

(*reference <https://mathematica.stackexchange.com/questions/311157/does-mathematica-have-weaker-symbolic-computation-especially-symbolic-polynomial>*)

TIMING only (see end of notebook for OUTPUT only)

Example 1

```
In[*]:= Quit[]

In[*]:= Expand[Times @@ (x - Tuples[{-1, 1}, Length@#] .# &[Sqrt@{2, 3, 5, 7, 11}])];
Factor[%, Extension -> {Sqrt[2], Sqrt[3], Sqrt[5]}]; // AbsoluteTiming

Out[*]=
{7.06011, Null}
```

Example 2

```
In[*]:= Quit[]

In[*]:= ClearSystemCache[];
MinimalPolynomial[2^(1/2) + 2^(1/3) + 2^(1/5) + 2^(1/7), x]; // AbsoluteTiming

Out[*]=
{3.65126, Null}
```

Example 3 (*Aborted after 10 minutes. Did not want to wait more...*)

```
In[*]:= Quit[]

In[*]:= Minimize[Sqrt[2 a^2 - 8 a + 10] + Sqrt[b^2 - 6 b + 10] + Sqrt[2 a^2 - 2 a b + b^2], {a, b}]; //
AbsoluteTiming
Minimize[Sqrt[x^2 + (y - 1)^2] + Sqrt[(x - 1)^2 + y^2] + Sqrt[(x + 1)^2 + y^2], {x, y}]; //
AbsoluteTiming

Out[*]=
$Aborted

Out[*]=
$Aborted
```

Example 4 a

```
In[*]:= Quit[]

In[*]:= Expand[Sum[i x[i], {i, 1, 12}]^12]; // AbsoluteTiming

Out[*]=
{2.34869, Null}
```

Example 4 b

```
In[*]:= Quit[]
In[*]:= Expand[(2 x + 3 y + 4 z) ^50 (5 x + 6 y + 7 z) ^51 (8 x + 9 y + 10 z) ^52]; // AbsoluteTiming
Out[*]=
{0.311462, Null}
```

Example 5

```
In[*]:= Quit[]
In[*]:= PolynomialMod[
  (-2826 * x^12 - 8790 + x^6 * (6631 - 4230 * y) - 8419 * y + 5267 * y^2 + 8182 * y^3 - 3940 * y^4 -
  2386 * y^5 + x * (-6026 * y^5 + 1622 * y^4 - 6940 * y^3 - 1813 * y^2 - 6030 * y - 7923)) ^2 *
  (((9756 * x^30 - 6880 * y^5 + 3699 * y^4 + 1340 * y^3 - 9362 * y^2 + 2794 * y + 5059) - 2578) +
  x * (-3125 * y^5 + 3550 * y^4 - 6425 * y^3 + 127 * y^2 + 7206 * y - 899)),
  -84 568 - 10 331 y - 91 710 y^2 - 79 947 y^3 + 38 747 y^4 - 97 554 y^5 + 55 518 y^6] //
  Factor[#, Modulus -> -84 568 - 10 331 y - 91 710 y^2 - 79 947 y^3 +
  38 747 y^4 - 97 554 y^5 + 55 518 y^6] &; // AbsoluteTiming
Out[*]=
{40.5864, Null}
```

Example 6

```
In[*]:= Quit[]
```

```

In[ ]:= PolynomialGCD[2 140 562 698 718 732 140 814 186 601 134 305 108 361 857 809 490 -
  2 784 610 594 523 673 638 484 774 509 153 686 569 751 130 624 770 x +
  3 858 489 651 140 979 026 284 638 768 361 566 754 023 704 101 963 x^2 -
  286 771 157 043 724 591 851 751 736 007 270 812 641 162 273 605 x^3 +
  1 040 070 357 631 236 974 043 798 352 460 064 492 096 759 420 031 x^4 +
  527 961 813 592 576 990 225 122 032 458 485 764 714 353 948 184 x^5 -
  223 657 724 243 176 368 425 265 508 444 964 464 071 309 890 719 x^6 -
  3 733 131 181 727 340 801 070 133 191 959 530 969 316 170 794 800 x^30 +
  10 745 497 692 902 796 595 974 970 291 342 011 573 796 136 255 136 x^31 -
  7 905 771 579 741 612 232 706 547 483 087 384 561 886 791 433 680 x^32 +
  1 395 228 177 175 535 320 672 887 161 804 454 665 548 111 711 824 x^33 -
  5 230 961 365 618 779 655 379 147 055 541 516 478 070 736 328 364 x^34 -
  3 706 780 992 584 927 498 391 542 297 913 941 634 360 275 844 528 x^35 -
  11 678 244 373 494 087 080 209 876 359 797 173 892 928 099 938 592 x^60 -
  5 514 859 882 567 035 785 577 006 631 456 739 844 414 574 289 760 x^61 -
  21 295 212 021 140 539 667 882 396 747 720 054 113 720 182 080 880 x^62 -
  21 789 643 204 816 141 848 070 620 182 749 478 757 088 623 737 712 x^63 -
  4 720 988 527 381 715 377 218 961 193 451 142 849 496 347 654 320 x^64 -
  27 444 066 359 527 368 757 788 068 859 219 000 443 573 584 940 160 x^90 -
  33 565 578 213 248 720 198 933 124 767 949 411 665 123 563 970 688 x^91 -
  26 098 925 852 445 627 183 784 692 986 363 859 867 173 544 281 664 x^92 -
  7 417 015 118 303 371 350 476 354 681 923 558 088 324 668 065 600 x^93 -
  26 712 940 739 577 401 605 838 204 365 261 893 824 578 131 755 520 x^120 -
  21 489 656 182 324 275 196 520 668 674 864 724 640 109 707 593 728 x^121 -
  3 537 371 331 357 954 585 225 002 235 363 286 524 220 979 284 224 x^122 -
  7 370 705 120 862 668 705 991 302 948 457 148 370 726 768 590 848 x^150 -
  3 449 161 085 875 451 377 293 133 065 291 672 477 113 785 516 032 x^151 +
  454 778 382 410 716 370 295 091 942 238 975 349 531 795 652 608 x^180,
  (-2826 * x^12 - 8790 + x^6 * (6631 - 4230 * y) - 8419 * y + 5267 * y^2 + 8182 * y^3 - 3940 * y^4 -
    2386 * y^5 + x * (-6026 * y^5 + 1622 * y^4 - 6940 * y^3 - 1813 * y^2 - 6030 * y - 7923)) ^2 *
    ((9756 * x^30 - 6880 * y^5 + 3699 * y^4 + 1340 * y^3 - 9362 * y^2 + 2794 * y + 5059) - 2578) +
    x * (-3125 * y^5 + 3550 * y^4 - 6425 * y^3 + 127 * y^2 + 7206 * y - 899) // Expand,
  Modulus -> -84 568 - 10 331 y - 91 710 y^2 - 79 947 y^3 + 38 747 y^4 - 97 554 y^5 + 55 518 y^6] //
  AbsoluteTiming

```

```
Out[ ]:=
```

```
{207.387, 2481 - 899 x + 9756 x^30 + 2794 y + 7206 x y - 9362 y^2 +
  127 x y^2 + 1340 y^3 - 6425 x y^3 + 3699 y^4 + 3550 x y^4 - 6880 y^5 - 3125 x y^5}
```

