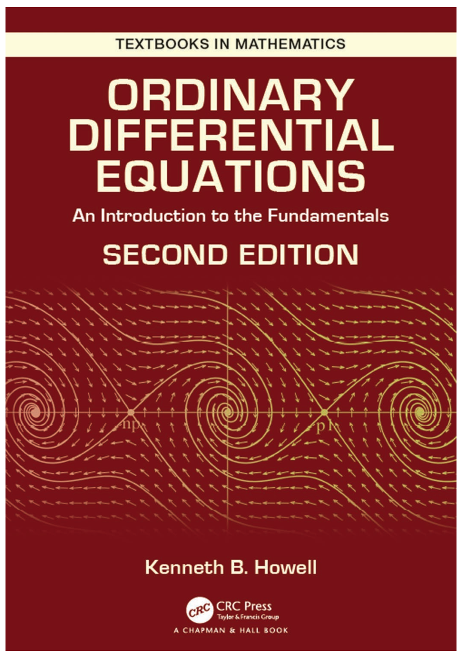
A Solution Manual For

Ordinary Differential Equations. An introduction to the fundamentals. Kenneth B. Howell. second edition. CRC Press. FL, USA. 2020



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1 Chapter 2. Integration and differential equations. Additional exercises. page 32

1.1 problem 2.2 (a)

Internal problem ID [13243]

Book: Ordinary Differential Equations. An introduction to the fundamentals. Kenneth B. Howell. second edition. CRC Press. FL, USA. 2020
Section: Chapter 2. Integration and differential equations. Additional exercises. page 32
Problem number: 2.2 (a).
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_quadrature]`

$$\boxed{y' = 3 - \sin(x)}$$

✓ Solution by Maple
Time used: 0.0 (sec). Leaf size: 11

```
dsolve(diff(y(x),x)=3-sin(x),y(x), singsol=all)
```

$$y(x) = \cos(x) + 3x + c_1$$

✓ Solution by Mathematica
Time used: 0.018 (sec). Leaf size: 13

```
DSolve[y'[x]==3-Sin[x],y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \rightarrow 3x + \cos(x) + c_1$$

1.2 problem 2.2 (b)

Internal problem ID [13244]

Book: Ordinary Differential Equations. An introduction to the fundamentals. Kenneth B. Howell. second edition. CRC Press. FL, USA. 2020
Section: Chapter 2. Integration and differential equations. Additional exercises. page 32
Problem number: 2.2 (b).
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_quadrature]`

$$\boxed{y' + \sin(y) = 3}$$

✓ Solution by Maple

Time used: 0.016 (sec). Leaf size: 22

```
dsolve(diff(y(x),x)=3-sin(y(x)),y(x), singsol=all)
```

$$y(x) = 2\arctan\left(\frac{1}{3} + \frac{2\sqrt{2}\,\tan\left((c_1 + x)\sqrt{2}\right)}{3}\right)$$

✓ Solution by Mathematica

Time used: 5.504 (sec). Leaf size: 83

```
DSolve[y'[x]==3-Sin[y[x]],y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to -2\arctan\left(\frac{1}{3}\left(-1 - 2\sqrt{2}\tan\left(\sqrt{2}(x - c_1)\right)\right)\right)$$
$$y(x) \to 2\arctan\left(\frac{1}{3}\left(1 + 2\sqrt{2}\tan\left(\sqrt{2}(x - c_1)\right)\right)\right)$$
$$y(x) \to \arcsin(3)$$
$$y(x) \to \text{Interval}[\{-\pi, \pi\}]$$

1.3 problem 2.2 (c)

Internal problem ID [13245]

Book: Ordinary Differential Equations. An introduction to the fundamentals. Kenneth B. Howell. second edition. CRC Press. FL, USA. 2020
Section: Chapter 2. Integration and differential equations. Additional exercises. page 32
Problem number: 2.2 (c).
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[[_linear, 'class A']]`

$$\boxed{y' + 4y = e^{2x}}$$

✓ Solution by Maple
Time used: 0.016 (sec). Leaf size: 18

```
dsolve(diff(y(x),x)+4*y(x)=exp(2*x),y(x), singsol=all)
```

$$y(x) = \frac{(e^{6x} + 6c_1)\, e^{-4x}}{6}$$

✓ Solution by Mathematica
Time used: 0.047 (sec). Leaf size: 23

```
DSolve[y'[x]+4*y[x]==Exp[2*x],y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to \frac{e^{2x}}{6} + c_1 e^{-4x}$$

1.4 problem 2.2 (d)

Internal problem ID [13246]

Book: Ordinary Differential Equations. An introduction to the fundamentals. Kenneth B. Howell. second edition. CRC Press. FL, USA. 2020
Section: Chapter 2. Integration and differential equations. Additional exercises. page 32
Problem number: 2.2 (d).
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_quadrature]`

$$\boxed{y'x = \arcsin(x^2)}$$

✓ Solution by Maple
Time used: 0.0 (sec). Leaf size: 88

```
dsolve(x*diff(y(x),x)=arcsin(x^2),y(x), singsol=all)
```

$$y(x) = -\frac{i \arcsin(x^2)^2}{4} + \frac{\arcsin(x^2) \ln\left(ix^2 + \sqrt{-x^4+1} + 1\right)}{2} - \frac{i \operatorname{polylog}\left(2, \left(ix^2 + \sqrt{-x^4+1}\right)^2\right)}{4} + \frac{\arcsin(x^2) \ln\left(1 - ix^2 - \sqrt{-x^4+1}\right)}{2} + c_1$$

✓ Solution by Mathematica
Time used: 0.031 (sec). Leaf size: 56

```
DSolve[x*y'[x]==ArcSin[x^2],y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to -\frac{1}{4} i \left(\arcsin(x^2)^2 + \operatorname{PolyLog}\left(2, e^{2i \arcsin(x^2)}\right)\right) + \frac{1}{2} \arcsin(x^2) \log\left(1 - e^{2i \arcsin(x^2)}\right) + c_1$$

1.5 problem 2.2 (e)

Internal problem ID [13247]

Book: Ordinary Differential Equations. An introduction to the fundamentals. Kenneth B. Howell. second edition. CRC Press. FL, USA. 2020
Section: Chapter 2. Integration and differential equations. Additional exercises. page 32
Problem number: 2.2 (e).
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_separable]`

$$\boxed{yy' = 2x}$$

✓ Solution by Maple
Time used: 0.015 (sec). Leaf size: 27

```
dsolve(y(x)*diff(y(x),x)=2*x,y(x), singsol=all)
```

$$y(x) = \sqrt{2x^2 + c_1}$$
$$y(x) = -\sqrt{2x^2 + c_1}$$

✓ Solution by Mathematica
Time used: 0.075 (sec). Leaf size: 42

```
DSolve[y[x]*y'[x]==2*x,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to -\sqrt{2}\sqrt{x^2 + c_1}$$
$$y(x) \to \sqrt{2}\sqrt{x^2 + c_1}$$

1.6 problem 2.2 (f)

Internal problem ID [13248]

Book: Ordinary Differential Equations. An introduction to the fundamentals. Kenneth B. Howell. second edition. CRC Press. FL, USA. 2020
Section: Chapter 2. Integration and differential equations. Additional exercises. page 32
Problem number: 2.2 (f).
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _quadrature]]`

$$\boxed{y'' = \frac{x+1}{x-1}}$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 26

```
dsolve(diff(y(x),x$2)=(x+1)/(x-1),y(x), singsol=all)
```

$$y(x) = 2 + 2\ln(-1+x)(-1+x) + \frac{x^2}{2} + (c_1 - 2)x + c_2$$

✓ Solution by Mathematica

Time used: 0.007 (sec). Leaf size: 30

```
DSolve[y''[x]==(x+1)/(x-1),y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to \frac{x^2}{2} + 2(x-1)\log(x-1) + (-2+c_2)x + c_1$$

1.7 problem 2.2 (g)

Internal problem ID [13249]

Book: Ordinary Differential Equations. An introduction to the fundamentals. Kenneth B. Howell. second edition. CRC Press. FL, USA. 2020
Section: Chapter 2. Integration and differential equations. Additional exercises. page 32
Problem number: 2.2 (g).
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _quadrature]]`

$$\boxed{x^2y'' = 1}$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 13

```
dsolve(x^2*diff(y(x),x$2)=1,y(x), singsol=all)
```

$$y(x) = -\ln(x) + c_1x + c_2$$

✓ Solution by Mathematica

Time used: 0.002 (sec). Leaf size: 16

```
DSolve[x^2*y''[x]==1,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to -\log(x) + c_2x + c_1$$

1.8 problem 2.2 (h)

Internal problem ID [13250]

Book: Ordinary Differential Equations. An introduction to the fundamentals. Kenneth B. Howell. second edition. CRC Press. FL, USA. 2020
Section: Chapter 2. Integration and differential equations. Additional exercises. page 32
Problem number: 2.2 (h).
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_Emden, _Fowler], [_2nd_order, _with_linear_symmetries]]`

$$\boxed{y^2y'' = 8x^2}$$

✗ Solution by Maple

```
dsolve(y(x)^2*diff(y(x),x$2)=8*x^2,y(x), singsol=all)
```

No solution found

✗ Solution by Mathematica
Time used: 0.0 (sec). Leaf size: 0

```
DSolve[y[x]^2*y''[x]==8*x^2,y[x],x,IncludeSingularSolutions -> True]
```

Not solved

1.9 problem 2.2 (i)

Internal problem ID [13251]

Book: Ordinary Differential Equations. An introduction to the fundamentals. Kenneth B. Howell. second edition. CRC Press. FL, USA. 2020
Section: Chapter 2. Integration and differential equations. Additional exercises. page 32
Problem number: 2.2 (i).
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _linear, _nonhomogeneous]]`

$$\boxed{y'' + 3y' + 8y = \mathrm{e}^{-x^2}}$$

✓ Solution by Maple
Time used: 0.0 (sec). Leaf size: 127

```
dsolve(diff(y(x),x$2)+3*diff(y(x),x)+8*y(x)=exp(-x^2),y(x), singsol=all)
```

$$
\begin{aligned}
y(x) = &-\frac{\operatorname{erf}\left(x-\frac{3}{4}+\frac{i\sqrt{23}}{4}\right)\left(i\cos\left(\frac{\sqrt{23}\,x}{2}\right)-\sin\left(\frac{\sqrt{23}\,x}{2}\right)\right)\sqrt{23}\,\sqrt{\pi}\,\mathrm{e}^{-\frac{3x}{2}-\frac{7}{8}-\frac{3i\sqrt{23}}{8}}}{46} \\
&+\frac{\sqrt{23}\,\operatorname{erf}\left(x-\frac{3}{4}-\frac{i\sqrt{23}}{4}\right)\left(i\cos\left(\frac{\sqrt{23}\,x}{2}\right)+\sin\left(\frac{\sqrt{23}\,x}{2}\right)\right)\sqrt{\pi}\,\mathrm{e}^{-\frac{3x}{2}-\frac{7}{8}+\frac{3i\sqrt{23}}{8}}}{46} \\
&+\mathrm{e}^{-\frac{3x}{2}}\left(c_1\cos\left(\frac{\sqrt{23}\,x}{2}\right)+c_2\sin\left(\frac{\sqrt{23}\,x}{2}\right)\right)
\end{aligned}
$$

✓ Solution by Mathematica

Time used: 0.519 (sec). Leaf size: 205

```
DSolve[y''[x]+3*y'[x]+8*y[x]==Exp[-x^2],y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to \frac{1}{46} e^{\frac{1}{8}\left(-12x-3i\sqrt{23}-7\right)} \left(-ie^{\frac{3i\sqrt{23}}{4}} \sqrt{23\pi}\,\text{erf}\left(-x+\frac{i\sqrt{23}}{4}+\frac{3}{4}\right) \left(\cos\left(\frac{\sqrt{23}x}{2}\right) - i\sin\left(\frac{\sqrt{23}x}{2}\right)\right) + i\sqrt{23\pi}\,\text{erf}\left(\frac{1}{4}\left(-4x-i\sqrt{23}+3\right)\right) \left(\cos\left(\frac{\sqrt{23}x}{2}\right) + i\sin\left(\frac{\sqrt{23}x}{2}\right)\right) + 46e^{\frac{7}{8}+\frac{3i\sqrt{23}}{8}} \left(c_2\cos\left(\frac{\sqrt{23}x}{2}\right) + c_1\sin\left(\frac{\sqrt{23}x}{2}\right)\right)\right)$$

1.10 problem 2.2 (j)

Internal problem ID [13252]

Book: Ordinary Differential Equations. An introduction to the fundamentals. Kenneth B. Howell. second edition. CRC Press. FL, USA. 2020
Section: Chapter 2. Integration and differential equations. Additional exercises. page 32
Problem number: 2.2 (j).
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _missing_y]]`

$$\boxed{x^2 y'' + 3y'x = 0}$$

✓ Solution by Maple
Time used: 0.0 (sec). Leaf size: 11

```
dsolve(x^2*diff(y(x),x$2)+3*x*diff(y(x),x)=0,y(x), singsol=all)
```

$$y(x) = c_1 + \frac{c_2}{x^2}$$

✓ Solution by Mathematica
Time used: 0.025 (sec). Leaf size: 17

```
DSolve[x^2*y''[x]+3*x*y'[x]==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to c_2 - \frac{c_1}{2x^2}$$

1.11 problem 2.3 (a)

Internal problem ID [13253]

Book: Ordinary Differential Equations. An introduction to the fundamentals. Kenneth B. Howell. second edition. CRC Press. FL, USA. 2020
Section: Chapter 2. Integration and differential equations. Additional exercises. page 32
Problem number: 2.3 (a).
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type **[_quadrature]**

$$\boxed{y' = 4x^3}$$

✓ Solution by Maple
Time used: 0.0 (sec). Leaf size: 9

```
dsolve(diff(y(x),x)=4*x^3,y(x), singsol=all)
```

$$y(x) = x^4 + c_1$$

✓ Solution by Mathematica
Time used: 0.002 (sec). Leaf size: 11

```
DSolve[y'[x]==4*x^3,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to x^4 + c_1$$

1.12 problem 2.3 (b)

Internal problem ID [13254]

Book: Ordinary Differential Equations. An introduction to the fundamentals. Kenneth B. Howell. second edition. CRC Press. FL, USA. 2020
Section: Chapter 2. Integration and differential equations. Additional exercises. page 32
Problem number: 2.3 (b).
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_quadrature]`

$$\boxed{y' = 20\,\mathrm{e}^{-4x}}$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 12

```
dsolve(diff(y(x),x)=20*exp(-4*x),y(x), singsol=all)
```

$$y(x) = -5\,\mathrm{e}^{-4x} + c_1$$

✓ Solution by Mathematica

Time used: 0.008 (sec). Leaf size: 15

```
DSolve[y'[x]==20*Exp[-4*x],y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to -5e^{-4x} + c_1$$

1.13 problem 2.3 (c)

Internal problem ID [13255]

Book: Ordinary Differential Equations. An introduction to the fundamentals. Kenneth B. Howell. second edition. CRC Press. FL, USA. 2020
Section: Chapter 2. Integration and differential equations. Additional exercises. page 32
Problem number: 2.3 (c).
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_quadrature]`

$$\boxed{y'x = -\sqrt{x} + 2}$$

✓ Solution by Maple
Time used: 0.0 (sec). Leaf size: 15

```
dsolve(x*diff(y(x),x)+sqrt(x)=2,y(x), singsol=all)
```

$$y(x) = 2\ln(x) - 2\sqrt{x} + c_1$$

✓ Solution by Mathematica
Time used: 0.011 (sec). Leaf size: 19

```
DSolve[x*y'[x]+Sqrt[x]==2,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to -2\sqrt{x} + 2\log(x) + c_1$$

1.14 problem 2.3 (d)

Internal problem ID [13256]

Book: Ordinary Differential Equations. An introduction to the fundamentals. Kenneth B. Howell. second edition. CRC Press. FL, USA. 2020
Section: Chapter 2. Integration and differential equations. Additional exercises. page 32
Problem number: 2.3 (d).
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_quadrature]`

$$\boxed{\sqrt{x+4}\, y' = 1}$$

✓ Solution by Maple
Time used: 0.0 (sec). Leaf size: 13

```
dsolve(sqrt(x+4)*diff(y(x),x)=1,y(x), singsol=all)
```

$$y(x) = 2\sqrt{x+4} + c_1$$

✓ Solution by Mathematica
Time used: 0.003 (sec). Leaf size: 17

```
DSolve[Sqrt[x+4]*y'[x]==1,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to 2\sqrt{x+4} + c_1$$

1.15 problem 2.3 (e)

Internal problem ID [13257]

Book: Ordinary Differential Equations. An introduction to the fundamentals. Kenneth B. Howell. second edition. CRC Press. FL, USA. 2020
Section: Chapter 2. Integration and differential equations. Additional exercises. page 32
Problem number: 2.3 (e).
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_quadrature]`

$$\boxed{y' = x\cos(x^2)}$$

✓ Solution by Maple
Time used: 0.0 (sec). Leaf size: 12

```
dsolve(diff(y(x),x)=x*cos(x^2),y(x), singsol=all)
```

$$y(x) = \frac{\sin(x^2)}{2} + c_1$$

✓ Solution by Mathematica
Time used: 0.007 (sec). Leaf size: 16

```
DSolve[y'[x]==x*Cos[x^2],y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to \frac{\sin(x^2)}{2} + c_1$$

1.16 problem 2.3 (f)

Internal problem ID [13258]

Book: Ordinary Differential Equations. An introduction to the fundamentals. Kenneth B. Howell. second edition. CRC Press. FL, USA. 2020
Section: Chapter 2. Integration and differential equations. Additional exercises. page 32
Problem number: 2.3 (f).
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_quadrature]`

$$\boxed{y' = \cos(x)\, x}$$

✓ Solution by Maple
Time used: 0.0 (sec). Leaf size: 12

```
dsolve(diff(y(x),x)=x*cos(x),y(x), singsol=all)
```

$$y(x) = x\sin(x) + \cos(x) + c_1$$

✓ Solution by Mathematica
Time used: 0.012 (sec). Leaf size: 14

```
DSolve[y'[x]==x*Cos[x],y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to x\sin(x) + \cos(x) + c_1$$

1.17 problem 2.3 (g)

Internal problem ID [13259]

Book: Ordinary Differential Equations. An introduction to the fundamentals. Kenneth B. Howell. second edition. CRC Press. FL, USA. 2020
Section: Chapter 2. Integration and differential equations. Additional exercises. page 32
Problem number: 2.3 (g).
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_quadrature]`

$$\boxed{-(x^2-9)\,y' = -x}$$

✓ Solution by Maple
Time used: 0.0 (sec). Leaf size: 14

```
dsolve(x=(x^2-9)*diff(y(x),x),y(x), singsol=all)
```

$$y(x) = \frac{\ln(x^2-9)}{2} + c_1$$

✓ Solution by Mathematica
Time used: 0.004 (sec). Leaf size: 18

```
DSolve[x==(x^2-9)*y'[x],y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to \frac{1}{2}\log(x^2-9) + c_1$$

1.18 problem 2.3 (h)

Internal problem ID [13260]

Book: Ordinary Differential Equations. An introduction to the fundamentals. Kenneth B. Howell. second edition. CRC Press. FL, USA. 2020
Section: Chapter 2. Integration and differential equations. Additional exercises. page 32
Problem number: 2.3 (h).
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_quadrature]`

$$\boxed{-\left(x^2-9\right)y' = -1}$$

✓ Solution by Maple
Time used: 0.016 (sec). Leaf size: 18

```
dsolve(1=(x^2-9)*diff(y(x),x),y(x), singsol=all)
```

$$y(x) = \frac{\ln\left(-3+x\right)}{6} - \frac{\ln\left(x+3\right)}{6} + c_1$$

✓ Solution by Mathematica
Time used: 0.004 (sec). Leaf size: 26

```
DSolve[1==(x^2-9)*y'[x],y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to \frac{1}{6}(\log(3-x) - \log(x+3) + 6c_1)$$

1.19 problem 2.3 (i)

Internal problem ID [13261]

Book: Ordinary Differential Equations. An introduction to the fundamentals. Kenneth B. Howell. second edition. CRC Press. FL, USA. 2020
Section: Chapter 2. Integration and differential equations. Additional exercises. page 32
Problem number: 2.3 (i).
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_quadrature]`

$$\boxed{9y' = x^2 - 1}$$

✓ Solution by Maple
Time used: 0.0 (sec). Leaf size: 14

```
dsolve(1=x^2-9*diff(y(x),x),y(x), singsol=all)
```

$$y(x) = \frac{1}{27}x^3 - \frac{1}{9}x + c_1$$

✓ Solution by Mathematica
Time used: 0.002 (sec). Leaf size: 20

```
DSolve[1==x^2-9*y'[x],y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to \frac{x^3}{27} - \frac{x}{9} + c_1$$

1.20 problem 2.3 (j)

Internal problem ID [13262]

Book: Ordinary Differential Equations. An introduction to the fundamentals. Kenneth B. Howell. second edition. CRC Press. FL, USA. 2020
Section: Chapter 2. Integration and differential equations. Additional exercises. page 32
Problem number: 2.3 (j).
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _quadrature]]`

$$\boxed{y'' = \sin(2x)}$$

✓ Solution by Maple
Time used: 0.0 (sec). Leaf size: 15

```
dsolve(diff(y(x),x$2)=sin(2*x),y(x), singsol=all)
```

$$y(x) = -\frac{\sin(2x)}{4} + c_1 x + c_2$$

✓ Solution by Mathematica
Time used: 0.036 (sec). Leaf size: 20

```
DSolve[y''[x]==Sin[2*x],y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to -\frac{1}{4}\sin(2x) + c_2 x + c_1$$

1.21 problem 2.3 (k)

Internal problem ID [13263]

Book: Ordinary Differential Equations. An introduction to the fundamentals. Kenneth B. Howell. second edition. CRC Press. FL, USA. 2020
Section: Chapter 2. Integration and differential equations. Additional exercises. page 32
Problem number: 2.3 (k).
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _quadrature]]`

$$\boxed{y'' = x + 3}$$

✓ Solution by Maple
Time used: 0.0 (sec). Leaf size: 19

```
dsolve(diff(y(x),x$2)-3=x,y(x), singsol=all)
```

$$y(x) = \frac{1}{6}x^3 + \frac{3}{2}x^2 + c_1 x + c_2$$

✓ Solution by Mathematica
Time used: 0.002 (sec). Leaf size: 26

```
DSolve[y''[x]-3==x,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to \frac{x^3}{6} + \frac{3x^2}{2} + c_2 x + c_1$$

1.22 problem 2.3 (L)

Internal problem ID [13264]

Book: Ordinary Differential Equations. An introduction to the fundamentals. Kenneth B. Howell. second edition. CRC Press. FL, USA. 2020
Section: Chapter 2. Integration and differential equations. Additional exercises. page 32
Problem number: 2.3 (L).
ODE order: 4.
ODE degree: 1.

CAS Maple gives this as type `[[_high_order, _quadrature]]`

$$\boxed{y'''' = 1}$$

✓ Solution by Maple
Time used: 0.016 (sec). Leaf size: 26

```
dsolve(diff(y(x),x$4)=1,y(x), singsol=all)
```

$$y(x) = \frac{1}{24}x^4 + \frac{1}{6}c_1x^3 + \frac{1}{2}c_2x^2 + xc_3 + c_4$$

✓ Solution by Mathematica
Time used: 0.002 (sec). Leaf size: 31

```
DSolve[y''''[x]==1,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to \frac{x^4}{24} + c_4x^3 + c_3x^2 + c_2x + c_1$$

1.23 problem 2.4 (a)

Internal problem ID [13265]

Book: Ordinary Differential Equations. An introduction to the fundamentals. Kenneth B. Howell. second edition. CRC Press. FL, USA. 2020
Section: Chapter 2. Integration and differential equations. Additional exercises. page 32
Problem number: 2.4 (a).
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type **[_quadrature]**

$$\boxed{y' = 40\,e^{2x}x}$$

With initial conditions

$$[y(0) = 4]$$

✓ Solution by Maple
Time used: 0.015 (sec). Leaf size: 16

```
dsolve([diff(y(x),x)=4*x*10*exp(2*x),y(0) = 4],y(x), singsol=all)
```

$$y(x) = 14 + (20x - 10)\,e^{2x}$$

✓ Solution by Mathematica
Time used: 0.014 (sec). Leaf size: 21

```
DSolve[{y'[x]==4*x*10*Exp[2*x],{y[0]==4}},y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to 2\left(5e^{2x}(2x-1)+7\right)$$

1.24 problem 2.4 (b)

Internal problem ID [13266]

Book: Ordinary Differential Equations. An introduction to the fundamentals. Kenneth B. Howell. second edition. CRC Press. FL, USA. 2020
Section: Chapter 2. Integration and differential equations. Additional exercises. page 32
Problem number: 2.4 (b).
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_quadrature]`

$$
\boxed{(x+6)^{\frac{1}{3}} y' = 1}
$$

With initial conditions

$$
[y(2) = 10]
$$

✓ Solution by Maple
Time used: 0.031 (sec). Leaf size: 13

```
dsolve([(x+6)^(1/3)*diff(y(x),x)=1,y(2) = 10],y(x), singsol=all)
```

$$
y(x) = \frac{3(x+6)^{\frac{2}{3}}}{2} + 4
$$

✓ Solution by Mathematica
Time used: 0.005 (sec). Leaf size: 18

```
DSolve[{(x+6)^(1/3)*y'[x]==1,{y[2]==10}},y[x],x,IncludeSingularSolutions -> True]
```

$$
y(x) \to \frac{3}{2}(x+6)^{2/3} + 4
$$

1.25 problem 2.4 (c)

Internal problem ID [13267]

Book: Ordinary Differential Equations. An introduction to the fundamentals. Kenneth B. Howell. second edition. CRC Press. FL, USA. 2020
Section: Chapter 2. Integration and differential equations. Additional exercises. page 32
Problem number: 2.4 (c).
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_quadrature]`

$$\boxed{y' = \frac{x-1}{x+1}}$$

With initial conditions

$$[y(0) = 8]$$

✓ Solution by Maple
Time used: 0.016 (sec). Leaf size: 13

```
dsolve([diff(y(x),x)=(x-1)/(x+1),y(0) = 8],y(x), singsol=all)
```

$$y(x) = x - 2\ln(1+x) + 8$$

✓ Solution by Mathematica
Time used: 0.004 (sec). Leaf size: 14

```
DSolve[{y'[x]==(x-1)/(x+1),{y[0]==8}},y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to x - 2\log(x+1) + 8$$

1.26 problem 2.4 (d)

Internal problem ID [13268]

Book: Ordinary Differential Equations. An introduction to the fundamentals. Kenneth B. Howell. second edition. CRC Press. FL, USA. 2020
Section: Chapter 2. Integration and differential equations. Additional exercises. page 32
Problem number: 2.4 (d).
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_quadrature]`

$$\boxed{y'x = \sqrt{x} - 2}$$

With initial conditions

$$[y(1) = 6]$$

✓ Solution by Maple
Time used: 0.016 (sec). Leaf size: 15

```
dsolve([x*diff(y(x),x)+2=sqrt(x),y(1) = 6],y(x), singsol=all)
```

$$y(x) = -2\ln(x) + 2\sqrt{x} + 4$$

✓ Solution by Mathematica
Time used: 0.011 (sec). Leaf size: 18

```
DSolve[{x*y'[x]+2==Sqrt[x],{y[1]==6}},y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to 2\left(\sqrt{x} - \log(x) + 2\right)$$

1.27 problem 2.4 (e)

Internal problem ID [13269]

Book: Ordinary Differential Equations. An introduction to the fundamentals. Kenneth B. Howell. second edition. CRC Press. FL, USA. 2020
Section: Chapter 2. Integration and differential equations. Additional exercises. page 32
Problem number: 2.4 (e).
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_quadrature]`

$$\boxed{\cos(x)\, y' = \sin(x)}$$

With initial conditions

$$[y(0) = 3]$$

✓ Solution by Maple
Time used: 0.0 (sec). Leaf size: 11

```
dsolve([cos(x)*diff(y(x),x)-sin(x)=0,y(0) = 3],y(x), singsol=all)
```

$$y(x) = -\ln(\cos(x)) + 3$$

✓ Solution by Mathematica
Time used: 0.01 (sec). Leaf size: 12

```
DSolve[{Cos[x]*y'[x]-Sin[x]==0,{y[0]==3}},y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to 3 - \log(\cos(x))$$

1.28 problem 2.4 (f)

Internal problem ID [13270]

Book: Ordinary Differential Equations. An introduction to the fundamentals. Kenneth B. Howell. second edition. CRC Press. FL, USA. 2020
Section: Chapter 2. Integration and differential equations. Additional exercises. page 32
Problem number: 2.4 (f).
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_quadrature]`

$$\boxed{(x^2+1)\,y'=1}$$

With initial conditions

$$[y(0)=3]$$

✓ Solution by Maple
Time used: 0.016 (sec). Leaf size: 8

```
dsolve([(x^2+1)*diff(y(x),x)=1,y(0) = 3],y(x), singsol=all)
```

$$y(x)=\arctan(x)+3$$

✓ Solution by Mathematica
Time used: 0.005 (sec). Leaf size: 9

```
DSolve[{(x^2+1)*y'[x]==1,{y[0]==3}},y[x],x,IncludeSingularSolutions -> True]
```

$$y(x)\to \arctan(x)+3$$

1.29 problem 2.4 (g)

Internal problem ID [13271]

Book: Ordinary Differential Equations. An introduction to the fundamentals. Kenneth B. Howell. second edition. CRC Press. FL, USA. 2020
Section: Chapter 2. Integration and differential equations. Additional exercises. page 32
Problem number: 2.4 (g).
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _quadrature]]`

$$\boxed{y''x = \sqrt{x} - 2}$$

With initial conditions

$$[y(1) = 8, y'(1) = 6]$$

✓ Solution by Maple
Time used: 0.0 (sec). Leaf size: 19

```
dsolve([x*diff(y(x),x$2)+2=sqrt(x),y(1) = 8, D(y)(1) = 6],y(x), singsol=all)
```

$$y(x) = \frac{4x^{\frac{3}{2}}}{3} - 2\ln(x)\,x + 6x + \frac{2}{3}$$

✓ Solution by Mathematica
Time used: 0.039 (sec). Leaf size: 26

```
DSolve[{x*y''[x]+2==Sqrt[x],{y[1]==8,y'[1]==6}},y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to \frac{2}{3}\left(2x^{3/2} + 9x - 3x\log(x) + 1\right)$$

1.30 problem 2.5 (a)

Internal problem ID [13272]

Book: Ordinary Differential Equations. An introduction to the fundamentals. Kenneth B. Howell. second edition. CRC Press. FL, USA. 2020
Section: Chapter 2. Integration and differential equations. Additional exercises. page 32
Problem number: 2.5 (a).
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_quadrature]`

$$\boxed{y' = \sin\left(\frac{x}{2}\right)}$$

✓ Solution by Maple
Time used: 0.0 (sec). Leaf size: 12

```
dsolve(diff(y(x),x)=sin(x/2),y(x), singsol=all)
```

$$y(x) = -2\cos\left(\frac{x}{2}\right) + c_1$$

✓ Solution by Mathematica
Time used: 0.021 (sec). Leaf size: 16

```
DSolve[y'[x]==Sin[x/2],y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to -2\cos\left(\frac{x}{2}\right) + c_1$$

1.31 problem 2.5 (b i)

Internal problem ID [13273]

Book: Ordinary Differential Equations. An introduction to the fundamentals. Kenneth B. Howell. second edition. CRC Press. FL, USA. 2020
Section: Chapter 2. Integration and differential equations. Additional exercises. page 32
Problem number: 2.5 (b i).
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_quadrature]`

$$\boxed{y' = \sin\left(\frac{x}{2}\right)}$$

With initial conditions

$$[y(0) = 0]$$

✓ Solution by Maple
Time used: 0.0 (sec). Leaf size: 12

```
dsolve([diff(y(x),x)=sin(x/2),y(0) = 0],y(x), singsol=all)
```

$$y(x) = -2\cos\left(\frac{x}{2}\right) + 2$$

✓ Solution by Mathematica
Time used: 0.005 (sec). Leaf size: 15

```
DSolve[{y'[x]==Sin[x/2],{y[0]==0}},y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to 4\sin^2\left(\frac{x}{4}\right)$$

1.32 problem 2.5 (b ii)

Internal problem ID [13274]

Book: Ordinary Differential Equations. An introduction to the fundamentals. Kenneth B. Howell. second edition. CRC Press. FL, USA. 2020
Section: Chapter 2. Integration and differential equations. Additional exercises. page 32
Problem number: 2.5 (b ii).
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_quadrature]`

$$\boxed{y' = \sin\left(\frac{x}{2}\right)}$$

With initial conditions

$$[y(0) = 3]$$

✓ Solution by Maple
Time used: 0.015 (sec). Leaf size: 12

```
dsolve([diff(y(x),x)=sin(x/2),y(0) = 3],y(x), singsol=all)
```

$$y(x) = -2\cos\left(\frac{x}{2}\right) + 5$$

✓ Solution by Mathematica
Time used: 0.003 (sec). Leaf size: 15

```
DSolve[{y'[x]==Sin[x/2],{y[0]==3}},y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to 5 - 2\cos\left(\frac{x}{2}\right)$$

1.33 problem 2.6 (a)

Internal problem ID [13275]

Book: Ordinary Differential Equations. An introduction to the fundamentals. Kenneth B. Howell. second edition. CRC Press. FL, USA. 2020
Section: Chapter 2. Integration and differential equations. Additional exercises. page 32
Problem number: 2.6 (a).
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_quadrature]`

$$\boxed{y' = 3\sqrt{x+3}}$$

✓ Solution by Maple
Time used: 0.0 (sec). Leaf size: 17

```
dsolve(diff(y(x),x)=3*sqrt(x+3),y(x), singsol=all)
```

$$y(x) = (2x+6)\sqrt{x+3} + c_1$$

✓ Solution by Mathematica
Time used: 0.004 (sec). Leaf size: 17

```
DSolve[y'[x]==3*Sqrt[x+3],y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to 2(x+3)^{3/2} + c_1$$

1.34 problem 2.6 (b i)

Internal problem ID [13276]

Book: Ordinary Differential Equations. An introduction to the fundamentals. Kenneth B. Howell. second edition. CRC Press. FL, USA. 2020
Section: Chapter 2. Integration and differential equations. Additional exercises. page 32
Problem number: 2.6 (b i).
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_quadrature]`

$$\boxed{y' = 3\sqrt{x+3}}$$

With initial conditions

$$[y(1) = 16]$$

✓ Solution by Maple
Time used: 0.0 (sec). Leaf size: 11

```
dsolve([diff(y(x),x)=3*sqrt(x+3),y(1) = 16],y(x), singsol=all)
```

$$y(x) = 2(x+3)^{\frac{3}{2}}$$

✓ Solution by Mathematica
Time used: 0.004 (sec). Leaf size: 14

```
DSolve[{y'[x]==3*Sqrt[x+3],{y[1]==16}},y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to 2(x+3)^{3/2}$$

1.35 problem 2.6 (b ii)

Internal problem ID [13277]

Book: Ordinary Differential Equations. An introduction to the fundamentals. Kenneth B. Howell. second edition. CRC Press. FL, USA. 2020
Section: Chapter 2. Integration and differential equations. Additional exercises. page 32
Problem number: 2.6 (b ii).
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_quadrature]`

$$\boxed{y' = 3\sqrt{x+3}}$$

With initial conditions

$$[y(1) = 20]$$

✓ Solution by Maple
Time used: 0.016 (sec). Leaf size: 13

```
dsolve([diff(y(x),x)=3*sqrt(x+3),y(1) = 20],y(x), singsol=all)
```

$$y(x) = 4 + (2x+6)\sqrt{x+3}$$

✓ Solution by Mathematica
Time used: 0.003 (sec). Leaf size: 16

```
DSolve[{y'[x]==3*Sqrt[x+3],{y[1]==20}},y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to 2\left((x+3)^{3/2} + 2\right)$$

1.36 problem 2.6 (b iii)

Internal problem ID [13278]

Book: Ordinary Differential Equations. An introduction to the fundamentals. Kenneth B. Howell. second edition. CRC Press. FL, USA. 2020
Section: Chapter 2. Integration and differential equations. Additional exercises. page 32
Problem number: 2.6 (b iii).
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_quadrature]`

$$\boxed{y' = 3\sqrt{x+3}}$$

With initial conditions

$$[y(1) = 0]$$

✓ Solution by Maple
Time used: 0.0 (sec). Leaf size: 13

```
dsolve([diff(y(x),x)=3*sqrt(x+3),y(1) = 0],y(x), singsol=all)
```

$$y(x) = -16 + (2x + 6)\sqrt{x+3}$$

✓ Solution by Mathematica
Time used: 0.004 (sec). Leaf size: 16

```
DSolve[{y'[x]==3*Sqrt[x+3],{y[1]==0}},y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to 2\left((x+3)^{3/2} - 8\right)$$

1.37 problem 2.7 a

Internal problem ID [13279]

Book: Ordinary Differential Equations. An introduction to the fundamentals. Kenneth B. Howell. second edition. CRC Press. FL, USA. 2020
Section: Chapter 2. Integration and differential equations. Additional exercises. page 32
Problem number: 2.7 a.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_quadrature]`

$$\boxed{y' = x\,\mathrm{e}^{-x^2}}$$

With initial conditions

$$[y(0) = 3]$$

✓ Solution by Maple
Time used: 0.016 (sec). Leaf size: 14

```
dsolve([diff(y(x),x)=x*exp(-x^2),y(0) = 3],y(x), singsol=all)
```

$$y(x) = -\frac{\mathrm{e}^{-x^2}}{2} + \frac{7}{2}$$

✓ Solution by Mathematica
Time used: 0.013 (sec). Leaf size: 20

```
DSolve[{y'[x]==x*Exp[-x^2],{y[0]==3}},y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to \frac{7}{2} - \frac{e^{-x^2}}{2}$$

1.38 problem 2.7 b

Internal problem ID [13280]

Book: Ordinary Differential Equations. An introduction to the fundamentals. Kenneth B. Howell. second edition. CRC Press. FL, USA. 2020
Section: Chapter 2. Integration and differential equations. Additional exercises. page 32
Problem number: 2.7 b.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_quadrature]`

$$\boxed{y' = \frac{x}{\sqrt{x^2+5}}}$$

With initial conditions

$$[y(2) = 7]$$

✓ Solution by Maple
Time used: 0.031 (sec). Leaf size: 13

```
dsolve([diff(y(x),x)=x/sqrt(x^2+5),y(2) = 7],y(x), singsol=all)
```

$$y(x) = \sqrt{x^2+5}+4$$

✓ Solution by Mathematica
Time used: 0.005 (sec). Leaf size: 16

```
DSolve[{y'[x]==x/Sqrt[x^2+5],{y[2]==7}},y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to \sqrt{x^2+5}+4$$

1.39 problem 2.7 c

Internal problem ID [13281]

Book: Ordinary Differential Equations. An introduction to the fundamentals. Kenneth B. Howell. second edition. CRC Press. FL, USA. 2020
Section: Chapter 2. Integration and differential equations. Additional exercises. page 32
Problem number: 2.7 c.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_quadrature]`

$$\boxed{y' = \frac{1}{x^2+1}}$$

With initial conditions

$$[y(1) = 0]$$

✓ Solution by Maple
Time used: 0.016 (sec). Leaf size: 10

```
dsolve([diff(y(x),x)=1/(x^2+1),y(1) = 0],y(x), singsol=all)
```

$$y(x) = \arctan(x) - \frac{\pi}{4}$$

✓ Solution by Mathematica
Time used: 0.004 (sec). Leaf size: 13

```
DSolve[{y'[x]==1/(x^2+1),{y[1]==0}},y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to \arctan(x) - \frac{\pi}{4}$$

1.40 problem 2.7 d

Internal problem ID [13282]

Book: Ordinary Differential Equations. An introduction to the fundamentals. Kenneth B. Howell. second edition. CRC Press. FL, USA. 2020
Section: Chapter 2. Integration and differential equations. Additional exercises. page 32
Problem number: 2.7 d.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_quadrature]`

$$\boxed{y' = e^{-9x^2}}$$

With initial conditions

$$[y(0) = 1]$$

✓ Solution by Maple
Time used: 0.016 (sec). Leaf size: 15

```
dsolve([diff(y(x),x)=exp(-9*x^2),y(0) = 1],y(x), singsol=all)
```

$$y(x) = \frac{\sqrt{\pi}\,\operatorname{erf}(3x)}{6} + 1$$

✓ Solution by Mathematica
Time used: 0.018 (sec). Leaf size: 20

```
DSolve[{y'[x]==Exp[-9*x^2],{y[0]==1}},y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to \frac{1}{6}\sqrt{\pi}\text{erf}(3x) + 1$$

1.41 problem 2.7 e

Internal problem ID [13283]

Book: Ordinary Differential Equations. An introduction to the fundamentals. Kenneth B. Howell. second edition. CRC Press. FL, USA. 2020
Section: Chapter 2. Integration and differential equations. Additional exercises. page 32
Problem number: 2.7 e.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_quadrature]`

$$\boxed{y'x = \sin(x)}$$

With initial conditions

$$[y(0) = 4]$$

✓ Solution by Maple
Time used: 0.0 (sec). Leaf size: 8

```
dsolve([x*diff(y(x),x)=sin(x),y(0) = 4],y(x), singsol=all)
```

$$y(x) = \operatorname{Si}(x) + 4$$

✓ Solution by Mathematica
Time used: 0.014 (sec). Leaf size: 9

```
DSolve[{x*y'[x]==Sin[x],{y[0]==4}},y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to \operatorname{Si}(x) + 4$$

1.42 problem 2.7 f

Internal problem ID [13284]

Book: Ordinary Differential Equations. An introduction to the fundamentals. Kenneth B. Howell. second edition. CRC Press. FL, USA. 2020
Section: Chapter 2. Integration and differential equations. Additional exercises. page 32
Problem number: 2.7 f.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_quadrature]`

$$\boxed{y'x = \sin\left(x^2\right)}$$

With initial conditions

$$[y(0) = 0]$$

✓ Solution by Maple
Time used: 0.0 (sec). Leaf size: 10

```
dsolve([x*diff(y(x),x)=sin(x^2),y(0) = 0],y(x), singsol=all)
```

$$y(x) = \frac{\operatorname{Si}\left(x^2\right)}{2}$$

✓ Solution by Mathematica
Time used: 0.008 (sec). Leaf size: 13

```
DSolve[{x*y'[x]==Sin[x^2],{y[0]==0}},y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to \frac{\operatorname{Si}(x^2)}{2}$$

1.43 problem 2.9 a

Internal problem ID [13285]

Book: Ordinary Differential Equations. An introduction to the fundamentals. Kenneth B. Howell. second edition. CRC Press. FL, USA. 2020
Section: Chapter 2. Integration and differential equations. Additional exercises. page 32
Problem number: 2.9 a.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_quadrature]`

$$y' = \begin{cases} 0 & x < 0 \\ 1 & 0 \leq x \end{cases}$$

With initial conditions

$$[y(0) = 0]$$

✓ Solution by Maple
Time used: 0.015 (sec). Leaf size: 13

```
dsolve([diff(y(x),x)=piecewise(x<0,0,x>=0,1),y(0) = 0],y(x), singsol=all)
```

$$y(x) = \begin{cases} 0 & x < 0 \\ x & 0 \leq x \end{cases}$$

✓ Solution by Mathematica
Time used: 0.006 (sec). Leaf size: 9

```
DSolve[{y'[x]==UnitStep[x],{y[0]==0}},y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to x\theta(x)$$

1.44 problem 2.9 b

Internal problem ID [13286]

Book: Ordinary Differential Equations. An introduction to the fundamentals. Kenneth B. Howell. second edition. CRC Press. FL, USA. 2020
Section: Chapter 2. Integration and differential equations. Additional exercises. page 32
Problem number: 2.9 b.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_quadrature]`

$$y' = \begin{cases} 0 & x < 1 \\ 1 & 1 \leq x \end{cases}$$

With initial conditions

$$[y(0) = 2]$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 15

```
dsolve([diff(y(x),x)=piecewise(x<1,0,x>=1,1),y(0) = 2],y(x), singsol=all)
```

$$y(x) = \begin{cases} 2 & x < 1 \\ 1 + x & 1 \leq x \end{cases}$$

✓ Solution by Mathematica

Time used: 0.005 (sec). Leaf size: 15

```
DSolve[{y'[x]==Piecewise[{{0,x<1},{1,x>=1}}],{y[0]==2}},y[x],x,IncludeSingularSolutions -> T
```

$$y(x) \to \left\{ \begin{array}{cc} 2 & x \leq 1 \\ x + 1 & \text{True} \end{array} \right.$$

1.45 problem 2.9 c

Internal problem ID [13287]

Book: Ordinary Differential Equations. An introduction to the fundamentals. Kenneth B. Howell. second edition. CRC Press. FL, USA. 2020
Section: Chapter 2. Integration and differential equations. Additional exercises. page 32
Problem number: 2.9 c.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_quadrature]`

$$y' = \begin{cases} 0 & x < 1 \\ 1 & 1 \leq x < 2 \\ 0 & 2 \leq x \end{cases}$$

With initial conditions

$$[y(0) = 0]$$

✓ Solution by Maple
Time used: 0.016 (sec). Leaf size: 19

```
dsolve([diff(y(x),x)=piecewise(x<1,0,1<=x and x<2,1,2<=x,0),y(0) = 0],y(x), singsol=all)
```

$$y(x) = \begin{cases} 0 & x < 1 \\ -1 + x & x < 2 \\ 1 & 2 \leq x \end{cases}$$

✓ Solution by Mathematica

Time used: 0.006 (sec). Leaf size: 23

```
DSolve[{y'[x]==Piecewise[{{0,x<1},{1,1<=x<2},{0,2<=x}}],{y[0]==0}},y[x],x,IncludeSingularSol
```

$$y(x) \to \begin{cases} 0 & x \leq 1 \\ x-1 & 1<x \leq 2 \\ 1 & \text{True} \end{cases}$$

2 Chapter 3. Some basics about First order equations. Additional exercises. page 63

2.1 problem 3.4 a

Internal problem ID [13288]

Book: Ordinary Differential Equations. An introduction to the fundamentals. Kenneth B. Howell. second edition. CRC Press. FL, USA. 2020
Section: Chapter 3. Some basics about First order equations. Additional exercises. page 63
Problem number: 3.4 a.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_separable]`

$$\boxed{y' + 3yx = 6x}$$

✓ Solution by Maple
Time used: 0.0 (sec). Leaf size: 14

```
dsolve(diff(y(x),x)+3*x*y(x)=6*x,y(x), singsol=all)
```

$$y(x) = 2 + \mathrm{e}^{-\frac{3x^2}{2}} c_1$$

✓ Solution by Mathematica
Time used: 0.052 (sec). Leaf size: 24

```
DSolve[y'[x]+3*x*y[x]==6*x,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to 2 + c_1 e^{-\frac{3x^2}{2}}$$
$$y(x) \to 2$$

2.2 problem 3.4 b

Internal problem ID [13289]

Book: Ordinary Differential Equations. An introduction to the fundamentals. Kenneth B. Howell. second edition. CRC Press. FL, USA. 2020
Section: Chapter 3. Some basics about First order equations. Additional exercises. page 63
Problem number: 3.4 b.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[‘y=_G(x,y’)‘]`

$$\boxed{\sin(y+x) - yy' = 0}$$

✗ Solution by Maple

```
dsolve(sin(x+y(x))-y(x)*diff(y(x),x)=0,y(x), singsol=all)
```

No solution found

✗ Solution by Mathematica
Time used: 0.0 (sec). Leaf size: 0

```
DSolve[Sin[x+y[x]]-y[x]*y'[x]==0,y[x],x,IncludeSingularSolutions -> True]
```

Not solved

2.3 problem 3.4 c

Internal problem ID [13290]

Book: Ordinary Differential Equations. An introduction to the fundamentals. Kenneth B. Howell. second edition. CRC Press. FL, USA. 2020
Section: Chapter 3. Some basics about First order equations. Additional exercises. page 63
Problem number: 3.4 c.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_quadrature]`

$$\boxed{y' - y^3 = 8}$$

✓ Solution by Maple
Time used: 0.516 (sec). Leaf size: 50

```
dsolve(diff(y(x),x)-y(x)^3=8,y(x), singsol=all)
```

$$y(x) = \sqrt{3}\,\tan\left(\operatorname{RootOf}\left(-\sqrt{3}\,\ln\left(\cos\left(\_Z\right)^2\right) - 2\sqrt{3}\,\ln\left(\sqrt{3} + \tan\left(\_Z\right)\right) + 24\sqrt{3}\,c_1 + 24\sqrt{3}\,x - 6\_Z\right)\right) + 1$$

✓ Solution by Mathematica
Time used: 0.206 (sec). Leaf size: 83

```
DSolve[y'[x]-y[x]^3==8,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to \text{InverseFunction}\left[-\frac{1}{24}\log\left(\#1^2 - 2\#1 + 4\right) + \frac{\arctan\left(\frac{\#1-1}{\sqrt{3}}\right)}{4\sqrt{3}} + \frac{1}{12}\log(\#1 + 2)\&\right][x + c_1]$$
$$y(x) \to -2$$
$$y(x) \to 2\sqrt[3]{-1}$$
$$y(x) \to -2(-1)^{2/3}$$

2.4 problem 3.4 d

Internal problem ID [13291]

Book: Ordinary Differential Equations. An introduction to the fundamentals. Kenneth B. Howell. second edition. CRC Press. FL, USA. 2020
Section: Chapter 3. Some basics about First order equations. Additional exercises. page 63
Problem number: 3.4 d.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type **[_separable]**

$$\boxed{x^2y' + y^2x = x}$$

✓ Solution by Maple
Time used: 0.0 (sec). Leaf size: 9

```
dsolve(x^2*diff(y(x),x)+x*y(x)^2=x,y(x), singsol=all)
```

$$y(x) = \tanh(\ln(x) + c_1)$$

✓ Solution by Mathematica
Time used: 1.054 (sec). Leaf size: 40

```
DSolve[x^2*y'[x]+x*y[x]^2==x,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to \frac{x^2 - e^{2c_1}}{x^2 + e^{2c_1}}$$
$$y(x) \to -1$$
$$y(x) \to 1$$

2.5 problem 3.4 e

Internal problem ID [13292]

Book: Ordinary Differential Equations. An introduction to the fundamentals. Kenneth B. Howell. second edition. CRC Press. FL, USA. 2020
Section: Chapter 3. Some basics about First order equations. Additional exercises. page 63
Problem number: 3.4 e.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[[_Riccati, _special]]`

$$\boxed{y' - y^2 = x}$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 31

```
dsolve(diff(y(x),x)-y(x)^2=x,y(x), singsol=all)
```

$$y(x) = \frac{c_1 \operatorname{AiryAi}(1, -x) + \operatorname{AiryBi}(1, -x)}{c_1 \operatorname{AiryAi}(-x) + \operatorname{AiryBi}(-x)}$$

✓ Solution by Mathematica

Time used: 0.114 (sec). Leaf size: 195

```
DSolve[y'[x]-y[x]^2==x,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to \frac{x^{3/2}\left(-2 \operatorname{BesselJ}\left(-\frac{2}{3}, \frac{2x^{3/2}}{3}\right) + c_1\left(\operatorname{BesselJ}\left(\frac{2}{3}, \frac{2x^{3/2}}{3}\right) - \operatorname{BesselJ}\left(-\frac{4}{3}, \frac{2x^{3/2}}{3}\right)\right)\right) - c_1 \operatorname{BesselJ}\left(-\frac{1}{3}, \frac{2x^{3/2}}{3}\right)}{2x\left(\operatorname{BesselJ}\left(\frac{1}{3}, \frac{2x^{3/2}}{3}\right) + c_1 \operatorname{BesselJ}\left(-\frac{1}{3}, \frac{2x^{3/2}}{3}\right)\right)}$$

$$y(x) \to -\frac{x^{3/2} \operatorname{BesselJ}\left(-\frac{4}{3}, \frac{2x^{3/2}}{3}\right) - x^{3/2} \operatorname{BesselJ}\left(\frac{2}{3}, \frac{2x^{3/2}}{3}\right) + \operatorname{BesselJ}\left(-\frac{1}{3}, \frac{2x^{3/2}}{3}\right)}{2x \operatorname{BesselJ}\left(-\frac{1}{3}, \frac{2x^{3/2}}{3}\right)}$$

2.6 problem 3.4 f

Internal problem ID [13293]

Book: Ordinary Differential Equations. An introduction to the fundamentals. Kenneth B. Howell. second edition. CRC Press. FL, USA. 2020
Section: Chapter 3. Some basics about First order equations. Additional exercises. page 63
Problem number: 3.4 f.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_quadrature]`

$$\boxed{y^3 - 25y + y' = 0}$$

✓ Solution by Maple
Time used: 0.0 (sec). Leaf size: 33

```
dsolve(y(x)^3-25*y(x)+diff(y(x),x)=0,y(x), singsol=all)
```

$$y(x) = -\frac{5}{\sqrt{25\,\mathrm{e}^{-50x} c_1 + 1}}$$
$$y(x) = \frac{5}{\sqrt{25\,\mathrm{e}^{-50x} c_1 + 1}}$$

✓ Solution by Mathematica

Time used: 0.68 (sec). Leaf size: 110

```
DSolve[y[x]^3-25*y[x]+y'[x]==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to -\frac{5e^{25x}}{\sqrt{e^{50x} + e^{50c_1}}}$$
$$y(x) \to \frac{5e^{25x}}{\sqrt{e^{50x} + e^{50c_1}}}$$
$$y(x) \to -5$$
$$y(x) \to 0$$
$$y(x) \to 5$$
$$y(x) \to -\frac{5e^{25x}}{\sqrt{e^{50x}}}$$
$$y(x) \to \frac{5e^{25x}}{\sqrt{e^{50x}}}$$

2.7 problem 3.4 g

Internal problem ID [13294]

Book: Ordinary Differential Equations. An introduction to the fundamentals. Kenneth B. Howell. second edition. CRC Press. FL, USA. 2020
Section: Chapter 3. Some basics about First order equations. Additional exercises. page 63
Problem number: 3.4 g.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_separable]`

$$\boxed{(x-2)\,y'-y=3}$$

✓ Solution by Maple
Time used: 0.0 (sec). Leaf size: 11

```
dsolve((x-2)*diff(y(x),x)=y(x)+3,y(x), singsol=all)
```

$$y(x)=-3+c_1(x-2)$$

✓ Solution by Mathematica
Time used: 0.026 (sec). Leaf size: 18

```
DSolve[(x-2)*y'[x]==y[x]+3,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x)\to -3+c_1(x-2)$$
$$y(x)\to -3$$

2.8 problem 3.4 h

Internal problem ID [13295]

Book: Ordinary Differential Equations. An introduction to the fundamentals. Kenneth B. Howell. second edition. CRC Press. FL, USA. 2020
Section: Chapter 3. Some basics about First order equations. Additional exercises. page 63
Problem number: 3.4 h.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_separable]`

$$\boxed{(y-2)\,y' = x-3}$$

✓ Solution by Maple
Time used: 0.016 (sec). Leaf size: 39

```
dsolve((y(x)-2)*diff(y(x),x)=x-3,y(x), singsol=all)
```

$$y(x) = 2-\sqrt{x^2+2c_1-6x+4}$$
$$y(x) = 2+\sqrt{x^2+2c_1-6x+4}$$

✓ Solution by Mathematica
Time used: 0.112 (sec). Leaf size: 47

```
DSolve[(y[x]-2)*y'[x]==x-3,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x)\to 2-\sqrt{x^2-6x+4+2c_1}$$
$$y(x)\to 2+\sqrt{x^2-6x+4+2c_1}$$

2.9 problem 3.4 i

Internal problem ID [13296]

Book: Ordinary Differential Equations. An introduction to the fundamentals. Kenneth B. Howell. second edition. CRC Press. FL, USA. 2020
Section: Chapter 3. Some basics about First order equations. Additional exercises. page 63
Problem number: 3.4 i.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_quadrature]`

$$\boxed{y' + 2y - y^2 = -2}$$

✓ Solution by Maple
Time used: 0.0 (sec). Leaf size: 19

```
dsolve(diff(y(x),x)+2*y(x)-y(x)^2=-2,y(x), singsol=all)
```

$$y(x) = 1 - \sqrt{3}\, \tanh\left((c_1 + x)\sqrt{3}\right)$$

✓ Solution by Mathematica
Time used: 0.394 (sec). Leaf size: 76

```
DSolve[y'[x]+2*y[x]-y[x]^2==-2,y[x],x,IncludeSingularSolutions -> True]
```

$$\begin{aligned} y(x) &\to \frac{-\left(\sqrt{3}-1\right) e^{2\sqrt{3}(x+c_1)} + 1 + \sqrt{3}}{1 + e^{2\sqrt{3}(x+c_1)}} \\ y(x) &\to 1 - \sqrt{3} \\ y(x) &\to 1 + \sqrt{3} \end{aligned}$$

2.10 problem 3.4 j

Internal problem ID [13297]

Book: Ordinary Differential Equations. An introduction to the fundamentals. Kenneth B. Howell. second edition. CRC Press. FL, USA. 2020
Section: Chapter 3. Some basics about First order equations. Additional exercises. page 63
Problem number: 3.4 j.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_Riccati]`

$$\boxed{y' + (8 - x)\, y - y^2 = -8x}$$

✓ Solution by Maple
Time used: 0.016 (sec). Leaf size: 66

```
dsolve(diff(y(x),x)+(8-x)*y(x)-y(x)^2=-8*x,y(x), singsol=all)
```

$$y(x) = \frac{8i\sqrt{\pi}\,\mathrm{e}^{-32}\sqrt{2}\,\operatorname{erf}\left(\frac{i\sqrt{2}\,(x+8)}{2}\right) + 2\,\mathrm{e}^{\frac{x(x+16)}{2}} + 16c_1}{i\sqrt{\pi}\,\mathrm{e}^{-32}\sqrt{2}\,\operatorname{erf}\left(\frac{i\sqrt{2}\,(x+8)}{2}\right) + 2c_1}$$

✓ Solution by Mathematica
Time used: 0.176 (sec). Leaf size: 54

```
DSolve[y'[x]+(8-x)*y[x]-y[x]^2==-8*x,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to 8 + \frac{2e^{\frac{1}{2}(x+8)^2}}{-\sqrt{2\pi}\operatorname{erfi}\left(\frac{x+8}{\sqrt{2}}\right) + 2e^{32}c_1}$$
$$y(x) \to 8$$

2.11 problem 3.6

Internal problem ID [13298]

Book: Ordinary Differential Equations. An introduction to the fundamentals. Kenneth B. Howell. second edition. CRC Press. FL, USA. 2020
Section: Chapter 3. Some basics about First order equations. Additional exercises. page 63
Problem number: 3.6.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_quadrature]`

$$\boxed{y' - 2\sqrt{y} = 0}$$

With initial conditions

$$[y(1) = 0]$$

✓ Solution by Maple
Time used: 0.0 (sec). Leaf size: 5

```
dsolve([diff(y(x),x)=2*sqrt(y(x)),y(1) = 0],y(x), singsol=all)
```

$$y(x) = 0$$

✓ Solution by Mathematica
Time used: 0.002 (sec). Leaf size: 6

```
DSolve[{y'[x]==2*Sqrt[y[x]],{y[1]==0}},y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to 0$$

3 Chapter 4. SEPARABLE FIRST ORDER EQUATIONS. Additional exercises. page 90

3.1 problem 4.3 (a)

Internal problem ID [13299]

Book: Ordinary Differential Equations. An introduction to the fundamentals. Kenneth B. Howell. second edition. CRC Press. FL, USA. 2020
Section: Chapter 4. SEPARABLE FIRST ORDER EQUATIONS. Additional exercises. page 90
Problem number: 4.3 (a).
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_separable]`

$$\boxed{y' - 3y^2 + \sin(x)\, y^2 = 0}$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 17

```
dsolve(diff(y(x),x)=3*y(x)^2-y(x)^2*sin(x),y(x), singsol=all)
```

$$y(x) = -\frac{1}{\cos(x) - c_1 + 3x}$$

✓ Solution by Mathematica

Time used: 0.156 (sec). Leaf size: 22

```
DSolve[y'[x]==3*y[x]^2-y[x]^2*Sin[x],y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to -\frac{1}{3x + \cos(x) + c_1}$$
$$y(x) \to 0$$

3.2 problem 4.3 (b)

Internal problem ID [13300]

Book: Ordinary Differential Equations. An introduction to the fundamentals. Kenneth B. Howell. second edition. CRC Press. FL, USA. 2020
Section: Chapter 4. SEPARABLE FIRST ORDER EQUATIONS. Additional exercises. page 90
Problem number: 4.3 (b).
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_linear]`

$$\boxed{y' + \sin(x)\, y = 3x}$$

✓ Solution by Maple
Time used: 0.0 (sec). Leaf size: 21

```
dsolve(diff(y(x),x)=3*x-y(x)*sin(x),y(x), singsol=all)
```

$$y(x) = \left(3\left(\int x\, \mathrm{e}^{-\cos(x)}dx\right) + c_1\right)\mathrm{e}^{\cos(x)}$$

✓ Solution by Mathematica
Time used: 0.785 (sec). Leaf size: 31

```
DSolve[y'[x]==3*x-y[x]*Sin[x],y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to e^{\cos(x)}\left(\int_1^x 3e^{-\cos(K[1])}K[1]dK[1] + c_1\right)$$

3.3 problem 4.3 (c)

Internal problem ID [13301]

Book: Ordinary Differential Equations. An introduction to the fundamentals. Kenneth B. Howell. second edition. CRC Press. FL, USA. 2020
Section: Chapter 4. SEPARABLE FIRST ORDER EQUATIONS. Additional exercises. page 90
Problem number: 4.3 (c).
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_rational, _Riccati]`

$$\boxed{y'x - (-y + x)^2 = 0}$$

✓ Solution by Maple
Time used: 0.0 (sec). Leaf size: 67

```
dsolve(x*diff(y(x),x)=(x-y(x))^2,y(x), singsol=all)
```

$$y(x) = \frac{\sqrt{x}\left(\left(\text{BesselK}\left(0,2\sqrt{x}\right)c_1+\text{BesselI}\left(0,2\sqrt{x}\right)\right)\sqrt{x}+\text{BesselK}\left(1,2\sqrt{x}\right)c_1-\text{BesselI}\left(1,2\sqrt{x}\right)\right)}{\text{BesselK}\left(0,2\sqrt{x}\right)c_1+\text{BesselI}\left(0,2\sqrt{x}\right)}$$

✓ Solution by Mathematica
Time used: 0.251 (sec). Leaf size: 121

```
DSolve[x*y'[x]==(x-y[x])^2,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to \frac{xK_0(2\sqrt{x})+\sqrt{x}K_1(2\sqrt{x})+c_1x\,\text{BesselI}\left(0,2\sqrt{x}\right)-c_1\sqrt{x}\,\text{BesselI}\left(1,2\sqrt{x}\right)}{K_0\left(2\sqrt{x}\right)+c_1\,\text{BesselI}\left(0,2\sqrt{x}\right)}$$

$$y(x) \to x - \frac{\sqrt{x}\,\text{BesselI}\left(1,2\sqrt{x}\right)}{\text{BesselI}\left(0,2\sqrt{x}\right)}$$

3.4 problem 4.3 (d)

Internal problem ID [13302]

Book: Ordinary Differential Equations. An introduction to the fundamentals. Kenneth B. Howell. second edition. CRC Press. FL, USA. 2020
Section: Chapter 4. SEPARABLE FIRST ORDER EQUATIONS. Additional exercises. page 90
Problem number: 4.3 (d).
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_quadrature]`

$$\boxed{y' = \sqrt{x^2+1}}$$

✓ Solution by Maple
Time used: 0.0 (sec). Leaf size: 20

```
dsolve(diff(y(x),x)=sqrt(1+x^2),y(x), singsol=all)
```

$$y(x) = \frac{x\sqrt{x^2+1}}{2} + \frac{\operatorname{arcsinh}(x)}{2} + c_1$$

✓ Solution by Mathematica
Time used: 0.024 (sec). Leaf size: 40

```
DSolve[y'[x]==Sqrt[1+x^2],y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to \frac{1}{2}\sqrt{x^2+1}x - \frac{1}{2}\log\left(\sqrt{x^2+1} - x\right) + c_1$$

3.5 problem 4.3 (e)

Internal problem ID [13303]

Book: Ordinary Differential Equations. An introduction to the fundamentals. Kenneth B. Howell. second edition. CRC Press. FL, USA. 2020
Section: Chapter 4. SEPARABLE FIRST ORDER EQUATIONS. Additional exercises. page 90
Problem number: 4.3 (e).
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_quadrature]`

$$\boxed{y' + 4y = 8}$$

✓ Solution by Maple
Time used: 0.0 (sec). Leaf size: 12

```
dsolve(diff(y(x),x)+4*y(x)=8,y(x), singsol=all)
```

$$y(x) = 2 + \mathrm{e}^{-4x} c_1$$

✓ Solution by Mathematica
Time used: 0.035 (sec). Leaf size: 20

```
DSolve[y'[x]+4*y[x]==8,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to 2 + c_1 e^{-4x}$$
$$y(x) \to 2$$

3.6 problem 4.3 (f)

Internal problem ID [13304]

Book: Ordinary Differential Equations. An introduction to the fundamentals. Kenneth B. Howell. second edition. CRC Press. FL, USA. 2020
Section: Chapter 4. SEPARABLE FIRST ORDER EQUATIONS. Additional exercises. page 90
Problem number: 4.3 (f).
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_separable]`

$$\boxed{y' + yx = 4x}$$

✓ Solution by Maple
Time used: 0.0 (sec). Leaf size: 14

```
dsolve(diff(y(x),x)+x*y(x)=4*x,y(x), singsol=all)
```

$$y(x) = 4 + \mathrm{e}^{-\frac{x^2}{2}} c_1$$

✓ Solution by Mathematica
Time used: 0.052 (sec). Leaf size: 24

```
DSolve[y'[x]+x*y[x]==4*x,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to 4 + c_1 e^{-\frac{x^2}{2}}$$
$$y(x) \to 4$$

3.7 problem 4.3 (g)

Internal problem ID [13305]

Book: Ordinary Differential Equations. An introduction to the fundamentals. Kenneth B. Howell. second edition. CRC Press. FL, USA. 2020
Section: Chapter 4. SEPARABLE FIRST ORDER EQUATIONS. Additional exercises. page 90
Problem number: 4.3 (g).
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[[_linear, 'class A']]`

$$\boxed{y' + 4y = x^2}$$

✓ Solution by Maple
Time used: 0.0 (sec). Leaf size: 20

```
dsolve(diff(y(x),x)+4*y(x)=x^2,y(x), singsol=all)
```

$$y(x) = \frac{x^2}{4} - \frac{x}{8} + \frac{1}{32} + \mathrm{e}^{-4x}c_1$$

✓ Solution by Mathematica
Time used: 0.047 (sec). Leaf size: 28

```
DSolve[y'[x]+4*y[x]==x^2,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to \frac{1}{32}\left(8x^2 - 4x + 1\right) + c_1e^{-4x}$$

3.8 problem 4.3 (h)

Internal problem ID [13306]

Book: Ordinary Differential Equations. An introduction to the fundamentals. Kenneth B. Howell. second edition. CRC Press. FL, USA. 2020
Section: Chapter 4. SEPARABLE FIRST ORDER EQUATIONS. Additional exercises. page 90
Problem number: 4.3 (h).
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_separable]`

$$\boxed{y' - yx + 2y = -3x + 6}$$

✓ Solution by Maple
Time used: 0.0 (sec). Leaf size: 15

```
dsolve(diff(y(x),x)=x*y(x)-3*x-2*y(x)+6,y(x), singsol=all)
```

$$y(x) = 3 + \mathrm{e}^{\frac{(x-4)x}{2}} c_1$$

✓ Solution by Mathematica
Time used: 0.079 (sec). Leaf size: 25

```
DSolve[y'[x]==x*y[x]-3*x-2*y[x]+6,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to 3 + c_1 e^{\frac{1}{2}(x-4)x}$$
$$y(x) \to 3$$

3.9 problem 4.3 (i)

Internal problem ID [13307]

Book: Ordinary Differential Equations. An introduction to the fundamentals. Kenneth B. Howell. second edition. CRC Press. FL, USA. 2020
Section: Chapter 4. SEPARABLE FIRST ORDER EQUATIONS. Additional exercises. page 90
Problem number: 4.3 (i).
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[[_homogeneous, ‘class C‘], _dAlembert]`

$$\boxed{y' - \sin(y + x) = 0}$$

✓ Solution by Maple
Time used: 0.062 (sec). Leaf size: 25

```
dsolve(diff(y(x),x)=sin(x+y(x)),y(x), singsol=all)
```

$$y(x) = -x - 2\arctan\left(\frac{c_1 - x - 2}{c_1 - x}\right)$$

✓ Solution by Mathematica

Time used: 35.052 (sec). Leaf size: 541

```
DSolve[y'[x]==Sin[x+y[x]],y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to -2\arccos\left(\frac{(x+c_1)\sin\left(\frac{x}{2}\right)-(x-2+c_1)\cos\left(\frac{x}{2}\right)}{\sqrt{2}\sqrt{x^2+2(-1+c_1)x+2+c_1{}^2-2c_1}}\right)$$
$$y(x) \to 2\arccos\left(\frac{(x+c_1)\sin\left(\frac{x}{2}\right)-(x-2+c_1)\cos\left(\frac{x}{2}\right)}{\sqrt{2}\sqrt{x^2+2(-1+c_1)x+2+c_1{}^2-2c_1}}\right)$$
$$y(x) \to -2\arccos\left(\frac{(x-2+c_1)\cos\left(\frac{x}{2}\right)-(x+c_1)\sin\left(\frac{x}{2}\right)}{\sqrt{2}\sqrt{x^2+2(-1+c_1)x+2+c_1{}^2-2c_1}}\right)$$
$$y(x) \to 2\arccos\left(\frac{(x-2+c_1)\cos\left(\frac{x}{2}\right)-(x+c_1)\sin\left(\frac{x}{2}\right)}{\sqrt{2}\sqrt{x^2+2(-1+c_1)x+2+c_1{}^2-2c_1}}\right)$$
$$y(x) \to -2\arccos\left(\frac{\cos\left(\frac{x}{2}\right)-\sin\left(\frac{x}{2}\right)}{\sqrt{2}}\right)$$
$$y(x) \to 2\arccos\left(\frac{\cos\left(\frac{x}{2}\right)-\sin\left(\frac{x}{2}\right)}{\sqrt{2}}\right)$$
$$y(x) \to -2\arccos\left(\frac{\sin\left(\frac{x}{2}\right)-\cos\left(\frac{x}{2}\right)}{\sqrt{2}}\right)$$
$$y(x) \to 2\arccos\left(\frac{\sin\left(\frac{x}{2}\right)-\cos\left(\frac{x}{2}\right)}{\sqrt{2}}\right)$$
$$y(x) \to -2\arccos\left(\frac{(x-2)\cos\left(\frac{x}{2}\right)-x\sin\left(\frac{x}{2}\right)}{\sqrt{2}\sqrt{x^2-2x+2}}\right)$$
$$y(x) \to 2\arccos\left(\frac{(x-2)\cos\left(\frac{x}{2}\right)-x\sin\left(\frac{x}{2}\right)}{\sqrt{2}\sqrt{x^2-2x+2}}\right)$$
$$y(x) \to -2\arccos\left(\frac{x\sin\left(\frac{x}{2}\right)-(x-2)\cos\left(\frac{x}{2}\right)}{\sqrt{2}\sqrt{x^2-2x+2}}\right)$$
$$y(x) \to 2\arccos\left(\frac{x\sin\left(\frac{x}{2}\right)-(x-2)\cos\left(\frac{x}{2}\right)}{\sqrt{2}\sqrt{x^2-2x+2}}\right)$$

3.10 problem 4.3 (j)

Internal problem ID [13308]

Book: Ordinary Differential Equations. An introduction to the fundamentals. Kenneth B. Howell. second edition. CRC Press. FL, USA. 2020
Section: Chapter 4. SEPARABLE FIRST ORDER EQUATIONS. Additional exercises. page 90
Problem number: 4.3 (j).
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_separable]`

$$\boxed{yy' - \mathrm{e}^{-3y^2+x} = 0}$$

✓ Solution by Maple
Time used: 0.031 (sec). Leaf size: 41

```
dsolve(y(x)*diff(y(x),x)=exp(x-3*y(x)^2),y(x), singsol=all)
```

$$y(x) = -\frac{\sqrt{3}\,\sqrt{\ln(2)+\ln(3)+\ln(\mathrm{e}^x+c_1)}}{3}$$
$$y(x) = \frac{\sqrt{3}\,\sqrt{\ln(2)+\ln(3)+\ln(\mathrm{e}^x+c_1)}}{3}$$

✓ Solution by Mathematica
Time used: 3.781 (sec). Leaf size: 48

```
DSolve[y[x]*y'[x]==Exp[x-3*y[x]^2],y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to -\frac{\sqrt{\log(6(e^x+c_1))}}{\sqrt{3}}$$
$$y(x) \to \frac{\sqrt{\log(6(e^x+c_1))}}{\sqrt{3}}$$

3.11 problem 4.4 (a)

Internal problem ID [13309]

Book: Ordinary Differential Equations. An introduction to the fundamentals. Kenneth B. Howell. second edition. CRC Press. FL, USA. 2020
Section: Chapter 4. SEPARABLE FIRST ORDER EQUATIONS. Additional exercises. page 90
Problem number: 4.4 (a).
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_separable]`

$$\boxed{y' - \frac{x}{y} = 0}$$

✓ Solution by Maple
Time used: 0.0 (sec). Leaf size: 23

```
dsolve(diff(y(x),x)=x/y(x),y(x), singsol=all)
```

$$y(x) = \sqrt{x^2 + c_1}$$
$$y(x) = -\sqrt{x^2 + c_1}$$

✓ Solution by Mathematica
Time used: 0.088 (sec). Leaf size: 35

```
DSolve[y'[x]==x/y[x],y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to -\sqrt{x^2 + 2c_1}$$
$$y(x) \to \sqrt{x^2 + 2c_1}$$

3.12 problem 4.4 (b)

Internal problem ID [13310]

Book: Ordinary Differential Equations. An introduction to the fundamentals. Kenneth B. Howell. second edition. CRC Press. FL, USA. 2020
Section: Chapter 4. SEPARABLE FIRST ORDER EQUATIONS. Additional exercises. page 90
Problem number: 4.4 (b).
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_quadrature]`

$$\boxed{y' - y^2 = 9}$$

✓ Solution by Maple
Time used: 0.015 (sec). Leaf size: 14

```
dsolve(diff(y(x),x)=y(x)^2+9,y(x), singsol=all)
```

$$y(x) = 3\tan(3c_1 + 3x)$$

✓ Solution by Mathematica
Time used: 0.199 (sec). Leaf size: 28

```
DSolve[y'[x]==y[x]^2+9,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to 3\tan(3(x + c_1))$$
$$y(x) \to -3i$$
$$y(x) \to 3i$$

3.13 problem 4.4 (c)

Internal problem ID [13311]

Book: Ordinary Differential Equations. An introduction to the fundamentals. Kenneth B. Howell. second edition. CRC Press. FL, USA. 2020
Section: Chapter 4. SEPARABLE FIRST ORDER EQUATIONS. Additional exercises. page 90
Problem number: 4.4 (c).
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_separable]`

$$\boxed{xyy' - y^2 = 9}$$

✓ Solution by Maple
Time used: 0.015 (sec). Leaf size: 27

```
dsolve(x*y(x)*diff(y(x),x)=y(x)^2+9,y(x), singsol=all)
```

$$y(x) = \sqrt{c_1 x^2 - 9}$$
$$y(x) = -\sqrt{c_1 x^2 - 9}$$

✓ Solution by Mathematica
Time used: 0.401 (sec). Leaf size: 57

```
DSolve[x*y[x]*y'[x]==y[x]^2+9,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to -\sqrt{-9 + e^{2c_1} x^2}$$
$$y(x) \to \sqrt{-9 + e^{2c_1} x^2}$$
$$y(x) \to -3i$$
$$y(x) \to 3i$$

3.14 problem 4.4 (d)

Internal problem ID [13312]

Book: Ordinary Differential Equations. An introduction to the fundamentals. Kenneth B. Howell. second edition. CRC Press. FL, USA. 2020
Section: Chapter 4. SEPARABLE FIRST ORDER EQUATIONS. Additional exercises. page 90
Problem number: 4.4 (d).
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_separable]`

$$\boxed{y' - \frac{1 + y^2}{x^2 + 1} = 0}$$

✓ Solution by Maple
Time used: 0.0 (sec). Leaf size: 9

```
dsolve(diff(y(x),x)=(y(x)^2+1)/(x^2+1),y(x), singsol=all)
```

$$y(x) = \tan\left(\arctan\left(x\right) + c_1\right)$$

✓ Solution by Mathematica
Time used: 0.225 (sec). Leaf size: 25

```
DSolve[y'[x]==(y[x]^2+1)/(x^2+1),y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to \tan(\arctan(x) + c_1)$$
$$y(x) \to -i$$
$$y(x) \to i$$

3.15 problem 4.4 (e)

Internal problem ID [13313]

Book: Ordinary Differential Equations. An introduction to the fundamentals. Kenneth B. Howell. second edition. CRC Press. FL, USA. 2020
Section: Chapter 4. SEPARABLE FIRST ORDER EQUATIONS. Additional exercises. page 90
Problem number: 4.4 (e).
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_separable]`

$$\boxed{\cos(y)\, y' = \sin(x)}$$

✓ Solution by Maple
Time used: 0.0 (sec). Leaf size: 11

```
dsolve(cos(y(x))*diff(y(x),x)=sin(x),y(x), singsol=all)
```

$$y(x) = \arcsin\left(-\cos(x) + c_1\right)$$

✓ Solution by Mathematica
Time used: 0.311 (sec). Leaf size: 13

```
DSolve[Cos[y[x]]*y'[x]==Sin[x],y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to \arcsin(-\cos(x) + c_1)$$

3.16 problem 4.4 (f)

Internal problem ID [13314]

Book: Ordinary Differential Equations. An introduction to the fundamentals. Kenneth B. Howell. second edition. CRC Press. FL, USA. 2020
Section: Chapter 4. SEPARABLE FIRST ORDER EQUATIONS. Additional exercises. page 90
Problem number: 4.4 (f).
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_separable]`

$$\boxed{y' - e^{2x-3y} = 0}$$

✓ Solution by Maple
Time used: 0.0 (sec). Leaf size: 24

```
dsolve(diff(y(x),x)=exp(2*x-3*y(x)),y(x), singsol=all)
```

$$y(x) = \frac{\ln(3)}{3} - \frac{\ln(2)}{3} + \frac{\ln(e^{2x} + 2c_1)}{3}$$

✓ Solution by Mathematica
Time used: 0.855 (sec). Leaf size: 24

```
DSolve[y'[x]==Exp[2*x-3*y[x]],y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to \frac{1}{3}\log\left(\frac{3}{2}(e^{2x} + 2c_1)\right)$$

3.17 problem 4.5 (a)

Internal problem ID [13315]

Book: Ordinary Differential Equations. An introduction to the fundamentals. Kenneth B. Howell. second edition. CRC Press. FL, USA. 2020
Section: Chapter 4. SEPARABLE FIRST ORDER EQUATIONS. Additional exercises. page 90
Problem number: 4.5 (a).
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_separable]`

$$\boxed{y' - \frac{x}{y} = 0}$$

With initial conditions

$$[y(1) = 3]$$

✓ Solution by Maple
Time used: 0.015 (sec). Leaf size: 11

```
dsolve([diff(y(x),x)=x/y(x),y(1) = 3],y(x), singsol=all)
```

$$y(x) = \sqrt{x^2 + 8}$$

✓ Solution by Mathematica
Time used: 0.11 (sec). Leaf size: 14

```
DSolve[{y'[x]==x/y[x],{y[1]==3}},y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to \sqrt{x^2 + 8}$$

3.18 problem 4.5 (b)

Internal problem ID [13316]

Book: Ordinary Differential Equations. An introduction to the fundamentals. Kenneth B. Howell. second edition. CRC Press. FL, USA. 2020
Section: Chapter 4. SEPARABLE FIRST ORDER EQUATIONS. Additional exercises. page 90
Problem number: 4.5 (b).
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type **[_separable]**

$$\boxed{y' - 2yx + y = 2x - 1}$$

With initial conditions

$$[y(0) = 2]$$

✓ Solution by Maple
Time used: 0.0 (sec). Leaf size: 14

```
dsolve([diff(y(x),x)=2*x-1+2*x*y(x)-y(x),y(0) = 2],y(x), singsol=all)
```

$$y(x) = -1 + 3\,\mathrm{e}^{x(-1+x)}$$

✓ Solution by Mathematica
Time used: 0.053 (sec). Leaf size: 16

```
DSolve[{y'[x]==2*x-1+2*x*y[x]-y[x],{y[0]==2}},y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to 3e^{(x-1)x} - 1$$

3.19 problem 4.5 (c)

Internal problem ID [13317]

Book: Ordinary Differential Equations. An introduction to the fundamentals. Kenneth B. Howell. second edition. CRC Press. FL, USA. 2020
Section: Chapter 4. SEPARABLE FIRST ORDER EQUATIONS. Additional exercises. page 90
Problem number: 4.5 (c).
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_separable]`

$$\boxed{yy' - y^2 x = x}$$

With initial conditions

$$[y(0) = -2]$$

✓ Solution by Maple
Time used: 0.062 (sec). Leaf size: 16

```
dsolve([y(x)*diff(y(x),x)=x*y(x)^2+x,y(0) = -2],y(x), singsol=all)
```

$$y(x) = -\sqrt{5\,\mathrm{e}^{x^2} - 1}$$

✓ Solution by Mathematica
Time used: 7.0 (sec). Leaf size: 20

```
DSolve[{y[x]*y'[x]==x*y[x]^2+x,{y[0]==-2}},y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to -\sqrt{5e^{x^2} - 1}$$

3.20 problem 4.5 (d)

Internal problem ID [13318]

Book: Ordinary Differential Equations. An introduction to the fundamentals. Kenneth B. Howell. second edition. CRC Press. FL, USA. 2020
Section: Chapter 4. SEPARABLE FIRST ORDER EQUATIONS. Additional exercises. page 90
Problem number: 4.5 (d).
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[[_1st_order, ‘_with_symmetry_[F(x),G(x)*y+H(x)]‘]]`

$$\boxed{yy' - 3\sqrt{y^2x + 9x} = 0}$$

With initial conditions

$$[y(1) = 4]$$

✓ Solution by Maple
Time used: 0.594 (sec). Leaf size: 17

```
dsolve([y(x)*diff(y(x),x)=3*sqrt(x*y(x)^2+9*x),y(1) = 4],y(x), singsol=all)
```

$$y(x) = 2\sqrt{x^{\frac{3}{2}}\left(x^{\frac{3}{2}} + 3\right)}$$

✓ Solution by Mathematica
Time used: 0.294 (sec). Leaf size: 44

```
DSolve[{y[x]*y'[x]==3*Sqrt[x*y[x]^2+9*x],{y[1]==4}},y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to 2\sqrt{3x^{3/2} + x^3}$$
$$y(x) \to 2\sqrt{-7x^{3/2} + x^3 + 10}$$

3.21 problem 4.6 (a)

Internal problem ID [13319]

Book: Ordinary Differential Equations. An introduction to the fundamentals. Kenneth B. Howell. second edition. CRC Press. FL, USA. 2020
Section: Chapter 4. SEPARABLE FIRST ORDER EQUATIONS. Additional exercises. page 90
Problem number: 4.6 (a).
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_separable]`

$$\boxed{y' - yx = -4x}$$

✓ Solution by Maple
Time used: 0.0 (sec). Leaf size: 14

```
dsolve(diff(y(x),x)=x*y(x)-4*x,y(x), singsol=all)
```

$$y(x) = 4 + c_1 \mathrm{e}^{\frac{x^2}{2}}$$

✓ Solution by Mathematica
Time used: 0.062 (sec). Leaf size: 24

```
DSolve[y'[x]==x*y[x]-4*x,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to 4 + c_1 e^{\frac{x^2}{2}}$$
$$y(x) \to 4$$

3.22 problem 4.6 (b)

Internal problem ID [13320]

Book: Ordinary Differential Equations. An introduction to the fundamentals. Kenneth B. Howell. second edition. CRC Press. FL, USA. 2020
Section: Chapter 4. SEPARABLE FIRST ORDER EQUATIONS. Additional exercises. page 90
Problem number: 4.6 (b).
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_quadrature]`

$$\boxed{y' - 4y = 2}$$

✓ Solution by Maple
Time used: 0.0 (sec). Leaf size: 12

```
dsolve(diff(y(x),x)-4*y(x)=2,y(x), singsol=all)
```

$$y(x) = -\frac{1}{2} + c_1 e^{4x}$$

✓ Solution by Mathematica
Time used: 0.03 (sec). Leaf size: 24

```
DSolve[y'[x]-4*y[x]==2,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to -\frac{1}{2} + c_1 e^{4x}$$
$$y(x) \to -\frac{1}{2}$$

3.23 problem 4.6 (c)

Internal problem ID [13321]

Book: Ordinary Differential Equations. An introduction to the fundamentals. Kenneth B. Howell. second edition. CRC Press. FL, USA. 2020
Section: Chapter 4. SEPARABLE FIRST ORDER EQUATIONS. Additional exercises. page 90
Problem number: 4.6 (c).
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_separable]`

$$\boxed{yy' - y^2x = -9x}$$

✓ Solution by Maple
Time used: 0.015 (sec). Leaf size: 29

```
dsolve(y(x)*diff(y(x),x)=x*y(x)^2-9*x,y(x), singsol=all)
```

$$y(x) = \sqrt{e^{x^2} c_1 + 9}$$
$$y(x) = -\sqrt{e^{x^2} c_1 + 9}$$

✓ Solution by Mathematica
Time used: 1.856 (sec). Leaf size: 53

```
DSolve[y[x]*y'[x]==x*y[x]^2-9*x,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to -\sqrt{9 + e^{x^2+2c_1}}$$
$$y(x) \to \sqrt{9 + e^{x^2+2c_1}}$$
$$y(x) \to -3$$
$$y(x) \to 3$$

3.24 problem 4.6 (d)

Internal problem ID [13322]

Book: Ordinary Differential Equations. An introduction to the fundamentals. Kenneth B. Howell. second edition. CRC Press. FL, USA. 2020
Section: Chapter 4. SEPARABLE FIRST ORDER EQUATIONS. Additional exercises. page 90
Problem number: 4.6 (d).
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_quadrature]`

$$\boxed{y' - \sin(y) = 0}$$

✓ Solution by Maple
Time used: 0.016 (sec). Leaf size: 46

```
dsolve(diff(y(x),x)=sin(y(x)),y(x), singsol=all)
```

$$y(x) = \arctan\left(\frac{2c_1 e^x}{c_1^2 e^{2x} + 1}, \frac{-c_1^2 e^{2x} + 1}{c_1^2 e^{2x} + 1}\right)$$

✓ Solution by Mathematica
Time used: 0.293 (sec). Leaf size: 44

```
DSolve[y'[x]==Sin[y[x]],y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to -\arccos(-\tanh(x + c_1))$$
$$y(x) \to \arccos(-\tanh(x + c_1))$$
$$y(x) \to 0$$
$$y(x) \to -\pi$$
$$y(x) \to \pi$$

3.25 problem 4.6 (e)

Internal problem ID [13323]

Book: Ordinary Differential Equations. An introduction to the fundamentals. Kenneth B. Howell. second edition. CRC Press. FL, USA. 2020
Section: Chapter 4. SEPARABLE FIRST ORDER EQUATIONS. Additional exercises. page 90
Problem number: 4.6 (e).
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_separable]`

$$\boxed{y' - e^{x+y^2} = 0}$$

✓ Solution by Maple
Time used: 0.0 (sec). Leaf size: 15

```
dsolve(diff(y(x),x)=exp(x+y(x)^2),y(x), singsol=all)
```

$$e^x - \frac{\sqrt{\pi}\,\operatorname{erf}(y(x))}{2} + c_1 = 0$$

✓ Solution by Mathematica
Time used: 0.439 (sec). Leaf size: 19

```
DSolve[y'[x]==Exp[x+y[x]^2],y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to \operatorname{erf}^{-1}\left(\frac{2(e^x + c_1)}{\sqrt{\pi}}\right)$$

3.26 problem 4.6 (f)

Internal problem ID [13324]

Book: Ordinary Differential Equations. An introduction to the fundamentals. Kenneth B. Howell. second edition. CRC Press. FL, USA. 2020
Section: Chapter 4. SEPARABLE FIRST ORDER EQUATIONS. Additional exercises. page 90
Problem number: 4.6 (f).
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_quadrature]`

$$\boxed{y' - 200y + 2y^2 = 0}$$

✓ Solution by Maple
Time used: 0.0 (sec). Leaf size: 17

```
dsolve(diff(y(x),x)=200*y(x)-2*y(x)^2,y(x), singsol=all)
```

$$y(x) = \frac{100}{1 + 100\,\mathrm{e}^{-200x} c_1}$$

✓ Solution by Mathematica
Time used: 0.285 (sec). Leaf size: 36

```
DSolve[y'[x]==200*y[x]-2*y[x]^2,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to \frac{100e^{200x}}{e^{200x} + e^{100c_1}}$$
$$y(x) \to 0$$
$$y(x) \to 100$$

3.27 problem 4.7 (a)

Internal problem ID [13325]

Book: Ordinary Differential Equations. An introduction to the fundamentals. Kenneth B. Howell. second edition. CRC Press. FL, USA. 2020
Section: Chapter 4. SEPARABLE FIRST ORDER EQUATIONS. Additional exercises. page 90
Problem number: 4.7 (a).
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_separable]`

$$\boxed{y' - yx = -4x}$$

✓ Solution by Maple
Time used: 0.0 (sec). Leaf size: 14

```
dsolve(diff(y(x),x)=x*y(x)-4*x,y(x), singsol=all)
```

$$y(x) = 4 + c_1 e^{\frac{x^2}{2}}$$

✓ Solution by Mathematica
Time used: 0.111 (sec). Leaf size: 24

```
DSolve[y'[x]==x*y[x]-4*x,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to 4 + c_1 e^{\frac{x^2}{2}}$$
$$y(x) \to 4$$

3.28 problem 4.7 (b)

Internal problem ID [13326]

Book: Ordinary Differential Equations. An introduction to the fundamentals. Kenneth B. Howell. second edition. CRC Press. FL, USA. 2020
Section: Chapter 4. SEPARABLE FIRST ORDER EQUATIONS. Additional exercises. page 90
Problem number: 4.7 (b).
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_separable]`

$$\boxed{y' - yx + 2y = -3x + 6}$$

✓ Solution by Maple
Time used: 0.0 (sec). Leaf size: 15

```
dsolve(diff(y(x),x)=x*y(x)-3*x-2*y(x)+6,y(x), singsol=all)
```

$$y(x) = 3 + \mathrm{e}^{\frac{(x-4)x}{2}} c_1$$

✓ Solution by Mathematica
Time used: 0.077 (sec). Leaf size: 25

```
DSolve[y'[x]==x*y[x]-3*x-2*y[x]+6,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to 3 + c_1 e^{\frac{1}{2}(x-4)x}$$
$$y(x) \to 3$$

3.29 problem 4.7 (c)

Internal problem ID [13327]

Book: Ordinary Differential Equations. An introduction to the fundamentals. Kenneth B. Howell. second edition. CRC Press. FL, USA. 2020
Section: Chapter 4. SEPARABLE FIRST ORDER EQUATIONS. Additional exercises. page 90
Problem number: 4.7 (c).
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_separable]`

$$\boxed{y' - 3y^2 + \sin(x)\, y^2 = 0}$$

✓ Solution by Maple
Time used: 0.0 (sec). Leaf size: 17

```
dsolve(diff(y(x),x)=3*y(x)^2-y(x)^2*sin(x),y(x), singsol=all)
```

$$y(x) = -\frac{1}{\cos(x) - c_1 + 3x}$$

✓ Solution by Mathematica
Time used: 0.173 (sec). Leaf size: 22

```
DSolve[y'[x]==3*y[x]^2-y[x]^2*Sin[x],y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to -\frac{1}{3x + \cos(x) + c_1}$$
$$y(x) \to 0$$

3.30 problem 4.7 (d)

Internal problem ID [13328]

Book: Ordinary Differential Equations. An introduction to the fundamentals. Kenneth B. Howell. second edition. CRC Press. FL, USA. 2020
Section: Chapter 4. SEPARABLE FIRST ORDER EQUATIONS. Additional exercises. page 90
Problem number: 4.7 (d).
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_quadrature]`

$$\boxed{y' - \tan(y) = 0}$$

✓ Solution by Maple
Time used: 0.0 (sec). Leaf size: 9

```
dsolve(diff(y(x),x)=tan(y(x)),y(x), singsol=all)
```

$$y(x) = \arcsin(c_1 e^x)$$

✓ Solution by Mathematica
Time used: 40.257 (sec). Leaf size: 17

```
DSolve[y'[x]==Tan[y[x]],y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to \arcsin\left(e^{x+c_1}\right)$$
$$y(x) \to 0$$

3.31 problem 4.7 (e)

Internal problem ID [13329]

Book: Ordinary Differential Equations. An introduction to the fundamentals. Kenneth B. Howell. second edition. CRC Press. FL, USA. 2020
Section: Chapter 4. SEPARABLE FIRST ORDER EQUATIONS. Additional exercises. page 90
Problem number: 4.7 (e).
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_separable]`

$$\boxed{y' - \frac{y}{x} = 0}$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 7

```
dsolve(diff(y(x),x)=y(x)/x,y(x), singsol=all)
```

$$y(x) = c_1 x$$

✓ Solution by Mathematica

Time used: 0.019 (sec). Leaf size: 14

```
DSolve[y'[x]==y[x]/x,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to c_1 x$$
$$y(x) \to 0$$

3.32 problem 4.7 (f)

Internal problem ID [13330]

Book: Ordinary Differential Equations. An introduction to the fundamentals. Kenneth B. Howell. second edition. CRC Press. FL, USA. 2020

Section: Chapter 4. SEPARABLE FIRST ORDER EQUATIONS. Additional exercises. page 90

Problem number: 4.7 (f).

ODE order: 1.

ODE degree: 1.

CAS Maple gives this as type `[_separable]`

$$\boxed{y' - \frac{6x^2 + 4}{3y^2 - 4y} = 0}$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 418

```
dsolve(diff(y(x),x)=(6*x^2+4)/(3*y(x)^2-4*y(x)),y(x), singsol=all)
```

$$y(x) = \frac{\left(8 + 27x^3 + 27c_1 + 54x + 3\sqrt{81}\sqrt{\left(x^3 + c_1 + 2x + \frac{16}{27}\right)\left(x^3 + c_1 + 2x\right)}\right)^{\frac{2}{3}} + 2\left(8 + 27x^3 + 27c_1 + 54x + \right.}{3\left(8 + 27x^3 + 27c_1 + 54x + 3\sqrt{81}\sqrt{\left(x^3 + c_1 + 2x + \frac{16}{27}\right)\left(x^3 + c\right.}\right.}$$

$$y(x) = -\frac{\left(1 + i\sqrt{3}\right)\left(8 + 27x^3 + 27c_1 + 54x + 3\sqrt{81}\sqrt{\left(x^3 + c_1 + 2x + \frac{16}{27}\right)\left(x^3 + c_1 + 2x\right)}\right)^{\frac{2}{3}} - 4i\sqrt{3} - 4\left(8 + \right.}{6\left(8 + 27x^3 + 27c_1 + 54x + 3\sqrt{81}\sqrt{\left(x^3 + c_1 + 2x \right.}\right.}$$

$$y(x) = \frac{\left(i\sqrt{3} - 1\right)\left(8 + 27x^3 + 27c_1 + 54x + 3\sqrt{81}\sqrt{\left(x^3 + c_1 + 2x + \frac{16}{27}\right)\left(x^3 + c_1 + 2x\right)}\right)^{\frac{2}{3}} - 4i\sqrt{3} + 4\left(8 + 27\right.}{6\left(8 + 27x^3 + 27c_1 + 54x + 3\sqrt{81}\sqrt{\left(x^3 + c_1 + 2x + \frac{1}{2}\right.}\right.}$$

✓ Solution by Mathematica

Time used: 2.967 (sec). Leaf size: 356

```
DSolve[y'[x]==(6*x^2+4)/(3*y[x]^2-4*y[x]),y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to \frac{1}{6}\left(2^{2/3}\sqrt[3]{54x^3+\sqrt{-256+(54x^3+108x+16+27c_1)^2}+108x+16+27c_1} + \frac{8\sqrt[3]{2}}{\sqrt[3]{54x^3+\sqrt{-256+(54x^3+108x+16+27c_1)^2}+108x+16+27c_1}}+4\right)$$

$$y(x) \to \frac{1}{12}\left(i2^{2/3}\left(\sqrt{3}+i\right)\sqrt[3]{54x^3+\sqrt{-256+(54x^3+108x+16+27c_1)^2}+108x+16+27c_1} - \frac{8\sqrt[3]{2}(1+i\sqrt{3})}{\sqrt[3]{54x^3+\sqrt{-256+(54x^3+108x+16+27c_1)^2}+108x+16+27c_1}}+8\right)$$

$$y(x) \to \frac{1}{12}\left(-2^{2/3}\left(1+i\sqrt{3}\right)\sqrt[3]{54x^3+\sqrt{-256+(54x^3+108x+16+27c_1)^2}+108x+16+27c_1} + \frac{8i\sqrt[3]{2}(\sqrt{3}+i)}{\sqrt[3]{54x^3+\sqrt{-256+(54x^3+108x+16+27c_1)^2}+108x+16+27c_1}}+8\right)$$

3.33 problem 4.7 (g)

Internal problem ID [13331]

Book: Ordinary Differential Equations. An introduction to the fundamentals. Kenneth B. Howell. second edition. CRC Press. FL, USA. 2020
Section: Chapter 4. SEPARABLE FIRST ORDER EQUATIONS. Additional exercises. page 90
Problem number: 4.7 (g).
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_separable]`

$$\boxed{(x^2+1)\,y'-y^2=1}$$

✓ Solution by Maple
Time used: 0.0 (sec). Leaf size: 9

```
dsolve((x^2+1)*diff(y(x),x)=y(x)^2+1,y(x), singsol=all)
```

$$y(x)=\tan\left(\arctan\left(x\right)+c_1\right)$$

✓ Solution by Mathematica
Time used: 0.208 (sec). Leaf size: 25

```
DSolve[(x^2+1)*y'[x]==y[x]^2+1,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x)\to \tan(\arctan(x)+c_1)$$
$$y(x)\to -i$$
$$y(x)\to i$$

3.34 problem 4.7 (h)

Internal problem ID [13332]

Book: Ordinary Differential Equations. An introduction to the fundamentals. Kenneth B. Howell. second edition. CRC Press. FL, USA. 2020
Section: Chapter 4. SEPARABLE FIRST ORDER EQUATIONS. Additional exercises. page 90
Problem number: 4.7 (h).
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_separable]`

$$\boxed{(y^2-1)\,y'-4y^2x=0}$$

✓ Solution by Maple
Time used: 0.0 (sec). Leaf size: 59

```
dsolve((y(x)^2-1)*diff(y(x),x)=4*x*y(x)^2,y(x), singsol=all)
```

$$y(x)=x^2+2c_1-\sqrt{x^4+4c_1x^2+4c_1^2-1}$$
$$y(x)=x^2+2c_1+\sqrt{x^4+4c_1x^2+4c_1^2-1}$$

✓ Solution by Mathematica
Time used: 0.273 (sec). Leaf size: 84

```
DSolve[(y[x]^2-1)*y'[x]==4*x*y[x]^2,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x)\to\frac{1}{2}\left(2x^2-\sqrt{4x^4+4c_1x^2-4+c_1{}^2}+c_1\right)$$
$$y(x)\to\frac{1}{2}\left(2x^2+\sqrt{4x^4+4c_1x^2-4+c_1{}^2}+c_1\right)$$
$$y(x)\to 0$$

3.35 problem 4.7 (i)

Internal problem ID [13333]

Book: Ordinary Differential Equations. An introduction to the fundamentals. Kenneth B. Howell. second edition. CRC Press. FL, USA. 2020
Section: Chapter 4. SEPARABLE FIRST ORDER EQUATIONS. Additional exercises. page 90
Problem number: 4.7 (i).
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_quadrature]`

$$\boxed{y' - e^{-y} = 0}$$

✓ Solution by Maple
Time used: 0.0 (sec). Leaf size: 8

```
dsolve(diff(y(x),x)=exp(-y(x)),y(x), singsol=all)
```

$$y(x) = \ln(c_1 + x)$$

✓ Solution by Mathematica
Time used: 0.196 (sec). Leaf size: 10

```
DSolve[y'[x]==Exp[-y[x]],y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \rightarrow \log(x + c_1)$$

3.36 problem 4.7 (j)

Internal problem ID [13334]

Book: Ordinary Differential Equations. An introduction to the fundamentals. Kenneth B. Howell. second edition. CRC Press. FL, USA. 2020
Section: Chapter 4. SEPARABLE FIRST ORDER EQUATIONS. Additional exercises. page 90
Problem number: 4.7 (j).
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type **[_quadrature]**

$$\boxed{y' - \mathrm{e}^{-y} = 1}$$

✓ Solution by Maple
Time used: 0.047 (sec). Leaf size: 11

```
dsolve(diff(y(x),x)=exp(-y(x))+1,y(x), singsol=all)
```

$$y(x) = \ln\left(-1 + c_1 \mathrm{e}^{x}\right)$$

✓ Solution by Mathematica
Time used: 1.163 (sec). Leaf size: 32

```
DSolve[y'[x]==Exp[-y[x]]+1,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to \log\left(-1 + e^{x+c_1}\right)$$
$$y(x) \to -i\pi$$
$$y(x) \to i\pi$$

3.37 problem 4.7 (k)

Internal problem ID [13335]

Book: Ordinary Differential Equations. An introduction to the fundamentals. Kenneth B. Howell. second edition. CRC Press. FL, USA. 2020
Section: Chapter 4. SEPARABLE FIRST ORDER EQUATIONS. Additional exercises. page 90
Problem number: 4.7 (k).
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_separable]`

$$\boxed{y' - 3y^3x = 0}$$

✓ Solution by Maple
Time used: 0.0 (sec). Leaf size: 27

```
dsolve(diff(y(x),x)=3*x*y(x)^3,y(x), singsol=all)
```

$$y(x) = \frac{1}{\sqrt{-3x^2 + c_1}}$$
$$y(x) = -\frac{1}{\sqrt{-3x^2 + c_1}}$$

✓ Solution by Mathematica
Time used: 0.158 (sec). Leaf size: 44

```
DSolve[y'[x]==3*x*y[x]^3,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to -\frac{1}{\sqrt{-3x^2 - 2c_1}}$$
$$y(x) \to \frac{1}{\sqrt{-3x^2 - 2c_1}}$$
$$y(x) \to 0$$

3.38 problem 4.7 (L)

Internal problem ID [13336]

Book: Ordinary Differential Equations. An introduction to the fundamentals. Kenneth B. Howell. second edition. CRC Press. FL, USA. 2020
Section: Chapter 4. SEPARABLE FIRST ORDER EQUATIONS. Additional exercises. page 90
Problem number: 4.7 (L).
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_separable]`

$$y' - \frac{2+\sqrt{x}}{2+\sqrt{y}} = 0$$

✓ Solution by Maple
Time used: 0.0 (sec). Leaf size: 23

```
dsolve(diff(y(x),x)=(2+sqrt(x))/(2+sqrt(y(x))),y(x), singsol=all)
```

$$2x + \frac{2x^{\frac{3}{2}}}{3} - 2y(x) - \frac{2y(x)^{\frac{3}{2}}}{3} + c_1 = 0$$

✓ Solution by Mathematica

Time used: 3.607 (sec). Leaf size: 1162

```
DSolve[y'[x]==(2+Sqrt[x])/(2+Sqrt[y[x]]),y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to \frac{-8x^{3/2} + \left(24x^{5/2} + 12(-6 + c_1)x^{3/2} + \sqrt{(2x^{3/2} + 6x - 8 + 3c_1)\left(2x^{3/2} + 6x + 3c_1\right)^3} + 4x^3 + 36x^2 + 3 \cdots\right.}{2\sqrt[3]{24x^{5/2} + 12(-6 + \cdots}}$$

$$y(x) \to \frac{\left(8 + 8i\sqrt{3}\right)x^{3/2} + i\sqrt{3}\left(24x^{5/2} + 12(-6 + c_1)x^{3/2} + \sqrt{(2x^{3/2} + 6x - 8 + 3c_1)\left(2x^{3/2} + 6x + 3c_1\right)^3} + 4x \cdots\right.}{\cdots}$$

$$y(x) \to \frac{\left(8 - 8i\sqrt{3}\right)x^{3/2} - i\sqrt{3}\left(24x^{5/2} + 12(-6 + c_1)x^{3/2} + \sqrt{(2x^{3/2} + 6x - 8 + 3c_1)\left(2x^{3/2} + 6x + 3c_1\right)^3} + 4x \cdots\right.}{\cdots}$$

3.39 problem 4.7 (m)

Internal problem ID [13337]

Book: Ordinary Differential Equations. An introduction to the fundamentals. Kenneth B. Howell. second edition. CRC Press. FL, USA. 2020
Section: Chapter 4. SEPARABLE FIRST ORDER EQUATIONS. Additional exercises. page 90
Problem number: 4.7 (m).
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_separable]`

$$\boxed{y' - 3x^2y^2 = -3x^2}$$

✓ Solution by Maple
Time used: 0.0 (sec). Leaf size: 14

```
dsolve(diff(y(x),x)-3*x^2*y(x)^2=-3*x^2,y(x), singsol=all)
```

$$y(x) = -\tanh\left(x^3 + 3c_1\right)$$

✓ Solution by Mathematica
Time used: 0.45 (sec). Leaf size: 44

```
DSolve[y'[x]-3*x^2*y[x]^2==-3*x^2,y[x],x,IncludeSingularSolutions -> True]
```

$$\begin{aligned} y(x) &\to \frac{1 - e^{2(x^3+c_1)}}{1 + e^{2(x^3+c_1)}} \\ y(x) &\to -1 \\ y(x) &\to 1 \end{aligned}$$

3.40 problem 4.7 (n)

Internal problem ID [13338]

Book: Ordinary Differential Equations. An introduction to the fundamentals. Kenneth B. Howell. second edition. CRC Press. FL, USA. 2020
Section: Chapter 4. SEPARABLE FIRST ORDER EQUATIONS. Additional exercises. page 90
Problem number: 4.7 (n).
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_separable]`

$$\boxed{y' - 3x^2y^2 = 3x^2}$$

✓ Solution by Maple
Time used: 0.0 (sec). Leaf size: 12

```
dsolve(diff(y(x),x)-3*x^2*y(x)^2=3*x^2,y(x), singsol=all)
```

$$y(x) = \tan\left(x^3 + 3c_1\right)$$

✓ Solution by Mathematica
Time used: 0.152 (sec). Leaf size: 26

```
DSolve[y'[x]-3*x^2*y[x]^2==3*x^2,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to \tan\left(x^3 + c_1\right)$$
$$y(x) \to -i$$
$$y(x) \to i$$

3.41 problem 4.7 (o)

Internal problem ID [13339]

Book: Ordinary Differential Equations. An introduction to the fundamentals. Kenneth B. Howell. second edition. CRC Press. FL, USA. 2020
Section: Chapter 4. SEPARABLE FIRST ORDER EQUATIONS. Additional exercises. page 90
Problem number: 4.7 (o).
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_quadrature]`

$$\boxed{y' - 200y + 2y^2 = 0}$$

✓ Solution by Maple
Time used: 0.0 (sec). Leaf size: 17

```
dsolve(diff(y(x),x)=200*y(x)-2*y(x)^2,y(x), singsol=all)
```

$$y(x) = \frac{100}{1 + 100\,\mathrm{e}^{-200x} c_1}$$

✓ Solution by Mathematica
Time used: 0.258 (sec). Leaf size: 36

```
DSolve[y'[x]==200*y[x]-2*y[x]^2,y[x],x,IncludeSingularSolutions -> True]
```

$$\begin{aligned} y(x) &\to \frac{100e^{200x}}{e^{200x} + e^{100c_1}} \\ y(x) &\to 0 \\ y(x) &\to 100 \end{aligned}$$

3.42 problem 4.8 (a)

Internal problem ID [13340]

Book: Ordinary Differential Equations. An introduction to the fundamentals. Kenneth B. Howell. second edition. CRC Press. FL, USA. 2020
Section: Chapter 4. SEPARABLE FIRST ORDER EQUATIONS. Additional exercises. page 90
Problem number: 4.8 (a).
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_quadrature]`

$$\boxed{y' - 2y = -10}$$

With initial conditions

$$[y(0) = 8]$$

✓ Solution by Maple
Time used: 0.0 (sec). Leaf size: 12

```
dsolve([diff(y(x),x)-2*y(x)=-10,y(0) = 8],y(x), singsol=all)
```

$$y(x) = 5 + 3\,\mathrm{e}^{2x}$$

✓ Solution by Mathematica
Time used: 0.055 (sec). Leaf size: 14

```
DSolve[{y'[x]-2*y[x]==-10,{y[0]==8}},y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to 3e^{2x} + 5$$

3.43 problem 4.8 (b)

Internal problem ID [13341]

Book: Ordinary Differential Equations. An introduction to the fundamentals. Kenneth B. Howell. second edition. CRC Press. FL, USA. 2020
Section: Chapter 4. SEPARABLE FIRST ORDER EQUATIONS. Additional exercises. page 90
Problem number: 4.8 (b).
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_separable]`

$$\boxed{yy' = \sin(x)}$$

With initial conditions

$$[y(0) = -4]$$

✓ Solution by Maple
Time used: 0.188 (sec). Leaf size: 14

```
dsolve([y(x)*diff(y(x),x)=sin(x),y(0) = -4],y(x), singsol=all)
```

$$y(x) = -\sqrt{18 - 2\cos(x)}$$

✓ Solution by Mathematica
Time used: 0.11 (sec). Leaf size: 22

```
DSolve[{y[x]*y'[x]==Sin[x],{y[0]==-4}},y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to -\sqrt{2}\sqrt{9 - \cos(x)}$$

3.44 problem 4.8 (c)

Internal problem ID [13342]

Book: Ordinary Differential Equations. An introduction to the fundamentals. Kenneth B. Howell. second edition. CRC Press. FL, USA. 2020
Section: Chapter 4. SEPARABLE FIRST ORDER EQUATIONS. Additional exercises. page 90
Problem number: 4.8 (c).
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_separable]`

$$\boxed{y' - 2yx + y = 2x - 1}$$

With initial conditions

$$[y(0) = -1]$$

✓ Solution by Maple
Time used: 0.016 (sec). Leaf size: 5

```
dsolve([diff(y(x),x)=2*x-1+2*x*y(x)-y(x),y(0) = -1],y(x), singsol=all)
```

$$y(x) = -1$$

✓ Solution by Mathematica
Time used: 0.001 (sec). Leaf size: 6

```
DSolve[{y'[x]==2*x-1+2*x*y[x]-y[x],{y[0]==-1}},y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to -1$$

3.45 problem 4.8 (d)

Internal problem ID [13343]

Book: Ordinary Differential Equations. An introduction to the fundamentals. Kenneth B. Howell. second edition. CRC Press. FL, USA. 2020
Section: Chapter 4. SEPARABLE FIRST ORDER EQUATIONS. Additional exercises. page 90
Problem number: 4.8 (d).
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_separable]`

$$\boxed{y'x - y^2 + y = 0}$$

With initial conditions

$$[y(2) = 1]$$

✓ Solution by Maple
Time used: 0.0 (sec). Leaf size: 5

```
dsolve([x*diff(y(x),x)=y(x)^2-y(x),y(2) = 1],y(x), singsol=all)
```

$$y(x) = 1$$

✓ Solution by Mathematica
Time used: 0.001 (sec). Leaf size: 6

```
DSolve[{x*y'[x]==y[x]^2-y[x],{y[2]==1}},y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to 1$$

3.46 problem 4.8 (e)

Internal problem ID [13344]

Book: Ordinary Differential Equations. An introduction to the fundamentals. Kenneth B. Howell. second edition. CRC Press. FL, USA. 2020
Section: Chapter 4. SEPARABLE FIRST ORDER EQUATIONS. Additional exercises. page 90
Problem number: 4.8 (e).
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_separable]`

$$\boxed{y'x - y^2 + y = 0}$$

With initial conditions

$$[y(1) = 2]$$

✓ Solution by Maple
Time used: 0.031 (sec). Leaf size: 11

```
dsolve([x*diff(y(x),x)=y(x)^2-y(x),y(1) = 2],y(x), singsol=all)
```

$$y(x) = -\frac{2}{x-2}$$

✓ Solution by Mathematica
Time used: 0.229 (sec). Leaf size: 12

```
DSolve[{x*y'[x]==y[x]^2-y[x],{y[1]==2}},y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to -\frac{2}{x-2}$$

3.47 problem 4.8 (f)

Internal problem ID [13345]

Book: Ordinary Differential Equations. An introduction to the fundamentals. Kenneth B. Howell. second edition. CRC Press. FL, USA. 2020
Section: Chapter 4. SEPARABLE FIRST ORDER EQUATIONS. Additional exercises. page 90
Problem number: 4.8 (f).
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_separable]`

$$\boxed{y' - \frac{y^2 - 1}{yx} = 0}$$

With initial conditions

$$[y(1) = -2]$$

✓ Solution by Maple
Time used: 0.032 (sec). Leaf size: 15

```
dsolve([diff(y(x),x)=(y(x)^2-1)/(x*y(x)),y(1) = -2],y(x), singsol=all)
```

$$y(x) = -\sqrt{3x^2 + 1}$$

✓ Solution by Mathematica
Time used: 0.248 (sec). Leaf size: 18

```
DSolve[{y'[x]==(y[x]^2-1)/(x*y[x]),{y[1]==-2}},y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to -\sqrt{3x^2 + 1}$$

3.48 problem 4.8 (g)

Internal problem ID [13346]

Book: Ordinary Differential Equations. An introduction to the fundamentals. Kenneth B. Howell. second edition. CRC Press. FL, USA. 2020
Section: Chapter 4. SEPARABLE FIRST ORDER EQUATIONS. Additional exercises. page 90
Problem number: 4.8 (g).
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_separable]`

$$\boxed{(y^2-1)\,y'-4yx=0}$$

With initial conditions

$$[y(0)=1]$$

✓ Solution by Maple

Time used: 0.203 (sec). Leaf size: 25

```
dsolve([(y(x)^2-1)*diff(y(x),x)=4*x*y(x),y(0) = 1],y(x), singsol=all)
```

$$y(x)=\frac{e^{-2x^2-\frac{1}{2}}}{\sqrt{-\frac{e^{-4x^2-1}}{\operatorname{LambertW}\left(-e^{-4x^2-1}\right)}}}$$

✓ Solution by Mathematica

Time used: 4.197 (sec). Leaf size: 25

```
DSolve[{(y[x]^2-1)*y'[x]==4*x*y[x],{y[0]==1}},y[x],x,IncludeSingularSolutions -> True]
```

$$y(x)\to -i\sqrt{W\left(-e^{-4x^2-1}\right)}$$

4 Chapter 5. LINEAR FIRST ORDER EQUATIONS. Additional exercises. page 103

4.1 problem 5.1 (a)

Internal problem ID [13347]

Book: Ordinary Differential Equations. An introduction to the fundamentals. Kenneth B. Howell. second edition. CRC Press. FL, USA. 2020
Section: Chapter 5. LINEAR FIRST ORDER EQUATIONS. Additional exercises. page 103
Problem number: 5.1 (a).
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[[_linear, ‘class A‘]]`

$$\boxed{x^2y' + 3x^2y = \sin(x)}$$

✓ Solution by Maple
Time used: 0.0 (sec). Leaf size: 60

```
dsolve(x^2*diff(y(x),x)+3*x^2*y(x)=sin(x),y(x), singsol=all)
```

$$y(x) = \frac{3\left(\left(-\frac{1}{3}+i\right)x\ \mathrm{expIntegral}_1\left((-3-i)\,x\right)+\left(-\frac{1}{3}-i\right)x\ \mathrm{expIntegral}_1\left((-3+i)\,x\right)-\frac{i e^{(3-i)x}}{3}+\frac{i e^{(3+i)x}}{3}+\frac{2c_1}{3}\right.}{2x}$$

✓ Solution by Mathematica
Time used: 0.104 (sec). Leaf size: 68

```
DSolve[x^2*y'[x]+3*x^2*y[x]==Sin[x],y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to \frac{1}{2}e^{-3x}\left((1+3i)\,\mathrm{ExpIntegralEi}((3-i)x) + \frac{(1-3i)x\,\mathrm{ExpIntegralEi}((3+i)x) - ie^{(3-i)x} + ie^{(3+i)x} + 2c_1x}{x}\right)$$

4.2 problem 5.1 (b)

Internal problem ID [13348]

Book: Ordinary Differential Equations. An introduction to the fundamentals. Kenneth B. Howell. second edition. CRC Press. FL, USA. 2020
Section: Chapter 5. LINEAR FIRST ORDER EQUATIONS. Additional exercises. page 103
Problem number: 5.1 (b).
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[‘y=_G(x,y’)‘]`

$$\boxed{y'y^2 + 3x^2y = \sin(x)}$$

✗ Solution by Maple

```
dsolve(y(x)^2*diff(y(x),x)+3*x^2*y(x)=sin(x),y(x), singsol=all)
```

No solution found

✗ Solution by Mathematica
Time used: 0.0 (sec). Leaf size: 0

```
DSolve[y[x]^2*y'[x]+3*x^2*y[x]==Sin[x],y[x],x,IncludeSingularSolutions -> True]
```

Not solved

4.3 problem 5.1 (c)

Internal problem ID [13349]

Book: Ordinary Differential Equations. An introduction to the fundamentals. Kenneth B. Howell. second edition. CRC Press. FL, USA. 2020
Section: Chapter 5. LINEAR FIRST ORDER EQUATIONS. Additional exercises. page 103
Problem number: 5.1 (c).
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_Riccati]`

$$\boxed{y' - y^2 x = \sqrt{x}}$$

✓ Solution by Maple

Time used: 0.015 (sec). Leaf size: 45

```
dsolve(diff(y(x),x)-x*y(x)^2=sqrt(x),y(x), singsol=all)
```

$$y(x) = -\frac{\text{BesselY}\left(-\frac{3}{7}, \frac{4x^{\frac{7}{4}}}{7}\right) c_1 + \text{BesselJ}\left(-\frac{3}{7}, \frac{4x^{\frac{7}{4}}}{7}\right)}{x^{\frac{1}{4}}\left(\text{BesselY}\left(\frac{4}{7}, \frac{4x^{\frac{7}{4}}}{7}\right) c_1 + \text{BesselJ}\left(\frac{4}{7}, \frac{4x^{\frac{7}{4}}}{7}\right)\right)}$$

✓ Solution by Mathematica

Time used: 0.242 (sec). Leaf size: 273

```
DSolve[y'[x]-x*y[x]^2==Sqrt[x],y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to$$
$$-\frac{x^{7/4}\,\text{Gamma}\left(\frac{11}{7}\right)\text{BesselJ}\left(-\frac{3}{7}, \frac{4x^{7/4}}{7}\right) - x^{7/4}\,\text{Gamma}\left(\frac{11}{7}\right)\text{BesselJ}\left(\frac{11}{7}, \frac{4x^{7/4}}{7}\right) + 2\,\text{Gamma}\left(\frac{11}{7}\right)\text{Bessel}}{2x^2\left(\text{Gamma}\left(\frac{11}{7}\right)\text{Bes}\right.}$$

$$y(x) \to -\frac{x^{7/4}\,\text{BesselJ}\left(-\frac{11}{7}, \frac{4x^{7/4}}{7}\right) - x^{7/4}\,\text{BesselJ}\left(\frac{3}{7}, \frac{4x^{7/4}}{7}\right) + 2\,\text{BesselJ}\left(-\frac{4}{7}, \frac{4x^{7/4}}{7}\right)}{2x^2\,\text{BesselJ}\left(-\frac{4}{7}, \frac{4x^{7/4}}{7}\right)}$$

4.4 problem 5.1 (d)

Internal problem ID [13350]

Book: Ordinary Differential Equations. An introduction to the fundamentals. Kenneth B. Howell. second edition. CRC Press. FL, USA. 2020
Section: Chapter 5. LINEAR FIRST ORDER EQUATIONS. Additional exercises. page 103
Problem number: 5.1 (d).
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_Riccati]`

$$\boxed{y' - (yx + 3y)^2 = 1}$$

✓ Solution by Maple

Time used: 0.015 (sec). Leaf size: 57

```
dsolve(diff(y(x),x)=1+(x*y(x)+3*y(x))^2,y(x), singsol=all)
```

$$y(x) = \frac{-\operatorname{BesselY}\left(-\frac{1}{4}, \frac{(x+3)^2}{2}\right) c_1 - \operatorname{BesselJ}\left(-\frac{1}{4}, \frac{(x+3)^2}{2}\right)}{\left(\operatorname{BesselY}\left(\frac{3}{4}, \frac{(x+3)^2}{2}\right) c_1 + \operatorname{BesselJ}\left(\frac{3}{4}, \frac{(x+3)^2}{2}\right)\right)(x+3)}$$

✓ Solution by Mathematica

Time used: 0.45 (sec). Leaf size: 351

```
DSolve[y'[x]==1+(x*y[x]+3*y[x])^2,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to -\frac{((x+3)^3)^{2/3}\operatorname{Gamma}\left(\frac{7}{4}\right)\operatorname{BesselJ}\left(-\frac{1}{4}, \frac{1}{2}((x+3)^3)^{2/3}\right) + 3\operatorname{Gamma}\left(\frac{7}{4}\right)\operatorname{BesselJ}\left(\frac{3}{4}, \frac{1}{2}((x+3)^3)^{2/3}\right) -}{}$$

$$y(x) \to \frac{-((x+3)^3)^{2/3}\operatorname{BesselJ}\left(-\frac{7}{4}, \frac{1}{2}((x+3)^3)^{2/3}\right) - 3\operatorname{BesselJ}\left(-\frac{3}{4}, \frac{1}{2}((x+3)^3)^{2/3}\right) + ((x+3)^3)^{2/3}\operatorname{BesselJ}\left(\right.}{2(x+3)^3\operatorname{BesselJ}\left(-\frac{3}{4}, \frac{1}{2}((x+3)^3)^{2/3}\right)}$$

4.5 problem 5.1 (e)

Internal problem ID [13351]

Book: Ordinary Differential Equations. An introduction to the fundamentals. Kenneth B. Howell. second edition. CRC Press. FL, USA. 2020
Section: Chapter 5. LINEAR FIRST ORDER EQUATIONS. Additional exercises. page 103
Problem number: 5.1 (e).
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_linear]`

$$\boxed{y' - yx - 3y = 1}$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 35

```
dsolve(diff(y(x),x)=1+x*y(x)+3*y(x),y(x), singsol=all)
```

$$y(x) = \frac{\left(\sqrt{\pi}\,\mathrm{e}^{\frac{9}{2}}\sqrt{2}\,\mathrm{erf}\left(\frac{\sqrt{2}\,(x+3)}{2}\right) + 2c_1\right)\mathrm{e}^{\frac{x(x+6)}{2}}}{2}$$

✓ Solution by Mathematica

Time used: 0.054 (sec). Leaf size: 47

```
DSolve[y'[x]==1+x*y[x]+3*y[x],y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to \frac{1}{2}e^{\frac{1}{2}x(x+6)}\left(e^{9/2}\sqrt{2\pi}\mathrm{erf}\left(\frac{x+3}{\sqrt{2}}\right) + 2c_1\right)$$

4.6 problem 5.1 (f)

Internal problem ID [13352]

Book: Ordinary Differential Equations. An introduction to the fundamentals. Kenneth B. Howell. second edition. CRC Press. FL, USA. 2020
Section: Chapter 5. LINEAR FIRST ORDER EQUATIONS. Additional exercises. page 103
Problem number: 5.1 (f).
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_quadrature]`

$$\boxed{y' - 4y = 8}$$

✓ Solution by Maple
Time used: 0.015 (sec). Leaf size: 12

```
dsolve(diff(y(x),x)=4*y(x)+8,y(x), singsol=all)
```

$$y(x) = -2 + c_1 \mathrm{e}^{4x}$$

✓ Solution by Mathematica
Time used: 0.032 (sec). Leaf size: 20

```
DSolve[y'[x]==4*y[x]+8,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to -2 + c_1 e^{4x}$$
$$y(x) \to -2$$

4.7 problem 5.1 (g)

Internal problem ID [13353]

Book: Ordinary Differential Equations. An introduction to the fundamentals. Kenneth B. Howell. second edition. CRC Press. FL, USA. 2020
Section: Chapter 5. LINEAR FIRST ORDER EQUATIONS. Additional exercises. page 103
Problem number: 5.1 (g).
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_quadrature]`

$$\boxed{y' = e^{2x}}$$

✓ Solution by Maple
Time used: 0.0 (sec). Leaf size: 12

```
dsolve(diff(y(x),x)-exp(2*x)=0,y(x), singsol=all)
```

$$y(x) = \frac{e^{2x}}{2} + c_1$$

✓ Solution by Mathematica
Time used: 0.008 (sec). Leaf size: 17

```
DSolve[y'[x]-Exp[2*x]==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to \frac{e^{2x}}{2} + c_1$$

4.8 problem 5.1 (h)

Internal problem ID [13354]

Book: Ordinary Differential Equations. An introduction to the fundamentals. Kenneth B. Howell. second edition. CRC Press. FL, USA. 2020
Section: Chapter 5. LINEAR FIRST ORDER EQUATIONS. Additional exercises. page 103
Problem number: 5.1 (h).
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_separable]`

$$\boxed{y' - \sin(x)\, y = 0}$$

✓ Solution by Maple
Time used: 0.0 (sec). Leaf size: 11

```
dsolve(diff(y(x),x)=y(x)*sin(x),y(x), singsol=all)
```

$$y(x) = \mathrm{e}^{-\cos(x)} c_1$$

✓ Solution by Mathematica
Time used: 0.029 (sec). Leaf size: 19

```
DSolve[y'[x]==y[x]*Sin[x],y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to c_1 e^{-\cos(x)}$$
$$y(x) \to 0$$

4.9 problem 5.1 (i)

Internal problem ID [13355]

Book: Ordinary Differential Equations. An introduction to the fundamentals. Kenneth B. Howell. second edition. CRC Press. FL, USA. 2020
Section: Chapter 5. LINEAR FIRST ORDER EQUATIONS. Additional exercises. page 103
Problem number: 5.1 (i).
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_quadrature]`

$$\boxed{y' + 4y - y^3 = 0}$$

✓ Solution by Maple
Time used: 0.016 (sec). Leaf size: 33

