Computer Algebra Independent Integration Tests

Summer 2024

1-Algebraic-functions/1.1-Binomial/1.1.6-Improper-linear-binomial/80-1.1.6.2

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Contents

CHAPTER 1

INTRODUCTION

This report gives the result of running the computer algebra independent integration test.The download section in on the main webpage contains links to download the problems in plain text format used for all CAS systems. The number of integrals in this report is [**462**]. This is test number [80].

1.1 Listing of CAS systems tested

The following are the CAS systems tested:

1. Mathematica 14 (January 9, 2024) on windows 10 pro.
2. Rubi 4.17.3 (Sept 25, 2023) on Mathematica 14 on windows 10m pro.
3. Maple 2024 (March 1, 2024) on windows 10 pro.
4. Maxima 5.47 (June 1, 2023) using Lisp SBCL 2.4.0 on Linux Manjaro 23.1.2 KDE via sagemath 10.3.
5. FriCAS 1.3.10 built with sbcl 2.3.11 (January 10, 2024) on Linux Manjaro 23.1.2 KDE via sagemath 10.3.
6. Giac/Xcas 1.9.0-99 on Linux via sagemath 10.3.
7. Sympy 1.12 using Python 3.11.6 (Nov 14 2023, 09:36:21) [GCC 13.2.1 20230801] on Linux Manjaro 23.1.2 KDE.
8. Mupad using Matlab 2021a with Symbolic Math Toolbox Version 8.7 on windows 10.
9. Reduce CSL rev 6687 (January 9, 2024) on Linux Manjaro 23.1.2 KDE.

Maxima and Fricas and Giac are called using Sagemath. This was done using Sagemath `integrate` command by changing the name of the algorithm to use the different CAS systems.

Sympy was run directly in Python not via sagemath.

Reduce was called directly.

1.2 Results

Important note: A number of problems in this test suite have no antiderivative in closed form. This means the antiderivative of these integrals can not be expressed in terms of elementary, special functions or `Hypergeometric2F1` functions. `RootSum` and `RootOf` are not allowed. If a CAS returns the above integral unevaluated within the time limit, then the result is counted as passed and assigned an A grade.

However, if CAS times out, then it is assigned an F grade even if the integral is not integrable, as this implies CAS could not determine that the integral is not integrable in the time limit.

If a CAS returns an antiderivative to such an integral, it is assigned an A grade automatically and this special result is listed in the introduction section of each individual test report to make it easy to identify as this can be important result to investigate.

The results given in in the table below reflects the above.

System	% solved	% Failed
Rubi	100.00 (462)	0.00 (0)
Mathematica	100.00 (462)	0.00 (0)
Maple	64.94 (300)	35.06 (162)
Fricas	62.34 (288)	37.66 (174)
Reduce	58.44 (270)	41.56 (192)
Giac	54.11 (250)	45.89 (212)
Mupad	38.53 (178)	61.47 (284)
Maxima	29.44 (136)	70.56 (326)
Sympy	8.66 (40)	91.34 (422)

Table 1.1: Percentage solved for each CAS

The table below gives additional break down of the grading of quality of the antiderivatives generated by each CAS. The grading is given using the letters A,B,C and F with A being the best quality. The grading is accomplished by comparing the antiderivative generated with the optimal antiderivatives included in the test suite. The following table describes the meaning of these grades.

grade	description
A	Integral was solved and antiderivative is optimal in quality and leaf size.
B	Integral was solved and antiderivative is optimal in quality but leaf size is larger than twice the optimal antiderivatives leaf size.
C	Integral was solved and antiderivative is non-optimal in quality. This can be due to one or more of the following reasons 1. antiderivative contains a hypergeometric function and the optimal antiderivative does not. 2. antiderivative contains a special function and the optimal antiderivative does not. 3. antiderivative contains the imaginary unit and the optimal antiderivative does not.
F	Integral was not solved. Either the integral was returned unevaluated within the time limit, or it timed out, or CAS hanged or crashed or an exception was raised.

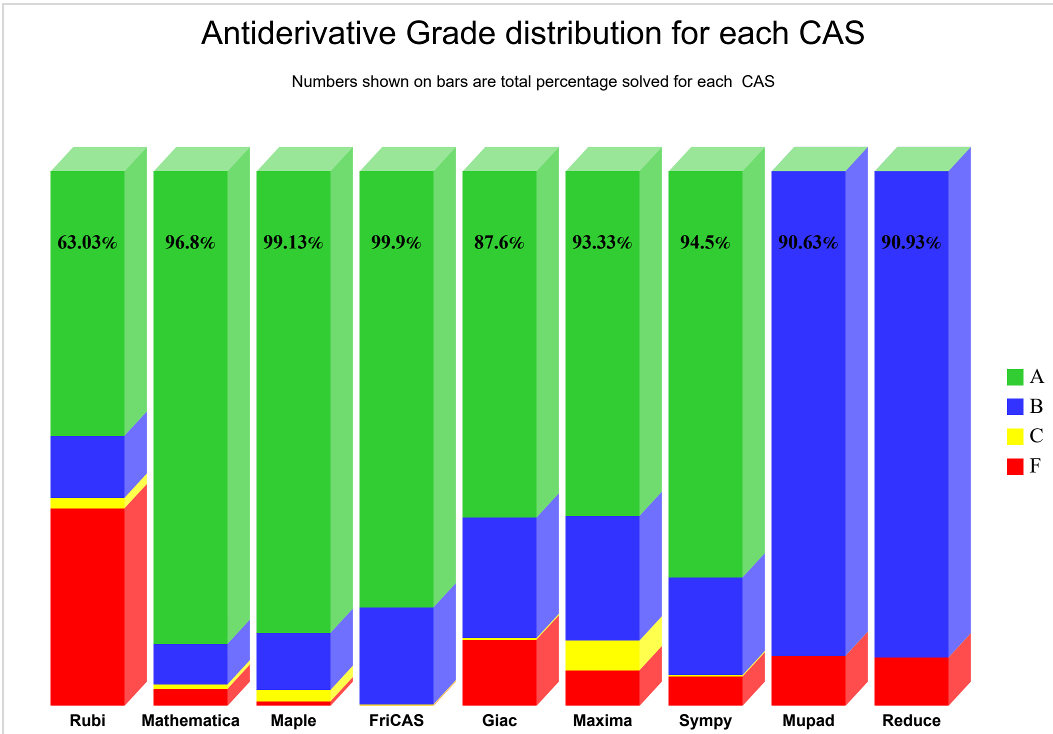
Table 1.2: Description of grading applied to integration result

Grading is implemented for all CAS systems. Based on the above, the following table summarizes the grading for this test suite.

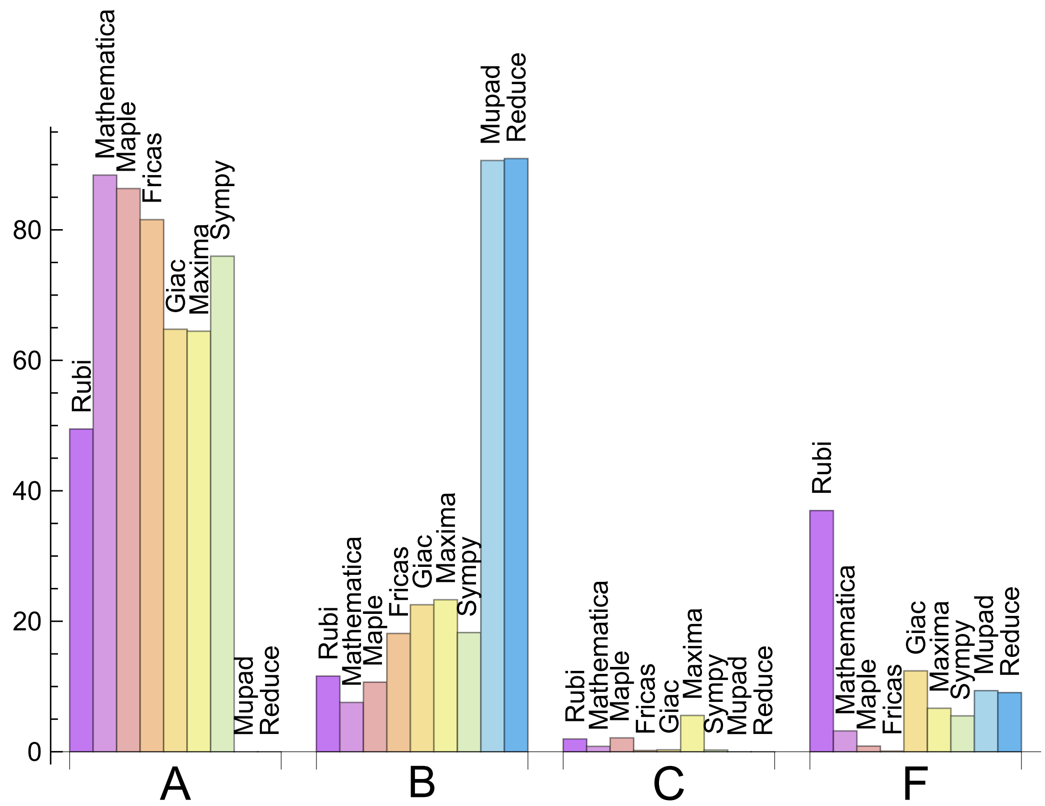
System	% A grade	% B grade	% C grade	% F grade
Rubi	96.753	3.247	0.000	0.000
Mathematica	80.087	0.216	19.697	0.000
Maple	60.823	1.299	2.814	35.065
Fricas	60.823	1.515	0.000	37.662
Giac	42.857	11.255	0.000	45.887
Maxima	27.056	2.381	0.000	70.563
Sympy	5.844	2.381	0.433	91.342
Mupad	0.000	38.528	0.000	61.472
Reduce	0.000	58.442	0.000	41.558

Table 1.3: Antiderivative Grade distribution of each CAS

The following is a Bar chart illustration of the data in the above table.



The figure below compares the grades of the CAS systems.



The following table shows the distribution of the different types of failures for each CAS. There are 3 types failures. The first is when CAS returns the input within the time limit, which means it could not solve it. This is the typical failure and given as **F**.

The second failure is due to time out. CAS could not solve the integral within the 3 minutes time limit which is assigned. This is assigned **F(-1)**.

The third is due to an exception generated, indicated as **F(-2)**. This most likely indicates an interface problem between sagemath and the CAS (applicable only to FriCAS, Maxima and Giac) or it could be an indication of an internal error in the CAS itself. This type of error requires more investigation to determine the cause.

System	Number failed	Percentage normal failure	Percentage timeout failure	Percentage exception failure
Rubi	0	0.00	0.00	0.00
Mathematica	0	0.00	0.00	0.00
Maple	162	100.00	0.00	0.00
Fricas	174	97.13	0.00	2.87
Reduce	192	100.00	0.00	0.00
Giac	212	97.17	2.83	0.00
Mupad	284	0.00	100.00	0.00
Maxima	326	100.00	0.00	0.00
Sympy	422	98.10	1.90	0.00

Table 1.4: Failure statistics for each CAS

1.3 Time and leaf size Performance

The table below summarizes the performance of each CAS system in terms of time used and leaf size of results.

Mean size is the average leaf size produced by the CAS (before any normalization). The Normalized mean is relative to the mean size of the optimal anti-derivative given in the input files.

For example, if CAS has **Normalized mean** of 3, then the mean size of its leaf size is 3 times as large as the mean size of the optimal leaf size.

Median size is value of leaf size where half the values are larger than this and half are smaller (before any normalization). i.e. The Middle value.

Similarly the **Normalized median** is relative to the median leaf size of the optimal.

For example, if a CAS has Normalized median of 1.2, then its median is 1.2 as large as the median leaf size of the optimal.

System	Mean time (sec)
Maxima	0.04
Fricas	0.09
Reduce	0.36
Rubi	0.41
Maple	0.43
Giac	1.37
Sympy	1.66
Mathematica	1.96
Mupad	8.64

Table 1.5: Time performance for each CAS

System	Mean size	Normalized mean	Median size	Normalized median
Maple	62.57	0.72	49.00	0.66
Mathematica	64.40	0.77	53.00	0.75
Mupad	64.93	0.91	49.00	0.75
Maxima	72.54	0.97	62.00	0.83
Reduce	76.63	0.85	66.50	0.77
Rubi	104.39	1.14	95.00	1.06
Giac	113.77	1.33	93.50	0.95
Fricas	133.80	1.30	95.00	1.18
Sympy	308.90	3.59	123.00	1.22

Table 1.6: Leaf size performance for each CAS

1.4 Performance based on number of rules Rubi used

This section shows how each CAS performed based on the number of rules Rubi needed to solve the same integral. One diagram is given for each CAS.

On the y axis is the percentage solved which Rubi itself needed the number of rules given the x axis. These plots show that as more rules are needed then most CAS system percentage of solving decreases which indicates the integral is becoming more complicated to solve.

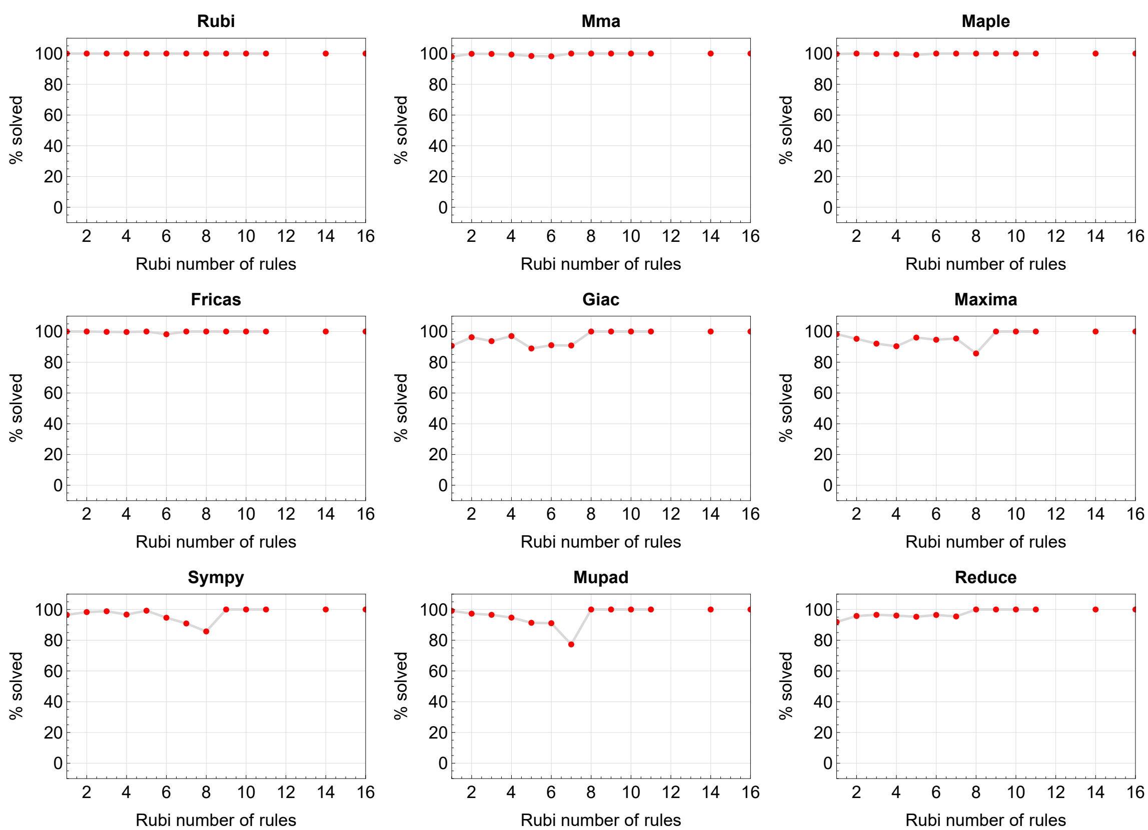


Figure 1.1: Solving statistics per number of Rubi rules used

1.5 Performance based on number of steps Rubi used

This section shows how each CAS performed based on the number of steps Rubi needed to solve the same integral. Note that the number of steps Rubi needed can be much higher than the number of rules, as the same rule could be used more than once.

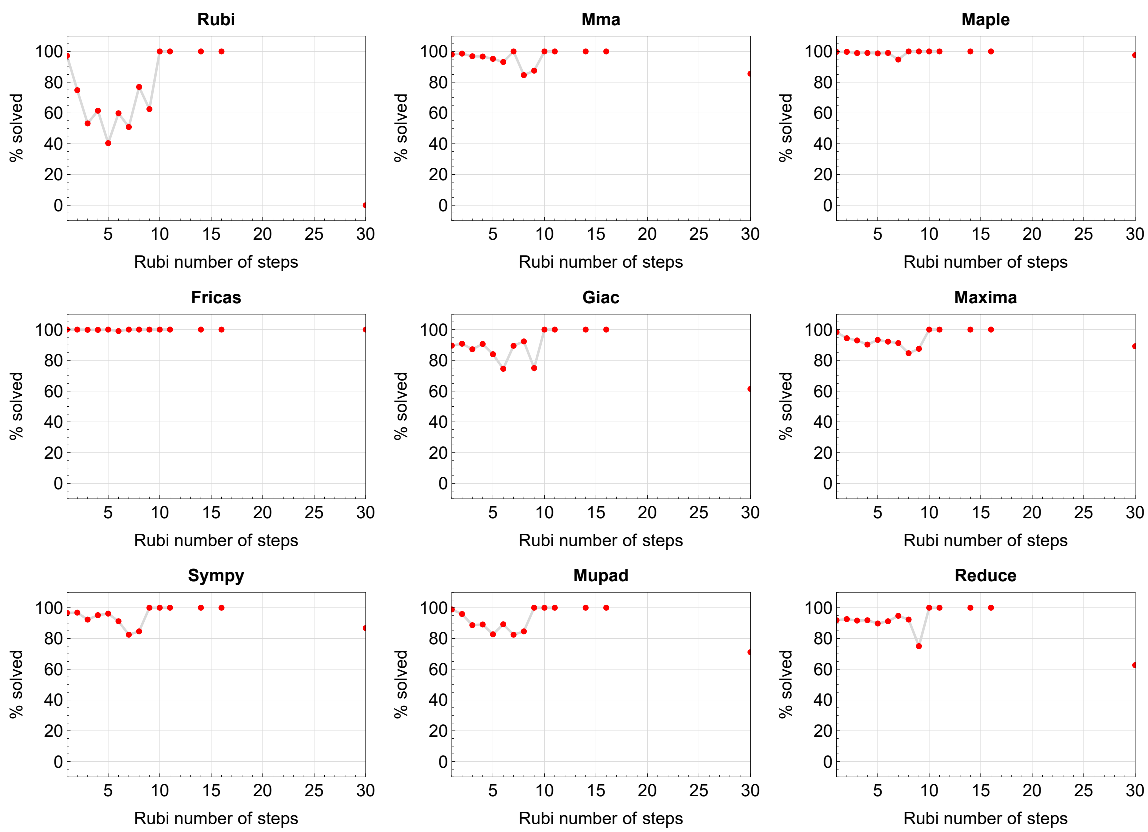


Figure 1.2: Solving statistics per number of Rubi steps used

The above diagram show that the precentage of solved intergals decreases for most CAS systems as the number of steps increases. As expected, for integrals that required less steps by Rubi, CAS systems had more success which indicates the integral was not as hard to solve. As Rubi needed more steps to solve the integral, the solved percentage decreased for most CAS systems which indicates the integral is becoming harder to solve.

1.6 Solved integrals histogram based on leaf size of result

The following shows the distribution of solved integrals for each CAS system based on leaf size of the antiderivatives produced by each CAS. It shows that most integrals solved produced leaf size less than about 100 to 150. The bin size used is 40.

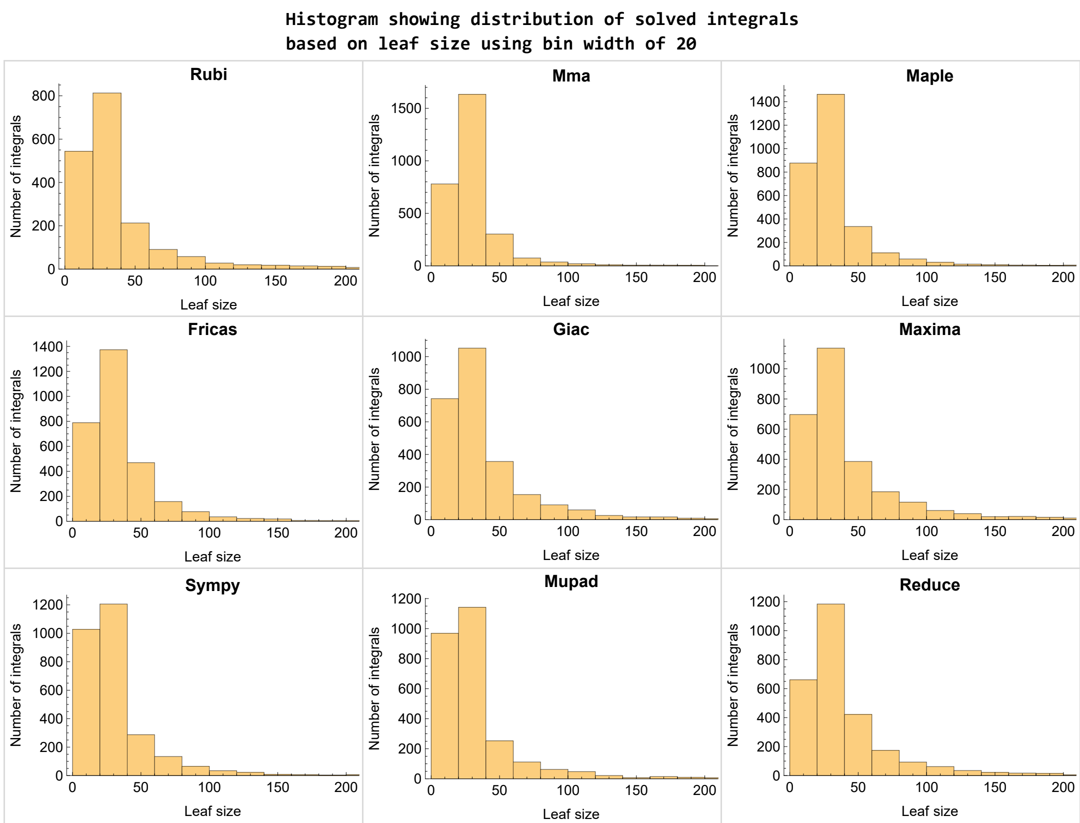


Figure 1.3: Solved integrals based on leaf size distribution

1.7 Solved integrals histogram based on CPU time used

The following shows the distribution of solved integrals for each CAS system based on CPU time used in seconds. The bin size used is 0.1 second.

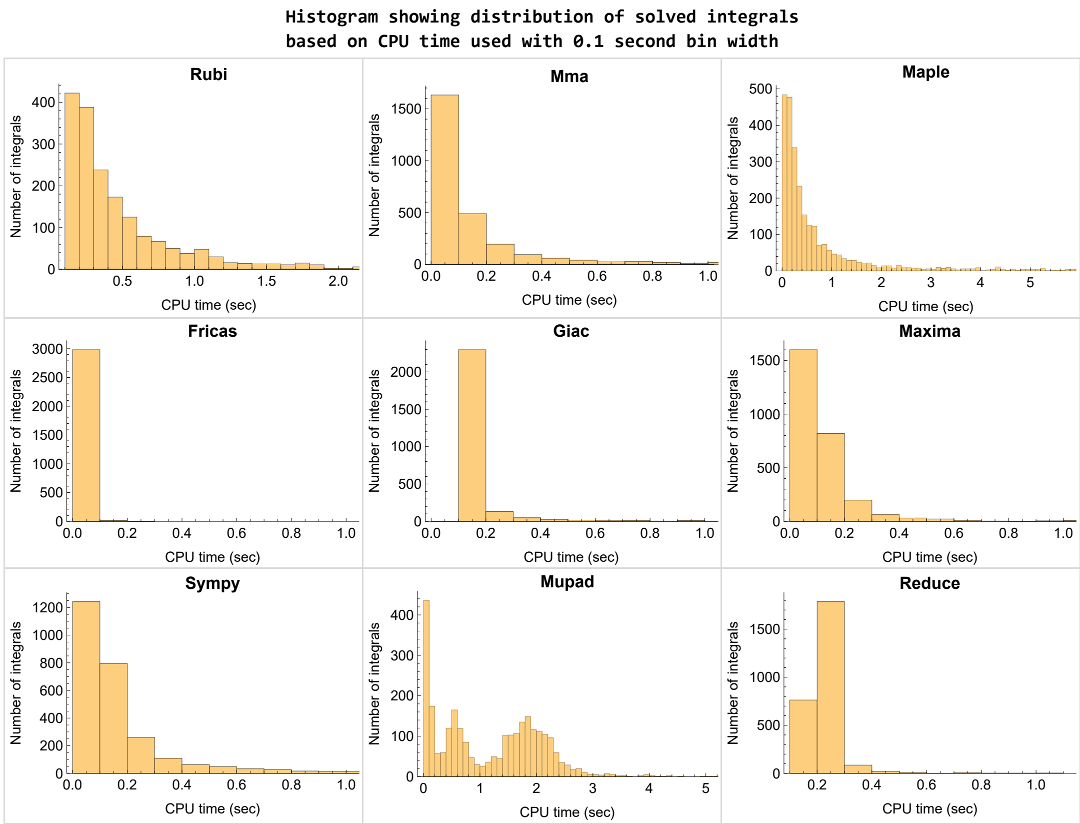


Figure 1.4: Solved integrals histogram based on CPU time used

1.8 Leaf size vs. CPU time used

The following shows the relation between the CPU time used to solve an integral and the leaf size of the antiderivative.

The result for Fricas, Maxima and Giac is shifted more to the right than the other CAS system due to the use of sagemath to call them, which causes an initial slight delay in the timing to start the integration due to overhead of starting a new process each time. This should also be taken into account when looking at the timing of these three CAS systems. Direct calls not using sagemath would result in faster timings, but current implementation uses sagemath as this makes testing much easier to do.

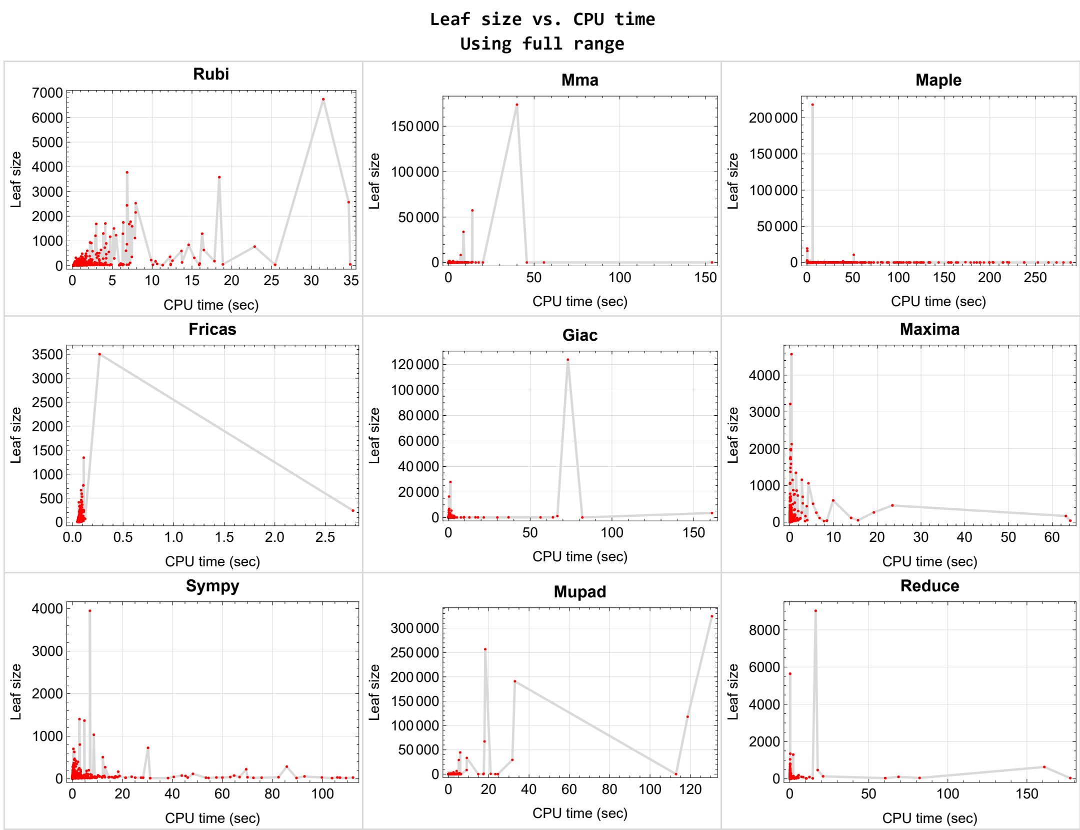


Figure 1.5: Leaf size vs. CPU time. Full range

1.9 list of integrals with no known antiderivative

{}

1.10 List of integrals solved by CAS but has no known antiderivative

Rubi {}

Mathematica {}

Maple {}

Maxima {}

Fricas {}

Sympy {}

Giac {}

Reduce {}

Mupad {}

1.11 list of integrals solved by CAS but failed verification

The following are integrals solved by CAS but the verification phase failed to verify the anti-derivative produced is correct. This does not necessarily mean that the anti-derivative is wrong as additional methods of verification might be needed, or more time is needed (3 minutes time limit was used). These integrals are listed here to make it possible to do further investigation to determine why the result could not be verified.

Rubi {145, 146, 149, 150, 151, 152, 153, 154, 155, 158, 159, 160, 161, 162, 163, 166, 167, 168, 169, 170, 171, 172, 173, 174, 175, 178, 179, 180, 181, 182, 183, 186, 187, 188, 189, 190, 191, 195, 196, 197, 198, 199, 200, 201, 202, 203, 204, 205, 206, 207, 214, 215, 216}

Mathematica {}

Maple $\{203, 209, 214\}$

Maxima Verification phase not currently implemented.

Fricas Verification phase not currently implemented.

Sympy Verification phase not currently implemented.

Giac Verification phase not currently implemented.

Reduce Verification phase not currently implemented.

Mupad Verification phase not currently implemented.

1.12 Timing

The command `AbsoluteTiming[]` was used in Mathematica to obtain the elapsed time for each integrate call. In Maple, the command `Usage` was used as in the following example

```
cpu_time := Usage(assign ('result_of_int',int(expr,x)),output='realtime'
```

For all other CAS systems, the elapsed time to complete each integral was found by taking the difference between the time after the call completed from the time before the call was made. This was done using Python's `time.time()` call.

All elapsed times shown are in seconds. A time limit of 3 CPU minutes was used for each integral. If the integrate command did not complete within this time limit, the integral was aborted and considered to have failed and assigned an F grade. The time used by failed integrals due to time out was not counted in the final statistics.

1.13 Verification

A verification phase was applied on the result of integration for `Rubi` and `Mathematica` and `Maple`.

Future version of this report will implement verification for the other CAS systems. For the integrals whose result was not run through a verification phase, it is assumed that the antiderivative was correct.

Verification phase also had 3 minutes time out. An integral whose result was not verified could still be correct, but further investigation is needed on those integrals.

These integrals were marked in the summary table below and also in each integral separate section so they are easy to identify and locate.

1.14 Important notes about some of the results

Important note about Maxima results

Since tests were run in a batch mode, and using an automated script, then any integral where Maxima needed an interactive response from the user to answer a question during the evaluation of the integral will fail.

The exception raised is `ValueError`. Therefore Maxima results is lower than what would result if Maxima was run directly and each question was answered correctly.

The percentage of such failures were not counted for each test file, but for an example, for the `Timofeev` test file, there were about 14 such integrals out of total 705, or about 2 percent. This percentage can be higher or lower depending on the specific input test file.

Such integrals can be identified by looking at the output of the integration in each section for Maxima. The exception message will indicate the cause of error.

Maxima integrate was run using SageMath with the following settings set by default

```
'besselexpand : true'
'display2d : false'
'domain : complex'
'keepfloat : true'
'load(to_poly_solve)'
'load(simplify_sum)'
'load(abs_integrate)' 'load(diag)'
```

SageMath automatic loading of Maxima `abs_integrate` was found to cause some problems. So the following code was added to disable this effect.

```
from sage.interfaces.maxima_lib import maxima_lib
maxima_lib.set('extra_definite_integration_methods', '[]')
maxima_lib.set('extra_integration_methods', '[]')
```

See https://ask.sagemath.org/question/43088/integrate-results-that-are-different-from-using-maxima/ for reference.

Important note about FriCAS result

There were few integrals which failed due to SageMath interface and not because FriCAS system could not do the integration.

These will fail With error `Exception raised: NotImplementedError`.

The number of such cases seems to be very small. About 1 or 2 percent of all integrals. These can be identified by looking at the exception message given in the result.

Important note about finding leaf size of antiderivative

For Mathematica, Rubi, and Maple, the builtin system function `LeafSize` was used to find the leaf size of each antiderivative.

The other CAS systems (SageMath and Sympy) do not have special builtin function for this purpose at this time. Therefore the leaf size for Fricas and Sympy antiderivative was determined using the following function, thanks to user `slelievre` at https://ask.sagemath.org/question/57123/could-we-have-a-leaf_count-function-in-base-sagemath/

```
def tree_size(expr):
    r"""
    Return the tree size of this expression.
    """
    if expr not in SR:
        # deal with lists, tuples, vectors
        return 1 + sum(tree_size(a) for a in expr)
    expr = SR(expr)
    x, aa = expr.operator(), expr.operands()
    if x is None:
        return 1
    else:
        return 1 + sum(tree_size(a) for a in aa)
```

For Sympy, which was called directly from Python, the following code was used to obtain the leafsize of its result

```
try:
  # 1.7 is a fudge factor since it is low side from actual leaf count
  leafCount = round(1.7*count_ops(anti))

  except Exception as ee:
        leafCount =1
```

Important note about Mupad results

Matlab's symbolic toolbox does not have a leaf count function to measure the size of the antiderivative. Maple was used to determine the leaf size of Mupad output by post processing Mupad result.

Currently no grading of the antiderivative for Mupad is implemented. If it can integrate the problem, it was assigned a B grade automatically as a placeholder. In the future, when grading function is implemented for Mupad, the tests will be rerun again.

The following is an example of using Matlab's symbolic toolbox (Mupad) to solve an integral

```
integrand = evalin(symengine,'cos(x)*sin(x)')
the_variable = evalin(symengine,'x')
anti = int(integrand,the_variable)
```

Which gives `sin(x)^2/2`

CHAPTER 2

DETAILED SUMMARY TABLES OF RESULTS

2.1 List of integrals sorted by grade for each CAS

Rubi

A grade { 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17, 18, 19, 20, 21, 22, 23, 24, 25, 26, 27, 28, 29, 30, 31, 32, 33, 34, 35, 36, 37, 38, 39, 40, 41, 42, 43, 44, 45, 46, 47, 48, 49, 50, 51, 52, 53, 54, 55, 56, 57, 58, 59, 60, 61, 62, 63, 64, 65, 66, 67, 68, 69, 70, 71, 72, 73, 74, 75, 76, 77, 78, 79, 80, 81, 82, 83, 84, 85, 86, 87, 88, 89, 90, 91, 92, 93, 94, 95, 96, 97, 98, 99, 100, 101, 102, 103, 104, 105, 106, 107, 108, 109, 110, 111, 112, 113, 114, 115, 116, 117, 118, 119, 120, 121, 122, 123, 124, 125, 126, 127, 128, 129, 130, 131, 132, 133, 134, 135, 136, 137, 138, 139, 140, 141, 142, 143, 144, 145, 146, 147, 148, 149, 150, 151, 152, 153, 154, 155, 156, 157, 158, 159, 160, 161, 162, 163, 164, 165, 166, 167, 168, 169, 170, 171, 172, 173, 174, 175, 176, 177, 178, 179, 180, 181, 182, 183, 184, 185, 186, 187, 188, 189, 190, 191, 192, 193, 194, 195, 196, 197, 198, 200, 202, 204, 207, 208, 209, 210, 211, 212, 213, 215, 216, 217, 218, 219, 220, 221, 222, 223, 224, 225, 226, 227, 228, 230, 231, 232, 233, 234, 236, 237, 238, 239, 240, 241, 243, 244, 245, 246, 247, 248, 249, 250, 251, 252, 253, 254, 255, 256, 257, 258, 259, 260, 261, 262, 263, 264, 265, 266, 267, 268, 269, 270, 271, 272, 273, 274, 275, 276, 277, 278, 279, 280, 281, 282, 283, 284, 285, 286, 287, 288, 289, 290, 291, 292, 293, 294, 295, 296, 297, 298, 299, 300, 301, 302, 303, 304, 305, 306, 307, 308, 309, 310, 311, 312, 313, 314, 315, 316, 317, 318, 319, 320, 321, 322, 323, 324, 325, 326, 327, 328, 329, 330, 331, 332, 333, 334, 335, 336, 337, 338, 339, 340, 341, 342, 343, 344, 345, 346, 347, 348, 349, 350, 351, 352, 353, 354, 355, 356, 357, 358, 359, 360, 361, 362, 363, 364, 365, 366, 367, 368, 369, 370, 371, 372, 373, 374, 375, 376, 377, 378, 379, 380, 381, 382, 383, 384, 385, 386, 387, 388, 389, 391, 392, 393, 394, 395, 396, 397, 398, 399, 400, 401, 402, 406, 407, 408, 409, 410, 411, 412, 414, 415, 416, 418, 419, 420, 421, 422, 423, 424, 425, 426, 427, 428, 429, 430, 431, 432, 433, 434, 435, 436, 437, 438, 439, 440, 441, 442, 443, 444, 445, 446, 447, 448, 449, 450, 451, 452, 453, 454, 455, 456, 457, 458, 459, 460, 461, 462 }

B grade { 199, 201, 203, 205, 206, 214, 229, 235, 242, 390, 403, 404, 405, 413, 417 }

C grade { }

F normal fail { }

F(-1) timedout fail { }

F(-2) exception fail { }

Mma

A grade { 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17, 18, 19, 20, 21, 22, 23, 24, 25, 26, 27, 28, 29, 30, 31, 32, 33, 34, 35, 36, 37, 38, 39, 40, 41, 42, 43, 44, 45, 46, 47, 48, 49, 50, 51, 52, 53, 54, 55, 56, 57, 58, 59, 60, 61, 62, 63, 64, 65, 66, 67, 68, 69, 70, 72, 73, 74, 75, 76, 77, 78, 79, 80, 81, 82, 83, 84, 85, 86, 87, 88, 89, 90, 91, 92, 93, 94, 95, 96, 97, 98, 99, 100, 101, 102, 103, 104, 105, 106, 107, 108, 109, 110, 111, 112, 113, 114, 115, 116, 117, 118, 119, 120, 121, 122, 123, 124, 125, 126, 127, 128, 129, 130, 131, 132, 133, 134, 135, 136, 137, 138, 139, 140, 141, 142, 143, 144, 215, 216, 217, 218, 219, 220, 221, 222, 223, 224, 225, 226, 227, 228, 229, 230, 231, 232, 233, 234, 235, 236, 237, 238, 239, 240, 241, 242, 243, 244, 245, 246, 247, 248, 249, 250, 251, 252, 253, 254, 255, 256, 257, 258, 259, 260, 261, 262, 263, 264, 265, 266, 267, 268, 269, 270, 271, 272, 273, 274, 275, 276, 277, 278, 279, 280, 281, 282, 283, 284, 285, 286, 287, 288, 289, 290, 291, 292, 293, 294, 295, 296, 297, 298, 299, 300, 301, 302, 303, 304, 305, 306, 307, 308, 309, 310, 311, 312, 313, 314, 315, 316, 317, 318, 319, 320, 321, 322, 323, 324, 325, 326, 327, 328, 329, 330, 331, 332, 333, 334, 335, 336, 337, 338, 339, 340, 341, 342, 343, 344, 345, 346, 347, 348, 349, 350, 351, 352, 353, 354, 355, 356, 357, 358, 359, 360, 361, 362, 363, 364, 365, 366, 367, 368, 369, 370, 371, 372, 373, 374, 375, 376, 377, 378, 379, 380, 381, 382, 383, 384, 385, 407, 408, 409, 410, 411, 412, 413, 414, 415, 416, 417, 418, 419, 420, 421, 422, 423, 424, 425, 426, 427, 428, 429, 430, 431, 432, 433, 434, 435, 436, 437, 438, 439, 440, 441, 442, 443, 444, 445, 446, 447, 448, 449, 450, 451, 452, 453, 454, 455, 456, 457, 458, 459, 460, 461, 462 }

B grade { 71 }

C grade { 145, 146, 147, 148, 149, 150, 151, 152, 153, 154, 155, 156, 157, 158, 159, 160, 161, 162, 163, 164, 165, 166, 167, 168, 169, 170, 171, 172, 173, 174, 175, 176, 177, 178, 179, 180, 181, 182, 183, 184, 185, 186, 187, 188, 189, 190, 191, 192, 193, 194, 195, 196, 197, 198, 199, 200, 201, 202, 203, 204, 205, 206, 207, 208, 209, 210, 211, 212, 213, 214, 386, 387, 388, 389, 390, 391, 392, 393, 394, 395, 396, 397, 398, 399, 400, 401, 402, 403, 404, 405, 406 }

F normal fail { }

F(-1) timedout fail { }

F(-2) exception fail { }

Maple

A grade { 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17, 18, 19, 20, 21, 22, 23, 24, 25, 26, 27, 28, 29, 30, 31, 32, 33, 34, 35, 36, 37, 38, 39, 40, 41, 42, 43, 44, 45, 46, 47, 48, 49, 50, 51, 52, 53, 54, 55, 56, 57, 58, 59, 60, 61, 62, 63, 64, 65, 66, 67, 68, 69, 70, 71, 72, 73, 74, 75, 76, 77, 78, 79, 80, 81, 82, 83, 84, 85, 86, 87, 88, 89, 90, 91, 92, 93, 94, 95, 96, 97, 98, 99, 100, 101, 102, 103, 104, 105, 106, 107, 108, 109, 110, 111, 112, 113, 114, 115, 116, 117, 118, 119, 120, 121, 122, 123, 124, 125, 126, 131, 132, 133, 134, 135, 136, 137, 138, 217, 218, 219, 234, 235, 236, 237, 238, 239, 240, 251, 252, 253, 254, 256, 257, 258, 259, 263, 264, 265, 266, 267, 268, 269, 270, 271, 272, 273, 274, 275, 276, 277, 279, 280, 281, 282, 283, 284, 285, 286, 287, 289, 290, 291, 292, 293, 294, 295, 296, 297, 298, 299, 300, 301, 302, 303, 304, 305, 306, 307, 308, 309, 310, 311, 312, 313, 314, 315, 316, 317, 318, 319, 320, 321, 322, 323, 324, 325, 326, 327, 328, 329, 330, 331, 332, 333, 334, 335, 336, 337, 338, 339, 340, 341, 342, 343, 344, 345, 346, 347, 348, 349, 351, 352, 353, 354, 355, 356, 358, 359, 360, 361, 362, 363, 365, 366, 367, 368, 369, 370, 371, 372, 373, 374, 375, 376, 377, 378, 379, 380, 381, 382, 383, 384, 385, 408, 409, 428, 429, 430, 431, 437, 456, 457, 458, 459 }

B grade { 350, 357, 364, 407, 435, 436 }

C grade { 202, 203, 205, 206, 208, 209, 211, 212, 214, 215, 216, 278, 288 }

F normal fail { 127, 128, 129, 130, 139, 140, 141, 142, 143, 144, 145, 146, 147, 148, 149, 150, 151, 152, 153, 154, 155, 156, 157, 158, 159, 160, 161, 162, 163, 164, 165, 166, 167, 168, 169, 170, 171, 172, 173, 174, 175, 176, 177, 178, 179, 180, 181, 182, 183, 184, 185, 186, 187, 188, 189, 190, 191, 192, 193, 194, 195, 196, 197, 198, 199, 200, 201, 204, 207, 210, 213, 220, 221, 222, 223, 224, 225, 226, 227, 228, 229, 230, 231, 232, 233, 241, 242, 243, 244, 245, 246, 247, 248, 249, 250, 255, 260, 261, 262, 386, 387, 388, 389, 390, 391, 392, 393, 394, 395, 396, 397, 398, 399, 400, 401, 402, 403, 404, 405, 406, 410, 411, 412, 413, 414, 415, 416, 417, 418, 419, 420, 421, 422, 423, 424, 425, 426, 427, 432, 433, 434, 438, 439, 440, 441, 442, 443, 444, 445, 446, 447, 448, 449, 450, 451, 452, 453, 454, 455, 460, 461, 462 }

F(-1) timedout fail { }

F(-2) exception fail { }

Fricas

A grade { 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17, 18, 19, 20, 21, 22, 23, 24, 25, 26, 27, 28, 29, 30, 32, 33, 34, 35, 36, 37, 38, 39, 40, 41, 42, 43, 44, 45, 46, 47, 48, 49, 50, 51, 52, 53, 54, 55, 56, 57, 58, 59, 60, 61, 62, 63, 64, 65, 66, 67, 68, 69, 70, 71, 72, 73, 74, 75, 76, 77, 78, 79, 80, 81, 82, 83, 84, 85, 86, 87, 88, 89, 90, 91, 92, 93, 94, 95, 96, 97, 98, 99, 100, 101, 102, 103, 104, 105, 106, 107, 108, 109, 110, 111, 112, 113, 114, 115, 117, 118, 119, 120, 121, 122, 123, 124, 125, 126, 127, 128, 129, 130, 131, 132, 133, 134, 135, 136, 137, 138, 139, 140, 141, 142, 143, 144, 217, 218, 219, 256, 257, 258, 259, 265, 266, 267, 268, 269, 270, 271, 272, 273, 274, 275, 276, 277, 278, 279, 280, 281, 282, 283, 284, 285, 286, 287, 288, 289, 290, 291, 293, 294, 295, 296, 297, 298, 299, 300, 301, 302, 303, 304, 305, 306, 307, 308, 309, 310, 311, 312, 313, 314, 315, 316, 317, 318, 319, 320, 321, 322, 323, 324, 325, 326, 327, 328, 329, 330, 331, 332, 333, 334, 335, 336, 337, 338, 339, 340, 341, 342, 343, 344, 345, 346, 347, 348, 349, 350, 351, 352, 353, 354, 355, 356, 357, 358, 359, 360, 361, 362, 363, 364, 365, 366, 367, 368, 369, 370, 371, 372, 373, 374, 375, 376, 377, 378, 379, 380, 381, 382, 383, 384, 385, 407, 408, 409, 428, 429, 430, 431, 437, 456, 457, 458, 459 }

B grade { 31, 116, 215, 216, 292, 435, 436 }

C grade { }

F normal fail { 145, 146, 147, 148, 149, 150, 151, 152, 153, 154, 155, 156, 157, 158, 159, 160, 161, 162, 163, 164, 165, 166, 167, 168, 169, 170, 171, 172, 173, 174, 175, 176, 177, 178, 179, 180, 181, 182, 183, 184, 185, 186, 187, 188, 189, 190, 191, 192, 193, 194, 195, 196, 197, 198, 199, 200, 201, 202, 203, 204, 205, 206, 207, 208, 209, 210, 211, 212, 213, 214, 220, 221, 222, 223, 224, 225, 226, 227, 228, 229, 230, 231, 232, 233, 234, 235, 236, 237, 238, 239, 240, 241, 242, 243, 244, 245, 246, 247, 248, 249, 250, 251, 252, 253, 254, 255, 260, 261, 262, 263, 264, 386, 387, 388, 389, 390, 391, 392, 393, 394, 395, 396, 397, 398, 399, 400, 401, 402, 403, 404, 405, 406, 410, 411, 412, 413, 414, 415, 416, 417, 418, 419, 420, 421, 422, 423, 424, 425, 426, 427, 432, 433, 434, 438, 439, 440, 446, 447, 448, 449, 450, 451, 452, 453, 454, 455, 460, 461, 462 }

F(-1) timedout fail { }

F(-2) exception fail { 441, 442, 443, 444, 445 }

Maxima

A grade { 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17, 20, 21, 22, 23, 24, 25, 26, 27, 28, 29, 30, 35, 36, 37, 38, 39, 40, 41, 42, 43, 44, 45, 46, 47, 48, 49, 50, 51, 52, 53, 54, 55, 56, 60, 61, 62, 63, 64, 65, 67, 68, 69, 70, 71, 72, 73, 74, 75, 76, 77, 78, 79, 80, 81, 82, 88, 89, 90, 97, 98, 99, 100, 121, 122, 123, 133, 134, 135, 215, 217, 218, 219, 265, 266, 267, 268, 269, 275, 276, 277, 278, 279, 280, 286, 287, 288, 289, 290, 291, 292, 300, 301, 302, 303, 308, 309, 310, 311, 317, 318, 319, 320, 407, 408, 409, 435, 436, 437 }

B grade { 18, 19, 31, 32, 33, 34, 57, 58, 59, 66, 91 }

C grade { }

F normal fail { 83, 84, 85, 86, 87, 92, 93, 94, 95, 96, 101, 102, 103, 104, 105, 106, 107, 108, 109, 110, 111, 112, 113, 114, 115, 116, 117, 118, 119, 120, 124, 125, 126, 127, 128, 129, 130, 131, 132, 136, 137, 138, 139, 140, 141, 142, 143, 144, 145, 146, 147, 148, 149, 150, 151, 152, 153, 154, 155, 156, 157, 158, 159, 160, 161, 162, 163, 164, 165, 166, 167, 168, 169, 170, 171, 172, 173, 174, 175, 176, 177, 178, 179, 180, 181, 182, 183, 184, 185, 186, 187, 188, 189, 190, 191, 192, 193, 194, 195, 196, 197, 198, 199, 200, 201, 202, 203, 204, 205, 206, 207, 208, 209, 210, 211, 212, 213, 214, 216, 220, 221, 222, 223, 224, 225, 226, 227, 228, 229, 230, 231, 232, 233, 234, 235, 236, 237, 238, 239, 240, 241, 242, 243, 244, 245, 246, 247, 248, 249, 250, 251, 252, 253, 254, 255, 256, 257, 258, 259, 260, 261, 262, 263, 264, 270, 271, 272, 273, 274, 281, 282, 283, 284, 285, 293, 294, 295, 296, 297, 298, 299, 304, 305, 306, 307, 312, 313, 314, 315, 316, 321, 322, 323, 324, 325, 326, 327, 328, 329, 330, 331, 332, 333, 334, 335, 336, 337, 338, 339, 340, 341, 342, 343, 344, 345, 346, 347, 348, 349, 350, 351, 352, 353, 354, 355, 356, 357, 358, 359, 360, 361, 362, 363, 364, 365, 366, 367, 368, 369, 370, 371, 372, 373, 374, 375, 376, 377, 378, 379, 380, 381, 382, 383, 384, 385, 386, 387, 388, 389, 390, 391, 392, 393, 394, 395, 396, 397, 398, 399, 400, 401, 402, 403, 404, 405, 406, 410, 411, 412, 413, 414, 415, 416, 417, 418, 419, 420, 421, 422, 423, 424, 425, 426, 427, 428, 429, 430, 431, 432, 433, 434, 438, 439, 440, 441, 442, 443, 444, 445, 446, 447, 448, 449, 450, 451, 452, 453, 454, 455, 456, 457, 458, 459, 460, 461, 462 }

F(-1) timedout fail { }

F(-2) exception fail { }

Giac

A grade { 1, 2, 3, 4, 5, 6, 10, 11, 12, 13, 14, 15, 16, 22, 23, 24, 25, 26, 27, 28, 29, 35, 36, 37, 38, 39, 40, 41, 43, 44, 45, 46, 47, 48, 49, 50, 51, 52, 56, 57, 60, 62, 67, 68, 69, 70, 71, 76, 77, 78, 79, 80, 81, 82, 83, 84, 85, 86, 87, 92, 93, 94, 95, 96, 97, 98, 99, 100, 101, 102, 103, 104, 105, 106, 107, 108, 110, 111, 112, 113, 114, 115, 117, 118, 119, 120, 124, 125, 126, 127, 128, 129, 136, 137, 138, 139, 140, 141, 216, 219, 265, 266, 267, 268, 270, 271, 272, 273, 274, 281, 282, 283, 284, 285, 293, 294, 295, 296, 297, 298, 299, 300, 301, 302, 303, 304, 305, 306, 307, 308, 309, 310, 311, 312, 313, 314, 315, 316, 317, 318, 319, 320, 321, 322, 323, 324, 325, 326, 327, 328, 329, 330, 331, 332, 333, 334, 335, 336, 337, 338, 339, 340, 341, 342, 343, 344, 345, 346, 347, 348, 349, 352, 353, 354, 355, 356, 357, 358, 363, 364, 365, 366, 371, 372, 373, 374, 375, 376, 377, 378, 379, 380, 381, 382, 383, 384, 385, 409 }

B grade { 7, 8, 9, 17, 18, 19, 20, 21, 30, 31, 32, 33, 34, 42, 58, 59, 66, 72, 73, 74, 75, 88, 89, 90, 91, 109, 116, 217, 218, 269, 275, 276, 277, 278, 279, 280, 286, 287, 288, 289, 290, 291, 292, 350, 360, 368, 369, 407, 408, 435, 436, 437 }

C grade { }

F normal fail { 53, 54, 55, 61, 63, 64, 65, 121, 122, 123, 130, 131, 132, 133, 134, 135, 142, 143, 144, 145, 146, 147, 148, 149, 150, 151, 152, 153, 154, 155, 156, 157, 158, 159, 160, 161, 162, 163, 164, 165, 166, 167, 168, 169, 170, 171, 172, 173, 174, 175, 176, 177, 178, 179, 180, 181, 182, 183, 184, 185, 186, 187, 188, 189, 190, 191, 192, 193, 194, 195, 196, 197, 198, 199, 200, 201, 202, 203, 204, 205, 206, 207, 208, 209, 210, 211, 212, 213, 214, 215, 220, 221, 222, 223, 224, 225, 226, 227, 228, 229, 230, 231, 232, 233, 234, 235, 236, 237, 238, 239, 240, 241, 242, 243, 244, 245, 246, 247, 248, 249, 250, 251, 252, 253, 254, 255, 256, 257, 258, 259, 260, 261, 262, 263, 264, 386, 387, 388, 389, 390, 391, 392, 393, 394, 395, 396, 397, 398, 399, 400, 401, 402, 403, 404, 405, 406, 410, 411, 412, 413, 414, 415, 416, 417, 418, 419, 420, 421, 422, 423, 424, 425, 426, 427, 428, 429, 430, 431, 432, 433, 434, 438, 439, 440, 441, 442, 443, 444, 445, 446, 447, 448, 449, 450, 451, 452, 453, 454, 455, 456, 457, 458, 459, 460, 461, 462 }

F(-1) timedout fail { 351, 359, 361, 362, 367, 370 }

F(-2) exception fail { }

Mupad

A grade { }

B grade { 1, 2, 3, 4, 5, 7, 8, 9, 10, 12, 13, 18, 19, 20, 21, 24, 31, 32, 33, 34, 35, 36, 37, 41, 42, 43, 44, 45, 46, 50, 51, 52, 53, 54, 55, 59, 60, 61, 62, 63, 64, 65, 70, 71, 72, 73, 74, 75, 76, 77, 79, 80, 81, 82, 88, 89, 90, 91, 97, 98, 99, 100, 105, 106, 107, 108, 113, 114, 115, 116, 121, 122, 123, 124, 125, 126, 133, 134, 135, 148, 157, 165, 177, 185, 194, 217, 218, 219, 230, 236, 243, 256, 257, 258, 259, 265, 266, 267, 268, 270, 275, 276, 277, 278, 279, 280, 286, 287, 288, 289, 290, 291, 292, 300, 301, 302, 303, 306, 308, 309, 310, 311, 314, 316, 317, 318, 319, 320, 331, 332, 333, 334, 343, 344, 345, 346, 351, 352, 353, 354, 358, 359, 360, 361, 362, 365, 366, 367, 368, 369, 370, 371, 375, 376, 377, 381, 382, 383, 384, 385, 389, 391, 397, 399, 405, 407, 408, 409, 420, 422, 428, 429, 430, 431, 435, 436, 437, 448 }

C grade { }

F normal fail { }

F(-1) timedout fail { 6, 11, 14, 15, 16, 17, 22, 23, 25, 26, 27, 28, 29, 30, 38, 39, 40, 47, 48, 49, 56, 57, 58, 66, 67, 68, 69, 78, 83, 84, 85, 86, 87, 92, 93, 94, 95, 96, 101, 102, 103, 104, 109, 110, 111, 112, 117, 118, 119, 120, 127, 128, 129, 130, 131, 132, 136, 137, 138, 139, 140, 141, 142, 143, 144, 145, 146, 147, 149, 150, 151, 152, 153, 154, 155, 156, 158, 159, 160, 161, 162, 163, 164, 166, 167, 168, 169, 170, 171, 172, 173, 174, 175, 176, 178, 179, 180, 181, 182, 183, 184, 186, 187, 188, 189, 190, 191, 192, 193, 195, 196, 197, 198, 199, 200, 201, 202, 203, 204, 205, 206, 207, 208, 209, 210, 211, 212, 213, 214, 215, 216, 220, 221, 222, 223, 224, 225, 226, 227, 228, 229, 231, 232, 233, 234, 235, 237, 238, 239, 240, 241, 242, 244, 245, 246, 247, 248, 249, 250, 251, 252, 253, 254, 255, 260, 261, 262, 263, 264, 269, 271, 272, 273, 274, 281, 282, 283, 284, 285, 293, 294, 295, 296, 297, 298, 299, 304, 305, 307, 312, 313, 315, 321, 322, 323, 324, 325, 326, 327, 328, 329, 330, 335, 336, 337, 338, 339, 340, 341, 342, 347, 348, 349, 350, 355, 356, 357, 363, 364, 372, 373, 374, 378, 379, 380, 386, 387, 388, 390, 392, 393, 394, 395, 396, 398, 400, 401, 402, 403, 404, 406, 410, 411, 412, 413, 414, 415, 416, 417, 418, 419, 421, 423, 424, 425, 426, 427, 432, 433, 434, 438, 439, 440, 441, 442, 443, 444, 445, 446, 447, 449, 450, 451, 452, 453, 454, 455, 456, 457, 458, 459, 460, 461, 462 }

F(-2) exception fail { }

Sympy

A grade { 1, 2, 3, 4, 11, 12, 13, 14, 22, 23, 24, 25, 26, 35, 36, 37, 38, 39, 40, 67, 68, 69, 70, 71, 76, 77, 78 }

B grade { 41, 42, 217, 218, 219, 407, 408, 409, 435, 436, 437 }

C grade { 215, 263 }

F normal fail { 5, 6, 7, 8, 9, 10, 15, 16, 17, 18, 19, 20, 21, 27, 28, 29, 30, 31, 32, 33, 34, 43, 44, 45, 46, 47, 48, 49, 50, 51, 52, 53, 54, 55, 56, 57, 58, 59, 60, 61, 62, 63, 64, 65, 66, 72, 73, 74, 75, 79, 80, 81, 82, 83, 84, 85, 86, 87, 88, 89, 90, 91, 92, 93, 94, 95, 97, 98, 99, 100, 101, 102, 103, 104, 105, 106, 107, 108, 109, 110, 111, 112, 113, 114, 115, 116, 117, 118, 119, 120, 121, 122, 123, 124, 125, 126, 127, 128, 129, 130, 131, 132, 133, 134, 135, 136, 137, 138, 139, 140, 141, 142, 143, 144, 145, 146, 147, 148, 149, 150, 151, 152, 153, 154, 155, 156, 157, 158, 159, 160, 161, 162, 163, 164, 165, 166, 167, 168, 169, 170, 171, 172, 173, 174, 175, 176, 177, 178, 179, 180, 181, 182, 183, 184, 185, 186, 187, 188, 189, 190, 191, 192, 193, 194, 195, 196, 197, 198, 199, 200, 201, 202, 203, 204, 205, 206, 207, 208, 209, 210, 211, 212, 213, 214, 216, 220, 221, 222, 223, 224, 225, 226, 227, 228, 229, 230, 231, 232, 233, 234, 235, 236, 237, 238, 239, 240, 241, 242, 243, 244, 245, 246, 247, 248, 249, 250, 251, 252, 253, 254, 255, 256, 257, 258, 259, 260, 261, 262, 264, 265, 266, 267, 268, 269, 270, 271, 272, 273, 274, 275, 276, 277, 278, 279, 280, 281, 282, 283, 284, 285, 286, 287, 288, 289, 290, 291, 292, 293, 294, 295, 296, 297, 298, 299, 300, 301, 302, 303, 304, 305, 306, 307, 308, 309, 310, 311, 312, 313, 314, 315, 316, 317, 318, 319, 320, 321, 322, 323, 324, 325, 326, 327, 328, 329, 330, 331, 332, 333, 334, 335, 336, 337, 338, 339, 340, 341, 342, 347, 348, 349, 350, 351, 352, 353, 354, 355, 356, 357, 358, 359, 360, 361, 362, 365, 366, 367, 368, 369, 370, 371, 372, 373, 374, 375, 376, 377, 378, 379, 380, 381, 382, 383, 384, 385, 386, 387, 388, 389, 390, 391, 392, 393, 394, 395, 396, 397, 398, 399, 400, 401, 402, 403, 404, 405, 406, 410, 411, 412, 413, 414, 415, 416, 417, 418, 419, 420, 421, 422, 423, 424, 425, 426, 427, 428, 429, 430, 431, 432, 433, 434, 438, 439, 440, 441, 442, 443, 444, 445, 446, 447, 448, 449, 450, 452, 453, 454, 455, 456, 457, 458, 459, 460, 461, 462 }

F(-1) timedout fail { 96, 343, 344, 345, 346, 363, 364, 451 }

F(-2) exception fail { }

Reduce

A grade { }

B grade { 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17, 18, 19, 20, 21, 22, 23, 24, 25, 26, 27, 28, 29, 30, 31, 32, 33, 34, 35, 36, 37, 38, 39, 40, 41, 42, 43, 44, 45, 46, 47, 48, 49, 50, 51, 52, 53, 54, 55, 56, 57, 58, 59, 60, 61, 62, 63, 64, 65, 66, 67, 68, 69, 70, 71, 72, 73, 74, 75, 76, 77, 78, 79, 80, 81, 82, 83, 84, 85, 86, 87, 88, 89, 90, 91, 92, 93, 94, 95, 96, 97, 98, 99, 100, 101, 102, 103, 104, 105, 106, 107, 108, 109, 110, 111, 112, 113, 114, 115, 116, 117, 118, 119, 120, 121, 122, 123, 130, 131, 132, 133, 134, 135, 142, 143, 144, 179, 180, 186, 187, 188, 194, 195, 196, 197, 216, 217, 218, 219, 256, 257, 258, 259, 265, 266, 267, 268, 269, 270, 271, 272, 273, 274, 275, 276, 277, 278, 279, 280, 281, 282, 283, 284, 285, 286, 287, 288, 289, 290, 291, 292, 293, 294, 295, 296, 297, 298, 299, 300, 301, 302, 303, 304, 305, 306, 307, 308, 309, 310, 311, 312, 313, 314, 315, 316, 317, 318, 319, 320, 321, 322, 323, 324, 325, 326, 327, 328, 329, 330, 331, 332, 333, 334, 335, 336, 337, 338, 339, 340, 341, 342, 343, 344, 345, 346, 347, 348, 349, 350, 351, 352, 353, 354, 355, 356, 357, 358, 359, 360, 361, 362, 363, 364, 365, 366, 367, 368, 369, 370, 371, 407, 408, 409, 428, 429, 430, 431, 435, 436, 437, 456, 457, 458, 459 }

C grade { }

F normal fail { 124, 125, 126, 127, 128, 129, 136, 137, 138, 139, 140, 141, 145, 146, 147, 148, 149, 150, 151, 152, 153, 154, 155, 156, 157, 158, 159, 160, 161, 162, 163, 164, 165, 166, 167, 168, 169, 170, 171, 172, 173, 174, 175, 176, 177, 178, 181, 182, 183, 184, 185, 189, 190, 191, 192, 193, 198, 199, 200, 201, 202, 203, 204, 205, 206, 207, 208, 209, 210, 211, 212, 213, 214, 215, 220, 221, 222, 223, 224, 225, 226, 227, 228, 229, 230, 231, 232, 233, 234, 235, 236, 237, 238, 239, 240, 241, 242, 243, 244, 245, 246, 247, 248, 249, 250, 251, 252, 253, 254, 255, 260, 261, 262, 263, 264, 372, 373, 374, 375, 376, 377, 378, 379, 380, 381, 382, 383, 384, 385, 386, 387, 388, 389, 390, 391, 392, 393, 394, 395, 396, 397, 398, 399, 400, 401, 402, 403, 404, 405, 406, 410, 411, 412, 413, 414, 415, 416, 417, 418, 419, 420, 421, 422, 423, 424, 425, 426, 427, 432, 433, 434, 438, 439, 440, 441, 442, 443, 444, 445, 446, 447, 448, 449, 450, 451, 452, 453, 454, 455, 460, 461, 462 }

F(-1) timedout fail { }

F(-2) exception fail { }

2.2 Detailed conclusion table per each integral for all CAS systems

Detailed conclusion table per each integral is given by the table below. The elapsed time is in seconds. For failed result it is given as **F(-1)** if the failure was due to timeout. It is given as **F(-2)** if the failure was due to an exception being raised, which could indicate a bug in the system. If the failure was due to integral not being evaluated within the time limit, then it is given as **F**.

In this table, the column `N.S.` means **normalized size** and is defined as $\frac{\text{antiderivative leaf size}}{\text{optimal antiderivative leaf size}}$. To make the table fit the page, the name **`Mathematica`** was abbreviated to **`MMA`**.

Problem 1	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	151	151	118	84	127	193	148	95	114	119
N.S.	1	1.00	0.78	0.56	0.84	1.28	0.98	0.63	0.75	0.79
time (sec)	N/A	0.476	0.467	0.431	0.027	0.092	0.245	0.218	0.214	9.829

Problem 2	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	125	119	107	73	105	170	134	83	95	93
N.S.	1	0.95	0.86	0.58	0.84	1.36	1.07	0.66	0.76	0.74
time (sec)	N/A	0.399	0.383	0.382	0.026	0.098	0.243	0.224	0.220	0.209

Problem 3	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	99	89	96	73	85	149	117	71	76	69
N.S.	1	0.90	0.97	0.74	0.86	1.51	1.18	0.72	0.77	0.70
time (sec)	N/A	0.344	0.300	0.355	0.032	0.097	0.239	0.167	0.261	0.093

Problem 4	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	73	60	83	56	63	122	102	59	56	55
N.S.	1	0.82	1.14	0.77	0.86	1.67	1.40	0.81	0.77	0.75
time (sec)	N/A	0.294	0.227	0.358	0.030	0.087	0.207	0.141	0.212	9.514

Problem 5	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	F	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	42	42	59	38	41	101	0	46	37	42
N.S.	1	1.00	1.40	0.90	0.98	2.40	0.00	1.10	0.88	1.00
time (sec)	N/A	0.279	0.064	0.354	0.027	0.082	0.000	0.120	0.245	0.082

Problem 6	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	F	A	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	47	47	70	40	44	96	0	60	43	0
N.S.	1	1.00	1.49	0.85	0.94	2.04	0.00	1.28	0.91	0.00
time (sec)	N/A	0.290	0.108	0.355	0.030	0.079	0.000	0.125	0.256	0.000

Problem 7	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	F	B	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	23	23	21	20	37	24	0	76	40	24
N.S.	1	1.00	0.91	0.87	1.61	1.04	0.00	3.30	1.74	1.04
time (sec)	N/A	0.241	0.023	0.346	0.030	0.081	0.000	0.236	0.235	9.407

Problem 8	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	F	B	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	48	48	39	29	59	38	0	107	61	37
N.S.	1	1.00	0.81	0.60	1.23	0.79	0.00	2.23	1.27	0.77
time (sec)	N/A	0.283	0.074	0.372	0.029	0.079	0.000	0.242	0.227	9.197

Problem 9	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	F	B	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	74	80	51	40	81	49	0	136	80	81
N.S.	1	1.08	0.69	0.54	1.09	0.66	0.00	1.84	1.08	1.09
time (sec)	N/A	0.324	0.082	0.378	0.032	0.075	0.000	0.144	0.225	8.905

Problem 10	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	F	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	100	112	62	55	103	60	0	165	99	103
N.S.	1	1.12	0.62	0.55	1.03	0.60	0.00	1.65	0.99	1.03
time (sec)	N/A	0.377	0.099	0.403	0.027	0.112	0.000	0.121	0.242	9.039

Problem 11	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	175	156	129	95	144	215	160	106	133	0
N.S.	1	0.89	0.74	0.54	0.82	1.23	0.91	0.61	0.76	0.00
time (sec)	N/A	0.444	0.603	0.392	0.028	0.084	0.291	0.132	0.215	0.000

Problem 12	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	149	126	118	84	124	192	144	93	114	118
N.S.	1	0.85	0.79	0.56	0.83	1.29	0.97	0.62	0.77	0.79
time (sec)	N/A	0.385	0.496	0.387	0.030	0.084	0.283	0.123	0.196	9.031

Problem 13	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	123	97	107	73	102	171	257	81	95	87
N.S.	1	0.79	0.87	0.59	0.83	1.39	2.09	0.66	0.77	0.71
time (sec)	N/A	0.340	0.379	0.362	0.026	0.083	0.348	0.125	0.209	9.338

Problem 14	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	97	83	96	73	79	148	224	70	76	0
N.S.	1	0.86	0.99	0.75	0.81	1.53	2.31	0.72	0.78	0.00
time (sec)	N/A	0.326	0.319	0.358	0.032	0.086	0.947	0.125	0.192	0.000

Problem 15	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	F	A	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	71	68	70	59	62	127	0	58	57	0
N.S.	1	0.96	0.99	0.83	0.87	1.79	0.00	0.82	0.80	0.00
time (sec)	N/A	0.319	0.123	0.367	0.025	0.082	0.000	0.365	0.196	0.000

Problem 16	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	F	A	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	64	65	83	56	62	117	0	76	59	0
N.S.	1	1.02	1.30	0.88	0.97	1.83	0.00	1.19	0.92	0.00
time (sec)	N/A	0.328	0.186	0.369	0.027	0.108	0.000	0.298	0.202	0.000

Problem 17	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	F	B	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	71	70	79	49	78	117	0	115	55	0
N.S.	1	0.99	1.11	0.69	1.10	1.65	0.00	1.62	0.77	0.00
time (sec)	N/A	0.333	0.131	0.379	0.026	0.087	0.000	0.168	0.219	0.000

Problem 18	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	B	A	F	B	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	23	23	21	20	73	35	0	134	61	26
N.S.	1	1.00	0.91	0.87	3.17	1.52	0.00	5.83	2.65	1.13
time (sec)	N/A	0.231	0.041	0.374	0.030	0.073	0.000	0.106	0.243	9.014

Problem 19	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	B	A	F	B	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	48	48	29	31	95	49	0	165	80	79
N.S.	1	1.00	0.60	0.65	1.98	1.02	0.00	3.44	1.67	1.65
time (sec)	N/A	0.277	0.118	0.384	0.027	0.080	0.000	0.267	0.246	9.642

Problem 20	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	F	B	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	74	80	40	42	117	60	0	194	99	101
N.S.	1	1.08	0.54	0.57	1.58	0.81	0.00	2.62	1.34	1.36
time (sec)	N/A	0.333	0.130	0.399	0.033	0.081	0.000	0.284	0.237	10.227

Problem 21	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	F	B	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	100	112	51	55	139	71	0	223	118	123
N.S.	1	1.12	0.51	0.55	1.39	0.71	0.00	2.23	1.18	1.23
time (sec)	N/A	0.388	0.152	0.431	0.035	0.079	0.000	0.143	0.252	10.599

Problem 22	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	227	193	151	128	183	258	187	129	171	0
N.S.	1	0.85	0.67	0.56	0.81	1.14	0.82	0.57	0.75	0.00
time (sec)	N/A	0.514	0.952	0.366	0.032	0.100	0.394	0.126	0.261	0.000

Problem 23	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	201	163	140	117	163	237	172	118	152	0
N.S.	1	0.81	0.70	0.58	0.81	1.18	0.86	0.59	0.76	0.00
time (sec)	N/A	0.473	0.749	0.359	0.032	0.113	0.374	0.136	0.220	0.000

Problem 24	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	175	134	129	106	141	214	459	105	133	119
N.S.	1	0.77	0.74	0.61	0.81	1.22	2.62	0.60	0.76	0.68
time (sec)	N/A	0.395	0.661	0.339	0.026	0.103	0.465	0.246	0.255	10.145

Problem 25	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	149	120	118	84	118	193	415	94	114	0
N.S.	1	0.81	0.79	0.56	0.79	1.30	2.79	0.63	0.77	0.00
time (sec)	N/A	0.386	0.505	0.387	0.026	0.091	1.491	0.288	0.279	0.000

Problem 26	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	123	109	107	73	98	170	369	82	95	0
N.S.	1	0.89	0.87	0.59	0.80	1.38	3.00	0.67	0.77	0.00
time (sec)	N/A	0.380	0.429	0.396	0.033	0.106	0.523	0.170	0.322	0.000

Problem 27	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	F	A	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	97	105	81	64	81	149	0	70	76	0
N.S.	1	1.08	0.84	0.66	0.84	1.54	0.00	0.72	0.78	0.00
time (sec)	N/A	0.377	0.124	0.380	0.026	0.099	0.000	0.209	0.213	0.000

Problem 28	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	F	A	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	95	94	100	69	84	145	0	89	82	0
N.S.	1	0.99	1.05	0.73	0.88	1.53	0.00	0.94	0.86	0.00
time (sec)	N/A	0.444	0.276	0.395	0.027	0.098	0.000	0.145	0.212	0.000

Problem 29	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	F	A	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	92	91	101	70	99	146	0	133	84	0
N.S.	1	0.99	1.10	0.76	1.08	1.59	0.00	1.45	0.91	0.00
time (sec)	N/A	0.375	0.216	0.408	0.033	0.102	0.000	0.145	0.200	0.000

Problem 30	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	F	B	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	97	93	92	62	134	145	0	175	86	0
N.S.	1	0.96	0.95	0.64	1.38	1.49	0.00	1.80	0.89	0.00
time (sec)	N/A	0.371	0.184	0.414	0.053	0.090	0.000	0.142	0.247	0.000

Problem 31	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	B	B	F	B	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	23	23	21	20	112	46	0	192	80	79
N.S.	1	1.00	0.91	0.87	4.87	2.00	0.00	8.35	3.48	3.43
time (sec)	N/A	0.225	0.046	0.418	0.027	0.081	0.000	0.172	0.222	9.632

Problem 32	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	B	A	F	B	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	48	48	29	31	134	60	0	223	99	101
N.S.	1	1.00	0.60	0.65	2.79	1.25	0.00	4.65	2.06	2.10
time (sec)	N/A	0.275	0.120	0.442	0.031	0.078	0.000	0.112	0.210	9.910

Problem 33	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	B	A	F	B	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	74	80	40	42	156	71	0	252	118	123
N.S.	1	1.08	0.54	0.57	2.11	0.96	0.00	3.41	1.59	1.66
time (sec)	N/A	0.334	0.151	0.486	0.034	0.077	0.000	0.124	0.211	10.208

Problem 34	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	B	A	F	B	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	100	112	51	55	178	82	0	281	137	145
N.S.	1	1.12	0.51	0.55	1.78	0.82	0.00	2.81	1.37	1.45
time (sec)	N/A	0.390	0.145	0.526	0.035	0.085	0.000	0.123	0.263	10.696

Problem 35	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	67	54	54	32	49	44	27	29	55	42
N.S.	1	0.81	0.81	0.48	0.73	0.66	0.40	0.43	0.82	0.63
time (sec)	N/A	0.293	0.079	0.651	0.106	0.093	0.240	0.119	0.222	0.098

Problem 36	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	72	59	75	38	49	47	34	32	56	44
N.S.	1	0.82	1.04	0.53	0.68	0.65	0.47	0.44	0.78	0.61
time (sec)	N/A	0.294	0.079	0.342	0.104	0.076	0.243	0.114	0.211	9.177

Problem 37	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	66	53	53	37	46	37	37	38	39	33
N.S.	1	0.80	0.80	0.56	0.70	0.56	0.56	0.58	0.59	0.50
time (sec)	N/A	0.283	0.068	0.341	0.029	0.071	0.196	0.373	0.212	0.090

Problem 38	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	128	141	119	73	110	171	143	87	95	0
N.S.	1	1.10	0.93	0.57	0.86	1.34	1.12	0.68	0.74	0.00
time (sec)	N/A	0.457	0.320	0.389	0.027	0.093	0.264	0.342	0.214	0.000

Problem 39	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	102	109	108	73	88	148	129	75	76	0
N.S.	1	1.07	1.06	0.72	0.86	1.45	1.26	0.74	0.75	0.00
time (sec)	N/A	0.393	0.275	0.369	0.033	0.086	0.249	0.186	0.217	0.000

Problem 40	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	76	77	97	59	66	127	116	63	57	0
N.S.	1	1.01	1.28	0.78	0.87	1.67	1.53	0.83	0.75	0.00
time (sec)	N/A	0.329	0.197	0.365	0.026	0.091	0.249	0.133	0.199	0.000

Problem 41	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	B	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	47	47	78	39	45	99	99	50	38	46
N.S.	1	1.00	1.66	0.83	0.96	2.11	2.11	1.06	0.81	0.98
time (sec)	N/A	0.277	0.149	0.361	0.027	0.083	0.232	0.122	0.212	0.101

Problem 42	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	B	B	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	28	28	55	23	27	63	75	59	25	28
N.S.	1	1.00	1.96	0.82	0.96	2.25	2.68	2.11	0.89	1.00
time (sec)	N/A	0.240	0.045	0.339	0.030	0.081	0.291	0.126	0.224	9.342

Problem 43	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	F	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	21	21	21	18	19	19	0	23	23	19
N.S.	1	1.00	1.00	0.86	0.90	0.90	0.00	1.10	1.10	0.90
time (sec)	N/A	0.230	0.021	0.339	0.027	0.069	0.000	0.122	0.213	9.267

Problem 44	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	F	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	48	48	29	24	40	27	0	49	40	25
N.S.	1	1.00	0.60	0.50	0.83	0.56	0.00	1.02	0.83	0.52
time (sec)	N/A	0.267	0.064	0.344	0.026	0.074	0.000	0.161	0.217	9.173

Problem 45	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	F	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	74	80	40	37	62	38	0	78	61	38
N.S.	1	1.08	0.54	0.50	0.84	0.51	0.00	1.05	0.82	0.51
time (sec)	N/A	0.327	0.085	0.355	0.026	0.080	0.000	0.391	0.209	8.884

Problem 46	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	F	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	100	112	51	48	84	49	0	107	80	84
N.S.	1	1.12	0.51	0.48	0.84	0.49	0.00	1.07	0.80	0.84
time (sec)	N/A	0.374	0.086	0.366	0.028	0.074	0.000	0.136	0.228	8.982

Problem 47	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	F	A	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	124	135	108	84	113	204	0	112	98	0
N.S.	1	1.09	0.87	0.68	0.91	1.65	0.00	0.90	0.79	0.00
time (sec)	N/A	0.619	0.414	0.402	0.031	0.086	0.000	0.321	0.246	0.000

Problem 48	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	F	A	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	98	97	97	73	91	183	0	100	85	0
N.S.	1	0.99	0.99	0.74	0.93	1.87	0.00	1.02	0.87	0.00
time (sec)	N/A	0.452	0.256	0.395	0.026	0.088	0.000	0.322	0.228	0.000

Problem 49	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	F	A	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	67	71	80	58	66	153	0	87	68	0
N.S.	1	1.06	1.19	0.87	0.99	2.28	0.00	1.30	1.01	0.00
time (sec)	N/A	0.351	0.226	0.385	0.027	0.082	0.000	0.132	0.199	0.000

Problem 50	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	F	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	48	48	73	39	45	127	0	67	56	46
N.S.	1	1.00	1.52	0.81	0.94	2.65	0.00	1.40	1.17	0.96
time (sec)	N/A	0.297	0.124	0.371	0.026	0.082	0.000	0.180	0.192	0.101

Problem 51	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	F	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	19	19	17	16	17	23	0	32	30	15
N.S.	1	1.00	0.89	0.84	0.89	1.21	0.00	1.68	1.58	0.79
time (sec)	N/A	0.241	0.022	0.336	0.026	0.074	0.000	0.133	0.232	0.035

Problem 52	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	F	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	40	24	22	21	35	35	0	24	39	24
N.S.	1	0.60	0.55	0.52	0.88	0.88	0.00	0.60	0.98	0.60
time (sec)	N/A	0.234	0.070	0.345	0.026	0.074	0.000	0.115	0.211	0.046

Problem 53	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	F	F	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	69	51	40	35	57	50	0	0	55	45
N.S.	1	0.74	0.58	0.51	0.83	0.72	0.00	0.00	0.80	0.65
time (sec)	N/A	0.276	0.093	0.356	0.027	0.076	0.000	0.000	0.223	9.228

Problem 54	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	F	F	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	95	83	49	46	79	59	0	0	70	54
N.S.	1	0.87	0.52	0.48	0.83	0.62	0.00	0.00	0.74	0.57
time (sec)	N/A	0.335	0.122	0.366	0.033	0.076	0.000	0.000	0.230	9.230

Problem 55	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	F	F	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	121	115	62	59	101	72	0	0	83	102
N.S.	1	0.95	0.51	0.49	0.83	0.60	0.00	0.00	0.69	0.84
time (sec)	N/A	0.396	0.113	0.381	0.028	0.080	0.000	0.000	0.248	9.422

Problem 56	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	F	A	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	128	128	109	98	190	249	0	157	153	0
N.S.	1	1.00	0.85	0.77	1.48	1.95	0.00	1.23	1.20	0.00
time (sec)	N/A	0.516	0.370	0.438	0.037	0.090	0.000	0.166	0.358	0.000

Problem 57	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	B	A	F	A	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	95	102	96	82	162	222	0	144	136	0
N.S.	1	1.07	1.01	0.86	1.71	2.34	0.00	1.52	1.43	0.00
time (sec)	N/A	0.404	0.293	0.414	0.038	0.089	0.000	0.129	0.499	0.000

Problem 58	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	B	A	F	B	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	74	76	84	73	140	194	0	124	94	0
N.S.	1	1.03	1.14	0.99	1.89	2.62	0.00	1.68	1.27	0.00
time (sec)	N/A	0.352	0.271	0.376	0.037	0.090	0.000	0.137	0.412	0.000

Problem 59	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	B	A	F	B	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	23	23	21	25	74	35	0	85	52	24
N.S.	1	1.00	0.91	1.09	3.22	1.52	0.00	3.70	2.26	1.04
time (sec)	N/A	0.233	0.037	0.356	0.027	0.075	0.000	0.150	0.187	9.544

Problem 60	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	F	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	45	51	29	29	54	44	0	60	61	31
N.S.	1	1.13	0.64	0.64	1.20	0.98	0.00	1.33	1.36	0.69
time (sec)	N/A	0.283	0.075	0.349	0.027	0.087	0.000	0.133	0.342	9.793

Problem 61	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	F	F	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	65	48	38	39	52	59	0	0	74	45
N.S.	1	0.74	0.58	0.60	0.80	0.91	0.00	0.00	1.14	0.69
time (sec)	N/A	0.269	0.091	0.362	0.031	0.079	0.000	0.000	0.516	10.017

Problem 62	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	F	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	89	54	48	47	72	72	0	50	92	43
N.S.	1	0.61	0.54	0.53	0.81	0.81	0.00	0.56	1.03	0.48
time (sec)	N/A	0.272	0.106	0.361	0.027	0.082	0.000	0.295	0.461	0.044

Problem 63	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	F	F	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	120	86	64	63	96	83	0	0	107	67
N.S.	1	0.72	0.53	0.52	0.80	0.69	0.00	0.00	0.89	0.56
time (sec)	N/A	0.331	0.132	0.370	0.026	0.088	0.000	0.000	0.237	10.325

Problem 64	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	F	F	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	146	118	73	75	118	94	0	0	120	121
N.S.	1	0.81	0.50	0.51	0.81	0.64	0.00	0.00	0.82	0.83
time (sec)	N/A	0.397	0.132	0.378	0.030	0.078	0.000	0.000	0.604	10.091

Problem 65	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	F	F	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	170	150	84	88	140	105	0	0	133	143
N.S.	1	0.88	0.49	0.52	0.82	0.62	0.00	0.00	0.78	0.84
time (sec)	N/A	0.453	0.157	0.388	0.027	0.087	0.000	0.000	0.632	10.286

Problem 66	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	B	A	F	B	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	100	104	101	83	357	260	0	182	197	0
N.S.	1	1.04	1.01	0.83	3.57	2.60	0.00	1.82	1.97	0.00
time (sec)	N/A	0.411	0.324	0.443	0.051	0.151	0.000	0.166	0.256	0.000

Problem 67	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	111	120	83	46	68	51	42	51	78	0
N.S.	1	1.08	0.75	0.41	0.61	0.46	0.38	0.46	0.70	0.00
time (sec)	N/A	0.460	0.090	0.355	0.106	0.097	0.485	0.147	0.350	0.000

Problem 68	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	88	95	78	41	52	46	36	42	63	0
N.S.	1	1.08	0.89	0.47	0.59	0.52	0.41	0.48	0.72	0.00
time (sec)	N/A	0.401	0.078	0.331	0.107	0.077	0.388	0.143	0.519	0.000

Problem 69	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	65	70	71	34	36	39	27	31	48	0
N.S.	1	1.08	1.09	0.52	0.55	0.60	0.42	0.48	0.74	0.00
time (sec)	N/A	0.331	0.079	0.336	0.108	0.078	0.326	0.134	0.329	0.000

Problem 70	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	42	37	68	23	22	36	20	25	35	37
N.S.	1	0.88	1.62	0.55	0.52	0.86	0.48	0.60	0.83	0.88
time (sec)	N/A	0.278	0.054	0.330	0.103	0.103	0.291	0.387	0.261	9.323

Problem 71	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	B	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	18	14	53	9	8	22	7	8	22	8
N.S.	1	0.78	2.94	0.50	0.44	1.22	0.39	0.44	1.22	0.44
time (sec)	N/A	0.246	0.009	0.341	0.103	0.075	0.278	0.202	0.326	9.732

Problem 72	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	F	B	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	21	23	19	17	16	16	0	31	22	16
N.S.	1	1.10	0.90	0.81	0.76	0.76	0.00	1.48	1.05	0.76
time (sec)	N/A	0.244	0.017	0.307	0.102	0.071	0.000	0.148	0.498	9.496

Problem 73	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	F	B	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	45	49	30	22	33	21	0	87	37	21
N.S.	1	1.09	0.67	0.49	0.73	0.47	0.00	1.93	0.82	0.47
time (sec)	N/A	0.286	0.054	0.333	0.102	0.096	0.000	0.136	0.363	9.274

Problem 74	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	F	B	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	70	74	35	27	49	26	0	143	52	26
N.S.	1	1.06	0.50	0.39	0.70	0.37	0.00	2.04	0.74	0.37
time (sec)	N/A	0.329	0.058	0.375	0.105	0.076	0.000	0.135	0.255	8.849

Problem 75	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	F	B	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	93	99	40	32	65	31	0	199	67	65
N.S.	1	1.06	0.43	0.34	0.70	0.33	0.00	2.14	0.72	0.70
time (sec)	N/A	0.363	0.070	0.321	0.103	0.076	0.000	0.126	0.291	8.926

Problem 76	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	29	28	47	23	22	34	17	22	29	22
N.S.	1	0.97	1.62	0.79	0.76	1.17	0.59	0.76	1.00	0.76
time (sec)	N/A	0.261	0.046	0.652	0.104	0.077	0.245	0.112	0.488	0.054

Problem 77	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	35	28	46	26	29	27	29	28	22	23
N.S.	1	0.80	1.31	0.74	0.83	0.77	0.83	0.80	0.63	0.66
time (sec)	N/A	0.260	0.006	0.336	0.026	0.073	0.206	0.112	0.493	9.026

Problem 78	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	51	51	53	25	36	37	26	23	42	0
N.S.	1	1.00	1.04	0.49	0.71	0.73	0.51	0.45	0.82	0.00
time (sec)	N/A	0.309	0.052	0.449	0.103	0.105	0.229	0.113	0.256	0.000

Problem 79	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	F	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	128	136	58	60	68	80	0	90	57	68
N.S.	1	1.06	0.45	0.47	0.53	0.62	0.00	0.70	0.45	0.53
time (sec)	N/A	0.434	0.046	0.358	0.036	0.074	0.000	0.162	0.291	9.394

Problem 80	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	F	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	95	98	47	48	54	58	0	74	44	55
N.S.	1	1.03	0.49	0.51	0.57	0.61	0.00	0.78	0.46	0.58
time (sec)	N/A	0.364	0.038	0.350	0.038	0.075	0.000	0.114	0.489	9.247

Problem 81	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	F	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	62	60	46	36	39	42	0	56	31	42
N.S.	1	0.97	0.74	0.58	0.63	0.68	0.00	0.90	0.50	0.68
time (sec)	N/A	0.298	0.011	0.358	0.041	0.098	0.000	0.110	0.445	9.268

Problem 82	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	F	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	28	28	26	25	27	32	0	34	21	27
N.S.	1	1.00	0.93	0.89	0.96	1.14	0.00	1.21	0.75	0.96
time (sec)	N/A	0.247	0.011	0.332	0.034	0.081	0.000	0.102	0.252	9.600

Problem 83	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F	A	F	A	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	70	70	62	70	0	147	0	82	44	0
N.S.	1	1.00	0.89	1.00	0.00	2.10	0.00	1.17	0.63	0.00
time (sec)	N/A	0.347	0.057	0.331	0.000	0.124	0.000	0.280	0.276	0.000

Problem 84	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F	A	F	A	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	71	71	69	74	0	157	0	54	56	0
N.S.	1	1.00	0.97	1.04	0.00	2.21	0.00	0.76	0.79	0.00
time (sec)	N/A	0.347	0.077	0.344	0.000	0.088	0.000	0.131	0.484	0.000

Problem 85	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F	A	F	A	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	108	106	83	96	0	179	0	94	77	0
N.S.	1	0.98	0.77	0.89	0.00	1.66	0.00	0.87	0.71	0.00
time (sec)	N/A	0.398	0.115	0.346	0.000	0.089	0.000	0.149	0.563	0.000

Problem 86	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F	A	F	A	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	141	145	101	108	0	205	0	102	95	0
N.S.	1	1.03	0.72	0.77	0.00	1.45	0.00	0.72	0.67	0.00
time (sec)	N/A	0.465	0.139	0.360	0.000	0.087	0.000	0.149	0.236	0.000

Problem 87	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F	A	F	A	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	174	184	113	119	0	227	0	139	112	0
N.S.	1	1.06	0.65	0.68	0.00	1.30	0.00	0.80	0.64	0.00
time (sec)	N/A	0.546	0.152	0.395	0.000	0.087	0.000	0.313	0.302	0.000

Problem 88	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	F	B	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	128	136	65	60	157	82	0	209	66	62
N.S.	1	1.06	0.51	0.47	1.23	0.64	0.00	1.63	0.52	0.48
time (sec)	N/A	0.415	0.019	0.329	0.042	0.080	0.000	0.330	0.453	9.324

Problem 89	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	F	B	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	95	98	52	49	129	65	0	170	54	50
N.S.	1	1.03	0.55	0.52	1.36	0.68	0.00	1.79	0.57	0.53
time (sec)	N/A	0.368	0.035	0.331	0.040	0.084	0.000	0.305	0.450	9.126

Problem 90	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	F	B	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	62	60	41	35	80	56	0	138	45	47
N.S.	1	0.97	0.66	0.56	1.29	0.90	0.00	2.23	0.73	0.76
time (sec)	N/A	0.293	0.024	0.337	0.040	0.073	0.000	0.255	0.232	9.222

Problem 91	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	B	A	F	B	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	28	28	26	27	52	43	0	98	32	45
N.S.	1	1.00	0.93	0.96	1.86	1.54	0.00	3.50	1.14	1.61
time (sec)	N/A	0.242	0.019	0.326	0.040	0.074	0.000	0.185	0.362	9.385

Problem 92	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F	A	F	A	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	98	103	75	92	0	166	0	123	59	0
N.S.	1	1.05	0.77	0.94	0.00	1.69	0.00	1.26	0.60	0.00
time (sec)	N/A	0.378	0.044	0.352	0.000	0.093	0.000	0.145	0.513	0.000

Problem 93	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F	A	F	A	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	98	104	74	94	0	171	0	71	64	0
N.S.	1	1.06	0.76	0.96	0.00	1.74	0.00	0.72	0.65	0.00
time (sec)	N/A	0.373	0.076	0.368	0.000	0.096	0.000	0.159	0.458	0.000

Problem 94	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F	A	F	A	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	106	107	87	91	0	184	0	87	78	0
N.S.	1	1.01	0.82	0.86	0.00	1.74	0.00	0.82	0.74	0.00
time (sec)	N/A	0.367	0.124	0.364	0.000	0.083	0.000	0.149	0.240	0.000

Problem 95	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F	A	F	A	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	139	142	102	108	0	205	0	108	95	0
N.S.	1	1.02	0.73	0.78	0.00	1.47	0.00	0.78	0.68	0.00
time (sec)	N/A	0.428	0.146	0.501	0.000	0.080	0.000	0.398	0.361	0.000

Problem 96	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F	A	F(-1)	A	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	172	181	112	119	0	227	0	139	112	0
N.S.	1	1.05	0.65	0.69	0.00	1.32	0.00	0.81	0.65	0.00
time (sec)	N/A	0.503	0.168	0.378	0.000	0.092	0.000	0.206	0.522	0.000

Problem 97	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	F	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	124	136	58	56	68	66	0	95	46	57
N.S.	1	1.10	0.47	0.45	0.55	0.53	0.00	0.77	0.37	0.46
time (sec)	N/A	0.410	0.041	0.345	0.040	0.069	0.000	0.112	0.394	9.559

Problem 98	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	F	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	93	98	47	45	54	52	0	82	35	46
N.S.	1	1.05	0.51	0.48	0.58	0.56	0.00	0.88	0.38	0.49
time (sec)	N/A	0.360	0.034	0.402	0.038	0.068	0.000	0.107	0.220	9.181

Problem 99	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	F	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	60	60	35	32	39	33	0	52	21	41
N.S.	1	1.00	0.58	0.53	0.65	0.55	0.00	0.87	0.35	0.68
time (sec)	N/A	0.286	0.022	0.342	0.038	0.070	0.000	0.110	0.339	9.196

Problem 100	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	F	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	26	26	24	23	24	24	0	32	13	24
N.S.	1	1.00	0.92	0.88	0.92	0.92	0.00	1.23	0.50	0.92
time (sec)	N/A	0.236	0.008	0.339	0.036	0.072	0.000	0.125	0.476	9.392

Problem 101	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F	A	F	A	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	44	44	51	48	0	101	0	54	36	0
N.S.	1	1.00	1.16	1.09	0.00	2.30	0.00	1.23	0.82	0.00
time (sec)	N/A	0.265	0.014	0.343	0.000	0.085	0.000	0.162	0.525	0.000

Problem 102	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F	A	F	A	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	70	70	66	76	0	157	0	63	56	0
N.S.	1	1.00	0.94	1.09	0.00	2.24	0.00	0.90	0.80	0.00
time (sec)	N/A	0.309	0.075	0.354	0.000	0.077	0.000	0.167	0.262	0.000

Problem 103	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F	A	F	A	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	108	109	86	97	0	183	0	93	78	0
N.S.	1	1.01	0.80	0.90	0.00	1.69	0.00	0.86	0.72	0.00
time (sec)	N/A	0.378	0.089	0.716	0.000	0.090	0.000	0.163	0.501	0.000

Problem 104	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F	A	F	A	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	141	148	99	108	0	205	0	101	95	0
N.S.	1	1.05	0.70	0.77	0.00	1.45	0.00	0.72	0.67	0.00
time (sec)	N/A	0.433	0.099	0.538	0.000	0.079	0.000	0.156	0.541	0.000

Problem 105	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F	A	F	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	122	132	57	59	0	75	0	121	47	91
N.S.	1	1.08	0.47	0.48	0.00	0.61	0.00	0.99	0.39	0.75
time (sec)	N/A	0.406	0.030	0.624	0.000	0.076	0.000	0.129	0.349	9.285

Problem 106	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F	A	F	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	91	94	46	49	0	61	0	106	36	53
N.S.	1	1.03	0.51	0.54	0.00	0.67	0.00	1.16	0.40	0.58
time (sec)	N/A	0.364	0.027	0.351	0.000	0.095	0.000	0.129	0.232	9.143

Problem 107	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F	A	F	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	58	56	33	37	0	47	0	66	25	53
N.S.	1	0.97	0.57	0.64	0.00	0.81	0.00	1.14	0.43	0.91
time (sec)	N/A	0.284	0.020	0.539	0.000	0.074	0.000	0.390	0.516	9.015

Problem 108	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F	A	F	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	26	26	24	27	0	33	0	40	16	22
N.S.	1	1.00	0.92	1.04	0.00	1.27	0.00	1.54	0.62	0.85
time (sec)	N/A	0.235	0.004	0.431	0.000	0.078	0.000	0.228	0.512	8.933

Problem 109	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F	A	F	B	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	70	70	58	69	0	188	0	116	59	0
N.S.	1	1.00	0.83	0.99	0.00	2.69	0.00	1.66	0.84	0.00
time (sec)	N/A	0.318	0.036	0.499	0.000	0.097	0.000	0.130	0.272	0.000

Problem 110	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F	A	F	A	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	100	107	67	81	0	217	0	99	78	0
N.S.	1	1.07	0.67	0.81	0.00	2.17	0.00	0.99	0.78	0.00
time (sec)	N/A	0.374	0.069	0.369	0.000	0.092	0.000	0.157	0.223	0.000

Problem 111	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F	A	F	A	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	141	146	86	105	0	255	0	121	97	0
N.S.	1	1.04	0.61	0.74	0.00	1.81	0.00	0.86	0.69	0.00
time (sec)	N/A	0.446	0.022	0.372	0.000	0.087	0.000	0.150	0.333	0.000

Problem 112	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F	A	F	A	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	174	185	98	121	0	277	0	141	108	0
N.S.	1	1.06	0.56	0.70	0.00	1.59	0.00	0.81	0.62	0.00
time (sec)	N/A	0.545	0.131	0.372	0.000	0.088	0.000	0.172	0.447	0.000

Problem 113	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F	A	F	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	124	128	57	60	0	86	0	121	54	103
N.S.	1	1.03	0.46	0.48	0.00	0.69	0.00	0.98	0.44	0.83
time (sec)	N/A	0.428	0.042	0.369	0.000	0.094	0.000	0.132	0.457	9.534

Problem 114	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F	A	F	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	91	92	47	49	0	73	0	88	44	53
N.S.	1	1.01	0.52	0.54	0.00	0.80	0.00	0.97	0.48	0.58
time (sec)	N/A	0.363	0.042	0.362	0.000	0.077	0.000	0.209	0.351	9.915

Problem 115	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F	A	F	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	60	58	36	38	0	59	0	52	33	65
N.S.	1	0.97	0.60	0.63	0.00	0.98	0.00	0.87	0.55	1.08
time (sec)	N/A	0.300	0.035	0.351	0.000	0.089	0.000	0.112	0.494	9.509

Problem 116	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F	B	F	B	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	28	28	26	27	0	46	0	45	25	48
N.S.	1	1.00	0.93	0.96	0.00	1.64	0.00	1.61	0.89	1.71
time (sec)	N/A	0.243	0.014	0.357	0.000	0.087	0.000	0.126	0.398	9.722

Problem 117	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F	A	F	A	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	98	146	69	111	0	265	0	131	124	0
N.S.	1	1.49	0.70	1.13	0.00	2.70	0.00	1.34	1.27	0.00
time (sec)	N/A	0.470	0.074	0.345	0.000	0.105	0.000	0.120	0.225	0.000

Problem 118	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F	A	F	A	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	128	143	83	109	0	291	0	106	149	0
N.S.	1	1.12	0.65	0.85	0.00	2.27	0.00	0.83	1.16	0.00
time (sec)	N/A	0.453	0.099	0.370	0.000	0.093	0.000	0.151	0.416	0.000

Problem 119	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F	A	F	A	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	174	182	97	120	0	315	0	138	171	0
N.S.	1	1.05	0.56	0.69	0.00	1.81	0.00	0.79	0.98	0.00
time (sec)	N/A	0.519	0.119	0.365	0.000	0.090	0.000	0.158	0.539	0.000

Problem 120	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F	A	F	A	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	207	221	109	137	0	349	0	168	182	0
N.S.	1	1.07	0.53	0.66	0.00	1.69	0.00	0.81	0.88	0.00
time (sec)	N/A	0.596	0.142	0.363	0.000	0.124	0.000	0.160	0.401	0.000

Problem 121	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	F	F	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	95	98	47	47	54	52	0	0	34	46
N.S.	1	1.03	0.49	0.49	0.57	0.55	0.00	0.00	0.36	0.48
time (sec)	N/A	0.349	0.033	0.326	0.038	0.087	0.000	0.000	0.339	10.244

Problem 122	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	F	F	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	62	60	36	34	40	34	0	0	23	33
N.S.	1	0.97	0.58	0.55	0.65	0.55	0.00	0.00	0.37	0.53
time (sec)	N/A	0.289	0.025	0.332	0.039	0.107	0.000	0.000	0.504	10.131

Problem 123	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	F	F	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	28	28	26	25	24	24	0	0	15	24
N.S.	1	1.00	0.93	0.89	0.86	0.86	0.00	0.00	0.54	0.86
time (sec)	N/A	0.236	0.008	0.332	0.040	0.070	0.000	0.000	0.525	10.000

Problem 124	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F	A	F	A	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	27	27	25	28	0	27	0	20	17	27
N.S.	1	1.00	0.93	1.04	0.00	1.00	0.00	0.74	0.63	1.00
time (sec)	N/A	0.235	0.025	0.335	0.000	0.075	0.000	0.139	0.315	10.676

Problem 125	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F	A	F	A	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	59	59	33	36	0	35	0	47	17	40
N.S.	1	1.00	0.56	0.61	0.00	0.59	0.00	0.80	0.29	0.68
time (sec)	N/A	0.290	0.154	0.346	0.000	0.083	0.000	0.159	0.780	10.330

Problem 126	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F	A	F	A	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	92	98	44	47	0	46	0	65	17	54
N.S.	1	1.07	0.48	0.51	0.00	0.50	0.00	0.71	0.18	0.59
time (sec)	N/A	0.354	0.169	0.336	0.000	0.109	0.000	0.200	0.698	9.896

Problem 127	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F	A	F	A	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	278	181	227	0	0	295	0	206	17	0
N.S.	1	0.65	0.82	0.00	0.00	1.06	0.00	0.74	0.06	0.00
time (sec)	N/A	0.388	0.400	0.000	0.000	0.133	0.000	0.161	0.435	0.000

Problem 128	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F	A	F	A	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	235	151	206	0	0	268	0	163	17	0
N.S.	1	0.64	0.88	0.00	0.00	1.14	0.00	0.69	0.07	0.00
time (sec)	N/A	0.356	0.333	0.000	0.000	0.088	0.000	0.192	0.541	0.000

Problem 129	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F	A	F	A	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	208	124	158	0	0	547	0	133	17	0
N.S.	1	0.60	0.76	0.00	0.00	2.63	0.00	0.64	0.08	0.00
time (sec)	N/A	0.321	0.024	0.000	0.000	0.094	0.000	0.170	0.530	0.000

Problem 130	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F	A	F	F	B	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	175	106	122	0	0	562	0	0	144	0
N.S.	1	0.61	0.70	0.00	0.00	3.21	0.00	0.00	0.82	0.00
time (sec)	N/A	0.373	0.084	0.000	0.000	0.108	0.000	0.000	0.478	0.000

Problem 131	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F	A	F	F	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	204	134	163	194	0	652	0	0	169	0
N.S.	1	0.66	0.80	0.95	0.00	3.20	0.00	0.00	0.83	0.00
time (sec)	N/A	0.402	0.167	0.368	0.000	0.119	0.000	0.000	0.577	0.000

Problem 132	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F	A	F	F	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	245	162	188	207	0	667	0	0	209	0
N.S.	1	0.66	0.77	0.84	0.00	2.72	0.00	0.00	0.85	0.00
time (sec)	N/A	0.421	0.200	0.374	0.000	0.105	0.000	0.000	0.329	0.000

Problem 133	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	F	F	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	93	98	47	48	54	52	0	0	34	46
N.S.	1	1.05	0.51	0.52	0.58	0.56	0.00	0.00	0.37	0.49
time (sec)	N/A	0.367	0.035	0.595	0.038	0.074	0.000	0.000	0.285	9.571

Problem 134	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	F	F	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	60	60	35	37	39	33	0	0	22	41
N.S.	1	1.00	0.58	0.62	0.65	0.55	0.00	0.00	0.37	0.68
time (sec)	N/A	0.303	0.022	0.445	0.036	0.079	0.000	0.000	0.524	9.358

Problem 135	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	F	F	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	26	26	24	27	24	24	0	0	15	24
N.S.	1	1.00	0.92	1.04	0.92	0.92	0.00	0.00	0.58	0.92
time (sec)	N/A	0.240	0.007	0.346	0.036	0.077	0.000	0.000	0.470	8.965

Problem 136	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F	A	F	A	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	25	25	23	26	0	27	0	20	17	0
N.S.	1	1.00	0.92	1.04	0.00	1.08	0.00	0.80	0.68	0.00
time (sec)	N/A	0.238	0.008	0.392	0.000	0.078	0.000	0.145	0.707	0.000

Problem 137	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F	A	F	A	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	59	59	31	34	0	35	0	47	17	0
N.S.	1	1.00	0.53	0.58	0.00	0.59	0.00	0.80	0.29	0.00
time (sec)	N/A	0.293	0.175	0.362	0.000	0.088	0.000	0.347	0.650	0.000

Problem 138	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F	A	F	A	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	92	98	44	47	0	46	0	68	17	0
N.S.	1	1.07	0.48	0.51	0.00	0.50	0.00	0.74	0.18	0.00
time (sec)	N/A	0.352	0.224	0.405	0.000	0.081	0.000	0.156	0.461	0.000

Problem 139	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F	A	F	A	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	280	185	227	0	0	235	0	206	17	0
N.S.	1	0.66	0.81	0.00	0.00	0.84	0.00	0.74	0.06	0.00
time (sec)	N/A	0.388	0.441	0.000	0.000	0.116	0.000	0.157	0.678	0.000

Problem 140	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F	A	F	A	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	235	155	207	0	0	203	0	162	17	0
N.S.	1	0.66	0.88	0.00	0.00	0.86	0.00	0.69	0.07	0.00
time (sec)	N/A	0.358	0.365	0.000	0.000	0.083	0.000	0.176	0.440	0.000

Problem 141	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F	A	F	A	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	210	128	158	0	0	179	0	135	17	0
N.S.	1	0.61	0.75	0.00	0.00	0.85	0.00	0.64	0.08	0.00
time (sec)	N/A	0.325	0.243	0.000	0.000	0.087	0.000	0.167	0.274	0.000

Problem 142	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F	A	F	F	B	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	176	107	120	0	0	468	0	0	144	0
N.S.	1	0.61	0.68	0.00	0.00	2.66	0.00	0.00	0.82	0.00
time (sec)	N/A	0.363	0.100	0.000	0.000	0.092	0.000	0.000	0.538	0.000

Problem 143	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F	A	F	F	B	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	202	135	162	0	0	567	0	0	169	0
N.S.	1	0.67	0.80	0.00	0.00	2.81	0.00	0.00	0.84	0.00
time (sec)	N/A	0.409	0.168	0.000	0.000	0.093	0.000	0.000	0.481	0.000

Problem 144	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F	A	F	F	B	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	245	163	188	0	0	574	0	0	209	0
N.S.	1	0.67	0.77	0.00	0.00	2.34	0.00	0.00	0.85	0.00
time (sec)	N/A	0.432	0.177	0.000	0.000	0.114	0.000	0.000	0.268	0.000

Problem 145	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	F	F	F	F	F	F	F(-1)
verified	N/A	No	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	191	225	47	0	0	0	0	0	114	0
N.S.	1	1.18	0.25	0.00	0.00	0.00	0.00	0.00	0.60	0.00
time (sec)	N/A	0.561	10.015	0.000	0.000	0.000	0.000	0.000	0.390	0.000

Problem 146	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	F	F	F	F	F	F	F(-1)
verified	N/A	No	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	165	195	47	0	0	0	0	0	96	0
N.S.	1	1.18	0.28	0.00	0.00	0.00	0.00	0.00	0.58	0.00
time (sec)	N/A	0.557	10.013	0.000	0.000	0.000	0.000	0.000	0.657	0.000

Problem 147	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	F	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	139	131	47	0	0	0	0	0	78	0
N.S.	1	0.94	0.34	0.00	0.00	0.00	0.00	0.00	0.56	0.00
time (sec)	N/A	0.447	10.012	0.000	0.000	0.000	0.000	0.000	0.391	0.000

Problem 148	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	F	F	F	F	F	F	B
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	113	102	45	0	0	0	0	0	60	36
N.S.	1	0.90	0.40	0.00	0.00	0.00	0.00	0.00	0.53	0.32
time (sec)	N/A	0.397	10.013	0.000	0.000	0.000	0.000	0.000	0.257	9.199

Problem 149	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	F	F	F	F	F	F	F(-1)
verified	N/A	No	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	87	108	42	0	0	0	0	0	39	0
N.S.	1	1.24	0.48	0.00	0.00	0.00	0.00	0.00	0.45	0.00
time (sec)	N/A	0.437	10.011	0.000	0.000	0.000	0.000	0.000	0.592	0.000

Problem 150	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	F	F	F	F	F	F	F(-1)
verified	N/A	No	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	91	112	47	0	0	0	0	0	44	0
N.S.	1	1.23	0.52	0.00	0.00	0.00	0.00	0.00	0.48	0.00
time (sec)	N/A	0.430	10.012	0.000	0.000	0.000	0.000	0.000	0.564	0.000

Problem 151	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	F	F	F	F	F	F	F(-1)
verified	N/A	No	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	116	139	47	0	0	0	0	0	44	0
N.S.	1	1.20	0.41	0.00	0.00	0.00	0.00	0.00	0.38	0.00
time (sec)	N/A	0.467	10.013	0.000	0.000	0.000	0.000	0.000	0.285	0.000

Problem 152	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	F	F	F	F	F	F	F(-1)
verified	N/A	No	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	144	169	47	0	0	0	0	0	44	0
N.S.	1	1.17	0.33	0.00	0.00	0.00	0.00	0.00	0.31	0.00
time (sec)	N/A	0.497	10.014	0.000	0.000	0.000	0.000	0.000	0.514	0.000

Problem 153	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	F	F	F	F	F	F	F(-1)
verified	N/A	No	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	170	199	47	0	0	0	0	0	44	0
N.S.	1	1.17	0.28	0.00	0.00	0.00	0.00	0.00	0.26	0.00
time (sec)	N/A	0.518	10.013	0.000	0.000	0.000	0.000	0.000	0.718	0.000

Problem 154	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	F	F	F	F	F	F	F(-1)
verified	N/A	No	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	185	254	47	0	0	0	0	0	114	0
N.S.	1	1.37	0.25	0.00	0.00	0.00	0.00	0.00	0.62	0.00
time (sec)	N/A	0.631	10.014	0.000	0.000	0.000	0.000	0.000	0.355	0.000

Problem 155	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	F	F	F	F	F	F	F(-1)
verified	N/A	No	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	159	224	47	0	0	0	0	0	96	0
N.S.	1	1.41	0.30	0.00	0.00	0.00	0.00	0.00	0.60	0.00
time (sec)	N/A	0.597	10.012	0.000	0.000	0.000	0.000	0.000	0.504	0.000

Problem 156	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	F	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	133	131	47	0	0	0	0	0	78	0
N.S.	1	0.98	0.35	0.00	0.00	0.00	0.00	0.00	0.59	0.00
time (sec)	N/A	0.445	10.011	0.000	0.000	0.000	0.000	0.000	0.661	0.000

Problem 157	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	F	F	F	F	F	F	B
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	107	102	45	0	0	0	0	0	60	36
N.S.	1	0.95	0.42	0.00	0.00	0.00	0.00	0.00	0.56	0.34
time (sec)	N/A	0.388	10.010	0.000	0.000	0.000	0.000	0.000	0.352	9.346

Problem 158	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	F	F	F	F	F	F	F(-1)
verified	N/A	No	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	82	134	44	0	0	0	0	0	39	0
N.S.	1	1.63	0.54	0.00	0.00	0.00	0.00	0.00	0.48	0.00
time (sec)	N/A	0.470	10.015	0.000	0.000	0.000	0.000	0.000	0.642	0.000

Problem 159	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	F	F	F	F	F	F	F(-1)
verified	N/A	No	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	79	132	45	0	0	0	0	0	44	0
N.S.	1	1.67	0.57	0.00	0.00	0.00	0.00	0.00	0.56	0.00
time (sec)	N/A	0.473	10.011	0.000	0.000	0.000	0.000	0.000	0.561	0.000

Problem 160	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	F	F	F	F	F	F	F(-1)
verified	N/A	No	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	110	162	47	0	0	0	0	0	44	0
N.S.	1	1.47	0.43	0.00	0.00	0.00	0.00	0.00	0.40	0.00
time (sec)	N/A	0.510	10.012	0.000	0.000	0.000	0.000	0.000	0.286	0.000

Problem 161	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	F	F	F	F	F	F	F(-1)
verified	N/A	No	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	141	192	47	0	0	0	0	0	44	0
N.S.	1	1.36	0.33	0.00	0.00	0.00	0.00	0.00	0.31	0.00
time (sec)	N/A	0.533	10.014	0.000	0.000	0.000	0.000	0.000	0.663	0.000

Problem 162	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	F	F	F	F	F	F	F(-1)
verified	N/A	No	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	167	222	47	0	0	0	0	0	44	0
N.S.	1	1.33	0.28	0.00	0.00	0.00	0.00	0.00	0.26	0.00
time (sec)	N/A	0.582	10.014	0.000	0.000	0.000	0.000	0.000	0.644	0.000

Problem 163	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	F	F	F	F	F	F	F(-1)
verified	N/A	No	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	214	249	48	0	0	0	0	0	132	0
N.S.	1	1.16	0.22	0.00	0.00	0.00	0.00	0.00	0.62	0.00
time (sec)	N/A	0.616	10.013	0.000	0.000	0.000	0.000	0.000	0.330	0.000

Problem 164	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	F	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	188	168	48	0	0	0	0	0	114	0
N.S.	1	0.89	0.26	0.00	0.00	0.00	0.00	0.00	0.61	0.00
time (sec)	N/A	0.521	10.011	0.000	0.000	0.000	0.000	0.000	0.552	0.000

Problem 165	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	F	F	F	F	F	F	B
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	160	139	48	0	0	0	0	0	96	36
N.S.	1	0.87	0.30	0.00	0.00	0.00	0.00	0.00	0.60	0.22
time (sec)	N/A	0.458	10.010	0.000	0.000	0.000	0.000	0.000	0.595	9.207

Problem 166	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	F	F	F	F	F	F	F(-1)
verified	N/A	No	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	133	159	46	0	0	0	0	0	78	0
N.S.	1	1.20	0.35	0.00	0.00	0.00	0.00	0.00	0.59	0.00
time (sec)	N/A	0.490	10.013	0.000	0.000	0.000	0.000	0.000	0.337	0.000

Problem 167	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	F	F	F	F	F	F	F(-1)
verified	N/A	No	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	112	132	43	0	0	0	0	0	55	0
N.S.	1	1.18	0.38	0.00	0.00	0.00	0.00	0.00	0.49	0.00
time (sec)	N/A	0.465	10.013	0.000	0.000	0.000	0.000	0.000	0.635	0.000

Problem 168	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	F	F	F	F	F	F	F(-1)
verified	N/A	No	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	108	132	48	0	0	0	0	0	58	0
N.S.	1	1.22	0.44	0.00	0.00	0.00	0.00	0.00	0.54	0.00
time (sec)	N/A	0.469	10.012	0.000	0.000	0.000	0.000	0.000	0.536	0.000

Problem 169	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	F	F	F	F	F	F	F(-1)
verified	N/A	No	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	114	136	48	0	0	0	0	0	58	0
N.S.	1	1.19	0.42	0.00	0.00	0.00	0.00	0.00	0.51	0.00
time (sec)	N/A	0.464	10.015	0.000	0.000	0.000	0.000	0.000	0.329	0.000

Problem 170	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	F	F	F	F	F	F	F(-1)
verified	N/A	No	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	142	163	48	0	0	0	0	0	57	0
N.S.	1	1.15	0.34	0.00	0.00	0.00	0.00	0.00	0.40	0.00
time (sec)	N/A	0.493	10.012	0.000	0.000	0.000	0.000	0.000	0.692	0.000

Problem 171	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	F	F	F	F	F	F	F(-1)
verified	N/A	No	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	167	193	48	0	0	0	0	0	58	0
N.S.	1	1.16	0.29	0.00	0.00	0.00	0.00	0.00	0.35	0.00
time (sec)	N/A	0.519	10.012	0.000	0.000	0.000	0.000	0.000	0.463	0.000

Problem 172	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	F	F	F	F	F	F	F(-1)
verified	N/A	No	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	193	223	48	0	0	0	0	0	58	0
N.S.	1	1.16	0.25	0.00	0.00	0.00	0.00	0.00	0.30	0.00
time (sec)	N/A	0.589	10.013	0.000	0.000	0.000	0.000	0.000	0.362	0.000

Problem 173	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	F	F	F	F	F	F	F(-1)
verified	N/A	No	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	162	230	47	0	0	0	0	0	13	0
N.S.	1	1.42	0.29	0.00	0.00	0.00	0.00	0.00	0.08	0.00
time (sec)	N/A	0.586	10.011	0.000	0.000	0.000	0.000	0.000	0.760	0.000

Problem 174	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	F	F	F	F	F	F	F(-1)
verified	N/A	No	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	136	200	47	0	0	0	0	0	13	0
N.S.	1	1.47	0.35	0.00	0.00	0.00	0.00	0.00	0.10	0.00
time (sec)	N/A	0.564	10.010	0.000	0.000	0.000	0.000	0.000	0.566	0.000

Problem 175	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	F	F	F	F	F	F	F(-1)
verified	N/A	No	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	110	170	47	0	0	0	0	0	13	0
N.S.	1	1.55	0.43	0.00	0.00	0.00	0.00	0.00	0.12	0.00
time (sec)	N/A	0.526	10.010	0.000	0.000	0.000	0.000	0.000	0.411	0.000

Problem 176	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	F	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	86	93	47	0	0	0	0	0	13	0
N.S.	1	1.08	0.55	0.00	0.00	0.00	0.00	0.00	0.15	0.00
time (sec)	N/A	0.412	10.011	0.000	0.000	0.000	0.000	0.000	0.639	0.000

Problem 177	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	F	F	F	F	F	F	B
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	62	71	45	0	0	0	0	0	13	36
N.S.	1	1.15	0.73	0.00	0.00	0.00	0.00	0.00	0.21	0.58
time (sec)	N/A	0.342	10.010	0.000	0.000	0.000	0.000	0.000	0.347	9.585

Problem 178	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	F	F	F	F	F	F	F(-1)
verified	N/A	No	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	81	138	42	0	0	0	0	0	13	0
N.S.	1	1.70	0.52	0.00	0.00	0.00	0.00	0.00	0.16	0.00
time (sec)	N/A	0.483	10.015	0.000	0.000	0.000	0.000	0.000	0.481	0.000

Problem 179	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	F	F	F	F	F	B	F(-1)
verified	N/A	No	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	113	168	47	0	0	0	0	0	19	0
N.S.	1	1.49	0.42	0.00	0.00	0.00	0.00	0.00	0.17	0.00
time (sec)	N/A	0.541	10.015	0.000	0.000	0.000	0.000	0.000	0.570	0.000

Problem 180	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	F	F	F	F	F	B	F(-1)
verified	N/A	No	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	141	198	47	0	0	0	0	0	27	0
N.S.	1	1.40	0.33	0.00	0.00	0.00	0.00	0.00	0.19	0.00
time (sec)	N/A	0.565	10.012	0.000	0.000	0.000	0.000	0.000	0.394	0.000

Problem 181	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	F	F	F	F	F	F	F(-1)
verified	N/A	No	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	165	201	47	0	0	0	0	0	13	0
N.S.	1	1.22	0.28	0.00	0.00	0.00	0.00	0.00	0.08	0.00
time (sec)	N/A	0.545	10.012	0.000	0.000	0.000	0.000	0.000	0.330	0.000

Problem 182	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	F	F	F	F	F	F	F(-1)
verified	N/A	No	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	139	171	47	0	0	0	0	0	13	0
N.S.	1	1.23	0.34	0.00	0.00	0.00	0.00	0.00	0.09	0.00
time (sec)	N/A	0.486	10.011	0.000	0.000	0.000	0.000	0.000	0.662	0.000

Problem 183	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	F	F	F	F	F	F	F(-1)
verified	N/A	No	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	113	141	47	0	0	0	0	0	13	0
N.S.	1	1.25	0.42	0.00	0.00	0.00	0.00	0.00	0.12	0.00
time (sec)	N/A	0.442	10.010	0.000	0.000	0.000	0.000	0.000	0.493	0.000

Problem 184	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	F	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	87	91	47	0	0	0	0	0	13	0
N.S.	1	1.05	0.54	0.00	0.00	0.00	0.00	0.00	0.15	0.00
time (sec)	N/A	0.377	10.017	0.000	0.000	0.000	0.000	0.000	0.273	0.000

Problem 185	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	F	F	F	F	F	F	B
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	68	71	43	0	0	0	0	0	13	36
N.S.	1	1.04	0.63	0.00	0.00	0.00	0.00	0.00	0.19	0.53
time (sec)	N/A	0.328	10.011	0.000	0.000	0.000	0.000	0.000	0.505	8.732