Example 7

```
In[ ]:= Quit[]
```

```
In[ ]:= f = -10 - 8 x + x^2 + 3 x^3 - 6 x^4 - 8 x^5 - 2 x^6 + 7 x^7 +
2 x^8 + x^33 + 5 x^34 + 9 x^35 - x^36 - 8 x^37 - x^47 - 2 x^48 + x^49 +
(6 + x - 9 x^3 + 7 x^4 - 10 x^5 - 5 x^6 + 9 x^7 + 6 x^8 + x^9 + 2 x^10 - 10 x^11 + 3 x^12 - 5 x^38 -
6 x^39 - 7 x^40 + 10 x^41 + x^42 + 6 x^43 - 9 x^44 - 3 x^45 + 9 x^46 + 6 x^47 + 3 x^48) y +
(1 + 3 x - 9 x^2 - 9 x^3 + 9 x^4 - 8 x^5 + 5 x^6 - x^7 + 9 x^35 - 7 x^36 + 10 x^37 +
6 x^41 - 6 x^42 - 4 x^43 + 7 x^44 - 8 x^46 - 4 x^47) y^2 +
(2 - 6 x + 9 x^2 + 3 x^4 + 9 x^5 - x^6 + 3 x^7 - 10 x^8 + 3 x^9 + 10 x^38 + 9 x^39 - 5 x^40 + 5 x^41)
y^8 + (-2 + 8 x - 3 x^2 + 6 x^3 - 2 x^4 - 3 x^5 - 4 x^7 - x^8 +
2 x^9 - 2 x^10 - x^36 - 7 x^37 + 9 x^38 - 7 x^39 - 6 x^40) y^9 +
(6 - 10 x - 2 x^2 - 9 x^3 + 2 x^4 + 10 x^5 - 2 x^7 - 4 x^8 + 7 x^9 - 4 x^35 + 6 x^36) y^13 +
(-7 + x + 10 x^2 - 7 x^3 - 6 x^4 - 9 x^5 + 3 x^6 - 7 x^7) y^42 +
(3 + 3 x - 9 x^2 + 7 x^4 + 2 x^5 - 8 x^6) y^43 + (-1 + x - 4 x^2 - 6 x^3 - x^4 + 3 x^5) y^44 +
(1 + 2 x + 3 x^2 + 6 x^3 - 7 x^4) y^45 + (9 - 9 x - 7 x^2 + 9 x^3) y^46 +
(-6 - 9 x + 5 x^2) y^47 + (-3 - 2 x) y^48 + 6 y^49;
Discriminant[f, y]; // AbsoluteTiming
```

```
Out[ ]:=
{13.2427, Null}
```

Example 8

```
In[ ]:= Quit[]
In[ ]:= f = 32 x^10 + 400 x^9 y + 176 x^10 y + 2240 x^8 y^2 + 2272 x^9 y^2 + 648 x^10 y^2 + 7400 x^7 y^3 +
12776 x^8 y^3 + 6484 x^9 y^3 - 48 x^10 y^3 + 15970 x^6 y^4 + 41192 x^7 y^4 + 28070 x^8 y^4 -
1020 x^9 y^4 - 550 x^10 y^4 + 23525 x^5 y^5 + 84091 x^6 y^5 + 68643 x^7 y^5 - 6112 x^8 y^5 -
4289 x^9 y^5 - 149 x^10 y^5 + 23955 x^4 y^6 + 112892 x^5 y^6 + 103672 x^6 y^6 -
17447 x^7 y^6 - 13730 x^8 y^6 - 786 x^9 y^6 + 81 x^10 y^6 + 16650 x^3 y^7 + 99786 x^4 y^7 +
98987 x^5 y^7 - 27472 x^6 y^7 - 23093 x^7 y^7 - 1538 x^8 y^7 + 504 x^9 y^7 + 66 x^10 y^7 +
7560 x^2 y^8 + 56052 x^3 y^8 + 58329 x^4 y^8 - 24561 x^5 y^8 - 21524 x^6 y^8 - 1313 x^7 y^8 +
1171 x^8 y^8 + 318 x^9 y^8 + 36 x^10 y^8 + 2025 x y^9 + 18171 x^2 y^9 + 19386 x^3 y^9 -
11718 x^4 y^9 - 10543 x^5 y^9 - 396 x^6 y^9 + 1204 x^7 y^9 + 508 x^8 y^9 + 117 x^9 y^9 +
11 x^10 y^9 + 243 y^10 + 2592 x y^10 + 2781 x^2 y^10 - 2322 x^3 y^10 - 2121 x^4 y^10 +
17 x^5 y^10 + 461 x^6 y^10 + 266 x^7 y^10 + 91 x^8 y^10 + 16 x^9 y^10 + x^10 y^10;
Solve[{f, D[f, x], D[f, y]} == 0, {x, y}] // AbsoluteTiming
```

```
Out[ ]:=
{41.1658, {{x -> 0, y -> 0}, {x -> -1/2, y -> 1/3}, {x -> -1, y -> 1}}}
```

Example 9

```
In[ ]:= Quit[]
In[ ]:= CoefficientList[
(a1 x1 + a10 x10 + a11 x11 + a2 x2 + a3 x3 + a4 x4 + a5 x5 + a6 x6 + a7 x7 + a8 x8 + a9 x9) ^11 //
Expand, {x1, x2, x3, x4, x5, x6, x7, x8, x9, x10, x11}]; // AbsoluteTiming
```

```
Out[ ]:=
{4.84414, Null}
```

Example 10

```
In[ ]:= Quit[]

In[ ]:= ClearSystemCache[];
f = Function[{x, y}, Evaluate@Expand[(x + y + 1)^60]];
a = ToNumberField@Root[
  -12992 + 40546 #1 - 59434 #1^2 - 28068 #1^3 + 10027 #1^4 + 74288 #1^5 + 95220 #1^6 &, 1];
f[55424 + 10799 a + 32924 a^2 - 94123 a^3 - 57476 a^4 + 16513 a^5,
  49641 - 47101 a - 37888 a^2 - 93639 a^3 - 63032 a^4 - 33804 a^5] // AbsoluteTiming
```

```
Out[ ]:= {9.18008, 1.34... × 10324}
```

```
In[ ]:= data = {"Problem number", "Mathematica time (sec)", "Maple time (sec)"},
  {1, 7.060114`, 0.694}, {2, 3.6512579`, 0.190}, {3, Infinity, 1.118},
  {4 a, 2.3486905`, 0.227}, {4 b, 0.3114615`, 0.113},
  {5, 40.5864421`, 0.137}, {6, 207.386756`, 0.023}, {7, 13.2426646`, 1.805},
  {8, 41.1657519`, 0.088}, {9, 4.8441381`, 1.198}, {10, 9.1800838`, 0.651}};
Grid[data, Frame -> All]
```

```
Out[ ]:=
```

Problem number	Mathematica time (sec)	Maple time (sec)
1	7.06011	0.694
2	3.65126	0.19
3	∞	1.118
4 a	2.34869	0.227
4 b	0.311462	0.113
5	40.5864	0.137
6	207.387	0.023
7	13.2427	1.805
8	41.1658	0.088
9	4.84414	1.198
10	9.18008	0.651

OUTPUT only (no timing)

