Example 1 RREF

In[359]:=

```
mat = {{1, 2, 3, 0}, {4, 5, 6, 3}, {7, 8, 9, 6}};
{result, pivots} = displayREF[mat];
MatrixForm[result];
RowReduce[mat] // MatrixForm
```

$$\begin{pmatrix} 1 & 2 & 3 & 0 \\ 4 & 5 & 6 & 3 \\ 7 & 8 & 9 & 6 \end{pmatrix}$$

Swapping row 1 with row 3 since row 3 has larger pivot

$$\begin{pmatrix} \boxed{7} & 8 & 9 & 6 \\ 4 & 5 & 6 & 3 \\ 1 & 2 & 3 & 0 \end{pmatrix}$$

Making the pivot entry as 1 by dividing row 1 by 7

$$\begin{pmatrix} \boxed{1} & \frac{8}{7} & \frac{9}{7} & \frac{6}{7} \\ 4 & 5 & 6 & 3 \\ 1 & 2 & 3 & 0 \end{pmatrix}$$

Zeroing out all entries in pivot column below pivot entry

$$\begin{pmatrix} \boxed{1} & \frac{8}{7} & \frac{9}{7} & \frac{6}{7} \\ 0 & \frac{3}{7} & \frac{6}{7} & -\frac{3}{7} \\ 0 & \frac{6}{7} & \frac{12}{7} & -\frac{6}{7} \end{pmatrix}$$

Swapping row 2 with row 3 since row 3 has larger pivot

$$\begin{pmatrix} 1 & \frac{8}{7} & \frac{9}{7} & \frac{6}{7} \\ 0 & \boxed{\frac{6}{7}} & \frac{12}{7} & -\frac{6}{7} \\ 0 & \frac{3}{7} & \frac{6}{7} & -\frac{3}{7} \end{pmatrix}$$

Making the pivot entry as 1 by dividing row 2 by $\frac{6}{7}$

$$\begin{pmatrix} 1 & \frac{8}{7} & \frac{9}{7} & \frac{6}{7} \\ 0 & \boxed{1} & 2 & -1 \\ 0 & \frac{3}{7} & \frac{6}{7} & -\frac{3}{7} \end{pmatrix}$$

Zeroing out all entries in pivot column below pivot entry

$$\begin{pmatrix} 1 & 0 & -1 & 2 \\ 0 & \boxed{1} & 2 & -1 \\ 0 & 0 & 0 & 0 \end{pmatrix}$$

Out[362]//MatrixForm=

$$\begin{pmatrix} 1 & 0 & -1 & 2 \\ 0 & 1 & 2 & -1 \\ 0 & 0 & 0 & 0 \end{pmatrix}$$

In[20]:=
```
RowReduce[mat] // MatrixForm
```

Out[20]//MatrixForm=

$$\begin{pmatrix} 1 & 0 & -1 & 2 \\ 0 & 1 & 2 & -1 \\ 0 & 0 & 0 & 0 \end{pmatrix}$$

In[29]:=
```
MatrixRank[mat]
```

Out[29]=

```
2
```

Example 2 RREF

```
mat = {{0, 2, 3, 0}, {4, 5, 6, 3}, {7, 8, 9, 6}};
displaymat = True; normalizePivot = True;
{result, pivots} = displayREF[mat];
MatrixForm[result];
RowReduce[mat] // MatrixForm
```

$$\begin{pmatrix} 0 & 2 & 3 & 0 \\ 4 & 5 & 6 & 3 \\ 7 & 8 & 9 & 6 \end{pmatrix}$$

Swapping row 1 with row 3 since row 3 has larger pivot

$$\begin{pmatrix} \boxed{7} & 8 & 9 & 6 \\ 4 & 5 & 6 & 3 \\ 0 & 2 & 3 & 0 \end{pmatrix}$$

Making the pivot entry as 1 by dividing row 1 by 7

$$\begin{pmatrix} \boxed{1} & \frac{8}{7} & \frac{9}{7} & \frac{6}{7} \\ 4 & 5 & 6 & 3 \\ 0 & 2 & 3 & 0 \end{pmatrix}$$

Zeroing out all entries in pivot column below pivot entry

$$\begin{pmatrix} \boxed{1} & \frac{8}{7} & \frac{9}{7} & \frac{6}{7} \\ 0 & \frac{3}{7} & \frac{6}{7} & -\frac{3}{7} \\ 0 & 2 & 3 & 0 \end{pmatrix}$$

Swapping row 2 with row 3 since row 3 has larger pivot

$$\begin{pmatrix} 1 & \frac{8}{7} & \frac{9}{7} & \frac{6}{7} \\ 0 & \boxed{2} & 3 & 0 \\ 0 & \frac{3}{7} & \frac{6}{7} & -\frac{3}{7} \end{pmatrix}$$

Making the pivot entry as 1 by dividing row 2 by 2

$$\begin{pmatrix} 1 & \frac{8}{7} & \frac{9}{7} & \frac{6}{7} \\ 0 & \boxed{1} & \frac{3}{2} & 0 \\ 0 & \frac{3}{7} & \frac{6}{7} & -\frac{3}{7} \end{pmatrix}$$

Zeroing out all entries in pivot column below pivot entry

$$\begin{pmatrix} 1 & 0 & -\frac{3}{7} & \frac{6}{7} \\ 0 & \boxed{1} & \frac{3}{2} & 0 \\ 0 & 0 & \frac{3}{14} & -\frac{3}{7} \end{pmatrix}$$

Making the pivot entry as 1 by dividing row 3 by $\frac{3}{14}$

$$\begin{pmatrix} 1 & 0 & -\frac{3}{7} & \frac{6}{7} \\ 0 & 1 & \frac{3}{2} & 0 \\ 0 & 0 & \boxed{1} & -2 \end{pmatrix}$$

Zeroing out all entries in pivot column below pivot entry

$$\begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 3 \\ 0 & 0 & \boxed{1} & -2 \end{pmatrix}$$

Out[367]//MatrixForm=

$$\begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 3 \\ 0 & 0 & 1 & -2 \end{pmatrix}$$

In[38]:=
```
RowReduce[mat] // MatrixForm
```

Out[38]//MatrixForm=

$$\begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 3 \\ 0 & 0 & 1 & -2 \end{pmatrix}$$

In[39]:=
```
MatrixRank[mat]
```

Out[39]=
```
3
```

Example 3 REF only with normalization. Row exchange example

In[372]:=
```
mat = {{0, 2, 3, 0}, {4, 5, 6, 3}, {7, 8, 9, 6}};
{result, pivots} = displayREF[mat];
MatrixForm[result]
RowReduce[mat] // MatrixForm
```

$$\begin{pmatrix} 0 & 2 & 3 & 0 \\ 4 & 5 & 6 & 3 \\ 7 & 8 & 9 & 6 \end{pmatrix}$$

Swapping row 1 with row 3 since row 3 has larger pivot

$$\begin{pmatrix} \boxed{7} & 8 & 9 & 6 \\ 4 & 5 & 6 & 3 \\ 0 & 2 & 3 & 0 \end{pmatrix}$$

Making the pivot entry as 1 by dividing row 1 by 7

$$\begin{pmatrix} \boxed{1} & \frac{8}{7} & \frac{9}{7} & \frac{6}{7} \\ 4 & 5 & 6 & 3 \\ 0 & 2 & 3 & 0 \end{pmatrix}$$

Zeroing out all entries in pivot column below pivot entry

$$\begin{pmatrix} \boxed{1} & \frac{8}{7} & \frac{9}{7} & \frac{6}{7} \\ 0 & \frac{3}{7} & \frac{6}{7} & -\frac{3}{7} \\ 0 & 2 & 3 & 0 \end{pmatrix}$$

Swapping row 2 with row 3 since row 3 has larger pivot

$$\begin{pmatrix} 1 & \frac{8}{7} & \frac{9}{7} & \frac{6}{7} \\ 0 & \boxed{2} & 3 & 0 \\ 0 & \frac{3}{7} & \frac{6}{7} & -\frac{3}{7} \end{pmatrix}$$

Making the pivot entry as 1 by dividing row 2 by 2

$$\begin{pmatrix} 1 & \frac{8}{7} & \frac{9}{7} & \frac{6}{7} \\ 0 & \boxed{1} & \frac{3}{2} & 0 \\ 0 & \frac{3}{7} & \frac{6}{7} & -\frac{3}{7} \end{pmatrix}$$

Zeroing out all entries in pivot column below pivot entry

$$\begin{pmatrix} 1 & 0 & -\frac{3}{7} & \frac{6}{7} \\ 0 & \boxed{1} & \frac{3}{2} & 0 \\ 0 & 0 & \frac{3}{14} & -\frac{3}{7} \end{pmatrix}$$

Making the pivot entry as 1 by dividing row 3 by $\frac{3}{14}$

$$\begin{pmatrix} 1 & 0 & -\frac{3}{7} & \frac{6}{7} \\ 0 & 1 & \frac{3}{2} & 0 \\ 0 & 0 & \boxed{1} & -2 \end{pmatrix}$$