Problem 186	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	F	F	F	F	F	B	F(-1)
verified	N/A	No	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	91	115	44	0	0	0	0	0	19	0
N.S.	1	1.26	0.48	0.00	0.00	0.00	0.00	0.00	0.21	0.00
time (sec)	N/A	0.425	10.012	0.000	0.000	0.000	0.000	0.000	0.541	0.000

Problem 187	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	F	F	F	F	F	B	F(-1)
verified	N/A	No	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	116	145	47	0	0	0	0	0	27	0
N.S.	1	1.25	0.41	0.00	0.00	0.00	0.00	0.00	0.23	0.00
time (sec)	N/A	0.466	10.014	0.000	0.000	0.000	0.000	0.000	0.273	0.000

Problem 188	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	F	F	F	F	F	B	F(-1)
verified	N/A	No	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	144	175	47	0	0	0	0	0	38	0
N.S.	1	1.22	0.33	0.00	0.00	0.00	0.00	0.00	0.26	0.00
time (sec)	N/A	0.488	10.010	0.000	0.000	0.000	0.000	0.000	0.431	0.000

Problem 189	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	F	F	F	F	F	F	F(-1)
verified	N/A	No	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	160	225	50	0	0	0	0	0	33	0
N.S.	1	1.41	0.31	0.00	0.00	0.00	0.00	0.00	0.21	0.00
time (sec)	N/A	0.588	10.023	0.000	0.000	0.000	0.000	0.000	0.803	0.000

Problem 190	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	F	F	F	F	F	F	F(-1)
verified	N/A	No	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	132	195	50	0	0	0	0	0	33	0
N.S.	1	1.48	0.38	0.00	0.00	0.00	0.00	0.00	0.25	0.00
time (sec)	N/A	0.566	10.013	0.000	0.000	0.000	0.000	0.000	0.341	0.000

Problem 191	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	F	F	F	F	F	F	F(-1)
verified	N/A	No	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	106	165	50	0	0	0	0	0	33	0
N.S.	1	1.56	0.47	0.00	0.00	0.00	0.00	0.00	0.31	0.00
time (sec)	N/A	0.530	10.014	0.000	0.000	0.000	0.000	0.000	0.670	0.000

Problem 192	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	F	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	83	91	50	0	0	0	0	0	31	0
N.S.	1	1.10	0.60	0.00	0.00	0.00	0.00	0.00	0.37	0.00
time (sec)	N/A	0.407	10.011	0.000	0.000	0.000	0.000	0.000	0.566	0.000

Problem 193	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	F	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	69	90	48	0	0	0	0	0	29	0
N.S.	1	1.30	0.70	0.00	0.00	0.00	0.00	0.00	0.42	0.00
time (sec)	N/A	0.393	10.016	0.000	0.000	0.000	0.000	0.000	0.258	0.000

Problem 194	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	F	F	F	F	F	B	B
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	84	95	45	0	0	0	0	0	27	36
N.S.	1	1.13	0.54	0.00	0.00	0.00	0.00	0.00	0.32	0.43
time (sec)	N/A	0.393	10.011	0.000	0.000	0.000	0.000	0.000	0.506	8.746

Problem 195	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	F	F	F	F	F	B	F(-1)
verified	N/A	No	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	134	193	52	0	0	0	0	0	35	0
N.S.	1	1.44	0.39	0.00	0.00	0.00	0.00	0.00	0.26	0.00
time (sec)	N/A	0.559	10.022	0.000	0.000	0.000	0.000	0.000	0.615	0.000

Problem 196	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	F	F	F	F	F	B	F(-1)
verified	N/A	No	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	162	223	50	0	0	0	0	0	46	0
N.S.	1	1.38	0.31	0.00	0.00	0.00	0.00	0.00	0.28	0.00
time (sec)	N/A	0.609	10.012	0.000	0.000	0.000	0.000	0.000	0.265	0.000

Problem 197	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	F	F	F	F	F	B	F(-1)
verified	N/A	No	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	188	253	50	0	0	0	0	0	57	0
N.S.	1	1.35	0.27	0.00	0.00	0.00	0.00	0.00	0.30	0.00
time (sec)	N/A	0.657	10.014	0.000	0.000	0.000	0.000	0.000	0.422	0.000

Problem 198	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	F	F	F	F	F	F	F(-1)
verified	N/A	No	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	120	147	43	0	0	0	0	0	14	0
N.S.	1	1.22	0.36	0.00	0.00	0.00	0.00	0.00	0.12	0.00
time (sec)	N/A	0.514	10.019	0.000	0.000	0.000	0.000	0.000	0.536	0.000

Problem 199	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	B	C	F	F	F	F	F	F	F(-1)
verified	N/A	No	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	72	151	44	0	0	0	0	0	15	0
N.S.	1	2.10	0.61	0.00	0.00	0.00	0.00	0.00	0.21	0.00
time (sec)	N/A	0.525	10.011	0.000	0.000	0.000	0.000	0.000	0.435	0.000

Problem 200	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	F	F	F	F	F	F	F(-1)
verified	N/A	No	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	81	138	42	0	0	0	0	0	13	0
N.S.	1	1.70	0.52	0.00	0.00	0.00	0.00	0.00	0.16	0.00
time (sec)	N/A	0.490	0.001	0.000	0.000	0.000	0.000	0.000	0.597	0.000

Problem 201	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	B	C	F	F	F	F	F	F	F(-1)
verified	N/A	No	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	73	152	43	0	0	0	0	0	18	0
N.S.	1	2.08	0.59	0.00	0.00	0.00	0.00	0.00	0.25	0.00
time (sec)	N/A	0.505	10.011	0.000	0.000	0.000	0.000	0.000	0.575	0.000

Problem 202	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	C	F	F	F	F	F	F(-1)
verified	N/A	No	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	77	122	42	18	0	0	0	0	13	0
N.S.	1	1.58	0.55	0.23	0.00	0.00	0.00	0.00	0.17	0.00
time (sec)	N/A	0.458	10.010	0.376	0.000	0.000	0.000	0.000	0.434	0.000

Problem 203	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	B	C	C	F	F	F	F	F	F(-1)
verified	N/A	No	Yes	No	TBD	TBD	TBD	TBD	TBD	TBD
size	59	122	41	32	0	0	0	0	13	0
N.S.	1	2.07	0.69	0.54	0.00	0.00	0.00	0.00	0.22	0.00
time (sec)	N/A	0.468	10.013	0.396	0.000	0.000	0.000	0.000	0.574	0.000

Problem 204	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	F	F	F	F	F	F	F(-1)
verified	N/A	No	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	85	134	41	0	0	0	0	0	13	0
N.S.	1	1.58	0.48	0.00	0.00	0.00	0.00	0.00	0.15	0.00
time (sec)	N/A	0.504	10.013	0.000	0.000	0.000	0.000	0.000	0.451	0.000

Problem 205	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	B	C	C	F	F	F	F	F	F(-1)
verified	N/A	No	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	36	138	43	18	0	0	0	0	13	0
N.S.	1	3.83	1.19	0.50	0.00	0.00	0.00	0.00	0.36	0.00
time (sec)	N/A	0.477	10.014	0.385	0.000	0.000	0.000	0.000	0.351	0.000

Problem 206	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	B	C	C	F	F	F	F	F	F(-1)
verified	N/A	No	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	38	138	43	21	0	0	0	0	13	0
N.S.	1	3.63	1.13	0.55	0.00	0.00	0.00	0.00	0.34	0.00
time (sec)	N/A	0.472	10.007	0.384	0.000	0.000	0.000	0.000	0.621	0.000

Problem 207	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	F	F	F	F	F	F	F(-1)
verified	N/A	No	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	80	148	41	0	0	0	0	0	13	0
N.S.	1	1.85	0.51	0.00	0.00	0.00	0.00	0.00	0.16	0.00
time (sec)	N/A	0.489	10.015	0.000	0.000	0.000	0.000	0.000	0.422	0.000

Problem 208	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	C	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	61	70	41	18	0	0	0	0	29	0
N.S.	1	1.15	0.67	0.30	0.00	0.00	0.00	0.00	0.48	0.00
time (sec)	N/A	0.335	10.013	0.320	0.000	0.000	0.000	0.000	0.355	0.000

Problem 209	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	C	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	No	TBD	TBD	TBD	TBD	TBD	TBD
size	77	70	42	32	0	0	0	0	29	0
N.S.	1	0.91	0.55	0.42	0.00	0.00	0.00	0.00	0.38	0.00
time (sec)	N/A	0.336	10.010	0.316	0.000	0.000	0.000	0.000	0.624	0.000

Problem 210	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	F	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	66	87	47	0	0	0	0	0	29	0
N.S.	1	1.32	0.71	0.00	0.00	0.00	0.00	0.00	0.44	0.00
time (sec)	N/A	0.381	10.015	0.000	0.000	0.000	0.000	0.000	0.425	0.000

Problem 211	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	C	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	36	40	43	18	0	0	0	0	31	0
N.S.	1	1.11	1.19	0.50	0.00	0.00	0.00	0.00	0.86	0.00
time (sec)	N/A	0.291	10.011	0.320	0.000	0.000	0.000	0.000	0.352	0.000

Problem 212	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	C	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	36	40	42	21	0	0	0	0	31	0
N.S.	1	1.11	1.17	0.58	0.00	0.00	0.00	0.00	0.86	0.00
time (sec)	N/A	0.285	10.010	0.358	0.000	0.000	0.000	0.000	0.666	0.000

Problem 213	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	F	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	78	86	47	0	0	0	0	0	29	0
N.S.	1	1.10	0.60	0.00	0.00	0.00	0.00	0.00	0.37	0.00
time (sec)	N/A	0.374	10.014	0.000	0.000	0.000	0.000	0.000	0.430	0.000

Problem 214	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	B	C	C	F	F	F	F	F	F(-1)
verified	N/A	No	Yes	No	TBD	TBD	TBD	TBD	TBD	TBD
size	37	99	33	27	0	0	0	0	11	0
N.S.	1	2.68	0.89	0.73	0.00	0.00	0.00	0.00	0.30	0.00
time (sec)	N/A	0.427	10.012	0.400	0.000	0.000	0.000	0.000	0.404	0.000

Problem 215	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	C	A	B	C	F	F	F(-1)
verified	N/A	No	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	112	161	88	18	126	170	39	0	13	0
N.S.	1	1.44	0.79	0.16	1.12	1.52	0.35	0.00	0.12	0.00
time (sec)	N/A	0.541	0.340	0.329	0.107	0.095	0.747	0.000	0.587	0.000

Problem 216	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	C	F	B	F	A	B	F(-1)
verified	N/A	No	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	112	188	75	18	0	224	0	118	32	0
N.S.	1	1.68	0.67	0.16	0.00	2.00	0.00	1.05	0.29	0.00
time (sec)	N/A	0.614	0.215	0.329	0.000	0.096	0.000	0.198	0.431	0.000

Problem 217	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	B	B	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	81	85	57	82	77	161	711	264	173	171
N.S.	1	1.05	0.70	1.01	0.95	1.99	8.78	3.26	2.14	2.11
time (sec)	N/A	0.387	0.069	0.303	0.038	0.084	0.394	0.233	0.240	9.003

Problem 218	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	B	B	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	58	62	41	59	55	89	330	141	90	97
N.S.	1	1.07	0.71	1.02	0.95	1.53	5.69	2.43	1.55	1.67
time (sec)	N/A	0.345	0.051	0.274	0.038	0.091	0.289	0.127	0.524	9.203

Problem 219	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	B	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	35	39	25	35	33	39	105	56	35	34
N.S.	1	1.11	0.71	1.00	0.94	1.11	3.00	1.60	1.00	0.97
time (sec)	N/A	0.296	0.035	0.039	0.035	0.109	0.203	0.109	0.448	8.832

Problem 220	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	25	25	25	0	0	0	0	0	21	0
N.S.	1	1.00	1.00	0.00	0.00	0.00	0.00	0.00	0.84	0.00
time (sec)	N/A	0.246	0.032	0.000	0.000	0.000	0.000	0.000	0.296	0.000

Problem 221	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	33	33	30	0	0	0	0	0	34	0
N.S.	1	1.00	0.91	0.00	0.00	0.00	0.00	0.00	1.03	0.00
time (sec)	N/A	0.256	0.031	0.000	0.000	0.000	0.000	0.000	0.506	0.000

Problem 222	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	37	37	32	0	0	0	0	0	45	0
N.S.	1	1.00	0.86	0.00	0.00	0.00	0.00	0.00	1.22	0.00
time (sec)	N/A	0.264	0.034	0.000	0.000	0.000	0.000	0.000	0.504	0.000

Problem 223	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	65	71	60	0	0	0	0	0	41	0
N.S.	1	1.09	0.92	0.00	0.00	0.00	0.00	0.00	0.63	0.00
time (sec)	N/A	0.320	0.181	0.000	0.000	0.000	0.000	0.000	0.628	0.000

Problem 224	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	65	67	60	0	0	0	0	0	18	0
N.S.	1	1.03	0.92	0.00	0.00	0.00	0.00	0.00	0.28	0.00
time (sec)	N/A	0.297	0.109	0.000	0.000	0.000	0.000	0.000	0.606	0.000

Problem 225	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	63	65	58	0	0	0	0	0	22	0
N.S.	1	1.03	0.92	0.00	0.00	0.00	0.00	0.00	0.35	0.00
time (sec)	N/A	0.299	0.100	0.000	0.000	0.000	0.000	0.000	0.445	0.000

Problem 226	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	63	66	58	0	0	0	0	0	37	0
N.S.	1	1.05	0.92	0.00	0.00	0.00	0.00	0.00	0.59	0.00
time (sec)	N/A	0.304	0.156	0.000	0.000	0.000	0.000	0.000	0.315	0.000

Problem 227	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	65	71	60	0	0	0	0	0	58	0
N.S.	1	1.09	0.92	0.00	0.00	0.00	0.00	0.00	0.89	0.00
time (sec)	N/A	0.301	0.157	0.000	0.000	0.000	0.000	0.000	0.637	0.000

Problem 228	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	42	49	47	0	0	0	0	0	532	0
N.S.	1	1.17	1.12	0.00	0.00	0.00	0.00	0.00	12.67	0.00
time (sec)	N/A	0.282	0.032	0.000	0.000	0.000	0.000	0.000	0.497	0.000

Problem 229	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	B	A	F	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	40	83	47	0	0	0	0	0	239	0
N.S.	1	2.08	1.18	0.00	0.00	0.00	0.00	0.00	5.98	0.00
time (sec)	N/A	0.326	0.025	0.000	0.000	0.000	0.000	0.000	0.252	0.000

Problem 230	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F	F	F	F	F	B
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	39	55	45	0	0	0	0	0	129	48
N.S.	1	1.41	1.15	0.00	0.00	0.00	0.00	0.00	3.31	1.23
time (sec)	N/A	0.270	0.034	0.000	0.000	0.000	0.000	0.000	0.369	9.144

Problem 231	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	40	42	40	0	0	0	0	0	45	0
N.S.	1	1.05	1.00	0.00	0.00	0.00	0.00	0.00	1.12	0.00
time (sec)	N/A	0.275	0.036	0.000	0.000	0.000	0.000	0.000	0.524	0.000

Problem 232	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	41	50	45	0	0	0	0	0	121	0
N.S.	1	1.22	1.10	0.00	0.00	0.00	0.00	0.00	2.95	0.00
time (sec)	N/A	0.291	0.026	0.000	0.000	0.000	0.000	0.000	0.395	0.000

Problem 233	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	45	52	47	0	0	0	0	0	119	0
N.S.	1	1.16	1.04	0.00	0.00	0.00	0.00	0.00	2.64	0.00
time (sec)	N/A	0.292	0.028	0.000	0.000	0.000	0.000	0.000	0.230	0.000

Problem 234	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	29	47	45	30	0	0	0	0	450	0
N.S.	1	1.62	1.55	1.03	0.00	0.00	0.00	0.00	15.52	0.00
time (sec)	N/A	0.282	0.026	0.465	0.000	0.000	0.000	0.000	0.561	0.000

Problem 235	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	B	A	A	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	29	60	45	30	0	0	0	0	207	0
N.S.	1	2.07	1.55	1.03	0.00	0.00	0.00	0.00	7.14	0.00
time (sec)	N/A	0.303	0.023	0.444	0.000	0.000	0.000	0.000	0.504	0.000

Problem 236	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F	F	F	F	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	29	37	43	30	0	0	0	0	114	41
N.S.	1	1.28	1.48	1.03	0.00	0.00	0.00	0.00	3.93	1.41
time (sec)	N/A	0.249	0.005	0.345	0.000	0.000	0.000	0.000	0.240	5.819

Problem 237	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	23	40	38	24	0	0	0	0	45	0
N.S.	1	1.74	1.65	1.04	0.00	0.00	0.00	0.00	1.96	0.00
time (sec)	N/A	0.263	0.022	0.346	0.000	0.000	0.000	0.000	0.309	0.000

Problem 238	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	30	48	43	28	0	0	0	0	111	0
N.S.	1	1.60	1.43	0.93	0.00	0.00	0.00	0.00	3.70	0.00
time (sec)	N/A	0.265	0.029	0.353	0.000	0.000	0.000	0.000	0.534	0.000

Problem 239	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	32	50	45	30	0	0	0	0	114	0
N.S.	1	1.56	1.41	0.94	0.00	0.00	0.00	0.00	3.56	0.00
time (sec)	N/A	0.272	0.024	0.410	0.000	0.000	0.000	0.000	0.438	0.000

Problem 240	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	29	47	46	30	0	0	0	0	449	0
N.S.	1	1.62	1.59	1.03	0.00	0.00	0.00	0.00	15.48	0.00
time (sec)	N/A	0.276	0.022	0.458	0.000	0.000	0.000	0.000	0.237	0.000

Problem 241	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	35	52	49	0	0	0	0	0	544	0
N.S.	1	1.49	1.40	0.00	0.00	0.00	0.00	0.00	15.54	0.00
time (sec)	N/A	0.285	0.034	0.000	0.000	0.000	0.000	0.000	0.436	0.000

Problem 242	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	B	A	F	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	35	88	49	0	0	0	0	0	256	0
N.S.	1	2.51	1.40	0.00	0.00	0.00	0.00	0.00	7.31	0.00
time (sec)	N/A	0.341	0.033	0.000	0.000	0.000	0.000	0.000	0.492	0.000

Problem 243	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F	F	F	F	F	B
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	35	54	47	0	0	0	0	0	139	47
N.S.	1	1.54	1.34	0.00	0.00	0.00	0.00	0.00	3.97	1.34
time (sec)	N/A	0.266	0.032	0.000	0.000	0.000	0.000	0.000	0.341	9.204

Problem 244	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	26	45	42	0	0	0	0	0	54	0
N.S.	1	1.73	1.62	0.00	0.00	0.00	0.00	0.00	2.08	0.00
time (sec)	N/A	0.274	0.025	0.000	0.000	0.000	0.000	0.000	0.383	0.000

Problem 245	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	34	53	47	0	0	0	0	0	127	0
N.S.	1	1.56	1.38	0.00	0.00	0.00	0.00	0.00	3.74	0.00
time (sec)	N/A	0.288	0.029	0.000	0.000	0.000	0.000	0.000	0.484	0.000

Problem 246	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	38	55	49	0	0	0	0	0	130	0
N.S.	1	1.45	1.29	0.00	0.00	0.00	0.00	0.00	3.42	0.00
time (sec)	N/A	0.288	0.030	0.000	0.000	0.000	0.000	0.000	0.426	0.000

Problem 247	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	53	61	58	0	0	0	0	0	657	0
N.S.	1	1.15	1.09	0.00	0.00	0.00	0.00	0.00	12.40	0.00
time (sec)	N/A	0.301	0.037	0.000	0.000	0.000	0.000	0.000	0.234	0.000

Problem 248	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	53	61	58	0	0	0	0	0	329	0
N.S.	1	1.15	1.09	0.00	0.00	0.00	0.00	0.00	6.21	0.00
time (sec)	N/A	0.300	0.035	0.000	0.000	0.000	0.000	0.000	0.230	0.000

Problem 249	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	53	61	58	0	0	0	0	0	116	0
N.S.	1	1.15	1.09	0.00	0.00	0.00	0.00	0.00	2.19	0.00
time (sec)	N/A	0.299	0.031	0.000	0.000	0.000	0.000	0.000	0.522	0.000

Problem 250	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	53	61	58	0	0	0	0	0	134	0
N.S.	1	1.15	1.09	0.00	0.00	0.00	0.00	0.00	2.53	0.00
time (sec)	N/A	0.312	0.033	0.000	0.000	0.000	0.000	0.000	0.532	0.000

Problem 251	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	45	60	56	39	0	0	0	0	569	0
N.S.	1	1.33	1.24	0.87	0.00	0.00	0.00	0.00	12.64	0.00
time (sec)	N/A	0.290	0.033	0.314	0.000	0.000	0.000	0.000	0.317	0.000

Problem 252	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	44	60	56	39	0	0	0	0	288	0
N.S.	1	1.36	1.27	0.89	0.00	0.00	0.00	0.00	6.55	0.00
time (sec)	N/A	0.289	0.030	0.304	0.000	0.000	0.000	0.000	0.443	0.000

Problem 253	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	44	60	56	39	0	0	0	0	111	0
N.S.	1	1.36	1.27	0.89	0.00	0.00	0.00	0.00	2.52	0.00
time (sec)	N/A	0.279	0.029	0.312	0.000	0.000	0.000	0.000	0.490	0.000

Problem 254	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	46	61	56	39	0	0	0	0	124	0
N.S.	1	1.33	1.22	0.85	0.00	0.00	0.00	0.00	2.70	0.00
time (sec)	N/A	0.279	0.032	0.316	0.000	0.000	0.000	0.000	0.447	0.000

Problem 255	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	47	55	53	0	0	0	0	0	746	0
N.S.	1	1.17	1.13	0.00	0.00	0.00	0.00	0.00	15.87	0.00
time (sec)	N/A	0.295	0.029	0.000	0.000	0.000	0.000	0.000	0.258	0.000

Problem 256	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F	A	F	F	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	175	167	102	131	0	160	0	0	155	239
N.S.	1	0.95	0.58	0.75	0.00	0.91	0.00	0.00	0.89	1.37
time (sec)	N/A	0.549	0.064	0.477	0.000	0.088	0.000	0.000	0.554	9.009

Problem 257	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F	A	F	F	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	122	120	73	85	0	111	0	0	106	163
N.S.	1	0.98	0.60	0.70	0.00	0.91	0.00	0.00	0.87	1.34
time (sec)	N/A	0.434	0.049	0.468	0.000	0.083	0.000	0.000	0.570	8.841

Problem 258	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F	A	F	F	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	73	73	48	51	0	70	0	0	70	102
N.S.	1	1.00	0.66	0.70	0.00	0.96	0.00	0.00	0.96	1.40
time (sec)	N/A	0.319	0.038	0.470	0.000	0.083	0.000	0.000	0.240	8.933

Problem 259	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F	A	F	F	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	32	32	30	37	0	38	0	0	46	41
N.S.	1	1.00	0.94	1.16	0.00	1.19	0.00	0.00	1.44	1.28
time (sec)	N/A	0.251	0.036	0.430	0.000	0.085	0.000	0.000	0.341	9.196

Problem 260	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	45	57	55	0	0	0	0	0	35	0
N.S.	1	1.27	1.22	0.00	0.00	0.00	0.00	0.00	0.78	0.00
time (sec)	N/A	0.294	0.028	0.000	0.000	0.000	0.000	0.000	0.539	0.000

Problem 261	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	46	60	58	0	0	0	0	0	69	0
N.S.	1	1.30	1.26	0.00	0.00	0.00	0.00	0.00	1.50	0.00
time (sec)	N/A	0.298	0.027	0.000	0.000	0.000	0.000	0.000	0.484	0.000

Problem 262	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	48	62	61	0	0	0	0	0	139	0
N.S.	1	1.29	1.27	0.00	0.00	0.00	0.00	0.00	2.90	0.00
time (sec)	N/A	0.310	0.028	0.000	0.000	0.000	0.000	0.000	0.398	0.000

Problem 263	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F	F	C	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	33	33	33	34	0	0	44	0	660	0
N.S.	1	1.00	1.00	1.03	0.00	0.00	1.33	0.00	20.00	0.00
time (sec)	N/A	0.238	0.024	0.392	0.000	0.000	1.227	0.000	0.496	0.000

Problem 264	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	33	52	50	34	0	0	0	0	680	0
N.S.	1	1.58	1.52	1.03	0.00	0.00	0.00	0.00	20.61	0.00
time (sec)	N/A	0.274	0.010	0.389	0.000	0.000	0.000	0.000	0.451	0.000

Problem 265	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	F	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	136	149	64	43	64	73	0	156	63	73
N.S.	1	1.10	0.47	0.32	0.47	0.54	0.00	1.15	0.46	0.54
time (sec)	N/A	0.536	0.032	0.440	0.035	0.068	0.000	0.120	0.257	9.197

Problem 266	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	F	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	108	117	53	32	53	62	0	131	52	62
N.S.	1	1.08	0.49	0.30	0.49	0.57	0.00	1.21	0.48	0.57
time (sec)	N/A	0.459	0.010	0.413	0.037	0.093	0.000	0.141	0.380	8.990

Problem 267	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	F	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	80	86	42	21	42	51	0	108	41	51
N.S.	1	1.08	0.52	0.26	0.52	0.64	0.00	1.35	0.51	0.64
time (sec)	N/A	0.367	0.008	0.409	0.035	0.077	0.000	0.137	0.556	9.090

Problem 268	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	F	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	52	52	41	13	30	39	0	81	29	39
N.S.	1	1.00	0.79	0.25	0.58	0.75	0.00	1.56	0.56	0.75
time (sec)	N/A	0.298	0.004	0.380	0.036	0.077	0.000	0.125	0.434	9.106

Problem 269	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	F	B	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	25	25	23	25	12	26	0	50	16	0
N.S.	1	1.00	0.92	1.00	0.48	1.04	0.00	2.00	0.64	0.00
time (sec)	N/A	0.250	0.005	0.392	0.037	0.066	0.000	0.141	0.523	0.000

Problem 270	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F	A	F	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	53	51	53	36	0	116	0	67	38	73
N.S.	1	0.96	1.00	0.68	0.00	2.19	0.00	1.26	0.72	1.38
time (sec)	N/A	0.318	0.023	0.416	0.000	0.078	0.000	0.148	0.536	9.249

Problem 271	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F	A	F	A	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	54	52	64	50	0	132	0	42	51	0
N.S.	1	0.96	1.19	0.93	0.00	2.44	0.00	0.78	0.94	0.00
time (sec)	N/A	0.318	0.017	0.447	0.000	0.080	0.000	0.209	0.301	0.000

Problem 272	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F	A	F	A	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	86	82	81	61	0	154	0	72	72	0
N.S.	1	0.95	0.94	0.71	0.00	1.79	0.00	0.84	0.84	0.00
time (sec)	N/A	0.378	0.026	0.458	0.000	0.083	0.000	0.209	0.536	0.000

Problem 273	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F	A	F	A	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	114	116	93	72	0	180	0	80	90	0
N.S.	1	1.02	0.82	0.63	0.00	1.58	0.00	0.70	0.79	0.00
time (sec)	N/A	0.478	0.028	0.487	0.000	0.099	0.000	0.225	0.524	0.000

Problem 274	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F	A	F	A	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	142	150	105	82	0	202	0	109	107	0
N.S.	1	1.06	0.74	0.58	0.00	1.42	0.00	0.77	0.75	0.00
time (sec)	N/A	0.553	0.050	0.504	0.000	0.085	0.000	0.245	0.243	0.000

Problem 275	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	F	B	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	164	185	80	32	86	95	0	282	85	80
N.S.	1	1.13	0.49	0.20	0.52	0.58	0.00	1.72	0.52	0.49
time (sec)	N/A	0.614	0.039	0.422	0.037	0.071	0.000	0.138	0.484	9.320

Problem 276	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	F	B	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	136	154	69	21	75	84	0	246	74	69
N.S.	1	1.13	0.51	0.15	0.55	0.62	0.00	1.81	0.54	0.51
time (sec)	N/A	0.563	0.032	0.398	0.039	0.068	0.000	0.125	0.484	9.062

Problem 277	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	F	B	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	108	120	58	13	64	73	0	210	63	58
N.S.	1	1.11	0.54	0.12	0.59	0.68	0.00	1.94	0.58	0.54
time (sec)	N/A	0.448	0.010	0.368	0.040	0.086	0.000	0.124	0.377	8.702

Problem 278	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	C	A	A	F	B	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	80	86	47	35	53	62	0	173	52	47
N.S.	1	1.08	0.59	0.44	0.66	0.78	0.00	2.16	0.65	0.59
time (sec)	N/A	0.389	0.011	0.385	0.039	0.074	0.000	0.271	0.526	8.543

Problem 279	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	F	B	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	52	52	31	35	41	50	0	136	40	36
N.S.	1	1.00	0.60	0.67	0.79	0.96	0.00	2.62	0.77	0.69
time (sec)	N/A	0.315	0.014	0.418	0.038	0.089	0.000	0.131	0.497	8.714

Problem 280	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	F	B	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	25	25	23	27	28	37	0	89	27	28
N.S.	1	1.00	0.92	1.08	1.12	1.48	0.00	3.56	1.08	1.12
time (sec)	N/A	0.255	0.012	0.411	0.038	0.076	0.000	0.122	0.298	9.135

Problem 281	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F	A	F	A	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	76	76	68	61	0	135	0	85	51	0
N.S.	1	1.00	0.89	0.80	0.00	1.78	0.00	1.12	0.67	0.00
time (sec)	N/A	0.385	0.048	0.430	0.000	0.111	0.000	0.131	0.254	0.000

Problem 282	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F	A	F	A	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	76	77	66	70	0	141	0	56	59	0
N.S.	1	1.01	0.87	0.92	0.00	1.86	0.00	0.74	0.78	0.00
time (sec)	N/A	0.386	0.029	0.468	0.000	0.083	0.000	0.149	0.533	0.000

Problem 283	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F	A	F	A	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	84	80	82	67	0	159	0	70	73	0
N.S.	1	0.95	0.98	0.80	0.00	1.89	0.00	0.83	0.87	0.00
time (sec)	N/A	0.371	0.025	0.496	0.000	0.084	0.000	0.147	0.511	0.000

Problem 284	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F	A	F	A	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	112	110	94	81	0	180	0	80	90	0
N.S.	1	0.98	0.84	0.72	0.00	1.61	0.00	0.71	0.80	0.00
time (sec)	N/A	0.449	0.034	0.521	0.000	0.087	0.000	0.137	0.253	0.000

Problem 285	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F	A	F	A	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	140	144	104	92	0	202	0	109	107	0
N.S.	1	1.03	0.74	0.66	0.00	1.44	0.00	0.78	0.76	0.00
time (sec)	N/A	0.542	0.039	0.558	0.000	0.091	0.000	0.155	0.498	0.000

Problem 286	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	F	B	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	190	222	91	21	108	117	0	444	107	91
N.S.	1	1.17	0.48	0.11	0.57	0.62	0.00	2.34	0.56	0.48
time (sec)	N/A	0.745	0.044	0.406	0.039	0.099	0.000	0.128	0.566	8.778

Problem 287	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	F	B	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	164	188	80	13	97	106	0	396	96	80
N.S.	1	1.15	0.49	0.08	0.59	0.65	0.00	2.41	0.59	0.49
time (sec)	N/A	0.660	0.014	0.372	0.037	0.078	0.000	0.116	0.258	8.824

Problem 288	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	C	A	A	F	B	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	136	154	69	47	86	95	0	348	85	69
N.S.	1	1.13	0.51	0.35	0.63	0.70	0.00	2.56	0.62	0.51
time (sec)	N/A	0.567	0.034	0.391	0.040	0.071	0.000	0.128	0.342	8.997

Problem 289	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	F	B	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	108	120	58	57	75	84	0	300	74	58
N.S.	1	1.11	0.54	0.53	0.69	0.78	0.00	2.78	0.69	0.54
time (sec)	N/A	0.475	0.033	0.417	0.038	0.070	0.000	0.122	0.593	8.903

Problem 290	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	F	B	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	80	86	47	46	64	73	0	252	63	47
N.S.	1	1.08	0.59	0.58	0.80	0.91	0.00	3.15	0.79	0.59
time (sec)	N/A	0.404	0.037	0.430	0.038	0.073	0.000	0.201	0.354	9.045

Problem 291	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	F	B	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	52	52	31	35	52	61	0	201	51	36
N.S.	1	1.00	0.60	0.67	1.00	1.17	0.00	3.87	0.98	0.69
time (sec)	N/A	0.323	0.028	0.433	0.039	0.069	0.000	0.123	0.206	8.882

Problem 292	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	B	F	B	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	25	25	23	27	39	48	0	140	38	82
N.S.	1	1.00	0.92	1.08	1.56	1.92	0.00	5.60	1.52	3.28
time (sec)	N/A	0.260	0.009	0.464	0.036	0.067	0.000	0.120	0.545	9.605

Problem 293	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F	A	F	A	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	101	101	80	75	0	161	0	99	72	0
N.S.	1	1.00	0.79	0.74	0.00	1.59	0.00	0.98	0.71	0.00
time (sec)	N/A	0.467	0.061	0.497	0.000	0.098	0.000	0.127	0.441	0.000

Problem 294	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F	A	F	A	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	102	102	81	82	0	165	0	73	78	0
N.S.	1	1.00	0.79	0.80	0.00	1.62	0.00	0.72	0.76	0.00
time (sec)	N/A	0.449	0.098	0.533	0.000	0.093	0.000	0.228	0.303	0.000

Problem 295	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F	A	F	A	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	110	105	88	80	0	168	0	88	84	0
N.S.	1	0.95	0.80	0.73	0.00	1.53	0.00	0.80	0.76	0.00
time (sec)	N/A	0.456	0.123	0.575	0.000	0.082	0.000	0.131	0.321	0.000

Problem 296	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F	A	F	A	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	112	108	93	78	0	181	0	75	90	0
N.S.	1	0.96	0.83	0.70	0.00	1.62	0.00	0.67	0.80	0.00
time (sec)	N/A	0.452	0.157	0.622	0.000	0.088	0.000	0.146	0.522	0.000

Problem 297	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F	A	F	A	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	140	138	105	92	0	202	0	109	107	0
N.S.	1	0.99	0.75	0.66	0.00	1.44	0.00	0.78	0.76	0.00
time (sec)	N/A	0.544	0.182	0.658	0.000	0.082	0.000	0.154	0.490	0.000

Problem 298	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F	A	F	A	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	168	172	115	103	0	224	0	108	124	0
N.S.	1	1.02	0.68	0.61	0.00	1.33	0.00	0.64	0.74	0.00
time (sec)	N/A	0.645	0.193	0.828	0.000	0.094	0.000	0.180	0.231	0.000

Problem 299	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F	A	F	A	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	196	206	127	114	0	246	0	143	141	0
N.S.	1	1.05	0.65	0.58	0.00	1.26	0.00	0.73	0.72	0.00
time (sec)	N/A	0.740	0.239	0.845	0.000	0.087	0.000	0.218	0.438	0.000

Problem 300	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	F	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	104	115	53	52	53	51	0	64	41	51
N.S.	1	1.11	0.51	0.50	0.51	0.49	0.00	0.62	0.39	0.49
time (sec)	N/A	0.484	0.011	0.429	0.037	0.082	0.000	0.128	0.529	8.755

Problem 301	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	F	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	78	81	42	41	42	40	0	52	30	40
N.S.	1	1.04	0.54	0.53	0.54	0.51	0.00	0.67	0.38	0.51
time (sec)	N/A	0.383	0.010	0.412	0.035	0.078	0.000	0.123	0.351	8.699

Problem 302	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	F	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	50	49	30	30	30	28	0	38	18	31
N.S.	1	0.98	0.60	0.60	0.60	0.56	0.00	0.76	0.36	0.62
time (sec)	N/A	0.303	0.009	0.389	0.038	0.073	0.000	0.271	0.227	8.591

Problem 303	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	F	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	23	23	21	21	12	21	0	27	11	17
N.S.	1	1.00	0.91	0.91	0.52	0.91	0.00	1.17	0.48	0.74
time (sec)	N/A	0.245	0.004	0.384	0.036	0.067	0.000	0.140	0.406	8.553

Problem 304	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F	A	F	A	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	32	30	46	13	0	79	0	45	31	0
N.S.	1	0.94	1.44	0.41	0.00	2.47	0.00	1.41	0.97	0.00
time (sec)	N/A	0.259	0.005	0.374	0.000	0.076	0.000	0.116	0.436	0.000

Problem 305	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F	A	F	A	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	56	54	60	18	0	132	0	48	51	0
N.S.	1	0.96	1.07	0.32	0.00	2.36	0.00	0.86	0.91	0.00
time (sec)	N/A	0.320	0.015	0.394	0.000	0.080	0.000	0.123	0.429	0.000

Problem 306	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F	A	F	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	89	88	83	36	0	158	0	73	73	44
N.S.	1	0.99	0.93	0.40	0.00	1.78	0.00	0.82	0.82	0.49
time (sec)	N/A	0.423	0.023	0.421	0.000	0.086	0.000	0.126	0.249	9.057

Problem 307	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F	A	F	A	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	117	122	96	56	0	180	0	76	90	0
N.S.	1	1.04	0.82	0.48	0.00	1.54	0.00	0.65	0.77	0.00
time (sec)	N/A	0.478	0.021	0.425	0.000	0.084	0.000	0.132	0.474	0.000

Problem 308	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	F	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	100	110	50	56	41	60	0	79	42	57
N.S.	1	1.10	0.50	0.56	0.41	0.60	0.00	0.79	0.42	0.57
time (sec)	N/A	0.447	0.010	0.471	0.040	0.076	0.000	0.111	0.525	8.907

Problem 309	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	F	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	74	78	39	46	30	49	0	70	31	47
N.S.	1	1.05	0.53	0.62	0.41	0.66	0.00	0.95	0.42	0.64
time (sec)	N/A	0.377	0.007	0.445	0.039	0.074	0.000	0.119	0.309	9.466

Problem 310	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	F	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	46	47	26	34	19	38	0	48	20	35
N.S.	1	1.02	0.57	0.74	0.41	0.83	0.00	1.04	0.43	0.76
time (sec)	N/A	0.320	0.006	0.443	0.042	0.082	0.000	0.117	0.485	9.008

Problem 311	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	F	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	21	21	19	27	12	29	0	27	13	28
N.S.	1	1.00	0.90	1.29	0.57	1.38	0.00	1.29	0.62	1.33
time (sec)	N/A	0.254	0.004	0.424	0.037	0.091	0.000	0.118	0.614	8.756

Problem 312	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F	A	F	A	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	54	52	54	31	0	161	0	77	57	0
N.S.	1	0.96	1.00	0.57	0.00	2.98	0.00	1.43	1.06	0.00
time (sec)	N/A	0.323	0.015	0.399	0.000	0.077	0.000	0.169	0.304	0.000

Problem 313	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F	A	F	A	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	76	80	62	20	0	194	0	72	73	0
N.S.	1	1.05	0.82	0.26	0.00	2.55	0.00	0.95	0.96	0.00
time (sec)	N/A	0.388	0.014	0.411	0.000	0.123	0.000	0.160	0.390	0.000

Problem 314	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F	A	F	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	112	117	84	13	0	224	0	92	92	42
N.S.	1	1.04	0.75	0.12	0.00	2.00	0.00	0.82	0.82	0.38
time (sec)	N/A	0.455	0.015	0.372	0.000	0.088	0.000	0.156	0.542	9.004

Problem 315	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F	A	F	A	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	140	151	96	31	0	246	0	107	103	0
N.S.	1	1.08	0.69	0.22	0.00	1.76	0.00	0.76	0.74	0.00
time (sec)	N/A	0.572	0.020	0.406	0.000	0.087	0.000	0.144	0.492	0.000

Problem 316	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F	A	F	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	168	185	106	50	0	268	0	122	114	44
N.S.	1	1.10	0.63	0.30	0.00	1.60	0.00	0.73	0.68	0.26
time (sec)	N/A	0.678	0.165	0.416	0.000	0.087	0.000	0.153	0.423	9.339

Problem 317	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	F	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	102	109	52	57	51	71	0	77	49	58
N.S.	1	1.07	0.51	0.56	0.50	0.70	0.00	0.75	0.48	0.57
time (sec)	N/A	0.466	0.013	0.496	0.040	0.072	0.000	0.243	0.489	9.186

Problem 318	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	F	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	74	80	42	46	41	61	0	62	39	82
N.S.	1	1.08	0.57	0.62	0.55	0.82	0.00	0.84	0.53	1.11
time (sec)	N/A	0.388	0.009	0.474	0.041	0.072	0.000	0.199	0.435	8.780

Problem 319	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	F	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	48	49	31	35	30	50	0	35	28	36
N.S.	1	1.02	0.65	0.73	0.62	1.04	0.00	0.73	0.58	0.75
time (sec)	N/A	0.308	0.008	0.445	0.039	0.072	0.000	0.196	0.241	8.680

Problem 320	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	F	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	25	25	23	27	20	40	0	27	20	28
N.S.	1	1.00	0.92	1.08	0.80	1.60	0.00	1.08	0.80	1.12
time (sec)	N/A	0.248	0.006	0.428	0.038	0.080	0.000	0.110	0.484	8.642

Problem 321	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F	A	F	A	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	79	82	67	54	0	228	0	88	121	0
N.S.	1	1.04	0.85	0.68	0.00	2.89	0.00	1.11	1.53	0.00
time (sec)	N/A	0.385	0.030	0.433	0.000	0.081	0.000	0.114	0.494	0.000

Problem 322	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F	A	F	A	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	105	113	81	42	0	264	0	77	147	0
N.S.	1	1.08	0.77	0.40	0.00	2.51	0.00	0.73	1.40	0.00
time (sec)	N/A	0.463	0.024	0.421	0.000	0.094	0.000	0.143	0.256	0.000

Problem 323	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F	A	F	A	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	141	148	93	32	0	290	0	105	166	0
N.S.	1	1.05	0.66	0.23	0.00	2.06	0.00	0.74	1.18	0.00
time (sec)	N/A	0.548	0.047	0.410	0.000	0.116	0.000	0.378	0.310	0.000

Problem 324	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F	A	F	A	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	166	179	104	21	0	312	0	123	177	0
N.S.	1	1.08	0.63	0.13	0.00	1.88	0.00	0.74	1.07	0.00
time (sec)	N/A	0.642	0.037	0.405	0.000	0.088	0.000	0.301	0.497	0.000

Problem 325	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F	A	F	A	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	214	242	141	149	0	291	0	162	119	0
N.S.	1	1.13	0.66	0.70	0.00	1.36	0.00	0.76	0.56	0.00
time (sec)	N/A	0.847	0.067	0.372	0.000	0.085	0.000	0.306	0.478	0.000

Problem 326	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F	A	F	A	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	179	200	130	138	0	242	0	147	98	0
N.S.	1	1.12	0.73	0.77	0.00	1.35	0.00	0.82	0.55	0.00
time (sec)	N/A	0.698	0.070	0.369	0.000	0.090	0.000	0.144	0.270	0.000

Problem 327	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F	A	F	A	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	144	158	115	123	0	211	0	128	78	0
N.S.	1	1.10	0.80	0.85	0.00	1.47	0.00	0.89	0.54	0.00
time (sec)	N/A	0.572	0.076	0.362	0.000	0.085	0.000	0.156	0.347	0.000

Problem 328	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F	A	F	A	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	111	116	106	108	0	186	0	133	61	0
N.S.	1	1.05	0.95	0.97	0.00	1.68	0.00	1.20	0.55	0.00
time (sec)	N/A	0.468	0.068	0.365	0.000	0.082	0.000	0.167	0.556	0.000

Problem 329	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F	A	F	A	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	73	76	72	94	0	165	0	112	42	0
N.S.	1	1.04	0.99	1.29	0.00	2.26	0.00	1.53	0.58	0.00
time (sec)	N/A	0.376	0.029	0.359	0.000	0.094	0.000	0.429	0.469	0.000

Problem 330	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F	A	F	A	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	74	77	85	99	0	163	0	80	48	0
N.S.	1	1.04	1.15	1.34	0.00	2.20	0.00	1.08	0.65	0.00
time (sec)	N/A	0.387	0.024	0.360	0.000	0.081	0.000	0.298	0.330	0.000

Problem 331	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F	A	F	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	30	30	28	30	0	34	0	40	42	39
N.S.	1	1.00	0.93	1.00	0.00	1.13	0.00	1.33	1.40	1.30
time (sec)	N/A	0.249	0.013	0.354	0.000	0.076	0.000	0.174	0.575	8.827

Problem 332	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F	A	F	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	61	61	46	38	0	48	0	65	66	53
N.S.	1	1.00	0.75	0.62	0.00	0.79	0.00	1.07	1.08	0.87
time (sec)	N/A	0.318	0.009	0.344	0.000	0.074	0.000	0.182	0.508	8.963

Problem 333	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F	A	F	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	96	103	58	49	0	59	0	88	85	67
N.S.	1	1.07	0.60	0.51	0.00	0.61	0.00	0.92	0.89	0.70
time (sec)	N/A	0.403	0.010	0.356	0.000	0.088	0.000	0.152	0.260	9.601

Problem 334	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F	A	F	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	131	145	69	60	0	70	0	107	104	81
N.S.	1	1.11	0.53	0.46	0.00	0.53	0.00	0.82	0.79	0.62
time (sec)	N/A	0.499	0.012	0.351	0.000	0.078	0.000	0.170	0.499	9.230

Problem 335	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F	A	F	A	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	282	322	163	171	0	314	0	347	155	0
N.S.	1	1.14	0.58	0.61	0.00	1.11	0.00	1.23	0.55	0.00
time (sec)	N/A	1.070	0.751	0.364	0.000	0.090	0.000	0.178	0.469	0.000

Problem 336	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F	A	F	A	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	247	280	152	156	0	277	0	312	135	0
N.S.	1	1.13	0.62	0.63	0.00	1.12	0.00	1.26	0.55	0.00
time (sec)	N/A	0.921	0.092	0.349	0.000	0.100	0.000	0.187	0.315	0.000

Problem 337	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F	A	F	A	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	214	238	141	143	0	256	0	200	119	0
N.S.	1	1.11	0.66	0.67	0.00	1.20	0.00	0.93	0.56	0.00
time (sec)	N/A	0.802	0.088	0.344	0.000	0.091	0.000	0.170	0.229	0.000

Problem 338	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F	A	F	A	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	182	196	130	138	0	235	0	184	100	0
N.S.	1	1.08	0.71	0.76	0.00	1.29	0.00	1.01	0.55	0.00
time (sec)	N/A	0.676	0.049	0.372	0.000	0.091	0.000	0.273	0.420	0.000

Problem 339	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F	A	F	A	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	147	154	119	127	0	212	0	161	81	0
N.S.	1	1.05	0.81	0.86	0.00	1.44	0.00	1.10	0.55	0.00
time (sec)	N/A	0.575	0.051	0.414	0.000	0.090	0.000	0.179	0.503	0.000

Problem 340	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F	A	F	A	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	112	114	103	110	0	191	0	139	62	0
N.S.	1	1.02	0.92	0.98	0.00	1.71	0.00	1.24	0.55	0.00
time (sec)	N/A	0.467	0.034	0.376	0.000	0.085	0.000	0.428	0.345	0.000

Problem 341	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F	A	F	A	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	103	110	95	112	0	183	0	95	64	0
N.S.	1	1.07	0.92	1.09	0.00	1.78	0.00	0.92	0.62	0.00
time (sec)	N/A	0.450	0.027	0.378	0.000	0.091	0.000	0.223	0.206	0.000

Problem 342	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F	A	F	A	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	107	112	88	110	0	180	0	98	60	0
N.S.	1	1.05	0.82	1.03	0.00	1.68	0.00	0.92	0.56	0.00
time (sec)	N/A	0.469	0.023	0.381	0.000	0.095	0.000	0.469	0.569	0.000

Problem 343	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F	A	F(-1)	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	30	30	28	30	0	45	0	40	66	51
N.S.	1	1.00	0.93	1.00	0.00	1.50	0.00	1.33	2.20	1.70
time (sec)	N/A	0.246	0.018	0.379	0.000	0.109	0.000	0.179	0.503	8.879

Problem 344	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F	A	F(-1)	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	61	61	36	38	0	59	0	67	85	65
N.S.	1	1.00	0.59	0.62	0.00	0.97	0.00	1.10	1.39	1.07
time (sec)	N/A	0.333	0.015	0.379	0.000	0.076	0.000	0.181	0.274	8.742

Problem 345	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F	A	F(-1)	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	96	103	47	49	0	70	0	86	104	79
N.S.	1	1.07	0.49	0.51	0.00	0.73	0.00	0.90	1.08	0.82
time (sec)	N/A	0.420	0.014	0.377	0.000	0.073	0.000	0.182	0.459	8.791

Problem 346	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F	A	F(-1)	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	131	145	58	60	0	81	0	109	123	93
N.S.	1	1.11	0.44	0.46	0.00	0.62	0.00	0.83	0.94	0.71
time (sec)	N/A	0.523	0.019	0.387	0.000	0.073	0.000	0.173	0.491	9.066

Problem 347	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F	A	F	A	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	152	162	124	126	0	234	0	141	81	0
N.S.	1	1.07	0.82	0.83	0.00	1.54	0.00	0.93	0.53	0.00
time (sec)	N/A	0.598	0.046	0.398	0.000	0.088	0.000	0.140	0.454	0.000

Problem 348	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F	A	F	A	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	117	120	110	115	0	207	0	123	62	0
N.S.	1	1.03	0.94	0.98	0.00	1.77	0.00	1.05	0.53	0.00
time (sec)	N/A	0.492	0.040	0.391	0.000	0.089	0.000	0.129	0.285	0.000

Problem 349	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F	A	F	A	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	77	78	92	101	0	169	0	108	41	0
N.S.	1	1.01	1.19	1.31	0.00	2.19	0.00	1.40	0.53	0.00
time (sec)	N/A	0.398	0.031	0.393	0.000	0.093	0.000	0.126	0.512	0.000

Problem 350	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	B	F	A	F	B	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	46	46	64	76	0	106	0	70	27	0
N.S.	1	1.00	1.39	1.65	0.00	2.30	0.00	1.52	0.59	0.00
time (sec)	N/A	0.310	0.009	0.382	0.000	0.078	0.000	0.143	0.460	0.000

Problem 351	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F	A	F	F(-1)	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	28	28	26	28	0	29	0	0	27	22
N.S.	1	1.00	0.93	1.00	0.00	1.04	0.00	0.00	0.96	0.79
time (sec)	N/A	0.254	0.010	0.373	0.000	0.077	0.000	0.000	0.240	9.725

Problem 352	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F	A	F	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	61	61	34	36	0	37	0	59	45	42
N.S.	1	1.00	0.56	0.59	0.00	0.61	0.00	0.97	0.74	0.69
time (sec)	N/A	0.319	0.012	0.387	0.000	0.077	0.000	0.282	0.304	9.945

Problem 353	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F	A	F	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	96	103	47	49	0	48	0	137	66	56
N.S.	1	1.07	0.49	0.51	0.00	0.50	0.00	1.43	0.69	0.58
time (sec)	N/A	0.394	0.018	0.384	0.000	0.086	0.000	33.842	0.471	9.764

Problem 354	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F	A	F	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	131	145	58	60	0	59	0	175	85	70
N.S.	1	1.11	0.44	0.46	0.00	0.45	0.00	1.34	0.65	0.53
time (sec)	N/A	0.484	0.019	0.392	0.000	0.078	0.000	37.293	0.498	9.510

Problem 355	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F	A	F	A	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	150	159	113	163	0	281	0	163	90	0
N.S.	1	1.06	0.75	1.09	0.00	1.87	0.00	1.09	0.60	0.00
time (sec)	N/A	0.578	0.047	0.401	0.000	0.091	0.000	0.192	0.237	0.000

Problem 356	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F	A	F	A	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	108	117	96	150	0	245	0	138	73	0
N.S.	1	1.08	0.89	1.39	0.00	2.27	0.00	1.28	0.68	0.00
time (sec)	N/A	0.489	0.038	0.409	0.000	0.087	0.000	0.174	0.414	0.000

Problem 357	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	B	F	A	F	A	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	77	80	89	145	0	215	0	115	61	0
N.S.	1	1.04	1.16	1.88	0.00	2.79	0.00	1.49	0.79	0.00
time (sec)	N/A	0.382	0.024	0.378	0.000	0.098	0.000	0.178	0.585	0.000

Problem 358	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F	A	F	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	28	28	26	30	0	37	0	35	35	37
N.S.	1	1.00	0.93	1.07	0.00	1.32	0.00	1.25	1.25	1.32
time (sec)	N/A	0.248	0.008	0.368	0.000	0.093	0.000	0.164	0.344	8.610

Problem 359	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F	A	F	F(-1)	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	59	59	32	36	0	47	0	0	42	56
N.S.	1	1.00	0.54	0.61	0.00	0.80	0.00	0.00	0.71	0.95
time (sec)	N/A	0.317	0.009	0.380	0.000	0.078	0.000	0.000	0.215	8.589

Problem 360	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F	A	F	B	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	94	98	47	47	0	57	0	190	57	67
N.S.	1	1.04	0.50	0.50	0.00	0.61	0.00	2.02	0.61	0.71
time (sec)	N/A	0.400	0.013	0.384	0.000	0.076	0.000	36.928	0.464	8.893

Problem 361	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F	A	F	F(-1)	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	127	140	56	58	0	68	0	0	75	75
N.S.	1	1.10	0.44	0.46	0.00	0.54	0.00	0.00	0.59	0.59
time (sec)	N/A	0.483	0.013	0.390	0.000	0.078	0.000	0.000	0.563	9.389

Problem 362	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F	A	F	F(-1)	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	159	182	69	71	0	85	0	0	88	70
N.S.	1	1.14	0.43	0.45	0.00	0.53	0.00	0.00	0.55	0.44
time (sec)	N/A	0.589	0.016	0.400	0.000	0.075	0.000	0.000	0.300	9.339

Problem 363	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F	A	F(-1)	A	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	145	160	111	196	0	326	0	163	141	0
N.S.	1	1.10	0.77	1.35	0.00	2.25	0.00	1.12	0.97	0.00
time (sec)	N/A	0.594	0.051	0.414	0.000	0.092	0.000	0.177	0.212	0.000

Problem 364	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	B	F	A	F(-1)	A	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	112	120	99	211	0	292	0	137	99	0
N.S.	1	1.07	0.88	1.88	0.00	2.61	0.00	1.22	0.88	0.00
time (sec)	N/A	0.480	0.050	0.380	0.000	0.092	0.000	0.149	0.549	0.000

Problem 365	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F	A	F	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	30	30	28	30	0	44	0	36	57	47
N.S.	1	1.00	0.93	1.00	0.00	1.47	0.00	1.20	1.90	1.57
time (sec)	N/A	0.259	0.011	0.372	0.000	0.078	0.000	0.167	0.547	8.530

Problem 366	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F	A	F	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	63	63	36	38	0	61	0	59	66	71
N.S.	1	1.00	0.57	0.60	0.00	0.97	0.00	0.94	1.05	1.13
time (sec)	N/A	0.330	0.015	0.381	0.000	0.080	0.000	0.133	0.230	8.702

Problem 367	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F	A	F	F(-1)	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	95	100	47	49	0	77	0	0	79	90
N.S.	1	1.05	0.49	0.52	0.00	0.81	0.00	0.00	0.83	0.95
time (sec)	N/A	0.411	0.016	0.394	0.000	0.082	0.000	0.000	0.363	9.017

Problem 368	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F	A	F	B	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	126	137	58	58	0	91	0	213	97	106
N.S.	1	1.09	0.46	0.46	0.00	0.72	0.00	1.69	0.77	0.84
time (sec)	N/A	0.495	0.021	0.402	0.000	0.080	0.000	33.450	0.541	8.896

Problem 369	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F	A	F	B	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	163	181	69	71	0	95	0	310	110	116
N.S.	1	1.11	0.42	0.44	0.00	0.58	0.00	1.90	0.67	0.71
time (sec)	N/A	0.589	0.019	0.398	0.000	0.081	0.000	104.842	0.388	8.990

Problem 370	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F	A	F	F(-1)	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	198	223	80	82	0	101	0	0	122	126
N.S.	1	1.13	0.40	0.41	0.00	0.51	0.00	0.00	0.62	0.64
time (sec)	N/A	0.684	0.020	0.402	0.000	0.084	0.000	0.000	0.229	8.793

Problem 371	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F	A	F	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	229	263	91	93	0	115	0	311	138	124
N.S.	1	1.15	0.40	0.41	0.00	0.50	0.00	1.36	0.60	0.54
time (sec)	N/A	0.822	0.019	0.404	0.000	0.085	0.000	2.563	0.510	9.059

Problem 372	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F	A	F	A	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	238	182	222	153	0	194	0	142	13	0
N.S.	1	0.76	0.93	0.64	0.00	0.82	0.00	0.60	0.05	0.00
time (sec)	N/A	0.481	0.068	0.508	0.000	0.088	0.000	4.289	0.697	0.000

Problem 373	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F	A	F	A	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	200	150	201	129	0	177	0	107	13	0
N.S.	1	0.75	1.00	0.64	0.00	0.88	0.00	0.54	0.06	0.00
time (sec)	N/A	0.410	0.034	0.475	0.000	0.077	0.000	4.163	0.364	0.000

Problem 374	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F	A	F	A	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	181	123	153	108	0	157	0	86	13	0
N.S.	1	0.68	0.85	0.60	0.00	0.87	0.00	0.48	0.07	0.00
time (sec)	N/A	0.328	0.022	0.448	0.000	0.082	0.000	3.487	0.690	0.000

Problem 375	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F	A	F	A	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	23	23	21	20	0	21	0	14	13	21
N.S.	1	1.00	0.91	0.87	0.00	0.91	0.00	0.61	0.57	0.91
time (sec)	N/A	0.243	0.004	0.398	0.000	0.068	0.000	0.138	0.531	9.454

Problem 376	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F	A	F	A	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	52	52	31	26	0	29	0	34	13	42
N.S.	1	1.00	0.60	0.50	0.00	0.56	0.00	0.65	0.25	0.81
time (sec)	N/A	0.308	0.008	0.414	0.000	0.075	0.000	0.113	0.375	9.409

Problem 377	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F	A	F	A	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	80	86	42	39	0	40	0	43	13	66
N.S.	1	1.08	0.52	0.49	0.00	0.50	0.00	0.54	0.16	0.82
time (sec)	N/A	0.378	0.011	0.432	0.000	0.073	0.000	0.399	0.737	9.553

Problem 378	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F	A	F	A	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	260	209	222	187	0	506	0	154	33	0
N.S.	1	0.80	0.85	0.72	0.00	1.95	0.00	0.59	0.13	0.00
time (sec)	N/A	0.552	0.576	0.750	0.000	0.094	0.000	7.320	0.354	0.000

Problem 379	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F	A	F	A	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	222	178	201	170	0	470	0	133	33	0
N.S.	1	0.80	0.91	0.77	0.00	2.12	0.00	0.60	0.15	0.00
time (sec)	N/A	0.475	0.401	0.746	0.000	0.101	0.000	7.409	0.298	0.000

Problem 380	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F	A	F	A	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	198	144	184	158	0	432	0	101	31	0
N.S.	1	0.73	0.93	0.80	0.00	2.18	0.00	0.51	0.16	0.00
time (sec)	N/A	0.379	0.300	0.486	0.000	0.085	0.000	8.324	0.608	0.000

Problem 381	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F	A	F	A	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	21	21	19	18	0	29	0	14	29	28
N.S.	1	1.00	0.90	0.86	0.00	1.38	0.00	0.67	1.38	1.33
time (sec)	N/A	0.249	0.027	0.424	0.000	0.071	0.000	3.331	0.575	9.004

Problem 382	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F	A	F	A	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	46	46	26	23	0	39	0	34	29	26
N.S.	1	1.00	0.57	0.50	0.00	0.85	0.00	0.74	0.63	0.57
time (sec)	N/A	0.304	0.221	0.437	0.000	0.071	0.000	3.939	0.297	9.177

Problem 383	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F	A	F	A	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	75	81	43	37	0	52	0	57	29	47
N.S.	1	1.08	0.57	0.49	0.00	0.69	0.00	0.76	0.39	0.63
time (sec)	N/A	0.380	0.006	0.417	0.000	0.071	0.000	3.484	0.689	9.037

Problem 384	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F	A	F	A	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	103	115	55	50	0	63	0	70	29	58
N.S.	1	1.12	0.53	0.49	0.00	0.61	0.00	0.68	0.28	0.56
time (sec)	N/A	0.461	0.320	0.454	0.000	0.070	0.000	3.643	0.571	8.512

Problem 385	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F	A	F	A	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	131	149	64	61	0	74	0	92	29	110
N.S.	1	1.14	0.49	0.47	0.00	0.56	0.00	0.70	0.22	0.84
time (sec)	N/A	0.541	0.421	0.472	0.000	0.076	0.000	3.310	0.344	8.557

Problem 386	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	F	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	173	248	49	0	0	0	0	0	124	0
N.S.	1	1.43	0.28	0.00	0.00	0.00	0.00	0.00	0.72	0.00
time (sec)	N/A	0.844	10.017	0.000	0.000	0.000	0.000	0.000	0.698	0.000

Problem 387	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	F	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	145	216	49	0	0	0	0	0	102	0
N.S.	1	1.49	0.34	0.00	0.00	0.00	0.00	0.00	0.70	0.00
time (sec)	N/A	0.743	10.016	0.000	0.000	0.000	0.000	0.000	0.598	0.000

Problem 388	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	F	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	117	185	49	0	0	0	0	0	84	0
N.S.	1	1.58	0.42	0.00	0.00	0.00	0.00	0.00	0.72	0.00
time (sec)	N/A	0.613	10.012	0.000	0.000	0.000	0.000	0.000	0.504	0.000

Problem 389	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	F	F	F	F	F	F	B
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	87	156	45	0	0	0	0	0	64	38
N.S.	1	1.79	0.52	0.00	0.00	0.00	0.00	0.00	0.74	0.44
time (sec)	N/A	0.568	0.005	0.000	0.000	0.000	0.000	0.000	0.676	9.033

Problem 390	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	B	C	F	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	86	180	44	0	0	0	0	0	70	0
N.S.	1	2.09	0.51	0.00	0.00	0.00	0.00	0.00	0.81	0.00
time (sec)	N/A	0.660	10.010	0.000	0.000	0.000	0.000	0.000	0.349	0.000

Problem 391	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	F	F	F	F	F	F	B
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	113	214	49	0	0	0	0	0	73	44
N.S.	1	1.89	0.43	0.00	0.00	0.00	0.00	0.00	0.65	0.39
time (sec)	N/A	0.763	10.011	0.000	0.000	0.000	0.000	0.000	0.665	9.413

Problem 392	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	F	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	144	248	49	0	0	0	0	0	73	0
N.S.	1	1.72	0.34	0.00	0.00	0.00	0.00	0.00	0.51	0.00
time (sec)	N/A	0.857	10.014	0.000	0.000	0.000	0.000	0.000	0.659	0.000

Problem 393	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	F	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	144	184	49	0	0	0	0	0	104	0
N.S.	1	1.28	0.34	0.00	0.00	0.00	0.00	0.00	0.72	0.00
time (sec)	N/A	0.612	10.013	0.000	0.000	0.000	0.000	0.000	0.345	0.000

Problem 394	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	F	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	116	150	49	0	0	0	0	0	84	0
N.S.	1	1.29	0.42	0.00	0.00	0.00	0.00	0.00	0.72	0.00
time (sec)	N/A	0.500	10.012	0.000	0.000	0.000	0.000	0.000	0.759	0.000

Problem 395	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	F	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	92	118	49	0	0	0	0	0	68	0
N.S.	1	1.28	0.53	0.00	0.00	0.00	0.00	0.00	0.74	0.00
time (sec)	N/A	0.444	10.014	0.000	0.000	0.000	0.000	0.000	0.569	0.000

Problem 396	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	F	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	67	93	47	0	0	0	0	0	35	0
N.S.	1	1.39	0.70	0.00	0.00	0.00	0.00	0.00	0.52	0.00
time (sec)	N/A	0.368	10.014	0.000	0.000	0.000	0.000	0.000	0.254	0.000

Problem 397	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	F	F	F	F	F	F	B
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	91	111	45	0	0	0	0	0	37	38
N.S.	1	1.22	0.49	0.00	0.00	0.00	0.00	0.00	0.41	0.42
time (sec)	N/A	0.394	0.008	0.000	0.000	0.000	0.000	0.000	0.597	9.055

Problem 398	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	F	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	121	144	46	0	0	0	0	0	37	0
N.S.	1	1.19	0.38	0.00	0.00	0.00	0.00	0.00	0.31	0.00
time (sec)	N/A	0.458	10.011	0.000	0.000	0.000	0.000	0.000	0.522	0.000

Problem 399	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	F	F	F	F	F	F	B
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	149	178	49	0	0	0	0	0	37	44
N.S.	1	1.19	0.33	0.00	0.00	0.00	0.00	0.00	0.25	0.30
time (sec)	N/A	0.551	10.011	0.000	0.000	0.000	0.000	0.000	0.295	8.890

Problem 400	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	F	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	148	245	52	0	0	0	0	0	104	0
N.S.	1	1.66	0.35	0.00	0.00	0.00	0.00	0.00	0.70	0.00
time (sec)	N/A	0.860	10.013	0.000	0.000	0.000	0.000	0.000	0.693	0.000

Problem 401	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	F	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	120	214	52	0	0	0	0	0	82	0
N.S.	1	1.78	0.43	0.00	0.00	0.00	0.00	0.00	0.68	0.00
time (sec)	N/A	0.753	10.011	0.000	0.000	0.000	0.000	0.000	0.560	0.000

Problem 402	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	F	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	90	178	52	0	0	0	0	0	68	0
N.S.	1	1.98	0.58	0.00	0.00	0.00	0.00	0.00	0.76	0.00
time (sec)	N/A	0.637	10.013	0.000	0.000	0.000	0.000	0.000	0.329	0.000

Problem 403	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	B	C	F	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	68	181	48	0	0	0	0	0	35	0
N.S.	1	2.66	0.71	0.00	0.00	0.00	0.00	0.00	0.51	0.00
time (sec)	N/A	0.652	10.012	0.000	0.000	0.000	0.000	0.000	0.567	0.000

Problem 404	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	B	C	F	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	89	206	47	0	0	0	0	0	37	0
N.S.	1	2.31	0.53	0.00	0.00	0.00	0.00	0.00	0.42	0.00
time (sec)	N/A	0.744	10.011	0.000	0.000	0.000	0.000	0.000	0.572	0.000

Problem 405	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	B	C	F	F	F	F	F	F	B
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	117	243	52	0	0	0	0	0	37	38
N.S.	1	2.08	0.44	0.00	0.00	0.00	0.00	0.00	0.32	0.32
time (sec)	N/A	0.820	0.010	0.000	0.000	0.000	0.000	0.000	0.294	8.921

Problem 406	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	F	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	147	277	54	0	0	0	0	0	37	0
N.S.	1	1.88	0.37	0.00	0.00	0.00	0.00	0.00	0.25	0.00
time (sec)	N/A	0.966	10.016	0.000	0.000	0.000	0.000	0.000	0.570	0.000

Problem 407	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	B	A	A	B	B	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	81	85	57	173	77	161	711	264	173	171
N.S.	1	1.05	0.70	2.14	0.95	1.99	8.78	3.26	2.14	2.11
time (sec)	N/A	0.391	0.070	0.362	0.039	0.077	0.532	0.121	0.471	9.075

Problem 408	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	B	B	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	58	62	41	59	55	89	330	141	90	97
N.S.	1	1.07	0.71	1.02	0.95	1.53	5.69	2.43	1.55	1.67
time (sec)	N/A	0.347	0.053	0.315	0.039	0.079	0.349	0.294	0.258	8.815

Problem 409	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	B	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	35	39	25	35	33	39	105	56	35	34
N.S.	1	1.11	0.71	1.00	0.94	1.11	3.00	1.60	1.00	0.97
time (sec)	N/A	0.296	0.036	0.046	0.037	0.082	0.235	0.221	0.240	8.993

Problem 410	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	33	33	30	0	0	0	0	0	23	0
N.S.	1	1.00	0.91	0.00	0.00	0.00	0.00	0.00	0.70	0.00
time (sec)	N/A	0.262	0.038	0.000	0.000	0.000	0.000	0.000	0.559	0.000

Problem 411	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	37	37	32	0	0	0	0	0	34	0
N.S.	1	1.00	0.86	0.00	0.00	0.00	0.00	0.00	0.92	0.00
time (sec)	N/A	0.264	0.031	0.000	0.000	0.000	0.000	0.000	0.521	0.000

Problem 412	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	37	37	32	0	0	0	0	0	45	0
N.S.	1	1.00	0.86	0.00	0.00	0.00	0.00	0.00	1.22	0.00
time (sec)	N/A	0.263	0.037	0.000	0.000	0.000	0.000	0.000	0.232	0.000

Problem 413	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	B	A	F	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	68	158	68	0	0	0	0	0	0	0
N.S.	1	2.32	1.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.548	0.163	0.000	0.000	0.000	0.000	0.000	0.525	0.000

Problem 414	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	68	107	60	0	0	0	0	0	521	0
N.S.	1	1.57	0.88	0.00	0.00	0.00	0.00	0.00	7.66	0.00
time (sec)	N/A	0.417	0.101	0.000	0.000	0.000	0.000	0.000	0.549	0.000

Problem 415	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	59	62	57	0	0	0	0	0	27	0
N.S.	1	1.05	0.97	0.00	0.00	0.00	0.00	0.00	0.46	0.00
time (sec)	N/A	0.315	0.102	0.000	0.000	0.000	0.000	0.000	0.306	0.000

Problem 416	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	64	99	58	0	0	0	0	0	40	0
N.S.	1	1.55	0.91	0.00	0.00	0.00	0.00	0.00	0.62	0.00
time (sec)	N/A	0.405	0.143	0.000	0.000	0.000	0.000	0.000	0.276	0.000

Problem 417	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	B	A	F	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	68	148	62	0	0	0	0	0	51	0
N.S.	1	2.18	0.91	0.00	0.00	0.00	0.00	0.00	0.75	0.00
time (sec)	N/A	0.514	0.150	0.000	0.000	0.000	0.000	0.000	0.626	0.000

Problem 418	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	46	57	55	0	0	0	0	0	563	0
N.S.	1	1.24	1.20	0.00	0.00	0.00	0.00	0.00	12.24	0.00
time (sec)	N/A	0.323	0.045	0.000	0.000	0.000	0.000	0.000	0.410	0.000

Problem 419	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	46	66	55	0	0	0	0	0	375	0
N.S.	1	1.43	1.20	0.00	0.00	0.00	0.00	0.00	8.15	0.00
time (sec)	N/A	0.327	0.050	0.000	0.000	0.000	0.000	0.000	0.280	0.000

Problem 420	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F	F	F	F	F	B
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	48	55	53	0	0	0	0	0	137	56
N.S.	1	1.15	1.10	0.00	0.00	0.00	0.00	0.00	2.85	1.17
time (sec)	N/A	0.310	0.006	0.000	0.000	0.000	0.000	0.000	0.564	8.737

Problem 421	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	47	51	49	0	0	0	0	0	49	0
N.S.	1	1.09	1.04	0.00	0.00	0.00	0.00	0.00	1.04	0.00
time (sec)	N/A	0.311	0.026	0.000	0.000	0.000	0.000	0.000	0.636	0.000

Problem 422	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F	F	F	F	F	B
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	45	56	53	0	0	0	0	0	127	63
N.S.	1	1.24	1.18	0.00	0.00	0.00	0.00	0.00	2.82	1.40
time (sec)	N/A	0.321	0.035	0.000	0.000	0.000	0.000	0.000	0.388	8.785

Problem 423	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	58	70	66	0	0	0	0	0	683	0
N.S.	1	1.21	1.14	0.00	0.00	0.00	0.00	0.00	11.78	0.00
time (sec)	N/A	0.336	0.034	0.000	0.000	0.000	0.000	0.000	0.363	0.000

Problem 424	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	58	67	66	0	0	0	0	0	335	0
N.S.	1	1.16	1.14	0.00	0.00	0.00	0.00	0.00	5.78	0.00
time (sec)	N/A	0.332	0.031	0.000	0.000	0.000	0.000	0.000	0.590	0.000

Problem 425	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	58	67	66	0	0	0	0	0	122	0
N.S.	1	1.16	1.14	0.00	0.00	0.00	0.00	0.00	2.10	0.00
time (sec)	N/A	0.329	0.047	0.000	0.000	0.000	0.000	0.000	0.486	0.000

Problem 426	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	58	69	66	0	0	0	0	0	140	0
N.S.	1	1.19	1.14	0.00	0.00	0.00	0.00	0.00	2.41	0.00
time (sec)	N/A	0.327	0.037	0.000	0.000	0.000	0.000	0.000	0.377	0.000

Problem 427	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	56	63	61	0	0	0	0	0	766	0
N.S.	1	1.12	1.09	0.00	0.00	0.00	0.00	0.00	13.68	0.00
time (sec)	N/A	0.331	0.031	0.000	0.000	0.000	0.000	0.000	0.649	0.000

Problem 428	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F	A	F	F	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	181	175	104	133	0	162	0	0	157	241
N.S.	1	0.97	0.57	0.73	0.00	0.90	0.00	0.00	0.87	1.33
time (sec)	N/A	0.674	0.048	0.522	0.000	0.091	0.000	0.000	0.488	8.874

Problem 429	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F	A	F	F	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	126	125	75	87	0	113	0	0	108	165
N.S.	1	0.99	0.60	0.69	0.00	0.90	0.00	0.00	0.86	1.31
time (sec)	N/A	0.526	0.050	0.522	0.000	0.086	0.000	0.000	0.270	8.800

Problem 430	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F	A	F	F	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	75	75	50	53	0	72	0	0	72	104
N.S.	1	1.00	0.67	0.71	0.00	0.96	0.00	0.00	0.96	1.39
time (sec)	N/A	0.379	0.051	0.513	0.000	0.088	0.000	0.000	0.253	9.121

Problem 431	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F	A	F	F	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	35	35	37	39	0	40	0	0	48	43
N.S.	1	1.00	1.06	1.11	0.00	1.14	0.00	0.00	1.37	1.23
time (sec)	N/A	0.282	0.031	0.473	0.000	0.123	0.000	0.000	0.218	9.092

Problem 432	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	48	59	57	0	0	0	0	0	37	0
N.S.	1	1.23	1.19	0.00	0.00	0.00	0.00	0.00	0.77	0.00
time (sec)	N/A	0.324	0.012	0.000	0.000	0.000	0.000	0.000	0.241	0.000

Problem 433	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	51	62	60	0	0	0	0	0	73	0
N.S.	1	1.22	1.18	0.00	0.00	0.00	0.00	0.00	1.43	0.00
time (sec)	N/A	0.315	0.012	0.000	0.000	0.000	0.000	0.000	0.234	0.000

Problem 434	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	49	65	63	0	0	0	0	0	147	0
N.S.	1	1.33	1.29	0.00	0.00	0.00	0.00	0.00	3.00	0.00
time (sec)	N/A	0.324	0.013	0.000	0.000	0.000	0.000	0.000	0.252	0.000

Problem 435	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	B	A	B	B	B	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	101	96	73	447	119	419	4519	1090	471	541
N.S.	1	0.95	0.72	4.43	1.18	4.15	44.74	10.79	4.66	5.36
time (sec)	N/A	0.420	0.068	1.680	0.043	0.182	38.090	0.145	0.234	9.294

Problem 436	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	B	A	B	B	B	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	73	72	54	191	86	188	1428	470	193	243
N.S.	1	0.99	0.74	2.62	1.18	2.58	19.56	6.44	2.64	3.33
time (sec)	N/A	0.384	0.053	0.629	0.038	0.143	10.914	0.136	0.266	9.063

Problem 437	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	B	B	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	37	40	29	61	47	56	282	129	51	73
N.S.	1	1.08	0.78	1.65	1.27	1.51	7.62	3.49	1.38	1.97
time (sec)	N/A	0.329	0.034	0.116	0.038	0.118	2.879	0.132	0.230	9.071

Problem 438	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	45	45	45	0	0	0	0	0	82	0
N.S.	1	1.00	1.00	0.00	0.00	0.00	0.00	0.00	1.82	0.00
time (sec)	N/A	0.325	0.042	0.000	0.000	0.000	0.000	0.000	0.235	0.000

Problem 439	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	45	45	45	0	0	0	0	0	1273	0
N.S.	1	1.00	1.00	0.00	0.00	0.00	0.00	0.00	28.29	0.00
time (sec)	N/A	0.313	0.036	0.000	0.000	0.000	0.000	0.000	0.250	0.000

Problem 440	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	45	45	45	0	0	0	0	0	0	0
N.S.	1	1.00	1.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.311	0.035	0.000	0.000	0.000	0.000	0.000	0.238	0.000

Problem 441	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F	F(-2)	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	75	80	71	0	0	0	0	0	42	0
N.S.	1	1.07	0.95	0.00	0.00	0.00	0.00	0.00	0.56	0.00
time (sec)	N/A	0.346	0.166	0.000	0.000	0.000	0.000	0.000	0.834	0.000

Problem 442	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F	F(-2)	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	75	75	71	0	0	0	0	0	20	0
N.S.	1	1.00	0.95	0.00	0.00	0.00	0.00	0.00	0.27	0.00
time (sec)	N/A	0.332	0.110	0.000	0.000	0.000	0.000	0.000	0.343	0.000

Problem 443	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F	F(-2)	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	73	91	69	0	0	0	0	0	25	0
N.S.	1	1.25	0.95	0.00	0.00	0.00	0.00	0.00	0.34	0.00
time (sec)	N/A	0.351	0.113	0.000	0.000	0.000	0.000	0.000	0.253	0.000

Problem 444	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F	F(-2)	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	73	86	69	0	0	0	0	0	40	0
N.S.	1	1.18	0.95	0.00	0.00	0.00	0.00	0.00	0.55	0.00
time (sec)	N/A	0.351	0.161	0.000	0.000	0.000	0.000	0.000	0.229	0.000

Problem 445	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F	F(-2)	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	75	95	71	0	0	0	0	0	62	0
N.S.	1	1.27	0.95	0.00	0.00	0.00	0.00	0.00	0.83	0.00
time (sec)	N/A	0.357	0.166	0.000	0.000	0.000	0.000	0.000	0.226	0.000

Problem 446	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	55	59	55	0	0	0	0	0	0	0
N.S.	1	1.07	1.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.322	0.040	0.000	0.000	0.000	0.000	0.000	0.257	0.000

Problem 447	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	55	59	55	0	0	0	0	0	0	0
N.S.	1	1.07	1.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.306	0.030	0.000	0.000	0.000	0.000	0.000	0.214	0.000

Problem 448	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F	F	F	F	F	B
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	55	57	53	0	0	0	0	0	508	58
N.S.	1	1.04	0.96	0.00	0.00	0.00	0.00	0.00	9.24	1.05
time (sec)	N/A	0.307	0.006	0.000	0.000	0.000	0.000	0.000	0.244	8.474

Problem 449	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	52	53	49	0	0	0	0	0	107	0
N.S.	1	1.02	0.94	0.00	0.00	0.00	0.00	0.00	2.06	0.00
time (sec)	N/A	0.313	0.039	0.000	0.000	0.000	0.000	0.000	0.226	0.000

Problem 450	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	54	59	53	0	0	0	0	0	219	0
N.S.	1	1.09	0.98	0.00	0.00	0.00	0.00	0.00	4.06	0.00
time (sec)	N/A	0.314	0.033	0.000	0.000	0.000	0.000	0.000	0.222	0.000

Problem 451	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F	F	F(-1)	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	69	73	66	0	0	0	0	0	0	0
N.S.	1	1.06	0.96	0.00	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.327	0.046	0.000	0.000	0.000	0.000	0.000	0.268	0.000

Problem 452	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	68	70	66	0	0	0	0	0	1228	0
N.S.	1	1.03	0.97	0.00	0.00	0.00	0.00	0.00	18.06	0.00
time (sec)	N/A	0.336	0.039	0.000	0.000	0.000	0.000	0.000	0.255	0.000

Problem 453	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	68	70	66	0	0	0	0	0	220	0
N.S.	1	1.03	0.97	0.00	0.00	0.00	0.00	0.00	3.24	0.00
time (sec)	N/A	0.327	0.046	0.000	0.000	0.000	0.000	0.000	0.282	0.000

Problem 454	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	69	72	66	0	0	0	0	0	252	0
N.S.	1	1.04	0.96	0.00	0.00	0.00	0.00	0.00	3.65	0.00
time (sec)	N/A	0.329	0.037	0.000	0.000	0.000	0.000	0.000	0.292	0.000

Problem 455	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	63	65	61	0	0	0	0	0	0	0
N.S.	1	1.03	0.97	0.00	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.332	0.035	0.000	0.000	0.000	0.000	0.000	0.247	0.000

Problem 456	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F	A	F	F	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	226	208	110	138	0	180	0	0	162	0
N.S.	1	0.92	0.49	0.61	0.00	0.80	0.00	0.00	0.72	0.00
time (sec)	N/A	0.746	0.144	2.447	0.000	0.134	0.000	0.000	0.211	0.000

Problem 457	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F	A	F	F	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	161	156	81	92	0	131	0	0	113	0
N.S.	1	0.97	0.50	0.57	0.00	0.81	0.00	0.00	0.70	0.00
time (sec)	N/A	0.558	0.126	2.431	0.000	0.126	0.000	0.000	0.242	0.000

Problem 458	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F	A	F	F	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	100	104	56	58	0	90	0	0	77	0
N.S.	1	1.04	0.56	0.58	0.00	0.90	0.00	0.00	0.77	0.00
time (sec)	N/A	0.429	0.084	2.418	0.000	0.131	0.000	0.000	0.214	0.000

Problem 459	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F	A	F	F	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	47	46	39	44	0	58	0	0	53	0
N.S.	1	0.98	0.83	0.94	0.00	1.23	0.00	0.00	1.13	0.00
time (sec)	N/A	0.289	0.064	2.440	0.000	0.139	0.000	0.000	0.202	0.000

Problem 460	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	58	64	60	0	0	0	0	0	42	0
N.S.	1	1.10	1.03	0.00	0.00	0.00	0.00	0.00	0.72	0.00
time (sec)	N/A	0.328	0.065	0.000	0.000	0.000	0.000	0.000	0.201	0.000

Problem 461	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	60	67	62	0	0	0	0	0	85	0
N.S.	1	1.12	1.03	0.00	0.00	0.00	0.00	0.00	1.42	0.00
time (sec)	N/A	0.321	0.050	0.000	0.000	0.000	0.000	0.000	0.246	0.000

Problem 462	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	61	70	65	0	0	0	0	0	167	0
N.S.	1	1.15	1.07	0.00	0.00	0.00	0.00	0.00	2.74	0.00
time (sec)	N/A	0.334	0.058	0.000	0.000	0.000	0.000	0.000	0.211	0.000

2.3 Detailed conclusion table specific for Rubi results

The following table is specific to Rubi only. It gives additional statistics for each integral. the column **steps** is the number of steps used by Rubi to obtain the antiderivative. The **rules** column is the number of unique rules used. The **integrand size** column is the leaf size of the integrand. Finally the ratio $\frac{\text{number of rules}}{\text{integrand size}}$ is also given. The larger this ratio is, the harder the integral is to solve. In this test file, problem number [405] had the largest ratio of [.8000000000000000044]

Table 2.1: Rubi specific breakdown of results for each integral

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
1	A	7	6	1.00	17	0.353
2	A	6	5	0.95	17	0.294
3	A	5	4	0.90	15	0.267
4	A	4	3	0.82	13	0.231
5	A	4	3	1.00	17	0.176
6	A	6	5	1.00	17	0.294
7	A	1	1	1.00	17	0.059
8	A	2	2	1.00	17	0.118
9	A	3	3	1.08	17	0.176
10	A	4	4	1.12	17	0.235
11	A	7	6	0.89	17	0.353
12	A	6	5	0.85	15	0.333
13	A	5	4	0.79	13	0.308
14	A	5	4	0.86	17	0.235
15	A	5	4	0.96	17	0.235
16	A	7	6	1.02	17	0.353
17	A	7	6	0.99	17	0.353
18	A	1	1	1.00	17	0.059
19	A	2	2	1.00	17	0.118
20	A	3	3	1.08	17	0.176
21	A	4	4	1.12	17	0.235

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
22	A	8	7	0.85	17	0.412
23	A	7	6	0.81	15	0.400
24	A	6	5	0.77	13	0.385
25	A	6	5	0.81	17	0.294
26	A	6	5	0.89	17	0.294
27	A	6	5	1.08	17	0.294
28	A	8	7	0.99	17	0.412
29	A	8	7	0.99	17	0.412
30	A	8	7	0.96	17	0.412
31	A	1	1	1.00	17	0.059
32	A	2	2	1.00	17	0.118
33	A	3	3	1.08	17	0.176
34	A	4	4	1.12	17	0.235
35	A	5	4	0.81	15	0.267
36	A	5	4	0.82	15	0.267
37	A	5	4	0.80	11	0.364
38	A	7	6	1.10	17	0.353
39	A	6	5	1.07	17	0.294
40	A	5	4	1.01	17	0.235
41	A	4	3	1.00	15	0.200
42	A	3	2	1.00	13	0.154
43	A	1	1	1.00	17	0.059
44	A	2	2	1.00	17	0.118
45	A	3	3	1.08	17	0.176
46	A	4	4	1.12	17	0.235
47	A	10	9	1.09	17	0.529
48	A	7	6	0.99	17	0.353
49	A	6	5	1.06	17	0.294
50	A	4	3	1.00	17	0.176
51	A	2	2	1.00	15	0.133
52	A	1	1	0.60	13	0.077
53	A	2	2	0.74	17	0.118

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
54	A	3	3	0.87	17	0.176
55	A	4	4	0.95	17	0.235
56	A	8	7	1.00	17	0.412
57	A	7	6	1.07	17	0.353
58	A	5	4	1.03	17	0.235
59	A	1	1	1.00	17	0.059
60	A	2	2	1.13	17	0.118
61	A	2	2	0.74	15	0.133
62	A	2	2	0.61	13	0.154
63	A	3	3	0.72	17	0.176
64	A	4	4	0.81	17	0.235
65	A	5	5	0.88	17	0.294
66	A	6	5	1.04	17	0.294
67	A	8	7	1.08	17	0.412
68	A	7	6	1.08	17	0.353
69	A	6	5	1.08	17	0.294
70	A	4	3	0.88	15	0.200
71	A	3	2	0.78	13	0.154
72	A	1	1	1.10	17	0.059
73	A	3	3	1.09	17	0.176
74	A	4	4	1.06	17	0.235
75	A	5	5	1.06	17	0.294
76	A	4	3	0.97	15	0.200
77	A	4	3	0.80	13	0.231
78	A	5	4	1.00	17	0.235
79	A	4	4	1.06	21	0.190
80	A	3	3	1.03	21	0.143
81	A	2	2	0.97	21	0.095
82	A	1	1	1.00	21	0.048
83	A	4	3	1.00	21	0.143
84	A	4	3	1.00	21	0.143
85	A	5	4	0.98	21	0.190

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
86	A	6	5	1.03	21	0.238
87	A	7	6	1.06	21	0.286
88	A	4	4	1.06	21	0.190
89	A	3	3	1.03	21	0.143
90	A	2	2	0.97	21	0.095
91	A	1	1	1.00	21	0.048
92	A	5	4	1.05	21	0.190
93	A	5	4	1.06	21	0.190
94	A	5	4	1.01	21	0.190
95	A	6	5	1.02	21	0.238
96	A	7	6	1.05	21	0.286
97	A	4	4	1.10	21	0.190
98	A	3	3	1.05	21	0.143
99	A	2	2	1.00	21	0.095
100	A	1	1	1.00	21	0.048
101	A	3	2	1.00	21	0.095
102	A	4	3	1.00	21	0.143
103	A	5	4	1.01	21	0.190
104	A	6	5	1.05	21	0.238
105	A	4	4	1.08	21	0.190
106	A	3	3	1.03	21	0.143
107	A	2	2	0.97	21	0.095
108	A	1	1	1.00	21	0.048
109	A	4	3	1.00	21	0.143
110	A	5	4	1.07	21	0.190
111	A	6	5	1.04	21	0.238
112	A	7	6	1.06	21	0.286
113	A	4	4	1.03	21	0.190
114	A	3	3	1.01	21	0.143
115	A	2	2	0.97	21	0.095
116	A	1	1	1.00	21	0.048
117	A	6	5	1.49	21	0.238

Continued on next page

Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
118	A	6	5	1.12	21	0.238
119	A	7	6	1.05	21	0.286
120	A	8	7	1.07	21	0.333
121	A	3	3	1.03	21	0.143
122	A	2	2	0.97	21	0.095
123	A	1	1	1.00	21	0.048
124	A	1	1	1.00	21	0.048
125	A	2	2	1.00	21	0.095
126	A	3	3	1.07	21	0.143
127	A	4	4	0.65	21	0.190
128	A	3	3	0.64	21	0.143
129	A	2	2	0.60	21	0.095
130	A	6	5	0.61	21	0.238
131	A	7	6	0.66	21	0.286
132	A	8	7	0.66	21	0.333
133	A	3	3	1.05	21	0.143
134	A	2	2	1.00	21	0.095
135	A	1	1	1.00	21	0.048
136	A	1	1	1.00	21	0.048
137	A	2	2	1.00	21	0.095
138	A	3	3	1.07	21	0.143
139	A	4	4	0.66	21	0.190
140	A	3	3	0.66	21	0.143
141	A	2	2	0.61	21	0.095
142	A	6	5	0.61	21	0.238
143	A	7	6	0.67	21	0.286
144	A	8	7	0.67	21	0.333
145	A	12	11	1.18	17	0.647
146	A	11	10	1.18	17	0.588
147	A	6	5	0.94	15	0.333
148	A	5	4	0.90	13	0.308
149	A	8	7	1.24	17	0.412

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
150	A	8	7	1.23	17	0.412
151	A	9	8	1.20	17	0.471
152	A	10	9	1.17	17	0.529
153	A	11	10	1.17	17	0.588
154	A	13	12	1.37	17	0.706
155	A	12	11	1.41	17	0.647
156	A	6	5	0.98	15	0.333
157	A	5	4	0.95	13	0.308
158	A	9	8	1.63	17	0.471
159	A	9	8	1.67	17	0.471
160	A	10	9	1.47	17	0.529
161	A	11	10	1.36	17	0.588
162	A	12	11	1.33	17	0.647
163	A	13	12	1.16	17	0.706
164	A	7	6	0.89	15	0.400
165	A	6	5	0.87	13	0.385
166	A	10	9	1.20	17	0.529
167	A	9	8	1.18	17	0.471
168	A	9	8	1.22	17	0.471
169	A	9	8	1.19	17	0.471
170	A	10	9	1.15	17	0.529
171	A	11	10	1.16	17	0.588
172	A	12	11	1.16	17	0.647
173	A	12	11	1.42	17	0.647
174	A	11	10	1.47	17	0.588
175	A	10	9	1.55	17	0.529
176	A	5	4	1.08	15	0.267
177	A	4	3	1.15	13	0.231
178	A	9	8	1.70	17	0.471
179	A	10	9	1.49	17	0.529
180	A	11	10	1.40	17	0.588
181	A	11	10	1.22	17	0.588

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
182	A	10	9	1.23	17	0.529
183	A	9	8	1.25	17	0.471
184	A	5	4	1.05	15	0.267
185	A	4	3	1.04	13	0.231
186	A	8	7	1.26	17	0.412
187	A	9	8	1.25	17	0.471
188	A	10	9	1.22	17	0.529
189	A	12	11	1.41	17	0.647
190	A	11	10	1.48	17	0.588
191	A	10	9	1.56	17	0.529
192	A	5	4	1.10	17	0.235
193	A	5	4	1.30	15	0.267
194	A	5	4	1.13	13	0.308
195	A	11	10	1.44	17	0.588
196	A	12	11	1.38	17	0.647
197	A	13	12	1.35	17	0.706
198	A	9	8	1.22	18	0.444
199	B	9	8	2.10	18	0.444
200	A	9	8	1.70	17	0.471
201	B	9	8	2.08	19	0.421
202	A	10	9	1.58	17	0.529
203	B	10	9	2.07	17	0.529
204	A	10	9	1.58	17	0.529
205	B	10	9	3.83	17	0.529
206	B	10	9	3.63	17	0.529
207	A	10	9	1.85	17	0.529
208	A	5	4	1.15	15	0.267
209	A	5	4	0.91	15	0.267
210	A	5	4	1.32	15	0.267
211	A	4	3	1.11	15	0.200
212	A	4	3	1.11	15	0.200
213	A	5	4	1.10	15	0.267

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
214	B	9	8	2.68	15	0.533
215	A	12	11	1.44	15	0.733
216	A	13	12	1.68	19	0.632
217	A	3	3	1.05	17	0.176
218	A	3	3	1.07	17	0.176
219	A	3	3	1.11	15	0.200
220	A	2	2	1.00	17	0.118
221	A	2	2	1.00	17	0.118
222	A	2	2	1.00	17	0.118
223	A	3	3	1.09	19	0.158
224	A	3	3	1.03	19	0.158
225	A	3	3	1.03	19	0.158
226	A	3	3	1.05	19	0.158
227	A	3	3	1.09	19	0.158
228	A	3	3	1.17	15	0.200
229	B	2	2	2.08	13	0.154
230	A	1	1	1.41	11	0.091
231	A	3	3	1.05	15	0.200
232	A	3	3	1.22	15	0.200
233	A	3	3	1.16	15	0.200
234	A	2	2	1.62	15	0.133
235	B	4	3	2.07	13	0.231
236	A	3	2	1.28	11	0.182
237	A	2	2	1.74	15	0.133
238	A	2	2	1.60	15	0.133
239	A	2	2	1.56	15	0.133
240	A	2	2	1.62	15	0.133
241	A	3	3	1.49	19	0.158
242	B	4	3	2.51	17	0.176
243	A	3	2	1.54	15	0.133
244	A	3	3	1.73	19	0.158
245	A	3	3	1.56	19	0.158

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
246	A	3	3	1.45	19	0.158
247	A	3	3	1.15	19	0.158
248	A	3	3	1.15	19	0.158
249	A	3	3	1.15	19	0.158
250	A	3	3	1.15	19	0.158
251	A	2	2	1.33	19	0.105
252	A	2	2	1.36	19	0.105
253	A	2	2	1.36	19	0.105
254	A	2	2	1.33	19	0.105
255	A	3	3	1.17	17	0.176
256	A	4	4	0.95	21	0.190
257	A	3	3	0.98	21	0.143
258	A	2	2	1.00	21	0.095
259	A	1	1	1.00	21	0.048
260	A	3	3	1.27	21	0.143
261	A	3	3	1.30	19	0.158
262	A	3	3	1.29	21	0.143
263	A	1	1	1.00	13	0.077
264	A	2	2	1.58	15	0.133
265	A	5	5	1.10	19	0.263
266	A	4	4	1.08	19	0.211
267	A	3	3	1.08	17	0.176
268	A	2	2	1.00	15	0.133
269	A	1	1	1.00	19	0.053
270	A	4	3	0.96	19	0.158
271	A	4	3	0.96	19	0.158
272	A	5	4	0.95	19	0.211
273	A	6	5	1.02	19	0.263
274	A	7	6	1.06	19	0.316
275	A	6	6	1.13	19	0.316
276	A	5	5	1.13	17	0.294
277	A	4	4	1.11	15	0.267

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
278	A	3	3	1.08	19	0.158
279	A	2	2	1.00	19	0.105
280	A	1	1	1.00	19	0.053
281	A	5	4	1.00	19	0.211
282	A	5	4	1.01	19	0.211
283	A	5	4	0.95	19	0.211
284	A	6	5	0.98	19	0.263
285	A	7	6	1.03	19	0.316
286	A	7	7	1.17	17	0.412
287	A	6	6	1.15	15	0.400
288	A	5	5	1.13	19	0.263
289	A	4	4	1.11	19	0.211
290	A	3	3	1.08	19	0.158
291	A	2	2	1.00	19	0.105
292	A	1	1	1.00	19	0.053
293	A	6	5	1.00	19	0.263
294	A	6	5	1.00	19	0.263
295	A	6	5	0.95	19	0.263
296	A	6	5	0.96	19	0.263
297	A	7	6	0.99	19	0.316
298	A	8	7	1.02	19	0.368
299	A	9	8	1.05	19	0.421
300	A	4	4	1.11	19	0.211
301	A	3	3	1.04	19	0.158
302	A	2	2	0.98	19	0.105
303	A	1	1	1.00	17	0.059
304	A	3	2	0.94	15	0.133
305	A	4	3	0.96	19	0.158
306	A	5	4	0.99	19	0.211
307	A	6	5	1.04	19	0.263
308	A	4	4	1.10	19	0.211
309	A	3	3	1.05	19	0.158

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
310	A	2	2	1.02	19	0.105
311	A	1	1	1.00	19	0.053
312	A	4	3	0.96	19	0.158
313	A	5	4	1.05	17	0.235
314	A	6	5	1.04	15	0.333
315	A	7	6	1.08	19	0.316
316	A	8	7	1.10	19	0.368
317	A	4	4	1.07	19	0.211
318	A	3	3	1.08	19	0.158
319	A	2	2	1.02	19	0.105
320	A	1	1	1.00	19	0.053
321	A	5	4	1.04	19	0.211
322	A	6	5	1.08	19	0.263
323	A	7	6	1.05	19	0.316
324	A	8	7	1.08	17	0.412
325	A	9	8	1.13	23	0.348
326	A	8	7	1.12	23	0.304
327	A	7	6	1.10	23	0.261
328	A	6	5	1.05	23	0.217
329	A	5	4	1.04	23	0.174
330	A	5	4	1.04	23	0.174
331	A	1	1	1.00	23	0.043
332	A	2	2	1.00	23	0.087
333	A	3	3	1.07	23	0.130
334	A	4	4	1.11	23	0.174
335	A	11	10	1.14	23	0.435
336	A	10	9	1.13	23	0.391
337	A	9	8	1.11	23	0.348
338	A	8	7	1.08	23	0.304
339	A	7	6	1.05	23	0.261
340	A	6	5	1.02	23	0.217
341	A	6	5	1.07	23	0.217

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
342	A	6	5	1.05	23	0.217
343	A	1	1	1.00	23	0.043
344	A	2	2	1.00	23	0.087
345	A	3	3	1.07	23	0.130
346	A	4	4	1.11	23	0.174
347	A	7	6	1.07	23	0.261
348	A	6	5	1.03	23	0.217
349	A	5	4	1.01	23	0.174
350	A	4	3	1.00	23	0.130
351	A	1	1	1.00	23	0.043
352	A	2	2	1.00	23	0.087
353	A	3	3	1.07	23	0.130
354	A	4	4	1.11	23	0.174
355	A	7	6	1.06	23	0.261
356	A	6	5	1.08	23	0.217
357	A	5	4	1.04	23	0.174
358	A	1	1	1.00	23	0.043
359	A	2	2	1.00	23	0.087
360	A	3	3	1.04	23	0.130
361	A	4	4	1.10	23	0.174
362	A	5	5	1.14	23	0.217
363	A	7	6	1.10	23	0.261
364	A	6	5	1.07	23	0.217
365	A	1	1	1.00	23	0.043
366	A	2	2	1.00	23	0.087
367	A	3	3	1.05	23	0.130
368	A	4	4	1.09	23	0.174
369	A	5	5	1.11	23	0.217
370	A	6	6	1.13	23	0.261
371	A	7	7	1.15	23	0.304
372	A	4	4	0.76	19	0.211
373	A	3	3	0.75	19	0.158

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
374	A	2	2	0.68	17	0.118
375	A	1	1	1.00	15	0.067
376	A	2	2	1.00	19	0.105
377	A	3	3	1.08	19	0.158
378	A	5	5	0.80	19	0.263
379	A	4	4	0.80	19	0.211
380	A	3	3	0.73	19	0.158
381	A	1	1	1.00	19	0.053
382	A	2	2	1.00	17	0.118
383	A	3	3	1.08	15	0.200
384	A	4	4	1.12	19	0.211
385	A	5	5	1.14	19	0.263
386	A	13	12	1.43	19	0.632
387	A	12	11	1.49	19	0.579
388	A	11	10	1.58	17	0.588
389	A	10	9	1.79	15	0.600
390	B	11	10	2.09	19	0.526
391	A	12	11	1.89	19	0.579
392	A	13	12	1.72	19	0.632
393	A	8	7	1.28	19	0.368
394	A	7	6	1.29	19	0.316
395	A	6	5	1.28	19	0.263
396	A	5	4	1.39	17	0.235
397	A	6	5	1.22	15	0.333
398	A	7	6	1.19	19	0.316
399	A	8	7	1.19	19	0.368
400	A	13	12	1.66	19	0.632
401	A	12	11	1.78	19	0.579
402	A	11	10	1.98	19	0.526
403	B	11	10	2.66	19	0.526
404	B	12	11	2.31	17	0.647
405	B	13	12	2.08	15	0.800

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
406	A	14	13	1.88	19	0.684
407	A	3	3	1.05	19	0.158
408	A	3	3	1.07	19	0.158
409	A	3	3	1.11	17	0.176
410	A	2	2	1.00	19	0.105
411	A	2	2	1.00	19	0.105
412	A	2	2	1.00	19	0.105
413	B	5	5	2.32	21	0.238
414	A	4	4	1.57	21	0.190
415	A	3	3	1.05	21	0.143
416	A	4	4	1.55	21	0.190
417	B	5	5	2.18	21	0.238
418	A	3	3	1.24	17	0.176
419	A	3	3	1.43	15	0.200
420	A	3	3	1.15	13	0.231
421	A	3	3	1.09	17	0.176
422	A	3	3	1.24	17	0.176
423	A	3	3	1.21	21	0.143
424	A	3	3	1.16	21	0.143
425	A	3	3	1.16	21	0.143
426	A	3	3	1.19	21	0.143
427	A	3	3	1.12	19	0.158
428	A	4	4	0.97	23	0.174
429	A	3	3	0.99	23	0.130
430	A	2	2	1.00	23	0.087
431	A	1	1	1.00	23	0.043
432	A	3	3	1.23	23	0.130
433	A	3	3	1.22	21	0.143
434	A	3	3	1.33	23	0.130
435	A	5	4	0.95	21	0.190
436	A	5	4	0.99	21	0.190
437	A	5	4	1.08	19	0.211

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
438	A	4	3	1.00	21	0.143
439	A	4	3	1.00	21	0.143
440	A	4	3	1.00	21	0.143
441	A	3	3	1.07	23	0.130
442	A	3	3	1.00	23	0.130
443	A	3	3	1.25	23	0.130
444	A	3	3	1.18	23	0.130
445	A	3	3	1.27	23	0.130
446	A	3	3	1.07	19	0.158
447	A	3	3	1.07	17	0.176
448	A	3	3	1.04	15	0.200
449	A	3	3	1.02	19	0.158
450	A	3	3	1.09	19	0.158
451	A	3	3	1.06	23	0.130
452	A	3	3	1.03	23	0.130
453	A	3	3	1.03	23	0.130
454	A	3	3	1.04	23	0.130
455	A	3	3	1.03	21	0.143
456	A	5	5	0.92	28	0.179
457	A	4	4	0.97	28	0.143
458	A	3	3	1.04	28	0.107
459	A	2	2	0.98	28	0.071
460	A	3	3	1.10	28	0.107
461	A	3	3	1.12	26	0.115
462	A	3	3	1.15	28	0.107

CHAPTER 3

Listing of integrals

3.1 $\int x^3\sqrt{ax+bx^2}\,dx$

Optimal result

Integrand size = 17, antiderivative size = 151

$$\int x^3\sqrt{ax+bx^2}\,dx = -\frac{7a^4\sqrt{ax+bx^2}}{128b^4}+\frac{7a^3x\sqrt{ax+bx^2}}{192b^3}-\frac{7a^2x^2\sqrt{ax+bx^2}}{240b^2}+\frac{ax^3\sqrt{ax+bx^2}}{40b}+\frac{1}{5}x^4\sqrt{ax+bx^2}+\frac{7a^5\operatorname{arctanh}\left(\frac{\sqrt{b}x}{\sqrt{ax+bx^2}}\right)}{128b^{9/2}}$$

```
-7/128*a^4*(b*x^2+a*x)^(1/2)/b^4+7/192*a^3*x*(b*x^2+a*x)^(1/2)/b^3-7/240*a
^2*x^2*(b*x^2+a*x)^(1/2)/b^2+1/40*a*x^3*(b*x^2+a*x)^(1/2)/b+1/5*x^4*(b*x^2
+a*x)^(1/2)+7/128*a^5*arctanh(b^(1/2)*x/(b*x^2+a*x)^(1/2))/b^(9/2)
```

Mathematica [A] (verified)

Time = 0.47 (sec) , antiderivative size = 118, normalized size of antiderivative = 0.78

$$\int x^3\sqrt{ax+bx^2}\,dx = \frac{\sqrt{x(a+bx)}\left(\sqrt{b}(-105a^4+70a^3bx-56a^2b^2x^2+48ab^3x^3+384b^4x^4)+\frac{210a^5\operatorname{arctanh}\left(\frac{\sqrt{b}\sqrt{x}}{-\sqrt{a}+\sqrt{a+bx}}\right)}{\sqrt{x}\sqrt{a+bx}}\right)}{1920b^{9/2}}$$

```
Integrate[x^3*Sqrt[a*x + b*x^2],x]
```

```
(Sqrt[x*(a + b*x)]*(Sqrt[b]*(-105*a^4 + 70*a^3*b*x - 56*a^2*b^2*x^2 + 48*a
*b^3*x^3 + 384*b^4*x^4) + (210*a^5*ArcTanh[(Sqrt[b]*Sqrt[x])/(-Sqrt[a] + S
qrt[a + b*x])])/(Sqrt[x]*Sqrt[a + b*x])))/(1920*b^(9/2))
```

Rubi [A] (verified)

Time = 0.48 (sec) , antiderivative size = 151, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.353$, Rules used = {1134, 1134, 1160, 1087, 1091, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^3 \sqrt{ax+bx^2} \, dx$$

$$\Big\downarrow \text{1134}$$

$$\frac{x^2 \left(ax+bx^2\right)^{3/2}}{5b} - \frac{7a \int x^2 \sqrt{bx^2+ax}dx}{10b}$$

$$\Big\downarrow \text{1134}$$

$$\frac{x^2 \left(ax+bx^2\right)^{3/2}}{5b} - \frac{7a \left(\frac{x\left(ax+bx^2\right)^{3/2}}{4b} - \frac{5a \int x\sqrt{bx^2+ax}dx}{8b}\right)}{10b}$$

$$\Big\downarrow \text{1160}$$

$$\frac{x^2 \left(ax+bx^2\right)^{3/2}}{5b} - \frac{7a \left(\frac{x\left(ax+bx^2\right)^{3/2}}{4b} - \frac{5a \left(\frac{\left(ax+bx^2\right)^{3/2}}{3b} - \frac{a \int \sqrt{bx^2+ax}dx}{2b}\right)}{8b}\right)}{10b}$$

$$\Big\downarrow \text{1087}$$

$$\frac{x^2\left(ax+bx^2\right)^{3/2}}{5b}-\frac{7a\left(\frac{x\left(ax+bx^2\right)^{3/2}}{4b}-\frac{5a\left(\frac{\left(ax+bx^2\right)^{3/2}}{3b}-\frac{a\left(\frac{(a+2bx)\sqrt{ax+bx^2}}{4b}-\frac{a^2\int\frac{1}{\sqrt{bx^2+ax}}dx}{8b}\right)}{2b}\right)}{8b}\right)}{10b}$$

$$\Big\downarrow \; 1091$$

$$\frac{x^2\left(ax+bx^2\right)^{3/2}}{5b}-\frac{7a\left(\frac{x\left(ax+bx^2\right)^{3/2}}{4b}-\frac{5a\left(\frac{\left(ax+bx^2\right)^{3/2}}{3b}-\frac{a\left(\frac{(a+2bx)\sqrt{ax+bx^2}}{4b}-\frac{a^2\int\frac{1}{1-\frac{bx^2}{bx^2+ax}}d\frac{x}{\sqrt{bx^2+ax}}}{4b}\right)}{2b}\right)}{8b}\right)}{10b}$$

$$\Big\downarrow \; 219$$

$$\frac{x^2\left(ax+bx^2\right)^{3/2}}{5b}-\frac{7a\left(\frac{x\left(ax+bx^2\right)^{3/2}}{4b}-\frac{5a\left(\frac{\left(ax+bx^2\right)^{3/2}}{3b}-\frac{a\left(\frac{(a+2bx)\sqrt{ax+bx^2}}{4b}-\frac{a^2\text{arctanh}\left(\frac{\sqrt{b}x}{\sqrt{ax+bx^2}}\right)}{4b^{3/2}}\right)}{2b}\right)}{8b}\right)}{10b}$$

```
Int[x^3*Sqrt[a*x + b*x^2],x]
```

```
(x^2*(a*x + b*x^2)^(3/2))/(5*b) - (7*a*((x*(a*x + b*x^2)^(3/2))/(4*b) - (5
*a*((a*x + b*x^2)^(3/2)/(3*b) - (a*(((a + 2*b*x)*Sqrt[a*x + b*x^2])/(4*b)
- (a^2*ArcTanh[(Sqrt[b]*x)/Sqrt[a*x + b*x^2]])/(4*b^(3/2))))/(2*b)))/(8*b)
))/(10*b)
```

Defintions of rubi rules used

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(b + 2*c*x)
*((a + b*x + c*x^2)^p/(2*c*(2*p + 1))), x] - Simp[p*((b^2 - 4*a*c)/(2*c*(2*
p + 1)))   Int[(a + b*x + c*x^2)^(p - 1), x], x] /; FreeQ[{a, b, c}, x] &&
GtQ[p, 0] && (IntegerQ[4*p] || IntegerQ[3*p])
```

```
Int[1/Sqrt[(b_.)*(x_) + (c_.)*(x_)^2], x_Symbol] :> Simp[2   Subst[Int[1/(1
 - c*x^2), x], x, x/Sqrt[b*x + c*x^2]], x] /; FreeQ[{b, c}, x]
```

```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] :> Simp[e*(d + e*x)^(m - 1)*((a + b*x + c*x^2)^(p + 1)/(c*(m + 2*p +
 1))), x] + Simp[(m + p)*((2*c*d - b*e)/(c*(m + 2*p + 1)))   Int[(d + e*x)^
(m - 1)*(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, p}, x] && EqQ[
c*d^2 - b*d*e + a*e^2, 0] && GtQ[m, 1] && NeQ[m + 2*p + 1, 0] && IntegerQ[2
*p]
```

```
Int[((d_.) + (e_.)*(x_))*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol
] :> Simp[e*((a + b*x + c*x^2)^(p + 1)/(2*c*(p + 1))), x] + Simp[(2*c*d - b
*e)/(2*c)   Int[(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, p}, x]
 && NeQ[p, -1]
```

Maple [A] (verified)

Time = 0.43 (sec) , antiderivative size = 84, normalized size of antiderivative = 0.56

method	result	size
pseudoelliptic	$\frac{\frac{7\,\mathrm{arctanh}\left(\frac{\sqrt{x(bx+a)}}{x\sqrt{b}}\right)a^{5}}{128}-\frac{7\left(\sqrt{b}\,a^{4}-\frac{2b^{\frac{3}{2}}a^{3}x}{3}+\frac{8b^{\frac{5}{2}}a^{2}x^{2}}{15}-\frac{16b^{\frac{7}{2}}a\,x^{3}}{35}-\frac{128b^{\frac{9}{2}}x^{4}}{35}\right)\sqrt{x(bx+a)}}{128}}{b^{\frac{9}{2}}}$	84
risch	$-\frac{(-384b^{4}x^{4}-48a\,b^{3}x^{3}+56a^{2}b^{2}x^{2}-70a^{3}bx+105a^{4})x(bx+a)}{1920b^{4}\sqrt{x(bx+a)}}+\frac{7a^{5}\ln\left(\frac{\frac{a}{2}+bx}{\sqrt{b}}+\sqrt{b\,x^{2}+ax}\right)}{256b^{\frac{9}{2}}}$	95
default	$\frac{x^{2}\left(b\,x^{2}+ax\right)^{\frac{3}{2}}}{5b}-\frac{7a\left(\frac{x\left(b\,x^{2}+ax\right)^{\frac{3}{2}}}{4b}-\frac{5a\left(\frac{\left(b\,x^{2}+ax\right)^{\frac{3}{2}}}{3b}-\frac{a\left(\frac{(2bx+a)\sqrt{b\,x^{2}+ax}}{4b}-\frac{a^{2}\ln\left(\frac{\frac{a}{2}+bx}{\sqrt{b}}+\sqrt{b\,x^{2}+ax}\right)}{8b^{\frac{3}{2}}}\right)}{2b}\right)}{8b}\right)}{10b}$	129

```
int(x^3*(b*x^2+a*x)^(1/2),x,method=_RETURNVERBOSE)
```

```
7/128/b^(9/2)*(arctanh((x*(b*x+a))^(1/2)/x/b^(1/2))*a^5-(b^(1/2)*a^4-2/3*b
^(3/2)*a^3*x+8/15*b^(5/2)*a^2*x^2-16/35*b^(7/2)*a*x^3-128/35*b^(9/2)*x^4)*
(x*(b*x+a))^(1/2))
```

Fricas [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 193, normalized size of antiderivative = 1.28

$$\int x^3\sqrt{ax+bx^2}\,dx$$
$$= \left[\frac{105\,a^5\sqrt{b}\log\left(2\,bx+a+2\,\sqrt{bx^2+ax}\sqrt{b}\right)+2\,(384\,b^5x^4+48\,ab^4x^3-56\,a^2b^3x^2+70\,a^3b^2x-105\,a^4b)\sqrt{}}{3840\,b^5}\right.$$
$$-\frac{105\,a^5\sqrt{-b}\arctan\left(\frac{\sqrt{bx^2+ax}\sqrt{-b}}{bx+a}\right)-(384\,b^5x^4+48\,ab^4x^3-56\,a^2b^3x^2+70\,a^3b^2x-105\,a^4b)\sqrt{bx^2+ax}}{1920\,b^5}$$

```
integrate(x^3*(b*x^2+a*x)^(1/2),x, algorithm="fricas")
```

```
[1/3840*(105*a^5*sqrt(b)*log(2*b*x + a + 2*sqrt(b*x^2 + a*x)*sqrt(b)) + 2*
(384*b^5*x^4 + 48*a*b^4*x^3 - 56*a^2*b^3*x^2 + 70*a^3*b^2*x - 105*a^4*b)*s
qrt(b*x^2 + a*x))/b^5, -1/1920*(105*a^5*sqrt(-b)*arctan(sqrt(b*x^2 + a*x)*
sqrt(-b)/(b*x + a)) - (384*b^5*x^4 + 48*a*b^4*x^3 - 56*a^2*b^3*x^2 + 70*a^
3*b^2*x - 105*a^4*b)*sqrt(b*x^2 + a*x))/b^5]
```

Sympy [A] (verification not implemented)

Time = 0.25 (sec) , antiderivative size = 148, normalized size of antiderivative = 0.98

$$\int x^3\sqrt{ax+bx^2}\,dx$$
$$= \begin{cases} \frac{7a^5\left(\begin{cases}\frac{\log\left(a+2\sqrt{b}\sqrt{ax+bx^2}+2bx\right)}{\sqrt{b}} & \text{for } \frac{a^2}{b}\neq 0\\ \frac{\left(\frac{a}{2b}+x\right)\log\left(\frac{a}{2b}+x\right)}{\sqrt{b\left(\frac{a}{2b}+x\right)^2}} & \text{otherwise}\end{cases}\right)}{256b^4}+\sqrt{ax+bx^2}\left(-\frac{7a^4}{128b^4}+\frac{7a^3x}{192b^3}-\frac{7a^2x^2}{240b^2}+\frac{ax^3}{40b}+\frac{x^4}{5}\right) & \text{for } b\neq 0\\ \frac{2(ax)^{\frac{9}{2}}}{9a^4} & \text{for } a\neq 0\\ 0 & \text{otherwis} \end{cases}$$

```
integrate(x**3*(b*x**2+a*x)**(1/2),x)
```

```
Piecewise((7*a**5*Piecewise((log(a + 2*sqrt(b)*sqrt(a*x + b*x**2) + 2*b*x)
/sqrt(b), Ne(a**2/b, 0)), ((a/(2*b) + x)*log(a/(2*b) + x)/sqrt(b*(a/(2*b)
+ x)**2), True))/(256*b**4) + sqrt(a*x + b*x**2)*(-7*a**4/(128*b**4) + 7*a
**3*x/(192*b**3) - 7*a**2*x**2/(240*b**2) + a*x**3/(40*b) + x**4/5), Ne(b,
 0)), (2*(a*x)**(9/2)/(9*a**4), Ne(a, 0)), (0, True))
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 127, normalized size of antiderivative = 0.84

$$\int x^3\sqrt{ax+bx^2}\,dx = \frac{(bx^2+ax)^{\frac{3}{2}}x^2}{5\,b} - \frac{7\sqrt{bx^2+ax}a^3x}{64\,b^3} - \frac{7\,(bx^2+ax)^{\frac{3}{2}}ax}{40\,b^2} + \frac{7\,a^5\log\left(2\,bx+a+2\,\sqrt{bx^2+ax}\sqrt{b}\right)}{256\,b^{\frac{9}{2}}} - \frac{7\sqrt{bx^2+ax}a^4}{128\,b^4} + \frac{7\,(bx^2+ax)^{\frac{3}{2}}a^2}{48\,b^3}$$

```
integrate(x^3*(b*x^2+a*x)^(1/2),x, algorithm="maxima")
```

```
1/5*(b*x^2 + a*x)^(3/2)*x^2/b - 7/64*sqrt(b*x^2 + a*x)*a^3*x/b^3 - 7/40*(b
*x^2 + a*x)^(3/2)*a*x/b^2 + 7/256*a^5*log(2*b*x + a + 2*sqrt(b*x^2 + a*x)*
sqrt(b))/b^(9/2) - 7/128*sqrt(b*x^2 + a*x)*a^4/b^4 + 7/48*(b*x^2 + a*x)^(3
/2)*a^2/b^3
```

Giac [A] (verification not implemented)

Time = 0.22 (sec) , antiderivative size = 95, normalized size of antiderivative = 0.63

$$\int x^3\sqrt{ax+bx^2}\,dx = \frac{1}{1920}\sqrt{bx^2+ax}\left(2\left(4\left(6\left(8\,x+\frac{a}{b}\right)x-\frac{7\,a^2}{b^2}\right)x+\frac{35\,a^3}{b^3}\right)x-\frac{105\,a^4}{b^4}\right) - \frac{7\,a^5\log\left(\left|2\left(\sqrt{b}x-\sqrt{bx^2+ax}\right)\sqrt{b}+a\right|\right)}{256\,b^{\frac{9}{2}}}$$

```
integrate(x^3*(b*x^2+a*x)^(1/2),x, algorithm="giac")
```

```
1/1920*sqrt(b*x^2 + a*x)*(2*(4*(6*(8*x + a/b)*x - 7*a^2/b^2)*x + 35*a^3/b^
3)*x - 105*a^4/b^4) - 7/256*a^5*log(abs(2*(sqrt(b)*x - sqrt(b*x^2 + a*x))*
sqrt(b) + a))/b^(9/2)
```

Mupad [B] (verification not implemented)

Time = 9.83 (sec) , antiderivative size = 119, normalized size of antiderivative = 0.79

$$\int x^3\sqrt{ax+bx^2}\,dx = \frac{x^2\,{\left(b\,x^2+a\,x\right)}^{3/2}}{5\,b} - \frac{7\,a\,\left(\frac{x\,{\left(b\,x^2+a\,x\right)}^{3/2}}{4\,b}-\frac{5\,a\,\left(\frac{a^3\,\ln\left(\frac{a+2\,b\,x}{\sqrt{b}}+2\,\sqrt{b\,x^2+a\,x}\right)}{16\,b^{5/2}}+\frac{\sqrt{b\,x^2+a\,x}\,\left(-3\,a^2+2\,a\,b\,x+8\,b^2\,x^2\right)}{24\,b^2}\right)}{8\,b}\right)}{10\,b}$$

```
int(x^3*(a*x + b*x^2)^(1/2),x)
```

```
(x^2*(a*x + b*x^2)^(3/2))/(5*b) - (7*a*((x*(a*x + b*x^2)^(3/2))/(4*b) - (5
*a*((a^3*log((a + 2*b*x)/b^(1/2) + 2*(a*x + b*x^2)^(1/2)))/(16*b^(5/2)) +
((a*x + b*x^2)^(1/2)*(8*b^2*x^2 - 3*a^2 + 2*a*b*x))/(24*b^2)))/(8*b)))/(10
*b)
```

Reduce [B] (verification not implemented)

Time = 0.21 (sec) , antiderivative size = 114, normalized size of antiderivative = 0.75

$$\int x^3\sqrt{ax+bx^2}\,dx = \frac{-105\sqrt{x}\,\sqrt{bx+a}\,a^4b+70\sqrt{x}\,\sqrt{bx+a}\,a^3b^2x-56\sqrt{x}\,\sqrt{bx+a}\,a^2b^3x^2+48\sqrt{x}\,\sqrt{bx+a}\,a\,b^4x^3+384\sqrt{x}}{1920b^5}$$

```
int(x^3*(b*x^2+a*x)^(1/2),x)
```

```
( - 105*sqrt(x)*sqrt(a + b*x)*a**4*b + 70*sqrt(x)*sqrt(a + b*x)*a**3*b**2*
x - 56*sqrt(x)*sqrt(a + b*x)*a**2*b**3*x**2 + 48*sqrt(x)*sqrt(a + b*x)*a*b
**4*x**3 + 384*sqrt(x)*sqrt(a + b*x)*b**5*x**4 + 105*sqrt(b)*log((sqrt(a +
 b*x) + sqrt(x)*sqrt(b))/sqrt(a))*a**5)/(1920*b**5)
```

