{BesselJ}\left(-\frac{1}{4}, \frac{it^2}{2}\right)}{2t \operatorname{BesselJ}\left(-\frac{1}{4}, \frac{it^2}{2}\right)}$$

3.2 problem 1 (b)

Internal problem ID [14126]

Book: INTRODUCTORY DIFFERENTIAL EQUATIONS. Martha L. Abell, James P. Braselton. Fourth edition 2014. ElScAe. 2014
Section: Chapter 2. First Order Equations. Exercises 2.1, page 32
Problem number: 1 (b).
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_rational]`

$$\boxed{y' - \frac{1}{y^2} = -t^2}$$

With initial conditions

$$[y(0) = 0]$$

✗ Solution by Maple

```
dsolve([diff(y(t),t)+t^2=1/y(t)^2,y(0) = 0],y(t), singsol=all)
```

No solution found

✗ Solution by Mathematica
Time used: 0.0 (sec). Leaf size: 0

```
DSolve[{y'[t]+t^2==1/y[t]^2,{y[0]==0}},y[t],t,IncludeSingularSolutions -> True]
```

Not solved

3.3 problem 1 (c)

Internal problem ID [14127]

Book: INTRODUCTORY DIFFERENTIAL EQUATIONS. Martha L. Abell, James P. Braselton. Fourth edition 2014. ElScAe. 2014
Section: Chapter 2. First Order Equations. Exercises 2.1, page 32
Problem number: 1 (c).
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_linear]`

$$\boxed{y' - y = \frac{1}{1-t}}$$

✓ Solution by Maple
Time used: 0.0 (sec). Leaf size: 17

```
dsolve(diff(y(t),t)=y(t)+1/(1-t),y(t), singsol=all)
```

$$y(t) = \left(\mathrm{e}^{-1} \operatorname{expIntegral}_1(t-1) + c_1\right) \mathrm{e}^t$$

✓ Solution by Mathematica
Time used: 0.117 (sec). Leaf size: 24

```
DSolve[y'[t]==y[t]+1/(1-t),y[t],t,IncludeSingularSolutions -> True]
```

$$y(t) \to e^{t-1}(-\operatorname{ExpIntegralEi}(1-t) + ec_1)$$

3.4 problem 3

Internal problem ID [14128]

Book: INTRODUCTORY DIFFERENTIAL EQUATIONS. Martha L. Abell, James P. Braselton. Fourth edition 2014. ElScAe. 2014
Section: Chapter 2. First Order Equations. Exercises 2.1, page 32
Problem number: 3.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_quadrature]`

$$\boxed{y' - y^{\frac{1}{5}} = 0}$$

With initial conditions

$$[y(0) = 0]$$

✓ Solution by Maple
Time used: 0.016 (sec). Leaf size: 5

```
dsolve([diff(y(t),t)=y(t)^(1/5),y(0) = 0],y(t), singsol=all)
```

$$y(t) = 0$$

✓ Solution by Mathematica
Time used: 0.004 (sec). Leaf size: 24

```
DSolve[{y'[t]==y[t]^(1/5),{y[0]==0}},y[t],t,IncludeSingularSolutions -> True]
```

$$y(t) \to \frac{4\sqrt{2}t^{5/4}}{5\sqrt[4]{5}}$$

3.5 problem 4

Internal problem ID [14129]

Book: INTRODUCTORY DIFFERENTIAL EQUATIONS. Martha L. Abell, James P. Braselton. Fourth edition 2014. ElScAe. 2014
Section: Chapter 2. First Order Equations. Exercises 2.1, page 32
Problem number: 4.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_separable]`

$$\boxed{\frac{y'}{t} - \sqrt{y} = 0}$$

With initial conditions

$$[y(0) = 0]$$

✓ Solution by Maple
Time used: 0.0 (sec). Leaf size: 5

```
dsolve([1/t*diff(y(t),t)=sqrt(y(t)),y(0) = 0],y(t), singsol=all)
```

$$y(t) = 0$$

✓ Solution by Mathematica
Time used: 0.111 (sec). Leaf size: 17

```
DSolve[{1/t*y'[t]==Sqrt[y[t]],{y[0]==0}},y[t],t,IncludeSingularSolutions -> True]
```

$$y(t) \to 0$$
$$y(t) \to \frac{t^4}{16}$$

3.6 problem 6

Internal problem ID [14130]

Book: INTRODUCTORY DIFFERENTIAL EQUATIONS. Martha L. Abell, James P. Braselton. Fourth edition 2014. ElScAe. 2014
Section: Chapter 2. First Order Equations. Exercises 2.1, page 32
Problem number: 6.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_Riccati]`

$$\boxed{y' + ty^2 = 4t^2}$$

With initial conditions

$$[y(2) = 1]$$

✓ Solution by Maple
Time used: 0.219 (sec). Leaf size: 127

```
dsolve([diff(y(t),t)=4*t^2-t*y(t)^2,y(2) = 1],y(t), singsol=all)
```

$$y(t) = -\frac{2\left(\left(2\sqrt{2}\,\mathrm{BesselK}\left(\frac{3}{5}, \frac{16\sqrt{2}}{5}\right) + \mathrm{BesselK}\left(\frac{2}{5}, \frac{16\sqrt{2}}{5}\right)\right)\mathrm{BesselI}\left(-\frac{3}{5}, \frac{4t^{\frac{5}{2}}}{5}\right) + \mathrm{BesselK}\left(\frac{3}{5}, \frac{4t^{\frac{5}{2}}}{5}\right)\left(-2\,\mathrm{BesselI}\right.\right.}{\left(-2\sqrt{2}\,\mathrm{BesselK}\left(\frac{3}{5}, \frac{16\sqrt{2}}{5}\right) - \mathrm{BesselK}\left(\frac{2}{5}, \frac{16\sqrt{2}}{5}\right)\right)\mathrm{BesselI}\left(\frac{2}{5}, \frac{4t^{\frac{5}{2}}}{5}\right) + \mathrm{BesselK}\left(\frac{2}{5}, \frac{4t^{\frac{5}{2}}}{5}\right)\left(-2\,\mathrm{Bessel}\right.}$$

✓ Solution by Mathematica
Time used: 0.268 (sec). Leaf size: 709

```
DSolve[{y'[t]==4*t^2-t*y[t]^2,{y[2]==1}},y[t],t,IncludeSingularSolutions -> True]
```

$$y(t) \to \frac{-3t^{5/2}\,\mathrm{BesselI}\left(\frac{2}{5}, \frac{16\sqrt{2}}{5}\right)\mathrm{BesselI}\left(\frac{3}{5}, \frac{4t^{5/2}}{5}\right) + 4\sqrt{2}t^{5/2}\,\mathrm{BesselI}\left(-\frac{3}{5}, \frac{16\sqrt{2}}{5}\right)\mathrm{BesselI}\left(\frac{3}{5}, \frac{4t^{5/2}}{5}\right) - 4\sqrt{2}t^{5/2}\,\mathrm{Be}}{}$$

3.7 problem 7

Internal problem ID [14131]

Book: INTRODUCTORY DIFFERENTIAL EQUATIONS. Martha L. Abell, James P. Braselton. Fourth edition 2014. ElScAe. 2014
Section: Chapter 2. First Order Equations. Exercises 2.1, page 32
Problem number: 7.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_separable]`

$$\boxed{y' - y\sqrt{t} = 0}$$

With initial conditions

$$[y(1) = 1]$$

✓ Solution by Maple
Time used: 0.031 (sec). Leaf size: 13

```
dsolve([diff(y(t),t)=y(t)*sqrt(t),y(1) = 1],y(t), singsol=all)
```

$$y(t) = \mathrm{e}^{\frac{2(\sqrt{t}-1)(t+\sqrt{t}+1)}{3}}$$

✓ Solution by Mathematica
Time used: 0.028 (sec). Leaf size: 18

```
DSolve[{y'[t]==y[t]*Sqrt[t],{y[1]==1}},y[t],t,IncludeSingularSolutions -> True]
```

$$y(t) \to e^{\frac{2}{3}(t^{3/2}-1)}$$

3.8 problem 8

Internal problem ID [14132]

Book: INTRODUCTORY DIFFERENTIAL EQUATIONS. Martha L. Abell, James P. Braselton. Fourth edition 2014. ElScAe. 2014
Section: Chapter 2. First Order Equations. Exercises 2.1, page 32
Problem number: 8.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_quadrature]`

$$\boxed{y' - 6y^{\frac{2}{3}} = 0}$$

With initial conditions

$$[y(1) = 0]$$

✓ Solution by Maple
Time used: 0.015 (sec). Leaf size: 5

```
dsolve([diff(y(t),t)=6*y(t)^(2/3),y(1) = 0],y(t), singsol=all)
```

$$y(t) = 0$$

✓ Solution by Mathematica
Time used: 0.001 (sec). Leaf size: 6

```
DSolve[{y'[t]==6*y[t]^(2/3),{y[1]==0}},y[t],t,IncludeSingularSolutions -> True]
```

$$y(t) \to 0$$

3.9 problem 9

Internal problem ID [14133]

Book: INTRODUCTORY DIFFERENTIAL EQUATIONS. Martha L. Abell, James P. Braselton. Fourth edition 2014. ElScAe. 2014
Section: Chapter 2. First Order Equations. Exercises 2.1, page 32
Problem number: 9.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[‘y=_G(x,y’)‘]`

$$\boxed{y' - \sin(y) = -\cos(t)}$$

With initial conditions

$$[y(\pi) = 0]$$

✗ Solution by Maple

```
dsolve([diff(y(t),t)=sin(y(t))-cos(t),y(Pi) = 0],y(t), singsol=all)
```

No solution found

✗ Solution by Mathematica
Time used: 0.0 (sec). Leaf size: 0

```
DSolve[{y'[t]==Sin[y[t]]-Cos[t],{y[Pi]==0}},y[t],t,IncludeSingularSolutions -> True]
```

Not solved

3.10 problem 10

Internal problem ID [14134]

Book: INTRODUCTORY DIFFERENTIAL EQUATIONS. Martha L. Abell, James P. Braselton. Fourth edition 2014. ElScAe. 2014
Section: Chapter 2. First Order Equations. Exercises 2.1, page 32
Problem number: 10.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_separable]`

$$\boxed{y't - y = 0}$$

✓ Solution by Maple
Time used: 0.0 (sec). Leaf size: 7

```
dsolve(t*diff(y(t),t)=y(t),y(t), singsol=all)
```

$$y(t) = c_1 t$$

✓ Solution by Mathematica
Time used: 0.019 (sec). Leaf size: 14

```
DSolve[t*y'[t]==y[t],y[t],t,IncludeSingularSolutions -> True]
```

$$y(t) \to c_1 t$$
$$y(t) \to 0$$

3.11 problem 11

Internal problem ID [14135]

Book: INTRODUCTORY DIFFERENTIAL EQUATIONS. Martha L. Abell, James P. Braselton. Fourth edition 2014. ElScAe. 2014
Section: Chapter 2. First Order Equations. Exercises 2.1, page 32
Problem number: 11.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_separable]`

$$\boxed{y' - y\tan(t) = 0}$$

With initial conditions

$$[y(0) = 1]$$

✓ Solution by Maple
Time used: 0.0 (sec). Leaf size: 6

```
dsolve([diff(y(t),t)=y(t)*tan(t),y(0) = 1],y(t), singsol=all)
```

$$y(t) = \sec(t)$$

✓ Solution by Mathematica
Time used: 0.042 (sec). Leaf size: 7

```
DSolve[{y'[t]==y[t]*Tan[t],{y[0]==1}},y[t],t,IncludeSingularSolutions -> True]
```

$$y(t) \to \sec(t)$$

3.12 problem 12

Internal problem ID [14136]

Book: INTRODUCTORY DIFFERENTIAL EQUATIONS. Martha L. Abell, James P. Braselton. Fourth edition 2014. ElScAe. 2014
Section: Chapter 2. First Order Equations. Exercises 2.1, page 32
Problem number: 12.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_quadrature]`

$$\boxed{y' = \frac{1}{t^2 + 1}}$$

With initial conditions

$$[y(0) = 0]$$

✓ Solution by Maple
Time used: 0.015 (sec). Leaf size: 6

```
dsolve([diff(y(t),t)=1/(1+t^2),y(0) = 0],y(t), singsol=all)
```

$$y(t) = \arctan(t)$$

✓ Solution by Mathematica
Time used: 0.005 (sec). Leaf size: 7

```
DSolve[{y'[t]==1/(1+t^2),{y[0]==0}},y[t],t,IncludeSingularSolutions -> True]
```

$$y(t) \to \arctan(t)$$

3.13 problem 13 (a)

Internal problem ID [14137]

Book: INTRODUCTORY DIFFERENTIAL EQUATIONS. Martha L. Abell, James P. Braselton. Fourth edition 2014. ElScAe. 2014
Section: Chapter 2. First Order Equations. Exercises 2.1, page 32
Problem number: 13 (a).
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_quadrature]`

$$\boxed{y' - \sqrt{y^2 - 1} = 0}$$

With initial conditions

$$[y(0) = 2]$$

✓ Solution by Maple
Time used: 0.734 (sec). Leaf size: 27

```
dsolve([diff(y(t),t)=sqrt(y(t)^2-1),y(0) = 2],y(t), singsol=all)
```

$$y(t) = \frac{e^t \sqrt{3}}{2} + e^t - \frac{\sqrt{3}\, e^{-t}}{2} + e^{-t}$$

✓ Solution by Mathematica
Time used: 0.067 (sec). Leaf size: 49

```
DSolve[{y'[t]==Sqrt[y[t]^2-1],{y[0]==2}},y[t],t,IncludeSingularSolutions -> True]
```

$$y(t) \to \frac{1}{2} e^{-t} \sqrt{2e^{2t} + \left(7 + 4\sqrt{3}\right) e^{4t} + 7 - 4\sqrt{3}}$$

3.14 problem 13 (b)

Internal problem ID [14138]

Book: INTRODUCTORY DIFFERENTIAL EQUATIONS. Martha L. Abell, James P. Braselton. Fourth edition 2014. ElScAe. 2014
Section: Chapter 2. First Order Equations. Exercises 2.1, page 32
Problem number: 13 (b).
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_quadrature]`

$$\boxed{y' - \sqrt{y^2 - 1} = 0}$$

With initial conditions

$$[y(4) = -1]$$

✓ Solution by Maple
Time used: 0.0 (sec). Leaf size: 5

```
dsolve([diff(y(t),t)=sqrt(y(t)^2-1),y(4) = -1],y(t), singsol=all)
```

$$y(t) = -1$$

✓ Solution by Mathematica
Time used: 0.002 (sec). Leaf size: 6

```
DSolve[{y'[t]==Sqrt[y[t]^2-1],{y[4]==-1}},y[t],t,IncludeSingularSolutions -> True]
```

$$y(t) \to -1$$

3.15 problem 13 (c)

Internal problem ID [14139]

Book: INTRODUCTORY DIFFERENTIAL EQUATIONS. Martha L. Abell, James P. Braselton. Fourth edition 2014. ElScAe. 2014
Section: Chapter 2. First Order Equations. Exercises 2.1, page 32
Problem number: 13 (c).
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_quadrature]`

$$\boxed{y' - \sqrt{y^2 - 1} = 0}$$

With initial conditions

$$\left[y(0) = \frac{1}{2}\right]$$

✓ Solution by Maple
Time used: 0.344 (sec). Leaf size: 26

```
dsolve([diff(y(t),t)=sqrt(y(t)^2-1),y(0) = 1/2],y(t), singsol=all)
```

$$y(t) = \frac{i\sqrt{3}\,\mathrm{e}^{t}}{4} - \frac{i\sqrt{3}\,\mathrm{e}^{-t}}{4} + \frac{\mathrm{e}^{t}}{4} + \frac{\mathrm{e}^{-t}}{4}$$

✓ Solution by Mathematica
Time used: 0.076 (sec). Leaf size: 59

```
DSolve[{y'[t]==Sqrt[y[t]^2-1],{y[0]==1/2}},y[t],t,IncludeSingularSolutions -> True]
```

$$y(t) \to \frac{e^{-t}\sqrt{4e^{2t} + i\left(\sqrt{3} + i\right)e^{4t} - 1 - i\sqrt{3}}}{2\sqrt{2}}$$

3.16 problem 13 (d)

Internal problem ID [14140]

Book: INTRODUCTORY DIFFERENTIAL EQUATIONS. Martha L. Abell, James P. Braselton. Fourth edition 2014. ElScAe. 2014
Section: Chapter 2. First Order Equations. Exercises 2.1, page 32
Problem number: 13 (d).
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_quadrature]`

$$\boxed{y' - \sqrt{y^2 - 1} = 0}$$

With initial conditions

$$[y(2) = 1]$$

✓ Solution by Maple
Time used: 0.0 (sec). Leaf size: 5

```
dsolve([diff(y(t),t)=sqrt(y(t)^2-1),y(2) = 1],y(t), singsol=all)
```

$$y(t) = 1$$

✓ Solution by Mathematica
Time used: 0.002 (sec). Leaf size: 6

```
DSolve[{y'[t]==Sqrt[y[t]^2-1],{y[2]==1}},y[t],t,IncludeSingularSolutions -> True]
```

$$y(t) \to 1$$

3.17 problem 14 (a)

Internal problem ID [14141]

Book: INTRODUCTORY DIFFERENTIAL EQUATIONS. Martha L. Abell, James P. Braselton. Fourth edition 2014. ElScAe. 2014
Section: Chapter 2. First Order Equations. Exercises 2.1, page 32
Problem number: 14 (a).
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_quadrature]`

$$\boxed{y' - \sqrt{25 - y^2} = 0}$$

With initial conditions

$$[y(-4) = 3]$$

✓ Solution by Maple
Time used: 0.047 (sec). Leaf size: 12

```
dsolve([diff(y(t),t)=sqrt(25-y(t)^2),y(-4) = 3],y(t), singsol=all)
```

$$y(t) = 5\sin\left(t + 4 + \arcsin\left(\frac{3}{5}\right)\right)$$

✓ Solution by Mathematica
Time used: 0.047 (sec). Leaf size: 17

```
DSolve[{y'[t]==Sqrt[25-y[t]^2],{y[-4]==3}},y[t],t,IncludeSingularSolutions -> True]
```

$$y(t) \to 5\cos\left(-2\arctan\left(\frac{1}{2}\right) + t + 4\right)$$

3.18 problem 14 (b)

Internal problem ID [14142]

Book: INTRODUCTORY DIFFERENTIAL EQUATIONS. Martha L. Abell, James P. Braselton. Fourth edition 2014. ElScAe. 2014
Section: Chapter 2. First Order Equations. Exercises 2.1, page 32
Problem number: 14 (b).
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_quadrature]`

$$\boxed{y' - \sqrt{25 - y^2} = 0}$$

With initial conditions

$$[y(0) = 5]$$

✓ Solution by Maple
Time used: 0.0 (sec). Leaf size: 5

```
dsolve([diff(y(t),t)=sqrt(25-y(t)^2),y(0) = 5],y(t), singsol=all)
```

$$y(t) = 5$$

✓ Solution by Mathematica
Time used: 0.001 (sec). Leaf size: 6

```
DSolve[{y'[t]==Sqrt[25-y[t]^2],{y[0]==5}},y[t],t,IncludeSingularSolutions -> True]
```

$$y(t) \to 5$$

3.19 problem 14 (c)

Internal problem ID [14143]

Book: INTRODUCTORY DIFFERENTIAL EQUATIONS. Martha L. Abell, James P. Braselton. Fourth edition 2014. ElScAe. 2014
Section: Chapter 2. First Order Equations. Exercises 2.1, page 32
Problem number: 14 (c).
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_quadrature]`

$$\boxed{y' - \sqrt{25 - y^2} = 0}$$

With initial conditions

$$[y(3) = -6]$$

✓ Solution by Maple
Time used: 0.094 (sec). Leaf size: 14

```
dsolve([diff(y(t),t)=sqrt(25-y(t)^2),y(3) = -6],y(t), singsol=all)
```

$$y(t) = 5\sin\left(t - 3 - \arcsin\left(\frac{6}{5}\right)\right)$$

✓ Solution by Mathematica
Time used: 0.043 (sec). Leaf size: 23

```
DSolve[{y'[t]==Sqrt[25-y[t]^2],{y[3]==-6}},y[t],t,IncludeSingularSolutions -> True]
```

$$y(t) \to 5\cos\left(-2i\operatorname{arctanh}\left(\sqrt{11}\right) - t + 3\right)$$

3.20 problem 14 (c)

Internal problem ID [14144]

Book: INTRODUCTORY DIFFERENTIAL EQUATIONS. Martha L. Abell, James P. Braselton. Fourth edition 2014. ElScAe. 2014
Section: Chapter 2. First Order Equations. Exercises 2.1, page 32
Problem number: 14 (c).
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_quadrature]`

$$\boxed{y' - \sqrt{25 - y^2} = 0}$$

With initial conditions

$$[y(4) = -5]$$

✓ Solution by Maple
Time used: 0.0 (sec). Leaf size: 5

```
dsolve([diff(y(t),t)=sqrt(25-y(t)^2),y(4) = -5],y(t), singsol=all)
```

$$y(t) = -5$$

✓ Solution by Mathematica
Time used: 0.001 (sec). Leaf size: 6

```
DSolve[{y'[t]==Sqrt[25-y[t]^2],{y[4]==-5}},y[t],t,IncludeSingularSolutions -> True]
```

$$y(t) \to -5$$

3.21 problem 15

Internal problem ID [14145]

Book: INTRODUCTORY DIFFERENTIAL EQUATIONS. Martha L. Abell, James P. Braselton. Fourth edition 2014. ElScAe. 2014
Section: Chapter 2. First Order Equations. Exercises 2.1, page 32
Problem number: 15.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_linear]`

$$\boxed{y't + y = t^3}$$

With initial conditions

$$[y(1) = 0]$$

✓ Solution by Maple
Time used: 0.0 (sec). Leaf size: 14

```
dsolve([t*diff(y(t),t)+y(t)=t^3,y(1) = 0],y(t), singsol=all)
```

$$y(t) = \frac{t^4 - 1}{4t}$$

✓ Solution by Mathematica
Time used: 0.025 (sec). Leaf size: 17

```
DSolve[{t*y'[t]+y[t]==t^3,{y[1]==0}},y[t],t,IncludeSingularSolutions -> True]
```

$$y(t) \to \frac{t^4 - 1}{4t}$$

3.22 problem 16

Internal problem ID [14146]

Book: INTRODUCTORY DIFFERENTIAL EQUATIONS. Martha L. Abell, James P. Braselton. Fourth edition 2014. ElScAe. 2014
Section: Chapter 2. First Order Equations. Exercises 2.1, page 32
Problem number: 16.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_linear]`

$$\boxed{y' t^3 + t^4 y = 2t^3}$$

With initial conditions

$$[y(0) = 0]$$

✓ Solution by Maple
Time used: 0.016 (sec). Leaf size: 27

```
dsolve([t^3*diff(y(t),t)+t^4*y(t)=2*t^3,y(0) = 0],y(t), singsol=all)
```

$$y(t) = -i\sqrt{2}\,\mathrm{e}^{-\frac{t^2}{2}}\sqrt{\pi}\ \mathrm{erf}\left(\frac{i\sqrt{2}\,t}{2}\right)$$

✓ Solution by Mathematica
Time used: 0.067 (sec). Leaf size: 30

```
DSolve[{t^3*y'[t]+t^4*y[t]==2*t^3,{y[0]==0}},y[t],t,IncludeSingularSolutions -> True]
```

$$y(t) \to \sqrt{2\pi}e^{-\frac{t^2}{2}}\mathrm{erfi}\left(\frac{t}{\sqrt{2}}\right)$$

3.23 problem 17

Internal problem ID [14147]

Book: INTRODUCTORY DIFFERENTIAL EQUATIONS. Martha L. Abell, James P. Braselton. Fourth edition 2014. ElScAe. 2014
Section: Chapter 2. First Order Equations. Exercises 2.1, page 32
Problem number: 17.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_linear]`

$$\boxed{2y' + ty = \ln(t)}$$

With initial conditions

$$[y(\mathrm{e}) = 0]$$

✓ Solution by Maple
Time used: 0.062 (sec). Leaf size: 28

```
dsolve([2*diff(y(t),t)+t*y(t)=ln(t),y(exp(1)) = 0],y(t), singsol=all)
```

$$y(t) = \frac{\left(\int_{\mathrm{e}}^{t} \ln(\_z1)\, \mathrm{e}^{\frac{\_z1^2}{4}} d\_z1\right) \mathrm{e}^{-\frac{t^2}{4}}}{2}$$

✓ Solution by Mathematica
Time used: 0.052 (sec). Leaf size: 95

```
DSolve[{2*y'[t]+t*y[t]==Log[t],{y[Exp[1]]==0}},y[t],t,IncludeSingularSolutions -> True]
```

$$y(t) \to \frac{1}{2} e^{-\frac{t^2}{4}} \left(-t\, {}_2F_2\left(\frac{1}{2}, \frac{1}{2}; \frac{3}{2}, \frac{3}{2}; \frac{t^2}{4}\right) + e\, {}_2F_2\left(\frac{1}{2}, \frac{1}{2}; \frac{3}{2}, \frac{3}{2}; \frac{e^2}{4}\right) + \sqrt{\pi}\,\mathrm{erfi}\left(\frac{t}{2}\right) \log(t) - \sqrt{\pi}\,\mathrm{erfi}\left(\frac{e}{2}\right)\right)$$

3.24 problem 18

Internal problem ID [14148]

Book: INTRODUCTORY DIFFERENTIAL EQUATIONS. Martha L. Abell, James P. Braselton. Fourth edition 2014. ElScAe. 2014
Section: Chapter 2. First Order Equations. Exercises 2.1, page 32
Problem number: 18.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_linear]`

$$\boxed{y' + y\sec(t) = t}$$

With initial conditions

$$[y(0) = 0]$$

✓ Solution by Maple
Time used: 0.094 (sec). Leaf size: 59

```
dsolve([diff(y(t),t)+y(t)*sec(t)=t,y(0) = 0],y(t), singsol=all)
```

$$y(t) = \frac{i\pi^2 + 12it^2 + 48i\,\mathrm{polylog}\left(2, -ie^{it}\right) - 48t\ln\left(1 + ie^{it}\right) - 48\,\mathrm{Catalan}}{24\sec(t) + 24\tan(t)}$$

✓ Solution by Mathematica
Time used: 0.209 (sec). Leaf size: 146

```
DSolve[{y'[t]+y[t]*Sec[t]==t,{y[0]==0}},y[t],t,IncludeSingularSolutions -> True]
```

$$\begin{aligned}y(t) \to \frac{1}{24}e^{-2\operatorname{arctanh}\left(\tan\left(\frac{t}{2}\right)\right)}\Bigg(&-48C + 48i\,\mathrm{PolyLog}\left(2, -ie^{it}\right) + 12it^2 - 36i\pi t \\ &- 48t\log\left(1 + ie^{it}\right) - 48\pi\log\left(1 + e^{-it}\right) + 24\pi\log\left(1 + ie^{it}\right) + 48\pi\log\left(\cos\left(\frac{t}{2}\right)\right) \\ &- 24\pi\log\left(-\cos\left(\frac{1}{4}(2t + \pi)\right)\right) + 25i\pi^2 + 36\pi\log(2) - 24\pi\log(1 + i)\Bigg)\end{aligned}$$

3.25 problem 19

Internal problem ID [14149]

Book: INTRODUCTORY DIFFERENTIAL EQUATIONS. Martha L. Abell, James P. Braselton. Fourth edition 2014. ElScAe. 2014
Section: Chapter 2. First Order Equations. Exercises 2.1, page 32
Problem number: 19.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_linear]`

$$\boxed{y' + \frac{y}{-3+t} = \frac{1}{t-1}}$$

With initial conditions

$$[y(-1) = 0]$$

✓ Solution by Maple
Time used: 0.031 (sec). Leaf size: 27

```
dsolve([diff(y(t),t)+1/(t-3)*y(t)=1/(t-1),y(-1) = 0],y(t), singsol=all)
```

$$y(t) = \frac{t - 2\ln(t-1) + 1 + 2\ln(2) + 2i\pi}{t-3}$$

✓ Solution by Mathematica
Time used: 0.034 (sec). Leaf size: 27

```
DSolve[{y'[t]+1/(t-3)*y[t]==1/(t-1),{y[-1]==0}},y[t],t,IncludeSingularSolutions -> True]
```

$$y(t) \to \frac{t - 2\log(t-1) + 2i\pi + 1 + \log(4)}{t-3}$$

3.26 problem 20

Internal problem ID [14150]

Book: INTRODUCTORY DIFFERENTIAL EQUATIONS. Martha L. Abell, James P. Braselton. Fourth edition 2014. ElScAe. 2014
Section: Chapter 2. First Order Equations. Exercises 2.1, page 32
Problem number: 20.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_linear]`

$$\boxed{(-2+t)\,y' + (t^2-4)\,y = \frac{1}{t+2}}$$

With initial conditions

$$[y(0) = 3]$$

✓ Solution by Maple
Time used: 0.047 (sec). Leaf size: 35

```
dsolve([(t-2)*diff(y(t),t)+(t^2-4)*y(t)=1/(t+2),y(0) = 3],y(t), singsol=all)
```

$$y(t) = \left(\int_0^t \frac{\mathrm{e}^{\frac{\_z1(\_z1+4)}{2}}}{\_z1^2-4}d\_z1 + 3\right)\mathrm{e}^{-\frac{t(t+4)}{2}}$$

✓ Solution by Mathematica
Time used: 0.435 (sec). Leaf size: 46

```
DSolve[{(t-2)*y'[t]+(t^2-4)*y[t]==1/(t+2),{y[0]==3}},y[t],t,IncludeSingularSolutions -> True
```

$$y(t) \to e^{-\frac{1}{2}t(t+4)}\left(\int_0^t \frac{e^{\frac{1}{2}K[1](K[1]+4)}}{K[1]^2-4}dK[1] + 3\right)$$

3.27 problem 21

Internal problem ID [14151]

Book: INTRODUCTORY DIFFERENTIAL EQUATIONS. Martha L. Abell, James P. Braselton. Fourth edition 2014. ElScAe. 2014
Section: Chapter 2. First Order Equations. Exercises 2.1, page 32
Problem number: 21.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_linear]`

$$\boxed{y' + \frac{y}{\sqrt{-t^2+4}} = t}$$

With initial conditions

$$[y(0) = 0]$$

✓ Solution by Maple
Time used: 0.047 (sec). Leaf size: 25

```
dsolve([diff(y(t),t)+y(t)/sqrt(4-t^2)=t,y(0) = 0],y(t), singsol=all)
```

$$y(t) = \left(\int_0^t \_z1\, \mathrm{e}^{\arcsin\left(\frac{\_z1}{2}\right)} d\_z1\right) \mathrm{e}^{-\arcsin\left(\frac{t}{2}\right)}$$

✓ Solution by Mathematica
Time used: 0.159 (sec). Leaf size: 62

```
DSolve[{y'[t]+y[t]/Sqrt[4-t^2]==t,{y[0]==0}},y[t],t,IncludeSingularSolutions -> True]
```

$$y(t) \to e^{2\arctan\left(\frac{\sqrt{4-t^2}}{t+2}\right)} \int_0^t e^{-2\arctan\left(\frac{\sqrt{4-K[1]^2}}{K[1]+2}\right)} K[1]dK[1]$$

3.28 problem 22

Internal problem ID [14152]

Book: INTRODUCTORY DIFFERENTIAL EQUATIONS. Martha L. Abell, James P. Braselton. Fourth edition 2014. ElScAe. 2014
Section: Chapter 2. First Order Equations. Exercises 2.1, page 32
Problem number: 22.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_linear]`

$$\boxed{y' + \frac{y}{\sqrt{-t^2+4}} = t}$$

With initial conditions

$$[y(3) = -1]$$

✓ Solution by Maple
Time used: 0.203 (sec). Leaf size: 31

```
dsolve([diff(y(t),t)+y(t)/sqrt(4-t^2)=t,y(3) = -1],y(t), singsol=all)
```

$$y(t) = \left(\int_3^t \_z1\, \mathrm{e}^{\arcsin\left(\frac{\_z1}{2}\right)} d\_z1 - \mathrm{e}^{\arcsin\left(\frac{3}{2}\right)}\right) \mathrm{e}^{-\arcsin\left(\frac{t}{2}\right)}$$

✓ Solution by Mathematica
Time used: 0.085 (sec). Leaf size: 88

```
DSolve[{y'[t]+y[t]/Sqrt[4-t^2]==t,{y[3]==-1}},y[t],t,IncludeSingularSolutions -> True]
```

$$y(t) \to e^{2\arctan\left(\frac{\sqrt{4-t^2}}{t+2}\right) - 2i\operatorname{arctanh}\left(\frac{1}{\sqrt{5}}\right)} \left(-1 + e^{2i\operatorname{arctanh}\left(\frac{1}{\sqrt{5}}\right)} \int_3^t e^{-2\arctan\left(\frac{\sqrt{4-K[1]^2}}{K[1]+2}\right)} K[1] dK[1]\right)$$

3.29 problem 23

Internal problem ID [14153]

Book: INTRODUCTORY DIFFERENTIAL EQUATIONS. Martha L. Abell, James P. Braselton. Fourth edition 2014. ElScAe. 2014
Section: Chapter 2. First Order Equations. Exercises 2.1, page 32
Problem number: 23.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_linear]`

$$\boxed{y't + y = \sin(t)\, t}$$

With initial conditions

$$[y(\pi) = 1]$$

✓ Solution by Maple
Time used: 0.016 (sec). Leaf size: 16

```
dsolve([t*diff(y(t),t)+y(t)=t*sin(t),y(Pi) = 1],y(t), singsol=all)
```

$$y(t) = \frac{-t\cos(t) + \sin(t)}{t}$$

✓ Solution by Mathematica
Time used: 0.05 (sec). Leaf size: 16

```
DSolve[{t*y'[t]+y[t]==t*Sin[t],{y[Pi]==1}},y[t],t,IncludeSingularSolutions -> True]
```

$$y(t) \to \frac{\sin(t)}{t} - \cos(t)$$

3.30 problem 24

Internal problem ID [14154]

Book: INTRODUCTORY DIFFERENTIAL EQUATIONS. Martha L. Abell, James P. Braselton. Fourth edition 2014. ElScAe. 2014
Section: Chapter 2. First Order Equations. Exercises 2.1, page 32
Problem number: 24.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_linear]`

$$\boxed{y' + y\tan(t) = \sin(t)}$$

With initial conditions

$$[y(0) = 0]$$

✓ Solution by Maple
Time used: 0.016 (sec). Leaf size: 11

```
dsolve([diff(y(t),t)+y(t)*tan(t)=sin(t),y(0) = 0],y(t), singsol=all)
```

$$y(t) = -\cos(t)\ln(\cos(t))$$

✓ Solution by Mathematica
Time used: 0.054 (sec). Leaf size: 12

```
DSolve[{y'[t]+y[t]*Tan[t]==Sin[t],{y[0]==0}},y[t],t,IncludeSingularSolutions -> True]
```

$$y(t) \to -\cos(t)\log(\cos(t))$$

3.31 problem 25

Internal problem ID [14155]

Book: INTRODUCTORY DIFFERENTIAL EQUATIONS. Martha L. Abell, James P. Braselton. Fourth edition 2014. ElScAe. 2014
Section: Chapter 2. First Order Equations. Exercises 2.1, page 32
Problem number: 25.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_quadrature]`

$$\boxed{y' - y^2 = 0}$$

With initial conditions

$$\left[y(0) = \frac{1}{2}\right]$$

✓ Solution by Maple
Time used: 0.015 (sec). Leaf size: 11

```
dsolve([diff(y(t),t)=y(t)^2,y(0) = 1/2],y(t), singsol=all)
```

$$y(t) = -\frac{1}{t-2}$$

✓ Solution by Mathematica
Time used: 0.003 (sec). Leaf size: 12

```
DSolve[{y'[t]==y[t]^2,{y[0]==1/2}},y[t],t,IncludeSingularSolutions -> True]
```

$$y(t) \to \frac{1}{2-t}$$

3.32 problem 26

Internal problem ID [14156]

Book: INTRODUCTORY DIFFERENTIAL EQUATIONS. Martha L. Abell, James P. Braselton. Fourth edition 2014. ElScAe. 2014
Section: Chapter 2. First Order Equations. Exercises 2.1, page 32
Problem number: 26.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_separable]`

$$\boxed{y' - t y^2 = 0}$$

With initial conditions

$$\left[y(0) = \frac{1}{2}\right]$$

✓ Solution by Maple
Time used: 0.063 (sec). Leaf size: 13

```
dsolve([diff(y(t),t)=t*y(t)^2,y(0) = 1/2],y(t), singsol=all)
```

$$y(t) = -\frac{2}{t^2 - 4}$$

✓ Solution by Mathematica
Time used: 0.101 (sec). Leaf size: 14

```
DSolve[{y'[t]==t*y[t]^2,{y[0]==1/2}},y[t],t,IncludeSingularSolutions -> True]
```

$$y(t) \to -\frac{2}{t^2 - 4}$$

3.33 problem 27

Internal problem ID [14157]

Book: INTRODUCTORY DIFFERENTIAL EQUATIONS. Martha L. Abell, James P. Braselton. Fourth edition 2014. ElScAe. 2014
Section: Chapter 2. First Order Equations. Exercises 2.1, page 32
Problem number: 27.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_separable]`

$$\boxed{y' + \frac{t}{y} = 0}$$

With initial conditions

$$\left[y(0) = \frac{1}{2}\right]$$

✓ Solution by Maple
Time used: 0.032 (sec). Leaf size: 15

```
dsolve([diff(y(t),t)=-t/y(t),y(0) = 1/2],y(t), singsol=all)
```

$$y(t) = \frac{\sqrt{-4t^2 + 1}}{2}$$

✓ Solution by Mathematica
Time used: 0.084 (sec). Leaf size: 20

```
DSolve[{y'[t]==-t/y[t],{y[0]==1/2}},y[t],t,IncludeSingularSolutions -> True]
```

$$y(t) \to \frac{1}{2}\sqrt{1 - 4t^2}$$

3.34 problem 28

Internal problem ID [14158]

Book: INTRODUCTORY DIFFERENTIAL EQUATIONS. Martha L. Abell, James P. Braselton. Fourth edition 2014. ElScAe. 2014
Section: Chapter 2. First Order Equations. Exercises 2.1, page 32
Problem number: 28.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_quadrature]`

$$\boxed{y' + y^3 = 0}$$

With initial conditions

$$\left[y(0) = \frac{1}{2}\right]$$

✓ Solution by Maple
Time used: 0.015 (sec). Leaf size: 11

```
dsolve([diff(y(t),t)=-y(t)^3,y(0) = 1/2],y(t), singsol=all)
```

$$y(t) = \frac{1}{\sqrt{2t+4}}$$

✓ Solution by Mathematica
Time used: 0.003 (sec). Leaf size: 18

```
DSolve[{y'[t]==-y[t]^3,{y[0]==1/2}},y[t],t,IncludeSingularSolutions -> True]
```

$$y(t) \to \frac{i}{\sqrt{-2t-4}}$$

4 Chapter 2. First Order Equations. Exercises 2.2, page 39

4.1 problem 1

Internal problem ID [14159]

Book: INTRODUCTORY DIFFERENTIAL EQUATIONS. Martha L. Abell, James P. Braselton. Fourth edition 2014. ElScAe. 2014
Section: Chapter 2. First Order Equations. Exercises 2.2, page 39
Problem number: 1.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_separable]`

$$\boxed{y' - \frac{x}{y^2} = 0}$$

✓ Solution by Maple
Time used: 0.016 (sec). Leaf size: 65

```
dsolve(diff(y(x),x)=x/y(x)^2,y(x), singsol=all)
```

$$y(x) = \frac{(12x^2 + 8c_1)^{\frac{1}{3}}}{2}$$
$$y(x) = -\frac{(12x^2 + 8c_1)^{\frac{1}{3}}\left(1 + i\sqrt{3}\right)}{4}$$
$$y(x) = \frac{(12x^2 + 8c_1)^{\frac{1}{3}}\left(i\sqrt{3} - 1\right)}{4}$$

✓ Solution by Mathematica
Time used: 0.179 (sec). Leaf size: 79

```
DSolve[y'[x]==x/y[x]^2,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to -\sqrt[3]{-\frac{3}{2}}\sqrt[3]{x^2 + 2c_1}$$
$$y(x) \to \sqrt[3]{\frac{3}{2}}\sqrt[3]{x^2 + 2c_1}$$
$$y(x) \to (-1)^{2/3}\sqrt[3]{\frac{3}{2}}\sqrt[3]{x^2 + 2c_1}$$

4.2 problem 2

Internal problem ID [14160]

Book: INTRODUCTORY DIFFERENTIAL EQUATIONS. Martha L. Abell, James P. Braselton. Fourth edition 2014. ElScAe. 2014
Section: Chapter 2. First Order Equations. Exercises 2.2, page 39
Problem number: 2.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_separable]`

$$\boxed{y^2y' = -\frac{1}{2\sqrt{t}}}$$

✓ Solution by Maple
Time used: 0.0 (sec). Leaf size: 57

```
dsolve(1/2/t^(1/2)+y(t)^2*diff(y(t),t)=0,y(t), singsol=all)
```

$$y(t) = \left(-3\sqrt{t}+c_1\right)^{\frac{1}{3}}$$
$$y(t) = -\frac{\left(-3\sqrt{t}+c_1\right)^{\frac{1}{3}}\left(1+i\sqrt{3}\right)}{2}$$
$$y(t) = \frac{\left(-3\sqrt{t}+c_1\right)^{\frac{1}{3}}\left(i\sqrt{3}-1\right)}{2}$$

✓ Solution by Mathematica
Time used: 1.899 (sec). Leaf size: 79

```
DSolve[1/2/t^(1/2)+y[t]^2*y'[t]==0,y[t],t,IncludeSingularSolutions -> True]
```

$$y(t) \to -\sqrt[3]{-3}\sqrt[3]{-\sqrt{t}+c_1}$$
$$y(t) \to \sqrt[3]{3}\sqrt[3]{-\sqrt{t}+c_1}$$
$$y(t) \to (-1)^{2/3}\sqrt[3]{3}\sqrt[3]{-\sqrt{t}+c_1}$$

4.3 problem 3

Internal problem ID [14161]

Book: INTRODUCTORY DIFFERENTIAL EQUATIONS. Martha L. Abell, James P. Braselton. Fourth edition 2014. ElScAe. 2014
Section: Chapter 2. First Order Equations. Exercises 2.2, page 39
Problem number: 3.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_separable]`

$$\boxed{y' - \frac{\sqrt{y}}{x^2} = 0}$$

✓ Solution by Maple
Time used: 0.0 (sec). Leaf size: 21

```
dsolve(diff(y(x),x)=sqrt(y(x))/x^2,y(x), singsol=all)
```

$$\frac{2\sqrt{y(x)}\,x - 2c_1x + 1}{2x} = 0$$

✓ Solution by Mathematica
Time used: 0.125 (sec). Leaf size: 25

```
DSolve[y'[x]==Sqrt[y[x]]/x^2,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to \frac{(-1 + c_1x)^2}{4x^2}$$
$$y(x) \to 0$$

4.4 problem 4

Internal problem ID [14162]

Book: INTRODUCTORY DIFFERENTIAL EQUATIONS. Martha L. Abell, James P. Braselton. Fourth edition 2014. ElScAe. 2014
Section: Chapter 2. First Order Equations. Exercises 2.2, page 39
Problem number: 4.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_quadrature]`

$$\boxed{y' - \frac{1+y^2}{y} = 0}$$

✓ Solution by Maple

Time used: 0.016 (sec). Leaf size: 29

```
dsolve(diff(y(t),t)=(1+y(t)^2)/y(t),y(t), singsol=all)
```

$$y(t) = \sqrt{c_1 \mathrm{e}^{2t} - 1}$$
$$y(t) = -\sqrt{c_1 \mathrm{e}^{2t} - 1}$$

✓ Solution by Mathematica

Time used: 2.441 (sec). Leaf size: 53

```
DSolve[y'[t]==(1+y[t]^2)/y[t],y[t],t,IncludeSingularSolutions -> True]
```

$$y(t) \to -\sqrt{-1 + e^{2(t+c_1)}}$$
$$y(t) \to \sqrt{-1 + e^{2(t+c_1)}}$$
$$y(t) \to -i$$
$$y(t) \to i$$

4.5 problem 5

Internal problem ID [14163]

Book: INTRODUCTORY DIFFERENTIAL EQUATIONS. Martha L. Abell, James P. Braselton. Fourth edition 2014. ElScAe. 2014
Section: Chapter 2. First Order Equations. Exercises 2.2, page 39
Problem number: 5.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_separable]`

$$\boxed{\left(5+\frac{9}{y^8}\right)y' = -4t^3-6}$$

✓ Solution by Maple

Time used: 0.015 (sec). Leaf size: 23

```
dsolve((6+4*t^3)+(5+9/y(t)^8)*diff(y(t),t)=0,y(t), singsol=all)
```

$$\frac{t^4}{2}+3t+\frac{5y(t)}{2}-\frac{9}{14y\left(t\right)^7}+c_1=0$$

✓ Solution by Mathematica

Time used: 3.593 (sec). Leaf size: 265

```
DSolve[(6+4*t^3)+(5+9/y[t]^8)*y'[t]==0,y[t],t,IncludeSingularSolutions -> True]
```

$$\begin{aligned}
y(t) &\to \text{Root}\left[35\#1^8+\#1^7\left(7t^4+42t-7c_1\right)-9\&,1\right]\\
y(t) &\to \text{Root}\left[35\#1^8+\#1^7\left(7t^4+42t-7c_1\right)-9\&,2\right]\\
y(t) &\to \text{Root}\left[35\#1^8+\#1^7\left(7t^4+42t-7c_1\right)-9\&,3\right]\\
y(t) &\to \text{Root}\left[35\#1^8+\#1^7\left(7t^4+42t-7c_1\right)-9\&,4\right]\\
y(t) &\to \text{Root}\left[35\#1^8+\#1^7\left(7t^4+42t-7c_1\right)-9\&,5\right]\\
y(t) &\to \text{Root}\left[35\#1^8+\#1^7\left(7t^4+42t-7c_1\right)-9\&,6\right]\\
y(t) &\to \text{Root}\left[35\#1^8+\#1^7\left(7t^4+42t-7c_1\right)-9\&,7\right]\\
y(t) &\to \text{Root}\left[35\#1^8+\#1^7\left(7t^4+42t-7c_1\right)-9\&,8\right]
\end{aligned}$$

4.6 problem 6

Internal problem ID [14164]

Book: INTRODUCTORY DIFFERENTIAL EQUATIONS. Martha L. Abell, James P. Braselton. Fourth edition 2014. ElScAe. 2014
Section: Chapter 2. First Order Equations. Exercises 2.2, page 39
Problem number: 6.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_separable]`

$$\boxed{\left(9+\frac{1}{s^2}-4s^8\right)s' = -\frac{6}{t^9}+\frac{6}{t^3}-t^7}$$

✓ Solution by Maple

Time used: 0.016 (sec). Leaf size: 36

```
dsolve((6/t^9-6/t^3+t^7)+(9+1/s(t)^2-4*s(t)^8)*diff(s(t),t)=0,s(t), singsol=all)
```

$$\frac{t^8}{8}+\frac{3}{t^2}-\frac{3}{4t^8}-\frac{4s(t)^9}{9}+9s(t)-\frac{1}{s(t)}+c_1=0$$

✓ Solution by Mathematica

Time used: 17.496 (sec). Leaf size: 531

```
DSolve[(6/t^9-6/t^3+t^7)+(9+1/s[t]^2-4*s[t]^8)*s'[t]==0,s[t],t,IncludeSingularSolutions -> T
```

$$s(t) \to \text{Root}\left[32\#1^{10}t^8 - 648\#1^2t^8 + \#1\left(-9t^{16} - 72c_1t^8 - 216t^6 + 54\right) + 72t^8\&, 1\right]$$
$$s(t) \to \text{Root}\left[32\#1^{10}t^8 - 648\#1^2t^8 + \#1\left(-9t^{16} - 72c_1t^8 - 216t^6 + 54\right) + 72t^8\&, 2\right]$$
$$s(t) \to \text{Root}\left[32\#1^{10}t^8 - 648\#1^2t^8 + \#1\left(-9t^{16} - 72c_1t^8 - 216t^6 + 54\right) + 72t^8\&, 3\right]$$
$$s(t) \to \text{Root}\left[32\#1^{10}t^8 - 648\#1^2t^8 + \#1\left(-9t^{16} - 72c_1t^8 - 216t^6 + 54\right) + 72t^8\&, 4\right]$$
$$s(t) \to \text{Root}\left[32\#1^{10}t^8 - 648\#1^2t^8 + \#1\left(-9t^{16} - 72c_1t^8 - 216t^6 + 54\right) + 72t^8\&, 5\right]$$
$$s(t) \to \text{Root}\left[32\#1^{10}t^8 - 648\#1^2t^8 + \#1\left(-9t^{16} - 72c_1t^8 - 216t^6 + 54\right) + 72t^8\&, 6\right]$$
$$s(t) \to \text{Root}\left[32\#1^{10}t^8 - 648\#1^2t^8 + \#1\left(-9t^{16} - 72c_1t^8 - 216t^6 + 54\right) + 72t^8\&, 7\right]$$
$$s(t) \to \text{Root}\left[32\#1^{10}t^8 - 648\#1^2t^8 + \#1\left(-9t^{16} - 72c_1t^8 - 216t^6 + 54\right) + 72t^8\&, 8\right]$$
$$s(t) \to \text{Root}\left[32\#1^{10}t^8 - 648\#1^2t^8 + \#1\left(-9t^{16} - 72c_1t^8 - 216t^6 + 54\right) + 72t^8\&, 9\right]$$
$$s(t) \to \text{Root}\left[32\#1^{10}t^8 - 648\#1^2t^8 + \#1\left(-9t^{16} - 72c_1t^8 - 216t^6 + 54\right) + 72t^8\&, 10\right]$$

4.7 problem 7

Internal problem ID [14165]

Book: INTRODUCTORY DIFFERENTIAL EQUATIONS. Martha L. Abell, James P. Braselton. Fourth edition 2014. ElScAe. 2014
Section: Chapter 2. First Order Equations. Exercises 2.2, page 39
Problem number: 7.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_separable]`

$$\boxed{4 \sinh (4y) y' = 6 \cosh (3x)}$$

✓ Solution by Maple
Time used: 0.015 (sec). Leaf size: 17

```
dsolve(4*sinh(4*y(x))*diff(y(x),x)=6*cosh(3*x),y(x), singsol=all)
```

$$y(x) = \frac{\operatorname{arccosh}(2 \sinh (3x) + 6c_1)}{4}$$

✓ Solution by Mathematica
Time used: 2.75 (sec). Leaf size: 41

```
DSolve[4*Sinh[4*y[x]]*y'[x]==6*Cosh[3*x],y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to -\frac{1}{4}\operatorname{arccosh}(2(\sinh(3x) + 2c_1))$$
$$y(x) \to \frac{1}{4}\operatorname{arccosh}(2(\sinh(3x) + 2c_1))$$

4.8 problem 8

Internal problem ID [14166]

Book: INTRODUCTORY DIFFERENTIAL EQUATIONS. Martha L. Abell, James P. Braselton. Fourth edition 2014. ElScAe. 2014
Section: Chapter 2. First Order Equations. Exercises 2.2, page 39
Problem number: 8.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_separable]`

$$\boxed{y' - \frac{y+1}{1+t} = 0}$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 10

```
dsolve(diff(y(t),t)=(y(t)+1)/(t+1),y(t), singsol=all)
```

$$y(t) = c_1 t + c_1 - 1$$

✓ Solution by Mathematica

Time used: 0.027 (sec). Leaf size: 18

```
DSolve[y'[t]==(y[t]+1)/(t+1),y[t],t,IncludeSingularSolutions -> True]
```

$$y(t) \to -1 + c_1(t+1)$$
$$y(t) \to -1$$

4.9 problem 9

Internal problem ID [14167]

Book: INTRODUCTORY DIFFERENTIAL EQUATIONS. Martha L. Abell, James P. Braselton. Fourth edition 2014. ElScAe. 2014
Section: Chapter 2. First Order Equations. Exercises 2.2, page 39
Problem number: 9.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_separable]`

$$\boxed{y' - \frac{y+2}{2t+1} = 0}$$

✓ Solution by Maple
Time used: 0.0 (sec). Leaf size: 15

```
dsolve(diff(y(t),t)=(y(t)+2)/(2*t+1),y(t), singsol=all)
```

$$y(t) = \sqrt{2t+1}\, c_1 - 2$$

✓ Solution by Mathematica
Time used: 0.03 (sec). Leaf size: 24

```
DSolve[y'[t]==(y[t]+2)/(2*t+1),y[t],t,IncludeSingularSolutions -> True]
```

$$y(t) \to -2 + c_1\sqrt{2t+1}$$
$$y(t) \to -2$$

4.10 problem 10

Internal problem ID [14168]

Book: INTRODUCTORY DIFFERENTIAL EQUATIONS. Martha L. Abell, James P. Braselton. Fourth edition 2014. ElScAe. 2014
Section: Chapter 2. First Order Equations. Exercises 2.2, page 39
Problem number: 10.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_separable]`

$$\boxed{-\left(\frac{1}{\sqrt{y}}+\sqrt{y}\right)y' = -\frac{3}{t^2}}$$

✓ Solution by Maple
Time used: 0.0 (sec). Leaf size: 20

```
dsolve(3/t^2=(1/y(t)^(1/2)+y(t)^(1/2))*diff(y(t),t),y(t), singsol=all)
```

$$-\frac{1}{t}-\frac{2\sqrt{y(t)}\,(y(t)+3)}{9}+c_1=0$$

✓ Solution by Mathematica

Time used: 6.594 (sec). Leaf size: 445

```
DSolve[3/t^2==(1/y[t]^(1/2)+y[t]^(1/2))*y'[t],y[t],t,IncludeSingularSolutions -> True]
```

$$y(t) \to \frac{\left(-2+\sqrt[3]{\frac{81}{t^2}+3\sqrt{\frac{(-3+c_1t)^2\left((16+9c_1{}^2)\,t^2-54c_1t+81\right)}{t^4}}-\frac{54c_1}{t}+8+9c_1{}^2}\right)^2}{2\sqrt[3]{\frac{81}{t^2}+3\sqrt{\frac{(-3+c_1t)^2\left((16+9c_1{}^2)\,t^2-54c_1t+81\right)}{t^4}}-\frac{54c_1}{t}+8+9c_1{}^2}}$$

$$y(t) \to \frac{1}{4}i\left(\sqrt{3}+i\right)\sqrt[3]{\frac{81}{t^2}+3\sqrt{\frac{(-3+c_1t)^2\left((16+9c_1{}^2)\,t^2-54c_1t+81\right)}{t^4}}-\frac{54c_1}{t}+8+9c_1{}^2}+\frac{-1-i\sqrt{3}}{\sqrt[3]{\frac{81}{t^2}+3\sqrt{\frac{(-3+c_1t)^2\left((16+9c_1{}^2)\,t^2-54c_1t+81\right)}{t^4}}-\frac{54c_1}{t}+8+9c_1{}^2}}-2$$

$$y(t) \to -\frac{1}{4}i\left(\sqrt{3}-i\right)\sqrt[3]{\frac{81}{t^2}+3\sqrt{\frac{(-3+c_1t)^2\left((16+9c_1{}^2)\,t^2-54c_1t+81\right)}{t^4}}-\frac{54c_1}{t}+8+9c_1{}^2}+\frac{-1+i\sqrt{3}}{\sqrt[3]{\frac{81}{t^2}+3\sqrt{\frac{(-3+c_1t)^2\left((16+9c_1{}^2)\,t^2-54c_1t+81\right)}{t^4}}-\frac{54c_1}{t}+8+9c_1{}^2}}-2$$

4.11 problem 11

Internal problem ID [14169]

Book: INTRODUCTORY DIFFERENTIAL EQUATIONS. Martha L. Abell, James P. Braselton. Fourth edition 2014. ElScAe. 2014
Section: Chapter 2. First Order Equations. Exercises 2.2, page 39
Problem number: 11.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_separable]`

$$\boxed{-4\cos(y)\,y' = -3\sin(x)}$$

✓ Solution by Maple
Time used: 0.0 (sec). Leaf size: 13

```
dsolve(3*sin(x)-4*cos(y(x))*diff(y(x),x)=0,y(x), singsol=all)
```

$$y(x) = \arcsin\left(-\frac{3\cos(x)}{4} + \frac{3c_1}{4}\right)$$

✓ Solution by Mathematica
Time used: 0.563 (sec). Leaf size: 29

```
DSolve[3*Sin[x]-4*Cos[y[x]]*y'[x]==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to \arcsin\left(-\frac{3\cos(x)}{4} + c_1\right)$$
$$y(x) \to \arcsin\left(-\frac{3\cos(x)}{4} + c_1\right)$$

4.12 problem 12

Internal problem ID [14170]

Book: INTRODUCTORY DIFFERENTIAL EQUATIONS. Martha L. Abell, James P. Braselton. Fourth edition 2014. ElScAe. 2014
Section: Chapter 2. First Order Equations. Exercises 2.2, page 39
Problem number: 12.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_separable]`

$$\boxed{\cos(y)\,y' = 8\sin(8t)}$$

✓ Solution by Maple
Time used: 0.0 (sec). Leaf size: 15

```
dsolve(cos(y(t))*diff(y(t),t)=8*sin(8*t),y(t), singsol=all)
```

$$y(t) = \arcsin\left(-\cos(8t) + 64c_1\right)$$

✓ Solution by Mathematica
Time used: 0.509 (sec). Leaf size: 15

```
DSolve[Cos[y[t]]*y'[t]==8*Sin[8*t],y[t],t,IncludeSingularSolutions -> True]
```

$$y(t) \to \arcsin(-\cos(8t) + c_1)$$

4.13 problem 13

Internal problem ID [14171]

Book: INTRODUCTORY DIFFERENTIAL EQUATIONS. Martha L. Abell, James P. Braselton. Fourth edition 2014. ElScAe. 2014
Section: Chapter 2. First Order Equations. Exercises 2.2, page 39
Problem number: 13.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_quadrature]`

$$\boxed{y' + yk = 0}$$

✓ Solution by Maple
Time used: 0.0 (sec). Leaf size: 11

```
dsolve(diff(y(x),x)+k*y(x)=0,y(x), singsol=all)
```

$$y(x) = c_1 \mathrm{e}^{-kx}$$

✓ Solution by Mathematica
Time used: 0.025 (sec). Leaf size: 19

```
DSolve[y'[x]+k*y[x]==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to c_1 e^{-kx}$$
$$y(x) \to 0$$

4.14 problem 14

Internal problem ID [14172]

Book: INTRODUCTORY DIFFERENTIAL EQUATIONS. Martha L. Abell, James P. Braselton. Fourth edition 2014. ElScAe. 2014
Section: Chapter 2. First Order Equations. Exercises 2.2, page 39
Problem number: 14.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_separable]`

$$\boxed{(5x^5 - 4\cos(x))\, x' = -2\cos(9t) - 2\sin(7t)}$$

✓ Solution by Maple
Time used: 0.141 (sec). Leaf size: 28

```
dsolve((5*x(t)^5-4*cos(x(t)))*diff(x(t),t)+(2*cos(9*t)+2*sin(7*t))=0,x(t), singsol=all)
```

$$\frac{\sin(9t)}{9} - \frac{\cos(7t)}{7} + \frac{5x(t)^6}{12} - 2\sin(x(t)) + c_1 = 0$$

✓ Solution by Mathematica
Time used: 0.667 (sec). Leaf size: 40

```
DSolve[(5*x[t]^5-4*Cos[x[t]])*x'[t]+(2*Cos[9*t]+2*Sin[7*t])==0,x[t],t,IncludeSingularSolutio
```

$$x(t) \to \text{InverseFunction}\left[\frac{5\#1^6}{6} - 4\sin(\#1)\&\right]\left[-\frac{2}{9}\sin(9t) + \frac{2}{7}\cos(7t) + c_1\right]$$

4.15 problem 15

Internal problem ID [14173]

Book: INTRODUCTORY DIFFERENTIAL EQUATIONS. Martha L. Abell, James P. Braselton. Fourth edition 2014. ElScAe. 2014
Section: Chapter 2. First Order Equations. Exercises 2.2, page 39
Problem number: 15.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_separable]`

$$\boxed{20\sinh(y)\, y' = -\cosh(6t) - 5\sinh(4t)}$$

✓ Solution by Maple
Time used: 0.0 (sec). Leaf size: 21

```
dsolve((cosh(6*t)+5*sinh(4*t))+(20*sinh(y(t)))*diff(y(t),t)=0,y(t), singsol=all)
```

$$y(t) = \operatorname{arccosh}\left(-\frac{\cosh(4t)}{16} - \frac{\sinh(6t)}{120} - \frac{c_1}{20}\right)$$

✓ Solution by Mathematica
Time used: 3.811 (sec). Leaf size: 51

```
DSolve[(Cosh[6*t]+5*Sinh[4*t])+(20*Sinh[y[t]])*y'[t]==0,y[t],t,IncludeSingularSolutions -> T
```

$$y(t) \to -\operatorname{arccosh}\left(-\frac{1}{120}\sinh(6t) - \frac{1}{16}\cosh(4t) + c_1\right)$$
$$y(t) \to \operatorname{arccosh}\left(-\frac{1}{120}\sinh(6t) - \frac{1}{16}\cosh(4t) + c_1\right)$$

4.16 problem 16

Internal problem ID [14174]

Book: INTRODUCTORY DIFFERENTIAL EQUATIONS. Martha L. Abell, James P. Braselton. Fourth edition 2014. ElScAe. 2014
Section: Chapter 2. First Order Equations. Exercises 2.2, page 39
Problem number: 16.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_separable]`

$$\boxed{y' - e^{2y+10t} = 0}$$

✓ Solution by Maple
Time used: 0.0 (sec). Leaf size: 22

```
dsolve(diff(y(t),t)=exp(2*y(t)+10*t),y(t), singsol=all)
```

$$y(t) = \frac{\ln(5)}{2} - \frac{\ln(-e^{10t} - 10c_1)}{2}$$

✓ Solution by Mathematica
Time used: 0.928 (sec). Leaf size: 24

```
DSolve[y'[t]==Exp[2*y[t]+10*t],y[t],t,IncludeSingularSolutions -> True]
```

$$y(t) \to -\frac{1}{2}\log\left(-\frac{e^{10t}}{5} - 2c_1\right)$$

4.17 problem 17

Internal problem ID [14175]

Book: INTRODUCTORY DIFFERENTIAL EQUATIONS. Martha L. Abell, James P. Braselton. Fourth edition 2014. ElScAe. 2014
Section: Chapter 2. First Order Equations. Exercises 2.2, page 39
Problem number: 17.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_separable]`

$$\boxed{y' - e^{3y+2t} = 0}$$

✓ Solution by Maple
Time used: 0.0 (sec). Leaf size: 26

```
dsolve(diff(y(t),t)=exp(3*y(t)+2*t),y(t), singsol=all)
```

$$y(t) = -\frac{\ln(3)}{3} + \frac{\ln(2)}{3} - \frac{\ln(-e^{2t} - 2c_1)}{3}$$

✓ Solution by Mathematica
Time used: 0.927 (sec). Leaf size: 24

```
DSolve[y'[t]==Exp[3*y[t]+2*t],y[t],t,IncludeSingularSolutions -> True]
```

$$y(t) \to -\frac{1}{3}\log\left(-\frac{3}{2}(e^{2t} + 2c_1)\right)$$

4.18 problem 18

Internal problem ID [14176]

Book: INTRODUCTORY DIFFERENTIAL EQUATIONS. Martha L. Abell, James P. Braselton. Fourth edition 2014. ElScAe. 2014
Section: Chapter 2. First Order Equations. Exercises 2.2, page 39
Problem number: 18.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_separable]`

$$\boxed{-\cos(y)^2 y' = -\sin(t)^2}$$

✓ Solution by Maple
Time used: 0.094 (sec). Leaf size: 27

```
dsolve(sin(t)^2=cos(y(t))^2*diff(y(t),t),y(t), singsol=all)
```

$$y(t) = \frac{\operatorname{RootOf}(-\_Z + 2t + 4c_1 - \sin(2t) - \sin(\_Z))}{2}$$

✓ Solution by Mathematica
Time used: 0.492 (sec). Leaf size: 35

```
DSolve[Sin[t]^2==Cos[y[t]]^2*y'[t],y[t],t,IncludeSingularSolutions -> True]
```

$$y(t) \to \text{InverseFunction}\left[2\left(\frac{\#1}{2} + \frac{1}{4}\sin(2\#1)\right)\&\right][t - \sin(t)\cos(t) + c_1]$$

4.19 problem 19

Internal problem ID [14177]

Book: INTRODUCTORY DIFFERENTIAL EQUATIONS. Martha L. Abell, James P. Braselton. Fourth edition 2014. ElScAe. 2014
Section: Chapter 2. First Order Equations. Exercises 2.2, page 39
Problem number: 19.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_separable]`

$$\boxed{-(\cos(4y) - 4\cos(y))\,y' = -3\sin(t) + \sin(3t)}$$

✓ Solution by Maple
Time used: 0.156 (sec). Leaf size: 243

```
dsolve(3*sin(t)-sin(3*t)=(cos(4*y(t))-4*cos(y(t)))*diff(y(t),t),y(t), singsol=all)
```

$$y(t) = \arctan\left(\frac{}{6\,\mathrm{RootOf}\left(36\_Z^8 - 72\_Z^6 + 16\cos(t)^6 - 144\_Z^5 - 96c_1\cos(t)^3 + 45\_Z^4 - 96\cos(t)^4 + 216\_\right.}\right.$$
$$- 72\_Z^6 + 16\cos(t)^6 - 144\_Z^5 - 96c_1\cos(t)^3 + 45\_Z^4 - 96\cos(t)^4 + 216\_Z^3$$
$$\left. + 144c_1^2 + 288\cos(t)\,c_1 + 135\_Z^2 + 144\cos(t)^2 - 72\_Z - 144)\right)$$

✓ Solution by Mathematica
Time used: 60.315 (sec). Leaf size: 2033

```
DSolve[3*Sin[t]-Sin[3*t]==(Cos[4*y[t]]-4*Cos[y[t]])*y'[t],y[t],t,IncludeSingularSolutions ->
```

Too large to display

4.20 problem 20

Internal problem ID [14178]

Book: INTRODUCTORY DIFFERENTIAL EQUATIONS. Martha L. Abell, James P. Braselton. Fourth edition 2014. ElScAe. 2014
Section: Chapter 2. First Order Equations. Exercises 2.2, page 39
Problem number: 20.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_separable]`

$$\boxed{x' - \frac{\sec(t)^2}{\sec(x)\tan(x)} = 0}$$

✓ Solution by Maple
Time used: 0.0 (sec). Leaf size: 11

```
dsolve(diff(x(t),t)=sec(t)^2/(sec(x(t))*tan(x(t))),x(t), singsol=all)
```

$$x(t) = \arccos\left(\frac{1}{\tan(t) + c_1}\right)$$

✓ Solution by Mathematica
Time used: 0.848 (sec). Leaf size: 45

```
DSolve[x'[t]==Sec[t]^2/(Sec[x[t]]*Tan[x[t]]),x[t],t,IncludeSingularSolutions -> True]
```

$$x(t) \to -\sec^{-1}(\tan(t) + 2c_1)$$
$$x(t) \to \sec^{-1}(\tan(t) + 2c_1)$$
$$x(t) \to -\frac{\pi}{2}$$
$$x(t) \to \frac{\pi}{2}$$

4.21 problem 21

Internal problem ID [14179]

Book: INTRODUCTORY DIFFERENTIAL EQUATIONS. Martha L. Abell, James P. Braselton. Fourth edition 2014. ElScAe. 2014
Section: Chapter 2. First Order Equations. Exercises 2.2, page 39
Problem number: 21.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_separable]`

$$\boxed{\left(2-\frac{5}{y^2}\right) y' = -4\cos(x)^2}$$

✓ Solution by Maple
Time used: 0.047 (sec). Leaf size: 107

```
dsolve((2-5/y(x)^2)*diff(y(x),x)+4*cos(x)^2=0,y(x), singsol=all)
```

$$y(x) = -c_1 - \frac{x}{2} - \frac{\sin(2x)}{4} - \frac{\sqrt{-158 + 16(x+2c_1)\sin(2x) + 16x^2 + 64c_1x + 64c_1^2 - 2\cos(4x)}}{8}$$
$$y(x) = -c_1 - \frac{x}{2} - \frac{\sin(2x)}{4} + \frac{\sqrt{-158 + 16(x+2c_1)\sin(2x) + 16x^2 + 64c_1x + 64c_1^2 - 2\cos(4x)}}{8}$$

✓ Solution by Mathematica
Time used: 0.672 (sec). Leaf size: 88

```
DSolve[(2-5/y[x]^2)*y'[x]+4*Cos[x]^2==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to \frac{1}{4}\left(-2x - \sin(2x) - \sqrt{-40 + (2x+\sin(2x)-c_1)^2} + c_1\right)$$
$$y(x) \to \frac{1}{4}\left(-2x - \sin(2x) + \sqrt{-40 + (2x+\sin(2x)-c_1)^2} + c_1\right)$$
$$y(x) \to 0$$

4.22 problem 22

Internal problem ID [14180]

Book: INTRODUCTORY DIFFERENTIAL EQUATIONS. Martha L. Abell, James P. Braselton. Fourth edition 2014. ElScAe. 2014
Section: Chapter 2. First Order Equations. Exercises 2.2, page 39
Problem number: 22.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[[_1st_order, '_with_symmetry_[F(x),G(x)*y+H(x)]']]`

$$\boxed{y' - \frac{t^3}{y\sqrt{(1-y^2)\,(t^4+9)}} = 0}$$

✓ Solution by Maple
Time used: 0.0 (sec). Leaf size: 50

```
dsolve(diff(y(t),t)=t^3/(y(t)*sqrt((1-y(t)^2)*(t^4+9))),y(t), singsol=all)
```

$$-\left(-\frac{y(t)^2}{3} + \int^t \frac{\_a^3}{\sqrt{-\left(\_a^4+9\right)\left(y\left(t\right)^2-1\right)}}d\_a + \frac{1}{3}\right)\sqrt{y\left(t\right)+1}\,\sqrt{-1+y\left(t\right)} + c_1 = 0$$

✓ Solution by Mathematica

Time used: 1.286 (sec). Leaf size: 519

```
DSolve[y'[t]==t^3/(y[t]*Sqrt[(1-y[t]^2)*(t^4+9)]),y[t],t,IncludeSingularSolutions -> True]
```

$$y(t) \to -\sqrt{1+\left(\frac{3}{2}\right)^{2/3}\sqrt[3]{-t^4-4ic_1\sqrt{t^4+9}-9+4c_1{}^2}}$$

$$y(t) \to \sqrt{1+\left(\frac{3}{2}\right)^{2/3}\sqrt[3]{-t^4-4ic_1\sqrt{t^4+9}-9+4c_1{}^2}}$$

$$y(t) \to -\frac{1}{2}\sqrt{-\sqrt[3]{2}3^{2/3}\sqrt[3]{-t^4-4ic_1\sqrt{t^4+9}-9+4c_1{}^2}-3i\sqrt[3]{2}\sqrt[6]{3}\sqrt[3]{-t^4-4ic_1\sqrt{t^4+9}-9+4c_1{}^2}+4}$$

$$y(t) \to \frac{1}{2}\sqrt{-\sqrt[3]{2}3^{2/3}\sqrt[3]{-t^4-4ic_1\sqrt{t^4+9}-9+4c_1{}^2}-3i\sqrt[3]{2}\sqrt[6]{3}\sqrt[3]{-t^4-4ic_1\sqrt{t^4+9}-9+4c_1{}^2}+4}$$

$$y(t) \to -\frac{1}{2}\sqrt{-\sqrt[3]{2}3^{2/3}\sqrt[3]{-t^4-4ic_1\sqrt{t^4+9}-9+4c_1{}^2}+3i\sqrt[3]{2}\sqrt[6]{3}\sqrt[3]{-t^4-4ic_1\sqrt{t^4+9}-9+4c_1{}^2}+4}$$

$$y(t) \to \frac{1}{2}\sqrt{-\sqrt[3]{2}3^{2/3}\sqrt[3]{-t^4-4ic_1\sqrt{t^4+9}-9+4c_1{}^2}+3i\sqrt[3]{2}\sqrt[6]{3}\sqrt[3]{-t^4-4ic_1\sqrt{t^4+9}-9+4c_1{}^2}+4}$$

4.23 problem 23

Internal problem ID [14181]

Book: INTRODUCTORY DIFFERENTIAL EQUATIONS. Martha L. Abell, James P. Braselton. Fourth edition 2014. ElScAe. 2014
Section: Chapter 2. First Order Equations. Exercises 2.2, page 39
Problem number: 23.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_separable]`

$$\boxed{\tan(y)\sec(y)^2 y' = -\cos(2x)^3 \sin(2x)}$$

✓ Solution by Maple
Time used: 0.047 (sec). Leaf size: 59

```
dsolve(tan(y(x))*sec(y(x))^2*diff(y(x),x)+cos(2*x)^3*sin(2*x)=0,y(x), singsol=all)
```

$$y(x) = \operatorname{arccot}\left(\frac{8}{\sqrt{-2+4\cos(4x)^2-256c_1+8\cos(4x)}}\right)$$
$$y(x) = \frac{\pi}{2}+\arctan\left(\frac{8}{\sqrt{-2+4\cos(4x)^2-256c_1+8\cos(4x)}}\right)$$

✓ Solution by Mathematica

Time used: 2.982 (sec). Leaf size: 139

```
DSolve[Tan[y[x]]*Sec[y[x]]^2*y'[x]+Cos[2*x]^3*Sin[2*x]==0,y[x],x,IncludeSingularSolutions ->
```

$$y(x) \to -\sec^{-1}\left(-\frac{\sqrt{8\cos^4(2x)+c_1}}{4\sqrt{2}}\right)$$
$$y(x) \to \sec^{-1}\left(-\frac{\sqrt{8\cos^4(2x)+c_1}}{4\sqrt{2}}\right)$$
$$y(x) \to -\sec^{-1}\left(\frac{\sqrt{8\cos^4(2x)+c_1}}{4\sqrt{2}}\right)$$
$$y(x) \to \sec^{-1}\left(\frac{\sqrt{8\cos^4(2x)+c_1}}{4\sqrt{2}}\right)$$
$$y(x) \to -\frac{\pi}{2}$$
$$y(x) \to \frac{\pi}{2}$$

4.24 problem 24

Internal problem ID [14182]

Book: INTRODUCTORY DIFFERENTIAL EQUATIONS. Martha L. Abell, James P. Braselton. Fourth edition 2014. ElScAe. 2014
Section: Chapter 2. First Order Equations. Exercises 2.2, page 39
Problem number: 24.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_separable]`

$$\boxed{y' - \frac{(1 + 2\,e^{y})\,e^{-y}}{t\ln(t)} = 0}$$

✓ Solution by Maple

Time used: 10.859 (sec). Leaf size: 22

```
dsolve(diff(y(t),t)=(1+2*exp(y(t)))/(exp(y(t))*t*ln(t)),y(t), singsol=all)
```

$$y(t) = -\ln(2) - \ln\left(\frac{1}{\ln(t)^2 c_1 - 1}\right)$$

✓ Solution by Mathematica

Time used: 0.685 (sec). Leaf size: 51

```
DSolve[y'[t]==(1+2*Exp[y[t]])/(Exp[y[t]]*t*Log[t]),y[t],t,IncludeSingularSolutions -> True]
```

$$\begin{aligned}
y(t) &\to \log\left(\frac{1}{2}\left(-1 + e^{2c_1}\log^2(t)\right)\right)\\
y(t) &\to -\log(2) - i\pi\\
y(t) &\to -\log(2) + i\pi
\end{aligned}$$

4.25 problem 25

Internal problem ID [14183]

Book: INTRODUCTORY DIFFERENTIAL EQUATIONS. Martha L. Abell, James P. Braselton. Fourth edition 2014. ElScAe. 2014
Section: Chapter 2. First Order Equations. Exercises 2.2, page 39
Problem number: 25.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_separable]`

$$\boxed{-\frac{\cos\left(\sqrt{y}\right) y'}{\sqrt{y}} = -\sin\left(x^2\right) x}$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 18

```
dsolve(x*sin(x^2)=cos(sqrt(y(x)))/sqrt(y(x))*diff(y(x),x),y(x), singsol=all)
```

$$-\frac{\cos\left(x^2\right)}{2} - 2\sin\left(\sqrt{y\left(x\right)}\right) + c_1 = 0$$

✓ Solution by Mathematica

Time used: 1.996 (sec). Leaf size: 43

```
DSolve[x*Sin[x^2]==Cos[Sqrt[y[x]]]/Sqrt[y[x]]*y'[x],y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to \arcsin\left(\frac{1}{4}\left(-\cos\left(x^2\right) + 2c_1\right)\right)^2$$
$$y(x) \to \arcsin\left(\frac{1}{4}\left(\cos\left(x^2\right) - 2c_1\right)\right)^2$$

4.26 problem 26

Internal problem ID [14184]

Book: INTRODUCTORY DIFFERENTIAL EQUATIONS. Martha L. Abell, James P. Braselton. Fourth edition 2014. ElScAe. 2014
Section: Chapter 2. First Order Equations. Exercises 2.2, page 39
Problem number: 26.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_separable]`

$$\boxed{-\frac{\left(1-\frac{1}{y}\right)^{2} y'}{y^{2}}=-\frac{x-2}{x^{2}-4x+3}}$$

✓ Solution by Maple

Time used: 0.015 (sec). Leaf size: 977

```
dsolve((x-2)/(x^2-4*x+3)=(1-1/y(x))^2*1/y(x)^2*diff(y(x),x),y(x), singsol=all)
```

$$y(x) = \frac{\left((9\,\mathrm{csgn}\left(3\ln\left((-1+x)\left(-3+x\right)\right)+6c_1+2\right)\ln\left((-1+x)\left(-3+x\right)\right)^{2}+36\,\mathrm{csgn}\left(3\ln\left((-1+x)\left(-3+\right.\right.\right.}{}$$

$$y(x) = \frac{\left(-i36^{\frac{2}{3}}\left(\left(\left(\frac{\ln((-1+x)(-3+x))}{2}+c_1\right)\mathrm{csgn}\left(3\ln\left((-1+x)\left(-3+x\right)\right)+6c_1+2\right)-c_1-\frac{\ln((-1+x)(-3+x))}{2}-\frac{2}{3}\right)\right.\right.}{}$$

$$y(x) = \frac{\left(i36^{\frac{2}{3}}\left(\left(\left(\frac{\ln((-1+x)(-3+x))}{2}+c_1\right)\mathrm{csgn}\left(3\ln\left((-1+x)\left(-3+x\right)\right)+6c_1+2\right)-c_1-\frac{\ln((-1+x)(-3+x))}{2}-\frac{2}{3}\right)\left(\right.\right.\right.}{}$$

✓ Solution by Mathematica

Time used: 1.679 (sec). Leaf size: 1134

```
DSolve[(x-2)/(x^2-4*x+3)==(1-1/y[x])^2*1/y[x]^2*y'[x],y[x],x,IncludeSingularSolutions -> Tru
```

$$y(x) \to \frac{-6\log(x-3) - 6\log(x-1) - \left(9\log^2(x-3) + 9\log^2(x-1) + 18\log(x-3) + 18\log(x-3)\log(x-1}{}$$

$$y(x) \to \frac{\sqrt[3]{9\log^2(x-3) + 9\log^2(x-1) + 18\log(x-3) + 18\log(x-3)\log(x-1) + 18\log(x-1) + 3\sqrt{\left(3\log^2}}}{}$$

$$y(x) \to \frac{\sqrt[3]{9\log^2(x-3) + 9\log^2(x-1) + 18\log(x-3) + 18\log(x-3)\log(x-1) + 18\log(x-1) + 3\sqrt{\left(3\log^2}}}{}$$

$$y(x) \to 0$$

4.27 problem 27

Internal problem ID [14185]

Book: INTRODUCTORY DIFFERENTIAL EQUATIONS. Martha L. Abell, James P. Braselton. Fourth edition 2014. ElScAe. 2014
Section: Chapter 2. First Order Equations. Exercises 2.2, page 39
Problem number: 27.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_separable]`

$$\boxed{\frac{\cos(y)\, y'}{(1-\sin(y))^2} = \cos(x)\sin(x)^3}$$

✓ Solution by Maple
Time used: 0.0 (sec). Leaf size: 42

```
dsolve(cos(y(x))/(1-sin(y(x)))^2*diff(y(x),x)=sin(x)^3*cos(x),y(x), singsol=all)
```

$$y(x) = \arcsin\left(\frac{\cos(2x)^2 + 16c_1 - 2\cos(2x) - 15}{\cos(2x)^2 + 16c_1 - 2\cos(2x) + 1}\right)$$

✓ Solution by Mathematica

Time used: 28.843 (sec). Leaf size: 705

```
DSolve[Cos[y[x]]/(1-Sin[y[x]])^2*y'[x]==Sin[x]^3*Cos[x],y[x],x,IncludeSingularSolutions -> T
```

$$y(x) \to -2\arccos\left(-\frac{4+\sqrt{-4\cos(2x)+\cos(4x)-13+c_1}}{\sqrt{2}\sqrt{8\sin^4(x)+c_1}}\right)$$
$$y(x) \to 2\arccos\left(-\frac{4+\sqrt{-4\cos(2x)+\cos(4x)-13+c_1}}{\sqrt{2}\sqrt{8\sin^4(x)+c_1}}\right)$$
$$y(x) \to -2\arccos\left(\frac{-4+\sqrt{-4\cos(2x)+\cos(4x)-13+c_1}}{\sqrt{2}\sqrt{8\sin^4(x)+c_1}}\right)$$
$$y(x) \to 2\arccos\left(\frac{-4+\sqrt{-4\cos(2x)+\cos(4x)-13+c_1}}{\sqrt{2}\sqrt{8\sin^4(x)+c_1}}\right)$$
$$y(x) \to -2\arccos\left(-\frac{-4+\sqrt{-4\cos(2x)+\cos(4x)-13+c_1}}{\sqrt{2}\sqrt{8\sin^4(x)+c_1}}\right)$$
$$y(x) \to 2\arccos\left(-\frac{-4+\sqrt{-4\cos(2x)+\cos(4x)-13+c_1}}{\sqrt{2}\sqrt{8\sin^4(x)+c_1}}\right)$$
$$y(x) \to -2\arccos\left(\frac{4+\sqrt{-4\cos(2x)+\cos(4x)-13+c_1}}{\sqrt{2}\sqrt{8\sin^4(x)+c_1}}\right)$$
$$y(x) \to 2\arccos\left(\frac{4+\sqrt{-4\cos(2x)+\cos(4x)-13+c_1}}{\sqrt{2}\sqrt{8\sin^4(x)+c_1}}\right)$$
$$y(x) \to -\frac{3\pi}{2}$$
$$y(x) \to -\frac{\pi}{2}$$
$$y(x) \to \frac{\pi}{2}$$
$$y(x) \to \frac{3\pi}{2}$$
$$y(x) \to -2\arccos\left(-\frac{\sqrt{-4\cos(2x)+\cos(4x)-13}-4}{4\sqrt{\sin^4(x)}}\right)$$
$$y(x) \to 2\arccos\left(-\frac{\sqrt{-4\cos(2x)+\cos(4x)-13}-4}{4\sqrt{\sin^4(x)}}\right)$$
$$y(x) \to -2\arccos\left(\frac{\sqrt{-4\cos(2x)+\cos(4x)-13}-4}{4\sqrt{\sin^4(x)}}\right)$$
$$y(x) \to 2\arccos\left(\frac{\sqrt{-4\cos(2x)+\cos(4x)-13}-4}{4\sqrt{\sin^4(x)}}\right)$$

$$y(x) \to -2\arccos\left(-\frac{\sqrt{-4\cos(2x)+\cos(4x)-13}+4}{4\sqrt{\sin^4(x)}}\right)$$

4.28 problem 28

Internal problem ID [14186]

Book: INTRODUCTORY DIFFERENTIAL EQUATIONS. Martha L. Abell, James P. Braselton. Fourth edition 2014. ElScAe. 2014
Section: Chapter 2. First Order Equations. Exercises 2.2, page 39
Problem number: 28.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_separable]`

$$\boxed{y' - \frac{(5-2\cos(x))^3 \sin(x)\cos(y)^4}{\sin(y)} = 0}$$

✓ Solution by Maple

Time used: 0.015 (sec). Leaf size: 239

```
dsolve(diff(y(x),x)=((5-2*cos(x))^3*sin(x)*cos(y(x))^4)/sin(y(x)),y(x), singsol=all)
```

$$y(x) = \pi - \arccos\left(\frac{2\,3^{\frac{2}{3}}\left((8c_1 - 931 - 2\cos(4x) - 308\cos(2x) + 40\cos(3x) + 1120\cos(x))^2\right)^{\frac{1}{3}}}{-48\cos(x)^4 + 480\cos(x)^3 - 1800\cos(x)^2 + 24c_1 + 3000\cos(x) - 1875}\right)$$

$$y(x) = \frac{\pi}{2} - i\,\mathrm{arcsinh}\left(\frac{\left(\frac{i3^{\frac{2}{3}}}{3} + 3^{\frac{1}{6}}\right)\left((8c_1 - 931 - 2\cos(4x) - 308\cos(2x) + 40\cos(3x) + 1120\cos(x))^2\right)^{\frac{1}{3}}}{16\cos(x)^4 - 160\cos(x)^3 + 600\cos(x)^2 - 1000\cos(x) - 8c_1 + 625}\right)$$

$$y(x) = \frac{\pi}{2} + i\,\mathrm{arcsinh}\left(\frac{\left((8c_1 - 931 - 2\cos(4x) - 308\cos(2x) + 40\cos(3x) + 1120\cos(x))^2\right)^{\frac{1}{3}} 3^{\frac{2}{3}}\left(i - \sqrt{3}\right)}{-48\cos(x)^4 + 480\cos(x)^3 - 1800\cos(x)^2 + 24c_1 + 3000\cos(x) - 1875}\right)$$

✓ Solution by Mathematica

Time used: 9.673 (sec). Leaf size: 311

```
DSolve[y'[x]==((5-2*Cos[x])^3*Sin[x]*Cos[y[x]]^4)/Sin[y[x]],y[x],x,IncludeSingularSolutions
```

$$y(x) \to -\sec^{-1}\left(-\frac{1}{2}\sqrt[3]{-\frac{3}{2}}\sqrt[3]{-2240\cos(x)+616\cos(2x)-80\cos(3x)+4\cos(4x)+1862+c_1}\right)$$

$$y(x) \to \sec^{-1}\left(-\frac{1}{2}\sqrt[3]{-\frac{3}{2}}\sqrt[3]{-2240\cos(x)+616\cos(2x)-80\cos(3x)+4\cos(4x)+1862+c_1}\right)$$

$$y(x) \to -\sec^{-1}\left(\frac{1}{2}\sqrt[3]{\frac{3}{2}}\sqrt[3]{-2240\cos(x)+616\cos(2x)-80\cos(3x)+4\cos(4x)+1862+c_1}\right)$$

$$y(x) \to \sec^{-1}\left(\frac{1}{2}\sqrt[3]{\frac{3}{2}}\sqrt[3]{-2240\cos(x)+616\cos(2x)-80\cos(3x)+4\cos(4x)+1862+c_1}\right)$$

$$y(x) \to -\sec^{-1}\left(\frac{1}{2}(-1)^{2/3}\sqrt[3]{\frac{3}{2}}\sqrt[3]{-2240\cos(x)+616\cos(2x)-80\cos(3x)+4\cos(4x)+1862+c_1}\right)$$

$$y(x) \to \sec^{-1}\left(\frac{1}{2}(-1)^{2/3}\sqrt[3]{\frac{3}{2}}\sqrt[3]{-2240\cos(x)+616\cos(2x)-80\cos(3x)+4\cos(4x)+1862+c_1}\right)$$

$$y(x) \to -\frac{\pi}{2}$$

$$y(x) \to \frac{\pi}{2}$$

4.29 problem 29

Internal problem ID [14187]

Book: INTRODUCTORY DIFFERENTIAL EQUATIONS. Martha L. Abell, James P. Braselton. Fourth edition 2014. ElScAe. 2014
Section: Chapter 2. First Order Equations. Exercises 2.2, page 39
Problem number: 29.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_separable]`

$$\boxed{-\frac{e^{\frac{3}{y}} y'}{y} = -\frac{\sqrt{\ln(x)}}{x}}$$

✓ Solution by Maple
Time used: 0.031 (sec). Leaf size: 24

```
dsolve(sqrt(ln(x))/x=exp(3/y(x))/y(x)*diff(y(x),x),y(x), singsol=all)
```

$$y(x) = -\frac{3}{\operatorname{RootOf}\left(2\ln(x)^{\frac{3}{2}} - 3\operatorname{expIntegral}_1(\_Z) + 3c_1\right)}$$

✓ Solution by Mathematica
Time used: 0.425 (sec). Leaf size: 29

```
DSolve[Sqrt[Log[x]]/x==Exp[3/y[x]]/y[x]*y'[x],y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to \operatorname{InverseFunction}\left[-\operatorname{ExpIntegralEi}\left(\frac{3}{\#1}\right)\&\right]\left[\frac{2}{3}\log^{\frac{3}{2}}(x) + c_1\right]$$

4.30 problem 30

Internal problem ID [14188]

Book: INTRODUCTORY DIFFERENTIAL EQUATIONS. Martha L. Abell, James P. Braselton. Fourth edition 2014. ElScAe. 2014
Section: Chapter 2. First Order Equations. Exercises 2.2, page 39
Problem number: 30.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_separable]`

$$\boxed{y' - \frac{5^{-t}}{y^2} = 0}$$

✓ Solution by Maple
Time used: 0.016 (sec). Leaf size: 85

```
dsolve(diff(y(t),t)=5^(-t)/y(t)^2,y(t), singsol=all)
```

$$y(t) = \frac{\left(c_1 \ln(5) - 3\,5^{-t}\right)^{\frac{1}{3}}}{\ln(5)^{\frac{1}{3}}}$$
$$y(t) = -\frac{\left(c_1 \ln(5) - 3\,5^{-t}\right)^{\frac{1}{3}} \left(1 + i\sqrt{3}\right)}{2 \ln(5)^{\frac{1}{3}}}$$
$$y(t) = \frac{\left(c_1 \ln(5) - 3\,5^{-t}\right)^{\frac{1}{3}} \left(i\sqrt{3} - 1\right)}{2 \ln(5)^{\frac{1}{3}}}$$

✓ Solution by Mathematica

Time used: 4.979 (sec). Leaf size: 88

```
DSolve[y'[t]==5^(-t)/y[t]^2,y[t],t,IncludeSingularSolutions -> True]
```

$$y(t) \to -\sqrt[3]{-\frac{3}{\log(5)}}\sqrt[3]{-5^{-t}+c_1\log(5)}$$
$$y(t) \to \sqrt[3]{-\frac{3\ 5^{-t}}{\log(5)}+3c_1}$$
$$y(t) \to (-1)^{2/3}\sqrt[3]{-\frac{3\ 5^{-t}}{\log(5)}+3c_1}$$

4.31 problem 31

Internal problem ID [14189]

Book: INTRODUCTORY DIFFERENTIAL EQUATIONS. Martha L. Abell, James P. Braselton. Fourth edition 2014. ElScAe. 2014
Section: Chapter 2. First Order Equations. Exercises 2.2, page 39
Problem number: 31.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_separable]`

$$\boxed{y' - t^2y^2 - y^2 = -t^2 - 1}$$

✓ Solution by Maple
Time used: 0.016 (sec). Leaf size: 15

```
dsolve(diff(y(t),t)=t^2*y(t)^2+y(t)^2-t^2-1,y(t), singsol=all)
```

$$y(t) = -\tanh\left(\frac{1}{3}t^3 + c_1 + t\right)$$

✓ Solution by Mathematica
Time used: 0.287 (sec). Leaf size: 58

```
DSolve[y'[t]==t^2*y[t]^2+y[t]^2-t^2-1,y[t],t,IncludeSingularSolutions -> True]
```

$$y(t) \to \frac{1 - e^{\frac{2t^3}{3}+2t+2c_1}}{1 + e^{\frac{2t^3}{3}+2t+2c_1}}$$
$$y(t) \to -1$$
$$y(t) \to 1$$

4.32 problem 32

Internal problem ID [14190]

Book: INTRODUCTORY DIFFERENTIAL EQUATIONS. Martha L. Abell, James P. Braselton. Fourth edition 2014. ElScAe. 2014
Section: Chapter 2. First Order Equations. Exercises 2.2, page 39
Problem number: 32.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_quadrature]`

$$\boxed{y' - y^2 + 3y = 2}$$

✓ Solution by Maple
Time used: 0.047 (sec). Leaf size: 19

```
dsolve(diff(y(x),x)=y(x)^2-3*y(x)+2,y(x), singsol=all)
```

$$y(x) = \frac{-2 + c_1 \mathrm{e}^x}{-1 + c_1 \mathrm{e}^x}$$

✓ Solution by Mathematica
Time used: 0.873 (sec). Leaf size: 34

```
DSolve[y'[x]==y[x]^2-3*y[x]+2,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to \frac{-2 + e^{x+c_1}}{-1 + e^{x+c_1}}$$
$$y(x) \to 1$$
$$y(x) \to 2$$

4.33 problem 33

Internal problem ID [14191]

Book: INTRODUCTORY DIFFERENTIAL EQUATIONS. Martha L. Abell, James P. Braselton. Fourth edition 2014. ElScAe. 2014
Section: Chapter 2. First Order Equations. Exercises 2.2, page 39
Problem number: 33.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_separable]`

$$\boxed{4(x-1)^2 y' - 3(y+3)^2 = 0}$$

✓ Solution by Maple
Time used: 0.0 (sec). Leaf size: 28

```
dsolve(4*(x-1)^2*diff(y(x),x)-3*(y(x)+3)^2=0,y(x), singsol=all)
```

$$y(x) = \frac{(-9x+9)\,c_1 - 4x + 13}{-3 + (3x-3)\,c_1}$$

✓ Solution by Mathematica
Time used: 0.208 (sec). Leaf size: 37

```
DSolve[4*(x-1)^2*y'[x]-3*(y[x]+3)^2==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to \frac{-4(1+3c_1)x + 13 + 12c_1}{-3 + 4c_1(x-1)}$$
$$y(x) \to -3$$

4.34 problem 34

Internal problem ID [14192]

Book: INTRODUCTORY DIFFERENTIAL EQUATIONS. Martha L. Abell, James P. Braselton. Fourth edition 2014. ElScAe. 2014
Section: Chapter 2. First Order Equations. Exercises 2.2, page 39
Problem number: 34.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_separable]`

$$\boxed{y' - \sin(t-y) - \sin(y+t) = 0}$$

✓ Solution by Maple
Time used: 0.219 (sec). Leaf size: 51

```
dsolve(diff(y(t),t)=sin(t-y(t))+sin(t+y(t)),y(t), singsol=all)
```

$$y(t) = \arctan\left(\frac{\mathrm{e}^{-4\cos(t)}c_1^2 - 1}{\mathrm{e}^{-4\cos(t)}c_1^2 + 1}, \frac{2\,\mathrm{e}^{-2\cos(t)}c_1}{\mathrm{e}^{-4\cos(t)}c_1^2 + 1}\right)$$

✓ Solution by Mathematica
Time used: 0.53 (sec). Leaf size: 38

```
DSolve[y'[t]==Sin[t-y[t]]+Sin[t+y[t]],y[t],t,IncludeSingularSolutions -> True]
```

$$\begin{aligned} y(t) &\to 2\arctan\left(\tanh\left(\frac{1}{2}(-2\cos(t)+c_1)\right)\right) \\ y(t) &\to -\frac{\pi}{2} \\ y(t) &\to \frac{\pi}{2} \end{aligned}$$

4.35 problem 35

Internal problem ID [14193]

Book: INTRODUCTORY DIFFERENTIAL EQUATIONS. Martha L. Abell, James P. Braselton. Fourth edition 2014. ElScAe. 2014
Section: Chapter 2. First Order Equations. Exercises 2.2, page 39
Problem number: 35.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_quadrature]`

$$\boxed{y' - y^3 = 1}$$

✓ Solution by Maple

Time used: 0.563 (sec). Leaf size: 51

```
dsolve(diff(y(t),t)=y(t)^3+1,y(t), singsol=all)
```

$$y(t) = \frac{1}{2} + \frac{\sqrt{3}\,\tan\left(\operatorname{RootOf}\left(-\sqrt{3}\,\ln\left(\cos\left(\_Z\right)^2\right) - 2\sqrt{3}\,\ln\left(\sqrt{3} + \tan\left(\_Z\right)\right) + 6\sqrt{3}\,c_1 + 6\sqrt{3}\,t - 6\_Z\right)\right)}{2}$$

✓ Solution by Mathematica

Time used: 0.196 (sec). Leaf size: 80

```
DSolve[y'[t]==y[t]^3+1,y[t],t,IncludeSingularSolutions -> True]
```

$$y(t) \to \text{InverseFunction}\left[-\frac{1}{6}\log\left(\#1^2 - \#1 + 1\right) + \frac{\arctan\left(\frac{2\#1-1}{\sqrt{3}}\right)}{\sqrt{3}} + \frac{1}{3}\log(\#1 + 1)\&\right][t + c_1]$$
$$y(t) \to -1$$
$$y(t) \to \sqrt[3]{-1}$$
$$y(t) \to -(-1)^{2/3}$$

4.36 problem 36

Internal problem ID [14194]

Book: INTRODUCTORY DIFFERENTIAL EQUATIONS. Martha L. Abell, James P. Braselton. Fourth edition 2014. ElScAe. 2014
Section: Chapter 2. First Order Equations. Exercises 2.2, page 39
Problem number: 36.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_quadrature]`

$$\boxed{y' - y^3 = -1}$$

✓ Solution by Maple
Time used: 0.547 (sec). Leaf size: 53

```
dsolve(diff(y(t),t)=y(t)^3-1,y(t), singsol=all)
```

$$y(t) = -\frac{1}{2} + \frac{\sqrt{3}\tan\left(\mathrm{RootOf}\left(-\sqrt{3}\ln\left(\cos\left(\_Z\right)^2\right) - 2\sqrt{3}\ln\left(\tan\left(\_Z\right) - \sqrt{3}\right) + 6\sqrt{3}\,c_1 + 6\sqrt{3}\,t + 6\_Z\right)\right)}{2}$$

✓ Solution by Mathematica
Time used: 0.176 (sec). Leaf size: 81

```
DSolve[y'[t]==y[t]^3-1,y[t],t,IncludeSingularSolutions -> True]
```

$$y(t) \to \mathrm{InverseFunction}\left[-\frac{1}{6}\log\left(\#1^2 + \#1 + 1\right) - \frac{\arctan\left(\frac{2\#1+1}{\sqrt{3}}\right)}{\sqrt{3}} + \frac{1}{3}\log(1 - \#1)\&\right][t + c_1]$$

$$y(t) \to 1$$
$$y(t) \to -\sqrt[3]{-1}$$
$$y(t) \to (-1)^{2/3}$$

4.37 problem 37

Internal problem ID [14195]

Book: INTRODUCTORY DIFFERENTIAL EQUATIONS. Martha L. Abell, James P. Braselton. Fourth edition 2014. ElScAe. 2014
Section: Chapter 2. First Order Equations. Exercises 2.2, page 39
Problem number: 37.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_quadrature]`

$$\boxed{y' - y^3 - y = 0}$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 29

```
dsolve(diff(y(t),t)=y(t)^3+y(t),y(t), singsol=all)
```

$$y(t) = \frac{1}{\sqrt{\mathrm{e}^{-2t}c_1 - 1}}$$
$$y(t) = -\frac{1}{\sqrt{\mathrm{e}^{-2t}c_1 - 1}}$$

✓ Solution by Mathematica

Time used: 60.048 (sec). Leaf size: 57

```
DSolve[y'[t]==y[t]^3+y[t],y[t],t,IncludeSingularSolutions -> True]
```

$$y(t) \to -\frac{ie^{t+c_1}}{\sqrt{-1 + e^{2(t+c_1)}}}$$
$$y(t) \to \frac{ie^{t+c_1}}{\sqrt{-1 + e^{2(t+c_1)}}}$$

4.38 problem 38

Internal problem ID [14196]

Book: INTRODUCTORY DIFFERENTIAL EQUATIONS. Martha L. Abell, James P. Braselton. Fourth edition 2014. ElScAe. 2014
Section: Chapter 2. First Order Equations. Exercises 2.2, page 39
Problem number: 38.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_quadrature]`

$$\boxed{y' - y^3 + y^2 = 0}$$

✓ Solution by Maple
Time used: 0.172 (sec). Leaf size: 16

```
dsolve(diff(y(t),t)=y(t)^3-y(t)^2,y(t), singsol=all)
```

$$y(t) = \frac{1}{\operatorname{LambertW}\left(-c_1 e^{t-1}\right) + 1}$$

✓ Solution by Mathematica
Time used: 0.225 (sec). Leaf size: 38

```
DSolve[y'[t]==y[t]^3-y[t]^2,y[t],t,IncludeSingularSolutions -> True]
```

$$\begin{aligned}
y(t) &\to \text{InverseFunction}\left[\frac{1}{\#1} + \log(1-\#1) - \log(\#1)\&\right][t + c_1] \\
y(t) &\to 0 \\
y(t) &\to 1
\end{aligned}$$

4.39 problem 39

Internal problem ID [14197]

Book: INTRODUCTORY DIFFERENTIAL EQUATIONS. Martha L. Abell, James P. Braselton. Fourth edition 2014. ElScAe. 2014
Section: Chapter 2. First Order Equations. Exercises 2.2, page 39
Problem number: 39.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_quadrature]`

$$\boxed{y' - y^3 + y = 0}$$

✓ Solution by Maple
Time used: 0.0 (sec). Leaf size: 29

```
dsolve(diff(y(t),t)=y(t)^3-y(t),y(t), singsol=all)
```

$$y(t) = \frac{1}{\sqrt{c_1 e^{2t} + 1}}$$
$$y(t) = -\frac{1}{\sqrt{c_1 e^{2t} + 1}}$$

✓ Solution by Mathematica
Time used: 0.714 (sec). Leaf size: 54

```
DSolve[y'[t]==y[t]^3-y[t],y[t],t,IncludeSingularSolutions -> True]
```

$$y(t) \to -\frac{1}{\sqrt{1 + e^{2(t+c_1)}}}$$
$$y(t) \to \frac{1}{\sqrt{1 + e^{2(t+c_1)}}}$$
$$y(t) \to -1$$
$$y(t) \to 0$$
$$y(t) \to 1$$

4.40 problem 40

Internal problem ID [14198]

Book: INTRODUCTORY DIFFERENTIAL EQUATIONS. Martha L. Abell, James P. Braselton. Fourth edition 2014. ElScAe. 2014
Section: Chapter 2. First Order Equations. Exercises 2.2, page 39
Problem number: 40.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_quadrature]`

$$\boxed{y' - y^3 - y = 0}$$

✓ Solution by Maple
Time used: 0.0 (sec). Leaf size: 29

```
dsolve(diff(y(t),t)=y(t)^3+y(t),y(t), singsol=all)
```

$$y(t) = \frac{1}{\sqrt{e^{-2t}c_1 - 1}}$$
$$y(t) = -\frac{1}{\sqrt{e^{-2t}c_1 - 1}}$$

✓ Solution by Mathematica
Time used: 60.063 (sec). Leaf size: 57

```
DSolve[y'[t]==y[t]^3+y[t],y[t],t,IncludeSingularSolutions -> True]
```

$$y(t) \to -\frac{ie^{t+c_1}}{\sqrt{-1 + e^{2(t+c_1)}}}$$
$$y(t) \to \frac{ie^{t+c_1}}{\sqrt{-1 + e^{2(t+c_1)}}}$$

4.41 problem 41

Internal problem ID [14199]

Book: INTRODUCTORY DIFFERENTIAL EQUATIONS. Martha L. Abell, James P. Braselton. Fourth edition 2014. ElScAe. 2014
Section: Chapter 2. First Order Equations. Exercises 2.2, page 39
Problem number: 41.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_quadrature]`

$$\boxed{y' = x^3}$$

With initial conditions

$$[y(0) = 0]$$

✓ Solution by Maple
Time used: 0.016 (sec). Leaf size: 9

```
dsolve([diff(y(x),x)=x^3,y(0) = 0],y(x), singsol=all)
```

$$y(x) = \frac{x^4}{4}$$

✓ Solution by Mathematica
Time used: 0.002 (sec). Leaf size: 12

```
DSolve[{y'[x]==x^3,{y[0]==0}},y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to \frac{x^4}{4}$$

4.42 problem 42

Internal problem ID [14200]

Book: INTRODUCTORY DIFFERENTIAL EQUATIONS. Martha L. Abell, James P. Braselton. Fourth edition 2014. ElScAe. 2014
Section: Chapter 2. First Order Equations. Exercises 2.2, page 39
Problem number: 42.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_quadrature]`

$$\boxed{y' = \cos(t)}$$

With initial conditions

$$\left[y\left(\frac{\pi}{2}\right) = -1\right]$$

✓ Solution by Maple
Time used: 0.0 (sec). Leaf size: 8

```
dsolve([diff(y(t),t)=cos(t),y(1/2*Pi) = -1],y(t), singsol=all)
```

$$y(t) = \sin(t) - 2$$

✓ Solution by Mathematica
Time used: 0.01 (sec). Leaf size: 9

```
DSolve[{y'[t]==Cos[t],{y[Pi/2]==-1}},y[t],t,IncludeSingularSolutions -> True]
```

$$y(t) \to \sin(t) - 2$$

4.43 problem 43

Internal problem ID [14201]

Book: INTRODUCTORY DIFFERENTIAL EQUATIONS. Martha L. Abell, James P. Braselton. Fourth edition 2014. ElScAe. 2014
Section: Chapter 2. First Order Equations. Exercises 2.2, page 39
Problem number: 43.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_quadrature]`

$$\boxed{-\cos(y)\, y' = -1}$$

With initial conditions

$$[y(0) = 2]$$

✓ Solution by Maple
Time used: 0.078 (sec). Leaf size: 13

```
dsolve([1=cos(y(x))*diff(y(x),x),y(0) = 2],y(x), singsol=all)
```

$$y(x) = \pi - \arcsin(x + \sin(2))$$

✗ Solution by Mathematica
Time used: 0.0 (sec). Leaf size: 0

```
DSolve[{1==Cos[y[x]]*y'[x],{y[0]==2}},y[x],x,IncludeSingularSolutions -> True]
```

{}

4.44 problem 44

Internal problem ID [14202]

Book: INTRODUCTORY DIFFERENTIAL EQUATIONS. Martha L. Abell, James P. Braselton. Fourth edition 2014. ElScAe. 2014
Section: Chapter 2. First Order Equations. Exercises 2.2, page 39
Problem number: 44.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_quadrature]`

$$\boxed{-x' = -\sin(y)^2}$$

With initial conditions

$$[x(0) = 0]$$

✓ Solution by Maple
Time used: 0.016 (sec). Leaf size: 14

```
dsolve([sin(y)^2=diff(x(y),y),x(0) = 0],x(y), singsol=all)
```

$$x(y) = \frac{y}{2} - \frac{\sin(2y)}{4}$$

✓ Solution by Mathematica
Time used: 0.012 (sec). Leaf size: 17

```
DSolve[{Sin[y]^2==x'[y],{x[0]==0}},x[y],y,IncludeSingularSolutions -> True]
```

$$x(y) \to \frac{1}{2}(y - \sin(y)\cos(y))$$

4.45 problem 45

Internal problem ID [14203]

Book: INTRODUCTORY DIFFERENTIAL EQUATIONS. Martha L. Abell, James P. Braselton. Fourth edition 2014. ElScAe. 2014
Section: Chapter 2. First Order Equations. Exercises 2.2, page 39
Problem number: 45.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_separable]`

$$\boxed{y' - \frac{\sqrt{t}}{y} = 0}$$

With initial conditions

$$[y(0) = 2]$$

✓ Solution by Maple
Time used: 0.047 (sec). Leaf size: 15

```
dsolve([diff(y(t),t)=sqrt(t)/y(t),y(0) = 2],y(t), singsol=all)
```

$$y(t) = \frac{2\sqrt{9 + 3t^{\frac{3}{2}}}}{3}$$

✓ Solution by Mathematica
Time used: 1.687 (sec). Leaf size: 23

```
DSolve[{y'[t]==Sqrt[t]/y[t],{y[0]==2}},y[t],t,IncludeSingularSolutions -> True]
```

$$y(t) \to \frac{2\sqrt{t^{3/2} + 3}}{\sqrt{3}}$$

4.46 problem 46

Internal problem ID [14204]

Book: INTRODUCTORY DIFFERENTIAL EQUATIONS. Martha L. Abell, James P. Braselton. Fourth edition 2014. ElScAe. 2014
Section: Chapter 2. First Order Equations. Exercises 2.2, page 39
Problem number: 46.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[[_homogeneous, ‘class A‘], _dAlembert]`

$$\boxed{y' - \sqrt{\frac{y}{t}} = 0}$$

With initial conditions

$$[y(1) = 2]$$

✓ Solution by Maple
Time used: 0.407 (sec). Leaf size: 94

```
dsolve([diff(y(t),t)=sqrt(y(t)/t),y(1) = 2],y(t), singsol=all)
```

$$y(t) = \frac{\left((t^2)^{\frac{1}{4}} \left(\sqrt{2}-1\right)+t\right)^2}{t}$$
$$y(t) = \frac{\left((t^2)^{\frac{1}{4}} \sqrt{2}+(t^2)^{\frac{1}{4}}-t\right)^2}{t}$$
$$y(t) = \left(-2-2\sqrt{2}\right)\sqrt{t}+t+2\sqrt{2}+3$$
$$y(t) = \left(-2+2\sqrt{2}\right)\sqrt{t}+t-2\sqrt{2}+3$$

✓ Solution by Mathematica

Time used: 0.208 (sec). Leaf size: 57

```
DSolve[{y'[t]==Sqrt[y[t]/t],{y[1]==2}},y[t],t,IncludeSingularSolutions -> True]
```

$$y(t) \to t - 2\left(1 + \sqrt{2}\right)\sqrt{t} + 2\sqrt{2} + 3$$
$$y(t) \to t + 2\left(\sqrt{2} - 1\right)\sqrt{t} - 2\sqrt{2} + 3$$

4.47 problem 47

Internal problem ID [14205]

Book: INTRODUCTORY DIFFERENTIAL EQUATIONS. Martha L. Abell, James P. Braselton. Fourth edition 2014. ElScAe. 2014
Section: Chapter 2. First Order Equations. Exercises 2.2, page 39
Problem number: 47.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_separable]`

$$\boxed{y' - \frac{\mathrm{e}^{t}}{y+1} = 0}$$

With initial conditions

$$[y(0) = -2]$$

✓ Solution by Maple
Time used: 0.093 (sec). Leaf size: 16

```
dsolve([diff(y(t),t)=exp(t)/(y(t)+1),y(0) = -2],y(t), singsol=all)
```

$$y(t) = -1 - \sqrt{2\,\mathrm{e}^{t} - 1}$$

✓ Solution by Mathematica
Time used: 0.593 (sec). Leaf size: 20

```
DSolve[{y'[t]==Exp[t]/(y[t]+1),{y[0]==-2}},y[t],t,IncludeSingularSolutions -> True]
```

$$y(t) \to -\sqrt{2e^{t} - 1} - 1$$

4.48 problem 48

Internal problem ID [14206]

Book: INTRODUCTORY DIFFERENTIAL EQUATIONS. Martha L. Abell, James P. Braselton. Fourth edition 2014. ElScAe. 2014
Section: Chapter 2. First Order Equations. Exercises 2.2, page 39
Problem number: 48.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_separable]`

$$\boxed{y' - e^{t-y} = 0}$$

With initial conditions

$$[y(0) = 0]$$

✓ Solution by Maple
Time used: 0.032 (sec). Leaf size: 5

```
dsolve([diff(y(t),t)=exp(t-y(t)),y(0) = 0],y(t), singsol=all)
```

$$y(t) = t$$

✓ Solution by Mathematica
Time used: 0.729 (sec). Leaf size: 9

```
DSolve[{y'[t]==Exp[t-y[t]],{y[0]==0}},y[t],t,IncludeSingularSolutions -> True]
```

$$y(t) \to \log(e^t)$$

4.49 problem 49

Internal problem ID [14207]

Book: INTRODUCTORY DIFFERENTIAL EQUATIONS. Martha L. Abell, James P. Braselton. Fourth edition 2014. ElScAe. 2014
Section: Chapter 2. First Order Equations. Exercises 2.2, page 39
Problem number: 49.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_quadrature]`

$$\boxed{y' - \frac{y}{\ln(y)} = 0}$$

With initial conditions

$$[y(0) = \mathrm{e}]$$

✓ Solution by Maple

Time used: 0.062 (sec). Leaf size: 12

```
dsolve([diff(y(x),x)=y(x)/ln(y(x)),y(0) = exp(1)],y(x), singsol=all)
```

$$y(x) = \mathrm{e}^{\sqrt{2x+1}}$$

✓ Solution by Mathematica

Time used: 0.01 (sec). Leaf size: 16

```
DSolve[{y'[x]==y[x]/Log[y[x]],{y[0]==Exp[1]}},y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to e^{\sqrt{2x+1}}$$

4.50 problem 50

Internal problem ID [14208]

Book: INTRODUCTORY DIFFERENTIAL EQUATIONS. Martha L. Abell, James P. Braselton. Fourth edition 2014. ElScAe. 2014
Section: Chapter 2. First Order Equations. Exercises 2.2, page 39
Problem number: 50.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_quadrature]`

$$\boxed{y' = t\sin\left(t^2\right)}$$

With initial conditions

$$\left[y\left(\sqrt{\pi}\right) = 0\right]$$

✓ Solution by Maple
Time used: 0.016 (sec). Leaf size: 12

```
dsolve([diff(y(t),t)=t*sin(t^2),y(Pi^(1/2)) = 0],y(t), singsol=all)
```

$$y(t) = -\frac{\cos\left(t^2\right)}{2} - \frac{1}{2}$$

✓ Solution by Mathematica
Time used: 0.013 (sec). Leaf size: 17

```
DSolve[{y'[t]==t*Sin[t^2],{y[Sqrt[Pi]]==0}},y[t],t,IncludeSingularSolutions -> True]
```

$$y(t) \to -\cos^2\left(\frac{t^2}{2}\right)$$

4.51 problem 51

Internal problem ID [14209]

Book: INTRODUCTORY DIFFERENTIAL EQUATIONS. Martha L. Abell, James P. Braselton. Fourth edition 2014. ElScAe. 2014
Section: Chapter 2. First Order Equations. Exercises 2.2, page 39
Problem number: 51.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_quadrature]`

$$\boxed{y' = \frac{1}{x^2 + 1}}$$

With initial conditions

$$[y(0) = 1]$$

✓ Solution by Maple
Time used: 0.015 (sec). Leaf size: 8

```
dsolve([diff(y(x),x)=1/(1+x^2),y(0) = 1],y(x), singsol=all)
```

$$y(x) = \arctan(x) + 1$$

✓ Solution by Mathematica
Time used: 0.005 (sec). Leaf size: 9

```
DSolve[{y'[x]==1/(1+x^2),{y[0]==1}},y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to \arctan(x) + 1$$

4.52 problem 52

Internal problem ID [14210]

Book: INTRODUCTORY DIFFERENTIAL EQUATIONS. Martha L. Abell, James P. Braselton. Fourth edition 2014. ElScAe. 2014
Section: Chapter 2. First Order Equations. Exercises 2.2, page 39
Problem number: 52.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_separable]`

$$\boxed{y' - \frac{\sin(x)}{\cos(y) + 1} = 0}$$

With initial conditions

$$[y(0) = 0]$$

✓ Solution by Maple
Time used: 0.187 (sec). Leaf size: 12

```
dsolve([diff(y(x),x)=sin(x)/(cos(y(x))+1),y(0) = 0],y(x), singsol=all)
```

$$y(x) = \operatorname{RootOf}(-1 + \cos(x) + \_Z + \sin(\_Z))$$

✓ Solution by Mathematica
Time used: 0.258 (sec). Leaf size: 19

```
DSolve[{y'[x]==Sin[x]/(Cos[y[x]]+1),{y[0]==0}},y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to \text{InverseFunction}[\#1 + \sin(\#1)\&][1 - \cos(x)]$$

4.53 problem 53

Internal problem ID [14211]

Book: INTRODUCTORY DIFFERENTIAL EQUATIONS. Martha L. Abell, James P. Braselton. Fourth edition 2014. ElScAe. 2014
Section: Chapter 2. First Order Equations. Exercises 2.2, page 39
Problem number: 53.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_separable]`

$$\boxed{y' - \frac{y+3}{3x+1} = 0}$$

With initial conditions

$$[y(0) = 1]$$

✓ Solution by Maple

Time used: 0.016 (sec). Leaf size: 15

```
dsolve([diff(y(x),x)=(y(x)+3)/(3*x+1),y(0) = 1],y(x), singsol=all)
```

$$y(x) = 4(3x+1)^{\frac{1}{3}} - 3$$

✓ Solution by Mathematica

Time used: 0.03 (sec). Leaf size: 18

```
DSolve[{y'[x]==(y[x]+3)/(3*x+1),{y[0]==1}},y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to 4\sqrt[3]{3x+1} - 3$$

4.54 problem 54

Internal problem ID [14212]

Book: INTRODUCTORY DIFFERENTIAL EQUATIONS. Martha L. Abell, James P. Braselton. Fourth edition 2014. ElScAe. 2014
Section: Chapter 2. First Order Equations. Exercises 2.2, page 39
Problem number: 54.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_separable]`

$$\boxed{y' - \mathrm{e}^{-y+x} = 0}$$

With initial conditions

$$[y(0) = 1]$$

✓ Solution by Maple
Time used: 0.047 (sec). Leaf size: 11

```
dsolve([diff(y(x),x)=exp(x-y(x)),y(0) = 1],y(x), singsol=all)
```

$$y(x) = \ln\left(\mathrm{e}^{x} - 1 + \mathrm{e}\right)$$

✓ Solution by Mathematica
Time used: 0.687 (sec). Leaf size: 12

```
DSolve[{y'[x]==Exp[x-y[x]],{y[0]==1}},y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to \log\left(e^{x} - 1 + e\right)$$

4.55 problem 55

Internal problem ID [14213]

Book: INTRODUCTORY DIFFERENTIAL EQUATIONS. Martha L. Abell, James P. Braselton. Fourth edition 2014. ElScAe. 2014
Section: Chapter 2. First Order Equations. Exercises 2.2, page 39
Problem number: 55.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_separable]`

$$\boxed{y' - e^{2x-y} = 0}$$

With initial conditions

$$[y(0) = 1]$$

✓ Solution by Maple
Time used: 0.078 (sec). Leaf size: 20

```
dsolve([diff(y(x),x)=exp(2*x-y(x)),y(0) = 1],y(x), singsol=all)
```

$$y(x) = -\ln(2) + \ln\left(e^{2x} - 1 + 2e\right)$$

✓ Solution by Mathematica
Time used: 0.785 (sec). Leaf size: 20

```
DSolve[{y'[x]==Exp[2*x-y[x]],{y[0]==1}},y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to \log\left(\frac{1}{2}\left(e^{2x} - 1 + 2e\right)\right)$$

4.56 problem 56

Internal problem ID [14214]

Book: INTRODUCTORY DIFFERENTIAL EQUATIONS. Martha L. Abell, James P. Braselton. Fourth edition 2014. ElScAe. 2014
Section: Chapter 2. First Order Equations. Exercises 2.2, page 39
Problem number: 56.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_separable]`

$$\boxed{y' - \frac{3y+1}{x+3} = 0}$$

With initial conditions

$$[y(0) = 1]$$

✓ Solution by Maple
Time used: 0.0 (sec). Leaf size: 13

```
dsolve([diff(y(x),x)=(3*y(x)+1)/(x+3),y(0) = 1],y(x), singsol=all)
```

$$y(x) = -\frac{1}{3} + \frac{4(x+3)^3}{81}$$

✓ Solution by Mathematica
Time used: 0.027 (sec). Leaf size: 26

```
DSolve[{y'[x]==(3*y[x]+1)/(x+3),{y[0]==1}},y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to \frac{4x^3}{81} + \frac{4x^2}{9} + \frac{4x}{3} + 1$$

4.57 problem 57 (a)

Internal problem ID [14215]

Book: INTRODUCTORY DIFFERENTIAL EQUATIONS. Martha L. Abell, James P. Braselton. Fourth edition 2014. ElScAe. 2014
Section: Chapter 2. First Order Equations. Exercises 2.2, page 39
Problem number: 57 (a).
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_separable]`

$$\boxed{y' - \cos(t)\, y = 0}$$

With initial conditions

$$[y(0) = 1]$$

✓ Solution by Maple
Time used: 0.0 (sec). Leaf size: 7

```
dsolve([diff(y(t),t)=y(t)*cos(t),y(0) = 1],y(t), singsol=all)
```

$$y(t) = \mathrm{e}^{\sin(t)}$$

✓ Solution by Mathematica
Time used: 0.035 (sec). Leaf size: 9

```
DSolve[{y'[t]==y[t]*Cos[t],{y[0]==1}},y[t],t,IncludeSingularSolutions -> True]
```

$$y(t) \to e^{\sin(t)}$$

4.58 problem 57 (b)

Internal problem ID [14216]

Book: INTRODUCTORY DIFFERENTIAL EQUATIONS. Martha L. Abell, James P. Braselton. Fourth edition 2014. ElScAe. 2014
Section: Chapter 2. First Order Equations. Exercises 2.2, page 39
Problem number: 57 (b).
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_separable]`

$$\boxed{y' - y^2 \cos(t) = 0}$$

With initial conditions

$$[y(0) = 1]$$

✓ Solution by Maple
Time used: 0.062 (sec). Leaf size: 12

```
dsolve([diff(y(t),t)=y(t)^2*cos(t),y(0) = 1],y(t), singsol=all)
```

$$y(t) = -\frac{1}{-1 + \sin(t)}$$

✓ Solution by Mathematica
Time used: 0.131 (sec). Leaf size: 13

```
DSolve[{y'[t]==y[t]^2*Cos[t],{y[0]==1}},y[t],t,IncludeSingularSolutions -> True]
```

$$y(t) \to \frac{1}{1 - \sin(t)}$$

4.59 problem 57 (c)

Internal problem ID [14217]

Book: INTRODUCTORY DIFFERENTIAL EQUATIONS. Martha L. Abell, James P. Braselton. Fourth edition 2014. ElScAe. 2014
Section: Chapter 2. First Order Equations. Exercises 2.2, page 39
Problem number: 57 (c).
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_separable]`

$$\boxed{y' - \sqrt{y}\, \cos(t) = 0}$$

With initial conditions

$$[y(0) = 1]$$

✓ Solution by Maple
Time used: 0.484 (sec). Leaf size: 12

```
dsolve([diff(y(t),t)=sqrt(y(t))*cos(t),y(0) = 1],y(t), singsol=all)
```

$$y(t) = \frac{(\sin(t) + 2)^2}{4}$$

✓ Solution by Mathematica
Time used: 0.105 (sec). Leaf size: 29

```
DSolve[{y'[t]==Sqrt[y[t]]*Cos[t],{y[0]==1}},y[t],t,IncludeSingularSolutions -> True]
```

$$y(t) \to \frac{1}{4}(\sin(t) - 2)^2$$
$$y(t) \to \frac{1}{4}(\sin(t) + 2)^2$$

4.60 problem 58

Internal problem ID [14218]

Book: INTRODUCTORY DIFFERENTIAL EQUATIONS. Martha L. Abell, James P. Braselton. Fourth edition 2014. ElScAe. 2014
Section: Chapter 2. First Order Equations. Exercises 2.2, page 39
Problem number: 58.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_separable]`

$$\boxed{y' + f(t)\, y = 0}$$

With initial conditions

$$[y(0) = 0]$$

✓ Solution by Maple
Time used: 0.0 (sec). Leaf size: 5

```
dsolve([diff(y(t),t)+f(t)*y(t)=0,y(0) = 0],y(t), singsol=all)
```

$$y(t) = 0$$

✓ Solution by Mathematica
Time used: 0.001 (sec). Leaf size: 6

```
DSolve[{y'[t]+f[t]*y[t]==0,{y[0]==0}},y[t],t,IncludeSingularSolutions -> True]
```

$$y(t) \to 0$$

4.61 problem 59

Internal problem ID [14219]

Book: INTRODUCTORY DIFFERENTIAL EQUATIONS. Martha L. Abell, James P. Braselton. Fourth edition 2014. ElScAe. 2014
Section: Chapter 2. First Order Equations. Exercises 2.2, page 39
Problem number: 59.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_separable]`

$$\boxed{y' + \frac{y-2}{x-2} = 0}$$

With initial conditions

$$[y(0) = 0]$$

✓ Solution by Maple
Time used: 0.016 (sec). Leaf size: 12

```
dsolve([diff(y(x),x)=-(y(x)-2)/(x-2),y(0) = 0],y(x), singsol=all)
```

$$y(x) = \frac{2x}{x-2}$$

✓ Solution by Mathematica
Time used: 0.025 (sec). Leaf size: 13

```
DSolve[{y'[x]==-(y[x]-2)/(x-2),{y[0]==0}},y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to \frac{2x}{x-2}$$

4.62 problem 60 (a)

Internal problem ID [14220]

Book: INTRODUCTORY DIFFERENTIAL EQUATIONS. Martha L. Abell, James P. Braselton. Fourth edition 2014. ElScAe. 2014
Section: Chapter 2. First Order Equations. Exercises 2.2, page 39
Problem number: 60 (a).
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[[_homogeneous, ‘class C‘], _rational, [_Abel, ‘2nd type‘, ‘cl`

$$\boxed{y' - \frac{x + y + 3}{3x + 3y + 1} = 0}$$

✓ Solution by Maple
Time used: 0.031 (sec). Leaf size: 23

```
dsolve(diff(y(x),x)=(x+y(x)+3)/(3*x+3*y(x)+1),y(x), singsol=all)
```

$$y(x) = -\frac{2\,\mathrm{LambertW}\left(-\frac{3\,\mathrm{e}^{-2x-\frac{3}{2}+2c_1}}{2}\right)}{3} - x - 1$$

✓ Solution by Mathematica
Time used: 3.699 (sec). Leaf size: 35

```
DSolve[y'[x]==(x+y[x]+3)/(3*x+3*y[x]+1),y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to -\frac{2}{3}W\left(-e^{-2x-1+c_1}\right) - x - 1$$
$$y(x) \to -x - 1$$

4.63 problem 60 (b)

Internal problem ID [14221]

Book: INTRODUCTORY DIFFERENTIAL EQUATIONS. Martha L. Abell, James P. Braselton. Fourth edition 2014. ElScAe. 2014
Section: Chapter 2. First Order Equations. Exercises 2.2, page 39
Problem number: 60 (b).
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[[_homogeneous, ‘class C‘], _rational, [_Abel, ‘2nd type‘, ‘cl`

$$\boxed{y' - \frac{x - y + 2}{2x - 2y - 1} = 0}$$

✓ Solution by Maple
Time used: 0.032 (sec). Leaf size: 19

```
dsolve(diff(y(x),x)=(x-y(x)+2)/(2*x-2*y(x)-1),y(x), singsol=all)
```

$$y(x) = x - \frac{5\,\mathrm{LambertW}\left(-\frac{2c_1 \mathrm{e}^{\frac{x}{5}-\frac{6}{5}}}{5}\right)}{2} - 3$$

✓ Solution by Mathematica
Time used: 3.627 (sec). Leaf size: 33

```
DSolve[y'[x]==(x-y[x]+2)/(2*x-2*y[x]-1),y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to -\frac{5}{2}W\left(-e^{\frac{x}{5}-1+c_1}\right) + x - 3$$
$$y(x) \to x - 3$$

4.64 problem 61 (a)

Internal problem ID [14222]

Book: INTRODUCTORY DIFFERENTIAL EQUATIONS. Martha L. Abell, James P. Braselton. Fourth edition 2014. ElScAe. 2014
Section: Chapter 2. First Order Equations. Exercises 2.2, page 39
Problem number: 61 (a).
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[[_homogeneous, 'class C'], _Riccati]`

$$\boxed{y' - (x + y - 4)^2 = 0}$$

✓ Solution by Maple
Time used: 0.0 (sec). Leaf size: 17

```
dsolve(diff(y(x),x)=(x+y(x)-4)^2,y(x), singsol=all)
```

$$y(x) = -x + 4 - \tan(c_1 - x)$$

✓ Solution by Mathematica
Time used: 0.168 (sec). Leaf size: 41

```
DSolve[y'[x]==(x+y[x]-4)^2,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to -x + \frac{1}{c_1 e^{2ix} - \frac{i}{2}} + (4 - i)$$
$$y(x) \to -x + (4 - i)$$

4.65 problem 61 (b)

Internal problem ID [14223]

Book: INTRODUCTORY DIFFERENTIAL EQUATIONS. Martha L. Abell, James P. Braselton. Fourth edition 2014. ElScAe. 2014
Section: Chapter 2. First Order Equations. Exercises 2.2, page 39
Problem number: 61 (b).
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_quadrature]`

$$\boxed{y' - (3y+1)^4 = 0}$$

✓ Solution by Maple
Time used: 0.0 (sec). Leaf size: 118

```
dsolve(diff(y(x),x)=(3*y(x)+1)^4,y(x), singsol=all)
```

$$y(x) = \frac{3^{\frac{1}{3}}\left(-(c_1+x)^2\right)^{\frac{1}{3}} - 3c_1 - 3x}{9c_1 + 9x}$$
$$y(x) = \frac{\left(-i3^{\frac{5}{6}} - 3^{\frac{1}{3}}\right)\left(-(c_1+x)^2\right)^{\frac{1}{3}} - 6x - 6c_1}{18c_1 + 18x}$$
$$y(x) = \frac{\left(i3^{\frac{5}{6}} - 3^{\frac{1}{3}}\right)\left(-(c_1+x)^2\right)^{\frac{1}{3}} - 6x - 6c_1}{18c_1 + 18x}$$

✓ Solution by Mathematica

Time used: 0.928 (sec). Leaf size: 144

```
DSolve[y'[x]==(3*y[x]+1)^4,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to -\frac{3x + \sqrt[3]{3}\sqrt[3]{(x+c_1)^2} + 3c_1}{9(x+c_1)}$$

$$y(x) \to \frac{-6x + \sqrt[3]{3}(1-i\sqrt{3})\sqrt[3]{(x+c_1)^2} - 6c_1}{18(x+c_1)}$$

$$y(x) \to \frac{-6x + \sqrt[3]{3}(1+i\sqrt{3})\sqrt[3]{(x+c_1)^2} - 6c_1}{18(x+c_1)}$$

$$y(x) \to -\frac{1}{3}$$

4.66 problem 63

Internal problem ID [14224]

Book: INTRODUCTORY DIFFERENTIAL EQUATIONS. Martha L. Abell, James P. Braselton. Fourth edition 2014. ElScAe. 2014
Section: Chapter 2. First Order Equations. Exercises 2.2, page 39
Problem number: 63.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_quadrature]`

$$\boxed{y' - 3y = 0}$$

✓ Solution by Maple
Time used: 0.0 (sec). Leaf size: 10

```
dsolve(diff(y(t),t)=3*y(t),y(t), singsol=all)
```

$$y(t) = c_1 e^{3t}$$

✓ Solution by Mathematica
Time used: 0.021 (sec). Leaf size: 18

```
DSolve[y'[t]==3*y[t],y[t],t,IncludeSingularSolutions -> True]
```

$$y(t) \to c_1 e^{3t}$$
$$y(t) \to 0$$

4.67 problem 64

Internal problem ID [14225]

Book: INTRODUCTORY DIFFERENTIAL EQUATIONS. Martha L. Abell, James P. Braselton. Fourth edition 2014. ElScAe. 2014
Section: Chapter 2. First Order Equations. Exercises 2.2, page 39
Problem number: 64.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_quadrature]`

$$\boxed{y' + y = 0}$$

✓ Solution by Maple
Time used: 0.0 (sec). Leaf size: 10

```
dsolve(diff(y(t),t)=-y(t),y(t), singsol=all)
```

$$y(t) = \mathrm{e}^{-t} c_1$$

✓ Solution by Mathematica
Time used: 0.019 (sec). Leaf size: 18

```
DSolve[y'[t]==-y[t],y[t],t,IncludeSingularSolutions -> True]
```

$$y(t) \to c_1 e^{-t}$$
$$y(t) \to 0$$

4.68 problem 65

Internal problem ID [14226]

Book: INTRODUCTORY DIFFERENTIAL EQUATIONS. Martha L. Abell, James P. Braselton. Fourth edition 2014. ElScAe. 2014
Section: Chapter 2. First Order Equations. Exercises 2.2, page 39
Problem number: 65.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_quadrature]`

$$\boxed{y' - y^2 + y = 0}$$

✓ Solution by Maple
Time used: 0.0 (sec). Leaf size: 12