Zeroing out all entries in pivot column below pivot entry

$$\begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 3 \\ 0 & 0 & \boxed{1} & -2 \end{pmatrix}$$

Out[374]//MatrixForm=

$$\begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 3 \\ 0 & 0 & 1 & -2 \end{pmatrix}$$

Out[375]//MatrixForm=

$$\begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 3 \\ 0 & 0 & 1 & -2 \end{pmatrix}$$

Example 4 displayRREF with normalization

In[376]:=

```
mat = {{2, 3, 2, 1}, {0, 1, 2, -3}, {3, 3, 3, 2}, {-1, 1, 1, -4}};
{result, pivots} = displayRREF[mat];
MatrixForm[result]
RowReduce[mat] // MatrixForm
```

$$\begin{pmatrix} 2 & 3 & 2 & 1 \\ 0 & 1 & 2 & -3 \\ 3 & 3 & 3 & 2 \\ -1 & 1 & 1 & -4 \end{pmatrix}$$

Swapping row 1 with row 3 since row 3 has larger pivot

$$\begin{pmatrix} \boxed{3} & 3 & 3 & 2 \\ 0 & 1 & 2 & -3 \\ 2 & 3 & 2 & 1 \\ -1 & 1 & 1 & -4 \end{pmatrix}$$

Making the pivot entry as 1 by dividing row 1 by 3

$$\begin{pmatrix} \boxed{1} & 1 & 1 & \frac{2}{3} \\ 0 & 1 & 2 & -3 \\ 2 & 3 & 2 & 1 \\ -1 & 1 & 1 & -4 \end{pmatrix}$$

Zeroing out all entries in pivot column below pivot entry

$$\begin{pmatrix} \boxed{1} & 1 & 1 & \frac{2}{3} \\ 0 & 1 & 2 & -3 \\ 0 & 1 & 0 & -\frac{1}{3} \\ 0 & 2 & 2 & -\frac{10}{3} \end{pmatrix}$$

Swapping row 2 with row 4 since row 4 has larger pivot

$$\begin{pmatrix} 1 & 1 & 1 & \frac{2}{3} \\ 0 & \boxed{2} & 2 & -\frac{10}{3} \\ 0 & 1 & 0 & -\frac{1}{3} \\ 0 & 1 & 2 & -3 \end{pmatrix}$$

Making the pivot entry as 1 by dividing row 2 by 2

$$\begin{pmatrix} 1 & 1 & 1 & \frac{2}{3} \\ 0 & \boxed{1} & 1 & -\frac{5}{3} \\ 0 & 1 & 0 & -\frac{1}{3} \\ 0 & 1 & 2 & -3 \end{pmatrix}$$

Zeroing out all entries in pivot column below pivot entry

$$\begin{pmatrix} 1 & 0 & 0 & \frac{7}{3} \\ 0 & \boxed{1} & 1 & -\frac{5}{3} \\ 0 & 0 & -1 & \frac{4}{3} \\ 0 & 0 & 1 & -\frac{4}{3} \end{pmatrix}$$

Making the pivot entry as 1 by dividing row 3 by −1

$$\begin{pmatrix} 1 & 0 & 0 & \frac{7}{3} \\ 0 & 1 & 1 & -\frac{5}{3} \\ 0 & 0 & \boxed{1} & -\frac{4}{3} \\ 0 & 0 & 1 & -\frac{4}{3} \end{pmatrix}$$

Zeroing out all entries in pivot column below pivot entry

$$\begin{pmatrix} 1 & 0 & 0 & \frac{7}{3} \\ 0 & 1 & 0 & -\frac{1}{3} \\ 0 & 0 & \boxed{1} & -\frac{4}{3} \\ 0 & 0 & 0 & 0 \end{pmatrix}$$

>>>>>>Starting backward elimination phase. The pivots are {{1, 1}, {2, 2}, {3, 3}}

Out[378]//MatrixForm=

$$\begin{pmatrix} 1 & 0 & 0 & \frac{7}{3} \\ 0 & 1 & 0 & -\frac{1}{3} \\ 0 & 0 & 1 & -\frac{4}{3} \\ 0 & 0 & 0 & 0 \end{pmatrix}$$

Out[379]//MatrixForm=

$$\begin{pmatrix} 1 & 0 & 0 & \frac{7}{3} \\ 0 & 1 & 0 & -\frac{1}{3} \\ 0 & 0 & 1 & -\frac{4}{3} \\ 0 & 0 & 0 & 0 \end{pmatrix}$$

Example 5 displayRREF with no normalization

In[380]:=

```
mat = {{2, 3, 2, 1}, {0, 1, 2, -3}, {3, 3, 3, 2}, {-1, 1, 1, -4}};
{result, pivots} = displayREF[mat];
MatrixForm[result]
RowReduce[mat] // MatrixForm
```

$$\begin{pmatrix} 2 & 3 & 2 & 1 \\ 0 & 1 & 2 & -3 \\ 3 & 3 & 3 & 2 \\ -1 & 1 & 1 & -4 \end{pmatrix}$$

Swapping row 1 with row 3 since row 3 has larger pivot

$$\begin{pmatrix} \boxed{3} & 3 & 3 & 2 \\ 0 & 1 & 2 & -3 \\ 2 & 3 & 2 & 1 \\ -1 & 1 & 1 & -4 \end{pmatrix}$$

Making the pivot entry as 1 by dividing row 1 by 3

$$\begin{pmatrix} \boxed{1} & 1 & 1 & \frac{2}{3} \\ 0 & 1 & 2 & -3 \\ 2 & 3 & 2 & 1 \\ -1 & 1 & 1 & -4 \end{pmatrix}$$

Zeroing out all entries in pivot column below pivot entry

$$\begin{pmatrix} \boxed{1} & 1 & 1 & \frac{2}{3} \\ 0 & 1 & 2 & -3 \\ 0 & 1 & 0 & -\frac{1}{3} \\ 0 & 2 & 2 & -\frac{10}{3} \end{pmatrix}$$

Swapping row 2 with row 4 since row 4 has larger pivot

$$\begin{pmatrix} 1 & 1 & 1 & \frac{2}{3} \\ 0 & \boxed{2} & 2 & -\frac{10}{3} \\ 0 & 1 & 0 & -\frac{1}{3} \\ 0 & 1 & 2 & -3 \end{pmatrix}$$

Making the pivot entry as 1 by dividing row 2 by 2

$$\begin{pmatrix} 1 & 1 & 1 & \frac{2}{3} \\ 0 & \boxed{1} & 1 & -\frac{5}{3} \\ 0 & 1 & 0 & -\frac{1}{3} \\ 0 & 1 & 2 & -3 \end{pmatrix}$$

Zeroing out all entries in pivot column below pivot entry

$$\begin{pmatrix} 1 & 0 & 0 & \frac{7}{3} \\ 0 & \boxed{1} & 1 & -\frac{5}{3} \\ 0 & 0 & -1 & \frac{4}{3} \\ 0 & 0 & 1 & -\frac{4}{3} \end{pmatrix}$$

Making the pivot entry as 1 by dividing row 3 by -1

$$\begin{pmatrix} 1 & 0 & 0 & \frac{7}{3} \\ 0 & 1 & 1 & -\frac{5}{3} \\ 0 & 0 & \boxed{1} & -\frac{4}{3} \\ 0 & 0 & 1 & -\frac{4}{3} \end{pmatrix}$$

Zeroing out all entries in pivot column below pivot entry

$$\begin{pmatrix} 1 & 0 & 0 & \frac{7}{3} \\ 0 & 1 & 0 & -\frac{1}{3} \\ 0 & 0 & \boxed{1} & -\frac{4}{3} \\ 0 & 0 & 0 & 0 \end{pmatrix}$$

Out[382]//MatrixForm=

$$\begin{pmatrix} 1 & 0 & 0 & \frac{7}{3} \\ 0 & 1 & 0 & -\frac{1}{3} \\ 0 & 0 & 1 & -\frac{4}{3} \\ 0 & 0 & 0 & 0 \end{pmatrix}$$

Out[383]//MatrixForm=

$$\begin{pmatrix} 1 & 0 & 0 & \frac{7}{3} \\ 0 & 1 & 0 & -\frac{1}{3} \\ 0 & 0 & 1 & -\frac{4}{3} \\ 0 & 0 & 0 & 0 \end{pmatrix}$$

Example 6 displayRREF with normalization

In[384]:=

```
mat = {{1, 2, 2, 4}, {1, 3, 3, 5}, {2, 6, 5, 6}};
{result, pivots} = displayRREF[mat];
MatrixForm[result]
RowReduce[mat] // MatrixForm
```

$$\begin{pmatrix} 1 & 2 & 2 & 4 \\ 1 & 3 & 3 & 5 \\ 2 & 6 & 5 & 6 \end{pmatrix}$$

