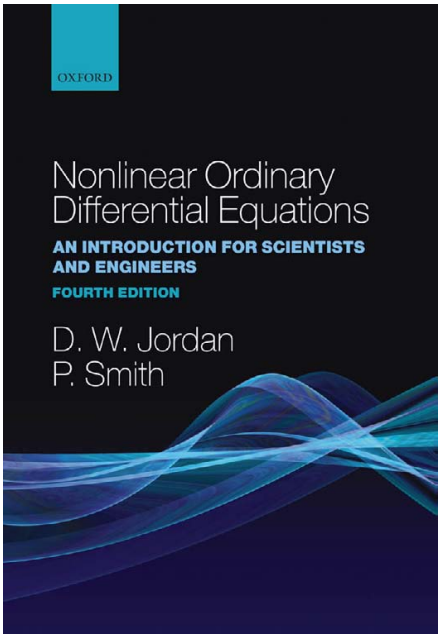
A Solution Manual For

Nonlinear Ordinary Differential Equations by D.W.Jordna and P.Smith. 4th edition 1999. Oxford Univ. Press. NY



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1.1 problem 2.1 (i)

Internal problem ID [12555]

Book: Nonlinear Ordinary Differential Equations by D.W.Jordna and P.Smith. 4th edition 1999. Oxford Univ. Press. NY
Section: Chapter 2. Plane autonomous systems and linearization. Problems page 79
Problem number: 2.1 (i).
ODE order: 1.
ODE degree: 1.

Solve

$$x'(t) = x(t) - 5y(t)$$
$$y'(t) = x(t) - y(t)$$

✓ Solution by Maple
Time used: 0.015 (sec). Leaf size: 50

```
dsolve([diff(x(t),t)=x(t)-5*y(t),diff(y(t),t)=x(t)-y(t)],singsol=all)
```

$$x(t) = c_1 \sin(2t) + c_2 \cos(2t)$$
$$y(t) = -\frac{2c_1 \cos(2t)}{5} + \frac{2c_2 \sin(2t)}{5} + \frac{c_1 \sin(2t)}{5} + \frac{c_2 \cos(2t)}{5}$$

✓ Solution by Mathematica
Time used: 0.009 (sec). Leaf size: 48

```
DSolve[{x'[t]==x[t]-5*y[t],y'[t]==x[t]-y[t]},{x[t],y[t]},t,IncludeSingularSolutions -> True]
```

$$x(t) \to c_1 \cos(2t) + (c_1 - 5c_2) \sin(t) \cos(t)$$
$$y(t) \to c_2 \cos(2t) + (c_1 - c_2) \sin(t) \cos(t)$$

1.2 problem 2.1 (ii)

Internal problem ID [12556]

Book: Nonlinear Ordinary Differential Equations by D.W.Jordna and P.Smith. 4th edition 1999. Oxford Univ. Press. NY
Section: Chapter 2. Plane autonomous systems and linearization. Problems page 79
Problem number: 2.1 (ii).
ODE order: 1.
ODE degree: 1.

Solve

$$x'(t) = x(t) + y(t)$$
$$y'(t) = x(t) - 2y(t)$$

✓ Solution by Maple
Time used: 0.031 (sec). Leaf size: 86

```
dsolve([diff(x(t),t)=x(t)+y(t),diff(y(t),t)=x(t)-2*y(t)],singsol=all)
```

$$x(t) = c_1 \mathrm{e}^{\frac{\left(-1+\sqrt{13}\right)t}{2}} + c_2 \mathrm{e}^{-\frac{\left(1+\sqrt{13}\right)t}{2}}$$
$$y(t) = \frac{c_1 \mathrm{e}^{\frac{\left(-1+\sqrt{13}\right)t}{2}}\sqrt{13}}{2} - \frac{c_2 \mathrm{e}^{-\frac{\left(1+\sqrt{13}\right)t}{2}}\sqrt{13}}{2} - \frac{3c_1 \mathrm{e}^{\frac{\left(-1+\sqrt{13}\right)t}{2}}}{2} - \frac{3c_2 \mathrm{e}^{-\frac{\left(1+\sqrt{13}\right)t}{2}}}{2}$$

