$$\boxed{e^{x} \sin(y) + \tan(y) + \left(e^{x} \cos(y) + x \sec(y)^{2}\right) y' = 0}$$

The first step is to write the ODE in standard form to check for exactness, which is

$$M(x,y)\,\mathrm{d}x + N(x,y)\,\mathrm{d}y = 0 \tag{1A}$$

Therefore

$$\begin{aligned}
\left(e^{x} \cos(y) + x \sec(y)^{2}\right) \mathrm{d}y &= \left(-e^{x} \sin(y) - \tan(y)\right) \mathrm{d}x \\
\left(e^{x} \sin(y) + \tan(y)\right) \mathrm{d}x + \left(e^{x} \cos(y) + x \sec(y)^{2}\right) \mathrm{d}y &= 0 \qquad (2A)
\end{aligned}$$

Comparing (1A) and (2A) shows that

$$\begin{aligned}
M(x,y) &= e^{x} \sin(y) + \tan(y) \\
N(x,y) &= e^{x} \cos(y) + x \sec(y)^{2}
\end{aligned}$$

The next step is to determine if the ODE is is exact or not. The ODE is exact when the following condition is satisfied

$$\frac{\partial M}{\partial y} = \frac{\partial N}{\partial x}$$

Using result found above gives

$$\begin{aligned}
\frac{\partial M}{\partial y} &= \frac{\partial}{\partial y}\left(e^{x} \sin(y) + \tan(y)\right) \\
&= e^{x} \cos(y) + \sec(y)^{2}
\end{aligned}$$

And

$$\begin{aligned}
\frac{\partial N}{\partial x} &= \frac{\partial}{\partial x}\left(e^{x} \cos(y) + x \sec(y)^{2}\right) \\
&= e^{x} \cos(y) + \sec(y)^{2}
\end{aligned}$$

Since $\frac{\partial M}{\partial y} = \frac{\partial N}{\partial x}$, then the ODE is exact The following equations are now set up to solve for the function $\phi(x,y)$

$$\frac{\partial \phi}{\partial x} = M \tag{1}$$

$$\frac{\partial \phi}{\partial y} = N \tag{2}$$

Integrating (1) w.r.t x gives

$$\begin{aligned}
\int \frac{\partial \phi}{\partial x}\,\mathrm{d}x &= \int M\,\mathrm{d}x \\
\int \frac{\partial \phi}{\partial x}\,\mathrm{d}x &= \int e^{x} \sin(y) + \tan(y)\,\mathrm{d}x \\
\phi &= e^{x} \sin(y) + x \tan(y) + f(y) \qquad (3)
\end{aligned}$$

Where $f(y)$ is used for the constant of integration since ϕ is a function of both x and y. Taking derivative of equation (3) w.r.t y gives

$$\begin{aligned}\frac{\partial \phi}{\partial y} &= e^{x}\cos(y) + x\left(1+\tan(y)^{2}\right) + f'(y) \qquad (4)\\ &= e^{x}\cos(y) + x\sec(y)^{2} + f'(y)\end{aligned}$$

But equation (2) says that $\frac{\partial \phi}{\partial y} = e^{x}\cos(y) + x\sec(y)^{2}$. Therefore equation (4) becomes

$$e^{x}\cos(y) + x\sec(y)^{2} = e^{x}\cos(y) + x\sec(y)^{2} + f'(y) \qquad (5)$$

Solving equation (5) for $f'(y)$ gives

$$f'(y) = 0$$

Therefore

$$f(y) = c_1$$

Where c_1 is constant of integration. Substituting this result for $f(y)$ into equation (3) gives ϕ

$$\phi = e^{x}\sin(y) + x\tan(y) + c_1$$

But since ϕ itself is a constant function, then let $\phi = c_0$ where c_0 is new constant and combining c_1 and c_0 constants into new constant c_1 gives the solution as

$$c_1 = e^{x}\sin(y) + x\tan(y)$$