Swapping row 1 with row 3 since row 3 has larger pivot

$$\begin{pmatrix} \boxed{2} & 6 & 5 & 6 \\ 1 & 3 & 3 & 5 \\ 1 & 2 & 2 & 4 \end{pmatrix}$$

Making the pivot entry as 1 by dividing row 1 by 2

$$\begin{pmatrix} \boxed{1} & 3 & \frac{5}{2} & 3 \\ 1 & 3 & 3 & 5 \\ 1 & 2 & 2 & 4 \end{pmatrix}$$

Zeroing out all entries in pivot column below pivot entry

$$\begin{pmatrix} \boxed{1} & 3 & \frac{5}{2} & 3 \\ 0 & 0 & \frac{1}{2} & 2 \\ 0 & -1 & -\frac{1}{2} & 1 \end{pmatrix}$$

Swapping row 2 with row 3 since row 3 has larger pivot

$$\begin{pmatrix} 1 & 3 & \frac{5}{2} & 3 \\ 0 & \boxed{-1} & -\frac{1}{2} & 1 \\ 0 & 0 & \frac{1}{2} & 2 \end{pmatrix}$$

Making the pivot entry as 1 by dividing row 2 by -1

$$\begin{pmatrix} 1 & 3 & \frac{5}{2} & 3 \\ 0 & \boxed{1} & \frac{1}{2} & -1 \\ 0 & 0 & \frac{1}{2} & 2 \end{pmatrix}$$

Zeroing out all entries in pivot column below pivot entry

$$\begin{pmatrix} 1 & 0 & 1 & 6 \\ 0 & \boxed{1} & \frac{1}{2} & -1 \\ 0 & 0 & \frac{1}{2} & 2 \end{pmatrix}$$

Making the pivot entry as 1 by dividing row 3 by $\frac{1}{2}$

$$\begin{pmatrix} 1 & 0 & 1 & 6 \\ 0 & 1 & \frac{1}{2} & -1 \\ 0 & 0 & \boxed{1} & 4 \end{pmatrix}$$

Zeroing out all entries in pivot column below pivot entry

$$\begin{pmatrix} 1 & 0 & 0 & 2 \\ 0 & 1 & 0 & -3 \\ 0 & 0 & \boxed{1} & 4 \end{pmatrix}$$

>>>>>>Starting backward elimination phase. The pivots are {{1, 1}, {2, 2}, {3, 3}}

Out[386]//MatrixForm=

$$\begin{pmatrix} 1 & 0 & 0 & 2 \\ 0 & 1 & 0 & -3 \\ 0 & 0 & 1 & 4 \end{pmatrix}$$

Out[387]//MatrixForm=

$$\begin{pmatrix} 1 & 0 & 0 & 2 \\ 0 & 1 & 0 & -3 \\ 0 & 0 & 1 & 4 \end{pmatrix}$$

Example 7 displayRREF with normalization

In[388]:=

```
mat = {{-7, -6, -12, -33}, {5, 5, 7, 24}, {1, 0, 4, 5}};
{result, pivots} = displayRREF[mat];
MatrixForm[result]
RowReduce[mat] // MatrixForm
```

$$\begin{pmatrix} -7 & -6 & -12 & -33 \\ 5 & 5 & 7 & 24 \\ 1 & 0 & 4 & 5 \end{pmatrix}$$

Making the pivot entry as 1 by dividing row 1 by -7

$$\begin{pmatrix} \boxed{1} & \frac{6}{7} & \frac{12}{7} & \frac{33}{7} \\ 5 & 5 & 7 & 24 \\ 1 & 0 & 4 & 5 \end{pmatrix}$$

Zeroing out all entries in pivot column below pivot entry

$$\begin{pmatrix} \boxed{1} & \frac{6}{7} & \frac{12}{7} & \frac{33}{7} \\ 0 & \frac{5}{7} & -\frac{11}{7} & \frac{3}{7} \\ 0 & -\frac{6}{7} & \frac{16}{7} & \frac{2}{7} \end{pmatrix}$$

Swapping row 2 with row 3 since row 3 has larger pivot

$$\begin{pmatrix} 1 & \frac{6}{7} & \frac{12}{7} & \frac{33}{7} \\ 0 & \boxed{-\frac{6}{7}} & \frac{16}{7} & \frac{2}{7} \\ 0 & \frac{5}{7} & -\frac{11}{7} & \frac{3}{7} \end{pmatrix}$$

Making the pivot entry as 1 by dividing row 2 by $-\frac{6}{7}$

$$\begin{pmatrix} 1 & \frac{6}{7} & \frac{12}{7} & \frac{33}{7} \\ 0 & \boxed{1} & -\frac{8}{3} & -\frac{1}{3} \\ 0 & \frac{5}{7} & -\frac{11}{7} & \frac{3}{7} \end{pmatrix}$$

Zeroing out all entries in pivot column below pivot entry

$$\begin{pmatrix} 1 & 0 & 4 & 5 \\ 0 & \boxed{1} & -\frac{8}{3} & -\frac{1}{3} \\ 0 & 0 & \frac{1}{3} & \frac{2}{3} \end{pmatrix}$$

Making the pivot entry as 1 by dividing row 3 by $\frac{1}{3}$

$$\begin{pmatrix} 1 & 0 & 4 & 5 \\ 0 & 1 & -\frac{8}{3} & -\frac{1}{3} \\ 0 & 0 & \boxed{1} & 2 \end{pmatrix}$$

Zeroing out all entries in pivot column below pivot entry

$$\begin{pmatrix} 1 & 0 & 0 & -3 \\ 0 & 1 & 0 & 5 \\ 0 & 0 & \boxed{1} & 2 \end{pmatrix}$$

>>>>>>Starting backward elimination phase. The pivots are {{1, 1}, {2, 2}, {3, 3}}

Out[390]//MatrixForm=

$$\begin{pmatrix} 1 & 0 & 0 & -3 \\ 0 & 1 & 0 & 5 \\ 0 & 0 & 1 & 2 \end{pmatrix}$$

Out[391]//MatrixForm=

$$\begin{pmatrix} 1 & 0 & 0 & -3 \\ 0 & 1 & 0 & 5 \\ 0 & 0 & 1 & 2 \end{pmatrix}$$

Example 8 displayRREF on symbolic matrix

In[412]:=

```
ClearAll[s];
mat = {{s, Sqrt[s], 3, 10, s^2},
   {1, s, 2 * s, 10, 1}, {0, 2, 3, 10, s^9}, {1 / s, 5, 3, 8, 2 + s}};
{result, pivots} = displayRREF[mat];
MatrixForm[result]
RowReduce[mat] // MatrixForm
```

$$\begin{pmatrix} s & \sqrt{s} & 3 & 10 & s^2 \\ 1 & s & 2\,s & 10 & 1 \\ 0 & 2 & 3 & 10 & s^9 \\ \frac{1}{s} & 5 & 3 & 8 & 2+s \end{pmatrix}$$

Making the pivot entry as 1 by dividing row 1 by s

$$\begin{pmatrix} \boxed{1} & \frac{1}{\sqrt{s}} & \frac{3}{s} & \frac{10}{s} & s \\ 1 & s & 2\,s & 10 & 1 \\ 0 & 2 & 3 & 10 & s^9 \\ \frac{1}{s} & 5 & 3 & 8 & 2+s \end{pmatrix}$$

Zeroing out all entries in pivot column below pivot entry

$$\begin{pmatrix} \boxed{1} & \frac{1}{\sqrt{s}} & \frac{3}{s} & \frac{10}{s} & s \\ 0 & -\frac{1}{\sqrt{s}}+s & -\frac{3}{s}+2\,s & \frac{10\,(-1+s)}{s} & 1-s \\ 0 & 2 & 3 & 10 & s^9 \\ 0 & 5-\frac{1}{s^{3/2}} & 3-\frac{3}{s^2} & 8-\frac{10}{s^2} & 1+s \end{pmatrix}$$

Making the pivot entry as 1 by dividing row 2 by $-\frac{1}{\sqrt{s}}+s$

$$\begin{pmatrix} 1 & \frac{1}{\sqrt{s}} & \frac{3}{s} & \frac{10}{s} & s \\ 0 & \boxed{1} & \frac{-3+2\,s^2}{-\sqrt{s}+s^2} & \frac{10\,(-1+s)}{-\sqrt{s}+s^2} & -\frac{\sqrt{s}+s}{1+\sqrt{s}+s} \\ 0 & 2 & 3 & 10 & s^9 \\ 0 & 5-\frac{1}{s^{3/2}} & 3-\frac{3}{s^2} & 8-\frac{10}{s^2} & 1+s \end{pmatrix}$$