```
dsolve(diff(y(t),t)=y(t)^2-y(t),y(t), singsol=all)
```

$$y(t) = \frac{1}{1 + \mathrm{e}^t c_1}$$

✓ Solution by Mathematica
Time used: 0.194 (sec). Leaf size: 25

```
DSolve[y'[t]==y[t]^2-y[t],y[t],t,IncludeSingularSolutions -> True]
```

$$y(t) \to \frac{1}{1 + e^{t+c_1}}$$
$$y(t) \to 0$$
$$y(t) \to 1$$

4.69 problem 66

Internal problem ID [14227]

Book: INTRODUCTORY DIFFERENTIAL EQUATIONS. Martha L. Abell, James P. Braselton. Fourth edition 2014. ElScAe. 2014
Section: Chapter 2. First Order Equations. Exercises 2.2, page 39
Problem number: 66.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_quadrature]`

$$\boxed{y' - 16y + 8y^2 = 0}$$

✓ Solution by Maple
Time used: 0.016 (sec). Leaf size: 17

```
dsolve(diff(y(t),t)=16*y(t)-8*y(t)^2,y(t), singsol=all)
```

$$y(t) = \frac{2}{1 + 2\,\mathrm{e}^{-16t} c_1}$$

✓ Solution by Mathematica
Time used: 0.278 (sec). Leaf size: 36

```
DSolve[y'[t]==16*y[t]-8*y[t]^2,y[t],t,IncludeSingularSolutions -> True]
```

$$\begin{aligned} y(t) &\to \frac{2e^{16t}}{e^{16t} + e^{2c_1}} \\ y(t) &\to 0 \\ y(t) &\to 2 \end{aligned}$$

4.70 problem 67

Internal problem ID [14228]

Book: INTRODUCTORY DIFFERENTIAL EQUATIONS. Martha L. Abell, James P. Braselton. Fourth edition 2014. ElScAe. 2014
Section: Chapter 2. First Order Equations. Exercises 2.2, page 39
Problem number: 67.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_quadrature]`

$$\boxed{y' - 4y + y^2 = 12}$$

✓ Solution by Maple
Time used: 0.047 (sec). Leaf size: 24

```
dsolve(diff(y(t),t)=12+4*y(t)-y(t)^2,y(t), singsol=all)
```

$$y(t) = \frac{6c_1 e^{8t} + 2}{c_1 e^{8t} - 1}$$

✓ Solution by Mathematica
Time used: 0.463 (sec). Leaf size: 47

```
DSolve[y'[t]==12+4*y[t]-y[t]^2,y[t],t,IncludeSingularSolutions -> True]
```

$$y(t) \to \frac{2(3e^{8t} + e^{8c_1})}{e^{8t} - e^{8c_1}}$$
$$y(t) \to -2$$
$$y(t) \to 6$$

4.71 problem 74 (a)

Internal problem ID [14229]

Book: INTRODUCTORY DIFFERENTIAL EQUATIONS. Martha L. Abell, James P. Braselton. Fourth edition 2014. ElScAe. 2014
Section: Chapter 2. First Order Equations. Exercises 2.2, page 39
Problem number: 74 (a).
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_separable]`

$$\boxed{y' - f(t)\, y = 0}$$

With initial conditions

$$[y(1) = 1]$$

✓ Solution by Maple
Time used: 0.063 (sec). Leaf size: 13

```
dsolve([diff(y(t),t)=f(t)*y(t),y(1) = 1],y(t), singsol=all)
```

$$y(t) = e^{\int_1^t f(\_z1)d\_z1}$$

✓ Solution by Mathematica
Time used: 0.028 (sec). Leaf size: 17

```
DSolve[{y'[t]==f[t]*y[t],{y[1]==1}},y[t],t,IncludeSingularSolutions -> True]
```

$$y(t) \to \exp\left(\int_1^t f(K[1])dK[1]\right)$$

5 Chapter 2. First Order Equations. Exercises 2.3, page 49

5.1 problem 1

Internal problem ID [14230]

Book: INTRODUCTORY DIFFERENTIAL EQUATIONS. Martha L. Abell, James P. Braselton. Fourth edition 2014. ElScAe. 2014
Section: Chapter 2. First Order Equations. Exercises 2.3, page 49
Problem number: 1.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_quadrature]`

$$\boxed{y' - y = 10}$$

✓ Solution by Maple
Time used: 0.0 (sec). Leaf size: 10

```
dsolve(diff(y(t),t)-y(t)=10,y(t), singsol=all)
```

$$y(t) = -10 + \mathrm{e}^{t} c_1$$

✓ Solution by Mathematica
Time used: 0.032 (sec). Leaf size: 18

```
DSolve[y'[t]-y[t]==10,y[t],t,IncludeSingularSolutions -> True]
```

$$y(t) \to -10 + c_1 e^t$$
$$y(t) \to -10$$

5.2 problem 2

Internal problem ID [14231]

Book: INTRODUCTORY DIFFERENTIAL EQUATIONS. Martha L. Abell, James P. Braselton. Fourth edition 2014. ElScAe. 2014
Section: Chapter 2. First Order Equations. Exercises 2.3, page 49
Problem number: 2.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[[_linear, ‘class A‘]]`

$$\boxed{y' - y = 2\,\mathrm{e}^{-t}}$$

✓ Solution by Maple
Time used: 0.0 (sec). Leaf size: 15

```
dsolve(diff(y(t),t)-y(t)=2*exp(-t),y(t), singsol=all)
```

$$y(t) = -\mathrm{e}^{-t} + \mathrm{e}^{t} c_1$$

✓ Solution by Mathematica
Time used: 0.056 (sec). Leaf size: 19

```
DSolve[y'[t]-y[t]==2*Exp[-t],y[t],t,IncludeSingularSolutions -> True]
```

$$y(t) \to -e^{-t} + c_1 e^{t}$$

5.3 problem 3

Internal problem ID [14232]

Book: INTRODUCTORY DIFFERENTIAL EQUATIONS. Martha L. Abell, James P. Braselton. Fourth edition 2014. ElScAe. 2014
Section: Chapter 2. First Order Equations. Exercises 2.3, page 49
Problem number: 3.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[[_linear, 'class A']]`

$$\boxed{y' - y = 2\cos(t)}$$

✓ Solution by Maple
Time used: 0.0 (sec). Leaf size: 15

```
dsolve(diff(y(t),t)-y(t)=2*cos(t),y(t), singsol=all)
```

$$y(t) = \sin(t) - \cos(t) + e^t c_1$$

✓ Solution by Mathematica
Time used: 0.048 (sec). Leaf size: 18

```
DSolve[y'[t]-y[t]==2*Cos[t],y[t],t,IncludeSingularSolutions -> True]
```

$$y(t) \to \sin(t) - \cos(t) + c_1 e^t$$

5.4 problem 4

Internal problem ID [14233]

Book: INTRODUCTORY DIFFERENTIAL EQUATIONS. Martha L. Abell, James P. Braselton. Fourth edition 2014. ElScAe. 2014
Section: Chapter 2. First Order Equations. Exercises 2.3, page 49
Problem number: 4.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[[_linear, 'class A']]`

$$\boxed{y' - y = t^2 - 2t}$$

✓ Solution by Maple
Time used: 0.0 (sec). Leaf size: 14

```
dsolve(diff(y(t),t)-y(t)=t^2-2*t,y(t), singsol=all)
```

$$y(t) = -t^2 + e^t c_1$$

✓ Solution by Mathematica
Time used: 0.088 (sec). Leaf size: 17

```
DSolve[y'[t]-y[t]==t^2-2*t,y[t],t,IncludeSingularSolutions -> True]
```

$$y(t) \to -t^2 + c_1 e^t$$

5.5 problem 5

Internal problem ID [14234]

Book: INTRODUCTORY DIFFERENTIAL EQUATIONS. Martha L. Abell, James P. Braselton. Fourth edition 2014. ElScAe. 2014
Section: Chapter 2. First Order Equations. Exercises 2.3, page 49
Problem number: 5.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[[_linear, 'class A']]`

$$\boxed{y' - y = 4t\,\mathrm{e}^{-t}}$$

✓ Solution by Maple
Time used: 0.0 (sec). Leaf size: 20

```
dsolve(diff(y(t),t)-y(t)=4*t*exp(-t),y(t), singsol=all)
```

$$y(t) = \left(c_1 \mathrm{e}^{2t} - 2t - 1\right) \mathrm{e}^{-t}$$

✓ Solution by Mathematica
Time used: 0.105 (sec). Leaf size: 24

```
DSolve[y'[t]-y[t]==4*t*Exp[-t],y[t],t,IncludeSingularSolutions -> True]
```

$$y(t) \to e^{-t}\left(-2t + c_1 e^{2t} - 1\right)$$

5.6 problem 6

Internal problem ID [14235]

Book: INTRODUCTORY DIFFERENTIAL EQUATIONS. Martha L. Abell, James P. Braselton. Fourth edition 2014. ElScAe. 2014
Section: Chapter 2. First Order Equations. Exercises 2.3, page 49
Problem number: 6.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_linear]`

$$\boxed{y't + y = t^2}$$

✓ Solution by Maple
Time used: 0.0 (sec). Leaf size: 16

```
dsolve(t*diff(y(t),t)+y(t)=t^2,y(t), singsol=all)
```

$$y(t) = \frac{t^3 + 3c_1}{3t}$$

✓ Solution by Mathematica
Time used: 0.04 (sec). Leaf size: 19

```
DSolve[t*y'[t]+y[t]==t^2,y[t],t,IncludeSingularSolutions -> True]
```

$$y(t) \to \frac{t^2}{3} + \frac{c_1}{t}$$

5.7 problem 7

Internal problem ID [14236]

Book: INTRODUCTORY DIFFERENTIAL EQUATIONS. Martha L. Abell, James P. Braselton. Fourth edition 2014. ElScAe. 2014
Section: Chapter 2. First Order Equations. Exercises 2.3, page 49
Problem number: 7.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_linear]`

$$\boxed{y't + y = t}$$

✓ Solution by Maple
Time used: 0.0 (sec). Leaf size: 13

```
dsolve(t*diff(y(t),t)+y(t)=t,y(t), singsol=all)
```

$$y(t) = \frac{t}{2} + \frac{c_1}{t}$$

✓ Solution by Mathematica
Time used: 0.037 (sec). Leaf size: 17

```
DSolve[t*y'[t]+y[t]==t,y[t],t,IncludeSingularSolutions -> True]
```

$$y(t) \to \frac{t}{2} + \frac{c_1}{t}$$

5.8 problem 8

Internal problem ID [14237]

Book: INTRODUCTORY DIFFERENTIAL EQUATIONS. Martha L. Abell, James P. Braselton. Fourth edition 2014. ElScAe. 2014
Section: Chapter 2. First Order Equations. Exercises 2.3, page 49
Problem number: 8.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_linear]`

$$\boxed{y'x + y = x\,\mathrm{e}^{x}}$$

✓ Solution by Maple
Time used: 0.0 (sec). Leaf size: 16

```
dsolve(x*diff(y(x),x)+y(x)=x*exp(x),y(x), singsol=all)
```

$$y(x) = \frac{(-1+x)\,\mathrm{e}^{x} + c_1}{x}$$

✓ Solution by Mathematica
Time used: 0.078 (sec). Leaf size: 19

```
DSolve[x*y'[x]+y[x]==x*Exp[x],y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to \frac{e^{x}(x-1) + c_1}{x}$$

5.9 problem 9

Internal problem ID [14238]

Book: INTRODUCTORY DIFFERENTIAL EQUATIONS. Martha L. Abell, James P. Braselton. Fourth edition 2014. ElScAe. 2014
Section: Chapter 2. First Order Equations. Exercises 2.3, page 49
Problem number: 9.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_linear]`

$$\boxed{y'x + y = \mathrm{e}^{-x}}$$

✓ Solution by Maple
Time used: 0.0 (sec). Leaf size: 16

```
dsolve(x*diff(y(x),x)+y(x)=exp(-x),y(x), singsol=all)
```

$$y(x) = \frac{-\mathrm{e}^{-x} + c_1}{x}$$

✓ Solution by Mathematica
Time used: 0.055 (sec). Leaf size: 20

```
DSolve[x*y'[x]+y[x]==Exp[-x],y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to -\frac{e^{-x} - c_1}{x}$$

5.10 problem 10

Internal problem ID [14239]

Book: INTRODUCTORY DIFFERENTIAL EQUATIONS. Martha L. Abell, James P. Braselton. Fourth edition 2014. ElScAe. 2014
Section: Chapter 2. First Order Equations. Exercises 2.3, page 49
Problem number: 10.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_linear]`

$$\boxed{y' - \frac{2ty}{t^2 + 1} = 2}$$

✓ Solution by Maple
Time used: 0.0 (sec). Leaf size: 16

```
dsolve(diff(y(t),t)-2*t/(1+t^2)*y(t)=2,y(t), singsol=all)
```

$$y(t) = (2\arctan(t) + c_1)\left(t^2 + 1\right)$$

✓ Solution by Mathematica
Time used: 0.037 (sec). Leaf size: 18

```
DSolve[y'[t]-2*t/(1+t^2)*y[t]==2,y[t],t,IncludeSingularSolutions -> True]
```

$$y(t) \to \left(t^2 + 1\right)(2\arctan(t) + c_1)$$

5.11 problem 11

Internal problem ID [14240]

Book: INTRODUCTORY DIFFERENTIAL EQUATIONS. Martha L. Abell, James P. Braselton. Fourth edition 2014. ElScAe. 2014
Section: Chapter 2. First Order Equations. Exercises 2.3, page 49
Problem number: 11.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_linear]`

$$\boxed{y' - \frac{4ty}{4t^2+1} = 4t}$$

✓ Solution by Maple
Time used: 0.015 (sec). Leaf size: 22

```
dsolve(diff(y(t),t)-4*t/(1+4*t^2)*y(t)=4*t,y(t), singsol=all)
```

$$y(t) = 4t^2 + 1 + \sqrt{4t^2+1}\, c_1$$

✓ Solution by Mathematica
Time used: 0.038 (sec). Leaf size: 26

```
DSolve[y'[t]-4*t/(1+4*t^2)*y[t]==4*t,y[t],t,IncludeSingularSolutions -> True]
```

$$y(t) \to 4t^2 + c_1\sqrt{4t^2+1} + 1$$

5.12 problem 12

Internal problem ID [14241]

Book: INTRODUCTORY DIFFERENTIAL EQUATIONS. Martha L. Abell, James P. Braselton. Fourth edition 2014. ElScAe. 2014
Section: Chapter 2. First Order Equations. Exercises 2.3, page 49
Problem number: 12.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_linear]`

$$\boxed{y' - \frac{xy}{x^2 - 1} = 2x}$$

✓ Solution by Maple
Time used: 0.0 (sec). Leaf size: 23

```
dsolve(diff(y(x),x)=2*x+x*y(x)/(x^2-1),y(x), singsol=all)
```

$$y(x) = 2x^2 - 2 + \sqrt{-1+x}\,\sqrt{1+x}\,c_1$$

✓ Solution by Mathematica
Time used: 0.034 (sec). Leaf size: 24

```
DSolve[y'[x]==2*x+x*y[x]/(x^2-1),y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to 2x^2 + c_1\sqrt{x^2 - 1} - 2$$

5.13 problem 13

Internal problem ID [14242]

Book: INTRODUCTORY DIFFERENTIAL EQUATIONS. Martha L. Abell, James P. Braselton. Fourth edition 2014. ElScAe. 2014
Section: Chapter 2. First Order Equations. Exercises 2.3, page 49
Problem number: 13.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_linear]`

$$\boxed{y' + y\cot(t) = \cos(t)}$$

✓ Solution by Maple
Time used: 0.0 (sec). Leaf size: 19

```
dsolve(diff(y(t),t)+y(t)*cot(t)=cos(t),y(t), singsol=all)
```

$$y(t) = -\frac{\csc(t)\left(2\cos(t)^2 - 4c_1 - 1\right)}{4}$$

✓ Solution by Mathematica
Time used: 0.049 (sec). Leaf size: 19

```
DSolve[y'[t]+y[t]*Cot[t]==Cos[t],y[t],t,IncludeSingularSolutions -> True]
```

$$y(t) \to -\frac{1}{2}\cos(t)\cot(t) + c_1\csc(t)$$

5.14 problem 14

Internal problem ID [14243]

Book: INTRODUCTORY DIFFERENTIAL EQUATIONS. Martha L. Abell, James P. Braselton. Fourth edition 2014. ElScAe. 2014
Section: Chapter 2. First Order Equations. Exercises 2.3, page 49
Problem number: 14.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_linear]`

$$\boxed{y' - \frac{3ty}{t^2-4} = t}$$

✓ Solution by Maple
Time used: 0.0 (sec). Leaf size: 22

```
dsolve(diff(y(t),t)-3*t/(t^2-4)*y(t)=t,y(t), singsol=all)
```

$$y(t) = (t-2)(t+2)\left(\sqrt{t^2-4}\, c_1 - 1\right)$$

✓ Solution by Mathematica
Time used: 0.04 (sec). Leaf size: 24

```
DSolve[y'[t]-3*t/(t^2-4)*y[t]==t,y[t],t,IncludeSingularSolutions -> True]
```

$$y(t) \to -t^2 + c_1 (t^2-4)^{3/2} + 4$$

5.15 problem 15

Internal problem ID [14244]

Book: INTRODUCTORY DIFFERENTIAL EQUATIONS. Martha L. Abell, James P. Braselton. Fourth edition 2014. ElScAe. 2014
Section: Chapter 2. First Order Equations. Exercises 2.3, page 49
Problem number: 15.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_linear]`

$$\boxed{y' - \frac{4ty}{4t^2 - 9} = t}$$

✓ Solution by Maple
Time used: 0.015 (sec). Leaf size: 20

```
dsolve(diff(y(t),t)-4*t/(4*t^2-9)*y(t)=t,y(t), singsol=all)
```

$$y(t) = t^2 - \frac{9}{4} + \sqrt{4t^2 - 9}\, c_1$$

✓ Solution by Mathematica
Time used: 0.037 (sec). Leaf size: 26

```
DSolve[y'[t]-4*t/(4*t^2-9)*y[t]==t,y[t],t,IncludeSingularSolutions -> True]
```

$$y(t) \to t^2 + c_1\sqrt{4t^2 - 9} - \frac{9}{4}$$

5.16 problem 16

Internal problem ID [14245]

Book: INTRODUCTORY DIFFERENTIAL EQUATIONS. Martha L. Abell, James P. Braselton. Fourth edition 2014. ElScAe. 2014
Section: Chapter 2. First Order Equations. Exercises 2.3, page 49
Problem number: 16.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_linear]`

$$\boxed{y' - \frac{9xy}{9x^2+49} = x}$$

✓ Solution by Maple
Time used: 0.0 (sec). Leaf size: 20

```
dsolve(diff(y(x),x)-9*x/(9*x^2+49)*y(x)=x,y(x), singsol=all)
```

$$y(x) = x^2 + \frac{49}{9} + \sqrt{9x^2+49}\, c_1$$

✓ Solution by Mathematica
Time used: 0.038 (sec). Leaf size: 26

```
DSolve[y'[x]-9*x/(9*x^2+49)*y[x]==x,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to x^2 + c_1\sqrt{9x^2+49} + \frac{49}{9}$$

5.17 problem 17

Internal problem ID [14246]

Book: INTRODUCTORY DIFFERENTIAL EQUATIONS. Martha L. Abell, James P. Braselton. Fourth edition 2014. ElScAe. 2014
Section: Chapter 2. First Order Equations. Exercises 2.3, page 49
Problem number: 17.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_linear]`

$$\boxed{y' + 2y \cot(x) = \cos(x)}$$

✓ Solution by Maple
Time used: 0.0 (sec). Leaf size: 15

```
dsolve(diff(y(x),x)+2*y(x)*cot(x)=cos(x),y(x), singsol=all)
```

$$y(x) = \frac{\sin(x)}{3} + \csc(x)^2 c_1$$

✓ Solution by Mathematica
Time used: 0.047 (sec). Leaf size: 19

```
DSolve[y'[x]+2*y[x]*Cot[x]==Cos[x],y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to \frac{\sin(x)}{3} + c_1 \csc^2(x)$$

5.18 problem 18

Internal problem ID [14247]

Book: INTRODUCTORY DIFFERENTIAL EQUATIONS. Martha L. Abell, James P. Braselton. Fourth edition 2014. ElScAe. 2014
Section: Chapter 2. First Order Equations. Exercises 2.3, page 49
Problem number: 18.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_linear]`

$$\boxed{yx + y' = x^3}$$

✓ Solution by Maple
Time used: 0.0 (sec). Leaf size: 17

```
dsolve(diff(y(x),x)+y(x)*x=x^3,y(x), singsol=all)
```

$$y(x) = x^2 - 2 + e^{-\frac{x^2}{2}} c_1$$

✓ Solution by Mathematica
Time used: 0.063 (sec). Leaf size: 22

```
DSolve[y'[x]+y[x]*x==x^3,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to x^2 + c_1 e^{-\frac{x^2}{2}} - 2$$

5.19 problem 19

Internal problem ID [14248]

Book: INTRODUCTORY DIFFERENTIAL EQUATIONS. Martha L. Abell, James P. Braselton. Fourth edition 2014. ElScAe. 2014
Section: Chapter 2. First Order Equations. Exercises 2.3, page 49
Problem number: 19.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_separable]`

$$\boxed{y' - yx = x}$$

✓ Solution by Maple
Time used: 0.015 (sec). Leaf size: 14

```
dsolve(diff(y(x),x)-y(x)*x=x,y(x), singsol=all)
```

$$y(x) = -1 + c_1 \mathrm{e}^{\frac{x^2}{2}}$$

✓ Solution by Mathematica
Time used: 0.057 (sec). Leaf size: 24

```
DSolve[y'[x]-y[x]*x==x,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to -1 + c_1 e^{\frac{x^2}{2}}$$
$$y(x) \to -1$$

5.20 problem 20

Internal problem ID [14249]

Book: INTRODUCTORY DIFFERENTIAL EQUATIONS. Martha L. Abell, James P. Braselton. Fourth edition 2014. ElScAe. 2014
Section: Chapter 2. First Order Equations. Exercises 2.3, page 49
Problem number: 20.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[[_1st_order, _with_exponential_symmetries]]`

$$\boxed{y' - \frac{1}{x + y^2} = 0}$$

✓ Solution by Maple
Time used: 0.015 (sec). Leaf size: 20

```
dsolve(diff(y(x),x)=1/(y(x)^2+x),y(x), singsol=all)
```

$$x + y(x)^2 + 2y(x) + 2 - e^{y(x)} c_1 = 0$$

✓ Solution by Mathematica
Time used: 0.121 (sec). Leaf size: 24

```
DSolve[y'[x]==1/(y[x]^2+x),y[x],x,IncludeSingularSolutions -> True]
```

$$\text{Solve}\left[x = -y(x)^2 - 2y(x) + c_1 e^{y(x)} - 2, y(x)\right]$$

5.21 problem 21

Internal problem ID [14250]

Book: INTRODUCTORY DIFFERENTIAL EQUATIONS. Martha L. Abell, James P. Braselton. Fourth edition 2014. ElScAe. 2014
Section: Chapter 2. First Order Equations. Exercises 2.3, page 49
Problem number: 21.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[[_linear, 'class A']]`

$$\boxed{y' - y = x}$$

✓ Solution by Maple
Time used: 0.0 (sec). Leaf size: 13

```
dsolve(diff(y(x),x)-x=y(x),y(x), singsol=all)
```

$$y(x) = -x - 1 + c_1 e^x$$

✓ Solution by Mathematica
Time used: 0.039 (sec). Leaf size: 16

```
DSolve[y'[x]-x==y[x],y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to -x + c_1 e^x - 1$$

5.22 problem 22

Internal problem ID [14251]

Book: INTRODUCTORY DIFFERENTIAL EQUATIONS. Martha L. Abell, James P. Braselton. Fourth edition 2014. ElScAe. 2014
Section: Chapter 2. First Order Equations. Exercises 2.3, page 49
Problem number: 22.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[[_homogeneous, 'class G'], _rational]`

$$\boxed{y - (x + 3y^2)\, y' = 0}$$

✓ Solution by Maple
Time used: 0.0 (sec). Leaf size: 37

```
dsolve(y(x)-(x+3*y(x)^2)*diff(y(x),x)=0,y(x), singsol=all)
```

$$y(x) = -\frac{c_1}{6} - \frac{\sqrt{c_1^2 + 12x}}{6}$$
$$y(x) = -\frac{c_1}{6} + \frac{\sqrt{c_1^2 + 12x}}{6}$$

✓ Solution by Mathematica
Time used: 0.254 (sec). Leaf size: 58

```
DSolve[y[x]-(x+3*y[x]^2)*y'[x]==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to \frac{1}{6}\left(-\sqrt{12x + c_1{}^2} - c_1\right)$$
$$y(x) \to \frac{1}{6}\left(\sqrt{12x + c_1{}^2} - c_1\right)$$
$$y(x) \to 0$$

5.23 problem 23

Internal problem ID [14252]

Book: INTRODUCTORY DIFFERENTIAL EQUATIONS. Martha L. Abell, James P. Braselton. Fourth edition 2014. ElScAe. 2014
Section: Chapter 2. First Order Equations. Exercises 2.3, page 49
Problem number: 23.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_separable]`

$$\boxed{x' - \frac{3x t^2}{-t^3 + 1} = 0}$$

✓ Solution by Maple
Time used: 0.0 (sec). Leaf size: 13

```
dsolve(diff(x(t),t)=3*x(t)*t^2/(1-t^3),x(t), singsol=all)
```

$$x(t) = \frac{c_1}{t^3 - 1}$$

✓ Solution by Mathematica
Time used: 0.031 (sec). Leaf size: 20

```
DSolve[x'[t]==3*x[t]*t^2/(1-t^3),x[t],t,IncludeSingularSolutions -> True]
```

$$x(t) \to \frac{c_1}{t^3 - 1}$$
$$x(t) \to 0$$

5.24 problem 24

Internal problem ID [14253]

Book: INTRODUCTORY DIFFERENTIAL EQUATIONS. Martha L. Abell, James P. Braselton. Fourth edition 2014. ElScAe. 2014
Section: Chapter 2. First Order Equations. Exercises 2.3, page 49
Problem number: 24.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_linear]`

$$\boxed{p' - \frac{p}{t} = t^3}$$

✓ Solution by Maple
Time used: 0.0 (sec). Leaf size: 14

```
dsolve(diff(p(t),t)=t^3+p(t)/t,p(t), singsol=all)
```

$$p(t) = \frac{(t^3 + 3c_1)\, t}{3}$$

✓ Solution by Mathematica
Time used: 0.024 (sec). Leaf size: 17

```
DSolve[p'[t]==t^3+p[t]/t,p[t],t,IncludeSingularSolutions -> True]
```

$$p(t) \to \frac{t^4}{3} + c_1 t$$

5.25 problem 25

Internal problem ID [14254]

Book: INTRODUCTORY DIFFERENTIAL EQUATIONS. Martha L. Abell, James P. Braselton. Fourth edition 2014. ElScAe. 2014
Section: Chapter 2. First Order Equations. Exercises 2.3, page 49
Problem number: 25.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[[_linear, 'class A']]`

$$\boxed{v' + v = \mathrm{e}^{-s}}$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 12

```
dsolve(diff(v(s),s)+v(s)=exp(-s),v(s), singsol=all)
```

$$v(s) = (s + c_1)\,\mathrm{e}^{-s}$$

✓ Solution by Mathematica

Time used: 0.044 (sec). Leaf size: 15

```
DSolve[v'[s]+v[s]==Exp[-s],v[s],s,IncludeSingularSolutions -> True]
```

$$v(s) \to e^{-s}(s + c_1)$$

5.26 problem 26

Internal problem ID [14255]

Book: INTRODUCTORY DIFFERENTIAL EQUATIONS. Martha L. Abell, James P. Braselton. Fourth edition 2014. ElScAe. 2014
Section: Chapter 2. First Order Equations. Exercises 2.3, page 49
Problem number: 26.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[[_linear, 'class A']]`

$$\boxed{y' - y = 4\,\mathrm{e}^{t}}$$

With initial conditions

$$[y(0) = 4]$$

✓ Solution by Maple
Time used: 0.0 (sec). Leaf size: 11

```
dsolve([diff(y(t),t)-y(t)=4*exp(t),y(0) = 4],y(t), singsol=all)
```

$$y(t) = 4(t+1)\,\mathrm{e}^{t}$$

✓ Solution by Mathematica
Time used: 0.038 (sec). Leaf size: 13

```
DSolve[{y'[t]-y[t]==4*Exp[t],{y[0]==4}},y[t],t,IncludeSingularSolutions -> True]
```

$$y(t) \to 4e^{t}(t+1)$$

5.27 problem 27

Internal problem ID [14256]

Book: INTRODUCTORY DIFFERENTIAL EQUATIONS. Martha L. Abell, James P. Braselton. Fourth edition 2014. ElScAe. 2014
Section: Chapter 2. First Order Equations. Exercises 2.3, page 49
Problem number: 27.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[[_linear, ‘class A‘]]`

$$\boxed{y' + y = e^{-t}}$$

With initial conditions

$$[y(0) = -1]$$

✓ Solution by Maple
Time used: 0.0 (sec). Leaf size: 12

```
dsolve([diff(y(t),t)+y(t)=exp(-t),y(0) = -1],y(t), singsol=all)
```

$$y(t) = e^{-t}(t - 1)$$

✓ Solution by Mathematica
Time used: 0.045 (sec). Leaf size: 14

```
DSolve[{y'[t]+y[t]==Exp[-t],{y[0]==-1}},y[t],t,IncludeSingularSolutions -> True]
```

$$y(t) \to e^{-t}(t - 1)$$

5.28 problem 28

Internal problem ID [14257]

Book: INTRODUCTORY DIFFERENTIAL EQUATIONS. Martha L. Abell, James P. Braselton. Fourth edition 2014. ElScAe. 2014
Section: Chapter 2. First Order Equations. Exercises 2.3, page 49
Problem number: 28.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_linear]`

$$\boxed{y' + 3t^2 y = e^{-t^3}}$$

With initial conditions

$$[y(0) = 2]$$

✓ Solution by Maple
Time used: 0.016 (sec). Leaf size: 14

```
dsolve([diff(y(t),t)+3*t^2*y(t)=exp(-t^3),y(0) = 2],y(t), singsol=all)
```

$$y(t) = (t + 2) e^{-t^3}$$

✓ Solution by Mathematica
Time used: 0.057 (sec). Leaf size: 16

```
DSolve[{y'[t]+3*t^2*y[t]==Exp[-t^3],{y[0]==2}},y[t],t,IncludeSingularSolutions -> True]
```

$$y(t) \to e^{-t^3}(t + 2)$$

5.29 problem 29

Internal problem ID [14258]

Book: INTRODUCTORY DIFFERENTIAL EQUATIONS. Martha L. Abell, James P. Braselton. Fourth edition 2014. ElScAe. 2014
Section: Chapter 2. First Order Equations. Exercises 2.3, page 49
Problem number: 29.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_separable]`

$$\boxed{y' + 2ty = 2t}$$

With initial conditions

$$[y(0) = -1]$$

✓ Solution by Maple
Time used: 0.015 (sec). Leaf size: 14

```
dsolve([diff(y(t),t)+2*t*y(t)=2*t,y(0) = -1],y(t), singsol=all)
```

$$y(t) = 1 - 2\,\mathrm{e}^{-t^2}$$

✓ Solution by Mathematica
Time used: 0.043 (sec). Leaf size: 16

```
DSolve[{y'[t]+2*t*y[t]==2*t,{y[0]==-1}},y[t],t,IncludeSingularSolutions -> True]
```

$$y(t) \to 1 - 2e^{-t^2}$$

5.30 problem 30

Internal problem ID [14259]

Book: INTRODUCTORY DIFFERENTIAL EQUATIONS. Martha L. Abell, James P. Braselton. Fourth edition 2014. ElScAe. 2014
Section: Chapter 2. First Order Equations. Exercises 2.3, page 49
Problem number: 30.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_linear]`

$$\boxed{y' t + y = \cos(t)}$$

With initial conditions

$$\left[y\left(\frac{\pi}{2}\right) = \frac{4}{\pi}\right]$$

✓ Solution by Maple
Time used: 0.016 (sec). Leaf size: 12

```
dsolve([t*diff(y(t),t)+y(t)=cos(t),y(1/2*Pi) = 4/Pi],y(t), singsol=all)
```

$$y(t) = \frac{1 + \sin(t)}{t}$$

✓ Solution by Mathematica
Time used: 0.034 (sec). Leaf size: 13

```
DSolve[{t*y'[t]+y[t]==Cos[t],{y[Pi/2]==4/Pi}},y[t],t,IncludeSingularSolutions -> True]
```

$$y(t) \to \frac{\sin(t) + 1}{t}$$

5.31 problem 31

Internal problem ID [14260]

Book: INTRODUCTORY DIFFERENTIAL EQUATIONS. Martha L. Abell, James P. Braselton. Fourth edition 2014. ElScAe. 2014
Section: Chapter 2. First Order Equations. Exercises 2.3, page 49
Problem number: 31.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_linear]`

$$\boxed{y't + y = 2t\,\mathrm{e}^{t}}$$

With initial conditions

$$[y(1) = -1]$$

✓ Solution by Maple
Time used: 0.016 (sec). Leaf size: 17

```
dsolve([t*diff(y(t),t)+y(t)=2*t*exp(t),y(1) = -1],y(t), singsol=all)
```

$$y(t) = \frac{-1 + (2t - 2)\,\mathrm{e}^{t}}{t}$$

✓ Solution by Mathematica
Time used: 0.052 (sec). Leaf size: 19

```
DSolve[{t*y'[t]+y[t]==2*t*Exp[t],{y[1]==-1}},y[t],t,IncludeSingularSolutions -> True]
```

$$y(t) \to \frac{2e^{t}(t-1) - 1}{t}$$

5.32 problem 32

Internal problem ID [14261]

Book: INTRODUCTORY DIFFERENTIAL EQUATIONS. Martha L. Abell, James P. Braselton. Fourth edition 2014. ElScAe. 2014
Section: Chapter 2. First Order Equations. Exercises 2.3, page 49
Problem number: 32.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_linear]`

$$\boxed{(1+e^{t})\, y' + e^{t} y = t}$$

With initial conditions

$$[y(0) = -1]$$

✓ Solution by Maple
Time used: 0.016 (sec). Leaf size: 18

```
dsolve([(exp(t)+1)*diff(y(t),t)+exp(t)*y(t)=t,y(0) = -1],y(t), singsol=all)
```

$$y(t) = \frac{t^2 - 4}{2 + 2\,e^{t}}$$

✓ Solution by Mathematica
Time used: 0.071 (sec). Leaf size: 21

```
DSolve[{(Exp[t]+1)*y'[t]+Exp[t]*y[t]==t,{y[0]==-1}},y[t],t,IncludeSingularSolutions -> True]
```

$$y(t) \to \frac{t^2 - 4}{2\,(e^{t} + 1)}$$

5.33 problem 33

Internal problem ID [14262]

Book: INTRODUCTORY DIFFERENTIAL EQUATIONS. Martha L. Abell, James P. Braselton. Fourth edition 2014. ElScAe. 2014
Section: Chapter 2. First Order Equations. Exercises 2.3, page 49
Problem number: 33.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_separable]`

$$\boxed{(t^2+4)\,y'+2ty=2t}$$

With initial conditions

$$[y(0)=-4]$$

✓ Solution by Maple
Time used: 0.015 (sec). Leaf size: 17

```
dsolve([(t^2+4)*diff(y(t),t)+2*t*y(t)=2*t,y(0) = -4],y(t), singsol=all)
```

$$y(t)=\frac{t^2-16}{t^2+4}$$

✓ Solution by Mathematica
Time used: 0.031 (sec). Leaf size: 18

```
DSolve[{(t^2+4)*y'[t]+2*t*y[t]==2*t,{y[0]==-4}},y[t],t,IncludeSingularSolutions -> True]
```

$$y(t)\to\frac{t^2-16}{t^2+4}$$

5.34 problem 34

Internal problem ID [14263]

Book: INTRODUCTORY DIFFERENTIAL EQUATIONS. Martha L. Abell, James P. Braselton. Fourth edition 2014. ElScAe. 2014
Section: Chapter 2. First Order Equations. Exercises 2.3, page 49
Problem number: 34.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[[_linear, ‘class A‘]]`

$$\boxed{-x + x' = 1 + t}$$

With initial conditions

$$[x(0) = 2]$$

✓ Solution by Maple
Time used: 0.016 (sec). Leaf size: 13

```
dsolve([diff(x(t),t)=x(t)+t+1,x(0) = 2],x(t), singsol=all)
```

$$x(t) = -t - 2 + 4\,\mathrm{e}^{t}$$

✓ Solution by Mathematica
Time used: 0.067 (sec). Leaf size: 15

```
DSolve[{x'[t]==x[t]+t+1,{x[0]==2}},x[t],t,IncludeSingularSolutions -> True]
```

$$x(t) \to -t + 4e^{t} - 2$$

5.35 problem 35

Internal problem ID [14264]

Book: INTRODUCTORY DIFFERENTIAL EQUATIONS. Martha L. Abell, James P. Braselton. Fourth edition 2014. ElScAe. 2014
Section: Chapter 2. First Order Equations. Exercises 2.3, page 49
Problem number: 35.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type **[[_linear, ‘class A‘]]**

$$\boxed{y' - 2y = \mathrm{e}^{2t}}$$

With initial conditions

$$[y(0) = 0]$$

✓ Solution by Maple
Time used: 0.0 (sec). Leaf size: 10

```
dsolve([diff(y(t),t)=exp(2*t)+2*y(t),y(0) = 0],y(t), singsol=all)
```

$$y(t) = \mathrm{e}^{2t}t$$

✓ Solution by Mathematica
Time used: 0.041 (sec). Leaf size: 12

```
DSolve[{y'[t]==Exp[2*t]+2*y[t],{y[0]==0}},y[t],t,IncludeSingularSolutions -> True]
```

$$y(t) \to e^{2t}t$$

5.36 problem 38 (b)

Internal problem ID [14265]

Book: INTRODUCTORY DIFFERENTIAL EQUATIONS. Martha L. Abell, James P. Braselton. Fourth edition 2014. ElScAe. 2014
Section: Chapter 2. First Order Equations. Exercises 2.3, page 49
Problem number: 38 (b).
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_linear]`

$$\boxed{y' - \frac{y}{t} = \ln(t)}$$

✓ Solution by Maple
Time used: 0.0 (sec). Leaf size: 14

```
dsolve(diff(y(t),t)-1/t*y(t)=ln(t),y(t), singsol=all)
```

$$y(t) = \left(\frac{\ln(t)^2}{2} + c_1\right) t$$

✓ Solution by Mathematica
Time used: 0.029 (sec). Leaf size: 19

```
DSolve[y'[t]-1/t*y[t]==Log[t],y[t],t,IncludeSingularSolutions -> True]
```

$$y(t) \to \frac{1}{2} t\left(\log^2(t) + 2c_1\right)$$

5.37 problem 38 (c)

Internal problem ID [14266]

Book: INTRODUCTORY DIFFERENTIAL EQUATIONS. Martha L. Abell, James P. Braselton. Fourth edition 2014. ElScAe. 2014
Section: Chapter 2. First Order Equations. Exercises 2.3, page 49
Problem number: 38 (c).
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _exact, _linear, _nonhomogeneous]]`

$$\boxed{y'' - \frac{y'}{t} + \frac{y}{t^2} = \frac{1}{t}}$$

✓ Solution by Maple
Time used: 0.0 (sec). Leaf size: 18

```
dsolve(diff(y(t),t$2)-1/t*diff(y(t),t)+1/t^2*y(t)=1/t,y(t), singsol=all)
```

$$y(t) = \left(c_2 + c_1 \ln(t) + \frac{\ln(t)^2}{2}\right) t$$

✓ Solution by Mathematica
Time used: 0.019 (sec). Leaf size: 25

```
DSolve[y''[t]-1/t*y'[t]+1/t^2*y[t]==1/t,y[t],t,IncludeSingularSolutions -> True]
```

$$y(t) \to \frac{1}{2} t\left(\log^2(t) + 2c_2 \log(t) + 2c_1\right)$$

5.38 problem 42

Internal problem ID [14267]

Book: INTRODUCTORY DIFFERENTIAL EQUATIONS. Martha L. Abell, James P. Braselton. Fourth edition 2014. ElScAe. 2014
Section: Chapter 2. First Order Equations. Exercises 2.3, page 49
Problem number: 42.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[[_linear, ‘class A‘]]`

$$\boxed{y' + y = \begin{cases} 4 & 0 \leq t < 2 \\ 0 & 2 \leq t \end{cases}}$$

With initial conditions

$$[y(0) = 0]$$

✓ Solution by Maple
Time used: 0.422 (sec). Leaf size: 38

```
dsolve([diff(y(t),t)+y(t)=piecewise(0<=t and t<2,4,t>=2,0),y(0) = 0],y(t), singsol=all)
```

$$y(t) = \begin{cases} 0 & t < 0 \\ 4 - 4\,\mathrm{e}^{-t} & t < 2 \\ 4\,\mathrm{e}^{2-t} - 4\,\mathrm{e}^{-t} & 2 \leq t \end{cases}$$

✓ Solution by Mathematica

Time used: 0.061 (sec). Leaf size: 40

```
DSolve[{y'[t]+y[t]==Piecewise[{{4,0<=t<2},{0,t>=2}}],{y[0]==0}},y[t],t,IncludeSingularSoluti
```

$$y(t) \to \begin{cases} 0 & t \leq 0 \\ 4 - 4e^{-t} & 0 < t \leq 2 \\ 4e^{-t}(-1 + e^2) & \text{True} \end{cases}$$

5.39 problem 43

Internal problem ID [14268]

Book: INTRODUCTORY DIFFERENTIAL EQUATIONS. Martha L. Abell, James P. Braselton. Fourth edition 2014. ElScAe. 2014
Section: Chapter 2. First Order Equations. Exercises 2.3, page 49
Problem number: 43.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[[_linear, 'class A']]`

$$\boxed{y' + y = \begin{cases} t & 0 \leq t < 1 \\ 0 & 1 \leq t \end{cases}}$$

With initial conditions

$$[y(0) = 1]$$

✓ Solution by Maple

Time used: 0.344 (sec). Leaf size: 33

```
dsolve([diff(y(t),t)+y(t)=piecewise(0<=t and t<1,t,t>=1,0),y(0) = 1],y(t), singsol=all)
```

$$y(t) = \begin{cases} e^{-t} & t < 0 \\ 2\,e^{-t} + t - 1 & t < 1 \\ 2\,e^{-t} & 1 \leq t \end{cases}$$

✓ Solution by Mathematica

Time used: 0.065 (sec). Leaf size: 37

```
DSolve[{y'[t]+y[t]==Piecewise[{{t,0<=t<1},{0,t>=1}}],{y[0]==1}},y[t],t,IncludeSingularSoluti
```

$$y(t) \to \begin{cases} e^{-t} & t \leq 0 \\ 2e^{-t} & t > 1 \\ t + 2e^{-t} - 1 & \text{True} \end{cases}$$

5.40 problem 47

Internal problem ID [14269]

Book: INTRODUCTORY DIFFERENTIAL EQUATIONS. Martha L. Abell, James P. Braselton. Fourth edition 2014. ElScAe. 2014
Section: Chapter 2. First Order Equations. Exercises 2.3, page 49
Problem number: 47.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[[_linear, 'class A']]`

$$\boxed{y' - y = \sin(2t)}$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 21

```
dsolve(diff(y(t),t)-y(t)=sin(2*t),y(t), singsol=all)
```

$$y(t) = -\frac{2\cos(2t)}{5} - \frac{\sin(2t)}{5} + e^t c_1$$

✓ Solution by Mathematica

Time used: 0.085 (sec). Leaf size: 28

```
DSolve[y'[t]-y[t]==Sin[2*t],y[t],t,IncludeSingularSolutions -> True]
```

$$y(t) \to -\frac{1}{5}\sin(2t) - \frac{2}{5}\cos(2t) + c_1 e^t$$

5.41 problem 48

Internal problem ID [14270]

Book: INTRODUCTORY DIFFERENTIAL EQUATIONS. Martha L. Abell, James P. Braselton. Fourth edition 2014. ElScAe. 2014
Section: Chapter 2. First Order Equations. Exercises 2.3, page 49
Problem number: 48.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[[_linear, 'class A']]`

$$\boxed{y' + y = 5\,\mathrm{e}^{2t}}$$

✓ Solution by Maple
Time used: 0.0 (sec). Leaf size: 17

```
dsolve(diff(y(t),t)+y(t)=5*exp(2*t),y(t), singsol=all)
```

$$y(t) = \frac{5\,\mathrm{e}^{2t}}{3} + \mathrm{e}^{-t} c_1$$

✓ Solution by Mathematica
Time used: 0.051 (sec). Leaf size: 23

```
DSolve[y'[t]+y[t]==5*Exp[2*t],y[t],t,IncludeSingularSolutions -> True]
```

$$y(t) \to \frac{5e^{2t}}{3} + c_1 e^{-t}$$

5.42 problem 49

Internal problem ID [14271]

Book: INTRODUCTORY DIFFERENTIAL EQUATIONS. Martha L. Abell, James P. Braselton. Fourth edition 2014. ElScAe. 2014
Section: Chapter 2. First Order Equations. Exercises 2.3, page 49
Problem number: 49.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[[_linear, 'class A']]`

$$\boxed{y' + y = e^{-t}}$$

✓ Solution by Maple
Time used: 0.0 (sec). Leaf size: 12

```
dsolve(diff(y(t),t)+y(t)=exp(-t),y(t), singsol=all)
```

$$y(t) = (t + c_1)\,e^{-t}$$

✓ Solution by Mathematica
Time used: 0.044 (sec). Leaf size: 15

```
DSolve[y'[t]+y[t]==Exp[-t],y[t],t,IncludeSingularSolutions -> True]
```

$$y(t) \to e^{-t}(t + c_1)$$

5.43 problem 50

Internal problem ID [14272]

Book: INTRODUCTORY DIFFERENTIAL EQUATIONS. Martha L. Abell, James P. Braselton. Fourth edition 2014. ElScAe. 2014
Section: Chapter 2. First Order Equations. Exercises 2.3, page 49
Problem number: 50.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[[_linear, 'class A']]`

$$\boxed{y' + y = 2 - e^{2t}}$$

✓ Solution by Maple
Time used: 0.0 (sec). Leaf size: 18

```
dsolve(diff(y(t),t)+y(t)=2-exp(2*t),y(t), singsol=all)
```

$$y(t) = 2 - \frac{e^{2t}}{3} + e^{-t}c_1$$

✓ Solution by Mathematica
Time used: 0.063 (sec). Leaf size: 24

```
DSolve[y'[t]+y[t]==2-Exp[2*t],y[t],t,IncludeSingularSolutions -> True]
```

$$y(t) \to -\frac{e^{2t}}{3} + c_1 e^{-t} + 2$$

5.44 problem 51

Internal problem ID [14273]

Book: INTRODUCTORY DIFFERENTIAL EQUATIONS. Martha L. Abell, James P. Braselton. Fourth edition 2014. ElScAe. 2014
Section: Chapter 2. First Order Equations. Exercises 2.3, page 49
Problem number: 51.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[[_linear, 'class A']]`

$$\boxed{y' - 5y = t}$$

✓ Solution by Maple
Time used: 0.0 (sec). Leaf size: 15

```
dsolve(diff(y(t),t)-5*y(t)=t,y(t), singsol=all)
```

$$y(t) = -\frac{t}{5} - \frac{1}{25} + c_1 e^{5t}$$

✓ Solution by Mathematica
Time used: 0.044 (sec). Leaf size: 22

```
DSolve[y'[t]-5*y[t]==t,y[t],t,IncludeSingularSolutions -> True]
```

$$y(t) \to -\frac{t}{5} + c_1 e^{5t} - \frac{1}{25}$$

5.45 problem 52

Internal problem ID [14274]

Book: INTRODUCTORY DIFFERENTIAL EQUATIONS. Martha L. Abell, James P. Braselton. Fourth edition 2014. ElScAe. 2014
Section: Chapter 2. First Order Equations. Exercises 2.3, page 49
Problem number: 52.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[[_linear, ‘class A‘]]`

$$\boxed{y' + 3y = 27t^2 + 9}$$

✓ Solution by Maple
Time used: 0.0 (sec). Leaf size: 20

```
dsolve(diff(y(t),t)+3*y(t)=27*t^2+9,y(t), singsol=all)
```

$$y(t) = 9t^2 - 6t + 5 + c_1 \mathrm{e}^{-3t}$$

✓ Solution by Mathematica
Time used: 0.072 (sec). Leaf size: 23

```
DSolve[y'[t]+3*y[t]==27*t^2+9,y[t],t,IncludeSingularSolutions -> True]
```

$$y(t) \to 9t^2 - 6t + c_1 e^{-3t} + 5$$

5.46 problem 53

Internal problem ID [14275]

Book: INTRODUCTORY DIFFERENTIAL EQUATIONS. Martha L. Abell, James P. Braselton. Fourth edition 2014. ElScAe. 2014
Section: Chapter 2. First Order Equations. Exercises 2.3, page 49
Problem number: 53.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[[_linear, 'class A']]`

$$\boxed{y' - \frac{y}{2} = 5\cos(t) + 2\,\mathrm{e}^{t}}$$

✓ Solution by Maple
Time used: 0.0 (sec). Leaf size: 23

```
dsolve(diff(y(t),t)-1/2*y(t)=5*cos(t)+2*exp(t),y(t), singsol=all)
```

$$y(t) = 4\sin(t) - 2\cos(t) + 4\,\mathrm{e}^{t} + \mathrm{e}^{\frac{t}{2}} c_1$$

✓ Solution by Mathematica
Time used: 0.124 (sec). Leaf size: 28

```
DSolve[y'[t]-1/2*y[t]==5*Cos[t]+2*Exp[t],y[t],t,IncludeSingularSolutions -> True]
```

$$y(t) \to 4\left(e^{t} + \sin(t)\right) - 2\cos(t) + c_1 e^{t/2}$$

5.47 problem 54

Internal problem ID [14276]

Book: INTRODUCTORY DIFFERENTIAL EQUATIONS. Martha L. Abell, James P. Braselton. Fourth edition 2014. ElScAe. 2014
Section: Chapter 2. First Order Equations. Exercises 2.3, page 49
Problem number: 54.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[[_linear, ‘class A‘]]`

$$\boxed{y' + 4y = 8\cos(4t)}$$

✓ Solution by Maple
Time used: 0.0 (sec). Leaf size: 19

```
dsolve(diff(y(t),t)+4*y(t)=8*cos(4*t),y(t), singsol=all)
```

$$y(t) = \sin(4t) + \cos(4t) + \mathrm{e}^{-4t}c_1$$

✓ Solution by Mathematica
Time used: 0.084 (sec). Leaf size: 22

```
DSolve[y'[t]+4*y[t]==8*Cos[4*t],y[t],t,IncludeSingularSolutions -> True]
```

$$y(t) \to \sin(4t) + \cos(4t) + c_1 e^{-4t}$$

5.48 problem 55

Internal problem ID [14277]

Book: INTRODUCTORY DIFFERENTIAL EQUATIONS. Martha L. Abell, James P. Braselton. Fourth edition 2014. ElScAe. 2014
Section: Chapter 2. First Order Equations. Exercises 2.3, page 49
Problem number: 55.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[[_linear, ‘class A‘]]`

$$\boxed{y' + 10y = 2\,\mathrm{e}^{t}}$$

✓ Solution by Maple
Time used: 0.016 (sec). Leaf size: 20

```
dsolve(diff(y(t),t)+10*y(t)=2*exp(t),y(t), singsol=all)
```

$$y(t) = \frac{(2\,\mathrm{e}^{11t} + 11c_1)\,\mathrm{e}^{-10t}}{11}$$

✓ Solution by Mathematica
Time used: 0.049 (sec). Leaf size: 21

```
DSolve[y'[t]+10*y[t]==2*Exp[t],y[t],t,IncludeSingularSolutions -> True]
```

$$y(t) \to \frac{2e^t}{11} + c_1 e^{-10t}$$

5.49 problem 56

Internal problem ID [14278]

Book: INTRODUCTORY DIFFERENTIAL EQUATIONS. Martha L. Abell, James P. Braselton. Fourth edition 2014. ElScAe. 2014
Section: Chapter 2. First Order Equations. Exercises 2.3, page 49
Problem number: 56.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[[_linear, 'class A']]`

$$\boxed{y' - 3y = 27t^2}$$

✓ Solution by Maple
Time used: 0.0 (sec). Leaf size: 20

```
dsolve(diff(y(t),t)-3*y(t)=27*t^2,y(t), singsol=all)
```

$$y(t) = -9t^2 - 6t - 2 + c_1 e^{3t}$$

✓ Solution by Mathematica
Time used: 0.049 (sec). Leaf size: 23

```
DSolve[y'[t]-3*y[t]==27*t^2,y[t],t,IncludeSingularSolutions -> True]
```

$$y(t) \to -9t^2 - 6t + c_1 e^{3t} - 2$$

5.50 problem 57

Internal problem ID [14279]

Book: INTRODUCTORY DIFFERENTIAL EQUATIONS. Martha L. Abell, James P. Braselton. Fourth edition 2014. ElScAe. 2014
Section: Chapter 2. First Order Equations. Exercises 2.3, page 49
Problem number: 57.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[[_linear, 'class A']]`

$$\boxed{y' - y = 2\,\mathrm{e}^{t}}$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 12

```
dsolve(diff(y(t),t)-y(t)=2*exp(t),y(t), singsol=all)
```

$$y(t) = (2t + c_1)\,\mathrm{e}^{t}$$

✓ Solution by Mathematica

Time used: 0.038 (sec). Leaf size: 15

```
DSolve[y'[t]-y[t]==2*Exp[t],y[t],t,IncludeSingularSolutions -> True]
```

$$y(t) \to e^{t}(2t + c_1)$$

5.51 problem 58

Internal problem ID [14280]

Book: INTRODUCTORY DIFFERENTIAL EQUATIONS. Martha L. Abell, James P. Braselton. Fourth edition 2014. ElScAe. 2014
Section: Chapter 2. First Order Equations. Exercises 2.3, page 49
Problem number: 58.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[[_linear, 'class A']]`

$$\boxed{y' + y = 4 + 3\,\mathrm{e}^{t}}$$

✓ Solution by Maple
Time used: 0.0 (sec). Leaf size: 16

```
dsolve(diff(y(t),t)+y(t)=4+3*exp(t),y(t), singsol=all)
```

$$y(t) = 4 + \frac{3\,\mathrm{e}^{t}}{2} + \mathrm{e}^{-t} c_1$$

✓ Solution by Mathematica
Time used: 0.055 (sec). Leaf size: 26

```
DSolve[y'[t]+y[t]==4+3*Exp[t],y[t],t,IncludeSingularSolutions -> True]
```

$$y(t) \to \frac{3e^{t}}{2} + \left(\frac{8}{3} + c_1\right) e^{-t} + 4$$

5.52 problem 59

Internal problem ID [14281]

Book: INTRODUCTORY DIFFERENTIAL EQUATIONS. Martha L. Abell, James P. Braselton. Fourth edition 2014. ElScAe. 2014
Section: Chapter 2. First Order Equations. Exercises 2.3, page 49
Problem number: 59.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[[_linear, 'class A']]`

$$\boxed{y' + y = 2\cos(t) + t}$$

✓ Solution by Maple
Time used: 0.0 (sec). Leaf size: 17

```
dsolve(diff(y(t),t)+y(t)=2*cos(t)+t,y(t), singsol=all)
```

$$y(t) = \sin(t) + \cos(t) + t - 1 + \mathrm{e}^{-t} c_1$$

✓ Solution by Mathematica
Time used: 0.074 (sec). Leaf size: 20

```
DSolve[y'[t]+y[t]==2*Cos[t]+t,y[t],t,IncludeSingularSolutions -> True]
```

$$y(t) \to t + \sin(t) + \cos(t) + c_1 e^{-t} - 1$$

5.53 problem 60 (a)

Internal problem ID [14282]

Book: INTRODUCTORY DIFFERENTIAL EQUATIONS. Martha L. Abell, James P. Braselton. Fourth edition 2014. ElScAe. 2014
Section: Chapter 2. First Order Equations. Exercises 2.3, page 49
Problem number: 60 (a).
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[[_linear, 'class A']]`

$$\boxed{y' + \frac{y}{2} = \sin(t)}$$

With initial conditions

$$[y(0) = a]$$

✓ Solution by Maple
Time used: 0.015 (sec). Leaf size: 21

```
dsolve([diff(y(t),t)+y(t)/2=sin(t),y(0) = a],y(t), singsol=all)
```

$$y(t) = \mathrm{e}^{-\frac{t}{2}}a + \frac{2\sin(t)}{5} + \frac{4\,\mathrm{e}^{-\frac{t}{2}}}{5} - \frac{4\cos(t)}{5}$$

✓ Solution by Mathematica
Time used: 0.053 (sec). Leaf size: 37

```
DSolve[{y'[t]+y[t]/2==Sin[t],{y[0]==a}},y[t],t,IncludeSingularSolutions -> True]
```

$$y(t) \to \frac{1}{5}\left(5ae^{-t/2} + 4e^{-t/2} + 2\sin(t) - 4\cos(t)\right)$$

5.54 problem 60 (b)

Internal problem ID [14283]

Book: INTRODUCTORY DIFFERENTIAL EQUATIONS. Martha L. Abell, James P. Braselton. Fourth edition 2014. ElScAe. 2014
Section: Chapter 2. First Order Equations. Exercises 2.3, page 49
Problem number: 60 (b).
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[[_linear, 'class A']]`

$$\boxed{y' - \frac{y}{2} = \sin(t)}$$

With initial conditions

$$[y(0) = a]$$

✓ Solution by Maple
Time used: 0.016 (sec). Leaf size: 21

```
dsolve([diff(y(t),t)-y(t)/2=sin(t),y(0) = a],y(t), singsol=all)
```

$$y(t) = -\frac{4\cos(t)}{5} - \frac{2\sin(t)}{5} + e^{\frac{t}{2}} a + \frac{4\,e^{\frac{t}{2}}}{5}$$

✓ Solution by Mathematica
Time used: 0.048 (sec). Leaf size: 31

```
DSolve[{y'[t]-y[t]/2==Sin[t],{y[0]==a}},y[t],t,IncludeSingularSolutions -> True]
```

$$y(t) \to \frac{1}{5}\left((5a+4)e^{t/2} - 2\sin(t) - 4\cos(t)\right)$$

5.55 problem 62

Internal problem ID [14284]

Book: INTRODUCTORY DIFFERENTIAL EQUATIONS. Martha L. Abell, James P. Braselton. Fourth edition 2014. ElScAe. 2014
Section: Chapter 2. First Order Equations. Exercises 2.3, page 49
Problem number: 62.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_linear]`

$$\boxed{y't + y = \cos(t)\, t}$$

✓ Solution by Maple
Time used: 0.0 (sec). Leaf size: 16

```
dsolve(t*diff(y(t),t)+y(t)=t*cos(t),y(t), singsol=all)
```

$$y(t) = \frac{\sin(t)\, t + \cos(t) + c_1}{t}$$

✓ Solution by Mathematica
Time used: 0.046 (sec). Leaf size: 18

```
DSolve[t*y'[t]+y[t]==t*Cos[t],y[t],t,IncludeSingularSolutions -> True]
```

$$y(t) \to \frac{t\sin(t) + \cos(t) + c_1}{t}$$

5.56 problem 63 (a)

Internal problem ID [14285]

Book: INTRODUCTORY DIFFERENTIAL EQUATIONS. Martha L. Abell, James P. Braselton. Fourth edition 2014. ElScAe. 2014
Section: Chapter 2. First Order Equations. Exercises 2.3, page 49
Problem number: 63 (a).
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[[_linear, 'class A']]`

$$\boxed{y' + y = t}$$

With initial conditions

$$[y(0) = 0]$$

✓ Solution by Maple
Time used: 0.0 (sec). Leaf size: 11

```
dsolve([diff(y(t),t)+y(t)=t,y(0) = 0],y(t), singsol=all)
```

$$y(t) = t - 1 + \mathrm{e}^{-t}$$

✓ Solution by Mathematica
Time used: 0.025 (sec). Leaf size: 13

```
DSolve[{y'[t]+y[t]==t,{y[0]==0}},y[t],t,IncludeSingularSolutions -> True]
```

$$y(t) \to t + e^{-t} - 1$$

5.57 problem 63 (b)

Internal problem ID [14286]

Book: INTRODUCTORY DIFFERENTIAL EQUATIONS. Martha L. Abell, James P. Braselton. Fourth edition 2014. ElScAe. 2014
Section: Chapter 2. First Order Equations. Exercises 2.3, page 49
Problem number: 63 (b).
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[[_linear, ‘class A‘]]`

$$\boxed{y' + y = \sin(t)}$$

With initial conditions

$$[y(0) = 0]$$

✓ Solution by Maple
Time used: 0.015 (sec). Leaf size: 19

```
dsolve([diff(y(t),t)+y(t)=sin(t),y(0) = 0],y(t), singsol=all)
```

$$y(t) = -\frac{\cos(t)}{2} + \frac{\sin(t)}{2} + \frac{e^{-t}}{2}$$

✓ Solution by Mathematica
Time used: 0.051 (sec). Leaf size: 21

```
DSolve[{y'[t]+y[t]==Sin[t],{y[0]==0}},y[t],t,IncludeSingularSolutions -> True]
```

$$y(t) \to \frac{1}{2}\left(e^{-t} + \sin(t) - \cos(t)\right)$$

5.58 problem 63 (c)

Internal problem ID [14287]

Book: INTRODUCTORY DIFFERENTIAL EQUATIONS. Martha L. Abell, James P. Braselton. Fourth edition 2014. ElScAe. 2014
Section: Chapter 2. First Order Equations. Exercises 2.3, page 49
Problem number: 63 (c).
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[[_linear, 'class A']]`

$$\boxed{y' + y = \cos(t)}$$

With initial conditions

$$[y(0) = 0]$$

✓ Solution by Maple
Time used: 0.016 (sec). Leaf size: 19

```
dsolve([diff(y(t),t)+y(t)=cos(t),y(0) = 0],y(t), singsol=all)
```

$$y(t) = \frac{\cos(t)}{2} + \frac{\sin(t)}{2} - \frac{e^{-t}}{2}$$

✓ Solution by Mathematica
Time used: 0.043 (sec). Leaf size: 21

```
DSolve[{y'[t]+y[t]==Cos[t],{y[0]==0}},y[t],t,IncludeSingularSolutions -> True]
```

$$y(t) \to \frac{1}{2}\left(-e^{-t} + \sin(t) + \cos(t)\right)$$

5.59 problem 63 (d)

Internal problem ID [14288]

Book: INTRODUCTORY DIFFERENTIAL EQUATIONS. Martha L. Abell, James P. Braselton. Fourth edition 2014. ElScAe. 2014
Section: Chapter 2. First Order Equations. Exercises 2.3, page 49
Problem number: 63 (d).
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[[_linear, 'class A']]`

$$\boxed{y' + y = \mathrm{e}^{t}}$$

With initial conditions

$$[y(0) = 0]$$

✓ Solution by Maple
Time used: 0.016 (sec). Leaf size: 15

```
dsolve([diff(y(t),t)+y(t)=exp(t),y(0) = 0],y(t), singsol=all)
```

$$y(t) = \frac{\mathrm{e}^{t}}{2} - \frac{\mathrm{e}^{-t}}{2}$$

✓ Solution by Mathematica
Time used: 0.043 (sec). Leaf size: 21

```
DSolve[{y'[t]+y[t]==Exp[t],{y[0]==0}},y[t],t,IncludeSingularSolutions -> True]
```

$$y(t) \to \frac{1}{2}e^{-t}\left(e^{2t} - 1\right)$$

6 Chapter 2. First Order Equations. Exercises 2.4, page 57

6.1 problem 1

Internal problem ID [14289]

Book: INTRODUCTORY DIFFERENTIAL EQUATIONS. Martha L. Abell, James P. Braselton. Fourth edition 2014. ElScAe. 2014
Section: Chapter 2. First Order Equations. Exercises 2.4, page 57
Problem number: 1.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_exact, _rational, [_Abel, '2nd type', 'class B']]`

$$\boxed{y^2 - \frac{y}{2\sqrt{t}} + \left(2ty - \sqrt{t} + 1\right) y' = 0}$$

✓ Solution by Maple
Time used: 0.0 (sec). Leaf size: 57

```
dsolve((y(t)^2-y(t)/(2*sqrt(t)))+(2*t*y(t)-sqrt(t)+1)*diff(y(t),t)=0,y(t), singsol=all)
```

$$y(t) = \frac{\sqrt{t} - 1 + \sqrt{t - 4c_1 t - 2\sqrt{t} + 1}}{2t}$$
$$y(t) = \frac{\sqrt{t} - 1 - \sqrt{t - 4c_1 t - 2\sqrt{t} + 1}}{2t}$$

✓ Solution by Mathematica
Time used: 15.797 (sec). Leaf size: 105

```
DSolve[(y[t]^2-y[t]/(2*Sqrt[t]))+(2*t*y[t]-Sqrt[t]+1)*y'[t]==0,y[t],t,IncludeSingularSolutio
```

$$y(t) \to \frac{t^{3/4} - \sqrt[4]{t} - \sqrt{\sqrt{t}\left(t - 2\sqrt{t} + 4c_1 t + 1\right)}}{2t^{5/4}}$$
$$y(t) \to \frac{t^{3/4} - \sqrt[4]{t} + \sqrt{\sqrt{t}\left(t - 2\sqrt{t} + 4c_1 t + 1\right)}}{2t^{5/4}}$$

6.2 problem 2

Internal problem ID [14290]

Book: INTRODUCTORY DIFFERENTIAL EQUATIONS. Martha L. Abell, James P. Braselton. Fourth edition 2014. ElScAe. 2014
Section: Chapter 2. First Order Equations. Exercises 2.4, page 57
Problem number: 2.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_separable]`

$$\boxed{\frac{t}{\sqrt{y^2+t^2}}+\frac{yy'}{\sqrt{y^2+t^2}}=0}$$

✓ Solution by Maple
Time used: 0.016 (sec). Leaf size: 27

```
dsolve(t/sqrt(t^2+y(t)^2)+y(t)/sqrt(t^2+y(t)^2)*diff(y(t),t)=0,y(t), singsol=all)
```

$$y(t)=\sqrt{-t^2+c_1}$$
$$y(t)=-\sqrt{-t^2+c_1}$$

✓ Solution by Mathematica
Time used: 0.087 (sec). Leaf size: 39

```
DSolve[t/Sqrt[t^2+y[t]^2]+y[t]/Sqrt[t^2+y[t]^2]*y'[t]==0,y[t],t,IncludeSingularSolutions ->
```

$$y(t)\to -\sqrt{-t^2+2c_1}$$
$$y(t)\to \sqrt{-t^2+2c_1}$$

6.3 problem 3

Internal problem ID [14291]

Book: INTRODUCTORY DIFFERENTIAL EQUATIONS. Martha L. Abell, James P. Braselton. Fourth edition 2014. ElScAe. 2014
Section: Chapter 2. First Order Equations. Exercises 2.4, page 57
Problem number: 3.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_separable]`

$$\boxed{y\cos(ty) + t\cos(ty)\,y' = 0}$$

✓ Solution by Maple
Time used: 0.016 (sec). Leaf size: 19

```
dsolve(y(t)*cos(t*y(t))+t*cos(t*y(t))*diff(y(t),t)=0,y(t), singsol=all)
```

$$y(t) = \frac{\pi}{2t}$$
$$y(t) = -\frac{c_1}{t}$$

✓ Solution by Mathematica
Time used: 0.03 (sec). Leaf size: 59

```
DSolve[y[t]*Cos[t*y[t]]+t*Cos[t*y[t]]*y'[t]==0,y[t],t,IncludeSingularSolutions -> True]
```

$$y(t) \to -\frac{\pi}{2t}$$
$$y(t) \to \frac{\pi}{2t}$$
$$y(t) \to \frac{c_1}{t}$$
$$y(t) \to -\frac{\pi}{2t}$$
$$y(t) \to \frac{\pi}{2t}$$

6.4 problem 4

Internal problem ID [14292]

Book: INTRODUCTORY DIFFERENTIAL EQUATIONS. Martha L. Abell, James P. Braselton. Fourth edition 2014. ElScAe. 2014
Section: Chapter 2. First Order Equations. Exercises 2.4, page 57
Problem number: 4.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_linear]`

$$\boxed{y \sec(t)^2 + \tan(t) y' = -2t}$$

✓ Solution by Maple
Time used: 0.0 (sec). Leaf size: 14

```
dsolve((y(t)*sec(t)^2+2*t)+tan(t)*diff(y(t),t)=0,y(t), singsol=all)
```

$$y(t) = \cot(t) \left(-t^2 + c_1\right)$$

✓ Solution by Mathematica
Time used: 0.056 (sec). Leaf size: 16

```
DSolve[(y[t]*Sec[t]^2+2*t)+Tan[t]*y'[t]==0,y[t],t,IncludeSingularSolutions -> True]
```

$$y(t) \to \left(-t^2 + c_1\right) \cot(t)$$

6.5 problem 5

Internal problem ID [14293]

Book: INTRODUCTORY DIFFERENTIAL EQUATIONS. Martha L. Abell, James P. Braselton. Fourth edition 2014. ElScAe. 2014
Section: Chapter 2. First Order Equations. Exercises 2.4, page 57
Problem number: 5.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_separable]`

$$\boxed{3ty^2 + y^3y' = 0}$$

✓ Solution by Maple
Time used: 0.0 (sec). Leaf size: 31

```
dsolve(3*t*y(t)^2+y(t)^3*diff(y(t),t)=0,y(t), singsol=all)
```

$$\begin{aligned} y(t) &= 0 \\ y(t) &= \sqrt{-3t^2 + c_1} \\ y(t) &= -\sqrt{-3t^2 + c_1} \end{aligned}$$

✓ Solution by Mathematica
Time used: 0.072 (sec). Leaf size: 49

```
DSolve[3*t*y[t]^2+y[t]^3*y'[t]==0,y[t],t,IncludeSingularSolutions -> True]
```

$$\begin{aligned} y(t) &\to 0 \\ y(t) &\to -\sqrt{-3t^2 + 2c_1} \\ y(t) &\to \sqrt{-3t^2 + 2c_1} \\ y(t) &\to 0 \end{aligned}$$

6.6 problem 6

Internal problem ID [14294]

Book: INTRODUCTORY DIFFERENTIAL EQUATIONS. Martha L. Abell, James P. Braselton. Fourth edition 2014. ElScAe. 2014
Section: Chapter 2. First Order Equations. Exercises 2.4, page 57
Problem number: 6.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_exact]`

$$\boxed{-\sin(t)\, y+\left(y^{6}+\cos(t)\right) y' = -t}$$

✓ Solution by Maple
Time used: 0.0 (sec). Leaf size: 21

```
dsolve((t-y(t)*sin(t))+(y(t)^6+cos(t))*diff(y(t),t)=0,y(t), singsol=all)
```

$$\frac{t^{2}}{2}+y(t)\cos(t)+\frac{y(t)^{7}}{7}+c_{1}=0$$

✓ Solution by Mathematica
Time used: 6.808 (sec). Leaf size: 204

```
DSolve[(t-y[t]*Sin[t])+(y[t]^6+Cos[t])*y'[t]==0,y[t],t,IncludeSingularSolutions -> True]
```

$$\begin{aligned}
y(t) &\to \mathrm{Root}\left[2\#1^{7}+14\#1\cos(t)+7t^{2}-14c_{1}\&,1\right]\\
y(t) &\to \mathrm{Root}\left[2\#1^{7}+14\#1\cos(t)+7t^{2}-14c_{1}\&,2\right]\\
y(t) &\to \mathrm{Root}\left[2\#1^{7}+14\#1\cos(t)+7t^{2}-14c_{1}\&,3\right]\\
y(t) &\to \mathrm{Root}\left[2\#1^{7}+14\#1\cos(t)+7t^{2}-14c_{1}\&,4\right]\\
y(t) &\to \mathrm{Root}\left[2\#1^{7}+14\#1\cos(t)+7t^{2}-14c_{1}\&,5\right]\\
y(t) &\to \mathrm{Root}\left[2\#1^{7}+14\#1\cos(t)+7t^{2}-14c_{1}\&,6\right]\\
y(t) &\to \mathrm{Root}\left[2\#1^{7}+14\#1\cos(t)+7t^{2}-14c_{1}\&,7\right]
\end{aligned}$$

6.7 problem 7

Internal problem ID [14295]

Book: INTRODUCTORY DIFFERENTIAL EQUATIONS. Martha L. Abell, James P. Braselton. Fourth edition 2014. ElScAe. 2014
Section: Chapter 2. First Order Equations. Exercises 2.4, page 57
Problem number: 7.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[[_1st_order, '_with_symmetry_[F(x)*G(y),0]']]`

$$\boxed{\sin(2t)\, y + \left(\sqrt{y} + \cos(2t)\right) y' = 0}$$

✓ Solution by Maple
Time used: 0.0 (sec). Leaf size: 21

```
dsolve(y(t)*sin(2*t)+(sqrt(y(t))+cos(2*t))*diff(y(t),t)=0,y(t), singsol=all)
```

$$-\frac{\cos(2t)}{2y(t)^2} - \frac{2}{3y(t)^{\frac{3}{2}}} + c_1 = 0$$

✓ Solution by Mathematica

Time used: 60.322 (sec). Leaf size: 1897

```
DSolve[y[t]*Sin[2*t]+(Sqrt[y[t]]+Cos[2*t])*y'[t]==0,y[t],t,IncludeSingularSolutions -> True]
```

$$y(t) \to \frac{1}{6}\left(-\sqrt{\frac{6^{2/3}\sqrt[3]{-6c_1{}^3\cos^3(2t)+\sqrt{2}\sqrt{-c_1{}^4(9c_1\cos(2t)+3c_1\cos(6t)-2)}+2c_1{}^2}}{c_1{}^2}+\frac{}{\sqrt[3]{-6c_1{}^3\cos^3(2t)+}}}\right.$$

$$-\sqrt{-\frac{6^{2/3}\sqrt[3]{-6c_1{}^3\cos^3(2t)+\sqrt{2}\sqrt{-c_1{}^4(9c_1\cos(2t)+3c_1\cos(6t)-2)}+2c_1{}^2}}{c_1{}^2}-\frac{}{\sqrt[3]{-6c_1{}^3\cos^3(2t)+\sqrt{2}}}}$$

$$y(t) \to \frac{1}{6}\left(\sqrt{-\frac{6^{2/3}\sqrt[3]{-6c_1{}^3\cos^3(2t)+\sqrt{2}\sqrt{-c_1{}^4(9c_1\cos(2t)+3c_1\cos(6t)-2)}+2c_1{}^2}}{c_1{}^2}-\frac{}{\sqrt[3]{-6c_1{}^3\cos^3(2t)+}}}\right.$$

$$-\sqrt{\frac{6^{2/3}\sqrt[3]{-6c_1{}^3\cos^3(2t)+\sqrt{2}\sqrt{-c_1{}^4(9c_1\cos(2t)+3c_1\cos(6t)-2)}+2c_1{}^2}}{c_1{}^2}+\frac{}{\sqrt[3]{-6c_1{}^3\cos^3(2t)+\sqrt{2}}}}$$

$$y(t) \to \frac{1}{6}\left(\sqrt{\frac{6^{2/3}\sqrt[3]{-6c_1{}^3\cos^3(2t)+\sqrt{2}\sqrt{-c_1{}^4(9c_1\cos(2t)+3c_1\cos(6t)-2)}+2c_1{}^2}}{c_1{}^2}+\frac{}{\sqrt[3]{-6c_1{}^3\cos^3(2t)+\sqrt{2}}}}\right.$$

6.8 problem 8

Internal problem ID [14296]

Book: INTRODUCTORY DIFFERENTIAL EQUATIONS. Martha L. Abell, James P. Braselton. Fourth edition 2014. ElScAe. 2014
Section: Chapter 2. First Order Equations. Exercises 2.4, page 57
Problem number: 8.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[‘y=_G(x,y’)‘]`

$$\boxed{-y - (e^{y} - t)\, y' = -e^{2t}}$$

✗ Solution by Maple

```
dsolve((exp(2*t)-y(t))-(exp(y(t))-t)*diff(y(t),t)=0,y(t), singsol=all)
```

No solution found

✗ Solution by Mathematica

Time used: 0.0 (sec). Leaf size: 0

```
DSolve[(Exp[2*t]-y[t])-(Exp[y[t]]-t)*y'[t]==0,y[t],t,IncludeSingularSolutions -> True]
```

Not solved

6.9 problem 9

Internal problem ID [14297]

Book: INTRODUCTORY DIFFERENTIAL EQUATIONS. Martha L. Abell, James P. Braselton. Fourth edition 2014. ElScAe. 2014
Section: Chapter 2. First Order Equations. Exercises 2.4, page 57
Problem number: 9.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[[_homogeneous, ‘class G‘], _exact]`

$$\boxed{\ln(ty) = -\frac{ty'}{y}}$$

✓ Solution by Maple
Time used: 0.031 (sec). Leaf size: 16

```
dsolve(ln(t*y(t))+t/y(t)*diff(y(t),t)=0,y(t), singsol=all)
```

$$y(t) = \frac{e^{\frac{t+c_1}{t}}}{t}$$

✓ Solution by Mathematica
Time used: 0.195 (sec). Leaf size: 19

```
DSolve[Log[t*y[t]]+t/y[t]*y'[t]==0,y[t],t,IncludeSingularSolutions -> True]
```

$$y(t) \to \frac{e^{1+\frac{c_1}{t}}}{t}$$

6.10 problem 10

Internal problem ID [14298]

Book: INTRODUCTORY DIFFERENTIAL EQUATIONS. Martha L. Abell, James P. Braselton. Fourth edition 2014. ElScAe. 2014
Section: Chapter 2. First Order Equations. Exercises 2.4, page 57
Problem number: 10.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_separable]`

$$\boxed{e^{ty} + \frac{t\,e^{ty}y'}{y} = 0}$$

✓ Solution by Maple
Time used: 0.0 (sec). Leaf size: 9

```
dsolve(exp(t*y(t))+t*exp(t*y(t))/y(t)*diff(y(t),t)=0,y(t), singsol=all)
```

$$y(t) = \frac{c_1}{t}$$

✓ Solution by Mathematica
Time used: 0.038 (sec). Leaf size: 11

```
DSolve[Exp[t*y[t]]+t*Exp[t*y[t]]/y[t]*y'[t]==0,y[t],t,IncludeSingularSolutions -> True]
```

$$y(t) \to \frac{c_1}{t}$$

6.11 problem 11

Internal problem ID [14299]

Book: INTRODUCTORY DIFFERENTIAL EQUATIONS. Martha L. Abell, James P. Braselton. Fourth edition 2014. ElScAe. 2014
Section: Chapter 2. First Order Equations. Exercises 2.4, page 57
Problem number: 11.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_quadrature]`

$$\boxed{-y' = -3t^2}$$

✓ Solution by Maple
Time used: 0.0 (sec). Leaf size: 9

```
dsolve(3*t^2-diff(y(t),t)=0,y(t), singsol=all)
```

$$y(t) = t^3 + c_1$$

✓ Solution by Mathematica
Time used: 0.002 (sec). Leaf size: 11

```
DSolve[3*t^2-y'[t]==0,y[t],t,IncludeSingularSolutions -> True]
```

$$y(t) \to t^3 + c_1$$

6.12 problem 12

Internal problem ID [14300]

Book: INTRODUCTORY DIFFERENTIAL EQUATIONS. Martha L. Abell, James P. Braselton. Fourth edition 2014. ElScAe. 2014
Section: Chapter 2. First Order Equations. Exercises 2.4, page 57
Problem number: 12.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_quadrature]`

$$\boxed{3y^2y' = 1}$$

✓ Solution by Maple
Time used: 0.016 (sec). Leaf size: 45

```
dsolve(-1+3*y(t)^2*diff(y(t),t)=0,y(t), singsol=all)
```

$$
\begin{aligned}
y(t) &= (t+c_1)^{\frac{1}{3}} \\
y(t) &= -\frac{(t+c_1)^{\frac{1}{3}}\left(1+i\sqrt{3}\right)}{2} \\
y(t) &= \frac{(t+c_1)^{\frac{1}{3}}\left(i\sqrt{3}-1\right)}{2}
\end{aligned}
$$

✓ Solution by Mathematica
Time used: 0.036 (sec). Leaf size: 56

```
DSolve[-1+3*y[t]^2*y'[t]==0,y[t],t,IncludeSingularSolutions -> True]
```

$$
\begin{aligned}
y(t) &\to \sqrt[3]{t+3c_1} \\
y(t) &\to -\sqrt[3]{-1}\sqrt[3]{t+3c_1} \\
y(t) &\to (-1)^{2/3}\sqrt[3]{t+3c_1}
\end{aligned}
$$

6.13 problem 13

Internal problem ID [14301]

Book: INTRODUCTORY DIFFERENTIAL EQUATIONS. Martha L. Abell, James P. Braselton. Fourth edition 2014. ElScAe. 2014
Section: Chapter 2. First Order Equations. Exercises 2.4, page 57
Problem number: 13.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_separable]`

$$\boxed{y^2 + 2tyy' = 0}$$

✓ Solution by Maple
Time used: 0.0 (sec). Leaf size: 32

```
dsolve(y(t)^2+2*t*y(t)*diff(y(t),t)=0,y(t), singsol=all)
```

$$y(t) = 0$$
$$y(t) = \frac{\sqrt{-c_1 t}}{t}$$
$$y(t) = -\frac{\sqrt{-c_1 t}}{t}$$

✓ Solution by Mathematica
Time used: 0.024 (sec). Leaf size: 23

```
DSolve[y[t]^2+2*t*y[t]*y'[t]==0,y[t],t,IncludeSingularSolutions -> True]
```

$$y(t) \to 0$$
$$y(t) \to \frac{c_1}{\sqrt{t}}$$
$$y(t) \to 0$$

6.14 problem 14

Internal problem ID [14302]

Book: INTRODUCTORY DIFFERENTIAL EQUATIONS. Martha L. Abell, James P. Braselton. Fourth edition 2014. ElScAe. 2014
Section: Chapter 2. First Order Equations. Exercises 2.4, page 57
Problem number: 14.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type **[_separable]**

$$\boxed{\frac{3t^2}{y} - \frac{t^3 y'}{y^2} = 0}$$

✓ Solution by Maple

Time used: 0.015 (sec). Leaf size: 12

```
dsolve(3*t^2/y(t)-t^3/y(t)^2*diff(y(t),t)=0,y(t), singsol=all)
```

$$y(t) = -\frac{t^3}{c_1}$$

✓ Solution by Mathematica

Time used: 0.04 (sec). Leaf size: 11

```
DSolve[3*t^2/y[t]-t^3/y[t]^2*y'[t]==0,y[t],t,IncludeSingularSolutions -> True]
```

$$y(t) \to c_1 t^3$$

6.15 problem 15

Internal problem ID [14303]

Book: INTRODUCTORY DIFFERENTIAL EQUATIONS. Martha L. Abell, James P. Braselton. Fourth edition 2014. ElScAe. 2014
Section: Chapter 2. First Order Equations. Exercises 2.4, page 57
Problem number: 15.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_exact, _rational]`

$$\boxed{y^3 + \left(3ty^2 + 4\right) y' = -2t}$$

✓ Solution by Maple

Time used: 0.015 (sec). Leaf size: 394

```
dsolve((2*t+y(t)^3)+(3*t*y(t)^2+4)*diff(y(t),t)=0,y(t), singsol=all)
```

$$y(t) = -\frac{2\,12^{\frac{1}{3}}\left(12^{\frac{1}{3}}t - \frac{\left(-9t^2\left(t^2-\frac{\sqrt{3}\sqrt{\frac{27t^5+54c_1t^3+27c_1^2t+256}{t}}}{9}+c_1\right)\right)^{\frac{2}{3}}}{4}\right)}{3\left(-9t^2\left(t^2-\frac{\sqrt{3}\sqrt{\frac{27t^5+54c_1t^3+27c_1^2t+256}{t}}}{9}+c_1\right)\right)^{\frac{1}{3}}t}$$

$$y(t) = -\frac{3^{\frac{1}{3}}2^{\frac{2}{3}}\left(4i2^{\frac{2}{3}}3^{\frac{5}{6}}t + i\left(-9t^2\left(t^2-\frac{\sqrt{3}\sqrt{\frac{27t^5+54c_1t^3+27c_1^2t+256}{t}}}{9}+c_1\right)\right)^{\frac{2}{3}}\sqrt{3} - 4\,3^{\frac{1}{3}}2^{\frac{2}{3}}t + \left(-9t^2\left(t^2-\frac{\sqrt{3}\sqrt{27}}{}\right.\right.\right.}{12t\left(-9t^2\left(t^2-\frac{\sqrt{3}\sqrt{\frac{27t^5+54c_1t^3+27c_1^2t+256}{t}}}{9}+c_1\right)\right)^{\frac{1}{3}}}$$

$$y(t) = \frac{\left(4\left(i3^{\frac{5}{6}}+3^{\frac{1}{3}}\right)2^{\frac{2}{3}}t + \left(i\sqrt{3}-1\right)\left(-9t^2\left(t^2-\frac{\sqrt{3}\sqrt{\frac{27t^5+54c_1t^3+27c_1^2t+256}{t}}}{9}+c_1\right)\right)^{\frac{2}{3}}\right)2^{\frac{2}{3}}3^{\frac{1}{3}}}{12\left(-9t^2\left(t^2-\frac{\sqrt{3}\sqrt{\frac{27t^5+54c_1t^3+27c_1^2t+256}{t}}}{9}+c_1\right)\right)^{\frac{1}{3}}t}$$

✓ Solution by Mathematica

Time used: 23.63 (sec). Leaf size: 369

```
DSolve[(2*t+y[t]^3)+(3*t*y[t]^2+4)*y'[t]==0,y[t],t,IncludeSingularSolutions -> True]
```

$$y(t) \to \frac{\sqrt[3]{-27t^4+27c_1t^2+\sqrt{6912t^3+729\left(t^4-c_1t^2\right)^2}}}{3\sqrt[3]{2}t} - \frac{4\sqrt[3]{2}}{\sqrt[3]{-27t^4+27c_1t^2+\sqrt{6912t^3+729\left(t^4-c_1t^2\right)^2}}}$$

$$y(t) \to \frac{2\sqrt[3]{2}\left(1+i\sqrt{3}\right)}{\sqrt[3]{-27t^4+27c_1t^2+\sqrt{6912t^3+729\left(t^4-c_1t^2\right)^2}}} - \frac{\left(1-i\sqrt{3}\right)\sqrt[3]{-27t^4+27c_1t^2+\sqrt{6912t^3+729\left(t^4-c_1t^2\right)^2}}}{6\sqrt[3]{2}t}$$

$$y(t) \to \frac{2\sqrt[3]{2}\left(1-i\sqrt{3}\right)}{\sqrt[3]{-27t^4+27c_1t^2+\sqrt{6912t^3+729\left(t^4-c_1t^2\right)^2}}} - \frac{\left(1+i\sqrt{3}\right)\sqrt[3]{-27t^4+27c_1t^2+\sqrt{6912t^3+729\left(t^4-c_1t^2\right)^2}}}{6\sqrt[3]{2}t}$$

6.16 problem 16

Internal problem ID [14304]

Book: INTRODUCTORY DIFFERENTIAL EQUATIONS. Martha L. Abell, James P. Braselton. Fourth edition 2014. ElScAe. 2014
Section: Chapter 2. First Order Equations. Exercises 2.4, page 57
Problem number: 16.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[[_homogeneous, ‘class G‘], _exact, _rational]`

$$\boxed{-\frac{1}{y}+\left(\frac{t}{y^{2}}+3y^{2}\right)y'=0}$$

✓ Solution by Maple
Time used: 0.016 (sec). Leaf size: 16

```
dsolve(-1/y(t)+(t/y(t)^2+3*y(t)^2)*diff(y(t),t)=0,y(t), singsol=all)
```

$$-y(t)^{4}-c_{1}y(t)+t=0$$

✓ Solution by Mathematica

Time used: 60.122 (sec). Leaf size: 1073

```
DSolve[-1/y[t]+(t/y[t]^2+3*y[t]^2)*y'[t]==0,y[t],t,IncludeSingularSolutions -> True]
```

$$y(t)\to \frac{\sqrt{\frac{-8\sqrt[3]{3}t+\sqrt[3]{2}\left(9c_1{}^2-\sqrt{768t^3+81c_1{}^4}\right)^{2/3}}{\sqrt[3]{9c_1{}^2-\sqrt{768t^3+81c_1{}^4}}}}}{2\sqrt[3]{6}} -\frac{1}{2}\sqrt{\frac{4\sqrt[3]{\frac{2}{3}}t}{\sqrt[3]{9c_1{}^2-\sqrt{768t^3+81c_1{}^4}}}-\frac{\sqrt[3]{9c_1{}^2-\sqrt{768t^3+81c_1{}^4}}}{\sqrt[3]{2}3^{2/3}}-\frac{2\sqrt[3]{6}c_1}{\sqrt{\frac{-8\sqrt[3]{3}t+\sqrt[3]{2}\left(9c_1{}^2-\sqrt{768t^3+81c_1{}^4}\right)^{2/3}}{\sqrt[3]{9c_1{}^2-\sqrt{768t^3+81c_1{}^4}}}}}}$$

$$y(t)\to \frac{\sqrt{\frac{-8\sqrt[3]{3}t+\sqrt[3]{2}\left(9c_1{}^2-\sqrt{768t^3+81c_1{}^4}\right)^{2/3}}{\sqrt[3]{9c_1{}^2-\sqrt{768t^3+81c_1{}^4}}}}}{2\sqrt[3]{6}} +\frac{1}{2}\sqrt{\frac{4\sqrt[3]{\frac{2}{3}}t}{\sqrt[3]{9c_1{}^2-\sqrt{768t^3+81c_1{}^4}}}-\frac{\sqrt[3]{9c_1{}^2-\sqrt{768t^3+81c_1{}^4}}}{\sqrt[3]{2}3^{2/3}}-\frac{2\sqrt[3]{6}c_1}{\sqrt{\frac{-8\sqrt[3]{3}t+\sqrt[3]{2}\left(9c_1{}^2-\sqrt{768t^3+81c_1{}^4}\right)^{2/3}}{\sqrt[3]{9c_1{}^2-\sqrt{768t^3+81c_1{}^4}}}}}}$$

$$y(t)\to -\frac{\sqrt{\frac{-8\sqrt[3]{3}t+\sqrt[3]{2}\left(9c_1{}^2-\sqrt{768t^3+81c_1{}^4}\right)^{2/3}}{\sqrt[3]{9c_1{}^2-\sqrt{768t^3+81c_1{}^4}}}}}{2\sqrt[3]{6}} -\frac{1}{2}\sqrt{\frac{4\sqrt[3]{\frac{2}{3}}t}{\sqrt[3]{9c_1{}^2-\sqrt{768t^3+81c_1{}^4}}}-\frac{\sqrt[3]{9c_1{}^2-\sqrt{768t^3+81c_1{}^4}}}{\sqrt[3]{2}3^{2/3}}+\frac{2\sqrt[3]{6}c_1}{\sqrt{\frac{-8\sqrt[3]{3}t+\sqrt[3]{2}\left(9c_1{}^2-\sqrt{768t^3+81c_1{}^4}\right)^{2/3}}{\sqrt[3]{9c_1{}^2-\sqrt{768t^3+81c_1{}^4}}}}}}$$

$$y(t)\to \frac{1}{2}\sqrt{\frac{4\sqrt[3]{\frac{2}{3}}t}{\sqrt[3]{9c_1{}^2-\sqrt{768t^3+81c_1{}^4}}}-\frac{\sqrt[3]{9c_1{}^2-\sqrt{768t^3+81c_1{}^4}}}{\sqrt[3]{2}3^{2/3}}+\frac{2\sqrt[3]{6}c_1}{\sqrt{\frac{-8\sqrt[3]{3}t+\sqrt[3]{2}\left(9c_1{}^2-\sqrt{768t^3+81c_1{}^4}\right)^{2/3}}{\sqrt[3]{9c_1{}^2-\sqrt{768t^3+81c_1{}^4}}}}}} -\frac{\sqrt{\frac{-8\sqrt[3]{3}t+\sqrt[3]{2}\left(9c_1{}^2-\sqrt{768t^3+81c_1{}^4}\right)^{2/3}}{\sqrt[3]{9c_1{}^2-\sqrt{768t^3+81c_1{}^4}}}}}{2\sqrt[3]{6}}$$

6.17 problem 17

Internal problem ID [14305]

Book: INTRODUCTORY DIFFERENTIAL EQUATIONS. Martha L. Abell, James P. Braselton. Fourth edition 2014. ElScAe. 2014
Section: Chapter 2. First Order Equations. Exercises 2.4, page 57
Problem number: 17.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[[_homogeneous, ‘class A‘], _exact, _rational, _dAlembert]`

$$\boxed{2ty + (y^2 + t^2)\, y' = 0}$$

✓ Solution by Maple
Time used: 0.078 (sec). Leaf size: 209

```
dsolve(2*t*y(t)+(t^2+y(t)^2)*diff(y(t),t)=0,y(t), singsol=all)
```

$$y(t) = -\frac{2\left(c_1 t^2 - \frac{\left(4+4\sqrt{4c_1^3 t^6+1}\right)^{\frac{2}{3}}}{4}\right)}{\sqrt{c_1}\,\left(4+4\sqrt{4c_1^3 t^6+1}\right)^{\frac{1}{3}}}$$

$$y(t) = -\frac{\left(1+i\sqrt{3}\right)\left(4+4\sqrt{4c_1^3 t^6+1}\right)^{\frac{1}{3}}}{4\sqrt{c_1}} - \frac{\sqrt{c_1}\, t^2 \left(i\sqrt{3}-1\right)}{\left(4+4\sqrt{4c_1^3 t^6+1}\right)^{\frac{1}{3}}}$$

$$y(t) = \frac{4i\sqrt{3}\, c_1 t^2 + i\sqrt{3}\left(4+4\sqrt{4c_1^3 t^6+1}\right)^{\frac{2}{3}} + 4c_1 t^2 - \left(4+4\sqrt{4c_1^3 t^6+1}\right)^{\frac{2}{3}}}{4\left(4+4\sqrt{4c_1^3 t^6+1}\right)^{\frac{1}{3}}\sqrt{c_1}}$$

✓ Solution by Mathematica

Time used: 14.781 (sec). Leaf size: 406

```
DSolve[2*t*y[t]+(t^2+y[t]^2)*y'[t]==0,y[t],t,IncludeSingularSolutions -> True]
```

$$y(t) \to \frac{\sqrt[3]{\sqrt{4t^6+e^{6c_1}}+e^{3c_1}}}{\sqrt[3]{2}} - \frac{\sqrt[3]{2}t^2}{\sqrt[3]{\sqrt{4t^6+e^{6c_1}}+e^{3c_1}}}$$

$$y(t) \to \frac{i2^{2/3}(\sqrt{3}+i)\left(\sqrt{4t^6+e^{6c_1}}+e^{3c_1}\right)^{2/3}+\sqrt[3]{2}(2+2i\sqrt{3})\,t^2}{4\sqrt[3]{\sqrt{4t^6+e^{6c_1}}+e^{3c_1}}}$$

$$y(t) \to \frac{(1-i\sqrt{3})\,t^2}{2^{2/3}\sqrt[3]{\sqrt{4t^6+e^{6c_1}}+e^{3c_1}}} - \frac{(1+i\sqrt{3})\sqrt[3]{\sqrt{4t^6+e^{6c_1}}+e^{3c_1}}}{2\sqrt[3]{2}}$$

$$y(t) \to 0$$

$$y(t) \to \frac{(-1-i\sqrt{3})\sqrt[3]{t^6}+(1-i\sqrt{3})\,t^2}{2\sqrt[6]{t^6}}$$

$$y(t) \to \frac{i(\sqrt{3}+i)\sqrt[3]{t^6}+(1+i\sqrt{3})\,t^2}{2\sqrt[6]{t^6}}$$

$$y(t) \to \sqrt[6]{t^6} - \frac{(t^6)^{5/6}}{t^4}$$

6.18 problem 18

Internal problem ID [14306]

Book: INTRODUCTORY DIFFERENTIAL EQUATIONS. Martha L. Abell, James P. Braselton. Fourth edition 2014. ElScAe. 2014
Section: Chapter 2. First Order Equations. Exercises 2.4, page 57
Problem number: 18.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[[_homogeneous, ‘class G‘], _exact, _rational]`

$$\boxed{2y^3t + (1 + 3t^2y^2)\, y' = 0}$$

✓ Solution by Maple
Time used: 0.016 (sec). Leaf size: 252

```
dsolve(2*t*y(t)^3+(1+3*t^2*y(t)^2)*diff(y(t),t)=0,y(t), singsol=all)
```

$$y(t) = \frac{12^{\frac{1}{3}}\left(-c_1^2 12^{\frac{1}{3}} + \left(\left(\sqrt{12c_1^2 + 81t^2} + 9t\right)c_1^2\right)^{\frac{2}{3}}\right)}{6c_1 t\left(\left(\sqrt{12c_1^2 + 81t^2} + 9t\right)c_1^2\right)^{\frac{1}{3}}}$$

$$y(t) = -\frac{\left((1 + i\sqrt{3})\left(\left(\sqrt{12c_1^2 + 81t^2} + 9t\right)c_1^2\right)^{\frac{2}{3}} + \left(i3^{\frac{5}{6}} - 3^{\frac{1}{3}}\right)c_1^2 2^{\frac{2}{3}}\right)2^{\frac{2}{3}}3^{\frac{1}{3}}}{12\left(\left(\sqrt{12c_1^2 + 81t^2} + 9t\right)c_1^2\right)^{\frac{1}{3}}c_1 t}$$

$$y(t) = \frac{\left((i\sqrt{3} - 1)\left(\left(\sqrt{12c_1^2 + 81t^2} + 9t\right)c_1^2\right)^{\frac{2}{3}} + \left(i3^{\frac{5}{6}} + 3^{\frac{1}{3}}\right)c_1^2 2^{\frac{2}{3}}\right)2^{\frac{2}{3}}3^{\frac{1}{3}}}{12\left(\left(\sqrt{12c_1^2 + 81t^2} + 9t\right)c_1^2\right)^{\frac{1}{3}}c_1 t}$$

✓ Solution by Mathematica

Time used: 28.553 (sec). Leaf size: 364

```
DSolve[2*t*y[t]^3+(1+3*t^2*y[t]^2)*y'[t]==0,y[t],t,IncludeSingularSolutions -> True]
```

$$y(t) \to \frac{-2\sqrt[3]{3} + \frac{\sqrt[3]{2}\left(9c_1t^4+\sqrt{3}\sqrt{t^6(4+27c_1{}^2t^2)}\right)^{2/3}}{t^2}}{6^{2/3}\sqrt[3]{9c_1t^4+\sqrt{3}\sqrt{t^6\left(4+27c_1{}^2t^2\right)}}}$$

$$y(t) \to \frac{2\sqrt[3]{2}\sqrt[6]{3}\left(\sqrt{3}+3i\right)t^2 + i\sqrt[3]{3}\left(\sqrt{3}+i\right)\left(18c_1t^4+2\sqrt{3}\sqrt{t^6\left(4+27c_1{}^2t^2\right)}\right)^{2/3}}{12t^2\sqrt[3]{9c_1t^4+\sqrt{3}\sqrt{t^6\left(4+27c_1{}^2t^2\right)}}}$$

$$y(t) \to \frac{2\sqrt[3]{2}\sqrt[6]{3}\left(\sqrt{3}-3i\right)t^2 + \sqrt[3]{3}\left(-1-i\sqrt{3}\right)\left(18c_1t^4+2\sqrt{3}\sqrt{t^6\left(4+27c_1{}^2t^2\right)}\right)^{2/3}}{12t^2\sqrt[3]{9c_1t^4+\sqrt{3}\sqrt{t^6\left(4+27c_1{}^2t^2\right)}}}$$

$$y(t) \to 0$$

6.19 problem 19

Internal problem ID [14307]

Book: INTRODUCTORY DIFFERENTIAL EQUATIONS. Martha L. Abell, James P. Braselton. Fourth edition 2014. ElScAe. 2014
Section: Chapter 2. First Order Equations. Exercises 2.4, page 57
Problem number: 19.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_separable]`

$$\boxed{\sin(y)^2 + t\sin(2y)\, y' = 0}$$

✓ Solution by Maple
Time used: 0.453 (sec). Leaf size: 21

```
dsolve(sin(y(t))^2+(t*sin(2*y(t)))*diff(y(t),t)=0,y(t), singsol=all)
```

$$y(t) = \arcsin\left(\frac{1}{\sqrt{c_1 t}}\right)$$
$$y(t) = -\arcsin\left(\frac{1}{\sqrt{c_1 t}}\right)$$

✓ Solution by Mathematica
Time used: 6.191 (sec). Leaf size: 26

```
DSolve[Sin[y[t]]^2+(t*Sin[2*y[t]])*y'[t]==0,y[t],t,IncludeSingularSolutions -> True]
```

$$y(t) \to 0$$
$$y(t) \to \arcsin\left(\frac{e^{c_1}}{\sqrt{t}}\right)$$
$$y(t) \to 0$$

6.20 problem 20

Internal problem ID [14308]

Book: INTRODUCTORY DIFFERENTIAL EQUATIONS. Martha L. Abell, James P. Braselton. Fourth edition 2014. ElScAe. 2014
Section: Chapter 2. First Order Equations. Exercises 2.4, page 57
Problem number: 20.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[[_homogeneous, 'class A'], _exact, _rational, _Bernoulli]`

$$\boxed{3y^2 + 6tyy' = -3t^2}$$

✓ Solution by Maple
Time used: 0.0 (sec). Leaf size: 47

```
dsolve((3*t^2+3*y(t)^2)+6*t*y(t)*diff(y(t),t)=0,y(t), singsol=all)
```

$$y(t) = -\frac{\sqrt{3}\,\sqrt{-t\,(t^3 - 3c_1)}}{3t}$$
$$y(t) = \frac{\sqrt{3}\,\sqrt{-t\,(t^3 - 3c_1)}}{3t}$$

✓ Solution by Mathematica
Time used: 0.196 (sec). Leaf size: 60

```
DSolve[(3*t^2+3*y[t]^2)+6*t*y[t]*y'[t]==0,y[t],t,IncludeSingularSolutions -> True]
```

$$y(t) \to -\frac{\sqrt{-t^3 + 3c_1}}{\sqrt{3}\sqrt{t}}$$
$$y(t) \to \frac{\sqrt{-t^3 + 3c_1}}{\sqrt{3}\sqrt{t}}$$

6.21 problem 21

Internal problem ID [14309]

Book: INTRODUCTORY DIFFERENTIAL EQUATIONS. Martha L. Abell, James P. Braselton. Fourth edition 2014. ElScAe. 2014
Section: Chapter 2. First Order Equations. Exercises 2.4, page 57
Problem number: 21.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_exact, [_1st_order, '_with_symmetry_[F(x)*G(y),0]']]`

$$\boxed{e^t \sin(y) + (1 + e^t \cos(y))\, y' = 0}$$

✓ Solution by Maple
Time used: 0.0 (sec). Leaf size: 13

```
dsolve((exp(t)*sin(y(t)))+(1+exp(t)*cos(y(t)))*diff(y(t),t)=0,y(t), singsol=all)
```

$$e^t \sin(y(t)) + y(t) + c_1 = 0$$

✓ Solution by Mathematica
Time used: 0.176 (sec). Leaf size: 16

```
DSolve[(Exp[t]*Sin[y[t]])+(1+Exp[t]*Cos[y[t]])*y'[t]==0,y[t],t,IncludeSingularSolutions -> T
```

$$\text{Solve}\left[y(t) + e^t \sin(y(t)) = c_1, y(t)\right]$$

6.22 problem 22

Internal problem ID [14310]

Book: INTRODUCTORY DIFFERENTIAL EQUATIONS. Martha L. Abell, James P. Braselton. Fourth edition 2014. ElScAe. 2014
Section: Chapter 2. First Order Equations. Exercises 2.4, page 57
Problem number: 22.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_exact, _rational, [_Abel, '2nd type', 'class B']]`

$$\boxed{3t^2y + 3y^2 + (t^3 + 6ty)\, y' = 1}$$

✓ Solution by Maple
Time used: 0.0 (sec). Leaf size: 59

```
dsolve((3*t^2*y(t)+3*y(t)^2-1)+(t^3+6*t*y(t))*diff(y(t),t)=0,y(t), singsol=all)
```

$$y(t) = \frac{-t^3 + \sqrt{t\,(t^5 - 12c_1 + 12t)}}{6t}$$
$$y(t) = \frac{-t^3 - \sqrt{t\,(t^5 - 12c_1 + 12t)}}{6t}$$

✓ Solution by Mathematica
Time used: 0.462 (sec). Leaf size: 67

```
DSolve[(3*t^2*y[t]+3*y[t]^2-1)+(t^3+6*t*y[t])*y'[t]==0,y[t],t,IncludeSingularSolutions -> Tr
```

$$y(t) \to -\frac{t^3 + \sqrt{t\,(t^5 + 12t + 36c_1)}}{6t}$$
$$y(t) \to \frac{-t^3 + \sqrt{t\,(t^5 + 12t + 36c_1)}}{6t}$$

6.23 problem 23

Internal problem ID [14311]

Book: INTRODUCTORY DIFFERENTIAL EQUATIONS. Martha L. Abell, James P. Braselton. Fourth edition 2014. ElScAe. 2014
Section: Chapter 2. First Order Equations. Exercises 2.4, page 57
Problem number: 23.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_separable]`

$$\boxed{-2ty^2 \sin(t^2) + 2y \cos(t^2) y' = 0}$$

✓ Solution by Maple
Time used: 0.047 (sec). Leaf size: 40

```
dsolve(-2*t*y(t)^2*sin(t^2)+2*y(t)*cos(t^2)*diff(y(t),t)=0,y(t), singsol=all)
```

$$\begin{aligned} y(t) &= 0 \\ y(t) &= \sec(t^2)\sqrt{-\cos(t^2)\, c_1} \\ y(t) &= -\sec(t^2)\sqrt{-\cos(t^2)\, c_1} \end{aligned}$$

✓ Solution by Mathematica
Time used: 0.057 (sec). Leaf size: 26

```
DSolve[-2*t*y[t]^2*Sin[t^2]+2*y[t]*Cos[t^2]*y'[t]==0,y[t],t,IncludeSingularSolutions -> True
```

$$\begin{aligned} y(t) &\to 0 \\ y(t) &\to \frac{c_1}{\sqrt{\cos(t^2)}} \\ y(t) &\to 0 \end{aligned}$$

6.24 problem 24

Internal problem ID [14312]

Book: INTRODUCTORY DIFFERENTIAL EQUATIONS. Martha L. Abell, James P. Braselton. Fourth edition 2014. ElScAe. 2014
Section: Chapter 2. First Order Equations. Exercises 2.4, page 57
Problem number: 24.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_exact]`

$$\boxed{-y^2 \sin(ty) + (\cos(ty) - ty\sin(ty))\, y' = -2t}$$

✓ Solution by Maple
Time used: 0.031 (sec). Leaf size: 20

```
dsolve((2*t-y(t)^2*sin(t*y(t)))+(cos(t*y(t))-t*y(t)*sin(t*y(t)))*diff(y(t),t)=0,y(t), singso
```

$$y(t) = \frac{\operatorname{RootOf}(t^3 + \_Z\cos(\_Z) + c_1 t)}{t}$$

✓ Solution by Mathematica
Time used: 0.261 (sec). Leaf size: 18

```
DSolve[(2*t-y[t]^2*Sin[t*y[t]])+(Cos[t*y[t]]-t*y[t]*Sin[t*y[t]])*y'[t]==0,y[t],t,IncludeSing
```

$$\text{Solve}\left[t^2 + y(t)\cos(ty(t)) = c_1, y(t)\right]$$

6.25 problem 25

Internal problem ID [14313]

Book: INTRODUCTORY DIFFERENTIAL EQUATIONS. Martha L. Abell, James P. Braselton. Fourth edition 2014. ElScAe. 2014
Section: Chapter 2. First Order Equations. Exercises 2.4, page 57
Problem number: 25.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[‘y=_G(x,y’)‘]`

$$\boxed{-y^2\cos(ty) + (t\cos(ty)\,y + \sin(ty))\,y' = -1}$$

✗ Solution by Maple

```
dsolve((1-y(t)^2*cos(t*y(t)))+(t*y(t)*cos(t*y(t))+sin(t*y(t)))*diff(y(t),t)=0,y(t), singsol=
```

No solution found

✗ Solution by Mathematica
Time used: 0.0 (sec). Leaf size: 0

```
DSolve[(1-y[t]^2*Cos[t*y[t]])+(t*y[t]*Cos[t*y[t]]+Sin[t*y[t]])*y'[t]==0,y[t],t,IncludeSingul
```

Not solved

6.26 problem 26

Internal problem ID [14314]

Book: INTRODUCTORY DIFFERENTIAL EQUATIONS. Martha L. Abell, James P. Braselton. Fourth edition 2014. ElScAe. 2014
Section: Chapter 2. First Order Equations. Exercises 2.4, page 57
Problem number: 26.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_exact]`

$$\boxed{2t \sin(y) - 2\sin(t^2)\, yt + (t^2 \cos(y) + \cos(t^2))\, y' = 0}$$

✓ Solution by Maple
Time used: 0.078 (sec). Leaf size: 19

```
dsolve((2*t*sin(y(t))-2*t*y(t)*sin(t^2))+(t^2*cos(y(t))+cos(t^2) )*diff(y(t),t)=0,y(t), sing
```

$$y(t)\cos(t^2) + t^2 \sin(y(t)) + c_1 = 0$$

✓ Solution by Mathematica
Time used: 0.365 (sec). Leaf size: 21

```
DSolve[(2*t*Sin[y[t]]-2*t*y[t]*Sin[t^2])+(t^2*Cos[y[t]]+Cos[t^2] )*y'[t]==0,y[t],t,IncludeSi
```

$$\text{Solve}\left[t^2 \sin(y(t)) + y(t)\cos(t^2) = c_1, y(t)\right]$$

6.27 problem 27

Internal problem ID [14315]

Book: INTRODUCTORY DIFFERENTIAL EQUATIONS. Martha L. Abell, James P. Braselton. Fourth edition 2014. ElScAe. 2014
Section: Chapter 2. First Order Equations. Exercises 2.4, page 57
Problem number: 27.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[[_1st_order, _with_linear_symmetries], _exact]`

$$\boxed{(3+t)\cos(y+t)+\sin(y+t)+(3+t)\cos(y+t)\,y'=0}$$

✓ Solution by Maple
Time used: 0.016 (sec). Leaf size: 16

```
dsolve(( (3+t)*cos(t+y(t))+sin(t+y(t)) )+( (3+t)*cos(t+y(t)) )*diff(y(t),t)=0,y(t), singsol=
```

$$y(t)=-t+\arcsin\left(\frac{c_1}{3+t}\right)$$

✓ Solution by Mathematica
Time used: 6.0 (sec). Leaf size: 18

```
DSolve[( (3+t)*Cos[t+y[t]]+Sin[t+y[t]] )+( (3+t)*Cos[t+y[t]] )*y'[t]==0,y[t],t,IncludeSingul
```

$$y(t)\to -t+\arcsin\left(\frac{c_1}{t+3}\right)$$

6.28 problem 28

Internal problem ID [14316]

Book: INTRODUCTORY DIFFERENTIAL EQUATIONS. Martha L. Abell, James P. Braselton. Fourth edition 2014. ElScAe. 2014
Section: Chapter 2. First Order Equations. Exercises 2.4, page 57
Problem number: 28.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_exact, [_1st_order, '_with_symmetry_[F(x)*G(y),0]'], [_Abel,`

$$\boxed{\frac{2t^2 y \cos(t^2) - \sin(t^2) y}{t^2} + \frac{(2ty + \sin(t^2)) y'}{t} = 0}$$

✓ Solution by Maple
Time used: 0.031 (sec). Leaf size: 63

```
dsolve(1/t^2*(2*t^2*y(t)*cos(t^2)-y(t)*sin(t^2)  )+1/t*(2*t*y(t)+sin(t^2))*diff(y(t),t)=0,y(
```

$$y(t) = \frac{-\sin(t^2) + \sqrt{\sin(t^2)^2 - 4c_1 t^2}}{2t}$$
$$y(t) = \frac{-\sin(t^2) - \sqrt{\sin(t^2)^2 - 4c_1 t^2}}{2t}$$

✓ Solution by Mathematica
Time used: 0.559 (sec). Leaf size: 94

```
DSolve[1/t^2*(2*t^2*y[t]*Cos[t^2]-y[t]*Sin[t^2]  )+1/t*(2*t*y[t]+Sin[t^2])*y'[t]==0,y[t],t,I
```

$$y(t) \to -\frac{\sin(t^2) + \sqrt{\frac{1}{t^2}} t \sqrt{\sin^2(t^2) + 4c_1 t^2}}{2t}$$
$$y(t) \to \frac{-\sin(t^2) + \sqrt{\frac{1}{t^2}} t \sqrt{\sin^2(t^2) + 4c_1 t^2}}{2t}$$
$$y(t) \to 0$$

6.29 problem 29

Internal problem ID [14317]

Book: INTRODUCTORY DIFFERENTIAL EQUATIONS. Martha L. Abell, James P. Braselton. Fourth edition 2014. ElScAe. 2014
Section: Chapter 2. First Order Equations. Exercises 2.4, page 57
Problem number: 29.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[[_homogeneous, ‘class A‘], _exact, _dAlembert]`

$$\boxed{-\frac{y^2 \mathrm{e}^{\frac{y}{t}}}{t^2}+\mathrm{e}^{\frac{y}{t}}\left(1+\frac{y}{t}\right) y'=-1}$$

✓ Solution by Maple
Time used: 0.047 (sec). Leaf size: 20

```
dsolve((-t^(-2)*y(t)^2*exp(y(t)/t)+1 )+exp(y(t)/t)*(1+y(t)/t )*diff(y(t),t)=0,y(t), singsol=
```

$$y(t)=\operatorname{LambertW}\left(\frac{-c_1 t+1}{c_1 t}\right) t$$

✓ Solution by Mathematica
Time used: 34.75 (sec). Leaf size: 18

```
DSolve[(-t^(-2)*y[t]^2*Exp[y[t]/t]+1 )+Exp[y[t]/t]*(1+y[t]/t )*y'[t]==0,y[t],t,IncludeSingul
```

$$y(t)\to tW\left(-1+\frac{e^{c_1}}{t}\right)$$

6.30 problem 30

Internal problem ID [14318]

Book: INTRODUCTORY DIFFERENTIAL EQUATIONS. Martha L. Abell, James P. Braselton. Fourth edition 2014. ElScAe. 2014
Section: Chapter 2. First Order Equations. Exercises 2.4, page 57
Problem number: 30.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[[_homogeneous, 'class A'], _exact, _dAlembert]`

$$\boxed{2t\sin\left(\frac{y}{t}\right)-y\cos\left(\frac{y}{t}\right)+t\cos\left(\frac{y}{t}\right)y'=0}$$

✓ Solution by Maple
Time used: 0.031 (sec). Leaf size: 12

```
dsolve((2*t*sin(y(t)/t)-y(t)*cos(y(t)/t))+t*cos(y(t)/t)*diff(y(t),t)=0,y(t), singsol=all)
```

$$y(t)=\arcsin\left(\frac{c_1}{t^2}\right)t$$

✓ Solution by Mathematica
Time used: 14.41 (sec). Leaf size: 21

```
DSolve[(2*t*Sin[y[t]/t]-y[t]*Cos[y[t]/t])+t*Cos[y[t]/t]*y'[t]==0,y[t],t,IncludeSingularSolut
```

$$y(t)\to t\arcsin\left(\frac{e^{c_1}}{t^2}\right)$$
$$y(t)\to 0$$

6.31 problem 31

Internal problem ID [14319]

Book: INTRODUCTORY DIFFERENTIAL EQUATIONS. Martha L. Abell, James P. Braselton. Fourth edition 2014. ElScAe. 2014
Section: Chapter 2. First Order Equations. Exercises 2.4, page 57
Problem number: 31.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_separable]`

$$\boxed{2ty^2 + 2y'yt^2 = 0}$$

With initial conditions

$$[y(1) = 1]$$

✓ Solution by Maple
Time used: 0.016 (sec). Leaf size: 7

```
dsolve([(2*t*y(t)^2)+(2*t^2*y(t))*diff(y(t),t)=0,y(1) = 1],y(t), singsol=all)
```

$$y(t) = \frac{1}{t}$$

✓ Solution by Mathematica
Time used: 0.024 (sec). Leaf size: 8

```
DSolve[{(2*t*y[t]^2)+(2*t^2*y[t])*y'[t]==0,{y[1]==1}},y[t],t,IncludeSingularSolutions -> Tru
```

$$y(t) \to \frac{1}{t}$$

6.32 problem 32

Internal problem ID [14320]

Book: INTRODUCTORY DIFFERENTIAL EQUATIONS. Martha L. Abell, James P. Braselton. Fourth edition 2014. ElScAe. 2014
Section: Chapter 2. First Order Equations. Exercises 2.4, page 57
Problem number: 32.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_linear]`

$$\boxed{\frac{y}{t^2} - \frac{y'}{t} = -1}$$

With initial conditions

$$[y(2) = 1]$$

✓ Solution by Maple
Time used: 0.0 (sec). Leaf size: 9

```
dsolve([(1+y(t)/t^2)-1/t*diff(y(t),t)=0,y(2) = 1],y(t), singsol=all)
```

$$y(t) = t^2 - \frac{3}{2}t$$

✓ Solution by Mathematica
Time used: 0.027 (sec). Leaf size: 14

```
DSolve[{(1+y[t]/t^2)-1/t*y'[t]==0,{y[2]==1}},y[t],t,IncludeSingularSolutions -> True]
```

$$y(t) \to t^2 - \frac{3t}{2}$$

6.33 problem 33

Internal problem ID [14321]

Book: INTRODUCTORY DIFFERENTIAL EQUATIONS. Martha L. Abell, James P. Braselton. Fourth edition 2014. ElScAe. 2014
Section: Chapter 2. First Order Equations. Exercises 2.4, page 57
Problem number: 33.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_linear]`

$$\boxed{2ty + (t^2 - 1) y' = -3t^2}$$

With initial conditions

$$[y(0) = 1]$$

✓ Solution by Maple
Time used: 0.016 (sec). Leaf size: 18

```
dsolve([(2*t*y(t)+3*t^2)+(t^2-1)*diff(y(t),t)=0,y(0) = 1],y(t), singsol=all)
```

$$y(t) = \frac{-t^2 + t - 1}{t - 1}$$

✓ Solution by Mathematica
Time used: 0.072 (sec). Leaf size: 19

```
DSolve[{(2*t*y[t]+3*t^2)+(t^2-1)*y'[t]==0,{y[0]==1}},y[t],t,IncludeSingularSolutions -> True
```

$$y(t) \to \frac{-t^2 + t - 1}{t - 1}$$

6.34 problem 34

Internal problem ID [14322]

Book: INTRODUCTORY DIFFERENTIAL EQUATIONS. Martha L. Abell, James P. Braselton. Fourth edition 2014. ElScAe. 2014
Section: Chapter 2. First Order Equations. Exercises 2.4, page 57
Problem number: 34.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[[_homogeneous, 'class C'], _exact, _rational, [_Abel, '2nd ty`

$$\boxed{-y-(t+2y)\,y'=-5t-1}$$

With initial conditions

$$[y(0)=0]$$

✓ Solution by Maple
Time used: 1.266 (sec). Leaf size: 41

```
dsolve([(1+5*t-y(t))-(t+2*y(t))*diff(y(t),t)=0,y(0) = 0],y(t), singsol=all)
```

$$y(t)=-\frac{t}{2}-\frac{\sqrt{11t^2+4t}}{2}$$
$$y(t)=-\frac{t}{2}+\frac{\sqrt{11t^2+4t}}{2}$$

✓ Solution by Mathematica
Time used: 0.135 (sec). Leaf size: 49

```
DSolve[{(1+5*t-y[t])-(t+2*y[t])*y'[t]==0,{y[0]==0}},y[t],t,IncludeSingularSolutions -> True]
```

$$y(t)\to -\frac{t}{2}-\frac{1}{2}\sqrt{t(11t+4)}$$
$$y(t)\to \frac{1}{2}\left(\sqrt{t(11t+4)}-t\right)$$

6.35 problem 35

Internal problem ID [14323]

Book: INTRODUCTORY DIFFERENTIAL EQUATIONS. Martha L. Abell, James P. Braselton. Fourth edition 2014. ElScAe. 2014
Section: Chapter 2. First Order Equations. Exercises 2.4, page 57
Problem number: 35.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_exact]`

$$\boxed{e^{y} - 2ty + \left(t\, e^{y} - t^{2}\right) y' = 0}$$

With initial conditions

$$[y(0) = 0]$$

✗ Solution by Maple

```
dsolve([(exp(y(t))-2*t*y(t))+(t*exp(y(t))-t^2)*diff(y(t),t)=0,y(0) = 0],y(t), singsol=all)
```

No solution found

✗ Solution by Mathematica
Time used: 0.0 (sec). Leaf size: 0

```
DSolve[{(Exp[y[t]]-2*t*y[t])+(t*Exp[y[t]]-t^2)*y'[t]==0,{y[0]==0}},y[t],t,IncludeSingularSol
```

{}

6.36 problem 36

Internal problem ID [14324]

Book: INTRODUCTORY DIFFERENTIAL EQUATIONS. Martha L. Abell, James P. Braselton. Fourth edition 2014. ElScAe. 2014
Section: Chapter 2. First Order Equations. Exercises 2.4, page 57
Problem number: 36.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_exact]`

$$\boxed{2ty\,\mathrm{e}^{t^2}+2t\,\mathrm{e}^{-y}+\left(\mathrm{e}^{t^2}-t^2\mathrm{e}^{-y}+1\right)y'=0}$$

With initial conditions

$$[y(0)=0]$$

✓ Solution by Maple
Time used: 0.203 (sec). Leaf size: 18

```
dsolve([(2*t*y(t)*exp(t^2)+2*t*exp(-y(t)))+(exp(t^2)-t^2*exp(-y(t))+1)*diff(y(t),t)=0,y(0) =
```

$$y(t)=\mathrm{LambertW}\left(-\frac{t^2}{\mathrm{e}^{t^2}+1}\right)$$

✓ Solution by Mathematica
Time used: 60.369 (sec). Leaf size: 20

```
DSolve[{(2*t*y[t]*Exp[t^2]+2*t*Exp[-y[t]])+(Exp[t^2]-t^2*Exp[-y[t]]+1)*y'[t]==0,{y[0]==0}},y
```

$$y(t)\to W\left(-\frac{t^2}{e^{t^2}+1}\right)$$

6.37 problem 37

Internal problem ID [14325]

Book: INTRODUCTORY DIFFERENTIAL EQUATIONS. Martha L. Abell, James P. Braselton. Fourth edition 2014. ElScAe. 2014
Section: Chapter 2. First Order Equations. Exercises 2.4, page 57
Problem number: 37.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_exact, [_Abel, '2nd type', 'class B']]`

$$\boxed{y^2 + (1 + 2ty)\, y' = 2\sin(2t)}$$

With initial conditions

$$[y(0) = 1]$$

✓ Solution by Maple

Time used: 0.312 (sec). Leaf size: 25

```
dsolve([(y(t)^2-2*sin(2*t))+(1+2*t*y(t))*diff(y(t),t)=0,y(0) = 1],y(t), singsol=all)
```

$$y(t) = \frac{-1 + \sqrt{-4t\cos(2t) + 8t + 1}}{2t}$$

✓ Solution by Mathematica

Time used: 1.457 (sec). Leaf size: 30

```
DSolve[{(y[t]^2-2*Sin[2*t])+(1+2*t*y[t])*y'[t]==0,{y[0]==1}},y[t],t,IncludeSingularSolutions
```

$$y(t) \to \frac{\sqrt{8t - 4t\cos(2t) + 1} - 1}{2t}$$

6.38 problem 38

Internal problem ID [14326]

Book: INTRODUCTORY DIFFERENTIAL EQUATIONS. Martha L. Abell, James P. Braselton. Fourth edition 2014. ElScAe. 2014
Section: Chapter 2. First Order Equations. Exercises 2.4, page 57
Problem number: 38.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_exact]`

$$\boxed{y + (\sec(y)\tan(y) + t)\, y' = -\cos(t)^2 + \sin(t)^2}$$

With initial conditions

$$[y(0) = 0]$$

✓ Solution by Maple
Time used: 0.454 (sec). Leaf size: 19

```
dsolve([(cos(t)^2-sin(t)^2+y(t))+(sec(y(t))*tan(y(t))+t)*diff(y(t),t)=0,y(0) = 0],y(t), sing
```

$$y(t) = \mathrm{RootOf}\left(2t\_Z + \sin(2t) + 2\sec(\_Z) - 2\right)$$

✓ Solution by Mathematica
Time used: 0.664 (sec). Leaf size: 22

```
DSolve[{(Cos[t]^2-Sin[t]^2+y[t])+(Sec[y[t]]*Tan[y[t]]+t)*y'[t]==0,{y[0]==0}},y[t],t,IncludeS
```

$$\mathrm{Solve}[4ty(t) + 4\sec(y(t)) + 2\sin(2t) = 4, y(t)]$$

6.39 problem 39

Internal problem ID [14327]

Book: INTRODUCTORY DIFFERENTIAL EQUATIONS. Martha L. Abell, James P. Braselton. Fourth edition 2014. ElScAe. 2014
Section: Chapter 2. First Order Equations. Exercises 2.4, page 57
Problem number: 39.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_exact, _rational, _Bernoulli]`

$$\boxed{-y^2 - 2tyy' = -\frac{1}{t^2+1}}$$

With initial conditions

$$[y(0) = 0]$$

✗ Solution by Maple

```
dsolve([(1/(1+t^2)-y(t)^2)-(2*t*y(t))*diff(y(t),t)=0,y(0) = 0],y(t), singsol=all)
```

No solution found

✗ Solution by Mathematica
Time used: 0.0 (sec). Leaf size: 0

```
DSolve[{(1/(1+t^2)-y[t]^2)-(2*t*y[t])*y'[t]==0,{y[0]==0}},y[t],t,IncludeSingularSolutions ->
```

Not solved

6.40 problem 40

Internal problem ID [14328]

Book: INTRODUCTORY DIFFERENTIAL EQUATIONS. Martha L. Abell, James P. Braselton. Fourth edition 2014. ElScAe. 2014
Section: Chapter 2. First Order Equations. Exercises 2.4, page 57
Problem number: 40.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_exact]`

$$\boxed{y + (e^{y} + t)\, y' = -\frac{2t}{t^{2}+1}}$$

With initial conditions

$$[y(0) = 0]$$

✗ Solution by Maple

```
dsolve([(2*t/(1+t^2)+y(t))+(exp(y(t))+t)*diff(y(t),t)=0,y(0) = 0],y(t), singsol=all)
```

No solution found

✗ Solution by Mathematica
Time used: 0.0 (sec). Leaf size: 0

```
DSolve[{(2*t/(1+t^2)+y[t])+(Exp[y[t]]+t)*y'[t]==0,{y[0]==0}},y[t],t,IncludeSingularSolutions
```

{}

6.41 problem 41

Internal problem ID [14329]

Book: INTRODUCTORY DIFFERENTIAL EQUATIONS. Martha L. Abell, James P. Braselton. Fourth edition 2014. ElScAe. 2014
Section: Chapter 2. First Order Equations. Exercises 2.4, page 57
Problem number: 41.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_exact]`

$$\boxed{-y\cos(yx)+(2y-x\cos(yx))\,y'=2x}$$

With initial conditions

$$[y(0)=0]$$

✓ Solution by Maple
Time used: 0.313 (sec). Leaf size: 25

```
dsolve([(-2*x-y(x)*cos(x*y(x)))+(2*y(x)-x*cos(x*y(x)))*diff(y(x),x)=0,y(0) = 0],y(x), singso
```

$$y(x)=\frac{\operatorname{RootOf}\left(-x^4-\sin(\_Z)\,x^2+\_Z^2\right)}{x}$$

✓ Solution by Mathematica
Time used: 0.234 (sec). Leaf size: 20

```
DSolve[{(-2*x-y[x]*Cos[x*y[x]])+(2*y[x]-x*Cos[x*y[x]])*y'[x]==0,{y[0]==0}},y[x],x,IncludeSin
```

$$\operatorname{Solve}\left[x^2-y(x)^2+\sin(xy(x))=0,y(x)\right]$$

6.42 problem 42

Internal problem ID [14330]

Book: INTRODUCTORY DIFFERENTIAL EQUATIONS. Martha L. Abell, James P. Braselton. Fourth edition 2014. ElScAe. 2014
Section: Chapter 2. First Order Equations. Exercises 2.4, page 57
Problem number: 42.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_exact]`

$$\boxed{6y\sin(6yx) + \left(4y^3 + 6x\sin(6yx)\right)y' = 4x^3}$$

With initial conditions

$$[y(0) = 0]$$

✓ Solution by Maple
Time used: 0.297 (sec). Leaf size: 31

```
dsolve([(-4*x^3+6*y(x)*sin(6*x*y(x)))+(4*y(x)^3+6*x*sin(6*x*y(x)))*diff(y(x),x)=0,y(0) = 0],
```

$$y(x) = \frac{\text{RootOf}\left(-1296x^8 - 1296\cos(\_Z)\,x^4 + 1296x^4 + \_Z^4\right)}{6x}$$

✓ Solution by Mathematica
Time used: 0.379 (sec). Leaf size: 33

```
DSolve[{(-4*x^3+6*y[x]*Sin[6*x*y[x]])+(4*y[x]^3+6*x*Sin[6*x*y[x]])*y'[x]==0,{y[0]==0}},y[x],
```

$$\text{Solve}\left[-\frac{x^4}{2} + \frac{y(x)^4}{2} - \frac{1}{2}\cos(6xy(x)) = -\frac{1}{2}, y(x)\right]$$

6.43 problem 48

Internal problem ID [14331]

Book: INTRODUCTORY DIFFERENTIAL EQUATIONS. Martha L. Abell, James P. Braselton. Fourth edition 2014. ElScAe. 2014
Section: Chapter 2. First Order Equations. Exercises 2.4, page 57
Problem number: 48.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_separable]`

$$\boxed{t^2y + y't^3 = 0}$$

✓ Solution by Maple
Time used: 0.0 (sec). Leaf size: 9

```
dsolve((t^2*y(t))+(t^3)*diff(y(t),t)=0,y(t), singsol=all)
```

$$y(t) = \frac{c_1}{t}$$

✓ Solution by Mathematica
Time used: 0.035 (sec). Leaf size: 16

```
DSolve[(t^2*y[t])+(t^3)*y'[t]==0,y[t],t,IncludeSingularSolutions -> True]
```

$$y(t) \to \frac{c_1}{t}$$
$$y(t) \to 0$$

6.44 problem 49

Internal problem ID [14332]

Book: INTRODUCTORY DIFFERENTIAL EQUATIONS. Martha L. Abell, James P. Braselton. Fourth edition 2014. ElScAe. 2014
Section: Chapter 2. First Order Equations. Exercises 2.4, page 57
Problem number: 49.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_separable]`

$$\boxed{y(2\,e^t + 4t) + 3(e^t + t^2)\,y' = 0}$$

✓ Solution by Maple
Time used: 0.0 (sec). Leaf size: 14

```
dsolve(y(t)*(2*exp(t)+4*t)+3*(exp(t)+t^2)*diff(y(t),t)=0,y(t), singsol=all)
```

$$y(t) = \frac{c_1}{(e^t + t^2)^{\frac{2}{3}}}$$

✓ Solution by Mathematica
Time used: 0.084 (sec). Leaf size: 24

```
DSolve[y[t]*(2*Exp[t]+4*t)+3*(Exp[t]+t^2)*y'[t]==0,y[t],t,IncludeSingularSolutions -> True]
```

$$y(t) \to \frac{c_1}{(t^2 + e^t)^{2/3}}$$
$$y(t) \to 0$$

6.45 problem 50

Internal problem ID [14333]

Book: INTRODUCTORY DIFFERENTIAL EQUATIONS. Martha L. Abell, James P. Braselton. Fourth edition 2014. ElScAe. 2014
Section: Chapter 2. First Order Equations. Exercises 2.4, page 57
Problem number: 50.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[[_1st_order, '_with_symmetry_[F(x)*G(y),0]']]`

$$\boxed{y + (2t - e^{y} y)\, y' = 0}$$

✓ Solution by Maple
Time used: 0.125 (sec). Leaf size: 34

```
dsolve(y(t)+(2*t-y(t)*exp(y(t)))*diff(y(t),t)=0,y(t), singsol=all)
```

$$\frac{\left(-y(t)^2 + 2y(t) - 2\right) e^{y(t)} + t y(t)^2 - c_1}{y(t)^2} = 0$$