Example 1

```
In[ ]:= Quit[]
```

```
In[*]:= Expand[Times @@ (x - Tuples[{-1, 1}, Length@#].# &[Sqrt@{2, 3, 5, 7, 11}]]];
Factor[%, Extension -> {Sqrt[2], Sqrt[3], Sqrt[5]}]
```

```
Out[*]=
```

$$\begin{aligned}
& (120 + 8\sqrt{6} - 8\sqrt{10} + 16\sqrt{15} + (32\sqrt{2} - 24\sqrt{3} - 8\sqrt{5} + 24\sqrt{30})x + \\
& \quad (-24 + 12\sqrt{6} + 12\sqrt{10} - 12\sqrt{15})x^2 + (4\sqrt{2} - 4\sqrt{3} - 4\sqrt{5})x^3 - x^4) \\
& (120 + 8\sqrt{6} + 8\sqrt{10} - 16\sqrt{15} + (-32\sqrt{2} + 24\sqrt{3} - 8\sqrt{5} + 24\sqrt{30})x + \\
& \quad (-24 + 12\sqrt{6} - 12\sqrt{10} + 12\sqrt{15})x^2 + (-4\sqrt{2} + 4\sqrt{3} - 4\sqrt{5})x^3 - x^4) \\
& (120 - 8\sqrt{6} - 8\sqrt{10} - 16\sqrt{15} + (-32\sqrt{2} - 24\sqrt{3} + 8\sqrt{5} + 24\sqrt{30})x + \\
& \quad (-24 - 12\sqrt{6} + 12\sqrt{10} + 12\sqrt{15})x^2 + (-4\sqrt{2} - 4\sqrt{3} + 4\sqrt{5})x^3 - x^4) \\
& (120 - 8\sqrt{6} + 8\sqrt{10} + 16\sqrt{15} + (32\sqrt{2} + 24\sqrt{3} + 8\sqrt{5} + 24\sqrt{30})x + \\
& \quad (-24 - 12\sqrt{6} - 12\sqrt{10} - 12\sqrt{15})x^2 + (4\sqrt{2} + 4\sqrt{3} + 4\sqrt{5})x^3 - x^4) \\
& (-120 - 8\sqrt{6} + 8\sqrt{10} - 16\sqrt{15} + (32\sqrt{2} - 24\sqrt{3} - 8\sqrt{5} + 24\sqrt{30})x + \\
& \quad (24 - 12\sqrt{6} - 12\sqrt{10} + 12\sqrt{15})x^2 + (4\sqrt{2} - 4\sqrt{3} - 4\sqrt{5})x^3 + x^4) \\
& (-120 - 8\sqrt{6} - 8\sqrt{10} + 16\sqrt{15} + (-32\sqrt{2} + 24\sqrt{3} - 8\sqrt{5} + 24\sqrt{30})x + \\
& \quad (24 - 12\sqrt{6} + 12\sqrt{10} - 12\sqrt{15})x^2 + (-4\sqrt{2} + 4\sqrt{3} - 4\sqrt{5})x^3 + x^4) \\
& (-120 + 8\sqrt{6} + 8\sqrt{10} + 16\sqrt{15} + (-32\sqrt{2} - 24\sqrt{3} + 8\sqrt{5} + 24\sqrt{30})x + \\
& \quad (24 + 12\sqrt{6} - 12\sqrt{10} - 12\sqrt{15})x^2 + (-4\sqrt{2} - 4\sqrt{3} + 4\sqrt{5})x^3 + x^4) \\
& (-120 + 8\sqrt{6} - 8\sqrt{10} - 16\sqrt{15} + (32\sqrt{2} + 24\sqrt{3} + 8\sqrt{5} + 24\sqrt{30})x + \\
& \quad (24 + 12\sqrt{6} + 12\sqrt{10} + 12\sqrt{15})x^2 + (4\sqrt{2} + 4\sqrt{3} + 4\sqrt{5})x^3 + x^4)
\end{aligned}$$

Example 2

```
In[*]:= Quit[]
```

```
In[*]:= ClearSystemCache[];
```

```
MinimalPolynomial[2^(1/2) + 2^(1/3) + 2^(1/5) + 2^(1/7), x]
```

```
Out[*]=
```

$$\begin{aligned}
& 8451763286190672190086742831401747098680754649962257580032 + \\
& 375051818338454624599221250957569363399277485632681001615360x + \\
& 5489618847053041239864296035268530846336059477262673054269440x^2 + \\
& 23997099804007302936740473294089072856647647939745383249346560x^3 + \\
& 7107369409137688808377787562208479525012228189701899928207360x^4 + \\
& 14996184930134090641294334277455448439285284787835824415965184x^5 + \\
& 352740013277718085405327506940608989003915701766061010955796480x^6 + \\
& 430837078726660169808675230280769018113414580688803384440913920x^7 + \\
& 62130356817274660244459961421980006776210887007019859590512640x^8 + \\
& 392678120217320912633006439187837818148985398855736204470517760x^9 - \\
& 318609776087716438291145715416958893575044527178328669899718656x^{10} - \\
& 4506713604123596559533443321913845788409865173297645300104560640x^{11} + \\
& 69155363631801423854257431436322212947885589158972488679424000x^{12} + \\
& 4631644764658898360015459848162150052836891979125372461388922880x^{13} - \\
& 20401642145820198832769745953805884210945439984315621895191920640x^{14} + \\
& 43220433919122509422237287647887326739980869145404029059312123904x^{15} - \\
& 26667489397924652662606610683382814314378284638791777434982154240x^{16} -
\end{aligned}$$

73 084 843 686 327 490 597 156 841 954 422 280 944 134 232 554 440 990 744 893 194 240 $\times^{17} +$
 70 260 645 523 600 823 542 032 203 189 702 936 980 584 290 932 679 798 655 413 125 120 $\times^{18} +$
 63 409 809 027 746 529 242 913 369 019 310 641 924 953 809 994 847 287 925 486 387 200 $\times^{19} +$
 147 096 906 858 886 789 921 461 563 103 206 362 162 841 124 550 978 812 573 760 094 208 $\times^{20} -$
 284 190 163 007 215 266 897 638 941 788 135 133 867 432 018 379 278 966 905 320 243 200 $\times^{21} +$
 84 285 351 926 665 828 588 272 247 258 187 249 618 452 915 470 341 030 036 184 760 320 $\times^{22} +$
 12 627 565 408 906 114 607 787 550 369 733 029 176 546 159 962 004 073 771 696 128 000 $\times^{23} -$
 589 182 548 391 680 302 650 559 881 253 818 389 065 340 823 350 852 621 634 780 528 640 $\times^{24} +$
 946 176 638 838 595 036 317 731 314 487 979 553 974 818 353 984 706 372 582 768 640 000 $\times^{25} +$
 923 771 902 602 444 791 122 632 569 592 252 013 387 021 489 910 754 929 483 490 263 040 $\times^{26} -$
 1 964 974 763 885 352 722 378 103 167 804 869 257 507 682 437 934 410 299 869 149 265 920 $\times^{27} -$
 1 511 099 585 539 152 310 448 120 211 601 789 068 607 013 839 249 744 259 916 245 237 760 $\times^{28} +$
 3 520 059 076 273 643 971 203 452 339 240 690 096 435 175 348 294 711 271 336 024 473 600 $\times^{29} +$
 2 132 377 821 090 794 542 006 165 401 193 486 962 264 304 157 542 270 604 401 904 189 440 $\times^{30} -$
 5 682 387 196 814 098 959 691 298 463 094 478 800 132 726 174 985 642 146 098 944 409 600 $\times^{31} -$
 2 235 433 198 645 518 599 104 291 115 190 548 544 336 060 354 538 526 801 544 358 133 760 $\times^{32} +$
 8 523 284 249 078 509 253 190 004 425 779 223 385 708 514 276 384 827 134 405 603 491 840 $\times^{33} +$
 723 721 097 681 740 026 339 935 805 327 903 426 143 050 281 568 053 635 385 875 496 960 $\times^{34} -$
 10 915 792 942 095 132 347 460 450 989 298 207 127 168 353 809 547 876 674 891 723 833 344 $\times^{35} +$
 3 127 985 853 061 419 891 539 300 711 047 424 005 651 206 422 507 615 361 779 630 080 000 $\times^{36} +$
 10 653 060 526 503 965 412 894 449 352 510 371 885 684 898 319 981 817 169 304 942 018 560 $\times^{37} -$
 7 302 072 998 711 692 969 180 710 796 770 352 402 649 845 367 642 293 301 087 871 959 040 $\times^{38} -$
 7 309 999 477 779 685 306 435 365 966 291 425 590 059 924 683 442 494 768 029 431 234 560 $\times^{39} +$
 9 413 537 441 850 592 173 932 574 551 043 205 086 294 143 739 869 721 797 576 594 817 024 $\times^{40} +$
 2 754 705 116 517 994 181 712 950 406 229 843 143 451 611 979 816 773 179 893 245 542 400 $\times^{41} -$
 8 701 009 338 204 285 434 344 631 999 081 673 575 667 999 055 979 328 065 131 700 551 680 $\times^{42} +$
 886 292 210 635 999 870 634 563 477 380 793 927 650 373 017 688 426 537 974 371 450 880 $\times^{43} +$
 6 175 585 363 224 637 286 864 467 408 445 553 415 076 758 545 777 639 910 031 417 671 680 $\times^{44} -$
 2 515 662 666 824 282 477 885 556 270 375 429 028 949 447 682 097 099 291 096 392 925 184 $\times^{45} -$
 3 452 205 855 946 091 582 297 058 228 700 841 004 981 424 619 708 715 595 942 770 769 920 $\times^{46} +$
 2 488 561 684 881 790 095 400 179 592 279 865 766 438 582 893 411 825 091 501 537 361 920 $\times^{47} +$
 1 542 571 314 879 374 158 824 022 388 660 840 043 757 275 807 961 369 873 121 185 628 160 $\times^{48} -$
 1 747 920 665 322 236 022 573 271 725 916 061 284 064 146 183 604 225 541 391 956 049 920 $\times^{49} -$
 565 922 200 810 303 958 044 791 864 982 008 219 166 073 198 594 338 796 720 848 109 568 $\times^{50} +$
 997 529 436 140 473 036 315 247 732 627 956 109 079 309 453 224 722 934 485 504 491 520 $\times^{51} +$
 193 301 572 942 715 967 622 398 083 345 753 096 442 951 504 246 803 864 451 243 048 960 $\times^{52} -$
 501 997 464 466 384 192 333 663 662 567 159 061 149 807 722 953 732 971 171 812 474 880 $\times^{53} -$
 84 224 229 838 171 683 714 639 518 680 687 131 332 511 177 228 753 888 342 840 442 880 $\times^{54} +$
 241 262 792 636 205 320 006 571 914 839 829 376 567 600 754 544 623 129 320 512 028 672 $\times^{55} +$
 51 194 579 299 759 059 929 961 730 284 691 916 482 417 940 617 852 680 459 119 493 120 $\times^{56} -$
 117 345 637 894 393 275 906 832 944 165 045 191 769 250 266 438 535 584 602 726 072 320 $\times^{57} -$
 32 247 632 452 079 412 773 263 378 055 644 260 551 166 671 858 957 317 295 623 372 800 $\times^{58} +$
 58 040 895 406 546 976 277 584 386 272 289 813 293 936 867 239 612 927 556 357 980 160 $\times^{59} +$
 17 938 385 447 112 239 018 717 225 033 814 679 916 433 777 372 658 828 701 702 553 600 $\times^{60} -$
 28 172 465 677 868 368 510 981 393 411 449 702 891 068 046 816 619 754 118 897 991 680 $\times^{61} -$
 8 674 770 517 862 458 944 816 205 831 234 384 132 215 897 765 917 855 988 559 380 480 $\times^{62} +$
 12 844 579 281 676 768 005 813 673 063 340 204 333 522 829 072 227 126 940 794 880 000 $\times^{63} +$