Zeroing out all entries in pivot column below pivot entry

$$\begin{pmatrix} 1 & 0 & \frac{3\sqrt{s}-2\,s}{-1+s^{3/2}} & \frac{10}{1+\sqrt{s}+s} & s+\frac{1+\sqrt{s}}{1+\sqrt{s}+s} \\ 0 & \boxed{1} & \frac{-3+2\,s^2}{-\sqrt{s}+s^2} & \frac{10\,(-1+s)}{-\sqrt{s}+s^2} & -\frac{\sqrt{s}+s}{1+\sqrt{s}+s} \\ 0 & 0 & 3+\frac{6-4\,s^2}{-\sqrt{s}+s^2} & 10-\frac{20\,(-1+s)}{-\sqrt{s}+s^2} & s^9+\frac{2\,(\sqrt{s}+s)}{1+\sqrt{s}+s} \\ 0 & 0 & -\frac{-12+\sqrt{s}+7\,s^2}{-\sqrt{s}+s^2} & 8-\frac{10}{s^2}-\frac{10\left(5-\frac{1}{s^{3/2}}\right)\,(-1+s)}{-\sqrt{s}+s^2} & 1+s+\frac{\left(5-\frac{1}{s^{3/2}}\right)\,(\sqrt{s}+s)}{1+\sqrt{s}+s} \end{pmatrix}$$

Making the pivot entry as 1 by dividing row 3 by $3+\frac{6-4\,s^2}{-\sqrt{s}+s^2}$

$$\begin{pmatrix} 1 & 0 & \frac{3\sqrt{s}-2s}{-1+s^{3/2}} & \frac{10}{1+\sqrt{s}+s} & s+\frac{1+\sqrt{s}}{1+\sqrt{s}+s} \\ 0 & 1 & \frac{-3+2s^2}{-\sqrt{s}+s^2} & \frac{10\,(-1+s)}{-\sqrt{s}+s^2} & -\frac{\sqrt{s}+s}{1+\sqrt{s}+s} \\ 0 & 0 & \boxed{1} & -\frac{10\left(2-\sqrt{s}-2s+s^2\right)}{-6+3\sqrt{s}+s^2} & \frac{s\left(2-2s+s^{17/2}-s^{10}\right)}{-6+3\sqrt{s}+s^2} \\ 0 & 0 & -\frac{-12+\sqrt{s}+7s^2}{-\sqrt{s}+s^2} & 8-\frac{10}{s^2}-\frac{10\left(5-\frac{1}{s^{3/2}}\right)(-1+s)}{-\sqrt{s}+s^2} & 1+s+\frac{\left(5-\frac{1}{s^{3/2}}\right)\left(\sqrt{s}+s\right)}{1+\sqrt{s}+s} \end{pmatrix}$$

Zeroing out all entries in pivot column below pivot entry

$$\begin{pmatrix} 1 & 0 & 0 & -\frac{10\left(6-3\sqrt{s}-4s+2s^{3/2}\right)}{-6+3\sqrt{s}+s^2} & \frac{-6+3\sqrt{s}+s^3+3s^{10}-2s^{21/2}}{-6+3\sqrt{s}+s^2} \\ 0 & 1 & 0 & \frac{10s\,(-3+2s)}{-6+3\sqrt{s}+s^2} & \frac{s\left(-3+3s-3s^8+2s^{10}\right)}{-6+3\sqrt{s}+s^2} \\ 0 & 0 & \boxed{1} & -\frac{10\left(2-\sqrt{s}-2s+s^2\right)}{-6+3\sqrt{s}+s^2} & \frac{s\left(2-2s+s^{17/2}-s^{10}\right)}{-6+3\sqrt{s}+s^2} \\ 0 & 0 & 0 & \frac{60-30\sqrt{s}-28s+14s^{3/2}+90s^2-62s^3}{s\left(-6+3\sqrt{s}+s^2\right)} & \frac{6-3\sqrt{s}-12s+6s^{3/2}+3s^2+3s^{5/2}-8s^3+s^4+12s^{10}-s^{21/2}-7s^{12}}{s\left(-6+3\sqrt{s}+s^2\right)} \end{pmatrix}$$

Making the pivot entry as 1 by dividing row 4 by $\frac{60-30\sqrt{s}-28s+14s^{3/2}+90s^2-62s^3}{s\left(-6+3\sqrt{s}+s^2\right)}$

$$\begin{pmatrix} 1 & 0 & 0 & -\frac{10\left(6-3\sqrt{s}-4s+2s^{3/2}\right)}{-6+3\sqrt{s}+s^2} & \frac{-6+3\sqrt{s}+s^3+3s^{10}-2s^{21/2}}{-6+3\sqrt{s}+s^2} \\ 0 & 1 & 0 & \frac{10s\,(-3+2s)}{-6+3\sqrt{s}+s^2} & \frac{s\left(-3+3s-3s^8+2s^{10}\right)}{-6+3\sqrt{s}+s^2} \\ 0 & 0 & 1 & -\frac{10\left(2-\sqrt{s}-2s+s^2\right)}{-6+3\sqrt{s}+s^2} & \frac{s\left(2-2s+s^{17/2}-s^{10}\right)}{-6+3\sqrt{s}+s^2} \\ 0 & 0 & 0 & \boxed{1} & \frac{6-3\sqrt{s}-12s+6s^{3/2}+3s^2+3s^{5/2}-8s^3+s^4+12s^{10}-s^{21/2}-7s^{12}}{60-30\sqrt{s}-28s+14s^{3/2}+90s^2-62s^3} \end{pmatrix}$$

Zeroing out all entries in pivot column below pivot entry

$$\begin{pmatrix} 1 & 0 & 0 & 0 & \frac{s\left(-66+33\sqrt{s}+10s-5s^{3/2}-25s^2-10s^{5/2}+31s^3+75s^9-15s^{19/2}-47s^{10}+8s^{21/2}\right)}{-30+15\sqrt{s}+14s-7s^{3/2}-45s^2+31s^3} \\ 0 & 1 & 0 & 0 & -\frac{s^2\left(18+2s-10s^2+15s^7-7s^8-15s^9+8s^{10}\right)}{-30+15\sqrt{s}+14s-7s^{3/2}-45s^2+31s^3} \\ 0 & 0 & 1 & 0 & \frac{10-5\sqrt{s}-20s+10s^{3/2}+17s^2+5s^{5/2}-12s^3-5s^4+5s^{19/2}+20s^{10}-4s^{21/2}-25s^{11}+4s^{12}}{-30+15\sqrt{s}+14s-7s^{3/2}-45s^2+31s^3} \\ 0 & 0 & 0 & \boxed{1} & \frac{6-3\sqrt{s}-12s+6s^{3/2}+3s^2+3s^{5/2}-8s^3+s^4+12s^{10}-s^{21/2}-7s^{12}}{60-30\sqrt{s}-28s+14s^{3/2}+90s^2-62s^3} \end{pmatrix}$$

>>>>>>Starting backward elimination phase. The pivots are {{1, 1}, {2, 2}, {3, 3}, {4, 4}}

Out[415]//MatrixForm=

$$\begin{pmatrix} 1 & 0 & 0 & 0 & \frac{s\left(-66+33\sqrt{s}+10s-5s^{3/2}-25s^2-10s^{5/2}+31s^3+75s^9-15s^{19/2}-47s^{10}+8s^{21/2}\right)}{-30+15\sqrt{s}+14s-7s^{3/2}-45s^2+31s^3} \\ 0 & 1 & 0 & 0 & -\frac{s^2\left(18+2s-10s^2+15s^7-7s^8-15s^9+8s^{10}\right)}{-30+15\sqrt{s}+14s-7s^{3/2}-45s^2+31s^3} \\ 0 & 0 & 1 & 0 & \frac{10-5\sqrt{s}-20s+10s^{3/2}+17s^2+5s^{5/2}-12s^3-5s^4+5s^{19/2}+20s^{10}-4s^{21/2}-25s^{11}+4s^{12}}{-30+15\sqrt{s}+14s-7s^{3/2}-45s^2+31s^3} \\ 0 & 0 & 0 & 1 & \frac{6-3\sqrt{s}-12s+6s^{3/2}+3s^2+3s^{5/2}-8s^3+s^4+12s^{10}-s^{21/2}-7s^{12}}{60-30\sqrt{s}-28s+14s^{3/2}+90s^2-62s^3} \end{pmatrix}$$