✓ Solution by Mathematica
Time used: 0.218 (sec). Leaf size: 32

```
DSolve[y[t]+(2*t-y[t]*Exp[y[t]])*y'[t]==0,y[t],t,IncludeSingularSolutions -> True]
```

$$\text{Solve}\left[t = \frac{e^{y(t)}(y(t)^2 - 2y(t) + 2)}{y(t)^2} + \frac{c_1}{y(t)^2}, y(t)\right]$$

6.46 problem 51

Internal problem ID [14334]

Book: INTRODUCTORY DIFFERENTIAL EQUATIONS. Martha L. Abell, James P. Braselton. Fourth edition 2014. ElScAe. 2014
Section: Chapter 2. First Order Equations. Exercises 2.4, page 57
Problem number: 51.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[[_homogeneous, ‘class A‘], _rational, _Bernoulli]`

$$\boxed{2ty + y^2 - t^2 y' = 0}$$

✓ Solution by Maple
Time used: 0.015 (sec). Leaf size: 15

```
dsolve((2*t*y(t)+y(t)^2)-(t^2)*diff(y(t),t)=0,y(t), singsol=all)
```

$$y(t) = \frac{t^2}{-t + c_1}$$

✓ Solution by Mathematica
Time used: 0.127 (sec). Leaf size: 23

```
DSolve[(2*t*y[t]+y[t]^2)-(t^2)*y'[t]==0,y[t],t,IncludeSingularSolutions -> True]
```

$$y(t) \to -\frac{t^2}{t - c_1}$$
$$y(t) \to 0$$

6.47 problem 52

Internal problem ID [14335]

Book: INTRODUCTORY DIFFERENTIAL EQUATIONS. Martha L. Abell, James P. Braselton. Fourth edition 2014. ElScAe. 2014
Section: Chapter 2. First Order Equations. Exercises 2.4, page 57
Problem number: 52.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_rational, [_1st_order, '_with_symmetry_[F(x),G(x)]'], [_Abel`

$$\boxed{y + (t^2 y - t) y' = -2t^2}$$

✓ Solution by Maple
Time used: 0.0 (sec). Leaf size: 51

```
dsolve((y(t)+2*t^2)+(t^2*y(t)-t)*diff(y(t),t)=0,y(t), singsol=all)
```

$$y(t) = \frac{1 + \sqrt{-2c_1 t^2 - 4t^3 + 1}}{t}$$
$$y(t) = \frac{1 - \sqrt{-2c_1 t^2 - 4t^3 + 1}}{t}$$

✓ Solution by Mathematica
Time used: 0.47 (sec). Leaf size: 68

```
DSolve[(y[t]+2*t^2)+(t^2*y[t]-t)*y'[t]==0,y[t],t,IncludeSingularSolutions -> True]
```

$$y(t) \to \frac{1}{t} - \sqrt{\frac{1}{t^2}}\sqrt{-4t^3 + c_1 t^2 + 1}$$
$$y(t) \to \frac{1}{t} + \sqrt{\frac{1}{t^2}}\sqrt{-4t^3 + c_1 t^2 + 1}$$

6.48 problem 53

Internal problem ID [14336]

Book: INTRODUCTORY DIFFERENTIAL EQUATIONS. Martha L. Abell, James P. Braselton. Fourth edition 2014. ElScAe. 2014
Section: Chapter 2. First Order Equations. Exercises 2.4, page 57
Problem number: 53.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_rational, [_Abel, ‘2nd type‘, ‘class B‘]]`

$$\boxed{5ty + 4y^2 + \left(t^2 + 2ty\right) y' = -1}$$

✓ Solution by Maple
Time used: 0.0 (sec). Leaf size: 59

```
dsolve((5*t*y(t)+4*y(t)^2+1)+(t^2+2*t*y(t))*diff(y(t),t)=0,y(t), singsol=all)
```

$$y(t) = \frac{-t^3 - \sqrt{t^6 - t^4 - 4c_1}}{2t^2}$$
$$y(t) = \frac{-t^3 + \sqrt{t^6 - t^4 - 4c_1}}{2t^2}$$

✓ Solution by Mathematica
Time used: 0.611 (sec). Leaf size: 84

```
DSolve[(5*t*y[t]+4*y[t]^2+1)+(t^2+2*t*y[t])*y'[t]==0,y[t],t,IncludeSingularSolutions -> True
```

$$y(t) \to -\frac{t^5 + \sqrt{t^3}\sqrt{t^7 - t^5 + 4c_1 t}}{2t^4}$$
$$y(t) \to -\frac{t}{2} + \frac{\sqrt{t^3}\sqrt{t^7 - t^5 + 4c_1 t}}{2t^4}$$

6.49 problem 54

Internal problem ID [14337]

Book: INTRODUCTORY DIFFERENTIAL EQUATIONS. Martha L. Abell, James P. Braselton. Fourth edition 2014. ElScAe. 2014
Section: Chapter 2. First Order Equations. Exercises 2.4, page 57
Problem number: 54.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[[_homogeneous, ‘class D‘], _rational, _Bernoulli]`

$$\boxed{5ty^2 + y + (2t^3 - t)\, y' = 0}$$

✓ Solution by Maple
Time used: 0.015 (sec). Leaf size: 23

```
dsolve((5*t*y(t)^2+y(t))+(2*t^3-t)*diff(y(t),t)=0,y(t), singsol=all)
```

$$y(t) = \frac{2t}{-5 + 2\sqrt{2t^2 - 1}\, c_1}$$

✓ Solution by Mathematica
Time used: 0.196 (sec). Leaf size: 32

```
DSolve[(5*t*y[t]^2+y[t])+(2*t^3-t)*y'[t]==0,y[t],t,IncludeSingularSolutions -> True]
```

$$y(t) \to \frac{2t}{-5 + 2c_1\sqrt{1 - 2t^2}}$$
$$y(t) \to 0$$

6.50 problem 55

Internal problem ID [14338]

Book: INTRODUCTORY DIFFERENTIAL EQUATIONS. Martha L. Abell, James P. Braselton. Fourth edition 2014. ElScAe. 2014
Section: Chapter 2. First Order Equations. Exercises 2.4, page 57
Problem number: 55.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[[_1st_order, '_with_symmetry_[F(x),G(x)]']]`

$$\boxed{\tan(y) + \left(t - t^2 \tan(y)\right) y' = -2t}$$

✓ Solution by Maple
Time used: 0.047 (sec). Leaf size: 134

```
dsolve((2*t+tan(y(t)))+(t-t^2*tan(y(t)))*diff(y(t),t)=0,y(t), singsol=all)
```

$$y(t) = \arctan\left(\frac{-\sqrt{t^4 - c_1^2 + t^2}\, t - c_1}{t\,(t^2+1)}, \frac{-c_1 t + \sqrt{t^4 - c_1^2 + t^2}}{(t^2+1)\, t}\right)$$
$$y(t) = \arctan\left(\frac{\sqrt{t^4 - c_1^2 + t^2}\, t - c_1}{t\,(t^2+1)}, \frac{-c_1 t - \sqrt{t^4 - c_1^2 + t^2}}{t\,(t^2+1)}\right)$$

✓ Solution by Mathematica

Time used: 47.448 (sec). Leaf size: 177

```
DSolve[(2*t+Tan[y[t]])+(t-t^2*Tan[y[t]])*y'[t]==0,y[t],t,IncludeSingularSolutions -> True]
```

$$y(t) \to -\arccos\left(-\frac{c_1t^2+\sqrt{t^6+t^4-{c_1}^2t^2}}{t^4+t^2}\right)$$
$$y(t) \to \arccos\left(-\frac{c_1t^2+\sqrt{t^6+t^4-{c_1}^2t^2}}{t^4+t^2}\right)$$
$$y(t) \to -\arccos\left(\frac{\sqrt{t^6+t^4-{c_1}^2t^2}-c_1t^2}{t^4+t^2}\right)$$
$$y(t) \to \arccos\left(\frac{\sqrt{t^6+t^4-{c_1}^2t^2}-c_1t^2}{t^4+t^2}\right)$$

6.51 problem 57

Internal problem ID [14339]

Book: INTRODUCTORY DIFFERENTIAL EQUATIONS. Martha L. Abell, James P. Braselton. Fourth edition 2014. ElScAe. 2014
Section: Chapter 2. First Order Equations. Exercises 2.4, page 57
Problem number: 57.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_exact]`

$$\boxed{-y^2 \sin(ty) + (\cos(ty) - ty\sin(ty))\, y' = -2t}$$

✓ Solution by Maple
Time used: 0.047 (sec). Leaf size: 20

```
dsolve((2*t-y(t)^2*sin(t*y(t)))+(cos(t*y(t))-t*y(t)*sin(t*y(t)))*diff(y(t),t)=0,y(t), singso
```

$$y(t) = \frac{\operatorname{RootOf}(t^3 + \_Z\cos(\_Z) + c_1 t)}{t}$$

✓ Solution by Mathematica
Time used: 0.013 (sec). Leaf size: 43

```
DSolve[(2*t-y[t]^2*Sin[t*y[t]])+(Cos[t*y[t]]-t*y[t]*Sin[t*y[t]])*y'[x]==0,y[t],t,IncludeSing
```

$$\text{Solve}\left[\left\{y'(x)(\cos(ty(t)) - ty(t)\sin(ty(t))) + y(t)^2(-\sin(ty(t))) + 2t = 0\right\}, \{y(t)\}\right]$$

6.52 problem 58

Internal problem ID [14340]

Book: INTRODUCTORY DIFFERENTIAL EQUATIONS. Martha L. Abell, James P. Braselton. Fourth edition 2014. ElScAe. 2014
Section: Chapter 2. First Order Equations. Exercises 2.4, page 57
Problem number: 58.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_exact]`

$$\boxed{e^{ty}y + y\cos(ty) + \left(1 + e^{ty}t + \cos(ty)\, t\right) y' = 1}$$

✓ Solution by Maple

Time used: 0.047 (sec). Leaf size: 27

```
dsolve((-1+exp(t*y(t))*y(t)+y(t)*cos(t*y(t)))+(1+exp(t*y(t))*t+t*cos(t*y(t)))*diff(y(t),t)=0
```

$$y(t) = \frac{\operatorname{RootOf}\left(t\, e^{\_Z} + t\sin(\_Z) + c_1 t - t^2 + \_Z\right)}{t}$$

✓ Solution by Mathematica

Time used: 0.475 (sec). Leaf size: 23

```
DSolve[(-1+Exp[t*y[t]]*y[t]+y[t]*Cos[t*y[t]])+(1+Exp[t*y[t]]*t+t*Cos[t*y[t]])*y'[t]==0,y[t],
```

$$\text{Solve}\left[e^{ty(t)} + y(t) + \sin(ty(t)) - t = c_1, y(t)\right]$$

6.53 problem 59 (i)

Internal problem ID [14341]

Book: INTRODUCTORY DIFFERENTIAL EQUATIONS. Martha L. Abell, James P. Braselton. Fourth edition 2014. ElScAe. 2014
Section: Chapter 2. First Order Equations. Exercises 2.4, page 57
Problem number: 59 (i).
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_quadrature]`

$$\boxed{2y + (2t + 2y)\, y' = -2t}$$

✓ Solution by Maple
Time used: 0.0 (sec). Leaf size: 25

```
dsolve((2*t+2*y(t))+(2*t+2*y(t))*diff(y(t),t)=0,y(t), singsol=all)
```

$$y(t) = -t$$
$$y(t) = -c_1 - t$$
$$y(t) = -t + c_1$$

✓ Solution by Mathematica
Time used: 0.002 (sec). Leaf size: 18

```
DSolve[(2*t+2*y[t])+(2*t+2*y[t])*y'[t]==0,y[t],t,IncludeSingularSolutions -> True]
```

$$y(t) \to -t$$
$$y(t) \to -t + c_1$$

6.54 problem 59 (ii)

Internal problem ID [14342]

Book: INTRODUCTORY DIFFERENTIAL EQUATIONS. Martha L. Abell, James P. Braselton. Fourth edition 2014. ElScAe. 2014
Section: Chapter 2. First Order Equations. Exercises 2.4, page 57
Problem number: 59 (ii).
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[[_homogeneous, ‘class A‘], _exact, _rational, [_Abel, ‘2nd ty`

$$\boxed{2y + (2t + 2y)\, y' = -\frac{9t}{5}}$$

✓ Solution by Maple
Time used: 0.078 (sec). Leaf size: 53

```
dsolve((18/10*t+2*y(t))+(2*t+2*y(t))*diff(y(t),t)=0,y(t), singsol=all)
```

$$y(t) = \frac{-10c_1 t - \sqrt{10c_1^2 t^2 + 10}}{10c_1}$$
$$y(t) = \frac{-10c_1 t + \sqrt{10c_1^2 t^2 + 10}}{10c_1}$$

✓ Solution by Mathematica
Time used: 0.549 (sec). Leaf size: 101

```
DSolve[(18/10*t+2*y[t])+(2*t+2*y[t])*y'[t]==0,y[t],t,IncludeSingularSolutions -> True]
```

$$y(t) \to -t - \frac{\sqrt{t^2 + e^{20c_1}}}{\sqrt{10}}$$
$$y(t) \to -t + \frac{\sqrt{t^2 + e^{20c_1}}}{\sqrt{10}}$$
$$y(t) \to -\frac{\sqrt{t^2}}{\sqrt{10}} - t$$
$$y(t) \to \frac{\sqrt{t^2}}{\sqrt{10}} - t$$

6.55 problem 59 (iii)

Internal problem ID [14343]

Book: INTRODUCTORY DIFFERENTIAL EQUATIONS. Martha L. Abell, James P. Braselton. Fourth edition 2014. ElScAe. 2014
Section: Chapter 2. First Order Equations. Exercises 2.4, page 57
Problem number: 59 (iii).
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[[_homogeneous, ‘class A‘], _exact, _rational, [_Abel, ‘2nd ty`

$$\boxed{\frac{19y}{10}+\left(\frac{19t}{10}+2y\right)y'=-2t}$$

✓ Solution by Maple
Time used: 0.094 (sec). Leaf size: 53

```
dsolve((2*t+19/10*y(t))+(19/10*t+2*y(t))*diff(y(t),t)=0,y(t), singsol=all)
```

$$y(t)=\frac{-19c_1t-\sqrt{-39c_1^2t^2+40}}{20c_1}$$
$$y(t)=\frac{-19c_1t+\sqrt{-39c_1^2t^2+40}}{20c_1}$$

✓ Solution by Mathematica
Time used: 0.461 (sec). Leaf size: 114

```
DSolve[(2*t+19/10*y[t])+(19/10*t+2*y[t])*y'[t]==0,y[t],t,IncludeSingularSolutions -> True]
```

$$y(t)\to\frac{1}{20}\left(-19t-\sqrt{-39t^2+40e^{c_1}}\right)$$
$$y(t)\to\frac{1}{20}\left(-19t+\sqrt{-39t^2+40e^{c_1}}\right)$$
$$y(t)\to\frac{1}{20}\left(-\sqrt{39}\sqrt{-t^2}-19t\right)$$
$$y(t)\to\frac{1}{20}\left(\sqrt{39}\sqrt{-t^2}-19t\right)$$

7 Chapter 2. First Order Equations. Exercises 2.5, page 64

7.1 problem 1

Internal problem ID [14344]

Book: INTRODUCTORY DIFFERENTIAL EQUATIONS. Martha L. Abell, James P. Braselton. Fourth edition 2014. ElScAe. 2014
Section: Chapter 2. First Order Equations. Exercises 2.5, page 64
Problem number: 1.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_rational, _Bernoulli]`

$$\boxed{y' - \frac{y}{2} - \frac{t}{y} = 0}$$

✓ Solution by Maple
Time used: 0.016 (sec). Leaf size: 31

```
dsolve(diff(y(t),t)-1/2*y(t)=t/y(t),y(t), singsol=all)
```

$$y(t) = \sqrt{e^t c_1 - 2t - 2}$$
$$y(t) = -\sqrt{e^t c_1 - 2t - 2}$$

✓ Solution by Mathematica
Time used: 3.39 (sec). Leaf size: 41

```
DSolve[y'[t]-1/2*y[t]==t/y[t],y[t],t,IncludeSingularSolutions -> True]
```

$$y(t) \to -\sqrt{-2t + c_1 e^t - 2}$$
$$y(t) \to \sqrt{-2t + c_1 e^t - 2}$$

7.2 problem 2

Internal problem ID [14345]

Book: INTRODUCTORY DIFFERENTIAL EQUATIONS. Martha L. Abell, James P. Braselton. Fourth edition 2014. ElScAe. 2014
Section: Chapter 2. First Order Equations. Exercises 2.5, page 64
Problem number: 2.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_Bernoulli]`

$$\boxed{y' + y - ty^2 = 0}$$

✓ Solution by Maple
Time used: 0.0 (sec). Leaf size: 13

```
dsolve(diff(y(t),t)+y(t)=t*y(t)^2,y(t), singsol=all)
```

$$y(t) = \frac{1}{1 + e^t c_1 + t}$$

✓ Solution by Mathematica
Time used: 0.156 (sec). Leaf size: 21

```
DSolve[y'[t]+y[t]==t*y[t]^2,y[t],t,IncludeSingularSolutions -> True]
```

$$y(t) \to \frac{1}{t + c_1 e^t + 1}$$
$$y(t) \to 0$$

7.3 problem 3

Internal problem ID [14346]

Book: INTRODUCTORY DIFFERENTIAL EQUATIONS. Martha L. Abell, James P. Braselton. Fourth edition 2014. ElScAe. 2014
Section: Chapter 2. First Order Equations. Exercises 2.5, page 64
Problem number: 3.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_Bernoulli]`

$$\boxed{2y't - y - 2ty^3 \cos(t) = 0}$$

✓ Solution by Maple
Time used: 0.109 (sec). Leaf size: 70

```
dsolve(2*t*diff(y(t),t)-y(t)=2*t*y(t)^3*cos(t),y(t), singsol=all)
```

$$y(t) = \frac{\sqrt{-2\left(\sin(t)\,t - \frac{c_1}{2} + \cos(t)\right)t}}{2\sin(t)\,t + 2\cos(t) - c_1}$$
$$y(t) = -\frac{\sqrt{-2\left(\sin(t)\,t - \frac{c_1}{2} + \cos(t)\right)t}}{2\sin(t)\,t + 2\cos(t) - c_1}$$

✓ Solution by Mathematica
Time used: 0.276 (sec). Leaf size: 59

```
DSolve[2*t*y'[t]-y[t]==2*t*y[t]^3*Cos[t],y[t],t,IncludeSingularSolutions -> True]
```

$$y(t) \to -\frac{\sqrt{t}}{\sqrt{-2t\sin(t) - 2\cos(t) + c_1}}$$
$$y(t) \to \frac{\sqrt{t}}{\sqrt{-2t\sin(t) - 2\cos(t) + c_1}}$$
$$y(t) \to 0$$

7.4 problem 4

Internal problem ID [14347]

Book: INTRODUCTORY DIFFERENTIAL EQUATIONS. Martha L. Abell, James P. Braselton. Fourth edition 2014. ElScAe. 2014
Section: Chapter 2. First Order Equations. Exercises 2.5, page 64
Problem number: 4.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[[_homogeneous, ‘class D‘], _Bernoulli]`

$$\boxed{y't - y - y^3 \sin(t)\, t = 0}$$

✓ Solution by Maple
Time used: 0.094 (sec). Leaf size: 52

```
dsolve(t*diff(y(t),t)-y(t)=t*y(t)^3*sin(t),y(t), singsol=all)
```

$$y(t) = \frac{t}{\sqrt{2t^2 \cos(t) - 4\cos(t) - 4\sin(t)\, t + c_1}}$$
$$y(t) = -\frac{t}{\sqrt{2t^2 \cos(t) - 4\cos(t) - 4\sin(t)\, t + c_1}}$$

✓ Solution by Mathematica
Time used: 0.335 (sec). Leaf size: 61

```
DSolve[t*y'[t]-y[t]==t*y[t]^3*Sin[t],y[t],t,IncludeSingularSolutions -> True]
```

$$y(t) \to -\frac{t}{\sqrt{2(t^2-2)\cos(t) - 4t\sin(t) + c_1}}$$
$$y(t) \to \frac{t}{\sqrt{2(t^2-2)\cos(t) - 4t\sin(t) + c_1}}$$
$$y(t) \to 0$$

7.5 problem 5

Internal problem ID [14348]

Book: INTRODUCTORY DIFFERENTIAL EQUATIONS. Martha L. Abell, James P. Braselton. Fourth edition 2014. ElScAe. 2014
Section: Chapter 2. First Order Equations. Exercises 2.5, page 64
Problem number: 5.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_Bernoulli]`

$$\boxed{y' - 2y - \frac{\cos(t)}{\sqrt{y}} = 0}$$

✓ Solution by Maple
Time used: 0.0 (sec). Leaf size: 23

```
dsolve(diff(y(t),t)-2*y(t)=y(t)^(-1/2)*cos(t),y(t), singsol=all)
```

$$y(t)^{\frac{3}{2}} + \frac{9\cos(t)}{20} - \frac{3\sin(t)}{20} - c_1 e^{3t} = 0$$

✓ Solution by Mathematica
Time used: 3.757 (sec). Leaf size: 40

```
DSolve[y'[t]-2*y[t]==y[t]^(-1/2)*Cos[t],y[t],t,IncludeSingularSolutions -> True]
```

$$y(t) \to \frac{\left(\frac{3\sin(t)}{5} - \frac{9\cos(t)}{5} + 4c_1 e^{3t}\right)^{2/3}}{2\sqrt[3]{2}}$$

7.6 problem 6

Internal problem ID [14349]

Book: INTRODUCTORY DIFFERENTIAL EQUATIONS. Martha L. Abell, James P. Braselton. Fourth edition 2014. ElScAe. 2014
Section: Chapter 2. First Order Equations. Exercises 2.5, page 64
Problem number: 6.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_Bernoulli]`

$$\boxed{y' + 3y - \sqrt{y}\,\sin(t) = 0}$$

✓ Solution by Maple
Time used: 0.0 (sec). Leaf size: 23

```
dsolve(diff(y(t),t)+3*y(t)=sqrt(y(t))*sin(t),y(t), singsol=all)
```

$$\sqrt{y(t)} + \frac{2\cos(t)}{13} - \frac{3\sin(t)}{13} - \mathrm{e}^{-\frac{3t}{2}} c_1 = 0$$

✓ Solution by Mathematica
Time used: 0.193 (sec). Leaf size: 43

```
DSolve[y'[t]+3*y[t]==Sqrt[y[t]]*Sin[t],y[t],t,IncludeSingularSolutions -> True]
```

$$y(t) \to \frac{1}{169} e^{-3t} \left(3e^{3t/2}\sin(t) - 2e^{3t/2}\cos(t) + 13c_1\right)^2$$

7.7 problem 7

Internal problem ID [14350]

Book: INTRODUCTORY DIFFERENTIAL EQUATIONS. Martha L. Abell, James P. Braselton. Fourth edition 2014. ElScAe. 2014
Section: Chapter 2. First Order Equations. Exercises 2.5, page 64
Problem number: 7.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[[_homogeneous, ‘class D‘], _rational, _Bernoulli]`

$$\boxed{y' - \frac{y}{t} - t y^2 = 0}$$

✓ Solution by Maple
Time used: 0.0 (sec). Leaf size: 16

```
dsolve(diff(y(t),t)-1/t*y(t)=t*y(t)^2,y(t), singsol=all)
```

$$y(t) = -\frac{3t}{t^3 - 3c_1}$$

✓ Solution by Mathematica
Time used: 0.131 (sec). Leaf size: 23

```
DSolve[y'[t]-1/t*y[t]==t*y[t]^2,y[t],t,IncludeSingularSolutions -> True]
```

$$y(t) \to -\frac{3t}{t^3 - 3c_1}$$
$$y(t) \to 0$$

7.8 problem 8

Internal problem ID [14351]

Book: INTRODUCTORY DIFFERENTIAL EQUATIONS. Martha L. Abell, James P. Braselton. Fourth edition 2014. ElScAe. 2014
Section: Chapter 2. First Order Equations. Exercises 2.5, page 64
Problem number: 8.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[[_homogeneous, ‘class A‘], _rational, _Bernoulli]`

$$\boxed{y' - \frac{y}{t} - \frac{y^2}{t^2} = 0}$$

✓ Solution by Maple
Time used: 0.0 (sec). Leaf size: 14

```
dsolve(diff(y(t),t)-1/t*y(t)=y(t)^2/t^2,y(t), singsol=all)
```

$$y(t) = \frac{t}{-\ln(t) + c_1}$$

✓ Solution by Mathematica
Time used: 0.191 (sec). Leaf size: 21

```
DSolve[y'[t]-1/t*y[t]==y[t]^2/t^2,y[t],t,IncludeSingularSolutions -> True]
```

$$y(t) \to \frac{t}{-\log(t) + c_1}$$
$$y(t) \to 0$$

7.9 problem 9

Internal problem ID [14352]

Book: INTRODUCTORY DIFFERENTIAL EQUATIONS. Martha L. Abell, James P. Braselton. Fourth edition 2014. ElScAe. 2014
Section: Chapter 2. First Order Equations. Exercises 2.5, page 64
Problem number: 9.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_separable]`

$$\boxed{y' - \frac{y}{t} - \frac{y^2}{t} = 0}$$

✓ Solution by Maple
Time used: 0.015 (sec). Leaf size: 13

```
dsolve(diff(y(t),t)-1/t*y(t)=y(t)^2/t,y(t), singsol=all)
```

$$y(t) = \frac{t}{-t + c_1}$$

✓ Solution by Mathematica
Time used: 0.383 (sec). Leaf size: 32

```
DSolve[y'[t]-1/t*y[t]==y[t]^2/t,y[t],t,IncludeSingularSolutions -> True]
```

$$y(t) \to \frac{e^{c_1}t}{1 - e^{c_1}t}$$
$$y(t) \to -1$$
$$y(t) \to 0$$

7.10 problem 10

Internal problem ID [14353]

Book: INTRODUCTORY DIFFERENTIAL EQUATIONS. Martha L. Abell, James P. Braselton. Fourth edition 2014. ElScAe. 2014
Section: Chapter 2. First Order Equations. Exercises 2.5, page 64
Problem number: 10.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[[_homogeneous, 'class G'], _rational, _Bernoulli]`

$$\boxed{y' - \frac{y}{t} - t^2 y^{\frac{3}{2}} = 0}$$

✓ Solution by Maple
Time used: 0.0 (sec). Leaf size: 19

```
dsolve(diff(y(t),t)-1/t*y(t)=t^2*y(t)^(3/2),y(t), singsol=all)
```

$$\frac{1}{\sqrt{y(t)}} + \frac{t^3}{7} - \frac{c_1}{\sqrt{t}} = 0$$

✓ Solution by Mathematica
Time used: 0.245 (sec). Leaf size: 25

```
DSolve[y'[t]-1/t*y[t]==t^2*y[t]^(3/2),y[t],t,IncludeSingularSolutions -> True]
```

$$y(t) \to \frac{49t}{(t^{7/2} - 7c_1)^2}$$
$$y(t) \to 0$$

7.11 problem 11

Internal problem ID [14354]

Book: INTRODUCTORY DIFFERENTIAL EQUATIONS. Martha L. Abell, James P. Braselton. Fourth edition 2014. ElScAe. 2014
Section: Chapter 2. First Order Equations. Exercises 2.5, page 64
Problem number: 11.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[[_homogeneous, ‘class A‘], _dAlembert]`

$$\boxed{\cos\left(\frac{t}{y+t}\right)+\mathrm{e}^{\frac{2y}{t}}y'=0}$$

✓ Solution by Maple

Time used: 0.016 (sec). Leaf size: 35

```
dsolve(( cos(t/(t+y(t))) )+( exp(2*y(t)/t))*diff(y(t),t)=0,y(t), singsol=all)
```

$$y(t)=\operatorname{RootOf}\left(\int^{\_Z}\frac{\mathrm{e}^{2\_a}}{\mathrm{e}^{2\_a}\_a+\cos\left(\frac{1}{\_a+1}\right)}d\_a+\ln(t)+c_1\right)t$$

✓ Solution by Mathematica

Time used: 0.828 (sec). Leaf size: 49

```
DSolve[( Cos[t/(t+y[t])] )+( Exp[2*y[t]/t] )*y'[t]==0,y[t],t,IncludeSingularSolutions -> Tru
```

$$\text{Solve}\left[\int_1^{\frac{y(t)}{t}}\frac{e^{2K[1]}}{\cos\left(\frac{1}{K[1]+1}\right)+e^{2K[1]}K[1]}dK[1]=-\log(t)+c_1,y(t)\right]$$

7.12 problem 12

Internal problem ID [14355]

Book: INTRODUCTORY DIFFERENTIAL EQUATIONS. Martha L. Abell, James P. Braselton. Fourth edition 2014. ElScAe. 2014
Section: Chapter 2. First Order Equations. Exercises 2.5, page 64
Problem number: 12.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[[_homogeneous, ‘class A‘], _dAlembert]`

$$\boxed{y\ln\left(\frac{t}{y}\right)+\frac{t^2y'}{y+t}=0}$$

✓ Solution by Maple
Time used: 0.015 (sec). Leaf size: 33

```
dsolve(( y(t)*ln(t/y(t)) )+( t^2/(t+y(t)))*diff(y(t),t)=0,y(t), singsol=all)
```

$$y(t)=\operatorname{RootOf}\left(\int^{\_Z}\frac{1}{\_a\left(\ln\left(\frac{1}{\_a}\right)\_a+\ln\left(\frac{1}{\_a}\right)+1\right)}d\_a+\ln(t)+c_1\right)t$$

✓ Solution by Mathematica
Time used: 0.142 (sec). Leaf size: 39

```
DSolve[( y[t]*Log[t/y[t]] )+( t^2/(t+y[t]))*y'[t]==0,y[t],t,IncludeSingularSolutions -> True
```

$$\text{Solve}\left[\int_1^{\frac{y(t)}{t}}\frac{1}{K[1](K[1]\log(K[1])+\log(K[1])-1)}dK[1]=\log(t)+c_1,y(t)\right]$$

7.13 problem 13

Internal problem ID [14356]

Book: INTRODUCTORY DIFFERENTIAL EQUATIONS. Martha L. Abell, James P. Braselton. Fourth edition 2014. ElScAe. 2014
Section: Chapter 2. First Order Equations. Exercises 2.5, page 64
Problem number: 13.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_separable]`

$$\boxed{-\ln\left(4y^2\right) y' = -2\ln(t)}$$

✓ Solution by Maple
Time used: 0.172 (sec). Leaf size: 59

```
dsolve(( 2*ln(t))-( ln(4*y(t)^2) )*diff(y(t),t)=0,y(t), singsol=all)
```

$$y(t) = \frac{t\ln(t) + c_1 - t}{\operatorname{LambertW}\left(-2\left(t\ln(t) + c_1 - t\right) e^{-1}\right)}$$
$$y(t) = \frac{t\ln(t) + c_1 - t}{\operatorname{LambertW}\left(2\left(t\ln(t) + c_1 - t\right) e^{-1}\right)}$$

✓ Solution by Mathematica
Time used: 60.066 (sec). Leaf size: 76

```
DSolve[( 2*Log[t])-( Log[4*y[t]^2] )*y'[t]==0,y[t],t,IncludeSingularSolutions -> True]
```

$$y(t) \to \frac{-2t + 2t\log(t) + c_1}{2W\left(-\frac{-2t+2t\log(t)+c_1}{e}\right)}$$
$$y(t) \to \frac{-2t + 2t\log(t) + c_1}{2W\left(\frac{-2t+2t\log(t)+c_1}{e}\right)}$$

7.14 problem 14

Internal problem ID [14357]

Book: INTRODUCTORY DIFFERENTIAL EQUATIONS. Martha L. Abell, James P. Braselton. Fourth edition 2014. ElScAe. 2014
Section: Chapter 2. First Order Equations. Exercises 2.5, page 64
Problem number: 14.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[[_homogeneous, 'class A'], _rational, _Bernoulli]`

$$\boxed{\frac{1}{y}+\frac{ty'}{y^2}=-\frac{2}{t}}$$

✓ Solution by Maple
Time used: 0.0 (sec). Leaf size: 15

```
dsolve(( 2/t+1/y(t))+(t/y(t)^2)*diff(y(t),t)=0,y(t), singsol=all)
```

$$y(t)=\frac{t}{c_1t^2-1}$$

✓ Solution by Mathematica
Time used: 0.132 (sec). Leaf size: 22

```
DSolve[( 2/t+1/y[t])+(t/y[t]^2)*y'[t]==0,y[t],t,IncludeSingularSolutions -> True]
```

$$y(t)\to \frac{t}{-1+c_1t^2}$$
$$y(t)\to 0$$

7.15 problem 15

Internal problem ID [14358]

Book: INTRODUCTORY DIFFERENTIAL EQUATIONS. Martha L. Abell, James P. Braselton. Fourth edition 2014. ElScAe. 2014
Section: Chapter 2. First Order Equations. Exercises 2.5, page 64
Problem number: 15.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_separable]`

$$\boxed{\frac{\sin(2t)}{\cos(2y)}+\frac{\ln(y)\, y'}{\ln(t)}=0}$$

✓ Solution by Maple
Time used: 0.687 (sec). Leaf size: 55

```
dsolve(( sin(2*t)/cos(2*y(t))  )+( ln(y(t))/ln(t) )*diff(y(t),t)=0,y(t), singsol=all)
```

$$-\frac{i\pi(\operatorname{csgn}(t)-1)\operatorname{csgn}(it)}{8}+\frac{\pi\operatorname{csgn}(y(t))}{8}+\frac{\sin(2y(t))\ln(y(t))}{4}-\frac{\cos(2t)\ln(t)}{4}-\frac{\operatorname{Si}(2y(t))}{4}+c_1+\frac{\operatorname{Ci}(2t)}{4}=0$$

✓ Solution by Mathematica
Time used: 0.697 (sec). Leaf size: 50

```
DSolve[( Sin[2*t]/Cos[2*y[t]]  )+( Log[y[t]]/Log[t] )*y'[t]==0,y[t],t,IncludeSingularSolutio
```

$$y(t)\to \text{InverseFunction}\left[\frac{1}{2}\log(\#1)\sin(2\#1)-\frac{\text{Si}(2\#1)}{2}\&\right]\left[-\frac{\text{CosIntegral}(2t)}{2}+\frac{1}{2}\log(t)\cos(2t)+c_1\right]$$

7.16 problem 16

Internal problem ID [14359]

Book: INTRODUCTORY DIFFERENTIAL EQUATIONS. Martha L. Abell, James P. Braselton. Fourth edition 2014. ElScAe. 2014
Section: Chapter 2. First Order Equations. Exercises 2.5, page 64
Problem number: 16.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_separable]`

$$\boxed{y'y = -\sqrt{t^2+1}}$$

✓ Solution by Maple

Time used: 0.015 (sec). Leaf size: 45

```
dsolve(( sqrt(t^2+1)  )+( y(t) )*diff(y(t),t)=0,y(t), singsol=all)
```

$$y(t) = \sqrt{-t\sqrt{t^2+1} - \operatorname{arcsinh}(t) + c_1}$$
$$y(t) = -\sqrt{-t\sqrt{t^2+1} - \operatorname{arcsinh}(t) + c_1}$$

✓ Solution by Mathematica

Time used: 2.194 (sec). Leaf size: 81

```
DSolve[( Sqrt[t^2+1]  )+( y[t] )*y'[t]==0,y[t],t,IncludeSingularSolutions -> True]
```

$$y(t) \to -\sqrt{-\sqrt{t^2+1}t + \log\left(\sqrt{t^2+1} - t\right) + 2c_1}$$
$$y(t) \to \sqrt{-\sqrt{t^2+1}t + \log\left(\sqrt{t^2+1} - t\right) + 2c_1}$$

7.17 problem 17

Internal problem ID [14360]

Book: INTRODUCTORY DIFFERENTIAL EQUATIONS. Martha L. Abell, James P. Braselton. Fourth edition 2014. ElScAe. 2014
Section: Chapter 2. First Order Equations. Exercises 2.5, page 64
Problem number: 17.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[[_homogeneous, 'class A'], _rational, [_Abel, '2nd type', 'cl`

$$\boxed{(y-3t)\,y' = -2t}$$

✓ Solution by Maple
Time used: 0.047 (sec). Leaf size: 47

```
dsolve(( 2*t )+( y(t)-3*t )*diff(y(t),t)=0,y(t), singsol=all)
```

$$y(t) = \frac{2c_1t - \sqrt{-4c_1t+1}+1}{2c_1}$$
$$y(t) = \frac{2c_1t + 1 + \sqrt{-4c_1t+1}}{2c_1}$$

✓ Solution by Mathematica
Time used: 1.268 (sec). Leaf size: 76

```
DSolve[( 2*t  )+( y[t]-3*t )*y'[t]==0,y[t],t,IncludeSingularSolutions -> True]
```

$$y(t) \to \frac{1}{2}\left(2t - e^{\frac{c_1}{2}}\sqrt{-4t+e^{c_1}} + e^{c_1}\right)$$
$$y(t) \to \frac{1}{2}\left(2t + e^{\frac{c_1}{2}}\sqrt{-4t+e^{c_1}} + e^{c_1}\right)$$

7.18 problem 18

Internal problem ID [14361]

Book: INTRODUCTORY DIFFERENTIAL EQUATIONS. Martha L. Abell, James P. Braselton. Fourth edition 2014. ElScAe. 2014
Section: Chapter 2. First Order Equations. Exercises 2.5, page 64
Problem number: 18.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_linear]`

$$\boxed{y't + 2y = 3t}$$

✓ Solution by Maple
Time used: 0.0 (sec). Leaf size: 11

```
dsolve(( 2*y(t)-3*t )+( t )*diff(y(t),t)=0,y(t), singsol=all)
```

$$y(t) = t + \frac{c_1}{t^2}$$

✓ Solution by Mathematica
Time used: 0.036 (sec). Leaf size: 13

```
DSolve[( 2*y[t]-3*t  )+( t )*y'[t]==0,y[t],t,IncludeSingularSolutions -> True]
```

$$y(t) \to t + \frac{c_1}{t^2}$$

7.19 problem 19

Internal problem ID [14362]

Book: INTRODUCTORY DIFFERENTIAL EQUATIONS. Martha L. Abell, James P. Braselton. Fourth edition 2014. ElScAe. 2014
Section: Chapter 2. First Order Equations. Exercises 2.5, page 64
Problem number: 19.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[[_homogeneous, ‘class A‘], _rational, [_Abel, ‘2nd type‘, ‘cl`

$$\boxed{ty - y^2 + t(t - 3y)\, y' = 0}$$

✓ Solution by Maple
Time used: 1.063 (sec). Leaf size: 239

```
dsolve(( t*y(t)-y(t)^2 )+( t*(t-3*y(t)) )*diff(y(t),t)=0,y(t), singsol=all)
```

$$y(t) = \frac{\left(\left(\sqrt{3}\,\sqrt{c_1^4 t^4 + 27} + 9\right) t c_1\right)^{\frac{1}{3}} 3^{\frac{2}{3}}}{c_1 \left(-t^2 c_1^2 3^{\frac{1}{3}} + \left(\left(\sqrt{3}\,\sqrt{c_1^4 t^4 + 27} + 9\right) t c_1\right)^{\frac{2}{3}}\right)}$$

$$y(t) = -\frac{2\left(\left(\sqrt{3}\,\sqrt{c_1^4 t^4 + 27} + 9\right) t c_1\right)^{\frac{1}{3}} 3^{\frac{2}{3}}}{\left(\left(1 + i\sqrt{3}\right)\left(\left(\sqrt{3}\,\sqrt{c_1^4 t^4 + 27} + 9\right) t c_1\right)^{\frac{2}{3}} + c_1^2 t^2 \left(i 3^{\frac{5}{6}} - 3^{\frac{1}{3}}\right)\right) c_1}$$

$$y(t) = \frac{2\left(\left(\sqrt{3}\,\sqrt{c_1^4 t^4 + 27} + 9\right) t c_1\right)^{\frac{1}{3}} 3^{\frac{2}{3}}}{\left(\left(i\sqrt{3} - 1\right)\left(\left(\sqrt{3}\,\sqrt{c_1^4 t^4 + 27} + 9\right) t c_1\right)^{\frac{2}{3}} + \left(i 3^{\frac{5}{6}} + 3^{\frac{1}{3}}\right) c_1^2 t^2\right) c_1}$$

✓ Solution by Mathematica

Time used: 60.223 (sec). Leaf size: 373

```
DSolve[( t*y[t]-y[t]^2 )+( t*(t-3*y[t]) )*y'[t]==0,y[t],t,IncludeSingularSolutions -> True]
```

$$y(t) \to \frac{1}{6}\left(\sqrt[3]{\frac{t^4+6\sqrt{81e^{4c_1}-3e^{2c_1}t^4}-54e^{2c_1}}{t}}+\frac{t^2}{\sqrt[3]{\frac{t^4+6\sqrt{81e^{4c_1}-3e^{2c_1}t^4}-54e^{2c_1}}{t}}}+t\right)$$

$$y(t) \to \frac{1}{12}\left(i\left(\sqrt{3}+i\right)\sqrt[3]{\frac{t^4+6\sqrt{81e^{4c_1}-3e^{2c_1}t^4}-54e^{2c_1}}{t}}-\frac{i\left(\sqrt{3}-i\right)t^2}{\sqrt[3]{\frac{t^4+6\sqrt{81e^{4c_1}-3e^{2c_1}t^4}-54e^{2c_1}}{t}}}+2t\right)$$

$$y(t) \to \frac{1}{12}\left(-\left(1+i\sqrt{3}\right)\sqrt[3]{\frac{t^4+6\sqrt{81e^{4c_1}-3e^{2c_1}t^4}-54e^{2c_1}}{t}}+\frac{i\left(\sqrt{3}+i\right)t^2}{\sqrt[3]{\frac{t^4+6\sqrt{81e^{4c_1}-3e^{2c_1}t^4}-54e^{2c_1}}{t}}}+2t\right)$$

7.20 problem 20

Internal problem ID [14363]

Book: INTRODUCTORY DIFFERENTIAL EQUATIONS. Martha L. Abell, James P. Braselton. Fourth edition 2014. ElScAe. 2014
Section: Chapter 2. First Order Equations. Exercises 2.5, page 64
Problem number: 20.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[[_homogeneous, ‘class A‘], _rational, [_Abel, ‘2nd type‘, ‘cl`

$$\boxed{y^2 + ty - tyy' = -t^2}$$

✓ Solution by Maple
Time used: 0.047 (sec). Leaf size: 22

```
dsolve(( t^2+t*y(t)+y(t)^2 )-( t*y(t) )*diff(y(t),t)=0,y(t), singsol=all)
```

$$y(t) = t\left(-\operatorname{LambertW}\left(-\frac{e^{-c_1-1}}{t}\right) - 1\right)$$

✓ Solution by Mathematica
Time used: 5.831 (sec). Leaf size: 31

```
DSolve[( t^2+t*y[t]+y[t]^2 )-( t*y[t] )*y'[t]==0,y[t],t,IncludeSingularSolutions -> True]
```

$$y(t) \to -t\left(1 + W\left(-\frac{e^{-1-c_1}}{t}\right)\right)$$
$$y(t) \to -t$$

7.21 problem 21

Internal problem ID [14364]

Book: INTRODUCTORY DIFFERENTIAL EQUATIONS. Martha L. Abell, James P. Braselton. Fourth edition 2014. ElScAe. 2014
Section: Chapter 2. First Order Equations. Exercises 2.5, page 64
Problem number: 21.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[[_homogeneous, 'class A'], _rational, _Bernoulli]`

$$\boxed{y^3 - y^2 y' t = -t^3}$$

✓ Solution by Maple
Time used: 0.031 (sec). Leaf size: 58

```
dsolve(( t^3+y(t)^3 )-( t*y(t)^2 )*diff(y(t),t)=0,y(t), singsol=all)
```

$$y(t) = (3\ln(t) + c_1)^{\frac{1}{3}} t$$
$$y(t) = -\frac{(3\ln(t) + c_1)^{\frac{1}{3}} (1 + i\sqrt{3}) t}{2}$$
$$y(t) = \frac{(3\ln(t) + c_1)^{\frac{1}{3}} (i\sqrt{3} - 1) t}{2}$$

✓ Solution by Mathematica
Time used: 0.187 (sec). Leaf size: 63

```
DSolve[( t^3+y[t]^3 )-( t*y[t]^2 )*y'[t]==0,y[t],t,IncludeSingularSolutions -> True]
```

$$y(t) \to t\sqrt[3]{3\log(t) + c_1}$$
$$y(t) \to -\sqrt[3]{-1}t\sqrt[3]{3\log(t) + c_1}$$
$$y(t) \to (-1)^{2/3}t\sqrt[3]{3\log(t) + c_1}$$

7.22 problem 22

Internal problem ID [14365]

Book: INTRODUCTORY DIFFERENTIAL EQUATIONS. Martha L. Abell, James P. Braselton. Fourth edition 2014. ElScAe. 2014
Section: Chapter 2. First Order Equations. Exercises 2.5, page 64
Problem number: 22.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[[_homogeneous, 'class A'], _rational, [_Abel, '2nd type', 'cl`

$$\boxed{y' - \frac{t + 4y}{4t + y} = 0}$$

✓ Solution by Maple
Time used: 1.141 (sec). Leaf size: 36

```
dsolve(diff(y(t),t)=(t+4*y(t))/(4*t+y(t)),y(t), singsol=all)
```

$$y(t) = \left(\operatorname{RootOf}\left(\_Z^{10}c_1t^2 - \_Z^6 - 6\_Z^4 - 12\_Z^2 - 8\right)^2 + 1\right)t$$

✓ Solution by Mathematica

Time used: 5.494 (sec). Leaf size: 456

```
DSolve[y'[t]==(t+4*y[t])/(4*t+y[t]),y[t],t,IncludeSingularSolutions -> True]
```

$$y(t) \to \text{Root}\left[\#1^5 - 5\#1^4 t + \#1^3\left(10t^2 + e^{2c_1}\right) + \#1^2\left(-10t^3 + 3e^{2c_1}t\right) + \#1\left(5t^4 + 3e^{2c_1}t^2\right) - t^5 + e^{2c_1}t^3\&, 1\right]$$

$$y(t) \to \text{Root}\left[\#1^5 - 5\#1^4 t + \#1^3\left(10t^2 + e^{2c_1}\right) + \#1^2\left(-10t^3 + 3e^{2c_1}t\right) + \#1\left(5t^4 + 3e^{2c_1}t^2\right) - t^5 + e^{2c_1}t^3\&, 2\right]$$

$$y(t) \to \text{Root}\left[\#1^5 - 5\#1^4 t + \#1^3\left(10t^2 + e^{2c_1}\right) + \#1^2\left(-10t^3 + 3e^{2c_1}t\right) + \#1\left(5t^4 + 3e^{2c_1}t^2\right) - t^5 + e^{2c_1}t^3\&, 3\right]$$

$$y(t) \to \text{Root}\left[\#1^5 - 5\#1^4 t + \#1^3\left(10t^2 + e^{2c_1}\right) + \#1^2\left(-10t^3 + 3e^{2c_1}t\right) + \#1\left(5t^4 + 3e^{2c_1}t^2\right) - t^5 + e^{2c_1}t^3\&, 4\right]$$

$$y(t) \to \text{Root}\left[\#1^5 - 5\#1^4 t + \#1^3\left(10t^2 + e^{2c_1}\right) + \#1^2\left(-10t^3 + 3e^{2c_1}t\right) + \#1\left(5t^4 + 3e^{2c_1}t^2\right) - t^5 + e^{2c_1}t^3\&, 5\right]$$

7.23 problem 23

Internal problem ID [14366]

Book: INTRODUCTORY DIFFERENTIAL EQUATIONS. Martha L. Abell, James P. Braselton. Fourth edition 2014. ElScAe. 2014
Section: Chapter 2. First Order Equations. Exercises 2.5, page 64
Problem number: 23.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_linear]`

$$\boxed{y't - y = -t}$$

✓ Solution by Maple
Time used: 0.0 (sec). Leaf size: 12

```
dsolve((t-y(t))+t*diff(y(t),t)=0,y(t), singsol=all)
```

$$y(t) = (-\ln(t) + c_1)\, t$$

✓ Solution by Mathematica
Time used: 0.035 (sec). Leaf size: 14

```
DSolve[(t-y[t])+t*y'[t]==0,y[t],t,IncludeSingularSolutions -> True]
```

$$y(t) \to t(-\log(t) + c_1)$$

7.24 problem 24

Internal problem ID [14367]

Book: INTRODUCTORY DIFFERENTIAL EQUATIONS. Martha L. Abell, James P. Braselton. Fourth edition 2014. ElScAe. 2014
Section: Chapter 2. First Order Equations. Exercises 2.5, page 64
Problem number: 24.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[[_homogeneous, ‘class A‘], _exact, _rational, [_Abel, ‘2nd ty`

$$\boxed{y + (y + t)\, y' = 0}$$

✓ Solution by Maple
Time used: 0.0 (sec). Leaf size: 35

```
dsolve((y(t))+(t+y(t))*diff(y(t),t)=0,y(t), singsol=all)
```

$$y(t) = -t - \sqrt{t^2 + 2c_1}$$
$$y(t) = -t + \sqrt{t^2 + 2c_1}$$

✓ Solution by Mathematica
Time used: 0.735 (sec). Leaf size: 84

```
DSolve[y[t]+(t+y[t])*y'[t]==0,y[t],t,IncludeSingularSolutions -> True]
```

$$y(t) \to -t - \sqrt{t^2 + e^{2c_1}}$$
$$y(t) \to -t + \sqrt{t^2 + e^{2c_1}}$$
$$y(t) \to 0$$
$$y(t) \to -\sqrt{t^2} - t$$
$$y(t) \to \sqrt{t^2} - t$$

7.25 problem 25

Internal problem ID [14368]

Book: INTRODUCTORY DIFFERENTIAL EQUATIONS. Martha L. Abell, James P. Braselton. Fourth edition 2014. ElScAe. 2014
Section: Chapter 2. First Order Equations. Exercises 2.5, page 64
Problem number: 25.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[[_homogeneous, ‘class A‘], _rational, [_Abel, ‘2nd type‘, ‘cl`

$$\boxed{-7ty + 5y^2 + tyy' = -2t^2}$$

✓ Solution by Maple
Time used: 1.031 (sec). Leaf size: 52

```
dsolve((2*t^2-7*t*y(t)+5*y(t)^2)+(t*y(t))*diff(y(t),t)=0,y(t), singsol=all)
```

$$y(t) = \frac{t\left(-2c_1t^2 + \operatorname{RootOf}\left(\_Z^4 - 3c_1t^2 + 2\_Z\right)\right)}{-3c_1t^2 + 2\operatorname{RootOf}\left(\_Z^4 - 3c_1t^2 + 2\_Z\right)}$$

✓ Solution by Mathematica

Time used: 60.175 (sec). Leaf size: 1935

```
DSolve[(2*t^2-7*t*y[t]+5*y[t]^2)+(t*y[t])*y'[t]==0,y[t],t,IncludeSingularSolutions -> True]
```

$$y(t) \rightarrow -\frac{-108t^6 + t^5\sqrt{\frac{54e^{6c_1}t^6\left(-4+\frac{3\ 2^{2/3}t^7}{\sqrt[3]{e^{12c_1}t^9+\sqrt{e^{18c_1}t^{18}\left(-16t^6+e^{6c_1}\right)}}}\right)+81\sqrt[3]{2}t^5\sqrt[3]{e^{12c_1}t^9+\sqrt{e^{18c_1}t^{18}\left(-16t^6 \cdots\right.}}}{t^{10}}}\cdots}{\cdots}$$

$$y(t) \rightarrow \frac{108t^6 - t^5\sqrt{\frac{54e^{6c_1}t^6\left(-4+\frac{3\ 2^{2/3}t^7}{\sqrt[3]{e^{12c_1}t^9+\sqrt{e^{18c_1}t^{18}\left(-16t^6+e^{6c_1}\right)}}}\right)+81\sqrt[3]{2}t^5\sqrt[3]{e^{12c_1}t^9+\sqrt{e^{18c_1}t^{18}\left(-16t^6+e \cdots\right.}}}{t^{10}}}\cdots}{\cdots}$$

$$y(t) \rightarrow \frac{108t^6 + t^5\sqrt{\frac{54e^{6c_1}t^6\left(-4+\frac{3\ 2^{2/3}t^7}{\sqrt[3]{e^{12c_1}t^9+\sqrt{e^{18c_1}t^{18}\left(-16t^6+e^{6c_1}\right)}}}\right)+81\sqrt[3]{2}t^5\sqrt[3]{e^{12c_1}t^9+\sqrt{e^{18c_1}t^{18}\left(-16t^6+e \cdots\right.}}}{t^{10}}}\cdots}{\cdots}$$

$$y(t) \rightarrow \frac{108t^6 + t^5\sqrt{\frac{54e^{6c_1}t^6\left(-4+\frac{3\ 2^{2/3}t^7}{\sqrt[3]{e^{12c_1}t^9+\sqrt{e^{18c_1}t^{18}\left(-16t^6+e^{6c_1}\right)}}}\right)+81\sqrt[3]{2}t^5\sqrt[3]{e^{12c_1}t^9+\sqrt{e^{18c_1}t^{18}\left(-16t^6+e \cdots\right.}}}{t^{10}}}\cdots}{\cdots}$$

7.26 problem 26

Internal problem ID [14369]

Book: INTRODUCTORY DIFFERENTIAL EQUATIONS. Martha L. Abell, James P. Braselton. Fourth edition 2014. ElScAe. 2014
Section: Chapter 2. First Order Equations. Exercises 2.5, page 64
Problem number: 26.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[[_homogeneous, ‘class A‘], _dAlembert]`

$$\boxed{y + 2\sqrt{y^2 + t^2} - y't = 0}$$

✓ Solution by Maple
Time used: 0.016 (sec). Leaf size: 26

```
dsolve((y(t)+2*sqrt(t^2+y(t)^2))-(t)*diff(y(t),t)=0,y(t), singsol=all)
```

$$\frac{-c_1 t^3 + \sqrt{t^2 + y(t)^2} + y(t)}{t^3} = 0$$

✓ Solution by Mathematica
Time used: 0.25 (sec). Leaf size: 28

```
DSolve[(y[t]+2*Sqrt[t^2+y[t]^2])-(t)*y'[t]==0,y[t],t,IncludeSingularSolutions -> True]
```

$$y(t) \to -\frac{e^{-c_1} - e^{c_1} t^4}{2t}$$

7.27 problem 27

Internal problem ID [14370]

Book: INTRODUCTORY DIFFERENTIAL EQUATIONS. Martha L. Abell, James P. Braselton. Fourth edition 2014. ElScAe. 2014
Section: Chapter 2. First Order Equations. Exercises 2.5, page 64
Problem number: 27.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[[_homogeneous, ‘class A‘], _rational, [_Abel, ‘2nd type‘, ‘cl`

$$\boxed{y^2 - (ty - 4t^2)\,y' = 0}$$

✓ Solution by Maple
Time used: 0.063 (sec). Leaf size: 17

```
dsolve(y(t)^2=(t*y(t)-4*t^2)*diff(y(t),t),y(t), singsol=all)
```

$$y(t) = -4t\,\mathrm{LambertW}\left(-\frac{\mathrm{e}^{-c_1}}{4t}\right)$$

✓ Solution by Mathematica
Time used: 3.085 (sec). Leaf size: 29

```
DSolve[y[t]^2==(t*y[t]-4*t^2)*y'[t],y[t],t,IncludeSingularSolutions -> True]
```

$$y(t) \to -4tW\left(-\frac{e^{-\frac{c_1}{4}}}{4t}\right)$$
$$y(t) \to 0$$

7.28 problem 28

Internal problem ID [14371]

Book: INTRODUCTORY DIFFERENTIAL EQUATIONS. Martha L. Abell, James P. Braselton. Fourth edition 2014. ElScAe. 2014
Section: Chapter 2. First Order Equations. Exercises 2.5, page 64
Problem number: 28.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[[_homogeneous, ‘class A‘], _dAlembert]`

$$\boxed{y - \left(3\sqrt{ty} + t\right) y' = 0}$$

✓ Solution by Maple

Time used: 0.015 (sec). Leaf size: 21

```
dsolve(y(t)-(3*sqrt(t*y(t))+t)*diff(y(t),t)=0,y(t), singsol=all)
```

$$3\ln\left(y(t)\right) - \frac{2t}{\sqrt{y\left(t\right)t}} - c_1 = 0$$

✓ Solution by Mathematica

Time used: 0.238 (sec). Leaf size: 33

```
DSolve[y[t]-(3*Sqrt[t*y[t]]+t)*y'[t]==0,y[t],t,IncludeSingularSolutions -> True]
```

$$\text{Solve}\left[3\log\left(\frac{y(t)}{t}\right) - \frac{2}{\sqrt{\frac{y(t)}{t}}} = -3\log(t) + c_1, y(t)\right]$$

7.29 problem 29

Internal problem ID [14372]

Book: INTRODUCTORY DIFFERENTIAL EQUATIONS. Martha L. Abell, James P. Braselton. Fourth edition 2014. ElScAe. 2014
Section: Chapter 2. First Order Equations. Exercises 2.5, page 64
Problem number: 29.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[[_homogeneous, ‘class A‘], _rational, [_Abel, ‘2nd type‘, ‘cl`

$$\boxed{(t^2 - y^2)\, y' + y^2 + ty = 0}$$

✓ Solution by Maple
Time used: 0.015 (sec). Leaf size: 37

```
dsolve((t^2-y(t)^2)*diff(y(t),t)+(y(t)^2+t*y(t))=0,y(t), singsol=all)
```

$$\begin{aligned} y(t) &= -t \\ y(t) &= t - \sqrt{t^2 - 2c_1} \\ y(t) &= t + \sqrt{t^2 - 2c_1} \end{aligned}$$

✓ Solution by Mathematica
Time used: 0.515 (sec). Leaf size: 89

```
DSolve[(t^2-y[t]^2)*y'[t]+(y[t]^2+t*y[t])==0,y[t],t,IncludeSingularSolutions -> True]
```

$$\begin{aligned} y(t) &\to -t \\ y(t) &\to t - \sqrt{t^2 - e^{2c_1}} \\ y(t) &\to t + \sqrt{t^2 - e^{2c_1}} \\ y(t) &\to -t \\ y(t) &\to t - \sqrt{t^2} \\ y(t) &\to \sqrt{t^2} + t \end{aligned}$$

7.30 problem 30

Internal problem ID [14373]

Book: INTRODUCTORY DIFFERENTIAL EQUATIONS. Martha L. Abell, James P. Braselton. Fourth edition 2014. ElScAe. 2014
Section: Chapter 2. First Order Equations. Exercises 2.5, page 64
Problem number: 30.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[[_homogeneous, ‘class A‘], _dAlembert]`

$$\boxed{tyy' - t^2 e^{-\frac{y}{t}} - y^2 = 0}$$

✓ Solution by Maple
Time used: 0.0 (sec). Leaf size: 16

```
dsolve((t*y(t))*diff(y(t),t)-(t^2*exp(-y(t)/t)+y(t)^2)=0,y(t), singsol=all)
```

$$y(t) = \left(\text{LambertW}\left((\ln(t) + c_1)\, e^{-1}\right) + 1\right) t$$

✓ Solution by Mathematica
Time used: 60.23 (sec). Leaf size: 19

```
DSolve[(t*y[t])*y'[t]-(t^2*Exp[-y[t]/t]+y[t]^2)==0,y[t],t,IncludeSingularSolutions -> True]
```

$$y(t) \to t\left(1 + W\left(\frac{\log(t) + c_1}{e}\right)\right)$$

7.31 problem 31

Internal problem ID [14374]

Book: INTRODUCTORY DIFFERENTIAL EQUATIONS. Martha L. Abell, James P. Braselton. Fourth edition 2014. ElScAe. 2014
Section: Chapter 2. First Order Equations. Exercises 2.5, page 64
Problem number: 31.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[[_homogeneous, ‘class A‘], _dAlembert]`

$$\boxed{y' - \frac{1}{\frac{2y\,\mathrm{e}^{-\frac{t}{y}}}{t} + \frac{t}{y}} = 0}$$

✓ Solution by Maple

Time used: 0.39 (sec). Leaf size: 31

```
dsolve(diff(y(t),t)=1/( 2*y(t)*exp(-t/y(t))/t+t/y(t) ),y(t), singsol=all)
```

$$y(t) = t\,\mathrm{e}^{-\operatorname{RootOf}\left(-2\_Z - \mathrm{e}^{\mathrm{e}^{-\_Z}+\_Z} + \mathrm{e}^{\mathrm{e}^{-\_Z}} + 2\ln(t) + 2c_1\right)}$$

✓ Solution by Mathematica

Time used: 0.259 (sec). Leaf size: 43

```
DSolve[y'[t]==1/( 2*y[t]*Exp[-t/y[t]]/t+t/y[t] ),y[t],t,IncludeSingularSolutions -> True]
```

$$\text{Solve}\left[\frac{te^{\frac{t}{y(t)}}\left(\frac{y(t)}{t}-1\right)}{y(t)} + 2\log\left(\frac{y(t)}{t}\right) = -2\log(t) + c_1, y(t)\right]$$

7.32 problem 32

Internal problem ID [14375]

Book: INTRODUCTORY DIFFERENTIAL EQUATIONS. Martha L. Abell, James P. Braselton. Fourth edition 2014. ElScAe. 2014
Section: Chapter 2. First Order Equations. Exercises 2.5, page 64
Problem number: 32.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[[_homogeneous, ‘class A‘], _dAlembert]`

$$\boxed{t(\ln(t) - \ln(y))\, y' - y = 0}$$

✓ Solution by Maple
Time used: 0.062 (sec). Leaf size: 14

```
dsolve(t*(ln(t)-ln(y(t)) )*diff(y(t),t)=y(t),y(t), singsol=all)
```

$$y(t) = \frac{\operatorname{LambertW}\left(\mathrm{e}^{-1} c_1 t\right)}{c_1}$$

✓ Solution by Mathematica
Time used: 5.377 (sec). Leaf size: 37

```
DSolve[t*(Log[t]-Log[y[t]] )*y'[t]==y[t],y[t],t,IncludeSingularSolutions -> True]
```

$$\begin{aligned}
y(t) &\to -e^{c_1} W\left(-e^{-1-c_1} t\right) \\
y(t) &\to 0 \\
y(t) &\to \frac{t}{e}
\end{aligned}$$

7.33 problem 33

Internal problem ID [14376]

Book: INTRODUCTORY DIFFERENTIAL EQUATIONS. Martha L. Abell, James P. Braselton. Fourth edition 2014. ElScAe. 2014
Section: Chapter 2. First Order Equations. Exercises 2.5, page 64
Problem number: 33.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_Bernoulli]`

$$\boxed{y' + 2y - t^2\sqrt{y} = 0}$$

With initial conditions

$$[y(0) = 1]$$

✓ Solution by Maple
Time used: 0.047 (sec). Leaf size: 16

```
dsolve([diff(y(t),t)+2*y(t)=t^2*sqrt(y(t)),y(0) = 1],y(t), singsol=all)
```

$$y(t) = \frac{(t^2 - 2t + 2)^2}{4}$$

✓ Solution by Mathematica
Time used: 0.186 (sec). Leaf size: 48

```
DSolve[{y'[t]+2*y[t]==t^2*Sqrt[y[t]],{y[0]==1}},y[t],t,IncludeSingularSolutions -> True]
```

$$y(t) \to \frac{1}{4}(t^2 - 2t + 2)^2$$
$$y(t) \to \frac{1}{4}e^{-2t}\left(e^t\left(t^2 - 2t + 2\right) - 4\right)^2$$

7.34 problem 34

Internal problem ID [14377]

Book: INTRODUCTORY DIFFERENTIAL EQUATIONS. Martha L. Abell, James P. Braselton. Fourth edition 2014. ElScAe. 2014
Section: Chapter 2. First Order Equations. Exercises 2.5, page 64
Problem number: 34.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_Bernoulli]`

$$\boxed{y' - 2y - t^2\sqrt{y} = 0}$$

With initial conditions

$$[y(0) = 1]$$

✓ Solution by Maple
Time used: 0.062 (sec). Leaf size: 44

```
dsolve([diff(y(t),t)-2*y(t)=t^2*sqrt(y(t)),y(0) = 1],y(t), singsol=all)
```

$$y(t) = 1 + 4\,\mathrm{e}^{2t} + 2\left(-t^2 - 2t - 2\right)\mathrm{e}^{t} + \frac{t^4}{4} + t^3 + 2t^2 + 2t$$

✓ Solution by Mathematica
Time used: 0.164 (sec). Leaf size: 42

```
DSolve[{y'[t]-2*y[t]==t^2*Sqrt[y[t]],{y[0]==1}},y[t],t,IncludeSingularSolutions -> True]
```

$$y(t) \to \frac{1}{4}\left(t^2 + 2t - 4e^t + 2\right)^2$$
$$y(t) \to \frac{1}{4}\left(t^2 + 2t + 2\right)^2$$

7.35 problem 35

Internal problem ID [14378]

Book: INTRODUCTORY DIFFERENTIAL EQUATIONS. Martha L. Abell, James P. Braselton. Fourth edition 2014. ElScAe. 2014
Section: Chapter 2. First Order Equations. Exercises 2.5, page 64
Problem number: 35.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[[_homogeneous, 'class A'], _rational, _Bernoulli]`

$$\boxed{y' - \frac{4y^2 - t^2}{2ty} = 0}$$

With initial conditions

$$[y(1) = 1]$$

✓ Solution by Maple
Time used: 0.047 (sec). Leaf size: 16

```
dsolve([diff(y(t),t)=(4*y(t)^2-t^2)/(2*t*y(t)),y(1) = 1],y(t), singsol=all)
```

$$y(t) = \frac{\sqrt{2t^2 + 2}\, t}{2}$$

✗ Solution by Mathematica
Time used: 0.0 (sec). Leaf size: 0

```
DSolve[{y'[t]==(4*y[t]^2-t^2)/(2*t*y[t]),{y[1]==1}},y[t],t,IncludeSingularSolutions -> True]
```

{}

7.36 problem 36

Internal problem ID [14379]

Book: INTRODUCTORY DIFFERENTIAL EQUATIONS. Martha L. Abell, James P. Braselton. Fourth edition 2014. ElScAe. 2014
Section: Chapter 2. First Order Equations. Exercises 2.5, page 64
Problem number: 36.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_linear]`

$$\boxed{y - y't = -t}$$

With initial conditions

$$[y(1) = 1]$$

✓ Solution by Maple
Time used: 0.016 (sec). Leaf size: 10

```
dsolve([(t+y(t))-t*diff(y(t),t)=0,y(1) = 1],y(t), singsol=all)
```

$$y(t) = t(\ln(t) + 1)$$

✓ Solution by Mathematica
Time used: 0.026 (sec). Leaf size: 11

```
DSolve[{(t+y[t])-t*y'[t]==0,{y[1]==1}},y[t],t,IncludeSingularSolutions -> True]
```

$$y(t) \to t(\log(t) + 1)$$

7.37 problem 37

Internal problem ID [14380]

Book: INTRODUCTORY DIFFERENTIAL EQUATIONS. Martha L. Abell, James P. Braselton. Fourth edition 2014. ElScAe. 2014
Section: Chapter 2. First Order Equations. Exercises 2.5, page 64
Problem number: 37.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[[_homogeneous, ‘class A‘], _rational, _dAlembert]`

$$\boxed{y't - y - \sqrt{y^2 + t^2} = 0}$$

With initial conditions

$$[y(1) = 0]$$

✓ Solution by Maple
Time used: 8.969 (sec). Leaf size: 21

```
dsolve([t*diff(y(t),t)-(y(t)+sqrt(t^2+y(t)^2))=0,y(1) = 0],y(t), singsol=all)
```

$$y(t) = -\frac{t^2}{2} + \frac{1}{2}$$
$$y(t) = \frac{t^2}{2} - \frac{1}{2}$$

✓ Solution by Mathematica
Time used: 0.346 (sec). Leaf size: 14

```
DSolve[{t*y'[t]-(y[t]+Sqrt[t^2+y[t]^2])==0,{y[1]==0}},y[t],t,IncludeSingularSolutions -> Tru
```

$$y(t) \to \frac{1}{2}(t^2 - 1)$$

7.38 problem 38

Internal problem ID [14381]

Book: INTRODUCTORY DIFFERENTIAL EQUATIONS. Martha L. Abell, James P. Braselton. Fourth edition 2014. ElScAe. 2014
Section: Chapter 2. First Order Equations. Exercises 2.5, page 64
Problem number: 38.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[[_homogeneous, ‘class A‘], _dAlembert]`

$$\boxed{y^2\sqrt{y^2+t^2}-ty\sqrt{y^2+t^2}\,y'=-t^3}$$

With initial conditions

$$[y(1)=1]$$

✓ Solution by Maple
Time used: 1.391 (sec). Leaf size: 22

```
dsolve([(t^3+y(t)^2*sqrt(t^2+y(t)^2))-(t*y(t)*sqrt(t^2+y(t)^2))*diff(y(t),t)=0,y(1) = 1],y(t
```

$$y(t)=\sqrt{-1+\left(3\ln(t)+2\sqrt{2}\right)^{\frac{2}{3}}}\,t$$

✓ Solution by Mathematica
Time used: 19.956 (sec). Leaf size: 80

```
DSolve[{(t^3+y[t]^2*Sqrt[t^2+y[t]^2])-(t*y[t]*Sqrt[t^2+y[t]^2])*y'[t]==0,{y[1]==1}},y[t],t,I
```

$$y(t)\to\sqrt{\sqrt[3]{-t^6\left(-9\log^2(t)+12\sqrt{2}\log(t)-8\right)}-t^2}$$
$$y(t)\to\sqrt{\sqrt[3]{t^6\left(9\log^2(t)+12\sqrt{2}\log(t)+8\right)}-t^2}$$

7.39 problem 39

Internal problem ID [14382]

Book: INTRODUCTORY DIFFERENTIAL EQUATIONS. Martha L. Abell, James P. Braselton. Fourth edition 2014. ElScAe. 2014
Section: Chapter 2. First Order Equations. Exercises 2.5, page 64
Problem number: 39.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[[_homogeneous, 'class A'], _rational, _Bernoulli]`

$$\boxed{y^3 - y^2 y' t = t^3}$$

With initial conditions

$$[y(1) = 3]$$

✓ Solution by Maple
Time used: 0.078 (sec). Leaf size: 14

```
dsolve([(y(t)^3-t^3)-(t*y(t)^2)*diff(y(t),t)=0,y(1) = 3],y(t), singsol=all)
```

$$y(t) = (-3\ln(t) + 27)^{\frac{1}{3}} t$$

✓ Solution by Mathematica
Time used: 0.212 (sec). Leaf size: 22

```
DSolve[{(y[t]^3-t^3)-(t*y[t]^2)*y'[t]==0,{y[1]==3}},y[t],t,IncludeSingularSolutions -> True]
```

$$y(t) \to \sqrt[3]{3} t \sqrt[3]{9 - \log(t)}$$

7.40 problem 40

Internal problem ID [14383]

Book: INTRODUCTORY DIFFERENTIAL EQUATIONS. Martha L. Abell, James P. Braselton. Fourth edition 2014. ElScAe. 2014
Section: Chapter 2. First Order Equations. Exercises 2.5, page 64
Problem number: 40.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[[_homogeneous, ‘class A‘], _rational, _dAlembert]`

$$\boxed{y^3 t - (t^4 + y^4)\, y' = 0}$$

With initial conditions

$$[y(1) = 1]$$

✓ Solution by Maple

Time used: 1.172 (sec). Leaf size: 63

```
dsolve([(t*y(t)^3)-(t^4+y(t)^4)*diff(y(t),t)=0,y(1) = 1],y(t), singsol=all)
```

$$y(t) = \mathrm{e}^{\frac{\sqrt{3}\left(6\,\mathrm{RootOf}\left(\sqrt{3}\, t^2 \mathrm{e}^{-\frac{\sqrt{3}\,\pi}{9}} - 3\tan(\_Z)\, t^2 \mathrm{e}^{-\frac{\sqrt{3}\,\pi}{9}} - 2\sqrt{3}\,\mathrm{e}^{\frac{2\_Z\sqrt{3}}{3}}\right) + \pi\right)}{18}}$$

✓ Solution by Mathematica

Time used: 0.173 (sec). Leaf size: 107

```
DSolve[{(t*y[t]^3)-(t^4+y[t]^4)*y'[t]==0,{y[1]==1}},y[t],t,IncludeSingularSolutions -> True]
```

$$\mathrm{Solve}\left[\log\left(\frac{y(t)}{t}\right) + \frac{i\left(\log\left(-\frac{2iy(t)^2}{t^2} + \sqrt{3} + i\right) - \log\left(\frac{2iy(t)^2}{t^2} + \sqrt{3} - i\right)\right)}{2\sqrt{3}} = -\log(t) + \frac{i\left(\log\left(\sqrt{3} - i\right) - \log\left(\sqrt{3} + i\right)\right)}{2\sqrt{3}}, y(t)\right]$$

7.41 problem 41

Internal problem ID [14384]

Book: INTRODUCTORY DIFFERENTIAL EQUATIONS. Martha L. Abell, James P. Braselton. Fourth edition 2014. ElScAe. 2014
Section: Chapter 2. First Order Equations. Exercises 2.5, page 64
Problem number: 41.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[[_homogeneous, 'class A'], _rational, _dAlembert]`

$$\boxed{y^4 + (t^4 - y^3 t)\, y' = 0}$$

With initial conditions

$$[y(1) = 2]$$

✓ Solution by Maple
Time used: 3.047 (sec). Leaf size: 103

```
dsolve([y(t)^4+(t^4-t*y(t)^3)*diff(y(t),t)=0,y(1) = 2],y(t), singsol=all)
```

$$y(t) = \frac{e^{-\operatorname{RootOf}\left(6i\pi\_Z220+\operatorname{LambertW}\left(\_Z222,-e^{-3\_Z}\right)+3\_Z+3\ln(2)+6i\pi\_Z223\right)}}{\left(-\frac{e^{-3\operatorname{RootOf}\left(6i\pi\_Z220+\operatorname{LambertW}\left(\_Z222,-e^{-3\_Z}\right)+3\_Z+3\ln(2)+6i\pi\_Z223\right)}}{t^3\operatorname{LambertW}\left(\_Z222,-\frac{e^{-3\operatorname{RootOf}\left(6i\pi\_Z220+\operatorname{LambertW}\left(\_Z222,-e^{-3\_Z}\right)+3\_Z+3\ln(2)+6i\pi\_Z223\right)}}{t^3}\right)}\right)^{\frac{1}{3}}}$$

✗ Solution by Mathematica
Time used: 0.0 (sec). Leaf size: 0

```
DSolve[{y[t]^4+(t^4-t*y[t]^3)*y'[t]==0,{y[1]==2}},y[t],t,IncludeSingularSolutions -> True]
```

{}

7.42 problem 42 (a)

Internal problem ID [14385]

Book: INTRODUCTORY DIFFERENTIAL EQUATIONS. Martha L. Abell, James P. Braselton. Fourth edition 2014. ElScAe. 2014
Section: Chapter 2. First Order Equations. Exercises 2.5, page 64
Problem number: 42 (a).
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[[_homogeneous, ‘class C‘], _rational, [_Abel, ‘2nd type‘, ‘cl`

$$\boxed{-2y + (4t - 3y - 6)\, y' = -1 - t}$$

✓ Solution by Maple

Time used: 1.204 (sec). Leaf size: 56

```
dsolve((t-2*y(t)+1)+(4*t-3*y(t)-6)*diff(y(t),t)=0,y(t), singsol=all)
```

$$y(t) = \frac{(-t+3)\operatorname{RootOf}\left(-4 + (3c_1t^4 - 36c_1t^3 + 162c_1t^2 - 324c_1t + 243c_1)\_Z^{20} - \_Z^4\right)^4}{3} - \frac{t}{3} + 3$$

✓ Solution by Mathematica

Time used: 60.07 (sec). Leaf size: 1511

```
DSolve[(t-2*y[t]+1)+(4*t-3*y[t]-6)*y'[t]==0,y[t],t,IncludeSingularSolutions -> True]
```

$$y(t) \to \frac{2}{3}(2t-3)$$

$$-\frac{}{3\text{Root}\left[\#1^5\left(3125e^{\frac{5c_1}{9}}t^5 - 46875e^{\frac{5c_1}{9}}t^4 + 281250e^{\frac{5c_1}{9}}t^3 - 843750e^{\frac{5c_1}{9}}t^2 + 3125t + 1265625e^{\frac{5c_1}{9}}t - 937\right.\right.}$$

$$y(t) \to \frac{2}{3}(2t-3)$$

$$-\frac{}{3\text{Root}\left[\#1^5\left(3125e^{\frac{5c_1}{9}}t^5 - 46875e^{\frac{5c_1}{9}}t^4 + 281250e^{\frac{5c_1}{9}}t^3 - 843750e^{\frac{5c_1}{9}}t^2 + 3125t + 1265625e^{\frac{5c_1}{9}}t - 937\right.\right.}$$

$$y(t) \to \frac{2}{3}(2t-3)$$

$$-\frac{}{3\text{Root}\left[\#1^5\left(3125e^{\frac{5c_1}{9}}t^5 - 46875e^{\frac{5c_1}{9}}t^4 + 281250e^{\frac{5c_1}{9}}t^3 - 843750e^{\frac{5c_1}{9}}t^2 + 3125t + 1265625e^{\frac{5c_1}{9}}t - 937\right.\right.}$$

$$y(t) \to \frac{2}{3}(2t-3)$$

$$-\frac{}{3\text{Root}\left[\#1^5\left(3125e^{\frac{5c_1}{9}}t^5 - 46875e^{\frac{5c_1}{9}}t^4 + 281250e^{\frac{5c_1}{9}}t^3 - 843750e^{\frac{5c_1}{9}}t^2 + 3125t + 1265625e^{\frac{5c_1}{9}}t - 937\right.\right.}$$

$$y(t) \to \frac{2}{3}(2t-3)$$

$$-\frac{}{3\text{Root}\left[\#1^5\left(3125e^{\frac{5c_1}{9}}t^5 - 46875e^{\frac{5c_1}{9}}t^4 + 281250e^{\frac{5c_1}{9}}t^3 - 843750e^{\frac{5c_1}{9}}t^2 + 3125t + 1265625e^{\frac{5c_1}{9}}t - 937\right.\right.}$$

7.43 problem 42 (b)

Internal problem ID [14386]

Book: INTRODUCTORY DIFFERENTIAL EQUATIONS. Martha L. Abell, James P. Braselton. Fourth edition 2014. ElScAe. 2014
Section: Chapter 2. First Order Equations. Exercises 2.5, page 64
Problem number: 42 (b).
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[[_homogeneous, ‘class C‘], _exact, _rational, [_Abel, ‘2nd ty`

$$\boxed{2y + (2t + y + 1)\, y' = -5t - 1}$$

✓ Solution by Maple
Time used: 0.828 (sec). Leaf size: 32

```
dsolve((5*t+2*y(t)+1)+(2*t+y(t)+1)*diff(y(t),t)=0,y(t), singsol=all)
```

$$y(t) = \frac{-\sqrt{-\left(t-1\right)^2 c_1^2 + 1} + \left(-2t - 1\right) c_1}{c_1}$$

✓ Solution by Mathematica
Time used: 0.228 (sec). Leaf size: 53

```
DSolve[(5*t+2*y[t]+1)+(2*t+y[t]+1)*y'[t]==0,y[t],t,IncludeSingularSolutions -> True]
```

$$y(t) \to -\sqrt{-t^2 + 2t + 1 + c_1} - 2t - 1$$
$$y(t) \to \sqrt{-t^2 + 2t + 1 + c_1} - 2t - 1$$

7.44 problem 42 (c)

Internal problem ID [14387]

Book: INTRODUCTORY DIFFERENTIAL EQUATIONS. Martha L. Abell, James P. Braselton. Fourth edition 2014. ElScAe. 2014
Section: Chapter 2. First Order Equations. Exercises 2.5, page 64
Problem number: 42 (c).
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[[_homogeneous, ‘class C‘], _rational, [_Abel, ‘2nd type‘, ‘cl`

$$\boxed{-y-(6t-2y-3)\,y'=-3t-1}$$

✓ Solution by Maple
Time used: 0.047 (sec). Leaf size: 23

```
dsolve((3*t-y(t)+1)-(6*t-2*y(t)-3)*diff(y(t),t)=0,y(t), singsol=all)
```

$$y(t)=-\frac{\operatorname{LambertW}\left(-2\,\mathrm{e}^{5t-4-5c_1}\right)}{2}+3t-2$$

✓ Solution by Mathematica
Time used: 3.885 (sec). Leaf size: 35

```
DSolve[(3*t-y[t]+1)-(6*t-2*y[t]-3)*y'[t]==0,y[t],t,IncludeSingularSolutions -> True]
```

$$y(t)\to -\frac{1}{2}W\left(-e^{5t-1+c_1}\right)+3t-2$$
$$y(t)\to 3t-2$$

7.45 problem 42 (d)

Internal problem ID [14388]

Book: INTRODUCTORY DIFFERENTIAL EQUATIONS. Martha L. Abell, James P. Braselton. Fourth edition 2014. ElScAe. 2014
Section: Chapter 2. First Order Equations. Exercises 2.5, page 64
Problem number: 42 (d).
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[[_homogeneous, 'class C'], _rational, [_Abel, '2nd type', 'cl`

$$\boxed{3y + (4t + 6y + 1)\, y' = -2t - 1}$$

✓ Solution by Maple
Time used: 0.047 (sec). Leaf size: 23

```
dsolve((2*t+3*y(t)+1)+(4*t+6*y(t)+1)*diff(y(t),t)=0,y(t), singsol=all)
```

$$y(t) = \frac{\operatorname{LambertW}\left(\frac{2\,\mathrm{e}^{-\frac{2}{3}+\frac{t}{3}-\frac{c_1}{3}}}{3}\right)}{2} + \frac{1}{3} - \frac{2t}{3}$$

✓ Solution by Mathematica
Time used: 5.178 (sec). Leaf size: 43

```
DSolve[(2*t+3*y[t]+1)+(4*t+6*y[t]+1)*y'[t]==0,y[t],t,IncludeSingularSolutions -> True]
```

$$y(t) \to \frac{1}{6}\left(3W\left(-e^{\frac{t}{3}-1+c_1}\right) - 4t + 2\right)$$
$$y(t) \to \frac{1}{3}(1 - 2t)$$

7.46 problem 46

Internal problem ID [14389]

Book: INTRODUCTORY DIFFERENTIAL EQUATIONS. Martha L. Abell, James P. Braselton. Fourth edition 2014. ElScAe. 2014
Section: Chapter 2. First Order Equations. Exercises 2.5, page 64
Problem number: 46.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_separable]`

$$\boxed{y' - \frac{2y}{x} + x^2 y = 0}$$

✓ Solution by Maple
Time used: 0.0 (sec). Leaf size: 15

```
dsolve(diff(y(x),x)-(2/x)*y(x)=-x^2*y(x),y(x), singsol=all)
```

$$y(x) = c_1 x^2 \mathrm{e}^{-\frac{x^3}{3}}$$

✓ Solution by Mathematica
Time used: 0.033 (sec). Leaf size: 25

```
DSolve[y'[x]-(2/x)*y[x]==-x^2*y[x],y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to c_1 e^{-\frac{x^3}{3}} x^2$$
$$y(x) \to 0$$

7.47 problem 47

Internal problem ID [14390]

Book: INTRODUCTORY DIFFERENTIAL EQUATIONS. Martha L. Abell, James P. Braselton. Fourth edition 2014. ElScAe. 2014
Section: Chapter 2. First Order Equations. Exercises 2.5, page 64
Problem number: 47.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_Bernoulli]`

$$\boxed{y' + y\cot(x) - y^4 = 0}$$

With initial conditions

$$[y(0) = 0]$$

✓ Solution by Maple
Time used: 0.0 (sec). Leaf size: 5

```
dsolve([diff(y(x),x)+y(x)*cot(x)=y(x)^4,y(0) = 0],y(x), singsol=all)
```

$$y(x) = 0$$

✓ Solution by Mathematica
Time used: 0.001 (sec). Leaf size: 6

```
DSolve[{y'[x]+y[x]*Cot[x]==y[x]^4,{y[0]==0}},y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to 0$$

7.48 problem 51

Internal problem ID [14391]

Book: INTRODUCTORY DIFFERENTIAL EQUATIONS. Martha L. Abell, James P. Braselton. Fourth edition 2014. ElScAe. 2014
Section: Chapter 2. First Order Equations. Exercises 2.5, page 64
Problem number: 51.
ODE order: 1.
ODE degree: 3.

CAS Maple gives this as type `[[_1st_order, _with_linear_symmetries], _Clairaut]`

$$\boxed{y't - y'^3 - y = 0}$$

✓ Solution by Maple
Time used: 0.11 (sec). Leaf size: 35

```
dsolve(t*diff(y(t),t)-diff(y(t),t)^3=y(t),y(t), singsol=all)
```

$$y(t) = -\frac{2\sqrt{3}\, t^{\frac{3}{2}}}{9}$$
$$y(t) = \frac{2\sqrt{3}\, t^{\frac{3}{2}}}{9}$$
$$y(t) = c_1\left(-c_1^2 + t\right)$$

✓ Solution by Mathematica
Time used: 0.006 (sec). Leaf size: 52

```
DSolve[t*y'[t]-y'[t]^3==y[t],y[t],t,IncludeSingularSolutions -> True]
```

$$y(t) \to c_1\left(t - c_1{}^2\right)$$
$$y(t) \to -\frac{2t^{3/2}}{3\sqrt{3}}$$
$$y(t) \to \frac{2t^{3/2}}{3\sqrt{3}}$$

7.49 problem 52

Internal problem ID [14392]

Book: INTRODUCTORY DIFFERENTIAL EQUATIONS. Martha L. Abell, James P. Braselton. Fourth edition 2014. ElScAe. 2014
Section: Chapter 2. First Order Equations. Exercises 2.5, page 64
Problem number: 52.
ODE order: 1.
ODE degree: 2.

CAS Maple gives this as type `[[_1st_order, _with_linear_symmetries], _rational, _Clairaut]`

$$\boxed{y't - y - 2(y't - y)^2 - y' = 1}$$

✓ Solution by Maple
Time used: 0.079 (sec). Leaf size: 53

```
dsolve(t*diff(y(t),t)-y(t)-2*(t*diff(y(t),t)-y(t))^2=diff(y(t),t)+1,y(t), singsol=all)
```

$$y(t) = \frac{-7t^2 - 2t + 1}{8t}$$
$$y(t) = c_1 t - \frac{1}{4} - \frac{\sqrt{-8c_1 - 7}}{4}$$
$$y(t) = c_1 t - \frac{1}{4} + \frac{\sqrt{-8c_1 - 7}}{4}$$

✓ Solution by Mathematica
Time used: 2.048 (sec). Leaf size: 90

```
DSolve[t*y'[t]-y[t]-2*(t*y'[t]-y[t])^2==y'[t]+1,y[t],t,IncludeSingularSolutions -> True]
```

$$y(t) \to \frac{1}{8}\left(-7t - e^{-2c_1}t - 2 + 2e^{-c_1}\right)$$
$$y(t) \to \frac{1}{8}\left(-7t - e^{-2c_1}t - 2 - 2e^{-c_1}\right)$$
$$y(t) \to \frac{1}{8}(-7t - 2)$$
$$y(t) \to \frac{1}{8}\left(-7t + \frac{1}{t} - 2\right)$$

7.50 problem 53

Internal problem ID [14393]

Book: INTRODUCTORY DIFFERENTIAL EQUATIONS. Martha L. Abell, James P. Braselton. Fourth edition 2014. ElScAe. 2014
Section: Chapter 2. First Order Equations. Exercises 2.5, page 64
Problem number: 53.
ODE order: 1.
ODE degree: 2.

CAS Maple gives this as type `[[_1st_order, _with_linear_symmetries], _Clairaut]`

$$\boxed{y't - y - y'^2 + y' = 1}$$

✓ Solution by Maple
Time used: 0.047 (sec). Leaf size: 28

```
dsolve(t*diff(y(t),t)-y(t)-1=diff(y(t),t)^2-diff(y(t),t),y(t), singsol=all)
```

$$y(t) = \frac{1}{4}t^2 + \frac{1}{2}t - \frac{3}{4}$$
$$y(t) = -c_1^2 + c_1 t + c_1 - 1$$

✓ Solution by Mathematica
Time used: 0.007 (sec). Leaf size: 35

```
DSolve[t*y'[t]-y[t]-1==y'[t]^2-y'[t],y[t],t,IncludeSingularSolutions -> True]
```

$$y(t) \to c_1(t+1) - 1 - c_1{}^2$$
$$y(t) \to \frac{1}{4}(t^2 + 2t - 3)$$

7.51 problem 54

Internal problem ID [14394]

Book: INTRODUCTORY DIFFERENTIAL EQUATIONS. Martha L. Abell, James P. Braselton. Fourth edition 2014. ElScAe. 2014
Section: Chapter 2. First Order Equations. Exercises 2.5, page 64
Problem number: 54.
ODE order: 1.
ODE degree: 0.

CAS Maple gives this as type `[[_1st_order, _with_linear_symmetries], _Clairaut]`

$$\boxed{y - y't - \ln(y') = -1}$$

✓ Solution by Maple
Time used: 0.0 (sec). Leaf size: 22

```
dsolve(1+y(t)-t*diff(y(t),t)=ln(diff(y(t),t)),y(t), singsol=all)
```

$$y(t) = \ln\left(-\frac{1}{t}\right) - 2$$
$$y(t) = -1 + c_1 t + \ln(c_1)$$

✓ Solution by Mathematica
Time used: 0.055 (sec). Leaf size: 26

```
DSolve[1+y[t]-t*y'[t]==Log[y'[t]],y[t],t,IncludeSingularSolutions -> True]
```

$$y(t) \to c_1 t - 1 + \log(c_1)$$
$$y(t) \to \log\left(-\frac{1}{t}\right) - 2$$

7.52 problem 55

Internal problem ID [14395]

Book: INTRODUCTORY DIFFERENTIAL EQUATIONS. Martha L. Abell, James P. Braselton. Fourth edition 2014. ElScAe. 2014
Section: Chapter 2. First Order Equations. Exercises 2.5, page 64
Problem number: 55.
ODE order: 1.
ODE degree: 3.

CAS Maple gives this as type `[[_1st_order, _with_linear_symmetries], _Clairaut]`

$$\boxed{-2y't + 2y - \frac{1}{y'^2} = -1}$$

✓ Solution by Maple
Time used: 0.078 (sec). Leaf size: 62

```
dsolve(1-2*(t*diff(y(t),t)-y(t))=1/diff(y(t),t)^2,y(t), singsol=all)
```

$$\begin{aligned}
y(t) &= \frac{3t^{\frac{2}{3}}}{2} - \frac{1}{2} \\
y(t) &= -\frac{3t^{\frac{2}{3}}}{4} - \frac{3i\sqrt{3}\, t^{\frac{2}{3}}}{4} - \frac{1}{2} \\
y(t) &= -\frac{3t^{\frac{2}{3}}}{4} + \frac{3i\sqrt{3}\, t^{\frac{2}{3}}}{4} - \frac{1}{2} \\
y(t) &= -\frac{1}{2} + c_1 t + \frac{1}{2c_1^2}
\end{aligned}$$

✓ Solution by Mathematica

Time used: 0.016 (sec). Leaf size: 93

```
DSolve[1-2*(t*y'[t]-y[t])==1/y'[t]^2,y[t],t,IncludeSingularSolutions -> True]
```

$$y(t) \rightarrow \frac{1}{2}\left(2c_1t - 1 + \frac{1}{c_1{}^2}\right)$$
$$y(t) \rightarrow \frac{1}{2}\left(3t^{2/3} - 1\right)$$
$$y(t) \rightarrow -\frac{1}{2} + \frac{3}{4}i\left(\sqrt{3} + i\right)t^{2/3}$$
$$y(t) \rightarrow -\frac{1}{2} - \frac{3}{4}\left(1 + i\sqrt{3}\right)t^{2/3}$$

7.53 problem 58

Internal problem ID [14396]

Book: INTRODUCTORY DIFFERENTIAL EQUATIONS. Martha L. Abell, James P. Braselton. Fourth edition 2014. ElScAe. 2014
Section: Chapter 2. First Order Equations. Exercises 2.5, page 64
Problem number: 58.
ODE order: 1.
ODE degree: 5.

CAS Maple gives this as type `[_dAlembert]`

$$\boxed{y + y't - \frac{y'^5}{5} = 0}$$

✓ Solution by Maple
Time used: 0.109 (sec). Leaf size: 32

```
dsolve(y(t)=-t*diff(y(t),t)+1/5*diff(y(t),t)^5,y(t), singsol=all)
```

$$\left[t(\_T) = \frac{\_T^{\frac{9}{2}} + 9c_1}{9\sqrt{\_T}}, y(\_T) = \frac{4\_T^5}{45} - \sqrt{\_T}\, c_1\right]$$

✓ Solution by Mathematica
Time used: 2.223 (sec). Leaf size: 2076

```
DSolve[y[t]==-t*y'[t]+1/5*y'[t]^5,y[t],t,IncludeSingularSolutions -> True]
```

Too large to display

7.54 problem 59

Internal problem ID [14397]

Book: INTRODUCTORY DIFFERENTIAL EQUATIONS. Martha L. Abell, James P. Braselton. Fourth edition 2014. ElScAe. 2014
Section: Chapter 2. First Order Equations. Exercises 2.5, page 64
Problem number: 59.
ODE order: 1.
ODE degree: 3.

CAS Maple gives this as type `[_dAlembert]`

$$\boxed{y - t{y'}^2 - 3{y'}^2 + 2{y'}^3 = 0}$$

✓ Solution by Maple
Time used: 0.094 (sec). Leaf size: 742

```
dsolve(y(t)=t*diff(y(t),t)^2+3*diff(y(t),t)^2-2*diff(y(t),t)^3,y(t), singsol=all)
```

$$y(t) = 0$$

$$y(t) = -\frac{\left(\left(t^3+6\sqrt{3}\,\sqrt{-c_1\,(t^3-27c_1)}-54c_1\right)^{\frac{2}{3}}+(t+6)\left(t^3+6\sqrt{3}\,\sqrt{-c_1\,(t^3-27c_1)}-54c_1\right)^{\frac{1}{3}}+t^2\right)^2\left(\left(t^3\right.\right.}{108t^3+648\sqrt{3}\,\sqrt{-c_1\,(t^3}}$$

$$y(t) = \frac{\left(i\left(t^3+6\sqrt{3}\,\sqrt{-c_1\,(t^3-27c_1)}-54c_1\right)^{\frac{2}{3}}\sqrt{3}-i\sqrt{3}\,t^2+\left(t^3+6\sqrt{3}\,\sqrt{-c_1\,(t^3-27c_1)}-54c_1\right)^{\frac{2}{3}}-2t\left(t^3\right.\right.}{}$$

$$y(t) = \frac{\left(i\sqrt{3}\,t^2-i\left(t^3+6\sqrt{3}\,\sqrt{-c_1\,(t^3-27c_1)}-54c_1\right)^{\frac{2}{3}}\sqrt{3}+t^2-2t\left(t^3+6\sqrt{3}\,\sqrt{-c_1\,(t^3-27c_1)}-54c_1\right)^{\frac{1}{3}}-\right.}{}$$

✓ Solution by Mathematica

Time used: 135.31 (sec). Leaf size: 875

```
DSolve[y[t]==t*y'[t]^2+3*y'[t]^2-2*y'[t]^3,y[t],t,IncludeSingularSolutions -> True]
```

$$
\begin{aligned}
y(t) \to \frac{1}{12}\Bigg(& -\frac{2t^3}{\sqrt[3]{-t^3+6\left(\sqrt{3}\sqrt{(2+c_1)\left(-t^3+54+27c_1\right)}+18+9c_1\right)}} \\
& - 2t\left(-6+\sqrt[3]{-t^3+6\left(\sqrt{3}\sqrt{(2+c_1)\left(-t^3+54+27c_1\right)}+18+9c_1\right)}\right) \\
& - \left(-t^3+6\sqrt{3}\sqrt{(2+c_1)\left(-t^3+54+27c_1\right)}+108+54c_1\right)^{2/3}+t^2 \\
& - \frac{t^4}{\left(-t^3+6\left(\sqrt{3}\sqrt{(2+c_1)\left(-t^3+54+27c_1\right)}+18+9c_1\right)\right)^{2/3}}+36+12c_1\Bigg)
\end{aligned}
$$

$$
\begin{aligned}
y(t) \to \frac{1}{24}\Bigg(& \frac{2\left(1-i\sqrt{3}\right)t^3}{\sqrt[3]{-t^3+6\left(\sqrt{3}\sqrt{(2+c_1)\left(-t^3+54+27c_1\right)}+18+9c_1\right)}} \\
& + 2t\Bigg(i\sqrt{3}\sqrt[3]{-t^3+6\left(\sqrt{3}\sqrt{(2+c_1)\left(-t^3+54+27c_1\right)}+18+9c_1\right)} \\
& + \sqrt[3]{-t^3+6\left(\sqrt{3}\sqrt{(2+c_1)\left(-t^3+54+27c_1\right)}+18+9c_1\right)}+12\Bigg) \\
& - i\sqrt{3}\left(-t^3+6\sqrt{3}\sqrt{(2+c_1)\left(-t^3+54+27c_1\right)}+108+54c_1\right)^{2/3} \\
& + \left(-t^3+6\sqrt{3}\sqrt{(2+c_1)\left(-t^3+54+27c_1\right)}+108+54c_1\right)^{2/3}+2t^2 \\
& + \frac{\left(1+i\sqrt{3}\right)t^4}{\left(-t^3+6\left(\sqrt{3}\sqrt{(2+c_1)\left(-t^3+54+27c_1\right)}+18+9c_1\right)\right)^{2/3}}+72+24c_1\Bigg)
\end{aligned}
$$

$$
\begin{aligned}
y(t) \to \frac{1}{24}\Bigg(& \frac{2\left(1+i\sqrt{3}\right)t^3}{\sqrt[3]{-t^3+6\left(\sqrt{3}\sqrt{(2+c_1)\left(-t^3+54+27c_1\right)}+18+9c_1\right)}} \\
& + 2t\Bigg(-i\sqrt{3}\sqrt[3]{-t^3+6\left(\sqrt{3}\sqrt{(2+c_1)\left(-t^3+54+27c_1\right)}+18+9c_1\right)} \\
& + \sqrt[3]{-t^3+6\left(\sqrt{3}\sqrt{(2+c_1)\left(-t^3+54+27c_1\right)}+18+9c_1\right)}+12\Bigg) \\
& + i\sqrt{3}\left(-t^3+6\sqrt{3}\sqrt{(2+c_1)\left(-t^3+54+27c_1\right)}+108+54c_1\right)^{2/3}
\end{aligned}
$$

7.55 problem 60

Internal problem ID [14398]

Book: INTRODUCTORY DIFFERENTIAL EQUATIONS. Martha L. Abell, James P. Braselton. Fourth edition 2014. ElScAe. 2014
Section: Chapter 2. First Order Equations. Exercises 2.5, page 64
Problem number: 60.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_linear]`

$$\boxed{y - t(y' + 1) - 2y' = 1}$$

✓ Solution by Maple
Time used: 0.0 (sec). Leaf size: 21

```
dsolve(y(t)=t*(diff(y(t),t)+1)+(2*diff(y(t),t)+1),y(t), singsol=all)
```

$$y(t) = (-t - 2)\ln(t + 2) - 1 + (t + 2)\, c_1$$

✓ Solution by Mathematica
Time used: 0.038 (sec). Leaf size: 22

```
DSolve[y[t]==t*(y'[t]+1)+(2*y'[t]+1),y[t],t,IncludeSingularSolutions -> True]
```

$$y(t) \to -(t + 2)\log(t + 2) + c_1(t + 2) - 1$$

7.56 problem 61

Internal problem ID [14399]

Book: INTRODUCTORY DIFFERENTIAL EQUATIONS. Martha L. Abell, James P. Braselton. Fourth edition 2014. ElScAe. 2014
Section: Chapter 2. First Order Equations. Exercises 2.5, page 64
Problem number: 61.
ODE order: 1.
ODE degree: 2.

CAS Maple gives this as type `[[_1st_order, _with_linear_symmetries], _dAlembert]`

$$\boxed{y - t(2 - y') - 2{y'}^2 = 1}$$

✓ Solution by Maple

Time used: 0.094 (sec). Leaf size: 670

```
dsolve(y(t)=t*(2-diff(y(t),t))+(2*diff(y(t),t)^2+1),y(t), singsol=all)
```

$$y(t) = -\frac{\left(\frac{t-4}{\left(-3c_1+\sqrt{-t^3+9c_1^2+12t^2-48t+64}\right)^{\frac{1}{3}}} + \left(-3c_1 + \sqrt{-t^3 + 9c_1^2 + 12t^2 - 48t + 64}\right)^{\frac{1}{3}} - 2\right) t \left(\frac{t-}{\left(-3c_1+\sqrt{-t^3+9c_1^2}\right.}\right.}{4}$$
$$+ \frac{\left(\frac{(t-4)^2}{\left(-3c_1+\sqrt{-t^3+9c_1^2+12t^2-48t+64}\right)^{\frac{2}{3}}} + 2t - 4 + \left(-3c_1 + \sqrt{-t^3 + 9c_1^2 + 12t^2 - 48t + 64}\right)^{\frac{2}{3}}\right)^2}{8}$$
$$+ 1$$

$$y(t) = -\frac{\left(1 + i\sqrt{3}\right)\left(t^3 - 6c_1\sqrt{-t^3 + 9c_1^2 + 12t^2 - 48t + 64} + 18c_1^2 - 12t^2 + 48t - 64\right)\left(-3c_1 + \sqrt{-t^3 + 9c_1^2 -\right.}{}$$

$$y(t) = \frac{\left(t^3 - 6c_1\sqrt{-t^3 + 9c_1^2 + 12t^2 - 48t + 64} + 18c_1^2 - 12t^2 + 48t - 64\right)\left(i\sqrt{3} - 1\right)\left(-3c_1 + \sqrt{-t^3 + 9c_1^2 + 1\right.}{}$$

✓ Solution by Mathematica

Time used: 61.119 (sec). Leaf size: 2369

```
DSolve[y[t]==t*(2-y'[t])+(2*y'[t]^2+1),y[t],t,IncludeSingularSolutions -> True]
```

$$y(t) \to \frac{2\sqrt[3]{2}t^4 - 32\sqrt[3]{2}t^3 + 192\sqrt[3]{2}t^2 + 4t^2\sqrt[3]{-2t^6 + 48t^5 - 480t^4 + 2560t^3 - 10e^{3c_1}t^3 - 7680t^2 + 120e^{3c_1}t^2 + 12}}{}$$

$$y(t) \to \frac{t^2}{4} + \frac{i(\sqrt{3}+i)\sqrt[3]{-2t^6 + 48t^5 - 480t^4 + 2560t^3 - 10e^{3c_1}t^3 - 7680t^2 + 120e^{3c_1}t^2 + 12288t - 480e^{3c_1}t + \sqrt{e}}}{16\sqrt[3]{2}} + \frac{(1+i\sqrt{3})(t-4)(-(t-4)^3 + 2e^{3c_1})}{8\,2^{2/3}\sqrt[3]{-2t^6 + 48t^5 - 480t^4 + 2560t^3 - 10e^{3c_1}t^3 - 7680t^2 + 120e^{3c_1}t^2 + 12288t - 480e^{3c_1}t + \sqrt{e^{3c_1}(}}} - t + 7$$

$$y(t) \to \frac{t^2}{4} - \frac{i(\sqrt{3}-i)\sqrt[3]{-2t^6 + 48t^5 - 480t^4 + 2560t^3 - 10e^{3c_1}t^3 - 7680t^2 + 120e^{3c_1}t^2 + 12288t - 480e^{3c_1}t + \sqrt{e}}}{16\sqrt[3]{2}} + \frac{i(\sqrt{3}+i)(t-4)((t-4)^3 - 2e^{3c_1})}{8\,2^{2/3}\sqrt[3]{-2t^6 + 48t^5 - 480t^4 + 2560t^3 - 10e^{3c_1}t^3 - 7680t^2 + 120e^{3c_1}t^2 + 12288t - 480e^{3c_1}t + \sqrt{e^{3c_1}(}}} - t + 7$$

$$y(t) \to \frac{t^4 - 16t^3 + 96t^2 + 2t^2\sqrt[3]{-t^6 + 24t^5 - 240t^4 + 1280t^3 + 20e^{3c_1}t^3 - 3840t^2 - 240e^{3c_1}t^2 + 6144t + 960e^{3c}}}{}$$

$$y(t) \to \frac{t^2}{4} + \frac{1}{16}i\Big(\sqrt{3} + i\Big)\sqrt[3]{-t^6 + 24t^5 - 240t^4 + 1280t^3 + 20e^{3c_1}t^3 - 3840t^2 - 240e^{3c_1}t^2 + 6144t + 960e^{3c_1}t + 8\sqrt{e^{3c_1}(-(t}} - \frac{i(\sqrt{3}-i)(t-4)((t-4)^3 + 8e^{3c_1})}{16\sqrt[3]{-t^6 + 24t^5 - 240t^4 + 1280t^3 + 20e^{3c_1}t^3 - 3840t^2 - 240e^{3c_1}t^2 + 6144t + 960e^{3c_1}t + 8\sqrt{e^{3c_1}(-(t}}} - t + 7$$

$$y(t) \to \frac{t^2}{4} - \frac{1}{16}i\Big(\sqrt{3} - i\Big)\sqrt[3]{-t^6 + 24t^5 - 240t^4 + 1280t^3 + 20e^{3c_1}t^3 - 3840t^2 - 240e^{3c_1}t^2 + 6144t + 960e^{3c_1}t + 8\sqrt{e^{3c_1}(-(t}} + \frac{i(\sqrt{3}+i)(t-4)((t-4)^3 + 8e^{3c_1})}{16\sqrt[3]{-t^6 + 24t^5 - 240t^4 + 1280t^3 + 20e^{3c_1}t^3 - 3840t^2 - 240e^{3c_1}t^2 + 6144t + 960e^{3c_1}t + 8\sqrt{e^{3c_1}(-(t}}} - t + 7$$

7.57 problem 64

Internal problem ID [14400]

Book: INTRODUCTORY DIFFERENTIAL EQUATIONS. Martha L. Abell, James P. Braselton. Fourth edition 2014. ElScAe. 2014
Section: Chapter 2. First Order Equations. Exercises 2.5, page 64
Problem number: 64.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[[_homogeneous, 'class G'], _rational]`

$$\boxed{t^{\frac{1}{3}} y^{\frac{2}{3}} + \left(t^{\frac{2}{3}} y^{\frac{1}{3}} + y\right) y' = -t}$$

✓ Solution by Maple

Time used: 0.046 (sec). Leaf size: 171

```
dsolve((t^(1/3)*y(t)^(2/3)+t)+(t^(2/3)*y(t)^(1/3)+y(t))*diff(y(t),t)=0,y(t), singsol=all)
```

$$-\left(\int_{\_b}^{t} \frac{y(t)^{\frac{8}{3}} \_a^{\frac{1}{3}} - y(t)^{\frac{4}{3}} \_a^{\frac{5}{3}} + \_a^{3}}{\_a^{4} + y\left(t\right)^{4}} d\_a\right)$$
$$-\frac{\left(\int^{y(t)} \frac{\left(4t^{2}+4\_f^{\frac{2}{3}} t^{\frac{4}{3}}+4\_f^{\frac{4}{3}} t^{\frac{2}{3}}+4\_f^{2}\right)\left(\int_{\_b}^{t} \frac{\_a^{\frac{17}{3}}\_f^{\frac{1}{3}}-2\_a^{\frac{13}{3}}\_f^{\frac{5}{3}}-2\_a^{\frac{5}{3}}\_f^{\frac{13}{3}}+\_a^{\frac{1}{3}}\_f^{\frac{17}{3}}+3\_a^{3}\_f^{3}}{\left(\_a^{4}+\_f^{4}\right)^{2}} d\_a\right)+3t^{\frac{2}{3}}\_f^{\frac{1}{3}}+3\_}{\_f^{\frac{4}{3}} t^{\frac{2}{3}}+\_f^{\frac{2}{3}} t^{\frac{4}{3}}+t^{2}+\_f^{2}}\right.}{3}$$
$$+ c_1 = 0$$

✓ Solution by Mathematica

Time used: 1.792 (sec). Leaf size: 53

```
DSolve[(t^(1/3)*y[t]^(2/3)+t)+(t^(2/3)*y[t]^(1/3)+y[t])*y'[t]==0,y[t],t,IncludeSingularSolut
```

$$y(t) \to \left(-t^{2/3}\right)^{3/2}$$
$$y(t) \to \left(-t^{4/3} + \frac{4c_1}{3}\right)^{3/4}$$
$$y(t) \to \left(-t^{2/3}\right)^{3/2}$$

7.58 problem 65

Internal problem ID [14401]

Book: INTRODUCTORY DIFFERENTIAL EQUATIONS. Martha L. Abell, James P. Braselton. Fourth edition 2014. ElScAe. 2014
Section: Chapter 2. First Order Equations. Exercises 2.5, page 64
Problem number: 65.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[[_homogeneous, ‘class A‘], _rational, _Bernoulli]`

$$\boxed{y' - \frac{-t^2 + y^2}{ty} = 0}$$

With initial conditions

$$[y(4) = 0]$$

✓ Solution by Maple
Time used: 0.094 (sec). Leaf size: 34

```
dsolve([diff(y(t),t)=(y(t)^2-t^2)/(t*y(t)),y(4) = 0],y(t), singsol=all)
```

$$y(t) = \sqrt{-2\ln(t) + 4\ln(2)}\, t$$
$$y(t) = -\sqrt{-2\ln(t) + 4\ln(2)}\, t$$

✓ Solution by Mathematica
Time used: 0.21 (sec). Leaf size: 36

```
DSolve[{y'[t]==(y[t]^2-t^2)/(t*y[t]),{y[4]==0}},y[t],t,IncludeSingularSolutions -> True]
```

$$y(t) \to -t\sqrt{\log(16) - 2\log(t)}$$
$$y(t) \to t\sqrt{\log(16) - 2\log(t)}$$

7.59 problem 66

Internal problem ID [14402]

Book: INTRODUCTORY DIFFERENTIAL EQUATIONS. Martha L. Abell, James P. Braselton. Fourth edition 2014. ElScAe. 2014
Section: Chapter 2. First Order Equations. Exercises 2.5, page 64
Problem number: 66.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[[_homogeneous, ‘class A‘], _dAlembert]`

$$\boxed{y \sin\left(\frac{t}{y}\right) - \left(t + t \sin\left(\frac{t}{y}\right)\right) y' = 0}$$

With initial conditions

$$[y(1) = 2]$$

✓ Solution by Maple
Time used: 0.469 (sec). Leaf size: 23

```
dsolve([y(t)*sin(t/y(t))-(t+t*sin(t/y(t)))*diff(y(t),t)=0,y(1) = 2],y(t), singsol=all)
```

$$y(t) = e^{\operatorname{RootOf}\left(\_Z - \operatorname{Si}\left(t\, e^{-\_Z}\right) - \ln(2) + \operatorname{Si}\left(\frac{1}{2}\right)\right)}$$

✓ Solution by Mathematica
Time used: 0.204 (sec). Leaf size: 34

```
DSolve[{y[t]*Sin[t/y[t]]-(t+t*Sin[t/y[t]])*y'[t]==0,{y[1]==2}},y[t],t,IncludeSingularSolutio
```

$$\text{Solve}\left[\log\left(\frac{y(t)}{t}\right) - \operatorname{Si}\left(\frac{t}{y(t)}\right) = -\operatorname{Si}\left(\frac{1}{2}\right) - \log(t) + \log(2), y(t)\right]$$

8 Chapter 2. First Order Equations. Review exercises, page 80

8.1 problem 1

Internal problem ID [14403]

Book: INTRODUCTORY DIFFERENTIAL EQUATIONS. Martha L. Abell, James P. Braselton. Fourth edition 2014. ElScAe. 2014
Section: Chapter 2. First Order Equations. Review exercises, page 80
Problem number: 1.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_separable]`

$$\boxed{y' - \frac{2t^5}{5y^2} = 0}$$

✓ Solution by Maple
Time used: 0.0 (sec). Leaf size: 65

```
dsolve(diff(y(t),t)=2*t^5/(5*y(t)^2),y(t), singsol=all)
```

$$y(t) = \frac{(25t^6 + 125c_1)^{\frac{1}{3}}}{5}$$
$$y(t) = -\frac{(25t^6 + 125c_1)^{\frac{1}{3}}\left(1 + i\sqrt{3}\right)}{10}$$
$$y(t) = \frac{(25t^6 + 125c_1)^{\frac{1}{3}}\left(i\sqrt{3} - 1\right)}{10}$$

✓ Solution by Mathematica

Time used: 0.209 (sec). Leaf size: 72

```
DSolve[y'[t]==2*t^5/(5*y[t]^2),y[t],t,IncludeSingularSolutions -> True]
```

$$y(t) \to -\sqrt[3]{-\frac{1}{5}}\sqrt[3]{t^6 + 15c_1}$$
$$y(t) \to \sqrt[3]{\frac{t^6}{5} + 3c_1}$$
$$y(t) \to (-1)^{2/3}\sqrt[3]{\frac{t^6}{5} + 3c_1}$$

8.2 problem 2

Internal problem ID [14404]

Book: INTRODUCTORY DIFFERENTIAL EQUATIONS. Martha L. Abell, James P. Braselton. Fourth edition 2014. ElScAe. 2014
Section: Chapter 2. First Order Equations. Review exercises, page 80
Problem number: 2.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_separable]`

$$\boxed{-8\sin(y)\, y' = -\cos(4x)}$$

✓ Solution by Maple
Time used: 0.0 (sec). Leaf size: 19

```
dsolve(cos(4*x)-8*sin(y(x))*diff(y(x),x)=0,y(x), singsol=all)
```

$$y(x) = \frac{\pi}{2} + \arcsin\left(\frac{\sin(4x)}{32} + \frac{c_1}{8}\right)$$

✓ Solution by Mathematica
Time used: 0.724 (sec). Leaf size: 39

```
DSolve[Cos[4*x]-8*Sin[y[x]]*y'[x]==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to -\arccos\left(-\frac{1}{32}\sin(4x) - c_1\right)$$
$$y(x) \to \arccos\left(-\frac{1}{32}\sin(4x) - c_1\right)$$

8.3 problem 3

Internal problem ID [14405]

Book: INTRODUCTORY DIFFERENTIAL EQUATIONS. Martha L. Abell, James P. Braselton. Fourth edition 2014. ElScAe. 2014
Section: Chapter 2. First Order Equations. Review exercises, page 80
Problem number: 3.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_separable]`

$$\boxed{y' - \frac{y}{t} - \frac{y^2}{t} = 0}$$

✓ Solution by Maple
Time used: 0.0 (sec). Leaf size: 13

```
dsolve(diff(y(t),t)-y(t)/t=y(t)^2/t,y(t), singsol=all)
```

$$y(t) = \frac{t}{-t + c_1}$$

✓ Solution by Mathematica
Time used: 0.262 (sec). Leaf size: 32

```
DSolve[y'[t]-y[t]/t==y[t]^2/t,y[t],t,IncludeSingularSolutions -> True]
```

$$y(t) \to \frac{e^{c_1} t}{1 - e^{c_1} t}$$
$$y(t) \to -1$$
$$y(t) \to 0$$

8.4 problem 4

Internal problem ID [14406]

Book: INTRODUCTORY DIFFERENTIAL EQUATIONS. Martha L. Abell, James P. Braselton. Fourth edition 2014. ElScAe. 2014
Section: Chapter 2. First Order Equations. Review exercises, page 80
Problem number: 4.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type **[_separable]**

$$\boxed{y' - \frac{e^{8y}}{t} = 0}$$

✓ Solution by Maple
Time used: 0.0 (sec). Leaf size: 20

```
dsolve(diff(y(t),t)=exp(8*y(t))/t,y(t), singsol=all)
```

$$y(t) = -\frac{3\ln(2)}{8} - \frac{\ln(-\ln(t) - c_1)}{8}$$

✓ Solution by Mathematica
Time used: 0.346 (sec). Leaf size: 17

```
DSolve[y'[t]==Exp[8*y[t]]/t,y[t],t,IncludeSingularSolutions -> True]
```

$$y(t) \to -\frac{1}{8}\log(-8(\log(t) + c_1))$$

8.5 problem 5

Internal problem ID [14407]

Book: INTRODUCTORY DIFFERENTIAL EQUATIONS. Martha L. Abell, James P. Braselton. Fourth edition 2014. ElScAe. 2014
Section: Chapter 2. First Order Equations. Review exercises, page 80
Problem number: 5.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_separable]`

$$\boxed{y' - \frac{e^{5t}}{y^4} = 0}$$

✓ Solution by Maple
Time used: 0.016 (sec). Leaf size: 144

```
dsolve(diff(y(t),t)=exp(5*t)/y(t)^4,y(t), singsol=all)
```

$$y(t) = \left(e^{5t} + c_1\right)^{\frac{1}{5}}$$
$$y(t) = -\frac{\left(i\sqrt{2}\,\sqrt{5-\sqrt{5}} + \sqrt{5} + 1\right)\left(e^{5t} + c_1\right)^{\frac{1}{5}}}{4}$$
$$y(t) = \frac{\left(i\sqrt{2}\,\sqrt{5-\sqrt{5}} - \sqrt{5} - 1\right)\left(e^{5t} + c_1\right)^{\frac{1}{5}}}{4}$$
$$y(t) = -\frac{\left(i\sqrt{2}\,\sqrt{5+\sqrt{5}} - \sqrt{5} + 1\right)\left(e^{5t} + c_1\right)^{\frac{1}{5}}}{4}$$
$$y(t) = \frac{\left(i\sqrt{2}\,\sqrt{5+\sqrt{5}} + \sqrt{5} - 1\right)\left(e^{5t} + c_1\right)^{\frac{1}{5}}}{4}$$

✓ Solution by Mathematica

Time used: 0.921 (sec). Leaf size: 117

```
DSolve[y'[t]==Exp[5*t]/y[t]^4,y[t],t,IncludeSingularSolutions -> True]
```

$$y(t) \to \sqrt[5]{e^{5t} + 5c_1}$$
$$y(t) \to -\sqrt[5]{-1}\sqrt[5]{e^{5t} + 5c_1}$$
$$y(t) \to (-1)^{2/5}\sqrt[5]{e^{5t} + 5c_1}$$
$$y(t) \to -(-1)^{3/5}\sqrt[5]{e^{5t} + 5c_1}$$
$$y(t) \to (-1)^{4/5}\sqrt[5]{e^{5t} + 5c_1}$$

8.6 problem 6

Internal problem ID [14408]

Book: INTRODUCTORY DIFFERENTIAL EQUATIONS. Martha L. Abell, James P. Braselton. Fourth edition 2014. ElScAe. 2014
Section: Chapter 2. First Order Equations. Review exercises, page 80
Problem number: 6.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_separable]`

$$\boxed{-\left(2y^4-6y^9\right)y'=\frac{1}{x^5}-\frac{1}{x^3}}$$

✓ Solution by Maple

Time used: 0.047 (sec). Leaf size: 530

```
dsolve((-x^(-5)+x^(-3))=(2*y(x)^4-6*y(x)^9)*diff(y(x),x),y(x), singsol=all)
```

$$y(x) = \frac{6^{\frac{4}{5}} \left(2x^5 + x^3\sqrt{-60c_1x^4 + 30x^2 - 15}\right)^{\frac{1}{5}}}{6x}$$

$$y(x) = \frac{6^{\frac{4}{5}} \left(2x^5 - x^3\sqrt{-60c_1x^4 + 30x^2 - 15}\right)^{\frac{1}{5}}}{6x}$$

$$y(x) = -\frac{\left(i\sqrt{10-2\sqrt{5}} + \sqrt{5} + 1\right) 6^{\frac{4}{5}} \left(2x^5 + x^3\sqrt{-60c_1x^4 + 30x^2 - 15}\right)^{\frac{1}{5}}}{24x}$$

$$y(x) = \frac{\left(i\sqrt{10-2\sqrt{5}} - \sqrt{5} - 1\right) 6^{\frac{4}{5}} \left(2x^5 + x^3\sqrt{-60c_1x^4 + 30x^2 - 15}\right)^{\frac{1}{5}}}{24x}$$

$$y(x) = -\frac{\left(i\sqrt{10+2\sqrt{5}} - \sqrt{5} + 1\right) 6^{\frac{4}{5}} \left(2x^5 + x^3\sqrt{-60c_1x^4 + 30x^2 - 15}\right)^{\frac{1}{5}}}{24x}$$

$$y(x) = \frac{\left(i\sqrt{10+2\sqrt{5}} + \sqrt{5} - 1\right) 6^{\frac{4}{5}} \left(2x^5 + x^3\sqrt{-60c_1x^4 + 30x^2 - 15}\right)^{\frac{1}{5}}}{24x}$$

$$y(x) = -\frac{\left(i\sqrt{10-2\sqrt{5}} + \sqrt{5} + 1\right) 6^{\frac{4}{5}} \left(2x^5 - x^3\sqrt{-60c_1x^4 + 30x^2 - 15}\right)^{\frac{1}{5}}}{24x}$$

$$y(x) = \frac{\left(i\sqrt{10-2\sqrt{5}} - \sqrt{5} - 1\right) 6^{\frac{4}{5}} \left(2x^5 - x^3\sqrt{-60c_1x^4 + 30x^2 - 15}\right)^{\frac{1}{5}}}{24x}$$

$$y(x) = -\frac{\left(i\sqrt{10+2\sqrt{5}} - \sqrt{5} + 1\right) 6^{\frac{4}{5}} \left(2x^5 - x^3\sqrt{-60c_1x^4 + 30x^2 - 15}\right)^{\frac{1}{5}}}{24x}$$

$$y(x) = \frac{\left(i\sqrt{10+2\sqrt{5}} + \sqrt{5} - 1\right) 6^{\frac{4}{5}} \left(2x^5 - x^3\sqrt{-60c_1x^4 + 30x^2 - 15}\right)^{\frac{1}{5}}}{24x}$$

✓ Solution by Mathematica

Time used: 16.779 (sec). Leaf size: 409

```
DSolve[(-x^(-5)+x^(-3))==(2*y[x]^4-6*y[x]^9)*y'[x],y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to \frac{\sqrt[5]{2-\sqrt{-\frac{15}{x^4}+\frac{30}{x^2}+4+120c_1}}}{\sqrt[5]{6}}$$

$$y(x) \to -\sqrt[5]{-\frac{1}{6}}\sqrt[5]{2-\sqrt{-\frac{15}{x^4}+\frac{30}{x^2}+4+120c_1}}$$

$$y(x) \to \frac{(-1)^{2/5}\sqrt[5]{2-\sqrt{-\frac{15}{x^4}+\frac{30}{x^2}+4+120c_1}}}{\sqrt[5]{6}}$$

$$y(x) \to -\frac{(-1)^{3/5}\sqrt[5]{2-\sqrt{-\frac{15}{x^4}+\frac{30}{x^2}+4+120c_1}}}{\sqrt[5]{6}}$$

$$y(x) \to \frac{(-1)^{4/5}\sqrt[5]{2-\sqrt{-\frac{15}{x^4}+\frac{30}{x^2}+4+120c_1}}}{\sqrt[5]{6}}$$

$$y(x) \to \frac{\sqrt[5]{2+\sqrt{-\frac{15}{x^4}+\frac{30}{x^2}+4+120c_1}}}{\sqrt[5]{6}}$$

$$y(x) \to -\sqrt[5]{-\frac{1}{6}}\sqrt[5]{2+\sqrt{-\frac{15}{x^4}+\frac{30}{x^2}+4+120c_1}}$$

$$y(x) \to \frac{(-1)^{2/5}\sqrt[5]{2+\sqrt{-\frac{15}{x^4}+\frac{30}{x^2}+4+120c_1}}}{\sqrt[5]{6}}$$

$$y(x) \to -\frac{(-1)^{3/5}\sqrt[5]{2+\sqrt{-\frac{15}{x^4}+\frac{30}{x^2}+4+120c_1}}}{\sqrt[5]{6}}$$

$$y(x) \to \frac{(-1)^{4/5}\sqrt[5]{2+\sqrt{-\frac{15}{x^4}+\frac{30}{x^2}+4+120c_1}}}{\sqrt[5]{6}}$$

8.7 problem 7

Internal problem ID [14409]

Book: INTRODUCTORY DIFFERENTIAL EQUATIONS. Martha L. Abell, James P. Braselton. Fourth edition 2014. ElScAe. 2014
Section: Chapter 2. First Order Equations. Review exercises, page 80
Problem number: 7.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_separable]`

$$\boxed{y' - \frac{y\,\mathrm{e}^{-2t}}{\ln(y)} = 0}$$

✓ Solution by Maple

Time used: 0.063 (sec). Leaf size: 35

```
dsolve(diff(y(t),t)=y(t)/(exp(2*t)*ln(y(t))),y(t), singsol=all)
```

$$y(t) = \mathrm{e}^{\sqrt{2c_1-\mathrm{e}^{-2t}}}$$
$$y(t) = \mathrm{e}^{-\sqrt{2c_1-\mathrm{e}^{-2t}}}$$

✓ Solution by Mathematica

Time used: 10.553 (sec). Leaf size: 61

```
DSolve[y'[t]==y[t]/(Exp[2*t]*Log[y[t]]),y[t],t,IncludeSingularSolutions -> True]
```

$$y(t) \to e^{-e^{-t}\sqrt{-1+2c_1e^{2t}}}$$
$$y(t) \to e^{e^{-t}\sqrt{-1+2c_1e^{2t}}}$$
$$y(t) \to 0$$

8.8 problem 8

Internal problem ID [14410]

Book: INTRODUCTORY DIFFERENTIAL EQUATIONS. Martha L. Abell, James P. Braselton. Fourth edition 2014. ElScAe. 2014
Section: Chapter 2. First Order Equations. Review exercises, page 80
Problem number: 8.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_separable]`

$$\boxed{y' - \frac{(4-7x)(2y-3)}{(x-1)(2x-5)} = 0}$$

✓ Solution by Maple
Time used: 0.0 (sec). Leaf size: 69

```
dsolve(diff(y(x),x)=((4-7*x)*(2*y(x)-3))/((x-1)*(2*x-5)),y(x), singsol=all)
```

$$y(x) = \frac{1536x^9 - 34560x^8 + 345600x^7 - 2016000x^6 + 7560000x^5 - 18900000x^4 + 31500000x^3 + (2c_1 - 283039}{2(2x-5)^9}$$

✓ Solution by Mathematica
Time used: 0.063 (sec). Leaf size: 82

```
DSolve[y'[x]==((4-7*x)*(2*y[x]-3))/((x-1)*(2*x-5)),y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to \frac{1536x^9 - 34560x^8 + 345600x^7 - 2016000x^6 + 7560000x^5 - 18900000x^4 + 31500000x^3 - 2(16402608 +}{2(2x-5)^9}$$

$$y(x) \to \frac{3}{2}$$

8.9 problem 9

Internal problem ID [14411]

Book: INTRODUCTORY DIFFERENTIAL EQUATIONS. Martha L. Abell, James P. Braselton. Fourth edition 2014. ElScAe. 2014
Section: Chapter 2. First Order Equations. Review exercises, page 80
Problem number: 9.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[[_linear, ‘class A‘]]`

$$\boxed{y' + 3y = -10\sin(t)}$$

✓ Solution by Maple
Time used: 0.0 (sec). Leaf size: 17

```
dsolve(diff(y(t),t)+3*y(t)=-10*sin(t),y(t), singsol=all)
```

$$y(t) = \cos(t) - 3\sin(t) + c_1 e^{-3t}$$

✓ Solution by Mathematica
Time used: 0.114 (sec). Leaf size: 20

```
DSolve[y'[t]+3*y[t]==-10*Sin[t],y[t],t,IncludeSingularSolutions -> True]
```

$$y(t) \to -3\sin(t) + \cos(t) + c_1 e^{-3t}$$

8.10 problem 10

Internal problem ID [14412]

Book: INTRODUCTORY DIFFERENTIAL EQUATIONS. Martha L. Abell, James P. Braselton. Fourth edition 2014. ElScAe. 2014
Section: Chapter 2. First Order Equations. Review exercises, page 80
Problem number: 10.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[[_homogeneous, ‘class A‘], _rational, [_Abel, ‘2nd type‘, ‘cl`

$$\boxed{(-4y+t)\,y' = -3t}$$

✓ Solution by Maple
Time used: 1.11 (sec). Leaf size: 49

```
dsolve(3*t+(t-4*y(t))*diff(y(t),t)=0,y(t), singsol=all)
```

$$y(t) = \frac{t\left(\operatorname{RootOf}\left(\_Z^{49}c_1t^7 - 21\_Z^{42}c_1t^7 + 147\_Z^{35}c_1t^7 - 343\_Z^{28}c_1t^7 - 64\right)^7 - 3\right)}{4}$$

✓ Solution by Mathematica

Time used: 3.627 (sec). Leaf size: 477

```
DSolve[3*t+(t-4*y[t])*y'[t]==0,y[t],t,IncludeSingularSolutions -> True]
```

$$y(t) \to \text{Root}\left[256\#1^7 - 672\#1^5t^2 - 112\#1^4t^3 + 609\#1^3t^4 + 189\#1^2t^5 - 189\#1t^6 - 81t^7 + e^{7c_1}\&, 1\right]$$
$$y(t) \to \text{Root}\left[256\#1^7 - 672\#1^5t^2 - 112\#1^4t^3 + 609\#1^3t^4 + 189\#1^2t^5 - 189\#1t^6 - 81t^7 + e^{7c_1}\&, 2\right]$$
$$y(t) \to \text{Root}\left[256\#1^7 - 672\#1^5t^2 - 112\#1^4t^3 + 609\#1^3t^4 + 189\#1^2t^5 - 189\#1t^6 - 81t^7 + e^{7c_1}\&, 3\right]$$
$$y(t) \to \text{Root}\left[256\#1^7 - 672\#1^5t^2 - 112\#1^4t^3 + 609\#1^3t^4 + 189\#1^2t^5 - 189\#1t^6 - 81t^7 + e^{7c_1}\&, 4\right]$$
$$y(t) \to \text{Root}\left[256\#1^7 - 672\#1^5t^2 - 112\#1^4t^3 + 609\#1^3t^4 + 189\#1^2t^5 - 189\#1t^6 - 81t^7 + e^{7c_1}\&, 5\right]$$
$$y(t) \to \text{Root}\left[256\#1^7 - 672\#1^5t^2 - 112\#1^4t^3 + 609\#1^3t^4 + 189\#1^2t^5 - 189\#1t^6 - 81t^7 + e^{7c_1}\&, 6\right]$$
$$y(t) \to \text{Root}\left[256\#1^7 - 672\#1^5t^2 - 112\#1^4t^3 + 609\#1^3t^4 + 189\#1^2t^5 - 189\#1t^6 - 81t^7 + e^{7c_1}\&, 7\right]$$

8.11 problem 11

Internal problem ID [14413]

Book: INTRODUCTORY DIFFERENTIAL EQUATIONS. Martha L. Abell, James P. Braselton. Fourth edition 2014. ElScAe. 2014
Section: Chapter 2. First Order Equations. Review exercises, page 80
Problem number: 11.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[[_homogeneous, 'class A'], _exact, _rational, [_Abel, '2nd ty`

$$\boxed{y + (y + t)\, y' = t}$$

✓ Solution by Maple
Time used: 0.078 (sec). Leaf size: 51

```
dsolve((y(t)-t)+(t+y(t))*diff(y(t),t)=0,y(t), singsol=all)
```

$$y(t) = \frac{-c_1 t - \sqrt{2c_1^2 t^2 + 1}}{c_1}$$
$$y(t) = \frac{-c_1 t + \sqrt{2c_1^2 t^2 + 1}}{c_1}$$

✓ Solution by Mathematica
Time used: 0.465 (sec). Leaf size: 94

```
DSolve[(y[t]-t)+(t+y[t])*y'[t]==0,y[t],t,IncludeSingularSolutions -> True]
```

$$y(t) \to -t - \sqrt{2t^2 + e^{2c_1}}$$
$$y(t) \to -t + \sqrt{2t^2 + e^{2c_1}}$$
$$y(t) \to -\sqrt{2}\sqrt{t^2} - t$$
$$y(t) \to \sqrt{2}\sqrt{t^2} - t$$

8.12 problem 12

Internal problem ID [14414]

Book: INTRODUCTORY DIFFERENTIAL EQUATIONS. Martha L. Abell, James P. Braselton. Fourth edition 2014. ElScAe. 2014
Section: Chapter 2. First Order Equations. Review exercises, page 80
Problem number: 12.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[[_linear, 'class A']]`

$$\boxed{y' + y = x}$$

✓ Solution by Maple
Time used: 0.0 (sec). Leaf size: 13

```
dsolve((y(x)-x)+diff(y(x),x)=0,y(x), singsol=all)
```

$$y(x) = x - 1 + c_1 e^{-x}$$

✓ Solution by Mathematica
Time used: 0.039 (sec). Leaf size: 16

```
DSolve[(y[x]-x)+y'[x]==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to x + c_1 e^{-x} - 1$$

8.13 problem 13

Internal problem ID [14415]

Book: INTRODUCTORY DIFFERENTIAL EQUATIONS. Martha L. Abell, James P. Braselton. Fourth edition 2014. ElScAe. 2014
Section: Chapter 2. First Order Equations. Review exercises, page 80
Problem number: 13.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[[_homogeneous, ‘class A‘], _rational, [_Abel, ‘2nd type‘, ‘cl`

$$\boxed{y^2 + (ty + t^2)\, y' = 0}$$

✓ Solution by Maple
Time used: 0.157 (sec). Leaf size: 45

```
dsolve(y(t)^2+(t*y(t)+t^2)*diff(y(t),t)=0,y(t), singsol=all)
```

$$y(t) = \frac{1 + \sqrt{c_1 t^2 + 1}}{c_1 t}$$
$$y(t) = \frac{1 - \sqrt{c_1 t^2 + 1}}{c_1 t}$$

✓ Solution by Mathematica
Time used: 2.71 (sec). Leaf size: 80

```
DSolve[y[t]^2+(t*y[t]+t^2)*y'[t]==0,y[t],t,IncludeSingularSolutions -> True]
```

$$y(t) \to \frac{e^{2c_1} - \sqrt{e^{2c_1}\left(t^2 + e^{2c_1}\right)}}{t}$$
$$y(t) \to \frac{\sqrt{e^{2c_1}\left(t^2 + e^{2c_1}\right)} + e^{2c_1}}{t}$$
$$y(t) \to 0$$

8.14 problem 14

Internal problem ID [14416]

Book: INTRODUCTORY DIFFERENTIAL EQUATIONS. Martha L. Abell, James P. Braselton. Fourth edition 2014. ElScAe. 2014
Section: Chapter 2. First Order Equations. Review exercises, page 80
Problem number: 14.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[[_homogeneous, ‘class A‘], _rational, _Bernoulli]`

$$\boxed{r' - \frac{r^2 + t^2}{rt} = 0}$$

✓ Solution by Maple
Time used: 0.016 (sec). Leaf size: 28

```
dsolve(diff(r(t),t)=(r(t)^2+t^2)/(r(t)*t),r(t), singsol=all)
```

$$r(t) = \sqrt{2\ln(t) + c_1}\, t$$
$$r(t) = -\sqrt{2\ln(t) + c_1}\, t$$

✓ Solution by Mathematica
Time used: 0.177 (sec). Leaf size: 36

```
DSolve[r'[t]==(r[t]^2+t^2)/(r[t]*t),r[t],t,IncludeSingularSolutions -> True]
```

$$r(t) \to -t\sqrt{2\log(t) + c_1}$$
$$r(t) \to t\sqrt{2\log(t) + c_1}$$

8.15 problem 15

Internal problem ID [14417]

Book: INTRODUCTORY DIFFERENTIAL EQUATIONS. Martha L. Abell, James P. Braselton. Fourth edition 2014. ElScAe. 2014
Section: Chapter 2. First Order Equations. Review exercises, page 80
Problem number: 15.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[[_homogeneous, ‘class A‘], _rational, _dAlembert]`

$$\boxed{x' - \frac{5tx}{x^2 + t^2} = 0}$$

✓ Solution by Maple
Time used: 14.282 (sec). Leaf size: 522

```
dsolve(diff(x(t),t)=5*t*x(t)/(x(t)^2+t^2),x(t), singsol=all)
```

$x(t)$

$= \frac{4 \operatorname{RootOf}\left(4\_Z^{80} - 4\_Z^{72} + \_Z^{64} + 1024\_Z^{40} c_1 t^8 - 1280\_Z^{32} c_1 t^8 + 640\_Z^{24} c_1 t^8 - 160\_Z^{16} c_1 t^8 + 20\_\right.}{}$

✓ Solution by Mathematica

Time used: 60.117 (sec). Leaf size: 641

```
DSolve[x'[t]==5*t*x[t]/(x[t]^2+t^2),x[t],t,IncludeSingularSolutions -> True]
```

$$x(t) \to -\sqrt{\text{Root}\left[\#1^5-20\#1^4t^2+160\#1^3t^4-640\#1^2t^6+\#1\left(1280t^8+e^{8c_1}\right)-1024t^{10}\&,1\right]}$$

$$x(t) \to \sqrt{\text{Root}\left[\#1^5-20\#1^4t^2+160\#1^3t^4-640\#1^2t^6+\#1\left(1280t^8+e^{8c_1}\right)-1024t^{10}\&,1\right]}$$

$$x(t) \to -\sqrt{\text{Root}\left[\#1^5-20\#1^4t^2+160\#1^3t^4-640\#1^2t^6+\#1\left(1280t^8+e^{8c_1}\right)-1024t^{10}\&,2\right]}$$

$$x(t) \to \sqrt{\text{Root}\left[\#1^5-20\#1^4t^2+160\#1^3t^4-640\#1^2t^6+\#1\left(1280t^8+e^{8c_1}\right)-1024t^{10}\&,2\right]}$$

$$x(t) \to -\sqrt{\text{Root}\left[\#1^5-20\#1^4t^2+160\#1^3t^4-640\#1^2t^6+\#1\left(1280t^8+e^{8c_1}\right)-1024t^{10}\&,3\right]}$$

$$x(t) \to \sqrt{\text{Root}\left[\#1^5-20\#1^4t^2+160\#1^3t^4-640\#1^2t^6+\#1\left(1280t^8+e^{8c_1}\right)-1024t^{10}\&,3\right]}$$

$$x(t) \to -\sqrt{\text{Root}\left[\#1^5-20\#1^4t^2+160\#1^3t^4-640\#1^2t^6+\#1\left(1280t^8+e^{8c_1}\right)-1024t^{10}\&,4\right]}$$

$$x(t) \to \sqrt{\text{Root}\left[\#1^5-20\#1^4t^2+160\#1^3t^4-640\#1^2t^6+\#1\left(1280t^8+e^{8c_1}\right)-1024t^{10}\&,4\right]}$$

$$x(t) \to -\sqrt{\text{Root}\left[\#1^5-20\#1^4t^2+160\#1^3t^4-640\#1^2t^6+\#1\left(1280t^8+e^{8c_1}\right)-1024t^{10}\&,5\right]}$$

$$x(t) \to \sqrt{\text{Root}\left[\#1^5-20\#1^4t^2+160\#1^3t^4-640\#1^2t^6+\#1\left(1280t^8+e^{8c_1}\right)-1024t^{10}\&,5\right]}$$

8.16 problem 16

Internal problem ID [14418]

Book: INTRODUCTORY DIFFERENTIAL EQUATIONS. Martha L. Abell, James P. Braselton. Fourth edition 2014. ElScAe. 2014
Section: Chapter 2. First Order Equations. Review exercises, page 80
Problem number: 16.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_exact, _rational, [_1st_order, '_with_symmetry_[F(x),G(x)]']`

$$\boxed{-y + (-t + y)\, y' = -t^2}$$

✓ Solution by Maple
Time used: 0.0 (sec). Leaf size: 47

```
dsolve((t^2-y(t))+(y(t)-t)*diff(y(t),t)=0,y(t), singsol=all)
```

$$y(t) = t - \frac{\sqrt{-6t^3 + 9t^2 - 18c_1}}{3}$$
$$y(t) = t + \frac{\sqrt{-6t^3 + 9t^2 - 18c_1}}{3}$$

✓ Solution by Mathematica
Time used: 0.143 (sec). Leaf size: 63

```
DSolve[(t^2-y[t])+(y[t]-t)*y'[t]==0,y[t],t,IncludeSingularSolutions -> True]
```

$$y(t) \to t - i\sqrt{\frac{2t^3}{3} - t^2 - c_1}$$
$$y(t) \to t + i\sqrt{\frac{2t^3}{3} - t^2 - c_1}$$

8.17 problem 17

Internal problem ID [14419]

Book: INTRODUCTORY DIFFERENTIAL EQUATIONS. Martha L. Abell, James P. Braselton. Fourth edition 2014. ElScAe. 2014
Section: Chapter 2. First Order Equations. Review exercises, page 80
Problem number: 17.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_exact]`

$$\boxed{t^2 y + \left(\frac{t^3}{3} - \cos(y)\right) y' = -\sin(t)}$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 21

```
dsolve((t^2*y(t)+sin(t))+(1/3*t^3-cos(y(t)))*diff(y(t),t)=0,y(t), singsol=all)
```

$$\frac{t^3 y(t)}{3} - \cos(t) - \sin(y(t)) + c_1 = 0$$

✓ Solution by Mathematica

Time used: 0.204 (sec). Leaf size: 22

```
DSolve[(t^2*y[t]+Sin[t])+(1/3*t^3-Cos[y[t]])*y'[t]==0,y[t],t,IncludeSingularSolutions -> Tru
```

$$\text{Solve}\left[t^3 y(t) - 3\sin(y(t)) - 3\cos(t) = c_1, y(t)\right]$$

8.18 problem 18

Internal problem ID [14420]

Book: INTRODUCTORY DIFFERENTIAL EQUATIONS. Martha L. Abell, James P. Braselton. Fourth edition 2014. ElScAe. 2014
Section: Chapter 2. First Order Equations. Review exercises, page 80
Problem number: 18.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_exact]`

$$\boxed{\tan(y) + \left(t \sec(y)^2 + 1\right) y' = t}$$

✓ Solution by Maple
Time used: 0.0 (sec). Leaf size: 17

```
dsolve((tan(y(t))-t)+(t*sec(y(t))^2+1)*diff(y(t),t)=0,y(t), singsol=all)
```

$$t\tan(y(t)) - \frac{t^2}{2} + y(t) + c_1 = 0$$

✓ Solution by Mathematica
Time used: 0.181 (sec). Leaf size: 52

```
DSolve[(Tan[y[t]]-t)+(t*Sec[y[t]]^2+1)*y'[t]==0,y[t],t,IncludeSingularSolutions -> True]
```

$$\text{Solve}\left[-\frac{1}{2}t^2\sec^2(y(t)) - \frac{1}{2}t^2\cos(2y(t))\sec^2(y(t)) + 2y(t) + t\sin(2y(t))\sec^2(y(t)) = c_1, y(t)\right]$$

8.19 problem 19

Internal problem ID [14421]

Book: INTRODUCTORY DIFFERENTIAL EQUATIONS. Martha L. Abell, James P. Braselton. Fourth edition 2014. ElScAe. 2014
Section: Chapter 2. First Order Equations. Review exercises, page 80
Problem number: 19.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_exact, [_1st_order, '_with_symmetry_[F(x)*G(y),0]']]`

$$\boxed{t \ln(y) + \left(\frac{t^2}{2y} + 1\right) y' = 0}$$

✓ Solution by Maple

Time used: 0.015 (sec). Leaf size: 31

```
dsolve((t*ln(y(t)))+(t^2/(2*y(t))+1)*diff(y(t),t)=0,y(t), singsol=all)
```

$$y(t) = \mathrm{e}^{\frac{-t^2 \operatorname{LambertW}\left(\frac{2\,\mathrm{e}^{-\frac{2c_1}{t^2}}}{t^2}\right) - 2c_1}{t^2}}$$

✓ Solution by Mathematica

Time used: 1.013 (sec). Leaf size: 31

```
DSolve[(t*Log[y[t]])+(t^2/(2*y[t])+1)*y'[t]==0,y[t],t,IncludeSingularSolutions -> True]
```

$$y(t) \to \frac{1}{2} t^2 W\left(\frac{2e^{\frac{c_1}{t^2}}}{t^2}\right)$$
$$y(t) \to 1$$

8.20 problem 20

Internal problem ID [14422]

Book: INTRODUCTORY DIFFERENTIAL EQUATIONS. Martha L. Abell, James P. Braselton. Fourth edition 2014. ElScAe. 2014
Section: Chapter 2. First Order Equations. Review exercises, page 80
Problem number: 20.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_quadrature]`

$$\boxed{y' + y = 5}$$

✓ Solution by Maple
Time used: 0.0 (sec). Leaf size: 12

```
dsolve(diff(y(t),t)+y(t)=5,y(t), singsol=all)
```

$$y(t) = 5 + e^{-t} c_1$$

✓ Solution by Mathematica
Time used: 0.021 (sec). Leaf size: 20

```
DSolve[y'[t]+y[t]==5,y[t],t,IncludeSingularSolutions -> True]
```

$$y(t) \to 5 + c_1 e^{-t}$$
$$y(t) \to 5$$

8.21 problem 21

Internal problem ID [14423]

Book: INTRODUCTORY DIFFERENTIAL EQUATIONS. Martha L. Abell, James P. Braselton. Fourth edition 2014. ElScAe. 2014
Section: Chapter 2. First Order Equations. Review exercises, page 80
Problem number: 21.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type **[_separable]**

$$\boxed{y' + ty = t}$$

✓ Solution by Maple
Time used: 0.0 (sec). Leaf size: 14

```
dsolve(diff(y(t),t)+t*y(t)=t,y(t), singsol=all)
```

$$y(t) = 1 + \mathrm{e}^{-\frac{t^2}{2}} c_1$$

✓ Solution by Mathematica
Time used: 0.05 (sec). Leaf size: 24

```
DSolve[y'[t]+t*y[t]==t,y[t],t,IncludeSingularSolutions -> True]
```

$$y(t) \to 1 + c_1 e^{-\frac{t^2}{2}}$$
$$y(t) \to 1$$

8.22 problem 22

Internal problem ID [14424]

Book: INTRODUCTORY DIFFERENTIAL EQUATIONS. Martha L. Abell, James P. Braselton. Fourth edition 2014. ElScAe. 2014
Section: Chapter 2. First Order Equations. Review exercises, page 80
Problem number: 22.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_linear]`

$$\boxed{x' + \frac{x}{y} = y^2}$$

✓ Solution by Maple
Time used: 0.0 (sec). Leaf size: 16

```
dsolve(diff(x(y),y)+x(y)/y=y^2,x(y), singsol=all)
```

$$x(y) = \frac{y^4 + 4c_1}{4y}$$

✓ Solution by Mathematica
Time used: 0.025 (sec). Leaf size: 19

```
DSolve[x'[y]+x[y]/y==y^2,x[y],y,IncludeSingularSolutions -> True]
```

$$x(y) \to \frac{y^3}{4} + \frac{c_1}{y}$$

8.23 problem 23

Internal problem ID [14425]

Book: INTRODUCTORY DIFFERENTIAL EQUATIONS. Martha L. Abell, James P. Braselton. Fourth edition 2014. ElScAe. 2014
Section: Chapter 2. First Order Equations. Review exercises, page 80
Problem number: 23.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_linear]`

$$\boxed{tr' + r = \cos(t)\, t}$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 16

```
dsolve(t*diff(r(t),t)+r(t)=t*cos(t),r(t), singsol=all)
```

$$r(t) = \frac{\sin(t)\, t + \cos(t) + c_1}{t}$$

✓ Solution by Mathematica

Time used: 0.046 (sec). Leaf size: 18

```
DSolve[t*r'[t]+r[t]==t*Cos[t],r[t],t,IncludeSingularSolutions -> True]
```

$$r(t) \to \frac{t\sin(t) + \cos(t) + c_1}{t}$$

8.24 problem 24

Internal problem ID [14426]

Book: INTRODUCTORY DIFFERENTIAL EQUATIONS. Martha L. Abell, James P. Braselton. Fourth edition 2014. ElScAe. 2014
Section: Chapter 2. First Order Equations. Review exercises, page 80
Problem number: 24.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_Bernoulli]`

$$\boxed{y' - y - y^3 t = 0}$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 39

```
dsolve(diff(y(t),t)-y(t)=t*y(t)^3,y(t), singsol=all)
```

$$y(t) = -\frac{2}{\sqrt{2 + 4\,\mathrm{e}^{-2t} c_1 - 4t}}$$
$$y(t) = \frac{2}{\sqrt{2 + 4\,\mathrm{e}^{-2t} c_1 - 4t}}$$

✓ Solution by Mathematica

Time used: 1.817 (sec). Leaf size: 68

```
DSolve[y'[t]-y[t]==t*y[t]^3,y[t],t,IncludeSingularSolutions -> True]
```

$$y(t) \to -\frac{ie^t}{\sqrt{e^{2t}\left(t - \frac{1}{2}\right) - c_1}}$$
$$y(t) \to \frac{ie^t}{\sqrt{e^{2t}\left(t - \frac{1}{2}\right) - c_1}}$$
$$y(t) \to 0$$

8.25 problem 25

Internal problem ID [14427]

Book: INTRODUCTORY DIFFERENTIAL EQUATIONS. Martha L. Abell, James P. Braselton. Fourth edition 2014. ElScAe. 2014
Section: Chapter 2. First Order Equations. Review exercises, page 80
Problem number: 25.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[[_1st_order, _with_linear_symmetries], _Bernoulli]`

$$\boxed{y' + y - \frac{e^t}{y^2} = 0}$$

✓ Solution by Maple
Time used: 0.031 (sec). Leaf size: 92

```
dsolve(diff(y(t),t)+y(t)=exp(t)/y(t)^2,y(t), singsol=all)
```

$$y(t) = \frac{2^{\frac{1}{3}}\left(\left(3\,e^{4t}+4c_1\right)e^{-3t}\right)^{\frac{1}{3}}}{2}$$
$$y(t) = -\frac{2^{\frac{1}{3}}\left(\left(3\,e^{4t}+4c_1\right)e^{-3t}\right)^{\frac{1}{3}}\left(1+i\sqrt{3}\right)}{4}$$
$$y(t) = \frac{2^{\frac{1}{3}}\left(\left(3\,e^{4t}+4c_1\right)e^{-3t}\right)^{\frac{1}{3}}\left(i\sqrt{3}-1\right)}{4}$$

✓ Solution by Mathematica

Time used: 7.499 (sec). Leaf size: 96

```
DSolve[y'[t]+y[t]==Exp[t]/y[t]^2,y[t],t,IncludeSingularSolutions -> True]
```

$$y(t) \rightarrow \frac{\sqrt[3]{3e^t + 4c_1e^{-3t}}}{2^{2/3}}$$
$$y(t) \rightarrow -\frac{\sqrt[3]{-1}\sqrt[3]{3e^t + 4c_1e^{-3t}}}{2^{2/3}}$$
$$y(t) \rightarrow \left(-\frac{1}{2}\right)^{2/3} \sqrt[3]{3e^t + 4c_1e^{-3t}}$$

8.26 problem 26

Internal problem ID [14428]

Book: INTRODUCTORY DIFFERENTIAL EQUATIONS. Martha L. Abell, James P. Braselton. Fourth edition 2014. ElScAe. 2014
Section: Chapter 2. First Order Equations. Review exercises, page 80
Problem number: 26.
ODE order: 1.
ODE degree: 4.