✓ Solution by Mathematica
Time used: 0.017 (sec). Leaf size: 149

```
DSolve[{x'[t]==x[t]+y[t],y'[t]==x[t]-2*y[t]},{x[t],y[t]},t,IncludeSingularSolutions -> True]
```

$$x(t) \to \frac{1}{26} e^{-\frac{1}{2}\left(1+\sqrt{13}\right)t}\left(c_1\left(\left(13+3\sqrt{13}\right)e^{\sqrt{13}t}+13-3\sqrt{13}\right)+2\sqrt{13}c_2\left(e^{\sqrt{13}t}-1\right)\right)$$
$$y(t) \to \frac{1}{26} e^{-\frac{1}{2}\left(1+\sqrt{13}\right)t}\left(2\sqrt{13}c_1\left(e^{\sqrt{13}t}-1\right)-c_2\left(\left(3\sqrt{13}-13\right)e^{\sqrt{13}t}-13-3\sqrt{13}\right)\right)$$

1.3 problem 2.1 (iii)

Internal problem ID [12557]

Book: Nonlinear Ordinary Differential Equations by D.W.Jordna and P.Smith. 4th edition 1999. Oxford Univ. Press. NY
Section: Chapter 2. Plane autonomous systems and linearization. Problems page 79
Problem number: 2.1 (iii).
ODE order: 1.
ODE degree: 1.

Solve

$$x'(t) = -4x(t) + 2y(t)$$
$$y'(t) = 3x(t) - 2y(t)$$

✓ Solution by Maple
Time used: 0.015 (sec). Leaf size: 83

```
dsolve([diff(x(t),t)=-4*x(t)+2*y(t),diff(y(t),t)=3*x(t)-2*y(t)],singsol=all)
```

$$x(t) = c_1 \mathrm{e}^{\left(-3+\sqrt{7}\right)t} + c_2 \mathrm{e}^{-\left(\sqrt{7}+3\right)t}$$
$$y(t) = \frac{c_1 \mathrm{e}^{\left(-3+\sqrt{7}\right)t}\sqrt{7}}{2} - \frac{c_2 \mathrm{e}^{-\left(\sqrt{7}+3\right)t}\sqrt{7}}{2} + \frac{c_1 \mathrm{e}^{\left(-3+\sqrt{7}\right)t}}{2} + \frac{c_2 \mathrm{e}^{-\left(\sqrt{7}+3\right)t}}{2}$$

✓ Solution by Mathematica
Time used: 0.015 (sec). Leaf size: 143

```
DSolve[{x'[t]==-4*x[t]+2*y[t],y'[t]==3*x[t]-2*y[t]},{x[t],y[t]},t,IncludeSingularSolutions -
```

$$x(t) \to \frac{1}{14} e^{-\left(\left(3+\sqrt{7}\right)t\right)} \left(c_1 \left(-\left(\sqrt{7}-7\right) e^{2\sqrt{7}t} + 7 + \sqrt{7}\right) + 2\sqrt{7} c_2 \left(e^{2\sqrt{7}t} - 1\right)\right)$$
$$y(t) \to \frac{1}{14} e^{-\left(\left(3+\sqrt{7}\right)t\right)} \left(3\sqrt{7} c_1 \left(e^{2\sqrt{7}t} - 1\right) + c_2 \left(\left(7+\sqrt{7}\right) e^{2\sqrt{7}t} + 7 - \sqrt{7}\right)\right)$$

1.4 problem 2.1 (iv)

Internal problem ID [12558]

Book: Nonlinear Ordinary Differential Equations by D.W.Jordna and P.Smith. 4th edition 1999. Oxford Univ. Press. NY
Section: Chapter 2. Plane autonomous systems and linearization. Problems page 79
Problem number: 2.1 (iv).
ODE order: 1.
ODE degree: 1.

Solve

$$x'(t) = x(t) + 2y(t)$$
$$y'(t) = 2x(t) + 2y(t)$$

✓ Solution by Maple
Time used: 0.031 (sec). Leaf size: 86

```
dsolve([diff(x(t),t)=x(t)+2*y(t),diff(y(t),t)=2*x(t)+2*y(t)],singsol=all)
```

$$x(t) = c_1 \mathrm{e}^{\frac{\left(3+\sqrt{17}\right)t}{2}} + c_2 \mathrm{e}^{-\frac{\left(-3+\sqrt{17}\right)t}{2}}$$
$$y(t) = \frac{c_1 \mathrm{e}^{\frac{\left(3+\sqrt{17}\right)t}{2}}\sqrt{17}}{4} - \frac{c_2 \mathrm{e}^{-\frac{\left(-3+\sqrt{17}\right)t}{2}}\sqrt{17}}{4} + \frac{c_1 \mathrm{e}^{\frac{\left(3+\sqrt{17}\right)t}{2}}}{4} + \frac{c_2 \mathrm{e}^{-\frac{\left(-3+\sqrt{17}\right)t}{2}}}{4}$$