Out[416]//MatrixForm=

$$\begin{pmatrix} 1 & 0 & 0 & 0 & \frac{-66\,s+33\,s^{3/2}+10\,s^2-5\,s^{5/2}-25\,s^3-10\,s^{7/2}+31\,s^4+75\,s^{10}-15\,s^{21/2}-47\,s^{11}+8\,s^{23/2}}{-30+15\,\sqrt{s}+14\,s-7\,s^{3/2}-45\,s^2+31\,s^3} \\ 0 & 1 & 0 & 0 & \frac{-18\,s^2-2\,s^3+10\,s^4-15\,s^9+7\,s^{10}+15\,s^{11}-8\,s^{12}}{-30+15\,\sqrt{s}+14\,s-7\,s^{3/2}-45\,s^2+31\,s^3} \\ 0 & 0 & 1 & 0 & \frac{10-5\,\sqrt{s}-20\,s+10\,s^{3/2}+17\,s^2+5\,s^{5/2}-12\,s^3-5\,s^4+5\,s^{19/2}+20\,s^{10}-4\,s^{21/2}-25\,s^{11}+4\,s^{12}}{-30+15\,\sqrt{s}+14\,s-7\,s^{3/2}-45\,s^2+31\,s^3} \\ 0 & 0 & 0 & 1 & \frac{-6+3\,\sqrt{s}+12\,s-6\,s^{3/2}-3\,s^2-3\,s^{5/2}+8\,s^3-s^4-12\,s^{10}+s^{21/2}+7\,s^{12}}{2\left(-30+15\,\sqrt{s}+14\,s-7\,s^{3/2}-45\,s^2+31\,s^3\right)} \end{pmatrix}$$

Example 9 displayRREF with normalization, row exchange is needed

In[417]:=

```
mat = {{0, -6, -12, -33}, {5, 5, 7, 24}, {1, 0, 4, 5}};
{result, pivots} = displayRREF[mat];
MatrixForm[result]
RowReduce[mat] // MatrixForm
```

$$\begin{pmatrix} 0 & -6 & -12 & -33 \\ 5 & 5 & 7 & 24 \\ 1 & 0 & 4 & 5 \end{pmatrix}$$

Swapping row 1 with row 2 since row 2 has larger pivot

$$\begin{pmatrix} \boxed{5} & 5 & 7 & 24 \\ 0 & -6 & -12 & -33 \\ 1 & 0 & 4 & 5 \end{pmatrix}$$

Making the pivot entry as 1 by dividing row 1 by 5

$$\begin{pmatrix} \boxed{1} & 1 & \frac{7}{5} & \frac{24}{5} \\ 0 & -6 & -12 & -33 \\ 1 & 0 & 4 & 5 \end{pmatrix}$$

Zeroing out all entries in pivot column below pivot entry

$$\begin{pmatrix} \boxed{1} & 1 & \frac{7}{5} & \frac{24}{5} \\ 0 & -6 & -12 & -33 \\ 0 & -1 & \frac{13}{5} & \frac{1}{5} \end{pmatrix}$$

Making the pivot entry as 1 by dividing row 2 by -6

$$\begin{pmatrix} 1 & 1 & \frac{7}{5} & \frac{24}{5} \\ 0 & \boxed{1} & 2 & \frac{11}{2} \\ 0 & -1 & \frac{13}{5} & \frac{1}{5} \end{pmatrix}$$

Zeroing out all entries in pivot column below pivot entry

$$\begin{pmatrix} 1 & 0 & -\frac{3}{5} & -\frac{7}{10} \\ 0 & \boxed{1} & 2 & \frac{11}{2} \\ 0 & 0 & \frac{23}{5} & \frac{57}{10} \end{pmatrix}$$

Making the pivot entry as 1 by dividing row 3 by $\frac{23}{5}$

$$\begin{pmatrix} 1 & 0 & -\frac{3}{5} & -\frac{7}{10} \\ 0 & 1 & 2 & \frac{11}{2} \\ 0 & 0 & \boxed{1} & \frac{57}{46} \end{pmatrix}$$

Zeroing out all entries in pivot column below pivot entry

$$\begin{pmatrix} 1 & 0 & 0 & \frac{1}{23} \\ 0 & 1 & 0 & \frac{139}{46} \\ 0 & 0 & \boxed{1} & \frac{57}{46} \end{pmatrix}$$

>>>>>>Starting backward elimination phase. The pivots are {{1, 1}, {2, 2}, {3, 3}}

Out[419]//MatrixForm=

$$\begin{pmatrix} 1 & 0 & 0 & \frac{1}{23} \\ 0 & 1 & 0 & \frac{139}{46} \\ 0 & 0 & 1 & \frac{57}{46} \end{pmatrix}$$

Out[420]//MatrixForm=

$$\begin{pmatrix} 1 & 0 & 0 & \frac{1}{23} \\ 0 & 1 & 0 & \frac{139}{46} \\ 0 & 0 & 1 & \frac{57}{46} \end{pmatrix}$$

Example 10 Added Dec 21,2024

In[421]:=

```
mat = {{1, 2, 5}, {3, 9, 21}};
{result, pivots} = displayRREF[mat];
MatrixForm[result]
RowReduce[mat] // MatrixForm
```

$$\begin{pmatrix} 1 & 2 & 5 \\ 3 & 9 & 21 \end{pmatrix}$$

Swapping row 1 with row 2 since row 2 has larger pivot

$$\begin{pmatrix} \boxed{3} & 9 & 21 \\ 1 & 2 & 5 \end{pmatrix}$$

Making the pivot entry as 1 by dividing row 1 by 3

$$\begin{pmatrix} \boxed{1} & 3 & 7 \\ 1 & 2 & 5 \end{pmatrix}$$

Zeroing out all entries in pivot column below pivot entry

$$\begin{pmatrix} \boxed{1} & 3 & 7 \\ 0 & -1 & -2 \end{pmatrix}$$

Making the pivot entry as 1 by dividing row 2 by -1

$$\begin{pmatrix} 1 & 3 & 7 \\ 0 & \boxed{1} & 2 \end{pmatrix}$$

Zeroing out all entries in pivot column below pivot entry

$$\begin{pmatrix} 1 & 0 & 1 \\ 0 & \boxed{1} & 2 \end{pmatrix}$$

>>>>>>Starting backward elimination phase. The pivots are {{1, 1}, {2, 2}}

Out[423]//MatrixForm=

$$\begin{pmatrix} 1 & 0 & 1 \\ 0 & 1 & 2 \end{pmatrix}$$

Out[424]//MatrixForm=

$$\begin{pmatrix} 1 & 0 & 1 \\ 0 & 1 & 2 \end{pmatrix}$$

Example 11 Added Oct 16,2025

In[425]:=

```
mat = {{5, -15, 10, 10}, {-3, 9, -1, -5}, {2, -6, 1, 3}};
{result, pivots} = displayRREF[mat];
MatrixForm[result]
RowReduce[mat] // MatrixForm
```

$$\begin{pmatrix} 5 & -15 & 10 & 10 \\ -3 & 9 & -1 & -5 \\ 2 & -6 & 1 & 3 \end{pmatrix}$$

Making the pivot entry as 1 by dividing row 1 by 5

$$\begin{pmatrix} \boxed{1} & -3 & 2 & 2 \\ -3 & 9 & -1 & -5 \\ 2 & -6 & 1 & 3 \end{pmatrix}$$

Zeroing out all entries in pivot column below pivot entry

$$\begin{pmatrix} \boxed{1} & -3 & 2 & 2 \\ 0 & 0 & 5 & 1 \\ 0 & 0 & -3 & -1 \end{pmatrix}$$

Making the pivot entry as 1 by dividing row 2 by 5

$$\begin{pmatrix} 1 & -3 & 2 & 2 \\ 0 & 0 & \boxed{1} & \frac{1}{5} \\ 0 & 0 & -3 & -1 \end{pmatrix}$$

Zeroing out all entries in pivot column below pivot entry

$$\begin{pmatrix} 1 & -3 & 0 & \frac{8}{5} \\ 0 & 0 & \boxed{1} & \frac{1}{5} \\ 0 & 0 & 0 & -\frac{2}{5} \end{pmatrix}$$

Making the pivot entry as 1 by dividing row 3 by $-\frac{2}{5}$

$$\begin{pmatrix} 1 & -3 & 0 & \frac{8}{5} \\ 0 & 0 & 1 & \frac{1}{5} \\ 0 & 0 & 0 & \boxed{1} \end{pmatrix}$$

Zeroing out all entries in pivot column below pivot entry

$$\begin{pmatrix} 1 & -3 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & \boxed{1} \end{pmatrix}$$

>>>>>>Starting backward elimination phase. The pivots are {{1, 1}, {2, 3}, {3, 4}}

Out[427]//MatrixForm=

$$\begin{pmatrix} 1 & -3 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix}$$

Out[428]//MatrixForm=

$$\begin{pmatrix} 1 & -3 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix}$$

Example 12 Added Feb 17,2026

In[429]:=

```
mat = {{2, -1, -1, 1, 3}, {4, 1, 1, 1, 2}, {6, 3, -1, 1, -1}, {8, 5, -1, 1, -1}};
{result, pivots} = displayRREF[mat];
MatrixForm[result]
RowReduce[mat] // MatrixForm
```

$$\begin{pmatrix} 2 & -1 & -1 & 1 & 3 \\ 4 & 1 & 1 & 1 & 2 \\ 6 & 3 & -1 & 1 & -1 \\ 8 & 5 & -1 & 1 & -1 \end{pmatrix}$$

Swapping row 1 with row 4 since row 4 has larger pivot

$$\begin{pmatrix} \boxed{8} & 5 & -1 & 1 & -1 \\ 4 & 1 & 1 & 1 & 2 \\ 6 & 3 & -1 & 1 & -1 \\ 2 & -1 & -1 & 1 & 3 \end{pmatrix}$$