```
dsolve(diff(y(x),x)+4*y(x)=y(x)^3,y(x), singsol=all)
```

$$y(x) = -\frac{2}{\sqrt{4\,e^{8x}c_1 + 1}}$$
$$y(x) = \frac{2}{\sqrt{4\,e^{8x}c_1 + 1}}$$

✓ Solution by Mathematica
Time used: 0.728 (sec). Leaf size: 56

```
DSolve[y'[x]+4*y[x]==y[x]^3,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to -\frac{2}{\sqrt{1 + e^{8(x+c_1)}}}$$
$$y(x) \to \frac{2}{\sqrt{1 + e^{8(x+c_1)}}}$$
$$y(x) \to -2$$
$$y(x) \to 0$$
$$y(x) \to 2$$

4.10 problem 5.1 (j)

Internal problem ID [13356]

Book: Ordinary Differential Equations. An introduction to the fundamentals. Kenneth B. Howell. second edition. CRC Press. FL, USA. 2020
Section: Chapter 5. LINEAR FIRST ORDER EQUATIONS. Additional exercises. page 103
Problem number: 5.1 (j).
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_linear]`

$$\boxed{y'x - 827y = -\cos\left(x^2\right)}$$

✓ Solution by Maple
Time used: 0.0 (sec). Leaf size: 22

```
dsolve(x*diff(y(x),x)+cos(x^2)=827*y(x),y(x), singsol=all)
```

$$y(x) = \left(-\left(\int \frac{\cos\left(x^2\right)}{x^{828}}dx\right) + c_1\right)x^{827}$$

✓ Solution by Mathematica
Time used: 6.501 (sec). Leaf size: 2119

```
DSolve[x*y'[x]+Cos[x^2]==827*y[x],y[x],x,IncludeSingularSolutions -> True]
```

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4.11 problem 5.2 (a)

Internal problem ID [13357]

Book: Ordinary Differential Equations. An introduction to the fundamentals. Kenneth B. Howell. second edition. CRC Press. FL, USA. 2020
Section: Chapter 5. LINEAR FIRST ORDER EQUATIONS. Additional exercises. page 103
Problem number: 5.2 (a).
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type **[_quadrature]**

$$\boxed{y' + 2y = 6}$$

✓ Solution by Maple
Time used: 0.0 (sec). Leaf size: 12

```
dsolve(diff(y(x),x)+2*y(x)=6,y(x), singsol=all)
```

$$y(x) = 3 + \mathrm{e}^{-2x} c_1$$

✓ Solution by Mathematica
Time used: 0.03 (sec). Leaf size: 20

```
DSolve[y'[x]+2*y[x]==6,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to 3 + c_1 e^{-2x}$$
$$y(x) \to 3$$

4.12 problem 5.2 (b)

Internal problem ID [13358]

Book: Ordinary Differential Equations. An introduction to the fundamentals. Kenneth B. Howell. second edition. CRC Press. FL, USA. 2020
Section: Chapter 5. LINEAR FIRST ORDER EQUATIONS. Additional exercises. page 103
Problem number: 5.2 (b).
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[[_linear, 'class A']]`

$$\boxed{y' + 2y = 20\,\mathrm{e}^{3x}}$$

✓ Solution by Maple
Time used: 0.0 (sec). Leaf size: 17

```
dsolve(diff(y(x),x)+2*y(x)=20*exp(3*x),y(x), singsol=all)
```

$$y(x) = \left(4\,\mathrm{e}^{5x} + c_1\right)\mathrm{e}^{-2x}$$

✓ Solution by Mathematica
Time used: 0.051 (sec). Leaf size: 21

```
DSolve[y'[x]+2*y[x]==20*Exp[3*x],y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to e^{-2x}\left(4e^{5x} + c_1\right)$$

4.13 problem 5.2 (c)

Internal problem ID [13359]

Book: Ordinary Differential Equations. An introduction to the fundamentals. Kenneth B. Howell. second edition. CRC Press. FL, USA. 2020
Section: Chapter 5. LINEAR FIRST ORDER EQUATIONS. Additional exercises. page 103
Problem number: 5.2 (c).
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[[_linear, 'class A']]`

$$\boxed{y' - 4y = 16x}$$

✓ Solution by Maple
Time used: 0.0 (sec). Leaf size: 15

```
dsolve(diff(y(x),x)=4*y(x)+16*x,y(x), singsol=all)
```

$$y(x) = -4x - 1 + c_1 e^{4x}$$

✓ Solution by Mathematica
Time used: 0.042 (sec). Leaf size: 18

```
DSolve[y'[x]==4*y[x]+16*x,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to -4x + c_1 e^{4x} - 1$$

4.14 problem 5.2 (d)

Internal problem ID [13360]

Book: Ordinary Differential Equations. An introduction to the fundamentals. Kenneth B. Howell. second edition. CRC Press. FL, USA. 2020
Section: Chapter 5. LINEAR FIRST ORDER EQUATIONS. Additional exercises. page 103
Problem number: 5.2 (d).
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_separable]`

$$\boxed{y' - 2yx = x}$$

✓ Solution by Maple
Time used: 0.0 (sec). Leaf size: 12

```
dsolve(diff(y(x),x)-2*x*y(x)=x,y(x), singsol=all)
```

$$y(x) = -\frac{1}{2} + \mathrm{e}^{x^2} c_1$$

✓ Solution by Mathematica
Time used: 0.041 (sec). Leaf size: 24

```
DSolve[y'[x]-2*x*y[x]==x,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to -\frac{1}{2} + c_1 e^{x^2}$$
$$y(x) \to -\frac{1}{2}$$

4.15 problem 5.2 (e)

Internal problem ID [13361]

Book: Ordinary Differential Equations. An introduction to the fundamentals. Kenneth B. Howell. second edition. CRC Press. FL, USA. 2020
Section: Chapter 5. LINEAR FIRST ORDER EQUATIONS. Additional exercises. page 103
Problem number: 5.2 (e).
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_linear]`

$$\boxed{y'x + 3y = 10x^2}$$

✓ Solution by Maple
Time used: 0.0 (sec). Leaf size: 15

```
dsolve(x*diff(y(x),x)+3*y(x)-10*x^2=0,y(x), singsol=all)
```

$$y(x) = \frac{2x^5 + c_1}{x^3}$$

✓ Solution by Mathematica
Time used: 0.026 (sec). Leaf size: 17

```
DSolve[x*y'[x]+3*y[x]-10*x^2==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to \frac{2x^5 + c_1}{x^3}$$

4.16 problem 5.2 (f)

Internal problem ID [13362]

Book: Ordinary Differential Equations. An introduction to the fundamentals. Kenneth B. Howell. second edition. CRC Press. FL, USA. 2020
Section: Chapter 5. LINEAR FIRST ORDER EQUATIONS. Additional exercises. page 103
Problem number: 5.2 (f).
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_linear]`

$$\boxed{x^2y' + 2yx = \sin(x)}$$

✓ Solution by Maple
Time used: 0.0 (sec). Leaf size: 14

```
dsolve(x^2*diff(y(x),x)+2*x*y(x)=sin(x),y(x), singsol=all)
```

$$y(x) = \frac{-\cos(x) + c_1}{x^2}$$

✓ Solution by Mathematica
Time used: 0.033 (sec). Leaf size: 16

```
DSolve[x^2*y'[x]+2*x*y[x]==Sin[x],y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to \frac{-\cos(x) + c_1}{x^2}$$

4.17 problem 5.2 (g)

Internal problem ID [13363]

Book: Ordinary Differential Equations. An introduction to the fundamentals. Kenneth B. Howell. second edition. CRC Press. FL, USA. 2020
Section: Chapter 5. LINEAR FIRST ORDER EQUATIONS. Additional exercises. page 103
Problem number: 5.2 (g).
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_linear]`

$$\boxed{y'x - 3y = \sqrt{x}}$$

✓ Solution by Maple
Time used: 0.0 (sec). Leaf size: 15

```
dsolve(x*diff(y(x),x)=sqrt(x)+3*y(x),y(x), singsol=all)
```

$$y(x) = -\frac{2\sqrt{x}}{5} + c_1 x^3$$

✓ Solution by Mathematica
Time used: 0.027 (sec). Leaf size: 21

```
DSolve[x*y'[x]==Sqrt[x]+3*y[x],y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to -\frac{2\sqrt{x}}{5} + c_1 x^3$$

4.18 problem 5.2 (h)

Internal problem ID [13364]

Book: Ordinary Differential Equations. An introduction to the fundamentals. Kenneth B. Howell. second edition. CRC Press. FL, USA. 2020
Section: Chapter 5. LINEAR FIRST ORDER EQUATIONS. Additional exercises. page 103
Problem number: 5.2 (h).
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_linear]`

$$\boxed{\cos(x)\,y' + \sin(x)\,y = \cos(x)^2}$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 10

```
dsolve(cos(x)*diff(y(x),x)+sin(x)*y(x)=cos(x)^2,y(x), singsol=all)
```

$$y(x) = (c_1 + x)\cos(x)$$

✓ Solution by Mathematica

Time used: 0.058 (sec). Leaf size: 12

```
DSolve[Cos[x]*y'[x]+Sin[x]*y[x]==Cos[x]^2,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to (x + c_1)\cos(x)$$

4.19 problem 5.2 (i)

Internal problem ID [13365]

Book: Ordinary Differential Equations. An introduction to the fundamentals. Kenneth B. Howell. second edition. CRC Press. FL, USA. 2020
Section: Chapter 5. LINEAR FIRST ORDER EQUATIONS. Additional exercises. page 103
Problem number: 5.2 (i).
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_linear]`

$$\boxed{y'x + (2+5x)\,y = \frac{20}{x}}$$

✓ Solution by Maple
Time used: 0.0 (sec). Leaf size: 16

```
dsolve(x*diff(y(x),x)+(5*x+2)*y(x)=20/x,y(x), singsol=all)
```

$$y(x) = \frac{e^{-5x}c_1 + 4}{x^2}$$

✓ Solution by Mathematica
Time used: 0.04 (sec). Leaf size: 12

```
DSolve[Cos[x]*y'[x]+Sin[x]*y[x]==Cos[x]^2,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to (x + c_1)\cos(x)$$

4.20 problem 5.2 (j)

Internal problem ID [13366]

Book: Ordinary Differential Equations. An introduction to the fundamentals. Kenneth B. Howell. second edition. CRC Press. FL, USA. 2020
Section: Chapter 5. LINEAR FIRST ORDER EQUATIONS. Additional exercises. page 103
Problem number: 5.2 (j).
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_linear]`

$$2\sqrt{x}\, y' + y = 2x\,\mathrm{e}^{-\sqrt{x}}$$

✓ Solution by Maple
Time used: 0.0 (sec). Leaf size: 21

```
dsolve(2*sqrt(x)*diff(y(x),x)+y(x)=2*x*exp(-sqrt(x)),y(x), singsol=all)
```

$$y(x) = \frac{\left(2x^{\frac{3}{2}} + 3c_1\right)\mathrm{e}^{-\sqrt{x}}}{3}$$

✓ Solution by Mathematica
Time used: 0.074 (sec). Leaf size: 30

```
DSolve[2*Sqrt[x]*y'[x]+y[x]==2*x*Exp[-Sqrt[x]],y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to \frac{1}{3}e^{-\sqrt{x}}\left(2x^{3/2} + 3c_1\right)$$

4.21 problem 5.3 (a)

Internal problem ID [13367]

Book: Ordinary Differential Equations. An introduction to the fundamentals. Kenneth B. Howell. second edition. CRC Press. FL, USA. 2020
Section: Chapter 5. LINEAR FIRST ORDER EQUATIONS. Additional exercises. page 103
Problem number: 5.3 (a).
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_quadrature]`

$$\boxed{y' - 3y = 6}$$

With initial conditions

$$[y(0) = 5]$$

✓ Solution by Maple
Time used: 0.016 (sec). Leaf size: 12

```
dsolve([diff(y(x),x)-3*y(x)=6,y(0) = 5],y(x), singsol=all)
```

$$y(x) = -2 + 7\,\mathrm{e}^{3x}$$

✓ Solution by Mathematica
Time used: 0.033 (sec). Leaf size: 14

```
DSolve[{y'[x]-3*y[x]==6,{y[0]==5}},y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to 7e^{3x} - 2$$

4.22 problem 5.3 (b)

Internal problem ID [13368]

Book: Ordinary Differential Equations. An introduction to the fundamentals. Kenneth B. Howell. second edition. CRC Press. FL, USA. 2020
Section: Chapter 5. LINEAR FIRST ORDER EQUATIONS. Additional exercises. page 103
Problem number: 5.3 (b).
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_quadrature]`

$$\boxed{y' - 3y = 6}$$

With initial conditions

$$[y(0) = -2]$$

✓ Solution by Maple
Time used: 0.016 (sec). Leaf size: 5

```
dsolve([diff(y(x),x)-3*y(x)=6,y(0) = -2],y(x), singsol=all)
```

$$y(x) = -2$$

✓ Solution by Mathematica
Time used: 0.001 (sec). Leaf size: 6

```
DSolve[{y'[x]-3*y[x]==6,{y[0]==-2}},y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to -2$$

4.23 problem 5.3 (c)

Internal problem ID [13369]

Book: Ordinary Differential Equations. An introduction to the fundamentals. Kenneth B. Howell. second edition. CRC Press. FL, USA. 2020
Section: Chapter 5. LINEAR FIRST ORDER EQUATIONS. Additional exercises. page 103
Problem number: 5.3 (c).
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[[_linear, 'class A']]`

$$\boxed{y' + 5y = \mathrm{e}^{-3x}}$$

With initial conditions

$$[y(0) = 0]$$

✓ Solution by Maple
Time used: 0.015 (sec). Leaf size: 16

```
dsolve([diff(y(x),x)+5*y(x)=exp(-3*x),y(0) = 0],y(x), singsol=all)
```

$$y(x) = \frac{(\mathrm{e}^{2x} - 1)\,\mathrm{e}^{-5x}}{2}$$

✓ Solution by Mathematica
Time used: 0.051 (sec). Leaf size: 21

```
DSolve[{y'[x]+5*y[x]==Exp[-3*x],{y[0]==0}},y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to \frac{1}{2}e^{-5x}\left(e^{2x} - 1\right)$$

4.24 problem 5.3 (d)

Internal problem ID [13370]

Book: Ordinary Differential Equations. An introduction to the fundamentals. Kenneth B. Howell. second edition. CRC Press. FL, USA. 2020
Section: Chapter 5. LINEAR FIRST ORDER EQUATIONS. Additional exercises. page 103
Problem number: 5.3 (d).
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_linear]`

$$\boxed{y'x + 3y = 20x^2}$$

With initial conditions

$$[y(1) = 10]$$

✓ Solution by Maple
Time used: 0.016 (sec). Leaf size: 15

```
dsolve([x*diff(y(x),x)+3*y(x)=20*x^2,y(1) = 10],y(x), singsol=all)
```

$$y(x) = \frac{4x^5 + 6}{x^3}$$

✓ Solution by Mathematica
Time used: 0.027 (sec). Leaf size: 16

```
DSolve[{x*y'[x]+3*y[x]==20*x^2,{y[1]==10}},y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to \frac{4x^5 + 6}{x^3}$$

4.25 problem 5.3 (e)

Internal problem ID [13371]

Book: Ordinary Differential Equations. An introduction to the fundamentals. Kenneth B. Howell. second edition. CRC Press. FL, USA. 2020
Section: Chapter 5. LINEAR FIRST ORDER EQUATIONS. Additional exercises. page 103
Problem number: 5.3 (e).
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_linear]`

$$\boxed{y'x - y = \cos(x)\, x^2}$$

With initial conditions

$$\left[y\left(\frac{\pi}{2}\right) = 0\right]$$

✓ Solution by Maple
Time used: 0.015 (sec). Leaf size: 10

```
dsolve([x*diff(y(x),x)=y(x)+x^2*cos(x),y(1/2*Pi) = 0],y(x), singsol=all)
```

$$y(x) = (\sin(x) - 1)\, x$$

✓ Solution by Mathematica
Time used: 0.043 (sec). Leaf size: 11

```
DSolve[{x*y'[x]==y[x]+x^2*Cos[x],{y[Pi/2]==0}},y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to x(\sin(x) - 1)$$

4.26 problem 5.3 (f)

Internal problem ID [13372]

Book: Ordinary Differential Equations. An introduction to the fundamentals. Kenneth B. Howell. second edition. CRC Press. FL, USA. 2020
Section: Chapter 5. LINEAR FIRST ORDER EQUATIONS. Additional exercises. page 103
Problem number: 5.3 (f).
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_linear]`

$$\boxed{(x^2+1)\, y' - x(3+3x^2-y) = 0}$$

With initial conditions

$$[y(2) = 8]$$

✓ Solution by Maple
Time used: 0.016 (sec). Leaf size: 21

```
dsolve([(1+x^2)*diff(y(x),x)=x*(3+3*x^2-y(x)),y(2) = 8],y(x), singsol=all)
```

$$y(x) = x^2 + 1 + \frac{3\sqrt{5}}{\sqrt{x^2+1}}$$

✓ Solution by Mathematica
Time used: 0.038 (sec). Leaf size: 26

```
DSolve[{(1+x^2)*y'[x]==x*(3+3*x^2-y[x]),{y[2]==8}},y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to x^2 + \frac{3\sqrt{5}}{\sqrt{x^2+1}} + 1$$

4.27 problem 5.4 (a)

Internal problem ID [13373]

Book: Ordinary Differential Equations. An introduction to the fundamentals. Kenneth B. Howell. second edition. CRC Press. FL, USA. 2020
Section: Chapter 5. LINEAR FIRST ORDER EQUATIONS. Additional exercises. page 103
Problem number: 5.4 (a).
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_linear]`

$$\boxed{y' + 6yx = \sin(x)}$$

With initial conditions

$$[y(0) = 4]$$

✓ Solution by Maple

Time used: 0.11 (sec). Leaf size: 77

```
dsolve([diff(y(x),x)+6*x*y(x)=sin(x),y(0) = 4],y(x), singsol=all)
```

$$y(x) = 4\,\mathrm{e}^{-3x^2} - \frac{\sqrt{3}\,\sqrt{\pi}\,\mathrm{e}^{-3x^2+\frac{1}{12}}\,\mathrm{erf}\left(\frac{\sqrt{3}}{6}\right)}{6} + \frac{\sqrt{3}\,\sqrt{\pi}\,\mathrm{e}^{-3x^2+\frac{1}{12}}\,\mathrm{erf}\left(\frac{\sqrt{3}\,(6ix+1)}{6}\right)}{12} - \frac{\sqrt{3}\,\sqrt{\pi}\,\mathrm{e}^{-3x^2+\frac{1}{12}}\,\mathrm{erf}\left(\frac{\sqrt{3}\,(6ix-1)}{6}\right)}{12}$$

✓ Solution by Mathematica

Time used: 0.072 (sec). Leaf size: 105

```
DSolve[{y'[x]+6*x*y[x]==Sin[x],{y[0]==4}},y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to \frac{1}{12}e^{-3x^2}\left(\sqrt[12]{e}\sqrt{3\pi}\mathrm{erf}\left(\frac{1+6ix}{2\sqrt{3}}\right) - 2\sqrt[12]{e}\sqrt{3\pi}\mathrm{erf}\left(\frac{1}{2\sqrt{3}}\right) - i\sqrt[12]{e}\sqrt{3\pi}\mathrm{erfi}\left(\frac{6x+i}{2\sqrt{3}}\right) + 48\right)$$

4.28 problem 5.4 (b)

Internal problem ID [13374]

Book: Ordinary Differential Equations. An introduction to the fundamentals. Kenneth B. Howell. second edition. CRC Press. FL, USA. 2020
Section: Chapter 5. LINEAR FIRST ORDER EQUATIONS. Additional exercises. page 103
Problem number: 5.4 (b).
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_linear]`

$$x^2y' + yx = \sqrt{x}\,\sin(x)$$

With initial conditions

$$[y(2) = 5]$$

✓ Solution by Maple

Time used: 0.031 (sec). Leaf size: 42

```
dsolve([x^2*diff(y(x),x)+x*y(x)=sqrt(x)*sin(x),y(2) = 5],y(x), singsol=all)
```

$$y(x) = \frac{\sqrt{\pi}\,\sqrt{2}\ \mathrm{FresnelS}\left(\frac{\sqrt{2}\,\sqrt{x}}{\sqrt{\pi}}\right) + 10 - \sqrt{\pi}\,\sqrt{2}\ \mathrm{FresnelS}\left(\frac{2}{\sqrt{\pi}}\right)}{x}$$

✓ Solution by Mathematica

Time used: 0.093 (sec). Leaf size: 185

```
DSolve[{x^2*y'[x]+x*y[x]==Sqrt[x]*Sin[x],{y[2]==5}},y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to \frac{2i\sqrt{\pi}x\mathrm{erf}\left(\sqrt{ix}\right) - (1+i)\sqrt{2\pi}\mathrm{erf}(1+i)\sqrt{ix}\sqrt{x} - 2i\sqrt{\pi}x\mathrm{erfi}\left(\sqrt{ix}\right) + (1+i)\sqrt{2\pi}\mathrm{erfi}(1+i)\sqrt{ix}\sqrt{x} - 2}{4\sqrt{ix}x^{3/2}}$$

4.29 problem 5.4 (c)

Internal problem ID [13375]

Book: Ordinary Differential Equations. An introduction to the fundamentals. Kenneth B. Howell. second edition. CRC Press. FL, USA. 2020
Section: Chapter 5. LINEAR FIRST ORDER EQUATIONS. Additional exercises. page 103
Problem number: 5.4 (c).
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_linear]`

$$\boxed{y'x - y = x^2 e^{-x^2}}$$

With initial conditions

$$[y(3) = 8]$$

✓ Solution by Maple
Time used: 0.094 (sec). Leaf size: 20

```
dsolve([x*diff(y(x),x)-y(x)=x^2*exp(-x^2),y(3) = 8],y(x), singsol=all)
```

$$y(x) = -\frac{\left(-\frac{16}{3} + (\operatorname{erf}(3) - \operatorname{erf}(x))\sqrt{\pi}\right)x}{2}$$

✓ Solution by Mathematica
Time used: 0.079 (sec). Leaf size: 30

```
DSolve[{x*y'[x]-y[x]==x^2*Exp[-x^2],{y[3]==8}},y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to \frac{1}{6}x\left(3\sqrt{\pi}\operatorname{erf}(x) - 3\sqrt{\pi}\operatorname{erf}(3) + 16\right)$$

5 Chapter 6. Simplifying through simplifiction. Additional exercises. page 114

5.1 problem 6.1 (a)

Internal problem ID [13376]

Book: Ordinary Differential Equations. An introduction to the fundamentals. Kenneth B. Howell. second edition. CRC Press. FL, USA. 2020
Section: Chapter 6. Simplifying through simplifiction. Additional exercises. page 114
Problem number: 6.1 (a).
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[[_homogeneous, 'class C'], _dAlembert]`

$$\boxed{y' - \frac{1}{(3x+3y+2)^2} = 0}$$

✓ Solution by Maple
Time used: 0.062 (sec). Leaf size: 24

```
dsolve(diff(y(x),x)=1/(3*x+3*y(x)+2)^2,y(x), singsol=all)
```

$$y(x) = -c_1 + \frac{\operatorname{RootOf}\left(-\_Z + 3c_1 - 3x - 2 + \tan\left(\_Z\right)\right)}{3}$$

✓ Solution by Mathematica
Time used: 0.162 (sec). Leaf size: 23

```
DSolve[y'[x]==1/(3*x+3*y[x]+2)^2,y[x],x,IncludeSingularSolutions -> True]
```

$$\text{Solve}\left[y(x) - \frac{1}{3}\arctan(3y(x) + 3x + 2) = c_1, y(x)\right]$$

5.2 problem 6.1 (b)

Internal problem ID [13377]

Book: Ordinary Differential Equations. An introduction to the fundamentals. Kenneth B. Howell. second edition. CRC Press. FL, USA. 2020
Section: Chapter 6. Simplifying through simplifiction. Additional exercises. page 114
Problem number: 6.1 (b).
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[[_homogeneous, ‘class C‘], _rational, [_Abel, ‘2nd type‘, ‘cl`

$$y' - \frac{(3x-2y)^2+1}{3x-2y} = \frac{3}{2}$$

✓ Solution by Maple
Time used: 0.031 (sec). Leaf size: 39

```
dsolve(diff(y(x),x)=( (3*x-2*y(x))^2+1 )/(3*x-2*y(x))+3/2,y(x), singsol=all)
```

$$y(x) = \frac{3x}{2} - \frac{\sqrt{\mathrm{e}^{-4x}c_1 - 1}}{2}$$
$$y(x) = \frac{3x}{2} + \frac{\sqrt{\mathrm{e}^{-4x}c_1 - 1}}{2}$$

✓ Solution by Mathematica
Time used: 11.283 (sec). Leaf size: 78

```
DSolve[y'[x]==( (3*x-2*y[x])^2+1 )/(3*x-2*y[x])+3/2,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to \frac{1}{2}\left(3x - \frac{\sqrt{e^{4x} - 4c_1}}{\sqrt{-e^{4x}}}\right)$$
$$y(x) \to \frac{1}{2}\left(3x + \frac{\sqrt{e^{4x} - 4c_1}}{\sqrt{-e^{4x}}}\right)$$

5.3 problem 6.1 (c)

Internal problem ID [13378]

Book: Ordinary Differential Equations. An introduction to the fundamentals. Kenneth B. Howell. second edition. CRC Press. FL, USA. 2020
Section: Chapter 6. Simplifying through simplifiction. Additional exercises. page 114
Problem number: 6.1 (c).
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[[_homogeneous, ‘class C‘], _exact, _dAlembert]`

$$\boxed{\cos \left(-4y+8x-3\right) y'-2\cos \left(-4y+8x-3\right)=2}$$

✓ Solution by Maple
Time used: 0.032 (sec). Leaf size: 19

```
dsolve(cos(4*y(x)-8*x+3)*diff(y(x),x)=2+2*cos(4*y(x)-8*x+3),y(x), singsol=all)
```

$$y(x)=2x-\frac{3}{4}-\frac{\arcsin \left(-8x+8c_1\right)}{4}$$

✓ Solution by Mathematica

Time used: 64.647 (sec). Leaf size: 1165

```
DSolve[Cos[4*y[x]-8*x+3]*y'[x]==2+2*Cos[4*y[x]-8*x+3],y[x],x,IncludeSingularSolutions -> Tru
```

$$y(x) \to -\arccos\left(-\frac{1}{2}\sqrt{2-\sqrt{-2\sqrt{-\left((64x^2+64c_1x-1+16c_1{}^2)\cos^2(3-8x)\right)}+8(2x+c_1)\sin(3-8x)+2}}\right)$$

$$y(x) \to \arccos\left(-\frac{1}{2}\sqrt{2-\sqrt{-2\sqrt{-\left((64x^2+64c_1x-1+16c_1{}^2)\cos^2(3-8x)\right)}+8(2x+c_1)\sin(3-8x)+2}}\right)$$

$$y(x) \to -\arccos\left(\frac{1}{2}\sqrt{2-\sqrt{-2\sqrt{-\left((64x^2+64c_1x-1+16c_1{}^2)\cos^2(3-8x)\right)}+8(2x+c_1)\sin(3-8x)+2}}\right)$$

$$y(x) \to \arccos\left(\frac{1}{2}\sqrt{2-\sqrt{-2\sqrt{-\left((64x^2+64c_1x-1+16c_1{}^2)\cos^2(3-8x)\right)}+8(2x+c_1)\sin(3-8x)+2}}\right)$$

$$y(x) \to -\arccos\left(-\frac{1}{2}\sqrt{2+\sqrt{-2\sqrt{-\left((64x^2+64c_1x-1+16c_1{}^2)\cos^2(3-8x)\right)}+8(2x+c_1)\sin(3-8x)+2}}\right)$$

$$y(x) \to \arccos\left(-\frac{1}{2}\sqrt{2+\sqrt{-2\sqrt{-\left((64x^2+64c_1x-1+16c_1{}^2)\cos^2(3-8x)\right)}+8(2x+c_1)\sin(3-8x)+2}}\right)$$

$$y(x) \to -\arccos\left(\frac{1}{2}\sqrt{2+\sqrt{-2\sqrt{-\left((64x^2+64c_1x-1+16c_1{}^2)\cos^2(3-8x)\right)}+8(2x+c_1)\sin(3-8x)+2}}\right)$$

$$y(x) \to \arccos\left(\frac{1}{2}\sqrt{2+\sqrt{-2\sqrt{-\left((64x^2+64c_1x-1+16c_1{}^2)\cos^2(3-8x)\right)}+8(2x+c_1)\sin(3-8x)+2}}\right)$$

$$y(x) \to -\arccos\left(-\frac{1}{2}\sqrt{2-\sqrt{2}\sqrt{\sqrt{-\left((64x^2+64c_1x-1+16c_1{}^2)\cos^2(3-8x)\right)}+4(2x+c_1)\sin(3-8x)+1}}\right)$$

$$y(x) \to \arccos\left(-\frac{1}{2}\sqrt{2-\sqrt{2}\sqrt{\sqrt{-\left((64x^2+64c_1x-1+16c_1{}^2)\cos^2(3-8x)\right)}+4(2x+c_1)\sin(3-8x)+1}}\right)$$

$$y(x) \to -\arccos\left(\frac{1}{2}\sqrt{2-\sqrt{2}\sqrt{\sqrt{-\left((64x^2+64c_1x-1+16c_1{}^2)\cos^2(3-8x)\right)}+4(2x+c_1)\sin(3-8x)+1}}\right)$$

$y(x)$

5.4 problem 6.2

Internal problem ID [13379]

Book: Ordinary Differential Equations. An introduction to the fundamentals. Kenneth B. Howell. second edition. CRC Press. FL, USA. 2020
Section: Chapter 6. Simplifying through simplifiction. Additional exercises. page 114
Problem number: 6.2.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[[_homogeneous, ‘class C‘], _Riccati]`

$$\boxed{y' - (y - x)^2 = 1}$$

With initial conditions

$$\left[y(0) = \frac{1}{4}\right]$$

✓ Solution by Maple
Time used: 0.187 (sec). Leaf size: 18

```
dsolve([diff(y(x),x)=1+(y(x)-x)^2,y(0) = 1/4],y(x), singsol=all)
```

$$y(x) = \frac{x^2 - 4x - 1}{x - 4}$$

✓ Solution by Mathematica
Time used: 0.134 (sec). Leaf size: 19

```
DSolve[{y'[x]==1+(y[x]-x)^2,{y[0]==1/4}},y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to \frac{x^2 - 4x - 1}{x - 4}$$

5.5 problem 6.3 (a)

Internal problem ID [13380]

Book: Ordinary Differential Equations. An introduction to the fundamentals. Kenneth B. Howell. second edition. CRC Press. FL, USA. 2020
Section: Chapter 6. Simplifying through simplifiction. Additional exercises. page 114
Problem number: 6.3 (a).
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[[_homogeneous, 'class A'], _rational, _Bernoulli]`

$$\boxed{x^2y' - yx - y^2 = 0}$$

✓ Solution by Maple
Time used: 0.0 (sec). Leaf size: 14

```
dsolve(x^2*diff(y(x),x)-x*y(x)=y(x)^2,y(x), singsol=all)
```

$$y(x) = \frac{x}{-\ln(x) + c_1}$$

✓ Solution by Mathematica
Time used: 0.125 (sec). Leaf size: 21

```
DSolve[x^2*y'[x]-x*y[x]==y[x]^2,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to \frac{x}{-\log(x) + c_1}$$
$$y(x) \to 0$$

5.6 problem 6.3 (b)

Internal problem ID [13381]

Book: Ordinary Differential Equations. An introduction to the fundamentals. Kenneth B. Howell. second edition. CRC Press. FL, USA. 2020
Section: Chapter 6. Simplifying through simplifiction. Additional exercises. page 114
Problem number: 6.3 (b).
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[[_homogeneous, 'class A'], _rational, _Bernoulli]`

$$y' - \frac{y}{x} - \frac{x}{y} = 0$$

✓ Solution by Maple

Time used: 0.031 (sec). Leaf size: 28

```
dsolve(diff(y(x),x)=y(x)/x+x/y(x),y(x), singsol=all)
```

$$y(x) = \sqrt{2\ln(x) + c_1}\, x$$
$$y(x) = -\sqrt{2\ln(x) + c_1}\, x$$

✓ Solution by Mathematica

Time used: 0.16 (sec). Leaf size: 36

```
DSolve[y'[x]==y[x]/x+x/y[x],y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to -x\sqrt{2\log(x) + c_1}$$
$$y(x) \to x\sqrt{2\log(x) + c_1}$$

5.7 problem 6.3 (c)

Internal problem ID [13382]

Book: Ordinary Differential Equations. An introduction to the fundamentals. Kenneth B. Howell. second edition. CRC Press. FL, USA. 2020
Section: Chapter 6. Simplifying through simplifiction. Additional exercises. page 114
Problem number: 6.3 (c).
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[[_homogeneous, ‘class A‘], _dAlembert]`

$$\cos\left(\frac{y}{x}\right)\left(y' - \frac{y}{x}\right) - \sin\left(\frac{y}{x}\right) = 1$$

✓ Solution by Maple
Time used: 0.0 (sec). Leaf size: 12

```
dsolve(cos(y(x)/x)*(diff(y(x),x)-y(x)/x)=1+sin(y(x)/x),y(x), singsol=all)
```

$$y(x) = \arcsin\left(c_1 x - 1\right) x$$

✓ Solution by Mathematica
Time used: 60.351 (sec). Leaf size: 185

```
DSolve[Cos[y[x]/x]*(y'[x]-y[x]/x)==1+Sin[y[x]/x],y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to -\frac{\pi x}{2}$$
$$y(x) \to \frac{3\pi x}{2}$$
$$y(x) \to -2x \arccos\left(\frac{1}{2}\left(e^{\frac{c_1}{2}}\sqrt{x} - \sqrt{2 - e^{c_1} x}\right)\right)$$
$$y(x) \to 2x \arccos\left(\frac{1}{2}\left(e^{\frac{c_1}{2}}\sqrt{x} - \sqrt{2 - e^{c_1} x}\right)\right)$$
$$y(x) \to -2x \arccos\left(\frac{1}{2}\left(e^{\frac{c_1}{2}}\sqrt{x} + \sqrt{2 - e^{c_1} x}\right)\right)$$
$$y(x) \to 2x \arccos\left(\frac{1}{2}\left(e^{\frac{c_1}{2}}\sqrt{x} + \sqrt{2 - e^{c_1} x}\right)\right)$$

5.8 problem 6.4

Internal problem ID [13383]

Book: Ordinary Differential Equations. An introduction to the fundamentals. Kenneth B. Howell. second edition. CRC Press. FL, USA. 2020
Section: Chapter 6. Simplifying through simplifiction. Additional exercises. page 114
Problem number: 6.4.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[[_homogeneous, 'class A'], _rational, [_Abel, '2nd type', 'cl`

$$\boxed{y' - \frac{-y+x}{y+x} = 0}$$

With initial conditions

$$[y(0) = 3]$$

✓ Solution by Maple
Time used: 0.765 (sec). Leaf size: 19

```
dsolve([diff(y(x),x)=(x-y(x))/(x+y(x)),y(0) = 3],y(x), singsol=all)
```

$$y(x) = -x + \sqrt{2x^2 + 9}$$

✓ Solution by Mathematica
Time used: 0.434 (sec). Leaf size: 20

```
DSolve[{y'[x]==(x-y[x])/(x+y[x]),{y[0]==3}},y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to \sqrt{2x^2 + 9} - x$$

5.9 problem 6.5 (a)

Internal problem ID [13384]

Book: Ordinary Differential Equations. An introduction to the fundamentals. Kenneth B. Howell. second edition. CRC Press. FL, USA. 2020
Section: Chapter 6. Simplifying through simplifiction. Additional exercises. page 114
Problem number: 6.5 (a).
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_quadrature]`

$$\boxed{y' + 3y - 3y^3 = 0}$$

✓ Solution by Maple
Time used: 0.015 (sec). Leaf size: 29

```
dsolve(diff(y(x),x)+3*y(x)=3*y(x)^3,y(x), singsol=all)
```

$$y(x) = \frac{1}{\sqrt{e^{6x}c_1 + 1}}$$
$$y(x) = -\frac{1}{\sqrt{e^{6x}c_1 + 1}}$$

✓ Solution by Mathematica
Time used: 0.675 (sec). Leaf size: 58

```
DSolve[y'[x]+3*y[x]==3*y[x]^3,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to -\frac{1}{\sqrt{1 + e^{6x+2c_1}}}$$
$$y(x) \to \frac{1}{\sqrt{1 + e^{6x+2c_1}}}$$
$$y(x) \to -1$$
$$y(x) \to 0$$
$$y(x) \to 1$$

5.10 problem 6.5 (b)

Internal problem ID [13385]

Book: Ordinary Differential Equations. An introduction to the fundamentals. Kenneth B. Howell. second edition. CRC Press. FL, USA. 2020
Section: Chapter 6. Simplifying through simplifiction. Additional exercises. page 114
Problem number: 6.5 (b).
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[[_homogeneous, ‘class A‘], _rational, _Bernoulli]`

$$\boxed{y' - \frac{3y}{x} - \frac{y^2}{x^2} = 0}$$

✓ Solution by Maple
Time used: 0.016 (sec). Leaf size: 18

```
dsolve(diff(y(x),x)-3*y(x)/x=(y(x)/x)^2,y(x), singsol=all)
```

$$y(x) = -\frac{2x^3}{x^2 - 2c_1}$$

✓ Solution by Mathematica
Time used: 0.141 (sec). Leaf size: 25

```
DSolve[y'[x]-3*y[x]/x==(y[x]/x)^2,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to -\frac{2x^3}{x^2 - 2c_1}$$
$$y(x) \to 0$$

5.11 problem 6.5 (c)

Internal problem ID [13386]

Book: Ordinary Differential Equations. An introduction to the fundamentals. Kenneth B. Howell. second edition. CRC Press. FL, USA. 2020
Section: Chapter 6. Simplifying through simplifiction. Additional exercises. page 114
Problem number: 6.5 (c).
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_Bernoulli]`

$$\boxed{y' + 3y\cot(x) - 6\cos(x)\, y^{\frac{2}{3}} = 0}$$

✓ Solution by Maple
Time used: 0.0 (sec). Leaf size: 17

```
dsolve(diff(y(x),x)+3*cot(x)*y(x)=6*cos(x)*y(x)^(2/3),y(x), singsol=all)
```

$$-\sin(x) + y(x)^{\frac{1}{3}} - \csc(x)\, c_1 = 0$$

✓ Solution by Mathematica
Time used: 0.305 (sec). Leaf size: 24

```
DSolve[y'[x]+3*Cot[x]*y[x]==6*Cos[x]*y[x]^(2/3),y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to -\frac{1}{8}\csc^3(x)(\cos(2x) - 2c_1)^3$$

5.12 problem 6.6

Internal problem ID [13387]

Book: Ordinary Differential Equations. An introduction to the fundamentals. Kenneth B. Howell. second edition. CRC Press. FL, USA. 2020
Section: Chapter 6. Simplifying through simplifiction. Additional exercises. page 114
Problem number: 6.6.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[[_homogeneous, ‘class D‘], _rational, _Bernoulli]`

$$\boxed{y' - \frac{y}{x} - \frac{1}{y} = 0}$$

With initial conditions

$$[y(1) = 3]$$

✓ Solution by Maple
Time used: 0.063 (sec). Leaf size: 13

```
dsolve([diff(y(x),x)-1/x*y(x)=1/y(x),y(1) = 3],y(x), singsol=all)
```

$$y(x) = \sqrt{x\,(11x - 2)}$$

✓ Solution by Mathematica
Time used: 0.251 (sec). Leaf size: 20

```
DSolve[{y'[x]-1/x*y[x]==1/y[x],{y[1]==3}},y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to \sqrt{x}\sqrt{11x - 2}$$

5.13 problem 6.7 (a)

Internal problem ID [13388]

Book: Ordinary Differential Equations. An introduction to the fundamentals. Kenneth B. Howell. second edition. CRC Press. FL, USA. 2020
Section: Chapter 6. Simplifying through simplifiction. Additional exercises. page 114
Problem number: 6.7 (a).
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[[_homogeneous, 'class A'], _rational, _Bernoulli]`

$$\boxed{y' - \frac{y}{x} - \frac{x^2}{y^2} = 0}$$

✓ Solution by Maple
Time used: 0.016 (sec). Leaf size: 58

```
dsolve(diff(y(x),x)=y(x)/x+(x/y(x))^2,y(x), singsol=all)
```

$$y(x) = (3\ln(x) + c_1)^{\frac{1}{3}} x$$
$$y(x) = -\frac{(3\ln(x) + c_1)^{\frac{1}{3}} \left(1 + i\sqrt{3}\right) x}{2}$$
$$y(x) = \frac{(3\ln(x) + c_1)^{\frac{1}{3}} \left(i\sqrt{3} - 1\right) x}{2}$$

✓ Solution by Mathematica
Time used: 0.174 (sec). Leaf size: 63

```
DSolve[y'[x]==y[x]/x+(x/y[x])^2,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to x\sqrt[3]{3\log(x) + c_1}$$
$$y(x) \to -\sqrt[3]{-1}x\sqrt[3]{3\log(x) + c_1}$$
$$y(x) \to (-1)^{2/3}x\sqrt[3]{3\log(x) + c_1}$$

5.14 problem 6.7 (b)

Internal problem ID [13389]

Book: Ordinary Differential Equations. An introduction to the fundamentals. Kenneth B. Howell. second edition. CRC Press. FL, USA. 2020
Section: Chapter 6. Simplifying through simplifiction. Additional exercises. page 114
Problem number: 6.7 (b).
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[[_homogeneous, ‘class C‘], _dAlembert]`

$$\boxed{3y' - \sqrt{2x + 3y + 4} = -2}$$

✓ Solution by Maple
Time used: 0.031 (sec). Leaf size: 21

```
dsolve(3*diff(y(x),x)=-2+sqrt(2*x+3*y(x)+4),y(x), singsol=all)
```

$$x - 2\sqrt{2x + 3y(x) + 4} - c_1 = 0$$

✓ Solution by Mathematica
Time used: 0.329 (sec). Leaf size: 51

```
DSolve[3*y'[x]==-2+Sqrt[2*x+3*y[x]+4],y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to \frac{1}{48}\left(4\left(x^2 - 6x - 15\right) - 4e^{c_1}(x + 1) + e^{2c_1}\right)$$
$$y(x) \to \frac{1}{12}\left(x^2 - 6x - 15\right)$$

5.15 problem 6.7 (c)

Internal problem ID [13390]

Book: Ordinary Differential Equations. An introduction to the fundamentals. Kenneth B. Howell. second edition. CRC Press. FL, USA. 2020
Section: Chapter 6. Simplifying through simplifiction. Additional exercises. page 114
Problem number: 6.7 (c).
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[[_homogeneous, ‘class G‘], _rational, _Bernoulli]`

$$\boxed{3y' + \frac{2y}{x} - 4\sqrt{y} = 0}$$

✓ Solution by Maple
Time used: 0.0 (sec). Leaf size: 17

```
dsolve(3*diff(y(x),x)+2/x*y(x)=4*sqrt(y(x)),y(x), singsol=all)
```

$$\sqrt{y(x)} - \frac{x}{2} - \frac{c_1}{x^{\frac{1}{3}}} = 0$$

✓ Solution by Mathematica
Time used: 0.158 (sec). Leaf size: 26

```
DSolve[3*y'[x]+2/x*y[x]==4*Sqrt[y[x]],y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to \frac{(x^{4/3} + 2c_1)^2}{4x^{2/3}}$$

5.16 problem 6.7 (d)

Internal problem ID [13391]

Book: Ordinary Differential Equations. An introduction to the fundamentals. Kenneth B. Howell. second edition. CRC Press. FL, USA. 2020
Section: Chapter 6. Simplifying through simplifiction. Additional exercises. page 114
Problem number: 6.7 (d).
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[[_homogeneous, ‘class C‘], _dAlembert]`

$$\boxed{y' - \frac{1}{\sin(4x - y)} = 4}$$

✓ Solution by Maple
Time used: 0.032 (sec). Leaf size: 19

```
dsolve(diff(y(x),x)=4+1/sin(4*x-y(x)),y(x), singsol=all)
```

$$y(x) = 4x - \frac{\pi}{2} - \arcsin(c_1 - x)$$

✓ Solution by Mathematica
Time used: 0.624 (sec). Leaf size: 33

```
DSolve[y'[x]==4+1/Sin[4*x-y[x]],y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to 4x - \arccos(x - c_1)$$
$$y(x) \to 4x + \arccos(x - c_1)$$

5.17 problem 6.7 (e)

Internal problem ID [13392]

Book: Ordinary Differential Equations. An introduction to the fundamentals. Kenneth B. Howell. second edition. CRC Press. FL, USA. 2020
Section: Chapter 6. Simplifying through simplifiction. Additional exercises. page 114
Problem number: 6.7 (e).
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[[_homogeneous, ‘class C‘], [_Abel, ‘2nd type‘, ‘class C‘], _d`

$$\boxed{(y - x)\, y' = 1}$$

✓ Solution by Maple
Time used: 0.0 (sec). Leaf size: 17

```
dsolve((y(x)-x)*diff(y(x),x)=1,y(x), singsol=all)
```

$$y(x) = \operatorname{LambertW}\left(-c_1 \mathrm{e}^{-x-1}\right) + x + 1$$

✓ Solution by Mathematica
Time used: 0.022 (sec). Leaf size: 20

```
DSolve[(y[x]-x)*y'[x]==1,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to W\left(c_1\left(-e^{-x-1}\right)\right) + x + 1$$

5.18 problem 6.7 (f)

Internal problem ID [13393]

Book: Ordinary Differential Equations. An introduction to the fundamentals. Kenneth B. Howell. second edition. CRC Press. FL, USA. 2020
Section: Chapter 6. Simplifying through simplifiction. Additional exercises. page 114
Problem number: 6.7 (f).
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[[_homogeneous, ‘class A‘], _rational, [_Abel, ‘2nd type‘, ‘cl`

$$\boxed{(y+x)\,y'-y=0}$$

✓ Solution by Maple
Time used: 0.0 (sec). Leaf size: 13

```
dsolve((y(x)+x)*diff(y(x),x)=y(x),y(x), singsol=all)
```

$$y(x)=\frac{x}{\operatorname{LambertW}\left(x\,\mathrm{e}^{c_1}\right)}$$

✓ Solution by Mathematica
Time used: 3.407 (sec). Leaf size: 23

```
DSolve[(y[x]+x)*y'[x]==y[x],y[x],x,IncludeSingularSolutions -> True]
```

$$y(x)\to\frac{x}{W\left(e^{-c_1}x\right)}$$
$$y(x)\to 0$$

5.19 problem 6.7 (g)

Internal problem ID [13394]

Book: Ordinary Differential Equations. An introduction to the fundamentals. Kenneth B. Howell. second edition. CRC Press. FL, USA. 2020
Section: Chapter 6. Simplifying through simplifiction. Additional exercises. page 114
Problem number: 6.7 (g).
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[[_homogeneous, ‘class A‘], _rational, [_Abel, ‘2nd type‘, ‘cl`

$$\boxed{(2yx + 2x^2)\, y' - 2yx - 2y^2 = x^2}$$

✓ Solution by Maple
Time used: 0.0 (sec). Leaf size: 31

```
dsolve((2*x*y(x)+2*x^2)*diff(y(x),x)=x^2+2*x*y(x)+2*y(x)^2,y(x), singsol=all)
```

$$y(x) = \left(-1 - \sqrt{1 + \ln(x) + c_1}\right) x$$
$$y(x) = \left(-1 + \sqrt{1 + \ln(x) + c_1}\right) x$$

✓ Solution by Mathematica
Time used: 0.243 (sec). Leaf size: 42

```
DSolve[(2*x*y[x]+2*x^2)*y'[x]==x^2+2*x*y[x]+2*y[x]^2,y[x],x,IncludeSingularSolutions -> True
```

$$y(x) \to -x\left(1 + \sqrt{\log(x) + 1 + 2c_1}\right)$$
$$y(x) \to x\left(-1 + \sqrt{\log(x) + 1 + 2c_1}\right)$$

5.20 problem 6.7 (h)

Internal problem ID [13395]

Book: Ordinary Differential Equations. An introduction to the fundamentals. Kenneth B. Howell. second edition. CRC Press. FL, USA. 2020
Section: Chapter 6. Simplifying through simplifiction. Additional exercises. page 114
Problem number: 6.7 (h).
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[[_homogeneous, ‘class G‘], _rational, _Bernoulli]`

$$\boxed{y' + \frac{y}{x} - y^3x^2 = 0}$$

✓ Solution by Maple
Time used: 0.015 (sec). Leaf size: 30

```
dsolve(diff(y(x),x)+1/x*y(x)=x^2*y(x)^3,y(x), singsol=all)
```

$$y(x) = \frac{1}{\sqrt{-2x + c_1}\, x}$$
$$y(x) = -\frac{1}{\sqrt{-2x + c_1}\, x}$$

✓ Solution by Mathematica
Time used: 0.336 (sec). Leaf size: 44

```
DSolve[y'[x]+1/x*y[x]==x^2*y[x]^3,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to -\frac{1}{\sqrt{x^2(-2x + c_1)}}$$
$$y(x) \to \frac{1}{\sqrt{x^2(-2x + c_1)}}$$
$$y(x) \to 0$$

5.21 problem 6.7 (i)

Internal problem ID [13396]

Book: Ordinary Differential Equations. An introduction to the fundamentals. Kenneth B. Howell. second edition. CRC Press. FL, USA. 2020
Section: Chapter 6. Simplifying through simplifiction. Additional exercises. page 114
Problem number: 6.7 (i).
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[[_homogeneous, ‘class C‘], _dAlembert]`

$$\boxed{y' - 2\sqrt{2x + y - 3} = -2}$$

✓ Solution by Maple
Time used: 0.032 (sec). Leaf size: 19

```
dsolve(diff(y(x),x)=2*sqrt(2*x+y(x)-3)-2,y(x), singsol=all)
```

$$x - \sqrt{2x + y(x) - 3} - c_1 = 0$$

✓ Solution by Mathematica
Time used: 0.256 (sec). Leaf size: 38

```
DSolve[y'[x]==2*Sqrt[2*x+y[x]-3]-2,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to x^2 - e^{c_1}(x+1) + 4 + \frac{e^{2c_1}}{4}$$
$$y(x) \to x^2 + 4$$

5.22 problem 6.7 (j)

Internal problem ID [13397]

Book: Ordinary Differential Equations. An introduction to the fundamentals. Kenneth B. Howell. second edition. CRC Press. FL, USA. 2020
Section: Chapter 6. Simplifying through simplifiction. Additional exercises. page 114
Problem number: 6.7 (j).
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[[_homogeneous, ‘class C‘], _dAlembert]`

$$\boxed{y' - 2\sqrt{2x + y - 3} = 0}$$

✓ Solution by Maple
Time used: 0.032 (sec). Leaf size: 57

```
dsolve(diff(y(x),x)=2*sqrt(2*x+y(x)-3),y(x), singsol=all)
```

$$x - \sqrt{2x + y(x) - 3} - \frac{\ln\left(-1 + \sqrt{2x + y(x) - 3}\right)}{2} + \frac{\ln\left(\sqrt{2x + y(x) - 3} + 1\right)}{2} + \frac{\ln(2x + y(x) - 4)}{2} - c_1 = 0$$

✓ Solution by Mathematica
Time used: 8.176 (sec). Leaf size: 51

```
DSolve[y'[x]==2*Sqrt[2*x+y[x]-3],y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to W\left(-e^{-x+\frac{1}{2}+c_1}\right)^2 + 2W\left(-e^{-x+\frac{1}{2}+c_1}\right) - 2x + 4$$
$$y(x) \to 4 - 2x$$

5.23 problem 6.7 (k)

Internal problem ID [13398]

Book: Ordinary Differential Equations. An introduction to the fundamentals. Kenneth B. Howell. second edition. CRC Press. FL, USA. 2020
Section: Chapter 6. Simplifying through simplifiction. Additional exercises. page 114
Problem number: 6.7 (k).
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[[_homogeneous, ‘class A‘], _rational, _dAlembert]`

$$\boxed{y'x - y - \sqrt{yx + x^2} = 0}$$

✓ Solution by Maple

Time used: 0.015 (sec). Leaf size: 25

```
dsolve(x*diff(y(x),x)-y(x)=sqrt(x*y(x)+x^2),y(x), singsol=all)
```

$$-\frac{x + y(x)}{\sqrt{x\,(x + y\,(x))}} + \frac{\ln\,(x)}{2} - c_1 = 0$$

✓ Solution by Mathematica

Time used: 0.194 (sec). Leaf size: 26

```
DSolve[x*y'[x]-y[x]==Sqrt[x*y[x]+x^2],y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to \frac{1}{4}x\left(\log^2(x) + 2c_1\log(x) - 4 + c_1{}^2\right)$$

5.24 problem 6.7 (L)

Internal problem ID [13399]

Book: Ordinary Differential Equations. An introduction to the fundamentals. Kenneth B. Howell. second edition. CRC Press. FL, USA. 2020
Section: Chapter 6. Simplifying through simplifiction. Additional exercises. page 114
Problem number: 6.7 (L).
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[[_1st_order, _with_linear_symmetries], _Bernoulli]`

$$\boxed{y' + 3y - \frac{28\,\mathrm{e}^{2x}}{y^3} = 0}$$

✓ Solution by Maple
Time used: 0.032 (sec). Leaf size: 78

```
dsolve(diff(y(x),x)+3*y(x)=28*exp(2*x)*1/(y(x)^3),y(x), singsol=all)
```

$$y(x) = \left(8\,\mathrm{e}^{14x} + c_1\right)^{\frac{1}{4}} \mathrm{e}^{-3x}$$
$$y(x) = -\left(8\,\mathrm{e}^{14x} + c_1\right)^{\frac{1}{4}} \mathrm{e}^{-3x}$$
$$y(x) = -i\left(8\,\mathrm{e}^{14x} + c_1\right)^{\frac{1}{4}} \mathrm{e}^{-3x}$$
$$y(x) = i\left(8\,\mathrm{e}^{14x} + c_1\right)^{\frac{1}{4}} \mathrm{e}^{-3x}$$

✓ Solution by Mathematica
Time used: 0.741 (sec). Leaf size: 104

```
DSolve[y'[x]+3*y[x]==28*Exp[2*x]*1/(y[x]^3),y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to -e^{-3x}\sqrt[4]{8e^{14x} + c_1}$$
$$y(x) \to -ie^{-3x}\sqrt[4]{8e^{14x} + c_1}$$
$$y(x) \to ie^{-3x}\sqrt[4]{8e^{14x} + c_1}$$
$$y(x) \to e^{-3x}\sqrt[4]{8e^{14x} + c_1}$$

5.25 problem 6.7 (m)

Internal problem ID [13400]

Book: Ordinary Differential Equations. An introduction to the fundamentals. Kenneth B. Howell. second edition. CRC Press. FL, USA. 2020
Section: Chapter 6. Simplifying through simplifiction. Additional exercises. page 114
Problem number: 6.7 (m).
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[[_homogeneous, ‘class C‘], _Riccati]`

$$\boxed{y' - (-y + 3 + x)^2 = 0}$$

✓ Solution by Maple
Time used: 0.031 (sec). Leaf size: 29

```
dsolve(diff(y(x),x)=(x-y(x)+3)^2,y(x), singsol=all)
```

$$y(x) = \frac{c_1(x + 2)\,\mathrm{e}^{2x} - x - 4}{-1 + \mathrm{e}^{2x} c_1}$$

✓ Solution by Mathematica
Time used: 0.163 (sec). Leaf size: 29

```
DSolve[y'[x]==(x-y[x]+3)^2,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to x + \frac{1}{\frac{1}{2} + c_1 e^{2x}} + 2$$
$$y(x) \to x + 2$$

5.26 problem 6.7 (n)

Internal problem ID [13401]

Book: Ordinary Differential Equations. An introduction to the fundamentals. Kenneth B. Howell. second edition. CRC Press. FL, USA. 2020
Section: Chapter 6. Simplifying through simplifiction. Additional exercises. page 114
Problem number: 6.7 (n).
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[[_1st_order, _with_linear_symmetries], _Clairaut]`

$$\boxed{y' - 2\sqrt{x^2 + y} = -2x}$$

✓ Solution by Maple
Time used: 0.031 (sec). Leaf size: 18

```
dsolve(diff(y(x),x)+2*x=2*sqrt(y(x)+x^2),y(x), singsol=all)
```

$$x - \sqrt{x^2 + y(x)} - c_1 = 0$$

✓ Solution by Mathematica
Time used: 0.736 (sec). Leaf size: 35

```
DSolve[y'[x]+2*x==2*Sqrt[y[x]+x^2],y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to (1 + e^{c_1})(2x + 1 + e^{c_1})$$
$$y(x) \to 0$$
$$y(x) \to 2x + 1$$

5.27 problem 6.7 (o)

Internal problem ID [13402]

Book: Ordinary Differential Equations. An introduction to the fundamentals. Kenneth B. Howell. second edition. CRC Press. FL, USA. 2020
Section: Chapter 6. Simplifying through simplifiction. Additional exercises. page 114
Problem number: 6.7 (o).
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[‘y=_G(x,y’)‘]`

$$\boxed{\cos(y)\, y' + \sin(y) = e^{-x}}$$

✓ Solution by Maple
Time used: 0.0 (sec). Leaf size: 17

```
dsolve(cos(y(x))*diff(y(x),x)=exp(-x)-sin(y(x)),y(x), singsol=all)
```

$$y(x) = -\arcsin\left((c_1 - x)\, e^{-x}\right)$$

✓ Solution by Mathematica
Time used: 11.73 (sec). Leaf size: 16

```
DSolve[Cos[y[x]]*y'[x]==Exp[-x]-Sin[y[x]],y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to \arcsin\left(e^{-x}(x + c_1)\right)$$

5.28 problem 6.7 (p)

Internal problem ID [13403]

Book: Ordinary Differential Equations. An introduction to the fundamentals. Kenneth B. Howell. second edition. CRC Press. FL, USA. 2020
Section: Chapter 6. Simplifying through simplifiction. Additional exercises. page 114
Problem number: 6.7 (p).
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[[_homogeneous, 'class G'], _rational, _Riccati]`

$$y' - x\left(1 + \frac{2y}{x^2} + \frac{y^2}{x^4}\right) = 0$$

✓ Solution by Maple
Time used: 0.016 (sec). Leaf size: 16

```
dsolve(diff(y(x),x)=x*(1+2*y(x)/x^2+y(x)^2/x^4),y(x), singsol=all)
```

$$y(x) = -\tan\left(-\ln\left(x\right) + c_1\right) x^2$$

✓ Solution by Mathematica
Time used: 0.187 (sec). Leaf size: 15

```
DSolve[y'[x]==x*(1+2*y[x]/x^2+y[x]^2/x^4),y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to x^2 \tan(\log(x) + c_1)$$

6 Chapter 7. The exact form and general integrating fators. Additional exercises. page 141

6.1 problem 7.2

Internal problem ID [13404]

Book: Ordinary Differential Equations. An introduction to the fundamentals. Kenneth B. Howell. second edition. CRC Press. FL, USA. 2020
Section: Chapter 7. The exact form and general integrating fators. Additional exercises. page 141
Problem number: 7.2.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[[_homogeneous, 'class G'], _rational, _Bernoulli]`

$$\boxed{y' - \frac{1}{y} + \frac{y}{2x} = 0}$$

✓ Solution by Maple
Time used: 0.016 (sec). Leaf size: 34

```
dsolve(diff(y(x),x)=1/y(x)-y(x)/(2*x),y(x), singsol=all)
```

$$y(x) = \frac{\sqrt{x\,(x^2+c_1)}}{x}$$
$$y(x) = -\frac{\sqrt{x\,(x^2+c_1)}}{x}$$

✓ Solution by Mathematica
Time used: 0.185 (sec). Leaf size: 42

```
DSolve[y'[x]==1/y[x]-y[x]/(2*x),y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to -\frac{\sqrt{x^2+c_1}}{\sqrt{x}}$$
$$y(x) \to \frac{\sqrt{x^2+c_1}}{\sqrt{x}}$$

6.2 problem 7.2 (c)

Internal problem ID [13405]

Book: Ordinary Differential Equations. An introduction to the fundamentals. Kenneth B. Howell. second edition. CRC Press. FL, USA. 2020
Section: Chapter 7. The exact form and general integrating fators. Additional exercises. page 141
Problem number: 7.2 (c).
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[[_homogeneous, ‘class G‘], _exact, _rational, _Bernoulli]`

$$\boxed{e^{y^2x-x^2}\left(y^2-2x\right)+2\,e^{y^2x-x^2}xyy'=0}$$

✓ Solution by Maple
Time used: 0.016 (sec). Leaf size: 38

```
dsolve(exp(x*y(x)^2-x^2)*(y(x)^2-2*x)+exp(x*y(x)^2-x^2)*2*x*y(x)*diff(y(x),x)=0,y(x), singso
```

$$y(x)=\frac{\sqrt{x\left(x^2-c_1\right)}}{x}$$
$$y(x)=-\frac{\sqrt{x\left(x^2-c_1\right)}}{x}$$

✓ Solution by Mathematica
Time used: 0.12 (sec). Leaf size: 42

```
DSolve[Exp[x*y[x]^2-x^2]*(y[x]^2-2*x)+Exp[x*y[x]^2-x^2]*2*x*y[x]*y'[x]==0,y[x],x,IncludeSing
```

$$y(x)\to -\frac{\sqrt{x^2+c_1}}{\sqrt{x}}$$
$$y(x)\to \frac{\sqrt{x^2+c_1}}{\sqrt{x}}$$

6.3 problem 7.4 (a)

Internal problem ID [13406]

Book: Ordinary Differential Equations. An introduction to the fundamentals. Kenneth B. Howell. second edition. CRC Press. FL, USA. 2020
Section: Chapter 7. The exact form and general integrating fators. Additional exercises. page 141
Problem number: 7.4 (a).
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[[_homogeneous, ‘class A‘], _exact, _rational, [_Abel, ‘2nd ty`

$$\boxed{2yx + y^2 + (2yx + x^2)\, y' = 0}$$

✓ Solution by Maple
Time used: 0.062 (sec). Leaf size: 71

```
dsolve(2*x*y(x)+y(x)^2+(2*x*y(x)+x^2)*diff(y(x),x)=0,y(x), singsol=all)
```

$$y(x) = \frac{-c_1^2x^2 + \sqrt{c_1x\,(c_1^3x^3 + 4)}}{2c_1^2x}$$
$$y(x) = \frac{-c_1^2x^2 - \sqrt{c_1x\,(c_1^3x^3 + 4)}}{2c_1^2x}$$

✓ Solution by Mathematica

Time used: 0.579 (sec). Leaf size: 118

```
DSolve[2*x*y[x]+y[x]^2+(2*x*y[x]+x^2)*y'[x]==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to \frac{1}{2}\left(-x - \frac{\sqrt{x^3 + 4e^{c_1}}}{\sqrt{x}}\right)$$
$$y(x) \to \frac{1}{2}\left(-x + \frac{\sqrt{x^3 + 4e^{c_1}}}{\sqrt{x}}\right)$$
$$y(x) \to -\frac{x^{3/2} + \sqrt{x^3}}{2\sqrt{x}}$$
$$y(x) \to \frac{\sqrt{x^3}}{2\sqrt{x}} - \frac{x}{2}$$

6.4 problem 7.4 (b)

Internal problem ID [13407]

Book: Ordinary Differential Equations. An introduction to the fundamentals. Kenneth B. Howell. second edition. CRC Press. FL, USA. 2020
Section: Chapter 7. The exact form and general integrating fators. Additional exercises. page 141
Problem number: 7.4 (b).
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[[_homogeneous, ‘class G‘], _exact, _rational, _Bernoulli]`

$$\boxed{2y^3x + 3y'y^2x^2 = -4x^3}$$

✓ Solution by Maple
Time used: 0.0 (sec). Leaf size: 73

```
dsolve(2*x*y(x)^3+4*x^3+3*x^2*y(x)^2*diff(y(x),x)=0,y(x), singsol=all)
```

$$y(x) = \frac{(x(-x^4 + c_1))^{\frac{1}{3}}}{x}$$
$$y(x) = -\frac{(x(-x^4 + c_1))^{\frac{1}{3}}(1 + i\sqrt{3})}{2x}$$
$$y(x) = \frac{(x(-x^4 + c_1))^{\frac{1}{3}}(i\sqrt{3} - 1)}{2x}$$

✓ Solution by Mathematica
Time used: 0.195 (sec). Leaf size: 78

```
DSolve[2*x*y[x]^3+4*x^3+3*x^2*y[x]^2*y'[x]==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to \frac{\sqrt[3]{-x^4 + c_1}}{x^{2/3}}$$
$$y(x) \to -\frac{\sqrt[3]{-1}\sqrt[3]{-x^4 + c_1}}{x^{2/3}}$$
$$y(x) \to \frac{(-1)^{2/3}\sqrt[3]{-x^4 + c_1}}{x^{2/3}}$$

6.5 problem 7.4 (c)

Internal problem ID [13408]

Book: Ordinary Differential Equations. An introduction to the fundamentals. Kenneth B. Howell. second edition. CRC Press. FL, USA. 2020
Section: Chapter 7. The exact form and general integrating fators. Additional exercises. page 141
Problem number: 7.4 (c).
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_separable]`

$$\boxed{3y'y^2 = 2x - 2}$$

✓ Solution by Maple
Time used: 0.0 (sec). Leaf size: 60

```
dsolve(2-2*x+3*y(x)^2*diff(y(x),x)=0,y(x), singsol=all)
```

$$y(x) = \left(x^2 + c_1 - 2x\right)^{\frac{1}{3}}$$
$$y(x) = -\frac{\left(x^2 + c_1 - 2x\right)^{\frac{1}{3}} \left(1 + i\sqrt{3}\right)}{2}$$
$$y(x) = \frac{\left(x^2 + c_1 - 2x\right)^{\frac{1}{3}} \left(i\sqrt{3} - 1\right)}{2}$$

✓ Solution by Mathematica
Time used: 0.179 (sec). Leaf size: 71

```
DSolve[2-2*x+3*y[x]^2*y'[x]==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to \sqrt[3]{x^2 - 2x + 3c_1}$$
$$y(x) \to -\sqrt[3]{-1}\sqrt[3]{x^2 - 2x + 3c_1}$$
$$y(x) \to (-1)^{2/3}\sqrt[3]{x^2 - 2x + 3c_1}$$

6.6 problem 7.4 (d)

Internal problem ID [13409]

Book: Ordinary Differential Equations. An introduction to the fundamentals. Kenneth B. Howell. second edition. CRC Press. FL, USA. 2020
Section: Chapter 7. The exact form and general integrating fators. Additional exercises. page 141
Problem number: 7.4 (d).
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_exact, _rational, _Bernoulli]`

$$3x^2y^2 + \left(2yx^3 + 6y\right) y' = -1$$

✓ Solution by Maple
Time used: 0.0 (sec). Leaf size: 50

```
dsolve(1+3*x^2*y(x)^2+(2*x^3*y(x)+6*y(x))*diff(y(x),x)=0,y(x), singsol=all)
```

$$y(x) = \frac{\sqrt{(x^3+3)(c_1-x)}}{x^3+3}$$
$$y(x) = -\frac{\sqrt{(x^3+3)(c_1-x)}}{x^3+3}$$

✓ Solution by Mathematica
Time used: 0.212 (sec). Leaf size: 50

```
DSolve[1+3*x^2*y[x]^2+(2*x^3*y[x]+6*y[x])*y'[x]==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to -\frac{\sqrt{-x+c_1}}{\sqrt{x^3+3}}$$
$$y(x) \to \frac{\sqrt{-x+c_1}}{\sqrt{x^3+3}}$$

6.7 problem 7.4 (e)

Internal problem ID [13410]

Book: Ordinary Differential Equations. An introduction to the fundamentals. Kenneth B. Howell. second edition. CRC Press. FL, USA. 2020
Section: Chapter 7. The exact form and general integrating fators. Additional exercises. page 141
Problem number: 7.4 (e).
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[[_homogeneous, ‘class A‘], _exact, _rational, _dAlembert]`

$$4yx^3 + \left(x^4 - y^4\right) y' = 0$$

✓ Solution by Maple

Time used: 0.078 (sec). Leaf size: 23

```
dsolve(4*x^3*y(x)+(x^4-y(x)^4)*diff(y(x),x)=0,y(x), singsol=all)
```

$$y(x) = \frac{\operatorname{RootOf}\left(-5\_Z c_1^4 x^4 + \_Z^5 - 1\right)}{c_1}$$

✓ Solution by Mathematica

Time used: 1.472 (sec). Leaf size: 131

```
DSolve[4*x^3*y[x]+(x^4-y[x]^4)*y'[x]==0,y[x],x,IncludeSingularSolutions -> True]
```

$$\begin{aligned} y(x) &\to \operatorname{Root}\left[\#1^5 - 5\#1x^4 + e^{5c_1}\&, 1\right] \\ y(x) &\to \operatorname{Root}\left[\#1^5 - 5\#1x^4 + e^{5c_1}\&, 2\right] \\ y(x) &\to \operatorname{Root}\left[\#1^5 - 5\#1x^4 + e^{5c_1}\&, 3\right] \\ y(x) &\to \operatorname{Root}\left[\#1^5 - 5\#1x^4 + e^{5c_1}\&, 4\right] \\ y(x) &\to \operatorname{Root}\left[\#1^5 - 5\#1x^4 + e^{5c_1}\&, 5\right] \\ y(x) &\to 0 \end{aligned}$$

6.8 problem 7.4 (f)

Internal problem ID [13411]

Book: Ordinary Differential Equations. An introduction to the fundamentals. Kenneth B. Howell. second edition. CRC Press. FL, USA. 2020
Section: Chapter 7. The exact form and general integrating fators. Additional exercises. page 141
Problem number: 7.4 (f).
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[[_homogeneous, 'class G'], _exact]`

$$\boxed{\ln(yx) = -1 - \frac{xy'}{y}}$$

✓ Solution by Maple
Time used: 0.031 (sec). Leaf size: 14

```
dsolve(1+ln(x*y(x))+x/y(x)*diff(y(x),x)=0,y(x), singsol=all)
```

$$y(x) = \frac{e^{\frac{c_1}{x}}}{x}$$

✓ Solution by Mathematica
Time used: 0.171 (sec). Leaf size: 17

```
DSolve[1+Log[x*y[x]]+x/y[x]*y'[x]==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to \frac{e^{\frac{c_1}{x}}}{x}$$

6.9 problem 7.4 (g)

Internal problem ID [13412]

Book: Ordinary Differential Equations. An introduction to the fundamentals. Kenneth B. Howell. second edition. CRC Press. FL, USA. 2020
Section: Chapter 7. The exact form and general integrating fators. Additional exercises. page 141
Problem number: 7.4 (g).
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_separable]`

$$\boxed{e^{y} + x\, e^{y} y' = -1}$$

✓ Solution by Maple
Time used: 0.015 (sec). Leaf size: 22

```
dsolve(1+exp(y(x))+x*exp(y(x))*diff(y(x),x)=0,y(x), singsol=all)
```

$$y(x) = -\ln\left(-\frac{x}{x\, e^{c_1} - 1}\right) - c_1$$

✓ Solution by Mathematica
Time used: 0.745 (sec). Leaf size: 25

```
DSolve[1+Exp[y[x]]+x*Exp[y[x]]*y'[x]==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to \log\left(-1 + \frac{e^{c_1}}{x}\right)$$
$$y(x) \to i\pi$$

6.10 problem 7.4 (h)

Internal problem ID [13413]

Book: Ordinary Differential Equations. An introduction to the fundamentals. Kenneth B. Howell. second edition. CRC Press. FL, USA. 2020
Section: Chapter 7. The exact form and general integrating fators. Additional exercises. page 141
Problem number: 7.4 (h).
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[[_1st_order, _with_exponential_symmetries], _exact]`

$$\boxed{e^{y} + (x\,e^{y} + 1)\,y' = 0}$$

✓ Solution by Maple
Time used: 0.0 (sec). Leaf size: 13

```
dsolve(exp(y(x))+(x*exp(y(x))+1)*diff(y(x),x)=0,y(x), singsol=all)
```

$$y(x) = -\operatorname{LambertW}\left(x\,e^{c_1}\right) + c_1$$

✓ Solution by Mathematica
Time used: 4.529 (sec). Leaf size: 17

```
DSolve[Exp[y[x]]+(x*Exp[y[x]]+1)*y'[x]==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to c_1 - W(e^{c_1}x)$$

6.11 problem 7.5 (a)

Internal problem ID [13414]

Book: Ordinary Differential Equations. An introduction to the fundamentals. Kenneth B. Howell. second edition. CRC Press. FL, USA. 2020
Section: Chapter 7. The exact form and general integrating fators. Additional exercises. page 141
Problem number: 7.5 (a).
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_separable]`

$$\boxed{y^4 + y^3 y' x = -1}$$

✓ Solution by Maple
Time used: 0.016 (sec). Leaf size: 70

```
dsolve(1+y(x)^4+x*y(x)^3*diff(y(x),x)=0,y(x), singsol=all)
```

$$y(x) = \frac{(-x^4 + c_1)^{\frac{1}{4}}}{x}$$
$$y(x) = -\frac{(-x^4 + c_1)^{\frac{1}{4}}}{x}$$
$$y(x) = -\frac{i(-x^4 + c_1)^{\frac{1}{4}}}{x}$$
$$y(x) = \frac{i(-x^4 + c_1)^{\frac{1}{4}}}{x}$$

✓ Solution by Mathematica

Time used: 0.295 (sec). Leaf size: 218

```
DSolve[1+y[x]^4+x*y[x]^3*y'[x]==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to -\frac{\sqrt[4]{-x^4 + e^{4c_1}}}{x}$$
$$y(x) \to -\frac{i\sqrt[4]{-x^4 + e^{4c_1}}}{x}$$
$$y(x) \to \frac{i\sqrt[4]{-x^4 + e^{4c_1}}}{x}$$
$$y(x) \to \frac{\sqrt[4]{-x^4 + e^{4c_1}}}{x}$$
$$y(x) \to -\sqrt[4]{-1}$$
$$y(x) \to \sqrt[4]{-1}$$
$$y(x) \to -(-1)^{3/4}$$
$$y(x) \to (-1)^{3/4}$$
$$y(x) \to \frac{ix^3}{(-x^4)^{3/4}}$$
$$y(x) \to \frac{x^3}{(-x^4)^{3/4}}$$
$$y(x) \to \frac{i\sqrt[4]{-x^4}}{x}$$
$$y(x) \to \frac{\sqrt[4]{-x^4}}{x}$$

6.12 problem 7.5 (b)

Internal problem ID [13415]

Book: Ordinary Differential Equations. An introduction to the fundamentals. Kenneth B. Howell. second edition. CRC Press. FL, USA. 2020
Section: Chapter 7. The exact form and general integrating fators. Additional exercises. page 141
Problem number: 7.5 (b).
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[[_homogeneous, 'class G'], _rational]`

$$\boxed{y + (y^4 - 3x)\, y' = 0}$$

✓ Solution by Maple
Time used: 0.0 (sec). Leaf size: 16

```
dsolve(y(x)+(y(x)^4-3*x)*diff(y(x),x)=0,y(x), singsol=all)
```

$$y(x)^4 - y(x)^3 c_1 + x = 0$$

✓ Solution by Mathematica

Time used: 43.447 (sec). Leaf size: 1270

```
DSolve[y[x]+(y[x]^4-3*x)*y'[x]==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to -\frac{1}{2}\sqrt{\frac{\sqrt[3]{\sqrt{-768x^3+81c_1{}^4x^2}+9c_1{}^2x}}{\sqrt[3]{2}3^{2/3}}+\frac{4\sqrt[3]{\frac{2}{3}}x}{\sqrt[3]{\sqrt{-768x^3+81c_1{}^4x^2}+9c_1{}^2x}}+\frac{c_1{}^2}{4}}$$
$$-\frac{1}{2}\sqrt{-\frac{c_1{}^3}{4\sqrt{\frac{\sqrt[3]{\sqrt{-768x^3+81c_1{}^4x^2}+9c_1{}^2x}}{\sqrt[3]{2}3^{2/3}}+\frac{4\sqrt[3]{\frac{2}{3}}x}{\sqrt[3]{\sqrt{-768x^3+81c_1{}^4x^2}+9c_1{}^2x}}+\frac{c_1{}^2}{4}}}-\frac{\sqrt[3]{\sqrt{-768x^3+\cdots}}}{\sqrt[3]{2}\cdots}}$$
$$+\frac{c_1}{4}$$

$$y(x) \to -\frac{1}{2}\sqrt{\frac{\sqrt[3]{\sqrt{-768x^3+81c_1{}^4x^2}+9c_1{}^2x}}{\sqrt[3]{2}3^{2/3}}+\frac{4\sqrt[3]{\frac{2}{3}}x}{\sqrt[3]{\sqrt{-768x^3+81c_1{}^4x^2}+9c_1{}^2x}}+\frac{c_1{}^2}{4}}$$
$$+\frac{1}{2}\sqrt{-\frac{c_1{}^3}{4\sqrt{\frac{\sqrt[3]{\sqrt{-768x^3+81c_1{}^4x^2}+9c_1{}^2x}}{\sqrt[3]{2}3^{2/3}}+\frac{4\sqrt[3]{\frac{2}{3}}x}{\sqrt[3]{\sqrt{-768x^3+81c_1{}^4x^2}+9c_1{}^2x}}+\frac{c_1{}^2}{4}}}-\frac{\sqrt[3]{\sqrt{-768x^3+\cdots}}}{\sqrt[3]{2}\cdots}}$$
$$+\frac{c_1}{4}$$

$$y(x) \to \frac{1}{2}\sqrt{\frac{\sqrt[3]{\sqrt{-768x^3+81c_1{}^4x^2}+9c_1{}^2x}}{\sqrt[3]{2}3^{2/3}}+\frac{4\sqrt[3]{\frac{2}{3}}x}{\sqrt[3]{\sqrt{-768x^3+81c_1{}^4x^2}+9c_1{}^2x}}+\frac{c_1{}^2}{4}}$$
$$-\frac{1}{2}\sqrt{\frac{c_1{}^3}{4\sqrt{\frac{\sqrt[3]{\sqrt{-768x^3+81c_1{}^4x^2}+9c_1{}^2x}}{\sqrt[3]{2}3^{2/3}}+\frac{4\sqrt[3]{\frac{2}{3}}x}{\sqrt[3]{\sqrt{-768x^3+81c_1{}^4x^2}+9c_1{}^2x}}+\frac{c_1{}^2}{4}}}-\frac{\sqrt[3]{\sqrt{-768x^3+8\cdots}}}{\sqrt[3]{2}3\cdots}}$$
$$+\frac{c_1}{4}$$

$$y(x) \to \frac{1}{2}\sqrt{\frac{\sqrt[3]{\sqrt{-768x^3+81c_1{}^4x^2}+9c_1{}^2x}}{\sqrt[3]{2}3^{2/3}}+\frac{4\sqrt[3]{\frac{2}{3}}x}{\sqrt[3]{\sqrt{-768x^3+81c_1{}^4x^2}+9c_1{}^2x}}+\frac{c_1{}^2}{4}}$$

6.13 problem 7.5 (c)

Internal problem ID [13416]

Book: Ordinary Differential Equations. An introduction to the fundamentals. Kenneth B. Howell. second edition. CRC Press. FL, USA. 2020
Section: Chapter 7. The exact form and general integrating fators. Additional exercises. page 141
Problem number: 7.5 (c).
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[[_homogeneous, ‘class G‘], _rational, [_Abel, ‘2nd type‘, ‘cl`

$$\boxed{\frac{2y}{x} + \left(4x^2y - 3\right) y' = 0}$$

✓ Solution by Maple
Time used: 3.234 (sec). Leaf size: 28

```
dsolve(2*y(x)/x+(4*x^2*y(x)-3)*diff(y(x),x)=0,y(x), singsol=all)
```

$$y(x) = \frac{\operatorname{RootOf}\left(\_Z^{32}c_1 - \_Z^{24}c_1 - x^8\right)^8}{x^2}$$

✓ Solution by Mathematica

Time used: 60.256 (sec). Leaf size: 1985

```
DSolve[2*y[x]/x+(4*x^2*y[x]-3)*y'[x]==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to \frac{1}{4x^2} - \frac{1}{2}\sqrt{\frac{1}{4x^4} + \frac{4\sqrt[3]{\frac{2}{3}}e^{-8c_1}x^2}{\sqrt[3]{9e^{-8c_1}x^2 + \sqrt{3}e^{-24c_1}\sqrt{e^{24c_1}x^4\left(-256x^8 + 27e^{8c_1}\right)}}} + \frac{\sqrt[3]{9e^{-8c_1}x^2 + \sqrt{3}e^{-24c_1}\sqrt{e^{24c_1}x^4\left(-2 \cdots\right.}}}{\sqrt[3]{2}3^{2/3}x^2}} - \frac{1}{2}\sqrt{\frac{1}{2x^4} - \frac{4\sqrt[3]{\frac{2}{3}}e^{-8c_1}x^2}{\sqrt[3]{9e^{-8c_1}x^2 + \sqrt{3}e^{-24c_1}\sqrt{e^{24c_1}x^4\left(-256x^8 + 27e^{8c_1}\right)}}} - \frac{\sqrt[3]{9e^{-8c_1}x^2 + \sqrt{3}e^{-24c_1}\sqrt{e^{24c_1}x^4\left(-2 \cdots\right.}}}{\sqrt[3]{2}3^{2/3}x^2} \cdots}$$

$$y(x) \to \frac{1}{4x^2} - \frac{1}{2}\sqrt{\frac{1}{4x^4} + \frac{4\sqrt[3]{\frac{2}{3}}e^{-8c_1}x^2}{\sqrt[3]{9e^{-8c_1}x^2 + \sqrt{3}e^{-24c_1}\sqrt{e^{24c_1}x^4\left(-256x^8 + 27e^{8c_1}\right)}}} + \frac{\sqrt[3]{9e^{-8c_1}x^2 + \sqrt{3}e^{-24c_1}\sqrt{e^{24c_1}x^4\left(-2 \cdots\right.}}}{\sqrt[3]{2}3^{2/3}x^2}} + \frac{1}{2}\sqrt{\frac{1}{2x^4} - \frac{4\sqrt[3]{\frac{2}{3}}e^{-8c_1}x^2}{\sqrt[3]{9e^{-8c_1}x^2 + \sqrt{3}e^{-24c_1}\sqrt{e^{24c_1}x^4\left(-256x^8 + 27e^{8c_1}\right)}}} - \frac{\sqrt[3]{9e^{-8c_1}x^2 + \sqrt{3}e^{-24c_1}\sqrt{e^{24c_1}x^4\left(-2 \cdots\right.}}}{\sqrt[3]{2}3^{2/3}x^2} \cdots}$$

$$y(x) \to \frac{1}{4x^2} + \frac{1}{2}\sqrt{\frac{1}{4x^4} + \frac{4\sqrt[3]{\frac{2}{3}}e^{-8c_1}x^2}{\sqrt[3]{9e^{-8c_1}x^2 + \sqrt{3}e^{-24c_1}\sqrt{e^{24c_1}x^4\left(-256x^8 + 27e^{8c_1}\right)}}} + \frac{\sqrt[3]{9e^{-8c_1}x^2 + \sqrt{3}e^{-24c_1}\sqrt{e^{24c_1}x^4\left(-2 \cdots\right.}}}{\sqrt[3]{2}3^{2/3}x^2}} - \frac{1}{2}\sqrt{\frac{1}{2x^4} - \frac{4\sqrt[3]{\frac{2}{3}}e^{-8c_1}x^2}{\sqrt[3]{9e^{-8c_1}x^2 + \sqrt{3}e^{-24c_1}\sqrt{e^{24c_1}x^4\left(-256x^8 + 27e^{8c_1}\right)}}} - \frac{\sqrt[3]{9e^{-8c_1}x^2 + \sqrt{3}e^{-24c_1}\sqrt{e^{24c_1}x^4\left(-2 \cdots\right.}}}{\sqrt[3]{2}3^{2/3}x^2} \cdots}$$

$$y(x) \to \frac{1}{4x^2} \cdots \frac{1}{\cdots} \quad \frac{4\sqrt[3]{\frac{2}{3}}e^{-8c_1}x^2}{\cdots} \quad \sqrt[3]{9e^{-8c_1}x^2 + \sqrt{3}e^{-24c_1}\sqrt{e^{24c_1}x^4\left(-2 \cdots\right.}}$$

6.14 problem 7.5 (d)

Internal problem ID [13417]

Book: Ordinary Differential Equations. An introduction to the fundamentals. Kenneth B. Howell. second edition. CRC Press. FL, USA. 2020
Section: Chapter 7. The exact form and general integrating fators. Additional exercises. page 141
Problem number: 7.5 (d).
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[[_1st_order, '_with_symmetry_[F(x)*G(y),0]']]`

$$\boxed{(1-\tan(y)\,x)\,y' = -1}$$

✓ Solution by Maple

Time used: 0.015 (sec). Leaf size: 108

```
dsolve(1+(1-x*tan(y(x)))*diff(y(x),x)=0,y(x), singsol=all)
```

$$y(x) = \arctan\left(\frac{-\sqrt{-c_1^2+x^2+1}\,x+c_1}{x^2+1}, \frac{c_1x+\sqrt{-c_1^2+x^2+1}}{x^2+1}\right)$$
$$y(x) = \arctan\left(\frac{\sqrt{-c_1^2+x^2+1}\,x+c_1}{x^2+1}, \frac{c_1x-\sqrt{-c_1^2+x^2+1}}{x^2+1}\right)$$

✓ Solution by Mathematica

Time used: 0.108 (sec). Leaf size: 145

```
DSolve[1+(1-x*Tan[y[x]])*y'[x]==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \rightarrow -\sec^{-1}\left(\frac{c_1x-\sqrt{x^2+1-c_1{}^2}}{-1+c_1{}^2}\right)$$
$$y(x) \rightarrow \sec^{-1}\left(\frac{c_1x-\sqrt{x^2+1-c_1{}^2}}{-1+c_1{}^2}\right)$$
$$y(x) \rightarrow -\sec^{-1}\left(\frac{\sqrt{x^2+1-c_1{}^2}+c_1x}{-1+c_1{}^2}\right)$$
$$y(x) \rightarrow \sec^{-1}\left(\frac{\sqrt{x^2+1-c_1{}^2}+c_1x}{-1+c_1{}^2}\right)$$

6.15 problem 7.5 (e)

Internal problem ID [13418]

Book: Ordinary Differential Equations. An introduction to the fundamentals. Kenneth B. Howell. second edition. CRC Press. FL, USA. 2020
Section: Chapter 7. The exact form and general integrating fators. Additional exercises. page 141
Problem number: 7.5 (e).
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_separable]`

$$3y + 3y^2 + (2x + 4yx)\, y' = 0$$

✓ Solution by Maple
Time used: 0.859 (sec). Leaf size: 137

```
dsolve(3*y(x)+3*y(x)^2+(2*x+4*x*y(x))*diff(y(x),x)=0,y(x), singsol=all)
```

$$y(x) = \frac{-c_1 x - \sqrt{c_1^2 x^2 - 4\sqrt{c_1 x}}}{2c_1 x}$$
$$y(x) = \frac{-c_1 x + \sqrt{c_1^2 x^2 - 4\sqrt{c_1 x}}}{2c_1 x}$$
$$y(x) = \frac{-c_1 x - \sqrt{c_1^2 x^2 + 4\sqrt{c_1 x}}}{2c_1 x}$$
$$y(x) = \frac{-c_1 x + \sqrt{c_1^2 x^2 + 4\sqrt{c_1 x}}}{2c_1 x}$$

✓ Solution by Mathematica

Time used: 6.61 (sec). Leaf size: 67

```
DSolve[3*y[x]+3*y[x]^2+(2*x+4*x*y[x])*y'[x]==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to \frac{1}{2}\left(-1-\sqrt{1+\frac{4e^{c_1}}{x^{3/2}}}\right)$$
$$y(x) \to \frac{1}{2}\left(-1+\sqrt{1+\frac{4e^{c_1}}{x^{3/2}}}\right)$$
$$y(x) \to -1$$
$$y(x) \to 0$$

6.16 problem 7.5 (f)

Internal problem ID [13419]

Book: Ordinary Differential Equations. An introduction to the fundamentals. Kenneth B. Howell. second edition. CRC Press. FL, USA. 2020
Section: Chapter 7. The exact form and general integrating fators. Additional exercises. page 141
Problem number: 7.5 (f).
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_separable]`

$$\boxed{2(y+1)\,x - y' = 0}$$

✓ Solution by Maple
Time used: 0.0 (sec). Leaf size: 12

```
dsolve(2*x*(y(x)+1)-diff(y(x),x)=0,y(x), singsol=all)
```

$$y(x) = \mathrm{e}^{x^2} c_1 - 1$$

✓ Solution by Mathematica
Time used: 0.025 (sec). Leaf size: 20

```
DSolve[2*x*(y[x]+1)-y'[x]==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to -1 + c_1 e^{x^2}$$
$$y(x) \to -1$$

6.17 problem 7.5 (g)

Internal problem ID [13420]

Book: Ordinary Differential Equations. An introduction to the fundamentals. Kenneth B. Howell. second edition. CRC Press. FL, USA. 2020
Section: Chapter 7. The exact form and general integrating fators. Additional exercises. page 141
Problem number: 7.5 (g).
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[[_homogeneous, ‘class G‘], _rational, [_Abel, ‘2nd type‘, ‘cl`

$$\boxed{2y^3 + (4y^3x^3 - 3y^2x)\, y' = 0}$$

✓ Solution by Maple
Time used: 0.015 (sec). Leaf size: 32

```
dsolve(2*y(x)^3+(4*x^3*y(x)^3-3*x*y(x)^2)*diff(y(x),x)=0,y(x), singsol=all)
```

$$y(x) = 0$$
$$y(x) = \frac{\operatorname{RootOf}\left(\_Z^{32}c_1 - \_Z^{24}c_1 - x^8\right)^8}{x^2}$$

✓ Solution by Mathematica

Time used: 60.187 (sec). Leaf size: 1990

```
DSolve[2*y[x]^3+(4*x^3*y[x]^3-3*x*y[x]^2)*y'[x]==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x)\to 0$$

$$y(x)\to \frac{1}{4x^2} - \frac{1}{2}\sqrt{\frac{1}{4x^4}+\frac{4\sqrt[3]{\frac{2}{3}}e^{-8c_1}x^2}{\sqrt[3]{9e^{-8c_1}x^2+\sqrt{3}e^{-24c_1}\sqrt{e^{24c_1}x^4\left(-256x^8+27e^{8c_1}\right)}}}+\frac{\sqrt[3]{9e^{-8c_1}x^2+\sqrt{3}e^{-24c_1}\sqrt{e^{24c_1}x^4\left(-2\cdots\right.}}}{\sqrt[3]{2}3^{2/3}x^2}\cdots} - \frac{1}{2}\sqrt{\frac{1}{2x^4}-\frac{4\sqrt[3]{\frac{2}{3}}e^{-8c_1}x^2}{\sqrt[3]{9e^{-8c_1}x^2+\sqrt{3}e^{-24c_1}\sqrt{e^{24c_1}x^4\left(-256x^8+27e^{8c_1}\right)}}}-\frac{\sqrt[3]{9e^{-8c_1}x^2+\sqrt{3}e^{-24c_1}\sqrt{e^{24c_1}x^4\left(-2\cdots\right.}}}{\sqrt[3]{2}3^{2/3}x^2}\cdots}$$

$$y(x)\to \frac{1}{4x^2} - \frac{1}{2}\sqrt{\frac{1}{4x^4}+\frac{4\sqrt[3]{\frac{2}{3}}e^{-8c_1}x^2}{\sqrt[3]{9e^{-8c_1}x^2+\sqrt{3}e^{-24c_1}\sqrt{e^{24c_1}x^4\left(-256x^8+27e^{8c_1}\right)}}}+\frac{\sqrt[3]{9e^{-8c_1}x^2+\sqrt{3}e^{-24c_1}\sqrt{e^{24c_1}x^4\left(-2\cdots\right.}}}{\sqrt[3]{2}3^{2/3}x^2}\cdots} + \frac{1}{2}\sqrt{\frac{1}{2x^4}-\frac{4\sqrt[3]{\frac{2}{3}}e^{-8c_1}x^2}{\sqrt[3]{9e^{-8c_1}x^2+\sqrt{3}e^{-24c_1}\sqrt{e^{24c_1}x^4\left(-256x^8+27e^{8c_1}\right)}}}-\frac{\sqrt[3]{9e^{-8c_1}x^2+\sqrt{3}e^{-24c_1}\sqrt{e^{24c_1}x^4\left(-2\cdots\right.}}}{\sqrt[3]{2}3^{2/3}x^2}\cdots}$$

$$y(x)\to \frac{1}{4x^2} + \frac{1}{2}\sqrt{\frac{1}{4x^4}+\frac{4\sqrt[3]{\frac{2}{3}}e^{-8c_1}x^2}{\sqrt[3]{9e^{-8c_1}x^2+\sqrt{3}e^{-24c_1}\sqrt{e^{24c_1}x^4\left(-256x^8+27e^{8c_1}\right)}}}+\frac{\sqrt[3]{9e^{-8c_1}x^2+\sqrt{3}e^{-24c_1}\sqrt{e^{24c_1}x^4\left(-2\cdots\right.}}}{\sqrt[3]{2}3^{2/3}x^2}\cdots} - \frac{1}{2}\sqrt{\frac{1}{2x^4}-\frac{4\sqrt[3]{\frac{2}{3}}e^{-8c_1}x^2}{\sqrt[3]{9e^{-8c_1}x^2+\sqrt{3}e^{-24c_1}\sqrt{e^{24c_1}x^4\left(-256x^8+27e^{8c_1}\right)}}}-\frac{\sqrt[3]{9e^{-8c_1}x^2+\sqrt{3}e^{-24c_1}\sqrt{e^{24c_1}x^4\left(-2\cdots\right.}}}{\sqrt[3]{2}3^{2/3}x^2}\cdots}$$

$$y(x)\to \frac{1}{4x^2} \cdots$$

6.18 problem 7.5 (h)

Internal problem ID [13421]

Book: Ordinary Differential Equations. An introduction to the fundamentals. Kenneth B. Howell. second edition. CRC Press. FL, USA. 2020
Section: Chapter 7. The exact form and general integrating fators. Additional exercises. page 141
Problem number: 7.5 (h).
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[[_homogeneous, ‘class G‘], _rational, [_Abel, ‘2nd type‘, ‘cl`

$$\boxed{4yx + (3x^2 + 5y)\, y' = 0}$$

✓ Solution by Maple
Time used: 0.922 (sec). Leaf size: 29

```
dsolve(4*x*y(x)+(3*x^2+5*y(x))*diff(y(x),x)=0,y(x), singsol=all)
```

$$y(x) = \operatorname{RootOf}\left(x^5\_Z^{25} + x^5\_Z^{15} - c_1\right)^{10} x^2$$

✓ Solution by Mathematica

Time used: 60.064 (sec). Leaf size: 1121

```
DSolve[4*x*y[x]+(3*x^2+5*y[x])*y'[x]==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to -\frac{3x^2}{5} + \frac{1}{5\text{Root}\left[\#1^{10}\left(11664x^{20}+11664e^{60c_1}\right)-9720\#1^8x^{16}+1080\#1^7x^{14}+3105\#1^6x^{12}-666\#1^5x^{10}-425\right.}$$

$$y(x) \to -\frac{3x^2}{5} + \frac{1}{5\text{Root}\left[\#1^{10}\left(11664x^{20}+11664e^{60c_1}\right)-9720\#1^8x^{16}+1080\#1^7x^{14}+3105\#1^6x^{12}-666\#1^5x^{10}-425\right.}$$

$$y(x) \to -\frac{3x^2}{5} + \frac{1}{5\text{Root}\left[\#1^{10}\left(11664x^{20}+11664e^{60c_1}\right)-9720\#1^8x^{16}+1080\#1^7x^{14}+3105\#1^6x^{12}-666\#1^5x^{10}-425\right.}$$

$$y(x) \to -\frac{3x^2}{5} + \frac{1}{5\text{Root}\left[\#1^{10}\left(11664x^{20}+11664e^{60c_1}\right)-9720\#1^8x^{16}+1080\#1^7x^{14}+3105\#1^6x^{12}-666\#1^5x^{10}-425\right.}$$

$$y(x) \to -\frac{3x^2}{5} + \frac{1}{5\text{Root}\left[\#1^{10}\left(11664x^{20}+11664e^{60c_1}\right)-9720\#1^8x^{16}+1080\#1^7x^{14}+3105\#1^6x^{12}-666\#1^5x^{10}-425\right.}$$

$$y(x) \to -\frac{3x^2}{5} + \frac{1}{5\text{Root}\left[\#1^{10}\left(11664x^{20}+11664e^{60c_1}\right)-9720\#1^8x^{16}+1080\#1^7x^{14}+3105\#1^6x^{12}-666\#1^5x^{10}-425\right.}$$

$$y(x) \to -\frac{3x^2}{5} + \frac{1}{5\text{Root}\left[\#1^{10}\left(11664x^{20}+11664e^{60c_1}\right)-9720\#1^8x^{16}+1080\#1^7x^{14}+3105\#1^6x^{12}-666\#1^5x^{10}-425\right.}$$

$$y(x) \to -\frac{3x^2}{5} + \frac{1}{5\text{Root}\left[\#1^{10}\left(11664x^{20}+11664e^{60c_1}\right)-9720\#1^8x^{16}+1080\#1^7x^{14}+3105\#1^6x^{12}-666\#1^5x^{10}-425\right.}$$

$$y(x) \to -\frac{3x^2}{5} + \frac{1}{5\text{Root}\left[\#1^{10}\left(11664x^{20}+11664e^{60c_1}\right)-9720\#1^8x^{16}+1080\#1^7x^{14}+3105\#1^6x^{12}-666\#1^5x^{10}-425\right.}$$

$$y(x) \to -\frac{3x^2}{5} + \frac{1}{5\text{Root}\left[\#1^{10}\left(11664x^{20}+11664e^{60c_1}\right)-9720\#1^8x^{16}+1080\#1^7x^{14}+3105\#1^6x^{12}-666\#1^5x^{10}-425\right.}$$

6.19 problem 7.5 (i)

Internal problem ID [13422]

Book: Ordinary Differential Equations. An introduction to the fundamentals. Kenneth B. Howell. second edition. CRC Press. FL, USA. 2020
Section: Chapter 7. The exact form and general integrating fators. Additional exercises. page 141
Problem number: 7.5 (i).
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[[_homogeneous, 'class G'], _rational]`

$$12x^2y^2 + \left(7yx^3 + \frac{x}{y}\right)y' = -6$$

✓ Solution by Maple

Time used: 1.015 (sec). Leaf size: 63

```
dsolve(6+12*x^2*y(x)^2+(7*x^3*y(x)+x/y(x))*diff(y(x),x)=0,y(x), singsol=all)
```

$$y(x) = \operatorname{RootOf}\left(\_Z^{35}c_1^2x^{10} - \_Z^{30}c_1^2x^{10} - 1\right)^{15} x^4 \left(\operatorname{RootOf}\left(\_Z^{35}c_1^2x^{10} - \_Z^{30}c_1^2x^{10} - 1\right)^5 - 1\right) c_1$$

✓ Solution by Mathematica

Time used: 3.003 (sec). Leaf size: 330

```
DSolve[6+12*x^2*y[x]^2+(7*x^3*y[x]+x/y[x])*y'[x]==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to \text{Root}\left[-\#1^7 - \frac{3\#1^5}{x^2} - \frac{3\#1^3}{x^4} - \frac{\#1}{x^6} + \frac{e^{c_1}}{x^{12}}\&, 1\right]$$
$$y(x) \to \text{Root}\left[-\#1^7 - \frac{3\#1^5}{x^2} - \frac{3\#1^3}{x^4} - \frac{\#1}{x^6} + \frac{e^{c_1}}{x^{12}}\&, 2\right]$$
$$y(x) \to \text{Root}\left[-\#1^7 - \frac{3\#1^5}{x^2} - \frac{3\#1^3}{x^4} - \frac{\#1}{x^6} + \frac{e^{c_1}}{x^{12}}\&, 3\right]$$
$$y(x) \to \text{Root}\left[-\#1^7 - \frac{3\#1^5}{x^2} - \frac{3\#1^3}{x^4} - \frac{\#1}{x^6} + \frac{e^{c_1}}{x^{12}}\&, 4\right]$$
$$y(x) \to \text{Root}\left[-\#1^7 - \frac{3\#1^5}{x^2} - \frac{3\#1^3}{x^4} - \frac{\#1}{x^6} + \frac{e^{c_1}}{x^{12}}\&, 5\right]$$
$$y(x) \to \text{Root}\left[-\#1^7 - \frac{3\#1^5}{x^2} - \frac{3\#1^3}{x^4} - \frac{\#1}{x^6} + \frac{e^{c_1}}{x^{12}}\&, 6\right]$$
$$y(x) \to \text{Root}\left[-\#1^7 - \frac{3\#1^5}{x^2} - \frac{3\#1^3}{x^4} - \frac{\#1}{x^6} + \frac{e^{c_1}}{x^{12}}\&, 7\right]$$

7 Chapter 8. Review exercises for part of part II. page 143

7.1 problem 1

Internal problem ID [13423]

Book: Ordinary Differential Equations. An introduction to the fundamentals. Kenneth B. Howell. second edition. CRC Press. FL, USA. 2020
Section: Chapter 8. Review exercises for part of part II. page 143
Problem number: 1.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_linear]`

$$\boxed{y'x - 2y = -6x^3}$$

✓ Solution by Maple
Time used: 0.0 (sec). Leaf size: 15

```
dsolve(x*diff(y(x),x)=2*y(x)-6*x^3,y(x), singsol=all)
```

$$y(x) = c_1 x^2 - 6x^3$$

✓ Solution by Mathematica
Time used: 0.028 (sec). Leaf size: 15

```
DSolve[x*y'[x]==2*y[x]-6*x^3,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to x^2(-6x + c_1)$$

7.2 problem 2

Internal problem ID [13424]

Book: Ordinary Differential Equations. An introduction to the fundamentals. Kenneth B. Howell. second edition. CRC Press. FL, USA. 2020
Section: Chapter 8. Review exercises for part of part II. page 143
Problem number: 2.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_separable]`

$$\boxed{y'x - 2y^2 + 6y = 0}$$

✓ Solution by Maple
Time used: 0.016 (sec). Leaf size: 16

```
dsolve(x*diff(y(x),x)=2*y(x)^2-6*y(x),y(x), singsol=all)
```

$$y(x) = \frac{3}{3c_1x^6 + 1}$$

✓ Solution by Mathematica
Time used: 1.886 (sec). Leaf size: 31

```
DSolve[x*y'[x]==2*y[x]^2-6*y[x],y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to \frac{3}{1 + e^{3c_1}x^6}$$
$$y(x) \to 0$$
$$y(x) \to 3$$

7.3 problem 3

Internal problem ID [13425]

Book: Ordinary Differential Equations. An introduction to the fundamentals. Kenneth B. Howell. second edition. CRC Press. FL, USA. 2020
Section: Chapter 8. Review exercises for part of part II. page 143
Problem number: 3.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_separable]`

$$\boxed{4y^2 - x^2y^2 + y' = 0}$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 18

```
dsolve(4*y(x)^2-x^2*y(x)^2+diff(y(x),x)=0,y(x), singsol=all)
```

$$y(x) = -\frac{3}{x^3 - 3c_1 - 12x}$$

✓ Solution by Mathematica

Time used: 0.125 (sec). Leaf size: 25

```
DSolve[4*y[x]^2-x^2*y[x]^2+y'[x]==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to -\frac{3}{x^3 - 12x + 3c_1}$$
$$y(x) \to 0$$

7.4 problem 4

Internal problem ID [13426]

Book: Ordinary Differential Equations. An introduction to the fundamentals. Kenneth B. Howell. second edition. CRC Press. FL, USA. 2020
Section: Chapter 8. Review exercises for part of part II. page 143
Problem number: 4.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[[_homogeneous, 'class C'], _dAlembert]`

$$\boxed{y' - \sqrt{y + x} = 0}$$

✓ Solution by Maple
Time used: 0.032 (sec). Leaf size: 42

```
dsolve(diff(y(x),x)=sqrt(x+y(x)),y(x), singsol=all)
```

$$x - 2\sqrt{x + y(x)} - \ln\left(\sqrt{x + y(x)} - 1\right) + \ln\left(1 + \sqrt{x + y(x)}\right) + \ln(x + y(x) - 1) - c_1 = 0$$

✓ Solution by Mathematica
Time used: 8.647 (sec). Leaf size: 59

```
DSolve[y'[x]==Sqrt[x+y[x]],y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to W\left(-e^{-\frac{x}{2}-1-\frac{c_1}{2}}\right)^2 + 2W\left(-e^{-\frac{x}{2}-1-\frac{c_1}{2}}\right) - x + 1$$
$$y(x) \to 1 - x$$

7.5 problem 5

Internal problem ID [13427]

Book: Ordinary Differential Equations. An introduction to the fundamentals. Kenneth B. Howell. second edition. CRC Press. FL, USA. 2020
Section: Chapter 8. Review exercises for part of part II. page 143
Problem number: 5.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_quadrature]`

$$\boxed{x^2y' = \sqrt{x} + 3}$$

✓ Solution by Maple
Time used: 0.0 (sec). Leaf size: 16

```
dsolve(x^2*diff(y(x),x)-sqrt(x)=3,y(x), singsol=all)
```

$$y(x) = -\frac{3}{x} - \frac{2}{\sqrt{x}} + c_1$$

✓ Solution by Mathematica
Time used: 0.011 (sec). Leaf size: 20

```
DSolve[x^2*y'[x]-Sqrt[x]==3,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to -\frac{2}{\sqrt{x}} - \frac{3}{x} + c_1$$

7.6 problem 6

Internal problem ID [13428]

Book: Ordinary Differential Equations. An introduction to the fundamentals. Kenneth B. Howell. second edition. CRC Press. FL, USA. 2020
Section: Chapter 8. Review exercises for part of part II. page 143
Problem number: 6.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[[_homogeneous, ‘class A‘], _rational, _dAlembert]`

$$\boxed{xyy' - y^2 - \sqrt{x^2y^2 + x^4} = 0}$$

✓ Solution by Maple
Time used: 0.015 (sec). Leaf size: 33

```
dsolve(x*y(x)*diff(y(x),x)-y(x)^2=sqrt(x^4+x^2*y(x)^2),y(x), singsol=all)
```

$$-\frac{y(x)^2 + x^2}{\sqrt{x^2 \left(y(x)^2 + x^2\right)}} + \ln(x) - c_1 = 0$$

✓ Solution by Mathematica
Time used: 0.278 (sec). Leaf size: 54

```
DSolve[x*y[x]*y'[x]-y[x]^2==Sqrt[x^4+x^2*y[x]^2],y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to -x\sqrt{\log^2(x) + 2c_1 \log(x) - 1 + {c_1}^2}$$
$$y(x) \to x\sqrt{\log^2(x) + 2c_1 \log(x) - 1 + {c_1}^2}$$

7.7 problem 7

Internal problem ID [13429]

Book: Ordinary Differential Equations. An introduction to the fundamentals. Kenneth B. Howell. second edition. CRC Press. FL, USA. 2020
Section: Chapter 8. Review exercises for part of part II. page 143
Problem number: 7.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[[_homogeneous, ‘class C‘], _Riccati]`

$$\boxed{y' - y^2 + 2yx = x^2}$$

✓ Solution by Maple
Time used: 0.047 (sec). Leaf size: 29

```
dsolve(diff(y(x),x)=y(x)^2-2*x*y(x)+x^2,y(x), singsol=all)
```

$$y(x) = \frac{c_1(-1+x)\,\mathrm{e}^{2x} - x - 1}{-1 + \mathrm{e}^{2x} c_1}$$

✓ Solution by Mathematica
Time used: 0.128 (sec). Leaf size: 29

```
DSolve[y'[x]==y[x]^2-2*x*y[x]+x^2,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to x + \frac{1}{\frac{1}{2} + c_1 e^{2x}} - 1$$
$$y(x) \to x - 1$$

7.8 problem 8

Internal problem ID [13430]

Book: Ordinary Differential Equations. An introduction to the fundamentals. Kenneth B. Howell. second edition. CRC Press. FL, USA. 2020
Section: Chapter 8. Review exercises for part of part II. page 143
Problem number: 8.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_linear]`

$$\boxed{4yx + x^2y' = 6}$$

✓ Solution by Maple
Time used: 0.0 (sec). Leaf size: 15

```
dsolve(4*x*y(x)-6+x^2*diff(y(x),x)=0,y(x), singsol=all)
```

$$y(x) = \frac{2x^3 + c_1}{x^4}$$

✓ Solution by Mathematica
Time used: 0.027 (sec). Leaf size: 17

```
DSolve[4*x*y[x]-6+x^2*y'[x]==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to \frac{2x^3 + c_1}{x^4}$$

7.9 problem 9

Internal problem ID [13431]

Book: Ordinary Differential Equations. An introduction to the fundamentals. Kenneth B. Howell. second edition. CRC Press. FL, USA. 2020
Section: Chapter 8. Review exercises for part of part II. page 143
Problem number: 9.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[[_homogeneous, ‘class G‘], _exact, _rational, _Bernoulli]`

$$\boxed{y^2x + y'yx^2 = 6}$$

✓ Solution by Maple
Time used: 0.0 (sec). Leaf size: 30

```
dsolve(x*y(x)^2-6+x^2*y(x)*diff(y(x),x)=0,y(x), singsol=all)
```

$$y(x) = \frac{\sqrt{12x + c_1}}{x}$$
$$y(x) = -\frac{\sqrt{12x + c_1}}{x}$$

✓ Solution by Mathematica
Time used: 0.19 (sec). Leaf size: 38

```
DSolve[x*y[x]^2-6+x^2*y[x]*y'[x]==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to -\frac{\sqrt{12x + c_1}}{x}$$
$$y(x) \to \frac{\sqrt{12x + c_1}}{x}$$

7.10 problem 10

Internal problem ID [13432]

Book: Ordinary Differential Equations. An introduction to the fundamentals. Kenneth B. Howell. second edition. CRC Press. FL, USA. 2020
Section: Chapter 8. Review exercises for part of part II. page 143
Problem number: 10.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[[_homogeneous, ‘class A‘], _rational, _Bernoulli]`

$$\boxed{y^3 + y'xy^2 = -x^3}$$

✓ Solution by Maple
Time used: 0.0 (sec). Leaf size: 74

```
dsolve(x^3+y(x)^3+x*y(x)^2*diff(y(x),x)=0,y(x), singsol=all)
```

$$y(x) = \frac{(-4x^6 + 8c_1)^{\frac{1}{3}}}{2x}$$
$$y(x) = -\frac{(-4x^6 + 8c_1)^{\frac{1}{3}} \left(1 + i\sqrt{3}\right)}{4x}$$
$$y(x) = \frac{(-4x^6 + 8c_1)^{\frac{1}{3}} \left(i\sqrt{3} - 1\right)}{4x}$$

✓ Solution by Mathematica

Time used: 0.232 (sec). Leaf size: 80

```
DSolve[x^3+y[x]^3+x*y[x]^2*y'[x]==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to -\frac{\sqrt[3]{-\frac{1}{2}}\sqrt[3]{-x^6+2c_1}}{x}$$
$$y(x) \to \frac{\sqrt[3]{-\frac{x^6}{2}+c_1}}{x}$$
$$y(x) \to \frac{(-1)^{2/3}\sqrt[3]{-\frac{x^6}{2}+c_1}}{x}$$

7.11 problem 11

Internal problem ID [13433]

Book: Ordinary Differential Equations. An introduction to the fundamentals. Kenneth B. Howell. second edition. CRC Press. FL, USA. 2020
Section: Chapter 8. Review exercises for part of part II. page 143
Problem number: 11.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_linear]`

$$\boxed{y'x + 3y = x^3}$$

✓ Solution by Maple
Time used: 0.0 (sec). Leaf size: 16

```
dsolve(3*y(x)-x^3+x*diff(y(x),x)=0,y(x), singsol=all)
```

$$y(x) = \frac{x^6 + 6c_1}{6x^3}$$

✓ Solution by Mathematica
Time used: 0.031 (sec). Leaf size: 19

```
DSolve[3*y[x]-x^3+x*y'[x]==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to \frac{x^3}{6} + \frac{c_1}{x^3}$$

7.12 problem 12

Internal problem ID [13434]

Book: Ordinary Differential Equations. An introduction to the fundamentals. Kenneth B. Howell. second edition. CRC Press. FL, USA. 2020
Section: Chapter 8. Review exercises for part of part II. page 143
Problem number: 12.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_exact, _rational, _Bernoulli]`

$$\boxed{2y^2x + (2x^2y + 2y)\, y' = -1}$$

✓ Solution by Maple
Time used: 0.0 (sec). Leaf size: 50

```
dsolve(1+2*x*y(x)^2+(2*x^2*y(x)+2*y(x))*diff(y(x),x)=0,y(x), singsol=all)
```

$$y(x) = \frac{\sqrt{(x^2+1)\,(c_1 - x)}}{x^2+1}$$
$$y(x) = -\frac{\sqrt{(x^2+1)\,(c_1 - x)}}{x^2+1}$$

✓ Solution by Mathematica
Time used: 0.212 (sec). Leaf size: 50

```
DSolve[1+2*x*y[x]^2+(2*x^2*y[x]+2*y[x])*y'[x]==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to -\frac{\sqrt{-x+c_1}}{\sqrt{x^2+1}}$$
$$y(x) \to \frac{\sqrt{-x+c_1}}{\sqrt{x^2+1}}$$

7.13 problem 13

Internal problem ID [13435]

Book: Ordinary Differential Equations. An introduction to the fundamentals. Kenneth B. Howell. second edition. CRC Press. FL, USA. 2020
Section: Chapter 8. Review exercises for part of part II. page 143
Problem number: 13.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[[_homogeneous, ‘class D‘], _rational, _Bernoulli]`

$$\boxed{3y^3x - y + y'x = 0}$$

✓ Solution by Maple
Time used: 0.0 (sec). Leaf size: 30

```
dsolve(3*x*y(x)^3-y(x)+x*diff(y(x),x)=0,y(x), singsol=all)
```

$$y(x) = \frac{x}{\sqrt{2x^3 + c_1}}$$
$$y(x) = -\frac{x}{\sqrt{2x^3 + c_1}}$$

✓ Solution by Mathematica
Time used: 0.182 (sec). Leaf size: 43

```
DSolve[3*x*y[x]^3-y[x]+x*y'[x]==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to -\frac{x}{\sqrt{2x^3 + c_1}}$$
$$y(x) \to \frac{x}{\sqrt{2x^3 + c_1}}$$
$$y(x) \to 0$$

7.14 problem 14

Internal problem ID [13436]

Book: Ordinary Differential Equations. An introduction to the fundamentals. Kenneth B. Howell. second edition. CRC Press. FL, USA. 2020
Section: Chapter 8. Review exercises for part of part II. page 143
Problem number: 14.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_linear]`

$$\boxed{-2yx + (x^2+1)\, y' = -2x^2 - 2}$$

✓ Solution by Maple
Time used: 0.0 (sec). Leaf size: 16

```
dsolve(2+2*x^2-2*x*y(x)+(x^2+1)*diff(y(x),x)=0,y(x), singsol=all)
```

$$y(x) = (-2\arctan(x) + c_1)\,(x^2+1)$$

✓ Solution by Mathematica
Time used: 0.035 (sec). Leaf size: 18

```
DSolve[2+2*x^2-2*x*y[x]+(x^2+1)*y'[x]==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to (x^2+1)\,(-2\arctan(x) + c_1)$$

7.15 problem 15

Internal problem ID [13437]

Book: Ordinary Differential Equations. An introduction to the fundamentals. Kenneth B. Howell. second edition. CRC Press. FL, USA. 2020
Section: Chapter 8. Review exercises for part of part II. page 143
Problem number: 15.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_quadrature]`

$$\boxed{(y^2 - 4)\, y' - y = 0}$$

✓ Solution by Maple
Time used: 0.015 (sec). Leaf size: 39

```
dsolve((y(x)^2-4)*diff(y(x),x)=y(x),y(x), singsol=all)
```

$$y(x) = \frac{2\,\mathrm{e}^{-\frac{c_1}{4}-\frac{x}{4}}}{\sqrt{-\frac{\mathrm{e}^{-\frac{c_1}{2}-\frac{x}{2}}}{\mathrm{LambertW}\left(-\frac{\mathrm{e}^{-\frac{c_1}{2}-\frac{x}{2}}}{4}\right)}}}$$

✓ Solution by Mathematica

Time used: 32.653 (sec). Leaf size: 246

```
DSolve[(y[x]^2-4)*y'[x]==y[x],y[x],x,IncludeSingularSolutions -> True]
```

$$y(x)\to -2i\sqrt{W\left(-\frac{1}{4}\sqrt[4]{e^{-2(x+c_1)}}\right)}$$
$$y(x)\to 2i\sqrt{W\left(-\frac{1}{4}\sqrt[4]{e^{-2(x+c_1)}}\right)}$$
$$y(x)\to -2i\sqrt{W\left(-\frac{1}{4}i\sqrt[4]{e^{-2(x+c_1)}}\right)}$$
$$y(x)\to 2i\sqrt{W\left(-\frac{1}{4}i\sqrt[4]{e^{-2(x+c_1)}}\right)}$$
$$y(x)\to -2i\sqrt{W\left(\frac{1}{4}i\sqrt[4]{e^{-2(x+c_1)}}\right)}$$
$$y(x)\to 2i\sqrt{W\left(\frac{1}{4}i\sqrt[4]{e^{-2(x+c_1)}}\right)}$$
$$y(x)\to -2i\sqrt{W\left(\frac{1}{4}\sqrt[4]{e^{-2(x+c_1)}}\right)}$$
$$y(x)\to 2i\sqrt{W\left(\frac{1}{4}\sqrt[4]{e^{-2(x+c_1)}}\right)}$$
$$y(x)\to 0$$

7.16 problem 16

Internal problem ID [13438]

Book: Ordinary Differential Equations. An introduction to the fundamentals. Kenneth B. Howell. second edition. CRC Press. FL, USA. 2020
Section: Chapter 8. Review exercises for part of part II. page 143
Problem number: 16.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_quadrature]`

$$\boxed{(x^2-4)\,y'=x}$$

✓ Solution by Maple
Time used: 0.0 (sec). Leaf size: 14

```
dsolve((x^2-4)*diff(y(x),x)=x,y(x), singsol=all)
```

$$y(x)=\frac{\ln(x^2-4)}{2}+c_1$$

✓ Solution by Mathematica
Time used: 0.004 (sec). Leaf size: 18

```
DSolve[(x^2-4)*y'[x]==x,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x)\to\frac{1}{2}\log\left(x^2-4\right)+c_1$$

7.17 problem 17

Internal problem ID [13439]

Book: Ordinary Differential Equations. An introduction to the fundamentals. Kenneth B. Howell. second edition. CRC Press. FL, USA. 2020
Section: Chapter 8. Review exercises for part of part II. page 143
Problem number: 17.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_separable]`

$$\boxed{y' - \frac{1}{yx - 3x} = 0}$$

✓ Solution by Maple
Time used: 0.0 (sec). Leaf size: 35

```
dsolve(diff(y(x),x)=1/(x*y(x)-3*x),y(x), singsol=all)
```

$$y(x) = 3 - \sqrt{9 + 2\ln(x) + 2c_1}$$
$$y(x) = 3 + \sqrt{9 + 2\ln(x) + 2c_1}$$

✓ Solution by Mathematica
Time used: 0.099 (sec). Leaf size: 43

```
DSolve[y'[x]==1/(x*y[x]-3*x),y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to 3 - \sqrt{2\log(x) + 9 + 2c_1}$$
$$y(x) \to 3 + \sqrt{2\log(x) + 9 + 2c_1}$$

7.18 problem 18

Internal problem ID [13440]

Book: Ordinary Differential Equations. An introduction to the fundamentals. Kenneth B. Howell. second edition. CRC Press. FL, USA. 2020
Section: Chapter 8. Review exercises for part of part II. page 143
Problem number: 18.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[[_1st_order, _with_linear_symmetries], _rational, _Bernoulli]`

$$y' - \frac{3y}{x+1} + y^2 = 0$$

✓ Solution by Maple
Time used: 0.0 (sec). Leaf size: 34

```
dsolve(diff(y(x),x)=3*y(x)/(1+x)-y(x)^2,y(x), singsol=all)
```

$$y(x) = \frac{4(1+x)^3}{x^4 + 4x^3 + 6x^2 + 4c_1 + 4x + 1}$$

✓ Solution by Mathematica
Time used: 0.258 (sec). Leaf size: 41

```
DSolve[y'[x]==3*y[x]/(1+x)-y[x]^2,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to \frac{4(x+1)^3}{x^4 + 4x^3 + 6x^2 + 4x + 1 + 4c_1}$$
$$y(x) \to 0$$

7.19 problem 19

Internal problem ID [13441]

Book: Ordinary Differential Equations. An introduction to the fundamentals. Kenneth B. Howell. second edition. CRC Press. FL, USA. 2020
Section: Chapter 8. Review exercises for part of part II. page 143
Problem number: 19.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_exact, [_1st_order, '_with_symmetry_[F(x)*G(y),0]']]`

$$\boxed{\sin(y) + (y + x)\cos(y)\, y' = 0}$$

✓ Solution by Maple
Time used: 0.312 (sec). Leaf size: 16

```
dsolve(sin(y(x))+(x+y(x))*cos(y(x))*diff(y(x),x)=0,y(x), singsol=all)
```

$$y(x) + x + \cot(y(x)) - \csc(y(x))\, c_1 = 0$$

✓ Solution by Mathematica
Time used: 0.193 (sec). Leaf size: 29

```
DSolve[Sin[y[x]]+(x+y[x])*Cos[y[x]]*y'[x]==0,y[x],x,IncludeSingularSolutions -> True]
```

$$\text{Solve}[x = \csc(y(x))(-y(x)\sin(y(x)) - \cos(y(x))) + c_1\csc(y(x)), y(x)]$$

7.20 problem 20

Internal problem ID [13442]

Book: Ordinary Differential Equations. An introduction to the fundamentals. Kenneth B. Howell. second edition. CRC Press. FL, USA. 2020
Section: Chapter 8. Review exercises for part of part II. page 143
Problem number: 20.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_separable]`

$$\boxed{\sin(y) + (x+1)\cos(y)\, y' = 0}$$

✓ Solution by Maple
Time used: 0.016 (sec). Leaf size: 14

```
dsolve(sin(y(x))+(1+x)*cos(y(x))*diff(y(x),x)=0,y(x), singsol=all)
```

$$y(x) = \arcsin\left(\frac{1}{c_1 (1+x)}\right)$$

✓ Solution by Mathematica
Time used: 19.016 (sec). Leaf size: 21

```
DSolve[Sin[y[x]]+(1+x)*Cos[y[x]]*y'[x]==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to \arcsin\left(\frac{e^{c_1}}{x+1}\right)$$
$$y(x) \to 0$$

7.21 problem 21

Internal problem ID [13443]

Book: Ordinary Differential Equations. An introduction to the fundamentals. Kenneth B. Howell. second edition. CRC Press. FL, USA. 2020
Section: Chapter 8. Review exercises for part of part II. page 143
Problem number: 21.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_quadrature]`

$$\boxed{2\cos(x)\, y' = -\sin(x)}$$

✓ Solution by Maple
Time used: 0.016 (sec). Leaf size: 11

```
dsolve(sin(x)+2*cos(x)*diff(y(x),x)=0,y(x), singsol=all)
```

$$y(x) = \frac{\ln(\cos(x))}{2} + c_1$$

✓ Solution by Mathematica
Time used: 0.009 (sec). Leaf size: 15

```
DSolve[Sin[x]+2*Cos[x]*y'[x]==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to \frac{1}{2}\log(\cos(x)) + c_1$$

7.22 problem 22

Internal problem ID [13444]

Book: Ordinary Differential Equations. An introduction to the fundamentals. Kenneth B. Howell. second edition. CRC Press. FL, USA. 2020
Section: Chapter 8. Review exercises for part of part II. page 143
Problem number: 22.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[[_homogeneous, ‘class A‘], _rational, _Bernoulli]`

$$\boxed{xyy' - 2y^2 = 2x^2}$$

✓ Solution by Maple
Time used: 0.015 (sec). Leaf size: 30

```
dsolve(x*y(x)*diff(y(x),x)=2*(x^2+y(x)^2),y(x), singsol=all)
```

$$y(x) = \sqrt{c_1x^2 - 2}\,x$$
$$y(x) = -\sqrt{c_1x^2 - 2}\,x$$

✓ Solution by Mathematica
Time used: 0.42 (sec). Leaf size: 38

```
DSolve[x*y[x]*y'[x]==2*(x^2+y[x]^2),y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to -x\sqrt{-2 + c_1x^2}$$
$$y(x) \to x\sqrt{-2 + c_1x^2}$$

7.23 problem 23

Internal problem ID [13445]

Book: Ordinary Differential Equations. An introduction to the fundamentals. Kenneth B. Howell. second edition. CRC Press. FL, USA. 2020
Section: Chapter 8. Review exercises for part of part II. page 143
Problem number: 23.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[[_homogeneous, 'class C'], _rational, [_Abel, '2nd type', 'cl`

$$\boxed{y' - \frac{x+2y}{x+2y+3} = 0}$$

✓ Solution by Maple
Time used: 0.047 (sec). Leaf size: 19

```
dsolve(diff(y(x),x)=(x+2*y(x))/(x+2*y(x)+3),y(x), singsol=all)
```

$$y(x) = -\frac{x}{2} + \operatorname{LambertW}\left(\frac{c_1 \mathrm{e}^{\frac{3x}{2}+\frac{1}{2}}}{2}\right) - \frac{1}{2}$$