3.2 $\int x^2\sqrt{ax+bx^2}\,dx$

Optimal result

Integrand size = 17, antiderivative size = 125

$$\int x^2\sqrt{ax+bx^2}\,dx=\frac{5a^3\sqrt{ax+bx^2}}{64b^3}-\frac{5a^2x\sqrt{ax+bx^2}}{96b^2}+\frac{ax^2\sqrt{ax+bx^2}}{24b}+\frac{1}{4}x^3\sqrt{ax+bx^2}-\frac{5a^4\operatorname{arctanh}\left(\frac{\sqrt{b}x}{\sqrt{ax+bx^2}}\right)}{64b^{7/2}}$$

```
5/64*a^3*(b*x^2+a*x)^(1/2)/b^3-5/96*a^2*x*(b*x^2+a*x)^(1/2)/b^2+1/24*a*x^2
*(b*x^2+a*x)^(1/2)/b+1/4*x^3*(b*x^2+a*x)^(1/2)-5/64*a^4*arctanh(b^(1/2)*x/
(b*x^2+a*x)^(1/2))/b^(7/2)
```

Mathematica [A] (verified)

Time = 0.38 (sec) , antiderivative size = 107, normalized size of antiderivative = 0.86

$$\int x^2\sqrt{ax+bx^2}\,dx=\frac{\sqrt{x(a+bx)}\left(\sqrt{b}(15a^3-10a^2bx+8ab^2x^2+48b^3x^3)+\frac{30a^4\operatorname{arctanh}\left(\frac{\sqrt{b}\sqrt{x}}{\sqrt{a}-\sqrt{a+bx}}\right)}{\sqrt{x}\sqrt{a+bx}}\right)}{192b^{7/2}}$$

```
Integrate[x^2*Sqrt[a*x + b*x^2],x]
```

```
(Sqrt[x*(a + b*x)]*(Sqrt[b]*(15*a^3 - 10*a^2*b*x + 8*a*b^2*x^2 + 48*b^3*x^
3) + (30*a^4*ArcTanh[(Sqrt[b]*Sqrt[x])/(Sqrt[a] - Sqrt[a + b*x])])/(Sqrt[x
]*Sqrt[a + b*x])))/(192*b^(7/2))
```

Rubi [A] (verified)

Time = 0.40 (sec) , antiderivative size = 119, normalized size of antiderivative = 0.95, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.294, Rules used = {1134, 1160, 1087, 1091, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^2 \sqrt{ax+bx^2} \, dx$$

$$\Big\downarrow \text{1134}$$

$$\frac{x\left(ax+bx^2\right)^{3/2}}{4b}-\frac{5a\int x\sqrt{bx^2+ax}dx}{8b}$$

$$\Big\downarrow \text{1160}$$

$$\frac{x\left(ax+bx^2\right)^{3/2}}{4b}-\frac{5a\left(\frac{\left(ax+bx^2\right)^{3/2}}{3b}-\frac{a\int \sqrt{bx^2+ax}dx}{2b}\right)}{8b}$$

$$\Big\downarrow \text{1087}$$

$$\frac{x\left(ax+bx^2\right)^{3/2}}{4b}-\frac{5a\left(\frac{\left(ax+bx^2\right)^{3/2}}{3b}-\frac{a\left(\frac{(a+2bx)\sqrt{ax+bx^2}}{4b}-\frac{a^2\int \frac{1}{\sqrt{bx^2+ax}}dx}{8b}\right)}{2b}\right)}{8b}$$

$$\Big\downarrow \text{1091}$$

$$\frac{x\left(ax+bx^2\right)^{3/2}}{4b}-\frac{5a\left(\frac{\left(ax+bx^2\right)^{3/2}}{3b}-\frac{a\left(\frac{(a+2bx)\sqrt{ax+bx^2}}{4b}-\frac{a^2\int \frac{1}{1-\frac{bx^2}{bx^2+ax}}d\frac{x}{\sqrt{bx^2+ax}}}{4b}\right)}{2b}\right)}{8b}$$

$$\Big\downarrow \text{219}$$

$$\frac{x\left(ax+bx^2\right)^{3/2}}{4b}-\frac{5a\left(\frac{\left(ax+bx^2\right)^{3/2}}{3b}-\frac{a\left(\frac{(a+2bx)\sqrt{ax+bx^2}}{4b}-\frac{a^2\text{arctanh}\left(\frac{\sqrt{b}x}{\sqrt{ax+bx^2}}\right)}{4b^{3/2}}\right)}{2b}\right)}{8b}$$

```
Int[x^2*Sqrt[a*x + b*x^2],x]
```

```
(x*(a*x + b*x^2)^(3/2))/(4*b) - (5*a*((a*x + b*x^2)^(3/2)/(3*b) - (a*(((a
+ 2*b*x)*Sqrt[a*x + b*x^2])/(4*b) - (a^2*ArcTanh[(Sqrt[b]*x)/Sqrt[a*x + b*
x^2]])/(4*b^(3/2))))/(2*b)))/(8*b)
```

Defintions of rubi rules used

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(b + 2*c*x)
*((a + b*x + c*x^2)^p/(2*c*(2*p + 1))), x] - Simp[p*((b^2 - 4*a*c)/(2*c*(2*
p + 1)))   Int[(a + b*x + c*x^2)^(p - 1), x], x] /; FreeQ[{a, b, c}, x] &&
GtQ[p, 0] && (IntegerQ[4*p] || IntegerQ[3*p])
```

```
Int[1/Sqrt[(b_.)*(x_) + (c_.)*(x_)^2], x_Symbol] :> Simp[2   Subst[Int[1/(1
 - c*x^2), x], x, x/Sqrt[b*x + c*x^2]], x] /; FreeQ[{b, c}, x]
```

```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] :> Simp[e*(d + e*x)^(m - 1)*((a + b*x + c*x^2)^(p + 1)/(c*(m + 2*p +
 1))), x] + Simp[(m + p)*((2*c*d - b*e)/(c*(m + 2*p + 1)))   Int[(d + e*x)^
(m - 1)*(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, p}, x] && EqQ[
c*d^2 - b*d*e + a*e^2, 0] && GtQ[m, 1] && NeQ[m + 2*p + 1, 0] && IntegerQ[2
*p]
```

```
Int[((d_.) + (e_.)*(x_))*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol
] :> Simp[e*((a + b*x + c*x^2)^(p + 1)/(2*c*(p + 1))), x] + Simp[(2*c*d - b
*e)/(2*c)   Int[(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, p}, x]
 && NeQ[p, -1]
```

Maple [A] (verified)

Time = 0.38 (sec) , antiderivative size = 73, normalized size of antiderivative = 0.58

method	result	size
pseudoelliptic	$-\frac{5\left(\operatorname{arctanh}\left(\frac{\sqrt{x(bx+a)}}{x\sqrt{b}}\right)a^4-\sqrt{x(bx+a)}\left(\sqrt{b}\,a^3-\frac{2b^{\frac{3}{2}}a^2x}{3}+\frac{8a\,x^2b^{\frac{5}{2}}}{15}+\frac{16b^{\frac{7}{2}}x^3}{5}\right)\right)}{64b^{\frac{7}{2}}}$	73
risch	$\frac{(48b^3x^3+8a\,b^2x^2-10a^2bx+15a^3)x(bx+a)}{192b^3\sqrt{x(bx+a)}}-\frac{5a^4\ln\left(\frac{\frac{a}{2}+bx}{\sqrt{b}}+\sqrt{b\,x^2+ax}\right)}{128b^{\frac{7}{2}}}$	84
default	$\frac{x(b\,x^2+ax)^{\frac{3}{2}}}{4b}-\frac{5a\left(\frac{(b\,x^2+ax)^{\frac{3}{2}}}{3b}-\frac{a\left(\frac{(2bx+a)\sqrt{b\,x^2+ax}}{4b}-\frac{a^2\ln\left(\frac{\frac{a}{2}+bx}{\sqrt{b}}+\sqrt{b\,x^2+ax}\right)}{8b^{\frac{3}{2}}}\right)}{2b}\right)}{8b}$	103

```
int(x^2*(b*x^2+a*x)^(1/2),x,method=_RETURNVERBOSE)
```

```
-5/64/b^(7/2)*(arctanh((x*(b*x+a))^(1/2)/x/b^(1/2))*a^4-(x*(b*x+a))^(1/2)*
(b^(1/2)*a^3-2/3*b^(3/2)*a^2*x+8/15*a*x^2*b^(5/2)+16/5*b^(7/2)*x^3))
```

Fricas [A] (verification not implemented)

Time = 0.10 (sec) , antiderivative size = 170, normalized size of antiderivative = 1.36

$$\int x^2\sqrt{ax+bx^2}\,dx$$

$$= \left[\frac{15\,a^4\sqrt{b}\log\left(2\,bx+a-2\,\sqrt{bx^2+ax}\sqrt{b}\right)+2\,(48\,b^4x^3+8\,ab^3x^2-10\,a^2b^2x+15\,a^3b)\sqrt{bx^2+ax}}{384\,b^4},\frac{15\,a^4}{}\right.$$

```
integrate(x^2*(b*x^2+a*x)^(1/2),x, algorithm="fricas")
```

```
[1/384*(15*a^4*sqrt(b)*log(2*b*x + a - 2*sqrt(b*x^2 + a*x)*sqrt(b)) + 2*(4
8*b^4*x^3 + 8*a*b^3*x^2 - 10*a^2*b^2*x + 15*a^3*b)*sqrt(b*x^2 + a*x))/b^4,
 1/192*(15*a^4*sqrt(-b)*arctan(sqrt(b*x^2 + a*x)*sqrt(-b)/(b*x + a)) + (48
*b^4*x^3 + 8*a*b^3*x^2 - 10*a^2*b^2*x + 15*a^3*b)*sqrt(b*x^2 + a*x))/b^4]
```

Sympy [A] (verification not implemented)

Time = 0.24 (sec) , antiderivative size = 134, normalized size of antiderivative = 1.07

$$\int x^2\sqrt{ax+bx^2}\,dx$$
$$= \begin{cases} -\frac{5a^4\left(\begin{cases}\frac{\log\left(a+2\sqrt{b}\sqrt{ax+bx^2}+2bx\right)}{\sqrt{b}} & \text{for } \frac{a^2}{b}\neq 0\\ \frac{\left(\frac{a}{2b}+x\right)\log\left(\frac{a}{2b}+x\right)}{\sqrt{b\left(\frac{a}{2b}+x\right)^2}} & \text{otherwise}\end{cases}\right)}{128b^3} + \sqrt{ax+bx^2}\cdot\left(\frac{5a^3}{64b^3}-\frac{5a^2x}{96b^2}+\frac{ax^2}{24b}+\frac{x^3}{4}\right) & \text{for } b\neq 0\\ \frac{2(ax)^{\frac{7}{2}}}{7a^3} & \text{for } a\neq 0\\ 0 & \text{otherwise}\end{cases}$$

```
integrate(x**2*(b*x**2+a*x)**(1/2),x)
```

```
Piecewise((-5*a**4*Piecewise((log(a + 2*sqrt(b)*sqrt(a*x + b*x**2) + 2*b*x
)/sqrt(b), Ne(a**2/b, 0)), ((a/(2*b) + x)*log(a/(2*b) + x)/sqrt(b*(a/(2*b)
 + x)**2), True))/(128*b**3) + sqrt(a*x + b*x**2)*(5*a**3/(64*b**3) - 5*a*
*2*x/(96*b**2) + a*x**2/(24*b) + x**3/4), Ne(b, 0)), (2*(a*x)**(7/2)/(7*a*
*3), Ne(a, 0)), (0, True))
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 105, normalized size of antiderivative = 0.84

$$\int x^2\sqrt{ax+bx^2}\,dx = \frac{5\sqrt{bx^2+ax}a^2x}{32\,b^2} + \frac{(bx^2+ax)^{\frac{3}{2}}x}{4\,b} - \frac{5\,a^4\log\left(2\,bx+a+2\sqrt{bx^2+ax}\sqrt{b}\right)}{128\,b^{\frac{7}{2}}} + \frac{5\sqrt{bx^2+ax}a^3}{64\,b^3} - \frac{5\,(bx^2+ax)^{\frac{3}{2}}a}{24\,b^2}$$

```
integrate(x^2*(b*x^2+a*x)^(1/2),x, algorithm="maxima")
```

```
5/32*sqrt(b*x^2 + a*x)*a^2*x/b^2 + 1/4*(b*x^2 + a*x)^(3/2)*x/b - 5/128*a^4
*log(2*b*x + a + 2*sqrt(b*x^2 + a*x)*sqrt(b))/b^(7/2) + 5/64*sqrt(b*x^2 +
a*x)*a^3/b^3 - 5/24*(b*x^2 + a*x)^(3/2)*a/b^2
```

Giac [A] (verification not implemented)

Time = 0.22 (sec) , antiderivative size = 83, normalized size of antiderivative = 0.66

$$\int x^2\sqrt{ax+bx^2}\,dx = \frac{1}{192}\sqrt{bx^2+ax}\left(2\left(4\left(6\,x+\frac{a}{b}\right)x-\frac{5\,a^2}{b^2}\right)x+\frac{15\,a^3}{b^3}\right) + \frac{5\,a^4\log\left(\left|2\left(\sqrt{b}x-\sqrt{bx^2+ax}\right)\sqrt{b}+a\right|\right)}{128\,b^{\frac{7}{2}}}$$

```
integrate(x^2*(b*x^2+a*x)^(1/2),x, algorithm="giac")
```

```
1/192*sqrt(b*x^2 + a*x)*(2*(4*(6*x + a/b)*x - 5*a^2/b^2)*x + 15*a^3/b^3) +
 5/128*a^4*log(abs(2*(sqrt(b)*x - sqrt(b*x^2 + a*x))*sqrt(b) + a))/b^(7/2)
```

Mupad [B] (verification not implemented)

Time = 0.21 (sec) , antiderivative size = 93, normalized size of antiderivative = 0.74

$$\int x^2\sqrt{ax+bx^2}\,dx = \frac{x\,{\left(b\,x^2+a\,x\right)}^{3/2}}{4\,b}-\frac{5\,a\,\left(\frac{a^3\,\ln\left(\frac{a+2\,b\,x}{\sqrt{b}}+2\,\sqrt{b\,x^2+a\,x}\right)}{16\,b^{5/2}}+\frac{\sqrt{b\,x^2+a\,x}\,\left(-3\,a^2+2\,a\,b\,x+8\,b^2\,x^2\right)}{24\,b^2}\right)}{8\,b}$$

```
int(x^2*(a*x + b*x^2)^(1/2),x)
```

```
(x*(a*x + b*x^2)^(3/2))/(4*b) - (5*a*((a^3*log((a + 2*b*x)/b^(1/2) + 2*(a*
x + b*x^2)^(1/2)))/(16*b^(5/2)) + ((a*x + b*x^2)^(1/2)*(8*b^2*x^2 - 3*a^2
+ 2*a*b*x))/(24*b^2)))/(8*b)
```

Reduce [B] (verification not implemented)

Time = 0.22 (sec) , antiderivative size = 95, normalized size of antiderivative = 0.76

$$\int x^2\sqrt{ax+bx^2}\,dx$$

$$= \frac{15\sqrt{x}\,\sqrt{bx+a}\,a^3b-10\sqrt{x}\,\sqrt{bx+a}\,a^2b^2x+8\sqrt{x}\,\sqrt{bx+a}\,a\,b^3x^2+48\sqrt{x}\,\sqrt{bx+a}\,b^4x^3-15\sqrt{b}\log\left(\frac{\sqrt{bx}\cdots}{}\right.}{192b^4}$$

```
int(x^2*(b*x^2+a*x)^(1/2),x)
```

```
(15*sqrt(x)*sqrt(a + b*x)*a**3*b - 10*sqrt(x)*sqrt(a + b*x)*a**2*b**2*x +
8*sqrt(x)*sqrt(a + b*x)*a*b**3*x**2 + 48*sqrt(x)*sqrt(a + b*x)*b**4*x**3 -
 15*sqrt(b)*log((sqrt(a + b*x) + sqrt(x)*sqrt(b))/sqrt(a))*a**4)/(192*b**4
)
```

3.3 $\int x\sqrt{ax+bx^2}\,dx$

Optimal result

Integrand size = 15, antiderivative size = 99

$$\int x\sqrt{ax+bx^2}\,dx = -\frac{a^2\sqrt{ax+bx^2}}{8b^2}+\frac{ax\sqrt{ax+bx^2}}{12b}+\frac{1}{3}x^2\sqrt{ax+bx^2}+\frac{a^3\text{arctanh}\left(\frac{\sqrt{b}x}{\sqrt{ax+bx^2}}\right)}{8b^{5/2}}$$

```
-1/8*a^2*(b*x^2+a*x)^(1/2)/b^2+1/12*a*x*(b*x^2+a*x)^(1/2)/b+1/3*x^2*(b*x^2
+a*x)^(1/2)+1/8*a^3*arctanh(b^(1/2)*x/(b*x^2+a*x)^(1/2))/b^(5/2)
```

Mathematica [A] (verified)

Time = 0.30 (sec) , antiderivative size = 96, normalized size of antiderivative = 0.97

$$\int x\sqrt{ax+bx^2}\,dx = \frac{\sqrt{x(a+bx)}\left(\sqrt{b}(-3a^2+2abx+8b^2x^2)+\frac{6a^3\text{arctanh}\left(\frac{\sqrt{b}\sqrt{x}}{-\sqrt{a}+\sqrt{a+bx}}\right)}{\sqrt{x}\sqrt{a+bx}}\right)}{24b^{5/2}}$$

```
Integrate[x*Sqrt[a*x + b*x^2],x]
```

```
(Sqrt[x*(a + b*x)]*(Sqrt[b]*(-3*a^2 + 2*a*b*x + 8*b^2*x^2) + (6*a^3*ArcTan
h[(Sqrt[b]*Sqrt[x])/(-Sqrt[a] + Sqrt[a + b*x])])/(Sqrt[x]*Sqrt[a + b*x])))
/(24*b^(5/2))
```

Rubi [A] (verified)

Time = 0.34 (sec) , antiderivative size = 89, normalized size of antiderivative = 0.90, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.267$, Rules used = {1160, 1087, 1091, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x\sqrt{ax+bx^2}\,dx$$

$$\Big\downarrow \text{1160}$$

$$\frac{(ax+bx^2)^{3/2}}{3b}-\frac{a\int \sqrt{bx^2+ax}dx}{2b}$$

$$\Big\downarrow \text{1087}$$

$$\frac{(ax+bx^2)^{3/2}}{3b}-\frac{a\left(\frac{(a+2bx)\sqrt{ax+bx^2}}{4b}-\frac{a^2\int \frac{1}{\sqrt{bx^2+ax}}dx}{8b}\right)}{2b}$$

$$\Big\downarrow \text{1091}$$

$$\frac{(ax+bx^2)^{3/2}}{3b}-\frac{a\left(\frac{(a+2bx)\sqrt{ax+bx^2}}{4b}-\frac{a^2\int \frac{1}{1-\frac{bx^2}{bx^2+ax}}d\frac{x}{\sqrt{bx^2+ax}}}{4b}\right)}{2b}$$

$$\Big\downarrow \text{219}$$

$$\frac{(ax+bx^2)^{3/2}}{3b}-\frac{a\left(\frac{(a+2bx)\sqrt{ax+bx^2}}{4b}-\frac{a^2\text{arctanh}\left(\frac{\sqrt{b}x}{\sqrt{ax+bx^2}}\right)}{4b^{3/2}}\right)}{2b}$$

```
Int[x*Sqrt[a*x + b*x^2],x]
```

```
(a*x + b*x^2)^(3/2)/(3*b) - (a*(((a + 2*b*x)*Sqrt[a*x + b*x^2])/(4*b) - (a
^2*ArcTanh[(Sqrt[b]*x)/Sqrt[a*x + b*x^2]])/(4*b^(3/2))))/(2*b)
```

Defintions of rubi rules used

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(b + 2*c*x)
*((a + b*x + c*x^2)^p/(2*c*(2*p + 1))), x] - Simp[p*((b^2 - 4*a*c)/(2*c*(2*
p + 1)))   Int[(a + b*x + c*x^2)^(p - 1), x], x] /; FreeQ[{a, b, c}, x] &&
GtQ[p, 0] && (IntegerQ[4*p] || IntegerQ[3*p])
```

```
Int[1/Sqrt[(b_.)*(x_) + (c_.)*(x_)^2], x_Symbol] :> Simp[2   Subst[Int[1/(1
 - c*x^2), x], x, x/Sqrt[b*x + c*x^2]], x] /; FreeQ[{b, c}, x]
```

```
Int[((d_.) + (e_.)*(x_))*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol
] :> Simp[e*((a + b*x + c*x^2)^(p + 1)/(2*c*(p + 1))), x] + Simp[(2*c*d - b
*e)/(2*c)   Int[(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, p}, x]
 && NeQ[p, -1]
```

Maple [A] (verified)

Time = 0.36 (sec) , antiderivative size = 73, normalized size of antiderivative = 0.74

method	result	size
risch	$-\frac{\left(-8 b^{2} x^{2}-2 a b x+3 a^{2}\right) x \left(b x+a \right)}{24 b^{2} \sqrt{x \left(b x+a \right)}}+\frac{a^{3} \ln \left(\frac{\frac{a}{2}+b x}{\sqrt{b}}+\sqrt{b \,x^{2}+a x}\right)}{16 b^{\frac{5}{2}}}$	73
default	$\frac{\left(b \,x^{2}+a x \right)^{\frac{3}{2}}}{3 b}-\frac{a \left(\frac{\left(2 b x+a \right) \sqrt{b \,x^{2}+a x}}{4 b}-\frac{a^{2} \ln \left(\frac{\frac{a}{2}+b x}{\sqrt{b}}+\sqrt{b \,x^{2}+a x}\right)}{8 b^{\frac{3}{2}}}\right)}{2 b}$	79
pseudoelliptic	$\frac{8 b^{\frac{5}{2}} \sqrt{x \left(b x+a \right)}\, x^{2}+2 a \,b^{\frac{3}{2}} x \sqrt{x \left(b x+a \right)}+3 \,\operatorname{arctanh}\left(\frac{\sqrt{x \left(b x+a \right)}}{x \sqrt{b}}\right) a^{3}-3 a^{2} \sqrt{b}\, \sqrt{x \left(b x+a \right)}}{24 b^{\frac{5}{2}}}$	79

```
int(x*(b*x^2+a*x)^(1/2),x,method=_RETURNVERBOSE)
```

```
-1/24*(-8*b^2*x^2-2*a*b*x+3*a^2)*x*(b*x+a)/b^2/(x*(b*x+a))^(1/2)+1/16*a^3/
b^(5/2)*ln((1/2*a+b*x)/b^(1/2)+(b*x^2+a*x)^(1/2))
```

Fricas [A] (verification not implemented)

Time = 0.10 (sec) , antiderivative size = 149, normalized size of antiderivative = 1.51

$$\int x\sqrt{ax+bx^2}\,dx = \left[\frac{3\,a^3\sqrt{b}\log\left(2\,bx+a+2\,\sqrt{bx^2+ax}\sqrt{b}\right)+2\,(8\,b^3x^2+2\,ab^2x-3\,a^2b)\sqrt{bx^2+ax}}{48\,b^3},\right.$$
$$\left. -\frac{3\,a^3\sqrt{-b}\arctan\left(\frac{\sqrt{bx^2+ax}\sqrt{-b}}{bx+a}\right)-(8\,b^3x^2+2\,ab^2x-3\,a^2b)\sqrt{bx^2+ax}}{24\,b^3}\right]$$

```
integrate(x*(b*x^2+a*x)^(1/2),x, algorithm="fricas")
```

```
[1/48*(3*a^3*sqrt(b)*log(2*b*x + a + 2*sqrt(b*x^2 + a*x)*sqrt(b)) + 2*(8*b
^3*x^2 + 2*a*b^2*x - 3*a^2*b)*sqrt(b*x^2 + a*x))/b^3, -1/24*(3*a^3*sqrt(-b
)*arctan(sqrt(b*x^2 + a*x)*sqrt(-b)/(b*x + a)) - (8*b^3*x^2 + 2*a*b^2*x -
3*a^2*b)*sqrt(b*x^2 + a*x))/b^3]
```

Sympy [A] (verification not implemented)

Time = 0.24 (sec) , antiderivative size = 117, normalized size of antiderivative = 1.18

$$\int x\sqrt{ax+bx^2}\,dx = \begin{cases} \frac{a^3\left(\begin{cases} \frac{\log\left(a+2\sqrt{b}\sqrt{ax+bx^2}+2bx\right)}{\sqrt{b}} & \text{for } \frac{a^2}{b}\neq 0 \\ \frac{\left(\frac{a}{2b}+x\right)\log\left(\frac{a}{2b}+x\right)}{\sqrt{b\left(\frac{a}{2b}+x\right)^2}} & \text{otherwise} \end{cases}\right)}{16b^2} + \sqrt{ax+bx^2}\left(-\frac{a^2}{8b^2}+\frac{ax}{12b}+\frac{x^2}{3}\right) & \text{for } b\neq 0 \\ \frac{2(ax)^{\frac{5}{2}}}{5a^2} & \text{for } a\neq 0 \\ 0 & \text{otherwise} \end{cases}$$

```
integrate(x*(b*x**2+a*x)**(1/2),x)
```

```
Piecewise((a**3*Piecewise((log(a + 2*sqrt(b)*sqrt(a*x + b*x**2) + 2*b*x)/s
qrt(b), Ne(a**2/b, 0)), ((a/(2*b) + x)*log(a/(2*b) + x)/sqrt(b*(a/(2*b) +
x)**2), True))/(16*b**2) + sqrt(a*x + b*x**2)*(-a**2/(8*b**2) + a*x/(12*b)
 + x**2/3), Ne(b, 0)), (2*(a*x)**(5/2)/(5*a**2), Ne(a, 0)), (0, True))
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 85, normalized size of antiderivative = 0.86

$$\int x\sqrt{ax+bx^2}\,dx = -\frac{\sqrt{bx^2+ax}ax}{4\,b} + \frac{a^3\log\left(2\,bx+a+2\,\sqrt{bx^2+ax}\sqrt{b}\right)}{16\,b^{\frac{5}{2}}} - \frac{\sqrt{bx^2+ax}a^2}{8\,b^2} + \frac{(bx^2+ax)^{\frac{3}{2}}}{3\,b}$$

```
integrate(x*(b*x^2+a*x)^(1/2),x, algorithm="maxima")
```

```
-1/4*sqrt(b*x^2 + a*x)*a*x/b + 1/16*a^3*log(2*b*x + a + 2*sqrt(b*x^2 + a*x
)*sqrt(b))/b^(5/2) - 1/8*sqrt(b*x^2 + a*x)*a^2/b^2 + 1/3*(b*x^2 + a*x)^(3/
2)/b
```

Giac [A] (verification not implemented)

Time = 0.17 (sec) , antiderivative size = 71, normalized size of antiderivative = 0.72

$$\int x\sqrt{ax+bx^2}\,dx = \frac{1}{24}\sqrt{bx^2+ax}\left(2\left(4x+\frac{a}{b}\right)x-\frac{3a^2}{b^2}\right) - \frac{a^3\log\left(\left|2\left(\sqrt{b}x-\sqrt{bx^2+ax}\right)\sqrt{b}+a\right|\right)}{16b^{\frac{5}{2}}}$$

```
integrate(x*(b*x^2+a*x)^(1/2),x, algorithm="giac")
```

```
1/24*sqrt(b*x^2 + a*x)*(2*(4*x + a/b)*x - 3*a^2/b^2) - 1/16*a^3*log(abs(2*
(sqrt(b)*x - sqrt(b*x^2 + a*x))*sqrt(b) + a))/b^(5/2)
```

Mupad [B] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 69, normalized size of antiderivative = 0.70

$$\int x\sqrt{ax+bx^2}\,dx = \frac{a^3\ln\left(\frac{a+2bx}{\sqrt{b}}+2\sqrt{bx^2+ax}\right)}{16b^{5/2}} + \frac{\sqrt{bx^2+ax}\left(-3a^2+2abx+8b^2x^2\right)}{24b^2}$$

```
int(x*(a*x + b*x^2)^(1/2),x)
```

```
(a^3*log((a + 2*b*x)/b^(1/2) + 2*(a*x + b*x^2)^(1/2)))/(16*b^(5/2)) + ((a*
x + b*x^2)^(1/2)*(8*b^2*x^2 - 3*a^2 + 2*a*b*x))/(24*b^2)
```

Reduce [B] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 76, normalized size of antiderivative = 0.77

$$\int x\sqrt{ax+bx^2}\,dx$$
$$=\frac{-3\sqrt{x}\,\sqrt{bx+a}\,a^2b+2\sqrt{x}\,\sqrt{bx+a}\,a\,b^2x+8\sqrt{x}\,\sqrt{bx+a}\,b^3x^2+3\sqrt{b}\log\left(\frac{\sqrt{bx+a}+\sqrt{x}\,\sqrt{b}}{\sqrt{a}}\right)a^3}{24b^3}$$

```
int(x*(b*x^2+a*x)^(1/2),x)
```

```
( - 3*sqrt(x)*sqrt(a + b*x)*a**2*b + 2*sqrt(x)*sqrt(a + b*x)*a*b**2*x + 8*
sqrt(x)*sqrt(a + b*x)*b**3*x**2 + 3*sqrt(b)*log((sqrt(a + b*x) + sqrt(x)*s
qrt(b))/sqrt(a))*a**3)/(24*b**3)
```

3.4 $\int \sqrt{ax+bx^2}\,dx$

Optimal result

Integrand size = 13, antiderivative size = 73

$$\int \sqrt{ax+bx^2}\,dx=\frac{a\sqrt{ax+bx^2}}{4b}+\frac{1}{2}x\sqrt{ax+bx^2}-\frac{a^2\text{arctanh}\left(\frac{\sqrt{b}x}{\sqrt{ax+bx^2}}\right)}{4b^{3/2}}$$

```
1/4*a*(b*x^2+a*x)^(1/2)/b+1/2*x*(b*x^2+a*x)^(1/2)-1/4*a^2*arctanh(b^(1/2)*
x/(b*x^2+a*x)^(1/2))/b^(3/2)
```

Mathematica [A] (verified)

Time = 0.23 (sec) , antiderivative size = 83, normalized size of antiderivative = 1.14

$$\int \sqrt{ax+bx^2}\,dx=\frac{\sqrt{x(a+bx)}\left(\sqrt{b}(a+2bx)+\frac{2a^2\text{arctanh}\left(\frac{\sqrt{b}\sqrt{x}}{\sqrt{a}-\sqrt{a+bx}}\right)}{\sqrt{x}\sqrt{a+bx}}\right)}{4b^{3/2}}$$

```
Integrate[Sqrt[a*x + b*x^2],x]
```

```
(Sqrt[x*(a + b*x)]*(Sqrt[b]*(a + 2*b*x) + (2*a^2*ArcTanh[(Sqrt[b]*Sqrt[x])
/(Sqrt[a] - Sqrt[a + b*x])])/(Sqrt[x]*Sqrt[a + b*x])))/(4*b^(3/2))
```

Rubi [A] (verified)

Time = 0.29 (sec) , antiderivative size = 60, normalized size of antiderivative = 0.82, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.231, Rules used = {1087, 1091, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sqrt{ax+bx^2}\,dx$$

$$\Big\downarrow \text{1087}$$

$$\frac{(a+2bx)\sqrt{ax+bx^2}}{4b}-\frac{a^2\int \frac{1}{\sqrt{bx^2+ax}}dx}{8b}$$

$$\Big\downarrow \text{1091}$$

$$\frac{(a+2bx)\sqrt{ax+bx^2}}{4b}-\frac{a^2\int \frac{1}{1-\frac{bx^2}{bx^2+ax}}d\frac{x}{\sqrt{bx^2+ax}}}{4b}$$

$$\Big\downarrow \text{219}$$

$$\frac{(a+2bx)\sqrt{ax+bx^2}}{4b}-\frac{a^2\text{arctanh}\left(\frac{\sqrt{b}x}{\sqrt{ax+bx^2}}\right)}{4b^{3/2}}$$

```
Int[Sqrt[a*x + b*x^2],x]
```

```
((a + 2*b*x)*Sqrt[a*x + b*x^2])/(4*b) - (a^2*ArcTanh[(Sqrt[b]*x)/Sqrt[a*x
+ b*x^2]])/(4*b^(3/2))
```

Defintions of rubi rules used

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(b + 2*c*x)
*((a + b*x + c*x^2)^p/(2*c*(2*p + 1))), x] - Simp[p*((b^2 - 4*a*c)/(2*c*(2*
p + 1)))   Int[(a + b*x + c*x^2)^(p - 1), x], x] /; FreeQ[{a, b, c}, x] &&
GtQ[p, 0] && (IntegerQ[4*p] || IntegerQ[3*p])
```

```
Int[1/Sqrt[(b_.)*(x_) + (c_.)*(x_)^2], x_Symbol] :> Simp[2   Subst[Int[1/(1
 - c*x^2), x], x, x/Sqrt[b*x + c*x^2]], x] /; FreeQ[{b, c}, x]
```

Maple [A] (verified)

Time = 0.36 (sec) , antiderivative size = 56, normalized size of antiderivative = 0.77

method	result	size
default	$\frac{(2bx+a)\sqrt{b\,x^{2}+ax}}{4b}-\frac{a^{2}\ln\left(\frac{\frac{a}{2}+bx}{\sqrt{b}}+\sqrt{b\,x^{2}+ax}\right)}{8b^{\frac{3}{2}}}$	56
pseudoelliptic	$\frac{2b^{\frac{3}{2}}\sqrt{x(bx+a)}\,x+a\sqrt{b}\,\sqrt{x(bx+a)}-\operatorname{arctanh}\left(\frac{\sqrt{x(bx+a)}}{x\sqrt{b}}\right)a^{2}}{4b^{\frac{3}{2}}}$	58
risch	$\frac{(2bx+a)x(bx+a)}{4b\sqrt{x(bx+a)}}-\frac{a^{2}\ln\left(\frac{\frac{a}{2}+bx}{\sqrt{b}}+\sqrt{b\,x^{2}+ax}\right)}{8b^{\frac{3}{2}}}$	60

```
int((b*x^2+a*x)^(1/2),x,method=_RETURNVERBOSE)
```

```
1/4*(2*b*x+a)/b*(b*x^2+a*x)^(1/2)-1/8*a^2/b^(3/2)*ln((1/2*a+b*x)/b^(1/2)+(
b*x^2+a*x)^(1/2))
```

Fricas [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 122, normalized size of antiderivative = 1.67

$$\int \sqrt{ax+bx^2}\,dx$$

$$= \left[\frac{a^2\sqrt{b}\log\left(2\,bx+a-2\,\sqrt{bx^2+ax}\sqrt{b}\right)+2\,(2\,b^2x+ab)\sqrt{bx^2+ax}}{8\,b^2}, \frac{a^2\sqrt{-b}\arctan\left(\frac{\sqrt{bx^2+ax}\sqrt{-b}}{bx+a}\right)+(2\,b}{4\,b^2}\right.$$

```
integrate((b*x^2+a*x)^(1/2),x, algorithm="fricas")
```

```
[1/8*(a^2*sqrt(b)*log(2*b*x + a - 2*sqrt(b*x^2 + a*x)*sqrt(b)) + 2*(2*b^2*
x + a*b)*sqrt(b*x^2 + a*x))/b^2, 1/4*(a^2*sqrt(-b)*arctan(sqrt(b*x^2 + a*x
)*sqrt(-b)/(b*x + a)) + (2*b^2*x + a*b)*sqrt(b*x^2 + a*x))/b^2]
```

Sympy [A] (verification not implemented)

Time = 0.21 (sec) , antiderivative size = 102, normalized size of antiderivative = 1.40

$$\int \sqrt{ax+bx^2}\,dx = \begin{cases} -\frac{a^2\left(\begin{cases} \frac{\log\left(a+2\sqrt{b}\sqrt{ax+bx^2}+2bx\right)}{\sqrt{b}} & \text{for } \frac{a^2}{b} \neq 0 \\ \frac{\left(\frac{a}{2b}+x\right)\log\left(\frac{a}{2b}+x\right)}{\sqrt{b\left(\frac{a}{2b}+x\right)^2}} & \text{otherwise} \end{cases}\right)}{8b} + \left(\frac{a}{4b}+\frac{x}{2}\right)\sqrt{ax+bx^2} & \text{for } b \neq 0 \\ \frac{2(ax)^{\frac{3}{2}}}{3a} & \text{for } a \neq 0 \\ 0 & \text{otherwise} \end{cases}$$

```
integrate((b*x**2+a*x)**(1/2),x)
```

```
Piecewise((-a**2*Piecewise((log(a + 2*sqrt(b)*sqrt(a*x + b*x**2) + 2*b*x)/
sqrt(b), Ne(a**2/b, 0)), ((a/(2*b) + x)*log(a/(2*b) + x)/sqrt(b*(a/(2*b) +
 x)**2), True))/(8*b) + (a/(4*b) + x/2)*sqrt(a*x + b*x**2), Ne(b, 0)), (2*
(a*x)**(3/2)/(3*a), Ne(a, 0)), (0, True))
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 63, normalized size of antiderivative = 0.86

$$\int \sqrt{ax+bx^2}\,dx = \frac{1}{2}\sqrt{bx^2+ax}x - \frac{a^2\log\left(2\,bx+a+2\sqrt{bx^2+ax}\sqrt{b}\right)}{8\,b^{\frac{3}{2}}} + \frac{\sqrt{bx^2+ax}a}{4\,b}$$

```
integrate((b*x^2+a*x)^(1/2),x, algorithm="maxima")
```

```
1/2*sqrt(b*x^2 + a*x)*x - 1/8*a^2*log(2*b*x + a + 2*sqrt(b*x^2 + a*x)*sqrt
(b))/b^(3/2) + 1/4*sqrt(b*x^2 + a*x)*a/b
```

Giac [A] (verification not implemented)

Time = 0.14 (sec) , antiderivative size = 59, normalized size of antiderivative = 0.81

$$\int \sqrt{ax+bx^2}\,dx = \frac{1}{4}\sqrt{bx^2+ax}\left(2\,x+\frac{a}{b}\right) + \frac{a^2\log\left(\left|2\left(\sqrt{b}x-\sqrt{bx^2+ax}\right)\sqrt{b}+a\right|\right)}{8\,b^{\frac{3}{2}}}$$

```
integrate((b*x^2+a*x)^(1/2),x, algorithm="giac")
```

```
1/4*sqrt(b*x^2 + a*x)*(2*x + a/b) + 1/8*a^2*log(abs(2*(sqrt(b)*x - sqrt(b*
x^2 + a*x))*sqrt(b) + a))/b^(3/2)
```

Mupad [B] (verification not implemented)

Time = 9.51 (sec) , antiderivative size = 55, normalized size of antiderivative = 0.75

$$\int \sqrt{ax+bx^2}\,dx = \sqrt{b\,x^2+a\,x}\left(\frac{x}{2}+\frac{a}{4\,b}\right) - \frac{a^2\,\ln\left(\frac{\frac{a}{2}+b\,x}{\sqrt{b}}+\sqrt{b\,x^2+a\,x}\right)}{8\,b^{3/2}}$$

```
int((a*x + b*x^2)^(1/2),x)
```

```
(a*x + b*x^2)^(1/2)*(x/2 + a/(4*b)) - (a^2*log((a/2 + b*x)/b^(1/2) + (a*x
+ b*x^2)^(1/2)))/(8*b^(3/2))
```

Reduce [B] (verification not implemented)

Time = 0.21 (sec) , antiderivative size = 56, normalized size of antiderivative = 0.77

$$\int \sqrt{ax+bx^2}\,dx = \frac{\sqrt{x}\,\sqrt{bx+a}\,ab+2\sqrt{x}\,\sqrt{bx+a}\,b^2x-\sqrt{b}\log\left(\frac{\sqrt{bx+a}+\sqrt{x}\,\sqrt{b}}{\sqrt{a}}\right)a^2}{4b^2}$$

```
int((b*x^2+a*x)^(1/2),x)
```

```
(sqrt(x)*sqrt(a + b*x)*a*b + 2*sqrt(x)*sqrt(a + b*x)*b**2*x - sqrt(b)*log(
(sqrt(a + b*x) + sqrt(x)*sqrt(b))/sqrt(a))*a**2)/(4*b**2)
```

3.5 $\int \frac{\sqrt{ax+bx^2}}{x}\,dx$

Optimal result

Integrand size = 17, antiderivative size = 42

$$\int \frac{\sqrt{ax+bx^2}}{x}\,dx=\sqrt{ax+bx^2}+\frac{a\operatorname{arctanh}\left(\frac{\sqrt{b}x}{\sqrt{ax+bx^2}}\right)}{\sqrt{b}}$$

```
(b*x^2+a*x)^(1/2)+a*arctanh(b^(1/2)*x/(b*x^2+a*x)^(1/2))/b^(1/2)
```

Mathematica [A] (verified)

Time = 0.06 (sec) , antiderivative size = 59, normalized size of antiderivative = 1.40

$$\int \frac{\sqrt{ax+bx^2}}{x}\,dx=\sqrt{x(a+bx)}\left(1-\frac{a\log\left(-\sqrt{b}\sqrt{x}+\sqrt{a+bx}\right)}{\sqrt{b}\sqrt{x}\sqrt{a+bx}}\right)$$

```
Integrate[Sqrt[a*x + b*x^2]/x,x]
```

```
Sqrt[x*(a + b*x)]*(1 - (a*Log[-(Sqrt[b]*Sqrt[x]) + Sqrt[a + b*x]])/(Sqrt[b
]*Sqrt[x]*Sqrt[a + b*x]))
```

Rubi [A] (verified)

Time = 0.28 (sec) , antiderivative size = 42, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.176$, Rules used = {1131, 1091, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sqrt{ax+bx^2}}{x} \, dx$$

$$\Big\downarrow \text{1131}$$

$$\frac{1}{2}a\int \frac{1}{\sqrt{bx^2+ax}}dx+\sqrt{ax+bx^2}$$

$$\Big\downarrow \text{1091}$$

$$a\int \frac{1}{1-\frac{bx^2}{bx^2+ax}}d\frac{x}{\sqrt{bx^2+ax}}+\sqrt{ax+bx^2}$$

$$\Big\downarrow \text{219}$$

$$\frac{a\text{arctanh}\left(\frac{\sqrt{b}x}{\sqrt{ax+bx^2}}\right)}{\sqrt{b}}+\sqrt{ax+bx^2}$$

```
Int[Sqrt[a*x + b*x^2]/x,x]
```

```
Sqrt[a*x + b*x^2] + (a*ArcTanh[(Sqrt[b]*x)/Sqrt[a*x + b*x^2]])/Sqrt[b]
```

Defintions of rubi rules used

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

```
Int[1/Sqrt[(b_.)*(x_) + (c_.)*(x_)^2], x_Symbol] :> Simp[2   Subst[Int[1/(1
 - c*x^2), x], x, x/Sqrt[b*x + c*x^2]], x] /; FreeQ[{b, c}, x]
```

```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] :> Simp[(d + e*x)^(m + 1)*((a + b*x + c*x^2)^p/(e*(m + 2*p + 1))), x
] - Simp[p*((2*c*d - b*e)/(e^2*(m + 2*p + 1)))   Int[(d + e*x)^(m + 1)*(a +
 b*x + c*x^2)^(p - 1), x], x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[c*d^2 - b
*d*e + a*e^2, 0] && GtQ[p, 0] && (LeQ[-2, m, 0] || EqQ[m + p + 1, 0]) && Ne
Q[m + 2*p + 1, 0] && IntegerQ[2*p]
```

Maple [A] (verified)

Time = 0.35 (sec) , antiderivative size = 38, normalized size of antiderivative = 0.90

method	result	size
pseudoelliptic	$\frac{\sqrt{x(bx+a)}\,\sqrt{b}+\operatorname{arctanh}\left(\frac{\sqrt{x(bx+a)}}{x\sqrt{b}}\right)a}{\sqrt{b}}$	38
default	$\sqrt{b\,x^2+ax}+\frac{a\ln\left(\frac{\frac{a}{2}+bx}{\sqrt{b}}+\sqrt{b\,x^2+ax}\right)}{2\sqrt{b}}$	43
risch	$\frac{x(bx+a)}{\sqrt{x(bx+a)}}+\frac{a\ln\left(\frac{\frac{a}{2}+bx}{\sqrt{b}}+\sqrt{b\,x^2+ax}\right)}{2\sqrt{b}}$	48

```
int((b*x^2+a*x)^(1/2)/x,x,method=_RETURNVERBOSE)
```

```
((x*(b*x+a))^(1/2)*b^(1/2)+arctanh((x*(b*x+a))^(1/2)/x/b^(1/2))*a)/b^(1/2)
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 101, normalized size of antiderivative = 2.40

$$\int \frac{\sqrt{ax+bx^2}}{x}\,dx = \left[\frac{a\sqrt{b}\log\left(2\,bx+a+2\,\sqrt{bx^2+ax}\sqrt{b}\right)+2\,\sqrt{bx^2+ax}b}{2\,b},\right.$$
$$\left.-\frac{a\sqrt{-b}\arctan\left(\frac{\sqrt{bx^2+ax}\sqrt{-b}}{bx+a}\right)-\sqrt{bx^2+ax}b}{b}\right]$$

```
integrate((b*x^2+a*x)^(1/2)/x,x, algorithm="fricas")
```

```
[1/2*(a*sqrt(b)*log(2*b*x + a + 2*sqrt(b*x^2 + a*x)*sqrt(b)) + 2*sqrt(b*x^
2 + a*x)*b)/b, -(a*sqrt(-b)*arctan(sqrt(b*x^2 + a*x)*sqrt(-b)/(b*x + a)) -
 sqrt(b*x^2 + a*x)*b)/b]
```

Sympy [F]

$$\int \frac{\sqrt{ax+bx^2}}{x}\,dx = \int \frac{\sqrt{x\,(a+bx)}}{x}\,dx$$

```
integrate((b*x**2+a*x)**(1/2)/x,x)
```

```
Integral(sqrt(x*(a + b*x))/x, x)
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 41, normalized size of antiderivative = 0.98

$$\int \frac{\sqrt{ax+bx^2}}{x}\,dx = \frac{a\log\left(2\,bx+a+2\,\sqrt{bx^2+ax}\sqrt{b}\right)}{2\,\sqrt{b}} + \sqrt{bx^2+ax}$$

```
integrate((b*x^2+a*x)^(1/2)/x,x, algorithm="maxima")
```

```
1/2*a*log(2*b*x + a + 2*sqrt(b*x^2 + a*x)*sqrt(b))/sqrt(b) + sqrt(b*x^2 +
a*x)
```

Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 46, normalized size of antiderivative = 1.10

$$\int \frac{\sqrt{ax+bx^2}}{x}\,dx = -\frac{a\log\left(\left|2\left(\sqrt{b}x-\sqrt{bx^2+ax}\right)\sqrt{b}+a\right|\right)}{2\sqrt{b}}+\sqrt{bx^2+ax}$$

```
integrate((b*x^2+a*x)^(1/2)/x,x, algorithm="giac")
```

```
-1/2*a*log(abs(2*(sqrt(b)*x - sqrt(b*x^2 + a*x))*sqrt(b) + a))/sqrt(b) + s
qrt(b*x^2 + a*x)
```

Mupad [B] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 42, normalized size of antiderivative = 1.00

$$\int \frac{\sqrt{ax+bx^2}}{x}\,dx = \sqrt{b\,x^2+a\,x}+\frac{a\,\ln\left(\frac{\frac{a}{2}+b\,x}{\sqrt{b}}+\sqrt{b\,x^2+a\,x}\right)}{2\sqrt{b}}$$

```
int((a*x + b*x^2)^(1/2)/x,x)
```

```
(a*x + b*x^2)^(1/2) + (a*log((a/2 + b*x)/b^(1/2) + (a*x + b*x^2)^(1/2)))/(
2*b^(1/2))
```

Reduce [B] (verification not implemented)

Time = 0.24 (sec) , antiderivative size = 37, normalized size of antiderivative = 0.88

$$\int \frac{\sqrt{ax+bx^2}}{x}\,dx = \frac{\sqrt{x}\,\sqrt{bx+a}\,b+\sqrt{b}\log\left(\frac{\sqrt{bx+a}+\sqrt{x}\,\sqrt{b}}{\sqrt{a}}\right)a}{b}$$

```
int((b*x^2+a*x)^(1/2)/x,x)
```

```
(sqrt(x)*sqrt(a + b*x)*b + sqrt(b)*log((sqrt(a + b*x) + sqrt(x)*sqrt(b))/s
qrt(a))*a)/b
```

3.6 $\int \frac{\sqrt{ax+bx^2}}{x^2} dx$

Optimal result

Integrand size = 17, antiderivative size = 47

$$\int \frac{\sqrt{ax+bx^2}}{x^2} dx = -\frac{2\sqrt{ax+bx^2}}{x} + 2\sqrt{b}\operatorname{arctanh}\left(\frac{\sqrt{b}x}{\sqrt{ax+bx^2}}\right)$$

```
-2*(b*x^2+a*x)^(1/2)/x+2*b^(1/2)*arctanh(b^(1/2)*x/(b*x^2+a*x)^(1/2))
```

Mathematica [A] (verified)

Time = 0.11 (sec) , antiderivative size = 70, normalized size of antiderivative = 1.49

$$\int \frac{\sqrt{ax+bx^2}}{x^2} dx = -\frac{2\left(a+bx+2\sqrt{b}\sqrt{x}\sqrt{a+bx}\operatorname{arctanh}\left(\frac{\sqrt{b}\sqrt{x}}{\sqrt{a}-\sqrt{a+bx}}\right)\right)}{\sqrt{x(a+bx)}}$$

```
Integrate[Sqrt[a*x + b*x^2]/x^2,x]
```

```
(-2*(a + b*x + 2*Sqrt[b]*Sqrt[x]*Sqrt[a + b*x]*ArcTanh[(Sqrt[b]*Sqrt[x])/(
Sqrt[a] - Sqrt[a + b*x])]))/Sqrt[x*(a + b*x)]
```

Rubi [A] (verified)

Time = 0.29 (sec) , antiderivative size = 47, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.294, Rules used = {1125, 25, 27, 1091, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sqrt{ax+bx^2}}{x^2} \, dx$$

$$\Big\downarrow \text{1125}$$

$$-\int -\frac{b}{\sqrt{bx^2+ax}}dx-\frac{2\sqrt{ax+bx^2}}{x}$$

$$\Big\downarrow \text{25}$$

$$\int \frac{b}{\sqrt{bx^2+ax}}dx-\frac{2\sqrt{ax+bx^2}}{x}$$

$$\Big\downarrow \text{27}$$

$$b\int \frac{1}{\sqrt{bx^2+ax}}dx-\frac{2\sqrt{ax+bx^2}}{x}$$

$$\Big\downarrow \text{1091}$$

$$2b\int \frac{1}{1-\frac{bx^2}{bx^2+ax}}d\frac{x}{\sqrt{bx^2+ax}}-\frac{2\sqrt{ax+bx^2}}{x}$$

$$\Big\downarrow \text{219}$$

$$2\sqrt{b}\text{arctanh}\left(\frac{\sqrt{b}x}{\sqrt{ax+bx^2}}\right)-\frac{2\sqrt{ax+bx^2}}{x}$$

```
Int[Sqrt[a*x + b*x^2]/x^2,x]
```

```
(-2*Sqrt[a*x + b*x^2])/x + 2*Sqrt[b]*ArcTanh[(Sqrt[b]*x)/Sqrt[a*x + b*x^2]
]
```

Defintions of rubi rules used

```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

```
Int[1/Sqrt[(b_.)*(x_) + (c_.)*(x_)^2], x_Symbol] :> Simp[2   Subst[Int[1/(1
 - c*x^2), x], x, x/Sqrt[b*x + c*x^2]], x] /; FreeQ[{b, c}, x]
```

```
Int[((d_.) + (e_.)*(x_))^(m_.)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_
Symbol] :> Simp[-2*e^(2*m + 3)*(Sqrt[a + b*x + c*x^2]/((-2*c*d + b*e)^(m +
2)*(d + e*x))), x] - Simp[e^(2*m + 2)   Int[(1/Sqrt[a + b*x + c*x^2])*Expan
dToSum[((-2*c*d + b*e)^(-m - 1) - ((-c)*d + b*e + c*e*x)^(-m - 1))/(d + e*x
), x], x], x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[c*d^2 - b*d*e + a*e^2, 0]
 && ILtQ[m, 0] && EqQ[m + p, -3/2]
```

Maple [A] (verified)

Time = 0.36 (sec) , antiderivative size = 40, normalized size of antiderivative = 0.85

method	result	size
pseudoelliptic	$\frac{2\sqrt{b}\,\operatorname{arctanh}\left(\frac{\sqrt{x(bx+a)}}{x\sqrt{b}}\right)x-2\sqrt{x(bx+a)}}{x}$	40
risch	$-\frac{2(bx+a)}{\sqrt{x(bx+a)}}+\sqrt{b}\,\ln\left(\frac{\frac{a}{2}+bx}{\sqrt{b}}+\sqrt{b\,x^{2}+ax}\right)$	46
default	$-\frac{2\left(b\,x^{2}+ax\right)^{\frac{3}{2}}}{a\,x^{2}}+\frac{2b\left(\sqrt{b\,x^{2}+ax}+\frac{a\ln\left(\frac{\frac{a}{2}+bx}{\sqrt{b}}+\sqrt{b\,x^{2}+ax}\right)}{2\sqrt{b}}\right)}{a}$	69

```
int((b*x^2+a*x)^(1/2)/x^2,x,method=_RETURNVERBOSE)
```

```
(2*b^(1/2)*arctanh((x*(b*x+a))^(1/2)/x/b^(1/2))*x-2*(x*(b*x+a))^(1/2))/x
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 96, normalized size of antiderivative = 2.04

$$\int \frac{\sqrt{ax+bx^2}}{x^2}\,dx = \left[\frac{\sqrt{b}x\log\left(2\,bx+a+2\,\sqrt{bx^2+ax}\sqrt{b}\right)-2\,\sqrt{bx^2+ax}}{x}, -\frac{2\left(\sqrt{-b}x\arctan\left(\frac{\sqrt{bx^2+ax}\sqrt{-b}}{bx+a}\right)+\sqrt{bx^2+ax}\right)}{x}\right]$$

```
integrate((b*x^2+a*x)^(1/2)/x^2,x, algorithm="fricas")
```

```
[(sqrt(b)*x*log(2*b*x + a + 2*sqrt(b*x^2 + a*x)*sqrt(b)) - 2*sqrt(b*x^2 +
a*x))/x, -2*(sqrt(-b)*x*arctan(sqrt(b*x^2 + a*x)*sqrt(-b)/(b*x + a)) + sqr
t(b*x^2 + a*x))/x]
```

Sympy [F]

$$\int \frac{\sqrt{ax+bx^2}}{x^2}\,dx = \int \frac{\sqrt{x\,(a+bx)}}{x^2}\,dx$$

```
integrate((b*x**2+a*x)**(1/2)/x**2,x)
```

```
Integral(sqrt(x*(a + b*x))/x**2, x)
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 44, normalized size of antiderivative = 0.94

$$\int \frac{\sqrt{ax+bx^2}}{x^2}\,dx = \sqrt{b}\log\left(2\,bx+a+2\,\sqrt{bx^2+ax}\sqrt{b}\right) - \frac{2\,\sqrt{bx^2+ax}}{x}$$

```
integrate((b*x^2+a*x)^(1/2)/x^2,x, algorithm="maxima")
```

```
sqrt(b)*log(2*b*x + a + 2*sqrt(b*x^2 + a*x)*sqrt(b)) - 2*sqrt(b*x^2 + a*x)
/x
```

Giac [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 60, normalized size of antiderivative = 1.28

$$\int \frac{\sqrt{ax+bx^2}}{x^2}\,dx = -\sqrt{b}\log\left(\left|-2\left(\sqrt{b}x-\sqrt{bx^2+ax}\right)\sqrt{b}-a\right|\right) + \frac{2\,a}{\sqrt{b}x-\sqrt{bx^2+ax}}$$

```
integrate((b*x^2+a*x)^(1/2)/x^2,x, algorithm="giac")
```

```
-sqrt(b)*log(abs(-2*(sqrt(b)*x - sqrt(b*x^2 + a*x))*sqrt(b) - a)) + 2*a/(s
qrt(b)*x - sqrt(b*x^2 + a*x))
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\sqrt{ax+bx^2}}{x^2}\,dx = \int \frac{\sqrt{b\,x^2+a\,x}}{x^2}\,dx$$

```
int((a*x + b*x^2)^(1/2)/x^2,x)
```

```
int((a*x + b*x^2)^(1/2)/x^2, x)
```

Reduce [B] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 43, normalized size of antiderivative = 0.91

$$\int \frac{\sqrt{ax+bx^2}}{x^2}\,dx = \frac{-2\sqrt{x}\,\sqrt{bx+a}+2\sqrt{b}\log\left(\frac{\sqrt{bx+a}+\sqrt{x}\,\sqrt{b}}{\sqrt{a}}\right)x-2\sqrt{b}\,x}{x}$$

```
int((b*x^2+a*x)^(1/2)/x^2,x)
```

```
(2*( - sqrt(x)*sqrt(a + b*x) + sqrt(b)*log((sqrt(a + b*x) + sqrt(x)*sqrt(b
))/sqrt(a))*x - sqrt(b)*x))/x
```

3.7 $\int \frac{\sqrt{ax+bx^2}}{x^3}\,dx$

Optimal result

Integrand size = 17, antiderivative size = 23

$$\int \frac{\sqrt{ax+bx^2}}{x^3}\,dx = -\frac{2(ax+bx^2)^{3/2}}{3ax^3}$$

```
-2/3*(b*x^2+a*x)^(3/2)/a/x^3
```

Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 21, normalized size of antiderivative = 0.91

$$\int \frac{\sqrt{ax+bx^2}}{x^3}\,dx = -\frac{2(x(a+bx))^{3/2}}{3ax^3}$$

```
Integrate[Sqrt[a*x + b*x^2]/x^3,x]
```

```
(-2*(x*(a + b*x))^(3/2))/(3*a*x^3)
```

Rubi [A] (verified)

Time = 0.24 (sec) , antiderivative size = 23, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}} = 0.059$, Rules used = {1123}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sqrt{ax+bx^2}}{x^3} \, dx$$

$$\Big\downarrow \color{blue}{1123}$$

$$-\frac{2(ax+bx^2)^{3/2}}{3ax^3}$$

```
Int[Sqrt[a*x + b*x^2]/x^3,x]
```

```
(-2*(a*x + b*x^2)^(3/2))/(3*a*x^3)
```

Defintions of rubi rules used

```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] :> Simp[e*(d + e*x)^m*((a + b*x + c*x^2)^(p + 1)/((p + 1)*(2*c*d - b
*e))), x] /; FreeQ[{a, b, c, d, e, m, p}, x] && EqQ[c*d^2 - b*d*e + a*e^2,
0] && EqQ[m + 2*p + 2, 0]
```

Maple [A] (verified)

Time = 0.35 (sec) , antiderivative size = 20, normalized size of antiderivative = 0.87

method	result	size
default	$-\frac{2(b\,x^2+ax)^{\frac{3}{2}}}{3a\,x^3}$	20
pseudoelliptic	$-\frac{2(bx+a)\sqrt{x(bx+a)}}{3x^2a}$	23
gosper	$-\frac{2(bx+a)\sqrt{b\,x^2+ax}}{3x^2a}$	25
trager	$-\frac{2(bx+a)\sqrt{b\,x^2+ax}}{3x^2a}$	25
risch	$-\frac{2(bx+a)^2}{3x\sqrt{x(bx+a)}\,a}$	25
orering	$-\frac{2(bx+a)\sqrt{b\,x^2+ax}}{3x^2a}$	25

```
int((b*x^2+a*x)^(1/2)/x^3,x,method=_RETURNVERBOSE)
```

```
-2/3*(b*x^2+a*x)^(3/2)/a/x^3
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.04

$$\int \frac{\sqrt{ax+bx^2}}{x^3}\,dx = -\frac{2\sqrt{bx^2+ax}(bx+a)}{3\,ax^2}$$

```
integrate((b*x^2+a*x)^(1/2)/x^3,x, algorithm="fricas")
```

```
-2/3*sqrt(b*x^2 + a*x)*(b*x + a)/(a*x^2)
```

Sympy [F]

$$\int \frac{\sqrt{ax+bx^2}}{x^3}\,dx = \int \frac{\sqrt{x\,(a+bx)}}{x^3}\,dx$$

```
integrate((b*x**2+a*x)**(1/2)/x**3,x)
```

```
Integral(sqrt(x*(a + b*x))/x**3, x)
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 37, normalized size of antiderivative = 1.61

$$\int \frac{\sqrt{ax+bx^2}}{x^3}\,dx = -\frac{2\,\sqrt{bx^2+ax}b}{3\,ax} - \frac{2\,\sqrt{bx^2+ax}}{3\,x^2}$$

```
integrate((b*x^2+a*x)^(1/2)/x^3,x, algorithm="maxima")
```

```
-2/3*sqrt(b*x^2 + a*x)*b/(a*x) - 2/3*sqrt(b*x^2 + a*x)/x^2
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 76 vs. 2(19) = 38.

Time = 0.24 (sec) , antiderivative size = 76, normalized size of antiderivative = 3.30

$$\int \frac{\sqrt{ax+bx^2}}{x^3}\,dx = \frac{2\left(3\left(\sqrt{b}x-\sqrt{bx^2+ax}\right)^2b+3\left(\sqrt{b}x-\sqrt{bx^2+ax}\right)a\sqrt{b}+a^2\right)}{3\left(\sqrt{b}x-\sqrt{bx^2+ax}\right)^3}$$

```
integrate((b*x^2+a*x)^(1/2)/x^3,x, algorithm="giac")
```

```
2/3*(3*(sqrt(b)*x - sqrt(b*x^2 + a*x))^2*b + 3*(sqrt(b)*x - sqrt(b*x^2 + a
*x))*a*sqrt(b) + a^2)/(sqrt(b)*x - sqrt(b*x^2 + a*x))^3
```

Mupad [B] (verification not implemented)

Time = 9.41 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.04

$$\int \frac{\sqrt{ax+bx^2}}{x^3}\,dx = -\frac{2\sqrt{b\,x^2+a\,x}\,(a+b\,x)}{3\,a\,x^2}$$

```
int((a*x + b*x^2)^(1/2)/x^3,x)
```

```
-(2*(a*x + b*x^2)^(1/2)*(a + b*x))/(3*a*x^2)
```

Reduce [B] (verification not implemented)

Time = 0.23 (sec) , antiderivative size = 40, normalized size of antiderivative = 1.74

$$\int \frac{\sqrt{ax+bx^2}}{x^3}\,dx = \frac{-2\sqrt{x}\,\sqrt{bx+a}\,a-2\sqrt{x}\,\sqrt{bx+a}\,bx-2\sqrt{b}\,b\,x^2}{3a\,x^2}$$

```
int((b*x^2+a*x)^(1/2)/x^3,x)
```

```
( - 2*(sqrt(x)*sqrt(a + b*x)*a + sqrt(x)*sqrt(a + b*x)*b*x + sqrt(b)*b*x**
2))/(3*a*x**2)
```

3.8 $\int \frac{\sqrt{ax+bx^2}}{x^4}\,dx$

Optimal result

Integrand size = 17, antiderivative size = 48

$$\int \frac{\sqrt{ax+bx^2}}{x^4}\,dx = -\frac{2(ax+bx^2)^{3/2}}{5ax^4} + \frac{4b(ax+bx^2)^{3/2}}{15a^2x^3}$$

```
-2/5*(b*x^2+a*x)^(3/2)/a/x^4+4/15*b*(b*x^2+a*x)^(3/2)/a^2/x^3
```

Mathematica [A] (verified)

Time = 0.07 (sec) , antiderivative size = 39, normalized size of antiderivative = 0.81

$$\int \frac{\sqrt{ax+bx^2}}{x^4}\,dx = -\frac{2\sqrt{x(a+bx)}(3a^2+abx-2b^2x^2)}{15a^2x^3}$$

```
Integrate[Sqrt[a*x + b*x^2]/x^4,x]
```

```
(-2*Sqrt[x*(a + b*x)]*(3*a^2 + a*b*x - 2*b^2*x^2))/(15*a^2*x^3)
```

Rubi [A] (verified)

Time = 0.28 (sec) , antiderivative size = 48, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.118$, Rules used = {1129, 1123}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sqrt{ax+bx^2}}{x^4} \, dx$$
$$\Big\downarrow \; 1129$$
$$-\frac{2b\int \frac{\sqrt{bx^2+ax}}{x^3}dx}{5a}-\frac{2(ax+bx^2)^{3/2}}{5ax^4}$$
$$\Big\downarrow \; 1123$$
$$\frac{4b(ax+bx^2)^{3/2}}{15a^2x^3}-\frac{2(ax+bx^2)^{3/2}}{5ax^4}$$

```
Int[Sqrt[a*x + b*x^2]/x^4,x]
```

```
(-2*(a*x + b*x^2)^(3/2))/(5*a*x^4) + (4*b*(a*x + b*x^2)^(3/2))/(15*a^2*x^3
)
```

Defintions of rubi rules used

```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] :> Simp[e*(d + e*x)^m*((a + b*x + c*x^2)^(p + 1)/((p + 1)*(2*c*d - b
*e))), x] /; FreeQ[{a, b, c, d, e, m, p}, x] && EqQ[c*d^2 - b*d*e + a*e^2,
0] && EqQ[m + 2*p + 2, 0]
```

```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] :> Simp[(-e)*(d + e*x)^m*((a + b*x + c*x^2)^(p + 1)/((m + p + 1)*(2*
c*d - b*e))), x] + Simp[c*(Simplify[m + 2*p + 2]/((m + p + 1)*(2*c*d - b*e)
))   Int[(d + e*x)^(m + 1)*(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d
, e, m, p}, x] && EqQ[c*d^2 - b*d*e + a*e^2, 0] && ILtQ[Simplify[m + 2*p +
2], 0]
```

Maple [A] (verified)

Time = 0.37 (sec) , antiderivative size = 29, normalized size of antiderivative = 0.60

method	result	size
pseudoelliptic	$-\frac{2\sqrt{x(bx+a)}\,(bx+a)\left(-\frac{2bx}{3}+a\right)}{5x^{3}a^{2}}$	29
gosper	$-\frac{2(bx+a)(-2bx+3a)\sqrt{b\,x^{2}+ax}}{15a^{2}x^{3}}$	33
orering	$-\frac{2(bx+a)(-2bx+3a)\sqrt{b\,x^{2}+ax}}{15a^{2}x^{3}}$	33
trager	$-\frac{2\left(-2b^{2}x^{2}+abx+3a^{2}\right)\sqrt{b\,x^{2}+ax}}{15a^{2}x^{3}}$	38
default	$-\frac{2\left(b\,x^{2}+ax\right)^{\frac{3}{2}}}{5a\,x^{4}}+\frac{4b\left(b\,x^{2}+ax\right)^{\frac{3}{2}}}{15a^{2}x^{3}}$	41
risch	$-\frac{2(bx+a)\left(-2b^{2}x^{2}+abx+3a^{2}\right)}{15x^{2}\sqrt{x(bx+a)}\,a^{2}}$	41

```
int((b*x^2+a*x)^(1/2)/x^4,x,method=_RETURNVERBOSE)
```

```
-2/5*(x*(b*x+a))^(1/2)*(b*x+a)*(-2/3*b*x+a)/x^3/a^2
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 38, normalized size of antiderivative = 0.79

$$\int \frac{\sqrt{ax+bx^2}}{x^4}\,dx = \frac{2\,(2\,b^2x^2-abx-3\,a^2)\sqrt{bx^2+ax}}{15\,a^2x^3}$$

```
integrate((b*x^2+a*x)^(1/2)/x^4,x, algorithm="fricas")
```

```
2/15*(2*b^2*x^2 - a*b*x - 3*a^2)*sqrt(b*x^2 + a*x)/(a^2*x^3)
```

Sympy [F]

$$\int \frac{\sqrt{ax + bx^2}}{x^4}\,dx = \int \frac{\sqrt{x\,(a + bx)}}{x^4}\,dx$$

```
integrate((b*x**2+a*x)**(1/2)/x**4,x)
```

```
Integral(sqrt(x*(a + b*x))/x**4, x)
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 59, normalized size of antiderivative = 1.23

$$\int \frac{\sqrt{ax + bx^2}}{x^4}\,dx = \frac{4\,\sqrt{bx^2 + ax}b^2}{15\,a^2x} - \frac{2\,\sqrt{bx^2 + ax}b}{15\,ax^2} - \frac{2\,\sqrt{bx^2 + ax}}{5\,x^3}$$

```
integrate((b*x^2+a*x)^(1/2)/x^4,x, algorithm="maxima")
```

```
4/15*sqrt(b*x^2 + a*x)*b^2/(a^2*x) - 2/15*sqrt(b*x^2 + a*x)*b/(a*x^2) - 2/
5*sqrt(b*x^2 + a*x)/x^3
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 107 vs. $2(40) = 80$.

Time = 0.24 (sec) , antiderivative size = 107, normalized size of antiderivative = 2.23

$$\int \frac{\sqrt{ax + bx^2}}{x^4}\,dx = \frac{2\left(15\left(\sqrt{b}x - \sqrt{bx^2 + ax}\right)^3 b^{\frac{3}{2}} + 25\left(\sqrt{b}x - \sqrt{bx^2 + ax}\right)^2 ab + 15\left(\sqrt{b}x - \sqrt{bx^2 + ax}\right)a^2\sqrt{b} + 3\,a^3\right)}{15\left(\sqrt{b}x - \sqrt{bx^2 + ax}\right)^5}$$

```
integrate((b*x^2+a*x)^(1/2)/x^4,x, algorithm="giac")
```

```
2/15*(15*(sqrt(b)*x - sqrt(b*x^2 + a*x))^3*b^(3/2) + 25*(sqrt(b)*x - sqrt(
b*x^2 + a*x))^2*a*b + 15*(sqrt(b)*x - sqrt(b*x^2 + a*x))*a^2*sqrt(b) + 3*a
^3)/(sqrt(b)*x - sqrt(b*x^2 + a*x))^5
```

Mupad [B] (verification not implemented)

Time = 9.20 (sec) , antiderivative size = 37, normalized size of antiderivative = 0.77

$$\int \frac{\sqrt{ax+bx^2}}{x^4}\,dx = -\frac{2\,\sqrt{b\,x^2+a\,x}\,\left(3\,a^2+a\,b\,x-2\,b^2\,x^2\right)}{15\,a^2\,x^3}$$

```
int((a*x + b*x^2)^(1/2)/x^4,x)
```

```
-(2*(a*x + b*x^2)^(1/2)*(3*a^2 - 2*b^2*x^2 + a*b*x))/(15*a^2*x^3)
```

Reduce [B] (verification not implemented)

Time = 0.23 (sec) , antiderivative size = 61, normalized size of antiderivative = 1.27

$$\int \frac{\sqrt{ax+bx^2}}{x^4}\,dx = \frac{-\frac{2\sqrt{x}\,\sqrt{bx+a}\,a^2}{5}-\frac{2\sqrt{x}\,\sqrt{bx+a}\,abx}{15}+\frac{4\sqrt{x}\,\sqrt{bx+a}\,b^2x^2}{15}-\frac{4\sqrt{b}\,b^2x^3}{15}}{a^2x^3}$$

```
int((b*x^2+a*x)^(1/2)/x^4,x)
```

```
(2*( - 3*sqrt(x)*sqrt(a + b*x)*a**2 - sqrt(x)*sqrt(a + b*x)*a*b*x + 2*sqrt
(x)*sqrt(a + b*x)*b**2*x**2 - 2*sqrt(b)*b**2*x**3))/(15*a**2*x**3)
```

3.9 $\int \frac{\sqrt{ax+bx^2}}{x^5}\,dx$

Optimal result

Integrand size = 17, antiderivative size = 74

$$\int \frac{\sqrt{ax+bx^2}}{x^5}\,dx = -\frac{2(ax+bx^2)^{3/2}}{7ax^5}+\frac{8b(ax+bx^2)^{3/2}}{35a^2x^4}-\frac{16b^2(ax+bx^2)^{3/2}}{105a^3x^3}$$

```
-2/7*(b*x^2+a*x)^(3/2)/a/x^5+8/35*b*(b*x^2+a*x)^(3/2)/a^2/x^4-16/105*b^2*(
b*x^2+a*x)^(3/2)/a^3/x^3
```

Mathematica [A] (verified)

Time = 0.08 (sec) , antiderivative size = 51, normalized size of antiderivative = 0.69

$$\int \frac{\sqrt{ax+bx^2}}{x^5}\,dx = -\frac{2\sqrt{x(a+bx)}(15a^3+3a^2bx-4ab^2x^2+8b^3x^3)}{105a^3x^4}$$

```
Integrate[Sqrt[a*x + b*x^2]/x^5,x]
```

```
(-2*Sqrt[x*(a + b*x)]*(15*a^3 + 3*a^2*b*x - 4*a*b^2*x^2 + 8*b^3*x^3))/(105
*a^3*x^4)
```

Rubi [A] (verified)

Time = 0.32 (sec) , antiderivative size = 80, normalized size of antiderivative = 1.08, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.176$, Rules used = {1129, 1129, 1123}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sqrt{ax+bx^2}}{x^5}\,dx$$

$$\Big\downarrow \text{1129}$$

$$-\frac{4b\int \frac{\sqrt{bx^2+ax}}{x^4}dx}{7a}-\frac{2\left(ax+bx^2\right)^{3/2}}{7ax^5}$$

$$\Big\downarrow \text{1129}$$

$$-\frac{4b\left(-\frac{2b\int \frac{\sqrt{bx^2+ax}}{x^3}dx}{5a}-\frac{2\left(ax+bx^2\right)^{3/2}}{5ax^4}\right)}{7a}-\frac{2\left(ax+bx^2\right)^{3/2}}{7ax^5}$$

$$\Big\downarrow \text{1123}$$

$$-\frac{4b\left(\frac{4b\left(ax+bx^2\right)^{3/2}}{15a^2x^3}-\frac{2\left(ax+bx^2\right)^{3/2}}{5ax^4}\right)}{7a}-\frac{2\left(ax+bx^2\right)^{3/2}}{7ax^5}$$

```
Int[Sqrt[a*x + b*x^2]/x^5,x]
```

```
(-2*(a*x + b*x^2)^(3/2))/(7*a*x^5) - (4*b*((-2*(a*x + b*x^2)^(3/2))/(5*a*x
^4) + (4*b*(a*x + b*x^2)^(3/2))/(15*a^2*x^3)))/(7*a)
```

Defintions of rubi rules used

```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] :> Simp[e*(d + e*x)^m*((a + b*x + c*x^2)^(p + 1)/((p + 1)*(2*c*d - b
*e))), x] /; FreeQ[{a, b, c, d, e, m, p}, x] && EqQ[c*d^2 - b*d*e + a*e^2,
0] && EqQ[m + 2*p + 2, 0]
```

```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] :> Simp[(-e)*(d + e*x)^m*((a + b*x + c*x^2)^(p + 1)/((m + p + 1)*(2*
c*d - b*e))), x] + Simp[c*(Simplify[m + 2*p + 2]/((m + p + 1)*(2*c*d - b*e)
))   Int[(d + e*x)^(m + 1)*(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d
, e, m, p}, x] && EqQ[c*d^2 - b*d*e + a*e^2, 0] && ILtQ[Simplify[m + 2*p +
2], 0]
```

Maple [A] (verified)

Time = 0.38 (sec) , antiderivative size = 40, normalized size of antiderivative = 0.54

method	result	size
pseudoelliptic	$-\frac{2\sqrt{x(bx+a)}\,(bx+a)\left(\frac{8}{15}b^2x^2-\frac{4}{5}abx+a^2\right)}{7x^4a^3}$	40
gosper	$-\frac{2(bx+a)\left(8b^2x^2-12abx+15a^2\right)\sqrt{b\,x^2+ax}}{105x^4a^3}$	44
orering	$-\frac{2(bx+a)\left(8b^2x^2-12abx+15a^2\right)\sqrt{b\,x^2+ax}}{105x^4a^3}$	44
trager	$-\frac{2\left(8b^3x^3-4a\,b^2x^2+3a^2bx+15a^3\right)\sqrt{b\,x^2+ax}}{105x^4a^3}$	50
risch	$-\frac{2(bx+a)\left(8b^3x^3-4a\,b^2x^2+3a^2bx+15a^3\right)}{105x^3\sqrt{x(bx+a)}\,a^3}$	53
default	$-\frac{2\left(b\,x^2+ax\right)^{\frac{3}{2}}}{7a\,x^5}-\frac{4b\left(-\frac{2\left(b\,x^2+ax\right)^{\frac{3}{2}}}{5a\,x^4}+\frac{4b\left(b\,x^2+ax\right)^{\frac{3}{2}}}{15a^2x^3}\right)}{7a}$	67

```
int((b*x^2+a*x)^(1/2)/x^5,x,method=_RETURNVERBOSE)
```

```
-2/7*(x*(b*x+a))^(1/2)*(b*x+a)*(8/15*b^2*x^2-4/5*a*b*x+a^2)/x^4/a^3
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 49, normalized size of antiderivative = 0.66

$$\int \frac{\sqrt{ax+bx^2}}{x^5}\,dx = -\frac{2\,(8\,b^3x^3-4\,ab^2x^2+3\,a^2bx+15\,a^3)\sqrt{bx^2+ax}}{105\,a^3x^4}$$

```
integrate((b*x^2+a*x)^(1/2)/x^5,x, algorithm="fricas")
```

```
-2/105*(8*b^3*x^3 - 4*a*b^2*x^2 + 3*a^2*b*x + 15*a^3)*sqrt(b*x^2 + a*x)/(a
^3*x^4)
```

Sympy [F]

$$\int \frac{\sqrt{ax+bx^2}}{x^5}\,dx = \int \frac{\sqrt{x\,(a+bx)}}{x^5}\,dx$$

```
integrate((b*x**2+a*x)**(1/2)/x**5,x)
```

```
Integral(sqrt(x*(a + b*x))/x**5, x)
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 81, normalized size of antiderivative = 1.09

$$\int \frac{\sqrt{ax+bx^2}}{x^5}\,dx = -\frac{16\,\sqrt{bx^2+ax}b^3}{105\,a^3x}+\frac{8\,\sqrt{bx^2+ax}b^2}{105\,a^2x^2}-\frac{2\,\sqrt{bx^2+ax}b}{35\,ax^3}-\frac{2\,\sqrt{bx^2+ax}}{7\,x^4}$$

```
integrate((b*x^2+a*x)^(1/2)/x^5,x, algorithm="maxima")
```

```
-16/105*sqrt(b*x^2 + a*x)*b^3/(a^3*x) + 8/105*sqrt(b*x^2 + a*x)*b^2/(a^2*x
^2) - 2/35*sqrt(b*x^2 + a*x)*b/(a*x^3) - 2/7*sqrt(b*x^2 + a*x)/x^4
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 136 vs. $2(62) = 124$.

Time = 0.14 (sec) , antiderivative size = 136, normalized size of antiderivative = 1.84

$$\int \frac{\sqrt{ax+bx^2}}{x^5}\,dx = \frac{2\left(140\left(\sqrt{b}x-\sqrt{bx^2+ax}\right)^4b^2+315\left(\sqrt{b}x-\sqrt{bx^2+ax}\right)^3ab^{\frac{3}{2}}+273\left(\sqrt{b}x-\sqrt{bx^2+ax}\right)^2a^2b+105\left(\cdots\right.\right.}{105\left(\sqrt{b}x-\sqrt{bx^2+ax}\right)^7}$$

```
integrate((b*x^2+a*x)^(1/2)/x^5,x, algorithm="giac")
```

```
2/105*(140*(sqrt(b)*x - sqrt(b*x^2 + a*x))^4*b^2 + 315*(sqrt(b)*x - sqrt(b
*x^2 + a*x))^3*a*b^(3/2) + 273*(sqrt(b)*x - sqrt(b*x^2 + a*x))^2*a^2*b + 1
05*(sqrt(b)*x - sqrt(b*x^2 + a*x))*a^3*sqrt(b) + 15*a^4)/(sqrt(b)*x - sqrt
(b*x^2 + a*x))^7
```

Mupad [B] (verification not implemented)

Time = 8.91 (sec) , antiderivative size = 81, normalized size of antiderivative = 1.09

$$\int \frac{\sqrt{ax+bx^2}}{x^5}\,dx = \frac{8\,b^2\,\sqrt{b\,x^2+a\,x}}{105\,a^2\,x^2} - \frac{2\,\sqrt{b\,x^2+a\,x}}{7\,x^4} - \frac{16\,b^3\,\sqrt{b\,x^2+a\,x}}{105\,a^3\,x} - \frac{2\,b\,\sqrt{b\,x^2+a\,x}}{35\,a\,x^3}$$

```
int((a*x + b*x^2)^(1/2)/x^5,x)
```

```
(8*b^2*(a*x + b*x^2)^(1/2))/(105*a^2*x^2) - (2*(a*x + b*x^2)^(1/2))/(7*x^4
) - (16*b^3*(a*x + b*x^2)^(1/2))/(105*a^3*x) - (2*b*(a*x + b*x^2)^(1/2))/(
35*a*x^3)
```

Reduce [B] (verification not implemented)

Time = 0.22 (sec) , antiderivative size = 80, normalized size of antiderivative = 1.08

$$\int \frac{\sqrt{ax+bx^2}}{x^5}\,dx$$
$$= \frac{-\frac{2\sqrt{x}\,\sqrt{bx+a}\,a^3}{7} - \frac{2\sqrt{x}\,\sqrt{bx+a}\,a^2bx}{35} + \frac{8\sqrt{x}\,\sqrt{bx+a}\,a\,b^2x^2}{105} - \frac{16\sqrt{x}\,\sqrt{bx+a}\,b^3x^3}{105} + \frac{16\sqrt{b}\,b^3x^4}{105}}{a^3x^4}$$

```
int((b*x^2+a*x)^(1/2)/x^5,x)
```

```
(2*( - 15*sqrt(x)*sqrt(a + b*x)*a**3 - 3*sqrt(x)*sqrt(a + b*x)*a**2*b*x +
4*sqrt(x)*sqrt(a + b*x)*a*b**2*x**2 - 8*sqrt(x)*sqrt(a + b*x)*b**3*x**3 +
8*sqrt(b)*b**3*x**4))/(105*a**3*x**4)
```

3.10 $\int \frac{\sqrt{ax+bx^2}}{x^6} dx$

Optimal result

Integrand size = 17, antiderivative size = 100

$$\int \frac{\sqrt{ax+bx^2}}{x^6} dx = -\frac{2(ax+bx^2)^{3/2}}{9ax^6} + \frac{4b(ax+bx^2)^{3/2}}{21a^2x^5} - \frac{16b^2(ax+bx^2)^{3/2}}{105a^3x^4} + \frac{32b^3(ax+bx^2)^{3/2}}{315a^4x^3}$$

```
-2/9*(b*x^2+a*x)^(3/2)/a/x^6+4/21*b*(b*x^2+a*x)^(3/2)/a^2/x^5-16/105*b^2*(
b*x^2+a*x)^(3/2)/a^3/x^4+32/315*b^3*(b*x^2+a*x)^(3/2)/a^4/x^3
```

Mathematica [A] (verified)

Time = 0.10 (sec) , antiderivative size = 62, normalized size of antiderivative = 0.62

$$\int \frac{\sqrt{ax+bx^2}}{x^6} dx = -\frac{2\sqrt{x(a+bx)}(35a^4+5a^3bx-6a^2b^2x^2+8ab^3x^3-16b^4x^4)}{315a^4x^5}$$

```
Integrate[Sqrt[a*x + b*x^2]/x^6,x]
```

```
(-2*Sqrt[x*(a + b*x)]*(35*a^4 + 5*a^3*b*x - 6*a^2*b^2*x^2 + 8*a*b^3*x^3 -
16*b^4*x^4))/(315*a^4*x^5)
```

Rubi [A] (verified)

Time = 0.38 (sec) , antiderivative size = 112, normalized size of antiderivative = 1.12, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.235, Rules used = {1129, 1129, 1129, 1123}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sqrt{ax+bx^2}}{x^6}dx$$

$$\Big\downarrow \text{1129}$$

$$-\frac{2b\int \frac{\sqrt{bx^2+ax}}{x^5}dx}{3a}-\frac{2(ax+bx^2)^{3/2}}{9ax^6}$$

$$\Big\downarrow \text{1129}$$

$$-\frac{2b\left(-\frac{4b\int \frac{\sqrt{bx^2+ax}}{x^4}dx}{7a}-\frac{2(ax+bx^2)^{3/2}}{7ax^5}\right)}{3a}-\frac{2(ax+bx^2)^{3/2}}{9ax^6}$$

$$\Big\downarrow \text{1129}$$

$$-\frac{2b\left(-\frac{4b\left(-\frac{2b\int \frac{\sqrt{bx^2+ax}}{x^3}dx}{5a}-\frac{2(ax+bx^2)^{3/2}}{5ax^4}\right)}{7a}-\frac{2(ax+bx^2)^{3/2}}{7ax^5}\right)}{3a}-\frac{2(ax+bx^2)^{3/2}}{9ax^6}$$

$$\Big\downarrow \text{1123}$$

$$-\frac{2b\left(-\frac{4b\left(\frac{4b(ax+bx^2)^{3/2}}{15a^2x^3}-\frac{2(ax+bx^2)^{3/2}}{5ax^4}\right)}{7a}-\frac{2(ax+bx^2)^{3/2}}{7ax^5}\right)}{3a}-\frac{2(ax+bx^2)^{3/2}}{9ax^6}$$

```
Int[Sqrt[a*x + b*x^2]/x^6,x]
```

```
(-2*(a*x + b*x^2)^(3/2))/(9*a*x^6) - (2*b*((-2*(a*x + b*x^2)^(3/2))/(7*a*x
^5) - (4*b*((-2*(a*x + b*x^2)^(3/2))/(5*a*x^4) + (4*b*(a*x + b*x^2)^(3/2))
/(15*a^2*x^3)))/(7*a)))/(3*a)
```

Defintions of rubi rules used

```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] :> Simp[e*(d + e*x)^m*((a + b*x + c*x^2)^(p + 1)/((p + 1)*(2*c*d - b
*e))), x] /; FreeQ[{a, b, c, d, e, m, p}, x] && EqQ[c*d^2 - b*d*e + a*e^2,
0] && EqQ[m + 2*p + 2, 0]
```

```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] :> Simp[(-e)*(d + e*x)^m*((a + b*x + c*x^2)^(p + 1)/((m + p + 1)*(2*
c*d - b*e))), x] + Simp[c*(Simplify[m + 2*p + 2]/((m + p + 1)*(2*c*d - b*e)
))   Int[(d + e*x)^(m + 1)*(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d
, e, m, p}, x] && EqQ[c*d^2 - b*d*e + a*e^2, 0] && ILtQ[Simplify[m + 2*p +
2], 0]
```

Maple [A] (verified)

Time = 0.40 (sec) , antiderivative size = 55, normalized size of antiderivative = 0.55

method	result	size
gosper	$-\frac{2(bx+a)\left(-16b^3x^3+24a\,b^2x^2-30a^2bx+35a^3\right)\sqrt{b\,x^2+ax}}{315x^5a^4}$	55
orering	$-\frac{2(bx+a)\left(-16b^3x^3+24a\,b^2x^2-30a^2bx+35a^3\right)\sqrt{b\,x^2+ax}}{315x^5a^4}$	55
pseudoelliptic	$\frac{2\left(16b^4x^4-8a\,b^3x^3+6a^2b^2x^2-5a^3bx-35a^4\right)\sqrt{x(bx+a)}}{315x^5a^4}$	59
trager	$-\frac{2\left(-16b^4x^4+8a\,b^3x^3-6a^2b^2x^2+5a^3bx+35a^4\right)\sqrt{b\,x^2+ax}}{315x^5a^4}$	61
risch	$-\frac{2(bx+a)\left(-16b^4x^4+8a\,b^3x^3-6a^2b^2x^2+5a^3bx+35a^4\right)}{315x^4\sqrt{x(bx+a)}\,a^4}$	64
default	$-\frac{2\left(b\,x^2+ax\right)^{\frac{3}{2}}}{9a\,x^6}-\frac{2b\left(-\frac{2\left(b\,x^2+ax\right)^{\frac{3}{2}}}{7a\,x^5}-\frac{4b\left(-\frac{2\left(b\,x^2+ax\right)^{\frac{3}{2}}}{5a\,x^4}+\frac{4b\left(b\,x^2+ax\right)^{\frac{3}{2}}}{15a^2x^3}\right)}{7a}\right)}{3a}$	93

```
int((b*x^2+a*x)^(1/2)/x^6,x,method=_RETURNVERBOSE)
```

```
-2/315*(b*x+a)*(-16*b^3*x^3+24*a*b^2*x^2-30*a^2*b*x+35*a^3)*(b*x^2+a*x)^(1
/2)/x^5/a^4
```

Fricas [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 60, normalized size of antiderivative = 0.60

$$\int \frac{\sqrt{ax+bx^2}}{x^6}\,dx = \frac{2\,(16\,b^4x^4 - 8\,ab^3x^3 + 6\,a^2b^2x^2 - 5\,a^3bx - 35\,a^4)\sqrt{bx^2+ax}}{315\,a^4x^5}$$

```
integrate((b*x^2+a*x)^(1/2)/x^6,x, algorithm="fricas")
```

```
2/315*(16*b^4*x^4 - 8*a*b^3*x^3 + 6*a^2*b^2*x^2 - 5*a^3*b*x - 35*a^4)*sqrt
(b*x^2 + a*x)/(a^4*x^5)
```

Sympy [F]

$$\int \frac{\sqrt{ax+bx^2}}{x^6}\,dx = \int \frac{\sqrt{x\,(a+bx)}}{x^6}\,dx$$

```
integrate((b*x**2+a*x)**(1/2)/x**6,x)
```

```
Integral(sqrt(x*(a + b*x))/x**6, x)
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 103, normalized size of antiderivative = 1.03

$$\int \frac{\sqrt{ax+bx^2}}{x^6}\,dx = \frac{32\sqrt{bx^2+ax}b^4}{315\,a^4x} - \frac{16\sqrt{bx^2+ax}b^3}{315\,a^3x^2} + \frac{4\sqrt{bx^2+ax}b^2}{105\,a^2x^3} - \frac{2\sqrt{bx^2+ax}b}{63\,ax^4} - \frac{2\sqrt{bx^2+ax}}{9\,x^5}$$

```
integrate((b*x^2+a*x)^(1/2)/x^6,x, algorithm="maxima")
```

```
32/315*sqrt(b*x^2 + a*x)*b^4/(a^4*x) - 16/315*sqrt(b*x^2 + a*x)*b^3/(a^3*x
^2) + 4/105*sqrt(b*x^2 + a*x)*b^2/(a^2*x^3) - 2/63*sqrt(b*x^2 + a*x)*b/(a*
x^4) - 2/9*sqrt(b*x^2 + a*x)/x^5
```

Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 165, normalized size of antiderivative = 1.65

$$\int \frac{\sqrt{ax+bx^2}}{x^6}\,dx = \frac{2\left(630\left(\sqrt{b}x-\sqrt{bx^2+ax}\right)^5 b^{\frac{5}{2}} + 1764\left(\sqrt{b}x-\sqrt{bx^2+ax}\right)^4 ab^2 + 1995\left(\sqrt{b}x-\sqrt{bx^2+ax}\right)^3 a^2b^{\frac{3}{2}} + 1}{315\left(\sqrt{b}x-\sqrt{bx^2+ax}\right)^9}$$

```
integrate((b*x^2+a*x)^(1/2)/x^6,x, algorithm="giac")
```

```
2/315*(630*(sqrt(b)*x - sqrt(b*x^2 + a*x))^5*b^(5/2) + 1764*(sqrt(b)*x - s
qrt(b*x^2 + a*x))^4*a*b^2 + 1995*(sqrt(b)*x - sqrt(b*x^2 + a*x))^3*a^2*b^(
3/2) + 1125*(sqrt(b)*x - sqrt(b*x^2 + a*x))^2*a^3*b + 315*(sqrt(b)*x - sqr
t(b*x^2 + a*x))*a^4*sqrt(b) + 35*a^5)/(sqrt(b)*x - sqrt(b*x^2 + a*x))^9
```

Mupad [B] (verification not implemented)

Time = 9.04 (sec) , antiderivative size = 103, normalized size of antiderivative = 1.03

$$\int \frac{\sqrt{ax+bx^2}}{x^6}\,dx = \frac{4\,b^2\,\sqrt{b\,x^2+a\,x}}{105\,a^2\,x^3} - \frac{2\,\sqrt{b\,x^2+a\,x}}{9\,x^5} - \frac{16\,b^3\,\sqrt{b\,x^2+a\,x}}{315\,a^3\,x^2} + \frac{32\,b^4\,\sqrt{b\,x^2+a\,x}}{315\,a^4\,x} - \frac{2\,b\,\sqrt{b\,x^2+a\,x}}{63\,a\,x^4}$$

```
int((a*x + b*x^2)^(1/2)/x^6,x)
```

```
(4*b^2*(a*x + b*x^2)^(1/2))/(105*a^2*x^3) - (2*(a*x + b*x^2)^(1/2))/(9*x^5
) - (16*b^3*(a*x + b*x^2)^(1/2))/(315*a^3*x^2) + (32*b^4*(a*x + b*x^2)^(1/
2))/(315*a^4*x) - (2*b*(a*x + b*x^2)^(1/2))/(63*a*x^4)
```

Reduce [B] (verification not implemented)

Time = 0.24 (sec) , antiderivative size = 99, normalized size of antiderivative = 0.99

$$\int \frac{\sqrt{ax+bx^2}}{x^6}\,dx = \frac{-\frac{2\sqrt{x}\,\sqrt{bx+a}\,a^4}{9} - \frac{2\sqrt{x}\,\sqrt{bx+a}\,a^3bx}{63} + \frac{4\sqrt{x}\,\sqrt{bx+a}\,a^2b^2x^2}{105} - \frac{16\sqrt{x}\,\sqrt{bx+a}\,a\,b^3x^3}{315} + \frac{32\sqrt{x}\,\sqrt{bx+a}\,b^4x^4}{315} - \frac{32\sqrt{b}\,b^4x^5}{315}}{a^4x^5}$$

```
int((b*x^2+a*x)^(1/2)/x^6,x)
```

```
(2*( - 35*sqrt(x)*sqrt(a + b*x)*a**4 - 5*sqrt(x)*sqrt(a + b*x)*a**3*b*x +
6*sqrt(x)*sqrt(a + b*x)*a**2*b**2*x**2 - 8*sqrt(x)*sqrt(a + b*x)*a*b**3*x*
*3 + 16*sqrt(x)*sqrt(a + b*x)*b**4*x**4 - 16*sqrt(b)*b**4*x**5))/(315*a**4
*x**5)
```

3.11 $\int x^2(ax+bx^2)^{3/2}\,dx$

Optimal result

Integrand size = 17, antiderivative size = 175

$$\int x^2\left(ax+bx^2\right)^{3/2}\,dx=-\frac{7a^5\sqrt{ax+bx^2}}{512b^4}+\frac{7a^4x\sqrt{ax+bx^2}}{768b^3}-\frac{7a^3x^2\sqrt{ax+bx^2}}{960b^2}+\frac{a^2x^3\sqrt{ax+bx^2}}{160b}+\frac{13}{60}ax^4\sqrt{ax+bx^2}+\frac{1}{6}bx^5\sqrt{ax+bx^2}+\frac{7a^6\operatorname{arctanh}\left(\frac{\sqrt{b}x}{\sqrt{ax+bx^2}}\right)}{512b^{9/2}}$$

```
-7/512*a^5*(b*x^2+a*x)^(1/2)/b^4+7/768*a^4*x*(b*x^2+a*x)^(1/2)/b^3-7/960*a
^3*x^2*(b*x^2+a*x)^(1/2)/b^2+1/160*a^2*x^3*(b*x^2+a*x)^(1/2)/b+13/60*a*x^4
*(b*x^2+a*x)^(1/2)+1/6*b*x^5*(b*x^2+a*x)^(1/2)+7/512*a^6*arctanh(b^(1/2)*x
/(b*x^2+a*x)^(1/2))/b^(9/2)
```

Mathematica [A] (verified)

Time = 0.60 (sec) , antiderivative size = 129, normalized size of antiderivative = 0.74

$$\int x^2\left(ax+bx^2\right)^{3/2}\,dx=\frac{\sqrt{x(a+bx)}\left(\sqrt{b}(-105a^5+70a^4bx-56a^3b^2x^2+48a^2b^3x^3+1664ab^4x^4+1280b^5x^5)+\frac{210a}{\cdots}\right.}{7680b^{9/2}}$$

```
Integrate[x^2*(a*x + b*x^2)^(3/2),x]
```

```
(Sqrt[x*(a + b*x)]*(Sqrt[b]*(-105*a^5 + 70*a^4*b*x - 56*a^3*b^2*x^2 + 48*a
^2*b^3*x^3 + 1664*a*b^4*x^4 + 1280*b^5*x^5) + (210*a^6*ArcTanh[(Sqrt[b]*Sq
rt[x])/(-Sqrt[a] + Sqrt[a + b*x])])/(Sqrt[x]*Sqrt[a + b*x])))/(7680*b^(9/2
))
```

Rubi [A] (verified)

Time = 0.44 (sec) , antiderivative size = 156, normalized size of antiderivative = 0.89, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.353$, Rules used = {1134, 1160, 1087, 1087, 1091, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^2 \left(ax+bx^2\right)^{3/2} \, dx$$

$$\Big\downarrow \text{1134}$$

$$\frac{x \left(ax+bx^2\right)^{5/2}}{6b}-\frac{7a \int x \left(bx^2+ax\right)^{3/2}dx}{12b}$$

$$\Big\downarrow \text{1160}$$

$$\frac{x \left(ax+bx^2\right)^{5/2}}{6b}-\frac{7a \left(\frac{\left(ax+bx^2\right)^{5/2}}{5b}-\frac{a \int \left(bx^2+ax\right)^{3/2}dx}{2b}\right)}{12b}$$

$$\Big\downarrow \text{1087}$$

$$\frac{x \left(ax+bx^2\right)^{5/2}}{6b}-\frac{7a \left(\frac{\left(ax+bx^2\right)^{5/2}}{5b}-\frac{a \left(\frac{(a+2bx)\left(ax+bx^2\right)^{3/2}}{8b}-\frac{3a^2 \int \sqrt{bx^2+ax}dx}{16b}\right)}{2b}\right)}{12b}$$

$$\Big\downarrow \text{1087}$$

$$\frac{x\left(ax+bx^2\right)^{5/2}}{6b}-\frac{7a\left(\frac{\left(ax+bx^2\right)^{5/2}}{5b}-\frac{a\left(\frac{(a+2bx)\left(ax+bx^2\right)^{3/2}}{8b}-\frac{3a^2\left(\frac{(a+2bx)\sqrt{ax+bx^2}}{4b}-\frac{a^2\int \frac{1}{\sqrt{bx^2+ax}}dx}{8b}\right)}{16b}\right)}{2b}\right)}{12b}$$

$$\Big\downarrow \ 1091$$

$$\frac{x\left(ax+bx^2\right)^{5/2}}{6b}-\frac{7a\left(\frac{\left(ax+bx^2\right)^{5/2}}{5b}-\frac{a\left(\frac{(a+2bx)\left(ax+bx^2\right)^{3/2}}{8b}-\frac{3a^2\left(\frac{(a+2bx)\sqrt{ax+bx^2}}{4b}-\frac{a^2\int \frac{1}{1-\frac{bx^2}{bx^2+ax}}d\frac{x}{\sqrt{bx^2+ax}}}{4b}\right)}{16b}\right)}{2b}\right)}{12b}$$

$$\Big\downarrow \ 219$$

$$\frac{x\left(ax+bx^2\right)^{5/2}}{6b}-\frac{7a\left(\frac{\left(ax+bx^2\right)^{5/2}}{5b}-\frac{a\left(\frac{(a+2bx)\left(ax+bx^2\right)^{3/2}}{8b}-\frac{3a^2\left(\frac{(a+2bx)\sqrt{ax+bx^2}}{4b}-\frac{a^2\text{arctanh}\left(\frac{\sqrt{b}x}{\sqrt{ax+bx^2}}\right)}{4b^{3/2}}\right)}{16b}\right)}{2b}\right)}{12b}$$

```
Int[x^2*(a*x + b*x^2)^(3/2),x]
```

```
(x*(a*x + b*x^2)^(5/2))/(6*b) - (7*a*((a*x + b*x^2)^(5/2)/(5*b) - (a*(((a
+ 2*b*x)*(a*x + b*x^2)^(3/2))/(8*b) - (3*a^2*(((a + 2*b*x)*Sqrt[a*x + b*x^
2])/(4*b) - (a^2*ArcTanh[(Sqrt[b]*x)/Sqrt[a*x + b*x^2]])/(4*b^(3/2))))/(16
*b)))/(2*b)))/(12*b)
```

Defintions of rubi rules used

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(b + 2*c*x)
*((a + b*x + c*x^2)^p/(2*c*(2*p + 1))), x] - Simp[p*((b^2 - 4*a*c)/(2*c*(2*
p + 1)))   Int[(a + b*x + c*x^2)^(p - 1), x], x] /; FreeQ[{a, b, c}, x] &&
GtQ[p, 0] && (IntegerQ[4*p] || IntegerQ[3*p])
```

```
Int[1/Sqrt[(b_.)*(x_) + (c_.)*(x_)^2], x_Symbol] :> Simp[2   Subst[Int[1/(1
 - c*x^2), x], x, x/Sqrt[b*x + c*x^2]], x] /; FreeQ[{b, c}, x]
```

```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] :> Simp[e*(d + e*x)^(m - 1)*((a + b*x + c*x^2)^(p + 1)/(c*(m + 2*p +
 1))), x] + Simp[(m + p)*((2*c*d - b*e)/(c*(m + 2*p + 1)))   Int[(d + e*x)^
(m - 1)*(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, p}, x] && EqQ[
c*d^2 - b*d*e + a*e^2, 0] && GtQ[m, 1] && NeQ[m + 2*p + 1, 0] && IntegerQ[2
*p]
```

```
Int[((d_.) + (e_.)*(x_))*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol
] :> Simp[e*((a + b*x + c*x^2)^(p + 1)/(2*c*(p + 1))), x] + Simp[(2*c*d - b
*e)/(2*c)   Int[(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, p}, x]
 && NeQ[p, -1]
```

Maple [A] (verified)

Time = 0.39 (sec) , antiderivative size = 95, normalized size of antiderivative = 0.54

method	result	s
pseudoelliptic	$\frac{\frac{7\,\mathrm{arctanh}\left(\frac{\sqrt{x(bx+a)}}{x\sqrt{b}}\right)a^{6}}{512}-\frac{7\sqrt{x(bx+a)}\left(-\frac{256b^{\frac{11}{2}}x^{5}}{21}+a\left(\sqrt{b}\,a^{4}-\frac{2b^{\frac{3}{2}}a^{3}x}{3}+\frac{8b^{\frac{5}{2}}a^{2}x^{2}}{15}-\frac{16b^{\frac{7}{2}}a\,x^{3}}{35}-\frac{1664b^{\frac{9}{2}}x^{4}}{105}\right)\right)}{512}}{b^{\frac{9}{2}}}$	9
risch	$-\frac{\left(-1280b^{5}x^{5}-1664a\,b^{4}x^{4}-48a^{2}b^{3}x^{3}+56a^{3}b^{2}x^{2}-70a^{4}bx+105a^{5}\right)x(bx+a)}{7680b^{4}\sqrt{x(bx+a)}}+\frac{7a^{6}\ln\left(\frac{\frac{a}{2}+bx}{\sqrt{b}}+\sqrt{b\,x^{2}+ax}\right)}{1024b^{\frac{9}{2}}}$	1
default	$\frac{x\left(b\,x^{2}+ax\right)^{\frac{5}{2}}}{6b}-\frac{7a\left(\frac{\left(b\,x^{2}+ax\right)^{\frac{5}{2}}}{5b}-\frac{a\left(\frac{(2bx+a)\left(b\,x^{2}+ax\right)^{\frac{3}{2}}}{8b}-\frac{3a^{2}\left(\frac{(2bx+a)\sqrt{b\,x^{2}+ax}}{4b}-\frac{a^{2}\ln\left(\frac{\frac{a}{2}+bx}{\sqrt{b}}+\sqrt{b\,x^{2}+ax}\right)}{8b^{\frac{3}{2}}}\right)}{16b}\right)}{2b}\right)}{12b}$	1

```
int(x^2*(b*x^2+a*x)^(3/2),x,method=_RETURNVERBOSE)
```

```
7/512/b^(9/2)*(arctanh((x*(b*x+a))^(1/2)/x/b^(1/2))*a^6-(x*(b*x+a))^(1/2)*
(-256/21*b^(11/2)*x^5+a*(b^(1/2)*a^4-2/3*b^(3/2)*a^3*x+8/15*b^(5/2)*a^2*x^
2-16/35*b^(7/2)*a*x^3-1664/105*b^(9/2)*x^4)))
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 215, normalized size of antiderivative = 1.23

$$\int x^2(ax+bx^2)^{3/2}\,dx = \left[\frac{105\,a^6\sqrt{b}\log\left(2\,bx+a+2\,\sqrt{bx^2+ax}\sqrt{b}\right)+2\,(1280\,b^6x^5+1664\,ab^5x^4+48\,a^2b^4x^3-56\,a}{15360\,b^5}\right.$$
$$-\frac{105\,a^6\sqrt{-b}\arctan\left(\frac{\sqrt{bx^2+ax}\sqrt{-b}}{bx+a}\right)-(1280\,b^6x^5+1664\,ab^5x^4+48\,a^2b^4x^3-56\,a^3b^3x^2+70\,a^4b^2x-105\,a^5}{7680\,b^5}$$

```
integrate(x^2*(b*x^2+a*x)^(3/2),x, algorithm="fricas")
```

```
[1/15360*(105*a^6*sqrt(b)*log(2*b*x + a + 2*sqrt(b*x^2 + a*x)*sqrt(b)) + 2
*(1280*b^6*x^5 + 1664*a*b^5*x^4 + 48*a^2*b^4*x^3 - 56*a^3*b^3*x^2 + 70*a^4
*b^2*x - 105*a^5*b)*sqrt(b*x^2 + a*x))/b^5, -1/7680*(105*a^6*sqrt(-b)*arct
an(sqrt(b*x^2 + a*x)*sqrt(-b)/(b*x + a)) - (1280*b^6*x^5 + 1664*a*b^5*x^4
+ 48*a^2*b^4*x^3 - 56*a^3*b^3*x^2 + 70*a^4*b^2*x - 105*a^5*b)*sqrt(b*x^2 +
 a*x))/b^5]
```

Sympy [A] (verification not implemented)

Time = 0.29 (sec) , antiderivative size = 160, normalized size of antiderivative = 0.91

$$\int x^2(ax+bx^2)^{3/2}\,dx = \begin{cases} \frac{7a^6\left(\begin{cases}\frac{\log\left(a+2\sqrt{b}\sqrt{ax+bx^2}+2bx\right)}{\sqrt{b}} & \text{for}\ \frac{a^2}{b}\neq 0\\ \frac{\left(\frac{a}{2b}+x\right)\log\left(\frac{a}{2b}+x\right)}{\sqrt{b\left(\frac{a}{2b}+x\right)^2}} & \text{otherwise}\end{cases}\right)}{1024b^4}+\sqrt{ax+bx^2}\left(-\frac{7a^5}{512b^4}+\frac{7a^4x}{768b^3}-\frac{7a^3x^2}{960b^2}+\frac{a^2x^3}{160b}+\right. \\ \frac{2(ax)^{\frac{9}{2}}}{9a^3} \\ 0 \end{cases}$$

```
integrate(x**2*(b*x**2+a*x)**(3/2),x)
```

```
Piecewise((7*a**6*Piecewise((log(a + 2*sqrt(b)*sqrt(a*x + b*x**2) + 2*b*x)
/sqrt(b), Ne(a**2/b, 0)), ((a/(2*b) + x)*log(a/(2*b) + x)/sqrt(b*(a/(2*b)
+ x)**2), True))/(1024*b**4) + sqrt(a*x + b*x**2)*(-7*a**5/(512*b**4) + 7*
a**4*x/(768*b**3) - 7*a**3*x**2/(960*b**2) + a**2*x**3/(160*b) + 13*a*x**4
/60 + b*x**5/6), Ne(b, 0)), (2*(a*x)**(9/2)/(9*a**3), Ne(a, 0)), (0, True)
)
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 144, normalized size of antiderivative = 0.82

$$\int x^2\left(ax+bx^2\right)^{3/2}\,dx=-\frac{7\,\sqrt{bx^2+ax}a^4x}{256\,b^3}+\frac{7\,(bx^2+ax)^{\frac{3}{2}}a^2x}{96\,b^2}+\frac{(bx^2+ax)^{\frac{5}{2}}x}{6\,b}+\frac{7\,a^6\log\left(2\,bx+a+2\,\sqrt{bx^2+ax}\sqrt{b}\right)}{1024\,b^{\frac{9}{2}}}-\frac{7\,\sqrt{bx^2+ax}a^5}{512\,b^4}+\frac{7\,(bx^2+ax)^{\frac{3}{2}}a^3}{192\,b^3}-\frac{7\,(bx^2+ax)^{\frac{5}{2}}a}{60\,b^2}$$

```
integrate(x^2*(b*x^2+a*x)^(3/2),x, algorithm="maxima")
```

```
-7/256*sqrt(b*x^2 + a*x)*a^4*x/b^3 + 7/96*(b*x^2 + a*x)^(3/2)*a^2*x/b^2 +
1/6*(b*x^2 + a*x)^(5/2)*x/b + 7/1024*a^6*log(2*b*x + a + 2*sqrt(b*x^2 + a*
x)*sqrt(b))/b^(9/2) - 7/512*sqrt(b*x^2 + a*x)*a^5/b^4 + 7/192*(b*x^2 + a*x
)^(3/2)*a^3/b^3 - 7/60*(b*x^2 + a*x)^(5/2)*a/b^2
```

Giac [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 106, normalized size of antiderivative = 0.61

$$\int x^2\left(ax+bx^2\right)^{3/2}\,dx=-\frac{7\,a^6\log\left(\left|2\left(\sqrt{b}x-\sqrt{bx^2+ax}\right)\sqrt{b}+a\right|\right)}{1024\,b^{\frac{9}{2}}}+\frac{1}{7680}\,\sqrt{bx^2+ax}\left(2\left(4\left(2\left(8\,(10\,bx+13\,a)x+\frac{3\,a^2}{b}\right)x-\frac{7\,a^3}{b^2}\right)x+\frac{35\,a^4}{b^3}\right)x-\frac{105\,a^5}{b^4}\right)$$

```
integrate(x^2*(b*x^2+a*x)^(3/2),x, algorithm="giac")
```

```
-7/1024*a^6*log(abs(2*(sqrt(b)*x - sqrt(b*x^2 + a*x))*sqrt(b) + a))/b^(9/2
) + 1/7680*sqrt(b*x^2 + a*x)*(2*(4*(2*(8*(10*b*x + 13*a)*x + 3*a^2/b)*x -
7*a^3/b^2)*x + 35*a^4/b^3)*x - 105*a^5/b^4)
```

Mupad [F(-1)]

Timed out.

$$\int x^2\left(ax+bx^2\right)^{3/2}\,dx=\int x^2\,\left(b\,x^2+a\,x\right)^{3/2}\,dx$$

```
int(x^2*(a*x + b*x^2)^(3/2),x)
```

```
int(x^2*(a*x + b*x^2)^(3/2), x)
```

Reduce [B] (verification not implemented)

Time = 0.22 (sec) , antiderivative size = 133, normalized size of antiderivative = 0.76

$$\int x^2\left(ax+bx^2\right)^{3/2}\,dx=\frac{-105\sqrt{x}\,\sqrt{bx+a}\,a^5b+70\sqrt{x}\,\sqrt{bx+a}\,a^4b^2x-56\sqrt{x}\,\sqrt{bx+a}\,a^3b^3x^2+48\sqrt{x}\,\sqrt{bx+a}\,a^2b^4}{7680b^5}$$

```
int(x^2*(b*x^2+a*x)^(3/2),x)
```

```
( - 105*sqrt(x)*sqrt(a + b*x)*a**5*b + 70*sqrt(x)*sqrt(a + b*x)*a**4*b**2*
x - 56*sqrt(x)*sqrt(a + b*x)*a**3*b**3*x**2 + 48*sqrt(x)*sqrt(a + b*x)*a**
2*b**4*x**3 + 1664*sqrt(x)*sqrt(a + b*x)*a*b**5*x**4 + 1280*sqrt(x)*sqrt(a
 + b*x)*b**6*x**5 + 105*sqrt(b)*log((sqrt(a + b*x) + sqrt(x)*sqrt(b))/sqrt
(a))*a**6)/(7680*b**5)
```

3.12 $\int x(ax+bx^2)^{3/2}\,dx$

Optimal result

Integrand size = 15, antiderivative size = 149

$$\int x\left(ax+bx^2\right)^{3/2}\,dx=\frac{3a^4\sqrt{ax+bx^2}}{128b^3}-\frac{a^3x\sqrt{ax+bx^2}}{64b^2}+\frac{a^2x^2\sqrt{ax+bx^2}}{80b}+\frac{11}{40}ax^3\sqrt{ax+bx^2}+\frac{1}{5}bx^4\sqrt{ax+bx^2}-\frac{3a^5\text{arctanh}\left(\frac{\sqrt{b}x}{\sqrt{ax+bx^2}}\right)}{128b^{7/2}}$$

```
3/128*a^4*(b*x^2+a*x)^(1/2)/b^3-1/64*a^3*x*(b*x^2+a*x)^(1/2)/b^2+1/80*a^2*
x^2*(b*x^2+a*x)^(1/2)/b+11/40*a*x^3*(b*x^2+a*x)^(1/2)+1/5*b*x^4*(b*x^2+a*x
)^(1/2)-3/128*a^5*arctanh(b^(1/2)*x/(b*x^2+a*x)^(1/2))/b^(7/2)
```

Mathematica [A] (verified)

Time = 0.50 (sec) , antiderivative size = 118, normalized size of antiderivative = 0.79

$$\int x(ax+bx^2)^{3/2}\,dx=\frac{\sqrt{x(a+bx)}\Bigg(\sqrt{b}(15a^4-10a^3bx+8a^2b^2x^2+176ab^3x^3+128b^4x^4)+\frac{30a^5\text{arctanh}\left(\frac{\sqrt{b}\sqrt{x}}{\sqrt{a}-\sqrt{a+bx}}\right)}{\sqrt{x}\sqrt{a+bx}}}{640b^{7/2}}$$

```
Integrate[x*(a*x + b*x^2)^(3/2),x]
```

```
(Sqrt[x*(a + b*x)]*(Sqrt[b]*(15*a^4 - 10*a^3*b*x + 8*a^2*b^2*x^2 + 176*a*b
^3*x^3 + 128*b^4*x^4) + (30*a^5*ArcTanh[(Sqrt[b]*Sqrt[x])/(Sqrt[a] - Sqrt[
a + b*x])])/(Sqrt[x]*Sqrt[a + b*x])))/(640*b^(7/2))
```

Rubi [A] (verified)

Time = 0.38 (sec) , antiderivative size = 126, normalized size of antiderivative = 0.85, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.333, Rules used = {1160, 1087, 1087, 1091, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x\left(ax+bx^2\right)^{3/2}\,dx$$

$$\Big\downarrow \text{1160}$$

$$\frac{\left(ax+bx^2\right)^{5/2}}{5b}-\frac{a\int \left(bx^2+ax\right)^{3/2}dx}{2b}$$

$$\Big\downarrow \text{1087}$$

$$\frac{\left(ax+bx^2\right)^{5/2}}{5b}-\frac{a\left(\frac{(a+2bx)\left(ax+bx^2\right)^{3/2}}{8b}-\frac{3a^2\int \sqrt{bx^2+ax}dx}{16b}\right)}{2b}$$

$$\Big\downarrow \text{1087}$$

$$\frac{\left(ax+bx^2\right)^{5/2}}{5b}-\frac{a\left(\frac{(a+2bx)\left(ax+bx^2\right)^{3/2}}{8b}-\frac{3a^2\left(\frac{(a+2bx)\sqrt{ax+bx^2}}{4b}-\frac{a^2\int \frac{1}{\sqrt{bx^2+ax}}dx}{8b}\right)}{16b}\right)}{2b}$$

$$\Big\downarrow \text{1091}$$

$$\frac{\left(ax+bx^2\right)^{5/2}}{5b}-\frac{a\left(\frac{(a+2bx)\left(ax+bx^2\right)^{3/2}}{8b}-\frac{3a^2\left(\frac{(a+2bx)\sqrt{ax+bx^2}}{4b}-\frac{a^2\int \frac{1}{1-\frac{bx^2}{bx^2+ax}}d\frac{x}{\sqrt{bx^2+ax}}}{4b}\right)}{16b}\right)}{2b}$$

$$\Big\downarrow \, 219$$

$$\frac{(ax+bx^2)^{5/2}}{5b}-\frac{a\left(\frac{(a+2bx)(ax+bx^2)^{3/2}}{8b}-\frac{3a^2\left(\frac{(a+2bx)\sqrt{ax+bx^2}}{4b}-\frac{a^2\text{arctanh}\left(\frac{\sqrt{b}x}{\sqrt{ax+bx^2}}\right)}{4b^{3/2}}\right)}{16b}\right)}{2b}$$

```
Int[x*(a*x + b*x^2)^(3/2),x]
```

```
(a*x + b*x^2)^(5/2)/(5*b) - (a*(((a + 2*b*x)*(a*x + b*x^2)^(3/2))/(8*b) -
(3*a^2*(((a + 2*b*x)*Sqrt[a*x + b*x^2])/(4*b) - (a^2*ArcTanh[(Sqrt[b]*x)/S
qrt[a*x + b*x^2]])/(4*b^(3/2))))/(16*b)))/(2*b)
```

Defintions of rubi rules used

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(b + 2*c*x)
*((a + b*x + c*x^2)^p/(2*c*(2*p + 1))), x] - Simp[p*((b^2 - 4*a*c)/(2*c*(2*
p + 1)))   Int[(a + b*x + c*x^2)^(p - 1), x], x] /; FreeQ[{a, b, c}, x] &&
GtQ[p, 0] && (IntegerQ[4*p] || IntegerQ[3*p])
```

```
Int[1/Sqrt[(b_.)*(x_) + (c_.)*(x_)^2], x_Symbol] :> Simp[2   Subst[Int[1/(1
 - c*x^2), x], x, x/Sqrt[b*x + c*x^2]], x] /; FreeQ[{b, c}, x]
```

```
Int[((d_.) + (e_.)*(x_))*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol
] :> Simp[e*((a + b*x + c*x^2)^(p + 1)/(2*c*(p + 1))), x] + Simp[(2*c*d - b
*e)/(2*c)   Int[(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, p}, x]
 && NeQ[p, -1]
```

Maple [A] (verified)

Time = 0.39 (sec) , antiderivative size = 84, normalized size of antiderivative = 0.56

method	result	size
pseudoelliptic	$-\frac{3\left(\operatorname{arctanh}\left(\frac{\sqrt{x(bx+a)}}{x\sqrt{b}}\right)a^5-\left(\sqrt{b}\,a^4-\frac{2b^{\frac{3}{2}}a^3x}{3}+\frac{8b^{\frac{5}{2}}a^2x^2}{15}+\frac{176b^{\frac{7}{2}}a\,x^3}{15}+\frac{128b^{\frac{9}{2}}x^4}{15}\right)\sqrt{x(bx+a)}\right)}{128b^{\frac{7}{2}}}$	84
risch	$\frac{(128b^4x^4+176a\,b^3x^3+8a^2b^2x^2-10a^3bx+15a^4)x(bx+a)}{640b^3\sqrt{x(bx+a)}}-\frac{3a^5\ln\left(\frac{\frac{a}{2}+bx}{\sqrt{b}}+\sqrt{b\,x^2+ax}\right)}{256b^{\frac{7}{2}}}$	95
default	$\frac{(b\,x^2+ax)^{\frac{5}{2}}}{5b}-\frac{a\left(\frac{(2bx+a)\left(b\,x^2+ax\right)^{\frac{3}{2}}}{8b}-\frac{3a^2\left(\frac{(2bx+a)\sqrt{b\,x^2+ax}}{4b}-\frac{a^2\ln\left(\frac{\frac{a}{2}+bx}{\sqrt{b}}+\sqrt{b\,x^2+ax}\right)}{8b^{\frac{3}{2}}}\right)}{16b}\right)}{2b}$	110

```
int(x*(b*x^2+a*x)^(3/2),x,method=_RETURNVERBOSE)
```

```
-3/128/b^(7/2)*(arctanh((x*(b*x+a))^(1/2)/x/b^(1/2))*a^5-(b^(1/2)*a^4-2/3*
b^(3/2)*a^3*x+8/15*b^(5/2)*a^2*x^2+176/15*b^(7/2)*a*x^3+128/15*b^(9/2)*x^4
)*(x*(b*x+a))^(1/2))
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 192, normalized size of antiderivative = 1.29

$$\int x(ax+bx^2)^{3/2}\,dx = \left[\frac{15\,a^5\sqrt{b}\log\left(2\,bx+a-2\,\sqrt{bx^2+ax}\sqrt{b}\right)+2\,(128\,b^5x^4+176\,ab^4x^3+8\,a^2b^3x^2-10\,a^3b^2x}{1280\,b^4}\right.$$