$$\begin{aligned}
& 3\ 794\ 759\ 981\ 283\ 931\ 257\ 868\ 882\ 552\ 613\ 837\ 807\ 024\ 724\ 877\ 788\ 845\ 528\ 076\ 779\ 520\ x^{64} - \\
& 5\ 373\ 261\ 763\ 691\ 605\ 219\ 575\ 970\ 069\ 926\ 972\ 069\ 589\ 034\ 423\ 063\ 365\ 526\ 495\ 428\ 608\ x^{65} - \\
& 1\ 576\ 382\ 343\ 366\ 187\ 215\ 825\ 853\ 622\ 270\ 572\ 778\ 644\ 713\ 447\ 786\ 956\ 662\ 232\ 842\ 240\ x^{66} + \\
& 2\ 047\ 709\ 958\ 318\ 799\ 104\ 020\ 332\ 955\ 352\ 886\ 410\ 731\ 527\ 915\ 033\ 568\ 661\ 083\ 258\ 880\ x^{67} + \\
& 644\ 510\ 170\ 120\ 578\ 168\ 183\ 233\ 534\ 523\ 829\ 202\ 976\ 095\ 165\ 520\ 530\ 269\ 620\ 142\ 080\ x^{68} - \\
& 710\ 805\ 817\ 388\ 744\ 471\ 672\ 554\ 369\ 482\ 521\ 003\ 559\ 139\ 987\ 888\ 508\ 681\ 909\ 698\ 560\ x^{69} - \\
& 261\ 238\ 673\ 830\ 369\ 732\ 516\ 148\ 656\ 907\ 879\ 335\ 356\ 026\ 605\ 166\ 012\ 825\ 431\ 179\ 264\ x^{70} + \\
& 226\ 243\ 052\ 230\ 106\ 207\ 892\ 280\ 199\ 212\ 473\ 611\ 368\ 509\ 569\ 183\ 909\ 149\ 944\ 053\ 760\ x^{71} + \\
& 101\ 851\ 492\ 599\ 873\ 219\ 845\ 018\ 474\ 456\ 583\ 711\ 142\ 616\ 036\ 273\ 087\ 588\ 964\ 761\ 600\ x^{72} - \\
& 66\ 948\ 623\ 423\ 164\ 660\ 900\ 125\ 119\ 424\ 405\ 427\ 651\ 877\ 189\ 241\ 316\ 870\ 623\ 068\ 160\ x^{73} - \\
& 36\ 058\ 685\ 156\ 349\ 660\ 046\ 904\ 509\ 180\ 705\ 840\ 941\ 293\ 080\ 900\ 460\ 365\ 181\ 091\ 840\ x^{74} + \\
& 18\ 699\ 763\ 464\ 291\ 067\ 716\ 724\ 825\ 611\ 093\ 453\ 404\ 909\ 944\ 891\ 399\ 572\ 698\ 431\ 488\ x^{75} + \\
& 10\ 733\ 560\ 014\ 650\ 223\ 418\ 935\ 327\ 110\ 831\ 872\ 924\ 416\ 786\ 265\ 096\ 843\ 878\ 727\ 680\ x^{76} - \\
& 5\ 020\ 993\ 711\ 048\ 798\ 138\ 509\ 908\ 290\ 290\ 060\ 318\ 937\ 183\ 989\ 879\ 644\ 828\ 467\ 200\ x^{77} - \\
& 2\ 268\ 167\ 794\ 078\ 002\ 613\ 251\ 047\ 590\ 153\ 154\ 675\ 632\ 763\ 716\ 560\ 850\ 460\ 344\ 320\ x^{78} + \\
& 1\ 351\ 405\ 813\ 382\ 446\ 157\ 265\ 573\ 194\ 028\ 521\ 823\ 193\ 386\ 951\ 079\ 382\ 184\ 099\ 840\ x^{79} + \\
& 41\ 290\ 532\ 247\ 901\ 731\ 030\ 243\ 925\ 949\ 229\ 953\ 427\ 474\ 759\ 989\ 077\ 358\ 936\ 064\ x^{80} - \\
& 396\ 345\ 862\ 166\ 163\ 271\ 097\ 367\ 325\ 555\ 659\ 557\ 624\ 067\ 186\ 826\ 666\ 332\ 651\ 520\ x^{81} + \\
& 279\ 382\ 075\ 917\ 544\ 313\ 988\ 965\ 040\ 273\ 752\ 641\ 406\ 493\ 876\ 944\ 068\ 678\ 778\ 880\ x^{82} + \\
& 136\ 830\ 206\ 754\ 670\ 038\ 512\ 763\ 705\ 700\ 747\ 600\ 807\ 007\ 724\ 251\ 712\ 816\ 414\ 720\ x^{83} - \\
& 188\ 388\ 108\ 880\ 862\ 332\ 898\ 667\ 394\ 464\ 895\ 567\ 891\ 852\ 134\ 866\ 227\ 092\ 520\ 960\ x^{84} - \\
& 54\ 319\ 077\ 128\ 834\ 620\ 305\ 698\ 108\ 580\ 310\ 231\ 653\ 816\ 292\ 551\ 168\ 557\ 580\ 288\ x^{85} + \\
& 86\ 278\ 747\ 650\ 469\ 349\ 539\ 680\ 100\ 243\ 111\ 329\ 290\ 102\ 975\ 115\ 898\ 351\ 779\ 840\ x^{86} + \\
& 22\ 446\ 735\ 886\ 666\ 919\ 401\ 499\ 334\ 953\ 384\ 758\ 166\ 386\ 195\ 374\ 753\ 253\ 949\ 440\ x^{87} - \\
& 32\ 007\ 240\ 788\ 129\ 770\ 572\ 937\ 250\ 319\ 444\ 654\ 732\ 457\ 899\ 262\ 766\ 949\ 335\ 040\ x^{88} - \\
& 8\ 921\ 991\ 317\ 121\ 014\ 460\ 658\ 962\ 237\ 587\ 989\ 197\ 777\ 263\ 898\ 648\ 953\ 487\ 360\ x^{89} + \\
& 10\ 019\ 900\ 508\ 090\ 776\ 225\ 259\ 150\ 005\ 796\ 410\ 182\ 177\ 798\ 525\ 748\ 775\ 485\ 440\ x^{90} + \\
& 3\ 304\ 961\ 943\ 748\ 102\ 445\ 165\ 736\ 502\ 773\ 287\ 561\ 864\ 788\ 006\ 943\ 151\ 882\ 240\ x^{91} - \\
& 2\ 627\ 157\ 469\ 943\ 540\ 520\ 309\ 904\ 223\ 204\ 423\ 026\ 870\ 179\ 780\ 281\ 303\ 040\ 000\ x^{92} - \\
& 1\ 138\ 644\ 609\ 895\ 790\ 377\ 088\ 391\ 445\ 919\ 947\ 711\ 098\ 571\ 845\ 794\ 216\ 017\ 920\ x^{93} + \\
& 541\ 271\ 785\ 765\ 315\ 884\ 909\ 466\ 175\ 896\ 562\ 050\ 963\ 829\ 921\ 104\ 684\ 646\ 400\ x^{94} + \\
& 367\ 304\ 266\ 168\ 194\ 356\ 942\ 394\ 558\ 607\ 173\ 887\ 689\ 765\ 076\ 202\ 097\ 999\ 872\ x^{95} - \\
& 66\ 230\ 530\ 256\ 825\ 786\ 065\ 894\ 153\ 824\ 779\ 887\ 039\ 383\ 523\ 987\ 966\ 525\ 440\ x^{96} - \\
& 111\ 083\ 855\ 906\ 361\ 975\ 986\ 290\ 820\ 497\ 669\ 090\ 161\ 943\ 695\ 877\ 322\ 833\ 920\ x^{97} - \\
& 8\ 295\ 210\ 665\ 105\ 669\ 218\ 091\ 657\ 535\ 821\ 961\ 430\ 313\ 242\ 379\ 097\ 538\ 560\ x^{98} + \\
& 31\ 208\ 184\ 480\ 490\ 443\ 284\ 138\ 637\ 978\ 186\ 092\ 857\ 621\ 328\ 584\ 875\ 704\ 320\ x^{99} + \\
& 8\ 843\ 168\ 360\ 915\ 823\ 554\ 795\ 517\ 258\ 284\ 599\ 219\ 913\ 988\ 081\ 701\ 093\ 376\ x^{100} - \\
& 7\ 994\ 508\ 943\ 159\ 007\ 746\ 617\ 762\ 141\ 077\ 728\ 787\ 173\ 965\ 490\ 700\ 943\ 360\ x^{101} - \\
& 3\ 658\ 079\ 015\ 561\ 930\ 965\ 650\ 287\ 628\ 899\ 772\ 276\ 671\ 544\ 125\ 002\ 219\ 520\ x^{102} + \\
& 1\ 814\ 988\ 940\ 357\ 744\ 439\ 088\ 061\ 247\ 803\ 419\ 394\ 590\ 152\ 965\ 139\ 988\ 480\ x^{103} + \\
& 1\ 142\ 785\ 694\ 141\ 037\ 640\ 224\ 098\ 191\ 023\ 235\ 442\ 607\ 803\ 431\ 694\ 499\ 840\ x^{104} - \\
& 347\ 168\ 268\ 691\ 361\ 252\ 088\ 999\ 333\ 811\ 268\ 194\ 975\ 320\ 462\ 165\ 016\ 576\ x^{105} - \\
& 301\ 865\ 244\ 875\ 520\ 647\ 744\ 080\ 273\ 189\ 390\ 352\ 424\ 876\ 010\ 429\ 808\ 640\ x^{106} + \\
& 48\ 919\ 169\ 112\ 075\ 049\ 220\ 160\ 245\ 619\ 321\ 417\ 404\ 231\ 130\ 407\ 239\ 680\ x^{107} + \\
& 69\ 919\ 997\ 378\ 653\ 136\ 490\ 626\ 351\ 102\ 125\ 917\ 217\ 142\ 964\ 708\ 966\ 400\ x^{108} - \\
& 1\ 886\ 879\ 519\ 019\ 149\ 033\ 515\ 639\ 589\ 257\ 985\ 214\ 703\ 652\ 567\ 777\ 280\ x^{109} - \\
& 14\ 287\ 057\ 354\ 788\ 291\ 556\ 074\ 581\ 405\ 289\ 908\ 427\ 846\ 972\ 213\ 166\ 080\ x^{110} -
\end{aligned}$$