✓ Solution by Mathematica
Time used: 0.017 (sec). Leaf size: 143

```
DSolve[{x'[t]==x[t]+2*y[t],y'[t]==2*x[t]+2*y[t]},{x[t],y[t]},t,IncludeSingularSolutions -> T
```

$$x(t) \to \frac{1}{34} e^{-\frac{1}{2}\left(\sqrt{17}-3\right)t}\left(c_1\left(-\left(\sqrt{17}-17\right)e^{\sqrt{17}t} + 17 + \sqrt{17}\right) + 4\sqrt{17}c_2\left(e^{\sqrt{17}t}-1\right)\right)$$
$$y(t) \to \frac{1}{34} e^{-\frac{1}{2}\left(\sqrt{17}-3\right)t}\left(4\sqrt{17}c_1\left(e^{\sqrt{17}t}-1\right) + c_2\left(\left(17+\sqrt{17}\right)e^{\sqrt{17}t} + 17 - \sqrt{17}\right)\right)$$

1.5 problem 2.1 (v)

Internal problem ID [12559]

Book: Nonlinear Ordinary Differential Equations by D.W.Jordna and P.Smith. 4th edition 1999. Oxford Univ. Press. NY
Section: Chapter 2. Plane autonomous systems and linearization. Problems page 79
Problem number: 2.1 (v).
ODE order: 1.
ODE degree: 1.

Solve

$$x'(t) = 4x(t) - 2y(t)$$
$$y'(t) = 3x(t) - y(t)$$

✓ Solution by Maple
Time used: 0.015 (sec). Leaf size: 31

```
dsolve([diff(x(t),t)=4*x(t)-2*y(t),diff(y(t),t)=3*x(t)-y(t)],singsol=all)
```

$$x(t) = c_1 e^t + c_2 e^{2t}$$
$$y(t) = \frac{3c_1 e^t}{2} + c_2 e^{2t}$$

✓ Solution by Mathematica
Time used: 0.004 (sec). Leaf size: 56

```
DSolve[{x'[t]==4*x[t]-2*y[t],y'[t]==3*x[t]-y[t]},{x[t],y[t]},t,IncludeSingularSolutions -> T
```

$$x(t) \to e^t\left(c_1\left(3e^t - 2\right) - 2c_2\left(e^t - 1\right)\right)$$
$$y(t) \to e^t\left(3c_1\left(e^t - 1\right) + c_2\left(3 - 2e^t\right)\right)$$

1.6 problem 2.1 (vi)

Internal problem ID [12560]

Book: Nonlinear Ordinary Differential Equations by D.W.Jordna and P.Smith. 4th edition 1999. Oxford Univ. Press. NY
Section: Chapter 2. Plane autonomous systems and linearization. Problems page 79
Problem number: 2.1 (vi).
ODE order: 1.
ODE degree: 1.

Solve

$$x'(t) = 2x(t) + y(t)$$
$$y'(t) = -x(t) + y(t)$$

✓ Solution by Maple
Time used: 0.031 (sec). Leaf size: 82

```
dsolve([diff(x(t),t)=2*x(t)+y(t),diff(y(t),t)=-x(t)+y(t)],singsol=all)
```

$$x(t) = \mathrm{e}^{\frac{3t}{2}} \left(\sin\left(\frac{\sqrt{3}\, t}{2} \right) c_1 + \cos\left(\frac{\sqrt{3}\, t}{2} \right) c_2 \right)$$
$$y(t) = -\frac{\mathrm{e}^{\frac{3t}{2}} \left(\sqrt{3} \sin\left(\frac{\sqrt{3}\, t}{2} \right) c_2 - \sqrt{3} \cos\left(\frac{\sqrt{3}\, t}{2} \right) c_1 + \sin\left(\frac{\sqrt{3}\, t}{2} \right) c_1 + \cos\left(\frac{\sqrt{3}\, t}{2} \right) c_2 \right)}{2}$$