✓ Solution by Mathematica
Time used: 3.703 (sec). Leaf size: 41

```
DSolve[y'[x]==(x+2*y[x])/(x+2*y[x]+3),y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to W\left(-e^{\frac{3x}{2}-1+c_1}\right) - \frac{x}{2} - \frac{1}{2}$$
$$y(x) \to \frac{1}{2}(-x-1)$$

7.24 problem 24

Internal problem ID [13446]

Book: Ordinary Differential Equations. An introduction to the fundamentals. Kenneth B. Howell. second edition. CRC Press. FL, USA. 2020
Section: Chapter 8. Review exercises for part of part II. page 143
Problem number: 24.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[[_homogeneous, ‘class A‘], _rational, [_Abel, ‘2nd type‘, ‘cl`

$$y' - \frac{x+2y}{2x-y} = 0$$

✓ Solution by Maple
Time used: 0.016 (sec). Leaf size: 24

```
dsolve(diff(y(x),x)=(x+2*y(x))/(2*x-y(x)),y(x), singsol=all)
```

$$y(x) = \tan\left(\operatorname{RootOf}\left(-4\_Z + \ln\left(\sec\left(\_Z\right)^2\right) + 2\ln\left(x\right) + 2c_1\right)\right) x$$

✓ Solution by Mathematica
Time used: 0.033 (sec). Leaf size: 36

```
DSolve[y'[x]==(x+2*y[x])/(2*x-y[x]),y[x],x,IncludeSingularSolutions -> True]
```

$$\text{Solve}\left[\frac{1}{2}\log\left(\frac{y(x)^2}{x^2}+1\right) - 2\arctan\left(\frac{y(x)}{x}\right) = -\log(x) + c_1, y(x)\right]$$

7.25 problem 25

Internal problem ID [13447]

Book: Ordinary Differential Equations. An introduction to the fundamentals. Kenneth B. Howell. second edition. CRC Press. FL, USA. 2020
Section: Chapter 8. Review exercises for part of part II. page 143
Problem number: 25.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[[_homogeneous, 'class A'], _dAlembert]`

$$\boxed{y' - \frac{y}{x} - \tan\left(\frac{y}{x}\right) = 0}$$

✓ Solution by Maple
Time used: 0.015 (sec). Leaf size: 10

```
dsolve(diff(y(x),x)=y(x)/x+tan(y(x)/x),y(x), singsol=all)
```

$$y(x) = \arcsin(c_1 x)\, x$$

✓ Solution by Mathematica
Time used: 4.007 (sec). Leaf size: 19

```
DSolve[y'[x]==y[x]/x+Tan[y[x]/x],y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to x \arcsin\left(e^{c_1} x\right)$$
$$y(x) \to 0$$

7.26 problem 26

Internal problem ID [13448]

Book: Ordinary Differential Equations. An introduction to the fundamentals. Kenneth B. Howell. second edition. CRC Press. FL, USA. 2020
Section: Chapter 8. Review exercises for part of part II. page 143
Problem number: 26.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_separable]`

$$\boxed{y' - y^2x - 3y^2 = x + 3}$$

✓ Solution by Maple
Time used: 0.0 (sec). Leaf size: 15

```
dsolve(diff(y(x),x)=x*y(x)^2+3*y(x)^2+x+3,y(x), singsol=all)
```

$$y(x) = \tan\left(\frac{1}{2}x^2 + c_1 + 3x\right)$$

✓ Solution by Mathematica
Time used: 0.206 (sec). Leaf size: 19

```
DSolve[y'[x]==x*y[x]^2+3*y[x]^2+x+3,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to \tan\left(\frac{x^2}{2} + 3x + c_1\right)$$

7.27 problem 27

Internal problem ID [13449]

Book: Ordinary Differential Equations. An introduction to the fundamentals. Kenneth B. Howell. second edition. CRC Press. FL, USA. 2020
Section: Chapter 8. Review exercises for part of part II. page 143
Problem number: 27.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[[_homogeneous, ‘class C‘], [_Abel, ‘2nd type‘, ‘class C‘], _d`

$$\boxed{-(x+2y)\,y' = -1}$$

✓ Solution by Maple
Time used: 0.0 (sec). Leaf size: 21

```
dsolve(1-(x+2*y(x))*diff(y(x),x)=0,y(x), singsol=all)
```

$$y(x) = -\operatorname{LambertW}\left(-\frac{c_1 \mathrm{e}^{-\frac{x}{2}-1}}{2}\right) - \frac{x}{2} - 1$$

✓ Solution by Mathematica
Time used: 0.025 (sec). Leaf size: 30

```
DSolve[1-(x+2*y[x])*y'[x]==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to -W\left(-\frac{1}{2}c_1 e^{-\frac{x}{2}-1}\right) - \frac{x}{2} - 1$$

7.28 problem 28

Internal problem ID [13450]

Book: Ordinary Differential Equations. An introduction to the fundamentals. Kenneth B. Howell. second edition. CRC Press. FL, USA. 2020
Section: Chapter 8. Review exercises for part of part II. page 143
Problem number: 28.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_exact, [_1st_order, '_with_symmetry_[F(x)*G(y),0]']]`

$$\ln(y) + \left(\frac{x}{y} + 3\right) y' = 0$$

✓ Solution by Maple

Time used: 0.016 (sec). Leaf size: 26

```
dsolve(ln(y(x))+(x/y(x)+3)*diff(y(x),x)=0,y(x), singsol=all)
```

$$y(x) = e^{\frac{-x \,\mathrm{LambertW}\left(\frac{3 e^{\frac{c_1}{x}}}{x}\right)+c_1}{x}}$$

✓ Solution by Mathematica

Time used: 0.925 (sec). Leaf size: 29

```
DSolve[Log[y[x]]+(x/y[x]+3)*y'[x]==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to \frac{1}{3} x W\left(\frac{3 e^{\frac{c_1}{x}}}{x}\right)$$
$$y(x) \to 1$$

7.29 problem 29

Internal problem ID [13451]

Book: Ordinary Differential Equations. An introduction to the fundamentals. Kenneth B. Howell. second edition. CRC Press. FL, USA. 2020
Section: Chapter 8. Review exercises for part of part II. page 143
Problem number: 29.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_quadrature]`

$$\boxed{y^2 - y' = -1}$$

✓ Solution by Maple
Time used: 0.0 (sec). Leaf size: 8

```
dsolve(y(x)^2+1-diff(y(x),x)=0,y(x), singsol=all)
```

$$y(x) = \tan(c_1 + x)$$

✓ Solution by Mathematica
Time used: 0.113 (sec). Leaf size: 24

```
DSolve[y[x]^2+1-y'[x]==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to \tan(x + c_1)$$
$$y(x) \to -i$$
$$y(x) \to i$$

7.30 problem 30

Internal problem ID [13452]

Book: Ordinary Differential Equations. An introduction to the fundamentals. Kenneth B. Howell. second edition. CRC Press. FL, USA. 2020
Section: Chapter 8. Review exercises for part of part II. page 143
Problem number: 30.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[[_linear, ‘class A‘]]`

$$\boxed{y' - 3y = 12\,e^{2x}}$$

✓ Solution by Maple
Time used: 0.0 (sec). Leaf size: 15

```
dsolve(diff(y(x),x)-3*y(x)=12*exp(2*x),y(x), singsol=all)
```

$$y(x) = (c_1 e^{x} - 12)\,e^{2x}$$

✓ Solution by Mathematica
Time used: 0.05 (sec). Leaf size: 19

```
DSolve[y'[x]-3*y[x]==12*Exp[2*x],y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to e^{2x}(-12 + c_1 e^{x})$$

7.31 problem 31

Internal problem ID [13453]

Book: Ordinary Differential Equations. An introduction to the fundamentals. Kenneth B. Howell. second edition. CRC Press. FL, USA. 2020
Section: Chapter 8. Review exercises for part of part II. page 143
Problem number: 31.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[[_homogeneous, 'class A'], _rational, [_Abel, '2nd type', 'cl`

$$\boxed{xyy' - yx - y^2 = x^2}$$

✓ Solution by Maple
Time used: 0.031 (sec). Leaf size: 22

```
dsolve(x*y(x)*diff(y(x),x)=x^2+x*y(x)+y(x)^2,y(x), singsol=all)
```

$$y(x) = x\left(-\operatorname{LambertW}\left(-\frac{e^{-c_1-1}}{x}\right)-1\right)$$

✓ Solution by Mathematica
Time used: 4.224 (sec). Leaf size: 31

```
DSolve[x*y[x]*y'[x]==x^2+x*y[x]+y[x]^2,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to -x\left(1+W\left(-\frac{e^{-1-c_1}}{x}\right)\right)$$
$$y(x) \to -x$$

7.32 problem 32

Internal problem ID [13454]

Book: Ordinary Differential Equations. An introduction to the fundamentals. Kenneth B. Howell. second edition. CRC Press. FL, USA. 2020
Section: Chapter 8. Review exercises for part of part II. page 143
Problem number: 32.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_quadrature]`

$$\boxed{(x+2)\,y' = x^3}$$

✓ Solution by Maple
Time used: 0.0 (sec). Leaf size: 25

```
dsolve((x+2)*diff(y(x),x)-x^3=0,y(x), singsol=all)
```

$$y(x) = \frac{x^3}{3} - x^2 + 4x - 8\ln(x+2) + c_1$$

✓ Solution by Mathematica
Time used: 0.005 (sec). Leaf size: 32

```
DSolve[(x+2)*y'[x]-x^3==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to \frac{x^3}{3} - x^2 + 4x - 8\log(x+2) + \frac{44}{3} + c_1$$

7.33 problem 33

Internal problem ID [13455]

Book: Ordinary Differential Equations. An introduction to the fundamentals. Kenneth B. Howell. second edition. CRC Press. FL, USA. 2020
Section: Chapter 8. Review exercises for part of part II. page 143
Problem number: 33.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[[_homogeneous, 'class D'], _rational, _Bernoulli]`

$$\boxed{y^3y'x - y^4 = -x^2}$$

✓ Solution by Maple
Time used: 0.016 (sec). Leaf size: 73

```
dsolve(x*y(x)^3*diff(y(x),x)=y(x)^4-x^2,y(x), singsol=all)
```

$$y(x) = \left(x^2\left(c_1x^2+2\right)\right)^{\frac{1}{4}}$$
$$y(x) = -\left(x^2\left(c_1x^2+2\right)\right)^{\frac{1}{4}}$$
$$y(x) = -i\left(x^2\left(c_1x^2+2\right)\right)^{\frac{1}{4}}$$
$$y(x) = i\left(x^2\left(c_1x^2+2\right)\right)^{\frac{1}{4}}$$

✓ Solution by Mathematica
Time used: 0.341 (sec). Leaf size: 96

```
DSolve[x*y[x]^3*y'[x]==y[x]^4-x^2,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to -\sqrt{x}\sqrt[4]{2+c_1x^2}$$
$$y(x) \to -i\sqrt{x}\sqrt[4]{2+c_1x^2}$$
$$y(x) \to i\sqrt{x}\sqrt[4]{2+c_1x^2}$$
$$y(x) \to \sqrt{x}\sqrt[4]{2+c_1x^2}$$

7.34 problem 34

Internal problem ID [13456]

Book: Ordinary Differential Equations. An introduction to the fundamentals. Kenneth B. Howell. second edition. CRC Press. FL, USA. 2020
Section: Chapter 8. Review exercises for part of part II. page 143
Problem number: 34.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[[_1st_order, _with_linear_symmetries], _Bernoulli]`

$$\boxed{y' - 4y + \frac{16\,\mathrm{e}^{4x}}{y^2} = 0}$$

✓ Solution by Maple
Time used: 0.031 (sec). Leaf size: 75

```
dsolve(diff(y(x),x)=4*y(x)-16*exp(4*x)/y(x)^2,y(x), singsol=all)
```

$$y(x) = \left(\mathrm{e}^{4x}\left(\mathrm{e}^{8x}c_1 + 6\right)\right)^{\frac{1}{3}}$$
$$y(x) = -\frac{\left(\mathrm{e}^{4x}(\mathrm{e}^{8x}c_1 + 6)\right)^{\frac{1}{3}}\left(1 + i\sqrt{3}\right)}{2}$$
$$y(x) = \frac{\left(\mathrm{e}^{4x}(\mathrm{e}^{8x}c_1 + 6)\right)^{\frac{1}{3}}\left(i\sqrt{3} - 1\right)}{2}$$

✓ Solution by Mathematica
Time used: 3.622 (sec). Leaf size: 90

```
DSolve[y'[x]==4*y[x]-16*Exp[4*x]/y[x]^2,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to e^{4x/3}\sqrt[3]{6 + c_1 e^{8x}}$$
$$y(x) \to -\sqrt[3]{-1}e^{4x/3}\sqrt[3]{6 + c_1 e^{8x}}$$
$$y(x) \to (-1)^{2/3}e^{4x/3}\sqrt[3]{6 + c_1 e^{8x}}$$

7.35 problem 35

Internal problem ID [13457]

Book: Ordinary Differential Equations. An introduction to the fundamentals. Kenneth B. Howell. second edition. CRC Press. FL, USA. 2020
Section: Chapter 8. Review exercises for part of part II. page 143
Problem number: 35.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_linear]`

$$\boxed{2y + (x+1)\,y' = 6x}$$

✓ Solution by Maple
Time used: 0.0 (sec). Leaf size: 22

```
dsolve((2*y(x)-6*x)+(x+1)*diff(y(x),x)=0,y(x), singsol=all)
```

$$y(x) = \frac{2x^3 + 3x^2 + c_1}{(1+x)^2}$$

✓ Solution by Mathematica
Time used: 0.027 (sec). Leaf size: 24

```
DSolve[(2*y[x]-6*x)+(x+1)*y'[x]==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to \frac{2x^3 + 3x^2 + c_1}{(x+1)^2}$$

7.36 problem 36

Internal problem ID [13458]

Book: Ordinary Differential Equations. An introduction to the fundamentals. Kenneth B. Howell. second edition. CRC Press. FL, USA. 2020
Section: Chapter 8. Review exercises for part of part II. page 143
Problem number: 36.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[[_homogeneous, ‘class G‘], _exact, _rational]`

$$\boxed{y^2x + (x^2y + 10y^4)\, y' = 0}$$

✓ Solution by Maple
Time used: 0.0 (sec). Leaf size: 24

```
dsolve(x*y(x)^2+(x^2*y(x)+10*y(x)^4)*diff(y(x),x)=0,y(x), singsol=all)
```

$$y(x) = 0$$
$$\frac{y(x)^2\, x^2}{2} + 2y(x)^5 + c_1 = 0$$

✓ Solution by Mathematica
Time used: 3.953 (sec). Leaf size: 141

```
DSolve[x*y[x]^2+(x^2*y[x]+10*y[x]^4)*y'[x]==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to 0$$
$$y(x) \to \text{Root}\left[4\#1^5 + \#1^2x^2 - 2c_1\&, 1\right]$$
$$y(x) \to \text{Root}\left[4\#1^5 + \#1^2x^2 - 2c_1\&, 2\right]$$
$$y(x) \to \text{Root}\left[4\#1^5 + \#1^2x^2 - 2c_1\&, 3\right]$$
$$y(x) \to \text{Root}\left[4\#1^5 + \#1^2x^2 - 2c_1\&, 4\right]$$
$$y(x) \to \text{Root}\left[4\#1^5 + \#1^2x^2 - 2c_1\&, 5\right]$$
$$y(x) \to 0$$

7.37 problem 37

Internal problem ID [13459]

Book: Ordinary Differential Equations. An introduction to the fundamentals. Kenneth B. Howell. second edition. CRC Press. FL, USA. 2020
Section: Chapter 8. Review exercises for part of part II. page 143
Problem number: 37.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_Bernoulli]`

$$\boxed{yy' - y^2x = 6x\,\mathrm{e}^{4x^2}}$$

✓ Solution by Maple
Time used: 0.031 (sec). Leaf size: 43

```
dsolve(y(x)*diff(y(x),x)-x*y(x)^2=6*x*exp(4*x^2),y(x), singsol=all)
```

$$y(x) = \sqrt{\mathrm{e}^{x^2}\left(2\,\mathrm{e}^{3x^2} + c_1\right)}$$
$$y(x) = -\sqrt{\mathrm{e}^{x^2}\left(2\,\mathrm{e}^{3x^2} + c_1\right)}$$

✓ Solution by Mathematica
Time used: 1.953 (sec). Leaf size: 62

```
DSolve[y[x]*y'[x]-x*y[x]^2==6*x*Exp[4*x^2],y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to -e^{\frac{x^2}{2}}\sqrt{2e^{3x^2} + c_1}$$
$$y(x) \to e^{\frac{x^2}{2}}\sqrt{2e^{3x^2} + c_1}$$

7.38 problem 38

Internal problem ID [13460]

Book: Ordinary Differential Equations. An introduction to the fundamentals. Kenneth B. Howell. second edition. CRC Press. FL, USA. 2020
Section: Chapter 8. Review exercises for part of part II. page 143
Problem number: 38.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[[_homogeneous, 'class C'], _exact, _rational, _dAlembert]`

$$\boxed{(y - x + 3)^2 (y' - 1) = 1}$$

✓ Solution by Maple
Time used: 0.016 (sec). Leaf size: 80

```
dsolve((y(x)-x+3)^2*(diff(y(x),x)-1)=1,y(x), singsol=all)
```

$$y(x) = (-3c_1 + 3x)^{\frac{1}{3}} + x - 3$$
$$y(x) = -\frac{(-3c_1 + 3x)^{\frac{1}{3}}}{2} - \frac{i\sqrt{3}\,(-3c_1 + 3x)^{\frac{1}{3}}}{2} + x - 3$$
$$y(x) = -\frac{(-3c_1 + 3x)^{\frac{1}{3}}}{2} + \frac{i\sqrt{3}\,(-3c_1 + 3x)^{\frac{1}{3}}}{2} + x - 3$$

✓ Solution by Mathematica
Time used: 0.421 (sec). Leaf size: 95

```
DSolve[(y[x]-x+3)^2*(y'[x]-1)==1,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to x + \sqrt[3]{3}\sqrt[3]{x + 9 + c_1} - 3$$
$$y(x) \to x + \frac{1}{2} i\sqrt[3]{3}\left(\sqrt{3} + i\right) \sqrt[3]{x + 9 + c_1} - 3$$
$$y(x) \to x - \frac{1}{2}\sqrt[3]{3}\left(1 + i\sqrt{3}\right) \sqrt[3]{x + 9 + c_1} - 3$$

7.39 problem 39

Internal problem ID [13461]

Book: Ordinary Differential Equations. An introduction to the fundamentals. Kenneth B. Howell. second edition. CRC Press. FL, USA. 2020
Section: Chapter 8. Review exercises for part of part II. page 143
Problem number: 39.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_exact, [_1st_order, '_with_symmetry_[F(x),G(x)*y+H(x)]']]`

$$\boxed{y\,\mathrm{e}^{yx} + x\,\mathrm{e}^{yx} y' = -x}$$

✓ Solution by Maple
Time used: 0.0 (sec). Leaf size: 23

```
dsolve(x+y(x)*exp(x*y(x))+x*exp(x*y(x))*diff(y(x),x)=0,y(x), singsol=all)
```

$$y(x) = \frac{-\ln(2) + \ln(-x^2 - 2c_1)}{x}$$

✓ Solution by Mathematica
Time used: 0.412 (sec). Leaf size: 20

```
DSolve[x+y[x]*Exp[x*y[x]]+x*Exp[x*y[x]]*y'[x]==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to \frac{\log\left(-\frac{x^2}{2} + c_1\right)}{x}$$

7.40 problem 40

Internal problem ID [13462]

Book: Ordinary Differential Equations. An introduction to the fundamentals. Kenneth B. Howell. second edition. CRC Press. FL, USA. 2020
Section: Chapter 8. Review exercises for part of part II. page 143
Problem number: 40.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_separable]`

$$\boxed{y^2 - y^2\cos(x) + y' = 0}$$

✓ Solution by Maple
Time used: 0.016 (sec). Leaf size: 13

```
dsolve(y(x)^2-y(x)^2*cos(x)+diff(y(x),x)=0,y(x), singsol=all)
```

$$y(x) = \frac{1}{-\sin(x) + c_1 + x}$$

✓ Solution by Mathematica
Time used: 0.123 (sec). Leaf size: 22

```
DSolve[y[x]^2-y[x]^2*Cos[x]+y'[x]==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to \frac{1}{x - \sin(x) - c_1}$$
$$y(x) \to 0$$

7.41 problem 41

Internal problem ID [13463]

Book: Ordinary Differential Equations. An introduction to the fundamentals. Kenneth B. Howell. second edition. CRC Press. FL, USA. 2020
Section: Chapter 8. Review exercises for part of part II. page 143
Problem number: 41.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[[_linear, 'class A']]`

$$\boxed{y' + 2y = \sin(x)}$$

✓ Solution by Maple
Time used: 0.0 (sec). Leaf size: 19

```
dsolve(diff(y(x),x)+2*y(x)=sin(x),y(x), singsol=all)
```

$$y(x) = -\frac{\cos(x)}{5} + \frac{2\sin(x)}{5} + \mathrm{e}^{-2x} c_1$$

✓ Solution by Mathematica
Time used: 0.048 (sec). Leaf size: 26

```
DSolve[y'[x]+2*y[x]==Sin[x],y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to \frac{2\sin(x)}{5} - \frac{\cos(x)}{5} + c_1 e^{-2x}$$

7.42 problem 42

Internal problem ID [13464]

Book: Ordinary Differential Equations. An introduction to the fundamentals. Kenneth B. Howell. second edition. CRC Press. FL, USA. 2020
Section: Chapter 8. Review exercises for part of part II. page 143
Problem number: 42.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_quadrature]`

$$\boxed{y' = \sin(x) - 2x}$$

✓ Solution by Maple
Time used: 0.0 (sec). Leaf size: 15

```
dsolve(diff(y(x),x)+2*x=sin(x),y(x), singsol=all)
```

$$y(x) = -x^2 - \cos(x) + c_1$$

✓ Solution by Mathematica
Time used: 0.009 (sec). Leaf size: 17

```
DSolve[y'[x]+2*x==Sin[x],y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to -x^2 - \cos(x) + c_1$$

7.43 problem 43

Internal problem ID [13465]

Book: Ordinary Differential Equations. An introduction to the fundamentals. Kenneth B. Howell. second edition. CRC Press. FL, USA. 2020
Section: Chapter 8. Review exercises for part of part II. page 143
Problem number: 43.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_separable]`

$$\boxed{y' - y^3 + y^3 \cos(x) = 0}$$

✓ Solution by Maple
Time used: 0.016 (sec). Leaf size: 31

```
dsolve(diff(y(x),x)=y(x)^3-y(x)^3*cos(x),y(x), singsol=all)
```

$$y(x) = \frac{1}{\sqrt{c_1 - 2x + 2\sin(x)}}$$
$$y(x) = -\frac{1}{\sqrt{c_1 - 2x + 2\sin(x)}}$$

✓ Solution by Mathematica
Time used: 0.219 (sec). Leaf size: 55

```
DSolve[y'[x]==y[x]^3-y[x]^3*Cos[x],y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to -\frac{1}{\sqrt{2}\sqrt{-x + \sin(x) - c_1}}$$
$$y(x) \to \frac{1}{\sqrt{2}\sqrt{-x + \sin(x) - c_1}}$$
$$y(x) \to 0$$

7.44 problem 44

Internal problem ID [13466]

Book: Ordinary Differential Equations. An introduction to the fundamentals. Kenneth B. Howell. second edition. CRC Press. FL, USA. 2020
Section: Chapter 8. Review exercises for part of part II. page 143
Problem number: 44.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_exact, [_1st_order, '_with_symmetry_[F(x),G(x)*y+H(x)]']]`

$$\boxed{y^2 e^{y^2 x} + 2xy\, e^{y^2 x} y' = 2x}$$

✓ Solution by Maple
Time used: 0.015 (sec). Leaf size: 40

```
dsolve(y(x)^2*exp(x*y(x)^2)-2*x+2*x*y(x)*exp(x*y(x)^2)*diff(y(x),x)=0,y(x), singsol=all)
```

$$y(x) = \frac{\sqrt{x \ln \left(x^2 - c_1\right)}}{x}$$
$$y(x) = -\frac{\sqrt{x \ln \left(x^2 - c_1\right)}}{x}$$

✓ Solution by Mathematica
Time used: 1.468 (sec). Leaf size: 44

```
DSolve[y[x]^2*Exp[x*y[x]^2]-2*x+2*x*y[x]*Exp[x*y[x]^2]*y'[x]==0,y[x],x,IncludeSingularSoluti
```

$$y(x) \to -\frac{\sqrt{\log \left(x^2 + c_1\right)}}{\sqrt{x}}$$
$$y(x) \to \frac{\sqrt{\log \left(x^2 + c_1\right)}}{\sqrt{x}}$$

7.45 problem 45

Internal problem ID [13467]

Book: Ordinary Differential Equations. An introduction to the fundamentals. Kenneth B. Howell. second edition. CRC Press. FL, USA. 2020
Section: Chapter 8. Review exercises for part of part II. page 143
Problem number: 45.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_separable]`

$$\boxed{y' - e^{4x+3y} = 0}$$

✓ Solution by Maple
Time used: 0.0 (sec). Leaf size: 26

```
dsolve(diff(y(x),x)=exp(4*x+3*y(x)),y(x), singsol=all)
```

$$y(x) = -\frac{\ln(3)}{3} + \frac{2\ln(2)}{3} - \frac{\ln(-e^{4x} - 4c_1)}{3}$$

✓ Solution by Mathematica
Time used: 0.884 (sec). Leaf size: 24

```
DSolve[y'[x]==Exp[4*x+3*y[x]],y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to -\frac{1}{3}\log\left(-\frac{3}{4}(e^{4x} + 4c_1)\right)$$

7.46 problem 46

Internal problem ID [13468]

Book: Ordinary Differential Equations. An introduction to the fundamentals. Kenneth B. Howell. second edition. CRC Press. FL, USA. 2020
Section: Chapter 8. Review exercises for part of part II. page 143
Problem number: 46.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[[_homogeneous, ‘class C‘], _dAlembert]`

$$y' - \tan(6x + 3y + 1) = -2$$

✓ Solution by Maple
Time used: 0.109 (sec). Leaf size: 35

```
dsolve(diff(y(x),x)=tan(6*x+3*y(x)+1)-2,y(x), singsol=all)
```

$$y(x) = -2x - \frac{1}{3} - \frac{\operatorname{arccsc}(c_1 e^{-3x})}{3}$$
$$y(x) = -2x - \frac{1}{3} + \frac{\operatorname{arccsc}(c_1 e^{-3x})}{3}$$

✓ Solution by Mathematica
Time used: 60.483 (sec). Leaf size: 25

```
DSolve[y'[x]==Tan[6*x+3*y[x]+1]-2,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to \frac{1}{3}\left(\arcsin\left(e^{3x-3c_1}\right) - 6x - 1\right)$$

7.47 problem 47

Internal problem ID [13469]

Book: Ordinary Differential Equations. An introduction to the fundamentals. Kenneth B. Howell. second edition. CRC Press. FL, USA. 2020
Section: Chapter 8. Review exercises for part of part II. page 143
Problem number: 47.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_separable]`

$$\boxed{y' - e^{4x+3y} = 0}$$

✓ Solution by Maple
Time used: 0.0 (sec). Leaf size: 26

```
dsolve(diff(y(x),x)=exp(4*x+3*y(x)),y(x), singsol=all)
```

$$y(x) = -\frac{\ln(3)}{3} + \frac{2\ln(2)}{3} - \frac{\ln(-e^{4x} - 4c_1)}{3}$$

✓ Solution by Mathematica
Time used: 0.872 (sec). Leaf size: 24

```
DSolve[y'[x]==Exp[4*x+3*y[x]],y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to -\frac{1}{3}\log\left(-\frac{3}{4}(e^{4x} + 4c_1)\right)$$

7.48 problem 48

Internal problem ID [13470]

Book: Ordinary Differential Equations. An introduction to the fundamentals. Kenneth B. Howell. second edition. CRC Press. FL, USA. 2020
Section: Chapter 8. Review exercises for part of part II. page 143
Problem number: 48.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_linear]`

$$y' - x\left(6y + e^{x^2}\right) = 0$$

✓ Solution by Maple
Time used: 0.0 (sec). Leaf size: 19

```
dsolve(diff(y(x),x)=x*(6*y(x)+exp(x^2)),y(x), singsol=all)
```

$$y(x) = -\frac{e^{x^2}}{4} + e^{3x^2} c_1$$

✓ Solution by Mathematica
Time used: 0.071 (sec). Leaf size: 25

```
DSolve[y'[x]==x*(6*y[x]+Exp[x^2]),y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to -\frac{e^{x^2}}{4} + c_1 e^{3x^2}$$

7.49 problem 49

Internal problem ID [13471]

Book: Ordinary Differential Equations. An introduction to the fundamentals. Kenneth B. Howell. second edition. CRC Press. FL, USA. 2020
Section: Chapter 8. Review exercises for part of part II. page 143
Problem number: 49.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_exact, _rational, [_1st_order, '_with_symmetry_[F(x)*G(y),0]`

$$\boxed{x(1-2y)+\left(-x^2+y\right)y'=0}$$

✓ Solution by Maple
Time used: 0.015 (sec). Leaf size: 45

```
dsolve(x*(1-2*y(x))+(y(x)-x^2)*diff(y(x),x)=0,y(x), singsol=all)
```

$$y(x)=x^2-\sqrt{x^4-x^2-2c_1}$$
$$y(x)=x^2+\sqrt{x^4-x^2-2c_1}$$

✓ Solution by Mathematica
Time used: 0.147 (sec). Leaf size: 66

```
DSolve[x*(1-2*y[x])+(y[x]-x^2)*y'[x]==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x)\to x^2-i\sqrt{-x^4+x^2-c_1}$$
$$y(x)\to x^2+i\sqrt{-x^4+x^2-c_1}$$
$$y(x)\to \frac{1}{2}$$

7.50 problem 50

Internal problem ID [13472]

Book: Ordinary Differential Equations. An introduction to the fundamentals. Kenneth B. Howell. second edition. CRC Press. FL, USA. 2020
Section: Chapter 8. Review exercises for part of part II. page 143
Problem number: 50.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_linear]`

$$\boxed{x^2y' + 3yx = 6\,\mathrm{e}^{-x^2}}$$

✓ Solution by Maple
Time used: 0.0 (sec). Leaf size: 18

```
dsolve(x^2*diff(y(x),x)+3*x*y(x)=6*exp(-x^2),y(x), singsol=all)
```

$$y(x) = \frac{-3\,\mathrm{e}^{-x^2} + c_1}{x^3}$$

✓ Solution by Mathematica
Time used: 0.079 (sec). Leaf size: 21

```
DSolve[x^2*y'[x]+3*x*y[x]==6*Exp[-x^2],y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to \frac{-3e^{-x^2} + c_1}{x^3}$$

8 Chapter 13. Higher order equations: Extending first order concepts. Additional exercises page 259

8.1 problem 13.1 (a)

Internal problem ID [13473]

Book: Ordinary Differential Equations. An introduction to the fundamentals. Kenneth B. Howell. second edition. CRC Press. FL, USA. 2020
Section: Chapter 13. Higher order equations: Extending first order concepts. Additional exercises page 259
Problem number: 13.1 (a).
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _missing_y]]`

$$\boxed{xy'' + 4y' = 18x^2}$$

✓ Solution by Maple
Time used: 0.0 (sec). Leaf size: 15

```
dsolve(x*diff(y(x),x$2)+4*diff(y(x),x)=18*x^2,y(x), singsol=all)
```

$$y(x) = x^3 - \frac{c_1}{3x^3} + c_2$$

✓ Solution by Mathematica
Time used: 0.032 (sec). Leaf size: 20

```
DSolve[x*y''[x]+4*y'[x]==18*x^2,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to x^3 - \frac{c_1}{3x^3} + c_2$$

8.2 problem 13.1 (b)

Internal problem ID [13474]

Book: Ordinary Differential Equations. An introduction to the fundamentals. Kenneth B. Howell. second edition. CRC Press. FL, USA. 2020
Section: Chapter 13. Higher order equations: Extending first order concepts. Additional exercises page 259
Problem number: 13.1 (b).
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _missing_y]]`

$$\boxed{xy'' - 2y' = 0}$$

✓ Solution by Maple
Time used: 0.0 (sec). Leaf size: 11

```
dsolve(x*diff(y(x),x$2)=2*diff(y(x),x),y(x), singsol=all)
```

$$y(x) = c_2x^3 + c_1$$

✓ Solution by Mathematica
Time used: 0.015 (sec). Leaf size: 19

```
DSolve[y''[x]==2*y'[x],y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to \frac{1}{2}c_1e^{2x} + c_2$$

8.3 problem 13.1 (c)

Internal problem ID [13475]

Book: Ordinary Differential Equations. An introduction to the fundamentals. Kenneth B. Howell. second edition. CRC Press. FL, USA. 2020
Section: Chapter 13. Higher order equations: Extending first order concepts. Additional exercises page 259
Problem number: 13.1 (c).
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _missing_x]]`

$$\boxed{y'' - y' = 0}$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 10

```
dsolve(diff(y(x),x$2)=diff(y(x),x),y(x), singsol=all)
```

$$y(x) = c_1 + c_2 e^x$$

✓ Solution by Mathematica

Time used: 0.009 (sec). Leaf size: 14

```
DSolve[y''[x]==y'[x],y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to c_1 e^x + c_2$$

8.4 problem 13.1 (d)

Internal problem ID [13476]

Book: Ordinary Differential Equations. An introduction to the fundamentals. Kenneth B. Howell. second edition. CRC Press. FL, USA. 2020
Section: Chapter 13. Higher order equations: Extending first order concepts. Additional exercises page 259
Problem number: 13.1 (d).
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _missing_y]]`

$$\boxed{y'' + 2y' = 8\,\mathrm{e}^{2x}}$$

✓ Solution by Maple
Time used: 0.0 (sec). Leaf size: 17

```
dsolve(diff(y(x),x$2)+2*diff(y(x),x)=8*exp(2*x),y(x), singsol=all)
```

$$y(x) = -\frac{\mathrm{e}^{-2x}c_1}{2} + \mathrm{e}^{2x} + c_2$$

✓ Solution by Mathematica
Time used: 0.059 (sec). Leaf size: 24

```
DSolve[y''[x]+2*y'[x]==8*Exp[2*x],y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to e^{2x} - \frac{1}{2}c_1 e^{-2x} + c_2$$

8.5 problem 13.1 (e)

Internal problem ID [13477]

Book: Ordinary Differential Equations. An introduction to the fundamentals. Kenneth B. Howell. second edition. CRC Press. FL, USA. 2020
Section: Chapter 13. Higher order equations: Extending first order concepts. Additional exercises page 259
Problem number: 13.1 (e).
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _missing_y]]`

$$\boxed{xy'' - y' + 2x^2y' = 0}$$

✓ Solution by Maple
Time used: 0.015 (sec). Leaf size: 14

```
dsolve(x*diff(y(x),x$2)=diff(y(x),x)-2*x^2*diff(y(x),x),y(x), singsol=all)
```

$$y(x) = c_1 + c_2 e^{-x^2}$$

✓ Solution by Mathematica
Time used: 0.015 (sec). Leaf size: 21

```
DSolve[x*y''[x]==y'[x]-2*x^2*y'[x],y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to c_2 - \frac{1}{2}c_1 e^{-x^2}$$

8.6 problem 13.1 (f)

Internal problem ID [13478]

Book: Ordinary Differential Equations. An introduction to the fundamentals. Kenneth B. Howell. second edition. CRC Press. FL, USA. 2020
Section: Chapter 13. Higher order equations: Extending first order concepts. Additional exercises page 259
Problem number: 13.1 (f).
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _missing_y]]`

$$\boxed{(x^2+1)\,y''+2y'x=0}$$

✓ Solution by Maple
Time used: 0.0 (sec). Leaf size: 10

```
dsolve((x^2+1)*diff(y(x),x$2)+2*x*diff(y(x),x)=0,y(x), singsol=all)
```

$$y(x)=c_1+\arctan(x)\,c_2$$

✓ Solution by Mathematica
Time used: 0.015 (sec). Leaf size: 13

```
DSolve[(x^2+1)*y''[x]+2*x*y'[x]==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x)\to c_1\arctan(x)+c_2$$

8.7 problem 13.2 (a)

Internal problem ID [13479]

Book: Ordinary Differential Equations. An introduction to the fundamentals. Kenneth B. Howell. second edition. CRC Press. FL, USA. 2020
Section: Chapter 13. Higher order equations: Extending first order concepts. Additional exercises page 259
Problem number: 13.2 (a).
ODE order: 2.
ODE degree: 2.

CAS Maple gives this as type `[[_2nd_order, _missing_y], [_2nd_order, _reducible, _mu_y_y1]]`

$$\boxed{y'' - 4x\sqrt{y'} = 0}$$

✓ Solution by Maple
Time used: 0.281 (sec). Leaf size: 51

```
dsolve(diff(y(x),x$2)=4*x*sqrt(diff(y(x),x)),y(x), singsol=all)
```

$$y(x) = c_1$$
$$y(x) = \frac{x^5}{5} - \frac{2x^3}{3c_1} + \frac{x}{c_1^2} + c_2$$
$$y(x) = \frac{x^5}{5} + \frac{2x^3}{3c_1} + \frac{x}{c_1^2} + c_2$$

✓ Solution by Mathematica
Time used: 0.095 (sec). Leaf size: 33

```
DSolve[y''[x]==4*x*Sqrt[y'[x]],y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to \frac{x^5}{5} + \frac{c_1 x^3}{3} + \frac{c_1{}^2 x}{4} + c_2$$

8.8 problem 13.2 (b)

Internal problem ID [13480]

Book: Ordinary Differential Equations. An introduction to the fundamentals. Kenneth B. Howell. second edition. CRC Press. FL, USA. 2020
Section: Chapter 13. Higher order equations: Extending first order concepts. Additional exercises page 259
Problem number: 13.2 (b).
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _missing_x], [_2nd_order, _exact, _nonlinear], [`

$$\boxed{y'y'' = 1}$$

✓ Solution by Maple
Time used: 0.094 (sec). Leaf size: 40

```
dsolve(diff(y(x),x)*diff(y(x),x$2)=1,y(x), singsol=all)
```

$$y(x) = \frac{(2x + 2c_1)^{\frac{3}{2}}}{3} + c_2$$
$$y(x) = \frac{(-2c_1 - 2x)\sqrt{2x + 2c_1}}{3} + c_2$$

✓ Solution by Mathematica
Time used: 0.014 (sec). Leaf size: 49

```
DSolve[y'[x]*y''[x]==1,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to c_2 - \frac{2}{3}\sqrt{2}(x + c_1)^{3/2}$$
$$y(x) \to \frac{2}{3}\sqrt{2}(x + c_1)^{3/2} + c_2$$

8.9 problem 13.2 (c)

Internal problem ID [13481]

Book: Ordinary Differential Equations. An introduction to the fundamentals. Kenneth B. Howell. second edition. CRC Press. FL, USA. 2020
Section: Chapter 13. Higher order equations: Extending first order concepts. Additional exercises page 259
Problem number: 13.2 (c).
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _missing_x], [_2nd_order, _exact, _nonlinear], _`

$$\boxed{yy'' + y'^2 = 0}$$

✓ Solution by Maple
Time used: 0.047 (sec). Leaf size: 33

```
dsolve(y(x)*diff(y(x),x$2)=-(diff(y(x),x)^2),y(x), singsol=all)
```

$$y(x) = 0$$
$$y(x) = \sqrt{2c_1x + 2c_2}$$
$$y(x) = -\sqrt{2c_1x + 2c_2}$$

✓ Solution by Mathematica
Time used: 0.181 (sec). Leaf size: 20

```
DSolve[y[x]*y''[x]==-(y'[x]^2),y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to c_2\sqrt{2x - c_1}$$

8.10 problem 13.2 (d)

Internal problem ID [13482]

Book: Ordinary Differential Equations. An introduction to the fundamentals. Kenneth B. Howell. second edition. CRC Press. FL, USA. 2020
Section: Chapter 13. Higher order equations: Extending first order concepts. Additional exercises page 259
Problem number: 13.2 (d).
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _missing_y], [_2nd_order, _reducible, _mu_y_y1]]`

$$\boxed{xy'' + y' - y'^2 = 0}$$

✓ Solution by Maple
Time used: 0.031 (sec). Leaf size: 17

```
dsolve(x*diff(y(x),x$2)=diff(y(x),x)^2-diff(y(x),x),y(x), singsol=all)
```

$$y(x) = -\frac{\ln(c_1 x - 1)}{c_1} + c_2$$

✓ Solution by Mathematica
Time used: 0.301 (sec). Leaf size: 38

```
DSolve[x*y''[x]==(y'[x])^2-y'[x],y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to e^{-c_1} \log(1 + e^{c_1} x) + c_2$$
$$y(x) \to c_2$$
$$y(x) \to x + c_2$$

8.11 problem 13.2 (e)

Internal problem ID [13483]

Book: Ordinary Differential Equations. An introduction to the fundamentals. Kenneth B. Howell. second edition. CRC Press. FL, USA. 2020
Section: Chapter 13. Higher order equations: Extending first order concepts. Additional exercises page 259
Problem number: 13.2 (e).
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _missing_y]]`

$$\boxed{xy'' - y'^2 = 6x^5}$$

✓ Solution by Maple

Time used: 0.032 (sec). Leaf size: 64

```
dsolve(x*diff(y(x),x$2)-diff(y(x),x)^2=6*x^5,y(x), singsol=all)
```

$$y(x) = \sqrt{6}\left(\int \frac{x^{\frac{5}{2}}\left(\text{BesselY}\left(1, \frac{2x^{\frac{5}{2}}\sqrt{6}}{5}\right)c_1 + \text{BesselJ}\left(1, \frac{2x^{\frac{5}{2}}\sqrt{6}}{5}\right)\right)}{c_1\,\text{BesselY}\left(0, \frac{2x^{\frac{5}{2}}\sqrt{6}}{5}\right) + \text{BesselJ}\left(0, \frac{2x^{\frac{5}{2}}\sqrt{6}}{5}\right)}dx\right) + c_2$$

✓ Solution by Mathematica

Time used: 60.384 (sec). Leaf size: 109

```
DSolve[x*y''[x]-y'[x]^2==6*x^5,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to \int_1^x \frac{\sqrt{6}\left(2\,\text{BesselY}\left(1, \frac{2}{5}\sqrt{6}K[1]^{5/2}\right) + \text{BesselJ}\left(1, \frac{2}{5}\sqrt{6}K[1]^{5/2}\right)c_1\right)K[1]^{5/2}}{2\,\text{BesselY}\left(0, \frac{2}{5}\sqrt{6}K[1]^{5/2}\right) + \text{BesselJ}\left(0, \frac{2}{5}\sqrt{6}K[1]^{5/2}\right)c_1}dK[1] + c_2$$

8.12 problem 13.2 (f)

Internal problem ID [13484]

Book: Ordinary Differential Equations. An introduction to the fundamentals. Kenneth B. Howell. second edition. CRC Press. FL, USA. 2020
Section: Chapter 13. Higher order equations: Extending first order concepts. Additional exercises page 259
Problem number: 13.2 (f).
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _missing_x], [_2nd_order, _reducible, _mu_x_y1],`

$$\boxed{yy'' - y'^2 - y' = 0}$$

✓ Solution by Maple
Time used: 0.094 (sec). Leaf size: 20

```
dsolve(y(x)*diff(y(x),x$2)-(diff(y(x),x)^2)=diff(y(x),x),y(x), singsol=all)
```

$$y(x) = 0$$
$$y(x) = \frac{e^{c_1(c_2+x)} + 1}{c_1}$$

✓ Solution by Mathematica
Time used: 1.7 (sec). Leaf size: 26

```
DSolve[y[x]*y''[x]-(y'[x]^2)==y'[x],y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to \frac{1 + e^{c_1(x+c_2)}}{c_1}$$
$$y(x) \to \text{Indeterminate}$$

8.13 problem 13.2 (g)

Internal problem ID [13485]

Book: Ordinary Differential Equations. An introduction to the fundamentals. Kenneth B. Howell. second edition. CRC Press. FL, USA. 2020
Section: Chapter 13. Higher order equations: Extending first order concepts. Additional exercises page 259
Problem number: 13.2 (g).
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _missing_x]]`

$$\boxed{y'' - 2y' = -6}$$

✓ Solution by Maple
Time used: 0.0 (sec). Leaf size: 16

```
dsolve(diff(y(x),x$2)=2*diff(y(x),x)-6,y(x), singsol=all)
```

$$y(x) = \frac{e^{2x}c_1}{2} + 3x + c_2$$

✓ Solution by Mathematica
Time used: 0.019 (sec). Leaf size: 22

```
DSolve[y''[x]==2*y'[x]-6,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to 3x + \frac{1}{2}c_1e^{2x} + c_2$$

8.14 problem 13.2 (h)

Internal problem ID [13486]

Book: Ordinary Differential Equations. An introduction to the fundamentals. Kenneth B. Howell. second edition. CRC Press. FL, USA. 2020
Section: Chapter 13. Higher order equations: Extending first order concepts. Additional exercises page 259
Problem number: 13.2 (h).
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _missing_x], _Liouville, [_2nd_order, _reducible`

$$\boxed{(y-3)\,y''-2{y'}^2=0}$$

✓ Solution by Maple
Time used: 0.047 (sec). Leaf size: 25

```
dsolve((y(x)-3)*diff(y(x),x$2)=2*diff(y(x),x)^2,y(x), singsol=all)
```

$$y(x)=3$$
$$y(x)=\frac{3c_1x+3c_2-1}{c_1x+c_2}$$

✓ Solution by Mathematica
Time used: 0.183 (sec). Leaf size: 44

```
DSolve[(y[x]-3)*y''[x]==2*y'[x]^2,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x)\to\frac{3c_1x-1+3c_2c_1}{c_1(x+c_2)}$$
$$y(x)\to 3$$
$$y(x)\to \text{Indeterminate}$$
$$y(x)\to 3$$

8.15 problem 13.2 (i)

Internal problem ID [13487]

Book: Ordinary Differential Equations. An introduction to the fundamentals. Kenneth B. Howell. second edition. CRC Press. FL, USA. 2020
Section: Chapter 13. Higher order equations: Extending first order concepts. Additional exercises page 259
Problem number: 13.2 (i).
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _missing_y]]`

$$\boxed{y'' + 4y' = 9\,\mathrm{e}^{-3x}}$$

✓ Solution by Maple
Time used: 0.0 (sec). Leaf size: 19

```
dsolve(diff(y(x),x$2)+4*diff(y(x),x)=9*exp(-3*x),y(x), singsol=all)
```

$$y(x) = -\frac{\mathrm{e}^{-4x} c_1}{4} - 3\,\mathrm{e}^{-3x} + c_2$$

✓ Solution by Mathematica
Time used: 0.074 (sec). Leaf size: 26

```
DSolve[y''[x]+4*y'[x]==9*Exp[-3*x],y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to -3e^{-3x} - \frac{1}{4}c_1 e^{-4x} + c_2$$

8.16 problem 13.3 (a)

Internal problem ID [13488]

Book: Ordinary Differential Equations. An introduction to the fundamentals. Kenneth B. Howell. second edition. CRC Press. FL, USA. 2020
Section: Chapter 13. Higher order equations: Extending first order concepts. Additional exercises page 259
Problem number: 13.3 (a).
ODE order: 3.
ODE degree: 1.

CAS Maple gives this as type `[[_3rd_order, _missing_x]]`

$$\boxed{y''' - y'' = 0}$$

✓ Solution by Maple
Time used: 0.015 (sec). Leaf size: 13

```
dsolve(diff(y(x),x$3)=diff(y(x),x$2),y(x), singsol=all)
```

$$y(x) = c_1 + c_2 x + c_3 \mathrm{e}^x$$

✓ Solution by Mathematica
Time used: 0.025 (sec). Leaf size: 18

```
DSolve[y'''[x]==y''[x],y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to c_1 e^x + c_3 x + c_2$$

8.17 problem 13.3 (b)

Internal problem ID [13489]

Book: Ordinary Differential Equations. An introduction to the fundamentals. Kenneth B. Howell. second edition. CRC Press. FL, USA. 2020
Section: Chapter 13. Higher order equations: Extending first order concepts. Additional exercises page 259
Problem number: 13.3 (b).
ODE order: 3.
ODE degree: 1.

CAS Maple gives this as type `[[_3rd_order, _missing_y]]`

$$\boxed{xy''' + 2y'' = 6x}$$

✓ Solution by Maple
Time used: 0.0 (sec). Leaf size: 19

```
dsolve(x*diff(y(x),x$3)+2*diff(y(x),x$2)=6*x,y(x), singsol=all)
```

$$y(x) = \frac{x^3}{3} - c_1 \ln(x) + c_2 x + c_3$$

✓ Solution by Mathematica
Time used: 0.048 (sec). Leaf size: 25

```
DSolve[x*y'''[x]+2*y''[x]==6*x,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to \frac{x^3}{3} + c_3 x - c_1 \log(x) + c_2$$

8.18 problem 13.3 (c)

Internal problem ID [13490]

Book: Ordinary Differential Equations. An introduction to the fundamentals. Kenneth B. Howell. second edition. CRC Press. FL, USA. 2020
Section: Chapter 13. Higher order equations: Extending first order concepts. Additional exercises page 259
Problem number: 13.3 (c).
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_3rd_order, _missing_x], [_3rd_order, _missing_y], [_3rd_ord`

✓ Solution by Maple
Time used: 0.14 (sec). Leaf size: 36

```
dsolve(diff(y(x),x$3)=2*sqrt(diff(y(x),x$2)),y(x), singsol=all)
```

$$y(x) = c_1 x + c_2$$
$$y(x) = \frac{1}{12}x^4 + \frac{1}{3}c_1 x^3 + \frac{1}{2}c_1^2 x^2 + c_2 x + c_3$$

✓ Solution by Mathematica
Time used: 0.13 (sec). Leaf size: 39

```
DSolve[y'''[x]==2*Sqrt[y''[x]],y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to \frac{x^4}{12} + \frac{c_1 x^3}{6} + \frac{{c_1}^2 x^2}{8} + c_3 x + c_2$$

8.19 problem 13.3 (d)

Internal problem ID [13491]

Book: Ordinary Differential Equations. An introduction to the fundamentals. Kenneth B. Howell. second edition. CRC Press. FL, USA. 2020
Section: Chapter 13. Higher order equations: Extending first order concepts. Additional exercises page 259
Problem number: 13.3 (d).
ODE order: 4.
ODE degree: 1.

CAS Maple gives this as type `[[_high_order, _missing_x]]`

$$\boxed{y'''' + 2y''' = 0}$$

✓ Solution by Maple
Time used: 0.0 (sec). Leaf size: 20

```
dsolve(diff(y(x),x$4)=-2*diff(y(x),x$3),y(x), singsol=all)
```

$$y(x) = c_1 + c_2x + x^2c_3 + c_4e^{-2x}$$

✓ Solution by Mathematica
Time used: 0.043 (sec). Leaf size: 28

```
DSolve[y''''[x]==-2*y'''[x],y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to -\frac{1}{8}c_1e^{-2x} + x(c_4x + c_3) + c_2$$

8.20 problem 13.4 (a)

Internal problem ID [13492]

Book: Ordinary Differential Equations. An introduction to the fundamentals. Kenneth B. Howell. second edition. CRC Press. FL, USA. 2020
Section: Chapter 13. Higher order equations: Extending first order concepts. Additional exercises page 259
Problem number: 13.4 (a).
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _missing_x], _Liouville, [_2nd_order, _reducible`

$$\boxed{yy'' - y'^2 = 0}$$

With initial conditions

$$[y(0) = 5, y'(0) = 15]$$

✓ Solution by Maple
Time used: 0.078 (sec). Leaf size: 10

```
dsolve([y(x)*diff(y(x),x$2)=diff(y(x),x)^2,y(0) = 5, D(y)(0) = 15],y(x), singsol=all)
```

$$y(x) = 5\,\mathrm{e}^{3x}$$

✓ Solution by Mathematica
Time used: 0.129 (sec). Leaf size: 12

```
DSolve[{y[x]*y''[x]==y'[x]^2,{y[0]==5,y'[0]==15}},y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to 5e^{3x}$$

8.21 problem 13.4 (b)

Internal problem ID [13493]

Book: Ordinary Differential Equations. An introduction to the fundamentals. Kenneth B. Howell. second edition. CRC Press. FL, USA. 2020
Section: Chapter 13. Higher order equations: Extending first order concepts. Additional exercises page 259
Problem number: 13.4 (b).
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _missing_x], _Liouville, [_2nd_order, _reducible`

$$3yy'' - 2y'^2 = 0$$

With initial conditions

$$[y(0) = 8, y'(0) = 6]$$

✓ Solution by Maple
Time used: 0.125 (sec). Leaf size: 11

```
dsolve([3*y(x)*diff(y(x),x$2)=2*diff(y(x),x)^2,y(0) = 8, D(y)(0) = 6],y(x), singsol=all)
```

$$y(x) = \frac{(x+4)^3}{8}$$

✓ Solution by Mathematica
Time used: 0.029 (sec). Leaf size: 14

```
DSolve[{3*y[x]*y''[x]==2*y'[x]^2,{y[0]==8,y'[0]==6}},y[x],x,IncludeSingularSolutions -> True
```

$$y(x) \to \frac{1}{8}(x+4)^3$$

8.22 problem 13.4 (c)

Internal problem ID [13494]

Book: Ordinary Differential Equations. An introduction to the fundamentals. Kenneth B. Howell. second edition. CRC Press. FL, USA. 2020
Section: Chapter 13. Higher order equations: Extending first order concepts. Additional exercises page 259
Problem number: 13.4 (c).
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _missing_x], [_2nd_order, _exact, _nonlinear], _`

$$\boxed{\sin (y)\, y'' + \cos (y)\, y'^2 = 0}$$

✓ Solution by Maple
Time used: 0.0 (sec). Leaf size: 14

```
dsolve(sin(y(x))*diff(y(x),x$2)+cos(y(x))*diff(y(x),x)^2=0,y(x), singsol=all)
```

$$y(x) = \frac{\pi}{2} + \arcsin (c_1 x + c_2)$$

✓ Solution by Mathematica
Time used: 11.859 (sec). Leaf size: 29

```
DSolve[Sin[y[x]]*y''[x]+Cos[y[x]]*y'[x]^2==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to -\arccos(-c_1(x + c_2))$$
$$y(x) \to \arccos(-c_1(x + c_2))$$

8.23 problem 13.4 (d)

Internal problem ID [13495]

Book: Ordinary Differential Equations. An introduction to the fundamentals. Kenneth B. Howell. second edition. CRC Press. FL, USA. 2020
Section: Chapter 13. Higher order equations: Extending first order concepts. Additional exercises page 259
Problem number: 13.4 (d).
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _missing_x]]`

$$\boxed{y'' - y' = 0}$$

✓ Solution by Maple
Time used: 0.0 (sec). Leaf size: 10

```
dsolve(diff(y(x),x$2)=diff(y(x),x),y(x), singsol=all)
```

$$y(x) = c_1 + c_2 e^x$$

✓ Solution by Mathematica
Time used: 0.01 (sec). Leaf size: 14

```
DSolve[y''[x]==y'[x],y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to c_1 e^x + c_2$$

8.24 problem 13.4 (e)

Internal problem ID [13496]

Book: Ordinary Differential Equations. An introduction to the fundamentals. Kenneth B. Howell. second edition. CRC Press. FL, USA. 2020
Section: Chapter 13. Higher order equations: Extending first order concepts. Additional exercises page 259
Problem number: 13.4 (e).
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _missing_x], [_2nd_order, _exact, _nonlinear], _`

$$\boxed{yy'' + y'^2 - 2yy' = 0}$$

✓ Solution by Maple
Time used: 0.063 (sec). Leaf size: 37

```
dsolve(diff(y(x),x)^2+y(x)*diff(y(x),x$2)=2*y(x)*diff(y(x),x),y(x), singsol=all)
```

$$y(x) = 0$$
$$y(x) = \sqrt{e^{2x}c_1 + 2c_2}$$
$$y(x) = -\sqrt{e^{2x}c_1 + 2c_2}$$

✓ Solution by Mathematica
Time used: 0.788 (sec). Leaf size: 38

```
DSolve[y'[x]^2+y[x]*y''[x]==2*y[x]*y'[x],y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to c_2\sqrt{e^{2x} + e^{c_1}}$$
$$y(x) \to c_2\sqrt{e^{2x}}$$

8.25 problem 13.4 (f)

Internal problem ID [13497]

Book: Ordinary Differential Equations. An introduction to the fundamentals. Kenneth B. Howell. second edition. CRC Press. FL, USA. 2020
Section: Chapter 13. Higher order equations: Extending first order concepts. Additional exercises page 259
Problem number: 13.4 (f).
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _missing_x], [_2nd_order, _exact, _nonlinear], _`

$$\boxed{y^2y'' + y'' + 2y{y'}^2 = 0}$$

✓ Solution by Maple

Time used: 0.093 (sec). Leaf size: 299

```
dsolve(y(x)^2*diff(y(x),x$2)+diff(y(x),x$2)+2*y(x)*diff(y(x),x)^2=0,y(x), singsol=all)
```

$$y(x) = -i$$
$$y(x) = i$$
$$y(x) = \frac{\left(12c_1x + 12c_2 + 4\sqrt{9c_1^2x^2 + 18c_1c_2x + 9c_2^2 + 4}\right)^{\frac{2}{3}} - 4}{2\left(12c_1x + 12c_2 + 4\sqrt{9c_1^2x^2 + 18c_1c_2x + 9c_2^2 + 4}\right)^{\frac{1}{3}}}$$
$$y(x) = -\frac{\left(1 + i\sqrt{3}\right)\left(12c_1x + 12c_2 + 4\sqrt{9c_1^2x^2 + 18c_1c_2x + 9c_2^2 + 4}\right)^{\frac{2}{3}} + 4i\sqrt{3} - 4}{4\left(12c_1x + 12c_2 + 4\sqrt{9c_1^2x^2 + 18c_1c_2x + 9c_2^2 + 4}\right)^{\frac{1}{3}}}$$
$$y(x) = \frac{i\left(12c_1x + 12c_2 + 4\sqrt{9c_1^2x^2 + 18c_1c_2x + 9c_2^2 + 4}\right)^{\frac{2}{3}}\sqrt{3} - \left(12c_1x + 12c_2 + 4\sqrt{9c_1^2x^2 + 18c_1c_2x + 9c_2^2 + 4}\right.}{4\left(12c_1x + 12c_2 + 4\sqrt{9c_1^2x^2 + 18c_1c_2x + 9c_2^2 + 4}\right)^{\frac{1}{3}}}$$

✓ Solution by Mathematica

Time used: 54.871 (sec). Leaf size: 307

```
DSolve[y[x]^2*y''[x]+y''[x]+2*y[x]*y'[x]^2==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to \frac{-2+\sqrt[3]{2}\left(3c_1x+\sqrt{4+9{c_1}^2(x+c_2)^2}+3c_2c_1\right)^{2/3}}{2^{2/3}\sqrt[3]{3c_1x+\sqrt{4+9{c_1}^2(x+c_2)^2}+3c_2c_1}}$$

$$y(x) \to \frac{i\left(\sqrt{3}+i\right)\sqrt[3]{3c_1x+\sqrt{4+9{c_1}^2(x+c_2)^2}+3c_2c_1}}{2\sqrt[3]{2}} + \frac{1+i\sqrt{3}}{2^{2/3}\sqrt[3]{3c_1x+\sqrt{4+9{c_1}^2(x+c_2)^2}+3c_2c_1}}$$

$$y(x) \to \frac{1-i\sqrt{3}}{2^{2/3}\sqrt[3]{3c_1x+\sqrt{4+9{c_1}^2(x+c_2)^2}+3c_2c_1}} - \frac{i\left(\sqrt{3}-i\right)\sqrt[3]{3c_1x+\sqrt{4+9{c_1}^2(x+c_2)^2}+3c_2c_1}}{2\sqrt[3]{2}}$$

8.26 problem 13.5 (a)

Internal problem ID [13498]

Book: Ordinary Differential Equations. An introduction to the fundamentals. Kenneth B. Howell. second edition. CRC Press. FL, USA. 2020
Section: Chapter 13. Higher order equations: Extending first order concepts. Additional exercises page 259
Problem number: 13.5 (a).
ODE order: 2.
ODE degree: 2.

CAS Maple gives this as type `[[_2nd_order, _missing_y], [_2nd_order, _reducible, _mu_y_y1]]`

$$\boxed{y'' - 4x\sqrt{y'} = 0}$$

✓ Solution by Maple
Time used: 0.219 (sec). Leaf size: 51

```
dsolve(diff(y(x),x$2)=4*x*sqrt(diff(y(x),x)),y(x), singsol=all)
```

$$y(x) = c_1$$
$$y(x) = \frac{x^5}{5} - \frac{2x^3}{3c_1} + \frac{x}{c_1^2} + c_2$$
$$y(x) = \frac{x^5}{5} + \frac{2x^3}{3c_1} + \frac{x}{c_1^2} + c_2$$

✓ Solution by Mathematica
Time used: 0.071 (sec). Leaf size: 33

```
DSolve[y''[x]==4*x*Sqrt[y'[x]],y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to \frac{x^5}{5} + \frac{c_1 x^3}{3} + \frac{{c_1}^2 x}{4} + c_2$$

8.27 problem 13.5 (c)

Internal problem ID [13499]

Book: Ordinary Differential Equations. An introduction to the fundamentals. Kenneth B. Howell. second edition. CRC Press. FL, USA. 2020
Section: Chapter 13. Higher order equations: Extending first order concepts. Additional exercises page 259
Problem number: 13.5 (c).
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _missing_x], [_2nd_order, _exact, _nonlinear], [`

$$\boxed{y'y''=1}$$

✓ Solution by Maple
Time used: 0.032 (sec). Leaf size: 40

```
dsolve(diff(y(x),x)*diff(y(x),x$2)=1,y(x), singsol=all)
```

$$y(x) = \frac{(2x+2c_1)^{\frac{3}{2}}}{3} + c_2$$
$$y(x) = \frac{(-2c_1-2x)\sqrt{2x+2c_1}}{3} + c_2$$

✓ Solution by Mathematica
Time used: 0.016 (sec). Leaf size: 49

```
DSolve[y'[x]*y''[x]==1,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to c_2 - \frac{2}{3}\sqrt{2}(x+c_1)^{3/2}$$
$$y(x) \to \frac{2}{3}\sqrt{2}(x+c_1)^{3/2} + c_2$$

8.28 problem 13.5 (d)

Internal problem ID [13500]

Book: Ordinary Differential Equations. An introduction to the fundamentals. Kenneth B. Howell. second edition. CRC Press. FL, USA. 2020
Section: Chapter 13. Higher order equations: Extending first order concepts. Additional exercises page 259
Problem number: 13.5 (d).
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _missing_y], [_2nd_order, _reducible, _mu_y_y1]]`

$$\boxed{xy'' + y' - y'^2 = 0}$$

✓ Solution by Maple

Time used: 0.016 (sec). Leaf size: 17

```
dsolve(x*diff(y(x),x$2)=diff(y(x),x)^2-diff(y(x),x),y(x), singsol=all)
```

$$y(x) = -\frac{\ln (c_1 x - 1)}{c_1} + c_2$$

✓ Solution by Mathematica

Time used: 0.338 (sec). Leaf size: 38

```
DSolve[x*y''[x]==y'[x]^2-y'[x],y[x],x,IncludeSingularSolutions -> True]
```

$$\begin{aligned} y(x) &\to e^{-c_1} \log (1 + e^{c_1} x) + c_2 \\ y(x) &\to c_2 \\ y(x) &\to x + c_2 \end{aligned}$$

8.29 problem 13.5 (e)

Internal problem ID [13501]

Book: Ordinary Differential Equations. An introduction to the fundamentals. Kenneth B. Howell. second edition. CRC Press. FL, USA. 2020
Section: Chapter 13. Higher order equations: Extending first order concepts. Additional exercises page 259
Problem number: 13.5 (e).
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _missing_y]]`

$$\boxed{xy'' - y' = 6x^5}$$

✓ Solution by Maple
Time used: 0.0 (sec). Leaf size: 17

```
dsolve(x*diff(y(x),x$2)-diff(y(x),x)=6*x^5,y(x), singsol=all)
```

$$y(x) = \frac{1}{4}x^6 + \frac{1}{2}c_1x^2 + c_2$$

✓ Solution by Mathematica
Time used: 0.031 (sec). Leaf size: 24

```
DSolve[x*y''[x]-y'[x]==6*x^5,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to \frac{1}{4}(x^6 + 2c_1x^2 + 4c_2)$$

8.30 problem 13.5 (f)

Internal problem ID [13502]

Book: Ordinary Differential Equations. An introduction to the fundamentals. Kenneth B. Howell. second edition. CRC Press. FL, USA. 2020
Section: Chapter 13. Higher order equations: Extending first order concepts. Additional exercises page 259
Problem number: 13.5 (f).
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _missing_x], [_2nd_order, _reducible, _mu_x_y1],`

$$\boxed{yy'' - y'^2 - y' = 0}$$

✓ Solution by Maple
Time used: 0.031 (sec). Leaf size: 20

```
dsolve(y(x)*diff(y(x),x$2)-diff(y(x),x)^2=diff(y(x),x),y(x), singsol=all)
```

$$y(x) = 0$$
$$y(x) = \frac{e^{c_1(c_2+x)} + 1}{c_1}$$

✓ Solution by Mathematica
Time used: 1.761 (sec). Leaf size: 26

```
DSolve[y[x]*y''[x]-y'[x]^2==y'[x],y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to \frac{1 + e^{c_1(x+c_2)}}{c_1}$$
$$y(x) \to \text{Indeterminate}$$

8.31 problem 13.5 (g)

Internal problem ID [13503]

Book: Ordinary Differential Equations. An introduction to the fundamentals. Kenneth B. Howell. second edition. CRC Press. FL, USA. 2020
Section: Chapter 13. Higher order equations: Extending first order concepts. Additional exercises page 259
Problem number: 13.5 (g).
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _missing_x], _Liouville, [_2nd_order, _reducible`

$$\boxed{yy'' - 2y'^2 = 0}$$

✓ Solution by Maple
Time used: 0.047 (sec). Leaf size: 17

```
dsolve(y(x)*diff(y(x),x$2)=2*diff(y(x),x)^2,y(x), singsol=all)
```

$$y(x) = 0$$
$$y(x) = -\frac{1}{c_1 x + c_2}$$

✓ Solution by Mathematica
Time used: 0.14 (sec). Leaf size: 19

```
DSolve[y[x]*y''[x]==2*y'[x]^2,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to \frac{c_2}{x + c_1}$$
$$y(x) \to 0$$

8.32 problem 13.5 (h)

Internal problem ID [13504]

Book: Ordinary Differential Equations. An introduction to the fundamentals. Kenneth B. Howell. second edition. CRC Press. FL, USA. 2020
Section: Chapter 13. Higher order equations: Extending first order concepts. Additional exercises page 259
Problem number: 13.5 (h).
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _missing_x], _Liouville, [_2nd_order, _reducible`

$$(y-3)\,y''-y'^2=0$$

✓ Solution by Maple
Time used: 0.047 (sec). Leaf size: 16

```
dsolve((y(x)-3)*diff(y(x),x$2)=diff(y(x),x)^2,y(x), singsol=all)
```

$$y(x)=3$$
$$y(x)=\mathrm{e}^{c_1 x}c_2+3$$

✓ Solution by Mathematica
Time used: 0.599 (sec). Leaf size: 16

```
DSolve[(y[x]-3)*y''[x]==y'[x]^2,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x)\to 3+e^{c_1(x+c_2)}$$

8.33 problem 13.5 (i)

Internal problem ID [13505]

Book: Ordinary Differential Equations. An introduction to the fundamentals. Kenneth B. Howell. second edition. CRC Press. FL, USA. 2020
Section: Chapter 13. Higher order equations: Extending first order concepts. Additional exercises page 259
Problem number: 13.5 (i).
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _missing_y]]`

$$\boxed{y'' + 4y' = 9\,\mathrm{e}^{-3x}}$$

✓ Solution by Maple
Time used: 0.0 (sec). Leaf size: 19

```
dsolve(diff(y(x),x$2)+4*diff(y(x),x)=9*exp(-3*x),y(x), singsol=all)
```

$$y(x) = -\frac{\mathrm{e}^{-4x} c_1}{4} - 3\,\mathrm{e}^{-3x} + c_2$$

✓ Solution by Mathematica
Time used: 0.058 (sec). Leaf size: 26

```
DSolve[y''[x]+4*y'[x]==9*Exp[-3*x],y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to -3e^{-3x} - \frac{1}{4}c_1 e^{-4x} + c_2$$

8.34 problem 13.5 (j)

Internal problem ID [13506]

Book: Ordinary Differential Equations. An introduction to the fundamentals. Kenneth B. Howell. second edition. CRC Press. FL, USA. 2020
Section: Chapter 13. Higher order equations: Extending first order concepts. Additional exercises page 259
Problem number: 13.5 (j).
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _missing_x], _Liouville, [_2nd_order, _reducible`

$$\boxed{y'' - y'(y' - 2) = 0}$$

✓ Solution by Maple
Time used: 0.032 (sec). Leaf size: 20

```
dsolve(diff(y(x),x$2)=diff(y(x),x)*(diff(y(x),x)-2),y(x), singsol=all)
```

$$y(x) = \ln(2) - \ln\left(\mathrm{e}^{-2x}c_1 - 2c_2\right)$$

✓ Solution by Mathematica
Time used: 60.076 (sec). Leaf size: 23

```
DSolve[y''[x]==y'[x]*(y'[x]-2),y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to c_2 - 2\operatorname{arctanh}\left(1 + 2e^{2(x+c_1)}\right)$$

8.35 problem 13.6 (a)

Internal problem ID [13507]

Book: Ordinary Differential Equations. An introduction to the fundamentals. Kenneth B. Howell. second edition. CRC Press. FL, USA. 2020
Section: Chapter 13. Higher order equations: Extending first order concepts. Additional exercises page 259
Problem number: 13.6 (a).
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _missing_y]]`

$$\boxed{xy'' + 4y' = 18x^2}$$

With initial conditions

$$[y(1) = 8, y'(1) = -3]$$

✓ Solution by Maple
Time used: 0.0 (sec). Leaf size: 14

```
dsolve([x*diff(y(x),x$2)+4*diff(y(x),x)=18*x^2,y(1) = 8, D(y)(1) = -3],y(x), singsol=all)
```

$$y(x) = x^3 + \frac{2}{x^3} + 5$$

✓ Solution by Mathematica
Time used: 0.035 (sec). Leaf size: 15

```
DSolve[{x*y''[x]+4*y'[x]==18*x^2,{y[1]==8,y'[1]==-3}},y[x],x,IncludeSingularSolutions -> Tru
```

$$y(x) \to x^3 + \frac{2}{x^3} + 5$$

8.36 problem 13.6 (b)

Internal problem ID [13508]

Book: Ordinary Differential Equations. An introduction to the fundamentals. Kenneth B. Howell. second edition. CRC Press. FL, USA. 2020
Section: Chapter 13. Higher order equations: Extending first order concepts. Additional exercises page 259
Problem number: 13.6 (b).
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _missing_y]]`

$$\boxed{xy'' - 2y' = 0}$$

With initial conditions

$$[y(-1) = 4, y'(-1) = 12]$$

✓ Solution by Maple
Time used: 0.015 (sec). Leaf size: 11

```
dsolve([x*diff(y(x),x$2)=2*diff(y(x),x),y(-1) = 4, D(y)(-1) = 12],y(x), singsol=all)
```

$$y(x) = 4x^3 + 8$$

✓ Solution by Mathematica
Time used: 0.011 (sec). Leaf size: 12

```
DSolve[{x*y''[x]==2*y'[x],{y[-1]==4,y'[-1]==12}},y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to 4(x^3 + 2)$$

8.37 problem 13.6 (c)

Internal problem ID [13509]

Book: Ordinary Differential Equations. An introduction to the fundamentals. Kenneth B. Howell. second edition. CRC Press. FL, USA. 2020
Section: Chapter 13. Higher order equations: Extending first order concepts. Additional exercises page 259
Problem number: 13.6 (c).
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _missing_x]]`

$$\boxed{y'' - y' = 0}$$

With initial conditions

$$[y(0) = 8, y'(0) = 5]$$

✓ Solution by Maple
Time used: 0.0 (sec). Leaf size: 10

```
dsolve([diff(y(x),x$2)=diff(y(x),x),y(0) = 8, D(y)(0) = 5],y(x), singsol=all)
```

$$y(x) = 3 + 5\,\mathrm{e}^{x}$$

✓ Solution by Mathematica
Time used: 0.01 (sec). Leaf size: 12

```
DSolve[{y''[x]==y'[x],{y[0]==8,y'[0]==5}},y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to 5e^{x} + 3$$

8.38 problem 13.6 (d)

Internal problem ID [13510]

Book: Ordinary Differential Equations. An introduction to the fundamentals. Kenneth B. Howell. second edition. CRC Press. FL, USA. 2020
Section: Chapter 13. Higher order equations: Extending first order concepts. Additional exercises page 259
Problem number: 13.6 (d).
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _missing_y]]`

$$\boxed{y'' + 2y' = 8\,\mathrm{e}^{2x}}$$

With initial conditions

$$[y(0) = 0, y'(0) = 0]$$

✓ Solution by Maple
Time used: 0.0 (sec). Leaf size: 14

```
dsolve([diff(y(x),x$2)+2*diff(y(x),x)=8*exp(2*x),y(0) = 0, D(y)(0) = 0],y(x), singsol=all)
```

$$y(x) = \mathrm{e}^{-2x} + \mathrm{e}^{2x} - 2$$

✓ Solution by Mathematica
Time used: 0.054 (sec). Leaf size: 17

```
DSolve[{y''[x]+2*y'[x]==8*Exp[2*x],{y[0]==0,y'[0]==0}},y[x],x,IncludeSingularSolutions -> Tr
```

$$y(x) \to e^{-2x} + e^{2x} - 2$$

8.39 problem 13.6 (e)

Internal problem ID [13511]

Book: Ordinary Differential Equations. An introduction to the fundamentals. Kenneth B. Howell. second edition. CRC Press. FL, USA. 2020
Section: Chapter 13. Higher order equations: Extending first order concepts. Additional exercises page 259
Problem number: 13.6 (e).
ODE order: 3.
ODE degree: 1.

CAS Maple gives this as type `[[_3rd_order, _missing_x]]`

$$\boxed{y''' - y'' = 0}$$

With initial conditions

$$[y(0) = 10, y'(0) = 5, y''(0) = 2]$$

✓ Solution by Maple
Time used: 0.016 (sec). Leaf size: 13

```
dsolve([diff(y(x),x$3)=diff(y(x),x$2),y(0) = 10, D(y)(0) = 5, (D@@2)(y)(0) = 2],y(x), singso
```

$$y(x) = 8 + 3x + 2\,e^{x}$$

✓ Solution by Mathematica
Time used: 0.024 (sec). Leaf size: 15

```
DSolve[{y'''[x]==y''[x],{y[0]==10,y'[0]==5,y''[0]==2}},y[x],x,IncludeSingularSolutions -> Tr
```

$$y(x) \to 3x + 2e^{x} + 8$$

8.40 problem 13.6 (f)

Internal problem ID [13512]

Book: Ordinary Differential Equations. An introduction to the fundamentals. Kenneth B. Howell. second edition. CRC Press. FL, USA. 2020
Section: Chapter 13. Higher order equations: Extending first order concepts. Additional exercises page 259
Problem number: 13.6 (f).
ODE order: 3.
ODE degree: 1.

CAS Maple gives this as type `[[_3rd_order, _missing_y]]`

$$\boxed{xy''' + 2y'' = 6x}$$

With initial conditions

$$[y(1) = 2, y'(1) = 1, y''(1) = 4]$$

✓ Solution by Maple

Time used: 0.015 (sec). Leaf size: 18

```
dsolve([x*diff(y(x),x$3)+2*diff(y(x),x$2)=6*x,y(1) = 2, D(y)(1) = 1, (D@@2)(y)(1) = 4],y(x),
```

$$y(x) = \frac{x^3}{3} - 2\ln(x) + 2x - \frac{1}{3}$$

✓ Solution by Mathematica

Time used: 0.045 (sec). Leaf size: 21

```
DSolve[{x*y'''[x]+2*y''[x]==6*x,{y[1]==2,y'[1]==1,y''[1]==4}},y[x],x,IncludeSingularSolution
```

$$y(x) \to \frac{1}{3}\left(x^3 + 6x - 6\log(x) - 1\right)$$

8.41 problem 13.6 (g)

Internal problem ID [13513]

Book: Ordinary Differential Equations. An introduction to the fundamentals. Kenneth B. Howell. second edition. CRC Press. FL, USA. 2020
Section: Chapter 13. Higher order equations: Extending first order concepts. Additional exercises page 259
Problem number: 13.6 (g).
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _missing_y]]`

$$\boxed{xy'' + 2y' = 6}$$

With initial conditions

$$[y(1) = 4, y'(1) = 5]$$

✓ Solution by Maple
Time used: 0.016 (sec). Leaf size: 14

```
dsolve([x*diff(y(x),x$2)+2*diff(y(x),x)=6,y(1) = 4, D(y)(1) = 5],y(x), singsol=all)
```

$$y(x) = -\frac{2}{x} + 3x + 3$$

✓ Solution by Mathematica
Time used: 0.011 (sec). Leaf size: 15

```
DSolve[{x*y''[x]+2*y'[x]==6,{y[1]==4,y'[1]==5}},y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to 3x - \frac{2}{x} + 3$$

8.42 problem 13.6 (h)

Internal problem ID [13514]

Book: Ordinary Differential Equations. An introduction to the fundamentals. Kenneth B. Howell. second edition. CRC Press. FL, USA. 2020
Section: Chapter 13. Higher order equations: Extending first order concepts. Additional exercises page 259
Problem number: 13.6 (h).
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _missing_y], [_2nd_order, _reducible, _mu_y_y1],`

$$\boxed{2xy'y'' - y'^2 = -1}$$

With initial conditions

$$\left[y(1) = 0, y'(1) = \sqrt{3}\right]$$

✓ Solution by Maple

Time used: 0.188 (sec). Leaf size: 19

```
dsolve([2*x*diff(y(x),x)*diff(y(x),x$2)=diff(y(x),x)^2-1,y(1) = 0, D(y)(1) = 3^(1/2)],y(x),
```

$$y(x) = \frac{(2x+1)^{\frac{3}{2}}}{3} - \sqrt{3}$$

✓ Solution by Mathematica

Time used: 0.121 (sec). Leaf size: 26

```
DSolve[{2*x*y'[x]*y''[x]==y'[x]^2-1,{y[1]==0,y'[1]==Sqrt[3]}},y[x],x,IncludeSingularSolution
```

$$y(x) \to \frac{1}{3}\left((2x+1)^{3/2} - 3\sqrt{3}\right)$$

8.43 problem 13.7 (c)

Internal problem ID [13515]

Book: Ordinary Differential Equations. An introduction to the fundamentals. Kenneth B. Howell. second edition. CRC Press. FL, USA. 2020
Section: Chapter 13. Higher order equations: Extending first order concepts. Additional exercises page 259
Problem number: 13.7 (c).
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _missing_x], _Liouville, [_2nd_order, _reducible`

$$\boxed{3yy'' - 2y'^2 = 0}$$

With initial conditions

$$[y(1) = 1, y'(1) = 9]$$

✓ Solution by Maple
Time used: 0.031 (sec). Leaf size: 11

```
dsolve([3*y(x)*diff(y(x),x$2)=2*diff(y(x),x)^2,y(1) = 1, D(y)(1) = 9],y(x), singsol=all)
```

$$y(x) = (3x - 2)^3$$

✓ Solution by Mathematica
Time used: 0.033 (sec). Leaf size: 12

```
DSolve[{3*y[x]*y''[x]==2*y'[x]^2,{y[1]==1,y'[1]==9}},y[x],x,IncludeSingularSolutions -> True
```

$$y(x) \to (3x - 2)^3$$

8.44 problem 13.7 (d)

Internal problem ID [13516]

Book: Ordinary Differential Equations. An introduction to the fundamentals. Kenneth B. Howell. second edition. CRC Press. FL, USA. 2020
Section: Chapter 13. Higher order equations: Extending first order concepts. Additional exercises page 259
Problem number: 13.7 (d).
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _missing_x], _Liouville, [_2nd_order, _reducible`

$$\boxed{yy'' + 2{y'}^2 - 3yy' = 0}$$

With initial conditions

$$\left[y(0) = 2, y'(0) = \frac{3}{4}\right]$$

✓ Solution by Maple

Time used: 0.125 (sec). Leaf size: 14

```
dsolve([y(x)*diff(y(x),x$2)+2*diff(y(x),x)^2=3*y(x)*diff(y(x),x),y(0) = 2, D(y)(0) = 3/4],y(
```

$$y(x) = \left(3\,\mathrm{e}^{3x} + 5\right)^{\frac{1}{3}}$$

✓ Solution by Mathematica

Time used: 1.151 (sec). Leaf size: 18

```
DSolve[{y[x]*y''[x]+2*y'[x]^2==3*y[x]*y'[x],{y[0]==2,y'[0]==3/4}},y[x],x,IncludeSingularSolu
```

$$y(x) \rightarrow \sqrt[3]{3e^{3x} + 5}$$

8.45 problem 13.7 (e)

Internal problem ID [13517]

Book: Ordinary Differential Equations. An introduction to the fundamentals. Kenneth B. Howell. second edition. CRC Press. FL, USA. 2020
Section: Chapter 13. Higher order equations: Extending first order concepts. Additional exercises page 259
Problem number: 13.7 (e).
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _missing_x], [_2nd_order, _exact, _nonlinear], [`

$$\boxed{y'' + y' e^{-y} = 0}$$

With initial conditions

$$[y(0) = 0, y'(0) = 2]$$

✓ Solution by Maple
Time used: 0.407 (sec). Leaf size: 11

```
dsolve([diff(y(x),x$2)=-diff(y(x),x)*exp(-y(x)),y(0) = 0, D(y)(0) = 2],y(x), singsol=all)
```

$$y(x) = \ln(2 e^{x} - 1)$$

✓ Solution by Mathematica
Time used: 5.75 (sec). Leaf size: 13

```
DSolve[{y''[x]==-y'[x]*Exp[-y[x]],{y[0]==0,y'[0]==2}},y[x],x,IncludeSingularSolutions -> Tru
```

$$y(x) \to \log(2e^x - 1)$$

8.46 problem 13.8 (i)

Internal problem ID [13518]

Book: Ordinary Differential Equations. An introduction to the fundamentals. Kenneth B. Howell. second edition. CRC Press. FL, USA. 2020
Section: Chapter 13. Higher order equations: Extending first order concepts. Additional exercises page 259
Problem number: 13.8 (i).
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _missing_y], [_2nd_order, _reducible, _mu_y_y1]]`

$$\boxed{y'' + 2xy'^2 = 0}$$

With initial conditions

$$[y(0) = 3, y'(0) = 4]$$

✓ Solution by Maple
Time used: 0.141 (sec). Leaf size: 12

```
dsolve([diff(y(x),x$2)=-2*x*diff(y(x),x)^2,y(0) = 3, D(y)(0) = 4],y(x), singsol=all)
```

$$y(x) = 2\arctan(2x) + 3$$

✓ Solution by Mathematica
Time used: 0.981 (sec). Leaf size: 13

```
DSolve[{y''[x]==-2*x*y'[x]^2,{y[0]==3,y'[0]==4}},y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to 2\arctan(2x) + 3$$

8.47 problem 13.8 (ii)

Internal problem ID [13519]

Book: Ordinary Differential Equations. An introduction to the fundamentals. Kenneth B. Howell. second edition. CRC Press. FL, USA. 2020
Section: Chapter 13. Higher order equations: Extending first order concepts. Additional exercises page 259
Problem number: 13.8 (ii).
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _missing_y], [_2nd_order, _reducible, _mu_y_y1]]`

$$\boxed{y'' + 2xy'^2 = 0}$$

With initial conditions

$$[y(0) = 3, y'(0) = 0]$$

✓ Solution by Maple
Time used: 0.016 (sec). Leaf size: 5

```
dsolve([diff(y(x),x$2)=-2*x*diff(y(x),x)^2,y(0) = 3, D(y)(0) = 0],y(x), singsol=all)
```

$$y(x) = 3$$

✓ Solution by Mathematica
Time used: 0.949 (sec). Leaf size: 6

```
DSolve[{y''[x]==-2*x*y'[x]^2,{y[0]==3,y'[0]==0}},y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to 3$$

8.48 problem 13.8 (iii)

Internal problem ID [13520]

Book: Ordinary Differential Equations. An introduction to the fundamentals. Kenneth B. Howell. second edition. CRC Press. FL, USA. 2020
Section: Chapter 13. Higher order equations: Extending first order concepts. Additional exercises page 259
Problem number: 13.8 (iii).
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _missing_y], [_2nd_order, _reducible, _mu_y_y1]]`

$$\boxed{y'' + 2xy'^2 = 0}$$

With initial conditions

$$[y(1) = 0, y'(1) = 1]$$

✓ Solution by Maple
Time used: 0.015 (sec). Leaf size: 11

```
dsolve([diff(y(x),x$2)=-2*x*diff(y(x),x)^2,y(1) = 0, D(y)(1) = 1],y(x), singsol=all)
```

$$y(x) = \frac{-1 + x}{x}$$

✗ Solution by Mathematica
Time used: 0.0 (sec). Leaf size: 0

```
DSolve[{y''[x]==-2*x*y'[x]^2,{y[1]==0,y'[1]==1}},y[x],x,IncludeSingularSolutions -> True]
```

{}

8.49 problem 13.8 (iv)

Internal problem ID [13521]

Book: Ordinary Differential Equations. An introduction to the fundamentals. Kenneth B. Howell. second edition. CRC Press. FL, USA. 2020
Section: Chapter 13. Higher order equations: Extending first order concepts. Additional exercises page 259
Problem number: 13.8 (iv).
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _missing_y], [_2nd_order, _reducible, _mu_y_y1]]`

$$\boxed{y'' + 2xy'^2 = 0}$$

With initial conditions

$$\left[y(1) = -\frac{1}{4}, y'(1) = 5\right]$$

✓ Solution by Maple
Time used: 0.062 (sec). Leaf size: 29

```
dsolve([diff(y(x),x$2)=-2*x*diff(y(x),x)^2,y(1) = -1/4, D(y)(1) = 5],y(x), singsol=all)
```

$$y(x) = -\frac{\sqrt{5}\,\operatorname{arctanh}\left(\frac{\sqrt{5}\,x}{2}\right)}{2} + \frac{\sqrt{5}\,\operatorname{arctanh}\left(\frac{\sqrt{5}}{2}\right)}{2} - \frac{1}{4}$$

✓ Solution by Mathematica
Time used: 0.185 (sec). Leaf size: 46

```
DSolve[{y''[x]==-2*x*y'[x]^2,{y[1]==-1/4,y'[1]==5}},y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to \frac{1}{4}\left(-2\sqrt{5}\operatorname{arctanh}\left(\frac{\sqrt{5}x}{2}\right) + 2\sqrt{5}\operatorname{arctanh}\left(\frac{\sqrt{5}}{2}\right) - 1\right)$$

8.50 problem 13.9 (i)

Internal problem ID [13522]

Book: Ordinary Differential Equations. An introduction to the fundamentals. Kenneth B. Howell. second edition. CRC Press. FL, USA. 2020
Section: Chapter 13. Higher order equations: Extending first order concepts. Additional exercises page 259
Problem number: 13.9 (i).
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _missing_x], [_2nd_order, _exact, _nonlinear], _`

$$\boxed{y'' - 2yy' = 0}$$

With initial conditions

$$[y(0) = 0, y'(0) = 1]$$

✓ Solution by Maple
Time used: 0.156 (sec). Leaf size: 6

```
dsolve([diff(y(x),x$2)=2*y(x)*diff(y(x),x),y(0) = 0, D(y)(0) = 1],y(x), singsol=all)
```

$$y(x) = \tan(x)$$

✓ Solution by Mathematica
Time used: 10.217 (sec). Leaf size: 7

```
DSolve[{y''[x]==2*y[x]*y'[x],{y[0]==0,y'[0]==1}},y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to \tan(x)$$

8.51 problem 13.9 (ii)

Internal problem ID [13523]

Book: Ordinary Differential Equations. An introduction to the fundamentals. Kenneth B. Howell. second edition. CRC Press. FL, USA. 2020
Section: Chapter 13. Higher order equations: Extending first order concepts. Additional exercises page 259
Problem number: 13.9 (ii).
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _missing_x], [_2nd_order, _exact, _nonlinear], _`

$$\boxed{y'' - 2yy' = 0}$$

With initial conditions

$$[y(0) = 1, y'(0) = 1]$$

✓ Solution by Maple
Time used: 0.016 (sec). Leaf size: 11

```
dsolve([diff(y(x),x$2)=2*y(x)*diff(y(x),x),y(0) = 1, D(y)(0) = 1],y(x), singsol=all)
```

$$y(x) = -\frac{1}{-1+x}$$

✗ Solution by Mathematica
Time used: 0.0 (sec). Leaf size: 0

```
DSolve[{y''[x]==2*y[x]*y'[x],{y[0]==1,y'[0]==1}},y[x],x,IncludeSingularSolutions -> True]
```

{}

8.52 problem 13.9 (iii)

Internal problem ID [13524]

Book: Ordinary Differential Equations. An introduction to the fundamentals. Kenneth B. Howell. second edition. CRC Press. FL, USA. 2020
Section: Chapter 13. Higher order equations: Extending first order concepts. Additional exercises page 259
Problem number: 13.9 (iii).
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _missing_x], [_2nd_order, _exact, _nonlinear], _`

$$\boxed{y'' - 2yy' = 0}$$

With initial conditions

$$[y(0) = 1, y'(0) = 0]$$

✓ Solution by Maple
Time used: 0.016 (sec). Leaf size: 5

```
dsolve([diff(y(x),x$2)=2*y(x)*diff(y(x),x),y(0) = 1, D(y)(0) = 0],y(x), singsol=all)
```

$$y(x) = 1$$

✗ Solution by Mathematica
Time used: 0.0 (sec). Leaf size: 0

```
DSolve[{y''[x]==2*y[x]*y'[x],{y[0]==1,y'[0]==0}},y[x],x,IncludeSingularSolutions -> True]
```

{}

8.53 problem 13.9 (iv)

Internal problem ID [13525]

Book: Ordinary Differential Equations. An introduction to the fundamentals. Kenneth B. Howell. second edition. CRC Press. FL, USA. 2020
Section: Chapter 13. Higher order equations: Extending first order concepts. Additional exercises page 259
Problem number: 13.9 (iv).
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _missing_x], [_2nd_order, _exact, _nonlinear], _`

$$y'' - 2yy' = 0$$

With initial conditions

$$[y(0) = 0, y'(0) = -1]$$

✓ Solution by Maple
Time used: 0.063 (sec). Leaf size: 8

```
dsolve([diff(y(x),x$2)=2*y(x)*diff(y(x),x),y(0) = 0, D(y)(0) = -1],y(x), singsol=all)
```

$$y(x) = -\tanh(x)$$

✓ Solution by Mathematica
Time used: 0.684 (sec). Leaf size: 9

```
DSolve[{y''[x]==2*y[x]*y'[x],{y[0]==0,y'[0]==-1}},y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to -\tanh(x)$$

9 Chapter 14. Higher order equations and the reduction of order method. Additional exercises page 277

9.1 problem 14.1 (a)

Internal problem ID [13526]

Book: Ordinary Differential Equations. An introduction to the fundamentals. Kenneth B. Howell. second edition. CRC Press. FL, USA. 2020
Section: Chapter 14. Higher order equations and the reduction of order method. Additional exercises page 277
Problem number: 14.1 (a).
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _linear, _nonhomogeneous]]`

$$\boxed{y'' + x^2 y' - 4y = x^3}$$

✓ Solution by Maple
Time used: 0.375 (sec). Leaf size: 127

```
dsolve(diff(y(x),x$2)+x^2*diff(y(x),x)-4*y(x)=x^3,y(x), singsol=all)
```

$$y(x) = -\mathrm{e}^{-\frac{x^3}{3}}\left(\left(\int \operatorname{HeunT}\left(-4\,3^{\frac{2}{3}}, -3, 0, \frac{3^{\frac{2}{3}}x}{3}\right)\left(\int \frac{\mathrm{e}^{\frac{x^3}{3}}}{\operatorname{HeunT}\left(-4\,3^{\frac{2}{3}}, -3, 0, \frac{3^{\frac{2}{3}}x}{3}\right)^2}dx\right)x^3dx + \left(-c_1 - \left(\int \operatorname{HeunT}\left(-4\,3^{\frac{2}{3}}, -3, 0, \frac{3^{\frac{2}{3}}x}{3}\right)x^3dx\right)\right)\left(\int \frac{\mathrm{e}^{\frac{x^3}{3}}}{\operatorname{HeunT}\left(-4\,3^{\frac{2}{3}}, -3, 0, \frac{3^{\frac{2}{3}}x}{3}\right)^2}dx\right) - c_2\right)\operatorname{HeunT}\left(-4\,3^{\frac{2}{3}}, -3, 0, \frac{3^{\frac{2}{3}}x}{3}\right)$$

✓ Solution by Mathematica

Time used: 0.589 (sec). Leaf size: 194

```
DSolve[y''[x]+x^2*y'[x]-4*y[x]==x^3,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to e^{-\frac{x^3}{3}}\text{HeunT}[4,-2,0,0,-1,x]\left(\int_1^x -\frac{e^{\frac{K[2]^3}{3}}\text{HeunT}[4,0,0,0,1,K[2]]K[2]^3}{\text{HeunT}[4,-2,0,0,-1,K[2]]\text{HeunTPrime}[4,0,0,0,1,K[2]]+\text{HeunT}[4,0,0,0,1,K[2]]\,(\text{HeunT}[4,-2,} \ldots + c_2\right) + \text{HeunT}[4,0,0,0,1,x]\left(\int_1^x \frac{\text{HeunT}[4,-}{\text{HeunT}[4,-2,0,0,-1,K[1]]\text{HeunTPrime}[4,0,0,0,1,K[1]]+\text{HeunT}[4,0,(} \ldots + c_1\right)$$

9.2 problem 14.1 (b)

Internal problem ID [13527]

Book: Ordinary Differential Equations. An introduction to the fundamentals. Kenneth B. Howell. second edition. CRC Press. FL, USA. 2020
Section: Chapter 14. Higher order equations and the reduction of order method. Additional exercises page 277
Problem number: 14.1 (b).
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$\boxed{y'' + x^2 y' - 4y = 0}$$

✓ Solution by Maple
Time used: 0.359 (sec). Leaf size: 54

```
dsolve(diff(y(x),x$2)+x^2*diff(y(x),x)-4*y(x)=0,y(x), singsol=all)
```

$$y(x) = \mathrm{e}^{-\frac{x^3}{3}} \operatorname{HeunT}\left(-4\,3^{\frac{2}{3}}, -3, 0, \frac{3^{\frac{2}{3}} x}{3}\right) \left(c_1 + c_2 \left(\int \frac{\mathrm{e}^{\frac{x^3}{3}}}{\operatorname{HeunT}\left(-4\,3^{\frac{2}{3}}, -3, 0, \frac{3^{\frac{2}{3}} x}{3}\right)^2} dx\right)\right)$$

✓ Solution by Mathematica
Time used: 0.061 (sec). Leaf size: 35

```
DSolve[y''[x]+x^2*y'[x]-4*y[x]==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to c_2 e^{-\frac{x^3}{3}} \operatorname{HeunT}[4, -2, 0, 0, -1, x] + c_1 \operatorname{HeunT}[4, 0, 0, 0, 1, x]$$

9.3 problem 14.1 (c)

Internal problem ID [13528]

Book: Ordinary Differential Equations. An introduction to the fundamentals. Kenneth B. Howell. second edition. CRC Press. FL, USA. 2020
Section: Chapter 14. Higher order equations and the reduction of order method. Additional exercises page 277
Problem number: 14.1 (c).
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$\boxed{y'' + x^2 y' - 4y = 0}$$

✓ Solution by Maple
Time used: 0.375 (sec). Leaf size: 54

```
dsolve(diff(y(x),x$2)+x^2*diff(y(x),x)=4*y(x),y(x), singsol=all)
```

$$y(x) = \mathrm{e}^{-\frac{x^3}{3}} \operatorname{HeunT}\left(-4\,3^{\frac{2}{3}}, -3, 0, \frac{3^{\frac{2}{3}} x}{3}\right)\left(c_1 + c_2\left(\int \frac{\mathrm{e}^{\frac{x^3}{3}}}{\operatorname{HeunT}\left(-4\,3^{\frac{2}{3}}, -3, 0, \frac{3^{\frac{2}{3}} x}{3}\right)^2} dx\right)\right)$$

✓ Solution by Mathematica
Time used: 0.06 (sec). Leaf size: 35

```
DSolve[y''[x]+x^2*y'[x]==4*y[x],y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to c_2 e^{-\frac{x^3}{3}} \operatorname{HeunT}[4, -2, 0, 0, -1, x] + c_1 \operatorname{HeunT}[4, 0, 0, 0, 1, x]$$

9.4 problem 14.1 (d)

Internal problem ID [13529]

Book: Ordinary Differential Equations. An introduction to the fundamentals. Kenneth B. Howell. second edition. CRC Press. FL, USA. 2020
Section: Chapter 14. Higher order equations and the reduction of order method. Additional exercises page 277
Problem number: 14.1 (d).
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[NONE]`

$$y'' + x^2y' + 4y - y^3 = 0$$

✗ Solution by Maple

```
dsolve(diff(y(x),x$2)+x^2*diff(y(x),x)+4*y(x)=y(x)^3,y(x), singsol=all)
```

No solution found

✗ Solution by Mathematica
Time used: 0.0 (sec). Leaf size: 0

```
DSolve[y''[x]+x^2*y'[x]+4*y[x]==y[x]^3,y[x],x,IncludeSingularSolutions -> True]
```

Not solved

9.5 problem 14.1 (e)

Internal problem ID [13530]

Book: Ordinary Differential Equations. An introduction to the fundamentals. Kenneth B. Howell. second edition. CRC Press. FL, USA. 2020
Section: Chapter 14. Higher order equations and the reduction of order method. Additional exercises page 277
Problem number: 14.1 (e).
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_linear]`

$$\boxed{y'x + 3y = e^{2x}}$$

✓ Solution by Maple
Time used: 0.0 (sec). Leaf size: 28

```
dsolve(x*diff(y(x),x)+3*y(x)=exp(2*x),y(x), singsol=all)
```

$$y(x) = \frac{(2x^2 - 2x + 1)\,e^{2x} + 4c_1}{4x^3}$$

✓ Solution by Mathematica
Time used: 0.069 (sec). Leaf size: 33

```
DSolve[x*y'[x]+3*y[x]==Exp[2*x],y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to \frac{e^{2x}(2x^2 - 2x + 1) + 4c_1}{4x^3}$$

9.6 problem 14.1 (f)

Internal problem ID [13531]

Book: Ordinary Differential Equations. An introduction to the fundamentals. Kenneth B. Howell. second edition. CRC Press. FL, USA. 2020
Section: Chapter 14. Higher order equations and the reduction of order method. Additional exercises page 277
Problem number: 14.1 (f).
ODE order: 3.
ODE degree: 1.

CAS Maple gives this as type `[[_3rd_order, _missing_x]]`

$$\boxed{y''' + y = 0}$$

✓ Solution by Maple
Time used: 0.016 (sec). Leaf size: 37

```
dsolve(diff(y(x),x$3)+y(x)=0,y(x), singsol=all)
```

$$y(x) = \left(c_2 \mathrm{e}^{\frac{3x}{2}} \sin\left(\frac{\sqrt{3}\, x}{2}\right) + c_3 \mathrm{e}^{\frac{3x}{2}} \cos\left(\frac{\sqrt{3}\, x}{2}\right) + c_1\right) \mathrm{e}^{-x}$$

✓ Solution by Mathematica
Time used: 0.009 (sec). Leaf size: 56

```
DSolve[y'''[x]+y[x]==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to e^{-x}\left(c_3 e^{3x/2} \cos\left(\frac{\sqrt{3}x}{2}\right) + c_2 e^{3x/2} \sin\left(\frac{\sqrt{3}x}{2}\right) + c_1\right)$$

9.7 problem 14.1 (g)

Internal problem ID [13532]

Book: Ordinary Differential Equations. An introduction to the fundamentals. Kenneth B. Howell. second edition. CRC Press. FL, USA. 2020
Section: Chapter 14. Higher order equations and the reduction of order method. Additional exercises page 277
Problem number: 14.1 (g).
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _missing_x], [_2nd_order, _reducible, _mu_x_y1],`

$$\boxed{(y+1)\,y''-y'^3=0}$$

✓ Solution by Maple

Time used: 0.047 (sec). Leaf size: 53

```
dsolve((y(x)+1)*diff(y(x),x$2)=diff(y(x),x)^3,y(x), singsol=all)
```

$$y(x)=-1$$
$$y(x)=c_1$$
$$y(x)=\frac{-c_1-c_2-x-\operatorname{LambertW}\left(-(c_1+c_2+x)\,\mathrm{e}^{-c_1-1}\right)}{\operatorname{LambertW}\left(-\left(c_1+c_2+x\right)\mathrm{e}^{-c_1-1}\right)}$$

✓ Solution by Mathematica

Time used: 0.438 (sec). Leaf size: 93

```
DSolve[(y[x]+1)*y''[x]==y'[x]^3,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x)\to \text{InverseFunction}[\#1-(\#1+1)\log(\#1+1)+\#1(-c_1)\&][x+c_2]$$
$$y(x)\to \text{InverseFunction}[\#1-(\#1+1)\log(\#1+1)+\#1(-(-c_1))\&][x+c_2]$$
$$y(x)\to \text{InverseFunction}[\#1-(\#1+1)\log(\#1+1)+\#1(-c_1)\&][x+c_2]$$

9.8 problem 14.1 (h)

Internal problem ID [13533]

Book: Ordinary Differential Equations. An introduction to the fundamentals. Kenneth B. Howell. second edition. CRC Press. FL, USA. 2020
Section: Chapter 14. Higher order equations and the reduction of order method. Additional exercises page 277
Problem number: 14.1 (h).
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$\boxed{y'' - 2y' + 5y = 30\,\mathrm{e}^{3x}}$$

✓ Solution by Maple
Time used: 0.016 (sec). Leaf size: 27

```
dsolve(diff(y(x),x$2)=2*diff(y(x),x)-5*y(x)+30*exp(3*x),y(x), singsol=all)
```

$$y(x) = \mathrm{e}^{x} \sin(2x)\, c_2 + \mathrm{e}^{x} \cos(2x)\, c_1 + \frac{15\,\mathrm{e}^{3x}}{4}$$

✓ Solution by Mathematica
Time used: 0.022 (sec). Leaf size: 35

```
DSolve[y''[x]==2*y'[x]-5*y[x]+30*Exp[3*x],y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to \frac{15e^{3x}}{4} + c_2 e^x \cos(2x) + c_1 e^x \sin(2x)$$

9.9 problem 14.1 (i)

Internal problem ID [13534]

Book: Ordinary Differential Equations. An introduction to the fundamentals. Kenneth B. Howell. second edition. CRC Press. FL, USA. 2020
Section: Chapter 14. Higher order equations and the reduction of order method. Additional exercises page 277
Problem number: 14.1 (i).
ODE order: 4.
ODE degree: 1.

CAS Maple gives this as type `[[_high_order, _missing_x]]`

$$\boxed{y'''' + 6y'' + 3y' - 83y = 25}$$

✓ Solution by Maple
Time used: 0.016 (sec). Leaf size: 94

```
dsolve(diff(y(x),x$4)+6*diff(y(x),x$2)+3*diff(y(x),x)-83*y(x)-25=0,y(x), singsol=all)
```

$$y(x) = -\frac{25}{83} + c_1 e^{\mathrm{RootOf}(\_Z^4+6\_Z^2+3\_Z-83,\mathrm{index}=1)x} + c_2 e^{\mathrm{RootOf}(\_Z^4+6\_Z^2+3\_Z-83,\mathrm{index}=2)x} + c_3 e^{\mathrm{RootOf}(\_Z^4+6\_Z^2+3\_Z-83,\mathrm{index}=3)x} + c_4 e^{\mathrm{RootOf}(\_Z^4+6\_Z^2+3\_Z-83,\mathrm{index}=4)x}$$

✓ Solution by Mathematica
Time used: 0.003 (sec). Leaf size: 117

```
DSolve[y''''[x]+6*y''[x]+3*y'[x]-83*y[x]-25==0,y[x],x,IncludeSingularSolutions -> True]
```

$$\begin{aligned} y(x) \to\ & c_3 \exp\left(x\mathrm{Root}\left[\#1^4 + 6\#1^2 + 3\#1 - 83\&, 3\right]\right) \\ & + c_4 \exp\left(x\mathrm{Root}\left[\#1^4 + 6\#1^2 + 3\#1 - 83\&, 4\right]\right) \\ & + c_2 \exp\left(x\mathrm{Root}\left[\#1^4 + 6\#1^2 + 3\#1 - 83\&, 2\right]\right) \\ & + c_1 \exp\left(x\mathrm{Root}\left[\#1^4 + 6\#1^2 + 3\#1 - 83\&, 1\right]\right) - \frac{25}{83} \end{aligned}$$

9.10 problem 14.1 (j)

Internal problem ID [13535]

Book: Ordinary Differential Equations. An introduction to the fundamentals. Kenneth B. Howell. second edition. CRC Press. FL, USA. 2020
Section: Chapter 14. Higher order equations and the reduction of order method. Additional exercises page 277
Problem number: 14.1 (j).
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_3rd_order, _missing_x], [_3rd_order, _with_linear_symmetrie`

✗ Solution by Maple

```
dsolve(y(x)*diff(y(x),x$3)+6*diff(y(x),x$2)+3*diff(y(x),x)=y(x),y(x), singsol=all)
```

No solution found

✗ Solution by Mathematica
Time used: 0.0 (sec). Leaf size: 0

```
DSolve[y[x]*y'''[x]+6*y''[x]+3*y'[x]==y[x],y[x],x,IncludeSingularSolutions -> True]
```

Not solved

9.11 problem 14.2 (a)

Internal problem ID [13536]

Book: Ordinary Differential Equations. An introduction to the fundamentals. Kenneth B. Howell. second edition. CRC Press. FL, USA. 2020
Section: Chapter 14. Higher order equations and the reduction of order method. Additional exercises page 277
Problem number: 14.2 (a).
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _missing_x]]`

$$\boxed{y'' - 5y' + 6y = 0}$$

Given that one solution of the ode is

$$y_1 = e^{2x}$$

✓ Solution by Maple
Time used: 0.016 (sec). Leaf size: 17

```
dsolve([diff(y(x),x$2)-5*diff(y(x),x)+6*y(x)=0,exp(2*x)],singsol=all)
```

$$y(x) = c_1 e^{3x} + c_2 e^{2x}$$

✓ Solution by Mathematica
Time used: 0.013 (sec). Leaf size: 20

```
DSolve[y''[x]-5*y'[x]+6*y[x]==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to e^{2x}(c_2 e^x + c_1)$$

9.12 problem 14.2 (b)

Internal problem ID [13537]

Book: Ordinary Differential Equations. An introduction to the fundamentals. Kenneth B. Howell. second edition. CRC Press. FL, USA. 2020
Section: Chapter 14. Higher order equations and the reduction of order method. Additional exercises page 277
Problem number: 14.2 (b).
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _missing_x]]`

$$\boxed{y'' - 10y' + 25y = 0}$$

Given that one solution of the ode is

$$y_1 = e^{5x}$$

✓ Solution by Maple
Time used: 0.0 (sec). Leaf size: 14

```
dsolve([diff(y(x),x$2)-10*diff(y(x),x)+25*y(x)=0,exp(5*x)],singsol=all)
```

$$y(x) = e^{5x}(c_2x + c_1)$$

✓ Solution by Mathematica
Time used: 0.015 (sec). Leaf size: 18

```
DSolve[y''[x]-10*y'[x]+25*y[x]==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to e^{5x}(c_2x + c_1)$$

9.13 problem 14.2 (c)

Internal problem ID [13538]

Book: Ordinary Differential Equations. An introduction to the fundamentals. Kenneth B. Howell. second edition. CRC Press. FL, USA. 2020
Section: Chapter 14. Higher order equations and the reduction of order method. Additional exercises page 277
Problem number: 14.2 (c).
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_Emden, _Fowler]]`

$$\boxed{x^2y'' - 6y'x + 12y = 0}$$

Given that one solution of the ode is

$$y_1 = x^3$$

✓ Solution by Maple
Time used: 0.0 (sec). Leaf size: 13

```
dsolve([x^2*diff(y(x),x$2)-6*x*diff(y(x),x)+12*y(x)=0,x^3],singsol=all)
```

$$y(x) = x^3(c_1x + c_2)$$

✓ Solution by Mathematica
Time used: 0.011 (sec). Leaf size: 16

```
DSolve[x^2*y''[x]-6*x*y'[x]+12*y[x]==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to x^3(c_2x + c_1)$$

9.14 problem 14.2 (d)

Internal problem ID [13539]

Book: Ordinary Differential Equations. An introduction to the fundamentals. Kenneth B. Howell. second edition. CRC Press. FL, USA. 2020
Section: Chapter 14. Higher order equations and the reduction of order method. Additional exercises page 277
Problem number: 14.2 (d).
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_Emden, _Fowler], [_2nd_order, _linear, '_with_symmetry_[0,F`

$$\boxed{2x^2y'' - y'x + y = 0}$$

Given that one solution of the ode is

$$y_1 = x$$

✓ Solution by Maple

Time used: 0.016 (sec). Leaf size: 13

```
dsolve([2*x^2*diff(y(x),x$2)-x*diff(y(x),x)+y(x)=0,x],singsol=all)
```

$$y(x) = c_1\sqrt{x} + c_2x$$

✓ Solution by Mathematica

Time used: 0.012 (sec). Leaf size: 18

```
DSolve[2*x^2*y''[x]-x*y'[x]+y[x]==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to c_1\sqrt{x} + c_2x$$

9.15 problem 14.2 (e)

Internal problem ID [13540]

Book: Ordinary Differential Equations. An introduction to the fundamentals. Kenneth B. Howell. second edition. CRC Press. FL, USA. 2020
Section: Chapter 14. Higher order equations and the reduction of order method. Additional exercises page 277
Problem number: 14.2 (e).
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_Emden, _Fowler]]`

$$\boxed{4x^2y'' + y = 0}$$

Given that one solution of the ode is

$$y_1 = \sqrt{x}$$

✓ Solution by Maple
Time used: 0.0 (sec). Leaf size: 14

```
dsolve([4*x^2*diff(y(x),x$2)+y(x)=0,sqrt(x)],singsol=all)
```

$$y(x) = (c_1 + c_2 \ln(x))\sqrt{x}$$

✓ Solution by Mathematica
Time used: 0.018 (sec). Leaf size: 24

```
DSolve[4*x^2*y''[x]+y[x]==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to \frac{1}{2}\sqrt{x}(c_2 \log(x) + 2c_1)$$

9.16 problem 14.2 (f)

Internal problem ID [13541]

Book: Ordinary Differential Equations. An introduction to the fundamentals. Kenneth B. Howell. second edition. CRC Press. FL, USA. 2020
Section: Chapter 14. Higher order equations and the reduction of order method. Additional exercises page 277
Problem number: 14.2 (f).
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$y'' - \left(4 + \frac{2}{x}\right) y' + \left(4 + \frac{4}{x}\right) y = 0$$

Given that one solution of the ode is

$$y_1 = e^{2x}$$

✓ Solution by Maple
Time used: 0.0 (sec). Leaf size: 16

```
dsolve([diff(y(x),x$2)-(4+2/x)*diff(y(x),x)+(4+4/x)*y(x)=0,exp(2*x)],singsol=all)
```

$$y(x) = e^{2x}(c_2 x^3 + c_1)$$

✓ Solution by Mathematica
Time used: 0.028 (sec). Leaf size: 25

```
DSolve[y''[x]-(4+2/x)*y'[x]+(4+4/x)*y[x]==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to \frac{1}{3} e^{2x}(c_2 x^3 + 3c_1)$$

9.17 problem 14.2 (g)

Internal problem ID [13542]

Book: Ordinary Differential Equations. An introduction to the fundamentals. Kenneth B. Howell. second edition. CRC Press. FL, USA. 2020
Section: Chapter 14. Higher order equations and the reduction of order method. Additional exercises page 277
Problem number: 14.2 (g).
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$\boxed{(x+1)\,y''+y'x-y=0}$$

Given that one solution of the ode is

$$y_1=\mathrm{e}^{-x}$$

✓ Solution by Maple
Time used: 0.0 (sec). Leaf size: 14

```
dsolve([(x+1)*diff(y(x),x$2)+x*diff(y(x),x)-y(x)=0,exp(-x)],singsol=all)
```

$$y(x)=c_1x+c_2\mathrm{e}^{-x}$$

✓ Solution by Mathematica
Time used: 0.113 (sec). Leaf size: 31

```
DSolve[(x+1)*y''[x]+x*y'[x]-y[x]==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x)\to\frac{2ec_2x+c_1\cosh(x)-c_1\sinh(x)}{\sqrt{2e}}$$

9.18 problem 14.2 (h)

Internal problem ID [13543]

Book: Ordinary Differential Equations. An introduction to the fundamentals. Kenneth B. Howell. second edition. CRC Press. FL, USA. 2020
Section: Chapter 14. Higher order equations and the reduction of order method. Additional exercises page 277
Problem number: 14.2 (h).
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_Emden, _Fowler], [_2nd_order, _linear, '_with_symmetry_[0,F`

$$\boxed{y'' - \frac{y'}{x} - 4x^2 y = 0}$$

Given that one solution of the ode is

$$y_1 = e^{-x^2}$$

✓ Solution by Maple
Time used: 0.0 (sec). Leaf size: 17

```
dsolve([diff(y(x),x$2)-1/x*diff(y(x),x)-4*x^2*y(x)=0,exp(-x^2)],singsol=all)
```

$$y(x) = c_1 \sinh\left(x^2\right) + c_2 \cosh\left(x^2\right)$$

✓ Solution by Mathematica
Time used: 0.019 (sec). Leaf size: 23

```
DSolve[y''[x]-1/x*y'[x]-4*x^2*y[x]==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to c_1 \cosh\left(x^2\right) + ic_2 \sinh\left(x^2\right)$$

9.19 problem 14.2 (i)

Internal problem ID [13544]

Book: Ordinary Differential Equations. An introduction to the fundamentals. Kenneth B. Howell. second edition. CRC Press. FL, USA. 2020
Section: Chapter 14. Higher order equations and the reduction of order method. Additional exercises page 277
Problem number: 14.2 (i).
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _missing_x]]`

$$\boxed{y'' + y = 0}$$

Given that one solution of the ode is

$$y_1 = \sin(x)$$

✓ Solution by Maple
Time used: 0.0 (sec). Leaf size: 13

```
dsolve([diff(y(x),x$2)+y(x)=0,sin(x)],singsol=all)
```

$$y(x) = c_1 \sin(x) + c_2 \cos(x)$$

✓ Solution by Mathematica
Time used: 0.013 (sec). Leaf size: 16

```
DSolve[y''[x]+y[x]==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to c_1 \cos(x) + c_2 \sin(x)$$

9.20 problem 14.2 (j)

Internal problem ID [13545]

Book: Ordinary Differential Equations. An introduction to the fundamentals. Kenneth B. Howell. second edition. CRC Press. FL, USA. 2020
Section: Chapter 14. Higher order equations and the reduction of order method. Additional exercises page 277
Problem number: 14.2 (j).
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _exact, _linear, _homogeneous]]`

$$\boxed{xy'' + (2x+2)\, y' + 2y = 0}$$

Given that one solution of the ode is

$$y_1 = \frac{1}{x}$$

✓ Solution by Maple
Time used: 0.0 (sec). Leaf size: 16

```
dsolve([x*diff(y(x),x$2)+(2+2*x)*diff(y(x),x)+2*y(x)=0,1/x],singsol=all)
```

$$y(x) = \frac{c_1 + \mathrm{e}^{-2x} c_2}{x}$$

✓ Solution by Mathematica
Time used: 0.035 (sec). Leaf size: 24

```
DSolve[x*y''[x]+(2+2*x)*y'[x]+2*y[x]==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to \frac{2c_1 e^{-2x} + c_2}{2x}$$

9.21 problem 14.2 (k)

Internal problem ID [13546]

Book: Ordinary Differential Equations. An introduction to the fundamentals. Kenneth B. Howell. second edition. CRC Press. FL, USA. 2020
Section: Chapter 14. Higher order equations and the reduction of order method. Additional exercises page 277
Problem number: 14.2 (k).
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$\boxed{\sin(x)^2 y'' - 2\cos(x)\sin(x)\, y' + \left(\cos(x)^2 + 1\right) y = 0}$$

Given that one solution of the ode is

$$y_1 = \sin(x)$$

✓ Solution by Maple
Time used: 0.0 (sec). Leaf size: 12

```
dsolve([sin(x)^2*diff(y(x),x$2)-2*cos(x)*sin(x)*diff(y(x),x)+(1+cos(x)^2)*y(x)=0,sin(x)],sin
```

$$y(x) = \sin(x)(c_2 x + c_1)$$

✓ Solution by Mathematica
Time used: 0.079 (sec). Leaf size: 15

```
DSolve[Sin[x]^2*y''[x]-2*Cos[x]*Sin[x]*y'[x]+(1+Cos[x]^2)*y[x]==0,y[x],x,IncludeSingularSolu
```

$$y(x) \rightarrow (c_2 x + c_1)\sin(x)$$

9.22 problem 14.2 (L)

Internal problem ID [13547]

Book: Ordinary Differential Equations. An introduction to the fundamentals. Kenneth B. Howell. second edition. CRC Press. FL, USA. 2020
Section: Chapter 14. Higher order equations and the reduction of order method. Additional exercises page 277
Problem number: 14.2 (L).
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$\boxed{x^2y'' - 2y'x + (x^2+2)\,y = 0}$$

Given that one solution of the ode is

$$y_1 = x\sin(x)$$

✓ Solution by Maple
Time used: 0.0 (sec). Leaf size: 15

```
dsolve([x^2*diff(y(x),x$2)-2*x*diff(y(x),x)+(x^2+2)*y(x)=0,x*sin(x)],singsol=all)
```

$$y(x) = x(c_1\sin(x) + c_2\cos(x))$$

✓ Solution by Mathematica
Time used: 0.04 (sec). Leaf size: 33

```
DSolve[x^2*y''[x]-2*x*y'[x]+(x^2+2)*y[x]==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to c_1 e^{-ix}x - \frac{1}{2}ic_2e^{ix}x$$

9.23 problem 14.2 (m)

Internal problem ID [13548]

Book: Ordinary Differential Equations. An introduction to the fundamentals. Kenneth B. Howell. second edition. CRC Press. FL, USA. 2020
Section: Chapter 14. Higher order equations and the reduction of order method. Additional exercises page 277
Problem number: 14.2 (m).
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_Emden, _Fowler], [_2nd_order, _linear, '_with_symmetry_[0,F`

$$\boxed{x^2y'' + y'x + y = 0}$$

Given that one solution of the ode is

$$y_1 = \frac{x}{2} - \frac{1}{2x}$$

✓ Solution by Maple
Time used: 0.0 (sec). Leaf size: 15

```
dsolve([x^2*diff(y(x),x$2)+x*diff(y(x),x)+y(x)=0,sinh(ln(x))],singsol=all)
```

$$y(x) = c_1 \sin(\ln(x)) + c_2 \cos(\ln(x))$$

✓ Solution by Mathematica
Time used: 0.022 (sec). Leaf size: 18

```
DSolve[x^2*y''[x]+x*y'[x]+y[x]==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to c_1 \cos(\log(x)) + c_2 \sin(\log(x))$$

9.24 problem 14.2 (n)

Internal problem ID [13549]

Book: Ordinary Differential Equations. An introduction to the fundamentals. Kenneth B. Howell. second edition. CRC Press. FL, USA. 2020
Section: Chapter 14. Higher order equations and the reduction of order method. Additional exercises page 277
Problem number: 14.2 (n).
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$\boxed{x^2y'' + y'x + \left(x^2 - \frac{1}{4}\right)y = 0}$$

Given that one solution of the ode is

$$y_1 = \frac{\cos(x)}{\sqrt{x}}$$

✓ Solution by Maple
Time used: 0.063 (sec). Leaf size: 17

```
dsolve([x^2*diff(y(x),x$2)+x*diff(y(x),x)+(x^2-1/4)*y(x)=0,1/sqrt(x)*cos(x)],singsol=all)
```

$$y(x) = \frac{c_1 \sin(x) + c_2 \cos(x)}{\sqrt{x}}$$

✓ Solution by Mathematica
Time used: 0.033 (sec). Leaf size: 39

```
DSolve[x^2*y''[x]+x*y'[x]+(x^2-1/4)*y[x]==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to \frac{e^{-ix}(2c_1 - ic_2e^{2ix})}{2\sqrt{x}}$$

9.25 problem 14.3 (a)

Internal problem ID [13550]

Book: Ordinary Differential Equations. An introduction to the fundamentals. Kenneth B. Howell. second edition. CRC Press. FL, USA. 2020
Section: Chapter 14. Higher order equations and the reduction of order method. Additional exercises page 277
Problem number: 14.3 (a).
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$\boxed{y'' - 4y' + 3y = 9\,\mathrm{e}^{2x}}$$

Given that one solution of the ode is

$$y_1 = \mathrm{e}^{3x}$$

✓ Solution by Maple
Time used: 0.0 (sec). Leaf size: 21

```
dsolve([diff(y(x),x$2)-4*diff(y(x),x)+3*y(x)=9*exp(2*x),exp(3*x)],singsol=all)
```

$$y(x) = c_2\mathrm{e}^{3x} + c_1\mathrm{e}^{x} - 9\,\mathrm{e}^{2x}$$

✓ Solution by Mathematica
Time used: 0.069 (sec). Leaf size: 25

```
DSolve[y''[x]-4*y'[x]+3*y[x]==9*Exp[2*x],y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to e^x\left(-9e^x + c_2e^{2x} + c_1\right)$$

9.26 problem 14.3 (b)

Internal problem ID [13551]

Book: Ordinary Differential Equations. An introduction to the fundamentals. Kenneth B. Howell. second edition. CRC Press. FL, USA. 2020
Section: Chapter 14. Higher order equations and the reduction of order method. Additional exercises page 277
Problem number: 14.3 (b).
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$\boxed{y'' - 6y' + 8y = e^{4x}}$$

Given that one solution of the ode is

$$y_1 = e^{2x}$$

✓ Solution by Maple
Time used: 0.0 (sec). Leaf size: 23

```
dsolve([diff(y(x),x$2)-6*diff(y(x),x)+8*y(x)=exp(4*x),exp(2*x)],singsol=all)
```

$$y(x) = \frac{\left(\left(x + c_1 - \frac{1}{2}\right) e^{2x} + 2c_2\right) e^{2x}}{2}$$

✓ Solution by Mathematica
Time used: 0.047 (sec). Leaf size: 31

```
DSolve[y''[x]-6*y'[x]+8*y[x]==Exp[4*x],y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to c_1 e^{2x} + e^{4x}\left(\frac{x}{2} - \frac{1}{4} + c_2\right)$$

9.27 problem 14.3 (c)

Internal problem ID [13552]

Book: Ordinary Differential Equations. An introduction to the fundamentals. Kenneth B. Howell. second edition. CRC Press. FL, USA. 2020
Section: Chapter 14. Higher order equations and the reduction of order method. Additional exercises page 277
Problem number: 14.3 (c).
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _exact, _linear, _nonhomogeneous]]`

$$\boxed{x^2y'' + y'x - y = \sqrt{x}}$$

Given that one solution of the ode is

$$y_1 = x$$

✓ Solution by Maple
Time used: 0.0 (sec). Leaf size: 24

```
dsolve([x^2*diff(y(x),x$2)+x*diff(y(x),x)-y(x)=sqrt(x),x],singsol=all)
```

$$y(x) = \frac{3c_2x^2 - 4x^{\frac{3}{2}} + 3c_1}{3x}$$

✓ Solution by Mathematica
Time used: 0.019 (sec). Leaf size: 25

```
DSolve[x^2*y''[x]+x*y'[x]-y[x]==Sqrt[x],y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to -\frac{4\sqrt{x}}{3} + \frac{c_1}{x} + c_2x$$

9.28 problem 14.3 (d)

Internal problem ID [13553]

Book: Ordinary Differential Equations. An introduction to the fundamentals. Kenneth B. Howell. second edition. CRC Press. FL, USA. 2020
Section: Chapter 14. Higher order equations and the reduction of order method. Additional exercises page 277
Problem number: 14.3 (d).
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type **[[_2nd_order, _with_linear_symmetries]]**

$$\boxed{x^2y'' - 20y = 27x^5}$$

Given that one solution of the ode is

$$y_1 = x^5$$

✓ Solution by Maple
Time used: 0.016 (sec). Leaf size: 29

```
dsolve([x^2*diff(y(x),x$2)-20*y(x)=27*x^5,x^5],singsol=all)
```

$$y(x) = \frac{9x^9 \ln(x) + (3c_2 - 1)x^9 + 3c_1}{3x^4}$$

✓ Solution by Mathematica
Time used: 0.012 (sec). Leaf size: 29

```
DSolve[x^2*y''[x]-20*y[x]==27*x^5,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to 3x^5 \log(x) + \left(-\frac{1}{3} + c_2\right)x^5 + \frac{c_1}{x^4}$$

9.29 problem 14.3 (e)

Internal problem ID [13554]

Book: Ordinary Differential Equations. An introduction to the fundamentals. Kenneth B. Howell. second edition. CRC Press. FL, USA. 2020
Section: Chapter 14. Higher order equations and the reduction of order method. Additional exercises page 277
Problem number: 14.3 (e).
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _exact, _linear, _nonhomogeneous]]`

$$\boxed{xy'' + (2x+2)\,y' + 2y = 8\,\mathrm{e}^{2x}}$$

Given that one solution of the ode is

$$y_1 = \frac{1}{x}$$

✓ Solution by Maple
Time used: 0.0 (sec). Leaf size: 20

```
dsolve([x*diff(y(x),x$2)+(2+2*x)*diff(y(x),x)+2*y(x)=8*exp(2*x),1/x],singsol=all)
```

$$y(x) = \frac{\mathrm{e}^{-2x}c_2 + \mathrm{e}^{2x} + c_1}{x}$$

✓ Solution by Mathematica
Time used: 0.066 (sec). Leaf size: 31

```
DSolve[x*y''[x]+(2+2*x)*y'[x]+2*y[x]==8*Exp[2*x],y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to \frac{2e^{2x} + 2c_1e^{-2x} + c_2}{2x}$$

9.30 problem 14.3 (f)

Internal problem ID [13555]

Book: Ordinary Differential Equations. An introduction to the fundamentals. Kenneth B. Howell. second edition. CRC Press. FL, USA. 2020
Section: Chapter 14. Higher order equations and the reduction of order method. Additional exercises page 277
Problem number: 14.3 (f).
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$\boxed{(x+1)\,y''+y'x-y=(x+1)^2}$$

Given that one solution of the ode is

$$y_1=\mathrm{e}^{-x}$$

✓ Solution by Maple
Time used: 0.015 (sec). Leaf size: 18

```
dsolve([(x+1)*diff(y(x),x$2)+x*diff(y(x),x)-y(x)=(1+x)^2,exp(-x)],singsol=all)
```

$$y(x)=c_2x+c_1\mathrm{e}^{-x}+x^2+1$$

✓ Solution by Mathematica
Time used: 0.156 (sec). Leaf size: 41

```
DSolve[(x+1)*y''[x]+x*y'[x]-y[x]==(1+x)^2,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x)\to x^2+\left(-1+\sqrt{2e}c_2\right)x+\frac{c_1e^{-x-\frac{1}{2}}}{\sqrt{2}}+1$$

9.31 problem 14.5 (a)

Internal problem ID [13556]

Book: Ordinary Differential Equations. An introduction to the fundamentals. Kenneth B. Howell. second edition. CRC Press. FL, USA. 2020
Section: Chapter 14. Higher order equations and the reduction of order method. Additional exercises page 277
Problem number: 14.5 (a).
ODE order: 3.
ODE degree: 1.