CAS Maple gives this as type `[[_1st_order, _with_linear_symmetries], _Clairaut]`

$$\boxed{y - y't - 3y'^4 = 0}$$

✓ Solution by Maple
Time used: 0.125 (sec). Leaf size: 68

```
dsolve(y(t)=t*diff(y(t),t)+3*diff(y(t),t)^4,y(t), singsol=all)
```

$$y(t) = -\frac{18^{\frac{1}{3}}(-t)^{\frac{4}{3}}}{8}$$
$$y(t) = \frac{18^{\frac{1}{3}}(-t)^{\frac{4}{3}}\left(1+i\sqrt{3}\right)}{16}$$
$$y(t) = -\frac{18^{\frac{1}{3}}(-t)^{\frac{4}{3}}\left(i\sqrt{3}-1\right)}{16}$$
$$y(t) = c_1\left(3c_1^3+t\right)$$

✓ Solution by Mathematica

Time used: 0.006 (sec). Leaf size: 81

```
DSolve[y[t]==t*y'[t]+3*y'[t]^4,y[t],t,IncludeSingularSolutions -> True]
```

$$
\begin{aligned}
y(t) &\to c_1\left(t+3c_1{}^3\right)\\
y(t) &\to -\frac{1}{4}\left(-\frac{3}{2}\right)^{2/3} t^{4/3}\\
y(t) &\to -\frac{1}{4}\left(\frac{3}{2}\right)^{2/3} t^{4/3}\\
y(t) &\to \frac{1}{4}\sqrt[3]{-1}\left(\frac{3}{2}\right)^{2/3} t^{4/3}
\end{aligned}
$$

8.27 problem 27

Internal problem ID [14429]

Book: INTRODUCTORY DIFFERENTIAL EQUATIONS. Martha L. Abell, James P. Braselton. Fourth edition 2014. ElScAe. 2014
Section: Chapter 2. First Order Equations. Review exercises, page 80
Problem number: 27.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[[_homogeneous, 'class D'], _Bernoulli]`

$$\boxed{y - y't - 2y^2 \ln(t) = 0}$$

✓ Solution by Maple
Time used: 0.0 (sec). Leaf size: 18

```
dsolve(y(t)-t*diff(y(t),t)=2*y(t)^2*ln(t),y(t), singsol=all)
```

$$y(t) = \frac{t}{2t\ln(t) - 2t + c_1}$$

✓ Solution by Mathematica
Time used: 0.152 (sec). Leaf size: 25

```
DSolve[y[t]-t*y'[t]==2*y[t]^2*Log[t],y[t],t,IncludeSingularSolutions -> True]
```

$$y(t) \to \frac{t}{-2t + 2t\log(t) + c_1}$$
$$y(t) \to 0$$

8.28 problem 28

Internal problem ID [14430]

Book: INTRODUCTORY DIFFERENTIAL EQUATIONS. Martha L. Abell, James P. Braselton. Fourth edition 2014. ElScAe. 2014
Section: Chapter 2. First Order Equations. Review exercises, page 80
Problem number: 28.
ODE order: 1.
ODE degree: 3.

CAS Maple gives this as type `[[_1st_order, _with_linear_symmetries], _Clairaut]`

$$\boxed{y - y't + 2{y'}^3 = 0}$$

✓ Solution by Maple
Time used: 0.125 (sec). Leaf size: 35

```
dsolve(y(t)-t*diff(y(t),t)=-2*diff(y(t),t)^3,y(t), singsol=all)
```

$$y(t) = -\frac{\sqrt{6}\, t^{\frac{3}{2}}}{9}$$
$$y(t) = \frac{\sqrt{6}\, t^{\frac{3}{2}}}{9}$$
$$y(t) = c_1\left(-2c_1^2 + t\right)$$

✓ Solution by Mathematica
Time used: 0.006 (sec). Leaf size: 56

```
DSolve[y[t]-t*y'[t]==-2*y'[t]^3,y[t],t,IncludeSingularSolutions -> True]
```

$$y(t) \to c_1\left(t - 2{c_1}^2\right)$$
$$y(t) \to -\frac{1}{3}\sqrt{\frac{2}{3}}t^{3/2}$$
$$y(t) \to \frac{1}{3}\sqrt{\frac{2}{3}}t^{3/2}$$

8.29 problem 29

Internal problem ID [14431]

Book: INTRODUCTORY DIFFERENTIAL EQUATIONS. Martha L. Abell, James P. Braselton. Fourth edition 2014. ElScAe. 2014
Section: Chapter 2. First Order Equations. Review exercises, page 80
Problem number: 29.
ODE order: 1.
ODE degree: 2.

CAS Maple gives this as type `[[_1st_order, _with_linear_symmetries], _Clairaut]`

$$\boxed{y - y't + 4y'^2 = 0}$$

✓ Solution by Maple
Time used: 0.078 (sec). Leaf size: 19

```
dsolve(y(t)-t*diff(y(t),t)=-4*diff(y(t),t)^2,y(t), singsol=all)
```

$$y(t) = \frac{t^2}{16}$$
$$y(t) = c_1(-4c_1 + t)$$

✓ Solution by Mathematica
Time used: 0.005 (sec). Leaf size: 25

```
DSolve[y[t]-t*y'[t]==-4*y'[t]^2,y[t],t,IncludeSingularSolutions -> True]
```

$$y(t) \to c_1(t - 4c_1)$$
$$y(t) \to \frac{t^2}{16}$$

8.30 problem 30

Internal problem ID [14432]

Book: INTRODUCTORY DIFFERENTIAL EQUATIONS. Martha L. Abell, James P. Braselton. Fourth edition 2014. ElScAe. 2014
Section: Chapter 2. First Order Equations. Review exercises, page 80
Problem number: 30.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[[_homogeneous, ‘class C‘], _exact, _rational, [_Abel, ‘2nd ty`

$$\boxed{-y + (-x + 2y)\, y' = -2x + 2}$$

With initial conditions

$$[y(0) = 1]$$

✓ Solution by Maple
Time used: 0.531 (sec). Leaf size: 22

```
dsolve([(2*x-y(x)-2)+(2*y(x)-x)*diff(y(x),x)=0,y(0) = 1],y(x), singsol=all)
```

$$y(x) = \frac{x}{2} + \frac{\sqrt{-3x^2 + 8x + 4}}{2}$$

✓ Solution by Mathematica
Time used: 0.144 (sec). Leaf size: 29

```
DSolve[{(2*x-y[x]-2)+(2*y[x]-x)*y'[x]==0,{y[0]==1}},y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to \frac{1}{2}\left(x - i\sqrt{3x^2 - 8x - 4}\right)$$

8.31 problem 31

Internal problem ID [14433]

Book: INTRODUCTORY DIFFERENTIAL EQUATIONS. Martha L. Abell, James P. Braselton. Fourth edition 2014. ElScAe. 2014
Section: Chapter 2. First Order Equations. Review exercises, page 80
Problem number: 31.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[[_homogeneous, `class C`], _exact, _dAlembert]`

$$\boxed{\cos(t-y)+(1-\cos(t-y))\,y'=0}$$

With initial conditions

$$[y(\pi)=\pi]$$

✓ Solution by Maple
Time used: 0.265 (sec). Leaf size: 19

```
dsolve([(cos(t-y(t)))+(1-cos(t-y(t)))*diff(y(t),t)=0,y(Pi) = Pi],y(t), singsol=all)
```

$$y(t)=t-\mathrm{RootOf}\left(\_Z-t+\pi-\sin\left(\_Z\right)\right)$$

✓ Solution by Mathematica
Time used: 0.105 (sec). Leaf size: 21

```
DSolve[{Cos[t-y[t]]+(1-Cos[t-y[t]])*y'[t]==0,{y[Pi]==Pi}},y[t],t,IncludeSingularSolutions ->
```

$$\mathrm{Solve}[-y(t)-\sin(t-y(t))=-\pi,y(t)]$$

8.32 problem 32

Internal problem ID [14434]

Book: INTRODUCTORY DIFFERENTIAL EQUATIONS. Martha L. Abell, James P. Braselton. Fourth edition 2014. ElScAe. 2014
Section: Chapter 2. First Order Equations. Review exercises, page 80
Problem number: 32.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_exact, [_1st_order, ‘_with_symmetry_[F(x),G(x)*y+H(x)]‘]]`

$$\boxed{e^{ty}y + t\,e^{ty}y' = 2t}$$

With initial conditions

$$[y(0) = 0]$$

✓ Solution by Maple
Time used: 0.172 (sec). Leaf size: 14

```
dsolve([(y(t)*exp(t*y(t))-2*t)+(t*exp(t*y(t)))*diff(y(t),t)=0,y(0) = 0],y(t), singsol=all)
```

$$y(t) = \frac{\ln(t^2+1)}{t}$$

✓ Solution by Mathematica
Time used: 0.498 (sec). Leaf size: 15

```
DSolve[{(y[t]*Exp[t*y[t]]-2*t)+(t*Exp[t*y[t]])*y'[t]==0,{y[0]==0}},y[t],t,IncludeSingularSol
```

$$y(t) \to \frac{\log(t^2+1)}{t}$$

8.33 problem 33

Internal problem ID [14435]

Book: INTRODUCTORY DIFFERENTIAL EQUATIONS. Martha L. Abell, James P. Braselton. Fourth edition 2014. ElScAe. 2014
Section: Chapter 2. First Order Equations. Review exercises, page 80
Problem number: 33.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_exact]`

$$\boxed{\sin(y) - \cos(t)\, y + (t\cos(y) - \sin(t))\, y' = 0}$$

With initial conditions

$$[y(\pi) = 0]$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 5

```
dsolve([(sin(y(t))-y(t)*cos(t))+(t*cos(y(t))-sin(t))*diff(y(t),t)=0,y(Pi) = 0],y(t), singsol
```

$$y(t) = 0$$

✓ Solution by Mathematica

Time used: 0.012 (sec). Leaf size: 6

```
DSolve[{(Sin[y[t]]-y[t]*Cos[t])+(t*Cos[y[t]]-Sin[t])*y'[t]==0,{y[Pi]==0}},y[t],t,IncludeSing
```

$$y(t) \to 0$$

8.34 problem 34

Internal problem ID [14436]

Book: INTRODUCTORY DIFFERENTIAL EQUATIONS. Martha L. Abell, James P. Braselton. Fourth edition 2014. ElScAe. 2014
Section: Chapter 2. First Order Equations. Review exercises, page 80
Problem number: 34.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_exact, [_1st_order, '_with_symmetry_[F(x)*G(y),0]']]`

$$\boxed{y^2 + (2ty - 2\cos(y)\sin(y))\, y' = 0}$$

With initial conditions

$$[y(0) = \pi]$$

✓ Solution by Maple
Time used: 0.109 (sec). Leaf size: 18

```
dsolve([(y(t)^2)+(2*t*y(t)-2*cos(y(t))*sin(y(t)))*diff(y(t),t)=0,y(0) = Pi],y(t), singsol=al
```

$$y(t) = \frac{\text{RootOf}\left(t\_Z^2 + 2\cos(\_Z) - 2\right)}{2}$$

✓ Solution by Mathematica
Time used: 0.242 (sec). Leaf size: 27

```
DSolve[{(y[t]^2)+(2*t*y[t]-2*Cos[y[t]]*Sin[y[t]])*y'[t]==0,{y[0]==Pi}},y[t],t,IncludeSingula
```

$$\text{Solve}\left[t = \frac{1}{2y(t)^2} - \frac{\cos(2y(t))}{2y(t)^2}, y(t)\right]$$

8.35 problem 35

Internal problem ID [14437]

Book: INTRODUCTORY DIFFERENTIAL EQUATIONS. Martha L. Abell, James P. Braselton. Fourth edition 2014. ElScAe. 2014
Section: Chapter 2. First Order Equations. Review exercises, page 80
Problem number: 35.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_exact]`

$$\boxed{\frac{y}{t} + \ln(y) + \left(\frac{t}{y} + \ln(t)\right) y' = 0}$$

With initial conditions

$$[y(1) = 1]$$

✓ Solution by Maple
Time used: 0.953 (sec). Leaf size: 14

```
dsolve([(y(t)/t+ln(y(t)))+(t/y(t)+ln(t))*diff(y(t),t)=0,y(1) = 1],y(t), singsol=all)
```

$$y(t) = \frac{t \,\mathrm{LambertW}\left(\frac{\ln(t)}{t}\right)}{\ln(t)}$$

✓ Solution by Mathematica
Time used: 1.922 (sec). Leaf size: 18

```
DSolve[{(y[t]/t+Log[y[t]])+(t/y[t]+Log[t])*y'[t]==0,{y[1]==1}},y[t],t,IncludeSingularSolutio
```

$$y(t) \to \frac{tW\left(\frac{\log(t)}{t}\right)}{\log(t)}$$

8.36 problem 36

Internal problem ID [14438]

Book: INTRODUCTORY DIFFERENTIAL EQUATIONS. Martha L. Abell, James P. Braselton. Fourth edition 2014. ElScAe. 2014
Section: Chapter 2. First Order Equations. Review exercises, page 80
Problem number: 36.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[[_Riccati, _special]]`

$$\boxed{y' - y^2 = -x}$$

With initial conditions

$$[y(0) = 0]$$

✓ Solution by Maple
Time used: 0.078 (sec). Leaf size: 28

```
dsolve([diff(y(x),x)=y(x)^2-x,y(0) = 0],y(x), singsol=all)
```

$$y(x) = \frac{-\sqrt{3}\,\mathrm{AiryAi}\,(1,x) - \mathrm{AiryBi}\,(1,x)}{\sqrt{3}\,\mathrm{AiryAi}\,(x) + \mathrm{AiryBi}\,(x)}$$

✓ Solution by Mathematica
Time used: 6.859 (sec). Leaf size: 93

```
DSolve[{y'[x]==y[x]^2-x,{y[0]==0}},y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to -\frac{ix^{3/2}\,\mathrm{BesselJ}\left(-\frac{4}{3}, \frac{2}{3}ix^{3/2}\right) - ix^{3/2}\,\mathrm{BesselJ}\left(\frac{2}{3}, \frac{2}{3}ix^{3/2}\right) + \mathrm{BesselJ}\left(-\frac{1}{3}, \frac{2}{3}ix^{3/2}\right)}{2x\,\mathrm{BesselJ}\left(-\frac{1}{3}, \frac{2}{3}ix^{3/2}\right)}$$

8.37 problem 37

Internal problem ID [14439]

Book: INTRODUCTORY DIFFERENTIAL EQUATIONS. Martha L. Abell, James P. Braselton. Fourth edition 2014. ElScAe. 2014
Section: Chapter 2. First Order Equations. Review exercises, page 80
Problem number: 37.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[[_homogeneous, 'class C'], _dAlembert]`

$$\boxed{y' - \sqrt{-y+x} = 0}$$

With initial conditions

$$[y(1) = 1]$$

✓ Solution by Maple
Time used: 1.985 (sec). Leaf size: 61

```
dsolve([diff(y(x),x)=sqrt(x-y(x)),y(1) = 1],y(x), singsol=all)
```

$$y(x) = -e^{2\operatorname{RootOf}\left(\_Z-x-2\,e^{-\_Z}+3-\ln\left(e^{-\_Z}\left(e^{-\_Z}-2\right)^2\right)\right)} + 2\,e^{\operatorname{RootOf}\left(\_Z-x-2\,e^{-\_Z}+3-\ln\left(e^{-\_Z}\left(e^{-\_Z}-2\right)^2\right)\right)} + x - 1$$

✗ Solution by Mathematica
Time used: 0.0 (sec). Leaf size: 0

```
DSolve[{y'[x]==Sqrt[x-y[x]],{y[1]==1}},y[x],x,IncludeSingularSolutions -> True]
```

{}

8.38 problem 38

Internal problem ID [14440]

Book: INTRODUCTORY DIFFERENTIAL EQUATIONS. Martha L. Abell, James P. Braselton. Fourth edition 2014. ElScAe. 2014
Section: Chapter 2. First Order Equations. Review exercises, page 80
Problem number: 38.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_Chini]`

$$\boxed{y' - y^{\frac{1}{3}} = x}$$

With initial conditions

$$[y(1) = 1]$$

✗ Solution by Maple

```
dsolve([diff(y(x),x)=x+y(x)^(1/3),y(1) = 1],y(x), singsol=all)
```

No solution found

✗ Solution by Mathematica
Time used: 0.0 (sec). Leaf size: 0

```
DSolve[{y'[x]==x+y[x]^(1/3),{y[1]==1}},y[x],x,IncludeSingularSolutions -> True]
```

Not solved

8.39 problem 39

Internal problem ID [14441]

Book: INTRODUCTORY DIFFERENTIAL EQUATIONS. Martha L. Abell, James P. Braselton. Fourth edition 2014. ElScAe. 2014
Section: Chapter 2. First Order Equations. Review exercises, page 80
Problem number: 39.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[‘y=_G(x,y’)‘]`

$$\boxed{y' - \sin(x^2 y) = 0}$$

With initial conditions

$$[y(0) = 1]$$

✗ Solution by Maple

```
dsolve([diff(y(x),x)=sin(x^2*y(x)),y(0) = 1],y(x), singsol=all)
```

No solution found

✗ Solution by Mathematica
Time used: 0.0 (sec). Leaf size: 0

```
DSolve[{y'[x]==Sin[x^2*y[x]],{y[0]==1}},y[x],x,IncludeSingularSolutions -> True]
```

Not solved

8.40 problem 40 (a)

Internal problem ID [14442]

Book: INTRODUCTORY DIFFERENTIAL EQUATIONS. Martha L. Abell, James P. Braselton. Fourth edition 2014. ElScAe. 2014
Section: Chapter 2. First Order Equations. Review exercises, page 80
Problem number: 40 (a).
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_separable]`

$$\boxed{y' - y^3 t = 0}$$

With initial conditions

$$[y(0) = 0]$$

✓ Solution by Maple
Time used: 0.0 (sec). Leaf size: 5

```
dsolve([diff(y(t),t)=t*y(t)^3,y(0) = 0],y(t), singsol=all)
```

$$y(t) = 0$$

✓ Solution by Mathematica
Time used: 0.001 (sec). Leaf size: 6

```
DSolve[{y'[t]==t*y[t]^3,{y[0]==0}},y[t],t,IncludeSingularSolutions -> True]
```

$$y(t) \to 0$$

8.41 problem 40 (b)

Internal problem ID [14443]

Book: INTRODUCTORY DIFFERENTIAL EQUATIONS. Martha L. Abell, James P. Braselton. Fourth edition 2014. ElScAe. 2014
Section: Chapter 2. First Order Equations. Review exercises, page 80
Problem number: 40 (b).
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_separable]`

$$\boxed{y' - \frac{t}{y^3} = 0}$$

With initial conditions

$$[y(0) = 0]$$

✓ Solution by Maple
Time used: 0.047 (sec). Leaf size: 46

```
dsolve([diff(y(t),t)=t/y(t)^3,y(0) = 0],y(t), singsol=all)
```

$$y(t) = 2^{\frac{1}{4}}\sqrt{t}$$
$$y(t) = -2^{\frac{1}{4}}\sqrt{t}$$
$$y(t) = -i2^{\frac{1}{4}}\sqrt{t}$$
$$y(t) = i2^{\frac{1}{4}}\sqrt{t}$$

✓ Solution by Mathematica
Time used: 0.187 (sec). Leaf size: 76

```
DSolve[{y'[t]==t/y[t]^3,{y[0]==0}},y[t],t,IncludeSingularSolutions -> True]
```

$$y(t) \to -\sqrt[4]{2}\sqrt[4]{t^2}$$
$$y(t) \to -i\sqrt[4]{2}\sqrt[4]{t^2}$$
$$y(t) \to i\sqrt[4]{2}\sqrt[4]{t^2}$$
$$y(t) \to \sqrt[4]{2}\sqrt[4]{t^2}$$

8.42 problem 40 (c)

Internal problem ID [14444]

Book: INTRODUCTORY DIFFERENTIAL EQUATIONS. Martha L. Abell, James P. Braselton. Fourth edition 2014. ElScAe. 2014
Section: Chapter 2. First Order Equations. Review exercises, page 80
Problem number: 40 (c).
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_separable]`

$$\boxed{y' + \frac{y}{-2+t} = 0}$$

With initial conditions

$$[y(2) = 0]$$

✓ Solution by Maple
Time used: 0.094 (sec). Leaf size: 5

```
dsolve([diff(y(t),t)=-y(t)/(t-2),y(2) = 0],y(t), singsol=all)
```

$$y(t) = 0$$

✓ Solution by Mathematica
Time used: 0.001 (sec). Leaf size: 6

```
DSolve[{y'[t]==-y[t]/(t-2),{y[2]==0}},y[t],t,IncludeSingularSolutions -> True]
```

$$y(t) \to 0$$

9 Chapter 4. Higher Order Equations. Exercises 4.1, page 141

9.1 problem 9

Internal problem ID [14445]

Book: INTRODUCTORY DIFFERENTIAL EQUATIONS. Martha L. Abell, James P. Braselton. Fourth edition 2014. ElScAe. 2014
Section: Chapter 4. Higher Order Equations. Exercises 4.1, page 141
Problem number: 9.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _missing_x]]`

$$\boxed{y'' - y = 0}$$

✓ Solution by Maple
Time used: 0.0 (sec). Leaf size: 15

```
dsolve(diff(y(t),t$2)-y(t)=0,y(t), singsol=all)
```

$$y(t) = c_2 \mathrm{e}^t + \mathrm{e}^{-t} c_1$$

✓ Solution by Mathematica
Time used: 0.017 (sec). Leaf size: 20

```
DSolve[y''[t]-y[t]==0,y[t],t,IncludeSingularSolutions -> True]
```

$$y(t) \to c_1 e^t + c_2 e^{-t}$$

9.2 problem 10

Internal problem ID [14446]

Book: INTRODUCTORY DIFFERENTIAL EQUATIONS. Martha L. Abell, James P. Braselton. Fourth edition 2014. ElScAe. 2014
Section: Chapter 4. Higher Order Equations. Exercises 4.1, page 141
Problem number: 10.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _missing_x]]`

$$\boxed{y'' + 2y' + y = 0}$$

✓ Solution by Maple
Time used: 0.0 (sec). Leaf size: 14

```
dsolve(diff(y(t),t$2)+2*diff(y(t),t)+y(t)=0,y(t), singsol=all)
```

$$y(t) = \mathrm{e}^{-t}(c_2 t + c_1)$$

✓ Solution by Mathematica
Time used: 0.013 (sec). Leaf size: 18

```
DSolve[y''[t]+2*y'[t]+y[t]==0,y[t],t,IncludeSingularSolutions -> True]
```

$$y(t) \to e^{-t}(c_2 t + c_1)$$

9.3 problem 11

Internal problem ID [14447]

Book: INTRODUCTORY DIFFERENTIAL EQUATIONS. Martha L. Abell, James P. Braselton. Fourth edition 2014. ElScAe. 2014
Section: Chapter 4. Higher Order Equations. Exercises 4.1, page 141
Problem number: 11.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_Emden, _Fowler]]`

$$\boxed{2t^2y'' - 3y't - 3y = 0}$$

✓ Solution by Maple
Time used: 0.0 (sec). Leaf size: 15

```
dsolve(2*t^2*diff(y(t),t$2)-3*t*diff(y(t),t)-3*y(t)=0,y(t), singsol=all)
```

$$y(t) = c_1 t^3 + \frac{c_2}{\sqrt{t}}$$

✓ Solution by Mathematica
Time used: 0.15 (sec). Leaf size: 124

```
DSolve[2*t^2*y''[t]-3*y'[t]-3*y[t]==0,y[t],t,IncludeSingularSolutions -> True]
```

$$y(t) \to \sqrt{\frac{2}{3}} t^{\frac{1}{2}-\frac{\sqrt{7}}{2}} \left(\left(\frac{2}{3}\right)^{\frac{\sqrt{7}}{2}} c_2 t^{\sqrt{7}} \text{Hypergeometric1F1}\left(-\frac{1}{2} - \frac{\sqrt{7}}{2}, 1 - \sqrt{7}, -\frac{3}{2t}\right) + \left(\frac{3}{2}\right)^{\frac{\sqrt{7}}{2}} c_1 \text{Hypergeometric1F1}\left(\frac{1}{2}\left(-1+\sqrt{7}\right), 1+\sqrt{7}, -\frac{3}{2t}\right)\right)$$

9.4 problem 12

Internal problem ID [14448]

Book: INTRODUCTORY DIFFERENTIAL EQUATIONS. Martha L. Abell, James P. Braselton. Fourth edition 2014. ElScAe. 2014
Section: Chapter 4. Higher Order Equations. Exercises 4.1, page 141
Problem number: 12.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _missing_x]]`

$$\boxed{y'' + 9y = 0}$$

✓ Solution by Maple
Time used: 0.0 (sec). Leaf size: 17

```
dsolve(diff(y(t),t$2)+9*y(t)=0,y(t), singsol=all)
```

$$y(t) = c_1 \sin(3t) + \cos(3t) c_2$$

✓ Solution by Mathematica
Time used: 0.014 (sec). Leaf size: 20

```
DSolve[y''[t]+9*y[t]==0,y[t],t,IncludeSingularSolutions -> True]
```

$$y(t) \to c_1 \cos(3t) + c_2 \sin(3t)$$

9.5 problem 13

Internal problem ID [14449]

Book: INTRODUCTORY DIFFERENTIAL EQUATIONS. Martha L. Abell, James P. Braselton. Fourth edition 2014. ElScAe. 2014
Section: Chapter 4. Higher Order Equations. Exercises 4.1, page 141
Problem number: 13.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _missing_x]]`

$$\boxed{y'' - y' - 2y = 0}$$

With initial conditions

$$[y(0) = -1, y'(0) = -5]$$

✓ Solution by Maple
Time used: 0.015 (sec). Leaf size: 15

```
dsolve([diff(y(t),t$2)-diff(y(t),t)-2*y(t)=0,y(0) = -1, D(y)(0) = -5],y(t), singsol=all)
```

$$y(t) = e^{-t} - 2e^{2t}$$

✓ Solution by Mathematica
Time used: 0.018 (sec). Leaf size: 18

```
DSolve[{y''[t]-y'[t]-2*y[t]==0,{y[0]==-1,y'[0]==-5}},y[t],t,IncludeSingularSolutions -> True
```

$$y(t) \to e^{-t} - 2e^{2t}$$

9.6 problem 14

Internal problem ID [14450]

Book: INTRODUCTORY DIFFERENTIAL EQUATIONS. Martha L. Abell, James P. Braselton. Fourth edition 2014. ElScAe. 2014
Section: Chapter 4. Higher Order Equations. Exercises 4.1, page 141
Problem number: 14.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _missing_x]]`

$$\boxed{y'' + 9y = 0}$$

With initial conditions

$$[y(0) = 1, y'(0) = -3]$$

✓ Solution by Maple
Time used: 0.0 (sec). Leaf size: 15

```
dsolve([diff(y(t),t$2)+9*y(t)=0,y(0) = 1, D(y)(0) = -3],y(t), singsol=all)
```

$$y(t) = -\sin(3t) + \cos(3t)$$

✓ Solution by Mathematica
Time used: 0.012 (sec). Leaf size: 16

```
DSolve[{y''[t]+9*y[t]==0,{y[0]==1,y'[0]==-3}},y[t],t,IncludeSingularSolutions -> True]
```

$$y(t) \to \cos(3t) - \sin(3t)$$

9.7 problem 15

Internal problem ID [14451]

Book: INTRODUCTORY DIFFERENTIAL EQUATIONS. Martha L. Abell, James P. Braselton. Fourth edition 2014. ElScAe. 2014
Section: Chapter 4. Higher Order Equations. Exercises 4.1, page 141
Problem number: 15.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_Emden, _Fowler]]`

$$\boxed{3t^2y'' - 5y't - 3y = 0}$$

With initial conditions

$$\left[y(1) = 1, y'(1) = \frac{17}{3}\right]$$

✓ Solution by Maple
Time used: 0.016 (sec). Leaf size: 15

```
dsolve([3*t^2*diff(y(t),t$2)-5*t*diff(y(t),t)-3*y(t)=0,y(1) = 1, D(y)(1) = 17/3],y(t), sings
```

$$y(t) = -\frac{4}{5t^{\frac{1}{3}}} + \frac{9t^3}{5}$$

✓ Solution by Mathematica
Time used: 0.012 (sec). Leaf size: 22

```
DSolve[{3*t^2*y''[t]-5*t*y'[t]-3*y[t]==0,{y[1]==1,y'[1]==17/3}},y[t],t,IncludeSingularSoluti
```

$$y(t) \to \frac{9t^3}{5} - \frac{4}{5\sqrt[3]{t}}$$

9.8 problem 16

Internal problem ID [14452]

Book: INTRODUCTORY DIFFERENTIAL EQUATIONS. Martha L. Abell, James P. Braselton. Fourth edition 2014. ElScAe. 2014
Section: Chapter 4. Higher Order Equations. Exercises 4.1, page 141
Problem number: 16.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_Emden, _Fowler]]`

$$\boxed{t^2y'' + 7y't - 7y = 0}$$

With initial conditions

$$[y(1) = 2, y'(1) = -22]$$

✓ Solution by Maple
Time used: 0.015 (sec). Leaf size: 13

```
dsolve([t^2*diff(y(t),t$2)+7*t*diff(y(t),t)-7*y(t)=0,y(1) = 2, D(y)(1) = -22],y(t), singsol=
```

$$y(t) = \frac{3}{t^7} - t$$

✓ Solution by Mathematica
Time used: 0.011 (sec). Leaf size: 14

```
DSolve[{t^2*y''[t]+7*t*y'[t]-7*y[t]==0,{y[1]==2,y'[1]==-22}},y[t],t,IncludeSingularSolutions
```

$$y(t) \to \frac{3}{t^7} - t$$

9.9 problem 17

Internal problem ID [14453]

Book: INTRODUCTORY DIFFERENTIAL EQUATIONS. Martha L. Abell, James P. Braselton. Fourth edition 2014. ElScAe. 2014
Section: Chapter 4. Higher Order Equations. Exercises 4.1, page 141
Problem number: 17.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _linear, _nonhomogeneous]]`

$$\boxed{y'' + y = 2\cos(t)}$$

With initial conditions

$$[y(0) = 1, y'(0) = 1]$$

✓ Solution by Maple

Time used: 0.015 (sec). Leaf size: 13

```
dsolve([diff(y(t),t$2)+y(t)=2*cos(t),y(0) = 1, D(y)(0) = 1],y(t), singsol=all)
```

$$y(t) = \sin(t) + \cos(t) + \sin(t)\, t$$

✓ Solution by Mathematica

Time used: 0.044 (sec). Leaf size: 14

```
DSolve[{y''[t]+y[t]==2*Cos[t],{y[0]==1,y'[0]==1}},y[t],t,IncludeSingularSolutions -> True]
```

$$y(t) \to (t+1)\sin(t) + \cos(t)$$

9.10 problem 20

Internal problem ID [14454]

Book: INTRODUCTORY DIFFERENTIAL EQUATIONS. Martha L. Abell, James P. Braselton. Fourth edition 2014. ElScAe. 2014
Section: Chapter 4. Higher Order Equations. Exercises 4.1, page 141
Problem number: 20.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _missing_x]]`

$$\boxed{y'' + 10y' + 24y = 0}$$

✓ Solution by Maple
Time used: 0.016 (sec). Leaf size: 17

```
dsolve(diff(y(t),t$2)+10*diff(y(t),t)+24*y(t)=0,y(t), singsol=all)
```

$$y(t) = c_1 \mathrm{e}^{-6t} + c_2 \mathrm{e}^{-4t}$$

✓ Solution by Mathematica
Time used: 0.013 (sec). Leaf size: 22

```
DSolve[y''[t]+10*y'[t]+24*y[t]==0,y[t],t,IncludeSingularSolutions -> True]
```

$$y(t) \to e^{-6t}\left(c_2 e^{2t} + c_1\right)$$

9.11 problem 21

Internal problem ID [14455]

Book: INTRODUCTORY DIFFERENTIAL EQUATIONS. Martha L. Abell, James P. Braselton. Fourth edition 2014. ElScAe. 2014
Section: Chapter 4. Higher Order Equations. Exercises 4.1, page 141
Problem number: 21.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _missing_x]]`

$$\boxed{y'' + 16y = 0}$$

✓ Solution by Maple
Time used: 0.0 (sec). Leaf size: 17

```
dsolve(diff(y(t),t$2)+16*y(t)=0,y(t), singsol=all)
```

$$y(t) = c_1 \sin(4t) + c_2 \cos(4t)$$

✓ Solution by Mathematica
Time used: 0.016 (sec). Leaf size: 20

```
DSolve[y''[t]+16*y[t]==0,y[t],t,IncludeSingularSolutions -> True]
```

$$y(t) \to c_1 \cos(4t) + c_2 \sin(4t)$$

9.12 problem 22

Internal problem ID [14456]

Book: INTRODUCTORY DIFFERENTIAL EQUATIONS. Martha L. Abell, James P. Braselton. Fourth edition 2014. ElScAe. 2014
Section: Chapter 4. Higher Order Equations. Exercises 4.1, page 141
Problem number: 22.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _missing_x]]`

$$\boxed{y'' + 6y' + 18y = 0}$$

✓ Solution by Maple
Time used: 0.0 (sec). Leaf size: 22

```
dsolve(diff(y(t),t$2)+6*diff(y(t),t)+18*y(t)=0,y(t), singsol=all)
```

$$y(t) = e^{-3t}(c_1 \sin(3t) + \cos(3t) c_2)$$

✓ Solution by Mathematica
Time used: 0.017 (sec). Leaf size: 26

```
DSolve[y''[t]+6*y'[t]+18*y[t]==0,y[t],t,IncludeSingularSolutions -> True]
```

$$y(t) \to e^{-3t}(c_2 \cos(3t) + c_1 \sin(3t))$$

9.13 problem 23

Internal problem ID [14457]

Book: INTRODUCTORY DIFFERENTIAL EQUATIONS. Martha L. Abell, James P. Braselton. Fourth edition 2014. ElScAe. 2014
Section: Chapter 4. Higher Order Equations. Exercises 4.1, page 141
Problem number: 23.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _exact, _linear, _homogeneous]]`

$$\boxed{t^2 y'' + y' t - y = 0}$$

✓ Solution by Maple
Time used: 0.0 (sec). Leaf size: 15

```
dsolve(t^2*diff(y(t),t$2)+t*diff(y(t),t)-y(t)=0,y(t), singsol=all)
```

$$y(t) = \frac{c_2 t^2 + c_1}{t}$$

✓ Solution by Mathematica
Time used: 0.012 (sec). Leaf size: 16

```
DSolve[t^2*y''[t]+t*y'[t]-y[t]==0,y[t],t,IncludeSingularSolutions -> True]
```

$$y(t) \to \frac{c_1}{t} + c_2 t$$

9.14 problem 25

Internal problem ID [14458]

Book: INTRODUCTORY DIFFERENTIAL EQUATIONS. Martha L. Abell, James P. Braselton. Fourth edition 2014. ElScAe. 2014
Section: Chapter 4. Higher Order Equations. Exercises 4.1, page 141
Problem number: 25.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _missing_x]]`

$$\boxed{y'' - 5y' + 6y = 0}$$

Given that one solution of the ode is

$$y_1 = \mathrm{e}^{3t}$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 17

```
dsolve([diff(y(t),t$2)-5*diff(y(t),t)+6*y(t)=0,exp(3*t)],singsol=all)
```

$$y(t) = c_1 \mathrm{e}^{3t} + c_2 \mathrm{e}^{2t}$$

✓ Solution by Mathematica

Time used: 0.012 (sec). Leaf size: 20

```
DSolve[y''[t]-5*y'[t]+6*y[t]==0,y[t],t,IncludeSingularSolutions -> True]
```

$$y(t) \to e^{2t}(c_2 e^t + c_1)$$

9.15 problem 26

Internal problem ID [14459]

Book: INTRODUCTORY DIFFERENTIAL EQUATIONS. Martha L. Abell, James P. Braselton. Fourth edition 2014. ElScAe. 2014
Section: Chapter 4. Higher Order Equations. Exercises 4.1, page 141
Problem number: 26.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _missing_x]]`

$$\boxed{y'' + 6y' + 8y = 0}$$

Given that one solution of the ode is

$$y_1 = \mathrm{e}^{-2t}$$

✓ Solution by Maple
Time used: 0.0 (sec). Leaf size: 17

```
dsolve([diff(y(t),t$2)+6*diff(y(t),t)+8*y(t)=0,exp(-2*t)],singsol=all)
```

$$y(t) = \mathrm{e}^{-2t} c_1 + c_2 \mathrm{e}^{-4t}$$

✓ Solution by Mathematica
Time used: 0.013 (sec). Leaf size: 22

```
DSolve[y''[t]+6*y'[t]+8*y[t]==0,y[t],t,IncludeSingularSolutions -> True]
```

$$y(t) \to e^{-4t} \left(c_2 e^{2t} + c_1\right)$$

9.16 problem 27

Internal problem ID [14460]

Book: INTRODUCTORY DIFFERENTIAL EQUATIONS. Martha L. Abell, James P. Braselton. Fourth edition 2014. ElScAe. 2014
Section: Chapter 4. Higher Order Equations. Exercises 4.1, page 141
Problem number: 27.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _missing_x]]`

$$\boxed{y'' - 4y' + 4y = 0}$$

Given that one solution of the ode is

$$y_1 = e^{2t}$$

With initial conditions

$$[y(0) = 0, y'(0) = 1]$$

✓ Solution by Maple
Time used: 0.0 (sec). Leaf size: 10

```
dsolve([diff(diff(y(t),t),t)-4*diff(y(t),t)+4*y(t) = 0, exp(2*t), y(0) = 0, D(y)(0) = 1], si
```

$$y(t) = e^{2t}t$$

✓ Solution by Mathematica
Time used: 0.014 (sec). Leaf size: 12

```
DSolve[y''[t]-4*y'[t]+4*y[t]==0,{y[0]==0,y'[0]==1},y[t],t,IncludeSingularSolutions -> True]
```

$$y(t) \to e^{2t}t$$

9.17 problem 28

Internal problem ID [14461]

Book: INTRODUCTORY DIFFERENTIAL EQUATIONS. Martha L. Abell, James P. Braselton. Fourth edition 2014. ElScAe. 2014
Section: Chapter 4. Higher Order Equations. Exercises 4.1, page 141
Problem number: 28.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _missing_x]]`

$$\boxed{y'' + 10y' + 25y = 0}$$

Given that one solution of the ode is

$$y_1 = \mathrm{e}^{-5t}$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 14

```
dsolve([diff(y(t),t$2)+10*diff(y(t),t)+25*y(t)=0,exp(-5*t)],singsol=all)
```

$$y(t) = \mathrm{e}^{-5t}(c_2 t + c_1)$$

✓ Solution by Mathematica

Time used: 0.014 (sec). Leaf size: 18

```
DSolve[y''[t]+10*y'[t]+25*y[t]==0,y[t],t,IncludeSingularSolutions -> True]
```

$$y(t) \to e^{-5t}(c_2 t + c_1)$$

9.18 problem 29

Internal problem ID [14462]

Book: INTRODUCTORY DIFFERENTIAL EQUATIONS. Martha L. Abell, James P. Braselton. Fourth edition 2014. ElScAe. 2014
Section: Chapter 4. Higher Order Equations. Exercises 4.1, page 141
Problem number: 29.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _missing_x]]`

$$\boxed{y'' + 9y = 0}$$

Given that one solution of the ode is

$$y_1 = \cos(3t)$$

With initial conditions

$$[y(0) = 1, y'(0) = -4]$$

✓ Solution by Maple
Time used: 0.016 (sec). Leaf size: 15

```
dsolve([diff(diff(y(t),t),t)+9*y(t) = 0, cos(3*t), y(0) = 1, D(y)(0) = -4], singsol=all)
```

$$y(t) = \cos(3t) - \frac{4\sin(3t)}{3}$$

✓ Solution by Mathematica
Time used: 0.012 (sec). Leaf size: 18

```
DSolve[y''[t]+9*y[t]==0,{y[0]==1,y'[0]==-4},y[t],t,IncludeSingularSolutions -> True]
```

$$y(t) \to \cos(3t) - \frac{4}{3}\sin(3t)$$

9.19 problem 30

Internal problem ID [14463]

Book: INTRODUCTORY DIFFERENTIAL EQUATIONS. Martha L. Abell, James P. Braselton. Fourth edition 2014. ElScAe. 2014
Section: Chapter 4. Higher Order Equations. Exercises 4.1, page 141
Problem number: 30.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _missing_x]]`

$$\boxed{y'' + 49y = 0}$$

Given that one solution of the ode is

$$y_1 = \sin(7t)$$

✓ Solution by Maple
Time used: 0.0 (sec). Leaf size: 17

```
dsolve([diff(y(t),t$2)+49*y(t)=0,sin(7*t)],singsol=all)
```

$$y(t) = c_1 \sin(7t) + c_2 \cos(7t)$$

✓ Solution by Mathematica
Time used: 0.014 (sec). Leaf size: 20

```
DSolve[y''[t]+49*y[t]==0,y[t],t,IncludeSingularSolutions -> True]
```

$$y(t) \to c_1 \cos(7t) + c_2 \sin(7t)$$

9.20 problem 31

Internal problem ID [14464]

Book: INTRODUCTORY DIFFERENTIAL EQUATIONS. Martha L. Abell, James P. Braselton. Fourth edition 2014. ElScAe. 2014
Section: Chapter 4. Higher Order Equations. Exercises 4.1, page 141
Problem number: 31.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_Emden, _Fowler]]`

$$\boxed{t^2y'' + 4y't - 4y = 0}$$

Given that one solution of the ode is

$$y_1 = \frac{1}{t^4}$$

✓ Solution by Maple
Time used: 0.0 (sec). Leaf size: 15

```
dsolve([t^2*diff(y(t),t$2)+4*t*diff(y(t),t)-4*y(t)=0,1/t^4],singsol=all)
```

$$y(t) = \frac{c_1t^5 + c_2}{t^4}$$

✓ Solution by Mathematica
Time used: 0.011 (sec). Leaf size: 16

```
DSolve[t^2*y''[t]+4*t*y'[t]-4*y[t]==0,y[t],t,IncludeSingularSolutions -> True]
```

$$y(t) \to \frac{c_1}{t^4} + c_2t$$

9.21 problem 32

Internal problem ID [14465]

Book: INTRODUCTORY DIFFERENTIAL EQUATIONS. Martha L. Abell, James P. Braselton. Fourth edition 2014. ElScAe. 2014
Section: Chapter 4. Higher Order Equations. Exercises 4.1, page 141
Problem number: 32.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_Emden, _Fowler]]`

$$\boxed{t^2y'' + 6y't + 6y = 0}$$

Given that one solution of the ode is

$$y_1 = \frac{1}{t^2}$$

✓ Solution by Maple
Time used: 0.0 (sec). Leaf size: 13

```
dsolve([t^2*diff(y(t),t$2)+6*t*diff(y(t),t)+6*y(t)=0,1/t^2],singsol=all)
```

$$y(t) = \frac{c_2 t + c_1}{t^3}$$

✓ Solution by Mathematica
Time used: 0.01 (sec). Leaf size: 16

```
DSolve[t^2*y''[t]+6*t*y'[t]+6*y[t]==0,y[t],t,IncludeSingularSolutions -> True]
```

$$y(t) \to \frac{c_2 t + c_1}{t^3}$$

9.22 problem 33

Internal problem ID [14466]

Book: INTRODUCTORY DIFFERENTIAL EQUATIONS. Martha L. Abell, James P. Braselton. Fourth edition 2014. ElScAe. 2014
Section: Chapter 4. Higher Order Equations. Exercises 4.1, page 141
Problem number: 33.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$\boxed{t^2 y'' + y' t + \left(t^2 - \frac{1}{4}\right) y = 0}$$

Given that one solution of the ode is

$$y_1 = \frac{\cos(t)}{\sqrt{t}}$$

✓ Solution by Maple
Time used: 0.062 (sec). Leaf size: 17

```
dsolve([t^2*diff(y(t),t$2)+t*diff(y(t),t)+(t^2-1/4)*y(t)=0,cos(t)/sqrt(t)],singsol=all)
```

$$y(t) = \frac{c_1 \sin(t) + c_2 \cos(t)}{\sqrt{t}}$$

✓ Solution by Mathematica
Time used: 0.041 (sec). Leaf size: 39

```
DSolve[t^2*y''[t]+t*y'[t]+(t^2-1/4)*y[t]==0,y[t],t,IncludeSingularSolutions -> True]
```

$$y(t) \to \frac{e^{-it}(2c_1 - ic_2 e^{2it})}{2\sqrt{t}}$$

9.23 problem 34

Internal problem ID [14467]

Book: INTRODUCTORY DIFFERENTIAL EQUATIONS. Martha L. Abell, James P. Braselton. Fourth edition 2014. ElScAe. 2014
Section: Chapter 4. Higher Order Equations. Exercises 4.1, page 141
Problem number: 34.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _exact, _linear, _homogeneous]]`

$$\boxed{t^2 y'' + 3y't + y = 0}$$

Given that one solution of the ode is

$$y_1 = \frac{1}{t}$$

✓ Solution by Maple
Time used: 0.016 (sec). Leaf size: 14

```
dsolve([t^2*diff(y(t),t$2)+3*t*diff(y(t),t)+y(t)=0,1/t],singsol=all)
```

$$y(t) = \frac{c_2 \ln(t) + c_1}{t}$$

✓ Solution by Mathematica
Time used: 0.016 (sec). Leaf size: 17

```
DSolve[t^2*y''[t]+3*t*y'[t]+y[t]==0,y[t],t,IncludeSingularSolutions -> True]
```

$$y(t) \to \frac{c_2 \log(t) + c_1}{t}$$

9.24 problem 35

Internal problem ID [14468]

Book: INTRODUCTORY DIFFERENTIAL EQUATIONS. Martha L. Abell, James P. Braselton. Fourth edition 2014. ElScAe. 2014
Section: Chapter 4. Higher Order Equations. Exercises 4.1, page 141
Problem number: 35.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _missing_x]]`

$$\boxed{ay'' + by' + cy = 0}$$

✓ Solution by Maple
Time used: 0.0 (sec). Leaf size: 49

```
dsolve(a*diff(y(t),t$2)+b*diff(y(t),t)+c*y(t)=0,y(t), singsol=all)
```

$$y(t) = c_1 \mathrm{e}^{\frac{\left(-b+\sqrt{-4ac+b^2}\right)t}{2a}} + c_2 \mathrm{e}^{-\frac{\left(b+\sqrt{-4ac+b^2}\right)t}{2a}}$$

✓ Solution by Mathematica
Time used: 0.038 (sec). Leaf size: 55

```
DSolve[a*y''[t]+b*y'[t]+c*y[t]==0,y[t],t,IncludeSingularSolutions -> True]
```

$$y(t) \to e^{-\frac{t\left(\sqrt{b^2-4ac}+b\right)}{2a}} \left(c_2 e^{\frac{t\sqrt{b^2-4ac}}{a}} + c_1\right)$$

9.25 problem 36

Internal problem ID [14469]

Book: INTRODUCTORY DIFFERENTIAL EQUATIONS. Martha L. Abell, James P. Braselton. Fourth edition 2014. ElScAe. 2014
Section: Chapter 4. Higher Order Equations. Exercises 4.1, page 141
Problem number: 36.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_Emden, _Fowler]]`

$$\boxed{t^2y'' + aty' + by = 0}$$

✓ Solution by Maple
Time used: 0.0 (sec). Leaf size: 52

```
dsolve(t^2*diff(y(t),t$2)+a*t*diff(y(t),t)+b*y(t)=0,y(t), singsol=all)
```

$$y(t) = \sqrt{t}\, t^{-\frac{a}{2}} \left(t^{\frac{\sqrt{a^2-2a-4b+1}}{2}} c_1 + t^{-\frac{\sqrt{a^2-2a-4b+1}}{2}} c_2\right)$$

✓ Solution by Mathematica
Time used: 0.039 (sec). Leaf size: 57

```
DSolve[t^2*y''[t]+a*t*y'[t]+b*y[t]==0,y[t],t,IncludeSingularSolutions -> True]
```

$$y(t) \to t^{\frac{1}{2}\left(-\sqrt{a^2-2a-4b+1}-a+1\right)} \left(c_2 t^{\sqrt{a^2-2a-4b+1}} + c_1\right)$$

9.26 problem 44

Internal problem ID [14470]

Book: INTRODUCTORY DIFFERENTIAL EQUATIONS. Martha L. Abell, James P. Braselton. Fourth edition 2014. ElScAe. 2014
Section: Chapter 4. Higher Order Equations. Exercises 4.1, page 141
Problem number: 44.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$\boxed{4t^2y'' + 4y't + (36t^2 - 1)\, y = 0}$$

Given that one solution of the ode is

$$y_1 = \frac{1}{\sqrt{t}}$$

✓ Solution by Maple
Time used: 0.063 (sec). Leaf size: 21

```
dsolve([4*t^2*diff(y(t),t$2)+4*t*diff(y(t),t)+(36*t^2-1)*y(t)=0,1/t^(1/2)],singsol=all)
```

$$y(t) = \frac{c_1 \sin(3t) + \cos(3t)\, c_2}{\sqrt{t}}$$

✓ Solution by Mathematica
Time used: 0.045 (sec). Leaf size: 39

```
DSolve[4*t^2*y''[t]+4*t*y'[t]+(36*t^2-1)*y[t]==0,y[t],t,IncludeSingularSolutions -> True]
```

$$y(t) \to \frac{e^{-3it}(6c_1 - ic_2e^{6it})}{6\sqrt{t}}$$

9.27 problem 45

Internal problem ID [14471]

Book: INTRODUCTORY DIFFERENTIAL EQUATIONS. Martha L. Abell, James P. Braselton. Fourth edition 2014. ElScAe. 2014
Section: Chapter 4. Higher Order Equations. Exercises 4.1, page 141
Problem number: 45.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$\boxed{ty'' + 2y' + 16ty = 0}$$

Given that one solution of the ode is

$$y_1 = \frac{\sin(4t)}{t}$$

✓ Solution by Maple
Time used: 0.016 (sec). Leaf size: 21

```
dsolve([t*diff(y(t),t$2)+2*diff(y(t),t)+16*t*y(t)=0,1/t*sin(4*t)],singsol=all)
```

$$y(t) = \frac{c_1 \sin(4t) + c_2 \cos(4t)}{t}$$

✓ Solution by Mathematica
Time used: 0.037 (sec). Leaf size: 37

```
DSolve[t*y''[t]+2*y'[t]+16*t*y[t]==0,y[t],t,IncludeSingularSolutions -> True]
```

$$y(t) \to \frac{8c_1 e^{-4it} - ic_2 e^{4it}}{8t}$$

9.28 problem 46

Internal problem ID [14472]

Book: INTRODUCTORY DIFFERENTIAL EQUATIONS. Martha L. Abell, James P. Braselton. Fourth edition 2014. ElScAe. 2014
Section: Chapter 4. Higher Order Equations. Exercises 4.1, page 141
Problem number: 46.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$\boxed{y'' + b(t)\, y' + c(t)\, y = 0}$$

Given that one solution of the ode is

$$y_1 = \frac{\sin(t)}{t^2}$$

✗ Solution by Maple

```
dsolve([diff(y(t),t$2)+b(t)*diff(y(t),t)+c(t)*y(t)=0,1/t^2*sin(t)],singsol=all)
```

No solution found

✗ Solution by Mathematica

Time used: 0.0 (sec). Leaf size: 0

```
DSolve[y''[t]+b[t]*y'[t]+c[t]*y[t]==0,y[t],t,IncludeSingularSolutions -> True]
```

Not solved

9.29 problem 47

Internal problem ID [14473]

Book: INTRODUCTORY DIFFERENTIAL EQUATIONS. Martha L. Abell, James P. Braselton. Fourth edition 2014. ElScAe. 2014
Section: Chapter 4. Higher Order Equations. Exercises 4.1, page 141
Problem number: 47.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$\boxed{y'' + b(t)\, y' + c(t)\, y = 0}$$

Given that one solution of the ode is

$$y_1 = \frac{\cos(t)}{t^3}$$

With initial conditions

$$[y(\pi) = 0, y'(2\pi) = 0]$$

✗ Solution by Maple

```
dsolve([diff(diff(y(t),t),t)+b(t)*diff(y(t),t)+c(t)*y(t) = 0, 1/t^3*cos(t), y(Pi) = 0, D(y)(
```

No solution found

✗ Solution by Mathematica

Time used: 0.0 (sec). Leaf size: 0

```
DSolve[y''[t]+b[t]*y'[t]+c[t]*y[t]==0,{y[Pi]==0,y'[2*Pi]==0},y[t],t,IncludeSingularSolutions
```

Not solved

10 Chapter 4. Higher Order Equations. Exercises 4.2, page 147

10.1 problem 1

Internal problem ID [14474]

Book: INTRODUCTORY DIFFERENTIAL EQUATIONS. Martha L. Abell, James P. Braselton. Fourth edition 2014. ElScAe. 2014
Section: Chapter 4. Higher Order Equations. Exercises 4.2, page 147
Problem number: 1.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _quadrature]]`

$$\boxed{y'' = 0}$$

✓ Solution by Maple
Time used: 0.0 (sec). Leaf size: 9

```
dsolve(diff(y(t),t$2)=0,y(t), singsol=all)
```

$$y(t) = c_1 t + c_2$$

✓ Solution by Mathematica
Time used: 0.002 (sec). Leaf size: 12

```
DSolve[y''[t]==0,y[t],t,IncludeSingularSolutions -> True]
```

$$y(t) \to c_2 t + c_1$$

10.2 problem 2

Internal problem ID [14475]

Book: INTRODUCTORY DIFFERENTIAL EQUATIONS. Martha L. Abell, James P. Braselton. Fourth edition 2014. ElScAe. 2014
Section: Chapter 4. Higher Order Equations. Exercises 4.2, page 147
Problem number: 2.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _missing_x]]`

$$\boxed{y'' - 4y' - 12y = 0}$$

✓ Solution by Maple
Time used: 0.0 (sec). Leaf size: 17

```
dsolve(diff(y(t),t$2)-4*diff(y(t),t)-12*y(t)=0,y(t), singsol=all)
```

$$y(t) = e^{6t}c_1 + c_2e^{-2t}$$

✓ Solution by Mathematica
Time used: 0.013 (sec). Leaf size: 22

```
DSolve[y''[t]-4*y'[t]-12*y[t]==0,y[t],t,IncludeSingularSolutions -> True]
```

$$y(t) \to e^{-2t}(c_2e^{8t} + c_1)$$

10.3 problem 3

Internal problem ID [14476]

Book: INTRODUCTORY DIFFERENTIAL EQUATIONS. Martha L. Abell, James P. Braselton. Fourth edition 2014. ElScAe. 2014
Section: Chapter 4. Higher Order Equations. Exercises 4.2, page 147
Problem number: 3.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _missing_x]]`

$$\boxed{y'' + y' = 0}$$

✓ Solution by Maple
Time used: 0.0 (sec). Leaf size: 12

```
dsolve(diff(y(t),t$2)+diff(y(t),t)=0,y(t), singsol=all)
```

$$y(t) = c_1 + c_2 \mathrm{e}^{-t}$$

✓ Solution by Mathematica
Time used: 0.015 (sec). Leaf size: 17

```
DSolve[y''[t]+y'[t]==0,y[t],t,IncludeSingularSolutions -> True]
```

$$y(t) \to c_2 - c_1 e^{-t}$$

10.4 problem 4

Internal problem ID [14477]

Book: INTRODUCTORY DIFFERENTIAL EQUATIONS. Martha L. Abell, James P. Braselton. Fourth edition 2014. ElScAe. 2014
Section: Chapter 4. Higher Order Equations. Exercises 4.2, page 147
Problem number: 4.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _missing_x]]`

$$\boxed{y'' + 3y' - 4y = 0}$$

✓ Solution by Maple
Time used: 0.016 (sec). Leaf size: 17

```
dsolve(diff(y(t),t$2)+3*diff(y(t),t)-4*y(t)=0,y(t), singsol=all)
```

$$y(t) = \left(c_1 \mathrm{e}^{5t} + c_2\right) \mathrm{e}^{-4t}$$

✓ Solution by Mathematica
Time used: 0.013 (sec). Leaf size: 20

```
DSolve[y''[t]+3*y'[t]-4*y[t]==0,y[t],t,IncludeSingularSolutions -> True]
```

$$y(t) \to c_1 e^{-4t} + c_2 e^{t}$$

10.5 problem 5

Internal problem ID [14478]

Book: INTRODUCTORY DIFFERENTIAL EQUATIONS. Martha L. Abell, James P. Braselton. Fourth edition 2014. ElScAe. 2014
Section: Chapter 4. Higher Order Equations. Exercises 4.2, page 147
Problem number: 5.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type **[[_2nd_order, _missing_x]]**

$$\boxed{y'' + 8y' + 12y = 0}$$

✓ Solution by Maple
Time used: 0.0 (sec). Leaf size: 17

```
dsolve(diff(y(t),t$2)+8*diff(y(t),t)+12*y(t)=0,y(t), singsol=all)
```

$$y(t) = c_1 e^{-6t} + c_2 e^{-2t}$$

✓ Solution by Mathematica
Time used: 0.013 (sec). Leaf size: 22

```
DSolve[y''[t]+8*y'[t]+12*y[t]==0,y[t],t,IncludeSingularSolutions -> True]
```

$$y(t) \to e^{-6t}\left(c_2 e^{4t} + c_1\right)$$

10.6 problem 6

Internal problem ID [14479]

Book: INTRODUCTORY DIFFERENTIAL EQUATIONS. Martha L. Abell, James P. Braselton. Fourth edition 2014. ElScAe. 2014
Section: Chapter 4. Higher Order Equations. Exercises 4.2, page 147
Problem number: 6.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _missing_x]]`

$$\boxed{y'' + 5y' + y = 0}$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 27

```
dsolve(diff(y(t),t$2)+5*diff(y(t),t)+y(t)=0,y(t), singsol=all)
```

$$y(t) = c_1 \mathrm{e}^{\frac{\left(-5+\sqrt{21}\right)t}{2}} + c_2 \mathrm{e}^{-\frac{\left(5+\sqrt{21}\right)t}{2}}$$

✓ Solution by Mathematica

Time used: 0.03 (sec). Leaf size: 35

```
DSolve[y''[t]+5*y'[t]+y[t]==0,y[t],t,IncludeSingularSolutions -> True]
```

$$y(t) \to e^{-\frac{1}{2}\left(5+\sqrt{21}\right)t}\left(c_2 e^{\sqrt{21}t} + c_1\right)$$

10.7 problem 7

Internal problem ID [14480]

Book: INTRODUCTORY DIFFERENTIAL EQUATIONS. Martha L. Abell, James P. Braselton. Fourth edition 2014. ElScAe. 2014
Section: Chapter 4. Higher Order Equations. Exercises 4.2, page 147
Problem number: 7.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _missing_x]]`

$$\boxed{8y'' + 6y' + y = 0}$$

✓ Solution by Maple
Time used: 0.0 (sec). Leaf size: 17

```
dsolve(8*diff(y(t),t$2)+6*diff(y(t),t)+y(t)=0,y(t), singsol=all)
```

$$y(t) = c_1 \mathrm{e}^{-\frac{t}{4}} + c_2 \mathrm{e}^{-\frac{t}{2}}$$

✓ Solution by Mathematica
Time used: 0.014 (sec). Leaf size: 26

```
DSolve[8*y''[t]+6*y'[t]+y[t]==0,y[t],t,IncludeSingularSolutions -> True]
```

$$y(t) \to e^{-t/2}\left(c_1 e^{t/4} + c_2\right)$$

10.8 problem 8

Internal problem ID [14481]

Book: INTRODUCTORY DIFFERENTIAL EQUATIONS. Martha L. Abell, James P. Braselton. Fourth edition 2014. ElScAe. 2014
Section: Chapter 4. Higher Order Equations. Exercises 4.2, page 147
Problem number: 8.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _missing_x]]`

$$\boxed{4y'' + 9y = 0}$$

✓ Solution by Maple
Time used: 0.0 (sec). Leaf size: 17

```
dsolve(4*diff(y(t),t$2)+9*y(t)=0,y(t), singsol=all)
```

$$y(t) = c_1 \sin\left(\frac{3t}{2}\right) + c_2 \cos\left(\frac{3t}{2}\right)$$

✓ Solution by Mathematica
Time used: 0.017 (sec). Leaf size: 24

```
DSolve[4*y''[t]+9*y[t]==0,y[t],t,IncludeSingularSolutions -> True]
```

$$y(t) \rightarrow c_1 \cos\left(\frac{3t}{2}\right) + c_2 \sin\left(\frac{3t}{2}\right)$$

10.9 problem 9

Internal problem ID [14482]

Book: INTRODUCTORY DIFFERENTIAL EQUATIONS. Martha L. Abell, James P. Braselton. Fourth edition 2014. ElScAe. 2014
Section: Chapter 4. Higher Order Equations. Exercises 4.2, page 147
Problem number: 9.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _missing_x]]`

$$\boxed{y'' + 16y = 0}$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 17

```
dsolve(diff(y(t),t$2)+16*y(t)=0,y(t), singsol=all)
```

$$y(t) = c_1 \sin(4t) + c_2 \cos(4t)$$

✓ Solution by Mathematica

Time used: 0.012 (sec). Leaf size: 20

```
DSolve[y''[t]+16*y[t]==0,y[t],t,IncludeSingularSolutions -> True]
```

$$y(t) \to c_1 \cos(4t) + c_2 \sin(4t)$$

10.10 problem 10

Internal problem ID [14483]

Book: INTRODUCTORY DIFFERENTIAL EQUATIONS. Martha L. Abell, James P. Braselton. Fourth edition 2014. ElScAe. 2014
Section: Chapter 4. Higher Order Equations. Exercises 4.2, page 147
Problem number: 10.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _missing_x]]`

$$\boxed{y'' + 8y = 0}$$

✓ Solution by Maple
Time used: 0.0 (sec). Leaf size: 23

```
dsolve(diff(y(t),t$2)+8*y(t)=0,y(t), singsol=all)
```

$$y(t) = c_1 \sin\left(2\sqrt{2}\, t\right) + c_2 \cos\left(2\sqrt{2}\, t\right)$$

✓ Solution by Mathematica
Time used: 0.018 (sec). Leaf size: 30

```
DSolve[y''[t]+8*y[t]==0,y[t],t,IncludeSingularSolutions -> True]
```

$$y(t) \to c_1 \cos\left(2\sqrt{2}t\right) + c_2 \sin\left(2\sqrt{2}t\right)$$

10.11 problem 11

Internal problem ID [14484]

Book: INTRODUCTORY DIFFERENTIAL EQUATIONS. Martha L. Abell, James P. Braselton. Fourth edition 2014. ElScAe. 2014
Section: Chapter 4. Higher Order Equations. Exercises 4.2, page 147
Problem number: 11.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _missing_x]]`

$$\boxed{y'' + 7y = 0}$$

✓ Solution by Maple
Time used: 0.0 (sec). Leaf size: 21

```
dsolve(diff(y(t),t$2)+7*y(t)=0,y(t), singsol=all)
```

$$y(t) = c_1 \sin\left(\sqrt{7}\, t\right) + c_2 \cos\left(\sqrt{7}\, t\right)$$

✓ Solution by Mathematica
Time used: 0.018 (sec). Leaf size: 28

```
DSolve[y''[t]+7*y[t]==0,y[t],t,IncludeSingularSolutions -> True]
```

$$y(t) \to c_1 \cos\left(\sqrt{7}t\right) + c_2 \sin\left(\sqrt{7}t\right)$$

10.12 problem 12

Internal problem ID [14485]

Book: INTRODUCTORY DIFFERENTIAL EQUATIONS. Martha L. Abell, James P. Braselton. Fourth edition 2014. ElScAe. 2014
Section: Chapter 4. Higher Order Equations. Exercises 4.2, page 147
Problem number: 12.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _missing_x]]`

$$\boxed{4y'' + 21y' + 5y = 0}$$

✓ Solution by Maple
Time used: 0.0 (sec). Leaf size: 17

```
dsolve(4*diff(y(t),t$2)+21*diff(y(t),t)+5*y(t)=0,y(t), singsol=all)
```

$$y(t) = c_1 \mathrm{e}^{-\frac{t}{4}} + c_2 \mathrm{e}^{-5t}$$

✓ Solution by Mathematica
Time used: 0.022 (sec). Leaf size: 35

```
DSolve[y''[t]+21*y'[t]+5*y[t]==0,y[t],t,IncludeSingularSolutions -> True]
```

$$y(t) \to e^{-\frac{1}{2}\left(21+\sqrt{421}\right)t}\left(c_2 e^{\sqrt{421}t} + c_1\right)$$

10.13 problem 13

Internal problem ID [14486]

Book: INTRODUCTORY DIFFERENTIAL EQUATIONS. Martha L. Abell, James P. Braselton. Fourth edition 2014. ElScAe. 2014
Section: Chapter 4. Higher Order Equations. Exercises 4.2, page 147
Problem number: 13.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _missing_x]]`

$$\boxed{7y'' + 4y' - 3y = 0}$$

✓ Solution by Maple
Time used: 0.0 (sec). Leaf size: 17

```
dsolve(7*diff(y(t),t$2)+4*diff(y(t),t)-3*y(t)=0,y(t), singsol=all)
```

$$y(t) = \left(c_1 \mathrm{e}^{\frac{10t}{7}} + c_2\right) \mathrm{e}^{-t}$$

✓ Solution by Mathematica
Time used: 0.013 (sec). Leaf size: 24

```
DSolve[7*y''[t]+4*y'[t]-3*y[t]==0,y[t],t,IncludeSingularSolutions -> True]
```

$$y(t) \to c_1 e^{3t/7} + c_2 e^{-t}$$

10.14 problem 14

Internal problem ID [14487]

Book: INTRODUCTORY DIFFERENTIAL EQUATIONS. Martha L. Abell, James P. Braselton. Fourth edition 2014. ElScAe. 2014
Section: Chapter 4. Higher Order Equations. Exercises 4.2, page 147
Problem number: 14.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _missing_x]]`

$$\boxed{4y'' + 4y' + y = 0}$$

✓ Solution by Maple
Time used: 0.0 (sec). Leaf size: 14

```
dsolve(4*diff(y(t),t$2)+4*diff(y(t),t)+y(t)=0,y(t), singsol=all)
```

$$y(t) = \mathrm{e}^{-\frac{t}{2}}(c_2 t + c_1)$$

✓ Solution by Mathematica
Time used: 0.013 (sec). Leaf size: 20

```
DSolve[4*y''[t]+4*y'[t]+y[t]==0,y[t],t,IncludeSingularSolutions -> True]
```

$$y(t) \to e^{-t/2}(c_2 t + c_1)$$

10.15 problem 15

Internal problem ID [14488]

Book: INTRODUCTORY DIFFERENTIAL EQUATIONS. Martha L. Abell, James P. Braselton. Fourth edition 2014. ElScAe. 2014
Section: Chapter 4. Higher Order Equations. Exercises 4.2, page 147
Problem number: 15.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _missing_x]]`

$$\boxed{y'' - 6y' + 9y = 0}$$

✓ Solution by Maple
Time used: 0.015 (sec). Leaf size: 14

```
dsolve(diff(y(t),t$2)-6*diff(y(t),t)+9*y(t)=0,y(t), singsol=all)
```

$$y(t) = \mathrm{e}^{3t}(c_2 t + c_1)$$

✓ Solution by Mathematica
Time used: 0.013 (sec). Leaf size: 18

```
DSolve[y''[t]-6*y'[t]+9*y[t]==0,y[t],t,IncludeSingularSolutions -> True]
```

$$y(t) \to e^{3t}(c_2 t + c_1)$$

10.16 problem 16

Internal problem ID [14489]

Book: INTRODUCTORY DIFFERENTIAL EQUATIONS. Martha L. Abell, James P. Braselton. Fourth edition 2014. ElScAe. 2014
Section: Chapter 4. Higher Order Equations. Exercises 4.2, page 147
Problem number: 16.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _missing_x]]`

$$\boxed{y'' - y' = 0}$$

With initial conditions

$$[y(0) = 3, y'(0) = 2]$$

✓ Solution by Maple
Time used: 0.016 (sec). Leaf size: 10

```
dsolve([diff(y(t),t$2)-diff(y(t),t)=0,y(0) = 3, D(y)(0) = 2],y(t), singsol=all)
```

$$y(t) = 1 + 2\,\mathrm{e}^{t}$$

✓ Solution by Mathematica
Time used: 0.009 (sec). Leaf size: 12

```
DSolve[{y''[t]-y'[t]==0,{y[0]==3,y'[0]==2}},y[t],t,IncludeSingularSolutions -> True]
```

$$y(t) \to 2e^{t} + 1$$

10.17 problem 17

Internal problem ID [14490]

Book: INTRODUCTORY DIFFERENTIAL EQUATIONS. Martha L. Abell, James P. Braselton. Fourth edition 2014. ElScAe. 2014
Section: Chapter 4. Higher Order Equations. Exercises 4.2, page 147
Problem number: 17.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _missing_x]]`

$$\boxed{3y'' - y' = 0}$$

With initial conditions

$$[y(0) = 0, y'(0) = 7]$$

✓ Solution by Maple
Time used: 0.015 (sec). Leaf size: 12

```
dsolve([3*diff(y(t),t$2)-diff(y(t),t)=0,y(0) = 0, D(y)(0) = 7],y(t), singsol=all)
```

$$y(t) = -21 + 21\,\mathrm{e}^{\frac{t}{3}}$$

✓ Solution by Mathematica
Time used: 0.016 (sec). Leaf size: 16

```
DSolve[{3*y''[t]-y'[t]==0,{y[0]==0,y'[0]==7}},y[t],t,IncludeSingularSolutions -> True]
```

$$y(t) \to 21\left(e^{t/3} - 1\right)$$

10.18 problem 18

Internal problem ID [14491]

Book: INTRODUCTORY DIFFERENTIAL EQUATIONS. Martha L. Abell, James P. Braselton. Fourth edition 2014. ElScAe. 2014
Section: Chapter 4. Higher Order Equations. Exercises 4.2, page 147
Problem number: 18.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _missing_x]]`

$$\boxed{y'' + y' - 12y = 0}$$

With initial conditions

$$[y(0) = 3, y'(0) = 7]$$

✓ Solution by Maple
Time used: 0.0 (sec). Leaf size: 18

```
dsolve([diff(y(t),t$2)+diff(y(t),t)-12*y(t)=0,y(0) = 3, D(y)(0) = 7],y(t), singsol=all)
```

$$y(t) = \frac{(19\,\mathrm{e}^{7t} + 2)\,\mathrm{e}^{-4t}}{7}$$

✓ Solution by Mathematica
Time used: 0.017 (sec). Leaf size: 23

```
DSolve[{y''[t]+y'[t]-12*y[t]==0,{y[0]==3,y'[0]==7}},y[t],t,IncludeSingularSolutions -> True]
```

$$y(t) \to \frac{1}{7}e^{-4t}(19e^{7t} + 2)$$

10.19 problem 19

Internal problem ID [14492]

Book: INTRODUCTORY DIFFERENTIAL EQUATIONS. Martha L. Abell, James P. Braselton. Fourth edition 2014. ElScAe. 2014
Section: Chapter 4. Higher Order Equations. Exercises 4.2, page 147
Problem number: 19.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _missing_x]]`

$$\boxed{y'' - 7y' + 12y = 0}$$

With initial conditions

$$[y(0) = 3, y'(0) = -2]$$

✓ Solution by Maple
Time used: 0.0 (sec). Leaf size: 17

```
dsolve([diff(y(t),t$2)-7*diff(y(t),t)+12*y(t)=0,y(0) = 3, D(y)(0) = -2],y(t), singsol=all)
```

$$y(t) = -11\,\mathrm{e}^{4t} + 14\,\mathrm{e}^{3t}$$

✓ Solution by Mathematica
Time used: 0.022 (sec). Leaf size: 18

```
DSolve[{y''[t]-7*y'[t]+12*y[t]==0,{y[0]==3,y'[0]==-2}},y[t],t,IncludeSingularSolutions -> Tr
```

$$y(t) \to e^{3t}(14 - 11e^t)$$

10.20 problem 20

Internal problem ID [14493]

Book: INTRODUCTORY DIFFERENTIAL EQUATIONS. Martha L. Abell, James P. Braselton. Fourth edition 2014. ElScAe. 2014
Section: Chapter 4. Higher Order Equations. Exercises 4.2, page 147
Problem number: 20.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _missing_x]]`

$$\boxed{2y'' - 7y' - 4y = 0}$$

With initial conditions

$$[y(0) = 0, y'(0) = 1]$$

✓ Solution by Maple
Time used: 0.016 (sec). Leaf size: 17

```
dsolve([2*diff(y(t),t$2)-7*diff(y(t),t)-4*y(t)=0,y(0) = 0, D(y)(0) = 1],y(t), singsol=all)
```

$$y(t) = \frac{2\,e^{4t}}{9} - \frac{2\,e^{-\frac{t}{2}}}{9}$$

✓ Solution by Mathematica
Time used: 0.014 (sec). Leaf size: 25

```
DSolve[{2*y''[t]-7*y'[t]-4*y[t]==0,{y[0]==0,y'[0]==1}},y[t],t,IncludeSingularSolutions -> Tr
```

$$y(t) \to \frac{2}{9}e^{-t/2}\left(e^{9t/2} - 1\right)$$

10.21 problem 21

Internal problem ID [14494]

Book: INTRODUCTORY DIFFERENTIAL EQUATIONS. Martha L. Abell, James P. Braselton. Fourth edition 2014. ElScAe. 2014
Section: Chapter 4. Higher Order Equations. Exercises 4.2, page 147
Problem number: 21.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _missing_x]]`

$$\boxed{y'' - 7y' + 10y = 0}$$

With initial conditions

$$[y(0) = 1, y'(0) = 5]$$

✓ Solution by Maple
Time used: 0.015 (sec). Leaf size: 8

```
dsolve([diff(y(t),t$2)-7*diff(y(t),t)+10*y(t)=0,y(0) = 1, D(y)(0) = 5],y(t), singsol=all)
```

$$y(t) = \mathrm{e}^{5t}$$

✓ Solution by Mathematica
Time used: 0.013 (sec). Leaf size: 10

```
DSolve[{y''[t]-7*y'[t]+10*y[t]==0,{y[0]==1,y'[0]==5}},y[t],t,IncludeSingularSolutions -> Tru
```

$$y(t) \to e^{5t}$$

10.22 problem 22

Internal problem ID [14495]

Book: INTRODUCTORY DIFFERENTIAL EQUATIONS. Martha L. Abell, James P. Braselton. Fourth edition 2014. ElScAe. 2014
Section: Chapter 4. Higher Order Equations. Exercises 4.2, page 147
Problem number: 22.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _missing_x]]`

$$\boxed{y'' + 36y = 0}$$

With initial conditions

$$[y(0) = 2, y'(0) = -6]$$

✓ Solution by Maple
Time used: 0.016 (sec). Leaf size: 17

```
dsolve([diff(y(t),t$2)+36*y(t)=0,y(0) = 2, D(y)(0) = -6],y(t), singsol=all)
```

$$y(t) = -\sin(6t) + 2\cos(6t)$$

✓ Solution by Mathematica
Time used: 0.014 (sec). Leaf size: 18

```
DSolve[{y''[t]+36*y[t]==0,{y[0]==2,y'[0]==-6}},y[t],t,IncludeSingularSolutions -> True]
```

$$y(t) \to 2\cos(6t) - \sin(6t)$$

10.23 problem 23

Internal problem ID [14496]

Book: INTRODUCTORY DIFFERENTIAL EQUATIONS. Martha L. Abell, James P. Braselton. Fourth edition 2014. ElScAe. 2014
Section: Chapter 4. Higher Order Equations. Exercises 4.2, page 147
Problem number: 23.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _missing_x]]`

$$\boxed{y'' + 100y = 0}$$

With initial conditions

$$[y(0) = 1, y'(0) = 10]$$

✓ Solution by Maple
Time used: 0.0 (sec). Leaf size: 13

```
dsolve([diff(y(t),t$2)+100*y(t)=0,y(0) = 1, D(y)(0) = 10],y(t), singsol=all)
```

$$y(t) = \sin(10t) + \cos(10t)$$

✓ Solution by Mathematica
Time used: 0.014 (sec). Leaf size: 14

```
DSolve[{y''[t]+100*y[t]==0,{y[0]==1,y'[0]==10}},y[t],t,IncludeSingularSolutions -> True]
```

$$y(t) \to \sin(10t) + \cos(10t)$$

10.24 problem 24

Internal problem ID [14497]

Book: INTRODUCTORY DIFFERENTIAL EQUATIONS. Martha L. Abell, James P. Braselton. Fourth edition 2014. ElScAe. 2014
Section: Chapter 4. Higher Order Equations. Exercises 4.2, page 147
Problem number: 24.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _missing_x]]`

$$\boxed{y'' - 2y' + y = 0}$$

With initial conditions

$$[y(0) = 4, y'(0) = 0]$$

✓ Solution by Maple
Time used: 0.016 (sec). Leaf size: 11

```
dsolve([diff(y(t),t$2)-2*diff(y(t),t)+y(t)=0,y(0) = 4, D(y)(0) = 0],y(t), singsol=all)
```

$$y(t) = -4\,\mathrm{e}^{t}(t-1)$$

✓ Solution by Mathematica
Time used: 0.012 (sec). Leaf size: 13

```
DSolve[{y''[t]-2*y'[t]+y[t]==0,{y[0]==4,y'[0]==0}},y[t],t,IncludeSingularSolutions -> True]
```

$$y(t) \rightarrow -4e^{t}(t-1)$$

10.25 problem 25

Internal problem ID [14498]

Book: INTRODUCTORY DIFFERENTIAL EQUATIONS. Martha L. Abell, James P. Braselton. Fourth edition 2014. ElScAe. 2014
Section: Chapter 4. Higher Order Equations. Exercises 4.2, page 147
Problem number: 25.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _missing_x]]`

$$\boxed{y'' + 4y' + 4y = 0}$$

With initial conditions

$$[y(0) = 1, y'(0) = 3]$$

✓ Solution by Maple
Time used: 0.016 (sec). Leaf size: 14

```
dsolve([diff(y(t),t$2)+4*diff(y(t),t)+4*y(t)=0,y(0) = 1, D(y)(0) = 3],y(t), singsol=all)
```

$$y(t) = \mathrm{e}^{-2t}(5t + 1)$$

✓ Solution by Mathematica
Time used: 0.014 (sec). Leaf size: 16

```
DSolve[{y''[t]+4*y'[t]+4*y[t]==0,{y[0]==1,y'[0]==3}},y[t],t,IncludeSingularSolutions -> True
```

$$y(t) \to e^{-2t}(5t + 1)$$

10.26 problem 26

Internal problem ID [14499]

Book: INTRODUCTORY DIFFERENTIAL EQUATIONS. Martha L. Abell, James P. Braselton. Fourth edition 2014. ElScAe. 2014
Section: Chapter 4. Higher Order Equations. Exercises 4.2, page 147
Problem number: 26.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _missing_x]]`

$$\boxed{y'' + 2y' + 5y = 0}$$

With initial conditions

$$[y(0) = 1, y'(0) = 0]$$

✓ Solution by Maple
Time used: 0.016 (sec). Leaf size: 21

```
dsolve([diff(y(t),t$2)+2*diff(y(t),t)+5*y(t)=0,y(0) = 1, D(y)(0) = 0],y(t), singsol=all)
```

$$y(t) = \frac{e^{-t}(2\cos(2t) + \sin(2t))}{2}$$

✓ Solution by Mathematica
Time used: 0.019 (sec). Leaf size: 25

```
DSolve[{y''[t]+2*y'[t]+5*y[t]==0,{y[0]==1,y'[0]==0}},y[t],t,IncludeSingularSolutions -> True
```

$$y(t) \to \frac{1}{2}e^{-t}(\sin(2t) + 2\cos(2t))$$

10.27 problem 27

Internal problem ID [14500]

Book: INTRODUCTORY DIFFERENTIAL EQUATIONS. Martha L. Abell, James P. Braselton. Fourth edition 2014. ElScAe. 2014
Section: Chapter 4. Higher Order Equations. Exercises 4.2, page 147
Problem number: 27.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _missing_x]]`

$$\boxed{y'' + 4y' + 20y = 0}$$

With initial conditions

$$[y(0) = 2, y'(0) = 0]$$

✓ Solution by Maple
Time used: 0.016 (sec). Leaf size: 20

```
dsolve([diff(y(t),t$2)+4*diff(y(t),t)+20*y(t)=0,y(0) = 2, D(y)(0) = 0],y(t), singsol=all)
```

$$y(t) = \mathrm{e}^{-2t}(\sin(4t) + 2\cos(4t))$$

✓ Solution by Mathematica
Time used: 0.017 (sec). Leaf size: 22

```
DSolve[{y''[t]+4*y'[t]+20*y[t]==0,{y[0]==2,y'[0]==0}},y[t],t,IncludeSingularSolutions -> Tru
```

$$y(t) \to e^{-2t}(\sin(4t) + 2\cos(4t))$$

10.28 problem 28

Internal problem ID [14501]

Book: INTRODUCTORY DIFFERENTIAL EQUATIONS. Martha L. Abell, James P. Braselton. Fourth edition 2014. ElScAe. 2014
Section: Chapter 4. Higher Order Equations. Exercises 4.2, page 147
Problem number: 28.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _missing_x]]`

$$\boxed{y'' + y' - y = 0}$$

With initial conditions

$$[y(0) = 1, y'(0) = 0]$$

✓ Solution by Maple

Time used: 0.046 (sec). Leaf size: 37

```
dsolve([diff(y(t),t$2)+diff(y(t),t)-y(t)=0,y(0) = 1, D(y)(0) = 0],y(t), singsol=all)
```

$$y(t) = \frac{(5+\sqrt{5})\,\mathrm{e}^{\frac{(\sqrt{5}-1)t}{2}}}{10} - \frac{\mathrm{e}^{-\frac{(\sqrt{5}+1)t}{2}}(-5+\sqrt{5})}{10}$$

✓ Solution by Mathematica

Time used: 0.024 (sec). Leaf size: 49

```
DSolve[{y''[t]+y'[t]-y[t]==0,{y[0]==1,y'[0]==0}},y[t],t,IncludeSingularSolutions -> True]
```

$$y(t) \to \frac{1}{10} e^{-\frac{1}{2}(1+\sqrt{5})t}\left(\left(5+\sqrt{5}\right)e^{\sqrt{5}t} + 5 - \sqrt{5}\right)$$

10.29 problem 29

Internal problem ID [14502]

Book: INTRODUCTORY DIFFERENTIAL EQUATIONS. Martha L. Abell, James P. Braselton. Fourth edition 2014. ElScAe. 2014
Section: Chapter 4. Higher Order Equations. Exercises 4.2, page 147
Problem number: 29.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _missing_x]]`

$$\boxed{y'' + y' + y = 0}$$

With initial conditions

$$[y(0) = 1, y'(0) = 0]$$

✓ Solution by Maple
Time used: 0.016 (sec). Leaf size: 31

```
dsolve([diff(y(t),t$2)+diff(y(t),t)+y(t)=0,y(0) = 1, D(y)(0) = 0],y(t), singsol=all)
```

$$y(t) = \frac{e^{-\frac{t}{2}}\left(\sqrt{3}\,\sin\left(\frac{\sqrt{3}\,t}{2}\right) + 3\cos\left(\frac{\sqrt{3}\,t}{2}\right)\right)}{3}$$

✓ Solution by Mathematica
Time used: 0.022 (sec). Leaf size: 47

```
DSolve[{y''[t]+y'[t]+y[t]==0,{y[0]==1,y'[0]==0}},y[t],t,IncludeSingularSolutions -> True]
```

$$y(t) \to \frac{1}{3}e^{-t/2}\left(\sqrt{3}\sin\left(\frac{\sqrt{3}t}{2}\right) + 3\cos\left(\frac{\sqrt{3}t}{2}\right)\right)$$

10.30 problem 30

Internal problem ID [14503]

Book: INTRODUCTORY DIFFERENTIAL EQUATIONS. Martha L. Abell, James P. Braselton. Fourth edition 2014. ElScAe. 2014
Section: Chapter 4. Higher Order Equations. Exercises 4.2, page 147
Problem number: 30.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _missing_x]]`

$$\boxed{y'' - y' + y = 0}$$

With initial conditions

$$[y(0) = 0, y'(0) = 1]$$

✓ Solution by Maple

Time used: 0.016 (sec). Leaf size: 20

```
dsolve([diff(y(t),t$2)-diff(y(t),t)+y(t)=0,y(0) = 0, D(y)(0) = 1],y(t), singsol=all)
```

$$y(t) = \frac{2\sqrt{3}\,\mathrm{e}^{\frac{t}{2}} \sin\left(\frac{\sqrt{3}\,t}{2}\right)}{3}$$

✓ Solution by Mathematica

Time used: 0.019 (sec). Leaf size: 30

```
DSolve[{y''[t]-y'[t]+y[t]==0,{y[0]==0,y'[0]==1}},y[t],t,IncludeSingularSolutions -> True]
```

$$y(t) \to \frac{2e^{t/2} \sin\left(\frac{\sqrt{3}t}{2}\right)}{\sqrt{3}}$$

10.31 problem 31

Internal problem ID [14504]

Book: INTRODUCTORY DIFFERENTIAL EQUATIONS. Martha L. Abell, James P. Braselton. Fourth edition 2014. ElScAe. 2014
Section: Chapter 4. Higher Order Equations. Exercises 4.2, page 147
Problem number: 31.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _missing_x]]`

$$\boxed{y'' - y' - y = 0}$$

With initial conditions

$$[y(0) = 0, y'(0) = 1]$$

✓ Solution by Maple
Time used: 0.047 (sec). Leaf size: 30

```
dsolve([diff(y(t),t$2)-diff(y(t),t)-y(t)=0,y(0) = 0, D(y)(0) = 1],y(t), singsol=all)
```

$$y(t) = -\frac{\left(-\mathrm{e}^{\frac{(\sqrt{5}+1)t}{2}} + \mathrm{e}^{-\frac{(\sqrt{5}-1)t}{2}}\right)\sqrt{5}}{5}$$

✓ Solution by Mathematica
Time used: 0.023 (sec). Leaf size: 38

```
DSolve[{y''[t]-y'[t]-y[t]==0,{y[0]==0,y'[0]==1}},y[t],t,IncludeSingularSolutions -> True]
```

$$y(t) \to \frac{e^{\frac{1}{2}(t-\sqrt{5}t)}\left(e^{\sqrt{5}t} - 1\right)}{\sqrt{5}}$$

10.32 problem 33

Internal problem ID [14505]

Book: INTRODUCTORY DIFFERENTIAL EQUATIONS. Martha L. Abell, James P. Braselton. Fourth edition 2014. ElScAe. 2014
Section: Chapter 4. Higher Order Equations. Exercises 4.2, page 147
Problem number: 33.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _missing_x]]`

$$\boxed{6y'' + 5y' + y = 0}$$

✓ Solution by Maple
Time used: 0.0 (sec). Leaf size: 17

```
dsolve(6*diff(y(t),t$2)+5*diff(y(t),t)+y(t)=0,y(t), singsol=all)
```

$$y(t) = c_1 e^{-\frac{t}{3}} + c_2 e^{-\frac{t}{2}}$$

✓ Solution by Mathematica
Time used: 0.013 (sec). Leaf size: 26

```
DSolve[6*y''[t]+5*y'[t]+y[t]==0,y[t],t,IncludeSingularSolutions -> True]
```

$$y(t) \to e^{-t/2}\left(c_1 e^{t/6} + c_2\right)$$

10.33 problem 34

Internal problem ID [14506]

Book: INTRODUCTORY DIFFERENTIAL EQUATIONS. Martha L. Abell, James P. Braselton. Fourth edition 2014. ElScAe. 2014
Section: Chapter 4. Higher Order Equations. Exercises 4.2, page 147
Problem number: 34.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _missing_x]]`

$$\boxed{9y'' + 6y' + y = 0}$$

✓ Solution by Maple
Time used: 0.0 (sec). Leaf size: 14

```
dsolve(9*diff(y(t),t$2)+6*diff(y(t),t)+y(t)=0,y(t), singsol=all)
```

$$y(t) = \mathrm{e}^{-\frac{t}{3}}(c_2 t + c_1)$$

✓ Solution by Mathematica
Time used: 0.013 (sec). Leaf size: 20

```
DSolve[9*y''[t]+6*y'[t]+y[t]==0,y[t],t,IncludeSingularSolutions -> True]
```

$$y(t) \to e^{-t/3}(c_2 t + c_1)$$

10.34 problem 35

Internal problem ID [14507]

Book: INTRODUCTORY DIFFERENTIAL EQUATIONS. Martha L. Abell, James P. Braselton. Fourth edition 2014. ElScAe. 2014
Section: Chapter 4. Higher Order Equations. Exercises 4.2, page 147
Problem number: 35.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _missing_x]]`

$$\boxed{y'' + 4y' + 20y = 0}$$

✓ Solution by Maple
Time used: 0.0 (sec). Leaf size: 22

```
dsolve(diff(y(t),t$2)+4*diff(y(t),t)+20*y(t)=0,y(t), singsol=all)
```

$$y(t) = \mathrm{e}^{-2t}(c_1 \sin(4t) + c_2 \cos(4t))$$

✓ Solution by Mathematica
Time used: 0.017 (sec). Leaf size: 26

```
DSolve[y''[t]+4*y'[t]+20*y[t]==0,y[t],t,IncludeSingularSolutions -> True]
```

$$y(t) \to e^{-2t}(c_2 \cos(4t) + c_1 \sin(4t))$$

10.35 problem 37 (a)

Internal problem ID [14508]

Book: INTRODUCTORY DIFFERENTIAL EQUATIONS. Martha L. Abell, James P. Braselton. Fourth edition 2014. ElScAe. 2014
Section: Chapter 4. Higher Order Equations. Exercises 4.2, page 147
Problem number: 37 (a).
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_Emden, _Fowler]]`

$$\boxed{3t^2y'' - 2y't + 2y = 0}$$

✓ Solution by Maple
Time used: 0.0 (sec). Leaf size: 13

```
dsolve(3*t^2*diff(y(t),t$2)-2*t*diff(y(t),t)+2*y(t)=0,y(t), singsol=all)
```

$$y(t) = c_1 t^{\frac{2}{3}} + c_2 t$$

✓ Solution by Mathematica
Time used: 0.011 (sec). Leaf size: 18

```
DSolve[3*t^2*y''[t]-2*t*y'[t]+2*y[t]==0,y[t],t,IncludeSingularSolutions -> True]
```

$$y(t) \to c_1 t^{2/3} + c_2 t$$

10.36 problem 37 (b)

Internal problem ID [14509]

Book: INTRODUCTORY DIFFERENTIAL EQUATIONS. Martha L. Abell, James P. Braselton. Fourth edition 2014. ElScAe. 2014
Section: Chapter 4. Higher Order Equations. Exercises 4.2, page 147
Problem number: 37 (b).
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_Emden, _Fowler]]`

$$\boxed{t^2y'' - y't + y = 0}$$

✓ Solution by Maple
Time used: 0.0 (sec). Leaf size: 12

```
dsolve(t^2*diff(y(t),t$2)-t*diff(y(t),t)+y(t)=0,y(t), singsol=all)
```

$$y(t) = t(c_2 \ln(t) + c_1)$$

✓ Solution by Mathematica
Time used: 0.016 (sec). Leaf size: 15

```
DSolve[t^2*y''[t]-t*y'[t]+y[t]==0,y[t],t,IncludeSingularSolutions -> True]
```

$$y(t) \to t(c_2 \log(t) + c_1)$$

10.37 problem 39

Internal problem ID [14510]

Book: INTRODUCTORY DIFFERENTIAL EQUATIONS. Martha L. Abell, James P. Braselton. Fourth edition 2014. ElScAe. 2014
Section: Chapter 4. Higher Order Equations. Exercises 4.2, page 147
Problem number: 39.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _missing_x]]`

$$\boxed{ay'' + 2by' + cy = 0}$$

✓ Solution by Maple
Time used: 0.0 (sec). Leaf size: 48

```
dsolve(a*diff(y(t),t$2)+2*b*diff(y(t),t)+c*y(t)=0,y(t), singsol=all)
```

$$y(t) = c_1 \mathrm{e}^{\frac{\left(-b+\sqrt{-ac+b^2}\right)t}{a}} + c_2 \mathrm{e}^{-\frac{\left(b+\sqrt{-ac+b^2}\right)t}{a}}$$

✓ Solution by Mathematica
Time used: 0.032 (sec). Leaf size: 54

```
DSolve[a*y''[t]+2*b*y'[t]+c*y[t]==0,y[t],t,IncludeSingularSolutions -> True]
```

$$y(t) \to e^{-\frac{t\left(\sqrt{b^2-ac}+b\right)}{a}} \left(c_2 e^{\frac{2t\sqrt{b^2-ac}}{a}} + c_1\right)$$

10.38 problem 40 (a)

Internal problem ID [14511]

Book: INTRODUCTORY DIFFERENTIAL EQUATIONS. Martha L. Abell, James P. Braselton. Fourth edition 2014. ElScAe. 2014
Section: Chapter 4. Higher Order Equations. Exercises 4.2, page 147
Problem number: 40 (a).
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _missing_x]]`

$$\boxed{y'' + 6y' + 2y = 0}$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 26

```
dsolve(diff(y(t),t$2)+6*diff(y(t),t)+2*y(t)=0,y(t), singsol=all)
```

$$y(t) = c_1 \mathrm{e}^{\left(-3+\sqrt{7}\right)t} + c_2 \mathrm{e}^{-\left(3+\sqrt{7}\right)t}$$

✓ Solution by Mathematica

Time used: 0.019 (sec). Leaf size: 34

```
DSolve[y''[t]+6*y'[t]+2*y[t]==0,y[t],t,IncludeSingularSolutions -> True]
```

$$y(t) \to e^{-\left(\left(3+\sqrt{7}\right)t\right)}\left(c_2 e^{2\sqrt{7}t} + c_1\right)$$

10.39 problem 40 (b)

Internal problem ID [14512]

Book: INTRODUCTORY DIFFERENTIAL EQUATIONS. Martha L. Abell, James P. Braselton. Fourth edition 2014. ElScAe. 2014
Section: Chapter 4. Higher Order Equations. Exercises 4.2, page 147
Problem number: 40 (b).
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _missing_x]]`

$$\boxed{y'' - 5y' + 6y = 0}$$

✓ Solution by Maple
Time used: 0.0 (sec). Leaf size: 17

```
dsolve(diff(y(t),t$2)-5*diff(y(t),t)+6*y(t)=0,y(t), singsol=all)
```

$$y(t) = c_1 e^{3t} + c_2 e^{2t}$$

✓ Solution by Mathematica
Time used: 0.012 (sec). Leaf size: 20

```
DSolve[y''[t]-5*y'[t]+6*y[t]==0,y[t],t,IncludeSingularSolutions -> True]
```

$$y(t) \to e^{2t}(c_2 e^t + c_1)$$

10.40 problem 40 (c)

Internal problem ID [14513]

Book: INTRODUCTORY DIFFERENTIAL EQUATIONS. Martha L. Abell, James P. Braselton. Fourth edition 2014. ElScAe. 2014
Section: Chapter 4. Higher Order Equations. Exercises 4.2, page 147
Problem number: 40 (c).
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _missing_x]]`

$$\boxed{y'' - 6y' - 16y = 0}$$

✓ Solution by Maple
Time used: 0.0 (sec). Leaf size: 17

```
dsolve(diff(y(t),t$2)-6*diff(y(t),t)-16*y(t)=0,y(t), singsol=all)
```

$$y(t) = c_1 \mathrm{e}^{8t} + c_2 \mathrm{e}^{-2t}$$

✓ Solution by Mathematica
Time used: 0.012 (sec). Leaf size: 22

```
DSolve[y''[t]-6*y'[t]-16*y[t]==0,y[t],t,IncludeSingularSolutions -> True]
```

$$y(t) \to c_1 e^{-2t} + c_2 e^{8t}$$

10.41 problem 40 (d)

Internal problem ID [14514]

Book: INTRODUCTORY DIFFERENTIAL EQUATIONS. Martha L. Abell, James P. Braselton. Fourth edition 2014. ElScAe. 2014
Section: Chapter 4. Higher Order Equations. Exercises 4.2, page 147
Problem number: 40 (d).
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _missing_x]]`

$$\boxed{y'' - 16y = 0}$$

✓ Solution by Maple
Time used: 0.0 (sec). Leaf size: 17

```
dsolve(diff(y(t),t$2)-16*y(t)=0,y(t), singsol=all)
```

$$y(t) = c_1 e^{4t} + c_2 e^{-4t}$$

✓ Solution by Mathematica
Time used: 0.012 (sec). Leaf size: 22

```
DSolve[y''[t]-16*y[t]==0,y[t],t,IncludeSingularSolutions -> True]
```

$$y(t) \to e^{-4t}\left(c_1 e^{8t} + c_2\right)$$

10.42 problem 41

Internal problem ID [14515]

Book: INTRODUCTORY DIFFERENTIAL EQUATIONS. Martha L. Abell, James P. Braselton. Fourth edition 2014. ElScAe. 2014
Section: Chapter 4. Higher Order Equations. Exercises 4.2, page 147
Problem number: 41.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type **[[_2nd_order, _missing_x]]**

$$\boxed{y'' + 2y' + 5y = 0}$$

With initial conditions

$$\left[y(0) = 0, y'\left(\frac{\pi}{2}\right) = 0\right]$$

✓ Solution by Maple
Time used: 0.016 (sec). Leaf size: 5

```
dsolve([diff(y(t),t$2)+2*diff(y(t),t)+5*y(t)=0,y(0) = 0, D(y)(1/2*Pi) = 0],y(t), singsol=all
```

$$y(t) = 0$$

✓ Solution by Mathematica
Time used: 0.02 (sec). Leaf size: 6

```
DSolve[{y''[t]+2*y'[t]+5*y[t]==0,{y[0]==0,y'[Pi/2]==0}},y[t],t,IncludeSingularSolutions -> T
```

$$y(t) \to 0$$

10.43 problem 42 (a)

Internal problem ID [14516]

Book: INTRODUCTORY DIFFERENTIAL EQUATIONS. Martha L. Abell, James P. Braselton. Fourth edition 2014. ElScAe. 2014
Section: Chapter 4. Higher Order Equations. Exercises 4.2, page 147
Problem number: 42 (a).
ODE order: 2.
ODE degree: 2.

CAS Maple gives this as type `[[_2nd_order, _missing_x]]`

$$\boxed{y''^2 - 5y'y'' + 4y^2 = 0}$$

✓ Solution by Maple
Time used: 0.172 (sec). Leaf size: 85

```
dsolve(diff(y(t),t$2)^2-5*diff(y(t),t$2)*diff(y(t),t)+4*y(t)^2=0,y(t), singsol=all)
```

$$y(t) = 0$$
$$y(t) = \mathrm{e}^{\int \mathrm{RootOf}\left(t+2\left(\int^{\_Z} \frac{1}{2\_f^2+\sqrt{25\_f^2-16}-5\_f} d\_f\right)+c_1\right)dt+c_2}$$
$$y(t) = \mathrm{e}^{\int \mathrm{RootOf}\left(t-2\left(\int^{\_Z} -\frac{1}{2\_f^2-\sqrt{25\_f^2-16}-5\_f} d\_f\right)+c_1\right)dt+c_2}$$

✗ Solution by Mathematica
Time used: 0.0 (sec). Leaf size: 0

```
DSolve[y''[t]^2-5*y''[t]*y'[t]+4*y[t]^2==0,y[t],t,IncludeSingularSolutions -> True]
```

Not solved

10.44 problem 42 (b)

Internal problem ID [14517]

Book: INTRODUCTORY DIFFERENTIAL EQUATIONS. Martha L. Abell, James P. Braselton. Fourth edition 2014. ElScAe. 2014
Section: Chapter 4. Higher Order Equations. Exercises 4.2, page 147
Problem number: 42 (b).
ODE order: 2.
ODE degree: 2.