```
integrate(x*(b*x^2+a*x)^(3/2),x, algorithm="fricas")
```

```
[1/1280*(15*a^5*sqrt(b)*log(2*b*x + a - 2*sqrt(b*x^2 + a*x)*sqrt(b)) + 2*(
128*b^5*x^4 + 176*a*b^4*x^3 + 8*a^2*b^3*x^2 - 10*a^3*b^2*x + 15*a^4*b)*sqr
t(b*x^2 + a*x))/b^4, 1/640*(15*a^5*sqrt(-b)*arctan(sqrt(b*x^2 + a*x)*sqrt(
-b)/(b*x + a)) + (128*b^5*x^4 + 176*a*b^4*x^3 + 8*a^2*b^3*x^2 - 10*a^3*b^2
*x + 15*a^4*b)*sqrt(b*x^2 + a*x))/b^4]
```

Sympy [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 144, normalized size of antiderivative = 0.97

$$\int x(ax+bx^2)^{3/2}\,dx = \begin{cases} -\frac{3a^5\left(\begin{cases}\frac{\log\left(a+2\sqrt{b}\sqrt{ax+bx^2}+2bx\right)}{\sqrt{b}} & \text{for } \frac{a^2}{b}\neq 0\\ \frac{\left(\frac{a}{2b}+x\right)\log\left(\frac{a}{2b}+x\right)}{\sqrt{b\left(\frac{a}{2b}+x\right)^2}} & \text{otherwise}\end{cases}\right)}{256b^3} + \sqrt{ax+bx^2}\cdot\left(\frac{3a^4}{128b^3}-\frac{a^3x}{64b^2}+\frac{a^2x^2}{80b}+\frac{11ax^3}{40}+\right. \\ \frac{2(ax)^{\frac{7}{2}}}{7a^2} \\ 0 \end{cases}$$

```
integrate(x*(b*x**2+a*x)**(3/2),x)
```

```
Piecewise((-3*a**5*Piecewise((log(a + 2*sqrt(b)*sqrt(a*x + b*x**2) + 2*b*x
)/sqrt(b), Ne(a**2/b, 0)), ((a/(2*b) + x)*log(a/(2*b) + x)/sqrt(b*(a/(2*b)
 + x)**2), True))/(256*b**3) + sqrt(a*x + b*x**2)*(3*a**4/(128*b**3) - a**
3*x/(64*b**2) + a**2*x**2/(80*b) + 11*a*x**3/40 + b*x**4/5), Ne(b, 0)), (2
*(a*x)**(7/2)/(7*a**2), Ne(a, 0)), (0, True))
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 124, normalized size of antiderivative = 0.83

$$\int x\left(ax+bx^2\right)^{3/2}\,dx = \frac{3\,\sqrt{bx^2+ax}a^3x}{64\,b^2} - \frac{\left(bx^2+ax\right)^{\frac{3}{2}}ax}{8\,b} - \frac{3\,a^5\log\left(2\,bx+a+2\,\sqrt{bx^2+ax}\sqrt{b}\right)}{256\,b^{\frac{7}{2}}} + \frac{3\,\sqrt{bx^2+ax}a^4}{128\,b^3} - \frac{\left(bx^2+ax\right)^{\frac{3}{2}}a^2}{16\,b^2} + \frac{\left(bx^2+ax\right)^{\frac{5}{2}}}{5\,b}$$

```
integrate(x*(b*x^2+a*x)^(3/2),x, algorithm="maxima")
```

```
3/64*sqrt(b*x^2 + a*x)*a^3*x/b^2 - 1/8*(b*x^2 + a*x)^(3/2)*a*x/b - 3/256*a
^5*log(2*b*x + a + 2*sqrt(b*x^2 + a*x)*sqrt(b))/b^(7/2) + 3/128*sqrt(b*x^2
 + a*x)*a^4/b^3 - 1/16*(b*x^2 + a*x)^(3/2)*a^2/b^2 + 1/5*(b*x^2 + a*x)^(5/
2)/b
```

Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 93, normalized size of antiderivative = 0.62

$$\int x\left(ax+bx^2\right)^{3/2}\,dx = \frac{3\,a^5\log\left(\left|2\,\left(\sqrt{b}x-\sqrt{bx^2+ax}\right)\sqrt{b}+a\right|\right)}{256\,b^{\frac{7}{2}}} + \frac{1}{640}\,\sqrt{bx^2+ax}\left(2\,\left(4\,\left(2\,(8\,bx+11\,a)x+\frac{a^2}{b}\right)x-\frac{5\,a^3}{b^2}\right)x+\frac{15\,a^4}{b^3}\right)$$

```
integrate(x*(b*x^2+a*x)^(3/2),x, algorithm="giac")
```

```
3/256*a^5*log(abs(2*(sqrt(b)*x - sqrt(b*x^2 + a*x))*sqrt(b) + a))/b^(7/2)
+ 1/640*sqrt(b*x^2 + a*x)*(2*(4*(2*(8*b*x + 11*a)*x + a^2/b)*x - 5*a^3/b^2
)*x + 15*a^4/b^3)
```

Mupad [B] (verification not implemented)

Time = 9.03 (sec) , antiderivative size = 118, normalized size of antiderivative = 0.79

$$\int x(ax+bx^2)^{3/2}\,dx = \frac{(b\,x^2+a\,x)^{5/2}}{5\,b} - \frac{a\left(\frac{x\,(b\,x^2+a\,x)^{3/2}}{4}+\frac{a\,(b\,x^2+a\,x)^{3/2}}{8\,b}-\frac{3\,a^2\left(\frac{\sqrt{b\,x^2+a\,x}\,(a+2\,b\,x)}{4\,b}-\frac{a^2\,\ln\left(\frac{\frac{a}{2}+b\,x}{\sqrt{b}}+\sqrt{b\,x^2+a\,x}\right)}{8\,b^{3/2}}\right)}{16\,b}\right)}{2\,b}$$

```
int(x*(a*x + b*x^2)^(3/2),x)
```

```
(a*x + b*x^2)^(5/2)/(5*b) - (a*((x*(a*x + b*x^2)^(3/2))/4 + (a*(a*x + b*x^
2)^(3/2))/(8*b) - (3*a^2*(((a*x + b*x^2)^(1/2)*(a + 2*b*x))/(4*b) - (a^2*l
og((a/2 + b*x)/b^(1/2) + (a*x + b*x^2)^(1/2)))/(8*b^(3/2))))/(16*b)))/(2*b
)
```

Reduce [B] (verification not implemented)

Time = 0.20 (sec) , antiderivative size = 114, normalized size of antiderivative = 0.77

$$\int x(ax+bx^2)^{3/2}\,dx = \frac{15\sqrt{x}\,\sqrt{bx+a}\,a^4b-10\sqrt{x}\,\sqrt{bx+a}\,a^3b^2x+8\sqrt{x}\,\sqrt{bx+a}\,a^2b^3x^2+176\sqrt{x}\,\sqrt{bx+a}\,a\,b^4x^3-}{640b^4}$$

```
int(x*(b*x^2+a*x)^(3/2),x)
```

```
(15*sqrt(x)*sqrt(a + b*x)*a**4*b - 10*sqrt(x)*sqrt(a + b*x)*a**3*b**2*x +
8*sqrt(x)*sqrt(a + b*x)*a**2*b**3*x**2 + 176*sqrt(x)*sqrt(a + b*x)*a*b**4*
x**3 + 128*sqrt(x)*sqrt(a + b*x)*b**5*x**4 - 15*sqrt(b)*log((sqrt(a + b*x)
 + sqrt(x)*sqrt(b))/sqrt(a))*a**5)/(640*b**4)
```

3.13 $\int (ax+bx^2)^{3/2}\,dx$

Optimal result

Integrand size = 13, antiderivative size = 123

$$\int (ax+bx^2)^{3/2}\,dx = -\frac{3a^3\sqrt{ax+bx^2}}{64b^2}+\frac{a^2x\sqrt{ax+bx^2}}{32b}+\frac{3}{8}ax^2\sqrt{ax+bx^2}+\frac{1}{4}bx^3\sqrt{ax+bx^2}+\frac{3a^4\operatorname{arctanh}\left(\frac{\sqrt{b}x}{\sqrt{ax+bx^2}}\right)}{64b^{5/2}}$$

```
-3/64*a^3*(b*x^2+a*x)^(1/2)/b^2+1/32*a^2*x*(b*x^2+a*x)^(1/2)/b+3/8*a*x^2*(
b*x^2+a*x)^(1/2)+1/4*b*x^3*(b*x^2+a*x)^(1/2)+3/64*a^4*arctanh(b^(1/2)*x/(b
*x^2+a*x)^(1/2))/b^(5/2)
```

Mathematica [A] (verified)

Time = 0.38 (sec) , antiderivative size = 107, normalized size of antiderivative = 0.87

$$\int (ax+bx^2)^{3/2}\,dx = \frac{\sqrt{x(a+bx)}\left(\sqrt{b}(-3a^3+2a^2bx+24ab^2x^2+16b^3x^3)+\frac{6a^4\operatorname{arctanh}\left(\frac{\sqrt{b}\sqrt{x}}{-\sqrt{a}+\sqrt{a+bx}}\right)}{\sqrt{x}\sqrt{a+bx}}\right)}{64b^{5/2}}$$

```
Integrate[(a*x + b*x^2)^(3/2),x]
```

```
(Sqrt[x*(a + b*x)]*(Sqrt[b]*(-3*a^3 + 2*a^2*b*x + 24*a*b^2*x^2 + 16*b^3*x^
3) + (6*a^4*ArcTanh[(Sqrt[b]*Sqrt[x])/(-Sqrt[a] + Sqrt[a + b*x])])/(Sqrt[x
]*Sqrt[a + b*x])))/(64*b^(5/2))
```

Rubi [A] (verified)

Time = 0.34 (sec) , antiderivative size = 97, normalized size of antiderivative = 0.79, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.308$, Rules used = {1087, 1087, 1091, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \left(ax+bx^2\right)^{3/2}dx$$

$$\Big\downarrow \text{ 1087}$$

$$\frac{(a+2bx)\left(ax+bx^2\right)^{3/2}}{8b}-\frac{3a^2\int \sqrt{bx^2+ax}dx}{16b}$$

$$\Big\downarrow \text{ 1087}$$

$$\frac{(a+2bx)\left(ax+bx^2\right)^{3/2}}{8b}-\frac{3a^2\left(\frac{(a+2bx)\sqrt{ax+bx^2}}{4b}-\frac{a^2\int \frac{1}{\sqrt{bx^2+ax}}dx}{8b}\right)}{16b}$$

$$\Big\downarrow \text{ 1091}$$

$$\frac{(a+2bx)\left(ax+bx^2\right)^{3/2}}{8b}-\frac{3a^2\left(\frac{(a+2bx)\sqrt{ax+bx^2}}{4b}-\frac{a^2\int \frac{1}{1-\frac{bx^2}{bx^2+ax}}d\frac{x}{\sqrt{bx^2+ax}}}{4b}\right)}{16b}$$

$$\Big\downarrow \text{ 219}$$

$$\frac{(a+2bx)\left(ax+bx^2\right)^{3/2}}{8b}-\frac{3a^2\left(\frac{(a+2bx)\sqrt{ax+bx^2}}{4b}-\frac{a^2\text{arctanh}\left(\frac{\sqrt{b}x}{\sqrt{ax+bx^2}}\right)}{4b^{3/2}}\right)}{16b}$$

```
Int[(a*x + b*x^2)^(3/2),x]
```

```
((a + 2*b*x)*(a*x + b*x^2)^(3/2))/(8*b) - (3*a^2*(((a + 2*b*x)*Sqrt[a*x +
b*x^2])/(4*b) - (a^2*ArcTanh[(Sqrt[b]*x)/Sqrt[a*x + b*x^2]])/(4*b^(3/2))))
/(16*b)
```

Defintions of rubi rules used

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(b + 2*c*x)
*((a + b*x + c*x^2)^p/(2*c*(2*p + 1))), x] - Simp[p*((b^2 - 4*a*c)/(2*c*(2*
p + 1)))   Int[(a + b*x + c*x^2)^(p - 1), x], x] /; FreeQ[{a, b, c}, x] &&
GtQ[p, 0] && (IntegerQ[4*p] || IntegerQ[3*p])
```

```
Int[1/Sqrt[(b_.)*(x_) + (c_.)*(x_)^2], x_Symbol] :> Simp[2   Subst[Int[1/(1
 - c*x^2), x], x, x/Sqrt[b*x + c*x^2]], x] /; FreeQ[{b, c}, x]
```

Maple [A] (verified)

Time = 0.36 (sec) , antiderivative size = 73, normalized size of antiderivative = 0.59

method	result	size
pseudoelliptic	$\frac{\frac{3\,\operatorname{arctanh}\left(\frac{\sqrt{x(bx+a)}}{x\sqrt{b}}\right)a^{4}}{64}-\frac{3\left(\sqrt{b}\,a^{3}-\frac{2b^{\frac{3}{2}}a^{2}x}{3}-8a\,x^{2}b^{\frac{5}{2}}-\frac{16b^{\frac{7}{2}}x^{3}}{3}\right)\sqrt{x(bx+a)}}{64}}{b^{\frac{5}{2}}}$	73
risch	$-\frac{\left(-16b^{3}x^{3}-24a\,b^{2}x^{2}-2a^{2}bx+3a^{3}\right)x(bx+a)}{64b^{2}\sqrt{x(bx+a)}}+\frac{3a^{4}\ln\left(\frac{\frac{a}{2}+bx}{\sqrt{b}}+\sqrt{b\,x^{2}+ax}\right)}{128b^{\frac{5}{2}}}$	84
default	$\frac{(2bx+a)\left(b\,x^{2}+ax\right)^{\frac{3}{2}}}{8b}-\frac{3a^{2}\left(\frac{(2bx+a)\sqrt{b\,x^{2}+ax}}{4b}-\frac{a^{2}\ln\left(\frac{\frac{a}{2}+bx}{\sqrt{b}}+\sqrt{b\,x^{2}+ax}\right)}{8b^{\frac{3}{2}}}\right)}{16b}$	87

```
int((b*x^2+a*x)^(3/2),x,method=_RETURNVERBOSE)
```

```
3/64*(arctanh((x*(b*x+a))^(1/2)/x/b^(1/2))*a^4-(b^(1/2)*a^3-2/3*b^(3/2)*a^
2*x-8*a*x^2*b^(5/2)-16/3*b^(7/2)*x^3)*(x*(b*x+a))^(1/2))/b^(5/2)
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 171, normalized size of antiderivative = 1.39

$$\int (ax+bx^2)^{3/2}\,dx = \left[\frac{3\,a^4\sqrt{b}\log\left(2\,bx+a+2\,\sqrt{bx^2+ax}\sqrt{b}\right)+2\,(16\,b^4x^3+24\,ab^3x^2+2\,a^2b^2x-3\,a^3b)\sqrt{bx^2+ax}}{128\,b^3}, -\frac{3\,a^4\sqrt{-b}\arctan\left(\frac{\sqrt{bx^2+ax}\sqrt{-b}}{bx+a}\right)-(16\,b^4x^3+24\,ab^3x^2+2\,a^2b^2x-3\,a^3b)\sqrt{bx^2+ax}}{64\,b^3}\right]$$

```
integrate((b*x^2+a*x)^(3/2),x, algorithm="fricas")
```

```
[1/128*(3*a^4*sqrt(b)*log(2*b*x + a + 2*sqrt(b*x^2 + a*x)*sqrt(b)) + 2*(16
*b^4*x^3 + 24*a*b^3*x^2 + 2*a^2*b^2*x - 3*a^3*b)*sqrt(b*x^2 + a*x))/b^3, -
1/64*(3*a^4*sqrt(-b)*arctan(sqrt(b*x^2 + a*x)*sqrt(-b)/(b*x + a)) - (16*b^
4*x^3 + 24*a*b^3*x^2 + 2*a^2*b^2*x - 3*a^3*b)*sqrt(b*x^2 + a*x))/b^3]
```

Sympy [A] (verification not implemented)

Time = 0.35 (sec) , antiderivative size = 257, normalized size of antiderivative = 2.09

$$\int (ax+bx^2)^{3/2}\,dx = a\left(\begin{cases} \frac{a^3\left(\begin{cases} \frac{\log\left(a+2\sqrt{b}\sqrt{ax+bx^2}+2bx\right)}{\sqrt{b}} & \text{for } \frac{a^2}{b}\neq 0 \\ \frac{\left(\frac{a}{2b}+x\right)\log\left(\frac{a}{2b}+x\right)}{\sqrt{b\left(\frac{a}{2b}+x\right)^2}} & \text{otherwise} \end{cases}\right)}{16b^2} + \sqrt{ax+bx^2}\left(-\frac{a^2}{8b^2}+\frac{ax}{12b}+\frac{x^2}{3}\right) & \text{for } b\neq 0 \\ \frac{2(ax)^{\frac{5}{2}}}{5a^2} & \text{for } a\neq 0 \\ 0 & \text{otherwise} \end{cases}\right) + b\left(\begin{cases} -\frac{5a^4\left(\begin{cases} \frac{\log\left(a+2\sqrt{b}\sqrt{ax+bx^2}+2bx\right)}{\sqrt{b}} & \text{for } \frac{a^2}{b}\neq 0 \\ \frac{\left(\frac{a}{2b}+x\right)\log\left(\frac{a}{2b}+x\right)}{\sqrt{b\left(\frac{a}{2b}+x\right)^2}} & \text{otherwise} \end{cases}\right)}{128b^3} + \sqrt{ax+bx^2}\cdot\left(\frac{5a^3}{64b^3}-\frac{5a^2x}{96b^2}+\frac{ax^2}{24b}+\frac{x^3}{4}\right) & \text{for } b\neq 0 \\ \frac{2(ax)^{\frac{7}{2}}}{7a^3} & \text{for } a\neq 0 \\ 0 & \text{otherwise} \end{cases}\right)$$

```
integrate((b*x**2+a*x)**(3/2),x)
```

```
a*Piecewise((a**3*Piecewise((log(a + 2*sqrt(b)*sqrt(a*x + b*x**2) + 2*b*x)
/sqrt(b), Ne(a**2/b, 0)), ((a/(2*b) + x)*log(a/(2*b) + x)/sqrt(b*(a/(2*b)
+ x)**2), True))/(16*b**2) + sqrt(a*x + b*x**2)*(-a**2/(8*b**2) + a*x/(12*
b) + x**2/3), Ne(b, 0)), (2*(a*x)**(5/2)/(5*a**2), Ne(a, 0)), (0, True)) +
 b*Piecewise((-5*a**4*Piecewise((log(a + 2*sqrt(b)*sqrt(a*x + b*x**2) + 2*
b*x)/sqrt(b), Ne(a**2/b, 0)), ((a/(2*b) + x)*log(a/(2*b) + x)/sqrt(b*(a/(2
*b) + x)**2), True))/(128*b**3) + sqrt(a*x + b*x**2)*(5*a**3/(64*b**3) - 5
*a**2*x/(96*b**2) + a*x**2/(24*b) + x**3/4), Ne(b, 0)), (2*(a*x)**(7/2)/(7
*a**3), Ne(a, 0)), (0, True))
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 102, normalized size of antiderivative = 0.83

$$\int \left(ax+bx^2\right)^{3/2} dx = \frac{1}{4}\left(bx^2+ax\right)^{\frac{3}{2}}x - \frac{3\sqrt{bx^2+ax}a^2x}{32\,b} + \frac{3\,a^4\log\left(2\,bx+a+2\,\sqrt{bx^2+ax}\sqrt{b}\right)}{128\,b^{\frac{5}{2}}} - \frac{3\,\sqrt{bx^2+ax}a^3}{64\,b^2} + \frac{\left(bx^2+ax\right)^{\frac{3}{2}}a}{8\,b}$$

```
integrate((b*x^2+a*x)^(3/2),x, algorithm="maxima")
```

```
1/4*(b*x^2 + a*x)^(3/2)*x - 3/32*sqrt(b*x^2 + a*x)*a^2*x/b + 3/128*a^4*log
(2*b*x + a + 2*sqrt(b*x^2 + a*x)*sqrt(b))/b^(5/2) - 3/64*sqrt(b*x^2 + a*x)
*a^3/b^2 + 1/8*(b*x^2 + a*x)^(3/2)*a/b
```

Giac [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 81, normalized size of antiderivative = 0.66

$$\int \left(ax+bx^2\right)^{3/2} dx = -\frac{3\,a^4\log\left(\left|2\left(\sqrt{b}x-\sqrt{bx^2+ax}\right)\sqrt{b}+a\right|\right)}{128\,b^{\frac{5}{2}}} + \frac{1}{64}\sqrt{bx^2+ax}\left(2\left(4\,(2\,bx+3\,a)x+\frac{a^2}{b}\right)x-\frac{3\,a^3}{b^2}\right)$$

```
integrate((b*x^2+a*x)^(3/2),x, algorithm="giac")
```

```
-3/128*a^4*log(abs(2*(sqrt(b)*x - sqrt(b*x^2 + a*x))*sqrt(b) + a))/b^(5/2)
 + 1/64*sqrt(b*x^2 + a*x)*(2*(4*(2*b*x + 3*a)*x + a^2/b)*x - 3*a^3/b^2)
```

Mupad [B] (verification not implemented)

Time = 9.34 (sec) , antiderivative size = 87, normalized size of antiderivative = 0.71

$$\int \left(ax+bx^2\right)^{3/2} \, dx = \frac{\left(b\,x^2+a\,x\right)^{3/2}\,\left(\frac{a}{2}+b\,x\right)}{4\,b} - \frac{3\,a^2\,\left(\sqrt{b\,x^2+a\,x}\,\left(\frac{x}{2}+\frac{a}{4\,b}\right)-\frac{a^2\,\ln\left(\frac{\frac{a}{2}+b\,x}{\sqrt{b}}+\sqrt{b\,x^2+a\,x}\right)}{8\,b^{3/2}}\right)}{16\,b}$$

```
int((a*x + b*x^2)^(3/2),x)
```

```
((a*x + b*x^2)^(3/2)*(a/2 + b*x))/(4*b) - (3*a^2*((a*x + b*x^2)^(1/2)*(x/2
 + a/(4*b)) - (a^2*log((a/2 + b*x)/b^(1/2) + (a*x + b*x^2)^(1/2)))/(8*b^(3
/2))))/(16*b)
```

Reduce [B] (verification not implemented)

Time = 0.21 (sec) , antiderivative size = 95, normalized size of antiderivative = 0.77

$$\int \left(ax+bx^2\right)^{3/2} \, dx = \frac{-3\sqrt{x}\,\sqrt{bx+a}\,a^3b+2\sqrt{x}\,\sqrt{bx+a}\,a^2b^2x+24\sqrt{x}\,\sqrt{bx+a}\,a\,b^3x^2+16\sqrt{x}\,\sqrt{bx+a}\,b^4x^3+3}{64b^3}$$

```
int((b*x^2+a*x)^(3/2),x)
```

```
( - 3*sqrt(x)*sqrt(a + b*x)*a**3*b + 2*sqrt(x)*sqrt(a + b*x)*a**2*b**2*x +
 24*sqrt(x)*sqrt(a + b*x)*a*b**3*x**2 + 16*sqrt(x)*sqrt(a + b*x)*b**4*x**3
 + 3*sqrt(b)*log((sqrt(a + b*x) + sqrt(x)*sqrt(b))/sqrt(a))*a**4)/(64*b**3
)
```

3.14 $\int \frac{(ax+bx^2)^{3/2}}{x} dx$

Optimal result

Integrand size = 17, antiderivative size = 97

$$\int \frac{(ax+bx^2)^{3/2}}{x} dx = \frac{a^2\sqrt{ax+bx^2}}{8b} + \frac{7}{12}ax\sqrt{ax+bx^2} + \frac{1}{3}bx^2\sqrt{ax+bx^2} - \frac{a^3\operatorname{arctanh}\left(\frac{\sqrt{b}x}{\sqrt{ax+bx^2}}\right)}{8b^{3/2}}$$

```
1/8*a^2*(b*x^2+a*x)^(1/2)/b+7/12*a*x*(b*x^2+a*x)^(1/2)+1/3*b*x^2*(b*x^2+a*
x)^(1/2)-1/8*a^3*arctanh(b^(1/2)*x/(b*x^2+a*x)^(1/2))/b^(3/2)
```

Mathematica [A] (verified)

Time = 0.32 (sec) , antiderivative size = 96, normalized size of antiderivative = 0.99

$$\int \frac{(ax+bx^2)^{3/2}}{x} dx = \frac{\sqrt{x(a+bx)}\left(\sqrt{b}(3a^2+14abx+8b^2x^2) + \frac{6a^3\operatorname{arctanh}\left(\frac{\sqrt{b}\sqrt{x}}{\sqrt{a}-\sqrt{a+bx}}\right)}{\sqrt{x}\sqrt{a+bx}}\right)}{24b^{3/2}}$$

```
Integrate[(a*x + b*x^2)^(3/2)/x,x]
```

```
(Sqrt[x*(a + b*x)]*(Sqrt[b]*(3*a^2 + 14*a*b*x + 8*b^2*x^2) + (6*a^3*ArcTan
h[(Sqrt[b]*Sqrt[x])/(Sqrt[a] - Sqrt[a + b*x])])/(Sqrt[x]*Sqrt[a + b*x])))/
(24*b^(3/2))
```

Rubi [A] (verified)

Time = 0.33 (sec) , antiderivative size = 83, normalized size of antiderivative = 0.86, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.235$, Rules used = {1131, 1087, 1091, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(ax+bx^2)^{3/2}}{x} \, dx$$

$$\Big\downarrow \text{1131}$$

$$\frac{1}{2}a\int \sqrt{bx^2+ax}dx+\frac{1}{3}\left(ax+bx^2\right)^{3/2}$$

$$\Big\downarrow \text{1087}$$

$$\frac{1}{2}a\left(\frac{(a+2bx)\sqrt{ax+bx^2}}{4b}-\frac{a^2\int \frac{1}{\sqrt{bx^2+ax}}dx}{8b}\right)+\frac{1}{3}\left(ax+bx^2\right)^{3/2}$$

$$\Big\downarrow \text{1091}$$

$$\frac{1}{2}a\left(\frac{(a+2bx)\sqrt{ax+bx^2}}{4b}-\frac{a^2\int \frac{1}{1-\frac{bx^2}{bx^2+ax}}d\frac{x}{\sqrt{bx^2+ax}}}{4b}\right)+\frac{1}{3}\left(ax+bx^2\right)^{3/2}$$

$$\Big\downarrow \text{219}$$

$$\frac{1}{2}a\left(\frac{(a+2bx)\sqrt{ax+bx^2}}{4b}-\frac{a^2\text{arctanh}\left(\frac{\sqrt{b}x}{\sqrt{ax+bx^2}}\right)}{4b^{3/2}}\right)+\frac{1}{3}\left(ax+bx^2\right)^{3/2}$$

```
Int[(a*x + b*x^2)^(3/2)/x,x]
```

```
(a*x + b*x^2)^(3/2)/3 + (a*(((a + 2*b*x)*Sqrt[a*x + b*x^2])/(4*b) - (a^2*A
rcTanh[(Sqrt[b]*x)/Sqrt[a*x + b*x^2]])/(4*b^(3/2))))/2
```

Defintions of rubi rules used

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(b + 2*c*x)
*((a + b*x + c*x^2)^p/(2*c*(2*p + 1))), x] - Simp[p*((b^2 - 4*a*c)/(2*c*(2*
p + 1)))   Int[(a + b*x + c*x^2)^(p - 1), x], x] /; FreeQ[{a, b, c}, x] &&
GtQ[p, 0] && (IntegerQ[4*p] || IntegerQ[3*p])
```

```
Int[1/Sqrt[(b_.)*(x_) + (c_.)*(x_)^2], x_Symbol] :> Simp[2   Subst[Int[1/(1
 - c*x^2), x], x, x/Sqrt[b*x + c*x^2]], x] /; FreeQ[{b, c}, x]
```

```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] :> Simp[(d + e*x)^(m + 1)*((a + b*x + c*x^2)^p/(e*(m + 2*p + 1))), x
] - Simp[p*((2*c*d - b*e)/(e^2*(m + 2*p + 1)))   Int[(d + e*x)^(m + 1)*(a +
 b*x + c*x^2)^(p - 1), x], x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[c*d^2 - b
*d*e + a*e^2, 0] && GtQ[p, 0] && (LeQ[-2, m, 0] || EqQ[m + p + 1, 0]) && Ne
Q[m + 2*p + 1, 0] && IntegerQ[2*p]
```

Maple [A] (verified)

Time = 0.36 (sec) , antiderivative size = 73, normalized size of antiderivative = 0.75

method	result	size
default	$\frac{\left(b\,x^{2}+a x \right)^{\frac{3}{2}}}{3}+\frac{a \left(\frac{\left(2 b x +a \right) \sqrt{b\,x^{2}+a x}}{4 b}-\frac{a^{2} \ln \left(\frac{\frac{a}{2}+b x}{\sqrt{b}}+\sqrt{b\,x^{2}+a x}\right)}{8 b^{\frac{3}{2}}}\right)}{2}$	73
risch	$\frac{\left(8 b^{2} x^{2}+14 a b x +3 a^{2}\right) x \left(b x +a \right)}{24 b \sqrt{x \left(b x +a \right)}}-\frac{a^{3} \ln \left(\frac{\frac{a}{2}+b x}{\sqrt{b}}+\sqrt{b\,x^{2}+a x}\right)}{16 b^{\frac{3}{2}}}$	73
pseudoelliptic	$\frac{8 b^{\frac{5}{2}} \sqrt{x \left(b x +a \right)}\, x^{2}+14 a\,b^{\frac{3}{2}} x \sqrt{x \left(b x +a \right)}+3 a^{2} \sqrt{b}\, \sqrt{x \left(b x +a \right)}-3 \,\operatorname{arctanh}\left(\frac{\sqrt{x \left(b x +a \right)}}{x \sqrt{b}}\right) a^{3}}{24 b^{\frac{3}{2}}}$	79

```
int((b*x^2+a*x)^(3/2)/x,x,method=_RETURNVERBOSE)
```

```
1/3*(b*x^2+a*x)^(3/2)+1/2*a*(1/4*(2*b*x+a)/b*(b*x^2+a*x)^(1/2)-1/8*a^2/b^(
3/2)*ln((1/2*a+b*x)/b^(1/2)+(b*x^2+a*x)^(1/2)))
```

Fricas [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 148, normalized size of antiderivative = 1.53

$$\int \frac{(ax+bx^2)^{3/2}}{x}\,dx = \left[\frac{3\,a^3\sqrt{b}\log\left(2\,bx+a-2\,\sqrt{bx^2+ax}\sqrt{b}\right)+2\,(8\,b^3x^2+14\,ab^2x+3\,a^2b)\sqrt{bx^2+ax}}{48\,b^2},\right.$$

```
integrate((b*x^2+a*x)^(3/2)/x,x, algorithm="fricas")
```

```
[1/48*(3*a^3*sqrt(b)*log(2*b*x + a - 2*sqrt(b*x^2 + a*x)*sqrt(b)) + 2*(8*b
^3*x^2 + 14*a*b^2*x + 3*a^2*b)*sqrt(b*x^2 + a*x))/b^2, 1/24*(3*a^3*sqrt(-b
)*arctan(sqrt(b*x^2 + a*x)*sqrt(-b)/(b*x + a)) + (8*b^3*x^2 + 14*a*b^2*x +
 3*a^2*b)*sqrt(b*x^2 + a*x))/b^2]
```

Sympy [A] (verification not implemented)

Time = 0.95 (sec) , antiderivative size = 224, normalized size of antiderivative = 2.31

$$\int \frac{(ax+bx^2)^{3/2}}{x}\,dx = a\left(\begin{cases} -\frac{a^2\left(\begin{cases}\frac{\log\left(a+2\sqrt{b}\sqrt{ax+bx^2}+2bx\right)}{\sqrt{b}} & \text{for } \frac{a^2}{b}\neq 0\\ \frac{\left(\frac{a}{2b}+x\right)\log\left(\frac{a}{2b}+x\right)}{\sqrt{b\left(\frac{a}{2b}+x\right)^2}} & \text{otherwise}\end{cases}\right)}{8b} + \left(\frac{a}{4b}+\frac{x}{2}\right)\sqrt{ax+bx^2} & \text{for } b\neq 0\\ \frac{2(ax)^{\frac{3}{2}}}{3a} & \text{for } a\neq 0\\ 0 & \text{otherwise}\end{cases}\right) + b\left(\begin{cases} \frac{a^3\left(\begin{cases}\frac{\log\left(a+2\sqrt{b}\sqrt{ax+bx^2}+2bx\right)}{\sqrt{b}} & \text{for } \frac{a^2}{b}\neq 0\\ \frac{\left(\frac{a}{2b}+x\right)\log\left(\frac{a}{2b}+x\right)}{\sqrt{b\left(\frac{a}{2b}+x\right)^2}} & \text{otherwise}\end{cases}\right)}{16b^2} + \sqrt{ax+bx^2}\left(-\frac{a^2}{8b^2}+\frac{ax}{12b}+\frac{x^2}{3}\right) & \text{for } b\neq 0\\ \frac{2(ax)^{\frac{5}{2}}}{5a^2} & \text{for } a\neq 0\\ 0 & \text{otherwise}\end{cases}\right)$$

```
integrate((b*x**2+a*x)**(3/2)/x,x)
```

```
a*Piecewise((-a**2*Piecewise((log(a + 2*sqrt(b)*sqrt(a*x + b*x**2) + 2*b*x
)/sqrt(b), Ne(a**2/b, 0)), ((a/(2*b) + x)*log(a/(2*b) + x)/sqrt(b*(a/(2*b)
 + x)**2), True))/(8*b) + (a/(4*b) + x/2)*sqrt(a*x + b*x**2), Ne(b, 0)), (
2*(a*x)**(3/2)/(3*a), Ne(a, 0)), (0, True)) + b*Piecewise((a**3*Piecewise(
(log(a + 2*sqrt(b)*sqrt(a*x + b*x**2) + 2*b*x)/sqrt(b), Ne(a**2/b, 0)), ((
a/(2*b) + x)*log(a/(2*b) + x)/sqrt(b*(a/(2*b) + x)**2), True))/(16*b**2) +
 sqrt(a*x + b*x**2)*(-a**2/(8*b**2) + a*x/(12*b) + x**2/3), Ne(b, 0)), (2*
(a*x)**(5/2)/(5*a**2), Ne(a, 0)), (0, True))
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 79, normalized size of antiderivative = 0.81

$$\int \frac{(ax+bx^2)^{3/2}}{x}\,dx = \frac{1}{4}\sqrt{bx^2+ax}ax - \frac{a^3\log\left(2\,bx+a+2\sqrt{bx^2+ax}\sqrt{b}\right)}{16\,b^{\frac{3}{2}}} + \frac{1}{3}\left(bx^2+ax\right)^{\frac{3}{2}} + \frac{\sqrt{bx^2+ax}a^2}{8\,b}$$

```
integrate((b*x^2+a*x)^(3/2)/x,x, algorithm="maxima")
```

```
1/4*sqrt(b*x^2 + a*x)*a*x - 1/16*a^3*log(2*b*x + a + 2*sqrt(b*x^2 + a*x)*s
qrt(b))/b^(3/2) + 1/3*(b*x^2 + a*x)^(3/2) + 1/8*sqrt(b*x^2 + a*x)*a^2/b
```

Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 70, normalized size of antiderivative = 0.72

$$\int \frac{(ax+bx^2)^{3/2}}{x}\,dx = \frac{a^3\log\left(\left|2\left(\sqrt{b}x-\sqrt{bx^2+ax}\right)\sqrt{b}+a\right|\right)}{16\,b^{\frac{3}{2}}} + \frac{1}{24}\sqrt{bx^2+ax}\left(2\,(4\,bx+7\,a)x+\frac{3\,a^2}{b}\right)$$

```
integrate((b*x^2+a*x)^(3/2)/x,x, algorithm="giac")
```

```
1/16*a^3*log(abs(2*(sqrt(b)*x - sqrt(b*x^2 + a*x))*sqrt(b) + a))/b^(3/2) +
 1/24*sqrt(b*x^2 + a*x)*(2*(4*b*x + 7*a)*x + 3*a^2/b)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(ax+bx^2)^{3/2}}{x}\,dx = \int \frac{(b\,x^2+a\,x)^{3/2}}{x}\,dx$$

```
int((a*x + b*x^2)^(3/2)/x,x)
```

```
int((a*x + b*x^2)^(3/2)/x, x)
```

Reduce [B] (verification not implemented)

Time = 0.19 (sec) , antiderivative size = 76, normalized size of antiderivative = 0.78

$$\int \frac{(ax+bx^2)^{3/2}}{x}\,dx = \frac{3\sqrt{x}\,\sqrt{bx+a}\,a^2b+14\sqrt{x}\,\sqrt{bx+a}\,a\,b^2x+8\sqrt{x}\,\sqrt{bx+a}\,b^3x^2-3\sqrt{b}\log\left(\frac{\sqrt{bx+a}+\sqrt{x}\,\sqrt{}}{\sqrt{a}}\right.}{24b^2}$$

```
int((b*x^2+a*x)^(3/2)/x,x)
```

```
(3*sqrt(x)*sqrt(a + b*x)*a**2*b + 14*sqrt(x)*sqrt(a + b*x)*a*b**2*x + 8*sq
rt(x)*sqrt(a + b*x)*b**3*x**2 - 3*sqrt(b)*log((sqrt(a + b*x) + sqrt(x)*sqr
t(b))/sqrt(a))*a**3)/(24*b**2)
```

3.15 $\int \frac{(ax+bx^2)^{3/2}}{x^2}\,dx$

Optimal result

Integrand size = 17, antiderivative size = 71

$$\int \frac{(ax+bx^2)^{3/2}}{x^2}\,dx = \frac{5}{4}a\sqrt{ax+bx^2}+\frac{1}{2}bx\sqrt{ax+bx^2}+\frac{3a^2\text{arctanh}\left(\frac{\sqrt{b}x}{\sqrt{ax+bx^2}}\right)}{4\sqrt{b}}$$

```
5/4*a*(b*x^2+a*x)^(1/2)+1/2*b*x*(b*x^2+a*x)^(1/2)+3/4*a^2*arctanh(b^(1/2)*
x/(b*x^2+a*x)^(1/2))/b^(1/2)
```

Mathematica [A] (verified)

Time = 0.12 (sec) , antiderivative size = 70, normalized size of antiderivative = 0.99

$$\int \frac{(ax+bx^2)^{3/2}}{x^2}\,dx = \frac{1}{4}\sqrt{x(a+bx)}\left(5a+2bx-\frac{3a^2\log\left(-\sqrt{b}\sqrt{x}+\sqrt{a+bx}\right)}{\sqrt{b}\sqrt{x}\sqrt{a+bx}}\right)$$

```
Integrate[(a*x + b*x^2)^(3/2)/x^2,x]
```

```
(Sqrt[x*(a + b*x)]*(5*a + 2*b*x - (3*a^2*Log[-(Sqrt[b]*Sqrt[x]) + Sqrt[a +
 b*x]])/(Sqrt[b]*Sqrt[x]*Sqrt[a + b*x])))/4
```

Rubi [A] (verified)

Time = 0.32 (sec) , antiderivative size = 68, normalized size of antiderivative = 0.96, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.235, Rules used = {1131, 1131, 1091, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(ax+bx^2)^{3/2}}{x^2}dx$$

$$\Big\downarrow \text{1131}$$

$$\frac{3}{4}a\int \frac{\sqrt{bx^2+ax}}{x}dx+\frac{(ax+bx^2)^{3/2}}{2x}$$

$$\Big\downarrow \text{1131}$$

$$\frac{3}{4}a\left(\frac{1}{2}a\int \frac{1}{\sqrt{bx^2+ax}}dx+\sqrt{ax+bx^2}\right)+\frac{(ax+bx^2)^{3/2}}{2x}$$

$$\Big\downarrow \text{1091}$$

$$\frac{3}{4}a\left(a\int \frac{1}{1-\frac{bx^2}{bx^2+ax}}d\frac{x}{\sqrt{bx^2+ax}}+\sqrt{ax+bx^2}\right)+\frac{(ax+bx^2)^{3/2}}{2x}$$

$$\Big\downarrow \text{219}$$

$$\frac{3}{4}a\left(\frac{a\text{arctanh}\left(\frac{\sqrt{b}x}{\sqrt{ax+bx^2}}\right)}{\sqrt{b}}+\sqrt{ax+bx^2}\right)+\frac{(ax+bx^2)^{3/2}}{2x}$$

```
Int[(a*x + b*x^2)^(3/2)/x^2,x]
```

```
(a*x + b*x^2)^(3/2)/(2*x) + (3*a*(Sqrt[a*x + b*x^2] + (a*ArcTanh[(Sqrt[b]*
x)/Sqrt[a*x + b*x^2]])/Sqrt[b]))/4
```

Defintions of rubi rules used

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

```
Int[1/Sqrt[(b_.)*(x_) + (c_.)*(x_)^2], x_Symbol] :> Simp[2   Subst[Int[1/(1
 - c*x^2), x], x, x/Sqrt[b*x + c*x^2]], x] /; FreeQ[{b, c}, x]
```

```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] :> Simp[(d + e*x)^(m + 1)*((a + b*x + c*x^2)^p/(e*(m + 2*p + 1))), x
] - Simp[p*((2*c*d - b*e)/(e^2*(m + 2*p + 1)))   Int[(d + e*x)^(m + 1)*(a +
 b*x + c*x^2)^(p - 1), x], x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[c*d^2 - b
*d*e + a*e^2, 0] && GtQ[p, 0] && (LeQ[-2, m, 0] || EqQ[m + p + 1, 0]) && Ne
Q[m + 2*p + 1, 0] && IntegerQ[2*p]
```

Maple [A] (verified)

Time = 0.37 (sec) , antiderivative size = 59, normalized size of antiderivative = 0.83

method	result	size
risch	$\frac{(2bx+5a)x(bx+a)}{4\sqrt{x(bx+a)}}+\frac{3a^{2}\ln\left(\frac{\frac{a}{2}+bx}{\sqrt{b}}+\sqrt{b\,x^{2}+ax}\right)}{8\sqrt{b}}$	59
pseudoelliptic	$\frac{2b^{\frac{3}{2}}\sqrt{x(bx+a)}\,x+3\,\operatorname{arctanh}\left(\frac{\sqrt{x(bx+a)}}{x\sqrt{b}}\right)a^{2}+5a\sqrt{b}\,\sqrt{x(bx+a)}}{4\sqrt{b}}$	59
default	$\frac{2(b\,x^{2}+ax)^{\frac{5}{2}}}{a\,x^{2}}-\frac{6b\left(\frac{(b\,x^{2}+ax)^{\frac{3}{2}}}{3}+\frac{a\left(\frac{(2bx+a)\sqrt{b\,x^{2}+ax}}{4b}-\frac{a^{2}\ln\left(\frac{\frac{a}{2}+bx}{\sqrt{b}}+\sqrt{b\,x^{2}+ax}\right)}{8b^{\frac{3}{2}}}\right)}{2}\right)}{a}$	99

```
int((b*x^2+a*x)^(3/2)/x^2,x,method=_RETURNVERBOSE)
```

```
1/4*(2*b*x+5*a)*x*(b*x+a)/(x*(b*x+a))^(1/2)+3/8*a^2*ln((1/2*a+b*x)/b^(1/2)
+(b*x^2+a*x)^(1/2))/b^(1/2)
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 127, normalized size of antiderivative = 1.79

$$\int \frac{(ax+bx^2)^{3/2}}{x^2}\,dx = \left[\frac{3\,a^2\sqrt{b}\log\left(2\,bx+a+2\,\sqrt{bx^2+ax}\sqrt{b}\right)+2\,(2\,b^2x+5\,ab)\sqrt{bx^2+ax}}{8\,b},\ -\frac{3\,a^2\sqrt{-b}\arctan\left(\frac{\sqrt{bx^2+ax}\sqrt{-b}}{bx+a}\right)-(2\,b^2x+5\,ab)\sqrt{bx^2+ax}}{4\,b}\right]$$

```
integrate((b*x^2+a*x)^(3/2)/x^2,x, algorithm="fricas")
```

```
[1/8*(3*a^2*sqrt(b)*log(2*b*x + a + 2*sqrt(b*x^2 + a*x)*sqrt(b)) + 2*(2*b^
2*x + 5*a*b)*sqrt(b*x^2 + a*x))/b, -1/4*(3*a^2*sqrt(-b)*arctan(sqrt(b*x^2
+ a*x)*sqrt(-b)/(b*x + a)) - (2*b^2*x + 5*a*b)*sqrt(b*x^2 + a*x))/b]
```

Sympy [F]

$$\int \frac{(ax+bx^2)^{3/2}}{x^2}\,dx = \int \frac{(x(a+bx))^{\frac{3}{2}}}{x^2}\,dx$$

```
integrate((b*x**2+a*x)**(3/2)/x**2,x)
```

```
Integral((x*(a + b*x))**(3/2)/x**2, x)
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 62, normalized size of antiderivative = 0.87

$$\int \frac{(ax+bx^2)^{3/2}}{x^2}\,dx = \frac{3\,a^2\log\left(2\,bx+a+2\,\sqrt{bx^2+ax}\sqrt{b}\right)}{8\,\sqrt{b}} + \frac{3}{4}\,\sqrt{bx^2+ax}a + \frac{(bx^2+ax)^{\frac{3}{2}}}{2\,x}$$

```
integrate((b*x^2+a*x)^(3/2)/x^2,x, algorithm="maxima")
```

```
3/8*a^2*log(2*b*x + a + 2*sqrt(b*x^2 + a*x)*sqrt(b))/sqrt(b) + 3/4*sqrt(b*
x^2 + a*x)*a + 1/2*(b*x^2 + a*x)^(3/2)/x
```

Giac [A] (verification not implemented)

Time = 0.36 (sec) , antiderivative size = 58, normalized size of antiderivative = 0.82

$$\int \frac{(ax+bx^2)^{3/2}}{x^2}\,dx = -\frac{3\,a^2\log\left(\left|2\left(\sqrt{b}x-\sqrt{bx^2+ax}\right)\sqrt{b}+a\right|\right)}{8\,\sqrt{b}} + \frac{1}{4}\,\sqrt{bx^2+ax}(2\,bx+5\,a)$$

```
integrate((b*x^2+a*x)^(3/2)/x^2,x, algorithm="giac")
```

```
-3/8*a^2*log(abs(2*(sqrt(b)*x - sqrt(b*x^2 + a*x))*sqrt(b) + a))/sqrt(b) +
 1/4*sqrt(b*x^2 + a*x)*(2*b*x + 5*a)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(ax+bx^2)^{3/2}}{x^2}\,dx = \int \frac{(b\,x^2+a\,x)^{3/2}}{x^2}\,dx$$

```
int((a*x + b*x^2)^(3/2)/x^2,x)
```

```
int((a*x + b*x^2)^(3/2)/x^2, x)
```

Reduce [B] (verification not implemented)

Time = 0.20 (sec) , antiderivative size = 57, normalized size of antiderivative = 0.80

$$\int \frac{(ax+bx^2)^{3/2}}{x^2}\,dx = \frac{5\sqrt{x}\,\sqrt{bx+a}\,ab+2\sqrt{x}\,\sqrt{bx+a}\,b^2x+3\sqrt{b}\log\left(\frac{\sqrt{bx+a}+\sqrt{x}\,\sqrt{b}}{\sqrt{a}}\right)a^2}{4b}$$

```
int((b*x^2+a*x)^(3/2)/x^2,x)
```

```
(5*sqrt(x)*sqrt(a + b*x)*a*b + 2*sqrt(x)*sqrt(a + b*x)*b**2*x + 3*sqrt(b)*
log((sqrt(a + b*x) + sqrt(x)*sqrt(b))/sqrt(a))*a**2)/(4*b)
```

3.16 $\int \frac{(ax+bx^2)^{3/2}}{x^3}\,dx$

Optimal result

Integrand size = 17, antiderivative size = 64

$$\int \frac{(ax+bx^2)^{3/2}}{x^3}\,dx = b\sqrt{ax+bx^2} - \frac{2a\sqrt{ax+bx^2}}{x} + 3a\sqrt{b}\operatorname{arctanh}\left(\frac{\sqrt{b}x}{\sqrt{ax+bx^2}}\right)$$

```
b*(b*x^2+a*x)^(1/2)-2*a*(b*x^2+a*x)^(1/2)/x+3*a*b^(1/2)*arctanh(b^(1/2)*x/
(b*x^2+a*x)^(1/2))
```

Mathematica [A] (verified)

Time = 0.19 (sec) , antiderivative size = 83, normalized size of antiderivative = 1.30

$$\int \frac{(ax+bx^2)^{3/2}}{x^3}\,dx = \frac{\sqrt{a+bx}\left((-2a+bx)\sqrt{a+bx} + 6a\sqrt{b}\sqrt{x}\operatorname{arctanh}\left(\frac{\sqrt{b}\sqrt{x}}{-\sqrt{a}+\sqrt{a+bx}}\right)\right)}{\sqrt{x(a+bx)}}$$

```
Integrate[(a*x + b*x^2)^(3/2)/x^3,x]
```

```
(Sqrt[a + b*x]*((-2*a + b*x)*Sqrt[a + b*x] + 6*a*Sqrt[b]*Sqrt[x]*ArcTanh[(
Sqrt[b]*Sqrt[x])/(-Sqrt[a] + Sqrt[a + b*x])]))/Sqrt[x*(a + b*x)]
```

Rubi [A] (verified)

Time = 0.33 (sec) , antiderivative size = 65, normalized size of antiderivative = 1.02, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.353, Rules used = {1125, 25, 27, 1160, 1091, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(ax+bx^2)^{3/2}}{x^3}dx$$
$$\Big\downarrow \text{1125}$$
$$-\int -\frac{b(2a+bx)}{\sqrt{bx^2+ax}}dx-\frac{2a\sqrt{ax+bx^2}}{x}$$
$$\Big\downarrow \text{25}$$
$$\int \frac{b(2a+bx)}{\sqrt{bx^2+ax}}dx-\frac{2a\sqrt{ax+bx^2}}{x}$$
$$\Big\downarrow \text{27}$$
$$b\int \frac{2a+bx}{\sqrt{bx^2+ax}}dx-\frac{2a\sqrt{ax+bx^2}}{x}$$
$$\Big\downarrow \text{1160}$$
$$b\left(\frac{3}{2}a\int \frac{1}{\sqrt{bx^2+ax}}dx+\sqrt{ax+bx^2}\right)-\frac{2a\sqrt{ax+bx^2}}{x}$$
$$\Big\downarrow \text{1091}$$
$$b\left(3a\int \frac{1}{1-\frac{bx^2}{bx^2+ax}}d\frac{x}{\sqrt{bx^2+ax}}+\sqrt{ax+bx^2}\right)-\frac{2a\sqrt{ax+bx^2}}{x}$$
$$\Big\downarrow \text{219}$$
$$b\left(\frac{3a\text{arctanh}\left(\frac{\sqrt{b}x}{\sqrt{ax+bx^2}}\right)}{\sqrt{b}}+\sqrt{ax+bx^2}\right)-\frac{2a\sqrt{ax+bx^2}}{x}$$

```
Int[(a*x + b*x^2)^(3/2)/x^3,x]
```

```
(-2*a*Sqrt[a*x + b*x^2])/x + b*(Sqrt[a*x + b*x^2] + (3*a*ArcTanh[(Sqrt[b]*
x)/Sqrt[a*x + b*x^2]])/Sqrt[b])
```

Defintions of rubi rules used

```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

```
Int[1/Sqrt[(b_.)*(x_) + (c_.)*(x_)^2], x_Symbol] :> Simp[2   Subst[Int[1/(1
 - c*x^2), x], x, x/Sqrt[b*x + c*x^2]], x] /; FreeQ[{b, c}, x]
```

```
Int[((d_.) + (e_.)*(x_))^(m_.)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_
Symbol] :> Simp[-2*e^(2*m + 3)*(Sqrt[a + b*x + c*x^2]/((-2*c*d + b*e)^(m +
2)*(d + e*x))), x] - Simp[e^(2*m + 2)   Int[(1/Sqrt[a + b*x + c*x^2])*Expan
dToSum[((-2*c*d + b*e)^(-m - 1) - ((-c)*d + b*e + c*e*x)^(-m - 1))/(d + e*x
), x], x], x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[c*d^2 - b*d*e + a*e^2, 0]
 && ILtQ[m, 0] && EqQ[m + p, -3/2]
```

```
Int[((d_.) + (e_.)*(x_))*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol
] :> Simp[e*((a + b*x + c*x^2)^(p + 1)/(2*c*(p + 1))), x] + Simp[(2*c*d - b
*e)/(2*c)   Int[(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, p}, x]
 && NeQ[p, -1]
```

Maple [A] (verified)

Time = 0.37 (sec) , antiderivative size = 56, normalized size of antiderivative = 0.88

method	result	size
risch	$-\frac{(bx+a)(-bx+2a)}{\sqrt{x(bx+a)}}+\frac{3a\sqrt{b}\,\ln\left(\frac{\frac{a}{2}+bx}{\sqrt{b}}+\sqrt{b\,x^{2}+ax}\right)}{2}$	56
pseudoelliptic	$\frac{b^{\frac{3}{2}}\sqrt{x(bx+a)}\,x+3\,\operatorname{arctanh}\left(\frac{\sqrt{x(bx+a)}}{x\sqrt{b}}\right)abx-2a\sqrt{b}\,\sqrt{x(bx+a)}}{x\sqrt{b}}$	60
default	$-\frac{2\left(b\,x^{2}+ax\right)^{\frac{5}{2}}}{a\,x^{3}}+\frac{4b\left(\frac{2\left(b\,x^{2}+ax\right)^{\frac{5}{2}}}{a\,x^{2}}-\frac{6b\left(\frac{\left(b\,x^{2}+ax\right)^{\frac{3}{2}}}{3}+\frac{a\left(\frac{(2bx+a)\sqrt{b\,x^{2}+ax}}{4b}-\frac{a^{2}\ln\left(\frac{\frac{a}{2}+bx}{\sqrt{b}}+\sqrt{b\,x^{2}+ax}\right)}{8b^{\frac{3}{2}}}\right)}{2}\right)}{a}\right)}{a}$	125

```
int((b*x^2+a*x)^(3/2)/x^3,x,method=_RETURNVERBOSE)
```

```
-(b*x+a)*(-b*x+2*a)/(x*(b*x+a))^(1/2)+3/2*a*b^(1/2)*ln((1/2*a+b*x)/b^(1/2)
+(b*x^2+a*x)^(1/2))
```

Fricas [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 117, normalized size of antiderivative = 1.83

$$\int\frac{(ax+bx^2)^{3/2}}{x^3}\,dx=\left[\frac{3\,a\sqrt{b}x\log\left(2\,bx+a+2\,\sqrt{bx^2+ax}\sqrt{b}\right)+2\,\sqrt{bx^2+ax}(bx-2\,a)}{2\,x},\right.$$
$$\left.-\frac{3\,a\sqrt{-b}x\arctan\left(\frac{\sqrt{bx^2+ax}\sqrt{-b}}{bx+a}\right)-\sqrt{bx^2+ax}(bx-2\,a)}{x}\right]$$

```
integrate((b*x^2+a*x)^(3/2)/x^3,x, algorithm="fricas")
```

```
[1/2*(3*a*sqrt(b)*x*log(2*b*x + a + 2*sqrt(b*x^2 + a*x)*sqrt(b)) + 2*sqrt(
b*x^2 + a*x)*(b*x - 2*a))/x, -(3*a*sqrt(-b)*x*arctan(sqrt(b*x^2 + a*x)*sqr
t(-b)/(b*x + a)) - sqrt(b*x^2 + a*x)*(b*x - 2*a))/x]
```

Sympy [F]

$$\int \frac{(ax+bx^2)^{3/2}}{x^3}\,dx = \int \frac{(x(a+bx))^{\frac{3}{2}}}{x^3}\,dx$$

```
integrate((b*x**2+a*x)**(3/2)/x**3,x)
```

```
Integral((x*(a + b*x))**(3/2)/x**3, x)
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 62, normalized size of antiderivative = 0.97

$$\int \frac{(ax+bx^2)^{3/2}}{x^3}\,dx = \frac{3}{2}\,a\sqrt{b}\log\left(2\,bx+a+2\,\sqrt{bx^2+ax}\sqrt{b}\right) - \frac{3\,\sqrt{bx^2+ax}a}{x} + \frac{(bx^2+ax)^{\frac{3}{2}}}{x^2}$$

```
integrate((b*x^2+a*x)^(3/2)/x^3,x, algorithm="maxima")
```

```
3/2*a*sqrt(b)*log(2*b*x + a + 2*sqrt(b*x^2 + a*x)*sqrt(b)) - 3*sqrt(b*x^2
+ a*x)*a/x + (b*x^2 + a*x)^(3/2)/x^2
```

Giac [A] (verification not implemented)

Time = 0.30 (sec) , antiderivative size = 76, normalized size of antiderivative = 1.19

$$\int \frac{(ax+bx^2)^{3/2}}{x^3}\,dx = -\frac{3}{2}\,a\sqrt{b}\log\left(\left|-2\left(\sqrt{b}x-\sqrt{bx^2+ax}\right)\sqrt{b}-a\right|\right)+\sqrt{bx^2+ax}b+\frac{2\,a^2}{\sqrt{b}x-\sqrt{bx^2+ax}}$$

```
integrate((b*x^2+a*x)^(3/2)/x^3,x, algorithm="giac")
```

```
-3/2*a*sqrt(b)*log(abs(-2*(sqrt(b)*x - sqrt(b*x^2 + a*x))*sqrt(b) - a)) +
sqrt(b*x^2 + a*x)*b + 2*a^2/(sqrt(b)*x - sqrt(b*x^2 + a*x))
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(ax+bx^2)^{3/2}}{x^3}\,dx = \int \frac{(b\,x^2+a\,x)^{3/2}}{x^3}\,dx$$

```
int((a*x + b*x^2)^(3/2)/x^3,x)
```

```
int((a*x + b*x^2)^(3/2)/x^3, x)
```

Reduce [B] (verification not implemented)

Time = 0.20 (sec) , antiderivative size = 59, normalized size of antiderivative = 0.92

$$\int \frac{(ax+bx^2)^{3/2}}{x^3}\,dx = \frac{-8\sqrt{x}\,\sqrt{bx+a}\,a+4\sqrt{x}\,\sqrt{bx+a}\,bx+12\sqrt{b}\log\left(\frac{\sqrt{bx+a}+\sqrt{x}\,\sqrt{b}}{\sqrt{a}}\right)ax-9\sqrt{b}\,ax}{4x}$$

```
int((b*x^2+a*x)^(3/2)/x^3,x)
```

```
( - 8*sqrt(x)*sqrt(a + b*x)*a + 4*sqrt(x)*sqrt(a + b*x)*b*x + 12*sqrt(b)*l
og((sqrt(a + b*x) + sqrt(x)*sqrt(b))/sqrt(a))*a*x - 9*sqrt(b)*a*x)/(4*x)
```

3.17 $\int \frac{(ax+bx^2)^{3/2}}{x^4} dx$

Optimal result

Integrand size = 17, antiderivative size = 71

$$\int \frac{(ax+bx^2)^{3/2}}{x^4} dx = -\frac{2a\sqrt{ax+bx^2}}{3x^2} - \frac{8b\sqrt{ax+bx^2}}{3x} + 2b^{3/2}\operatorname{arctanh}\left(\frac{\sqrt{b}x}{\sqrt{ax+bx^2}}\right)$$

```
-2/3*a*(b*x^2+a*x)^(1/2)/x^2-8/3*b*(b*x^2+a*x)^(1/2)/x+2*b^(3/2)*arctanh(b
^(1/2)*x/(b*x^2+a*x)^(1/2))
```

Mathematica [A] (verified)

Time = 0.13 (sec) , antiderivative size = 79, normalized size of antiderivative = 1.11

$$\int \frac{(ax+bx^2)^{3/2}}{x^4} dx = -\frac{2\sqrt{x(a+bx)}\left(\sqrt{a+bx}(a+4bx)+3b^{3/2}x^{3/2}\log\left(-\sqrt{b}\sqrt{x}+\sqrt{a+bx}\right)\right)}{3x^2\sqrt{a+bx}}$$

```
Integrate[(a*x + b*x^2)^(3/2)/x^4,x]
```

```
(-2*Sqrt[x*(a + b*x)]*(Sqrt[a + b*x]*(a + 4*b*x) + 3*b^(3/2)*x^(3/2)*Log[-
(Sqrt[b]*Sqrt[x]) + Sqrt[a + b*x]]))/(3*x^2*Sqrt[a + b*x])
```

Rubi [A] (verified)

Time = 0.33 (sec) , antiderivative size = 70, normalized size of antiderivative = 0.99, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.353, Rules used = {1130, 1125, 25, 27, 1091, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(ax+bx^2)^{3/2}}{x^4}dx$$

$$\Big\downarrow \text{1130}$$

$$b\int \frac{\sqrt{bx^2+ax}}{x^2}dx-\frac{2(ax+bx^2)^{3/2}}{3x^3}$$

$$\Big\downarrow \text{1125}$$

$$b\left(-\int -\frac{b}{\sqrt{bx^2+ax}}dx-\frac{2\sqrt{ax+bx^2}}{x}\right)-\frac{2(ax+bx^2)^{3/2}}{3x^3}$$

$$\Big\downarrow \text{25}$$

$$b\left(\int \frac{b}{\sqrt{bx^2+ax}}dx-\frac{2\sqrt{ax+bx^2}}{x}\right)-\frac{2(ax+bx^2)^{3/2}}{3x^3}$$

$$\Big\downarrow \text{27}$$

$$b\left(b\int \frac{1}{\sqrt{bx^2+ax}}dx-\frac{2\sqrt{ax+bx^2}}{x}\right)-\frac{2(ax+bx^2)^{3/2}}{3x^3}$$

$$\Big\downarrow \text{1091}$$

$$b\left(2b\int \frac{1}{1-\frac{bx^2}{bx^2+ax}}d\frac{x}{\sqrt{bx^2+ax}}-\frac{2\sqrt{ax+bx^2}}{x}\right)-\frac{2(ax+bx^2)^{3/2}}{3x^3}$$

$$\Big\downarrow \text{219}$$

$$b\left(2\sqrt{b}\text{arctanh}\left(\frac{\sqrt{b}x}{\sqrt{ax+bx^2}}\right)-\frac{2\sqrt{ax+bx^2}}{x}\right)-\frac{2(ax+bx^2)^{3/2}}{3x^3}$$

```
Int[(a*x + b*x^2)^(3/2)/x^4,x]
```

```
(-2*(a*x + b*x^2)^(3/2))/(3*x^3) + b*((-2*Sqrt[a*x + b*x^2])/x + 2*Sqrt[b]
*ArcTanh[(Sqrt[b]*x)/Sqrt[a*x + b*x^2]])
```

Defintions of rubi rules used

```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

```
Int[1/Sqrt[(b_.)*(x_) + (c_.)*(x_)^2], x_Symbol] :> Simp[2   Subst[Int[1/(1
 - c*x^2), x], x, x/Sqrt[b*x + c*x^2]], x] /; FreeQ[{b, c}, x]
```

```
Int[((d_.) + (e_.)*(x_))^(m_.)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_
Symbol] :> Simp[-2*e^(2*m + 3)*(Sqrt[a + b*x + c*x^2]/((-2*c*d + b*e)^(m +
2)*(d + e*x))), x] - Simp[e^(2*m + 2)   Int[(1/Sqrt[a + b*x + c*x^2])*Expan
dToSum[((-2*c*d + b*e)^(-m - 1) - ((-c)*d + b*e + c*e*x)^(-m - 1))/(d + e*x
), x], x], x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[c*d^2 - b*d*e + a*e^2, 0]
 && ILtQ[m, 0] && EqQ[m + p, -3/2]
```

```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] :> Simp[(d + e*x)^(m + 1)*((a + b*x + c*x^2)^p/(e*(m + p + 1))), x]
- Simp[c*(p/(e^2*(m + p + 1)))   Int[(d + e*x)^(m + 2)*(a + b*x + c*x^2)^(p
 - 1), x], x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[c*d^2 - b*d*e + a*e^2, 0]
 && GtQ[p, 0] && (LtQ[m, -2] || EqQ[m + 2*p + 1, 0]) && NeQ[m + p + 1, 0] &
& IntegerQ[2*p]
```

Maple [A] (verified)

Time = 0.38 (sec) , antiderivative size = 49, normalized size of antiderivative = 0.69

method	result
pseudoelliptic	$\frac{6b^{\frac{3}{2}}\operatorname{arctanh}\left(\frac{\sqrt{x(bx+a)}}{x\sqrt{b}}\right)x^2-2\sqrt{x(bx+a)}\,(4bx+a)}{3x^2}$
risch	$-\frac{2(bx+a)(4bx+a)}{3x\sqrt{x(bx+a)}}+b^{\frac{3}{2}}\ln\left(\frac{\frac{a}{2}+bx}{\sqrt{b}}+\sqrt{b\,x^2+ax}\right)$
default	$-\frac{2\left(b\,x^2+ax\right)^{\frac{5}{2}}}{3a\,x^4}+\frac{2b\left(-\frac{2\left(b\,x^2+ax\right)^{\frac{5}{2}}}{a\,x^3}+\frac{4b\left(\frac{2\left(b\,x^2+ax\right)^{\frac{5}{2}}}{a\,x^2}-\frac{6b\left(\frac{\left(b\,x^2+ax\right)^{\frac{3}{2}}}{3}+\frac{a\left(\frac{(2bx+a)\sqrt{b\,x^2+ax}}{4b}-\frac{a^2\ln\left(\frac{\frac{a}{2}+bx}{\sqrt{b}}+\sqrt{b\,x^2+}\right.}{8b^{\frac{3}{2}}}\right.}{2}\right.}{a}\right.}{a}\right.}{3a}$

```
int((b*x^2+a*x)^(3/2)/x^4,x,method=_RETURNVERBOSE)
```

```
1/3*(6*b^(3/2)*arctanh((x*(b*x+a))^(1/2)/x/b^(1/2))*x^2-2*(x*(b*x+a))^(1/2
)*(4*b*x+a))/x^2
```

Fricas [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 117, normalized size of antiderivative = 1.65

$$\int \frac{(ax+bx^2)^{3/2}}{x^4}\,dx = \left[\frac{3\,b^{\frac{3}{2}}x^2\log\left(2\,bx+a+2\,\sqrt{bx^2+ax}\sqrt{b}\right)-2\,\sqrt{bx^2+ax}(4\,bx+a)}{3\,x^2},\right.$$
$$\left.-\frac{2\left(3\,\sqrt{-b}bx^2\arctan\left(\frac{\sqrt{bx^2+ax}\sqrt{-b}}{bx+a}\right)+\sqrt{bx^2+ax}(4\,bx+a)\right)}{3\,x^2}\right]$$

```
integrate((b*x^2+a*x)^(3/2)/x^4,x, algorithm="fricas")
```

```
[1/3*(3*b^(3/2)*x^2*log(2*b*x + a + 2*sqrt(b*x^2 + a*x)*sqrt(b)) - 2*sqrt(
b*x^2 + a*x)*(4*b*x + a))/x^2, -2/3*(3*sqrt(-b)*b*x^2*arctan(sqrt(b*x^2 +
a*x)*sqrt(-b)/(b*x + a)) + sqrt(b*x^2 + a*x)*(4*b*x + a))/x^2]
```

Sympy [F]

$$\int \frac{(ax+bx^2)^{3/2}}{x^4}\,dx = \int \frac{(x(a+bx))^{\frac{3}{2}}}{x^4}\,dx$$

```
integrate((b*x**2+a*x)**(3/2)/x**4,x)
```

```
Integral((x*(a + b*x))**(3/2)/x**4, x)
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 78, normalized size of antiderivative = 1.10

$$\int \frac{(ax+bx^2)^{3/2}}{x^4}\,dx = b^{\frac{3}{2}}\log\left(2\,bx+a+2\,\sqrt{bx^2+ax}\sqrt{b}\right) - \frac{7\,\sqrt{bx^2+ax}b}{3\,x} - \frac{\sqrt{bx^2+ax}a}{3\,x^2} - \frac{(bx^2+ax)^{\frac{3}{2}}}{3\,x^3}$$

```
integrate((b*x^2+a*x)^(3/2)/x^4,x, algorithm="maxima")
```

```
b^(3/2)*log(2*b*x + a + 2*sqrt(b*x^2 + a*x)*sqrt(b)) - 7/3*sqrt(b*x^2 + a*
x)*b/x - 1/3*sqrt(b*x^2 + a*x)*a/x^2 - 1/3*(b*x^2 + a*x)^(3/2)/x^3
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 115 vs. 2(57) = 114.

Time = 0.17 (sec) , antiderivative size = 115, normalized size of antiderivative = 1.62

$$\int \frac{(ax+bx^2)^{3/2}}{x^4}\,dx = -b^{\frac{3}{2}}\log\left(\left|-2\left(\sqrt{b}x-\sqrt{bx^2+ax}\right)\sqrt{b}-a\right|\right) + \frac{2\left(6\left(\sqrt{b}x-\sqrt{bx^2+ax}\right)^2ab+3\left(\sqrt{b}x-\sqrt{bx^2+ax}\right)a^2\sqrt{b}+a^3\right)}{3\left(\sqrt{b}x-\sqrt{bx^2+ax}\right)^3}$$

```
integrate((b*x^2+a*x)^(3/2)/x^4,x, algorithm="giac")
```

```
-b^(3/2)*log(abs(-2*(sqrt(b)*x - sqrt(b*x^2 + a*x))*sqrt(b) - a)) + 2/3*(6
*(sqrt(b)*x - sqrt(b*x^2 + a*x))^2*a*b + 3*(sqrt(b)*x - sqrt(b*x^2 + a*x))
*a^2*sqrt(b) + a^3)/(sqrt(b)*x - sqrt(b*x^2 + a*x))^3
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(ax+bx^2)^{3/2}}{x^4}\,dx=\int \frac{(b\,x^2+a\,x)^{3/2}}{x^4}\,dx$$

```
int((a*x + b*x^2)^(3/2)/x^4,x)
```

```
int((a*x + b*x^2)^(3/2)/x^4, x)
```

Reduce [B] (verification not implemented)

Time = 0.22 (sec) , antiderivative size = 55, normalized size of antiderivative = 0.77

$$\int \frac{(ax+bx^2)^{3/2}}{x^4}\,dx=\frac{-\frac{2\sqrt{x}\,\sqrt{bx+a}\,a}{3}-\frac{8\sqrt{x}\,\sqrt{bx+a}\,bx}{3}+2\sqrt{b}\log\left(\frac{\sqrt{bx+a}+\sqrt{x}\,\sqrt{b}}{\sqrt{a}}\right)b\,x^2}{x^2}$$

```
int((b*x^2+a*x)^(3/2)/x^4,x)
```

```
(2*( - sqrt(x)*sqrt(a + b*x)*a - 4*sqrt(x)*sqrt(a + b*x)*b*x + 3*sqrt(b)*l
og((sqrt(a + b*x) + sqrt(x)*sqrt(b))/sqrt(a))*b*x**2))/(3*x**2)
```

3.18 $\int \frac{(ax+bx^2)^{3/2}}{x^5} dx$

Optimal result

Integrand size = 17, antiderivative size = 23

$$\int \frac{(ax+bx^2)^{3/2}}{x^5} dx = -\frac{2(ax+bx^2)^{5/2}}{5ax^5}$$

```
-2/5*(b*x^2+a*x)^(5/2)/a/x^5
```

Mathematica [A] (verified)

Time = 0.04 (sec) , antiderivative size = 21, normalized size of antiderivative = 0.91

$$\int \frac{(ax+bx^2)^{3/2}}{x^5} dx = -\frac{2(x(a+bx))^{5/2}}{5ax^5}$$

```
Integrate[(a*x + b*x^2)^(3/2)/x^5,x]
```

```
(-2*(x*(a + b*x))^(5/2))/(5*a*x^5)
```

Rubi [A] (verified)

Time = 0.23 (sec) , antiderivative size = 23, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}} = 0.059$, Rules used = $\{1123\}$

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(ax+bx^2)^{3/2}}{x^5} \, dx$$

$$\Big\downarrow \text{1123}$$

$$-\frac{2(ax+bx^2)^{5/2}}{5ax^5}$$

```
Int[(a*x + b*x^2)^(3/2)/x^5,x]
```

```
(-2*(a*x + b*x^2)^(5/2))/(5*a*x^5)
```

Defintions of rubi rules used

```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] :> Simp[e*(d + e*x)^m*((a + b*x + c*x^2)^(p + 1)/((p + 1)*(2*c*d - b
*e))), x] /; FreeQ[{a, b, c, d, e, m, p}, x] && EqQ[c*d^2 - b*d*e + a*e^2,
0] && EqQ[m + 2*p + 2, 0]
```

Maple [A] (verified)

Time = 0.37 (sec) , antiderivative size = 20, normalized size of antiderivative = 0.87

method	result	size
default	$-\frac{2(b\,x^{2}+a x)^{\frac{5}{2}}}{5 a\,x^{5}}$	20
gosper	$-\frac{2(bx+a)(b\,x^{2}+ax)^{\frac{3}{2}}}{5x^{4}a}$	25
pseudoelliptic	$-\frac{2(bx+a)^{2}\sqrt{x(bx+a)}}{5x^{3}a}$	25
orering	$-\frac{2(bx+a)(b\,x^{2}+ax)^{\frac{3}{2}}}{5x^{4}a}$	25
trager	$-\frac{2(b^{2}x^{2}+2abx+a^{2})\sqrt{b\,x^{2}+ax}}{5a\,x^{3}}$	36
risch	$-\frac{2(bx+a)(b^{2}x^{2}+2abx+a^{2})}{5x^{2}\sqrt{x(bx+a)}\,a}$	39

```
int((b*x^2+a*x)^(3/2)/x^5,x,method=_RETURNVERBOSE)
```

```
-2/5*(b*x^2+a*x)^(5/2)/a/x^5
```

Fricas [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 35, normalized size of antiderivative = 1.52

$$\int \frac{(ax+bx^2)^{3/2}}{x^5}\,dx = -\frac{2\,(b^2x^2+2\,abx+a^2)\sqrt{bx^2+ax}}{5\,ax^3}$$

```
integrate((b*x^2+a*x)^(3/2)/x^5,x, algorithm="fricas")
```

```
-2/5*(b^2*x^2 + 2*a*b*x + a^2)*sqrt(b*x^2 + a*x)/(a*x^3)
```

Sympy [F]

$$\int \frac{(ax+bx^2)^{3/2}}{x^5}\,dx = \int \frac{(x(a+bx))^{\frac{3}{2}}}{x^5}\,dx$$

```
integrate((b*x**2+a*x)**(3/2)/x**5,x)
```

```
Integral((x*(a + b*x))**(3/2)/x**5, x)
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 73 vs. $2(19) = 38$.

Time = 0.03 (sec) , antiderivative size = 73, normalized size of antiderivative = 3.17

$$\int \frac{(ax+bx^2)^{3/2}}{x^5}\,dx = -\frac{2\sqrt{bx^2+ax}b^2}{5\,ax} + \frac{\sqrt{bx^2+ax}b}{5\,x^2} + \frac{3\sqrt{bx^2+ax}a}{5\,x^3} - \frac{(bx^2+ax)^{\frac{3}{2}}}{x^4}$$

```
integrate((b*x^2+a*x)^(3/2)/x^5,x, algorithm="maxima")
```

```
-2/5*sqrt(b*x^2 + a*x)*b^2/(a*x) + 1/5*sqrt(b*x^2 + a*x)*b/x^2 + 3/5*sqrt(
b*x^2 + a*x)*a/x^3 - (b*x^2 + a*x)^(3/2)/x^4
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 134 vs. $2(19) = 38$.

Time = 0.11 (sec) , antiderivative size = 134, normalized size of antiderivative = 5.83

$$\int \frac{(ax+bx^2)^{3/2}}{x^5}\,dx = \frac{2\left(5\left(\sqrt{b}x-\sqrt{bx^2+ax}\right)^4 b^2 + 10\left(\sqrt{b}x-\sqrt{bx^2+ax}\right)^3 ab^{\frac{3}{2}} + 10\left(\sqrt{b}x-\sqrt{bx^2+a\right.}\right.}{5\left(\sqrt{b}x-\sqrt{bx^2+ax}\right)^5}$$

```
integrate((b*x^2+a*x)^(3/2)/x^5,x, algorithm="giac")
```

```
2/5*(5*(sqrt(b)*x - sqrt(b*x^2 + a*x))^4*b^2 + 10*(sqrt(b)*x - sqrt(b*x^2
+ a*x))^3*a*b^(3/2) + 10*(sqrt(b)*x - sqrt(b*x^2 + a*x))^2*a^2*b + 5*(sqrt
(b)*x - sqrt(b*x^2 + a*x))*a^3*sqrt(b) + a^4)/(sqrt(b)*x - sqrt(b*x^2 + a*
x))^5
```

Mupad [B] (verification not implemented)

Time = 9.01 (sec) , antiderivative size = 26, normalized size of antiderivative = 1.13

$$\int \frac{(ax+bx^2)^{3/2}}{x^5}\,dx = -\frac{2\sqrt{b\,x^2+a\,x}\,(a+b\,x)^2}{5\,a\,x^3}$$

```
int((a*x + b*x^2)^(3/2)/x^5,x)
```

```
-(2*(a*x + b*x^2)^(1/2)*(a + b*x)^2)/(5*a*x^3)
```

Reduce [B] (verification not implemented)

Time = 0.24 (sec) , antiderivative size = 61, normalized size of antiderivative = 2.65

$$\int \frac{(ax+bx^2)^{3/2}}{x^5}\,dx = \frac{-\frac{2\sqrt{x}\,\sqrt{bx+a}\,a^2}{5}-\frac{4\sqrt{x}\,\sqrt{bx+a}\,abx}{5}-\frac{2\sqrt{x}\,\sqrt{bx+a}\,b^2x^2}{5}-\frac{2\sqrt{b}\,b^2x^3}{5}}{a\,x^3}$$

```
int((b*x^2+a*x)^(3/2)/x^5,x)
```

```
(2*( - sqrt(x)*sqrt(a + b*x)*a**2 - 2*sqrt(x)*sqrt(a + b*x)*a*b*x - sqrt(x
)*sqrt(a + b*x)*b**2*x**2 - sqrt(b)*b**2*x**3))/(5*a*x**3)
```

3.19 $\int \frac{(ax+bx^2)^{3/2}}{x^6}\,dx$

Optimal result

Integrand size = 17, antiderivative size = 48

$$\int \frac{(ax+bx^2)^{3/2}}{x^6}\,dx = -\frac{2(ax+bx^2)^{5/2}}{7ax^6} + \frac{4b(ax+bx^2)^{5/2}}{35a^2x^5}$$

```
-2/7*(b*x^2+a*x)^(5/2)/a/x^6+4/35*b*(b*x^2+a*x)^(5/2)/a^2/x^5
```

Mathematica [A] (verified)

Time = 0.12 (sec) , antiderivative size = 29, normalized size of antiderivative = 0.60

$$\int \frac{(ax+bx^2)^{3/2}}{x^6}\,dx = -\frac{2(5a-2bx)(x(a+bx))^{5/2}}{35a^2x^6}$$

```
Integrate[(a*x + b*x^2)^(3/2)/x^6,x]
```

```
(-2*(5*a - 2*b*x)*(x*(a + b*x))^(5/2))/(35*a^2*x^6)
```

Rubi [A] (verified)

Time = 0.28 (sec) , antiderivative size = 48, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.118$, Rules used = {1129, 1123}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(ax+bx^2)^{3/2}}{x^6}\,dx$$
$$\Big\downarrow \text{1129}$$
$$-\frac{2b\int \frac{(bx^2+ax)^{3/2}}{x^5}dx}{7a}-\frac{2(ax+bx^2)^{5/2}}{7ax^6}$$
$$\Big\downarrow \text{1123}$$
$$\frac{4b(ax+bx^2)^{5/2}}{35a^2x^5}-\frac{2(ax+bx^2)^{5/2}}{7ax^6}$$

```
Int[(a*x + b*x^2)^(3/2)/x^6,x]
```

```
(-2*(a*x + b*x^2)^(5/2))/(7*a*x^6) + (4*b*(a*x + b*x^2)^(5/2))/(35*a^2*x^5
)
```

Defintions of rubi rules used

```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] :> Simp[e*(d + e*x)^m*((a + b*x + c*x^2)^(p + 1)/((p + 1)*(2*c*d - b
*e))), x] /; FreeQ[{a, b, c, d, e, m, p}, x] && EqQ[c*d^2 - b*d*e + a*e^2,
0] && EqQ[m + 2*p + 2, 0]
```

```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] :> Simp[(-e)*(d + e*x)^m*((a + b*x + c*x^2)^(p + 1)/((m + p + 1)*(2*
c*d - b*e))), x] + Simp[c*(Simplify[m + 2*p + 2]/((m + p + 1)*(2*c*d - b*e)
))   Int[(d + e*x)^(m + 1)*(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d
, e, m, p}, x] && EqQ[c*d^2 - b*d*e + a*e^2, 0] && ILtQ[Simplify[m + 2*p +
2], 0]
```

Maple [A] (verified)

Time = 0.38 (sec) , antiderivative size = 31, normalized size of antiderivative = 0.65

method	result	size
pseudoelliptic	$-\frac{2\sqrt{x(bx+a)}\,(bx+a)^{2}\left(-\frac{2bx}{5}+a\right)}{7x^{4}a^{2}}$	31
gosper	$-\frac{2(bx+a)(-2bx+5a)\left(b\,x^{2}+ax\right)^{\frac{3}{2}}}{35a^{2}x^{5}}$	33
orering	$-\frac{2(bx+a)(-2bx+5a)\left(b\,x^{2}+ax\right)^{\frac{3}{2}}}{35a^{2}x^{5}}$	33
default	$-\frac{2\left(b\,x^{2}+ax\right)^{\frac{5}{2}}}{7a\,x^{6}}+\frac{4b\left(b\,x^{2}+ax\right)^{\frac{5}{2}}}{35a^{2}x^{5}}$	41
trager	$-\frac{2\left(-2b^{3}x^{3}+a\,b^{2}x^{2}+8a^{2}bx+5a^{3}\right)\sqrt{b\,x^{2}+ax}}{35x^{4}a^{2}}$	49
risch	$-\frac{2(bx+a)\left(-2b^{3}x^{3}+a\,b^{2}x^{2}+8a^{2}bx+5a^{3}\right)}{35x^{3}\sqrt{x(bx+a)}\,a^{2}}$	52

```
int((b*x^2+a*x)^(3/2)/x^6,x,method=_RETURNVERBOSE)
```

```
-2/7*(x*(b*x+a))^(1/2)*(b*x+a)^2*(-2/5*b*x+a)/x^4/a^2
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 49, normalized size of antiderivative = 1.02

$$\int \frac{(ax+bx^2)^{3/2}}{x^6}\,dx = \frac{2\,(2\,b^3x^3 - ab^2x^2 - 8\,a^2bx - 5\,a^3)\sqrt{bx^2+ax}}{35\,a^2x^4}$$

```
integrate((b*x^2+a*x)^(3/2)/x^6,x, algorithm="fricas")
```

```
2/35*(2*b^3*x^3 - a*b^2*x^2 - 8*a^2*b*x - 5*a^3)*sqrt(b*x^2 + a*x)/(a^2*x^
4)
```

Sympy [F]

$$\int \frac{(ax + bx^2)^{3/2}}{x^6}\, dx = \int \frac{(x(a + bx))^{\frac{3}{2}}}{x^6}\, dx$$

```
integrate((b*x**2+a*x)**(3/2)/x**6,x)
```

```
Integral((x*(a + b*x))**(3/2)/x**6, x)
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 95 vs. $2(40) = 80$.

Time = 0.03 (sec) , antiderivative size = 95, normalized size of antiderivative = 1.98

$$\int \frac{(ax + bx^2)^{3/2}}{x^6}\, dx = \frac{4\sqrt{bx^2 + ax}b^3}{35\, a^2 x} - \frac{2\sqrt{bx^2 + ax}b^2}{35\, ax^2} + \frac{3\sqrt{bx^2 + ax}b}{70\, x^3} + \frac{3\sqrt{bx^2 + ax}a}{14\, x^4} - \frac{(bx^2 + ax)^{\frac{3}{2}}}{2\, x^5}$$

```
integrate((b*x^2+a*x)^(3/2)/x^6,x, algorithm="maxima")
```

```
4/35*sqrt(b*x^2 + a*x)*b^3/(a^2*x) - 2/35*sqrt(b*x^2 + a*x)*b^2/(a*x^2) +
3/70*sqrt(b*x^2 + a*x)*b/x^3 + 3/14*sqrt(b*x^2 + a*x)*a/x^4 - 1/2*(b*x^2 +
 a*x)^(3/2)/x^5
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 165 vs. $2(40) = 80$.

Time = 0.27 (sec) , antiderivative size = 165, normalized size of antiderivative = 3.44

$$\int \frac{(ax+bx^2)^{3/2}}{x^6}\,dx = \frac{2\left(35\left(\sqrt{b}x-\sqrt{bx^2+ax}\right)^5 b^{\frac{5}{2}} + 105\left(\sqrt{b}x-\sqrt{bx^2+ax}\right)^4 ab^2 + 140\left(\sqrt{b}x-\sqrt{bx^2}\right.\right.}{35\left(\sqrt{b}x-\right.}$$

```
integrate((b*x^2+a*x)^(3/2)/x^6,x, algorithm="giac")
```

```
2/35*(35*(sqrt(b)*x - sqrt(b*x^2 + a*x))^5*b^(5/2) + 105*(sqrt(b)*x - sqrt
(b*x^2 + a*x))^4*a*b^2 + 140*(sqrt(b)*x - sqrt(b*x^2 + a*x))^3*a^2*b^(3/2)
 + 98*(sqrt(b)*x - sqrt(b*x^2 + a*x))^2*a^3*b + 35*(sqrt(b)*x - sqrt(b*x^2
 + a*x))*a^4*sqrt(b) + 5*a^5)/(sqrt(b)*x - sqrt(b*x^2 + a*x))^7
```

Mupad [B] (verification not implemented)

Time = 9.64 (sec) , antiderivative size = 79, normalized size of antiderivative = 1.65

$$\int \frac{(ax+bx^2)^{3/2}}{x^6}\,dx = \frac{4\,b^3\,\sqrt{b\,x^2+a\,x}}{35\,a^2\,x} - \frac{16\,b\,\sqrt{b\,x^2+a\,x}}{35\,x^3} - \frac{2\,b^2\,\sqrt{b\,x^2+a\,x}}{35\,a\,x^2} - \frac{2\,a\,\sqrt{b\,x^2+a\,x}}{7\,x^4}$$

```
int((a*x + b*x^2)^(3/2)/x^6,x)
```

```
(4*b^3*(a*x + b*x^2)^(1/2))/(35*a^2*x) - (16*b*(a*x + b*x^2)^(1/2))/(35*x^
3) - (2*b^2*(a*x + b*x^2)^(1/2))/(35*a*x^2) - (2*a*(a*x + b*x^2)^(1/2))/(7
*x^4)
```

Reduce [B] (verification not implemented)

Time = 0.25 (sec) , antiderivative size = 80, normalized size of antiderivative = 1.67

$$\int \frac{(ax+bx^2)^{3/2}}{x^6}\,dx = \frac{-\frac{2\sqrt{x}\,\sqrt{bx+a}\,a^3}{7}-\frac{16\sqrt{x}\,\sqrt{bx+a}\,a^2bx}{35}-\frac{2\sqrt{x}\,\sqrt{bx+a}\,a\,b^2x^2}{35}+\frac{4\sqrt{x}\,\sqrt{bx+a}\,b^3x^3}{35}-\frac{4\sqrt{b}\,b^3x^4}{35}}{a^2x^4}$$

```
int((b*x^2+a*x)^(3/2)/x^6,x)
```

```
(2*( - 5*sqrt(x)*sqrt(a + b*x)*a**3 - 8*sqrt(x)*sqrt(a + b*x)*a**2*b*x - s
qrt(x)*sqrt(a + b*x)*a*b**2*x**2 + 2*sqrt(x)*sqrt(a + b*x)*b**3*x**3 - 2*s
qrt(b)*b**3*x**4))/(35*a**2*x**4)
```

3.20 $\int \frac{(ax+bx^2)^{3/2}}{x^7} dx$

Optimal result

Integrand size = 17, antiderivative size = 74

$$\int \frac{(ax+bx^2)^{3/2}}{x^7} dx = -\frac{2(ax+bx^2)^{5/2}}{9ax^7} + \frac{8b(ax+bx^2)^{5/2}}{63a^2x^6} - \frac{16b^2(ax+bx^2)^{5/2}}{315a^3x^5}$$

```
-2/9*(b*x^2+a*x)^(5/2)/a/x^7+8/63*b*(b*x^2+a*x)^(5/2)/a^2/x^6-16/315*b^2*(
b*x^2+a*x)^(5/2)/a^3/x^5
```

Mathematica [A] (verified)

Time = 0.13 (sec) , antiderivative size = 40, normalized size of antiderivative = 0.54

$$\int \frac{(ax+bx^2)^{3/2}}{x^7} dx = -\frac{2(x(a+bx))^{5/2}(35a^2-20abx+8b^2x^2)}{315a^3x^7}$$

```
Integrate[(a*x + b*x^2)^(3/2)/x^7,x]
```

```
(-2*(x*(a + b*x))^(5/2)*(35*a^2 - 20*a*b*x + 8*b^2*x^2))/(315*a^3*x^7)
```

Rubi [A] (verified)

Time = 0.33 (sec) , antiderivative size = 80, normalized size of antiderivative = 1.08, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.176, Rules used = {1129, 1129, 1123}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(ax+bx^2)^{3/2}}{x^7} \, dx$$

$$\Big\downarrow \ 1129$$

$$-\frac{4b\int \frac{(bx^2+ax)^{3/2}}{x^6}dx}{9a}-\frac{2(ax+bx^2)^{5/2}}{9ax^7}$$

$$\Big\downarrow \ 1129$$

$$-\frac{4b\left(-\frac{2b\int \frac{(bx^2+ax)^{3/2}}{x^5}dx}{7a}-\frac{2(ax+bx^2)^{5/2}}{7ax^6}\right)}{9a}-\frac{2(ax+bx^2)^{5/2}}{9ax^7}$$

$$\Big\downarrow \ 1123$$

$$-\frac{4b\left(\frac{4b(ax+bx^2)^{5/2}}{35a^2x^5}-\frac{2(ax+bx^2)^{5/2}}{7ax^6}\right)}{9a}-\frac{2(ax+bx^2)^{5/2}}{9ax^7}$$

```
Int[(a*x + b*x^2)^(3/2)/x^7,x]
```

```
(-2*(a*x + b*x^2)^(5/2))/(9*a*x^7) - (4*b*((-2*(a*x + b*x^2)^(5/2))/(7*a*x
^6) + (4*b*(a*x + b*x^2)^(5/2))/(35*a^2*x^5)))/(9*a)
```

Defintions of rubi rules used

```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] :> Simp[e*(d + e*x)^m*((a + b*x + c*x^2)^(p + 1)/((p + 1)*(2*c*d - b
*e))), x] /; FreeQ[{a, b, c, d, e, m, p}, x] && EqQ[c*d^2 - b*d*e + a*e^2,
0] && EqQ[m + 2*p + 2, 0]
```

```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] :> Simp[(-e)*(d + e*x)^m*((a + b*x + c*x^2)^(p + 1)/((m + p + 1)*(2*
c*d - b*e))), x] + Simp[c*(Simplify[m + 2*p + 2]/((m + p + 1)*(2*c*d - b*e)
))   Int[(d + e*x)^(m + 1)*(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d
, e, m, p}, x] && EqQ[c*d^2 - b*d*e + a*e^2, 0] && ILtQ[Simplify[m + 2*p +
2], 0]
```

Maple [A] (verified)

Time = 0.40 (sec) , antiderivative size = 42, normalized size of antiderivative = 0.57

method	result	size
pseudoelliptic	$-\frac{2\sqrt{x(bx+a)}\,(bx+a)^2\left(\frac{8}{35}b^2x^2-\frac{4}{7}abx+a^2\right)}{9x^5a^3}$	42
gosper	$-\frac{2(bx+a)\left(8b^2x^2-20abx+35a^2\right)\left(b\,x^2+ax\right)^{\frac{3}{2}}}{315x^6a^3}$	44
orering	$-\frac{2(bx+a)\left(8b^2x^2-20abx+35a^2\right)\left(b\,x^2+ax\right)^{\frac{3}{2}}}{315x^6a^3}$	44
trager	$-\frac{2\left(8b^4x^4-4a\,b^3x^3+3a^2b^2x^2+50a^3bx+35a^4\right)\sqrt{b\,x^2+ax}}{315a^3x^5}$	61
risch	$-\frac{2(bx+a)\left(8b^4x^4-4a\,b^3x^3+3a^2b^2x^2+50a^3bx+35a^4\right)}{315x^4\sqrt{x(bx+a)}\,a^3}$	64
default	$-\frac{2\left(b\,x^2+ax\right)^{\frac{5}{2}}}{9a\,x^7}-\frac{4b\left(-\frac{2\left(b\,x^2+ax\right)^{\frac{5}{2}}}{7a\,x^6}+\frac{4b\left(b\,x^2+ax\right)^{\frac{5}{2}}}{35a^2x^5}\right)}{9a}$	67

```
int((b*x^2+a*x)^(3/2)/x^7,x,method=_RETURNVERBOSE)
```

```
-2/9*(x*(b*x+a))^(1/2)*(b*x+a)^2*(8/35*b^2*x^2-4/7*a*b*x+a^2)/x^5/a^3
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 60, normalized size of antiderivative = 0.81

$$\int \frac{(ax+bx^2)^{3/2}}{x^7}\,dx = -\frac{2\,(8\,b^4x^4-4\,ab^3x^3+3\,a^2b^2x^2+50\,a^3bx+35\,a^4)\sqrt{bx^2+ax}}{315\,a^3x^5}$$

```
integrate((b*x^2+a*x)^(3/2)/x^7,x, algorithm="fricas")
```

```
-2/315*(8*b^4*x^4 - 4*a*b^3*x^3 + 3*a^2*b^2*x^2 + 50*a^3*b*x + 35*a^4)*sqr
t(b*x^2 + a*x)/(a^3*x^5)
```

Sympy [F]

$$\int \frac{(ax+bx^2)^{3/2}}{x^7}\,dx = \int \frac{(x(a+bx))^{\frac{3}{2}}}{x^7}\,dx$$

```
integrate((b*x**2+a*x)**(3/2)/x**7,x)
```

```
Integral((x*(a + b*x))**(3/2)/x**7, x)
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 117, normalized size of antiderivative = 1.58

$$\begin{aligned}\int \frac{(ax+bx^2)^{3/2}}{x^7}\,dx &= -\frac{16\,\sqrt{bx^2+ax}b^4}{315\,a^3x}+\frac{8\,\sqrt{bx^2+ax}b^3}{315\,a^2x^2}\\ &\quad-\frac{2\,\sqrt{bx^2+ax}b^2}{105\,ax^3}+\frac{\sqrt{bx^2+ax}b}{63\,x^4}+\frac{\sqrt{bx^2+ax}a}{9\,x^5}-\frac{(bx^2+ax)^{\frac{3}{2}}}{3\,x^6}\end{aligned}$$

```
integrate((b*x^2+a*x)^(3/2)/x^7,x, algorithm="maxima")
```

```
-16/315*sqrt(b*x^2 + a*x)*b^4/(a^3*x) + 8/315*sqrt(b*x^2 + a*x)*b^3/(a^2*x
^2) - 2/105*sqrt(b*x^2 + a*x)*b^2/(a*x^3) + 1/63*sqrt(b*x^2 + a*x)*b/x^4 +
 1/9*sqrt(b*x^2 + a*x)*a/x^5 - 1/3*(b*x^2 + a*x)^(3/2)/x^6
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 194 vs. $2(62) = 124$.

Time = 0.28 (sec) , antiderivative size = 194, normalized size of antiderivative = 2.62

$$\int \frac{(ax+bx^2)^{3/2}}{x^7}\,dx = \frac{2\left(420\left(\sqrt{b}x-\sqrt{bx^2+ax}\right)^6 b^3 + 1575\left(\sqrt{b}x-\sqrt{bx^2+ax}\right)^5 ab^{\frac{5}{2}} + 2583\left(\sqrt{b}x-\sqrt{\cdots}\right.\right.}{}$$

```
integrate((b*x^2+a*x)^(3/2)/x^7,x, algorithm="giac")
```

```
2/315*(420*(sqrt(b)*x - sqrt(b*x^2 + a*x))^6*b^3 + 1575*(sqrt(b)*x - sqrt(
b*x^2 + a*x))^5*a*b^(5/2) + 2583*(sqrt(b)*x - sqrt(b*x^2 + a*x))^4*a^2*b^2
 + 2310*(sqrt(b)*x - sqrt(b*x^2 + a*x))^3*a^3*b^(3/2) + 1170*(sqrt(b)*x -
sqrt(b*x^2 + a*x))^2*a^4*b + 315*(sqrt(b)*x - sqrt(b*x^2 + a*x))*a^5*sqrt(
b) + 35*a^6)/(sqrt(b)*x - sqrt(b*x^2 + a*x))^9
```

Mupad [B] (verification not implemented)

Time = 10.23 (sec) , antiderivative size = 101, normalized size of antiderivative = 1.36

$$\int \frac{(ax+bx^2)^{3/2}}{x^7}\,dx = \frac{8\,b^3\,\sqrt{b\,x^2+a\,x}}{315\,a^2\,x^2} - \frac{20\,b\,\sqrt{b\,x^2+a\,x}}{63\,x^4} - \frac{2\,b^2\,\sqrt{b\,x^2+a\,x}}{105\,a\,x^3} - \frac{2\,a\,\sqrt{b\,x^2+a\,x}}{9\,x^5} - \frac{16\,b^4\,\sqrt{b\,x^2+a\,x}}{315\,a^3\,x}$$

```
int((a*x + b*x^2)^(3/2)/x^7,x)
```

```
(8*b^3*(a*x + b*x^2)^(1/2))/(315*a^2*x^2) - (20*b*(a*x + b*x^2)^(1/2))/(63
*x^4) - (2*b^2*(a*x + b*x^2)^(1/2))/(105*a*x^3) - (2*a*(a*x + b*x^2)^(1/2)
)/(9*x^5) - (16*b^4*(a*x + b*x^2)^(1/2))/(315*a^3*x)
```

Reduce [B] (verification not implemented)

Time = 0.24 (sec) , antiderivative size = 99, normalized size of antiderivative = 1.34

$$\int \frac{(ax+bx^2)^{3/2}}{x^7}\,dx = \frac{-\frac{2\sqrt{x}\,\sqrt{bx+a}\,a^4}{9} - \frac{20\sqrt{x}\,\sqrt{bx+a}\,a^3bx}{63} - \frac{2\sqrt{x}\,\sqrt{bx+a}\,a^2b^2x^2}{105} + \frac{8\sqrt{x}\,\sqrt{bx+a}\,a\,b^3x^3}{315} - \frac{16\sqrt{x}\,\sqrt{bx+a}\,b^4x^4}{315} + }{a^3x^5}$$

```
int((b*x^2+a*x)^(3/2)/x^7,x)
```

```
(2*( - 35*sqrt(x)*sqrt(a + b*x)*a**4 - 50*sqrt(x)*sqrt(a + b*x)*a**3*b*x -
 3*sqrt(x)*sqrt(a + b*x)*a**2*b**2*x**2 + 4*sqrt(x)*sqrt(a + b*x)*a*b**3*x
**3 - 8*sqrt(x)*sqrt(a + b*x)*b**4*x**4 + 8*sqrt(b)*b**4*x**5))/(315*a**3*
x**5)
```

3.21 $\int \frac{(ax+bx^2)^{3/2}}{x^8} dx$

Optimal result

Integrand size = 17, antiderivative size = 100

$$\int \frac{(ax+bx^2)^{3/2}}{x^8} dx = -\frac{2(ax+bx^2)^{5/2}}{11ax^8} + \frac{4b(ax+bx^2)^{5/2}}{33a^2x^7} - \frac{16b^2(ax+bx^2)^{5/2}}{231a^3x^6} + \frac{32b^3(ax+bx^2)^{5/2}}{1155a^4x^5}$$

```
-2/11*(b*x^2+a*x)^(5/2)/a/x^8+4/33*b*(b*x^2+a*x)^(5/2)/a^2/x^7-16/231*b^2*
(b*x^2+a*x)^(5/2)/a^3/x^6+32/1155*b^3*(b*x^2+a*x)^(5/2)/a^4/x^5
```

Mathematica [A] (verified)

Time = 0.15 (sec) , antiderivative size = 51, normalized size of antiderivative = 0.51

$$\int \frac{(ax+bx^2)^{3/2}}{x^8} dx = -\frac{2(x(a+bx))^{5/2}(105a^3-70a^2bx+40ab^2x^2-16b^3x^3)}{1155a^4x^8}$$

```
Integrate[(a*x + b*x^2)^(3/2)/x^8,x]
```

```
(-2*(x*(a + b*x))^(5/2)*(105*a^3 - 70*a^2*b*x + 40*a*b^2*x^2 - 16*b^3*x^3)
)/(1155*a^4*x^8)
```

Rubi [A] (verified)

Time = 0.39 (sec) , antiderivative size = 112, normalized size of antiderivative = 1.12, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.235$, Rules used = {1129, 1129, 1129, 1123}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\left(ax+bx^2\right)^{3/2}}{x^8}dx$$

$$\Big\downarrow \text{ 1129}$$

$$-\frac{6b\int \frac{\left(bx^2+ax\right)^{3/2}}{x^7}dx}{11a}-\frac{2\left(ax+bx^2\right)^{5/2}}{11ax^8}$$

$$\Big\downarrow \text{ 1129}$$

$$-\frac{6b\left(-\frac{4b\int \frac{\left(bx^2+ax\right)^{3/2}}{x^6}dx}{9a}-\frac{2\left(ax+bx^2\right)^{5/2}}{9ax^7}\right)}{11a}-\frac{2\left(ax+bx^2\right)^{5/2}}{11ax^8}$$

$$\Big\downarrow \text{ 1129}$$

$$-\frac{6b\left(-\frac{4b\left(-\frac{2b\int \frac{\left(bx^2+ax\right)^{3/2}}{x^5}dx}{7a}-\frac{2\left(ax+bx^2\right)^{5/2}}{7ax^6}\right)}{9a}-\frac{2\left(ax+bx^2\right)^{5/2}}{9ax^7}\right)}{11a}-\frac{2\left(ax+bx^2\right)^{5/2}}{11ax^8}$$

$$\Big\downarrow \text{ 1123}$$

$$-\frac{6b\left(-\frac{4b\left(\frac{4b\left(ax+bx^2\right)^{5/2}}{35a^2x^5}-\frac{2\left(ax+bx^2\right)^{5/2}}{7ax^6}\right)}{9a}-\frac{2\left(ax+bx^2\right)^{5/2}}{9ax^7}\right)}{11a}-\frac{2\left(ax+bx^2\right)^{5/2}}{11ax^8}$$

```
Int[(a*x + b*x^2)^(3/2)/x^8,x]
```

```
(-2*(a*x + b*x^2)^(5/2))/(11*a*x^8) - (6*b*((-2*(a*x + b*x^2)^(5/2))/(9*a*
x^7) - (4*b*((-2*(a*x + b*x^2)^(5/2))/(7*a*x^6) + (4*b*(a*x + b*x^2)^(5/2)
)/(35*a^2*x^5)))/(9*a)))/(11*a)
```

Defintions of rubi rules used

```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] :> Simp[e*(d + e*x)^m*((a + b*x + c*x^2)^(p + 1)/((p + 1)*(2*c*d - b
*e))), x] /; FreeQ[{a, b, c, d, e, m, p}, x] && EqQ[c*d^2 - b*d*e + a*e^2,
0] && EqQ[m + 2*p + 2, 0]
```

```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] :> Simp[(-e)*(d + e*x)^m*((a + b*x + c*x^2)^(p + 1)/((m + p + 1)*(2*
c*d - b*e))), x] + Simp[c*(Simplify[m + 2*p + 2]/((m + p + 1)*(2*c*d - b*e)
))   Int[(d + e*x)^(m + 1)*(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d
, e, m, p}, x] && EqQ[c*d^2 - b*d*e + a*e^2, 0] && ILtQ[Simplify[m + 2*p +
2], 0]
```

Maple [A] (verified)

Time = 0.43 (sec) , antiderivative size = 55, normalized size of antiderivative = 0.55

method	result	size
gosper	$-\frac{2(bx+a)\left(-16b^3x^3+40a\,b^2x^2-70a^2bx+105a^3\right)\left(b\,x^2+ax\right)^{\frac{3}{2}}}{1155x^7a^4}$	55
pseudoelliptic	$-\frac{2(bx+a)^2\sqrt{x(bx+a)}\,\left(-16b^3x^3+40a\,b^2x^2-70a^2bx+105a^3\right)}{1155x^6a^4}$	55
orering	$-\frac{2(bx+a)\left(-16b^3x^3+40a\,b^2x^2-70a^2bx+105a^3\right)\left(b\,x^2+ax\right)^{\frac{3}{2}}}{1155x^7a^4}$	55
trager	$-\frac{2\left(-16b^5x^5+8a\,b^4x^4-6a^2b^3x^3+5a^3b^2x^2+140a^4bx+105a^5\right)\sqrt{b\,x^2+ax}}{1155a^4x^6}$	72
risch	$-\frac{2(bx+a)\left(-16b^5x^5+8a\,b^4x^4-6a^2b^3x^3+5a^3b^2x^2+140a^4bx+105a^5\right)}{1155x^5\sqrt{x(bx+a)}\,a^4}$	75
default	$-\frac{2\left(b\,x^2+ax\right)^{\frac{5}{2}}}{11a\,x^8}-\frac{6b\left(-\frac{2\left(b\,x^2+ax\right)^{\frac{5}{2}}}{9a\,x^7}-\frac{4b\left(-\frac{2\left(b\,x^2+ax\right)^{\frac{5}{2}}}{7a\,x^6}+\frac{4b\left(b\,x^2+ax\right)^{\frac{5}{2}}}{35a^2x^5}\right)}{9a}\right)}{11a}$	93

```
int((b*x^2+a*x)^(3/2)/x^8,x,method=_RETURNVERBOSE)
```

```
-2/1155*(b*x+a)*(-16*b^3*x^3+40*a*b^2*x^2-70*a^2*b*x+105*a^3)*(b*x^2+a*x)^
(3/2)/x^7/a^4
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 71, normalized size of antiderivative = 0.71

$$\int \frac{(ax+bx^2)^{3/2}}{x^8}\,dx = \frac{2\,(16\,b^5x^5-8\,ab^4x^4+6\,a^2b^3x^3-5\,a^3b^2x^2-140\,a^4bx-105\,a^5)\sqrt{bx^2+ax}}{1155\,a^4x^6}$$

```
integrate((b*x^2+a*x)^(3/2)/x^8,x, algorithm="fricas")
```

```
2/1155*(16*b^5*x^5 - 8*a*b^4*x^4 + 6*a^2*b^3*x^3 - 5*a^3*b^2*x^2 - 140*a^4
*b*x - 105*a^5)*sqrt(b*x^2 + a*x)/(a^4*x^6)
```

Sympy [F]

$$\int \frac{(ax+bx^2)^{3/2}}{x^8}\,dx = \int \frac{(x(a+bx))^{\frac{3}{2}}}{x^8}\,dx$$

```
integrate((b*x**2+a*x)**(3/2)/x**8,x)
```

```
Integral((x*(a + b*x))**(3/2)/x**8, x)
```

Maxima [A] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 139, normalized size of antiderivative = 1.39

$$\int \frac{(ax+bx^2)^{3/2}}{x^8}\,dx = \frac{32\sqrt{bx^2+ax}b^5}{1155\,a^4x} - \frac{16\sqrt{bx^2+ax}b^4}{1155\,a^3x^2} + \frac{4\sqrt{bx^2+ax}b^3}{385\,a^2x^3} - \frac{2\sqrt{bx^2+ax}b^2}{231\,ax^4} + \frac{\sqrt{bx^2+ax}b}{132\,x^5} + \frac{3\sqrt{bx^2+ax}a}{44\,x^6} - \frac{(bx^2+ax)^{\frac{3}{2}}}{4\,x^7}$$

```
integrate((b*x^2+a*x)^(3/2)/x^8,x, algorithm="maxima")
```

```
32/1155*sqrt(b*x^2 + a*x)*b^5/(a^4*x) - 16/1155*sqrt(b*x^2 + a*x)*b^4/(a^3
*x^2) + 4/385*sqrt(b*x^2 + a*x)*b^3/(a^2*x^3) - 2/231*sqrt(b*x^2 + a*x)*b^
2/(a*x^4) + 1/132*sqrt(b*x^2 + a*x)*b/x^5 + 3/44*sqrt(b*x^2 + a*x)*a/x^6 -
 1/4*(b*x^2 + a*x)^(3/2)/x^7
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 223 vs. 2(84) = 168.