✓ Solution by Mathematica
Time used: 0.029 (sec). Leaf size: 111

```
DSolve[{x'[t]==2*x[t]+y[t],y'[t]==-x[t]+y[t]},{x[t],y[t]},t,IncludeSingularSolutions -> True
```

$$x(t) \to \frac{1}{3} e^{3t/2} \left(3c_1 \cos\left(\frac{\sqrt{3}t}{2} \right) + \sqrt{3}(c_1 + 2c_2) \sin\left(\frac{\sqrt{3}t}{2} \right) \right)$$
$$y(t) \to \frac{1}{3} e^{3t/2} \left(3c_2 \cos\left(\frac{\sqrt{3}t}{2} \right) - \sqrt{3}(2c_1 + c_2) \sin\left(\frac{\sqrt{3}t}{2} \right) \right)$$

1.7 problem 2.2 (i)

Internal problem ID [12561]

Book: Nonlinear Ordinary Differential Equations by D.W.Jordna and P.Smith. 4th edition 1999. Oxford Univ. Press. NY
Section: Chapter 2. Plane autonomous systems and linearization. Problems page 79
Problem number: 2.2 (i).
ODE order: 1.
ODE degree: 1.

Solve

$$x'(t) = 3x(t) - y(t)$$
$$y'(t) = x(t) + y(t)$$

✓ Solution by Maple
Time used: 0.016 (sec). Leaf size: 31

```
dsolve([diff(x(t),t)=3*x(t)-y(t),diff(y(t),t)=x(t)+y(t)],singsol=all)
```

$$x(t) = \mathrm{e}^{2t}(c_2 t + c_1)$$
$$y(t) = \mathrm{e}^{2t}(c_2 t + c_1 - c_2)$$

✓ Solution by Mathematica
Time used: 0.004 (sec). Leaf size: 44

```
DSolve[{x'[t]==3*x[t]-y[t],y'[t]==x[t]+y[t]},{x[t],y[t]},t,IncludeSingularSolutions -> True]
```

$$x(t) \to e^{2t}(c_1(t+1) - c_2 t)$$
$$y(t) \to e^{2t}((c_1 - c_2)t + c_2)$$

1.8 problem 2.2 (ii)

Internal problem ID [12562]

Book: Nonlinear Ordinary Differential Equations by D.W.Jordna and P.Smith. 4th edition 1999. Oxford Univ. Press. NY
Section: Chapter 2. Plane autonomous systems and linearization. Problems page 79
Problem number: 2.2 (ii).
ODE order: 1.
ODE degree: 1.

Solve

$$x'(t) = x(t) - y(t)$$
$$y'(t) = 2x(t) - 2y(t)$$

✓ Solution by Maple
Time used: 0.015 (sec). Leaf size: 25

```
dsolve([diff(x(t),t)=x(t)-y(t),diff(y(t),t)=2*x(t)-2*y(t)],singsol=all)
```

$$x(t) = c_1 + c_2 \mathrm{e}^{-t}$$
$$y(t) = 2c_2 \mathrm{e}^{-t} + c_1$$

✓ Solution by Mathematica
Time used: 0.004 (sec). Leaf size: 59

```
DSolve[{x'[t]==x[t]-y[t],y'[t]==2*x[t]-2*y[t]},{x[t],y[t]},t,IncludeSingularSolutions -> Tru
```

$$x(t) \to e^{-t}\left(c_1\left(2e^t - 1\right) - c_2\left(e^t - 1\right)\right)$$
$$y(t) \to e^{-t}\left(2c_1\left(e^t - 1\right) - c_2\left(e^t - 2\right)\right)$$

1.9 problem 2.2 (iii)

Internal problem ID [12563]

Book: Nonlinear Ordinary Differential Equations by D.W.Jordna and P.Smith. 4th edition 1999. Oxford Univ. Press. NY
Section: Chapter 2. Plane autonomous systems and linearization. Problems page 79
Problem number: 2.2 (iii).
ODE order: 1.
ODE degree: 1.