CAS Maple gives this as type `[[_2nd_order, _missing_x]]`

$$\boxed{y''^2 - 2y'y'' + y^2 = 0}$$

✓ Solution by Maple

Time used: 0.172 (sec). Leaf size: 82

```
dsolve(diff(y(t),t$2)^2-2*diff(y(t),t$2)*diff(y(t),t)+y(t)^2=0,y(t), singsol=all)
```

$$
\begin{aligned}
y(t) &= 0 \\
y(t) &= \mathrm{e}^{t} c_1 \\
y(t) &= \mathrm{e}^{\int \operatorname{RootOf}\left(t+\int^{\_Z} \frac{1}{\_f^2+\sqrt{\_f^2-1}-\_f} d\_f+c_1\right) dt+c_2} \\
y(t) &= \mathrm{e}^{\int \operatorname{RootOf}\left(t-\left(\int^{\_Z} -\frac{1}{\_f^2-\sqrt{\_f^2-1}-\_f} d\_f\right)+c_1\right) dt+c_2}
\end{aligned}
$$

✗ Solution by Mathematica

Time used: 0.0 (sec). Leaf size: 0

```
DSolve[y''[t]^2-2*y''[t]*y'[t]+y[t]^2==0,y[t],t,IncludeSingularSolutions -> True]
```

Not solved

10.45 problem 43

Internal problem ID [14518]

Book: INTRODUCTORY DIFFERENTIAL EQUATIONS. Martha L. Abell, James P. Braselton. Fourth edition 2014. ElScAe. 2014
Section: Chapter 4. Higher Order Equations. Exercises 4.2, page 147
Problem number: 43.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _missing_x]]`

$$\boxed{y'' + 4y' + 3y = 0}$$

With initial conditions

$$[y(0) = a, y'(0) = b]$$

✓ Solution by Maple
Time used: 0.015 (sec). Leaf size: 29

```
dsolve([diff(y(t),t$2)+4*diff(y(t),t)+3*y(t)=0,y(0) = a, D(y)(0) = b],y(t), singsol=all)
```

$$y(t) = \frac{(-a-b)\,\mathrm{e}^{-3t}}{2} + \frac{3\left(a+\frac{b}{3}\right)\mathrm{e}^{-t}}{2}$$

✓ Solution by Mathematica
Time used: 0.015 (sec). Leaf size: 35

```
DSolve[{y''[t]+4*y'[t]+3*y[t]==0,{y[0]==a,y'[0]==b}},y[t],t,IncludeSingularSolutions -> True
```

$$y(t) \to \frac{1}{2}e^{-3t}\left(a\left(3e^{2t}-1\right)+b\left(e^{2t}-1\right)\right)$$

11 Chapter 4. Higher Order Equations. Exercises 4.3, page 156

11.1 problem 13

Internal problem ID [14519]

Book: INTRODUCTORY DIFFERENTIAL EQUATIONS. Martha L. Abell, James P. Braselton. Fourth edition 2014. ElScAe. 2014
Section: Chapter 4. Higher Order Equations. Exercises 4.3, page 156
Problem number: 13.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$\boxed{y'' + y = 8\,\mathrm{e}^{2t}}$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 19

```
dsolve(diff(y(t),t$2)+y(t)=8*exp(2*t),y(t), singsol=all)
```

$$y(t) = \sin(t)\, c_2 + \cos(t)\, c_1 + \frac{8\,\mathrm{e}^{2t}}{5}$$

✓ Solution by Mathematica

Time used: 0.016 (sec). Leaf size: 25

```
DSolve[y''[t]+y[t]==8*Exp[2*t],y[t],t,IncludeSingularSolutions -> True]
```

$$y(t) \to \frac{8e^{2t}}{5} + c_1 \cos(t) + c_2 \sin(t)$$

11.2 problem 14

Internal problem ID [14520]

Book: INTRODUCTORY DIFFERENTIAL EQUATIONS. Martha L. Abell, James P. Braselton. Fourth edition 2014. ElScAe. 2014
Section: Chapter 4. Higher Order Equations. Exercises 4.3, page 156
Problem number: 14.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$\boxed{y'' - 4y' + 3y = -\mathrm{e}^{-9t}}$$

✓ Solution by Maple
Time used: 0.0 (sec). Leaf size: 23

```
dsolve(diff(y(t),t$2)-4*diff(y(t),t)+3*y(t)=-exp(-9*t),y(t), singsol=all)
```

$$y(t) = \mathrm{e}^{-9t}\left(\mathrm{e}^{10t}c_1 + c_2\mathrm{e}^{12t} - \frac{1}{120}\right)$$

✓ Solution by Mathematica
Time used: 0.019 (sec). Leaf size: 29

```
DSolve[y''[t]-4*y'[t]+3*y[t]==-Exp[-9*t],y[t],t,IncludeSingularSolutions -> True]
```

$$y(t) \to -\frac{e^{-9t}}{120} + c_1e^t + c_2e^{3t}$$

11.3 problem 15

Internal problem ID [14521]

Book: INTRODUCTORY DIFFERENTIAL EQUATIONS. Martha L. Abell, James P. Braselton. Fourth edition 2014. ElScAe. 2014
Section: Chapter 4. Higher Order Equations. Exercises 4.3, page 156
Problem number: 15.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$\boxed{y'' - 4y' + 3y = 2\,\mathrm{e}^{3t}}$$

✓ Solution by Maple
Time used: 0.015 (sec). Leaf size: 17

```
dsolve(diff(y(t),t$2)-4*diff(y(t),t)+3*y(t)=2*exp(3*t),y(t), singsol=all)
```

$$y(t) = (c_2 + t)\,\mathrm{e}^{3t} + \mathrm{e}^{t} c_1$$

✓ Solution by Mathematica
Time used: 0.034 (sec). Leaf size: 25

```
DSolve[y''[t]-4*y'[t]+3*y[t]==2*Exp[3*t],y[t],t,IncludeSingularSolutions -> True]
```

$$y(t) \to c_1 e^t + e^{3t}\left(t - \frac{1}{2} + c_2\right)$$

11.4 problem 16

Internal problem ID [14522]

Book: INTRODUCTORY DIFFERENTIAL EQUATIONS. Martha L. Abell, James P. Braselton. Fourth edition 2014. ElScAe. 2014
Section: Chapter 4. Higher Order Equations. Exercises 4.3, page 156
Problem number: 16.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$\boxed{y'' - y = -4 + 2t}$$

✓ Solution by Maple
Time used: 0.0 (sec). Leaf size: 19

```
dsolve(diff(y(t),t$2)-y(t)=2*t-4,y(t), singsol=all)
```

$$y(t) = c_2 \mathrm{e}^{-t} + \mathrm{e}^{t} c_1 - 2t + 4$$

✓ Solution by Mathematica
Time used: 0.013 (sec). Leaf size: 24

```
DSolve[y''[t]-y[t]==2*t-4,y[t],t,IncludeSingularSolutions -> True]
```

$$y(t) \to -2t + c_1 e^t + c_2 e^{-t} + 4$$

11.5 problem 17

Internal problem ID [14523]

Book: INTRODUCTORY DIFFERENTIAL EQUATIONS. Martha L. Abell, James P. Braselton. Fourth edition 2014. ElScAe. 2014
Section: Chapter 4. Higher Order Equations. Exercises 4.3, page 156
Problem number: 17.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$\boxed{y'' - 2y' + y = t^2}$$

✓ Solution by Maple
Time used: 0.0 (sec). Leaf size: 20

```
dsolve(diff(y(t),t$2)-2*diff(y(t),t)+y(t)=t^2,y(t), singsol=all)
```

$$y(t) = (c_1 t + c_2) e^t + t^2 + 4t + 6$$

✓ Solution by Mathematica
Time used: 0.014 (sec). Leaf size: 26

```
DSolve[y''[t]-2*y'[t]+y[t]==t^2,y[t],t,IncludeSingularSolutions -> True]
```

$$y(t) \to t^2 + t(4 + c_2 e^t) + c_1 e^t + 6$$

11.6 problem 18

Internal problem ID [14524]

Book: INTRODUCTORY DIFFERENTIAL EQUATIONS. Martha L. Abell, James P. Braselton. Fourth edition 2014. ElScAe. 2014
Section: Chapter 4. Higher Order Equations. Exercises 4.3, page 156
Problem number: 18.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _missing_y]]`

$$\boxed{y'' + 2y' = -4t + 3}$$

✓ Solution by Maple
Time used: 0.016 (sec). Leaf size: 21

```
dsolve(diff(y(t),t$2)+2*diff(y(t),t)=3-4*t,y(t), singsol=all)
```

$$y(t) = -\frac{e^{-2t}c_1}{2} - t^2 + \frac{5t}{2} + c_2$$

✓ Solution by Mathematica
Time used: 0.109 (sec). Leaf size: 29

```
DSolve[y''[t]+2*y'[t]==3-4*t,y[t],t,IncludeSingularSolutions -> True]
```

$$y(t) \to -t^2 + \frac{5t}{2} - \frac{1}{2}c_1e^{-2t} + c_2$$

11.7 problem 19

Internal problem ID [14525]

Book: INTRODUCTORY DIFFERENTIAL EQUATIONS. Martha L. Abell, James P. Braselton. Fourth edition 2014. ElScAe. 2014
Section: Chapter 4. Higher Order Equations. Exercises 4.3, page 156
Problem number: 19.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _linear, _nonhomogeneous]]`

$$\boxed{y'' + y = \cos(2t)}$$

✓ Solution by Maple
Time used: 0.016 (sec). Leaf size: 19

```
dsolve(diff(y(t),t$2)+y(t)=cos(2*t),y(t), singsol=all)
```

$$y(t) = \sin(t)\, c_2 + \cos(t)\, c_1 - \frac{\cos(2t)}{3}$$

✓ Solution by Mathematica
Time used: 0.106 (sec). Leaf size: 24

```
DSolve[y''[t]+y[t]==Cos[2*t],y[t],t,IncludeSingularSolutions -> True]
```

$$y(t) \to -\frac{1}{3}\cos(2t) + c_1 \cos(t) + c_2 \sin(t)$$

11.8 problem 20

Internal problem ID [14526]

Book: INTRODUCTORY DIFFERENTIAL EQUATIONS. Martha L. Abell, James P. Braselton. Fourth edition 2014. ElScAe. 2014
Section: Chapter 4. Higher Order Equations. Exercises 4.3, page 156
Problem number: 20.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _linear, _nonhomogeneous]]`

$$\boxed{y'' + 4y = 4\cos(t) - \sin(t)}$$

✓ Solution by Maple
Time used: 0.0 (sec). Leaf size: 25

```
dsolve(diff(y(t),t$2)+4*y(t)=4*cos(t)-sin(t),y(t), singsol=all)
```

$$y(t) = \sin(2t)\, c_2 + \cos(2t)\, c_1 + \frac{4\cos(t)}{3} - \frac{\sin(t)}{3}$$

✓ Solution by Mathematica
Time used: 0.022 (sec). Leaf size: 32

```
DSolve[y''[t]+4*y[t]==4*Cos[t]-Sin[t],y[t],t,IncludeSingularSolutions -> True]
```

$$y(t) \to -\frac{\sin(t)}{3} + \frac{4\cos(t)}{3} + c_1\cos(2t) + c_2\sin(2t)$$

11.9 problem 21

Internal problem ID [14527]

Book: INTRODUCTORY DIFFERENTIAL EQUATIONS. Martha L. Abell, James P. Braselton. Fourth edition 2014. ElScAe. 2014
Section: Chapter 4. Higher Order Equations. Exercises 4.3, page 156
Problem number: 21.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _linear, _nonhomogeneous]]`

$$\boxed{y'' + 4y = \cos(2t) + t}$$

✓ Solution by Maple
Time used: 0.0 (sec). Leaf size: 32

```
dsolve(diff(y(t),t$2)+4*y(t)=cos(2*t)+t,y(t), singsol=all)
```

$$y(t) = \frac{(8c_1 + 1)\cos(2t)}{8} + \frac{(8c_2 + 2t)\sin(2t)}{8} + \frac{t}{4}$$

✓ Solution by Mathematica
Time used: 0.223 (sec). Leaf size: 36

```
DSolve[y''[t]+4*y[t]==Cos[2*t]+t,y[t],t,IncludeSingularSolutions -> True]
```

$$y(t) \to \frac{1}{8}((1 + 8c_1)\cos(2t) + 2(t + (t + 4c_2)\sin(2t)))$$

11.10 problem 22

Internal problem ID [14528]

Book: INTRODUCTORY DIFFERENTIAL EQUATIONS. Martha L. Abell, James P. Braselton. Fourth edition 2014. ElScAe. 2014
Section: Chapter 4. Higher Order Equations. Exercises 4.3, page 156
Problem number: 22.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _linear, _nonhomogeneous]]`

$$\boxed{y'' + 4y = 3t\,\mathrm{e}^{-t}}$$

✓ Solution by Maple
Time used: 0.0 (sec). Leaf size: 28

```
dsolve(diff(y(t),t$2)+4*y(t)=3*t*exp(-t),y(t), singsol=all)
```

$$y(t) = \sin(2t)\,c_2 + \cos(2t)\,c_1 + \frac{3(5t+2)\,\mathrm{e}^{-t}}{25}$$

✓ Solution by Mathematica
Time used: 0.02 (sec). Leaf size: 34

```
DSolve[y''[t]+4*y[t]==3*t*Exp[-t],y[t],t,IncludeSingularSolutions -> True]
```

$$y(t) \to \frac{3}{25}e^{-t}(5t+2) + c_1\cos(2t) + c_2\sin(2t)$$

11.11 problem 23

Internal problem ID [14529]

Book: INTRODUCTORY DIFFERENTIAL EQUATIONS. Martha L. Abell, James P. Braselton. Fourth edition 2014. ElScAe. 2014
Section: Chapter 4. Higher Order Equations. Exercises 4.3, page 156
Problem number: 23.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _quadrature]]`

$$\boxed{y'' = 3t^4 - 2t}$$

✓ Solution by Maple
Time used: 0.0 (sec). Leaf size: 20

```
dsolve(diff(y(t),t$2)=3*t^4-2*t,y(t), singsol=all)
```

$$y(t) = \frac{1}{10}t^6 - \frac{1}{3}t^3 + \frac{5}{18} + c_1 t + c_2$$

✓ Solution by Mathematica
Time used: 0.002 (sec). Leaf size: 26

```
DSolve[y''[t]==3*t^4-2*t,y[t],t,IncludeSingularSolutions -> True]
```

$$y(t) \to \frac{t^6}{10} - \frac{t^3}{3} + c_2 t + c_1$$

11.12 problem 24

Internal problem ID [14530]

Book: INTRODUCTORY DIFFERENTIAL EQUATIONS. Martha L. Abell, James P. Braselton. Fourth edition 2014. ElScAe. 2014
Section: Chapter 4. Higher Order Equations. Exercises 4.3, page 156
Problem number: 24.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _linear, _nonhomogeneous]]`

$$\boxed{y'' - 4y' + 13y = 2t\,\mathrm{e}^{-2t} \sin(3t)}$$

✓ Solution by Maple
Time used: 0.0 (sec). Leaf size: 50

```
dsolve(diff(y(t),t$2)-4*diff(y(t),t)+13*y(t)=2*t*exp(-2*t)*sin(3*t),y(t), singsol=all)
```

$$y(t) = \frac{((156t + 63)\cos(3t) + (104t + 16)\sin(3t))\,\mathrm{e}^{-2t}}{2704} + \mathrm{e}^{2t}(c_1 \cos(3t) + c_2 \sin(3t))$$

✓ Solution by Mathematica
Time used: 0.042 (sec). Leaf size: 54

```
DSolve[y''[t]-4*y'[t]+13*y[t]==2*t*Exp[-2*t]*Sin[3*t],y[t],t,IncludeSingularSolutions -> Tru
```

$$y(t) \to \frac{e^{-2t}((156t + 2704c_2e^{4t} + 63)\cos(3t) + 8(13t + 338c_1e^{4t} + 2)\sin(3t))}{2704}$$

11.13 problem 25

Internal problem ID [14531]

Book: INTRODUCTORY DIFFERENTIAL EQUATIONS. Martha L. Abell, James P. Braselton. Fourth edition 2014. ElScAe. 2014
Section: Chapter 4. Higher Order Equations. Exercises 4.3, page 156
Problem number: 25.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _missing_x]]`

$$\boxed{y'' + y' - 2y = -1}$$

✓ Solution by Maple
Time used: 0.0 (sec). Leaf size: 25

```
dsolve(diff(y(t),t$2)+diff(y(t),t)-2*y(t)=-1,y(t), singsol=all)
```

$$y(t) = \frac{(2c_2 e^{3t} + e^{2t} + 2c_1)\, e^{-2t}}{2}$$

✓ Solution by Mathematica
Time used: 0.012 (sec). Leaf size: 23

```
DSolve[y''[t]+y'[t]-2*y[t]==-1,y[t],t,IncludeSingularSolutions -> True]
```

$$y(t) \to c_1 e^{-2t} + c_2 e^t + \frac{1}{2}$$

11.14 problem 26

Internal problem ID [14532]

Book: INTRODUCTORY DIFFERENTIAL EQUATIONS. Martha L. Abell, James P. Braselton. Fourth edition 2014. ElScAe. 2014
Section: Chapter 4. Higher Order Equations. Exercises 4.3, page 156
Problem number: 26.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _missing_x]]`

$$\boxed{5y'' + y' - 4y = -3}$$

✓ Solution by Maple
Time used: 0.016 (sec). Leaf size: 25

```
dsolve(5*diff(y(t),t$2)+diff(y(t),t)-4*y(t)=-3,y(t), singsol=all)
```

$$y(t) = \frac{\left(4\,\mathrm{e}^{\frac{9t}{5}} c_1 + 3\,\mathrm{e}^{t} + 4c_2\right)\mathrm{e}^{-t}}{4}$$

✓ Solution by Mathematica
Time used: 0.014 (sec). Leaf size: 27

```
DSolve[5*y''[t]+y'[t]-4*y[t]==-3,y[t],t,IncludeSingularSolutions -> True]
```

$$y(t) \to c_1 e^{4t/5} + c_2 e^{-t} + \frac{3}{4}$$

11.15 problem 27

Internal problem ID [14533]

Book: INTRODUCTORY DIFFERENTIAL EQUATIONS. Martha L. Abell, James P. Braselton. Fourth edition 2014. ElScAe. 2014
Section: Chapter 4. Higher Order Equations. Exercises 4.3, page 156
Problem number: 27.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$\boxed{y'' - 2y' - 8y = 32t}$$

✓ Solution by Maple
Time used: 0.0 (sec). Leaf size: 21

```
dsolve(diff(y(t),t$2)-2*diff(y(t),t)-8*y(t)=32*t,y(t), singsol=all)
```

$$y(t) = e^{4t}c_2 + e^{-2t}c_1 - 4t + 1$$

✓ Solution by Mathematica
Time used: 0.014 (sec). Leaf size: 26

```
DSolve[y''[t]-2*y'[t]-8*y[t]==32*t,y[t],t,IncludeSingularSolutions -> True]
```

$$y(t) \to -4t + c_1e^{-2t} + c_2e^{4t} + 1$$

11.16 problem 28

Internal problem ID [14534]

Book: INTRODUCTORY DIFFERENTIAL EQUATIONS. Martha L. Abell, James P. Braselton. Fourth edition 2014. ElScAe. 2014
Section: Chapter 4. Higher Order Equations. Exercises 4.3, page 156
Problem number: 28.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$\boxed{16y'' - 8y' - 15y = 75t}$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 34

```
dsolve(16*diff(y(t),t$2)-8*diff(y(t),t)-15*y(t)=75*t,y(t), singsol=all)
```

$$y(t) = \frac{\left(3c_2 e^{2t} - 15t\, e^{\frac{3t}{4}} + 8\, e^{\frac{3t}{4}} + 3c_1\right) e^{-\frac{3t}{4}}}{3}$$

✓ Solution by Mathematica

Time used: 0.015 (sec). Leaf size: 32

```
DSolve[16*y''[t]-8*y'[t]-15*y[t]==75*t,y[t],t,IncludeSingularSolutions -> True]
```

$$y(t) \to -5t + c_1 e^{-3t/4} + c_2 e^{5t/4} + \frac{8}{3}$$

11.17 problem 29

Internal problem ID [14535]

Book: INTRODUCTORY DIFFERENTIAL EQUATIONS. Martha L. Abell, James P. Braselton. Fourth edition 2014. ElScAe. 2014
Section: Chapter 4. Higher Order Equations. Exercises 4.3, page 156
Problem number: 29.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$\boxed{y'' + 2y' + 26y = -338t}$$

✓ Solution by Maple
Time used: 0.0 (sec). Leaf size: 29

```
dsolve(diff(y(t),t$2)+2*diff(y(t),t)+26*y(t)=-338*t,y(t), singsol=all)
```

$$y(t) = \mathrm{e}^{-t} \sin(5t) c_2 + \mathrm{e}^{-t} \cos(5t) c_1 - 13t + 1$$

✓ Solution by Mathematica
Time used: 0.02 (sec). Leaf size: 34

```
DSolve[y''[t]+2*y'[t]+26*y[t]==-338*t,y[t],t,IncludeSingularSolutions -> True]
```

$$y(t) \to -13t + c_2 e^{-t} \cos(5t) + c_1 e^{-t} \sin(5t) + 1$$

11.18 problem 30

Internal problem ID [14536]

Book: INTRODUCTORY DIFFERENTIAL EQUATIONS. Martha L. Abell, James P. Braselton. Fourth edition 2014. ElScAe. 2014
Section: Chapter 4. Higher Order Equations. Exercises 4.3, page 156
Problem number: 30.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$\boxed{y'' + 3y' - 4y = -32t^2}$$

✓ Solution by Maple
Time used: 0.016 (sec). Leaf size: 37

```
dsolve(diff(y(t),t$2)+3*diff(y(t),t)-4*y(t)=-32*t^2,y(t), singsol=all)
```

$$y(t) = \left(8t^2 + 12t + 13\right) \mathrm{e}^{-4t}\mathrm{e}^{4t} + \left(c_2\mathrm{e}^{5t} + c_1\right) \mathrm{e}^{-4t}$$

✓ Solution by Mathematica
Time used: 0.016 (sec). Leaf size: 29

```
DSolve[y''[t]+3*y'[t]-4*y[t]==-32*t^2,y[t],t,IncludeSingularSolutions -> True]
```

$$y(t) \to 8t^2 + 12t + c_1e^{-4t} + c_2e^t + 13$$

11.19 problem 31

Internal problem ID [14537]

Book: INTRODUCTORY DIFFERENTIAL EQUATIONS. Martha L. Abell, James P. Braselton. Fourth edition 2014. ElScAe. 2014
Section: Chapter 4. Higher Order Equations. Exercises 4.3, page 156
Problem number: 31.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$\boxed{8y'' + 6y' + y = 5t^2}$$

✓ Solution by Maple
Time used: 0.015 (sec). Leaf size: 27

```
dsolve(8*diff(y(t),t$2)+6*diff(y(t),t)+y(t)=5*t^2,y(t), singsol=all)
```

$$y(t) = -4\,\mathrm{e}^{-\frac{t}{2}}c_1 + \mathrm{e}^{-\frac{t}{4}}c_2 + 5t^2 - 60t + 280$$

✓ Solution by Mathematica
Time used: 0.014 (sec). Leaf size: 35

```
DSolve[8*y''[t]+6*y'[t]+y[t]==5*t^2,y[t],t,IncludeSingularSolutions -> True]
```

$$y(t) \rightarrow 5t^2 - 60t + c_1e^{-t/4} + c_2e^{-t/2} + 280$$

11.20 problem 32

Internal problem ID [14538]

Book: INTRODUCTORY DIFFERENTIAL EQUATIONS. Martha L. Abell, James P. Braselton. Fourth edition 2014. ElScAe. 2014
Section: Chapter 4. Higher Order Equations. Exercises 4.3, page 156
Problem number: 32.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _linear, _nonhomogeneous]]`

$$\boxed{y'' - 6y' + 8y = -256t^3}$$

✓ Solution by Maple
Time used: 0.016 (sec). Leaf size: 32

```
dsolve(diff(y(t),t$2)-6*diff(y(t),t)+8*y(t)=-256*t^3,y(t), singsol=all)
```

$$y(t) = \frac{c_1 e^{4t}}{2} - 45 - 84t - 72t^2 - 32t^3 + c_2 e^{2t}$$

✓ Solution by Mathematica
Time used: 0.015 (sec). Leaf size: 36

```
DSolve[y''[t]-6*y'[t]+8*y[t]==-256*t^3,y[t],t,IncludeSingularSolutions -> True]
```

$$y(t) \to -32t^3 - 72t^2 - 84t + c_1 e^{2t} + c_2 e^{4t} - 45$$

11.21 problem 33

Internal problem ID [14539]

Book: INTRODUCTORY DIFFERENTIAL EQUATIONS. Martha L. Abell, James P. Braselton. Fourth edition 2014. ElScAe. 2014
Section: Chapter 4. Higher Order Equations. Exercises 4.3, page 156
Problem number: 33.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _missing_y]]`

$$\boxed{y'' - 2y' = 52 \sin(3t)}$$

✓ Solution by Maple
Time used: 0.0 (sec). Leaf size: 25

```
dsolve(diff(y(t),t$2)-2*diff(y(t),t)=52*sin(3*t),y(t), singsol=all)
```

$$y(t) = \frac{c_1 e^{2t}}{2} - 4\sin(3t) + \frac{8\cos(3t)}{3} + c_2$$

✓ Solution by Mathematica
Time used: 0.171 (sec). Leaf size: 33

```
DSolve[y''[t]-2*y'[t]==52*Sin[3*t],y[t],t,IncludeSingularSolutions -> True]
```

$$y(t) \to -4\sin(3t) + \frac{8}{3}\cos(3t) + \frac{1}{2}c_1 e^{2t} + c_2$$

11.22 problem 34

Internal problem ID [14540]

Book: INTRODUCTORY DIFFERENTIAL EQUATIONS. Martha L. Abell, James P. Braselton. Fourth edition 2014. ElScAe. 2014
Section: Chapter 4. Higher Order Equations. Exercises 4.3, page 156
Problem number: 34.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _linear, _nonhomogeneous]]`

$$\boxed{y'' - 6y' + 13y = 25\sin(2t)}$$

✓ Solution by Maple
Time used: 0.015 (sec). Leaf size: 36

```
dsolve(diff(y(t),t$2)-6*diff(y(t),t)+13*y(t)=25*sin(2*t),y(t), singsol=all)
```

$$y(t) = \frac{(3c_2 e^{3t} + 3)\sin(2t)}{3} + \cos(2t)\, e^{3t} c_1 + \frac{4\cos(2t)}{3}$$

✓ Solution by Mathematica
Time used: 0.027 (sec). Leaf size: 38

```
DSolve[y''[t]-6*y'[t]+13*y[t]==25*Sin[2*t],y[t],t,IncludeSingularSolutions -> True]
```

$$y(t) \to \left(\frac{4}{3} + c_2 e^{3t}\right)\cos(2t) + \left(1 + c_1 e^{3t}\right)\sin(2t)$$

11.23 problem 35

Internal problem ID [14541]

Book: INTRODUCTORY DIFFERENTIAL EQUATIONS. Martha L. Abell, James P. Braselton. Fourth edition 2014. ElScAe. 2014
Section: Chapter 4. Higher Order Equations. Exercises 4.3, page 156
Problem number: 35.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _linear, _nonhomogeneous]]`

$$\boxed{y'' - 9y = 54 \sin(2t)\, t}$$

✓ Solution by Maple
Time used: 0.0 (sec). Leaf size: 30

```
dsolve(diff(y(t),t$2)-9*y(t)=54*t*sin(2*t),y(t), singsol=all)
```

$$y(t) = c_2 \mathrm{e}^{-3t} + c_1 \mathrm{e}^{3t} - \frac{216\cos(2t)}{169} - \frac{54t\sin(2t)}{13}$$

✓ Solution by Mathematica
Time used: 0.022 (sec). Leaf size: 40

```
DSolve[y''[t]-9*y[t]==54*t*Sin[2*t],y[t],t,IncludeSingularSolutions -> True]
```

$$y(t) \to c_1 e^{3t} + c_2 e^{-3t} - \frac{54}{169}(13t\sin(2t) + 4\cos(2t))$$

11.24 problem 36

Internal problem ID [14542]

Book: INTRODUCTORY DIFFERENTIAL EQUATIONS. Martha L. Abell, James P. Braselton. Fourth edition 2014. ElScAe. 2014
Section: Chapter 4. Higher Order Equations. Exercises 4.3, page 156
Problem number: 36.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _linear, _nonhomogeneous]]`

$$\boxed{y'' - 5y' + 6y = -78\cos(3t)}$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 27

```
dsolve(diff(y(t),t$2)-5*diff(y(t),t)+6*y(t)=-78*cos(3*t),y(t), singsol=all)
```

$$y(t) = c_2 e^{3t} + c_1 e^{2t} + 5\sin(3t) + \cos(3t)$$

✓ Solution by Mathematica

Time used: 0.022 (sec). Leaf size: 31

```
DSolve[y''[t]-5*y'[t]+6*y[t]==-78*Cos[3*t],y[t],t,IncludeSingularSolutions -> True]
```

$$y(t) \to 5\sin(3t) + \cos(3t) + e^{2t}(c_2 e^t + c_1)$$

11.25 problem 37

Internal problem ID [14543]

Book: INTRODUCTORY DIFFERENTIAL EQUATIONS. Martha L. Abell, James P. Braselton. Fourth edition 2014. ElScAe. 2014
Section: Chapter 4. Higher Order Equations. Exercises 4.3, page 156
Problem number: 37.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _linear, _nonhomogeneous]]`

$$\boxed{y'' + 4y' + 4y = -32t^2 \cos(2t)}$$

✓ Solution by Maple
Time used: 0.015 (sec). Leaf size: 35

```
dsolve(diff(y(t),t$2)+4*diff(y(t),t)+4*y(t)=-32*t^2*cos(2*t),y(t), singsol=all)
```

$$y(t) = (3 - 4t)\cos(2t) + (c_1 t + c_2)\,\mathrm{e}^{-2t} - 4t \sin(2t)(t - 1)$$

✓ Solution by Mathematica
Time used: 0.026 (sec). Leaf size: 44

```
DSolve[y''[t]+4*y'[t]+4*y[t]==-32*t^2*Cos[2*t],y[t],t,IncludeSingularSolutions -> True]
```

$$y(t) \to (3 - 4t)\cos(2t) + e^{-2t}\left(-4e^{2t}(t-1)t\sin(2t) + c_2 t + c_1\right)$$

11.26 problem 38

Internal problem ID [14544]

Book: INTRODUCTORY DIFFERENTIAL EQUATIONS. Martha L. Abell, James P. Braselton. Fourth edition 2014. ElScAe. 2014
Section: Chapter 4. Higher Order Equations. Exercises 4.3, page 156
Problem number: 38.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$\boxed{y'' - y' - 20y = -2\,\mathrm{e}^{t}}$$

✓ Solution by Maple
Time used: 0.0 (sec). Leaf size: 25

```
dsolve(diff(y(t),t$2)-diff(y(t),t)-20*y(t)=-2*exp(t),y(t), singsol=all)
```

$$y(t) = \frac{(10c_2\mathrm{e}^{9t} + \mathrm{e}^{5t} + 10c_1)\,\mathrm{e}^{-4t}}{10}$$

✓ Solution by Mathematica
Time used: 0.017 (sec). Leaf size: 29

```
DSolve[y''[t]-y'[t]-20*y[t]==-2*Exp[t],y[t],t,IncludeSingularSolutions -> True]
```

$$y(t) \to \frac{e^t}{10} + c_1 e^{-4t} + c_2 e^{5t}$$

11.27 problem 39

Internal problem ID [14545]

Book: INTRODUCTORY DIFFERENTIAL EQUATIONS. Martha L. Abell, James P. Braselton. Fourth edition 2014. ElScAe. 2014
Section: Chapter 4. Higher Order Equations. Exercises 4.3, page 156
Problem number: 39.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _linear, _nonhomogeneous]]`

$$\boxed{y'' - 4y' - 5y = -648t^2 \mathrm{e}^{5t}}$$

✓ Solution by Maple
Time used: 0.016 (sec). Leaf size: 31

```
dsolve(diff(y(t),t$2)-4*diff(y(t),t)-5*y(t)=-648*t^2*exp(5*t),y(t), singsol=all)
```

$$y(t) = \left(-36t^3 + 18t^2 + c_2 - 6t\right) \mathrm{e}^{5t} + \mathrm{e}^{-t} c_1$$

✓ Solution by Mathematica
Time used: 0.052 (sec). Leaf size: 37

```
DSolve[y''[t]-4*y'[t]-5*y[t]==-648*t^2*Exp[5*t],y[t],t,IncludeSingularSolutions -> True]
```

$$y(t) \to e^{-t}\left(e^{6t}\left(-36t^3 + 18t^2 - 6t + 1 + c_2\right) + c_1\right)$$

11.28 problem 40

Internal problem ID [14546]

Book: INTRODUCTORY DIFFERENTIAL EQUATIONS. Martha L. Abell, James P. Braselton. Fourth edition 2014. ElScAe. 2014
Section: Chapter 4. Higher Order Equations. Exercises 4.3, page 156
Problem number: 40.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _linear, _nonhomogeneous]]`

$$\boxed{y'' - 7y' + 12y = -2t^3 \mathrm{e}^{4t}}$$

✓ Solution by Maple
Time used: 0.0 (sec). Leaf size: 39

```
dsolve(diff(y(t),t$2)-7*diff(y(t),t)+12*y(t)=-2*t^3*exp(4*t),y(t), singsol=all)
```

$$y(t) = \frac{(-t^4 + 4t^3 - 12t^2 + 2c_2 + 24t)\,\mathrm{e}^{4t}}{2} + c_1 \mathrm{e}^{3t}$$

✓ Solution by Mathematica
Time used: 0.048 (sec). Leaf size: 42

```
DSolve[y''[t]-7*y'[t]+12*y[t]==-2*t^3*Exp[4*t],y[t],t,IncludeSingularSolutions -> True]
```

$$y(t) \to e^{3t}\left(e^t\left(-\frac{t^4}{2} + 2t^3 - 6t^2 + 12t - 12 + c_2\right) + c_1\right)$$

11.29 problem 41

Internal problem ID [14547]

Book: INTRODUCTORY DIFFERENTIAL EQUATIONS. Martha L. Abell, James P. Braselton. Fourth edition 2014. ElScAe. 2014
Section: Chapter 4. Higher Order Equations. Exercises 4.3, page 156
Problem number: 41.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _missing_y]]`

$$\boxed{y'' + 4y' = 8\,\mathrm{e}^{4t} - 4\,\mathrm{e}^{-4t}}$$

✓ Solution by Maple
Time used: 0.0 (sec). Leaf size: 26

```
dsolve(diff(y(t),t$2)+4*diff(y(t),t)=8*exp(4*t)-4*exp(-4*t),y(t), singsol=all)
```

$$y(t) = \frac{(4t - c_1 + 1)\,\mathrm{e}^{-4t}}{4} + c_2 + \frac{\mathrm{e}^{4t}}{4}$$

✓ Solution by Mathematica
Time used: 0.115 (sec). Leaf size: 31

```
DSolve[y''[t]+4*y'[t]==8*Exp[4*t]-4*Exp[-4*t],y[t],t,IncludeSingularSolutions -> True]
```

$$y(t) \to \frac{1}{4}e^{-4t}\left(4t + e^{8t} + 1 - c_1\right) + c_2$$

11.30 problem 42

Internal problem ID [14548]

Book: INTRODUCTORY DIFFERENTIAL EQUATIONS. Martha L. Abell, James P. Braselton. Fourth edition 2014. ElScAe. 2014
Section: Chapter 4. Higher Order Equations. Exercises 4.3, page 156
Problem number: 42.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _missing_y]]`

$$\boxed{y'' - 3y' = t^2 - e^{3t}}$$

✓ Solution by Maple
Time used: 0.0 (sec). Leaf size: 33

```
dsolve(diff(y(t),t$2)-3*diff(y(t),t)=t^2-exp(3*t),y(t), singsol=all)
```

$$y(t) = \frac{(1 - 3t + 3c_1)\,e^{3t}}{9} - \frac{t^3}{9} - \frac{t^2}{9} - \frac{2t}{27} + c_2$$

✓ Solution by Mathematica
Time used: 0.177 (sec). Leaf size: 42

```
DSolve[y''[t]-3*y'[t]==t^2-Exp[3*t],y[t],t,IncludeSingularSolutions -> True]
```

$$y(t) \to \frac{1}{27}\left(-3t^3 - 3t^2 - 2t + e^{3t}(-9t + 3 + 9c_1) + 27c_2\right)$$

11.31 problem 43

Internal problem ID [14549]

Book: INTRODUCTORY DIFFERENTIAL EQUATIONS. Martha L. Abell, James P. Braselton. Fourth edition 2014. ElScAe. 2014
Section: Chapter 4. Higher Order Equations. Exercises 4.3, page 156
Problem number: 43.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _missing_y]]`

$$\boxed{y'' + 4y' = -24t - 6 - 4t\,\mathrm{e}^{-4t} + \mathrm{e}^{-4t}}$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 26

```
dsolve(diff(y(t),t$2)+4*diff(y(t),t)=-24*t-6-4*t*exp(-4*t)+exp(-4*t),y(t), singsol=all)
```

$$y(t) = \frac{(2t^2 - c_1)\,\mathrm{e}^{-4t}}{4} - 3t^2 + c_2$$

✓ Solution by Mathematica

Time used: 0.24 (sec). Leaf size: 32

```
DSolve[y''[t]+4*y'[t]==-24*t-6-4*t*Exp[-4*t]+Exp[-4*t],y[t],t,IncludeSingularSolutions -> Tr
```

$$y(t) \to -3t^2 + \frac{1}{4}e^{-4t}\left(2t^2 - c_1\right) + c_2$$

11.32 problem 44

Internal problem ID [14550]

Book: INTRODUCTORY DIFFERENTIAL EQUATIONS. Martha L. Abell, James P. Braselton. Fourth edition 2014. ElScAe. 2014
Section: Chapter 4. Higher Order Equations. Exercises 4.3, page 156
Problem number: 44.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _missing_y]]`

$$\boxed{y'' - 3y' = t^2 - e^{3t}}$$

✓ Solution by Maple
Time used: 0.0 (sec). Leaf size: 33

```
dsolve(diff(y(t),t$2)-3*diff(y(t),t)=t^2-exp(3*t),y(t), singsol=all)
```

$$y(t) = \frac{(1 - 3t + 3c_1) e^{3t}}{9} - \frac{t^3}{9} - \frac{t^2}{9} - \frac{2t}{27} + c_2$$

✓ Solution by Mathematica
Time used: 0.062 (sec). Leaf size: 42

```
DSolve[y''[t]-3*y'[t]==t^2-Exp[3*t],y[t],t,IncludeSingularSolutions -> True]
```

$$y(t) \to \frac{1}{27}\left(-3t^3 - 3t^2 - 2t + e^{3t}(-9t + 3 + 9c_1) + 27c_2\right)$$

11.33 problem 45

Internal problem ID [14551]

Book: INTRODUCTORY DIFFERENTIAL EQUATIONS. Martha L. Abell, James P. Braselton. Fourth edition 2014. ElScAe. 2014
Section: Chapter 4. Higher Order Equations. Exercises 4.3, page 156
Problem number: 45.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _quadrature]]`

$$\boxed{y'' = t^2 + e^t + \sin(t)}$$

✓ Solution by Maple
Time used: 0.0 (sec). Leaf size: 20

```
dsolve(diff(y(t),t$2)=t^2+exp(t)+sin(t),y(t), singsol=all)
```

$$y(t) = \frac{t^4}{12} - \sin(t) + e^t + c_1 t + c_2$$

✓ Solution by Mathematica
Time used: 0.029 (sec). Leaf size: 26

```
DSolve[y''[t]==t^2+Exp[t]+Sin[t],y[t],t,IncludeSingularSolutions -> True]
```

$$y(t) \to \frac{t^4}{12} + e^t - \sin(t) + c_2 t + c_1$$

11.34 problem 46

Internal problem ID [14552]

Book: INTRODUCTORY DIFFERENTIAL EQUATIONS. Martha L. Abell, James P. Braselton. Fourth edition 2014. ElScAe. 2014
Section: Chapter 4. Higher Order Equations. Exercises 4.3, page 156
Problem number: 46.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _missing_x]]`

$$\boxed{y'' + 3y' = 18}$$

With initial conditions

$$[y(0) = 0, y'(0) = 3]$$

✓ Solution by Maple
Time used: 0.016 (sec). Leaf size: 13

```
dsolve([diff(y(t),t$2)+3*diff(y(t),t)=18,y(0) = 0, D(y)(0) = 3],y(t), singsol=all)
```

$$y(t) = \mathrm{e}^{-3t} + 6t - 1$$

✓ Solution by Mathematica
Time used: 0.02 (sec). Leaf size: 15

```
DSolve[{y''[t]+3*y'[t]==18,{y[0]==0,y'[0]==3}},y[t],t,IncludeSingularSolutions -> True]
```

$$y(t) \rightarrow 6t + e^{-3t} - 1$$

11.35 problem 47

Internal problem ID [14553]

Book: INTRODUCTORY DIFFERENTIAL EQUATIONS. Martha L. Abell, James P. Braselton. Fourth edition 2014. ElScAe. 2014
Section: Chapter 4. Higher Order Equations. Exercises 4.3, page 156
Problem number: 47.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _missing_x]]`

$$\boxed{y'' - y = 4}$$

With initial conditions

$$[y(0) = 0, y'(0) = 0]$$

✓ Solution by Maple
Time used: 0.0 (sec). Leaf size: 16

```
dsolve([diff(y(t),t$2)-y(t)=4,y(0) = 0, D(y)(0) = 0],y(t), singsol=all)
```

$$y(t) = 2\,\mathrm{e}^{-t} + 2\,\mathrm{e}^{t} - 4$$

✓ Solution by Mathematica
Time used: 0.012 (sec). Leaf size: 19

```
DSolve[{y''[t]-y[t]==4,{y[0]==0,y'[0]==0}},y[t],t,IncludeSingularSolutions -> True]
```

$$y(t) \to 2e^{-t}\left(e^{t} - 1\right)^2$$

11.36 problem 48

Internal problem ID [14554]

Book: INTRODUCTORY DIFFERENTIAL EQUATIONS. Martha L. Abell, James P. Braselton. Fourth edition 2014. ElScAe. 2014
Section: Chapter 4. Higher Order Equations. Exercises 4.3, page 156
Problem number: 48.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$\boxed{y'' - 4y = 32t}$$

With initial conditions

$$[y(0) = 0, y'(0) = 6]$$

✓ Solution by Maple
Time used: 0.016 (sec). Leaf size: 20

```
dsolve([diff(y(t),t$2)-4*y(t)=32*t,y(0) = 0, D(y)(0) = 6],y(t), singsol=all)
```

$$y(t) = \frac{7\,\mathrm{e}^{2t}}{2} - \frac{7\,\mathrm{e}^{-2t}}{2} - 8t$$

✓ Solution by Mathematica
Time used: 0.014 (sec). Leaf size: 27

```
DSolve[{y''[t]-4*y[t]==32*t,{y[0]==0,y'[0]==6}},y[t],t,IncludeSingularSolutions -> True]
```

$$y(t) \to -8t - \frac{7e^{-2t}}{2} + \frac{7e^{2t}}{2}$$

11.37 problem 49

Internal problem ID [14555]

Book: INTRODUCTORY DIFFERENTIAL EQUATIONS. Martha L. Abell, James P. Braselton. Fourth edition 2014. ElScAe. 2014
Section: Chapter 4. Higher Order Equations. Exercises 4.3, page 156
Problem number: 49.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _missing_x]]`

$$\boxed{y'' + 2y' - 3y = -2}$$

With initial conditions

$$\left[y(0) = \frac{2}{3}, y'(0) = 8\right]$$

✓ Solution by Maple

Time used: 0.015 (sec). Leaf size: 16

```
dsolve([diff(y(t),t$2)+2*diff(y(t),t)-3*y(t)=-2,y(0) = 2/3, D(y)(0) = 8],y(t), singsol=all)
```

$$y(t) = \frac{2(3\,e^{4t} + e^{3t} - 3)\,e^{-3t}}{3}$$

✓ Solution by Mathematica

Time used: 0.013 (sec). Leaf size: 21

```
DSolve[{y''[t]+2*y'[t]-3*y[t]==-2,{y[0]==2/3,y'[0]==8}},y[t],t,IncludeSingularSolutions -> T
```

$$y(t) \to -2e^{-3t} + 2e^t + \frac{2}{3}$$

11.38 problem 50

Internal problem ID [14556]

Book: INTRODUCTORY DIFFERENTIAL EQUATIONS. Martha L. Abell, James P. Braselton. Fourth edition 2014. ElScAe. 2014
Section: Chapter 4. Higher Order Equations. Exercises 4.3, page 156
Problem number: 50.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$\boxed{y'' + y' - 6y = 3t}$$

With initial conditions

$$\left[y(0) = \frac{23}{12}, y'(0) = -\frac{3}{2}\right]$$

✓ Solution by Maple

Time used: 0.016 (sec). Leaf size: 17

```
dsolve([diff(y(t),t$2)+diff(y(t),t)-6*y(t)=3*t,y(0) = 23/12, D(y)(0) = -3/2],y(t), singsol=a
```

$$y(t) = -\frac{\left(\left(t + \frac{1}{6}\right) e^{3t} - 2\, e^{5t} - 2\right) e^{-3t}}{2}$$

✓ Solution by Mathematica

Time used: 0.014 (sec). Leaf size: 24

```
DSolve[{y''[t]+y'[t]-6*y[t]==3*t,{y[0]==23/12,y'[0]==-3/2}},y[t],t,IncludeSingularSolutions
```

$$y(t) \to -\frac{t}{2} + e^{-3t} + e^{2t} - \frac{1}{12}$$

11.39 problem 51

Internal problem ID [14557]

Book: INTRODUCTORY DIFFERENTIAL EQUATIONS. Martha L. Abell, James P. Braselton. Fourth edition 2014. ElScAe. 2014
Section: Chapter 4. Higher Order Equations. Exercises 4.3, page 156
Problem number: 51.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _missing_x]]`

$$\boxed{y'' + 8y' + 16y = 4}$$

With initial conditions

$$\left[y(0) = \frac{5}{4}, y'(0) = 0\right]$$

✓ Solution by Maple
Time used: 0.0 (sec). Leaf size: 17

```
dsolve([diff(y(t),t$2)+8*diff(y(t),t)+16*y(t)=4,y(0) = 5/4, D(y)(0) = 0],y(t), singsol=all)
```

$$y(t) = \mathrm{e}^{-4t} + 4t\,\mathrm{e}^{-4t} + \frac{1}{4}$$

✓ Solution by Mathematica
Time used: 0.015 (sec). Leaf size: 20

```
DSolve[{y''[t]+8*y'[t]+16*y[t]==4,{y[0]==5/4,y'[0]==0}},y[t],t,IncludeSingularSolutions -> T
```

$$y(t) \to e^{-4t}(4t + 1) + \frac{1}{4}$$

11.40 problem 52

Internal problem ID [14558]

Book: INTRODUCTORY DIFFERENTIAL EQUATIONS. Martha L. Abell, James P. Braselton. Fourth edition 2014. ElScAe. 2014
Section: Chapter 4. Higher Order Equations. Exercises 4.3, page 156
Problem number: 52.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _linear, _nonhomogeneous]]`

$$\boxed{y'' + 7y' + 10y = t\,\mathrm{e}^{-t}}$$

With initial conditions

$$\left[y(0) = -\frac{5}{16}, y'(0) = \frac{9}{16}\right]$$

✓ Solution by Maple

Time used: 0.016 (sec). Leaf size: 15

```
dsolve([diff(y(t),t$2)+7*diff(y(t),t)+10*y(t)=t*exp(-t),y(0) = -5/16, D(y)(0) = 9/16],y(t),
```

$$y(t) = \frac{(4t-5)\,\mathrm{e}^{-t}}{16}$$

✓ Solution by Mathematica

Time used: 0.021 (sec). Leaf size: 19

```
DSolve[{y''[t]+7*y'[t]+10*y[t]==t*Exp[-t],{y[0]==-15/48,y'[0]==9/16}},y[t],t,IncludeSingular
```

$$y(t) \to \frac{1}{16}e^{-t}(4t-5)$$

11.41 problem 53

Internal problem ID [14559]

Book: INTRODUCTORY DIFFERENTIAL EQUATIONS. Martha L. Abell, James P. Braselton. Fourth edition 2014. ElScAe. 2014
Section: Chapter 4. Higher Order Equations. Exercises 4.3, page 156
Problem number: 53.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _missing_x]]`

$$\boxed{y'' + 6y' + 25y = -1}$$

With initial conditions

$$\left[y(0) = -\frac{1}{25}, y'(0) = 7\right]$$

✓ Solution by Maple

Time used: 0.016 (sec). Leaf size: 16

```
dsolve([diff(y(t),t$2)+6*diff(y(t),t)+25*y(t)=-1,y(0) = -1/25, D(y)(0) = 7],y(t), singsol=al
```

$$y(t) = \frac{7\,\mathrm{e}^{-3t}\sin(4t)}{4} - \frac{1}{25}$$

✓ Solution by Mathematica

Time used: 0.017 (sec). Leaf size: 22

```
DSolve[{y''[t]+6*y'[t]+25*y[t]==-1,{y[0]==-1/25,y'[0]==7}},y[t],t,IncludeSingularSolutions -
```

$$y(t) \to \frac{7}{4}e^{-3t}\sin(4t) - \frac{1}{25}$$

11.42 problem 54

Internal problem ID [14560]

Book: INTRODUCTORY DIFFERENTIAL EQUATIONS. Martha L. Abell, James P. Braselton. Fourth edition 2014. ElScAe. 2014
Section: Chapter 4. Higher Order Equations. Exercises 4.3, page 156
Problem number: 54.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _missing_y]]`

$$\boxed{y'' - 3y' = -e^{3t} - 2t}$$

With initial conditions

$$\left[y(0) = 0, y'(0) = \frac{8}{9}\right]$$

✓ Solution by Maple

Time used: 0.015 (sec). Leaf size: 25

```
dsolve([diff(y(t),t$2)-3*diff(y(t),t)=-exp(3*t)-2*t,y(0) = 0, D(y)(0) = 8/9],y(t), singsol=a
```

$$y(t) = \frac{(-3t + 3)\,e^{3t}}{9} + \frac{t^2}{3} + \frac{2t}{9} - \frac{1}{3}$$

✓ Solution by Mathematica

Time used: 0.148 (sec). Leaf size: 29

```
DSolve[{y''[t]-3*y'[t]==-Exp[3*t]-2*t,{y[0]==0,y'[0]==8/9}},y[t],t,IncludeSingularSolutions
```

$$y(t) \to \frac{1}{9}\left(3t^2 + 2t - 3e^{3t}(t-1) - 3\right)$$

11.43 problem 55

Internal problem ID [14561]

Book: INTRODUCTORY DIFFERENTIAL EQUATIONS. Martha L. Abell, James P. Braselton. Fourth edition 2014. ElScAe. 2014
Section: Chapter 4. Higher Order Equations. Exercises 4.3, page 156
Problem number: 55.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _missing_y]]`

$$\boxed{y'' - y' = -3t - 4\,\mathrm{e}^{2t}t^2}$$

With initial conditions

$$\left[y(0) = -\frac{7}{2}, y'(0) = 0\right]$$

✓ Solution by Maple

Time used: 0.015 (sec). Leaf size: 40

```
dsolve([diff(y(t),t$2)-diff(y(t),t)=-3*t-4*t^2*exp(2*t),y(0) = -7/2, D(y)(0) = 0],y(t), sing
```

$$y(t) = -\frac{3}{2} + \left(-2t^2 + 6t - 7\right)\mathrm{e}^{2t} + \frac{3t^2}{2} + 3t + 5\,\mathrm{e}^{t}$$

✓ Solution by Mathematica

Time used: 0.238 (sec). Leaf size: 42

```
DSolve[{y''[t]-y'[t]==-3*t-4*t^2*Exp[2*t],{y[0]==0,y'[0]==8/9}},y[t],t,IncludeSingularSoluti
```

$$y(t) \to \frac{3t^2}{2} + e^{2t}\left(-2t^2 + 6t - 7\right) + 3t + \frac{53e^t}{9} + \frac{10}{9}$$

11.44 problem 56

Internal problem ID [14562]

Book: INTRODUCTORY DIFFERENTIAL EQUATIONS. Martha L. Abell, James P. Braselton. Fourth edition 2014. ElScAe. 2014
Section: Chapter 4. Higher Order Equations. Exercises 4.3, page 156
Problem number: 56.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _missing_y]]`

$$\boxed{y'' - 2y' = 2t^2}$$

With initial conditions

$$\left[y(0) = 3, y'(0) = \frac{3}{2}\right]$$

✓ Solution by Maple

Time used: 0.015 (sec). Leaf size: 23

```
dsolve([diff(y(t),t$2)-2*diff(y(t),t)=2*t^2,y(0) = 3, D(y)(0) = 3/2],y(t), singsol=all)
```

$$y(t) = e^{2t} - \frac{t^2}{2} - \frac{t^3}{3} - \frac{t}{2} + 2$$

✓ Solution by Mathematica

Time used: 0.057 (sec). Leaf size: 30

```
DSolve[{y''[t]-2*y'[t]==2*t^2,{y[0]==3,y'[0]==3/2}},y[t],t,IncludeSingularSolutions -> True]
```

$$y(t) \to \frac{1}{6}\left(-2t^3 - 3t^2 - 3t + 12\right) + e^{2t}$$

11.45 problem 57

Internal problem ID [14563]

Book: INTRODUCTORY DIFFERENTIAL EQUATIONS. Martha L. Abell, James P. Braselton. Fourth edition 2014. ElScAe. 2014
Section: Chapter 4. Higher Order Equations. Exercises 4.3, page 156
Problem number: 57.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _missing_y]]`

$$\boxed{y'' + 4y' = -24t - 6 - 4t\,\mathrm{e}^{-4t} + \mathrm{e}^{-4t}}$$

With initial conditions

$$[y(0) = 0, y'(0) = 0]$$

✓ Solution by Maple
Time used: 0.016 (sec). Leaf size: 16

```
dsolve([diff(y(t),t$2)+4*diff(y(t),t)=-24*t-6-4*t*exp(-4*t)+exp(-4*t),y(0) = 0, D(y)(0) = 0]
```

$$y(t) = \frac{t^2(-6 + \mathrm{e}^{-4t})}{2}$$

✓ Solution by Mathematica
Time used: 0.082 (sec). Leaf size: 19

```
DSolve[{y''[t]+4*y'[t]==-24*t-6-4*t*Exp[-4*t]+Exp[-4*t],{y[0]==0,y'[0]==0}},y[t],t,IncludeSi
```

$$y(t) \to \frac{1}{2}\left(e^{-4t} - 6\right) t^2$$

11.46 problem 58

Internal problem ID [14564]

Book: INTRODUCTORY DIFFERENTIAL EQUATIONS. Martha L. Abell, James P. Braselton. Fourth edition 2014. ElScAe. 2014
Section: Chapter 4. Higher Order Equations. Exercises 4.3, page 156
Problem number: 58.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _missing_y]]`

$$\boxed{y'' - 3y' = \mathrm{e}^{-3t} - \mathrm{e}^{3t}}$$

With initial conditions

$$[y(0) = 1, y'(0) = 1]$$

✓ Solution by Maple
Time used: 0.016 (sec). Leaf size: 23

```
dsolve([diff(y(t),t$2)-3*diff(y(t),t)=exp(-3*t)-exp(3*t),y(0) = 1, D(y)(0) = 1],y(t), singso
```

$$y(t) = \frac{4}{9} + \frac{(3-2t)\,\mathrm{e}^{3t}}{6} + \frac{\mathrm{e}^{-3t}}{18}$$

✓ Solution by Mathematica
Time used: 0.137 (sec). Leaf size: 27

```
DSolve[{y''[t]-3*y'[t]==Exp[-3*t]-Exp[3*t],{y[0]==1,y'[0]==1}},y[t],t,IncludeSingularSolutio
```

$$y(t) \to \frac{1}{18}\left(e^{3t}(9-6t) + e^{-3t} + 8\right)$$

11.47 problem 59

Internal problem ID [14565]

Book: INTRODUCTORY DIFFERENTIAL EQUATIONS. Martha L. Abell, James P. Braselton. Fourth edition 2014. ElScAe. 2014
Section: Chapter 4. Higher Order Equations. Exercises 4.3, page 156
Problem number: 59.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _linear, _nonhomogeneous]]`

$$\boxed{y''+9y=\begin{cases} 2t & 0\le t<\pi \\ 0 & \pi\le t \end{cases}}$$

With initial conditions

$$[y(0)=0, y'(0)=0]$$

✓ Solution by Maple

Time used: 0.719 (sec). Leaf size: 41

```
dsolve([diff(y(t),t$2)+9*y(t)=piecewise(0<=t and t<Pi,2*t,t>=Pi,0),y(0) = 0, D(y)(0) = 0],y(
```

$$y(t)=\frac{2\left(\begin{cases} 0 & t<0 \\ 3t-\sin(3t) & t<\pi \\ -3\cos(3t)\pi-2\sin(3t) & \pi\le t \end{cases}\right)}{27}$$

✓ Solution by Mathematica

Time used: 0.038 (sec). Leaf size: 49

```
DSolve[{y''[t]+9*y[t]==Piecewise[{ {2*t,0<=t<Pi},{0,t>=Pi} }],{y[0]==0,y'[0]==0}},y[t],t,Inc
```

$$y(t) \to \begin{cases} 0 & t \leq 0 \\ -\frac{2}{27}(\sin(3t) - 3t) & 0 < t \leq \pi \\ -\frac{2}{27}(3\pi\cos(3t) + 2\sin(3t)) & \text{True} \end{cases}$$

11.48 problem 60

Internal problem ID [14566]

Book: INTRODUCTORY DIFFERENTIAL EQUATIONS. Martha L. Abell, James P. Braselton. Fourth edition 2014. ElScAe. 2014
Section: Chapter 4. Higher Order Equations. Exercises 4.3, page 156
Problem number: 60.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _linear, _nonhomogeneous]]`

$$\boxed{y''+9\pi^2 y=\begin{cases} 2t & 0\le t<\pi \\ 2t-2\pi & \pi\le t<2\pi \\ 0 & 2\pi\le t \end{cases}}$$

With initial conditions

$$[y(0)=0, y'(0)=0]$$

✓ Solution by Maple

Time used: 2.875 (sec). Leaf size: 121

```
dsolve([diff(y(t),t$2)+9*Pi^2*y(t)=piecewise(0<=t and t<Pi,2*t,t>=Pi and t<2*Pi,2*(t-Pi),t>=
```

$$y(t) = \frac{2\left(\begin{cases} 0 & t<0 \\ 3\pi t-\sin(3\pi t) & t<\pi \\ 3\cos(3\pi^2-3\pi t)\,\pi^2-3\pi^2+3\pi t-\sin(3\pi t) & t<2\pi \\ 3(\cos(3\pi^2-3\pi t)+\cos(6\pi^2-3\pi t))\,\pi^2-\sin(3\pi t)-\sin(6\pi^2-3\pi t) & 2\pi\le t \end{cases}\right)}{27\pi^3}$$

✓ Solution by Mathematica

Time used: 0.076 (sec). Leaf size: 146

```
DSolve[{y''[t]+9*Pi^2*y[t]==Piecewise[{ {2*t,0<=t<Pi},{2*(t-Pi),Pi<=t<2*Pi},{0,t>=2*Pi} }],{
```

$$y(t) \to \begin{cases} \frac{2(3\pi t-\sin(3\pi t))}{27\pi^3} & 0 < t \leq \pi \\ \frac{2(3\pi(t-\pi)+3\pi^2\cos(3\pi(\pi-t))-\sin(3\pi t))}{27\pi^3} & \pi < t \leq 2\pi \\ \frac{4\cos\left(\frac{3\pi^2}{2}\right)\left(3\pi^2\cos\left(\frac{3}{2}\pi(3\pi-2t)\right)+\sin\left(\frac{3}{2}\pi(\pi-2t)\right)-\sin\left(\frac{3}{2}\pi(3\pi-2t)\right)\right)}{27\pi^3} & t > 2\pi \end{cases}$$

11.49 problem 61

Internal problem ID [14567]

Book: INTRODUCTORY DIFFERENTIAL EQUATIONS. Martha L. Abell, James P. Braselton. Fourth edition 2014. ElScAe. 2014
Section: Chapter 4. Higher Order Equations. Exercises 4.3, page 156
Problem number: 61.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _linear, _nonhomogeneous]]`

$$\boxed{y'' + 4y = \begin{cases} 0 & 0 \leq t < \pi \\ 10 & \pi \leq t < 2\pi \\ 0 & 2\pi \leq t \end{cases}}$$

With initial conditions

$$[y(0) = 0, y'(0) = 2]$$

✓ Solution by Maple

Time used: 0.625 (sec). Leaf size: 33

```
dsolve([diff(y(t),t$2)+4*y(t)=piecewise(0<=t and t<Pi,0, t>=Pi and t<2*Pi,10,t>=2*Pi,0  ),y(
```

$$y(t) = \sin(2t) + \left(\begin{cases} 0 & t < \pi \\ \frac{5}{2} - \frac{5\cos(2t)}{2} & t < 2\pi \\ 0 & 2\pi \leq t \end{cases} \right)$$

✓ Solution by Mathematica

Time used: 0.037 (sec). Leaf size: 37

```
DSolve[{y''[t]+4*y[t]==Piecewise[{ {0,0<=t<Pi},{10,Pi<=t<2*Pi},{0,t>=2*Pi} }],{y[0]==0,y'[0]
```

$$y(t) \to \begin{cases} \sin(2t) & t > 2\pi \lor t \leq \pi \\ -\frac{5}{2}\cos(2t) + \sin(2t) + \frac{5}{2} & \text{True} \end{cases}$$

11.50 problem 62 (a)

Internal problem ID [14568]

Book: INTRODUCTORY DIFFERENTIAL EQUATIONS. Martha L. Abell, James P. Braselton. Fourth edition 2014. ElScAe. 2014
Section: Chapter 4. Higher Order Equations. Exercises 4.3, page 156
Problem number: 62 (a).
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[[_linear, 'class A']]`

$$\boxed{y' - 4y = t^2}$$

✓ Solution by Maple
Time used: 0.0 (sec). Leaf size: 20

```
dsolve(diff(y(t),t)-4*y(t)=t^2,y(t), singsol=all)
```

$$y(t) = -\frac{t^2}{4} - \frac{t}{8} - \frac{1}{32} + c_1 \mathrm{e}^{4t}$$

✓ Solution by Mathematica
Time used: 0.049 (sec). Leaf size: 28

```
DSolve[y'[t]-4*y[t]==t^2,y[t],t,IncludeSingularSolutions -> True]
```

$$y(t) \to \frac{1}{32}\left(-8t^2 - 4t - 1\right) + c_1 e^{4t}$$

11.51 problem 62 (b)

Internal problem ID [14569]

Book: INTRODUCTORY DIFFERENTIAL EQUATIONS. Martha L. Abell, James P. Braselton. Fourth edition 2014. ElScAe. 2014
Section: Chapter 4. Higher Order Equations. Exercises 4.3, page 156
Problem number: 62 (b).
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[[_linear, 'class A']]`

$$\boxed{y' + y = \cos(2t)}$$

With initial conditions

$$[y(0) = 0]$$

✓ Solution by Maple
Time used: 0.015 (sec). Leaf size: 23

```
dsolve([diff(y(t),t)+y(t)=cos(2*t),y(0) = 0],y(t), singsol=all)
```

$$y(t) = \frac{\cos(2t)}{5} + \frac{2\sin(2t)}{5} - \frac{e^{-t}}{5}$$

✓ Solution by Mathematica
Time used: 0.084 (sec). Leaf size: 27

```
DSolve[{y'[t]+y[t]==Cos[2*t],{y[0]==0}},y[t],t,IncludeSingularSolutions -> True]
```

$$y(t) \to \frac{1}{5}\left(-e^{-t} + 2\sin(2t) + \cos(2t)\right)$$

11.52 problem 62 (c)

Internal problem ID [14570]

Book: INTRODUCTORY DIFFERENTIAL EQUATIONS. Martha L. Abell, James P. Braselton. Fourth edition 2014. ElScAe. 2014
Section: Chapter 4. Higher Order Equations. Exercises 4.3, page 156
Problem number: 62 (c).
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[[_linear, 'class A']]`

$$\boxed{y' - y = e^{4t}}$$

With initial conditions

$$[y(0) = 0]$$

✓ Solution by Maple

Time used: 0.016 (sec). Leaf size: 14

```
dsolve([diff(y(t),t)-y(t)=exp(4*t),y(0) = 0],y(t), singsol=all)
```

$$y(t) = \frac{(e^{3t} - 1)\, e^{t}}{3}$$

✓ Solution by Mathematica

Time used: 0.042 (sec). Leaf size: 19

```
DSolve[{y'[t]-y[t]==Exp[4*t],{y[0]==0}},y[t],t,IncludeSingularSolutions -> True]
```

$$y(t) \to \frac{1}{3} e^{t} \left(e^{3t} - 1\right)$$

11.53 problem 62 (d)

Internal problem ID [14571]

Book: INTRODUCTORY DIFFERENTIAL EQUATIONS. Martha L. Abell, James P. Braselton. Fourth edition 2014. ElScAe. 2014
Section: Chapter 4. Higher Order Equations. Exercises 4.3, page 156
Problem number: 62 (d).
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[[_linear, 'class A']]`

$$\boxed{y' + 4y = \mathrm{e}^{-4t}}$$

With initial conditions

$$[y(0) = 1]$$

✓ Solution by Maple
Time used: 0.016 (sec). Leaf size: 12

```
dsolve([diff(y(t),t)+4*y(t)=exp(-4*t),y(0) = 1],y(t), singsol=all)
```

$$y(t) = (t+1)\,\mathrm{e}^{-4t}$$

✓ Solution by Mathematica
Time used: 0.055 (sec). Leaf size: 14

```
DSolve[{y'[t]+4*y[t]==Exp[-4*t],{y[0]==1}},y[t],t,IncludeSingularSolutions -> True]
```

$$y(t) \to e^{-4t}(t+1)$$

11.54 problem 62 (e)

Internal problem ID [14572]

Book: INTRODUCTORY DIFFERENTIAL EQUATIONS. Martha L. Abell, James P. Braselton. Fourth edition 2014. ElScAe. 2014
Section: Chapter 4. Higher Order Equations. Exercises 4.3, page 156
Problem number: 62 (e).
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[[_linear, 'class A']]`

$$\boxed{y' + 4y = t\,\mathrm{e}^{-4t}}$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 17

```
dsolve(diff(y(t),t)+4*y(t)=t*exp(-4*t),y(t), singsol=all)
```

$$y(t) = \frac{(t^2 + 2c_1)\,\mathrm{e}^{-4t}}{2}$$

✓ Solution by Mathematica

Time used: 0.053 (sec). Leaf size: 22

```
DSolve[y'[t]+4*y[t]==t*Exp[-4*t],y[t],t,IncludeSingularSolutions -> True]
```

$$y(t) \to \frac{1}{2}e^{-4t}(t^2 + 2c_1)$$

11.55 problem 68

Internal problem ID [14573]

Book: INTRODUCTORY DIFFERENTIAL EQUATIONS. Martha L. Abell, James P. Braselton. Fourth edition 2014. ElScAe. 2014
Section: Chapter 4. Higher Order Equations. Exercises 4.3, page 156
Problem number: 68.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _linear, _nonhomogeneous]]`

$$\boxed{y'' + y' - 2y = f(t)}$$

With initial conditions

$$[y(0) = 0, y'(0) = a]$$

✓ Solution by Maple

Time used: 0.094 (sec). Leaf size: 52

```
dsolve([diff(y(t),t$2)+diff(y(t),t)-2*y(t)=f(t),y(0) = 0, D(y)(0) = a],y(t), singsol=all)
```

$$y(t) = \frac{\left(-\left(\int_0^t f(\_z1)\,\mathrm{e}^{2\_z1} d\_z1\right) + \left(\int_0^t f(\_z1)\,\mathrm{e}^{-\_z1} d\_z1\right)\mathrm{e}^{3t} + a\left(\mathrm{e}^{3t} - 1\right)\right)\mathrm{e}^{-2t}}{3}$$

✓ Solution by Mathematica

Time used: 0.078 (sec). Leaf size: 123

```
DSolve[{y''[t]+y'[t]-2*y[t]==f[t],{y[0]==0,y'[0]==a}},y[t],t,IncludeSingularSolutions -> Tru
```

$$y(t) \to \frac{1}{3}e^{-2t}\left(3\int_1^t -\frac{1}{3}e^{2K[1]}f(K[1])dK[1] - 3e^{3t}\int_1^0 \frac{1}{3}e^{-K[2]}f(K[2])dK[2] + 3e^{3t}\int_1^t \frac{1}{3}e^{-K[2]}f(K[2])dK[2] - 3\int_1^0 -\frac{1}{3}e^{2K[1]}f(K[1])dK[1] + ae^{3t} - a\right)$$

11.56 problem 69

Internal problem ID [14574]

Book: INTRODUCTORY DIFFERENTIAL EQUATIONS. Martha L. Abell, James P. Braselton. Fourth edition 2014. ElScAe. 2014
Section: Chapter 4. Higher Order Equations. Exercises 4.3, page 156
Problem number: 69.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _linear, _nonhomogeneous]]`

$$\boxed{x'' + 9x = \sin(3t)}$$

With initial conditions

$$[x(0) = 0, x'(0) = 0]$$

✓ Solution by Maple
Time used: 0.015 (sec). Leaf size: 18

```
dsolve([diff(x(t),t$2)+9*x(t)=sin(3*t),x(0) = 0, D(x)(0) = 0],x(t), singsol=all)
```

$$x(t) = \frac{\sin(3t)}{18} - \frac{\cos(3t)\, t}{6}$$

✓ Solution by Mathematica
Time used: 0.117 (sec). Leaf size: 21

```
DSolve[{x''[t]+9*x[t]==Sin[3*t],{x[0]==0,x'[0]==0}},x[t],t,IncludeSingularSolutions -> True]
```

$$x(t) \to \frac{1}{18}(\sin(3t) - 3t\cos(3t))$$

11.57 problem 70 (a)

Internal problem ID [14575]

Book: INTRODUCTORY DIFFERENTIAL EQUATIONS. Martha L. Abell, James P. Braselton. Fourth edition 2014. ElScAe. 2014
Section: Chapter 4. Higher Order Equations. Exercises 4.3, page 156
Problem number: 70 (a).
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _linear, _nonhomogeneous]]`

$$\boxed{4y'' + 4y' + 37y = \cos(3t)}$$

With initial conditions

$$[y(0) = a, y'(\pi) = a]$$

✓ Solution by Maple

Time used: 0.125 (sec). Leaf size: 55

```
dsolve([4*diff(y(t),t$2)+4*diff(y(t),t)+37*y(t)=cos(3*t),y(0) = a, D(y)(Pi) = a],y(t), sings
```

$$y(t) = \frac{(-290a - 72)\sin(3t)\,\mathrm{e}^{-\frac{t}{2}+\frac{\pi}{2}}}{870} + \left(\frac{\sin(3t)}{6} + \cos(3t)\right)\left(a - \frac{1}{145}\right)\mathrm{e}^{-\frac{t}{2}} + \frac{\cos(3t)}{145} + \frac{12\sin(3t)}{145}$$

✓ Solution by Mathematica

Time used: 0.031 (sec). Leaf size: 75

```
DSolve[{4*y''[t]+4*y'[t]+37*y[t]==Cos[3*t],{y[0]==a,y'[Pi]==a}},y[t],t,IncludeSingularSoluti
```

$$y(t) \to \frac{1}{870}e^{-t/2}\left(6\left(145a + e^{t/2} - 1\right)\cos(3t) - \left(145\left(2e^{\pi/2} - 1\right)a - 72e^{t/2} + 72e^{\pi/2} + 1\right)\sin(3t)\right)$$

12 Chapter 4. Higher Order Equations. Exercises 4.4, page 163

12.1 problem 1

Internal problem ID [14576]

Book: INTRODUCTORY DIFFERENTIAL EQUATIONS. Martha L. Abell, James P. Braselton. Fourth edition 2014. ElScAe. 2014
Section: Chapter 4. Higher Order Equations. Exercises 4.4, page 163
Problem number: 1.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _missing_x]]`

$$\boxed{y'' + 4y = 1}$$

✓ Solution by Maple
Time used: 0.0 (sec). Leaf size: 18

```
dsolve(diff(y(t),t$2)+4*y(t)=1,y(t), singsol=all)
```

$$y(t) = \frac{1}{4} + \sin(2t)\, c_2 + \cos(2t)\, c_1$$

✓ Solution by Mathematica
Time used: 0.012 (sec). Leaf size: 23

```
DSolve[y''[t]+4*y[t]==1,y[t],t,IncludeSingularSolutions -> True]
```

$$y(t) \to c_1 \cos(2t) + c_2 \sin(2t) + \frac{1}{4}$$

12.2 problem 2

Internal problem ID [14577]

Book: INTRODUCTORY DIFFERENTIAL EQUATIONS. Martha L. Abell, James P. Braselton. Fourth edition 2014. ElScAe. 2014
Section: Chapter 4. Higher Order Equations. Exercises 4.4, page 163
Problem number: 2.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _missing_y]]`

$$\boxed{y'' + 16y' = t}$$

✓ Solution by Maple
Time used: 0.0 (sec). Leaf size: 21

```
dsolve(diff(y(t),t$2)+16*diff(y(t),t)=t,y(t), singsol=all)
```

$$y(t) = \frac{t^2}{32} - \frac{e^{-16t}c_1}{16} - \frac{t}{256} + c_2$$

✓ Solution by Mathematica
Time used: 0.069 (sec). Leaf size: 31

```
DSolve[y''[t]+16*y'[t]==t,y[t],t,IncludeSingularSolutions -> True]
```

$$y(t) \to \frac{t^2}{32} - \frac{t}{256} - \frac{1}{16}c_1 e^{-16t} + c_2$$

12.3 problem 3

Internal problem ID [14578]

Book: INTRODUCTORY DIFFERENTIAL EQUATIONS. Martha L. Abell, James P. Braselton. Fourth edition 2014. ElScAe. 2014
Section: Chapter 4. Higher Order Equations. Exercises 4.4, page 163
Problem number: 3.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$\boxed{y'' - 7y' + 10y = e^{3t}}$$

✓ Solution by Maple
Time used: 0.015 (sec). Leaf size: 23

```
dsolve(diff(y(t),t$2)-7*diff(y(t),t)+10*y(t)=exp(3*t),y(t), singsol=all)
```

$$y(t) = c_2 e^{5t} + c_1 e^{2t} - \frac{e^{3t}}{2}$$

✓ Solution by Mathematica
Time used: 0.029 (sec). Leaf size: 31

```
DSolve[y''[t]-7*y'[t]+10*y[t]==Exp[3*t],y[t],t,IncludeSingularSolutions -> True]
```

$$y(t) \to -\frac{e^{3t}}{2} + c_1 e^{2t} + c_2 e^{5t}$$

12.4 problem 4

Internal problem ID [14579]

Book: INTRODUCTORY DIFFERENTIAL EQUATIONS. Martha L. Abell, James P. Braselton. Fourth edition 2014. ElScAe. 2014
Section: Chapter 4. Higher Order Equations. Exercises 4.4, page 163
Problem number: 4.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _linear, _nonhomogeneous]]`

$$\boxed{y'' + 16y = 2\cos(4t)}$$

✓ Solution by Maple
Time used: 0.0 (sec). Leaf size: 27

```
dsolve(diff(y(t),t$2)+16*y(t)=2*cos(4*t),y(t), singsol=all)
```

$$y(t) = \frac{(16c_1 + 1)\cos(4t)}{16} + \frac{\sin(4t)(t + 4c_2)}{4}$$

✓ Solution by Mathematica
Time used: 0.107 (sec). Leaf size: 31

```
DSolve[y''[t]+16*y[t]==2*Cos[4*t],y[t],t,IncludeSingularSolutions -> True]
```

$$y(t) \to \left(\frac{1}{32} + c_1\right)\cos(4t) + \frac{1}{4}(t + 4c_2)\sin(4t)$$

12.5 problem 5

Internal problem ID [14580]

Book: INTRODUCTORY DIFFERENTIAL EQUATIONS. Martha L. Abell, James P. Braselton. Fourth edition 2014. ElScAe. 2014
Section: Chapter 4. Higher Order Equations. Exercises 4.4, page 163
Problem number: 5.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _linear, _nonhomogeneous]]`

$$\boxed{y'' + 4y' + 20y = 2t\,\mathrm{e}^{-2t}}$$

✓ Solution by Maple
Time used: 0.0 (sec). Leaf size: 26

```
dsolve(diff(y(t),t$2)+4*diff(y(t),t)+20*y(t)=2*t*exp(-2*t),y(t), singsol=all)
```

$$y(t) = \frac{\mathrm{e}^{-2t}(8c_1 \cos(4t) + 8c_2 \sin(4t) + t)}{8}$$

✓ Solution by Mathematica
Time used: 0.025 (sec). Leaf size: 32

```
DSolve[y''[t]+4*y'[t]+20*y[t]==2*t*Exp[-2*t],y[t],t,IncludeSingularSolutions -> True]
```

$$y(t) \to \frac{1}{8}e^{-2t}(t + 8c_2 \cos(4t) + 8c_1 \sin(4t))$$

12.6 problem 6

Internal problem ID [14581]

Book: INTRODUCTORY DIFFERENTIAL EQUATIONS. Martha L. Abell, James P. Braselton. Fourth edition 2014. ElScAe. 2014
Section: Chapter 4. Higher Order Equations. Exercises 4.4, page 163
Problem number: 6.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _linear, _nonhomogeneous]]`

$$\boxed{y'' + \frac{y}{4} = \sec\left(\frac{t}{2}\right) + \csc\left(\frac{t}{2}\right)}$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 48

```
dsolve(diff(y(t),t$2)+1/4*y(t)=sec(t/2)+csc(t/2),y(t), singsol=all)
```

$$y(t) = 4\cos\left(\frac{t}{2}\right)\ln\left(\cos\left(\frac{t}{2}\right)\right) + 4\sin\left(\frac{t}{2}\right)\ln\left(\sin\left(\frac{t}{2}\right)\right) + (c_1 - 2t)\cos\left(\frac{t}{2}\right) + 2\left(t + \frac{c_2}{2}\right)\sin\left(\frac{t}{2}\right)$$

✓ Solution by Mathematica

Time used: 0.066 (sec). Leaf size: 50

```
DSolve[y''[t]+1/4*y[t]==Sec[t/2]+Csc[t/2],y[t],t,IncludeSingularSolutions -> True]
```

$$y(t) \to \cos\left(\frac{t}{2}\right)\left(-2t + 4\log\left(\cos\left(\frac{t}{2}\right)\right) + c_1\right) + \sin\left(\frac{t}{2}\right)\left(2t + 4\log\left(\sin\left(\frac{t}{2}\right)\right) + c_2\right)$$

12.7 problem 7

Internal problem ID [14582]

Book: INTRODUCTORY DIFFERENTIAL EQUATIONS. Martha L. Abell, James P. Braselton. Fourth edition 2014. ElScAe. 2014
Section: Chapter 4. Higher Order Equations. Exercises 4.4, page 163
Problem number: 7.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _linear, _nonhomogeneous]]`

$$\boxed{y'' + 16y = \csc(4t)}$$

✓ Solution by Maple
Time used: 0.0 (sec). Leaf size: 35

```
dsolve(diff(y(t),t$2)+16*y(t)=csc(4*t),y(t), singsol=all)
```

$$y(t) = -\frac{\ln(\csc(4t))\sin(4t)}{16} + \frac{(-t + 4c_1)\cos(4t)}{4} + c_2\sin(4t)$$

✓ Solution by Mathematica
Time used: 0.036 (sec). Leaf size: 37

```
DSolve[y''[t]+16*y[t]==Csc[4*t],y[t],t,IncludeSingularSolutions -> True]
```

$$y(t) \to \left(-\frac{t}{4} + c_1\right)\cos(4t) + \frac{1}{16}\sin(4t)(\log(\sin(4t)) + 16c_2)$$

12.8 problem 8

Internal problem ID [14583]

Book: INTRODUCTORY DIFFERENTIAL EQUATIONS. Martha L. Abell, James P. Braselton. Fourth edition 2014. ElScAe. 2014
Section: Chapter 4. Higher Order Equations. Exercises 4.4, page 163
Problem number: 8.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _linear, _nonhomogeneous]]`

$$\boxed{y'' + 16y = \cot(4t)}$$

✓ Solution by Maple
Time used: 0.0 (sec). Leaf size: 35

```
dsolve(diff(y(t),t$2)+16*y(t)=cot(4*t),y(t), singsol=all)
```

$$y(t) = c_2 \sin(4t) + c_1 \cos(4t) + \frac{\sin(4t)\ln(\csc(4t) - \cot(4t))}{16}$$

✓ Solution by Mathematica
Time used: 0.146 (sec). Leaf size: 38

```
DSolve[y''[t]+16*y[t]==Cot[4*t],y[t],t,IncludeSingularSolutions -> True]
```

$$y(t) \to c_1 \cos(4t) + \frac{1}{16}\sin(4t)(\log(\sin(2t)) - \log(\cos(2t)) + 16c_2)$$

12.9 problem 9

Internal problem ID [14584]

Book: INTRODUCTORY DIFFERENTIAL EQUATIONS. Martha L. Abell, James P. Braselton. Fourth edition 2014. ElScAe. 2014
Section: Chapter 4. Higher Order Equations. Exercises 4.4, page 163
Problem number: 9.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _linear, _nonhomogeneous]]`

$$\boxed{y'' + 2y' + 50y = e^{-t} \csc(7t)}$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 39

```
dsolve(diff(y(t),t$2)+2*diff(y(t),t)+50*y(t)=exp(-t)*csc(7*t),y(t), singsol=all)
```

$$y(t) = -\frac{\left(\frac{\sin(7t)\ln(\csc(7t))}{7} + (t - 7c_1)\cos(7t) - 7\sin(7t)\, c_2\right) e^{-t}}{7}$$

✓ Solution by Mathematica

Time used: 0.082 (sec). Leaf size: 42

```
DSolve[y''[t]+2*y'[t]+50*y[t]==Exp[-t]*Csc[7*t],y[t],t,IncludeSingularSolutions -> True]
```

$$y(t) \to \frac{1}{49} e^{-t}(\sin(7t)(\log(\sin(7t)) + 49c_1) - 7(t - 7c_2)\cos(7t))$$

12.10 problem 10

Internal problem ID [14585]

Book: INTRODUCTORY DIFFERENTIAL EQUATIONS. Martha L. Abell, James P. Braselton. Fourth edition 2014. ElScAe. 2014
Section: Chapter 4. Higher Order Equations. Exercises 4.4, page 163
Problem number: 10.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _linear, _nonhomogeneous]]`

$$\boxed{y'' + 6y' + 25y = \mathrm{e}^{-3t}(\sec(4t) + \csc(4t))}$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 54

```
dsolve(diff(y(t),t$2)+6*diff(y(t),t)+25*y(t)=exp(-3*t)*(sec(4*t)+csc(4*t)),y(t), singsol=all
```

$$y(t) = -\frac{\left(-\frac{\ln(\cos(4t))\cos(4t)}{4} - \frac{\ln(\sin(4t))\sin(4t)}{4} + (-4c_1 + t)\cos(4t) - \sin(4t)(t + 4c_2)\right)\mathrm{e}^{-3t}}{4}$$

✓ Solution by Mathematica

Time used: 0.135 (sec). Leaf size: 52

```
DSolve[y''[t]+6*y'[t]+25*y[t]==Exp[-3*t]*(Sec[4*t]+Csc[4*t]),y[t],t,IncludeSingularSolutions
```

$$y(t) \to \frac{1}{16}e^{-3t}(\cos(4t)(-4t + \log(\cos(4t)) + 16c_2) + \sin(4t)(\log(\sin(4t)) + 4(t + 4c_1)))$$

12.11 problem 11

Internal problem ID [14586]

Book: INTRODUCTORY DIFFERENTIAL EQUATIONS. Martha L. Abell, James P. Braselton. Fourth edition 2014. ElScAe. 2014
Section: Chapter 4. Higher Order Equations. Exercises 4.4, page 163
Problem number: 11.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _linear, _nonhomogeneous]]`

$$\boxed{y'' - 2y' + 26y = \mathrm{e}^{t}(\sec(5t) + \csc(5t))}$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 52

```
dsolve(diff(y(t),t$2)-2*diff(y(t),t)+26*y(t)=exp(t)*(sec(5*t)+csc(5*t)),y(t), singsol=all)
```

$$y(t) = -\frac{\mathrm{e}^{t}\left(-\frac{\ln(\cos(5t))\cos(5t)}{5} - \frac{\ln(\sin(5t))\sin(5t)}{5} + (t - 5c_1)\cos(5t) - \sin(5t)(t + 5c_2)\right)}{5}$$

✓ Solution by Mathematica

Time used: 0.139 (sec). Leaf size: 50

```
DSolve[y''[t]-2*y'[t]+26*y[t]==Exp[t]*(Sec[5*t]+Csc[5*t]),y[t],t,IncludeSingularSolutions ->
```

$$y(t) \to \frac{1}{25}e^{t}(\cos(5t)(-5t + \log(\cos(5t)) + 25c_2) + \sin(5t)(\log(\sin(5t)) + 5(t + 5c_1)))$$

12.12 problem 12

Internal problem ID [14587]

Book: INTRODUCTORY DIFFERENTIAL EQUATIONS. Martha L. Abell, James P. Braselton. Fourth edition 2014. ElScAe. 2014
Section: Chapter 4. Higher Order Equations. Exercises 4.4, page 163
Problem number: 12.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _linear, _nonhomogeneous]]`

$$\boxed{y'' + 12y' + 37y = \mathrm{e}^{-6t} \csc(t)}$$

✓ Solution by Maple

Time used: 0.016 (sec). Leaf size: 30

```
dsolve(diff(y(t),t$2)+12*diff(y(t),t)+37*y(t)=exp(-6*t)*csc(t),y(t), singsol=all)
```

$$y(t) = -(\sin(t)\ln(\csc(t)) + (-c_1 + t)\cos(t) - \sin(t) c_2)\,\mathrm{e}^{-6t}$$

✓ Solution by Mathematica

Time used: 0.062 (sec). Leaf size: 30

```
DSolve[y''[t]+12*y'[t]+37*y[t]==Exp[-6*t]*Csc[t],y[t],t,IncludeSingularSolutions -> True]
```

$$y(t) \to e^{-6t}((-t + c_2)\cos(t) + \sin(t)(\log(\sin(t)) + c_1))$$

12.13 problem 13

Internal problem ID [14588]

Book: INTRODUCTORY DIFFERENTIAL EQUATIONS. Martha L. Abell, James P. Braselton. Fourth edition 2014. ElScAe. 2014
Section: Chapter 4. Higher Order Equations. Exercises 4.4, page 163
Problem number: 13.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _linear, _nonhomogeneous]]`

$$\boxed{y'' - 6y' + 34y = e^{3t} \tan(5t)}$$

✓ Solution by Maple

Time used: 0.016 (sec). Leaf size: 40

```
dsolve(diff(y(t),t$2)-6*diff(y(t),t)+34*y(t)=exp(3*t)*tan(5*t),y(t), singsol=all)
```

$$y(t) = -\frac{e^{3t}(\ln(\sec(5t) + \tan(5t))\cos(5t) - 25\cos(5t)\, c_1 - 25\sin(5t)\, c_2)}{25}$$

✓ Solution by Mathematica

Time used: 0.434 (sec). Leaf size: 318

```
DSolve[y''[t]-6*y'[t]+4*y[t]==Exp[3*t]*Tan[5*t],y[t],t,IncludeSingularSolutions -> True]
```

$$
\begin{aligned}
y(t) \to \frac{1}{210}e^{3t}\Bigg(&-21i\,\text{Hypergeometric2F1}\left(1,-\frac{i}{2\sqrt{5}},1-\frac{i}{2\sqrt{5}},-e^{10it}\right) \\
&- 21i\,\text{Hypergeometric2F1}\left(1,\frac{i}{2\sqrt{5}},1+\frac{i}{2\sqrt{5}},-e^{10it}\right) \\
&+ 2\sqrt{5}e^{10it}\,\text{Hypergeometric2F1}\left(1,1-\frac{i}{2\sqrt{5}},2-\frac{i}{2\sqrt{5}},-e^{10it}\right) \\
&+ ie^{10it}\,\text{Hypergeometric2F1}\left(1,1-\frac{i}{2\sqrt{5}},2-\frac{i}{2\sqrt{5}},-e^{10it}\right) \\
&- 2\sqrt{5}e^{10it}\,\text{Hypergeometric2F1}\left(1,1+\frac{i}{2\sqrt{5}},2+\frac{i}{2\sqrt{5}},-e^{10it}\right) \\
&+ ie^{10it}\,\text{Hypergeometric2F1}\left(1,1+\frac{i}{2\sqrt{5}},2+\frac{i}{2\sqrt{5}},-e^{10it}\right) + 210c_1e^{-\sqrt{5}t} \\
&+ 210c_2e^{\sqrt{5}t}\Bigg)
\end{aligned}
$$

12.14 problem 14

Internal problem ID [14589]

Book: INTRODUCTORY DIFFERENTIAL EQUATIONS. Martha L. Abell, James P. Braselton. Fourth edition 2014. ElScAe. 2014
Section: Chapter 4. Higher Order Equations. Exercises 4.4, page 163
Problem number: 14.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _linear, _nonhomogeneous]]`

$$\boxed{y'' - 10y' + 34y = e^{5t} \cot(3t)}$$

✓ Solution by Maple
Time used: 0.015 (sec). Leaf size: 42

```
dsolve(diff(y(t),t$2)-10*diff(y(t),t)+34*y(t)=exp(5*t)*cot(3*t),y(t), singsol=all)
```

$$y(t) = \frac{e^{5t}(\ln(\csc(3t) - \cot(3t)) \sin(3t) + 9c_2 \sin(3t) + 9c_1 \cos(3t))}{9}$$

✓ Solution by Mathematica
Time used: 0.147 (sec). Leaf size: 49

```
DSolve[y''[t]-10*y'[t]+34*y[t]==Exp[5*t]*Cot[3*t],y[t],t,IncludeSingularSolutions -> True]
```

$$y(t) \to \frac{1}{9}e^{5t}\left(9c_2 \cos(3t) + \sin(3t)\left(\log\left(\sin\left(\frac{3t}{2}\right)\right) - \log\left(\cos\left(\frac{3t}{2}\right)\right) + 9c_1\right)\right)$$

12.15 problem 15

Internal problem ID [14590]

Book: INTRODUCTORY DIFFERENTIAL EQUATIONS. Martha L. Abell, James P. Braselton. Fourth edition 2014. ElScAe. 2014
Section: Chapter 4. Higher Order Equations. Exercises 4.4, page 163
Problem number: 15.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _linear, _nonhomogeneous]]`

$$\boxed{y'' - 12y' + 37y = e^{6t} \sec(t)}$$

✓ Solution by Maple
Time used: 0.0 (sec). Leaf size: 27

```
dsolve(diff(y(t),t$2)-12*diff(y(t),t)+37*y(t)=exp(6*t)*sec(t),y(t), singsol=all)
```

$$y(t) = (-\cos(t)\ln(\sec(t)) + \cos(t) c_1 + \sin(t)(c_2 + t)) e^{6t}$$

✓ Solution by Mathematica
Time used: 0.054 (sec). Leaf size: 28

```
DSolve[y''[t]-12*y'[t]+37*y[t]==Exp[6*t]*Sec[t],y[t],t,IncludeSingularSolutions -> True]
```

$$y(t) \to e^{6t}((t + c_1)\sin(t) + \cos(t)(\log(\cos(t)) + c_2))$$

12.16 problem 16

Internal problem ID [14591]

Book: INTRODUCTORY DIFFERENTIAL EQUATIONS. Martha L. Abell, James P. Braselton. Fourth edition 2014. ElScAe. 2014
Section: Chapter 4. Higher Order Equations. Exercises 4.4, page 163
Problem number: 16.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _linear, _nonhomogeneous]]`

$$\boxed{y'' - 8y' + 17y = \mathrm{e}^{4t} \sec(t)}$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 27

```
dsolve(diff(y(t),t$2)-8*diff(y(t),t)+17*y(t)=exp(4*t)*sec(t),y(t), singsol=all)
```

$$y(t) = (-\cos(t)\ln(\sec(t)) + \cos(t)\, c_1 + \sin(t)\,(c_2 + t))\,\mathrm{e}^{4t}$$

✓ Solution by Mathematica

Time used: 0.048 (sec). Leaf size: 28

```
DSolve[y''[t]-8*y'[t]+17*y[t]==Exp[4*t]*Sec[t],y[t],t,IncludeSingularSolutions -> True]
```

$$y(t) \to e^{4t}((t + c_1)\sin(t) + \cos(t)(\log(\cos(t)) + c_2))$$

12.17 problem 17

Internal problem ID [14592]

Book: INTRODUCTORY DIFFERENTIAL EQUATIONS. Martha L. Abell, James P. Braselton. Fourth edition 2014. ElScAe. 2014
Section: Chapter 4. Higher Order Equations. Exercises 4.4, page 163
Problem number: 17.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _linear, _nonhomogeneous]]`

$$\boxed{y'' - 9y = \frac{1}{1 + \mathrm{e}^{3t}}}$$

✓ Solution by Maple
Time used: 0.015 (sec). Leaf size: 69

```
dsolve(diff(y(t),t$2)-9*y(t)=(1+exp(3*t))^(-1),y(t), singsol=all)
```

$$y(t) = \frac{\mathrm{e}^{-3t}\left(\ln\left(\mathrm{e}^{2t} - \mathrm{e}^{t} + 1\right)\mathrm{e}^{6t} - \ln\left(1 + \mathrm{e}^{3t}\right) + \ln\left(1 + \mathrm{e}^{t}\right)\mathrm{e}^{6t} + \left(18c_1 - 3\ln\left(\mathrm{e}^{t}\right)\right)\mathrm{e}^{6t} + 18c_2 - \mathrm{e}^{3t}\right)}{18}$$

✓ Solution by Mathematica
Time used: 0.184 (sec). Leaf size: 94

```
DSolve[y''[t]-9*y[t]==(1+Exp[3*t])^(-1),y[t],t,IncludeSingularSolutions -> True]
```

$$y(t) \to \frac{1}{18}e^{-3t}\left(-e^{3t} - 3e^{6t}\log\left(e^{t}\right) + \left(e^{6t} - 1\right)\log\left(e^{t} + 1\right) + e^{6t}\log\left(-e^{t} + e^{2t} + 1\right) - \log\left(-e^{t} + e^{2t} + 1\right) + 18c_1e^{6t} + 18c_2\right)$$

12.18 problem 18

Internal problem ID [14593]

Book: INTRODUCTORY DIFFERENTIAL EQUATIONS. Martha L. Abell, James P. Braselton. Fourth edition 2014. ElScAe. 2014
Section: Chapter 4. Higher Order Equations. Exercises 4.4, page 163
Problem number: 18.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _linear, _nonhomogeneous]]`

$$\boxed{y'' - 25y = \frac{1}{1 - e^{5t}}}$$

✓ Solution by Maple
Time used: 0.0 (sec). Leaf size: 77

```
dsolve(diff(y(t),t$2)-25*y(t)=(1-exp(5*t))^(-1),y(t), singsol=all)
```

$$y(t) = -\frac{\left(\ln\left(e^{4t}+e^{3t}+e^{2t}+e^{t}+1\right)e^{10t}+\ln\left(e^{t}-1\right)e^{10t}-5\ln\left(e^{t}\right)e^{10t}-50c_2e^{10t}+e^{5t}-\ln\left(1-e^{5t}\right)-50c_1\right)}{50}$$

✓ Solution by Mathematica
Time used: 0.226 (sec). Leaf size: 110

```
DSolve[y''[t]-25*y[t]==(1-Exp[5*t])^(-1),y[t],t,IncludeSingularSolutions -> True]
```

$$y(t) \to \frac{1}{50}e^{-5t}\left(-e^{5t}+5e^{10t}\log\left(e^{t}\right)-\left(e^{10t}-1\right)\log\left(e^{t}-1\right)-e^{10t}\log\left(e^{t}+e^{2t}+e^{3t}+e^{4t}+1\right) + \log\left(e^{t}+e^{2t}+e^{3t}+e^{4t}+1\right)+50c_1e^{10t}+50c_2\right)$$

12.19 problem 19

Internal problem ID [14594]

Book: INTRODUCTORY DIFFERENTIAL EQUATIONS. Martha L. Abell, James P. Braselton. Fourth edition 2014. ElScAe. 2014
Section: Chapter 4. Higher Order Equations. Exercises 4.4, page 163
Problem number: 19.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _linear, _nonhomogeneous]]`

$$\boxed{y'' - y = 2\sinh(t)}$$

✓ Solution by Maple
Time used: 0.0 (sec). Leaf size: 28

```
dsolve(diff(y(t),t$2)-y(t)=2*sinh(t),y(t), singsol=all)
```

$$y(t) = \frac{(2t + 4c_2)\,\mathrm{e}^{-t}}{4} + \frac{\left(t + 2c_1 - \frac{1}{2}\right)\mathrm{e}^{t}}{2}$$

✓ Solution by Mathematica
Time used: 0.052 (sec). Leaf size: 38

```
DSolve[y''[t]-y[t]==2*Sinh[t],y[t],t,IncludeSingularSolutions -> True]
```

$$y(t) \to \frac{1}{4}e^{-t}\left(2t + e^{2t}(2t - 1 + 4c_1) + 1 + 4c_2\right)$$

12.20 problem 20

Internal problem ID [14595]

Book: INTRODUCTORY DIFFERENTIAL EQUATIONS. Martha L. Abell, James P. Braselton. Fourth edition 2014. ElScAe. 2014
Section: Chapter 4. Higher Order Equations. Exercises 4.4, page 163
Problem number: 20.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _linear, _nonhomogeneous]]`

$$\boxed{y'' - 2y' + y = \frac{e^t}{t}}$$

✓ Solution by Maple
Time used: 0.015 (sec). Leaf size: 18

```
dsolve(diff(y(t),t$2)-2*diff(y(t),t)+y(t)=1/t*exp(t),y(t), singsol=all)
```

$$y(t) = (t \ln(t) + t(c_1 - 1) + c_2) e^t$$

✓ Solution by Mathematica
Time used: 0.02 (sec). Leaf size: 22

```
DSolve[y''[t]-2*y'[t]+y[t]==1/t*Exp[t],y[t],t,IncludeSingularSolutions -> True]
```

$$y(t) \to e^t(t \log(t) + (-1 + c_2)t + c_1)$$

12.21 problem 21

Internal problem ID [14596]

Book: INTRODUCTORY DIFFERENTIAL EQUATIONS. Martha L. Abell, James P. Braselton. Fourth edition 2014. ElScAe. 2014
Section: Chapter 4. Higher Order Equations. Exercises 4.4, page 163
Problem number: 21.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _linear, _nonhomogeneous]]`

$$\boxed{y'' - 4y' + 4y = \frac{e^{2t}}{t^2}}$$

✓ Solution by Maple
Time used: 0.016 (sec). Leaf size: 19

```
dsolve(diff(y(t),t$2)-4*diff(y(t),t)+4*y(t)=1/t^2*exp(2*t),y(t), singsol=all)
```

$$y(t) = e^{2t}(-1 + c_1 t - \ln(t) + c_2)$$

✓ Solution by Mathematica
Time used: 0.022 (sec). Leaf size: 23

```
DSolve[y''[t]-4*y'[t]+4*y[t]==1/t^2*Exp[2*t],y[t],t,IncludeSingularSolutions -> True]
```

$$y(t) \to e^{2t}(-\log(t) + c_2 t - 1 + c_1)$$

12.22 problem 22

Internal problem ID [14597]

Book: INTRODUCTORY DIFFERENTIAL EQUATIONS. Martha L. Abell, James P. Braselton. Fourth edition 2014. ElScAe. 2014
Section: Chapter 4. Higher Order Equations. Exercises 4.4, page 163
Problem number: 22.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _linear, _nonhomogeneous]]`

$$\boxed{y'' + 8y' + 16y = \frac{e^{-4t}}{t^4}}$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 27

```
dsolve(diff(y(t),t$2)+8*diff(y(t),t)+16*y(t)=1/t^4*exp(-4*t),y(t), singsol=all)
```

$$y(t) = \frac{e^{-4t}(6c_1t^3 + 6c_2t^2 + 1)}{6t^2}$$

✓ Solution by Mathematica

Time used: 0.031 (sec). Leaf size: 33

```
DSolve[y''[t]+8*y'[t]+16*y[t]==1/t^4*Exp[-4*t],y[t],t,IncludeSingularSolutions -> True]
```

$$y(t) \to \frac{e^{-4t}(6c_2t^3 + 6c_1t^2 + 1)}{6t^2}$$

12.23 problem 23

Internal problem ID [14598]

Book: INTRODUCTORY DIFFERENTIAL EQUATIONS. Martha L. Abell, James P. Braselton. Fourth edition 2014. ElScAe. 2014
Section: Chapter 4. Higher Order Equations. Exercises 4.4, page 163
Problem number: 23.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _linear, _nonhomogeneous]]`

$$\boxed{y'' + 6y' + 9y = \frac{\mathrm{e}^{-3t}}{t}}$$

✓ Solution by Maple
Time used: 0.015 (sec). Leaf size: 20

```
dsolve(diff(y(t),t$2)+6*diff(y(t),t)+9*y(t)=1/t*exp(-3*t),y(t), singsol=all)
```

$$y(t) = (t\ln(t) + t(c_1 - 1) + c_2)\,\mathrm{e}^{-3t}$$

✓ Solution by Mathematica
Time used: 0.029 (sec). Leaf size: 24

```
DSolve[y''[t]+6*y'[t]+9*y[t]==1/t*Exp[-3*t],y[t],t,IncludeSingularSolutions -> True]
```

$$y(t) \to e^{-3t}(t\log(t) + (-1 + c_2)t + c_1)$$

12.24 problem 24

Internal problem ID [14599]

Book: INTRODUCTORY DIFFERENTIAL EQUATIONS. Martha L. Abell, James P. Braselton. Fourth edition 2014. ElScAe. 2014
Section: Chapter 4. Higher Order Equations. Exercises 4.4, page 163
Problem number: 24.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _linear, _nonhomogeneous]]`

$$\boxed{y'' + 6y' + 9y = \mathrm{e}^{-3t} \ln(t)}$$

✓ Solution by Maple
Time used: 0.015 (sec). Leaf size: 30

```
dsolve(diff(y(t),t$2)+6*diff(y(t),t)+9*y(t)=exp(-3*t)*ln(t),y(t), singsol=all)
```

$$y(t) = \frac{\mathrm{e}^{-3t}(2\ln(t)\,t^2 + 4c_1t - 3t^2 + 4c_2)}{4}$$

✓ Solution by Mathematica
Time used: 0.031 (sec). Leaf size: 36

```
DSolve[y''[t]+6*y'[t]+9*y[t]==Exp[-3*t]*Log[t],y[t],t,IncludeSingularSolutions -> True]
```

$$y(t) \to \frac{1}{4}e^{-3t}\left(-3t^2 + 2t^2\log(t) + 4c_2t + 4c_1\right)$$

12.25 problem 25

Internal problem ID [14600]

Book: INTRODUCTORY DIFFERENTIAL EQUATIONS. Martha L. Abell, James P. Braselton. Fourth edition 2014. ElScAe. 2014
Section: Chapter 4. Higher Order Equations. Exercises 4.4, page 163
Problem number: 25.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _linear, _nonhomogeneous]]`

$$\boxed{y'' + 3y' + 2y = \cos\left(\mathrm{e}^{t}\right)}$$

✓ Solution by Maple
Time used: 0.0 (sec). Leaf size: 21

```
dsolve(diff(y(t),t$2)+3*diff(y(t),t)+2*y(t)=cos(exp(t)),y(t), singsol=all)
```

$$y(t) = -\left(-c_2 \mathrm{e}^{t} + \cos\left(\mathrm{e}^{t}\right) + c_1 + 1\right) \mathrm{e}^{-2t}$$

✓ Solution by Mathematica
Time used: 0.072 (sec). Leaf size: 26

```
DSolve[y''[t]+3*y'[t]+2*y[t]==Cos[Exp[t]],y[t],t,IncludeSingularSolutions -> True]
```

$$y(t) \to e^{-2t}\left(-\cos\left(e^{t}\right) + c_2 e^{t} + c_1\right)$$

12.26 problem 26

Internal problem ID [14601]

Book: INTRODUCTORY DIFFERENTIAL EQUATIONS. Martha L. Abell, James P. Braselton. Fourth edition 2014. ElScAe. 2014
Section: Chapter 4. Higher Order Equations. Exercises 4.4, page 163
Problem number: 26.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _linear, _nonhomogeneous]]`

$$\boxed{y'' + 4y' + 4y = e^{-2t}\sqrt{-t^2 + 1}}$$

✓ Solution by Maple
Time used: 0.0 (sec). Leaf size: 38

```
dsolve(diff(y(t),t$2)+4*diff(y(t),t)+4*y(t)=exp(-2*t)*sqrt(1-t^2),y(t), singsol=all)
```

$$y(t) = \frac{\left((t^2 + 2)\sqrt{-t^2 + 1} + 6c_1 t + 3\arcsin(t)\,t + 6c_2\right)e^{-2t}}{6}$$

✓ Solution by Mathematica
Time used: 0.11 (sec). Leaf size: 73

```
DSolve[y''[t]+4*y'[t]+4*y[t]==Exp[-2*t]*Sqrt[1-t^2],y[t],t,IncludeSingularSolutions -> True]
```

$$y(t) \to \frac{1}{6}e^{-2t}\left(-6t\arctan\left(\frac{\sqrt{1-t^2}}{t+1}\right) + \sqrt{1-t^2}t^2 + 2\sqrt{1-t^2} + 6c_2 t + 6c_1\right)$$

12.27 problem 27

Internal problem ID [14602]

Book: INTRODUCTORY DIFFERENTIAL EQUATIONS. Martha L. Abell, James P. Braselton. Fourth edition 2014. ElScAe. 2014
Section: Chapter 4. Higher Order Equations. Exercises 4.4, page 163
Problem number: 27.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _linear, _nonhomogeneous]]`

$$\boxed{y'' - 2y' + y = \mathrm{e}^{t}\sqrt{-t^{2}+1}}$$

✓ Solution by Maple
Time used: 0.0 (sec). Leaf size: 36

```
dsolve(diff(y(t),t$2)-2*diff(y(t),t)+y(t)=exp(t)*sqrt(1-t^2),y(t), singsol=all)
```

$$y(t) = \frac{\mathrm{e}^{t}\left(\left(t^{2}+2\right)\sqrt{-t^{2}+1} + 6c_{1}t + 3\arcsin\left(t\right)t + 6c_{2}\right)}{6}$$

✓ Solution by Mathematica
Time used: 0.03 (sec). Leaf size: 71

```
DSolve[y''[t]-2*y'[t]+y[t]==Exp[t]*Sqrt[1-t^2],y[t],t,IncludeSingularSolutions -> True]
```

$$y(t) \to \frac{1}{6}e^{t}\left(-6t\arctan\left(\frac{\sqrt{1-t^{2}}}{t+1}\right) + \sqrt{1-t^{2}}t^{2} + 2\sqrt{1-t^{2}} + 6c_{2}t + 6c_{1}\right)$$

12.28 problem 28

Internal problem ID [14603]

Book: INTRODUCTORY DIFFERENTIAL EQUATIONS. Martha L. Abell, James P. Braselton. Fourth edition 2014. ElScAe. 2014
Section: Chapter 4. Higher Order Equations. Exercises 4.4, page 163
Problem number: 28.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _linear, _nonhomogeneous]]`

$$\boxed{y'' - 10y' + 25y = e^{5t} \ln(2t)}$$

✓ Solution by Maple
Time used: 0.0 (sec). Leaf size: 37

```
dsolve(diff(y(t),t$2)-10*diff(y(t),t)+25*y(t)=exp(5*t)*ln(2*t),y(t), singsol=all)
```

$$y(t) = \frac{e^{5t}(2t^2 \ln(2) + 2\ln(t)\, t^2 + 4c_1 t - 3t^2 + 4c_2)}{4}$$

✓ Solution by Mathematica
Time used: 0.031 (sec). Leaf size: 38

```
DSolve[y''[t]-10*y'[t]+25*y[t]==Exp[5*t]*Log[2*t],y[t],t,IncludeSingularSolutions -> True]
```

$$y(t) \to \frac{1}{4} e^{5t}\left(-3t^2 + 2t^2 \log(2t) + 4c_2 t + 4c_1\right)$$

12.29 problem 29

Internal problem ID [14604]

Book: INTRODUCTORY DIFFERENTIAL EQUATIONS. Martha L. Abell, James P. Braselton. Fourth edition 2014. ElScAe. 2014
Section: Chapter 4. Higher Order Equations. Exercises 4.4, page 163
Problem number: 29.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _linear, _nonhomogeneous]]`

$$\boxed{y'' - 4y' + 4y = e^{2t} \arctan(t)}$$

✓ Solution by Maple
Time used: 0.015 (sec). Leaf size: 38

```
dsolve(diff(y(t),t$2)-4*diff(y(t),t)+4*y(t)=exp(2*t)*arctan(t),y(t), singsol=all)
```

$$y(t) = \frac{e^{2t}(\arctan(t)\, t^2 - \ln(t^2+1)\, t + 2c_1 t - \arctan(t) + 2c_2 + t)}{2}$$

✓ Solution by Mathematica
Time used: 0.037 (sec). Leaf size: 42

```
DSolve[y''[t]-4*y'[t]+4*y[t]==Exp[2*t]*ArcTan[t],y[t],t,IncludeSingularSolutions -> True]
```

$$y(t) \to \frac{1}{2} e^{2t} \left((t^2 - 1) \arctan(t) - t \log(t^2+1) + t + 2c_2 t + 2c_1 \right)$$

12.30 problem 30

Internal problem ID [14605]

Book: INTRODUCTORY DIFFERENTIAL EQUATIONS. Martha L. Abell, James P. Braselton. Fourth edition 2014. ElScAe. 2014
Section: Chapter 4. Higher Order Equations. Exercises 4.4, page 163
Problem number: 30.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _linear, _nonhomogeneous]]`

$$\boxed{y'' + 8y' + 16y = \frac{e^{-4t}}{t^2 + 1}}$$

✓ Solution by Maple

Time used: 0.016 (sec). Leaf size: 26

```
dsolve(diff(y(t),t$2)+8*diff(y(t),t)+16*y(t)=exp(-4*t)*1/(1+t^2),y(t), singsol=all)
```

$$y(t) = e^{-4t}\left(c_2 + c_1 t - \frac{\ln(t^2+1)}{2} + \arctan(t)\, t\right)$$

✓ Solution by Mathematica

Time used: 0.04 (sec). Leaf size: 37

```
DSolve[y''[t]+8*y'[t]+16*y[t]==Exp[-4*t]*1/(1+t^2),y[t],t,IncludeSingularSolutions -> True]
```

$$y(t) \to \frac{1}{2}e^{-4t}\left(2t\arctan(t) - \log\left(t^2+1\right) + 2(c_2 t + c_1)\right)$$

12.31 problem 31

Internal problem ID [14606]

Book: INTRODUCTORY DIFFERENTIAL EQUATIONS. Martha L. Abell, James P. Braselton. Fourth edition 2014. ElScAe. 2014
Section: Chapter 4. Higher Order Equations. Exercises 4.4, page 163
Problem number: 31.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _linear, _nonhomogeneous]]`

$$\boxed{y'' + y = \sec\left(\frac{t}{2}\right) + \csc\left(\frac{t}{2}\right)}$$

✓ Solution by Maple
Time used: 0.0 (sec). Leaf size: 55

```
dsolve(diff(y(t),t$2)+y(t)=sec(t/2)+csc(t/2),y(t), singsol=all)
```

$$y(t) = \sin(t)\, c_2 + \cos(t)\, c_1 - 2\sin(t)\ln\left(\sec\left(\frac{t}{2}\right) + \tan\left(\frac{t}{2}\right)\right) + 2\sin(t)\ln\left(\csc\left(\frac{t}{2}\right) - \cot\left(\frac{t}{2}\right)\right) + 4\cos\left(\frac{t}{2}\right) + 4\sin\left(\frac{t}{2}\right)$$

✓ Solution by Mathematica
Time used: 0.21 (sec). Leaf size: 65

```
DSolve[y''[t]+y[t]==Sec[t/2]+Csc[t/2],y[t],t,IncludeSingularSolutions -> True]
```

$$y(t) \to -2\sin(t)\text{arctanh}\left(\sin\left(\frac{t}{2}\right)\right) + 4\sin\left(\frac{t}{2}\right) + 4\cos\left(\frac{t}{2}\right) + 2\sin(t)\log\left(\sin\left(\frac{t}{4}\right)\right) + c_1\cos(t) + c_2\sin(t) - 2\sin(t)\log\left(\cos\left(\frac{t}{4}\right)\right)$$

12.32 problem 32

Internal problem ID [14607]

Book: INTRODUCTORY DIFFERENTIAL EQUATIONS. Martha L. Abell, James P. Braselton. Fourth edition 2014. ElScAe. 2014
Section: Chapter 4. Higher Order Equations. Exercises 4.4, page 163
Problem number: 32.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _linear, _nonhomogeneous]]`

$$\boxed{y'' + 9y = \tan(3t)^2}$$

✓ Solution by Maple
Time used: 0.0 (sec). Leaf size: 34

```
dsolve(diff(y(t),t$2)+9*y(t)=tan(3*t)^2,y(t), singsol=all)
```

$$y(t) = c_2 \sin(3t) + c_1 \cos(3t) + \frac{\sin(3t)\ln(\sec(3t) + \tan(3t))}{9} - \frac{2}{9}$$

✓ Solution by Mathematica
Time used: 0.224 (sec). Leaf size: 36

```
DSolve[y''[t]+9*y[t]==Tan[3*t]^2,y[t],t,IncludeSingularSolutions -> True]
```

$$y(t) \to \frac{1}{9}\sin(3t)\text{arctanh}(\sin(3t)) + c_1\cos(3t) + c_2\sin(3t) - \frac{2}{9}$$

12.33 problem 33

Internal problem ID [14608]

Book: INTRODUCTORY DIFFERENTIAL EQUATIONS. Martha L. Abell, James P. Braselton. Fourth edition 2014. ElScAe. 2014
Section: Chapter 4. Higher Order Equations. Exercises 4.4, page 163
Problem number: 33.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _linear, _nonhomogeneous]]`

$$\boxed{y'' + 9y = \sec(3t)}$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 33

```
dsolve(diff(y(t),t$2)+9*y(t)=sec(3*t),y(t), singsol=all)
```

$$y(t) = -\frac{\ln(\sec(3t))\cos(3t)}{9} + c_1\cos(3t) + \frac{\sin(3t)(t + 3c_2)}{3}$$

✓ Solution by Mathematica

Time used: 0.037 (sec). Leaf size: 37

```
DSolve[y''[t]+9*y[t]==Sec[3*t],y[t],t,IncludeSingularSolutions -> True]
```

$$y(t) \to \frac{1}{3}(t + 3c_2)\sin(3t) + \cos(3t)\left(\frac{1}{9}\log(\cos(3t)) + c_1\right)$$

12.34 problem 34

Internal problem ID [14609]

Book: INTRODUCTORY DIFFERENTIAL EQUATIONS. Martha L. Abell, James P. Braselton. Fourth edition 2014. ElScAe. 2014
Section: Chapter 4. Higher Order Equations. Exercises 4.4, page 163
Problem number: 34.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _linear, _nonhomogeneous]]`

$$\boxed{y'' + 9y = \tan(3t)}$$

✓ Solution by Maple
Time used: 0.0 (sec). Leaf size: 33

```
dsolve(diff(y(t),t$2)+9*y(t)=tan(3*t),y(t), singsol=all)
```

$$y(t) = c_2 \sin(3t) + c_1 \cos(3t) - \frac{\cos(3t)\ln(\sec(3t) + \tan(3t))}{9}$$

✓ Solution by Mathematica
Time used: 0.035 (sec). Leaf size: 33

```
DSolve[y''[t]+9*y[t]==Tan[3*t],y[t],t,IncludeSingularSolutions -> True]
```

$$y(t) \to -\frac{1}{9}\cos(3t)\operatorname{arctanh}(\sin(3t)) + c_1\cos(3t) + c_2\sin(3t)$$

12.35 problem 35

Internal problem ID [14610]

Book: INTRODUCTORY DIFFERENTIAL EQUATIONS. Martha L. Abell, James P. Braselton. Fourth edition 2014. ElScAe. 2014
Section: Chapter 4. Higher Order Equations. Exercises 4.4, page 163
Problem number: 35.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _linear, _nonhomogeneous]]`

$$\boxed{y'' + 4y = \tan(2t)}$$

✓ Solution by Maple
Time used: 0.016 (sec). Leaf size: 33

```
dsolve(diff(y(t),t$2)+4*y(t)=tan(2*t),y(t), singsol=all)
```

$$y(t) = \sin(2t)\, c_2 + \cos(2t)\, c_1 - \frac{\cos(2t)\ln(\sec(2t) + \tan(2t))}{4}$$

✓ Solution by Mathematica
Time used: 0.046 (sec). Leaf size: 40

```
DSolve[y''[t]+4*y[t]==Tan[2*t],y[t],t,IncludeSingularSolutions -> True]
```

$$y(t) \to -\frac{1}{4}\cos(2t)\text{arctanh}(\sin(2t)) + c_1\cos(2t) + \frac{1}{4}(-1 + 4c_2)\sin(2t)$$

12.36 problem 36

Internal problem ID [14611]

Book: INTRODUCTORY DIFFERENTIAL EQUATIONS. Martha L. Abell, James P. Braselton. Fourth edition 2014. ElScAe. 2014
Section: Chapter 4. Higher Order Equations. Exercises 4.4, page 163
Problem number: 36.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _linear, _nonhomogeneous]]`

$$\boxed{y'' + 16y = \tan(2t)}$$

✓ Solution by Maple
Time used: 0.0 (sec). Leaf size: 56

```
dsolve(diff(y(t),t$2)+16*y(t)=tan(2*t),y(t), singsol=all)
```

$$y(t) = \frac{\sin(2t)\cos(2t)\ln(\cos(2t))}{4} + \frac{(-t+4c_1)\cos(2t)^2}{2} + \frac{\sin(2t)(16c_2-1)\cos(2t)}{8} + \frac{t}{4} - c_1$$

✓ Solution by Mathematica
Time used: 0.057 (sec). Leaf size: 40

```
DSolve[y''[t]+16*y[t]==Tan[2*t],y[t],t,IncludeSingularSolutions -> True]
```

$$y(t) \to \left(-\frac{t}{4} + c_1\right)\cos(4t) + \frac{1}{16}\sin(4t)(2\log(\cos(2t)) - 1 + 16c_2)$$

12.37 problem 37

Internal problem ID [14612]

Book: INTRODUCTORY DIFFERENTIAL EQUATIONS. Martha L. Abell, James P. Braselton. Fourth edition 2014. ElScAe. 2014
Section: Chapter 4. Higher Order Equations. Exercises 4.4, page 163
Problem number: 37.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _linear, _nonhomogeneous]]`

$$\boxed{y'' + 4y = \tan(t)}$$

✓ Solution by Maple
Time used: 0.0 (sec). Leaf size: 42

```
dsolve(diff(y(t),t$2)+4*y(t)=tan(t),y(t), singsol=all)
```

$$y(t) = \sin(t)\cos(t)\ln(\cos(t)) + (2c_1 - t)\cos(t)^2 + \frac{\sin(t)(4c_2 + 1)\cos(t)}{2} - c_1 + \frac{t}{2}$$

✓ Solution by Mathematica
Time used: 0.05 (sec). Leaf size: 38

```
DSolve[y''[t]+4*y[t]==Tan[t],y[t],t,IncludeSingularSolutions -> True]
```

$$y(t) \to \frac{1}{4}(\sin(2t)(2\log(\cos(t)) - 1 + 4c_2) - 2(t - 2c_1)\cos(2t))$$

12.38 problem 38

Internal problem ID [14613]

Book: INTRODUCTORY DIFFERENTIAL EQUATIONS. Martha L. Abell, James P. Braselton. Fourth edition 2014. ElScAe. 2014
Section: Chapter 4. Higher Order Equations. Exercises 4.4, page 163
Problem number: 38.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _linear, _nonhomogeneous]]`

$$\boxed{y'' + 9y = \sec(3t)\tan(3t)}$$

✓ Solution by Maple
Time used: 0.0 (sec). Leaf size: 38

```
dsolve(diff(y(t),t$2)+9*y(t)=sec(3*t)*tan(3*t),y(t), singsol=all)
```

$$y(t) = \frac{\ln(\sec(3t))\sin(3t)}{9} + \frac{(9c_2 - 1)\sin(3t)}{9} + \frac{\cos(3t)(t + 3c_1)}{3}$$

✓ Solution by Mathematica
Time used: 0.054 (sec). Leaf size: 46

```
DSolve[y''[t]+9*y[t]==Sec[3*t]*Tan[3*t],y[t],t,IncludeSingularSolutions -> True]
```

$$y(t) \to \frac{1}{9}(\cos(3t)\arctan(\tan(3t)) + 9c_1\cos(3t) + \sin(3t)(-\log(\cos(3t)) - 1 + 9c_2))$$

12.39 problem 39

Internal problem ID [14614]

Book: INTRODUCTORY DIFFERENTIAL EQUATIONS. Martha L. Abell, James P. Braselton. Fourth edition 2014. ElScAe. 2014
Section: Chapter 4. Higher Order Equations. Exercises 4.4, page 163
Problem number: 39.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _linear, _nonhomogeneous]]`

$$\boxed{y'' + 4y = \sec(2t)\tan(2t)}$$

✓ Solution by Maple
Time used: 0.0 (sec). Leaf size: 38

```
dsolve(diff(y(t),t$2)+4*y(t)=sec(2*t)*tan(2*t),y(t), singsol=all)
```

$$y(t) = \frac{\ln(\sec(2t))\sin(2t)}{4} + \frac{(4c_2 - 1)\sin(2t)}{4} + \frac{\cos(2t)(2c_1 + t)}{2}$$

✓ Solution by Mathematica
Time used: 0.058 (sec). Leaf size: 46

```
DSolve[y''[t]+4*y[t]==Sec[2*t]*Tan[2*t],y[t],t,IncludeSingularSolutions -> True]
```

$$y(t) \to \frac{1}{4}(\cos(2t)\arctan(\tan(2t)) + 4c_1\cos(2t) + \sin(2t)(-\log(\cos(2t)) - 1 + 4c_2))$$

12.40 problem 40

Internal problem ID [14615]

Book: INTRODUCTORY DIFFERENTIAL EQUATIONS. Martha L. Abell, James P. Braselton. Fourth edition 2014. ElScAe. 2014
Section: Chapter 4. Higher Order Equations. Exercises 4.4, page 163
Problem number: 40.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _linear, _nonhomogeneous]]`

$$\boxed{y''+9y=\frac{\csc(3t)}{2}}$$

With initial conditions

$$\left[y\left(\frac{\pi}{4}\right)=\sqrt{2}, y'\left(\frac{\pi}{4}\right)=0\right]$$

✓ Solution by Maple
Time used: 0.11 (sec). Leaf size: 38

```
dsolve([diff(y(t),t$2)+9*y(t)=1/2*csc(3*t),y(1/4*Pi) = 2^(1/2), D(y)(1/4*Pi) = 0],y(t), sing
```

$$y(t)=-\frac{\ln(\csc(3t))\sin(3t)}{18}+\frac{(-24-4t+\pi)\cos(3t)}{24}+\frac{(\ln(2)+36)\sin(3t)}{36}$$

✓ Solution by Mathematica
Time used: 0.036 (sec). Leaf size: 40

```
DSolve[{y''[t]+9*y[t]==1/2*Csc[3*t],{y[Pi/4]==Sqrt[2],y'[Pi/4]==0}},y[t],t,IncludeSingularSo
```

$$y(t)\to\frac{1}{72}(3(\pi-4(t+6))\cos(3t)+2\sin(3t)(2\log(\sin(3t))+36+\log(2)))$$

12.41 problem 41

Internal problem ID [14616]

Book: INTRODUCTORY DIFFERENTIAL EQUATIONS. Martha L. Abell, James P. Braselton. Fourth edition 2014. ElScAe. 2014
Section: Chapter 4. Higher Order Equations. Exercises 4.4, page 163
Problem number: 41.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _linear, _nonhomogeneous]]`

$$\boxed{y'' + 4y = \sec(2t)^2}$$

With initial conditions

$$[y(0) = 0, y'(0) = 1]$$

✓ Solution by Maple
Time used: 0.016 (sec). Leaf size: 34

```
dsolve([diff(y(t),t$2)+4*y(t)=sec(2*t)^2,y(0) = 0, D(y)(0) = 1],y(t), singsol=all)
```

$$y(t) = \frac{\sin(2t)}{2} + \frac{\cos(2t)}{4} - \frac{1}{4} + \frac{\ln(\sec(2t) + \tan(2t))\sin(2t)}{4}$$

✓ Solution by Mathematica
Time used: 0.101 (sec). Leaf size: 35

```
DSolve[{y''[t]+4*y[t]==Sec[2*t]^2,{y[0]==0,y'[0]==1}},y[t],t,IncludeSingularSolutions -> Tru
```

$$y(t) \to \frac{1}{4}\sin(t)\left(-2\sin(t) + (4 - i\pi)\cos(t) + 2\cos(t)\coth^{-1}(\sin(2t))\right)$$

12.42 problem 42

Internal problem ID [14617]

Book: INTRODUCTORY DIFFERENTIAL EQUATIONS. Martha L. Abell, James P. Braselton. Fourth edition 2014. ElScAe. 2014
Section: Chapter 4. Higher Order Equations. Exercises 4.4, page 163
Problem number: 42.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _linear, _nonhomogeneous]]`

$$\boxed{y'' - 16y = 16t\,\mathrm{e}^{-4t}}$$

With initial conditions

$$[y(0) = 0, y'(0) = 0]$$

✓ Solution by Maple
Time used: 0.031 (sec). Leaf size: 29

```
dsolve([diff(y(t),t$2)-16*y(t)=16*t*exp(-4*t),y(0) = 0, D(y)(0) = 0],y(t), singsol=all)
```

$$y(t) = \frac{(-32t^2 - 8t - 1)\,\mathrm{e}^{-4t}}{32} + \frac{\mathrm{e}^{4t}}{32}$$

✓ Solution by Mathematica
Time used: 0.042 (sec). Leaf size: 29

```
DSolve[{y''[t]-16*y[t]==16*t*Exp[-4*t],{y[0]==0,y'[0]==0}},y[t],t,IncludeSingularSolutions -
```

$$y(t) \to \frac{1}{32}e^{-4t}\left(-32t^2 - 8t + e^{8t} - 1\right)$$

12.43 problem 43

Internal problem ID [14618]

Book: INTRODUCTORY DIFFERENTIAL EQUATIONS. Martha L. Abell, James P. Braselton. Fourth edition 2014. ElScAe. 2014
Section: Chapter 4. Higher Order Equations. Exercises 4.4, page 163
Problem number: 43.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _linear, _nonhomogeneous]]`

$$\boxed{y'' + y = \tan(t)^2}$$

With initial conditions

$$[y(0) = 0, y'(0) = 1]$$

✓ Solution by Maple
Time used: 0.016 (sec). Leaf size: 21

```
dsolve([diff(y(t),t$2)+y(t)=tan(t)^2,y(0) = 0, D(y)(0) = 1],y(t), singsol=all)
```

$$y(t) = \sin(t) + 2\cos(t) - 2 + \sin(t)\ln(\sec(t) + \tan(t))$$

✓ Solution by Mathematica
Time used: 0.117 (sec). Leaf size: 19

```
DSolve[{y''[t]+y[t]==Tan[t]^2,{y[0]==0,y'[0]==1}},y[t],t,IncludeSingularSolutions -> True]
```

$$y(t) \to \sin(t)\text{arctanh}(\sin(t)) + \sin(t) + 2\cos(t) - 2$$

12.44 problem 44

Internal problem ID [14619]

Book: INTRODUCTORY DIFFERENTIAL EQUATIONS. Martha L. Abell, James P. Braselton. Fourth edition 2014. ElScAe. 2014
Section: Chapter 4. Higher Order Equations. Exercises 4.4, page 163
Problem number: 44.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _linear, _nonhomogeneous]]`

$$\boxed{y'' + 4y = \sec(2t) + \tan(2t)}$$

With initial conditions

$$[y(0) = 1, y'(0) = 0]$$

✓ Solution by Maple
Time used: 0.047 (sec). Leaf size: 42

```
dsolve([diff(y(t),t$2)+4*y(t)=sec(2*t)+tan(2*t),y(0) = 1, D(y)(0) = 0],y(t), singsol=all)
```

$$y(t) = \frac{\sin(2t)}{4} - \frac{i\cos(2t)\pi}{4} + \cos(2t) + \frac{t\sin(2t)}{2} + \frac{\cos(2t)\ln(\sin(2t)-1)}{4}$$

✓ Solution by Mathematica
Time used: 0.509 (sec). Leaf size: 36

```
DSolve[{y''[t]+4*y[t]==Sec[2*t]+Tan[2*t],{y[0]==1,y'[0]==0}},y[t],t,IncludeSingularSolutions
```

$$y(t) \to \frac{1}{4}((2t+1)\sin(2t) + 2\cos(2t)(\log(\cos(t)-\sin(t)) + 2))$$

12.45 problem 45

Internal problem ID [14620]

Book: INTRODUCTORY DIFFERENTIAL EQUATIONS. Martha L. Abell, James P. Braselton. Fourth edition 2014. ElScAe. 2014
Section: Chapter 4. Higher Order Equations. Exercises 4.4, page 163
Problem number: 45.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _linear, _nonhomogeneous]]`

$$\boxed{y'' + 9y = \csc(3t)}$$

With initial conditions

$$\left[y\left(\frac{\pi}{12}\right) = 0, y'\left(\frac{\pi}{12}\right) = 1\right]$$

✓ Solution by Maple
Time used: 0.125 (sec). Leaf size: 46

```
dsolve([diff(y(t),t$2)+9*y(t)=csc(3*t),y(1/12*Pi) = 0, D(y)(1/12*Pi) = 1],y(t), singsol=all)
```

$$y(t) = -\frac{\ln(\csc(3t))\sin(3t)}{9} + \frac{(-12t + \pi - 6\sqrt{2})\cos(3t)}{36} + \frac{\sin(3t)\left(3\sqrt{2} + \ln(2)\right)}{18}$$

✓ Solution by Mathematica
Time used: 0.03 (sec). Leaf size: 51

```
DSolve[{y''[t]+9*y[t]==Csc[3*t],{y[Pi/12]==0,y'[Pi/12]==1}},y[t],t,IncludeSingularSolutions
```

$$y(t) \to \frac{1}{36}\left(\left(\pi - 6\left(2t + \sqrt{2}\right)\right)\cos(3t) + 2\sin(3t)\left(2\log(\sin(3t)) + 3\sqrt{2} + \log(2)\right)\right)$$

12.46 problem 46

Internal problem ID [14621]

Book: INTRODUCTORY DIFFERENTIAL EQUATIONS. Martha L. Abell, James P. Braselton. Fourth edition 2014. ElScAe. 2014
Section: Chapter 4. Higher Order Equations. Exercises 4.4, page 163
Problem number: 46.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _linear, _nonhomogeneous]]`

$$\boxed{y'' + 4y' + 3y = 65\cos(2t)}$$

✓ Solution by Maple
Time used: 0.0 (sec). Leaf size: 29

```
dsolve(diff(y(t),t$2)+4*diff(y(t),t)+3*y(t)=65*cos(2*t),y(t), singsol=all)
```

$$y(t) = c_2 e^{-3t} + e^{-t} c_1 - \cos(2t) + 8\sin(2t)$$

✓ Solution by Mathematica
Time used: 0.02 (sec). Leaf size: 35

```
DSolve[y''[t]+4*y'[t]+3*y[t]==65*Cos[2*t],y[t],t,IncludeSingularSolutions -> True]
```

$$y(t) \to 8\sin(2t) - \cos(2t) + e^{-3t}\left(c_2 e^{2t} + c_1\right)$$

12.47 problem 55

Internal problem ID [14622]

Book: INTRODUCTORY DIFFERENTIAL EQUATIONS. Martha L. Abell, James P. Braselton. Fourth edition 2014. ElScAe. 2014
Section: Chapter 4. Higher Order Equations. Exercises 4.4, page 163
Problem number: 55.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _exact, _linear, _nonhomogeneous]]`

$$\boxed{t^2 y'' + 3y't + y = \ln(t)}$$

✓ Solution by Maple
Time used: 0.0 (sec). Leaf size: 19

```
dsolve(t^2*diff(y(t),t$2)+3*t*diff(y(t),t)+y(t)=ln(t),y(t), singsol=all)
```

$$y(t) = \frac{(t + c_1)\ln(t) - 2t + c_2}{t}$$

✓ Solution by Mathematica
Time used: 0.118 (sec). Leaf size: 22

```
DSolve[t^2*y''[t]+3*t*y'[t]+y[t]==Log[t],y[t],t,IncludeSingularSolutions -> True]
```

$$y(t) \to \frac{-2t + (t + c_2)\log(t) + c_1}{t}$$

12.48 problem 56

Internal problem ID [14623]

Book: INTRODUCTORY DIFFERENTIAL EQUATIONS. Martha L. Abell, James P. Braselton. Fourth edition 2014. ElScAe. 2014
Section: Chapter 4. Higher Order Equations. Exercises 4.4, page 163
Problem number: 56.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type **[[_2nd_order, _with_linear_symmetries]]**

$$\boxed{t^2 y'' + y' t + 4y = t}$$

✓ Solution by Maple
Time used: 0.015 (sec). Leaf size: 22

```
dsolve(t^2*diff(y(t),t$2)+t*diff(y(t),t)+4*y(t)=t,y(t), singsol=all)
```

$$y(t) = \sin(2\ln(t))\, c_2 + \cos(2\ln(t))\, c_1 + \frac{t}{5}$$

✓ Solution by Mathematica
Time used: 0.037 (sec). Leaf size: 27

```
DSolve[t^2*y''[t]+t*y'[t]+4*y[t]==t,y[t],t,IncludeSingularSolutions -> True]
```

$$y(t) \to \frac{t}{5} + c_1 \cos(2\log(t)) + c_2 \sin(2\log(t))$$

12.49 problem 57

Internal problem ID [14624]

Book: INTRODUCTORY DIFFERENTIAL EQUATIONS. Martha L. Abell, James P. Braselton. Fourth edition 2014. ElScAe. 2014
Section: Chapter 4. Higher Order Equations. Exercises 4.4, page 163
Problem number: 57.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _exact, _linear, _nonhomogeneous]]`

$$\boxed{t^2 y'' - 4y't - 6y = 2\ln(t)}$$

✓ Solution by Maple
Time used: 0.0 (sec). Leaf size: 20

```
dsolve(t^2*diff(y(t),t$2)-4*t*diff(y(t),t)-6*y(t)=2*ln(t),y(t), singsol=all)
```

$$y(t) = t^6 c_2 - \frac{\ln(t)}{3} + \frac{5}{18} + \frac{c_1}{t}$$

✓ Solution by Mathematica
Time used: 0.019 (sec). Leaf size: 27

```
DSolve[t^2*y''[t]-4*t*y'[t]-6*y[t]==2*Log[t],y[t],t,IncludeSingularSolutions -> True]
```

$$y(t) \to c_2 t^6 - \frac{\log(t)}{3} + \frac{c_1}{t} + \frac{5}{18}$$

12.50 problem 58 (a)

Internal problem ID [14625]

Book: INTRODUCTORY DIFFERENTIAL EQUATIONS. Martha L. Abell, James P. Braselton. Fourth edition 2014. ElScAe. 2014
Section: Chapter 4. Higher Order Equations. Exercises 4.4, page 163
Problem number: 58 (a).
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$\boxed{4y'' + 4y' + y = \mathrm{e}^{-\frac{t}{2}}}$$

With initial conditions

$$[y(0) = a, y'(0) = b]$$

✓ Solution by Maple
Time used: 0.031 (sec). Leaf size: 26

```
dsolve([4*diff(y(t),t$2)+4*diff(y(t),t)+y(t)=exp(-t/2),y(0) = a, D(y)(0) = b],y(t), singsol=
```

$$y(t) = \frac{\left(\frac{t^2}{4} + t(a+2b) + 2a\right) \mathrm{e}^{-\frac{t}{2}}}{2}$$

✓ Solution by Mathematica
Time used: 0.031 (sec). Leaf size: 30

```
DSolve[{4*y''[t]+4*y'[t]+y[t]==Exp[-t/2],{y[0]==a,y'[0]==b}},y[t],t,IncludeSingularSolutions
```

$$y(t) \to \frac{1}{8} e^{-t/2} (4a(t+2) + t(8b+t))$$

12.51 problem 59

Internal problem ID [14626]

Book: INTRODUCTORY DIFFERENTIAL EQUATIONS. Martha L. Abell, James P. Braselton. Fourth edition 2014. ElScAe. 2014
Section: Chapter 4. Higher Order Equations. Exercises 4.4, page 163
Problem number: 59.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[NONE]`

$$\boxed{-2t(1+t)\,y = -\mathrm{e}^{-2t}\left(yy'' - y'^2\right)}$$

✗ Solution by Maple

```
dsolve(exp(-2*t)*(y(t)*diff(y(t),t$2)-diff(y(t),t)^2)-2*t*(1+t)*y(t)=0,y(t), singsol=all)
```

No solution found

✗ Solution by Mathematica
Time used: 0.0 (sec). Leaf size: 0

```
DSolve[Exp[-2*t]*(y[t]*y''[t]-y'[t]^2)-2*t*(1+t)*y[t]==0,y[t],t,IncludeSingularSolutions ->
```

Not solved

12.52 problem 60

Internal problem ID [14627]

Book: INTRODUCTORY DIFFERENTIAL EQUATIONS. Martha L. Abell, James P. Braselton. Fourth edition 2014. ElScAe. 2014
Section: Chapter 4. Higher Order Equations. Exercises 4.4, page 163
Problem number: 60.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _linear, _nonhomogeneous]]`

$$\boxed{y'' + 4y = f(t)}$$

With initial conditions

$$[y(0) = 0, y'(0) = 2]$$

✓ Solution by Maple

Time used: 0.093 (sec). Leaf size: 47

```
dsolve([diff(y(t),t$2)+4*y(t)=f(t),y(0) = 0, D(y)(0) = 2],y(t), singsol=all)
```

$$y(t) = \sin(2t) + \frac{\left(\int_0^t \cos(2\_z1)\, f(\_z1)\, d\_z1\right)\sin(2t)}{2} - \frac{\left(\int_0^t \sin(2\_z1)\, f(\_z1)\, d\_z1\right)\cos(2t)}{2}$$

✓ Solution by Mathematica

Time used: 0.089 (sec). Leaf size: 106

```
DSolve[{y''[t]+4*y[t]==f[t],{y[0]==0,y'[0]==2}},y[t],t,IncludeSingularSolutions -> True]
```

$$y(t) \to \sin(2t)\left(-\int_1^0 \frac{1}{2}\cos(2K[2])f(K[2])dK[2]\right) + \sin(2t)\int_1^t \frac{1}{2}\cos(2K[2])f(K[2])dK[2] - \cos(2t)\int_1^0 -\cos(K[1])f(K[1])\sin(K[1])dK[1] + \cos(2t)\int_1^t -\cos(K[1])f(K[1])\sin(K[1])dK[1] + \sin(2t)$$

12.53 problem 61

Internal problem ID [14628]

Book: INTRODUCTORY DIFFERENTIAL EQUATIONS. Martha L. Abell, James P. Braselton. Fourth edition 2014. ElScAe. 2014
Section: Chapter 4. Higher Order Equations. Exercises 4.4, page 163
Problem number: 61.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$\boxed{t^2y'' - 4y't + (t^2+6)\,y = 0}$$

Given that one solution of the ode is

$$y_1 = \cos(t)\,t^2$$

✓ Solution by Maple
Time used: 0.0 (sec). Leaf size: 17

```
dsolve([t^2*diff(y(t),t$2)-4*t*diff(y(t),t)+(t^2+6)*y(t)=0,t^2*cos(t)],singsol=all)
```

$$y(t) = t^2(c_1\sin(t) + c_2\cos(t))$$

✓ Solution by Mathematica
Time used: 0.053 (sec). Leaf size: 37

```
DSolve[t^2*y''[t]-4*t*y'[t]+(t^2+6)*y[t]==0,y[t],t,IncludeSingularSolutions -> True]
```

$$y(t) \to \frac{1}{2}e^{-it}t^2\left(2c_1 - ic_2e^{2it}\right)$$

12.54 problem 61 (a)

Internal problem ID [14629]

Book: INTRODUCTORY DIFFERENTIAL EQUATIONS. Martha L. Abell, James P. Braselton. Fourth edition 2014. ElScAe. 2014
Section: Chapter 4. Higher Order Equations. Exercises 4.4, page 163
Problem number: 61 (a).
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$\boxed{t^2y'' - 4y't + (t^2 + 6)\, y = t^3 + 2t}$$

With initial conditions

$$[y(0) = 0, y'(0) = 1]$$

✓ Solution by Maple
Time used: 0.031 (sec). Leaf size: 18

```
dsolve([t^2*diff(y(t),t$2)-4*t*diff(y(t),t)+(t^2+6)*y(t)=t^3+2*t,y(0) = 0, D(y)(0) = 1],y(t)
```

$$y(t) = t(\sin (t)\, tc_2 + t\cos (t)\, c_1 + 1)$$

✓ Solution by Mathematica
Time used: 0.22 (sec). Leaf size: 37

```
DSolve[{t^2*y''[t]-4*t*y'[t]+(t^2+6)*y[t]==t^3+2*t,{y[0]==0,y'[0]==1}},y[t],t,IncludeSingula
```

$$y(t) \to t + t^2\left(c_1e^{-it} - \frac{1}{2}ic_2e^{it}\right)$$

12.55 problem 62

Internal problem ID [14630]

Book: INTRODUCTORY DIFFERENTIAL EQUATIONS. Martha L. Abell, James P. Braselton. Fourth edition 2014. ElScAe. 2014
Section: Chapter 4. Higher Order Equations. Exercises 4.4, page 163
Problem number: 62.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[_Lienard]`

$$\boxed{ty'' + 2y' + ty = 0}$$

Given that one solution of the ode is

$$y_1 = \frac{\cos(t)}{t}$$

✓ Solution by Maple
Time used: 0.015 (sec). Leaf size: 17

```
dsolve([t*diff(y(t),t$2)+2*diff(y(t),t)+t*y(t)=0,1/t*cos(t)],singsol=all)
```

$$y(t) = \frac{c_1 \sin(t) + c_2 \cos(t)}{t}$$

✓ Solution by Mathematica
Time used: 0.023 (sec). Leaf size: 37

```
DSolve[t*y''[t]+2*y'[t]+t*y[t]==0,y[t],t,IncludeSingularSolutions -> True]
```

$$y(t) \to \frac{2c_1 e^{-it} - ic_2 e^{it}}{2t}$$

12.56 problem 62 (a)

Internal problem ID [14631]

Book: INTRODUCTORY DIFFERENTIAL EQUATIONS. Martha L. Abell, James P. Braselton. Fourth edition 2014. ElScAe. 2014
Section: Chapter 4. Higher Order Equations. Exercises 4.4, page 163
Problem number: 62 (a).
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$\boxed{ty'' + 2y' + ty = -t}$$

With initial conditions

$$\left[y(\pi) = -1, y'(\pi) = -\frac{1}{\pi}\right]$$

✓ Solution by Maple
Time used: 0.031 (sec). Leaf size: 12

```
dsolve([t*diff(y(t),t$2)+2*diff(y(t),t)+t*y(t)=-t,y(Pi) = -1, D(y)(Pi) = -1/Pi],y(t), singso
```

$$y(t) = \frac{-t + \sin(t)}{t}$$

✓ Solution by Mathematica
Time used: 0.025 (sec). Leaf size: 36

```
DSolve[{t*y''[t]+2*y'[t]+t*y[t]==-t,{y[Pi]==-1,y'[Pi]==-1/Pi}},y[t],t,IncludeSingularSolutio
```

$$y(t) \to \frac{i(2it + e^{-it} - e^{it})}{2t}$$

12.57 problem 63

Internal problem ID [14632]

Book: INTRODUCTORY DIFFERENTIAL EQUATIONS. Martha L. Abell, James P. Braselton. Fourth edition 2014. ElScAe. 2014
Section: Chapter 4. Higher Order Equations. Exercises 4.4, page 163
Problem number: 63.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$\boxed{4t^2y'' + 4y't + (16t^2 - 1)\, y = 0}$$

Given that one solution of the ode is

$$y_1 = \frac{\sin(2t)}{\sqrt{t}}$$

✓ Solution by Maple
Time used: 0.063 (sec). Leaf size: 21

```
dsolve([4*t^2*diff(y(t),t$2)+4*t*diff(y(t),t)+(16*t^2-1)*y(t)=0,1/sqrt(t)*sin(2*t)],singsol=
```

$$y(t) = \frac{\sin(2t)\, c_1 + c_2 \cos(2t)}{\sqrt{t}}$$

✓ Solution by Mathematica
Time used: 0.04 (sec). Leaf size: 39

```
DSolve[4*t^2*y''[t]+4*t*y'[t]+(16*t^2-1)*y[t]==0,y[t],t,IncludeSingularSolutions -> True]
```

$$y(t) \to \frac{e^{-2it}(4c_1 - ic_2 e^{4it})}{4\sqrt{t}}$$

12.58 problem 63 (b)

Internal problem ID [14633]

Book: INTRODUCTORY DIFFERENTIAL EQUATIONS. Martha L. Abell, James P. Braselton. Fourth edition 2014. ElScAe. 2014
Section: Chapter 4. Higher Order Equations. Exercises 4.4, page 163
Problem number: 63 (b).
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _linear, _nonhomogeneous]]`

$$\boxed{4t^2y'' + 4y't + (16t^2 - 1)\, y = 16t^{\frac{3}{2}}}$$

With initial conditions

$$\left[y(\pi) = 0, y\left(\frac{3\pi}{2}\right) = 0\right]$$

✗ Solution by Maple

```
dsolve([4*t^2*diff(y(t),t$2)+4*t*diff(y(t),t)+(16*t^2-1)*y(t)=16*t^(3/2),y(Pi) = 0, y(3/2*Pi
```

No solution found

✗ Solution by Mathematica

Time used: 0.0 (sec). Leaf size: 0

```
DSolve[{4*t^2*y''[t]+4*t*y'[t]+(16*t^2-1)*y[t]==16*t^(3/2),{y[Pi]==0,y[3*Pi/2]==0}},y[t],t,I
```

{}

12.59 problem 63 (c)

Internal problem ID [14634]

Book: INTRODUCTORY DIFFERENTIAL EQUATIONS. Martha L. Abell, James P. Braselton. Fourth edition 2014. ElScAe. 2014
Section: Chapter 4. Higher Order Equations. Exercises 4.4, page 163
Problem number: 63 (c).
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _linear, _nonhomogeneous]]`

$$\boxed{4t^2y'' + 4y't + \left(16t^2 - 1\right)y = 16t^{\frac{3}{2}}}$$

With initial conditions

$$[y(\pi) = 0, y'(2\pi) = 0]$$

✓ Solution by Maple
Time used: 0.094 (sec). Leaf size: 15

```
dsolve([4*t^2*diff(y(t),t$2)+4*t*diff(y(t),t)+(16*t^2-1)*y(t)=16*t^(3/2),y(Pi) = 0, D(y)(2*P
```

$$y(t) = -\frac{-1 + \cos(2t)}{\sqrt{t}}$$

✓ Solution by Mathematica
Time used: 0.06 (sec). Leaf size: 32

```
DSolve[{4*t^2*y''[t]+4*t*y'[t]+(16*t^2-1)*y[t]==16*t^(3/2),{y[Pi]==0,y'[2*Pi]==0}},y[t],t,In
```

$$y(t) \to -\frac{e^{-2it}(-1 + e^{2it})^2}{2\sqrt{t}}$$

12.60 problem 64

Internal problem ID [14635]

Book: INTRODUCTORY DIFFERENTIAL EQUATIONS. Martha L. Abell, James P. Braselton. Fourth edition 2014. ElScAe. 2014
Section: Chapter 4. Higher Order Equations. Exercises 4.4, page 163
Problem number: 64.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$\boxed{t^2(-1+\ln(t))\, y''-y't+y=-\frac{3(\ln(t)+1)}{4\sqrt{t}}}$$

With initial conditions

$$[y(1)=0, y'(1)=0]$$

✓ Solution by Maple

Time used: 0.078 (sec). Leaf size: 20

```
dsolve([t^2*(ln(t)-1)*diff(y(t),t$2)-t*diff(y(t),t)+y(t)=-3/4*(1+ln(t))*1/sqrt(t),y(1) = 0,
```

$$y(t)=\left(-\frac{1}{t^{\frac{3}{2}}}-\frac{3\ln(t)}{2t}+1\right)t$$

✓ Solution by Mathematica

Time used: 0.42 (sec). Leaf size: 20

```
DSolve[{t^2*(Log[t]-1)*y''[t]-t*y'[t]+y[t]==-3/4*(1+Log[t])*1/Sqrt[t],{y[1]==0,y'[1]==0}},y[
```

$$y(t)\to t-\frac{1}{\sqrt{t}}-\frac{3\log(t)}{2}$$

12.61 problem 65

Internal problem ID [14636]

Book: INTRODUCTORY DIFFERENTIAL EQUATIONS. Martha L. Abell, James P. Braselton. Fourth edition 2014. ElScAe. 2014
Section: Chapter 4. Higher Order Equations. Exercises 4.4, page 163
Problem number: 65.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$\boxed{(-\cos(t)\,t+\sin(t))\,y''-t\sin(t)\,y'+\sin(t)\,y=t}$$

With initial conditions

$$\left[y\left(\frac{\pi}{4}\right)=0, y'\left(\frac{\pi}{4}\right)=0\right]$$

✓ Solution by Maple
Time used: 0.156 (sec). Leaf size: 29

```
dsolve([(sin(t)-t*cos(t))*diff(y(t),t$2)-t*sin(t)*diff(y(t),t)+sin(t)*y(t)=t,y(1/4*Pi) = 0,
```

$$y(t)=\frac{-4\sqrt{2}\,t+(-4+\pi)\cos(t)+(\pi+4)\sin(t)}{-4+\pi}$$

✓ Solution by Mathematica
Time used: 1.047 (sec). Leaf size: 32

```
DSolve[{(Sin[t]-t*Cos[t])*y''[t]-t*Sin[t]*y'[t]+Sin[t]*y[t]==t,{y[Pi/4]==0,y'[Pi/4]==0}},y[t
```

$$y(t)\to\frac{-4\sqrt{2}t+(4+\pi)\sin(t)+(\pi-4)\cos(t)}{\pi-4}$$

13 Chapter 4. Higher Order Equations. Exercises 4.5, page 175

13.1 problem 17

Internal problem ID [14637]

Book: INTRODUCTORY DIFFERENTIAL EQUATIONS. Martha L. Abell, James P. Braselton. Fourth edition 2014. ElScAe. 2014
Section: Chapter 4. Higher Order Equations. Exercises 4.5, page 175
Problem number: 17.
ODE order: 3.
ODE degree: 1.