Time = 0.14 (sec) , antiderivative size = 223, normalized size of antiderivative = 2.23

$$\int \frac{(ax+bx^2)^{3/2}}{x^8}\,dx = \frac{2\left(2310\left(\sqrt{b}x-\sqrt{bx^2+ax}\right)^7 b^{\frac{7}{2}} + 10164\left(\sqrt{b}x-\sqrt{bx^2+ax}\right)^6 ab^3 + 19635\left(\sqrt{b}x-\right.\right.}{}$$

```
integrate((b*x^2+a*x)^(3/2)/x^8,x, algorithm="giac")
```

```
2/1155*(2310*(sqrt(b)*x - sqrt(b*x^2 + a*x))^7*b^(7/2) + 10164*(sqrt(b)*x
- sqrt(b*x^2 + a*x))^6*a*b^3 + 19635*(sqrt(b)*x - sqrt(b*x^2 + a*x))^5*a^2
*b^(5/2) + 21285*(sqrt(b)*x - sqrt(b*x^2 + a*x))^4*a^3*b^2 + 13860*(sqrt(b
)*x - sqrt(b*x^2 + a*x))^3*a^4*b^(3/2) + 5390*(sqrt(b)*x - sqrt(b*x^2 + a*
x))^2*a^5*b + 1155*(sqrt(b)*x - sqrt(b*x^2 + a*x))*a^6*sqrt(b) + 105*a^7)/
(sqrt(b)*x - sqrt(b*x^2 + a*x))^11
```

Mupad [B] (verification not implemented)

Time = 10.60 (sec) , antiderivative size = 123, normalized size of antiderivative = 1.23

$$\int \frac{(ax+bx^2)^{3/2}}{x^8}\,dx = \frac{4\,b^3\,\sqrt{b\,x^2+a\,x}}{385\,a^2\,x^3}-\frac{8\,b\,\sqrt{b\,x^2+a\,x}}{33\,x^5}-\frac{2\,b^2\,\sqrt{b\,x^2+a\,x}}{231\,a\,x^4} -\frac{2\,a\,\sqrt{b\,x^2+a\,x}}{11\,x^6}-\frac{16\,b^4\,\sqrt{b\,x^2+a\,x}}{1155\,a^3\,x^2}+\frac{32\,b^5\,\sqrt{b\,x^2+a\,x}}{1155\,a^4\,x}$$

```
int((a*x + b*x^2)^(3/2)/x^8,x)
```

```
(4*b^3*(a*x + b*x^2)^(1/2))/(385*a^2*x^3) - (8*b*(a*x + b*x^2)^(1/2))/(33*
x^5) - (2*b^2*(a*x + b*x^2)^(1/2))/(231*a*x^4) - (2*a*(a*x + b*x^2)^(1/2))
/(11*x^6) - (16*b^4*(a*x + b*x^2)^(1/2))/(1155*a^3*x^2) + (32*b^5*(a*x + b
*x^2)^(1/2))/(1155*a^4*x)
```

Reduce [B] (verification not implemented)

Time = 0.25 (sec) , antiderivative size = 118, normalized size of antiderivative = 1.18

$$\int \frac{(ax+bx^2)^{3/2}}{x^8}\,dx = \frac{-\frac{2\sqrt{x}\,\sqrt{bx+a}\,a^5}{11}-\frac{8\sqrt{x}\,\sqrt{bx+a}\,a^4bx}{33}-\frac{2\sqrt{x}\,\sqrt{bx+a}\,a^3b^2x^2}{231}+\frac{4\sqrt{x}\,\sqrt{bx+a}\,a^2b^3x^3}{385}-\frac{16\sqrt{x}\,\sqrt{bx+a}\,a\,b^4x^4}{1155}-}{a^4x^6}$$

```
int((b*x^2+a*x)^(3/2)/x^8,x)
```

```
(2*( - 105*sqrt(x)*sqrt(a + b*x)*a**5 - 140*sqrt(x)*sqrt(a + b*x)*a**4*b*x
 - 5*sqrt(x)*sqrt(a + b*x)*a**3*b**2*x**2 + 6*sqrt(x)*sqrt(a + b*x)*a**2*b
**3*x**3 - 8*sqrt(x)*sqrt(a + b*x)*a*b**4*x**4 + 16*sqrt(x)*sqrt(a + b*x)*
b**5*x**5 - 16*sqrt(b)*b**5*x**6))/(1155*a**4*x**6)
```

3.22 $\int x^2(ax+bx^2)^{5/2}\,dx$

Optimal result

Integrand size = 17, antiderivative size = 227

$$\int x^2\left(ax+bx^2\right)^{5/2}\,dx=\frac{45a^7\sqrt{ax+bx^2}}{16384b^5}-\frac{15a^6x\sqrt{ax+bx^2}}{8192b^4}+\frac{3a^5x^2\sqrt{ax+bx^2}}{2048b^3}-\frac{9a^4x^3\sqrt{ax+bx^2}}{7168b^2}+\frac{a^3x^4\sqrt{ax+bx^2}}{896b}+\frac{81}{448}a^2x^5\sqrt{ax+bx^2}+\frac{33}{112}abx^6\sqrt{ax+bx^2}+\frac{1}{8}b^2x^7\sqrt{ax+bx^2}-\frac{45a^8\operatorname{arctanh}\left(\frac{\sqrt{b}x}{\sqrt{ax+bx^2}}\right)}{16384b^{11/2}}$$

```
45/16384*a^7*(b*x^2+a*x)^(1/2)/b^5-15/8192*a^6*x*(b*x^2+a*x)^(1/2)/b^4+3/2
048*a^5*x^2*(b*x^2+a*x)^(1/2)/b^3-9/7168*a^4*x^3*(b*x^2+a*x)^(1/2)/b^2+1/8
96*a^3*x^4*(b*x^2+a*x)^(1/2)/b+81/448*a^2*x^5*(b*x^2+a*x)^(1/2)+33/112*a*b
*x^6*(b*x^2+a*x)^(1/2)+1/8*b^2*x^7*(b*x^2+a*x)^(1/2)-45/16384*a^8*arctanh(
b^(1/2)*x/(b*x^2+a*x)^(1/2))/b^(11/2)
```

Mathematica [A] (verified)

Time = 0.95 (sec) , antiderivative size = 151, normalized size of antiderivative = 0.67

$$\int x^2(ax+bx^2)^{5/2}\,dx = \frac{\sqrt{x(a+bx)}\Big(\sqrt{b}(315a^7-210a^6bx+168a^5b^2x^2-144a^4b^3x^3+128a^3b^4x^4+20736a^2b^5x^5+}{114688b^{11/2}}$$

```
Integrate[x^2*(a*x + b*x^2)^(5/2),x]
```

```
(Sqrt[x*(a + b*x)]*(Sqrt[b]*(315*a^7 - 210*a^6*b*x + 168*a^5*b^2*x^2 - 144
*a^4*b^3*x^3 + 128*a^3*b^4*x^4 + 20736*a^2*b^5*x^5 + 33792*a*b^6*x^6 + 143
36*b^7*x^7) + (630*a^8*ArcTanh[(Sqrt[b]*Sqrt[x])/(Sqrt[a] - Sqrt[a + b*x])
])/(Sqrt[x]*Sqrt[a + b*x])))/(114688*b^(11/2))
```

Rubi [A] (verified)

Time = 0.51 (sec) , antiderivative size = 193, normalized size of antiderivative = 0.85, number of steps used = 8, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}} = 0.412$, Rules used = {1134, 1160, 1087, 1087, 1087, 1091, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^2\left(ax+bx^2\right)^{5/2}dx$$

$$\Big\downarrow \text{1134}$$

$$\frac{x\left(ax+bx^2\right)^{7/2}}{8b}-\frac{9a\int x\left(bx^2+ax\right)^{5/2}dx}{16b}$$

$$\Big\downarrow \text{1160}$$

$$\frac{x\left(ax+bx^2\right)^{7/2}}{8b}-\frac{9a\left(\frac{\left(ax+bx^2\right)^{7/2}}{7b}-\frac{a\int\left(bx^2+ax\right)^{5/2}dx}{2b}\right)}{16b}$$

$$\Big\downarrow \text{1087}$$

$$\frac{x\left(ax+bx^2\right)^{7/2}}{8b}-\frac{9a\left(\frac{\left(ax+bx^2\right)^{7/2}}{7b}-\frac{a\left(\frac{(a+2bx)\left(ax+bx^2\right)^{5/2}}{12b}-\frac{5a^2\int\left(bx^2+ax\right)^{3/2}dx}{24b}\right)}{2b}\right)}{16b}$$

$$\Big\downarrow \text{ 1087}$$

$$\frac{x\left(ax+bx^2\right)^{7/2}}{8b}-\frac{9a\left(\frac{\left(ax+bx^2\right)^{7/2}}{7b}-\frac{a\left(\frac{(a+2bx)\left(ax+bx^2\right)^{5/2}}{12b}-\frac{5a^2\left(\frac{(a+2bx)\left(ax+bx^2\right)^{3/2}}{8b}-\frac{3a^2\int\sqrt{bx^2+ax}dx}{16b}\right)}{24b}\right)}{2b}\right)}{16b}$$

$$\Big\downarrow \text{ 1087}$$

$$\frac{x\left(ax+bx^2\right)^{7/2}}{8b}-\frac{9a\left(\frac{\left(ax+bx^2\right)^{7/2}}{7b}-\frac{a\left(\frac{(a+2bx)\left(ax+bx^2\right)^{5/2}}{12b}-\frac{5a^2\left(\frac{(a+2bx)\left(ax+bx^2\right)^{3/2}}{8b}-\frac{3a^2\left(\frac{(a+2bx)\sqrt{ax+bx^2}}{4b}-\frac{a^2\int\frac{1}{\sqrt{bx^2+ax}}dx}{8b}\right)}{16b}\right)}{24b}\right)}{2b}\right)}{16b}$$

$$\Big\downarrow \text{ 1091}$$

$$\frac{x\left(ax+bx^2\right)^{7/2}}{8b}-\frac{9a\left(\frac{\left(ax+bx^2\right)^{7/2}}{7b}-\frac{a\left(\frac{(a+2bx)\left(ax+bx^2\right)^{5/2}}{12b}-\frac{5a^2\left(\frac{(a+2bx)\left(ax+bx^2\right)^{3/2}}{8b}-\frac{3a^2\left(\frac{(a+2bx)\sqrt{ax+bx^2}}{4b}-\frac{a^2\int\frac{1}{1-\frac{bx^2}{bx^2+ax}}d\frac{x}{\sqrt{bx^2+ax}}}{4b}\right)}{16b}\right)}{24b}\right)}{2b}\right)}{16b}$$

$$\Big\downarrow \; 219$$

$$\frac{x(ax+bx^2)^{7/2}}{8b} - \frac{9a\left(\frac{(ax+bx^2)^{7/2}}{7b} - \frac{a\left(\frac{(a+2bx)(ax+bx^2)^{5/2}}{12b} - \frac{5a^2\left(\frac{(a+2bx)(ax+bx^2)^{3/2}}{8b} - \frac{3a^2\left(\frac{(a+2bx)\sqrt{ax+bx^2}}{4b} - \frac{a^2\operatorname{arctanh}\left(\frac{\sqrt{b}x}{\sqrt{ax+bx^2}}\right)}{4b^{3/2}}\right)}{16b}\right)}{24b}\right)}{2b}\right)}{16b}$$

```
Int[x^2*(a*x + b*x^2)^(5/2),x]
```

```
(x*(a*x + b*x^2)^(7/2))/(8*b) - (9*a*((a*x + b*x^2)^(7/2)/(7*b) - (a*(((a
+ 2*b*x)*(a*x + b*x^2)^(5/2))/(12*b) - (5*a^2*(((a + 2*b*x)*(a*x + b*x^2)^
(3/2))/(8*b) - (3*a^2*(((a + 2*b*x)*Sqrt[a*x + b*x^2])/(4*b) - (a^2*ArcTan
h[(Sqrt[b]*x)/Sqrt[a*x + b*x^2]])/(4*b^(3/2))))/(16*b)))/(24*b)))/(2*b)))/
(16*b)
```

Defintions of rubi rules used

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(b + 2*c*x)
*((a + b*x + c*x^2)^p/(2*c*(2*p + 1))), x] - Simp[p*((b^2 - 4*a*c)/(2*c*(2*
p + 1)))   Int[(a + b*x + c*x^2)^(p - 1), x], x] /; FreeQ[{a, b, c}, x] &&
GtQ[p, 0] && (IntegerQ[4*p] || IntegerQ[3*p])
```

```
Int[1/Sqrt[(b_.)*(x_) + (c_.)*(x_)^2], x_Symbol] :> Simp[2   Subst[Int[1/(1
 - c*x^2), x], x, x/Sqrt[b*x + c*x^2]], x] /; FreeQ[{b, c}, x]
```

```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] :> Simp[e*(d + e*x)^(m - 1)*((a + b*x + c*x^2)^(p + 1)/(c*(m + 2*p +
 1))), x] + Simp[(m + p)*((2*c*d - b*e)/(c*(m + 2*p + 1)))   Int[(d + e*x)^
(m - 1)*(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, p}, x] && EqQ[
c*d^2 - b*d*e + a*e^2, 0] && GtQ[m, 1] && NeQ[m + 2*p + 1, 0] && IntegerQ[2
*p]
```

```
Int[((d_.) + (e_.)*(x_))*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol
] :> Simp[e*((a + b*x + c*x^2)^(p + 1)/(2*c*(p + 1))), x] + Simp[(2*c*d - b
*e)/(2*c)   Int[(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, p}, x]
 && NeQ[p, -1]
```

Maple [A] (verified)

Time = 0.37 (sec) , antiderivative size = 128, normalized size of antiderivative = 0.56

method	result
risch	$\frac{(14336x^7b^7+33792b^6a\,x^6+20736a^2x^5b^5+128b^4x^4a^3-144b^3x^3a^4+168x^2b^2a^5-210xb\,a^6+315a^7)x(bx+a)}{114688b^5\sqrt{x(bx+a)}}-\frac{45a^8\ln\left(\frac{\frac{a}{2}+bx}{\sqrt{b}}+\sqrt{b}\right.}{32768b^{\frac{11}{2}}}$
default	$\frac{x(b\,x^2+ax)^{\frac{7}{2}}}{8b}-\frac{9a\left(\frac{(b\,x^2+ax)^{\frac{7}{2}}}{7b}-\frac{a\left(\frac{(2bx+a)(b\,x^2+ax)^{\frac{5}{2}}}{12b}-\frac{5a^2\left(\frac{(2bx+a)(b\,x^2+ax)^{\frac{3}{2}}}{8b}-\frac{3a^2\left(\frac{(2bx+a)\sqrt{b\,x^2+ax}}{4b}-\frac{a^2\ln\left(\frac{\frac{a}{2}+bx}{\sqrt{b}}+\sqrt{b\,x}\right.}{8b^{\frac{3}{2}}}\right.}{16b}\right.}{24b}\right.}{2b}\right.}{16b}$

```
int(x^2*(b*x^2+a*x)^(5/2),x,method=_RETURNVERBOSE)
```

```
1/114688*(14336*b^7*x^7+33792*a*b^6*x^6+20736*a^2*b^5*x^5+128*a^3*b^4*x^4-
144*a^4*b^3*x^3+168*a^5*b^2*x^2-210*a^6*b*x+315*a^7)*x*(b*x+a)/b^5/(x*(b*x
+a))^(1/2)-45/32768*a^8/b^(11/2)*ln((1/2*a+b*x)/b^(1/2)+(b*x^2+a*x)^(1/2))
```

Fricas [A] (verification not implemented)

Time = 0.10 (sec) , antiderivative size = 258, normalized size of antiderivative = 1.14

$$\int x^2(ax+bx^2)^{5/2}\,dx = \left[\frac{315\,a^8\sqrt{b}\log\left(2\,bx+a-2\,\sqrt{bx^2+ax}\sqrt{b}\right)+2\,(14336\,b^8x^7+33792\,ab^7x^6+20736\,a^2b^6x^5}{229376\,b^6}\right.$$

```
integrate(x^2*(b*x^2+a*x)^(5/2),x, algorithm="fricas")
```

```
[1/229376*(315*a^8*sqrt(b)*log(2*b*x + a - 2*sqrt(b*x^2 + a*x)*sqrt(b)) +
2*(14336*b^8*x^7 + 33792*a*b^7*x^6 + 20736*a^2*b^6*x^5 + 128*a^3*b^5*x^4 -
 144*a^4*b^4*x^3 + 168*a^5*b^3*x^2 - 210*a^6*b^2*x + 315*a^7*b)*sqrt(b*x^2
 + a*x))/b^6, 1/114688*(315*a^8*sqrt(-b)*arctan(sqrt(b*x^2 + a*x)*sqrt(-b)
/(b*x + a)) + (14336*b^8*x^7 + 33792*a*b^7*x^6 + 20736*a^2*b^6*x^5 + 128*a
^3*b^5*x^4 - 144*a^4*b^4*x^3 + 168*a^5*b^3*x^2 - 210*a^6*b^2*x + 315*a^7*b
)*sqrt(b*x^2 + a*x))/b^6]
```

Sympy [A] (verification not implemented)

Time = 0.39 (sec) , antiderivative size = 187, normalized size of antiderivative = 0.82

$$\int x^2(ax+bx^2)^{5/2}\,dx = \begin{cases} -\frac{45a^8\left(\begin{cases}\frac{\log\left(a+2\sqrt{b}\sqrt{ax+bx^2}+2bx\right)}{\sqrt{b}} & \text{for}\ \frac{a^2}{b}\neq 0\\ \frac{\left(\frac{a}{2b}+x\right)\log\left(\frac{a}{2b}+x\right)}{\sqrt{b\left(\frac{a}{2b}+x\right)^2}} & \text{otherwise}\end{cases}\right)}{32768b^5}+\sqrt{ax+bx^2}\cdot\left(\frac{45a^7}{16384b^5}-\frac{15a^6x}{8192b^4}+\frac{3a^5x^2}{2048b^3}-\frac{9a}{71}\right. \\ \frac{2(ax)^{\frac{11}{2}}}{11a^3} \\ 0 \end{cases}$$

```
integrate(x**2*(b*x**2+a*x)**(5/2),x)
```

```
Piecewise((-45*a**8*Piecewise((log(a + 2*sqrt(b)*sqrt(a*x + b*x**2) + 2*b*
x)/sqrt(b), Ne(a**2/b, 0)), ((a/(2*b) + x)*log(a/(2*b) + x)/sqrt(b*(a/(2*b
) + x)**2), True))/(32768*b**5) + sqrt(a*x + b*x**2)*(45*a**7/(16384*b**5)
 - 15*a**6*x/(8192*b**4) + 3*a**5*x**2/(2048*b**3) - 9*a**4*x**3/(7168*b**
2) + a**3*x**4/(896*b) + 81*a**2*x**5/448 + 33*a*b*x**6/112 + b**2*x**7/8)
, Ne(b, 0)), (2*(a*x)**(11/2)/(11*a**3), Ne(a, 0)), (0, True))
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 183, normalized size of antiderivative = 0.81

$$\int x^2(ax+bx^2)^{5/2}\,dx = \frac{45\,\sqrt{bx^2+ax}a^6x}{8192\,b^4} - \frac{15\,(bx^2+ax)^{\frac{3}{2}}a^4x}{1024\,b^3} + \frac{3\,(bx^2+ax)^{\frac{5}{2}}a^2x}{64\,b^2} + \frac{(bx^2+ax)^{\frac{7}{2}}x}{8\,b} - \frac{45\,a^8\log\left(2\,bx+a+2\,\sqrt{bx^2+ax}\sqrt{b}\right)}{32768\,b^{\frac{11}{2}}} + \frac{45\,\sqrt{bx^2+ax}a^7}{16384\,b^5} - \frac{15\,(bx^2+ax)^{\frac{3}{2}}a^5}{2048\,b^4} + \frac{3\,(bx^2+ax)^{\frac{5}{2}}a^3}{128\,b^3} - \frac{9\,(bx^2+ax)^{\frac{7}{2}}a}{112\,b^2}$$

```
integrate(x^2*(b*x^2+a*x)^(5/2),x, algorithm="maxima")
```

```
45/8192*sqrt(b*x^2 + a*x)*a^6*x/b^4 - 15/1024*(b*x^2 + a*x)^(3/2)*a^4*x/b^
3 + 3/64*(b*x^2 + a*x)^(5/2)*a^2*x/b^2 + 1/8*(b*x^2 + a*x)^(7/2)*x/b - 45/
32768*a^8*log(2*b*x + a + 2*sqrt(b*x^2 + a*x)*sqrt(b))/b^(11/2) + 45/16384
*sqrt(b*x^2 + a*x)*a^7/b^5 - 15/2048*(b*x^2 + a*x)^(3/2)*a^5/b^4 + 3/128*(
b*x^2 + a*x)^(5/2)*a^3/b^3 - 9/112*(b*x^2 + a*x)^(7/2)*a/b^2
```

Giac [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 129, normalized size of antiderivative = 0.57

$$\int x^2(ax+bx^2)^{5/2}\,dx = \frac{45\,a^8\log\left(\left|2\left(\sqrt{b}x-\sqrt{bx^2+ax}\right)\sqrt{b}+a\right|\right)}{32768\,b^{\frac{11}{2}}} + \frac{1}{114688}\,\sqrt{bx^2+ax}\left(\frac{315\,a^7}{b^5} - 2\left(\frac{105\,a^6}{b^4} - 4\left(\frac{21\,a^5}{b^3} - 2\left(\frac{9\,a^4}{b^2} - 8\left(\frac{a^3}{b} + 2\,(81\,a^2 + 4\,(14\,b^2x + 33\,ab)x)\right.\right.\right.\right.\right.$$

```
integrate(x^2*(b*x^2+a*x)^(5/2),x, algorithm="giac")
```

```
45/32768*a^8*log(abs(2*(sqrt(b)*x - sqrt(b*x^2 + a*x))*sqrt(b) + a))/b^(11
/2) + 1/114688*sqrt(b*x^2 + a*x)*(315*a^7/b^5 - 2*(105*a^6/b^4 - 4*(21*a^5
/b^3 - 2*(9*a^4/b^2 - 8*(a^3/b + 2*(81*a^2 + 4*(14*b^2*x + 33*a*b)*x)*x)*x
)*x)*x)*x)
```

Mupad [F(-1)]

Timed out.

$$\int x^2(ax+bx^2)^{5/2}\,dx=\int x^2\,\left(b\,x^2+a\,x\right)^{5/2}\,dx$$

```
int(x^2*(a*x + b*x^2)^(5/2),x)
```

```
int(x^2*(a*x + b*x^2)^(5/2), x)
```

Reduce [B] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 171, normalized size of antiderivative = 0.75

$$\int x^2(ax+bx^2)^{5/2}\,dx=\frac{315\sqrt{x}\,\sqrt{bx+a}\,a^7b-210\sqrt{x}\,\sqrt{bx+a}\,a^6b^2x+168\sqrt{x}\,\sqrt{bx+a}\,a^5b^3x^2-144\sqrt{x}\,\sqrt{bx+a}\,a^4b}{}$$

```
int(x^2*(b*x^2+a*x)^(5/2),x)
```

```
(315*sqrt(x)*sqrt(a + b*x)*a**7*b - 210*sqrt(x)*sqrt(a + b*x)*a**6*b**2*x
+ 168*sqrt(x)*sqrt(a + b*x)*a**5*b**3*x**2 - 144*sqrt(x)*sqrt(a + b*x)*a**
4*b**4*x**3 + 128*sqrt(x)*sqrt(a + b*x)*a**3*b**5*x**4 + 20736*sqrt(x)*sqr
t(a + b*x)*a**2*b**6*x**5 + 33792*sqrt(x)*sqrt(a + b*x)*a*b**7*x**6 + 1433
6*sqrt(x)*sqrt(a + b*x)*b**8*x**7 - 315*sqrt(b)*log((sqrt(a + b*x) + sqrt(
x)*sqrt(b))/sqrt(a))*a**8)/(114688*b**6)
```

3.23 $\int x(ax+bx^2)^{5/2}\,dx$

Optimal result

Integrand size = 15, antiderivative size = 201

$$\int x(ax+bx^2)^{5/2}\,dx = -\frac{5a^6\sqrt{ax+bx^2}}{1024b^4}+\frac{5a^5x\sqrt{ax+bx^2}}{1536b^3} - \frac{a^4x^2\sqrt{ax+bx^2}}{384b^2}+\frac{a^3x^3\sqrt{ax+bx^2}}{448b}+\frac{37}{168}a^2x^4\sqrt{ax+bx^2} + \frac{29}{84}abx^5\sqrt{ax+bx^2}+\frac{1}{7}b^2x^6\sqrt{ax+bx^2}+\frac{5a^7\operatorname{arctanh}\left(\frac{\sqrt{b}x}{\sqrt{ax+bx^2}}\right)}{1024b^{9/2}}$$

```
-5/1024*a^6*(b*x^2+a*x)^(1/2)/b^4+5/1536*a^5*x*(b*x^2+a*x)^(1/2)/b^3-1/384
*a^4*x^2*(b*x^2+a*x)^(1/2)/b^2+1/448*a^3*x^3*(b*x^2+a*x)^(1/2)/b+37/168*a^
2*x^4*(b*x^2+a*x)^(1/2)+29/84*a*b*x^5*(b*x^2+a*x)^(1/2)+1/7*b^2*x^6*(b*x^2
+a*x)^(1/2)+5/1024*a^7*arctanh(b^(1/2)*x/(b*x^2+a*x)^(1/2))/b^(9/2)
```

Mathematica [A] (verified)

Time = 0.75 (sec) , antiderivative size = 140, normalized size of antiderivative = 0.70

$$\int x(ax+bx^2)^{5/2}\,dx = \frac{\sqrt{x(a+bx)}\left(\sqrt{b}(-105a^6+70a^5bx-56a^4b^2x^2+48a^3b^3x^3+4736a^2b^4x^4+7424ab^5x^5+30\right.}{21504b^{9/2}}$$

```
Integrate[x*(a*x + b*x^2)^(5/2),x]
```

```
(Sqrt[x*(a + b*x)]*(Sqrt[b]*(-105*a^6 + 70*a^5*b*x - 56*a^4*b^2*x^2 + 48*a
^3*b^3*x^3 + 4736*a^2*b^4*x^4 + 7424*a*b^5*x^5 + 3072*b^6*x^6) + (210*a^7*
ArcTanh[(Sqrt[b]*Sqrt[x])/(-Sqrt[a] + Sqrt[a + b*x])])/(Sqrt[x]*Sqrt[a + b
*x])))/(21504*b^(9/2))
```

Rubi [A] (verified)

Time = 0.47 (sec) , antiderivative size = 163, normalized size of antiderivative = 0.81, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.400$, Rules used = {1160, 1087, 1087, 1087, 1091, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x(ax+bx^2)^{5/2}\,dx$$

$$\Big\downarrow \text{1160}$$

$$\frac{(ax+bx^2)^{7/2}}{7b}-\frac{a\int (bx^2+ax)^{5/2}dx}{2b}$$

$$\Big\downarrow \text{1087}$$

$$\frac{(ax+bx^2)^{7/2}}{7b}-\frac{a\left(\frac{(a+2bx)(ax+bx^2)^{5/2}}{12b}-\frac{5a^2\int (bx^2+ax)^{3/2}dx}{24b}\right)}{2b}$$

$$\Big\downarrow \text{1087}$$

$$\frac{\left(ax+bx^2\right)^{7/2}}{7b}-\frac{a\left(\frac{(a+2bx)\left(ax+bx^2\right)^{5/2}}{12b}-\frac{5a^2\left(\frac{(a+2bx)\left(ax+bx^2\right)^{3/2}}{8b}-\frac{3a^2\int\sqrt{bx^2+ax}dx}{16b}\right)}{24b}\right)}{2b}$$

$$\Big\downarrow \text{1087}$$

$$\frac{\left(ax+bx^2\right)^{7/2}}{7b}-\frac{a\left(\frac{(a+2bx)\left(ax+bx^2\right)^{5/2}}{12b}-\frac{5a^2\left(\frac{(a+2bx)\left(ax+bx^2\right)^{3/2}}{8b}-\frac{3a^2\left(\frac{(a+2bx)\sqrt{ax+bx^2}}{4b}-\frac{a^2\int\frac{1}{\sqrt{bx^2+ax}}dx}{8b}\right)}{16b}\right)}{24b}\right)}{2b}$$

$$\Big\downarrow \text{1091}$$

$$\frac{\left(ax+bx^2\right)^{7/2}}{7b}-\frac{a\left(\frac{(a+2bx)\left(ax+bx^2\right)^{5/2}}{12b}-\frac{5a^2\left(\frac{(a+2bx)\left(ax+bx^2\right)^{3/2}}{8b}-\frac{3a^2\left(\frac{(a+2bx)\sqrt{ax+bx^2}}{4b}-\frac{a^2\int\frac{1}{1-\frac{bx^2}{bx^2+ax}}d\frac{x}{\sqrt{bx^2+ax}}}{4b}\right)}{16b}\right)}{24b}\right)}{2b}$$

$$\Big\downarrow \text{219}$$

$$\frac{\left(ax+bx^2\right)^{7/2}}{7b}-\frac{a\left(\frac{(a+2bx)\left(ax+bx^2\right)^{5/2}}{12b}-\frac{5a^2\left(\frac{(a+2bx)\left(ax+bx^2\right)^{3/2}}{8b}-\frac{3a^2\left(\frac{(a+2bx)\sqrt{ax+bx^2}}{4b}-\frac{a^2\operatorname{arctanh}\left(\frac{\sqrt{b}x}{\sqrt{ax+bx^2}}\right)}{4b^{3/2}}\right)}{16b}\right)}{24b}\right)}{2b}$$

```
Int[x*(a*x + b*x^2)^(5/2),x]
```

```
(a*x + b*x^2)^(7/2)/(7*b) - (a*(((a + 2*b*x)*(a*x + b*x^2)^(5/2))/(12*b) -
 (5*a^2*(((a + 2*b*x)*(a*x + b*x^2)^(3/2))/(8*b) - (3*a^2*(((a + 2*b*x)*Sq
rt[a*x + b*x^2])/(4*b) - (a^2*ArcTanh[(Sqrt[b]*x)/Sqrt[a*x + b*x^2]])/(4*b
^(3/2))))/(16*b)))/(24*b)))/(2*b)
```

Defintions of rubi rules used

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(b + 2*c*x)
*((a + b*x + c*x^2)^p/(2*c*(2*p + 1))), x] - Simp[p*((b^2 - 4*a*c)/(2*c*(2*
p + 1)))   Int[(a + b*x + c*x^2)^(p - 1), x], x] /; FreeQ[{a, b, c}, x] &&
GtQ[p, 0] && (IntegerQ[4*p] || IntegerQ[3*p])
```

```
Int[1/Sqrt[(b_.)*(x_) + (c_.)*(x_)^2], x_Symbol] :> Simp[2   Subst[Int[1/(1
 - c*x^2), x], x, x/Sqrt[b*x + c*x^2]], x] /; FreeQ[{b, c}, x]
```

```
Int[((d_.) + (e_.)*(x_))*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol
] :> Simp[e*((a + b*x + c*x^2)^(p + 1)/(2*c*(p + 1))), x] + Simp[(2*c*d - b
*e)/(2*c)   Int[(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, p}, x]
 && NeQ[p, -1]
```

Maple [A] (verified)

Time = 0.36 (sec) , antiderivative size = 117, normalized size of antiderivative = 0.58

method	result	siz
risch	$-\frac{\left(-3072 b^{6} x^{6}-7424 a\, b^{5} x^{5}-4736 a^{2} b^{4} x^{4}-48 a^{3} b^{3} x^{3}+56 a^{4} b^{2} x^{2}-70 a^{5} b x+105 a^{6}\right) x (b x+a)}{21504 b^{4} \sqrt{x (b x+a)}}+\frac{5 a^{7} \ln\left(\frac{\frac{a}{2}+b x}{\sqrt{b}}+\sqrt{b\, x^{2}+a x}\right)}{2048 b^{\frac{9}{2}}}$	11
default	$\frac{\left(b\, x^{2}+a x\right)^{\frac{7}{2}}}{7 b}-\frac{a \left(\frac{(2 b x+a) \left(b\, x^{2}+a x\right)^{\frac{5}{2}}}{12 b}-\frac{5 a^{2} \left(\frac{(2 b x+a) \left(b\, x^{2}+a x\right)^{\frac{3}{2}}}{8 b}-\frac{3 a^{2} \left(\frac{(2 b x+a) \sqrt{b\, x^{2}+a x}}{4 b}-\frac{a^{2} \ln\left(\frac{\frac{a}{2}+b x}{\sqrt{b}}+\sqrt{b\, x^{2}+a x}\right)}{8 b^{\frac{3}{2}}}\right)}{16 b}\right)}{24 b}\right)}{2 b}$	14

```
int(x*(b*x^2+a*x)^(5/2),x,method=_RETURNVERBOSE)
```

```
-1/21504*(-3072*b^6*x^6-7424*a*b^5*x^5-4736*a^2*b^4*x^4-48*a^3*b^3*x^3+56*
a^4*b^2*x^2-70*a^5*b*x+105*a^6)*x*(b*x+a)/b^4/(x*(b*x+a))^(1/2)+5/2048*a^7
/b^(9/2)*ln((1/2*a+b*x)/b^(1/2)+(b*x^2+a*x)^(1/2))
```

Fricas [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 237, normalized size of antiderivative = 1.18

$$\int x(ax+bx^2)^{5/2}\,dx = \left[\frac{105\,a^7\sqrt{b}\log\left(2\,bx+a+2\,\sqrt{bx^2+ax}\sqrt{b}\right)+2\,(3072\,b^7x^6+7424\,ab^6x^5+4736\,a^2b^5x^4+48\right.}{43008\,b^5}$$
$$-\frac{105\,a^7\sqrt{-b}\arctan\left(\frac{\sqrt{bx^2+ax}\sqrt{-b}}{bx+a}\right)-(3072\,b^7x^6+7424\,ab^6x^5+4736\,a^2b^5x^4+48\,a^3b^4x^3-56\,a^4b^3x^2+70\,a}{21504\,b^5}$$

```
integrate(x*(b*x^2+a*x)^(5/2),x, algorithm="fricas")
```

```
[1/43008*(105*a^7*sqrt(b)*log(2*b*x + a + 2*sqrt(b*x^2 + a*x)*sqrt(b)) + 2
*(3072*b^7*x^6 + 7424*a*b^6*x^5 + 4736*a^2*b^5*x^4 + 48*a^3*b^4*x^3 - 56*a
^4*b^3*x^2 + 70*a^5*b^2*x - 105*a^6*b)*sqrt(b*x^2 + a*x))/b^5, -1/21504*(1
05*a^7*sqrt(-b)*arctan(sqrt(b*x^2 + a*x)*sqrt(-b)/(b*x + a)) - (3072*b^7*x
^6 + 7424*a*b^6*x^5 + 4736*a^2*b^5*x^4 + 48*a^3*b^4*x^3 - 56*a^4*b^3*x^2 +
 70*a^5*b^2*x - 105*a^6*b)*sqrt(b*x^2 + a*x))/b^5]
```

Sympy [A] (verification not implemented)

Time = 0.37 (sec) , antiderivative size = 172, normalized size of antiderivative = 0.86

$$\int x(ax+bx^2)^{5/2}\,dx = \begin{cases} \frac{5a^7\left(\begin{cases} \frac{\log\left(a+2\sqrt{b}\sqrt{ax+bx^2}+2bx\right)}{\sqrt{b}} & \text{for}\ \frac{a^2}{b}\neq 0 \\ \frac{\left(\frac{a}{2b}+x\right)\log\left(\frac{a}{2b}+x\right)}{\sqrt{b\left(\frac{a}{2b}+x\right)^2}} & \text{otherwise} \end{cases}\right)}{2048b^4}+\sqrt{ax+bx^2}\left(-\frac{5a^6}{1024b^4}+\frac{5a^5x}{1536b^3}-\frac{a^4x^2}{384b^2}+\frac{a^3x^3}{448b}+\right. \\ \frac{2(ax)^{\frac{9}{2}}}{9a^2} \\ 0 \end{cases}$$

```
integrate(x*(b*x**2+a*x)**(5/2),x)
```

```
Piecewise((5*a**7*Piecewise((log(a + 2*sqrt(b)*sqrt(a*x + b*x**2) + 2*b*x)
/sqrt(b), Ne(a**2/b, 0)), ((a/(2*b) + x)*log(a/(2*b) + x)/sqrt(b*(a/(2*b)
+ x)**2), True))/(2048*b**4) + sqrt(a*x + b*x**2)*(-5*a**6/(1024*b**4) + 5
*a**5*x/(1536*b**3) - a**4*x**2/(384*b**2) + a**3*x**3/(448*b) + 37*a**2*x
**4/168 + 29*a*b*x**5/84 + b**2*x**6/7), Ne(b, 0)), (2*(a*x)**(9/2)/(9*a**
2), Ne(a, 0)), (0, True))
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 163, normalized size of antiderivative = 0.81

$$\begin{aligned}\int x\left(ax+bx^2\right)^{5/2}\,dx =& -\frac{5\sqrt{bx^2+ax}a^5x}{512\,b^3}+\frac{5\left(bx^2+ax\right)^{\frac{3}{2}}a^3x}{192\,b^2}\\&-\frac{\left(bx^2+ax\right)^{\frac{5}{2}}ax}{12\,b}+\frac{5\,a^7\log\left(2\,bx+a+2\sqrt{bx^2+ax}\sqrt{b}\right)}{2048\,b^{\frac{9}{2}}}\\&-\frac{5\sqrt{bx^2+ax}a^6}{1024\,b^4}+\frac{5\left(bx^2+ax\right)^{\frac{3}{2}}a^4}{384\,b^3}-\frac{\left(bx^2+ax\right)^{\frac{5}{2}}a^2}{24\,b^2}+\frac{\left(bx^2+ax\right)^{\frac{7}{2}}}{7\,b}\end{aligned}$$

```
integrate(x*(b*x^2+a*x)^(5/2),x, algorithm="maxima")
```

```
-5/512*sqrt(b*x^2 + a*x)*a^5*x/b^3 + 5/192*(b*x^2 + a*x)^(3/2)*a^3*x/b^2 -
 1/12*(b*x^2 + a*x)^(5/2)*a*x/b + 5/2048*a^7*log(2*b*x + a + 2*sqrt(b*x^2
+ a*x)*sqrt(b))/b^(9/2) - 5/1024*sqrt(b*x^2 + a*x)*a^6/b^4 + 5/384*(b*x^2
+ a*x)^(3/2)*a^4/b^3 - 1/24*(b*x^2 + a*x)^(5/2)*a^2/b^2 + 1/7*(b*x^2 + a*x
)^(7/2)/b
```

Giac [A] (verification not implemented)

Time = 0.14 (sec) , antiderivative size = 118, normalized size of antiderivative = 0.59

$$\begin{aligned}\int x\left(ax+bx^2\right)^{5/2}\,dx =& -\frac{5\,a^7\log\left(\left|2\left(\sqrt{b}x-\sqrt{bx^2+ax}\right)\sqrt{b}+a\right|\right)}{2048\,b^{\frac{9}{2}}}\\&-\frac{1}{21504}\sqrt{bx^2+ax}\left(\frac{105\,a^6}{b^4}-2\left(\frac{35\,a^5}{b^3}-4\left(\frac{7\,a^4}{b^2}-2\left(\frac{3\,a^3}{b}+8\left(37\,a^2+2\left(12\,b^2x+29\,ab\right)x\right)x\right)x\right)x\right)x\right.\end{aligned}$$

```
integrate(x*(b*x^2+a*x)^(5/2),x, algorithm="giac")
```

```
-5/2048*a^7*log(abs(2*(sqrt(b)*x - sqrt(b*x^2 + a*x))*sqrt(b) + a))/b^(9/2
) - 1/21504*sqrt(b*x^2 + a*x)*(105*a^6/b^4 - 2*(35*a^5/b^3 - 4*(7*a^4/b^2
- 2*(3*a^3/b + 8*(37*a^2 + 2*(12*b^2*x + 29*a*b)*x)*x)*x)*x)*x)
```

Mupad [F(-1)]

Timed out.

$$\int x(ax+bx^2)^{5/2}\,dx=\int x\left(b\,x^2+a\,x\right)^{5/2}dx$$

```
int(x*(a*x + b*x^2)^(5/2),x)
```

```
int(x*(a*x + b*x^2)^(5/2), x)
```

Reduce [B] (verification not implemented)

Time = 0.22 (sec) , antiderivative size = 152, normalized size of antiderivative = 0.76

$$\int x(ax+bx^2)^{5/2}\,dx=\frac{-105\sqrt{x}\,\sqrt{bx+a}\,a^6b+70\sqrt{x}\,\sqrt{bx+a}\,a^5b^2x-56\sqrt{x}\,\sqrt{bx+a}\,a^4b^3x^2+48\sqrt{x}\,\sqrt{bx+a}\,a^3b^4}{}$$

```
int(x*(b*x^2+a*x)^(5/2),x)
```

```
( - 105*sqrt(x)*sqrt(a + b*x)*a**6*b + 70*sqrt(x)*sqrt(a + b*x)*a**5*b**2*
x - 56*sqrt(x)*sqrt(a + b*x)*a**4*b**3*x**2 + 48*sqrt(x)*sqrt(a + b*x)*a**
3*b**4*x**3 + 4736*sqrt(x)*sqrt(a + b*x)*a**2*b**5*x**4 + 7424*sqrt(x)*sqr
t(a + b*x)*a*b**6*x**5 + 3072*sqrt(x)*sqrt(a + b*x)*b**7*x**6 + 105*sqrt(b
)*log((sqrt(a + b*x) + sqrt(x)*sqrt(b))/sqrt(a))*a**7)/(21504*b**5)
```

3.24 $\int (ax + bx^2)^{5/2} \, dx$

Optimal result

Integrand size = 13, antiderivative size = 175

$$\int (ax+bx^2)^{5/2}\,dx = \frac{5a^5\sqrt{ax+bx^2}}{512b^3} - \frac{5a^4x\sqrt{ax+bx^2}}{768b^2} + \frac{a^3x^2\sqrt{ax+bx^2}}{192b} + \frac{9}{32}a^2x^3\sqrt{ax+bx^2} + \frac{5}{12}abx^4\sqrt{ax+bx^2} + \frac{1}{6}b^2x^5\sqrt{ax+bx^2} - \frac{5a^6\operatorname{arctanh}\left(\frac{\sqrt{b}x}{\sqrt{ax+bx^2}}\right)}{512b^{7/2}}$$

```
5/512*a^5*(b*x^2+a*x)^(1/2)/b^3-5/768*a^4*x*(b*x^2+a*x)^(1/2)/b^2+1/192*a^
3*x^2*(b*x^2+a*x)^(1/2)/b+9/32*a^2*x^3*(b*x^2+a*x)^(1/2)+5/12*a*b*x^4*(b*x
^2+a*x)^(1/2)+1/6*b^2*x^5*(b*x^2+a*x)^(1/2)-5/512*a^6*arctanh(b^(1/2)*x/(b
*x^2+a*x)^(1/2))/b^(7/2)
```

Mathematica [A] (verified)

Time = 0.66 (sec) , antiderivative size = 129, normalized size of antiderivative = 0.74

$$\int (ax+bx^2)^{5/2}\,dx = \frac{\sqrt{x(a+bx)}\left(\sqrt{b}(15a^5-10a^4bx+8a^3b^2x^2+432a^2b^3x^3+640ab^4x^4+256b^5x^5)+\frac{30a^6\operatorname{arcta}}{\sqrt{a}}\right.}{1536b^{7/2}}$$

```
Integrate[(a*x + b*x^2)^(5/2),x]
```

```
(Sqrt[x*(a + b*x)]*(Sqrt[b]*(15*a^5 - 10*a^4*b*x + 8*a^3*b^2*x^2 + 432*a^2
*b^3*x^3 + 640*a*b^4*x^4 + 256*b^5*x^5) + (30*a^6*ArcTanh[(Sqrt[b]*Sqrt[x]
)/(Sqrt[a] - Sqrt[a + b*x])])/(Sqrt[x]*Sqrt[a + b*x])))/(1536*b^(7/2))
```

Rubi [A] (verified)

Time = 0.40 (sec) , antiderivative size = 134, normalized size of antiderivative = 0.77, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.385$, Rules used = $\{1087, 1087, 1087, 1091, 219\}$

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \left(ax+bx^2\right)^{5/2}dx$$

$$\Big\downarrow \text{1087}$$

$$\frac{(a+2bx)\left(ax+bx^2\right)^{5/2}}{12b}-\frac{5a^2\int \left(bx^2+ax\right)^{3/2}dx}{24b}$$

$$\Big\downarrow \text{1087}$$

$$\frac{(a+2bx)\left(ax+bx^2\right)^{5/2}}{12b}-\frac{5a^2\left(\frac{(a+2bx)\left(ax+bx^2\right)^{3/2}}{8b}-\frac{3a^2\int \sqrt{bx^2+ax}dx}{16b}\right)}{24b}$$

$$\Big\downarrow \text{1087}$$

$$\frac{(a+2bx)\left(ax+bx^2\right)^{5/2}}{12b}-\frac{5a^2\left(\frac{(a+2bx)\left(ax+bx^2\right)^{3/2}}{8b}-\frac{3a^2\left(\frac{(a+2bx)\sqrt{ax+bx^2}}{4b}-\frac{a^2\int \frac{1}{\sqrt{bx^2+ax}}dx}{8b}\right)}{16b}\right)}{24b}$$

$$\Big\downarrow \text{1091}$$

$$\frac{(a+2bx)\left(ax+bx^2\right)^{5/2}}{12b}-\frac{5a^2\left(\frac{(a+2bx)\left(ax+bx^2\right)^{3/2}}{8b}-\frac{3a^2\left(\frac{(a+2bx)\sqrt{ax+bx^2}}{4b}-\frac{a^2\int\frac{1}{1-\frac{bx^2}{bx^2+ax}}d\frac{x}{\sqrt{bx^2+ax}}}{4b}\right)}{16b}\right)}{24b}$$

$$\Big\downarrow \ 219$$

$$\frac{(a+2bx)\left(ax+bx^2\right)^{5/2}}{12b}-\frac{5a^2\left(\frac{(a+2bx)\left(ax+bx^2\right)^{3/2}}{8b}-\frac{3a^2\left(\frac{(a+2bx)\sqrt{ax+bx^2}}{4b}-\frac{a^2\text{arctanh}\left(\frac{\sqrt{b}x}{\sqrt{ax+bx^2}}\right)}{4b^{3/2}}\right)}{16b}\right)}{24b}$$

```
Int[(a*x + b*x^2)^(5/2),x]
```

```
((a + 2*b*x)*(a*x + b*x^2)^(5/2))/(12*b) - (5*a^2*(((a + 2*b*x)*(a*x + b*x
^2)^(3/2))/(8*b) - (3*a^2*(((a + 2*b*x)*Sqrt[a*x + b*x^2])/(4*b) - (a^2*Ar
cTanh[(Sqrt[b]*x)/Sqrt[a*x + b*x^2]])/(4*b^(3/2))))/(16*b)))/(24*b)
```

Defintions of rubi rules used

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(b + 2*c*x)
*((a + b*x + c*x^2)^p/(2*c*(2*p + 1))), x] - Simp[p*((b^2 - 4*a*c)/(2*c*(2*
p + 1)))   Int[(a + b*x + c*x^2)^(p - 1), x], x] /; FreeQ[{a, b, c}, x] &&
GtQ[p, 0] && (IntegerQ[4*p] || IntegerQ[3*p])
```

```
Int[1/Sqrt[(b_.)*(x_) + (c_.)*(x_)^2], x_Symbol] :> Simp[2   Subst[Int[1/(1
 - c*x^2), x], x, x/Sqrt[b*x + c*x^2]], x] /; FreeQ[{b, c}, x]
```

Maple [A] (verified)

Time = 0.34 (sec) , antiderivative size = 106, normalized size of antiderivative = 0.61

method	result	size
risch	$\frac{\left(256 b^{5} x^{5}+640 a \,b^{4} x^{4}+432 a^{2} b^{3} x^{3}+8 a^{3} b^{2} x^{2}-10 a^{4} b x +15 a^{5}\right) x \left(b x +a \right)}{1536 b^{3} \sqrt{x \left(b x +a \right)}}-\frac{5 a^{6} \ln \left(\frac{\frac{a}{2}+b x}{\sqrt{b}}+\sqrt{b \,x^{2}+a x}\right)}{1024 b^{\frac{7}{2}}}$	106
default	$\frac{\left(2 b x +a \right) \left(b \,x^{2}+a x \right)^{\frac{5}{2}}}{12 b}-\frac{5 a^{2} \left(\frac{\left(2 b x +a \right) \left(b \,x^{2}+a x \right)^{\frac{3}{2}}}{8 b}-\frac{3 a^{2} \left(\frac{\left(2 b x +a \right) \sqrt{b \,x^{2}+a x}}{4 b}-\frac{a^{2} \ln \left(\frac{\frac{a}{2}+b x}{\sqrt{b}}+\sqrt{b \,x^{2}+a x}\right)}{8 b^{\frac{3}{2}}}\right)}{16 b}\right)}{24 b}$	118

```
int((b*x^2+a*x)^(5/2),x,method=_RETURNVERBOSE)
```

```
1/1536*(256*b^5*x^5+640*a*b^4*x^4+432*a^2*b^3*x^3+8*a^3*b^2*x^2-10*a^4*b*x
+15*a^5)*x*(b*x+a)/b^3/(x*(b*x+a))^(1/2)-5/1024*a^6/b^(7/2)*ln((1/2*a+b*x)
/b^(1/2)+(b*x^2+a*x)^(1/2))
```

Fricas [A] (verification not implemented)

Time = 0.10 (sec) , antiderivative size = 214, normalized size of antiderivative = 1.22

$$\int (ax+bx^2)^{5/2}\, dx = \left[\frac{15\, a^6 \sqrt{b} \log\left(2\, bx + a - 2\, \sqrt{bx^2 + ax}\sqrt{b}\right) + 2\, (256\, b^6 x^5 + 640\, ab^5 x^4 + 432\, a^2 b^4 x^3 + 8\, a^3 b^3 x}{3072\, b^4}\right.$$

```
integrate((b*x^2+a*x)^(5/2),x, algorithm="fricas")
```

```
[1/3072*(15*a^6*sqrt(b)*log(2*b*x + a - 2*sqrt(b*x^2 + a*x)*sqrt(b)) + 2*(
256*b^6*x^5 + 640*a*b^5*x^4 + 432*a^2*b^4*x^3 + 8*a^3*b^3*x^2 - 10*a^4*b^2
*x + 15*a^5*b)*sqrt(b*x^2 + a*x))/b^4, 1/1536*(15*a^6*sqrt(-b)*arctan(sqrt
(b*x^2 + a*x)*sqrt(-b)/(b*x + a)) + (256*b^6*x^5 + 640*a*b^5*x^4 + 432*a^2
*b^4*x^3 + 8*a^3*b^3*x^2 - 10*a^4*b^2*x + 15*a^5*b)*sqrt(b*x^2 + a*x))/b^4
]
```

Sympy [A] (verification not implemented)

Time = 0.47 (sec) , antiderivative size = 459, normalized size of antiderivative = 2.62

$$\int \left(ax + bx^2\right)^{5/2} \, dx = \text{Too large to display}$$

```
integrate((b*x**2+a*x)**(5/2),x)
```

```
a**2*Piecewise((-5*a**4*Piecewise((log(a + 2*sqrt(b)*sqrt(a*x + b*x**2) +
2*b*x)/sqrt(b), Ne(a**2/b, 0)), ((a/(2*b) + x)*log(a/(2*b) + x)/sqrt(b*(a/
(2*b) + x)**2), True))/(128*b**3) + sqrt(a*x + b*x**2)*(5*a**3/(64*b**3) -
 5*a**2*x/(96*b**2) + a*x**2/(24*b) + x**3/4), Ne(b, 0)), (2*(a*x)**(7/2)/
(7*a**3), Ne(a, 0)), (0, True)) + 2*a*b*Piecewise((7*a**5*Piecewise((log(a
 + 2*sqrt(b)*sqrt(a*x + b*x**2) + 2*b*x)/sqrt(b), Ne(a**2/b, 0)), ((a/(2*b
) + x)*log(a/(2*b) + x)/sqrt(b*(a/(2*b) + x)**2), True))/(256*b**4) + sqrt
(a*x + b*x**2)*(-7*a**4/(128*b**4) + 7*a**3*x/(192*b**3) - 7*a**2*x**2/(24
0*b**2) + a*x**3/(40*b) + x**4/5), Ne(b, 0)), (2*(a*x)**(9/2)/(9*a**4), Ne
(a, 0)), (0, True)) + b**2*Piecewise((-21*a**6*Piecewise((log(a + 2*sqrt(b
)*sqrt(a*x + b*x**2) + 2*b*x)/sqrt(b), Ne(a**2/b, 0)), ((a/(2*b) + x)*log(
a/(2*b) + x)/sqrt(b*(a/(2*b) + x)**2), True))/(1024*b**5) + sqrt(a*x + b*x
**2)*(21*a**5/(512*b**5) - 7*a**4*x/(256*b**4) + 7*a**3*x**2/(320*b**3) -
3*a**2*x**3/(160*b**2) + a*x**4/(60*b) + x**5/6), Ne(b, 0)), (2*(a*x)**(11
/2)/(11*a**5), Ne(a, 0)), (0, True))
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 141, normalized size of antiderivative = 0.81

$$\int \left(ax+bx^2\right)^{5/2}\,dx = \frac{1}{6}\left(bx^2+ax\right)^{\frac{5}{2}}x + \frac{5\sqrt{bx^2+ax}a^4x}{256\,b^2} - \frac{5\left(bx^2+ax\right)^{\frac{3}{2}}a^2x}{96\,b} - \frac{5\,a^6\log\left(2\,bx+a+2\sqrt{bx^2+ax}\sqrt{b}\right)}{1024\,b^{\frac{7}{2}}} + \frac{5\sqrt{bx^2+ax}a^5}{512\,b^3} - \frac{5\left(bx^2+ax\right)^{\frac{3}{2}}a^3}{192\,b^2} + \frac{\left(bx^2+ax\right)^{\frac{5}{2}}a}{12\,b}$$

```
integrate((b*x^2+a*x)^(5/2),x, algorithm="maxima")
```

```
1/6*(b*x^2 + a*x)^(5/2)*x + 5/256*sqrt(b*x^2 + a*x)*a^4*x/b^2 - 5/96*(b*x^
2 + a*x)^(3/2)*a^2*x/b - 5/1024*a^6*log(2*b*x + a + 2*sqrt(b*x^2 + a*x)*sq
rt(b))/b^(7/2) + 5/512*sqrt(b*x^2 + a*x)*a^5/b^3 - 5/192*(b*x^2 + a*x)^(3/
2)*a^3/b^2 + 1/12*(b*x^2 + a*x)^(5/2)*a/b
```

Giac [A] (verification not implemented)

Time = 0.25 (sec) , antiderivative size = 105, normalized size of antiderivative = 0.60

$$\int \left(ax+bx^2\right)^{5/2}\,dx = \frac{5\,a^6\log\left(\left|2\left(\sqrt{b}x-\sqrt{bx^2+ax}\right)\sqrt{b}+a\right|\right)}{1024\,b^{\frac{7}{2}}} + \frac{1}{1536}\sqrt{bx^2+ax}\left(\frac{15\,a^5}{b^3}-2\left(\frac{5\,a^4}{b^2}-4\left(\frac{a^3}{b}+2\left(27\,a^2+8\left(2\,b^2x+5\,ab\right)x\right)x\right)x\right)x\right)$$

```
integrate((b*x^2+a*x)^(5/2),x, algorithm="giac")
```

```
5/1024*a^6*log(abs(2*(sqrt(b)*x - sqrt(b*x^2 + a*x))*sqrt(b) + a))/b^(7/2)
 + 1/1536*sqrt(b*x^2 + a*x)*(15*a^5/b^3 - 2*(5*a^4/b^2 - 4*(a^3/b + 2*(27*
a^2 + 8*(2*b^2*x + 5*a*b)*x)*x)*x)*x)
```

Mupad [B] (verification not implemented)

Time = 10.14 (sec) , antiderivative size = 119, normalized size of antiderivative = 0.68

$$\int \left(ax+bx^2\right)^{5/2} dx = \frac{\left(b\,x^2+a\,x\right)^{5/2}\,\left(\frac{a}{2}+b\,x\right)}{6\,b} - \frac{5\,a^2\,\left(\frac{\left(b\,x^2+a\,x\right)^{3/2}\,\left(\frac{a}{2}+b\,x\right)}{4\,b}-\frac{3\,a^2\,\left(\sqrt{b\,x^2+a\,x}\,\left(\frac{x}{2}+\frac{a}{4\,b}\right)-\frac{a^2\,\ln\left(\frac{\frac{a}{2}+b\,x}{\sqrt{b}}+\sqrt{b\,x^2+a\,x}\right)}{8\,b^{3/2}}\right)}{16\,b}\right)}{24\,b}$$

```
int((a*x + b*x^2)^(5/2),x)
```

```
((a*x + b*x^2)^(5/2)*(a/2 + b*x))/(6*b) - (5*a^2*(((a*x + b*x^2)^(3/2)*(a/
2 + b*x))/(4*b) - (3*a^2*((a*x + b*x^2)^(1/2)*(x/2 + a/(4*b)) - (a^2*log((
a/2 + b*x)/b^(1/2) + (a*x + b*x^2)^(1/2)))/(8*b^(3/2))))/(16*b)))/(24*b)
```

Reduce [B] (verification not implemented)

Time = 0.25 (sec) , antiderivative size = 133, normalized size of antiderivative = 0.76

$$\int \left(ax+bx^2\right)^{5/2} dx = \frac{15\sqrt{x}\,\sqrt{bx+a}\,a^5b-10\sqrt{x}\,\sqrt{bx+a}\,a^4b^2x+8\sqrt{x}\,\sqrt{bx+a}\,a^3b^3x^2+432\sqrt{x}\,\sqrt{bx+a}\,a^2b^4x^3}{1536b^4}$$

```
int((b*x^2+a*x)^(5/2),x)
```

```
(15*sqrt(x)*sqrt(a + b*x)*a**5*b - 10*sqrt(x)*sqrt(a + b*x)*a**4*b**2*x +
8*sqrt(x)*sqrt(a + b*x)*a**3*b**3*x**2 + 432*sqrt(x)*sqrt(a + b*x)*a**2*b*
*4*x**3 + 640*sqrt(x)*sqrt(a + b*x)*a*b**5*x**4 + 256*sqrt(x)*sqrt(a + b*x
)*b**6*x**5 - 15*sqrt(b)*log((sqrt(a + b*x) + sqrt(x)*sqrt(b))/sqrt(a))*a*
*6)/(1536*b**4)
```

3.25 $\int \frac{(ax+bx^2)^{5/2}}{x}\,dx$

Optimal result

Integrand size = 17, antiderivative size = 149

$$\int \frac{(ax+bx^2)^{5/2}}{x}\,dx = -\frac{3a^4\sqrt{ax+bx^2}}{128b^2}+\frac{a^3x\sqrt{ax+bx^2}}{64b}+\frac{31}{80}a^2x^2\sqrt{ax+bx^2}+\frac{21}{40}abx^3\sqrt{ax+bx^2}+\frac{1}{5}b^2x^4\sqrt{ax+bx^2}+\frac{3a^5\operatorname{arctanh}\left(\frac{\sqrt{b}x}{\sqrt{ax+bx^2}}\right)}{128b^{5/2}}$$

```
-3/128*a^4*(b*x^2+a*x)^(1/2)/b^2+1/64*a^3*x*(b*x^2+a*x)^(1/2)/b+31/80*a^2*
x^2*(b*x^2+a*x)^(1/2)+21/40*a*b*x^3*(b*x^2+a*x)^(1/2)+1/5*b^2*x^4*(b*x^2+a
*x)^(1/2)+3/128*a^5*arctanh(b^(1/2)*x/(b*x^2+a*x)^(1/2))/b^(5/2)
```

Mathematica [A] (verified)

Time = 0.50 (sec) , antiderivative size = 118, normalized size of antiderivative = 0.79

$$\int \frac{(ax+bx^2)^{5/2}}{x}\,dx = \frac{\sqrt{x(a+bx)}\left(\sqrt{b}(-15a^4+10a^3bx+248a^2b^2x^2+336ab^3x^3+128b^4x^4)+\frac{30a^5\operatorname{arctan}}{\sqrt{x}}\right.}{640b^{5/2}}$$

```
Integrate[(a*x + b*x^2)^(5/2)/x,x]
```

```
(Sqrt[x*(a + b*x)]*(Sqrt[b]*(-15*a^4 + 10*a^3*b*x + 248*a^2*b^2*x^2 + 336*
a*b^3*x^3 + 128*b^4*x^4) + (30*a^5*ArcTanh[(Sqrt[b]*Sqrt[x])/(-Sqrt[a] + S
qrt[a + b*x])])/(Sqrt[x]*Sqrt[a + b*x])))/(640*b^(5/2))
```

Rubi [A] (verified)

Time = 0.39 (sec) , antiderivative size = 120, normalized size of antiderivative = 0.81, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.294$, Rules used = {1131, 1087, 1087, 1091, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(ax+bx^2)^{5/2}}{x} dx$$

$$\Big\downarrow \text{1131}$$

$$\frac{1}{2}a\int (bx^2+ax)^{3/2}dx+\frac{1}{5}(ax+bx^2)^{5/2}$$

$$\Big\downarrow \text{1087}$$

$$\frac{1}{2}a\left(\frac{(a+2bx)(ax+bx^2)^{3/2}}{8b}-\frac{3a^2\int \sqrt{bx^2+ax}dx}{16b}\right)+\frac{1}{5}(ax+bx^2)^{5/2}$$

$$\Big\downarrow \text{1087}$$

$$\frac{1}{2}a\left(\frac{(a+2bx)(ax+bx^2)^{3/2}}{8b}-\frac{3a^2\left(\frac{(a+2bx)\sqrt{ax+bx^2}}{4b}-\frac{a^2\int \frac{1}{\sqrt{bx^2+ax}}dx}{8b}\right)}{16b}\right)+\frac{1}{5}(ax+bx^2)^{5/2}$$

$$\Big\downarrow \text{1091}$$

$$\frac{1}{2}a\left(\frac{(a+2bx)(ax+bx^2)^{3/2}}{8b}-\frac{3a^2\left(\frac{(a+2bx)\sqrt{ax+bx^2}}{4b}-\frac{a^2\int \frac{1}{1-\frac{bx^2}{bx^2+ax}}d\frac{x}{\sqrt{bx^2+ax}}}{4b}\right)}{16b}\right)+$$

$$\frac{1}{5}(ax+bx^2)^{5/2}$$

$$\Big\downarrow \text{219}$$

$$\frac{1}{2}a\left(\frac{(a+2bx)\left(ax+bx^2\right)^{3/2}}{8b}-\frac{3a^2\left(\frac{(a+2bx)\sqrt{ax+bx^2}}{4b}-\frac{a^2\text{arctanh}\left(\frac{\sqrt{b}x}{\sqrt{ax+bx^2}}\right)}{4b^{3/2}}\right)}{16b}\right)+$$

$$\frac{1}{5}\left(ax+bx^2\right)^{5/2}$$

```
Int[(a*x + b*x^2)^(5/2)/x,x]
```

```
(a*x + b*x^2)^(5/2)/5 + (a*(((a + 2*b*x)*(a*x + b*x^2)^(3/2))/(8*b) - (3*a
^2*(((a + 2*b*x)*Sqrt[a*x + b*x^2])/(4*b) - (a^2*ArcTanh[(Sqrt[b]*x)/Sqrt[
a*x + b*x^2]])/(4*b^(3/2))))/(16*b)))/2
```

Defintions of rubi rules used

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(b + 2*c*x)
*((a + b*x + c*x^2)^p/(2*c*(2*p + 1))), x] - Simp[p*((b^2 - 4*a*c)/(2*c*(2*
p + 1)))   Int[(a + b*x + c*x^2)^(p - 1), x], x] /; FreeQ[{a, b, c}, x] &&
GtQ[p, 0] && (IntegerQ[4*p] || IntegerQ[3*p])
```

```
Int[1/Sqrt[(b_.)*(x_) + (c_.)*(x_)^2], x_Symbol] :> Simp[2   Subst[Int[1/(1
 - c*x^2), x], x, x/Sqrt[b*x + c*x^2]], x] /; FreeQ[{b, c}, x]
```

```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] :> Simp[(d + e*x)^(m + 1)*((a + b*x + c*x^2)^p/(e*(m + 2*p + 1))), x
] - Simp[p*((2*c*d - b*e)/(e^2*(m + 2*p + 1)))   Int[(d + e*x)^(m + 1)*(a +
 b*x + c*x^2)^(p - 1), x], x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[c*d^2 - b
*d*e + a*e^2, 0] && GtQ[p, 0] && (LeQ[-2, m, 0] || EqQ[m + p + 1, 0]) && Ne
Q[m + 2*p + 1, 0] && IntegerQ[2*p]
```

Maple [A] (verified)

Time = 0.39 (sec) , antiderivative size = 84, normalized size of antiderivative = 0.56

method	result	size
pseudoelliptic	$\frac{\frac{3\,\operatorname{arctanh}\left(\frac{\sqrt{x(bx+a)}}{x\sqrt{b}}\right)a^{5}}{128}-\frac{3\sqrt{x(bx+a)}\left(\sqrt{b}\,a^{4}-\frac{2b^{\frac{3}{2}}a^{3}x}{3}-\frac{248b^{\frac{5}{2}}a^{2}x^{2}}{15}-\frac{112b^{\frac{7}{2}}a\,x^{3}}{5}-\frac{128b^{\frac{9}{2}}x^{4}}{15}\right)}{128}}{b^{\frac{5}{2}}}$	84
risch	$-\frac{\left(-128b^{4}x^{4}-336a\,b^{3}x^{3}-248a^{2}b^{2}x^{2}-10a^{3}bx+15a^{4}\right)x(bx+a)}{640b^{2}\sqrt{x(bx+a)}}+\frac{3a^{5}\ln\left(\frac{\frac{a}{2}+bx}{\sqrt{b}}+\sqrt{b\,x^{2}+ax}\right)}{256b^{\frac{5}{2}}}$	95
default	$\frac{\left(b\,x^{2}+ax\right)^{\frac{5}{2}}}{5}+\frac{a\left(\frac{(2bx+a)\left(b\,x^{2}+ax\right)^{\frac{3}{2}}}{8b}-\frac{3a^{2}\left(\frac{(2bx+a)\sqrt{b\,x^{2}+ax}}{4b}-\frac{a^{2}\ln\left(\frac{\frac{a}{2}+bx}{\sqrt{b}}+\sqrt{b\,x^{2}+ax}\right)}{8b^{\frac{3}{2}}}\right)}{16b}\right)}{2}$	104

```
int((b*x^2+a*x)^(5/2)/x,x,method=_RETURNVERBOSE)
```

```
3/128/b^(5/2)*(arctanh((x*(b*x+a))^(1/2)/x/b^(1/2))*a^5-(x*(b*x+a))^(1/2)*
(b^(1/2)*a^4-2/3*b^(3/2)*a^3*x-248/15*b^(5/2)*a^2*x^2-112/5*b^(7/2)*a*x^3-
128/15*b^(9/2)*x^4))
```

Fricas [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 193, normalized size of antiderivative = 1.30

$$\int \frac{(ax+bx^2)^{5/2}}{x}\,dx = \left[\frac{15\,a^5\sqrt{b}\log\left(2\,bx+a+2\,\sqrt{bx^2+ax}\sqrt{b}\right)+2\,(128\,b^5x^4+336\,ab^4x^3+248\,a^2b^3x^2+}{1280\,b^3}\right.$$
$$\left.-\frac{15\,a^5\sqrt{-b}\arctan\left(\frac{\sqrt{bx^2+ax}\sqrt{-b}}{bx+a}\right)-(128\,b^5x^4+336\,ab^4x^3+248\,a^2b^3x^2+10\,a^3b^2x-15\,a^4b)\sqrt{bx^2+ax}}{640\,b^3}\right]$$

```
integrate((b*x^2+a*x)^(5/2)/x,x, algorithm="fricas")
```

```
[1/1280*(15*a^5*sqrt(b)*log(2*b*x + a + 2*sqrt(b*x^2 + a*x)*sqrt(b)) + 2*(
128*b^5*x^4 + 336*a*b^4*x^3 + 248*a^2*b^3*x^2 + 10*a^3*b^2*x - 15*a^4*b)*s
qrt(b*x^2 + a*x))/b^3, -1/640*(15*a^5*sqrt(-b)*arctan(sqrt(b*x^2 + a*x)*sq
rt(-b)/(b*x + a)) - (128*b^5*x^4 + 336*a*b^4*x^3 + 248*a^2*b^3*x^2 + 10*a^
3*b^2*x - 15*a^4*b)*sqrt(b*x^2 + a*x))/b^3]
```

Sympy [A] (verification not implemented)

Time = 1.49 (sec) , antiderivative size = 415, normalized size of antiderivative = 2.79

$$\int \frac{(ax+bx^2)^{5/2}}{x}\,dx = a^2\left(\begin{cases} \frac{a^3\left(\begin{cases} \frac{\log\left(a+2\sqrt{b}\sqrt{ax+bx^2}+2bx\right)}{\sqrt{b}} & \text{for } \frac{a^2}{b}\neq 0 \\ \frac{\left(\frac{a}{2b}+x\right)\log\left(\frac{a}{2b}+x\right)}{\sqrt{b\left(\frac{a}{2b}+x\right)^2}} & \text{otherwise} \end{cases}\right)}{16b^2} + \sqrt{ax+bx^2}\left(-\frac{a^2}{8b^2}+\frac{ax}{12b}+\frac{x^2}{3}\right) & \text{for } b \neq 0 \\ \frac{2(ax)^{\frac{5}{2}}}{5a^2} & \text{for } a \neq 0 \\ 0 & \text{otherwise} \end{cases}\right)$$
$$+2ab\left(\begin{cases} -\frac{5a^4\left(\begin{cases} \frac{\log\left(a+2\sqrt{b}\sqrt{ax+bx^2}+2bx\right)}{\sqrt{b}} & \text{for } \frac{a^2}{b}\neq 0 \\ \frac{\left(\frac{a}{2b}+x\right)\log\left(\frac{a}{2b}+x\right)}{\sqrt{b\left(\frac{a}{2b}+x\right)^2}} & \text{otherwise} \end{cases}\right)}{128b^3} + \sqrt{ax+bx^2}\cdot\left(\frac{5a^3}{64b^3}-\frac{5a^2x}{96b^2}+\frac{ax^2}{24b}+\frac{x^3}{4}\right) & \text{for } b \neq 0 \\ \frac{2(ax)^{\frac{7}{2}}}{7a^3} & \text{for } a \neq 0 \\ 0 & \text{otherwise} \end{cases}\right)$$
$$+b^2\left(\begin{cases} \frac{7a^5\left(\begin{cases} \frac{\log\left(a+2\sqrt{b}\sqrt{ax+bx^2}+2bx\right)}{\sqrt{b}} & \text{for } \frac{a^2}{b}\neq 0 \\ \frac{\left(\frac{a}{2b}+x\right)\log\left(\frac{a}{2b}+x\right)}{\sqrt{b\left(\frac{a}{2b}+x\right)^2}} & \text{otherwise} \end{cases}\right)}{256b^4} + \sqrt{ax+bx^2}\left(-\frac{7a^4}{128b^4}+\frac{7a^3x}{192b^3}-\frac{7a^2x^2}{240b^2}+\frac{ax^3}{40b}+\frac{x^4}{5}\right) & \text{for } b \neq 0 \\ \frac{2(ax)^{\frac{9}{2}}}{9a^4} & \text{for } a \neq 0 \\ 0 & \text{otherwise} \end{cases}\right)$$

```
integrate((b*x**2+a*x)**(5/2)/x,x)
```

```
a**2*Piecewise((a**3*Piecewise((log(a + 2*sqrt(b)*sqrt(a*x + b*x**2) + 2*b
*x)/sqrt(b), Ne(a**2/b, 0)), ((a/(2*b) + x)*log(a/(2*b) + x)/sqrt(b*(a/(2*
b) + x)**2), True))/(16*b**2) + sqrt(a*x + b*x**2)*(-a**2/(8*b**2) + a*x/(
12*b) + x**2/3), Ne(b, 0)), (2*(a*x)**(5/2)/(5*a**2), Ne(a, 0)), (0, True)
) + 2*a*b*Piecewise((-5*a**4*Piecewise((log(a + 2*sqrt(b)*sqrt(a*x + b*x**
2) + 2*b*x)/sqrt(b), Ne(a**2/b, 0)), ((a/(2*b) + x)*log(a/(2*b) + x)/sqrt(
b*(a/(2*b) + x)**2), True))/(128*b**3) + sqrt(a*x + b*x**2)*(5*a**3/(64*b*
*3) - 5*a**2*x/(96*b**2) + a*x**2/(24*b) + x**3/4), Ne(b, 0)), (2*(a*x)**(
7/2)/(7*a**3), Ne(a, 0)), (0, True)) + b**2*Piecewise((7*a**5*Piecewise((l
og(a + 2*sqrt(b)*sqrt(a*x + b*x**2) + 2*b*x)/sqrt(b), Ne(a**2/b, 0)), ((a/
(2*b) + x)*log(a/(2*b) + x)/sqrt(b*(a/(2*b) + x)**2), True))/(256*b**4) +
sqrt(a*x + b*x**2)*(-7*a**4/(128*b**4) + 7*a**3*x/(192*b**3) - 7*a**2*x**2
/(240*b**2) + a*x**3/(40*b) + x**4/5), Ne(b, 0)), (2*(a*x)**(9/2)/(9*a**4)
, Ne(a, 0)), (0, True))
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 118, normalized size of antiderivative = 0.79

$$\int \frac{(ax+bx^2)^{5/2}}{x}\,dx = \frac{1}{8}\left(bx^2+ax\right)^{\frac{3}{2}}ax - \frac{3\sqrt{bx^2+ax}a^3x}{64\,b} + \frac{3\,a^5\log\left(2\,bx+a+2\,\sqrt{bx^2+ax}\sqrt{b}\right)}{256\,b^{\frac{5}{2}}} + \frac{1}{5}\left(bx^2+ax\right)^{\frac{5}{2}} - \frac{3\sqrt{bx^2+ax}a^4}{128\,b^2} + \frac{\left(bx^2+ax\right)^{\frac{3}{2}}a^2}{16\,b}$$

```
integrate((b*x^2+a*x)^(5/2)/x,x, algorithm="maxima")
```

```
1/8*(b*x^2 + a*x)^(3/2)*a*x - 3/64*sqrt(b*x^2 + a*x)*a^3*x/b + 3/256*a^5*l
og(2*b*x + a + 2*sqrt(b*x^2 + a*x)*sqrt(b))/b^(5/2) + 1/5*(b*x^2 + a*x)^(5
/2) - 3/128*sqrt(b*x^2 + a*x)*a^4/b^2 + 1/16*(b*x^2 + a*x)^(3/2)*a^2/b
```

Giac [A] (verification not implemented)

Time = 0.29 (sec) , antiderivative size = 94, normalized size of antiderivative = 0.63

$$\int \frac{\left(ax+bx^2\right)^{5/2}}{x}\,dx = -\frac{3\,a^5\log\left(\left|2\left(\sqrt{b}x-\sqrt{bx^2+ax}\right)\sqrt{b}+a\right|\right)}{256\,b^{\frac{5}{2}}} - \frac{1}{640}\,\sqrt{bx^2+ax}\left(\frac{15\,a^4}{b^2}-2\left(\frac{5\,a^3}{b}+4\left(31\,a^2+2\left(8\,b^2x+21\,ab\right)x\right)x\right)x\right)$$

```
integrate((b*x^2+a*x)^(5/2)/x,x, algorithm="giac")
```

```
-3/256*a^5*log(abs(2*(sqrt(b)*x - sqrt(b*x^2 + a*x))*sqrt(b) + a))/b^(5/2)
 - 1/640*sqrt(b*x^2 + a*x)*(15*a^4/b^2 - 2*(5*a^3/b + 4*(31*a^2 + 2*(8*b^2
*x + 21*a*b)*x)*x)*x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\left(ax+bx^2\right)^{5/2}}{x}\,dx = \int \frac{\left(b\,x^2+a\,x\right)^{5/2}}{x}\,dx$$

```
int((a*x + b*x^2)^(5/2)/x,x)
```

```
int((a*x + b*x^2)^(5/2)/x, x)
```

Reduce [B] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 114, normalized size of antiderivative = 0.77

$$\int \frac{\left(ax+bx^2\right)^{5/2}}{x}\,dx = \frac{-15\sqrt{x}\,\sqrt{bx+a}\,a^4b+10\sqrt{x}\,\sqrt{bx+a}\,a^3b^2x+248\sqrt{x}\,\sqrt{bx+a}\,a^2b^3x^2+336\sqrt{x}\,\sqrt{bx}}{640b^3}$$

```
int((b*x^2+a*x)^(5/2)/x,x)
```

```
( - 15*sqrt(x)*sqrt(a + b*x)*a**4*b + 10*sqrt(x)*sqrt(a + b*x)*a**3*b**2*x
 + 248*sqrt(x)*sqrt(a + b*x)*a**2*b**3*x**2 + 336*sqrt(x)*sqrt(a + b*x)*a*
b**4*x**3 + 128*sqrt(x)*sqrt(a + b*x)*b**5*x**4 + 15*sqrt(b)*log((sqrt(a +
 b*x) + sqrt(x)*sqrt(b))/sqrt(a))*a**5)/(640*b**3)
```

3.26 $\int \frac{(ax+bx^2)^{5/2}}{x^2}\,dx$

Optimal result

Integrand size = 17, antiderivative size = 123

$$\int \frac{(ax+bx^2)^{5/2}}{x^2}\,dx = \frac{5a^3\sqrt{ax+bx^2}}{64b}+\frac{59}{96}a^2x\sqrt{ax+bx^2}+\frac{17}{24}abx^2\sqrt{ax+bx^2}+\frac{1}{4}b^2x^3\sqrt{ax+bx^2}-\frac{5a^4\text{arctanh}\left(\frac{\sqrt{b}x}{\sqrt{ax+bx^2}}\right)}{64b^{3/2}}$$

```
5/64*a^3*(b*x^2+a*x)^(1/2)/b+59/96*a^2*x*(b*x^2+a*x)^(1/2)+17/24*a*b*x^2*(
b*x^2+a*x)^(1/2)+1/4*b^2*x^3*(b*x^2+a*x)^(1/2)-5/64*a^4*arctanh(b^(1/2)*x/
(b*x^2+a*x)^(1/2))/b^(3/2)
```

Mathematica [A] (verified)

Time = 0.43 (sec) , antiderivative size = 107, normalized size of antiderivative = 0.87

$$\int \frac{(ax+bx^2)^{5/2}}{x^2}\,dx = \frac{\sqrt{x(a+bx)}\left(\sqrt{b}(15a^3+118a^2bx+136ab^2x^2+48b^3x^3)+\frac{30a^4\text{arctanh}\left(\frac{\sqrt{b}\sqrt{x}}{\sqrt{a}-\sqrt{a+bx}}\right)}{\sqrt{x}\sqrt{a+bx}}\right)}{192b^{3/2}}$$

```
Integrate[(a*x + b*x^2)^(5/2)/x^2,x]
```

```
(Sqrt[x*(a + b*x)]*(Sqrt[b]*(15*a^3 + 118*a^2*b*x + 136*a*b^2*x^2 + 48*b^3
*x^3) + (30*a^4*ArcTanh[(Sqrt[b]*Sqrt[x])/(Sqrt[a] - Sqrt[a + b*x])])/(Sqr
t[x]*Sqrt[a + b*x])))/(192*b^(3/2))
```

Rubi [A] (verified)

Time = 0.38 (sec) , antiderivative size = 109, normalized size of antiderivative = 0.89, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.294$, Rules used = {1131, 1131, 1087, 1091, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(ax+bx^2)^{5/2}}{x^2} \, dx$$

$$\Big\downarrow \text{1131}$$

$$\frac{5}{8}a\int \frac{(bx^2+ax)^{3/2}}{x}dx+\frac{(ax+bx^2)^{5/2}}{4x}$$

$$\Big\downarrow \text{1131}$$

$$\frac{5}{8}a\left(\frac{1}{2}a\int \sqrt{bx^2+ax}dx+\frac{1}{3}(ax+bx^2)^{3/2}\right)+\frac{(ax+bx^2)^{5/2}}{4x}$$

$$\Big\downarrow \text{1087}$$

$$\frac{5}{8}a\left(\frac{1}{2}a\left(\frac{(a+2bx)\sqrt{ax+bx^2}}{4b}-\frac{a^2\int \frac{1}{\sqrt{bx^2+ax}}dx}{8b}\right)+\frac{1}{3}(ax+bx^2)^{3/2}\right)+\frac{(ax+bx^2)^{5/2}}{4x}$$

$$\Big\downarrow \text{1091}$$

$$\frac{5}{8}a\left(\frac{1}{2}a\left(\frac{(a+2bx)\sqrt{ax+bx^2}}{4b}-\frac{a^2\int \frac{1}{1-\frac{bx^2}{bx^2+ax}}d\frac{x}{\sqrt{bx^2+ax}}}{4b}\right)+\frac{1}{3}(ax+bx^2)^{3/2}\right)+$$
$$\frac{(ax+bx^2)^{5/2}}{4x}$$

$$\Big\downarrow \text{219}$$

$$\frac{5}{8}a\left(\frac{1}{2}a\left(\frac{(a+2bx)\sqrt{ax+bx^2}}{4b}-\frac{a^2\text{arctanh}\left(\frac{\sqrt{b}x}{\sqrt{ax+bx^2}}\right)}{4b^{3/2}}\right)+\frac{1}{3}\left(ax+bx^2\right)^{3/2}\right)+\frac{\left(ax+bx^2\right)^{5/2}}{4x}$$

```
Int[(a*x + b*x^2)^(5/2)/x^2,x]
```

```
(a*x + b*x^2)^(5/2)/(4*x) + (5*a*((a*x + b*x^2)^(3/2)/3 + (a*(((a + 2*b*x)
*Sqrt[a*x + b*x^2])/(4*b) - (a^2*ArcTanh[(Sqrt[b]*x)/Sqrt[a*x + b*x^2]])/(
4*b^(3/2))))/2))/8
```

Defintions of rubi rules used

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(b + 2*c*x)
*((a + b*x + c*x^2)^p/(2*c*(2*p + 1))), x] - Simp[p*((b^2 - 4*a*c)/(2*c*(2*
p + 1)))   Int[(a + b*x + c*x^2)^(p - 1), x], x] /; FreeQ[{a, b, c}, x] &&
GtQ[p, 0] && (IntegerQ[4*p] || IntegerQ[3*p])
```

```
Int[1/Sqrt[(b_.)*(x_) + (c_.)*(x_)^2], x_Symbol] :> Simp[2   Subst[Int[1/(1
 - c*x^2), x], x, x/Sqrt[b*x + c*x^2]], x] /; FreeQ[{b, c}, x]
```

```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] :> Simp[(d + e*x)^(m + 1)*((a + b*x + c*x^2)^p/(e*(m + 2*p + 1))), x
] - Simp[p*((2*c*d - b*e)/(e^2*(m + 2*p + 1)))   Int[(d + e*x)^(m + 1)*(a +
 b*x + c*x^2)^(p - 1), x], x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[c*d^2 - b
*d*e + a*e^2, 0] && GtQ[p, 0] && (LeQ[-2, m, 0] || EqQ[m + p + 1, 0]) && Ne
Q[m + 2*p + 1, 0] && IntegerQ[2*p]
```

Maple [A] (verified)

Time = 0.40 (sec) , antiderivative size = 73, normalized size of antiderivative = 0.59

method	result
pseudoelliptic	$-\frac{5\left(\operatorname{arctanh}\left(\frac{\sqrt{x(bx+a)}}{x\sqrt{b}}\right)a^4-\left(\sqrt{b}\,a^3+\frac{118b^{\frac{3}{2}}a^2x}{15}+\frac{136a\,x^2b^{\frac{5}{2}}}{15}+\frac{16b^{\frac{7}{2}}x^3}{5}\right)\sqrt{x(bx+a)}\right)}{64b^{\frac{3}{2}}}$
risch	$\frac{(48b^3x^3+136a\,b^2x^2+118a^2bx+15a^3)x(bx+a)}{192b\sqrt{x(bx+a)}}-\frac{5a^4\ln\left(\frac{\frac{a}{2}+bx}{\sqrt{b}}+\sqrt{b\,x^2+ax}\right)}{128b^{\frac{3}{2}}}$
default	$\frac{2(b\,x^2+ax)^{\frac{7}{2}}}{3a\,x^2}-\frac{10b\left(\frac{(b\,x^2+ax)^{\frac{5}{2}}}{5}+\frac{a\left(\frac{(2bx+a)(b\,x^2+ax)^{\frac{3}{2}}}{8b}-\frac{3a^2\left(\frac{(2bx+a)\sqrt{b\,x^2+ax}}{4b}-\frac{a^2\ln\left(\frac{\frac{a}{2}+bx}{\sqrt{b}}+\sqrt{b\,x^2+ax}\right)}{8b^{\frac{3}{2}}}\right)}{16b}\right)}{2}\right)}{3a}$

```
int((b*x^2+a*x)^(5/2)/x^2,x,method=_RETURNVERBOSE)
```

```
-5/64/b^(3/2)*(arctanh((x*(b*x+a))^(1/2)/x/b^(1/2))*a^4-(b^(1/2)*a^3+118/1
5*b^(3/2)*a^2*x+136/15*a*x^2*b^(5/2)+16/5*b^(7/2)*x^3)*(x*(b*x+a))^(1/2))
```

Fricas [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 170, normalized size of antiderivative = 1.38

$$\int \frac{(ax+bx^2)^{5/2}}{x^2}\,dx = \left[\frac{15\,a^4\sqrt{b}\log\left(2\,bx+a-2\,\sqrt{bx^2+ax}\sqrt{b}\right)+2\,(48\,b^4x^3+136\,ab^3x^2+118\,a^2b^2x+1}{384\,b^2}\right.$$

```
integrate((b*x^2+a*x)^(5/2)/x^2,x, algorithm="fricas")
```

```
[1/384*(15*a^4*sqrt(b)*log(2*b*x + a - 2*sqrt(b*x^2 + a*x)*sqrt(b)) + 2*(4
8*b^4*x^3 + 136*a*b^3*x^2 + 118*a^2*b^2*x + 15*a^3*b)*sqrt(b*x^2 + a*x))/b
^2, 1/192*(15*a^4*sqrt(-b)*arctan(sqrt(b*x^2 + a*x)*sqrt(-b)/(b*x + a)) +
(48*b^4*x^3 + 136*a*b^3*x^2 + 118*a^2*b^2*x + 15*a^3*b)*sqrt(b*x^2 + a*x))
/b^2]
```

Sympy [A] (verification not implemented)

Time = 0.52 (sec) , antiderivative size = 369, normalized size of antiderivative = 3.00

$$\int \frac{(ax+bx^2)^{5/2}}{x^2}\,dx = a^2\left(\begin{cases} -\frac{a^2\left(\begin{cases} \frac{\log\left(a+2\sqrt{b}\sqrt{ax+bx^2}+2bx\right)}{\sqrt{b}} & \text{for}\ \frac{a^2}{b}\neq 0 \\ \frac{\left(\frac{a}{2b}+x\right)\log\left(\frac{a}{2b}+x\right)}{\sqrt{b\left(\frac{a}{2b}+x\right)^2}} & \text{otherwise} \end{cases}\right)}{8b} + \left(\frac{a}{4b}+\frac{x}{2}\right)\sqrt{ax+bx^2} & \text{for}\ b\neq 0 \\ \frac{2(ax)^{\frac{3}{2}}}{3a} & \text{for}\ a\neq 0 \\ 0 & \text{otherwise} \end{cases}\right)$$
$$+2ab\left(\begin{cases} \frac{a^3\left(\begin{cases} \frac{\log\left(a+2\sqrt{b}\sqrt{ax+bx^2}+2bx\right)}{\sqrt{b}} & \text{for}\ \frac{a^2}{b}\neq 0 \\ \frac{\left(\frac{a}{2b}+x\right)\log\left(\frac{a}{2b}+x\right)}{\sqrt{b\left(\frac{a}{2b}+x\right)^2}} & \text{otherwise} \end{cases}\right)}{16b^2} + \sqrt{ax+bx^2}\left(-\frac{a^2}{8b^2}+\frac{ax}{12b}+\frac{x^2}{3}\right) & \text{for}\ b\neq 0 \\ \frac{2(ax)^{\frac{5}{2}}}{5a^2} & \text{for}\ a\neq 0 \\ 0 & \text{otherwise} \end{cases}\right)$$
$$+b^2\left(\begin{cases} -\frac{5a^4\left(\begin{cases} \frac{\log\left(a+2\sqrt{b}\sqrt{ax+bx^2}+2bx\right)}{\sqrt{b}} & \text{for}\ \frac{a^2}{b}\neq 0 \\ \frac{\left(\frac{a}{2b}+x\right)\log\left(\frac{a}{2b}+x\right)}{\sqrt{b\left(\frac{a}{2b}+x\right)^2}} & \text{otherwise} \end{cases}\right)}{128b^3} + \sqrt{ax+bx^2}\cdot\left(\frac{5a^3}{64b^3}-\frac{5a^2x}{96b^2}+\frac{ax^2}{24b}+\frac{x^3}{4}\right) & \text{for}\ b\neq 0 \\ \frac{2(ax)^{\frac{7}{2}}}{7a^3} & \text{for}\ a\neq 0 \\ 0 & \text{otherwise} \end{cases}\right)$$

```
integrate((b*x**2+a*x)**(5/2)/x**2,x)
```

```
a**2*Piecewise((-a**2*Piecewise((log(a + 2*sqrt(b)*sqrt(a*x + b*x**2) + 2*
b*x)/sqrt(b), Ne(a**2/b, 0)), ((a/(2*b) + x)*log(a/(2*b) + x)/sqrt(b*(a/(2
*b) + x)**2), True))/(8*b) + (a/(4*b) + x/2)*sqrt(a*x + b*x**2), Ne(b, 0))
, (2*(a*x)**(3/2)/(3*a), Ne(a, 0)), (0, True)) + 2*a*b*Piecewise((a**3*Pie
cewise((log(a + 2*sqrt(b)*sqrt(a*x + b*x**2) + 2*b*x)/sqrt(b), Ne(a**2/b,
0)), ((a/(2*b) + x)*log(a/(2*b) + x)/sqrt(b*(a/(2*b) + x)**2), True))/(16*
b**2) + sqrt(a*x + b*x**2)*(-a**2/(8*b**2) + a*x/(12*b) + x**2/3), Ne(b, 0
)), (2*(a*x)**(5/2)/(5*a**2), Ne(a, 0)), (0, True)) + b**2*Piecewise((-5*a
**4*Piecewise((log(a + 2*sqrt(b)*sqrt(a*x + b*x**2) + 2*b*x)/sqrt(b), Ne(a
**2/b, 0)), ((a/(2*b) + x)*log(a/(2*b) + x)/sqrt(b*(a/(2*b) + x)**2), True
))/(128*b**3) + sqrt(a*x + b*x**2)*(5*a**3/(64*b**3) - 5*a**2*x/(96*b**2)
+ a*x**2/(24*b) + x**3/4), Ne(b, 0)), (2*(a*x)**(7/2)/(7*a**3), Ne(a, 0)),
 (0, True))
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 98, normalized size of antiderivative = 0.80

$$\int \frac{(ax+bx^2)^{5/2}}{x^2}\,dx = \frac{5}{32}\,\sqrt{bx^2+ax}a^2x - \frac{5\,a^4\log\left(2\,bx+a+2\,\sqrt{bx^2+ax}\sqrt{b}\right)}{128\,b^{\frac{3}{2}}} + \frac{5}{24}\left(bx^2+ax\right)^{\frac{3}{2}}a + \frac{5\,\sqrt{bx^2+ax}a^3}{64\,b} + \frac{\left(bx^2+ax\right)^{\frac{5}{2}}}{4\,x}$$

```
integrate((b*x^2+a*x)^(5/2)/x^2,x, algorithm="maxima")
```

```
5/32*sqrt(b*x^2 + a*x)*a^2*x - 5/128*a^4*log(2*b*x + a + 2*sqrt(b*x^2 + a*
x)*sqrt(b))/b^(3/2) + 5/24*(b*x^2 + a*x)^(3/2)*a + 5/64*sqrt(b*x^2 + a*x)*
a^3/b + 1/4*(b*x^2 + a*x)^(5/2)/x
```

Giac [A] (verification not implemented)

Time = 0.17 (sec) , antiderivative size = 82, normalized size of antiderivative = 0.67

$$\int \frac{(ax+bx^2)^{5/2}}{x^2}\,dx = \frac{5\,a^4\log\left(\left|2\left(\sqrt{b}x-\sqrt{bx^2+ax}\right)\sqrt{b}+a\right|\right)}{128\,b^{\frac{3}{2}}} + \frac{1}{192}\sqrt{bx^2+ax}\left(\frac{15\,a^3}{b}+2\left(59\,a^2+4\left(6\,b^2x+17\,ab\right)x\right)x\right)$$

```
integrate((b*x^2+a*x)^(5/2)/x^2,x, algorithm="giac")
```

```
5/128*a^4*log(abs(2*(sqrt(b)*x - sqrt(b*x^2 + a*x))*sqrt(b) + a))/b^(3/2)
+ 1/192*sqrt(b*x^2 + a*x)*(15*a^3/b + 2*(59*a^2 + 4*(6*b^2*x + 17*a*b)*x)*
x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(ax+bx^2)^{5/2}}{x^2}\,dx = \int \frac{(b\,x^2+a\,x)^{5/2}}{x^2}\,dx$$

```
int((a*x + b*x^2)^(5/2)/x^2,x)
```

```
int((a*x + b*x^2)^(5/2)/x^2, x)
```

Reduce [B] (verification not implemented)

Time = 0.32 (sec) , antiderivative size = 95, normalized size of antiderivative = 0.77

$$\int \frac{(ax+bx^2)^{5/2}}{x^2}\,dx = \frac{15\sqrt{x}\,\sqrt{bx+a}\,a^3b+118\sqrt{x}\,\sqrt{bx+a}\,a^2b^2x+136\sqrt{x}\,\sqrt{bx+a}\,a\,b^3x^2+48\sqrt{x}\,\sqrt{bx+}}{192b^2}$$

```
int((b*x^2+a*x)^(5/2)/x^2,x)
```

```
(15*sqrt(x)*sqrt(a + b*x)*a**3*b + 118*sqrt(x)*sqrt(a + b*x)*a**2*b**2*x +
 136*sqrt(x)*sqrt(a + b*x)*a*b**3*x**2 + 48*sqrt(x)*sqrt(a + b*x)*b**4*x**
3 - 15*sqrt(b)*log((sqrt(a + b*x) + sqrt(x)*sqrt(b))/sqrt(a))*a**4)/(192*b
**2)
```

3.27 $\int \frac{(ax+bx^2)^{5/2}}{x^3}\,dx$

Optimal result

Integrand size = 17, antiderivative size = 97

$$\int \frac{(ax+bx^2)^{5/2}}{x^3}\,dx = \frac{11}{8}a^2\sqrt{ax+bx^2}+\frac{13}{12}abx\sqrt{ax+bx^2} + \frac{1}{3}b^2x^2\sqrt{ax+bx^2}+\frac{5a^3\operatorname{arctanh}\left(\frac{\sqrt{b}x}{\sqrt{ax+bx^2}}\right)}{8\sqrt{b}}$$

```
11/8*a^2*(b*x^2+a*x)^(1/2)+13/12*a*b*x*(b*x^2+a*x)^(1/2)+1/3*b^2*x^2*(b*x^
2+a*x)^(1/2)+5/8*a^3*arctanh(b^(1/2)*x/(b*x^2+a*x)^(1/2))/b^(1/2)
```

Mathematica [A] (verified)

Time = 0.12 (sec) , antiderivative size = 81, normalized size of antiderivative = 0.84

$$\int \frac{(ax+bx^2)^{5/2}}{x^3}\,dx = \frac{1}{24}\sqrt{x(a+bx)}\left(33a^2+26abx + 8b^2x^2-\frac{15a^3\log\left(-\sqrt{b}\sqrt{x}+\sqrt{a+bx}\right)}{\sqrt{b}\sqrt{x}\sqrt{a+bx}}\right)$$

```
Integrate[(a*x + b*x^2)^(5/2)/x^3,x]
```

```
(Sqrt[x*(a + b*x)]*(33*a^2 + 26*a*b*x + 8*b^2*x^2 - (15*a^3*Log[-(Sqrt[b]*
Sqrt[x]) + Sqrt[a + b*x]])/(Sqrt[b]*Sqrt[x]*Sqrt[a + b*x])))/24
```

Rubi [A] (verified)

Time = 0.38 (sec) , antiderivative size = 105, normalized size of antiderivative = 1.08, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.294$, Rules used = {1130, 1131, 1087, 1091, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(ax+bx^2)^{5/2}}{x^3}dx$$

$$\Big\downarrow \text{1130}$$

$$\frac{2(ax+bx^2)^{5/2}}{x^2}-5b\int \frac{(bx^2+ax)^{3/2}}{x}dx$$

$$\Big\downarrow \text{1131}$$

$$\frac{2(ax+bx^2)^{5/2}}{x^2}-5b\left(\frac{1}{2}a\int \sqrt{bx^2+ax}dx+\frac{1}{3}(ax+bx^2)^{3/2}\right)$$

$$\Big\downarrow \text{1087}$$

$$\frac{2(ax+bx^2)^{5/2}}{x^2}-5b\left(\frac{1}{2}a\left(\frac{(a+2bx)\sqrt{ax+bx^2}}{4b}-\frac{a^2\int \frac{1}{\sqrt{bx^2+ax}}dx}{8b}\right)+\frac{1}{3}(ax+bx^2)^{3/2}\right)$$

$$\Big\downarrow \text{1091}$$

$$\frac{2(ax+bx^2)^{5/2}}{x^2}-5b\left(\frac{1}{2}a\left(\frac{(a+2bx)\sqrt{ax+bx^2}}{4b}-\frac{a^2\int \frac{1}{1-\frac{bx^2}{bx^2+ax}}d\frac{x}{\sqrt{bx^2+ax}}}{4b}\right)+\frac{1}{3}(ax+bx^2)^{3/2}\right)$$

$$\Big\downarrow \text{219}$$

$$\frac{2(ax+bx^2)^{5/2}}{x^2}-5b\left(\frac{1}{2}a\left(\frac{(a+2bx)\sqrt{ax+bx^2}}{4b}-\frac{a^2\operatorname{arctanh}\left(\frac{\sqrt{b}x}{\sqrt{ax+bx^2}}\right)}{4b^{3/2}}\right)+\frac{1}{3}(ax+bx^2)^{3/2}\right)$$

```
Int[(a*x + b*x^2)^(5/2)/x^3,x]
```

```
(2*(a*x + b*x^2)^(5/2))/x^2 - 5*b*((a*x + b*x^2)^(3/2)/3 + (a*(((a + 2*b*x
)*Sqrt[a*x + b*x^2])/(4*b) - (a^2*ArcTanh[(Sqrt[b]*x)/Sqrt[a*x + b*x^2]])/
(4*b^(3/2))))/2)
```

Defintions of rubi rules used

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(b + 2*c*x)
*((a + b*x + c*x^2)^p/(2*c*(2*p + 1))), x] - Simp[p*((b^2 - 4*a*c)/(2*c*(2*
p + 1)))   Int[(a + b*x + c*x^2)^(p - 1), x], x] /; FreeQ[{a, b, c}, x] &&
GtQ[p, 0] && (IntegerQ[4*p] || IntegerQ[3*p])
```

```
Int[1/Sqrt[(b_.)*(x_) + (c_.)*(x_)^2], x_Symbol] :> Simp[2   Subst[Int[1/(1
 - c*x^2), x], x, x/Sqrt[b*x + c*x^2]], x] /; FreeQ[{b, c}, x]
```

```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] :> Simp[(d + e*x)^(m + 1)*((a + b*x + c*x^2)^p/(e*(m + p + 1))), x]
- Simp[c*(p/(e^2*(m + p + 1)))   Int[(d + e*x)^(m + 2)*(a + b*x + c*x^2)^(p
 - 1), x], x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[c*d^2 - b*d*e + a*e^2, 0]
 && GtQ[p, 0] && (LtQ[m, -2] || EqQ[m + 2*p + 1, 0]) && NeQ[m + p + 1, 0] &
& IntegerQ[2*p]
```

```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] :> Simp[(d + e*x)^(m + 1)*((a + b*x + c*x^2)^p/(e*(m + 2*p + 1))), x
] - Simp[p*((2*c*d - b*e)/(e^2*(m + 2*p + 1)))   Int[(d + e*x)^(m + 1)*(a +
 b*x + c*x^2)^(p - 1), x], x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[c*d^2 - b
*d*e + a*e^2, 0] && GtQ[p, 0] && (LeQ[-2, m, 0] || EqQ[m + p + 1, 0]) && Ne
Q[m + 2*p + 1, 0] && IntegerQ[2*p]
```

Maple [A] (verified)

Time = 0.38 (sec) , antiderivative size = 64, normalized size of antiderivative = 0.66

method	result
pseudoelliptic	$\frac{5 \operatorname{arctanh}\left(\frac{\sqrt{x(bx+a)}}{x\sqrt{b}}\right)a^{3}}{8\sqrt{b}}+\frac{11\sqrt{x(bx+a)}\left(\sqrt{b}\,a^{2}+\frac{26b^{\frac{3}{2}}ax}{33}+\frac{8b^{\frac{5}{2}}x^{2}}{33}\right)}{8\sqrt{b}}$
risch	$\frac{(8b^{2}x^{2}+26abx+33a^{2})x(bx+a)}{24\sqrt{x(bx+a)}}+\frac{5a^{3}\ln\left(\frac{\frac{a}{2}+bx}{\sqrt{b}}+\sqrt{b\,x^{2}+ax}\right)}{16\sqrt{b}}$
default	$\frac{2(b\,x^{2}+ax)^{\frac{7}{2}}}{a\,x^{3}}-\frac{8b\left(\frac{2(b\,x^{2}+ax)^{\frac{7}{2}}}{3a\,x^{2}}-\frac{10b\left(\frac{(b\,x^{2}+ax)^{\frac{5}{2}}}{5}+\frac{a\left(\frac{(2bx+a)(b\,x^{2}+ax)^{\frac{3}{2}}}{8b}-\frac{3a^{2}\left(\frac{(2bx+a)\sqrt{b\,x^{2}+ax}}{4b}-\frac{a^{2}\ln\left(\frac{\frac{a}{2}+bx}{\sqrt{b}}+\sqrt{b}\right.}{8b^{\frac{3}{2}}}\right.}{16b}\right.}{2}\right.}{3a}\right.}{a}$

```
int((b*x^2+a*x)^(5/2)/x^3,x,method=_RETURNVERBOSE)
```

```
5/8/b^(1/2)*arctanh((x*(b*x+a))^(1/2)/x/b^(1/2))*a^3+11/8/b^(1/2)*(x*(b*x+
a))^(1/2)*(b^(1/2)*a^2+26/33*b^(3/2)*a*x+8/33*b^(5/2)*x^2)
```

Fricas [A] (verification not implemented)

Time = 0.10 (sec) , antiderivative size = 149, normalized size of antiderivative = 1.54

$$\int \frac{(ax+bx^2)^{5/2}}{x^3}\,dx = \left[\frac{15\,a^3\sqrt{b}\log\left(2\,bx+a+2\,\sqrt{bx^2+ax}\sqrt{b}\right)+2\,(8\,b^3x^2+26\,ab^2x+33\,a^2b)\sqrt{bx^2+ax}}{48\,b}, -\frac{15\,a^3\sqrt{-b}\arctan\left(\frac{\sqrt{bx^2+ax}\sqrt{-b}}{bx+a}\right)-(8\,b^3x^2+26\,ab^2x+33\,a^2b)\sqrt{bx^2+ax}}{24\,b}\right]$$

```
integrate((b*x^2+a*x)^(5/2)/x^3,x, algorithm="fricas")
```

```
[1/48*(15*a^3*sqrt(b)*log(2*b*x + a + 2*sqrt(b*x^2 + a*x)*sqrt(b)) + 2*(8*
b^3*x^2 + 26*a*b^2*x + 33*a^2*b)*sqrt(b*x^2 + a*x))/b, -1/24*(15*a^3*sqrt(
-b)*arctan(sqrt(b*x^2 + a*x)*sqrt(-b)/(b*x + a)) - (8*b^3*x^2 + 26*a*b^2*x
 + 33*a^2*b)*sqrt(b*x^2 + a*x))/b]
```

Sympy [F]

$$\int \frac{(ax+bx^2)^{5/2}}{x^3}\,dx = \int \frac{(x(a+bx))^{\frac{5}{2}}}{x^3}\,dx$$

```
integrate((b*x**2+a*x)**(5/2)/x**3,x)
```

```
Integral((x*(a + b*x))**(5/2)/x**3, x)
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 81, normalized size of antiderivative = 0.84

$$\int \frac{(ax+bx^2)^{5/2}}{x^3}\,dx = \frac{5\,a^3\log\left(2\,bx+a+2\,\sqrt{bx^2+ax}\sqrt{b}\right)}{16\,\sqrt{b}} + \frac{5}{8}\,\sqrt{bx^2+ax}a^2 + \frac{5\,(bx^2+ax)^{\frac{3}{2}}a}{12\,x} + \frac{(bx^2+ax)^{\frac{5}{2}}}{3\,x^2}$$

```
integrate((b*x^2+a*x)^(5/2)/x^3,x, algorithm="maxima")
```

```
5/16*a^3*log(2*b*x + a + 2*sqrt(b*x^2 + a*x)*sqrt(b))/sqrt(b) + 5/8*sqrt(b
*x^2 + a*x)*a^2 + 5/12*(b*x^2 + a*x)^(3/2)*a/x + 1/3*(b*x^2 + a*x)^(5/2)/x
^2
```

Giac [A] (verification not implemented)

Time = 0.21 (sec) , antiderivative size = 70, normalized size of antiderivative = 0.72

$$\int \frac{(ax+bx^2)^{5/2}}{x^3}\,dx = -\frac{5\,a^3\log\left(\left|2\left(\sqrt{b}x-\sqrt{bx^2+ax}\right)\sqrt{b}+a\right|\right)}{16\,\sqrt{b}} + \frac{1}{24}\,\sqrt{bx^2+ax}\left(33\,a^2+2\,\left(4\,b^2x+13\,ab\right)x\right)$$

```
integrate((b*x^2+a*x)^(5/2)/x^3,x, algorithm="giac")
```

```
-5/16*a^3*log(abs(2*(sqrt(b)*x - sqrt(b*x^2 + a*x))*sqrt(b) + a))/sqrt(b)
+ 1/24*sqrt(b*x^2 + a*x)*(33*a^2 + 2*(4*b^2*x + 13*a*b)*x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(ax+bx^2)^{5/2}}{x^3}\,dx = \int \frac{(b\,x^2+a\,x)^{5/2}}{x^3}\,dx$$

```
int((a*x + b*x^2)^(5/2)/x^3,x)
```

```
int((a*x + b*x^2)^(5/2)/x^3, x)
```

Reduce [B] (verification not implemented)

Time = 0.21 (sec) , antiderivative size = 76, normalized size of antiderivative = 0.78

$$\int \frac{(ax+bx^2)^{5/2}}{x^3}\,dx = \frac{33\sqrt{x}\,\sqrt{bx+a}\,a^2b+26\sqrt{x}\,\sqrt{bx+a}\,a\,b^2x+8\sqrt{x}\,\sqrt{bx+a}\,b^3x^2+15\sqrt{b}\log\left(\frac{\sqrt{bx+a}+\sqrt{}}{\sqrt{a}}\right.}{24b}$$

```
int((b*x^2+a*x)^(5/2)/x^3,x)
```

```
(33*sqrt(x)*sqrt(a + b*x)*a**2*b + 26*sqrt(x)*sqrt(a + b*x)*a*b**2*x + 8*s
qrt(x)*sqrt(a + b*x)*b**3*x**2 + 15*sqrt(b)*log((sqrt(a + b*x) + sqrt(x)*s
qrt(b))/sqrt(a))*a**3)/(24*b)
```

3.28 $\int \frac{(ax+bx^2)^{5/2}}{x^4} \, dx$

Optimal result

Integrand size = 17, antiderivative size = 95

$$\int \frac{(ax+bx^2)^{5/2}}{x^4} \, dx = \frac{9}{4}ab\sqrt{ax+bx^2} - \frac{2a^2\sqrt{ax+bx^2}}{x} + \frac{1}{2}b^2x\sqrt{ax+bx^2} + \frac{15}{4}a^2\sqrt{b}\text{arctanh}\left(\frac{\sqrt{b}x}{\sqrt{ax+bx^2}}\right)$$

```
9/4*a*b*(b*x^2+a*x)^(1/2)-2*a^2*(b*x^2+a*x)^(1/2)/x+1/2*b^2*x*(b*x^2+a*x)^
(1/2)+15/4*a^2*b^(1/2)*arctanh(b^(1/2)*x/(b*x^2+a*x)^(1/2))
```

Mathematica [A] (verified)

Time = 0.28 (sec) , antiderivative size = 100, normalized size of antiderivative = 1.05

$$\int \frac{(ax+bx^2)^{5/2}}{x^4} \, dx = \frac{\sqrt{a+bx}\left(\sqrt{a+bx}(-8a^2+9abx+2b^2x^2)+30a^2\sqrt{b}\sqrt{x}\text{arctanh}\left(\frac{\sqrt{b}\sqrt{x}}{-\sqrt{a}+\sqrt{a+bx}}\right)\right)}{4\sqrt{x(a+bx)}}$$

```
Integrate[(a*x + b*x^2)^(5/2)/x^4,x]
```

```
(Sqrt[a + b*x]*(Sqrt[a + b*x]*(-8*a^2 + 9*a*b*x + 2*b^2*x^2) + 30*a^2*Sqrt
[b]*Sqrt[x]*ArcTanh[(Sqrt[b]*Sqrt[x])/(-Sqrt[a] + Sqrt[a + b*x])]))/(4*Sqr
t[x*(a + b*x)])
```

Rubi [A] (verified)

Time = 0.44 (sec) , antiderivative size = 94, normalized size of antiderivative = 0.99, number of steps used = 8, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.412, Rules used = {1125, 25, 2192, 27, 1160, 1091, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(ax+bx^2)^{5/2}}{x^4}\,dx$$

$$\Big\downarrow \text{1125}$$

$$-\int -\frac{x^2b^3+3axb^2+3a^2b}{\sqrt{bx^2+ax}}dx-\frac{2a^2\sqrt{ax+bx^2}}{x}$$

$$\Big\downarrow \text{25}$$

$$\int \frac{x^2b^3+3axb^2+3a^2b}{\sqrt{bx^2+ax}}dx-\frac{2a^2\sqrt{ax+bx^2}}{x}$$

$$\Big\downarrow \text{2192}$$

$$\frac{\int \frac{3ab^2(4a+3bx)}{2\sqrt{bx^2+ax}}dx}{2b}-\frac{2a^2\sqrt{ax+bx^2}}{x}+\frac{1}{2}b^2x\sqrt{ax+bx^2}$$

$$\Big\downarrow \text{27}$$

$$\frac{3}{4}ab\int \frac{4a+3bx}{\sqrt{bx^2+ax}}dx-\frac{2a^2\sqrt{ax+bx^2}}{x}+\frac{1}{2}b^2x\sqrt{ax+bx^2}$$

$$\Big\downarrow \text{1160}$$

$$\frac{3}{4}ab\left(\frac{5}{2}a\int \frac{1}{\sqrt{bx^2+ax}}dx+3\sqrt{ax+bx^2}\right)-\frac{2a^2\sqrt{ax+bx^2}}{x}+\frac{1}{2}b^2x\sqrt{ax+bx^2}$$

$$\Big\downarrow \text{1091}$$

$$\frac{3}{4}ab\left(5a\int \frac{1}{1-\frac{bx^2}{bx^2+ax}}d\frac{x}{\sqrt{bx^2+ax}}+3\sqrt{ax+bx^2}\right)-\frac{2a^2\sqrt{ax+bx^2}}{x}+\frac{1}{2}b^2x\sqrt{ax+bx^2}$$

$$\Big\downarrow \text{219}$$

$$-\frac{2a^2\sqrt{ax+bx^2}}{x}+\frac{3}{4}ab\left(\frac{5a\text{arctanh}\left(\frac{\sqrt{b}x}{\sqrt{ax+bx^2}}\right)}{\sqrt{b}}+3\sqrt{ax+bx^2}\right)+\frac{1}{2}b^2x\sqrt{ax+bx^2}$$

```
Int[(a*x + b*x^2)^(5/2)/x^4,x]
```

```
(-2*a^2*Sqrt[a*x + b*x^2])/x + (b^2*x*Sqrt[a*x + b*x^2])/2 + (3*a*b*(3*Sqr
t[a*x + b*x^2] + (5*a*ArcTanh[(Sqrt[b]*x)/Sqrt[a*x + b*x^2]])/Sqrt[b]))/4
```

Defintions of rubi rules used

```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

```
Int[1/Sqrt[(b_.)*(x_) + (c_.)*(x_)^2], x_Symbol] :> Simp[2   Subst[Int[1/(1
 - c*x^2), x], x, x/Sqrt[b*x + c*x^2]], x] /; FreeQ[{b, c}, x]
```

```
Int[((d_.) + (e_.)*(x_))^(m_.)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_
Symbol] :> Simp[-2*e^(2*m + 3)*(Sqrt[a + b*x + c*x^2]/((-2*c*d + b*e)^(m +
2)*(d + e*x))), x] - Simp[e^(2*m + 2)   Int[(1/Sqrt[a + b*x + c*x^2])*Expan
dToSum[((-2*c*d + b*e)^(-m - 1) - ((-c)*d + b*e + c*e*x)^(-m - 1))/(d + e*x
), x], x], x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[c*d^2 - b*d*e + a*e^2, 0]
 && ILtQ[m, 0] && EqQ[m + p, -3/2]
```

```
Int[((d_.) + (e_.)*(x_))*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol
] :> Simp[e*((a + b*x + c*x^2)^(p + 1)/(2*c*(p + 1))), x] + Simp[(2*c*d - b
*e)/(2*c)   Int[(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, p}, x]
 && NeQ[p, -1]
```

```
Int[(Pq_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> With[{q =
Expon[Pq, x], e = Coeff[Pq, x, Expon[Pq, x]]}, Simp[e*x^(q - 1)*((a + b*x +
 c*x^2)^(p + 1)/(c*(q + 2*p + 1))), x] + Simp[1/(c*(q + 2*p + 1))   Int[(a
+ b*x + c*x^2)^p*ExpandToSum[c*(q + 2*p + 1)*Pq - a*e*(q - 1)*x^(q - 2) - b
*e*(q + p)*x^(q - 1) - c*e*(q + 2*p + 1)*x^q, x], x], x]] /; FreeQ[{a, b, c
, p}, x] && PolyQ[Pq, x] && NeQ[b^2 - 4*a*c, 0] &&  !LeQ[p, -1]
```

Maple [A] (verified)

Time = 0.40 (sec) , antiderivative size = 69, normalized size of antiderivative = 0.73

method	result
risch	$-\frac{(bx+a)\left(-2b^{2}x^{2}-9abx+8a^{2}\right)}{4\sqrt{x(bx+a)}}+\frac{15a^{2}\sqrt{b}\,\ln\left(\frac{\frac{a}{2}+bx}{\sqrt{b}}+\sqrt{b\,x^{2}+ax}\right)}{8}$
pseudoelliptic	$\frac{2b^{\frac{5}{2}}\sqrt{x(bx+a)}\,x^{2}+15\,\mathrm{arctanh}\left(\frac{\sqrt{x(bx+a)}}{x\sqrt{b}}\right)bx\,a^{2}+9a\,b^{\frac{3}{2}}x\sqrt{x(bx+a)}-8a^{2}\sqrt{b}\,\sqrt{x(bx+a)}}{4x\sqrt{b}}$
default	$-\frac{2\left(b\,x^{2}+ax\right)^{\frac{7}{2}}}{a\,x^{4}}+\frac{6b\left(\frac{2\left(b\,x^{2}+ax\right)^{\frac{7}{2}}}{a\,x^{3}}-\frac{8b\left(\frac{2\left(b\,x^{2}+ax\right)^{\frac{7}{2}}}{3a\,x^{2}}-\frac{10b\left(\frac{\left(b\,x^{2}+ax\right)^{\frac{5}{2}}}{5}+\frac{a\left(\frac{(2bx+a)\left(b\,x^{2}+ax\right)^{\frac{3}{2}}}{8b}-\frac{3a^{2}\left(\frac{(2bx+a)\sqrt{b\,x^{2}+\cdots}}{4b}\cdots\right)}{\cdots}\right)}{2}\cdots\right)}{3a}\cdots\right)}{a}\cdots\right)}{a}\cdots$

```
int((b*x^2+a*x)^(5/2)/x^4,x,method=_RETURNVERBOSE)
```

```
-1/4*(b*x+a)*(-2*b^2*x^2-9*a*b*x+8*a^2)/(x*(b*x+a))^(1/2)+15/8*a^2*b^(1/2)
*ln((1/2*a+b*x)/b^(1/2)+(b*x^2+a*x)^(1/2))
```

Fricas [A] (verification not implemented)

Time = 0.10 (sec) , antiderivative size = 145, normalized size of antiderivative = 1.53

$$\int \frac{(ax+bx^2)^{5/2}}{x^4}\,dx = \left[\frac{15\,a^2\sqrt{b}x\log\left(2\,bx+a+2\,\sqrt{bx^2+ax}\sqrt{b}\right)+2\,(2\,b^2x^2+9\,abx-8\,a^2)\sqrt{bx^2+ax}}{8\,x},\ -\frac{15\,a^2\sqrt{-b}x\arctan\left(\frac{\sqrt{bx^2+ax}\sqrt{-b}}{bx+a}\right)-(2\,b^2x^2+9\,abx-8\,a^2)\sqrt{bx^2+ax}}{4\,x}\right]$$

```
integrate((b*x^2+a*x)^(5/2)/x^4,x, algorithm="fricas")
```

```
[1/8*(15*a^2*sqrt(b)*x*log(2*b*x + a + 2*sqrt(b*x^2 + a*x)*sqrt(b)) + 2*(2
*b^2*x^2 + 9*a*b*x - 8*a^2)*sqrt(b*x^2 + a*x))/x, -1/4*(15*a^2*sqrt(-b)*x*
arctan(sqrt(b*x^2 + a*x)*sqrt(-b)/(b*x + a)) - (2*b^2*x^2 + 9*a*b*x - 8*a^
2)*sqrt(b*x^2 + a*x))/x]
```

Sympy [F]

$$\int \frac{(ax+bx^2)^{5/2}}{x^4}\,dx = \int \frac{(x(a+bx))^{\frac{5}{2}}}{x^4}\,dx$$

```
integrate((b*x**2+a*x)**(5/2)/x**4,x)
```

```
Integral((x*(a + b*x))**(5/2)/x**4, x)
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 84, normalized size of antiderivative = 0.88

$$\int \frac{(ax+bx^2)^{5/2}}{x^4}\,dx = \frac{15}{8}\,a^2\sqrt{b}\log\left(2\,bx+a+2\,\sqrt{bx^2+ax}\sqrt{b}\right) - \frac{15\,\sqrt{bx^2+ax}a^2}{4\,x} + \frac{5\,(bx^2+ax)^{\frac{3}{2}}a}{4\,x^2} + \frac{(bx^2+ax)^{\frac{5}{2}}}{2\,x^3}$$

```
integrate((b*x^2+a*x)^(5/2)/x^4,x, algorithm="maxima")
```

```
15/8*a^2*sqrt(b)*log(2*b*x + a + 2*sqrt(b*x^2 + a*x)*sqrt(b)) - 15/4*sqrt(
b*x^2 + a*x)*a^2/x + 5/4*(b*x^2 + a*x)^(3/2)*a/x^2 + 1/2*(b*x^2 + a*x)^(5/
2)/x^3
```

Giac [A] (verification not implemented)

Time = 0.15 (sec) , antiderivative size = 89, normalized size of antiderivative = 0.94

$$\int \frac{(ax+bx^2)^{5/2}}{x^4}\,dx = -\frac{15}{8}\,a^2\sqrt{b}\log\left(\left|-2\left(\sqrt{b}x-\sqrt{bx^2+ax}\right)\sqrt{b}-a\right|\right) + \frac{2\,a^3}{\sqrt{b}x-\sqrt{bx^2+ax}} + \frac{1}{4}\left(2\,b^2x+9\,ab\right)\sqrt{bx^2+ax}$$

```
integrate((b*x^2+a*x)^(5/2)/x^4,x, algorithm="giac")
```

```
-15/8*a^2*sqrt(b)*log(abs(-2*(sqrt(b)*x - sqrt(b*x^2 + a*x))*sqrt(b) - a))
 + 2*a^3/(sqrt(b)*x - sqrt(b*x^2 + a*x)) + 1/4*(2*b^2*x + 9*a*b)*sqrt(b*x^
2 + a*x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(ax+bx^2)^{5/2}}{x^4}\,dx = \int \frac{\left(b\,x^2+a\,x\right)^{5/2}}{x^4}\,dx$$

```
int((a*x + b*x^2)^(5/2)/x^4,x)
```

```
int((a*x + b*x^2)^(5/2)/x^4, x)
```

Reduce [B] (verification not implemented)

Time = 0.21 (sec) , antiderivative size = 82, normalized size of antiderivative = 0.86

$$\int \frac{(ax+bx^2)^{5/2}}{x^4}\,dx = \frac{-8\sqrt{x}\,\sqrt{bx+a}\,a^2+9\sqrt{x}\,\sqrt{bx+a}\,abx+2\sqrt{x}\,\sqrt{bx+a}\,b^2x^2+15\sqrt{b}\log\left(\frac{\sqrt{bx+a}+\sqrt{x}\,\sqrt{}}{\sqrt{a}}\right.}{4x}$$

```
int((b*x^2+a*x)^(5/2)/x^4,x)
```

```
( - 8*sqrt(x)*sqrt(a + b*x)*a**2 + 9*sqrt(x)*sqrt(a + b*x)*a*b*x + 2*sqrt(
x)*sqrt(a + b*x)*b**2*x**2 + 15*sqrt(b)*log((sqrt(a + b*x) + sqrt(x)*sqrt(
b))/sqrt(a))*a**2*x - 10*sqrt(b)*a**2*x)/(4*x)
```

3.29 $\int \frac{(ax+bx^2)^{5/2}}{x^5} dx$

Optimal result

Integrand size = 17, antiderivative size = 92

$$\int \frac{(ax+bx^2)^{5/2}}{x^5} dx = b^2\sqrt{ax+bx^2} - \frac{2a^2\sqrt{ax+bx^2}}{3x^2} - \frac{14ab\sqrt{ax+bx^2}}{3x} + 5ab^{3/2}\text{arctanh}\left(\frac{\sqrt{b}x}{\sqrt{ax+bx^2}}\right)$$

```
b^2*(b*x^2+a*x)^(1/2)-2/3*a^2*(b*x^2+a*x)^(1/2)/x^2-14/3*a*b*(b*x^2+a*x)^(
1/2)/x+5*a*b^(3/2)*arctanh(b^(1/2)*x/(b*x^2+a*x)^(1/2))
```

Mathematica [A] (verified)

Time = 0.22 (sec) , antiderivative size = 101, normalized size of antiderivative = 1.10

$$\int \frac{(ax+bx^2)^{5/2}}{x^5} dx = \frac{\sqrt{x(a+bx)}\left(\sqrt{a+bx}(-2a^2-14abx+3b^2x^2)+30ab^{3/2}x^{3/2}\text{arctanh}\left(\frac{\sqrt{b}\sqrt{x}}{-\sqrt{a}+\sqrt{a+bx}}\right)\right)}{3x^2\sqrt{a+bx}}$$

```
Integrate[(a*x + b*x^2)^(5/2)/x^5,x]
```

```
(Sqrt[x*(a + b*x)]*(Sqrt[a + b*x]*(-2*a^2 - 14*a*b*x + 3*b^2*x^2) + 30*a*b
^(3/2)*x^(3/2)*ArcTanh[(Sqrt[b]*Sqrt[x])/(-Sqrt[a] + Sqrt[a + b*x])]))/(3*
x^2*Sqrt[a + b*x])
```

Rubi [A] (verified)

Time = 0.38 (sec) , antiderivative size = 91, normalized size of antiderivative = 0.99, number of steps used = 8, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}} = 0.412$, Rules used = {1130, 1125, 25, 27, 1160, 1091, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\left(ax+bx^2\right)^{5/2}}{x^5}dx$$

$$\Big\downarrow \text{1130}$$

$$\frac{5}{3}b\int \frac{\left(bx^2+ax\right)^{3/2}}{x^3}dx-\frac{2\left(ax+bx^2\right)^{5/2}}{3x^4}$$

$$\Big\downarrow \text{1125}$$

$$\frac{5}{3}b\left(-\int -\frac{b(2a+bx)}{\sqrt{bx^2+ax}}dx-\frac{2a\sqrt{ax+bx^2}}{x}\right)-\frac{2\left(ax+bx^2\right)^{5/2}}{3x^4}$$

$$\Big\downarrow \text{25}$$

$$\frac{5}{3}b\left(\int \frac{b(2a+bx)}{\sqrt{bx^2+ax}}dx-\frac{2a\sqrt{ax+bx^2}}{x}\right)-\frac{2\left(ax+bx^2\right)^{5/2}}{3x^4}$$

$$\Big\downarrow \text{27}$$

$$\frac{5}{3}b\left(b\int \frac{2a+bx}{\sqrt{bx^2+ax}}dx-\frac{2a\sqrt{ax+bx^2}}{x}\right)-\frac{2\left(ax+bx^2\right)^{5/2}}{3x^4}$$

$$\Big\downarrow \text{1160}$$

$$\frac{5}{3}b\left(b\left(\frac{3}{2}a\int \frac{1}{\sqrt{bx^2+ax}}dx+\sqrt{ax+bx^2}\right)-\frac{2a\sqrt{ax+bx^2}}{x}\right)-\frac{2\left(ax+bx^2\right)^{5/2}}{3x^4}$$

$$\Big\downarrow \text{1091}$$

$$\frac{5}{3}b\left(b\left(3a\int\frac{1}{1-\frac{bx^2}{bx^2+ax}}d\frac{x}{\sqrt{bx^2+ax}}+\sqrt{ax+bx^2}\right)-\frac{2a\sqrt{ax+bx^2}}{x}\right)-\frac{2(ax+bx^2)^{5/2}}{3x^4}$$

$$\Big\downarrow \text{219}$$

$$\frac{5}{3}b\left(b\left(\frac{3a\text{arctanh}\left(\frac{\sqrt{b}x}{\sqrt{ax+bx^2}}\right)}{\sqrt{b}}+\sqrt{ax+bx^2}\right)-\frac{2a\sqrt{ax+bx^2}}{x}\right)-\frac{2(ax+bx^2)^{5/2}}{3x^4}$$

```
Int[(a*x + b*x^2)^(5/2)/x^5,x]
```

```
(-2*(a*x + b*x^2)^(5/2))/(3*x^4) + (5*b*((-2*a*Sqrt[a*x + b*x^2])/x + b*(S
qrt[a*x + b*x^2] + (3*a*ArcTanh[(Sqrt[b]*x)/Sqrt[a*x + b*x^2]])/Sqrt[b])))
/3
```

Defintions of rubi rules used

```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

```
Int[1/Sqrt[(b_.)*(x_) + (c_.)*(x_)^2], x_Symbol] :> Simp[2   Subst[Int[1/(1
 - c*x^2), x], x, x/Sqrt[b*x + c*x^2]], x] /; FreeQ[{b, c}, x]
```

```
Int[((d_.) + (e_.)*(x_))^(m_.)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_
Symbol] :> Simp[-2*e^(2*m + 3)*(Sqrt[a + b*x + c*x^2]/((-2*c*d + b*e)^(m +
2)*(d + e*x))), x] - Simp[e^(2*m + 2)   Int[(1/Sqrt[a + b*x + c*x^2])*Expan
dToSum[((-2*c*d + b*e)^(-m - 1) - ((-c)*d + b*e + c*e*x)^(-m - 1))/(d + e*x
), x], x], x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[c*d^2 - b*d*e + a*e^2, 0]
 && ILtQ[m, 0] && EqQ[m + p, -3/2]
```

```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] :> Simp[(d + e*x)^(m + 1)*((a + b*x + c*x^2)^p/(e*(m + p + 1))), x]
- Simp[c*(p/(e^2*(m + p + 1)))   Int[(d + e*x)^(m + 2)*(a + b*x + c*x^2)^(p
 - 1), x], x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[c*d^2 - b*d*e + a*e^2, 0]
 && GtQ[p, 0] && (LtQ[m, -2] || EqQ[m + 2*p + 1, 0]) && NeQ[m + p + 1, 0] &
& IntegerQ[2*p]
```

```
Int[((d_.) + (e_.)*(x_))*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol
] :> Simp[e*((a + b*x + c*x^2)^(p + 1)/(2*c*(p + 1))), x] + Simp[(2*c*d - b
*e)/(2*c)   Int[(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, p}, x]
 && NeQ[p, -1]
```

Maple [A] (verified)

Time = 0.41 (sec) , antiderivative size = 70, normalized size of antiderivative = 0.76

method	result
risch	$-\frac{(bx+a)\left(-3b^2x^2+14abx+2a^2\right)}{3x\sqrt{x(bx+a)}}+\frac{5a\,b^{\frac{3}{2}}\ln\left(\frac{\frac{a}{2}+bx}{\sqrt{b}}+\sqrt{b\,x^2+ax}\right)}{2}$
pseudoelliptic	$\frac{15\,\operatorname{arctanh}\left(\frac{\sqrt{x(bx+a)}}{x\sqrt{b}}\right)a\,b^2x^2+3b^{\frac{5}{2}}\sqrt{x(bx+a)}\,x^2-14a\,b^{\frac{3}{2}}x\sqrt{x(bx+a)}-2a^2\sqrt{b}\,\sqrt{x(bx+a)}}{3x^2\sqrt{b}}$
	$a\left(\frac{(2bx+a)\left(b\,x^2+ax\right)^{\frac{3}{2}}}{8b}\right.$ $10b\left(\frac{\left(b\,x^2+ax\right)^{\frac{5}{2}}}{5}+\right.$ $8b\left(\frac{2\left(b\,x^2+ax\right)^{\frac{7}{2}}}{3a\,x^2}-\right.$ $6b\left(\frac{2\left(b\,x^2+ax\right)^{\frac{7}{2}}}{a\,x^3}-\right.$ a $4b\left(-\frac{2\left(b\,x^2+ax\right)^{\frac{7}{2}}}{a\,x^4}+\right.$ a

```
int((b*x^2+a*x)^(5/2)/x^5,x,method=_RETURNVERBOSE)
```

```
-1/3*(b*x+a)*(-3*b^2*x^2+14*a*b*x+2*a^2)/x/(x*(b*x+a))^(1/2)+5/2*a*b^(3/2)
*ln((1/2*a+b*x)/b^(1/2)+(b*x^2+a*x)^(1/2))
```

Fricas [A] (verification not implemented)

Time = 0.10 (sec) , antiderivative size = 146, normalized size of antiderivative = 1.59

$$\int \frac{(ax+bx^2)^{5/2}}{x^5}\,dx = \left[\frac{15\,ab^{\frac{3}{2}}x^2\log\left(2\,bx+a+2\,\sqrt{bx^2+ax}\sqrt{b}\right)+2\,(3\,b^2x^2-14\,abx-2\,a^2)\sqrt{bx^2+ax}}{6\,x^2},\right.$$
$$\left.-\frac{15\,a\sqrt{-b}bx^2\arctan\left(\frac{\sqrt{bx^2+ax}\sqrt{-b}}{bx+a}\right)-(3\,b^2x^2-14\,abx-2\,a^2)\sqrt{bx^2+ax}}{3\,x^2}\right]$$

```
integrate((b*x^2+a*x)^(5/2)/x^5,x, algorithm="fricas")
```

```
[1/6*(15*a*b^(3/2)*x^2*log(2*b*x + a + 2*sqrt(b*x^2 + a*x)*sqrt(b)) + 2*(3
*b^2*x^2 - 14*a*b*x - 2*a^2)*sqrt(b*x^2 + a*x))/x^2, -1/3*(15*a*sqrt(-b)*b
*x^2*arctan(sqrt(b*x^2 + a*x)*sqrt(-b)/(b*x + a)) - (3*b^2*x^2 - 14*a*b*x
- 2*a^2)*sqrt(b*x^2 + a*x))/x^2]
```

Sympy [F]

$$\int \frac{(ax+bx^2)^{5/2}}{x^5}\,dx = \int \frac{(x(a+bx))^{\frac{5}{2}}}{x^5}\,dx$$

```
integrate((b*x**2+a*x)**(5/2)/x**5,x)
```

```
Integral((x*(a + b*x))**(5/2)/x**5, x)
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 99, normalized size of antiderivative = 1.08

$$\int \frac{(ax+bx^2)^{5/2}}{x^5}\,dx = \frac{5}{2}\,ab^{\frac{3}{2}}\log\left(2\,bx+a+2\,\sqrt{bx^2+ax}\sqrt{b}\right) - \frac{35\,\sqrt{bx^2+ax}ab}{6\,x} - \frac{5\,\sqrt{bx^2+ax}a^2}{6\,x^2} - \frac{5\,(bx^2+ax)^{\frac{3}{2}}a}{6\,x^3} + \frac{(bx^2+ax)^{\frac{5}{2}}}{x^4}$$

```
integrate((b*x^2+a*x)^(5/2)/x^5,x, algorithm="maxima")
```

```
5/2*a*b^(3/2)*log(2*b*x + a + 2*sqrt(b*x^2 + a*x)*sqrt(b)) - 35/6*sqrt(b*x
^2 + a*x)*a*b/x - 5/6*sqrt(b*x^2 + a*x)*a^2/x^2 - 5/6*(b*x^2 + a*x)^(3/2)*
a/x^3 + (b*x^2 + a*x)^(5/2)/x^4
```

Giac [A] (verification not implemented)

Time = 0.15 (sec) , antiderivative size = 133, normalized size of antiderivative = 1.45

$$\int \frac{(ax+bx^2)^{5/2}}{x^5}\,dx = -\frac{5}{2}\,ab^{\frac{3}{2}}\log\left(\left|-2\left(\sqrt{b}x-\sqrt{bx^2+ax}\right)\sqrt{b}-a\right|\right) + \sqrt{bx^2+ax}b^2 + \frac{2\left(9\left(\sqrt{b}x-\sqrt{bx^2+ax}\right)^2a^2b+3\left(\sqrt{b}x-\sqrt{bx^2+ax}\right)a^3\sqrt{b}+a^4\right)}{3\left(\sqrt{b}x-\sqrt{bx^2+ax}\right)^3}$$

```
integrate((b*x^2+a*x)^(5/2)/x^5,x, algorithm="giac")
```

```
-5/2*a*b^(3/2)*log(abs(-2*(sqrt(b)*x - sqrt(b*x^2 + a*x))*sqrt(b) - a)) +
sqrt(b*x^2 + a*x)*b^2 + 2/3*(9*(sqrt(b)*x - sqrt(b*x^2 + a*x))^2*a^2*b + 3
*(sqrt(b)*x - sqrt(b*x^2 + a*x))*a^3*sqrt(b) + a^4)/(sqrt(b)*x - sqrt(b*x^
2 + a*x))^3
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(ax+bx^2)^{5/2}}{x^5}\,dx=\int \frac{(b\,x^2+a\,x)^{5/2}}{x^5}\,dx$$