$$\begin{aligned}
& 1\ 780\ 195\ 225\ 609\ 750\ 110\ 170\ 752\ 229\ 562\ 664\ 842\ 678\ 103\ 288\ 709\ 120\ x^{111} + \\
& 2\ 522\ 852\ 640\ 233\ 547\ 611\ 114\ 847\ 283\ 501\ 745\ 082\ 790\ 407\ 402\ 209\ 280\ x^{112} + \\
& 819\ 782\ 734\ 429\ 798\ 620\ 166\ 277\ 811\ 138\ 823\ 455\ 145\ 979\ 638\ 579\ 200\ x^{113} - \\
& 357\ 240\ 130\ 203\ 182\ 970\ 054\ 939\ 726\ 752\ 854\ 005\ 281\ 754\ 011\ 238\ 400\ x^{114} - \\
& 241\ 996\ 028\ 867\ 110\ 903\ 463\ 324\ 627\ 628\ 693\ 705\ 521\ 095\ 652\ 474\ 880\ x^{115} + \\
& 28\ 833\ 320\ 421\ 960\ 924\ 641\ 303\ 063\ 939\ 895\ 062\ 194\ 927\ 793\ 848\ 320\ x^{116} + \\
& 57\ 400\ 560\ 904\ 677\ 861\ 644\ 561\ 351\ 591\ 685\ 310\ 329\ 182\ 073\ 487\ 360\ x^{117} + \\
& 4\ 194\ 733\ 164\ 894\ 157\ 066\ 626\ 682\ 273\ 711\ 042\ 229\ 343\ 044\ 075\ 520\ x^{118} - \\
& 11\ 575\ 581\ 284\ 929\ 952\ 000\ 569\ 479\ 883\ 271\ 408\ 836\ 542\ 218\ 649\ 600\ x^{119} - \\
& 2\ 798\ 589\ 416\ 545\ 086\ 538\ 038\ 377\ 009\ 216\ 089\ 694\ 742\ 307\ 504\ 128\ x^{120} + \\
& 2\ 003\ 982\ 959\ 217\ 417\ 625\ 349\ 039\ 704\ 731\ 638\ 962\ 040\ 968\ 396\ 800\ x^{121} + \\
& 877\ 617\ 876\ 868\ 449\ 488\ 510\ 131\ 868\ 357\ 590\ 866\ 473\ 602\ 990\ 080\ x^{122} - \\
& 290\ 126\ 528\ 648\ 129\ 104\ 521\ 842\ 309\ 455\ 741\ 427\ 353\ 373\ 655\ 040\ x^{123} - \\
& 213\ 301\ 647\ 571\ 008\ 678\ 961\ 770\ 245\ 849\ 648\ 429\ 679\ 487\ 918\ 080\ x^{124} + \\
& 31\ 758\ 682\ 026\ 133\ 507\ 468\ 155\ 222\ 770\ 589\ 557\ 833\ 082\ 978\ 304\ x^{125} + \\
& 43\ 959\ 100\ 614\ 079\ 032\ 092\ 605\ 494\ 684\ 783\ 399\ 880\ 652\ 984\ 320\ x^{126} - \\
& 1\ 507\ 112\ 562\ 976\ 486\ 160\ 141\ 401\ 374\ 729\ 412\ 754\ 602\ 721\ 280\ x^{127} - \\
& 7\ 873\ 660\ 666\ 195\ 139\ 233\ 932\ 990\ 750\ 553\ 019\ 825\ 575\ 034\ 880\ x^{128} - \\
& 380\ 038\ 806\ 781\ 090\ 765\ 153\ 073\ 044\ 880\ 486\ 077\ 850\ 173\ 440\ x^{129} + \\
& 1\ 226\ 163\ 805\ 565\ 960\ 522\ 269\ 584\ 211\ 376\ 399\ 451\ 772\ 084\ 224\ x^{130} + \\
& 134\ 288\ 054\ 809\ 995\ 042\ 066\ 883\ 625\ 836\ 139\ 127\ 875\ 420\ 160\ x^{131} - \\
& 163\ 399\ 589\ 492\ 089\ 723\ 875\ 279\ 571\ 955\ 976\ 036\ 763\ 975\ 680\ x^{132} - \\
& 24\ 138\ 965\ 019\ 560\ 503\ 956\ 005\ 761\ 573\ 339\ 076\ 183\ 408\ 640\ x^{133} + \\
& 17\ 976\ 727\ 138\ 305\ 931\ 502\ 543\ 761\ 574\ 447\ 911\ 967\ 109\ 120\ x^{134} + \\
& 2\ 530\ 189\ 515\ 081\ 475\ 929\ 463\ 185\ 861\ 158\ 995\ 460\ 091\ 904\ x^{135} - \\
& 1\ 513\ 748\ 985\ 035\ 818\ 632\ 534\ 079\ 583\ 450\ 923\ 854\ 090\ 240\ x^{136} + \\
& 18\ 234\ 761\ 312\ 195\ 760\ 792\ 334\ 529\ 129\ 294\ 025\ 400\ 320\ x^{137} + \\
& 80\ 139\ 302\ 022\ 382\ 805\ 040\ 699\ 078\ 183\ 065\ 149\ 665\ 280\ x^{138} - \\
& 77\ 286\ 267\ 157\ 913\ 876\ 758\ 479\ 299\ 200\ 971\ 330\ 723\ 840\ x^{139} - \\
& 1\ 003\ 688\ 067\ 851\ 208\ 919\ 199\ 808\ 676\ 842\ 985\ 276\ 416\ x^{140} + \\
& 20\ 593\ 222\ 330\ 810\ 081\ 110\ 433\ 526\ 365\ 594\ 375\ 454\ 720\ x^{141} + \\
& 181\ 467\ 028\ 143\ 402\ 986\ 808\ 073\ 700\ 981\ 914\ 091\ 520\ x^{142} - \\
& 3\ 573\ 748\ 718\ 635\ 325\ 441\ 294\ 804\ 939\ 535\ 085\ 547\ 520\ x^{143} - \\
& 153\ 994\ 235\ 836\ 064\ 378\ 608\ 841\ 305\ 036\ 504\ 898\ 560\ x^{144} + \\
& 459\ 133\ 954\ 606\ 470\ 527\ 893\ 897\ 453\ 511\ 503\ 583\ 232\ x^{145} + \\
& 34\ 231\ 194\ 679\ 034\ 327\ 482\ 725\ 019\ 709\ 285\ 406\ 720\ x^{146} - \\
& 42\ 256\ 641\ 220\ 940\ 049\ 719\ 961\ 125\ 237\ 136\ 522\ 240\ x^{147} - \\
& 3\ 206\ 238\ 763\ 968\ 287\ 520\ 678\ 615\ 931\ 348\ 638\ 720\ x^{148} + \\
& 2\ 066\ 605\ 299\ 356\ 818\ 496\ 937\ 495\ 258\ 061\ 747\ 200\ x^{149} - \\
& 293\ 886\ 194\ 125\ 156\ 537\ 084\ 111\ 501\ 395\ 116\ 032\ x^{150} + 106\ 857\ 920\ 179\ 301\ 338\ 192\ 404\ 100\ 411\ 100\ 160\ x^{151} + \\
& 177\ 466\ 428\ 487\ 129\ 851\ 591\ 409\ 305\ 087\ 290\ 880\ x^{152} - 28\ 348\ 722\ 409\ 594\ 249\ 081\ 037\ 549\ 906\ 805\ 760\ x^{153} - \\
& 39\ 937\ 806\ 413\ 152\ 114\ 466\ 299\ 828\ 645\ 912\ 320\ x^{154} + 37\ 393\ 287\ 287\ 674\ 454\ 636\ 382\ 382\ 832\ 640\ x^{155} + \\
& 6\ 278\ 855\ 006\ 394\ 100\ 703\ 242\ 023\ 761\ 021\ 440\ x^{156} + 974\ 252\ 969\ 705\ 256\ 543\ 119\ 126\ 835\ 025\ 920\ x^{157} - \\
& 759\ 681\ 418\ 175\ 900\ 785\ 145\ 726\ 791\ 107\ 840\ x^{158} - 264\ 264\ 062\ 153\ 239\ 963\ 389\ 372\ 853\ 529\ 600\ x^{159} + \\
& 70\ 390\ 458\ 281\ 297\ 839\ 254\ 440\ 477\ 225\ 472\ x^{160} + 46\ 874\ 396\ 726\ 445\ 924\ 407\ 922\ 335\ 493\ 120\ x^{161} - \\
& 4\ 340\ 280\ 723\ 950\ 510\ 981\ 531\ 922\ 878\ 720\ x^{162} - 6\ 614\ 137\ 951\ 506\ 471\ 808\ 205\ 764\ 861\ 440\ x^{163} + \\
& 8\ 654\ 385\ 777\ 352\ 412\ 727\ 360\ 998\ 400\ x^{164} + 799\ 069\ 073\ 633\ 673\ 083\ 962\ 992\ 911\ 360\ x^{165} +
\end{aligned}$$