CAS Maple gives this as type `[[_3rd_order, _quadrature]]`

$$\boxed{y''' = 0}$$

✓ Solution by Maple
Time used: 0.0 (sec). Leaf size: 15

```
dsolve(diff(y(t),t$3)=0,y(t), singsol=all)
```

$$y(t) = \frac{1}{2}c_1 t^2 + tc_2 + c_3$$

✓ Solution by Mathematica
Time used: 0.035 (sec). Leaf size: 17

```
DSolve[y'''[t]==0,y[t],t,IncludeSingularSolutions -> True]
```

$$y(t) \to t(c_3 t + c_2) + c_1$$

13.2 problem 19

Internal problem ID [14638]

Book: INTRODUCTORY DIFFERENTIAL EQUATIONS. Martha L. Abell, James P. Braselton. Fourth edition 2014. ElScAe. 2014
Section: Chapter 4. Higher Order Equations. Exercises 4.5, page 175
Problem number: 19.
ODE order: 3.
ODE degree: 1.

CAS Maple gives this as type `[[_3rd_order, _missing_x]]`

$$\boxed{y''' - 10y'' + 25y' = 0}$$

✓ Solution by Maple
Time used: 0.0 (sec). Leaf size: 16

```
dsolve(diff(y(t),t$3)-10*diff(y(t),t$2)+25*diff(y(t),t)=0,y(t), singsol=all)
```

$$y(t) = (c_3 t + c_2) e^{5t} + c_1$$

✓ Solution by Mathematica
Time used: 0.071 (sec). Leaf size: 30

```
DSolve[y'''[t]-10*y''[t]+25*y'[t]==0,y[t],t,IncludeSingularSolutions -> True]
```

$$y(t) \to \frac{1}{25} e^{5t}(c_2(5t - 1) + 5c_1) + c_3$$

13.3 problem 20

Internal problem ID [14639]

Book: INTRODUCTORY DIFFERENTIAL EQUATIONS. Martha L. Abell, James P. Braselton. Fourth edition 2014. ElScAe. 2014
Section: Chapter 4. Higher Order Equations. Exercises 4.5, page 175
Problem number: 20.
ODE order: 3.
ODE degree: 1.

CAS Maple gives this as type `[[_3rd_order, _missing_x]]`

$$\boxed{8y''' + y'' = 0}$$

✓ Solution by Maple
Time used: 0.0 (sec). Leaf size: 15

```
dsolve(8*diff(y(t),t$3)+diff(y(t),t$2)=0,y(t), singsol=all)
```

$$y(t) = c_1 + tc_2 + c_3 e^{-\frac{t}{8}}$$

✓ Solution by Mathematica
Time used: 0.059 (sec). Leaf size: 23

```
DSolve[8*y'''[t]+y''[t]==0,y[t],t,IncludeSingularSolutions -> True]
```

$$y(t) \to 64c_1 e^{-t/8} + c_3 t + c_2$$

13.4 problem 21

Internal problem ID [14640]

Book: INTRODUCTORY DIFFERENTIAL EQUATIONS. Martha L. Abell, James P. Braselton. Fourth edition 2014. ElScAe. 2014
Section: Chapter 4. Higher Order Equations. Exercises 4.5, page 175
Problem number: 21.
ODE order: 4.
ODE degree: 1.

CAS Maple gives this as type `[[_high_order, _missing_x]]`

$$\boxed{y'''' + 16y'' = 0}$$

✓ Solution by Maple
Time used: 0.0 (sec). Leaf size: 21

```
dsolve(diff(y(t),t$4)+16*diff(y(t),t$2)=0,y(t), singsol=all)
```

$$y(t) = c_1 + tc_2 + c_3 \sin(4t) + c_4 \cos(4t)$$

✓ Solution by Mathematica
Time used: 0.17 (sec). Leaf size: 32

```
DSolve[y''''[t]+16*y''[t]==0,y[t],t,IncludeSingularSolutions -> True]
```

$$y(t) \to c_4 t - \frac{1}{16} c_1 \cos(4t) - \frac{1}{16} c_2 \sin(4t) + c_3$$

13.5 problem 22

Internal problem ID [14641]

Book: INTRODUCTORY DIFFERENTIAL EQUATIONS. Martha L. Abell, James P. Braselton. Fourth edition 2014. ElScAe. 2014
Section: Chapter 4. Higher Order Equations. Exercises 4.5, page 175
Problem number: 22.
ODE order: 3.
ODE degree: 1.

CAS Maple gives this as type `[[_3rd_order, _missing_x]]`

$$\boxed{y''' - 2y'' - y' + 2y = 0}$$

✓ Solution by Maple
Time used: 0.0 (sec). Leaf size: 21

```
dsolve(diff(y(t),t$3)-2*diff(y(t),t$2)-diff(y(t),t)+2*y(t)=0,y(t), singsol=all)
```

$$y(t) = \mathrm{e}^{-t} c_1 + c_2 \mathrm{e}^{t} + c_3 \mathrm{e}^{2t}$$

✓ Solution by Mathematica
Time used: 0.009 (sec). Leaf size: 28

```
DSolve[y'''[t]-2*y''[t]-y'[t]+2*y[t]==0,y[t],t,IncludeSingularSolutions -> True]
```

$$y(t) \to c_1 e^{-t} + c_2 e^{t} + c_3 e^{2t}$$

13.6 problem 23

Internal problem ID [14642]

Book: INTRODUCTORY DIFFERENTIAL EQUATIONS. Martha L. Abell, James P. Braselton. Fourth edition 2014. ElScAe. 2014
Section: Chapter 4. Higher Order Equations. Exercises 4.5, page 175
Problem number: 23.
ODE order: 3.
ODE degree: 1.

CAS Maple gives this as type `[[_3rd_order, _missing_x]]`

$$\boxed{3y''' - 4y'' - 5y' + 2y = 0}$$

✓ Solution by Maple
Time used: 0.0 (sec). Leaf size: 23

```
dsolve(3*diff(y(t),t$3)-4*diff(y(t),t$2)-5*diff(y(t),t)+2*y(t)=0,y(t), singsol=all)
```

$$y(t) = \left(c_3 e^{3t} + e^{\frac{4t}{3}} c_1 + c_2\right) e^{-t}$$

✓ Solution by Mathematica
Time used: 0.003 (sec). Leaf size: 32

```
DSolve[3*y'''[t]-4*y''[t]-5*y'[t]+2*y[t]==0,y[t],t,IncludeSingularSolutions -> True]
```

$$y(t) \to e^{-t}\left(c_1 e^{4t/3} + c_3 e^{3t} + c_2\right)$$

13.7 problem 24

Internal problem ID [14643]

Book: INTRODUCTORY DIFFERENTIAL EQUATIONS. Martha L. Abell, James P. Braselton. Fourth edition 2014. ElScAe. 2014
Section: Chapter 4. Higher Order Equations. Exercises 4.5, page 175
Problem number: 24.
ODE order: 3.
ODE degree: 1.

CAS Maple gives this as type `[[_3rd_order, _missing_x]]`

$$\boxed{6y''' - 5y'' - 2y' + y = 0}$$

✓ Solution by Maple
Time used: 0.0 (sec). Leaf size: 23

```
dsolve(6*diff(y(t),t$3)-5*diff(y(t),t$2)-2*diff(y(t),t)+y(t)=0,y(t), singsol=all)
```

$$y(t) = \left(c_3 e^{\frac{3t}{2}} + c_2 e^{\frac{5t}{6}} + c_1\right) e^{-\frac{t}{2}}$$

✓ Solution by Mathematica
Time used: 0.003 (sec). Leaf size: 32

```
DSolve[6*y'''[t]-5*y''[t]-2*y'[t]+y[t]==0,y[t],t,IncludeSingularSolutions -> True]
```

$$y(t) \to c_1 e^{t/3} + c_2 e^{-t/2} + c_3 e^t$$

13.8 problem 25

Internal problem ID [14644]

Book: INTRODUCTORY DIFFERENTIAL EQUATIONS. Martha L. Abell, James P. Braselton. Fourth edition 2014. ElScAe. 2014
Section: Chapter 4. Higher Order Equations. Exercises 4.5, page 175
Problem number: 25.
ODE order: 3.
ODE degree: 1.

CAS Maple gives this as type `[[_3rd_order, _missing_x]]`

$$\boxed{y''' - 5y' + 2y = 0}$$

✓ Solution by Maple
Time used: 0.0 (sec). Leaf size: 32

```
dsolve(diff(y(t),t$3)-5*diff(y(t),t)+2*y(t)=0,y(t), singsol=all)
```

$$y(t) = c_1 e^{2t} + c_2 e^{(\sqrt{2}-1)t} + c_3 e^{-(1+\sqrt{2})t}$$

✓ Solution by Mathematica
Time used: 0.003 (sec). Leaf size: 43

```
DSolve[y'''[t]-5*y'[t]+2*y[t]==0,y[t],t,IncludeSingularSolutions -> True]
```

$$y(t) \to c_1 e^{-((1+\sqrt{2})t)} + c_2 e^{(\sqrt{2}-1)t} + c_3 e^{2t}$$

13.9 problem 26

Internal problem ID [14645]

Book: INTRODUCTORY DIFFERENTIAL EQUATIONS. Martha L. Abell, James P. Braselton. Fourth edition 2014. ElScAe. 2014
Section: Chapter 4. Higher Order Equations. Exercises 4.5, page 175
Problem number: 26.
ODE order: 3.
ODE degree: 1.

CAS Maple gives this as type `[[_3rd_order, _missing_x]]`

$$\boxed{5y''' - 15y' + 11y = 0}$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 149

```
dsolve(5*diff(y(t),t$3)-15*diff(y(t),t)+11*y(t)=0,y(t), singsol=all)
```

$$y(t) = \left(c_2 \mathrm{e}^{\frac{3\left(\left(1100+100\sqrt{21}\right)^{\frac{2}{3}}+100\right)t}{20\left(1100+100\sqrt{21}\right)^{\frac{1}{3}}}} \sin\left(\frac{\sqrt{3}\left(\left(1100+100\sqrt{3}\,\sqrt{7}\right)^{\frac{2}{3}}-100\right)t}{20\left(1100+100\sqrt{3}\,\sqrt{7}\right)^{\frac{1}{3}}}\right) + c_3 \mathrm{e}^{\frac{3\left(\left(1100+100\sqrt{21}\right)^{\frac{2}{3}}+100\right)t}{20\left(1100+100\sqrt{21}\right)^{\frac{1}{3}}}} \cos\left(\frac{\sqrt{3}\left(\left(1100+100\sqrt{3}\,\sqrt{7}\right)^{\frac{2}{3}}-100\right)t}{20\left(1100+100\sqrt{3}\,\sqrt{7}\right)^{\frac{1}{3}}}\right) + c_1\right) \mathrm{e}^{-\frac{\left(\left(1100+100\sqrt{21}\right)^{\frac{2}{3}}+100\right)t}{10\left(1100+100\sqrt{21}\right)^{\frac{1}{3}}}}$$

✓ Solution by Mathematica

Time used: 0.002 (sec). Leaf size: 75

```
DSolve[5*y'''[t]-15*y'[t]+11*y[t]==0,y[t],t,IncludeSingularSolutions -> True]
```

$$y(t) \to c_2 \exp\left(t\text{Root}\left[5\#1^3 - 15\#1 + 11\&, 2\right]\right) + c_3 \exp\left(t\text{Root}\left[5\#1^3 - 15\#1 + 11\&, 3\right]\right) + c_1 \exp\left(t\text{Root}\left[5\#1^3 - 15\#1 + 11\&, 1\right]\right)$$

13.10 problem 27

Internal problem ID [14646]

Book: INTRODUCTORY DIFFERENTIAL EQUATIONS. Martha L. Abell, James P. Braselton. Fourth edition 2014. ElScAe. 2014
Section: Chapter 4. Higher Order Equations. Exercises 4.5, page 175
Problem number: 27.
ODE order: 4.
ODE degree: 1.

CAS Maple gives this as type `[[_high_order, _missing_x]]`

$$\boxed{y'''' + y''' = 0}$$

✓ Solution by Maple
Time used: 0.0 (sec). Leaf size: 20

```
dsolve(diff(y(t),t$4)+diff(y(t),t$3)=0,y(t), singsol=all)
```

$$y(t) = c_1 + tc_2 + c_3t^2 + c_4\mathrm{e}^{-t}$$

✓ Solution by Mathematica
Time used: 0.032 (sec). Leaf size: 26

```
DSolve[y''''[t]+y'''[t]==0,y[t],t,IncludeSingularSolutions -> True]
```

$$y(t) \to c_1\left(-e^{-t}\right) + t(c_4t + c_3) + c_2$$

13.11 problem 28

Internal problem ID [14647]

Book: INTRODUCTORY DIFFERENTIAL EQUATIONS. Martha L. Abell, James P. Braselton. Fourth edition 2014. ElScAe. 2014
Section: Chapter 4. Higher Order Equations. Exercises 4.5, page 175
Problem number: 28.
ODE order: 4.
ODE degree: 1.

CAS Maple gives this as type `[[_high_order, _missing_x]]`

$$\boxed{y'''' - 9y'' = 0}$$

✓ Solution by Maple
Time used: 0.0 (sec). Leaf size: 21

```
dsolve(diff(y(t),t$4)-9*diff(y(t),t$2)=0,y(t), singsol=all)
```

$$y(t) = c_1 + tc_2 + c_3 \mathrm{e}^{-3t} + c_4 \mathrm{e}^{3t}$$

✓ Solution by Mathematica
Time used: 0.036 (sec). Leaf size: 32

```
DSolve[y''''[t]-9*y''[t]==0,y[t],t,IncludeSingularSolutions -> True]
```

$$y(t) \to \frac{1}{9} e^{-3t} \left(c_1 e^{6t} + c_2\right) + c_4 t + c_3$$

13.12 problem 29

Internal problem ID [14648]

Book: INTRODUCTORY DIFFERENTIAL EQUATIONS. Martha L. Abell, James P. Braselton. Fourth edition 2014. ElScAe. 2014
Section: Chapter 4. Higher Order Equations. Exercises 4.5, page 175
Problem number: 29.
ODE order: 4.
ODE degree: 1.

CAS Maple gives this as type `[[_high_order, _missing_x]]`

$$\boxed{y'''' - 16y = 0}$$

✓ Solution by Maple
Time used: 0.015 (sec). Leaf size: 29

```
dsolve(diff(y(t),t$4)-16*y(t)=0,y(t), singsol=all)
```

$$y(t) = c_1 \mathrm{e}^{2t} + c_2 \mathrm{e}^{-2t} + c_3 \sin(2t) + c_4 \cos(2t)$$

✓ Solution by Mathematica
Time used: 0.002 (sec). Leaf size: 36

```
DSolve[y''''[t]-16*y[t]==0,y[t],t,IncludeSingularSolutions -> True]
```

$$y(t) \to c_1 e^{2t} + c_3 e^{-2t} + c_2 \cos(2t) + c_4 \sin(2t)$$

13.13 problem 30

Internal problem ID [14649]

Book: INTRODUCTORY DIFFERENTIAL EQUATIONS. Martha L. Abell, James P. Braselton. Fourth edition 2014. ElScAe. 2014
Section: Chapter 4. Higher Order Equations. Exercises 4.5, page 175
Problem number: 30.
ODE order: 4.
ODE degree: 1.

CAS Maple gives this as type `[[_high_order, _missing_x]]`

$$\boxed{y'''' - 6y''' - y'' + 54y' - 72y = 0}$$

✓ Solution by Maple

Time used: 0.016 (sec). Leaf size: 29

```
dsolve(diff(y(t),t$4)-6*diff(y(t),t$3)-diff(y(t),t$2)+54*diff(y(t),t)-72*y(t)=0,y(t), singso
```

$$y(t) = \left(e^{7t}c_2 + c_3e^{6t} + c_4e^{5t} + c_1\right) e^{-3t}$$

✓ Solution by Mathematica

Time used: 0.003 (sec). Leaf size: 36

```
DSolve[y''''[t]-6*y'''[t]-y''[t]+54*y'[t]-72*y[t]==0,y[t],t,IncludeSingularSolutions -> True
```

$$y(t) \to e^{-3t}\left(e^{5t}\left(e^t\left(c_4e^t + c_3\right) + c_2\right) + c_1\right)$$

13.14 problem 31

Internal problem ID [14650]

Book: INTRODUCTORY DIFFERENTIAL EQUATIONS. Martha L. Abell, James P. Braselton. Fourth edition 2014. ElScAe. 2014
Section: Chapter 4. Higher Order Equations. Exercises 4.5, page 175
Problem number: 31.
ODE order: 4.
ODE degree: 1.

CAS Maple gives this as type `[[_high_order, _missing_x]]`

$$\boxed{y'''' + 7y''' + 6y'' - 32y' - 32y = 0}$$

✓ Solution by Maple
Time used: 0.0 (sec). Leaf size: 26

```
dsolve(diff(y(t),t$4)+7*diff(y(t),t$3)+6*diff(y(t),t$2)-32*diff(y(t),t)-32*y(t)=0,y(t), sing
```

$$y(t) = \left(c_2 \mathrm{e}^{6t} + c_1 \mathrm{e}^{3t} + t c_4 + c_3\right) \mathrm{e}^{-4t}$$

✓ Solution by Mathematica
Time used: 0.003 (sec). Leaf size: 34

```
DSolve[y''''[t]+7*y'''[t]+6*y''[t]-32*y'[t]-32*y[t]==0,y[t],t,IncludeSingularSolutions -> Tr
```

$$y(t) \to e^{-4t}\left(c_2 t + c_3 e^{3t} + c_4 e^{6t} + c_1\right)$$

13.15 problem 32

Internal problem ID [14651]

Book: INTRODUCTORY DIFFERENTIAL EQUATIONS. Martha L. Abell, James P. Braselton. Fourth edition 2014. ElScAe. 2014
Section: Chapter 4. Higher Order Equations. Exercises 4.5, page 175
Problem number: 32.
ODE order: 4.
ODE degree: 1.

CAS Maple gives this as type `[[_high_order, _missing_x]]`

$$\boxed{y'''' + 2y''' - 2y'' + 8y = 0}$$

✓ Solution by Maple
Time used: 0.016 (sec). Leaf size: 30

```
dsolve(diff(y(t),t$4)+2*diff(y(t),t$3)-2*diff(y(t),t$2)+8*y(t)=0,y(t), singsol=all)
```

$$y(t) = \left(c_3 e^{3t} \sin(t) + c_4 e^{3t} \cos(t) + t c_2 + c_1\right) e^{-2t}$$

✓ Solution by Mathematica
Time used: 0.003 (sec). Leaf size: 38

```
DSolve[y''''[t]+2*y'''[t]-2*y''[t]+8*y[t]==0,y[t],t,IncludeSingularSolutions -> True]
```

$$y(t) \to e^{-2t}\left(c_4 t + c_2 e^{3t} \cos(t) + c_1 e^{3t} \sin(t) + c_3\right)$$

13.16 problem 33

Internal problem ID [14652]

Book: INTRODUCTORY DIFFERENTIAL EQUATIONS. Martha L. Abell, James P. Braselton. Fourth edition 2014. ElScAe. 2014
Section: Chapter 4. Higher Order Equations. Exercises 4.5, page 175
Problem number: 33.
ODE order: 5.
ODE degree: 1.

CAS Maple gives this as type `[[_high_order, _missing_x]]`

$$\boxed{y^{(5)} + 4y'''' = 0}$$

✓ Solution by Maple
Time used: 0.0 (sec). Leaf size: 25

```
dsolve(diff(y(t),t$5)+4*diff(y(t),t$4)=0,y(t), singsol=all)
```

$$y(t) = c_1 + tc_2 + c_3t^2 + c_4t^3 + c_5\mathrm{e}^{-4t}$$

✓ Solution by Mathematica
Time used: 0.04 (sec). Leaf size: 33

```
DSolve[y'''''[t]+4*y''''[t]==0,y[t],t,IncludeSingularSolutions -> True]
```

$$y(t) \to \frac{1}{256}c_1e^{-4t} + t(t(c_5t + c_4) + c_3) + c_2$$

13.17 problem 34

Internal problem ID [14653]

Book: INTRODUCTORY DIFFERENTIAL EQUATIONS. Martha L. Abell, James P. Braselton. Fourth edition 2014. ElScAe. 2014
Section: Chapter 4. Higher Order Equations. Exercises 4.5, page 175
Problem number: 34.
ODE order: 5.
ODE degree: 1.

CAS Maple gives this as type `[[_high_order, _missing_x]]`

$$\boxed{y^{(5)} + 4y''' = 0}$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 26

```
dsolve(diff(y(t),t$5)+4*diff(y(t),t$3)=0,y(t), singsol=all)
```

$$y(t) = c_1 + tc_2 + c_3t^2 + c_4 \sin(2t) + c_5 \cos(2t)$$

✓ Solution by Mathematica

Time used: 0.204 (sec). Leaf size: 38

```
DSolve[y'''''[t]+4*y'''[t]==0,y[t],t,IncludeSingularSolutions -> True]
```

$$y(t) \to c_5t^2 + c_4t + \frac{1}{8}c_2 \cos(2t) - \frac{1}{8}c_1 \sin(2t) + c_3$$

13.18 problem 35

Internal problem ID [14654]

Book: INTRODUCTORY DIFFERENTIAL EQUATIONS. Martha L. Abell, James P. Braselton. Fourth edition 2014. ElScAe. 2014
Section: Chapter 4. Higher Order Equations. Exercises 4.5, page 175
Problem number: 35.
ODE order: 5.
ODE degree: 1.

CAS Maple gives this as type `[[_high_order, _missing_x]]`

$$\boxed{y^{(5)} + 3y'''' + 3y''' + y'' = 0}$$

✓ Solution by Maple
Time used: 0.0 (sec). Leaf size: 24

```
dsolve(diff(y(t),t$5)+3*diff(y(t),t$4)+3*diff(y(t),t$3)+diff(y(t),t$2)=0,y(t), singsol=all)
```

$$y(t) = \left(c_5 t^2 + t c_4 + c_3\right) \mathrm{e}^{-t} + t c_2 + c_1$$

✓ Solution by Mathematica
Time used: 0.056 (sec). Leaf size: 38

```
DSolve[y'''''[t]+3*y''''[t]+3*y'''[t]+y''[t]==0,y[t],t,IncludeSingularSolutions -> True]
```

$$y(t) \to e^{-t}\left(c_3\left(t^2 + 4t + 6\right) + c_2(t + 2) + c_1\right) + c_5 t + c_4$$

13.19 problem 36

Internal problem ID [14655]

Book: INTRODUCTORY DIFFERENTIAL EQUATIONS. Martha L. Abell, James P. Braselton. Fourth edition 2014. ElScAe. 2014
Section: Chapter 4. Higher Order Equations. Exercises 4.5, page 175
Problem number: 36.
ODE order: 4.
ODE degree: 1.

CAS Maple gives this as type `[[_high_order, _missing_x]]`

$$\boxed{y'''' + 2y'' + y = 0}$$

✓ Solution by Maple
Time used: 0.0 (sec). Leaf size: 21

```
dsolve(diff(y(t),t$4)+2*diff(y(t),t$2)+y(t)=0,y(t), singsol=all)
```

$$y(t) = (tc_4 + c_2) \cos (t) + \sin (t) (c_3 t + c_1)$$

✓ Solution by Mathematica
Time used: 0.003 (sec). Leaf size: 26

```
DSolve[y''''[t]+2*y''[t]+y[t]==0,y[t],t,IncludeSingularSolutions -> True]
```

$$y(t) \to (c_2 t + c_1) \cos(t) + (c_4 t + c_3) \sin(t)$$

13.20 problem 37

Internal problem ID [14656]

Book: INTRODUCTORY DIFFERENTIAL EQUATIONS. Martha L. Abell, James P. Braselton. Fourth edition 2014. ElScAe. 2014
Section: Chapter 4. Higher Order Equations. Exercises 4.5, page 175
Problem number: 37.
ODE order: 4.
ODE degree: 1.

CAS Maple gives this as type `[[_high_order, _missing_x]]`

$$\boxed{y'''' + 8y'' + 16y = 0}$$

✓ Solution by Maple
Time used: 0.0 (sec). Leaf size: 25

```
dsolve(diff(y(t),t$4)+8*diff(y(t),t$2)+16*y(t)=0,y(t), singsol=all)
```

$$y(t) = (tc_4 + c_2)\cos(2t) + \sin(2t)(c_3t + c_1)$$

✓ Solution by Mathematica
Time used: 0.003 (sec). Leaf size: 30

```
DSolve[y''''[t]+8*y''[t]+16*y[t]==0,y[t],t,IncludeSingularSolutions -> True]
```

$$y(t) \to (c_2t + c_1)\cos(2t) + (c_4t + c_3)\sin(2t)$$

13.21 problem 38

Internal problem ID [14657]

Book: INTRODUCTORY DIFFERENTIAL EQUATIONS. Martha L. Abell, James P. Braselton. Fourth edition 2014. ElScAe. 2014
Section: Chapter 4. Higher Order Equations. Exercises 4.5, page 175
Problem number: 38.
ODE order: 6.
ODE degree: 1.

CAS Maple gives this as type `[[_high_order, _missing_x]]`

$$\boxed{y^{(6)} + 3y'''' + 3y'' + y = 0}$$

✓ Solution by Maple
Time used: 0.0 (sec). Leaf size: 31

```
dsolve(diff(y(t),t$6)+3*diff(y(t),t$4)+3*diff(y(t),t$2)+y(t)=0,y(t), singsol=all)
```

$$y(t) = \left(c_6 t^2 + t c_4 + c_2\right) \cos(t) + \sin(t) \left(c_5 t^2 + c_3 t + c_1\right)$$

✓ Solution by Mathematica
Time used: 0.003 (sec). Leaf size: 36

```
DSolve[y''''''[t]+3*y''''[t]+3*y''[t]+y[t]==0,y[t],t,IncludeSingularSolutions -> True]
```

$$y(t) \to (t(c_3 t + c_2) + c_1) \cos(t) + (t(c_6 t + c_5) + c_4) \sin(t)$$

13.22 problem 39

Internal problem ID [14658]

Book: INTRODUCTORY DIFFERENTIAL EQUATIONS. Martha L. Abell, James P. Braselton. Fourth edition 2014. ElScAe. 2014
Section: Chapter 4. Higher Order Equations. Exercises 4.5, page 175
Problem number: 39.
ODE order: 6.
ODE degree: 1.

CAS Maple gives this as type `[[_high_order, _missing_x]]`

$$\boxed{y^{(6)} + 12y'''' + 48y'' + 64y = 0}$$

✓ Solution by Maple
Time used: 0.015 (sec). Leaf size: 35

```
dsolve(diff(y(t),t$6)+12*diff(y(t),t$4)+48*diff(y(t),t$2)+64*y(t)=0,y(t), singsol=all)
```

$$y(t) = \left(c_6 t^2 + t c_4 + c_2\right) \cos\left(2t\right) + \sin\left(2t\right) \left(c_5 t^2 + c_3 t + c_1\right)$$

✓ Solution by Mathematica
Time used: 0.003 (sec). Leaf size: 40

```
DSolve[y''''''[t]+12*y''''[t]+48*y''[t]+64*y[t]==0,y[t],t,IncludeSingularSolutions -> True]
```

$$y(t) \to (t(c_3 t + c_2) + c_1) \cos(2t) + (t(c_6 t + c_5) + c_4) \sin(2t)$$

13.23 problem 40

Internal problem ID [14659]

Book: INTRODUCTORY DIFFERENTIAL EQUATIONS. Martha L. Abell, James P. Braselton. Fourth edition 2014. ElScAe. 2014
Section: Chapter 4. Higher Order Equations. Exercises 4.5, page 175
Problem number: 40.
ODE order: 3.
ODE degree: 1.

CAS Maple gives this as type `[[_3rd_order, _missing_x]]`

$$\boxed{y''' - 2y'' = 0}$$

With initial conditions

$$[y(0) = 1, y'(0) = 2, y''(0) = 0]$$

✓ Solution by Maple
Time used: 0.0 (sec). Leaf size: 9

```
dsolve([diff(y(t),t$3)-2*diff(y(t),t$2)=0,y(0) = 1, D(y)(0) = 2, (D@@2)(y)(0) = 0],y(t), sin
```

$$y(t) = 2t + 1$$

✓ Solution by Mathematica
Time used: 0.036 (sec). Leaf size: 10

```
DSolve[{y'''[t]-2*y''[t]==0,{y[0]==1,y'[0]==2,y''[0]==0}},y[t],t,IncludeSingularSolutions ->
```

$$y(t) \to 2t + 1$$

13.24 problem 41

Internal problem ID [14660]

Book: INTRODUCTORY DIFFERENTIAL EQUATIONS. Martha L. Abell, James P. Braselton. Fourth edition 2014. ElScAe. 2014
Section: Chapter 4. Higher Order Equations. Exercises 4.5, page 175
Problem number: 41.
ODE order: 3.
ODE degree: 1.

CAS Maple gives this as type `[[_3rd_order, _missing_x]]`

$$\boxed{y''' - y = 0}$$

With initial conditions

$$[y(0) = 0, y'(0) = 0, y''(0) = 3]$$

✓ Solution by Maple

Time used: 0.031 (sec). Leaf size: 36

```
dsolve([diff(y(t),t$3)-y(t)=0,y(0) = 0, D(y)(0) = 0, (D@@2)(y)(0) = 3],y(t), singsol=all)
```

$$y(t) = \mathrm{e}^{t} - \sqrt{3}\,\mathrm{e}^{-\frac{t}{2}} \sin\left(\frac{\sqrt{3}\,t}{2}\right) - \mathrm{e}^{-\frac{t}{2}} \cos\left(\frac{\sqrt{3}\,t}{2}\right)$$

✓ Solution by Mathematica

Time used: 0.004 (sec). Leaf size: 52

```
DSolve[{y'''[t]-y[t]==0,{y[0]==0,y'[0]==0,y''[0]==3}},y[t],t,IncludeSingularSolutions -> Tru
```

$$y(t) \to e^{-t/2}\left(e^{3t/2} - \sqrt{3}\sin\left(\frac{\sqrt{3}t}{2}\right) - \cos\left(\frac{\sqrt{3}t}{2}\right)\right)$$

13.25 problem 42

Internal problem ID [14661]

Book: INTRODUCTORY DIFFERENTIAL EQUATIONS. Martha L. Abell, James P. Braselton. Fourth edition 2014. ElScAe. 2014
Section: Chapter 4. Higher Order Equations. Exercises 4.5, page 175
Problem number: 42.
ODE order: 4.
ODE degree: 1.

CAS Maple gives this as type `[[_high_order, _missing_x]]`

$$\boxed{y'''' + 16y''' = 0}$$

With initial conditions

$$[y(0) = 0, y'(0) = 1, y''(0) = 0, y'''(0) = 1]$$

✓ Solution by Maple
Time used: 0.016 (sec). Leaf size: 20

```
dsolve([diff(y(t),t$4)+16*diff(y(t),t$3)=0,y(0) = 0, D(y)(0) = 1, (D@@2)(y)(0) = 0, (D@@3)(y
```

$$y(t) = \frac{1}{4096} + \frac{255t}{256} + \frac{t^2}{32} - \frac{e^{-16t}}{4096}$$

✓ Solution by Mathematica
Time used: 0.037 (sec). Leaf size: 26

```
DSolve[{y''''[t]+16*y'''[t]==0,{y[0]==0,y'[0]==1,y''[0]==0,y'''[0]==1}},y[t],t,IncludeSingul
```

$$y(t) \to \frac{128t^2 + 4080t - e^{-16t} + 1}{4096}$$

13.26 problem 43

Internal problem ID [14662]

Book: INTRODUCTORY DIFFERENTIAL EQUATIONS. Martha L. Abell, James P. Braselton. Fourth edition 2014. ElScAe. 2014
Section: Chapter 4. Higher Order Equations. Exercises 4.5, page 175
Problem number: 43.
ODE order: 4.
ODE degree: 1.

CAS Maple gives this as type `[[_high_order, _missing_x]]`

$$\boxed{y'''' - 8y'' + 16y = 0}$$

With initial conditions

$$[y(0) = 0, y'(0) = 0, y''(0) = 8, y'''(0) = 0]$$

✓ Solution by Maple

Time used: 0.016 (sec). Leaf size: 18

```
dsolve([diff(y(t),t$4)-8*diff(y(t),t$2)+16*y(t)=0,y(0) = 0, D(y)(0) = 0, (D@@2)(y)(0) = 8, (
```

$$y(t) = -t\left(-e^{2t} + e^{-2t}\right)$$

✓ Solution by Mathematica

Time used: 0.003 (sec). Leaf size: 19

```
DSolve[{y''''[t]-8*y''[t]+16*y[t]==0,{y[0]==0,y'[0]==0,y''[0]==8,y'''[0]==0}},y[t],t,Include
```

$$y(t) \to e^{-2t}\left(e^{4t} - 1\right) t$$

13.27 problem 44

Internal problem ID [14663]

Book: INTRODUCTORY DIFFERENTIAL EQUATIONS. Martha L. Abell, James P. Braselton. Fourth edition 2014. ElScAe. 2014
Section: Chapter 4. Higher Order Equations. Exercises 4.5, page 175
Problem number: 44.
ODE order: 3.
ODE degree: 1.

CAS Maple gives this as type `[[_3rd_order, _missing_x]]`

$$\boxed{24y''' - 26y'' + 9y' - y = 0}$$

With initial conditions

$$[y(0) = 0, y'(0) = 1, y''(0) = 0]$$

✓ Solution by Maple

Time used: 0.015 (sec). Leaf size: 23

```
dsolve([24*diff(y(t),t$3)-26*diff(y(t),t$2)+9*diff(y(t),t)-y(t)=0,y(0) = 0, D(y)(0) = 1, (D@
```

$$y(t) = 54\,\mathrm{e}^{\frac{t}{3}} - 14\,\mathrm{e}^{\frac{t}{2}} - 40\,\mathrm{e}^{\frac{t}{4}}$$

✓ Solution by Mathematica

Time used: 0.003 (sec). Leaf size: 33

```
DSolve[{24*y'''[t]-26*y''[t]+9*y'[t]-y[t]==0,{y[0]==0,y'[0]==1,y''[0]==0}},y[t],t,IncludeSin
```

$$y(t) \to -40e^{t/4} + 54e^{t/3} - 14e^{t/2}$$

13.28 problem 45

Internal problem ID [14664]

Book: INTRODUCTORY DIFFERENTIAL EQUATIONS. Martha L. Abell, James P. Braselton. Fourth edition 2014. ElScAe. 2014
Section: Chapter 4. Higher Order Equations. Exercises 4.5, page 175
Problem number: 45.
ODE order: 4.
ODE degree: 1.

CAS Maple gives this as type `[[_high_order, _missing_x]]`

$$\boxed{y'''' - 5y'' + 4y = 0}$$

With initial conditions

$$[y(0) = -1, y'(0) = 3, y''(0) = -7, y'''(0) = 15]$$

✓ Solution by Maple

Time used: 0.016 (sec). Leaf size: 15

```
dsolve([diff(y(t),t$4)-5*diff(y(t),t$2)+4*y(t)=0,y(0) = -1, D(y)(0) = 3, (D@@2)(y)(0) = -7,
```

$$y(t) = \mathrm{e}^{-t} - 2\,\mathrm{e}^{-2t}$$

✓ Solution by Mathematica

Time used: 0.003 (sec). Leaf size: 16

```
DSolve[{y''''[t]-5*y''[t]+4*y[t]==0,{y[0]==-1,y'[0]==3,y''[0]==-7,y'''[0]==15}},y[t],t,Inclu
```

$$y(t) \to e^{-2t}(e^t - 2)$$

13.29 problem 46

Internal problem ID [14665]

Book: INTRODUCTORY DIFFERENTIAL EQUATIONS. Martha L. Abell, James P. Braselton. Fourth edition 2014. ElScAe. 2014
Section: Chapter 4. Higher Order Equations. Exercises 4.5, page 175
Problem number: 46.
ODE order: 4.
ODE degree: 1.

CAS Maple gives this as type `[[_high_order, _missing_x]]`

$$\boxed{y'''' - 16y = 0}$$

With initial conditions

$$[y(0) = 1, y'(0) = 2, y''(0) = 4, y'''(0) = -24]$$

✓ Solution by Maple

Time used: 0.015 (sec). Leaf size: 15

```
dsolve([diff(y(t),t$4)-16*y(t)=0,y(0) = 1, D(y)(0) = 2, (D@@2)(y)(0) = 4, (D@@3)(y)(0) = -24
```

$$y(t) = \mathrm{e}^{-2t} + 2\sin(2t)$$

✓ Solution by Mathematica

Time used: 0.003 (sec). Leaf size: 17

```
DSolve[{y''''[t]-16*y[t]==0,{y[0]==1,y'[0]==2,y''[0]==4,y'''[0]==-24}},y[t],t,IncludeSingula
```

$$y(t) \to e^{-2t} + 2\sin(2t)$$

13.30 problem 47

Internal problem ID [14666]

Book: INTRODUCTORY DIFFERENTIAL EQUATIONS. Martha L. Abell, James P. Braselton. Fourth edition 2014. ElScAe. 2014
Section: Chapter 4. Higher Order Equations. Exercises 4.5, page 175
Problem number: 47.
ODE order: 5.
ODE degree: 1.

CAS Maple gives this as type `[[_high_order, _missing_x]]`

$$\boxed{8y^{(5)} + 4y'''' + 66y''' - 41y'' - 37y' = 0}$$

With initial conditions

$$\left[y(0) = 4, y'(0) = -14, y''(0) = -14, y'''(0) = 139, y''''(0) = -\frac{29}{4}\right]$$

✓ Solution by Maple
Time used: 0.031 (sec). Leaf size: 23

```
dsolve([8*diff(y(t),t$5)+4*diff(y(t),t$4)+66*diff(y(t),t$3)-41*diff(y(t),t$2)-37*diff(y(t),t
```

$$y(t) = e^{-\frac{t}{2}}(1 - 4\sin(3t) + 3\cos(3t))$$

✓ Solution by Mathematica
Time used: 0.326 (sec). Leaf size: 27

```
DSolve[{8*y'''''[t]+4*y''''[t]+66*y'''[t]-41*y''[t]-37*y'[t]==0,{y[0]==4,y'[0]==-14,y''[0]==
```

$$y(t) \to e^{-t/2}(-4\sin(3t) + 3\cos(3t) + 1)$$

13.31 problem 48

Internal problem ID [14667]

Book: INTRODUCTORY DIFFERENTIAL EQUATIONS. Martha L. Abell, James P. Braselton. Fourth edition 2014. ElScAe. 2014
Section: Chapter 4. Higher Order Equations. Exercises 4.5, page 175
Problem number: 48.
ODE order: 5.
ODE degree: 1.

CAS Maple gives this as type `[[_high_order, _missing_x]]`

$$\boxed{2y^{(5)} + 7y'''' + 17y''' + 17y'' + 5y' = 0}$$

With initial conditions

$$\left[y(0) = -3, y'(0) = \frac{15}{2}, y''(0) = \frac{17}{4}, y'''(0) = -\frac{385}{8}, y''''(0) = \frac{1217}{16}\right]$$

✓ Solution by Maple
Time used: 0.031 (sec). Leaf size: 29

```
dsolve([2*diff(y(t),t$5)+7*diff(y(t),t$4)+17*diff(y(t),t$3)+17*diff(y(t),t$2)+5*diff(y(t),t)
```

$$y(t) = (2\sin(2t) - 4\cos(2t))\,\mathrm{e}^{-t} + \mathrm{e}^{-\frac{t}{2}}$$

✓ Solution by Mathematica
Time used: 0.298 (sec). Leaf size: 31

```
DSolve[{2*y'''''[t]+7*y''''[t]+17*y'''[t]+17*y''[t]+5*y'[t]==0,{y[0]==-3,y'[0]==15/2,y''[0]=
```

$$y(t) \to e^{-t}\left(e^{t/2} + 2\sin(2t) - 4\cos(2t)\right)$$

13.32 problem 49

Internal problem ID [14668]

Book: INTRODUCTORY DIFFERENTIAL EQUATIONS. Martha L. Abell, James P. Braselton. Fourth edition 2014. ElScAe. 2014
Section: Chapter 4. Higher Order Equations. Exercises 4.5, page 175
Problem number: 49.
ODE order: 5.
ODE degree: 1.

CAS Maple gives this as type `[[_high_order, _missing_x]]`

$$\boxed{y^{(5)} + 8y'''' = 0}$$

With initial conditions

$$[y(0) = 8, y'(0) = 4, y''(0) = 0, y'''(0) = 48, y''''(0) = 0]$$

✓ Solution by Maple

Time used: 0.031 (sec). Leaf size: 14

```
dsolve([diff(y(t),t$5)+8*diff(y(t),t$4)=0,y(0) = 8, D(y)(0) = 4, (D@@2)(y)(0) = 0, (D@@3)(y)
```

$$y(t) = 8t^3 + 4t + 8$$

✓ Solution by Mathematica

Time used: 0.037 (sec). Leaf size: 15

```
DSolve[{y'''''[t]+8*y''''[t]==0,{y[0]==8,y'[0]==4,y''[0]==0,y'''[0]==48,y''''[0]==0}},y[t],t
```

$$y(t) \to 4(2t^3 + t + 2)$$

13.33 problem 50

Internal problem ID [14669]

Book: INTRODUCTORY DIFFERENTIAL EQUATIONS. Martha L. Abell, James P. Braselton. Fourth edition 2014. ElScAe. 2014
Section: Chapter 4. Higher Order Equations. Exercises 4.5, page 175
Problem number: 50.
ODE order: 6.
ODE degree: 1.

CAS Maple gives this as type `[[_high_order, _missing_x]]`

$$\boxed{y^{(6)} - 3y'''' + 3y'' - y = 0}$$

With initial conditions

$$[y(0) = 16, y'(0) = 0, y''(0) = 0, y'''(0) = 0, y''''(0) = 0, y^{(5)}(0) = 0]$$

✓ Solution by Maple
Time used: 0.032 (sec). Leaf size: 29

```
dsolve([diff(y(t),t$6)-3*diff(y(t),t$4)+3*diff(y(t),t$2)-y(t)=0,y(0) = 16, D(y)(0) = 0, (D@@
```

$$y(t) = \left(t^2 + 5t + 8\right) e^{-t} + e^t \left(t^2 - 5t + 8\right)$$

✓ Solution by Mathematica
Time used: 0.004 (sec). Leaf size: 33

```
DSolve[{y''''''[t]-3*y''''[t]+3*y''[t]-y[t]==0,{y[0]==16,y'[0]==0,y''[0]==0,y'''[0]==0,y''''
```

$$y(t) \to e^{-t}\left(t^2 + e^{2t}\left(t^2 - 5t + 8\right) + 5t + 8\right)$$

13.34 problem 58

Internal problem ID [14670]

Book: INTRODUCTORY DIFFERENTIAL EQUATIONS. Martha L. Abell, James P. Braselton. Fourth edition 2014. ElScAe. 2014
Section: Chapter 4. Higher Order Equations. Exercises 4.5, page 175
Problem number: 58.
ODE order: 3.
ODE degree: 1.

CAS Maple gives this as type `[[_3rd_order, _missing_x]]`

$$\boxed{y''' + 9y'' + 16y' - 26y = 0}$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 24

```
dsolve(diff(y(t),t$3)+9*diff(y(t),t$2)+16*diff(y(t),t)-26*y(t)=0,y(t), singsol=all)
```

$$y(t) = \left(e^{6t}c_1 + \sin(t)\, c_2 + \cos(t)\, c_3\right) e^{-5t}$$

✓ Solution by Mathematica

Time used: 0.003 (sec). Leaf size: 30

```
DSolve[y'''[t]+9*y''[t]+16*y'[t]-26*y[t]==0,y[t],t,IncludeSingularSolutions -> True]
```

$$y(t) \to e^{-5t}\left(c_3 e^{6t} + c_2 \cos(t) + c_1 \sin(t)\right)$$

13.35 problem 59

Internal problem ID [14671]

Book: INTRODUCTORY DIFFERENTIAL EQUATIONS. Martha L. Abell, James P. Braselton. Fourth edition 2014. ElScAe. 2014
Section: Chapter 4. Higher Order Equations. Exercises 4.5, page 175
Problem number: 59.
ODE order: 4.
ODE degree: 1.

CAS Maple gives this as type `[[_high_order, _missing_x]]`

$$\boxed{y'''' + 12y''' + 60y'' + 124y' + 75y = 0}$$

✓ Solution by Maple
Time used: 0.0 (sec). Leaf size: 37

```
dsolve(diff(y(t),t$4)+12*diff(y(t),t$3)+60*diff(y(t),t$2)+124*diff(y(t),t)+75*y(t)=0,y(t), s
```

$$y(t) = c_1 e^{-3t} + c_2 e^{-t} + c_3 e^{-4t} \sin(3t) + c_4 e^{-4t} \cos(3t)$$

✓ Solution by Mathematica
Time used: 0.003 (sec). Leaf size: 40

```
DSolve[y''''[t]+12*y'''[t]+60*y''[t]+124*y'[t]+75*y[t]==0,y[t],t,IncludeSingularSolutions ->
```

$$y(t) \to e^{-4t}\left(c_3 e^t + c_4 e^{3t} + c_2 \cos(3t) + c_1 \sin(3t)\right)$$

13.36 problem 63 (a)

Internal problem ID [14672]

Book: INTRODUCTORY DIFFERENTIAL EQUATIONS. Martha L. Abell, James P. Braselton. Fourth edition 2014. ElScAe. 2014
Section: Chapter 4. Higher Order Equations. Exercises 4.5, page 175
Problem number: 63 (a).
ODE order: 3.
ODE degree: 1.

CAS Maple gives this as type `[[_3rd_order, _missing_x]]`

$$\boxed{y''' + 3y'' + 2y' + 6y = 0}$$

With initial conditions

$$[y(0) = 0, y'(0) = 1, y''(0) = -1]$$

✓ Solution by Maple
Time used: 0.031 (sec). Leaf size: 30

```
dsolve([diff(y(t),t$3)+3*diff(y(t),t$2)+2*diff(y(t),t)+6*y(t)=0,y(0) = 0, D(y)(0) = 1, (D@@2
```

$$y(t) = -\frac{e^{-3t}}{11} + \frac{4\sqrt{2}\,\sin\left(\sqrt{2}\,t\right)}{11} + \frac{\cos\left(\sqrt{2}\,t\right)}{11}$$

✓ Solution by Mathematica
Time used: 0.004 (sec). Leaf size: 40

```
DSolve[{y'''[t]+3*y''[t]+2*y'[t]+6*y[t]==0,{y[0]==0,y'[0]==1,y''[0]==-1}},y[t],t,IncludeSing
```

$$y(t) \to \frac{1}{11}\left(-e^{-3t} + 4\sqrt{2}\sin\left(\sqrt{2}t\right) + \cos\left(\sqrt{2}t\right)\right)$$

13.37 problem 63 (b)

Internal problem ID [14673]

Book: INTRODUCTORY DIFFERENTIAL EQUATIONS. Martha L. Abell, James P. Braselton. Fourth edition 2014. ElScAe. 2014
Section: Chapter 4. Higher Order Equations. Exercises 4.5, page 175
Problem number: 63 (b).
ODE order: 4.
ODE degree: 1.

CAS Maple gives this as type `[[_high_order, _missing_x]]`

$$\boxed{y'''' - 8y''' + 30y'' - 56y' + 49y = 0}$$

With initial conditions

$$[y(0) = 1, y'(0) = 2, y''(0) = -1, y'''(0) = -1]$$

✓ Solution by Maple
Time used: 0.047 (sec). Leaf size: 36

```
dsolve([diff(y(t),t$4)-8*diff(y(t),t$3)+30*diff(y(t),t$2)-56*diff(y(t),t)+49*y(t)=0,y(0) = 1
```

$$y(t) = -\frac{\left(\left(\frac{21t}{2} - 3\right)\cos\left(\sqrt{3}\,t\right) + \sqrt{3}\,\sin\left(\sqrt{3}\,t\right)\left(t - \frac{7}{2}\right)\right)\mathrm{e}^{2t}}{3}$$

✓ Solution by Mathematica
Time used: 0.005 (sec). Leaf size: 49

```
DSolve[{y''''[t]-8*y'''[t]+30*y''[t]-56*y'[t]+49*y[t]==0,{y[0]==1,y'[0]==2,y''[0]==-1,y'''[0
```

$$y(t) \to -\frac{1}{6}e^{2t}\left(\sqrt{3}(2t-7)\sin\left(\sqrt{3}t\right) + 3(7t-2)\cos\left(\sqrt{3}t\right)\right)$$

13.38 problem 63 (c)

Internal problem ID [14674]

Book: INTRODUCTORY DIFFERENTIAL EQUATIONS. Martha L. Abell, James P. Braselton. Fourth edition 2014. ElScAe. 2014
Section: Chapter 4. Higher Order Equations. Exercises 4.5, page 175
Problem number: 63 (c).
ODE order: 3.
ODE degree: 1.

CAS Maple gives this as type `[[_3rd_order, _missing_x]]`

$$\boxed{\frac{31y'''}{100}+\frac{56y''}{5}-\frac{49y'}{5}+\frac{53y}{10}=0}$$

With initial conditions

$$[y(0)=-1, y'(0)=-1, y''(0)=0]$$

✓ Solution by Maple

Time used: 0.75 (sec). Leaf size: 320

```
dsolve([31/100*diff(y(t),t$3)+112/10*diff(y(t),t$2)-98/10*diff(y(t),t)+53/10*y(t)=0,y(0) = -
```

$y(t)$

$$= \frac{\left(\left(\left(-1228360\sqrt{3}\,\sqrt{19889065283}+588872235000\right)\left(1564919155+465\sqrt{3}\,\sqrt{19889065283}\right)^{\frac{1}{3}}+\left(-1213\right.\right.\right.}{}$$

✓ Solution by Mathematica

Time used: 0.009 (sec). Leaf size: 905

```
DSolve[{31/100*y'''[t]+112/10*y''[t]-98/10*y'[t]+53/10*y[t]==0,{y[0]==-1,y'[0]==-1,y''[0]==0
```

$y(t) \rightarrow$

$$-\frac{e^{t\text{Root}\left[31\#1^3+1120\#1^2-980\#1+530\&,2\right]}\text{Root}\left[31\#1^3+1120\#1^2-980\#1+530\&,1\right]^2-e^{t\text{Root}\left[31\#1^3+1120\#1^2-\right.}}{}$$

13.39 problem 67

Internal problem ID [14675]

Book: INTRODUCTORY DIFFERENTIAL EQUATIONS. Martha L. Abell, James P. Braselton. Fourth edition 2014. ElScAe. 2014
Section: Chapter 4. Higher Order Equations. Exercises 4.5, page 175
Problem number: 67.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _missing_x], [_2nd_order, _reducible, _mu_xy]]`

$$\boxed{2yy'' + y^2 - y'^2 = 0}$$

✓ Solution by Maple
Time used: 0.063 (sec). Leaf size: 26

```
dsolve(2*y(t)*diff(y(t),t$2)+y(t)^2=diff(y(t),t)^2,y(t), singsol=all)
```

$$y(t) = 0$$
$$y(t) = \sqrt{c_1^2 + c_2^2} + c_1 \sin(t) + c_2 \cos(t)$$

✓ Solution by Mathematica
Time used: 0.254 (sec). Leaf size: 32

```
DSolve[2*y[t]*y''[t]+y[t]^2==y'[t]^2,y[t],t,IncludeSingularSolutions -> True]
```

$$y(t) \to c_2 \cos^2\left(\frac{1}{2}(t - 2c_1)\right)$$
$$y(t) \to c_2 \text{Interval}[\{0, 1\}]$$

14 Chapter 4. Higher Order Equations. Exercises 4.6, page 187

14.1 problem 1

Internal problem ID [14676]

Book: INTRODUCTORY DIFFERENTIAL EQUATIONS. Martha L. Abell, James P. Braselton. Fourth edition 2014. ElScAe. 2014
Section: Chapter 4. Higher Order Equations. Exercises 4.6, page 187
Problem number: 1.
ODE order: 3.
ODE degree: 1.

CAS Maple gives this as type `[[_3rd_order, _missing_y]]`

$$\boxed{y''' + y'' = \mathrm{e}^t}$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 19

```
dsolve(diff(y(t),t$3)+diff(y(t),t$2)=exp(t),y(t), singsol=all)
```

$$y(t) = \frac{\mathrm{e}^t}{2} + \mathrm{e}^{-t} c_1 + t c_2 + c_3$$

✓ Solution by Mathematica

Time used: 0.093 (sec). Leaf size: 27

```
DSolve[y'''[t]+y''[t]==Exp[t],y[t],t,IncludeSingularSolutions -> True]
```

$$y(t) \to \frac{e^t}{2} + c_1 e^{-t} + c_3 t + c_2$$

14.2 problem 2

Internal problem ID [14677]

Book: INTRODUCTORY DIFFERENTIAL EQUATIONS. Martha L. Abell, James P. Braselton. Fourth edition 2014. ElScAe. 2014
Section: Chapter 4. Higher Order Equations. Exercises 4.6, page 187
Problem number: 2.
ODE order: 4.
ODE degree: 1.

CAS Maple gives this as type `[[_high_order, _missing_x]]`

$$\boxed{y'''' - 16y = 1}$$

✓ Solution by Maple
Time used: 0.016 (sec). Leaf size: 30

```
dsolve(diff(y(t),t$4)-16*y(t)=1,y(t), singsol=all)
```

$$y(t) = -\frac{1}{16} + \cos(2t) c_1 + c_2 e^{-2t} + c_3 e^{2t} + c_4 \sin(2t)$$

✓ Solution by Mathematica
Time used: 0.002 (sec). Leaf size: 39

```
DSolve[y''''[t]-16*y[t]==1,y[t],t,IncludeSingularSolutions -> True]
```

$$y(t) \to c_1 e^{2t} + c_3 e^{-2t} + c_2 \cos(2t) + c_4 \sin(2t) - \frac{1}{16}$$

14.3 problem 3

Internal problem ID [14678]

Book: INTRODUCTORY DIFFERENTIAL EQUATIONS. Martha L. Abell, James P. Braselton. Fourth edition 2014. ElScAe. 2014
Section: Chapter 4. Higher Order Equations. Exercises 4.6, page 187
Problem number: 3.
ODE order: 5.
ODE degree: 1.

CAS Maple gives this as type `[[_high_order, _missing_x]]`

$$\boxed{y^{(5)} - y'''' = 1}$$

✓ Solution by Maple
Time used: 0.0 (sec). Leaf size: 30

```
dsolve(diff(y(t),t$5)-diff(y(t),t$4)=1,y(t), singsol=all)
```

$$y(t) = -\frac{t^4}{24} + \frac{c_3 t^2}{2} + \frac{c_2 t^3}{6} + \mathrm{e}^t c_1 + t c_4 + c_5$$

✓ Solution by Mathematica
Time used: 0.034 (sec). Leaf size: 37

```
DSolve[y'''''[t]-y''''[t]==1,y[t],t,IncludeSingularSolutions -> True]
```

$$y(t) \to -\frac{t^4}{24} + c_5 t^3 + c_4 t^2 + c_3 t + c_1 e^t + c_2$$

14.4 problem 4

Internal problem ID [14679]

Book: INTRODUCTORY DIFFERENTIAL EQUATIONS. Martha L. Abell, James P. Braselton. Fourth edition 2014. ElScAe. 2014
Section: Chapter 4. Higher Order Equations. Exercises 4.6, page 187
Problem number: 4.
ODE order: 4.
ODE degree: 1.

CAS Maple gives this as type `[[_high_order, _missing_x]]`

$$\boxed{y'''' + 9y'' = 1}$$

✓ Solution by Maple
Time used: 0.0 (sec). Leaf size: 28

```
dsolve(diff(y(t),t$4)+9*diff(y(t),t$2)=1,y(t), singsol=all)
```

$$y(t) = \frac{t^2}{18} - \frac{c_1 \cos(3t)}{9} - \frac{c_2 \sin(3t)}{9} + c_3 t + c_4$$

✓ Solution by Mathematica
Time used: 0.16 (sec). Leaf size: 39

```
DSolve[y''''[t]+9*y''[t]==1,y[t],t,IncludeSingularSolutions -> True]
```

$$y(t) \to \frac{t^2}{18} + c_4 t - \frac{1}{9} c_1 \cos(3t) - \frac{1}{9} c_2 \sin(3t) + c_3$$

14.5 problem 5

Internal problem ID [14680]

Book: INTRODUCTORY DIFFERENTIAL EQUATIONS. Martha L. Abell, James P. Braselton. Fourth edition 2014. ElScAe. 2014
Section: Chapter 4. Higher Order Equations. Exercises 4.6, page 187
Problem number: 5.
ODE order: 4.
ODE degree: 1.

CAS Maple gives this as type `[[_high_order, _missing_y]]`

$$\boxed{y'''' + 9y'' = 9\,\mathrm{e}^{3t}}$$

✓ Solution by Maple
Time used: 0.0 (sec). Leaf size: 29

```
dsolve(diff(y(t),t$4)+9*diff(y(t),t$2)=9*exp(3*t),y(t), singsol=all)
```

$$y(t) = -\frac{c_1 \cos(3t)}{9} - \frac{c_2 \sin(3t)}{9} + \frac{\mathrm{e}^{3t}}{18} + c_3 t + c_4$$

✓ Solution by Mathematica
Time used: 0.128 (sec). Leaf size: 41

```
DSolve[y''''[t]+9*y''[t]==9*Exp[3*t],y[t],t,IncludeSingularSolutions -> True]
```

$$y(t) \to \frac{e^{3t}}{18} + c_4 t - \frac{1}{9}c_1 \cos(3t) - \frac{1}{9}c_2 \sin(3t) + c_3$$

14.6 problem 6

Internal problem ID [14681]

Book: INTRODUCTORY DIFFERENTIAL EQUATIONS. Martha L. Abell, James P. Braselton. Fourth edition 2014. ElScAe. 2014
Section: Chapter 4. Higher Order Equations. Exercises 4.6, page 187
Problem number: 6.
ODE order: 3.
ODE degree: 1.

CAS Maple gives this as type `[[_3rd_order, _linear, _nonhomogeneous]]`

$$\boxed{y''' + 10y'' + 34y' + 40y = t\,\mathrm{e}^{-4t} + 2\,\mathrm{e}^{-3t}\cos(t)}$$

✓ Solution by Maple
Time used: 0.0 (sec). Leaf size: 50

```
dsolve(diff(y(t),t$3)+10*diff(y(t),t$2)+34*diff(y(t),t)+40*y(t)=t*exp(-4*t)+2*exp(-3*t)*cos(
```

$$y(t) = \frac{((-2t + 4c_2 + 4)\cos(t) + 2\sin(t)(t + 2c_3 + 1))\,\mathrm{e}^{-3t}}{4} + \frac{\mathrm{e}^{-4t}(t^2 + 4c_1 + 2t + 1)}{4}$$

✓ Solution by Mathematica
Time used: 0.209 (sec). Leaf size: 57

```
DSolve[y'''[t]+10*y''[t]+34*y'[t]+40*y[t]==t*Exp[-4*t]+2*Exp[-3*t]*Cos[t],y[t],t,IncludeSing
```

$$y(t) \to \frac{1}{4}e^{-4t}\left(t^2 + 2t - e^t(2t - 3 - 4c_2)\cos(t) + e^t(2t + 1 + 4c_1)\sin(t) + 1 + 4c_3\right)$$

14.7 problem 7

Internal problem ID [14682]

Book: INTRODUCTORY DIFFERENTIAL EQUATIONS. Martha L. Abell, James P. Braselton. Fourth edition 2014. ElScAe. 2014
Section: Chapter 4. Higher Order Equations. Exercises 4.6, page 187
Problem number: 7.
ODE order: 3.
ODE degree: 1.

CAS Maple gives this as type `[[_3rd_order, _linear, _nonhomogeneous]]`

$$\boxed{y''' + 6y'' + 11y' + 6y = 2\,\mathrm{e}^{-3t} - t\,\mathrm{e}^{-t}}$$

✓ Solution by Maple
Time used: 0.0 (sec). Leaf size: 44

```
dsolve(diff(y(t),t$3)+6*diff(y(t),t$2)+11*diff(y(t),t)+6*y(t)=2*exp(-3*t)-t*exp(-t),y(t), si
```

$$y(t) = \frac{(-2t^2 + 8c_3 + 6t - 7)\,\mathrm{e}^{-t}}{8} + \frac{(3 + 2t + 2c_1)\,\mathrm{e}^{-3t}}{2} + c_2\mathrm{e}^{-2t}$$

✓ Solution by Mathematica
Time used: 0.06 (sec). Leaf size: 50

```
DSolve[y'''[t]+6*y''[t]+11*y'[t]+6*y[t]==2*Exp[-3*t]-t*Exp[-t],y[t],t,IncludeSingularSolutio
```

$$y(t) \to \frac{1}{8}e^{-3t}\left(e^{2t}\left(-2t^2 + 6t - 7 + 8c_3\right) + 8t + 8c_2e^t + 12 + 8c_1\right)$$

14.8 problem 8

Internal problem ID [14683]

Book: INTRODUCTORY DIFFERENTIAL EQUATIONS. Martha L. Abell, James P. Braselton. Fourth edition 2014. ElScAe. 2014
Section: Chapter 4. Higher Order Equations. Exercises 4.6, page 187
Problem number: 8.
ODE order: 4.
ODE degree: 1.

CAS Maple gives this as type `[[_high_order, _with_linear_symmetries]]`

$$\boxed{y'''' - 6y''' + 13y'' - 24y' + 36y = 108t}$$

✓ Solution by Maple
Time used: 0.016 (sec). Leaf size: 31

```
dsolve(diff(y(t),t$4)-6*diff(y(t),t$3)+13*diff(y(t),t$2)-24*diff(y(t),t)+36*y(t)=108*t,y(t),
```

$$y(t) = (tc_4 + c_2)\,\mathrm{e}^{3t} + \cos(2t)\,c_1 + c_3 \sin(2t) + 3t + 2$$

✓ Solution by Mathematica
Time used: 0.003 (sec). Leaf size: 41

```
DSolve[y''''[t]-6*y'''[t]+13*y''[t]-24*y'[t]+36*y[t]==108*t,y[t],t,IncludeSingularSolutions
```

$$y(t) \to 3t + c_4 e^{3t} t + c_3 e^{3t} + c_1 \cos(2t) + c_2 \sin(2t) + 2$$

14.9 problem 9

Internal problem ID [14684]

Book: INTRODUCTORY DIFFERENTIAL EQUATIONS. Martha L. Abell, James P. Braselton. Fourth edition 2014. ElScAe. 2014
Section: Chapter 4. Higher Order Equations. Exercises 4.6, page 187
Problem number: 9.
ODE order: 3.
ODE degree: 1.

CAS Maple gives this as type `[[_3rd_order, _with_linear_symmetries]]`

$$\boxed{y''' + 6y'' - 14y' - 104y = -111\,\mathrm{e}^{t}}$$

✓ Solution by Maple
Time used: 0.016 (sec). Leaf size: 28

```
dsolve(diff(y(t),t$3)+6*diff(y(t),t$2)-14*diff(y(t),t)-104*y(t)=-111*exp(t),y(t), singsol=al
```

$$y(t) = \left(\mathrm{e}^{9t} c_1 + \mathrm{e}^{6t} + c_3 \sin(t) + c_2 \cos(t)\right) \mathrm{e}^{-5t}$$

✓ Solution by Mathematica
Time used: 0.004 (sec). Leaf size: 35

```
DSolve[y'''[t]+6*y''[t]-14*y'[t]-104*y[t]==-111*Exp[t],y[t],t,IncludeSingularSolutions -> Tr
```

$$y(t) \to e^{-5t}\left(e^{6t} + c_3 e^{9t} + c_2 \cos(t) + c_1 \sin(t)\right)$$

14.10 problem 10

Internal problem ID [14685]

Book: INTRODUCTORY DIFFERENTIAL EQUATIONS. Martha L. Abell, James P. Braselton. Fourth edition 2014. ElScAe. 2014
Section: Chapter 4. Higher Order Equations. Exercises 4.6, page 187
Problem number: 10.
ODE order: 4.
ODE degree: 1.

CAS Maple gives this as type `[[_high_order, _with_linear_symmetries]]`

$$\boxed{y'''' - 10y''' + 38y'' - 64y' + 40y = 153\,\mathrm{e}^{-t}}$$

✓ Solution by Maple
Time used: 0.0 (sec). Leaf size: 33

```
dsolve(diff(y(t),t$4)-10*diff(y(t),t$3)+38*diff(y(t),t$2)-64*diff(y(t),t)+40*y(t)=153*exp(-t
```

$$y(t) = (tc_2 + c_1)\,\mathrm{e}^{2t} + (\cos(t)\,c_3 + \sin(t)\,c_4)\,\mathrm{e}^{3t} + \mathrm{e}^{-t}$$

✓ Solution by Mathematica
Time used: 0.006 (sec). Leaf size: 48

```
DSolve[y''''[t]-10*y'''[t]+38*y''[t]-64*y'[t]+40*y[t]==153*Exp[-t],y[t],t,IncludeSingularSol
```

$$y(t) \to e^{-t} + c_3e^{2t} + c_4e^{2t}t + c_2e^{3t}\cos(t) + c_1e^{3t}\sin(t)$$

14.11 problem 11

Internal problem ID [14686]

Book: INTRODUCTORY DIFFERENTIAL EQUATIONS. Martha L. Abell, James P. Braselton. Fourth edition 2014. ElScAe. 2014
Section: Chapter 4. Higher Order Equations. Exercises 4.6, page 187
Problem number: 11.
ODE order: 3.
ODE degree: 1.

CAS Maple gives this as type `[[_3rd_order, _missing_y]]`

$$\boxed{y''' + 4y' = \tan(2t)}$$

✓ Solution by Maple

Time used: 0.016 (sec). Leaf size: 90

```
dsolve(diff(y(t),t$3)+4*diff(y(t),t)=tan(2*t),y(t), singsol=all)
```

$$y(t) = \frac{i(\mathrm{e}^{2it} - \mathrm{e}^{-2it}) \ln\left(\frac{i\mathrm{e}^{2it}-1}{-\mathrm{e}^{2it}+i}\right)}{16} + \frac{\sin(2t)\, c_1}{2} - \frac{c_2 \cos(2t)}{2} + \frac{\ln(\mathrm{e}^{it})}{4} - \frac{\ln(\mathrm{e}^{2it} - i)}{8} - \frac{\ln(\mathrm{e}^{2it} + i)}{8} + c_3$$

✓ Solution by Mathematica

Time used: 0.167 (sec). Leaf size: 53

```
DSolve[y'''[t]+4*y'[t]==Tan[2*t],y[t],t,IncludeSingularSolutions -> True]
```

$$y(t) \to \frac{1}{16}\left(-2\sin(2t)\operatorname{arctanh}(\sin(2t)) - \log\left(\cos^2(2t)\right) + (2 - 8c_2)\cos(2t) + 8c_1 \sin(2t) + 16c_3\right)$$

14.12 problem 12

Internal problem ID [14687]

Book: INTRODUCTORY DIFFERENTIAL EQUATIONS. Martha L. Abell, James P. Braselton. Fourth edition 2014. ElScAe. 2014
Section: Chapter 4. Higher Order Equations. Exercises 4.6, page 187
Problem number: 12.
ODE order: 3.
ODE degree: 1.

CAS Maple gives this as type `[[_3rd_order, _missing_y]]`

$$\boxed{y''' + 4y' = \sec(2t)\tan(2t)}$$

✓ Solution by Maple
Time used: 0.0 (sec). Leaf size: 41

```
dsolve(diff(y(t),t$3)+4*diff(y(t),t)=sec(2*t)*tan(2*t),y(t), singsol=all)
```

$$y(t) = -\frac{\ln(\sec(2t))\cos(2t)}{8} + \frac{(1-4c_2)\cos(2t)}{8} + \frac{(2t+4c_1)\sin(2t)}{8} + c_3$$

✓ Solution by Mathematica
Time used: 0.241 (sec). Leaf size: 47

```
DSolve[y'''[t]+4*y'[t]==Sec[2*t]*Tan[2*t],y[t],t,IncludeSingularSolutions -> True]
```

$$y(t) \to \frac{1}{8}\sin(2t)\arctan(\tan(2t)) + \frac{1}{8}\cos(2t)(\log(\cos(2t)) + 1 - 4c_2) + c_1\sin(t)\cos(t) + c_3$$

14.13 problem 13

Internal problem ID [14688]

Book: INTRODUCTORY DIFFERENTIAL EQUATIONS. Martha L. Abell, James P. Braselton. Fourth edition 2014. ElScAe. 2014
Section: Chapter 4. Higher Order Equations. Exercises 4.6, page 187
Problem number: 13.
ODE order: 4.
ODE degree: 1.

CAS Maple gives this as type `[[_high_order, _missing_y]]`

$$\boxed{y'''' + 4y'' = \sec(2t)^2}$$

✓ Solution by Maple
Time used: 0.016 (sec). Leaf size: 364

```
dsolve(diff(y(t),t$4)+4*diff(y(t),t$2)=sec(2*t)^2,y(t), singsol=all)
```

$$
\begin{aligned}
y(t) = &-\frac{\pi\,\mathrm{e}^{-2it}\left(\operatorname{csgn}\left(\frac{i}{\mathrm{e}^{-4it}+1}\right)\operatorname{csgn}\left(2i\mathrm{e}^{-2it}-\mathrm{e}^{-4it}+1\right)+1\right)\operatorname{csgn}\left(\frac{2i\mathrm{e}^{-2it}-\mathrm{e}^{-4it}+1}{\mathrm{e}^{-4it}+1}\right)}{64} \\
&+\frac{\mathrm{e}^{2it}\pi\left(\operatorname{csgn}\left(\frac{i}{\mathrm{e}^{4it}+1}\right)\operatorname{csgn}\left(\mathrm{e}^{4it}+2i\mathrm{e}^{2it}-1\right)+1\right)\operatorname{csgn}\left(\frac{\mathrm{e}^{4it}+2i\mathrm{e}^{2it}-1}{\mathrm{e}^{4it}+1}\right)}{64} \\
&+\frac{\pi\,\operatorname{csgn}\left(2i\mathrm{e}^{-2it}-\mathrm{e}^{-4it}+1\right)\mathrm{e}^{-2it}}{64}+\frac{\pi\,\operatorname{csgn}\left(\mathrm{e}^{4it}+2i\mathrm{e}^{2it}-1\right)\mathrm{e}^{2it}}{64} \\
&+\frac{i\mathrm{e}^{2it}\ln\left(i(\mathrm{e}^{2it}+i)^2\right)}{32}-\frac{i\ln\left(\mathrm{e}^{it}\right)t}{4}+\frac{\pi\,\operatorname{csgn}\left(\frac{i}{\mathrm{e}^{-4it}+1}\right)\mathrm{e}^{-2it}}{64} \\
&-\frac{\pi\,\operatorname{csgn}\left(\frac{i}{\mathrm{e}^{4it}+1}\right)\mathrm{e}^{2it}}{64}+\frac{\left(-1+i\mathrm{e}^{-2it}\right)\ln\left(\mathrm{e}^{-4it}+1\right)}{32}-\frac{i\mathrm{e}^{-2it}\ln\left(i(\mathrm{e}^{-2it}-i)^2\right)}{32} \\
&-\frac{\mathrm{e}^{-2it}(c_2 i+c_1)}{8}+\frac{\left(-i\mathrm{e}^{2it}-1\right)\ln\left(\mathrm{e}^{4it}+1\right)}{32}+\frac{(c_2 i-c_1)\,\mathrm{e}^{2it}}{8}-\frac{t^2}{4}+c_3 t+c_4
\end{aligned}
$$

✓ Solution by Mathematica

Time used: 0.256 (sec). Leaf size: 55

```
DSolve[y''''[t]+4*y''[t]==Sec[2*t]^2,y[t],t,IncludeSingularSolutions -> True]
```

$$y(t) \to \frac{1}{32}\left(32c_4t - \log\left(\cos^2(2t)\right) - 8c_1\cos(2t) - 2\sin(2t)\coth^{-1}(\sin(2t)) - 8c_2\sin(2t) + 32c_3\right)$$

14.14 problem 14

Internal problem ID [14689]

Book: INTRODUCTORY DIFFERENTIAL EQUATIONS. Martha L. Abell, James P. Braselton. Fourth edition 2014. ElScAe. 2014
Section: Chapter 4. Higher Order Equations. Exercises 4.6, page 187
Problem number: 14.
ODE order: 4.
ODE degree: 1.

CAS Maple gives this as type `[[_high_order, _missing_y]]`

$$\boxed{y'''' + 4y'' = \tan(2t)^2}$$

✓ Solution by Maple

Time used: 0.016 (sec). Leaf size: 364

```
dsolve(diff(y(t),t$4)+4*diff(y(t),t$2)=tan(2*t)^2,y(t), singsol=all)
```

$$\begin{aligned}
y(t) = &-\frac{\pi\,\mathrm{e}^{-2it}\left(\mathrm{csgn}\left(\frac{i}{\mathrm{e}^{-4it}+1}\right)\mathrm{csgn}\left(2i\mathrm{e}^{-2it}-\mathrm{e}^{-4it}+1\right)+1\right)\mathrm{csgn}\left(\frac{2i\mathrm{e}^{-2it}-\mathrm{e}^{-4it}+1}{\mathrm{e}^{-4it}+1}\right)}{64} \\
&+\frac{\mathrm{e}^{2it}\pi\left(\mathrm{csgn}\left(\frac{i}{\mathrm{e}^{4it}+1}\right)\mathrm{csgn}\left(\mathrm{e}^{4it}+2i\mathrm{e}^{2it}-1\right)+1\right)\mathrm{csgn}\left(\frac{\mathrm{e}^{4it}+2i\mathrm{e}^{2it}-1}{\mathrm{e}^{4it}+1}\right)}{64} \\
&+\frac{\pi\ \mathrm{csgn}\left(2i\mathrm{e}^{-2it}-\mathrm{e}^{-4it}+1\right)\mathrm{e}^{-2it}}{64}+\frac{\pi\ \mathrm{csgn}\left(\mathrm{e}^{4it}+2i\mathrm{e}^{2it}-1\right)\mathrm{e}^{2it}}{64} \\
&+\frac{i\mathrm{e}^{2it}\ln\left(i(\mathrm{e}^{2it}+i)^2\right)}{32}-\frac{i\ln\left(\mathrm{e}^{it}\right)t}{4}+\frac{\pi\ \mathrm{csgn}\left(\frac{i}{\mathrm{e}^{-4it}+1}\right)\mathrm{e}^{-2it}}{64}-\frac{\pi\ \mathrm{csgn}\left(\frac{i}{\mathrm{e}^{4it}+1}\right)\mathrm{e}^{2it}}{64} \\
&+\frac{\left(-1+i\mathrm{e}^{-2it}\right)\ln\left(\mathrm{e}^{-4it}+1\right)}{32}-\frac{i\mathrm{e}^{-2it}\ln\left(i(\mathrm{e}^{-2it}-i)^2\right)}{32}-\frac{\mathrm{e}^{-2it}(c_2 i+c_1)}{8} \\
&+\frac{\left(-i\mathrm{e}^{2it}-1\right)\ln\left(\mathrm{e}^{4it}+1\right)}{32}+\frac{(c_2 i-c_1)\,\mathrm{e}^{2it}}{8}-\frac{3t^2}{8}+c_3 t+c_4
\end{aligned}$$

✓ Solution by Mathematica

Time used: 0.389 (sec). Leaf size: 60

```
DSolve[y''''[t]+4*y''[t]==Tan[2*t]^2,y[t],t,IncludeSingularSolutions -> True]
```

$$y(t) \to \frac{1}{32}\left(-2\sin(2t)\operatorname{arctanh}(\sin(2t)) - 4t^2 + 32c_4t - \log\left(\cos^2(2t)\right) - 8c_1\cos(2t) - 8c_2\sin(2t) + 32c_3\right)$$

14.15 problem 15

Internal problem ID [14690]

Book: INTRODUCTORY DIFFERENTIAL EQUATIONS. Martha L. Abell, James P. Braselton. Fourth edition 2014. ElScAe. 2014
Section: Chapter 4. Higher Order Equations. Exercises 4.6, page 187
Problem number: 15.
ODE order: 3.
ODE degree: 1.

CAS Maple gives this as type `[[_3rd_order, _missing_y]]`

$$\boxed{y''' + 9y' = \sec(3t)}$$

✓ Solution by Maple
Time used: 0.0 (sec). Leaf size: 132

```
dsolve(diff(y(t),t$3)+9*diff(y(t),t)=sec(3*t),y(t), singsol=all)
```

$$y(t) = \frac{i(e^{3it} - e^{-3it}) \ln\left(\frac{e^{3it}}{e^{6it}+1}\right)}{54} - \frac{i \arctan\left(2\,e^{it} - \sqrt{3}\right)}{27} - \frac{i \arctan\left(2\,e^{it} + \sqrt{3}\right)}{27} - \frac{i \arctan\left(e^{3it}\right)}{27} - \frac{i e^{-3it}}{54} + \frac{i e^{3it}}{54} + \frac{i \arctan\left(e^{it}\right)}{27} + \frac{(1 + 9c_1 - \ln(2)) \sin(3t)}{27} + \frac{(-t - 3c_2) \cos(3t)}{9} + c_3$$

✓ Solution by Mathematica
Time used: 0.245 (sec). Leaf size: 73

```
DSolve[y'''[t]+9*y'[t]==Sec[3*t],y[t],t,IncludeSingularSolutions -> True]
```

$$y(t) \to \frac{1}{27}\left(-3(t + 3c_2)\cos(3t) - \log\left(\cos\left(\frac{3t}{2}\right) - \sin\left(\frac{3t}{2}\right)\right) + \log\left(\sin\left(\frac{3t}{2}\right) + \cos\left(\frac{3t}{2}\right)\right) + \sin(3t)(\log(\cos(3t)) + 9c_1) + 27c_3\right)$$

14.16 problem 16

Internal problem ID [14691]

Book: INTRODUCTORY DIFFERENTIAL EQUATIONS. Martha L. Abell, James P. Braselton. Fourth edition 2014. ElScAe. 2014
Section: Chapter 4. Higher Order Equations. Exercises 4.6, page 187
Problem number: 16.
ODE order: 3.
ODE degree: 1.

CAS Maple gives this as type `[[_3rd_order, _missing_y]]`

$$\boxed{y''' + y' = -\sec(t)\tan(t)}$$

✓ Solution by Maple
Time used: 0.0 (sec). Leaf size: 28

```
dsolve(diff(y(t),t$3)+diff(y(t),t)=-sec(t)*tan(t),y(t), singsol=all)
```

$$y(t) = \cos(t)\ln(\sec(t)) + (-c_2 - 1)\cos(t) + (-t + c_1)\sin(t) + c_3$$

✓ Solution by Mathematica
Time used: 0.141 (sec). Leaf size: 31

```
DSolve[y'''[t]+y'[t]==-Sec[t]*Tan[t],y[t],t,IncludeSingularSolutions -> True]
```

$$y(t) \to -\sin(t)\arctan(\tan(t)) + c_1\sin(t) - \cos(t)(\log(\cos(t)) + 1 + c_2) + c_3$$

14.17 problem 17

Internal problem ID [14692]

Book: INTRODUCTORY DIFFERENTIAL EQUATIONS. Martha L. Abell, James P. Braselton. Fourth edition 2014. ElScAe. 2014
Section: Chapter 4. Higher Order Equations. Exercises 4.6, page 187
Problem number: 17.
ODE order: 3.
ODE degree: 1.

CAS Maple gives this as type `[[_3rd_order, _missing_y]]`

$$\boxed{y''' + 4y' = \sec(2t)}$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 97

```
dsolve(diff(y(t),t$3)+4*diff(y(t),t)=sec(2*t),y(t), singsol=all)
```

$$y(t) = \frac{i(\mathrm{e}^{2it} - \mathrm{e}^{-2it}) \ln\left(\frac{\mathrm{e}^{2it}}{\mathrm{e}^{4it}+1}\right)}{16} - \frac{i \arctan(\mathrm{e}^{2it})}{4} + \frac{i(-1-\ln(2))\,\mathrm{e}^{-2it}}{16} + \frac{i(1+\ln(2))\,\mathrm{e}^{2it}}{16} + \frac{(-t-2c_2)\cos(2t)}{4} + \frac{(4c_1+1)\sin(2t)}{8} + c_3$$

✓ Solution by Mathematica

Time used: 0.118 (sec). Leaf size: 63

```
DSolve[y'''[t]+4*y'[t]==Sec[2*t],y[t],t,IncludeSingularSolutions -> True]
```

$$y(t) \to \frac{1}{8}\left(-2t\cos(2t) - 8c_2\cos^2(t) + 4c_1\sin(2t) - \log(\cos(t)-\sin(t)) + \log(\sin(t)+\cos(t)) + \sin(2t)\log(\cos(2t)) + 8c_3\right)$$

14.18 problem 18

Internal problem ID [14693]

Book: INTRODUCTORY DIFFERENTIAL EQUATIONS. Martha L. Abell, James P. Braselton. Fourth edition 2014. ElScAe. 2014
Section: Chapter 4. Higher Order Equations. Exercises 4.6, page 187
Problem number: 18.
ODE order: 3.
ODE degree: 1.

CAS Maple gives this as type `[[_3rd_order, _missing_y]]`

$$\boxed{y''' - 2y'' = -\frac{1}{t^2} - \frac{2}{t}}$$

✓ Solution by Maple
Time used: 0.015 (sec). Leaf size: 26

```
dsolve(diff(y(t),t$3)-2*diff(y(t),t$2)=-1/t^2-2/t,y(t), singsol=all)
```

$$y(t) = \frac{c_1 \mathrm{e}^{2t}}{4} + t\ln(2) + t\ln(t) + (c_2 - 1)\, t + c_3$$

✓ Solution by Mathematica
Time used: 0.116 (sec). Leaf size: 29

```
DSolve[y'''[t]-2*y''[t]==-1/t^2-2/t,y[t],t,IncludeSingularSolutions -> True]
```

$$y(t) \to t\log(t) + \frac{1}{4}c_1 e^{2t} + (-1 + c_3)t + c_2$$

14.19 problem 19

Internal problem ID [14694]

Book: INTRODUCTORY DIFFERENTIAL EQUATIONS. Martha L. Abell, James P. Braselton. Fourth edition 2014. ElScAe. 2014
Section: Chapter 4. Higher Order Equations. Exercises 4.6, page 187
Problem number: 19.
ODE order: 3.
ODE degree: 1.

CAS Maple gives this as type `[[_3rd_order, _linear, _nonhomogeneous]]`

$$\boxed{y''' - 3y'' + 3y' - y = \frac{e^t}{t}}$$

✓ Solution by Maple
Time used: 0.0 (sec). Leaf size: 32

```
dsolve(diff(y(t),t$3)-3*diff(y(t),t$2)+3*diff(y(t),t)-y(t)=1/t*exp(t),y(t), singsol=all)
```

$$y(t) = \frac{(2\ln(t)\,t^2 + (4c_3 - 3)\,t^2 + 4tc_2 + 4c_1)\,e^t}{4}$$

✓ Solution by Mathematica
Time used: 0.008 (sec). Leaf size: 39

```
DSolve[y'''[t]-3*y''[t]+3*y'[t]-y[t]==1/t*Exp[t],y[t],t,IncludeSingularSolutions -> True]
```

$$y(t) \to \frac{1}{4}e^t\left(2t^2\log(t) + (-3+4c_3)t^2 + 4c_2t + 4c_1\right)$$

14.20 problem 20

Internal problem ID [14695]

Book: INTRODUCTORY DIFFERENTIAL EQUATIONS. Martha L. Abell, James P. Braselton. Fourth edition 2014. ElScAe. 2014
Section: Chapter 4. Higher Order Equations. Exercises 4.6, page 187
Problem number: 20.
ODE order: 3.
ODE degree: 1.

CAS Maple gives this as type `[[_3rd_order, _with_linear_symmetries]]`

$$\boxed{y''' - 4y'' - 11y' + 30y = \mathrm{e}^{4t}}$$

✓ Solution by Maple
Time used: 0.016 (sec). Leaf size: 32

```
dsolve(diff(y(t),t$3)-4*diff(y(t),t$2)-11*diff(y(t),t)+30*y(t)=exp(4*t),y(t), singsol=all)
```

$$y(t) = -\frac{(-14c_3\mathrm{e}^{8t} + \mathrm{e}^{7t} - 14c_2\mathrm{e}^{5t} - 14c_1)\,\mathrm{e}^{-3t}}{14}$$

✓ Solution by Mathematica
Time used: 0.013 (sec). Leaf size: 39

```
DSolve[y'''[t]-4*y''[t]-11*y'[t]+30*y[t]==Exp[4*t],y[t],t,IncludeSingularSolutions -> True]
```

$$y(t) \to -\frac{e^{4t}}{14} + c_1e^{-3t} + c_2e^{2t} + c_3e^{5t}$$

14.21 problem 21

Internal problem ID [14696]

Book: INTRODUCTORY DIFFERENTIAL EQUATIONS. Martha L. Abell, James P. Braselton. Fourth edition 2014. ElScAe. 2014
Section: Chapter 4. Higher Order Equations. Exercises 4.6, page 187
Problem number: 21.
ODE order: 3.
ODE degree: 1.

CAS Maple gives this as type `[[_3rd_order, _with_linear_symmetries]]`

$$\boxed{y''' + 3y'' - 10y' - 24y = e^{-3t}}$$

✓ Solution by Maple

Time used: 0.016 (sec). Leaf size: 30

```
dsolve(diff(y(t),t$3)+3*diff(y(t),t$2)-10*diff(y(t),t)-24*y(t)=exp(-3*t),y(t), singsol=all)
```

$$y(t) = \frac{(6c_3 e^{7t} + 6c_2 e^{2t} + e^t + 6c_1) e^{-4t}}{6}$$

✓ Solution by Mathematica

Time used: 0.015 (sec). Leaf size: 40

```
DSolve[y'''[t]+3*y''[t]-10*y'[t]-24*y[t]==Exp[-3*t],y[t],t,IncludeSingularSolutions -> True]
```

$$y(t) \to \frac{1}{6} e^{-4t} \left(e^t + 6c_2 e^{2t} + 6c_3 e^{7t} + 6c_1\right)$$

14.22 problem 22

Internal problem ID [14697]

Book: INTRODUCTORY DIFFERENTIAL EQUATIONS. Martha L. Abell, James P. Braselton. Fourth edition 2014. ElScAe. 2014
Section: Chapter 4. Higher Order Equations. Exercises 4.6, page 187
Problem number: 22.
ODE order: 3.
ODE degree: 1.

CAS Maple gives this as type `[[_3rd_order, _linear, _nonhomogeneous]]`

$$\boxed{y''' - 13y' + 12y = \cos(t)}$$

✓ Solution by Maple
Time used: 0.0 (sec). Leaf size: 37

```
dsolve(diff(y(t),t$3)-13*diff(y(t),t)+12*y(t)=cos(t),y(t), singsol=all)
```

$$y(t) = \left(\left(\frac{3\cos(t)}{85} - \frac{7\sin(t)}{170}\right)e^{4t} + c_3 e^{7t} + c_1 e^{5t} + c_2\right)e^{-4t}$$

✓ Solution by Mathematica
Time used: 0.033 (sec). Leaf size: 40

```
DSolve[y'''[t]-13*y'[t]+12*y[t]==Cos[t],y[t],t,IncludeSingularSolutions -> True]
```

$$y(t) \to -\frac{7\sin(t)}{170} + \frac{3\cos(t)}{85} + c_1 e^{-4t} + c_2 e^{t} + c_3 e^{3t}$$

14.23 problem 23

Internal problem ID [14698]

Book: INTRODUCTORY DIFFERENTIAL EQUATIONS. Martha L. Abell, James P. Braselton. Fourth edition 2014. ElScAe. 2014
Section: Chapter 4. Higher Order Equations. Exercises 4.6, page 187
Problem number: 23.
ODE order: 3.
ODE degree: 1.

CAS Maple gives this as type `[[_3rd_order, _missing_y]]`

$$\boxed{y''' + 3y'' + 2y' = \cos(t)}$$

✓ Solution by Maple
Time used: 0.0 (sec). Leaf size: 28

```
dsolve(diff(y(t),t$3)+3*diff(y(t),t$2)+2*diff(y(t),t)=cos(t),y(t), singsol=all)
```

$$y(t) = -c_2 e^{-t} + \frac{e^{-2t} c_1}{2} - \frac{3\cos(t)}{10} + \frac{\sin(t)}{10} + c_3$$

✓ Solution by Mathematica
Time used: 0.218 (sec). Leaf size: 36

```
DSolve[y'''[t]+3*y''[t]+2*y'[t]==Cos[t],y[t],t,IncludeSingularSolutions -> True]
```

$$y(t) \to \frac{1}{10}\left(\sin(t) - 3\cos(t) - 5e^{-2t}\left(2c_2 e^t + c_1\right)\right) + c_3$$

14.24 problem 24

Internal problem ID [14699]

Book: INTRODUCTORY DIFFERENTIAL EQUATIONS. Martha L. Abell, James P. Braselton. Fourth edition 2014. ElScAe. 2014
Section: Chapter 4. Higher Order Equations. Exercises 4.6, page 187
Problem number: 24.
ODE order: 6.
ODE degree: 1.

CAS Maple gives this as type `[[_high_order, _missing_x]]`

$$\boxed{y^{(6)} + y'''' = -24}$$

✓ Solution by Maple
Time used: 0.0 (sec). Leaf size: 34

```
dsolve(diff(y(t),t$6)+diff(y(t),t$4)=-24,y(t), singsol=all)
```

$$y(t) = \frac{c_3 t^3}{6} - t^4 + \frac{c_4 t^2}{2} + \cos(t)\, c_1 + \sin(t)\, c_2 + c_5 t + c_6$$

✓ Solution by Mathematica
Time used: 0.149 (sec). Leaf size: 39

```
DSolve[y''''''[t]+y''''[t]==-24,y[t],t,IncludeSingularSolutions -> True]
```

$$y(t) \to -t^4 + c_6 t^3 + c_5 t^2 + c_4 t + c_1 \cos(t) + c_2 \sin(t) + c_3$$

14.25 problem 25

Internal problem ID [14700]

Book: INTRODUCTORY DIFFERENTIAL EQUATIONS. Martha L. Abell, James P. Braselton. Fourth edition 2014. ElScAe. 2014
Section: Chapter 4. Higher Order Equations. Exercises 4.6, page 187
Problem number: 25.
ODE order: 4.
ODE degree: 1.

CAS Maple gives this as type `[[_high_order, _missing_y]]`

$$\boxed{y'''' + y'' = \tan(t)^2}$$

✓ Solution by Maple
Time used: 0.015 (sec). Leaf size: 76

```
dsolve(diff(y(t),t$4)+diff(y(t),t$2)=tan(t)^2,y(t), singsol=all)
```

$$y(t) = \frac{\left(\int \left(\left(-\mathrm{e}^{-it} - \mathrm{e}^{it}\right) \ln\left(\frac{i\mathrm{e}^{it}-1}{-\mathrm{e}^{it}+i}\right) - 2i\ln\left(\mathrm{e}^{it}\right) + 2c_1 \sin(t) - 2c_2 \cos(t) - 4t\right) dt\right)}{2} + c_3 t + c_4$$

✓ Solution by Mathematica
Time used: 0.169 (sec). Leaf size: 47

```
DSolve[y''''[t]+y''[t]==Tan[t]^2,y[t],t,IncludeSingularSolutions -> True]
```

$$y(t) \to -\sin(t)\operatorname{arctanh}(\sin(t)) - \frac{t^2}{2} + c_4 t - \frac{1}{2}\log\left(\cos^2(t)\right) - c_1\cos(t) - c_2\sin(t) + c_3$$

14.26 problem 26

Internal problem ID [14701]

Book: INTRODUCTORY DIFFERENTIAL EQUATIONS. Martha L. Abell, James P. Braselton. Fourth edition 2014. ElScAe. 2014
Section: Chapter 4. Higher Order Equations. Exercises 4.6, page 187
Problem number: 26.
ODE order: 3.
ODE degree: 1.

CAS Maple gives this as type `[[_3rd_order, _missing_y]]`

$$\boxed{y''' - y'' = 3t^2}$$

With initial conditions

$$[y(0) = 0, y'(0) = 0, y''(0) = 0]$$

✓ Solution by Maple

Time used: 0.015 (sec). Leaf size: 28

```
dsolve([diff(y(t),t$3)-diff(y(t),t$2)=3*t^2,y(0) = 0, D(y)(0) = 0, (D@@2)(y)(0) = 0],y(t), s
```

$$y(t) = 6\,\mathrm{e}^{t} - 3t^2 - t^3 - \frac{t^4}{4} - 6t - 6$$

✓ Solution by Mathematica

Time used: 0.07 (sec). Leaf size: 32

```
DSolve[{y'''[t]-y''[t]==3*t^2,{y[0]==0,y'[0]==0,y''[0]==0}},y[t],t,IncludeSingularSolutions
```

$$y(t) \to -\frac{t^4}{4} - t^3 - 3t^2 - 6t + 6e^t - 6$$

14.27 problem 27

Internal problem ID [14702]

Book: INTRODUCTORY DIFFERENTIAL EQUATIONS. Martha L. Abell, James P. Braselton. Fourth edition 2014. ElScAe. 2014
Section: Chapter 4. Higher Order Equations. Exercises 4.6, page 187
Problem number: 27.
ODE order: 4.
ODE degree: 1.

CAS Maple gives this as type `[[_high_order, _missing_y]]`

$$\boxed{y'''' + y'' = \sec(t)^2}$$

With initial conditions

$$[y(0) = 0, y'(0) = 1, y''(0) = 0, y'''(0) = 0]$$

✓ Solution by Maple
Time used: 0.5 (sec). Leaf size: 71

```
dsolve([diff(y(t),t$4)+diff(y(t),t$2)=sec(t)^2,y(0) = 0, D(y)(0) = 1, (D@@2)(y)(0) = 0, (D@@
```

$$y(t) = \frac{\left(\int_0^t \left(\left(-\mathrm{e}^{i\_z1} - \mathrm{e}^{-i\_z1}\right)\ln\left(\frac{i\mathrm{e}^{i\_z1}-1}{-\mathrm{e}^{i\_z1}+i}\right) - 2i\ln\left(\mathrm{e}^{i\_z1}\right) - 2\_z1 + 2\sin(\_z1)\right) d\_z1\right)}{2} + t$$

✓ Solution by Mathematica
Time used: 0.129 (sec). Leaf size: 28

```
DSolve[{y''''[t]+y''[t]==Sec[t]^2,{y[0]==0,y'[0]==1,y''[0]==0,y'''[0]==0}},y[t],t,IncludeSin
```

$$y(t) \to -2\sin(t)\operatorname{arctanh}\left(\tan\left(\frac{t}{2}\right)\right) + t - \cos(t) - \log(\cos(t)) + 1$$

14.28 problem 28

Internal problem ID [14703]

Book: INTRODUCTORY DIFFERENTIAL EQUATIONS. Martha L. Abell, James P. Braselton. Fourth edition 2014. ElScAe. 2014
Section: Chapter 4. Higher Order Equations. Exercises 4.6, page 187
Problem number: 28.
ODE order: 3.
ODE degree: 1.

CAS Maple gives this as type `[[_3rd_order, _missing_y]]`

$$\boxed{y''' + y' = \sec(t)}$$

With initial conditions

$$[y(0) = 0, y'(0) = 0, y''(0) = 1]$$

✓ Solution by Maple
Time used: 0.094 (sec). Leaf size: 84

```
dsolve([diff(y(t),t$3)+diff(y(t),t)=sec(t),y(0) = 0, D(y)(0) = 0, (D@@2)(y)(0) = 1],y(t), si
```

$$y(t) = 1 - \sin(t)\ln\left(\frac{e^{it}}{e^{2it}+1}\right) - \frac{ie^{-it}}{2} - 2i\arctan\left(e^{it}\right) + \frac{ie^{it}}{2} - t\cos(t) - \cos(t) - \ln(2)\sin(t) + \sin(t) + \frac{i\pi}{2}$$

✓ Solution by Mathematica
Time used: 0.065 (sec). Leaf size: 52

```
DSolve[{y'''[t]+y'[t]==Sec[t],{y[0]==0,y'[0]==0,y''[0]==1}},y[t],t,IncludeSingularSolutions
```

$$y(t) \to -((t+1)\cos(t)) - \log\left(\cos\left(\frac{t}{2}\right) - \sin\left(\frac{t}{2}\right)\right) + \log\left(\sin\left(\frac{t}{2}\right) + \cos\left(\frac{t}{2}\right)\right) + \sin(t)\log(\cos(t)) + 1$$

14.29 problem 29

Internal problem ID [14704]

Book: INTRODUCTORY DIFFERENTIAL EQUATIONS. Martha L. Abell, James P. Braselton. Fourth edition 2014. ElScAe. 2014
Section: Chapter 4. Higher Order Equations. Exercises 4.6, page 187
Problem number: 29.
ODE order: 4.
ODE degree: 1.

CAS Maple gives this as type `[[_high_order, _missing_y]]`

$$\boxed{y'''' + y'' = \cos(t)}$$

With initial conditions

$$[y(0) = 0, y'(0) = 0, y''(0) = 1, y'''(0) = 0]$$

✓ Solution by Maple

Time used: 0.016 (sec). Leaf size: 15

```
dsolve([diff(y(t),t$4)+diff(y(t),t$2)=cos(t),y(0) = 0, D(y)(0) = 0, (D@@2)(y)(0) = 1, (D@@3)
```

$$y(t) = -\frac{\sin(t)\, t}{2} - 2\cos(t) + 2$$

✓ Solution by Mathematica

Time used: 0.114 (sec). Leaf size: 18

```
DSolve[{y''''[t]+y''[t]==Cos[t],{y[0]==0,y'[0]==0,y''[0]==1,y'''[0]==0}},y[t],t,IncludeSingu
```

$$y(t) \to -\frac{1}{2} t \sin(t) - 2\cos(t) + 2$$

14.30 problem 30

Internal problem ID [14705]

Book: INTRODUCTORY DIFFERENTIAL EQUATIONS. Martha L. Abell, James P. Braselton. Fourth edition 2014. ElScAe. 2014
Section: Chapter 4. Higher Order Equations. Exercises 4.6, page 187
Problem number: 30.
ODE order: 4.
ODE degree: 1.

CAS Maple gives this as type `[[_high_order, _missing_y]]`

$$\boxed{y'''' + y'' = t}$$

With initial conditions

$$[y(0) = 0, y'(0) = 0, y''(0) = 1, y'''(0) = 0]$$

✓ Solution by Maple
Time used: 0.015 (sec). Leaf size: 20

```
dsolve([diff(y(t),t$4)+diff(y(t),t$2)=t,y(0) = 0, D(y)(0) = 0, (D@@2)(y)(0) = 1, (D@@3)(y)(0
```

$$y(t) = \frac{t^3}{6} - \cos(t) + \sin(t) - t + 1$$

✓ Solution by Mathematica
Time used: 0.025 (sec). Leaf size: 23

```
DSolve[{y''''[t]+y''[t]==t,{y[0]==0,y'[0]==0,y''[0]==1,y'''[0]==0}},y[t],t,IncludeSingularSo
```

$$y(t) \to \frac{t^3}{6} - t + \sin(t) - \cos(t) + 1$$

14.31 problem 31

Internal problem ID [14706]

Book: INTRODUCTORY DIFFERENTIAL EQUATIONS. Martha L. Abell, James P. Braselton. Fourth edition 2014. ElScAe. 2014
Section: Chapter 4. Higher Order Equations. Exercises 4.6, page 187
Problem number: 31.
ODE order: 3.
ODE degree: 1.

CAS Maple gives this as type `[[_3rd_order, _missing_y]]`

$$\boxed{t^2 \ln(t)\, y''' - t y'' + y' = 1}$$

✓ Solution by Maple
Time used: 0.016 (sec). Leaf size: 19

```
dsolve(t^2*ln(t)*diff(y(t),t$3)-t*diff(y(t),t$2)+diff(y(t),t)=1,y(t), singsol=all)
```

$$y(t) = \frac{c_2 t^2}{2} - \ln(t)\, c_1 t + t + c_3$$

✓ Solution by Mathematica
Time used: 0.082 (sec). Leaf size: 25

```
DSolve[t^2*Log[t]*y'''[t]-t*y''[t]+y'[t]==1,y[t],t,IncludeSingularSolutions -> True]
```

$$y(t) \to \frac{c_1 t^2}{2} + t - c_2 t \log(t) + c_3$$

14.32 problem 32

Internal problem ID [14707]

Book: INTRODUCTORY DIFFERENTIAL EQUATIONS. Martha L. Abell, James P. Braselton. Fourth edition 2014. ElScAe. 2014
Section: Chapter 4. Higher Order Equations. Exercises 4.6, page 187
Problem number: 32.
ODE order: 3.
ODE degree: 1.

CAS Maple gives this as type `[[_3rd_order, _missing_y]]`

$$\boxed{(t^2+t)\,y''' + (-t^2+2)\,y'' - (t+2)\,y' = -t-2}$$

✓ Solution by Maple
Time used: 0.0 (sec). Leaf size: 15

```
dsolve((t^2+t)*diff(y(t),t$3)+(2-t^2)*diff(y(t),t$2)-(t+2)*diff(y(t),t)=-2-t,y(t), singsol=a
```

$$y(t) = c_2 \ln(t) + e^t c_1 + t + c_3$$

✓ Solution by Mathematica
Time used: 0.479 (sec). Leaf size: 38

```
DSolve[(t^2+t)*y'''[t]+(2-t^2)*y''[t]-(t+2)*y'[t]==-2-t,y[t],t,IncludeSingularSolutions -> T
```

$$y(t) \to t + \log(t) + \sqrt{2}c_2 e^{t+\frac{1}{2}} + \frac{c_1 \log(t)}{\sqrt{2e}} + c_3$$

14.33 problem 33

Internal problem ID [14708]

Book: INTRODUCTORY DIFFERENTIAL EQUATIONS. Martha L. Abell, James P. Braselton. Fourth edition 2014. ElScAe. 2014
Section: Chapter 4. Higher Order Equations. Exercises 4.6, page 187
Problem number: 33.
ODE order: 3.
ODE degree: 1.

CAS Maple gives this as type `[[_3rd_order, _with_linear_symmetries]]`

$$\boxed{2t^3y''' + t^2y'' + y't - y = -3t^2}$$

With initial conditions

$$[y(1) = 0, y'(1) = 1, y''(1) = 0]$$

✓ Solution by Maple
Time used: 0.031 (sec). Leaf size: 14

```
dsolve([2*t^3*diff(y(t),t$3)+t^2*diff(y(t),t$2)+t*diff(y(t),t)-y(t)=-3*t^2,y(1) = 0, D(y)(1)
```

$$y(t) = -t(-2\ln(t) + t - 1)$$

✓ Solution by Mathematica
Time used: 0.008 (sec). Leaf size: 16

```
DSolve[{2*t^3*y'''[t]+t^2*y''[t]+t*y'[t]-y[t]==-3*t^2,{y[1]==0,y'[1]==1,y''[1]==0}},y[t],t,I
```

$$y(t) \to t(-t + 2\log(t) + 1)$$

14.34 problem 34

Internal problem ID [14709]

Book: INTRODUCTORY DIFFERENTIAL EQUATIONS. Martha L. Abell, James P. Braselton. Fourth edition 2014. ElScAe. 2014
Section: Chapter 4. Higher Order Equations. Exercises 4.6, page 187
Problem number: 34.
ODE order: 4.
ODE degree: 1.

CAS Maple gives this as type `[[_high_order, _missing_y]]`

$$\boxed{ty'''' + 2y''' = \frac{45}{8t^{\frac{7}{2}}}}$$

With initial conditions

$$[y(1) = 0, y'(1) = 0, y''(1) = 1, y'''(1) = 0]$$

✓ Solution by Maple
Time used: 0.047 (sec). Leaf size: 24

```
dsolve([t*diff(y(t),t$4)+2*diff(y(t),t$3)=45/8*1/t^(7/2),y(1) = 0, D(y)(1) = 0, (D@@2)(y)(1)
```

$$y(t) = \frac{13t^2}{8} + \frac{2}{\sqrt{t}} - \frac{15t\ln(t)}{4} + \frac{3t}{2} - \frac{41}{8}$$

✓ Solution by Mathematica
Time used: 0.098 (sec). Leaf size: 31

```
DSolve[{t*y''''[t]+2*y'''[t]==45/8*1/t^(7/2),{y[1]==0,y'[1]==0,y''[1]==1,y'''[1]==0}},y[t],t
```

$$y(t) \to \frac{1}{8}\left(13t^2 + 12t + \frac{16}{\sqrt{t}} - 30t\log(t) - 41\right)$$

15 Chapter 4. Higher Order Equations. Exercises 4.7, page 195

15.1 problem 1

Internal problem ID [14710]

Book: INTRODUCTORY DIFFERENTIAL EQUATIONS. Martha L. Abell, James P. Braselton. Fourth edition 2014. ElScAe. 2014
Section: Chapter 4. Higher Order Equations. Exercises 4.7, page 195
Problem number: 1.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_Emden, _Fowler], [_2nd_order, _linear, '_with_symmetry_[0,F`

$$\boxed{4x^2y'' - 8y'x + 5y = 0}$$

✓ Solution by Maple

Time used: 0.015 (sec). Leaf size: 15

```
dsolve(4*x^2*diff(y(x),x$2)-8*x*diff(y(x),x)+5*y(x)=0,y(x), singsol=all)
```

$$y(x) = \sqrt{x}\,(c_1x^2 + c_2)$$

✓ Solution by Mathematica

Time used: 0.011 (sec). Leaf size: 20

```
DSolve[4*x^2*y''[x]-8*x*y'[x]+5*y[x]==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to \sqrt{x}(c_2x^2 + c_1)$$

15.2 problem 2

Internal problem ID [14711]

Book: INTRODUCTORY DIFFERENTIAL EQUATIONS. Martha L. Abell, James P. Braselton. Fourth edition 2014. ElScAe. 2014
Section: Chapter 4. Higher Order Equations. Exercises 4.7, page 195
Problem number: 2.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_Emden, _Fowler], [_2nd_order, _linear, '_with_symmetry_[0,F`

$$\boxed{3x^2y'' - 4y'x + 2y = 0}$$

✓ Solution by Maple
Time used: 0.016 (sec). Leaf size: 15

```
dsolve(3*x^2*diff(y(x),x$2)-4*x*diff(y(x),x)+2*y(x)=0,y(x), singsol=all)
```

$$y(x) = c_1 x^{\frac{1}{3}} + c_2 x^2$$

✓ Solution by Mathematica
Time used: 0.012 (sec). Leaf size: 20

```
DSolve[3*x^2*y''[x]-4*x*y'[x]+2*y[x]==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to c_2 x^2 + c_1 \sqrt[3]{x}$$

15.3 problem 3

Internal problem ID [14712]

Book: INTRODUCTORY DIFFERENTIAL EQUATIONS. Martha L. Abell, James P. Braselton. Fourth edition 2014. ElScAe. 2014
Section: Chapter 4. Higher Order Equations. Exercises 4.7, page 195
Problem number: 3.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_Emden, _Fowler]]`

$$\boxed{2x^2y'' - 8y'x + 8y = 0}$$

✓ Solution by Maple
Time used: 0.0 (sec). Leaf size: 13

```
dsolve(2*x^2*diff(y(x),x$2)-8*x*diff(y(x),x)+8*y(x)=0,y(x), singsol=all)
```

$$y(x) = x(c_1x^3 + c_2)$$

✓ Solution by Mathematica
Time used: 0.009 (sec). Leaf size: 16

```
DSolve[2*x^2*y''[x]-8*x*y'[x]+8*y[x]==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to x(c_2x^3 + c_1)$$

15.4 problem 4

Internal problem ID [14713]

Book: INTRODUCTORY DIFFERENTIAL EQUATIONS. Martha L. Abell, James P. Braselton. Fourth edition 2014. ElScAe. 2014
Section: Chapter 4. Higher Order Equations. Exercises 4.7, page 195
Problem number: 4.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_Emden, _Fowler]]`

$$\boxed{2x^2y'' - 7y'x + 7y = 0}$$

✓ Solution by Maple
Time used: 0.0 (sec). Leaf size: 13

```
dsolve(2*x^2*diff(y(x),x$2)-7*x*diff(y(x),x)+7*y(x)=0,y(x), singsol=all)
```

$$y(x) = x^{\frac{7}{2}}c_1 + c_2x$$

✓ Solution by Mathematica
Time used: 0.01 (sec). Leaf size: 18

```
DSolve[2*x^2*y''[x]-7*x*y'[x]+7*y[x]==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to x\left(c_2x^{5/2} + c_1\right)$$

15.5 problem 5

Internal problem ID [14714]

Book: INTRODUCTORY DIFFERENTIAL EQUATIONS. Martha L. Abell, James P. Braselton. Fourth edition 2014. ElScAe. 2014
Section: Chapter 4. Higher Order Equations. Exercises 4.7, page 195
Problem number: 5.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_Emden, _Fowler]]`

$$\boxed{4x^2y'' + 17y = 0}$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 23

```
dsolve(4*x^2*diff(y(x),x$2)+17*y(x)=0,y(x), singsol=all)
```

$$y(x) = \sqrt{x}\,(c_1 \sin(2\ln(x)) + c_2 \cos(2\ln(x)))$$

✓ Solution by Mathematica

Time used: 0.027 (sec). Leaf size: 28

```
DSolve[4*x^2*y''[x]+17*y[x]==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to \sqrt{x}(c_2 \cos(2\log(x)) + c_1 \sin(2\log(x)))$$

15.6 problem 6

Internal problem ID [14715]

Book: INTRODUCTORY DIFFERENTIAL EQUATIONS. Martha L. Abell, James P. Braselton. Fourth edition 2014. ElScAe. 2014
Section: Chapter 4. Higher Order Equations. Exercises 4.7, page 195
Problem number: 6.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_Emden, _Fowler]]`

$$\boxed{9x^2y'' - 9y'x + 10y = 0}$$

✓ Solution by Maple
Time used: 0.0 (sec). Leaf size: 21

```
dsolve(9*x^2*diff(y(x),x$2)-9*x*diff(y(x),x)+10*y(x)=0,y(x), singsol=all)
```

$$y(x) = x\left(c_1 \sin\left(\frac{\ln(x)}{3}\right) + c_2 \cos\left(\frac{\ln(x)}{3}\right)\right)$$

✓ Solution by Mathematica
Time used: 0.026 (sec). Leaf size: 28

```
DSolve[9*x^2*y''[x]-9*x*y'[x]+10*y[x]==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to x\left(c_2 \cos\left(\frac{\log(x)}{3}\right) + c_1 \sin\left(\frac{\log(x)}{3}\right)\right)$$

15.7 problem 7

Internal problem ID [14716]

Book: INTRODUCTORY DIFFERENTIAL EQUATIONS. Martha L. Abell, James P. Braselton. Fourth edition 2014. ElScAe. 2014
Section: Chapter 4. Higher Order Equations. Exercises 4.7, page 195
Problem number: 7.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_Emden, _Fowler]]`

$$\boxed{2x^2y'' - 2y'x + 20y = 0}$$

✓ Solution by Maple
Time used: 0.0 (sec). Leaf size: 21

```
dsolve(2*x^2*diff(y(x),x$2)-2*x*diff(y(x),x)+20*y(x)=0,y(x), singsol=all)
```

$$y(x) = x(c_1 \sin(3\ln(x)) + c_2 \cos(3\ln(x)))$$

✓ Solution by Mathematica
Time used: 0.025 (sec). Leaf size: 24

```
DSolve[2*x^2*y''[x]-2*x*y'[x]+20*y[x]==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to x(c_2 \cos(3\log(x)) + c_1 \sin(3\log(x)))$$

15.8 problem 8

Internal problem ID [14717]

Book: INTRODUCTORY DIFFERENTIAL EQUATIONS. Martha L. Abell, James P. Braselton. Fourth edition 2014. ElScAe. 2014
Section: Chapter 4. Higher Order Equations. Exercises 4.7, page 195
Problem number: 8.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_Emden, _Fowler]]`

$$\boxed{x^2y'' - 5y'x + 10y = 0}$$

✓ Solution by Maple
Time used: 0.0 (sec). Leaf size: 19

```
dsolve(x^2*diff(y(x),x$2)-5*x*diff(y(x),x)+10*y(x)=0,y(x), singsol=all)
```

$$y(x) = x^3(c_1 \sin(\ln(x)) + c_2 \cos(\ln(x)))$$

✓ Solution by Mathematica
Time used: 0.023 (sec). Leaf size: 22

```
DSolve[x^2*y''[x]-5*x*y'[x]+10*y[x]==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to x^3(c_2 \cos(\log(x)) + c_1 \sin(\log(x)))$$

15.9 problem 9

Internal problem ID [14718]

Book: INTRODUCTORY DIFFERENTIAL EQUATIONS. Martha L. Abell, James P. Braselton. Fourth edition 2014. ElScAe. 2014
Section: Chapter 4. Higher Order Equations. Exercises 4.7, page 195
Problem number: 9.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_Emden, _Fowler]]`

$$\boxed{4x^2y'' + 8y'x + y = 0}$$

✓ Solution by Maple
Time used: 0.0 (sec). Leaf size: 14

```
dsolve(4*x^2*diff(y(x),x$2)+8*x*diff(y(x),x)+y(x)=0,y(x), singsol=all)
```

$$y(x) = \frac{c_1 + c_2 \ln(x)}{\sqrt{x}}$$

✓ Solution by Mathematica
Time used: 0.017 (sec). Leaf size: 24

```
DSolve[4*x^2*y''[x]+8*x*y'[x]+y[x]==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to \frac{c_2 \log(x) + 2c_1}{2\sqrt{x}}$$

15.10 problem 10

Internal problem ID [14719]

Book: INTRODUCTORY DIFFERENTIAL EQUATIONS. Martha L. Abell, James P. Braselton. Fourth edition 2014. ElScAe. 2014
Section: Chapter 4. Higher Order Equations. Exercises 4.7, page 195
Problem number: 10.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_Emden, _Fowler]]`

$$\boxed{4x^2y'' + y = 0}$$

✓ Solution by Maple
Time used: 0.016 (sec). Leaf size: 14

```
dsolve(4*x^2*diff(y(x),x$2)+y(x)=0,y(x), singsol=all)
```

$$y(x) = (c_1 + c_2 \ln(x)) \sqrt{x}$$

✓ Solution by Mathematica
Time used: 0.017 (sec). Leaf size: 24

```
DSolve[4*x^2*y''[x]+y[x]==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to \frac{1}{2}\sqrt{x}(c_2 \log(x) + 2c_1)$$

15.11 problem 11

Internal problem ID [14720]

Book: INTRODUCTORY DIFFERENTIAL EQUATIONS. Martha L. Abell, James P. Braselton. Fourth edition 2014. ElScAe. 2014
Section: Chapter 4. Higher Order Equations. Exercises 4.7, page 195
Problem number: 11.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_Emden, _Fowler]]`

$$\boxed{x^2y'' - 5y'x + 9y = 0}$$

✓ Solution by Maple
Time used: 0.0 (sec). Leaf size: 14

```
dsolve(x^2*diff(y(x),x$2)-5*x*diff(y(x),x)+9*y(x)=0,y(x), singsol=all)
```

$$y(x) = x^3(c_1 + c_2 \ln(x))$$

✓ Solution by Mathematica
Time used: 0.015 (sec). Leaf size: 18

```
DSolve[x^2*y''[x]-5*x*y'[x]+9*y[x]==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to x^3(3c_2 \log(x) + c_1)$$

15.12 problem 12

Internal problem ID [14721]

Book: INTRODUCTORY DIFFERENTIAL EQUATIONS. Martha L. Abell, James P. Braselton. Fourth edition 2014. ElScAe. 2014
Section: Chapter 4. Higher Order Equations. Exercises 4.7, page 195
Problem number: 12.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_Emden, _Fowler]]`

$$\boxed{x^2y'' + 7y'x + 9y = 0}$$

✓ Solution by Maple
Time used: 0.0 (sec). Leaf size: 14

```
dsolve(x^2*diff(y(x),x$2)+7*x*diff(y(x),x)+9*y(x)=0,y(x), singsol=all)
```

$$y(x) = \frac{c_1 + c_2 \ln (x)}{x^3}$$

✓ Solution by Mathematica
Time used: 0.015 (sec). Leaf size: 18

```
DSolve[x^2*y''[x]+7*x*y'[x]+9*y[x]==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to \frac{3c_2 \log(x) + c_1}{x^3}$$

15.13 problem 13

Internal problem ID [14722]

Book: INTRODUCTORY DIFFERENTIAL EQUATIONS. Martha L. Abell, James P. Braselton. Fourth edition 2014. ElScAe. 2014
Section: Chapter 4. Higher Order Equations. Exercises 4.7, page 195
Problem number: 13.
ODE order: 3.
ODE degree: 1.

CAS Maple gives this as type `[[_3rd_order, _with_linear_symmetries]]`

$$\boxed{x^3y''' + 22x^2y'' + 124y'x + 140y = 0}$$

✓ Solution by Maple
Time used: 0.015 (sec). Leaf size: 20

```
dsolve(x^3*diff(y(x),x$3)+22*x^2*diff(y(x),x$2)+124*x*diff(y(x),x)+140*y(x)=0,y(x), singsol=
```

$$y(x) = \frac{c_3x^8 + c_1x^3 + c_2}{x^{10}}$$

✓ Solution by Mathematica
Time used: 0.005 (sec). Leaf size: 24

```
DSolve[x^3*y'''[x]+22*x^2*y''[x]+124*x*y'[x]+140*y[x]==0,y[x],x,IncludeSingularSolutions ->
```

$$y(x) \to \frac{c_3x^8 + c_2x^3 + c_1}{x^{10}}$$

15.14 problem 14

Internal problem ID [14723]

Book: INTRODUCTORY DIFFERENTIAL EQUATIONS. Martha L. Abell, James P. Braselton. Fourth edition 2014. ElScAe. 2014
Section: Chapter 4. Higher Order Equations. Exercises 4.7, page 195
Problem number: 14.
ODE order: 3.
ODE degree: 1.

CAS Maple gives this as type `[[_3rd_order, _with_linear_symmetries]]`

$$\boxed{x^3y''' - 4x^2y'' - 46y'x + 100y = 0}$$

✓ Solution by Maple
Time used: 0.0 (sec). Leaf size: 20

```
dsolve(x^3*diff(y(x),x$3)-4*x^2*diff(y(x),x$2)-46*x*diff(y(x),x)+100*y(x)=0,y(x), singsol=al
```

$$y(x) = \frac{c_2x^{15} + c_3x^7 + c_1}{x^5}$$

✓ Solution by Mathematica
Time used: 0.004 (sec). Leaf size: 24

```
DSolve[x^3*y'''[x]-4*x^2*y''[x]-46*x*y'[x]+100*y[x]==0,y[x],x,IncludeSingularSolutions -> Tr
```

$$y(x) \to \frac{c_3x^{15} + c_2x^7 + c_1}{x^5}$$

15.15 problem 15

Internal problem ID [14724]

Book: INTRODUCTORY DIFFERENTIAL EQUATIONS. Martha L. Abell, James P. Braselton. Fourth edition 2014. ElScAe. 2014
Section: Chapter 4. Higher Order Equations. Exercises 4.7, page 195
Problem number: 15.
ODE order: 3.
ODE degree: 1.

CAS Maple gives this as type `[[_3rd_order, _with_linear_symmetries]]`

$$\boxed{x^3y''' + 2x^2y'' - 4y'x + 4y = 0}$$

✓ Solution by Maple
Time used: 0.0 (sec). Leaf size: 20

```
dsolve(x^3*diff(y(x),x$3)+2*x^2*diff(y(x),x$2)-4*x*diff(y(x),x)+4*y(x)=0,y(x), singsol=all)
```

$$y(x) = \frac{c_2x^4 + c_1x^3 + c_3}{x^2}$$

✓ Solution by Mathematica
Time used: 0.004 (sec). Leaf size: 22

```
DSolve[x^3*y'''[x]+2*x^2*y''[x]-4*x*y'[x]+4*y[x]==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to c_3x^2 + \frac{c_1}{x^2} + c_2x$$

15.16 problem 16

Internal problem ID [14725]

Book: INTRODUCTORY DIFFERENTIAL EQUATIONS. Martha L. Abell, James P. Braselton. Fourth edition 2014. ElScAe. 2014
Section: Chapter 4. Higher Order Equations. Exercises 4.7, page 195
Problem number: 16.
ODE order: 3.
ODE degree: 1.

CAS Maple gives this as type `[[_3rd_order, _exact, _linear, _homogeneous]]`

$$\boxed{x^3y''' + 4x^2y'' + 6y'x + 4y = 0}$$

✓ Solution by Maple
Time used: 0.0 (sec). Leaf size: 24

```
dsolve(x^3*diff(y(x),x$3)+4*x^2*diff(y(x),x$2)+6*x*diff(y(x),x)+4*y(x)=0,y(x), singsol=all)
```

$$y(x) = \frac{c_1}{x} + c_2 \sin(2\ln(x)) + c_3 \cos(2\ln(x))$$

✓ Solution by Mathematica
Time used: 0.004 (sec). Leaf size: 28

```
DSolve[x^3*y'''[x]+4*x^2*y''[x]+6*x*y'[x]+4*y[x]==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to \frac{c_3}{x} + c_1 \cos(2\log(x)) + c_2 \sin(2\log(x))$$

15.17 problem 17

Internal problem ID [14726]

Book: INTRODUCTORY DIFFERENTIAL EQUATIONS. Martha L. Abell, James P. Braselton. Fourth edition 2014. ElScAe. 2014
Section: Chapter 4. Higher Order Equations. Exercises 4.7, page 195
Problem number: 17.
ODE order: 3.
ODE degree: 1.

CAS Maple gives this as type `[[_3rd_order, _with_linear_symmetries]]`

$$\boxed{x^3y''' + 2y'x - 2y = 0}$$

✓ Solution by Maple
Time used: 0.0 (sec). Leaf size: 18

```
dsolve(x^3*diff(y(x),x$3)+2*x*diff(y(x),x)-2*y(x)=0,y(x), singsol=all)
```

$$y(x) = x(c_1 + c_2 \sin(\ln(x)) + c_3 \cos(\ln(x)))$$

✓ Solution by Mathematica
Time used: 0.003 (sec). Leaf size: 22

```
DSolve[x^3*y'''[x]+2*x*y'[x]-2*y[x]==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to x(c_2 \cos(\log(x)) + c_1 \sin(\log(x)) + c_3)$$

15.18 problem 18

Internal problem ID [14727]

Book: INTRODUCTORY DIFFERENTIAL EQUATIONS. Martha L. Abell, James P. Braselton. Fourth edition 2014. ElScAe. 2014
Section: Chapter 4. Higher Order Equations. Exercises 4.7, page 195
Problem number: 18.
ODE order: 3.
ODE degree: 1.

CAS Maple gives this as type `[[_3rd_order, _exact, _linear, _homogeneous]]`

$$\boxed{x^3y''' + 3x^2y'' - 2y'x - 2y = 0}$$

✓ Solution by Maple
Time used: 0.015 (sec). Leaf size: 19

```
dsolve(x^3*diff(y(x),x$3)+3*x^2*diff(y(x),x$2)-2*x*diff(y(x),x)-2*y(x)=0,y(x), singsol=all)
```

$$y(x) = \frac{c_1x^3 + c_3\ln(x) + c_2}{x}$$

✓ Solution by Mathematica
Time used: 0.003 (sec). Leaf size: 23

```
DSolve[x^3*y'''[x]+3*x^2*y''[x]-2*x*y'[x]-2*y[x]==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to \frac{c_3x^3 + c_2\log(x) + c_1}{x}$$

15.19 problem 19

Internal problem ID [14728]

Book: INTRODUCTORY DIFFERENTIAL EQUATIONS. Martha L. Abell, James P. Braselton. Fourth edition 2014. ElScAe. 2014
Section: Chapter 4. Higher Order Equations. Exercises 4.7, page 195
Problem number: 19.
ODE order: 3.
ODE degree: 1.