```
int((a*x + b*x^2)^(5/2)/x^5,x)
```

```
int((a*x + b*x^2)^(5/2)/x^5, x)
```

Reduce [B] (verification not implemented)

Time = 0.20 (sec) , antiderivative size = 84, normalized size of antiderivative = 0.91

$$\int \frac{(ax+bx^2)^{5/2}}{x^5}\,dx=\frac{-4\sqrt{x}\,\sqrt{bx+a}\,a^2-28\sqrt{x}\,\sqrt{bx+a}\,abx+6\sqrt{x}\,\sqrt{bx+a}\,b^2x^2+30\sqrt{b}\log\left(\frac{\sqrt{bx+a}+\sqrt{x}}{\sqrt{a}}\right.}{6x^2}$$

```
int((b*x^2+a*x)^(5/2)/x^5,x)
```

```
( - 4*sqrt(x)*sqrt(a + b*x)*a**2 - 28*sqrt(x)*sqrt(a + b*x)*a*b*x + 6*sqrt
(x)*sqrt(a + b*x)*b**2*x**2 + 30*sqrt(b)*log((sqrt(a + b*x) + sqrt(x)*sqrt
(b))/sqrt(a))*a*b*x**2 + 5*sqrt(b)*a*b*x**2)/(6*x**2)
```

3.30 $\int \frac{(ax+bx^2)^{5/2}}{x^6}\,dx$

Optimal result

Integrand size = 17, antiderivative size = 97

$$\int \frac{(ax+bx^2)^{5/2}}{x^6}\,dx = -\frac{2a^2\sqrt{ax+bx^2}}{5x^3} - \frac{22ab\sqrt{ax+bx^2}}{15x^2} - \frac{46b^2\sqrt{ax+bx^2}}{15x} + 2b^{5/2}\text{arctanh}\left(\frac{\sqrt{b}x}{\sqrt{ax+bx^2}}\right)$$

```
-2/5*a^2*(b*x^2+a*x)^(1/2)/x^3-22/15*a*b*(b*x^2+a*x)^(1/2)/x^2-46/15*b^2*(
b*x^2+a*x)^(1/2)/x+2*b^(5/2)*arctanh(b^(1/2)*x/(b*x^2+a*x)^(1/2))
```

Mathematica [A] (verified)

Time = 0.18 (sec) , antiderivative size = 92, normalized size of antiderivative = 0.95

$$\int \frac{(ax+bx^2)^{5/2}}{x^6}\,dx = -\frac{2\sqrt{x(a+bx)}\left(\sqrt{a+bx}(3a^2+11abx+23b^2x^2)+15b^{5/2}x^{5/2}\log\left(-\sqrt{b}\sqrt{x}+\sqrt{a+bx}\right)\right)}{15x^3\sqrt{a+bx}}$$

```
Integrate[(a*x + b*x^2)^(5/2)/x^6,x]
```

```
(-2*Sqrt[x*(a + b*x)]*(Sqrt[a + b*x]*(3*a^2 + 11*a*b*x + 23*b^2*x^2) + 15*
b^(5/2)*x^(5/2)*Log[-(Sqrt[b]*Sqrt[x]) + Sqrt[a + b*x]]))/(15*x^3*Sqrt[a +
 b*x])
```

Rubi [A] (verified)

Time = 0.37 (sec) , antiderivative size = 93, normalized size of antiderivative = 0.96, number of steps used = 8, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}} = 0.412$, Rules used = {1130, 1130, 1125, 25, 27, 1091, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(ax+bx^2)^{5/2}}{x^6}dx$$

$$\Big\downarrow \text{1130}$$

$$b\int \frac{(bx^2+ax)^{3/2}}{x^4}dx-\frac{2(ax+bx^2)^{5/2}}{5x^5}$$

$$\Big\downarrow \text{1130}$$

$$b\left(b\int \frac{\sqrt{bx^2+ax}}{x^2}dx-\frac{2(ax+bx^2)^{3/2}}{3x^3}\right)-\frac{2(ax+bx^2)^{5/2}}{5x^5}$$

$$\Big\downarrow \text{1125}$$

$$b\left(b\left(-\int -\frac{b}{\sqrt{bx^2+ax}}dx-\frac{2\sqrt{ax+bx^2}}{x}\right)-\frac{2(ax+bx^2)^{3/2}}{3x^3}\right)-\frac{2(ax+bx^2)^{5/2}}{5x^5}$$

$$\Big\downarrow \text{25}$$

$$b\left(b\left(\int \frac{b}{\sqrt{bx^2+ax}}dx-\frac{2\sqrt{ax+bx^2}}{x}\right)-\frac{2(ax+bx^2)^{3/2}}{3x^3}\right)-\frac{2(ax+bx^2)^{5/2}}{5x^5}$$

$$\Big\downarrow \text{27}$$

$$b\left(b\left(b\int \frac{1}{\sqrt{bx^2+ax}}dx-\frac{2\sqrt{ax+bx^2}}{x}\right)-\frac{2(ax+bx^2)^{3/2}}{3x^3}\right)-\frac{2(ax+bx^2)^{5/2}}{5x^5}$$

$$\Big\downarrow \text{1091}$$

$$b\left(b\left(2b\int \frac{1}{1-\frac{bx^2}{bx^2+ax}}d\frac{x}{\sqrt{bx^2+ax}}-\frac{2\sqrt{ax+bx^2}}{x}\right)-\frac{2\left(ax+bx^2\right)^{3/2}}{3x^3}\right)-\frac{2\left(ax+bx^2\right)^{5/2}}{5x^5}$$

$$\Big\downarrow \text{219}$$

$$b\left(b\left(2\sqrt{b}\text{arctanh}\left(\frac{\sqrt{b}x}{\sqrt{ax+bx^2}}\right)-\frac{2\sqrt{ax+bx^2}}{x}\right)-\frac{2\left(ax+bx^2\right)^{3/2}}{3x^3}\right)-\frac{2\left(ax+bx^2\right)^{5/2}}{5x^5}$$

```
Int[(a*x + b*x^2)^(5/2)/x^6,x]
```

```
(-2*(a*x + b*x^2)^(5/2))/(5*x^5) + b*((-2*(a*x + b*x^2)^(3/2))/(3*x^3) + b
*((-2*Sqrt[a*x + b*x^2])/x + 2*Sqrt[b]*ArcTanh[(Sqrt[b]*x)/Sqrt[a*x + b*x^
2]]))
```

Defintions of rubi rules used

```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

```
Int[1/Sqrt[(b_.)*(x_) + (c_.)*(x_)^2], x_Symbol] :> Simp[2   Subst[Int[1/(1
 - c*x^2), x], x, x/Sqrt[b*x + c*x^2]], x] /; FreeQ[{b, c}, x]
```

```
Int[((d_.) + (e_.)*(x_))^(m_.)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_
Symbol] :> Simp[-2*e^(2*m + 3)*(Sqrt[a + b*x + c*x^2]/((-2*c*d + b*e)^(m +
2)*(d + e*x))), x] - Simp[e^(2*m + 2)   Int[(1/Sqrt[a + b*x + c*x^2])*Expan
dToSum[((-2*c*d + b*e)^(-m - 1) - ((-c)*d + b*e + c*e*x)^(-m - 1))/(d + e*x
), x], x], x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[c*d^2 - b*d*e + a*e^2, 0]
 && ILtQ[m, 0] && EqQ[m + p, -3/2]
```

```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] :> Simp[(d + e*x)^(m + 1)*((a + b*x + c*x^2)^p/(e*(m + p + 1))), x]
- Simp[c*(p/(e^2*(m + p + 1)))   Int[(d + e*x)^(m + 2)*(a + b*x + c*x^2)^(p
 - 1), x], x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[c*d^2 - b*d*e + a*e^2, 0]
 && GtQ[p, 0] && (LtQ[m, -2] || EqQ[m + 2*p + 1, 0]) && NeQ[m + p + 1, 0] &
& IntegerQ[2*p]
```

Maple [A] (verified)

Time = 0.41 (sec) , antiderivative size = 62, normalized size of antiderivative = 0.64

method	result
pseudoelliptic	$\frac{30b^{\frac{5}{2}}\operatorname{arctanh}\left(\frac{\sqrt{x(bx+a)}}{x\sqrt{b}}\right)x^3-2(23b^2x^2+11abx+3a^2)\sqrt{x(bx+a)}}{15x^3}$
risch	$-\frac{2(bx+a)(23b^2x^2+11abx+3a^2)}{15x^2\sqrt{x(bx+a)}}+b^{\frac{5}{2}}\ln\left(\frac{\frac{a}{2}+bx}{\sqrt{b}}+\sqrt{b\,x^2+ax}\right)$
	$4b\left(-\frac{2\left(b\,x^2+ax\right)^{\frac{7}{2}}}{a\,x^4}+\frac{6b\left(\frac{2\left(b\,x^2+ax\right)^{\frac{7}{2}}}{a\,x^3}-\frac{8b\left(\frac{2\left(b\,x^2+ax\right)^{\frac{7}{2}}}{3a\,x^2}-\frac{10b\left(\frac{\left(b\,x^2+ax\right)^{\frac{5}{2}}}{5}+\frac{a(\ldots)}{\ldots}\right)}{\ldots}\right)}{\ldots}\right)}{\ldots}\right)$

```
int((b*x^2+a*x)^(5/2)/x^6,x,method=_RETURNVERBOSE)
```

```
1/15*(30*b^(5/2)*arctanh((x*(b*x+a))^(1/2)/x/b^(1/2))*x^3-2*(23*b^2*x^2+11
*a*b*x+3*a^2)*(x*(b*x+a))^(1/2))/x^3
```

Fricas [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 145, normalized size of antiderivative = 1.49

$$\int \frac{(ax+bx^2)^{5/2}}{x^6}\,dx = \left[\frac{15\,b^{\frac{5}{2}}x^3\log\left(2\,bx+a+2\,\sqrt{bx^2+ax}\sqrt{b}\right)-2\,(23\,b^2x^2+11\,abx+3\,a^2)\sqrt{bx^2+ax}}{15\,x^3}, -\frac{2\left(15\,\sqrt{-b}b^2x^3\arctan\left(\frac{\sqrt{bx^2+ax}\sqrt{-b}}{bx+a}\right)+(23\,b^2x^2+11\,abx+3\,a^2)\sqrt{bx^2+ax}\right)}{15\,x^3}\right]$$

```
integrate((b*x^2+a*x)^(5/2)/x^6,x, algorithm="fricas")
```

```
[1/15*(15*b^(5/2)*x^3*log(2*b*x + a + 2*sqrt(b*x^2 + a*x)*sqrt(b)) - 2*(23
*b^2*x^2 + 11*a*b*x + 3*a^2)*sqrt(b*x^2 + a*x))/x^3, -2/15*(15*sqrt(-b)*b^
2*x^3*arctan(sqrt(b*x^2 + a*x)*sqrt(-b)/(b*x + a)) + (23*b^2*x^2 + 11*a*b*
x + 3*a^2)*sqrt(b*x^2 + a*x))/x^3]
```

Sympy [F]

$$\int \frac{(ax+bx^2)^{5/2}}{x^6}\,dx = \int \frac{(x(a+bx))^{\frac{5}{2}}}{x^6}\,dx$$

```
integrate((b*x**2+a*x)**(5/2)/x**6,x)
```

```
Integral((x*(a + b*x))**(5/2)/x**6, x)
```

Maxima [A] (verification not implemented)

Time = 0.05 (sec) , antiderivative size = 134, normalized size of antiderivative = 1.38

$$\int \frac{(ax+bx^2)^{5/2}}{x^6}\,dx = b^{\frac{5}{2}}\log\left(2\,bx+a+2\,\sqrt{bx^2+ax}\sqrt{b}\right)-\frac{38\,\sqrt{bx^2+ax}b^2}{15\,x}-\frac{7\,\sqrt{bx^2+ax}ab}{30\,x^2}+\frac{3\,\sqrt{bx^2+ax}a^2}{10\,x^3}-\frac{(bx^2+ax)^{\frac{3}{2}}b}{3\,x^3}-\frac{(bx^2+ax)^{\frac{3}{2}}a}{2\,x^4}-\frac{(bx^2+ax)^{\frac{5}{2}}}{5\,x^5}$$

```
integrate((b*x^2+a*x)^(5/2)/x^6,x, algorithm="maxima")
```

```
b^(5/2)*log(2*b*x + a + 2*sqrt(b*x^2 + a*x)*sqrt(b)) - 38/15*sqrt(b*x^2 +
a*x)*b^2/x - 7/30*sqrt(b*x^2 + a*x)*a*b/x^2 + 3/10*sqrt(b*x^2 + a*x)*a^2/x
^3 - 1/3*(b*x^2 + a*x)^(3/2)*b/x^3 - 1/2*(b*x^2 + a*x)^(3/2)*a/x^4 - 1/5*(
b*x^2 + a*x)^(5/2)/x^5
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 175 vs. 2(79) = 158.

Time = 0.14 (sec) , antiderivative size = 175, normalized size of antiderivative = 1.80

$$\int \frac{(ax+bx^2)^{5/2}}{x^6}\,dx = -b^{\frac{5}{2}}\log\left(\left|-2\left(\sqrt{b}x-\sqrt{bx^2+ax}\right)\sqrt{b}-a\right|\right)+\frac{2\left(45\left(\sqrt{b}x-\sqrt{bx^2+ax}\right)^4ab^2+45\left(\sqrt{b}x-\sqrt{bx^2+ax}\right)^3a^2b^{\frac{3}{2}}+35\left(\sqrt{b}x-\sqrt{bx^2+ax}\right)^2a^3b+15\left(\sqrt{b}\right.\right.}{15\left(\sqrt{b}x-\sqrt{bx^2+ax}\right)^5}$$

```
integrate((b*x^2+a*x)^(5/2)/x^6,x, algorithm="giac")
```

```
-b^(5/2)*log(abs(-2*(sqrt(b)*x - sqrt(b*x^2 + a*x))*sqrt(b) - a)) + 2/15*(
45*(sqrt(b)*x - sqrt(b*x^2 + a*x))^4*a*b^2 + 45*(sqrt(b)*x - sqrt(b*x^2 +
a*x))^3*a^2*b^(3/2) + 35*(sqrt(b)*x - sqrt(b*x^2 + a*x))^2*a^3*b + 15*(sqr
t(b)*x - sqrt(b*x^2 + a*x))*a^4*sqrt(b) + 3*a^5)/(sqrt(b)*x - sqrt(b*x^2 +
 a*x))^5
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(ax+bx^2)^{5/2}}{x^6}\,dx = \int \frac{\left(b\,x^2+a\,x\right)^{5/2}}{x^6}\,dx$$

```
int((a*x + b*x^2)^(5/2)/x^6,x)
```

```
int((a*x + b*x^2)^(5/2)/x^6, x)
```

Reduce [B] (verification not implemented)

Time = 0.25 (sec) , antiderivative size = 86, normalized size of antiderivative = 0.89

$$\int \frac{(ax+bx^2)^{5/2}}{x^6}\,dx = \frac{-\frac{2\sqrt{x}\,\sqrt{bx+a}\,a^2}{5}-\frac{22\sqrt{x}\,\sqrt{bx+a}\,abx}{15}-\frac{46\sqrt{x}\,\sqrt{bx+a}\,b^2x^2}{15}+2\sqrt{b}\log\left(\frac{\sqrt{bx+a}+\sqrt{x}\,\sqrt{b}}{\sqrt{a}}\right)b^2x^3+\frac{2\sqrt{b}\,b}{3}}{x^3}$$

```
int((b*x^2+a*x)^(5/2)/x^6,x)
```

```
(2*( - 3*sqrt(x)*sqrt(a + b*x)*a**2 - 11*sqrt(x)*sqrt(a + b*x)*a*b*x - 23*
sqrt(x)*sqrt(a + b*x)*b**2*x**2 + 15*sqrt(b)*log((sqrt(a + b*x) + sqrt(x)*
sqrt(b))/sqrt(a))*b**2*x**3 + 5*sqrt(b)*b**2*x**3))/(15*x**3)
```

3.31 $\int \frac{(ax+bx^2)^{5/2}}{x^7} dx$

Optimal result

Integrand size = 17, antiderivative size = 23

$$\int \frac{(ax+bx^2)^{5/2}}{x^7} dx = -\frac{2(ax+bx^2)^{7/2}}{7ax^7}$$

```
-2/7*(b*x^2+a*x)^(7/2)/a/x^7
```

Mathematica [A] (verified)

Time = 0.05 (sec) , antiderivative size = 21, normalized size of antiderivative = 0.91

$$\int \frac{(ax+bx^2)^{5/2}}{x^7} dx = -\frac{2(x(a+bx))^{7/2}}{7ax^7}$$

```
Integrate[(a*x + b*x^2)^(5/2)/x^7,x]
```

```
(-2*(x*(a + b*x))^(7/2))/(7*a*x^7)
```

Rubi [A] (verified)

Time = 0.23 (sec) , antiderivative size = 23, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}} = 0.059$, Rules used = $\{1123\}$

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(ax+bx^2)^{5/2}}{x^7}\,dx$$

$$\Big\downarrow \; 1123$$

$$-\frac{2(ax+bx^2)^{7/2}}{7ax^7}$$

```
Int[(a*x + b*x^2)^(5/2)/x^7,x]
```

```
(-2*(a*x + b*x^2)^(7/2))/(7*a*x^7)
```

Defintions of rubi rules used

```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] :> Simp[e*(d + e*x)^m*((a + b*x + c*x^2)^(p + 1)/((p + 1)*(2*c*d - b
*e))), x] /; FreeQ[{a, b, c, d, e, m, p}, x] && EqQ[c*d^2 - b*d*e + a*e^2,
0] && EqQ[m + 2*p + 2, 0]
```

Maple [A] (verified)

Time = 0.42 (sec) , antiderivative size = 20, normalized size of antiderivative = 0.87

method	result	size
default	$-\frac{2\left(b\,x^{2}+a x \right)^{\frac{7}{2}}}{7 a\,x^{7}}$	20
gosper	$-\frac{2 \left(b x +a \right) \left(b\,x^{2}+a x \right)^{\frac{5}{2}}}{7 x^{6} a}$	25
pseudoelliptic	$-\frac{2 \left(b x +a \right)^{3} \sqrt{x \left(b x +a \right)}}{7 x^{4} a}$	25
orering	$-\frac{2 \left(b x +a \right) \left(b\,x^{2}+a x \right)^{\frac{5}{2}}}{7 x^{6} a}$	25
trager	$-\frac{2 \left(b^{3} x^{3}+3 a\,b^{2} x^{2}+3 a^{2} b x +a^{3}\right) \sqrt{b\,x^{2}+a x}}{7 a\,x^{4}}$	47
risch	$-\frac{2 \left(b x +a \right) \left(b^{3} x^{3}+3 a\,b^{2} x^{2}+3 a^{2} b x +a^{3}\right)}{7 x^{3} \sqrt{x \left(b x +a \right)}\, a}$	50

```
int((b*x^2+a*x)^(5/2)/x^7,x,method=_RETURNVERBOSE)
```

```
-2/7*(b*x^2+a*x)^(7/2)/a/x^7
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 46 vs. $2(19) = 38$.

Time = 0.08 (sec) , antiderivative size = 46, normalized size of antiderivative = 2.00

$$\int \frac{(ax+bx^2)^{5/2}}{x^7}\,dx = -\frac{2\,(b^3x^3+3\,ab^2x^2+3\,a^2bx+a^3)\sqrt{bx^2+ax}}{7\,ax^4}$$

```
integrate((b*x^2+a*x)^(5/2)/x^7,x, algorithm="fricas")
```

```
-2/7*(b^3*x^3 + 3*a*b^2*x^2 + 3*a^2*b*x + a^3)*sqrt(b*x^2 + a*x)/(a*x^4)
```

Sympy [F]

$$\int \frac{(ax+bx^2)^{5/2}}{x^7}\,dx = \int \frac{(x(a+bx))^{\frac{5}{2}}}{x^7}\,dx$$

```
integrate((b*x**2+a*x)**(5/2)/x**7,x)
```

```
Integral((x*(a + b*x))**(5/2)/x**7, x)
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 112 vs. 2(19) = 38.

Time = 0.03 (sec) , antiderivative size = 112, normalized size of antiderivative = 4.87

$$\int \frac{(ax+bx^2)^{5/2}}{x^7}\,dx = -\frac{2\sqrt{bx^2+ax}b^3}{7\,ax} + \frac{\sqrt{bx^2+ax}b^2}{7\,x^2} - \frac{3\sqrt{bx^2+ax}ab}{28\,x^3} - \frac{15\sqrt{bx^2+ax}a^2}{28\,x^4} + \frac{5\,(bx^2+ax)^{\frac{3}{2}}a}{4\,x^5} - \frac{(bx^2+ax)^{\frac{5}{2}}}{x^6}$$

```
integrate((b*x^2+a*x)^(5/2)/x^7,x, algorithm="maxima")
```

```
-2/7*sqrt(b*x^2 + a*x)*b^3/(a*x) + 1/7*sqrt(b*x^2 + a*x)*b^2/x^2 - 3/28*sq
rt(b*x^2 + a*x)*a*b/x^3 - 15/28*sqrt(b*x^2 + a*x)*a^2/x^4 + 5/4*(b*x^2 + a
*x)^(3/2)*a/x^5 - (b*x^2 + a*x)^(5/2)/x^6
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 192 vs. 2(19) = 38.

Time = 0.17 (sec) , antiderivative size = 192, normalized size of antiderivative = 8.35

$$\int \frac{(ax+bx^2)^{5/2}}{x^7}\,dx = \frac{2\left(7\left(\sqrt{b}x-\sqrt{bx^2+ax}\right)^6 b^3 + 21\left(\sqrt{b}x-\sqrt{bx^2+ax}\right)^5 ab^{\frac{5}{2}} + 35\left(\sqrt{b}x-\sqrt{bx^2+a\right.}\right.}{}$$

```
integrate((b*x^2+a*x)^(5/2)/x^7,x, algorithm="giac")
```

```
2/7*(7*(sqrt(b)*x - sqrt(b*x^2 + a*x))^6*b^3 + 21*(sqrt(b)*x - sqrt(b*x^2
+ a*x))^5*a*b^(5/2) + 35*(sqrt(b)*x - sqrt(b*x^2 + a*x))^4*a^2*b^2 + 35*(s
qrt(b)*x - sqrt(b*x^2 + a*x))^3*a^3*b^(3/2) + 21*(sqrt(b)*x - sqrt(b*x^2 +
 a*x))^2*a^4*b + 7*(sqrt(b)*x - sqrt(b*x^2 + a*x))*a^5*sqrt(b) + a^6)/(sqr
t(b)*x - sqrt(b*x^2 + a*x))^7
```

Mupad [B] (verification not implemented)

Time = 9.63 (sec) , antiderivative size = 79, normalized size of antiderivative = 3.43

$$\int \frac{(ax+bx^2)^{5/2}}{x^7}\,dx = -\frac{2\,a^2\,\sqrt{b\,x^2+a\,x}}{7\,x^4} - \frac{6\,b^2\,\sqrt{b\,x^2+a\,x}}{7\,x^2} - \frac{2\,b^3\,\sqrt{b\,x^2+a\,x}}{7\,a\,x} - \frac{6\,a\,b\,\sqrt{b\,x^2+a\,x}}{7\,x^3}$$

```
int((a*x + b*x^2)^(5/2)/x^7,x)
```

```
- (2*a^2*(a*x + b*x^2)^(1/2))/(7*x^4) - (6*b^2*(a*x + b*x^2)^(1/2))/(7*x^2
) - (2*b^3*(a*x + b*x^2)^(1/2))/(7*a*x) - (6*a*b*(a*x + b*x^2)^(1/2))/(7*x
^3)
```

Reduce [B] (verification not implemented)

Time = 0.22 (sec) , antiderivative size = 80, normalized size of antiderivative = 3.48

$$\int \frac{(ax+bx^2)^{5/2}}{x^7}\,dx = \frac{-\frac{2\sqrt{x}\,\sqrt{bx+a}\,a^3}{7} - \frac{6\sqrt{x}\,\sqrt{bx+a}\,a^2bx}{7} - \frac{6\sqrt{x}\,\sqrt{bx+a}\,a\,b^2x^2}{7} - \frac{2\sqrt{x}\,\sqrt{bx+a}\,b^3x^3}{7} - \frac{2\sqrt{b}\,b^3x^4}{7}}{a\,x^4}$$

```
int((b*x^2+a*x)^(5/2)/x^7,x)
```

```
(2*( - sqrt(x)*sqrt(a + b*x)*a**3 - 3*sqrt(x)*sqrt(a + b*x)*a**2*b*x - 3*s
qrt(x)*sqrt(a + b*x)*a*b**2*x**2 - sqrt(x)*sqrt(a + b*x)*b**3*x**3 - sqrt(
b)*b**3*x**4))/(7*a*x**4)
```

3.32 $\int \frac{(ax+bx^2)^{5/2}}{x^8} dx$

Optimal result

Integrand size = 17, antiderivative size = 48

$$\int \frac{(ax+bx^2)^{5/2}}{x^8} dx = -\frac{2(ax+bx^2)^{7/2}}{9ax^8} + \frac{4b(ax+bx^2)^{7/2}}{63a^2x^7}$$

```
-2/9*(b*x^2+a*x)^(7/2)/a/x^8+4/63*b*(b*x^2+a*x)^(7/2)/a^2/x^7
```

Mathematica [A] (verified)

Time = 0.12 (sec) , antiderivative size = 29, normalized size of antiderivative = 0.60

$$\int \frac{(ax+bx^2)^{5/2}}{x^8} dx = -\frac{2(7a-2bx)(x(a+bx))^{7/2}}{63a^2x^8}$$

```
Integrate[(a*x + b*x^2)^(5/2)/x^8,x]
```

```
(-2*(7*a - 2*b*x)*(x*(a + b*x))^(7/2))/(63*a^2*x^8)
```

Rubi [A] (verified)

Time = 0.27 (sec) , antiderivative size = 48, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.118, Rules used = {1129, 1123}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(ax+bx^2)^{5/2}}{x^8}dx$$

$$\Big\downarrow \text{1129}$$

$$-\frac{2b\int \frac{(bx^2+ax)^{5/2}}{x^7}dx}{9a}-\frac{2(ax+bx^2)^{7/2}}{9ax^8}$$

$$\Big\downarrow \text{1123}$$

$$\frac{4b(ax+bx^2)^{7/2}}{63a^2x^7}-\frac{2(ax+bx^2)^{7/2}}{9ax^8}$$

```
Int[(a*x + b*x^2)^(5/2)/x^8,x]
```

```
(-2*(a*x + b*x^2)^(7/2))/(9*a*x^8) + (4*b*(a*x + b*x^2)^(7/2))/(63*a^2*x^7
)
```

Defintions of rubi rules used

```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] :> Simp[e*(d + e*x)^m*((a + b*x + c*x^2)^(p + 1)/((p + 1)*(2*c*d - b
*e))), x] /; FreeQ[{a, b, c, d, e, m, p}, x] && EqQ[c*d^2 - b*d*e + a*e^2,
0] && EqQ[m + 2*p + 2, 0]
```

```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] :> Simp[(-e)*(d + e*x)^m*((a + b*x + c*x^2)^(p + 1)/((m + p + 1)*(2*
c*d - b*e))), x] + Simp[c*(Simplify[m + 2*p + 2]/((m + p + 1)*(2*c*d - b*e)
))   Int[(d + e*x)^(m + 1)*(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d
, e, m, p}, x] && EqQ[c*d^2 - b*d*e + a*e^2, 0] && ILtQ[Simplify[m + 2*p +
2], 0]
```

Maple [A] (verified)

Time = 0.44 (sec) , antiderivative size = 31, normalized size of antiderivative = 0.65

method	result	size
pseudoelliptic	$-\frac{2\sqrt{x(bx+a)}\,(bx+a)^3\left(-\frac{2bx}{7}+a\right)}{9x^5a^2}$	31
gosper	$-\frac{2(bx+a)(-2bx+7a)\left(b\,x^2+ax\right)^{\frac{5}{2}}}{63x^7a^2}$	33
orering	$-\frac{2(bx+a)(-2bx+7a)\left(b\,x^2+ax\right)^{\frac{5}{2}}}{63x^7a^2}$	33
default	$-\frac{2\left(b\,x^2+ax\right)^{\frac{7}{2}}}{9a\,x^8}+\frac{4b\left(b\,x^2+ax\right)^{\frac{7}{2}}}{63a^2x^7}$	41
trager	$-\frac{2\left(-2b^4x^4+a\,b^3x^3+15a^2b^2x^2+19a^3bx+7a^4\right)\sqrt{b\,x^2+ax}}{63a^2x^5}$	60
risch	$-\frac{2(bx+a)\left(-2b^4x^4+a\,b^3x^3+15a^2b^2x^2+19a^3bx+7a^4\right)}{63x^4\sqrt{x(bx+a)}\,a^2}$	63

```
int((b*x^2+a*x)^(5/2)/x^8,x,method=_RETURNVERBOSE)
```

```
-2/9*(x*(b*x+a))^(1/2)*(b*x+a)^3*(-2/7*b*x+a)/x^5/a^2
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 60, normalized size of antiderivative = 1.25

$$\int \frac{\left(ax+bx^2\right)^{5/2}}{x^8}\,dx = \frac{2\left(2\,b^4x^4-ab^3x^3-15\,a^2b^2x^2-19\,a^3bx-7\,a^4\right)\sqrt{bx^2+ax}}{63\,a^2x^5}$$

```
integrate((b*x^2+a*x)^(5/2)/x^8,x, algorithm="fricas")
```

```
2/63*(2*b^4*x^4 - a*b^3*x^3 - 15*a^2*b^2*x^2 - 19*a^3*b*x - 7*a^4)*sqrt(b*
x^2 + a*x)/(a^2*x^5)
```

Sympy [F]

$$\int \frac{(ax+bx^2)^{5/2}}{x^8}\,dx = \int \frac{(x(a+bx))^{\frac{5}{2}}}{x^8}\,dx$$

```
integrate((b*x**2+a*x)**(5/2)/x**8,x)
```

```
Integral((x*(a + b*x))**(5/2)/x**8, x)
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 134 vs. 2(40) = 80.

Time = 0.03 (sec) , antiderivative size = 134, normalized size of antiderivative = 2.79

$$\begin{aligned}\int \frac{(ax+bx^2)^{5/2}}{x^8}\,dx =& \frac{4\sqrt{bx^2+ax}b^4}{63\,a^2x} - \frac{2\sqrt{bx^2+ax}b^3}{63\,ax^2} + \frac{\sqrt{bx^2+ax}b^2}{42\,x^3} \\ &- \frac{5\sqrt{bx^2+ax}ab}{252\,x^4} - \frac{5\sqrt{bx^2+ax}a^2}{36\,x^5} + \frac{5\,(bx^2+ax)^{\frac{3}{2}}a}{12\,x^6} - \frac{(bx^2+ax)^{\frac{5}{2}}}{2\,x^7}\end{aligned}$$

```
integrate((b*x^2+a*x)^(5/2)/x^8,x, algorithm="maxima")
```

```
4/63*sqrt(b*x^2 + a*x)*b^4/(a^2*x) - 2/63*sqrt(b*x^2 + a*x)*b^3/(a*x^2) +
1/42*sqrt(b*x^2 + a*x)*b^2/x^3 - 5/252*sqrt(b*x^2 + a*x)*a*b/x^4 - 5/36*sq
rt(b*x^2 + a*x)*a^2/x^5 + 5/12*(b*x^2 + a*x)^(3/2)*a/x^6 - 1/2*(b*x^2 + a*
x)^(5/2)/x^7
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 223 vs. $2(40) = 80$.

Time = 0.11 (sec) , antiderivative size = 223, normalized size of antiderivative = 4.65

$$\int \frac{(ax+bx^2)^{5/2}}{x^8}\,dx = \frac{2\left(63\left(\sqrt{b}x-\sqrt{bx^2+ax}\right)^7 b^{\frac{7}{2}} + 273\left(\sqrt{b}x-\sqrt{bx^2+ax}\right)^6 ab^3 + 567\left(\sqrt{b}x-\sqrt{bx^2}\right.\right.}{}$$

```
integrate((b*x^2+a*x)^(5/2)/x^8,x, algorithm="giac")
```

```
2/63*(63*(sqrt(b)*x - sqrt(b*x^2 + a*x))^7*b^(7/2) + 273*(sqrt(b)*x - sqrt
(b*x^2 + a*x))^6*a*b^3 + 567*(sqrt(b)*x - sqrt(b*x^2 + a*x))^5*a^2*b^(5/2)
 + 693*(sqrt(b)*x - sqrt(b*x^2 + a*x))^4*a^3*b^2 + 525*(sqrt(b)*x - sqrt(b
*x^2 + a*x))^3*a^4*b^(3/2) + 243*(sqrt(b)*x - sqrt(b*x^2 + a*x))^2*a^5*b +
 63*(sqrt(b)*x - sqrt(b*x^2 + a*x))*a^6*sqrt(b) + 7*a^7)/(sqrt(b)*x - sqrt
(b*x^2 + a*x))^9
```

Mupad [B] (verification not implemented)

Time = 9.91 (sec) , antiderivative size = 101, normalized size of antiderivative = 2.10

$$\int \frac{(ax+bx^2)^{5/2}}{x^8}\,dx = \frac{4\,b^4\,\sqrt{b\,x^2+a\,x}}{63\,a^2\,x} - \frac{10\,b^2\,\sqrt{b\,x^2+a\,x}}{21\,x^3} - \frac{2\,b^3\,\sqrt{b\,x^2+a\,x}}{63\,a\,x^2} - \frac{2\,a^2\,\sqrt{b\,x^2+a\,x}}{9\,x^5} - \frac{38\,a\,b\,\sqrt{b\,x^2+a\,x}}{63\,x^4}$$

```
int((a*x + b*x^2)^(5/2)/x^8,x)
```

```
(4*b^4*(a*x + b*x^2)^(1/2))/(63*a^2*x) - (10*b^2*(a*x + b*x^2)^(1/2))/(21*
x^3) - (2*b^3*(a*x + b*x^2)^(1/2))/(63*a*x^2) - (2*a^2*(a*x + b*x^2)^(1/2)
)/(9*x^5) - (38*a*b*(a*x + b*x^2)^(1/2))/(63*x^4)
```

Reduce [B] (verification not implemented)

Time = 0.21 (sec) , antiderivative size = 99, normalized size of antiderivative = 2.06

$$\int \frac{(ax+bx^2)^{5/2}}{x^8}\,dx = \frac{-\frac{2\sqrt{x}\,\sqrt{bx+a}\,a^4}{9} - \frac{38\sqrt{x}\,\sqrt{bx+a}\,a^3bx}{63} - \frac{10\sqrt{x}\,\sqrt{bx+a}\,a^2b^2x^2}{21} - \frac{2\sqrt{x}\,\sqrt{bx+a}\,a\,b^3x^3}{63} + \frac{4\sqrt{x}\,\sqrt{bx+a}\,b^4x^4}{63} -}{a^2x^5}$$

```
int((b*x^2+a*x)^(5/2)/x^8,x)
```

```
(2*( - 7*sqrt(x)*sqrt(a + b*x)*a**4 - 19*sqrt(x)*sqrt(a + b*x)*a**3*b*x -
15*sqrt(x)*sqrt(a + b*x)*a**2*b**2*x**2 - sqrt(x)*sqrt(a + b*x)*a*b**3*x**
3 + 2*sqrt(x)*sqrt(a + b*x)*b**4*x**4 - 2*sqrt(b)*b**4*x**5))/(63*a**2*x**
5)
```

3.33 $\int \frac{(ax+bx^2)^{5/2}}{x^9}\,dx$

Optimal result

Integrand size = 17, antiderivative size = 74

$$\int \frac{(ax+bx^2)^{5/2}}{x^9}\,dx = -\frac{2(ax+bx^2)^{7/2}}{11ax^9} + \frac{8b(ax+bx^2)^{7/2}}{99a^2x^8} - \frac{16b^2(ax+bx^2)^{7/2}}{693a^3x^7}$$

```
-2/11*(b*x^2+a*x)^(7/2)/a/x^9+8/99*b*(b*x^2+a*x)^(7/2)/a^2/x^8-16/693*b^2*
(b*x^2+a*x)^(7/2)/a^3/x^7
```

Mathematica [A] (verified)

Time = 0.15 (sec) , antiderivative size = 40, normalized size of antiderivative = 0.54

$$\int \frac{(ax+bx^2)^{5/2}}{x^9}\,dx = -\frac{2(x(a+bx))^{7/2}\,(63a^2-28abx+8b^2x^2)}{693a^3x^9}$$

```
Integrate[(a*x + b*x^2)^(5/2)/x^9,x]
```

```
(-2*(x*(a + b*x))^(7/2)*(63*a^2 - 28*a*b*x + 8*b^2*x^2))/(693*a^3*x^9)
```

Rubi [A] (verified)

Time = 0.33 (sec) , antiderivative size = 80, normalized size of antiderivative = 1.08, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.176$, Rules used = {1129, 1129, 1123}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(ax+bx^2)^{5/2}}{x^9}dx$$

$$\Big\downarrow \textbf{1129}$$

$$-\frac{4b\int \frac{(bx^2+ax)^{5/2}}{x^8}dx}{11a}-\frac{2(ax+bx^2)^{7/2}}{11ax^9}$$

$$\Big\downarrow \textbf{1129}$$

$$-\frac{4b\left(-\frac{2b\int \frac{(bx^2+ax)^{5/2}}{x^7}dx}{9a}-\frac{2(ax+bx^2)^{7/2}}{9ax^8}\right)}{11a}-\frac{2(ax+bx^2)^{7/2}}{11ax^9}$$

$$\Big\downarrow \textbf{1123}$$

$$-\frac{4b\left(\frac{4b(ax+bx^2)^{7/2}}{63a^2x^7}-\frac{2(ax+bx^2)^{7/2}}{9ax^8}\right)}{11a}-\frac{2(ax+bx^2)^{7/2}}{11ax^9}$$

```
Int[(a*x + b*x^2)^(5/2)/x^9,x]
```

```
(-2*(a*x + b*x^2)^(7/2))/(11*a*x^9) - (4*b*((-2*(a*x + b*x^2)^(7/2))/(9*a*
x^8) + (4*b*(a*x + b*x^2)^(7/2))/(63*a^2*x^7)))/(11*a)
```

Defintions of rubi rules used

```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] :> Simp[e*(d + e*x)^m*((a + b*x + c*x^2)^(p + 1)/((p + 1)*(2*c*d - b
*e))), x] /; FreeQ[{a, b, c, d, e, m, p}, x] && EqQ[c*d^2 - b*d*e + a*e^2,
0] && EqQ[m + 2*p + 2, 0]
```

```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] :> Simp[(-e)*(d + e*x)^m*((a + b*x + c*x^2)^(p + 1)/((m + p + 1)*(2*
c*d - b*e))), x] + Simp[c*(Simplify[m + 2*p + 2]/((m + p + 1)*(2*c*d - b*e)
))   Int[(d + e*x)^(m + 1)*(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d
, e, m, p}, x] && EqQ[c*d^2 - b*d*e + a*e^2, 0] && ILtQ[Simplify[m + 2*p +
2], 0]
```

Maple [A] (verified)

Time = 0.49 (sec) , antiderivative size = 42, normalized size of antiderivative = 0.57

method	result	size
pseudoelliptic	$-\frac{2\left(\frac{8}{63}b^{2}x^{2}-\frac{4}{9}abx+a^{2}\right)\sqrt{x\left(bx+a\right)}\,\left(bx+a\right)^{3}}{11x^{6}a^{3}}$	42
gosper	$-\frac{2\left(bx+a\right)\left(8b^{2}x^{2}-28abx+63a^{2}\right)\left(b\,x^{2}+ax\right)^{\frac{5}{2}}}{693x^{8}a^{3}}$	44
orering	$-\frac{2\left(bx+a\right)\left(8b^{2}x^{2}-28abx+63a^{2}\right)\left(b\,x^{2}+ax\right)^{\frac{5}{2}}}{693x^{8}a^{3}}$	44
default	$-\frac{2\left(b\,x^{2}+ax\right)^{\frac{7}{2}}}{11a\,x^{9}}-\frac{4b\left(-\frac{2\left(b\,x^{2}+ax\right)^{\frac{7}{2}}}{9a\,x^{8}}+\frac{4b\left(b\,x^{2}+ax\right)^{\frac{7}{2}}}{63a^{2}x^{7}}\right)}{11a}$	67
trager	$-\frac{2\left(8b^{5}x^{5}-4a\,b^{4}x^{4}+3a^{2}b^{3}x^{3}+113a^{3}b^{2}x^{2}+161a^{4}bx+63a^{5}\right)\sqrt{b\,x^{2}+ax}}{693a^{3}x^{6}}$	72
risch	$-\frac{2\left(bx+a\right)\left(8b^{5}x^{5}-4a\,b^{4}x^{4}+3a^{2}b^{3}x^{3}+113a^{3}b^{2}x^{2}+161a^{4}bx+63a^{5}\right)}{693x^{5}\sqrt{x\left(bx+a\right)}\,a^{3}}$	75

```
int((b*x^2+a*x)^(5/2)/x^9,x,method=_RETURNVERBOSE)
```

```
-2/11*(8/63*b^2*x^2-4/9*a*b*x+a^2)*(x*(b*x+a))^(1/2)*(b*x+a)^3/x^6/a^3
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 71, normalized size of antiderivative = 0.96

$$\int \frac{(ax+bx^2)^{5/2}}{x^9}\,dx = -\frac{2\,(8\,b^5x^5-4\,ab^4x^4+3\,a^2b^3x^3+113\,a^3b^2x^2+161\,a^4bx+63\,a^5)\sqrt{bx^2+ax}}{693\,a^3x^6}$$

```
integrate((b*x^2+a*x)^(5/2)/x^9,x, algorithm="fricas")
```

```
-2/693*(8*b^5*x^5 - 4*a*b^4*x^4 + 3*a^2*b^3*x^3 + 113*a^3*b^2*x^2 + 161*a^
4*b*x + 63*a^5)*sqrt(b*x^2 + a*x)/(a^3*x^6)
```

Sympy [F]

$$\int \frac{(ax+bx^2)^{5/2}}{x^9}\,dx = \int \frac{(x(a+bx))^{\frac{5}{2}}}{x^9}\,dx$$

```
integrate((b*x**2+a*x)**(5/2)/x**9,x)
```

```
Integral((x*(a + b*x))**(5/2)/x**9, x)
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 156 vs. 2(62) = 124.

Time = 0.03 (sec) , antiderivative size = 156, normalized size of antiderivative = 2.11

$$\int \frac{(ax+bx^2)^{5/2}}{x^9}\,dx = -\frac{16\,\sqrt{bx^2+ax}b^5}{693\,a^3x} + \frac{8\,\sqrt{bx^2+ax}b^4}{693\,a^2x^2} - \frac{2\,\sqrt{bx^2+ax}b^3}{231\,ax^3} + \frac{5\,\sqrt{bx^2+ax}b^2}{693\,x^4} - \frac{5\,\sqrt{bx^2+ax}ab}{792\,x^5} - \frac{5\,\sqrt{bx^2+ax}a^2}{88\,x^6} + \frac{5\,(bx^2+ax)^{\frac{3}{2}}a}{24\,x^7} - \frac{(bx^2+ax)^{\frac{5}{2}}}{3\,x^8}$$

```
integrate((b*x^2+a*x)^(5/2)/x^9,x, algorithm="maxima")
```

```
-16/693*sqrt(b*x^2 + a*x)*b^5/(a^3*x) + 8/693*sqrt(b*x^2 + a*x)*b^4/(a^2*x
^2) - 2/231*sqrt(b*x^2 + a*x)*b^3/(a*x^3) + 5/693*sqrt(b*x^2 + a*x)*b^2/x^
4 - 5/792*sqrt(b*x^2 + a*x)*a*b/x^5 - 5/88*sqrt(b*x^2 + a*x)*a^2/x^6 + 5/2
4*(b*x^2 + a*x)^(3/2)*a/x^7 - 1/3*(b*x^2 + a*x)^(5/2)/x^8
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 252 vs. $2(62) = 124$.

Time $= 0.12$ (sec) , antiderivative size $= 252$, normalized size of antiderivative $= 3.41$

$$\int \frac{(ax+bx^2)^{5/2}}{x^9}\,dx = \frac{2\left(924\left(\sqrt{b}x-\sqrt{bx^2+ax}\right)^8 b^4 + 4851\left(\sqrt{b}x-\sqrt{bx^2+ax}\right)^7 ab^{\frac{7}{2}} + 11781\left(\sqrt{b}x-\sqrt{\right.\right.}}{}$$

```
integrate((b*x^2+a*x)^(5/2)/x^9,x, algorithm="giac")
```

```
2/693*(924*(sqrt(b)*x - sqrt(b*x^2 + a*x))^8*b^4 + 4851*(sqrt(b)*x - sqrt(
b*x^2 + a*x))^7*a*b^(7/2) + 11781*(sqrt(b)*x - sqrt(b*x^2 + a*x))^6*a^2*b^
3 + 16863*(sqrt(b)*x - sqrt(b*x^2 + a*x))^5*a^3*b^(5/2) + 15345*(sqrt(b)*x
 - sqrt(b*x^2 + a*x))^4*a^4*b^2 + 9009*(sqrt(b)*x - sqrt(b*x^2 + a*x))^3*a
^5*b^(3/2) + 3311*(sqrt(b)*x - sqrt(b*x^2 + a*x))^2*a^6*b + 693*(sqrt(b)*x
 - sqrt(b*x^2 + a*x))*a^7*sqrt(b) + 63*a^8)/(sqrt(b)*x - sqrt(b*x^2 + a*x)
)^11
```

Mupad [B] (verification not implemented)

Time $= 10.21$ (sec) , antiderivative size $= 123$, normalized size of antiderivative $= 1.66$

$$\int \frac{(ax+bx^2)^{5/2}}{x^9}\,dx = \frac{8\,b^4\,\sqrt{b\,x^2+a\,x}}{693\,a^2\,x^2} - \frac{226\,b^2\,\sqrt{b\,x^2+a\,x}}{693\,x^4} - \frac{2\,b^3\,\sqrt{b\,x^2+a\,x}}{231\,a\,x^3} - \frac{2\,a^2\,\sqrt{b\,x^2+a\,x}}{11\,x^6} - \frac{16\,b^5\,\sqrt{b\,x^2+a\,x}}{693\,a^3\,x} - \frac{46\,a\,b\,\sqrt{b\,x^2+a\,x}}{99\,x^5}$$

```
int((a*x + b*x^2)^(5/2)/x^9,x)
```

```
(8*b^4*(a*x + b*x^2)^(1/2))/(693*a^2*x^2) - (226*b^2*(a*x + b*x^2)^(1/2))/
(693*x^4) - (2*b^3*(a*x + b*x^2)^(1/2))/(231*a*x^3) - (2*a^2*(a*x + b*x^2)
^(1/2))/(11*x^6) - (16*b^5*(a*x + b*x^2)^(1/2))/(693*a^3*x) - (46*a*b*(a*x
 + b*x^2)^(1/2))/(99*x^5)
```

Reduce [B] (verification not implemented)

Time = 0.21 (sec) , antiderivative size = 118, normalized size of antiderivative = 1.59

$$\int \frac{(ax+bx^2)^{5/2}}{x^9}\,dx = \frac{-\frac{2\sqrt{x}\,\sqrt{bx+a}\,a^5}{11} - \frac{46\sqrt{x}\,\sqrt{bx+a}\,a^4bx}{99} - \frac{226\sqrt{x}\,\sqrt{bx+a}\,a^3b^2x^2}{693} - \frac{2\sqrt{x}\,\sqrt{bx+a}\,a^2b^3x^3}{231} + \frac{8\sqrt{x}\,\sqrt{bx+a}\,a\,b^4x^4}{693}}{a^3x^6}$$

```
int((b*x^2+a*x)^(5/2)/x^9,x)
```

```
(2*( - 63*sqrt(x)*sqrt(a + b*x)*a**5 - 161*sqrt(x)*sqrt(a + b*x)*a**4*b*x
- 113*sqrt(x)*sqrt(a + b*x)*a**3*b**2*x**2 - 3*sqrt(x)*sqrt(a + b*x)*a**2*
b**3*x**3 + 4*sqrt(x)*sqrt(a + b*x)*a*b**4*x**4 - 8*sqrt(x)*sqrt(a + b*x)*
b**5*x**5 + 8*sqrt(b)*b**5*x**6))/(693*a**3*x**6)
```

3.34 $\int \frac{(ax+bx^2)^{5/2}}{x^{10}} dx$

Optimal result

Integrand size = 17, antiderivative size = 100

$$\int \frac{(ax+bx^2)^{5/2}}{x^{10}} dx = -\frac{2(ax+bx^2)^{7/2}}{13ax^{10}} + \frac{12b(ax+bx^2)^{7/2}}{143a^2x^9} - \frac{16b^2(ax+bx^2)^{7/2}}{429a^3x^8} + \frac{32b^3(ax+bx^2)^{7/2}}{3003a^4x^7}$$

```
-2/13*(b*x^2+a*x)^(7/2)/a/x^10+12/143*b*(b*x^2+a*x)^(7/2)/a^2/x^9-16/429*b
^2*(b*x^2+a*x)^(7/2)/a^3/x^8+32/3003*b^3*(b*x^2+a*x)^(7/2)/a^4/x^7
```

Mathematica [A] (verified)

Time = 0.14 (sec) , antiderivative size = 51, normalized size of antiderivative = 0.51

$$\int \frac{(ax+bx^2)^{5/2}}{x^{10}} dx = -\frac{2(x(a+bx))^{7/2}(231a^3-126a^2bx+56ab^2x^2-16b^3x^3)}{3003a^4x^{10}}$$

```
Integrate[(a*x + b*x^2)^(5/2)/x^10,x]
```

```
(-2*(x*(a + b*x))^(7/2)*(231*a^3 - 126*a^2*b*x + 56*a*b^2*x^2 - 16*b^3*x^3
))/(3003*a^4*x^10)
```

Rubi [A] (verified)

Time = 0.39 (sec) , antiderivative size = 112, normalized size of antiderivative = 1.12, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.235$, Rules used = {1129, 1129, 1129, 1123}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(ax+bx^2)^{5/2}}{x^{10}} \, dx$$

$$\Big\downarrow \ 1129$$

$$-\frac{6b\int \frac{(bx^2+ax)^{5/2}}{x^9}dx}{13a}-\frac{2(ax+bx^2)^{7/2}}{13ax^{10}}$$

$$\Big\downarrow \ 1129$$

$$-\frac{6b\left(-\frac{4b\int \frac{(bx^2+ax)^{5/2}}{x^8}dx}{11a}-\frac{2(ax+bx^2)^{7/2}}{11ax^9}\right)}{13a}-\frac{2(ax+bx^2)^{7/2}}{13ax^{10}}$$

$$\Big\downarrow \ 1129$$

$$-\frac{6b\left(-\frac{4b\left(-\frac{2b\int \frac{(bx^2+ax)^{5/2}}{x^7}dx}{9a}-\frac{2(ax+bx^2)^{7/2}}{9ax^8}\right)}{11a}-\frac{2(ax+bx^2)^{7/2}}{11ax^9}\right)}{13a}-\frac{2(ax+bx^2)^{7/2}}{13ax^{10}}$$

$$\Big\downarrow \ 1123$$

$$-\frac{6b\left(-\frac{4b\left(\frac{4b(ax+bx^2)^{7/2}}{63a^2x^7}-\frac{2(ax+bx^2)^{7/2}}{9ax^8}\right)}{11a}-\frac{2(ax+bx^2)^{7/2}}{11ax^9}\right)}{13a}-\frac{2(ax+bx^2)^{7/2}}{13ax^{10}}$$

```
Int[(a*x + b*x^2)^(5/2)/x^10,x]
```

```
(-2*(a*x + b*x^2)^(7/2))/(13*a*x^10) - (6*b*((-2*(a*x + b*x^2)^(7/2))/(11*
a*x^9) - (4*b*((-2*(a*x + b*x^2)^(7/2))/(9*a*x^8) + (4*b*(a*x + b*x^2)^(7/
2))/(63*a^2*x^7)))/(11*a)))/(13*a)
```

Defintions of rubi rules used

```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] :> Simp[e*(d + e*x)^m*((a + b*x + c*x^2)^(p + 1)/((p + 1)*(2*c*d - b
*e))), x] /; FreeQ[{a, b, c, d, e, m, p}, x] && EqQ[c*d^2 - b*d*e + a*e^2,
0] && EqQ[m + 2*p + 2, 0]
```

```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] :> Simp[(-e)*(d + e*x)^m*((a + b*x + c*x^2)^(p + 1)/((m + p + 1)*(2*
c*d - b*e))), x] + Simp[c*(Simplify[m + 2*p + 2]/((m + p + 1)*(2*c*d - b*e)
))   Int[(d + e*x)^(m + 1)*(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d
, e, m, p}, x] && EqQ[c*d^2 - b*d*e + a*e^2, 0] && ILtQ[Simplify[m + 2*p +
2], 0]
```

Maple [A] (verified)

Time = 0.53 (sec) , antiderivative size = 55, normalized size of antiderivative = 0.55

method	result	size
gosper	$-\frac{2(bx+a)\left(-16b^{3}x^{3}+56a\,b^{2}x^{2}-126a^{2}bx+231a^{3}\right)\left(b\,x^{2}+ax\right)^{\frac{5}{2}}}{3003a^{4}x^{9}}$	55
pseudoelliptic	$-\frac{2(bx+a)^{3}\sqrt{x(bx+a)}\,\left(-16b^{3}x^{3}+56a\,b^{2}x^{2}-126a^{2}bx+231a^{3}\right)}{3003x^{7}a^{4}}$	55
orering	$-\frac{2(bx+a)\left(-16b^{3}x^{3}+56a\,b^{2}x^{2}-126a^{2}bx+231a^{3}\right)\left(b\,x^{2}+ax\right)^{\frac{5}{2}}}{3003a^{4}x^{9}}$	55
trager	$-\frac{2\left(-16b^{6}x^{6}+8a\,b^{5}x^{5}-6a^{2}b^{4}x^{4}+5a^{3}b^{3}x^{3}+371a^{4}b^{2}x^{2}+567a^{5}bx+231a^{6}\right)\sqrt{b\,x^{2}+ax}}{3003a^{4}x^{7}}$	83
risch	$-\frac{2(bx+a)\left(-16b^{6}x^{6}+8a\,b^{5}x^{5}-6a^{2}b^{4}x^{4}+5a^{3}b^{3}x^{3}+371a^{4}b^{2}x^{2}+567a^{5}bx+231a^{6}\right)}{3003x^{6}\sqrt{x(bx+a)}\,a^{4}}$	86
default	$-\frac{2\left(b\,x^{2}+ax\right)^{\frac{7}{2}}}{13a\,x^{10}}-\frac{6b\left(-\frac{2\left(b\,x^{2}+ax\right)^{\frac{7}{2}}}{11a\,x^{9}}-\frac{4b\left(-\frac{2\left(b\,x^{2}+ax\right)^{\frac{7}{2}}}{9a\,x^{8}}+\frac{4b\left(b\,x^{2}+ax\right)^{\frac{7}{2}}}{63a^{2}x^{7}}\right)}{11a}\right)}{13a}$	93

```
int((b*x^2+a*x)^(5/2)/x^10,x,method=_RETURNVERBOSE)
```

```
-2/3003*(b*x+a)*(-16*b^3*x^3+56*a*b^2*x^2-126*a^2*b*x+231*a^3)*(b*x^2+a*x)
^(5/2)/a^4/x^9
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 82, normalized size of antiderivative = 0.82

$$\int \frac{(ax+bx^2)^{5/2}}{x^{10}}\,dx = \frac{2\,(16\,b^6x^6-8\,ab^5x^5+6\,a^2b^4x^4-5\,a^3b^3x^3-371\,a^4b^2x^2-567\,a^5bx-231\,a^6)\sqrt{bx^2+}}{3003\,a^4x^7}$$

```
integrate((b*x^2+a*x)^(5/2)/x^10,x, algorithm="fricas")
```

```
2/3003*(16*b^6*x^6 - 8*a*b^5*x^5 + 6*a^2*b^4*x^4 - 5*a^3*b^3*x^3 - 371*a^4
*b^2*x^2 - 567*a^5*b*x - 231*a^6)*sqrt(b*x^2 + a*x)/(a^4*x^7)
```

Sympy [F]

$$\int \frac{(ax + bx^2)^{5/2}}{x^{10}}\,dx = \int \frac{(x(a + bx))^{\frac{5}{2}}}{x^{10}}\,dx$$

```
integrate((b*x**2+a*x)**(5/2)/x**10,x)
```

```
Integral((x*(a + b*x))**(5/2)/x**10, x)
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 178 vs. 2(84) = 168.

Time = 0.04 (sec) , antiderivative size = 178, normalized size of antiderivative = 1.78

$$\begin{aligned}\int \frac{(ax + bx^2)^{5/2}}{x^{10}}\,dx &= \frac{32\sqrt{bx^2 + ax}b^6}{3003\,a^4x} - \frac{16\sqrt{bx^2 + ax}b^5}{3003\,a^3x^2} \\ &+ \frac{4\sqrt{bx^2 + ax}b^4}{1001\,a^2x^3} - \frac{10\sqrt{bx^2 + ax}b^3}{3003\,ax^4} + \frac{5\sqrt{bx^2 + ax}b^2}{1716\,x^5} \\ &- \frac{3\sqrt{bx^2 + ax}ab}{1144\,x^6} - \frac{3\sqrt{bx^2 + ax}a^2}{104\,x^7} + \frac{(bx^2 + ax)^{\frac{3}{2}}a}{8\,x^8} - \frac{(bx^2 + ax)^{\frac{5}{2}}}{4\,x^9}\end{aligned}$$

```
integrate((b*x^2+a*x)^(5/2)/x^10,x, algorithm="maxima")
```

```
32/3003*sqrt(b*x^2 + a*x)*b^6/(a^4*x) - 16/3003*sqrt(b*x^2 + a*x)*b^5/(a^3
*x^2) + 4/1001*sqrt(b*x^2 + a*x)*b^4/(a^2*x^3) - 10/3003*sqrt(b*x^2 + a*x)
*b^3/(a*x^4) + 5/1716*sqrt(b*x^2 + a*x)*b^2/x^5 - 3/1144*sqrt(b*x^2 + a*x)
*a*b/x^6 - 3/104*sqrt(b*x^2 + a*x)*a^2/x^7 + 1/8*(b*x^2 + a*x)^(3/2)*a/x^8
 - 1/4*(b*x^2 + a*x)^(5/2)/x^9
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 281 vs. $2(84) = 168$.

Time = 0.12 (sec) , antiderivative size = 281, normalized size of antiderivative = 2.81

$$\int \frac{(ax+bx^2)^{5/2}}{x^{10}}\,dx = \frac{2\left(6006\left(\sqrt{b}x-\sqrt{bx^2+ax}\right)^9 b^{\frac{9}{2}} + 36036\left(\sqrt{b}x-\sqrt{bx^2+ax}\right)^8 ab^4 + 99099\left(\sqrt{b}x-\right.\right.}{}$$

```
integrate((b*x^2+a*x)^(5/2)/x^10,x, algorithm="giac")
```

```
2/3003*(6006*(sqrt(b)*x - sqrt(b*x^2 + a*x))^9*b^(9/2) + 36036*(sqrt(b)*x
- sqrt(b*x^2 + a*x))^8*a*b^4 + 99099*(sqrt(b)*x - sqrt(b*x^2 + a*x))^7*a^2
*b^(7/2) + 161733*(sqrt(b)*x - sqrt(b*x^2 + a*x))^6*a^3*b^3 + 171171*(sqrt
(b)*x - sqrt(b*x^2 + a*x))^5*a^4*b^(5/2) + 121121*(sqrt(b)*x - sqrt(b*x^2
+ a*x))^4*a^5*b^2 + 57057*(sqrt(b)*x - sqrt(b*x^2 + a*x))^3*a^6*b^(3/2) +
17199*(sqrt(b)*x - sqrt(b*x^2 + a*x))^2*a^7*b + 3003*(sqrt(b)*x - sqrt(b*x
^2 + a*x))*a^8*sqrt(b) + 231*a^9)/(sqrt(b)*x - sqrt(b*x^2 + a*x))^13
```

Mupad [B] (verification not implemented)

Time = 10.70 (sec) , antiderivative size = 145, normalized size of antiderivative = 1.45

$$\begin{aligned}\int \frac{(ax+bx^2)^{5/2}}{x^{10}}\,dx &= \frac{4\,b^4\,\sqrt{b\,x^2+a\,x}}{1001\,a^2\,x^3} - \frac{106\,b^2\,\sqrt{b\,x^2+a\,x}}{429\,x^5} - \frac{10\,b^3\,\sqrt{b\,x^2+a\,x}}{3003\,a\,x^4} \\ &\quad - \frac{2\,a^2\,\sqrt{b\,x^2+a\,x}}{13\,x^7} - \frac{16\,b^5\,\sqrt{b\,x^2+a\,x}}{3003\,a^3\,x^2} + \frac{32\,b^6\,\sqrt{b\,x^2+a\,x}}{3003\,a^4\,x} - \frac{54\,a\,b\,\sqrt{b\,x^2+a\,x}}{143\,x^6}\end{aligned}$$

```
int((a*x + b*x^2)^(5/2)/x^10,x)
```

```
(4*b^4*(a*x + b*x^2)^(1/2))/(1001*a^2*x^3) - (106*b^2*(a*x + b*x^2)^(1/2))
/(429*x^5) - (10*b^3*(a*x + b*x^2)^(1/2))/(3003*a*x^4) - (2*a^2*(a*x + b*x
^2)^(1/2))/(13*x^7) - (16*b^5*(a*x + b*x^2)^(1/2))/(3003*a^3*x^2) + (32*b^
6*(a*x + b*x^2)^(1/2))/(3003*a^4*x) - (54*a*b*(a*x + b*x^2)^(1/2))/(143*x^
6)
```

Reduce [B] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 137, normalized size of antiderivative = 1.37

$$\int \frac{(ax+bx^2)^{5/2}}{x^{10}}\,dx = \frac{-\frac{2\sqrt{x}\,\sqrt{bx+a}\,a^6}{13} - \frac{54\sqrt{x}\,\sqrt{bx+a}\,a^5bx}{143} - \frac{106\sqrt{x}\,\sqrt{bx+a}\,a^4b^2x^2}{429} - \frac{10\sqrt{x}\,\sqrt{bx+a}\,a^3b^3x^3}{3003} + \frac{4\sqrt{x}\,\sqrt{bx+a}\,a^2b^4}{1001}}{a^4x^7}$$

```
int((b*x^2+a*x)^(5/2)/x^10,x)
```

```
(2*( - 231*sqrt(x)*sqrt(a + b*x)*a**6 - 567*sqrt(x)*sqrt(a + b*x)*a**5*b*x
 - 371*sqrt(x)*sqrt(a + b*x)*a**4*b**2*x**2 - 5*sqrt(x)*sqrt(a + b*x)*a**3
*b**3*x**3 + 6*sqrt(x)*sqrt(a + b*x)*a**2*b**4*x**4 - 8*sqrt(x)*sqrt(a + b
*x)*a*b**5*x**5 + 16*sqrt(x)*sqrt(a + b*x)*b**6*x**6 - 16*sqrt(b)*b**6*x**
7))/(3003*a**4*x**7)
```

3.35 $\int x\sqrt{2x - x^2}\,dx$

Optimal result

Integrand size = 15, antiderivative size = 67

$$\int x\sqrt{2x-x^2}\,dx=\frac{1}{2}\sqrt{2-x}\sqrt{x}-\frac{1}{2}(2-x)^{3/2}\sqrt{x}-\frac{1}{3}(2-x)^{3/2}x^{3/2}+\arcsin\left(\frac{\sqrt{x}}{\sqrt{2}}\right)$$

```
1/2*(2-x)^(1/2)*x^(1/2)-1/2*(2-x)^(3/2)*x^(1/2)-1/3*(2-x)^(3/2)*x^(3/2)+ar
csin(1/2*2^(1/2)*x^(1/2))
```

Mathematica [A] (verified)

Time = 0.08 (sec) , antiderivative size = 54, normalized size of antiderivative = 0.81

$$\int x\sqrt{2x-x^2}\,dx=\frac{1}{6}\sqrt{-((-2+x)x)}\left(-3-x+2x^2+\frac{6\log\left(\sqrt{-2+x}-\sqrt{x}\right)}{\sqrt{-2+x}\sqrt{x}}\right)$$

```
Integrate[x*Sqrt[2*x - x^2],x]
```

```
(Sqrt[-((-2 + x)*x)]*(-3 - x + 2*x^2 + (6*Log[Sqrt[-2 + x] - Sqrt[x]])/(Sq
rt[-2 + x]*Sqrt[x])))/6
```

Rubi [A] (verified)

Time = 0.29 (sec) , antiderivative size = 54, normalized size of antiderivative = 0.81, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.267, Rules used = {1160, 1087, 1090, 223}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x\sqrt{2x-x^2}\,dx$$

$$\Big\downarrow \text{1160}$$

$$\int \sqrt{2x-x^2}dx-\frac{1}{3}\left(2x-x^2\right)^{3/2}$$

$$\Big\downarrow \text{1087}$$

$$\frac{1}{2}\int \frac{1}{\sqrt{2x-x^2}}dx-\frac{1}{3}\left(2x-x^2\right)^{3/2}-\frac{1}{2}(1-x)\sqrt{2x-x^2}$$

$$\Big\downarrow \text{1090}$$

$$-\frac{1}{4}\int \frac{1}{\sqrt{1-\frac{1}{4}(2-2x)^2}}d(2-2x)-\frac{1}{3}\left(2x-x^2\right)^{3/2}-\frac{1}{2}(1-x)\sqrt{2x-x^2}$$

$$\Big\downarrow \text{223}$$

$$-\frac{1}{2}\arcsin\left(\frac{1}{2}(2-2x)\right)-\frac{1}{3}\left(2x-x^2\right)^{3/2}-\frac{1}{2}(1-x)\sqrt{2x-x^2}$$

```
Int[x*Sqrt[2*x - x^2],x]
```

```
-1/2*((1 - x)*Sqrt[2*x - x^2]) - (2*x - x^2)^(3/2)/3 - ArcSin[(2 - 2*x)/2]
/2
```

Defintions of rubi rules used

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSin[Rt[-b, 2]*(x/Sqrt
[a])]/Rt[-b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && NegQ[b]
```

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(b + 2*c*x)
*((a + b*x + c*x^2)^p/(2*c*(2*p + 1))), x] - Simp[p*((b^2 - 4*a*c)/(2*c*(2*
p + 1)))   Int[(a + b*x + c*x^2)^(p - 1), x], x] /; FreeQ[{a, b, c}, x] &&
GtQ[p, 0] && (IntegerQ[4*p] || IntegerQ[3*p])
```

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[1/(2*c*(-4*
(c/(b^2 - 4*a*c)))^p)   Subst[Int[Simp[1 - x^2/(b^2 - 4*a*c), x]^p, x], x,
b + 2*c*x], x] /; FreeQ[{a, b, c, p}, x] && GtQ[4*a - b^2/c, 0]
```

```
Int[((d_.) + (e_.)*(x_))*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol
] :> Simp[e*((a + b*x + c*x^2)^(p + 1)/(2*c*(p + 1))), x] + Simp[(2*c*d - b
*e)/(2*c)   Int[(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, p}, x]
 && NeQ[p, -1]
```

Maple [A] (verified)

Time = 0.65 (sec) , antiderivative size = 32, normalized size of antiderivative = 0.48

method	result	size
risch	$-\frac{(2x^2-x-3)x(x-2)}{6\sqrt{-x(x-2)}}+\frac{\arcsin(x-1)}{2}$	32
pseudoelliptic	$-\arctan\left(\frac{\sqrt{-x(x-2)}}{x}\right)+\frac{(2x^2-x-3)\sqrt{-x(x-2)}}{6}$	37
default	$-\frac{(-x^2+2x)^{\frac{3}{2}}}{3}-\frac{(-2x+2)\sqrt{-x^2+2x}}{4}+\frac{\arcsin(x-1)}{2}$	39
meijerg	$\frac{4i\left(\frac{i\sqrt{\pi}\,\sqrt{x}\,\sqrt{2}\,(-10x^2+5x+15)\sqrt{-\frac{x}{2}+1}}{120}-\frac{i\sqrt{\pi}\,\arcsin\left(\frac{\sqrt{2}\,\sqrt{x}}{2}\right)}{4}\right)}{\sqrt{\pi}}$	52
trager	$\left(\frac{1}{3}x^2-\frac{1}{6}x-\frac{1}{2}\right)\sqrt{-x^2+2x}+\frac{\operatorname{RootOf}(\_Z^2+1)\ln\left(\operatorname{RootOf}(\_Z^2+1)\sqrt{-x^2+2x}+x-1\right)}{2}$	54

```
int(x*(-x^2+2*x)^(1/2),x,method=_RETURNVERBOSE)
```

```
-1/6*(2*x^2-x-3)*x*(x-2)/(-x*(x-2))^(1/2)+1/2*arcsin(x-1)
```

Fricas [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 44, normalized size of antiderivative = 0.66

$$\int x\sqrt{2x-x^2}\,dx = \frac{1}{6}\left(2\,x^2-x-3\right)\sqrt{-x^2+2\,x}-\arctan\left(\frac{\sqrt{-x^2+2\,x}}{x-2}\right)$$

```
integrate(x*(-x^2+2*x)^(1/2),x, algorithm="fricas")
```

```
1/6*(2*x^2 - x - 3)*sqrt(-x^2 + 2*x) - arctan(sqrt(-x^2 + 2*x)/(x - 2))
```

Sympy [A] (verification not implemented)

Time = 0.24 (sec) , antiderivative size = 27, normalized size of antiderivative = 0.40

$$\int x\sqrt{2x-x^2}\,dx = \sqrt{-x^2+2x}\left(\frac{x^2}{3}-\frac{x}{6}-\frac{1}{2}\right)+\frac{\operatorname{asin}\left(x-1\right)}{2}$$

```
integrate(x*(-x**2+2*x)**(1/2),x)
```

```
sqrt(-x**2 + 2*x)*(x**2/3 - x/6 - 1/2) + asin(x - 1)/2
```

Maxima [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 49, normalized size of antiderivative = 0.73

$$\int x\sqrt{2x-x^2}\,dx = -\frac{1}{3}\left(-x^2+2\,x\right)^{\frac{3}{2}} + \frac{1}{2}\sqrt{-x^2+2\,x}x - \frac{1}{2}\sqrt{-x^2+2\,x} - \frac{1}{2}\arcsin\left(-x+1\right)$$

```
integrate(x*(-x^2+2*x)^(1/2),x, algorithm="maxima")
```

```
-1/3*(-x^2 + 2*x)^(3/2) + 1/2*sqrt(-x^2 + 2*x)*x - 1/2*sqrt(-x^2 + 2*x) -
1/2*arcsin(-x + 1)
```

Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 29, normalized size of antiderivative = 0.43

$$\int x\sqrt{2x-x^2}\,dx = \frac{1}{6}\left((2\,x-1)x-3\right)\sqrt{-x^2+2\,x} + \frac{1}{2}\arcsin\left(x-1\right)$$

```
integrate(x*(-x^2+2*x)^(1/2),x, algorithm="giac")
```

```
1/6*((2*x - 1)*x - 3)*sqrt(-x^2 + 2*x) + 1/2*arcsin(x - 1)
```

Mupad [B] (verification not implemented)

Time = 0.10 (sec) , antiderivative size = 42, normalized size of antiderivative = 0.63

$$\int x\sqrt{2x-x^2}\,dx = -\frac{\sqrt{2\,x-x^2}\left(-8\,x^2+4\,x+12\right)}{24} - \frac{\ln\left(x-1-\sqrt{-x\,\left(x-2\right)}\,1{}\mathrm{i}\right)\,1{}\mathrm{i}}{2}$$

```
int(x*(2*x - x^2)^(1/2),x)
```

```
- (log(x - (-x*(x - 2))^(1/2)*1i - 1)*1i)/2 - ((2*x - x^2)^(1/2)*(4*x - 8*
x^2 + 12))/24
```

Reduce [B] (verification not implemented)

Time = 0.22 (sec) , antiderivative size = 55, normalized size of antiderivative = 0.82

$$\int x\sqrt{2x-x^2}\,dx = \frac{\sqrt{x}\,\sqrt{-x+2}\,x^2}{3} - \frac{\sqrt{x}\,\sqrt{-x+2}\,x}{6} - \frac{\sqrt{x}\,\sqrt{-x+2}}{2} - \log\left(\frac{\sqrt{-x+2}+\sqrt{x}\,i}{\sqrt{2}}\right)i$$

```
int(x*(-x^2+2*x)^(1/2),x)
```

```
(2*sqrt(x)*sqrt( - x + 2)*x**2 - sqrt(x)*sqrt( - x + 2)*x - 3*sqrt(x)*sqrt
( - x + 2) - 6*log((sqrt( - x + 2) + sqrt(x)*i)/sqrt(2))*i)/6
```

3.36 $\int x\sqrt{3x-4x^2}\,dx$

Optimal result

Integrand size = 15, antiderivative size = 72

$$\int x\sqrt{3x-4x^2}\,dx = \frac{9}{128}\sqrt{3-4x}\sqrt{x}-\frac{3}{64}(3-4x)^{3/2}\sqrt{x} - \frac{1}{12}(3-4x)^{3/2}x^{3/2}+\frac{27}{256}\arcsin\left(\frac{2\sqrt{x}}{\sqrt{3}}\right)$$

```
9/128*(3-4*x)^(1/2)*x^(1/2)-3/64*(3-4*x)^(3/2)*x^(1/2)-1/12*(3-4*x)^(3/2)*
x^(3/2)+27/256*arcsin(2/3*x^(1/2)*3^(1/2))
```

Mathematica [A] (verified)

Time = 0.08 (sec) , antiderivative size = 75, normalized size of antiderivative = 1.04

$$\int x\sqrt{3x-4x^2}\,dx = \frac{1}{384}\sqrt{-x(-3+4x)}\left(-27-24x+128x^2\right) + \frac{27\sqrt{-x(-3+4x)}\log\left(-2\sqrt{x}+\sqrt{-3+4x}\right)}{256\sqrt{x}\sqrt{-3+4x}}$$

```
Integrate[x*Sqrt[3*x - 4*x^2],x]
```

```
(Sqrt[-(x*(-3 + 4*x))]*(-27 - 24*x + 128*x^2))/384 + (27*Sqrt[-(x*(-3 + 4*
x))]*Log[-2*Sqrt[x] + Sqrt[-3 + 4*x]])/(256*Sqrt[x]*Sqrt[-3 + 4*x])
```

Rubi [A] (verified)

Time = 0.29 (sec) , antiderivative size = 59, normalized size of antiderivative = 0.82, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.267$, Rules used = {1160, 1087, 1090, 223}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x\sqrt{3x-4x^2}\,dx$$

$$\Big\downarrow \text{1160}$$

$$\frac{3}{8}\int \sqrt{3x-4x^2}dx-\frac{1}{12}\left(3x-4x^2\right)^{3/2}$$

$$\Big\downarrow \text{1087}$$

$$\frac{3}{8}\left(\frac{9}{32}\int \frac{1}{\sqrt{3x-4x^2}}dx-\frac{1}{16}(3-8x)\sqrt{3x-4x^2}\right)-\frac{1}{12}\left(3x-4x^2\right)^{3/2}$$

$$\Big\downarrow \text{1090}$$

$$\frac{3}{8}\left(-\frac{3}{64}\int \frac{1}{\sqrt{1-\frac{1}{9}(3-8x)^2}}d(3-8x)-\frac{1}{16}\sqrt{3x-4x^2}(3-8x)\right)-\frac{1}{12}\left(3x-4x^2\right)^{3/2}$$

$$\Big\downarrow \text{223}$$

$$\frac{3}{8}\left(-\frac{9}{64}\arcsin\left(\frac{1}{3}(3-8x)\right)-\frac{1}{16}\sqrt{3x-4x^2}(3-8x)\right)-\frac{1}{12}\left(3x-4x^2\right)^{3/2}$$

```
Int[x*Sqrt[3*x - 4*x^2],x]
```

```
-1/12*(3*x - 4*x^2)^(3/2) + (3*(-1/16*((3 - 8*x)*Sqrt[3*x - 4*x^2]) - (9*A
rcSin[(3 - 8*x)/3])/64))/8
```

Defintions of rubi rules used

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSin[Rt[-b, 2]*(x/Sqrt
[a])]/Rt[-b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && NegQ[b]
```

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(b + 2*c*x)
*((a + b*x + c*x^2)^p/(2*c*(2*p + 1))), x] - Simp[p*((b^2 - 4*a*c)/(2*c*(2*
p + 1)))   Int[(a + b*x + c*x^2)^(p - 1), x], x] /; FreeQ[{a, b, c}, x] &&
GtQ[p, 0] && (IntegerQ[4*p] || IntegerQ[3*p])
```

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[1/(2*c*(-4*
(c/(b^2 - 4*a*c)))^p)   Subst[Int[Simp[1 - x^2/(b^2 - 4*a*c), x]^p, x], x,
b + 2*c*x], x] /; FreeQ[{a, b, c, p}, x] && GtQ[4*a - b^2/c, 0]
```

```
Int[((d_.) + (e_.)*(x_))*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol
] :> Simp[e*((a + b*x + c*x^2)^(p + 1)/(2*c*(p + 1))), x] + Simp[(2*c*d - b
*e)/(2*c)   Int[(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, p}, x]
 && NeQ[p, -1]
```

Maple [A] (verified)

Time = 0.34 (sec) , antiderivative size = 38, normalized size of antiderivative = 0.53

method	result
risch	$-\frac{(128x^2-24x-27)x(4x-3)}{384\sqrt{-x(4x-3)}}+\frac{27\arcsin\left(-1+\frac{8x}{3}\right)}{512}$
default	$-\frac{(-4x^2+3x)^{\frac{3}{2}}}{12}-\frac{3(-8x+3)\sqrt{-4x^2+3x}}{128}+\frac{27\arcsin\left(-1+\frac{8x}{3}\right)}{512}$
pseudoelliptic	$-\frac{27\arctan\left(\frac{\sqrt{-4x^2+3x}}{2x}\right)}{256}+\frac{(256x^2-48x-54)\sqrt{-4x^2+3x}}{768}$
meijerg	$\frac{27i\left(\frac{i\sqrt{\pi}\,\sqrt{x}\,\sqrt{3}\,\left(-\frac{640}{9}x^2+\frac{40}{3}x+15\right)\sqrt{-\frac{4x}{3}+1}}{90}-\frac{i\sqrt{\pi}\,\arcsin\left(\frac{2\sqrt{x}\,\sqrt{3}}{3}\right)}{4}\right)}{64\sqrt{\pi}}$
trager	$\left(\frac{1}{3}x^2-\frac{1}{16}x-\frac{9}{128}\right)\sqrt{-4x^2+3x}+\frac{27\,\mathrm{RootOf}\left(\_Z^2+1\right)\ln\left(-8\,\mathrm{RootOf}\left(\_Z^2+1\right)x+4\sqrt{-4x^2+3x}+3\,\mathrm{RootO}\right.}{512}$

```
int(x*(-4*x^2+3*x)^(1/2),x,method=_RETURNVERBOSE)
```

```
-1/384*(128*x^2-24*x-27)*x*(4*x-3)/(-x*(4*x-3))^(1/2)+27/512*arcsin(-1+8/3
*x)
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 47, normalized size of antiderivative = 0.65

$$\int x\sqrt{3x-4x^2}\,dx = \frac{1}{384}\left(128\,x^2-24\,x-27\right)\sqrt{-4\,x^2+3\,x} - \frac{27}{256}\arctan\left(\frac{2\,\sqrt{-4\,x^2+3\,x}}{4\,x-3}\right)$$

```
integrate(x*(-4*x^2+3*x)^(1/2),x, algorithm="fricas")
```

```
1/384*(128*x^2 - 24*x - 27)*sqrt(-4*x^2 + 3*x) - 27/256*arctan(2*sqrt(-4*x
^2 + 3*x)/(4*x - 3))
```

Sympy [A] (verification not implemented)

Time = 0.24 (sec) , antiderivative size = 34, normalized size of antiderivative = 0.47

$$\int x\sqrt{3x-4x^2}\,dx = \sqrt{-4x^2+3x}\left(\frac{x^2}{3}-\frac{x}{16}-\frac{9}{128}\right)+\frac{27\,\mathrm{asin}\left(\frac{8x}{3}-1\right)}{512}$$

```
integrate(x*(-4*x**2+3*x)**(1/2),x)
```

```
sqrt(-4*x**2 + 3*x)*(x**2/3 - x/16 - 9/128) + 27*asin(8*x/3 - 1)/512
```

Maxima [A] (verification not implemented)

Time = 0.10 (sec) , antiderivative size = 49, normalized size of antiderivative = 0.68

$$\int x\sqrt{3x-4x^2}\,dx = -\frac{1}{12}\left(-4\,x^2+3\,x\right)^{\frac{3}{2}} + \frac{3}{16}\sqrt{-4\,x^2+3\,x}x - \frac{9}{128}\sqrt{-4\,x^2+3\,x} - \frac{27}{512}\arcsin\left(-\frac{8}{3}\,x+1\right)$$

```
integrate(x*(-4*x^2+3*x)^(1/2),x, algorithm="maxima")
```

```
-1/12*(-4*x^2 + 3*x)^(3/2) + 3/16*sqrt(-4*x^2 + 3*x)*x - 9/128*sqrt(-4*x^2
 + 3*x) - 27/512*arcsin(-8/3*x + 1)
```

Giac [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 32, normalized size of antiderivative = 0.44

$$\int x\sqrt{3x-4x^2}\,dx = \frac{1}{384}\left(8\left(16\,x-3\right)x-27\right)\sqrt{-4\,x^2+3\,x} + \frac{27}{512}\arcsin\left(\frac{8}{3}\,x-1\right)$$

```
integrate(x*(-4*x^2+3*x)^(1/2),x, algorithm="giac")
```

```
1/384*(8*(16*x - 3)*x - 27)*sqrt(-4*x^2 + 3*x) + 27/512*arcsin(8/3*x - 1)
```

Mupad [B] (verification not implemented)

Time = 9.18 (sec) , antiderivative size = 44, normalized size of antiderivative = 0.61

$$\int x\sqrt{3x-4x^2}\,dx = -\frac{\sqrt{3\,x-4\,x^2}\left(-128\,x^2+24\,x+27\right)}{384} - \frac{\ln\left(x-\frac{3}{8}-\frac{\sqrt{-x\,\left(4\,x-3\right)}\,1\mathrm{i}}{2}\right)\,27\mathrm{i}}{512}$$

```
int(x*(3*x - 4*x^2)^(1/2),x)
```

```
- (log(x - ((-x*(4*x - 3))^(1/2)*1i)/2 - 3/8)*27i)/512 - ((3*x - 4*x^2)^(1
/2)*(24*x - 128*x^2 + 27))/384
```

Reduce [B] (verification not implemented)

Time = 0.21 (sec) , antiderivative size = 56, normalized size of antiderivative = 0.78

$$\int x\sqrt{3x-4x^2}\,dx = \frac{\sqrt{x}\,\sqrt{-4x+3}\,x^2}{3} - \frac{\sqrt{x}\,\sqrt{-4x+3}\,x}{16} - \frac{9\sqrt{x}\,\sqrt{-4x+3}}{128} - \frac{27\log\left(\frac{\sqrt{-4x+3}+2\sqrt{x}\,i}{\sqrt{3}}\right)i}{256}$$

```
int(x*(-4*x^2+3*x)^(1/2),x)
```

```
(256*sqrt(x)*sqrt( - 4*x + 3)*x**2 - 48*sqrt(x)*sqrt( - 4*x + 3)*x - 54*sq
rt(x)*sqrt( - 4*x + 3) - 81*log((sqrt( - 4*x + 3) + 2*sqrt(x)*i)/sqrt(3))*
i)/768
```

3.37 $\int x\sqrt{x+x^2}\,dx$

Optimal result

Integrand size = 11, antiderivative size = 66

$$\int x\sqrt{x+x^2}\,dx = -\frac{1}{8}\sqrt{x+x^2}+\frac{1}{12}x\sqrt{x+x^2}+\frac{1}{3}x^2\sqrt{x+x^2}+\frac{1}{8}\text{arcsinh}\left(\frac{\sqrt{x+x^2}}{\sqrt{1+x}}\right)$$

```
-1/8*(x^2+x)^(1/2)+1/12*x*(x^2+x)^(1/2)+1/3*x^2*(x^2+x)^(1/2)+1/8*arcsinh(
(x^2+x)^(1/2)/(1+x)^(1/2))
```

Mathematica [A] (verified)

Time = 0.07 (sec) , antiderivative size = 53, normalized size of antiderivative = 0.80

$$\int x\sqrt{x+x^2}\,dx = \frac{1}{24}\sqrt{x(1+x)}\left(-3+2x+8x^2-\frac{3\log\left(-\sqrt{x}+\sqrt{1+x}\right)}{\sqrt{x}\sqrt{1+x}}\right)$$

```
Integrate[x*Sqrt[x + x^2],x]
```

```
(Sqrt[x*(1 + x)]*(-3 + 2*x + 8*x^2 - (3*Log[-Sqrt[x] + Sqrt[1 + x]])/(Sqrt
[x]*Sqrt[1 + x])))/24
```

Rubi [A] (verified)

Time = 0.28 (sec) , antiderivative size = 53, normalized size of antiderivative = 0.80, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.364$, Rules used = {1160, 1087, 1091, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x\sqrt{x^2+x}\,dx$$

$$\Big\downarrow \text{1160}$$

$$\frac{1}{3}\left(x^2+x\right)^{3/2}-\frac{1}{2}\int \sqrt{x^2+x}dx$$

$$\Big\downarrow \text{1087}$$

$$\frac{1}{2}\left(\frac{1}{8}\int \frac{1}{\sqrt{x^2+x}}dx-\frac{1}{4}(2x+1)\sqrt{x^2+x}\right)+\frac{1}{3}\left(x^2+x\right)^{3/2}$$

$$\Big\downarrow \text{1091}$$

$$\frac{1}{2}\left(\frac{1}{4}\int \frac{1}{1-\frac{x^2}{x^2+x}}d\frac{x}{\sqrt{x^2+x}}-\frac{1}{4}(2x+1)\sqrt{x^2+x}\right)+\frac{1}{3}\left(x^2+x\right)^{3/2}$$

$$\Big\downarrow \text{219}$$

$$\frac{1}{2}\left(\frac{1}{4}\text{arctanh}\left(\frac{x}{\sqrt{x^2+x}}\right)-\frac{1}{4}(2x+1)\sqrt{x^2+x}\right)+\frac{1}{3}\left(x^2+x\right)^{3/2}$$

```
Int[x*Sqrt[x + x^2],x]
```

```
(x + x^2)^(3/2)/3 + (-1/4*((1 + 2*x)*Sqrt[x + x^2]) + ArcTanh[x/Sqrt[x + x
^2]]/4)/2
```

Defintions of rubi rules used

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(b + 2*c*x)
*((a + b*x + c*x^2)^p/(2*c*(2*p + 1))), x] - Simp[p*((b^2 - 4*a*c)/(2*c*(2*
p + 1)))   Int[(a + b*x + c*x^2)^(p - 1), x], x] /; FreeQ[{a, b, c}, x] &&
GtQ[p, 0] && (IntegerQ[4*p] || IntegerQ[3*p])
```

```
Int[1/Sqrt[(b_.)*(x_) + (c_.)*(x_)^2], x_Symbol] :> Simp[2   Subst[Int[1/(1
 - c*x^2), x], x, x/Sqrt[b*x + c*x^2]], x] /; FreeQ[{b, c}, x]
```

```
Int[((d_.) + (e_.)*(x_))*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol
] :> Simp[e*((a + b*x + c*x^2)^(p + 1)/(2*c*(p + 1))), x] + Simp[(2*c*d - b
*e)/(2*c)   Int[(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, p}, x]
 && NeQ[p, -1]
```

Maple [A] (verified)

Time = 0.34 (sec) , antiderivative size = 37, normalized size of antiderivative = 0.56

method	result	size
trager	$\left(\frac{1}{3}x^{2}+\frac{1}{12}x-\frac{1}{8}\right)\sqrt{x^{2}+x}-\frac{\ln\left(2\sqrt{x^{2}+x}-1-2x\right)}{16}$	37
default	$\frac{\left(x^{2}+x\right)^{\frac{3}{2}}}{3}-\frac{\left(1+2x\right)\sqrt{x^{2}+x}}{8}+\frac{\ln\left(\frac{1}{2}+x+\sqrt{x^{2}+x}\right)}{16}$	38
risch	$\frac{\left(8x^{2}+2x-3\right)\left(x+1\right)x}{24\sqrt{\left(x+1\right)x}}+\frac{\ln\left(\frac{1}{2}+x+\sqrt{x^{2}+x}\right)}{16}$	38
meijerg	$-\frac{\frac{\sqrt{\pi}\,\sqrt{x}\,\left(-40x^{2}-10x+15\right)\sqrt{x+1}}{60}-\frac{\sqrt{\pi}\,\operatorname{arcsinh}\left(\sqrt{x}\right)}{4}}{2\sqrt{\pi}}$	39
pseudoelliptic	$\frac{x^{3}\left(16\sqrt{\left(x+1\right)x}\,x^{2}+4x\sqrt{\left(x+1\right)x}+3\ln\left(\frac{x+\sqrt{\left(x+1\right)x}}{x}\right)-3\ln\left(\frac{\sqrt{\left(x+1\right)x}-x}{x}\right)-6\sqrt{\left(x+1\right)x}\right)}{48\left(x+\sqrt{\left(x+1\right)x}\right)^{3}\left(\sqrt{\left(x+1\right)x}-x\right)^{3}}$	96

```
int(x*(x^2+x)^(1/2),x,method=_RETURNVERBOSE)
```

```
(1/3*x^2+1/12*x-1/8)*(x^2+x)^(1/2)-1/16*ln(2*(x^2+x)^(1/2)-1-2*x)
```

Fricas [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 37, normalized size of antiderivative = 0.56

$$\int x\sqrt{x+x^2}\,dx = \frac{1}{24}\left(8\,x^2+2\,x-3\right)\sqrt{x^2+x}-\frac{1}{16}\,\log\left(-2\,x+2\,\sqrt{x^2+x}-1\right)$$

```
integrate(x*(x^2+x)^(1/2),x, algorithm="fricas")
```

```
1/24*(8*x^2 + 2*x - 3)*sqrt(x^2 + x) - 1/16*log(-2*x + 2*sqrt(x^2 + x) - 1
)
```

Sympy [A] (verification not implemented)

Time = 0.20 (sec) , antiderivative size = 37, normalized size of antiderivative = 0.56

$$\int x\sqrt{x+x^2}\,dx = \sqrt{x^2+x}\left(\frac{x^2}{3}+\frac{x}{12}-\frac{1}{8}\right)+\frac{\log\left(2x+2\sqrt{x^2+x}+1\right)}{16}$$

```
integrate(x*(x**2+x)**(1/2),x)
```

```
sqrt(x**2 + x)*(x**2/3 + x/12 - 1/8) + log(2*x + 2*sqrt(x**2 + x) + 1)/16
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 46, normalized size of antiderivative = 0.70

$$\int x\sqrt{x+x^2}\,dx = \frac{1}{3}\left(x^2+x\right)^{\frac{3}{2}} - \frac{1}{4}\sqrt{x^2+x}x - \frac{1}{8}\sqrt{x^2+x} + \frac{1}{16}\log\left(2x+2\sqrt{x^2+x}+1\right)$$

```
integrate(x*(x^2+x)^(1/2),x, algorithm="maxima")
```

```
1/3*(x^2 + x)^(3/2) - 1/4*sqrt(x^2 + x)*x - 1/8*sqrt(x^2 + x) + 1/16*log(2
*x + 2*sqrt(x^2 + x) + 1)
```

Giac [A] (verification not implemented)

Time = 0.37 (sec) , antiderivative size = 38, normalized size of antiderivative = 0.58

$$\int x\sqrt{x+x^2}\,dx = \frac{1}{24}(2(4x+1)x-3)\sqrt{x^2+x} - \frac{1}{16}\log\left(\left|-2x+2\sqrt{x^2+x}-1\right|\right)$$

```
integrate(x*(x^2+x)^(1/2),x, algorithm="giac")
```

```
1/24*(2*(4*x + 1)*x - 3)*sqrt(x^2 + x) - 1/16*log(abs(-2*x + 2*sqrt(x^2 +
x) - 1))
```

Mupad [B] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 33, normalized size of antiderivative = 0.50

$$\int x\sqrt{x+x^2}\,dx = \frac{\ln\left(x+\sqrt{x\,(x+1)}+\frac{1}{2}\right)}{16} + \frac{\sqrt{x^2+x}\,(8x^2+2x-3)}{24}$$

```
int(x*(x + x^2)^(1/2),x)
```

```
log(x + (x*(x + 1))^(1/2) + 1/2)/16 + ((x + x^2)^(1/2)*(2*x + 8*x^2 - 3))/
24
```

Reduce [B] (verification not implemented)

Time = 0.21 (sec) , antiderivative size = 39, normalized size of antiderivative = 0.59

$$\int x\sqrt{x+x^2}\,dx = \frac{\sqrt{x}\,\sqrt{x+1}\,x^2}{3} + \frac{\sqrt{x}\,\sqrt{x+1}\,x}{12} - \frac{\sqrt{x}\,\sqrt{x+1}}{8} + \frac{\log\left(\sqrt{x+1}+\sqrt{x}\right)}{8}$$

```
int(x*(x^2+x)^(1/2),x)
```

```
(8*sqrt(x)*sqrt(x + 1)*x**2 + 2*sqrt(x)*sqrt(x + 1)*x - 3*sqrt(x)*sqrt(x +
 1) + 3*log(sqrt(x + 1) + sqrt(x)))/24
```

3.38 $\int \frac{x^4}{\sqrt{ax+bx^2}}\,dx$

Optimal result

Integrand size = 17, antiderivative size = 128

$$\int \frac{x^4}{\sqrt{ax+bx^2}}\,dx=-\frac{35a^3\sqrt{ax+bx^2}}{64b^4}+\frac{35a^2x\sqrt{ax+bx^2}}{96b^3}-\frac{7ax^2\sqrt{ax+bx^2}}{24b^2}+\frac{x^3\sqrt{ax+bx^2}}{4b}+\frac{35a^4\operatorname{arctanh}\left(\frac{\sqrt{b}x}{\sqrt{ax+bx^2}}\right)}{64b^{9/2}}$$

```
-35/64*a^3*(b*x^2+a*x)^(1/2)/b^4+35/96*a^2*x*(b*x^2+a*x)^(1/2)/b^3-7/24*a*
x^2*(b*x^2+a*x)^(1/2)/b^2+1/4*x^3*(b*x^2+a*x)^(1/2)/b+35/64*a^4*arctanh(b^
(1/2)*x/(b*x^2+a*x)^(1/2))/b^(9/2)
```

Mathematica [A] (verified)

Time = 0.32 (sec) , antiderivative size = 119, normalized size of antiderivative = 0.93

$$\int \frac{x^4}{\sqrt{ax+bx^2}}\,dx=\frac{\sqrt{b}x(-105a^4-35a^3bx+14a^2b^2x^2-8ab^3x^3+48b^4x^4)+210a^4\sqrt{x}\sqrt{a+bx}\operatorname{arctanh}\left(\frac{\sqrt{b}\sqrt{x}}{-\sqrt{a}+\sqrt{a+bx}}\right)}{192b^{9/2}\sqrt{x(a+bx)}}$$

```
Integrate[x^4/Sqrt[a*x + b*x^2],x]
```

```
(Sqrt[b]*x*(-105*a^4 - 35*a^3*b*x + 14*a^2*b^2*x^2 - 8*a*b^3*x^3 + 48*b^4*
x^4) + 210*a^4*Sqrt[x]*Sqrt[a + b*x]*ArcTanh[(Sqrt[b]*Sqrt[x])/(-Sqrt[a] +
 Sqrt[a + b*x])])/(192*b^(9/2)*Sqrt[x*(a + b*x)])
```

Rubi [A] (verified)

Time = 0.46 (sec) , antiderivative size = 141, normalized size of antiderivative = 1.10, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.353$, Rules used = {1134, 1134, 1134, 1160, 1091, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^4}{\sqrt{ax+bx^2}}dx$$

$$\Big\downarrow \text{1134}$$

$$\frac{x^3\sqrt{ax+bx^2}}{4b}-\frac{7a\int \frac{x^3}{\sqrt{bx^2+ax}}dx}{8b}$$

$$\Big\downarrow \text{1134}$$

$$\frac{x^3\sqrt{ax+bx^2}}{4b}-\frac{7a\left(\frac{x^2\sqrt{ax+bx^2}}{3b}-\frac{5a\int \frac{x^2}{\sqrt{bx^2+ax}}dx}{6b}\right)}{8b}$$

$$\Big\downarrow \text{1134}$$

$$\frac{x^3\sqrt{ax+bx^2}}{4b}-\frac{7a\left(\frac{x^2\sqrt{ax+bx^2}}{3b}-\frac{5a\left(\frac{x\sqrt{ax+bx^2}}{2b}-\frac{3a\int \frac{x}{\sqrt{bx^2+ax}}dx}{4b}\right)}{6b}\right)}{8b}$$

$$\Big\downarrow \text{1160}$$

$$\frac{x^3\sqrt{ax+bx^2}}{4b}-\frac{7a\left(\frac{x^2\sqrt{ax+bx^2}}{3b}-\frac{5a\left(\frac{x\sqrt{ax+bx^2}}{2b}-\frac{3a\left(\frac{\sqrt{ax+bx^2}}{b}-\frac{a\int\frac{1}{\sqrt{bx^2+ax}}dx}{2b}\right)}{4b}\right)}{6b}\right)}{8b}$$

$$\Big\downarrow \text{ 1091}$$

$$\frac{x^3\sqrt{ax+bx^2}}{4b}-\frac{7a\left(\frac{x^2\sqrt{ax+bx^2}}{3b}-\frac{5a\left(\frac{x\sqrt{ax+bx^2}}{2b}-\frac{3a\left(\frac{\sqrt{ax+bx^2}}{b}-\frac{a\int\frac{1}{1-\frac{bx^2}{bx^2+ax}}d\frac{x}{\sqrt{bx^2+ax}}}{b}\right)}{4b}\right)}{6b}\right)}{8b}$$

$$\Big\downarrow \text{ 219}$$

$$\frac{x^3\sqrt{ax+bx^2}}{4b}-\frac{7a\left(\frac{x^2\sqrt{ax+bx^2}}{3b}-\frac{5a\left(\frac{x\sqrt{ax+bx^2}}{2b}-\frac{3a\left(\frac{\sqrt{ax+bx^2}}{b}-\frac{a\text{arctanh}\left(\frac{\sqrt{b}x}{\sqrt{ax+bx^2}}\right)}{b^{3/2}}\right)}{4b}\right)}{6b}\right)}{8b}$$

```
Int[x^4/Sqrt[a*x + b*x^2],x]
```

```
(x^3*Sqrt[a*x + b*x^2])/(4*b) - (7*a*((x^2*Sqrt[a*x + b*x^2])/(3*b) - (5*a
*((x*Sqrt[a*x + b*x^2])/(2*b) - (3*a*(Sqrt[a*x + b*x^2]/b - (a*ArcTanh[(Sq
rt[b]*x)/Sqrt[a*x + b*x^2]])/b^(3/2)))/(4*b)))/(6*b)))/(8*b)
```

Defintions of rubi rules used

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

```
Int[1/Sqrt[(b_.)*(x_) + (c_.)*(x_)^2], x_Symbol] :> Simp[2   Subst[Int[1/(1
 - c*x^2), x], x, x/Sqrt[b*x + c*x^2]], x] /; FreeQ[{b, c}, x]
```

```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] :> Simp[e*(d + e*x)^(m - 1)*((a + b*x + c*x^2)^(p + 1)/(c*(m + 2*p +
 1))), x] + Simp[(m + p)*((2*c*d - b*e)/(c*(m + 2*p + 1)))   Int[(d + e*x)^
(m - 1)*(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, p}, x] && EqQ[
c*d^2 - b*d*e + a*e^2, 0] && GtQ[m, 1] && NeQ[m + 2*p + 1, 0] && IntegerQ[2
*p]
```

```
Int[((d_.) + (e_.)*(x_))*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol
] :> Simp[e*((a + b*x + c*x^2)^(p + 1)/(2*c*(p + 1))), x] + Simp[(2*c*d - b
*e)/(2*c)   Int[(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, p}, x]
 && NeQ[p, -1]
```

Maple [A] (verified)

Time = 0.39 (sec) , antiderivative size = 73, normalized size of antiderivative = 0.57

method	result	size
pseudoelliptic	$\frac{\frac{35\,\mathrm{arctanh}\left(\frac{\sqrt{x(bx+a)}}{x\sqrt{b}}\right)a^4}{64}-\frac{35\left(\sqrt{b}\,a^3-\frac{2b^{\frac{3}{2}}a^2x}{3}+\frac{8a\,x^2b^{\frac{5}{2}}}{15}-\frac{16b^{\frac{7}{2}}x^3}{35}\right)\sqrt{x(bx+a)}}{64}}{b^{\frac{9}{2}}}$	73
risch	$-\frac{(-48b^3x^3+56a\,b^2x^2-70a^2bx+105a^3)x(bx+a)}{192b^4\sqrt{x(bx+a)}}+\frac{35a^4\ln\left(\frac{\frac{a}{2}+bx}{\sqrt{b}}+\sqrt{b\,x^2+ax}\right)}{128b^{\frac{9}{2}}}$	84
default	$\frac{x^3\sqrt{b\,x^2+ax}}{4b}-\frac{7a\left(\frac{x^2\sqrt{b\,x^2+ax}}{3b}-\frac{5a\left(\frac{x\sqrt{b\,x^2+ax}}{2b}-\frac{3a\left(\frac{\sqrt{b\,x^2+ax}}{b}-\frac{a\ln\left(\frac{\frac{a}{2}+bx}{\sqrt{b}}+\sqrt{b\,x^2+ax}\right)}{2b^{\frac{3}{2}}}\right)}{4b}\right)}{6b}\right)}{8b}$	123

```
int(x^4/(b*x^2+a*x)^(1/2),x,method=_RETURNVERBOSE)
```

```
35/64/b^(9/2)*(arctanh((x*(b*x+a))^(1/2)/x/b^(1/2))*a^4-(b^(1/2)*a^3-2/3*b
^(3/2)*a^2*x+8/15*a*x^2*b^(5/2)-16/35*b^(7/2)*x^3)*(x*(b*x+a))^(1/2))
```

Fricas [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 171, normalized size of antiderivative = 1.34

$$\int \frac{x^4}{\sqrt{ax+bx^2}}\,dx = \left[\frac{105\,a^4\sqrt{b}\log\left(2\,bx+a+2\,\sqrt{bx^2+ax}\sqrt{b}\right)+2\,(48\,b^4x^3-56\,ab^3x^2+70\,a^2b^2x-105\,a^3b)\sqrt{bx^2+ax}}{384\,b^5},\right.$$
$$\left.-\frac{105\,a^4\sqrt{-b}\arctan\left(\frac{\sqrt{bx^2+ax}\sqrt{-b}}{bx+a}\right)-(48\,b^4x^3-56\,ab^3x^2+70\,a^2b^2x-105\,a^3b)\sqrt{bx^2+ax}}{192\,b^5}\right]$$

```
integrate(x^4/(b*x^2+a*x)^(1/2),x, algorithm="fricas")
```

```
[1/384*(105*a^4*sqrt(b)*log(2*b*x + a + 2*sqrt(b*x^2 + a*x)*sqrt(b)) + 2*(
48*b^4*x^3 - 56*a*b^3*x^2 + 70*a^2*b^2*x - 105*a^3*b)*sqrt(b*x^2 + a*x))/b
^5, -1/192*(105*a^4*sqrt(-b)*arctan(sqrt(b*x^2 + a*x)*sqrt(-b)/(b*x + a))
- (48*b^4*x^3 - 56*a*b^3*x^2 + 70*a^2*b^2*x - 105*a^3*b)*sqrt(b*x^2 + a*x)
)/b^5]
```

Sympy [A] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 143, normalized size of antiderivative = 1.12

$$\int \frac{x^4}{\sqrt{ax+bx^2}}\,dx$$
$$= \begin{cases} \frac{35a^4\left(\begin{cases} \frac{\log\left(a+2\sqrt{b}\sqrt{ax+bx^2}+2bx\right)}{\sqrt{b}} & \text{for } \frac{a^2}{b} \neq 0 \\ \frac{\left(\frac{a}{2b}+x\right)\log\left(\frac{a}{2b}+x\right)}{\sqrt{b\left(\frac{a}{2b}+x\right)^2}} & \text{otherwise} \end{cases}\right)}{128b^4} + \sqrt{ax+bx^2}\left(-\frac{35a^3}{64b^4}+\frac{35a^2x}{96b^3}-\frac{7ax^2}{24b^2}+\frac{x^3}{4b}\right) & \text{for } b \neq 0 \\ \frac{2(ax)^{\frac{9}{2}}}{9a^5} & \text{for } a \neq 0 \\ \tilde{\infty}x^5 & \text{otherwise} \end{cases}$$

```
integrate(x**4/(b*x**2+a*x)**(1/2),x)
```

```
Piecewise((35*a**4*Piecewise((log(a + 2*sqrt(b)*sqrt(a*x + b*x**2) + 2*b*x
)/sqrt(b), Ne(a**2/b, 0)), ((a/(2*b) + x)*log(a/(2*b) + x)/sqrt(b*(a/(2*b)
 + x)**2), True))/(128*b**4) + sqrt(a*x + b*x**2)*(-35*a**3/(64*b**4) + 35
*a**2*x/(96*b**3) - 7*a*x**2/(24*b**2) + x**3/(4*b)), Ne(b, 0)), (2*(a*x)*
*(9/2)/(9*a**5), Ne(a, 0)), (zoo*x**5, True))
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 110, normalized size of antiderivative = 0.86

$$\int \frac{x^4}{\sqrt{ax+bx^2}}\,dx = \frac{\sqrt{bx^2+ax}x^3}{4\,b} - \frac{7\,\sqrt{bx^2+ax}ax^2}{24\,b^2} + \frac{35\,\sqrt{bx^2+ax}a^2x}{96\,b^3} + \frac{35\,a^4\log\left(2\,bx+a+2\,\sqrt{bx^2+ax}\sqrt{b}\right)}{128\,b^{\frac{9}{2}}} - \frac{35\,\sqrt{bx^2+ax}a^3}{64\,b^4}$$

```
integrate(x^4/(b*x^2+a*x)^(1/2),x, algorithm="maxima")
```

```
1/4*sqrt(b*x^2 + a*x)*x^3/b - 7/24*sqrt(b*x^2 + a*x)*a*x^2/b^2 + 35/96*sqr
t(b*x^2 + a*x)*a^2*x/b^3 + 35/128*a^4*log(2*b*x + a + 2*sqrt(b*x^2 + a*x)*
sqrt(b))/b^(9/2) - 35/64*sqrt(b*x^2 + a*x)*a^3/b^4
```

Giac [A] (verification not implemented)

Time = 0.34 (sec) , antiderivative size = 87, normalized size of antiderivative = 0.68

$$\int \frac{x^4}{\sqrt{ax+bx^2}}\,dx = \frac{1}{192}\,\sqrt{bx^2+ax}\left(2\left(4\,x\left(\frac{6\,x}{b}-\frac{7\,a}{b^2}\right)+\frac{35\,a^2}{b^3}\right)x-\frac{105\,a^3}{b^4}\right) - \frac{35\,a^4\log\left(\left|2\left(\sqrt{b}x-\sqrt{bx^2+ax}\right)\sqrt{b}+a\right|\right)}{128\,b^{\frac{9}{2}}}$$

```
integrate(x^4/(b*x^2+a*x)^(1/2),x, algorithm="giac")
```

```
1/192*sqrt(b*x^2 + a*x)*(2*(4*x*(6*x/b - 7*a/b^2) + 35*a^2/b^3)*x - 105*a^
3/b^4) - 35/128*a^4*log(abs(2*(sqrt(b)*x - sqrt(b*x^2 + a*x))*sqrt(b) + a)
)/b^(9/2)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{x^4}{\sqrt{ax+bx^2}}\,dx = \int \frac{x^4}{\sqrt{b\,x^2+a\,x}}\,dx$$

```
int(x^4/(a*x + b*x^2)^(1/2),x)
```

```
int(x^4/(a*x + b*x^2)^(1/2), x)
```

Reduce [B] (verification not implemented)

Time = 0.21 (sec) , antiderivative size = 95, normalized size of antiderivative = 0.74

$$\int \frac{x^4}{\sqrt{ax+bx^2}}\,dx = \frac{-105\sqrt{x}\,\sqrt{bx+a}\,a^3b+70\sqrt{x}\,\sqrt{bx+a}\,a^2b^2x-56\sqrt{x}\,\sqrt{bx+a}\,a\,b^3x^2+48\sqrt{x}\,\sqrt{bx+a}\,b^4x^3+105\sqrt{b}\log}{192b^5}$$

```
int(x^4/(b*x^2+a*x)^(1/2),x)
```

```
( - 105*sqrt(x)*sqrt(a + b*x)*a**3*b + 70*sqrt(x)*sqrt(a + b*x)*a**2*b**2*
x - 56*sqrt(x)*sqrt(a + b*x)*a*b**3*x**2 + 48*sqrt(x)*sqrt(a + b*x)*b**4*x
**3 + 105*sqrt(b)*log((sqrt(a + b*x) + sqrt(x)*sqrt(b))/sqrt(a))*a**4)/(19
2*b**5)
```

3.39 $\int \frac{x^3}{\sqrt{ax+bx^2}}\,dx$

Optimal result

Integrand size = 17, antiderivative size = 102

$$\int \frac{x^3}{\sqrt{ax+bx^2}}\,dx = \frac{5a^2\sqrt{ax+bx^2}}{8b^3} - \frac{5ax\sqrt{ax+bx^2}}{12b^2} + \frac{x^2\sqrt{ax+bx^2}}{3b} - \frac{5a^3\operatorname{arctanh}\left(\frac{\sqrt{b}x}{\sqrt{ax+bx^2}}\right)}{8b^{7/2}}$$

```
5/8*a^2*(b*x^2+a*x)^(1/2)/b^3-5/12*a*x*(b*x^2+a*x)^(1/2)/b^2+1/3*x^2*(b*x^
2+a*x)^(1/2)/b-5/8*a^3*arctanh(b^(1/2)*x/(b*x^2+a*x)^(1/2))/b^(7/2)
```

Mathematica [A] (verified)

Time = 0.27 (sec) , antiderivative size = 108, normalized size of antiderivative = 1.06

$$\int \frac{x^3}{\sqrt{ax+bx^2}}\,dx = \frac{\sqrt{b}x(15a^3+5a^2bx-2ab^2x^2+8b^3x^3)+30a^3\sqrt{x}\sqrt{a+bx}\operatorname{arctanh}\left(\frac{\sqrt{b}\sqrt{x}}{\sqrt{a}-\sqrt{a+bx}}\right)}{24b^{7/2}\sqrt{x(a+bx)}}$$

```
Integrate[x^3/Sqrt[a*x + b*x^2],x]
```

```
(Sqrt[b]*x*(15*a^3 + 5*a^2*b*x - 2*a*b^2*x^2 + 8*b^3*x^3) + 30*a^3*Sqrt[x]
*Sqrt[a + b*x]*ArcTanh[(Sqrt[b]*Sqrt[x])/(Sqrt[a] - Sqrt[a + b*x])])/(24*b
^(7/2)*Sqrt[x*(a + b*x)])
```

Rubi [A] (verified)

Time = 0.39 (sec) , antiderivative size = 109, normalized size of antiderivative = 1.07, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.294$, Rules used = {1134, 1134, 1160, 1091, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^3}{\sqrt{ax+bx^2}}dx$$

$$\Big\downarrow \text{1134}$$

$$\frac{x^2\sqrt{ax+bx^2}}{3b}-\frac{5a\int \frac{x^2}{\sqrt{bx^2+ax}}dx}{6b}$$

$$\Big\downarrow \text{1134}$$

$$\frac{x^2\sqrt{ax+bx^2}}{3b}-\frac{5a\left(\frac{x\sqrt{ax+bx^2}}{2b}-\frac{3a\int \frac{x}{\sqrt{bx^2+ax}}dx}{4b}\right)}{6b}$$

$$\Big\downarrow \text{1160}$$

$$\frac{x^2\sqrt{ax+bx^2}}{3b}-\frac{5a\left(\frac{x\sqrt{ax+bx^2}}{2b}-\frac{3a\left(\frac{\sqrt{ax+bx^2}}{b}-\frac{a\int \frac{1}{\sqrt{bx^2+ax}}dx}{2b}\right)}{4b}\right)}{6b}$$

$$\Big\downarrow \text{1091}$$

$$\frac{x^2\sqrt{ax+bx^2}}{3b}-\frac{5a\left(\frac{x\sqrt{ax+bx^2}}{2b}-\frac{3a\left(\frac{\sqrt{ax+bx^2}}{b}-\frac{a\int \frac{1}{1-\frac{bx^2}{bx^2+ax}}d\frac{x}{\sqrt{bx^2+ax}}}{b}\right)}{4b}\right)}{6b}$$

$$\Big\downarrow \textcolor{blue}{219}$$

$$\frac{x^2\sqrt{ax+bx^2}}{3b}-\frac{5a\left(\frac{x\sqrt{ax+bx^2}}{2b}-\frac{3a\left(\frac{\sqrt{ax+bx^2}}{b}-\frac{a\text{arctanh}\left(\frac{\sqrt{b}x}{\sqrt{ax+bx^2}}\right)}{b^{3/2}}\right)}{4b}\right)}{6b}$$

```
Int[x^3/Sqrt[a*x + b*x^2],x]
```

```
(x^2*Sqrt[a*x + b*x^2])/(3*b) - (5*a*((x*Sqrt[a*x + b*x^2])/(2*b) - (3*a*(
Sqrt[a*x + b*x^2]/b - (a*ArcTanh[(Sqrt[b]*x)/Sqrt[a*x + b*x^2]])/b^(3/2)))
/(4*b)))/(6*b)
```

Defintions of rubi rules used

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

```
Int[1/Sqrt[(b_.)*(x_) + (c_.)*(x_)^2], x_Symbol] :> Simp[2   Subst[Int[1/(1
 - c*x^2), x], x, x/Sqrt[b*x + c*x^2]], x] /; FreeQ[{b, c}, x]
```

```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] :> Simp[e*(d + e*x)^(m - 1)*((a + b*x + c*x^2)^(p + 1)/(c*(m + 2*p +
 1))), x] + Simp[(m + p)*((2*c*d - b*e)/(c*(m + 2*p + 1)))   Int[(d + e*x)^
(m - 1)*(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, p}, x] && EqQ[
c*d^2 - b*d*e + a*e^2, 0] && GtQ[m, 1] && NeQ[m + 2*p + 1, 0] && IntegerQ[2
*p]
```

```
Int[((d_.) + (e_.)*(x_))*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol
] :> Simp[e*((a + b*x + c*x^2)^(p + 1)/(2*c*(p + 1))), x] + Simp[(2*c*d - b
*e)/(2*c)   Int[(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, p}, x]
 && NeQ[p, -1]
```

Maple [A] (verified)

Time = 0.37 (sec) , antiderivative size = 73, normalized size of antiderivative = 0.72

method	result	size
risch	$\frac{(8b^2x^2-10abx+15a^2)x(bx+a)}{24b^3\sqrt{x(bx+a)}}-\frac{5a^3\ln\left(\frac{\frac{a}{2}+bx}{\sqrt{b}}+\sqrt{b\,x^2+ax}\right)}{16b^{\frac{7}{2}}}$	73
pseudoelliptic	$\frac{8b^{\frac{5}{2}}\sqrt{x(bx+a)}\,x^2-10a\,b^{\frac{3}{2}}x\sqrt{x(bx+a)}+15a^2\sqrt{b}\,\sqrt{x(bx+a)}-15\,\operatorname{arctanh}\left(\frac{\sqrt{x(bx+a)}}{x\sqrt{b}}\right)a^3}{24b^{\frac{7}{2}}}$	79
default	$\frac{x^2\sqrt{b\,x^2+ax}}{3b}-\frac{5a\left(\frac{x\sqrt{b\,x^2+ax}}{2b}-\frac{3a\left(\frac{\sqrt{b\,x^2+ax}}{b}-\frac{a\ln\left(\frac{\frac{a}{2}+bx}{\sqrt{b}}+\sqrt{b\,x^2+ax}\right)}{2b^{\frac{3}{2}}}\right)}{4b}\right)}{6b}$	97

```
int(x^3/(b*x^2+a*x)^(1/2),x,method=_RETURNVERBOSE)
```

```
1/24*(8*b^2*x^2-10*a*b*x+15*a^2)*x*(b*x+a)/b^3/(x*(b*x+a))^(1/2)-5/16*a^3/
b^(7/2)*ln((1/2*a+b*x)/b^(1/2)+(b*x^2+a*x)^(1/2))
```

Fricas [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 148, normalized size of antiderivative = 1.45

$$\int \frac{x^3}{\sqrt{ax+bx^2}}\,dx$$
$$= \left[\frac{15\,a^3\sqrt{b}\log\left(2\,bx+a-2\,\sqrt{bx^2+ax}\sqrt{b}\right)+2\,(8\,b^3x^2-10\,ab^2x+15\,a^2b)\sqrt{bx^2+ax}}{48\,b^4}, \frac{15\,a^3\sqrt{-b}\arctan}{}\right.$$

```
integrate(x^3/(b*x^2+a*x)^(1/2),x, algorithm="fricas")
```

```
[1/48*(15*a^3*sqrt(b)*log(2*b*x + a - 2*sqrt(b*x^2 + a*x)*sqrt(b)) + 2*(8*
b^3*x^2 - 10*a*b^2*x + 15*a^2*b)*sqrt(b*x^2 + a*x))/b^4, 1/24*(15*a^3*sqrt
(-b)*arctan(sqrt(b*x^2 + a*x)*sqrt(-b)/(b*x + a)) + (8*b^3*x^2 - 10*a*b^2*
x + 15*a^2*b)*sqrt(b*x^2 + a*x))/b^4]
```

Sympy [A] (verification not implemented)

Time = 0.25 (sec) , antiderivative size = 129, normalized size of antiderivative = 1.26

$$\int \frac{x^3}{\sqrt{ax+bx^2}}\,dx = \begin{cases} -\frac{5a^3\left(\begin{cases} \frac{\log\left(a+2\sqrt{b}\sqrt{ax+bx^2}+2bx\right)}{\sqrt{b}} & \text{for } \frac{a^2}{b}\neq 0 \\ \frac{\left(\frac{a}{2b}+x\right)\log\left(\frac{a}{2b}+x\right)}{\sqrt{b\left(\frac{a}{2b}+x\right)^2}} & \text{otherwise} \end{cases}\right)}{16b^3} + \sqrt{ax+bx^2}\cdot\left(\frac{5a^2}{8b^3}-\frac{5ax}{12b^2}+\frac{x^2}{3b}\right) & \text{for } b\neq 0 \\ \frac{2(ax)^{\frac{7}{2}}}{7a^4} & \text{for } a\neq 0 \\ \tilde{\infty}x^4 & \text{otherwise} \end{cases}$$

```
integrate(x**3/(b*x**2+a*x)**(1/2),x)
```

```
Piecewise((-5*a**3*Piecewise((log(a + 2*sqrt(b)*sqrt(a*x + b*x**2) + 2*b*x
)/sqrt(b), Ne(a**2/b, 0)), ((a/(2*b) + x)*log(a/(2*b) + x)/sqrt(b*(a/(2*b)
 + x)**2), True))/(16*b**3) + sqrt(a*x + b*x**2)*(5*a**2/(8*b**3) - 5*a*x/
(12*b**2) + x**2/(3*b)), Ne(b, 0)), (2*(a*x)**(7/2)/(7*a**4), Ne(a, 0)), (
zoo*x**4, True))
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 88, normalized size of antiderivative = 0.86

$$\int \frac{x^3}{\sqrt{ax+bx^2}}\,dx = \frac{\sqrt{bx^2+ax}x^2}{3\,b} - \frac{5\,\sqrt{bx^2+ax}ax}{12\,b^2} - \frac{5\,a^3\log\left(2\,bx+a+2\,\sqrt{bx^2+ax}\sqrt{b}\right)}{16\,b^{\frac{7}{2}}} + \frac{5\,\sqrt{bx^2+ax}a^2}{8\,b^3}$$

```
integrate(x^3/(b*x^2+a*x)^(1/2),x, algorithm="maxima")
```

```
1/3*sqrt(b*x^2 + a*x)*x^2/b - 5/12*sqrt(b*x^2 + a*x)*a*x/b^2 - 5/16*a^3*lo
g(2*b*x + a + 2*sqrt(b*x^2 + a*x)*sqrt(b))/b^(7/2) + 5/8*sqrt(b*x^2 + a*x)
*a^2/b^3
```

Giac [A] (verification not implemented)

Time = 0.19 (sec) , antiderivative size = 75, normalized size of antiderivative = 0.74

$$\int \frac{x^3}{\sqrt{ax+bx^2}}\,dx = \frac{1}{24}\sqrt{bx^2+ax}\left(2\,x\left(\frac{4\,x}{b}-\frac{5\,a}{b^2}\right)+\frac{15\,a^2}{b^3}\right) + \frac{5\,a^3\log\left(\left|2\left(\sqrt{b}x-\sqrt{bx^2+ax}\right)\sqrt{b}+a\right|\right)}{16\,b^{\frac{7}{2}}}$$

```
integrate(x^3/(b*x^2+a*x)^(1/2),x, algorithm="giac")
```

```
1/24*sqrt(b*x^2 + a*x)*(2*x*(4*x/b - 5*a/b^2) + 15*a^2/b^3) + 5/16*a^3*log
(abs(2*(sqrt(b)*x - sqrt(b*x^2 + a*x))*sqrt(b) + a))/b^(7/2)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{x^3}{\sqrt{ax+bx^2}}\,dx = \int \frac{x^3}{\sqrt{b\,x^2+a\,x}}\,dx$$