Solve

$$x'(t) = x(t)$$
$$y'(t) = 2x(t) - 3y(t)$$

✓ Solution by Maple
Time used: 0.016 (sec). Leaf size: 24

```
dsolve([diff(x(t),t)=x(t),diff(y(t),t)=2*x(t)-3*y(t)],singsol=all)
```

$$x(t) = c_2 \mathrm{e}^{t}$$
$$y(t) = \frac{c_2 \mathrm{e}^{t}}{2} + c_1 \mathrm{e}^{-3t}$$

✓ Solution by Mathematica
Time used: 0.011 (sec). Leaf size: 38

```
DSolve[{x'[t]==x[t],y'[t]==2*x[t]-3*y[t]},{x[t],y[t]},t,IncludeSingularSolutions -> True]
```

$$x(t) \to c_1 e^{t}$$
$$y(t) \to \frac{1}{2} e^{-3t} \left(c_1 \left(e^{4t} - 1 \right) + 2c_2 \right)$$

1.10 problem 2.2 (iv)

Internal problem ID [12564]

Book: Nonlinear Ordinary Differential Equations by D.W.Jordna and P.Smith. 4th edition 1999. Oxford Univ. Press. NY
Section: Chapter 2. Plane autonomous systems and linearization. Problems page 79
Problem number: 2.2 (iv).
ODE order: 1.
ODE degree: 1.

Solve

$$x'(t) = x(t)$$
$$y'(t) = x(t) + 3y(t)$$

✓ Solution by Maple
Time used: 0.031 (sec). Leaf size: 24

```
dsolve([diff(x(t),t)=x(t),diff(y(t),t)=x(t)+3*y(t)],singsol=all)
```

$$x(t) = c_2 \mathrm{e}^{t}$$
$$y(t) = -\frac{c_2 \mathrm{e}^{t}}{2} + c_1 \mathrm{e}^{3t}$$

✓ Solution by Mathematica
Time used: 0.004 (sec). Leaf size: 39

```
DSolve[{x'[t]==x[t],y'[t]==x[t]+3*y[t]},{x[t],y[t]},t,IncludeSingularSolutions -> True]
```

$$x(t) \to c_1 e^{t}$$
$$y(t) \to \left(\frac{c_1}{2} + c_2\right) e^{3t} - \frac{c_1 e^{t}}{2}$$

1.11 problem 2.2 (v)

Internal problem ID [12565]

Book: Nonlinear Ordinary Differential Equations by D.W.Jordna and P.Smith. 4th edition 1999. Oxford Univ. Press. NY
Section: Chapter 2. Plane autonomous systems and linearization. Problems page 79
Problem number: 2.2 (v).
ODE order: 1.
ODE degree: 1.

Solve

$$x'(t) = -y(t)$$
$$y'(t) = 2x(t) - 4y(t)$$

✓ Solution by Maple
Time used: 0.016 (sec). Leaf size: 64

```
dsolve([diff(x(t),t)=-y(t),diff(y(t),t)=2*x(t)-4*y(t)],singsol=all)
```

$$x(t) = c_1 \mathrm{e}^{\left(-2+\sqrt{2}\right)t} + c_2 \mathrm{e}^{-\left(2+\sqrt{2}\right)t}$$
$$y(t) = \left(2+\sqrt{2}\right) c_2 \mathrm{e}^{-\left(2+\sqrt{2}\right)t} + \left(2-\sqrt{2}\right) c_1 \mathrm{e}^{\left(-2+\sqrt{2}\right)t}$$

✓ Solution by Mathematica
Time used: 0.019 (sec). Leaf size: 143

```
DSolve[{x'[t]==-y[t],y'[t]==2*x[t]-4*y[t]},{x[t],y[t]},t,IncludeSingularSolutions -> True]
```

$$x(t) \to \frac{1}{4} e^{-\left(\left(2+\sqrt{2}\right)t\right)} \left(2c_1\left(\left(1+\sqrt{2}\right)e^{2\sqrt{2}t} + 1 - \sqrt{2}\right) - \sqrt{2}c_2\left(e^{2\sqrt{2}t} - 1\right)\right)$$
$$y(t) \to \frac{1}{2} e^{-\left(\left(2+\sqrt{2}\right)t\right)} \left(\sqrt{2}c_1\left(e^{2\sqrt{2}t} - 1\right) + c_2\left(-\left(\sqrt{2}-1\right)e^{2\sqrt{2}t} + 1 + \sqrt{2}\right)\right)$$

1.12 problem 2.2 (vi)

Internal problem ID [12566]

Book: Nonlinear Ordinary Differential Equations by D.W.Jordna and P.Smith. 4th edition 1999. Oxford Univ. Press. NY
Section: Chapter 2. Plane autonomous systems and linearization. Problems page 79
Problem number: 2.2 (vi).
ODE order: 1.
ODE degree: 1.