CAS Maple gives this as type `[[_3rd_order, _exact, _linear, _homogeneous]]`

$$\boxed{x^3 y''' + 6x^2 y'' + 7y'x + y = 0}$$

✓ Solution by Maple
Time used: 0.0 (sec). Leaf size: 20

```
dsolve(x^3*diff(y(x),x$3)+6*x^2*diff(y(x),x$2)+7*x*diff(y(x),x)+y(x)=0,y(x), singsol=all)
```

$$y(x) = \frac{c_1 + c_2 \ln(x) + c_3 \ln(x)^2}{x}$$

✓ Solution by Mathematica
Time used: 0.003 (sec). Leaf size: 24

```
DSolve[x^3*y'''[x]+6*x^2*y''[x]+7*x*y'[x]+y[x]==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to \frac{c_3 \log^2(x) + c_2 \log(x) + c_1}{x}$$

15.20 problem 20

Internal problem ID [14729]

Book: INTRODUCTORY DIFFERENTIAL EQUATIONS. Martha L. Abell, James P. Braselton. Fourth edition 2014. ElScAe. 2014
Section: Chapter 4. Higher Order Equations. Exercises 4.7, page 195
Problem number: 20.
ODE order: 4.
ODE degree: 1.

CAS Maple gives this as type `[[_high_order, _missing_y]]`

$$\boxed{x^3y'''' + 6x^2y''' + 7xy'' + y' = 0}$$

✓ Solution by Maple

Time used: 0.015 (sec). Leaf size: 22

```
dsolve(x^3*diff(y(x),x$4)+6*x^2*diff(y(x),x$3)+7*x*diff(y(x),x$2)+diff(y(x),x)=0,y(x), sings
```

$$y(x) = c_4 \ln(x)^3 + c_3 \ln(x)^2 + c_2 \ln(x) + c_1$$

✓ Solution by Mathematica

Time used: 0.007 (sec). Leaf size: 33

```
DSolve[x^3*y''''[x]+6*x^2*y'''[x]+7*x*y''[x]+y'[x]==0,y[x],x,IncludeSingularSolutions -> Tru
```

$$y(x) \to \frac{1}{3}c_3 \log^3(x) + \frac{1}{2}c_2 \log^2(x) + c_1 \log(x) + c_4$$

15.21 problem 21

Internal problem ID [14730]

Book: INTRODUCTORY DIFFERENTIAL EQUATIONS. Martha L. Abell, James P. Braselton. Fourth edition 2014. ElScAe. 2014
Section: Chapter 4. Higher Order Equations. Exercises 4.7, page 195
Problem number: 21.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$\boxed{x^2y'' + 5y'x + 4y = \frac{1}{x^5}}$$

✓ Solution by Maple
Time used: 0.0 (sec). Leaf size: 22

```
dsolve(x^2*diff(y(x),x$2)+5*x*diff(y(x),x)+4*y(x)=1/x^5,y(x), singsol=all)
```

$$y(x) = \frac{c_2}{x^2} + \frac{\ln(x)\, c_1}{x^2} + \frac{1}{9x^5}$$

✓ Solution by Mathematica
Time used: 0.105 (sec). Leaf size: 28

```
DSolve[x^2*y''[x]+5*x*y'[x]+4*y[x]==1/x^5,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to \frac{1}{9x^5} + \frac{c_1}{x^2} + \frac{2c_2\log(x)}{x^2}$$

15.22 problem 22

Internal problem ID [14731]

Book: INTRODUCTORY DIFFERENTIAL EQUATIONS. Martha L. Abell, James P. Braselton. Fourth edition 2014. ElScAe. 2014
Section: Chapter 4. Higher Order Equations. Exercises 4.7, page 195
Problem number: 22.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$\boxed{x^2y'' - 5y'x + 9y = x^3}$$

✓ Solution by Maple
Time used: 0.015 (sec). Leaf size: 20

```
dsolve(x^2*diff(y(x),x$2)-5*x*diff(y(x),x)+9*y(x)=x^3,y(x), singsol=all)
```

$$y(x) = x^3\left(c_2 + c_1\ln(x) + \frac{\ln(x)^2}{2}\right)$$

✓ Solution by Mathematica
Time used: 0.021 (sec). Leaf size: 27

```
DSolve[x^2*y''[x]-5*x*y'[x]+9*y[x]==x^3,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to \frac{1}{2}x^3\left(\log^2(x) + 6c_2\log(x) + 2c_1\right)$$

15.23 problem 23

Internal problem ID [14732]

Book: INTRODUCTORY DIFFERENTIAL EQUATIONS. Martha L. Abell, James P. Braselton. Fourth edition 2014. ElScAe. 2014
Section: Chapter 4. Higher Order Equations. Exercises 4.7, page 195
Problem number: 23.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$\boxed{x^2 y'' + y'x + y = \frac{1}{x^2}}$$

✓ Solution by Maple
Time used: 0.0 (sec). Leaf size: 20

```
dsolve(x^2*diff(y(x),x$2)+x*diff(y(x),x)+y(x)=1/x^2,y(x), singsol=all)
```

$$y(x) = c_2 \sin(\ln(x)) + \cos(\ln(x)) c_1 + \frac{1}{5x^2}$$

✓ Solution by Mathematica
Time used: 0.054 (sec). Leaf size: 25

```
DSolve[x^2*y''[x]+x*y'[x]+y[x]==1/x^2,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to \frac{1}{5x^2} + c_1 \cos(\log(x)) + c_2 \sin(\log(x))$$

15.24 problem 24

Internal problem ID [14733]

Book: INTRODUCTORY DIFFERENTIAL EQUATIONS. Martha L. Abell, James P. Braselton. Fourth edition 2014. ElScAe. 2014
Section: Chapter 4. Higher Order Equations. Exercises 4.7, page 195
Problem number: 24.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$\boxed{x^2y'' + y'x + 4y = \frac{1}{x^2}}$$

✓ Solution by Maple
Time used: 0.0 (sec). Leaf size: 24

```
dsolve(x^2*diff(y(x),x$2)+x*diff(y(x),x)+4*y(x)=1/x^2,y(x), singsol=all)
```

$$y(x) = c_2 \sin(2\ln(x)) + \cos(2\ln(x))\, c_1 + \frac{1}{8x^2}$$

✓ Solution by Mathematica
Time used: 0.053 (sec). Leaf size: 29

```
DSolve[x^2*y''[x]+x*y'[x]+4*y[x]==1/x^2,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to \frac{1}{8x^2} + c_1 \cos(2\log(x)) + c_2 \sin(2\log(x))$$

15.25 problem 25

Internal problem ID [14734]

Book: INTRODUCTORY DIFFERENTIAL EQUATIONS. Martha L. Abell, James P. Braselton. Fourth edition 2014. ElScAe. 2014
Section: Chapter 4. Higher Order Equations. Exercises 4.7, page 195
Problem number: 25.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$\boxed{x^2 y'' + 2y'x - 6y = 2x}$$

✓ Solution by Maple
Time used: 0.016 (sec). Leaf size: 18

```
dsolve(x^2*diff(y(x),x$2)+2*x*diff(y(x),x)-6*y(x)=2*x,y(x), singsol=all)
```

$$y(x) = \frac{c_2}{x^3} + c_1 x^2 - \frac{x}{2}$$

✓ Solution by Mathematica
Time used: 0.014 (sec). Leaf size: 23

```
DSolve[x^2*y''[x]+2*x*y'[x]-6*y[x]==2*x,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to \frac{c_1}{x^3} + c_2 x^2 - \frac{x}{2}$$

15.26 problem 26

Internal problem ID [14735]

Book: INTRODUCTORY DIFFERENTIAL EQUATIONS. Martha L. Abell, James P. Braselton. Fourth edition 2014. ElScAe. 2014
Section: Chapter 4. Higher Order Equations. Exercises 4.7, page 195
Problem number: 26.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$\boxed{x^2 y'' + y'x - 16y = \ln(x)}$$

✓ Solution by Maple
Time used: 0.0 (sec). Leaf size: 19

```
dsolve(x^2*diff(y(x),x$2)+x*diff(y(x),x)-16*y(x)=ln(x),y(x), singsol=all)
```

$$y(x) = \frac{c_2}{x^4} + x^4 c_1 - \frac{\ln(x)}{16}$$

✓ Solution by Mathematica
Time used: 0.022 (sec). Leaf size: 24

```
DSolve[x^2*y''[x]+x*y'[x]-16*y[x]==Log[x],y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to c_2 x^4 + \frac{c_1}{x^4} - \frac{\log(x)}{16}$$

15.27 problem 27

Internal problem ID [14736]

Book: INTRODUCTORY DIFFERENTIAL EQUATIONS. Martha L. Abell, James P. Braselton. Fourth edition 2014. ElScAe. 2014
Section: Chapter 4. Higher Order Equations. Exercises 4.7, page 195
Problem number: 27.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$\boxed{x^2y'' + y'x + 4y = 8}$$

✓ Solution by Maple
Time used: 0.016 (sec). Leaf size: 20

```
dsolve(x^2*diff(y(x),x$2)+x*diff(y(x),x)+4*y(x)=8,y(x), singsol=all)
```

$$y(x) = c_2 \sin(2\ln(x)) + \cos(2\ln(x))\, c_1 + 2$$

✓ Solution by Mathematica
Time used: 0.017 (sec). Leaf size: 23

```
DSolve[x^2*y''[x]+x*y'[x]+4*y[x]==8,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to c_1 \cos(2\log(x)) + c_2 \sin(2\log(x)) + 2$$

15.28 problem 28

Internal problem ID [14737]

Book: INTRODUCTORY DIFFERENTIAL EQUATIONS. Martha L. Abell, James P. Braselton. Fourth edition 2014. ElScAe. 2014
Section: Chapter 4. Higher Order Equations. Exercises 4.7, page 195
Problem number: 28.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$\boxed{x^2 y'' + y'x + 36y = x^2}$$

✓ Solution by Maple
Time used: 0.015 (sec). Leaf size: 24

```
dsolve(x^2*diff(y(x),x$2)+x*diff(y(x),x)+36*y(x)=x^2,y(x), singsol=all)
```

$$y(x) = \sin(6\ln(x))\, c_2 + \cos(6\ln(x))\, c_1 + \frac{x^2}{40}$$

✓ Solution by Mathematica
Time used: 0.057 (sec). Leaf size: 29

```
DSolve[x^2*y''[x]+x*y'[x]+36*y[x]==x^2,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to \frac{x^2}{40} + c_1 \cos(6\log(x)) + c_2 \sin(6\log(x))$$

15.29 problem 29

Internal problem ID [14738]

Book: INTRODUCTORY DIFFERENTIAL EQUATIONS. Martha L. Abell, James P. Braselton. Fourth edition 2014. ElScAe. 2014
Section: Chapter 4. Higher Order Equations. Exercises 4.7, page 195
Problem number: 29.
ODE order: 3.
ODE degree: 1.

CAS Maple gives this as type `[[_3rd_order, _with_linear_symmetries]]`

$$\boxed{x^3y''' + 3x^2y'' - 11y'x + 16y = \frac{1}{x^3}}$$

✓ Solution by Maple
Time used: 0.015 (sec). Leaf size: 28

```
dsolve(x^3*diff(y(x),x$3)+3*x^2*diff(y(x),x$2)-11*x*diff(y(x),x)+16*y(x)=1/x^3,y(x), singsol
```

$$y(x) = \frac{25c_3 \ln(x)\, x^6 + 25c_1x^6 + 25c_2 + x}{25x^4}$$

✓ Solution by Mathematica
Time used: 0.013 (sec). Leaf size: 33

```
DSolve[x^3*y'''[x]+3*x^2*y''[x]-11*x*y'[x]+16*y[x]==1/x^3,y[x],x,IncludeSingularSolutions ->
```

$$y(x) \to \frac{c_1}{x^4} + \frac{1}{25x^3} + c_2x^2 + c_3x^2 \log(x)$$

15.30 problem 30

Internal problem ID [14739]

Book: INTRODUCTORY DIFFERENTIAL EQUATIONS. Martha L. Abell, James P. Braselton. Fourth edition 2014. ElScAe. 2014
Section: Chapter 4. Higher Order Equations. Exercises 4.7, page 195
Problem number: 30.
ODE order: 3.
ODE degree: 1.

CAS Maple gives this as type `[[_3rd_order, _with_linear_symmetries]]`

$$\boxed{x^3 y''' + 16x^2 y'' + 70y'x + 80y = \frac{1}{x^{13}}}$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 27

```
dsolve(x^3*diff(y(x),x$3)+16*x^2*diff(y(x),x$2)+70*x*diff(y(x),x)+80*y(x)=1/x^(13),y(x), sin
```

$$y(x) = -\frac{1}{648x^{13}} + \frac{c_1}{x^4} + \frac{c_2}{x^5} + \frac{c_3 \ln(x)}{x^4}$$

✓ Solution by Mathematica

Time used: 0.008 (sec). Leaf size: 33

```
DSolve[x^3*y'''[x]+16*x^2*y''[x]+70*x*y'[x]+80*y[x]==1/x^(13),y[x],x,IncludeSingularSolution
```

$$y(x) \to -\frac{1}{648x^{13}} + \frac{c_1}{x^5} + \frac{c_2}{x^4} + \frac{c_3 \log(x)}{x^4}$$

15.31 problem 31

Internal problem ID [14740]

Book: INTRODUCTORY DIFFERENTIAL EQUATIONS. Martha L. Abell, James P. Braselton. Fourth edition 2014. ElScAe. 2014
Section: Chapter 4. Higher Order Equations. Exercises 4.7, page 195
Problem number: 31.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_Emden, _Fowler], [_2nd_order, _linear, '_with_symmetry_[0,F`

$$\boxed{3x^2y'' - 4y'x + 2y = 0}$$

With initial conditions

$$[y(1) = 2, y'(1) = 1]$$

✓ Solution by Maple

Time used: 0.015 (sec). Leaf size: 15

```
dsolve([3*x^2*diff(y(x),x$2)-4*x*diff(y(x),x)+2*y(x)=0,y(1) = 2, D(y)(1) = 1],y(x), singsol=
```

$$y(x) = \frac{9x^{\frac{1}{3}}}{5} + \frac{x^2}{5}$$

✓ Solution by Mathematica

Time used: 0.018 (sec). Leaf size: 20

```
DSolve[{3*x^2*y''[x]-4*x*y'[x]+2*y[x]==0,{y[1]==2,y'[1]==1}},y[x],x,IncludeSingularSolutions
```

$$y(x) \to \frac{1}{5}\left(x^2 + 9\sqrt[3]{x}\right)$$

15.32 problem 32

Internal problem ID [14741]

Book: INTRODUCTORY DIFFERENTIAL EQUATIONS. Martha L. Abell, James P. Braselton. Fourth edition 2014. ElScAe. 2014
Section: Chapter 4. Higher Order Equations. Exercises 4.7, page 195
Problem number: 32.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_Emden, _Fowler]]`

$$\boxed{2x^2y'' - 7y'x + 7y = 0}$$

With initial conditions

$$[y(1) = -1, y'(1) = 1]$$

✓ Solution by Maple
Time used: 0.015 (sec). Leaf size: 13

```
dsolve([2*x^2*diff(y(x),x$2)-7*x*diff(y(x),x)+7*y(x)=0,y(1) = -1, D(y)(1) = 1],y(x), singsol
```

$$y(x) = \frac{4x^{\frac{7}{2}}}{5} - \frac{9x}{5}$$

✓ Solution by Mathematica
Time used: 0.011 (sec). Leaf size: 19

```
DSolve[{2*x^2*y''[x]-7*x*y'[x]+7*y[x]==0,{y[1]==-1,y'[1]==1}},y[x],x,IncludeSingularSolution
```

$$y(x) \to \frac{1}{5}x\left(4x^{5/2} - 9\right)$$

15.33 problem 33

Internal problem ID [14742]

Book: INTRODUCTORY DIFFERENTIAL EQUATIONS. Martha L. Abell, James P. Braselton. Fourth edition 2014. ElScAe. 2014
Section: Chapter 4. Higher Order Equations. Exercises 4.7, page 195
Problem number: 33.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_Emden, _Fowler], [_2nd_order, _linear, '_with_symmetry_[0,F`

$$\boxed{x^2y'' + y'x + 4y = 0}$$

With initial conditions

$$[y(1) = -1, y'(1) = 0]$$

✓ Solution by Maple
Time used: 0.016 (sec). Leaf size: 11

```
dsolve([x^2*diff(y(x),x$2)+x*diff(y(x),x)+4*y(x)=0,y(1) = -1, D(y)(1) = 0],y(x), singsol=all
```

$$y(x) = -\cos(2\ln(x))$$

✓ Solution by Mathematica
Time used: 0.017 (sec). Leaf size: 12

```
DSolve[{x^2*y''[x]+x*y'[x]+4*y[x]==0,{y[1]==-1,y'[1]==0}},y[x],x,IncludeSingularSolutions ->
```

$$y(x) \to -\cos(2\log(x))$$

15.34 problem 34

Internal problem ID [14743]

Book: INTRODUCTORY DIFFERENTIAL EQUATIONS. Martha L. Abell, James P. Braselton. Fourth edition 2014. ElScAe. 2014
Section: Chapter 4. Higher Order Equations. Exercises 4.7, page 195
Problem number: 34.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_Emden, _Fowler], [_2nd_order, _linear, '_with_symmetry_[0,F`

$$\boxed{x^2y'' + y'x + 2y = 0}$$

With initial conditions

$$[y(1) = 0, y'(1) = 2]$$

✓ Solution by Maple
Time used: 0.016 (sec). Leaf size: 15

```
dsolve([x^2*diff(y(x),x$2)+x*diff(y(x),x)+2*y(x)=0,y(1) = 0, D(y)(1) = 2],y(x), singsol=all)
```

$$y(x) = \sqrt{2}\,\sin\left(\ln\left(x\right)\sqrt{2}\right)$$

✓ Solution by Mathematica
Time used: 0.023 (sec). Leaf size: 20

```
DSolve[{x^2*y''[x]+x*y'[x]+2*y[x]==0,{y[1]==0,y'[1]==2}},y[x],x,IncludeSingularSolutions ->
```

$$y(x) \to \sqrt{2}\sin\left(\sqrt{2}\log(x)\right)$$

15.35 problem 35

Internal problem ID [14744]

Book: INTRODUCTORY DIFFERENTIAL EQUATIONS. Martha L. Abell, James P. Braselton. Fourth edition 2014. ElScAe. 2014
Section: Chapter 4. Higher Order Equations. Exercises 4.7, page 195
Problem number: 35.
ODE order: 3.
ODE degree: 1.

CAS Maple gives this as type `[[_3rd_order, _with_linear_symmetries]]`

$$\boxed{x^3y''' + 10x^2y'' - 20y'x + 20y = 0}$$

With initial conditions

$$[y(1) = 0, y'(1) = -1, y''(1) = 1]$$

✓ Solution by Maple
Time used: 0.031 (sec). Leaf size: 18

```
dsolve([x^3*diff(y(x),x$3)+10*x^2*diff(y(x),x$2)-20*x*diff(y(x),x)+20*y(x)=0,y(1) = 0, D(y)(
```

$$y(x) = \frac{8x}{11} + \frac{1}{44x^{10}} - \frac{3x^2}{4}$$

✓ Solution by Mathematica
Time used: 0.004 (sec). Leaf size: 25

```
DSolve[{x^3*y'''[x]+10*x^2*y''[x]-20*x*y'[x]+20*y[x]==0,{y[1]==0,y'[1]==-1,y''[1]==1}},y[x],
```

$$y(x) \to \frac{1}{44x^{10}} - \frac{3x^2}{4} + \frac{8x}{11}$$

15.36 problem 36

Internal problem ID [14745]

Book: INTRODUCTORY DIFFERENTIAL EQUATIONS. Martha L. Abell, James P. Braselton. Fourth edition 2014. ElScAe. 2014
Section: Chapter 4. Higher Order Equations. Exercises 4.7, page 195
Problem number: 36.
ODE order: 3.
ODE degree: 1.

CAS Maple gives this as type `[[_3rd_order, _with_linear_symmetries]]`

$$\boxed{x^3 y''' + 15x^2 y'' + 54y'x + 42y = 0}$$

With initial conditions

$$[y(1) = 5, y'(1) = 0, y''(1) = 0]$$

✓ Solution by Maple
Time used: 0.031 (sec). Leaf size: 20

```
dsolve([x^3*diff(y(x),x$3)+15*x^2*diff(y(x),x$2)+54*x*diff(y(x),x)+42*y(x)=0,y(1) = 5, D(y)(
```

$$y(x) = -\frac{35}{2x^3} + \frac{3}{2x^7} + \frac{21}{x^2}$$

✓ Solution by Mathematica
Time used: 0.004 (sec). Leaf size: 24

```
DSolve[{x^3*y'''[x]+15*x^2*y''[x]+54*x*y'[x]+42*y[x]==0,{y[1]==5,y'[1]==0,y''[1]==0}},y[x],x
```

$$y(x) \to \frac{42x^5 - 35x^4 + 3}{2x^7}$$

15.37 problem 37

Internal problem ID [14746]

Book: INTRODUCTORY DIFFERENTIAL EQUATIONS. Martha L. Abell, James P. Braselton. Fourth edition 2014. ElScAe. 2014
Section: Chapter 4. Higher Order Equations. Exercises 4.7, page 195
Problem number: 37.
ODE order: 3.
ODE degree: 1.

CAS Maple gives this as type `[[_3rd_order, _with_linear_symmetries]]`

$$\boxed{x^3y''' - 2x^2y'' + 5y'x - 5y = 0}$$

With initial conditions

$$[y(1) = 0, y'(1) = -1, y''(1) = 0]$$

✓ Solution by Maple

Time used: 0.016 (sec). Leaf size: 20

```
dsolve([x^3*diff(y(x),x$3)-2*x^2*diff(y(x),x$2)+5*x*diff(y(x),x)-5*y(x)=0,y(1) = 0, D(y)(1)
```

$$y(x) = -\frac{3x\left(x\cos(\ln(x)) - \frac{\sin(\ln(x))x}{3} - 1\right)}{2}$$

✓ Solution by Mathematica

Time used: 0.004 (sec). Leaf size: 24

```
DSolve[{x^3*y'''[x]-2*x^2*y''[x]+5*x*y'[x]-5*y[x]==0,{y[1]==0,y'[1]==-1,y''[1]==0}},y[x],x,I
```

$$y(x) \to -\frac{1}{2}x(-x\sin(\log(x)) + 3x\cos(\log(x)) - 3)$$

15.38 problem 38

Internal problem ID [14747]

Book: INTRODUCTORY DIFFERENTIAL EQUATIONS. Martha L. Abell, James P. Braselton. Fourth edition 2014. ElScAe. 2014
Section: Chapter 4. Higher Order Equations. Exercises 4.7, page 195
Problem number: 38.
ODE order: 3.
ODE degree: 1.

CAS Maple gives this as type `[[_3rd_order, _with_linear_symmetries]]`

$$\boxed{x^3y''' - 6x^2y'' + 17y'x - 17y = 0}$$

With initial conditions

$$[y(1) = -2, y'(1) = 0, y''(1) = 0]$$

✓ Solution by Maple
Time used: 0.046 (sec). Leaf size: 24

```
dsolve([x^3*diff(y(x),x$3)-6*x^2*diff(y(x),x$2)+17*x*diff(y(x),x)-17*y(x)=0,y(1) = -2, D(y)(
```

$$y(x) = \frac{(-11\sin(\ln(x)) + 7\cos(\ln(x)))\,x^4}{5} - \frac{17x}{5}$$

✓ Solution by Mathematica
Time used: 0.004 (sec). Leaf size: 28

```
DSolve[{x^3*y'''[x]-6*x^2*y''[x]+17*x*y'[x]-17*y[x]==0,{y[1]==-2,y'[1]==0,y''[1]==0}},y[x],x
```

$$y(x) \to \frac{1}{5}x\left(-11x^3\sin(\log(x)) + 7x^3\cos(\log(x)) - 17\right)$$

15.39 problem 39

Internal problem ID [14748]

Book: INTRODUCTORY DIFFERENTIAL EQUATIONS. Martha L. Abell, James P. Braselton. Fourth edition 2014. ElScAe. 2014
Section: Chapter 4. Higher Order Equations. Exercises 4.7, page 195
Problem number: 39.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _exact, _linear, _nonhomogeneous]]`

$$\boxed{2x^2y'' + 3y'x - y = \frac{1}{x^2}}$$

With initial conditions

$$[y(1) = 0, y'(1) = 2]$$

✓ Solution by Maple

Time used: 0.016 (sec). Leaf size: 19

```
dsolve([2*x^2*diff(y(x),x$2)+3*x*diff(y(x),x)-y(x)=1/x^2,y(1) = 0, D(y)(1) = 2],y(x), singso
```

$$y(x) = \frac{22x^{\frac{5}{2}} - 25x + 3}{15x^2}$$

✓ Solution by Mathematica

Time used: 0.021 (sec). Leaf size: 24

```
DSolve[{2*x^2*y''[x]+3*x*y'[x]-y[x]==1/x^2,{y[1]==0,y'[1]==2}},y[x],x,IncludeSingularSolutio
```

$$y(x) \to \frac{22x^{5/2} - 25x + 3}{15x^2}$$

15.40 problem 40

Internal problem ID [14749]

Book: INTRODUCTORY DIFFERENTIAL EQUATIONS. Martha L. Abell, James P. Braselton. Fourth edition 2014. ElScAe. 2014
Section: Chapter 4. Higher Order Equations. Exercises 4.7, page 195
Problem number: 40.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _exact, _linear, _nonhomogeneous]]`

$$\boxed{x^2y'' + 4y'x + 2y = \ln(x)}$$

With initial conditions

$$[y(1) = 2, y'(1) = 0]$$

✓ Solution by Maple
Time used: 0.015 (sec). Leaf size: 20

```
dsolve([x^2*diff(y(x),x$2)+4*x*diff(y(x),x)+2*y(x)=ln(x),y(1) = 2, D(y)(1) = 0],y(x), singso
```

$$y(x) = -\frac{9}{4x^2} + \frac{5}{x} + \frac{\ln(x)}{2} - \frac{3}{4}$$

✓ Solution by Mathematica
Time used: 0.02 (sec). Leaf size: 25

```
DSolve[{x^2*y''[x]+4*x*y'[x]+2*y[x]==Log[x],{y[1]==2,y'[1]==0}},y[x],x,IncludeSingularSoluti
```

$$y(x) \to \frac{1}{4}\left(-\frac{9}{x^2} + \frac{20}{x} + 2\log(x) - 3\right)$$

15.41 problem 41

Internal problem ID [14750]

Book: INTRODUCTORY DIFFERENTIAL EQUATIONS. Martha L. Abell, James P. Braselton. Fourth edition 2014. ElScAe. 2014
Section: Chapter 4. Higher Order Equations. Exercises 4.7, page 195
Problem number: 41.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$\boxed{4x^2y'' + y = x^3}$$

With initial conditions

$$[y(1) = 1, y'(1) = -1]$$

✓ Solution by Maple
Time used: 0.031 (sec). Leaf size: 21

```
dsolve([4*x^2*diff(y(x),x$2)+y(x)=x^3,y(1) = 1, D(y)(1) = -1],y(x), singsol=all)
```

$$y(x) = \frac{8(3-5\ln(x))\sqrt{x}}{25} + \frac{x^3}{25}$$

✓ Solution by Mathematica
Time used: 0.025 (sec). Leaf size: 25

```
DSolve[{4*x^2*y''[x]+y[x]==x^3,{y[1]==1,y'[1]==-1}},y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to \frac{1}{25}\sqrt{x}\left(x^{5/2} - 40\log(x) + 24\right)$$

15.42 problem 42

Internal problem ID [14751]

Book: INTRODUCTORY DIFFERENTIAL EQUATIONS. Martha L. Abell, James P. Braselton. Fourth edition 2014. ElScAe. 2014
Section: Chapter 4. Higher Order Equations. Exercises 4.7, page 195
Problem number: 42.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$\boxed{9x^2y'' + 27y'x + 10y = \frac{1}{x}}$$

With initial conditions

$$[y(1) = 0, y'(1) = -1]$$

✓ Solution by Maple

Time used: 0.031 (sec). Leaf size: 24

```
dsolve([9*x^2*diff(y(x),x$2)+27*x*diff(y(x),x)+10*y(x)=1/x,y(1) = 0, D(y)(1) = -1],y(x), sin
```

$$y(x) = \frac{-3\sin\left(\frac{\ln(x)}{3}\right) - \cos\left(\frac{\ln(x)}{3}\right) + 1}{x}$$

✓ Solution by Mathematica

Time used: 0.129 (sec). Leaf size: 28

```
DSolve[{9*x^2*y''[x]+27*x*y'[x]+10*y[x]==1/x,{y[1]==0,y'[1]==-1}},y[x],x,IncludeSingularSolu
```

$$y(x) \to -\frac{3\sin\left(\frac{\log(x)}{3}\right) + \cos\left(\frac{\log(x)}{3}\right) - 1}{x}$$

15.43 problem 46 (a)

Internal problem ID [14752]

Book: INTRODUCTORY DIFFERENTIAL EQUATIONS. Martha L. Abell, James P. Braselton. Fourth edition 2014. ElScAe. 2014
Section: Chapter 4. Higher Order Equations. Exercises 4.7, page 195
Problem number: 46 (a).
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_Emden, _Fowler]]`

$$\boxed{x^2y'' - y'x + 2y = 0}$$

✓ Solution by Maple
Time used: 0.0 (sec). Leaf size: 17

```
dsolve(x^2*diff(y(x),x$2)-x*diff(y(x),x)+2*y(x)=0,y(x), singsol=all)
```

$$y(x) = x(c_1 \sin(\ln(x)) + c_2 \cos(\ln(x)))$$

✓ Solution by Mathematica
Time used: 0.02 (sec). Leaf size: 20

```
DSolve[x^2*y''[x]-x*y'[x]+2*y[x]==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to x(c_2 \cos(\log(x)) + c_1 \sin(\log(x)))$$

15.44 problem 46 (b)

Internal problem ID [14753]

Book: INTRODUCTORY DIFFERENTIAL EQUATIONS. Martha L. Abell, James P. Braselton. Fourth edition 2014. ElScAe. 2014
Section: Chapter 4. Higher Order Equations. Exercises 4.7, page 195
Problem number: 46 (b).
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _exact, _linear, _homogeneous]]`

$$\boxed{x^2y'' + 4y'x + 2y = 0}$$

✓ Solution by Maple
Time used: 0.0 (sec). Leaf size: 13

```
dsolve(x^2*diff(y(x),x$2)+4*x*diff(y(x),x)+2*y(x)=0,y(x), singsol=all)
```

$$y(x) = \frac{c_1x + c_2}{x^2}$$

✓ Solution by Mathematica
Time used: 0.011 (sec). Leaf size: 16

```
DSolve[x^2*y''[x]+4*x*y'[x]+2*y[x]==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to \frac{c_2x + c_1}{x^2}$$

15.45 problem 46 (c)

Internal problem ID [14754]

Book: INTRODUCTORY DIFFERENTIAL EQUATIONS. Martha L. Abell, James P. Braselton. Fourth edition 2014. ElScAe. 2014
Section: Chapter 4. Higher Order Equations. Exercises 4.7, page 195
Problem number: 46 (c).
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_Emden, _Fowler], [_2nd_order, _linear, '_with_symmetry_[0,F`

$$\boxed{x^2 y'' + y' x + y = 0}$$

✓ Solution by Maple
Time used: 0.016 (sec). Leaf size: 15

```
dsolve(x^2*diff(y(x),x$2)+x*diff(y(x),x)+y(x)=0,y(x), singsol=all)
```

$$y(x) = c_1 \sin(\ln(x)) + c_2 \cos(\ln(x))$$

✓ Solution by Mathematica
Time used: 0.014 (sec). Leaf size: 18

```
DSolve[x^2*y''[x]+x*y'[x]+y[x]==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to c_1 \cos(\log(x)) + c_2 \sin(\log(x))$$

15.46 problem 49

Internal problem ID [14755]

Book: INTRODUCTORY DIFFERENTIAL EQUATIONS. Martha L. Abell, James P. Braselton. Fourth edition 2014. ElScAe. 2014
Section: Chapter 4. Higher Order Equations. Exercises 4.7, page 195
Problem number: 49.
ODE order: 3.
ODE degree: 1.

CAS Maple gives this as type `[[_3rd_order, _missing_y]]`

$$\boxed{x^3y''' + 3x^2y'' + 37y'x = 0}$$

✓ Solution by Maple
Time used: 0.0 (sec). Leaf size: 20

```
dsolve(x^3*diff(y(x),x$3)+3*x^2*diff(y(x),x$2)+37*x*diff(y(x),x)=0,y(x), singsol=all)
```

$$y(x) = c_1 + \sin(6\ln(x))\, c_2 + c_3 \cos(6\ln(x))$$

✓ Solution by Mathematica
Time used: 0.164 (sec). Leaf size: 30

```
DSolve[x^3*y'''[x]+3*x^2*y''[x]+37*x*y'[x]==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to -\frac{1}{6}c_1\cos(6\log(x)) + \frac{1}{6}c_2\sin(6\log(x)) + c_3$$

15.47 problem 50

Internal problem ID [14756]

Book: INTRODUCTORY DIFFERENTIAL EQUATIONS. Martha L. Abell, James P. Braselton. Fourth edition 2014. ElScAe. 2014
Section: Chapter 4. Higher Order Equations. Exercises 4.7, page 195
Problem number: 50.
ODE order: 3.
ODE degree: 1.

CAS Maple gives this as type `[[_3rd_order, _missing_y]]`

$$\boxed{x^3y''' + 3x^2y'' - 3y'x = 0}$$

✓ Solution by Maple
Time used: 0.0 (sec). Leaf size: 16

```
dsolve(x^3*diff(y(x),x$3)+3*x^2*diff(y(x),x$2)-3*x*diff(y(x),x)=0,y(x), singsol=all)
```

$$y(x) = c_1 + c_2x^2 + \frac{c_3}{x^2}$$

✓ Solution by Mathematica
Time used: 0.013 (sec). Leaf size: 26

```
DSolve[x^3*y'''[x]+3*x^2*y''[x]-3*x*y'[x]==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to \frac{c_2x^2}{2} - \frac{c_1}{2x^2} + c_3$$

15.48 problem 51

Internal problem ID [14757]

Book: INTRODUCTORY DIFFERENTIAL EQUATIONS. Martha L. Abell, James P. Braselton. Fourth edition 2014. ElScAe. 2014
Section: Chapter 4. Higher Order Equations. Exercises 4.7, page 195
Problem number: 51.
ODE order: 3.
ODE degree: 1.

CAS Maple gives this as type `[[_3rd_order, _with_linear_symmetries]]`

$$\boxed{x^3 y''' + y'x - y = 0}$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 18

```
dsolve(x^3*diff(y(x),x$3)+x*diff(y(x),x)-y(x)=0,y(x), singsol=all)
```

$$y(x) = x\left(c_1 + c_2 \ln(x) + c_3 \ln(x)^2\right)$$

✓ Solution by Mathematica

Time used: 0.003 (sec). Leaf size: 22

```
DSolve[x^3*y'''[x]+x*y'[x]-y[x]==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to x\left(c_3 \log^2(x) + c_2 \log(x) + c_1\right)$$

15.49 problem 52

Internal problem ID [14758]

Book: INTRODUCTORY DIFFERENTIAL EQUATIONS. Martha L. Abell, James P. Braselton. Fourth edition 2014. ElScAe. 2014
Section: Chapter 4. Higher Order Equations. Exercises 4.7, page 195
Problem number: 52.
ODE order: 3.
ODE degree: 1.

CAS Maple gives this as type `[[_3rd_order, _missing_y]]`

$$\boxed{x^3y''' + 3x^2y'' - 3y'x = -8}$$

✓ Solution by Maple
Time used: 0.0 (sec). Leaf size: 24

```
dsolve(x^3*diff(y(x),x$3)+3*x^2*diff(y(x),x$2)-3*x*diff(y(x),x)=-8,y(x), singsol=all)
```

$$y(x) = \frac{c_1x^2}{2} - \frac{c_1 + c_2}{2x^2} + 2\ln(x) + c_3$$

✓ Solution by Mathematica
Time used: 0.016 (sec). Leaf size: 30

```
DSolve[x^3*y'''[x]+3*x^2*y''[x]-3*x*y'[x]==-8,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to \frac{c_2x^2}{2} - \frac{c_1}{2x^2} + 2\log(x) + c_3$$

15.50 problem 53 (c)

Internal problem ID [14759]

Book: INTRODUCTORY DIFFERENTIAL EQUATIONS. Martha L. Abell, James P. Braselton. Fourth edition 2014. ElScAe. 2014
Section: Chapter 4. Higher Order Equations. Exercises 4.7, page 195
Problem number: 53 (c).
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries], [_2nd_order, _linear,`

$$\boxed{(x^2+1)^2 y'' + 2x(x^2+1) y' + 4y = 0}$$

✓ Solution by Maple
Time used: 0.0 (sec). Leaf size: 24

```
dsolve((1+x^2)^2*diff(y(x),x$2)+2*x*(1+x^2)*diff(y(x),x)+4*y(x)=0,y(x), singsol=all)
```

$$y(x) = \frac{c_2x^2 + c_1x - c_2}{x^2+1}$$

✓ Solution by Mathematica
Time used: 2.034 (sec). Leaf size: 22

```
DSolve[(1+x^2)^2*y''[x]+2*x*(1+x^2)*y'[x]+4*y[x]==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to c_1 \cos(2\arctan(x)) + c_2 \sin(2\arctan(x))$$

15.51 problem 53 (d)

Internal problem ID [14760]

Book: INTRODUCTORY DIFFERENTIAL EQUATIONS. Martha L. Abell, James P. Braselton. Fourth edition 2014. ElScAe. 2014
Section: Chapter 4. Higher Order Equations. Exercises 4.7, page 195
Problem number: 53 (d).
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _linear, _nonhomogeneous]]`

$$\boxed{(x^2+1)^2 y'' + 2x(x^2+1) y' + 4y = \arctan(x)}$$

✓ Solution by Maple
Time used: 0.0 (sec). Leaf size: 37

```
dsolve((1+x^2)^2*diff(y(x),x$2)+2*x*(1+x^2)*diff(y(x),x)+4*y(x)=arctan(x),y(x), singsol=all)
```

$$y(x) = \frac{x^2 \arctan(x) + 4c_1 x^2 + 4c_2 x + \arctan(x) - 4c_1 + x}{4x^2 + 4}$$

✓ Solution by Mathematica
Time used: 0.37 (sec). Leaf size: 28

```
DSolve[(1+x^2)^2*y''[x]+2*x*(1+x^2)*y'[x]+4*y[x]==ArcTan[x],y[x],x,IncludeSingularSolutions
```

$$y(x) \to \frac{\arctan(x)}{4} + c_1 \cos(2\arctan(x)) + c_2 \sin(2\arctan(x))$$

15.52 problem 53 (e)

Internal problem ID [14761]

Book: INTRODUCTORY DIFFERENTIAL EQUATIONS. Martha L. Abell, James P. Braselton. Fourth edition 2014. ElScAe. 2014
Section: Chapter 4. Higher Order Equations. Exercises 4.7, page 195
Problem number: 53 (e).
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries], [_2nd_order, _linear,`

$$\boxed{(x^2+1)^2 y'' + 2x(x^2+1)\, y' + 4y = 0}$$

With initial conditions

$$[y(0) = 0, y'(0) = 1]$$

✓ Solution by Maple
Time used: 0.0 (sec). Leaf size: 13

```
dsolve([(1+x^2)^2*diff(y(x),x$2)+2*x*(1+x^2)*diff(y(x),x)+4*y(x)=0,y(0) = 0, D(y)(0) = 1],y(
```

$$y(x) = \frac{x}{x^2+1}$$

✓ Solution by Mathematica
Time used: 0.025 (sec). Leaf size: 14

```
DSolve[{(1+x^2)^2*y''[x]+2*x*(1+x^2)*y'[x]+4*y[x]==0,{y[0]==0,y'[0]==1}},y[x],x,IncludeSingu
```

$$y(x) \to \frac{1}{2}\sin(2\arctan(x))$$

15.53 problem 53 (f)

Internal problem ID [14762]

Book: INTRODUCTORY DIFFERENTIAL EQUATIONS. Martha L. Abell, James P. Braselton. Fourth edition 2014. ElScAe. 2014
Section: Chapter 4. Higher Order Equations. Exercises 4.7, page 195
Problem number: 53 (f).
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _linear, _nonhomogeneous]]`

$$\boxed{(x^2+1)^2 y'' + 2x(x^2+1) y' + 4y = \arctan(x)}$$

With initial conditions

$$[y(0) = 0, y'(0) = 1]$$

✓ Solution by Maple
Time used: 0.031 (sec). Leaf size: 26

```
dsolve([(1+x^2)^2*diff(y(x),x$2)+2*x*(1+x^2)*diff(y(x),x)+4*y(x)=arctan(x),y(0) = 0, D(y)(0)
```

$$y(x) = \frac{x^2 \arctan(x) + \arctan(x) + 3x}{4x^2 + 4}$$

✓ Solution by Mathematica
Time used: 0.045 (sec). Leaf size: 24

```
DSolve[{(1+x^2)^2*y''[x]+2*x*(1+x^2)*y'[x]+4*y[x]==ArcTan[x],{y[0]==0,y'[0]==1}},y[x],x,Incl
```

$$y(x) \to \frac{1}{8}\left(2\arctan(x) + \frac{6x}{x^2+1}\right)$$

15.54 problem 54 (c)

Internal problem ID [14763]

Book: INTRODUCTORY DIFFERENTIAL EQUATIONS. Martha L. Abell, James P. Braselton. Fourth edition 2014. ElScAe. 2014
Section: Chapter 4. Higher Order Equations. Exercises 4.7, page 195
Problem number: 54 (c).
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries], [_2nd_order, _linear,`

$$\boxed{(x^4-1)\,y''+(x^3-x)\,y'+y(x^2-1)=0}$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 15

```
dsolve((x^4-1)*diff(y(x),x$2)+(x^3-x)*diff(y(x),x)+(x^2-1)*y(x)=0,y(x), singsol=all)
```

$$y(x)=c_1\sin(\operatorname{arcsinh}(x))+c_2\cos(\operatorname{arcsinh}(x))$$

✓ Solution by Mathematica

Time used: 0.062 (sec). Leaf size: 43

```
DSolve[(x^4-1)*y''[x]+(x^3-x)*y'[x]+(x^2-1)*y[x]==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x)\to c_1\cos\left(\log\left(\sqrt{x^2+1}-x\right)\right)-c_2\sin\left(\log\left(\sqrt{x^2+1}-x\right)\right)$$

15.55 problem 54 (d)

Internal problem ID [14764]

Book: INTRODUCTORY DIFFERENTIAL EQUATIONS. Martha L. Abell, James P. Braselton. Fourth edition 2014. ElScAe. 2014
Section: Chapter 4. Higher Order Equations. Exercises 4.7, page 195
Problem number: 54 (d).
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries], [_2nd_order, _linear,`

$$\boxed{(x^4-1)\,y''+(x^3-x)\,y'+(4x^2-4)\,y=0}$$

✓ Solution by Maple
Time used: 0.0 (sec). Leaf size: 19

```
dsolve((x^4-1)*diff(y(x),x$2)+(x^3-x)*diff(y(x),x)+(4*x^2-4)*y(x)=0,y(x), singsol=all)
```

$$y(x)=c_1\sin(2\operatorname{arcsinh}(x))+c_2\cos(2\operatorname{arcsinh}(x))$$

✓ Solution by Mathematica
Time used: 0.035 (sec). Leaf size: 47

```
DSolve[(x^4-1)*y''[x]+(x^3-x)*y'[x]+(4*x^2-4)*y[x]==0,y[x],x,IncludeSingularSolutions -> Tru
```

$$y(x)\to c_1\cos\left(2\log\left(\sqrt{x^2+1}-x\right)\right)-c_2\sin\left(2\log\left(\sqrt{x^2+1}-x\right)\right)$$

15.56 problem 54 (e)

Internal problem ID [14765]

Book: INTRODUCTORY DIFFERENTIAL EQUATIONS. Martha L. Abell, James P. Braselton. Fourth edition 2014. ElScAe. 2014
Section: Chapter 4. Higher Order Equations. Exercises 4.7, page 195
Problem number: 54 (e).
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries], [_2nd_order, _linear,`

$$\boxed{(x^4-1)\,y''+(x^3-x)\,y'+y(x^2-1)=0}$$

With initial conditions

$$[y(0)=0, y'(0)=-1]$$

✓ Solution by Maple
Time used: 0.0 (sec). Leaf size: 9

```
dsolve([(x^4-1)*diff(y(x),x$2)+(x^3-x)*diff(y(x),x)+(x^2-1)*y(x)=0,y(0) = 0, D(y)(0) = -1],y
```

$$y(x) = -\sin(\operatorname{arcsinh}(x))$$

✓ Solution by Mathematica
Time used: 0.028 (sec). Leaf size: 20

```
DSolve[{(x^4-1)*y''[x]+(x^3-x)*y'[x]+(x^2-1)*y[x]==0,{y[0]==0,y'[0]==-1}},y[x],x,IncludeSing
```

$$y(x) \to \sin\left(\log\left(\sqrt{x^2+1}-x\right)\right)$$

15.57 problem 57

Internal problem ID [14766]

Book: INTRODUCTORY DIFFERENTIAL EQUATIONS. Martha L. Abell, James P. Braselton. Fourth edition 2014. ElScAe. 2014
Section: Chapter 4. Higher Order Equations. Exercises 4.7, page 195
Problem number: 57.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _exact, _linear, _homogeneous]]`

$$\boxed{x^2y'' + 4y'x + 2y = 0}$$

✓ Solution by Maple
Time used: 0.0 (sec). Leaf size: 13

```
dsolve(x^2*diff(y(x),x$2)+4*x*diff(y(x),x)+2*y(x)=0,y(x), singsol=all)
```

$$y(x) = \frac{c_1x + c_2}{x^2}$$

✓ Solution by Mathematica
Time used: 0.01 (sec). Leaf size: 16

```
DSolve[x^2*y''[x]+4*x*y'[x]+2*y[x]==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to \frac{c_2x + c_1}{x^2}$$

15.58 problem 58

Internal problem ID [14767]

Book: INTRODUCTORY DIFFERENTIAL EQUATIONS. Martha L. Abell, James P. Braselton. Fourth edition 2014. ElScAe. 2014
Section: Chapter 4. Higher Order Equations. Exercises 4.7, page 195
Problem number: 58.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$\boxed{x^2y'' + y'x + y = x^2}$$

✓ Solution by Maple
Time used: 0.016 (sec). Leaf size: 20

```
dsolve(x^2*diff(y(x),x$2)+x*diff(y(x),x)+y(x)=x^2,y(x), singsol=all)
```

$$y(x) = c_2 \sin(\ln(x)) + \cos(\ln(x))\, c_1 + \frac{x^2}{5}$$

✓ Solution by Mathematica
Time used: 0.04 (sec). Leaf size: 25

```
DSolve[x^2*y''[x]+x*y'[x]+y[x]==x^2,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to \frac{x^2}{5} + c_1 \cos(\log(x)) + c_2 \sin(\log(x))$$

15.59 problem 59

Internal problem ID [14768]

Book: INTRODUCTORY DIFFERENTIAL EQUATIONS. Martha L. Abell, James P. Braselton. Fourth edition 2014. ElScAe. 2014
Section: Chapter 4. Higher Order Equations. Exercises 4.7, page 195
Problem number: 59.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_Emden, _Fowler], [_2nd_order, _linear, '_with_symmetry_[0,F`

$$\boxed{x^2y'' + y'x + 4y = 0}$$

With initial conditions

$$[y(-1) = 0, y'(-1) = 2]$$

✓ Solution by Maple
Time used: 0.094 (sec). Leaf size: 28

```
dsolve([x^2*diff(y(x),x$2)+x*diff(y(x),x)+4*y(x)=0,y(-1) = 0, D(y)(-1) = 2],y(x), singsol=al
```

$$y(x) = -\cosh(2\pi)\sin(2\ln(x)) + i\sinh(2\pi)\cos(2\ln(x))$$

✓ Solution by Mathematica
Time used: 0.018 (sec). Leaf size: 20

```
DSolve[{x^2*y''[x]+x*y'[x]+4*y[x]==0,{y[-1]==0,y'[-1]==2}},y[x],x,IncludeSingularSolutions -
```

$$y(x) \to i\sinh(2(\pi + i\log(x)))$$

15.60 problem 60

Internal problem ID [14769]

Book: INTRODUCTORY DIFFERENTIAL EQUATIONS. Martha L. Abell, James P. Braselton. Fourth edition 2014. ElScAe. 2014
Section: Chapter 4. Higher Order Equations. Exercises 4.7, page 195
Problem number: 60.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_Emden, _Fowler]]`

$$\boxed{x^2y'' - y'x + y = 0}$$

With initial conditions

$$[y(-1) = 0, y'(-1) = 1]$$

✓ Solution by Maple
Time used: 0.016 (sec). Leaf size: 13

```
dsolve([x^2*diff(y(x),x$2)-x*diff(y(x),x)+y(x)=0,y(-1) = 0, D(y)(-1) = 1],y(x), singsol=all)
```

$$y(x) = (-i\pi + \ln(x))\,x$$

✓ Solution by Mathematica
Time used: 0.015 (sec). Leaf size: 15

```
DSolve[{x^2*y''[x]-x*y'[x]+y[x]==0,{y[-1]==0,y'[-1]==1}},y[x],x,IncludeSingularSolutions ->
```

$$y(x) \to x(\log(x) - i\pi)$$

15.61 problem 64 (a)

Internal problem ID [14770]

Book: INTRODUCTORY DIFFERENTIAL EQUATIONS. Martha L. Abell, James P. Braselton. Fourth edition 2014. ElScAe. 2014
Section: Chapter 4. Higher Order Equations. Exercises 4.7, page 195
Problem number: 64 (a).
ODE order: 3.
ODE degree: 1.

CAS Maple gives this as type `[[_3rd_order, _with_linear_symmetries]]`

$$\boxed{x^3 y''' + 16x^2 y'' + 79y'x + 125y = 0}$$

✓ Solution by Maple
Time used: 0.0 (sec). Leaf size: 26

```
dsolve(x^3*diff(y(x),x$3)+16*x^2*diff(y(x),x$2)+79*x*diff(y(x),x)+125*y(x)=0,y(x), singsol=a
```

$$y(x) = \frac{c_2 \sin\left(3\ln\left(x\right)\right) x + c_3 \cos\left(3\ln\left(x\right)\right) x + c_1}{x^5}$$

✓ Solution by Mathematica
Time used: 0.004 (sec). Leaf size: 30

```
DSolve[x^3*y'''[x]+16*x^2*y''[x]+79*x*y'[x]+125*y[x]==0,y[x],x,IncludeSingularSolutions -> T
```

$$y(x) \to \frac{c_2 x \cos(3\log(x)) + c_1 x \sin(3\log(x)) + c_3}{x^5}$$

15.62 problem 64 (b)

Internal problem ID [14771]

Book: INTRODUCTORY DIFFERENTIAL EQUATIONS. Martha L. Abell, James P. Braselton. Fourth edition 2014. ElScAe. 2014
Section: Chapter 4. Higher Order Equations. Exercises 4.7, page 195
Problem number: 64 (b).
ODE order: 4.
ODE degree: 1.

CAS Maple gives this as type `[[_high_order, _with_linear_symmetries]]`

$$\boxed{x^4y'''' + 5x^3y''' - 12x^2y'' - 12y'x + 48y = 0}$$

✓ Solution by Maple
Time used: 0.016 (sec). Leaf size: 23

```
dsolve(x^4*diff(y(x),x$4)+5*x^3*diff(y(x),x$3)-12*x^2*diff(y(x),x$2)-12*x*diff(y(x),x)+48*y(
```

$$y(x) = \frac{c_2x^7 + c_3x^5 + c_4x + c_1}{x^3}$$

✓ Solution by Mathematica
Time used: 0.004 (sec). Leaf size: 28

```
DSolve[x^4*y''''[x]+5*x^3*y'''[x]-12*x^2*y''[x]-12*x*y'[x]+48*y[x]==0,y[x],x,IncludeSingular
```

$$y(x) \to \frac{c_4x^7 + c_3x^5 + c_2x + c_1}{x^3}$$

15.63 problem 64 (c)

Internal problem ID [14772]

Book: INTRODUCTORY DIFFERENTIAL EQUATIONS. Martha L. Abell, James P. Braselton. Fourth edition 2014. ElScAe. 2014
Section: Chapter 4. Higher Order Equations. Exercises 4.7, page 195
Problem number: 64 (c).
ODE order: 4.
ODE degree: 1.

CAS Maple gives this as type `[[_high_order, _exact, _linear, _homogeneous]]`

$$\boxed{x^4y'''' + 14x^3y''' + 55x^2y'' + 65y'x + 15y = 0}$$

✓ Solution by Maple
Time used: 0.0 (sec). Leaf size: 27

```
dsolve(x^4*diff(y(x),x$4)+14*x^3*diff(y(x),x$3)+55*x^2*diff(y(x),x$2)+65*x*diff(y(x),x)+15*y
```

$$y(x) = \frac{c_3 \sin(\ln(x))\, x + c_4 \cos(\ln(x))\, x + c_2 x^2 + c_1}{x^3}$$

✓ Solution by Mathematica
Time used: 0.004 (sec). Leaf size: 32

```
DSolve[x^4*y''''[x]+14*x^3*y'''[x]+55*x^2*y''[x]+65*x*y'[x]+15*y[x]==0,y[x],x,IncludeSingula
```

$$y(x) \to \frac{c_4 x^2 + c_2 x \cos(\log(x)) + c_1 x \sin(\log(x)) + c_3}{x^3}$$

15.64 problem 64 (d)

Internal problem ID [14773]

Book: INTRODUCTORY DIFFERENTIAL EQUATIONS. Martha L. Abell, James P. Braselton. Fourth edition 2014. ElScAe. 2014
Section: Chapter 4. Higher Order Equations. Exercises 4.7, page 195
Problem number: 64 (d).
ODE order: 4.
ODE degree: 1.

CAS Maple gives this as type `[[_high_order, _with_linear_symmetries]]`

$$\boxed{x^4y'''' + 8x^3y''' + 27x^2y'' + 35y'x + 45y = 0}$$

✓ Solution by Maple
Time used: 0.0 (sec). Leaf size: 39

```
dsolve(x^4*diff(y(x),x$4)+8*x^3*diff(y(x),x$3)+27*x^2*diff(y(x),x$2)+35*x*diff(y(x),x)+45*y(
```

$$y(x) = \frac{c_1 \sin(2\ln(x))}{x} + \frac{c_2 \cos(2\ln(x))}{x} + c_3 \sin(3\ln(x)) + c_4 \cos(3\ln(x))$$

✓ Solution by Mathematica
Time used: 0.004 (sec). Leaf size: 44

```
DSolve[x^4*y''''[x]+8*x^3*y'''[x]+27*x^2*y''[x]+35*x*y'[x]+45*y[x]==0,y[x],x,IncludeSingular
```

$$y(x) \to \frac{c_2 \cos(2\log(x)) + c_3 x \cos(3\log(x)) + c_1 \sin(2\log(x)) + c_4 x \sin(3\log(x))}{x}$$

15.65 problem 64 (e)

Internal problem ID [14774]

Book: INTRODUCTORY DIFFERENTIAL EQUATIONS. Martha L. Abell, James P. Braselton. Fourth edition 2014. ElScAe. 2014
Section: Chapter 4. Higher Order Equations. Exercises 4.7, page 195
Problem number: 64 (e).
ODE order: 4.
ODE degree: 1.

CAS Maple gives this as type `[[_high_order, _with_linear_symmetries]]`

$$\boxed{x^4y'''' + 10x^3y''' + 27x^2y'' + 21y'x + 4y = 0}$$

✓ Solution by Maple
Time used: 0.0 (sec). Leaf size: 29

```
dsolve(x^4*diff(y(x),x$4)+10*x^3*diff(y(x),x$3)+27*x^2*diff(y(x),x$2)+21*x*diff(y(x),x)+4*y(
```

$$y(x) = \frac{(c_4 \ln(x) + c_2) \cos(\ln(x)) + \sin(\ln(x)) (c_3 \ln(x) + c_1)}{x}$$

✓ Solution by Mathematica
Time used: 0.004 (sec). Leaf size: 34

```
DSolve[x^4*y''''[x]+10*x^3*y'''[x]+27*x^2*y''[x]+21*x*y'[x]+4*y[x]==0,y[x],x,IncludeSingular
```

$$y(x) \to \frac{(c_4 \log(x) + c_3) \cos(\log(x)) + (c_2 \log(x) + c_1) \sin(\log(x))}{x}$$

15.66 problem 65

Internal problem ID [14775]

Book: INTRODUCTORY DIFFERENTIAL EQUATIONS. Martha L. Abell, James P. Braselton. Fourth edition 2014. ElScAe. 2014
Section: Chapter 4. Higher Order Equations. Exercises 4.7, page 195
Problem number: 65.
ODE order: 3.
ODE degree: 1.

CAS Maple gives this as type `[[_3rd_order, _with_linear_symmetries]]`

$$\boxed{x^3y''' + 9x^2y'' + 44y'x + 58y = 0}$$

With initial conditions

$$[y(1) = 2, y'(1) = 10, y''(1) = -2]$$

✓ Solution by Maple

Time used: 0.015 (sec). Leaf size: 25

```
dsolve([x^3*diff(y(x),x$3)+9*x^2*diff(y(x),x$2)+44*x*diff(y(x),x)+58*y(x)=0,y(1) = 2, D(y)(1
```

$$y(x) = \frac{\frac{106}{25} + \frac{14\sin(5\ln(x))}{5} - \frac{56\cos(5\ln(x))}{25}}{x^2}$$

✓ Solution by Mathematica

Time used: 0.004 (sec). Leaf size: 28

```
DSolve[{x^3*y'''[x]+9*x^2*y''[x]+44*x*y'[x]+58*y[x]==0,{y[1]==2,y'[1]==10,y''[1]==-2}},y[x],
```

$$y(x) \to \frac{70\sin(5\log(x)) - 56\cos(5\log(x)) + 106}{25x^2}$$

15.67 problem 66 (a)

Internal problem ID [14776]

Book: INTRODUCTORY DIFFERENTIAL EQUATIONS. Martha L. Abell, James P. Braselton. Fourth edition 2014. ElScAe. 2014
Section: Chapter 4. Higher Order Equations. Exercises 4.7, page 195
Problem number: 66 (a).
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_Emden, _Fowler], [_2nd_order, _linear, '_with_symmetry_[0,F`

$$\boxed{6x^2y'' + 5y'x - y = 0}$$

With initial conditions

$$[y(1) = a, y'(1) = b]$$

✓ Solution by Maple
Time used: 0.047 (sec). Leaf size: 25

```
dsolve([6*x^2*diff(y(x),x$2)+5*x*diff(y(x),x)-y(x)=0,y(1) = a, D(y)(1) = b],y(x), singsol=al
```

$$y(x) = \frac{\frac{2(a+3b)x^{\frac{5}{6}}}{5} + \frac{3a}{5} - \frac{6b}{5}}{x^{\frac{1}{3}}}$$

✓ Solution by Mathematica
Time used: 0.012 (sec). Leaf size: 36

```
DSolve[{6*x^2*y''[x]+5*x*y'[x]-y[x]==0,{y[1]==a,y'[1]==b}},y[x],x,IncludeSingularSolutions -
```

$$y(x) \to \frac{a(2x^{5/6} + 3) + 6b(x^{5/6} - 1)}{5\sqrt[3]{x}}$$

16 Chapter 4. Higher Order Equations. Exercises 4.8, page 203

16.1 problem 1

Internal problem ID [14777]

Book: INTRODUCTORY DIFFERENTIAL EQUATIONS. Martha L. Abell, James P. Braselton. Fourth edition 2014. ElScAe. 2014
Section: Chapter 4. Higher Order Equations. Exercises 4.8, page 203
Problem number: 1.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_Emden, _Fowler]]`

$$\boxed{x^2y'' - 2y'x + 7y = 0}$$

With the expansion point for the power series method at $x = 1$.

✓ Solution by Maple
Time used: 0.0 (sec). Leaf size: 42

```
Order:=6;
dsolve(x^2*diff(y(x),x$2)-2*x*diff(y(x),x)+7*y(x)=0,y(x),type='series',x=1);
```

$$y(x) = \left(1 - \frac{7(-1+x)^2}{2} + \frac{35(-1+x)^4}{24} - \frac{7(-1+x)^5}{6}\right) y(1) + \left(-1 + x + (-1+x)^2 - \frac{5(-1+x)^3}{6} + \frac{7(-1+x)^5}{24}\right) D(y)(1) + O(x^6)$$

✓ Solution by Mathematica
Time used: 0.015 (sec). Leaf size: 65

```
AsymptoticDSolveValue[x^2*y''[x]-2*x*y'[x]+7*y[x]==0,y[x],{x,1,5}]
```

$$y(x) \to c_1\left(-\frac{7}{6}(x-1)^5 + \frac{35}{24}(x-1)^4 - \frac{7}{2}(x-1)^2 + 1\right) + c_2\left(\frac{7}{24}(x-1)^5 - \frac{5}{6}(x-1)^3 + (x-1)^2 + x - 1\right)$$

16.2 problem 2

Internal problem ID [14778]

Book: INTRODUCTORY DIFFERENTIAL EQUATIONS. Martha L. Abell, James P. Braselton. Fourth edition 2014. ElScAe. 2014
Section: Chapter 4. Higher Order Equations. Exercises 4.8, page 203
Problem number: 2.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$\boxed{(x-2)\,y''+y'-y=0}$$

With the expansion point for the power series method at $x=-2$.

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 49

```
Order:=6;
dsolve((x-2)*diff(y(x),x$2)+diff(y(x),x)-y(x)=0,y(x),type='series',x=-2);
```

$$y(x)=\left(1-\frac{(x+2)^{2}}{8}-\frac{(x+2)^{3}}{48}-\frac{(x+2)^{4}}{768}\right)y(-2)+\left(x+2+\frac{(x+2)^{2}}{8}-\frac{(x+2)^{3}}{48}-\frac{5(x+2)^{4}}{768}-\frac{(x+2)^{5}}{960}\right)D(y)\,(-2)+O\left(x^{6}\right)$$

✓ Solution by Mathematica

Time used: 0.002 (sec). Leaf size: 78

```
AsymptoticDSolveValue[(x-2)*y''[x]+y'[x]-y[x]==0,y[x],{x,-2,5}]
```

$$y(x)\to c_{1}\left(-\frac{1}{768}(x+2)^{4}-\frac{1}{48}(x+2)^{3}-\frac{1}{8}(x+2)^{2}+1\right)+c_{2}\left(-\frac{1}{960}(x+2)^{5}-\frac{5}{768}(x+2)^{4}-\frac{1}{48}(x+2)^{3}+\frac{1}{8}(x+2)^{2}+x+2\right)$$

16.3 problem 3

Internal problem ID [14779]

Book: INTRODUCTORY DIFFERENTIAL EQUATIONS. Martha L. Abell, James P. Braselton. Fourth edition 2014. ElScAe. 2014
Section: Chapter 4. Higher Order Equations. Exercises 4.8, page 203
Problem number: 3.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$\boxed{(x^2-4)\,y''+16(x+2)\,y'-y=0}$$

With the expansion point for the power series method at $x=1$.

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 54

```
Order:=6;
dsolve((x^2-4)*diff(y(x),x$2)+16*(x+2)*diff(y(x),x)-y(x)=0,y(x),type='series',x=1);
```

$$y(x)=\left(1-\frac{(-1+x)^2}{6}-\frac{25(-1+x)^3}{27}-\frac{2699(-1+x)^4}{648}-\frac{6404(-1+x)^5}{405}\right)y(1)+\left(-1+x+8(-1+x)^2+\frac{815(-1+x)^3}{18}+\frac{10991(-1+x)^4}{54}+\frac{834547(-1+x)^5}{1080}\right)D(y)(1)+O(x^6)$$

✓ Solution by Mathematica

Time used: 0.001 (sec). Leaf size: 85

```
AsymptoticDSolveValue[(x^2-4)*y''[x]+16*(x+2)*y'[x]-y[x]==0,y[x],{x,1,5}]
```

$$y(x) \to c_1\left(-\frac{6404}{405}(x-1)^5 - \frac{2699}{648}(x-1)^4 - \frac{25}{27}(x-1)^3 - \frac{1}{6}(x-1)^2 + 1\right) + c_2\left(\frac{834547(x-1)^5}{1080} + \frac{10991}{54}(x-1)^4 + \frac{815}{18}(x-1)^3 + 8(x-1)^2 + x - 1\right)$$

16.4 problem 4

Internal problem ID [14780]

Book: INTRODUCTORY DIFFERENTIAL EQUATIONS. Martha L. Abell, James P. Braselton. Fourth edition 2014. ElScAe. 2014
Section: Chapter 4. Higher Order Equations. Exercises 4.8, page 203
Problem number: 4.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _missing_x]]`

$$\boxed{y'' + 3y' - 18y = 0}$$

With the expansion point for the power series method at $x = 0$.

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 54

```
Order:=6;
dsolve(diff(y(x),x$2)+3*diff(y(x),x)-18*y(x)=0,y(x),type='series',x=0);
```

$$\begin{aligned} y(x) = &\left(1 + 9x^2 - 9x^3 + \frac{81}{4}x^4 - \frac{81}{4}x^5\right) y(0) \\ &+ \left(x - \frac{3}{2}x^2 + \frac{9}{2}x^3 - \frac{45}{8}x^4 + \frac{297}{40}x^5\right) D(y)(0) + O(x^6) \end{aligned}$$

✓ Solution by Mathematica

Time used: 0.001 (sec). Leaf size: 66

```
AsymptoticDSolveValue[y''[x]+3*y'[x]-18*y[x]==0,y[x],{x,0,5}]
```

$$y(x) \to c_1 \left(-\frac{81x^5}{4} + \frac{81x^4}{4} - 9x^3 + 9x^2 + 1\right) + c_2 \left(\frac{297x^5}{40} - \frac{45x^4}{8} + \frac{9x^3}{2} - \frac{3x^2}{2} + x\right)$$

16.5 problem 5

Internal problem ID [14781]

Book: INTRODUCTORY DIFFERENTIAL EQUATIONS. Martha L. Abell, James P. Braselton. Fourth edition 2014. ElScAe. 2014
Section: Chapter 4. Higher Order Equations. Exercises 4.8, page 203
Problem number: 5.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _missing_x]]`

$$\boxed{y'' - 11y' + 30y = 0}$$

With the expansion point for the power series method at $x = 0$.

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 54

```
Order:=6;
dsolve(diff(y(x),x$2)-11*diff(y(x),x)+30*y(x)=0,y(x),type='series',x=0);
```

$$y(x) = \left(1 - 15x^2 - 55x^3 - \frac{455}{4}x^4 - \frac{671}{4}x^5\right) y(0) + \left(x + \frac{11}{2}x^2 + \frac{91}{6}x^3 + \frac{671}{24}x^4 + \frac{4651}{120}x^5\right) D(y)(0) + O(x^6)$$

✓ Solution by Mathematica

Time used: 0.001 (sec). Leaf size: 66

```
AsymptoticDSolveValue[y''[x]-11*y'[x]+30*y[x]==0,y[x],{x,0,5}]
```

$$y(x) \to c_1\left(-\frac{671x^5}{4} - \frac{455x^4}{4} - 55x^3 - 15x^2 + 1\right) + c_2\left(\frac{4651x^5}{120} + \frac{671x^4}{24} + \frac{91x^3}{6} + \frac{11x^2}{2} + x\right)$$

16.6 problem 6

Internal problem ID [14782]

Book: INTRODUCTORY DIFFERENTIAL EQUATIONS. Martha L. Abell, James P. Braselton. Fourth edition 2014. ElScAe. 2014
Section: Chapter 4. Higher Order Equations. Exercises 4.8, page 203
Problem number: 6.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _missing_x]]`

$$\boxed{y'' + y = 0}$$

With the expansion point for the power series method at $x = 0$.

✓ Solution by Maple
Time used: 0.0 (sec). Leaf size: 34

```
Order:=6;
dsolve(diff(y(x),x$2)+y(x)=0,y(x),type='series',x=0);
```

$$y(x) = \left(1 - \frac{1}{2}x^2 + \frac{1}{24}x^4\right) y(0) + \left(x - \frac{1}{6}x^3 + \frac{1}{120}x^5\right) D(y)(0) + O(x^6)$$

✓ Solution by Mathematica
Time used: 0.001 (sec). Leaf size: 42

```
AsymptoticDSolveValue[y''[x]+y[x]==0,y[x],{x,0,5}]
```

$$y(x) \to c_2\left(\frac{x^5}{120} - \frac{x^3}{6} + x\right) + c_1\left(\frac{x^4}{24} - \frac{x^2}{2} + 1\right)$$

16.7 problem 7

Internal problem ID [14783]

Book: INTRODUCTORY DIFFERENTIAL EQUATIONS. Martha L. Abell, James P. Braselton. Fourth edition 2014. ElScAe. 2014
Section: Chapter 4. Higher Order Equations. Exercises 4.8, page 203
Problem number: 7.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$\boxed{y'' - y' - 2y = \mathrm{e}^{-x}}$$

With the expansion point for the power series method at $x = 0$.

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 55

```
Order:=6;
dsolve(diff(y(x),x$2)-diff(y(x),x)-2*y(x)=exp(-x),y(x),type='series',x=0);
```

$$y(x) = \left(1 + x^2 + \frac{1}{3}x^3 + \frac{1}{4}x^4 + \frac{1}{12}x^5\right) y(0) + \left(x + \frac{1}{2}x^2 + \frac{1}{2}x^3 + \frac{5}{24}x^4 + \frac{11}{120}x^5\right) D(y)(0) + \frac{x^2}{2} + \frac{x^4}{8} + \frac{x^5}{60} + O(x^6)$$

✓ Solution by Mathematica

Time used: 0.02 (sec). Leaf size: 87

```
AsymptoticDSolveValue[y''[x]-y'[x]-2*y[x]==Exp[-x],y[x],{x,0,5}]
```

$$y(x) \to \frac{x^5}{60} + \frac{x^4}{8} + \frac{x^2}{2} + c_1\left(\frac{x^5}{12} + \frac{x^4}{4} + \frac{x^3}{3} + x^2 + 1\right) + c_2\left(\frac{11x^5}{120} + \frac{5x^4}{24} + \frac{x^3}{2} + \frac{x^2}{2} + x\right)$$

16.8 problem 8

Internal problem ID [14784]

Book: INTRODUCTORY DIFFERENTIAL EQUATIONS. Martha L. Abell, James P. Braselton. Fourth edition 2014. ElScAe. 2014
Section: Chapter 4. Higher Order Equations. Exercises 4.8, page 203
Problem number: 8.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$\boxed{(-2x-2)\,y''+2y'+4y=0}$$

With the expansion point for the power series method at $x=0$.

✓ Solution by Maple
Time used: 0.0 (sec). Leaf size: 42

```
Order:=6;
dsolve((-2-2*x)*diff(y(x),x$2)+2*diff(y(x),x)+4*y(x)=0,y(x),type='series',x=0);
```

$$y(x)=\left(1+x^2+\frac{1}{6}x^4-\frac{1}{15}x^5\right)y(0)+\left(x+\frac{1}{2}x^2+\frac{1}{3}x^3+\frac{1}{30}x^5\right)D(y)\,(0)+O\left(x^6\right)$$

✓ Solution by Mathematica
Time used: 0.001 (sec). Leaf size: 52

```
AsymptoticDSolveValue[(-2-2*x)*y''[x]+2*y'[x]+4*y[x]==0,y[x],{x,0,5}]
```

$$y(x)\to c_1\left(-\frac{x^5}{15}+\frac{x^4}{6}+x^2+1\right)+c_2\left(\frac{x^5}{30}+\frac{x^3}{3}+\frac{x^2}{2}+x\right)$$

16.9 problem 9

Internal problem ID [14785]

Book: INTRODUCTORY DIFFERENTIAL EQUATIONS. Martha L. Abell, James P. Braselton. Fourth edition 2014. ElScAe. 2014
Section: Chapter 4. Higher Order Equations. Exercises 4.8, page 203
Problem number: 9.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _missing_y]]`

$$\boxed{(3x+2)\,y''+3y'x=0}$$

With the expansion point for the power series method at $x=0$.

✓ Solution by Maple
Time used: 0.0 (sec). Leaf size: 29

```
Order:=6;
dsolve((2+3*x)*diff(y(x),x$2)+3*x*diff(y(x),x)=0,y(x),type='series',x=0);
```

$$y(x)=y(0)+\left(x-\frac{1}{4}x^3+\frac{3}{16}x^4-\frac{9}{80}x^5\right)D(y)\,(0)+O\!\left(x^6\right)$$

✓ Solution by Mathematica
Time used: 0.001 (sec). Leaf size: 32

```
AsymptoticDSolveValue[(2+3*x)*y''[x]+3*x*y'[x]==0,y[x],{x,0,5}]
```

$$y(x)\to c_2\left(-\frac{9x^5}{80}+\frac{3x^4}{16}-\frac{x^3}{4}+x\right)+c_1$$

16.10 problem 10

Internal problem ID [14786]

Book: INTRODUCTORY DIFFERENTIAL EQUATIONS. Martha L. Abell, James P. Braselton. Fourth edition 2014. ElScAe. 2014
Section: Chapter 4. Higher Order Equations. Exercises 4.8, page 203
Problem number: 10.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$\boxed{(3x+1)\,y''-3y'-2y=0}$$

With the expansion point for the power series method at $x=0$.

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 42

```
Order:=6;
dsolve((1+3*x)*diff(y(x),x$2)-3*diff(y(x),x)-2*y(x)=0,y(x),type='series',x=0);
```

$$y(x)=\left(1+x^2+\frac{1}{6}x^4-\frac{1}{5}x^5\right)y(0)+\left(x+\frac{3}{2}x^2+\frac{1}{3}x^3+\frac{1}{30}x^5\right)D(y)\,(0)+O\left(x^6\right)$$

✓ Solution by Mathematica

Time used: 0.001 (sec). Leaf size: 52

```
AsymptoticDSolveValue[(1+3*x)*y''[x]-3*y'[x]-2*y[x]==0,y[x],{x,0,5}]
```

$$y(x)\to c_1\left(-\frac{x^5}{5}+\frac{x^4}{6}+x^2+1\right)+c_2\left(\frac{x^5}{30}+\frac{x^3}{3}+\frac{3x^2}{2}+x\right)$$

16.11 problem 11

Internal problem ID [14787]

Book: INTRODUCTORY DIFFERENTIAL EQUATIONS. Martha L. Abell, James P. Braselton. Fourth edition 2014. ElScAe. 2014
Section: Chapter 4. Higher Order Equations. Exercises 4.8, page 203
Problem number: 11.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _exact, _linear, _homogeneous]]`

$$\boxed{(-x^2+2)\,y''+2y'(x-1)+4y=0}$$

With the expansion point for the power series method at $x=0$.

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 54

```
Order:=6;
dsolve((2-x^2)*diff(y(x),x$2)+2*(x-1)*diff(y(x),x)+4*y(x)=0,y(x),type='series',x=0);
```

$$y(x)=\left(1-x^2-\frac{1}{3}x^3+\frac{1}{6}x^4+\frac{1}{15}x^5\right)y(0)+\left(x+\frac{1}{2}x^2-\frac{1}{3}x^3-\frac{5}{24}x^4-\frac{1}{120}x^5\right)D(y)(0)+O(x^6)$$

✓ Solution by Mathematica

Time used: 0.001 (sec). Leaf size: 68

```
AsymptoticDSolveValue[(2-x^2)*y''[x]+2*(x-1)*y'[x]+4*y[x]==0,y[x],{x,0,5}]
```

$$y(x)\to c_1\left(\frac{x^5}{15}+\frac{x^4}{6}-\frac{x^3}{3}-x^2+1\right)+c_2\left(-\frac{x^5}{120}-\frac{5x^4}{24}-\frac{x^3}{3}+\frac{x^2}{2}+x\right)$$

16.12 problem 12

Internal problem ID [14788]

Book: INTRODUCTORY DIFFERENTIAL EQUATIONS. Martha L. Abell, James P. Braselton. Fourth edition 2014. ElScAe. 2014
Section: Chapter 4. Higher Order Equations. Exercises 4.8, page 203
Problem number: 12.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[_Hermite]`

$$\boxed{y'' - xy' + 4y = 0}$$

With the expansion point for the power series method at $x = 0$.

✓ Solution by Maple
Time used: 0.0 (sec). Leaf size: 34

```
Order:=6;
dsolve(diff(y(x),x$2)-x*diff(y(x),x)+4*y(x)=0,y(x),type='series',x=0);
```

$$y(x) = \left(1 - 2x^2 + \frac{1}{3}x^4\right) y(0) + \left(x - \frac{1}{2}x^3 + \frac{1}{40}x^5\right) D(y)(0) + O(x^6)$$

✓ Solution by Mathematica
Time used: 0.001 (sec). Leaf size: 40

```
AsymptoticDSolveValue[y''[x]-x*y'[x]+4*y[x]==0,y[x],{x,0,5}]
```

$$y(x) \to c_2\left(\frac{x^5}{40} - \frac{x^3}{2} + x\right) + c_1\left(\frac{x^4}{3} - 2x^2 + 1\right)$$

16.13 problem 13

Internal problem ID [14789]

Book: INTRODUCTORY DIFFERENTIAL EQUATIONS. Martha L. Abell, James P. Braselton. Fourth edition 2014. ElScAe. 2014
Section: Chapter 4. Higher Order Equations. Exercises 4.8, page 203
Problem number: 13.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries], [_2nd_order, _linear,`

$$\boxed{(2x^2+2)\,y''+2y'x-3y=0}$$

With the expansion point for the power series method at $x=0$.

✓ Solution by Maple
Time used: 0.016 (sec). Leaf size: 34

```
Order:=6;
dsolve((2+2*x^2)*diff(y(x),x$2)+2*x*diff(y(x),x)-3*y(x)=0,y(x),type='series',x=0);
```

$$y(x)=\left(1+\frac{3}{4}x^2-\frac{5}{32}x^4\right)y(0)+\left(x+\frac{1}{12}x^3-\frac{1}{32}x^5\right)D(y)\,(0)+O\big(x^6\big)$$

✓ Solution by Mathematica
Time used: 0.001 (sec). Leaf size: 42

```
AsymptoticDSolveValue[(2+2*x^2)*y''[x]+2*x*y'[x]-3*y[x]==0,y[x],{x,0,5}]
```

$$y(x)\to c_2\left(-\frac{x^5}{32}+\frac{x^3}{12}+x\right)+c_1\left(-\frac{5x^4}{32}+\frac{3x^2}{4}+1\right)$$

16.14 problem 14

Internal problem ID [14790]

Book: INTRODUCTORY DIFFERENTIAL EQUATIONS. Martha L. Abell, James P. Braselton. Fourth edition 2014. ElScAe. 2014
Section: Chapter 4. Higher Order Equations. Exercises 4.8, page 203
Problem number: 14.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$\boxed{(-2x+3)\,y''+2y'-2y=0}$$

With initial conditions

$$[y(0)=3, y'(0)=-2]$$

With the expansion point for the power series method at $x=0$.

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 20

```
Order:=6;
dsolve([(3-2*x)*diff(y(x),x$2)+2*diff(y(x),x)-2*y(x)=0,y(0) = 3, D(y)(0) = -2],y(x),type='se
```

$$y(x)=3-2x+\frac{5}{3}x^2-\frac{2}{9}x^3+\frac{1}{18}x^4+\frac{1}{135}x^5+\mathrm{O}\left(x^6\right)$$

✓ Solution by Mathematica

Time used: 0.001 (sec). Leaf size: 36

```
AsymptoticDSolveValue[{(3-2*x)*y''[x]+2*y'[x]-2*y[x]==0,{y[0]==3,y'[0]==-2}},y[x],{x,0,5}]
```

$$y(x)\to \frac{x^5}{135}+\frac{x^4}{18}-\frac{2x^3}{9}+\frac{5x^2}{3}-2x+3$$

16.15 problem 15

Internal problem ID [14791]

Book: INTRODUCTORY DIFFERENTIAL EQUATIONS. Martha L. Abell, James P. Braselton. Fourth edition 2014. ElScAe. 2014
Section: Chapter 4. Higher Order Equations. Exercises 4.8, page 203
Problem number: 15.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_Emden, _Fowler]]`

$$\boxed{y'' - 4x^2y = 0}$$

With initial conditions

$$[y(0) = 1, y'(0) = 0]$$

With the expansion point for the power series method at $x = 0$.

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 12

```
Order:=6;
dsolve([diff(y(x),x$2)-4*x^2*y(x)=0,y(0) = 1, D(y)(0) = 0],y(x),type='series',x=0);
```

$$y(x) = 1 + \frac{1}{3}x^4 + \mathrm{O}\left(x^6\right)$$

✓ Solution by Mathematica

Time used: 0.001 (sec). Leaf size: 12

```
AsymptoticDSolveValue[{y''[x]-4*x^2*y[x]==0,{y[0]==1,y'[0]==0}},y[x],{x,0,5}]
```

$$y(x) \to \frac{x^4}{3} + 1$$

16.16 problem 16

Internal problem ID [14792]

Book: INTRODUCTORY DIFFERENTIAL EQUATIONS. Martha L. Abell, James P. Braselton. Fourth edition 2014. ElScAe. 2014
Section: Chapter 4. Higher Order Equations. Exercises 4.8, page 203
Problem number: 16.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[_Gegenbauer, [_2nd_order, _linear, '_with_symmetry_[0,F(x)]']`

$$\boxed{(2x^2-1)\,y''+2y'x-3y=0}$$

With initial conditions

$$[y(0)=-2, y'(0)=2]$$

With the expansion point for the power series method at $x=0$.

✓ Solution by Maple
Time used: 0.0 (sec). Leaf size: 20

```
Order:=6;
dsolve([(2*x^2-1)*diff(y(x),x$2)+2*x*diff(y(x),x)-3*y(x)=0,y(0) = -2, D(y)(0) = 2],y(x),type
```

$$y(x)=-2+2x+3x^2-\frac{1}{3}x^3+\frac{5}{4}x^4-\frac{1}{4}x^5+\mathrm{O}\left(x^6\right)$$

✓ Solution by Mathematica
Time used: 0.001 (sec). Leaf size: 34

```
AsymptoticDSolveValue[{(2*x^2-1)*y''[x]+2*x*y'[x]-3*y[x]==0,{y[0]==-2,y'[0]==2}},y[x],{x,0,5
```

$$y(x)\to -\frac{x^5}{4}+\frac{5x^4}{4}-\frac{x^3}{3}+3x^2+2x-2$$

16.17 problem 19

Internal problem ID [14793]

Book: INTRODUCTORY DIFFERENTIAL EQUATIONS. Martha L. Abell, James P. Braselton. Fourth edition 2014. ElScAe. 2014
Section: Chapter 4. Higher Order Equations. Exercises 4.8, page 203
Problem number: 19.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _missing_y]]`

$$\boxed{y'' + y'x = \sin(x)}$$

With initial conditions

$$[y(0) = 1, y'(0) = 0]$$

With the expansion point for the power series method at $x = 0$.

✓ Solution by Maple
Time used: 0.0 (sec). Leaf size: 14

```
Order:=6;
dsolve([diff(y(x),x$2)+x*diff(y(x),x)=sin(x),y(0) = 1, D(y)(0) = 0],y(x),type='series',x=0);
```

$$y(x) = 1 + \frac{1}{6}x^3 - \frac{1}{30}x^5 + \mathrm{O}\left(x^6\right)$$

✓ Solution by Mathematica
Time used: 0.03 (sec). Leaf size: 19

```
AsymptoticDSolveValue[{y''[x]+2*x*y'[x]==Sin[x],{y[0]==1,y'[0]==0}},y[x],{x,0,5}]
```

$$y(x) \to -\frac{7x^5}{120} + \frac{x^3}{6} + 1$$

16.18 problem 20

Internal problem ID [14794]

Book: INTRODUCTORY DIFFERENTIAL EQUATIONS. Martha L. Abell, James P. Braselton. Fourth edition 2014. ElScAe. 2014
Section: Chapter 4. Higher Order Equations. Exercises 4.8, page 203
Problem number: 20.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _linear, _nonhomogeneous]]`

$$\boxed{y'' + y' + yx = \cos(x)}$$

With initial conditions

$$[y(0) = 0, y'(0) = 1]$$

With the expansion point for the power series method at $x = 0$.

✓ Solution by Maple
Time used: 0.015 (sec). Leaf size: 14

```
Order:=6;
dsolve([diff(y(x),x$2)+diff(y(x),x)+x*y(x)=cos(x),y(0) = 0, D(y)(0) = 1],y(x),type='series',
```

$$y(x) = x - \frac{1}{8}x^4 + \frac{1}{40}x^5 + O\left(x^6\right)$$

✓ Solution by Mathematica
Time used: 0.019 (sec). Leaf size: 19

```
AsymptoticDSolveValue[{y''[x]+y'[x]+x*y[x]==Cos[x],{y[0]==0,y'[0]==1}},y[x],{x,0,5}]
```

$$y(x) \to \frac{x^5}{40} - \frac{x^4}{8} + x$$

16.19 problem 21

Internal problem ID [14795]

Book: INTRODUCTORY DIFFERENTIAL EQUATIONS. Martha L. Abell, James P. Braselton. Fourth edition 2014. ElScAe. 2014
Section: Chapter 4. Higher Order Equations. Exercises 4.8, page 203
Problem number: 21.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _missing_x], _Van_der_Pol]`

$$\boxed{y'' + (y^2 - 1)\, y' + y = 0}$$

With initial conditions

$$[y(0) = 0, y'(0) = 1]$$

With the expansion point for the power series method at $x = 0$.

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 16

```
Order:=6;
dsolve([diff(y(x),x$2)+(y(x)^2-1)*diff(y(x),x)+y(x)=0,y(0) = 0, D(y)(0) = 1],y(x),type='seri
```

$$y(x) = x + \frac{1}{2}x^2 - \frac{1}{8}x^4 - \frac{1}{8}x^5 + \mathrm{O}\left(x^6\right)$$

✓ Solution by Mathematica

Time used: 0.021 (sec). Leaf size: 26

```
AsymptoticDSolveValue[{y''[x]+(y[x]^2-1)*y'[x]+y[x]==0,{y[0]==0,y'[0]==1}},y[x],{x,0,5}]
```

$$y(x) \to -\frac{x^5}{8} - \frac{x^4}{8} + \frac{x^2}{2} + x$$

16.20 problem 22

Internal problem ID [14796]

Book: INTRODUCTORY DIFFERENTIAL EQUATIONS. Martha L. Abell, James P. Braselton. Fourth edition 2014. ElScAe. 2014
Section: Chapter 4. Higher Order Equations. Exercises 4.8, page 203
Problem number: 22.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _missing_x]]`

$$\boxed{y'' + \left(\frac{y'^2}{3} - 1\right) y' + y = 0}$$

With initial conditions

$$[y(0) = 1, y'(0) = 0]$$

With the expansion point for the power series method at $x = 0$.

✓ Solution by Maple
Time used: 0.0 (sec). Leaf size: 16

```
Order:=6;
dsolve([diff(y(x),x$2)+(1/3*diff(y(x),x)^2-1)*diff(y(x),x)+y(x)=0,y(0) = 1, D(y)(0) = 0],y(x
```

$$y(x) = 1 - \frac{1}{2}x^2 - \frac{1}{6}x^3 + \frac{1}{40}x^5 + O\left(x^6\right)$$

✓ Solution by Mathematica
Time used: 0.015 (sec). Leaf size: 26

```
AsymptoticDSolveValue[{y''[x]+(1/3*y'[x]^2-1)*y'[x]+y[x]==0,{y[0]==1,y'[0]==0}},y[x],{x,0,5}
```

$$y(x) \to \frac{x^5}{40} - \frac{x^3}{6} - \frac{x^2}{2} + 1$$

16.21 problem 23 (a)

Internal problem ID [14797]

Book: INTRODUCTORY DIFFERENTIAL EQUATIONS. Martha L. Abell, James P. Braselton. Fourth edition 2014. ElScAe. 2014
Section: Chapter 4. Higher Order Equations. Exercises 4.8, page 203
Problem number: 23 (a).
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$\boxed{y'' - 2y'x + 2y = 0}$$

With the expansion point for the power series method at $x = 0$.

✓ Solution by Maple
Time used: 0.0 (sec). Leaf size: 24

```
Order:=6;
dsolve(diff(y(x),x$2)-2*x*diff(y(x),x)+2*y(x)=0,y(x),type='series',x=0);
```

$$y(x) = \left(1 - x^2 - \frac{1}{6}x^4\right) y(0) + xD(y)(0) + O(x^6)$$

✓ Solution by Mathematica
Time used: 0.001 (sec). Leaf size: 25

```
AsymptoticDSolveValue[y''[x]-2*x*y'[x]+2*y[x]==0,y[x],{x,0,5}]
```

$$y(x) \to c_1\left(-\frac{x^4}{6} - x^2 + 1\right) + c_2 x$$

16.22 problem 23 (b)

Internal problem ID [14798]

Book: INTRODUCTORY DIFFERENTIAL EQUATIONS. Martha L. Abell, James P. Braselton. Fourth edition 2014. ElScAe. 2014
Section: Chapter 4. Higher Order Equations. Exercises 4.8, page 203
Problem number: 23 (b).
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$\boxed{y'' - 2y'x + 6y = 0}$$

With the expansion point for the power series method at $x = 0$.

✓ Solution by Maple
Time used: 0.0 (sec). Leaf size: 29

```
Order:=6;
dsolve(diff(y(x),x$2)-2*x*diff(y(x),x)+6*y(x)=0,y(x),type='series',x=0);
```

$$y(x) = \left(1 - 3x^2 + \frac{1}{2}x^4\right) y(0) + \left(-\frac{2}{3}x^3 + x\right) D(y)(0) + O(x^6)$$

✓ Solution by Mathematica
Time used: 0.001 (sec). Leaf size: 33

```
AsymptoticDSolveValue[y''[x]-2*x*y'[x]+6*y[x]==0,y[x],{x,0,5}]
```

$$y(x) \to c_2\left(x - \frac{2x^3}{3}\right) + c_1\left(\frac{x^4}{2} - 3x^2 + 1\right)$$

16.23 problem 24 (a)

Internal problem ID [14799]

Book: INTRODUCTORY DIFFERENTIAL EQUATIONS. Martha L. Abell, James P. Braselton. Fourth edition 2014. ElScAe. 2014
Section: Chapter 4. Higher Order Equations. Exercises 4.8, page 203
Problem number: 24 (a).
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _exact, _linear, _homogeneous]]`

$$\boxed{(-x^2+1)\,y''-y'x+y=0}$$

With the expansion point for the power series method at $x=0$.

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 24

```
Order:=6;
dsolve((1-x^2)*diff(y(x),x$2)-x*diff(y(x),x)+y(x)=0,y(x),type='series',x=0);
```

$$y(x)=\left(1-\frac{1}{2}x^2-\frac{1}{8}x^4\right)y(0)+xD(y)\,(0)+O\left(x^6\right)$$

✓ Solution by Mathematica

Time used: 0.001 (sec). Leaf size: 27

```
AsymptoticDSolveValue[(1-x^2)*y''[x]-x*y'[x]+y[x]==0,y[x],{x,0,5}]
```

$$y(x)\to c_1\left(-\frac{x^4}{8}-\frac{x^2}{2}+1\right)+c_2x$$

16.24 problem 24 (b)

Internal problem ID [14800]

Book: INTRODUCTORY DIFFERENTIAL EQUATIONS. Martha L. Abell, James P. Braselton. Fourth edition 2014. ElScAe. 2014
Section: Chapter 4. Higher Order Equations. Exercises 4.8, page 203
Problem number: 24 (b).
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[_Gegenbauer, [_2nd_order, _linear, '_with_symmetry_[0,F(x)]']]`

$$\boxed{(-x^2+1)\,y''-y'x+9y=0}$$

With the expansion point for the power series method at $x=0$.

✓ Solution by Maple
Time used: 0.0 (sec). Leaf size: 29

```
Order:=6;
dsolve((1-x^2)*diff(y(x),x$2)-x*diff(y(x),x)+9*y(x)=0,y(x),type='series',x=0);
```

$$y(x)=\left(1-\frac{9}{2}x^2+\frac{15}{8}x^4\right)y(0)+\left(x-\frac{4}{3}x^3\right)D(y)\,(0)+O(x^6)$$

✓ Solution by Mathematica
Time used: 0.001 (sec). Leaf size: 35

```
AsymptoticDSolveValue[(1-x^2)*y''[x]-x*y'[x]+9*y[x]==0,y[x],{x,0,5}]
```

$$y(x)\to c_2\left(x-\frac{4x^3}{3}\right)+c_1\left(\frac{15x^4}{8}-\frac{9x^2}{2}+1\right)$$

16.25 problem 27 (a)

Internal problem ID [14801]

Book: INTRODUCTORY DIFFERENTIAL EQUATIONS. Martha L. Abell, James P. Braselton. Fourth edition 2014. ElScAe. 2014
Section: Chapter 4. Higher Order Equations. Exercises 4.8, page 203
Problem number: 27 (a).
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _linear, _nonhomogeneous]]`

$$\boxed{y'' - y\cos(x) = \sin(x)}$$

With initial conditions

$$[y(0) = 1, y'(0) = 0]$$

With the expansion point for the power series method at $x = 0$.

✓ Solution by Maple
Time used: 0.0 (sec). Leaf size: 14

```
Order:=6;
dsolve([diff(y(x),x$2)-y(x)*cos(x)=sin(x),y(0) = 1, D(y)(0) = 0],y(x),type='series',x=0);
```

$$y(x) = 1 + \frac{1}{2}x^2 + \frac{1}{6}x^3 + \mathrm{O}\left(x^6\right)$$

✓ Solution by Mathematica
Time used: 0.018 (sec). Leaf size: 19

```
AsymptoticDSolveValue[{y''[x]-y[x]*Cos[x]==Sin[x],{y[0]==1,y'[0]==0}},y[x],{x,0,5}]
```

$$y(x) \to \frac{x^3}{6} + \frac{x^2}{2} + 1$$

17 Chapter 4. Higher Order Equations. Exercises 4.9, page 215

17.1 problem 1

Internal problem ID [14802]

Book: INTRODUCTORY DIFFERENTIAL EQUATIONS. Martha L. Abell, James P. Braselton. Fourth edition 2014. ElScAe. 2014
Section: Chapter 4. Higher Order Equations. Exercises 4.9, page 215
Problem number: 1.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_Emden, _Fowler]]`

$$\boxed{x^2y'' + 6y = 0}$$

With the expansion point for the power series method at $x = 0$.

✓ Solution by Maple
Time used: 0.047 (sec). Leaf size: 41

```
Order:=6;
dsolve(x^2*diff(y(x),x$2)+6*y(x)=0,y(x),type='series',x=0);
```

$$y(x) = \sqrt{x}\left(c_1x^{-\frac{i\sqrt{23}}{2}} + c_2x^{\frac{i\sqrt{23}}{2}}\right) + O\left(x^6\right)$$

✓ Solution by Mathematica
Time used: 0.003 (sec). Leaf size: 44

```
AsymptoticDSolveValue[x^2*y''[x]+6*y[x]==0,y[x],{x,0,5}]
```

$$y(x) \to c_1x^{\frac{1}{2}\left(1+i\sqrt{23}\right)} + c_2x^{\frac{1}{2}\left(1-i\sqrt{23}\right)}$$

17.2 problem 2

Internal problem ID [14803]

Book: INTRODUCTORY DIFFERENTIAL EQUATIONS. Martha L. Abell, James P. Braselton. Fourth edition 2014. ElScAe. 2014
Section: Chapter 4. Higher Order Equations. Exercises 4.9, page 215
Problem number: 2.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$\boxed{x(x+1)\, y'' + \frac{y'}{x^2} + 5y = 0}$$

With the expansion point for the power series method at $x = 0$.

✗ Solution by Maple

```
Order:=6;
dsolve(x*(x+1)*diff(y(x),x$2)+diff(y(x),x)/x^2+5*y(x)=0,y(x),type='series',x=0);
```

No solution found

✓ Solution by Mathematica

Time used: 0.055 (sec). Leaf size: 73

```
AsymptoticDSolveValue[x*(1+x)*y''[x]+y'[x]/x^2+5*y[x]==0,y[x],{x,0,5}]
```

$$y(x) \to c_1 \left(2x^5 - \frac{5x^3}{3} + 1\right) + c_2 e^{\frac{1}{2x^2} - \frac{1}{x}} \left(\frac{272x^5}{3} + \frac{88x^4}{3} + \frac{35x^3}{3} + 4x^2 + 2x + 1\right) x^2$$

17.3 problem 3

Internal problem ID [14804]

Book: INTRODUCTORY DIFFERENTIAL EQUATIONS. Martha L. Abell, James P. Braselton. Fourth edition 2014. ElScAe. 2014
Section: Chapter 4. Higher Order Equations. Exercises 4.9, page 215
Problem number: 3.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$\boxed{(x^2-3x-4)\,y''-(x+1)\,y'+y(x^2-1)=0}$$

With the expansion point for the power series method at $x=0$.

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 54

```
Order:=6;
dsolve((x^2-3*x-4)*diff(y(x),x$2)-(x+1)*diff(y(x),x)+(x^2-1)*y(x)=0,y(x),type='series',x=0);
```

$$\begin{aligned}y(x) = &\left(1-\frac{1}{8}x^2+\frac{1}{24}x^3+\frac{1}{192}x^4-\frac{1}{640}x^5\right)y(0)\\&+\left(x-\frac{1}{8}x^2-\frac{1}{24}x^3+\frac{1}{48}x^4+\frac{1}{960}x^5\right)D(y)\,(0)+O(x^6)\end{aligned}$$

✓ Solution by Mathematica

Time used: 0.001 (sec). Leaf size: 70

```
AsymptoticDSolveValue[(x^2-3*x-4)*y''[x]-(x+1)*y'[x]+(x^2-1)*y[x]==0,y[x],{x,0,5}]
```

$$y(x)\to c_1\left(-\frac{x^5}{640}+\frac{x^4}{192}+\frac{x^3}{24}-\frac{x^2}{8}+1\right)+c_2\left(\frac{x^5}{960}+\frac{x^4}{48}-\frac{x^3}{24}-\frac{x^2}{8}+x\right)$$

17.4 problem 4

Internal problem ID [14805]

Book: INTRODUCTORY DIFFERENTIAL EQUATIONS. Martha L. Abell, James P. Braselton. Fourth edition 2014. ElScAe. 2014
Section: Chapter 4. Higher Order Equations. Exercises 4.9, page 215
Problem number: 4.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$\boxed{(x^2-25)^2 y''-(x+5)\, y'+10y=0}$$

With the expansion point for the power series method at $x=0$.

✓ Solution by Maple
Time used: 0.0 (sec). Leaf size: 54

```
Order:=6;
dsolve((x^2-25)^2*diff(y(x),x$2)-(x+5)*diff(y(x),x)+10*y(x)=0,y(x),type='series',x=0);
```

$$y(x)=\left(1-\frac{1}{125}x^2-\frac{1}{46875}x^3-\frac{767}{7812500}x^4-\frac{4813}{7324218750}x^5\right)y(0)+\left(x+\frac{1}{250}x^2-\frac{112}{46875}x^3+\frac{173}{3906250}x^4-\frac{409681}{7324218750}x^5\right)D(y)(0)+O(x^6)$$

✓ Solution by Mathematica
Time used: 0.001 (sec). Leaf size: 70

```
AsymptoticDSolveValue[(x^2-25)^2*y''[x]-(x+5)*y'[x]+10*y[x]==0,y[x],{x,0,5}]
```

$$y(x)\to c_1\left(-\frac{4813x^5}{7324218750}-\frac{767x^4}{7812500}-\frac{x^3}{46875}-\frac{x^2}{125}+1\right)+c_2\left(-\frac{409681x^5}{7324218750}+\frac{173x^4}{3906250}-\frac{112x^3}{46875}+\frac{x^2}{250}+x\right)$$

17.5 problem 5

Internal problem ID [14806]

Book: INTRODUCTORY DIFFERENTIAL EQUATIONS. Martha L. Abell, James P. Braselton. Fourth edition 2014. ElScAe. 2014
Section: Chapter 4. Higher Order Equations. Exercises 4.9, page 215
Problem number: 5.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_Emden, _Fowler]]`

$$\boxed{2xy'' - 5y' - 3y = 0}$$

With the expansion point for the power series method at $x = 0$.

✓ Solution by Maple

Time used: 0.078 (sec). Leaf size: 44

```
Order:=6;
dsolve(2*x*diff(y(x),x$2)-5*diff(y(x),x)-3*y(x)=0,y(x),type='series',x=0);
```

$$y(x) = c_1 x^{\frac{7}{2}} \left(1 + \frac{1}{3}x + \frac{1}{22}x^2 + \frac{1}{286}x^3 + \frac{1}{5720}x^4 + \frac{3}{486200}x^5 + \mathrm{O}\left(x^6\right)\right) + c_2 \left(1 - \frac{3}{5}x + \frac{3}{10}x^2 - \frac{3}{10}x^3 - \frac{9}{40}x^4 - \frac{9}{200}x^5 + \mathrm{O}\left(x^6\right)\right)$$

✓ Solution by Mathematica

Time used: 0.002 (sec). Leaf size: 85

```
AsymptoticDSolveValue[2*x*y''[x]-5*y'[x]-3*y[x]==0,y[x],{x,0,5}]
```

$$y(x) \to c_2 \left(-\frac{9x^5}{200} - \frac{9x^4}{40} - \frac{3x^3}{10} + \frac{3x^2}{10} - \frac{3x}{5} + 1\right) + c_1 \left(\frac{3x^5}{486200} + \frac{x^4}{5720} + \frac{x^3}{286} + \frac{x^2}{22} + \frac{x}{3} + 1\right) x^{7/2}$$

17.6 problem 6

Internal problem ID [14807]

Book: INTRODUCTORY DIFFERENTIAL EQUATIONS. Martha L. Abell, James P. Braselton. Fourth edition 2014. ElScAe. 2014
Section: Chapter 4. Higher Order Equations. Exercises 4.9, page 215
Problem number: 6.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$\boxed{5xy'' + 8y' - yx = 0}$$

With the expansion point for the power series method at $x = 0$.

✓ Solution by Maple
Time used: 0.031 (sec). Leaf size: 32

```
Order:=6;
dsolve(5*x*diff(y(x),x$2)+8*diff(y(x),x)-x*y(x)=0,y(x),type='series',x=0);
```

$$y(x) = \frac{c_1\left(1 + \frac{1}{14}x^2 + \frac{1}{952}x^4 + \mathrm{O}\left(x^6\right)\right)}{x^{\frac{3}{5}}} + c_2\left(1 + \frac{1}{26}x^2 + \frac{1}{2392}x^4 + \mathrm{O}\left(x^6\right)\right)$$

✓ Solution by Mathematica
Time used: 0.002 (sec). Leaf size: 47

```
AsymptoticDSolveValue[5*x*y''[x]+8*y'[x]-x*y[x]==0,y[x],{x,0,5}]
```

$$y(x) \to c_1\left(\frac{x^4}{2392} + \frac{x^2}{26} + 1\right) + \frac{c_2\left(\frac{x^4}{952} + \frac{x^2}{14} + 1\right)}{x^{3/5}}$$

17.7 problem 7

Internal problem ID [14808]

Book: INTRODUCTORY DIFFERENTIAL EQUATIONS. Martha L. Abell, James P. Braselton. Fourth edition 2014. ElScAe. 2014
Section: Chapter 4. Higher Order Equations. Exercises 4.9, page 215
Problem number: 7.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$\boxed{9xy'' + 14y' + y(x-1) = 0}$$

With the expansion point for the power series method at $x = 0$.

✓ Solution by Maple

Time used: 0.031 (sec). Leaf size: 44

```
Order:=6;
dsolve(9*x*diff(y(x),x$2)+14*diff(y(x),x)+(x-1)*y(x)=0,y(x),type='series',x=0);
```

$$y(x) = \frac{c_1\left(1 + \frac{1}{4}x - \frac{3}{104}x^2 - \frac{29}{6864}x^3 + \frac{13}{65472}x^4 + \frac{251}{11348480}x^5 + \mathrm{O}\left(x^6\right)\right)}{x^{\frac{5}{9}}} + c_2\left(1 + \frac{1}{14}x - \frac{13}{644}x^2 - \frac{59}{61824}x^3 + \frac{29}{247296}x^4 + \frac{53}{12364800}x^5 + \mathrm{O}\left(x^6\right)\right)$$

✓ Solution by Mathematica

Time used: 0.002 (sec). Leaf size: 85

```
AsymptoticDSolveValue[9*x*y''[x]+14*y'[x]+(x-1)*y[x]==0,y[x],{x,0,5}]
```

$$y(x) \to c_1\left(\frac{53x^5}{12364800} + \frac{29x^4}{247296} - \frac{59x^3}{61824} - \frac{13x^2}{644} + \frac{x}{14} + 1\right) + \frac{c_2\left(\frac{251x^5}{11348480} + \frac{13x^4}{65472} - \frac{29x^3}{6864} - \frac{3x^2}{104} + \frac{x}{4} + 1\right)}{x^{5/9}}$$

17.8 problem 8

Internal problem ID [14809]

Book: INTRODUCTORY DIFFERENTIAL EQUATIONS. Martha L. Abell, James P. Braselton. Fourth edition 2014. ElScAe. 2014
Section: Chapter 4. Higher Order Equations. Exercises 4.9, page 215
Problem number: 8.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$\boxed{7xy'' + 10y' + \left(-x^2 + 1\right) y = 0}$$

With the expansion point for the power series method at $x = 0$.

✓ Solution by Maple
Time used: 0.047 (sec). Leaf size: 44

```
Order:=6;
dsolve(7*x*diff(y(x),x$2)+10*diff(y(x),x)+(1-x^2)*y(x)=0,y(x),type='series',x=0);
```

$$y(x) = \frac{c_1\left(1 - \frac{1}{4}x + \frac{1}{88}x^2 + \frac{29}{1584}x^3 - \frac{17}{6336}x^4 + \frac{89}{1013760}x^5 + \mathrm{O}\left(x^6\right)\right)}{x^{\frac{3}{7}}} + c_2\left(1 - \frac{1}{10}x + \frac{1}{340}x^2 + \frac{113}{8160}x^3 - \frac{929}{1011840}x^4 + \frac{781}{38449920}x^5 + \mathrm{O}\left(x^6\right)\right)$$

✓ Solution by Mathematica
Time used: 0.002 (sec). Leaf size: 85

```
AsymptoticDSolveValue[7*x*y''[x]+10*y'[x]+(1-x^2)*y[x]==0,y[x],{x,0,5}]
```

$$y(x) \to c_1\left(\frac{781x^5}{38449920} - \frac{929x^4}{1011840} + \frac{113x^3}{8160} + \frac{x^2}{340} - \frac{x}{10} + 1\right) + \frac{c_2\left(\frac{89x^5}{1013760} - \frac{17x^4}{6336} + \frac{29x^3}{1584} + \frac{x^2}{88} - \frac{x}{4} + 1\right)}{x^{3/7}}$$

17.9 problem 9

Internal problem ID [14810]

Book: INTRODUCTORY DIFFERENTIAL EQUATIONS. Martha L. Abell, James P. Braselton. Fourth edition 2014. ElScAe. 2014
Section: Chapter 4. Higher Order Equations. Exercises 4.9, page 215
Problem number: 9.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$\boxed{x^2y'' + y'x + y(x-1) = 0}$$

With the expansion point for the power series method at $x = 0$.

✓ Solution by Maple

Time used: 0.047 (sec). Leaf size: 63

```
Order:=6;
dsolve(x^2*diff(y(x),x$2)+x*diff(y(x),x)+(x-1)*y(x)=0,y(x),type='series',x=0);
```

$$y(x) = \frac{c_1 x^2\left(1 - \frac{1}{3}x + \frac{1}{24}x^2 - \frac{1}{360}x^3 + \frac{1}{8640}x^4 - \frac{1}{302400}x^5 + \mathrm{O}\left(x^6\right)\right) + c_2\left(\ln\left(x\right)\left(x^2 - \frac{1}{3}x^3 + \frac{1}{24}x^4 - \frac{1}{360}x^5 + \mathrm{O}\left(x^6\right.\right.\right.}{x}$$

✓ Solution by Mathematica

Time used: 0.017 (sec). Leaf size: 83

```
AsymptoticDSolveValue[x^2*y''[x]+x*y'[x]+(x-1)*y[x]==0,y[x],{x,0,5}]
```

$$y(x) \to c_1\left(\frac{31x^4 - 176x^3 + 144x^2 + 576x + 576}{576x} - \frac{1}{48}x\left(x^2 - 8x + 24\right)\log(x)\right) + c_2\left(\frac{x^5}{8640} - \frac{x^4}{360} + \frac{x^3}{24} - \frac{x^2}{3} + x\right)$$

17.10 problem 10

Internal problem ID [14811]

Book: INTRODUCTORY DIFFERENTIAL EQUATIONS. Martha L. Abell, James P. Braselton. Fourth edition 2014. ElScAe. 2014
Section: Chapter 4. Higher Order Equations. Exercises 4.9, page 215
Problem number: 10.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$\boxed{xy'' + 2y'x + y = 0}$$

With the expansion point for the power series method at $x = 0$.

✓ Solution by Maple

Time used: 0.047 (sec). Leaf size: 60

```
Order:=6;
dsolve(x*diff(y(x),x$2)+2*x*diff(y(x),x)+y(x)=0,y(x),type='series',x=0);
```

$$y(x) = c_1 x\left(1 - \frac{3}{2}x + \frac{5}{4}x^2 - \frac{35}{48}x^3 + \frac{21}{64}x^4 - \frac{77}{640}x^5 + \mathrm{O}\left(x^6\right)\right) + c_2\left(\ln\left(x\right)\left(-x + \frac{3}{2}x^2 - \frac{5}{4}x^3 + \frac{35}{48}x^4 - \frac{21}{64}x^5 + \mathrm{O}\left(x^6\right)\right) + \left(1 - 2x + \frac{7}{4}x^2 - \frac{11}{12}x^3 + \frac{61}{192}x^4 - \frac{131}{1920}x^5 + \mathrm{O}\left(x^6\right)\right)\right)$$

✓ Solution by Mathematica

Time used: 0.024 (sec). Leaf size: 87

```
AsymptoticDSolveValue[x*y''[x]+2*x*y'[x]+y[x]==0,y[x],{x,0,5}]
```

$$y(x) \to c_1\left(\frac{1}{48}x\left(35x^3 - 60x^2 + 72x - 48\right)\log(x) + \frac{1}{192}\left(-79x^4 + 64x^3 + 48x^2 - 192x + 192\right)\right) + c_2\left(\frac{21x^5}{64} - \frac{35x^4}{48} + \frac{5x^3}{4} - \frac{3x^2}{2} + x\right)$$

17.11 problem 11

Internal problem ID [14812]

Book: INTRODUCTORY DIFFERENTIAL EQUATIONS. Martha L. Abell, James P. Braselton. Fourth edition 2014. ElScAe. 2014
Section: Chapter 4. Higher Order Equations. Exercises 4.9, page 215
Problem number: 11.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$\boxed{y'' + \frac{8y'}{3x} - \left(\frac{2}{3x^2} - 1\right) y = 0}$$

With the expansion point for the power series method at $x = 0$.

✓ Solution by Maple
Time used: 0.031 (sec). Leaf size: 35

```
Order:=6;
dsolve(diff(y(x),x$2)+8/3*1/x*diff(y(x),x)-(2/3*1/x^2-1)*y(x)=0,y(x),type='series',x=0);
```

$$y(x) = \frac{c_2 x^{\frac{7}{3}} \left(1 - \frac{3}{26}x^2 + \frac{9}{1976}x^4 + \mathrm{O}\left(x^6\right)\right) + c_1 \left(1 + \frac{3}{2}x^2 - \frac{9}{40}x^4 + \mathrm{O}\left(x^6\right)\right)}{x^2}$$

✓ Solution by Mathematica
Time used: 0.002 (sec). Leaf size: 50

```
AsymptoticDSolveValue[y''[x]+8/3*1/x*y'[x]-(2/3*1/x^2-1)*y[x]==0,y[x],{x,0,5}]
```

$$y(x) \to c_1 \sqrt[3]{x} \left(\frac{9x^4}{1976} - \frac{3x^2}{26} + 1\right) + \frac{c_2 \left(-\frac{9x^4}{40} + \frac{3x^2}{2} + 1\right)}{x^2}$$

17.12 problem 12

Internal problem ID [14813]

Book: INTRODUCTORY DIFFERENTIAL EQUATIONS. Martha L. Abell, James P. Braselton. Fourth edition 2014. ElScAe. 2014
Section: Chapter 4. Higher Order Equations. Exercises 4.9, page 215
Problem number: 12.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _exact, _linear, _homogeneous]]`

$$\boxed{y'' + \left(\frac{16}{3x} - 1\right) y' - \frac{16y}{3x^2} = 0}$$

With the expansion point for the power series method at $x = 0$.

✓ Solution by Maple

Time used: 0.031 (sec). Leaf size: 45

```
Order:=6;
dsolve(diff(y(x),x$2)+(16/3*1/x-1)*diff(y(x),x)-16/3*1/x^2*y(x)=0,y(x),type='series',x=0);
```

$$y(x) = \frac{c_1\left(1 + x + \frac{1}{2}x^2 + \frac{1}{6}x^3 + \frac{1}{24}x^4 + \frac{1}{120}x^5 + \mathrm{O}\left(x^6\right)\right)}{x^{\frac{16}{3}}} + c_2 x\left(1 + \frac{3}{22}x + \frac{9}{550}x^2 + \frac{27}{15400}x^3 + \frac{81}{477400}x^4 + \frac{243}{16231600}x^5 + \mathrm{O}\left(x^6\right)\right)$$

✓ Solution by Mathematica

Time used: 0.003 (sec). Leaf size: 82

```
AsymptoticDSolveValue[y''[x]+(16/3*1/x-1)*y'[x]-16/3*1/x^2*y[x]==0,y[x],{x,0,5}]
```

$$y(x) \to c_1 x\left(\frac{243x^5}{16231600} + \frac{81x^4}{477400} + \frac{27x^3}{15400} + \frac{9x^2}{550} + \frac{3x}{22} + 1\right) + \frac{c_2\left(\frac{x^5}{120} + \frac{x^4}{24} + \frac{x^3}{6} + \frac{x^2}{2} + x + 1\right)}{x^{16/3}}$$

17.13 problem 13

Internal problem ID [14814]

Book: INTRODUCTORY DIFFERENTIAL EQUATIONS. Martha L. Abell, James P. Braselton. Fourth edition 2014. ElScAe. 2014
Section: Chapter 4. Higher Order Equations. Exercises 4.9, page 215
Problem number: 13.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$\boxed{y'' + \left(\frac{1}{2x} - 2\right) y' - \frac{35y}{16x^2} = 0}$$

With the expansion point for the power series method at $x = 0$.

✓ Solution by Maple

Time used: 0.031 (sec). Leaf size: 65

```
Order:=6;
dsolve(diff(y(x),x$2)+(1/2*1/x-2)*diff(y(x),x)-35/16*1/x^2*y(x)=0,y(x),type='series',x=0);
```

$$y(x) = \frac{c_1 x^3 \left(1 + \frac{7}{8}x + \frac{77}{160}x^2 + \frac{77}{384}x^3 + \frac{209}{3072}x^4 + \frac{4807}{245760}x^5 + \mathrm{O}\left(x^6\right)\right) + c_2 \left(\ln\left(x\right)\left(\frac{15}{8}x^3 + \frac{105}{64}x^4 + \frac{231}{256}x^5 + \mathrm{O}\left(x^6\right)\right)\right.}{x^{\frac{5}{4}}}$$

✓ Solution by Mathematica

Time used: 0.028 (sec). Leaf size: 98

```
AsymptoticDSolveValue[y''[x]+(1/2*1/x-2)*y'[x]-35/16*1/x^2*y[x]==0,y[x],{x,0,5}]
```

$$y(x) \to c_2 \left(\frac{209x^{23/4}}{3072} + \frac{77x^{19/4}}{384} + \frac{77x^{15/4}}{160} + \frac{7x^{11/4}}{8} + x^{7/4}\right) + c_1 \left(\frac{5}{256}x^{7/4}(7x+8)\log(x) - \frac{627x^4 + 608x^3 - 320x^2 - 1280x - 1024}{1024x^{5/4}}\right)$$

17.14 problem 14

Internal problem ID [14815]

Book: INTRODUCTORY DIFFERENTIAL EQUATIONS. Martha L. Abell, James P. Braselton. Fourth edition 2014. ElScAe. 2014
Section: Chapter 4. Higher Order Equations. Exercises 4.9, page 215
Problem number: 14.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$\boxed{y''-\left(\frac{1}{x}+2\right)y'+\left(x+\frac{1}{x^2}\right)y=0}$$

With the expansion point for the power series method at $x=0$.

✓ Solution by Maple

Time used: 0.031 (sec). Leaf size: 63

```
Order:=6;
dsolve(diff(y(x),x$2)-(1/x+2)*diff(y(x),x)+(x+1/x^2)*y(x)=0,y(x),type='series',x=0);
```

$$y(x)=\left((c_1+c_2\ln(x))\left(1+2x+2x^2+\frac{11}{9}x^3+\frac{35}{72}x^4+\frac{103}{900}x^5+\mathrm{O}\left(x^6\right)\right)+\left((-2)x-3x^2-\frac{64}{27}x^3-\frac{497}{432}x^4-\frac{9371}{27000}x^5+\mathrm{O}\left(x^6\right)\right)c_2\right)x$$

✓ Solution by Mathematica

Time used: 0.004 (sec). Leaf size: 110

```
AsymptoticDSolveValue[y''[x]-(1/x+2)*y'[x]+(x+1/x^2)*y[x]==0,y[x],{x,0,5}]
```

$$y(x)\to c_1x\left(\frac{103x^5}{900}+\frac{35x^4}{72}+\frac{11x^3}{9}+2x^2+2x+1\right)+c_2\left(x\left(-\frac{9371x^5}{27000}-\frac{497x^4}{432}-\frac{64x^3}{27}-3x^2-2x\right)+x\left(\frac{103x^5}{900}+\frac{35x^4}{72}+\frac{11x^3}{9}+2x^2+2x+1\right)\log(x)\right)$$

17.15 problem 15

Internal problem ID [14816]

Book: INTRODUCTORY DIFFERENTIAL EQUATIONS. Martha L. Abell, James P. Braselton. Fourth edition 2014. ElScAe. 2014
Section: Chapter 4. Higher Order Equations. Exercises 4.9, page 215
Problem number: 15.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_Emden, _Fowler]]`

$$\boxed{x^2y'' + 7y'x - 7y = 0}$$

With the expansion point for the power series method at $x = 0$.

✓ Solution by Maple
Time used: 0.047 (sec). Leaf size: 25

```
Order:=6;
dsolve(x^2*diff(y(x),x$2)+7*x*diff(y(x),x)-7*y(x)=0,y(x),type='series',x=0);
```

$$y(x) = c_1x\left(1 + \mathrm{O}\left(x^6\right)\right) + \frac{c_2(-203212800 + \mathrm{O}\left(x^6\right))}{x^7}$$

✓ Solution by Mathematica
Time used: 0.003 (sec). Leaf size: 14

```
AsymptoticDSolveValue[x^2*y''[x]+7*x*y'[x]-7*y[x]==0,y[x],{x,0,5}]
```

$$y(x) \to \frac{c_1}{x^7} + c_2x$$

17.16 problem 16

Internal problem ID [14817]

Book: INTRODUCTORY DIFFERENTIAL EQUATIONS. Martha L. Abell, James P. Braselton. Fourth edition 2014. ElScAe. 2014
Section: Chapter 4. Higher Order Equations. Exercises 4.9, page 215
Problem number: 16.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _exact, _linear, _homogeneous]]`

$$\boxed{x^2y'' + 3y'x + y = 0}$$

With the expansion point for the power series method at $x = 0$.

✓ Solution by Maple

Time used: 0.031 (sec). Leaf size: 29

```
Order:=6;
dsolve(x^2*diff(y(x),x$2)+3*x*diff(y(x),x)+y(x)=0,y(x),type='series',x=0);
```

$$y(x) = \frac{c_1 + c_2 \ln(x)}{x} + O(x^6)$$

✓ Solution by Mathematica

Time used: 0.002 (sec). Leaf size: 18

```
AsymptoticDSolveValue[x^2*y''[x]+3*x*y'[x]+y[x]==0,y[x],{x,0,5}]
```

$$y(x) \to \frac{c_1}{x} + \frac{c_2 \log(x)}{x}$$

17.17 problem 17

Internal problem ID [14818]

Book: INTRODUCTORY DIFFERENTIAL EQUATIONS. Martha L. Abell, James P. Braselton. Fourth edition 2014. ElScAe. 2014
Section: Chapter 4. Higher Order Equations. Exercises 4.9, page 215
Problem number: 17.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_Emden, _Fowler], [_2nd_order, _linear, '_with_symmetry_[0,F`

$$\boxed{x^2y'' - 3y'x + 4y = 0}$$

With the expansion point for the power series method at $x = 0$.

✓ Solution by Maple
Time used: 0.032 (sec). Leaf size: 29

```
Order:=6;
dsolve(x^2*diff(y(x),x$2)-3*x*diff(y(x),x)+4*y(x)=0,y(x),type='series',x=0);
```

$$y(x) = x^2(c_1 + c_2 \ln(x)) + O(x^6)$$

✓ Solution by Mathematica
Time used: 0.002 (sec). Leaf size: 18

```
AsymptoticDSolveValue[x^2*y''[x]-3*x*y'[x]+4*y[x]==0,y[x],{x,0,5}]
```

$$y(x) \to c_1x^2 + c_2x^2 \log(x)$$

17.18 problem 19 (a)

Internal problem ID [14819]

Book: INTRODUCTORY DIFFERENTIAL EQUATIONS. Martha L. Abell, James P. Braselton. Fourth edition 2014. ElScAe. 2014
Section: Chapter 4. Higher Order Equations. Exercises 4.9, page 215
Problem number: 19 (a).
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_Emden, _Fowler]]`

$$\boxed{y'' + yx = 0}$$

With the expansion point for the power series method at $x = 0$.

✓ Solution by Maple
Time used: 0.0 (sec). Leaf size: 24

```
Order:=6;
dsolve(diff(y(x),x$2)+x*y(x)=0,y(x),type='series',x=0);
```

$$y(x) = \left(1 - \frac{x^3}{6}\right) y(0) + \left(x - \frac{1}{12}x^4\right) D(y)(0) + O(x^6)$$

✓ Solution by Mathematica
Time used: 0.001 (sec). Leaf size: 28

```
AsymptoticDSolveValue[y''[x]+x*y[x]==0,y[x],{x,0,5}]
```

$$y(x) \to c_2\left(x - \frac{x^4}{12}\right) + c_1\left(1 - \frac{x^3}{6}\right)$$

17.19 problem 19 (b)

Internal problem ID [14820]

Book: INTRODUCTORY DIFFERENTIAL EQUATIONS. Martha L. Abell, James P. Braselton. Fourth edition 2014. ElScAe. 2014
Section: Chapter 4. Higher Order Equations. Exercises 4.9, page 215
Problem number: 19 (b).
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[_Bessel]`

$$\boxed{x^2 y'' + y'x + \left(-k^2 + x^2\right) y = 0}$$

With the expansion point for the power series method at $x = 0$.

✓ Solution by Maple
Time used: 0.032 (sec). Leaf size: 77

```
Order:=6;
dsolve(x^2*diff(y(x),x$2)+x*diff(y(x),x)+(x^2-k^2)*y(x)=0,y(x),type='series',x=0);
```

$$y(x) = c_1 x^{-k}\left(1 + \frac{1}{4k-4}x^2 + \frac{1}{32}\frac{1}{(-2+k)(-1+k)}x^4 + \mathrm{O}\left(x^6\right)\right) + c_2 x^k\left(1 - \frac{1}{4k+4}x^2 + \frac{1}{32}\frac{1}{(k+2)(1+k)}x^4 + \mathrm{O}\left(x^6\right)\right)$$

✓ Solution by Mathematica

Time used: 0.003 (sec). Leaf size: 160

```
AsymptoticDSolveValue[x^2*y''[x]+x*y'[x]+(x^2-k^2)*y[x]==0,y[x],{x,0,5}]
```

$$y(x) \to c_2\left(\frac{x^4}{(-k^2-k+(1-k)(2-k)+2)(-k^2-k+(3-k)(4-k)+4)} - \frac{x^2}{-k^2-k+(1-k)(2-k)+2}+1\right)x^{-k} + c_1\left(\frac{x^4}{(-k^2+k+(k+1)(k+2)+2)(-k^2+k+(k+3)(k+4)+4)} - \frac{x^2}{-k^2+k+(k+1)(k+2)+2}+1\right)x^k$$

17.20 problem 19 (c)

Internal problem ID [14821]

Book: INTRODUCTORY DIFFERENTIAL EQUATIONS. Martha L. Abell, James P. Braselton. Fourth edition 2014. ElScAe. 2014
Section: Chapter 4. Higher Order Equations. Exercises 4.9, page 215
Problem number: 19 (c).
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[_Gegenbauer]`

$$\boxed{(-x^2+1)\,y''-2y'x+k(1+k)\,y=0}$$

With the expansion point for the power series method at $x=0$.

✓ Solution by Maple
Time used: 0.0 (sec). Leaf size: 101

```
Order:=6;
dsolve((1-x^2)*diff(y(x),x$2)-2*x*diff(y(x),x)+k*(k+1)*y(x)=0,y(x),type='series',x=0);
```

$$y(x)=\left(1-\frac{k(1+k)\,x^2}{2}+\frac{k(k^3+2k^2-5k-6)\,x^4}{24}\right)y(0)+\left(x-\frac{(k^2+k-2)\,x^3}{6}+\frac{(k^4+2k^3-13k^2-14k+24)\,x^5}{120}\right)D(y)\,(0)+O(x^6)$$

✓ Solution by Mathematica
Time used: 0.001 (sec). Leaf size: 120

```
AsymptoticDSolveValue[(1-x^2)*y''[x]-2*x*y'[x]+k*(k+1)*y[x]==0,y[x],{x,0,5}]
```

$$y(x)\to c_2\left(\frac{1}{120}(k^2+k)^2x^5+\frac{7}{60}(-k^2-k)\,x^5+\frac{1}{6}(-k^2-k)\,x^3+\frac{x^5}{5}+\frac{x^3}{3}+x\right)+c_1\left(\frac{1}{24}(k^2+k)^2x^4+\frac{1}{4}(-k^2-k)\,x^4+\frac{1}{2}(-k^2-k)\,x^2+1\right)$$

17.21 problem 21

Internal problem ID [14822]

Book: INTRODUCTORY DIFFERENTIAL EQUATIONS. Martha L. Abell, James P. Braselton. Fourth edition 2014. ElScAe. 2014
Section: Chapter 4. Higher Order Equations. Exercises 4.9, page 215
Problem number: 21.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _exact, _linear, _homogeneous]]`

$$\boxed{x(1-x)\,y''+\left(\frac{1}{2}-3x\right)y'-y=0}$$

With the expansion point for the power series method at $x=0$.

✓ Solution by Maple

Time used: 0.047 (sec). Leaf size: 44

```
Order:=6;
dsolve(x*(1-x)*diff(y(x),x$2)+(1/2-3*x)*diff(y(x),x)-y(x)=0,y(x),type='series',x=0);
```

$$y(x)=c_1\sqrt{x}\left(1+\frac{3}{2}x+\frac{15}{8}x^2+\frac{35}{16}x^3+\frac{315}{128}x^4+\frac{693}{256}x^5+\mathrm{O}\left(x^6\right)\right)+c_2\left(1+2x+\frac{8}{3}x^2+\frac{16}{5}x^3+\frac{128}{35}x^4+\frac{256}{63}x^5+\mathrm{O}\left(x^6\right)\right)$$

✓ Solution by Mathematica

Time used: 0.004 (sec). Leaf size: 83

```
AsymptoticDSolveValue[x*(1-x)*y''[x]+(1/2-3*x)*y'[x]-y[x]==0,y[x],{x,0,5}]
```

$$y(x)\to c_1\sqrt{x}\left(\frac{693x^5}{256}+\frac{315x^4}{128}+\frac{35x^3}{16}+\frac{15x^2}{8}+\frac{3x}{2}+1\right)+c_2\left(\frac{256x^5}{63}+\frac{128x^4}{35}+\frac{16x^3}{5}+\frac{8x^2}{3}+2x+1\right)$$

17.22 problem 22

Internal problem ID [14823]

Book: INTRODUCTORY DIFFERENTIAL EQUATIONS. Martha L. Abell, James P. Braselton. Fourth edition 2014. ElScAe. 2014
Section: Chapter 4. Higher Order Equations. Exercises 4.9, page 215
Problem number: 22.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _exact, _linear, _homogeneous]]`

$$\boxed{x(1-x)\,y''+y'+2y=0}$$

With the expansion point for the power series method at $x=0$.

✓ Solution by Maple

Time used: 0.031 (sec). Leaf size: 47

```
Order:=6;
dsolve(x*(1-x)*diff(y(x),x$2)+diff(y(x),x)+2*y(x)=0,y(x),type='series',x=0);
```

$$y(x)=(c_1+c_2\ln(x))\left(1-2x+x^2+\mathrm{O}\left(x^6\right)\right)+\left(3x-3x^2+\frac{1}{3}x^3+\frac{1}{12}x^4+\frac{1}{30}x^5+\mathrm{O}\left(x^6\right)\right)c_2$$

✓ Solution by Mathematica

Time used: 0.004 (sec). Leaf size: 59

```
AsymptoticDSolveValue[x*(1-x)*y''[x]+y'[x]+2*y[x]==0,y[x],{x,0,5}]
```

$$y(x)\to c_1\left(x^2-2x+1\right)+c_2\left(\frac{x^5}{30}+\frac{x^4}{12}+\frac{x^3}{3}-3x^2+\left(x^2-2x+1\right)\log(x)+3x\right)$$

17.23 problem 23

Internal problem ID [14824]

Book: INTRODUCTORY DIFFERENTIAL EQUATIONS. Martha L. Abell, James P. Braselton. Fourth edition 2014. ElScAe. 2014
Section: Chapter 4. Higher Order Equations. Exercises 4.9, page 215
Problem number: 23.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[_Jacobi]`

$$\boxed{x(1-x)\,y''+(-2x+1)\,y'+2y=0}$$

With the expansion point for the power series method at $x=0$.

✓ Solution by Maple
Time used: 0.031 (sec). Leaf size: 43

```
Order:=6;
dsolve(x*(1-x)*diff(y(x),x$2)+(1-2*x)*diff(y(x),x)+2*y(x)=0,y(x),type='series',x=0);
```

$$y(x)=(c_1+c_2\ln(x))\left(1-2x+\mathrm{O}\left(x^6\right)\right)+\left(5x-\frac{3}{2}x^2-\frac{2}{3}x^3-\frac{5}{12}x^4-\frac{3}{10}x^5+\mathrm{O}\left(x^6\right)\right)c_2$$

✓ Solution by Mathematica
Time used: 0.005 (sec). Leaf size: 55

```
AsymptoticDSolveValue[x*(1-x)*y''[x]+(1-2*x)*y'[x]+2*y[x]==0,y[x],{x,0,5}]
```

$$y(x)\to c_2\left(-\frac{3x^5}{10}-\frac{5x^4}{12}-\frac{2x^3}{3}-\frac{3x^2}{2}+5x+(1-2x)\log(x)\right)+c_1(1-2x)$$

17.24 problem 24

Internal problem ID [14825]

Book: INTRODUCTORY DIFFERENTIAL EQUATIONS. Martha L. Abell, James P. Braselton. Fourth edition 2014. ElScAe. 2014
Section: Chapter 4. Higher Order Equations. Exercises 4.9, page 215
Problem number: 24.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[_Laguerre]`

$$\boxed{xy'' + (1-x)\,y' + yk = 0}$$

With the expansion point for the power series method at $x = 0$.

✓ Solution by Maple
Time used: 0.031 (sec). Leaf size: 309

```
Order:=6;
dsolve(x*diff(y(x),x$2)+(1-x)*diff(y(x),x)+k*y(x)=0,y(x),type='series',x=0);
```

$$y(x) = \left((2k+1)\,x + \left(\frac{1}{4}k + \frac{1}{4} - \frac{3}{4}k^2\right)x^2 + \left(-\frac{2}{9}k^2 + \frac{1}{27}k + \frac{1}{18} + \frac{11}{108}k^3\right)x^3 + \left(\frac{7}{192}k^3 - \frac{167}{3456}k^2 + \frac{1}{192}k + \frac{1}{96} - \frac{25}{3456}k^4\right)x^4 + \left(\frac{1}{1500}k - \frac{61}{21600}k^4 + \frac{1}{600} + \frac{719}{86400}k^3 - \frac{37}{4320}k^2 + \frac{137}{432000}k^5\right)x^5 + \mathrm{O}\left(x^6\right)\right)c_2 + \left(1 - kx + \frac{1}{4}(-1+k)\,kx^2 - \frac{1}{36}(-2+k)\,(-1+k)\,kx^3 + \frac{1}{576}(k-3)\,(-2+k)\,(-1+k)\,kx^4 - \frac{1}{14400}(-4+k)\,(k-3)\,(-2+k)\,(-1+k)\,kx^5 + \mathrm{O}\left(x^6\right)\right)(c_1 + c_2\ln(x))$$

✓ Solution by Mathematica

Time used: 0.004 (sec). Leaf size: 415

```
AsymptoticDSolveValue[x*y''[x]+(1-x)*y'[x]+k*y[x]==0,y[x],{x,0,5}]
```

$$
\begin{aligned}
y(x) \to\ & c_1\left(-\frac{(k-4)(k-3)(k-2)(k-1)kx^5}{14400}+\frac{1}{576}(k-3)(k-2)(k-1)kx^4\right. \\
& \left.-\frac{1}{36}(k-2)(k-1)kx^3+\frac{1}{4}(k-1)kx^2-kx+1\right) \\
& +c_2\left(\frac{(k-4)(k-3)(k-2)(k-1)x^5}{14400}+\frac{(k-4)(k-3)(k-2)kx^5}{14400}\right. \\
& +\frac{(k-4)(k-3)(k-1)kx^5}{14400}+\frac{(k-4)(k-2)(k-1)kx^5}{14400} \\
& +\frac{137(k-4)(k-3)(k-2)(k-1)kx^5}{432000}+\frac{(k-3)(k-2)(k-1)kx^5}{14400} \\
& -\frac{1}{576}(k-3)(k-2)(k-1)x^4-\frac{1}{576}(k-3)(k-2)kx^4-\frac{1}{576}(k-3)(k-1)kx^4 \\
& -\frac{25(k-3)(k-2)(k-1)kx^4}{3456}-\frac{1}{576}(k-2)(k-1)kx^4+\frac{1}{36}(k-2)(k-1)x^3 \\
& +\frac{1}{36}(k-2)kx^3+\frac{11}{108}(k-2)(k-1)kx^3+\frac{1}{36}(k-1)kx^3-\frac{1}{4}(k-1)x^2-\frac{3}{4}(k-1)kx^2 \\
& -\frac{kx^2}{4}+\left(-\frac{(k-4)(k-3)(k-2)(k-1)kx^5}{14400}+\frac{1}{576}(k-3)(k-2)(k-1)kx^4\right. \\
& \left.\left.-\frac{1}{36}(k-2)(k-1)kx^3+\frac{1}{4}(k-1)kx^2-kx+1\right)\log(x)+2kx+x\right)
\end{aligned}
$$

17.25 problem 27 (a)

Internal problem ID [14826]

Book: INTRODUCTORY DIFFERENTIAL EQUATIONS. Martha L. Abell, James P. Braselton. Fourth edition 2014. ElScAe. 2014
Section: Chapter 4. Higher Order Equations. Exercises 4.9, page 215
Problem number: 27 (a).
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$\boxed{x^2y'' + y'x + \left(x^2 - \frac{1}{4}\right)y = 0}$$

With the expansion point for the power series method at $x = 0$.

✓ Solution by Maple
Time used: 0.047 (sec). Leaf size: 35

```
Order:=6;
dsolve(x^2*diff(y(x),x$2)+x*diff(y(x),x)+(x^2-1/4)*y(x)=0,y(x),type='series',x=0);
```

$$y(x) = \frac{c_1x\left(1 - \frac{1}{6}x^2 + \frac{1}{120}x^4 + \mathrm{O}\left(x^6\right)\right) + c_2\left(1 - \frac{1}{2}x^2 + \frac{1}{24}x^4 + \mathrm{O}\left(x^6\right)\right)}{\sqrt{x}}$$

✓ Solution by Mathematica
Time used: 0.009 (sec). Leaf size: 58

```
AsymptoticDSolveValue[x^2*y''[x]+x*y'[x]+(x^2-1/4)*y[x]==0,y[x],{x,0,5}]
```

$$y(x) \to c_1\left(\frac{x^{7/2}}{24} - \frac{x^{3/2}}{2} + \frac{1}{\sqrt{x}}\right) + c_2\left(\frac{x^{9/2}}{120} - \frac{x^{5/2}}{6} + \sqrt{x}\right)$$

17.26 problem 27 (b)

Internal problem ID [14827]

Book: INTRODUCTORY DIFFERENTIAL EQUATIONS. Martha L. Abell, James P. Braselton. Fourth edition 2014. ElScAe. 2014
Section: Chapter 4. Higher Order Equations. Exercises 4.9, page 215
Problem number: 27 (b).
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$\boxed{x^2y'' + y'x + (16x^2 - 25)\, y = 0}$$

With the expansion point for the power series method at $x = 0$.

✓ Solution by Maple

Time used: 0.047 (sec). Leaf size: 35

```
Order:=6;
dsolve(x^2*diff(y(x),x$2)+x*diff(y(x),x)+(16*x^2-25)*y(x)=0,y(x),type='series',x=0);
```

$$y(x) = c_1 x^5 \left(1 - \frac{2}{3}x^2 + \frac{4}{21}x^4 + \mathrm{O}\left(x^6\right)\right) + \frac{c_2 (-1316818944000 - 1316818944000x^2 - 877879296000x^4 + \mathrm{O}\left(x^6\right))}{x^5}$$

✓ Solution by Mathematica

Time used: 0.009 (sec). Leaf size: 42

```
AsymptoticDSolveValue[x^2*y''[x]+x*y'[x]+(16*x^2-25)*y[x]==0,y[x],{x,0,5}]
```

$$y(x) \to c_1 \left(\frac{1}{x^5} + \frac{1}{x^3} + \frac{2}{3x}\right) + c_2 \left(\frac{4x^9}{21} - \frac{2x^7}{3} + x^5\right)$$

18 Chapter 4. Higher Order Equations. Chapter 4 review exercises, page 219

18.1 problem 7

Internal problem ID [14828]

Book: INTRODUCTORY DIFFERENTIAL EQUATIONS. Martha L. Abell, James P. Braselton. Fourth edition 2014. ElScAe. 2014
Section: Chapter 4. Higher Order Equations. Chapter 4 review exercises, page 219
Problem number: 7.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _missing_x]]`

$$\boxed{y'' - 7y' + 10y = 0}$$

✓ Solution by Maple

Time used: 0.015 (sec). Leaf size: 17

```
dsolve(diff(y(x),x$2)-7*diff(y(x),x)+10*y(x)=0,y(x), singsol=all)
```

$$y(x) = c_1 e^{5x} + c_2 e^{2x}$$

✓ Solution by Mathematica

Time used: 0.013 (sec). Leaf size: 22

```
DSolve[y''[x]-7*y'[x]+10*y[x]==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to e^{2x}\left(c_2 e^{3x} + c_1\right)$$

18.2 problem 8

Internal problem ID [14829]

Book: INTRODUCTORY DIFFERENTIAL EQUATIONS. Martha L. Abell, James P. Braselton. Fourth edition 2014. ElScAe. 2014
Section: Chapter 4. Higher Order Equations. Chapter 4 review exercises, page 219
Problem number: 8.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _missing_x]]`

$$\boxed{y'' - y' - 2y = 0}$$

✓ Solution by Maple
Time used: 0.0 (sec). Leaf size: 17

```
dsolve(diff(y(x),x$2)-diff(y(x),x)-2*y(x)=0,y(x), singsol=all)
```

$$y(x) = c_1 \mathrm{e}^{-x} + c_2 \mathrm{e}^{2x}$$

✓ Solution by Mathematica
Time used: 0.012 (sec). Leaf size: 22

```
DSolve[y''[x]-y'[x]-2*y[x]==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \rightarrow e^{-x}\left(c_2 e^{3x} + c_1\right)$$

18.3 problem 9

Internal problem ID [14830]

Book: INTRODUCTORY DIFFERENTIAL EQUATIONS. Martha L. Abell, James P. Braselton. Fourth edition 2014. ElScAe. 2014
Section: Chapter 4. Higher Order Equations. Chapter 4 review exercises, page 219
Problem number: 9.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _missing_x]]`

$$\boxed{y'' - 2y' + 2y = 0}$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 16

```
dsolve(diff(y(x),x$2)-2*diff(y(x),x)+2*y(x)=0,y(x), singsol=all)
```

$$y(x) = e^{x}(c_1 \sin(x) + c_2 \cos(x))$$

✓ Solution by Mathematica

Time used: 0.012 (sec). Leaf size: 20

```
DSolve[y''[x]-2*y'[x]+2*y[x]==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to e^{x}(c_2 \cos(x) + c_1 \sin(x))$$

18.4 problem 10

Internal problem ID [14831]

Book: INTRODUCTORY DIFFERENTIAL EQUATIONS. Martha L. Abell, James P. Braselton. Fourth edition 2014. ElScAe. 2014
Section: Chapter 4. Higher Order Equations. Chapter 4 review exercises, page 219
Problem number: 10.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries], [_2nd_order, _linear,`

$$\boxed{(1+t)^2 y'' - 2y'(1+t) + 2y = 0}$$

Given that one solution of the ode is

$$y_1 = 1 + t$$

✓ Solution by Maple
Time used: 0.0 (sec). Leaf size: 15

```
dsolve([(t+1)^2*diff(y(t),t$2)-2*(t+1)*diff(y(t),t)+2*y(t)=0,t+1],singsol=all)
```

$$y(t) = (t+1)(c_1 + c_2(t+1))$$

✓ Solution by Mathematica
Time used: 0.028 (sec). Leaf size: 18

```
DSolve[(t+1)^2*y''[t]-2*(t+1)*y'[t]+2*y[t]==0,y[t],t,IncludeSingularSolutions -> True]
```

$$y(t) \to (t+1)(c_2(t+1) + c_1)$$

18.5 problem 11

Internal problem ID [14832]

Book: INTRODUCTORY DIFFERENTIAL EQUATIONS. Martha L. Abell, James P. Braselton. Fourth edition 2014. ElScAe. 2014
Section: Chapter 4. Higher Order Equations. Chapter 4 review exercises, page 219
Problem number: 11.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[_Lienard]`

$$\boxed{ty'' + 2y' + ty = 0}$$

Given that one solution of the ode is

$$y_1 = \frac{\sin(t)}{t}$$

✓ Solution by Maple
Time used: 0.0 (sec). Leaf size: 17

```
dsolve([t*diff(y(t),t$2)+2*diff(y(t),t)+t*y(t)=0,sin(t)/t],singsol=all)
```

$$y(t) = \frac{c_1 \sin(t) + c_2 \cos(t)}{t}$$

✓ Solution by Mathematica
Time used: 0.044 (sec). Leaf size: 37

```
DSolve[t*y''[t]+2*y'[t]+t*y[t]==0,y[t],t,IncludeSingularSolutions -> True]
```

$$y(t) \to \frac{2c_1 e^{-it} - ic_2 e^{it}}{2t}$$

18.6 problem 12

Internal problem ID [14833]

Book: INTRODUCTORY DIFFERENTIAL EQUATIONS. Martha L. Abell, James P. Braselton. Fourth edition 2014. ElScAe. 2014
Section: Chapter 4. Higher Order Equations. Chapter 4 review exercises, page 219
Problem number: 12.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _missing_x]]`

$$\boxed{y'' + 7y' + 10y = 0}$$

✓ Solution by Maple
Time used: 0.0 (sec). Leaf size: 17

```
dsolve(diff(y(x),x$2)+7*diff(y(x),x)+10*y(x)=0,y(x), singsol=all)
```

$$y(x) = \mathrm{e}^{-5x} c_1 + \mathrm{e}^{-2x} c_2$$

✓ Solution by Mathematica
Time used: 0.013 (sec). Leaf size: 22

```
DSolve[y''[x]+7*y'[x]+10*y[x]==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to e^{-5x}\left(c_2 e^{3x} + c_1\right)$$

18.7 problem 13

Internal problem ID [14834]

Book: INTRODUCTORY DIFFERENTIAL EQUATIONS. Martha L. Abell, James P. Braselton. Fourth edition 2014. ElScAe. 2014
Section: Chapter 4. Higher Order Equations. Chapter 4 review exercises, page 219
Problem number: 13.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _missing_x]]`

$$\boxed{6y'' + 5y' - 4y = 0}$$

✓ Solution by Maple
Time used: 0.0 (sec). Leaf size: 17

```
dsolve(6*diff(y(x),x$2)+5*diff(y(x),x)-4*y(x)=0,y(x), singsol=all)
```

$$y(x) = \left(c_1 \mathrm{e}^{\frac{11x}{6}} + c_2\right) \mathrm{e}^{-\frac{4x}{3}}$$

✓ Solution by Mathematica
Time used: 0.013 (sec). Leaf size: 26

```
DSolve[6*y''[x]+5*y'[x]-4*y[x]==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to c_1 e^{-4x/3} + c_2 e^{x/2}$$

18.8 problem 14

Internal problem ID [14835]

Book: INTRODUCTORY DIFFERENTIAL EQUATIONS. Martha L. Abell, James P. Braselton. Fourth edition 2014. ElScAe. 2014
Section: Chapter 4. Higher Order Equations. Chapter 4 review exercises, page 219
Problem number: 14.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _missing_x]]`

$$\boxed{y'' + 2y' + y = 0}$$

✓ Solution by Maple
Time used: 0.0 (sec). Leaf size: 14

```
dsolve(diff(y(x),x$2)+2*diff(y(x),x)+y(x)=0,y(x), singsol=all)
```

$$y(x) = e^{-x}(c_2x + c_1)$$

✓ Solution by Mathematica
Time used: 0.013 (sec). Leaf size: 18

```
DSolve[y''[x]+2*y'[x]+y[x]==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to e^{-x}(c_2x + c_1)$$

18.9 problem 15

Internal problem ID [14836]

Book: INTRODUCTORY DIFFERENTIAL EQUATIONS. Martha L. Abell, James P. Braselton. Fourth edition 2014. ElScAe. 2014
Section: Chapter 4. Higher Order Equations. Chapter 4 review exercises, page 219
Problem number: 15.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _missing_x]]`

$$\boxed{y'' + 3y' + 2y = 0}$$

✓ Solution by Maple
Time used: 0.0 (sec). Leaf size: 17

```
dsolve(diff(y(x),x$2)+3*diff(y(x),x)+2*y(x)=0,y(x), singsol=all)
```

$$y(x) = c_1 \mathrm{e}^{-x} + \mathrm{e}^{-2x} c_2$$

✓ Solution by Mathematica
Time used: 0.012 (sec). Leaf size: 20

```
DSolve[y''[x]+3*y'[x]+2*y[x]==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to e^{-2x}(c_2 e^x + c_1)$$

18.10 problem 16

Internal problem ID [14837]

Book: INTRODUCTORY DIFFERENTIAL EQUATIONS. Martha L. Abell, James P. Braselton. Fourth edition 2014. ElScAe. 2014
Section: Chapter 4. Higher Order Equations. Chapter 4 review exercises, page 219
Problem number: 16.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _missing_x]]`

$$\boxed{y'' - 10y' + 34y = 0}$$

✓ Solution by Maple
Time used: 0.0 (sec). Leaf size: 22

```
dsolve(diff(y(x),x$2)-10*diff(y(x),x)+34*y(x)=0,y(x), singsol=all)
```

$$y(x) = e^{5x}(c_1 \sin(3x) + c_2 \cos(3x))$$

✓ Solution by Mathematica
Time used: 0.018 (sec). Leaf size: 26

```
DSolve[y''[x]-10*y'[x]+34*y[x]==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to e^{5x}(c_2 \cos(3x) + c_1 \sin(3x))$$

18.11 problem 17

Internal problem ID [14838]

Book: INTRODUCTORY DIFFERENTIAL EQUATIONS. Martha L. Abell, James P. Braselton. Fourth edition 2014. ElScAe. 2014
Section: Chapter 4. Higher Order Equations. Chapter 4 review exercises, page 219
Problem number: 17.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _missing_x]]`

$$\boxed{2y'' - 5y' + 2y = 0}$$

✓ Solution by Maple
Time used: 0.0 (sec). Leaf size: 17

```
dsolve(2*diff(y(x),x$2)-5*diff(y(x),x)+2*y(x)=0,y(x), singsol=all)
```

$$y(x) = c_1 e^{\frac{x}{2}} + c_2 e^{2x}$$

✓ Solution by Mathematica
Time used: 0.012 (sec). Leaf size: 24

```
DSolve[2*y''[x]-5*y'[x]+2*y[x]==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to c_1 e^{x/2} + c_2 e^{2x}$$

18.12 problem 18

Internal problem ID [14839]

Book: INTRODUCTORY DIFFERENTIAL EQUATIONS. Martha L. Abell, James P. Braselton. Fourth edition 2014. ElScAe. 2014
Section: Chapter 4. Higher Order Equations. Chapter 4 review exercises, page 219
Problem number: 18.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _missing_x]]`

$$\boxed{15y'' - 11y' + 2y = 0}$$

✓ Solution by Maple
Time used: 0.016 (sec). Leaf size: 17

```
dsolve(15*diff(y(x),x$2)-11*diff(y(x),x)+2*y(x)=0,y(x), singsol=all)
```

$$y(x) = c_1 e^{\frac{x}{3}} + c_2 e^{\frac{2x}{5}}$$

✓ Solution by Mathematica
Time used: 0.013 (sec). Leaf size: 26

```
DSolve[15*y''[x]-11*y'[x]+2*y[x]==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to c_1 e^{2x/5} + c_2 e^{x/3}$$

18.13 problem 19

Internal problem ID [14840]

Book: INTRODUCTORY DIFFERENTIAL EQUATIONS. Martha L. Abell, James P. Braselton. Fourth edition 2014. ElScAe. 2014
Section: Chapter 4. Higher Order Equations. Chapter 4 review exercises, page 219
Problem number: 19.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _missing_x]]`

$$\boxed{20y'' + y' - y = 0}$$

✓ Solution by Maple
Time used: 0.0 (sec). Leaf size: 17

```
dsolve(20*diff(y(x),x$2)+diff(y(x),x)-y(x)=0,y(x), singsol=all)
```

$$y(x) = \left(c_1 \mathrm{e}^{\frac{9x}{20}} + c_2\right) \mathrm{e}^{-\frac{x}{4}}$$

✓ Solution by Mathematica
Time used: 0.013 (sec). Leaf size: 26

```
DSolve[20*y''[x]+y'[x]-y[x]==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to c_1 e^{x/5} + c_2 e^{-x/4}$$

18.14 problem 20

Internal problem ID [14841]

Book: INTRODUCTORY DIFFERENTIAL EQUATIONS. Martha L. Abell, James P. Braselton. Fourth edition 2014. ElScAe. 2014
Section: Chapter 4. Higher Order Equations. Chapter 4 review exercises, page 219
Problem number: 20.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _missing_x]]`

$$\boxed{12y'' + 8y' + y = 0}$$

✓ Solution by Maple
Time used: 0.015 (sec). Leaf size: 17

```
dsolve(12*diff(y(x),x$2)+8*diff(y(x),x)+y(x)=0,y(x), singsol=all)
```

$$y(x) = c_1 \mathrm{e}^{-\frac{x}{2}} + c_2 \mathrm{e}^{-\frac{x}{6}}$$

✓ Solution by Mathematica
Time used: 0.012 (sec). Leaf size: 26

```
DSolve[12*y''[x]+8*y'[x]+y[x]==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to e^{-x/2}\left(c_1 e^{x/3} + c_2\right)$$

18.15 problem 21

Internal problem ID [14842]

Book: INTRODUCTORY DIFFERENTIAL EQUATIONS. Martha L. Abell, James P. Braselton. Fourth edition 2014. ElScAe. 2014
Section: Chapter 4. Higher Order Equations. Chapter 4 review exercises, page 219
Problem number: 21.
ODE order: 3.
ODE degree: 1.

CAS Maple gives this as type `[[_3rd_order, _missing_x]]`

$$\boxed{2y''' + 3y'' + y' = 0}$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 18

```
dsolve(2*diff(y(x),x$3)+3*diff(y(x),x$2)+diff(y(x),x)=0,y(x), singsol=all)
```

$$y(x) = c_1 + c_2 \mathrm{e}^{-\frac{x}{2}} + c_3 \mathrm{e}^{-x}$$

✓ Solution by Mathematica

Time used: 0.033 (sec). Leaf size: 28

```
DSolve[2*y'''[x]+3*y''[x]+y'[x]==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to -2c_1 e^{-x/2} - c_2 e^{-x} + c_3$$

18.16 problem 22

Internal problem ID [14843]

Book: INTRODUCTORY DIFFERENTIAL EQUATIONS. Martha L. Abell, James P. Braselton. Fourth edition 2014. ElScAe. 2014
Section: Chapter 4. Higher Order Equations. Chapter 4 review exercises, page 219
Problem number: 22.
ODE order: 3.
ODE degree: 1.