CAS Maple gives this as type `[[_3rd_order, _missing_x]]`

$$\boxed{y''' - 9y'' + 27y' - 27y = 0}$$

✓ Solution by Maple
Time used: 0.0 (sec). Leaf size: 19

```
dsolve([diff(y(x),x$3)-9*diff(y(x),x$2)+27*diff(y(x),x)-27*y(x)=0,exp(3*x)],singsol=all)
```

$$y(x) = e^{3x}(x^2 c_3 + c_2 x + c_1)$$

✓ Solution by Mathematica
Time used: 0.009 (sec). Leaf size: 23

```
DSolve[y'''[x]-9*y''[x]+27*y'[x]-27*y[x]==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to e^{3x}(x(c_3 x + c_2) + c_1)$$

9.32 problem 14.5 (b)

Internal problem ID [13557]

Book: Ordinary Differential Equations. An introduction to the fundamentals. Kenneth B. Howell. second edition. CRC Press. FL, USA. 2020
Section: Chapter 14. Higher order equations and the reduction of order method. Additional exercises page 277
Problem number: 14.5 (b).
ODE order: 3.
ODE degree: 1.

CAS Maple gives this as type `[[_3rd_order, _linear, _nonhomogeneous]]`

$$y''' - 9y'' + 27y' - 27y = e^{3x} \sin(x)$$

✓ Solution by Maple
Time used: 0.0 (sec). Leaf size: 21

```
dsolve([diff(y(x),x$3)-9*diff(y(x),x$2)+27*diff(y(x),x)-27*y(x)=exp(3*x)*sin(x),exp(3*x)],si
```

$$y(x) = e^{3x}(\cos(x) + c_1 + c_2x + x^2c_3)$$

✓ Solution by Mathematica
Time used: 0.04 (sec). Leaf size: 25

```
DSolve[y'''[x]-9*y''[x]+27*y'[x]-27*y[x]==Exp[3*x]*Sin[x],y[x],x,IncludeSingularSolutions ->
```

$$y(x) \to e^{3x}(\cos(x) + x(c_3x + c_2) + c_1)$$

9.33 problem 14.5 (c)

Internal problem ID [13558]

Book: Ordinary Differential Equations. An introduction to the fundamentals. Kenneth B. Howell. second edition. CRC Press. FL, USA. 2020
Section: Chapter 14. Higher order equations and the reduction of order method. Additional exercises page 277
Problem number: 14.5 (c).
ODE order: 4.
ODE degree: 1.

CAS Maple gives this as type `[[_high_order, _missing_x]]`

$$y'''' - 8y''' + 24y'' - 32y' + 16y = 0$$

✓ Solution by Maple
Time used: 0.0 (sec). Leaf size: 24

```
dsolve([diff(y(x),x$4)-8*diff(y(x),x$3)+24*diff(y(x),x$2)-32*diff(y(x),x)+16*y(x)=0,exp(2*x)
```

$$y(x) = \mathrm{e}^{2x}\left(c_4x^3 + x^2c_3 + c_2x + c_1\right)$$

✓ Solution by Mathematica
Time used: 0.003 (sec). Leaf size: 28

```
DSolve[y''''[x]-8*y'''[x]+24*y''[x]-32*y'[x]+16*y[x]==0,y[x],x,IncludeSingularSolutions -> T
```

$$y(x) \to e^{2x}(x(x(c_4x + c_3) + c_2) + c_1)$$

9.34 problem 14.5 (d)

Internal problem ID [13559]

Book: Ordinary Differential Equations. An introduction to the fundamentals. Kenneth B. Howell. second edition. CRC Press. FL, USA. 2020
Section: Chapter 14. Higher order equations and the reduction of order method. Additional exercises page 277
Problem number: 14.5 (d).
ODE order: 3.
ODE degree: 1.

CAS Maple gives this as type `[[_3rd_order, _with_linear_symmetries]]`

$$x^3y''' - 4y'' + 10y' - 12y = 0$$

✗ Solution by Maple

```
dsolve([x^3*diff(y(x),x$3)-4*diff(y(x),x$2)+10*diff(y(x),x)-12*y(x)=0,x^2],singsol=all)
```

No solution found

✗ Solution by Mathematica

Time used: 0.0 (sec). Leaf size: 0

```
DSolve[x^3*y'''[x]-4*y''[x]+10*y'[x]-12*y[x]==0,y[x],x,IncludeSingularSolutions -> True]
```

Not solved

10 Chapter 15. General solutions to Homogeneous linear differential equations. Additional exercises page 294

10.1 problem 15.2 (a)

Internal problem ID [13560]

Book: Ordinary Differential Equations. An introduction to the fundamentals. Kenneth B. Howell. second edition. CRC Press. FL, USA. 2020
Section: Chapter 15. General solutions to Homogeneous linear differential equations. Additional exercises page 294
Problem number: 15.2 (a).
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _missing_x]]`

$$\boxed{y'' + 4y = 0}$$

With initial conditions

$$[y(0) = 2, y'(0) = 6]$$

✓ Solution by Maple
Time used: 0.0 (sec). Leaf size: 17

```
dsolve([diff(y(x),x$2)+4*y(x)=0,y(0) = 2, D(y)(0) = 6],y(x), singsol=all)
```

$$y(x) = 3\sin(2x) + 2\cos(2x)$$

✓ Solution by Mathematica
Time used: 0.02 (sec). Leaf size: 18

```
DSolve[{y''[x]+4*y[x]==0,{y[0]==2,y'[0]==6}},y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to 3\sin(2x) + 2\cos(2x)$$

10.2 problem 15.2 (b)

Internal problem ID [13561]

Book: Ordinary Differential Equations. An introduction to the fundamentals. Kenneth B. Howell. second edition. CRC Press. FL, USA. 2020
Section: Chapter 15. General solutions to Homogeneous linear differential equations. Additional exercises page 294
Problem number: 15.2 (b).
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _missing_x]]`

$$\boxed{y'' - 4y = 0}$$

With initial conditions

$$[y(0) = 0, y'(0) = 12]$$

✓ Solution by Maple
Time used: 0.015 (sec). Leaf size: 17

```
dsolve([diff(y(x),x$2)-4*y(x)=0,y(0) = 0, D(y)(0) = 12],y(x), singsol=all)
```

$$y(x) = 3\,\mathrm{e}^{2x} - 3\,\mathrm{e}^{-2x}$$

✓ Solution by Mathematica
Time used: 0.017 (sec). Leaf size: 19

```
DSolve[{y''[x]-4*y[x]==0,{y[0]==0,y'[0]==12}},y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to 3e^{-2x}\left(e^{4x} - 1\right)$$

10.3 problem 15.2 (c)

Internal problem ID [13562]

Book: Ordinary Differential Equations. An introduction to the fundamentals. Kenneth B. Howell. second edition. CRC Press. FL, USA. 2020
Section: Chapter 15. General solutions to Homogeneous linear differential equations. Additional exercises page 294
Problem number: 15.2 (c).
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _missing_x]]`

$$\boxed{y'' + y' - 6y = 0}$$

With initial conditions

$$[y(0) = 8, y'(0) = -9]$$

✓ Solution by Maple
Time used: 0.016 (sec). Leaf size: 17

```
dsolve([diff(y(x),x$2)+diff(y(x),x)-6*y(x)=0,y(0) = 8, D(y)(0) = -9],y(x), singsol=all)
```

$$y(x) = \left(3\,\mathrm{e}^{5x} + 5\right)\mathrm{e}^{-3x}$$

✓ Solution by Mathematica
Time used: 0.013 (sec). Leaf size: 20

```
DSolve[{y''[x]+y'[x]-6*y[x]==0,{y[0]==8,y'[0]==-9}},y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to e^{-3x}\left(3e^{5x} + 5\right)$$

10.4 problem 15.2 (d)

Internal problem ID [13563]

Book: Ordinary Differential Equations. An introduction to the fundamentals. Kenneth B. Howell. second edition. CRC Press. FL, USA. 2020
Section: Chapter 15. General solutions to Homogeneous linear differential equations. Additional exercises page 294
Problem number: 15.2 (d).
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _missing_x]]`

$$\boxed{y'' - 4y' + 4y = 0}$$

With initial conditions

$$[y(0) = 1, y'(0) = 6]$$

✓ Solution by Maple
Time used: 0.0 (sec). Leaf size: 14

```
dsolve([diff(y(x),x$2)-4*diff(y(x),x)+4*y(x)=0,y(0) = 1, D(y)(0) = 6],y(x), singsol=all)
```

$$y(x) = \mathrm{e}^{2x}(4x + 1)$$

✓ Solution by Mathematica
Time used: 0.014 (sec). Leaf size: 16

```
DSolve[{y''[x]-4*y'[x]+4*y[x]==0,{y[0]==1,y'[0]==6}},y[x],x,IncludeSingularSolutions -> True
```

$$y(x) \to e^{2x}(4x + 1)$$

10.5 problem 15.2 (e)

Internal problem ID [13564]

Book: Ordinary Differential Equations. An introduction to the fundamentals. Kenneth B. Howell. second edition. CRC Press. FL, USA. 2020
Section: Chapter 15. General solutions to Homogeneous linear differential equations. Additional exercises page 294
Problem number: 15.2 (e).
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_Emden, _Fowler], [_2nd_order, _linear, '_with_symmetry_[0,F`

$$\boxed{x^2y'' - 4y'x + 6y = 0}$$

With initial conditions

$$[y(1) = 0, y'(1) = 4]$$

✓ Solution by Maple
Time used: 0.016 (sec). Leaf size: 15

```
dsolve([x^2*diff(y(x),x$2)-4*x*diff(y(x),x)+6*y(x)=0,y(1) = 0, D(y)(1) = 4],y(x), singsol=al
```

$$y(x) = 4x^3 - 4x^2$$

✓ Solution by Mathematica
Time used: 0.011 (sec). Leaf size: 13

```
DSolve[{x^2*y''[x]-4*x*y'[x]+6*y[x]==0,{y[1]==0,y'[1]==4}},y[x],x,IncludeSingularSolutions -
```

$$y(x) \to 4(x-1)x^2$$

10.6 problem 15.2 (f)

Internal problem ID [13565]

Book: Ordinary Differential Equations. An introduction to the fundamentals. Kenneth B. Howell. second edition. CRC Press. FL, USA. 2020
Section: Chapter 15. General solutions to Homogeneous linear differential equations. Additional exercises page 294
Problem number: 15.2 (f).
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_Emden, _Fowler], [_2nd_order, _linear, '_with_symmetry_[0,F`

$$\boxed{4x^2y'' + 4y'x - y = 0}$$

With initial conditions

$$[y(1) = 8, y'(1) = 1]$$

✓ Solution by Maple
Time used: 0.016 (sec). Leaf size: 13

```
dsolve([4*x^2*diff(y(x),x$2)+4*x*diff(y(x),x)-y(x)=0,y(1) = 8, D(y)(1) = 1],y(x), singsol=al
```

$$y(x) = \frac{5x + 3}{\sqrt{x}}$$

✓ Solution by Mathematica
Time used: 0.012 (sec). Leaf size: 16

```
DSolve[{4*x^2*y''[x]+4*x*y'[x]-y[x]==0,{y[1]==8,y'[1]==1}},y[x],x,IncludeSingularSolutions -
```

$$y(x) \to \frac{5x + 3}{\sqrt{x}}$$

10.7 problem 15.2 (g)

Internal problem ID [13566]

Book: Ordinary Differential Equations. An introduction to the fundamentals. Kenneth B. Howell. second edition. CRC Press. FL, USA. 2020
Section: Chapter 15. General solutions to Homogeneous linear differential equations. Additional exercises page 294
Problem number: 15.2 (g).
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_Emden, _Fowler]]`

$$\boxed{x^2y'' - y'x + y = 0}$$

With initial conditions

$$[y(1) = 5, y'(1) = 3]$$

✓ Solution by Maple
Time used: 0.0 (sec). Leaf size: 12

```
dsolve([x^2*diff(y(x),x$2)-x*diff(y(x),x)+y(x)=0,y(1) = 5, D(y)(1) = 3],y(x), singsol=all)
```

$$y(x) = x(5 - 2\ln(x))$$

✓ Solution by Mathematica
Time used: 0.016 (sec). Leaf size: 13

```
DSolve[{x^2*y''[x]-x*y'[x]+y[x]==0,{y[1]==5,y'[1]==3}},y[x],x,IncludeSingularSolutions -> Tr
```

$$y(x) \to x(5 - 2\log(x))$$

10.8 problem 15.2 (h)

Internal problem ID [13567]

Book: Ordinary Differential Equations. An introduction to the fundamentals. Kenneth B. Howell. second edition. CRC Press. FL, USA. 2020
Section: Chapter 15. General solutions to Homogeneous linear differential equations. Additional exercises page 294
Problem number: 15.2 (h).
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_Emden, _Fowler], [_2nd_order, _linear, '_with_symmetry_[0,F`

$$\boxed{xy'' - y' + 4yx^3 = 0}$$

With initial conditions

$$\left[y\left(\sqrt{\pi}\right) = 3, y'\left(\sqrt{\pi}\right) = 4\right]$$

✓ Solution by Maple
Time used: 0.078 (sec). Leaf size: 20

```
dsolve([x*diff(y(x),x$2)-diff(y(x),x)+4*x^3*y(x)=0,y(Pi^(1/2)) = 3, D(y)(Pi^(1/2)) = 4],y(x)
```

$$y(x) = \frac{-3\cos\left(x^2\right)\sqrt{\pi} - 2\sin\left(x^2\right)}{\sqrt{\pi}}$$

✓ Solution by Mathematica
Time used: 0.02 (sec). Leaf size: 23

```
DSolve[{x*y''[x]-y'[x]+4*x^3*y[x]==0,{y[Sqrt[Pi]]==3,y'[Sqrt[Pi]]==4}},y[x],x,IncludeSingula
```

$$y(x) \to -\frac{2\sin\left(x^2\right)}{\sqrt{\pi}} - 3\cos\left(x^2\right)$$

10.9 problem 15.2 (i)

Internal problem ID [13568]

Book: Ordinary Differential Equations. An introduction to the fundamentals. Kenneth B. Howell. second edition. CRC Press. FL, USA. 2020
Section: Chapter 15. General solutions to Homogeneous linear differential equations. Additional exercises page 294
Problem number: 15.2 (i).
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries], [_2nd_order, _linear,`

$$\boxed{(x+1)^2 y'' - 2(x+1) y' + 2y = 0}$$

With initial conditions

$$[y(0) = 0, y'(0) = 4]$$

✓ Solution by Maple
Time used: 0.016 (sec). Leaf size: 13

```
dsolve([(x+1)^2*diff(y(x),x$2)-2*(x+1)*diff(y(x),x)+2*y(x)=0,y(0) = 0, D(y)(0) = 4],y(x), si
```

$$y(x) = 4x^2 + 4x$$

✓ Solution by Mathematica
Time used: 0.025 (sec). Leaf size: 11

```
DSolve[{(x+1)^2*y''[x]-2*(x+1)*y'[x]+2*y[x]==0,{y[0]==0,y'[0]==4}},y[x],x,IncludeSingularSol
```

$$y(x) \to 4x(x+1)$$

10.10 problem 15.3

Internal problem ID [13569]

Book: Ordinary Differential Equations. An introduction to the fundamentals. Kenneth B. Howell. second edition. CRC Press. FL, USA. 2020
Section: Chapter 15. General solutions to Homogeneous linear differential equations. Additional exercises page 294
Problem number: 15.3.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_Emden, _Fowler], [_2nd_order, _linear, '_with_symmetry_[0,F`

$$\boxed{x^2y'' - 4y'x + 6y = 0}$$

With initial conditions

$$[y(0) = 0, y'(0) = -4]$$

✗ Solution by Maple

```
dsolve([x^2*diff(y(x),x$2)-4*x*diff(y(x),x)+6*y(x)=0,y(0) = 0, D(y)(0) = -4],y(x), singsol=a
```

No solution found

✗ Solution by Mathematica
Time used: 0.0 (sec). Leaf size: 0

```
DSolve[{x^2*y''[x]-4*x*y'[x]+6*y[x]==0,{y[0]==0,y'[0]==-4}},y[x],x,IncludeSingularSolutions
```

{}

10.11 problem 15.4

Internal problem ID [13570]

Book: Ordinary Differential Equations. An introduction to the fundamentals. Kenneth B. Howell. second edition. CRC Press. FL, USA. 2020
Section: Chapter 15. General solutions to Homogeneous linear differential equations. Additional exercises page 294
Problem number: 15.4.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_Emden, _Fowler], [_2nd_order, _linear, '_with_symmetry_[0,F`

$$\boxed{xy'' - y' + 4yx^3 = 0}$$

With initial conditions

$$[y(0) = 1, y'(0) = 4]$$

✗ Solution by Maple

```
dsolve([x*diff(y(x),x$2)-diff(y(x),x)+4*x^3*y(x)=0,y(0) = 1, D(y)(0) = 4],y(x), singsol=all)
```

No solution found

✗ Solution by Mathematica
Time used: 0.0 (sec). Leaf size: 0

```
DSolve[{x*y''[x]-y'[x]+4*x^3*y[x]==0,{y[0]==1,y'[0]==4}},y[x],x,IncludeSingularSolutions ->
```

{}

10.12 problem 15.5 (a)

Internal problem ID [13571]

Book: Ordinary Differential Equations. An introduction to the fundamentals. Kenneth B. Howell. second edition. CRC Press. FL, USA. 2020
Section: Chapter 15. General solutions to Homogeneous linear differential equations. Additional exercises page 294
Problem number: 15.5 (a).
ODE order: 3.
ODE degree: 1.

CAS Maple gives this as type `[[_3rd_order, _missing_x]]`

$$\boxed{y''' + 4y' = 0}$$

With initial conditions

$$[y(0) = 3, y'(0) = 8, y''(0) = 4]$$

✓ Solution by Maple
Time used: 0.0 (sec). Leaf size: 18

```
dsolve([diff(y(x),x$3)+4*diff(y(x),x)=0,y(0) = 3, D(y)(0) = 8, (D@@2)(y)(0) = 4],y(x), sings
```

$$y(x) = 4 + 4\sin(2x) - \cos(2x)$$

✓ Solution by Mathematica
Time used: 0.111 (sec). Leaf size: 19

```
DSolve[{y'''[x]+4*y'[x]==0,{y[0]==3,y'[0]==8,y''[0]==4}},y[x],x,IncludeSingularSolutions ->
```

$$y(x) \to 4\sin(2x) - \cos(2x) + 4$$

10.13 problem 15.5 (c)

Internal problem ID [13572]

Book: Ordinary Differential Equations. An introduction to the fundamentals. Kenneth B. Howell. second edition. CRC Press. FL, USA. 2020
Section: Chapter 15. General solutions to Homogeneous linear differential equations. Additional exercises page 294
Problem number: 15.5 (c).
ODE order: 4.
ODE degree: 1.

CAS Maple gives this as type `[[_high_order, _missing_x]]`

$$\boxed{y'''' - y = 0}$$

With initial conditions

$$[y(0) = 0, y'(0) = 4, y''(0) = 0, y'''(0) = 0]$$

✓ Solution by Maple
Time used: 0.031 (sec). Leaf size: 17

```
dsolve([diff(y(x),x$4)-y(x)=0,y(0) = 0, D(y)(0) = 4, (D@@2)(y)(0) = 0, (D@@3)(y)(0) = 0],y(x
```

$$y(x) = -\mathrm{e}^{-x} + \mathrm{e}^{x} + 2\sin(x)$$

✓ Solution by Mathematica
Time used: 0.004 (sec). Leaf size: 20

```
DSolve[{y''''[x]-y[x]==0,{y[0]==0,y'[0]==4,y''[0]==0,y'''[0]==0}},y[x],x,IncludeSingularSolu
```

$$y(x) \to -e^{-x} + e^{x} + 2\sin(x)$$

10.14 problem 15.6 (a)

Internal problem ID [13573]

Book: Ordinary Differential Equations. An introduction to the fundamentals. Kenneth B. Howell. second edition. CRC Press. FL, USA. 2020
Section: Chapter 15. General solutions to Homogeneous linear differential equations. Additional exercises page 294
Problem number: 15.6 (a).
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _missing_x]]`

$$\boxed{y'' - 4y = 0}$$

With initial conditions

$$[y(0) = 1, y'(0) = 0]$$

✓ Solution by Maple
Time used: 0.0 (sec). Leaf size: 17

```
dsolve([diff(y(x),x$2)-4*y(x)=0,y(0) = 1, D(y)(0) = 0],y(x), singsol=all)
```

$$y(x) = \frac{e^{2x}}{2} + \frac{e^{-2x}}{2}$$

✓ Solution by Mathematica
Time used: 0.012 (sec). Leaf size: 21

```
DSolve[{y''[x]-4*y[x]==0,{y[0]==1,y'[0]==0}},y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to \frac{1}{2}e^{-2x}\left(e^{4x} + 1\right)$$

10.15 problem 15.6 (b)

Internal problem ID [13574]

Book: Ordinary Differential Equations. An introduction to the fundamentals. Kenneth B. Howell. second edition. CRC Press. FL, USA. 2020
Section: Chapter 15. General solutions to Homogeneous linear differential equations. Additional exercises page 294
Problem number: 15.6 (b).
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _missing_x]]`

$$\boxed{y'' + 2y' - 3y = 0}$$

With initial conditions

$$[y(0) = 0, y'(0) = 1]$$

✓ Solution by Maple
Time used: 0.015 (sec). Leaf size: 16

```
dsolve([diff(y(x),x$2)+2*diff(y(x),x)-3*y(x)=0,y(0) = 0, D(y)(0) = 1],y(x), singsol=all)
```

$$y(x) = \frac{(e^{4x} - 1)\, e^{-3x}}{4}$$

✓ Solution by Mathematica
Time used: 0.012 (sec). Leaf size: 21

```
DSolve[{y''[x]+2*y'[x]-3*y[x]==0,{y[0]==0,y'[0]==1}},y[x],x,IncludeSingularSolutions -> True
```

$$y(x) \to \frac{1}{4} e^{-3x} \left(e^{4x} - 1\right)$$

10.16 problem 15.6 (c)

Internal problem ID [13575]

Book: Ordinary Differential Equations. An introduction to the fundamentals. Kenneth B. Howell. second edition. CRC Press. FL, USA. 2020
Section: Chapter 15. General solutions to Homogeneous linear differential equations. Additional exercises page 294
Problem number: 15.6 (c).
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _missing_x]]`

$$\boxed{y'' - 10y' + 9y = 0}$$

With initial conditions

$$[y(0) = 8, y'(0) = -24]$$

✓ Solution by Maple
Time used: 0.015 (sec). Leaf size: 15

```
dsolve([diff(y(x),x$2)-10*diff(y(x),x)+9*y(x)=0,y(0) = 8, D(y)(0) = -24],y(x), singsol=all)
```

$$y(x) = 12\,e^{x} - 4\,e^{9x}$$

✓ Solution by Mathematica
Time used: 0.012 (sec). Leaf size: 17

```
DSolve[{y''[x]-10*y'[x]+9*y[x]==0,{y[0]==8,y'[0]==-24}},y[x],x,IncludeSingularSolutions -> T
```

$$y(x) \to -4e^{x}\left(e^{8x} - 3\right)$$

10.17 problem 15.6 (d)

Internal problem ID [13576]

Book: Ordinary Differential Equations. An introduction to the fundamentals. Kenneth B. Howell. second edition. CRC Press. FL, USA. 2020
Section: Chapter 15. General solutions to Homogeneous linear differential equations. Additional exercises page 294
Problem number: 15.6 (d).
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _missing_x]]`

$$\boxed{y'' + 5y' = 0}$$

With initial conditions

$$[y(0) = 1, y'(0) = 0]$$

✓ Solution by Maple
Time used: 0.0 (sec). Leaf size: 5

```
dsolve([diff(y(x),x$2)+5*diff(y(x),x)=0,y(0) = 1, D(y)(0) = 0],y(x), singsol=all)
```

$$y(x) = 1$$

✓ Solution by Mathematica
Time used: 0.026 (sec). Leaf size: 6

```
DSolve[{y''[x]+5*y'[x]==0,{y[0]==1,y'[0]==0}},y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to 1$$

10.18 problem 15.7 (a)

Internal problem ID [13577]

Book: Ordinary Differential Equations. An introduction to the fundamentals. Kenneth B. Howell. second edition. CRC Press. FL, USA. 2020
Section: Chapter 15. General solutions to Homogeneous linear differential equations. Additional exercises page 294
Problem number: 15.7 (a).
ODE order: 3.
ODE degree: 1.

CAS Maple gives this as type `[[_3rd_order, _missing_x]]`

$$\boxed{y''' - 9y' = 0}$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 18

```
dsolve(diff(y(x),x$3)-9*diff(y(x),x)=0,y(x), singsol=all)
```

$$y(x) = c_1 + e^{-3x}c_2 + c_3e^{3x}$$

✓ Solution by Mathematica

Time used: 0.027 (sec). Leaf size: 30

```
DSolve[y'''[x]-9*y'[x]==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to \frac{1}{3}c_1e^{3x} - \frac{1}{3}c_2e^{-3x} + c_3$$

10.19 problem 15.7 (b)

Internal problem ID [13578]

Book: Ordinary Differential Equations. An introduction to the fundamentals. Kenneth B. Howell. second edition. CRC Press. FL, USA. 2020
Section: Chapter 15. General solutions to Homogeneous linear differential equations. Additional exercises page 294
Problem number: 15.7 (b).
ODE order: 4.
ODE degree: 1.

CAS Maple gives this as type `[[_high_order, _missing_x]]`

$$\boxed{y'''' - 10y'' + 9y = 0}$$

✓ Solution by Maple
Time used: 0.0 (sec). Leaf size: 29

```
dsolve(diff(y(x),x$4)-10*diff(y(x),x$2)+9*y(x)=0,y(x), singsol=all)
```

$$y(x) = \left(\mathrm{e}^{6x} c_2 + c_4 \mathrm{e}^{4x} + c_3 \mathrm{e}^{2x} + c_1\right) \mathrm{e}^{-3x}$$

✓ Solution by Mathematica
Time used: 0.003 (sec). Leaf size: 36

```
DSolve[y''''[x]-10*y''[x]+9*y[x]==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to c_1 e^{-3x} + c_2 e^{-x} + c_3 e^{x} + c_4 e^{3x}$$

11 Chapter 17. Second order Homogeneous equations with constant coefficients. Additional exercises page 334

11.1 problem 17.1 (a)

Internal problem ID [13579]

Book: Ordinary Differential Equations. An introduction to the fundamentals. Kenneth B. Howell. second edition. CRC Press. FL, USA. 2020
Section: Chapter 17. Second order Homogeneous equations with constant coefficients. Additional exercises page 334
Problem number: 17.1 (a).
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _missing_x]]`

$$\boxed{y'' - 7y' + 10y = 0}$$

✓ Solution by Maple
Time used: 0.0 (sec). Leaf size: 17

```
dsolve(diff(y(x),x$2)-7*diff(y(x),x)+10*y(x)=0,y(x), singsol=all)
```

$$y(x) = c_1 e^{5x} + c_2 e^{2x}$$

✓ Solution by Mathematica
Time used: 0.013 (sec). Leaf size: 22

```
DSolve[y''[x]-7*y'[x]+10*y[x]==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to e^{2x}(c_2 e^{3x} + c_1)$$

11.2 problem 17.1 (b)

Internal problem ID [13580]

Book: Ordinary Differential Equations. An introduction to the fundamentals. Kenneth B. Howell. second edition. CRC Press. FL, USA. 2020
Section: Chapter 17. Second order Homogeneous equations with constant coefficients. Additional exercises page 334
Problem number: 17.1 (b).
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _missing_x]]`

$$\boxed{y'' + 2y' - 24y = 0}$$

✓ Solution by Maple
Time used: 0.0 (sec). Leaf size: 17

```
dsolve(diff(y(x),x$2)+2*diff(y(x),x)-24*y(x)=0,y(x), singsol=all)
```

$$y(x) = \left(\mathrm{e}^{10x} c_1 + c_2\right) \mathrm{e}^{-6x}$$

✓ Solution by Mathematica
Time used: 0.013 (sec). Leaf size: 22

```
DSolve[y''[x]+2*y'[x]-24*y[x]==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to c_1 e^{-6x} + c_2 e^{4x}$$

11.3 problem 17.1 (c)

Internal problem ID [13581]

Book: Ordinary Differential Equations. An introduction to the fundamentals. Kenneth B. Howell. second edition. CRC Press. FL, USA. 2020
Section: Chapter 17. Second order Homogeneous equations with constant coefficients. Additional exercises page 334
Problem number: 17.1 (c).
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _missing_x]]`

$$\boxed{y'' - 25y = 0}$$

✓ Solution by Maple
Time used: 0.016 (sec). Leaf size: 17

```
dsolve(diff(y(x),x$2)-25*y(x)=0,y(x), singsol=all)
```

$$y(x) = c_1 e^{5x} + c_2 e^{-5x}$$

✓ Solution by Mathematica
Time used: 0.012 (sec). Leaf size: 22

```
DSolve[y''[x]-25*y[x]==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to c_1 e^{5x} + c_2 e^{-5x}$$

11.4 problem 17.1 (d)

Internal problem ID [13582]

Book: Ordinary Differential Equations. An introduction to the fundamentals. Kenneth B. Howell. second edition. CRC Press. FL, USA. 2020
Section: Chapter 17. Second order Homogeneous equations with constant coefficients. Additional exercises page 334
Problem number: 17.1 (d).
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _missing_x]]`

$$\boxed{y'' + 3y' = 0}$$

✓ Solution by Maple
Time used: 0.0 (sec). Leaf size: 12

```
dsolve(diff(y(x),x$2)+3*diff(y(x),x)=0,y(x), singsol=all)
```

$$y(x) = c_1 + e^{-3x} c_2$$

✓ Solution by Mathematica
Time used: 0.014 (sec). Leaf size: 19

```
DSolve[y''[x]+3*y'[x]==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to c_2 - \frac{1}{3} c_1 e^{-3x}$$

11.5 problem 17.1 (e)

Internal problem ID [13583]

Book: Ordinary Differential Equations. An introduction to the fundamentals. Kenneth B. Howell. second edition. CRC Press. FL, USA. 2020
Section: Chapter 17. Second order Homogeneous equations with constant coefficients. Additional exercises page 334
Problem number: 17.1 (e).
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _missing_x]]`

$$\boxed{4y'' - y = 0}$$

✓ Solution by Maple
Time used: 0.0 (sec). Leaf size: 17

```
dsolve(4*diff(y(x),x$2)-y(x)=0,y(x), singsol=all)
```

$$y(x) = c_1 \mathrm{e}^{-\frac{x}{2}} + c_2 \mathrm{e}^{\frac{x}{2}}$$

✓ Solution by Mathematica
Time used: 0.013 (sec). Leaf size: 22

```
DSolve[4*y''[x]-y[x]==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to e^{-x/2}(c_1 e^x + c_2)$$

11.6 problem 17.1 (f)

Internal problem ID [13584]

Book: Ordinary Differential Equations. An introduction to the fundamentals. Kenneth B. Howell. second edition. CRC Press. FL, USA. 2020
Section: Chapter 17. Second order Homogeneous equations with constant coefficients. Additional exercises page 334
Problem number: 17.1 (f).
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _missing_x]]`

$$\boxed{3y'' + 7y' - 6y = 0}$$

✓ Solution by Maple
Time used: 0.0 (sec). Leaf size: 17

```
dsolve(3*diff(y(x),x$2)+7*diff(y(x),x)-6*y(x)=0,y(x), singsol=all)
```

$$y(x) = \left(c_2 e^{\frac{11x}{3}} + c_1\right) e^{-3x}$$

✓ Solution by Mathematica
Time used: 0.013 (sec). Leaf size: 24

```
DSolve[3*y''[x]+7*y'[x]-6*y[x]==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to c_1 e^{2x/3} + c_2 e^{-3x}$$

11.7 problem 17.2 (a)

Internal problem ID [13585]

Book: Ordinary Differential Equations. An introduction to the fundamentals. Kenneth B. Howell. second edition. CRC Press. FL, USA. 2020
Section: Chapter 17. Second order Homogeneous equations with constant coefficients. Additional exercises page 334
Problem number: 17.2 (a).
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _missing_x]]`

$$\boxed{y'' - 8y' + 15y = 0}$$

With initial conditions

$$[y(0) = 1, y'(0) = 0]$$

✓ Solution by Maple
Time used: 0.015 (sec). Leaf size: 17

```
dsolve([diff(y(x),x$2)-8*diff(y(x),x)+15*y(x)=0,y(0) = 1, D(y)(0) = 0],y(x), singsol=all)
```

$$y(x) = -\frac{3\,\mathrm{e}^{5x}}{2} + \frac{5\,\mathrm{e}^{3x}}{2}$$

✓ Solution by Mathematica
Time used: 0.013 (sec). Leaf size: 23

```
DSolve[{y''[x]-8*y'[x]+15*y[x]==0,{y[0]==1,y'[0]==0}},y[x],x,IncludeSingularSolutions -> Tru
```

$$y(x) \to \frac{1}{2}e^{3x}\left(5 - 3e^{2x}\right)$$

11.8 problem 17.2 (b)

Internal problem ID [13586]

Book: Ordinary Differential Equations. An introduction to the fundamentals. Kenneth B. Howell. second edition. CRC Press. FL, USA. 2020
Section: Chapter 17. Second order Homogeneous equations with constant coefficients. Additional exercises page 334
Problem number: 17.2 (b).
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _missing_x]]`

$$\boxed{y'' - 8y' + 15y = 0}$$

With initial conditions

$$[y(0) = 0, y'(0) = 1]$$

✓ Solution by Maple
Time used: 0.016 (sec). Leaf size: 17

```
dsolve([diff(y(x),x$2)-8*diff(y(x),x)+15*y(x)=0,y(0) = 0, D(y)(0) = 1],y(x), singsol=all)
```

$$y(x) = \frac{e^{5x}}{2} - \frac{e^{3x}}{2}$$

✓ Solution by Mathematica
Time used: 0.013 (sec). Leaf size: 21

```
DSolve[{y''[x]-8*y'[x]+15*y[x]==0,{y[0]==0,y'[0]==1}},y[x],x,IncludeSingularSolutions -> Tru
```

$$y(x) \to \frac{1}{2}e^{3x}\left(e^{2x} - 1\right)$$

11.9 problem 17.2 (c)

Internal problem ID [13587]

Book: Ordinary Differential Equations. An introduction to the fundamentals. Kenneth B. Howell. second edition. CRC Press. FL, USA. 2020
Section: Chapter 17. Second order Homogeneous equations with constant coefficients. Additional exercises page 334
Problem number: 17.2 (c).
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _missing_x]]`

$$\boxed{y'' - 8y' + 15y = 0}$$

With initial conditions

$$[y(0) = 5, y'(0) = 19]$$

✓ Solution by Maple
Time used: 0.0 (sec). Leaf size: 17

```
dsolve([diff(y(x),x$2)-8*diff(y(x),x)+15*y(x)=0,y(0) = 5, D(y)(0) = 19],y(x), singsol=all)
```

$$y(x) = 2\,e^{5x} + 3\,e^{3x}$$

✓ Solution by Mathematica
Time used: 0.014 (sec). Leaf size: 20

```
DSolve[{y''[x]-8*y'[x]+15*y[x]==0,{y[0]==5,y'[0]==19}},y[x],x,IncludeSingularSolutions -> Tr
```

$$y(x) \to e^{3x}(2e^{2x} + 3)$$

11.10 problem 17.2 (d)

Internal problem ID [13588]

Book: Ordinary Differential Equations. An introduction to the fundamentals. Kenneth B. Howell. second edition. CRC Press. FL, USA. 2020
Section: Chapter 17. Second order Homogeneous equations with constant coefficients. Additional exercises page 334
Problem number: 17.2 (d).
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _missing_x]]`

$$\boxed{y'' - 9y = 0}$$

With initial conditions

$$[y(0) = 1, y'(0) = 0]$$

✓ Solution by Maple
Time used: 0.015 (sec). Leaf size: 17

```
dsolve([diff(y(x),x$2)-9*y(x)=0,y(0) = 1, D(y)(0) = 0],y(x), singsol=all)
```

$$y(x) = \frac{e^{-3x}}{2} + \frac{e^{3x}}{2}$$

✓ Solution by Mathematica
Time used: 0.012 (sec). Leaf size: 21

```
DSolve[{y''[x]-9*y[x]==0,{y[0]==1,y'[0]==0}},y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to \frac{1}{2}e^{-3x}\left(e^{6x} + 1\right)$$

11.11 problem 17.2 (e)

Internal problem ID [13589]

Book: Ordinary Differential Equations. An introduction to the fundamentals. Kenneth B. Howell. second edition. CRC Press. FL, USA. 2020
Section: Chapter 17. Second order Homogeneous equations with constant coefficients. Additional exercises page 334
Problem number: 17.2 (e).
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _missing_x]]`

$$\boxed{y'' - 9y = 0}$$

With initial conditions

$$[y(0) = 0, y'(0) = 1]$$

✓ Solution by Maple
Time used: 0.016 (sec). Leaf size: 17

```
dsolve([diff(y(x),x$2)-9*y(x)=0,y(0) = 0, D(y)(0) = 1],y(x), singsol=all)
```

$$y(x) = -\frac{e^{-3x}}{6} + \frac{e^{3x}}{6}$$

✓ Solution by Mathematica
Time used: 0.012 (sec). Leaf size: 21

```
DSolve[{y''[x]-9*y[x]==0,{y[0]==0,y'[0]==1}},y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to \frac{1}{6}e^{-3x}\left(e^{6x} - 1\right)$$

11.12 problem 17.2 (f)

Internal problem ID [13590]

Book: Ordinary Differential Equations. An introduction to the fundamentals. Kenneth B. Howell. second edition. CRC Press. FL, USA. 2020
Section: Chapter 17. Second order Homogeneous equations with constant coefficients. Additional exercises page 334
Problem number: 17.2 (f).
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _missing_x]]`

$$\boxed{y'' - 9y = 0}$$

With initial conditions

$$[y(0) = 3, y'(0) = -3]$$

✓ Solution by Maple
Time used: 0.015 (sec). Leaf size: 15

```
dsolve([diff(y(x),x$2)-9*y(x)=0,y(0) = 3, D(y)(0) = -3],y(x), singsol=all)
```

$$y(x) = 2\,\mathrm{e}^{-3x} + \mathrm{e}^{3x}$$

✓ Solution by Mathematica
Time used: 0.012 (sec). Leaf size: 18

```
DSolve[{y''[x]-9*y[x]==0,{y[0]==3,y'[0]==-3}},y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to e^{-3x}\left(e^{6x} + 2\right)$$

11.13 problem 17.3 (a)

Internal problem ID [13591]

Book: Ordinary Differential Equations. An introduction to the fundamentals. Kenneth B. Howell. second edition. CRC Press. FL, USA. 2020
Section: Chapter 17. Second order Homogeneous equations with constant coefficients. Additional exercises page 334
Problem number: 17.3 (a).
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _missing_x]]`

$$\boxed{y'' - 10y' + 25y = 0}$$

✓ Solution by Maple
Time used: 0.0 (sec). Leaf size: 14

```
dsolve(diff(y(x),x$2)-10*diff(y(x),x)+25*y(x)=0,y(x), singsol=all)
```

$$y(x) = e^{5x}(c_2x + c_1)$$

✓ Solution by Mathematica
Time used: 0.013 (sec). Leaf size: 18

```
DSolve[y''[x]-10*y'[x]+25*y[x]==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to e^{5x}(c_2x + c_1)$$

11.14 problem 17.3 (b)

Internal problem ID [13592]

Book: Ordinary Differential Equations. An introduction to the fundamentals. Kenneth B. Howell. second edition. CRC Press. FL, USA. 2020
Section: Chapter 17. Second order Homogeneous equations with constant coefficients. Additional exercises page 334
Problem number: 17.3 (b).
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _missing_x]]`

$$\boxed{y'' + 2y' + y = 0}$$

✓ Solution by Maple
Time used: 0.0 (sec). Leaf size: 14

```
dsolve(diff(y(x),x$2)+2*diff(y(x),x)+y(x)=0,y(x), singsol=all)
```

$$y(x) = \mathrm{e}^{-x}(c_2x + c_1)$$

✓ Solution by Mathematica
Time used: 0.014 (sec). Leaf size: 18

```
DSolve[y''[x]+2*y'[x]+y[x]==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to e^{-x}(c_2x + c_1)$$

11.15 problem 17.3 (c)

Internal problem ID [13593]

Book: Ordinary Differential Equations. An introduction to the fundamentals. Kenneth B. Howell. second edition. CRC Press. FL, USA. 2020
Section: Chapter 17. Second order Homogeneous equations with constant coefficients. Additional exercises page 334
Problem number: 17.3 (c).
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _missing_x]]`

$$\boxed{4y'' - 4y' + y = 0}$$

✓ Solution by Maple
Time used: 0.0 (sec). Leaf size: 14

```
dsolve(4*diff(y(x),x$2)-4*diff(y(x),x)+y(x)=0,y(x), singsol=all)
```

$$y(x) = \mathrm{e}^{\frac{x}{2}}(c_2 x + c_1)$$

✓ Solution by Mathematica
Time used: 0.013 (sec). Leaf size: 20

```
DSolve[4*y''[x]-4*y'[x]+y[x]==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \rightarrow e^{x/2}(c_2 x + c_1)$$

11.16 problem 17.3 (d)

Internal problem ID [13594]

Book: Ordinary Differential Equations. An introduction to the fundamentals. Kenneth B. Howell. second edition. CRC Press. FL, USA. 2020
Section: Chapter 17. Second order Homogeneous equations with constant coefficients. Additional exercises page 334
Problem number: 17.3 (d).
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _missing_x]]`

$$\boxed{25y'' - 10y' + y = 0}$$

✓ Solution by Maple
Time used: 0.0 (sec). Leaf size: 14

```
dsolve(25*diff(y(x),x$2)-10*diff(y(x),x)+y(x)=0,y(x), singsol=all)
```

$$y(x) = e^{\frac{x}{5}}(c_2x + c_1)$$

✓ Solution by Mathematica
Time used: 0.014 (sec). Leaf size: 20

```
DSolve[25*y''[x]-10*y'[x]+y[x]==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to e^{x/5}(c_2x + c_1)$$

11.17 problem 17.3 (e)

Internal problem ID [13595]

Book: Ordinary Differential Equations. An introduction to the fundamentals. Kenneth B. Howell. second edition. CRC Press. FL, USA. 2020
Section: Chapter 17. Second order Homogeneous equations with constant coefficients. Additional exercises page 334
Problem number: 17.3 (e).
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _missing_x]]`

$$\boxed{16y'' - 24y' + 9y = 0}$$

✓ Solution by Maple
Time used: 0.0 (sec). Leaf size: 14

```
dsolve(16*diff(y(x),x$2)-24*diff(y(x),x)+9*y(x)=0,y(x), singsol=all)
```

$$y(x) = \mathrm{e}^{\frac{3x}{4}}(c_2 x + c_1)$$

✓ Solution by Mathematica
Time used: 0.013 (sec). Leaf size: 20

```
DSolve[16*y''[x]-24*y'[x]+9*y[x]==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to e^{3x/4}(c_2 x + c_1)$$

11.18 problem 17.3 (f)

Internal problem ID [13596]

Book: Ordinary Differential Equations. An introduction to the fundamentals. Kenneth B. Howell. second edition. CRC Press. FL, USA. 2020
Section: Chapter 17. Second order Homogeneous equations with constant coefficients. Additional exercises page 334
Problem number: 17.3 (f).
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _missing_x]]`

$$\boxed{9y'' + 12y' + 4y = 0}$$

✓ Solution by Maple
Time used: 0.0 (sec). Leaf size: 14

```
dsolve(9*diff(y(x),x$2)+12*diff(y(x),x)+4*y(x)=0,y(x), singsol=all)
```

$$y(x) = \mathrm{e}^{-\frac{2x}{3}}(c_2x + c_1)$$

✓ Solution by Mathematica
Time used: 0.014 (sec). Leaf size: 20

```
DSolve[9*y''[x]+12*y'[x]+4*y[x]==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to e^{-2x/3}(c_2x + c_1)$$

11.19 problem 17.4 (a)

Internal problem ID [13597]

Book: Ordinary Differential Equations. An introduction to the fundamentals. Kenneth B. Howell. second edition. CRC Press. FL, USA. 2020
Section: Chapter 17. Second order Homogeneous equations with constant coefficients. Additional exercises page 334
Problem number: 17.4 (a).
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _missing_x]]`

$$\boxed{y'' - 8y' + 16y = 0}$$

With initial conditions

$$[y(0) = 1, y'(0) = 0]$$

✓ Solution by Maple
Time used: 0.0 (sec). Leaf size: 14

```
dsolve([diff(y(x),x$2)-8*diff(y(x),x)+16*y(x)=0,y(0) = 1, D(y)(0) = 0],y(x), singsol=all)
```

$$y(x) = e^{4x}(-4x + 1)$$

✓ Solution by Mathematica
Time used: 0.014 (sec). Leaf size: 16

```
DSolve[{y''[x]-8*y'[x]+16*y[x]==0,{y[0]==1,y'[0]==0}},y[x],x,IncludeSingularSolutions -> Tru
```

$$y(x) \to e^{4x}(1 - 4x)$$

11.20 problem 17.4 (b)

Internal problem ID [13598]

Book: Ordinary Differential Equations. An introduction to the fundamentals. Kenneth B. Howell. second edition. CRC Press. FL, USA. 2020
Section: Chapter 17. Second order Homogeneous equations with constant coefficients. Additional exercises page 334
Problem number: 17.4 (b).
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _missing_x]]`

$$\boxed{y'' - 8y' + 16y = 0}$$

With initial conditions

$$[y(0) = 0, y'(0) = 1]$$

✓ Solution by Maple
Time used: 0.015 (sec). Leaf size: 10

```
dsolve([diff(y(x),x$2)-8*diff(y(x),x)+16*y(x)=0,y(0) = 0, D(y)(0) = 1],y(x), singsol=all)
```

$$y(x) = e^{4x}x$$

✓ Solution by Mathematica
Time used: 0.013 (sec). Leaf size: 12

```
DSolve[{y''[x]-8*y'[x]+16*y[x]==0,{y[0]==0,y'[0]==1}},y[x],x,IncludeSingularSolutions -> Tru
```

$$y(x) \to e^{4x}x$$

11.21 problem 17.4 (c)

Internal problem ID [13599]

Book: Ordinary Differential Equations. An introduction to the fundamentals. Kenneth B. Howell. second edition. CRC Press. FL, USA. 2020
Section: Chapter 17. Second order Homogeneous equations with constant coefficients. Additional exercises page 334
Problem number: 17.4 (c).
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _missing_x]]`

$$\boxed{y'' - 8y' + 16y = 0}$$

With initial conditions

$$[y(0) = 3, y'(0) = 14]$$

✓ Solution by Maple
Time used: 0.016 (sec). Leaf size: 14

```
dsolve([diff(y(x),x$2)-8*diff(y(x),x)+16*y(x)=0,y(0) = 3, D(y)(0) = 14],y(x), singsol=all)
```

$$y(x) = e^{4x}(3 + 2x)$$

✓ Solution by Mathematica
Time used: 0.014 (sec). Leaf size: 16

```
DSolve[{y''[x]-8*y'[x]+16*y[x]==0,{y[0]==3,y'[0]==14}},y[x],x,IncludeSingularSolutions -> Tr
```

$$y(x) \to e^{4x}(2x + 3)$$

11.22 problem 17.4 (d)

Internal problem ID [13600]

Book: Ordinary Differential Equations. An introduction to the fundamentals. Kenneth B. Howell. second edition. CRC Press. FL, USA. 2020
Section: Chapter 17. Second order Homogeneous equations with constant coefficients. Additional exercises page 334
Problem number: 17.4 (d).
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _missing_x]]`

$$\boxed{4y'' + 4y' + y = 0}$$

With initial conditions

$$[y(0) = 1, y'(0) = 0]$$

✓ Solution by Maple
Time used: 0.0 (sec). Leaf size: 14

```
dsolve([4*diff(y(x),x$2)+4*diff(y(x),x)+y(x)=0,y(0) = 1, D(y)(0) = 0],y(x), singsol=all)
```

$$y(x) = \frac{e^{-\frac{x}{2}}(x+2)}{2}$$

✓ Solution by Mathematica
Time used: 0.014 (sec). Leaf size: 19

```
DSolve[{4*y''[x]+4*y'[x]+y[x]==0,{y[0]==1,y'[0]==0}},y[x],x,IncludeSingularSolutions -> True
```

$$y(x) \to \frac{1}{2}e^{-x/2}(x+2)$$

11.23 problem 17.4 (e)

Internal problem ID [13601]

Book: Ordinary Differential Equations. An introduction to the fundamentals. Kenneth B. Howell. second edition. CRC Press. FL, USA. 2020
Section: Chapter 17. Second order Homogeneous equations with constant coefficients. Additional exercises page 334
Problem number: 17.4 (e).
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _missing_x]]`

$$\boxed{4y'' + 4y' + y = 0}$$

With initial conditions

$$[y(0) = 0, y'(0) = 1]$$

✓ Solution by Maple
Time used: 0.015 (sec). Leaf size: 10

```
dsolve([4*diff(y(x),x$2)+4*diff(y(x),x)+y(x)=0,y(0) = 0, D(y)(0) = 1],y(x), singsol=all)
```

$$y(x) = e^{-\frac{x}{2}} x$$

✓ Solution by Mathematica
Time used: 0.014 (sec). Leaf size: 14

```
DSolve[{4*y''[x]+4*y'[x]+y[x]==0,{y[0]==0,y'[0]==1}},y[x],x,IncludeSingularSolutions -> True
```

$$y(x) \to e^{-x/2} x$$

11.24 problem 17.4 (f)

Internal problem ID [13602]

Book: Ordinary Differential Equations. An introduction to the fundamentals. Kenneth B. Howell. second edition. CRC Press. FL, USA. 2020
Section: Chapter 17. Second order Homogeneous equations with constant coefficients. Additional exercises page 334
Problem number: 17.4 (f).
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _missing_x]]`

$$\boxed{4y'' + 4y' + y = 0}$$

With initial conditions

$$[y(0) = 6, y'(0) = -5]$$

✓ Solution by Maple
Time used: 0.016 (sec). Leaf size: 13

```
dsolve([4*diff(y(x),x$2)+4*diff(y(x),x)+y(x)=0,y(0) = 6, D(y)(0) = -5],y(x), singsol=all)
```

$$y(x) = -2\,e^{-\frac{x}{2}}(-3 + x)$$

✓ Solution by Mathematica
Time used: 0.014 (sec). Leaf size: 17

```
DSolve[{4*y''[x]+4*y'[x]+y[x]==0,{y[0]==6,y'[0]==-5}},y[x],x,IncludeSingularSolutions -> Tru
```

$$y(x) \to -2e^{-x/2}(x - 3)$$

11.25 problem 17.5 (a)

Internal problem ID [13603]

Book: Ordinary Differential Equations. An introduction to the fundamentals. Kenneth B. Howell. second edition. CRC Press. FL, USA. 2020
Section: Chapter 17. Second order Homogeneous equations with constant coefficients. Additional exercises page 334
Problem number: 17.5 (a).
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _missing_x]]`

$$\boxed{y'' + 25y = 0}$$

✓ Solution by Maple
Time used: 0.016 (sec). Leaf size: 17

```
dsolve(diff(y(x),x$2)+25*y(x)=0,y(x), singsol=all)
```

$$y(x) = c_1 \sin(5x) + c_2 \cos(5x)$$

✓ Solution by Mathematica
Time used: 0.013 (sec). Leaf size: 20

```
DSolve[y''[x]+25*y[x]==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to c_1 \cos(5x) + c_2 \sin(5x)$$

11.26 problem 17.5 (b)

Internal problem ID [13604]

Book: Ordinary Differential Equations. An introduction to the fundamentals. Kenneth B. Howell. second edition. CRC Press. FL, USA. 2020
Section: Chapter 17. Second order Homogeneous equations with constant coefficients. Additional exercises page 334
Problem number: 17.5 (b).
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _missing_x]]`

$$\boxed{y'' + 2y' + 5y = 0}$$

✓ Solution by Maple
Time used: 0.0 (sec). Leaf size: 22

```
dsolve(diff(y(x),x$2)+2*diff(y(x),x)+5*y(x)=0,y(x), singsol=all)
```

$$y(x) = e^{-x}(c_1 \sin(2x) + c_2 \cos(2x))$$

✓ Solution by Mathematica
Time used: 0.016 (sec). Leaf size: 26

```
DSolve[y''[x]+2*y'[x]+5*y[x]==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to e^{-x}(c_2 \cos(2x) + c_1 \sin(2x))$$

11.27 problem 17.5 (c)

Internal problem ID [13605]

Book: Ordinary Differential Equations. An introduction to the fundamentals. Kenneth B. Howell. second edition. CRC Press. FL, USA. 2020
Section: Chapter 17. Second order Homogeneous equations with constant coefficients. Additional exercises page 334
Problem number: 17.5 (c).
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _missing_x]]`

$$\boxed{y'' - 2y' + 5y = 0}$$

✓ Solution by Maple
Time used: 0.0 (sec). Leaf size: 20

```
dsolve(diff(y(x),x$2)-2*diff(y(x),x)+5*y(x)=0,y(x), singsol=all)
```

$$y(x) = e^{x}(c_1 \sin(2x) + c_2 \cos(2x))$$

✓ Solution by Mathematica
Time used: 0.014 (sec). Leaf size: 24

```
DSolve[y''[x]-2*y'[x]+5*y[x]==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \rightarrow e^{x}(c_2 \cos(2x) + c_1 \sin(2x))$$

11.28 problem 17.5 (d)

Internal problem ID [13606]

Book: Ordinary Differential Equations. An introduction to the fundamentals. Kenneth B. Howell. second edition. CRC Press. FL, USA. 2020
Section: Chapter 17. Second order Homogeneous equations with constant coefficients. Additional exercises page 334
Problem number: 17.5 (d).
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _missing_x]]`

$$\boxed{y'' - 4y' + 29y = 0}$$

✓ Solution by Maple
Time used: 0.0 (sec). Leaf size: 22

```
dsolve(diff(y(x),x$2)-4*diff(y(x),x)+29*y(x)=0,y(x), singsol=all)
```

$$y(x) = e^{2x}(c_1 \sin(5x) + c_2 \cos(5x))$$

✓ Solution by Mathematica
Time used: 0.016 (sec). Leaf size: 26

```
DSolve[y''[x]-4*y'[x]+29*y[x]==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to e^{2x}(c_2 \cos(5x) + c_1 \sin(5x))$$

11.29 problem 17.5 (e)

Internal problem ID [13607]

Book: Ordinary Differential Equations. An introduction to the fundamentals. Kenneth B. Howell. second edition. CRC Press. FL, USA. 2020
Section: Chapter 17. Second order Homogeneous equations with constant coefficients. Additional exercises page 334
Problem number: 17.5 (e).
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _missing_x]]`

$$\boxed{9y'' + 18y' + 10y = 0}$$

✓ Solution by Maple
Time used: 0.0 (sec). Leaf size: 22

```
dsolve(9*diff(y(x),x$2)+18*diff(y(x),x)+10*y(x)=0,y(x), singsol=all)
```

$$y(x) = \mathrm{e}^{-x}\left(c_1 \sin\left(\frac{x}{3}\right) + c_2 \cos\left(\frac{x}{3}\right)\right)$$

✓ Solution by Mathematica
Time used: 0.018 (sec). Leaf size: 30

```
DSolve[9*y''[x]+18*y'[x]+10*y[x]==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to e^{-x}\left(c_2 \cos\left(\frac{x}{3}\right) + c_1 \sin\left(\frac{x}{3}\right)\right)$$

11.30 problem 17.5 (f)

Internal problem ID [13608]

Book: Ordinary Differential Equations. An introduction to the fundamentals. Kenneth B. Howell. second edition. CRC Press. FL, USA. 2020
Section: Chapter 17. Second order Homogeneous equations with constant coefficients. Additional exercises page 334
Problem number: 17.5 (f).
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _missing_x]]`

$$\boxed{4y'' + y = 0}$$

✓ Solution by Maple
Time used: 0.0 (sec). Leaf size: 17

```
dsolve(4*diff(y(x),x$2)+y(x)=0,y(x), singsol=all)
```

$$y(x) = c_1 \sin\left(\frac{x}{2}\right) + c_2 \cos\left(\frac{x}{2}\right)$$

✓ Solution by Mathematica
Time used: 0.014 (sec). Leaf size: 24

```
DSolve[4*y''[x]+y[x]==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to c_1 \cos\left(\frac{x}{2}\right) + c_2 \sin\left(\frac{x}{2}\right)$$

11.31 problem 17.6 (a)

Internal problem ID [13609]

Book: Ordinary Differential Equations. An introduction to the fundamentals. Kenneth B. Howell. second edition. CRC Press. FL, USA. 2020
Section: Chapter 17. Second order Homogeneous equations with constant coefficients. Additional exercises page 334
Problem number: 17.6 (a).
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _missing_x]]`

$$\boxed{y'' + 16y = 0}$$

With initial conditions

$$[y(0) = 1, y'(0) = 0]$$

✓ Solution by Maple
Time used: 0.0 (sec). Leaf size: 8

```
dsolve([diff(y(x),x$2)+16*y(x)=0,y(0) = 1, D(y)(0) = 0],y(x), singsol=all)
```

$$y(x) = \cos(4x)$$

✓ Solution by Mathematica
Time used: 0.014 (sec). Leaf size: 9

```
DSolve[{y''[x]+16*y[x]==0,{y[0]==1,y'[0]==0}},y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to \cos(4x)$$

11.32 problem 17.6 (b)

Internal problem ID [13610]

Book: Ordinary Differential Equations. An introduction to the fundamentals. Kenneth B. Howell. second edition. CRC Press. FL, USA. 2020
Section: Chapter 17. Second order Homogeneous equations with constant coefficients. Additional exercises page 334
Problem number: 17.6 (b).
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _missing_x]]`

$$\boxed{y'' + 16y = 0}$$

With initial conditions

$$[y(0) = 0, y'(0) = 1]$$

✓ Solution by Maple
Time used: 0.0 (sec). Leaf size: 10

```
dsolve([diff(y(x),x$2)+16*y(x)=0,y(0) = 0, D(y)(0) = 1],y(x), singsol=all)
```

$$y(x) = \frac{\sin(4x)}{4}$$

✓ Solution by Mathematica
Time used: 0.012 (sec). Leaf size: 13

```
DSolve[{y''[x]+16*y[x]==0,{y[0]==0,y'[0]==1}},y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to \frac{1}{4}\sin(4x)$$

11.33 problem 17.6 (c)

Internal problem ID [13611]

Book: Ordinary Differential Equations. An introduction to the fundamentals. Kenneth B. Howell. second edition. CRC Press. FL, USA. 2020
Section: Chapter 17. Second order Homogeneous equations with constant coefficients. Additional exercises page 334
Problem number: 17.6 (c).
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _missing_x]]`

$$\boxed{y'' + 16y = 0}$$

With initial conditions

$$[y(0) = 4, y'(0) = 12]$$

✓ Solution by Maple
Time used: 0.016 (sec). Leaf size: 17

```
dsolve([diff(y(x),x$2)+16*y(x)=0,y(0) = 4, D(y)(0) = 12],y(x), singsol=all)
```

$$y(x) = 3\sin(4x) + 4\cos(4x)$$

✓ Solution by Mathematica
Time used: 0.012 (sec). Leaf size: 18

```
DSolve[{y''[x]+16*y[x]==0,{y[0]==4,y'[0]==12}},y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to 3\sin(4x) + 4\cos(4x)$$

11.34 problem 17.6 (d)

Internal problem ID [13612]

Book: Ordinary Differential Equations. An introduction to the fundamentals. Kenneth B. Howell. second edition. CRC Press. FL, USA. 2020
Section: Chapter 17. Second order Homogeneous equations with constant coefficients. Additional exercises page 334
Problem number: 17.6 (d).
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _missing_x]]`

$$\boxed{y'' - 4y' + 13y = 0}$$

With initial conditions

$$[y(0) = 1, y'(0) = 0]$$

✓ Solution by Maple
Time used: 0.015 (sec). Leaf size: 24

```
dsolve([diff(y(x),x$2)-4*diff(y(x),x)+13*y(x)=0,y(0) = 1, D(y)(0) = 0],y(x), singsol=all)
```

$$y(x) = \frac{e^{2x}(-2\sin(3x) + 3\cos(3x))}{3}$$

✓ Solution by Mathematica
Time used: 0.018 (sec). Leaf size: 27

```
DSolve[{y''[x]-4*y'[x]+13*y[x]==0,{y[0]==1,y'[0]==0}},y[x],x,IncludeSingularSolutions -> Tru
```

$$y(x) \to \frac{1}{3}e^{2x}(3\cos(3x) - 2\sin(3x))$$

11.35 problem 17.6 (e)

Internal problem ID [13613]

Book: Ordinary Differential Equations. An introduction to the fundamentals. Kenneth B. Howell. second edition. CRC Press. FL, USA. 2020
Section: Chapter 17. Second order Homogeneous equations with constant coefficients. Additional exercises page 334
Problem number: 17.6 (e).
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _missing_x]]`

$$\boxed{y'' - 4y' + 13y = 0}$$

With initial conditions

$$[y(0) = 0, y'(0) = 1]$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 14

```
dsolve([diff(y(x),x$2)-4*diff(y(x),x)+13*y(x)=0,y(0) = 0, D(y)(0) = 1],y(x), singsol=all)
```

$$y(x) = \frac{e^{2x} \sin(3x)}{3}$$

✓ Solution by Mathematica

Time used: 0.016 (sec). Leaf size: 18

```
DSolve[{y''[x]-4*y'[x]+13*y[x]==0,{y[0]==0,y'[0]==1}},y[x],x,IncludeSingularSolutions -> Tru
```

$$y(x) \to \frac{1}{3}e^{2x} \sin(3x)$$

11.36 problem 17.6 (f)

Internal problem ID [13614]

Book: Ordinary Differential Equations. An introduction to the fundamentals. Kenneth B. Howell. second edition. CRC Press. FL, USA. 2020
Section: Chapter 17. Second order Homogeneous equations with constant coefficients. Additional exercises page 334
Problem number: 17.6 (f).
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _missing_x]]`

$$\boxed{y'' - 4y' + 13y = 0}$$

With initial conditions

$$[y(0) = 5, y'(0) = 31]$$

✓ Solution by Maple
Time used: 0.015 (sec). Leaf size: 22

```
dsolve([diff(y(x),x$2)-4*diff(y(x),x)+13*y(x)=0,y(0) = 5, D(y)(0) = 31],y(x), singsol=all)
```

$$y(x) = e^{2x}(7\sin(3x) + 5\cos(3x))$$

✓ Solution by Mathematica
Time used: 0.017 (sec). Leaf size: 24

```
DSolve[{y''[x]-4*y'[x]+13*y[x]==0,{y[0]==5,y'[0]==31}},y[x],x,IncludeSingularSolutions -> Tr
```

$$y(x) \to e^{2x}(7\sin(3x) + 5\cos(3x))$$

11.37 problem 17.7 (a)

Internal problem ID [13615]

Book: Ordinary Differential Equations. An introduction to the fundamentals. Kenneth B. Howell. second edition. CRC Press. FL, USA. 2020
Section: Chapter 17. Second order Homogeneous equations with constant coefficients. Additional exercises page 334
Problem number: 17.7 (a).
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _missing_x]]`

$$\boxed{y'' - y' + \left(\frac{1}{4} + 4\pi^2\right) y = 0}$$

With initial conditions

$$\left[y(0) = 1, y'(0) = \frac{1}{2}\right]$$

✓ Solution by Maple
Time used: 0.0 (sec). Leaf size: 14

```
dsolve([diff(y(x),x$2)-diff(y(x),x)+(1/4+4*Pi^2)*y(x)=0,y(0) = 1, D(y)(0) = 1/2],y(x), sings
```

$$y(x) = e^{\frac{x}{2}} \cos (2\pi x)$$

✓ Solution by Mathematica
Time used: 0.021 (sec). Leaf size: 18

```
DSolve[{y''[x]-y'[x]+(1/4+4*Pi^2)*y[x]==0,{y[0]==1,y'[0]==1/2}},y[x],x,IncludeSingularSoluti
```

$$y(x) \to e^{x/2} \cos(2\pi x)$$

11.38 problem 17.7 (b)

Internal problem ID [13616]

Book: Ordinary Differential Equations. An introduction to the fundamentals. Kenneth B. Howell. second edition. CRC Press. FL, USA. 2020
Section: Chapter 17. Second order Homogeneous equations with constant coefficients. Additional exercises page 334
Problem number: 17.7 (b).
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _missing_x]]`

$$\boxed{y'' - y' + \left(\frac{1}{4} + 4\pi^2\right) y = 0}$$

With initial conditions

$$\left[y(0) = 1, y'(0) = -\frac{1}{2}\right]$$

✓ Solution by Maple
Time used: 0.016 (sec). Leaf size: 29

```
dsolve([diff(y(x),x$2)-diff(y(x),x)+(1/4+4*Pi^2)*y(x)=0,y(0) = 1, D(y)(0) = -1/2],y(x), sing
```

$$y(x) = \frac{e^{\frac{x}{2}} (2\pi \cos(2\pi x) - \sin(2\pi x))}{2\pi}$$

✓ Solution by Mathematica
Time used: 0.02 (sec). Leaf size: 35

```
DSolve[{y''[x]-y'[x]+(1/4+4*Pi^2)*y[x]==0,{y[0]==1,y'[0]==-1/2}},y[x],x,IncludeSingularSolut
```

$$y(x) \to \frac{e^{x/2}(2\pi \cos(2\pi x) - \sin(2\pi x))}{2\pi}$$

12 Chapter 19. Arbitrary Homogeneous linear equations with constant coefficients. Additional exercises page 369

12.1 problem 19.1 (a)

Internal problem ID [13617]

Book: Ordinary Differential Equations. An introduction to the fundamentals. Kenneth B. Howell. second edition. CRC Press. FL, USA. 2020
Section: Chapter 19. Arbitrary Homogeneous linear equations with constant coefficients. Additional exercises page 369
Problem number: 19.1 (a).
ODE order: 4.
ODE degree: 1.

CAS Maple gives this as type `[[_high_order, _missing_x]]`

$$\boxed{y'''' - 4y''' = 0}$$

✓ Solution by Maple
Time used: 0.016 (sec). Leaf size: 20

```
dsolve(diff(y(x),x$4)-4*diff(y(x),x$3)=0,y(x), singsol=all)
```

$$y(x) = c_1 + c_2 x + x^2 c_3 + c_4 e^{4x}$$

✓ Solution by Mathematica
Time used: 0.026 (sec). Leaf size: 28

```
DSolve[y''''[x]-4*y'''[x]==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to \frac{1}{64} c_1 e^{4x} + x(c_4 x + c_3) + c_2$$

12.2 problem 19.1 (b)

Internal problem ID [13618]

Book: Ordinary Differential Equations. An introduction to the fundamentals. Kenneth B. Howell. second edition. CRC Press. FL, USA. 2020
Section: Chapter 19. Arbitrary Homogeneous linear equations with constant coefficients. Additional exercises page 369
Problem number: 19.1 (b).
ODE order: 4.
ODE degree: 1.

CAS Maple gives this as type `[[_high_order, _missing_x]]`

$$\boxed{y'''' + 4y'' = 0}$$

✓ Solution by Maple
Time used: 0.0 (sec). Leaf size: 21

```
dsolve(diff(y(x),x$4)+4*diff(y(x),x$2)=0,y(x), singsol=all)
```

$$y(x) = c_1 + c_2 x + c_3 \sin(2x) + c_4 \cos(2x)$$

✓ Solution by Mathematica
Time used: 0.077 (sec). Leaf size: 32

```
DSolve[y''''[x]+4*y''[x]==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to c_4 x - \frac{1}{4} c_1 \cos(2x) - \frac{1}{4} c_2 \sin(2x) + c_3$$

12.3 problem 19.1 (c)

Internal problem ID [13619]

Book: Ordinary Differential Equations. An introduction to the fundamentals. Kenneth B. Howell. second edition. CRC Press. FL, USA. 2020
Section: Chapter 19. Arbitrary Homogeneous linear equations with constant coefficients. Additional exercises page 369
Problem number: 19.1 (c).
ODE order: 4.
ODE degree: 1.

CAS Maple gives this as type `[[_high_order, _missing_x]]`

$$\boxed{y'''' - 34y'' + 225y = 0}$$

✓ Solution by Maple
Time used: 0.0 (sec). Leaf size: 29

```
dsolve(diff(y(x),x$4)-34*diff(y(x),x$2)+225*y(x)=0,y(x), singsol=all)
```

$$y(x) = \left(\mathrm{e}^{10x} c_1 + c_3 \mathrm{e}^{8x} + c_2 \mathrm{e}^{2x} + c_4\right) \mathrm{e}^{-5x}$$

✓ Solution by Mathematica
Time used: 0.003 (sec). Leaf size: 39

```
DSolve[y''''[x]-34*y''[x]+225*y[x]==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to e^{-5x}\left(e^{2x}\left(c_3 e^{6x} + c_4 e^{8x} + c_2\right) + c_1\right)$$

12.4 problem 19.1 (d)

Internal problem ID [13620]

Book: Ordinary Differential Equations. An introduction to the fundamentals. Kenneth B. Howell. second edition. CRC Press. FL, USA. 2020
Section: Chapter 19. Arbitrary Homogeneous linear equations with constant coefficients. Additional exercises page 369
Problem number: 19.1 (d).
ODE order: 4.
ODE degree: 1.

CAS Maple gives this as type `[[_high_order, _missing_x]]`

$$\boxed{y'''' - 81y = 0}$$

✓ Solution by Maple
Time used: 0.0 (sec). Leaf size: 29

```
dsolve(diff(y(x),x$4)-81*y(x)=0,y(x), singsol=all)
```

$$y(x) = c_1 e^{-3x} + c_2 e^{3x} + c_3 \sin(3x) + c_4 \cos(3x)$$

✓ Solution by Mathematica
Time used: 0.002 (sec). Leaf size: 36

```
DSolve[y''''[x]-81*y[x]==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to c_1 e^{3x} + c_3 e^{-3x} + c_2 \cos(3x) + c_4 \sin(3x)$$

12.5 problem 19.1 (e)

Internal problem ID [13621]

Book: Ordinary Differential Equations. An introduction to the fundamentals. Kenneth B. Howell. second edition. CRC Press. FL, USA. 2020
Section: Chapter 19. Arbitrary Homogeneous linear equations with constant coefficients. Additional exercises page 369
Problem number: 19.1 (e).
ODE order: 4.
ODE degree: 1.

CAS Maple gives this as type `[[_high_order, _missing_x]]`

$$\boxed{y'''' - 18y'' + 81y = 0}$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 25

```
dsolve(diff(y(x),x$4)-18*diff(y(x),x$2)+81*y(x)=0,y(x), singsol=all)
```

$$y(x) = (c_2x + c_1)\,e^{-3x} + e^{3x}(c_4x + c_3)$$

✓ Solution by Mathematica

Time used: 0.003 (sec). Leaf size: 35

```
DSolve[y''''[x]-18*y''[x]+81*y[x]==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to e^{-3x}\left(c_3e^{6x} + x\left(c_4e^{6x} + c_2\right) + c_1\right)$$

12.6 problem 19.1 (f)

Internal problem ID [13622]

Book: Ordinary Differential Equations. An introduction to the fundamentals. Kenneth B. Howell. second edition. CRC Press. FL, USA. 2020
Section: Chapter 19. Arbitrary Homogeneous linear equations with constant coefficients. Additional exercises page 369
Problem number: 19.1 (f).
ODE order: 5.
ODE degree: 1.

CAS Maple gives this as type `[[_high_order, _missing_x]]`

$$\boxed{y^{(5)} + 18y''' + 81y' = 0}$$

✓ Solution by Maple
Time used: 0.0 (sec). Leaf size: 26

```
dsolve(diff(y(x),x$5)+18*diff(y(x),x$3)+81*diff(y(x),x)=0,y(x), singsol=all)
```

$$y(x) = (c_5x + c_3)\cos(3x) + (c_4x + c_2)\sin(3x) + c_1$$

✓ Solution by Mathematica
Time used: 0.11 (sec). Leaf size: 48

```
DSolve[y'''''[x]+18*y'''[x]+81*y'[x]==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to \frac{1}{9}((c_2 - 3(c_4x + c_3))\cos(3x) + (3c_2x + 3c_1 + c_4)\sin(3x) + 9c_5)$$

12.7 problem 19.2 (a)

Internal problem ID [13623]

Book: Ordinary Differential Equations. An introduction to the fundamentals. Kenneth B. Howell. second edition. CRC Press. FL, USA. 2020
Section: Chapter 19. Arbitrary Homogeneous linear equations with constant coefficients. Additional exercises page 369
Problem number: 19.2 (a).
ODE order: 3.
ODE degree: 1.

CAS Maple gives this as type `[[_3rd_order, _missing_x]]`

$$\boxed{y''' - y'' + y' - y = 0}$$

✓ Solution by Maple
Time used: 0.0 (sec). Leaf size: 17

```
dsolve(diff(y(x),x$3)-diff(y(x),x$2)+diff(y(x),x)-y(x)=0,y(x), singsol=all)
```

$$y(x) = c_1 e^x + \sin(x) c_2 + c_3 \cos(x)$$

✓ Solution by Mathematica
Time used: 0.003 (sec). Leaf size: 22

```
DSolve[y'''[x]-y''[x]+y'[x]-y[x]==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to c_3 e^x + c_1 \cos(x) + c_2 \sin(x)$$

12.8 problem 19.2 (b)

Internal problem ID [13624]

Book: Ordinary Differential Equations. An introduction to the fundamentals. Kenneth B. Howell. second edition. CRC Press. FL, USA. 2020
Section: Chapter 19. Arbitrary Homogeneous linear equations with constant coefficients. Additional exercises page 369
Problem number: 19.2 (b).
ODE order: 3.
ODE degree: 1.

CAS Maple gives this as type `[[_3rd_order, _missing_x]]`

$$\boxed{y''' - 6y'' + 11y' - 6y = 0}$$

✓ Solution by Maple
Time used: 0.0 (sec). Leaf size: 21

```
dsolve(diff(y(x),x$3)-6*diff(y(x),x$2)+11*diff(y(x),x)-6*y(x)=0,y(x), singsol=all)
```

$$y(x) = c_1 e^{3x} + c_2 e^{x} + c_3 e^{2x}$$

✓ Solution by Mathematica
Time used: 0.003 (sec). Leaf size: 25

```
DSolve[y'''[x]-6*y''[x]+11*y'[x]-6*y[x]==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to e^x (e^x (c_3 e^x + c_2) + c_1)$$

12.9 problem 19.2 (c)

Internal problem ID [13625]

Book: Ordinary Differential Equations. An introduction to the fundamentals. Kenneth B. Howell. second edition. CRC Press. FL, USA. 2020
Section: Chapter 19. Arbitrary Homogeneous linear equations with constant coefficients. Additional exercises page 369
Problem number: 19.2 (c).
ODE order: 3.
ODE degree: 1.

CAS Maple gives this as type `[[_3rd_order, _missing_x]]`

$$\boxed{y''' - 8y'' + 37y' - 50y = 0}$$

✓ Solution by Maple
Time used: 0.0 (sec). Leaf size: 31

```
dsolve(diff(y(x),x$3)-8*diff(y(x),x$2)+37*diff(y(x),x)-50*y(x)=0,y(x), singsol=all)
```

$$y(x) = \mathrm{e}^{2x}c_1 + c_2\mathrm{e}^{3x}\sin(4x) + c_3\mathrm{e}^{3x}\cos(4x)$$

✓ Solution by Mathematica
Time used: 0.003 (sec). Leaf size: 34

```
DSolve[y'''[x]-8*y''[x]+37*y'[x]-50*y[x]==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to e^{2x}(c_2e^x\cos(4x) + c_1e^x\sin(4x) + c_3)$$

12.10 problem 19.2 (d)

Internal problem ID [13626]

Book: Ordinary Differential Equations. An introduction to the fundamentals. Kenneth B. Howell. second edition. CRC Press. FL, USA. 2020
Section: Chapter 19. Arbitrary Homogeneous linear equations with constant coefficients. Additional exercises page 369
Problem number: 19.2 (d).
ODE order: 3.
ODE degree: 1.

CAS Maple gives this as type `[[_3rd_order, _missing_x]]`

$$\boxed{y''' - 9y'' + 31y' - 39y = 0}$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 23

```
dsolve(diff(y(x),x$3)-9*diff(y(x),x$2)+31*diff(y(x),x)-39*y(x)=0,y(x), singsol=all)
```

$$y(x) = e^{3x}(c_1 + c_2 \sin(2x) + c_3 \cos(2x))$$

✓ Solution by Mathematica

Time used: 0.003 (sec). Leaf size: 28

```
DSolve[y'''[x]-9*y''[x]+31*y'[x]-39*y[x]==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to e^{3x}(c_2 \cos(2x) + c_1 \sin(2x) + c_3)$$

12.11 problem 19.2 (e)

Internal problem ID [13627]

Book: Ordinary Differential Equations. An introduction to the fundamentals. Kenneth B. Howell. second edition. CRC Press. FL, USA. 2020
Section: Chapter 19. Arbitrary Homogeneous linear equations with constant coefficients. Additional exercises page 369
Problem number: 19.2 (e).
ODE order: 4.
ODE degree: 1.

CAS Maple gives this as type `[[_high_order, _missing_x]]`

$$y'''' + y''' + 2y'' + 4y' - 8y = 0$$

✓ Solution by Maple
Time used: 0.0 (sec). Leaf size: 37

```
dsolve(diff(y(x),x$4)+diff(y(x),x$3)+2*diff(y(x),x$2)+4*diff(y(x),x)-8*y(x)=0,y(x), singsol=
```

$$y(x) = \left(c_3 \sin(2x)\, e^{2x} + c_4 \cos(2x)\, e^{2x} + c_1 e^{3x} + c_2\right) e^{-2x}$$

✓ Solution by Mathematica
Time used: 0.003 (sec). Leaf size: 34

```
DSolve[y''''[x]+y'''[x]+2*y''[x]+4*y'[x]-8*y[x]==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to c_3 e^{-2x} + c_4 e^x + c_1 \cos(2x) + c_2 \sin(2x)$$

12.12 problem 19.2 (f)

Internal problem ID [13628]

Book: Ordinary Differential Equations. An introduction to the fundamentals. Kenneth B. Howell. second edition. CRC Press. FL, USA. 2020
Section: Chapter 19. Arbitrary Homogeneous linear equations with constant coefficients. Additional exercises page 369
Problem number: 19.2 (f).
ODE order: 4.
ODE degree: 1.

CAS Maple gives this as type `[[_high_order, _missing_x]]`

$$\boxed{y'''' + 2y''' + 10y'' + 18y' + 9y = 0}$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 27

```
dsolve(diff(y(x),x$4)+2*diff(y(x),x$3)+10*diff(y(x),x$2)+18*diff(y(x),x)+9*y(x)=0,y(x), sing
```

$$y(x) = e^{-x}(c_2x + c_1) + c_3 \sin(3x) + c_4 \cos(3x)$$

✓ Solution by Mathematica

Time used: 0.003 (sec). Leaf size: 38

```
DSolve[y''''[x]+2*y'''[x]+10*y''[x]+18*y'[x]+9*y[x]==0,y[x],x,IncludeSingularSolutions -> Tr
```

$$y(x) \to e^{-x}(c_4x + c_1e^x \cos(3x) + c_2e^x \sin(3x) + c_3)$$

12.13 problem 19.3 (a)

Internal problem ID [13629]

Book: Ordinary Differential Equations. An introduction to the fundamentals. Kenneth B. Howell. second edition. CRC Press. FL, USA. 2020
Section: Chapter 19. Arbitrary Homogeneous linear equations with constant coefficients. Additional exercises page 369
Problem number: 19.3 (a).
ODE order: 3.
ODE degree: 1.

CAS Maple gives this as type `[[_3rd_order, _missing_x]]`

$$\boxed{y''' + 4y' = 0}$$

With initial conditions

$$[y(0) = 4, y'(0) = 6, y''(0) = 8]$$

✓ Solution by Maple

Time used: 0.016 (sec). Leaf size: 18

```
dsolve([diff(y(x),x$3)+4*diff(y(x),x)=0,y(0) = 4, D(y)(0) = 6, (D@@2)(y)(0) = 8],y(x), sings
```

$$y(x) = 6 + 3\sin(2x) - 2\cos(2x)$$

✓ Solution by Mathematica

Time used: 0.021 (sec). Leaf size: 19

```
DSolve[{y'''[x]+4*y'[x]==0,{y[0]==4,y'[0]==6,y''[0]==8}},y[x],x,IncludeSingularSolutions ->
```

$$y(x) \to 3\sin(2x) - 2\cos(2x) + 6$$

12.14 problem 19.3 (b)

Internal problem ID [13630]

Book: Ordinary Differential Equations. An introduction to the fundamentals. Kenneth B. Howell. second edition. CRC Press. FL, USA. 2020
Section: Chapter 19. Arbitrary Homogeneous linear equations with constant coefficients. Additional exercises page 369
Problem number: 19.3 (b).
ODE order: 3.
ODE degree: 1.

CAS Maple gives this as type `[[_3rd_order, _missing_x]]`

$$\boxed{y''' - 6y'' + 12y' - 8y = 0}$$

With initial conditions

$$[y(0) = 5, y'(0) = 13, y''(0) = 86]$$

✓ Solution by Maple
Time used: 0.016 (sec). Leaf size: 19

```
dsolve([diff(y(x),x$3)-6*diff(y(x),x$2)+12*diff(y(x),x)-8*y(x)=0,y(0) = 5, D(y)(0) = 13, (D@
```

$$y(x) = e^{2x}(27x^2 + 3x + 5)$$

✓ Solution by Mathematica
Time used: 0.003 (sec). Leaf size: 21

```
DSolve[{y'''[x]-6*y''[x]+12*y'[x]-8*y[x]==0,{y[0]==5,y'[0]==13,y''[0]==86}},y[x],x,IncludeSi
```

$$y(x) \to e^{2x}(27x^2 + 3x + 5)$$

12.15 problem 19.3 (c)

Internal problem ID [13631]

Book: Ordinary Differential Equations. An introduction to the fundamentals. Kenneth B. Howell. second edition. CRC Press. FL, USA. 2020
Section: Chapter 19. Arbitrary Homogeneous linear equations with constant coefficients. Additional exercises page 369
Problem number: 19.3 (c).
ODE order: 4.
ODE degree: 1.

CAS Maple gives this as type `[[_high_order, _missing_x]]`

$$\boxed{y'''' + 26y'' + 25y = 0}$$

With initial conditions

$$[y(0) = 6, y'(0) = -28, y''(0) = -102, y'''(0) = 622]$$

✓ Solution by Maple
Time used: 0.047 (sec). Leaf size: 25

```
dsolve([diff(y(x),x$4)+26*diff(y(x),x$2)+25*y(x)=0,y(0) = 6, D(y)(0) = -28, (D@@2)(y)(0) = -
```

$$y(x) = -\frac{13 \sin (x)}{4} + 2 \cos (x) - \frac{99 \sin (5x)}{20} + 4 \cos (5x)$$

✓ Solution by Mathematica
Time used: 0.003 (sec). Leaf size: 30

```
DSolve[{y''''[x]+26*y''[x]+25*y[x]==0,{y[0]==6,y'[0]==-28,y''[0]==-102,y'''[0]==622}},y[x],x
```

$$y(x) \to -\frac{13 \sin(x)}{4} - \frac{99}{20} \sin(5x) + 2 \cos(x) + 4 \cos(5x)$$

12.16 problem 19.3 (d)

Internal problem ID [13632]

Book: Ordinary Differential Equations. An introduction to the fundamentals. Kenneth B. Howell. second edition. CRC Press. FL, USA. 2020
Section: Chapter 19. Arbitrary Homogeneous linear equations with constant coefficients. Additional exercises page 369
Problem number: 19.3 (d).
ODE order: 4.
ODE degree: 1.

CAS Maple gives this as type `[[_high_order, _missing_x]]`

$$\boxed{y'''' + y''' + 9y'' + 9y' = 0}$$

With initial conditions

$$[y(0) = 10, y'(0) = 0, y''(0) = 6, y'''(0) = -60]$$

✓ Solution by Maple
Time used: 0.016 (sec). Leaf size: 18

```
dsolve([diff(y(x),x$4)+diff(y(x),x$3)+9*diff(y(x),x$2)+9*diff(y(x),x)=0,y(0) = 10, D(y)(0) =
```

$$y(x) = 4 + 6\,\mathrm{e}^{-x} + 2\sin(3x)$$

✓ Solution by Mathematica
Time used: 0.055 (sec). Leaf size: 20

```
DSolve[{y''''[x]+y'''[x]+9*y''[x]+9*y'[x]==0,{y[0]==10,y'[0]==0,y''[0]==6,y'''[0]==-60}},y[x
```

$$y(x) \to 6e^{-x} + 2\sin(3x) + 4$$

12.17 problem 19.4 (a)

Internal problem ID [13633]

Book: Ordinary Differential Equations. An introduction to the fundamentals. Kenneth B. Howell. second edition. CRC Press. FL, USA. 2020
Section: Chapter 19. Arbitrary Homogeneous linear equations with constant coefficients. Additional exercises page 369
Problem number: 19.4 (a).
ODE order: 3.
ODE degree: 1.

CAS Maple gives this as type `[[_3rd_order, _missing_x]]`

$$\boxed{y''' - 8y = 0}$$

✓ Solution by Maple
Time used: 0.0 (sec). Leaf size: 35

```
dsolve(diff(y(x),x$3)-8*y(x)=0,y(x), singsol=all)
```

$$y(x) = e^{2x} c_1 + e^{-x} \sin\left(\sqrt{3}\, x\right) c_2 + c_3 e^{-x} \cos\left(\sqrt{3}\, x\right)$$

✓ Solution by Mathematica
Time used: 0.003 (sec). Leaf size: 42

```
DSolve[y'''[x]-8*y[x]==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to e^{-x}\left(c_1 e^{3x} + c_2 \cos\left(\sqrt{3}x\right) + c_3 \sin\left(\sqrt{3}x\right)\right)$$

12.18 problem 19.4 (b)

Internal problem ID [13634]

Book: Ordinary Differential Equations. An introduction to the fundamentals. Kenneth B. Howell. second edition. CRC Press. FL, USA. 2020
Section: Chapter 19. Arbitrary Homogeneous linear equations with constant coefficients. Additional exercises page 369
Problem number: 19.4 (b).
ODE order: 3.
ODE degree: 1.

CAS Maple gives this as type `[[_3rd_order, _missing_x]]`

$$\boxed{y''' + 216y = 0}$$

✓ Solution by Maple
Time used: 0.016 (sec). Leaf size: 37

```
dsolve(diff(y(x),x$3)+216*y(x)=0,y(x), singsol=all)
```

$$y(x) = \left(c_2 e^{9x} \sin\left(3\sqrt{3}\,x\right) + c_3 e^{9x} \cos\left(3\sqrt{3}\,x\right) + c_1\right) e^{-6x}$$

✓ Solution by Mathematica
Time used: 0.003 (sec). Leaf size: 48

```
DSolve[y'''[x]+216*y[x]==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to e^{-6x}\left(c_3 e^{9x} \cos\left(3\sqrt{3}x\right) + c_2 e^{9x} \sin\left(3\sqrt{3}x\right) + c_1\right)$$

12.19 problem 19.4 (c)

Internal problem ID [13635]

Book: Ordinary Differential Equations. An introduction to the fundamentals. Kenneth B. Howell. second edition. CRC Press. FL, USA. 2020
Section: Chapter 19. Arbitrary Homogeneous linear equations with constant coefficients. Additional exercises page 369
Problem number: 19.4 (c).
ODE order: 4.
ODE degree: 1.

CAS Maple gives this as type `[[_high_order, _missing_x]]`

$$\boxed{y'''' - 3y'' - 4y = 0}$$

✓ Solution by Maple
Time used: 0.0 (sec). Leaf size: 25

```
dsolve(diff(y(x),x$4)-3*diff(y(x),x$2)-4*y(x)=0,y(x), singsol=all)
```

$$y(x) = \mathrm{e}^{2x}c_1 + \mathrm{e}^{-2x}c_2 + c_3\sin(x) + c_4\cos(x)$$

✓ Solution by Mathematica
Time used: 0.003 (sec). Leaf size: 32

```
DSolve[y''''[x]-3*y''[x]-4*y[x]==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to c_3e^{-2x} + c_4e^{2x} + c_1\cos(x) + c_2\sin(x)$$

12.20 problem 19.4 (d)

Internal problem ID [13636]

Book: Ordinary Differential Equations. An introduction to the fundamentals. Kenneth B. Howell. second edition. CRC Press. FL, USA. 2020
Section: Chapter 19. Arbitrary Homogeneous linear equations with constant coefficients. Additional exercises page 369
Problem number: 19.4 (d).
ODE order: 4.
ODE degree: 1.

CAS Maple gives this as type `[[_high_order, _missing_x]]`

$$\boxed{y'''' + 13y'' + 36y = 0}$$

✓ Solution by Maple
Time used: 0.016 (sec). Leaf size: 29

```
dsolve(diff(y(x),x$4)+13*diff(y(x),x$2)+36*y(x)=0,y(x), singsol=all)
```

$$y(x) = c_1 \sin(3x) + c_2 \cos(3x) + c_3 \sin(2x) + c_4 \cos(2x)$$

✓ Solution by Mathematica
Time used: 0.003 (sec). Leaf size: 34

```
DSolve[y''''[x]+13*y''[x]+36*y[x]==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to c_3 \cos(2x) + c_1 \cos(3x) + c_4 \sin(2x) + c_2 \sin(3x)$$

12.21 problem 19.4 (e)

Internal problem ID [13637]

Book: Ordinary Differential Equations. An introduction to the fundamentals. Kenneth B. Howell. second edition. CRC Press. FL, USA. 2020
Section: Chapter 19. Arbitrary Homogeneous linear equations with constant coefficients. Additional exercises page 369
Problem number: 19.4 (e).
ODE order: 6.
ODE degree: 1.

CAS Maple gives this as type `[[_high_order, _missing_x]]`

$$\boxed{y^{(6)} - 3y'''' + 3y'' - y = 0}$$

✓ Solution by Maple
Time used: 0.0 (sec). Leaf size: 33

```
dsolve(diff(y(x),x$6)-3*diff(y(x),x$4)+3*diff(y(x),x$2)-y(x)=0,y(x), singsol=all)
```

$$y(x) = \left(x^2 c_3 + c_2 x + c_1\right) \mathrm{e}^{-x} + \mathrm{e}^{x}\left(c_6 x^2 + c_5 x + c_4\right)$$

✓ Solution by Mathematica
Time used: 0.003 (sec). Leaf size: 50

```
DSolve[y''''''[x]-3*y''''[x]+3*y''[x]-y[x]==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to e^{-x}\left(x^2\left(c_6 e^{2x} + c_3\right) + x\left(c_5 e^{2x} + c_2\right) + c_4 e^{2x} + c_1\right)$$

12.22 problem 19.4 (f)

Internal problem ID [13638]

Book: Ordinary Differential Equations. An introduction to the fundamentals. Kenneth B. Howell. second edition. CRC Press. FL, USA. 2020
Section: Chapter 19. Arbitrary Homogeneous linear equations with constant coefficients. Additional exercises page 369
Problem number: 19.4 (f).
ODE order: 6.
ODE degree: 1.

CAS Maple gives this as type `[[_high_order, _missing_x]]`

$$\boxed{y^{(6)} - 2y''' + y = 0}$$

✓ Solution by Maple
Time used: 0.0 (sec). Leaf size: 47

```
dsolve(diff(y(x),x$6)-2*diff(y(x),x$3)+y(x)=0,y(x), singsol=all)
```

$$y(x) = \mathrm{e}^{-\frac{x}{2}}(c_6x + c_4)\cos\left(\frac{\sqrt{3}\,x}{2}\right) + \mathrm{e}^{-\frac{x}{2}}(c_5x + c_3)\sin\left(\frac{\sqrt{3}\,x}{2}\right) + \mathrm{e}^{x}(c_2x + c_1)$$

✓ Solution by Mathematica
Time used: 0.003 (sec). Leaf size: 67

```
DSolve[y''''''[x]-2*y'''[x]+y[x]==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to e^{-x/2}\left(e^{3x/2}(c_6x + c_5) + (c_4x + c_3)\cos\left(\frac{\sqrt{3}x}{2}\right) + (c_2x + c_1)\sin\left(\frac{\sqrt{3}x}{2}\right)\right)$$

12.23 problem 19.4 (g)

Internal problem ID [13639]

Book: Ordinary Differential Equations. An introduction to the fundamentals. Kenneth B. Howell. second edition. CRC Press. FL, USA. 2020
Section: Chapter 19. Arbitrary Homogeneous linear equations with constant coefficients. Additional exercises page 369
Problem number: 19.4 (g).
ODE order: 4.
ODE degree: 1.

CAS Maple gives this as type `[[_high_order, _missing_x]]`

$$\boxed{16y'''' - y = 0}$$

✓ Solution by Maple
Time used: 0.015 (sec). Leaf size: 29

```
dsolve(16*diff(y(x),x$4)-y(x)=0,y(x), singsol=all)
```

$$y(x) = c_1 e^{-\frac{x}{2}} + c_2 e^{\frac{x}{2}} + c_3 \sin\left(\frac{x}{2}\right) + c_4 \cos\left(\frac{x}{2}\right)$$

✓ Solution by Mathematica
Time used: 0.002 (sec). Leaf size: 41

```
DSolve[16*y''''[x]-y[x]==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to e^{-x/2}(c_1 e^x + c_3) + c_2 \cos\left(\frac{x}{2}\right) + c_4 \sin\left(\frac{x}{2}\right)$$

12.24 problem 19.4 (h)

Internal problem ID [13640]

Book: Ordinary Differential Equations. An introduction to the fundamentals. Kenneth B. Howell. second edition. CRC Press. FL, USA. 2020
Section: Chapter 19. Arbitrary Homogeneous linear equations with constant coefficients. Additional exercises page 369
Problem number: 19.4 (h).
ODE order: 4.
ODE degree: 1.

CAS Maple gives this as type `[[_high_order, _missing_x]]`

$$\boxed{4y'''' + 15y'' - 4y = 0}$$

✓ Solution by Maple

Time used: 0.016 (sec). Leaf size: 29

```
dsolve(4*diff(y(x),x$4)+15*diff(y(x),x$2)-4*y(x)=0,y(x), singsol=all)
```

$$y(x) = c_1 e^{-\frac{x}{2}} + c_2 e^{\frac{x}{2}} + c_3 \sin(2x) + c_4 \cos(2x)$$

✓ Solution by Mathematica

Time used: 0.003 (sec). Leaf size: 37

```
DSolve[4*y''''[x]+15*y''[x]-4*y[x]==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to e^{-x/2}(c_4 e^x + c_3) + c_1 \cos(2x) + c_2 \sin(2x)$$

12.25 problem 19.4 (i)

Internal problem ID [13641]

Book: Ordinary Differential Equations. An introduction to the fundamentals. Kenneth B. Howell. second edition. CRC Press. FL, USA. 2020
Section: Chapter 19. Arbitrary Homogeneous linear equations with constant coefficients. Additional exercises page 369
Problem number: 19.4 (i).
ODE order: 4.
ODE degree: 1.

CAS Maple gives this as type `[[_high_order, _missing_x]]`

$$\boxed{y'''' - 4y''' + 16y' - 16y = 0}$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 26

```
dsolve(diff(y(x),x$4)-4*diff(y(x),x$3)+16*diff(y(x),x)-16*y(x)=0,y(x), singsol=all)
```

$$y(x) = (c_4x^2 + c_3x + c_2)\,e^{2x} + e^{-2x}c_1$$

✓ Solution by Mathematica

Time used: 0.003 (sec). Leaf size: 32

```
DSolve[y''''[x]-4*y'''[x]+16*y'[x]-16*y[x]==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to e^{-2x}\left(e^{4x}(x(c_4x + c_3) + c_2) + c_1\right)$$

12.26 problem 19.4 (j)

Internal problem ID [13642]

Book: Ordinary Differential Equations. An introduction to the fundamentals. Kenneth B. Howell. second edition. CRC Press. FL, USA. 2020
Section: Chapter 19. Arbitrary Homogeneous linear equations with constant coefficients. Additional exercises page 369
Problem number: 19.4 (j).
ODE order: 6.
ODE degree: 1.

CAS Maple gives this as type `[[_high_order, _missing_x]]`

$$\boxed{y^{(6)} + 16y''' + 64y = 0}$$

✓ Solution by Maple
Time used: 0.0 (sec). Leaf size: 46

```
dsolve(diff(y(x),x$6)+16*diff(y(x),x$3)+64*y(x)=0,y(x), singsol=all)
```

$$y(x) = \left(\mathrm{e}^{3x}(c_6x + c_4)\cos\left(\sqrt{3}\,x\right) + \mathrm{e}^{3x}(c_5x + c_3)\sin\left(\sqrt{3}\,x\right) + c_2x + c_1\right)\mathrm{e}^{-2x}$$

✓ Solution by Mathematica
Time used: 0.004 (sec). Leaf size: 60

```
DSolve[y''''''[x]+16*y'''[x]+64*y[x]==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to e^{-2x}\left(c_6x + e^{3x}(c_4x + c_3)\cos\left(\sqrt{3}x\right) + e^{3x}(c_2x + c_1)\sin\left(\sqrt{3}x\right) + c_5\right)$$

13 Chapter 20. Euler equations. Additional exercises page 382

13.1 problem 20.1 (a)

Internal problem ID [13643]

Book: Ordinary Differential Equations. An introduction to the fundamentals. Kenneth B. Howell. second edition. CRC Press. FL, USA. 2020
Section: Chapter 20. Euler equations. Additional exercises page 382
Problem number: 20.1 (a).
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_Emden, _Fowler], [_2nd_order, _linear, '_with_symmetry_[0,F`

$$x^2y'' - 5y'x + 8y = 0$$

✓ Solution by Maple
Time used: 0.0 (sec). Leaf size: 15

```
dsolve(x^2*diff(y(x),x$2)-5*x*diff(y(x),x)+8*y(x)=0,y(x), singsol=all)
```

$$y(x) = x^2(c_1x^2 + c_2)$$

✓ Solution by Mathematica
Time used: 0.009 (sec). Leaf size: 18

```
DSolve[x^2*y''[x]-5*x*y'[x]+8*y[x]==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to x^2(c_2x^2 + c_1)$$

13.2 problem 20.1 (b)

Internal problem ID [13644]

Book: Ordinary Differential Equations. An introduction to the fundamentals. Kenneth B. Howell. second edition. CRC Press. FL, USA. 2020
Section: Chapter 20. Euler equations. Additional exercises page 382
Problem number: 20.1 (b).
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _exact, _linear, _homogeneous]]`

$$\boxed{x^2y'' - 2y = 0}$$

✓ Solution by Maple
Time used: 0.0 (sec). Leaf size: 15

```
dsolve(x^2*diff(y(x),x$2)-2*y(x)=0,y(x), singsol=all)
```

$$y(x) = \frac{c_2 x^3 + c_1}{x}$$

✓ Solution by Mathematica
Time used: 0.009 (sec). Leaf size: 18

```
DSolve[x^2*y''[x]-2*y[x]==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to \frac{c_2 x^3 + c_1}{x}$$

13.3 problem 20.1 (c)

Internal problem ID [13645]

Book: Ordinary Differential Equations. An introduction to the fundamentals. Kenneth B. Howell. second edition. CRC Press. FL, USA. 2020
Section: Chapter 20. Euler equations. Additional exercises page 382
Problem number: 20.1 (c).
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _missing_y]]`

$$\boxed{x^2y'' - 2y'x = 0}$$

✓ Solution by Maple
Time used: 0.0 (sec). Leaf size: 11

```
dsolve(x^2*diff(y(x),x$2)-2*x*diff(y(x),x)=0,y(x), singsol=all)
```

$$y(x) = c_2x^3 + c_1$$

✓ Solution by Mathematica
Time used: 0.024 (sec). Leaf size: 17

```
DSolve[x^2*y''[x]-2*x*y'[x]==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to \frac{c_1x^3}{3} + c_2$$

13.4 problem 20.1 (d)

Internal problem ID [13646]

Book: Ordinary Differential Equations. An introduction to the fundamentals. Kenneth B. Howell. second edition. CRC Press. FL, USA. 2020
Section: Chapter 20. Euler equations. Additional exercises page 382
Problem number: 20.1 (d).
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_Emden, _Fowler], [_2nd_order, _linear, '_with_symmetry_[0,F`

$$2x^2y'' - y'x + y = 0$$

✓ Solution by Maple
Time used: 0.0 (sec). Leaf size: 13

```
dsolve(2*x^2*diff(y(x),x$2)-x*diff(y(x),x)+y(x)=0,y(x), singsol=all)
```

$$y(x) = c_1\sqrt{x} + c_2x$$

✓ Solution by Mathematica
Time used: 0.011 (sec). Leaf size: 18

```
DSolve[2*x^2*y''[x]-x*y'[x]+y[x]==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to c_1\sqrt{x} + c_2x$$

13.5 problem 20.1 (e)

Internal problem ID [13647]

Book: Ordinary Differential Equations. An introduction to the fundamentals. Kenneth B. Howell. second edition. CRC Press. FL, USA. 2020
Section: Chapter 20. Euler equations. Additional exercises page 382
Problem number: 20.1 (e).
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_Emden, _Fowler]]`

$$\boxed{x^2y'' - 5y'x + 9y = 0}$$

✓ Solution by Maple
Time used: 0.0 (sec). Leaf size: 14

```
dsolve(x^2*diff(y(x),x$2)-5*x*diff(y(x),x)+9*y(x)=0,y(x), singsol=all)
```

$$y(x) = x^3(c_1 + c_2 \ln(x))$$

✓ Solution by Mathematica
Time used: 0.015 (sec). Leaf size: 18

```
DSolve[x^2*y''[x]-5*x*y'[x]+9*y[x]==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to x^3(3c_2 \log(x) + c_1)$$

13.6 problem 20.1 (f)

Internal problem ID [13648]

Book: Ordinary Differential Equations. An introduction to the fundamentals. Kenneth B. Howell. second edition. CRC Press. FL, USA. 2020
Section: Chapter 20. Euler equations. Additional exercises page 382
Problem number: 20.1 (f).
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_Emden, _Fowler]]`

$$\boxed{x^2y'' + 5y'x + 4y = 0}$$

✓ Solution by Maple
Time used: 0.0 (sec). Leaf size: 14

```
dsolve(x^2*diff(y(x),x$2)+5*x*diff(y(x),x)+4*y(x)=0,y(x), singsol=all)
```

$$y(x) = \frac{c_1 + c_2 \ln (x)}{x^2}$$

✓ Solution by Mathematica
Time used: 0.016 (sec). Leaf size: 18

```
DSolve[x^2*y''[x]+5*x*y'[x]+4*y[x]==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to \frac{2c_2 \log(x) + c_1}{x^2}$$

13.7 problem 20.1 (g)

Internal problem ID [13649]

Book: Ordinary Differential Equations. An introduction to the fundamentals. Kenneth B. Howell. second edition. CRC Press. FL, USA. 2020
Section: Chapter 20. Euler equations. Additional exercises page 382
Problem number: 20.1 (g).
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_Emden, _Fowler]]`

$$\boxed{4x^2y'' + y = 0}$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 14

```
dsolve(4*x^2*diff(y(x),x$2)+y(x)=0,y(x), singsol=all)
```

$$y(x) = (c_1 + c_2 \ln(x))\sqrt{x}$$

✓ Solution by Mathematica

Time used: 0.017 (sec). Leaf size: 24

```
DSolve[4*x^2*y''[x]+y[x]==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to \frac{1}{2}\sqrt{x}(c_2 \log(x) + 2c_1)$$

13.8 problem 20.1 (h)

Internal problem ID [13650]

Book: Ordinary Differential Equations. An introduction to the fundamentals. Kenneth B. Howell. second edition. CRC Press. FL, USA. 2020
Section: Chapter 20. Euler equations. Additional exercises page 382
Problem number: 20.1 (h).
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_Emden, _Fowler]]`

$$\boxed{x^2y'' - 19y'x + 100y = 0}$$

✓ Solution by Maple
Time used: 0.0 (sec). Leaf size: 14

```
dsolve(x^2*diff(y(x),x$2)-19*x*diff(y(x),x)+100*y(x)=0,y(x), singsol=all)
```

$$y(x) = x^{10}(c_1 + c_2 \ln(x))$$

✓ Solution by Mathematica
Time used: 0.016 (sec). Leaf size: 18

```
DSolve[x^2*y''[x]-19*x*y'[x]+100*y[x]==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to x^{10}(10c_2 \log(x) + c_1)$$

13.9 problem 20.1 (i)

Internal problem ID [13651]

Book: Ordinary Differential Equations. An introduction to the fundamentals. Kenneth B. Howell. second edition. CRC Press. FL, USA. 2020
Section: Chapter 20. Euler equations. Additional exercises page 382
Problem number: 20.1 (i).
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_Emden, _Fowler]]`

$$\boxed{x^2y'' - 5y'x + 29y = 0}$$

✓ Solution by Maple
Time used: 0.0 (sec). Leaf size: 29

```
dsolve(x^2*diff(y(x),x$2)-5*x*diff(y(x),x)+29*y(x)=0,y(x), singsol=all)
```

$$y(x) = x^3\left(c_1 \sin\left(2\sqrt{5}\,\ln(x)\right) + c_2 \cos\left(2\sqrt{5}\,\ln(x)\right)\right)$$

✓ Solution by Mathematica
Time used: 0.029 (sec). Leaf size: 36

```
DSolve[x^2*y''[x]-5*x*y'[x]+29*y[x]==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to x^3\left(c_2 \cos\left(2\sqrt{5}\log(x)\right) + c_1 \sin\left(2\sqrt{5}\log(x)\right)\right)$$

13.10 problem 20.1 (j)

Internal problem ID [13652]

Book: Ordinary Differential Equations. An introduction to the fundamentals. Kenneth B. Howell. second edition. CRC Press. FL, USA. 2020
Section: Chapter 20. Euler equations. Additional exercises page 382
Problem number: 20.1 (j).
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_Emden, _Fowler]]`

$$\boxed{x^2y'' - y'x + 10y = 0}$$

✓ Solution by Maple
Time used: 0.0 (sec). Leaf size: 21

```
dsolve(x^2*diff(y(x),x$2)-x*diff(y(x),x)+10*y(x)=0,y(x), singsol=all)
```

$$y(x) = x(c_1 \sin(3\ln(x)) + c_2 \cos(3\ln(x)))$$

✓ Solution by Mathematica
Time used: 0.026 (sec). Leaf size: 24

```
DSolve[x^2*y''[x]-x*y'[x]+10*y[x]==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \rightarrow x(c_2 \cos(3\log(x)) + c_1 \sin(3\log(x)))$$

13.11 problem 20.1 (k)

Internal problem ID [13653]

Book: Ordinary Differential Equations. An introduction to the fundamentals. Kenneth B. Howell. second edition. CRC Press. FL, USA. 2020
Section: Chapter 20. Euler equations. Additional exercises page 382
Problem number: 20.1 (k).
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_Emden, _Fowler]]`

$$\boxed{x^2 y'' + 5y'x + 29y = 0}$$

✓ Solution by Maple
Time used: 0.0 (sec). Leaf size: 23

```
dsolve(x^2*diff(y(x),x$2)+5*x*diff(y(x),x)+29*y(x)=0,y(x), singsol=all)
```

$$y(x) = \frac{c_1 \sin(5\ln(x)) + c_2 \cos(5\ln(x))}{x^2}$$

✓ Solution by Mathematica
Time used: 0.024 (sec). Leaf size: 26

```
DSolve[x^2*y''[x]+5*x*y'[x]+29*y[x]==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to \frac{c_2 \cos(5\log(x)) + c_1 \sin(5\log(x))}{x^2}$$

13.12 problem 20.1 (L)

Internal problem ID [13654]

Book: Ordinary Differential Equations. An introduction to the fundamentals. Kenneth B. Howell. second edition. CRC Press. FL, USA. 2020
Section: Chapter 20. Euler equations. Additional exercises page 382
Problem number: 20.1 (L).
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_Emden, _Fowler], [_2nd_order, _linear, '_with_symmetry_[0,F`

$$\boxed{x^2 y'' + y'x + y = 0}$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 15

```
dsolve(x^2*diff(y(x),x$2)+x*diff(y(x),x)+y(x)=0,y(x), singsol=all)
```

$$y(x) = c_1 \sin(\ln(x)) + c_2 \cos(\ln(x))$$

✓ Solution by Mathematica

Time used: 0.016 (sec). Leaf size: 18

```
DSolve[x^2*y''[x]+x*y'[x]+y[x]==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to c_1 \cos(\log(x)) + c_2 \sin(\log(x))$$

13.13 problem 20.1 (m)

Internal problem ID [13655]

Book: Ordinary Differential Equations. An introduction to the fundamentals. Kenneth B. Howell. second edition. CRC Press. FL, USA. 2020
Section: Chapter 20. Euler equations. Additional exercises page 382
Problem number: 20.1 (m).
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _exact, _linear, _homogeneous]]`

$$\boxed{2x^2y'' + 5y'x + y = 0}$$

✓ Solution by Maple
Time used: 0.016 (sec). Leaf size: 15

```
dsolve(2*x^2*diff(y(x),x$2)+5*x*diff(y(x),x)+y(x)=0,y(x), singsol=all)
```

$$y(x) = \frac{c_1}{\sqrt{x}} + \frac{c_2}{x}$$

✓ Solution by Mathematica
Time used: 0.011 (sec). Leaf size: 20

```
DSolve[2*x^2*y''[x]+5*x*y'[x]+y[x]==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to \frac{c_2\sqrt{x} + c_1}{x}$$

13.14 problem 20.1 (n)

Internal problem ID [13656]

Book: Ordinary Differential Equations. An introduction to the fundamentals. Kenneth B. Howell. second edition. CRC Press. FL, USA. 2020
Section: Chapter 20. Euler equations. Additional exercises page 382
Problem number: 20.1 (n).
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_Emden, _Fowler]]`

$$\boxed{4x^2y'' + 37y = 0}$$

✓ Solution by Maple
Time used: 0.015 (sec). Leaf size: 23

```
dsolve(4*x^2*diff(y(x),x$2)+37*y(x)=0,y(x), singsol=all)
```

$$y(x) = \sqrt{x}\,(c_1 \sin(3\ln(x)) + c_2 \cos(3\ln(x)))$$

✓ Solution by Mathematica
Time used: 0.023 (sec). Leaf size: 28

```
DSolve[4*x^2*y''[x]+37*y[x]==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to \sqrt{x}(c_2 \cos(3\log(x)) + c_1 \sin(3\log(x)))$$

13.15 problem 20.1 (o)

Internal problem ID [13657]

Book: Ordinary Differential Equations. An introduction to the fundamentals. Kenneth B. Howell. second edition. CRC Press. FL, USA. 2020
Section: Chapter 20. Euler equations. Additional exercises page 382
Problem number: 20.1 (o).
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _missing_y]]`

$$\boxed{x^2y'' + y'x = 0}$$

✓ Solution by Maple
Time used: 0.0 (sec). Leaf size: 10

```
dsolve(x^2*diff(y(x),x$2)+x*diff(y(x),x)=0,y(x), singsol=all)
```

$$y(x) = c_1 + c_2 \ln (x)$$

✓ Solution by Mathematica
Time used: 0.022 (sec). Leaf size: 13

```
DSolve[x^2*y''[x]+x*y'[x]==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to c_1 \log(x) + c_2$$

13.16 problem 20.1 (p)

Internal problem ID [13658]

Book: Ordinary Differential Equations. An introduction to the fundamentals. Kenneth B. Howell. second edition. CRC Press. FL, USA. 2020
Section: Chapter 20. Euler equations. Additional exercises page 382
Problem number: 20.1 (p).
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_Emden, _Fowler], [_2nd_order, _linear, '_with_symmetry_[0,F`

$$\boxed{x^2y'' + y'x - 25y = 0}$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 15

```
dsolve(x^2*diff(y(x),x$2)+x*diff(y(x),x)-25*y(x)=0,y(x), singsol=all)
```

$$y(x) = x^5c_1 + \frac{c_2}{x^5}$$

✓ Solution by Mathematica

Time used: 0.01 (sec). Leaf size: 18

```
DSolve[x^2*y''[x]+x*y'[x]-25*y[x]==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to c_2x^5 + \frac{c_1}{x^5}$$

13.17 problem 20.1 (q)

Internal problem ID [13659]

Book: Ordinary Differential Equations. An introduction to the fundamentals. Kenneth B. Howell. second edition. CRC Press. FL, USA. 2020
Section: Chapter 20. Euler equations. Additional exercises page 382
Problem number: 20.1 (q).
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_Emden, _Fowler]]`

$$4x^2y'' + 8y'x + 5y = 0$$

✓ Solution by Maple
Time used: 0.0 (sec). Leaf size: 19

```
dsolve(4*x^2*diff(y(x),x$2)+8*x*diff(y(x),x)+5*y(x)=0,y(x), singsol=all)
```

$$y(x) = \frac{c_1 \sin(\ln(x)) + c_2 \cos(\ln(x))}{\sqrt{x}}$$

✓ Solution by Mathematica
Time used: 0.023 (sec). Leaf size: 24

```
DSolve[4*x^2*y''[x]+8*x*y'[x]+5*y[x]==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to \frac{c_2 \cos(\log(x)) + c_1 \sin(\log(x))}{\sqrt{x}}$$

13.18 problem 20.1 (r)

Internal problem ID [13660]

Book: Ordinary Differential Equations. An introduction to the fundamentals. Kenneth B. Howell. second edition. CRC Press. FL, USA. 2020
Section: Chapter 20. Euler equations. Additional exercises page 382
Problem number: 20.1 (r).
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_Emden, _Fowler]]`

$$\boxed{3x^2y'' - 7y'x + 3y = 0}$$

✓ Solution by Maple
Time used: 0.0 (sec). Leaf size: 15

```
dsolve(3*x^2*diff(y(x),x$2)-7*x*diff(y(x),x)+3*y(x)=0,y(x), singsol=all)
```

$$y(x) = c_1 x^{\frac{1}{3}} + c_2 x^3$$

✓ Solution by Mathematica
Time used: 0.011 (sec). Leaf size: 20

```
DSolve[3*x^2*y''[x]-7*x*y'[x]+3*y[x]==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to c_2 x^3 + c_1 \sqrt[3]{x}$$

13.19 problem 20.2 (a)

Internal problem ID [13661]

Book: Ordinary Differential Equations. An introduction to the fundamentals. Kenneth B. Howell. second edition. CRC Press. FL, USA. 2020
Section: Chapter 20. Euler equations. Additional exercises page 382
Problem number: 20.2 (a).
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_Emden, _Fowler], [_2nd_order, _linear, '_with_symmetry_[0,F`

$$x^2y'' - 2y'x - 10y = 0$$

With initial conditions

$$[y(1) = 5, y'(1) = 4]$$

✓ Solution by Maple

Time used: 0.016 (sec). Leaf size: 15

```
dsolve([x^2*diff(y(x),x$2)-2*x*diff(y(x),x)-10*y(x)=0,y(1) = 5, D(y)(1) = 4],y(x), singsol=a
```

$$y(x) = 2x^5 + \frac{3}{x^2}$$

✓ Solution by Mathematica

Time used: 0.011 (sec). Leaf size: 16

```
DSolve[{x^2*y''[x]-2*x*y'[x]-10*y[x]==0,{y[1]==5,y'[1]==4}},y[x],x,IncludeSingularSolutions
```

$$y(x) \to \frac{2x^7 + 3}{x^2}$$

13.20 problem 20.2 (b)

Internal problem ID [13662]

Book: Ordinary Differential Equations. An introduction to the fundamentals. Kenneth B. Howell. second edition. CRC Press. FL, USA. 2020
Section: Chapter 20. Euler equations. Additional exercises page 382
Problem number: 20.2 (b).
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_Emden, _Fowler], [_2nd_order, _linear, '_with_symmetry_[0,F`

$$4x^2y'' + 4y'x - y = 0$$

With initial conditions

$$[y(4) = 0, y'(4) = 2]$$

✓ Solution by Maple
Time used: 0.016 (sec). Leaf size: 12

```
dsolve([4*x^2*diff(y(x),x$2)+4*x*diff(y(x),x)-y(x)=0,y(4) = 0, D(y)(4) = 2],y(x), singsol=al
```

$$y(x) = \frac{4x - 16}{\sqrt{x}}$$

✓ Solution by Mathematica
Time used: 0.012 (sec). Leaf size: 15

```
DSolve[{4*x^2*y''[x]+4*x*y'[x]-y[x]==0,{y[4]==0,y'[4]==2}},y[x],x,IncludeSingularSolutions -
```

$$y(x) \to \frac{4(x - 4)}{\sqrt{x}}$$

13.21 problem 20.2 (c)

Internal problem ID [13663]

Book: Ordinary Differential Equations. An introduction to the fundamentals. Kenneth B. Howell. second edition. CRC Press. FL, USA. 2020
Section: Chapter 20. Euler equations. Additional exercises page 382
Problem number: 20.2 (c).
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_Emden, _Fowler]]`

$$\boxed{x^2y'' - 11y'x + 36y = 0}$$

With initial conditions

$$\left[y(1) = \frac{1}{2}, y'(1) = 2\right]$$

✓ Solution by Maple
Time used: 0.015 (sec). Leaf size: 14

```
dsolve([x^2*diff(y(x),x$2)-11*x*diff(y(x),x)+36*y(x)=0,y(1) = 1/2, D(y)(1) = 2],y(x), singso
```

$$y(x) = x^6\left(\frac{1}{2} - \ln(x)\right)$$

✓ Solution by Mathematica
Time used: 0.016 (sec). Leaf size: 18

```
DSolve[{x^2*y''[x]-11*x*y'[x]+36*y[x]==0,{y[1]==1/2,y'[1]==2}},y[x],x,IncludeSingularSolutio
```

$$y(x) \to \frac{1}{2}x^6(1 - 2\log(x))$$

13.22 problem 20.2 (d)

Internal problem ID [13664]

Book: Ordinary Differential Equations. An introduction to the fundamentals. Kenneth B. Howell. second edition. CRC Press. FL, USA. 2020
Section: Chapter 20. Euler equations. Additional exercises page 382
Problem number: 20.2 (d).
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_Emden, _Fowler]]`

$$\boxed{x^2y'' - y'x + y = 0}$$

With initial conditions

$$[y(1) = 3, y'(1) = 0]$$

✓ Solution by Maple
Time used: 0.0 (sec). Leaf size: 13

```
dsolve([x^2*diff(y(x),x$2)-x*diff(y(x),x)+y(x)=0,y(1) = 3, D(y)(1) = 0],y(x), singsol=all)
```

$$y(x) = 3x - 3\ln(x)\,x$$

✓ Solution by Mathematica
Time used: 0.015 (sec). Leaf size: 12

```
DSolve[{x^2*y''[x]-x*y'[x]+y[x]==0,{y[1]==3,y'[1]==0}},y[x],x,IncludeSingularSolutions -> Tr
```

$$y(x) \to -3x(\log(x) - 1)$$

13.23 problem 20.2 (e)

Internal problem ID [13665]

Book: Ordinary Differential Equations. An introduction to the fundamentals. Kenneth B. Howell. second edition. CRC Press. FL, USA. 2020
Section: Chapter 20. Euler equations. Additional exercises page 382
Problem number: 20.2 (e).
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_Emden, _Fowler]]`

$$\boxed{x^2y'' - y'x + 2y = 0}$$

With initial conditions

$$[y(1) = 3, y'(1) = 0]$$

✓ Solution by Maple
Time used: 0.031 (sec). Leaf size: 17

```
dsolve([x^2*diff(y(x),x$2)-x*diff(y(x),x)+2*y(x)=0,y(1) = 3, D(y)(1) = 0],y(x), singsol=all)
```

$$y(x) = (-3\sin(\ln(x)) + 3\cos(\ln(x)))\, x$$

✓ Solution by Mathematica
Time used: 0.022 (sec). Leaf size: 17

```
DSolve[{x^2*y''[x]-x*y'[x]+2*y[x]==0,{y[1]==3,y'[1]==0}},y[x],x,IncludeSingularSolutions ->
```

$$y(x) \to 3x(\cos(\log(x)) - \sin(\log(x)))$$

13.24 problem 20.2 (f)

Internal problem ID [13666]

Book: Ordinary Differential Equations. An introduction to the fundamentals. Kenneth B. Howell. second edition. CRC Press. FL, USA. 2020
Section: Chapter 20. Euler equations. Additional exercises page 382
Problem number: 20.2 (f).
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_Emden, _Fowler]]`

$$\boxed{x^2y'' - 3y'x + 13y = 0}$$

With initial conditions

$$[y(1) = 9, y'(1) = 3]$$

✓ Solution by Maple
Time used: 0.016 (sec). Leaf size: 23

```
dsolve([x^2*diff(y(x),x$2)-3*x*diff(y(x),x)+13*y(x)=0,y(1) = 9, D(y)(1) = 3],y(x), singsol=a
```

$$y(x) = x^2(-5\sin(3\ln(x)) + 9\cos(3\ln(x)))$$

✓ Solution by Mathematica
Time used: 0.024 (sec). Leaf size: 24

```
DSolve[{x^2*y''[x]-3*x*y'[x]+13*y[x]==0,{y[1]==9,y'[1]==3}},y[x],x,IncludeSingularSolutions
```

$$y(x) \to x^2(9\cos(3\log(x)) - 5\sin(3\log(x)))$$

13.25 problem 20.4 (a)

Internal problem ID [13667]

Book: Ordinary Differential Equations. An introduction to the fundamentals. Kenneth B. Howell. second edition. CRC Press. FL, USA. 2020
Section: Chapter 20. Euler equations. Additional exercises page 382
Problem number: 20.4 (a).
ODE order: 3.
ODE degree: 1.

CAS Maple gives this as type `[[_3rd_order, _with_linear_symmetries]]`

$$\boxed{x^3y''' + 2x^2y'' - 4y'x + 4y = 0}$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 20

```
dsolve(x^3*diff(y(x),x$3)+2*x^2*diff(y(x),x$2)-4*x*diff(y(x),x)+4*y(x)=0,y(x), singsol=all)
```

$$y(x) = \frac{c_2x^4 + c_1x^3 + c_3}{x^2}$$

✓ Solution by Mathematica

Time used: 0.004 (sec). Leaf size: 22

```
DSolve[x^3*y'''[x]+2*x^2*y''[x]-4*x*y'[x]+4*y[x]==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to c_3x^2 + \frac{c_1}{x^2} + c_2x$$

13.26 problem 20.4 (b)

Internal problem ID [13668]

Book: Ordinary Differential Equations. An introduction to the fundamentals. Kenneth B. Howell. second edition. CRC Press. FL, USA. 2020
Section: Chapter 20. Euler equations. Additional exercises page 382
Problem number: 20.4 (b).
ODE order: 3.
ODE degree: 1.