Making the pivot entry as 1 by dividing row 1 by 8

$$\begin{pmatrix} \boxed{1} & \frac{5}{8} & -\frac{1}{8} & \frac{1}{8} & -\frac{1}{8} \\ 4 & 1 & 1 & 1 & 2 \\ 6 & 3 & -1 & 1 & -1 \\ 2 & -1 & -1 & 1 & 3 \end{pmatrix}$$

Zeroing out all entries in pivot column below pivot entry

$$\begin{pmatrix} \boxed{1} & \frac{5}{8} & -\frac{1}{8} & \frac{1}{8} & -\frac{1}{8} \\ 0 & -\frac{3}{2} & \frac{3}{2} & \frac{1}{2} & \frac{5}{2} \\ 0 & -\frac{3}{4} & -\frac{1}{4} & \frac{1}{4} & -\frac{1}{4} \\ 0 & -\frac{9}{4} & -\frac{3}{4} & \frac{3}{4} & \frac{13}{4} \end{pmatrix}$$

Swapping row 2 with row 4 since row 4 has larger pivot

$$\begin{pmatrix} 1 & \frac{5}{8} & -\frac{1}{8} & \frac{1}{8} & -\frac{1}{8} \\ 0 & \boxed{-\frac{9}{4}} & -\frac{3}{4} & \frac{3}{4} & \frac{13}{4} \\ 0 & -\frac{3}{4} & -\frac{1}{4} & \frac{1}{4} & -\frac{1}{4} \\ 0 & -\frac{3}{2} & \frac{3}{2} & \frac{1}{2} & \frac{5}{2} \end{pmatrix}$$

Making the pivot entry as 1 by dividing row 2 by $-\frac{9}{4}$

$$\begin{pmatrix} 1 & \frac{5}{8} & -\frac{1}{8} & \frac{1}{8} & -\frac{1}{8} \\ 0 & \boxed{1} & \frac{1}{3} & -\frac{1}{3} & -\frac{13}{9} \\ 0 & -\frac{3}{4} & -\frac{1}{4} & \frac{1}{4} & -\frac{1}{4} \\ 0 & -\frac{3}{2} & \frac{3}{2} & \frac{1}{2} & \frac{5}{2} \end{pmatrix}$$

Zeroing out all entries in pivot column below pivot entry

$$\begin{pmatrix} 1 & 0 & -\frac{1}{3} & \frac{1}{3} & \frac{7}{9} \\ 0 & \boxed{1} & \frac{1}{3} & -\frac{1}{3} & -\frac{13}{9} \\ 0 & 0 & 0 & 0 & -\frac{4}{3} \\ 0 & 0 & 2 & 0 & \frac{1}{3} \end{pmatrix}$$

Swapping row 3 with row 4 since row 4 has larger pivot

$$\begin{pmatrix} 1 & 0 & -\frac{1}{3} & \frac{1}{3} & \frac{7}{9} \\ 0 & 1 & \frac{1}{3} & -\frac{1}{3} & -\frac{13}{9} \\ 0 & 0 & \boxed{2} & 0 & \frac{1}{3} \\ 0 & 0 & 0 & 0 & -\frac{4}{3} \end{pmatrix}$$

Making the pivot entry as 1 by dividing row 3 by 2

$$\begin{pmatrix} 1 & 0 & -\frac{1}{3} & \frac{1}{3} & \frac{7}{9} \\ 0 & 1 & \frac{1}{3} & -\frac{1}{3} & -\frac{13}{9} \\ 0 & 0 & \boxed{1} & 0 & \frac{1}{6} \\ 0 & 0 & 0 & 0 & -\frac{4}{3} \end{pmatrix}$$

Zeroing out all entries in pivot column below pivot entry

$$\begin{pmatrix} 1 & 0 & 0 & \frac{1}{3} & \frac{5}{6} \\ 0 & 1 & 0 & -\frac{1}{3} & -\frac{3}{2} \\ 0 & 0 & \boxed{1} & 0 & \frac{1}{6} \\ 0 & 0 & 0 & 0 & -\frac{4}{3} \end{pmatrix}$$

Making the pivot entry as 1 by dividing row 4 by $-\frac{4}{3}$

$$\begin{pmatrix} 1 & 0 & 0 & \frac{1}{3} & \frac{5}{6} \\ 0 & 1 & 0 & -\frac{1}{3} & -\frac{3}{2} \\ 0 & 0 & 1 & 0 & \frac{1}{6} \\ 0 & 0 & 0 & 0 & \boxed{1} \end{pmatrix}$$

Zeroing out all entries in pivot column below pivot entry

$$\begin{pmatrix} 1 & 0 & 0 & \frac{1}{3} & 0 \\ 0 & 1 & 0 & -\frac{1}{3} & 0 \\ 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & \boxed{1} \end{pmatrix}$$

>>>>>>Starting backward elimination phase. The pivots are {{1, 1}, {2, 2}, {3, 3}, {4, 5}}

Out[431]//MatrixForm=

$$\begin{pmatrix} 1 & 0 & 0 & \frac{1}{3} & 0 \\ 0 & 1 & 0 & -\frac{1}{3} & 0 \\ 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 \end{pmatrix}$$

Out[432]//MatrixForm=

$$\begin{pmatrix} 1 & 0 & 0 & \frac{1}{3} & 0 \\ 0 & 1 & 0 & -\frac{1}{3} & 0 \\ 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 \end{pmatrix}$$

In[433]:=

```
pivots
```

Out[433]=

```
{{1, 1}, {2, 2}, {3, 3}, {4, 5}}
```

In[46]:=

```
MatrixRank[mat]
```

Out[46]=

```
4
```

Example 13 Added Feb 17,2026

In[434]:=

```
mat = {{0.4898, 0.2760, 0.4984, 0.7513, 0.9593, 0.8407, 0.3500, 0.3517, 0.2858},
{0.4456, 0.6797, 0.9597, 0.2551, 0.5472, 0.2543, 0.1966, 0.830, 0.7572},
{0.6463, 0.6551, 0.3404, 0.5060, 0.1386, 0.8143, 0.2511, 0.5853, 0.7537},
{0.7094, 0.1626, 0.5853, 0.6991, 0.1493, 0.2435, 0.6160, 0.5497, 0.3804},
{0.7547, 0.1190, 0.2238, 0.8909, 0.2575, 0.9293, 0.4733, 0.9172, 0.5678}};
{result, pivots} = displayRREF[mat];
MatrixForm[Chop@result]
RowReduce[mat] // MatrixForm
```

$$\begin{pmatrix} 0.4898 & 0.276 & 0.4984 & 0.7513 & 0.9593 & 0.8407 & 0.35 & 0.3517 & 0.2858 \\ 0.4456 & 0.6797 & 0.9597 & 0.2551 & 0.5472 & 0.2543 & 0.1966 & 0.83 & 0.7572 \\ 0.6463 & 0.6551 & 0.3404 & 0.506 & 0.1386 & 0.8143 & 0.2511 & 0.5853 & 0.7537 \\ 0.7094 & 0.1626 & 0.5853 & 0.6991 & 0.1493 & 0.2435 & 0.616 & 0.5497 & 0.3804 \\ 0.7547 & 0.119 & 0.2238 & 0.8909 & 0.2575 & 0.9293 & 0.4733 & 0.9172 & 0.5678 \end{pmatrix}$$

Swapping row 1 with row 5 since row 5 has larger pivot

$$\begin{pmatrix} \boxed{0.7547} & 0.119 & 0.2238 & 0.8909 & 0.2575 & 0.9293 & 0.4733 & 0.9172 & 0.5678 \\ 0.4456 & 0.6797 & 0.9597 & 0.2551 & 0.5472 & 0.2543 & 0.1966 & 0.83 & 0.7572 \\ 0.6463 & 0.6551 & 0.3404 & 0.506 & 0.1386 & 0.8143 & 0.2511 & 0.5853 & 0.7537 \\ 0.7094 & 0.1626 & 0.5853 & 0.6991 & 0.1493 & 0.2435 & 0.616 & 0.5497 & 0.3804 \\ 0.4898 & 0.276 & 0.4984 & 0.7513 & 0.9593 & 0.8407 & 0.35 & 0.3517 & 0.2858 \end{pmatrix}$$

Making the pivot entry as 1 by dividing row 1 by 0.7547

$$\begin{pmatrix} \boxed{1.} & 0.157679 & 0.296542 & 1.18047 & 0.341195 & 1.23135 & 0.627137 & 1.21532 & 0.752352 \\ 0.4456 & 0.6797 & 0.9597 & 0.2551 & 0.5472 & 0.2543 & 0.1966 & 0.83 & 0.7572 \\ 0.6463 & 0.6551 & 0.3404 & 0.506 & 0.1386 & 0.8143 & 0.2511 & 0.5853 & 0.7537 \\ 0.7094 & 0.1626 & 0.5853 & 0.6991 & 0.1493 & 0.2435 & 0.616 & 0.5497 & 0.3804 \\ 0.4898 & 0.276 & 0.4984 & 0.7513 & 0.9593 & 0.8407 & 0.35 & 0.3517 & 0.2858 \end{pmatrix}$$

