$$\boxed{y''y'-y^n=0}$$

This is solved by reduction of order by using substitution which makes the dependent variable y an independent variable. Using $v(y)=y'(x)$. Applying this transformation the ode becomes

$$v\left(vv'\right)-y^n=0$$

Where from now on v' is understood as derivative w.r.t. y. The above is now solved as first order ode in $v\left(y\right)$. This is separable first order ODE.

$$\begin{aligned}
v^2\,\mathrm{d}v&=y^n\,\mathrm{d}y\\
\int v^2\,\mathrm{d}v&=\int y^n\,\mathrm{d}y\\
\frac{v^3}{3}&=\frac{y^{1+n}}{1+n}+c_1
\end{aligned}$$

Substititing $v=y'(x)$ gives the following ode

$$\frac{\left(y'\right)^3}{3}=\frac{y^{1+n}}{1+n}+c_1$$

Solving for y' gives

$$\begin{aligned}
y'&=\frac{\left(\left(3c_2n+3y^{1+n}+3c_2\right)\left(1+n\right)^2\right)^{\frac{1}{3}}}{1+n} &(1)\\
y'&=-\frac{\left(\left(3c_2n+3y^{1+n}+3c_2\right)\left(1+n\right)^2\right)^{\frac{1}{3}}}{2\left(1+n\right)}+\frac{i\sqrt{3}\,\left(\left(3c_2n+3y^{1+n}+3c_2\right)\left(1+n\right)^2\right)^{\frac{1}{3}}}{2+2n} &(2)\\
y'&=-\frac{\left(\left(3c_2n+3y^{1+n}+3c_2\right)\left(1+n\right)^2\right)^{\frac{1}{3}}}{2\left(1+n\right)}-\frac{i\sqrt{3}\,\left(\left(3c_2n+3y^{1+n}+3c_2\right)\left(1+n\right)^2\right)^{\frac{1}{3}}}{2\left(1+n\right)} &(3)
\end{aligned}$$

Each one of the above ode's is separable. For example, for the first ode above, integrating gives

$$\int_0^y\frac{\left(1+n\right)3^{\frac{2}{3}}}{3\left(\left(1+n\right)^2\left(c_2\left(1+n\right)+\_a^{1+n}\right)\right)^{\frac{1}{3}}}d\_a-x-c_1=0$$

Similarly for the second and third ode's. At the end, the following are the solutions found

$$\int_0^y\frac{\left(1+n\right)3^{\frac{2}{3}}}{3\left(\left(1+n\right)^2\left(c_2\left(1+n\right)+\_a^{1+n}\right)\right)^{\frac{1}{3}}}d\_a-x-c_1=0$$

$$\int_0^y\frac{2+2n}{\left(\left(1+n\right)^2\left(c_2\left(1+n\right)+\_a^{1+n}\right)\right)^{\frac{1}{3}}\left(i3^{\frac{5}{6}}-3^{\frac{1}{3}}\right)}d\_a-x-c_1=0$$

$$\int_0^y -\frac{2+2n}{\left((1+n)^2\left(c_2\left(1+n\right)+\_a^{1+n}\right)\right)^{\frac{1}{3}}\left(i3^{\frac{5}{6}}+3^{\frac{1}{3}}\right)}d\_a - x - c_1 = 0$$