CAS Maple gives this as type `[[_3rd_order, _missing_x]]`

$$\boxed{9y''' + 36y'' + 40y' = 0}$$

✓ Solution by Maple
Time used: 0.016 (sec). Leaf size: 26

```
dsolve(9*diff(y(x),x$3)+36*diff(y(x),x$2)+40*diff(y(x),x)=0,y(x), singsol=all)
```

$$y(x) = c_1 + c_2 \mathrm{e}^{-2x} \sin\left(\frac{2x}{3}\right) + c_3 \mathrm{e}^{-2x} \cos\left(\frac{2x}{3}\right)$$

✓ Solution by Mathematica
Time used: 0.242 (sec). Leaf size: 48

```
DSolve[9*y'''[x]+36*y''[x]+40*y'[x]==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to c_3 - \frac{3}{20} e^{-2x} \left((c_1 + 3c_2) \cos\left(\frac{2x}{3}\right) + (3c_1 - c_2) \sin\left(\frac{2x}{3}\right) \right)$$

18.17 problem 23

Internal problem ID [14844]

Book: INTRODUCTORY DIFFERENTIAL EQUATIONS. Martha L. Abell, James P. Braselton. Fourth edition 2014. ElScAe. 2014
Section: Chapter 4. Higher Order Equations. Chapter 4 review exercises, page 219
Problem number: 23.
ODE order: 3.
ODE degree: 1.

CAS Maple gives this as type `[[_3rd_order, _missing_x]]`

$$\boxed{9y''' + 12y'' + 13y' = 0}$$

✓ Solution by Maple
Time used: 0.015 (sec). Leaf size: 22

```
dsolve(9*diff(y(x),x$3)+12*diff(y(x),x$2)+13*diff(y(x),x)=0,y(x), singsol=all)
```

$$y(x) = c_1 + c_2 e^{-\frac{2x}{3}} \sin(x) + c_3 e^{-\frac{2x}{3}} \cos(x)$$

✓ Solution by Mathematica
Time used: 0.113 (sec). Leaf size: 44

```
DSolve[9*y'''[x]+12*y''[x]+13*y'[x]==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to c_3 - \frac{3}{13} e^{-2x/3} ((3c_1 + 2c_2) \cos(x) + (2c_1 - 3c_2) \sin(x))$$

18.18 problem 24

Internal problem ID [14845]

Book: INTRODUCTORY DIFFERENTIAL EQUATIONS. Martha L. Abell, James P. Braselton. Fourth edition 2014. ElScAe. 2014
Section: Chapter 4. Higher Order Equations. Chapter 4 review exercises, page 219
Problem number: 24.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$\boxed{y'' - 2y' - 8y = -t}$$

✓ Solution by Maple
Time used: 0.0 (sec). Leaf size: 21

```
dsolve(diff(y(t),t$2)-2*diff(y(t),t)-8*y(t)=-t,y(t), singsol=all)
```

$$y(t) = \mathrm{e}^{4t}c_2 + \mathrm{e}^{-2t}c_1 + \frac{t}{8} - \frac{1}{32}$$

✓ Solution by Mathematica
Time used: 0.013 (sec). Leaf size: 30

```
DSolve[y''[t]-2*y'[t]-8*y[t]==-t,y[t],t,IncludeSingularSolutions -> True]
```

$$y(t) \to \frac{t}{8} + c_1 e^{-2t} + c_2 e^{4t} - \frac{1}{32}$$

18.19 problem 25

Internal problem ID [14846]

Book: INTRODUCTORY DIFFERENTIAL EQUATIONS. Martha L. Abell, James P. Braselton. Fourth edition 2014. ElScAe. 2014
Section: Chapter 4. Higher Order Equations. Chapter 4 review exercises, page 219
Problem number: 25.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _missing_y]]`

$$\boxed{y'' + 5y' = 5t^2}$$

✓ Solution by Maple
Time used: 0.0 (sec). Leaf size: 26

```
dsolve(diff(y(t),t$2)+5*diff(y(t),t)=5*t^2,y(t), singsol=all)
```

$$y(t) = -\frac{t^2}{5} + \frac{t^3}{3} - \frac{\mathrm{e}^{-5t}c_1}{5} + \frac{2t}{25} + c_2$$

✓ Solution by Mathematica
Time used: 0.088 (sec). Leaf size: 38

```
DSolve[y''[t]+5*y'[t]==5*t^2,y[t],t,IncludeSingularSolutions -> True]
```

$$y(t) \to \frac{t^3}{3} - \frac{t^2}{5} + \frac{2t}{25} - \frac{1}{5}c_1e^{-5t} + c_2$$

18.20 problem 26

Internal problem ID [14847]

Book: INTRODUCTORY DIFFERENTIAL EQUATIONS. Martha L. Abell, James P. Braselton. Fourth edition 2014. ElScAe. 2014
Section: Chapter 4. Higher Order Equations. Chapter 4 review exercises, page 219
Problem number: 26.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _missing_y]]`

$$\boxed{y'' - 4y' = -3\sin(t)}$$

✓ Solution by Maple
Time used: 0.0 (sec). Leaf size: 21

```
dsolve(diff(y(t),t$2)-4*diff(y(t),t)=-3*sin(t),y(t), singsol=all)
```

$$y(t) = \frac{c_1 e^{4t}}{4} + \frac{3\sin(t)}{17} - \frac{12\cos(t)}{17} + c_2$$

✓ Solution by Mathematica
Time used: 0.082 (sec). Leaf size: 31

```
DSolve[y''[t]-4*y'[t]==-3*Sin[t],y[t],t,IncludeSingularSolutions -> True]
```

$$y(t) \to \frac{3\sin(t)}{17} - \frac{12\cos(t)}{17} + \frac{1}{4}c_1 e^{4t} + c_2$$

18.21 problem 27

Internal problem ID [14848]

Book: INTRODUCTORY DIFFERENTIAL EQUATIONS. Martha L. Abell, James P. Braselton. Fourth edition 2014. ElScAe. 2014
Section: Chapter 4. Higher Order Equations. Chapter 4 review exercises, page 219
Problem number: 27.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _linear, _nonhomogeneous]]`

$$\boxed{y'' + 2y' + 5y = 3\sin(2t)}$$

✓ Solution by Maple
Time used: 0.0 (sec). Leaf size: 36

```
dsolve(diff(y(t),t$2)+2*diff(y(t),t)+5*y(t)=3*sin(2*t),y(t), singsol=all)
```

$$y(t) = \frac{\sin(2t)\left(17c_2\mathrm{e}^{-t} + 3\right)}{17} + \mathrm{e}^{-t}\cos(2t)\, c_1 - \frac{12\cos(2t)}{17}$$

✓ Solution by Mathematica
Time used: 0.026 (sec). Leaf size: 45

```
DSolve[y''[t]+2*y'[t]+5*y[t]==3*Sin[2*t],y[t],t,IncludeSingularSolutions -> True]
```

$$y(t) \to \frac{1}{17}e^{-t}\left(\left(-12e^t + 17c_2\right)\cos(2t) + \left(3e^t + 17c_1\right)\sin(2t)\right)$$

18.22 problem 28

Internal problem ID [14849]

Book: INTRODUCTORY DIFFERENTIAL EQUATIONS. Martha L. Abell, James P. Braselton. Fourth edition 2014. ElScAe. 2014
Section: Chapter 4. Higher Order Equations. Chapter 4 review exercises, page 219
Problem number: 28.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _linear, _nonhomogeneous]]`

$$\boxed{y'' - 9y = \frac{1}{1 + \mathrm{e}^{3t}}}$$

✓ Solution by Maple
Time used: 0.016 (sec). Leaf size: 69

```
dsolve(diff(y(t),t$2)-9*y(t)=1/(1+exp(3*t)),y(t), singsol=all)
```

$$y(t) = \frac{\left(\ln\left(\mathrm{e}^{2t} - \mathrm{e}^{t} + 1\right)\mathrm{e}^{6t} - \ln\left(1 + \mathrm{e}^{3t}\right) + \ln\left(1 + \mathrm{e}^{t}\right)\mathrm{e}^{6t} + \left(18c_1 - 3\ln\left(\mathrm{e}^{t}\right)\right)\mathrm{e}^{6t} + 18c_2 - \mathrm{e}^{3t}\right)\mathrm{e}^{-3t}}{18}$$

✓ Solution by Mathematica
Time used: 0.181 (sec). Leaf size: 94

```
DSolve[y''[t]-9*y[t]==1/(1+Exp[3*t]),y[t],t,IncludeSingularSolutions -> True]
```

$$y(t) \to \frac{1}{18}e^{-3t}\left(-e^{3t} - 3e^{6t}\log\left(e^{t}\right) + \left(e^{6t} - 1\right)\log\left(e^{t} + 1\right) + e^{6t}\log\left(-e^{t} + e^{2t} + 1\right) - \log\left(-e^{t} + e^{2t} + 1\right) + 18c_1e^{6t} + 18c_2\right)$$

18.23 problem 29

Internal problem ID [14850]

Book: INTRODUCTORY DIFFERENTIAL EQUATIONS. Martha L. Abell, James P. Braselton. Fourth edition 2014. ElScAe. 2014
Section: Chapter 4. Higher Order Equations. Chapter 4 review exercises, page 219
Problem number: 29.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _missing_y]]`

$$\boxed{y'' - 2y' = \frac{1}{1 + \mathrm{e}^{2t}}}$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 40

```
dsolve(diff(y(t),t$2)-2*diff(y(t),t)=1/(1+exp(2*t)),y(t), singsol=all)
```

$$y(t) = \frac{\ln\left(\mathrm{e}^{2t}+1\right)\left(\mathrm{e}^{2t}+1\right)}{4} + \frac{\left(2c_1 - 2\ln\left(\mathrm{e}^{t}\right)\right)\mathrm{e}^{2t}}{4} - \frac{t}{2} + c_2 - \frac{1}{4}$$

✓ Solution by Mathematica

Time used: 0.15 (sec). Leaf size: 61

```
DSolve[y''[t]-2*y'[t]==1/(1+Exp[2*t]),y[t],t,IncludeSingularSolutions -> True]
```

$$y(t) \to \frac{1}{8}\left(\left(4e^{2t}+2\right)\operatorname{arctanh}\left(2e^{2t}+1\right) - 4t + \log\left(-4e^{2t}\left(e^{2t}+1\right)\right) + 4c_1e^{2t} + 8c_2\right)$$

18.24 problem 30

Internal problem ID [14851]

Book: INTRODUCTORY DIFFERENTIAL EQUATIONS. Martha L. Abell, James P. Braselton. Fourth edition 2014. ElScAe. 2014
Section: Chapter 4. Higher Order Equations. Chapter 4 review exercises, page 219
Problem number: 30.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$\boxed{y'' - 3y' + 2y = -4\,\mathrm{e}^{-2t}}$$

✓ Solution by Maple

Time used: 0.015 (sec). Leaf size: 26

```
dsolve(diff(y(t),t$2)-3*diff(y(t),t)+2*y(t)=-4*exp(-2*t),y(t), singsol=all)
```

$$y(t) = \frac{(3c_1\mathrm{e}^{4t} + 3c_2\mathrm{e}^{3t} - 1)\,\mathrm{e}^{-2t}}{3}$$

✓ Solution by Mathematica

Time used: 0.017 (sec). Leaf size: 29

```
DSolve[y''[t]-3*y'[t]+2*y[t]==-4*Exp[-2*t],y[t],t,IncludeSingularSolutions -> True]
```

$$y(t) \to -\frac{e^{-2t}}{3} + c_1e^t + c_2e^{2t}$$

18.25 problem 31

Internal problem ID [14852]

Book: INTRODUCTORY DIFFERENTIAL EQUATIONS. Martha L. Abell, James P. Braselton. Fourth edition 2014. ElScAe. 2014
Section: Chapter 4. Higher Order Equations. Chapter 4 review exercises, page 219
Problem number: 31.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$\boxed{y'' - 6y' + 13y = 3\,\mathrm{e}^{-2t}}$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 33

```
dsolve(diff(y(t),t$2)-6*diff(y(t),t)+13*y(t)=3*exp(-2*t),y(t), singsol=all)
```

$$y(t) = (\cos(2t)\,c_1 + \sin(2t)\,c_2)\,\mathrm{e}^{-2t}\mathrm{e}^{5t} + \frac{3\,\mathrm{e}^{-2t}}{29}$$

✓ Solution by Mathematica

Time used: 0.023 (sec). Leaf size: 39

```
DSolve[y''[t]-6*y'[t]+13*y[t]==3*Exp[-2*t],y[t],t,IncludeSingularSolutions -> True]
```

$$y(t) \to \frac{3e^{-2t}}{29} + c_2e^{3t}\cos(2t) + c_1e^{3t}\sin(2t)$$

18.26 problem 32

Internal problem ID [14853]

Book: INTRODUCTORY DIFFERENTIAL EQUATIONS. Martha L. Abell, James P. Braselton. Fourth edition 2014. ElScAe. 2014
Section: Chapter 4. Higher Order Equations. Chapter 4 review exercises, page 219
Problem number: 32.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _linear, _nonhomogeneous]]`

$$\boxed{y'' + 9y' + 20y = -2t\,\mathrm{e}^{t}}$$

✓ Solution by Maple
Time used: 0.0 (sec). Leaf size: 27

```
dsolve(diff(y(t),t$2)+9*diff(y(t),t)+20*y(t)=-2*t*exp(t),y(t), singsol=all)
```

$$y(t) = -\frac{\mathrm{e}^{-5t}\left(\left(t-\frac{11}{30}\right)\mathrm{e}^{6t} - 15\,\mathrm{e}^{t}c_1 - 15c_2\right)}{15}$$

✓ Solution by Mathematica
Time used: 0.017 (sec). Leaf size: 34

```
DSolve[y''[t]+9*y'[t]+20*y[t]==-2*t*Exp[t],y[t],t,IncludeSingularSolutions -> True]
```

$$y(t) \to \frac{1}{450}e^{t}(11-30t) + c_1e^{-5t} + c_2e^{-4t}$$

18.27 problem 33

Internal problem ID [14854]

Book: INTRODUCTORY DIFFERENTIAL EQUATIONS. Martha L. Abell, James P. Braselton. Fourth edition 2014. ElScAe. 2014
Section: Chapter 4. Higher Order Equations. Chapter 4 review exercises, page 219
Problem number: 33.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _linear, _nonhomogeneous]]`

$$\boxed{y'' + 7y' + 12y = 3t^2 \mathrm{e}^{-4t}}$$

✓ Solution by Maple
Time used: 0.015 (sec). Leaf size: 31

```
dsolve(diff(y(t),t$2)+7*diff(y(t),t)+12*y(t)=3*t^2*exp(-4*t),y(t), singsol=all)
```

$$y(t) = \left(-t^3 - 3t^2 + c_1 - 6t\right) \mathrm{e}^{-4t} + c_2 \mathrm{e}^{-3t}$$

✓ Solution by Mathematica
Time used: 0.051 (sec). Leaf size: 34

```
DSolve[y''[t]+7*y'[t]+12*y[t]==3*t^2*Exp[-4*t],y[t],t,IncludeSingularSolutions -> True]
```

$$y(t) \to e^{-4t}\left(-t^3 - 3t^2 - 6t + c_2 e^t - 6 + c_1\right)$$

18.28 problem 34

Internal problem ID [14855]

Book: INTRODUCTORY DIFFERENTIAL EQUATIONS. Martha L. Abell, James P. Braselton. Fourth edition 2014. ElScAe. 2014
Section: Chapter 4. Higher Order Equations. Chapter 4 review exercises, page 219
Problem number: 34.
ODE order: 3.
ODE degree: 1.

CAS Maple gives this as type `[[_3rd_order, _with_linear_symmetries]]`

$$\boxed{y''' + 3y'' - 9y' + 5y = \mathrm{e}^t}$$

✓ Solution by Maple
Time used: 0.0 (sec). Leaf size: 30

```
dsolve(diff(y(t),t$3)+3*diff(y(t),t$2)-9*diff(y(t),t)+5*y(t)=exp(t),y(t), singsol=all)
```

$$y(t) = \frac{\left(\left(12c_3t + t^2 + 12c_1\right)\mathrm{e}^{6t} + 12c_2\right)\mathrm{e}^{-5t}}{12}$$

✓ Solution by Mathematica
Time used: 0.013 (sec). Leaf size: 39

```
DSolve[y'''[t]+3*y''[t]-9*y'[t]+5*y[t]==Exp[t],y[t],t,IncludeSingularSolutions -> True]
```

$$y(t) \to e^t\left(\frac{t^2}{12} + \left(-\frac{1}{36} + c_3\right)t + \frac{1}{216} + c_2\right) + c_1e^{-5t}$$

18.29 problem 35

Internal problem ID [14856]

Book: INTRODUCTORY DIFFERENTIAL EQUATIONS. Martha L. Abell, James P. Braselton. Fourth edition 2014. ElScAe. 2014
Section: Chapter 4. Higher Order Equations. Chapter 4 review exercises, page 219
Problem number: 35.
ODE order: 3.
ODE degree: 1.

CAS Maple gives this as type `[[_3rd_order, _linear, _nonhomogeneous]]`

$$\boxed{y''' - 12y' - 16y = e^{4t} - e^{-2t}}$$

✓ Solution by Maple
Time used: 0.0 (sec). Leaf size: 40

```
dsolve(diff(y(t),t$3)-12*diff(y(t),t)-16*y(t)=exp(4*t)-exp(-2*t),y(t), singsol=all)
```

$$y(t) = \frac{(18t^2 + (216c_3 + 6)\,t + 216c_1 + 1)\,e^{-2t}}{216} + \frac{\left(t + 36c_2 - \frac{1}{3}\right)e^{4t}}{36}$$

✓ Solution by Mathematica
Time used: 0.104 (sec). Leaf size: 49

```
DSolve[y'''[t]-12*y'[t]-16*y[t]==Exp[4*t]-Exp[-2*t],y[t],t,IncludeSingularSolutions -> True]
```

$$y(t) \to \frac{1}{216}e^{-2t}\left(18t^2 + 6(1 + 36c_2)t + e^{6t}(6t - 2 + 216c_3) + 1 + 216c_1\right)$$

18.30 problem 36

Internal problem ID [14857]

Book: INTRODUCTORY DIFFERENTIAL EQUATIONS. Martha L. Abell, James P. Braselton. Fourth edition 2014. ElScAe. 2014
Section: Chapter 4. Higher Order Equations. Chapter 4 review exercises, page 219
Problem number: 36.
ODE order: 4.
ODE degree: 1.

CAS Maple gives this as type `[[_high_order, _linear, _nonhomogeneous]]`

$$\boxed{y'''' + 6y''' + 18y'' + 30y' + 25y = e^{-t}\cos(2t) + \sin(t)\, e^{-2t}}$$

✓ Solution by Maple
Time used: 0.0 (sec). Leaf size: 70

```
dsolve(diff(y(t),t$4)+6*diff(y(t),t$3)+18*diff(y(t),t$2)+30*diff(y(t),t)+25*y(t)=exp(-t)*cos
```

$$y(t) = \frac{\left((-20t + 400c_3 - 6)\cos(t)^2 - 10\left(t - 40c_4 - \frac{21}{5}\right)\sin(t)\cos(t) + 10t - 200c_3 + 3\right)e^{-t}}{200} - \frac{e^{-2t}\left(\left(t - 10c_1 + \frac{7}{10}\right)\cos(t) - \frac{\left(t+20c_2+\frac{1}{5}\right)\sin(t)}{2}\right)}{10}$$

✓ Solution by Mathematica
Time used: 0.438 (sec). Leaf size: 77

```
DSolve[y''''[t]+6*y'''[t]+18*y''[t]+30*y'[t]+25*y[t]==Exp[-t]*Cos[2*t]+Exp[-2*t]*Sin[t],y[t]
```

$$y(t) \to \frac{1}{800}e^{-2t}\big(-4(20t + 9 - 200c_2)\cos(t) - e^t(40t - 3 - 800c_4)\cos(2t) + 8(5t + 6 + 100c_1)\sin(t) - 2e^t(10t - 27 - 400c_3)\sin(2t)\big)$$

18.31 problem 37

Internal problem ID [14858]

Book: INTRODUCTORY DIFFERENTIAL EQUATIONS. Martha L. Abell, James P. Braselton. Fourth edition 2014. ElScAe. 2014
Section: Chapter 4. Higher Order Equations. Chapter 4 review exercises, page 219
Problem number: 37.
ODE order: 4.
ODE degree: 1.

CAS Maple gives this as type `[[_high_order, _with_linear_symmetries]]`

$$\boxed{y'''' + 4y''' + 14y'' + 20y' + 25y = t^2}$$

✓ Solution by Maple
Time used: 0.015 (sec). Leaf size: 40

```
dsolve(diff(y(t),t$4)+4*diff(y(t),t$3)+14*diff(y(t),t$2)+20*diff(y(t),t)+25*y(t)=t^2,y(t), s
```

$$y(t) = \frac{4}{625} + ((c_3t + c_1)\cos(2t) + \sin(2t)(tc_4 + c_2))\,\mathrm{e}^{-t} + \frac{t^2}{25} - \frac{8t}{125}$$

✓ Solution by Mathematica
Time used: 0.004 (sec). Leaf size: 54

```
DSolve[y''''[t]+4*y'''[t]+14*y''[t]+20*y'[t]+25*y[t]==t^2,y[t],t,IncludeSingularSolutions ->
```

$$y(t) \to \frac{1}{625}(25t^2 - 40t + 4) + e^{-t}(c_4t + c_3)\cos(2t) + e^{-t}(c_2t + c_1)\sin(2t)$$

18.32 problem 38

Internal problem ID [14859]

Book: INTRODUCTORY DIFFERENTIAL EQUATIONS. Martha L. Abell, James P. Braselton. Fourth edition 2014. ElScAe. 2014
Section: Chapter 4. Higher Order Equations. Chapter 4 review exercises, page 219
Problem number: 38.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _missing_x]]`

$$\boxed{y'' + 5y' + 6y = 0}$$

With initial conditions

$$[y(0) = 2, y'(0) = 0]$$

✓ Solution by Maple
Time used: 0.016 (sec). Leaf size: 17

```
dsolve([diff(y(t),t$2)+5*diff(y(t),t)+6*y(t)=0,y(0) = 2, D(y)(0) = 0],y(t), singsol=all)
```

$$y(t) = -4\,e^{-3t} + 6\,e^{-2t}$$

✓ Solution by Mathematica
Time used: 0.013 (sec). Leaf size: 18

```
DSolve[{y''[t]+5*y'[t]+6*y[t]==0,{y[0]==2,y'[0]==0}},y[t],t,IncludeSingularSolutions -> True
```

$$y(t) \to e^{-3t}(6e^t - 4)$$

18.33 problem 39

Internal problem ID [14860]

Book: INTRODUCTORY DIFFERENTIAL EQUATIONS. Martha L. Abell, James P. Braselton. Fourth edition 2014. ElScAe. 2014
Section: Chapter 4. Higher Order Equations. Chapter 4 review exercises, page 219
Problem number: 39.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _missing_x]]`

$$\boxed{y'' + 10y' + 16y = 0}$$

With initial conditions

$$[y(0) = 0, y'(0) = 4]$$

✓ Solution by Maple

Time used: 0.016 (sec). Leaf size: 17

```
dsolve([diff(y(t),t$2)+10*diff(y(t),t)+16*y(t)=0,y(0) = 0, D(y)(0) = 4],y(t), singsol=all)
```

$$y(t) = -\frac{2\,\mathrm{e}^{-8t}}{3} + \frac{2\,\mathrm{e}^{-2t}}{3}$$

✓ Solution by Mathematica

Time used: 0.013 (sec). Leaf size: 21

```
DSolve[{y''[t]+10*y'[t]+16*y[t]==0,{y[0]==0,y'[0]==4}},y[t],t,IncludeSingularSolutions -> Tr
```

$$y(t) \to \frac{2}{3} e^{-8t}\left(e^{6t} - 1\right)$$

18.34 problem 40

Internal problem ID [14861]

Book: INTRODUCTORY DIFFERENTIAL EQUATIONS. Martha L. Abell, James P. Braselton. Fourth edition 2014. ElScAe. 2014
Section: Chapter 4. Higher Order Equations. Chapter 4 review exercises, page 219
Problem number: 40.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _missing_x]]`

$$\boxed{y'' + 16y = 0}$$

With initial conditions

$$[y(0) = 0, y'(0) = -8]$$

✓ Solution by Maple
Time used: 0.0 (sec). Leaf size: 10

```
dsolve([diff(y(t),t$2)+16*y(t)=0,y(0) = 0, D(y)(0) = -8],y(t), singsol=all)
```

$$y(t) = -2\sin(4t)$$

✓ Solution by Mathematica
Time used: 0.013 (sec). Leaf size: 11

```
DSolve[{y''[t]+16*y[t]==0,{y[0]==0,y'[0]==-8}},y[t],t,IncludeSingularSolutions -> True]
```

$$y(t) \to -2\sin(4t)$$

18.35 problem 41

Internal problem ID [14862]

Book: INTRODUCTORY DIFFERENTIAL EQUATIONS. Martha L. Abell, James P. Braselton. Fourth edition 2014. ElScAe. 2014
Section: Chapter 4. Higher Order Equations. Chapter 4 review exercises, page 219
Problem number: 41.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _missing_x]]`

$$\boxed{y'' + 25y = 0}$$

With initial conditions

$$[y(0) = 1, y'(0) = 0]$$

✓ Solution by Maple
Time used: 0.0 (sec). Leaf size: 8

```
dsolve([diff(y(t),t$2)+25*y(t)=0,y(0) = 1, D(y)(0) = 0],y(t), singsol=all)
```

$$y(t) = \cos(5t)$$

✓ Solution by Mathematica
Time used: 0.013 (sec). Leaf size: 9

```
DSolve[{y''[t]+25*y[t]==0,{y[0]==1,y'[0]==0}},y[t],t,IncludeSingularSolutions -> True]
```

$$y(t) \to \cos(5t)$$

18.36 problem 42

Internal problem ID [14863]

Book: INTRODUCTORY DIFFERENTIAL EQUATIONS. Martha L. Abell, James P. Braselton. Fourth edition 2014. ElScAe. 2014
Section: Chapter 4. Higher Order Equations. Chapter 4 review exercises, page 219
Problem number: 42.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$\boxed{y'' - 4y = t}$$

With initial conditions

$$[y(0) = 2, y'(0) = 0]$$

✓ Solution by Maple
Time used: 0.015 (sec). Leaf size: 20

```
dsolve([diff(y(t),t$2)-4*y(t)=t,y(0) = 2, D(y)(0) = 0],y(t), singsol=all)
```

$$y(t) = \frac{17\,\mathrm{e}^{2t}}{16} + \frac{15\,\mathrm{e}^{-2t}}{16} - \frac{t}{4}$$

✓ Solution by Mathematica
Time used: 0.011 (sec). Leaf size: 16

```
DSolve[{y''[t]-4*y[t]==0,{y[0]==2,y'[0]==0}},y[t],t,IncludeSingularSolutions -> True]
```

$$y(t) \to e^{-2t} + e^{2t}$$

18.37 problem 43

Internal problem ID [14864]

Book: INTRODUCTORY DIFFERENTIAL EQUATIONS. Martha L. Abell, James P. Braselton. Fourth edition 2014. ElScAe. 2014
Section: Chapter 4. Higher Order Equations. Chapter 4 review exercises, page 219
Problem number: 43.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$\boxed{y'' + 3y' - 4y = \mathrm{e}^{t}}$$

With initial conditions

$$[y(0) = 0, y'(0) = 4]$$

✓ Solution by Maple
Time used: 0.015 (sec). Leaf size: 20

```
dsolve([diff(y(t),t$2)+3*diff(y(t),t)-4*y(t)=exp(t),y(0) = 0, D(y)(0) = 4],y(t), singsol=all
```

$$y(t) = \frac{(5t+19)\,\mathrm{e}^{-4t}\mathrm{e}^{5t}}{25} - \frac{19\,\mathrm{e}^{-4t}}{25}$$

✓ Solution by Mathematica
Time used: 0.031 (sec). Leaf size: 27

```
DSolve[{y''[t]+3*y'[t]-4*y[t]==Exp[t],{y[0]==0,y'[0]==4}},y[t],t,IncludeSingularSolutions ->
```

$$y(t) \to \frac{1}{25}e^{-4t}\left(e^{5t}(5t+19) - 19\right)$$

18.38 problem 44

Internal problem ID [14865]

Book: INTRODUCTORY DIFFERENTIAL EQUATIONS. Martha L. Abell, James P. Braselton. Fourth edition 2014. ElScAe. 2014
Section: Chapter 4. Higher Order Equations. Chapter 4 review exercises, page 219
Problem number: 44.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _linear, _nonhomogeneous]]`

$$\boxed{y'' + 9y = \sin(3t)}$$

With initial conditions

$$[y(0) = 6, y'(0) = 0]$$

✓ Solution by Maple

Time used: 0.015 (sec). Leaf size: 24

```
dsolve([diff(y(t),t$2)+9*y(t)=sin(3*t),y(0) = 6, D(y)(0) = 0],y(t), singsol=all)
```

$$y(t) = \frac{\sin(3t)}{18} + 6\cos(3t) - \frac{\cos(3t)\, t}{6}$$

✓ Solution by Mathematica

Time used: 0.09 (sec). Leaf size: 23

```
DSolve[{y''[t]+9*y[t]==Sin[3*t],{y[0]==6,y'[0]==0}},y[t],t,IncludeSingularSolutions -> True]
```

$$y(t) \to \frac{1}{18}(\sin(3t) - 3(t - 36)\cos(3t))$$

18.39 problem 45

Internal problem ID [14866]

Book: INTRODUCTORY DIFFERENTIAL EQUATIONS. Martha L. Abell, James P. Braselton. Fourth edition 2014. ElScAe. 2014
Section: Chapter 4. Higher Order Equations. Chapter 4 review exercises, page 219
Problem number: 45.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _linear, _nonhomogeneous]]`

$$\boxed{y'' + y = \cos(t)}$$

With initial conditions

$$[y(0) = 0, y'(0) = 0]$$

✓ Solution by Maple
Time used: 0.0 (sec). Leaf size: 9

```
dsolve([diff(y(t),t$2)+y(t)=cos(t),y(0) = 0, D(y)(0) = 0],y(t), singsol=all)
```

$$y(t) = \frac{\sin(t)\, t}{2}$$

✓ Solution by Mathematica
Time used: 0.032 (sec). Leaf size: 12

```
DSolve[{y''[t]+y[t]==Cos[t],{y[0]==0,y'[0]==0}},y[t],t,IncludeSingularSolutions -> True]
```

$$y(t) \to \frac{1}{2} t \sin(t)$$

18.40 problem 46

Internal problem ID [14867]

Book: INTRODUCTORY DIFFERENTIAL EQUATIONS. Martha L. Abell, James P. Braselton. Fourth edition 2014. ElScAe. 2014
Section: Chapter 4. Higher Order Equations. Chapter 4 review exercises, page 219
Problem number: 46.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _linear, _nonhomogeneous]]`

$$\boxed{y'' + 4y = \tan(2t)}$$

✓ Solution by Maple
Time used: 0.0 (sec). Leaf size: 33

```
dsolve(diff(y(t),t$2)+4*y(t)=tan(2*t),y(t), singsol=all)
```

$$y(t) = \sin(2t)\, c_2 + \cos(2t)\, c_1 - \frac{\cos(2t)\ln(\sec(2t) + \tan(2t))}{4}$$

✓ Solution by Mathematica
Time used: 0.042 (sec). Leaf size: 40

```
DSolve[y''[t]+4*y[t]==Tan[2*t],y[t],t,IncludeSingularSolutions -> True]
```

$$y(t) \to -\frac{1}{4}\cos(2t)\text{arctanh}(\sin(2t)) + c_1\cos(2t) + \frac{1}{4}(-1 + 4c_2)\sin(2t)$$

18.41 problem 47

Internal problem ID [14868]

Book: INTRODUCTORY DIFFERENTIAL EQUATIONS. Martha L. Abell, James P. Braselton. Fourth edition 2014. ElScAe. 2014
Section: Chapter 4. Higher Order Equations. Chapter 4 review exercises, page 219
Problem number: 47.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _linear, _nonhomogeneous]]`

$$\boxed{y'' + y = \csc(t)}$$

✓ Solution by Maple
Time used: 0.0 (sec). Leaf size: 24

```
dsolve(diff(y(t),t$2)+y(t)=csc(t),y(t), singsol=all)
```

$$y(t) = -\sin(t)\ln(\csc(t)) + (-t + c_1)\cos(t) + \sin(t)\, c_2$$

✓ Solution by Mathematica
Time used: 0.026 (sec). Leaf size: 24

```
DSolve[y''[t]+y[t]==Csc[t],y[t],t,IncludeSingularSolutions -> True]
```

$$y(t) \to (-t + c_1)\cos(t) + \sin(t)(\log(\sin(t)) + c_2)$$

18.42 problem 48

Internal problem ID [14869]

Book: INTRODUCTORY DIFFERENTIAL EQUATIONS. Martha L. Abell, James P. Braselton. Fourth edition 2014. ElScAe. 2014
Section: Chapter 4. Higher Order Equations. Chapter 4 review exercises, page 219
Problem number: 48.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _linear, _nonhomogeneous]]`

$$\boxed{y'' - 8y' + 16y = \frac{\mathrm{e}^{4t}}{t^3}}$$

✓ Solution by Maple
Time used: 0.0 (sec). Leaf size: 25

```
dsolve(diff(y(t),t$2)-8*diff(y(t),t)+16*y(t)=1/t^3*exp(4*t),y(t), singsol=all)
```

$$y(t) = \frac{\mathrm{e}^{4t}(2c_1t^2 + 2tc_2 + 1)}{2t}$$

✓ Solution by Mathematica
Time used: 0.025 (sec). Leaf size: 31

```
DSolve[y''[t]-8*y'[t]+16*y[t]==1/t^3*Exp[4*t],y[t],t,IncludeSingularSolutions -> True]
```

$$y(t) \to \frac{e^{4t}(2c_2t^2 + 2c_1t + 1)}{2t}$$

18.43 problem 49

Internal problem ID [14870]

Book: INTRODUCTORY DIFFERENTIAL EQUATIONS. Martha L. Abell, James P. Braselton. Fourth edition 2014. ElScAe. 2014
Section: Chapter 4. Higher Order Equations. Chapter 4 review exercises, page 219
Problem number: 49.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _linear, _nonhomogeneous]]`

$$\boxed{y'' - 8y' + 16y = \frac{e^{4t}}{t^3}}$$

With initial conditions

$$[y(0) = 0, y'(0) = 1]$$

✗ Solution by Maple

```
dsolve([diff(y(t),t$2)-8*diff(y(t),t)+16*y(t)=1/t^3*exp(4*t),y(0) = 0, D(y)(0) = 1],y(t), si
```

No solution found

✗ Solution by Mathematica
Time used: 0.0 (sec). Leaf size: 0

```
DSolve[{y''[t]-8*y'[t]+16*y[t]==1/t^3*Exp[4*t],{y[0]==0,y'[0]==1}},y[t],t,IncludeSingularSol
```

{}

18.44 problem 50

Internal problem ID [14871]

Book: INTRODUCTORY DIFFERENTIAL EQUATIONS. Martha L. Abell, James P. Braselton. Fourth edition 2014. ElScAe. 2014
Section: Chapter 4. Higher Order Equations. Chapter 4 review exercises, page 219
Problem number: 50.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type **[[_2nd_order, _linear, _nonhomogeneous]]**

$$\boxed{y'' - 2y' + y = e^t \ln(t)}$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 28

```
dsolve(diff(y(t),t$2)-2*diff(y(t),t)+y(t)=exp(t)*ln(t),y(t), singsol=all)
```

$$y(t) = \frac{e^t(2\ln(t)\, t^2 + 4c_1 t - 3t^2 + 4c_2)}{4}$$

✓ Solution by Mathematica

Time used: 0.022 (sec). Leaf size: 34

```
DSolve[y''[t]-2*y'[t]+y[t]==Exp[t]*Log[t],y[t],t,IncludeSingularSolutions -> True]
```

$$y(t) \to \frac{1}{4}e^t\left(-3t^2 + 2t^2\log(t) + 4c_2 t + 4c_1\right)$$

18.45 problem 51

Internal problem ID [14872]

Book: INTRODUCTORY DIFFERENTIAL EQUATIONS. Martha L. Abell, James P. Braselton. Fourth edition 2014. ElScAe. 2014
Section: Chapter 4. Higher Order Equations. Chapter 4 review exercises, page 219
Problem number: 51.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _linear, _nonhomogeneous]]`

$$\boxed{y'' - 2y' + y = \mathrm{e}^{t} \ln(t)}$$

With initial conditions

$$[y(1) = 1, y'(1) = 0]$$

✓ Solution by Maple
Time used: 0.063 (sec). Leaf size: 37

```
dsolve([diff(y(t),t$2)-2*diff(y(t),t)+y(t)=exp(t)*ln(t),y(1) = 1, D(y)(1) = 0],y(t), singsol
```

$$y(t) = \frac{\mathrm{e}^{t}(2\ln(t)\, t^{2} - 4\,\mathrm{e}^{-1} t - 3t^{2} + 8\,\mathrm{e}^{-1} + 4t - 1)}{4}$$

✓ Solution by Mathematica
Time used: 0.02 (sec). Leaf size: 39

```
DSolve[{y''[t]-2*y'[t]+y[t]==Exp[t]*Log[t],{y[1]==1,y'[1]==0}},y[t],t,IncludeSingularSolutio
```

$$y(t) \to \frac{1}{4} e^{t-1}\left(e\left(-3t^{2} + 4t - 1\right) + 2et^{2}\log(t) - 4t + 8\right)$$

18.46 problem 53

Internal problem ID [14873]

Book: INTRODUCTORY DIFFERENTIAL EQUATIONS. Martha L. Abell, James P. Braselton. Fourth edition 2014. ElScAe. 2014
Section: Chapter 4. Higher Order Equations. Chapter 4 review exercises, page 219
Problem number: 53.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$\boxed{y'' - 2y't + t^2 y = 0}$$

✓ Solution by Maple
Time used: 0.032 (sec). Leaf size: 20

```
dsolve(diff(y(t),t$2)-2*t*diff(y(t),t)+t^2*y(t)=0,y(t), singsol=all)
```

$$y(t) = \mathrm{e}^{\frac{t^2}{2}} (\sin(t)\, c_2 + \cos(t)\, c_1)$$

✓ Solution by Mathematica
Time used: 0.029 (sec). Leaf size: 39

```
DSolve[y''[t]-2*t*y'[t]+t^2*y[t]==0,y[t],t,IncludeSingularSolutions -> True]
```

$$y(t) \to \frac{1}{2} e^{\frac{1}{2}t(t-2i)} \left(2c_1 - ic_2 e^{2it}\right)$$

18.47 problem 54 (a)

Internal problem ID [14874]

Book: INTRODUCTORY DIFFERENTIAL EQUATIONS. Martha L. Abell, James P. Braselton. Fourth edition 2014. ElScAe. 2014
Section: Chapter 4. Higher Order Equations. Chapter 4 review exercises, page 219
Problem number: 54 (a).
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _missing_x]]`

$$\boxed{y'' + 3y' - 4y = 0}$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 17

```
dsolve(diff(y(t),t$2)+3*diff(y(t),t)-4*y(t)=0,y(t), singsol=all)
```

$$y(t) = \left(c_1 \mathrm{e}^{5t} + c_2\right) \mathrm{e}^{-4t}$$

✓ Solution by Mathematica

Time used: 0.011 (sec). Leaf size: 20

```
DSolve[y''[t]+3*y'[t]-4*y[t]==0,y[t],t,IncludeSingularSolutions -> True]
```

$$y(t) \to c_1 e^{-4t} + c_2 e^t$$

18.48 problem 54 (b)

Internal problem ID [14875]

Book: INTRODUCTORY DIFFERENTIAL EQUATIONS. Martha L. Abell, James P. Braselton. Fourth edition 2014. ElScAe. 2014
Section: Chapter 4. Higher Order Equations. Chapter 4 review exercises, page 219
Problem number: 54 (b).
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _missing_x]]`

$$\boxed{y'' + 4y' + 4y = 0}$$

✓ Solution by Maple
Time used: 0.0 (sec). Leaf size: 14

```
dsolve(diff(y(t),t$2)+4*diff(y(t),t)+4*y(t)=0,y(t), singsol=all)
```

$$y(t) = \mathrm{e}^{-2t}(tc_2 + c_1)$$

✓ Solution by Mathematica
Time used: 0.012 (sec). Leaf size: 18

```
DSolve[y''[t]+4*y'[t]+4*y[t]==0,y[t],t,IncludeSingularSolutions -> True]
```

$$y(t) \to e^{-2t}(c_2 t + c_1)$$

18.49 problem 54 (c)

Internal problem ID [14876]

Book: INTRODUCTORY DIFFERENTIAL EQUATIONS. Martha L. Abell, James P. Braselton. Fourth edition 2014. ElScAe. 2014
Section: Chapter 4. Higher Order Equations. Chapter 4 review exercises, page 219
Problem number: 54 (c).
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_Emden, _Fowler]]`

$$\boxed{t^2y'' - 5y't + 5y = 0}$$

✓ Solution by Maple
Time used: 0.0 (sec). Leaf size: 13

```
dsolve(t^2*diff(y(t),t$2)-5*t*diff(y(t),t)+5*y(t)=0,y(t), singsol=all)
```

$$y(t) = t\left(c_1t^4 + c_2\right)$$

✓ Solution by Mathematica
Time used: 0.01 (sec). Leaf size: 16

```
DSolve[t^2*y''[t]-5*t*y'[t]+5*y[t]==0,y[t],t,IncludeSingularSolutions -> True]
```

$$y(t) \to t\left(c_2t^4 + c_1\right)$$

18.50 problem 56

Internal problem ID [14877]

Book: INTRODUCTORY DIFFERENTIAL EQUATIONS. Martha L. Abell, James P. Braselton. Fourth edition 2014. ElScAe. 2014
Section: Chapter 4. Higher Order Equations. Chapter 4 review exercises, page 219
Problem number: 56.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_Emden, _Fowler]]`

$$\boxed{x^2y'' + 7y'x + 8y = 0}$$

✓ Solution by Maple
Time used: 0.0 (sec). Leaf size: 15

```
dsolve(x^2*diff(y(x),x$2)+7*x*diff(y(x),x)+8*y(x)=0,y(x), singsol=all)
```

$$y(x) = \frac{c_1x^2 + c_2}{x^4}$$

✓ Solution by Mathematica
Time used: 0.01 (sec). Leaf size: 18

```
DSolve[x^2*y''[x]+7*x*y'[x]+8*y[x]==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to \frac{c_2x^2 + c_1}{x^4}$$

18.51 problem 57

Internal problem ID [14878]

Book: INTRODUCTORY DIFFERENTIAL EQUATIONS. Martha L. Abell, James P. Braselton. Fourth edition 2014. ElScAe. 2014
Section: Chapter 4. Higher Order Equations. Chapter 4 review exercises, page 219
Problem number: 57.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_Emden, _Fowler], [_2nd_order, _linear, '_with_symmetry_[0,F`

$$\boxed{x^2y'' - 4y'x + 6y = 0}$$

✓ Solution by Maple
Time used: 0.0 (sec). Leaf size: 13

```
dsolve(x^2*diff(y(x),x$2)-4*x*diff(y(x),x)+6*y(x)=0,y(x), singsol=all)
```

$$y(x) = x^2(c_1x + c_2)$$

✓ Solution by Mathematica
Time used: 0.01 (sec). Leaf size: 16

```
DSolve[x^2*y''[x]-4*x*y'[x]+6*y[x]==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to x^2(c_2x + c_1)$$

18.52 problem 58

Internal problem ID [14879]

Book: INTRODUCTORY DIFFERENTIAL EQUATIONS. Martha L. Abell, James P. Braselton. Fourth edition 2014. ElScAe. 2014
Section: Chapter 4. Higher Order Equations. Chapter 4 review exercises, page 219
Problem number: 58.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_Emden, _Fowler], [_2nd_order, _linear, ‘_with_symmetry_[0,F`

$$\boxed{x^2 y'' + y' x + y = 0}$$

✓ Solution by Maple
Time used: 0.0 (sec). Leaf size: 15

```
dsolve(x^2*diff(y(x),x$2)+x*diff(y(x),x)+y(x)=0,y(x), singsol=all)
```

$$y(x) = c_1 \sin(\ln(x)) + c_2 \cos(\ln(x))$$

✓ Solution by Mathematica
Time used: 0.014 (sec). Leaf size: 18

```
DSolve[x^2*y''[x]+x*y'[x]+y[x]==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to c_1 \cos(\log(x)) + c_2 \sin(\log(x))$$

18.53 problem 59

Internal problem ID [14880]

Book: INTRODUCTORY DIFFERENTIAL EQUATIONS. Martha L. Abell, James P. Braselton. Fourth edition 2014. ElScAe. 2014
Section: Chapter 4. Higher Order Equations. Chapter 4 review exercises, page 219
Problem number: 59.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _exact, _linear, _homogeneous]]`

$$\boxed{2x^2y'' + 5y'x + y = 0}$$

With initial conditions

$$[y(1) = 1, y'(1) = 0]$$

✓ Solution by Maple

Time used: 0.015 (sec). Leaf size: 15

```
dsolve([2*x^2*diff(y(x),x$2)+5*x*diff(y(x),x)+y(x)=0,y(1) = 1, D(y)(1) = 0],y(x), singsol=al
```

$$y(x) = \frac{2}{\sqrt{x}} - \frac{1}{x}$$

✓ Solution by Mathematica

Time used: 0.012 (sec). Leaf size: 18

```
DSolve[{2*x^2*y''[x]+5*x*y'[x]+y[x]==0,{y[1]==1,y'[1]==0}},y[x],x,IncludeSingularSolutions -
```

$$y(x) \to \frac{2\sqrt{x} - 1}{x}$$

18.54 problem 60

Internal problem ID [14881]

Book: INTRODUCTORY DIFFERENTIAL EQUATIONS. Martha L. Abell, James P. Braselton. Fourth edition 2014. ElScAe. 2014
Section: Chapter 4. Higher Order Equations. Chapter 4 review exercises, page 219
Problem number: 60.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_Emden, _Fowler]]`

$$\boxed{5x^2y'' - y'x + 2y = 0}$$

✓ Solution by Maple
Time used: 0.0 (sec). Leaf size: 23

```
dsolve(5*x^2*diff(y(x),x$2)-x*diff(y(x),x)+2*y(x)=0,y(x), singsol=all)
```

$$y(x) = x^{\frac{3}{5}}\left(c_1 \sin\left(\frac{\ln(x)}{5}\right) + c_2 \cos\left(\frac{\ln(x)}{5}\right)\right)$$

✓ Solution by Mathematica
Time used: 0.028 (sec). Leaf size: 32

```
DSolve[5*x^2*y''[x]-x*y'[x]+2*y[x]==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to x^{3/5}\left(c_2 \cos\left(\frac{\log(x)}{5}\right) + c_1 \sin\left(\frac{\log(x)}{5}\right)\right)$$

18.55 problem 61

Internal problem ID [14882]

Book: INTRODUCTORY DIFFERENTIAL EQUATIONS. Martha L. Abell, James P. Braselton. Fourth edition 2014. ElScAe. 2014
Section: Chapter 4. Higher Order Equations. Chapter 4 review exercises, page 219
Problem number: 61.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_Emden, _Fowler]]`

$$\boxed{x^2y'' - 7y'x + 25y = 0}$$

✓ Solution by Maple
Time used: 0.0 (sec). Leaf size: 23

```
dsolve(x^2*diff(y(x),x$2)-7*x*diff(y(x),x)+25*y(x)=0,y(x), singsol=all)
```

$$y(x) = x^4(c_1 \sin(3\ln(x)) + c_2 \cos(3\ln(x)))$$

✓ Solution by Mathematica
Time used: 0.021 (sec). Leaf size: 26

```
DSolve[x^2*y''[x]-7*x*y'[x]+25*y[x]==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to x^4(c_2 \cos(3\log(x)) + c_1 \sin(3\log(x)))$$

18.56 problem 62

Internal problem ID [14883]

Book: INTRODUCTORY DIFFERENTIAL EQUATIONS. Martha L. Abell, James P. Braselton. Fourth edition 2014. ElScAe. 2014
Section: Chapter 4. Higher Order Equations. Chapter 4 review exercises, page 219
Problem number: 62.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$\boxed{x^2y'' - 7y'x + 15y = 8x}$$

✓ Solution by Maple
Time used: 0.016 (sec). Leaf size: 16

```
dsolve(x^2*diff(y(x),x$2)-7*x*diff(y(x),x)+15*y(x)=8*x,y(x), singsol=all)
```

$$y(x) = c_2x^5 + c_1x^3 + x$$

✓ Solution by Mathematica
Time used: 0.013 (sec). Leaf size: 19

```
DSolve[x^2*y''[x]-7*x*y'[x]+15*y[x]==8*x,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to c_2x^5 + c_1x^3 + x$$

18.57 problem 63

Internal problem ID [14884]

Book: INTRODUCTORY DIFFERENTIAL EQUATIONS. Martha L. Abell, James P. Braselton. Fourth edition 2014. ElScAe. 2014
Section: Chapter 4. Higher Order Equations. Chapter 4 review exercises, page 219
Problem number: 63.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _missing_x]]`

$$\boxed{y'' - 4y' + 4y = 0}$$

With the expansion point for the power series method at $x = 0$.

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 54

```
Order:=6;
dsolve(diff(y(x),x$2)-4*diff(y(x),x)+4*y(x)=0,y(x),type='series',x=0);
```

$$y(x) = \left(1 - 2x^2 - \frac{8}{3}x^3 - 2x^4 - \frac{16}{15}x^5\right) y(0) + \left(x + 2x^2 + 2x^3 + \frac{4}{3}x^4 + \frac{2}{3}x^5\right) D(y)(0) + O(x^6)$$

✓ Solution by Mathematica

Time used: 0.001 (sec). Leaf size: 62

```
AsymptoticDSolveValue[y''[x]-4*y'[x]+4*y[x]==0,y[x],{x,0,5}]
```

$$y(x) \to c_1 \left(-\frac{16x^5}{15} - 2x^4 - \frac{8x^3}{3} - 2x^2 + 1\right) + c_2 \left(\frac{2x^5}{3} + \frac{4x^4}{3} + 2x^3 + 2x^2 + x\right)$$

18.58 problem 64

Internal problem ID [14885]

Book: INTRODUCTORY DIFFERENTIAL EQUATIONS. Martha L. Abell, James P. Braselton. Fourth edition 2014. ElScAe. 2014
Section: Chapter 4. Higher Order Equations. Chapter 4 review exercises, page 219
Problem number: 64.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _linear, _nonhomogeneous]]`

$$\boxed{y'' + 2y' - 3y = x\,\mathrm{e}^{x}}$$

With the expansion point for the power series method at $x = 0$.

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 56

```
Order:=6;
dsolve(diff(y(x),x$2)+2*diff(y(x),x)-3*y(x)=x*exp(x),y(x),type='series',x=0);
```

$$y(x) = \left(1+\frac{3}{2}x^{2}-x^{3}+\frac{7}{8}x^{4}-\frac{1}{2}x^{5}\right)y(0) + \left(x-x^{2}+\frac{7}{6}x^{3}-\frac{5}{6}x^{4}+\frac{61}{120}x^{5}\right)D(y)(0)+\frac{x^{3}}{6}+\frac{x^{5}}{20}+O\left(x^{6}\right)$$

✓ Solution by Mathematica

Time used: 0.016 (sec). Leaf size: 66

```
AsymptoticDSolveValue[y''[x]+2*y'[x]-3*y[x]==x*Exp[x],y[x],{x,0,5}]
```

$$y(x) \to c_{1}\left(-\frac{x^{5}}{2}+\frac{7x^{4}}{8}-x^{3}+\frac{3x^{2}}{2}+1\right)+c_{2}\left(\frac{61x^{5}}{120}-\frac{5x^{4}}{6}+\frac{7x^{3}}{6}-x^{2}+x\right)$$

18.59 problem 65

Internal problem ID [14886]

Book: INTRODUCTORY DIFFERENTIAL EQUATIONS. Martha L. Abell, James P. Braselton. Fourth edition 2014. ElScAe. 2014
Section: Chapter 4. Higher Order Equations. Chapter 4 review exercises, page 219
Problem number: 65.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[_Gegenbauer, [_2nd_order, _linear, '_with_symmetry_[0,F(x)]']]`

$$\boxed{(2x^2-1)\,y''+2y'x-3y=0}$$

With the expansion point for the power series method at $x=0$.

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 34

```
Order:=6;
dsolve((2*x^2-1)*diff(y(x),x$2)+2*x*diff(y(x),x)-3*y(x)=0,y(x),type='series',x=0);
```

$$y(x)=\left(1-\frac{3}{2}x^2-\frac{5}{8}x^4\right)y(0)+\left(x-\frac{1}{6}x^3-\frac{1}{8}x^5\right)D(y)\,(0)+O\left(x^6\right)$$

✓ Solution by Mathematica

Time used: 0.001 (sec). Leaf size: 42

```
AsymptoticDSolveValue[(2*x^2-1)*y''[x]+2*x*y'[x]-3*y[x]==0,y[x],{x,0,5}]
```

$$y(x)\to c_2\left(-\frac{x^5}{8}-\frac{x^3}{6}+x\right)+c_1\left(-\frac{5x^4}{8}-\frac{3x^2}{2}+1\right)$$

18.60 problem 66

Internal problem ID [14887]

Book: INTRODUCTORY DIFFERENTIAL EQUATIONS. Martha L. Abell, James P. Braselton. Fourth edition 2014. ElScAe. 2014
Section: Chapter 4. Higher Order Equations. Chapter 4 review exercises, page 219
Problem number: 66.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_Emden, _Fowler]]`

$$\boxed{3xy'' + 11y' - y = 0}$$

With the expansion point for the power series method at $x = 0$.

✓ Solution by Maple

Time used: 0.062 (sec). Leaf size: 44

```
Order:=6;
dsolve(3*x*diff(y(x),x$2)+11*diff(y(x),x)-y(x)=0,y(x),type='series',x=0);
```

$$y(x) = \frac{c_1\left(1 - \frac{1}{5}x + \frac{1}{20}x^2 + \frac{1}{60}x^3 + \frac{1}{960}x^4 + \frac{1}{33600}x^5 + \mathrm{O}\left(x^6\right)\right)}{x^{\frac{8}{3}}} + c_2\left(1 + \frac{1}{11}x + \frac{1}{308}x^2 + \frac{1}{15708}x^3 + \frac{1}{1256640}x^4 + \frac{1}{144513600}x^5 + \mathrm{O}\left(x^6\right)\right)$$

✓ Solution by Mathematica

Time used: 0.002 (sec). Leaf size: 85

```
AsymptoticDSolveValue[3*x*y''[x]+11*y'[x]-y[x]==0,y[x],{x,0,5}]
```

$$y(x) \to c_1\left(\frac{x^5}{144513600} + \frac{x^4}{1256640} + \frac{x^3}{15708} + \frac{x^2}{308} + \frac{x}{11} + 1\right) + \frac{c_2\left(\frac{x^5}{33600} + \frac{x^4}{960} + \frac{x^3}{60} + \frac{x^2}{20} - \frac{x}{5} + 1\right)}{x^{8/3}}$$

18.61 problem 67

Internal problem ID [14888]

Book: INTRODUCTORY DIFFERENTIAL EQUATIONS. Martha L. Abell, James P. Braselton. Fourth edition 2014. ElScAe. 2014
Section: Chapter 4. Higher Order Equations. Chapter 4 review exercises, page 219
Problem number: 67.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_Emden, _Fowler], [_2nd_order, _linear, '_with_symmetry_[0,F`

$$\boxed{2x^2y'' + 5y'x - 2y = 0}$$

With the expansion point for the power series method at $x = 0$.

✓ Solution by Maple
Time used: 0.047 (sec). Leaf size: 27

```
Order:=6;
dsolve(2*x^2*diff(y(x),x$2)+5*x*diff(y(x),x)-2*y(x)=0,y(x),type='series',x=0);
```

$$y(x) = \frac{x^{\frac{5}{2}}c_2 + c_1}{x^2} + O(x^6)$$

✓ Solution by Mathematica
Time used: 0.002 (sec). Leaf size: 18

```
AsymptoticDSolveValue[2*x^2*y''[x]+5*x*y'[x]-2*y[x]==0,y[x],{x,0,5}]
```

$$y(x) \to \frac{c_2}{x^2} + c_1\sqrt{x}$$

18.62 problem 68

Internal problem ID [14889]

Book: INTRODUCTORY DIFFERENTIAL EQUATIONS. Martha L. Abell, James P. Braselton. Fourth edition 2014. ElScAe. 2014
Section: Chapter 4. Higher Order Equations. Chapter 4 review exercises, page 219
Problem number: 68.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$\boxed{x^2y'' - 7y'x + (-2x^2 + 7)\, y = 0}$$

With the expansion point for the power series method at $x = 0$.

✓ Solution by Maple
Time used: 0.047 (sec). Leaf size: 33

```
Order:=6;
dsolve(x^2*diff(y(x),x$2)-7*x*diff(y(x),x)+(7-2*x^2)*y(x)=0,y(x),type='series',x=0);
```

$$y(x) = c_1 x^7 \left(1 + \frac{1}{8}x^2 + \frac{1}{160}x^4 + \mathrm{O}\left(x^6\right)\right) + c_2 x\left(-86400 + 21600x^2 - 5400x^4 + \mathrm{O}\left(x^6\right)\right)$$

✓ Solution by Mathematica
Time used: 0.009 (sec). Leaf size: 44

```
AsymptoticDSolveValue[x^2*y''[x]-7*x*y'[x]+(7-2*x^2)*y[x]==0,y[x],{x,0,5}]
```

$$y(x) \to c_1\left(\frac{x^5}{16} - \frac{x^3}{4} + x\right) + c_2\left(\frac{x^{11}}{160} + \frac{x^9}{8} + x^7\right)$$

18.63 problem 69

Internal problem ID [14890]

Book: INTRODUCTORY DIFFERENTIAL EQUATIONS. Martha L. Abell, James P. Braselton. Fourth edition 2014. ElScAe. 2014
Section: Chapter 4. Higher Order Equations. Chapter 4 review exercises, page 219
Problem number: 69.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[_Jacobi]`

$$\boxed{x(1-x)\,y''+(2x+1)\,y'+10y=0}$$

With the expansion point for the power series method at $x=0$.

✓ Solution by Maple

Time used: 0.031 (sec). Leaf size: 59

```
Order:=6;
dsolve(x*(1-x)*diff(y(x),x$2)+(1+2*x)*diff(y(x),x)+10*y(x)=0,y(x),type='series',x=0);
```

$$y(x)=(c_1+c_2\ln(x))\left(1-10x+30x^2-40x^3+25x^4-6x^5+\mathrm{O}\left(x^6\right)\right)+\left(17x-\frac{157}{2}x^2+\frac{404}{3}x^3-\frac{625}{6}x^4+\frac{162}{5}x^5+\mathrm{O}\left(x^6\right)\right)c_2$$

✓ Solution by Mathematica

Time used: 0.005 (sec). Leaf size: 95

```
AsymptoticDSolveValue[x*(1-x)*y''[x]+(1+2*x)*y'[x]+10*y[x]==0,y[x],{x,0,5}]
```

$$y(x)\to c_1\left(-6x^5+25x^4-40x^3+30x^2-10x+1\right)+c_2\left(\frac{162x^5}{5}-\frac{625x^4}{6}+\frac{404x^3}{3}-\frac{157x^2}{2}+\left(-6x^5+25x^4-40x^3+30x^2-10x+1\right)\log(x)+17x\right)$$

18.64 problem 70

Internal problem ID [14891]

Book: INTRODUCTORY DIFFERENTIAL EQUATIONS. Martha L. Abell, James P. Braselton. Fourth edition 2014. ElScAe. 2014
Section: Chapter 4. Higher Order Equations. Chapter 4 review exercises, page 219
Problem number: 70.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$\boxed{x(x+1)\,y''+(-2x+1)\,y'-10y=0}$$

With the expansion point for the power series method at $x=0$.

✓ Solution by Maple

Time used: 0.062 (sec). Leaf size: 59

```
Order:=6;
dsolve(x*(1+x)*diff(y(x),x$2)+(1-2*x)*diff(y(x),x)-10*y(x)=0,y(x),type='series',x=0);
```

$$\begin{aligned}y(x)=&\,(c_1+c_2\ln(x))\left(1+10x+30x^2+40x^3+25x^4+6x^5+\mathrm{O}\left(x^6\right)\right)\\&+\left((-17)\,x-\frac{157}{2}x^2-\frac{404}{3}x^3-\frac{625}{6}x^4-\frac{162}{5}x^5+\mathrm{O}\left(x^6\right)\right)c_2\end{aligned}$$

✓ Solution by Mathematica

Time used: 0.005 (sec). Leaf size: 95

```
AsymptoticDSolveValue[x*(1+x)*y''[x]+(1-2*x)*y'[x]-10*y[x]==0,y[x],{x,0,5}]
```

$$\begin{aligned}y(x)\to&\,c_1\left(6x^5+25x^4+40x^3+30x^2+10x+1\right)+c_2\left(-\frac{162x^5}{5}-\frac{625x^4}{6}-\frac{404x^3}{3}\right.\\&\left.-\frac{157x^2}{2}+\left(6x^5+25x^4+40x^3+30x^2+10x+1\right)\log(x)-17x\right)\end{aligned}$$

18.65 problem 73

Internal problem ID [14892]

Book: INTRODUCTORY DIFFERENTIAL EQUATIONS. Martha L. Abell, James P. Braselton. Fourth edition 2014. ElScAe. 2014
Section: Chapter 4. Higher Order Equations. Chapter 4 review exercises, page 219
Problem number: 73.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _exact, _nonlinear], [_2nd_order, _with_linear_s`

$$\boxed{t\left(y''y+y'^2\right)+y'y=1}$$

With initial conditions

$$[y(1)=1, y'(1)=1]$$

✓ Solution by Maple
Time used: 0.047 (sec). Leaf size: 11

```
dsolve([t*(diff(y(t),t$2)*y(t)+diff(y(t),t)^2)+diff(y(t),t)*y(t)=1,y(1) = 1, D(y)(1) = 1],y(
```

$$y(t)=\sqrt{2t-1}$$

✓ Solution by Mathematica
Time used: 0.371 (sec). Leaf size: 14

```
DSolve[{t*(y''[t]*y[t]+y'[t]^2)+y'[t]*y[t]==1,{y[1]==1,y'[1]==1}},y[t],t,IncludeSingularSolu
```

$$y(t)\to\sqrt{2t-1}$$

19 Chapter 5. Applications of Higher Order Equations. Exercises 5.1, page 232

19.1 problem 1

Internal problem ID [14893]

Book: INTRODUCTORY DIFFERENTIAL EQUATIONS. Martha L. Abell, James P. Braselton. Fourth edition 2014. ElScAe. 2014
Section: Chapter 5. Applications of Higher Order Equations. Exercises 5.1, page 232
Problem number: 1.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _missing_x]]`

$$\boxed{4x'' + 9x = 0}$$

With initial conditions

$$[x(0) = -1, x'(0) = 0]$$

✓ Solution by Maple
Time used: 0.0 (sec). Leaf size: 10

```
dsolve([4*diff(x(t),t$2)+9*x(t)=0,x(0) = -1, D(x)(0) = 0],x(t), singsol=all)
```

$$x(t) = -\cos\left(\frac{3t}{2}\right)$$

✓ Solution by Mathematica
Time used: 0.013 (sec). Leaf size: 13

```
DSolve[{4*x''[t]+9*x[t]==0,{x[0]==-1,x'[0]==0}},x[t],t,IncludeSingularSolutions -> True]
```

$$x(t) \to -\cos\left(\frac{3t}{2}\right)$$

19.2 problem 2

Internal problem ID [14894]

Book: INTRODUCTORY DIFFERENTIAL EQUATIONS. Martha L. Abell, James P. Braselton. Fourth edition 2014. ElScAe. 2014
Section: Chapter 5. Applications of Higher Order Equations. Exercises 5.1, page 232
Problem number: 2.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _missing_x]]`

$$\boxed{9x'' + 4x = 0}$$

With initial conditions

$$\left[x(0) = -\frac{1}{2}, x'(0) = 1\right]$$

✓ Solution by Maple
Time used: 0.016 (sec). Leaf size: 17

```
dsolve([9*diff(x(t),t$2)+4*x(t)=0,x(0) = -1/2, D(x)(0) = 1],x(t), singsol=all)
```

$$x(t) = \frac{3 \sin\left(\frac{2t}{3}\right)}{2} - \frac{\cos\left(\frac{2t}{3}\right)}{2}$$

✓ Solution by Mathematica
Time used: 0.026 (sec). Leaf size: 26

```
DSolve[{9*x''[t]+4*x[t]==0,{x[0]==-1/2,x'[0]==1}},x[t],t,IncludeSingularSolutions -> True]
```

$$x(t) \to \frac{1}{2}\left(3 \sin\left(\frac{2t}{3}\right) - \cos\left(\frac{2t}{3}\right)\right)$$

19.3 problem 3

Internal problem ID [14895]

Book: INTRODUCTORY DIFFERENTIAL EQUATIONS. Martha L. Abell, James P. Braselton. Fourth edition 2014. ElScAe. 2014
Section: Chapter 5. Applications of Higher Order Equations. Exercises 5.1, page 232
Problem number: 3.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _missing_x]]`

$$\boxed{x'' + 64x = 0}$$

With initial conditions

$$\left[x(0) = \frac{3}{4}, x'(0) = -2\right]$$

✓ Solution by Maple
Time used: 0.016 (sec). Leaf size: 17

```
dsolve([diff(x(t),t$2)+64*x(t)=0,x(0) = 3/4, D(x)(0) = -2],x(t), singsol=all)
```

$$x(t) = -\frac{\sin(8t)}{4} + \frac{3\cos(8t)}{4}$$

✓ Solution by Mathematica
Time used: 0.014 (sec). Leaf size: 22

```
DSolve[{x''[t]+64*x[t]==0,{x[0]==3/4,x'[0]==-2}},x[t],t,IncludeSingularSolutions -> True]
```

$$x(t) \to \frac{1}{4}(3\cos(8t) - \sin(8t))$$

19.4 problem 4

Internal problem ID [14896]

Book: INTRODUCTORY DIFFERENTIAL EQUATIONS. Martha L. Abell, James P. Braselton. Fourth edition 2014. ElScAe. 2014
Section: Chapter 5. Applications of Higher Order Equations. Exercises 5.1, page 232
Problem number: 4.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _missing_x]]`

$$\boxed{x'' + 100x = 0}$$

With initial conditions

$$\left[x(0) = -\frac{1}{4}, x'(0) = 1\right]$$

✓ Solution by Maple
Time used: 0.016 (sec). Leaf size: 17

```
dsolve([diff(x(t),t$2)+100*x(t)=0,x(0) = -1/4, D(x)(0) = 1],x(t), singsol=all)
```

$$x(t) = \frac{\sin(10t)}{10} - \frac{\cos(10t)}{4}$$

✓ Solution by Mathematica
Time used: 0.014 (sec). Leaf size: 22

```
DSolve[{x''[t]+100*x[t]==0,{x[0]==-1/4,x'[0]==1}},x[t],t,IncludeSingularSolutions -> True]
```

$$x(t) \to \frac{1}{20}(2\sin(10t) - 5\cos(10t))$$

19.5 problem 5

Internal problem ID [14897]

Book: INTRODUCTORY DIFFERENTIAL EQUATIONS. Martha L. Abell, James P. Braselton. Fourth edition 2014. ElScAe. 2014
Section: Chapter 5. Applications of Higher Order Equations. Exercises 5.1, page 232
Problem number: 5.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _missing_x]]`

$$\boxed{x'' + x = 0}$$

With initial conditions

$$[x(0) = 3, x'(0) = -4]$$

✓ Solution by Maple
Time used: 0.015 (sec). Leaf size: 13

```
dsolve([diff(x(t),t$2)+x(t)=0,x(0) = 3, D(x)(0) = -4],x(t), singsol=all)
```

$$x(t) = -4\sin(t) + 3\cos(t)$$

✓ Solution by Mathematica
Time used: 0.011 (sec). Leaf size: 14

```
DSolve[{x''[t]+x[t]==0,{x[0]==3,x'[0]==-4}},x[t],t,IncludeSingularSolutions -> True]
```

$$x(t) \to 3\cos(t) - 4\sin(t)$$

19.6 problem 6

Internal problem ID [14898]

Book: INTRODUCTORY DIFFERENTIAL EQUATIONS. Martha L. Abell, James P. Braselton. Fourth edition 2014. ElScAe. 2014
Section: Chapter 5. Applications of Higher Order Equations. Exercises 5.1, page 232
Problem number: 6.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _missing_x]]`

$$\boxed{x'' + 4x = 0}$$

With initial conditions

$$[x(0) = 1, x'(0) = 1]$$

✓ Solution by Maple
Time used: 0.016 (sec). Leaf size: 15

```
dsolve([diff(x(t),t$2)+4*x(t)=0,x(0) = 1, D(x)(0) = 1],x(t), singsol=all)
```

$$x(t) = \frac{\sin(2t)}{2} + \cos(2t)$$

✓ Solution by Mathematica
Time used: 0.014 (sec). Leaf size: 15

```
DSolve[{x''[t]+4*x[t]==0,{x[0]==1,x'[0]==1}},x[t],t,IncludeSingularSolutions -> True]
```

$$x(t) \to \cos(2t) + \sin(t)\cos(t)$$

19.7 problem 7

Internal problem ID [14899]

Book: INTRODUCTORY DIFFERENTIAL EQUATIONS. Martha L. Abell, James P. Braselton. Fourth edition 2014. ElScAe. 2014
Section: Chapter 5. Applications of Higher Order Equations. Exercises 5.1, page 232
Problem number: 7.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _missing_x]]`

$$\boxed{x'' + 16x = 0}$$

With initial conditions

$$[x(0) = -2, x'(0) = 1]$$

✓ Solution by Maple
Time used: 0.016 (sec). Leaf size: 17

```
dsolve([diff(x(t),t$2)+16*x(t)=0,x(0) = -2, D(x)(0) = 1],x(t), singsol=all)
```

$$x(t) = \frac{\sin(4t)}{4} - 2\cos(4t)$$

✓ Solution by Mathematica
Time used: 0.015 (sec). Leaf size: 20

```
DSolve[{x''[t]+16*x[t]==0,{x[0]==-2,x'[0]==1}},x[t],t,IncludeSingularSolutions -> True]
```

$$x(t) \to \frac{1}{4}(\sin(4t) - 8\cos(4t))$$

19.8 problem 8

Internal problem ID [14900]

Book: INTRODUCTORY DIFFERENTIAL EQUATIONS. Martha L. Abell, James P. Braselton. Fourth edition 2014. ElScAe. 2014
Section: Chapter 5. Applications of Higher Order Equations. Exercises 5.1, page 232
Problem number: 8.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _missing_x]]`

$$\boxed{x'' + 256x = 0}$$

With initial conditions

$$[x(0) = 2, x'(0) = 4]$$

✓ Solution by Maple
Time used: 0.0 (sec). Leaf size: 17

```
dsolve([diff(x(t),t$2)+256*x(t)=0,x(0) = 2, D(x)(0) = 4],x(t), singsol=all)
```

$$x(t) = \frac{\sin(16t)}{4} + 2\cos(16t)$$

✓ Solution by Mathematica
Time used: 0.015 (sec). Leaf size: 20

```
DSolve[{x''[t]+256*x[t]==0,{x[0]==2,x'[0]==4}},x[t],t,IncludeSingularSolutions -> True]
```

$$x(t) \to \frac{1}{4}(\sin(16t) + 8\cos(16t))$$

19.9 problem 9

Internal problem ID [14901]

Book: INTRODUCTORY DIFFERENTIAL EQUATIONS. Martha L. Abell, James P. Braselton. Fourth edition 2014. ElScAe. 2014
Section: Chapter 5. Applications of Higher Order Equations. Exercises 5.1, page 232
Problem number: 9.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _missing_x]]`

$$\boxed{x'' + 9x = 0}$$

With initial conditions

$$\left[x(0) = \frac{1}{3}, x'(0) = -1\right]$$

✓ Solution by Maple
Time used: 0.016 (sec). Leaf size: 17

```
dsolve([diff(x(t),t$2)+9*x(t)=0,x(0) = 1/3, D(x)(0) = -1],x(t), singsol=all)
```

$$x(t) = -\frac{\sin(3t)}{3} + \frac{\cos(3t)}{3}$$

✓ Solution by Mathematica
Time used: 0.013 (sec). Leaf size: 20

```
DSolve[{x''[t]+9*x[t]==0,{x[0]==1/3,x'[0]==-1}},x[t],t,IncludeSingularSolutions -> True]
```

$$x(t) \to \frac{1}{3}(\cos(3t) - \sin(3t))$$

19.10 problem 10

Internal problem ID [14902]

Book: INTRODUCTORY DIFFERENTIAL EQUATIONS. Martha L. Abell, James P. Braselton. Fourth edition 2014. ElScAe. 2014
Section: Chapter 5. Applications of Higher Order Equations. Exercises 5.1, page 232
Problem number: 10.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _missing_x]]`

$$\boxed{10x'' + \frac{x}{10} = 0}$$

With initial conditions

$$[x(0) = -5, x'(0) = 1]$$

✓ Solution by Maple
Time used: 0.0 (sec). Leaf size: 17

```
dsolve([10*diff(x(t),t$2)+1/10*x(t)=0,x(0) = -5, D(x)(0) = 1],x(t), singsol=all)
```

$$x(t) = 10\sin\left(\frac{t}{10}\right) - 5\cos\left(\frac{t}{10}\right)$$

✓ Solution by Mathematica
Time used: 0.014 (sec). Leaf size: 22

```
DSolve[{10*x''[t]+1/10*x[t]==0,{x[0]==-5,x'[0]==1}},x[t],t,IncludeSingularSolutions -> True]
```

$$x(t) \to -5\left(\cos\left(\frac{t}{10}\right) - 2\sin\left(\frac{t}{10}\right)\right)$$

20 Chapter 5. Applications of Higher Order Equations. Exercises 5.2, page 241

20.1 problem 1

Internal problem ID [14903]

Book: INTRODUCTORY DIFFERENTIAL EQUATIONS. Martha L. Abell, James P. Braselton. Fourth edition 2014. ElScAe. 2014
Section: Chapter 5. Applications of Higher Order Equations. Exercises 5.2, page 241
Problem number: 1.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _missing_x]]`

$$\boxed{x'' + 4x' + 3x = 0}$$

With initial conditions

$$[x(0) = 0, x'(0) = -4]$$

✓ Solution by Maple
Time used: 0.016 (sec). Leaf size: 17

```
dsolve([diff(x(t),t$2)+4*diff(x(t),t)+3*x(t)=0,x(0) = 0, D(x)(0) = -4],x(t), singsol=all)
```

$$x(t) = 2\,\mathrm{e}^{-3t} - 2\,\mathrm{e}^{-t}$$

✓ Solution by Mathematica
Time used: 0.013 (sec). Leaf size: 19

```
DSolve[{x''[t]+4*x'[t]+3*x[t]==0,{x[0]==0,x'[0]==-4}},x[t],t,IncludeSingularSolutions -> Tru
```

$$x(t) \to -2e^{-3t}\left(e^{2t} - 1\right)$$

20.2 problem 2

Internal problem ID [14904]

Book: INTRODUCTORY DIFFERENTIAL EQUATIONS. Martha L. Abell, James P. Braselton. Fourth edition 2014. ElScAe. 2014
Section: Chapter 5. Applications of Higher Order Equations. Exercises 5.2, page 241
Problem number: 2.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _missing_x]]`

$$\boxed{\frac{x''}{32}+2x'+x=0}$$

With initial conditions

$$[x(0)=1, x'(0)=0]$$

✓ Solution by Maple
Time used: 0.078 (sec). Leaf size: 39

```
dsolve([1/32*diff(x(t),t$2)+2*diff(x(t),t)+x(t)=0,x(0) = 1, D(x)(0) = 0],x(t), singsol=all)
```

$$x(t)=\frac{\left(31+4\sqrt{62}\right)\mathrm{e}^{4\left(-8+\sqrt{62}\right)t}}{62}+\frac{\left(31-4\sqrt{62}\right)\mathrm{e}^{-4\left(8+\sqrt{62}\right)t}}{62}$$

✓ Solution by Mathematica
Time used: 0.028 (sec). Leaf size: 50

```
DSolve[{1/32*x''[t]+2*x'[t]+x[t]==0,{x[0]==1,x'[0]==0}},x[t],t,IncludeSingularSolutions -> T
```

$$x(t)\to \frac{1}{62}e^{-4\left(8+\sqrt{62}\right)t}\left(\left(31+4\sqrt{62}\right)e^{8\sqrt{62}t}+31-4\sqrt{62}\right)$$

20.3 problem 3

Internal problem ID [14905]

Book: INTRODUCTORY DIFFERENTIAL EQUATIONS. Martha L. Abell, James P. Braselton. Fourth edition 2014. ElScAe. 2014
Section: Chapter 5. Applications of Higher Order Equations. Exercises 5.2, page 241
Problem number: 3.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _missing_x]]`

$$\boxed{\frac{x''}{4}+2x'+x=0}$$

With initial conditions

$$\left[x(0)=-\frac{1}{2},x'(0)=1\right]$$

✓ Solution by Maple
Time used: 0.078 (sec). Leaf size: 39

```
dsolve([1/4*diff(x(t),t$2)+2*diff(x(t),t)+x(t)=0,x(0) = -1/2, D(x)(0) = 1],x(t), singsol=all
```

$$x(t)=\frac{\left(-3-\sqrt{3}\right)\mathrm{e}^{2\left(-2+\sqrt{3}\right)t}}{12}+\frac{\mathrm{e}^{-2\left(2+\sqrt{3}\right)t}\left(\sqrt{3}-3\right)}{12}$$

✓ Solution by Mathematica
Time used: 0.022 (sec). Leaf size: 47

```
DSolve[{1/4*x''[t]+2*x'[t]+x[t]==0,{x[0]==-1/2,x'[0]==1}},x[t],t,IncludeSingularSolutions ->
```

$$x(t)\to\frac{1}{12}e^{-2\left(2+\sqrt{3}\right)t}\left(-\left(3+\sqrt{3}\right)e^{4\sqrt{3}t}-3+\sqrt{3}\right)$$

20.4 problem 4

Internal problem ID [14906]

Book: INTRODUCTORY DIFFERENTIAL EQUATIONS. Martha L. Abell, James P. Braselton. Fourth edition 2014. ElScAe. 2014
Section: Chapter 5. Applications of Higher Order Equations. Exercises 5.2, page 241
Problem number: 4.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _missing_x]]`

$$\boxed{4x'' + 2x' + 8x = 0}$$

With initial conditions

$$[x(0) = 0, x'(0) = 2]$$

✓ Solution by Maple
Time used: 0.031 (sec). Leaf size: 20

```
dsolve([4*diff(x(t),t$2)+2*diff(x(t),t)+8*x(t)=0,x(0) = 0, D(x)(0) = 2],x(t), singsol=all)
```

$$x(t) = \frac{8\sqrt{31}\,\mathrm{e}^{-\frac{t}{4}} \sin\left(\frac{\sqrt{31}\,t}{4}\right)}{31}$$

✓ Solution by Mathematica
Time used: 0.025 (sec). Leaf size: 30

```
DSolve[{4*x''[t]+2*x'[t]+8*x[t]==0,{x[0]==0,x'[0]==2}},x[t],t,IncludeSingularSolutions -> Tr
```

$$x(t) \to \frac{8e^{-t/4} \sin\left(\frac{\sqrt{31}t}{4}\right)}{\sqrt{31}}$$

20.5 problem 5

Internal problem ID [14907]

Book: INTRODUCTORY DIFFERENTIAL EQUATIONS. Martha L. Abell, James P. Braselton. Fourth edition 2014. ElScAe. 2014
Section: Chapter 5. Applications of Higher Order Equations. Exercises 5.2, page 241
Problem number: 5.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _missing_x]]`

$$\boxed{x'' + 4x' + 13x = 0}$$

With initial conditions

$$[x(0) = 1, x'(0) = -1]$$

✓ Solution by Maple
Time used: 0.016 (sec). Leaf size: 21

```
dsolve([diff(x(t),t$2)+4*diff(x(t),t)+13*x(t)=0,x(0) = 1, D(x)(0) = -1],x(t), singsol=all)
```

$$x(t) = \frac{e^{-2t}(\sin(3t) + 3\cos(3t))}{3}$$

✓ Solution by Mathematica
Time used: 0.017 (sec). Leaf size: 25

```
DSolve[{x''[t]+4*x'[t]+13*x[t]==0,{x[0]==1,x'[0]==-1}},x[t],t,IncludeSingularSolutions -> Tr
```

$$x(t) \to \frac{1}{3}e^{-2t}(\sin(3t) + 3\cos(3t))$$

20.6 problem 6

Internal problem ID [14908]

Book: INTRODUCTORY DIFFERENTIAL EQUATIONS. Martha L. Abell, James P. Braselton. Fourth edition 2014. ElScAe. 2014
Section: Chapter 5. Applications of Higher Order Equations. Exercises 5.2, page 241
Problem number: 6.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _missing_x]]`

$$\boxed{x'' + 4x' + 20x = 0}$$

With initial conditions

$$[x(0) = 1, x'(0) = 2]$$

✓ Solution by Maple
Time used: 0.015 (sec). Leaf size: 18

```
dsolve([diff(x(t),t$2)+4*diff(x(t),t)+20*x(t)=0,x(0) = 1, D(x)(0) = 2],x(t), singsol=all)
```

$$x(t) = e^{-2t}(\sin(4t) + \cos(4t))$$

✓ Solution by Mathematica
Time used: 0.016 (sec). Leaf size: 20

```
DSolve[{x''[t]+4*x'[t]+20*x[t]==0,{x[0]==1,x'[0]==2}},x[t],t,IncludeSingularSolutions -> Tru
```

$$x(t) \to e^{-2t}(\sin(4t) + \cos(4t))$$

21 Chapter 5. Applications of Higher Order Equations. Exercises 5.3, page 249

21.1 problem 15

Internal problem ID [14909]

Book: INTRODUCTORY DIFFERENTIAL EQUATIONS. Martha L. Abell, James P. Braselton. Fourth edition 2014. ElScAe. 2014
Section: Chapter 5. Applications of Higher Order Equations. Exercises 5.3, page 249
Problem number: 15.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _linear, _nonhomogeneous]]`

$$\boxed{x'' + x = \begin{cases} 1 & 0 \le t < \pi \\ 0 & \pi \le t \end{cases}}$$

With initial conditions

$$[x(0) = 0, x'(0) = 0]$$

✓ Solution by Maple
Time used: 0.578 (sec). Leaf size: 25

```
dsolve([diff(x(t),t$2)+x(t)=piecewise(0<=t and t<Pi,1,t>=Pi,0),x(0) = 0, D(x)(0) = 0],x(t),
```

$$x(t) = \begin{cases} 0 & t < 0 \\ -\cos(t) + 1 & t < \pi \\ -2\cos(t) & \pi \le t \end{cases}$$

✓ Solution by Mathematica

Time used: 0.03 (sec). Leaf size: 29

```
DSolve[{x''[t]+x[t]==Piecewise[{{1,0<=t<Pi},{0,t>=Pi}}],{x[0]==0,x'[0]==0}},x[t],t,IncludeSi
```

$$x(t) \to \begin{cases} 0 & t \leq 0 \\ 1-\cos(t) & 0 < t \leq \pi \\ -2\cos(t) & \text{True} \end{cases}$$

21.2 problem 16

Internal problem ID [14910]

Book: INTRODUCTORY DIFFERENTIAL EQUATIONS. Martha L. Abell, James P. Braselton. Fourth edition 2014. ElScAe. 2014
Section: Chapter 5. Applications of Higher Order Equations. Exercises 5.3, page 249
Problem number: 16.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _linear, _nonhomogeneous]]`

$$\boxed{x'' + x = \begin{cases} \cos(t) & 0 \leq t < \pi \\ 0 & \pi \leq t \end{cases}}$$

With initial conditions

$$[x(0) = 0, x'(0) = 0]$$

✓ Solution by Maple
Time used: 0.61 (sec). Leaf size: 21

```
dsolve([diff(x(t),t$2)+x(t)=piecewise(0<=t and t<Pi,cos(t),t>=Pi,0),x(0) = 0, D(x)(0) = 0],x
```

$$x(t) = \frac{\sin(t)\left(\begin{cases} 0 & t < 0 \\ t & t < \pi \\ \pi & \pi \leq t \end{cases}\right)}{2}$$

✓ Solution by Mathematica

Time used: 0.052 (sec). Leaf size: 33

```
DSolve[{x''[t]+x[t]==Piecewise[{{Cos[t],0<=t<Pi},{0,t>=Pi}}],{x[0]==0,x'[0]==0}},x[t],t,Incl
```

$$x(t) \to \begin{cases} 0 & t \leq 0 \\ \frac{1}{2} t \sin(t) & 0 < t \leq \pi \\ \frac{1}{2} \pi \sin(t) & \text{True} \end{cases}$$

21.3 problem 17

Internal problem ID [14911]

Book: INTRODUCTORY DIFFERENTIAL EQUATIONS. Martha L. Abell, James P. Braselton. Fourth edition 2014. ElScAe. 2014
Section: Chapter 5. Applications of Higher Order Equations. Exercises 5.3, page 249
Problem number: 17.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _linear, _nonhomogeneous]]`

$$\boxed{x'' + x = \begin{cases} t & 0 \leq t < 1 \\ -t + 2 & 1 \leq t < 2 \\ 0 & 2 \leq t \end{cases}}$$

With initial conditions

$$[x(0) = 0, x'(0) = 0]$$

✓ Solution by Maple
Time used: 0.953 (sec). Leaf size: 56

```
dsolve([diff(x(t),t$2)+x(t)=piecewise(0<=t and t<1,t,t>=1 and t<2,2-t,t>=2,0),x(0) = 0, D(x)
```

$$x(t) = \begin{cases} 0 & t < 0 \\ t - \sin(t) & t < 1 \\ 2\sin(t-1) - \sin(t) - t + 2 & t < 2 \\ 2\sin(t-1) - \sin(t) - \sin(t-2) & 2 \leq t \end{cases}$$

✓ Solution by Mathematica

Time used: 0.049 (sec). Leaf size: 63

```
DSolve[{x''[t]+x[t]==Piecewise[{{t,0<=t<1},{2-t,1<=t<2},{0,t>=2}}],{x[0]==0,x'[0]==0}},x[t],
```

$$x(t) \to \begin{cases} 0 & t \leq 0 \\ t - \sin(t) & 0 < t \leq 1 \\ -t - 2\sin(1-t) - \sin(t) + 2 & 1 < t \leq 2 \\ -4\sin^2\left(\frac{1}{2}\right)\sin(1-t) & \text{True} \end{cases}$$

21.4 problem 18

Internal problem ID [14912]

Book: INTRODUCTORY DIFFERENTIAL EQUATIONS. Martha L. Abell, James P. Braselton. Fourth edition 2014. ElScAe. 2014
Section: Chapter 5. Applications of Higher Order Equations. Exercises 5.3, page 249
Problem number: 18.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _linear, _nonhomogeneous]]`

$$\boxed{x'' + 4x' + 13x = \begin{cases} 1 & 0 \leq t < \pi \\ 1 - t & \pi \leq t < 2\pi \\ 0 & 2\pi \leq t \end{cases}}$$

With initial conditions

$$[x(0) = 0, x'(0) = 0]$$

✓ Solution by Maple
Time used: 2.516 (sec). Leaf size: 180

```
dsolve([diff(x(t),t$2)+4*diff(x(t),t)+13*x(t)=piecewise(0<=t and t<Pi,1,t>=Pi and t<2*Pi,1-t
```

$$x(t) = -\frac{\left(\begin{cases} 0 \\ -3 + (3\cos(3t) + 2\sin(3t))\,\mathrm{e}^{-2t} \\ \frac{((39\pi-12)\cos(3t)+\sin(3t)(26\pi+5))\mathrm{e}^{-2t+2\pi}}{13} + 3\,\mathrm{e}^{-2t}\cos(3t) + 2\,\mathrm{e}^{-2t}\sin(3t) + 3t - \frac{51}{13} \\ \frac{((78\pi-51)\cos(3t)+\sin(3t)(52\pi-21))\mathrm{e}^{4\pi-2t}}{13} + \frac{((39\pi-12)\cos(3t)+\sin(3t)(26\pi+5))\mathrm{e}^{-2t+2\pi}}{13} + 3\,\mathrm{e}^{-2t}\left(\cos(3t) + \frac{2\sin(3t)}{3}\right) \end{cases}\right)}{39}$$

✓ Solution by Mathematica

Time used: 0.042 (sec). Leaf size: 60

```
DSolve[{x''[t]+x[t]==Piecewise[{{1,0<=t<Pi},{1-t,Pi<=t<2*Pi},{0,t>=2*Pi}}],{x[0]==0,x'[0]==0
```

$$x(t) \to \begin{cases} 0 & t \leq 0 \\ 1 - \cos(t) & 0 < t \leq \pi \\ -t - (1+\pi)\cos(t) - \sin(t) + 1 & \pi < t \leq 2\pi \\ -3\pi\cos(t) - 2\sin(t) & \text{True} \end{cases}$$

21.5 problem 21 (a)

Internal problem ID [14913]

Book: INTRODUCTORY DIFFERENTIAL EQUATIONS. Martha L. Abell, James P. Braselton. Fourth edition 2014. ElScAe. 2014
Section: Chapter 5. Applications of Higher Order Equations. Exercises 5.3, page 249
Problem number: 21 (a).
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _linear, _nonhomogeneous]]`

$$\boxed{x'' + x = \cos(t)}$$

With initial conditions

$$[x(0) = 0, x'(0) = 0]$$

✓ Solution by Maple
Time used: 0.0 (sec). Leaf size: 9

```
dsolve([diff(x(t),t$2)+x(t)=cos(t),x(0) = 0, D(x)(0) = 0],x(t), singsol=all)
```

$$x(t) = \frac{\sin(t)\, t}{2}$$

✓ Solution by Mathematica
Time used: 0.02 (sec). Leaf size: 12

```
DSolve[{x''[t]+x[t]==Cos[t],{x[0]==0,x'[0]==0}},x[t],t,IncludeSingularSolutions -> True]
```

$$x(t) \to \frac{1}{2} t \sin(t)$$

21.6 problem 21 (b)

Internal problem ID [14914]

Book: INTRODUCTORY DIFFERENTIAL EQUATIONS. Martha L. Abell, James P. Braselton. Fourth edition 2014. ElScAe. 2014
Section: Chapter 5. Applications of Higher Order Equations. Exercises 5.3, page 249
Problem number: 21 (b).
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _linear, _nonhomogeneous]]`

$$\boxed{x'' + x = \cos(t)}$$

With initial conditions

$$[x(0) = 0, x'(0) = 1]$$

✓ Solution by Maple
Time used: 0.0 (sec). Leaf size: 12

```
dsolve([diff(x(t),t$2)+x(t)=cos(t),x(0) = 0, D(x)(0) = 1],x(t), singsol=all)
```

$$x(t) = \sin(t)\left(1 + \frac{t}{2}\right)$$

✓ Solution by Mathematica
Time used: 0.021 (sec). Leaf size: 14

```
DSolve[{x''[t]+x[t]==Cos[t],{x[0]==0,x'[0]==1}},x[t],t,IncludeSingularSolutions -> True]
```

$$x(t) \to \frac{1}{2}(t+2)\sin(t)$$

21.7 problem 22 (a)

Internal problem ID [14915]

Book: INTRODUCTORY DIFFERENTIAL EQUATIONS. Martha L. Abell, James P. Braselton. Fourth edition 2014. ElScAe. 2014
Section: Chapter 5. Applications of Higher Order Equations. Exercises 5.3, page 249
Problem number: 22 (a).
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _linear, _nonhomogeneous]]`

$$\boxed{x'' + x = \cos\left(\frac{9t}{10}\right)}$$

With initial conditions

$$[x(0) = 0, x'(0) = 1]$$

✓ Solution by Maple

Time used: 0.062 (sec). Leaf size: 17

```
dsolve([diff(x(t),t$2)+x(t)=cos(9/10*t),x(0) = 0, D(x)(0) = 1],x(t), singsol=all)
```

$$x(t) = \sin(t) - \frac{100\cos(t)}{19} + \frac{100\cos\left(\frac{9t}{10}\right)}{19}$$

✓ Solution by Mathematica

Time used: 0.16 (sec). Leaf size: 24

```
DSolve[{x''[t]+x[t]==Cos[9/10*t],{x[0]==0,x'[0]==1}},x[t],t,IncludeSingularSolutions -> True
```

$$x(t) \to \sin(t) + \frac{100}{19}\cos\left(\frac{9t}{10}\right) - \frac{100\cos(t)}{19}$$

21.8 problem 22 (a)

Internal problem ID [14916]

Book: INTRODUCTORY DIFFERENTIAL EQUATIONS. Martha L. Abell, James P. Braselton. Fourth edition 2014. ElScAe. 2014
Section: Chapter 5. Applications of Higher Order Equations. Exercises 5.3, page 249
Problem number: 22 (a).
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type **[[_2nd_order, _linear, _nonhomogeneous]]**

$$\boxed{x'' + x = \cos\left(\frac{7t}{10}\right)}$$

With initial conditions

$$[x(0) = 0, x'(0) = 1]$$

✓ Solution by Maple
Time used: 0.063 (sec). Leaf size: 17

```
dsolve([diff(x(t),t$2)+x(t)=cos(7/10*t),x(0) = 0, D(x)(0) = 1],x(t), singsol=all)
```

$$x(t) = \sin(t) - \frac{100\cos(t)}{51} + \frac{100\cos\left(\frac{7t}{10}\right)}{51}$$

✓ Solution by Mathematica
Time used: 0.16 (sec). Leaf size: 24

```
DSolve[{x''[t]+x[t]==Cos[7/10*t],{x[0]==0,x'[0]==1}},x[t],t,IncludeSingularSolutions -> True
```

$$x(t) \to \sin(t) + \frac{100}{51}\cos\left(\frac{7t}{10}\right) - \frac{100\cos(t)}{51}$$

21.9 problem 24

Internal problem ID [14917]

Book: INTRODUCTORY DIFFERENTIAL EQUATIONS. Martha L. Abell, James P. Braselton. Fourth edition 2014. ElScAe. 2014
Section: Chapter 5. Applications of Higher Order Equations. Exercises 5.3, page 249
Problem number: 24.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _linear, _nonhomogeneous]]`

$$\boxed{x'' + \frac{x'}{10} + x = 3\cos(2t)}$$

With initial conditions

$$[x(0) = 0, x'(0) = 0]$$

✓ Solution by Maple
Time used: 0.046 (sec). Leaf size: 46

```
dsolve([diff(x(t),t$2)+1/10*diff(x(t),t)+x(t)=3*cos(2*t),x(0) = 0, D(x)(0) = 0],x(t), singso
```

$$x(t) = -\frac{125\,\mathrm{e}^{-\frac{t}{20}}\sqrt{399}\,\sin\left(\frac{\sqrt{399}\,t}{20}\right)}{30058} + \frac{225\,\mathrm{e}^{-\frac{t}{20}}\cos\left(\frac{\sqrt{399}\,t}{20}\right)}{226} - \frac{225\cos(2t)}{226} + \frac{15\sin(2t)}{226}$$

✓ Solution by Mathematica
Time used: 0.037 (sec). Leaf size: 74

```
DSolve[{x''[t]+1/10*x'[t]+x[t]==3*Cos[2*t],{x[0]==0,x'[0]==0}},x[t],t,IncludeSingularSolutio
```

$$x(t) \to -\frac{5e^{-t/20}\left(-399e^{t/20}\sin(2t) + 25\sqrt{399}\sin\left(\frac{\sqrt{399}t}{20}\right) + 5985e^{t/20}\cos(2t) - 5985\cos\left(\frac{\sqrt{399}t}{20}\right)\right)}{30058}$$

22 Chapter 6. Systems of Differential Equations. Exercises 6.1, page 282

22.1 problem 1

Internal problem ID [14918]

Book: INTRODUCTORY DIFFERENTIAL EQUATIONS. Martha L. Abell, James P. Braselton. Fourth edition 2014. ElScAe. 2014
Section: Chapter 6. Systems of Differential Equations. Exercises 6.1, page 282
Problem number: 1.
ODE order: 1.
ODE degree: 1.

Solve

$$
\begin{aligned}
x' &= 6 \\
y'(t) &= \cos(t)
\end{aligned}
$$

✓ Solution by Maple

Time used: 0.031 (sec). Leaf size: 17

```
dsolve([diff(x(t),t)=6,diff(y(t),t)=cos(t)],singsol=all)
```

$$
\begin{aligned}
x(t) &= 6t + c_2 \\
y(t) &= \sin(t) + c_1
\end{aligned}
$$

✓ Solution by Mathematica

Time used: 0.008 (sec). Leaf size: 19

```
DSolve[{x'[t]==6,y'[t]==Cos[t]},{x[t],y[t]},t,IncludeSingularSolutions -> True]
```

$$
\begin{aligned}
x(t) &\to 6t + c_1 \\
y(t) &\to \sin(t) + c_2
\end{aligned}
$$

22.2 problem 2

Internal problem ID [14919]

Book: INTRODUCTORY DIFFERENTIAL EQUATIONS. Martha L. Abell, James P. Braselton. Fourth edition 2014. ElScAe. 2014
Section: Chapter 6. Systems of Differential Equations. Exercises 6.1, page 282
Problem number: 2.
ODE order: 1.
ODE degree: 1.

Solve

$$
\begin{aligned}
x' &= x \\
y'(t) &= 1
\end{aligned}
$$

✓ Solution by Maple
Time used: 0.015 (sec). Leaf size: 15

```
dsolve([diff(x(t),t)=x(t),diff(y(t),t)=1],singsol=all)
```

$$
\begin{aligned}
x(t) &= c_1 e^t \\
y(t) &= c_2 + t
\end{aligned}
$$

✓ Solution by Mathematica
Time used: 0.019 (sec). Leaf size: 30

```
DSolve[{x'[t]==x[t],y'[t]==1},{x[t],y[t]},t,IncludeSingularSolutions -> True]
```

$$
\begin{aligned}
x(t) &\to c_1 e^t \\
y(t) &\to t + c_2 \\
x(t) &\to 0 \\
y(t) &\to t + c_2
\end{aligned}
$$

22.3 problem 3

Internal problem ID [14920]

Book: INTRODUCTORY DIFFERENTIAL EQUATIONS. Martha L. Abell, James P. Braselton. Fourth edition 2014. ElScAe. 2014
Section: Chapter 6. Systems of Differential Equations. Exercises 6.1, page 282
Problem number: 3.
ODE order: 1.
ODE degree: 1.

Solve

$$
\begin{aligned}
x' &= 0 \\
y'(t) &= -2y(t)
\end{aligned}
$$

✓ Solution by Maple
Time used: 0.015 (sec). Leaf size: 15

```
dsolve([diff(x(t),t)=0,diff(y(t),t)=-2*y(t)],singsol=all)
```

$$
\begin{aligned}
x(t) &= c_2 \\
y(t) &= c_1 \mathrm{e}^{-2t}
\end{aligned}
$$

✓ Solution by Mathematica
Time used: 0.021 (sec). Leaf size: 28

```
DSolve[{x'[t]==0,y'[t]==-2*y[t]},{x[t],y[t]},t,IncludeSingularSolutions -> True]
```

$$
\begin{aligned}
x(t) &\to c_1 \\
y(t) &\to c_2 e^{-2t} \\
x(t) &\to c_1 \\
y(t) &\to 0
\end{aligned}
$$

22.4 problem 4

Internal problem ID [14921]

Book: INTRODUCTORY DIFFERENTIAL EQUATIONS. Martha L. Abell, James P. Braselton. Fourth edition 2014. ElScAe. 2014
Section: Chapter 6. Systems of Differential Equations. Exercises 6.1, page 282
Problem number: 4.
ODE order: 1.
ODE degree: 1.

Solve

$$
\begin{aligned}
x' &= x^2 \\
y'(t) &= \mathrm{e}^t
\end{aligned}
$$

✓ Solution by Maple

Time used: 0.016 (sec). Leaf size: 21

```
dsolve([diff(x(t),t)=x(t)^2,diff(y(t),t)=exp(t)],singsol=all)
```

$$
\left\{x(t) = \frac{1}{-t + c_2}\right\}
$$
$$
\{y(t) = \mathrm{e}^t + c_1\}
$$

✓ Solution by Mathematica

Time used: 0.107 (sec). Leaf size: 36

```
DSolve[{x'[t]==x[t]^2,y'[t]==Exp[t]},{x[t],y[t]},t,IncludeSingularSolutions -> True]
```

$$
\begin{aligned}
x(t) &\to -\frac{1}{t + c_1} \\
y(t) &\to e^t + c_2 \\
x(t) &\to 0 \\
y(t) &\to e^t + c_2
\end{aligned}
$$

22.5 problem 5

Internal problem ID [14922]

Book: INTRODUCTORY DIFFERENTIAL EQUATIONS. Martha L. Abell, James P. Braselton. Fourth edition 2014. ElScAe. 2014
Section: Chapter 6. Systems of Differential Equations. Exercises 6.1, page 282
Problem number: 5.
ODE order: 1.
ODE degree: 1.

Solve

$$x_1'(t) = -3x_1(t)$$
$$x_2'(t) = 1$$

With initial conditions

$$[x_1(0) = -1, x_2(0) = 1]$$

✓ Solution by Maple
Time used: 0.031 (sec). Leaf size: 17

```
dsolve([diff(x__1(t),t) = -3*x__1(t), diff(x__2(t),t) = 1, x__1(0) = -1, x__2(0) = 1], sings
```

$$x_1(t) = -\mathrm{e}^{-3t}$$
$$x_2(t) = t + 1$$

✓ Solution by Mathematica
Time used: 0.045 (sec). Leaf size: 18

```
DSolve[{x1'[t]==-3*x1[t],x2'[t]==1},{x1[0]==-1,x2[0]==1},{x1[t],x2[t]},t,IncludeSingularSolu
```

$$\text{x1}(t) \to -e^{-3t}$$
$$\text{x2}(t) \to t + 1$$

22.6 problem 6

Internal problem ID [14923]

Book: INTRODUCTORY DIFFERENTIAL EQUATIONS. Martha L. Abell, James P. Braselton. Fourth edition 2014. ElScAe. 2014
Section: Chapter 6. Systems of Differential Equations. Exercises 6.1, page 282
Problem number: 6.
ODE order: 1.
ODE degree: 1.

Solve

$$
\begin{aligned}
x_1'(t) &= -x_1(t) + 1 \\
x_2'(t) &= x_2(t)
\end{aligned}
$$

With initial conditions

$$
[x_1(0) = 0, x_2(0) = 1]
$$

✓ Solution by Maple
Time used: 0.016 (sec). Leaf size: 18

```
dsolve([diff(x__1(t),t) = -x__1(t)+1, diff(x__2(t),t) = x__2(t), x__1(0) = 0, x__2(0) = 1],
```

$$
\begin{aligned}
x_1(t) &= 1 - \mathrm{e}^{-t} \\
x_2(t) &= \mathrm{e}^{t}
\end{aligned}
$$

✓ Solution by Mathematica
Time used: 0.067 (sec). Leaf size: 12

```
DSolve[{x1'[t]==-x1[t],x2'[t]==x2[t]},{x1[0]==0,x2[0]==1},{x1[t],x2[t]},t,IncludeSingularSol
```

$$
\begin{aligned}
\mathrm{x1}(t) &\to 0 \\
\mathrm{x2}(t) &\to e^{t}
\end{aligned}
$$

22.7 problem 7

Internal problem ID [14924]

Book: INTRODUCTORY DIFFERENTIAL EQUATIONS. Martha L. Abell, James P. Braselton. Fourth edition 2014. ElScAe. 2014
Section: Chapter 6. Systems of Differential Equations. Exercises 6.1, page 282
Problem number: 7.
ODE order: 1.
ODE degree: 1.

Solve

$$x' = -3x + 6y(t)$$
$$y'(t) = 4x - y(t)$$

✓ Solution by Maple
Time used: 0.031 (sec). Leaf size: 35

```
dsolve([diff(x(t),t)=-3*x(t)+6*y(t),diff(y(t),t)=4*x(t)-y(t)],singsol=all)
```

$$x(t) = c_1 e^{-7t} + c_2 e^{3t}$$
$$y(t) = -\frac{2c_1 e^{-7t}}{3} + c_2 e^{3t}$$

✓ Solution by Mathematica
Time used: 0.017 (sec). Leaf size: 74

```
DSolve[{x'[t]==-3*x[t]+6*y[t],y'[t]==4*x[t]-y[t]},{x[t],y[t]},t,IncludeSingularSolutions ->
```

$$x(t) \to \frac{1}{5}e^{-7t}\left(c_1\left(2e^{10t}+3\right)+3c_2\left(e^{10t}-1\right)\right)$$
$$y(t) \to \frac{1}{5}e^{-7t}\left(2c_1\left(e^{10t}-1\right)+c_2\left(3e^{10t}+2\right)\right)$$

22.8 problem 8

Internal problem ID [14925]

Book: INTRODUCTORY DIFFERENTIAL EQUATIONS. Martha L. Abell, James P. Braselton. Fourth edition 2014. ElScAe. 2014
Section: Chapter 6. Systems of Differential Equations. Exercises 6.1, page 282
Problem number: 8.
ODE order: 1.
ODE degree: 1.

Solve

$$
\begin{aligned}
x' &= 8x - y(t) \\
y'(t) &= x + 6y(t)
\end{aligned}
$$

✓ Solution by Maple
Time used: 0.016 (sec). Leaf size: 31

```
dsolve([diff(x(t),t)=8*x(t)-y(t),diff(y(t),t)=x(t)+6*y(t)],singsol=all)
```

$$
\begin{aligned}
x(t) &= \mathrm{e}^{7t}(c_2 t + c_1) \\
y(t) &= \mathrm{e}^{7t}(c_2 t + c_1 - c_2)
\end{aligned}
$$

✓ Solution by Mathematica
Time used: 0.002 (sec). Leaf size: 44

```
DSolve[{x'[t]==8*x[t]-y[t],y'[t]==x[t]+6*y[t]},{x[t],y[t]},t,IncludeSingularSolutions -> Tru
```

$$
\begin{aligned}
x(t) &\to e^{7t}(c_1(t+1) - c_2 t) \\
y(t) &\to e^{7t}((c_1 - c_2)t + c_2)
\end{aligned}
$$

22.9 problem 9

Internal problem ID [14926]

Book: INTRODUCTORY DIFFERENTIAL EQUATIONS. Martha L. Abell, James P. Braselton. Fourth edition 2014. ElScAe. 2014
Section: Chapter 6. Systems of Differential Equations. Exercises 6.1, page 282
Problem number: 9.
ODE order: 1.
ODE degree: 1.

Solve

$$x' = -x - 2y(t)$$
$$y'(t) = x + y(t)$$

✓ Solution by Maple
Time used: 0.015 (sec). Leaf size: 38

```
dsolve([diff(x(t),t)=-x(t)-2*y(t),diff(y(t),t)=x(t)+y(t)],singsol=all)
```

$$x(t) = c_1 \sin(t) + c_2 \cos(t)$$
$$y(t) = -\frac{c_1 \cos(t)}{2} + \frac{c_2 \sin(t)}{2} - \frac{c_1 \sin(t)}{2} - \frac{c_2 \cos(t)}{2}$$

✓ Solution by Mathematica
Time used: 0.004 (sec). Leaf size: 39

```
DSolve[{x'[t]==-x[t]-2*y[t],y'[t]==x[t]+y[t]},{x[t],y[t]},t,IncludeSingularSolutions -> True
```

$$x(t) \to c_1 \cos(t) - (c_1 + 2c_2) \sin(t)$$
$$y(t) \to c_2 \cos(t) + (c_1 + c_2) \sin(t)$$

22.10 problem 10

Internal problem ID [14927]

Book: INTRODUCTORY DIFFERENTIAL EQUATIONS. Martha L. Abell, James P. Braselton. Fourth edition 2014. ElScAe. 2014
Section: Chapter 6. Systems of Differential Equations. Exercises 6.1, page 282
Problem number: 10.
ODE order: 1.
ODE degree: 1.

Solve

$$x' = 4x + 2y(t)$$
$$y'(t) = -x + 2y(t)$$

✓ Solution by Maple
Time used: 0.031 (sec). Leaf size: 46

```
dsolve([diff(x(t),t)=4*x(t)+2*y(t),diff(y(t),t)=-x(t)+2*y(t)],singsol=all)
```

$$x(t) = e^{3t}(c_1 \sin(t) + c_2 \cos(t))$$
$$y(t) = -\frac{e^{3t}(c_1 \sin(t) + c_2 \sin(t) - c_1 \cos(t) + c_2 \cos(t))}{2}$$

✓ Solution by Mathematica
Time used: 0.005 (sec). Leaf size: 51

```
DSolve[{x'[t]==4*x[t]+2*y[t],y'[t]==-x[t]+2*y[t]},{x[t],y[t]},t,IncludeSingularSolutions ->
```

$$x(t) \to e^{3t}(c_1 \cos(t) + (c_1 + 2c_2) \sin(t))$$
$$y(t) \to e^{3t}(c_2 \cos(t) - (c_1 + c_2) \sin(t))$$

22.11 problem 11

Internal problem ID [14928]

Book: INTRODUCTORY DIFFERENTIAL EQUATIONS. Martha L. Abell, James P. Braselton. Fourth edition 2014. ElScAe. 2014
Section: Chapter 6. Systems of Differential Equations. Exercises 6.1, page 282
Problem number: 11.
ODE order: 1.
ODE degree: 1.

Solve

$$
\begin{aligned}
x' &= y(t) \\
y'(t) &= 1 - x
\end{aligned}
$$

✓ Solution by Maple

Time used: 0.016 (sec). Leaf size: 28

```
dsolve([diff(x(t),t)=y(t),diff(y(t),t)=-x(t)+1],singsol=all)
```

$$
\begin{aligned}
x(t) &= c_2 \sin(t) + c_1 \cos(t) + 1 \\
y(t) &= c_2 \cos(t) - c_1 \sin(t)
\end{aligned}
$$

✓ Solution by Mathematica

Time used: 0.007 (sec). Leaf size: 32

```
DSolve[{x'[t]==y[t],y'[t]==-x[t]+1},{x[t],y[t]},t,IncludeSingularSolutions -> True]
```

$$
\begin{aligned}
x(t) &\to c_1 \cos(t) + c_2 \sin(t) + 1 \\
y(t) &\to c_2 \cos(t) - c_1 \sin(t)
\end{aligned}
$$

22.12 problem 12

Internal problem ID [14929]

Book: INTRODUCTORY DIFFERENTIAL EQUATIONS. Martha L. Abell, James P. Braselton. Fourth edition 2014. ElScAe. 2014
Section: Chapter 6. Systems of Differential Equations. Exercises 6.1, page 282
Problem number: 12.
ODE order: 1.
ODE degree: 1.

Solve

$$
\begin{aligned}
x' &= y(t) \\
y'(t) &= -x + 2\cos(t)\sin(t)
\end{aligned}
$$

✓ Solution by Maple
Time used: 0.125 (sec). Leaf size: 39

```
dsolve([diff(x(t),t)=y(t),diff(y(t),t)=-x(t)+sin(2*t)],singsol=all)
```

$$
\begin{aligned}
x(t) &= c_2 \sin(t) + c_1 \cos(t) - \frac{\sin(2t)}{3} \\
y(t) &= c_2 \cos(t) - c_1 \sin(t) - \frac{2\cos(2t)}{3}
\end{aligned}
$$

✓ Solution by Mathematica
Time used: 0.022 (sec). Leaf size: 46

```
DSolve[{x'[t]==y[t],y'[t]==-x[t]+Sin[2*t]},{x[t],y[t]},t,IncludeSingularSolutions -> True]
```

$$
\begin{aligned}
x(t) &\to c_2 \sin(t) + \cos(t)\left(-\frac{2\sin(t)}{3} + c_1\right) \\
y(t) &\to -\frac{2}{3}\cos(2t) + c_2\cos(t) - c_1\sin(t)
\end{aligned}
$$

22.13 problem 13

Internal problem ID [14930]

Book: INTRODUCTORY DIFFERENTIAL EQUATIONS. Martha L. Abell, James P. Braselton. Fourth edition 2014. ElScAe. 2014
Section: Chapter 6. Systems of Differential Equations. Exercises 6.1, page 282
Problem number: 13.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _missing_x]]`

$$\boxed{x'' - 3x' + 4x = 0}$$

✓ Solution by Maple
Time used: 0.0 (sec). Leaf size: 28

```
dsolve(diff(x(t),t$2)-3*diff(x(t),t)+4*x(t)=0,x(t), singsol=all)
```

$$x(t) = \mathrm{e}^{\frac{3t}{2}} \left(c_1 \sin\left(\frac{\sqrt{7}\, t}{2}\right) + c_2 \cos\left(\frac{\sqrt{7}\, t}{2}\right)\right)$$

✓ Solution by Mathematica
Time used: 0.023 (sec). Leaf size: 42

```
DSolve[x''[t]-3*x'[t]+4*x[t]==0,x[t],t,IncludeSingularSolutions -> True]
```

$$x(t) \to e^{3t/2} \left(c_2 \cos\left(\frac{\sqrt{7}t}{2}\right) + c_1 \sin\left(\frac{\sqrt{7}t}{2}\right)\right)$$

22.14 problem 14

Internal problem ID [14931]

Book: INTRODUCTORY DIFFERENTIAL EQUATIONS. Martha L. Abell, James P. Braselton. Fourth edition 2014. ElScAe. 2014
Section: Chapter 6. Systems of Differential Equations. Exercises 6.1, page 282
Problem number: 14.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _missing_x]]`

$$\boxed{x'' + 6x' + 9x = 0}$$

✓ Solution by Maple
Time used: 0.0 (sec). Leaf size: 14

```
dsolve(diff(x(t),t$2)+6*diff(x(t),t)+9*x(t)=0,x(t), singsol=all)
```

$$x(t) = \mathrm{e}^{-3t}(tc_2 + c_1)$$

✓ Solution by Mathematica
Time used: 0.013 (sec). Leaf size: 18

```
DSolve[x''[t]+6*x'[t]+9*x[t]==0,x[t],t,IncludeSingularSolutions -> True]
```

$$x(t) \to e^{-3t}(c_2 t + c_1)$$

22.15 problem 15

Internal problem ID [14932]

Book: INTRODUCTORY DIFFERENTIAL EQUATIONS. Martha L. Abell, James P. Braselton. Fourth edition 2014. ElScAe. 2014
Section: Chapter 6. Systems of Differential Equations. Exercises 6.1, page 282
Problem number: 15.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _linear, _nonhomogeneous]]`

$$\boxed{x'' + 16x = \sin(t)\, t}$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 26

```
dsolve(diff(x(t),t$2)+16*x(t)=t*sin(t),x(t), singsol=all)
```

$$x(t) = c_2 \sin(4t) + c_1 \cos(4t) - \frac{2\cos(t)}{225} + \frac{\sin(t)\, t}{15}$$

✓ Solution by Mathematica

Time used: 0.021 (sec). Leaf size: 33

```
DSolve[x''[t]+16*x[t]==t*Sin[t],x[t],t,IncludeSingularSolutions -> True]
```

$$x(t) \to \frac{1}{15} t \sin(t) - \frac{2\cos(t)}{225} + c_1 \cos(4t) + c_2 \sin(4t)$$

22.16 problem 16

Internal problem ID [14933]

Book: INTRODUCTORY DIFFERENTIAL EQUATIONS. Martha L. Abell, James P. Braselton. Fourth edition 2014. ElScAe. 2014
Section: Chapter 6. Systems of Differential Equations. Exercises 6.1, page 282
Problem number: 16.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$\boxed{x'' + x = \mathrm{e}^t}$$

✓ Solution by Maple
Time used: 0.0 (sec). Leaf size: 17

```
dsolve(diff(x(t),t$2)+x(t)=exp(t),x(t), singsol=all)
```

$$x(t) = \sin(t)\, c_2 + \cos(t)\, c_1 + \frac{\mathrm{e}^t}{2}$$

✓ Solution by Mathematica
Time used: 0.055 (sec). Leaf size: 23

```
DSolve[x''[t]+x[t]==Exp[t],x[t],t,IncludeSingularSolutions -> True]
```

$$x(t) \to \frac{e^t}{2} + c_1 \cos(t) + c_2 \sin(t)$$