Solve

$$x'(t) = x(t)$$
$$y'(t) = y(t)$$

✓ Solution by Maple
Time used: 0.016 (sec). Leaf size: 16

```
dsolve([diff(x(t),t)=x(t),diff(y(t),t)=y(t)],singsol=all)
```

$$x(t) = c_2 \mathrm{e}^t$$
$$y(t) = c_1 \mathrm{e}^t$$

✓ Solution by Mathematica
Time used: 0.065 (sec). Leaf size: 57

```
DSolve[{x'[t]==x[t],y'[t]==y[t]},{x[t],y[t]},t,IncludeSingularSolutions -> True]
```

$$x(t) \to c_1 e^t$$
$$y(t) \to c_2 e^t$$
$$x(t) \to c_1 e^t$$
$$y(t) \to 0$$
$$x(t) \to 0$$
$$y(t) \to c_2 e^t$$
$$x(t) \to 0$$
$$y(t) \to 0$$

1.13 problem 2.2 (vii)

Internal problem ID [12567]

Book: Nonlinear Ordinary Differential Equations by D.W.Jordna and P.Smith. 4th edition 1999. Oxford Univ. Press. NY
Section: Chapter 2. Plane autonomous systems and linearization. Problems page 79
Problem number: 2.2 (vii).
ODE order: 1.
ODE degree: 1.

Solve

$$x'(t) = 0$$
$$y'(t) = x(t)$$

✓ Solution by Maple
Time used: 0.016 (sec). Leaf size: 14

```
dsolve([diff(x(t),t)=0,diff(y(t),t)=x(t)],singsol=all)
```

$$x(t) = c_2$$
$$y(t) = c_2 t + c_1$$

✓ Solution by Mathematica
Time used: 0.009 (sec). Leaf size: 17

```
DSolve[{x'[t]==0,y'[t]==x[t]},{x[t],y[t]},t,IncludeSingularSolutions -> True]
```

$$x(t) \to c_1$$
$$y(t) \to c_1 t + c_2$$

1.14 problem 2.4 (i)

Internal problem ID [12568]

Book: Nonlinear Ordinary Differential Equations by D.W.Jordna and P.Smith. 4th edition 1999. Oxford Univ. Press. NY
Section: Chapter 2. Plane autonomous systems and linearization. Problems page 79
Problem number: 2.4 (i).
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _missing_x], _Duffing, [_2nd_order, _reducible,`

$$\boxed{x'' + x - x^3 = 0}$$

✓ Solution by Maple
Time used: 0.031 (sec). Leaf size: 43

```
dsolve(diff(x(t),t$2)+x(t)-x(t)^3=0,x(t), singsol=all)
```

$$x(t) = c_2\sqrt{2}\sqrt{\frac{1}{c_2^2+1}}\,\text{JacobiSN}\left(\frac{\left(\sqrt{2}\,t + 2c_1\right)\sqrt{2}\sqrt{\frac{1}{c_2^2+1}}}{2}, c_2\right)$$

✓ Solution by Mathematica
Time used: 60.266 (sec). Leaf size: 171

```
DSolve[x''[t]+x[t]-x[t]^3==0,x[t],t,IncludeSingularSolutions -> True]
```

$$x(t) \to -\frac{i\,\text{sn}\left(\frac{\sqrt{\left(\sqrt{1-2c_1}+1\right)(t+c_2)^2}}{\sqrt{2}}\middle|\frac{1-\sqrt{1-2c_1}}{\sqrt{1-2c_1}+1}\right)}{\sqrt{\frac{1}{-1+\sqrt{1-2c_1}}}}$$

$$x(t) \to \frac{i\,\text{sn}\left(\frac{\sqrt{\left(\sqrt{1-2c_1}+1\right)(t+c_2)^2}}{\sqrt{2}}\middle|\frac{1-\sqrt{1-2c_1}}{\sqrt{1-2c_1}+1}\right)}{\sqrt{\frac{1}{-1+\sqrt{1-2c_1}}}}$$