```
int(x^3/(a*x + b*x^2)^(1/2),x)
```

```
int(x^3/(a*x + b*x^2)^(1/2), x)
```

Reduce [B] (verification not implemented)

Time = 0.22 (sec) , antiderivative size = 76, normalized size of antiderivative = 0.75

$$\int \frac{x^3}{\sqrt{ax+bx^2}}\,dx$$
$$= \frac{15\sqrt{x}\,\sqrt{bx+a}\,a^2b - 10\sqrt{x}\,\sqrt{bx+a}\,a\,b^2x + 8\sqrt{x}\,\sqrt{bx+a}\,b^3x^2 - 15\sqrt{b}\log\left(\frac{\sqrt{bx+a}+\sqrt{x}\,\sqrt{b}}{\sqrt{a}}\right)a^3}{24b^4}$$

```
int(x^3/(b*x^2+a*x)^(1/2),x)
```

```
(15*sqrt(x)*sqrt(a + b*x)*a**2*b - 10*sqrt(x)*sqrt(a + b*x)*a*b**2*x + 8*s
qrt(x)*sqrt(a + b*x)*b**3*x**2 - 15*sqrt(b)*log((sqrt(a + b*x) + sqrt(x)*s
qrt(b))/sqrt(a))*a**3)/(24*b**4)
```

3.40 $\int \frac{x^2}{\sqrt{ax+bx^2}}\,dx$

Optimal result

Integrand size = 17, antiderivative size = 76

$$\int \frac{x^2}{\sqrt{ax+bx^2}}\,dx = -\frac{3a\sqrt{ax+bx^2}}{4b^2}+\frac{x\sqrt{ax+bx^2}}{2b}+\frac{3a^2\operatorname{arctanh}\left(\frac{\sqrt{b}x}{\sqrt{ax+bx^2}}\right)}{4b^{5/2}}$$

```
-3/4*a*(b*x^2+a*x)^(1/2)/b^2+1/2*x*(b*x^2+a*x)^(1/2)/b+3/4*a^2*arctanh(b^(
1/2)*x/(b*x^2+a*x)^(1/2))/b^(5/2)
```

Mathematica [A] (verified)

Time = 0.20 (sec) , antiderivative size = 97, normalized size of antiderivative = 1.28

$$\int \frac{x^2}{\sqrt{ax+bx^2}}\,dx = \frac{\sqrt{b}x(-3a^2-abx+2b^2x^2)+6a^2\sqrt{x}\sqrt{a+bx}\operatorname{arctanh}\left(\frac{\sqrt{b}\sqrt{x}}{-\sqrt{a}+\sqrt{a+bx}}\right)}{4b^{5/2}\sqrt{x(a+bx)}}$$

```
Integrate[x^2/Sqrt[a*x + b*x^2],x]
```

```
(Sqrt[b]*x*(-3*a^2 - a*b*x + 2*b^2*x^2) + 6*a^2*Sqrt[x]*Sqrt[a + b*x]*ArcT
anh[(Sqrt[b]*Sqrt[x])/(-Sqrt[a] + Sqrt[a + b*x])])/(4*b^(5/2)*Sqrt[x*(a +
b*x)])
```

Rubi [A] (verified)

Time = 0.33 (sec) , antiderivative size = 77, normalized size of antiderivative = 1.01, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.235$, Rules used = {1134, 1160, 1091, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^2}{\sqrt{ax+bx^2}}\,dx$$

$$\Big\downarrow \text{1134}$$

$$\frac{x\sqrt{ax+bx^2}}{2b}-\frac{3a\int \frac{x}{\sqrt{bx^2+ax}}dx}{4b}$$

$$\Big\downarrow \text{1160}$$

$$\frac{x\sqrt{ax+bx^2}}{2b}-\frac{3a\left(\frac{\sqrt{ax+bx^2}}{b}-\frac{a\int \frac{1}{\sqrt{bx^2+ax}}dx}{2b}\right)}{4b}$$

$$\Big\downarrow \text{1091}$$

$$\frac{x\sqrt{ax+bx^2}}{2b}-\frac{3a\left(\frac{\sqrt{ax+bx^2}}{b}-\frac{a\int \frac{1}{1-\frac{bx^2}{bx^2+ax}}d\frac{x}{\sqrt{bx^2+ax}}}{b}\right)}{4b}$$

$$\Big\downarrow \text{219}$$

$$\frac{x\sqrt{ax+bx^2}}{2b}-\frac{3a\left(\frac{\sqrt{ax+bx^2}}{b}-\frac{a\text{arctanh}\left(\frac{\sqrt{b}x}{\sqrt{ax+bx^2}}\right)}{b^{3/2}}\right)}{4b}$$

```
Int[x^2/Sqrt[a*x + b*x^2],x]
```

```
(x*Sqrt[a*x + b*x^2])/(2*b) - (3*a*(Sqrt[a*x + b*x^2]/b - (a*ArcTanh[(Sqrt
[b]*x)/Sqrt[a*x + b*x^2]])/b^(3/2)))/(4*b)
```

Defintions of rubi rules used

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

```
Int[1/Sqrt[(b_.)*(x_) + (c_.)*(x_)^2], x_Symbol] :> Simp[2   Subst[Int[1/(1
 - c*x^2), x], x, x/Sqrt[b*x + c*x^2]], x] /; FreeQ[{b, c}, x]
```

```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] :> Simp[e*(d + e*x)^(m - 1)*((a + b*x + c*x^2)^(p + 1)/(c*(m + 2*p +
 1))), x] + Simp[(m + p)*((2*c*d - b*e)/(c*(m + 2*p + 1)))   Int[(d + e*x)^
(m - 1)*(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, p}, x] && EqQ[
c*d^2 - b*d*e + a*e^2, 0] && GtQ[m, 1] && NeQ[m + 2*p + 1, 0] && IntegerQ[2
*p]
```

```
Int[((d_.) + (e_.)*(x_))*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol
] :> Simp[e*((a + b*x + c*x^2)^(p + 1)/(2*c*(p + 1))), x] + Simp[(2*c*d - b
*e)/(2*c)   Int[(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, p}, x]
 && NeQ[p, -1]
```

Maple [A] (verified)

Time = 0.36 (sec) , antiderivative size = 59, normalized size of antiderivative = 0.78

method	result	size
pseudoelliptic	$\frac{2b^{\frac{3}{2}}\sqrt{x(bx+a)}\,x+3\,\operatorname{arctanh}\left(\frac{\sqrt{x(bx+a)}}{x\sqrt{b}}\right)a^{2}-3a\sqrt{b}\,\sqrt{x(bx+a)}}{4b^{\frac{5}{2}}}$	59
risch	$-\frac{(-2bx+3a)x(bx+a)}{4b^{2}\sqrt{x(bx+a)}}+\frac{3a^{2}\ln\left(\frac{\frac{a}{2}+bx}{\sqrt{b}}+\sqrt{b\,x^{2}+ax}\right)}{8b^{\frac{5}{2}}}$	62
default	$\frac{x\sqrt{b\,x^{2}+ax}}{2b}-\frac{3a\left(\frac{\sqrt{b\,x^{2}+ax}}{b}-\frac{a\ln\left(\frac{\frac{a}{2}+bx}{\sqrt{b}}+\sqrt{b\,x^{2}+ax}\right)}{2b^{\frac{3}{2}}}\right)}{4b}$	71

```
int(x^2/(b*x^2+a*x)^(1/2),x,method=_RETURNVERBOSE)
```

```
1/4/b^(5/2)*(2*b^(3/2)*(x*(b*x+a))^(1/2)*x+3*arctanh((x*(b*x+a))^(1/2)/x/b
^(1/2))*a^2-3*a*b^(1/2)*(x*(b*x+a))^(1/2))
```

Fricas [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 127, normalized size of antiderivative = 1.67

$$\int \frac{x^2}{\sqrt{ax+bx^2}}\,dx = \left[\frac{3\,a^2\sqrt{b}\log\left(2\,bx+a+2\,\sqrt{bx^2+ax}\sqrt{b}\right)+2\,(2\,b^2x-3\,ab)\sqrt{bx^2+ax}}{8\,b^3},\right.$$
$$\left.-\frac{3\,a^2\sqrt{-b}\arctan\left(\frac{\sqrt{bx^2+ax}\sqrt{-b}}{bx+a}\right)-(2\,b^2x-3\,ab)\sqrt{bx^2+ax}}{4\,b^3}\right]$$

```
integrate(x^2/(b*x^2+a*x)^(1/2),x, algorithm="fricas")
```

```
[1/8*(3*a^2*sqrt(b)*log(2*b*x + a + 2*sqrt(b*x^2 + a*x)*sqrt(b)) + 2*(2*b^
2*x - 3*a*b)*sqrt(b*x^2 + a*x))/b^3, -1/4*(3*a^2*sqrt(-b)*arctan(sqrt(b*x^
2 + a*x)*sqrt(-b)/(b*x + a)) - (2*b^2*x - 3*a*b)*sqrt(b*x^2 + a*x))/b^3]
```

Sympy [A] (verification not implemented)

Time = 0.25 (sec) , antiderivative size = 116, normalized size of antiderivative = 1.53

$$\int \frac{x^2}{\sqrt{ax+bx^2}}\,dx = \begin{cases} \frac{3a^2\left(\begin{cases} \frac{\log\left(a+2\sqrt{b}\sqrt{ax+bx^2}+2bx\right)}{\sqrt{b}} & \text{for } \frac{a^2}{b}\neq 0 \\ \frac{\left(\frac{a}{2b}+x\right)\log\left(\frac{a}{2b}+x\right)}{\sqrt{b\left(\frac{a}{2b}+x\right)^2}} & \text{otherwise} \end{cases}\right)}{8b^2} + \left(-\frac{3a}{4b^2}+\frac{x}{2b}\right)\sqrt{ax+bx^2} & \text{for } b\neq 0 \\ \frac{2(ax)^{\frac{5}{2}}}{5a^3} & \text{for } a\neq 0 \\ \tilde{\infty}x^3 & \text{otherwise} \end{cases}$$

```
integrate(x**2/(b*x**2+a*x)**(1/2),x)
```

```
Piecewise((3*a**2*Piecewise((log(a + 2*sqrt(b)*sqrt(a*x + b*x**2) + 2*b*x)
/sqrt(b), Ne(a**2/b, 0)), ((a/(2*b) + x)*log(a/(2*b) + x)/sqrt(b*(a/(2*b)
+ x)**2), True))/(8*b**2) + (-3*a/(4*b**2) + x/(2*b))*sqrt(a*x + b*x**2),
Ne(b, 0)), (2*(a*x)**(5/2)/(5*a**3), Ne(a, 0)), (zoo*x**3, True))
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 66, normalized size of antiderivative = 0.87

$$\int \frac{x^2}{\sqrt{ax+bx^2}}\,dx = \frac{\sqrt{bx^2+ax}x}{2b} + \frac{3a^2\log\left(2bx+a+2\sqrt{bx^2+ax}\sqrt{b}\right)}{8b^{\frac{5}{2}}} - \frac{3\sqrt{bx^2+ax}a}{4b^2}$$

```
integrate(x^2/(b*x^2+a*x)^(1/2),x, algorithm="maxima")
```

```
1/2*sqrt(b*x^2 + a*x)*x/b + 3/8*a^2*log(2*b*x + a + 2*sqrt(b*x^2 + a*x)*sq
rt(b))/b^(5/2) - 3/4*sqrt(b*x^2 + a*x)*a/b^2
```

Giac [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 63, normalized size of antiderivative = 0.83

$$\int \frac{x^2}{\sqrt{ax+bx^2}}\,dx$$
$$= \frac{1}{4}\sqrt{bx^2+ax}\left(\frac{2\,x}{b}-\frac{3\,a}{b^2}\right)-\frac{3\,a^2\log\left(\left|2\left(\sqrt{b}x-\sqrt{bx^2+ax}\right)\sqrt{b}+a\right|\right)}{8\,b^{\frac{5}{2}}}$$

```
integrate(x^2/(b*x^2+a*x)^(1/2),x, algorithm="giac")
```

```
1/4*sqrt(b*x^2 + a*x)*(2*x/b - 3*a/b^2) - 3/8*a^2*log(abs(2*(sqrt(b)*x - s
qrt(b*x^2 + a*x))*sqrt(b) + a))/b^(5/2)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{x^2}{\sqrt{ax+bx^2}}\,dx = \int \frac{x^2}{\sqrt{b\,x^2+a\,x}}\,dx$$

```
int(x^2/(a*x + b*x^2)^(1/2),x)
```

```
int(x^2/(a*x + b*x^2)^(1/2), x)
```

Reduce [B] (verification not implemented)

Time = 0.20 (sec) , antiderivative size = 57, normalized size of antiderivative = 0.75

$$\int \frac{x^2}{\sqrt{ax+bx^2}}\,dx = \frac{-3\sqrt{x}\,\sqrt{bx+a}\,ab+2\sqrt{x}\,\sqrt{bx+a}\,b^2x+3\sqrt{b}\log\left(\frac{\sqrt{bx+a}+\sqrt{x}\,\sqrt{b}}{\sqrt{a}}\right)a^2}{4b^3}$$

```
int(x^2/(b*x^2+a*x)^(1/2),x)
```

```
( - 3*sqrt(x)*sqrt(a + b*x)*a*b + 2*sqrt(x)*sqrt(a + b*x)*b**2*x + 3*sqrt(
b)*log((sqrt(a + b*x) + sqrt(x)*sqrt(b))/sqrt(a))*a**2)/(4*b**3)
```

3.41 $\int \frac{x}{\sqrt{ax+bx^2}}\,dx$

Optimal result

Integrand size = 15, antiderivative size = 47

$$\int \frac{x}{\sqrt{ax+bx^2}}\,dx = \frac{\sqrt{ax+bx^2}}{b} - \frac{a\text{arctanh}\left(\frac{\sqrt{b}x}{\sqrt{ax+bx^2}}\right)}{b^{3/2}}$$

```
(b*x^2+a*x)^(1/2)/b-a*arctanh(b^(1/2)*x/(b*x^2+a*x)^(1/2))/b^(3/2)
```

Mathematica [A] (verified)

Time = 0.15 (sec) , antiderivative size = 78, normalized size of antiderivative = 1.66

$$\int \frac{x}{\sqrt{ax+bx^2}}\,dx = \frac{\sqrt{b}x(a+bx) + 2a\sqrt{x}\sqrt{a+bx}\text{arctanh}\left(\frac{\sqrt{b}\sqrt{x}}{\sqrt{a}-\sqrt{a+bx}}\right)}{b^{3/2}\sqrt{x(a+bx)}}$$

```
Integrate[x/Sqrt[a*x + b*x^2],x]
```

```
(Sqrt[b]*x*(a + b*x) + 2*a*Sqrt[x]*Sqrt[a + b*x]*ArcTanh[(Sqrt[b]*Sqrt[x])
/(Sqrt[a] - Sqrt[a + b*x])])/(b^(3/2)*Sqrt[x*(a + b*x)])
```

Rubi [A] (verified)

Time = 0.28 (sec) , antiderivative size = 47, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.200, Rules used = {1160, 1091, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x}{\sqrt{ax+bx^2}}dx$$
$$\Big\downarrow \ 1160$$
$$\frac{\sqrt{ax+bx^2}}{b}-\frac{a\int \frac{1}{\sqrt{bx^2+ax}}dx}{2b}$$
$$\Big\downarrow \ 1091$$
$$\frac{\sqrt{ax+bx^2}}{b}-\frac{a\int \frac{1}{1-\frac{bx^2}{bx^2+ax}}d\frac{x}{\sqrt{bx^2+ax}}}{b}$$
$$\Big\downarrow \ 219$$
$$\frac{\sqrt{ax+bx^2}}{b}-\frac{a\text{arctanh}\left(\frac{\sqrt{b}x}{\sqrt{ax+bx^2}}\right)}{b^{3/2}}$$

```
Int[x/Sqrt[a*x + b*x^2],x]
```

```
Sqrt[a*x + b*x^2]/b - (a*ArcTanh[(Sqrt[b]*x)/Sqrt[a*x + b*x^2]])/b^(3/2)
```

Defintions of rubi rules used

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

```
Int[1/Sqrt[(b_.)*(x_) + (c_.)*(x_)^2], x_Symbol] :> Simp[2   Subst[Int[1/(1
 - c*x^2), x], x, x/Sqrt[b*x + c*x^2]], x] /; FreeQ[{b, c}, x]
```

```
Int[((d_.) + (e_.)*(x_))*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol
] :> Simp[e*((a + b*x + c*x^2)^(p + 1)/(2*c*(p + 1))), x] + Simp[(2*c*d - b
*e)/(2*c)   Int[(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, p}, x]
 && NeQ[p, -1]
```

Maple [A] (verified)

Time = 0.36 (sec) , antiderivative size = 39, normalized size of antiderivative = 0.83

method	result	size
pseudoelliptic	$\frac{-\operatorname{arctanh}\left(\frac{\sqrt{x(bx+a)}}{x\sqrt{b}}\right)a+\sqrt{x(bx+a)}\,\sqrt{b}}{b^{\frac{3}{2}}}$	39
default	$\frac{\sqrt{b\,x^2+ax}}{b}-\frac{a\ln\left(\frac{\frac{a}{2}+bx}{\sqrt{b}}+\sqrt{b\,x^2+ax}\right)}{2b^{\frac{3}{2}}}$	47
risch	$\frac{x(bx+a)}{b\sqrt{x(bx+a)}}-\frac{a\ln\left(\frac{\frac{a}{2}+bx}{\sqrt{b}}+\sqrt{b\,x^2+ax}\right)}{2b^{\frac{3}{2}}}$	51

```
int(x/(b*x^2+a*x)^(1/2),x,method=_RETURNVERBOSE)
```

```
(-arctanh((x*(b*x+a))^(1/2)/x/b^(1/2))*a+(x*(b*x+a))^(1/2)*b^(1/2))/b^(3/2
)
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 99, normalized size of antiderivative = 2.11

$$\int \frac{x}{\sqrt{ax+bx^2}}\,dx$$
$$=\left[\frac{a\sqrt{b}\log\left(2\,bx+a-2\,\sqrt{bx^2+ax}\sqrt{b}\right)+2\,\sqrt{bx^2+ax}b}{2\,b^2},\frac{a\sqrt{-b}\arctan\left(\frac{\sqrt{bx^2+ax}\sqrt{-b}}{bx+a}\right)+\sqrt{bx^2+ax}b}{b^2}\right]$$

```
integrate(x/(b*x^2+a*x)^(1/2),x, algorithm="fricas")
```

```
[1/2*(a*sqrt(b)*log(2*b*x + a - 2*sqrt(b*x^2 + a*x)*sqrt(b)) + 2*sqrt(b*x^
2 + a*x)*b)/b^2, (a*sqrt(-b)*arctan(sqrt(b*x^2 + a*x)*sqrt(-b)/(b*x + a))
+ sqrt(b*x^2 + a*x)*b)/b^2]
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 99 vs. $2(39) = 78$.

Time = 0.23 (sec) , antiderivative size = 99, normalized size of antiderivative = 2.11

$$\int \frac{x}{\sqrt{ax+bx^2}}\,dx = \begin{cases} -\frac{a\left(\begin{cases} \frac{\log\left(a+2\sqrt{b}\sqrt{ax+bx^2}+2bx\right)}{\sqrt{b}} & \text{for}\ \frac{a^2}{b}\neq 0 \\ \frac{\left(\frac{a}{2b}+x\right)\log\left(\frac{a}{2b}+x\right)}{\sqrt{b\left(\frac{a}{2b}+x\right)^2}} & \text{otherwise} \end{cases}\right)}{2b} + \frac{\sqrt{ax+bx^2}}{b} & \text{for}\ b\neq 0 \\ \frac{2(ax)^{\frac{3}{2}}}{3a^2} & \text{for}\ a\neq 0 \\ \tilde{\infty}x^2 & \text{otherwise} \end{cases}$$

```
integrate(x/(b*x**2+a*x)**(1/2),x)
```

```
Piecewise((-a*Piecewise((log(a + 2*sqrt(b)*sqrt(a*x + b*x**2) + 2*b*x)/sqr
t(b), Ne(a**2/b, 0)), ((a/(2*b) + x)*log(a/(2*b) + x)/sqrt(b*(a/(2*b) + x)
**2), True))/(2*b) + sqrt(a*x + b*x**2)/b, Ne(b, 0)), (2*(a*x)**(3/2)/(3*a
**2), Ne(a, 0)), (zoo*x**2, True))
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 45, normalized size of antiderivative = 0.96

$$\int \frac{x}{\sqrt{ax+bx^2}}\,dx = -\frac{a\log\left(2\,bx+a+2\,\sqrt{bx^2+ax}\sqrt{b}\right)}{2\,b^{\frac{3}{2}}} + \frac{\sqrt{bx^2+ax}}{b}$$

```
integrate(x/(b*x^2+a*x)^(1/2),x, algorithm="maxima")
```

```
-1/2*a*log(2*b*x + a + 2*sqrt(b*x^2 + a*x)*sqrt(b))/b^(3/2) + sqrt(b*x^2 +
 a*x)/b
```

Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 50, normalized size of antiderivative = 1.06

$$\int \frac{x}{\sqrt{ax+bx^2}}\,dx = \frac{a\log\left(\left|2\left(\sqrt{b}x-\sqrt{bx^2+ax}\right)\sqrt{b}+a\right|\right)}{2\,b^{\frac{3}{2}}}+\frac{\sqrt{bx^2+ax}}{b}$$

```
integrate(x/(b*x^2+a*x)^(1/2),x, algorithm="giac")
```

```
1/2*a*log(abs(2*(sqrt(b)*x - sqrt(b*x^2 + a*x))*sqrt(b) + a))/b^(3/2) + sq
rt(b*x^2 + a*x)/b
```

Mupad [B] (verification not implemented)

Time = 0.10 (sec) , antiderivative size = 46, normalized size of antiderivative = 0.98

$$\int \frac{x}{\sqrt{ax+bx^2}}\,dx = \frac{\sqrt{b\,x^2+a\,x}}{b}-\frac{a\,\ln\left(\frac{\frac{a}{2}+b\,x}{\sqrt{b}}+\sqrt{b\,x^2+a\,x}\right)}{2\,b^{3/2}}$$

```
int(x/(a*x + b*x^2)^(1/2),x)
```

```
(a*x + b*x^2)^(1/2)/b - (a*log((a/2 + b*x)/b^(1/2) + (a*x + b*x^2)^(1/2)))
/(2*b^(3/2))
```

Reduce [B] (verification not implemented)

Time = 0.21 (sec) , antiderivative size = 38, normalized size of antiderivative = 0.81

$$\int \frac{x}{\sqrt{ax+bx^2}}\,dx = \frac{\sqrt{x}\,\sqrt{bx+a}\,b-\sqrt{b}\log\left(\frac{\sqrt{bx+a}+\sqrt{x}\,\sqrt{b}}{\sqrt{a}}\right)a}{b^2}$$

```
int(x/(b*x^2+a*x)^(1/2),x)
```

```
(sqrt(x)*sqrt(a + b*x)*b - sqrt(b)*log((sqrt(a + b*x) + sqrt(x)*sqrt(b))/s
qrt(a))*a)/b**2
```

3.42 $\int \frac{1}{\sqrt{ax+bx^2}}\,dx$

Optimal result

Integrand size = 13, antiderivative size = 28

$$\int \frac{1}{\sqrt{ax+bx^2}}\,dx = \frac{2\text{arctanh}\left(\frac{\sqrt{b}x}{\sqrt{ax+bx^2}}\right)}{\sqrt{b}}$$

```
2*arctanh(b^(1/2)*x/(b*x^2+a*x)^(1/2))/b^(1/2)
```

Mathematica [A] (verified)

Time = 0.04 (sec) , antiderivative size = 55, normalized size of antiderivative = 1.96

$$\int \frac{1}{\sqrt{ax+bx^2}}\,dx = -\frac{2\sqrt{x}\sqrt{a+bx}\log\left(-\sqrt{b}\sqrt{x}+\sqrt{a+bx}\right)}{\sqrt{b}\sqrt{x(a+bx)}}$$

```
Integrate[1/Sqrt[a*x + b*x^2],x]
```

```
(-2*Sqrt[x]*Sqrt[a + b*x]*Log[-(Sqrt[b]*Sqrt[x]) + Sqrt[a + b*x]])/(Sqrt[b
]*Sqrt[x*(a + b*x)])
```

Rubi [A] (verified)

Time = 0.24 (sec) , antiderivative size = 28, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.154, Rules used = {1091, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{\sqrt{ax+bx^2}}\,dx$$
$$\Big\downarrow \text{ 1091}$$
$$2\int \frac{1}{1-\frac{bx^2}{bx^2+ax}}d\frac{x}{\sqrt{bx^2+ax}}$$
$$\Big\downarrow \text{ 219}$$
$$\frac{2\text{arctanh}\left(\frac{\sqrt{b}x}{\sqrt{ax+bx^2}}\right)}{\sqrt{b}}$$

```
Int[1/Sqrt[a*x + b*x^2],x]
```

```
(2*ArcTanh[(Sqrt[b]*x)/Sqrt[a*x + b*x^2]])/Sqrt[b]
```

Defintions of rubi rules used

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

```
Int[1/Sqrt[(b_.)*(x_) + (c_.)*(x_)^2], x_Symbol] :> Simp[2   Subst[Int[1/(1
 - c*x^2), x], x, x/Sqrt[b*x + c*x^2]], x] /; FreeQ[{b, c}, x]
```

Maple [A] (verified)

Time = 0.34 (sec) , antiderivative size = 23, normalized size of antiderivative = 0.82

method	result	size
pseudoelliptic	$\frac{2 \operatorname{arctanh}\left(\frac{\sqrt{x(bx+a)}}{x\sqrt{b}}\right)}{\sqrt{b}}$	23
default	$\frac{\ln\left(\frac{\frac{a}{2}+bx}{\sqrt{b}}+\sqrt{b\,x^2+ax}\right)}{\sqrt{b}}$	29

```
int(1/(b*x^2+a*x)^(1/2),x,method=_RETURNVERBOSE)
```

```
2/b^(1/2)*arctanh((x*(b*x+a))^(1/2)/x/b^(1/2))
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 63, normalized size of antiderivative = 2.25

$$\int \frac{1}{\sqrt{ax+bx^2}}\,dx = \left[\frac{\log\left(2\,bx+a+2\,\sqrt{bx^2+ax}\sqrt{b}\right)}{\sqrt{b}}, -\frac{2\,\sqrt{-b}\arctan\left(\frac{\sqrt{bx^2+ax}\sqrt{-b}}{bx+a}\right)}{b}\right]$$

```
integrate(1/(b*x^2+a*x)^(1/2),x, algorithm="fricas")
```

```
[log(2*b*x + a + 2*sqrt(b*x^2 + a*x)*sqrt(b))/sqrt(b), -2*sqrt(-b)*arctan(
sqrt(b*x^2 + a*x)*sqrt(-b)/(b*x + a))/b]
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 75 vs. $2(26) = 52$.

Time = 0.29 (sec) , antiderivative size = 75, normalized size of antiderivative = 2.68

$$\int \frac{1}{\sqrt{ax+bx^2}}\,dx = \begin{cases} \frac{\log\left(a+2\sqrt{b}\sqrt{ax+bx^2}+2bx\right)}{\sqrt{b}} & \text{for } b\neq 0 \wedge \frac{a^2}{b}\neq 0 \\ \frac{\left(\frac{a}{2b}+x\right)\log\left(\frac{a}{2b}+x\right)}{\sqrt{b\left(\frac{a}{2b}+x\right)^2}} & \text{for } b\neq 0 \\ \frac{2\sqrt{ax}}{a} & \text{for } a\neq 0 \\ \tilde{\infty}x & \text{otherwise} \end{cases}$$

```
integrate(1/(b*x**2+a*x)**(1/2),x)
```

```
Piecewise((log(a + 2*sqrt(b)*sqrt(a*x + b*x**2) + 2*b*x)/sqrt(b), Ne(b, 0)
 & Ne(a**2/b, 0)), ((a/(2*b) + x)*log(a/(2*b) + x)/sqrt(b*(a/(2*b) + x)**2
), Ne(b, 0)), (2*sqrt(a*x)/a, Ne(a, 0)), (zoo*x, True))
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 27, normalized size of antiderivative = 0.96

$$\int \frac{1}{\sqrt{ax+bx^2}}\,dx = \frac{\log\left(2\,bx+a+2\,\sqrt{bx^2+ax}\sqrt{b}\right)}{\sqrt{b}}$$

```
integrate(1/(b*x^2+a*x)^(1/2),x, algorithm="maxima")
```

```
log(2*b*x + a + 2*sqrt(b*x^2 + a*x)*sqrt(b))/sqrt(b)
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 59 vs. $2(22) = 44$.

Time = 0.13 (sec) , antiderivative size = 59, normalized size of antiderivative = 2.11

$$\int \frac{1}{\sqrt{ax+bx^2}}\,dx = \frac{1}{4}\sqrt{bx^2+ax}\left(2\,x+\frac{a}{b}\right)+\frac{a^2\log\left(\left|2\left(\sqrt{b}x-\sqrt{bx^2+ax}\right)\sqrt{b}+a\right|\right)}{8\,b^{\frac{3}{2}}}$$

```
integrate(1/(b*x^2+a*x)^(1/2),x, algorithm="giac")
```

```
1/4*sqrt(b*x^2 + a*x)*(2*x + a/b) + 1/8*a^2*log(abs(2*(sqrt(b)*x - sqrt(b*
x^2 + a*x))*sqrt(b) + a))/b^(3/2)
```

Mupad [B] (verification not implemented)

Time = 9.34 (sec) , antiderivative size = 28, normalized size of antiderivative = 1.00

$$\int \frac{1}{\sqrt{ax+bx^2}}\,dx = \frac{\ln\left(\frac{\frac{a}{2}+b\,x}{\sqrt{b}}+\sqrt{b\,x^2+a\,x}\right)}{\sqrt{b}}$$

```
int(1/(a*x + b*x^2)^(1/2),x)
```

```
log((a/2 + b*x)/b^(1/2) + (a*x + b*x^2)^(1/2))/b^(1/2)
```

Reduce [B] (verification not implemented)

Time = 0.22 (sec) , antiderivative size = 25, normalized size of antiderivative = 0.89

$$\int \frac{1}{\sqrt{ax+bx^2}}\,dx = \frac{2\sqrt{b}\log\left(\frac{\sqrt{bx+a}+\sqrt{x}\,\sqrt{b}}{\sqrt{a}}\right)}{b}$$

```
int(1/(b*x^2+a*x)^(1/2),x)
```

```
(2*sqrt(b)*log((sqrt(a + b*x) + sqrt(x)*sqrt(b))/sqrt(a)))/b
```

3.43 $\int \frac{1}{x\sqrt{ax+bx^2}}\,dx$

Optimal result

Integrand size = 17, antiderivative size = 21

$$\int \frac{1}{x\sqrt{ax+bx^2}}\,dx = -\frac{2\sqrt{ax+bx^2}}{ax}$$

```
-2*(b*x^2+a*x)^(1/2)/a/x
```

Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 21, normalized size of antiderivative = 1.00

$$\int \frac{1}{x\sqrt{ax+bx^2}}\,dx = -\frac{2(a+bx)}{a\sqrt{x(a+bx)}}$$

```
Integrate[1/(x*Sqrt[a*x + b*x^2]),x]
```

```
(-2*(a + b*x))/(a*Sqrt[x*(a + b*x)])
```

Rubi [A] (verified)

Time = 0.23 (sec) , antiderivative size = 21, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}} = 0.059$, Rules used = $\{1123\}$

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{x\sqrt{ax+bx^2}} \, dx$$
$$\Big\downarrow \; 1123$$
$$-\frac{2\sqrt{ax+bx^2}}{ax}$$

```
Int[1/(x*Sqrt[a*x + b*x^2]),x]
```

```
(-2*Sqrt[a*x + b*x^2])/(a*x)
```

Defintions of rubi rules used

```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] :> Simp[e*(d + e*x)^m*((a + b*x + c*x^2)^(p + 1)/((p + 1)*(2*c*d - b
*e))), x] /; FreeQ[{a, b, c, d, e, m, p}, x] && EqQ[c*d^2 - b*d*e + a*e^2,
0] && EqQ[m + 2*p + 2, 0]
```

Maple [A] (verified)

Time = 0.34 (sec) , antiderivative size = 18, normalized size of antiderivative = 0.86

method	result	size
pseudoelliptic	$-\frac{2\sqrt{x(bx+a)}}{ax}$	18
default	$-\frac{2\sqrt{b\,x^2+ax}}{ax}$	20
trager	$-\frac{2\sqrt{b\,x^2+ax}}{ax}$	20
risch	$-\frac{2(bx+a)}{a\sqrt{x(bx+a)}}$	20
gosper	$-\frac{2(bx+a)}{a\sqrt{b\,x^2+ax}}$	22
orering	$-\frac{2(bx+a)}{a\sqrt{b\,x^2+ax}}$	22

```
int(1/x/(b*x^2+a*x)^(1/2),x,method=_RETURNVERBOSE)
```

```
-2*(x*(b*x+a))^(1/2)/a/x
```

Fricas [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 19, normalized size of antiderivative = 0.90

$$\int \frac{1}{x\sqrt{ax+bx^2}}\,dx = -\frac{2\sqrt{bx^2+ax}}{ax}$$

```
integrate(1/x/(b*x^2+a*x)^(1/2),x, algorithm="fricas")
```

```
-2*sqrt(b*x^2 + a*x)/(a*x)
```

Sympy [F]

$$\int \frac{1}{x\sqrt{ax+bx^2}}\,dx = \int \frac{1}{x\sqrt{x\,(a+bx)}}\,dx$$

```
integrate(1/x/(b*x**2+a*x)**(1/2),x)
```

```
Integral(1/(x*sqrt(x*(a + b*x))), x)
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 19, normalized size of antiderivative = 0.90

$$\int \frac{1}{x\sqrt{ax+bx^2}}\,dx = -\frac{2\sqrt{bx^2+ax}}{ax}$$

```
integrate(1/x/(b*x^2+a*x)^(1/2),x, algorithm="maxima")
```

```
-2*sqrt(b*x^2 + a*x)/(a*x)
```

Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 23, normalized size of antiderivative = 1.10

$$\int \frac{1}{x\sqrt{ax+bx^2}}\,dx = \frac{2}{\sqrt{b}x-\sqrt{bx^2+ax}}$$

```
integrate(1/x/(b*x^2+a*x)^(1/2),x, algorithm="giac")
```

```
2/(sqrt(b)*x - sqrt(b*x^2 + a*x))
```

Mupad [B] (verification not implemented)

Time = 9.27 (sec) , antiderivative size = 19, normalized size of antiderivative = 0.90

$$\int \frac{1}{x\sqrt{ax+bx^2}}\,dx = -\frac{2\sqrt{b\,x^2+a\,x}}{a\,x}$$

```
int(1/(x*(a*x + b*x^2)^(1/2)),x)
```

```
-(2*(a*x + b*x^2)^(1/2))/(a*x)
```

Reduce [B] (verification not implemented)

Time = 0.21 (sec) , antiderivative size = 23, normalized size of antiderivative = 1.10

$$\int \frac{1}{x\sqrt{ax+bx^2}}\,dx = \frac{-2\sqrt{x}\,\sqrt{bx+a}-2\sqrt{b}\,x}{ax}$$

```
int(1/x/(b*x^2+a*x)^(1/2),x)
```

```
( - 2*(sqrt(x)*sqrt(a + b*x) + sqrt(b)*x))/(a*x)
```

3.44 $\int \frac{1}{x^2\sqrt{ax+bx^2}}\,dx$

Optimal result

Integrand size = 17, antiderivative size = 48

$$\int \frac{1}{x^2\sqrt{ax+bx^2}}\,dx = -\frac{2\sqrt{ax+bx^2}}{3ax^2}+\frac{4b\sqrt{ax+bx^2}}{3a^2x}$$

```
-2/3*(b*x^2+a*x)^(1/2)/a/x^2+4/3*b*(b*x^2+a*x)^(1/2)/a^2/x
```

Mathematica [A] (verified)

Time = 0.06 (sec) , antiderivative size = 29, normalized size of antiderivative = 0.60

$$\int \frac{1}{x^2\sqrt{ax+bx^2}}\,dx = \frac{2\sqrt{x(a+bx)}(-a+2bx)}{3a^2x^2}$$

```
Integrate[1/(x^2*Sqrt[a*x + b*x^2]),x]
```

```
(2*Sqrt[x*(a + b*x)]*(-a + 2*b*x))/(3*a^2*x^2)
```

Rubi [A] (verified)

Time = 0.27 (sec) , antiderivative size = 48, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.118, Rules used = {1129, 1123}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{x^2\sqrt{ax+bx^2}}dx$$
$$\Big\downarrow \text{1129}$$
$$-\frac{2b\int \frac{1}{x\sqrt{bx^2+ax}}dx}{3a}-\frac{2\sqrt{ax+bx^2}}{3ax^2}$$
$$\Big\downarrow \text{1123}$$
$$\frac{4b\sqrt{ax+bx^2}}{3a^2x}-\frac{2\sqrt{ax+bx^2}}{3ax^2}$$

```
Int[1/(x^2*Sqrt[a*x + b*x^2]),x]
```

```
(-2*Sqrt[a*x + b*x^2])/(3*a*x^2) + (4*b*Sqrt[a*x + b*x^2])/(3*a^2*x)
```

Defintions of rubi rules used

```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] :> Simp[e*(d + e*x)^m*((a + b*x + c*x^2)^(p + 1)/((p + 1)*(2*c*d - b
*e))), x] /; FreeQ[{a, b, c, d, e, m, p}, x] && EqQ[c*d^2 - b*d*e + a*e^2,
0] && EqQ[m + 2*p + 2, 0]
```

```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] :> Simp[(-e)*(d + e*x)^m*((a + b*x + c*x^2)^(p + 1)/((m + p + 1)*(2*
c*d - b*e))), x] + Simp[c*(Simplify[m + 2*p + 2]/((m + p + 1)*(2*c*d - b*e)
))   Int[(d + e*x)^(m + 1)*(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d
, e, m, p}, x] && EqQ[c*d^2 - b*d*e + a*e^2, 0] && ILtQ[Simplify[m + 2*p +
2], 0]
```

Maple [A] (verified)

Time = 0.34 (sec) , antiderivative size = 24, normalized size of antiderivative = 0.50

method	result	size
pseudoelliptic	$-\frac{2(-2bx+a)\sqrt{x(bx+a)}}{3a^2x^2}$	24
trager	$-\frac{2(-2bx+a)\sqrt{b\,x^2+ax}}{3a^2x^2}$	26
risch	$-\frac{2(bx+a)(-2bx+a)}{3a^2x\sqrt{x(bx+a)}}$	29
gosper	$-\frac{2(bx+a)(-2bx+a)}{3x\,a^2\sqrt{b\,x^2+ax}}$	31
orering	$-\frac{2(bx+a)(-2bx+a)}{3x\,a^2\sqrt{b\,x^2+ax}}$	31
default	$-\frac{2\sqrt{b\,x^2+ax}}{3a\,x^2}+\frac{4b\sqrt{b\,x^2+ax}}{3a^2x}$	41

```
int(1/x^2/(b*x^2+a*x)^(1/2),x,method=_RETURNVERBOSE)
```

```
-2/3*(-2*b*x+a)/a^2/x^2*(x*(b*x+a))^(1/2)
```

Fricas [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 27, normalized size of antiderivative = 0.56

$$\int \frac{1}{x^2\sqrt{ax+bx^2}}\,dx = \frac{2\sqrt{bx^2+ax}(2\,bx-a)}{3\,a^2x^2}$$

```
integrate(1/x^2/(b*x^2+a*x)^(1/2),x, algorithm="fricas")
```

```
2/3*sqrt(b*x^2 + a*x)*(2*b*x - a)/(a^2*x^2)
```

Sympy [F]

$$\int \frac{1}{x^2\sqrt{ax+bx^2}}\,dx = \int \frac{1}{x^2\sqrt{x\,(a+bx)}}\,dx$$

```
integrate(1/x**2/(b*x**2+a*x)**(1/2),x)
```

```
Integral(1/(x**2*sqrt(x*(a + b*x))), x)
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 40, normalized size of antiderivative = 0.83

$$\int \frac{1}{x^2\sqrt{ax+bx^2}}\,dx = \frac{4\,\sqrt{bx^2+ax}b}{3\,a^2x} - \frac{2\,\sqrt{bx^2+ax}}{3\,ax^2}$$

```
integrate(1/x^2/(b*x^2+a*x)^(1/2),x, algorithm="maxima")
```

```
4/3*sqrt(b*x^2 + a*x)*b/(a^2*x) - 2/3*sqrt(b*x^2 + a*x)/(a*x^2)
```

Giac [A] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 49, normalized size of antiderivative = 1.02

$$\int \frac{1}{x^2\sqrt{ax+bx^2}}\,dx = \frac{2\left(3\left(\sqrt{b}x-\sqrt{bx^2+ax}\right)\sqrt{b}+a\right)}{3\left(\sqrt{b}x-\sqrt{bx^2+ax}\right)^3}$$

```
integrate(1/x^2/(b*x^2+a*x)^(1/2),x, algorithm="giac")
```

```
2/3*(3*(sqrt(b)*x - sqrt(b*x^2 + a*x))*sqrt(b) + a)/(sqrt(b)*x - sqrt(b*x^
2 + a*x))^3
```

Mupad [B] (verification not implemented)

Time = 9.17 (sec) , antiderivative size = 25, normalized size of antiderivative = 0.52

$$\int \frac{1}{x^2\sqrt{ax+bx^2}}\,dx = -\frac{2\,\sqrt{b\,x^2+a\,x}\,(a-2\,b\,x)}{3\,a^2\,x^2}$$

```
int(1/(x^2*(a*x + b*x^2)^(1/2)),x)
```

```
-(2*(a*x + b*x^2)^(1/2)*(a - 2*b*x))/(3*a^2*x^2)
```

Reduce [B] (verification not implemented)

Time = 0.22 (sec) , antiderivative size = 40, normalized size of antiderivative = 0.83

$$\int \frac{1}{x^2\sqrt{ax+bx^2}}\,dx = \frac{-\frac{2\sqrt{x}\,\sqrt{bx+a}\,a}{3}+\frac{4\sqrt{x}\,\sqrt{bx+a}\,bx}{3}-\frac{4\sqrt{b}\,b\,x^2}{3}}{a^2x^2}$$

```
int(1/x^2/(b*x^2+a*x)^(1/2),x)
```

```
(2*( - sqrt(x)*sqrt(a + b*x)*a + 2*sqrt(x)*sqrt(a + b*x)*b*x - 2*sqrt(b)*b
*x**2))/(3*a**2*x**2)
```

3.45 $\int \frac{1}{x^3\sqrt{ax+bx^2}}\,dx$

Optimal result

Integrand size = 17, antiderivative size = 74

$$\int \frac{1}{x^3\sqrt{ax+bx^2}}\,dx = -\frac{2\sqrt{ax+bx^2}}{5ax^3}+\frac{8b\sqrt{ax+bx^2}}{15a^2x^2}-\frac{16b^2\sqrt{ax+bx^2}}{15a^3x}$$

```
-2/5*(b*x^2+a*x)^(1/2)/a/x^3+8/15*b*(b*x^2+a*x)^(1/2)/a^2/x^2-16/15*b^2*(b
*x^2+a*x)^(1/2)/a^3/x
```

Mathematica [A] (verified)

Time = 0.09 (sec) , antiderivative size = 40, normalized size of antiderivative = 0.54

$$\int \frac{1}{x^3\sqrt{ax+bx^2}}\,dx = -\frac{2\sqrt{x(a+bx)}(3a^2-4abx+8b^2x^2)}{15a^3x^3}$$

```
Integrate[1/(x^3*Sqrt[a*x + b*x^2]),x]
```

```
(-2*Sqrt[x*(a + b*x)]*(3*a^2 - 4*a*b*x + 8*b^2*x^2))/(15*a^3*x^3)
```

Rubi [A] (verified)

Time = 0.33 (sec) , antiderivative size = 80, normalized size of antiderivative = 1.08, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.176, Rules used = {1129, 1129, 1123}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{x^3\sqrt{ax+bx^2}}dx$$

$$\Big\downarrow \; 1129$$

$$-\frac{4b\int \frac{1}{x^2\sqrt{bx^2+ax}}dx}{5a}-\frac{2\sqrt{ax+bx^2}}{5ax^3}$$

$$\Big\downarrow \; 1129$$

$$-\frac{4b\left(-\frac{2b\int \frac{1}{x\sqrt{bx^2+ax}}dx}{3a}-\frac{2\sqrt{ax+bx^2}}{3ax^2}\right)}{5a}-\frac{2\sqrt{ax+bx^2}}{5ax^3}$$

$$\Big\downarrow \; 1123$$

$$-\frac{4b\left(\frac{4b\sqrt{ax+bx^2}}{3a^2x}-\frac{2\sqrt{ax+bx^2}}{3ax^2}\right)}{5a}-\frac{2\sqrt{ax+bx^2}}{5ax^3}$$

```
Int[1/(x^3*Sqrt[a*x + b*x^2]),x]
```

```
(-2*Sqrt[a*x + b*x^2])/(5*a*x^3) - (4*b*((-2*Sqrt[a*x + b*x^2])/(3*a*x^2)
+ (4*b*Sqrt[a*x + b*x^2])/(3*a^2*x)))/(5*a)
```

Defintions of rubi rules used

```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] :> Simp[e*(d + e*x)^m*((a + b*x + c*x^2)^(p + 1)/((p + 1)*(2*c*d - b
*e))), x] /; FreeQ[{a, b, c, d, e, m, p}, x] && EqQ[c*d^2 - b*d*e + a*e^2,
0] && EqQ[m + 2*p + 2, 0]
```

```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] :> Simp[(-e)*(d + e*x)^m*((a + b*x + c*x^2)^(p + 1)/((m + p + 1)*(2*
c*d - b*e))), x] + Simp[c*(Simplify[m + 2*p + 2]/((m + p + 1)*(2*c*d - b*e)
))   Int[(d + e*x)^(m + 1)*(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d
, e, m, p}, x] && EqQ[c*d^2 - b*d*e + a*e^2, 0] && ILtQ[Simplify[m + 2*p +
2], 0]
```

Maple [A] (verified)

Time = 0.36 (sec) , antiderivative size = 37, normalized size of antiderivative = 0.50

method	result	size
pseudoelliptic	$-\frac{2\left(8b^{2}x^{2}-4abx+3a^{2}\right)\sqrt{x\left(bx+a\right)}}{15a^{3}x^{3}}$	37
trager	$-\frac{2\left(8b^{2}x^{2}-4abx+3a^{2}\right)\sqrt{b\,x^{2}+ax}}{15a^{3}x^{3}}$	39
risch	$-\frac{2\left(bx+a\right)\left(8b^{2}x^{2}-4abx+3a^{2}\right)}{15a^{3}x^{2}\sqrt{x\left(bx+a\right)}}$	42
gosper	$-\frac{2\left(bx+a\right)\left(8b^{2}x^{2}-4abx+3a^{2}\right)}{15x^{2}a^{3}\sqrt{b\,x^{2}+ax}}$	44
orering	$-\frac{2\left(bx+a\right)\left(8b^{2}x^{2}-4abx+3a^{2}\right)}{15x^{2}a^{3}\sqrt{b\,x^{2}+ax}}$	44
default	$-\frac{2\sqrt{b\,x^{2}+ax}}{5a\,x^{3}}-\frac{4b\left(-\frac{2\sqrt{b\,x^{2}+ax}}{3a\,x^{2}}+\frac{4b\sqrt{b\,x^{2}+ax}}{3a^{2}x}\right)}{5a}$	67

```
int(1/x^3/(b*x^2+a*x)^(1/2),x,method=_RETURNVERBOSE)
```

```
-2/15*(8*b^2*x^2-4*a*b*x+3*a^2)/a^3/x^3*(x*(b*x+a))^(1/2)
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 38, normalized size of antiderivative = 0.51

$$\int \frac{1}{x^3\sqrt{ax+bx^2}}\,dx = -\frac{2\,(8\,b^2x^2-4\,abx+3\,a^2)\sqrt{bx^2+ax}}{15\,a^3x^3}$$

```
integrate(1/x^3/(b*x^2+a*x)^(1/2),x, algorithm="fricas")
```

```
-2/15*(8*b^2*x^2 - 4*a*b*x + 3*a^2)*sqrt(b*x^2 + a*x)/(a^3*x^3)
```

Sympy [F]

$$\int \frac{1}{x^3\sqrt{ax+bx^2}}\,dx = \int \frac{1}{x^3\sqrt{x\,(a+bx)}}\,dx$$

```
integrate(1/x**3/(b*x**2+a*x)**(1/2),x)
```

```
Integral(1/(x**3*sqrt(x*(a + b*x))), x)
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 62, normalized size of antiderivative = 0.84

$$\int \frac{1}{x^3\sqrt{ax+bx^2}}\,dx = -\frac{16\,\sqrt{bx^2+ax}b^2}{15\,a^3x} + \frac{8\,\sqrt{bx^2+ax}b}{15\,a^2x^2} - \frac{2\,\sqrt{bx^2+ax}}{5\,ax^3}$$

```
integrate(1/x^3/(b*x^2+a*x)^(1/2),x, algorithm="maxima")
```

```
-16/15*sqrt(b*x^2 + a*x)*b^2/(a^3*x) + 8/15*sqrt(b*x^2 + a*x)*b/(a^2*x^2)
- 2/5*sqrt(b*x^2 + a*x)/(a*x^3)
```

Giac [A] (verification not implemented)

Time = 0.39 (sec) , antiderivative size = 78, normalized size of antiderivative = 1.05

$$\int \frac{1}{x^3\sqrt{ax+bx^2}}\,dx$$
$$= \frac{2\left(20\left(\sqrt{b}x-\sqrt{bx^2+ax}\right)^2b+15\left(\sqrt{b}x-\sqrt{bx^2+ax}\right)a\sqrt{b}+3\,a^2\right)}{15\left(\sqrt{b}x-\sqrt{bx^2+ax}\right)^5}$$

```
integrate(1/x^3/(b*x^2+a*x)^(1/2),x, algorithm="giac")
```

```
2/15*(20*(sqrt(b)*x - sqrt(b*x^2 + a*x))^2*b + 15*(sqrt(b)*x - sqrt(b*x^2
+ a*x))*a*sqrt(b) + 3*a^2)/(sqrt(b)*x - sqrt(b*x^2 + a*x))^5
```

Mupad [B] (verification not implemented)

Time = 8.88 (sec) , antiderivative size = 38, normalized size of antiderivative = 0.51

$$\int \frac{1}{x^3\sqrt{ax+bx^2}}\,dx = -\frac{2\sqrt{b\,x^2+a\,x}\,(3\,a^2-4\,a\,b\,x+8\,b^2\,x^2)}{15\,a^3\,x^3}$$

```
int(1/(x^3*(a*x + b*x^2)^(1/2)),x)
```

```
-(2*(a*x + b*x^2)^(1/2)*(3*a^2 + 8*b^2*x^2 - 4*a*b*x))/(15*a^3*x^3)
```

Reduce [B] (verification not implemented)

Time = 0.21 (sec) , antiderivative size = 61, normalized size of antiderivative = 0.82

$$\int \frac{1}{x^3\sqrt{ax+bx^2}}\,dx = \frac{-\frac{2\sqrt{x}\,\sqrt{bx+a}\,a^2}{5}+\frac{8\sqrt{x}\,\sqrt{bx+a}\,abx}{15}-\frac{16\sqrt{x}\,\sqrt{bx+a}\,b^2x^2}{15}+\frac{16\sqrt{b}\,b^2x^3}{15}}{a^3x^3}$$

```
int(1/x^3/(b*x^2+a*x)^(1/2),x)
```

```
(2*( - 3*sqrt(x)*sqrt(a + b*x)*a**2 + 4*sqrt(x)*sqrt(a + b*x)*a*b*x - 8*sq
rt(x)*sqrt(a + b*x)*b**2*x**2 + 8*sqrt(b)*b**2*x**3))/(15*a**3*x**3)
```

3.46 $\int \frac{1}{x^4\sqrt{ax+bx^2}}\,dx$

Optimal result

Integrand size = 17, antiderivative size = 100

$$\int \frac{1}{x^4\sqrt{ax+bx^2}}\,dx = -\frac{2\sqrt{ax+bx^2}}{7ax^4}+\frac{12b\sqrt{ax+bx^2}}{35a^2x^3}-\frac{16b^2\sqrt{ax+bx^2}}{35a^3x^2}+\frac{32b^3\sqrt{ax+bx^2}}{35a^4x}$$

```
-2/7*(b*x^2+a*x)^(1/2)/a/x^4+12/35*b*(b*x^2+a*x)^(1/2)/a^2/x^3-16/35*b^2*(
b*x^2+a*x)^(1/2)/a^3/x^2+32/35*b^3*(b*x^2+a*x)^(1/2)/a^4/x
```

Mathematica [A] (verified)

Time = 0.09 (sec) , antiderivative size = 51, normalized size of antiderivative = 0.51

$$\int \frac{1}{x^4\sqrt{ax+bx^2}}\,dx = \frac{2\sqrt{x(a+bx)}(-5a^3+6a^2bx-8ab^2x^2+16b^3x^3)}{35a^4x^4}$$

```
Integrate[1/(x^4*Sqrt[a*x + b*x^2]),x]
```

```
(2*Sqrt[x*(a + b*x)]*(-5*a^3 + 6*a^2*b*x - 8*a*b^2*x^2 + 16*b^3*x^3))/(35*
a^4*x^4)
```

Rubi [A] (verified)

Time = 0.37 (sec) , antiderivative size = 112, normalized size of antiderivative = 1.12, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.235, Rules used = {1129, 1129, 1129, 1123}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{x^4\sqrt{ax+bx^2}}dx$$
$$\Big\downarrow \text{1129}$$
$$-\frac{6b\int \frac{1}{x^3\sqrt{bx^2+ax}}dx}{7a}-\frac{2\sqrt{ax+bx^2}}{7ax^4}$$
$$\Big\downarrow \text{1129}$$
$$-\frac{6b\left(-\frac{4b\int \frac{1}{x^2\sqrt{bx^2+ax}}dx}{5a}-\frac{2\sqrt{ax+bx^2}}{5ax^3}\right)}{7a}-\frac{2\sqrt{ax+bx^2}}{7ax^4}$$
$$\Big\downarrow \text{1129}$$
$$-\frac{6b\left(-\frac{4b\left(-\frac{2b\int \frac{1}{x\sqrt{bx^2+ax}}dx}{3a}-\frac{2\sqrt{ax+bx^2}}{3ax^2}\right)}{5a}-\frac{2\sqrt{ax+bx^2}}{5ax^3}\right)}{7a}-\frac{2\sqrt{ax+bx^2}}{7ax^4}$$
$$\Big\downarrow \text{1123}$$
$$-\frac{6b\left(-\frac{4b\left(\frac{4b\sqrt{ax+bx^2}}{3a^2x}-\frac{2\sqrt{ax+bx^2}}{3ax^2}\right)}{5a}-\frac{2\sqrt{ax+bx^2}}{5ax^3}\right)}{7a}-\frac{2\sqrt{ax+bx^2}}{7ax^4}$$

```
Int[1/(x^4*Sqrt[a*x + b*x^2]),x]
```

```
(-2*Sqrt[a*x + b*x^2])/(7*a*x^4) - (6*b*((-2*Sqrt[a*x + b*x^2])/(5*a*x^3)
- (4*b*((-2*Sqrt[a*x + b*x^2])/(3*a*x^2) + (4*b*Sqrt[a*x + b*x^2])/(3*a^2*
x)))/(5*a)))/(7*a)
```

Defintions of rubi rules used

```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] :> Simp[e*(d + e*x)^m*((a + b*x + c*x^2)^(p + 1)/((p + 1)*(2*c*d - b
*e))), x] /; FreeQ[{a, b, c, d, e, m, p}, x] && EqQ[c*d^2 - b*d*e + a*e^2,
0] && EqQ[m + 2*p + 2, 0]
```

```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] :> Simp[(-e)*(d + e*x)^m*((a + b*x + c*x^2)^(p + 1)/((m + p + 1)*(2*
c*d - b*e))), x] + Simp[c*(Simplify[m + 2*p + 2]/((m + p + 1)*(2*c*d - b*e)
))   Int[(d + e*x)^(m + 1)*(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d
, e, m, p}, x] && EqQ[c*d^2 - b*d*e + a*e^2, 0] && ILtQ[Simplify[m + 2*p +
2], 0]
```

Maple [A] (verified)

Time = 0.37 (sec) , antiderivative size = 48, normalized size of antiderivative = 0.48

method	result	size
pseudoelliptic	$-\frac{2\left(-16 b^{3} x^{3}+8 a \,b^{2} x^{2}-6 a^{2} b x +5 a^{3}\right) \sqrt{x \left(b x +a \right)}}{35 a^{4} x^{4}}$	48
trager	$-\frac{2\left(-16 b^{3} x^{3}+8 a \,b^{2} x^{2}-6 a^{2} b x +5 a^{3}\right) \sqrt{b \,x^{2}+a x}}{35 a^{4} x^{4}}$	50
risch	$-\frac{2 \left(b x +a \right) \left(-16 b^{3} x^{3}+8 a \,b^{2} x^{2}-6 a^{2} b x +5 a^{3}\right)}{35 a^{4} x^{3} \sqrt{x \left(b x +a \right)}}$	53
gosper	$-\frac{2 \left(b x +a \right) \left(-16 b^{3} x^{3}+8 a \,b^{2} x^{2}-6 a^{2} b x +5 a^{3}\right)}{35 x^{3} a^{4} \sqrt{b \,x^{2}+a x}}$	55
orering	$-\frac{2 \left(b x +a \right) \left(-16 b^{3} x^{3}+8 a \,b^{2} x^{2}-6 a^{2} b x +5 a^{3}\right)}{35 x^{3} a^{4} \sqrt{b \,x^{2}+a x}}$	55
default	$-\frac{2 \sqrt{b \,x^{2}+a x}}{7 a \,x^{4}}-\frac{6 b \left(-\frac{2 \sqrt{b \,x^{2}+a x}}{5 a \,x^{3}}-\frac{4 b \left(-\frac{2 \sqrt{b \,x^{2}+a x}}{3 a \,x^{2}}+\frac{4 b \sqrt{b \,x^{2}+a x}}{3 a^{2} x}\right)}{5 a}\right)}{7 a}$	93

```
int(1/x^4/(b*x^2+a*x)^(1/2),x,method=_RETURNVERBOSE)
```

```
-2/35*(-16*b^3*x^3+8*a*b^2*x^2-6*a^2*b*x+5*a^3)/a^4/x^4*(x*(b*x+a))^(1/2)
```

Fricas [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 49, normalized size of antiderivative = 0.49

$$\int \frac{1}{x^4\sqrt{ax+bx^2}}\,dx = \frac{2\,(16\,b^3x^3 - 8\,ab^2x^2 + 6\,a^2bx - 5\,a^3)\sqrt{bx^2+ax}}{35\,a^4x^4}$$

```
integrate(1/x^4/(b*x^2+a*x)^(1/2),x, algorithm="fricas")
```

```
2/35*(16*b^3*x^3 - 8*a*b^2*x^2 + 6*a^2*b*x - 5*a^3)*sqrt(b*x^2 + a*x)/(a^4
*x^4)
```

Sympy [F]

$$\int \frac{1}{x^4\sqrt{ax+bx^2}}\,dx = \int \frac{1}{x^4\sqrt{x\,(a+bx)}}\,dx$$

```
integrate(1/x**4/(b*x**2+a*x)**(1/2),x)
```

```
Integral(1/(x**4*sqrt(x*(a + b*x))), x)
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 84, normalized size of antiderivative = 0.84

$$\int \frac{1}{x^4\sqrt{ax+bx^2}}\,dx = \frac{32\,\sqrt{bx^2+ax}b^3}{35\,a^4x} - \frac{16\,\sqrt{bx^2+ax}b^2}{35\,a^3x^2} + \frac{12\,\sqrt{bx^2+ax}b}{35\,a^2x^3} - \frac{2\,\sqrt{bx^2+ax}}{7\,ax^4}$$

```
integrate(1/x^4/(b*x^2+a*x)^(1/2),x, algorithm="maxima")
```

```
32/35*sqrt(b*x^2 + a*x)*b^3/(a^4*x) - 16/35*sqrt(b*x^2 + a*x)*b^2/(a^3*x^2
) + 12/35*sqrt(b*x^2 + a*x)*b/(a^2*x^3) - 2/7*sqrt(b*x^2 + a*x)/(a*x^4)
```

Giac [A] (verification not implemented)

Time = 0.14 (sec) , antiderivative size = 107, normalized size of antiderivative = 1.07

$$\int \frac{1}{x^4\sqrt{ax+bx^2}}\,dx = \frac{2\left(70\left(\sqrt{b}x-\sqrt{bx^2+ax}\right)^3 b^{\frac{3}{2}}+84\left(\sqrt{b}x-\sqrt{bx^2+ax}\right)^2 ab+35\left(\sqrt{b}x-\sqrt{bx^2+ax}\right)a^2\sqrt{b}+5\,a^3\right)}{35\left(\sqrt{b}x-\sqrt{bx^2+ax}\right)^7}$$

```
integrate(1/x^4/(b*x^2+a*x)^(1/2),x, algorithm="giac")
```

```
2/35*(70*(sqrt(b)*x - sqrt(b*x^2 + a*x))^3*b^(3/2) + 84*(sqrt(b)*x - sqrt(
b*x^2 + a*x))^2*a*b + 35*(sqrt(b)*x - sqrt(b*x^2 + a*x))*a^2*sqrt(b) + 5*a
^3)/(sqrt(b)*x - sqrt(b*x^2 + a*x))^7
```

Mupad [B] (verification not implemented)

Time = 8.98 (sec) , antiderivative size = 84, normalized size of antiderivative = 0.84

$$\int \frac{1}{x^4\sqrt{ax+bx^2}}\,dx = \frac{32\,b^3\,\sqrt{b\,x^2+a\,x}}{35\,a^4\,x}-\frac{16\,b^2\,\sqrt{b\,x^2+a\,x}}{35\,a^3\,x^2} -\frac{2\,\sqrt{b\,x^2+a\,x}}{7\,a\,x^4}+\frac{12\,b\,\sqrt{b\,x^2+a\,x}}{35\,a^2\,x^3}$$

```
int(1/(x^4*(a*x + b*x^2)^(1/2)),x)
```

```
(32*b^3*(a*x + b*x^2)^(1/2))/(35*a^4*x) - (16*b^2*(a*x + b*x^2)^(1/2))/(35
*a^3*x^2) - (2*(a*x + b*x^2)^(1/2))/(7*a*x^4) + (12*b*(a*x + b*x^2)^(1/2))
/(35*a^2*x^3)
```

Reduce [B] (verification not implemented)

Time = 0.23 (sec) , antiderivative size = 80, normalized size of antiderivative = 0.80

$$\int \frac{1}{x^4\sqrt{ax+bx^2}}\,dx = \frac{-\frac{2\sqrt{x}\,\sqrt{bx+a}\,a^3}{7}+\frac{12\sqrt{x}\,\sqrt{bx+a}\,a^2bx}{35}-\frac{16\sqrt{x}\,\sqrt{bx+a}\,a\,b^2x^2}{35}+\frac{32\sqrt{x}\,\sqrt{bx+a}\,b^3x^3}{35}-\frac{32\sqrt{b}\,b^3x^4}{35}}{a^4x^4}$$

```
int(1/x^4/(b*x^2+a*x)^(1/2),x)
```

```
(2*( - 5*sqrt(x)*sqrt(a + b*x)*a**3 + 6*sqrt(x)*sqrt(a + b*x)*a**2*b*x - 8
*sqrt(x)*sqrt(a + b*x)*a*b**2*x**2 + 16*sqrt(x)*sqrt(a + b*x)*b**3*x**3 -
16*sqrt(b)*b**3*x**4))/(35*a**4*x**4)
```

3.47 $\int \frac{x^5}{(ax+bx^2)^{3/2}}\,dx$

Optimal result

Integrand size = 17, antiderivative size = 124

$$\int \frac{x^5}{(ax+bx^2)^{3/2}}\,dx = \frac{2a^3x}{b^4\sqrt{ax+bx^2}}+\frac{19a^2\sqrt{ax+bx^2}}{8b^4}-\frac{11ax\sqrt{ax+bx^2}}{12b^3}+\frac{x^2\sqrt{ax+bx^2}}{3b^2}-\frac{35a^3\text{arctanh}\left(\frac{\sqrt{b}x}{\sqrt{ax+bx^2}}\right)}{8b^{9/2}}$$

```
2*a^3*x/b^4/(b*x^2+a*x)^(1/2)+19/8*a^2*(b*x^2+a*x)^(1/2)/b^4-11/12*a*x*(b*
x^2+a*x)^(1/2)/b^3+1/3*x^2*(b*x^2+a*x)^(1/2)/b^2-35/8*a^3*arctanh(b^(1/2)*
x/(b*x^2+a*x)^(1/2))/b^(9/2)
```

Mathematica [A] (verified)

Time = 0.41 (sec) , antiderivative size = 108, normalized size of antiderivative = 0.87

$$\int \frac{x^5}{(ax+bx^2)^{3/2}}\,dx = \frac{\sqrt{b}x(105a^3+35a^2bx-14ab^2x^2+8b^3x^3)+210a^3\sqrt{x}\sqrt{a+bx}\text{arctanh}\left(\frac{\sqrt{b}\sqrt{x}}{\sqrt{a}-\sqrt{a+bx}}\right)}{24b^{9/2}\sqrt{x(a+bx)}}$$

```
Integrate[x^5/(a*x + b*x^2)^(3/2),x]
```

```
(Sqrt[b]*x*(105*a^3 + 35*a^2*b*x - 14*a*b^2*x^2 + 8*b^3*x^3) + 210*a^3*Sqr
t[x]*Sqrt[a + b*x]*ArcTanh[(Sqrt[b]*Sqrt[x])/(Sqrt[a] - Sqrt[a + b*x])])/(
24*b^(9/2)*Sqrt[x*(a + b*x)])
```

Rubi [A] (verified)

Time = 0.62 (sec) , antiderivative size = 135, normalized size of antiderivative = 1.09, number of steps used = 10, number of rules used = 9, $\frac{\text{number of rules}}{\text{integrand size}} = 0.529$, Rules used = {1124, 25, 2192, 27, 2192, 27, 1160, 1091, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^5}{(ax+bx^2)^{3/2}}\,dx$$

$$\Big\downarrow \text{1124}$$

$$\frac{\int -\frac{a^3-bxa^2+b^2x^2a-b^3x^3}{\sqrt{bx^2+ax}}dx}{b^4}+\frac{2a^3x}{b^4\sqrt{ax+bx^2}}$$

$$\Big\downarrow \text{25}$$

$$\frac{2a^3x}{b^4\sqrt{ax+bx^2}}-\frac{\int \frac{a^3-bxa^2+b^2x^2a-b^3x^3}{\sqrt{bx^2+ax}}dx}{b^4}$$

$$\Big\downarrow \text{2192}$$

$$\frac{2a^3x}{b^4\sqrt{ax+bx^2}}-\frac{\frac{\int \frac{6ba^3-6b^2xa^2+11b^3x^2a}{2\sqrt{bx^2+ax}}dx}{3b}-\frac{1}{3}b^2x^2\sqrt{ax+bx^2}}{b^4}$$

$$\Big\downarrow \text{27}$$

$$\frac{2a^3x}{b^4\sqrt{ax+bx^2}}-\frac{\frac{\int \frac{6ba^3-6b^2xa^2+11b^3x^2a}{\sqrt{bx^2+ax}}dx}{6b}-\frac{1}{3}b^2x^2\sqrt{ax+bx^2}}{b^4}$$

$$\Big\downarrow \text{2192}$$

$$\frac{2a^3x}{b^4\sqrt{ax+bx^2}}-\frac{\frac{\frac{\int \frac{3a^2b^2(8a-19bx)}{2\sqrt{bx^2+ax}}dx}{2b}+\frac{11}{2}ab^2x\sqrt{ax+bx^2}}{6b}-\frac{1}{3}b^2x^2\sqrt{ax+bx^2}}{b^4}$$

$$\Big\downarrow \text{27}$$

$$\frac{2a^3x}{b^4\sqrt{ax+bx^2}}-\frac{\frac{\frac{3}{4}a^2b\int\frac{8a-19bx}{\sqrt{bx^2+ax}}dx+\frac{11}{2}ab^2x\sqrt{ax+bx^2}}{6b}-\frac{1}{3}b^2x^2\sqrt{ax+bx^2}}{b^4}$$

$$\Big\downarrow \text{1160}$$

$$\frac{2a^3x}{b^4\sqrt{ax+bx^2}}-\frac{\frac{\frac{3}{4}a^2b\left(\frac{35}{2}a\int\frac{1}{\sqrt{bx^2+ax}}dx-19\sqrt{ax+bx^2}\right)+\frac{11}{2}ab^2x\sqrt{ax+bx^2}}{6b}-\frac{1}{3}b^2x^2\sqrt{ax+bx^2}}{b^4}$$

$$\Big\downarrow \text{1091}$$

$$\frac{2a^3x}{b^4\sqrt{ax+bx^2}}-\frac{\frac{\frac{3}{4}a^2b\left(35a\int\frac{1}{1-\frac{bx^2}{bx^2+ax}}d\frac{x}{\sqrt{bx^2+ax}}-19\sqrt{ax+bx^2}\right)+\frac{11}{2}ab^2x\sqrt{ax+bx^2}}{6b}-\frac{1}{3}b^2x^2\sqrt{ax+bx^2}}{b^4}$$

$$\Big\downarrow \text{219}$$

$$\frac{2a^3x}{b^4\sqrt{ax+bx^2}}-\frac{\frac{\frac{3}{4}a^2b\left(\frac{35a\,\text{arctanh}\left(\frac{\sqrt{b}x}{\sqrt{ax+bx^2}}\right)}{\sqrt{b}}-19\sqrt{ax+bx^2}\right)+\frac{11}{2}ab^2x\sqrt{ax+bx^2}}{6b}-\frac{1}{3}b^2x^2\sqrt{ax+bx^2}}{b^4}$$

```
Int[x^5/(a*x + b*x^2)^(3/2),x]
```

```
(2*a^3*x)/(b^4*Sqrt[a*x + b*x^2]) - (-1/3*(b^2*x^2*Sqrt[a*x + b*x^2]) + ((
11*a*b^2*x*Sqrt[a*x + b*x^2])/2 + (3*a^2*b*(-19*Sqrt[a*x + b*x^2] + (35*a*
ArcTanh[(Sqrt[b]*x)/Sqrt[a*x + b*x^2]])/Sqrt[b]))/4)/(6*b))/b^4
```

Defintions of rubi rules used

```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

```
Int[1/Sqrt[(b_.)*(x_) + (c_.)*(x_)^2], x_Symbol] :> Simp[2   Subst[Int[1/(1
 - c*x^2), x], x, x/Sqrt[b*x + c*x^2]], x] /; FreeQ[{b, c}, x]
```

```
Int[((d_.) + (e_.)*(x_))^(m_.)/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(3/2), x
_Symbol] :> Simp[-2*e*(2*c*d - b*e)^(m - 2)*((d + e*x)/(c^(m - 1)*Sqrt[a +
b*x + c*x^2])), x] + Simp[e^2/c^(m - 1)   Int[(1/Sqrt[a + b*x + c*x^2])*Exp
andToSum[((2*c*d - b*e)^(m - 1) - c^(m - 1)*(d + e*x)^(m - 1))/(c*d - b*e -
 c*e*x), x], x], x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[c*d^2 - b*d*e + a*e
^2, 0] && IGtQ[m, 0]
```

```
Int[((d_.) + (e_.)*(x_))*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol
] :> Simp[e*((a + b*x + c*x^2)^(p + 1)/(2*c*(p + 1))), x] + Simp[(2*c*d - b
*e)/(2*c)   Int[(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, p}, x]
 && NeQ[p, -1]
```

```
Int[(Pq_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> With[{q =
Expon[Pq, x], e = Coeff[Pq, x, Expon[Pq, x]]}, Simp[e*x^(q - 1)*((a + b*x +
 c*x^2)^(p + 1)/(c*(q + 2*p + 1))), x] + Simp[1/(c*(q + 2*p + 1))   Int[(a
+ b*x + c*x^2)^p*ExpandToSum[c*(q + 2*p + 1)*Pq - a*e*(q - 1)*x^(q - 2) - b
*e*(q + p)*x^(q - 1) - c*e*(q + 2*p + 1)*x^q, x], x], x]] /; FreeQ[{a, b, c
, p}, x] && PolyQ[Pq, x] && NeQ[b^2 - 4*a*c, 0] &&  !LeQ[p, -1]
```

Maple [A] (verified)

Time = 0.40 (sec) , antiderivative size = 84, normalized size of antiderivative = 0.68

method	result
pseudoelliptic	$\frac{8b^{\frac{7}{2}}x^4-14b^{\frac{5}{2}}a\,x^3+35b^{\frac{3}{2}}a^2x^2-105\sqrt{x(bx+a)}\,\operatorname{arctanh}\left(\frac{\sqrt{x(bx+a)}}{x\sqrt{b}}\right)a^3+105x\,a^3\sqrt{b}}{24b^{\frac{9}{2}}\sqrt{x(bx+a)}}$
risch	$\frac{(8b^2x^2-22abx+57a^2)x(bx+a)}{24b^4\sqrt{x(bx+a)}}-\frac{35a^3\ln\left(\frac{\frac{a}{2}+bx}{\sqrt{b}}+\sqrt{b\,x^2+ax}\right)}{16b^{\frac{9}{2}}}+\frac{2a^3\sqrt{\left(x+\frac{a}{b}\right)^2b-a\left(x+\frac{a}{b}\right)}}{b^5\left(x+\frac{a}{b}\right)}$
default	$\frac{x^4}{3b\sqrt{b\,x^2+ax}}-\frac{7a\left(\frac{x^3}{2b\sqrt{b\,x^2+ax}}-\frac{5a\left(\frac{x^2}{b\sqrt{b\,x^2+ax}}-\frac{3a\left(-\frac{x}{b\sqrt{b\,x^2+ax}}-\frac{a\left(-\frac{1}{b\sqrt{b\,x^2+ax}}+\frac{2bx+a}{ab\sqrt{b\,x^2+ax}}\right)}{2b}+\frac{\ln\left(\frac{\frac{a}{2}+bx}{\sqrt{b}}+\sqrt{b\,x^2+a\right.}}{b^{\frac{3}{2}}}\right.}{2b}\right.}{4b}\right.}{6b}$

```
int(x^5/(b*x^2+a*x)^(3/2),x,method=_RETURNVERBOSE)
```

```
1/24*(8*b^(7/2)*x^4-14*b^(5/2)*a*x^3+35*b^(3/2)*a^2*x^2-105*(x*(b*x+a))^(1
/2)*arctanh((x*(b*x+a))^(1/2)/x/b^(1/2))*a^3+105*x*a^3*b^(1/2))/b^(9/2)/(x
*(b*x+a))^(1/2)
```

Fricas [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 204, normalized size of antiderivative = 1.65

$$\int \frac{x^5}{(ax+bx^2)^{3/2}}\,dx = \left[\frac{105\,(a^3bx+a^4)\sqrt{b}\log\left(2\,bx+a-2\,\sqrt{bx^2+ax}\sqrt{b}\right)+2\,(8\,b^4x^3-14\,ab^3x^2+35\,a^2}{48\,(b^6x+ab^5)}\right.$$

```
integrate(x^5/(b*x^2+a*x)^(3/2),x, algorithm="fricas")
```

```
[1/48*(105*(a^3*b*x + a^4)*sqrt(b)*log(2*b*x + a - 2*sqrt(b*x^2 + a*x)*sqr
t(b)) + 2*(8*b^4*x^3 - 14*a*b^3*x^2 + 35*a^2*b^2*x + 105*a^3*b)*sqrt(b*x^2
 + a*x))/(b^6*x + a*b^5), 1/24*(105*(a^3*b*x + a^4)*sqrt(-b)*arctan(sqrt(b
*x^2 + a*x)*sqrt(-b)/(b*x + a)) + (8*b^4*x^3 - 14*a*b^3*x^2 + 35*a^2*b^2*x
 + 105*a^3*b)*sqrt(b*x^2 + a*x))/(b^6*x + a*b^5)]
```

Sympy [F]

$$\int \frac{x^5}{(ax+bx^2)^{3/2}}\,dx = \int \frac{x^5}{(x\,(a+bx))^{\frac{3}{2}}}\,dx$$

```
integrate(x**5/(b*x**2+a*x)**(3/2),x)
```

```
Integral(x**5/(x*(a + b*x))**(3/2), x)
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 113, normalized size of antiderivative = 0.91

$$\int \frac{x^5}{(ax+bx^2)^{3/2}}\,dx = \frac{x^4}{3\,\sqrt{bx^2+ax}b} - \frac{7\,ax^3}{12\,\sqrt{bx^2+ax}b^2} + \frac{35\,a^2x^2}{24\,\sqrt{bx^2+ax}b^3} + \frac{35\,a^3x}{8\,\sqrt{bx^2+ax}b^4} - \frac{35\,a^3\log\left(2\,bx+a+2\,\sqrt{bx^2+ax}\sqrt{b}\right)}{16\,b^{\frac{9}{2}}}$$

```
integrate(x^5/(b*x^2+a*x)^(3/2),x, algorithm="maxima")
```

```
1/3*x^4/(sqrt(b*x^2 + a*x)*b) - 7/12*a*x^3/(sqrt(b*x^2 + a*x)*b^2) + 35/24
*a^2*x^2/(sqrt(b*x^2 + a*x)*b^3) + 35/8*a^3*x/(sqrt(b*x^2 + a*x)*b^4) - 35
/16*a^3*log(2*b*x + a + 2*sqrt(b*x^2 + a*x)*sqrt(b))/b^(9/2)
```

Giac [A] (verification not implemented)

Time = 0.32 (sec) , antiderivative size = 112, normalized size of antiderivative = 0.90

$$\int \frac{x^5}{\left(ax+bx^2\right)^{3/2}}\,dx = \frac{1}{24}\sqrt{bx^2+ax}\left(2\,x\left(\frac{4\,x}{b^2}-\frac{11\,a}{b^3}\right)+\frac{57\,a^2}{b^4}\right) + \frac{35\,a^3\log\left(\left|-2\left(\sqrt{b}x-\sqrt{bx^2+ax}\right)\sqrt{b}-a\right|\right)}{16\,b^{\frac{9}{2}}} + \frac{2\,a^4}{\left(\left(\sqrt{b}x-\sqrt{bx^2+ax}\right)\sqrt{b}+a\right)b^{\frac{9}{2}}}$$

```
integrate(x^5/(b*x^2+a*x)^(3/2),x, algorithm="giac")
```

```
1/24*sqrt(b*x^2 + a*x)*(2*x*(4*x/b^2 - 11*a/b^3) + 57*a^2/b^4) + 35/16*a^3
*log(abs(-2*(sqrt(b)*x - sqrt(b*x^2 + a*x))*sqrt(b) - a))/b^(9/2) + 2*a^4/
(((sqrt(b)*x - sqrt(b*x^2 + a*x))*sqrt(b) + a)*b^(9/2))
```

Mupad [F(-1)]

Timed out.

$$\int \frac{x^5}{\left(ax+bx^2\right)^{3/2}}\,dx = \int \frac{x^5}{\left(b\,x^2+a\,x\right)^{3/2}}\,dx$$

```
int(x^5/(a*x + b*x^2)^(3/2),x)
```

```
int(x^5/(a*x + b*x^2)^(3/2), x)
```

Reduce [B] (verification not implemented)

Time = 0.25 (sec) , antiderivative size = 98, normalized size of antiderivative = 0.79

$$\int \frac{x^5}{\left(ax+bx^2\right)^{3/2}}\,dx = \frac{-840\sqrt{b}\,\sqrt{bx+a}\,\log\left(\frac{\sqrt{bx+a}+\sqrt{x}\,\sqrt{b}}{\sqrt{a}}\right)a^3+525\sqrt{b}\,\sqrt{bx+a}\,a^3+840\sqrt{x}\,a^3b+280\sqrt{x}\,a}{192\sqrt{bx+a}\,b^5}$$

```
int(x^5/(b*x^2+a*x)^(3/2),x)
```

```
( - 840*sqrt(b)*sqrt(a + b*x)*log((sqrt(a + b*x) + sqrt(x)*sqrt(b))/sqrt(a
))*a**3 + 525*sqrt(b)*sqrt(a + b*x)*a**3 + 840*sqrt(x)*a**3*b + 280*sqrt(x
)*a**2*b**2*x - 112*sqrt(x)*a*b**3*x**2 + 64*sqrt(x)*b**4*x**3)/(192*sqrt(
a + b*x)*b**5)
```

3.48 $\int \frac{x^4}{\left(ax+bx^2\right)^{3/2}}\,dx$

Optimal result

Integrand size = 17, antiderivative size = 98

$$\int \frac{x^4}{(ax+bx^2)^{3/2}}\,dx = -\frac{2a^2x}{b^3\sqrt{ax+bx^2}} - \frac{7a\sqrt{ax+bx^2}}{4b^3} + \frac{x\sqrt{ax+bx^2}}{2b^2} + \frac{15a^2\operatorname{arctanh}\left(\frac{\sqrt{b}x}{\sqrt{ax+bx^2}}\right)}{4b^{7/2}}$$

```
-2*a^2*x/b^3/(b*x^2+a*x)^(1/2)-7/4*a*(b*x^2+a*x)^(1/2)/b^3+1/2*x*(b*x^2+a*
x)^(1/2)/b^2+15/4*a^2*arctanh(b^(1/2)*x/(b*x^2+a*x)^(1/2))/b^(7/2)
```

Mathematica [A] (verified)

Time = 0.26 (sec) , antiderivative size = 97, normalized size of antiderivative = 0.99

$$\int \frac{x^4}{(ax+bx^2)^{3/2}}\,dx = \frac{\sqrt{b}x(-15a^2-5abx+2b^2x^2)+30a^2\sqrt{x}\sqrt{a+bx}\operatorname{arctanh}\left(\frac{\sqrt{b}\sqrt{x}}{-\sqrt{a}+\sqrt{a+bx}}\right)}{4b^{7/2}\sqrt{x(a+bx)}}$$

```
Integrate[x^4/(a*x + b*x^2)^(3/2),x]
```

```
(Sqrt[b]*x*(-15*a^2 - 5*a*b*x + 2*b^2*x^2) + 30*a^2*Sqrt[x]*Sqrt[a + b*x]*
ArcTanh[(Sqrt[b]*Sqrt[x])/(-Sqrt[a] + Sqrt[a + b*x])])/(4*b^(7/2)*Sqrt[x*(
a + b*x)])
```

Rubi [A] (verified)

Time = 0.45 (sec) , antiderivative size = 97, normalized size of antiderivative = 0.99, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.353$, Rules used = {1124, 2192, 27, 1160, 1091, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^4}{(ax+bx^2)^{3/2}}dx$$

$$\Big\downarrow \text{1124}$$

$$\frac{\int \frac{a^2-bxa+b^2x^2}{\sqrt{bx^2+ax}}dx}{b^3}-\frac{2a^2x}{b^3\sqrt{ax+bx^2}}$$

$$\Big\downarrow \text{2192}$$

$$\frac{\frac{\int \frac{ab(4a-7bx)}{2\sqrt{bx^2+ax}}dx}{2b}+\frac{1}{2}bx\sqrt{ax+bx^2}}{b^3}-\frac{2a^2x}{b^3\sqrt{ax+bx^2}}$$

$$\Big\downarrow \text{27}$$

$$\frac{\frac{1}{4}a\int \frac{4a-7bx}{\sqrt{bx^2+ax}}dx+\frac{1}{2}bx\sqrt{ax+bx^2}}{b^3}-\frac{2a^2x}{b^3\sqrt{ax+bx^2}}$$

$$\Big\downarrow \text{1160}$$

$$\frac{\frac{1}{4}a\left(\frac{15}{2}a\int \frac{1}{\sqrt{bx^2+ax}}dx-7\sqrt{ax+bx^2}\right)+\frac{1}{2}bx\sqrt{ax+bx^2}}{b^3}-\frac{2a^2x}{b^3\sqrt{ax+bx^2}}$$

$$\Big\downarrow \text{1091}$$

$$\frac{\frac{1}{4}a\left(15a\int \frac{1}{1-\frac{bx^2}{bx^2+ax}}d\frac{x}{\sqrt{bx^2+ax}}-7\sqrt{ax+bx^2}\right)+\frac{1}{2}bx\sqrt{ax+bx^2}}{b^3}-\frac{2a^2x}{b^3\sqrt{ax+bx^2}}$$

$$\Big\downarrow \text{219}$$

$$\frac{\frac{1}{4}a\left(\frac{15a\operatorname{arctanh}\left(\frac{\sqrt{b}x}{\sqrt{ax+bx^2}}\right)}{\sqrt{b}}-7\sqrt{ax+bx^2}\right)+\frac{1}{2}bx\sqrt{ax+bx^2}}{b^3}-\frac{2a^2x}{b^3\sqrt{ax+bx^2}}$$

```
Int[x^4/(a*x + b*x^2)^(3/2),x]
```

```
(-2*a^2*x)/(b^3*Sqrt[a*x + b*x^2]) + ((b*x*Sqrt[a*x + b*x^2])/2 + (a*(-7*S
qrt[a*x + b*x^2] + (15*a*ArcTanh[(Sqrt[b]*x)/Sqrt[a*x + b*x^2]])/Sqrt[b]))
/4)/b^3
```

Defintions of rubi rules used

```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

```
Int[1/Sqrt[(b_.)*(x_) + (c_.)*(x_)^2], x_Symbol] :> Simp[2   Subst[Int[1/(1
 - c*x^2), x], x, x/Sqrt[b*x + c*x^2]], x] /; FreeQ[{b, c}, x]
```

```
Int[((d_.) + (e_.)*(x_))^(m_.)/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(3/2), x
_Symbol] :> Simp[-2*e*(2*c*d - b*e)^(m - 2)*((d + e*x)/(c^(m - 1)*Sqrt[a +
b*x + c*x^2])), x] + Simp[e^2/c^(m - 1)   Int[(1/Sqrt[a + b*x + c*x^2])*Exp
andToSum[((2*c*d - b*e)^(m - 1) - c^(m - 1)*(d + e*x)^(m - 1))/(c*d - b*e -
 c*e*x), x], x], x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[c*d^2 - b*d*e + a*e
^2, 0] && IGtQ[m, 0]
```

```
Int[((d_.) + (e_.)*(x_))*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol
] :> Simp[e*((a + b*x + c*x^2)^(p + 1)/(2*c*(p + 1))), x] + Simp[(2*c*d - b
*e)/(2*c)   Int[(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, p}, x]
 && NeQ[p, -1]
```

```
Int[(Pq_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> With[{q =
Expon[Pq, x], e = Coeff[Pq, x, Expon[Pq, x]]}, Simp[e*x^(q - 1)*((a + b*x +
 c*x^2)^(p + 1)/(c*(q + 2*p + 1))), x] + Simp[1/(c*(q + 2*p + 1))   Int[(a
+ b*x + c*x^2)^p*ExpandToSum[c*(q + 2*p + 1)*Pq - a*e*(q - 1)*x^(q - 2) - b
*e*(q + p)*x^(q - 1) - c*e*(q + 2*p + 1)*x^q, x], x], x]] /; FreeQ[{a, b, c
, p}, x] && PolyQ[Pq, x] && NeQ[b^2 - 4*a*c, 0] &&  !LeQ[p, -1]
```

Maple [A] (verified)

Time = 0.40 (sec) , antiderivative size = 73, normalized size of antiderivative = 0.74

method	result	size
pseudoelliptic	$\frac{2 b^{\frac{5}{2}} x^{3}-5 b^{\frac{3}{2}} a \,x^{2}+15 \sqrt{x \left(b x +a \right)}\, \operatorname{arctanh}\left(\frac{\sqrt{x \left(b x +a \right)}}{x \sqrt{b}}\right) a^{2}-15 a^{2} x \sqrt{b}}{4 b^{\frac{7}{2}} \sqrt{x \left(b x +a \right)}}$	73
risch	$-\frac{\left(-2 b x +7 a \right) x \left(b x +a \right)}{4 b^{3} \sqrt{x \left(b x +a \right)}}+\frac{15 a^{2} \ln \left(\frac{\frac{a}{2}+b x}{\sqrt{b}}+\sqrt{b \,x^{2}+a x}\right)}{8 b^{\frac{7}{2}}}-\frac{2 a^{2} \sqrt{\left(x +\frac{a}{b}\right)^{2} b -a \left(x +\frac{a}{b}\right)}}{b^{4} \left(x +\frac{a}{b}\right)}$	103
default	$\frac{x^{3}}{2 b \sqrt{b \,x^{2}+a x}}-\frac{5 a \left(\frac{x^{2}}{b \sqrt{b \,x^{2}+a x}}-\frac{3 a \left(-\frac{x}{b \sqrt{b \,x^{2}+a x}}-\frac{a \left(-\frac{1}{b \sqrt{b \,x^{2}+a x}}+\frac{2 b x +a}{a b \sqrt{b \,x^{2}+a x}}\right)}{2 b}+\frac{\ln \left(\frac{\frac{a}{2}+b x}{\sqrt{b}}+\sqrt{b \,x^{2}+a x}\right)}{b^{\frac{3}{2}}}\right)}{2 b}\right)}{4 b}$	145

```
int(x^4/(b*x^2+a*x)^(3/2),x,method=_RETURNVERBOSE)
```

```
1/4*(2*b^(5/2)*x^3-5*b^(3/2)*a*x^2+15*(x*(b*x+a))^(1/2)*arctanh((x*(b*x+a)
)^(1/2)/x/b^(1/2))*a^2-15*a^2*x*b^(1/2))/b^(7/2)/(x*(b*x+a))^(1/2)
```

Fricas [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 183, normalized size of antiderivative = 1.87

$$\int \frac{x^4}{(ax+bx^2)^{3/2}}\,dx = \left[\frac{15\,(a^2bx+a^3)\sqrt{b}\log\left(2\,bx+a+2\,\sqrt{bx^2+ax}\sqrt{b}\right)+2\,(2\,b^3x^2-5\,ab^2x-15\,a^2b)\surd}{8\,(b^5x+ab^4)}\right.$$
$$\left.-\frac{15\,(a^2bx+a^3)\sqrt{-b}\arctan\left(\frac{\sqrt{bx^2+ax}\sqrt{-b}}{bx+a}\right)-(2\,b^3x^2-5\,ab^2x-15\,a^2b)\sqrt{bx^2+ax}}{4\,(b^5x+ab^4)}\right]$$

```
integrate(x^4/(b*x^2+a*x)^(3/2),x, algorithm="fricas")
```

```
[1/8*(15*(a^2*b*x + a^3)*sqrt(b)*log(2*b*x + a + 2*sqrt(b*x^2 + a*x)*sqrt(
b)) + 2*(2*b^3*x^2 - 5*a*b^2*x - 15*a^2*b)*sqrt(b*x^2 + a*x))/(b^5*x + a*b
^4), -1/4*(15*(a^2*b*x + a^3)*sqrt(-b)*arctan(sqrt(b*x^2 + a*x)*sqrt(-b)/(
b*x + a)) - (2*b^3*x^2 - 5*a*b^2*x - 15*a^2*b)*sqrt(b*x^2 + a*x))/(b^5*x +
 a*b^4)]
```

Sympy [F]

$$\int \frac{x^4}{(ax+bx^2)^{3/2}}\,dx = \int \frac{x^4}{(x\,(a+bx))^{\frac{3}{2}}}\,dx$$

```
integrate(x**4/(b*x**2+a*x)**(3/2),x)
```

```
Integral(x**4/(x*(a + b*x))**(3/2), x)
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 91, normalized size of antiderivative = 0.93

$$\int \frac{x^4}{(ax+bx^2)^{3/2}}\,dx = \frac{x^3}{2\sqrt{bx^2+ax}b} - \frac{5\,ax^2}{4\sqrt{bx^2+ax}b^2} - \frac{15\,a^2x}{4\sqrt{bx^2+ax}b^3} + \frac{15\,a^2\log\left(2\,bx+a+2\sqrt{bx^2+ax}\sqrt{b}\right)}{8\,b^{\frac{7}{2}}}$$

```
integrate(x^4/(b*x^2+a*x)^(3/2),x, algorithm="maxima")
```

```
1/2*x^3/(sqrt(b*x^2 + a*x)*b) - 5/4*a*x^2/(sqrt(b*x^2 + a*x)*b^2) - 15/4*a
^2*x/(sqrt(b*x^2 + a*x)*b^3) + 15/8*a^2*log(2*b*x + a + 2*sqrt(b*x^2 + a*x
)*sqrt(b))/b^(7/2)
```

Giac [A] (verification not implemented)

Time = 0.32 (sec) , antiderivative size = 100, normalized size of antiderivative = 1.02

$$\int \frac{x^4}{(ax+bx^2)^{3/2}}\,dx = \frac{1}{4}\sqrt{bx^2+ax}\left(\frac{2\,x}{b^2} - \frac{7\,a}{b^3}\right) - \frac{15\,a^2\log\left(\left|-2\left(\sqrt{b}x-\sqrt{bx^2+ax}\right)\sqrt{b}-a\right|\right)}{8\,b^{\frac{7}{2}}} - \frac{2\,a^3}{\left(\left(\sqrt{b}x-\sqrt{bx^2+ax}\right)\sqrt{b}+a\right)b^{\frac{7}{2}}}$$

```
integrate(x^4/(b*x^2+a*x)^(3/2),x, algorithm="giac")
```

```
1/4*sqrt(b*x^2 + a*x)*(2*x/b^2 - 7*a/b^3) - 15/8*a^2*log(abs(-2*(sqrt(b)*x
 - sqrt(b*x^2 + a*x))*sqrt(b) - a))/b^(7/2) - 2*a^3/(((sqrt(b)*x - sqrt(b*
x^2 + a*x))*sqrt(b) + a)*b^(7/2))
```

Mupad [F(-1)]

Timed out.

$$\int \frac{x^4}{(ax+bx^2)^{3/2}}\,dx=\int \frac{x^4}{{\left(b\,x^2+a\,x\right)}^{3/2}} \,d x$$

```
int(x^4/(a*x + b*x^2)^(3/2),x)
```

```
int(x^4/(a*x + b*x^2)^(3/2), x)
```

Reduce [B] (verification not implemented)

Time = 0.23 (sec) , antiderivative size = 85, normalized size of antiderivative = 0.87

$$\int \frac{x^4}{(ax+bx^2)^{3/2}}\,dx=\frac{15\sqrt{b}\,\sqrt{bx+a}\,\log\left(\frac{\sqrt{bx+a}+\sqrt{x}\,\sqrt{b}}{\sqrt{a}}\right)a^2-10\sqrt{b}\,\sqrt{bx+a}\,a^2-15\sqrt{x}\,a^2b-5\sqrt{x}\,a\,b^2x+2}{4\sqrt{bx+a}\,b^4}$$

```
int(x^4/(b*x^2+a*x)^(3/2),x)
```

```
(15*sqrt(b)*sqrt(a + b*x)*log((sqrt(a + b*x) + sqrt(x)*sqrt(b))/sqrt(a))*a
**2 - 10*sqrt(b)*sqrt(a + b*x)*a**2 - 15*sqrt(x)*a**2*b - 5*sqrt(x)*a*b**2
*x + 2*sqrt(x)*b**3*x**2)/(4*sqrt(a + b*x)*b**4)
```

3.49 $\int \frac{x^3}{(ax+bx^2)^{3/2}}\,dx$

Optimal result

Integrand size = 17, antiderivative size = 67

$$\int \frac{x^3}{(ax+bx^2)^{3/2}}\,dx=\frac{2ax}{b^2\sqrt{ax+bx^2}}+\frac{\sqrt{ax+bx^2}}{b^2}-\frac{3a\,\text{arctanh}\left(\frac{\sqrt{b}x}{\sqrt{ax+bx^2}}\right)}{b^{5/2}}$$

```
2*a*x/b^2/(b*x^2+a*x)^(1/2)+(b*x^2+a*x)^(1/2)/b^2-3*a*arctanh(b^(1/2)*x/(b
*x^2+a*x)^(1/2))/b^(5/2)
```

Mathematica [A] (verified)

Time = 0.23 (sec) , antiderivative size = 80, normalized size of antiderivative = 1.19

$$\int \frac{x^3}{(ax+bx^2)^{3/2}}\,dx=\frac{\sqrt{b}x(3a+bx)+6a\sqrt{x}\sqrt{a+bx}\,\text{arctanh}\left(\frac{\sqrt{b}\sqrt{x}}{\sqrt{a}-\sqrt{a+bx}}\right)}{b^{5/2}\sqrt{x(a+bx)}}$$

```
Integrate[x^3/(a*x + b*x^2)^(3/2),x]
```

```
(Sqrt[b]*x*(3*a + b*x) + 6*a*Sqrt[x]*Sqrt[a + b*x]*ArcTanh[(Sqrt[b]*Sqrt[x
])/(Sqrt[a] - Sqrt[a + b*x])])/(b^(5/2)*Sqrt[x*(a + b*x)])
```

Rubi [A] (verified)

Time = 0.35 (sec) , antiderivative size = 71, normalized size of antiderivative = 1.06, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.294, Rules used = {1124, 25, 1160, 1091, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^3}{(ax+bx^2)^{3/2}}\,dx$$

$$\Big\downarrow \ 1124$$

$$\frac{\int -\frac{a-bx}{\sqrt{bx^2+ax}}dx}{b^2}+\frac{2ax}{b^2\sqrt{ax+bx^2}}$$

$$\Big\downarrow \ 25$$

$$\frac{2ax}{b^2\sqrt{ax+bx^2}}-\frac{\int \frac{a-bx}{\sqrt{bx^2+ax}}dx}{b^2}$$

$$\Big\downarrow \ 1160$$

$$\frac{2ax}{b^2\sqrt{ax+bx^2}}-\frac{\frac{3}{2}a\int \frac{1}{\sqrt{bx^2+ax}}dx-\sqrt{ax+bx^2}}{b^2}$$

$$\Big\downarrow \ 1091$$

$$\frac{2ax}{b^2\sqrt{ax+bx^2}}-\frac{3a\int \frac{1}{1-\frac{bx^2}{bx^2+ax}}d\frac{x}{\sqrt{bx^2+ax}}-\sqrt{ax+bx^2}}{b^2}$$

$$\Big\downarrow \ 219$$

$$\frac{2ax}{b^2\sqrt{ax+bx^2}}-\frac{\frac{3a\text{arctanh}\left(\frac{\sqrt{b}x}{\sqrt{ax+bx^2}}\right)}{\sqrt{b}}-\sqrt{ax+bx^2}}{b^2}$$

```
Int[x^3/(a*x + b*x^2)^(3/2),x]
```

```
(2*a*x)/(b^2*Sqrt[a*x + b*x^2]) - (-Sqrt[a*x + b*x^2] + (3*a*ArcTanh[(Sqrt
[b]*x)/Sqrt[a*x + b*x^2]])/Sqrt[b])/b^2
```

Defintions of rubi rules used

```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

```
Int[1/Sqrt[(b_.)*(x_) + (c_.)*(x_)^2], x_Symbol] :> Simp[2   Subst[Int[1/(1
 - c*x^2), x], x, x/Sqrt[b*x + c*x^2]], x] /; FreeQ[{b, c}, x]
```

```
Int[((d_.) + (e_.)*(x_))^(m_.)/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(3/2), x
_Symbol] :> Simp[-2*e*(2*c*d - b*e)^(m - 2)*((d + e*x)/(c^(m - 1)*Sqrt[a +
b*x + c*x^2])), x] + Simp[e^2/c^(m - 1)   Int[(1/Sqrt[a + b*x + c*x^2])*Exp
andToSum[((2*c*d - b*e)^(m - 1) - c^(m - 1)*(d + e*x)^(m - 1))/(c*d - b*e -
 c*e*x), x], x], x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[c*d^2 - b*d*e + a*e
^2, 0] && IGtQ[m, 0]
```

```
Int[((d_.) + (e_.)*(x_))*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol
] :> Simp[e*((a + b*x + c*x^2)^(p + 1)/(2*c*(p + 1))), x] + Simp[(2*c*d - b
*e)/(2*c)   Int[(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, p}, x]
 && NeQ[p, -1]
```

Maple [A] (verified)

Time = 0.38 (sec) , antiderivative size = 58, normalized size of antiderivative = 0.87

method	result	size
pseudoelliptic	$\frac{b^{\frac{3}{2}} x^{2}-3 \sqrt{x \left(b x +a \right)}\, \operatorname{arctanh}\left(\frac{\sqrt{x \left(b x +a \right)}}{x \sqrt{b}}\right) a +3 x a \sqrt{b}}{b^{\frac{5}{2}} \sqrt{x \left(b x +a \right)}}$	58
risch	$\frac{x \left(b x +a \right)}{b^{2} \sqrt{x \left(b x +a \right)}}-\frac{3 a \ln \left(\frac{\frac{a}{2}+b x}{\sqrt{b}}+\sqrt{b \,x^{2}+a x}\right)}{2 b^{\frac{5}{2}}}+\frac{2 a \sqrt{\left(x +\frac{a}{b}\right)^{2} b -a \left(x +\frac{a}{b}\right)}}{b^{3} \left(x +\frac{a}{b}\right)}$	90
default	$\frac{x^{2}}{b \sqrt{b \,x^{2}+a x}}-\frac{3 a \left(-\frac{x}{b \sqrt{b \,x^{2}+a x}}-\frac{a \left(-\frac{1}{b \sqrt{b \,x^{2}+a x}}+\frac{2 b x +a}{a b \sqrt{b \,x^{2}+a x}}\right)}{2 b}+\frac{\ln \left(\frac{\frac{a}{2}+b x}{\sqrt{b}}+\sqrt{b \,x^{2}+a x}\right)}{b^{\frac{3}{2}}}\right)}{2 b}$	119

```
int(x^3/(b*x^2+a*x)^(3/2),x,method=_RETURNVERBOSE)
```

```
(b^(3/2)*x^2-3*(x*(b*x+a))^(1/2)*arctanh((x*(b*x+a))^(1/2)/x/b^(1/2))*a+3*
x*a*b^(1/2))/b^(5/2)/(x*(b*x+a))^(1/2)
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 153, normalized size of antiderivative = 2.28

$$\int \frac{x^3}{\left(ax+bx^2\right)^{3/2}}\,dx = \left[\frac{3\left(abx+a^2\right)\sqrt{b}\log\left(2\,bx+a-2\,\sqrt{bx^2+ax}\sqrt{b}\right)+2\left(b^2x+3\,ab\right)\sqrt{bx^2+ax}}{2\left(b^4x+ab^3\right)}, \frac{3\,(ab$$

```
integrate(x^3/(b*x^2+a*x)^(3/2),x, algorithm="fricas")
```

```
[1/2*(3*(a*b*x + a^2)*sqrt(b)*log(2*b*x + a - 2*sqrt(b*x^2 + a*x)*sqrt(b))
 + 2*(b^2*x + 3*a*b)*sqrt(b*x^2 + a*x))/(b^4*x + a*b^3), (3*(a*b*x + a^2)*
sqrt(-b)*arctan(sqrt(b*x^2 + a*x)*sqrt(-b)/(b*x + a)) + (b^2*x + 3*a*b)*sq
rt(b*x^2 + a*x))/(b^4*x + a*b^3)]
```

Sympy [F]

$$\int \frac{x^3}{(ax+bx^2)^{3/2}}\,dx = \int \frac{x^3}{(x(a+bx))^{\frac{3}{2}}}\,dx$$

```
integrate(x**3/(b*x**2+a*x)**(3/2),x)
```

```
Integral(x**3/(x*(a + b*x))**(3/2), x)
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 66, normalized size of antiderivative = 0.99

$$\int \frac{x^3}{(ax+bx^2)^{3/2}}\,dx = \frac{x^2}{\sqrt{bx^2+ax}b} + \frac{3\,ax}{\sqrt{bx^2+ax}b^2} - \frac{3\,a\log\left(2\,bx+a+2\,\sqrt{bx^2+ax}\sqrt{b}\right)}{2\,b^{\frac{5}{2}}}$$

```
integrate(x^3/(b*x^2+a*x)^(3/2),x, algorithm="maxima")
```

```
x^2/(sqrt(b*x^2 + a*x)*b) + 3*a*x/(sqrt(b*x^2 + a*x)*b^2) - 3/2*a*log(2*b*
x + a + 2*sqrt(b*x^2 + a*x)*sqrt(b))/b^(5/2)
```

Giac [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 87, normalized size of antiderivative = 1.30

$$\int \frac{x^3}{(ax+bx^2)^{3/2}}\,dx = \frac{3\,a\log\left(\left|-2\left(\sqrt{b}x-\sqrt{bx^2+ax}\right)\sqrt{b}-a\right|\right)}{2\,b^{\frac{5}{2}}} + \frac{2\,a^2}{\left(\left(\sqrt{b}x-\sqrt{bx^2+ax}\right)\sqrt{b}+a\right)b^{\frac{5}{2}}} + \frac{\sqrt{bx^2+ax}}{b^2}$$

```
integrate(x^3/(b*x^2+a*x)^(3/2),x, algorithm="giac")
```

```
3/2*a*log(abs(-2*(sqrt(b)*x - sqrt(b*x^2 + a*x))*sqrt(b) - a))/b^(5/2) + 2
*a^2/(((sqrt(b)*x - sqrt(b*x^2 + a*x))*sqrt(b) + a)*b^(5/2)) + sqrt(b*x^2
+ a*x)/b^2
```

Mupad [F(-1)]

Timed out.

$$\int \frac{x^3}{\left(ax+bx^2\right)^{3/2}}\,dx=\int \frac{x^3}{{\left(b\,x^2+a\,x\right)}^{3/2}} \,d x$$

```
int(x^3/(a*x + b*x^2)^(3/2),x)
```

```
int(x^3/(a*x + b*x^2)^(3/2), x)
```

Reduce [B] (verification not implemented)

Time = 0.20 (sec) , antiderivative size = 68, normalized size of antiderivative = 1.01

$$\int \frac{x^3}{\left(ax+bx^2\right)^{3/2}}\,dx=\frac{-12\sqrt{b}\,\sqrt{bx+a}\,\log\left(\frac{\sqrt{bx+a}+\sqrt{x}\,\sqrt{b}}{\sqrt{a}}\right)a+9\sqrt{b}\,\sqrt{bx+a}\,a+12\sqrt{x}\,ab+4\sqrt{x}\,b^2x}{4\sqrt{bx+a}\,b^3}$$

```
int(x^3/(b*x^2+a*x)^(3/2),x)
```

```
( - 12*sqrt(b)*sqrt(a + b*x)*log((sqrt(a + b*x) + sqrt(x)*sqrt(b))/sqrt(a)
)*a + 9*sqrt(b)*sqrt(a + b*x)*a + 12*sqrt(x)*a*b + 4*sqrt(x)*b**2*x)/(4*sq
rt(a + b*x)*b**3)
```

3.50 $\int \frac{x^2}{(ax+bx^2)^{3/2}}\,dx$

Optimal result

Integrand size = 17, antiderivative size = 48

$$\int \frac{x^2}{(ax+bx^2)^{3/2}}\,dx = -\frac{2x}{b\sqrt{ax+bx^2}}+\frac{2\,\mathrm{arctanh}\left(\frac{\sqrt{b}x}{\sqrt{ax+bx^2}}\right)}{b^{3/2}}$$

```
-2*x/b/(b*x^2+a*x)^(1/2)+2*arctanh(b^(1/2)*x/(b*x^2+a*x)^(1/2))/b^(3/2)
```

Mathematica [A] (verified)

Time = 0.12 (sec) , antiderivative size = 73, normalized size of antiderivative = 1.52

$$\int \frac{x^2}{(ax+bx^2)^{3/2}}\,dx = \frac{-2\sqrt{b}x+4\sqrt{x}\sqrt{a+bx}\,\mathrm{arctanh}\left(\frac{\sqrt{b}\sqrt{x}}{-\sqrt{a}+\sqrt{a+bx}}\right)}{b^{3/2}\sqrt{x(a+bx)}}$$

```
Integrate[x^2/(a*x + b*x^2)^(3/2),x]
```

```
(-2*Sqrt[b]*x + 4*Sqrt[x]*Sqrt[a + b*x]*ArcTanh[(Sqrt[b]*Sqrt[x])/(-Sqrt[a
] + Sqrt[a + b*x])])/(b^(3/2)*Sqrt[x*(a + b*x)])
```

Rubi [A] (verified)

Time = 0.30 (sec) , antiderivative size = 48, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.176, Rules used = {1124, 1091, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^2}{(ax+bx^2)^{3/2}}\,dx$$

$$\Big\downarrow \ 1124$$

$$\frac{\int \frac{1}{\sqrt{bx^2+ax}}dx}{b}-\frac{2x}{b\sqrt{ax+bx^2}}$$

$$\Big\downarrow \ 1091$$

$$\frac{2\int \frac{1}{1-\frac{bx^2}{bx^2+ax}}d\frac{x}{\sqrt{bx^2+ax}}}{b}-\frac{2x}{b\sqrt{ax+bx^2}}$$

$$\Big\downarrow \ 219$$

$$\frac{2\text{arctanh}\left(\frac{\sqrt{b}x}{\sqrt{ax+bx^2}}\right)}{b^{3/2}}-\frac{2x}{b\sqrt{ax+bx^2}}$$

```
Int[x^2/(a*x + b*x^2)^(3/2),x]
```

```
(-2*x)/(b*Sqrt[a*x + b*x^2]) + (2*ArcTanh[(Sqrt[b]*x)/Sqrt[a*x + b*x^2]])/
b^(3/2)
```

Defintions of rubi rules used

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

```
Int[1/Sqrt[(b_.)*(x_) + (c_.)*(x_)^2], x_Symbol] :> Simp[2   Subst[Int[1/(1
 - c*x^2), x], x, x/Sqrt[b*x + c*x^2]], x] /; FreeQ[{b, c}, x]
```

```
Int[((d_.) + (e_.)*(x_))^(m_.)/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(3/2), x
_Symbol] :> Simp[-2*e*(2*c*d - b*e)^(m - 2)*((d + e*x)/(c^(m - 1)*Sqrt[a +
b*x + c*x^2])), x] + Simp[e^2/c^(m - 1)   Int[(1/Sqrt[a + b*x + c*x^2])*Exp
andToSum[((2*c*d - b*e)^(m - 1) - c^(m - 1)*(d + e*x)^(m - 1))/(c*d - b*e -
 c*e*x), x], x], x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[c*d^2 - b*d*e + a*e
^2, 0] && IGtQ[m, 0]
```

Maple [A] (verified)

Time = 0.37 (sec) , antiderivative size = 39, normalized size of antiderivative = 0.81

method	result	size
pseudoelliptic	$-\frac{2x}{b\sqrt{x(bx+a)}}+\frac{2\,\operatorname{arctanh}\left(\frac{\sqrt{x(bx+a)}}{x\sqrt{b}}\right)}{b^{\frac{3}{2}}}$	39
default	$-\frac{x}{b\sqrt{b\,x^2+ax}}-\frac{a\left(-\frac{1}{b\sqrt{b\,x^2+ax}}+\frac{2bx+a}{ab\sqrt{b\,x^2+ax}}\right)}{2b}+\frac{\ln\left(\frac{\frac{a}{2}+bx}{\sqrt{b}}+\sqrt{b\,x^2+ax}\right)}{b^{\frac{3}{2}}}$	94

```
int(x^2/(b*x^2+a*x)^(3/2),x,method=_RETURNVERBOSE)
```

```
-2*x/b/(x*(b*x+a))^(1/2)+2/b^(3/2)*arctanh((x*(b*x+a))^(1/2)/x/b^(1/2))
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 127, normalized size of antiderivative = 2.65

$$\int \frac{x^2}{(ax+bx^2)^{3/2}}\,dx = \left[\frac{(bx+a)\sqrt{b}\log\left(2\,bx+a+2\,\sqrt{bx^2+ax}\sqrt{b}\right)-2\,\sqrt{bx^2+ax}b}{b^3x+ab^2},\right.$$
$$\left.-\frac{2\left((bx+a)\sqrt{-b}\arctan\left(\frac{\sqrt{bx^2+ax}\sqrt{-b}}{bx+a}\right)+\sqrt{bx^2+ax}b\right)}{b^3x+ab^2}\right]$$

```
integrate(x^2/(b*x^2+a*x)^(3/2),x, algorithm="fricas")
```

```
[((b*x + a)*sqrt(b)*log(2*b*x + a + 2*sqrt(b*x^2 + a*x)*sqrt(b)) - 2*sqrt(
b*x^2 + a*x)*b)/(b^3*x + a*b^2), -2*((b*x + a)*sqrt(-b)*arctan(sqrt(b*x^2
+ a*x)*sqrt(-b)/(b*x + a)) + sqrt(b*x^2 + a*x)*b)/(b^3*x + a*b^2)]
```

Sympy [F]

$$\int \frac{x^2}{(ax+bx^2)^{3/2}}\,dx = \int \frac{x^2}{(x\,(a+bx))^{\frac{3}{2}}}\,dx$$

```
integrate(x**2/(b*x**2+a*x)**(3/2),x)
```

```
Integral(x**2/(x*(a + b*x))**(3/2), x)
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 45, normalized size of antiderivative = 0.94

$$\int \frac{x^2}{(ax+bx^2)^{3/2}}\,dx = -\frac{2\,x}{\sqrt{bx^2+ax}b} + \frac{\log\left(2\,bx + a + 2\,\sqrt{bx^2+ax}\sqrt{b}\right)}{b^{\frac{3}{2}}}$$

```
integrate(x^2/(b*x^2+a*x)^(3/2),x, algorithm="maxima")
```

```
-2*x/(sqrt(b*x^2 + a*x)*b) + log(2*b*x + a + 2*sqrt(b*x^2 + a*x)*sqrt(b))/
b^(3/2)
```

Giac [A] (verification not implemented)

Time = 0.18 (sec) , antiderivative size = 67, normalized size of antiderivative = 1.40

$$\int \frac{x^2}{(ax+bx^2)^{3/2}}\,dx = -\frac{\log\left(\left|2\left(\sqrt{b}x - \sqrt{bx^2+ax}\right)\sqrt{b} + a\right|\right)}{b^{\frac{3}{2}}} - \frac{2\,a}{\left(\left(\sqrt{b}x - \sqrt{bx^2+ax}\right)\sqrt{b} + a\right)b^{\frac{3}{2}}}$$

```
integrate(x^2/(b*x^2+a*x)^(3/2),x, algorithm="giac")
```

```
-log(abs(2*(sqrt(b)*x - sqrt(b*x^2 + a*x))*sqrt(b) + a))/b^(3/2) - 2*a/(((
sqrt(b)*x - sqrt(b*x^2 + a*x))*sqrt(b) + a)*b^(3/2))
```

Mupad [B] (verification not implemented)

Time = 0.10 (sec) , antiderivative size = 46, normalized size of antiderivative = 0.96

$$\int \frac{x^2}{(ax+bx^2)^{3/2}}\,dx = \frac{\ln\left(\frac{\frac{a}{2}+b\,x}{\sqrt{b}}+\sqrt{b\,x^2+a\,x}\right)}{b^{3/2}} - \frac{2\,x}{b\,\sqrt{b\,x^2+a\,x}}$$

```
int(x^2/(a*x + b*x^2)^(3/2),x)
```

```
log((a/2 + b*x)/b^(1/2) + (a*x + b*x^2)^(1/2))/b^(3/2) - (2*x)/(b*(a*x + b
*x^2)^(1/2))
```

Reduce [B] (verification not implemented)

Time = 0.19 (sec) , antiderivative size = 56, normalized size of antiderivative = 1.17

$$\int \frac{x^2}{(ax+bx^2)^{3/2}}\,dx = \frac{2\sqrt{b}\,\sqrt{bx+a}\log\left(\frac{\sqrt{bx+a}+\sqrt{x}\,\sqrt{b}}{\sqrt{a}}\right) - 2\sqrt{b}\,\sqrt{bx+a} - 2\sqrt{x}\,b}{\sqrt{bx+a}\,b^2}$$

```
int(x^2/(b*x^2+a*x)^(3/2),x)
```

```
(2*(sqrt(b)*sqrt(a + b*x)*log((sqrt(a + b*x) + sqrt(x)*sqrt(b))/sqrt(a)) -
 sqrt(b)*sqrt(a + b*x) - sqrt(x)*b))/(sqrt(a + b*x)*b**2)
```

3.51 $\int \frac{x}{\left(ax+bx^2\right)^{3/2}}\,dx$

Optimal result

Integrand size = 15, antiderivative size = 19

$$\int \frac{x}{\left(ax+bx^2\right)^{3/2}}\,dx=\frac{2x}{a\sqrt{ax+bx^2}}$$

```
2*x/a/(b*x^2+a*x)^(1/2)
```

Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 17, normalized size of antiderivative = 0.89

$$\int \frac{x}{\left(ax+bx^2\right)^{3/2}}\,dx=\frac{2x}{a\sqrt{x(a+bx)}}$$

```
Integrate[x/(a*x + b*x^2)^(3/2),x]
```

```
(2*x)/(a*Sqrt[x*(a + b*x)])
```

Rubi [A] (verified)

Time = 0.24 (sec) , antiderivative size = 19, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.133$, Rules used = {1124, 24}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x}{(ax+bx^2)^{3/2}}dx$$
$$\Big\downarrow \text{1124}$$
$$\int 0dx+\frac{2x}{a\sqrt{ax+bx^2}}$$
$$\Big\downarrow \text{24}$$
$$\frac{2x}{a\sqrt{ax+bx^2}}$$

```
Int[x/(a*x + b*x^2)^(3/2),x]
```

```
(2*x)/(a*Sqrt[a*x + b*x^2])
```

Defintions of rubi rules used

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

```
Int[((d_.) + (e_.)*(x_))^(m_.)/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(3/2), x
_Symbol] :> Simp[-2*e*(2*c*d - b*e)^(m - 2)*((d + e*x)/(c^(m - 1)*Sqrt[a +
b*x + c*x^2])), x] + Simp[e^2/c^(m - 1)   Int[(1/Sqrt[a + b*x + c*x^2])*Exp
andToSum[((2*c*d - b*e)^(m - 1) - c^(m - 1)*(d + e*x)^(m - 1))/(c*d - b*e -
 c*e*x), x], x], x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[c*d^2 - b*d*e + a*e
^2, 0] && IGtQ[m, 0]
```

Maple [A] (verified)

Time = 0.34 (sec) , antiderivative size = 16, normalized size of antiderivative = 0.84

method	result	size
pseudoelliptic	$\frac{2x}{a\sqrt{x(bx+a)}}$	16
trager	$\frac{2\sqrt{b\,x^{2}+ax}}{(bx+a)a}$	24
gosper	$\frac{2x^{2}(bx+a)}{a(b\,x^{2}+ax)^{\frac{3}{2}}}$	25
orering	$\frac{2x^{2}(bx+a)}{a(b\,x^{2}+ax)^{\frac{3}{2}}}$	25
default	$-\frac{1}{b\sqrt{b\,x^{2}+ax}}+\frac{2bx+a}{ab\sqrt{b\,x^{2}+ax}}$	42

```
int(x/(b*x^2+a*x)^(3/2),x,method=_RETURNVERBOSE)
```

```
2*x/a/(x*(b*x+a))^(1/2)
```

Fricas [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 23, normalized size of antiderivative = 1.21

$$\int \frac{x}{(ax+bx^2)^{3/2}}\,dx = \frac{2\,\sqrt{bx^2+ax}}{abx+a^2}$$

```
integrate(x/(b*x^2+a*x)^(3/2),x, algorithm="fricas")
```

```
2*sqrt(b*x^2 + a*x)/(a*b*x + a^2)
```

Sympy [F]

$$\int \frac{x}{(ax + bx^2)^{3/2}}\,dx = \int \frac{x}{(x(a + bx))^{\frac{3}{2}}}\,dx$$

```
integrate(x/(b*x**2+a*x)**(3/2),x)
```

```
Integral(x/(x*(a + b*x))**(3/2), x)
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 17, normalized size of antiderivative = 0.89

$$\int \frac{x}{(ax + bx^2)^{3/2}}\,dx = \frac{2\,x}{\sqrt{bx^2 + ax}a}$$

```
integrate(x/(b*x^2+a*x)^(3/2),x, algorithm="maxima")
```

```
2*x/(sqrt(b*x^2 + a*x)*a)
```

Giac [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 32, normalized size of antiderivative = 1.68

$$\int \frac{x}{(ax + bx^2)^{3/2}}\,dx = \frac{2}{\left(\left(\sqrt{b}x - \sqrt{bx^2 + ax}\right)\sqrt{b} + a\right)\sqrt{b}}$$

```
integrate(x/(b*x^2+a*x)^(3/2),x, algorithm="giac")
```

```
2/(((sqrt(b)*x - sqrt(b*x^2 + a*x))*sqrt(b) + a)*sqrt(b))
```

Mupad [B] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 15, normalized size of antiderivative = 0.79

$$\int \frac{x}{\left(ax+bx^2\right)^{3/2}}\,dx=\frac{2\,x}{a\,\sqrt{x\,\left(a+b\,x\right)}}$$

```
int(x/(a*x + b*x^2)^(3/2),x)
```

```
(2*x)/(a*(x*(a + b*x))^(1/2))
```

Reduce [B] (verification not implemented)

Time = 0.23 (sec) , antiderivative size = 30, normalized size of antiderivative = 1.58

$$\int \frac{x}{\left(ax+bx^2\right)^{3/2}}\,dx=\frac{2\sqrt{b}\,\sqrt{bx+a}+2\sqrt{x}\,b}{\sqrt{bx+a}\,ab}$$