CAS Maple gives this as type `[[_3rd_order, _with_linear_symmetries]]`

$$x^3y''' + 2x^2y'' + y'x - y = 0$$

✓ Solution by Maple
Time used: 0.0 (sec). Leaf size: 18

```
dsolve(x^3*diff(y(x),x$3)+2*x^2*diff(y(x),x$2)+x*diff(y(x),x)-y(x)=0,y(x), singsol=all)
```

$$y(x) = c_1x + c_2 \sin(\ln(x)) + c_3 \cos(\ln(x))$$

✓ Solution by Mathematica
Time used: 0.004 (sec). Leaf size: 22

```
DSolve[x^3*y'''[x]+2*x^2*y''[x]+x*y'[x]-y[x]==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to c_3x + c_1 \cos(\log(x)) + c_2 \sin(\log(x))$$

13.27 problem 20.4 (c)

Internal problem ID [13669]

Book: Ordinary Differential Equations. An introduction to the fundamentals. Kenneth B. Howell. second edition. CRC Press. FL, USA. 2020
Section: Chapter 20. Euler equations. Additional exercises page 382
Problem number: 20.4 (c).
ODE order: 3.
ODE degree: 1.

CAS Maple gives this as type `[[_3rd_order, _with_linear_symmetries]]`

$$x^3y''' - 5x^2y'' + 14y'x - 18y = 0$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 18

```
dsolve(x^3*diff(y(x),x$3)-5*x^2*diff(y(x),x$2)+14*x*diff(y(x),x)-18*y(x)=0,y(x), singsol=all
```

$$y(x) = x^2(\ln(x)\, c_3x + c_2x + c_1)$$

✓ Solution by Mathematica

Time used: 0.004 (sec). Leaf size: 22

```
DSolve[x^3*y'''[x]-5*x^2*y''[x]+14*x*y'[x]-18*y[x]==0,y[x],x,IncludeSingularSolutions -> Tru
```

$$y(x) \to x^2(c_2x + c_3x\log(x) + c_1)$$

13.28 problem 20.4 (d)

Internal problem ID [13670]

Book: Ordinary Differential Equations. An introduction to the fundamentals. Kenneth B. Howell. second edition. CRC Press. FL, USA. 2020
Section: Chapter 20. Euler equations. Additional exercises page 382
Problem number: 20.4 (d).
ODE order: 3.
ODE degree: 1.

CAS Maple gives this as type `[[_3rd_order, _with_linear_symmetries]]`

$$\boxed{x^3y''' - 3x^2y'' + 7y'x - 8y = 0}$$

✓ Solution by Maple
Time used: 0.015 (sec). Leaf size: 20

```
dsolve(x^3*diff(y(x),x$3)-3*x^2*diff(y(x),x$2)+7*x*diff(y(x),x)-8*y(x)=0,y(x), singsol=all)
```

$$y(x) = x^2\left(c_1 + c_2 \ln(x) + c_3 \ln(x)^2\right)$$

✓ Solution by Mathematica
Time used: 0.004 (sec). Leaf size: 24

```
DSolve[x^3*y'''[x]-3*x^2*y''[x]+7*x*y'[x]-8*y[x]==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to x^2\left(c_3 \log^2(x) + c_2 \log(x) + c_1\right)$$

13.29 problem 20.4 (e)

Internal problem ID [13671]

Book: Ordinary Differential Equations. An introduction to the fundamentals. Kenneth B. Howell. second edition. CRC Press. FL, USA. 2020
Section: Chapter 20. Euler equations. Additional exercises page 382
Problem number: 20.4 (e).
ODE order: 4.
ODE degree: 1.

CAS Maple gives this as type `[[_high_order, _with_linear_symmetries]]`

$$x^4y'''' + 6x^3y''' + 15x^2y'' + 9y'x + 16y = 0$$

✓ Solution by Maple
Time used: 0.0 (sec). Leaf size: 29

```
dsolve(x^4*diff(y(x),x$4)+6*x^3*diff(y(x),x$3)+15*x^2*diff(y(x),x$2)+9*x*diff(y(x),x)+16*y(x
```

$$y(x) = (c_4 \ln(x) + c_2)\cos(2\ln(x)) + \sin(2\ln(x))(c_3 \ln(x) + c_1)$$

✓ Solution by Mathematica
Time used: 0.004 (sec). Leaf size: 34

```
DSolve[x^4*y''''[x]+6*x^3*y'''[x]+15*x^2*y''[x]+9*x*y'[x]+16*y[x]==0,y[x],x,IncludeSingularS
```

$$y(x) \to (c_2 \log(x) + c_1)\cos(2\log(x)) + (c_4 \log(x) + c_3)\sin(2\log(x))$$

13.30 problem 20.4 (f)

Internal problem ID [13672]

Book: Ordinary Differential Equations. An introduction to the fundamentals. Kenneth B. Howell. second edition. CRC Press. FL, USA. 2020
Section: Chapter 20. Euler equations. Additional exercises page 382
Problem number: 20.4 (f).
ODE order: 4.
ODE degree: 1.

CAS Maple gives this as type `[[_high_order, _exact, _linear, _homogeneous]]`

$$\boxed{x^4y'''' + 6x^3y''' - 3x^2y'' - 9y'x + 9y = 0}$$

✓ Solution by Maple

Time used: 0.016 (sec). Leaf size: 25

```
dsolve(x^4*diff(y(x),x$4)+6*x^3*diff(y(x),x$3)-3*x^2*diff(y(x),x$2)-9*x*diff(y(x),x)+9*y(x)=
```

$$y(x) = \frac{c_2x^6 + c_4x^4 + x^2c_3 + c_1}{x^3}$$

✓ Solution by Mathematica

Time used: 0.004 (sec). Leaf size: 28

```
DSolve[x^4*y''''[x]+6*x^3*y'''[x]-3*x^2*y''[x]-9*x*y'[x]+9*y[x]==0,y[x],x,IncludeSingularSol
```

$$y(x) \to c_4x^3 + \frac{c_1}{x^3} + c_3x + \frac{c_2}{x}$$

13.31 problem 20.4 (g)

Internal problem ID [13673]

Book: Ordinary Differential Equations. An introduction to the fundamentals. Kenneth B. Howell. second edition. CRC Press. FL, USA. 2020
Section: Chapter 20. Euler equations. Additional exercises page 382
Problem number: 20.4 (g).
ODE order: 4.
ODE degree: 1.

CAS Maple gives this as type `[[_high_order, _with_linear_symmetries]]`

$$x^4y'''' + 2x^3y''' + x^2y'' - y'x + y = 0$$

✓ Solution by Maple
Time used: 0.0 (sec). Leaf size: 24

```
dsolve(x^4*diff(y(x),x$4)+2*x^3*diff(y(x),x$3)+x^2*diff(y(x),x$2)-x*diff(y(x),x)+y(x)=0,y(x)
```

$$y(x) = x\left(c_1 + c_2 \ln(x) + c_3 \ln(x)^2 + c_4 \ln(x)^3\right)$$

✓ Solution by Mathematica
Time used: 0.004 (sec). Leaf size: 29

```
DSolve[x^4*y''''[x]+2*x^3*y'''[x]+x^2*y''[x]-x*y'[x]+y[x]==0,y[x],x,IncludeSingularSolutions
```

$$y(x) \to x\left(c_4 \log^3(x) + c_3 \log^2(x) + c_2 \log(x) + c_1\right)$$

13.32 problem 20.4 (h)

Internal problem ID [13674]

Book: Ordinary Differential Equations. An introduction to the fundamentals. Kenneth B. Howell. second edition. CRC Press. FL, USA. 2020
Section: Chapter 20. Euler equations. Additional exercises page 382
Problem number: 20.4 (h).
ODE order: 4.
ODE degree: 1.

CAS Maple gives this as type `[[_high_order, _exact, _linear, _homogeneous]]`

$$x^4y'''' + 6x^3y''' + 7x^2y'' + y'x - y = 0$$

✓ Solution by Maple
Time used: 0.0 (sec). Leaf size: 23

```
dsolve(x^4*diff(y(x),x$4)+6*x^3*diff(y(x),x$3)+7*x^2*diff(y(x),x$2)+x*diff(y(x),x)-y(x)=0,y(
```

$$y(x) = \frac{c_1}{x} + c_2x + c_3 \sin(\ln(x)) + c_4 \cos(\ln(x))$$

✓ Solution by Mathematica
Time used: 0.004 (sec). Leaf size: 28

```
DSolve[x^4*y''''[x]+6*x^3*y'''[x]+7*x^2*y''[x]+x*y'[x]-y[x]==0,y[x],x,IncludeSingularSolutio
```

$$y(x) \rightarrow c_1x + \frac{c_3}{x} + c_2 \cos(\log(x)) + c_4 \sin(\log(x))$$

14 Chapter 21. Nonhomogeneous equations in general. Additional exercises page 391

14.1 problem 21.5 (i)

Internal problem ID [13675]

Book: Ordinary Differential Equations. An introduction to the fundamentals. Kenneth B. Howell. second edition. CRC Press. FL, USA. 2020
Section: Chapter 21. Nonhomogeneous equations in general. Additional exercises page 391
Problem number: 21.5 (i).
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$\boxed{y'' + 4y = 24\,e^{2x}}$$

With initial conditions

$$[y(0) = 6, y'(0) = 6]$$

✓ Solution by Maple
Time used: 0.015 (sec). Leaf size: 17

```
dsolve([diff(y(x),x$2)+4*y(x)=24*exp(2*x),y(0) = 6, D(y)(0) = 6],y(x), singsol=all)
```

$$y(x) = 3\cos(2x) + 3\,e^{2x}$$

✓ Solution by Mathematica
Time used: 0.016 (sec). Leaf size: 17

```
DSolve[{y''[x]+4*y[x]==24*Exp[2*x],{y[0]==6,y'[0]==6}},y[x],x,IncludeSingularSolutions -> Tr
```

$$y(x) \to 3\left(e^{2x} + \cos(2x)\right)$$

14.2 problem 21.5 (ii)

Internal problem ID [13676]

Book: Ordinary Differential Equations. An introduction to the fundamentals. Kenneth B. Howell. second edition. CRC Press. FL, USA. 2020
Section: Chapter 21. Nonhomogeneous equations in general. Additional exercises page 391
Problem number: 21.5 (ii).
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$\boxed{y'' + 4y = 24\,\mathrm{e}^{2x}}$$

With initial conditions

$$[y(0) = -2, y'(0) = 2]$$

✓ Solution by Maple
Time used: 0.016 (sec). Leaf size: 23

```
dsolve([diff(y(x),x$2)+4*y(x)=24*exp(2*x),y(0) = -2, D(y)(0) = 2],y(x), singsol=all)
```

$$y(x) = -2\sin(2x) - 5\cos(2x) + 3\,\mathrm{e}^{2x}$$

✓ Solution by Mathematica
Time used: 0.016 (sec). Leaf size: 25

```
DSolve[{y''[x]+4*y[x]==24*Exp[2*x],{y[0]==-2,y'[0]==2}},y[x],x,IncludeSingularSolutions -> T
```

$$y(x) \to 3e^{2x} - 2\sin(2x) - 5\cos(2x)$$

14.3 problem 21.6 (i)

Internal problem ID [13677]

Book: Ordinary Differential Equations. An introduction to the fundamentals. Kenneth B. Howell. second edition. CRC Press. FL, USA. 2020
Section: Chapter 21. Nonhomogeneous equations in general. Additional exercises page 391
Problem number: 21.6 (i).
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$\boxed{y'' + 2y' - 8y = 8x^2 - 3}$$

With initial conditions

$$[y(0) = 0, y'(0) = 0]$$

✓ Solution by Maple
Time used: 0.016 (sec). Leaf size: 25

```
dsolve([diff(y(x),x$2)+2*diff(y(x),x)-8*y(x)=8*x^2-3,y(0) = 0, D(y)(0) = 0],y(x), singsol=al
```

$$y(x) = \frac{\left(-12\,\mathrm{e}^{4x} x^2 + \mathrm{e}^{6x} - 6\,\mathrm{e}^{4x} x - 1\right) \mathrm{e}^{-4x}}{12}$$

✓ Solution by Mathematica
Time used: 0.016 (sec). Leaf size: 34

```
DSolve[{y''[x]+2*y'[x]-8*y[x]==8*x^2-3,{y[0]==0,y'[0]==0}},y[x],x,IncludeSingularSolutions -
```

$$y(x) \to \frac{1}{12} e^{-4x} \left(-6e^{4x} x(2x+1) + e^{6x} - 1\right)$$

14.4 problem 21.6 (ii)

Internal problem ID [13678]

Book: Ordinary Differential Equations. An introduction to the fundamentals. Kenneth B. Howell. second edition. CRC Press. FL, USA. 2020
Section: Chapter 21. Nonhomogeneous equations in general. Additional exercises page 391
Problem number: 21.6 (ii).
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$\boxed{y'' + 2y' - 8y = 8x^2 - 3}$$

With initial conditions

$$[y(0) = 1, y'(0) = -3]$$

✓ Solution by Maple
Time used: 0.016 (sec). Leaf size: 25

```
dsolve([diff(y(x),x$2)+2*diff(y(x),x)-8*y(x)=8*x^2-3,y(0) = 1, D(y)(0) = -3],y(x), singsol=a
```

$$y(x) = \frac{(-4\,e^{4x}x^2 + e^{6x} - 2\,e^{4x}x + 3)\,e^{-4x}}{4}$$

✓ Solution by Mathematica
Time used: 0.015 (sec). Leaf size: 34

```
DSolve[{y''[x]+2*y'[x]-8*y[x]==8*x^2-3,{y[0]==1,y'[0]==-3}},y[x],x,IncludeSingularSolutions
```

$$y(x) \to \frac{1}{4}e^{-4x}\left(-2e^{4x}x(2x+1) + e^{6x} + 3\right)$$

14.5 problem 21.7

Internal problem ID [13679]

Book: Ordinary Differential Equations. An introduction to the fundamentals. Kenneth B. Howell. second edition. CRC Press. FL, USA. 2020
Section: Chapter 21. Nonhomogeneous equations in general. Additional exercises page 391
Problem number: 21.7.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _missing_x]]`

$$\boxed{y'' - 9y = 36}$$

With initial conditions

$$[y(0) = 8, y'(0) = 6]$$

✓ Solution by Maple
Time used: 0.016 (sec). Leaf size: 18

```
dsolve([diff(y(x),x$2)-9*y(x)=36,y(0) = 8, D(y)(0) = 6],y(x), singsol=all)
```

$$y(x) = 5\,\mathrm{e}^{-3x} + 7\,\mathrm{e}^{3x} - 4$$

✓ Solution by Mathematica
Time used: 0.013 (sec). Leaf size: 21

```
DSolve[{y''[x]-9*y[x]==36,{y[0]==8,y'[0]==6}},y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to 5e^{-3x} + 7e^{3x} - 4$$

14.6 problem 21.8

Internal problem ID [13680]

Book: Ordinary Differential Equations. An introduction to the fundamentals. Kenneth B. Howell. second edition. CRC Press. FL, USA. 2020
Section: Chapter 21. Nonhomogeneous equations in general. Additional exercises page 391
Problem number: 21.8.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$\boxed{y'' - 3y' - 10y = -6\,\mathrm{e}^{4x}}$$

With initial conditions

$$[y(0) = 6, y'(0) = 8]$$

✓ Solution by Maple
Time used: 0.016 (sec). Leaf size: 21

```
dsolve([diff(y(x),x$2)-3*diff(y(x),x)-10*y(x)=-6*exp(4*x),y(0) = 6, D(y)(0) = 8],y(x), sings
```

$$y(x) = \left(2\,\mathrm{e}^{7x} + \mathrm{e}^{6x} + 3\right)\mathrm{e}^{-2x}$$

✓ Solution by Mathematica
Time used: 0.017 (sec). Leaf size: 25

```
DSolve[{y''[x]-3*y'[x]-10*y[x]==-6*Exp[4*x],{y[0]==6,y'[0]==8}},y[x],x,IncludeSingularSoluti
```

$$y(x) \to e^{-2x}\left(e^{6x} + 2e^{7x} + 3\right)$$

14.7 problem 21.9

Internal problem ID [13681]

Book: Ordinary Differential Equations. An introduction to the fundamentals. Kenneth B. Howell. second edition. CRC Press. FL, USA. 2020
Section: Chapter 21. Nonhomogeneous equations in general. Additional exercises page 391
Problem number: 21.9.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$y'' - 3y' - 10y = 7\,\mathrm{e}^{5x}$$

With initial conditions

$$[y(0) = 12, y'(0) = -2]$$

✓ Solution by Maple

Time used: 0.031 (sec). Leaf size: 19

```
dsolve([diff(y(x),x$2)-3*diff(y(x),x)-10*y(x)=7*exp(5*x),y(0) = 12, D(y)(0) = -2],y(x), sing
```

$$y(x) = \left(9 + (x+3)\,\mathrm{e}^{7x}\right)\mathrm{e}^{-2x}$$

✓ Solution by Mathematica

Time used: 0.035 (sec). Leaf size: 22

```
DSolve[{y''[x]-3*y'[x]-10*y[x]==7*Exp[5*x],{y[0]==12,y'[0]==-2}},y[x],x,IncludeSingularSolut
```

$$y(x) \to e^{-2x}\left(e^{7x}(x+3) + 9\right)$$

14.8 problem 21.10

Internal problem ID [13682]

Book: Ordinary Differential Equations. An introduction to the fundamentals. Kenneth B. Howell. second edition. CRC Press. FL, USA. 2020
Section: Chapter 21. Nonhomogeneous equations in general. Additional exercises page 391
Problem number: 21.10.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _linear, _nonhomogeneous]]`

$$y'' + 6y' + 9y = 169\sin(2x)$$

With initial conditions

$$[y(0) = -10, y'(0) = 9]$$

✓ Solution by Maple
Time used: 0.016 (sec). Leaf size: 27

```
dsolve([diff(y(x),x$2)+6*diff(y(x),x)+9*y(x)=169*sin(2*x),y(0) = -10, D(y)(0) = 9],y(x), sin
```

$$y(x) = (5x + 2)\,\mathrm{e}^{-3x} - 12\cos(2x) + 5\sin(2x)$$

✓ Solution by Mathematica
Time used: 0.022 (sec). Leaf size: 34

```
DSolve[{y''[x]+6*y'[x]+9*y[x]==169*Sin[2*x],{y[0]==-10,y'[0]==9}},y[x],x,IncludeSingularSolu
```

$$y(x) \to e^{-3x}\left(5x + 5e^{3x}\sin(2x) + 2\right) - 12\cos(2x)$$

14.9 problem 21.11

Internal problem ID [13683]

Book: Ordinary Differential Equations. An introduction to the fundamentals. Kenneth B. Howell. second edition. CRC Press. FL, USA. 2020
Section: Chapter 21. Nonhomogeneous equations in general. Additional exercises page 391
Problem number: 21.11.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$\boxed{x^2y'' - 4y'x + 6y = 10x + 12}$$

With initial conditions

$$[y(1) = 6, y'(1) = 8]$$

✓ Solution by Maple

Time used: 0.016 (sec). Leaf size: 19

```
dsolve([x^2*diff(y(x),x$2)-4*x*diff(y(x),x)+6*y(x)=10*x+12,y(1) = 6, D(y)(1) = 8],y(x), sing
```

$$y(x) = 5x^3 - 6x^2 + 5x + 2$$

✓ Solution by Mathematica

Time used: 0.02 (sec). Leaf size: 20

```
DSolve[{x^2*y''[x]-4*x*y'[x]+6*y[x]==10*x+12,{y[1]==6,y'[1]==8}},y[x],x,IncludeSingularSolut
```

$$y(x) \to 5x^3 - 6x^2 + 5x + 2$$

14.10 problem 21.12

Internal problem ID [13684]

Book: Ordinary Differential Equations. An introduction to the fundamentals. Kenneth B. Howell. second edition. CRC Press. FL, USA. 2020
Section: Chapter 21. Nonhomogeneous equations in general. Additional exercises page 391
Problem number: 21.12.
ODE order: 4.
ODE degree: 1.

CAS Maple gives this as type `[[_high_order, _missing_x]]`

$$\boxed{y'''' + y'' = 1}$$

With initial conditions

$$[y(0) = 4, y'(0) = 3, y''(0) = 0, y'''(0) = 2]$$

✓ Solution by Maple
Time used: 0.031 (sec). Leaf size: 20

```
dsolve([diff(y(x),x$4)+diff(y(x),x$2)=1,y(0) = 4, D(y)(0) = 3, (D@@2)(y)(0) = 0, (D@@3)(y)(0
```

$$y(x) = \frac{x^2}{2} + \cos(x) - 2\sin(x) + 5x + 3$$

✓ Solution by Mathematica
Time used: 0.099 (sec). Leaf size: 23

```
DSolve[{y''''[x]+y''[x]==1,{y[0]==4,y'[0]==3,y''[0]==0,y'''[0]==2}},y[x],x,IncludeSingularSo
```

$$y(x) \to \frac{x^2}{2} + 5x - 2\sin(x) + \cos(x) + 3$$

14.11 problem 21.13 (a)

Internal problem ID [13685]

Book: Ordinary Differential Equations. An introduction to the fundamentals. Kenneth B. Howell. second edition. CRC Press. FL, USA. 2020
Section: Chapter 21. Nonhomogeneous equations in general. Additional exercises page 391
Problem number: 21.13 (a).
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$\boxed{y'' - 3y' - 10y = e^{4x}}$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 25

```
dsolve(diff(y(x),x$2)-3*diff(y(x),x)-10*y(x)=exp(4*x),y(x), singsol=all)
```

$$y(x) = -\frac{(-6c_2 e^{7x} + e^{6x} - 6c_1)\, e^{-2x}}{6}$$

✓ Solution by Mathematica

Time used: 0.039 (sec). Leaf size: 31

```
DSolve[y''[x]-3*y'[x]-10*y[x]==Exp[4*x],y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \rightarrow -\frac{e^{4x}}{6} + c_1 e^{-2x} + c_2 e^{5x}$$

14.12 problem 21.13 (b)

Internal problem ID [13686]

Book: Ordinary Differential Equations. An introduction to the fundamentals. Kenneth B. Howell. second edition. CRC Press. FL, USA. 2020
Section: Chapter 21. Nonhomogeneous equations in general. Additional exercises page 391
Problem number: 21.13 (b).
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$\boxed{y'' - 3y' - 10y = e^{5x}}$$

✓ Solution by Maple
Time used: 0.0 (sec). Leaf size: 24

```
dsolve(diff(y(x),x$2)-3*diff(y(x),x)-10*y(x)=exp(5*x),y(x), singsol=all)
```

$$y(x) = \frac{e^{-2x}\left((x + 7c_2)\,e^{7x} + 7c_1\right)}{7}$$

✓ Solution by Mathematica
Time used: 0.021 (sec). Leaf size: 31

```
DSolve[y''[x]-3*y'[x]-10*y[x]==Exp[5*x],y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to c_1 e^{-2x} + e^{5x}\left(\frac{x}{7} - \frac{1}{49} + c_2\right)$$

14.13 problem 21.13 (c)

Internal problem ID [13687]

Book: Ordinary Differential Equations. An introduction to the fundamentals. Kenneth B. Howell. second edition. CRC Press. FL, USA. 2020
Section: Chapter 21. Nonhomogeneous equations in general. Additional exercises page 391
Problem number: 21.13 (c).
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _linear, _nonhomogeneous]]`

$$\boxed{y'' - 3y' - 10y = -18\,\mathrm{e}^{4x} + 14\,\mathrm{e}^{5x}}$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 31

```
dsolve(diff(y(x),x$2)-3*diff(y(x),x)-10*y(x)=-18*exp(4*x)+14*exp(5*x),y(x), singsol=all)
```

$$y(x) = \mathrm{e}^{-2x}\left(\frac{(14x + 7c_2 - 2)\,\mathrm{e}^{7x}}{7} + c_1 + 3\,\mathrm{e}^{6x}\right)$$

✓ Solution by Mathematica

Time used: 0.057 (sec). Leaf size: 36

```
DSolve[y''[x]-3*y'[x]-10*y[x]==-18*Exp[4*x]+14*Exp[5*x],y[x],x,IncludeSingularSolutions -> T
```

$$y(x) \to 3e^{4x} + c_1 e^{-2x} + e^{5x}\left(2x - \frac{2}{7} + c_2\right)$$

14.14 problem 21.13 (d)

Internal problem ID [13688]

Book: Ordinary Differential Equations. An introduction to the fundamentals. Kenneth B. Howell. second edition. CRC Press. FL, USA. 2020
Section: Chapter 21. Nonhomogeneous equations in general. Additional exercises page 391
Problem number: 21.13 (d).
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _linear, _nonhomogeneous]]`

$$\boxed{y'' - 3y' - 10y = 35\,\mathrm{e}^{5x} + 12\,\mathrm{e}^{4x}}$$

✓ Solution by Maple

Time used: 0.016 (sec). Leaf size: 31

```
dsolve(diff(y(x),x$2)-3*diff(y(x),x)-10*y(x)=35*exp(5*x)+12*exp(4*x),y(x), singsol=all)
```

$$y(x) = \mathrm{e}^{-2x}\left(\frac{(35x + 7c_2 - 5)\,\mathrm{e}^{7x}}{7} + c_1 - 2\,\mathrm{e}^{6x}\right)$$

✓ Solution by Mathematica

Time used: 0.054 (sec). Leaf size: 36

```
DSolve[y''[x]-3*y'[x]-10*y[x]==35*Exp[5*x]+12*Exp[4*x],y[x],x,IncludeSingularSolutions -> Tr
```

$$y(x) \to -2e^{4x} + c_1e^{-2x} + e^{5x}\left(5x - \frac{5}{7} + c_2\right)$$

14.15 problem 21.14 (a)

Internal problem ID [13689]

Book: Ordinary Differential Equations. An introduction to the fundamentals. Kenneth B. Howell. second edition. CRC Press. FL, USA. 2020
Section: Chapter 21. Nonhomogeneous equations in general. Additional exercises page 391
Problem number: 21.14 (a).
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$\boxed{x^2y'' - 4y'x + 6y = 1}$$

✓ Solution by Maple
Time used: 0.016 (sec). Leaf size: 16

```
dsolve(x^2*diff(y(x),x$2)-4*x*diff(y(x),x)+6*y(x)=1,y(x), singsol=all)
```

$$y(x) = c_2x^3 + c_1x^2 + \frac{1}{6}$$

✓ Solution by Mathematica
Time used: 0.011 (sec). Leaf size: 21

```
DSolve[x^2*y''[x]-4*x*y'[x]+6*y[x]==1,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to c_2x^3 + c_1x^2 + \frac{1}{6}$$

14.16 problem 21.14 (b)

Internal problem ID [13690]

Book: Ordinary Differential Equations. An introduction to the fundamentals. Kenneth B. Howell. second edition. CRC Press. FL, USA. 2020
Section: Chapter 21. Nonhomogeneous equations in general. Additional exercises page 391
Problem number: 21.14 (b).
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$\boxed{x^2y'' - 4y'x + 6y = x}$$

✓ Solution by Maple
Time used: 0.0 (sec). Leaf size: 18

```
dsolve(x^2*diff(y(x),x$2)-4*x*diff(y(x),x)+6*y(x)=x,y(x), singsol=all)
```

$$y(x) = c_2x^3 + c_1x^2 + \frac{1}{2}x$$

✓ Solution by Mathematica
Time used: 0.012 (sec). Leaf size: 23

```
DSolve[x^2*y''[x]-4*x*y'[x]+6*y[x]==x,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to c_2x^3 + c_1x^2 + \frac{x}{2}$$

14.17 problem 21.14 (c)

Internal problem ID [13691]

Book: Ordinary Differential Equations. An introduction to the fundamentals. Kenneth B. Howell. second edition. CRC Press. FL, USA. 2020
Section: Chapter 21. Nonhomogeneous equations in general. Additional exercises page 391
Problem number: 21.14 (c).
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type **`[[_2nd_order, _with_linear_symmetries]]`**

$$x^2y'' - 4y'x + 6y = 22x + 24$$

✓ Solution by Maple
Time used: 0.016 (sec). Leaf size: 19

```
dsolve(x^2*diff(y(x),x$2)-4*x*diff(y(x),x)+6*y(x)=22*x+24,y(x), singsol=all)
```

$$y(x) = c_2x^3 + c_1x^2 + 11x + 4$$

✓ Solution by Mathematica
Time used: 0.02 (sec). Leaf size: 22

```
DSolve[x^2*y''[x]-4*x*y'[x]+6*y[x]==22*x+24,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to c_2x^3 + c_1x^2 + 11x + 4$$

14.18 problem 21.15 (i)

Internal problem ID [13692]

Book: Ordinary Differential Equations. An introduction to the fundamentals. Kenneth B. Howell. second edition. CRC Press. FL, USA. 2020
Section: Chapter 21. Nonhomogeneous equations in general. Additional exercises page 391
Problem number: 21.15 (i).
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$\boxed{x^2y'' - 7y'x + 15y = x^2}$$

✓ Solution by Maple
Time used: 0.015 (sec). Leaf size: 20

```
dsolve(x^2*diff(y(x),x$2)-7*x*diff(y(x),x)+15*y(x)=x^2,y(x), singsol=all)
```

$$y(x) = c_2x^5 + c_1x^3 + \frac{1}{3}x^2$$

✓ Solution by Mathematica
Time used: 0.013 (sec). Leaf size: 25

```
DSolve[x^2*y''[x]-7*x*y'[x]+15*y[x]==x^2,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to c_2x^5 + c_1x^3 + \frac{x^2}{3}$$

14.19 problem 21.15 (ii)

Internal problem ID [13693]

Book: Ordinary Differential Equations. An introduction to the fundamentals. Kenneth B. Howell. second edition. CRC Press. FL, USA. 2020
Section: Chapter 21. Nonhomogeneous equations in general. Additional exercises page 391
Problem number: 21.15 (ii).
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type **[[_2nd_order, _with_linear_symmetries]]**

$$\boxed{x^2y'' - 7y'x + 15y = x}$$

✓ Solution by Maple
Time used: 0.0 (sec). Leaf size: 18

```
dsolve(x^2*diff(y(x),x$2)-7*x*diff(y(x),x)+15*y(x)=x,y(x), singsol=all)
```

$$y(x) = c_2x^5 + c_1x^3 + \frac{1}{8}x$$

✓ Solution by Mathematica
Time used: 0.013 (sec). Leaf size: 23

```
DSolve[x^2*y''[x]-7*x*y'[x]+15*y[x]==x,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to c_2x^5 + c_1x^3 + \frac{x}{8}$$

14.20 problem 21.15 (iii)

Internal problem ID [13694]

Book: Ordinary Differential Equations. An introduction to the fundamentals. Kenneth B. Howell. second edition. CRC Press. FL, USA. 2020
Section: Chapter 21. Nonhomogeneous equations in general. Additional exercises page 391
Problem number: 21.15 (iii).
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$\boxed{x^2y'' - 7y'x + 15y = 1}$$

✓ Solution by Maple
Time used: 0.0 (sec). Leaf size: 16

```
dsolve(x^2*diff(y(x),x$2)-7*x*diff(y(x),x)+15*y(x)=1,y(x), singsol=all)
```

$$y(x) = c_2x^5 + c_1x^3 + \frac{1}{15}$$

✓ Solution by Mathematica
Time used: 0.01 (sec). Leaf size: 21

```
DSolve[x^2*y''[x]-7*x*y'[x]+15*y[x]==1,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to c_2x^5 + c_1x^3 + \frac{1}{15}$$

14.21 problem 21.15 (c)

Internal problem ID [13695]

Book: Ordinary Differential Equations. An introduction to the fundamentals. Kenneth B. Howell. second edition. CRC Press. FL, USA. 2020
Section: Chapter 21. Nonhomogeneous equations in general. Additional exercises page 391
Problem number: 21.15 (c).
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$\boxed{x^2y'' - 7y'x + 15y = 4x^2 + 2x + 3}$$

✓ Solution by Maple
Time used: 0.015 (sec). Leaf size: 24

```
dsolve(x^2*diff(y(x),x$2)-7*x*diff(y(x),x)+15*y(x)=4*x^2+2*x+3,y(x), singsol=all)
```

$$y(x) = c_2x^5 + c_1x^3 + \frac{1}{5} + \frac{4}{3}x^2 + \frac{1}{4}x$$

✓ Solution by Mathematica
Time used: 0.021 (sec). Leaf size: 33

```
DSolve[x^2*y''[x]-7*x*y'[x]+15*y[x]==4*x^2+2*x+3,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to c_2x^5 + c_1x^3 + \frac{4x^2}{3} + \frac{x}{4} + \frac{1}{5}$$

15 Chapter 22. Method of undetermined coefficients. Additional exercises page 412

15.1 problem 22.1 (a)

Internal problem ID [13696]

Book: Ordinary Differential Equations. An introduction to the fundamentals. Kenneth B. Howell. second edition. CRC Press. FL, USA. 2020
Section: Chapter 22. Method of undetermined coefficients. Additional exercises page 412
Problem number: 22.1 (a).
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$\boxed{y'' + 9y = 52\,\mathrm{e}^{2x}}$$

✓ Solution by Maple
Time used: 0.0 (sec). Leaf size: 23

```
dsolve(diff(y(x),x$2)+9*y(x)=52*exp(2*x),y(x), singsol=all)
```

$$y(x) = c_2 \sin(3x) + c_1 \cos(3x) + 4\,\mathrm{e}^{2x}$$

✓ Solution by Mathematica
Time used: 0.016 (sec). Leaf size: 27

```
DSolve[y''[x]+9*y[x]==52*Exp[2*x],y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to 4e^{2x} + c_1 \cos(3x) + c_2 \sin(3x)$$

15.2 problem 22.1 (b)

Internal problem ID [13697]

Book: Ordinary Differential Equations. An introduction to the fundamentals. Kenneth B. Howell. second edition. CRC Press. FL, USA. 2020
Section: Chapter 22. Method of undetermined coefficients. Additional exercises page 412
Problem number: 22.1 (b).
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$y'' - 6y' + 9y = 27\,\mathrm{e}^{6x}$$

✓ Solution by Maple
Time used: 0.016 (sec). Leaf size: 21

```
dsolve(diff(y(x),x$2)-6*diff(y(x),x)+9*y(x)=27*exp(6*x),y(x), singsol=all)
```

$$y(x) = (c_1 x + c_2)\,\mathrm{e}^{3x} + 3\,\mathrm{e}^{6x}$$

✓ Solution by Mathematica
Time used: 0.018 (sec). Leaf size: 25

```
DSolve[y''[x]-6*y'[x]+9*y[x]==27*Exp[6*x],y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to e^{3x}\left(3e^{3x} + c_2 x + c_1\right)$$

15.3 problem 22.1 (c)

Internal problem ID [13698]

Book: Ordinary Differential Equations. An introduction to the fundamentals. Kenneth B. Howell. second edition. CRC Press. FL, USA. 2020
Section: Chapter 22. Method of undetermined coefficients. Additional exercises page 412
Problem number: 22.1 (c).
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$y'' + 4y' - 5y = 30\,\mathrm{e}^{-4x}$$

✓ Solution by Maple
Time used: 0.016 (sec). Leaf size: 21

```
dsolve(diff(y(x),x$2)+4*diff(y(x),x)-5*y(x)=30*exp(-4*x),y(x), singsol=all)
```

$$y(x) = \left(\mathrm{e}^{6x} c_2 - 6\,\mathrm{e}^{x} + c_1\right) \mathrm{e}^{-5x}$$

✓ Solution by Mathematica
Time used: 0.018 (sec). Leaf size: 27

```
DSolve[y''[x]+4*y'[x]-5*y[x]==30*Exp[-4*x],y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to e^{-5x}\left(-6e^{x} + c_2 e^{6x} + c_1\right)$$

15.4 problem 22.1 (d)

Internal problem ID [13699]

Book: Ordinary Differential Equations. An introduction to the fundamentals. Kenneth B. Howell. second edition. CRC Press. FL, USA. 2020
Section: Chapter 22. Method of undetermined coefficients. Additional exercises page 412
Problem number: 22.1 (d).
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _missing_y]]`

$$\boxed{y'' + 3y' = e^{\frac{x}{2}}}$$

✓ Solution by Maple
Time used: 0.0 (sec). Leaf size: 25

```
dsolve(diff(y(x),x$2)+3*diff(y(x),x)=exp(x/2),y(x), singsol=all)
```

$$y(x) = -\frac{e^{-3x}\left(-3c_2 e^{3x} + c_1 - \frac{12\,e^{\frac{7x}{2}}}{7}\right)}{3}$$

✓ Solution by Mathematica
Time used: 0.078 (sec). Leaf size: 30

```
DSolve[y''[x]+3*y'[x]==30*Exp[x/2],y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to \frac{120e^{x/2}}{7} - \frac{1}{3}c_1 e^{-3x} + c_2$$

15.5 problem 22.2

Internal problem ID [13700]

Book: Ordinary Differential Equations. An introduction to the fundamentals. Kenneth B. Howell. second edition. CRC Press. FL, USA. 2020
Section: Chapter 22. Method of undetermined coefficients. Additional exercises page 412
Problem number: 22.2.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$y'' - 3y' - 10y = -5\,\mathrm{e}^{3x}$$

With initial conditions

$$[y(0) = 5, y'(0) = 3]$$

✓ Solution by Maple
Time used: 0.015 (sec). Leaf size: 22

```
dsolve([diff(y(x),x$2)-3*diff(y(x),x)-10*y(x)=-5*exp(3*x),y(0) = 5, D(y)(0) = 3],y(x), sings
```

$$y(x) = \frac{(3\,\mathrm{e}^{7x} + \mathrm{e}^{5x} + 6)\,\mathrm{e}^{-2x}}{2}$$

✓ Solution by Mathematica
Time used: 0.017 (sec). Leaf size: 28

```
DSolve[{y''[x]-3*y'[x]-10*y[x]==-5*Exp[3*x],{y[0]==5,y'[0]==3}},y[x],x,IncludeSingularSoluti
```

$$y(x) \to \frac{1}{2}e^{-2x}\left(e^{5x} + 3e^{7x} + 6\right)$$

15.6 problem 22.3 (a)

Internal problem ID [13701]

Book: Ordinary Differential Equations. An introduction to the fundamentals. Kenneth B. Howell. second edition. CRC Press. FL, USA. 2020
Section: Chapter 22. Method of undetermined coefficients. Additional exercises page 412
Problem number: 22.3 (a).
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _linear, _nonhomogeneous]]`

$$\boxed{y'' + 9y = 10\cos(2x) + 15\sin(2x)}$$

✓ Solution by Maple
Time used: 0.0 (sec). Leaf size: 29

```
dsolve(diff(y(x),x$2)+9*y(x)=10*cos(2*x)+15*sin(2*x),y(x), singsol=all)
```

$$y(x) = c_2 \sin(3x) + c_1 \cos(3x) + 3\sin(2x) + 2\cos(2x)$$

✓ Solution by Mathematica
Time used: 0.02 (sec). Leaf size: 32

```
DSolve[y''[x]+9*y[x]==10*Cos[2*x]+15*Sin[2*x],y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \rightarrow 3\sin(2x) + 2\cos(2x) + c_1 \cos(3x) + c_2 \sin(3x)$$

15.7 problem 22.3 (b)

Internal problem ID [13702]

Book: Ordinary Differential Equations. An introduction to the fundamentals. Kenneth B. Howell. second edition. CRC Press. FL, USA. 2020
Section: Chapter 22. Method of undetermined coefficients. Additional exercises page 412
Problem number: 22.3 (b).
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _linear, _nonhomogeneous]]`

$$\boxed{y'' - 6y' + 9y = 25\sin(6x)}$$

✓ Solution by Maple
Time used: 0.0 (sec). Leaf size: 27

```
dsolve(diff(y(x),x$2)-6*diff(y(x),x)+9*y(x)=25*sin(6*x),y(x), singsol=all)
```

$$y(x) = (c_1x + c_2)\,\mathrm{e}^{3x} + \frac{4\cos(6x)}{9} - \frac{\sin(6x)}{3}$$

✓ Solution by Mathematica
Time used: 0.022 (sec). Leaf size: 35

```
DSolve[y''[x]-6*y'[x]+9*y[x]==25*Sin[6*x],y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to -\frac{1}{3}\sin(6x) + \frac{4}{9}\cos(6x) + e^{3x}(c_2x + c_1)$$

15.8 problem 22.3 (c)

Internal problem ID [13703]

Book: Ordinary Differential Equations. An introduction to the fundamentals. Kenneth B. Howell. second edition. CRC Press. FL, USA. 2020
Section: Chapter 22. Method of undetermined coefficients. Additional exercises page 412
Problem number: 22.3 (c).
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _missing_y]]`

$$\boxed{y'' + 3y' = 26\cos\left(\frac{x}{3}\right) - 12\sin\left(\frac{x}{3}\right)}$$

✓ Solution by Maple
Time used: 0.0 (sec). Leaf size: 25

```
dsolve(diff(y(x),x$2)+3*diff(y(x),x)=26*cos(x/3)-12*sin(x/3),y(x), singsol=all)
```

$$y(x) = -\frac{c_1 e^{-3x}}{3} + 27\sin\left(\frac{x}{3}\right) + 9\cos\left(\frac{x}{3}\right) + c_2$$

✓ Solution by Mathematica
Time used: 0.29 (sec). Leaf size: 35

```
DSolve[y''[x]+3*y'[x]==26*Cos[x/3]-12*Sin[x/3],y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to 27\sin\left(\frac{x}{3}\right) + 9\cos\left(\frac{x}{3}\right) - \frac{1}{3}c_1 e^{-3x} + c_2$$

15.9 problem 22.3 (d)

Internal problem ID [13704]

Book: Ordinary Differential Equations. An introduction to the fundamentals. Kenneth B. Howell. second edition. CRC Press. FL, USA. 2020
Section: Chapter 22. Method of undetermined coefficients. Additional exercises page 412
Problem number: 22.3 (d).
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _linear, _nonhomogeneous]]`

$$\boxed{y'' + 4y' - 5y = \cos(x)}$$

✓ Solution by Maple
Time used: 0.0 (sec). Leaf size: 32

```
dsolve(diff(y(x),x$2)+4*diff(y(x),x)-5*y(x)=cos(x),y(x), singsol=all)
```

$$y(x) = \left(\frac{(-3\cos(x) + 2\sin(x))\,\mathrm{e}^{5x}}{26} + c_1 + \mathrm{e}^{6x}c_2\right)\mathrm{e}^{-5x}$$

✓ Solution by Mathematica
Time used: 0.066 (sec). Leaf size: 32

```
DSolve[y''[x]+4*y'[x]-5*y[x]==Cos[x],y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to \frac{\sin(x)}{13} - \frac{3\cos(x)}{26} + c_1e^{-5x} + c_2e^{x}$$

15.10 problem 22.4

Internal problem ID [13705]

Book: Ordinary Differential Equations. An introduction to the fundamentals. Kenneth B. Howell. second edition. CRC Press. FL, USA. 2020
Section: Chapter 22. Method of undetermined coefficients. Additional exercises page 412
Problem number: 22.4.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _linear, _nonhomogeneous]]`

$$y'' - 3y' - 10y = -4\cos(x) + 7\sin(x)$$

With initial conditions

$$[y(0) = 8, y'(0) = -5]$$

✓ Solution by Maple
Time used: 0.032 (sec). Leaf size: 25

```
dsolve([diff(y(x),x$2)-3*diff(y(x),x)-10*y(x)=-4*cos(x)+7*sin(x),y(0) = 8, D(y)(0) = -5],y(x
```

$$y(x) = \frac{e^{-2x}((\cos(x) - \sin(x))e^{2x} + 3e^{7x} + 12)}{2}$$

✓ Solution by Mathematica
Time used: 0.024 (sec). Leaf size: 30

```
DSolve[{y''[x]-3*y'[x]-10*y[x]==-4*Cos[x]+7*Sin[x],{y[0]==8,y'[0]==-5}},y[x],x,IncludeSingul
```

$$y(x) \to \frac{1}{2}\left(3e^{-2x}\left(e^{7x} + 4\right) - \sin(x) + \cos(x)\right)$$

15.11 problem 22.5 (a)

Internal problem ID [13706]

Book: Ordinary Differential Equations. An introduction to the fundamentals. Kenneth B. Howell. second edition. CRC Press. FL, USA. 2020
Section: Chapter 22. Method of undetermined coefficients. Additional exercises page 412
Problem number: 22.5 (a).
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _missing_x]]`

$$\boxed{y'' - 3y' - 10y = -200}$$

✓ Solution by Maple
Time used: 0.015 (sec). Leaf size: 23

```
dsolve(diff(y(x),x$2)-3*diff(y(x),x)-10*y(x)=-200,y(x), singsol=all)
```

$$y(x) = \left(c_2 e^{7x} + 20\, e^{2x} + c_1\right) e^{-2x}$$

✓ Solution by Mathematica
Time used: 0.013 (sec). Leaf size: 23

```
DSolve[y''[x]-3*y'[x]-10*y[x]==-200,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \rightarrow c_1 e^{-2x} + c_2 e^{5x} + 20$$

15.12 problem 22.5 (b)

Internal problem ID [13707]

Book: Ordinary Differential Equations. An introduction to the fundamentals. Kenneth B. Howell. second edition. CRC Press. FL, USA. 2020
Section: Chapter 22. Method of undetermined coefficients. Additional exercises page 412
Problem number: 22.5 (b).
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _linear, _nonhomogeneous]]`

$$\boxed{y''+4y'-5y=x^3}$$

✓ Solution by Maple
Time used: 0.016 (sec). Leaf size: 39

```
dsolve(diff(y(x),x$2)+4*diff(y(x),x)-5*y(x)=x^3,y(x), singsol=all)
```

$$y(x)=-\frac{e^{-5x}\left(\left(x^3+\frac{12}{5}x^2+\frac{126}{25}x+\frac{624}{125}\right)e^{5x}-5\,e^{6x}c_2-5c_1\right)}{5}$$

✓ Solution by Mathematica
Time used: 0.013 (sec). Leaf size: 39

```
DSolve[y''[x]+4*y'[x]-5*y[x]==x^3,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x)\to\frac{1}{625}\left(-125x^3-300x^2-630x-624\right)+c_1e^{-5x}+c_2e^x$$

15.13 problem 22.5 (c)

Internal problem ID [13708]

Book: Ordinary Differential Equations. An introduction to the fundamentals. Kenneth B. Howell. second edition. CRC Press. FL, USA. 2020
Section: Chapter 22. Method of undetermined coefficients. Additional exercises page 412
Problem number: 22.5 (c).
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$y'' - 6y' + 9y = 18x^2 + 3x + 4$$

✓ Solution by Maple
Time used: 0.0 (sec). Leaf size: 24

```
dsolve(diff(y(x),x$2)-6*diff(y(x),x)+9*y(x)=18*x^2+3*x+4,y(x), singsol=all)
```

$$y(x) = (c_1 x + c_2) e^{3x} + 2x^2 + 3x + 2$$

✓ Solution by Mathematica
Time used: 0.015 (sec). Leaf size: 32

```
DSolve[y''[x]-6*y'[x]+9*y[x]==18*x^2+3*x+4,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to 2x^2 + x(3 + c_2 e^{3x}) + c_1 e^{3x} + 2$$

15.14 problem 22.5 (d)

Internal problem ID [13709]

Book: Ordinary Differential Equations. An introduction to the fundamentals. Kenneth B. Howell. second edition. CRC Press. FL, USA. 2020
Section: Chapter 22. Method of undetermined coefficients. Additional exercises page 412
Problem number: 22.5 (d).
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _linear, _nonhomogeneous]]`

$$\boxed{y'' + 9y = 9x^4 - 9}$$

✓ Solution by Maple
Time used: 0.016 (sec). Leaf size: 26

```
dsolve(diff(y(x),x$2)+9*y(x)=9*x^4-9,y(x), singsol=all)
```

$$y(x) = c_2 \sin(3x) + c_1 \cos(3x) + x^4 - \frac{4x^2}{3} - \frac{19}{27}$$

✓ Solution by Mathematica
Time used: 0.014 (sec). Leaf size: 33

```
DSolve[y''[x]+9*y[x]==9*x^4-9,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to x^4 - \frac{4x^2}{3} + c_1 \cos(3x) + c_2 \sin(3x) - \frac{19}{27}$$

15.15 problem 22.6

Internal problem ID [13710]

Book: Ordinary Differential Equations. An introduction to the fundamentals. Kenneth B. Howell. second edition. CRC Press. FL, USA. 2020
Section: Chapter 22. Method of undetermined coefficients. Additional exercises page 412
Problem number: 22.6.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _linear, _nonhomogeneous]]`

$$\boxed{y'' + 9y = x^3}$$

With initial conditions

$$[y(0) = 0, y'(0) = 0]$$

✓ Solution by Maple
Time used: 0.016 (sec). Leaf size: 19

```
dsolve([diff(y(x),x$2)+9*y(x)=x^3,y(0) = 0, D(y)(0) = 0],y(x), singsol=all)
```

$$y(x) = \frac{2\sin(3x)}{81} + \frac{x^3}{9} - \frac{2x}{27}$$

✓ Solution by Mathematica
Time used: 0.014 (sec). Leaf size: 24

```
DSolve[{y''[x]+9*y[x]==x^3,{y[0]==0,y'[0]==0}},y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \rightarrow \frac{1}{81}\left(9x^3 - 6x + 2\sin(3x)\right)$$

15.16 problem 22.7 (a)

Internal problem ID [13711]

Book: Ordinary Differential Equations. An introduction to the fundamentals. Kenneth B. Howell. second edition. CRC Press. FL, USA. 2020
Section: Chapter 22. Method of undetermined coefficients. Additional exercises page 412
Problem number: 22.7 (a).
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _linear, _nonhomogeneous]]`

$$\boxed{y'' + 9y = 25x\cos(2x)}$$

✓ Solution by Maple
Time used: 0.0 (sec). Leaf size: 30

```
dsolve(diff(y(x),x$2)+9*y(x)=25*x*cos(2*x),y(x), singsol=all)
```

$$y(x) = c_2 \sin(3x) + c_1 \cos(3x) + 5\cos(2x)\, x + 4\sin(2x)$$

✓ Solution by Mathematica
Time used: 0.021 (sec). Leaf size: 33

```
DSolve[y''[x]+9*y[x]==25*x*Cos[2*x],y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to 4\sin(2x) + 5x\cos(2x) + c_1\cos(3x) + c_2\sin(3x)$$

15.17 problem 22.7 (b)

Internal problem ID [13712]

Book: Ordinary Differential Equations. An introduction to the fundamentals. Kenneth B. Howell. second edition. CRC Press. FL, USA. 2020
Section: Chapter 22. Method of undetermined coefficients. Additional exercises page 412
Problem number: 22.7 (b).
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _linear, _nonhomogeneous]]`

$$\boxed{y'' - 6y' + 9y = e^{2x} \sin(x)}$$

✓ Solution by Maple
Time used: 0.0 (sec). Leaf size: 23

```
dsolve(diff(y(x),x$2)-6*diff(y(x),x)+9*y(x)=exp(2*x)*sin(x),y(x), singsol=all)
```

$$y(x) = (c_1 x + c_2)\, e^{3x} + \frac{e^{2x} \cos(x)}{2}$$

✓ Solution by Mathematica
Time used: 0.085 (sec). Leaf size: 29

```
DSolve[y''[x]-6*y'[x]+9*y[x]==Exp[2*x]*Sin[x],y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to \frac{1}{2} e^{2x} (\cos(x) + 2e^{x}(c_2 x + c_1))$$

15.18 problem 22.7 (c)

Internal problem ID [13713]

Book: Ordinary Differential Equations. An introduction to the fundamentals. Kenneth B. Howell. second edition. CRC Press. FL, USA. 2020
Section: Chapter 22. Method of undetermined coefficients. Additional exercises page 412
Problem number: 22.7 (c).
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _linear, _nonhomogeneous]]`

$$\boxed{y'' + 9y = 54\,\mathrm{e}^{3x}x^2}$$

✓ Solution by Maple
Time used: 0.016 (sec). Leaf size: 28

```
dsolve(diff(y(x),x$2)+9*y(x)=54*x^2*exp(3*x),y(x), singsol=all)
```

$$y(x) = c_2 \sin(3x) + c_1 \cos(3x) + 3\left(x - \frac{1}{3}\right)^2 \mathrm{e}^{3x}$$

✓ Solution by Mathematica
Time used: 0.019 (sec). Leaf size: 36

```
DSolve[y''[x]+9*y[x]==54*x^2*Exp[3*x],y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to \frac{1}{3}e^{3x}(1-3x)^2 + c_1 \cos(3x) + c_2 \sin(3x)$$

15.19 problem 22.7 (d)

Internal problem ID [13714]

Book: Ordinary Differential Equations. An introduction to the fundamentals. Kenneth B. Howell. second edition. CRC Press. FL, USA. 2020
Section: Chapter 22. Method of undetermined coefficients. Additional exercises page 412
Problem number: 22.7 (d).
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _quadrature]]`

$$\boxed{y'' = 6x\,\mathrm{e}^{x} \sin(x)}$$

✓ Solution by Maple
Time used: 0.0 (sec). Leaf size: 25

```
dsolve(diff(y(x),x$2)=6*x*exp(x)*sin(x),y(x), singsol=all)
```

$$y(x) = ((-3x+3)\cos(x) + 3\sin(x))\,\mathrm{e}^{x} + c_1 x + c_2$$

✓ Solution by Mathematica
Time used: 0.104 (sec). Leaf size: 27

```
DSolve[y''[x]==6*x*Exp[x]*Sin[x],y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to c_2 x + 3e^{x}(\sin(x) - x\cos(x) + \cos(x)) + c_1$$

15.20 problem 22.7 (e)

Internal problem ID [13715]

Book: Ordinary Differential Equations. An introduction to the fundamentals. Kenneth B. Howell. second edition. CRC Press. FL, USA. 2020
Section: Chapter 22. Method of undetermined coefficients. Additional exercises page 412
Problem number: 22.7 (e).
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _linear, _nonhomogeneous]]`

$$y'' - 2y' + y = (-6x - 8) \cos (2x) + (8x - 11) \sin (2x)$$

✓ Solution by Maple
Time used: 0.0 (sec). Leaf size: 24

```
dsolve(diff(y(x),x$2)-2*diff(y(x),x)+y(x)=(-6*x-8)*cos(2*x)+(8*x-11)*sin(2*x),y(x), singsol=
```

$$y(x) = 2 \cos (2x) x + \sin (2x) + (c_1 x + c_2) e^x$$

✓ Solution by Mathematica
Time used: 0.027 (sec). Leaf size: 28

```
DSolve[y''[x]-2*y'[x]+y[x]==(-6*x-8)*Cos[2*x]+(8*x-11)*Sin[2*x],y[x],x,IncludeSingularSoluti
```

$$y(x) \to \sin(2x) + 2x \cos(2x) + e^x (c_2 x + c_1)$$

15.21 problem 22.7 (f)

Internal problem ID [13716]

Book: Ordinary Differential Equations. An introduction to the fundamentals. Kenneth B. Howell. second edition. CRC Press. FL, USA. 2020
Section: Chapter 22. Method of undetermined coefficients. Additional exercises page 412
Problem number: 22.7 (f).
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _linear, _nonhomogeneous]]`

$$y'' - 2y' + y = (12x - 4)\,e^{-5x}$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 23

```
dsolve(diff(y(x),x$2)-2*diff(y(x),x)+y(x)=(12*x-4)*exp(-5*x),y(x), singsol=all)
```

$$y(x) = \left((c_1x + c_2)\,e^{6x} + \frac{x}{3}\right)e^{-5x}$$

✓ Solution by Mathematica

Time used: 0.02 (sec). Leaf size: 27

```
DSolve[y''[x]-2*y'[x]+y[x]==(12*x-4)*Exp[-5*x],y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to \frac{1}{3}e^{-5x}x + e^x(c_2x + c_1)$$

15.22 problem 22.8

Internal problem ID [13717]

Book: Ordinary Differential Equations. An introduction to the fundamentals. Kenneth B. Howell. second edition. CRC Press. FL, USA. 2020
Section: Chapter 22. Method of undetermined coefficients. Additional exercises page 412
Problem number: 22.8.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _linear, _nonhomogeneous]]`

$$\boxed{y'' + 9y = 39\,\mathrm{e}^{2x} x}$$

With initial conditions

$$[y(0) = 1, y'(0) = 0]$$

✓ Solution by Maple
Time used: 0.031 (sec). Leaf size: 26

```
dsolve([diff(y(x),x$2)+9*y(x)=39*x*exp(2*x),y(0) = 1, D(y)(0) = 0],y(x), singsol=all)
```

$$y(x) = 3x\,\mathrm{e}^{2x} + \frac{25\cos(3x)}{13} - \frac{5\sin(3x)}{13} - \frac{12\,\mathrm{e}^{2x}}{13}$$

✓ Solution by Mathematica
Time used: 0.019 (sec). Leaf size: 34

```
DSolve[{y''[x]+9*y[x]==39*x*Exp[2*x],{y[0]==1,y'[0]==0}},y[x],x,IncludeSingularSolutions ->
```

$$y(x) \to \frac{1}{13}\left(3e^{2x}(13x - 4) - 5\sin(3x) + 25\cos(3x)\right)$$

15.23 problem 22.9 (a)

Internal problem ID [13718]

Book: Ordinary Differential Equations. An introduction to the fundamentals. Kenneth B. Howell. second edition. CRC Press. FL, USA. 2020
Section: Chapter 22. Method of undetermined coefficients. Additional exercises page 412
Problem number: 22.9 (a).
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$y'' - 3y' - 10y = -3\,\mathrm{e}^{-2x}$$

✓ Solution by Maple
Time used: 0.0 (sec). Leaf size: 24

```
dsolve(diff(y(x),x$2)-3*diff(y(x),x)-10*y(x)=-3*exp(-2*x),y(x), singsol=all)
```

$$y(x) = \frac{(7c_2\mathrm{e}^{7x} + 7c_1 + 3x)\,\mathrm{e}^{-2x}}{7}$$

✓ Solution by Mathematica
Time used: 0.035 (sec). Leaf size: 32

```
DSolve[y''[x]-3*y'[x]-10*y[x]==-3*Exp[-2*x],y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to \frac{1}{49}e^{-2x}\left(21x + 49c_2e^{7x} + 3 + 49c_1\right)$$

15.24 problem 22.9 (b)

Internal problem ID [13719]

Book: Ordinary Differential Equations. An introduction to the fundamentals. Kenneth B. Howell. second edition. CRC Press. FL, USA. 2020
Section: Chapter 22. Method of undetermined coefficients. Additional exercises page 412
Problem number: 22.9 (b).
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _missing_x]]`

$$\boxed{y'' + 4y' = 20}$$

✓ Solution by Maple
Time used: 0.016 (sec). Leaf size: 16

```
dsolve(diff(y(x),x$2)+4*diff(y(x),x)=20,y(x), singsol=all)
```

$$y(x) = -\frac{e^{-4x} c_1}{4} + 5x + c_2$$

✓ Solution by Mathematica
Time used: 0.017 (sec). Leaf size: 22

```
DSolve[y''[x]+4*y'[x]==20,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to 5x - \frac{1}{4} c_1 e^{-4x} + c_2$$

15.25 problem 22.9 (c)

Internal problem ID [13720]

Book: Ordinary Differential Equations. An introduction to the fundamentals. Kenneth B. Howell. second edition. CRC Press. FL, USA. 2020
Section: Chapter 22. Method of undetermined coefficients. Additional exercises page 412
Problem number: 22.9 (c).
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _missing_y]]`

$$\boxed{y'' + 4y' = x^2}$$

✓ Solution by Maple
Time used: 0.0 (sec). Leaf size: 26

```
dsolve(diff(y(x),x$2)+4*diff(y(x),x)=x^2,y(x), singsol=all)
```

$$y(x) = -\frac{x^2}{16} + \frac{x^3}{12} - \frac{e^{-4x}c_1}{4} + \frac{x}{32} + c_2$$

✓ Solution by Mathematica
Time used: 0.074 (sec). Leaf size: 36

```
DSolve[y''[x]+4*y'[x]==x^2,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to \frac{1}{96}\left(8x^3 - 6x^2 + 3x - 24c_1e^{-4x} + 96c_2\right)$$

15.26 problem 22.9 (d)

Internal problem ID [13721]

Book: Ordinary Differential Equations. An introduction to the fundamentals. Kenneth B. Howell. second edition. CRC Press. FL, USA. 2020
Section: Chapter 22. Method of undetermined coefficients. Additional exercises page 412
Problem number: 22.9 (d).
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _linear, _nonhomogeneous]]`

$$\boxed{y'' + 9y = 3\sin(3x)}$$

✓ Solution by Maple
Time used: 0.0 (sec). Leaf size: 24

```
dsolve(diff(y(x),x$2)+9*y(x)=3*sin(3*x),y(x), singsol=all)
```

$$y(x) = \frac{(2c_1 - x)\cos(3x)}{2} + c_2 \sin(3x)$$

✓ Solution by Mathematica
Time used: 0.108 (sec). Leaf size: 33

```
DSolve[y''[x]+9*y[x]==3*Sin[3*x],y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to \left(-\frac{x}{2} + c_1\right)\cos(3x) + \frac{1}{12}(1 + 12c_2)\sin(3x)$$

15.27 problem 22.9 (e)

Internal problem ID [13722]

Book: Ordinary Differential Equations. An introduction to the fundamentals. Kenneth B. Howell. second edition. CRC Press. FL, USA. 2020
Section: Chapter 22. Method of undetermined coefficients. Additional exercises page 412
Problem number: 22.9 (e).
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$y'' - 6y' + 9y = 10\,e^{3x}$$

✓ Solution by Maple
Time used: 0.0 (sec). Leaf size: 19

```
dsolve(diff(y(x),x$2)-6*diff(y(x),x)+9*y(x)=10*exp(3*x),y(x), singsol=all)
```

$$y(x) = e^{3x}(c_1 x + 5x^2 + c_2)$$

✓ Solution by Mathematica
Time used: 0.019 (sec). Leaf size: 23

```
DSolve[y''[x]-6*y'[x]+9*y[x]==10*Exp[3*x],y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to e^{3x}(5x^2 + c_2 x + c_1)$$

15.28 problem 22.10 (a)

Internal problem ID [13723]

Book: Ordinary Differential Equations. An introduction to the fundamentals. Kenneth B. Howell. second edition. CRC Press. FL, USA. 2020
Section: Chapter 22. Method of undetermined coefficients. Additional exercises page 412
Problem number: 22.10 (a).
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _linear, _nonhomogeneous]]`

$$y'' - 3y' - 10y = (72x^2 - 1)\, e^{2x}$$

✓ Solution by Maple
Time used: 0.0 (sec). Leaf size: 37

```
dsolve(diff(y(x),x$2)-3*diff(y(x),x)-10*y(x)=(72*x^2-1)*exp(2*x),y(x), singsol=all)
```

$$y(x) = (-6x^2 - x - 1)\, e^{-2x} e^{4x} + (c_2 e^{7x} + c_1)\, e^{-2x}$$

✓ Solution by Mathematica
Time used: 0.021 (sec). Leaf size: 37

```
DSolve[y''[x]-3*y'[x]-10*y[x]==(72*x^2-1)*Exp[2*x],y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to -e^{2x}(6x^2 + x + 1) + c_1 e^{-2x} + c_2 e^{5x}$$

15.29 problem 22.10 (b)

Internal problem ID [13724]

Book: Ordinary Differential Equations. An introduction to the fundamentals. Kenneth B. Howell. second edition. CRC Press. FL, USA. 2020
Section: Chapter 22. Method of undetermined coefficients. Additional exercises page 412
Problem number: 22.10 (b).
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _linear, _nonhomogeneous]]`

$$\boxed{y'' - 3y' - 10y = 4x\,e^{6x}}$$

✓ Solution by Maple

Time used: 0.015 (sec). Leaf size: 34

```
dsolve(diff(y(x),x$2)-3*diff(y(x),x)-10*y(x)=4*x*exp(6*x),y(x), singsol=all)
```

$$y(x) = \frac{\left(8\,e^{8x}x - 9\,e^{8x} + 16c_2e^{7x} + 16c_1\right)e^{-2x}}{16}$$

✓ Solution by Mathematica

Time used: 0.018 (sec). Leaf size: 36

```
DSolve[y''[x]-3*y'[x]-10*y[x]==4*x*Exp[6*x],y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to \frac{1}{16}e^{6x}(8x-9) + c_1e^{-2x} + c_2e^{5x}$$

15.30 problem 22.10 (c)

Internal problem ID [13725]

Book: Ordinary Differential Equations. An introduction to the fundamentals. Kenneth B. Howell. second edition. CRC Press. FL, USA. 2020
Section: Chapter 22. Method of undetermined coefficients. Additional exercises page 412
Problem number: 22.10 (c).
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$\boxed{y'' - 10y' + 25y = 6\,\mathrm{e}^{5x}}$$

✓ Solution by Maple
Time used: 0.016 (sec). Leaf size: 19

```
dsolve(diff(y(x),x$2)-10*diff(y(x),x)+25*y(x)=6*exp(5*x),y(x), singsol=all)
```

$$y(x) = \mathrm{e}^{5x}(c_1x + 3x^2 + c_2)$$

✓ Solution by Mathematica
Time used: 0.021 (sec). Leaf size: 23

```
DSolve[y''[x]-10*y'[x]+25*y[x]==6*Exp[5*x],y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to e^{5x}(3x^2 + c_2x + c_1)$$

15.31 problem 22.10 (d)

Internal problem ID [13726]

Book: Ordinary Differential Equations. An introduction to the fundamentals. Kenneth B. Howell. second edition. CRC Press. FL, USA. 2020
Section: Chapter 22. Method of undetermined coefficients. Additional exercises page 412
Problem number: 22.10 (d).
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$y'' - 10y' + 25y = 6\,e^{-5x}$$

✓ Solution by Maple
Time used: 0.0 (sec). Leaf size: 21

```
dsolve(diff(y(x),x$2)-10*diff(y(x),x)+25*y(x)=6*exp(-5*x),y(x), singsol=all)
```

$$y(x) = (c_1 x + c_2)\,e^{5x} + \frac{3\,e^{-5x}}{50}$$

✓ Solution by Mathematica
Time used: 0.019 (sec). Leaf size: 28

```
DSolve[y''[x]-10*y'[x]+25*y[x]==6*Exp[-5*x],y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to \frac{3e^{-5x}}{50} + e^{5x}(c_2 x + c_1)$$

15.32 problem 22.10 (e)

Internal problem ID [13727]

Book: Ordinary Differential Equations. An introduction to the fundamentals. Kenneth B. Howell. second edition. CRC Press. FL, USA. 2020
Section: Chapter 22. Method of undetermined coefficients. Additional exercises page 412
Problem number: 22.10 (e).
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _linear, _nonhomogeneous]]`

$$\boxed{y'' + 4y' + 5y = 24 \sin(3x)}$$

✓ Solution by Maple
Time used: 0.016 (sec). Leaf size: 33

```
dsolve(diff(y(x),x$2)+4*diff(y(x),x)+5*y(x)=24*sin(3*x),y(x), singsol=all)
```

$$y(x) = \mathrm{e}^{-2x} \sin(x) c_2 + \mathrm{e}^{-2x} \cos(x) c_1 - \frac{3\sin(3x)}{5} - \frac{9\cos(3x)}{5}$$

✓ Solution by Mathematica
Time used: 0.023 (sec). Leaf size: 41

```
DSolve[y''[x]+4*y'[x]+5*y[x]==24*Sin[3*x],y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to -\frac{3}{5}(\sin(3x) + 3\cos(3x)) + c_2 e^{-2x} \cos(x) + c_1 e^{-2x} \sin(x)$$

15.33 problem 22.10 (f)

Internal problem ID [13728]

Book: Ordinary Differential Equations. An introduction to the fundamentals. Kenneth B. Howell. second edition. CRC Press. FL, USA. 2020
Section: Chapter 22. Method of undetermined coefficients. Additional exercises page 412
Problem number: 22.10 (f).
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$y'' + 4y' + 5y = 8\,\mathrm{e}^{-3x}$$

✓ Solution by Maple
Time used: 0.0 (sec). Leaf size: 27

```
dsolve(diff(y(x),x$2)+4*diff(y(x),x)+5*y(x)=8*exp(-3*x),y(x), singsol=all)
```

$$y(x) = \mathrm{e}^{-2x}\sin(x)\,c_2 + \mathrm{e}^{-2x}\cos(x)\,c_1 + 4\,\mathrm{e}^{-3x}$$

✓ Solution by Mathematica
Time used: 0.021 (sec). Leaf size: 29

```
DSolve[y''[x]+4*y'[x]+5*y[x]==8*Exp[-3*x],y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to e^{-3x}(c_2e^x\cos(x) + c_1e^x\sin(x) + 4)$$

15.34 problem 22.10 (g)

Internal problem ID [13729]

Book: Ordinary Differential Equations. An introduction to the fundamentals. Kenneth B. Howell. second edition. CRC Press. FL, USA. 2020
Section: Chapter 22. Method of undetermined coefficients. Additional exercises page 412
Problem number: 22.10 (g).
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type **[[_2nd_order, _linear, _nonhomogeneous]]**

$$\boxed{y'' - 4y' + 5y = e^{2x} \sin(x)}$$

✓ Solution by Maple
Time used: 0.016 (sec). Leaf size: 27

```
dsolve(diff(y(x),x$2)-4*diff(y(x),x)+5*y(x)=exp(2*x)*sin(x),y(x), singsol=all)
```

$$y(x) = -\frac{((x - 2c_1)\cos(x) + (-2c_2 - 1)\sin(x))\,e^{2x}}{2}$$

✓ Solution by Mathematica
Time used: 0.052 (sec). Leaf size: 30

```
DSolve[y''[x]-4*y'[x]+5*y[x]==Exp[2*x]*Sin[x],y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to -\frac{1}{2}e^{2x}((x - 2c_2)\cos(x) - 2c_1\sin(x))$$

15.35 problem 22.10 (h)

Internal problem ID [13730]

Book: Ordinary Differential Equations. An introduction to the fundamentals. Kenneth B. Howell. second edition. CRC Press. FL, USA. 2020
Section: Chapter 22. Method of undetermined coefficients. Additional exercises page 412
Problem number: 22.10 (h).
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _linear, _nonhomogeneous]]`

$$y'' - 4y' + 5y = \mathrm{e}^{-x} \sin(x)$$

✓ Solution by Maple
Time used: 0.016 (sec). Leaf size: 34

```
dsolve(diff(y(x),x$2)-4*diff(y(x),x)+5*y(x)=exp(-x)*sin(x),y(x), singsol=all)
```

$$y(x) = \frac{\mathrm{e}^{-x}(2\cos(x) + 3\sin(x))}{39} + (\sin(x)\, c_2 + c_1 \cos(x))\, \mathrm{e}^{2x}$$

✓ Solution by Mathematica
Time used: 0.16 (sec). Leaf size: 44

```
DSolve[y''[x]-4*y'[x]+5*y[x]==Exp[-x]*Sin[x],y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to \frac{1}{78} e^{-x}\left(\left(4 + 78c_2 e^{3x}\right)\cos(x) + 6\left(1 + 13c_1 e^{3x}\right)\sin(x)\right)$$

15.36 problem 22.10 (i)

Internal problem ID [13731]

Book: Ordinary Differential Equations. An introduction to the fundamentals. Kenneth B. Howell. second edition. CRC Press. FL, USA. 2020
Section: Chapter 22. Method of undetermined coefficients. Additional exercises page 412
Problem number: 22.10 (i).
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _missing_x]]`

$$\boxed{y'' - 4y' + 5y = 100}$$

✓ Solution by Maple
Time used: 0.0 (sec). Leaf size: 22

```
dsolve(diff(y(x),x$2)-4*diff(y(x),x)+5*y(x)=100,y(x), singsol=all)
```

$$y(x) = e^{2x} \sin(x) c_2 + e^{2x} \cos(x) c_1 + 20$$

✓ Solution by Mathematica
Time used: 0.014 (sec). Leaf size: 27

```
DSolve[y''[x]-4*y'[x]+5*y[x]==100,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to c_2 e^{2x} \cos(x) + c_1 e^{2x} \sin(x) + 20$$

15.37 problem 22.10 (j)

Internal problem ID [13732]

Book: Ordinary Differential Equations. An introduction to the fundamentals. Kenneth B. Howell. second edition. CRC Press. FL, USA. 2020
Section: Chapter 22. Method of undetermined coefficients. Additional exercises page 412
Problem number: 22.10 (j).
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$\boxed{y'' - 4y' + 5y = \mathrm{e}^{-x}}$$

✓ Solution by Maple
Time used: 0.0 (sec). Leaf size: 27

```
dsolve(diff(y(x),x$2)-4*diff(y(x),x)+5*y(x)=exp(-x),y(x), singsol=all)
```

$$y(x) = \mathrm{e}^{2x} \sin(x) c_2 + \mathrm{e}^{2x} \cos(x) c_1 + \frac{\mathrm{e}^{-x}}{10}$$

✓ Solution by Mathematica
Time used: 0.067 (sec). Leaf size: 35

```
DSolve[y''[x]-4*y'[x]+5*y[x]==Exp[-x],y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to \frac{e^{-x}}{10} + c_2 e^{2x} \cos(x) + c_1 e^{2x} \sin(x)$$

15.38 problem 22.10 (k)

Internal problem ID [13733]

Book: Ordinary Differential Equations. An introduction to the fundamentals. Kenneth B. Howell. second edition. CRC Press. FL, USA. 2020
Section: Chapter 22. Method of undetermined coefficients. Additional exercises page 412
Problem number: 22.10 (k).
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$\boxed{y'' - 4y' + 5y = 10x^2 + 4x + 8}$$

✓ Solution by Maple
Time used: 0.0 (sec). Leaf size: 30

```
dsolve(diff(y(x),x$2)-4*diff(y(x),x)+5*y(x)=10*x^2+4*x+8,y(x), singsol=all)
```

$$y(x) = e^{2x} \sin(x) c_2 + e^{2x} \cos(x) c_1 + 2x^2 + 4x + 4$$

✓ Solution by Mathematica
Time used: 0.018 (sec). Leaf size: 35

```
DSolve[y''[x]-4*y'[x]+5*y[x]==10*x^2+4*x+8,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to 2x^2 + 4x + c_2 e^{2x} \cos(x) + c_1 e^{2x} \sin(x) + 4$$

15.39 problem 22.10 (L)

Internal problem ID [13734]

Book: Ordinary Differential Equations. An introduction to the fundamentals. Kenneth B. Howell. second edition. CRC Press. FL, USA. 2020
Section: Chapter 22. Method of undetermined coefficients. Additional exercises page 412
Problem number: 22.10 (L).
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _linear, _nonhomogeneous]]`

$$\boxed{y'' + 9y = e^{2x} \sin(x)}$$

✓ Solution by Maple
Time used: 0.015 (sec). Leaf size: 32

```
dsolve(diff(y(x),x$2)+9*y(x)=exp(2*x)*sin(x),y(x), singsol=all)
```

$$y(x) = \frac{e^{2x}(3\sin(x) - \cos(x))}{40} + c_1 \cos(3x) + c_2 \sin(3x)$$

✓ Solution by Mathematica
Time used: 0.255 (sec). Leaf size: 42

```
DSolve[y''[x]+9*y[x]==Exp[2*x]*Sin[x],y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to \frac{3}{40}e^{2x}\sin(x) - \frac{1}{40}e^{2x}\cos(x) + c_1\cos(3x) + c_2\sin(3x)$$

15.40 problem 22.10 (m)

Internal problem ID [13735]

Book: Ordinary Differential Equations. An introduction to the fundamentals. Kenneth B. Howell. second edition. CRC Press. FL, USA. 2020
Section: Chapter 22. Method of undetermined coefficients. Additional exercises page 412
Problem number: 22.10 (m).
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _linear, _nonhomogeneous]]`

$$\boxed{y'' + y = 6\cos(x) - 3\sin(x)}$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 26

```
dsolve(diff(y(x),x$2)+y(x)=6*cos(x)-3*sin(x),y(x), singsol=all)
```

$$y(x) = \frac{(2c_1 + 3x + 6)\cos(x)}{2} + 3\left(x + \frac{c_2}{3}\right)\sin(x)$$

✓ Solution by Mathematica

Time used: 0.096 (sec). Leaf size: 27

```
DSolve[y''[x]+y[x]==6*Cos[x]-3*Sin[x],y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to \left(\frac{3x}{2} + 3 + c_1\right)\cos(x) + (3x + c_2)\sin(x)$$

15.41 problem 22.10 (n)

Internal problem ID [13736]

Book: Ordinary Differential Equations. An introduction to the fundamentals. Kenneth B. Howell. second edition. CRC Press. FL, USA. 2020
Section: Chapter 22. Method of undetermined coefficients. Additional exercises page 412
Problem number: 22.10 (n).
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _linear, _nonhomogeneous]]`

$$y'' + y = -3\sin(2x) + 6\cos(2x)$$

✓ Solution by Maple
Time used: 0.0 (sec). Leaf size: 23

```
dsolve(diff(y(x),x$2)+y(x)=6*cos(2*x)-3*sin(2*x),y(x), singsol=all)
```

$$y(x) = \sin(x) c_2 + c_1 \cos(x) + \sin(2x) - 2\cos(2x)$$

✓ Solution by Mathematica
Time used: 0.017 (sec). Leaf size: 26

```
DSolve[y''[x]+y[x]==6*Cos[2*x]-3*Sin[2*x],y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to \sin(2x) - 2\cos(2x) + c_1 \cos(x) + c_2 \sin(x)$$

15.42 problem 22.11 (a)

Internal problem ID [13737]

Book: Ordinary Differential Equations. An introduction to the fundamentals. Kenneth B. Howell. second edition. CRC Press. FL, USA. 2020
Section: Chapter 22. Method of undetermined coefficients. Additional exercises page 412
Problem number: 22.11 (a).
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _linear, _nonhomogeneous]]`

$$y'' - 4y' + 5y = x^3 e^{-x} \sin(x)$$

✓ Solution by Maple
Time used: 0.016 (sec). Leaf size: 61

```
dsolve(diff(y(x),x$2)-4*diff(y(x),x)+5*y(x)=x^3*exp(-x)*sin(x),y(x), singsol=all)
```

$$y(x) = \frac{\left((39546x^3 + 94302x^2 + 95160x + 38200)\cos(x) + 59319\left(x^3 + \frac{18}{13}x^2 + \frac{138}{169}x + \frac{360}{2197}\right)\sin(x)\right)e^{-x}}{771147} + (\sin(x)c_2 + c_1\cos(x))e^{2x}$$

✓ Solution by Mathematica
Time used: 0.051 (sec). Leaf size: 70

```
DSolve[y''[x]-4*y'[x]+5*y[x]==x^3*Exp[-x]*Sin[x],y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to \frac{e^{-x}((39546x^3 + 94302x^2 + 95160x + 771147c_2e^{3x} + 38200)\cos(x) + 27(2197x^3 + 3042x^2 + 1794x + 28}{771147}$$

15.43 problem 22.11 (b)

Internal problem ID [13738]

Book: Ordinary Differential Equations. An introduction to the fundamentals. Kenneth B. Howell. second edition. CRC Press. FL, USA. 2020
Section: Chapter 22. Method of undetermined coefficients. Additional exercises page 412
Problem number: 22.11 (b).
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _linear, _nonhomogeneous]]`

$$y'' - 4y' + 5y = x^3 e^{2x} \sin(x)$$

✓ Solution by Maple
Time used: 0.016 (sec). Leaf size: 40

```
dsolve(diff(y(x),x$2)-4*diff(y(x),x)+5*y(x)=x^3*exp(2*x)*sin(x),y(x), singsol=all)
```

$$y(x) = -\frac{e^{2x}\left((x^4 - 3x^2 - 8c_1)\cos(x) - 2\left(x^3 - \frac{3}{2}x + 4c_2\right)\sin(x)\right)}{8}$$

✓ Solution by Mathematica
Time used: 0.116 (sec). Leaf size: 51

```
DSolve[y''[x]-4*y'[x]+5*y[x]==x^3*Exp[2*x]*Sin[x],y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to \frac{1}{16}e^{2x}\left(2\left(2x^3 - 3x + 8c_1\right)\sin(x) + \left(-2x^4 + 6x^2 - 3 + 16c_2\right)\cos(x)\right)$$

15.44 problem 22.11 (c)

Internal problem ID [13739]

Book: Ordinary Differential Equations. An introduction to the fundamentals. Kenneth B. Howell. second edition. CRC Press. FL, USA. 2020
Section: Chapter 22. Method of undetermined coefficients. Additional exercises page 412
Problem number: 22.11 (c).
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _linear, _nonhomogeneous]]`

$$y'' - 5y' + 6y = x^2 e^{-7x} + 2\,e^{-7x}$$

✓ Solution by Maple
Time used: 0.0 (sec). Leaf size: 32

```
dsolve(diff(y(x),x$2)-5*diff(y(x),x)+6*y(x)=x^2*exp(-7*x)+2*exp(-7*x),y(x), singsol=all)
```

$$y(x) = \frac{e^{-7x}\left(x^2 + 90\,e^{9x}c_1 + 90c_2 e^{10x} + \frac{19x}{45} + \frac{8371}{4050}\right)}{90}$$

✓ Solution by Mathematica
Time used: 0.024 (sec). Leaf size: 41

```
DSolve[y''[x]-5*y'[x]+6*y[x]==x^2*Exp[-7*x]+2*Exp[-7*x],y[x],x,IncludeSingularSolutions -> T
```

$$y(x) \to \frac{e^{-7x}(4050x^2 + 1710x + 8371)}{364500} + c_1 e^{2x} + c_2 e^{3x}$$

15.45 problem 22.11 (d)

Internal problem ID [13740]

Book: Ordinary Differential Equations. An introduction to the fundamentals. Kenneth B. Howell. second edition. CRC Press. FL, USA. 2020
Section: Chapter 22. Method of undetermined coefficients. Additional exercises page 412
Problem number: 22.11 (d).
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$\boxed{y'' - 5y' + 6y = x^2}$$

✓ Solution by Maple
Time used: 0.0 (sec). Leaf size: 26

```
dsolve(diff(y(x),x$2)-5*diff(y(x),x)+6*y(x)=x^2,y(x), singsol=all)
```

$$y(x) = c_2 e^{3x} + e^{2x} c_1 + \frac{x^2}{6} + \frac{5x}{18} + \frac{19}{108}$$

✓ Solution by Mathematica
Time used: 0.013 (sec). Leaf size: 37

```
DSolve[y''[x]-5*y'[x]+6*y[x]==x^2,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to \frac{x^2}{6} + \frac{5x}{18} + c_1 e^{2x} + c_2 e^{3x} + \frac{19}{108}$$

15.46 problem 22.11 (e)

Internal problem ID [13741]

Book: Ordinary Differential Equations. An introduction to the fundamentals. Kenneth B. Howell. second edition. CRC Press. FL, USA. 2020
Section: Chapter 22. Method of undetermined coefficients. Additional exercises page 412
Problem number: 22.11 (e).
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$y'' - 5y' + 6y = 4\,\mathrm{e}^{-8x}$$

✓ Solution by Maple

Time used: 0.016 (sec). Leaf size: 23

```
dsolve(diff(y(x),x$2)-5*diff(y(x),x)+6*y(x)=4*exp(-8*x),y(x), singsol=all)
```

$$y(x) = \mathrm{e}^{-8x}\left(\mathrm{e}^{10x}c_1 + c_2\mathrm{e}^{11x} + \frac{2}{55}\right)$$

✓ Solution by Mathematica

Time used: 0.02 (sec). Leaf size: 31

```
DSolve[y''[x]-5*y'[x]+6*y[x]==4*Exp[-8*x],y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to \frac{2e^{-8x}}{55} + c_1e^{2x} + c_2e^{3x}$$

15.47 problem 22.11 (f)

Internal problem ID [13742]

Book: Ordinary Differential Equations. An introduction to the fundamentals. Kenneth B. Howell. second edition. CRC Press. FL, USA. 2020
Section: Chapter 22. Method of undetermined coefficients. Additional exercises page 412
Problem number: 22.11 (f).
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$\boxed{y'' - 5y' + 6y = 4\,\mathrm{e}^{3x}}$$

✓ Solution by Maple
Time used: 0.015 (sec). Leaf size: 21

```
dsolve(diff(y(x),x$2)-5*diff(y(x),x)+6*y(x)=4*exp(3*x),y(x), singsol=all)
```

$$y(x) = (4x + c_2)\,\mathrm{e}^{3x} + \mathrm{e}^{2x}c_1$$

✓ Solution by Mathematica
Time used: 0.023 (sec). Leaf size: 25

```
DSolve[y''[x]-5*y'[x]+6*y[x]==4*Exp[3*x],y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to e^{2x}(e^x(4x - 4 + c_2) + c_1)$$

15.48 problem 22.11 (g)

Internal problem ID [13743]

Book: Ordinary Differential Equations. An introduction to the fundamentals. Kenneth B. Howell. second edition. CRC Press. FL, USA. 2020
Section: Chapter 22. Method of undetermined coefficients. Additional exercises page 412
Problem number: 22.11 (g).
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _linear, _nonhomogeneous]]`

$$\boxed{y'' - 5y' + 6y = e^{3x}x^2}$$

✓ Solution by Maple
Time used: 0.0 (sec). Leaf size: 32

```
dsolve(diff(y(x),x$2)-5*diff(y(x),x)+6*y(x)=x^2*exp(3*x),y(x), singsol=all)
```

$$y(x) = \frac{(x^3 - 3x^2 + 3c_2 + 6x)\,e^{3x}}{3} + e^{2x}c_1$$

✓ Solution by Mathematica
Time used: 0.047 (sec). Leaf size: 40

```
DSolve[y''[x]-5*y'[x]+6*y[x]==x^2*Exp[3*x],y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to \frac{1}{3}e^{2x}\left(e^x\left(x^3 - 3x^2 + 6x - 6 + 3c_2\right) + 3c_1\right)$$

15.49 problem 22.11 (h)

Internal problem ID [13744]

Book: Ordinary Differential Equations. An introduction to the fundamentals. Kenneth B. Howell. second edition. CRC Press. FL, USA. 2020
Section: Chapter 22. Method of undetermined coefficients. Additional exercises page 412
Problem number: 22.11 (h).
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _linear, _nonhomogeneous]]`

$$y'' - 5y' + 6y = x^2 \cos(2x)$$

✓ Solution by Maple

Time used: 0.015 (sec). Leaf size: 49

```
dsolve(diff(y(x),x$2)-5*diff(y(x),x)+6*y(x)=x^2*cos(2*x),y(x), singsol=all)
```

$$y(x) = c_2 e^{3x} + e^{2x} c_1 + \frac{(676x^2 - 2080x - 1909)\cos(2x)}{35152} + \frac{(-3380x^2 - 3796x - 725)\sin(2x)}{35152}$$

✓ Solution by Mathematica

Time used: 0.033 (sec). Leaf size: 58

```
DSolve[y''[x]-5*y'[x]+6*y[x]==x^2*Cos[2*x],y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to \frac{(676x^2 - 2080x - 1909)\cos(2x) - (3380x^2 + 3796x + 725)\sin(2x)}{35152} + c_1 e^{2x} + c_2 e^{3x}$$

15.50 problem 22.11 (i)

Internal problem ID [13745]

Book: Ordinary Differential Equations. An introduction to the fundamentals. Kenneth B. Howell. second edition. CRC Press. FL, USA. 2020
Section: Chapter 22. Method of undetermined coefficients. Additional exercises page 412
Problem number: 22.11 (i).
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _linear, _nonhomogeneous]]`

$$\boxed{y'' - 5y' + 6y = x^2 e^{3x} \sin(2x)}$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 51

```
dsolve(diff(y(x),x$2)-5*diff(y(x),x)+6*y(x)=x^2*exp(3*x)*sin(2*x),y(x), singsol=all)
```

$$y(x) = \frac{((-50x^2 - 160x + 109)\cos(2x) + (-100x^2 + 130x + 88)\sin(2x) + 500c_2) e^{3x}}{500} + e^{2x} c_1$$

✓ Solution by Mathematica

Time used: 0.037 (sec). Leaf size: 63

```
DSolve[y''[x]-5*y'[x]+6*y[x]==x^2*Exp[3*x]*Sin[2*x],y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to -\frac{1}{500} e^{3x} \left(2\left(50x^2 - 65x - 44\right)\sin(2x) + \left(50x^2 + 160x - 109\right)\cos(2x)\right) + c_1 e^{2x} + c_2 e^{3x}$$

15.51 problem 22.11 (j)

Internal problem ID [13746]

Book: Ordinary Differential Equations. An introduction to the fundamentals. Kenneth B. Howell. second edition. CRC Press. FL, USA. 2020
Section: Chapter 22. Method of undetermined coefficients. Additional exercises page 412
Problem number: 22.11 (j).
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _linear, _nonhomogeneous]]`

$$\boxed{y'' - 4y' + 20y = e^{4x} \sin(2x)}$$

✓ Solution by Maple
Time used: 0.0 (sec). Leaf size: 40

```
dsolve(diff(y(x),x$2)-4*diff(y(x),x)+20*y(x)=exp(4*x)*sin(2*x),y(x), singsol=all)
```

$$y(x) = (c_1 \cos(4x) + c_2 \sin(4x)) e^{2x} - \frac{e^{4x}(\cos(2x) - 2\sin(2x))}{40}$$

✓ Solution by Mathematica
Time used: 0.224 (sec). Leaf size: 54

```
DSolve[y''[x]-4*y'[x]+20*y[x]==Exp[4*x]*Sin[2*x],y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to -\frac{1}{40} e^{2x} \left(e^{2x} \cos(2x) - 2\left(e^{2x} \sin(2x) + 20c_2 \cos(4x) + 20c_1 \sin(4x)\right)\right)$$

15.52 problem 22.11 (k)

Internal problem ID [13747]

Book: Ordinary Differential Equations. An introduction to the fundamentals. Kenneth B. Howell. second edition. CRC Press. FL, USA. 2020
Section: Chapter 22. Method of undetermined coefficients. Additional exercises page 412
Problem number: 22.11 (k).
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _linear, _nonhomogeneous]]`

$$\boxed{y'' - 4y' + 20y = e^{2x} \sin(4x)}$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 28

```
dsolve(diff(y(x),x$2)-4*diff(y(x),x)+20*y(x)=exp(2*x)*sin(4*x),y(x), singsol=all)
```

$$y(x) = -\frac{((x - 8c_1)\cos(4x) - 8c_2 \sin(4x))\, e^{2x}}{8}$$

✓ Solution by Mathematica

Time used: 0.116 (sec). Leaf size: 38

```
DSolve[y''[x]-4*y'[x]+20*y[x]==Exp[2*x]*Sin[4*x],y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to \frac{1}{64} e^{2x}((1 + 64c_1)\sin(4x) - 8(x - 8c_2)\cos(4x))$$

15.53 problem 22.11 (L)

Internal problem ID [13748]

Book: Ordinary Differential Equations. An introduction to the fundamentals. Kenneth B. Howell. second edition. CRC Press. FL, USA. 2020
Section: Chapter 22. Method of undetermined coefficients. Additional exercises page 412
Problem number: 22.11 (L).
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _linear, _nonhomogeneous]]`

$$y'' - 4y' + 20y = x^3 \sin(4x)$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 59

```
dsolve(diff(y(x),x$2)-4*diff(y(x),x)+20*y(x)=x^3*sin(4*x),y(x), singsol=all)
```

$$y(x) = \frac{(9826x^3 + 16473x^2 + 167042\,\mathrm{e}^{2x}c_1 + 15810x + 7815)\cos(4x)}{167042} + \frac{\left(x^3 + \frac{3x^2}{17} + 68c_2\mathrm{e}^{2x} - \frac{39x}{578} - \frac{45}{4913}\right)\sin(4x)}{68}$$

✓ Solution by Mathematica

Time used: 0.041 (sec). Leaf size: 76

```
DSolve[y''[x]-4*y'[x]+20*y[x]==x^3*Sin[4*x],y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to \frac{(9826x^3 + 1734x^2 - 663x - 90)\sin(4x) + 4(9826x^3 + 16473x^2 + 15810x + 7815)\cos(4x)}{668168} + c_2e^{2x}\cos(4x) + c_1e^{2x}\sin(4x)$$

15.54 problem 22.11 (m)

Internal problem ID [13749]

Book: Ordinary Differential Equations. An introduction to the fundamentals. Kenneth B. Howell. second edition. CRC Press. FL, USA. 2020
Section: Chapter 22. Method of undetermined coefficients. Additional exercises page 412
Problem number: 22.11 (m).
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _linear, _nonhomogeneous]]`

$$\boxed{y'' - 10y' + 25y = 3x^2 e^{5x}}$$

✓ Solution by Maple
Time used: 0.016 (sec). Leaf size: 19

```
dsolve(diff(y(x),x$2)-10*diff(y(x),x)+25*y(x)=3*x^2*exp(5*x),y(x), singsol=all)
```

$$y(x) = e^{5x}\left(c_2 + c_1 x + \frac{1}{4}x^4\right)$$

✓ Solution by Mathematica
Time used: 0.021 (sec). Leaf size: 27

```
DSolve[y''[x]-10*y'[x]+25*y[x]==3*x^2*Exp[5*x],y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to \frac{1}{4}e^{5x}\left(x^4 + 4c_2 x + 4c_1\right)$$

15.55 problem 22.11 (n)

Internal problem ID [13750]

Book: Ordinary Differential Equations. An introduction to the fundamentals. Kenneth B. Howell. second edition. CRC Press. FL, USA. 2020
Section: Chapter 22. Method of undetermined coefficients. Additional exercises page 412
Problem number: 22.11 (n).
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _linear, _nonhomogeneous]]`

$$\boxed{y'' - 10y' + 25y = 3x^4}$$

✓ Solution by Maple
Time used: 0.0 (sec). Leaf size: 34

```
dsolve(diff(y(x),x$2)-10*diff(y(x),x)+25*y(x)=3*x^4,y(x), singsol=all)
```

$$y(x) = \frac{72}{3125} + (c_1x + c_2)\,\mathrm{e}^{5x} + \frac{3x^4}{25} + \frac{24x^3}{125} + \frac{108x^2}{625} + \frac{288x}{3125}$$

✓ Solution by Mathematica
Time used: 0.018 (sec). Leaf size: 47

```
DSolve[y''[x]-10*y'[x]+25*y[x]==3*x^4,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to \frac{3(125x^4 + 200x^3 + 180x^2 + 96x + 24)}{3125} + c_1e^{5x} + c_2e^{5x}x$$

15.56 problem 22.12 (a)

Internal problem ID [13751]

Book: Ordinary Differential Equations. An introduction to the fundamentals. Kenneth B. Howell. second edition. CRC Press. FL, USA. 2020
Section: Chapter 22. Method of undetermined coefficients. Additional exercises page 412
Problem number: 22.12 (a).
ODE order: 4.
ODE degree: 1.

CAS Maple gives this as type `[[_high_order, _missing_y]]`

$$y'''' - 4y''' = 12\,\mathrm{e}^{-2x}$$

✓ Solution by Maple
Time used: 0.0 (sec). Leaf size: 28

```
dsolve(diff(y(x),x$4)-4*diff(y(x),x$3)=12*exp(-2*x),y(x), singsol=all)
```

$$y(x) = \frac{c_1 \mathrm{e}^{4x}}{64} + \frac{c_2 x^2}{2} + \frac{\mathrm{e}^{-2x}}{4} + c_3 x + c_4$$

✓ Solution by Mathematica
Time used: 0.152 (sec). Leaf size: 37

```
DSolve[y''''[x]-4*y'''[x]==12*Exp[-2*x],y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to \frac{e^{-2x}}{4} + \frac{1}{64} c_1 e^{4x} + x(c_4 x + c_3) + c_2$$

15.57 problem 22.12 (b)

Internal problem ID [13752]

Book: Ordinary Differential Equations. An introduction to the fundamentals. Kenneth B. Howell. second edition. CRC Press. FL, USA. 2020
Section: Chapter 22. Method of undetermined coefficients. Additional exercises page 412
Problem number: 22.12 (b).
ODE order: 4.
ODE degree: 1.

CAS Maple gives this as type **[[_high_order, _missing_y]]**

$$y'''' - 4y''' = 10\sin(2x)$$

✓ Solution by Maple
Time used: 0.016 (sec). Leaf size: 34

```
dsolve(diff(y(x),x$4)-4*diff(y(x),x$3)=10*sin(2*x),y(x), singsol=all)
```

$$y(x) = \frac{c_1 e^{4x}}{64} + \frac{c_2 x^2}{2} + \frac{\sin(2x)}{8} - \frac{\cos(2x)}{4} + c_3 x + c_4$$

✓ Solution by Mathematica
Time used: 0.383 (sec). Leaf size: 46

```
DSolve[y''''[x]-4*y'''[x]==10*Sin[2*x],y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to c_4 x^2 + c_3 x + \frac{1}{64}\left(16x + 8\sin(2x) - 16\cos(2x) + c_1 e^{4x}\right) + c_2$$

15.58 problem 22.12 (c)

Internal problem ID [13753]

Book: Ordinary Differential Equations. An introduction to the fundamentals. Kenneth B. Howell. second edition. CRC Press. FL, USA. 2020
Section: Chapter 22. Method of undetermined coefficients. Additional exercises page 412
Problem number: 22.12 (c).
ODE order: 4.
ODE degree: 1.

CAS Maple gives this as type `[[_high_order, _missing_y]]`

$$\boxed{y'''' - 4y''' = 32\,\mathrm{e}^{4x}}$$

✓ Solution by Maple
Time used: 0.0 (sec). Leaf size: 27

```
dsolve(diff(y(x),x$4)-4*diff(y(x),x$3)=32*exp(4*x),y(x), singsol=all)
```

$$y(x) = \frac{(32x + c_1 - 24)\,\mathrm{e}^{4x}}{64} + \frac{c_2 x^2}{2} + c_3 x + c_4$$

✓ Solution by Mathematica
Time used: 0.137 (sec). Leaf size: 33

```
DSolve[y''''[x]-4*y'''[x]==32*Exp[4*x],y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to \frac{1}{64}e^{4x}(32x - 24 + c_1) + x(c_4 x + c_3) + c_2$$

15.59 problem 22.12 (d)

Internal problem ID [13754]

Book: Ordinary Differential Equations. An introduction to the fundamentals. Kenneth B. Howell. second edition. CRC Press. FL, USA. 2020
Section: Chapter 22. Method of undetermined coefficients. Additional exercises page 412
Problem number: 22.12 (d).
ODE order: 4.
ODE degree: 1.

CAS Maple gives this as type `[[_high_order, _missing_y]]`

$$y'''' - 4y''' = 32x$$

✓ Solution by Maple
Time used: 0.015 (sec). Leaf size: 32

```
dsolve(diff(y(x),x$4)-4*diff(y(x),x$3)=32*x,y(x), singsol=all)
```

$$y(x) = -\frac{x^4}{3} - \frac{x^3}{3} + \frac{c_1 e^{4x}}{64} + \frac{c_2 x^2}{2} + c_3 x + c_4$$

✓ Solution by Mathematica
Time used: 0.079 (sec). Leaf size: 43

```
DSolve[y''''[x]-4*y'''[x]==32*x,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to -\frac{x^4}{3} - \frac{x^3}{3} + c_4 x^2 + c_3 x + \frac{1}{64} c_1 e^{4x} + c_2$$

15.60 problem 22.12 (e)

Internal problem ID [13755]

Book: Ordinary Differential Equations. An introduction to the fundamentals. Kenneth B. Howell. second edition. CRC Press. FL, USA. 2020
Section: Chapter 22. Method of undetermined coefficients. Additional exercises page 412
Problem number: 22.12 (e).
ODE order: 3.
ODE degree: 1.

CAS Maple gives this as type `[[_3rd_order, _with_linear_symmetries]]`

$$y''' - y'' + y' - y = x^2$$

✓ Solution by Maple
Time used: 0.0 (sec). Leaf size: 25

```
dsolve(diff(y(x),x$3)-diff(y(x),x$2)+diff(y(x),x)-y(x)=x^2,y(x), singsol=all)
```

$$y(x) = -x^2 - 2x + c_1 \cos(x) + c_2 e^x + c_3 \sin(x)$$

✓ Solution by Mathematica
Time used: 0.01 (sec). Leaf size: 30

```
DSolve[y'''[x]-y''[x]+y'[x]-y[x]==x^2,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to -x^2 - 2x + c_3 e^x + c_1 \cos(x) + c_2 \sin(x)$$

15.61 problem 22.12 (f)

Internal problem ID [13756]

Book: Ordinary Differential Equations. An introduction to the fundamentals. Kenneth B. Howell. second edition. CRC Press. FL, USA. 2020
Section: Chapter 22. Method of undetermined coefficients. Additional exercises page 412
Problem number: 22.12 (f).
ODE order: 3.
ODE degree: 1.

CAS Maple gives this as type `[[_3rd_order, _linear, _nonhomogeneous]]`

$$\boxed{y''' - y'' + y' - y = 30\cos(2x)}$$

✓ Solution by Maple
Time used: 0.0 (sec). Leaf size: 29

```
dsolve(diff(y(x),x$3)-diff(y(x),x$2)+diff(y(x),x)-y(x)=30*cos(2*x),y(x), singsol=all)
```

$$y(x) = 2\cos(2x) - 4\sin(2x) + c_1\cos(x) + c_2 e^x + c_3\sin(x)$$

✓ Solution by Mathematica
Time used: 0.006 (sec). Leaf size: 34

```
DSolve[y'''[x]-y''[x]+y'[x]-y[x]==30*Cos[2*x],y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to -4\sin(2x) + 2\cos(2x) + c_3 e^x + c_1\cos(x) + c_2\sin(x)$$

15.62 problem 22.12 (g)

Internal problem ID [13757]

Book: Ordinary Differential Equations. An introduction to the fundamentals. Kenneth B. Howell. second edition. CRC Press. FL, USA. 2020
Section: Chapter 22. Method of undetermined coefficients. Additional exercises page 412
Problem number: 22.12 (g).
ODE order: 3.
ODE degree: 1.

CAS Maple gives this as type `[[_3rd_order, _with_linear_symmetries]]`

$$y''' - y'' + y' - y = 6\,e^{x}$$

✓ Solution by Maple
Time used: 0.0 (sec). Leaf size: 21

```
dsolve(diff(y(x),x$3)-diff(y(x),x$2)+diff(y(x),x)-y(x)=6*exp(x),y(x), singsol=all)
```

$$y(x) = (c_2 + 3x)\,e^{x} + c_1 \cos(x) + c_3 \sin(x)$$

✓ Solution by Mathematica
Time used: 0.016 (sec). Leaf size: 27

```
DSolve[y'''[x]-y''[x]+y'[x]-y[x]==6*Exp[x],y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to e^{x}(3x - 3 + c_3) + c_1 \cos(x) + c_2 \sin(x)$$

15.63 problem 22.13 (a)

Internal problem ID [13758]

Book: Ordinary Differential Equations. An introduction to the fundamentals. Kenneth B. Howell. second edition. CRC Press. FL, USA. 2020
Section: Chapter 22. Method of undetermined coefficients. Additional exercises page 412
Problem number: 22.13 (a).
ODE order: 5.
ODE degree: 1.

CAS Maple gives this as type `[[_high_order, _missing_y]]`

$$y^{(5)} + 18y''' + 81y' = \mathrm{e}^{3x}x^2$$

✓ Solution by Maple
Time used: 0.0 (sec). Leaf size: 52

```
dsolve(diff(y(x),x$5)+18*diff(y(x),x$3)+81*diff(y(x),x)=x^2*exp(3*x),y(x), singsol=all)
```

$$y(x) = \frac{(-3c_4x - 3c_2 + c_3)\cos(3x)}{9} + \frac{(9x^2 - 18x + 10)\,\mathrm{e}^{3x}}{8748} + \frac{(3c_3x + 3c_1 + c_4)\sin(3x)}{9} + c_5$$

✓ Solution by Mathematica
Time used: 0.497 (sec). Leaf size: 67

```
DSolve[y'''''[x]+18*y'''[x]+81*y'[x]==x^2*Exp[3*x],y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to \frac{e^{3x}(9x^2 - 18x + 10)}{8748} + \frac{1}{9}(c_2 - 3(c_4x + c_3))\cos(3x) + \frac{1}{9}(3c_2x + 3c_1 + c_4)\sin(3x) + c_5$$

15.64 problem 22.13 (b)

Internal problem ID [13759]

Book: Ordinary Differential Equations. An introduction to the fundamentals. Kenneth B. Howell. second edition. CRC Press. FL, USA. 2020
Section: Chapter 22. Method of undetermined coefficients. Additional exercises page 412
Problem number: 22.13 (b).
ODE order: 5.
ODE degree: 1.

CAS Maple gives this as type `[[_high_order, _missing_y]]`

$$y^{(5)} + 18y''' + 81y' = x^2 \sin(3x)$$

✓ Solution by Maple
Time used: 0.0 (sec). Leaf size: 59

```
dsolve(diff(y(x),x$5)+18*diff(y(x),x$3)+81*diff(y(x),x)=x^2*sin(3*x),y(x), singsol=all)
```

$$y(x) = \frac{(18x^4 - 7776c_4x - 66x^2 - 7776c_2 + 2592c_3 + 19)\cos(3x)}{23328} + \frac{(-12x^3 + (13 + 1944c_3)x + 1944c_1 + 648c_4)\sin(3x)}{5832} + c_5$$

✓ Solution by Mathematica
Time used: 0.411 (sec). Leaf size: 72

```
DSolve[y'''''[x]+18*y'''[x]+81*y'[x]==x^2*Sin[3*x],y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to \frac{4(-12x^3 + (13 + 1944c_2)x + 648(3c_1 + c_4))\sin(3x) + (18x^4 - 66x^2 - 7776c_4x + 19 + 2592c_2 - 7776c_3}{23328} + c_5$$

15.65 problem 22.13 (c)

Internal problem ID [13760]

Book: Ordinary Differential Equations. An introduction to the fundamentals. Kenneth B. Howell. second edition. CRC Press. FL, USA. 2020
Section: Chapter 22. Method of undetermined coefficients. Additional exercises page 412
Problem number: 22.13 (c).
ODE order: 5.
ODE degree: 1.

CAS Maple gives this as type `[[_high_order, _missing_y]]`

$$y^{(5)} + 18y''' + 81y' = \sin(3x)\,e^{3x}x^2$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 73

```
dsolve(diff(y(x),x$5)+18*diff(y(x),x$3)+81*diff(y(x),x)=x^2*exp(3*x)*sin(3*x),y(x), singsol=
```

$$y(x) = \frac{((-75x^2 - 310x + 417)\,e^{3x} - 303750c_4x - 303750c_2 + 101250c_3)\cos(3x)}{911250} + \frac{(-1575x^2 + 3240x - 1693)\sin(3x)\,e^{3x}}{2733750} + \frac{(3c_3x + 3c_1 + c_4)\sin(3x)}{9} + c_5$$

✓ Solution by Mathematica

Time used: 0.792 (sec). Leaf size: 94

```
DSolve[y'''''[x]+18*y'''[x]+81*y'[x]==x^2*Exp[3*x]*Sin[3*x],y[x],x,IncludeSingularSolutions
```

$$y(x) \to -\frac{e^{3x}(1575x^2 - 3240x + 1693)\sin(3x)}{2733750} - \frac{e^{3x}(75x^2 + 310x - 417)\cos(3x)}{911250} + \frac{1}{9}(c_2 - 3(c_4x + c_3))\cos(3x) + \frac{1}{9}(3c_2x + 3c_1 + c_4)\sin(3x) + c_5$$

15.66 problem 22.13 (d)

Internal problem ID [13761]

Book: Ordinary Differential Equations. An introduction to the fundamentals. Kenneth B. Howell. second edition. CRC Press. FL, USA. 2020
Section: Chapter 22. Method of undetermined coefficients. Additional exercises page 412
Problem number: 22.13 (d).
ODE order: 3.
ODE degree: 1.

CAS Maple gives this as type `[[_3rd_order, _linear, _nonhomogeneous]]`

$$\boxed{y''' - y'' + y' - y = 30x \cos(2x)}$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 39

```
dsolve(diff(y(x),x$3)-diff(y(x),x$2)+diff(y(x),x)-y(x)=30*x*cos(2*x),y(x), singsol=all)
```

$$y(x) = \frac{(30x - 98)\cos(2x)}{15} + \frac{(-60x - 64)\sin(2x)}{15} + c_1 \cos(x) + c_2 e^x + c_3 \sin(x)$$

✓ Solution by Mathematica

Time used: 0.009 (sec). Leaf size: 49

```
DSolve[y'''[x]-y''[x]+y'[x]-y[x]==30*x*Cos[2*x],y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to -4x\sin(2x) - \frac{64}{15}\sin(2x) + \left(2x - \frac{98}{15}\right)\cos(2x) + c_3 e^x + c_1 \cos(x) + c_2 \sin(x)$$

15.67 problem 22.13 (e)

Internal problem ID [13762]

Book: Ordinary Differential Equations. An introduction to the fundamentals. Kenneth B. Howell. second edition. CRC Press. FL, USA. 2020
Section: Chapter 22. Method of undetermined coefficients. Additional exercises page 412
Problem number: 22.13 (e).
ODE order: 3.
ODE degree: 1.

CAS Maple gives this as type `[[_3rd_order, _linear, _nonhomogeneous]]`

$$\boxed{y''' - y'' + y' - y = 3\cos(x)\, x}$$

✓ Solution by Maple
Time used: 0.016 (sec). Leaf size: 42

```
dsolve(diff(y(x),x$3)-diff(y(x),x$2)+diff(y(x),x)-y(x)=3*x*cos(x),y(x), singsol=all)
```

$$y(x) = \frac{(-3x^2 + 8c_3 + 3x + 9)\sin(x)}{8} + \frac{(-3x^2 + 8c_1 - 9x)\cos(x)}{8} + c_2 e^x$$

✓ Solution by Mathematica
Time used: 0.046 (sec). Leaf size: 52

```
DSolve[y'''[x]-y''[x]+y'[x]-y[x]==3*x*Cos[x],y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to \frac{1}{16}\left(-\left(6x^2 + 18x + 3 - 16c_1\right)\cos(x) + \left(-6x^2 + 6x + 15 + 16c_2\right)\sin(x) + 16c_3 e^x\right)$$

15.68 problem 22.13 (f)

Internal problem ID [13763]

Book: Ordinary Differential Equations. An introduction to the fundamentals. Kenneth B. Howell. second edition. CRC Press. FL, USA. 2020
Section: Chapter 22. Method of undetermined coefficients. Additional exercises page 412
Problem number: 22.13 (f).
ODE order: 3.
ODE degree: 1.

CAS Maple gives this as type `[[_3rd_order, _linear, _nonhomogeneous]]`

$$y''' - y'' + y' - y = 3x\,\mathrm{e}^{x} \cos(x)$$

✓ Solution by Maple
Time used: 0.0 (sec). Leaf size: 37

```
dsolve(diff(y(x),x$3)-diff(y(x),x$2)+diff(y(x),x)-y(x)=3*x*exp(x)*cos(x),y(x), singsol=all)
```

$$y(x) = \frac{((-30x+57)\cos(x) + (15x+24)\sin(x) + 25c_2)\,\mathrm{e}^{x}}{25} + c_1 \cos(x) + c_3 \sin(x)$$

✓ Solution by Mathematica
Time used: 0.011 (sec). Leaf size: 49

```
DSolve[y'''[x]-y''[x]+y'[x]-y[x]==3*x*Exp[x]*Cos[x],y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to c_3 e^{x} + \left(e^{x}\left(\frac{57}{25} - \frac{6x}{5}\right) + c_1\right)\cos(x) + \left(\frac{3}{25}e^{x}(5x+8) + c_2\right)\sin(x)$$

15.69 problem 22.13 (g)

Internal problem ID [13764]

Book: Ordinary Differential Equations. An introduction to the fundamentals. Kenneth B. Howell. second edition. CRC Press. FL, USA. 2020
Section: Chapter 22. Method of undetermined coefficients. Additional exercises page 412
Problem number: 22.13 (g).
ODE order: 3.
ODE degree: 1.