Zeroing out all entries in pivot column below pivot entry

$$\begin{pmatrix} \boxed{1.} & 0.157679 & 0.296542 & 1.18047 & 0.341195 & 1.23135 & 0.627137 & 1.21532 & 0.752352 \\ 0. & 0.609438 & 0.827561 & -0.270917 & 0.395163 & -0.29439 & -0.0828521 & 0.288455 & 0.421952 \\ 0. & 0.553192 & 0.148745 & -0.256937 & -0.0819144 & 0.0184784 & -0.154218 & -0.20016 & 0.267455 \\ 0. & 0.0507428 & 0.374933 & -0.138325 & -0.0927439 & -0.63002 & 0.171109 & -0.312446 & -0.153318 \\ 0. & 0.198769 & 0.353154 & 0.173106 & 0.792183 & 0.237585 & 0.0428285 & -0.243562 & -0.082702 \end{pmatrix}$$

Making the pivot entry as 1 by dividing row 2 by 0.609438

$$\begin{pmatrix} 1. & 0.157679 & 0.296542 & 1.18047 & 0.341195 & 1.23135 & 0.627137 & 1.21532 & 0.752352 \\ 0. & \boxed{1.} & 1.35791 & -0.444535 & 0.648406 & -0.483051 & -0.135948 & 0.473312 & 0.692362 \\ 0. & 0.553192 & 0.148745 & -0.256937 & -0.0819144 & 0.0184784 & -0.154218 & -0.20016 & 0.267455 \\ 0. & 0.0507428 & 0.374933 & -0.138325 & -0.0927439 & -0.63002 & 0.171109 & -0.312446 & -0.153318 \\ 0. & 0.198769 & 0.353154 & 0.173106 & 0.792183 & 0.237585 & 0.0428285 & -0.243562 & -0.082702 \end{pmatrix}$$

Zeroing out all entries in pivot column below pivot entry

$$\begin{pmatrix} 1. & 0. & 0.0824288 & 1.25056 & 0.238955 & 1.30752 & 0.648573 & 1.14069 & 0.643181 \\ 0. & \boxed{1.} & 1.35791 & -0.444535 & 0.648406 & -0.483051 & -0.135948 & 0.473312 & 0.692362 \\ 0. & 0. & -0.602439 & -0.0110235 & -0.440608 & 0.285698 & -0.0790129 & -0.461992 & -0.115554 \\ 0. & 0. & 0.306029 & -0.115768 & -0.125646 & -0.605508 & 0.178008 & -0.336463 & -0.188451 \\ 0. & 0. & 0.0832439 & 0.261466 & 0.6633 & 0.3336 & 0.0698508 & -0.337642 & -0.220322 \end{pmatrix}$$

Making the pivot entry as 1 by dividing row 3 by -0.602439

$$\begin{pmatrix} 1. & 0. & 0.0824288 & 1.25056 & 0.238955 & 1.30752 & 0.648573 & 1.14069 & 0.643181 \\ 0. & 1. & 1.35791 & -0.444535 & 0.648406 & -0.483051 & -0.135948 & 0.473312 & 0.692362 \\ 0. & 0. & \boxed{1.} & 0.0182982 & 0.731373 & -0.474236 & 0.131155 & 0.76687 & 0.191811 \\ 0. & 0. & 0.306029 & -0.115768 & -0.125646 & -0.605508 & 0.178008 & -0.336463 & -0.188451 \\ 0. & 0. & 0.0832439 & 0.261466 & 0.6633 & 0.3336 & 0.0698508 & -0.337642 & -0.220322 \end{pmatrix}$$

Zeroing out all entries in pivot column below pivot entry

$$\begin{pmatrix} 1. & 0. & 0. & 1.24905 & 0.178669 & 1.34661 & 0.637762 & 1.07747 & 0.627371 \\ 0. & 1. & 0. & -0.469383 & -0.344731 & 0.160918 & -0.314045 & -0.568026 & 0.4319 \\ 0. & 0. & \boxed{1.} & 0.0182982 & 0.731373 & -0.474236 & 0.131155 & 0.76687 & 0.191811 \\ 0. & 0. & 0. & -0.121368 & -0.349467 & -0.460378 & 0.13787 & -0.571148 & -0.247151 \\ 0. & 0. & 0. & 0.259943 & 0.602417 & 0.373077 & 0.0589329 & -0.401479 & -0.236289 \end{pmatrix}$$

Swapping row 4 with row 5 since row 5 has larger pivot

$$\begin{pmatrix} 1. & 0. & 0. & 1.24905 & 0.178669 & 1.34661 & 0.637762 & 1.07747 & 0.627371 \\ 0. & 1. & 0. & -0.469383 & -0.344731 & 0.160918 & -0.314045 & -0.568026 & 0.4319 \\ 0. & 0. & 1. & 0.0182982 & 0.731373 & -0.474236 & 0.131155 & 0.76687 & 0.191811 \\ 0. & 0. & 0. & \boxed{0.259943} & 0.602417 & 0.373077 & 0.0589329 & -0.401479 & -0.236289 \\ 0. & 0. & 0. & -0.121368 & -0.349467 & -0.460378 & 0.13787 & -0.571148 & -0.247151 \end{pmatrix}$$

Making the pivot entry as 1 by dividing row 4 by 0.259943

$$\begin{pmatrix} 1. & 0. & 0. & 1.24905 & 0.178669 & 1.34661 & 0.637762 & 1.07747 & 0.627371 \\ 0. & 1. & 0. & -0.469383 & -0.344731 & 0.160918 & -0.314045 & -0.568026 & 0.4319 \\ 0. & 0. & 1. & 0.0182982 & 0.731373 & -0.474236 & 0.131155 & 0.76687 & 0.191811 \\ 0. & 0. & 0. & \boxed{1.} & 2.3175 & 1.43523 & 0.226715 & -1.54449 & -0.909004 \\ 0. & 0. & 0. & -0.121368 & -0.349467 & -0.460378 & 0.13787 & -0.571148 & -0.247151 \end{pmatrix}$$

Zeroing out all entries in pivot column below pivot entry

$$\begin{pmatrix} 1. & 0. & 0. & 0. & -2.71601 & -0.446071 & 0.354583 & 3.00663 & 1.76277 \\ 0. & 1. & 0. & 0. & 0.743062 & 0.834589 & -0.207629 & -1.29298 & 0.00522964 \\ 0. & 0. & 1. & 0. & 0.688967 & -0.500498 & 0.127007 & 0.795131 & 0.208444 \\ 0. & 0. & 0. & \boxed{1.} & 2.3175 & 1.43523 & 0.226715 & -1.54449 & -0.909004 \\ 0. & 0. & 0. & 0. & -0.0681983 & -0.286188 & 0.165386 & -0.758599 & -0.357474 \end{pmatrix}$$

Making the pivot entry as 1 by dividing row 5 by -0.0681983

$$\begin{pmatrix} 1. & 0. & 0. & 0. & -2.71601 & -0.446071 & 0.354583 & 3.00663 & 1.76277 \\ 0. & 1. & 0. & 0. & 0.743062 & 0.834589 & -0.207629 & -1.29298 & 0.00522964 \\ 0. & 0. & 1. & 0. & 0.688967 & -0.500498 & 0.127007 & 0.795131 & 0.208444 \\ 0. & 0. & 0. & 1. & 2.3175 & 1.43523 & 0.226715 & -1.54449 & -0.909004 \\ 0. & 0. & 0. & 0. & \boxed{1.} & 4.19641 & -2.42508 & 11.1234 & 5.24168 \end{pmatrix}$$

Zeroing out all entries in pivot column below pivot entry

$$\begin{pmatrix} 1. & 0. & 0. & 0. & 0. & 10.9514 & -6.23196 & 33.218 & 15.9992 \\ 0. & 1. & 0. & 0. & 0. & -2.28361 & 1.59436 & -9.55838 & -3.88967 \\ 0. & 0. & 1. & 0. & 0. & -3.39169 & 1.7978 & -6.86854 & -3.4029 \\ 0. & 0. & 0. & 1. & 0. & -8.28994 & 5.84683 & -27.323 & -13.0566 \\ 0. & 0. & 0. & 0. & \boxed{1.} & 4.19641 & -2.42508 & 11.1234 & 5.24168 \end{pmatrix}$$

>>>>>>Starting backward elimination phase. The pivots are {{1, 1}, {2, 2}, {3, 3}, {4, 4}, {5, 5}}