$$\begin{aligned}
& 45\,485\,710\,243\,994\,792\,835\,042\,792\,960\,x^{166} - 86\,471\,789\,698\,734\,544\,218\,217\,420\,800\,x^{167} - \\
& 8\,377\,335\,326\,531\,863\,937\,410\,671\,680\,x^{168} + 8\,701\,223\,766\,821\,669\,162\,003\,904\,000\,x^{169} + \\
& 1\,071\,200\,500\,669\,419\,765\,558\,807\,936\,x^{170} - 841\,281\,933\,838\,868\,843\,605\,473\,280\,x^{171} - \\
& 115\,248\,009\,273\,691\,373\,796\,937\,920\,x^{172} + 79\,937\,290\,652\,607\,794\,849\,116\,800\,x^{173} + \\
& 11\,238\,024\,853\,544\,774\,238\,647\,680\,x^{174} - 7\,505\,981\,951\,878\,949\,618\,883\,648\,x^{175} - \\
& 1\,036\,856\,701\,264\,362\,387\,447\,360\,x^{176} + 689\,157\,610\,926\,725\,926\,717\,760\,x^{177} + \\
& 92\,751\,449\,138\,488\,790\,354\,880\,x^{178} - 60\,751\,033\,651\,804\,309\,907\,520\,x^{179} - \\
& 8\,104\,089\,179\,161\,038\,212\,576\,x^{180} + 5\,050\,519\,052\,149\,891\,904\,640\,x^{181} + \\
& 687\,199\,590\,102\,922\,818\,000\,x^{182} - 390\,264\,989\,454\,191\,225\,920\,x^{183} - 55\,748\,442\,956\,563\,200\,480\,x^{184} + \\
& 27\,702\,906\,083\,674\,070\,592\,x^{185} + 4\,253\,566\,332\,487\,331\,440\,x^{186} - 1\,787\,384\,934\,475\,969\,920\,x^{187} - \\
& 300\,092\,178\,198\,751\,680\,x^{188} + 103\,680\,235\,385\,998\,720\,x^{189} + 19\,262\,531\,195\,852\,784\,x^{190} - \\
& 5\,341\,680\,035\,003\,760\,x^{191} - 1\,107\,708\,361\,733\,680\,x^{192} + 240\,999\,922\,020\,480\,x^{193} + \\
& 56\,205\,611\,805\,840\,x^{194} - 9\,360\,664\,234\,208\,x^{195} - 2\,476\,844\,257\,620\,x^{196} + 306\,373\,357\,920\,x^{197} + \\
& 93\,136\,939\,280\,x^{198} - 8\,213\,662\,800\,x^{199} - 2\,925\,769\,308\,x^{200} + 173\,239\,080\,x^{201} + 74\,690\,280\,x^{202} - \\
& 2\,698\,140\,x^{203} - 1\,490\,020\,x^{204} + 27\,636\,x^{205} + 21\,840\,x^{206} - 140\,x^{207} - 210\,x^{208} + x^{210}
\end{aligned}$$