CAS Maple gives this as type `[[_3rd_order, _linear, _nonhomogeneous]]`

$$y''' - y'' + y' - y = 5x^5 e^{2x}$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 48

```
dsolve(diff(y(x),x$3)-diff(y(x),x$2)+diff(y(x),x)-y(x)=5*x^5*exp(2*x),y(x), singsol=all)
```

$$y(x) = \frac{(625x^5 - 5625x^4 + 28000x^3 - 91200x^2 + 187320x - 188376)\,e^{2x}}{625} + c_1 \cos(x) + c_2 e^x + c_3 \sin(x)$$

✓ Solution by Mathematica

Time used: 0.009 (sec). Leaf size: 58

```
DSolve[y'''[x]-y''[x]+y'[x]-y[x]==5*x^5*Exp[2*x],y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to e^x \left(e^x \left(x^5 - 9x^4 + \frac{224x^3}{5} - \frac{3648x^2}{25} + \frac{37464x}{125} - \frac{188376}{625} \right) + c_3 \right) + c_1 \cos(x) + c_2 \sin(x)$$

15.70 problem 22.14 (a)

Internal problem ID [13765]

Book: Ordinary Differential Equations. An introduction to the fundamentals. Kenneth B. Howell. second edition. CRC Press. FL, USA. 2020
Section: Chapter 22. Method of undetermined coefficients. Additional exercises page 412
Problem number: 22.14 (a).
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _linear, _nonhomogeneous]]`

$$y'' - 6y' + 9y = 27\,\mathrm{e}^{6x} + 25\sin(6x)$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 33

```
dsolve(diff(y(x),x$2)-6*diff(y(x),x)+9*y(x)=27*exp(6*x)+25*sin(6*x),y(x), singsol=all)
```

$$y(x) = (c_1 x + c_2)\,\mathrm{e}^{3x} + \frac{4\cos(6x)}{9} + 3\,\mathrm{e}^{6x} - \frac{\sin(6x)}{3}$$

✓ Solution by Mathematica

Time used: 0.409 (sec). Leaf size: 42

```
DSolve[y''[x]-6*y'[x]+9*y[x]==27*Exp[6*x]+25*Sin[6*x],y[x],x,IncludeSingularSolutions -> Tru
```

$$y(x) \to -\frac{1}{3}\sin(6x) + \frac{4}{9}\cos(6x) + e^{3x}\left(3e^{3x} + c_2 x + c_1\right)$$

15.71 problem 22.14 (b)

Internal problem ID [13766]

Book: Ordinary Differential Equations. An introduction to the fundamentals. Kenneth B. Howell. second edition. CRC Press. FL, USA. 2020
Section: Chapter 22. Method of undetermined coefficients. Additional exercises page 412
Problem number: 22.14 (b).
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _linear, _nonhomogeneous]]`

$$\boxed{y'' + 9y = 25x\cos(2x) + 3\sin(3x)}$$

✓ Solution by Maple
Time used: 0.0 (sec). Leaf size: 42

```
dsolve(diff(y(x),x$2)+9*y(x)=25*x*cos(2*x)+3*sin(3*x),y(x), singsol=all)
```

$$y(x) = \frac{(2c_1 - x)\cos(3x)}{2} + \frac{(1 + 12c_2)\sin(3x)}{12} + 5\cos(2x)\,x + 4\sin(2x)$$

✓ Solution by Mathematica
Time used: 0.274 (sec). Leaf size: 39

```
DSolve[y''[x]+9*y[x]==25*x*Cos[2*x]+3*Sin[3*x],y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to 4\sin(2x) + 5x\cos(2x) + \left(-\frac{x}{2} + c_1\right)\cos(3x) + c_2\sin(3x)$$

15.72 problem 22.14 (c)

Internal problem ID [13767]

Book: Ordinary Differential Equations. An introduction to the fundamentals. Kenneth B. Howell. second edition. CRC Press. FL, USA. 2020
Section: Chapter 22. Method of undetermined coefficients. Additional exercises page 412
Problem number: 22.14 (c).
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type **[[_2nd_order, _linear, _nonhomogeneous]]**

$$\boxed{y'' - 4y' + 5y = 5\sin(x)^2}$$

✓ Solution by Maple
Time used: 0.015 (sec). Leaf size: 34

```
dsolve(diff(y(x),x$2)-4*diff(y(x),x)+5*y(x)=5*sin(x)^2,y(x), singsol=all)
```

$$y(x) = e^{2x}\sin(x)\,c_2 + e^{2x}\cos(x)\,c_1 - \frac{\cos(2x)}{26} + \frac{1}{2} + \frac{4\sin(2x)}{13}$$

✓ Solution by Mathematica
Time used: 0.168 (sec). Leaf size: 45

```
DSolve[y''[x]-4*y'[x]+5*y[x]==5*Sin[x]^2,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to \frac{4}{13}\sin(2x) - \frac{1}{26}\cos(2x) + c_2e^{2x}\cos(x) + c_1e^{2x}\sin(x) + \frac{1}{2}$$

15.73 problem 22.14 (d)

Internal problem ID [13768]

Book: Ordinary Differential Equations. An introduction to the fundamentals. Kenneth B. Howell. second edition. CRC Press. FL, USA. 2020
Section: Chapter 22. Method of undetermined coefficients. Additional exercises page 412
Problem number: 22.14 (d).
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type **[[_2nd_order, _linear, _nonhomogeneous]]**

$$\boxed{y'' - 4y' + 5y = 20 \sinh(x)}$$

✓ Solution by Maple
Time used: 0.0 (sec). Leaf size: 31

```
dsolve(diff(y(x),x$2)-4*diff(y(x),x)+5*y(x)=20*sinh(x),y(x), singsol=all)
```

$$y(x) = e^{2x} \sin(x) c_2 + e^{2x} \cos(x) c_1 + 5 e^{x} - e^{-x}$$

✓ Solution by Mathematica
Time used: 0.105 (sec). Leaf size: 38

```
DSolve[y''[x]-4*y'[x]+5*y[x]==20*Sinh[x],y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to -e^{-x} + 5e^{x} + c_2 e^{2x} \cos(x) + c_1 e^{2x} \sin(x)$$

15.74 problem 22.15 (a)

Internal problem ID [13769]

Book: Ordinary Differential Equations. An introduction to the fundamentals. Kenneth B. Howell. second edition. CRC Press. FL, USA. 2020
Section: Chapter 22. Method of undetermined coefficients. Additional exercises page 412
Problem number: 22.15 (a).
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$x^2y'' - 5y'x + 8y = \frac{5}{x^3}$$

✓ Solution by Maple
Time used: 0.016 (sec). Leaf size: 20

```
dsolve(x^2*diff(y(x),x$2)-5*x*diff(y(x),x)+8*y(x)=5/x^3,y(x), singsol=all)
```

$$y(x) = c_2x^4 + c_1x^2 + \frac{1}{7x^3}$$

✓ Solution by Mathematica
Time used: 0.016 (sec). Leaf size: 25

```
DSolve[x^2*y''[x]-5*x*y'[x]+8*y[x]==5/x^3,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to c_2x^4 + \frac{1}{7x^3} + c_1x^2$$

15.75 problem 22.15 (b)

Internal problem ID [13770]

Book: Ordinary Differential Equations. An introduction to the fundamentals. Kenneth B. Howell. second edition. CRC Press. FL, USA. 2020
Section: Chapter 22. Method of undetermined coefficients. Additional exercises page 412
Problem number: 22.15 (b).
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$\boxed{2x^2y'' - y'x + y = \frac{50}{x^3}}$$

✓ Solution by Maple
Time used: 0.016 (sec). Leaf size: 23

```
dsolve(2*x^2*diff(y(x),x$2)-x*diff(y(x),x)+y(x)=50/x^3,y(x), singsol=all)
```

$$y(x) = \frac{14c_2x^{\frac{7}{2}} + 14x^4c_1 + 25}{14x^3}$$

✓ Solution by Mathematica
Time used: 0.018 (sec). Leaf size: 25

```
DSolve[2*x^2*y''[x]-x*y'[x]+y[x]==50/x^3,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to \frac{25}{14x^3} + c_1\sqrt{x} + c_2x$$

15.76 problem 22.15 (c)

Internal problem ID [13771]

Book: Ordinary Differential Equations. An introduction to the fundamentals. Kenneth B. Howell. second edition. CRC Press. FL, USA. 2020
Section: Chapter 22. Method of undetermined coefficients. Additional exercises page 412
Problem number: 22.15 (c).
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _exact, _linear, _nonhomogeneous]]`

$$\boxed{2x^2y'' + 5y'x + y = 85\cos(2\ln(x))}$$

✓ Solution by Maple
Time used: 0.0 (sec). Leaf size: 37

```
dsolve(2*x^2*diff(y(x),x$2)+5*x*diff(y(x),x)+y(x)=85*cos(2*ln(x)),y(x), singsol=all)
```

$$y(x) = \frac{2c_2 + \int \frac{c_1+17\cos(2\ln(x))x+34x\sin(2\ln(x))}{x^{\frac{3}{2}}}dx}{2\sqrt{x}}$$

✓ Solution by Mathematica
Time used: 0.061 (sec). Leaf size: 34

```
DSolve[2*x^2*y''[x]+5*x*y'[x]+y[x]==85*Cos[2*Log[x]],y[x],x,IncludeSingularSolutions -> True
```

$$y(x) \to \frac{c_1}{x} + \frac{c_2}{\sqrt{x}} + 6\sin(2\log(x)) - 7\cos(2\log(x))$$

15.77 problem 22.15 (d)

Internal problem ID [13772]

Book: Ordinary Differential Equations. An introduction to the fundamentals. Kenneth B. Howell. second edition. CRC Press. FL, USA. 2020
Section: Chapter 22. Method of undetermined coefficients. Additional exercises page 412
Problem number: 22.15 (d).
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _exact, _linear, _nonhomogeneous]]`

$$x^2y'' - 2y = 15\cos(3\ln(x)) - 10\sin(3\ln(x))$$

✓ Solution by Maple
Time used: 0.015 (sec). Leaf size: 29

```
dsolve(x^2*diff(y(x),x$2)-2*y(x)=15*cos(3*ln(x))-10*sin(3*ln(x)),y(x), singsol=all)
```

$$y(x) = \frac{c_1}{x} + c_2x^2 - \frac{3\cos(3\ln(x))}{2} + \frac{\sin(3\ln(x))}{2}$$

✓ Solution by Mathematica
Time used: 0.154 (sec). Leaf size: 35

```
DSolve[x^2*y''[x]-2*y[x]==15*Cos[3*Log[x]]-10*Sin[3*Log[x]],y[x],x,IncludeSingularSolutions
```

$$y(x) \to c_2x^2 + \frac{c_1}{x} + \frac{1}{2}(\sin(3\log(x)) - 3\cos(3\log(x)))$$

15.78 problem 22.15 (e)

Internal problem ID [13773]

Book: Ordinary Differential Equations. An introduction to the fundamentals. Kenneth B. Howell. second edition. CRC Press. FL, USA. 2020
Section: Chapter 22. Method of undetermined coefficients. Additional exercises page 412
Problem number: 22.15 (e).
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$\boxed{3x^2y'' - 7y'x + 3y = 4x^3}$$

✓ Solution by Maple
Time used: 0.0 (sec). Leaf size: 21

```
dsolve(3*x^2*diff(y(x),x$2)-7*x*diff(y(x),x)+3*y(x)=4*x^3,y(x), singsol=all)
```

$$y(x) = c_1 x^{\frac{1}{3}} + \left(c_2 + \frac{\ln(x)}{2} - \frac{3}{16}\right) x^3$$

✓ Solution by Mathematica
Time used: 0.018 (sec). Leaf size: 33

```
DSolve[3*x^2*y''[x]-7*x*y'[x]+3*y[x]==4*x^3,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to \frac{1}{2}x^3 \log(x) + \left(-\frac{3}{16} + c_2\right) x^3 + c_1 \sqrt[3]{x}$$

15.79 problem 22.15 (f)

Internal problem ID [13774]

Book: Ordinary Differential Equations. An introduction to the fundamentals. Kenneth B. Howell. second edition. CRC Press. FL, USA. 2020
Section: Chapter 22. Method of undetermined coefficients. Additional exercises page 412
Problem number: 22.15 (f).
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _exact, _linear, _nonhomogeneous]]`

$$2x^2y'' + 5y'x + y = \frac{10}{x}$$

✓ Solution by Maple
Time used: 0.016 (sec). Leaf size: 26

```
dsolve(2*x^2*diff(y(x),x$2)+5*x*diff(y(x),x)+y(x)=10/x,y(x), singsol=all)
```

$$y(x) = \frac{-10\sqrt{x}\,\ln(x) + c_2 x + (c_1 - 20)\sqrt{x}}{x^{\frac{3}{2}}}$$

✓ Solution by Mathematica
Time used: 0.015 (sec). Leaf size: 25

```
DSolve[2*x^2*y''[x]+5*x*y'[x]+y[x]==10/x,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to \frac{-10\log(x) + c_2\sqrt{x} - 20 + c_1}{x}$$

15.80 problem 22.15 (g)

Internal problem ID [13775]

Book: Ordinary Differential Equations. An introduction to the fundamentals. Kenneth B. Howell. second edition. CRC Press. FL, USA. 2020
Section: Chapter 22. Method of undetermined coefficients. Additional exercises page 412
Problem number: 22.15 (g).
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$x^2y'' - 5y'x + 9y = 6x^3$$

✓ Solution by Maple
Time used: 0.0 (sec). Leaf size: 20

```
dsolve(x^2*diff(y(x),x$2)-5*x*diff(y(x),x)+9*y(x)=6*x^3,y(x), singsol=all)
```

$$y(x) = x^3 (c_2 + c_1 \ln(x) + 3 \ln(x)^2)$$

✓ Solution by Mathematica
Time used: 0.023 (sec). Leaf size: 24

```
DSolve[x^2*y''[x]-5*x*y'[x]+9*y[x]==6*x^3,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to x^3 (3 \log^2(x) + 3c_2 \log(x) + c_1)$$

15.81 problem 22.15 (h)

Internal problem ID [13776]

Book: Ordinary Differential Equations. An introduction to the fundamentals. Kenneth B. Howell. second edition. CRC Press. FL, USA. 2020
Section: Chapter 22. Method of undetermined coefficients. Additional exercises page 412
Problem number: 22.15 (h).
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _linear, _nonhomogeneous]]`

$$\boxed{x^2y'' + 5y'x + 4y = 64x^2 \ln(x)}$$

✓ Solution by Maple
Time used: 0.016 (sec). Leaf size: 25

```
dsolve(x^2*diff(y(x),x$2)+5*x*diff(y(x),x)+4*y(x)=64*x^2*ln(x),y(x), singsol=all)
```

$$y(x) = \frac{(4x^4 + c_1)\ln(x) - 2x^4 + c_2}{x^2}$$

✓ Solution by Mathematica
Time used: 0.026 (sec). Leaf size: 29

```
DSolve[x^2*y''[x]+5*x*y'[x]+4*y[x]==64*x^2*Log[x],y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to \frac{-2x^4 + 2(2x^4 + c_2)\log(x) + c_1}{x^2}$$

16 Chapter 24. Variation of parameters. Additional exercises page 444

16.1 problem 24.1 (a)

Internal problem ID [13777]

Book: Ordinary Differential Equations. An introduction to the fundamentals. Kenneth B. Howell. second edition. CRC Press. FL, USA. 2020
Section: Chapter 24. Variation of parameters. Additional exercises page 444
Problem number: 24.1 (a).
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$x^2y'' - 2y'x + 2y = 3\sqrt{x}$$

✓ Solution by Maple
Time used: 0.016 (sec). Leaf size: 18

```
dsolve(x^2*diff(y(x),x$2)-2*x*diff(y(x),x)+2*y(x)=3*sqrt(x),y(x), singsol=all)
```

$$y(x) = c_2x + c_1x^2 + 4\sqrt{x}$$

✓ Solution by Mathematica
Time used: 0.015 (sec). Leaf size: 23

```
DSolve[x^2*y''[x]-2*x*y'[x]+2*y[x]==3*Sqrt[x],y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to c_2x^2 + 4\sqrt{x} + c_1x$$

16.2 problem 24.1 (b)

Internal problem ID [13778]

Book: Ordinary Differential Equations. An introduction to the fundamentals. Kenneth B. Howell. second edition. CRC Press. FL, USA. 2020
Section: Chapter 24. Variation of parameters. Additional exercises page 444
Problem number: 24.1 (b).
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _linear, _nonhomogeneous]]`

$$\boxed{y'' + y = \cot(x)}$$

✓ Solution by Maple
Time used: 0.0 (sec). Leaf size: 24

```
dsolve(diff(y(x),x$2)+y(x)=cot(x),y(x), singsol=all)
```

$$y(x) = \sin(x)\, c_2 + c_1 \cos(x) + \sin(x) \ln(\csc(x) - \cot(x))$$

✓ Solution by Mathematica
Time used: 0.055 (sec). Leaf size: 33

```
DSolve[y''[x]+y[x]==Cot[x],y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to c_1 \cos(x) + \sin(x)\left(\log\left(\sin\left(\frac{x}{2}\right)\right) - \log\left(\cos\left(\frac{x}{2}\right)\right) + c_2\right)$$

16.3 problem 24.1 (c)

Internal problem ID [13779]

Book: Ordinary Differential Equations. An introduction to the fundamentals. Kenneth B. Howell. second edition. CRC Press. FL, USA. 2020
Section: Chapter 24. Variation of parameters. Additional exercises page 444
Problem number: 24.1 (c).
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _linear, _nonhomogeneous]]`

$$y'' + 4y = \csc(2x)$$

✓ Solution by Maple
Time used: 0.0 (sec). Leaf size: 35

```
dsolve(diff(y(x),x$2)+4*y(x)=csc(2*x),y(x), singsol=all)
```

$$y(x) = -\frac{\ln(\csc(2x))\sin(2x)}{4} + \frac{(-2x + 4c_1)\cos(2x)}{4} + c_2\sin(2x)$$

✓ Solution by Mathematica
Time used: 0.031 (sec). Leaf size: 37

```
DSolve[y''[x]+4*y[x]==Csc[2*x],y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to \left(-\frac{x}{2} + c_1\right)\cos(2x) + \frac{1}{4}\sin(2x)(\log(\sin(2x)) + 4c_2)$$

16.4 problem 24.1 (d)

Internal problem ID [13780]

Book: Ordinary Differential Equations. An introduction to the fundamentals. Kenneth B. Howell. second edition. CRC Press. FL, USA. 2020
Section: Chapter 24. Variation of parameters. Additional exercises page 444
Problem number: 24.1 (d).
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$y'' - 7y' + 10y = 6\,\mathrm{e}^{3x}$$

✓ Solution by Maple
Time used: 0.0 (sec). Leaf size: 23

```
dsolve(diff(y(x),x$2)-7*diff(y(x),x)+10*y(x)=6*exp(3*x),y(x), singsol=all)
```

$$y(x) = \mathrm{e}^{5x}c_2 + \mathrm{e}^{2x}c_1 - 3\,\mathrm{e}^{3x}$$

✓ Solution by Mathematica
Time used: 0.016 (sec). Leaf size: 27

```
DSolve[y''[x]-7*y'[x]+10*y[x]==6*Exp[3*x],y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to e^{2x}\left(-3e^x + c_2e^{3x} + c_1\right)$$

16.5 problem 24.1 (e)

Internal problem ID [13781]

Book: Ordinary Differential Equations. An introduction to the fundamentals. Kenneth B. Howell. second edition. CRC Press. FL, USA. 2020
Section: Chapter 24. Variation of parameters. Additional exercises page 444
Problem number: 24.1 (e).
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _linear, _nonhomogeneous]]`

$$y'' - 4y' + 4y = \left(24x^2 + 2\right) e^{2x}$$

✓ Solution by Maple

Time used: 0.016 (sec). Leaf size: 22

```
dsolve(diff(y(x),x$2)-4*diff(y(x),x)+4*y(x)=(24*x^2+2)*exp(2*x),y(x), singsol=all)
```

$$y(x) = e^{2x}\left(2x^4 + c_1 x + x^2 + c_2\right)$$

✓ Solution by Mathematica

Time used: 0.022 (sec). Leaf size: 26

```
DSolve[y''[x]-4*y'[x]+4*y[x]==(24*x^2+2)*Exp[2*x],y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to e^{2x}\left(2x^4 + x^2 + c_2 x + c_1\right)$$

16.6 problem 24.1 (f)

Internal problem ID [13782]

Book: Ordinary Differential Equations. An introduction to the fundamentals. Kenneth B. Howell. second edition. CRC Press. FL, USA. 2020
Section: Chapter 24. Variation of parameters. Additional exercises page 444
Problem number: 24.1 (f).
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _linear, _nonhomogeneous]]`

$$y'' + 4y' + 4y = \frac{e^{-2x}}{x^2 + 1}$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 26

```
dsolve(diff(y(x),x$2)+4*diff(y(x),x)+4*y(x)=exp(-2*x)/(1+x^2),y(x), singsol=all)
```

$$y(x) = e^{-2x}\left(c_2 + c_1 x - \frac{\ln(x^2+1)}{2} + x \arctan(x)\right)$$

✓ Solution by Mathematica

Time used: 0.042 (sec). Leaf size: 37

```
DSolve[y''[x]+4*y'[x]+4*y[x]==Exp[-2*x]/(1+x^2),y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to \frac{1}{2}e^{-2x}\left(2x \arctan(x) - \log\left(x^2+1\right) + 2(c_2 x + c_1)\right)$$

16.7 problem 24.1 (g)

Internal problem ID [13783]

Book: Ordinary Differential Equations. An introduction to the fundamentals. Kenneth B. Howell. second edition. CRC Press. FL, USA. 2020
Section: Chapter 24. Variation of parameters. Additional exercises page 444
Problem number: 24.1 (g).
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _exact, _linear, _nonhomogeneous]]`

$$\boxed{x^2y'' + y'x - y = \sqrt{x}}$$

✓ Solution by Maple
Time used: 0.0 (sec). Leaf size: 24

```
dsolve(x^2*diff(y(x),x$2)+x*diff(y(x),x)-y(x)=sqrt(x),y(x), singsol=all)
```

$$y(x) = \frac{3c_2x^2 - 4x^{\frac{3}{2}} + 3c_1}{3x}$$

✓ Solution by Mathematica
Time used: 0.017 (sec). Leaf size: 25

```
DSolve[x^2*y''[x]+x*y'[x]-y[x]==Sqrt[x],y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to -\frac{4\sqrt{x}}{3} + \frac{c_1}{x} + c_2x$$

16.8 problem 24.1 (h)

Internal problem ID [13784]

Book: Ordinary Differential Equations. An introduction to the fundamentals. Kenneth B. Howell. second edition. CRC Press. FL, USA. 2020
Section: Chapter 24. Variation of parameters. Additional exercises page 444
Problem number: 24.1 (h).
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$x^2y'' + y'x - 9y = 12x^3$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 29

```
dsolve(x^2*diff(y(x),x$2)+x*diff(y(x),x)-9*y(x)=12*x^3,y(x), singsol=all)
```

$$y(x) = \frac{6x^6 \ln(x) + (3c_1 - 1)x^6 + 3c_2}{3x^3}$$

✓ Solution by Mathematica

Time used: 0.015 (sec). Leaf size: 29

```
DSolve[x^2*y''[x]+x*y'[x]-9*y[x]==12*x^3,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to 2x^3 \log(x) + \left(-\frac{1}{3} + c_2\right)x^3 + \frac{c_1}{x^3}$$

16.9 problem 24.1 (i)

Internal problem ID [13785]

Book: Ordinary Differential Equations. An introduction to the fundamentals. Kenneth B. Howell. second edition. CRC Press. FL, USA. 2020
Section: Chapter 24. Variation of parameters. Additional exercises page 444
Problem number: 24.1 (i).
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$\boxed{x^2y'' - 3y'x + 4y = x^2}$$

✓ Solution by Maple
Time used: 0.0 (sec). Leaf size: 20

```
dsolve(x^2*diff(y(x),x$2)-3*x*diff(y(x),x)+4*y(x)=x^2,y(x), singsol=all)
```

$$y(x) = x^2\left(c_2 + c_1 \ln(x) + \frac{\ln(x)^2}{2}\right)$$

✓ Solution by Mathematica
Time used: 0.019 (sec). Leaf size: 27

```
DSolve[x^2*y''[x]-3*x*y'[x]+4*y[x]==x^2,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to \frac{1}{2}x^2\left(\log^2(x) + 4c_2\log(x) + 2c_1\right)$$

16.10 problem 24.1 (j)

Internal problem ID [13786]

Book: Ordinary Differential Equations. An introduction to the fundamentals. Kenneth B. Howell. second edition. CRC Press. FL, USA. 2020
Section: Chapter 24. Variation of parameters. Additional exercises page 444
Problem number: 24.1 (j).
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$x^2y'' + 5y'x + 4y = \ln(x)$$

✓ Solution by Maple
Time used: 0.0 (sec). Leaf size: 28

```
dsolve(x^2*diff(y(x),x$2)+5*x*diff(y(x),x)+4*y(x)=ln(x),y(x), singsol=all)
```

$$y(x) = \frac{(x^2 + 4c_1)\ln(x) - x^2 + 4c_2}{4x^2}$$

✓ Solution by Mathematica
Time used: 0.026 (sec). Leaf size: 29

```
DSolve[x^2*y''[x]+5*x*y'[x]+4*y[x]==Log[x],y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to \frac{c_1}{x^2} + \left(\frac{1}{4} + \frac{2c_2}{x^2}\right)\log(x) - \frac{1}{4}$$

16.11 problem 24.1 (k)

Internal problem ID [13787]

Book: Ordinary Differential Equations. An introduction to the fundamentals. Kenneth B. Howell. second edition. CRC Press. FL, USA. 2020
Section: Chapter 24. Variation of parameters. Additional exercises page 444
Problem number: 24.1 (k).
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _exact, _linear, _nonhomogeneous]]`

$$x^2y'' - 2y = \frac{1}{x-2}$$

✓ Solution by Maple
Time used: 0.0 (sec). Leaf size: 44

```
dsolve(x^2*diff(y(x),x$2)-2*y(x)=1/(x-2),y(x), singsol=all)
```

$$y(x) = \frac{(x^3-8)\ln(x-2) + 24c_2x^3 - \ln(x)\,x^3 + 2x^2 + 2x + 24c_1}{24x}$$

✓ Solution by Mathematica
Time used: 0.02 (sec). Leaf size: 57

```
DSolve[x^2*y''[x]-2*y[x]==1/(x-2),y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to \frac{x^3\log(2-x) - x^3\log(x) + 24c_2x^3 + 2x^2 + 2x - 8\log(6-3x) + 24c_1}{24x}$$

16.12 problem 24.1 (L)

Internal problem ID [13788]

Book: Ordinary Differential Equations. An introduction to the fundamentals. Kenneth B. Howell. second edition. CRC Press. FL, USA. 2020
Section: Chapter 24. Variation of parameters. Additional exercises page 444
Problem number: 24.1 (L).
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _linear, _nonhomogeneous]]`

$$\boxed{xy'' - y' - 4yx^3 = e^{x^2}x^3}$$

✓ Solution by Maple
Time used: 0.0 (sec). Leaf size: 26

```
dsolve(x*diff(y(x),x$2)-diff(y(x),x)-4*x^3*y(x)=x^3*exp(x^2),y(x), singsol=all)
```

$$y(x) = \sinh\left(x^2\right) c_2 + \cosh\left(x^2\right) c_1 + \frac{x^2 e^{x^2}}{8}$$

✓ Solution by Mathematica
Time used: 0.069 (sec). Leaf size: 47

```
DSolve[x*y''[x]-y'[x]-4*x^3*y[x]==x^3*Exp[x^2],y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to \frac{1}{16}\left(\left(2x^2 - 1 + 16c_1\right)\cosh\left(x^2\right) + \sinh\left(x^2\right)\left(\log\left(e^{2x^2}\right) - 1 + 16ic_2\right)\right)$$

16.13 problem 24.1 (m)

Internal problem ID [13789]

Book: Ordinary Differential Equations. An introduction to the fundamentals. Kenneth B. Howell. second edition. CRC Press. FL, USA. 2020
Section: Chapter 24. Variation of parameters. Additional exercises page 444
Problem number: 24.1 (m).
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _exact, _linear, _nonhomogeneous]]`

$$xy'' + (2x + 2) y' + 2y = 8 e^{2x}$$

✓ Solution by Maple
Time used: 0.0 (sec). Leaf size: 20

```
dsolve(x*diff(y(x),x$2)+(2+2*x)*diff(y(x),x)+2*y(x)=8*exp(2*x),y(x), singsol=all)
```

$$y(x) = \frac{e^{-2x} c_2 + e^{2x} + c_1}{x}$$

✓ Solution by Mathematica
Time used: 0.068 (sec). Leaf size: 31

```
DSolve[x*y''[x]+(2+2*x)*y'[x]+2*y[x]==8*Exp[2*x],y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to \frac{2e^{2x} + 2c_1 e^{-2x} + c_2}{2x}$$

16.14 problem 24.1 (n)

Internal problem ID [13790]

Book: Ordinary Differential Equations. An introduction to the fundamentals. Kenneth B. Howell. second edition. CRC Press. FL, USA. 2020
Section: Chapter 24. Variation of parameters. Additional exercises page 444
Problem number: 24.1 (n).
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$\boxed{(x+1)\,y''+y'x-y=(x+1)^2}$$

✓ Solution by Maple
Time used: 0.016 (sec). Leaf size: 18

```
dsolve((x+1)*diff(y(x),x$2)+x*diff(y(x),x)-y(x)=(x+1)^2,y(x), singsol=all)
```

$$y(x)=c_2x+c_1\mathrm{e}^{-x}+x^2+1$$

✓ Solution by Mathematica
Time used: 0.163 (sec). Leaf size: 41

```
DSolve[(x+1)*y''[x]+x*y'[x]-y[x]==(x+1)^2,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x)\to x^2+\left(-1+\sqrt{2e}c_2\right)x+\frac{c_1e^{-x-\frac{1}{2}}}{\sqrt{2}}+1$$

16.15 problem 24.2 (a)

Internal problem ID [13791]

Book: Ordinary Differential Equations. An introduction to the fundamentals. Kenneth B. Howell. second edition. CRC Press. FL, USA. 2020
Section: Chapter 24. Variation of parameters. Additional exercises page 444
Problem number: 24.2 (a).
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _exact, _linear, _nonhomogeneous]]`

$$\boxed{x^2y'' - 2y'x - 4y = \frac{10}{x}}$$

With initial conditions

$$[y(1) = 3, y'(1) = -15]$$

✓ Solution by Maple
Time used: 0.016 (sec). Leaf size: 19

```
dsolve([x^2*diff(y(x),x$2)-2*x*diff(y(x),x)-4*y(x)=10/x,y(1) = 3, D(y)(1) = -15],y(x), sings
```

$$y(x) = \frac{-2x^5 - 2\ln(x) + 5}{x}$$

✓ Solution by Mathematica
Time used: 0.022 (sec). Leaf size: 20

```
DSolve[{x^2*y''[x]-2*x*y'[x]-4*y[x]==10/x,{y[1]==3,y'[1]==-15}},y[x],x,IncludeSingularSoluti
```

$$y(x) \to \frac{-2x^5 - 2\log(x) + 5}{x}$$

16.16 problem 24.2 (b)

Internal problem ID [13792]

Book: Ordinary Differential Equations. An introduction to the fundamentals. Kenneth B. Howell. second edition. CRC Press. FL, USA. 2020
Section: Chapter 24. Variation of parameters. Additional exercises page 444
Problem number: 24.2 (b).
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$\boxed{y'' - y' - 6y = 12\,\mathrm{e}^{2x}}$$

With initial conditions

$$[y(0) = 0, y'(0) = 8]$$

✓ Solution by Maple
Time used: 0.016 (sec). Leaf size: 23

```
dsolve([diff(y(x),x$2)-diff(y(x),x)-6*y(x)=12*exp(2*x),y(0) = 0, D(y)(0) = 8],y(x), singsol=
```

$$y(x) = \left(4\,\mathrm{e}^{5x} - 3\,\mathrm{e}^{4x} - 1\right)\mathrm{e}^{-2x}$$

✓ Solution by Mathematica
Time used: 0.016 (sec). Leaf size: 27

```
DSolve[{y''[x]-y'[x]-6*y[x]==12*Exp[2*x],{y[0]==0,y'[0]==8}},y[x],x,IncludeSingularSolutions
```

$$y(x) \to e^{-2x}\left(-3e^{4x} + 4e^{5x} - 1\right)$$

16.17 problem 24.3 (a)

Internal problem ID [13793]

Book: Ordinary Differential Equations. An introduction to the fundamentals. Kenneth B. Howell. second edition. CRC Press. FL, USA. 2020
Section: Chapter 24. Variation of parameters. Additional exercises page 444
Problem number: 24.3 (a).
ODE order: 3.
ODE degree: 1.

CAS Maple gives this as type `[[_3rd_order, _missing_y]]`

$$\boxed{y''' - 4y' = 30\,\mathrm{e}^{3x}}$$

✓ Solution by Maple
Time used: 0.0 (sec). Leaf size: 32

```
dsolve(diff(y(x),x$3)-4*diff(y(x),x)=30*exp(3*x),y(x), singsol=all)
```

$$y(x) = -\frac{(-4\,\mathrm{e}^{5x} - c_2\mathrm{e}^{4x} - 2c_3\mathrm{e}^{2x} + c_1)\,\mathrm{e}^{-2x}}{2}$$

✓ Solution by Mathematica
Time used: 0.046 (sec). Leaf size: 37

```
DSolve[y'''[x]-4*y'[x]==30*Exp[3*x],y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to 2e^{3x} + \frac{1}{2}c_1e^{2x} - \frac{1}{2}c_2e^{-2x} + c_3$$

16.18 problem 24.3 (b)

Internal problem ID [13794]

Book: Ordinary Differential Equations. An introduction to the fundamentals. Kenneth B. Howell. second edition. CRC Press. FL, USA. 2020
Section: Chapter 24. Variation of parameters. Additional exercises page 444
Problem number: 24.3 (b).
ODE order: 3.
ODE degree: 1.

CAS Maple gives this as type `[[_3rd_order, _with_linear_symmetries]]`

$$\boxed{x^3y''' - 3x^2y'' + 6y'x - 6y = x^3}$$

✓ Solution by Maple
Time used: 0.016 (sec). Leaf size: 30

```
dsolve(x^3*diff(y(x),x$3)-3*x^2*diff(y(x),x$2)+6*x*diff(y(x),x)-6*y(x)=x^3,y(x), singsol=all
```

$$y(x) = \frac{\ln(x)\, x^3}{2} + \frac{(4c_3 - 3)\, x^3}{4} + c_2x^2 + c_1x$$

✓ Solution by Mathematica
Time used: 0.005 (sec). Leaf size: 34

```
DSolve[x^3*y'''[x]-3*x^2*y''[x]+6*x*y'[x]-6*y[x]==x^3,y[x],x,IncludeSingularSolutions -> Tru
```

$$y(x) \to \frac{1}{2}x^3 \log(x) + x\left(\left(-\frac{3}{4} + c_3\right)x^2 + c_2x + c_1\right)$$

16.19 problem 24.4 (a)

Internal problem ID [13795]

Book: Ordinary Differential Equations. An introduction to the fundamentals. Kenneth B. Howell. second edition. CRC Press. FL, USA. 2020
Section: Chapter 24. Variation of parameters. Additional exercises page 444
Problem number: 24.4 (a).
ODE order: 3.
ODE degree: 1.

CAS Maple gives this as type `[[_3rd_order, _with_linear_symmetries]]`

$$x^3 y''' - 3x^2 y'' + 6y'x - 6y = e^{-x^2}$$

✓ Solution by Maple

Time used: 0.016 (sec). Leaf size: 53

```
dsolve(x^3*diff(y(x),x$3)-3*x^2*diff(y(x),x$2)+6*x*diff(y(x),x)-6*y(x)=exp(-x^2),y(x), sings
```

$$y(x) = \frac{(2x^2-1)\,e^{-x^2}}{6} + x\left(-\frac{x \operatorname{expIntegral}_1(x^2)}{2} + \operatorname{erf}(x)\left(\frac{x^2}{3}-\frac{1}{2}\right)\sqrt{\pi} + x^2 c_3 + c_2 x + c_1\right)$$

✓ Solution by Mathematica

Time used: 0.068 (sec). Leaf size: 77

```
DSolve[x^3*y'''[x]-3*x^2*y''[x]+6*x*y'[x]-6*y[x]==Exp[-x^2],y[x],x,IncludeSingularSolutions
```

$$y(x) \to \frac{1}{6}\left(\sqrt{\pi}(2x^2-3)\,x\operatorname{erf}(x) + 3x^2 \operatorname{ExpIntegralEi}(-x^2) + 6c_3x^3 + 2e^{-x^2}x^2 - e^{-x^2} + 6c_2x^2 + 6c_1x\right)$$

16.20 problem 24.4 (b)

Internal problem ID [13796]

Book: Ordinary Differential Equations. An introduction to the fundamentals. Kenneth B. Howell. second edition. CRC Press. FL, USA. 2020
Section: Chapter 24. Variation of parameters. Additional exercises page 444
Problem number: 24.4 (b).
ODE order: 3.
ODE degree: 1.

CAS Maple gives this as type `[[_3rd_order, _linear, _nonhomogeneous]]`

$$y''' - y'' + y' - y = \tan(x)$$

✓ Solution by Maple
Time used: 0.0 (sec). Leaf size: 46

```
dsolve(diff(y(x),x$3)-diff(y(x),x$2)+diff(y(x),x)-y(x)=tan(x),y(x), singsol=all)
```

$$y(x) = \frac{\left(\int \tan(x)\,\mathrm{e}^{-x}dx\right)\mathrm{e}^{x}}{2} + \frac{(\cos(x)-\sin(x))\ln(\sec(x)+\tan(x))}{2} + c_1\cos(x) + c_2\mathrm{e}^{x} + c_3\sin(x) + \frac{1}{2}$$

✓ Solution by Mathematica
Time used: 0.06 (sec). Leaf size: 106

```
DSolve[y'''[x]-y''[x]+y'[x]-y[x]==Tan[x],y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to -\frac{1}{2}\sin(x)\text{arctanh}(\sin(x)) + \frac{1}{2}\cos(x)\text{arctanh}(\sin(x)) - \frac{1}{2}i\,\text{Hypergeometric2F1}\left(\frac{i}{2},1,1+\frac{i}{2},-e^{2ix}\right) - \left(\frac{1}{5}-\frac{i}{10}\right)e^{2ix}\,\text{Hypergeometric2F1}\left(1,1+\frac{i}{2},2+\frac{i}{2},-e^{2ix}\right) + c_3e^{x} + c_1\cos(x) + c_2\sin(x) + \frac{1}{2}$$

16.21 problem 24.4 (c)

Internal problem ID [13797]

Book: Ordinary Differential Equations. An introduction to the fundamentals. Kenneth B. Howell. second edition. CRC Press. FL, USA. 2020
Section: Chapter 24. Variation of parameters. Additional exercises page 444
Problem number: 24.4 (c).
ODE order: 4.
ODE degree: 1.

CAS Maple gives this as type `[[_high_order, _linear, _nonhomogeneous]]`

$$\boxed{y'''' - 81y = \sinh(x)}$$

✓ Solution by Maple
Time used: 0.0 (sec). Leaf size: 53

```
dsolve(diff(y(x),x$4)-81*y(x)=sinh(x),y(x), singsol=all)
```

$$y(x) = \frac{\left(160c_3\mathrm{e}^{6x} + 160c_1\cos(3x)\,\mathrm{e}^{3x} + 160c_4\sin(3x)\,\mathrm{e}^{3x} - \mathrm{e}^{4x} + \mathrm{e}^{2x} + 160c_2\right)\mathrm{e}^{-3x}}{160}$$

✓ Solution by Mathematica
Time used: 0.033 (sec). Leaf size: 52

```
DSolve[y''''[x]-81*y[x]==Sinh[x],y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to \frac{e^{-x}}{160} - \frac{e^{x}}{160} + c_1e^{3x} + c_3e^{-3x} + c_2\cos(3x) + c_4\sin(3x)$$

16.22 problem 24.4 (d)

Internal problem ID [13798]

Book: Ordinary Differential Equations. An introduction to the fundamentals. Kenneth B. Howell. second edition. CRC Press. FL, USA. 2020
Section: Chapter 24. Variation of parameters. Additional exercises page 444
Problem number: 24.4 (d).
ODE order: 4.
ODE degree: 1.

CAS Maple gives this as type `[[_high_order, _exact, _linear, _nonhomogeneous]]`

$$x^4y'''' + 6x^3y''' - 3x^2y'' - 9y'x + 9y = 12\sin\left(x^2\right)x$$

✓ Solution by Maple
Time used: 0.0 (sec). Leaf size: 69

```
dsolve(x^4*diff(y(x),x$4)+6*x^3*diff(y(x),x$3)-3*x^2*diff(y(x),x$2)-9*x*diff(y(x),x)+9*y(x)=
```

$$y(x) = \frac{(-2x^4 - 2)\sin\left(x^2\right) + 16c_4x^6 + 2\ \mathrm{Ci}\left(x^2\right)x^6 + 16c_2x^4 - 6\ \mathrm{Si}\left(x^2\right)x^4 + c_1x^2 - 4x^2\cos\left(x^2\right) + 16c_3}{16x^3}$$

✓ Solution by Mathematica
Time used: 0.032 (sec). Leaf size: 79

```
DSolve[x^4*y''''[x]+6*x^3*y'''[x]-3*x^2*y''[x]-9*x*y'[x]+9*y[x]==12*x*Sin[x^2],y[x],x,Includ
```

$$y(x) \to \frac{x^6\,\mathrm{CosIntegral}\left(x^2\right) - 3x^4\mathrm{Si}(x^2) + 8c_4x^6 + 8c_3x^4 - \sin\left(x^2\right) - 2x^2\cos\left(x^2\right) + 8c_2x^2 - x^4\sin\left(x^2\right) + 8c_1}{8x^3}$$

17 Chapter 25. Review exercises for part III. page 447

17.1 problem 1

Internal problem ID [13799]

Book: Ordinary Differential Equations. An introduction to the fundamentals. Kenneth B. Howell. second edition. CRC Press. FL, USA. 2020
Section: Chapter 25. Review exercises for part III. page 447
Problem number: 1.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _missing_x]]`

$$\boxed{y'' + 36y = 0}$$

✓ Solution by Maple
Time used: 0.0 (sec). Leaf size: 17

```
dsolve(diff(y(x),x$2)+36*y(x)=0,y(x), singsol=all)
```

$$y(x) = c_1 \sin(6x) + c_2 \cos(6x)$$

✓ Solution by Mathematica
Time used: 0.011 (sec). Leaf size: 20

```
DSolve[y''[x]+36*y[x]==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to c_1 \cos(6x) + c_2 \sin(6x)$$

17.2 problem 2

Internal problem ID [13800]

Book: Ordinary Differential Equations. An introduction to the fundamentals. Kenneth B. Howell. second edition. CRC Press. FL, USA. 2020
Section: Chapter 25. Review exercises for part III. page 447
Problem number: 2.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _missing_x]]`

$$\boxed{y'' - 12y' + 36y = 0}$$

✓ Solution by Maple
Time used: 0.0 (sec). Leaf size: 14

```
dsolve(diff(y(x),x$2)-12*diff(y(x),x)+36*y(x)=0,y(x), singsol=all)
```

$$y(x) = e^{6x}(c_2 x + c_1)$$

✓ Solution by Mathematica
Time used: 0.013 (sec). Leaf size: 18

```
DSolve[y''[x]-12*y'[x]+36*y[x]==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to e^{6x}(c_2 x + c_1)$$

17.3 problem 3

Internal problem ID [13801]

Book: Ordinary Differential Equations. An introduction to the fundamentals. Kenneth B. Howell. second edition. CRC Press. FL, USA. 2020
Section: Chapter 25. Review exercises for part III. page 447
Problem number: 3.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_Emden, _Fowler], [_2nd_order, _linear, '_with_symmetry_[0,F`

$$x^2y'' + y'x - 9y = 0$$

✓ Solution by Maple
Time used: 0.015 (sec). Leaf size: 15

```
dsolve(x^2*diff(y(x),x$2)+x*diff(y(x),x)-9*y(x)=0,y(x), singsol=all)
```

$$y(x) = \frac{c_2x^6 + c_1}{x^3}$$

✓ Solution by Mathematica
Time used: 0.01 (sec). Leaf size: 18

```
DSolve[x^2*y''[x]+x*y'[x]-9*y[x]==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to \frac{c_2x^6 + c_1}{x^3}$$

17.4 problem 4

Internal problem ID [13802]

Book: Ordinary Differential Equations. An introduction to the fundamentals. Kenneth B. Howell. second edition. CRC Press. FL, USA. 2020
Section: Chapter 25. Review exercises for part III. page 447
Problem number: 4.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _missing_x]]`

$$\boxed{y'' - 36y = 0}$$

✓ Solution by Maple
Time used: 0.0 (sec). Leaf size: 17

```
dsolve(diff(y(x),x$2)-36*y(x)=0,y(x), singsol=all)
```

$$y(x) = \mathrm{e}^{6x} c_1 + \mathrm{e}^{-6x} c_2$$

✓ Solution by Mathematica
Time used: 0.011 (sec). Leaf size: 22

```
DSolve[y''[x]-36*y[x]==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to c_1 e^{6x} + c_2 e^{-6x}$$

17.5 problem 5

Internal problem ID [13803]

Book: Ordinary Differential Equations. An introduction to the fundamentals. Kenneth B. Howell. second edition. CRC Press. FL, USA. 2020
Section: Chapter 25. Review exercises for part III. page 447
Problem number: 5.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _missing_x]]`

$$\boxed{y'' - 9y' + 14y = 0}$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 17

```
dsolve(diff(y(x),x$2)-9*diff(y(x),x)+14*y(x)=0,y(x), singsol=all)
```

$$y(x) = c_1 e^{7x} + c_2 e^{2x}$$

✓ Solution by Mathematica

Time used: 0.012 (sec). Leaf size: 22

```
DSolve[y''[x]-9*y'[x]+14*y[x]==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to e^{2x}\left(c_2 e^{5x} + c_1\right)$$

17.6 problem 6

Internal problem ID [13804]

Book: Ordinary Differential Equations. An introduction to the fundamentals. Kenneth B. Howell. second edition. CRC Press. FL, USA. 2020
Section: Chapter 25. Review exercises for part III. page 447
Problem number: 6.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_Emden, _Fowler]]`

$$\boxed{x^2y'' - 7y'x + 16y = 0}$$

✓ Solution by Maple
Time used: 0.0 (sec). Leaf size: 14

```
dsolve(x^2*diff(y(x),x$2)-7*x*diff(y(x),x)+16*y(x)=0,y(x), singsol=all)
```

$$y(x) = x^4(c_1 + c_2 \ln(x))$$

✓ Solution by Mathematica
Time used: 0.016 (sec). Leaf size: 18

```
DSolve[x^2*y''[x]-7*x*y'[x]+16*y[x]==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to x^4(4c_2 \log(x) + c_1)$$

17.7 problem 7

Internal problem ID [13805]

Book: Ordinary Differential Equations. An introduction to the fundamentals. Kenneth B. Howell. second edition. CRC Press. FL, USA. 2020
Section: Chapter 25. Review exercises for part III. page 447
Problem number: 7.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _missing_y]]`

$$\boxed{2xy'' + y' = \sqrt{x}}$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 16

```
dsolve(2*x*diff(y(x),x$2)+diff(y(x),x)=sqrt(x),y(x), singsol=all)
```

$$y(x) = \frac{\sqrt{x}\,(x + 6c_1)}{3} + c_2$$

✓ Solution by Mathematica

Time used: 0.035 (sec). Leaf size: 23

```
DSolve[2*x*y''[x]+y'[x]==Sqrt[x],y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to \frac{1}{3}\sqrt{x}(x + 6c_1) + c_2$$

17.8 problem 8

Internal problem ID [13806]

Book: Ordinary Differential Equations. An introduction to the fundamentals. Kenneth B. Howell. second edition. CRC Press. FL, USA. 2020
Section: Chapter 25. Review exercises for part III. page 447
Problem number: 8.
ODE order: 4.
ODE degree: 1.

CAS Maple gives this as type `[[_high_order, _missing_x]]`

$$y'''' - 8y'' + 16y = 0$$

✓ Solution by Maple
Time used: 0.016 (sec). Leaf size: 25

```
dsolve(diff(y(x),x$4)-8*diff(y(x),x$2)+16*y(x)=0,y(x), singsol=all)
```

$$y(x) = (c_4x + c_3)\,\mathrm{e}^{-2x} + \mathrm{e}^{2x}(c_2x + c_1)$$

✓ Solution by Mathematica
Time used: 0.003 (sec). Leaf size: 35

```
DSolve[y''''[x]-8*y''[x]+16*y[x]==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to e^{-2x}\left(c_3e^{4x} + x\left(c_4e^{4x} + c_2\right) + c_1\right)$$

17.9 problem 9

Internal problem ID [13807]

Book: Ordinary Differential Equations. An introduction to the fundamentals. Kenneth B. Howell. second edition. CRC Press. FL, USA. 2020
Section: Chapter 25. Review exercises for part III. page 447
Problem number: 9.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _missing_x]]`

$$\boxed{y'' + 6y' + 9y = 0}$$

✓ Solution by Maple
Time used: 0.0 (sec). Leaf size: 14

```
dsolve(diff(y(x),x$2)+6*diff(y(x),x)+9*y(x)=0,y(x), singsol=all)
```

$$y(x) = (c_2x + c_1)\,\mathrm{e}^{-3x}$$

✓ Solution by Mathematica
Time used: 0.014 (sec). Leaf size: 18

```
DSolve[y''[x]+6*y'[x]+9*y[x]==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to e^{-3x}(c_2x + c_1)$$

17.10 problem 10

Internal problem ID [13808]

Book: Ordinary Differential Equations. An introduction to the fundamentals. Kenneth B. Howell. second edition. CRC Press. FL, USA. 2020
Section: Chapter 25. Review exercises for part III. page 447
Problem number: 10.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _missing_x]]`

$$\boxed{y'' + 3y = 0}$$

✓ Solution by Maple
Time used: 0.0 (sec). Leaf size: 21

```
dsolve(diff(y(x),x$2)+3*y(x)=0,y(x), singsol=all)
```

$$y(x) = c_1 \sin\left(\sqrt{3}\, x\right) + c_2 \cos\left(\sqrt{3}\, x\right)$$

✓ Solution by Mathematica
Time used: 0.017 (sec). Leaf size: 28

```
DSolve[y''[x]+3*y[x]==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to c_1 \cos\left(\sqrt{3}x\right) + c_2 \sin\left(\sqrt{3}x\right)$$

17.11 problem 11

Internal problem ID [13809]

Book: Ordinary Differential Equations. An introduction to the fundamentals. Kenneth B. Howell. second edition. CRC Press. FL, USA. 2020
Section: Chapter 25. Review exercises for part III. page 447
Problem number: 11.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_Emden, _Fowler]]`

$$\boxed{x^2y'' + 7y'x + 9y = 0}$$

✓ Solution by Maple
Time used: 0.0 (sec). Leaf size: 14

```
dsolve(x^2*diff(y(x),x$2)+7*x*diff(y(x),x)+9*y(x)=0,y(x), singsol=all)
```

$$y(x) = \frac{c_1 + c_2 \ln(x)}{x^3}$$

✓ Solution by Mathematica
Time used: 0.015 (sec). Leaf size: 18

```
DSolve[x^2*y''[x]+7*x*y'[x]+9*y[x]==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to \frac{3c_2 \log(x) + c_1}{x^3}$$

17.12 problem 12

Internal problem ID [13810]

Book: Ordinary Differential Equations. An introduction to the fundamentals. Kenneth B. Howell. second edition. CRC Press. FL, USA. 2020
Section: Chapter 25. Review exercises for part III. page 447
Problem number: 12.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_Emden, _Fowler]]`

$$\boxed{x^2y'' + \frac{5y}{2} = 0}$$

✓ Solution by Maple
Time used: 0.015 (sec). Leaf size: 23

```
dsolve(x^2*diff(y(x),x$2)+5/2*y(x)=0,y(x), singsol=all)
```

$$y(x) = \sqrt{x}\left(c_1 \sin\left(\frac{3\ln(x)}{2}\right) + c_2 \cos\left(\frac{3\ln(x)}{2}\right)\right)$$

✓ Solution by Mathematica
Time used: 0.034 (sec). Leaf size: 32

```
DSolve[x^2*y''[x]+5/2*y[x]==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to \sqrt{x}\left(c_2 \cos\left(\frac{3\log(x)}{2}\right) + c_1 \sin\left(\frac{3\log(x)}{2}\right)\right)$$

17.13 problem 13

Internal problem ID [13811]

Book: Ordinary Differential Equations. An introduction to the fundamentals. Kenneth B. Howell. second edition. CRC Press. FL, USA. 2020
Section: Chapter 25. Review exercises for part III. page 447
Problem number: 13.
ODE order: 5.
ODE degree: 1.

CAS Maple gives this as type `[[_high_order, _missing_x]]`

$$y^{(5)} - 6y'''' + 13y''' = 0$$

✓ Solution by Maple
Time used: 0.0 (sec). Leaf size: 34

```
dsolve(diff(y(x),x$5)-6*diff(y(x),x$4)+13*diff(y(x),x$3)=0,y(x), singsol=all)
```

$$y(x) = c_1 + c_2 x + x^2 c_3 + c_4 e^{3x} \sin(2x) + c_5 e^{3x} \cos(2x)$$

✓ Solution by Mathematica
Time used: 2.104 (sec). Leaf size: 56

```
DSolve[y'''''[x]-6*y''''[x]+13*y'''[x]==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to c_5 x^2 + c_4 x - \frac{e^{3x}((46c_1 + 9c_2)\cos(2x) + (9c_1 - 46c_2)\sin(2x))}{2197} + c_3$$

17.14 problem 14

Internal problem ID [13812]

Book: Ordinary Differential Equations. An introduction to the fundamentals. Kenneth B. Howell. second edition. CRC Press. FL, USA. 2020
Section: Chapter 25. Review exercises for part III. page 447
Problem number: 14.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_Emden, _Fowler]]`

$$\boxed{x^2y'' - 6y = 0}$$

✓ Solution by Maple
Time used: 0.0 (sec). Leaf size: 15

```
dsolve(x^2*diff(y(x),x$2)-6*y(x)=0,y(x), singsol=all)
```

$$y(x) = \frac{x^5c_1 + c_2}{x^2}$$

✓ Solution by Mathematica
Time used: 0.01 (sec). Leaf size: 18

```
DSolve[x^2*y''[x]-6*y[x]==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to \frac{c_2x^5 + c_1}{x^2}$$

17.15 problem 15

Internal problem ID [13813]

Book: Ordinary Differential Equations. An introduction to the fundamentals. Kenneth B. Howell. second edition. CRC Press. FL, USA. 2020
Section: Chapter 25. Review exercises for part III. page 447
Problem number: 15.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _missing_x]]`

$$\boxed{y'' - 6y' + 25y = 0}$$

✓ Solution by Maple
Time used: 0.0 (sec). Leaf size: 22

```
dsolve(diff(y(x),x$2)-6*diff(y(x),x)+25*y(x)=0,y(x), singsol=all)
```

$$y(x) = e^{3x}(c_1 \sin(4x) + c_2 \cos(4x))$$

✓ Solution by Mathematica
Time used: 0.016 (sec). Leaf size: 26

```
DSolve[y''[x]-6*y'[x]+25*y[x]==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to e^{3x}(c_2 \cos(4x) + c_1 \sin(4x))$$

17.16 problem 16

Internal problem ID [13814]

Book: Ordinary Differential Equations. An introduction to the fundamentals. Kenneth B. Howell. second edition. CRC Press. FL, USA. 2020
Section: Chapter 25. Review exercises for part III. page 447
Problem number: 16.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _missing_x], _Liouville, [_2nd_order, _reducible`

$$\boxed{y'' - {y'}^2 = 0}$$

✓ Solution by Maple
Time used: 0.031 (sec). Leaf size: 15

```
dsolve(diff(y(x),x$2)=diff(y(x),x)^2,y(x), singsol=all)
```

$$y(x) = -\ln(-c_1 x - c_2)$$

✓ Solution by Mathematica
Time used: 0.217 (sec). Leaf size: 15

```
DSolve[y''[x]==y'[x]^2,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to c_2 - \log(x + c_1)$$

17.17 problem 17

Internal problem ID [13815]

Book: Ordinary Differential Equations. An introduction to the fundamentals. Kenneth B. Howell. second edition. CRC Press. FL, USA. 2020
Section: Chapter 25. Review exercises for part III. page 447
Problem number: 17.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_Emden, _Fowler], [_2nd_order, _linear, '_with_symmetry_[0,F`

$$\boxed{x^2y'' + y'x + 9y = 0}$$

✓ Solution by Maple
Time used: 0.0 (sec). Leaf size: 19

```
dsolve(x^2*diff(y(x),x$2)+x*diff(y(x),x)+9*y(x)=0,y(x), singsol=all)
```

$$y(x) = c_1 \sin(3\ln(x)) + c_2 \cos(3\ln(x))$$

✓ Solution by Mathematica
Time used: 0.016 (sec). Leaf size: 22

```
DSolve[x^2*y''[x]+x*y'[x]+9*y[x]==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to c_1 \cos(3\log(x)) + c_2 \sin(3\log(x))$$

17.18 problem 18

Internal problem ID [13816]

Book: Ordinary Differential Equations. An introduction to the fundamentals. Kenneth B. Howell. second edition. CRC Press. FL, USA. 2020
Section: Chapter 25. Review exercises for part III. page 447
Problem number: 18.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _missing_x]]`

$$\boxed{y'' - 8y' + 25y = 0}$$

✓ Solution by Maple
Time used: 0.0 (sec). Leaf size: 22

```
dsolve(diff(y(x),x$2)-8*diff(y(x),x)+25*y(x)=0,y(x), singsol=all)
```

$$y(x) = e^{4x}(c_1 \sin(3x) + c_2 \cos(3x))$$

✓ Solution by Mathematica
Time used: 0.015 (sec). Leaf size: 26

```
DSolve[y''[x]-8*y'[x]+25*y[x]==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to e^{4x}(c_2 \cos(3x) + c_1 \sin(3x))$$

17.19 problem 19

Internal problem ID [13817]

Book: Ordinary Differential Equations. An introduction to the fundamentals. Kenneth B. Howell. second edition. CRC Press. FL, USA. 2020
Section: Chapter 25. Review exercises for part III. page 447
Problem number: 19.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_Emden, _Fowler]]`

$$\boxed{x^2y'' + 2y'x - 30y = 0}$$

✓ Solution by Maple
Time used: 0.0 (sec). Leaf size: 15

```
dsolve(x^2*diff(y(x),x$2)+2*x*diff(y(x),x)-30*y(x)=0,y(x), singsol=all)
```

$$y(x) = x^5c_1 + \frac{c_2}{x^6}$$

✓ Solution by Mathematica
Time used: 0.01 (sec). Leaf size: 18

```
DSolve[x^2*y''[x]+2*x*y'[x]-30*y[x]==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to \frac{c_1}{x^6} + c_2x^5$$

17.20 problem 20

Internal problem ID [13818]

Book: Ordinary Differential Equations. An introduction to the fundamentals. Kenneth B. Howell. second edition. CRC Press. FL, USA. 2020
Section: Chapter 25. Review exercises for part III. page 447
Problem number: 20.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _missing_x]]`

$$\boxed{y'' + y' - 30y = 0}$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 17

```
dsolve(diff(y(x),x$2)+diff(y(x),x)-30*y(x)=0,y(x), singsol=all)
```

$$y(x) = \left(e^{11x} c_1 + c_2\right) e^{-6x}$$

✓ Solution by Mathematica

Time used: 0.012 (sec). Leaf size: 22

```
DSolve[y''[x]+y'[x]-30*y[x]==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to c_1 e^{-6x} + c_2 e^{5x}$$

17.21 problem 21

Internal problem ID [13819]

Book: Ordinary Differential Equations. An introduction to the fundamentals. Kenneth B. Howell. second edition. CRC Press. FL, USA. 2020
Section: Chapter 25. Review exercises for part III. page 447
Problem number: 21.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _missing_x]]`

$$\boxed{16y'' - 8y' + y = 0}$$

✓ Solution by Maple
Time used: 0.0 (sec). Leaf size: 14

```
dsolve(16*diff(y(x),x$2)-8*diff(y(x),x)+y(x)=0,y(x), singsol=all)
```

$$y(x) = e^{\frac{x}{4}}(c_2 x + c_1)$$

✓ Solution by Mathematica
Time used: 0.014 (sec). Leaf size: 20

```
DSolve[16*y''[x]-8*y'[x]+y[x]==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to e^{x/4}(c_2 x + c_1)$$

17.22 problem 22

Internal problem ID [13820]

Book: Ordinary Differential Equations. An introduction to the fundamentals. Kenneth B. Howell. second edition. CRC Press. FL, USA. 2020
Section: Chapter 25. Review exercises for part III. page 447
Problem number: 22.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_Emden, _Fowler]]`

$$\boxed{4x^2y'' + 8y'x + y = 0}$$

✓ Solution by Maple
Time used: 0.0 (sec). Leaf size: 14

```
dsolve(4*x^2*diff(y(x),x$2)+8*x*diff(y(x),x)+y(x)=0,y(x), singsol=all)
```

$$y(x) = \frac{c_1 + c_2 \ln(x)}{\sqrt{x}}$$

✓ Solution by Mathematica
Time used: 0.018 (sec). Leaf size: 24

```
DSolve[4*x^2*y''[x]+8*x*y'[x]+y[x]==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to \frac{c_2 \log(x) + 2c_1}{2\sqrt{x}}$$

17.23 problem 23

Internal problem ID [13821]

Book: Ordinary Differential Equations. An introduction to the fundamentals. Kenneth B. Howell. second edition. CRC Press. FL, USA. 2020
Section: Chapter 25. Review exercises for part III. page 447
Problem number: 23.
ODE order: 3.
ODE degree: 1.

CAS Maple gives this as type `[[_3rd_order, _missing_x]]`

$$\boxed{y''' - 6y'' + 12y' = 8}$$

✓ Solution by Maple
Time used: 0.0 (sec). Leaf size: 50

```
dsolve(diff(y(x),x$3)-6*diff(y(x),x$2)+12*diff(y(x),x)=8,y(x), singsol=all)
```

$$y(x) = \frac{\left(-\frac{\sqrt{3}\,c_2}{3} + c_1\right) e^{3x} \cos\left(\sqrt{3}\,x\right)}{4} + \frac{e^{3x}\left(\sqrt{3}\,c_1 + 3c_2\right) \sin\left(\sqrt{3}\,x\right)}{12} + \frac{2x}{3} + c_3$$

✓ Solution by Mathematica
Time used: 0.312 (sec). Leaf size: 71

```
DSolve[y'''[x]-6*y''[x]+12*y'[x]==8,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to \frac{1}{12}\left(8x - \left(\sqrt{3}c_1 - 3c_2\right) e^{3x} \cos\left(\sqrt{3}x\right) + \left(3c_1 + \sqrt{3}c_2\right) e^{3x} \sin\left(\sqrt{3}x\right)\right) + c_3$$

17.24 problem 24

Internal problem ID [13822]

Book: Ordinary Differential Equations. An introduction to the fundamentals. Kenneth B. Howell. second edition. CRC Press. FL, USA. 2020
Section: Chapter 25. Review exercises for part III. page 447
Problem number: 24.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_Emden, _Fowler], [_2nd_order, _linear, '_with_symmetry_[0,F`

$$\boxed{2x^2y'' - 3y'x + 2y = 0}$$

✓ Solution by Maple
Time used: 0.0 (sec). Leaf size: 15

```
dsolve(2*x^2*diff(y(x),x$2)-3*x*diff(y(x),x)+2*y(x)=0,y(x), singsol=all)
```

$$y(x) = c_1\sqrt{x} + c_2x^2$$

✓ Solution by Mathematica
Time used: 0.01 (sec). Leaf size: 20

```
DSolve[2*x^2*y''[x]-3*x*y'[x]+2*y[x]==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to c_2x^2 + c_1\sqrt{x}$$

17.25 problem 25

Internal problem ID [13823]

Book: Ordinary Differential Equations. An introduction to the fundamentals. Kenneth B. Howell. second edition. CRC Press. FL, USA. 2020
Section: Chapter 25. Review exercises for part III. page 447
Problem number: 25.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_Emden, _Fowler], [_2nd_order, _linear, '_with_symmetry_[0,F`

$$\boxed{9x^2y'' + 3y'x + y = 0}$$

✓ Solution by Maple
Time used: 0.0 (sec). Leaf size: 14

```
dsolve(9*x^2*diff(y(x),x$2)+3*x*diff(y(x),x)+y(x)=0,y(x), singsol=all)
```

$$y(x) = (c_1 + c_2 \ln(x))\, x^{\frac{1}{3}}$$

✓ Solution by Mathematica
Time used: 0.016 (sec). Leaf size: 24

```
DSolve[9*x^2*y''[x]+3*x*y'[x]+y[x]==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to \frac{1}{3}\sqrt[3]{x}(c_2 \log(x) + 3c_1)$$

17.26 problem 26

Internal problem ID [13824]

Book: Ordinary Differential Equations. An introduction to the fundamentals. Kenneth B. Howell. second edition. CRC Press. FL, USA. 2020
Section: Chapter 25. Review exercises for part III. page 447
Problem number: 26.
ODE order: 4.
ODE degree: 1.

CAS Maple gives this as type `[[_high_order, _missing_x]]`

$$\boxed{y'''' - 16y = 0}$$

✓ Solution by Maple
Time used: 0.0 (sec). Leaf size: 29

```
dsolve(diff(y(x),x$4)-16*y(x)=0,y(x), singsol=all)
```

$$y(x) = \mathrm{e}^{2x} c_1 + \mathrm{e}^{-2x} c_2 + c_3 \sin(2x) + c_4 \cos(2x)$$

✓ Solution by Mathematica
Time used: 0.002 (sec). Leaf size: 36

```
DSolve[y''''[x]-16*y[x]==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to c_1 e^{2x} + c_3 e^{-2x} + c_2 \cos(2x) + c_4 \sin(2x)$$

17.27 problem 27

Internal problem ID [13825]

Book: Ordinary Differential Equations. An introduction to the fundamentals. Kenneth B. Howell. second edition. CRC Press. FL, USA. 2020
Section: Chapter 25. Review exercises for part III. page 447
Problem number: 27.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _missing_x]]`

$$\boxed{2y'' - 7y' = -3}$$

✓ Solution by Maple
Time used: 0.0 (sec). Leaf size: 16

```
dsolve(2*diff(y(x),x$2)-7*diff(y(x),x)+3=0,y(x), singsol=all)
```

$$y(x) = \frac{2\,\mathrm{e}^{\frac{7x}{2}} c_1}{7} + \frac{3x}{7} + c_2$$

✓ Solution by Mathematica
Time used: 0.022 (sec). Leaf size: 26

```
DSolve[2*y''[x]-7*y'[x]+3==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to \frac{3x}{7} + \frac{2}{7} c_1 e^{7x/2} + c_2$$

17.28 problem 28

Internal problem ID [13826]

Book: Ordinary Differential Equations. An introduction to the fundamentals. Kenneth B. Howell. second edition. CRC Press. FL, USA. 2020
Section: Chapter 25. Review exercises for part III. page 447
Problem number: 28.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _missing_x]]`

$$\boxed{y'' + 20y' + 100y = 0}$$

✓ Solution by Maple
Time used: 0.016 (sec). Leaf size: 14

```
dsolve(diff(y(x),x$2)+20*diff(y(x),x)+100*y(x)=0,y(x), singsol=all)
```

$$y(x) = \mathrm{e}^{-10x}(c_2 x + c_1)$$

✓ Solution by Mathematica
Time used: 0.015 (sec). Leaf size: 18

```
DSolve[y''[x]+20*y'[x]+100*y[x]==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to e^{-10x}(c_2 x + c_1)$$

17.29 problem 29

Internal problem ID [13827]

Book: Ordinary Differential Equations. An introduction to the fundamentals. Kenneth B. Howell. second edition. CRC Press. FL, USA. 2020
Section: Chapter 25. Review exercises for part III. page 447
Problem number: 29.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _missing_y]]`

$$\boxed{xy'' - 3y' = 0}$$

✓ Solution by Maple
Time used: 0.0 (sec). Leaf size: 11

```
dsolve(x*diff(y(x),x$2)=3*diff(y(x),x),y(x), singsol=all)
```

$$y(x) = c_2 x^4 + c_1$$

✓ Solution by Mathematica
Time used: 0.009 (sec). Leaf size: 17

```
DSolve[x*y''[x]==3*y'[x],y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to \frac{c_1 x^4}{4} + c_2$$

17.30 problem 30

Internal problem ID [13828]

Book: Ordinary Differential Equations. An introduction to the fundamentals. Kenneth B. Howell. second edition. CRC Press. FL, USA. 2020
Section: Chapter 25. Review exercises for part III. page 447
Problem number: 30.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _missing_x]]`

$$\boxed{y'' - 5y' = 0}$$

✓ Solution by Maple
Time used: 0.0 (sec). Leaf size: 12

```
dsolve(diff(y(x),x$2)-5*diff(y(x),x)=0,y(x), singsol=all)
```

$$y(x) = c_1 + e^{5x} c_2$$

✓ Solution by Mathematica
Time used: 0.014 (sec). Leaf size: 19

```
DSolve[y''[x]-5*y'[x]==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to \frac{1}{5} c_1 e^{5x} + c_2$$

17.31 problem 31

Internal problem ID [13829]

Book: Ordinary Differential Equations. An introduction to the fundamentals. Kenneth B. Howell. second edition. CRC Press. FL, USA. 2020
Section: Chapter 25. Review exercises for part III. page 447
Problem number: 31.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$\boxed{y'' - 9y' + 14y = 98x^2}$$

✓ Solution by Maple
Time used: 0.0 (sec). Leaf size: 26

```
dsolve(diff(y(x),x$2)-9*diff(y(x),x)+14*y(x)=98*x^2,y(x), singsol=all)
```

$$y(x) = c_2 e^{7x} + e^{2x} c_1 + 7x^2 + 9x + \frac{67}{14}$$

✓ Solution by Mathematica
Time used: 0.014 (sec). Leaf size: 33

```
DSolve[y''[x]-9*y'[x]+14*y[x]==98*x^2,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to 7x^2 + 9x + c_1 e^{2x} + c_2 e^{7x} + \frac{67}{14}$$

17.32 problem 32

Internal problem ID [13830]

Book: Ordinary Differential Equations. An introduction to the fundamentals. Kenneth B. Howell. second edition. CRC Press. FL, USA. 2020
Section: Chapter 25. Review exercises for part III. page 447
Problem number: 32.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _linear, _nonhomogeneous]]`

$$\boxed{y'' - 12y' + 36y = 25 \sin(3x)}$$

✓ Solution by Maple
Time used: 0.0 (sec). Leaf size: 27

```
dsolve(diff(y(x),x$2)-12*diff(y(x),x)+36*y(x)=25*sin(3*x),y(x), singsol=all)
```

$$y(x) = (c_1 x + c_2) e^{6x} + \frac{4\cos(3x)}{9} + \frac{\sin(3x)}{3}$$

✓ Solution by Mathematica
Time used: 0.092 (sec). Leaf size: 35

```
DSolve[y''[x]-12*y'[x]+36*y[x]==25*Sin[3*x],y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to \frac{1}{3}\sin(3x) + \frac{4}{9}\cos(3x) + e^{6x}(c_2 x + c_1)$$

17.33 problem 33

Internal problem ID [13831]

Book: Ordinary Differential Equations. An introduction to the fundamentals. Kenneth B. Howell. second edition. CRC Press. FL, USA. 2020
Section: Chapter 25. Review exercises for part III. page 447
Problem number: 33.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _linear, _nonhomogeneous]]`

$$\boxed{y'' - 9y' + 14y = 576x^2 e^{-x}}$$

✓ Solution by Maple
Time used: 0.016 (sec). Leaf size: 33

```
dsolve(diff(y(x),x$2)-9*diff(y(x),x)+14*y(x)=576*x^2*exp(-x),y(x), singsol=all)
```

$$y(x) = c_2 e^{7x} + e^{2x} c_1 + \frac{(288x^2 + 264x + 97) e^{-x}}{12}$$

✓ Solution by Mathematica
Time used: 0.034 (sec). Leaf size: 39

```
DSolve[y''[x]-9*y'[x]+14*y[x]==576*x^2*Exp[-x],y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to e^{-x} \left(24x^2 + 22x + c_1 e^{3x} + c_2 e^{8x} + \frac{97}{12}\right)$$

17.34 problem 34

Internal problem ID [13832]

Book: Ordinary Differential Equations. An introduction to the fundamentals. Kenneth B. Howell. second edition. CRC Press. FL, USA. 2020
Section: Chapter 25. Review exercises for part III. page 447
Problem number: 34.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$\boxed{y'' - 12y' + 36y = 81\,\mathrm{e}^{3x}}$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 21

```
dsolve(diff(y(x),x$2)-12*diff(y(x),x)+36*y(x)=81*exp(3*x),y(x), singsol=all)
```

$$y(x) = (c_1 x + c_2)\,\mathrm{e}^{6x} + 9\,\mathrm{e}^{3x}$$

✓ Solution by Mathematica

Time used: 0.017 (sec). Leaf size: 26

```
DSolve[y''[x]-12*y'[x]+36*y[x]==81*Exp[3*x],y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to e^{3x}\left(9 + e^{3x}(c_2 x + c_1)\right)$$

17.35 problem 35

Internal problem ID [13833]

Book: Ordinary Differential Equations. An introduction to the fundamentals. Kenneth B. Howell. second edition. CRC Press. FL, USA. 2020
Section: Chapter 25. Review exercises for part III. page 447
Problem number: 35.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$\boxed{x^2y'' + y'x - 9y = 3\sqrt{x}}$$

✓ Solution by Maple
Time used: 0.0 (sec). Leaf size: 24

```
dsolve(x^2*diff(y(x),x$2)+x*diff(y(x),x)-9*y(x)=3*sqrt(x),y(x), singsol=all)
```

$$y(x) = \frac{35c_1x^6 - 12x^{\frac{7}{2}} + 35c_2}{35x^3}$$

✓ Solution by Mathematica
Time used: 0.03 (sec). Leaf size: 27

```
DSolve[x^2*y''[x]+x*y'[x]-9*y[x]==3*Sqrt[x],y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to c_2x^3 + \frac{c_1}{x^3} - \frac{12\sqrt{x}}{35}$$

17.36 problem 36

Internal problem ID [13834]

Book: Ordinary Differential Equations. An introduction to the fundamentals. Kenneth B. Howell. second edition. CRC Press. FL, USA. 2020
Section: Chapter 25. Review exercises for part III. page 447
Problem number: 36.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _linear, _nonhomogeneous]]`

$$y'' - 12y' + 36y = 3x\,\mathrm{e}^{6x} - 2\,\mathrm{e}^{6x}$$

✓ Solution by Maple
Time used: 0.0 (sec). Leaf size: 27

```
dsolve(diff(y(x),x$2)-12*diff(y(x),x)+36*y(x)=3*x*exp(6*x)-2*exp(6*x),y(x), singsol=all)
```

$$y(x) = \mathrm{e}^{6x}\left(c_2 + c_1 x + \frac{1}{2}x^3 - x^2 + \frac{4}{9}x\right)$$

✓ Solution by Mathematica
Time used: 0.023 (sec). Leaf size: 32

```
DSolve[y''[x]-12*y'[x]+36*y[x]==3*x*Exp[6*x]-2*Exp[6*x],y[x],x,IncludeSingularSolutions -> T
```

$$y(x) \to \frac{1}{2}e^{6x}\left(x^3 - 2x^2 + 2c_2 x + 2c_1\right)$$

17.37 problem 37

Internal problem ID [13835]

Book: Ordinary Differential Equations. An introduction to the fundamentals. Kenneth B. Howell. second edition. CRC Press. FL, USA. 2020
Section: Chapter 25. Review exercises for part III. page 447
Problem number: 37.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _linear, _nonhomogeneous]]`

$$\boxed{y'' + 36y = 6 \sec(6x)}$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 30

```
dsolve(diff(y(x),x$2)+36*y(x)=6*sec(6*x),y(x), singsol=all)
```

$$y(x) = -\frac{\ln(\sec(6x))\cos(6x)}{6} + \cos(6x)\,c_1 + \sin(6x)\,(c_2 + x)$$

✓ Solution by Mathematica

Time used: 0.045 (sec). Leaf size: 32

```
DSolve[y''[x]+36*y[x]==6*Sec[6*x],y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to (x + c_2)\sin(6x) + \cos(6x)\left(\frac{1}{6}\log(\cos(6x)) + c_1\right)$$

17.38 problem 38

Internal problem ID [13836]

Book: Ordinary Differential Equations. An introduction to the fundamentals. Kenneth B. Howell. second edition. CRC Press. FL, USA. 2020
Section: Chapter 25. Review exercises for part III. page 447
Problem number: 38.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$x^2y'' + 2y'x - 6y = 18\ln(x)$$

✓ Solution by Maple
Time used: 0.0 (sec). Leaf size: 20

```
dsolve(x^2*diff(y(x),x$2)+2*x*diff(y(x),x)-6*y(x)=18*ln(x),y(x), singsol=all)
```

$$y(x) = \frac{c_2}{x^3} + c_1x^2 - 3\ln(x) - \frac{1}{2}$$

✓ Solution by Mathematica
Time used: 0.024 (sec). Leaf size: 25

```
DSolve[x^2*y''[x]+2*x*y'[x]-6*y[x]==18*Log[x],y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to \frac{c_1}{x^3} + c_2x^2 - 3\log(x) - \frac{1}{2}$$

17.39 problem 39

Internal problem ID [13837]

Book: Ordinary Differential Equations. An introduction to the fundamentals. Kenneth B. Howell. second edition. CRC Press. FL, USA. 2020
Section: Chapter 25. Review exercises for part III. page 447
Problem number: 39.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$y'' + 6y' + 9y = 10\,e^{-3x}$$

✓ Solution by Maple
Time used: 0.016 (sec). Leaf size: 19

```
dsolve(diff(y(x),x$2)+6*diff(y(x),x)+9*y(x)=10*exp(-3*x),y(x), singsol=all)
```

$$y(x) = e^{-3x}(c_1 x + 5x^2 + c_2)$$

✓ Solution by Mathematica
Time used: 0.027 (sec). Leaf size: 23

```
DSolve[y''[x]+6*y'[x]+9*y[x]==10*Exp[-3*x],y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to e^{-3x}(5x^2 + c_2 x + c_1)$$

17.40 problem 40

Internal problem ID [13838]

Book: Ordinary Differential Equations. An introduction to the fundamentals. Kenneth B. Howell. second edition. CRC Press. FL, USA. 2020
Section: Chapter 25. Review exercises for part III. page 447
Problem number: 40.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$\boxed{2x^2y'' - y'x - 2y = 10x^2}$$

✓ Solution by Maple
Time used: 0.0 (sec). Leaf size: 26

```
dsolve(2*x^2*diff(y(x),x$2)-x*diff(y(x),x)-2*y(x)=10*x^2,y(x), singsol=all)
```

$$y(x) = \frac{c_2}{\sqrt{x}} + c_1x^2 + \frac{2x^2(-2 + 5\ln(x))}{5}$$

✓ Solution by Mathematica
Time used: 0.017 (sec). Leaf size: 31

```
DSolve[2*x^2*y''[x]-x*y'[x]-2*y[x]==10*x^2,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to 2x^2\log(x) + \left(-\frac{4}{5} + c_2\right)x^2 + \frac{c_1}{\sqrt{x}}$$

17.41 problem 41

Internal problem ID [13839]

Book: Ordinary Differential Equations. An introduction to the fundamentals. Kenneth B. Howell. second edition. CRC Press. FL, USA. 2020
Section: Chapter 25. Review exercises for part III. page 447
Problem number: 41.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _linear, _nonhomogeneous]]`

$$\boxed{y'' + 6y' + 9y = 2\cos(2x)}$$

✓ Solution by Maple
Time used: 0.0 (sec). Leaf size: 27

```
dsolve(diff(y(x),x$2)+6*diff(y(x),x)+9*y(x)=2*cos(2*x),y(x), singsol=all)
```

$$y(x) = (c_1 x + c_2)\,\mathrm{e}^{-3x} + \frac{10\cos(2x)}{169} + \frac{24\sin(2x)}{169}$$

✓ Solution by Mathematica
Time used: 0.021 (sec). Leaf size: 35

```
DSolve[y''[x]+6*y'[x]+9*y[x]==2*Cos[2*x],y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to \frac{24}{169}\sin(2x) + \frac{10}{169}\cos(2x) + e^{-3x}(c_2 x + c_1)$$

17.42 problem 42

Internal problem ID [13840]

Book: Ordinary Differential Equations. An introduction to the fundamentals. Kenneth B. Howell. second edition. CRC Press. FL, USA. 2020
Section: Chapter 25. Review exercises for part III. page 447
Problem number: 42.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _missing_y]]`

$$\boxed{xy'' - y' + 3xy'^3 = 0}$$

✓ Solution by Maple
Time used: 0.047 (sec). Leaf size: 43

```
dsolve(x*diff(y(x),x$2)-diff(y(x),x)=-3*x* diff(y(x),x)^3,y(x), singsol=all)
```

$$y(x) = \int \frac{x}{\sqrt{2x^3 - c_1}} dx + c_2$$
$$y(x) = -\left(\int \frac{x}{\sqrt{2x^3 - c_1}} dx\right) + c_2$$

✓ Solution by Mathematica

Time used: 1.949 (sec). Leaf size: 195

```
DSolve[x*y''[x]-y'[x]==-3*x* (y'[x])^3,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to c_2 - \frac{x^2\sqrt{1+\frac{2x^3}{c_1}}\,\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{2}{3},\frac{5}{3},-\frac{2x^3}{c_1}\right)}{2\sqrt{2x^3+c_1}}$$
$$y(x) \to \frac{x^2\sqrt{1+\frac{2x^3}{c_1}}\,\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{2}{3},\frac{5}{3},-\frac{2x^3}{c_1}\right)}{2\sqrt{2x^3+c_1}} + c_2$$
$$y(x) \to c_2$$
$$y(x) \to -\frac{3\sqrt{x^3}\,\text{Gamma}\left(\frac{5}{3}\right)}{\sqrt{2}x\,\text{Gamma}\left(\frac{2}{3}\right)} + c_2$$
$$y(x) \to \frac{3\sqrt{x^3}\,\text{Gamma}\left(\frac{5}{3}\right)}{\sqrt{2}x\,\text{Gamma}\left(\frac{2}{3}\right)} + c_2$$

17.43 problem 43

Internal problem ID [13841]

Book: Ordinary Differential Equations. An introduction to the fundamentals. Kenneth B. Howell. second edition. CRC Press. FL, USA. 2020
Section: Chapter 25. Review exercises for part III. page 447
Problem number: 43.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$\boxed{x^2y'' + 3y'x + 2y = 6}$$

✓ Solution by Maple
Time used: 0.015 (sec). Leaf size: 22

```
dsolve(x^2*diff(y(x),x$2)+3*x*diff(y(x),x)+2*y(x)=6,y(x), singsol=all)
```

$$y(x) = \frac{\cos(\ln(x))\, c_1 + c_2 \sin(\ln(x)) + 3x}{x}$$

✓ Solution by Mathematica
Time used: 0.025 (sec). Leaf size: 25

```
DSolve[x^2*y''[x]+3*x*y'[x]+2*y[x]==6,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to \frac{3x + c_2 \cos(\log(x)) + c_1 \sin(\log(x))}{x}$$

17.44 problem 44

Internal problem ID [13842]

Book: Ordinary Differential Equations. An introduction to the fundamentals. Kenneth B. Howell. second edition. CRC Press. FL, USA. 2020
Section: Chapter 25. Review exercises for part III. page 447
Problem number: 44.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _exact, _linear, _nonhomogeneous]]`

$$\boxed{x^2y'' + y'x - y = \frac{1}{x^2+1}}$$

✓ Solution by Maple
Time used: 0.0 (sec). Leaf size: 33

```
dsolve(x^2*diff(y(x),x$2)+x*diff(y(x),x)-y(x)=1/(1+x^2),y(x), singsol=all)
```

$$y(x) = \frac{-\arctan(x)\,x^2 + 2c_2x^2 - \arctan(x) + 2c_1 - x}{2x}$$

✓ Solution by Mathematica
Time used: 0.031 (sec). Leaf size: 33

```
DSolve[x^2*y''[x]+x*y'[x]-y[x]==1/(1+x^2),y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to -\frac{x^2\arctan(x) + \arctan(x) - 2c_2x^2 + x - 2c_1}{2x}$$

17.45 problem 45

Internal problem ID [13843]

Book: Ordinary Differential Equations. An introduction to the fundamentals. Kenneth B. Howell. second edition. CRC Press. FL, USA. 2020
Section: Chapter 25. Review exercises for part III. page 447
Problem number: 45.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _linear, _nonhomogeneous]]`

$$4y'' - 12y' + 9y = x\,\mathrm{e}^{\frac{3x}{2}}$$

✓ Solution by Maple
Time used: 0.0 (sec). Leaf size: 19

```
dsolve(4*diff(y(x),x$2)-12*diff(y(x),x)+9*y(x)=x*exp(3*x/2),y(x), singsol=all)
```

$$y(x) = \mathrm{e}^{\frac{3x}{2}}\left(c_2 + c_1 x + \frac{1}{24}x^3\right)$$

✓ Solution by Mathematica
Time used: 0.022 (sec). Leaf size: 29

```
DSolve[4*y''[x]-12*y'[x]+9*y[x]==x*Exp[3*x/2],y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to \frac{1}{24}e^{3x/2}(x^3 + 24c_2 x + 24c_1)$$

17.46 problem 46

Internal problem ID [13844]

Book: Ordinary Differential Equations. An introduction to the fundamentals. Kenneth B. Howell. second edition. CRC Press. FL, USA. 2020
Section: Chapter 25. Review exercises for part III. page 447
Problem number: 46.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _linear, _nonhomogeneous]]`

$$3y'' + 8y' - 3y = 123x\sin(3x)$$

✓ Solution by Maple
Time used: 0.0 (sec). Leaf size: 45

```
dsolve(3*diff(y(x),x$2)+8*diff(y(x),x)-3*y(x)=123*x*sin(3*x),y(x), singsol=all)
```

$$y(x) = -2\,\mathrm{e}^{-3x}\left(-\frac{c_1\mathrm{e}^{\frac{10x}{3}}}{2} + \left(\left(x+\frac{241}{492}\right)\cos(3x) + \sin(3x)\left(\frac{5x}{4} - \frac{27}{41}\right)\right)\mathrm{e}^{3x} - \frac{c_2}{2}\right)$$

✓ Solution by Mathematica
Time used: 0.027 (sec). Leaf size: 50

```
DSolve[3*y''[x]+8*y'[x]-3*y[x]==123*x*Sin[3*x],y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to \left(\frac{54}{41} - \frac{5x}{2}\right)\sin(3x) + \left(-2x - \frac{241}{246}\right)\cos(3x) + c_1e^{x/3} + c_2e^{-3x}$$

17.47 problem 47

Internal problem ID [13845]

Book: Ordinary Differential Equations. An introduction to the fundamentals. Kenneth B. Howell. second edition. CRC Press. FL, USA. 2020
Section: Chapter 25. Review exercises for part III. page 447
Problem number: 47.
ODE order: 3.
ODE degree: 1.

CAS Maple gives this as type `[[_3rd_order, _with_linear_symmetries]]`

$$y''' + 8y = e^{-2x}$$

✓ Solution by Maple
Time used: 0.0 (sec). Leaf size: 41

```
dsolve(diff(y(x),x$3)+8*y(x)=exp(-2*x),y(x), singsol=all)
```

$$y(x) = \frac{\left(12c_3e^{3x}\sin\left(\sqrt{3}\,x\right) + 12c_2e^{3x}\cos\left(\sqrt{3}\,x\right) + 12c_1 + x\right)e^{-2x}}{12}$$

✓ Solution by Mathematica
Time used: 0.337 (sec). Leaf size: 57

```
DSolve[y'''[x]+8*y[x]==Exp[-2*x],y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to \frac{1}{24}e^{-2x}\left(2x + 24c_3e^{3x}\cos\left(\sqrt{3}x\right) + 24c_2e^{3x}\sin\left(\sqrt{3}x\right) + 1 + 24c_1\right)$$

17.48 problem 48

Internal problem ID [13846]

Book: Ordinary Differential Equations. An introduction to the fundamentals. Kenneth B. Howell. second edition. CRC Press. FL, USA. 2020
Section: Chapter 25. Review exercises for part III. page 447
Problem number: 48.
ODE order: 6.
ODE degree: 1.

CAS Maple gives this as type `[[_high_order, _with_linear_symmetries]]`

$$\boxed{y^{(6)} - 64y = e^{-2x}}$$

✓ Solution by Maple
Time used: 0.015 (sec). Leaf size: 62

```
dsolve(diff(y(x),x$6)-64*y(x)=exp(-2*x),y(x), singsol=all)
```

$$y(x) = -\frac{\left(\left(-192c_3 e^{3x} - 192c_5 e^{x}\right)\cos\left(\sqrt{3}\,x\right) + \left(-192c_4 e^{3x} - 192c_6 e^{x}\right)\sin\left(\sqrt{3}\,x\right) - 192c_2 e^{4x} + x - 192c_1\right) e^{-2x}}{192}$$

✓ Solution by Mathematica
Time used: 0.738 (sec). Leaf size: 80

```
DSolve[y''''''[x]-64*y[x]==Exp[-2*x],y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to \frac{1}{768}e^{-2x}\left(-4x + 768c_1e^{4x} + 768e^{x}\left(c_2e^{2x} + c_3\right)\cos\left(\sqrt{3}x\right) + 768e^{x}\left(c_6e^{2x} + c_5\right)\sin\left(\sqrt{3}x\right) - 5 + 768c_4\right)$$

17.49 problem 49

Internal problem ID [13847]

Book: Ordinary Differential Equations. An introduction to the fundamentals. Kenneth B. Howell. second edition. CRC Press. FL, USA. 2020
Section: Chapter 25. Review exercises for part III. page 447
Problem number: 49.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _exact, _linear, _nonhomogeneous]]`

$$\boxed{x^2y'' + 3y'x + y = \frac{1}{(x+1)^2}}$$

✓ Solution by Maple
Time used: 0.0 (sec). Leaf size: 20

```
dsolve(x^2*diff(y(x),x$2)+3*x*diff(y(x),x)+y(x)=1/(1+x)^2,y(x), singsol=all)
```

$$y(x) = \frac{c_2 + \ln(1+x) + (c_1 - 1)\ln(x)}{x}$$

✓ Solution by Mathematica
Time used: 0.035 (sec). Leaf size: 23

```
DSolve[x^2*y''[x]+3*x*y'[x]+y[x]==1/(1+x)^2,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to \frac{\log(x+1) + (-1+c_2)\log(x) + c_1}{x}$$

17.50 problem 50

Internal problem ID [13848]

Book: Ordinary Differential Equations. An introduction to the fundamentals. Kenneth B. Howell. second edition. CRC Press. FL, USA. 2020
Section: Chapter 25. Review exercises for part III. page 447
Problem number: 50.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _exact, _linear, _nonhomogeneous]]`

$$\boxed{x^2 y'' + 3y'x + y = \frac{1}{x}}$$

✓ Solution by Maple
Time used: 0.0 (sec). Leaf size: 20

```
dsolve(x^2*diff(y(x),x$2)+3*x*diff(y(x),x)+y(x)=1/x,y(x), singsol=all)
```

$$y(x) = \frac{c_2 + c_1 \ln(x) + \frac{\ln(x)^2}{2}}{x}$$

✓ Solution by Mathematica
Time used: 0.022 (sec). Leaf size: 27

```
DSolve[x^2*y''[x]+3*x*y'[x]+y[x]==1/x,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to \frac{\log^2(x) + 2c_2 \log(x) + 2c_1}{2x}$$

18 Chapter 27. Differentiation and the Laplace transform. Additional Exercises. page 496

18.1 problem 27.1 (a)

Internal problem ID [13849]

Book: Ordinary Differential Equations. An introduction to the fundamentals. Kenneth B. Howell. second edition. CRC Press. FL, USA. 2020
Section: Chapter 27. Differentiation and the Laplace transform. Additional Exercises. page 496
Problem number: 27.1 (a).
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type **[_quadrature]**

$$\boxed{y' + 4y = 0}$$

With initial conditions

$$[y(0) = 3]$$

✓ Solution by Maple
Time used: 3.953 (sec). Leaf size: 10

```
dsolve([diff(y(t),t)+4*y(t)=0,y(0) = 3],y(t), singsol=all)
```

$$y(t) = 3\,\mathrm{e}^{-4t}$$

✓ Solution by Mathematica
Time used: 0.047 (sec). Leaf size: 12

```
DSolve[{y'[t]+4*y[t]==0,{y[0]==3}},y[t],t,IncludeSingularSolutions -> True]
```

$$y(t) \to 3e^{-4t}$$

18.2 problem 27.1 (b)

Internal problem ID [13850]

Book: Ordinary Differential Equations. An introduction to the fundamentals. Kenneth B. Howell. second edition. CRC Press. FL, USA. 2020
Section: Chapter 27. Differentiation and the Laplace transform. Additional Exercises. page 496
Problem number: 27.1 (b).
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[[_linear, ‘class A‘]]`

$$\boxed{y' - 2y = t^3}$$

With initial conditions

$$[y(0) = 4]$$

✓ Solution by Maple
Time used: 4.172 (sec). Leaf size: 25

```
dsolve([diff(y(t),t)-2*y(t)=t^3,y(0) = 4],y(t), singsol=all)
```

$$y(t) = -\frac{3t}{4} - \frac{t^3}{2} - \frac{3t^2}{4} + \frac{35\,e^{2t}}{8} - \frac{3}{8}$$

✓ Solution by Mathematica
Time used: 0.052 (sec). Leaf size: 31

```
DSolve[{y'[t]+4*y[t]==t^3,{y[0]==4}},y[t],t,IncludeSingularSolutions -> True]
```

$$y(t) \to \frac{1}{128}\left(32t^3 - 24t^2 + 12t + 515e^{-4t} - 3\right)$$

18.3 problem 27.1 (c)

Internal problem ID [13851]

Book: Ordinary Differential Equations. An introduction to the fundamentals. Kenneth B. Howell. second edition. CRC Press. FL, USA. 2020
Section: Chapter 27. Differentiation and the Laplace transform. Additional Exercises. page 496
Problem number: 27.1 (c).
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[[_linear, ‘class A‘]]`

$$\boxed{y' + 3y = \text{Heaviside}(t-4)}$$

With initial conditions

$$[y(0) = 0]$$

✓ Solution by Maple
Time used: 4.937 (sec). Leaf size: 20

```
dsolve([diff(y(t),t)+3*y(t)=Heaviside(t-4),y(0) = 0],y(t), singsol=all)
```

$$y(t) = -\frac{\text{Heaviside}(t-4)\left(-1+\mathrm{e}^{-3t+12}\right)}{3}$$

✓ Solution by Mathematica
Time used: 0.065 (sec). Leaf size: 27

```
DSolve[{y'[t]+3*y[t]==UnitStep[t-4],{y[0]==0}},y[t],t,IncludeSingularSolutions -> True]
```

$$y(t) \rightarrow \begin{cases} \frac{1}{3} - \frac{1}{3}e^{12-3t} & t > 4 \\ 0 & \text{True} \end{cases}$$

18.4 problem 27.1 (d)

Internal problem ID [13852]

Book: Ordinary Differential Equations. An introduction to the fundamentals. Kenneth B. Howell. second edition. CRC Press. FL, USA. 2020
Section: Chapter 27. Differentiation and the Laplace transform. Additional Exercises. page 496
Problem number: 27.1 (d).
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _linear, _nonhomogeneous]]`

$$\boxed{y'' - 4y = t^3}$$

With initial conditions

$$[y(0) = 1, y'(0) = 0]$$

✓ Solution by Maple
Time used: 4.188 (sec). Leaf size: 25

```
dsolve([diff(y(t),t$2)-4*y(t)=t^3,y(0) = 1, D(y)(0) = 0],y(t), singsol=all)
```

$$y(t) = -\frac{3t}{8} - \frac{t^3}{4} + \frac{19\,\mathrm{e}^{2t}}{32} + \frac{13\,\mathrm{e}^{-2t}}{32}$$

✓ Solution by Mathematica
Time used: 0.02 (sec). Leaf size: 34

```
DSolve[{y''[t]-4*y[t]==t^3,{y[0]==1,y'[0]==3}},y[t],t,IncludeSingularSolutions -> True]
```

$$y(t) \to \frac{1}{32}\left(-4t\left(2t^2 + 3\right) - 11e^{-2t} + 43e^{2t}\right)$$

18.5 problem 27.1 (e)

Internal problem ID [13853]

Book: Ordinary Differential Equations. An introduction to the fundamentals. Kenneth B. Howell. second edition. CRC Press. FL, USA. 2020
Section: Chapter 27. Differentiation and the Laplace transform. Additional Exercises. page 496
Problem number: 27.1 (e).
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$\boxed{y'' + 4y = 20\,\mathrm{e}^{4t}}$$

With initial conditions

$$[y(0) = 3, y'(0) = 12]$$

✓ Solution by Maple
Time used: 4.954 (sec). Leaf size: 21

```
dsolve([diff(y(t),t$2)+4*y(t)=20*exp(4*t),y(0) = 3, D(y)(0) = 12],y(t), singsol=all)
```

$$y(t) = 2\cos(2t) + 4\sin(2t) + \mathrm{e}^{4t}$$

✓ Solution by Mathematica
Time used: 0.018 (sec). Leaf size: 23

```
DSolve[{y''[t]+4*y[t]==20*Exp[4*t],{y[0]==3,y'[0]==12}},y[t],t,IncludeSingularSolutions -> T
```

$$y(t) \to e^{4t} + 4\sin(2t) + 2\cos(2t)$$

18.6 problem 27.1 (f)

Internal problem ID [13854]

Book: Ordinary Differential Equations. An introduction to the fundamentals. Kenneth B. Howell. second edition. CRC Press. FL, USA. 2020
Section: Chapter 27. Differentiation and the Laplace transform. Additional Exercises. page 496
Problem number: 27.1 (f).
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _linear, _nonhomogeneous]]`

$$\boxed{y'' + 4y = \sin(2t)}$$

With initial conditions

$$[y(0) = 3, y'(0) = 5]$$

✓ Solution by Maple
Time used: 3.891 (sec). Leaf size: 20

```
dsolve([diff(y(t),t$2)+4*y(t)=sin(2*t),y(0) = 3, D(y)(0) = 5],y(t), singsol=all)
```

$$y(t) = \frac{21\sin(2t)}{8} - \frac{\cos(2t)(-12+t)}{4}$$

✓ Solution by Mathematica
Time used: 0.099 (sec). Leaf size: 26

```
DSolve[{y''[t]+4*y[t]==Sin[2*t],{y[0]==3,y'[0]==5}},y[t],t,IncludeSingularSolutions -> True]
```

$$y(t) \to \frac{21}{8}\sin(2t) + \left(3 - \frac{t}{4}\right)\cos(2t)$$

18.7 problem 27.1 (g)

Internal problem ID [13855]

Book: Ordinary Differential Equations. An introduction to the fundamentals. Kenneth B. Howell. second edition. CRC Press. FL, USA. 2020
Section: Chapter 27. Differentiation and the Laplace transform. Additional Exercises. page 496
Problem number: 27.1 (g).
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _linear, _nonhomogeneous]]`

$$\boxed{y'' + 4y = 3\,\text{Heaviside}\,(-2+t)}$$

With initial conditions

$$[y(0) = 0, y'(0) = 5]$$

✓ Solution by Maple
Time used: 4.594 (sec). Leaf size: 23

```
dsolve([diff(y(t),t$2)+4*y(t)=3*Heaviside(t-2),y(0) = 0, D(y)(0) = 5],y(t), singsol=all)
```

$$y(t) = \frac{3\,\text{Heaviside}\,(t-2)\sin(t-2)^2}{2} + \frac{5\sin(2t)}{2}$$

✓ Solution by Mathematica
Time used: 0.028 (sec). Leaf size: 37

```
DSolve[{y''[t]+4*y[t]==UnitStep[t-2],{y[0]==0,y'[0]==5}},y[t],t,IncludeSingularSolutions ->
```

$$y(t) \to \begin{cases} 5\cos(t)\sin(t) & t \leq 2 \\ \frac{1}{4}(-\cos(4-2t) + 10\sin(2t) + 1) & \text{True} \end{cases}$$

18.8 problem 27.1 (h)

Internal problem ID [13856]

Book: Ordinary Differential Equations. An introduction to the fundamentals. Kenneth B. Howell. second edition. CRC Press. FL, USA. 2020
Section: Chapter 27. Differentiation and the Laplace transform. Additional Exercises. page 496
Problem number: 27.1 (h).
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$\boxed{y'' + 5y' + 6y = e^{4t}}$$

With initial conditions

$$[y(0) = 1, y'(0) = 0]$$

✓ Solution by Maple
Time used: 4.953 (sec). Leaf size: 23

```
dsolve([diff(y(t),t$2)+5*diff(y(t),t)+6*y(t)=exp(4*t),y(0) = 1, D(y)(0) = 0],y(t), singsol=a
```

$$y(t) = \frac{(e^{7t} + 119\, e^{t} - 78)\, e^{-3t}}{42}$$

✓ Solution by Mathematica
Time used: 0.048 (sec). Leaf size: 26

```
DSolve[{y''[t]+5*y'[t]+6*y[t]==Exp[4*t],{y[0]==1,y'[0]==0}},y[t],t,IncludeSingularSolutions
```

$$y(t) \to \frac{1}{42} e^{-3t} \left(119 e^{t} + e^{7t} - 78\right)$$

18.9 problem 27.1 (i)

Internal problem ID [13857]

Book: Ordinary Differential Equations. An introduction to the fundamentals. Kenneth B. Howell. second edition. CRC Press. FL, USA. 2020
Section: Chapter 27. Differentiation and the Laplace transform. Additional Exercises. page 496
Problem number: 27.1 (i).
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _linear, _nonhomogeneous]]`

$$\boxed{y'' - 5y' + 6y = t^2 e^{4t}}$$

With initial conditions

$$[y(0) = 0, y'(0) = 2]$$

✓ Solution by Maple
Time used: 4.906 (sec). Leaf size: 27

```
dsolve([diff(y(t),t$2)-5*diff(y(t),t)+6*y(t)=t^2*exp(4*t),y(0) = 0, D(y)(0) = 2],y(t), sings
```

$$y(t) = -\frac{7\,e^{2t}}{4} + \frac{(2t^2 - 6t + 7)\,e^{4t}}{4}$$

✓ Solution by Mathematica
Time used: 0.021 (sec). Leaf size: 32

```
DSolve[{y''[t]-5*y'[t]+6*y[t]==t^2*Exp[4*t],{y[0]==0,y'[0]==2}},y[t],t,IncludeSingularSoluti
```

$$y(t) \to \frac{1}{4}e^{2t}\left(e^{2t}(2t^2 - 6t + 7) - 7\right)$$

18.10 problem 27.1 (j)

Internal problem ID [13858]

Book: Ordinary Differential Equations. An introduction to the fundamentals. Kenneth B. Howell. second edition. CRC Press. FL, USA. 2020
Section: Chapter 27. Differentiation and the Laplace transform. Additional Exercises. page 496
Problem number: 27.1 (j).
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _missing_x]]`

$$\boxed{y'' - 5y' + 6y = 7}$$

With initial conditions

$$[y(0) = 2, y'(0) = 4]$$

✓ Solution by Maple
Time used: 4.094 (sec). Leaf size: 18

```
dsolve([diff(y(t),t$2)-5*diff(y(t),t)+6*y(t)=7,y(0) = 2, D(y)(0) = 4],y(t), singsol=all)
```

$$y(t) = \frac{7\,e^{3t}}{3} - \frac{3\,e^{2t}}{2} + \frac{7}{6}$$

✓ Solution by Mathematica
Time used: 0.013 (sec). Leaf size: 25

```
DSolve[{y''[t]-5*y'[t]+6*y[t]==7,{y[0]==2,y'[0]==4}},y[t],t,IncludeSingularSolutions -> True
```

$$y(t) \to \frac{1}{6}\left(-9e^{2t} + 14e^{3t} + 7\right)$$

18.11 problem 27.1 (k)

Internal problem ID [13859]

Book: Ordinary Differential Equations. An introduction to the fundamentals. Kenneth B. Howell. second edition. CRC Press. FL, USA. 2020
Section: Chapter 27. Differentiation and the Laplace transform. Additional Exercises. page 496
Problem number: 27.1 (k).
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _linear, _nonhomogeneous]]`

$$\boxed{y'' - 4y' + 13y = e^{2t} \sin(3t)}$$

With initial conditions

$$[y(0) = 4, y'(0) = 3]$$

✓ Solution by Maple
Time used: 5.0 (sec). Leaf size: 26

```
dsolve([diff(y(t),t$2)-4*diff(y(t),t)+13*y(t)=exp(2*t)*sin(3*t),y(0) = 4, D(y)(0) = 3],y(t),
```

$$y(t) = -\frac{(-24 + t) e^{2t} \cos(3t)}{6} - \frac{29 e^{2t} \sin(3t)}{18}$$

✓ Solution by Mathematica
Time used: 0.139 (sec). Leaf size: 30

```
DSolve[{y''[t]-4*y'[t]+13*y[t]==Exp[2*t]*Sin[3*t],{y[0]==4,y'[0]==3}},y[t],t,IncludeSingular
```

$$y(t) \to -\frac{1}{18} e^{2t} (29 \sin(3t) + 3(t - 24) \cos(3t))$$

18.12 problem 27.1 (L)

Internal problem ID [13860]

Book: Ordinary Differential Equations. An introduction to the fundamentals. Kenneth B. Howell. second edition. CRC Press. FL, USA. 2020
Section: Chapter 27. Differentiation and the Laplace transform. Additional Exercises. page 496
Problem number: 27.1 (L).
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _linear, _nonhomogeneous]]`

$$\boxed{y'' + 4y' + 13y = 4t + 2\,\mathrm{e}^{2t}\sin(3t)}$$

With initial conditions

$$[y(0) = 4, y'(0) = 3]$$

✓ Solution by Maple
Time used: 5.204 (sec). Leaf size: 47

```
dsolve([diff(y(t),t$2)+4*diff(y(t),t)+13*y(t)=4*t+2*exp(2*t)*sin(3*t),y(0) = 4, D(y)(0) = 3]
```

$$y(t) = -\frac{16}{169} + \frac{2\cosh(2t)\,(346\cos(3t) + 313\sin(3t))}{169} + \frac{(-1423\cos(3t) - 1226\sin(3t))\sinh(2t)}{338} + \frac{4t}{13}$$

✓ Solution by Mathematica
Time used: 1.331 (sec). Leaf size: 55

```
DSolve[{y''[t]+4*y'[t]+13*y[t]==4*t+2*Exp[2*t]*Sin[3*t],{y[0]==4,y'[0]==3}},y[t],t,IncludeSi
```

$$y(t) \to \frac{1}{676}e^{-2t}\left(16e^{2t}(13t-4) + \left(26e^{4t} + 2478\right)\sin(3t) + \left(2807 - 39e^{4t}\right)\cos(3t)\right)$$

18.13 problem 27.1 (m)

Internal problem ID [13861]

Book: Ordinary Differential Equations. An introduction to the fundamentals. Kenneth B. Howell. second edition. CRC Press. FL, USA. 2020
Section: Chapter 27. Differentiation and the Laplace transform. Additional Exercises. page 496
Problem number: 27.1 (m).
ODE order: 3.
ODE degree: 1.

CAS Maple gives this as type `[[_3rd_order, _with_linear_symmetries]]`

$$\boxed{y''' - 27y = \mathrm{e}^{-3t}}$$

With initial conditions

$$[y(0) = 2, y'(0) = 3, y''(0) = 4]$$

✓ Solution by Maple

Time used: 5.672 (sec). Leaf size: 44

```
dsolve([diff(y(t),t$3)-27*y(t)=exp(-3*t),y(0) = 2, D(y)(0) = 3, (D@@2)(y)(0) = 4],y(t), sing
```

$$y(t) = \frac{14\sqrt{3}\,\mathrm{e}^{-\frac{3t}{2}} \sin\left(\frac{3\sqrt{3}\,t}{2}\right)}{81} + \frac{70\,\mathrm{e}^{-\frac{3t}{2}} \cos\left(\frac{3\sqrt{3}\,t}{2}\right)}{81} + \frac{92\cosh(3t)}{81} + \frac{95\sinh(3t)}{81}$$

✓ Solution by Mathematica

Time used: 0.405 (sec). Leaf size: 68

```
DSolve[{y'''[t]-27*y[t]==Exp[-3*t],{y[0]==2,y'[0]==3,y''[0]==4}},y[t],t,IncludeSingularSolut
```

$$y(t) \to \frac{1}{162}e^{-3t}\left(187e^{6t} + 28\sqrt{3}e^{3t/2}\sin\left(\frac{3\sqrt{3}t}{2}\right) + 140e^{3t/2}\cos\left(\frac{3\sqrt{3}t}{2}\right) - 3\right)$$

18.14 problem 27.4

Internal problem ID [13862]

Book: Ordinary Differential Equations. An introduction to the fundamentals. Kenneth B. Howell. second edition. CRC Press. FL, USA. 2020
Section: Chapter 27. Differentiation and the Laplace transform. Additional Exercises. page 496
Problem number: 27.4.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[_Lienard]`

✓ Solution by Maple
Time used: 4.516 (sec). Leaf size: 7

```
dsolve([t*diff(y(t),t$2)+diff(y(t),t)+t*y(t)=0,y(0) = 1, D(y)(0) = 0],y(t), singsol=all)
```

$$y(t) = \operatorname{BesselJ}(0, t)$$

✓ Solution by Mathematica
Time used: 0.108 (sec). Leaf size: 8

```
DSolve[{t*y''[t]+y'[t]+t*y[t]==0,{y[0]==1,y'[0]==0}},y[t],t,IncludeSingularSolutions -> True
```

$$y(t) \to \operatorname{BesselJ}(0, t)$$

19 Chapter 28. The inverse Laplace transform. Additional Exercises. page 509

19.1 problem 28.6 (a)

Internal problem ID [13863]

Book: Ordinary Differential Equations. An introduction to the fundamentals. Kenneth B. Howell. second edition. CRC Press. FL, USA. 2020
Section: Chapter 28. The inverse Laplace transform. Additional Exercises. page 509
Problem number: 28.6 (a).
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _missing_x]]`

$$\boxed{y'' - 9y = 0}$$

With initial conditions

$$[y(0) = 4, y'(0) = 9]$$

✓ Solution by Maple

Time used: 4.485 (sec). Leaf size: 17

```
dsolve([diff(y(t),t$2)-9*y(t)=0,y(0) = 4, D(y)(0) = 9],y(t), singsol=all)
```

$$y(t) = \frac{e^{-3t}}{2} + \frac{7\,e^{3t}}{2}$$

✓ Solution by Mathematica

Time used: 0.013 (sec). Leaf size: 23

```
DSolve[{y''[t]-9*y[t]==0,{y[0]==4,y'[0]==9}},y[t],t,IncludeSingularSolutions -> True]
```

$$y(t) \to \frac{1}{2}e^{-3t}\left(7e^{6t} + 1\right)$$

19.2 problem 28.6 (b)

Internal problem ID [13864]

Book: Ordinary Differential Equations. An introduction to the fundamentals. Kenneth B. Howell. second edition. CRC Press. FL, USA. 2020
Section: Chapter 28. The inverse Laplace transform. Additional Exercises. page 509
Problem number: 28.6 (b).
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _linear, _nonhomogeneous]]`

$$\boxed{y'' + 9y = 27t^3}$$

With initial conditions

$$[y(0) = 0, y'(0) = 0]$$

✓ Solution by Maple
Time used: 4.156 (sec). Leaf size: 19

```
dsolve([diff(y(t),t$2)+9*y(t)=27*t^3,y(0) = 0, D(y)(0) = 0],y(t), singsol=all)
```

$$y(t) = -2t + \frac{2\sin(3t)}{3} + 3t^3$$

✓ Solution by Mathematica
Time used: 0.015 (sec). Leaf size: 22

```
DSolve[{y''[t]+9*y[t]==27*t^3,{y[0]==0,y'[0]==0}},y[t],t,IncludeSingularSolutions -> True]
```

$$y(t) \rightarrow 3t^3 - 2t + \frac{2}{3}\sin(3t)$$

19.3 problem 28.6 (c)

Internal problem ID [13865]

Book: Ordinary Differential Equations. An introduction to the fundamentals. Kenneth B. Howell. second edition. CRC Press. FL, USA. 2020
Section: Chapter 28. The inverse Laplace transform. Additional Exercises. page 509
Problem number: 28.6 (c).
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$\boxed{y'' + 8y' + 7y = 165\,\mathrm{e}^{4t}}$$

With initial conditions

$$[y(0) = 8, y'(0) = 1]$$

✓ Solution by Maple

Time used: 4.906 (sec). Leaf size: 21

```
dsolve([diff(y(t),t$2)+8*diff(y(t),t)+7*y(t)=165*exp(4*t),y(0) = 8, D(y)(0) = 1],y(t), sings
```

$$y(t) = \left(3\,\mathrm{e}^{11t} + 4\,\mathrm{e}^{6t} + 1\right)\mathrm{e}^{-7t}$$

✓ Solution by Mathematica

Time used: 0.018 (sec). Leaf size: 25

```
DSolve[{y''[t]+8*y'[t]+7*y[t]==165*Exp[4*t],{y[0]==8,y'[0]==1}},y[t],t,IncludeSingularSoluti
```

$$y(t) \to e^{-7t} + 4e^{-t} + 3e^{4t}$$

19.4 problem 28.8 (a)

Internal problem ID [13866]

Book: Ordinary Differential Equations. An introduction to the fundamentals. Kenneth B. Howell. second edition. CRC Press. FL, USA. 2020
Section: Chapter 28. The inverse Laplace transform. Additional Exercises. page 509
Problem number: 28.8 (a).
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _missing_x]]`

$$\boxed{y'' - 8y' + 17y = 0}$$

With initial conditions

$$[y(0) = 3, y'(0) = 12]$$

✓ Solution by Maple
Time used: 4.593 (sec). Leaf size: 12

```
dsolve([diff(y(t),t$2)-8*diff(y(t),t)+17*y(t)=0,y(0) = 3, D(y)(0) = 12],y(t), singsol=all)
```

$$y(t) = 3\,\mathrm{e}^{4t} \cos(t)$$

✓ Solution by Mathematica
Time used: 0.015 (sec). Leaf size: 14

```
DSolve[{y''[t]-8*y'[t]+17*y[t]==0,{y[0]==3,y'[0]==12}},y[t],t,IncludeSingularSolutions -> Tr
```

$$y(t) \to 3e^{4t} \cos(t)$$

19.5 problem 28.8 (b)

Internal problem ID [13867]

Book: Ordinary Differential Equations. An introduction to the fundamentals. Kenneth B. Howell. second edition. CRC Press. FL, USA. 2020
Section: Chapter 28. The inverse Laplace transform. Additional Exercises. page 509
Problem number: 28.8 (b).
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _linear, _nonhomogeneous]]`

$$\boxed{y'' - 6y' + 9y = t^2 \mathrm{e}^{3t}}$$

With initial conditions

$$[y(0) = 0, y'(0) = 0]$$

✓ Solution by Maple
Time used: 4.828 (sec). Leaf size: 13

```
dsolve([diff(y(t),t$2)-6*diff(y(t),t)+9*y(t)=exp(3*t)*t^2,y(0) = 0, D(y)(0) = 0],y(t), sings
```

$$y(t) = \frac{t^4 \mathrm{e}^{3t}}{12}$$

✓ Solution by Mathematica
Time used: 0.024 (sec). Leaf size: 17

```
DSolve[{y''[t]-6*y'[t]+9*y[t]==Exp[3*t]*t^2,{y[0]==0,y'[0]==0}},y[t],t,IncludeSingularSoluti
```

$$y(t) \to \frac{1}{12} e^{3t} t^4$$

19.6 problem 28.8 (c)

Internal problem ID [13868]

Book: Ordinary Differential Equations. An introduction to the fundamentals. Kenneth B. Howell. second edition. CRC Press. FL, USA. 2020
Section: Chapter 28. The inverse Laplace transform. Additional Exercises. page 509
Problem number: 28.8 (c).
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _missing_x]]`

$$\boxed{y'' + 6y' + 13y = 0}$$

With initial conditions

$$[y(0) = 2, y'(0) = 8]$$

✓ Solution by Maple
Time used: 4.75 (sec). Leaf size: 22

```
dsolve([diff(y(t),t$2)+6*diff(y(t),t)+13*y(t)=0,y(0) = 2, D(y)(0) = 8],y(t), singsol=all)
```

$$y(t) = e^{-3t}(2\cos(2t) + 7\sin(2t))$$

✓ Solution by Mathematica
Time used: 0.017 (sec). Leaf size: 24

```
DSolve[{y''[t]+6*y'[t]+13*y[t]==0,{y[0]==2,y'[0]==8}},y[t],t,IncludeSingularSolutions -> Tru
```

$$y(t) \to e^{-3t}(7\sin(2t) + 2\cos(2t))$$

19.7 problem 28.8 (d)

Internal problem ID [13869]

Book: Ordinary Differential Equations. An introduction to the fundamentals. Kenneth B. Howell. second edition. CRC Press. FL, USA. 2020
Section: Chapter 28. The inverse Laplace transform. Additional Exercises. page 509
Problem number: 28.8 (d).
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _missing_x]]`

$$\boxed{y'' + 8y' + 17y = 0}$$

With initial conditions

$$[y(0) = 3, y'(0) = -12]$$

✓ Solution by Maple
Time used: 4.562 (sec). Leaf size: 12

```
dsolve([diff(y(t),t$2)+8*diff(y(t),t)+17*y(t)=0,y(0) = 3, D(y)(0) = -12],y(t), singsol=all)
```

$$y(t) = 3\,e^{-4t}\cos(t)$$

✓ Solution by Mathematica
Time used: 0.016 (sec). Leaf size: 14

```
DSolve[{y''[t]+8*y'[t]+17*y[t]==0,{y[0]==3,y'[0]==-12}},y[t],t,IncludeSingularSolutions -> T
```

$$y(t) \to 3e^{-4t}\cos(t)$$

19.8 problem 28.9 (a)

Internal problem ID [13870]

Book: Ordinary Differential Equations. An introduction to the fundamentals. Kenneth B. Howell. second edition. CRC Press. FL, USA. 2020
Section: Chapter 28. The inverse Laplace transform. Additional Exercises. page 509
Problem number: 28.9 (a).
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _quadrature]]`

$$\boxed{y'' = e^t \sin(t)}$$

With initial conditions

$$[y(0) = 0, y'(0) = 0]$$

✓ Solution by Maple

Time used: 4.703 (sec). Leaf size: 15

```
dsolve([diff(y(t),t$2)=exp(t)*sin(t),y(0) = 0, D(y)(0) = 0],y(t), singsol=all)
```

$$y(t) = -\frac{e^t \cos(t)}{2} + \frac{t}{2} + \frac{1}{2}$$

✓ Solution by Mathematica

Time used: 0.056 (sec). Leaf size: 19

```
DSolve[{y''[t]==Exp[t]*Sin[t],{y[0]==0,y'[0]==0}},y[t],t,IncludeSingularSolutions -> True]
```

$$y(t) \to \frac{1}{2}\left(t - e^t \cos(t) + 1\right)$$

19.9 problem 28.9 (b)

Internal problem ID [13871]

Book: Ordinary Differential Equations. An introduction to the fundamentals. Kenneth B. Howell. second edition. CRC Press. FL, USA. 2020
Section: Chapter 28. The inverse Laplace transform. Additional Exercises. page 509
Problem number: 28.9 (b).
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$\boxed{y'' - 4y' + 40y = 122\,\mathrm{e}^{-3t}}$$

With initial conditions

$$[y(0) = 0, y'(0) = 8]$$

✓ Solution by Maple
Time used: 5.25 (sec). Leaf size: 29

```
dsolve([diff(y(t),t$2)-4*diff(y(t),t)+40*y(t)=122*exp(-3*t),y(0) = 0, D(y)(0) = 8],y(t), sin
```

$$y(t) = -2\left(-1 + \left(\cos(6t) - \frac{3\sin(6t)}{2}\right)\mathrm{e}^{5t}\right)\mathrm{e}^{-3t}$$

✓ Solution by Mathematica
Time used: 0.027 (sec). Leaf size: 35

```
DSolve[{y''[t]-4*y'[t]+40*y[t]==122*Exp[-3*t],{y[0]==0,y'[0]==8}},y[t],t,IncludeSingularSolu
```

$$y(t) \to e^{-3t}\left(3e^{5t}\sin(6t) - 2e^{5t}\cos(6t) + 2\right)$$

19.10 problem 28.9 (c)

Internal problem ID [13872]

Book: Ordinary Differential Equations. An introduction to the fundamentals. Kenneth B. Howell. second edition. CRC Press. FL, USA. 2020
Section: Chapter 28. The inverse Laplace transform. Additional Exercises. page 509
Problem number: 28.9 (c).
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$\boxed{y'' - 9y = 24\,\mathrm{e}^{-3t}}$$

With initial conditions

$$[y(0) = 6, y'(0) = 2]$$

✓ Solution by Maple
Time used: 4.922 (sec). Leaf size: 22

```
dsolve([diff(y(t),t$2)-9*y(t)=24*exp(-3*t),y(0) = 6, D(y)(0) = 2],y(t), singsol=all)
```

$$y(t) = (-4t + 2)\,\mathrm{e}^{-3t} + 4\,\mathrm{e}^{3t}$$

✓ Solution by Mathematica
Time used: 0.032 (sec). Leaf size: 23

```
DSolve[{y''[t]-9*y[t]==24*Exp[-3*t],{y[0]==6,y'[0]==2}},y[t],t,IncludeSingularSolutions -> T
```

$$y(t) \to e^{-3t}\left(-4t + 4e^{6t} + 2\right)$$

19.11 problem 28.9 (d)

Internal problem ID [13873]

Book: Ordinary Differential Equations. An introduction to the fundamentals. Kenneth B. Howell. second edition. CRC Press. FL, USA. 2020
Section: Chapter 28. The inverse Laplace transform. Additional Exercises. page 509
Problem number: 28.9 (d).
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _linear, _nonhomogeneous]]`

$$\boxed{y'' - 4y' + 13y = e^{2t} \sin(3t)}$$

With initial conditions

$$[y(0) = 4, y'(0) = 3]$$

✓ Solution by Maple
Time used: 3.922 (sec). Leaf size: 26

```
dsolve([diff(y(t),t$2)-4*diff(y(t),t)+13*y(t)=exp(2*t)*sin(3*t),y(0) = 4, D(y)(0) = 3],y(t),
```

$$y(t) = -\frac{(-24 + t)\, e^{2t} \cos(3t)}{6} - \frac{29\, e^{2t} \sin(3t)}{18}$$

✓ Solution by Mathematica
Time used: 0.13 (sec). Leaf size: 30

```
DSolve[{y''[t]-4*y'[t]+13*y[t]==Exp[2*t]*Sin[3*t],{y[0]==4,y'[0]==3}},y[t],t,IncludeSingular
```

$$y(t) \to -\frac{1}{18} e^{2t} (29 \sin(3t) + 3(t - 24) \cos(3t))$$

20 Chapter 29. Convolution. Additional Exercises. page 523

20.1 problem 29.6 (a)

Internal problem ID [13874]

Book: Ordinary Differential Equations. An introduction to the fundamentals. Kenneth B. Howell. second edition. CRC Press. FL, USA. 2020
Section: Chapter 29. Convolution. Additional Exercises. page 523
Problem number: 29.6 (a).
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _missing_x]]`

$$\boxed{y'' + 4y = 1}$$

With initial conditions

$$[y(0) = 0, y'(0) = 0]$$

✓ Solution by Maple
Time used: 4.156 (sec). Leaf size: 12

```
dsolve([diff(y(t),t$2)+4*y(t)=1,y(0) = 0, D(y)(0) = 0],y(t), singsol=all)
```

$$y(t) = -\frac{\cos(2t)}{4} + \frac{1}{4}$$

✓ Solution by Mathematica
Time used: 0.015 (sec). Leaf size: 13

```
DSolve[{y''[t]+4*y[t]==1,{y[0]==0,y'[0]==0}},y[t],t,IncludeSingularSolutions -> True]
```

$$y(t) \to \frac{\sin^2(t)}{2}$$

20.2 problem 29.6 (b)

Internal problem ID [13875]

Book: Ordinary Differential Equations. An introduction to the fundamentals. Kenneth B. Howell. second edition. CRC Press. FL, USA. 2020
Section: Chapter 29. Convolution. Additional Exercises. page 523
Problem number: 29.6 (b).
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$\boxed{y'' + 4y = t}$$

With initial conditions

$$[y(0) = 0, y'(0) = 0]$$

✓ Solution by Maple
Time used: 4.141 (sec). Leaf size: 14

```
dsolve([diff(y(t),t$2)+4*y(t)=t,y(0) = 0, D(y)(0) = 0],y(t), singsol=all)
```

$$y(t) = -\frac{\sin(2t)}{8} + \frac{t}{4}$$

✓ Solution by Mathematica
Time used: 0.014 (sec). Leaf size: 17

```
DSolve[{y''[t]+4*y[t]==t,{y[0]==0,y'[0]==0}},y[t],t,IncludeSingularSolutions -> True]
```

$$y(t) \to \frac{1}{4}(t - \sin(t)\cos(t))$$

20.3 problem 29.6 (c)

Internal problem ID [13876]

Book: Ordinary Differential Equations. An introduction to the fundamentals. Kenneth B. Howell. second edition. CRC Press. FL, USA. 2020
Section: Chapter 29. Convolution. Additional Exercises. page 523
Problem number: 29.6 (c).
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$\boxed{y'' + 4y = e^{3t}}$$

With initial conditions

$$[y(0) = 0, y'(0) = 0]$$

✓ Solution by Maple
Time used: 4.985 (sec). Leaf size: 23

```
dsolve([diff(y(t),t$2)+4*y(t)=exp(3*t),y(0) = 0, D(y)(0) = 0],y(t), singsol=all)
```

$$y(t) = -\frac{\cos(2t)}{13} - \frac{3\sin(2t)}{26} + \frac{e^{3t}}{13}$$

✓ Solution by Mathematica
Time used: 0.1 (sec). Leaf size: 29

```
DSolve[{y''[t]+4*y[t]==Exp[3*t],{y[0]==0,y'[0]==0}},y[t],t,IncludeSingularSolutions -> True]
```

$$y(t) \to \frac{1}{26}\left(2e^{3t} - 3\sin(2t) - 2\cos(2t)\right)$$

20.4 problem 29.6 (d)

Internal problem ID [13877]

Book: Ordinary Differential Equations. An introduction to the fundamentals. Kenneth B. Howell. second edition. CRC Press. FL, USA. 2020
Section: Chapter 29. Convolution. Additional Exercises. page 523
Problem number: 29.6 (d).
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _linear, _nonhomogeneous]]`

$$\boxed{y'' + 4y = \sin(2t)}$$

With initial conditions

$$[y(0) = 0, y'(0) = 0]$$

✓ Solution by Maple

Time used: 3.953 (sec). Leaf size: 18

```
dsolve([diff(y(t),t$2)+4*y(t)=sin(2*t),y(0) = 0, D(y)(0) = 0],y(t), singsol=all)
```

$$y(t) = \frac{\sin(2t)}{8} - \frac{t\cos(2t)}{4}$$

✓ Solution by Mathematica

Time used: 0.029 (sec). Leaf size: 21

```
DSolve[{y''[t]+4*y[t]==Sin[2*t],{y[0]==0,y'[0]==0}},y[t],t,IncludeSingularSolutions -> True]
```

$$y(t) \to \frac{1}{8}(\sin(2t) - 2t\cos(2t))$$

20.5 problem 29.6 (e)

Internal problem ID [13878]

Book: Ordinary Differential Equations. An introduction to the fundamentals. Kenneth B. Howell. second edition. CRC Press. FL, USA. 2020
Section: Chapter 29. Convolution. Additional Exercises. page 523
Problem number: 29.6 (e).
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _linear, _nonhomogeneous]]`

$$\boxed{y'' + 4y = \sin(t)}$$

With initial conditions

$$[y(0) = 0, y'(0) = 0]$$

✓ Solution by Maple

Time used: 4.782 (sec). Leaf size: 15

```
dsolve([diff(y(t),t$2)+4*y(t)=sin(t),y(0) = 0, D(y)(0) = 0],y(t), singsol=all)
```

$$y(t) = \frac{\sin(t)}{3} - \frac{\sin(2t)}{6}$$

✓ Solution by Mathematica

Time used: 0.072 (sec). Leaf size: 15

```
DSolve[{y''[t]+4*y[t]==Sin[t],{y[0]==0,y'[0]==0}},y[t],t,IncludeSingularSolutions -> True]
```

$$y(t) \to -\frac{1}{3}\sin(t)(\cos(t) - 1)$$

20.6 problem 29.7 (a)

Internal problem ID [13879]

Book: Ordinary Differential Equations. An introduction to the fundamentals. Kenneth B. Howell. second edition. CRC Press. FL, USA. 2020
Section: Chapter 29. Convolution. Additional Exercises. page 523
Problem number: 29.7 (a).
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _missing_x]]`

$$\boxed{y'' - 6y' + 9y = 1}$$

With initial conditions

$$[y(0) = 0, y'(0) = 0]$$

✓ Solution by Maple
Time used: 4.125 (sec). Leaf size: 17

```
dsolve([diff(y(t),t$2)-6*diff(y(t),t)+9*y(t)=1,y(0) = 0, D(y)(0) = 0],y(t), singsol=all)
```

$$y(t) = \frac{1}{9} + \frac{e^{3t}(3t-1)}{9}$$

✓ Solution by Mathematica
Time used: 0.016 (sec). Leaf size: 22

```
DSolve[{y''[t]-6*y'[t]+9*y[t]==1,{y[0]==0,y'[0]==0}},y[t],t,IncludeSingularSolutions -> True
```

$$y(t) \to \frac{1}{9}\left(e^{3t}(3t-1)+1\right)$$

20.7 problem 29.7 (b)

Internal problem ID [13880]

Book: Ordinary Differential Equations. An introduction to the fundamentals. Kenneth B. Howell. second edition. CRC Press. FL, USA. 2020
Section: Chapter 29. Convolution. Additional Exercises. page 523
Problem number: 29.7 (b).
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$\boxed{y'' - 6y' + 9y = t}$$

With initial conditions

$$[y(0) = 0, y'(0) = 0]$$

✓ Solution by Maple
Time used: 4.141 (sec). Leaf size: 24

```
dsolve([diff(y(t),t$2)-6*diff(y(t),t)+9*y(t)=t,y(0) = 0, D(y)(0) = 0],y(t), singsol=all)
```

$$y(t) = \frac{2\,e^{\frac{3t}{2}}\left(t\cosh\left(\frac{3t}{2}\right) - \frac{2\sinh\left(\frac{3t}{2}\right)}{3}\right)}{9}$$

✓ Solution by Mathematica
Time used: 0.016 (sec). Leaf size: 25

```
DSolve[{y''[t]-6*y'[t]+9*y[t]==t,{y[0]==0,y'[0]==0}},y[t],t,IncludeSingularSolutions -> True
```

$$y(t) \to \frac{1}{27}\left(3t + e^{3t}(3t - 2) + 2\right)$$

20.8 problem 29.7 (c)

Internal problem ID [13881]

Book: Ordinary Differential Equations. An introduction to the fundamentals. Kenneth B. Howell. second edition. CRC Press. FL, USA. 2020
Section: Chapter 29. Convolution. Additional Exercises. page 523
Problem number: 29.7 (c).
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$y'' - 6y' + 9y = e^{3t}$$

With initial conditions

$$[y(0) = 0, y'(0) = 0]$$

✓ Solution by Maple
Time used: 4.656 (sec). Leaf size: 13

```
dsolve([diff(y(t),t$2)-6*diff(y(t),t)+9*y(t)=exp(3*t),y(0) = 0, D(y)(0) = 0],y(t), singsol=a
```

$$y(t) = \frac{t^2 e^{3t}}{2}$$

✓ Solution by Mathematica
Time used: 0.024 (sec). Leaf size: 17

```
DSolve[{y''[t]-6*y'[t]+9*y[t]==Exp[3*t],{y[0]==0,y'[0]==0}},y[t],t,IncludeSingularSolutions
```

$$y(t) \to \frac{1}{2} e^{3t} t^2$$

20.9 problem 29.7 (d)

Internal problem ID [13882]

Book: Ordinary Differential Equations. An introduction to the fundamentals. Kenneth B. Howell. second edition. CRC Press. FL, USA. 2020
Section: Chapter 29. Convolution. Additional Exercises. page 523
Problem number: 29.7 (d).
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$y'' - 6y' + 9y = e^{-3t}$$

With initial conditions

$$[y(0) = 0, y'(0) = 0]$$

✓ Solution by Maple
Time used: 4.688 (sec). Leaf size: 23

```
dsolve([diff(y(t),t$2)-6*diff(y(t),t)+9*y(t)=exp(-3*t),y(0) = 0, D(y)(0) = 0],y(t), singsol=
```

$$y(t) = \frac{t \cosh(3t)}{6} + \frac{\sinh(3t)(3t-1)}{18}$$

✓ Solution by Mathematica
Time used: 0.043 (sec). Leaf size: 27

```
DSolve[{y''[t]-6*y'[t]+9*y[t]==Exp[-3*t],{y[0]==0,y'[0]==0}},y[t],t,IncludeSingularSolutions
```

$$y(t) \to \frac{1}{36}e^{-3t}\left(e^{6t}(6t-1)+1\right)$$

20.10 problem 29.7 (e)

Internal problem ID [13883]

Book: Ordinary Differential Equations. An introduction to the fundamentals. Kenneth B. Howell. second edition. CRC Press. FL, USA. 2020
Section: Chapter 29. Convolution. Additional Exercises. page 523
Problem number: 29.7 (e).
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$\boxed{y'' - 6y' + 9y = e^t}$$

With initial conditions

$$[y(0) = 0, y'(0) = 0]$$

✓ Solution by Maple
Time used: 4.781 (sec). Leaf size: 20

```
dsolve([diff(y(t),t$2)-6*diff(y(t),t)+9*y(t)=exp(t),y(0) = 0, D(y)(0) = 0],y(t), singsol=all
```

$$y(t) = \frac{e^t}{4} + \frac{(2t-1)\,e^{3t}}{4}$$

✓ Solution by Mathematica
Time used: 0.052 (sec). Leaf size: 24

```
DSolve[{y''[t]-6*y'[t]+9*y[t]==Exp[t],{y[0]==0,y'[0]==0}},y[t],t,IncludeSingularSolutions ->
```

$$y(t) \to \frac{1}{4}\left(e^{3t}(2t-1) + e^t\right)$$

21 Chapter 30. Piecewise-defined functions and periodic functions. Additional Exercises. page 553

21.1 problem 30.6 (a)

Internal problem ID [13884]

Book: Ordinary Differential Equations. An introduction to the fundamentals. Kenneth B. Howell. second edition. CRC Press. FL, USA. 2020
Section: Chapter 30. Piecewise-defined functions and periodic functions. Additional Exercises. page 553
Problem number: 30.6 (a).
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_quadrature]`

$$\boxed{y' = \text{Heaviside}\,(-3+t)}$$

With initial conditions

$$[y(0) = 0]$$

✓ Solution by Maple
Time used: 4.735 (sec). Leaf size: 12

```
dsolve([diff(y(t),t)=Heaviside(t-3),y(0) = 0],y(t), singsol=all)
```

$$y(t) = \text{Heaviside}\,(t-3)\,(t-3)$$

✓ Solution by Mathematica
Time used: 0.005 (sec). Leaf size: 15

```
DSolve[{y'[t]==UnitStep[t-3],{y[0]==0}},y[t],t,IncludeSingularSolutions -> True]
```

$$y(t) \to \left\{ \begin{array}{cc} t-3 & t>3 \\ 0 & \text{True} \end{array} \right.$$

21.2 problem 30.6 (b)

Internal problem ID [13885]

Book: Ordinary Differential Equations. An introduction to the fundamentals. Kenneth B. Howell. second edition. CRC Press. FL, USA. 2020
Section: Chapter 30. Piecewise-defined functions and periodic functions. Additional Exercises. page 553
Problem number: 30.6 (b).
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_quadrature]`

$$\boxed{y' = \text{Heaviside}\,(-3 + t)}$$

With initial conditions

$$[y(0) = 4]$$

✓ Solution by Maple
Time used: 3.656 (sec). Leaf size: 14

```
dsolve([diff(y(t),t)=Heaviside(t-3),y(0) = 4],y(t), singsol=all)
```

$$y(t) = \text{Heaviside}\,(t-3)\,(t-3) + 4$$

✓ Solution by Mathematica
Time used: 0.005 (sec). Leaf size: 15

```
DSolve[{y'[t]==UnitStep[t-3],{y[0]==4}},y[t],t,IncludeSingularSolutions -> True]
```

$$y(t) \to \begin{cases} 4 & t \leq 3 \\ t+1 & \text{True} \end{cases}$$

21.3 problem 30.6 (c)

Internal problem ID [13886]

Book: Ordinary Differential Equations. An introduction to the fundamentals. Kenneth B. Howell. second edition. CRC Press. FL, USA. 2020
Section: Chapter 30. Piecewise-defined functions and periodic functions. Additional Exercises. page 553
Problem number: 30.6 (c).
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _quadrature]]`

$$\boxed{y'' = \text{Heaviside}\,(-2+t)}$$

With initial conditions

$$[y(0) = 0, y'(0) = 0]$$

✓ Solution by Maple

Time used: 4.656 (sec). Leaf size: 15

```
dsolve([diff(y(t),t$2)=Heaviside(t-2),y(0) = 0, D(y)(0) = 0],y(t), singsol=all)
```

$$y(t) = \frac{\text{Heaviside}\,(t-2)\,(t-2)^2}{2}$$

✓ Solution by Mathematica

Time used: 0.005 (sec). Leaf size: 21

```
DSolve[{y''[t]==UnitStep[t-2],{y[0]==0,y'[0]==0}},y[t],t,IncludeSingularSolutions -> True]
```

$$y(t) \to \begin{cases} \frac{1}{2}(t-2)^2 & t > 2 \\ 0 & \text{True} \end{cases}$$

21.4 problem 30.6 (d)

Internal problem ID [13887]

Book: Ordinary Differential Equations. An introduction to the fundamentals. Kenneth B. Howell. second edition. CRC Press. FL, USA. 2020
Section: Chapter 30. Piecewise-defined functions and periodic functions. Additional Exercises. page 553
Problem number: 30.6 (d).
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _quadrature]]`

$$\boxed{y'' = \text{Heaviside}\,(-2+t)}$$

With initial conditions

$$[y(0) = 4, y'(0) = 6]$$

✓ Solution by Maple
Time used: 3.625 (sec). Leaf size: 20

```
dsolve([diff(y(t),t$2)=Heaviside(t-2),y(0) = 4, D(y)(0) = 6],y(t), singsol=all)
```

$$y(t) = 4 + \frac{\text{Heaviside}\,(t-2)\,(t-2)^2}{2} + 6t$$

✓ Solution by Mathematica
Time used: 0.005 (sec). Leaf size: 28

```
DSolve[{y''[t]==UnitStep[t-2],{y[0]==4,y'[0]==6}},y[t],t,IncludeSingularSolutions -> True]
```

$$y(t) \to \begin{cases} 6t+4 & t \leq 2 \\ \frac{t^2}{2} + 4t + 6 & \text{True} \end{cases}$$

21.5 problem 30.6 (e)

Internal problem ID [13888]

Book: Ordinary Differential Equations. An introduction to the fundamentals. Kenneth B. Howell. second edition. CRC Press. FL, USA. 2020
Section: Chapter 30. Piecewise-defined functions and periodic functions. Additional Exercises. page 553
Problem number: 30.6 (e).
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _linear, _nonhomogeneous]]`

$$\boxed{y'' + 9y = \text{Heaviside}\,(-10 + t)}$$

With initial conditions

$$[y(0) = 0, y'(0) = 0]$$

✓ Solution by Maple
Time used: 4.859 (sec). Leaf size: 18

```
dsolve([diff(y(t),t$2)+9*y(t)=Heaviside(t-10),y(0) = 0, D(y)(0) = 0],y(t), singsol=all)
```

$$y(t) = \frac{2\,\text{Heaviside}\,(t-10)\sin\left(\frac{3t}{2} - 15\right)^2}{9}$$

✓ Solution by Mathematica
Time used: 0.027 (sec). Leaf size: 26

```
DSolve[{y''[t]+9*y[t]==UnitStep[t-10],{y[0]==0,y'[0]==0}},y[t],t,IncludeSingularSolutions ->
```

$$y(t) \to \begin{cases} \frac{2}{9}\sin^2\left(15 - \frac{3t}{2}\right) & t > 10 \\ 0 & \text{True} \end{cases}$$

21.6 problem 30.10 (a)

Internal problem ID [13889]

Book: Ordinary Differential Equations. An introduction to the fundamentals. Kenneth B. Howell. second edition. CRC Press. FL, USA. 2020
Section: Chapter 30. Piecewise-defined functions and periodic functions. Additional Exercises. page 553
Problem number: 30.10 (a).
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_quadrature]`

$$y' = \begin{cases} 0 & t < 1 \\ 1 & 1 < t < 3 \\ 0 & 3 < t \end{cases}$$

With initial conditions

$$[y(0) = 0]$$

✓ Solution by Maple
Time used: 5.203 (sec). Leaf size: 19

```
dsolve([diff(y(t),t)=piecewise(t<1,0,1<t and t<3,1,t>3,0),y(0) = 0],y(t), singsol=all)
```

$$y(t) = \begin{cases} 0 & t < 1 \\ t - 1 & t < 3 \\ 2 & 3 \leq t \end{cases}$$

✓ Solution by Mathematica

Time used: 0.006 (sec). Leaf size: 23

```
DSolve[{y'[t]==Piecewise[{ {0,t<1},{1,1<t<3},{0,t>3}}],{y[0]==0}},y[t],t,IncludeSingularSolu
```

$$y(t) \to \begin{cases} 0 & t \leq 1 \\ t-1 & 1 < t \leq 3 \\ 2 & \text{True} \end{cases}$$

21.7 problem 30.10 (b)

Internal problem ID [13890]