1.15 problem 2.4 (ii)

Internal problem ID [12569]

Book: Nonlinear Ordinary Differential Equations by D.W.Jordna and P.Smith. 4th edition 1999. Oxford Univ. Press. NY
Section: Chapter 2. Plane autonomous systems and linearization. Problems page 79
Problem number: 2.4 (ii).
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _missing_x], _Duffing, [_2nd_order, _reducible,`

$$\boxed{x'' + x + x^3 = 0}$$

✓ Solution by Maple
Time used: 0.047 (sec). Leaf size: 56

```
dsolve(diff(x(t),t$2)+x(t)+x(t)^3=0,x(t), singsol=all)
```

$$x(t) = c_2 \operatorname{JacobiSN}\left(\frac{\left(\sqrt{3}\,\sqrt{2}\,t + 2c_1\right)\sqrt{2}\,\sqrt{-\frac{1}{c_2^2-3}}}{2}, \frac{ic_2\sqrt{3}}{3}\right)\sqrt{2}\,\sqrt{-\frac{1}{c_2^2-3}}$$

✓ Solution by Mathematica
Time used: 60.261 (sec). Leaf size: 169

```
DSolve[x''[t]+x[t]+x[t]^3==0,x[t],t,IncludeSingularSolutions -> True]
```

$$x(t) \to -i\sqrt{1+\sqrt{1+2c_1}}\,\operatorname{sn}\left(\frac{\sqrt{-\left(\left(\sqrt{2c_1+1}-1\right)(t+c_2)^2\right)}}{\sqrt{2}}\middle|\frac{\sqrt{2c_1+1}+1}{1-\sqrt{2c_1+1}}\right)$$
$$x(t) \to i\sqrt{1+\sqrt{1+2c_1}}\,\operatorname{sn}\left(\frac{\sqrt{-\left(\left(\sqrt{2c_1+1}-1\right)(t+c_2)^2\right)}}{\sqrt{2}}\middle|\frac{\sqrt{2c_1+1}+1}{1-\sqrt{2c_1+1}}\right)$$

1.16 problem 2.4 (iii)

Internal problem ID [12570]

Book: Nonlinear Ordinary Differential Equations by D.W.Jordna and P.Smith. 4th edition 1999. Oxford Univ. Press. NY
Section: Chapter 2. Plane autonomous systems and linearization. Problems page 79
Problem number: 2.4 (iii).
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _missing_x]]`

$$\boxed{x'' + x' + x - x^3 = 0}$$

✗ Solution by Maple

```
dsolve(diff(x(t),t$2)+diff(x(t),t)+x(t)-x(t)^3=0,x(t), singsol=all)
```

No solution found

✗ Solution by Mathematica
Time used: 0.0 (sec). Leaf size: 0

```
DSolve[x''[t]+x'[t]+x[t]-x[t]^3==0,x[t],t,IncludeSingularSolutions -> True]
```

Not solved

1.17 problem 2.4 (iv)

Internal problem ID [12571]

Book: Nonlinear Ordinary Differential Equations by D.W.Jordna and P.Smith. 4th edition 1999. Oxford Univ. Press. NY
Section: Chapter 2. Plane autonomous systems and linearization. Problems page 79
Problem number: 2.4 (iv).
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _missing_x]]`

$$x'' + x' + x + x^3 = 0$$

✗ Solution by Maple

```
dsolve(diff(x(t),t$2)+diff(x(t),t)+x(t)+x(t)^3=0,x(t), singsol=all)
```

No solution found

✗ Solution by Mathematica
Time used: 0.0 (sec). Leaf size: 0

```
DSolve[x''[t]+x'[t]+x[t]+x[t]^3==0,x[t],t,IncludeSingularSolutions -> True]
```

Not solved

1.18 problem 2.4 (v)

Internal problem ID [12572]

Book: Nonlinear Ordinary Differential Equations by D.W.Jordna and P.Smith. 4th edition 1999. Oxford Univ. Press. NY
Section: Chapter 2. Plane autonomous systems and linearization. Problems page 79
Problem number: 2.4 (v).
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _missing_x], [_2nd_order, _reducible, _mu_x_y1]]`

$$\boxed{x'' - (2\cos(x) - 1)\sin(x) = 0}$$

✓ Solution by Maple
Time used: 0.015 (sec). Leaf size: 59

```
dsolve(diff(x(t),t$2)=(2*cos(x(t))-1)*sin(x(t)),x(t), singsol=all)
```

$$\int^{x(t)} \frac{1}{\sqrt{2\sin(\_a)^2 + 2\cos(\_a) + c_1}} d\_a - t - c_2 = 0$$
$$-\left(\int^{x(t)} \frac{1}{\sqrt{2\sin(\_a)^2 + 2\cos(\_a) + c_1}} d\_a\right) - t - c_2 = 0$$