Zeroing out element A(1,2) using row(1)=row(1)-A(1,2)*row(2)

$$\begin{pmatrix} 1. & 0. & 0. & 0. & 0. & 10.9514 & -6.23196 & 33.218 & 15.9992 \\ 0. & \boxed{1.} & 0. & 0. & 0. & -2.28361 & 1.59436 & -9.55838 & -3.88967 \\ 0. & 0. & 1. & 0. & 0. & -3.39169 & 1.7978 & -6.86854 & -3.4029 \\ 0. & 0. & 0. & 1. & 0. & -8.28994 & 5.84683 & -27.323 & -13.0566 \\ 0. & 0. & 0. & 0. & 1. & 4.19641 & -2.42508 & 11.1234 & 5.24168 \end{pmatrix}$$

Zeroing out element A(2,3) using row(2)=row(2)-A(2,3)*row(3)

$$\begin{pmatrix} 1. & 0. & 0. & 0. & 0. & 10.9514 & -6.23196 & 33.218 & 15.9992 \\ 0. & 1. & 0. & 0. & 0. & -2.28361 & 1.59436 & -9.55838 & -3.88967 \\ 0. & 0. & \boxed{1.} & 0. & 0. & -3.39169 & 1.7978 & -6.86854 & -3.4029 \\ 0. & 0. & 0. & 1. & 0. & -8.28994 & 5.84683 & -27.323 & -13.0566 \\ 0. & 0. & 0. & 0. & 1. & 4.19641 & -2.42508 & 11.1234 & 5.24168 \end{pmatrix}$$

Zeroing out element A(1,3) using row(1)=row(1)-A(1,3)*row(3)

$$\begin{pmatrix} 1. & 0. & 0. & 0. & 0. & 10.9514 & -6.23196 & 33.218 & 15.9992 \\ 0. & 1. & 0. & 0. & 0. & -2.28361 & 1.59436 & -9.55838 & -3.88967 \\ 0. & 0. & \boxed{1.} & 0. & 0. & -3.39169 & 1.7978 & -6.86854 & -3.4029 \\ 0. & 0. & 0. & 1. & 0. & -8.28994 & 5.84683 & -27.323 & -13.0566 \\ 0. & 0. & 0. & 0. & 1. & 4.19641 & -2.42508 & 11.1234 & 5.24168 \end{pmatrix}$$

Zeroing out element A(3,4) using row(3)=row(3)-A(3,4)*row(4)

$$\begin{pmatrix} 1. & 0. & 0. & 0. & 0. & 10.9514 & -6.23196 & 33.218 & 15.9992 \\ 0. & 1. & 0. & 0. & 0. & -2.28361 & 1.59436 & -9.55838 & -3.88967 \\ 0. & 0. & 1. & 0. & 0. & -3.39169 & 1.7978 & -6.86854 & -3.4029 \\ 0. & 0. & 0. & \boxed{1.} & 0. & -8.28994 & 5.84683 & -27.323 & -13.0566 \\ 0. & 0. & 0. & 0. & 1. & 4.19641 & -2.42508 & 11.1234 & 5.24168 \end{pmatrix}$$

Zeroing out element A(2,4) using row(2)=row(2)-A(2,4)*row(4)

$$\begin{pmatrix} 1. & 0. & 0. & 0. & 0. & 10.9514 & -6.23196 & 33.218 & 15.9992 \\ 0. & 1. & 0. & 0. & 0. & -2.28361 & 1.59436 & -9.55838 & -3.88967 \\ 0. & 0. & 1. & 0. & 0. & -3.39169 & 1.7978 & -6.86854 & -3.4029 \\ 0. & 0. & 0. & \boxed{1.} & 0. & -8.28994 & 5.84683 & -27.323 & -13.0566 \\ 0. & 0. & 0. & 0. & 1. & 4.19641 & -2.42508 & 11.1234 & 5.24168 \end{pmatrix}$$

Zeroing out element A(1,4) using row(1)=row(1)-A(1,4)*row(4)

$$\begin{pmatrix} 1. & 0. & 0. & 0. & 0. & 10.9514 & -6.23196 & 33.218 & 15.9992 \\ 0. & 1. & 0. & 0. & 0. & -2.28361 & 1.59436 & -9.55838 & -3.88967 \\ 0. & 0. & 1. & 0. & 0. & -3.39169 & 1.7978 & -6.86854 & -3.4029 \\ 0. & 0. & 0. & \boxed{1.} & 0. & -8.28994 & 5.84683 & -27.323 & -13.0566 \\ 0. & 0. & 0. & 0. & 1. & 4.19641 & -2.42508 & 11.1234 & 5.24168 \end{pmatrix}$$

Zeroing out element A(4,5) using row(4)=row(4)-A(4,5)*row(5)

$$\begin{pmatrix} 1. & 0. & 0. & 0. & 0. & 10.9514 & -6.23196 & 33.218 & 15.9992 \\ 0. & 1. & 0. & 0. & 0. & -2.28361 & 1.59436 & -9.55838 & -3.88967 \\ 0. & 0. & 1. & 0. & 0. & -3.39169 & 1.7978 & -6.86854 & -3.4029 \\ 0. & 0. & 0. & 1. & 0. & -8.28994 & 5.84683 & -27.323 & -13.0566 \\ 0. & 0. & 0. & 0. & \boxed{1.} & 4.19641 & -2.42508 & 11.1234 & 5.24168 \end{pmatrix}$$

Zeroing out element A(3,5) using row(3)=row(3)-A(3,5)*row(5)

$$\begin{pmatrix} 1. & 0. & 0. & 0. & 0. & 10.9514 & -6.23196 & 33.218 & 15.9992 \\ 0. & 1. & 0. & 0. & 0. & -2.28361 & 1.59436 & -9.55838 & -3.88967 \\ 0. & 0. & 1. & 0. & 0. & -3.39169 & 1.7978 & -6.86854 & -3.4029 \\ 0. & 0. & 0. & 1. & 0. & -8.28994 & 5.84683 & -27.323 & -13.0566 \\ 0. & 0. & 0. & 0. & \boxed{1.} & 4.19641 & -2.42508 & 11.1234 & 5.24168 \end{pmatrix}$$

Zeroing out element A(2,5) using row(2)=row(2)-A(2,5)*row(5)

$$\begin{pmatrix} 1. & 0. & 0. & 0. & 0. & 10.9514 & -6.23196 & 33.218 & 15.9992 \\ 0. & 1. & 0. & 0. & 0. & -2.28361 & 1.59436 & -9.55838 & -3.88967 \\ 0. & 0. & 1. & 0. & 0. & -3.39169 & 1.7978 & -6.86854 & -3.4029 \\ 0. & 0. & 0. & 1. & 0. & -8.28994 & 5.84683 & -27.323 & -13.0566 \\ 0. & 0. & 0. & 0. & \boxed{1.} & 4.19641 & -2.42508 & 11.1234 & 5.24168 \end{pmatrix}$$

Zeroing out element A(1,5) using row(1)=row(1)-A(1,5)*row(5)

$$\begin{pmatrix} 1. & 0. & 0. & 0. & 0. & 10.9514 & -6.23196 & 33.218 & 15.9992 \\ 0. & 1. & 0. & 0. & 0. & -2.28361 & 1.59436 & -9.55838 & -3.88967 \\ 0. & 0. & 1. & 0. & 0. & -3.39169 & 1.7978 & -6.86854 & -3.4029 \\ 0. & 0. & 0. & 1. & 0. & -8.28994 & 5.84683 & -27.323 & -13.0566 \\ 0. & 0. & 0. & 0. & \boxed{1.} & 4.19641 & -2.42508 & 11.1234 & 5.24168 \end{pmatrix}$$

Out[436]//MatrixForm=

$$\begin{pmatrix} 1. & 0 & 0 & 0 & 0 & 10.9514 & -6.23196 & 33.218 & 15.9992 \\ 0 & 1. & 0 & 0 & 0 & -2.28361 & 1.59436 & -9.55838 & -3.88967 \\ 0 & 0 & 1. & 0 & 0 & -3.39169 & 1.7978 & -6.86854 & -3.4029 \\ 0 & 0 & 0 & 1. & 0 & -8.28994 & 5.84683 & -27.323 & -13.0566 \\ 0 & 0 & 0 & 0 & 1. & 4.19641 & -2.42508 & 11.1234 & 5.24168 \end{pmatrix}$$

Out[437]//MatrixForm=

$$\begin{pmatrix} 1 & 0. & 0. & 0. & 0. & 10.9514 & -6.23196 & 33.218 & 15.9992 \\ 0 & 1 & 0. & 0. & 0. & -2.28361 & 1.59436 & -9.55838 & -3.88967 \\ 0 & 0 & 1 & 0. & 0. & -3.39169 & 1.7978 & -6.86854 & -3.4029 \\ 0 & 0 & 0 & 1 & 0. & -8.28994 & 5.84683 & -27.323 & -13.0566 \\ 0 & 0 & 0 & 0 & 1 & 4.19641 & -2.42508 & 11.1234 & 5.24168 \end{pmatrix}$$