Book: Ordinary Differential Equations. An introduction to the fundamentals. Kenneth B. Howell. second edition. CRC Press. FL, USA. 2020
Section: Chapter 30. Piecewise-defined functions and periodic functions. Additional Exercises. page 553
Problem number: 30.10 (b).
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _quadrature]]`

$$y'' = \begin{cases} 0 & t < 1 \\ 1 & 1 < t < 3 \\ 0 & 3 < t \end{cases}$$

With initial conditions

$$[y(0) = 0, y'(0) = 0]$$

✓ Solution by Maple
Time used: 4.844 (sec). Leaf size: 37

```
dsolve([diff(y(t),t$2)=piecewise(t<1,0,1<t and t<3,1,t>3,0),y(0) = 0, D(y)(0) = 0],y(t), sin
```

$$y(t) = \begin{cases} 0 & t < 1 \\ \frac{(t-1)^2}{2} & t < 3 \\ 2t - 4 & 3 \leq t \end{cases}$$

✓ Solution by Mathematica

Time used: 0.006 (sec). Leaf size: 33

```
DSolve[{y''[t]==Piecewise[{ {0,t<1},{1,1<t<3},{0,t>3}}],{y[0]==0,y'[0]==0}},y[t],t,IncludeSi
```

$$y(t) \to \begin{cases} 0 & t \leq 1 \\ \frac{1}{2}(t-1)^2 & 1 < t \leq 3 \\ 2(t-2) & \text{True} \end{cases}$$

21.8 problem 30.10 (c)

Internal problem ID [13891]

Book: Ordinary Differential Equations. An introduction to the fundamentals. Kenneth B. Howell. second edition. CRC Press. FL, USA. 2020
Section: Chapter 30. Piecewise-defined functions and periodic functions. Additional Exercises. page 553
Problem number: 30.10 (c).
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _linear, _nonhomogeneous]]`

$$y'' + 9y = \begin{cases} 0 & t < 1 \\ 1 & 1 < t < 3 \\ 0 & 3 < t \end{cases}$$

With initial conditions

$$[y(0) = 0, y'(0) = 0]$$

✓ Solution by Maple
Time used: 6.984 (sec). Leaf size: 46

```
dsolve([diff(y(t),t$2)+9*y(t)=piecewise(t<1,0,1<t and t<3,1,t>3,0),y(0) = 0, D(y)(0) = 0],y(
```

$$y(t) = \frac{\left(\begin{cases} 0 & t < 1 \\ 1 - \cos(3t - 3) & t < 3 \\ \cos(3t - 9) - \cos(3t - 3) & 3 \le t \end{cases}\right)}{9}$$

✓ Solution by Mathematica

Time used: 0.047 (sec). Leaf size: 47

```
DSolve[{y''[t]+9*y[t]==Piecewise[{ {0,t<1},{1,1<t<3},{0,t>3}}],{y[0]==0,y'[0]==0}},y[t],t,In
```

$$y(t) \to \begin{cases} \frac{2}{9}\sin^2\left(\frac{3}{2}-\frac{3t}{2}\right) & 1 < t \leq 3 \\ -\frac{2}{9}\sin(3)\sin(6-3t) & t > 3 \end{cases}$$

22 Chapter 31. Delta Functions. Additional Exercises. page 572

22.1 problem 31.6 (a)

Internal problem ID [13892]

Book: Ordinary Differential Equations. An introduction to the fundamentals. Kenneth B. Howell. second edition. CRC Press. FL, USA. 2020
Section: Chapter 31. Delta Functions. Additional Exercises. page 572
Problem number: 31.6 (a).
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_quadrature]`

$$\boxed{y' = 3\delta(-2+t)}$$

With initial conditions

$$[y(0) = 0]$$

✓ Solution by Maple
Time used: 5.0 (sec). Leaf size: 10

```
dsolve([diff(y(t),t)=3*Dirac(t-2),y(0) = 0],y(t), singsol=all)
```

$$y(t) = 3\,\mathrm{Heaviside}\,(t-2)$$

✓ Solution by Mathematica
Time used: 0.006 (sec). Leaf size: 11

```
DSolve[{y'[t]==3*DiracDelta[t-2],{y[0]==0}},y[t],t,IncludeSingularSolutions -> True]
```

$$y(t) \to 3\theta(t-2)$$

22.2 problem 31.6 (b)

Internal problem ID [13893]

Book: Ordinary Differential Equations. An introduction to the fundamentals. Kenneth B. Howell. second edition. CRC Press. FL, USA. 2020
Section: Chapter 31. Delta Functions. Additional Exercises. page 572
Problem number: 31.6 (b).
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_quadrature]`

$$\boxed{y' = \delta(-2+t) - \delta(t-4)}$$

With initial conditions

$$[y(0) = 0]$$

✓ Solution by Maple
Time used: 5.015 (sec). Leaf size: 15

```
dsolve([diff(y(t),t)=Dirac(t-2)-Dirac(t-4),y(0) = 0],y(t), singsol=all)
```

$$y(t) = -\text{Heaviside}(t-4) + \text{Heaviside}(t-2)$$

✓ Solution by Mathematica
Time used: 0.01 (sec). Leaf size: 16

```
DSolve[{y'[t]==DiracDelta[t-2]-DiracDelta[t-4],{y[0]==0}},y[t],t,IncludeSingularSolutions ->
```

$$y(t) \rightarrow \theta(t-2) - \theta(t-4)$$

22.3 problem 31.6 (c)

Internal problem ID [13894]

Book: Ordinary Differential Equations. An introduction to the fundamentals. Kenneth B. Howell. second edition. CRC Press. FL, USA. 2020
Section: Chapter 31. Delta Functions. Additional Exercises. page 572
Problem number: 31.6 (c).
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _quadrature]]`

$$\boxed{y'' = \delta(-3+t)}$$

With initial conditions

$$[y(0) = 0, y'(0) = 0]$$

✓ Solution by Maple
Time used: 4.875 (sec). Leaf size: 12

```
dsolve([diff(y(t),t$2)=Dirac(t-3),y(0) = 0, D(y)(0) = 0],y(t), singsol=all)
```

$$y(t) = \text{Heaviside}\,(t-3)\,(t-3)$$

✓ Solution by Mathematica
Time used: 0.006 (sec). Leaf size: 13

```
DSolve[{y''[t]==DiracDelta[t-3],{y[0]==0,y'[0]==0}},y[t],t,IncludeSingularSolutions -> True]
```

$$y(t) \to (t-3)\theta(t-3)$$

22.4 problem 31.6 (d)

Internal problem ID [13895]

Book: Ordinary Differential Equations. An introduction to the fundamentals. Kenneth B. Howell. second edition. CRC Press. FL, USA. 2020
Section: Chapter 31. Delta Functions. Additional Exercises. page 572
Problem number: 31.6 (d).
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _quadrature]]`

$$\boxed{y'' = \delta(t-1) - \delta(t-4)}$$

With initial conditions

$$[y(0) = 0, y'(0) = 0]$$

✓ Solution by Maple
Time used: 5.063 (sec). Leaf size: 22

```
dsolve([diff(y(t),t$2)=Dirac(t-1)-Dirac(t-4),y(0) = 0, D(y)(0) = 0],y(t), singsol=all)
```

$$y(t) = (4-t)\,\text{Heaviside}\,(t-4) + \text{Heaviside}\,(t-1)\,(t-1)$$

✓ Solution by Mathematica
Time used: 0.008 (sec). Leaf size: 23

```
DSolve[{y''[t]==DiracDelta[t-1]-DiracDelta[t-4],{y[0]==0,y'[0]==0}},y[t],t,IncludeSingularSo
```

$$y(t) \to (t-1)\theta(t-1) - (t-4)\theta(t-4)$$

22.5 problem 31.6 (e)

Internal problem ID [13896]

Book: Ordinary Differential Equations. An introduction to the fundamentals. Kenneth B. Howell. second edition. CRC Press. FL, USA. 2020
Section: Chapter 31. Delta Functions. Additional Exercises. page 572
Problem number: 31.6 (e).
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[[_linear, 'class A']]`

$$\boxed{y' + 2y = 4\delta(t - 1)}$$

With initial conditions

$$[y(0) = 0]$$

✓ Solution by Maple
Time used: 5.172 (sec). Leaf size: 16

```
dsolve([diff(y(t),t)+2*y(t)=4*Dirac(t-1),y(0) = 0],y(t), singsol=all)
```

$$y(t) = 4 \operatorname{Heaviside}(t - 1) e^{-2t+2}$$

✓ Solution by Mathematica
Time used: 0.031 (sec). Leaf size: 18

```
DSolve[{y'[t]+2*y[t]==4*DiracDelta[t-1],{y[0]==0}},y[t],t,IncludeSingularSolutions -> True]
```

$$y(t) \rightarrow 4e^{2-2t}\theta(t - 1)$$

22.6 problem 31.6 (f)

Internal problem ID [13897]

Book: Ordinary Differential Equations. An introduction to the fundamentals. Kenneth B. Howell. second edition. CRC Press. FL, USA. 2020
Section: Chapter 31. Delta Functions. Additional Exercises. page 572
Problem number: 31.6 (f).
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _linear, _nonhomogeneous]]`

$$y'' + y = \delta(t) + \delta(t - \pi)$$

✓ Solution by Maple
Time used: 5.531 (sec). Leaf size: 22

```
dsolve(diff(y(t),t$2)+y(t)=Dirac(t)+Dirac(t-Pi),y(t), singsol=all)
```

$$y(t) = \cos(t)\, y(0) + \sin(t)\left(\text{Heaviside}(\pi - t) + D(y)(0)\right)$$

✓ Solution by Mathematica
Time used: 0.118 (sec). Leaf size: 92

```
DSolve[y''[t]+2*y[t]==DiracDelta[t]+DiracDelta[t-Pi],y[t],t,IncludeSingularSolutions -> True
```

$$y(t) \to -\frac{\theta(t-\pi)\sin\left(\sqrt{2}(\pi-t)\right)}{\sqrt{2}} + \frac{\theta(t)\sin\left(\sqrt{2}t\right)}{\sqrt{2}} - \frac{\cos\left(\sqrt{2}\pi\right)\sin\left(\sqrt{2}t\right)}{\sqrt{2}} + c_1\cos\left(\sqrt{2}t\right) + c_2\sin\left(\sqrt{2}t\right)$$

22.7 problem 31.6 (g)

Internal problem ID [13898]

Book: Ordinary Differential Equations. An introduction to the fundamentals. Kenneth B. Howell. second edition. CRC Press. FL, USA. 2020
Section: Chapter 31. Delta Functions. Additional Exercises. page 572
Problem number: 31.6 (g).
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _linear, _nonhomogeneous]]`

$$\boxed{y'' + y = -2\delta\left(t - \frac{\pi}{2}\right)}$$

With initial conditions

$$[y(0) = 0, y'(0) = 0]$$

✓ Solution by Maple

Time used: 5.219 (sec). Leaf size: 14

```
dsolve([diff(y(t),t$2)+y(t)=-2*Dirac(t-Pi/2),y(0) = 0, D(y)(0) = 0],y(t), singsol=all)
```

$$y(t) = 2\cos(t)\,\text{Heaviside}\left(t - \frac{\pi}{2}\right)$$

✓ Solution by Mathematica

Time used: 0.025 (sec). Leaf size: 17

```
DSolve[{y''[t]+y[t]==-2*DiracDelta[t-Pi/2],{y[0]==0,y'[0]==0}},y[t],t,IncludeSingularSolutio
```

$$y(t) \to 2\theta(2t - \pi)\cos(t)$$

22.8 problem 31.7 (a)

Internal problem ID [13899]

Book: Ordinary Differential Equations. An introduction to the fundamentals. Kenneth B. Howell. second edition. CRC Press. FL, USA. 2020
Section: Chapter 31. Delta Functions. Additional Exercises. page 572
Problem number: 31.7 (a).
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[[_linear, ‘class A‘]]`

$$\boxed{y' + 3y = \delta(-2 + t)}$$

With initial conditions

$$[y(0) = 2]$$

✓ Solution by Maple

Time used: 4.141 (sec). Leaf size: 22

```
dsolve([diff(y(t),t)+3*y(t)=Dirac(t-2),y(0) = 2],y(t), singsol=all)
```

$$y(t) = \text{Heaviside}\,(t-2)\,\mathrm{e}^{6-3t} + 2\,\mathrm{e}^{-3t}$$

✓ Solution by Mathematica

Time used: 0.029 (sec). Leaf size: 21

```
DSolve[{y'[t]+3*y[t]==DiracDelta[t-2],{y[0]==2}},y[t],t,IncludeSingularSolutions -> True]
```

$$y(t) \to e^{-3t}\left(e^6\theta(t-2) + 2\right)$$

22.9 problem 31.7 (b)

Internal problem ID [13900]

Book: Ordinary Differential Equations. An introduction to the fundamentals. Kenneth B. Howell. second edition. CRC Press. FL, USA. 2020
Section: Chapter 31. Delta Functions. Additional Exercises. page 572
Problem number: 31.7 (b).
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _missing_y]]`

$$\boxed{y'' + 3y' = \delta(t)}$$

✓ Solution by Maple
Time used: 5.031 (sec). Leaf size: 22

```
dsolve(diff(y(t),t$2)+3*diff(y(t),t)=Dirac(t),y(t), singsol=all)
```

$$y(t) = \frac{1}{3} + \frac{D(y)(0)}{3} + y(0) - \frac{e^{-3t}(1 + D(y)(0))}{3}$$

✓ Solution by Mathematica
Time used: 0.138 (sec). Leaf size: 27

```
DSolve[y''[t]+3*y'[t]==DiracDelta[t],y[t],t,IncludeSingularSolutions -> True]
```

$$y(t) \to \frac{1}{3}\left(\theta(t) - e^{-3t}(\theta(t) + c_1)\right) + c_2$$

22.10 problem 31.7 (c)

Internal problem ID [13901]

Book: Ordinary Differential Equations. An introduction to the fundamentals. Kenneth B. Howell. second edition. CRC Press. FL, USA. 2020
Section: Chapter 31. Delta Functions. Additional Exercises. page 572
Problem number: 31.7 (c).
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _missing_y]]`

$$\boxed{y'' + 3y' = \delta(t-1)}$$

With initial conditions

$$[y(0) = 0, y'(0) = 1]$$

✓ Solution by Maple
Time used: 5.422 (sec). Leaf size: 28

```
dsolve([diff(y(t),t$2)+3*diff(y(t),t)=Dirac(t-1),y(0) = 0, D(y)(0) = 1],y(t), singsol=all)
```

$$y(t) = -\frac{\text{Heaviside}\,(t-1)\,\mathrm{e}^{-3t+3}}{3} + \frac{\text{Heaviside}\,(t-1)}{3} - \frac{\mathrm{e}^{-3t}}{3} + \frac{1}{3}$$

✓ Solution by Mathematica
Time used: 0.14 (sec). Leaf size: 37

```
DSolve[{y''[t]+3*y'[t]==DiracDelta[t-1],{y[0]==0,y'[0]==1}},y[t],t,IncludeSingularSolutions
```

$$y(t) \to \frac{1}{3}e^{-3t}\left(\left(e^{3t} - e^3\right)\theta(t-1) + e^{3t} - 1\right)$$

22.11 problem 31.7 (d)

Internal problem ID [13902]

Book: Ordinary Differential Equations. An introduction to the fundamentals. Kenneth B. Howell. second edition. CRC Press. FL, USA. 2020
Section: Chapter 31. Delta Functions. Additional Exercises. page 572
Problem number: 31.7 (d).
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _linear, _nonhomogeneous]]`

$$\boxed{y'' + 16y = \delta(-2 + t)}$$

With initial conditions

$$[y(0) = 0, y'(0) = 0]$$

✓ Solution by Maple
Time used: 5.125 (sec). Leaf size: 16

```
dsolve([diff(y(t),t$2)+16*y(t)=Dirac(t-2),y(0) = 0, D(y)(0) = 0],y(t), singsol=all)
```

$$y(t) = \frac{\text{Heaviside}\,(t-2)\sin\,(4t-8)}{4}$$

✓ Solution by Mathematica
Time used: 0.025 (sec). Leaf size: 19

```
DSolve[{y''[t]+16*y[t]==DiracDelta[t-2],{y[0]==0,y'[0]==0}},y[t],t,IncludeSingularSolutions
```

$$y(t) \to -\frac{1}{4}\theta(t-2)\sin(8-4t)$$

22.12 problem 31.7 (e)

Internal problem ID [13903]

Book: Ordinary Differential Equations. An introduction to the fundamentals. Kenneth B. Howell. second edition. CRC Press. FL, USA. 2020
Section: Chapter 31. Delta Functions. Additional Exercises. page 572
Problem number: 31.7 (e).
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _linear, _nonhomogeneous]]`

$$\boxed{y'' - 16y = \delta(-10 + t)}$$

With initial conditions

$$[y(0) = 0, y'(0) = 0]$$

✓ Solution by Maple

Time used: 5.656 (sec). Leaf size: 16

```
dsolve([diff(y(t),t$2)-16*y(t)=Dirac(t-10),y(0) = 0, D(y)(0) = 0],y(t), singsol=all)
```

$$y(t) = \frac{\text{Heaviside}\,(t-10)\sinh\,(-40+4t)}{4}$$

✓ Solution by Mathematica

Time used: 0.028 (sec). Leaf size: 31

```
DSolve[{y''[t]-16*y[t]==DiracDelta[t-10],{y[0]==0,y'[0]==0}},y[t],t,IncludeSingularSolutions
```

$$y(t) \to \frac{1}{8}e^{-4(t+10)}\left(e^{8t} - e^{80}\right)\theta(t-10)$$

22.13 problem 31.7 (f)

Internal problem ID [13904]

Book: Ordinary Differential Equations. An introduction to the fundamentals. Kenneth B. Howell. second edition. CRC Press. FL, USA. 2020
Section: Chapter 31. Delta Functions. Additional Exercises. page 572
Problem number: 31.7 (f).
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _linear, _nonhomogeneous]]`

$$\boxed{y'' + y = \delta(t)}$$

With initial conditions

$$[y(0) = 0, y'(0) = -1]$$

✓ Solution by Maple
Time used: 4.75 (sec). Leaf size: 5

```
dsolve([diff(y(t),t$2)+y(t)=Dirac(t),y(0) = 0, D(y)(0) = -1],y(t), singsol=all)
```

$$y(t) = 0$$

✓ Solution by Mathematica
Time used: 0.015 (sec). Leaf size: 16

```
DSolve[{y''[t]+y[t]==DiracDelta[t],{y[0]==0,y'[0]==-1}},y[t],t,IncludeSingularSolutions -> T
```

$$y(t) \to (\theta(t) - \theta(0) - 1) \sin(t)$$

22.14 problem 31.7 (g)

Internal problem ID [13905]

Book: Ordinary Differential Equations. An introduction to the fundamentals. Kenneth B. Howell. second edition. CRC Press. FL, USA. 2020
Section: Chapter 31. Delta Functions. Additional Exercises. page 572
Problem number: 31.7 (g).
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _linear, _nonhomogeneous]]`

$$\boxed{y'' + 4y' - 12y = \delta(t)}$$

With initial conditions

$$[y(0) = 0, y'(0) = 0]$$

✓ Solution by Maple
Time used: 5.0 (sec). Leaf size: 14

```
dsolve([diff(y(t),t$2)+4*diff(y(t),t)-12*y(t)=Dirac(t),y(0) = 0, D(y)(0) = 0],y(t), singsol=
```

$$y(t) = \frac{e^{-2t} \sinh(4t)}{4}$$

✓ Solution by Mathematica
Time used: 0.029 (sec). Leaf size: 28

```
DSolve[{y''[t]+4*y'[t]-12*y[t]==DiracDelta[t],{y[0]==0,y'[0]==0}},y[t],t,IncludeSingularSolu
```

$$y(t) \to -\frac{1}{8} e^{-6t} \left(e^{8t} - 1\right) (\theta(0) - \theta(t))$$

22.15 problem 31.7 (h)

Internal problem ID [13906]

Book: Ordinary Differential Equations. An introduction to the fundamentals. Kenneth B. Howell. second edition. CRC Press. FL, USA. 2020
Section: Chapter 31. Delta Functions. Additional Exercises. page 572
Problem number: 31.7 (h).
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _linear, _nonhomogeneous]]`

$$\boxed{y'' + 4y' - 12y = \delta(-3 + t)}$$

With initial conditions

$$[y(0) = 0, y'(0) = 0]$$

✓ Solution by Maple
Time used: 5.172 (sec). Leaf size: 22

```
dsolve([diff(y(t),t$2)+4*diff(y(t),t)-12*y(t)=Dirac(t-3),y(0) = 0, D(y)(0) = 0],y(t), singso
```

$$y(t) = \frac{\text{Heaviside}\,(t-3)\,\mathrm{e}^{-2t+6}\sinh\,(4t-12)}{4}$$

✓ Solution by Mathematica
Time used: 0.028 (sec). Leaf size: 31

```
DSolve[{y''[t]+4*y'[t]-12*y[t]==DiracDelta[t-3],{y[0]==0,y'[0]==0}},y[t],t,IncludeSingularSo
```

$$y(t) \to \frac{1}{8}e^{-6(t+1)}\left(e^{8t} - e^{24}\right)\theta(t-3)$$

22.16 problem 31.7 (i)

Internal problem ID [13907]

Book: Ordinary Differential Equations. An introduction to the fundamentals. Kenneth B. Howell. second edition. CRC Press. FL, USA. 2020
Section: Chapter 31. Delta Functions. Additional Exercises. page 572
Problem number: 31.7 (i).
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _linear, _nonhomogeneous]]`

$$\boxed{y'' + 6y' + 9y = \delta(t - 4)}$$

With initial conditions

$$[y(0) = 0, y'(0) = 0]$$

✓ Solution by Maple
Time used: 5.188 (sec). Leaf size: 18

```
dsolve([diff(y(t),t$2)+6*diff(y(t),t)+9*y(t)=Dirac(t-4),y(0) = 0, D(y)(0) = 0],y(t), singsol
```

$$y(t) = (t - 4)\,\mathrm{e}^{-3t+12}\,\mathrm{Heaviside}\,(t - 4)$$

✓ Solution by Mathematica
Time used: 0.032 (sec). Leaf size: 20

```
DSolve[{y''[t]+6*y'[t]+9*y[t]==DiracDelta[t-4],{y[0]==0,y'[0]==0}},y[t],t,IncludeSingularSol
```

$$y(t) \to e^{12-3t}(t - 4)\theta(t - 4)$$

22.17 problem 31.7 (j)

Internal problem ID [13908]

Book: Ordinary Differential Equations. An introduction to the fundamentals. Kenneth B. Howell. second edition. CRC Press. FL, USA. 2020
Section: Chapter 31. Delta Functions. Additional Exercises. page 572
Problem number: 31.7 (j).
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _linear, _nonhomogeneous]]`

$$\boxed{y'' - 12y' + 45y = \delta(t)}$$

With initial conditions

$$[y(0) = 0, y'(0) = 0]$$

✓ Solution by Maple
Time used: 5.047 (sec). Leaf size: 14

```
dsolve([diff(y(t),t$2)-12*diff(y(t),t)+45*y(t)=Dirac(t),y(0) = 0, D(y)(0) = 0],y(t), singsol
```

$$y(t) = \frac{e^{6t}\sin(3t)}{3}$$

✓ Solution by Mathematica
Time used: 0.035 (sec). Leaf size: 25

```
DSolve[{y''[t]-12*y'[t]+45*y[t]==DiracDelta[t],{y[0]==0,y'[0]==0}},y[t],t,IncludeSingularSol
```

$$y(t) \to -\frac{1}{3}e^{6t}(\theta(0) - \theta(t))\sin(3t)$$

22.18 problem 31.7 (k)

Internal problem ID [13909]

Book: Ordinary Differential Equations. An introduction to the fundamentals. Kenneth B. Howell. second edition. CRC Press. FL, USA. 2020
Section: Chapter 31. Delta Functions. Additional Exercises. page 572
Problem number: 31.7 (k).
ODE order: 3.
ODE degree: 1.

CAS Maple gives this as type `[[_3rd_order, _missing_y]]`

$$\boxed{y''' + 9y' = \delta(t-1)}$$

With initial conditions

$$[y(0) = 0, y'(0) = 0, y''(0) = 0]$$

✓ Solution by Maple

Time used: 5.234 (sec). Leaf size: 18

```
dsolve([diff(y(t),t$3)+9*diff(y(t),t)=Dirac(t-1),y(0) = 0, D(y)(0) = 0, (D@@2)(y)(0) = 0],y(
```

$$y(t) = -\frac{\text{Heaviside}\,(t-1)\,(-1+\cos\,(3t-3))}{9}$$

✓ Solution by Mathematica

Time used: 0.174 (sec). Leaf size: 21

```
DSolve[{y'''[t]+9*y'[t]==DiracDelta[t-1],{y[0]==0,y'[0]==0,y''[0]==0}},y[t],t,IncludeSingula
```

$$y(t) \to -\frac{1}{9}\theta(t-1)(\cos(3-3t)-1)$$

22.19 problem 31.7 (L)

Internal problem ID [13910]

Book: Ordinary Differential Equations. An introduction to the fundamentals. Kenneth B. Howell. second edition. CRC Press. FL, USA. 2020
Section: Chapter 31. Delta Functions. Additional Exercises. page 572
Problem number: 31.7 (L).
ODE order: 4.
ODE degree: 1.

CAS Maple gives this as type `[[_high_order, _linear, _nonhomogeneous]]`

$$\boxed{y'''' - 16y = \delta(t)}$$

With initial conditions

$$[y(0) = 0, y'(0) = 0, y''(0) = 0, y'''(0) = 0]$$

✓ Solution by Maple

Time used: 5.063 (sec). Leaf size: 17

```
dsolve([diff(y(t),t$4)-16*y(t)=Dirac(t),y(0) = 0, D(y)(0) = 0, (D@@2)(y)(0) = 0, (D@@3)(y)(0
```

$$y(t) = -\frac{\sin(2t)}{16} + \frac{\sinh(2t)}{16}$$

✓ Solution by Mathematica

Time used: 0.012 (sec). Leaf size: 39

```
DSolve[{y''''[t]-16*y[t]==DiracDelta[t],{y[0]==0,y'[0]==0,y''[0]==0,y'''[0]==0}},y[t],t,Incl
```

$$y(t) \to -\frac{1}{32}e^{-2t}(\theta(0) - \theta(t))\left(e^{4t} - 2e^{2t}\sin(2t) - 1\right)$$

23 Chapter 33. Power series solutions I: Basic computational methods. Additional Exercises. page 641

23.1 problem 33.3 (a)

Internal problem ID [13911]

Book: Ordinary Differential Equations. An introduction to the fundamentals. Kenneth B. Howell. second edition. CRC Press. FL, USA. 2020
Section: Chapter 33. Power series solutions I: Basic computational methods. Additional Exercises. page 641
Problem number: 33.3 (a).
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_quadrature]`

$$\boxed{y' - 2y = 0}$$

With the expansion point for the power series method at $x = 0$.

✓ Solution by Maple
Time used: 0.0 (sec). Leaf size: 36

```
Order:=6;
dsolve(diff(y(x),x)-2*y(x)=0,y(x),type='series',x=0);
```

$$y(x) = \left(1 + 2x + 2x^2 + \frac{4}{3}x^3 + \frac{2}{3}x^4 + \frac{4}{15}x^5\right) y(0) + O(x^6)$$

✓ Solution by Mathematica
Time used: 0.009 (sec). Leaf size: 37

```
AsymptoticDSolveValue[y'[x]-2*y[x]==0,y[x],{x,0,5}]
```

$$y(x) \to c_1 \left(\frac{4x^5}{15} + \frac{2x^4}{3} + \frac{4x^3}{3} + 2x^2 + 2x + 1\right)$$

23.2 problem 33.3 (b)

Internal problem ID [13912]

Book: Ordinary Differential Equations. An introduction to the fundamentals. Kenneth B. Howell. second edition. CRC Press. FL, USA. 2020
Section: Chapter 33. Power series solutions I: Basic computational methods. Additional Exercises. page 641
Problem number: 33.3 (b).
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_separable]`

$$\boxed{y' - 2yx = 0}$$

With the expansion point for the power series method at $x = 0$.

✓ Solution by Maple
Time used: 0.0 (sec). Leaf size: 19

```
Order:=6;
dsolve(diff(y(x),x)-2*x*y(x)=0,y(x),type='series',x=0);
```

$$y(x) = \left(1 + x^2 + \frac{1}{2}x^4\right) y(0) + O(x^6)$$

✓ Solution by Mathematica
Time used: 0.001 (sec). Leaf size: 18

```
AsymptoticDSolveValue[y'[x]-2*x*y[x]==0,y[x],{x,0,5}]
```

$$y(x) \to c_1\left(\frac{x^4}{2} + x^2 + 1\right)$$

23.3 problem 33.3 (c)

Internal problem ID [13913]

Book: Ordinary Differential Equations. An introduction to the fundamentals. Kenneth B. Howell. second edition. CRC Press. FL, USA. 2020
Section: Chapter 33. Power series solutions I: Basic computational methods. Additional Exercises. page 641
Problem number: 33.3 (c).
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_separable]`

$$\boxed{y' + \frac{2y}{2x - 1} = 0}$$

With the expansion point for the power series method at $x = 0$.

✓ Solution by Maple
Time used: 0.0 (sec). Leaf size: 36

```
Order:=6;
dsolve(diff(y(x),x)+2/(2*x-1)*y(x)=0,y(x),type='series',x=0);
```

$$y(x) = \left(32x^5 + 16x^4 + 8x^3 + 4x^2 + 2x + 1\right) y(0) + O\left(x^6\right)$$

✓ Solution by Mathematica
Time used: 0.001 (sec). Leaf size: 31

```
AsymptoticDSolveValue[y'[x]+2/(2*x-1)*y[x]==0,y[x],{x,0,5}]
```

$$y(x) \to c_1\left(32x^5 + 16x^4 + 8x^3 + 4x^2 + 2x + 1\right)$$

23.4 problem 33.3 (d)

Internal problem ID [13914]

Book: Ordinary Differential Equations. An introduction to the fundamentals. Kenneth B. Howell. second edition. CRC Press. FL, USA. 2020
Section: Chapter 33. Power series solutions I: Basic computational methods. Additional Exercises. page 641
Problem number: 33.3 (d).
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_separable]`

$$\boxed{(x-3)\,y'-2y=0}$$

With the expansion point for the power series method at $x=0$.

✓ Solution by Maple
Time used: 0.0 (sec). Leaf size: 19

```
Order:=6;
dsolve((x-3)*diff(y(x),x)-2*y(x)=0,y(x),type='series',x=0);
```

$$y(x)=\frac{y(0)\,(-3+x)^2}{9}$$

✓ Solution by Mathematica
Time used: 0.001 (sec). Leaf size: 20

```
AsymptoticDSolveValue[(x-3)*y'[x]-2*y[x]==0,y[x],{x,0,5}]
```

$$y(x)\to c_1\left(\frac{x^2}{9}-\frac{2x}{3}+1\right)$$

23.5 problem 33.3 (e)

Internal problem ID [13915]

Book: Ordinary Differential Equations. An introduction to the fundamentals. Kenneth B. Howell. second edition. CRC Press. FL, USA. 2020
Section: Chapter 33. Power series solutions I: Basic computational methods. Additional Exercises. page 641
Problem number: 33.3 (e).
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_separable]`

$$\boxed{(x^2+1)\,y' - 2yx = 0}$$

With the expansion point for the power series method at $x = 0$.

✓ Solution by Maple
Time used: 0.0 (sec). Leaf size: 13

```
Order:=6;
dsolve((1+x^2)*diff(y(x),x)-2*x*y(x)=0,y(x),type='series',x=0);
```

$$y(x) = y(0)\,(x^2+1)$$

✓ Solution by Mathematica
Time used: 0.001 (sec). Leaf size: 11

```
AsymptoticDSolveValue[(1+x^2)*y'[x]-2*x*y[x]==0,y[x],{x,0,5}]
```

$$y(x) \to c_1(x^2+1)$$

23.6 problem 33.3 (f)

Internal problem ID [13916]

Book: Ordinary Differential Equations. An introduction to the fundamentals. Kenneth B. Howell. second edition. CRC Press. FL, USA. 2020
Section: Chapter 33. Power series solutions I: Basic computational methods. Additional Exercises. page 641
Problem number: 33.3 (f).
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_separable]`

$$\boxed{y' + \frac{y}{x-1} = 0}$$

With the expansion point for the power series method at $x = 0$.

✓ Solution by Maple
Time used: 0.0 (sec). Leaf size: 26

```
Order:=6;
dsolve(diff(y(x),x)+1/(x-1)*y(x)=0,y(x),type='series',x=0);
```

$$y(x) = \left(x^5 + x^4 + x^3 + x^2 + x + 1\right) y(0) + O\left(x^6\right)$$

✓ Solution by Mathematica
Time used: 0.001 (sec). Leaf size: 21

```
AsymptoticDSolveValue[y'[x]+1/(x-1)*y[x]==0,y[x],{x,0,5}]
```

$$y(x) \rightarrow c_1\left(x^5 + x^4 + x^3 + x^2 + x + 1\right)$$

23.7 problem 33.3 (g)

Internal problem ID [13917]

Book: Ordinary Differential Equations. An introduction to the fundamentals. Kenneth B. Howell. second edition. CRC Press. FL, USA. 2020
Section: Chapter 33. Power series solutions I: Basic computational methods. Additional Exercises. page 641
Problem number: 33.3 (g).
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_separable]`

$$\boxed{y' + \frac{y}{x-1} = 0}$$

With the expansion point for the power series method at $x = 3$.

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 36

```
Order:=6;
dsolve(diff(y(x),x)+1/(x-1)*y(x)=0,y(x),type='series',x=3);
```

$$y(x) = \left(\frac{5}{2} - \frac{x}{2} + \frac{(-3+x)^2}{4} - \frac{(-3+x)^3}{8} + \frac{(-3+x)^4}{16} - \frac{(-3+x)^5}{32}\right) y(3) + O(x^6)$$

✓ Solution by Mathematica

Time used: 0.001 (sec). Leaf size: 53

```
AsymptoticDSolveValue[y'[x]+1/(x-1)*y[x]==0,y[x],{x,3,5}]
```

$$y(x) \to c_1\left(-\frac{1}{32}(x-3)^5 + \frac{1}{16}(x-3)^4 - \frac{1}{8}(x-3)^3 + \frac{1}{4}(x-3)^2 + \frac{3-x}{2} + 1\right)$$

23.8 problem 33.3 (h)

Internal problem ID [13918]

Book: Ordinary Differential Equations. An introduction to the fundamentals. Kenneth B. Howell. second edition. CRC Press. FL, USA. 2020
Section: Chapter 33. Power series solutions I: Basic computational methods. Additional Exercises. page 641
Problem number: 33.3 (h).
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_separable]`

$$\boxed{(1-x)\,y'-2y=0}$$

With the expansion point for the power series method at $x=5$.

✓ Solution by Maple
Time used: 0.0 (sec). Leaf size: 36

```
Order:=6;
dsolve((1-x)*diff(y(x),x)-2*y(x)=0,y(x),type='series',x=5);
```

$$y(x)=\left(\frac{7}{2}-\frac{x}{2}+\frac{3(x-5)^{2}}{16}-\frac{(x-5)^{3}}{16}+\frac{5(x-5)^{4}}{256}-\frac{3(x-5)^{5}}{512}\right)y(5)+O\left(x^{6}\right)$$

✓ Solution by Mathematica
Time used: 0.001 (sec). Leaf size: 53

```
AsymptoticDSolveValue[(1-x)*y'[x]-2*y[x]==0,y[x],{x,5,5}]
```

$$y(x)\to c_1\left(-\frac{3}{512}(x-5)^5+\frac{5}{256}(x-5)^4-\frac{1}{16}(x-5)^3+\frac{3}{16}(x-5)^2+\frac{5-x}{2}+1\right)$$

23.9 problem 33.3 (i)

Internal problem ID [13919]

Book: Ordinary Differential Equations. An introduction to the fundamentals. Kenneth B. Howell. second edition. CRC Press. FL, USA. 2020
Section: Chapter 33. Power series solutions I: Basic computational methods. Additional Exercises. page 641
Problem number: 33.3 (i).
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_separable]`

$$\boxed{(-x^3 + 2)\, y' - 3x^2 y = 0}$$

With the expansion point for the power series method at $x = 0$.

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 16

```
Order:=6;
dsolve((2-x^3)*diff(y(x),x)-3*x^2*y(x)=0,y(x),type='series',x=0);
```

$$y(x) = \left(1 + \frac{x^3}{2}\right) y(0) + O(x^6)$$

✓ Solution by Mathematica

Time used: 0.001 (sec). Leaf size: 15

```
AsymptoticDSolveValue[(2-x^3)*y'[x]-3*x^2*y[x]==0,y[x],{x,0,5}]
```

$$y(x) \to c_1\left(\frac{x^3}{2} + 1\right)$$

23.10 problem 33.3 (j)

Internal problem ID [13920]

Book: Ordinary Differential Equations. An introduction to the fundamentals. Kenneth B. Howell. second edition. CRC Press. FL, USA. 2020
Section: Chapter 33. Power series solutions I: Basic computational methods. Additional Exercises. page 641
Problem number: 33.3 (j).
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_separable]`

$$\boxed{(-x^3+2)\,y'+3x^2y=0}$$

With the expansion point for the power series method at $x=0$.

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 14

```
Order:=6;
dsolve((2-x^3)*diff(y(x),x)+3*x^2*y(x)=0,y(x),type='series',x=0);
```

$$y(x)=y(0)\left(1-\frac{x^3}{2}\right)$$

✓ Solution by Mathematica

Time used: 0.001 (sec). Leaf size: 15

```
AsymptoticDSolveValue[(2-x^3)*y'[x]+3*x^2*y[x]==0,y[x],{x,0,5}]
```

$$y(x)\to c_1\left(1-\frac{x^3}{2}\right)$$

23.11 problem 33.3 (k)

Internal problem ID [13921]

Book: Ordinary Differential Equations. An introduction to the fundamentals. Kenneth B. Howell. second edition. CRC Press. FL, USA. 2020
Section: Chapter 33. Power series solutions I: Basic computational methods. Additional Exercises. page 641
Problem number: 33.3 (k).
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_separable]`

$$\boxed{(x+1)\,y' - yx = 0}$$

With the expansion point for the power series method at $x = 0$.

✓ Solution by Maple
Time used: 0.0 (sec). Leaf size: 31

```
Order:=6;
dsolve((1+x)*diff(y(x),x)-x*y(x)=0,y(x),type='series',x=0);
```

$$y(x) = \left(1 + \frac{1}{2}x^2 - \frac{1}{3}x^3 + \frac{3}{8}x^4 - \frac{11}{30}x^5\right) y(0) + O\left(x^6\right)$$

✓ Solution by Mathematica
Time used: 0.001 (sec). Leaf size: 36

```
AsymptoticDSolveValue[(1+x)*y'[x]-x*y[x]==0,y[x],{x,0,5}]
```

$$y(x) \to c_1\left(-\frac{11x^5}{30} + \frac{3x^4}{8} - \frac{x^3}{3} + \frac{x^2}{2} + 1\right)$$

23.12 problem 33.3 (L)

Internal problem ID [13922]

Book: Ordinary Differential Equations. An introduction to the fundamentals. Kenneth B. Howell. second edition. CRC Press. FL, USA. 2020
Section: Chapter 33. Power series solutions I: Basic computational methods. Additional Exercises. page 641
Problem number: 33.3 (L).
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_separable]`

$$\boxed{(x+1)\,y'+(1-x)\,y=0}$$

With the expansion point for the power series method at $x=0$.

✓ Solution by Maple
Time used: 0.0 (sec). Leaf size: 36

```
Order:=6;
dsolve((1+x)*diff(y(x),x)+(1-x)*y(x)=0,y(x),type='series',x=0);
```

$$y(x)=\left(1-x+\frac{3}{2}x^2-\frac{11}{6}x^3+\frac{53}{24}x^4-\frac{103}{40}x^5\right)y(0)+O\left(x^6\right)$$

✓ Solution by Mathematica
Time used: 0.001 (sec). Leaf size: 39

```
AsymptoticDSolveValue[(1+x)*y'[x]+(1-x)*y[x]==0,y[x],{x,0,5}]
```

$$y(x)\to c_1\left(-\frac{103x^5}{40}+\frac{53x^4}{24}-\frac{11x^3}{6}+\frac{3x^2}{2}-x+1\right)$$

23.13 problem 33.5 (a)

Internal problem ID [13923]

Book: Ordinary Differential Equations. An introduction to the fundamentals. Kenneth B. Howell. second edition. CRC Press. FL, USA. 2020
Section: Chapter 33. Power series solutions I: Basic computational methods. Additional Exercises. page 641
Problem number: 33.5 (a).
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _exact, _linear, _homogeneous]]`

$$\boxed{(x^2+1)\, y'' - 2y = 0}$$

With the expansion point for the power series method at $x = 0$.

✓ Solution by Maple
Time used: 0.0 (sec). Leaf size: 27

```
Order:=6;
dsolve((1+x^2)*diff(y(x),x$2)-2*y(x)=0,y(x),type='series',x=0);
```

$$y(x) = y(0)\,(x^2+1) + \left(x + \frac{1}{3}x^3 - \frac{1}{15}x^5\right) D(y)\,(0) + O(x^6)$$

✓ Solution by Mathematica
Time used: 0.001 (sec). Leaf size: 31

```
AsymptoticDSolveValue[(1+x^2)*y''[x]-2*y[x]==0,y[x],{x,0,5}]
```

$$y(x) \to c_1(x^2+1) + c_2\left(-\frac{x^5}{15} + \frac{x^3}{3} + x\right)$$

23.14 problem 33.5 (b)

Internal problem ID [13924]

Book: Ordinary Differential Equations. An introduction to the fundamentals. Kenneth B. Howell. second edition. CRC Press. FL, USA. 2020
Section: Chapter 33. Power series solutions I: Basic computational methods. Additional Exercises. page 641
Problem number: 33.5 (b).
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _exact, _linear, _homogeneous]]`

$$\boxed{y'' + y'x + y = 0}$$

With the expansion point for the power series method at $x = 0$.

✓ Solution by Maple
Time used: 0.0 (sec). Leaf size: 34

```
Order:=6;
dsolve(diff(y(x),x$2)+x*diff(y(x),x)+y(x)=0,y(x),type='series',x=0);
```

$$y(x) = \left(1 - \frac{1}{2}x^2 + \frac{1}{8}x^4\right) y(0) + \left(x - \frac{1}{3}x^3 + \frac{1}{15}x^5\right) D(y)(0) + O(x^6)$$

✓ Solution by Mathematica
Time used: 0.001 (sec). Leaf size: 42

```
AsymptoticDSolveValue[y''[x]+x*y'[x]+y[x]==0,y[x],{x,0,5}]
```

$$y(x) \to c_2\left(\frac{x^5}{15} - \frac{x^3}{3} + x\right) + c_1\left(\frac{x^4}{8} - \frac{x^2}{2} + 1\right)$$

23.15 problem 33.5 (c)

Internal problem ID [13925]

Book: Ordinary Differential Equations. An introduction to the fundamentals. Kenneth B. Howell. second edition. CRC Press. FL, USA. 2020
Section: Chapter 33. Power series solutions I: Basic computational methods. Additional Exercises. page 641
Problem number: 33.5 (c).
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _missing_y]]`

$$\boxed{(x^2+4)\,y''+2y'x=0}$$

With the expansion point for the power series method at $x=0$.

✓ Solution by Maple
Time used: 0.0 (sec). Leaf size: 24

```
Order:=6;
dsolve((4+x^2)*diff(y(x),x$2)+2*x*diff(y(x),x)=0,y(x),type='series',x=0);
```

$$y(x)=y(0)+\left(x-\frac{1}{12}x^3+\frac{1}{80}x^5\right)D(y)\,(0)+O(x^6)$$

✓ Solution by Mathematica
Time used: 0.001 (sec). Leaf size: 25

```
AsymptoticDSolveValue[(4+x^2)*y''[x]+2*x*y'[x]==0,y[x],{x,0,5}]
```

$$y(x)\to c_2\left(\frac{x^5}{80}-\frac{x^3}{12}+x\right)+c_1$$

23.16 problem 33.5 (d)

Internal problem ID [13926]

Book: Ordinary Differential Equations. An introduction to the fundamentals. Kenneth B. Howell. second edition. CRC Press. FL, USA. 2020
Section: Chapter 33. Power series solutions I: Basic computational methods. Additional Exercises. page 641
Problem number: 33.5 (d).
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_Emden, _Fowler]]`

$$\boxed{y'' - 3x^2y = 0}$$

With the expansion point for the power series method at $x = 0$.

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 24

```
Order:=6;
dsolve(diff(y(x),x$2)-3*x^2*y(x)=0,y(x),type='series',x=0);
```

$$y(x) = \left(1 + \frac{x^4}{4}\right) y(0) + \left(x + \frac{3}{20}x^5\right) D(y)(0) + O(x^6)$$

✓ Solution by Mathematica

Time used: 0.001 (sec). Leaf size: 28

```
AsymptoticDSolveValue[y''[x]-3*x^2*y[x]==0,y[x],{x,0,5}]
```

$$y(x) \to c_2\left(\frac{3x^5}{20} + x\right) + c_1\left(\frac{x^4}{4} + 1\right)$$

23.17 problem 33.5 (e)

Internal problem ID [13927]

Book: Ordinary Differential Equations. An introduction to the fundamentals. Kenneth B. Howell. second edition. CRC Press. FL, USA. 2020
Section: Chapter 33. Power series solutions I: Basic computational methods. Additional Exercises. page 641
Problem number: 33.5 (e).
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _exact, _linear, _homogeneous]]`

$$\boxed{(-x^2+4)\,y''-5y'x-3y=0}$$

With the expansion point for the power series method at $x=0$.

✓ Solution by Maple
Time used: 0.0 (sec). Leaf size: 34

```
Order:=6;
dsolve((4-x^2)*diff(y(x),x$2)-5*x*diff(y(x),x)-3*y(x)=0,y(x),type='series',x=0);
```

$$y(x)=\left(1+\frac{3}{8}x^2+\frac{15}{128}x^4\right)y(0)+\left(x+\frac{1}{3}x^3+\frac{1}{10}x^5\right)D(y)(0)+O(x^6)$$

✓ Solution by Mathematica
Time used: 0.001 (sec). Leaf size: 42

```
AsymptoticDSolveValue[(4-x^2)*y''[x]-5*x*y'[x]-3*y[x]==0,y[x],{x,0,5}]
```

$$y(x)\to c_2\left(\frac{x^5}{10}+\frac{x^3}{3}+x\right)+c_1\left(\frac{15x^4}{128}+\frac{3x^2}{8}+1\right)$$

23.18 problem 33.5 (f)

Internal problem ID [13928]

Book: Ordinary Differential Equations. An introduction to the fundamentals. Kenneth B. Howell. second edition. CRC Press. FL, USA. 2020
Section: Chapter 33. Power series solutions I: Basic computational methods. Additional Exercises. page 641
Problem number: 33.5 (f).
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[_Gegenbauer, [_2nd_order, _linear, '_with_symmetry_[0,F(x)]']]`

$$\boxed{(-x^2+1)\, y'' - y'x + 4y = 0}$$

With the expansion point for the power series method at $x = 0$.

✓ Solution by Maple
Time used: 0.0 (sec). Leaf size: 29

```
Order:=6;
dsolve((1-x^2)*diff(y(x),x$2)-x*diff(y(x),x)+4*y(x)=0,y(x),type='series',x=0);
```

$$y(x) = (-2x^2+1)\, y(0) + \left(x - \frac{1}{2}x^3 - \frac{1}{8}x^5\right) D(y)\,(0) + O(x^6)$$

✓ Solution by Mathematica
Time used: 0.001 (sec). Leaf size: 33

```
AsymptoticDSolveValue[(1-x^2)*y''[x]-x*y'[x]+4*y[x]==0,y[x],{x,0,5}]
```

$$y(x) \to c_1(1-2x^2) + c_2\left(-\frac{x^5}{8} - \frac{x^3}{2} + x\right)$$

23.19 problem 33.5 (g)

Internal problem ID [13929]

Book: Ordinary Differential Equations. An introduction to the fundamentals. Kenneth B. Howell. second edition. CRC Press. FL, USA. 2020
Section: Chapter 33. Power series solutions I: Basic computational methods. Additional Exercises. page 641
Problem number: 33.5 (g).
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$\boxed{y'' - 2y'x + 6y = 0}$$

With the expansion point for the power series method at $x = 0$.

✓ Solution by Maple
Time used: 0.0 (sec). Leaf size: 29

```
Order:=6;
dsolve(diff(y(x),x$2)-2*x*diff(y(x),x)+6*y(x)=0,y(x),type='series',x=0);
```

$$y(x) = \left(1 - 3x^2 + \frac{1}{2}x^4\right) y(0) + \left(-\frac{2}{3}x^3 + x\right) D(y)(0) + O(x^6)$$

✓ Solution by Mathematica
Time used: 0.001 (sec). Leaf size: 33

```
AsymptoticDSolveValue[y''[x]-2*x*y'[x]+6*y[x]==0,y[x],{x,0,5}]
```

$$y(x) \to c_2\left(x - \frac{2x^3}{3}\right) + c_1\left(\frac{x^4}{2} - 3x^2 + 1\right)$$

23.20 problem 33.5 (h)

Internal problem ID [13930]

Book: Ordinary Differential Equations. An introduction to the fundamentals. Kenneth B. Howell. second edition. CRC Press. FL, USA. 2020
Section: Chapter 33. Power series solutions I: Basic computational methods. Additional Exercises. page 641
Problem number: 33.5 (h).
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _exact, _linear, _homogeneous]]`

$$\boxed{(x^2 - 6x)\, y'' + 4(x-3)\, y' + 2y = 0}$$

With the expansion point for the power series method at $x = 0$.

✓ Solution by Maple

Time used: 0.047 (sec). Leaf size: 44

```
Order:=6;
dsolve((x^2-6*x)*diff(y(x),x$2)+4*(x-3)*diff(y(x),x)+2*y(x)=0,y(x),type='series',x=0);
```

$$y(x) = c_1\left(1 + \frac{1}{6}x + \frac{1}{36}x^2 + \frac{1}{216}x^3 + \frac{1}{1296}x^4 + \frac{1}{7776}x^5 + \mathrm{O}\left(x^6\right)\right) + \frac{c_2\left(1 + \frac{1}{3}x + \frac{1}{18}x^2 + \frac{1}{108}x^3 + \frac{1}{648}x^4 + \frac{1}{3888}x^5 + \mathrm{O}\left(x^6\right)\right)}{x}$$

✓ Solution by Mathematica

Time used: 0.034 (sec). Leaf size: 64

```
AsymptoticDSolveValue[(x^2-6*x)*y''[x]+4*(x-3)*y'[x]+2*y[x]==0,y[x],{x,0,5}]
```

$$y(x) \to c_1\left(\frac{x^3}{1296} + \frac{x^2}{216} + \frac{x}{36} + \frac{1}{x} + \frac{1}{6}\right) + c_2\left(\frac{x^4}{1296} + \frac{x^3}{216} + \frac{x^2}{36} + \frac{x}{6} + 1\right)$$

23.21 problem 33.5 (i)

Internal problem ID [13931]

Book: Ordinary Differential Equations. An introduction to the fundamentals. Kenneth B. Howell. second edition. CRC Press. FL, USA. 2020
Section: Chapter 33. Power series solutions I: Basic computational methods. Additional Exercises. page 641
Problem number: 33.5 (i).
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$\boxed{y'' + (x+2)\, y' + 2y = 0}$$

With the expansion point for the power series method at $x = -2$.

✓ Solution by Maple
Time used: 0.0 (sec). Leaf size: 34

```
Order:=6;
dsolve(diff(y(x),x$2)+(x+2)*diff(y(x),x)+2*y(x)=0,y(x),type='series',x=-2);
```

$$y(x) = \left(1 - (x+2)^2 + \frac{(x+2)^4}{3}\right) y(-2) + \left(x + 2 - \frac{(x+2)^3}{2} + \frac{(x+2)^5}{8}\right) D(y)\,(-2) + O\left(x^6\right)$$

✓ Solution by Mathematica
Time used: 0.001 (sec). Leaf size: 49

```
AsymptoticDSolveValue[y''[x]+(x+2)*y'[x]+2*y[x]==0,y[x],{x,-2,5}]
```

$$y(x) \to c_1\left(\frac{1}{3}(x+2)^4 - (x+2)^2 + 1\right) + c_2\left(\frac{1}{8}(x+2)^5 - \frac{1}{2}(x+2)^3 + x + 2\right)$$

23.22 problem 33.5 (j)

Internal problem ID [13932]

Book: Ordinary Differential Equations. An introduction to the fundamentals. Kenneth B. Howell. second edition. CRC Press. FL, USA. 2020
Section: Chapter 33. Power series solutions I: Basic computational methods. Additional Exercises. page 641
Problem number: 33.5 (j).
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _exact, _linear, _homogeneous]]`

$$\boxed{(x^2-2x+2)\,y''+(1-x)\,y'-3y=0}$$

With the expansion point for the power series method at $x=1$.

✓ Solution by Maple
Time used: 0.016 (sec). Leaf size: 29

```
Order:=6;
dsolve((x^2-2*x+2)*diff(y(x),x$2)+(1-x)*diff(y(x),x)-3*y(x)=0,y(x),type='series',x=1);
```

$$y(x)=\left(1+\frac{3(-1+x)^2}{2}+\frac{3(-1+x)^4}{8}\right)y(1)+\left(-1+x+\frac{2(-1+x)^3}{3}\right)D(y)\,(1)+O\left(x^6\right)$$

✓ Solution by Mathematica
Time used: 0.001 (sec). Leaf size: 42

```
AsymptoticDSolveValue[(x^2-2*x+2)*y''[x]+(1-x)*y'[x]-3*y[x]==0,y[x],{x,1,5}]
```

$$y(x)\to c_1\left(\frac{3}{8}(x-1)^4+\frac{3}{2}(x-1)^2+1\right)+c_2\left(\frac{2}{3}(x-1)^3+x-1\right)$$

23.23 problem 33.5 (k)

Internal problem ID [13933]

Book: Ordinary Differential Equations. An introduction to the fundamentals. Kenneth B. Howell. second edition. CRC Press. FL, USA. 2020
Section: Chapter 33. Power series solutions I: Basic computational methods. Additional Exercises. page 641
Problem number: 33.5 (k).
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$\boxed{y'' - 2y' - yx = 0}$$

With the expansion point for the power series method at $x = 0$.

✓ Solution by Maple
Time used: 0.016 (sec). Leaf size: 47

```
Order:=6;
dsolve(diff(y(x),x$2)-2*diff(y(x),x)-x*y(x)=0,y(x),type='series',x=0);
```

$$y(x) = \left(1+\frac{1}{6}x^3+\frac{1}{12}x^4+\frac{1}{30}x^5\right)y(0)+\left(x+x^2+\frac{2}{3}x^3+\frac{5}{12}x^4+\frac{13}{60}x^5\right)D(y)(0)+O(x^6)$$

✓ Solution by Mathematica
Time used: 0.001 (sec). Leaf size: 59

```
AsymptoticDSolveValue[y''[x]-2*y'[x]-x*y[x]==0,y[x],{x,0,5}]
```

$$y(x) \to c_1\left(\frac{x^5}{30}+\frac{x^4}{12}+\frac{x^3}{6}+1\right)+c_2\left(\frac{13x^5}{60}+\frac{5x^4}{12}+\frac{2x^3}{3}+x^2+x\right)$$

23.24 problem 33.5 (L)

Internal problem ID [13934]

Book: Ordinary Differential Equations. An introduction to the fundamentals. Kenneth B. Howell. second edition. CRC Press. FL, USA. 2020
Section: Chapter 33. Power series solutions I: Basic computational methods. Additional Exercises. page 641
Problem number: 33.5 (L).
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$\boxed{y'' - y'x - 2yx = 0}$$

With the expansion point for the power series method at $x = 0$.

✓ Solution by Maple
Time used: 0.0 (sec). Leaf size: 39

```
Order:=6;
dsolve(diff(y(x),x$2)-x*diff(y(x),x)-2*x*y(x)=0,y(x),type='series',x=0);
```

$$y(x) = \left(1 + \frac{1}{3}x^3 + \frac{1}{20}x^5\right) y(0) + \left(x + \frac{1}{6}x^3 + \frac{1}{6}x^4 + \frac{1}{40}x^5\right) D(y)(0) + O(x^6)$$

✓ Solution by Mathematica
Time used: 0.001 (sec). Leaf size: 49

```
AsymptoticDSolveValue[y''[x]-x*y'[x]-2*x*y[x]==0,y[x],{x,0,5}]
```

$$y(x) \to c_1\left(\frac{x^5}{20} + \frac{x^3}{3} + 1\right) + c_2\left(\frac{x^5}{40} + \frac{x^4}{6} + \frac{x^3}{6} + x\right)$$

23.25 problem 33.9

Internal problem ID [13935]

Book: Ordinary Differential Equations. An introduction to the fundamentals. Kenneth B. Howell. second edition. CRC Press. FL, USA. 2020
Section: Chapter 33. Power series solutions I: Basic computational methods. Additional Exercises. page 641
Problem number: 33.9.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[_Gegenbauer, [_2nd_order, _linear, '_with_symmetry_[0,F(x)]']]`

$$\boxed{(-x^2+1)\,y''-y'x+y\lambda=0}$$

With the expansion point for the power series method at $x=0$.

✓ Solution by Maple
Time used: 0.0 (sec). Leaf size: 63

```
Order:=6;
dsolve((1-x^2)*diff(y(x),x$2)-x*diff(y(x),x)+lambda*y(x)=0,y(x),type='series',x=0);
```

$$y(x)=\left(1-\frac{x^2\lambda}{2}+\frac{\lambda(\lambda-4)\,x^4}{24}\right)y(0)+\left(x-\frac{(\lambda-1)\,x^3}{6}+\frac{(\lambda-1)\,(\lambda-9)\,x^5}{120}\right)D(y)\,(0)+O\left(x^6\right)$$

✓ Solution by Mathematica
Time used: 0.001 (sec). Leaf size: 80

```
AsymptoticDSolveValue[(1-x^2)*y''[x]-x*y'[x]+\[Lambda]*y[x]==0,y[x],{x,0,5}]
```

$$y(x)\to c_2\left(\frac{\lambda^2x^5}{120}-\frac{\lambda x^5}{12}+\frac{3x^5}{40}-\frac{\lambda x^3}{6}+\frac{x^3}{6}+x\right)+c_1\left(\frac{\lambda^2x^4}{24}-\frac{\lambda x^4}{6}-\frac{\lambda x^2}{2}+1\right)$$

23.26 problem 33.10

Internal problem ID [13936]

Book: Ordinary Differential Equations. An introduction to the fundamentals. Kenneth B. Howell. second edition. CRC Press. FL, USA. 2020
Section: Chapter 33. Power series solutions I: Basic computational methods. Additional Exercises. page 641
Problem number: 33.10.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[_Gegenbauer]`

$$\boxed{(-x^2+1)\,y''-2y'x+y\lambda=0}$$

With the expansion point for the power series method at $x=0$.

✓ Solution by Maple
Time used: 0.0 (sec). Leaf size: 63

```
Order:=6;
dsolve((1-x^2)*diff(y(x),x$2)-2*x*diff(y(x),x)+lambda*y(x)=0,y(x),type='series',x=0);
```

$$\begin{aligned}y(x) = &\left(1-\frac{x^2\lambda}{2}+\frac{\lambda(\lambda-6)\,x^4}{24}\right)y(0)\\&+\left(x-\frac{(\lambda-2)\,x^3}{6}+\frac{(\lambda-2)\,(-12+\lambda)\,x^5}{120}\right)D(y)\,(0)+O\left(x^6\right)\end{aligned}$$

✓ Solution by Mathematica
Time used: 0.001 (sec). Leaf size: 80

```
AsymptoticDSolveValue[(1-x^2)*y''[x]-2*x*y'[x]+\[Lambda]*y[x]==0,y[x],{x,0,5}]
```

$$y(x)\to c_2\left(\frac{\lambda^2x^5}{120}-\frac{7\lambda x^5}{60}+\frac{x^5}{5}-\frac{\lambda x^3}{6}+\frac{x^3}{3}+x\right)+c_1\left(\frac{\lambda^2x^4}{24}-\frac{\lambda x^4}{4}-\frac{\lambda x^2}{2}+1\right)$$

23.27 problem 33.11 (a)

Internal problem ID [13937]

Book: Ordinary Differential Equations. An introduction to the fundamentals. Kenneth B. Howell. second edition. CRC Press. FL, USA. 2020
Section: Chapter 33. Power series solutions I: Basic computational methods. Additional Exercises. page 641
Problem number: 33.11 (a).
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _missing_x]]`

$$\boxed{y'' + 4y = 0}$$

With the expansion point for the power series method at $x = 0$.

✓ Solution by Maple
Time used: 0.0 (sec). Leaf size: 29

```
Order:=5;
dsolve(diff(y(x),x$2)+4*y(x)=0,y(x),type='series',x=0);
```

$$y(x) = \left(1 - 2x^2 + \frac{2}{3}x^4\right) y(0) + \left(-\frac{2}{3}x^3 + x\right) D(y)(0) + O(x^5)$$

✓ Solution by Mathematica
Time used: 0.001 (sec). Leaf size: 33

```
AsymptoticDSolveValue[y''[x]+4*y[x]==0,y[x],{x,0,4}]
```

$$y(x) \to c_2\left(x - \frac{2x^3}{3}\right) + c_1\left(\frac{2x^4}{3} - 2x^2 + 1\right)$$

23.28 problem 33.11 (b)

Internal problem ID [13938]

Book: Ordinary Differential Equations. An introduction to the fundamentals. Kenneth B. Howell. second edition. CRC Press. FL, USA. 2020
Section: Chapter 33. Power series solutions I: Basic computational methods. Additional Exercises. page 641
Problem number: 33.11 (b).
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_Emden, _Fowler]]`

$$\boxed{y'' - x^2 y = 0}$$

With the expansion point for the power series method at $x = 0$.

✓ Solution by Maple
Time used: 0.0 (sec). Leaf size: 19

```
Order:=5;
dsolve(diff(y(x),x$2)-x^2*y(x)=0,y(x),type='series',x=0);
```

$$y(x) = \left(1 + \frac{x^4}{12}\right) y(0) + x D(y)(0) + O(x^5)$$

✓ Solution by Mathematica
Time used: 0.001 (sec). Leaf size: 20

```
AsymptoticDSolveValue[y''[x]-x^2*y[x]==0,y[x],{x,0,4}]
```

$$y(x) \to c_1 \left(\frac{x^4}{12} + 1\right) + c_2 x$$

23.29 problem 33.11 (c)

Internal problem ID [13939]

Book: Ordinary Differential Equations. An introduction to the fundamentals. Kenneth B. Howell. second edition. CRC Press. FL, USA. 2020
Section: Chapter 33. Power series solutions I: Basic computational methods. Additional Exercises. page 641
Problem number: 33.11 (c).
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$\boxed{y'' + y\,\mathrm{e}^{2x} = 0}$$

With the expansion point for the power series method at $x = 0$.

✓ Solution by Maple
Time used: 0.0 (sec). Leaf size: 29

```
Order:=4;
dsolve(diff(y(x),x$2)+exp(2*x)*y(x)=0,y(x),type='series',x=0);
```

$$y(x) = \left(1 - \frac{1}{2}x^2 - \frac{1}{3}x^3\right)y(0) + \left(x - \frac{1}{6}x^3\right)D(y)(0) + O(x^4)$$

✓ Solution by Mathematica
Time used: 0.001 (sec). Leaf size: 35

```
AsymptoticDSolveValue[y''[x]+Exp[2*x]*y[x]==0,y[x],{x,0,3}]
```

$$y(x) \to c_2\left(x - \frac{x^3}{6}\right) + c_1\left(-\frac{x^3}{3} - \frac{x^2}{2} + 1\right)$$

23.30 problem 33.11 (d)

Internal problem ID [13940]

Book: Ordinary Differential Equations. An introduction to the fundamentals. Kenneth B. Howell. second edition. CRC Press. FL, USA. 2020
Section: Chapter 33. Power series solutions I: Basic computational methods. Additional Exercises. page 641
Problem number: 33.11 (d).
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$\boxed{\sin(x)\, y'' - y = 0}$$

With the expansion point for the power series method at $x = \frac{\pi}{2}$.

✓ Solution by Maple
Time used: 0.0 (sec). Leaf size: 32

```
Order:=4;
dsolve(sin(x)*diff(y(x),x$2)-y(x)=0,y(x),type='series',x=Pi/2);
```

$$y(x) = \left(1 + \frac{\left(-\frac{\pi}{2} + x\right)^2}{2}\right) y\left(\frac{\pi}{2}\right) + \left(-\frac{\pi}{2} + x + \frac{\left(-\frac{\pi}{2} + x\right)^3}{6}\right) D(y)\left(\frac{\pi}{2}\right) + O\left(x^4\right)$$

✓ Solution by Mathematica
Time used: 0.001 (sec). Leaf size: 45

```
AsymptoticDSolveValue[Sin[x]*y''[x]-y[x]==0,y[x],{x,Pi/2,3}]
```

$$y(x) \to c_1\left(\frac{1}{2}\left(x - \frac{\pi}{2}\right)^2 + 1\right) + c_2\left(\frac{1}{6}\left(x - \frac{\pi}{2}\right)^3 + x - \frac{\pi}{2}\right)$$

23.31 problem 33.11 (e)

Internal problem ID [13941]

Book: Ordinary Differential Equations. An introduction to the fundamentals. Kenneth B. Howell. second edition. CRC Press. FL, USA. 2020
Section: Chapter 33. Power series solutions I: Basic computational methods. Additional Exercises. page 641
Problem number: 33.11 (e).
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _linear, _nonhomogeneous]]`

$$\boxed{y'' + yx = \sin(x)}$$

With the expansion point for the power series method at $x = 0$.

✓ Solution by Maple
Time used: 0.015 (sec). Leaf size: 26

```
Order:=5;
dsolve(diff(y(x),x$2)+x*y(x)=sin(x),y(x),type='series',x=0);
```

$$y(x) = \left(1 - \frac{x^3}{6}\right) y(0) + \left(x - \frac{1}{12}x^4\right) D(y)(0) + \frac{x^3}{6} + O(x^5)$$

✓ Solution by Mathematica
Time used: 0.045 (sec). Leaf size: 35

```
AsymptoticDSolveValue[y''[x]+x*y[x]==Sin[x],y[x],{x,0,4}]
```

$$y(x) \to c_2\left(x - \frac{x^4}{12}\right) + \frac{x^3}{6} + c_1\left(1 - \frac{x^3}{6}\right)$$

23.32 problem 33.11 (f)

Internal problem ID [13942]

Book: Ordinary Differential Equations. An introduction to the fundamentals. Kenneth B. Howell. second edition. CRC Press. FL, USA. 2020
Section: Chapter 33. Power series solutions I: Basic computational methods. Additional Exercises. page 641
Problem number: 33.11 (f).
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$\boxed{y'' - y' \sin(x) - yx = 0}$$

With the expansion point for the power series method at $x = 0$.

✓ Solution by Maple
Time used: 0.0 (sec). Leaf size: 29

```
Order:=5;
dsolve(diff(y(x),x$2)-sin(x)*diff(y(x),x)-x*y(x)=0,y(x),type='series',x=0);
```

$$y(x) = \left(1 + \frac{x^3}{6}\right) y(0) + \left(x + \frac{1}{6}x^3 + \frac{1}{12}x^4\right) D(y)(0) + O(x^5)$$

✓ Solution by Mathematica
Time used: 0.001 (sec). Leaf size: 35

```
AsymptoticDSolveValue[y''[x]-Sin[x]*y'[x]-x*y[x]==0,y[x],{x,0,4}]
```

$$y(x) \to c_1\left(\frac{x^3}{6} + 1\right) + c_2\left(\frac{x^4}{12} + \frac{x^3}{6} + x\right)$$

23.33 problem 33.11 (g)

Internal problem ID [13943]

Book: Ordinary Differential Equations. An introduction to the fundamentals. Kenneth B. Howell. second edition. CRC Press. FL, USA. 2020
Section: Chapter 33. Power series solutions I: Basic computational methods. Additional Exercises. page 641
Problem number: 33.11 (g).
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _missing_x], [_2nd_order, _reducible, _mu_x_y1]]`

$$\boxed{y'' - y^2 = 0}$$

With the expansion point for the power series method at $x = 0$.

✓ Solution by Maple
Time used: 0.0 (sec). Leaf size: 42

```
Order:=5;
dsolve(diff(y(x),x$2)-y(x)^2=0,y(x),type='series',x=0);
```

$$y(x) = \frac{x^4 y(0)^3}{12} + \frac{y(0)^2 x^2}{2} + \left(1 + \frac{D(y)(0)\, x^3}{3}\right) y(0) + x D(y)(0) + \frac{D(y)(0)^2 x^4}{12} + O(x^5)$$

✓ Solution by Mathematica
Time used: 0.018 (sec). Leaf size: 48

```
AsymptoticDSolveValue[y''[x]-y[x]^2==0,y[x],{x,0,4}]
```

$$y(x) \to \frac{1}{12}\left(c_1{}^3 + c_2{}^2\right) x^4 + \frac{1}{3} c_1 c_2 x^3 + \frac{c_1{}^2 x^2}{2} + c_2 x + c_1$$

23.34 problem 33.11 (h)

Internal problem ID [13944]

Book: Ordinary Differential Equations. An introduction to the fundamentals. Kenneth B. Howell. second edition. CRC Press. FL, USA. 2020
Section: Chapter 33. Power series solutions I: Basic computational methods. Additional Exercises. page 641
Problem number: 33.11 (h).
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_quadrature]`

$$\boxed{y' + \cos(y) = 0}$$

With the expansion point for the power series method at $x = 0$.

✓ Solution by Maple
Time used: 0.0 (sec). Leaf size: 67

```
Order:=5;
dsolve(diff(y(x),x)+cos(y(x))=0,y(x),type='series',x=0);
```

$$y(x) = y(0) - \cos(y(0))\,x - \frac{\sin(2y(0))\,x^2}{4} + \frac{\cos(y(0))\cos(2y(0))\,x^3}{6} + \left(\frac{\sin(4y(0))}{32} + \frac{\sin(2y(0))}{24}\right)x^4 + O(x^5)$$

✓ Solution by Mathematica
Time used: 0.074 (sec). Leaf size: 76

```
AsymptoticDSolveValue[y'[x]+Cos[y[x]]==0,y[x],{x,0,4}]
```

$$y(x) \to \frac{1}{24}x^4(5\sin(c_1)\cos^3(c_1) - \sin^3(c_1)\cos(c_1)) + \frac{1}{6}x^3(\cos^3(c_1) - \sin^2(c_1)\cos(c_1)) - \frac{1}{2}x^2\sin(c_1)\cos(c_1) - x\cos(c_1) + c_1$$

24 Chapter 34. Power series solutions II: Generalization and theory. Additional Exercises. page 678

24.1 problem 34.5 (a)

Internal problem ID [13945]

Book: Ordinary Differential Equations. An introduction to the fundamentals. Kenneth B. Howell. second edition. CRC Press. FL, USA. 2020
Section: Chapter 34. Power series solutions II: Generalization and theory. Additional Exercises. page 678
Problem number: 34.5 (a).
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_separable]`

$$\boxed{y' - y\,e^{x} = 0}$$

With the expansion point for the power series method at $x = 0$.

✓ Solution by Maple
Time used: 0.0 (sec). Leaf size: 32

```
Order:=6;
dsolve(diff(y(x),x)-exp(x)*y(x)=0,y(x),type='series',x=0);
```

$$y(x) = \left(1 + x + x^2 + \frac{5}{6}x^3 + \frac{5}{8}x^4 + \frac{13}{30}x^5\right) y(0) + O\left(x^6\right)$$

✓ Solution by Mathematica
Time used: 0.001 (sec). Leaf size: 33

```
AsymptoticDSolveValue[y'[x]-Exp[x]*y[x]==0,y[x],{x,0,5}]
```

$$y(x) \to c_1\left(\frac{13x^5}{30} + \frac{5x^4}{8} + \frac{5x^3}{6} + x^2 + x + 1\right)$$

24.2 problem 34.5 (b)

Internal problem ID [13946]

Book: Ordinary Differential Equations. An introduction to the fundamentals. Kenneth B. Howell. second edition. CRC Press. FL, USA. 2020
Section: Chapter 34. Power series solutions II: Generalization and theory. Additional Exercises. page 678
Problem number: 34.5 (b).
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_separable]`

$$\boxed{y' - \tan(x)\, y = 0}$$

With the expansion point for the power series method at $x = 0$.

✓ Solution by Maple
Time used: 0.0 (sec). Leaf size: 21

```
Order:=6;
dsolve(diff(y(x),x)-tan(x)*y(x)=0,y(x),type='series',x=0);
```

$$y(x) = \left(1 + \frac{1}{2}x^2 + \frac{5}{24}x^4\right) y(0) + O(x^6)$$

✓ Solution by Mathematica
Time used: 0.001 (sec). Leaf size: 22

```
AsymptoticDSolveValue[y'[x]-Tan[x]*y[x]==0,y[x],{x,0,5}]
```

$$y(x) \to c_1 \left(\frac{5x^4}{24} + \frac{x^2}{2} + 1\right)$$

24.3 problem 34.5 (c)

Internal problem ID [13947]

Book: Ordinary Differential Equations. An introduction to the fundamentals. Kenneth B. Howell. second edition. CRC Press. FL, USA. 2020
Section: Chapter 34. Power series solutions II: Generalization and theory. Additional Exercises. page 678
Problem number: 34.5 (c).
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$\boxed{\sin(x)\, y'' + x^2 y' - y\, e^x = 0}$$

With the expansion point for the power series method at $x = 2$.

✓ Solution by Maple

Time used: 0.031 (sec). Leaf size: 509

```
Order:=6;
dsolve(sin(x)*diff(y(x),x$2)+x^2*diff(y(x),x)-exp(x)*y(x)=0,y(x),type='series',x=2);
```

$$y(x) = \left(1 + \frac{\csc(2)\,e^2(x-2)^2}{2} - \frac{e^2(4+\cos(2)-\sin(2))\csc(2)^2(x-2)^3}{6} + \frac{\csc(2)^3\left(\left(\frac{3}{2}-\frac{\sin(4)}{12}-\sin(2)+\cos(2)\right)e^2+\frac{e^4\sin(2)}{12}\right)(x-2)^4}{2} + \frac{((-210+56\sin(4)+\sin(6)+201\sin(2)+\cos(6)-205\cos(2)-6\cos(4))\,e^2-4\,e^4(-1+\sin(4)+}{240}\right.$$

$$+\left(x-2-2\csc(2)(x-2)^2-\frac{(-e^2\sin(2)-4\cos(2)+4\sin(2)-16)\csc(2)^2(x-2)^3}{6} + \frac{\csc(2)^3\left(\left(-\frac{\cos(4)}{12}-\frac{2\sin(2)}{3}-\frac{\sin(4)}{12}+\frac{1}{12}\right)e^2-4\cos(2)-\frac{\cos(4)}{12}+4\sin(2)+\frac{\sin(4)}{3}-\frac{71}{12}\right)(x-2)^4}{2} + \frac{(((-12\cos(2)-72)\sin(2)^2+108\sin(2)+36\sin(4))\,e^2+2\sin(2)^2e^4-6\sin(6)+817\cos(2)+32}{240}\right.$$

$$+O\left(x^6\right)$$

✓ Solution by Mathematica

Time used: 0.004 (sec). Leaf size: 953

```
AsymptoticDSolveValue[Sin[x]*y''[x]+x^2*y'[x]-Exp[x]*y[x]==0,y[x],{x,2,5}]
```

$$
\begin{aligned}
y(x) \to c_2 \Bigg(& -\frac{1}{60}\left(6\csc(2)-13\cot(2)\csc(2)+12\cot^2(2)\csc(2)-12\cot^3(2)\csc(2)\right)(x-2)^5 \\
& -\frac{1}{20}\left(-e^2\csc(2)+e^2\cot(2)\csc(2)-e^2\cot^2(2)\csc(2)\right)(x-2)^5 \\
& +\frac{4}{15}\csc(2)\left(3\csc(2)-4\cot(2)\csc(2)+4\cot^2(2)\csc(2)\right)(x-2)^5 \\
& +\frac{1}{6}\csc(2)\left(-e^2\csc(2)+e^2\cot(2)\csc(2)\right)(x-2)^5 \\
& -\frac{1}{40}(4\csc(2)-4\cot(2)\csc(2))(-4\csc(2)+4\cot(2)\csc(2))(x-2)^5 \\
& +\frac{2}{5}\csc^2(2)(-4\csc(2)+4\cot(2)\csc(2))(x-2)^5 \\
& +\frac{1}{40}e^2\csc(2)(-4\csc(2)+4\cot(2)\csc(2))(x-2)^5 \\
& -\frac{2}{5}\csc^2(2)(4\csc(2)-4\cot(2)\csc(2))(x-2)^5 \\
& -\frac{1}{120}e^2\csc(2)(4\csc(2)-4\cot(2)\csc(2))(x-2)^5+\frac{32}{15}\csc^4(2)(x-2)^5 \\
& +\frac{2}{5}e^2\csc^3(2)(x-2)^5+\frac{1}{120}e^4\csc^2(2)(x-2)^5 \\
& -\frac{1}{12}\left(3\csc(2)-4\cot(2)\csc(2)+4\cot^2(2)\csc(2)\right)(x-2)^4 \\
& -\frac{1}{12}\left(-e^2\csc(2)+e^2\cot(2)\csc(2)\right)(x-2)^4 \\
& +\frac{1}{2}\csc(2)(4\csc(2)-4\cot(2)\csc(2))(x-2)^4-\frac{8}{3}\csc^3(2)(x-2)^4 \\
& -\frac{1}{3}e^2\csc^2(2)(x-2)^4-\frac{1}{6}(4\csc(2)-4\cot(2)\csc(2))(x-2)^3+\frac{8}{3}\csc^2(2)(x-2)^3 \\
& +\frac{1}{6}e^2\csc(2)(x-2)^3-2\csc(2)(x-2)^2+x-2\Bigg)+c_1\Bigg(-\frac{1}{60}\left(-2e^2\csc(2)\right. \\
& \left.+4e^2\cot(2)\csc(2)-3e^2\cot^2(2)\csc(2)+3e^2\cot^3(2)\csc(2)\right)(x-2)^5 \\
& +\frac{1}{15}\csc(2)\left(-e^2\csc(2)+e^2\cot(2)\csc(2)-e^2\cot^2(2)\csc(2)\right)(x-2)^5 \\
& -\frac{1}{20}e^2\csc(2)\left(3\csc(2)-4\cot(2)\csc(2)+4\cot^2(2)\csc(2)\right)(x-2)^5 \\
& -\frac{1}{40}(-4\csc(2)+4\cot(2)\csc(2))\left(-e^2\csc(2)+e^2\cot(2)\csc(2)\right)(x-2)^5 \\
& -\frac{2}{15}\csc^2(2)\left(-e^2\csc(2)+e^2\cot(2)\csc(2)\right)(x-2)^5 \\
& -\frac{1}{30}e^2\csc(2)\left(-e^2\csc(2)+e^2\cot(2)\csc(2)\right)(x-2)^5 \\
& -\frac{1}{10}e^2\csc^2(2)(-4\csc(2)+4\cot(2)\csc(2))(x-2)^5 \\
& +\frac{1}{\ldots}e^2\csc^2(2)(4\csc(2)-4\cot(2)\csc(2))(x-2)^5-\frac{8}{\ldots}e^2\csc^4(2)(x-2)^5
\end{aligned}
$$

24.4 problem 34.5 (d)

Internal problem ID [13948]

Book: Ordinary Differential Equations. An introduction to the fundamentals. Kenneth B. Howell. second edition. CRC Press. FL, USA. 2020
Section: Chapter 34. Power series solutions II: Generalization and theory. Additional Exercises. page 678
Problem number: 34.5 (d).
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$\sinh(x)\, y'' + x^2 y' - y\, e^x = 0$$

With the expansion point for the power series method at $x = 2$.

✓ Solution by Maple

Time used: 0.015 (sec). Leaf size: 275

```
Order:=6;
dsolve(sinh(x)*diff(y(x),x$2)+x^2*diff(y(x),x)-exp(x)*y(x)=0,y(x),type='series',x=2);
```

$$y(x) = \left(1 + \frac{e^4 (x-2)^2}{e^4 - 1} - \frac{2(e^2 + 4 e^4) e^2 (x-2)^3}{3 (e^4 - 1)^2} + \frac{\left(e^2 + 12 e^4 + \frac{33 e^6}{2} + \frac{e^{10}}{2}\right) e^2 (x-2)^4}{3 (e^4 - 1)^3} - \frac{2 e^2 \left(e^2 + \frac{53 e^4}{2} + 98 e^6 + 79 e^8 + 3 e^{10} + \frac{5 e^{12}}{2}\right) (x-2)^5}{15 (e^4 - 1)^4}\right) y(2) + \left(x - 2 - \frac{4 e^2 (x-2)^2}{e^4 - 1} - \frac{2\left(-\frac{31 e^2}{2} - \frac{e^6}{2} - 4\right) e^2 (x-2)^3}{3 (e^4 - 1)^2} + \frac{\left(-47 e^2 - 65 e^4 - e^6 - \frac{7 e^8}{2} - \frac{7}{2}\right) e^2 (x-2)^4}{3 (e^4 - 1)^3} - \frac{2 e^2 \left(-\frac{205 e^2}{2} - \frac{1537 e^4}{4} - \frac{1249 e^6}{4} - \frac{85 e^8}{4} - 17 e^{10} + \frac{e^{12}}{4} - \frac{e^{14}}{4} - \frac{11}{4}\right) (x-2)^5}{15 (e^4 - 1)^4}\right) D(y)(2) + O(x^6)$$

✓ Solution by Mathematica

Time used: 0.004 (sec). Leaf size: 931

```
AsymptoticDSolveValue[Sinh[x]*y''[x]+x^2*y'[x]-Exp[x]*y[x]==0,y[x],{x,2,5}]
```

$$
\begin{aligned}
y(x) \to c_2\Bigg(&-\frac{1}{60}\left(-6\operatorname{csch}(2)+7\coth(2)\operatorname{csch}(2)+12\coth^2(2)\operatorname{csch}(2)-12\coth^3(2)\operatorname{csch}(2)\right)(x-2)^5\\
&-\frac{1}{20}\left(e^2\coth(2)\operatorname{csch}(2)-e^2\coth^2(2)\operatorname{csch}(2)\right)(x-2)^5\\
&+\frac{4}{15}\operatorname{csch}(2)\left(-\operatorname{csch}(2)-4\coth(2)\operatorname{csch}(2)+4\coth^2(2)\operatorname{csch}(2)\right)(x-2)^5\\
&+\frac{1}{6}\operatorname{csch}(2)\left(-e^2\operatorname{csch}(2)+e^2\coth(2)\operatorname{csch}(2)\right)(x-2)^5\\
&-\frac{1}{40}(4\operatorname{csch}(2)-4\coth(2)\operatorname{csch}(2))(-4\operatorname{csch}(2)+4\coth(2)\operatorname{csch}(2))(x-2)^5\\
&+\frac{2}{5}\operatorname{csch}^2(2)(-4\operatorname{csch}(2)+4\coth(2)\operatorname{csch}(2))(x-2)^5\\
&+\frac{1}{40}e^2\operatorname{csch}(2)(-4\operatorname{csch}(2)+4\coth(2)\operatorname{csch}(2))(x-2)^5\\
&-\frac{2}{5}\operatorname{csch}^2(2)(4\operatorname{csch}(2)-4\coth(2)\operatorname{csch}(2))(x-2)^5\\
&-\frac{1}{120}e^2\operatorname{csch}(2)(4\operatorname{csch}(2)-4\coth(2)\operatorname{csch}(2))(x-2)^5+\frac{32}{15}\operatorname{csch}^4(2)(x-2)^5\\
&+\frac{2}{5}e^2\operatorname{csch}^3(2)(x-2)^5+\frac{1}{120}e^4\operatorname{csch}^2(2)(x-2)^5\\
&-\frac{1}{12}\left(-\operatorname{csch}(2)-4\coth(2)\operatorname{csch}(2)+4\coth^2(2)\operatorname{csch}(2)\right)(x-2)^4\\
&-\frac{1}{12}\left(-e^2\operatorname{csch}(2)+e^2\coth(2)\operatorname{csch}(2)\right)(x-2)^4\\
&+\frac{1}{2}\operatorname{csch}(2)(4\operatorname{csch}(2)-4\coth(2)\operatorname{csch}(2))(x-2)^4-\frac{8}{3}\operatorname{csch}^3(2)(x-2)^4\\
&-\frac{1}{3}e^2\operatorname{csch}^2(2)(x-2)^4-\frac{1}{6}(4\operatorname{csch}(2)-4\coth(2)\operatorname{csch}(2))(x-2)^3\\
&+\frac{8}{3}\operatorname{csch}^2(2)(x-2)^3+\frac{1}{6}e^2\operatorname{csch}(2)(x-2)^3-2\operatorname{csch}(2)(x-2)^2+x-2\Bigg)\\
+c_1\Bigg(&-\frac{1}{60}\left(e^2\operatorname{csch}(2)-e^2\coth(2)\operatorname{csch}(2)-3e^2\coth^2(2)\operatorname{csch}(2)+3e^2\coth^3(2)\operatorname{csch}(2)\right)(x-2)^5\\
&+\frac{1}{15}\operatorname{csch}(2)\left(e^2\coth(2)\operatorname{csch}(2)-e^2\coth^2(2)\operatorname{csch}(2)\right)(x-2)^5\\
&-\frac{1}{20}e^2\operatorname{csch}(2)\left(-\operatorname{csch}(2)-4\coth(2)\operatorname{csch}(2)+4\coth^2(2)\operatorname{csch}(2)\right)(x-2)^5\\
&-\frac{1}{40}(-4\operatorname{csch}(2)+4\coth(2)\operatorname{csch}(2))\left(-e^2\operatorname{csch}(2)+e^2\coth(2)\operatorname{csch}(2)\right)(x-2)^5\\
&-\frac{2}{15}\operatorname{csch}^2(2)\left(-e^2\operatorname{csch}(2)+e^2\coth(2)\operatorname{csch}(2)\right)(x-2)^5\\
&-\frac{1}{20}e^2\operatorname{csch}(2)\left(-e^2\operatorname{csch}(2)+e^2\coth(2)\operatorname{csch}(2)\right)(x-2)^5
\end{aligned}
$$

24.5 problem 34.5 (e)

Internal problem ID [13949]

Book: Ordinary Differential Equations. An introduction to the fundamentals. Kenneth B. Howell. second edition. CRC Press. FL, USA. 2020
Section: Chapter 34. Power series solutions II: Generalization and theory. Additional Exercises. page 678
Problem number: 34.5 (e).
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$\boxed{\sinh(x)\, y'' + x^2 y' - \sin(x)\, y = 0}$$

With the expansion point for the power series method at $x = 2$.

✓ Solution by Maple

Time used: 0.031 (sec). Leaf size: 710

```
Order:=6;
dsolve(sinh(x)*diff(y(x),x$2)+x^2*diff(y(x),x)-sin(x)*y(x)=0,y(x),type='series',x=2);
```

$$y(x) = \left(1 + \frac{\sin(2)\,\mathrm{e}^2(x-2)^2}{\mathrm{e}^4-1} + \frac{\mathrm{e}^2((-8\,\mathrm{e}^2-\mathrm{e}^4-1)\sin(2)+\cos(2)(\mathrm{e}^4-1))(x-2)^3}{3(\mathrm{e}^4-1)^2}\right.$$

$$+ \frac{2\left(\frac{(-\mathrm{e}^2+\mathrm{e}^6)\sin(2)^2}{4} + (5\,\mathrm{e}^2+9\,\mathrm{e}^4+\mathrm{e}^6)\sin(2) + \cos(2)\left(\mathrm{e}^2-\mathrm{e}^6-\frac{\mathrm{e}^8}{4}+\frac{1}{4}\right)\right)\mathrm{e}^2(x-2)^4}{3(\mathrm{e}^4-1)^3}$$

$$+ \frac{2\left(\left((\mathrm{e}^2-2\,\mathrm{e}^6+\mathrm{e}^{10})\sin(2)-8\,\mathrm{e}^2-\frac{41\,\mathrm{e}^4}{4}+6\,\mathrm{e}^6+\frac{41\,\mathrm{e}^8}{4}+2\,\mathrm{e}^{10}+\frac{\mathrm{e}^{12}}{4}-\frac{1}{4}\right)\cos(2)+\left((\mathrm{e}^2+4\,\mathrm{e}^4-4\,\mathrm{e}^8\right.\right.}{15(\mathrm{e}^4-1)^4}$$

$$+ \left(x-2-\frac{4\,\mathrm{e}^2(x-2)^2}{\mathrm{e}^4-1} + \frac{\mathrm{e}^2((\mathrm{e}^4-1)\sin(2)+32\,\mathrm{e}^2+8)(x-2)^3}{3(\mathrm{e}^4-1)^2}\right.$$

$$+ \frac{2\left(-\frac{7}{4}+\left(2\,\mathrm{e}^2-2\,\mathrm{e}^6-\frac{\mathrm{e}^8}{4}+\frac{1}{4}\right)\sin(2)+\frac{\left(\frac{1}{2}-\mathrm{e}^4+\frac{\mathrm{e}^8}{2}\right)\cos(2)}{2}-24\,\mathrm{e}^2-\frac{69\,\mathrm{e}^4}{2}+\frac{\mathrm{e}^8}{4}\right)\mathrm{e}^2(x-2)^4}{3(\mathrm{e}^4-1)^3}$$

$$+ \frac{2\left(\left(-\frac{\mathrm{e}^2}{4}-\frac{\mathrm{e}^{10}}{4}+\frac{\mathrm{e}^6}{2}\right)\cos(2)^2+\left(-\frac{3}{4}+\frac{3\,\mathrm{e}^4}{4}+\frac{3\,\mathrm{e}^8}{4}-\frac{3\,\mathrm{e}^{12}}{4}-5\,\mathrm{e}^{10}+10\,\mathrm{e}^6-5\,\mathrm{e}^2\right)\cos(2)+(-13\,\mathrm{e}^2-27\right.}{15(\mathrm{e}^4-1}$$

$$+ O(x^6)$$

✓ Solution by Mathematica

Time used: 0.004 (sec). Leaf size: 934

```
AsymptoticDSolveValue[Sinh[x]*y''[x]+x^2*y'[x]-Sin[x]*y[x]==0,y[x],{x,2,5}]
```

$$
\begin{aligned}
y(x) \\
\rightarrow c_2\Bigg(& -\frac{1}{20}\left(\cos(2)\coth(2)\text{csch}(2)+\text{csch}(2)\sin(2)-\coth^2(2)\text{csch}(2)\sin(2)\right)(x-2)^5 \\
& +\frac{1}{6}\text{csch}(2)(-\cos(2)\text{csch}(2)+\coth(2)\text{csch}(2)\sin(2))(x-2)^5 \\
& +\frac{1}{120}\text{csch}^2(2)\sin^2(2)(x-2)^5 \\
& +\frac{1}{40}\text{csch}(2)(-4\text{csch}(2)+4\coth(2)\text{csch}(2))\sin(2)(x-2)^5 \\
& -\frac{1}{120}\text{csch}(2)(4\text{csch}(2)-4\coth(2)\text{csch}(2))\sin(2)(x-2)^5+\frac{2}{5}\text{csch}^3(2)\sin(2)(x-2)^5 \\
& -\frac{1}{60}\left(-6\text{csch}(2)+7\coth(2)\text{csch}(2)+12\coth^2(2)\text{csch}(2)-12\coth^3(2)\text{csch}(2)\right)(x \\
& -2)^5+\frac{1}{15}\text{csch}(2)\left(-\text{csch}(2)-4\coth(2)\text{csch}(2)+4\coth^2(2)\text{csch}(2)\right)(x-2)^5 \\
& -\frac{1}{5}\text{csch}(2)\left(\text{csch}(2)+4\coth(2)\text{csch}(2)-4\coth^2(2)\text{csch}(2)\right)(x-2)^5 \\
& -\frac{1}{40}(4\text{csch}(2)-4\coth(2)\text{csch}(2))(-4\text{csch}(2)+4\coth(2)\text{csch}(2))(x-2)^5 \\
& +\frac{2}{3}\text{csch}^2(2)(-4\text{csch}(2)+4\coth(2)\text{csch}(2))(x-2)^5 \\
& -\frac{2}{15}\text{csch}^2(2)(4\text{csch}(2)-4\coth(2)\text{csch}(2))(x-2)^5+\frac{32}{15}\text{csch}^4(2)(x-2)^5 \\
& -\frac{1}{12}(-\cos(2)\text{csch}(2)+\coth(2)\text{csch}(2)\sin(2))(x-2)^4-\frac{1}{3}\text{csch}^2(2)\sin(2)(x-2)^4 \\
& -\frac{1}{12}\left(-\text{csch}(2)-4\coth(2)\text{csch}(2)+4\coth^2(2)\text{csch}(2)\right)(x-2)^4 \\
& -\frac{1}{3}\text{csch}(2)(-4\text{csch}(2)+4\coth(2)\text{csch}(2))(x-2)^4 \\
& +\frac{1}{6}\text{csch}(2)(4\text{csch}(2)-4\coth(2)\text{csch}(2))(x-2)^4-\frac{8}{3}\text{csch}^3(2)(x-2)^4 \\
& +\frac{1}{6}\text{csch}(2)\sin(2)(x-2)^3-\frac{1}{6}(4\text{csch}(2)-4\coth(2)\text{csch}(2))(x-2)^3 \\
& +\frac{8}{3}\text{csch}^2(2)(x-2)^3-2\text{csch}(2)(x-2)^2+x-2\Bigg) \\
+c_1\Bigg(& -\frac{1}{60}\left(2\cos(2)\text{csch}(2)-3\cos(2)\coth^2(2)\text{csch}(2)-4\coth(2)\text{csch}(2)\sin(2)\right. \\
& \left.+3\coth^3(2)\text{csch}(2)\sin(2)\right)(x-2)^5 \\
& +\frac{1}{15}\text{csch}(2)\left(\cos(2)\coth(2)\text{csch}(2)+\text{csch}(2)\sin(2)-\coth^2(2)\text{csch}(2)\sin(2)\right)(x-2)^5 \\
& -\frac{1}{30}\text{csch}(2)\sin(2)(-\cos(2)\text{csch}(2)+\coth(2)\text{csch}(2)\sin(2))(x-2)^5 \\
& -\frac{1}{40}(-4\text{csch}(2)+4\coth(2)\text{csch}(2))(-\cos(2)\text{csch}(2)+\coth(2)\text{csch}(2)\sin(2))(x-2)^5
\end{aligned}
$$

24.6 problem 34.5 (f)

Internal problem ID [13950]

Book: Ordinary Differential Equations. An introduction to the fundamentals. Kenneth B. Howell. second edition. CRC Press. FL, USA. 2020
Section: Chapter 34. Power series solutions II: Generalization and theory. Additional Exercises. page 678
Problem number: 34.5 (f).
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$\boxed{e^{3x}y'' + y' \sin(x) + \frac{2y}{x^2+4} = 0}$$

With the expansion point for the power series method at $x = 0$.

✓ Solution by Maple
Time used: 0.0 (sec). Leaf size: 49

```
Order:=6;
dsolve(exp(3*x)*diff(y(x),x$2)+sin(x)*diff(y(x),x)+2/(x^2+4)*y(x)=0,y(x),type='series',x=0);
```

$$y(x) = \left(1-\frac{1}{4}x^2+\frac{1}{4}x^3-\frac{1}{8}x^4-\frac{7}{160}x^5\right)y(0)+\left(x-\frac{1}{4}x^3+\frac{3}{8}x^4-\frac{67}{240}x^5\right)D(y)(0)+O(x^6)$$

✓ Solution by Mathematica
Time used: 0.002 (sec). Leaf size: 63

```
AsymptoticDSolveValue[Exp[3*x]*y''[x]+Sin[x]*y'[x]+2/(x^2+4)*y[x]==0,y[x],{x,0,5}]
```

$$y(x) \to c_2\left(-\frac{67x^5}{240}+\frac{3x^4}{8}-\frac{x^3}{4}+x\right)+c_1\left(-\frac{7x^5}{160}-\frac{x^4}{8}+\frac{x^3}{4}-\frac{x^2}{4}+1\right)$$

24.7 problem 34.5 (g)

Internal problem ID [13951]

Book: Ordinary Differential Equations. An introduction to the fundamentals. Kenneth B. Howell. second edition. CRC Press. FL, USA. 2020
Section: Chapter 34. Power series solutions II: Generalization and theory. Additional Exercises. page 678
Problem number: 34.5 (g).
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$y'' + \frac{(e^x + 1)\, y}{1 - e^x} = 0$$

With the expansion point for the power series method at $x = 3$.

✓ Solution by Maple
Time used: 0.0 (sec). Leaf size: 167

```
Order:=6;
dsolve(diff(y(x),x$2)+(1+exp(x))/(1-exp(x))*y(x)=0,y(x),type='series',x=3);
```

$$y(x) = \left(1 + \frac{(e^3+1)(-3+x)^2}{2\,e^3-2} - \frac{e^3(-3+x)^3}{3\,(e^3-1)^2} + \frac{(e^3+3\,e^6+e^9-1)(-3+x)^4}{24\,(e^3-1)^3} + \frac{(6\,e^3-8\,e^6-10\,e^9)(-3+x)^5}{120\,(e^3-1)^4}\right) y(3) + \left(-3+x+\frac{(e^6-1)(-3+x)^3}{6\,(e^3-1)^2} + \frac{(e^3-e^6)(-3+x)^4}{6\,(e^3-1)^3} + \frac{(-6\,e^3-2\,e^6+6\,e^9+e^{12}+1)(-3+x)^5}{120\,(e^3-1)^4}\right) D(y)(3) + O(x^6)$$

✓ Solution by Mathematica

Time used: 0.003 (sec). Leaf size: 275

```
AsymptoticDSolveValue[y''[x]+(1+Exp[x])/(1-Exp[x])*y[x]==0,y[x],{x,3,5}]
```

$$y(x) \to c_1\left(-\frac{(e^3+4e^6+e^9)(x-3)^5}{60(e^3-1)^4}-\frac{e^3(1+e^3)(x-3)^5}{60(e^3-1)^3}+\frac{e^3(-1-e^3)(x-3)^5}{20(e^3-1)^3}-\frac{(-e^3-e^6)(x-3)^4}{12(e^3-1)^3}+\frac{(-1-e^3)^2(x-3)^4}{24(e^3-1)^2}-\frac{e^3(x-3)^3}{3(e^3-1)^2}+\frac{(1+e^3)(x-3)^2}{2(e^3-1)}+1\right)+c_2\left(-\frac{(-e^3-e^6)(x-3)^5}{20(e^3-1)^3}+\frac{(1+e^3)^2(x-3)^5}{120(e^3-1)^2}-\frac{e^3(x-3)^4}{6(e^3-1)^2}+\frac{(1+e^3)(x-3)^3}{6(e^3-1)}+x-3\right)$$

24.8 problem 34.5 (h)

Internal problem ID [13952]

Book: Ordinary Differential Equations. An introduction to the fundamentals. Kenneth B. Howell. second edition. CRC Press. FL, USA. 2020
Section: Chapter 34. Power series solutions II: Generalization and theory. Additional Exercises. page 678
Problem number: 34.5 (h).
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$\boxed{(x^2-4)\,y''+(x^2+x-6)\,y=0}$$

With the expansion point for the power series method at $x=2$.

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 49

```
Order:=6;
dsolve((x^2-4)*diff(y(x),x$2)+(x^2+x-6)*y(x)=0,y(x),type='series',x=2);
```

$$y(x)=\left(1-\frac{5(x-2)^2}{8}+\frac{(x-2)^3}{96}+\frac{49(x-2)^4}{768}-\frac{37(x-2)^5}{15360}\right)y(2)+\left(x-2-\frac{5(x-2)^3}{24}+\frac{(x-2)^4}{192}+\frac{47(x-2)^5}{3840}\right)D(y)\,(2)+O(x^6)$$

✓ Solution by Mathematica

Time used: 0.001 (sec). Leaf size: 78

```
AsymptoticDSolveValue[(x^2-4)*y''[x]+(x^2+x-6)*y[x]==0,y[x],{x,2,5}]
```

$$y(x)\to c_1\left(-\frac{37(x-2)^5}{15360}+\frac{49}{768}(x-2)^4+\frac{1}{96}(x-2)^3-\frac{5}{8}(x-2)^2+1\right)+c_2\left(\frac{47(x-2)^5}{3840}+\frac{1}{192}(x-2)^4-\frac{5}{24}(x-2)^3+x-2\right)$$

24.9 problem 34.5 (i)

Internal problem ID [13953]

Book: Ordinary Differential Equations. An introduction to the fundamentals. Kenneth B. Howell. second edition. CRC Press. FL, USA. 2020
Section: Chapter 34. Power series solutions II: Generalization and theory. Additional Exercises. page 678
Problem number: 34.5 (i).
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$\boxed{xy'' + (1 - e^x)\, y = 0}$$

With the expansion point for the power series method at $x = 0$.

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 49

```
Order:=6;
dsolve(x*diff(y(x),x$2)+(1-exp(x))*y(x)=0,y(x),type='series',x=0);
```

$$y(x) = \left(1 + \frac{1}{2}x^2 + \frac{1}{12}x^3 + \frac{1}{18}x^4 + \frac{3}{160}x^5\right) y(0) + \left(x + \frac{1}{6}x^3 + \frac{1}{24}x^4 + \frac{1}{60}x^5\right) D(y)(0) + O(x^6)$$

✓ Solution by Mathematica

Time used: 0.001 (sec). Leaf size: 63

```
AsymptoticDSolveValue[x*y''[x]+(1-Exp[x])*y[x]==0,y[x],{x,0,5}]
```

$$y(x) \to c_2\left(\frac{x^5}{60} + \frac{x^4}{24} + \frac{x^3}{6} + x\right) + c_1\left(\frac{3x^5}{160} + \frac{x^4}{18} + \frac{x^3}{12} + \frac{x^2}{2} + 1\right)$$

24.10 problem 34.5 (j)

Internal problem ID [13954]

Book: Ordinary Differential Equations. An introduction to the fundamentals. Kenneth B. Howell. second edition. CRC Press. FL, USA. 2020
Section: Chapter 34. Power series solutions II: Generalization and theory. Additional Exercises. page 678
Problem number: 34.5 (j).
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$\boxed{\sin \left(\pi x^2\right) y'' + x^2 y = 0}$$

With the expansion point for the power series method at $x = 0$.

✓ Solution by Maple
Time used: 0.015 (sec). Leaf size: 46

```
Order:=6;
dsolve(sin(Pi*x^2)*diff(y(x),x$2)+x^2*y(x)=0,y(x),type='series',x=0);
```

$$y(x) = \left(1 - \frac{x^2}{2\pi} + \frac{x^4}{24\pi^2}\right) y(0) + \left(x - \frac{x^3}{6\pi} + \frac{x^5}{120\pi^2}\right) D(y)(0) + O(x^6)$$

✓ Solution by Mathematica
Time used: 0.002 (sec). Leaf size: 54

```
AsymptoticDSolveValue[Sin[Pi*x^2]*y''[x]+x^2*y[x]==0,y[x],{x,0,5}]
```

$$y(x) \to c_2\left(\frac{x^5}{120\pi^2} - \frac{x^3}{6\pi} + x\right) + c_1\left(\frac{x^4}{24\pi^2} - \frac{x^2}{2\pi} + 1\right)$$

24.11 problem 34.6 (a)

Internal problem ID [13955]

Book: Ordinary Differential Equations. An introduction to the fundamentals. Kenneth B. Howell. second edition. CRC Press. FL, USA. 2020
Section: Chapter 34. Power series solutions II: Generalization and theory. Additional Exercises. page 678
Problem number: 34.6 (a).
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_separable]`

$$\boxed{y' - y\,\mathrm{e}^{x} = 0}$$

With the expansion point for the power series method at $x = 0$.

✓ Solution by Maple
Time used: 0.0 (sec). Leaf size: 32

```
Order:=6;
dsolve(diff(y(x),x)-exp(x)*y(x)=0,y(x),type='series',x=0);
```

$$y(x) = \left(1 + x + x^2 + \frac{5}{6}x^3 + \frac{5}{8}x^4 + \frac{13}{30}x^5\right) y(0) + O(x^6)$$

✓ Solution by Mathematica
Time used: 0.001 (sec). Leaf size: 33

```
AsymptoticDSolveValue[y'[x]-Exp[x]*y[x]==0,y[x],{x,0,5}]
```

$$y(x) \to c_1\left(\frac{13x^5}{30} + \frac{5x^4}{8} + \frac{5x^3}{6} + x^2 + x + 1\right)$$

24.12 problem 34.6 (b)

Internal problem ID [13956]

Book: Ordinary Differential Equations. An introduction to the fundamentals. Kenneth B. Howell. second edition. CRC Press. FL, USA. 2020
Section: Chapter 34. Power series solutions II: Generalization and theory. Additional Exercises. page 678
Problem number: 34.6 (b).
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_separable]`

$$\boxed{y' + y\,e^{2x} = 0}$$

With the expansion point for the power series method at $x = 0$.

✓ Solution by Maple
Time used: 0.0 (sec). Leaf size: 36

```
Order:=6;
dsolve(diff(y(x),x)+exp(2*x)*y(x)=0,y(x),type='series',x=0);
```

$$y(x) = \left(1 - x - \frac{1}{2}x^2 + \frac{1}{6}x^3 + \frac{3}{8}x^4 + \frac{23}{120}x^5\right) y(0) + O(x^6)$$

✓ Solution by Mathematica
Time used: 0.001 (sec). Leaf size: 39

```
AsymptoticDSolveValue[y'[x]+Exp[2*x]*y[x]==0,y[x],{x,0,5}]
```

$$y(x) \to c_1\left(\frac{23x^5}{120} + \frac{3x^4}{8} + \frac{x^3}{6} - \frac{x^2}{2} - x + 1\right)$$

24.13 problem 34.6 (c)

Internal problem ID [13957]

Book: Ordinary Differential Equations. An introduction to the fundamentals. Kenneth B. Howell. second edition. CRC Press. FL, USA. 2020
Section: Chapter 34. Power series solutions II: Generalization and theory. Additional Exercises. page 678
Problem number: 34.6 (c).
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_separable]`

$$\boxed{y' + y\cos(x) = 0}$$

With the expansion point for the power series method at $x = 0$.

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 31

```
Order:=6;
dsolve(diff(y(x),x)+cos(x)*y(x)=0,y(x),type='series',x=0);
```

$$y(x) = \left(1 - x + \frac{1}{2}x^2 - \frac{1}{8}x^4 + \frac{1}{15}x^5\right)y(0) + O(x^6)$$

✓ Solution by Mathematica

Time used: 0.001 (sec). Leaf size: 32

```
AsymptoticDSolveValue[y'[x]+Cos[x]*y[x]==0,y[x],{x,0,5}]
```

$$y(x) \to c_1\left(\frac{x^5}{15} - \frac{x^4}{8} + \frac{x^2}{2} - x + 1\right)$$

24.14 problem 34.6 (d)

Internal problem ID [13958]

Book: Ordinary Differential Equations. An introduction to the fundamentals. Kenneth B. Howell. second edition. CRC Press. FL, USA. 2020
Section: Chapter 34. Power series solutions II: Generalization and theory. Additional Exercises. page 678
Problem number: 34.6 (d).
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_separable]`

$$\boxed{y' + y\ln(x) = 0}$$

With the expansion point for the power series method at $x = 1$.

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 31

```
Order:=6;
dsolve(diff(y(x),x)+ln(x)*y(x)=0,y(x),type='series',x=1);
```

$$y(x) = \left(1 - \frac{(-1+x)^2}{2} + \frac{(-1+x)^3}{6} + \frac{(-1+x)^4}{24} - \frac{(-1+x)^5}{30}\right) y(1) + O(x^6)$$

✓ Solution by Mathematica

Time used: 0.001 (sec). Leaf size: 44

```
AsymptoticDSolveValue[y'[x]+Log[x]*y[x]==0,y[x],{x,1,5}]
```

$$y(x) \to c_1\left(-\frac{1}{30}(x-1)^5 + \frac{1}{24}(x-1)^4 + \frac{1}{6}(x-1)^3 - \frac{1}{2}(x-1)^2 + 1\right)$$

24.15 problem 34.7 (a)

Internal problem ID [13959]

Book: Ordinary Differential Equations. An introduction to the fundamentals. Kenneth B. Howell. second edition. CRC Press. FL, USA. 2020
Section: Chapter 34. Power series solutions II: Generalization and theory. Additional Exercises. page 678
Problem number: 34.7 (a).
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$\boxed{y'' - y\,\mathrm{e}^{x} = 0}$$

With the expansion point for the power series method at $x = 1$.

✓ Solution by Maple
Time used: 0.016 (sec). Leaf size: 87

```
Order:=6;
dsolve(diff(y(x),x$2)-exp(x)*y(x)=0,y(x),type='series',x=1);
```

$$y(x) = \left(1 + \frac{\mathrm{e}(-1+x)^2}{2} + \frac{\mathrm{e}(-1+x)^3}{6} + \left(\frac{\mathrm{e}^2}{24} + \frac{\mathrm{e}}{24}\right)(-1+x)^4 + \left(\frac{\mathrm{e}^2}{30} + \frac{\mathrm{e}}{120}\right)(-1+x)^5\right) y(1) + \left(-1+x+\frac{\mathrm{e}(-1+x)^3}{6} + \frac{\mathrm{e}(-1+x)^4}{12} + \frac{(3\,\mathrm{e}+\mathrm{e}^2)(-1+x)^5}{120}\right) D(y)(1) + O\left(x^6\right)$$

✓ Solution by Mathematica

Time used: 0.001 (sec). Leaf size: 121

```
AsymptoticDSolveValue[y''[x]-Exp[x]*y[x]==0,y[x],{x,1,5}]
```

$$y(x) \to c_1 \left(\frac{1}{30}e^2(x-1)^5 + \frac{1}{120}e(x-1)^5 + \frac{1}{24}e^2(x-1)^4 + \frac{1}{24}e(x-1)^4 + \frac{1}{6}e(x-1)^3 + \frac{1}{2}e(x-1)^2 + 1\right) + c_2 \left(\frac{1}{120}e^2(x-1)^5 + \frac{1}{40}e(x-1)^5 + \frac{1}{12}e(x-1)^4 + \frac{1}{6}e(x-1)^3 + x - 1\right)$$

24.16 problem 34.7 (b)

Internal problem ID [13960]

Book: Ordinary Differential Equations. An introduction to the fundamentals. Kenneth B. Howell. second edition. CRC Press. FL, USA. 2020
Section: Chapter 34. Power series solutions II: Generalization and theory. Additional Exercises. page 678
Problem number: 34.7 (b).
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$\boxed{y'' + 3y'x - y\,\mathrm{e}^{x} = 0}$$

With the expansion point for the power series method at $x = 0$.

✓ Solution by Maple
Time used: 0.0 (sec). Leaf size: 49

```
Order:=6;
dsolve(diff(y(x),x$2)+3*x*diff(y(x),x)-exp(x)*y(x)=0,y(x),type='series',x=0);
```

$$y(x) = \left(1+\frac{1}{2}x^2+\frac{1}{6}x^3-\frac{1}{6}x^4-\frac{1}{30}x^5\right)y(0)+\left(x-\frac{1}{3}x^3+\frac{1}{12}x^4+\frac{19}{120}x^5\right)D(y)(0)+O(x^6)$$

✓ Solution by Mathematica
Time used: 0.001 (sec). Leaf size: 63

```
AsymptoticDSolveValue[y''[x]+3*x*y'[x]-Exp[x]*y[x]==0,y[x],{x,0,5}]
```

$$y(x) \to c_2\left(\frac{19x^5}{120}+\frac{x^4}{12}-\frac{x^3}{3}+x\right)+c_1\left(-\frac{x^5}{30}-\frac{x^4}{6}+\frac{x^3}{6}+\frac{x^2}{2}+1\right)$$

24.17 problem 34.7 (c)

Internal problem ID [13961]

Book: Ordinary Differential Equations. An introduction to the fundamentals. Kenneth B. Howell. second edition. CRC Press. FL, USA. 2020
Section: Chapter 34. Power series solutions II: Generalization and theory. Additional Exercises. page 678
Problem number: 34.7 (c).
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$\boxed{xy'' - 3y'x + \sin(x)\, y = 0}$$

With the expansion point for the power series method at $x = 0$.

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 34

```
Order:=4;
dsolve(x*diff(y(x),x$2)-3*x*diff(y(x),x)+sin(x)*y(x)=0,y(x),type='series',x=0);
```

$$y(x) = \left(1 - \frac{1}{2}x^2 - \frac{1}{2}x^3\right) y(0) + \left(x + \frac{3}{2}x^2 + \frac{4}{3}x^3\right) D(y)(0) + O(x^4)$$

✓ Solution by Mathematica

Time used: 0.001 (sec). Leaf size: 42

```
AsymptoticDSolveValue[x*y''[x]-3*x*y'[x]+Sin[x]*y[x]==0,y[x],{x,0,3}]
```

$$y(x) \to c_1\left(-\frac{x^3}{2} - \frac{x^2}{2} + 1\right) + c_2\left(\frac{4x^3}{3} + \frac{3x^2}{2} + x\right)$$

24.18 problem 34.7 (d)

Internal problem ID [13962]

Book: Ordinary Differential Equations. An introduction to the fundamentals. Kenneth B. Howell. second edition. CRC Press. FL, USA. 2020
Section: Chapter 34. Power series solutions II: Generalization and theory. Additional Exercises. page 678
Problem number: 34.7 (d).
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[_Titchmarsh]`

$$\boxed{y'' + y\ln(x) = 0}$$

With the expansion point for the power series method at $x = 1$.

✓ Solution by Maple
Time used: 0.0 (sec). Leaf size: 39

```
Order:=6;
dsolve(diff(y(x),x$2)+ln(x)*y(x)=0,y(x),type='series',x=1);
```

$$y(x) = \left(1 - \frac{(-1+x)^3}{6} + \frac{(-1+x)^4}{24} - \frac{(-1+x)^5}{60}\right) y(1) + \left(-1 + x - \frac{(-1+x)^4}{12} + \frac{(-1+x)^5}{40}\right) D(y)(1) + O(x^6)$$

✓ Solution by Mathematica
Time used: 0.001 (sec). Leaf size: 60

```
AsymptoticDSolveValue[y''[x]+Log[x]*y[x]==0,y[x],{x,1,5}]
```

$$y(x) \to c_1\left(-\frac{1}{60}(x-1)^5 + \frac{1}{24}(x-1)^4 - \frac{1}{6}(x-1)^3 + 1\right) + c_2\left(\frac{1}{40}(x-1)^5 - \frac{1}{12}(x-1)^4 + x - 1\right)$$

24.19 problem 34.7 (e)

Internal problem ID [13963]

Book: Ordinary Differential Equations. An introduction to the fundamentals. Kenneth B. Howell. second edition. CRC Press. FL, USA. 2020
Section: Chapter 34. Power series solutions II: Generalization and theory. Additional Exercises. page 678
Problem number: 34.7 (e).
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_Emden, _Fowler]]`

$$\boxed{\sqrt{x}\, y'' + y = 0}$$

With the expansion point for the power series method at $x = 1$.

✓ Solution by Maple
Time used: 0.0 (sec). Leaf size: 49

```
Order:=6;
dsolve(sqrt(x)*diff(y(x),x$2)+y(x)=0,y(x),type='series',x=1);
```

$$y(x) = \left(1 - \frac{(-1+x)^2}{2} + \frac{(-1+x)^3}{12} + \frac{(-1+x)^4}{96} - \frac{(-1+x)^5}{960}\right) y(1) + \left(-1 + x - \frac{(-1+x)^3}{6} + \frac{(-1+x)^4}{24} - \frac{(-1+x)^5}{96}\right) D(y)(1) + O(x^6)$$

✓ Solution by Mathematica
Time used: 0.001 (sec). Leaf size: 78

```
AsymptoticDSolveValue[Sqrt[x]*y''[x]+y[x]==0,y[x],{x,1,5}]
```

$$y(x) \to c_1\left(-\frac{1}{960}(x-1)^5 + \frac{1}{96}(x-1)^4 + \frac{1}{12}(x-1)^3 - \frac{1}{2}(x-1)^2 + 1\right) + c_2\left(-\frac{1}{96}(x-1)^5 + \frac{1}{24}(x-1)^4 - \frac{1}{6}(x-1)^3 + x - 1\right)$$

24.20 problem 34.7 (f)

Internal problem ID [13964]

Book: Ordinary Differential Equations. An introduction to the fundamentals. Kenneth B. Howell. second edition. CRC Press. FL, USA. 2020
Section: Chapter 34. Power series solutions II: Generalization and theory. Additional Exercises. page 678
Problem number: 34.7 (f).
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _exact, _linear, _homogeneous]]`

$$\boxed{y'' + (6x^2 + 2x + 1)\, y' + (2 + 12x)\, y = 0}$$

With the expansion point for the power series method at $x = 0$.

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 54

```
Order:=6;
dsolve(diff(y(x),x$2)+(1+2*x+6*x^2)*diff(y(x),x)+(2+12*x)*y(x)=0,y(x),type='series',x=0);
```

$$y(x) = \left(1 - x^2 - \frac{5}{3}x^3 + \frac{11}{12}x^4 + \frac{101}{60}x^5\right) y(0) + \left(x - \frac{1}{2}x^2 - \frac{1}{2}x^3 - \frac{9}{8}x^4 + \frac{41}{40}x^5\right) D(y)(0) + O(x^6)$$

✓ Solution by Mathematica

Time used: 0.001 (sec). Leaf size: 68

```
AsymptoticDSolveValue[y''[x]+(1+2*x+6*x^2)*y'[x]+(2+12*x)*y[x]==0,y[x],{x,0,5}]
```

$$y(x) \to c_1\left(\frac{101x^5}{60} + \frac{11x^4}{12} - \frac{5x^3}{3} - x^2 + 1\right) + c_2\left(\frac{41x^5}{40} - \frac{9x^4}{8} - \frac{x^3}{2} - \frac{x^2}{2} + x\right)$$

24.21 problem 34.8 b(i)

Internal problem ID [13965]

Book: Ordinary Differential Equations. An introduction to the fundamentals. Kenneth B. Howell. second edition. CRC Press. FL, USA. 2020
Section: Chapter 34. Power series solutions II: Generalization and theory. Additional Exercises. page 678
Problem number: 34.8 b(i).
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_separable]`

$$\boxed{y' - y\,\mathrm{e}^{x} = 0}$$

With the expansion point for the power series method at $x = 0$.

✓ Solution by Maple
Time used: 0.0 (sec). Leaf size: 52

```
Order:=10;
dsolve(diff(y(x),x)-exp(x)*y(x)=0,y(x),type='series',x=0);
```

$$y(x) = \left(1 + x + x^2 + \frac{5}{6}x^3 + \frac{5}{8}x^4 + \frac{13}{30}x^5 + \frac{203}{720}x^6 + \frac{877}{5040}x^7 + \frac{23}{224}x^8 + \frac{1007}{17280}x^9\right) y(0) + O\left(x^{10}\right)$$

✓ Solution by Mathematica
Time used: 0.001 (sec). Leaf size: 61

```
AsymptoticDSolveValue[y'[x]-Exp[x]*y[x]==0,y[x],{x,0,9}]
```

$$y(x) \to c_1\left(\frac{1007x^9}{17280} + \frac{23x^8}{224} + \frac{877x^7}{5040} + \frac{203x^6}{720} + \frac{13x^5}{30} + \frac{5x^4}{8} + \frac{5x^3}{6} + x^2 + x + 1\right)$$

24.22 problem 34.8 b(ii)

Internal problem ID [13966]

Book: Ordinary Differential Equations. An introduction to the fundamentals. Kenneth B. Howell. second edition. CRC Press. FL, USA. 2020
Section: Chapter 34. Power series solutions II: Generalization and theory. Additional Exercises. page 678
Problem number: 34.8 b(ii).
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_separable]`

$$\boxed{y' + \sqrt{x^2+1}\, y = 0}$$

With the expansion point for the power series method at $x = 0$.

✓ Solution by Maple
Time used: 0.0 (sec). Leaf size: 46

```
Order:=8;
dsolve(diff(y(x),x)+sqrt(1+x^2)*y(x)=0,y(x),type='series',x=0);
```

$$y(x) = \left(1 - x + \frac{1}{2}x^2 - \frac{1}{3}x^3 + \frac{5}{24}x^4 - \frac{1}{15}x^5 + \frac{13}{720}x^6 - \frac{11}{630}x^7\right) y(0) + O(x^8)$$

✓ Solution by Mathematica
Time used: 0.001 (sec). Leaf size: 53

```
AsymptoticDSolveValue[y'[x]+Sqrt[1+x^2]*y[x]==0,y[x],{x,0,7}]
```

$$y(x) \to c_1\left(-\frac{11x^7}{630} + \frac{13x^6}{720} - \frac{x^5}{15} + \frac{5x^4}{24} - \frac{x^3}{3} + \frac{x^2}{2} - x + 1\right)$$

24.23 problem 34.8 b(iii)

Internal problem ID [13967]

Book: Ordinary Differential Equations. An introduction to the fundamentals. Kenneth B. Howell. second edition. CRC Press. FL, USA. 2020
Section: Chapter 34. Power series solutions II: Generalization and theory. Additional Exercises. page 678
Problem number: 34.8 b(iii).
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_separable]`

$$\boxed{\cos(x)\, y' + y = 0}$$

With the expansion point for the power series method at $x = 0$.

✓ Solution by Maple
Time used: 0.0 (sec). Leaf size: 46

```
Order:=8;
dsolve(cos(x)*diff(y(x),x)+y(x)=0,y(x),type='series',x=0);
```

$$y(x) = \left(1 - x + \frac{1}{2}x^2 - \frac{1}{3}x^3 + \frac{5}{24}x^4 - \frac{2}{15}x^5 + \frac{61}{720}x^6 - \frac{17}{315}x^7\right) y(0) + O(x^8)$$

✓ Solution by Mathematica
Time used: 0.001 (sec). Leaf size: 53

```
AsymptoticDSolveValue[Cos[x]*y'[x]+y[x]==0,y[x],{x,0,7}]
```

$$y(x) \to c_1\left(-\frac{17x^7}{315} + \frac{61x^6}{720} - \frac{2x^5}{15} + \frac{5x^4}{24} - \frac{x^3}{3} + \frac{x^2}{2} - x + 1\right)$$

24.24 problem 34.8 b(iv)

Internal problem ID [13968]

Book: Ordinary Differential Equations. An introduction to the fundamentals. Kenneth B. Howell. second edition. CRC Press. FL, USA. 2020
Section: Chapter 34. Power series solutions II: Generalization and theory. Additional Exercises. page 678
Problem number: 34.8 b(iv).
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_separable]`

$$\boxed{y' + \sqrt{2x^2 + 1}\, y = 0}$$

With the expansion point for the power series method at $x = 0$.

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 46

```
Order:=8;
dsolve(diff(y(x),x)+sqrt(1+2*x^2)*y(x)=0,y(x),type='series',x=0);
```

$$y(x) = \left(1 - x + \frac{1}{2}x^2 - \frac{1}{2}x^3 + \frac{3}{8}x^4 - \frac{3}{40}x^5 + \frac{1}{80}x^6 - \frac{51}{560}x^7\right) y(0) + O(x^8)$$

✓ Solution by Mathematica

Time used: 0.001 (sec). Leaf size: 53

```
AsymptoticDSolveValue[y'[x]+Sqrt[1+2*x^2]*y[x]==0,y[x],{x,0,7}]
```

$$y(x) \to c_1 \left(-\frac{51x^7}{560} + \frac{x^6}{80} - \frac{3x^5}{40} + \frac{3x^4}{8} - \frac{x^3}{2} + \frac{x^2}{2} - x + 1\right)$$

24.25 problem 34.9 b(i)

Internal problem ID [13969]

Book: Ordinary Differential Equations. An introduction to the fundamentals. Kenneth B. Howell. second edition. CRC Press. FL, USA. 2020
Section: Chapter 34. Power series solutions II: Generalization and theory. Additional Exercises. page 678
Problem number: 34.9 b(i).
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$\boxed{y'' - y\,\mathrm{e}^{x} = 0}$$

With the expansion point for the power series method at $x = 0$.

✓ Solution by Maple
Time used: 0.0 (sec). Leaf size: 69

```
Order:=8;
dsolve(diff(y(x),x$2)-exp(x)*y(x)=0,y(x),type='series',x=0);
```

$$y(x) = \left(1+\frac{1}{2}x^2+\frac{1}{6}x^3+\frac{1}{12}x^4+\frac{1}{24}x^5+\frac{13}{720}x^6+\frac{1}{140}x^7\right)y(0) + \left(x+\frac{1}{6}x^3+\frac{1}{12}x^4+\frac{1}{30}x^5+\frac{1}{72}x^6+\frac{29}{5040}x^7\right)D(y)(0)+O(x^8)$$

✓ Solution by Mathematica
Time used: 0.001 (sec). Leaf size: 91

```
AsymptoticDSolveValue[y''[x]-Exp[x]*y[x]==0,y[x],{x,0,7}]
```

$$y(x) \to c_2\left(\frac{29x^7}{5040}+\frac{x^6}{72}+\frac{x^5}{30}+\frac{x^4}{12}+\frac{x^3}{6}+x\right)+c_1\left(\frac{x^7}{140}+\frac{13x^6}{720}+\frac{x^5}{24}+\frac{x^4}{12}+\frac{x^3}{6}+\frac{x^2}{2}+1\right)$$

24.26 problem 34.9 b(ii)

Internal problem ID [13970]

Book: Ordinary Differential Equations. An introduction to the fundamentals. Kenneth B. Howell. second edition. CRC Press. FL, USA. 2020
Section: Chapter 34. Power series solutions II: Generalization and theory. Additional Exercises. page 678
Problem number: 34.9 b(ii).
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$\boxed{y'' + y\cos(x) = 0}$$

With the expansion point for the power series method at $x = 0$.