✓ Solution by Mathematica

Time used: 61.831 (sec). Leaf size: 437

```
DSolve[x''[t]==(2*Cos[x[t]]-1)*Sin[x[t]],x[t],t,IncludeSingularSolutions -> True]
```

$$x(t) \to -2\arccos\left(-\frac{1}{2}\sqrt{3-\sqrt{3+2c_1}}\right)$$

$$x(t) \to 2\arccos\left(-\frac{1}{2}\sqrt{3-\sqrt{3+2c_1}}\right)$$

$$x(t) \to -2\arccos\left(\frac{1}{2}\sqrt{3-\sqrt{3+2c_1}}\right)$$

$$x(t) \to 2\arccos\left(\frac{1}{2}\sqrt{3-\sqrt{3+2c_1}}\right)$$

$$x(t) \to -2\arccos\left(-\frac{1}{2}\sqrt{3+\sqrt{3+2c_1}}\right)$$

$$x(t) \to 2\arccos\left(-\frac{1}{2}\sqrt{3+\sqrt{3+2c_1}}\right)$$

$$x(t) \to -2\arccos\left(\frac{1}{2}\sqrt{3+\sqrt{3+2c_1}}\right)$$

$$x(t) \to 2\arccos\left(\frac{1}{2}\sqrt{3+\sqrt{3+2c_1}}\right)$$

$$x(t) \to -2i\operatorname{arctanh}\left(\frac{\operatorname{sn}\left(\frac{1}{2}\sqrt{\left(-c_1+2\sqrt{2c_1+3}-3\right)(t+c_2)^2}\middle|\frac{c_1+2\sqrt{2c_1+3}+3}{c_1-2\sqrt{2c_1+3}+3}\right)}{\sqrt{\frac{-3+c_1}{3+c_1+2\sqrt{3+2c_1}}}}\right)$$

$$x(t) \to 2i\operatorname{arctanh}\left(\frac{\operatorname{sn}\left(\frac{1}{2}\sqrt{\left(-c_1+2\sqrt{2c_1+3}-3\right)(t+c_2)^2}\middle|\frac{c_1+2\sqrt{2c_1+3}+3}{c_1-2\sqrt{2c_1+3}+3}\right)}{\sqrt{\frac{-3+c_1}{3+c_1+2\sqrt{3+2c_1}}}}\right)$$

1.19 problem 2.5

Internal problem ID [12573]

Book: Nonlinear Ordinary Differential Equations by D.W.Jordna and P.Smith. 4th edition 1999. Oxford Univ. Press. NY
Section: Chapter 2. Plane autonomous systems and linearization. Problems page 79
Problem number: 2.5.
ODE order: 1.
ODE degree: 1.

Solve

$$x' = x - 5y(t)$$
$$y'(t) = x - y(t)$$

✓ Solution by Maple
Time used: 0.0 (sec). Leaf size: 50

```
dsolve([diff(x(t),t)=x(t)-5*y(t),diff(y(t),t)=x(t)-y(t)],singsol=all)
```

$$x(t) = c_1 \sin(2t) + c_2 \cos(2t)$$
$$y(t) = -\frac{2c_1 \cos(2t)}{5} + \frac{2c_2 \sin(2t)}{5} + \frac{c_1 \sin(2t)}{5} + \frac{c_2 \cos(2t)}{5}$$

✓ Solution by Mathematica
Time used: 0.009 (sec). Leaf size: 48

```
DSolve[{x'[t]==x[t]-5*y[t],y'[t]==x[t]-y[t]},{x[t],y[t]},t,IncludeSingularSolutions -> True]
```

$$x(t) \to c_1 \cos(2t) + (c_1 - 5c_2) \sin(t) \cos(t)$$
$$y(t) \to c_2 \cos(2t) + (c_1 - c_2) \sin(t) \cos(t)$$