Using Mathematica to study basic probability

by Nasser Abbasi, september 30, 2007

Basic relations. Let $f(x)$ be a PDF for some continuous random variable. Then the following is true

$P(X \leq a) = \int_{-\infty}^{a} f(t)\, dt$

As can be verified as follows (using Normal distribution as an example) by evaluating the above integral and see if it the same as CDF(X=a)

In[1]:=
```
μ = 0; σ = 1;
a = 1;
(∫_{-∞}^{a} PDF[NormalDistribution[μ, σ], x] ⅆx) // N
```

Out[3]=
```
0.841345
```

Now find F (a), it should be the same as above

```
CDF[NormalDistribution[μ, σ], a] // N
```

```
0.841345
```

Another important relation is probability of X being in some range. This is the same as the area under the curve of f(x) between the 2 points:

$P(a \leq X \leq b) = \int_{a}^{b} f(t)\, dt = \text{CDF}(b - a)$

The above is found as follows

```
μ = 0; σ = 1;
a = 1; b = 2;
(Integrate[PDF[NormalDistribution[μ, σ], x], {x, a, b}]) // N
```

```
0.135905
```

Now find F (b)-F(a), it should be the same as above

```
(CDF[NormalDistribution[μ, σ], b] - CDF[NormalDistribution[μ, σ], a]) // N
```

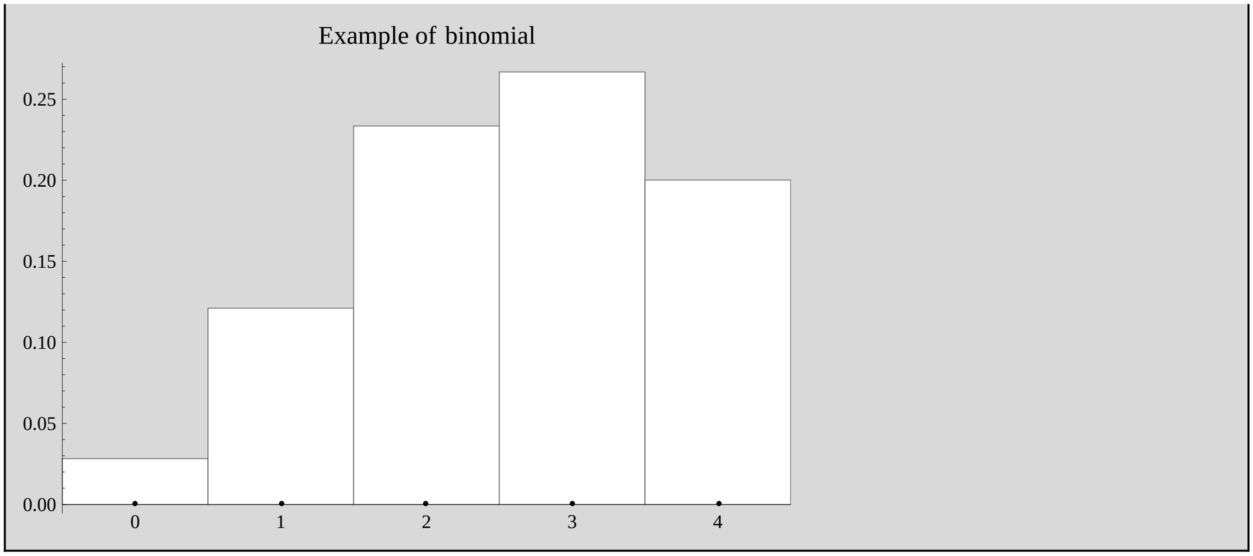
```
0.135905
```

```
Plot[PDF[NormalDistribution[μ, σ], x], {x, -3 σ, 3 σ}];
```

lets try to see how to find probabilty of X be in some range when X is discrete. Assume X is a discrete random variable, say a

Binomial. We need to do the same as above. Now we can not use Integrate, but need to us SUM

```
p = .3; n = 10; (*parameters for binomial*)
Needs["BarCharts`"];
BarChart[Table[PDF[BinomialDistribution[n, p], x], {x, 0, 4}],
 PlotLabel → "Example of binomial", BarLabels → Map[ToString, Range[0, 4, 1]],
 BarSpacing → 0, BarGroupSpacing → 0, BarStyle → White]
```



Let find P (X < 3) in the above. Now instead of integration, we use sum, we want to add the area under the PDF from 0 to 3. But the width is 1 for each interval. So we just sum the y values.

```
Sum[PDF[BinomialDistribution[n, p], x], {x, 0, 3}]
```

```
0.649611
```

Verify by checking the CDF at 3, it should be the same as above

```
CDF[BinomialDistribution[n, p], 3]
```

```
0.649611
```

To show that probability mass function adds to one. Say we have binomial distribution

```
Remove["Global`*"]
Simplify[Sum[ Binomial[n, k] p^k (1 - p)^(n-k), {k, 0, n}]]
```

$$\left(\frac{1}{1-p}\right)^n (1-p)^n$$

```
Assuming[Element[n, Integers], Simplify[%]]
```

```
1
```