✓ Solution by Maple
Time used: 0.0 (sec). Leaf size: 54

```
Order:=10;
dsolve(diff(y(x),x$2)+cos(x)*y(x)=0,y(x),type='series',x=0);
```

$$y(x) = \left(1 - \frac{1}{2}x^2 + \frac{1}{12}x^4 - \frac{1}{80}x^6 + \frac{11}{8064}x^8\right) y(0) + \left(x - \frac{1}{6}x^3 + \frac{1}{30}x^5 - \frac{19}{5040}x^7 + \frac{29}{72576}x^9\right) D(y)(0) + O(x^{10})$$

✓ Solution by Mathematica
Time used: 0.001 (sec). Leaf size: 70

```
AsymptoticDSolveValue[y''[x]+Cos[x]*y[x]==0,y[x],{x,0,9}]
```

$$y(x) \to c_2\left(\frac{29x^9}{72576} - \frac{19x^7}{5040} + \frac{x^5}{30} - \frac{x^3}{6} + x\right) + c_1\left(\frac{11x^8}{8064} - \frac{x^6}{80} + \frac{x^4}{12} - \frac{x^2}{2} + 1\right)$$

24.27 problem 34.9 b(iii)

Internal problem ID [13971]

Book: Ordinary Differential Equations. An introduction to the fundamentals. Kenneth B. Howell. second edition. CRC Press. FL, USA. 2020
Section: Chapter 34. Power series solutions II: Generalization and theory. Additional Exercises. page 678
Problem number: 34.9 b(iii).
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _exact, _linear, _homogeneous]]`

$$\boxed{y'' + y' \sin(x) + y \cos(x) = 0}$$

With the expansion point for the power series method at $x = 0$.

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 39

```
Order:=7;
dsolve(diff(y(x),x$2)+sin(x)*diff(y(x),x)+cos(x)*y(x)=0,y(x),type='series',x=0);
```

$$y(x) = \left(1 - \frac{1}{2}x^2 + \frac{1}{6}x^4 - \frac{31}{720}x^6\right) y(0) + \left(x - \frac{1}{3}x^3 + \frac{1}{10}x^5\right) D(y)(0) + O(x^7)$$

✓ Solution by Mathematica

Time used: 0.001 (sec). Leaf size: 49

```
AsymptoticDSolveValue[y''[x]+Sin[x]*y'[x]+Cos[x]*y[x]==0,y[x],{x,0,6}]
```

$$y(x) \to c_2\left(\frac{x^5}{10} - \frac{x^3}{3} + x\right) + c_1\left(-\frac{31x^6}{720} + \frac{x^4}{6} - \frac{x^2}{2} + 1\right)$$

24.28 problem 34.9 b(iv)

Internal problem ID [13972]

Book: Ordinary Differential Equations. An introduction to the fundamentals. Kenneth B. Howell. second edition. CRC Press. FL, USA. 2020
Section: Chapter 34. Power series solutions II: Generalization and theory. Additional Exercises. page 678
Problem number: 34.9 b(iv).
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$\boxed{\sqrt{x}\, y'' + y' + yx = 0}$$

With the expansion point for the power series method at $x = 1$.

✓ Solution by Maple

Time used: 0.015 (sec). Leaf size: 44

```
Order:=5;
dsolve(sqrt(x)*diff(y(x),x$2)+diff(y(x),x)+x*y(x)=0,y(x),type='series',x=1);
```

$$y(x) = \left(1 - \frac{(-1+x)^2}{2} + \frac{(-1+x)^3}{12} - \frac{(-1+x)^4}{96}\right) y(1) + \left(-1 + x - \frac{(-1+x)^2}{2} + \frac{(-1+x)^3}{12} - \frac{3(-1+x)^4}{32}\right) D(y)(1) + O(x^5)$$

✓ Solution by Mathematica

Time used: 0.002 (sec). Leaf size: 69

```
AsymptoticDSolveValue[Sqrt[x]*y''[x]+y'[x]+x*y[x]==0,y[x],{x,1,4}]
```

$$y(x) \to c_1\left(-\frac{1}{96}(x-1)^4 + \frac{1}{12}(x-1)^3 - \frac{1}{2}(x-1)^2 + 1\right) + c_2\left(-\frac{3}{32}(x-1)^4 + \frac{1}{12}(x-1)^3 - \frac{1}{2}(x-1)^2 + x - 1\right)$$

25 Chapter 35. Modified Power series solutions and basic method of Frobenius. Additional Exercises. page 715

25.1 problem 35.2 (a)

Internal problem ID [13973]

Book: Ordinary Differential Equations. An introduction to the fundamentals. Kenneth B. Howell. second edition. CRC Press. FL, USA. 2020
Section: Chapter 35. Modified Power series solutions and basic method of Frobenius. Additional Exercises. page 715
Problem number: 35.2 (a).
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries], [_2nd_order, _linear,`

$$\boxed{(x-3)^2 y'' - 2(x-3) y' + 2y = 0}$$

With the expansion point for the power series method at $x = 0$.

✓ Solution by Maple
Time used: 0.016 (sec). Leaf size: 24

```
Order:=6;
dsolve((x-3)^2*diff(y(x),x$2)-2*(x-3)*diff(y(x),x)+2*y(x)=0,y(x),type='series',x=0);
```

$$y(x) = \frac{(-x^2+9)\, y(0)}{9} - \frac{x D(y)(0)(-3+x)}{3}$$

✓ Solution by Mathematica
Time used: 0.001 (sec). Leaf size: 28

```
AsymptoticDSolveValue[(x-3)^2*y''[x]-2*(x-3)*y'[x]+2*y[x]==0,y[x],{x,0,5}]
```

$$y(x) \to c_1\left(1-\frac{x^2}{9}\right) + c_2\left(x-\frac{x^2}{3}\right)$$

25.2 problem 35.2 (b)

Internal problem ID [13974]

Book: Ordinary Differential Equations. An introduction to the fundamentals. Kenneth B. Howell. second edition. CRC Press. FL, USA. 2020
Section: Chapter 35. Modified Power series solutions and basic method of Frobenius. Additional Exercises. page 715
Problem number: 35.2 (b).
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _exact, _linear, _homogeneous]]`

$$\boxed{2x^2y'' + 5y'x + y = 0}$$

With the expansion point for the power series method at $x = 0$.

✓ Solution by Maple
Time used: 0.031 (sec). Leaf size: 27

```
Order:=6;
dsolve(2*x^2*diff(y(x),x$2)+5*x*diff(y(x),x)+y(x)=0,y(x),type='series',x=0);
```

$$y(x) = \frac{c_1\sqrt{x} + c_2x}{x^{\frac{3}{2}}} + O(x^6)$$

✓ Solution by Mathematica
Time used: 0.002 (sec). Leaf size: 18

```
AsymptoticDSolveValue[2*x^2*y''[x]+5*x*y'[x]+y[x]==0,y[x],{x,0,5}]
```

$$y(x) \to \frac{c_1}{\sqrt{x}} + \frac{c_2}{x}$$

25.3 problem 35.2 (c)

Internal problem ID [13975]

Book: Ordinary Differential Equations. An introduction to the fundamentals. Kenneth B. Howell. second edition. CRC Press. FL, USA. 2020
Section: Chapter 35. Modified Power series solutions and basic method of Frobenius. Additional Exercises. page 715
Problem number: 35.2 (c).
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$(x-1)^2 y'' - 5y'(x-1) + 9y = 0$$

With the expansion point for the power series method at $x = 0$.

✓ Solution by Maple
Time used: 0.016 (sec). Leaf size: 54

```
Order:=6;
dsolve((x-1)^2*diff(y(x),x$2)-5*(x-1)*diff(y(x),x)+9*y(x)=0,y(x),type='series',x=0);
```

$$y(x) = \left(1 - \frac{9}{2}x^2 + \frac{9}{2}x^3 - \frac{3}{4}x^4 - \frac{3}{20}x^5\right) y(0) + \left(x - \frac{5}{2}x^2 + \frac{11}{6}x^3 - \frac{1}{4}x^4 - \frac{1}{20}x^5\right) D(y)(0) + O(x^6)$$

✓ Solution by Mathematica
Time used: 0.002 (sec). Leaf size: 70

```
AsymptoticDSolveValue[(x-1)^2*y''[x]-5*(x-1)*y'[x]+9*y[x]==0,y[x],{x,0,5}]
```

$$y(x) \to c_1\left(-\frac{3x^5}{20} - \frac{3x^4}{4} + \frac{9x^3}{2} - \frac{9x^2}{2} + 1\right) + c_2\left(-\frac{x^5}{20} - \frac{x^4}{4} + \frac{11x^3}{6} - \frac{5x^2}{2} + x\right)$$

25.4 problem 35.2 (d)

Internal problem ID [13976]

Book: Ordinary Differential Equations. An introduction to the fundamentals. Kenneth B. Howell. second edition. CRC Press. FL, USA. 2020
Section: Chapter 35. Modified Power series solutions and basic method of Frobenius. Additional Exercises. page 715
Problem number: 35.2 (d).
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _missing_y]]`

$$\boxed{(x+2)^2 y'' + (x+2)\, y' = 0}$$

With the expansion point for the power series method at $x = 0$.

✓ Solution by Maple
Time used: 0.0 (sec). Leaf size: 34

```
Order:=6;
dsolve((x+2)^2*diff(y(x),x$2)+(x+2)*diff(y(x),x)=0,y(x),type='series',x=0);
```

$$y(x) = y(0) + \left(x - \frac{1}{4}x^2 + \frac{1}{12}x^3 - \frac{1}{32}x^4 + \frac{1}{80}x^5\right) D(y)(0) + O(x^6)$$

✓ Solution by Mathematica
Time used: 0.001 (sec). Leaf size: 39

```
AsymptoticDSolveValue[(x+2)^2*y''[x]+(x+2)*y'[x]==0,y[x],{x,0,5}]
```

$$y(x) \to c_2\left(\frac{x^5}{80} - \frac{x^4}{32} + \frac{x^3}{12} - \frac{x^2}{4} + x\right) + c_1$$

25.5 problem 35.2 (e)

Internal problem ID [13977]

Book: Ordinary Differential Equations. An introduction to the fundamentals. Kenneth B. Howell. second edition. CRC Press. FL, USA. 2020
Section: Chapter 35. Modified Power series solutions and basic method of Frobenius. Additional Exercises. page 715
Problem number: 35.2 (e).
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$3(x-2)^2 y'' - 4(-5+x) y' + 2y = 0$$

With the expansion point for the power series method at $x = 0$.

✓ Solution by Maple
Time used: 0.0 (sec). Leaf size: 54

```
Order:=6;
dsolve(3*(x-2)^2*diff(y(x),x$2)-4*(x-5)*diff(y(x),x)+2*y(x)=0,y(x),type='series',x=0);
```

$$y(x) = \left(1 - \frac{1}{12}x^2 + \frac{1}{54}x^3 + \frac{1}{648}x^4 - \frac{1}{4860}x^5\right) y(0) + \left(x - \frac{5}{6}x^2 + \frac{23}{108}x^3 + \frac{23}{1296}x^4 - \frac{23}{9720}x^5\right) D(y)(0) + O(x^6)$$

✓ Solution by Mathematica
Time used: 0.001 (sec). Leaf size: 70

```
AsymptoticDSolveValue[3*(x-2)^2*y''[x]-4*(x-5)*y'[x]+2*y[x]==0,y[x],{x,0,5}]
```

$$y(x) \to c_1\left(-\frac{x^5}{4860} + \frac{x^4}{648} + \frac{x^3}{54} - \frac{x^2}{12} + 1\right) + c_2\left(-\frac{23x^5}{9720} + \frac{23x^4}{1296} + \frac{23x^3}{108} - \frac{5x^2}{6} + x\right)$$

25.6 problem 35.2 (f)

Internal problem ID [13978]

Book: Ordinary Differential Equations. An introduction to the fundamentals. Kenneth B. Howell. second edition. CRC Press. FL, USA. 2020
Section: Chapter 35. Modified Power series solutions and basic method of Frobenius. Additional Exercises. page 715
Problem number: 35.2 (f).
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries], [_2nd_order, _linear,`

$$\boxed{(-5+x)^2 y'' + (-5+x) y' + 4y = 0}$$

With the expansion point for the power series method at $x = 0$.

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 54

```
Order:=6;
dsolve((x-5)^2*diff(y(x),x$2)+(x-5)*diff(y(x),x)+4*y(x)=0,y(x),type='series',x=0);
```

$$y(x) = \left(1 - \frac{2}{25}x^2 - \frac{2}{125}x^3 - \frac{7}{3750}x^4 - \frac{1}{9375}x^5\right) y(0) + \left(x + \frac{1}{10}x^2 - \frac{1}{75}x^3 - \frac{3}{500}x^4 - \frac{1}{750}x^5\right) D(y)(0) + O(x^6)$$

✓ Solution by Mathematica

Time used: 0.001 (sec). Leaf size: 70

```
AsymptoticDSolveValue[(x-5)^2*y''[x]+(x-5)*y'[x]+4*y[x]==0,y[x],{x,0,5}]
```

$$y(x) \to c_1\left(-\frac{x^5}{9375} - \frac{7x^4}{3750} - \frac{2x^3}{125} - \frac{2x^2}{25} + 1\right) + c_2\left(-\frac{x^5}{750} - \frac{3x^4}{500} - \frac{x^3}{75} + \frac{x^2}{10} + x\right)$$

25.7 problem 35.3 (a)

Internal problem ID [13979]

Book: Ordinary Differential Equations. An introduction to the fundamentals. Kenneth B. Howell. second edition. CRC Press. FL, USA. 2020
Section: Chapter 35. Modified Power series solutions and basic method of Frobenius. Additional Exercises. page 715
Problem number: 35.3 (a).
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$\boxed{x^2y'' + \frac{xy'}{x-2} + \frac{2y}{x+2} = 0}$$

With the expansion point for the power series method at $x = 0$.

✓ Solution by Maple

Time used: 0.031 (sec). Leaf size: 1283

```
Order:=6;
dsolve(x^2*diff(y(x),x$2)+x/(x-2)*diff(y(x),x)+2/(x+2)*y(x)=0,y(x),type='series',x=0);
```

$$y(x) = x^{\frac{3}{4}}\left(c_2 x^{\frac{i\sqrt{7}}{4}}\left(1+\frac{i\sqrt{7}+11}{8i\sqrt{7}+16}x+\frac{1}{64}\frac{7i\sqrt{7}+45}{(2+i\sqrt{7})(i\sqrt{7}+4)}x^2 + \frac{1}{256}\frac{223i\sqrt{7}-43}{(2+i\sqrt{7})(i\sqrt{7}+4)(i\sqrt{7}+6)}x^3 + \frac{1}{4096}\frac{7577i\sqrt{7}+979}{(2+i\sqrt{7})(i\sqrt{7}+4)(i\sqrt{7}+6)(i\sqrt{7}+8)}x^4 + \frac{1}{81920}\frac{553875\sqrt{7}+1249007i}{(-2i+\sqrt{7})(i\sqrt{7}+4)(i\sqrt{7}+6)(i\sqrt{7}+8)(i\sqrt{7}+10)}x^5+\mathrm{O}(x^6)\right) + c_1 x^{-\frac{i\sqrt{7}}{4}}\left(1+\frac{\sqrt{7}+11i}{8\sqrt{7}+16i}x+\frac{7i\sqrt{7}-45}{-64+384i\sqrt{7}}x^2+\frac{-223\sqrt{7}+43i}{9216i-9472\sqrt{7}}x^3 + \frac{7577\sqrt{7}+979i}{-2240512i+1064960\sqrt{7}}x^4 + \frac{1}{81920}\frac{553875\sqrt{7}-1249007i}{(4i+\sqrt{7})(\sqrt{7}+2i)(\sqrt{7}+6i)(\sqrt{7}+8i)(\sqrt{7}+10i)}x^5+\mathrm{O}(x^6)\right)\right)$$

✓ Solution by Mathematica

Time used: 0.006 (sec). Leaf size: 6290

```
AsymptoticDSolveValue[x^2*y''[x]+x/(x-2)*y'[x]+2/(x+2)*y[x]==0,y[x],{x,0,5}]
```

Too large to display

25.8 problem 35.3 (b)

Internal problem ID [13980]

Book: Ordinary Differential Equations. An introduction to the fundamentals. Kenneth B. Howell. second edition. CRC Press. FL, USA. 2020
Section: Chapter 35. Modified Power series solutions and basic method of Frobenius. Additional Exercises. page 715
Problem number: 35.3 (b).
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_Emden, _Fowler]]`

$$x^3y'' + x^2y' + y = 0$$

With the expansion point for the power series method at $x = 2$.

✓ Solution by Maple
Time used: 0.0 (sec). Leaf size: 54

```
Order:=6;
dsolve(x^3*diff(y(x),x$2)+x^2*diff(y(x),x)+y(x)=0,y(x),type='series',x=2);
```

$$y(x) = \left(1 - \frac{(x-2)^2}{16} + \frac{(x-2)^3}{24} - \frac{35(x-2)^4}{1536} + \frac{89(x-2)^5}{7680}\right) y(2) + \left(x - 2 - \frac{(x-2)^2}{4} + \frac{(x-2)^3}{16} - \frac{(x-2)^4}{96} - \frac{19(x-2)^5}{7680}\right) D(y)(2) + O(x^6)$$

✓ Solution by Mathematica
Time used: 0.001 (sec). Leaf size: 87

```
AsymptoticDSolveValue[x^3*y''[x]+x^2*y'[x]+y[x]==0,y[x],{x,2,5}]
```

$$y(x) \to c_1\left(\frac{89(x-2)^5}{7680} - \frac{35(x-2)^4}{1536} + \frac{1}{24}(x-2)^3 - \frac{1}{16}(x-2)^2 + 1\right) + c_2\left(-\frac{19(x-2)^5}{7680} - \frac{1}{96}(x-2)^4 + \frac{1}{16}(x-2)^3 - \frac{1}{4}(x-2)^2 + x - 2\right)$$

25.9 problem 35.3 (c)

Internal problem ID [13981]

Book: Ordinary Differential Equations. An introduction to the fundamentals. Kenneth B. Howell. second edition. CRC Press. FL, USA. 2020
Section: Chapter 35. Modified Power series solutions and basic method of Frobenius. Additional Exercises. page 715
Problem number: 35.3 (c).
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$(-x^4 + x^3)\, y'' + (3x - 1)\, y' + 827y = 0$$

With the expansion point for the power series method at $x = 1$.

✓ Solution by Maple
Time used: 0.046 (sec). Leaf size: 62

```
Order:=6;
dsolve((x^3-x^4)*diff(y(x),x$2)+(3*x-1)*diff(y(x),x)+827*y(x)=0,y(x),type='series',x=1);
```

$$y(x) = c_1(-1+x)^3 \left(1 + \frac{409}{2}(-1+x) + \frac{328391}{20}(-1+x)^2 + \frac{128327201}{180}(-1+x)^3 + \frac{19341852779}{1008}(-1+x)^4 + \frac{6949904889503}{20160}(-1+x)^5 + \mathrm{O}\left((-1+x)^6\right)\right) + c_2\left(\ln(-1+x)\left(567661070(-1+x)^3 + 116086688815(-1+x)^4 + \frac{18641478643837}{2}(-1+x)^5 + \mathrm{O}\left((-1+x)^6\right)\right) + \left(12 - 4962(-1+x) + 2059230(-1+x)^2 - 6162812(-1+x)^3 - \frac{592298912511}{4}(-1+x)^4 - \frac{744988601770307}{40}(-1+x)^5 + \mathrm{O}\left((-1+x)^6\right)\right)\right)$$

✓ Solution by Mathematica

Time used: 0.067 (sec). Leaf size: 105

```
AsymptoticDSolveValue[(x^3-x^4)*y''[x]+(3*x-1)*y'[x]+827*y[x]==0,y[x],{x,1,5}]
```

$$y(x) \to c_2 \left(\frac{19341852779(x-1)^7}{1008} + \frac{128327201}{180}(x-1)^6 + \frac{328391}{20}(x-1)^5 + \frac{409}{2}(x-1)^4 + (x-1)^3 \right) + c_1 \left(\frac{1}{144}(-2226119942329(x-1)^4 - 2270644232(x-1)^3 + 24710760(x-1)^2 - 59544(x-1) + 144) + \frac{283830535}{12}(409(x-1)+2)(x-1)^3 \log(x-1) \right)$$

25.10 problem 35.3 (d)

Internal problem ID [13982]

Book: Ordinary Differential Equations. An introduction to the fundamentals. Kenneth B. Howell. second edition. CRC Press. FL, USA. 2020
Section: Chapter 35. Modified Power series solutions and basic method of Frobenius. Additional Exercises. page 715
Problem number: 35.3 (d).
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$\boxed{y''+\frac{y'}{x-3}+\frac{y}{-4+x}=0}$$

With the expansion point for the power series method at $x=3$.

✓ Solution by Maple
Time used: 0.031 (sec). Leaf size: 55

```
Order:=6;
dsolve(diff(y(x),x$2)+1/(x-3)*diff(y(x),x)+1/(x-4)*y(x)=0,y(x),type='series',x=3);
```

$$y(x)=(\ln(-3+x)c_2+c_1)\left(1+\frac{1}{4}(-3+x)^2+\frac{1}{9}(-3+x)^3+\frac{5}{64}(-3+x)^4+\frac{49}{900}(-3+x)^5+\mathrm{O}\left((-3+x)^6\right)\right)+\left(-\frac{1}{4}(-3+x)^2-\frac{2}{27}(-3+x)^3-\frac{7}{128}(-3+x)^4-\frac{469}{13500}(-3+x)^5+\mathrm{O}\left((-3+x)^6\right)\right)c_2$$

✓ Solution by Mathematica

Time used: 0.005 (sec). Leaf size: 128

```
AsymptoticDSolveValue[y''[x]+1/(x-3)*y'[x]+1/(x-4)*y[x]==0,y[x],{x,3,5}]
```

$$y(x) \to c_1\left(\frac{49}{900}(x-3)^5+\frac{5}{64}(x-3)^4+\frac{1}{9}(x-3)^3+\frac{1}{4}(x-3)^2+1\right) + c_2\left(-\frac{469(x-3)^5}{13500}-\frac{7}{128}(x-3)^4-\frac{2}{27}(x-3)^3-\frac{1}{4}(x-3)^2 + \left(\frac{49}{900}(x-3)^5+\frac{5}{64}(x-3)^4+\frac{1}{9}(x-3)^3+\frac{1}{4}(x-3)^2+1\right)\log(x-3)\right)$$

25.11 problem 35.3 (e)

Internal problem ID [13983]

Book: Ordinary Differential Equations. An introduction to the fundamentals. Kenneth B. Howell. second edition. CRC Press. FL, USA. 2020
Section: Chapter 35. Modified Power series solutions and basic method of Frobenius. Additional Exercises. page 715
Problem number: 35.3 (e).
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$y'' + \frac{y'}{(x-3)^2} + \frac{y}{(-4+x)^2} = 0$$

With the expansion point for the power series method at $x = 4$.

✓ Solution by Maple
Time used: 0.031 (sec). Leaf size: 1199

```
Order:=6;
dsolve(diff(y(x),x$2)+1/(x-3)^2*diff(y(x),x)+1/(x-4)^2*y(x)=0,y(x),type='series',x=4);
```

$$y(x) = \sqrt{x-4}\left(c_2(x-4)^{\frac{i\sqrt{3}}{2}}\left(1-\frac{1}{2}(x-4)+\frac{5i\sqrt{3}+7}{8i\sqrt{3}+16}(x-4)^2 - \frac{1}{12}\frac{5+36i\sqrt{3}}{(i\sqrt{3}+3)(i\sqrt{3}+2)}(x-4)^3 + \frac{1}{96}\frac{1313i\sqrt{3}-865}{(i\sqrt{3}+4)(i\sqrt{3}+3)(i\sqrt{3}+2)}(x-4)^4 + \frac{1}{480}\frac{23995i+15978\sqrt{3}}{\left(-\frac{\sqrt{3}}{2}+i\right)(i\sqrt{3}+3)(i\sqrt{3}+4)(i\sqrt{3}+5)}(x-4)^5 + \mathrm{O}\left((x-4)^6\right)\right) + c_1(x-4)^{-\frac{i\sqrt{3}}{2}}\left(1-\frac{1}{2}(x-4)+\frac{5\sqrt{3}+7i}{8\sqrt{3}+16i}(x-4)^2+\frac{5-36i\sqrt{3}}{-36+60i\sqrt{3}}(x-4)^3 + \frac{-1313\sqrt{3}+865i}{288i-2208\sqrt{3}}(x-4)^4 + \frac{-23995-15978i\sqrt{3}}{26880i\sqrt{3}+20160}(x-4)^5 + \mathrm{O}\left((x-4)^6\right)\right)\right)$$

✓ Solution by Mathematica

Time used: 0.007 (sec). Leaf size: 2225

```
AsymptoticDSolveValue[y''[x]+1/(x-3)^2*y'[x]+1/(x-4)^2*y[x]==0,y[x],{x,4,5}]
```

Too large to display

25.12 problem 35.3 (f)

Internal problem ID [13984]

Book: Ordinary Differential Equations. An introduction to the fundamentals. Kenneth B. Howell. second edition. CRC Press. FL, USA. 2020
Section: Chapter 35. Modified Power series solutions and basic method of Frobenius. Additional Exercises. page 715
Problem number: 35.3 (f).
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$y'' + \left(\frac{1}{x} - \frac{1}{3}\right) y' + \left(\frac{1}{x} - \frac{1}{4}\right) y = 0$$

With the expansion point for the power series method at $x = 0$.

✓ Solution by Maple

Time used: 0.031 (sec). Leaf size: 59

```
Order:=6;
dsolve(diff(y(x),x$2)+(1/x-1/3)*diff(y(x),x)+(1/x-1/4)*y(x)=0,y(x),type='series',x=0);
```

$$y(x) = (c_1 + c_2 \ln(x)) \left(1 - x + \frac{11}{48}x^2 - \frac{47}{1296}x^3 + \frac{11}{3072}x^4 - \frac{653}{2073600}x^5 + \mathrm{O}\left(x^6\right)\right) + \left(\frac{7}{3}x - \frac{101}{144}x^2 + \frac{10}{81}x^3 - \frac{6721}{497664}x^4 + \frac{229213}{186624000}x^5 + \mathrm{O}\left(x^6\right)\right) c_2$$

✓ Solution by Mathematica

Time used: 0.004 (sec). Leaf size: 113

```
AsymptoticDSolveValue[y''[x]+(1/x-1/3)*y'[x]+(1/x-1/4)*y[x]==0,y[x],{x,0,5}]
```

$$y(x) \to c_1\left(-\frac{653x^5}{2073600} + \frac{11x^4}{3072} - \frac{47x^3}{1296} + \frac{11x^2}{48} - x + 1\right) + c_2\left(\frac{229213x^5}{186624000} - \frac{6721x^4}{497664} + \frac{10x^3}{81} - \frac{101x^2}{144} + \left(-\frac{653x^5}{2073600} + \frac{11x^4}{3072} - \frac{47x^3}{1296} + \frac{11x^2}{48} - x + 1\right)\log(x) + \frac{7x}{3}\right)$$

25.13 problem 35.3 (g)

Internal problem ID [13985]

Book: Ordinary Differential Equations. An introduction to the fundamentals. Kenneth B. Howell. second edition. CRC Press. FL, USA. 2020
Section: Chapter 35. Modified Power series solutions and basic method of Frobenius. Additional Exercises. page 715
Problem number: 35.3 (g).
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$(4x^2-1)\,y''+\left(4-\frac{2}{x}\right)y'+\frac{(-x^2+1)\,y}{x^2+1}=0$$

With the expansion point for the power series method at $x=0$.

✓ Solution by Maple

Time used: 0.032 (sec). Leaf size: 62

```
Order:=6;
dsolve((4*x^2-1)*diff(y(x),x$2)+(4-2/x)*diff(y(x),x)+(1-x^2)/(1+x^2)*y(x)=0,y(x),type='serie
```

$$y(x) = \frac{\ln(x)\left((-4)\,x-\frac{2}{3}x^3-\frac{4}{9}x^4-\frac{1}{6}x^5+\mathrm{O}(x^6)\right)c_2+c_1\left(1+\frac{1}{6}x^2+\frac{1}{9}x^3+\frac{1}{24}x^4+\frac{31}{270}x^5+\mathrm{O}(x^6)\right)x+(1+4}{x}$$

✓ Solution by Mathematica

Time used: 0.034 (sec). Leaf size: 79

```
AsymptoticDSolveValue[(4*x^2-1)*y''[x]+(4-2/x)*y'[x]+(1-x^2)/(1+x^2)*y[x]==0,y[x],{x,0,5}]
```

$$y(x)\to c_2\left(\frac{x^4}{24}+\frac{x^3}{9}+\frac{x^2}{6}+1\right)+c_1\left(\frac{229x^4+480x^3-756x^2+1728x+216}{216x}-\frac{2}{9}(2x^3+3x^2+18)\log(x)\right)$$

25.14 problem 35.3 (h)

Internal problem ID [13986]

Book: Ordinary Differential Equations. An introduction to the fundamentals. Kenneth B. Howell. second edition. CRC Press. FL, USA. 2020
Section: Chapter 35. Modified Power series solutions and basic method of Frobenius. Additional Exercises. page 715
Problem number: 35.3 (h).
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_Emden, _Fowler]]`

$$\boxed{(x^2+4)^2 y''+y=0}$$

With the expansion point for the power series method at $x=0$.

✓ Solution by Maple
Time used: 0.0 (sec). Leaf size: 34

```
Order:=6;
dsolve((4+x^2)^2*diff(y(x),x$2)+y(x)=0,y(x),type='series',x=0);
```

$$y(x)=\left(1-\frac{1}{32}x^2+\frac{17}{6144}x^4\right)y(0)+\left(x-\frac{1}{96}x^3+\frac{49}{30720}x^5\right)D(y)(0)+O(x^6)$$

✓ Solution by Mathematica
Time used: 0.001 (sec). Leaf size: 42

```
AsymptoticDSolveValue[(4+x^2)^2*y''[x]+y[x]==0,y[x],{x,0,5}]
```

$$y(x)\to c_2\left(\frac{49x^5}{30720}-\frac{x^3}{96}+x\right)+c_1\left(\frac{17x^4}{6144}-\frac{x^2}{32}+1\right)$$

25.15 problem 35.4 (a)

Internal problem ID [13987]

Book: Ordinary Differential Equations. An introduction to the fundamentals. Kenneth B. Howell. second edition. CRC Press. FL, USA. 2020
Section: Chapter 35. Modified Power series solutions and basic method of Frobenius. Additional Exercises. page 715
Problem number: 35.4 (a).
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$\boxed{x^2y'' - 2y'x + (x^2+2)\, y = 0}$$

With the expansion point for the power series method at $x = 0$.

✓ Solution by Maple
Time used: 0.047 (sec). Leaf size: 33

```
Order:=6;
dsolve(x^2*diff(y(x),x$2)-2*x*diff(y(x),x)+(x^2+2)*y(x)=0,y(x),type='series',x=0);
```

$$y(x) = c_1x^2\left(1 - \frac{1}{6}x^2 + \frac{1}{120}x^4 + \mathrm{O}\left(x^6\right)\right) + c_2x\left(1 - \frac{1}{2}x^2 + \frac{1}{24}x^4 + \mathrm{O}\left(x^6\right)\right)$$

✓ Solution by Mathematica
Time used: 0.01 (sec). Leaf size: 44

```
AsymptoticDSolveValue[x^2*y''[x]-2*x*y'[x]+(x^2+2)*y[x]==0,y[x],{x,0,5}]
```

$$y(x) \to c_1\left(\frac{x^5}{24} - \frac{x^3}{2} + x\right) + c_2\left(\frac{x^6}{120} - \frac{x^4}{6} + x^2\right)$$

25.16 problem 35.4 (b)

Internal problem ID [13988]

Book: Ordinary Differential Equations. An introduction to the fundamentals. Kenneth B. Howell. second edition. CRC Press. FL, USA. 2020
Section: Chapter 35. Modified Power series solutions and basic method of Frobenius. Additional Exercises. page 715
Problem number: 35.4 (b).
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$\boxed{4x^2y'' + (1-4x)\,y = 0}$$

With the expansion point for the power series method at $x = 0$.

✓ Solution by Maple
Time used: 0.032 (sec). Leaf size: 69

```
Order:=6;
dsolve(4*x^2*diff(y(x),x$2)+(1-4*x)*y(x)=0,y(x),type='series',x=0);
```

$$y(x) = \left((c_1 + c_2 \ln(x))\left(1 + x + \frac{1}{4}x^2 + \frac{1}{36}x^3 + \frac{1}{576}x^4 + \frac{1}{14400}x^5 + \mathrm{O}\left(x^6\right)\right) + \left((-2)\,x - \frac{3}{4}x^2 - \frac{11}{108}x^3 - \frac{25}{3456}x^4 - \frac{137}{432000}x^5 + \mathrm{O}\left(x^6\right)\right)c_2\right)\sqrt{x}$$

✓ Solution by Mathematica
Time used: 0.003 (sec). Leaf size: 124

```
AsymptoticDSolveValue[4*x^2*y''[x]+(1-4*x)*y[x]==0,y[x],{x,0,5}]
```

$$y(x) \to c_1\sqrt{x}\left(\frac{x^5}{14400} + \frac{x^4}{576} + \frac{x^3}{36} + \frac{x^2}{4} + x + 1\right) + c_2\left(\sqrt{x}\left(-\frac{137x^5}{432000} - \frac{25x^4}{3456} - \frac{11x^3}{108} - \frac{3x^2}{4} - 2x\right) + \sqrt{x}\left(\frac{x^5}{14400} + \frac{x^4}{576} + \frac{x^3}{36} + \frac{x^2}{4} + x + 1\right)\log(x)\right)$$

25.17 problem 35.4 (c)

Internal problem ID [13989]

Book: Ordinary Differential Equations. An introduction to the fundamentals. Kenneth B. Howell. second edition. CRC Press. FL, USA. 2020
Section: Chapter 35. Modified Power series solutions and basic method of Frobenius. Additional Exercises. page 715
Problem number: 35.4 (c).
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$x^2y'' + y'x + (4x - 4)\,y = 0$$

With the expansion point for the power series method at $x = 0$.

✓ Solution by Maple
Time used: 0.062 (sec). Leaf size: 61

```
Order:=6;
dsolve(x^2*diff(y(x),x$2)+x*diff(y(x),x)+(4*x-4)*y(x)=0,y(x),type='series',x=0);
```

$$y(x) = \frac{c_1x^4\left(1 - \frac{4}{5}x + \frac{4}{15}x^2 - \frac{16}{315}x^3 + \frac{2}{315}x^4 - \frac{8}{14175}x^5 + \mathrm{O}\left(x^6\right)\right) + c_2\left(\ln\left(x\right)\left(256x^4 - \frac{1024}{5}x^5 + \mathrm{O}\left(x^6\right)\right) + \left(-14\right.\right.}{x^2}$$

✓ Solution by Mathematica
Time used: 0.021 (sec). Leaf size: 79

```
AsymptoticDSolveValue[x^2*y''[x]+x*y'[x]+(4*x-4)*y[x]==0,y[x],{x,0,5}]
```

$$y(x) \to c_1\left(\frac{4x^4 + 16x^3 + 12x^2 + 12x + 9}{9x^2} - \frac{16}{9}x^2\log(x)\right) + c_2\left(\frac{2x^6}{315} - \frac{16x^5}{315} + \frac{4x^4}{15} - \frac{4x^3}{5} + x^2\right)$$

25.18 problem 35.4 (d)

Internal problem ID [13990]

Book: Ordinary Differential Equations. An introduction to the fundamentals. Kenneth B. Howell. second edition. CRC Press. FL, USA. 2020
Section: Chapter 35. Modified Power series solutions and basic method of Frobenius. Additional Exercises. page 715
Problem number: 35.4 (d).
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$\boxed{(-9x^4 + x^2)\, y'' - 6y'x + 10y = 0}$$

With the expansion point for the power series method at $x = 0$.

✓ Solution by Maple

Time used: 0.047 (sec). Leaf size: 35

```
Order:=6;
dsolve((x^2-9*x^4)*diff(y(x),x$2)-6*x*diff(y(x),x)+10*y(x)=0,y(x),type='series',x=0);
```

$$y(x) = c_1 x^5 (1 + 18x^2 + 243x^4 + \mathrm{O}(x^6)) + c_2 x^2 (12 - 108x^2 - 2916x^4 + \mathrm{O}(x^6))$$

✓ Solution by Mathematica

Time used: 0.013 (sec). Leaf size: 38

```
AsymptoticDSolveValue[(x^2-9*x^4)*y''[x]-6*x*y'[x]+10*y[x]==0,y[x],{x,0,5}]
```

$$y(x) \to c_2 (243x^9 + 18x^7 + x^5) + c_1 (-243x^6 - 9x^4 + x^2)$$

25.19 problem 35.4 (e)

Internal problem ID [13991]

Book: Ordinary Differential Equations. An introduction to the fundamentals. Kenneth B. Howell. second edition. CRC Press. FL, USA. 2020
Section: Chapter 35. Modified Power series solutions and basic method of Frobenius. Additional Exercises. page 715
Problem number: 35.4 (e).
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$x^2y'' - y'x + \frac{y}{1-x} = 0$$

With the expansion point for the power series method at $x = 0$.

✓ Solution by Maple

Time used: 0.031 (sec). Leaf size: 47

```
Order:=6;
dsolve(x^2*diff(y(x),x$2)-x*diff(y(x),x)+1/(1-x)*y(x)=0,y(x),type='series',x=0);
```

$$y(x) = \left((c_1+c_2\ln(x))\left(1-x+\mathrm{O}\left(x^6\right)\right)+\left(2x-\frac{1}{2}x^2-\frac{1}{6}x^3-\frac{1}{12}x^4-\frac{1}{20}x^5+\mathrm{O}\left(x^6\right)\right)c_2\right)x$$

✓ Solution by Mathematica

Time used: 0.006 (sec). Leaf size: 60

```
AsymptoticDSolveValue[x^2*y''[x]-x*y'[x]+1/(1-x)*y[x]==0,y[x],{x,0,5}]
```

$$y(x) \to c_2\left(x\left(-\frac{x^5}{20}-\frac{x^4}{12}-\frac{x^3}{6}-\frac{x^2}{2}+2x\right)+(1-x)x\log(x)\right)+c_1(1-x)x$$

25.20 problem 35.4 (f)

Internal problem ID [13992]

Book: Ordinary Differential Equations. An introduction to the fundamentals. Kenneth B. Howell. second edition. CRC Press. FL, USA. 2020
Section: Chapter 35. Modified Power series solutions and basic method of Frobenius. Additional Exercises. page 715
Problem number: 35.4 (f).
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[_Lienard]`

$$\boxed{y'' + \frac{y'}{x} + y = 0}$$

With the expansion point for the power series method at $x = 0$.

✓ Solution by Maple

Time used: 0.032 (sec). Leaf size: 41

```
Order:=6;
dsolve(diff(y(x),x$2)+1/x*diff(y(x),x)+y(x)=0,y(x),type='series',x=0);
```

$$y(x) = (c_1 + c_2 \ln(x)) \left(1 - \frac{1}{4}x^2 + \frac{1}{64}x^4 + \mathrm{O}(x^6)\right) + \left(\frac{1}{4}x^2 - \frac{3}{128}x^4 + \mathrm{O}(x^6)\right) c_2$$

✓ Solution by Mathematica

Time used: 0.002 (sec). Leaf size: 60

```
AsymptoticDSolveValue[y''[x]+1/x*y'[x]+y[x]==0,y[x],{x,0,5}]
```

$$y(x) \to c_1 \left(\frac{x^4}{64} - \frac{x^2}{4} + 1\right) + c_2 \left(-\frac{3x^4}{128} + \frac{x^2}{4} + \left(\frac{x^4}{64} - \frac{x^2}{4} + 1\right) \log(x)\right)$$

25.21 problem 35.4 (g)

Internal problem ID [13993]

Book: Ordinary Differential Equations. An introduction to the fundamentals. Kenneth B. Howell. second edition. CRC Press. FL, USA. 2020
Section: Chapter 35. Modified Power series solutions and basic method of Frobenius. Additional Exercises. page 715
Problem number: 35.4 (g).
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[_Bessel]`

$$\boxed{y'' + \frac{y'}{x} + \left(1 - \frac{1}{x^2}\right) y = 0}$$

With the expansion point for the power series method at $x = 0$.

✓ Solution by Maple

Time used: 0.047 (sec). Leaf size: 47

```
Order:=6;
dsolve(diff(y(x),x$2)+1/x*diff(y(x),x)+(1-1/x^2)*y(x)=0,y(x),type='series',x=0);
```

$$y(x) = \frac{c_1 x^2\left(1 - \frac{1}{8}x^2 + \frac{1}{192}x^4 + \mathrm{O}\left(x^6\right)\right) + c_2\left(\ln\left(x\right)\left(x^2 - \frac{1}{8}x^4 + \mathrm{O}\left(x^6\right)\right) + \left(-2 + \frac{3}{32}x^4 + \mathrm{O}\left(x^6\right)\right)\right)}{x}$$

✓ Solution by Mathematica

Time used: 0.01 (sec). Leaf size: 58

```
AsymptoticDSolveValue[y''[x]+1/x*y'[x]+(1-1/x^2)*y[x]==0,y[x],{x,0,5}]
```

$$y(x) \to c_2\left(\frac{x^5}{192} - \frac{x^3}{8} + x\right) + c_1\left(\frac{1}{16}x\left(x^2 - 8\right)\log(x) - \frac{5x^4 - 16x^2 - 64}{64x}\right)$$

25.22 problem 35.4 (h)

Internal problem ID [13994]

Book: Ordinary Differential Equations. An introduction to the fundamentals. Kenneth B. Howell. second edition. CRC Press. FL, USA. 2020
Section: Chapter 35. Modified Power series solutions and basic method of Frobenius. Additional Exercises. page 715
Problem number: 35.4 (h).
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$\boxed{2x^2y'' + \left(-2x^3 + 5x\right) y' + \left(-x^2 + 1\right) y = 0}$$

With the expansion point for the power series method at $x = 0$.

✓ Solution by Maple
Time used: 0.031 (sec). Leaf size: 31

```
Order:=6;
dsolve(2*x^2*diff(y(x),x$2)+(5*x-2*x^3)*diff(y(x),x)+(1-x^2)*y(x)=0,y(x),type='series',x=0);
```

$$y(x) = \frac{c_1\left(1 - \frac{1}{6}x^2 - \frac{1}{56}x^4 + \mathrm{O}\left(x^6\right)\right)}{x} + \frac{c_2\left(1 + \mathrm{O}\left(x^6\right)\right)}{\sqrt{x}}$$

✓ Solution by Mathematica
Time used: 0.004 (sec). Leaf size: 34

```
AsymptoticDSolveValue[2*x^2*y''[x]+(5*x-2*x^3)*y'[x]+(1-x^2)*y[x]==0,y[x],{x,0,5}]
```

$$y(x) \to \frac{c_2\left(-\frac{x^4}{56} - \frac{x^2}{6} + 1\right)}{x} + \frac{c_1}{\sqrt{x}}$$

25.23 problem 35.4 (i)

Internal problem ID [13995]

Book: Ordinary Differential Equations. An introduction to the fundamentals. Kenneth B. Howell. second edition. CRC Press. FL, USA. 2020
Section: Chapter 35. Modified Power series solutions and basic method of Frobenius. Additional Exercises. page 715
Problem number: 35.4 (i).
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$x^2y'' - (2x^2 + 5x)\, y' + (9 + 4x)\, y = 0$$

With the expansion point for the power series method at $x = 0$.

✓ Solution by Maple
Time used: 0.047 (sec). Leaf size: 69

```
Order:=6;
dsolve(x^2*diff(y(x),x$2)-(5*x+2*x^2)*diff(y(x),x)+(9+4*x)*y(x)=0,y(x),type='series',x=0);
```

$$y(x) = \left((c_1 + c_2 \ln(x)) \left(1 + 2x + 2x^2 + \frac{4}{3}x^3 + \frac{2}{3}x^4 + \frac{4}{15}x^5 + \mathrm{O}\left(x^6\right)\right) + \left((-2)\, x - 3x^2 - \frac{22}{9}x^3 - \frac{25}{18}x^4 - \frac{137}{225}x^5 + \mathrm{O}\left(x^6\right)\right) c_2\right) x^3$$

✓ Solution by Mathematica

Time used: 0.004 (sec). Leaf size: 116

```
AsymptoticDSolveValue[x^2*y''[x]-(5*x+2*x^2)*y'[x]+(9+4*x)*y[x]==0,y[x],{x,0,5}]
```

$$y(x) \to c_1 \left(\frac{4x^5}{15} + \frac{2x^4}{3} + \frac{4x^3}{3} + 2x^2 + 2x + 1\right) x^3 + c_2 \left(\left(-\frac{137x^5}{225} - \frac{25x^4}{18} - \frac{22x^3}{9} - 3x^2 - 2x\right) x^3 + \left(\frac{4x^5}{15} + \frac{2x^4}{3} + \frac{4x^3}{3} + 2x^2 + 2x + 1\right) x^3 \log(x) \right)$$

25.24 problem 35.4 (j)

Internal problem ID [13996]

Book: Ordinary Differential Equations. An introduction to the fundamentals. Kenneth B. Howell. second edition. CRC Press. FL, USA. 2020
Section: Chapter 35. Modified Power series solutions and basic method of Frobenius. Additional Exercises. page 715
Problem number: 35.4 (j).
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$\boxed{(-3x^3+3x^2)\,y''-(5x^2+4x)\,y'+2y=0}$$

With the expansion point for the power series method at $x=0$.

✓ Solution by Maple
Time used: 0.031 (sec). Leaf size: 47

```
Order:=6;
dsolve((3*x^2-3*x^3)*diff(y(x),x$2)-(4*x+5*x^2)*diff(y(x),x)+2*y(x)=0,y(x),type='series',x=0
```

$$y(x)=c_1x^{\frac{1}{3}}\left(1-\frac{1}{2}x-2x^2-\frac{7}{2}x^3-5x^4-\frac{13}{2}x^5+\mathrm{O}\left(x^6\right)\right)+c_2x^2\left(1+2x+3x^2+4x^3+5x^4+6x^5+\mathrm{O}\left(x^6\right)\right)$$

✓ Solution by Mathematica
Time used: 0.006 (sec). Leaf size: 74

```
AsymptoticDSolveValue[(3*x^2-3*x^3)*y''[x]-(4*x+5*x^2)*y'[x]+2*y[x]==0,y[x],{x,0,5}]
```

$$y(x)\to c_1\left(6x^5+5x^4+4x^3+3x^2+2x+1\right)x^2+c_2\left(-\frac{13x^5}{2}-5x^4-\frac{7x^3}{2}-2x^2-\frac{x}{2}+1\right)\sqrt[3]{x}$$

25.25 problem 35.4 (k)

Internal problem ID [13997]

Book: Ordinary Differential Equations. An introduction to the fundamentals. Kenneth B. Howell. second edition. CRC Press. FL, USA. 2020
Section: Chapter 35. Modified Power series solutions and basic method of Frobenius. Additional Exercises. page 715
Problem number: 35.4 (k).
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[_Laguerre]`

$$\boxed{x^2y'' - (x^2 + x)\, y' + 4yx = 0}$$

With the expansion point for the power series method at $x = 0$.

✓ Solution by Maple
Time used: 0.032 (sec). Leaf size: 52

```
Order:=6;
dsolve(x^2*diff(y(x),x$2)-(x+x^2)*diff(y(x),x)+4*x*y(x)=0,y(x),type='series',x=0);
```

$$y(x) = c_1x^2\left(1 - \frac{2}{3}x + \frac{1}{12}x^2 + \mathrm{O}\left(x^6\right)\right) + \ln(x)\left(12x^2 - 8x^3 + x^4 + \mathrm{O}\left(x^6\right)\right)c_2 + \left(-2 - 8x - 7x^2 + \frac{58}{3}x^3 - \frac{25}{6}x^4 + \frac{1}{15}x^5 + \mathrm{O}\left(x^6\right)\right)c_2$$

✓ Solution by Mathematica
Time used: 0.027 (sec). Leaf size: 70

```
AsymptoticDSolveValue[x^2*y''[x]-(x+x^2)*y'[x]+4*x*y[x]==0,y[x],{x,0,5}]
```

$$y(x) \to c_2\left(\frac{x^4}{12} - \frac{2x^3}{3} + x^2\right) + c_1\left(\frac{1}{6}\left(14x^4 - 70x^3 + 39x^2 + 24x + 6\right) - \frac{1}{2}x^2\left(x^2 - 8x + 12\right)\log(x)\right)$$

25.26 problem 35.4 (L)

Internal problem ID [13998]

Book: Ordinary Differential Equations. An introduction to the fundamentals. Kenneth B. Howell. second edition. CRC Press. FL, USA. 2020
Section: Chapter 35. Modified Power series solutions and basic method of Frobenius. Additional Exercises. page 715
Problem number: 35.4 (L).
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$\boxed{4x^2y'' + 8x^2y' + y = 0}$$

With the expansion point for the power series method at $x = 0$.

✓ Solution by Maple
Time used: 0.016 (sec). Leaf size: 67

```
Order:=6;
dsolve(4*x^2*diff(y(x),x$2)+8*x^2*diff(y(x),x)+y(x)=0,y(x),type='series',x=0);
```

$$y(x) = \left((c_1 + c_2 \ln(x)) \left(1 - x + \frac{3}{4}x^2 - \frac{5}{12}x^3 + \frac{35}{192}x^4 - \frac{21}{320}x^5 + \mathrm{O}\left(x^6\right)\right) + \left(-\frac{1}{4}x^2 + \frac{1}{4}x^3 - \frac{19}{128}x^4 + \frac{25}{384}x^5 + \mathrm{O}\left(x^6\right)\right) c_2\right)\sqrt{x}$$

✓ Solution by Mathematica
Time used: 0.003 (sec). Leaf size: 125

```
AsymptoticDSolveValue[4*x^2*y''[x]+8*x^2*y'[x]+y[x]==0,y[x],{x,0,5}]
```

$$y(x) \to c_1\sqrt{x}\left(-\frac{21x^5}{320} + \frac{35x^4}{192} - \frac{5x^3}{12} + \frac{3x^2}{4} - x + 1\right) + c_2\left(\sqrt{x}\left(\frac{25x^5}{384} - \frac{19x^4}{128} + \frac{x^3}{4} - \frac{x^2}{4}\right) + \sqrt{x}\left(-\frac{21x^5}{320} + \frac{35x^4}{192} - \frac{5x^3}{12} + \frac{3x^2}{4} - x + 1\right)\log(x)\right)$$

25.27 problem 35.4 (m)

Internal problem ID [13999]

Book: Ordinary Differential Equations. An introduction to the fundamentals. Kenneth B. Howell. second edition. CRC Press. FL, USA. 2020
Section: Chapter 35. Modified Power series solutions and basic method of Frobenius. Additional Exercises. page 715
Problem number: 35.4 (m).
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$\boxed{x^2y'' + \left(-x^4 + x\right) y' + 3yx^3 = 0}$$

With the expansion point for the power series method at $x = 0$.

✓ Solution by Maple

Time used: 0.031 (sec). Leaf size: 35

```
Order:=6;
dsolve(x^2*diff(y(x),x$2)+(x-x^4)*diff(y(x),x)+3*x^3*y(x)=0,y(x),type='series',x=0);
```

$$y(x) = (c_1 + c_2 \ln(x)) \left(1 - \frac{1}{3}x^3 + \mathrm{O}\left(x^6\right)\right) + \left(\frac{1}{3}x^3 + \mathrm{O}\left(x^6\right)\right) c_2$$

✓ Solution by Mathematica

Time used: 0.003 (sec). Leaf size: 39

```
AsymptoticDSolveValue[x^2*y''[x]+(x-x^4)*y'[x]+3*x^3*y[x]==0,y[x],{x,0,5}]
```

$$y(x) \to c_1\left(1 - \frac{x^3}{3}\right) + c_2\left(\frac{x^3}{3} + \left(1 - \frac{x^3}{3}\right)\log(x)\right)$$

25.28 problem 35.4 (n)

Internal problem ID [14000]

Book: Ordinary Differential Equations. An introduction to the fundamentals. Kenneth B. Howell. second edition. CRC Press. FL, USA. 2020
Section: Chapter 35. Modified Power series solutions and basic method of Frobenius. Additional Exercises. page 715
Problem number: 35.4 (n).
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$\boxed{(9x^3 + 9x^2)\, y'' + (27x^2 + 9x)\, y' + (8x - 1)\, y = 0}$$

With the expansion point for the power series method at $x = 0$.

✓ Solution by Maple
Time used: 0.047 (sec). Leaf size: 47

```
Order:=6;
dsolve((9*x^2+9*x^3)*diff(y(x),x$2)+(9*x+27*x^2)*diff(y(x),x)+(8*x-1)*y(x)=0,y(x),type='seri
```

$$y(x) = \frac{(-x^5 + x^4 - x^3 + x^2 - x + 1)\left(x^{\frac{2}{3}} c_2 + c_1\right)}{x^{\frac{1}{3}}} + O\left(x^6\right)$$

✓ Solution by Mathematica
Time used: 0.006 (sec). Leaf size: 62

```
AsymptoticDSolveValue[(9*x^2+9*x^3)*y''[x]+(9*x+27*x^2)*y'[x]+(8*x-1)*y[x]==0,y[x],{x,0,5}]
```

$$y(x) \rightarrow c_1 \sqrt[3]{x}(-x^5 + x^4 - x^3 + x^2 - x + 1) + \frac{c_2(-x^5 + x^4 - x^3 + x^2 - x + 1)}{\sqrt[3]{x}}$$

25.29 problem 35.5 (a)

Internal problem ID [14001]

Book: Ordinary Differential Equations. An introduction to the fundamentals. Kenneth B. Howell. second edition. CRC Press. FL, USA. 2020
Section: Chapter 35. Modified Power series solutions and basic method of Frobenius. Additional Exercises. page 715
Problem number: 35.5 (a).
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _exact, _linear, _homogeneous]]`

$$(x-3)\,y''+(x-3)\,y'+y=0$$

With the expansion point for the power series method at $x=3$.

✓ Solution by Maple
Time used: 0.047 (sec). Leaf size: 62

```
Order:=6;
dsolve((x-3)*diff(y(x),x$2)+(x-3)*diff(y(x),x)+y(x)=0,y(x),type='series',x=3);
```

$$y(x)=c_1(-3+x)\left(1-(-3+x)+\frac{1}{2}(-3+x)^2-\frac{1}{6}(-3+x)^3+\frac{1}{24}(-3+x)^4-\frac{1}{120}(-3+x)^5+\mathrm{O}\left((-3+x)^6\right)\right)+c_2\left(\ln(-3+x)\left(-(-3+x)+(-3+x)^2-\frac{1}{2}(-3+x)^3+\frac{1}{6}(-3+x)^4-\frac{1}{24}(-3+x)^5+\mathrm{O}\left((-3+x)^6\right)\right)+\left(1-(-3+x)+\frac{1}{4}(-3+x)^3-\frac{5}{36}(-3+x)^4+\frac{13}{288}(-3+x)^5+\mathrm{O}\left((-3+x)^6\right)\right)\right)$$

✓ Solution by Mathematica

Time used: 0.019 (sec). Leaf size: 105

```
AsymptoticDSolveValue[(x-3)*y''[x]+(x-3)*y'[x]+y[x]==0,y[x],{x,3,5}]
```

$$
\begin{aligned}
y(x) \to c_2\left(\frac{1}{24}(x-3)^5-\frac{1}{6}(x-3)^4+\frac{1}{2}(x-3)^3-(x-3)^2+x-3\right) \\
+c_1\left(\frac{1}{36}\left(-11(x-3)^4+27(x-3)^3-36(x-3)^2+36\right)\right. \\
\left.+\frac{1}{6}\left((x-3)^3-3(x-3)^2+6(x-3)-6\right)(x-3)\log(x-3)\right)
\end{aligned}
$$

25.30 problem 35.5 (b)

Internal problem ID [14002]

Book: Ordinary Differential Equations. An introduction to the fundamentals. Kenneth B. Howell. second edition. CRC Press. FL, USA. 2020
Section: Chapter 35. Modified Power series solutions and basic method of Frobenius. Additional Exercises. page 715
Problem number: 35.5 (b).
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$\boxed{y'' + \frac{2y'}{x+2} + y = 0}$$

With the expansion point for the power series method at $x = -2$.

✓ Solution by Maple
Time used: 0.031 (sec). Leaf size: 34

```
Order:=6;
dsolve(diff(y(x),x$2)+2/(x+2)*diff(y(x),x)+y(x)=0,y(x),type='series',x=-2);
```

$$y(x) = c_1\left(1 - \frac{1}{6}(x+2)^2 + \frac{1}{120}(x+2)^4 + \mathrm{O}\left((x+2)^6\right)\right) + \frac{c_2\left(1 - \frac{1}{2}(x+2)^2 + \frac{1}{24}(x+2)^4 + \mathrm{O}\left((x+2)^6\right)\right)}{x+2}$$

✓ Solution by Mathematica
Time used: 0.009 (sec). Leaf size: 54

```
AsymptoticDSolveValue[y''[x]+2/(x+2)*y'[x]+y[x]==0,y[x],{x,-2,5}]
```

$$y(x) \to c_1\left(\frac{1}{24}(x+2)^3 + \frac{1}{2}(-x-2) + \frac{1}{x+2}\right) + c_2\left(\frac{1}{120}(x+2)^4 - \frac{1}{6}(x+2)^2 + 1\right)$$

25.31 problem 35.5 (c)

Internal problem ID [14003]

Book: Ordinary Differential Equations. An introduction to the fundamentals. Kenneth B. Howell. second edition. CRC Press. FL, USA. 2020
Section: Chapter 35. Modified Power series solutions and basic method of Frobenius. Additional Exercises. page 715
Problem number: 35.5 (c).
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$4y'' + \frac{(4x-3)\,y}{(x-1)^2} = 0$$

With the expansion point for the power series method at $x = 1$.

✓ Solution by Maple
Time used: 0.032 (sec). Leaf size: 77

```
Order:=6;
dsolve(4*diff(y(x),x$2)+(4*x-3)/(x-1)^2*y(x)=0,y(x),type='series',x=1);
```

$$y(x) = \left((\ln(-1+x)\,c_2 + c_1) \left(1 - (-1+x) + \frac{1}{4}(-1+x)^2 - \frac{1}{36}(-1+x)^3 + \frac{1}{576}(-1+x)^4 - \frac{1}{14400}(-1+x)^5 + \mathrm{O}\left((-1+x)^6\right) \right) + \left(2(-1+x) - \frac{3}{4}(-1+x)^2 + \frac{11}{108}(-1+x)^3 - \frac{25}{3456}(-1+x)^4 + \frac{137}{432000}(-1+x)^5 + \mathrm{O}\left((-1+x)^6\right) \right) c_2 \right) \sqrt{-1+x}$$

✓ Solution by Mathematica

Time used: 0.005 (sec). Leaf size: 162

```
AsymptoticDSolveValue[4*y''[x]+(4*x-3)/(x-1)^2*y[x]==0,y[x],{x,1,5}]
```

$$y(x) \to c_1\left(-\frac{(x-1)^5}{14400}+\frac{1}{576}(x-1)^4-\frac{1}{36}(x-1)^3+\frac{1}{4}(x-1)^2-x+2\right)\sqrt{x-1}$$
$$+c_2\left(\sqrt{x-1}\left(\frac{137(x-1)^5}{432000}-\frac{25(x-1)^4}{3456}+\frac{11}{108}(x-1)^3-\frac{3}{4}(x-1)^2+2(x-1)\right)\right.$$
$$\left.+\left(-\frac{(x-1)^5}{14400}+\frac{1}{576}(x-1)^4-\frac{1}{36}(x-1)^3+\frac{1}{4}(x-1)^2-x+2\right)\sqrt{x-1}\log(x-1)\right)$$

25.32 problem 35.5 (d)

Internal problem ID [14004]

Book: Ordinary Differential Equations. An introduction to the fundamentals. Kenneth B. Howell. second edition. CRC Press. FL, USA. 2020
Section: Chapter 35. Modified Power series solutions and basic method of Frobenius. Additional Exercises. page 715
Problem number: 35.5 (d).
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$(x-3)^2 y'' + (x^2-3x) y' - 3y = 0$$

With the expansion point for the power series method at $x = 3$.

✓ Solution by Maple

Time used: 0.031 (sec). Leaf size: 49

```
Order:=6;
dsolve((x-3)^2*diff(y(x),x$2)+(x^2-3*x)*diff(y(x),x)-3*y(x)=0,y(x),type='series',x=3);
```

$$y(x) = \frac{c_1(-3+x)^4 \left(1 - \frac{1}{5}(-3+x) + \frac{1}{30}(-3+x)^2 - \frac{1}{210}(-3+x)^3 + \frac{1}{1680}(-3+x)^4 - \frac{1}{15120}(-3+x)^5 + \mathrm{O}\left((-3\right.\right.}{}$$

✓ Solution by Mathematica

Time used: 0.023 (sec). Leaf size: 81

```
AsymptoticDSolveValue[(x-3)^2*y''[x]+(x^2-3*x)*y'[x]-3*y[x]==0,y[x],{x,3,5}]
```

$$y(x) \to c_1\left(\frac{x-3}{24} + \frac{1}{2(x-3)} - \frac{1}{(x-3)^2} + \frac{1}{(x-3)^3} - \frac{1}{6}\right) + c_2\left(\frac{(x-3)^5}{1680} - \frac{1}{210}(x-3)^4 + \frac{1}{30}(x-3)^3 - \frac{1}{5}(x-3)^2 + x - 3\right)$$

26 Chapter 36. The big theorem on the the Frobenius method. Additional Exercises. page 739

26.1 problem 36.2 (a)

Internal problem ID [14005]

Book: Ordinary Differential Equations. An introduction to the fundamentals. Kenneth B. Howell. second edition. CRC Press. FL, USA. 2020
Section: Chapter 36. The big theorem on the the Frobenius method. Additional Exercises. page 739
Problem number: 36.2 (a).
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$\boxed{x^2y'' - 2y'x + (-x^2 + 2)\,y = 0}$$

With the expansion point for the power series method at $x = 0$.

✓ Solution by Maple

Time used: 0.047 (sec). Leaf size: 33

```
Order:=6;
dsolve(x^2*diff(y(x),x$2)-2*x*diff(y(x),x)+(2-x^2)*y(x)=0,y(x),type='series',x=0);
```

$$y(x) = c_1x^2\left(1 + \frac{1}{6}x^2 + \frac{1}{120}x^4 + \mathrm{O}\left(x^6\right)\right) + c_2x\left(1 + \frac{1}{2}x^2 + \frac{1}{24}x^4 + \mathrm{O}\left(x^6\right)\right)$$

✓ Solution by Mathematica

Time used: 0.011 (sec). Leaf size: 44

```
AsymptoticDSolveValue[x^2*y''[x]-2*x*y'[x]+(2-x^2)*y[x]==0,y[x],{x,0,5}]
```

$$y(x) \to c_1\left(\frac{x^5}{24} + \frac{x^3}{2} + x\right) + c_2\left(\frac{x^6}{120} + \frac{x^4}{6} + x^2\right)$$

26.2 problem 36.2 (b)

Internal problem ID [14006]

Book: Ordinary Differential Equations. An introduction to the fundamentals. Kenneth B. Howell. second edition. CRC Press. FL, USA. 2020
Section: Chapter 36. The big theorem on the the Frobenius method. Additional Exercises. page 739
Problem number: 36.2 (b).
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$\boxed{x^2y'' - 2x^2y' + (x^2 - 2)\,y = 0}$$

With the expansion point for the power series method at $x = 0$.

✓ Solution by Maple
Time used: 0.047 (sec). Leaf size: 47

```
Order:=6;
dsolve(x^2*diff(y(x),x$2)-2*x^2*diff(y(x),x)+(x^2-2)*y(x)=0,y(x),type='series',x=0);
```

$$y(x) = c_1x^2\left(1 + x + \frac{1}{2}x^2 + \frac{1}{6}x^3 + \frac{1}{24}x^4 + \frac{1}{120}x^5 + \mathrm{O}\left(x^6\right)\right) + \frac{c_2\left(12 + 12x + 6x^2 + 4x^3 + \frac{5}{2}x^4 + \frac{11}{10}x^5 + \mathrm{O}\left(x^6\right)\right)}{x}$$

✓ Solution by Mathematica
Time used: 0.028 (sec). Leaf size: 62

```
AsymptoticDSolveValue[x^2*y''[x]-2*x^2*y'[x]+(x^2-2)*y[x]==0,y[x],{x,0,5}]
```

$$y(x) \to c_1\left(\frac{5x^3}{24} + \frac{x^2}{3} + \frac{x}{2} + \frac{1}{x} + 1\right) + c_2\left(\frac{x^6}{24} + \frac{x^5}{6} + \frac{x^4}{2} + x^3 + x^2\right)$$

26.3 problem 36.2 (c)

Internal problem ID [14007]

Book: Ordinary Differential Equations. An introduction to the fundamentals. Kenneth B. Howell. second edition. CRC Press. FL, USA. 2020
Section: Chapter 36. The big theorem on the the Frobenius method. Additional Exercises. page 739
Problem number: 36.2 (c).
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[_Lienard]`

$$\boxed{y'' + \frac{y'}{x} + y = 0}$$

With the expansion point for the power series method at $x = 0$.

✓ Solution by Maple

Time used: 0.032 (sec). Leaf size: 41

```
Order:=6;
dsolve(diff(y(x),x$2)+1/x*diff(y(x),x)+y(x)=0,y(x),type='series',x=0);
```

$$y(x) = (c_1 + c_2 \ln(x)) \left(1 - \frac{1}{4}x^2 + \frac{1}{64}x^4 + \mathrm{O}\left(x^6\right)\right) + \left(\frac{1}{4}x^2 - \frac{3}{128}x^4 + \mathrm{O}\left(x^6\right)\right) c_2$$

✓ Solution by Mathematica

Time used: 0.002 (sec). Leaf size: 60

```
AsymptoticDSolveValue[y''[x]+1/x*y'[x]+y[x]==0,y[x],{x,0,5}]
```

$$y(x) \to c_1 \left(\frac{x^4}{64} - \frac{x^2}{4} + 1\right) + c_2 \left(-\frac{3x^4}{128} + \frac{x^2}{4} + \left(\frac{x^4}{64} - \frac{x^2}{4} + 1\right) \log(x)\right)$$

26.4 problem 36.2 (d)

Internal problem ID [14008]

Book: Ordinary Differential Equations. An introduction to the fundamentals. Kenneth B. Howell. second edition. CRC Press. FL, USA. 2020
Section: Chapter 36. The big theorem on the the Frobenius method. Additional Exercises. page 739
Problem number: 36.2 (d).
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$x^2(-x^2+2)\,y''+(4x^2+5x)\,y'+(x^2+1)\,y=0$$

With the expansion point for the power series method at $x=0$.

✓ Solution by Maple

Time used: 0.031 (sec). Leaf size: 47

```
Order:=6;
dsolve(x^2*(2-x^2)*diff(y(x),x$2)+(5*x+4*x^2)*diff(y(x),x)+(1+x^2)*y(x)=0,y(x),type='series'
```

$$y(x)=\frac{c_1\left(1+4x+\frac{1}{6}x^2-\frac{14}{45}x^3+\frac{209}{2520}x^4-\frac{823}{28350}x^5+\mathrm{O}(x^6)\right)}{x}+\frac{c_2\left(1+\frac{2}{3}x-\frac{19}{120}x^2+\frac{1}{180}x^3-\frac{23}{51840}x^4+\frac{557}{1425600}x^5+\mathrm{O}(x^6)\right)}{\sqrt{x}}$$

✓ Solution by Mathematica

Time used: 0.007 (sec). Leaf size: 86

```
AsymptoticDSolveValue[x^2*(2-x^2)*y''[x]+(5*x+4*x^2)*y'[x]+(1+x^2)*y[x]==0,y[x],{x,0,5}]
```

$$y(x)\to\frac{c_1\left(\frac{557x^5}{1425600}-\frac{23x^4}{51840}+\frac{x^3}{180}-\frac{19x^2}{120}+\frac{2x}{3}+1\right)}{\sqrt{x}}+\frac{c_2\left(-\frac{823x^5}{28350}+\frac{209x^4}{2520}-\frac{14x^3}{45}+\frac{x^2}{6}+4x+1\right)}{x}$$

26.5 problem 36.2 (e)

Internal problem ID [14009]

Book: Ordinary Differential Equations. An introduction to the fundamentals. Kenneth B. Howell. second edition. CRC Press. FL, USA. 2020
Section: Chapter 36. The big theorem on the the Frobenius method. Additional Exercises. page 739
Problem number: 36.2 (e).
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$\boxed{x^2y'' - (2x^2 + 5x)\, y' + 9y = 0}$$

With the expansion point for the power series method at $x = 0$.

✓ Solution by Maple
Time used: 0.032 (sec). Leaf size: 69

```
Order:=6;
dsolve(x^2*diff(y(x),x$2)-(5*x+2*x^2)*diff(y(x),x)+9*y(x)=0,y(x),type='series',x=0);
```

$$y(x) = \left((c_1 + c_2 \ln(x)) \left(1 + 6x + 12x^2 + \frac{40}{3}x^3 + 10x^4 + \frac{28}{5}x^5 + \mathrm{O}\left(x^6\right)\right) + \left((-10)\, x - 29x^2 - \frac{346}{9}x^3 - \frac{193}{6}x^4 - \frac{1459}{75}x^5 + \mathrm{O}\left(x^6\right)\right) c_2\right) x^3$$

✓ Solution by Mathematica

Time used: 0.005 (sec). Leaf size: 112

```
AsymptoticDSolveValue[x^2*y''[x]-(5*x+2*x^2)*y'[x]+9*y[x]==0,y[x],{x,0,5}]
```

$$
\begin{aligned}
y(x) \to\ & c_1\left(\frac{28x^5}{5}+10x^4+\frac{40x^3}{3}+12x^2+6x+1\right)x^3 \\
& + c_2\left(\left(-\frac{1459x^5}{75}-\frac{193x^4}{6}-\frac{346x^3}{9}-29x^2-10x\right)x^3\right. \\
& \left. + \left(\frac{28x^5}{5}+10x^4+\frac{40x^3}{3}+12x^2+6x+1\right)x^3\log(x)\right)
\end{aligned}
$$

26.6 problem 36.2 (f)

Internal problem ID [14010]

Book: Ordinary Differential Equations. An introduction to the fundamentals. Kenneth B. Howell. second edition. CRC Press. FL, USA. 2020
Section: Chapter 36. The big theorem on the the Frobenius method. Additional Exercises. page 739
Problem number: 36.2 (f).
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$x^2(2x+1)\,y''+y'x+\left(4x^3-4\right)y=0$$

With the expansion point for the power series method at $x=0$.

✓ Solution by Maple

Time used: 0.031 (sec). Leaf size: 63

```
Order:=6;
dsolve(x^2*(1+2*x)*diff(y(x),x$2)+x*diff(y(x),x)+(4*x^3-4)*y(x)=0,y(x),type='series',x=0);
```

$$y(x) = \frac{c_1x^4\left(1-\frac{4}{5}x+\frac{4}{5}x^2-\frac{116}{105}x^3+\frac{311}{210}x^4-\frac{358}{175}x^5+\mathrm{O}\left(x^6\right)\right)+c_2\left(\ln\left(x\right)\left(576x^4-\frac{2304}{5}x^5+\mathrm{O}\left(x^6\right)\right)+\left(-144-\right.\right.}{x^2}$$

✓ Solution by Mathematica

Time used: 0.044 (sec). Leaf size: 77

```
AsymptoticDSolveValue[x^2*(1+2*x)*y''[x]+x*y'[x]+(4*x^3-4)*y[x]==0,y[x],{x,0,5}]
```

$$y(x)\to c_1\left(\frac{19x^4+4x^3+12x^2+12x+3}{3x^2}-4x^2\log(x)\right)+c_2\left(\frac{311x^6}{210}-\frac{116x^5}{105}+\frac{4x^4}{5}-\frac{4x^3}{5}+x^2\right)$$

26.7 problem 36.2 (g)

Internal problem ID [14011]

Book: Ordinary Differential Equations. An introduction to the fundamentals. Kenneth B. Howell. second edition. CRC Press. FL, USA. 2020
Section: Chapter 36. The big theorem on the the Frobenius method. Additional Exercises. page 739
Problem number: 36.2 (g).
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$4x^2y'' + 8y'x + (1 - 4x)\,y = 0$$

With the expansion point for the power series method at $x = 0$.

✓ Solution by Maple

Time used: 0.047 (sec). Leaf size: 69

```
Order:=6;
dsolve(4*x^2*diff(y(x),x$2)+8*x*diff(y(x),x)+(1-4*x)*y(x)=0,y(x),type='series',x=0);
```

$$y(x) = \frac{(c_1 + c_2\ln(x))\left(1 + x + \frac{1}{4}x^2 + \frac{1}{36}x^3 + \frac{1}{576}x^4 + \frac{1}{14400}x^5 + \mathrm{O}(x^6)\right) + \left((-2)\,x - \frac{3}{4}x^2 - \frac{11}{108}x^3 - \frac{25}{3456}x^4 - \cdots\right.}{\sqrt{x}}$$

✓ Solution by Mathematica

Time used: 0.004 (sec). Leaf size: 124

```
AsymptoticDSolveValue[4*x^2*y''[x]+8*x*y'[x]+(1-4*x)*y[x]==0,y[x],{x,0,5}]
```

$$y(x) \to \frac{c_1\left(\frac{x^5}{14400} + \frac{x^4}{576} + \frac{x^3}{36} + \frac{x^2}{4} + x + 1\right)}{\sqrt{x}} + c_2\left(\frac{-\frac{137x^5}{432000} - \frac{25x^4}{3456} - \frac{11x^3}{108} - \frac{3x^2}{4} - 2x}{\sqrt{x}} + \frac{\left(\frac{x^5}{14400} + \frac{x^4}{576} + \frac{x^3}{36} + \frac{x^2}{4} + x + 1\right)\log(x)}{\sqrt{x}}\right)$$

26.8 problem 36.2 (h)

Internal problem ID [14012]

Book: Ordinary Differential Equations. An introduction to the fundamentals. Kenneth B. Howell. second edition. CRC Press. FL, USA. 2020
Section: Chapter 36. The big theorem on the the Frobenius method. Additional Exercises. page 739
Problem number: 36.2 (h).
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$\boxed{x^2y'' + y'x - (2x+1)\,y = 0}$$

With the expansion point for the power series method at $x = 0$.

✓ Solution by Maple

Time used: 0.047 (sec). Leaf size: 63

```
Order:=6;
dsolve(x^2*diff(y(x),x$2)+x*diff(y(x),x)-(1+2*x)*y(x)=0,y(x),type='series',x=0);
```

$$y(x) = \frac{c_1x^2\left(1+\frac{2}{3}x+\frac{1}{6}x^2+\frac{1}{45}x^3+\frac{1}{540}x^4+\frac{1}{9450}x^5+\mathrm{O}\left(x^6\right)\right)+c_2\left(\ln\left(x\right)\left(4x^2+\frac{8}{3}x^3+\frac{2}{3}x^4+\frac{4}{45}x^5+\mathrm{O}\left(x^6\right)\right)+\right.}{x}$$

✓ Solution by Mathematica

Time used: 0.02 (sec). Leaf size: 83

```
AsymptoticDSolveValue[x^2*y''[x]+x*y'[x]-(1+2*x)*y[x]==0,y[x],{x,0,5}]
```

$$y(x) \to c_1\left(\frac{31x^4+88x^3+36x^2-72x+36}{36x}-\frac{1}{3}x\left(x^2+4x+6\right)\log(x)\right) + c_2\left(\frac{x^5}{540}+\frac{x^4}{45}+\frac{x^3}{6}+\frac{2x^2}{3}+x\right)$$

26.9 problem 36.2 (i)

Internal problem ID [14013]

Book: Ordinary Differential Equations. An introduction to the fundamentals. Kenneth B. Howell. second edition. CRC Press. FL, USA. 2020
Section: Chapter 36. The big theorem on the the Frobenius method. Additional Exercises. page 739
Problem number: 36.2 (i).
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$\boxed{xy'' + 4y' + \frac{12y}{(x+2)^2} = 0}$$

With the expansion point for the power series method at $x = 0$.

✓ Solution by Maple

Time used: 0.047 (sec). Leaf size: 42

```
Order:=6;
dsolve(x*diff(y(x),x$2)+4*diff(y(x),x)+12/(x+2)^2*y(x)=0,y(x),type='series',x=0);
```

$$y(x) = c_1\left(1 - \frac{3}{4}x + \frac{21}{40}x^2 - \frac{27}{80}x^3 + \frac{33}{160}x^4 - \frac{39}{320}x^5 + \mathrm{O}\left(x^6\right)\right) + \frac{c_2\left(12 + 18x + 9x^2 + \frac{9}{8}x^4 - \frac{63}{80}x^5 + \mathrm{O}\left(x^6\right)\right)}{x^3}$$

✓ Solution by Mathematica

Time used: 0.065 (sec). Leaf size: 59

```
AsymptoticDSolveValue[x*y''[x]+4*y'[x]+12/(x+2)^2*y[x]==0,y[x],{x,0,5}]
```

$$y(x) \to c_1\left(\frac{1}{x^3} + \frac{3}{2x^2} + \frac{3}{4x} + \frac{1}{8}\right) + c_2\left(\frac{33x^4}{160} - \frac{27x^3}{80} + \frac{21x^2}{40} - \frac{3x}{4} + 1\right)$$

26.10 problem 36.2 (j)

Internal problem ID [14014]

Book: Ordinary Differential Equations. An introduction to the fundamentals. Kenneth B. Howell. second edition. CRC Press. FL, USA. 2020
Section: Chapter 36. The big theorem on the the Frobenius method. Additional Exercises. page 739
Problem number: 36.2 (j).
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$\boxed{xy'' + 4y' + \frac{12y}{(x+2)^2} = 0}$$

With the expansion point for the power series method at $x = -2$.

✓ Solution by Maple
Time used: 0.047 (sec). Leaf size: 47

```
Order:=6;
dsolve(x*diff(y(x),x$2)+4*diff(y(x),x)+12/(x+2)^2*y(x)=0,y(x),type='series',x=-2);
```

$$y(x) = \frac{c_1(x+2)^5\left(1+\frac{3}{2}(x+2)+\frac{3}{2}(x+2)^2+\frac{5}{4}(x+2)^3+\frac{15}{16}(x+2)^4+\frac{21}{32}(x+2)^5+\mathrm{O}\left((x+2)^6\right)\right)+c_2(2880}{(x+2)^2}$$

✓ Solution by Mathematica
Time used: 0.037 (sec). Leaf size: 70

```
AsymptoticDSolveValue[x*y''[x]+4*y'[x]+12/(x+2)^2*y[x]==0,y[x],{x,-2,5}]
```

$$y(x) \to c_1\left(\frac{1}{4(x+2)}+\frac{1}{(x+2)^2}+\frac{1}{24}\right) + c_2\left(\frac{15}{16}(x+2)^7+\frac{5}{4}(x+2)^6+\frac{3}{2}(x+2)^5+\frac{3}{2}(x+2)^4+(x+2)^3\right)$$

26.11 problem 36.2 (k)

Internal problem ID [14015]

Book: Ordinary Differential Equations. An introduction to the fundamentals. Kenneth B. Howell. second edition. CRC Press. FL, USA. 2020
Section: Chapter 36. The big theorem on the the Frobenius method. Additional Exercises. page 739
Problem number: 36.2 (k).
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _exact, _linear, _homogeneous]]`

$$\boxed{(x-3)\,y''+(x-3)\,y'+y=0}$$

With the expansion point for the power series method at $x=3$.

✓ Solution by Maple
Time used: 0.047 (sec). Leaf size: 62

```
Order:=6;
dsolve((x-3)*diff(y(x),x$2)+(x-3)*diff(y(x),x)+y(x)=0,y(x),type='series',x=3);
```

$$y(x) = c_1(-3+x)\left(1-(-3+x)+\frac{1}{2}(-3+x)^2-\frac{1}{6}(-3+x)^3+\frac{1}{24}(-3+x)^4-\frac{1}{120}(-3+x)^5+\mathrm{O}\left((-3+x)^6\right)\right)+c_2\left(\ln(-3+x)\left(-(-3+x)+(-3+x)^2-\frac{1}{2}(-3+x)^3+\frac{1}{6}(-3+x)^4-\frac{1}{24}(-3+x)^5+\mathrm{O}\left((-3+x)^6\right)\right)+\left(1-(-3+x)+\frac{1}{4}(-3+x)^3-\frac{5}{36}(-3+x)^4+\frac{13}{288}(-3+x)^5+\mathrm{O}\left((-3+x)^6\right)\right)\right)$$

✓ Solution by Mathematica

Time used: 0.02 (sec). Leaf size: 105

```
AsymptoticDSolveValue[(x-3)*y''[x]+(x-3)*y'[x]+y[x]==0,y[x],{x,3,5}]
```

$$
\begin{aligned}
y(x) \to\ & c_2\left(\frac{1}{24}(x-3)^5-\frac{1}{6}(x-3)^4+\frac{1}{2}(x-3)^3-(x-3)^2+x-3\right) \\
& + c_1\left(\frac{1}{36}\left(-11(x-3)^4+27(x-3)^3-36(x-3)^2+36\right)\right. \\
& \left. + \frac{1}{6}\left((x-3)^3-3(x-3)^2+6(x-3)-6\right)(x-3)\log(x-3)\right)
\end{aligned}
$$

26.12 problem 36.2 (L)

Internal problem ID [14016]

Book: Ordinary Differential Equations. An introduction to the fundamentals. Kenneth B. Howell. second edition. CRC Press. FL, USA. 2020
Section: Chapter 36. The big theorem on the the Frobenius method. Additional Exercises. page 739
Problem number: 36.2 (L).
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[_Gegenbauer, [_2nd_order, _linear, '_with_symmetry_[0,F(x)]']`

$$\boxed{(-x^2+1)\,y''-y'x+3y=0}$$

With the expansion point for the power series method at $x=1$.

✓ Solution by Maple
Time used: 0.031 (sec). Leaf size: 46

```
Order:=6;
dsolve((1-x^2)*diff(y(x),x$2)-x*diff(y(x),x)+3*y(x)=0,y(x),type='series',x=1);
```

$$y(x)=c_1\sqrt{-1+x}\left(1+\frac{11}{12}(-1+x)+\frac{11}{160}(-1+x)^2-\frac{143}{13440}(-1+x)^3+\frac{5291}{1935360}(-1+x)^4-\frac{11063}{12902400}(-1+x)^5+\mathrm{O}\left((-1+x)^6\right)\right)+c_2\left(1+3(-1+x)+(-1+x)^2-\frac{1}{15}(-1+x)^3+\frac{1}{70}(-1+x)^4-\frac{13}{3150}(-1+x)^5+\mathrm{O}\left((-1+x)^6\right)\right)$$

✓ Solution by Mathematica

Time used: 0.004 (sec). Leaf size: 101

```
AsymptoticDSolveValue[(1-x^2)*y''[x]-x*y'[x]+3*y[x]==0,y[x],{x,1,5}]
```

$$y(x) \to c_1 \left(-\frac{11063(x-1)^5}{12902400} + \frac{5291(x-1)^4}{1935360} - \frac{143(x-1)^3}{13440} + \frac{11}{160}(x-1)^2 + \frac{11(x-1)}{12} + 1 \right) \sqrt{x-1} + c_2 \left(-\frac{13(x-1)^5}{3150} + \frac{1}{70}(x-1)^4 - \frac{1}{15}(x-1)^3 + (x-1)^2 + 3(x-1) + 1 \right)$$

26.13 problem 36.6 (a)

Internal problem ID [14017]

Book: Ordinary Differential Equations. An introduction to the fundamentals. Kenneth B. Howell. second edition. CRC Press. FL, USA. 2020
Section: Chapter 36. The big theorem on the the Frobenius method. Additional Exercises. page 739
Problem number: 36.6 (a).
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$\boxed{4x^2y'' + (1-4x)\,y = 0}$$

With the expansion point for the power series method at $x = 0$.

✓ Solution by Maple
Time used: 0.016 (sec). Leaf size: 69

```
Order:=6;
dsolve(4*x^2*diff(y(x),x$2)+(1-4*x)*y(x)=0,y(x),type='series',x=0);
```

$$y(x) = \left((c_1 + c_2 \ln(x))\left(1 + x + \frac{1}{4}x^2 + \frac{1}{36}x^3 + \frac{1}{576}x^4 + \frac{1}{14400}x^5 + \mathrm{O}\left(x^6\right)\right) + \left((-2)\,x - \frac{3}{4}x^2 - \frac{11}{108}x^3 - \frac{25}{3456}x^4 - \frac{137}{432000}x^5 + \mathrm{O}\left(x^6\right)\right)c_2\right)\sqrt{x}$$

✓ Solution by Mathematica
Time used: 0.003 (sec). Leaf size: 124

```
AsymptoticDSolveValue[4*x^2*y''[x]+(1-4*x)*y[x]==0,y[x],{x,0,5}]
```

$$y(x) \to c_1\sqrt{x}\left(\frac{x^5}{14400} + \frac{x^4}{576} + \frac{x^3}{36} + \frac{x^2}{4} + x + 1\right) + c_2\left(\sqrt{x}\left(-\frac{137x^5}{432000} - \frac{25x^4}{3456} - \frac{11x^3}{108} - \frac{3x^2}{4} - 2x\right) + \sqrt{x}\left(\frac{x^5}{14400} + \frac{x^4}{576} + \frac{x^3}{36} + \frac{x^2}{4} + x + 1\right)\log(x)\right)$$

26.14 problem 36.6 (b)

Internal problem ID [14018]

Book: Ordinary Differential Equations. An introduction to the fundamentals. Kenneth B. Howell. second edition. CRC Press. FL, USA. 2020
Section: Chapter 36. The big theorem on the the Frobenius method. Additional Exercises. page 739
Problem number: 36.6 (b).
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[_Lienard]`

$$\boxed{y'' + \frac{y'}{x} + y = 0}$$

With the expansion point for the power series method at $x = 0$.

✓ Solution by Maple

Time used: 0.016 (sec). Leaf size: 41

```
Order:=6;
dsolve(diff(y(x),x$2)+1/x*diff(y(x),x)+y(x)=0,y(x),type='series',x=0);
```

$$y(x) = (c_1 + c_2 \ln(x)) \left(1 - \frac{1}{4}x^2 + \frac{1}{64}x^4 + \mathrm{O}\left(x^6\right)\right) + \left(\frac{1}{4}x^2 - \frac{3}{128}x^4 + \mathrm{O}\left(x^6\right)\right) c_2$$

✓ Solution by Mathematica

Time used: 0.002 (sec). Leaf size: 60

```
AsymptoticDSolveValue[y''[x]+1/x*y'[x]+y[x]==0,y[x],{x,0,5}]
```

$$y(x) \to c_1 \left(\frac{x^4}{64} - \frac{x^2}{4} + 1\right) + c_2 \left(-\frac{3x^4}{128} + \frac{x^2}{4} + \left(\frac{x^4}{64} - \frac{x^2}{4} + 1\right) \log(x)\right)$$

26.15 problem 36.6 (c)

Internal problem ID [14019]

Book: Ordinary Differential Equations. An introduction to the fundamentals. Kenneth B. Howell. second edition. CRC Press. FL, USA. 2020
Section: Chapter 36. The big theorem on the the Frobenius method. Additional Exercises. page 739
Problem number: 36.6 (c).
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[_Laguerre]`

$$\boxed{x^2y'' - (x^2+x)\,y' + 4yx = 0}$$

With the expansion point for the power series method at $x = 0$.

✓ Solution by Maple

Time used: 0.047 (sec). Leaf size: 52

```
Order:=6;
dsolve(x^2*diff(y(x),x$2)-(x+x^2)*diff(y(x),x)+4*x*y(x)=0,y(x),type='series',x=0);
```

$$y(x) = c_1x^2\left(1-\frac{2}{3}x+\frac{1}{12}x^2+\mathrm{O}\left(x^6\right)\right)+\ln(x)\left(12x^2-8x^3+x^4+\mathrm{O}\left(x^6\right)\right)c_2 + \left(-2-8x-7x^2+\frac{58}{3}x^3-\frac{25}{6}x^4+\frac{1}{15}x^5+\mathrm{O}\left(x^6\right)\right)c_2$$

✓ Solution by Mathematica

Time used: 0.025 (sec). Leaf size: 70

```
AsymptoticDSolveValue[x^2*y''[x]-(x+x^2)*y'[x]+4*x*y[x]==0,y[x],{x,0,5}]
```

$$y(x) \to c_2\left(\frac{x^4}{12}-\frac{2x^3}{3}+x^2\right) + c_1\left(\frac{1}{6}(14x^4-70x^3+39x^2+24x+6)-\frac{1}{2}x^2(x^2-8x+12)\log(x)\right)$$

26.16 problem 36.6 (d)

Internal problem ID [14020]

Book: Ordinary Differential Equations. An introduction to the fundamentals. Kenneth B. Howell. second edition. CRC Press. FL, USA. 2020
Section: Chapter 36. The big theorem on the the Frobenius method. Additional Exercises. page 739
Problem number: 36.6 (d).
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$\boxed{x^2y'' + y'x + (4x - 4)\, y = 0}$$

With the expansion point for the power series method at $x = 0$.

✓ Solution by Maple

Time used: 0.047 (sec). Leaf size: 61

```
Order:=6;
dsolve(x^2*diff(y(x),x$2)+x*diff(y(x),x)+(4*x-4)*y(x)=0,y(x),type='series',x=0);
```

$$y(x) = \frac{c_1 x^4 \left(1 - \frac{4}{5}x + \frac{4}{15}x^2 - \frac{16}{315}x^3 + \frac{2}{315}x^4 - \frac{8}{14175}x^5 + \mathrm{O}\left(x^6\right)\right) + c_2 \left(\ln\left(x\right)\left(256x^4 - \frac{1024}{5}x^5 + \mathrm{O}\left(x^6\right)\right) + (-14\cdots}{x^2}$$

✓ Solution by Mathematica

Time used: 0.019 (sec). Leaf size: 79

```
AsymptoticDSolveValue[x^2*y''[x]+x*y'[x]+(4*x-4)*y[x]==0,y[x],{x,0,5}]
```

$$y(x) \to c_1 \left(\frac{4x^4 + 16x^3 + 12x^2 + 12x + 9}{9x^2} - \frac{16}{9}x^2 \log(x)\right) + c_2 \left(\frac{2x^6}{315} - \frac{16x^5}{315} + \frac{4x^4}{15} - \frac{4x^3}{5} + x^2\right)$$

27 Chapter 38. Systems of differential equations. A starting point. Additional Exercises. page 786

27.1 problem 38.1

Internal problem ID [14021]

Book: Ordinary Differential Equations. An introduction to the fundamentals. Kenneth B. Howell. second edition. CRC Press. FL, USA. 2020
Section: Chapter 38. Systems of differential equations. A starting point. Additional Exercises. page 786
Problem number: 38.1.
ODE order: 1.
ODE degree: 1.

Solve

$$x'(t) = 2y(t)$$
$$y'(t) = 1 - 2x(t)$$

✓ Solution by Maple
Time used: 0.016 (sec). Leaf size: 36

```
dsolve([diff(x(t),t)=2*y(t),diff(y(t),t)=1-2*x(t)],singsol=all)
```

$$x(t) = c_2 \sin(2t) + c_1 \cos(2t) + \frac{1}{2}$$
$$y(t) = c_2 \cos(2t) - c_1 \sin(2t)$$

✓ Solution by Mathematica
Time used: 0.004 (sec). Leaf size: 42

```
DSolve[{x'[t]==2*y[t],y'[t]==1-2*x[t]},{x[t],y[t]},t,IncludeSingularSolutions -> True]
```

$$x(t) \to c_1 \cos(2t) + c_2 \sin(2t) + \frac{1}{2}$$
$$y(t) \to c_2 \cos(2t) - c_1 \sin(2t)$$

27.2 problem 38.2

Internal problem ID [14022]

Book: Ordinary Differential Equations. An introduction to the fundamentals. Kenneth B. Howell. second edition. CRC Press. FL, USA. 2020
Section: Chapter 38. Systems of differential equations. A starting point. Additional Exercises. page 786
Problem number: 38.2.
ODE order: 1.
ODE degree: 1.

Solve

$$x'(t) = 4x(t) - 3y(t)$$
$$y'(t) = 6x(t) - 7y(t)$$

✓ Solution by Maple
Time used: 0.016 (sec). Leaf size: 36

```
dsolve([diff(x(t),t)=4*x(t)-3*y(t),diff(y(t),t)=6*x(t)-7*y(t)],singsol=all)
```

$$x(t) = c_1 e^{-5t} + c_2 e^{2t}$$
$$y(t) = 3c_1 e^{-5t} + \frac{2c_2 e^{2t}}{3}$$

✓ Solution by Mathematica
Time used: 0.003 (sec). Leaf size: 74

```
DSolve[{x'[t]==4*x[t]-3*y[t],y'[t]==6*x[t]-7*y[t]},{x[t],y[t]},t,IncludeSingularSolutions ->
```

$$x(t) \to \frac{1}{7}e^{-5t}\left(c_1\left(9e^{7t}-2\right)-3c_2\left(e^{7t}-1\right)\right)$$
$$y(t) \to \frac{1}{7}e^{-5t}\left(6c_1\left(e^{7t}-1\right)+c_2\left(9-2e^{7t}\right)\right)$$

27.3 problem 38.3

Internal problem ID [14023]

Book: Ordinary Differential Equations. An introduction to the fundamentals. Kenneth B. Howell. second edition. CRC Press. FL, USA. 2020
Section: Chapter 38. Systems of differential equations. A starting point. Additional Exercises. page 786
Problem number: 38.3.
ODE order: 1.
ODE degree: 1.

Solve

$$x'(t) = \frac{15y(t)}{t} - \frac{2x(t)}{t}$$
$$y'(t) = \frac{x(t)}{t}$$

✓ Solution by Maple
Time used: 0.016 (sec). Leaf size: 33

```
dsolve([t*diff(x(t),t)+2*x(t)=15*y(t),t*diff(y(t),t)=x(t)],singsol=all)
```

$$x(t) = \frac{3c_1t^8 - 5c_2}{t^5}$$
$$y(t) = \frac{c_1t^8 + c_2}{t^5}$$

✓ Solution by Mathematica
Time used: 0.003 (sec). Leaf size: 36

```
DSolve[{t*x'[t]+2*x[t]==15*y[t],t*y'[t]==x[t]},{x[t],y[t]},t,IncludeSingularSolutions -> Tru
```

$$x(t) \to 3c_2t^3 - \frac{5c_1}{t^5}$$
$$y(t) \to \frac{c_2t^8 + c_1}{t^5}$$

27.4 problem 38.4

Internal problem ID [14024]

Book: Ordinary Differential Equations. An introduction to the fundamentals. Kenneth B. Howell. second edition. CRC Press. FL, USA. 2020
Section: Chapter 38. Systems of differential equations. A starting point. Additional Exercises. page 786
Problem number: 38.4.
ODE order: 1.
ODE degree: 1.

Solve

$$x'(t) = x(t) + 2y(t)$$
$$y'(t) = 5x(t) - 2y(t)$$

With initial conditions

$$[x(0) = 7, y(0) = -7]$$

✓ Solution by Maple

Time used: 0.031 (sec). Leaf size: 34

```
dsolve([diff(x(t),t) = x(t)+2*y(t), diff(y(t),t) = 5*x(t)-2*y(t), x(0) = 7, y(0) = -7], sing
```

$$x(t) = 3\,e^{3t} + 4\,e^{-4t}$$
$$y(t) = 3\,e^{3t} - 10\,e^{-4t}$$

✓ Solution by Mathematica

Time used: 0.004 (sec). Leaf size: 44

```
DSolve[{x'[t]==x[t]+2*y[t],y'[t]==5*x[t]-2*y[t]},{x[0]==8,y[0]==-7},{x[t],y[t]},t,IncludeSin
```

$$x(t) \to \frac{2}{7}e^{-4t}\left(13e^{7t} + 15\right)$$
$$y(t) \to \frac{1}{7}e^{-4t}\left(26e^{7t} - 75\right)$$

27.5 problem 38.5

Internal problem ID [14025]

Book: Ordinary Differential Equations. An introduction to the fundamentals. Kenneth B. Howell. second edition. CRC Press. FL, USA. 2020
Section: Chapter 38. Systems of differential equations. A starting point. Additional Exercises. page 786
Problem number: 38.5.
ODE order: 1.
ODE degree: 1.

Solve

$$x'(t) = 5x(t) + 4y(t)$$
$$y'(t) = 8x(t) + y(t)$$

With initial conditions

$$[x(0) = 0, y(0) = 9]$$

✓ Solution by Maple
Time used: 0.016 (sec). Leaf size: 34

```
dsolve([diff(x(t),t) = 5*x(t)+4*y(t), diff(y(t),t) = 8*x(t)+y(t), x(0) = 0, y(0) = 9], sings
```

$$x(t) = -3\,e^{-3t} + 3\,e^{9t}$$
$$y(t) = 6\,e^{-3t} + 3\,e^{9t}$$

✓ Solution by Mathematica
Time used: 0.004 (sec). Leaf size: 36

```
DSolve[{x'[t]==5*x[t]+4*y[t],y'[t]==8*x[t]+y[t]},{x[0]==0,y[0]==9},{x[t],y[t]},t,IncludeSing
```

$$x(t) \to 3e^{-3t}\left(e^{12t} - 1\right)$$
$$y(t) \to 3e^{-3t}\left(e^{12t} + 2\right)$$

27.6 problem 38.6

Internal problem ID [14026]

Book: Ordinary Differential Equations. An introduction to the fundamentals. Kenneth B. Howell. second edition. CRC Press. FL, USA. 2020
Section: Chapter 38. Systems of differential equations. A starting point. Additional Exercises. page 786
Problem number: 38.6.
ODE order: 1.
ODE degree: 1.

Solve

$$x'(t) = 4x(t) + 2y(t)$$
$$y'(t) = 3x(t) - y(t)$$

With initial conditions

$$[x(0) = 0, y(0) = -21]$$

✓ Solution by Maple
Time used: 0.031 (sec). Leaf size: 34

```
dsolve([diff(x(t),t) = 4*x(t)+2*y(t), diff(y(t),t) = 3*x(t)-y(t), x(0) = 0, y(0) = -21], sin
```

$$x(t) = 6\,e^{-2t} - 6\,e^{5t}$$
$$y(t) = -18\,e^{-2t} - 3\,e^{5t}$$

✓ Solution by Mathematica
Time used: 0.004 (sec). Leaf size: 36

```
DSolve[{x'[t]==4*x[t]+2*y[t],y'[t]==3*x[t]-y[t]},{x[0]==0,y[0]==-21},{x[t],y[t]},t,IncludeSi
```

$$x(t) \to -6e^{-2t}\left(e^{7t} - 1\right)$$
$$y(t) \to -3e^{-2t}\left(e^{7t} + 6\right)$$

27.7 problem 38.10 (a)

Internal problem ID [14027]

Book: Ordinary Differential Equations. An introduction to the fundamentals. Kenneth B. Howell. second edition. CRC Press. FL, USA. 2020
Section: Chapter 38. Systems of differential equations. A starting point. Additional Exercises. page 786
Problem number: 38.10 (a).
ODE order: 1.
ODE degree: 1.

Solve

$$x'(t) = x(t) + 2y(t)$$
$$y'(t) = 5x(t) - 2y(t)$$

With initial conditions

$$[x(0) = 1, y(0) = 15]$$

✓ Solution by Maple

Time used: 0.016 (sec). Leaf size: 34

```
dsolve([diff(x(t),t) = x(t)+2*y(t), diff(y(t),t) = 5*x(t)-2*y(t), x(0) = 1, y(0) = 15], sing
```

$$x(t) = 5\,e^{3t} - 4\,e^{-4t}$$
$$y(t) = 5\,e^{3t} + 10\,e^{-4t}$$

✓ Solution by Mathematica

Time used: 0.004 (sec). Leaf size: 37

```
DSolve[{x'[t]==x[t]+2*y[t],y'[t]==5*x[t]-2*y[t]},{x[0]==1,y[0]==15},{x[t],y[t]},t,IncludeSin
```

$$x(t) \to e^{-4t}(5e^{7t} - 4)$$
$$y(t) \to 5e^{-4t}(e^{7t} + 2)$$

27.8 problem 38.10 (b)

Internal problem ID [14028]

Book: Ordinary Differential Equations. An introduction to the fundamentals. Kenneth B. Howell. second edition. CRC Press. FL, USA. 2020
Section: Chapter 38. Systems of differential equations. A starting point. Additional Exercises. page 786
Problem number: 38.10 (b).
ODE order: 1.
ODE degree: 1.

Solve

$$x'(t) = 2y(t)$$
$$y'(t) = 2x(t)$$

✓ Solution by Maple
Time used: 0.016 (sec). Leaf size: 35

```
dsolve([diff(x(t),t)=2*y(t),diff(y(t),t)=2*x(t)],singsol=all)
```

$$x(t) = c_1 e^{2t} + c_2 e^{-2t}$$
$$y(t) = c_1 e^{2t} - c_2 e^{-2t}$$

✓ Solution by Mathematica
Time used: 0.003 (sec). Leaf size: 68

```
DSolve[{x'[t]==2*y[t],y'[t]==2*x[t]},{x[t],y[t]},t,IncludeSingularSolutions -> True]
```

$$x(t) \to \frac{1}{2}e^{-2t}\left(c_1\left(e^{4t}+1\right)+c_2\left(e^{4t}-1\right)\right)$$
$$y(t) \to \frac{1}{2}e^{-2t}\left(c_1\left(e^{4t}-1\right)+c_2\left(e^{4t}+1\right)\right)$$

27.9 problem 38.10 (c)

Internal problem ID [14029]

Book: Ordinary Differential Equations. An introduction to the fundamentals. Kenneth B. Howell. second edition. CRC Press. FL, USA. 2020
Section: Chapter 38. Systems of differential equations. A starting point. Additional Exercises. page 786
Problem number: 38.10 (c).
ODE order: 1.
ODE degree: 1.

Solve

$$
\begin{aligned}
x'(t) &= 2y(t) \\
y'(t) &= -2x(t)
\end{aligned}
$$

✓ Solution by Maple
Time used: 0.0 (sec). Leaf size: 35

```
dsolve([diff(x(t),t)=2*y(t),diff(y(t),t)=-2*x(t)],singsol=all)
```

$$
\begin{aligned}
x(t) &= c_1 \sin(2t) + c_2 \cos(2t) \\
y(t) &= c_1 \cos(2t) - c_2 \sin(2t)
\end{aligned}
$$

✓ Solution by Mathematica
Time used: 0.002 (sec). Leaf size: 39

```
DSolve[{x'[t]==2*y[t],y'[t]==-2*x[t]},{x[t],y[t]},t,IncludeSingularSolutions -> True]
```

$$
\begin{aligned}
x(t) &\to c_1 \cos(2t) + c_2 \sin(2t) \\
y(t) &\to c_2 \cos(2t) - c_1 \sin(2t)
\end{aligned}
$$

27.10 problem 38.10 (d)

Internal problem ID [14030]

Book: Ordinary Differential Equations. An introduction to the fundamentals. Kenneth B. Howell. second edition. CRC Press. FL, USA. 2020
Section: Chapter 38. Systems of differential equations. A starting point. Additional Exercises. page 786
Problem number: 38.10 (d).
ODE order: 1.
ODE degree: 1.

Solve

$$x'(t) = -2y(t)$$
$$y'(t) = 8x(t)$$

✓ Solution by Maple
Time used: 0.032 (sec). Leaf size: 36

```
dsolve([diff(x(t),t)=-2*y(t),diff(y(t),t)=8*x(t)],singsol=all)
```

$$x(t) = c_1 \sin(4t) + c_2 \cos(4t)$$
$$y(t) = -2c_1 \cos(4t) + 2c_2 \sin(4t)$$

✓ Solution by Mathematica
Time used: 0.005 (sec). Leaf size: 42

```
DSolve[{x'[t]==-2*y[t],y'[t]==8*x[t]},{x[t],y[t]},t,IncludeSingularSolutions -> True]
```

$$x(t) \to c_1 \cos(4t) - \frac{1}{2}c_2 \sin(4t)$$
$$y(t) \to c_2 \cos(4t) + 2c_1 \sin(4t)$$

27.11 problem 38.10 (e)

Internal problem ID [14031]

Book: Ordinary Differential Equations. An introduction to the fundamentals. Kenneth B. Howell. second edition. CRC Press. FL, USA. 2020
Section: Chapter 38. Systems of differential equations. A starting point. Additional Exercises. page 786
Problem number: 38.10 (e).
ODE order: 1.
ODE degree: 1.

Solve

$$x'(t) = 4x(t) - 13y(t)$$
$$y'(t) = x(t)$$

With initial conditions

$$[x(0) = 2, y(0) = 1]$$

✓ Solution by Maple
Time used: 0.032 (sec). Leaf size: 35

```
dsolve([diff(x(t),t) = 4*x(t)-13*y(t), diff(y(t),t) = x(t), x(0) = 2, y(0) = 1], singsol=all
```

$$x(t) = e^{2t}(-3\sin(3t) + 2\cos(3t))$$
$$y(t) = e^{2t}\cos(3t)$$

✓ Solution by Mathematica
Time used: 0.007 (sec). Leaf size: 37

```
DSolve[{x'[t]==4*x[t]-13*y[t],y'[t]==x[t]},{x[0]==2,y[0]==1},{x[t],y[t]},t,IncludeSingularSo
```

$$x(t) \to e^{2t}(2\cos(3t) - 3\sin(3t))$$
$$y(t) \to e^{2t}\cos(3t)$$

27.12 problem 38.10 (f)

Internal problem ID [14032]

Book: Ordinary Differential Equations. An introduction to the fundamentals. Kenneth B. Howell. second edition. CRC Press. FL, USA. 2020
Section: Chapter 38. Systems of differential equations. A starting point. Additional Exercises. page 786
Problem number: 38.10 (f).
ODE order: 1.
ODE degree: 1.

Solve

$$x'(t) = 3x(t) + 2y(t)$$
$$y'(t) = -2x(t) + 3y(t)$$

With initial conditions

$$[x(0) = a_1, y(0) = a_2]$$

✓ Solution by Maple
Time used: 0.016 (sec). Leaf size: 45

```
dsolve([diff(x(t),t) = 3*x(t)+2*y(t), diff(y(t),t) = -2*x(t)+3*y(t), x(0) = a__1, y(0) = a__
```

$$x(t) = e^{3t}(a_2 \sin(2t) + a_1 \cos(2t))$$
$$y(t) = e^{3t}(a_2 \cos(2t) - a_1 \sin(2t))$$

✓ Solution by Mathematica
Time used: 0.003 (sec). Leaf size: 47

```
DSolve[{x'[t]==3*x[t]+2*y[t],y'[t]==-2*x[t]+3*y[t]},{x[0]==a1,y[0]==a2},{x[t],y[t]},t,Includ
```

$$x(t) \to e^{3t}(\text{a1} \cos(2t) + \text{a2} \sin(2t))$$
$$y(t) \to e^{3t}(\text{a2} \cos(2t) - \text{a1} \sin(2t))$$

27.13 problem 38.10 (g)

Internal problem ID [14033]

Book: Ordinary Differential Equations. An introduction to the fundamentals. Kenneth B. Howell. second edition. CRC Press. FL, USA. 2020
Section: Chapter 38. Systems of differential equations. A starting point. Additional Exercises. page 786
Problem number: 38.10 (g).
ODE order: 1.
ODE degree: 1.

Solve

$$x'(t) = 8x(t) + 2y(t) - 17$$
$$y'(t) = 4x(t) + y(t) - 13$$

With initial conditions

$$[x(0) = 0, y(0) = 0]$$

✓ Solution by Maple

Time used: 0.032 (sec). Leaf size: 28

```
dsolve([diff(x(t),t) = 8*x(t)+2*y(t)-17, diff(y(t),t) = 4*x(t)+y(t)-13, x(0) = 0, y(0) = 0],
```

$$x(t) = -2\,\mathrm{e}^{9t} + t + 2$$
$$y(t) = -\mathrm{e}^{9t} + 1 - 4t$$

✓ Solution by Mathematica

Time used: 0.004 (sec). Leaf size: 30

```
DSolve[{x'[t]==8*x[t]+2*y[t]-17,y'[t]==4*x[t]+y[t]-13},{x[0]==0,y[0]==0},{x[t],y[t]},t,Inclu
```

$$x(t) \to t - 2e^{9t} + 2$$
$$y(t) \to -4t - e^{9t} + 1$$

27.14 problem 38.10 (h)

Internal problem ID [14034]

Book: Ordinary Differential Equations. An introduction to the fundamentals. Kenneth B. Howell. second edition. CRC Press. FL, USA. 2020
Section: Chapter 38. Systems of differential equations. A starting point. Additional Exercises. page 786
Problem number: 38.10 (h).
ODE order: 1.
ODE degree: 1.

Solve

$$x'(t) = 8x(t) + 2y(t) + 7\,e^{2t}$$
$$y'(t) = 4x(t) + y(t) - 7\,e^{2t}$$

With initial conditions

$$[x(0) = -1, y(0) = 1]$$

✓ Solution by Maple

Time used: 0.032 (sec). Leaf size: 24

```
dsolve([diff(x(t),t) = 8*x(t)+2*y(t)+7*exp(2*t), diff(y(t),t) = 4*x(t)+y(t)-7*exp(2*t), x(0)
```

$$x(t) = -\frac{3}{2} + \frac{e^{2t}}{2}$$
$$y(t) = 6 - 5\,e^{2t}$$

✓ Solution by Mathematica

Time used: 0.005 (sec). Leaf size: 28

```
DSolve[{x'[t]==8*x[t]+2*y[t]+7*Exp[2*t],y'[t]==4*x[t]+y[t]-7*Exp[2*t]},{x[0]==-1,y[0]==1},{x
```

$$x(t) \to \frac{1}{2}\left(e^{2t} - 3\right)$$
$$y(t) \to 6 - 5e^{2t}$$

27.15 problem 38.10 (i)

Internal problem ID [14035]

Book: Ordinary Differential Equations. An introduction to the fundamentals. Kenneth B. Howell. second edition. CRC Press. FL, USA. 2020
Section: Chapter 38. Systems of differential equations. A starting point. Additional Exercises. page 786
Problem number: 38.10 (i).
ODE order: 1.
ODE degree: 1.

Solve

$$x'(t) = 4x(t) + 3y(t) - 6\,e^{3t}$$
$$y'(t) = x(t) + 6y(t) + 2\,e^{3t}$$

With initial conditions

$$[x(0) = 4, y(0) = 0]$$

✓ Solution by Maple

Time used: 0.047 (sec). Leaf size: 44

```
dsolve([diff(x(t),t) = 4*x(t)+3*y(t)-6*exp(3*t), diff(y(t),t) = x(t)+6*y(t)+2*exp(3*t), x(0)
```

$$x(t) = 3\,e^{3t} + e^{7t} - 6\,e^{3t}t$$
$$y(t) = -e^{3t} + e^{7t} + 2\,e^{3t}t$$

✓ Solution by Mathematica

Time used: 0.006 (sec). Leaf size: 50

```
DSolve[{x'[t]==4*x[t]+3*y[t]+6*Exp[3*t],y'[t]==x[t]+6*y[t]+2*Exp[3*t]},{x[0]==4,y[0]==0},{x[
```

$$x(t) \to \frac{1}{4}e^{3t}\left(12t + 7e^{4t} + 9\right)$$
$$y(t) \to \frac{1}{4}e^{3t}\left(-4t + 7e^{4t} - 7\right)$$

27.16 problem 38.10 (j)

Internal problem ID [14036]

Book: Ordinary Differential Equations. An introduction to the fundamentals. Kenneth B. Howell. second edition. CRC Press. FL, USA. 2020
Section: Chapter 38. Systems of differential equations. A starting point. Additional Exercises. page 786
Problem number: 38.10 (j).
ODE order: 1.
ODE degree: 1.

Solve

$$x'(t) = -y(t)$$
$$y'(t) = 4x(t) + 24t$$

With initial conditions

$$[x(0) = 0, y(0) = 0]$$

✓ Solution by Maple
Time used: 0.015 (sec). Leaf size: 26

```
dsolve([diff(x(t),t) = -y(t), diff(y(t),t) = 4*x(t)+24*t, x(0) = 0, y(0) = 0], singsol=all)
```

$$x(t) = 3\sin(2t) - 6t$$
$$y(t) = -6\cos(2t) + 6$$

✓ Solution by Mathematica
Time used: 0.008 (sec). Leaf size: 24

```
DSolve[{x'[t]==-y[t],y'[t]==4*x[t]+24*t},{x[0]==0,y[0]==0},{x[t],y[t]},t,IncludeSingularSolu
```

$$x(t) \to 3\sin(2t) - 6t$$
$$y(t) \to 12\sin^2(t)$$

27.17 problem 38.10 (k)

Internal problem ID [14037]

Book: Ordinary Differential Equations. An introduction to the fundamentals. Kenneth B. Howell. second edition. CRC Press. FL, USA. 2020
Section: Chapter 38. Systems of differential equations. A starting point. Additional Exercises. page 786
Problem number: 38.10 (k).
ODE order: 1.
ODE degree: 1.

Solve

$$x'(t) = 4x(t) - 13y(t)$$
$$y'(t) = x(t) + 152\cos(t)^4 - 152\cos(t)^2 + 19 - 104\cos(t)^3\sin(t) + 52\cos(t)\sin(t)$$

With initial conditions

$$[x(0) = 13, y(0) = 0]$$

✓ Solution by Maple

Time used: 0.282 (sec). Leaf size: 27

```
dsolve([diff(x(t),t) = 4*x(t)-13*y(t), diff(y(t),t) = x(t)+19*cos(4*t)-13*sin(4*t), x(0) = 1
```

$$x(t) = 13\sin(4t) + 13\cos(4t)$$
$$y(t) = 8\sin(4t)$$

✓ Solution by Mathematica

Time used: 0.021 (sec). Leaf size: 25

```
DSolve[{x'[t]==4*x[t]-13*y[t],y'[t]==x[t]+19*Cos[4*t]-13*Sin[4*t]},{x[0]==13,y[0]==0},{x[t],
```

$$x(t) \to 13(\sin(4t) + \cos(4t))$$
$$y(t) \to 8\sin(4t)$$

27.18 problem 38.10 (L)

Internal problem ID [14038]

Book: Ordinary Differential Equations. An introduction to the fundamentals. Kenneth B. Howell. second edition. CRC Press. FL, USA. 2020
Section: Chapter 38. Systems of differential equations. A starting point. Additional Exercises. page 786
Problem number: 38.10 (L).
ODE order: 1.
ODE degree: 1.

Solve

$$x'(t) = 4x(t) + 3y(t) + 5\,\text{Heaviside}\,(-2+t)$$
$$y'(t) = x(t) + 6y(t) + 17\,\text{Heaviside}\,(-2+t)$$

With initial conditions

$$[x(0) = 0, y(0) = 0]$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 67

```
dsolve([diff(x(t),t) = 4*x(t)+3*y(t)+5*Heaviside(t-2), diff(y(t),t) = x(t)+6*y(t)+17*Heavisi
```

$$x(t) = \text{Heaviside}\,(t-2) + 2\,\text{Heaviside}\,(t-2)\,\mathrm{e}^{7t-14} - 3\,\text{Heaviside}\,(t-2)\,\mathrm{e}^{3t-6}$$
$$y(t) = -3\,\text{Heaviside}\,(t-2) + 2\,\text{Heaviside}\,(t-2)\,\mathrm{e}^{7t-14} + \text{Heaviside}\,(t-2)\,\mathrm{e}^{3t-6}$$

✓ Solution by Mathematica

Time used: 0.11 (sec). Leaf size: 60

```
DSolve[{x'[t]==4*x[t]+3*y[t]+5*UnitStep[t-2],y'[t]==x[t]+6*y[t]+17*UnitStep[t-2]},{x[0]==0,y
```

$$x(t) \to \begin{cases} 1 + 2e^{7(t-2)} - 3e^{3t-6} & t \geq 2 \\ 0 & \text{True} \end{cases}$$

$$y(t) \to \begin{cases} -3 + 2e^{7(t-2)} + e^{3t-6} & t \geq 2 \\ 0 & \text{True} \end{cases}$$

27.19 problem 38.11

Internal problem ID [14039]

Book: Ordinary Differential Equations. An introduction to the fundamentals. Kenneth B. Howell. second edition. CRC Press. FL, USA. 2020
Section: Chapter 38. Systems of differential equations. A starting point. Additional Exercises. page 786
Problem number: 38.11.
ODE order: 1.
ODE degree: 1.

Solve

$$x'(t) = 5x(t) + 4y(t)$$
$$y'(t) = 8x(t) + y(t)$$

✓ Solution by Maple
Time used: 0.0 (sec). Leaf size: 35

```
dsolve([diff(x(t),t)=5*x(t)+4*y(t),diff(y(t),t)=8*x(t)+y(t)],singsol=all)
```

$$x(t) = c_1 e^{-3t} + c_2 e^{9t}$$
$$y(t) = -2c_1 e^{-3t} + c_2 e^{9t}$$

✓ Solution by Mathematica
Time used: 0.003 (sec). Leaf size: 71

```
DSolve[{x'[t]==5*x[t]+4*y[t],y'[t]==8*x[t]+y[t]},{x[t],y[t]},t,IncludeSingularSolutions -> T
```

$$x(t) \to \frac{1}{3}e^{-3t}\left(c_1\left(2e^{12t}+1\right)+c_2\left(e^{12t}-1\right)\right)$$
$$y(t) \to \frac{1}{3}e^{-3t}\left(2c_1\left(e^{12t}-1\right)+c_2\left(e^{12t}+2\right)\right)$$

28 Chapter 39. Critical points, Direction fields and trajectories. Additional Exercises. page 815

28.1 problem 39.1 (a)

Internal problem ID [14040]

Book: Ordinary Differential Equations. An introduction to the fundamentals. Kenneth B. Howell. second edition. CRC Press. FL, USA. 2020
Section: Chapter 39. Critical points, Direction fields and trajectories. Additional Exercises. page 815
Problem number: 39.1 (a).
ODE order: 1.
ODE degree: 1.

Solve

$$x'(t) = 2x(t) - 5y(t)$$
$$y'(t) = 3x(t) - 7y(t)$$

✓ Solution by Maple
Time used: 0.015 (sec). Leaf size: 86

```
dsolve([diff(x(t),t)=2*x(t)-5*y(t),diff(y(t),t)=3*x(t)-7*y(t)],singsol=all)
```

$$x(t) = c_1 \mathrm{e}^{\frac{\left(-5+\sqrt{21}\right)t}{2}} + c_2 \mathrm{e}^{-\frac{\left(5+\sqrt{21}\right)t}{2}}$$
$$y(t) = \frac{c_2 \mathrm{e}^{-\frac{\left(5+\sqrt{21}\right)t}{2}}\sqrt{21}}{10} - \frac{c_1 \mathrm{e}^{\frac{\left(-5+\sqrt{21}\right)t}{2}}\sqrt{21}}{10} + \frac{9c_2 \mathrm{e}^{-\frac{\left(5+\sqrt{21}\right)t}{2}}}{10} + \frac{9c_1 \mathrm{e}^{\frac{\left(-5+\sqrt{21}\right)t}{2}}}{10}$$

✓ Solution by Mathematica
Time used: 0.011 (sec). Leaf size: 150

```
DSolve[{x'[t]==2*x[t]-5*y[t],y'[t]==3*x[t]-7*y[t]},{x[t],y[t]},t,IncludeSingularSolutions ->
```

$$x(t) \to \frac{1}{42} e^{-\frac{1}{2}\left(5+\sqrt{21}\right)t}\left(3c_1\left(\left(7+3\sqrt{21}\right)e^{\sqrt{21}t} + 7 - 3\sqrt{21}\right) - 10\sqrt{21}c_2\left(e^{\sqrt{21}t} - 1\right)\right)$$
$$y(t) \to \frac{1}{14} e^{-\frac{1}{2}\left(5+\sqrt{21}\right)t}\left(2\sqrt{21}c_1\left(e^{\sqrt{21}t} - 1\right) - c_2\left(\left(3\sqrt{21} - 7\right)e^{\sqrt{21}t} - 7 - 3\sqrt{21}\right)\right)$$

28.2 problem 39.1 (b)

Internal problem ID [14041]

Book: Ordinary Differential Equations. An introduction to the fundamentals. Kenneth B. Howell. second edition. CRC Press. FL, USA. 2020
Section: Chapter 39. Critical points, Direction fields and trajectories. Additional Exercises. page 815
Problem number: 39.1 (b).
ODE order: 1.
ODE degree: 1.

Solve

$$x'(t) = 2x(t) - 5y(t) + 4$$
$$y'(t) = 3x(t) - 7y(t) + 5$$

✓ Solution by Maple

Time used: 0.016 (sec). Leaf size: 88

```
dsolve([diff(x(t),t)=2*x(t)-5*y(t)+4,diff(y(t),t)=3*x(t)-7*y(t)+5],singsol=all)
```

$$x(t) = \mathrm{e}^{\frac{\left(-5+\sqrt{21}\right)t}{2}} c_2 + \mathrm{e}^{-\frac{\left(5+\sqrt{21}\right)t}{2}} c_1 + 3$$
$$y(t) = \frac{\mathrm{e}^{-\frac{\left(5+\sqrt{21}\right)t}{2}} c_1 \sqrt{21}}{10} - \frac{\mathrm{e}^{\frac{\left(-5+\sqrt{21}\right)t}{2}} c_2 \sqrt{21}}{10} + \frac{9\,\mathrm{e}^{-\frac{\left(5+\sqrt{21}\right)t}{2}} c_1}{10} + \frac{9\,\mathrm{e}^{\frac{\left(-5+\sqrt{21}\right)t}{2}} c_2}{10} + 2$$

✓ Solution by Mathematica

Time used: 0.016 (sec). Leaf size: 185

```
DSolve[{x'[t]==2*x[t]-5*y[t]+4,y'[t]==3*x[t]-7*y[t]+5},{x[t],y[t]},t,IncludeSingularSolution
```

$$x(t) \to \frac{1}{42} e^{-\frac{1}{2}\left(5+\sqrt{21}\right)t} \left(126 e^{\frac{1}{2}\left(5+\sqrt{21}\right)t} + \left(3\left(7+3\sqrt{21}\right) c_1 - 10\sqrt{21} c_2\right) e^{\sqrt{21}t} + \left(21 - 9\sqrt{21}\right) c_1 + 10\sqrt{21} c_2\right)$$
$$y(t) \to \frac{1}{14} e^{-\frac{1}{2}\left(5+\sqrt{21}\right)t} \left(28 e^{\frac{1}{2}\left(5+\sqrt{21}\right)t} + \left(2\sqrt{21} c_1 + \left(7-3\sqrt{21}\right) c_2\right) e^{\sqrt{21}t} - 2\sqrt{21} c_1 + \left(7+3\sqrt{21}\right) c_2\right)$$

28.3 problem 39.1 (c)

Internal problem ID [14042]

Book: Ordinary Differential Equations. An introduction to the fundamentals. Kenneth B. Howell. second edition. CRC Press. FL, USA. 2020
Section: Chapter 39. Critical points, Direction fields and trajectories. Additional Exercises. page 815
Problem number: 39.1 (c).
ODE order: 1.
ODE degree: 1.

Solve

$$x'(t) = 3x(t) + y(t)$$
$$y'(t) = 6x(t) + 2y(t)$$

✓ Solution by Maple
Time used: 0.015 (sec). Leaf size: 27

```
dsolve([diff(x(t),t)=3*x(t)+y(t),diff(y(t),t)=6*x(t)+2*y(t)],singsol=all)
```

$$x(t) = c_1 + c_2 e^{5t}$$
$$y(t) = 2c_2 e^{5t} - 3c_1$$

✓ Solution by Mathematica
Time used: 0.002 (sec). Leaf size: 63

```
DSolve[{x'[t]==3*x[t]+y[t],y'[t]==6*x[t]+2*y[t]},{x[t],y[t]},t,IncludeSingularSolutions -> T
```

$$x(t) \to \frac{1}{5}\left(c_1\left(3e^{5t}+2\right)+c_2\left(e^{5t}-1\right)\right)$$
$$y(t) \to \frac{1}{5}\left(6c_1\left(e^{5t}-1\right)+c_2\left(2e^{5t}+3\right)\right)$$

28.4 problem 39.1 (d)

Internal problem ID [14043]

Book: Ordinary Differential Equations. An introduction to the fundamentals. Kenneth B. Howell. second edition. CRC Press. FL, USA. 2020
Section: Chapter 39. Critical points, Direction fields and trajectories. Additional Exercises. page 815
Problem number: 39.1 (d).
ODE order: 1.
ODE degree: 1.

Solve

$$x'(t) = x(t)\,y(t) - 6y(t)$$
$$y'(t) = x(t) - y(t) - 5$$

✗ Solution by Maple

```
dsolve([diff(x(t),t)=x(t)*y(t)-6*y(t),diff(y(t),t)=x(t)-y(t)-5],singsol=all)
```

No solution found

✗ Solution by Mathematica

Time used: 0.0 (sec). Leaf size: 0

```
DSolve[{x'[t]==x[t]*y[t]-6*y[t],y'[t]==x[t]-y[t]-5},{x[t],y[t]},t,IncludeSingularSolutions -
```

Not solved

28.5 problem 39.2

Internal problem ID [14044]

Book: Ordinary Differential Equations. An introduction to the fundamentals. Kenneth B. Howell. second edition. CRC Press. FL, USA. 2020
Section: Chapter 39. Critical points, Direction fields and trajectories. Additional Exercises. page 815
Problem number: 39.2.
ODE order: 1.
ODE degree: 1.

Solve

$$x'(t) = -x(t) + 2y(t)$$
$$y'(t) = 2x(t) - y(t)$$

✓ Solution by Maple
Time used: 0.0 (sec). Leaf size: 31

```
dsolve([diff(x(t),t)=-x(t)+2*y(t),diff(y(t),t)=2*x(t)-y(t)],singsol=all)
```

$$x(t) = c_1 e^t + c_2 e^{-3t}$$
$$y(t) = c_1 e^t - c_2 e^{-3t}$$

✓ Solution by Mathematica
Time used: 0.007 (sec). Leaf size: 68

```
DSolve[{x'[t]==-x[t]+2*y[t],y'[t]==2*x[t]-y[t]},{x[t],y[t]},t,IncludeSingularSolutions -> Tr
```

$$x(t) \to \frac{1}{2}e^{-3t}\left(c_1\left(e^{4t}+1\right)+c_2\left(e^{4t}-1\right)\right)$$
$$y(t) \to \frac{1}{2}e^{-3t}\left(c_1\left(e^{4t}-1\right)+c_2\left(e^{4t}+1\right)\right)$$