Example 3 (*Aborted after 10 minutes. Did not want to wait more...*)

```
In[ ]:= Quit[]
```

```
In[ ]:= Minimize[Sqrt[2 a^2 - 8 a + 10] + Sqrt[b^2 - 6 b + 10] + Sqrt[2 a^2 - 2 a b + b^2], {a, b}]; //
AbsoluteTiming
Minimize[Sqrt[x^2 + (y - 1)^2] + Sqrt[(x - 1)^2 + y^2] + Sqrt[(x + 1)^2 + y^2], {x, y}]; //
AbsoluteTiming
```

```
Out[ ]:=
```

\$Aborted

```
Out[ ]:=
```

\$Aborted

Example 4 a

```
In[ ]:= Quit[]
```

```
In[ ]:= Expand[Sum[i x[i], {i, 1, 12}]^12]
```

```
Out[ ]:=
```

$$\begin{aligned}
& x[1]^{12} + 24 x[1]^{11} x[2] + 264 x[1]^{10} x[2]^2 + 1760 x[1]^9 x[2]^3 + 7920 x[1]^8 x[2]^4 + 25\,344 x[1]^7 x[2]^5 + \\
& 59\,136 x[1]^6 x[2]^6 + 101\,376 x[1]^5 x[2]^7 + 126\,720 x[1]^4 x[2]^8 + 112\,640 x[1]^3 x[2]^9 + 67\,584 x[1]^2 x[2]^{10} + \\
& 24\,576 x[1] x[2]^{11} + 4096 x[2]^{12} + 36 x[1]^{11} x[3] + \dots 1\,352\,051 \dots + 494\,472\,070\,692\,864 x[11]^2 x[12]^{10} + \\
& 8\,916\,100\,448\,256 x[1] x[12]^{11} + 17\,832\,200\,896\,512 x[2] x[12]^{11} + 26\,748\,301\,344\,768 x[3] x[12]^{11} + \\
& 35\,664\,401\,793\,024 x[4] x[12]^{11} + 44\,580\,502\,241\,280 x[5] x[12]^{11} + 53\,496\,602\,689\,536 x[6] x[12]^{11} + \\
& 62\,412\,703\,137\,792 x[7] x[12]^{11} + 71\,328\,803\,586\,048 x[8] x[12]^{11} + 80\,244\,904\,034\,304 x[9] x[12]^{11} + \\
& 89\,161\,004\,482\,560 x[10] x[12]^{11} + 98\,077\,104\,930\,816 x[11] x[12]^{11} + 8\,916\,100\,448\,256 x[12]^{12}
\end{aligned}$$

Full expression not available (original memory size: 1 GB)



Example 4 b

```
In[ ]:= Quit[]
```



```
In[ ]:= PolynomialGCD[2 140 562 698 718 732 140 814 186 601 134 305 108 361 857 809 490 -
2 784 610 594 523 673 638 484 774 509 153 686 569 751 130 624 770 x +
3 858 489 651 140 979 026 284 638 768 361 566 754 023 704 101 963 x^2 -
286 771 157 043 724 591 851 751 736 007 270 812 641 162 273 605 x^3 +
1 040 070 357 631 236 974 043 798 352 460 064 492 096 759 420 031 x^4 +
527 961 813 592 576 990 225 122 032 458 485 764 714 353 948 184 x^5 -
223 657 724 243 176 368 425 265 508 444 964 464 071 309 890 719 x^6 -
3 733 131 181 727 340 801 070 133 191 959 530 969 316 170 794 800 x^30 +
10 745 497 692 902 796 595 974 970 291 342 011 573 796 136 255 136 x^31 -
7 905 771 579 741 612 232 706 547 483 087 384 561 886 791 433 680 x^32 +
1 395 228 177 175 535 320 672 887 161 804 454 665 548 111 711 824 x^33 -
5 230 961 365 618 779 655 379 147 055 541 516 478 070 736 328 364 x^34 -
3 706 780 992 584 927 498 391 542 297 913 941 634 360 275 844 528 x^35 -
11 678 244 373 494 087 080 209 876 359 797 173 892 928 099 938 592 x^60 -
5 514 859 882 567 035 785 577 006 631 456 739 844 414 574 289 760 x^61 -
21 295 212 021 140 539 667 882 396 747 720 054 113 720 182 080 880 x^62 -
21 789 643 204 816 141 848 070 620 182 749 478 757 088 623 737 712 x^63 -
4 720 988 527 381 715 377 218 961 193 451 142 849 496 347 654 320 x^64 -
27 444 066 359 527 368 757 788 068 859 219 000 443 573 584 940 160 x^90 -
33 565 578 213 248 720 198 933 124 767 949 411 665 123 563 970 688 x^91 -
26 098 925 852 445 627 183 784 692 986 363 859 867 173 544 281 664 x^92 -
7 417 015 118 303 371 350 476 354 681 923 558 088 324 668 065 600 x^93 -
26 712 940 739 577 401 605 838 204 365 261 893 824 578 131 755 520 x^120 -
21 489 656 182 324 275 196 520 668 674 864 724 640 109 707 593 728 x^121 -
3 537 371 331 357 954 585 225 002 235 363 286 524 220 979 284 224 x^122 -
7 370 705 120 862 668 705 991 302 948 457 148 370 726 768 590 848 x^150 -
3 449 161 085 875 451 377 293 133 065 291 672 477 113 785 516 032 x^151 +
454 778 382 410 716 370 295 091 942 238 975 349 531 795 652 608 x^180,
(-2826 * x^12 - 8790 + x^6 * (6631 - 4230 * y) - 8419 * y + 5267 * y^2 + 8182 * y^3 - 3940 * y^4 -
2386 * y^5 + x * (-6026 * y^5 + 1622 * y^4 - 6940 * y^3 - 1813 * y^2 - 6030 * y - 7923)) ^2 *
((9756 * x^30 - 6880 * y^5 + 3699 * y^4 + 1340 * y^3 - 9362 * y^2 + 2794 * y + 5059) - 2578) +
x * (-3125 * y^5 + 3550 * y^4 - 6425 * y^3 + 127 * y^2 + 7206 * y - 899) // Expand,
Modulus -> -84 568 - 10 331 y - 91 710 y^2 - 79 947 y^3 + 38 747 y^4 - 97 554 y^5 + 55 518 y^6]
```

```
Out[ ]:=
2481 - 899 x + 9756 x^30 + 2794 y + 7206 x y - 9362 y^2 +
127 x y^2 + 1340 y^3 - 6425 x y^3 + 3699 y^4 + 3550 x y^4 - 6880 y^5 - 3125 x y^5
```

Example 7

```
In[ ]:= Quit[]
```

```
In[*]:= f = -10 - 8 x + x^2 + 3 x^3 - 6 x^4 - 8 x^5 - 2 x^6 + 7 x^7 +
2 x^8 + x^33 + 5 x^34 + 9 x^35 - x^36 - 8 x^37 - x^47 - 2 x^48 + x^49 +
(6 + x - 9 x^3 + 7 x^4 - 10 x^5 - 5 x^6 + 9 x^7 + 6 x^8 + x^9 + 2 x^10 - 10 x^11 + 3 x^12 - 5 x^38 -
6 x^39 - 7 x^40 + 10 x^41 + x^42 + 6 x^43 - 9 x^44 - 3 x^45 + 9 x^46 + 6 x^47 + 3 x^48) y +
(1 + 3 x - 9 x^2 - 9 x^3 + 9 x^4 - 8 x^5 + 5 x^6 - x^7 + 9 x^35 - 7 x^36 + 10 x^37 +
6 x^41 - 6 x^42 - 4 x^43 + 7 x^44 - 8 x^46 - 4 x^47) y^2 +
(2 - 6 x + 9 x^2 + 3 x^4 + 9 x^5 - x^6 + 3 x^7 - 10 x^8 + 3 x^9 + 10 x^38 + 9 x^39 - 5 x^40 + 5 x^41)
y^8 + (-2 + 8 x - 3 x^2 + 6 x^3 - 2 x^4 - 3 x^5 - 4 x^7 - x^8 +
2 x^9 - 2 x^10 - x^36 - 7 x^37 + 9 x^38 - 7 x^39 - 6 x^40) y^9 +
(6 - 10 x - 2 x^2 - 9 x^3 + 2 x^4 + 10 x^5 - 2 x^7 - 4 x^8 + 7 x^9 - 4 x^35 + 6 x^36) y^13 +
(-7 + x + 10 x^2 - 7 x^3 - 6 x^4 - 9 x^5 + 3 x^6 - 7 x^7) y^42 +
(3 + 3 x - 9 x^2 + 7 x^4 + 2 x^5 - 8 x^6) y^43 + (-1 + x - 4 x^2 - 6 x^3 - x^4 + 3 x^5) y^44 +
(1 + 2 x + 3 x^2 + 6 x^3 - 7 x^4) y^45 + (9 - 9 x - 7 x^2 + 9 x^3) y^46 +
(-6 - 9 x + 5 x^2) y^47 + (-3 - 2 x) y^48 + 6 y^49;
```

Discriminant[f, y]

Out[*]=

1 232 871 053 862 991 046 221 057 932 110 462 800 038 610 514 102 097 377 161 819 197 711 076 809 895 376 282 282 712 786 779 677 429 :
909 819 541 186 709 964 616 887 128 477 059 143 727 606 240 916 489 527 054 257 832 858 599 418 940 + ... 3539 ...

Full expression not available (original memory size: 0.6 MB)



Example 8

In[*]:= **Quit[]**

```
In[*]:= f = 32 x^10 + 400 x^9 y + 176 x^10 y + 2240 x^8 y^2 + 2272 x^9 y^2 + 648 x^10 y^2 + 7400 x^7 y^3 +
12 776 x^8 y^3 + 6484 x^9 y^3 - 48 x^10 y^3 + 15 970 x^6 y^4 + 41 192 x^7 y^4 + 28 070 x^8 y^4 -
10 20 x^9 y^4 - 550 x^10 y^4 + 23 525 x^5 y^5 + 84 091 x^6 y^5 + 68 643 x^7 y^5 - 6112 x^8 y^5 -
4 289 x^9 y^5 - 149 x^10 y^5 + 23 955 x^4 y^6 + 112 892 x^5 y^6 + 103 672 x^6 y^6 -
17 447 x^7 y^6 - 13 730 x^8 y^6 - 786 x^9 y^6 + 81 x^10 y^6 + 16 650 x^3 y^7 + 99 786 x^4 y^7 +
98 987 x^5 y^7 - 27 472 x^6 y^7 - 23 093 x^7 y^7 - 1538 x^8 y^7 + 504 x^9 y^7 + 66 x^10 y^7 +
75 60 x^2 y^8 + 56 052 x^3 y^8 + 58 329 x^4 y^8 - 24 561 x^5 y^8 - 21 524 x^6 y^8 - 1313 x^7 y^8 +
1171 x^8 y^8 + 318 x^9 y^8 + 36 x^10 y^8 + 2025 x y^9 + 18 171 x^2 y^9 + 19 386 x^3 y^9 -
11 718 x^4 y^9 - 10 543 x^5 y^9 - 396 x^6 y^9 + 1204 x^7 y^9 + 508 x^8 y^9 + 117 x^9 y^9 +
11 x^10 y^9 + 243 y^10 + 2592 x y^10 + 2781 x^2 y^10 - 2322 x^3 y^10 - 2121 x^4 y^10 +
17 x^5 y^10 + 461 x^6 y^10 + 266 x^7 y^10 + 91 x^8 y^10 + 16 x^9 y^10 + x^10 y^10;
```

Solve[{f, D[f, x], D[f, y]} == 0, {x, y}]

Out[*]=

$\left\{ \{x \rightarrow 0, y \rightarrow 0\}, \left\{ x \rightarrow -\frac{1}{2}, y \rightarrow \frac{1}{3} \right\}, \{x \rightarrow -1, y \rightarrow 1\} \right\}$

Example 9

In[*]:= **Quit[]**

```

CoefficientList[
  (a1 x1 + a10 x10 + a11 x11 + a2 x2 + a3 x3 + a4 x4 + a5 x5 + a6 x6 + a7 x7 + a8 x8 + a9 x9) ^11 // Expand,
  {x1, x2, x3, x4, x5, x6, x7, x8, x9, x10, x11}]; (*too slow when output*)

```

```

Out[*]=
$Aborted

```

Example 10


```

In[*]:= Quit[]

In[1]:= ClearSystemCache[];
f = Function[{x, y}, Evaluate@Expand[(x + y + 1) ^60]];
a = ToNumberField@Root[
  -12 992 + 40 546 #1 - 59 434 #1^2 - 28 068 #1^3 + 10 027 #1^4 + 74 288 #1^5 + 95 220 #1^6 &, 1];
f[55 424 + 10 799 a + 32 924 a^2 - 94 123 a^3 - 57 476 a^4 + 16 513 a^5,
  49 641 - 47 101 a - 37 888 a^2 - 93 639 a^3 - 63 032 a^4 - 33 804 a^5]

```

```

Out[4]=  1.34... × 10324

```