```
int(x/(b*x^2+a*x)^(3/2),x)
```

```
(2*(sqrt(b)*sqrt(a + b*x) + sqrt(x)*b))/(sqrt(a + b*x)*a*b)
```

3.52 $\int \frac{1}{(ax+bx^2)^{3/2}} \, dx$

Optimal result

Integrand size = 13, antiderivative size = 40

$$\int \frac{1}{(ax+bx^2)^{3/2}} \, dx = \frac{2}{a\sqrt{ax+bx^2}} - \frac{4\sqrt{ax+bx^2}}{a^2x}$$

```
2/a/(b*x^2+a*x)^(1/2)-4*(b*x^2+a*x)^(1/2)/a^2/x
```

Mathematica [A] (verified)

Time = 0.07 (sec) , antiderivative size = 22, normalized size of antiderivative = 0.55

$$\int \frac{1}{(ax+bx^2)^{3/2}} \, dx = -\frac{2(a+2bx)}{a^2\sqrt{x(a+bx)}}$$

```
Integrate[(a*x + b*x^2)^(-3/2),x]
```

```
(-2*(a + 2*b*x))/(a^2*Sqrt[x*(a + b*x)])
```

Rubi [A] (verified)

Time = 0.23 (sec) , antiderivative size = 24, normalized size of antiderivative = 0.60, number of steps used = 1, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}} = 0.077$, Rules used = $\{1088\}$

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{(ax+bx^2)^{3/2}}\,dx$$
$$\Big\downarrow \text{1088}$$
$$-\frac{2(a+2bx)}{a^2\sqrt{ax+bx^2}}$$

```
Int[(a*x + b*x^2)^(-3/2),x]
```

```
(-2*(a + 2*b*x))/(a^2*Sqrt[a*x + b*x^2])
```

Defintions of rubi rules used

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-3/2), x_Symbol] :> Simp[-2*((b +
2*c*x)/((b^2 - 4*a*c)*Sqrt[a + b*x + c*x^2])), x] /; FreeQ[{a, b, c}, x] &&
 NeQ[b^2 - 4*a*c, 0]
```

Maple [A] (verified)

Time = 0.34 (sec) , antiderivative size = 21, normalized size of antiderivative = 0.52

method	result	size
pseudoelliptic	$-\frac{2(2bx+a)}{a^{2}\sqrt{x(bx+a)}}$	21
default	$-\frac{2(2bx+a)}{a^{2}\sqrt{b\,x^{2}+ax}}$	23
gosper	$-\frac{2x(bx+a)(2bx+a)}{a^{2}\left(b\,x^{2}+ax\right)^{\frac{3}{2}}}$	29
orering	$-\frac{2x(bx+a)(2bx+a)}{a^{2}\left(b\,x^{2}+ax\right)^{\frac{3}{2}}}$	29
trager	$-\frac{2(2bx+a)\sqrt{b\,x^{2}+ax}}{(bx+a)a^{2}x}$	33
risch	$-\frac{2(bx+a)}{a^{2}\sqrt{x(bx+a)}}-\frac{2bx}{\sqrt{x(bx+a)}\,a^{2}}$	37

```
int(1/(b*x^2+a*x)^(3/2),x,method=_RETURNVERBOSE)
```

```
-2*(2*b*x+a)/a^2/(x*(b*x+a))^(1/2)
```

Fricas [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 35, normalized size of antiderivative = 0.88

$$\int \frac{1}{(ax+bx^2)^{3/2}}\,dx = -\frac{2\sqrt{bx^2+ax}(2\,bx+a)}{a^2bx^2+a^3x}$$

```
integrate(1/(b*x^2+a*x)^(3/2),x, algorithm="fricas")
```

```
-2*sqrt(b*x^2 + a*x)*(2*b*x + a)/(a^2*b*x^2 + a^3*x)
```

Sympy [F]

$$\int \frac{1}{(ax+bx^2)^{3/2}}\,dx = \int \frac{1}{(ax+bx^2)^{\frac{3}{2}}}\,dx$$

```
integrate(1/(b*x**2+a*x)**(3/2),x)
```

```
Integral((a*x + b*x**2)**(-3/2), x)
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 35, normalized size of antiderivative = 0.88

$$\int \frac{1}{(ax+bx^2)^{3/2}}\,dx = -\frac{4\,bx}{\sqrt{bx^2+ax}a^2} - \frac{2}{\sqrt{bx^2+ax}a}$$

```
integrate(1/(b*x^2+a*x)^(3/2),x, algorithm="maxima")
```

```
-4*b*x/(sqrt(b*x^2 + a*x)*a^2) - 2/(sqrt(b*x^2 + a*x)*a)
```

Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 24, normalized size of antiderivative = 0.60

$$\int \frac{1}{(ax+bx^2)^{3/2}}\,dx = -\frac{2\left(\frac{2\,bx}{a^2}+\frac{1}{a}\right)}{\sqrt{bx^2+ax}}$$

```
integrate(1/(b*x^2+a*x)^(3/2),x, algorithm="giac")
```

```
-2*(2*b*x/a^2 + 1/a)/sqrt(b*x^2 + a*x)
```

Mupad [B] (verification not implemented)

Time = 0.05 (sec) , antiderivative size = 24, normalized size of antiderivative = 0.60

$$\int \frac{1}{(ax+bx^2)^{3/2}}\,dx = -\frac{2\,a+4\,b\,x}{a^2\,\sqrt{b\,x^2+a\,x}}$$

```
int(1/(a*x + b*x^2)^(3/2),x)
```

```
-(2*a + 4*b*x)/(a^2*(a*x + b*x^2)^(1/2))
```

Reduce [B] (verification not implemented)

Time = 0.21 (sec) , antiderivative size = 39, normalized size of antiderivative = 0.98

$$\int \frac{1}{(ax+bx^2)^{3/2}}\,dx = \frac{-4\sqrt{b}\,\sqrt{bx+a}\,x-2\sqrt{x}\,a-4\sqrt{x}\,bx}{\sqrt{bx+a}\,a^2x}$$

```
int(1/(b*x^2+a*x)^(3/2),x)
```

```
(2*( - 2*sqrt(b)*sqrt(a + b*x)*x - sqrt(x)*a - 2*sqrt(x)*b*x))/(sqrt(a + b
*x)*a**2*x)
```

3.53 $\int \frac{1}{x(ax+bx^2)^{3/2}}\,dx$

Optimal result

Integrand size = 17, antiderivative size = 69

$$\int \frac{1}{x(ax+bx^2)^{3/2}}\,dx = \frac{2}{ax\sqrt{ax+bx^2}} - \frac{8\sqrt{ax+bx^2}}{3a^2x^2} + \frac{16b\sqrt{ax+bx^2}}{3a^3x}$$

```
2/a/x/(b*x^2+a*x)^(1/2)-8/3*(b*x^2+a*x)^(1/2)/a^2/x^2+16/3*b*(b*x^2+a*x)^(
1/2)/a^3/x
```

Mathematica [A] (verified)

Time = 0.09 (sec) , antiderivative size = 40, normalized size of antiderivative = 0.58

$$\int \frac{1}{x(ax+bx^2)^{3/2}}\,dx = -\frac{2(a+bx)(a^2-4abx-8b^2x^2)}{3a^3(x(a+bx))^{3/2}}$$

```
Integrate[1/(x*(a*x + b*x^2)^(3/2)),x]
```

```
(-2*(a + b*x)*(a^2 - 4*a*b*x - 8*b^2*x^2))/(3*a^3*(x*(a + b*x))^(3/2))
```

Rubi [A] (verified)

Time = 0.28 (sec) , antiderivative size = 51, normalized size of antiderivative = 0.74, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.118, Rules used = {1129, 1088}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{x\left(ax+bx^2\right)^{3/2}}dx$$
$$\Big\downarrow \text{1129}$$
$$-\frac{4b\int \frac{1}{\left(bx^2+ax\right)^{3/2}}dx}{3a}-\frac{2}{3ax\sqrt{ax+bx^2}}$$
$$\Big\downarrow \text{1088}$$
$$\frac{8b(a+2bx)}{3a^3\sqrt{ax+bx^2}}-\frac{2}{3ax\sqrt{ax+bx^2}}$$

```
Int[1/(x*(a*x + b*x^2)^(3/2)),x]
```

```
-2/(3*a*x*Sqrt[a*x + b*x^2]) + (8*b*(a + 2*b*x))/(3*a^3*Sqrt[a*x + b*x^2])
```

Defintions of rubi rules used

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-3/2), x_Symbol] :> Simp[-2*((b +
2*c*x)/((b^2 - 4*a*c)*Sqrt[a + b*x + c*x^2])), x] /; FreeQ[{a, b, c}, x] &&
 NeQ[b^2 - 4*a*c, 0]
```

```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] :> Simp[(-e)*(d + e*x)^m*((a + b*x + c*x^2)^(p + 1)/((m + p + 1)*(2*
c*d - b*e))), x] + Simp[c*(Simplify[m + 2*p + 2]/((m + p + 1)*(2*c*d - b*e)
))   Int[(d + e*x)^(m + 1)*(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d
, e, m, p}, x] && EqQ[c*d^2 - b*d*e + a*e^2, 0] && ILtQ[Simplify[m + 2*p +
2], 0]
```

Maple [A] (verified)

Time = 0.36 (sec) , antiderivative size = 35, normalized size of antiderivative = 0.51

method	result	size
pseudoelliptic	$-\frac{2\left(-8b^{2}x^{2}-4abx+a^{2}\right)}{3x\sqrt{x\left(bx+a\right)}\,a^{3}}$	35
gosper	$-\frac{2(bx+a)\left(-8b^{2}x^{2}-4abx+a^{2}\right)}{3a^{3}\left(b\,x^{2}+ax\right)^{\frac{3}{2}}}$	39
orering	$-\frac{2(bx+a)\left(-8b^{2}x^{2}-4abx+a^{2}\right)}{3a^{3}\left(b\,x^{2}+ax\right)^{\frac{3}{2}}}$	39
default	$-\frac{2}{3ax\sqrt{b\,x^{2}+ax}}+\frac{8b(2bx+a)}{3a^{3}\sqrt{b\,x^{2}+ax}}$	44
trager	$-\frac{2\left(-8b^{2}x^{2}-4abx+a^{2}\right)\sqrt{b\,x^{2}+ax}}{3(bx+a)a^{3}x^{2}}$	44
risch	$-\frac{2(bx+a)(-5bx+a)}{3a^{3}x\sqrt{x(bx+a)}}+\frac{2b^{2}x}{\sqrt{x(bx+a)}\,a^{3}}$	48

```
int(1/x/(b*x^2+a*x)^(3/2),x,method=_RETURNVERBOSE)
```

```
-2/3/x*(-8*b^2*x^2-4*a*b*x+a^2)/(x*(b*x+a))^(1/2)/a^3
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 50, normalized size of antiderivative = 0.72

$$\int \frac{1}{x\left(ax+bx^2\right)^{3/2}}\,dx = \frac{2\left(8\,b^2x^2+4\,abx-a^2\right)\sqrt{bx^2+ax}}{3\left(a^3bx^3+a^4x^2\right)}$$

```
integrate(1/x/(b*x^2+a*x)^(3/2),x, algorithm="fricas")
```

```
2/3*(8*b^2*x^2 + 4*a*b*x - a^2)*sqrt(b*x^2 + a*x)/(a^3*b*x^3 + a^4*x^2)
```

Sympy [F]

$$\int \frac{1}{x\,(ax+bx^2)^{3/2}}\,dx = \int \frac{1}{x\,(x\,(a+bx))^{\frac{3}{2}}}\,dx$$

```
integrate(1/x/(b*x**2+a*x)**(3/2),x)
```

```
Integral(1/(x*(x*(a + b*x))**(3/2)), x)
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 57, normalized size of antiderivative = 0.83

$$\int \frac{1}{x\,(ax+bx^2)^{3/2}}\,dx = \frac{16\,b^2x}{3\,\sqrt{bx^2+ax}a^3} + \frac{8\,b}{3\,\sqrt{bx^2+ax}a^2} - \frac{2}{3\,\sqrt{bx^2+ax}ax}$$

```
integrate(1/x/(b*x^2+a*x)^(3/2),x, algorithm="maxima")
```

```
16/3*b^2*x/(sqrt(b*x^2 + a*x)*a^3) + 8/3*b/(sqrt(b*x^2 + a*x)*a^2) - 2/3/(
sqrt(b*x^2 + a*x)*a*x)
```

Giac [F]

$$\int \frac{1}{x\,(ax+bx^2)^{3/2}}\,dx = \int \frac{1}{(bx^2+ax)^{\frac{3}{2}}x}\,dx$$

```
integrate(1/x/(b*x^2+a*x)^(3/2),x, algorithm="giac")
```

```
integrate(1/((b*x^2 + a*x)^(3/2)*x), x)
```

Mupad [B] (verification not implemented)

Time = 9.23 (sec) , antiderivative size = 45, normalized size of antiderivative = 0.65

$$\int \frac{1}{x\,(ax+bx^2)^{3/2}}\,dx = \frac{2\sqrt{b\,x^2+a\,x}\,\left(-a^2+4\,a\,b\,x+8\,b^2\,x^2\right)}{3\,a^3\,x^2\,\left(a+b\,x\right)}$$

```
int(1/(x*(a*x + b*x^2)^(3/2)),x)
```

```
(2*(a*x + b*x^2)^(1/2)*(8*b^2*x^2 - a^2 + 4*a*b*x))/(3*a^3*x^2*(a + b*x))
```

Reduce [B] (verification not implemented)

Time = 0.22 (sec) , antiderivative size = 55, normalized size of antiderivative = 0.80

$$\int \frac{1}{x\,(ax+bx^2)^{3/2}}\,dx = \frac{-\frac{16\sqrt{b}\,\sqrt{bx+a}\,b\,x^2}{3}-\frac{2\sqrt{x}\,a^2}{3}+\frac{8\sqrt{x}\,abx}{3}+\frac{16\sqrt{x}\,b^2x^2}{3}}{\sqrt{bx+a}\,a^3x^2}$$

```
int(1/x/(b*x^2+a*x)^(3/2),x)
```

```
(2*( - 8*sqrt(b)*sqrt(a + b*x)*b*x**2 - sqrt(x)*a**2 + 4*sqrt(x)*a*b*x + 8
*sqrt(x)*b**2*x**2))/(3*sqrt(a + b*x)*a**3*x**2)
```

3.54 $\int \frac{1}{x^2(ax+bx^2)^{3/2}}\,dx$

Optimal result

Integrand size = 17, antiderivative size = 95

$$\int \frac{1}{x^2(ax+bx^2)^{3/2}}\,dx = \frac{2}{ax^2\sqrt{ax+bx^2}} - \frac{12\sqrt{ax+bx^2}}{5a^2x^3} + \frac{16b\sqrt{ax+bx^2}}{5a^3x^2} - \frac{32b^2\sqrt{ax+bx^2}}{5a^4x}$$

```
2/a/x^2/(b*x^2+a*x)^(1/2)-12/5*(b*x^2+a*x)^(1/2)/a^2/x^3+16/5*b*(b*x^2+a*x
)^(1/2)/a^3/x^2-32/5*b^2*(b*x^2+a*x)^(1/2)/a^4/x
```

Mathematica [A] (verified)

Time = 0.12 (sec) , antiderivative size = 49, normalized size of antiderivative = 0.52

$$\int \frac{1}{x^2(ax+bx^2)^{3/2}}\,dx = -\frac{2(a^3-2a^2bx+8ab^2x^2+16b^3x^3)}{5a^4x^2\sqrt{x(a+bx)}}$$

```
Integrate[1/(x^2*(a*x + b*x^2)^(3/2)),x]
```

```
(-2*(a^3 - 2*a^2*b*x + 8*a*b^2*x^2 + 16*b^3*x^3))/(5*a^4*x^2*Sqrt[x*(a + b
*x)])
```

Rubi [A] (verified)

Time = 0.34 (sec) , antiderivative size = 83, normalized size of antiderivative = 0.87, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.176, Rules used = {1129, 1129, 1088}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{x^2 (ax+bx^2)^{3/2}} \, dx$$

$$\Big\downarrow \text{1129}$$

$$-\frac{6b \int \frac{1}{x(bx^2+ax)^{3/2}}dx}{5a}-\frac{2}{5ax^2\sqrt{ax+bx^2}}$$

$$\Big\downarrow \text{1129}$$

$$-\frac{6b \left(-\frac{4b \int \frac{1}{\left(bx^2+ax\right)^{3/2}}dx}{3a}-\frac{2}{3ax\sqrt{ax+bx^2}}\right)}{5a}-\frac{2}{5ax^2\sqrt{ax+bx^2}}$$

$$\Big\downarrow \text{1088}$$

$$-\frac{6b \left(\frac{8b(a+2bx)}{3a^3\sqrt{ax+bx^2}}-\frac{2}{3ax\sqrt{ax+bx^2}}\right)}{5a}-\frac{2}{5ax^2\sqrt{ax+bx^2}}$$

```
Int[1/(x^2*(a*x + b*x^2)^(3/2)),x]
```

```
-2/(5*a*x^2*Sqrt[a*x + b*x^2]) - (6*b*(-2/(3*a*x*Sqrt[a*x + b*x^2]) + (8*b
*(a + 2*b*x))/(3*a^3*Sqrt[a*x + b*x^2])))/(5*a)
```

Defintions of rubi rules used

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-3/2), x_Symbol] :> Simp[-2*((b +
2*c*x)/((b^2 - 4*a*c)*Sqrt[a + b*x + c*x^2])), x] /; FreeQ[{a, b, c}, x] &&
 NeQ[b^2 - 4*a*c, 0]
```

```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] :> Simp[(-e)*(d + e*x)^m*((a + b*x + c*x^2)^(p + 1)/((m + p + 1)*(2*
c*d - b*e))), x] + Simp[c*(Simplify[m + 2*p + 2]/((m + p + 1)*(2*c*d - b*e)
))   Int[(d + e*x)^(m + 1)*(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d
, e, m, p}, x] && EqQ[c*d^2 - b*d*e + a*e^2, 0] && ILtQ[Simplify[m + 2*p +
2], 0]
```

Maple [A] (verified)

Time = 0.37 (sec) , antiderivative size = 46, normalized size of antiderivative = 0.48

method	result	size
pseudoelliptic	$-\frac{2\left(16b^{3}x^{3}+8a\,b^{2}x^{2}-2a^{2}bx+a^{3}\right)}{5x^{2}\sqrt{x\left(bx+a\right)}\,a^{4}}$	46
gosper	$-\frac{2\left(bx+a\right)\left(16b^{3}x^{3}+8a\,b^{2}x^{2}-2a^{2}bx+a^{3}\right)}{5x\,a^{4}\left(b\,x^{2}+ax\right)^{\frac{3}{2}}}$	53
orering	$-\frac{2\left(bx+a\right)\left(16b^{3}x^{3}+8a\,b^{2}x^{2}-2a^{2}bx+a^{3}\right)}{5x\,a^{4}\left(b\,x^{2}+ax\right)^{\frac{3}{2}}}$	53
trager	$-\frac{2\left(16b^{3}x^{3}+8a\,b^{2}x^{2}-2a^{2}bx+a^{3}\right)\sqrt{b\,x^{2}+ax}}{5\left(bx+a\right)a^{4}x^{3}}$	55
risch	$-\frac{2\left(bx+a\right)\left(11b^{2}x^{2}-3abx+a^{2}\right)}{5a^{4}x^{2}\sqrt{x\left(bx+a\right)}}-\frac{2b^{3}x}{\sqrt{x\left(bx+a\right)}\,a^{4}}$	59
default	$-\frac{2}{5a\,x^{2}\sqrt{b\,x^{2}+ax}}-\frac{6b\left(-\frac{2}{3ax\sqrt{b\,x^{2}+ax}}+\frac{8b\left(2bx+a\right)}{3a^{3}\sqrt{b\,x^{2}+ax}}\right)}{5a}$	70

```
int(1/x^2/(b*x^2+a*x)^(3/2),x,method=_RETURNVERBOSE)
```

```
-2/5/x^2*(16*b^3*x^3+8*a*b^2*x^2-2*a^2*b*x+a^3)/(x*(b*x+a))^(1/2)/a^4
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 59, normalized size of antiderivative = 0.62

$$\int \frac{1}{x^2 (ax+bx^2)^{3/2}}\,dx = -\frac{2\,(16\,b^3x^3+8\,ab^2x^2-2\,a^2bx+a^3)\sqrt{bx^2+ax}}{5\,(a^4bx^4+a^5x^3)}$$

```
integrate(1/x^2/(b*x^2+a*x)^(3/2),x, algorithm="fricas")
```

```
-2/5*(16*b^3*x^3 + 8*a*b^2*x^2 - 2*a^2*b*x + a^3)*sqrt(b*x^2 + a*x)/(a^4*b
*x^4 + a^5*x^3)
```

Sympy [F]

$$\int \frac{1}{x^2 (ax+bx^2)^{3/2}}\,dx = \int \frac{1}{x^2\,(x\,(a+bx))^{\frac{3}{2}}}\,dx$$

```
integrate(1/x**2/(b*x**2+a*x)**(3/2),x)
```

```
Integral(1/(x**2*(x*(a + b*x))**(3/2)), x)
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 79, normalized size of antiderivative = 0.83

$$\int \frac{1}{x^2 (ax+bx^2)^{3/2}}\,dx = -\frac{32\,b^3x}{5\,\sqrt{bx^2+ax}a^4} - \frac{16\,b^2}{5\,\sqrt{bx^2+ax}a^3} + \frac{4\,b}{5\,\sqrt{bx^2+ax}a^2x} - \frac{2}{5\,\sqrt{bx^2+ax}ax^2}$$

```
integrate(1/x^2/(b*x^2+a*x)^(3/2),x, algorithm="maxima")
```

```
-32/5*b^3*x/(sqrt(b*x^2 + a*x)*a^4) - 16/5*b^2/(sqrt(b*x^2 + a*x)*a^3) + 4
/5*b/(sqrt(b*x^2 + a*x)*a^2*x) - 2/5/(sqrt(b*x^2 + a*x)*a*x^2)
```

Giac [F]

$$\int \frac{1}{x^2 (ax + bx^2)^{3/2}} \, dx = \int \frac{1}{(bx^2 + ax)^{\frac{3}{2}} x^2} \, dx$$

```
integrate(1/x^2/(b*x^2+a*x)^(3/2),x, algorithm="giac")
```

```
integrate(1/((b*x^2 + a*x)^(3/2)*x^2), x)
```

Mupad [B] (verification not implemented)

Time = 9.23 (sec) , antiderivative size = 54, normalized size of antiderivative = 0.57

$$\int \frac{1}{x^2 (ax + bx^2)^{3/2}} \, dx = -\frac{2\sqrt{b\,x^2 + a\,x}\,(a^3 - 2\,a^2\,b\,x + 8\,a\,b^2\,x^2 + 16\,b^3\,x^3)}{5\,a^4\,x^3\,(a + b\,x)}$$

```
int(1/(x^2*(a*x + b*x^2)^(3/2)),x)
```

```
-(2*(a*x + b*x^2)^(1/2)*(a^3 + 16*b^3*x^3 + 8*a*b^2*x^2 - 2*a^2*b*x))/(5*a
^4*x^3*(a + b*x))
```

Reduce [B] (verification not implemented)

Time = 0.23 (sec) , antiderivative size = 70, normalized size of antiderivative = 0.74

$$\int \frac{1}{x^2 (ax + bx^2)^{3/2}} \, dx = \frac{\frac{32\sqrt{b}\,\sqrt{bx+a}\,b^2x^3}{5} - \frac{2\sqrt{x}\,a^3}{5} + \frac{4\sqrt{x}\,a^2bx}{5} - \frac{16\sqrt{x}\,a\,b^2x^2}{5} - \frac{32\sqrt{x}\,b^3x^3}{5}}{\sqrt{bx+a}\,a^4x^3}$$

```
int(1/x^2/(b*x^2+a*x)^(3/2),x)
```

```
(2*(16*sqrt(b)*sqrt(a + b*x)*b**2*x**3 - sqrt(x)*a**3 + 2*sqrt(x)*a**2*b*x
 - 8*sqrt(x)*a*b**2*x**2 - 16*sqrt(x)*b**3*x**3))/(5*sqrt(a + b*x)*a**4*x**
*3)
```

3.55 $\int \frac{1}{x^3 (ax+bx^2)^{3/2}}\, dx$

Optimal result

Integrand size = 17, antiderivative size = 121

$$\int \frac{1}{x^3 (ax+bx^2)^{3/2}}\, dx = \frac{2}{ax^3\sqrt{ax+bx^2}} - \frac{16\sqrt{ax+bx^2}}{7a^2x^4} + \frac{96b\sqrt{ax+bx^2}}{35a^3x^3} - \frac{128b^2\sqrt{ax+bx^2}}{35a^4x^2} + \frac{256b^3\sqrt{ax+bx^2}}{35a^5x}$$

```
2/a/x^3/(b*x^2+a*x)^(1/2)-16/7*(b*x^2+a*x)^(1/2)/a^2/x^4+96/35*b*(b*x^2+a*
x)^(1/2)/a^3/x^3-128/35*b^2*(b*x^2+a*x)^(1/2)/a^4/x^2+256/35*b^3*(b*x^2+a*
x)^(1/2)/a^5/x
```

Mathematica [A] (verified)

Time = 0.11 (sec) , antiderivative size = 62, normalized size of antiderivative = 0.51

$$\int \frac{1}{x^3 (ax+bx^2)^{3/2}}\, dx = \frac{2(-5a^4 + 8a^3bx - 16a^2b^2x^2 + 64ab^3x^3 + 128b^4x^4)}{35a^5x^3\sqrt{x(a+bx)}}$$

```
Integrate[1/(x^3*(a*x + b*x^2)^(3/2)),x]
```

```
(2*(-5*a^4 + 8*a^3*b*x - 16*a^2*b^2*x^2 + 64*a*b^3*x^3 + 128*b^4*x^4))/(35
*a^5*x^3*Sqrt[x*(a + b*x)])
```

Rubi [A] (verified)

Time = 0.40 (sec) , antiderivative size = 115, normalized size of antiderivative = 0.95, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.235, Rules used = {1129, 1129, 1129, 1088}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{x^3 (ax+bx^2)^{3/2}} \, dx$$

$$\Big\downarrow \ 1129$$

$$-\frac{8b \int \frac{1}{x^2 (bx^2+ax)^{3/2}}dx}{7a}-\frac{2}{7ax^3\sqrt{ax+bx^2}}$$

$$\Big\downarrow \ 1129$$

$$-\frac{8b \left(-\frac{6b \int \frac{1}{x\left(bx^2+ax\right)^{3/2}}dx}{5a}-\frac{2}{5ax^2\sqrt{ax+bx^2}}\right)}{7a}-\frac{2}{7ax^3\sqrt{ax+bx^2}}$$

$$\Big\downarrow \ 1129$$

$$-\frac{8b \left(-\frac{6b \left(-\frac{4b \int \frac{1}{\left(bx^2+ax\right)^{3/2}}dx}{3a}-\frac{2}{3ax\sqrt{ax+bx^2}}\right)}{5a}-\frac{2}{5ax^2\sqrt{ax+bx^2}}\right)}{7a}-\frac{2}{7ax^3\sqrt{ax+bx^2}}$$

$$\Big\downarrow \ 1088$$

$$-\frac{8b \left(-\frac{6b \left(\frac{8b(a+2bx)}{3a^3\sqrt{ax+bx^2}}-\frac{2}{3ax\sqrt{ax+bx^2}}\right)}{5a}-\frac{2}{5ax^2\sqrt{ax+bx^2}}\right)}{7a}-\frac{2}{7ax^3\sqrt{ax+bx^2}}$$

```
Int[1/(x^3*(a*x + b*x^2)^(3/2)),x]
```

```
-2/(7*a*x^3*Sqrt[a*x + b*x^2]) - (8*b*(-2/(5*a*x^2*Sqrt[a*x + b*x^2]) - (6
*b*(-2/(3*a*x*Sqrt[a*x + b*x^2]) + (8*b*(a + 2*b*x))/(3*a^3*Sqrt[a*x + b*x
^2])))/(5*a)))/(7*a)
```

Defintions of rubi rules used

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-3/2), x_Symbol] :> Simp[-2*((b +
2*c*x)/((b^2 - 4*a*c)*Sqrt[a + b*x + c*x^2])), x] /; FreeQ[{a, b, c}, x] &&
 NeQ[b^2 - 4*a*c, 0]
```

```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] :> Simp[(-e)*(d + e*x)^m*((a + b*x + c*x^2)^(p + 1)/((m + p + 1)*(2*
c*d - b*e))), x] + Simp[c*(Simplify[m + 2*p + 2]/((m + p + 1)*(2*c*d - b*e)
))   Int[(d + e*x)^(m + 1)*(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d
, e, m, p}, x] && EqQ[c*d^2 - b*d*e + a*e^2, 0] && ILtQ[Simplify[m + 2*p +
2], 0]
```

Maple [A] (verified)

Time = 0.38 (sec) , antiderivative size = 59, normalized size of antiderivative = 0.49

method	result	size
pseudoelliptic	$\frac{\frac{256}{35}b^4x^4+\frac{128}{35}a\,b^3x^3-\frac{32}{35}a^2b^2x^2+\frac{16}{35}a^3bx-\frac{2}{7}a^4}{x^3\sqrt{x(bx+a)}\,a^5}$	59
gosper	$-\frac{2(bx+a)\left(-128b^4x^4-64a\,b^3x^3+16a^2b^2x^2-8a^3bx+5a^4\right)}{35x^2a^5\left(b\,x^2+ax\right)^{\frac{3}{2}}}$	66
orering	$-\frac{2(bx+a)\left(-128b^4x^4-64a\,b^3x^3+16a^2b^2x^2-8a^3bx+5a^4\right)}{35x^2a^5\left(b\,x^2+ax\right)^{\frac{3}{2}}}$	66
trager	$-\frac{2\left(-128b^4x^4-64a\,b^3x^3+16a^2b^2x^2-8a^3bx+5a^4\right)\sqrt{b\,x^2+ax}}{35(bx+a)a^5x^4}$	68
risch	$-\frac{2(bx+a)\left(-93b^3x^3+29a\,b^2x^2-13a^2bx+5a^3\right)}{35a^5x^3\sqrt{x(bx+a)}}+\frac{2b^4x}{\sqrt{x(bx+a)}\,a^5}$	72
default	$-\frac{2}{7a\,x^3\sqrt{b\,x^2+ax}}-\frac{8b\left(-\frac{2}{5a\,x^2\sqrt{b\,x^2+ax}}-\frac{6b\left(-\frac{2}{3ax\sqrt{b\,x^2+ax}}+\frac{8b(2bx+a)}{3a^3\sqrt{b\,x^2+ax}}\right)}{5a}\right)}{7a}$	96

```
int(1/x^3/(b*x^2+a*x)^(3/2),x,method=_RETURNVERBOSE)
```

```
2/35*(128*b^4*x^4+64*a*b^3*x^3-16*a^2*b^2*x^2+8*a^3*b*x-5*a^4)/x^3/(x*(b*x
+a))^(1/2)/a^5
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 72, normalized size of antiderivative = 0.60

$$\int \frac{1}{x^3 (ax+bx^2)^{3/2}}\,dx = \frac{2\,(128\,b^4x^4+64\,ab^3x^3-16\,a^2b^2x^2+8\,a^3bx-5\,a^4)\sqrt{bx^2+ax}}{35\,(a^5bx^5+a^6x^4)}$$

```
integrate(1/x^3/(b*x^2+a*x)^(3/2),x, algorithm="fricas")
```

```
2/35*(128*b^4*x^4 + 64*a*b^3*x^3 - 16*a^2*b^2*x^2 + 8*a^3*b*x - 5*a^4)*sqr
t(b*x^2 + a*x)/(a^5*b*x^5 + a^6*x^4)
```

Sympy [F]

$$\int \frac{1}{x^3 (ax+bx^2)^{3/2}}\,dx = \int \frac{1}{x^3 \left(x\,(a+bx)\right)^{\frac{3}{2}}}\,dx$$

```
integrate(1/x**3/(b*x**2+a*x)**(3/2),x)
```

```
Integral(1/(x**3*(x*(a + b*x))**(3/2)), x)
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 101, normalized size of antiderivative = 0.83

$$\int \frac{1}{x^3 (ax+bx^2)^{3/2}}\, dx = \frac{256\, b^4 x}{35\, \sqrt{bx^2+ax}a^5} + \frac{128\, b^3}{35\, \sqrt{bx^2+ax}a^4} - \frac{32\, b^2}{35\, \sqrt{bx^2+ax}a^3 x} + \frac{16\, b}{35\, \sqrt{bx^2+ax}a^2 x^2} - \frac{2}{7\, \sqrt{bx^2+ax}a x^3}$$

```
integrate(1/x^3/(b*x^2+a*x)^(3/2),x, algorithm="maxima")
```

```
256/35*b^4*x/(sqrt(b*x^2 + a*x)*a^5) + 128/35*b^3/(sqrt(b*x^2 + a*x)*a^4)
- 32/35*b^2/(sqrt(b*x^2 + a*x)*a^3*x) + 16/35*b/(sqrt(b*x^2 + a*x)*a^2*x^2
) - 2/7/(sqrt(b*x^2 + a*x)*a*x^3)
```

Giac [F]

$$\int \frac{1}{x^3 (ax+bx^2)^{3/2}}\, dx = \int \frac{1}{(bx^2+ax)^{\frac{3}{2}} x^3}\, dx$$

```
integrate(1/x^3/(b*x^2+a*x)^(3/2),x, algorithm="giac")
```

```
integrate(1/((b*x^2 + a*x)^(3/2)*x^3), x)
```

Mupad [B] (verification not implemented)

Time = 9.42 (sec) , antiderivative size = 102, normalized size of antiderivative = 0.84

$$\int \frac{1}{x^3 (ax+bx^2)^{3/2}}\, dx = \frac{\sqrt{b\,x^2+a\,x}\,\left(\frac{186\,b^3}{35\,a^4}+\frac{256\,b^4\,x}{35\,a^5}\right)}{x\,\left(a+b\,x\right)} - \frac{58\,b^2\,\sqrt{b\,x^2+a\,x}}{35\,a^4\,x^2} - \frac{2\,\sqrt{b\,x^2+a\,x}}{7\,a^2\,x^4} + \frac{26\,b\,\sqrt{b\,x^2+a\,x}}{35\,a^3\,x^3}$$

```
int(1/(x^3*(a*x + b*x^2)^(3/2)),x)
```

```
((a*x + b*x^2)^(1/2)*((186*b^3)/(35*a^4) + (256*b^4*x)/(35*a^5)))/(x*(a +
b*x)) - (58*b^2*(a*x + b*x^2)^(1/2))/(35*a^4*x^2) - (2*(a*x + b*x^2)^(1/2)
)/(7*a^2*x^4) + (26*b*(a*x + b*x^2)^(1/2))/(35*a^3*x^3)
```

Reduce [B] (verification not implemented)

Time = 0.25 (sec) , antiderivative size = 83, normalized size of antiderivative = 0.69

$$\int \frac{1}{x^3 (ax + bx^2)^{3/2}}\, dx = \frac{-\frac{256\sqrt{b}\,\sqrt{bx+a}\,b^3x^4}{35} - \frac{2\sqrt{x}\,a^4}{7} + \frac{16\sqrt{x}\,a^3bx}{35} - \frac{32\sqrt{x}\,a^2b^2x^2}{35} + \frac{128\sqrt{x}\,a\,b^3x^3}{35} + \frac{256\sqrt{x}\,b^4x^4}{35}}{\sqrt{bx+a}\,a^5x^4}$$

```
int(1/x^3/(b*x^2+a*x)^(3/2),x)
```

```
(2*( - 128*sqrt(b)*sqrt(a + b*x)*b**3*x**4 - 5*sqrt(x)*a**4 + 8*sqrt(x)*a*
*3*b*x - 16*sqrt(x)*a**2*b**2*x**2 + 64*sqrt(x)*a*b**3*x**3 + 128*sqrt(x)*
b**4*x**4))/(35*sqrt(a + b*x)*a**5*x**4)
```

3.56 $\int \frac{x^6}{(ax+bx^2)^{5/2}}\,dx$

Optimal result

Integrand size = 17, antiderivative size = 128

$$\int \frac{x^6}{(ax+bx^2)^{5/2}}\,dx = \frac{2a^3x^2}{3b^4(ax+bx^2)^{3/2}} - \frac{20a^2x}{3b^4\sqrt{ax+bx^2}} - \frac{13a\sqrt{ax+bx^2}}{4b^4} + \frac{(ax+bx^2)^{3/2}}{2b^4x} + \frac{35a^2\operatorname{arctanh}\left(\frac{\sqrt{b}x}{\sqrt{ax+bx^2}}\right)}{4b^{9/2}}$$

```
2/3*a^3*x^2/b^4/(b*x^2+a*x)^(3/2)-20/3*a^2*x/b^4/(b*x^2+a*x)^(1/2)-13/4*a*
(b*x^2+a*x)^(1/2)/b^4+1/2*(b*x^2+a*x)^(3/2)/b^4/x+35/4*a^2*arctanh(b^(1/2)
*x/(b*x^2+a*x)^(1/2))/b^(9/2)
```

Mathematica [A] (verified)

Time = 0.37 (sec) , antiderivative size = 109, normalized size of antiderivative = 0.85

$$\int \frac{x^6}{(ax+bx^2)^{5/2}}\,dx = \frac{x\left(\sqrt{b}x(-105a^3-140a^2bx-21ab^2x^2+6b^3x^3)+210a^2\sqrt{x}(a+bx)^{3/2}\operatorname{arctanh}\left(\frac{\sqrt{}}{-\sqrt{a}+}\right.\right.}{12b^{9/2}(x(a+bx))^{3/2}}$$

```
Integrate[x^6/(a*x + b*x^2)^(5/2),x]
```

```
(x*(Sqrt[b]*x*(-105*a^3 - 140*a^2*b*x - 21*a*b^2*x^2 + 6*b^3*x^3) + 210*a^
2*Sqrt[x]*(a + b*x)^(3/2)*ArcTanh[(Sqrt[b]*Sqrt[x])/(-Sqrt[a] + Sqrt[a + b
*x])]))/(12*b^(9/2)*(x*(a + b*x))^(3/2))
```

Rubi [A] (verified)

Time = 0.52 (sec) , antiderivative size = 128, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}} = 0.412$, Rules used = {1133, 1124, 2192, 27, 1160, 1091, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^6}{(ax+bx^2)^{5/2}}dx$$

$$\Big\downarrow \text{1133}$$

$$\frac{7\int \frac{x^4}{(bx^2+ax)^{3/2}}dx}{3b}-\frac{2x^5}{3b(ax+bx^2)^{3/2}}$$

$$\Big\downarrow \text{1124}$$

$$\frac{7\left(\frac{\int \frac{a^2-bxa+b^2x^2}{\sqrt{bx^2+ax}}dx}{b^3}-\frac{2a^2x}{b^3\sqrt{ax+bx^2}}\right)}{3b}-\frac{2x^5}{3b(ax+bx^2)^{3/2}}$$

$$\Big\downarrow \text{2192}$$

$$\frac{7\left(\frac{\frac{\int \frac{ab(4a-7bx)}{2\sqrt{bx^2+ax}}dx}{2b}+\frac{1}{2}bx\sqrt{ax+bx^2}}{b^3}-\frac{2a^2x}{b^3\sqrt{ax+bx^2}}\right)}{3b}-\frac{2x^5}{3b(ax+bx^2)^{3/2}}$$

$$\Big\downarrow \text{27}$$

$$\frac{7\left(\frac{\frac{1}{4}a\int \frac{4a-7bx}{\sqrt{bx^2+ax}}dx+\frac{1}{2}bx\sqrt{ax+bx^2}}{b^3}-\frac{2a^2x}{b^3\sqrt{ax+bx^2}}\right)}{3b}-\frac{2x^5}{3b(ax+bx^2)^{3/2}}$$

$$\Big\downarrow \text{1160}$$

$$\frac{7\left(\frac{\frac{1}{4}a\left(\frac{15}{2}a\int \frac{1}{\sqrt{bx^2+ax}}dx-7\sqrt{ax+bx^2}\right)+\frac{1}{2}bx\sqrt{ax+bx^2}}{b^3}-\frac{2a^2x}{b^3\sqrt{ax+bx^2}}\right)}{3b}-\frac{2x^5}{3b\left(ax+bx^2\right)^{3/2}}$$

$$\Big\downarrow \text{1091}$$

$$\frac{7\left(\frac{\frac{1}{4}a\left(15a\int \frac{1}{1-\frac{bx^2}{bx^2+ax}}d\frac{x}{\sqrt{bx^2+ax}}-7\sqrt{ax+bx^2}\right)+\frac{1}{2}bx\sqrt{ax+bx^2}}{b^3}-\frac{2a^2x}{b^3\sqrt{ax+bx^2}}\right)}{3b}-\frac{2x^5}{3b\left(ax+bx^2\right)^{3/2}}$$

$$\Big\downarrow \text{219}$$

$$\frac{7\left(\frac{\frac{1}{4}a\left(\frac{15a\,\text{arctanh}\left(\frac{\sqrt{b}x}{\sqrt{ax+bx^2}}\right)}{\sqrt{b}}-7\sqrt{ax+bx^2}\right)+\frac{1}{2}bx\sqrt{ax+bx^2}}{b^3}-\frac{2a^2x}{b^3\sqrt{ax+bx^2}}\right)}{3b}-\frac{2x^5}{3b\left(ax+bx^2\right)^{3/2}}$$

```
Int[x^6/(a*x + b*x^2)^(5/2),x]
```

```
(-2*x^5)/(3*b*(a*x + b*x^2)^(3/2)) + (7*((-2*a^2*x)/(b^3*Sqrt[a*x + b*x^2]
) + ((b*x*Sqrt[a*x + b*x^2])/2 + (a*(-7*Sqrt[a*x + b*x^2] + (15*a*ArcTanh[
(Sqrt[b]*x)/Sqrt[a*x + b*x^2]])/Sqrt[b]))/4)/b^3))/(3*b)
```

Defintions of rubi rules used

```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

```
Int[1/Sqrt[(b_.)*(x_) + (c_.)*(x_)^2], x_Symbol] :> Simp[2   Subst[Int[1/(1
 - c*x^2), x], x, x/Sqrt[b*x + c*x^2]], x] /; FreeQ[{b, c}, x]
```

```
Int[((d_.) + (e_.)*(x_))^(m_.)/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(3/2), x
_Symbol] :> Simp[-2*e*(2*c*d - b*e)^(m - 2)*((d + e*x)/(c^(m - 1)*Sqrt[a +
b*x + c*x^2])), x] + Simp[e^2/c^(m - 1)   Int[(1/Sqrt[a + b*x + c*x^2])*Exp
andToSum[((2*c*d - b*e)^(m - 1) - c^(m - 1)*(d + e*x)^(m - 1))/(c*d - b*e -
 c*e*x), x], x], x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[c*d^2 - b*d*e + a*e
^2, 0] && IGtQ[m, 0]
```

```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] :> Simp[e*(d + e*x)^(m - 1)*((a + b*x + c*x^2)^(p + 1)/(c*(p + 1))),
 x] - Simp[e^2*((m + p)/(c*(p + 1)))   Int[(d + e*x)^(m - 2)*(a + b*x + c*x
^2)^(p + 1), x], x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[c*d^2 - b*d*e + a*e
^2, 0] && LtQ[p, -1] && GtQ[m, 1] && IntegerQ[2*p]
```

```
Int[((d_.) + (e_.)*(x_))*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol
] :> Simp[e*((a + b*x + c*x^2)^(p + 1)/(2*c*(p + 1))), x] + Simp[(2*c*d - b
*e)/(2*c)   Int[(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, p}, x]
 && NeQ[p, -1]
```

```
Int[(Pq_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> With[{q =
Expon[Pq, x], e = Coeff[Pq, x, Expon[Pq, x]]}, Simp[e*x^(q - 1)*((a + b*x +
 c*x^2)^(p + 1)/(c*(q + 2*p + 1))), x] + Simp[1/(c*(q + 2*p + 1))   Int[(a
+ b*x + c*x^2)^p*ExpandToSum[c*(q + 2*p + 1)*Pq - a*e*(q - 1)*x^(q - 2) - b
*e*(q + p)*x^(q - 1) - c*e*(q + 2*p + 1)*x^q, x], x], x]] /; FreeQ[{a, b, c
, p}, x] && PolyQ[Pq, x] && NeQ[b^2 - 4*a*c, 0] &&  !LeQ[p, -1]
```

Maple [A] (verified)

Time = 0.44 (sec) , antiderivative size = 98, normalized size of antiderivative = 0.77

method	result
pseudoelliptic	$\frac{-140 b^{\frac{3}{2}} a^{2} x^{2}-21 b^{\frac{5}{2}} a\,x^{3}+6 b^{\frac{7}{2}} x^{4}+105 a^{2} \left(-x a \sqrt{b}+\sqrt{x (b x+a)}\, \operatorname{arctanh}\left(\frac{\sqrt{x (b x+a)}}{x \sqrt{b}}\right) (b x+a)\right)}{b^{\frac{9}{2}} \sqrt{x (b x+a)}\, (12 b x+12 a)}$
risch	$-\frac{(-2bx+11a)x(bx+a)}{4b^{4}\sqrt{x(bx+a)}}+\frac{35a^{2}\ln\left(\frac{\frac{a}{2}+bx}{\sqrt{b}}+\sqrt{b\,x^{2}+ax}\right)}{8b^{\frac{9}{2}}}-\frac{20a^{2}\sqrt{\left(x+\frac{a}{b}\right)^{2}b-a\left(x+\frac{a}{b}\right)}}{3b^{5}\left(x+\frac{a}{b}\right)}+\frac{2a^{3}\sqrt{\left(x+\frac{a}{b}\right)^{2}b-a\left(x+\frac{a}{b}\right)}}{3b^{6}\left(x+\frac{a}{b}\right)^{2}}$
	$7a\left(\frac{x^{4}}{b\left(b\,x^{2}+ax\right)^{\frac{3}{2}}}-\frac{5a\left(-\frac{x^{3}}{3b\left(b\,x^{2}+ax\right)^{\frac{3}{2}}}-\frac{a\left(-\frac{x^{2}}{b\left(b\,x^{2}+ax\right)^{\frac{3}{2}}}+\frac{a\left(-\frac{x}{2b\left(b\,x^{2}+ax\right)^{\frac{3}{2}}}-\frac{a\left(-\frac{1}{3b\left(b\,x^{2}+ax\right)^{\frac{3}{2}}}-\cdots\right)}{\cdots}\right)}{2b\cdots}\right)}{2b\cdots}\right)}{\cdots}\cdots\right.$

```
int(x^6/(b*x^2+a*x)^(5/2),x,method=_RETURNVERBOSE)
```

```
105/b^(9/2)/(x*(b*x+a))^(1/2)*(-4/3*b^(3/2)*a^2*x^2-1/5*b^(5/2)*a*x^3+2/35
*b^(7/2)*x^4+a^2*(-x*a*b^(1/2)+(x*(b*x+a))^(1/2)*arctanh((x*(b*x+a))^(1/2)
/x/b^(1/2))*(b*x+a)))/(12*b*x+12*a)
```

Fricas [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 249, normalized size of antiderivative = 1.95

$$\int \frac{x^6}{(ax+bx^2)^{5/2}}\,dx = \left[\frac{105\,(a^2b^2x^2+2\,a^3bx+a^4)\sqrt{b}\log\left(2\,bx+a+2\,\sqrt{bx^2+ax}\sqrt{b}\right)+2\,(6\,b^4x^3-21\,ab^3}{24\,(b^7x^2+2\,ab^6x+a^2b^5)}\right.$$
$$-\frac{105\,(a^2b^2x^2+2\,a^3bx+a^4)\sqrt{-b}\arctan\left(\frac{\sqrt{bx^2+ax}\sqrt{-b}}{bx+a}\right)-(6\,b^4x^3-21\,ab^3x^2-140\,a^2b^2x-105\,a^3b)\sqrt{bx^2+}}{12\,(b^7x^2+2\,ab^6x+a^2b^5)}$$

```
integrate(x^6/(b*x^2+a*x)^(5/2),x, algorithm="fricas")
```

```
[1/24*(105*(a^2*b^2*x^2 + 2*a^3*b*x + a^4)*sqrt(b)*log(2*b*x + a + 2*sqrt(
b*x^2 + a*x)*sqrt(b)) + 2*(6*b^4*x^3 - 21*a*b^3*x^2 - 140*a^2*b^2*x - 105*
a^3*b)*sqrt(b*x^2 + a*x))/(b^7*x^2 + 2*a*b^6*x + a^2*b^5), -1/12*(105*(a^2
*b^2*x^2 + 2*a^3*b*x + a^4)*sqrt(-b)*arctan(sqrt(b*x^2 + a*x)*sqrt(-b)/(b*
x + a)) - (6*b^4*x^3 - 21*a*b^3*x^2 - 140*a^2*b^2*x - 105*a^3*b)*sqrt(b*x^
2 + a*x))/(b^7*x^2 + 2*a*b^6*x + a^2*b^5)]
```

Sympy [F]

$$\int \frac{x^6}{(ax+bx^2)^{5/2}}\,dx = \int \frac{x^6}{(x\,(a+bx))^{\frac{5}{2}}}\,dx$$

```
integrate(x**6/(b*x**2+a*x)**(5/2),x)
```

```
Integral(x**6/(x*(a + b*x))**(5/2), x)
```

Maxima [A] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 190, normalized size of antiderivative = 1.48

$$\int \frac{x^6}{(ax+bx^2)^{5/2}}\,dx = \frac{x^5}{2\,(bx^2+ax)^{\frac{3}{2}}b} - \frac{35\,a^2x\left(\frac{3\,x^2}{(bx^2+ax)^{\frac{3}{2}}b} + \frac{ax}{(bx^2+ax)^{\frac{3}{2}}b^2} - \frac{2\,x}{\sqrt{bx^2+ax}ab} - \frac{1}{\sqrt{bx^2+ax}b^2}\right)}{24\,b^2} - \frac{7\,ax^4}{4\,(bx^2+ax)^{\frac{3}{2}}b^2} - \frac{35\,a^2x}{6\,\sqrt{bx^2+ax}b^4} + \frac{35\,a^2\log\left(2\,bx+a+2\,\sqrt{bx^2+ax}\sqrt{b}\right)}{8\,b^{\frac{9}{2}}} - \frac{35\,\sqrt{bx^2+ax}a}{12\,b^4}$$

```
integrate(x^6/(b*x^2+a*x)^(5/2),x, algorithm="maxima")
```

```
1/2*x^5/((b*x^2 + a*x)^(3/2)*b) - 35/24*a^2*x*(3*x^2/((b*x^2 + a*x)^(3/2)*
b) + a*x/((b*x^2 + a*x)^(3/2)*b^2) - 2*x/(sqrt(b*x^2 + a*x)*a*b) - 1/(sqrt
(b*x^2 + a*x)*b^2))/b^2 - 7/4*a*x^4/((b*x^2 + a*x)^(3/2)*b^2) - 35/6*a^2*x
/(sqrt(b*x^2 + a*x)*b^4) + 35/8*a^2*log(2*b*x + a + 2*sqrt(b*x^2 + a*x)*sq
rt(b))/b^(9/2) - 35/12*sqrt(b*x^2 + a*x)*a/b^4
```

Giac [A] (verification not implemented)

Time = 0.17 (sec) , antiderivative size = 157, normalized size of antiderivative = 1.23

$$\int \frac{x^6}{(ax+bx^2)^{5/2}}\,dx = \frac{1}{4}\,\sqrt{bx^2+ax}\left(\frac{2\,x}{b^3} - \frac{11\,a}{b^4}\right) - \frac{35\,a^2\log\left(\left|-2\,\left(\sqrt{b}x-\sqrt{bx^2+ax}\right)\sqrt{b}-a\right|\right)}{8\,b^{\frac{9}{2}}} - \frac{2\left(12\,\left(\sqrt{b}x-\sqrt{bx^2+ax}\right)^2a^3b + 21\,\left(\sqrt{b}x-\sqrt{bx^2+ax}\right)a^4\sqrt{b} + 10\,a^5\right)}{3\left(\left(\sqrt{b}x-\sqrt{bx^2+ax}\right)\sqrt{b}+a\right)^3b^{\frac{9}{2}}}$$

```
integrate(x^6/(b*x^2+a*x)^(5/2),x, algorithm="giac")
```

```
1/4*sqrt(b*x^2 + a*x)*(2*x/b^3 - 11*a/b^4) - 35/8*a^2*log(abs(-2*(sqrt(b)*
x - sqrt(b*x^2 + a*x))*sqrt(b) - a))/b^(9/2) - 2/3*(12*(sqrt(b)*x - sqrt(b
*x^2 + a*x))^2*a^3*b + 21*(sqrt(b)*x - sqrt(b*x^2 + a*x))*a^4*sqrt(b) + 10
*a^5)/(((sqrt(b)*x - sqrt(b*x^2 + a*x))*sqrt(b) + a)^3*b^(9/2))
```

Mupad [F(-1)]

Timed out.

$$\int \frac{x^6}{(ax+bx^2)^{5/2}}\,dx = \int \frac{x^6}{(b\,x^2+a\,x)^{5/2}}\,dx$$

```
int(x^6/(a*x + b*x^2)^(5/2),x)
```

```
int(x^6/(a*x + b*x^2)^(5/2), x)
```

Reduce [B] (verification not implemented)

Time = 0.36 (sec) , antiderivative size = 153, normalized size of antiderivative = 1.20

$$\int \frac{x^6}{(ax+bx^2)^{5/2}}\,dx = \frac{840\sqrt{b}\,\sqrt{bx+a}\log\left(\frac{\sqrt{bx+a}+\sqrt{x}\,\sqrt{b}}{\sqrt{a}}\right)a^3 + 840\sqrt{b}\,\sqrt{bx+a}\log\left(\frac{\sqrt{bx+a}+\sqrt{x}\,\sqrt{b}}{\sqrt{a}}\right)a^2bx + 17}{96}$$

```
int(x^6/(b*x^2+a*x)^(5/2),x)
```

```
(840*sqrt(b)*sqrt(a + b*x)*log((sqrt(a + b*x) + sqrt(x)*sqrt(b))/sqrt(a))*
a**3 + 840*sqrt(b)*sqrt(a + b*x)*log((sqrt(a + b*x) + sqrt(x)*sqrt(b))/sqr
t(a))*a**2*b*x + 175*sqrt(b)*sqrt(a + b*x)*a**3 + 175*sqrt(b)*sqrt(a + b*x
)*a**2*b*x - 840*sqrt(x)*a**3*b - 1120*sqrt(x)*a**2*b**2*x - 168*sqrt(x)*a
*b**3*x**2 + 48*sqrt(x)*b**4*x**3)/(96*sqrt(a + b*x)*b**5*(a + b*x))
```

3.57 $\int \frac{x^5}{(ax+bx^2)^{5/2}}\,dx$

Optimal result

Integrand size = 17, antiderivative size = 95

$$\int \frac{x^5}{(ax+bx^2)^{5/2}}\,dx = -\frac{2a^2x^2}{3b^3(ax+bx^2)^{3/2}} + \frac{14ax}{3b^3\sqrt{ax+bx^2}} + \frac{\sqrt{ax+bx^2}}{b^3} - \frac{5a\operatorname{arctanh}\left(\frac{\sqrt{b}x}{\sqrt{ax+bx^2}}\right)}{b^{7/2}}$$

```
-2/3*a^2*x^2/b^3/(b*x^2+a*x)^(3/2)+14/3*a*x/b^3/(b*x^2+a*x)^(1/2)+(b*x^2+a
*x)^(1/2)/b^3-5*a*arctanh(b^(1/2)*x/(b*x^2+a*x)^(1/2))/b^(7/2)
```

Mathematica [A] (verified)

Time = 0.29 (sec) , antiderivative size = 96, normalized size of antiderivative = 1.01

$$\int \frac{x^5}{(ax+bx^2)^{5/2}}\,dx = \frac{x\left(\sqrt{b}x(15a^2+20abx+3b^2x^2)+30a\sqrt{x}(a+bx)^{3/2}\operatorname{arctanh}\left(\frac{\sqrt{b}\sqrt{x}}{\sqrt{a}-\sqrt{a+bx}}\right)\right)}{3b^{7/2}(x(a+bx))^{3/2}}$$

```
Integrate[x^5/(a*x + b*x^2)^(5/2),x]
```

```
(x*(Sqrt[b]*x*(15*a^2 + 20*a*b*x + 3*b^2*x^2) + 30*a*Sqrt[x]*(a + b*x)^(3/
2)*ArcTanh[(Sqrt[b]*Sqrt[x])/(Sqrt[a] - Sqrt[a + b*x])]))/(3*b^(7/2)*(x*(a
 + b*x))^(3/2))
```

Rubi [A] (verified)

Time = 0.40 (sec) , antiderivative size = 102, normalized size of antiderivative = 1.07, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.353$, Rules used = {1133, 1124, 25, 1160, 1091, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^5}{(ax+bx^2)^{5/2}}dx$$

$$\Big\downarrow \text{1133}$$

$$\frac{5\int \frac{x^3}{(bx^2+ax)^{3/2}}dx}{3b}-\frac{2x^4}{3b(ax+bx^2)^{3/2}}$$

$$\Big\downarrow \text{1124}$$

$$\frac{5\left(\frac{\int -\frac{a-bx}{\sqrt{bx^2+ax}}dx}{b^2}+\frac{2ax}{b^2\sqrt{ax+bx^2}}\right)}{3b}-\frac{2x^4}{3b(ax+bx^2)^{3/2}}$$

$$\Big\downarrow \text{25}$$

$$\frac{5\left(\frac{2ax}{b^2\sqrt{ax+bx^2}}-\frac{\int \frac{a-bx}{\sqrt{bx^2+ax}}dx}{b^2}\right)}{3b}-\frac{2x^4}{3b(ax+bx^2)^{3/2}}$$

$$\Big\downarrow \text{1160}$$

$$\frac{5\left(\frac{2ax}{b^2\sqrt{ax+bx^2}}-\frac{\frac{3}{2}a\int \frac{1}{\sqrt{bx^2+ax}}dx-\sqrt{ax+bx^2}}{b^2}\right)}{3b}-\frac{2x^4}{3b(ax+bx^2)^{3/2}}$$

$$\Big\downarrow \text{1091}$$

$$\frac{5\left(\frac{2ax}{b^2\sqrt{ax+bx^2}}-\frac{3a\int \frac{1}{1-\frac{bx^2}{bx^2+ax}}d\frac{x}{\sqrt{bx^2+ax}}-\sqrt{ax+bx^2}}{b^2}\right)}{3b}-\frac{2x^4}{3b\left(ax+bx^2\right)^{3/2}}$$

$$\Big\downarrow \; 219$$

$$\frac{5\left(\frac{2ax}{b^2\sqrt{ax+bx^2}}-\frac{\frac{3a\text{arctanh}\left(\frac{\sqrt{b}x}{\sqrt{ax+bx^2}}\right)}{\sqrt{b}}-\sqrt{ax+bx^2}}{b^2}\right)}{3b}-\frac{2x^4}{3b\left(ax+bx^2\right)^{3/2}}$$

```
Int[x^5/(a*x + b*x^2)^(5/2),x]
```

```
(-2*x^4)/(3*b*(a*x + b*x^2)^(3/2)) + (5*((2*a*x)/(b^2*Sqrt[a*x + b*x^2]) -
 (-Sqrt[a*x + b*x^2] + (3*a*ArcTanh[(Sqrt[b]*x)/Sqrt[a*x + b*x^2]])/Sqrt[b
])/b^2))/(3*b)
```

Defintions of rubi rules used

```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

```
Int[1/Sqrt[(b_.)*(x_) + (c_.)*(x_)^2], x_Symbol] :> Simp[2   Subst[Int[1/(1
 - c*x^2), x], x, x/Sqrt[b*x + c*x^2]], x] /; FreeQ[{b, c}, x]
```

```
Int[((d_.) + (e_.)*(x_))^(m_.)/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(3/2), x
_Symbol] :> Simp[-2*e*(2*c*d - b*e)^(m - 2)*((d + e*x)/(c^(m - 1)*Sqrt[a +
b*x + c*x^2])), x] + Simp[e^2/c^(m - 1)   Int[(1/Sqrt[a + b*x + c*x^2])*Exp
andToSum[((2*c*d - b*e)^(m - 1) - c^(m - 1)*(d + e*x)^(m - 1))/(c*d - b*e -
 c*e*x), x], x], x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[c*d^2 - b*d*e + a*e
^2, 0] && IGtQ[m, 0]
```

```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] :> Simp[e*(d + e*x)^(m - 1)*((a + b*x + c*x^2)^(p + 1)/(c*(p + 1))),
 x] - Simp[e^2*((m + p)/(c*(p + 1)))   Int[(d + e*x)^(m - 2)*(a + b*x + c*x
^2)^(p + 1), x], x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[c*d^2 - b*d*e + a*e
^2, 0] && LtQ[p, -1] && GtQ[m, 1] && IntegerQ[2*p]
```

```
Int[((d_.) + (e_.)*(x_))*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol
] :> Simp[e*((a + b*x + c*x^2)^(p + 1)/(2*c*(p + 1))), x] + Simp[(2*c*d - b
*e)/(2*c)   Int[(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, p}, x]
 && NeQ[p, -1]
```

Maple [A] (verified)

Time = 0.41 (sec) , antiderivative size = 82, normalized size of antiderivative = 0.86

method	result
pseudoelliptic	$-\frac{5\left(a\sqrt{x(bx+a)}\,(bx+a)\operatorname{arctanh}\left(\frac{\sqrt{x(bx+a)}}{x\sqrt{b}}\right)-a^{2}x\sqrt{b}-\frac{4b^{\frac{3}{2}}a\,x^{2}}{3}-\frac{b^{\frac{5}{2}}x^{3}}{5}\right)}{b^{\frac{7}{2}}\sqrt{x(bx+a)}\,(bx+a)}$
risch	$\frac{x(bx+a)}{b^{3}\sqrt{x(bx+a)}}-\frac{5a\ln\left(\frac{\frac{a}{2}+bx}{\sqrt{b}}+\sqrt{b\,x^{2}+ax}\right)}{2b^{\frac{7}{2}}}+\frac{14a\sqrt{\left(x+\frac{a}{b}\right)^{2}b-a\left(x+\frac{a}{b}\right)}}{3b^{4}\left(x+\frac{a}{b}\right)}-\frac{2a^{2}\sqrt{\left(x+\frac{a}{b}\right)^{2}b-a\left(x+\frac{a}{b}\right)}}{3b^{5}\left(x+\frac{a}{b}\right)^{2}}$
default	$\frac{x^{4}}{b(b\,x^{2}+ax)^{\frac{3}{2}}}-\frac{5a\left(-\frac{x^{3}}{3b\left(b\,x^{2}+ax\right)^{\frac{3}{2}}}-\frac{a\left(-\frac{x^{2}}{b\left(b\,x^{2}+ax\right)^{\frac{3}{2}}}+\frac{a\left(-\frac{x}{2b\left(b\,x^{2}+ax\right)^{\frac{3}{2}}}-\frac{a\left(-\frac{1}{3b\left(b\,x^{2}+ax\right)^{\frac{3}{2}}}-\frac{a\left(-\frac{2(2bx+a)}{3a^{2}\left(b\,x^{2}+ax\right)^{\frac{3}{2}}}-\cdots\right)}{2b}\right)}{4b}\right)}{2b}\right)}{2b}\right)}{2b}$

```
int(x^5/(b*x^2+a*x)^(5/2),x,method=_RETURNVERBOSE)
```

```
-5/b^(7/2)*(a*(x*(b*x+a))^(1/2)*(b*x+a)*arctanh((x*(b*x+a))^(1/2)/x/b^(1/2
))-a^2*x*b^(1/2)-4/3*b^(3/2)*a*x^2-1/5*b^(5/2)*x^3)/(x*(b*x+a))^(1/2)/(b*x
+a)
```

Fricas [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 222, normalized size of antiderivative = 2.34

$$\int \frac{x^5}{(ax+bx^2)^{5/2}}\,dx = \left[\frac{15\,(ab^2x^2+2\,a^2bx+a^3)\sqrt{b}\log\left(2\,bx+a-2\,\sqrt{bx^2+ax}\sqrt{b}\right)+2\,(3\,b^3x^2+20\,ab^2x}{6\,(b^6x^2+2\,ab^5x+a^2b^4)}\right.$$

```
integrate(x^5/(b*x^2+a*x)^(5/2),x, algorithm="fricas")
```

```
[1/6*(15*(a*b^2*x^2 + 2*a^2*b*x + a^3)*sqrt(b)*log(2*b*x + a - 2*sqrt(b*x^
2 + a*x)*sqrt(b)) + 2*(3*b^3*x^2 + 20*a*b^2*x + 15*a^2*b)*sqrt(b*x^2 + a*x
))/(b^6*x^2 + 2*a*b^5*x + a^2*b^4), 1/3*(15*(a*b^2*x^2 + 2*a^2*b*x + a^3)*
sqrt(-b)*arctan(sqrt(b*x^2 + a*x)*sqrt(-b)/(b*x + a)) + (3*b^3*x^2 + 20*a*
b^2*x + 15*a^2*b)*sqrt(b*x^2 + a*x))/(b^6*x^2 + 2*a*b^5*x + a^2*b^4)]
```

Sympy [F]

$$\int \frac{x^5}{(ax+bx^2)^{5/2}}\,dx = \int \frac{x^5}{(x\,(a+bx))^{\frac{5}{2}}}\,dx$$

```
integrate(x**5/(b*x**2+a*x)**(5/2),x)
```

```
Integral(x**5/(x*(a + b*x))**(5/2), x)
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 162 vs. $2(79) = 158$.

Time = 0.04 (sec) , antiderivative size = 162, normalized size of antiderivative = 1.71

$$\int \frac{x^5}{(ax+bx^2)^{5/2}}\,dx = \frac{5\,ax\left(\frac{3\,x^2}{(bx^2+ax)^{\frac{3}{2}}b} + \frac{ax}{(bx^2+ax)^{\frac{3}{2}}b^2} - \frac{2\,x}{\sqrt{bx^2+ax}ab} - \frac{1}{\sqrt{bx^2+ax}b^2}\right)}{6\,b} + \frac{x^4}{(bx^2+ax)^{\frac{3}{2}}b} + \frac{10\,ax}{3\,\sqrt{bx^2+ax}b^3} - \frac{5\,a\log\left(2\,bx+a+2\,\sqrt{bx^2+ax}\sqrt{b}\right)}{2\,b^{\frac{7}{2}}} + \frac{5\,\sqrt{bx^2+ax}}{3\,b^3}$$

```
integrate(x^5/(b*x^2+a*x)^(5/2),x, algorithm="maxima")
```

```
5/6*a*x*(3*x^2/((b*x^2 + a*x)^(3/2)*b) + a*x/((b*x^2 + a*x)^(3/2)*b^2) - 2
*x/(sqrt(b*x^2 + a*x)*a*b) - 1/(sqrt(b*x^2 + a*x)*b^2))/b + x^4/((b*x^2 +
a*x)^(3/2)*b) + 10/3*a*x/(sqrt(b*x^2 + a*x)*b^3) - 5/2*a*log(2*b*x + a + 2
*sqrt(b*x^2 + a*x)*sqrt(b))/b^(7/2) + 5/3*sqrt(b*x^2 + a*x)/b^3
```

Giac [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 144, normalized size of antiderivative = 1.52

$$\int \frac{x^5}{(ax+bx^2)^{5/2}}\,dx = \frac{5\,a\log\left(\left|-2\left(\sqrt{b}x-\sqrt{bx^2+ax}\right)\sqrt{b}-a\right|\right)}{2\,b^{\frac{7}{2}}} + \frac{\sqrt{bx^2+ax}}{b^3} + \frac{2\left(9\left(\sqrt{b}x-\sqrt{bx^2+ax}\right)^2 a^2b + 15\left(\sqrt{b}x-\sqrt{bx^2+ax}\right)a^3\sqrt{b} + 7\,a^4\right)}{3\left(\left(\sqrt{b}x-\sqrt{bx^2+ax}\right)\sqrt{b}+a\right)^3 b^{\frac{7}{2}}}$$

```
integrate(x^5/(b*x^2+a*x)^(5/2),x, algorithm="giac")
```

```
5/2*a*log(abs(-2*(sqrt(b)*x - sqrt(b*x^2 + a*x))*sqrt(b) - a))/b^(7/2) + s
qrt(b*x^2 + a*x)/b^3 + 2/3*(9*(sqrt(b)*x - sqrt(b*x^2 + a*x))^2*a^2*b + 15
*(sqrt(b)*x - sqrt(b*x^2 + a*x))*a^3*sqrt(b) + 7*a^4)/(((sqrt(b)*x - sqrt(
b*x^2 + a*x))*sqrt(b) + a)^3*b^(7/2))
```

Mupad [F(-1)]

Timed out.

$$\int \frac{x^5}{(ax+bx^2)^{5/2}}\,dx=\int \frac{x^5}{{\left(b\,x^2+a\,x\right)}^{5/2}}\,dx$$

```
int(x^5/(a*x + b*x^2)^(5/2),x)
```

```
int(x^5/(a*x + b*x^2)^(5/2), x)
```

Reduce [B] (verification not implemented)

Time = 0.50 (sec) , antiderivative size = 136, normalized size of antiderivative = 1.43

$$\int \frac{x^5}{(ax+bx^2)^{5/2}}\,dx=\frac{-30\sqrt{b}\,\sqrt{bx+a}\,\log\left(\frac{\sqrt{bx+a}+\sqrt{x}\,\sqrt{b}}{\sqrt{a}}\right)a^2-30\sqrt{b}\,\sqrt{bx+a}\,\log\left(\frac{\sqrt{bx+a}+\sqrt{x}\,\sqrt{b}}{\sqrt{a}}\right)abx-5\sqrt{b}}{6\sqrt{bx+a}\,b^4\,(bx+}$$

```
int(x^5/(b*x^2+a*x)^(5/2),x)
```

```
( - 30*sqrt(b)*sqrt(a + b*x)*log((sqrt(a + b*x) + sqrt(x)*sqrt(b))/sqrt(a)
)*a**2 - 30*sqrt(b)*sqrt(a + b*x)*log((sqrt(a + b*x) + sqrt(x)*sqrt(b))/sq
rt(a))*a*b*x - 5*sqrt(b)*sqrt(a + b*x)*a**2 - 5*sqrt(b)*sqrt(a + b*x)*a*b*
x + 30*sqrt(x)*a**2*b + 40*sqrt(x)*a*b**2*x + 6*sqrt(x)*b**3*x**2)/(6*sqrt
(a + b*x)*b**4*(a + b*x))
```

3.58 $\int \frac{x^4}{(ax+bx^2)^{5/2}}\,dx$

Optimal result

Integrand size = 17, antiderivative size = 74

$$\int \frac{x^4}{(ax+bx^2)^{5/2}}\,dx = \frac{2ax^2}{3b^2(ax+bx^2)^{3/2}} - \frac{8x}{3b^2\sqrt{ax+bx^2}} + \frac{2\operatorname{arctanh}\left(\frac{\sqrt{b}x}{\sqrt{ax+bx^2}}\right)}{b^{5/2}}$$

```
2/3*a*x^2/b^2/(b*x^2+a*x)^(3/2)-8/3*x/b^2/(b*x^2+a*x)^(1/2)+2*arctanh(b^(1
/2)*x/(b*x^2+a*x)^(1/2))/b^(5/2)
```

Mathematica [A] (verified)

Time = 0.27 (sec) , antiderivative size = 84, normalized size of antiderivative = 1.14

$$\int \frac{x^4}{(ax+bx^2)^{5/2}}\,dx = -\frac{2x\left(\sqrt{b}x(3a+4bx)+6\sqrt{x}(a+bx)^{3/2}\operatorname{arctanh}\left(\frac{\sqrt{b}\sqrt{x}}{\sqrt{a}-\sqrt{a+bx}}\right)\right)}{3b^{5/2}(x(a+bx))^{3/2}}$$

```
Integrate[x^4/(a*x + b*x^2)^(5/2),x]
```

```
(-2*x*(Sqrt[b]*x*(3*a + 4*b*x) + 6*Sqrt[x]*(a + b*x)^(3/2)*ArcTanh[(Sqrt[b
]*Sqrt[x])/(Sqrt[a] - Sqrt[a + b*x])]))/(3*b^(5/2)*(x*(a + b*x))^(3/2))
```

Rubi [A] (verified)

Time = 0.35 (sec) , antiderivative size = 76, normalized size of antiderivative = 1.03, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.235$, Rules used = {1133, 1124, 1091, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^4}{(ax+bx^2)^{5/2}}\,dx$$

$$\Big\downarrow \text{1133}$$

$$\frac{\int \frac{x^2}{(bx^2+ax)^{3/2}}dx}{b}-\frac{2x^3}{3b\,(ax+bx^2)^{3/2}}$$

$$\Big\downarrow \text{1124}$$

$$\frac{\frac{\int \frac{1}{\sqrt{bx^2+ax}}dx}{b}-\frac{2x}{b\sqrt{ax+bx^2}}}{b}-\frac{2x^3}{3b\,(ax+bx^2)^{3/2}}$$

$$\Big\downarrow \text{1091}$$

$$\frac{\frac{2\int \frac{1}{1-\frac{bx^2}{bx^2+ax}}d\frac{x}{\sqrt{bx^2+ax}}}{b}-\frac{2x}{b\sqrt{ax+bx^2}}}{b}-\frac{2x^3}{3b\,(ax+bx^2)^{3/2}}$$

$$\Big\downarrow \text{219}$$

$$\frac{\frac{2\text{arctanh}\left(\frac{\sqrt{b}x}{\sqrt{ax+bx^2}}\right)}{b^{3/2}}-\frac{2x}{b\sqrt{ax+bx^2}}}{b}-\frac{2x^3}{3b\,(ax+bx^2)^{3/2}}$$

```
Int[x^4/(a*x + b*x^2)^(5/2),x]
```

```
(-2*x^3)/(3*b*(a*x + b*x^2)^(3/2)) + ((-2*x)/(b*Sqrt[a*x + b*x^2]) + (2*Ar
cTanh[(Sqrt[b]*x)/Sqrt[a*x + b*x^2]])/b^(3/2))/b
```

Defintions of rubi rules used

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

```
Int[1/Sqrt[(b_.)*(x_) + (c_.)*(x_)^2], x_Symbol] :> Simp[2   Subst[Int[1/(1
 - c*x^2), x], x, x/Sqrt[b*x + c*x^2]], x] /; FreeQ[{b, c}, x]
```

```
Int[((d_.) + (e_.)*(x_))^(m_.)/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(3/2), x
_Symbol] :> Simp[-2*e*(2*c*d - b*e)^(m - 2)*((d + e*x)/(c^(m - 1)*Sqrt[a +
b*x + c*x^2])), x] + Simp[e^2/c^(m - 1)   Int[(1/Sqrt[a + b*x + c*x^2])*Exp
andToSum[((2*c*d - b*e)^(m - 1) - c^(m - 1)*(d + e*x)^(m - 1))/(c*d - b*e -
 c*e*x), x], x], x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[c*d^2 - b*d*e + a*e
^2, 0] && IGtQ[m, 0]
```

```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] :> Simp[e*(d + e*x)^(m - 1)*((a + b*x + c*x^2)^(p + 1)/(c*(p + 1))),
 x] - Simp[e^2*((m + p)/(c*(p + 1)))   Int[(d + e*x)^(m - 2)*(a + b*x + c*x
^2)^(p + 1), x], x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[c*d^2 - b*d*e + a*e
^2, 0] && LtQ[p, -1] && GtQ[m, 1] && IntegerQ[2*p]
```

Maple [A] (verified)

Time = 0.38 (sec) , antiderivative size = 73, normalized size of antiderivative = 0.99

method	result
pseudoelliptic	$\frac{6\sqrt{x(bx+a)}\,\operatorname{arctanh}\left(\frac{\sqrt{x(bx+a)}}{x\sqrt{b}}\right)(bx+a)-6xa\sqrt{b}-8b^{\frac{3}{2}}x^{2}}{b^{\frac{5}{2}}\sqrt{x(bx+a)}\,(3bx+3a)}$
default	$-\frac{x^{3}}{3b(b\,x^{2}+ax)^{\frac{3}{2}}}-\frac{a\left(-\frac{x^{2}}{b\left(b\,x^{2}+ax\right)^{\frac{3}{2}}}+\frac{a\left(-\frac{x}{2b\left(b\,x^{2}+ax\right)^{\frac{3}{2}}}-\frac{a\left(-\frac{1}{3b\left(b\,x^{2}+ax\right)^{\frac{3}{2}}}-\frac{a\left(-\frac{2(2bx+a)}{3a^{2}\left(b\,x^{2}+ax\right)^{\frac{3}{2}}}+\frac{16b(2bx+a)}{3a^{4}\sqrt{b\,x^{2}+ax}}\right)}{2b}\right)}{4b}\right)}{2b}\right)}{2b}$

```
int(x^4/(b*x^2+a*x)^(5/2),x,method=_RETURNVERBOSE)
```

```
6/b^(5/2)/(x*(b*x+a))^(1/2)*((x*(b*x+a))^(1/2)*arctanh((x*(b*x+a))^(1/2)/x
/b^(1/2))*(b*x+a)-x*a*b^(1/2)-4/3*b^(3/2)*x^2)/(3*b*x+3*a)
```

Fricas [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 194, normalized size of antiderivative = 2.62

$$\int \frac{x^4}{(ax+bx^2)^{5/2}}\,dx = \left[\frac{3\,(b^2x^2+2\,abx+a^2)\sqrt{b}\log\left(2\,bx+a+2\,\sqrt{bx^2+ax}\sqrt{b}\right)-2\,(4\,b^2x+3\,ab)\sqrt{bx^2+}}{3\,(b^5x^2+2\,ab^4x+a^2b^3)}\right.$$
$$\left.-\frac{2\left(3\,(b^2x^2+2\,abx+a^2)\sqrt{-b}\arctan\left(\frac{\sqrt{bx^2+ax}\sqrt{-b}}{bx+a}\right)+(4\,b^2x+3\,ab)\sqrt{bx^2+ax}\right)}{3\,(b^5x^2+2\,ab^4x+a^2b^3)}\right]$$

```
integrate(x^4/(b*x^2+a*x)^(5/2),x, algorithm="fricas")
```

```
[1/3*(3*(b^2*x^2 + 2*a*b*x + a^2)*sqrt(b)*log(2*b*x + a + 2*sqrt(b*x^2 + a
*x)*sqrt(b)) - 2*(4*b^2*x + 3*a*b)*sqrt(b*x^2 + a*x))/(b^5*x^2 + 2*a*b^4*x
 + a^2*b^3), -2/3*(3*(b^2*x^2 + 2*a*b*x + a^2)*sqrt(-b)*arctan(sqrt(b*x^2
+ a*x)*sqrt(-b)/(b*x + a)) + (4*b^2*x + 3*a*b)*sqrt(b*x^2 + a*x))/(b^5*x^2
 + 2*a*b^4*x + a^2*b^3)]
```

Sympy [F]

$$\int \frac{x^4}{(ax+bx^2)^{5/2}}\,dx = \int \frac{x^4}{(x\,(a+bx))^{\frac{5}{2}}}\,dx$$

```
integrate(x**4/(b*x**2+a*x)**(5/2),x)
```

```
Integral(x**4/(x*(a + b*x))**(5/2), x)
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 140 vs. $2(60) = 120$.

Time = 0.04 (sec) , antiderivative size = 140, normalized size of antiderivative = 1.89

$$\int \frac{x^4}{(ax+bx^2)^{5/2}}\,dx = -\frac{1}{3}\,x\left(\frac{3\,x^2}{(bx^2+ax)^{\frac{3}{2}}b} + \frac{ax}{(bx^2+ax)^{\frac{3}{2}}b^2} - \frac{2\,x}{\sqrt{bx^2+ax}ab} - \frac{1}{\sqrt{bx^2+ax}b^2}\right) - \frac{4\,x}{3\,\sqrt{bx^2+ax}b^2} + \frac{\log\left(2\,bx+a+2\,\sqrt{bx^2+ax}\sqrt{b}\right)}{b^{\frac{5}{2}}} - \frac{2\,\sqrt{bx^2+ax}}{3\,ab^2}$$

```
integrate(x^4/(b*x^2+a*x)^(5/2),x, algorithm="maxima")
```

```
-1/3*x*(3*x^2/((b*x^2 + a*x)^(3/2)*b) + a*x/((b*x^2 + a*x)^(3/2)*b^2) - 2*
x/(sqrt(b*x^2 + a*x)*a*b) - 1/(sqrt(b*x^2 + a*x)*b^2)) - 4/3*x/(sqrt(b*x^2
 + a*x)*b^2) + log(2*b*x + a + 2*sqrt(b*x^2 + a*x)*sqrt(b))/b^(5/2) - 2/3*
sqrt(b*x^2 + a*x)/(a*b^2)
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 124 vs. $2(60) = 120$.

Time = 0.14 (sec) , antiderivative size = 124, normalized size of antiderivative = 1.68

$$\int \frac{x^4}{(ax+bx^2)^{5/2}}\,dx = -\frac{\log\left(\left|2\left(\sqrt{b}x-\sqrt{bx^2+ax}\right)\sqrt{b}+a\right|\right)}{b^{\frac{5}{2}}} - \frac{2\left(6\left(\sqrt{b}x-\sqrt{bx^2+ax}\right)^2ab+9\left(\sqrt{b}x-\sqrt{bx^2+ax}\right)a^2\sqrt{b}+4\,a^3\right)}{3\left(\left(\sqrt{b}x-\sqrt{bx^2+ax}\right)\sqrt{b}+a\right)^3b^{\frac{5}{2}}}$$

```
integrate(x^4/(b*x^2+a*x)^(5/2),x, algorithm="giac")
```

```
-log(abs(2*(sqrt(b)*x - sqrt(b*x^2 + a*x))*sqrt(b) + a))/b^(5/2) - 2/3*(6*
(sqrt(b)*x - sqrt(b*x^2 + a*x))^2*a*b + 9*(sqrt(b)*x - sqrt(b*x^2 + a*x))*
a^2*sqrt(b) + 4*a^3)/(((sqrt(b)*x - sqrt(b*x^2 + a*x))*sqrt(b) + a)^3*b^(5
/2))
```

Mupad [F(-1)]

Timed out.

$$\int \frac{x^4}{(ax+bx^2)^{5/2}}\,dx = \int \frac{x^4}{{\left(b\,x^2+a\,x\right)}^{5/2}}\,dx$$

```
int(x^4/(a*x + b*x^2)^(5/2),x)
```

```
int(x^4/(a*x + b*x^2)^(5/2), x)
```

Reduce [B] (verification not implemented)

Time = 0.41 (sec) , antiderivative size = 94, normalized size of antiderivative = 1.27

$$\int \frac{x^4}{(ax+bx^2)^{5/2}}\,dx = \frac{2\sqrt{b}\,\sqrt{bx+a}\log\left(\frac{\sqrt{bx+a}+\sqrt{x}\,\sqrt{b}}{\sqrt{a}}\right)a+2\sqrt{b}\,\sqrt{bx+a}\log\left(\frac{\sqrt{bx+a}+\sqrt{x}\,\sqrt{b}}{\sqrt{a}}\right)bx-2\sqrt{x}\,ab-}{\sqrt{bx+a}\,b^3\,(bx+a)}$$

```
int(x^4/(b*x^2+a*x)^(5/2),x)
```

```
(2*(3*sqrt(b)*sqrt(a + b*x)*log((sqrt(a + b*x) + sqrt(x)*sqrt(b))/sqrt(a))
*a + 3*sqrt(b)*sqrt(a + b*x)*log((sqrt(a + b*x) + sqrt(x)*sqrt(b))/sqrt(a)
)*b*x - 3*sqrt(x)*a*b - 4*sqrt(x)*b**2*x))/(3*sqrt(a + b*x)*b**3*(a + b*x)
)
```

3.59 $\int \frac{x^3}{(ax+bx^2)^{5/2}}\,dx$

Optimal result

Integrand size = 17, antiderivative size = 23

$$\int \frac{x^3}{(ax+bx^2)^{5/2}}\,dx = \frac{2x^3}{3a\,(ax+bx^2)^{3/2}}$$

```
2/3*x^3/a/(b*x^2+a*x)^(3/2)
```

Mathematica [A] (verified)

Time = 0.04 (sec) , antiderivative size = 21, normalized size of antiderivative = 0.91

$$\int \frac{x^3}{(ax+bx^2)^{5/2}}\,dx = \frac{2x^3}{3a(x(a+bx))^{3/2}}$$

```
Integrate[x^3/(a*x + b*x^2)^(5/2),x]
```

```
(2*x^3)/(3*a*(x*(a + b*x))^(3/2))
```

Rubi [A] (verified)

Time = 0.23 (sec) , antiderivative size = 23, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}} = 0.059$, Rules used = $\{1123\}$

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^3}{(ax+bx^2)^{5/2}}\,dx$$
$$\Big\downarrow \text{ 1123}$$
$$\frac{2x^3}{3a\,(ax+bx^2)^{3/2}}$$

```
Int[x^3/(a*x + b*x^2)^(5/2),x]
```

```
(2*x^3)/(3*a*(a*x + b*x^2)^(3/2))
```

Defintions of rubi rules used

```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] :> Simp[e*(d + e*x)^m*((a + b*x + c*x^2)^(p + 1)/((p + 1)*(2*c*d - b
*e))), x] /; FreeQ[{a, b, c, d, e, m, p}, x] && EqQ[c*d^2 - b*d*e + a*e^2,
0] && EqQ[m + 2*p + 2, 0]
```

Maple [A] (verified)

Time = 0.36 (sec) , antiderivative size = 25, normalized size of antiderivative = 1.09

method	result	size
gosper	$\frac{2x^{4}(bx+a)}{3a(b\,x^{2}+ax)^{\frac{5}{2}}}$	25
trager	$\frac{2x\sqrt{b\,x^{2}+ax}}{3(bx+a)^{2}a}$	25
pseudoelliptic	$\frac{2x^{2}}{3a(bx+a)\sqrt{x(bx+a)}}$	25
orering	$\frac{2x^{4}(bx+a)}{3a(b\,x^{2}+ax)^{\frac{5}{2}}}$	25
default	$-\frac{x^{2}}{b(b\,x^{2}+ax)^{\frac{3}{2}}}+\frac{a\left(-\frac{x}{2b\left(b\,x^{2}+ax\right)^{\frac{3}{2}}}-\frac{a\left(-\frac{1}{3b\left(b\,x^{2}+ax\right)^{\frac{3}{2}}}-\frac{a\left(-\frac{2(2bx+a)}{3a^{2}\left(b\,x^{2}+ax\right)^{\frac{3}{2}}}+\frac{16b(2bx+a)}{3a^{4}\sqrt{b\,x^{2}+ax}}\right)}{2b}\right)}{4b}\right)}{2b}$	120

```
int(x^3/(b*x^2+a*x)^(5/2),x,method=_RETURNVERBOSE)
```

```
2/3*x^4*(b*x+a)/a/(b*x^2+a*x)^(5/2)
```

Fricas [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 35, normalized size of antiderivative = 1.52

$$\int \frac{x^3}{(ax+bx^2)^{5/2}}\,dx = \frac{2\sqrt{bx^2+ax}\,x}{3\,(ab^2x^2+2\,a^2bx+a^3)}$$

```
integrate(x^3/(b*x^2+a*x)^(5/2),x, algorithm="fricas")
```

```
2/3*sqrt(b*x^2 + a*x)*x/(a*b^2*x^2 + 2*a^2*b*x + a^3)
```

Sympy [F]

$$\int \frac{x^3}{(ax+bx^2)^{5/2}}\,dx = \int \frac{x^3}{(x(a+bx))^{\frac{5}{2}}}\,dx$$

```
integrate(x**3/(b*x**2+a*x)**(5/2),x)
```

```
Integral(x**3/(x*(a + b*x))**(5/2), x)
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 74 vs. $2(19) = 38$.

Time = 0.03 (sec) , antiderivative size = 74, normalized size of antiderivative = 3.22

$$\int \frac{x^3}{(ax+bx^2)^{5/2}}\,dx = -\frac{x^2}{(bx^2+ax)^{\frac{3}{2}}b} - \frac{ax}{3\,(bx^2+ax)^{\frac{3}{2}}b^2} + \frac{2\,x}{3\,\sqrt{bx^2+ax}ab} + \frac{1}{3\,\sqrt{bx^2+ax}b^2}$$

```
integrate(x^3/(b*x^2+a*x)^(5/2),x, algorithm="maxima")
```

```
-x^2/((b*x^2 + a*x)^(3/2)*b) - 1/3*a*x/((b*x^2 + a*x)^(3/2)*b^2) + 2/3*x/(
sqrt(b*x^2 + a*x)*a*b) + 1/3/(sqrt(b*x^2 + a*x)*b^2)
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 85 vs. $2(19) = 38$.

Time = 0.15 (sec) , antiderivative size = 85, normalized size of antiderivative = 3.70

$$\int \frac{x^3}{(ax+bx^2)^{5/2}}\,dx = \frac{2\left(3\left(\sqrt{b}x-\sqrt{bx^2+ax}\right)^2b+3\left(\sqrt{b}x-\sqrt{bx^2+ax}\right)a\sqrt{b}+a^2\right)}{3\left(\left(\sqrt{b}x-\sqrt{bx^2+ax}\right)\sqrt{b}+a\right)^3b^{\frac{3}{2}}}$$

```
integrate(x^3/(b*x^2+a*x)^(5/2),x, algorithm="giac")
```

```
2/3*(3*(sqrt(b)*x - sqrt(b*x^2 + a*x))^2*b + 3*(sqrt(b)*x - sqrt(b*x^2 + a
*x))*a*sqrt(b) + a^2)/(((sqrt(b)*x - sqrt(b*x^2 + a*x))*sqrt(b) + a)^3*b^(
3/2))
```

Mupad [B] (verification not implemented)

Time = 9.54 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.04

$$\int \frac{x^3}{(ax+bx^2)^{5/2}}\,dx = \frac{2\,x\,\sqrt{b\,x^2+a\,x}}{3\,a\,{\left(a+b\,x\right)}^2}$$

```
int(x^3/(a*x + b*x^2)^(5/2),x)
```

```
(2*x*(a*x + b*x^2)^(1/2))/(3*a*(a + b*x)^2)
```

Reduce [B] (verification not implemented)

Time = 0.19 (sec) , antiderivative size = 52, normalized size of antiderivative = 2.26

$$\int \frac{x^3}{(ax+bx^2)^{5/2}}\,dx = \frac{\frac{2\sqrt{b}\,\sqrt{bx+a}\,a}{3}+\frac{2\sqrt{b}\,\sqrt{bx+a}\,bx}{3}+\frac{2\sqrt{x}\,b^2x}{3}}{\sqrt{bx+a}\,a\,b^2\,(bx+a)}$$

```
int(x^3/(b*x^2+a*x)^(5/2),x)
```

```
(2*(sqrt(b)*sqrt(a + b*x)*a + sqrt(b)*sqrt(a + b*x)*b*x + sqrt(x)*b**2*x))
/(3*sqrt(a + b*x)*a*b**2*(a + b*x))
```

3.60 $\int \frac{x^2}{(ax+bx^2)^{5/2}}\,dx$

Optimal result

Integrand size = 17, antiderivative size = 45

$$\int \frac{x^2}{(ax+bx^2)^{5/2}}\,dx=\frac{2x^2}{3a\,(ax+bx^2)^{3/2}}+\frac{4x}{3a^2\sqrt{ax+bx^2}}$$

```
2/3*x^2/a/(b*x^2+a*x)^(3/2)+4/3*x/a^2/(b*x^2+a*x)^(1/2)
```

Mathematica [A] (verified)

Time = 0.08 (sec) , antiderivative size = 29, normalized size of antiderivative = 0.64

$$\int \frac{x^2}{(ax+bx^2)^{5/2}}\,dx=\frac{2x^2(3a+2bx)}{3a^2(x(a+bx))^{3/2}}$$

```
Integrate[x^2/(a*x + b*x^2)^(5/2),x]
```

```
(2*x^2*(3*a + 2*b*x))/(3*a^2*(x*(a + b*x))^(3/2))
```

Rubi [A] (verified)

Time = 0.28 (sec) , antiderivative size = 51, normalized size of antiderivative = 1.13, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.118$, Rules used = {1126, 1088}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^2}{(ax+bx^2)^{5/2}}\,dx$$

$$\Big\downarrow \text{1126}$$

$$-\frac{\int \frac{1}{(bx^2+ax)^{3/2}}dx}{3b}-\frac{2x}{3b\left(ax+bx^2\right)^{3/2}}$$

$$\Big\downarrow \text{1088}$$

$$\frac{2(a+2bx)}{3a^2b\sqrt{ax+bx^2}}-\frac{2x}{3b\left(ax+bx^2\right)^{3/2}}$$

```
Int[x^2/(a*x + b*x^2)^(5/2),x]
```

```
(-2*x)/(3*b*(a*x + b*x^2)^(3/2)) + (2*(a + 2*b*x))/(3*a^2*b*Sqrt[a*x + b*x
^2])
```

Defintions of rubi rules used

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-3/2), x_Symbol] :> Simp[-2*((b +
2*c*x)/((b^2 - 4*a*c)*Sqrt[a + b*x + c*x^2])), x] /; FreeQ[{a, b, c}, x] &&
 NeQ[b^2 - 4*a*c, 0]
```

```
Int[((d_.) + (e_.)*(x_))^2*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symb
ol] :> Simp[e*(d + e*x)*((a + b*x + c*x^2)^(p + 1)/(c*(p + 1))), x] - Simp[
e^2*((p + 2)/(c*(p + 1)))   Int[(a + b*x + c*x^2)^(p + 1), x], x] /; FreeQ[
{a, b, c, d, e, p}, x] && EqQ[c*d^2 - b*d*e + a*e^2, 0] && LtQ[p, -1]
```

Maple [A] (verified)

Time = 0.35 (sec) , antiderivative size = 29, normalized size of antiderivative = 0.64

method	result	size
pseudoelliptic	$\frac{2\left(\frac{2bx}{3}+a\right)x}{\sqrt{x(bx+a)}\,(bx+a)a^{2}}$	29
trager	$\frac{2(2bx+3a)\sqrt{b\,x^{2}+ax}}{3a^{2}(bx+a)^{2}}$	32
gosper	$\frac{2x^{3}(bx+a)(2bx+3a)}{3a^{2}(b\,x^{2}+ax)^{\frac{5}{2}}}$	33
orering	$\frac{2x^{3}(bx+a)(2bx+3a)}{3a^{2}(b\,x^{2}+ax)^{\frac{5}{2}}}$	33
default	$-\frac{x}{2b(b\,x^{2}+ax)^{\frac{3}{2}}}-\frac{a\left(-\frac{1}{3b\left(b\,x^{2}+ax\right)^{\frac{3}{2}}}-\frac{a\left(-\frac{2(2bx+a)}{3a^{2}\left(b\,x^{2}+ax\right)^{\frac{3}{2}}}+\frac{16b(2bx+a)}{3a^{4}\sqrt{b\,x^{2}+ax}}\right)}{2b}\right)}{4b}$	94

```
int(x^2/(b*x^2+a*x)^(5/2),x,method=_RETURNVERBOSE)
```

```
2/(x*(b*x+a))^(1/2)*(2/3*b*x+a)*x/(b*x+a)/a^2
```

Fricas [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 44, normalized size of antiderivative = 0.98

$$\int \frac{x^2}{(ax+bx^2)^{5/2}}\,dx = \frac{2\sqrt{bx^2+ax}(2\,bx+3\,a)}{3\,(a^2b^2x^2+2\,a^3bx+a^4)}$$

```
integrate(x^2/(b*x^2+a*x)^(5/2),x, algorithm="fricas")
```

```
2/3*sqrt(b*x^2 + a*x)*(2*b*x + 3*a)/(a^2*b^2*x^2 + 2*a^3*b*x + a^4)
```

Sympy [F]

$$\int \frac{x^2}{(ax+bx^2)^{5/2}}\,dx = \int \frac{x^2}{(x(a+bx))^{\frac{5}{2}}}\,dx$$

```
integrate(x**2/(b*x**2+a*x)**(5/2),x)
```

```
Integral(x**2/(x*(a + b*x))**(5/2), x)
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 54, normalized size of antiderivative = 1.20

$$\int \frac{x^2}{(ax+bx^2)^{5/2}}\,dx = \frac{4\,x}{3\,\sqrt{bx^2+ax}a^2} - \frac{2\,x}{3\,(bx^2+ax)^{\frac{3}{2}}b} + \frac{2}{3\,\sqrt{bx^2+ax}ab}$$

```
integrate(x^2/(b*x^2+a*x)^(5/2),x, algorithm="maxima")
```

```
4/3*x/(sqrt(b*x^2 + a*x)*a^2) - 2/3*x/((b*x^2 + a*x)^(3/2)*b) + 2/3/(sqrt(
b*x^2 + a*x)*a*b)
```

Giac [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 60, normalized size of antiderivative = 1.33

$$\int \frac{x^2}{(ax+bx^2)^{5/2}}\,dx = \frac{2\left(3\left(\sqrt{b}x-\sqrt{bx^2+ax}\right)\sqrt{b}+2\,a\right)}{3\left(\left(\sqrt{b}x-\sqrt{bx^2+ax}\right)\sqrt{b}+a\right)^3\sqrt{b}}$$

```
integrate(x^2/(b*x^2+a*x)^(5/2),x, algorithm="giac")
```

```
2/3*(3*(sqrt(b)*x - sqrt(b*x^2 + a*x))*sqrt(b) + 2*a)/(((sqrt(b)*x - sqrt(
b*x^2 + a*x))*sqrt(b) + a)^3*sqrt(b))
```

Mupad [B] (verification not implemented)

Time = 9.79 (sec) , antiderivative size = 31, normalized size of antiderivative = 0.69

$$\int \frac{x^2}{(ax+bx^2)^{5/2}}\,dx = \frac{2\sqrt{b\,x^2+a\,x}\,(3\,a+2\,b\,x)}{3\,a^2\,{(a+b\,x)}^2}$$

```
int(x^2/(a*x + b*x^2)^(5/2),x)
```

```
(2*(a*x + b*x^2)^(1/2)*(3*a + 2*b*x))/(3*a^2*(a + b*x)^2)
```

Reduce [B] (verification not implemented)

Time = 0.34 (sec) , antiderivative size = 61, normalized size of antiderivative = 1.36

$$\int \frac{x^2}{(ax+bx^2)^{5/2}}\,dx = \frac{-\frac{4\sqrt{b}\,\sqrt{bx+a}\,a}{3}-\frac{4\sqrt{b}\,\sqrt{bx+a}\,bx}{3}+2\sqrt{x}\,ab+\frac{4\sqrt{x}\,b^2x}{3}}{\sqrt{bx+a}\,a^2b\,(bx+a)}$$

```
int(x^2/(b*x^2+a*x)^(5/2),x)
```

```
(2*( - 2*sqrt(b)*sqrt(a + b*x)*a - 2*sqrt(b)*sqrt(a + b*x)*b*x + 3*sqrt(x)
*a*b + 2*sqrt(x)*b**2*x))/(3*sqrt(a + b*x)*a**2*b*(a + b*x))
```

3.61 $\int \frac{x}{(ax+bx^2)^{5/2}} \, dx$

Optimal result

Integrand size = 15, antiderivative size = 65

$$\int \frac{x}{(ax+bx^2)^{5/2}} \, dx = \frac{2x}{3a\,(ax+bx^2)^{3/2}} + \frac{8}{3a^2\sqrt{ax+bx^2}} - \frac{16\sqrt{ax+bx^2}}{3a^3x}$$

```
2/3*x/a/(b*x^2+a*x)^(3/2)+8/3/a^2/(b*x^2+a*x)^(1/2)-16/3*(b*x^2+a*x)^(1/2)
/a^3/x
```

Mathematica [A] (verified)

Time = 0.09 (sec) , antiderivative size = 38, normalized size of antiderivative = 0.58

$$\int \frac{x}{(ax+bx^2)^{5/2}} \, dx = -\frac{2x(3a^2+12abx+8b^2x^2)}{3a^3(x(a+bx))^{3/2}}$$

```
Integrate[x/(a*x + b*x^2)^(5/2),x]
```

```
(-2*x*(3*a^2 + 12*a*b*x + 8*b^2*x^2))/(3*a^3*(x*(a + b*x))^(3/2))
```

Rubi [A] (verified)

Time = 0.27 (sec) , antiderivative size = 48, normalized size of antiderivative = 0.74, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.133$, Rules used = {1159, 1088}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x}{(ax+bx^2)^{5/2}}\,dx$$

$$\Big\downarrow \text{1159}$$

$$\frac{4\int \frac{1}{(bx^2+ax)^{3/2}}dx}{3a}+\frac{2x}{3a\,(ax+bx^2)^{3/2}}$$

$$\Big\downarrow \text{1088}$$

$$\frac{2x}{3a\,(ax+bx^2)^{3/2}}-\frac{8(a+2bx)}{3a^3\sqrt{ax+bx^2}}$$

```
Int[x/(a*x + b*x^2)^(5/2),x]
```

```
(2*x)/(3*a*(a*x + b*x^2)^(3/2)) - (8*(a + 2*b*x))/(3*a^3*Sqrt[a*x + b*x^2]
)
```

Defintions of rubi rules used

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-3/2), x_Symbol] :> Simp[-2*((b +
2*c*x)/((b^2 - 4*a*c)*Sqrt[a + b*x + c*x^2])), x] /; FreeQ[{a, b, c}, x] &&
 NeQ[b^2 - 4*a*c, 0]
```

```
Int[((d_.) + (e_.)*(x_))*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol
] :> Simp[((b*d - 2*a*e + (2*c*d - b*e)*x)/((p + 1)*(b^2 - 4*a*c)))*(a + b*
x + c*x^2)^(p + 1), x] - Simp[(2*p + 3)*((2*c*d - b*e)/((p + 1)*(b^2 - 4*a*
c)))   Int[(a + b*x + c*x^2)^(p + 1), x], x] /; FreeQ[{a, b, c, d, e}, x] &
& LtQ[p, -1] && NeQ[p, -3/2]
```

Maple [A] (verified)

Time = 0.36 (sec) , antiderivative size = 39, normalized size of antiderivative = 0.60

method	result	size
pseudoelliptic	$-\frac{2\left(\frac{8}{3}b^{2}x^{2}+4abx+a^{2}\right)}{\sqrt{x(bx+a)}\,(bx+a)a^{3}}$	39
gosper	$-\frac{2x^{2}(bx+a)\left(8b^{2}x^{2}+12abx+3a^{2}\right)}{3a^{3}\left(b\,x^{2}+ax\right)^{\frac{5}{2}}}$	44
orering	$-\frac{2x^{2}(bx+a)\left(8b^{2}x^{2}+12abx+3a^{2}\right)}{3a^{3}\left(b\,x^{2}+ax\right)^{\frac{5}{2}}}$	44
trager	$-\frac{2\left(8b^{2}x^{2}+12abx+3a^{2}\right)\sqrt{b\,x^{2}+ax}}{3a^{3}(bx+a)^{2}x}$	46
risch	$-\frac{2(bx+a)}{a^{3}\sqrt{x(bx+a)}}-\frac{2b(5bx+6a)x}{3\sqrt{x(bx+a)}\,(bx+a)a^{3}}$	52
default	$-\frac{1}{3b\left(b\,x^{2}+ax\right)^{\frac{3}{2}}}-\frac{a\left(-\frac{2(2bx+a)}{3a^{2}\left(b\,x^{2}+ax\right)^{\frac{3}{2}}}+\frac{16b(2bx+a)}{3a^{4}\sqrt{b\,x^{2}+ax}}\right)}{2b}$	70

```
int(x/(b*x^2+a*x)^(5/2),x,method=_RETURNVERBOSE)
```

```
-2/(x*(b*x+a))^(1/2)*(8/3*b^2*x^2+4*a*b*x+a^2)/(b*x+a)/a^3
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 59, normalized size of antiderivative = 0.91

$$\int \frac{x}{(ax+bx^2)^{5/2}}\,dx = -\frac{2\,(8\,b^2x^2+12\,abx+3\,a^2)\sqrt{bx^2+ax}}{3\,(a^3b^2x^3+2\,a^4bx^2+a^5x)}$$

```
integrate(x/(b*x^2+a*x)^(5/2),x, algorithm="fricas")
```

```
-2/3*(8*b^2*x^2 + 12*a*b*x + 3*a^2)*sqrt(b*x^2 + a*x)/(a^3*b^2*x^3 + 2*a^4
*b*x^2 + a^5*x)
```

Sympy [F]

$$\int \frac{x}{(ax+bx^2)^{5/2}}\,dx = \int \frac{x}{(x(a+bx))^{\frac{5}{2}}}\,dx$$

```
integrate(x/(b*x**2+a*x)**(5/2),x)
```

```
Integral(x/(x*(a + b*x))**(5/2), x)
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 52, normalized size of antiderivative = 0.80

$$\int \frac{x}{(ax+bx^2)^{5/2}}\,dx = \frac{2\,x}{3\,(bx^2+ax)^{\frac{3}{2}}a} - \frac{16\,bx}{3\,\sqrt{bx^2+ax}a^3} - \frac{8}{3\,\sqrt{bx^2+ax}a^2}$$

```
integrate(x/(b*x^2+a*x)^(5/2),x, algorithm="maxima")
```

```
2/3*x/((b*x^2 + a*x)^(3/2)*a) - 16/3*b*x/(sqrt(b*x^2 + a*x)*a^3) - 8/3/(sq
rt(b*x^2 + a*x)*a^2)
```

Giac [F]

$$\int \frac{x}{(ax+bx^2)^{5/2}}\,dx = \int \frac{x}{(bx^2+ax)^{\frac{5}{2}}}\,dx$$

```
integrate(x/(b*x^2+a*x)^(5/2),x, algorithm="giac")
```

```
integrate(x/(b*x^2 + a*x)^(5/2), x)
```

Mupad [B] (verification not implemented)

Time = 10.02 (sec) , antiderivative size = 45, normalized size of antiderivative = 0.69

$$\int \frac{x}{(ax+bx^2)^{5/2}}\,dx = -\frac{2\sqrt{b\,x^2+a\,x}\,(3\,a^2+12\,a\,b\,x+8\,b^2\,x^2)}{3\,a^3\,x\,(a+b\,x)^2}$$

```
int(x/(a*x + b*x^2)^(5/2),x)
```

```
-(2*(a*x + b*x^2)^(1/2)*(3*a^2 + 8*b^2*x^2 + 12*a*b*x))/(3*a^3*x*(a + b*x)
^2)
```

Reduce [B] (verification not implemented)

Time = 0.52 (sec) , antiderivative size = 74, normalized size of antiderivative = 1.14

$$\int \frac{x}{(ax+bx^2)^{5/2}}\,dx = \frac{\frac{16\sqrt{b}\,\sqrt{bx+a}\,ax}{3}+\frac{16\sqrt{b}\,\sqrt{bx+a}\,b\,x^2}{3}-2\sqrt{x}\,a^2-8\sqrt{x}\,abx-\frac{16\sqrt{x}\,b^2x^2}{3}}{\sqrt{bx+a}\,a^3x\,(bx+a)}$$

```
int(x/(b*x^2+a*x)^(5/2),x)
```

```
(2*(8*sqrt(b)*sqrt(a + b*x)*a*x + 8*sqrt(b)*sqrt(a + b*x)*b*x**2 - 3*sqrt(
x)*a**2 - 12*sqrt(x)*a*b*x - 8*sqrt(x)*b**2*x**2))/(3*sqrt(a + b*x)*a**3*x
*(a + b*x))
```

3.62 $\int \frac{1}{(ax+bx^2)^{5/2}}\,dx$

Optimal result

Integrand size = 13, antiderivative size = 89

$$\int \frac{1}{(ax+bx^2)^{5/2}}\,dx=\frac{2}{3a\,(ax+bx^2)^{3/2}}+\frac{4}{a^2x\sqrt{ax+bx^2}}-\frac{16\sqrt{ax+bx^2}}{3a^3x^2}+\frac{32b\sqrt{ax+bx^2}}{3a^4x}$$

```
2/3/a/(b*x^2+a*x)^(3/2)+4/a^2/x/(b*x^2+a*x)^(1/2)-16/3*(b*x^2+a*x)^(1/2)/a
^3/x^2+32/3*b*(b*x^2+a*x)^(1/2)/a^4/x
```

Mathematica [A] (verified)

Time = 0.11 (sec) , antiderivative size = 48, normalized size of antiderivative = 0.54

$$\int \frac{1}{(ax+bx^2)^{5/2}}\,dx=\frac{-2a^3+12a^2bx+48ab^2x^2+32b^3x^3}{3a^4(x(a+bx))^{3/2}}$$

```
Integrate[(a*x + b*x^2)^(-5/2),x]
```

```
(-2*a^3 + 12*a^2*b*x + 48*a*b^2*x^2 + 32*b^3*x^3)/(3*a^4*(x*(a + b*x))^(3/
2))
```

Rubi [A] (verified)

Time = 0.27 (sec) , antiderivative size = 54, normalized size of antiderivative = 0.61, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.154, Rules used = {1089, 1088}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{(ax+bx^2)^{5/2}}\,dx$$

$$\Big\downarrow \text{1089}$$

$$-\frac{8b\int \frac{1}{(bx^2+ax)^{3/2}}dx}{3a^2}-\frac{2(a+2bx)}{3a^2\,(ax+bx^2)^{3/2}}$$

$$\Big\downarrow \text{1088}$$

$$\frac{16b(a+2bx)}{3a^4\sqrt{ax+bx^2}}-\frac{2(a+2bx)}{3a^2\,(ax+bx^2)^{3/2}}$$

```
Int[(a*x + b*x^2)^(-5/2),x]
```

```
(-2*(a + 2*b*x))/(3*a^2*(a*x + b*x^2)^(3/2)) + (16*b*(a + 2*b*x))/(3*a^4*S
qrt[a*x + b*x^2])
```

Defintions of rubi rules used

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-3/2), x_Symbol] :> Simp[-2*((b +
2*c*x)/((b^2 - 4*a*c)*Sqrt[a + b*x + c*x^2])), x] /; FreeQ[{a, b, c}, x] &&
 NeQ[b^2 - 4*a*c, 0]
```

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(b + 2*c*x)
*((a + b*x + c*x^2)^(p + 1)/((p + 1)*(b^2 - 4*a*c))), x] - Simp[2*c*((2*p +
 3)/((p + 1)*(b^2 - 4*a*c)))   Int[(a + b*x + c*x^2)^(p + 1), x], x] /; Fre
eQ[{a, b, c}, x] && LtQ[p, -1] && (IntegerQ[4*p] || IntegerQ[3*p])
```

Maple [A] (verified)

Time = 0.36 (sec) , antiderivative size = 47, normalized size of antiderivative = 0.53

method	result	size
default	$-\frac{2(2bx+a)}{3a^{2}(b\,x^{2}+ax)^{\frac{3}{2}}}+\frac{16b(2bx+a)}{3a^{4}\sqrt{b\,x^{2}+ax}}$	47
pseudoelliptic	$-\frac{2(2bx+a)\left(-8b^{2}x^{2}-8abx+a^{2}\right)}{3\sqrt{x(bx+a)}\,x(bx+a)a^{4}}$	48
gosper	$-\frac{2x(bx+a)\left(-16b^{3}x^{3}-24a\,b^{2}x^{2}-6a^{2}bx+a^{3}\right)}{3a^{4}(b\,x^{2}+ax)^{\frac{5}{2}}}$	51
orering	$-\frac{2x(bx+a)\left(-16b^{3}x^{3}-24a\,b^{2}x^{2}-6a^{2}bx+a^{3}\right)}{3a^{4}(b\,x^{2}+ax)^{\frac{5}{2}}}$	51
trager	$-\frac{2\left(-16b^{3}x^{3}-24a\,b^{2}x^{2}-6a^{2}bx+a^{3}\right)\sqrt{b\,x^{2}+ax}}{3a^{4}x^{2}(bx+a)^{2}}$	55
risch	$-\frac{2(bx+a)(-8bx+a)}{3a^{4}x\sqrt{x(bx+a)}}+\frac{2b^{2}(8bx+9a)x}{3\sqrt{x(bx+a)}\,(bx+a)a^{4}}$	63

```
int(1/(b*x^2+a*x)^(5/2),x,method=_RETURNVERBOSE)
```

```
-2/3*(2*b*x+a)/a^2/(b*x^2+a*x)^(3/2)+16/3*b/a^4*(2*b*x+a)/(b*x^2+a*x)^(1/2
)
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 72, normalized size of antiderivative = 0.81

$$\int \frac{1}{(ax+bx^2)^{5/2}}\,dx = \frac{2\,(16\,b^3x^3+24\,ab^2x^2+6\,a^2bx-a^3)\sqrt{bx^2+ax}}{3\,(a^4b^2x^4+2\,a^5bx^3+a^6x^2)}$$

```
integrate(1/(b*x^2+a*x)^(5/2),x, algorithm="fricas")
```

```
2/3*(16*b^3*x^3 + 24*a*b^2*x^2 + 6*a^2*b*x - a^3)*sqrt(b*x^2 + a*x)/(a^4*b
^2*x^4 + 2*a^5*b*x^3 + a^6*x^2)
```

Sympy [F]

$$\int \frac{1}{(ax+bx^2)^{5/2}}\,dx = \int \frac{1}{(ax+bx^2)^{\frac{5}{2}}}\,dx$$

```
integrate(1/(b*x**2+a*x)**(5/2),x)
```

```
Integral((a*x + b*x**2)**(-5/2), x)
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 72, normalized size of antiderivative = 0.81

$$\int \frac{1}{(ax+bx^2)^{5/2}}\,dx = -\frac{4\,bx}{3\,(bx^2+ax)^{\frac{3}{2}}a^2} + \frac{32\,b^2x}{3\,\sqrt{bx^2+ax}a^4} - \frac{2}{3\,(bx^2+ax)^{\frac{3}{2}}a} + \frac{16\,b}{3\,\sqrt{bx^2+ax}a^3}$$

```
integrate(1/(b*x^2+a*x)^(5/2),x, algorithm="maxima")
```

```
-4/3*b*x/((b*x^2 + a*x)^(3/2)*a^2) + 32/3*b^2*x/(sqrt(b*x^2 + a*x)*a^4) -
2/3/((b*x^2 + a*x)^(3/2)*a) + 16/3*b/(sqrt(b*x^2 + a*x)*a^3)
```

Giac [A] (verification not implemented)

Time = 0.30 (sec) , antiderivative size = 50, normalized size of antiderivative = 0.56

$$\int \frac{1}{(ax+bx^2)^{5/2}}\,dx = \frac{2\left(2\left(4\,x\left(\frac{2\,b^3x}{a^4}+\frac{3\,b^2}{a^3}\right)+\frac{3\,b}{a^2}\right)x-\frac{1}{a}\right)}{3\,(bx^2+ax)^{\frac{3}{2}}}$$

```
integrate(1/(b*x^2+a*x)^(5/2),x, algorithm="giac")
```

```
2/3*(2*(4*x*(2*b^3*x/a^4 + 3*b^2/a^3) + 3*b/a^2)*x - 1/a)/(b*x^2 + a*x)^(3
/2)
```

Mupad [B] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 43, normalized size of antiderivative = 0.48

$$\int \frac{1}{(ax+bx^2)^{5/2}}\,dx = \frac{(2\,a+4\,b\,x)\,\left(-a^2+8\,a\,b\,x+8\,b^2\,x^2\right)}{3\,a^4\,{\left(b\,x^2+a\,x\right)}^{3/2}}$$

```
int(1/(a*x + b*x^2)^(5/2),x)
```

```
((2*a + 4*b*x)*(8*b^2*x^2 - a^2 + 8*a*b*x))/(3*a^4*(a*x + b*x^2)^(3/2))
```

Reduce [B] (verification not implemented)

Time = 0.46 (sec) , antiderivative size = 92, normalized size of antiderivative = 1.03

$$\int \frac{1}{(ax+bx^2)^{5/2}}\,dx = \frac{-\frac{32\sqrt{b}\,\sqrt{bx+a}\,ab\,x^2}{3}-\frac{32\sqrt{b}\,\sqrt{bx+a}\,b^2x^3}{3}-\frac{2\sqrt{x}\,a^3}{3}+4\sqrt{x}\,a^2bx+16\sqrt{x}\,a\,b^2x^2+\frac{32\sqrt{x}\,b^3x^3}{3}}{\sqrt{bx+a}\,a^4x^2\,(bx+a)}$$

```
int(1/(b*x^2+a*x)^(5/2),x)
```

```
(2*( - 16*sqrt(b)*sqrt(a + b*x)*a*b*x**2 - 16*sqrt(b)*sqrt(a + b*x)*b**2*x
**3 - sqrt(x)*a**3 + 6*sqrt(x)*a**2*b*x + 24*sqrt(x)*a*b**2*x**2 + 16*sqrt
(x)*b**3*x**3))/(3*sqrt(a + b*x)*a**4*x**2*(a + b*x))
```

3.63 $\int \frac{1}{x(ax+bx^2)^{5/2}}\,dx$

Optimal result

Integrand size = 17, antiderivative size = 120

$$\int \frac{1}{x\,(ax+bx^2)^{5/2}}\,dx = \frac{2}{3ax\,(ax+bx^2)^{3/2}} + \frac{16}{3a^2x^2\sqrt{ax+bx^2}} - \frac{32\sqrt{ax+bx^2}}{5a^3x^3} + \frac{128b\sqrt{ax+bx^2}}{15a^4x^2} - \frac{256b^2\sqrt{ax+bx^2}}{15a^5x}$$

```
2/3/a/x/(b*x^2+a*x)^(3/2)+16/3/a^2/x^2/(b*x^2+a*x)^(1/2)-32/5*(b*x^2+a*x)^
(1/2)/a^3/x^3+128/15*b*(b*x^2+a*x)^(1/2)/a^4/x^2-256/15*b^2*(b*x^2+a*x)^(1
/2)/a^5/x
```

Mathematica [A] (verified)

Time = 0.13 (sec) , antiderivative size = 64, normalized size of antiderivative = 0.53

$$\int \frac{1}{x\,(ax+bx^2)^{5/2}}\,dx = -\frac{2(a+bx)\,(3a^4-8a^3bx+48a^2b^2x^2+192ab^3x^3+128b^4x^4)}{15a^5(x(a+bx))^{5/2}}$$

```
Integrate[1/(x*(a*x + b*x^2)^(5/2)),x]
```

```
(-2*(a + b*x)*(3*a^4 - 8*a^3*b*x + 48*a^2*b^2*x^2 + 192*a*b^3*x^3 + 128*b^
4*x^4))/(15*a^5*(x*(a + b*x))^(5/2))
```

Rubi [A] (verified)

Time = 0.33 (sec) , antiderivative size = 86, normalized size of antiderivative = 0.72, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.176$, Rules used = {1129, 1089, 1088}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{x\left(ax+bx^2\right)^{5/2}}dx$$

$$\Big\downarrow \text{ 1129}$$

$$-\frac{8b\int \frac{1}{\left(bx^2+ax\right)^{5/2}}dx}{5a}-\frac{2}{5ax\left(ax+bx^2\right)^{3/2}}$$

$$\Big\downarrow \text{ 1089}$$

$$-\frac{8b\left(-\frac{8b\int \frac{1}{\left(bx^2+ax\right)^{3/2}}dx}{3a^2}-\frac{2(a+2bx)}{3a^2\left(ax+bx^2\right)^{3/2}}\right)}{5a}-\frac{2}{5ax\left(ax+bx^2\right)^{3/2}}$$

$$\Big\downarrow \text{ 1088}$$

$$-\frac{8b\left(\frac{16b(a+2bx)}{3a^4\sqrt{ax+bx^2}}-\frac{2(a+2bx)}{3a^2\left(ax+bx^2\right)^{3/2}}\right)}{5a}-\frac{2}{5ax\left(ax+bx^2\right)^{3/2}}$$

```
Int[1/(x*(a*x + b*x^2)^(5/2)),x]
```

```
-2/(5*a*x*(a*x + b*x^2)^(3/2)) - (8*b*((-2*(a + 2*b*x))/(3*a^2*(a*x + b*x^
2)^(3/2)) + (16*b*(a + 2*b*x))/(3*a^4*Sqrt[a*x + b*x^2])))/(5*a)
```

Defintions of rubi rules used

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-3/2), x_Symbol] :> Simp[-2*((b +
2*c*x)/((b^2 - 4*a*c)*Sqrt[a + b*x + c*x^2])), x] /; FreeQ[{a, b, c}, x] &&
 NeQ[b^2 - 4*a*c, 0]
```

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(b + 2*c*x)
*((a + b*x + c*x^2)^(p + 1)/((p + 1)*(b^2 - 4*a*c))), x] - Simp[2*c*((2*p +
 3)/((p + 1)*(b^2 - 4*a*c)))   Int[(a + b*x + c*x^2)^(p + 1), x], x] /; Fre
eQ[{a, b, c}, x] && LtQ[p, -1] && (IntegerQ[4*p] || IntegerQ[3*p])
```

```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] :> Simp[(-e)*(d + e*x)^m*((a + b*x + c*x^2)^(p + 1)/((m + p + 1)*(2*
c*d - b*e))), x] + Simp[c*(Simplify[m + 2*p + 2]/((m + p + 1)*(2*c*d - b*e)
))   Int[(d + e*x)^(m + 1)*(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d
, e, m, p}, x] && EqQ[c*d^2 - b*d*e + a*e^2, 0] && ILtQ[Simplify[m + 2*p +
2], 0]
```

Maple [A] (verified)

Time = 0.37 (sec) , antiderivative size = 63, normalized size of antiderivative = 0.52

method	result	size
gosper	$-\frac{2(bx+a)\left(128b^{4}x^{4}+192a\,b^{3}x^{3}+48a^{2}b^{2}x^{2}-8a^{3}bx+3a^{4}\right)}{15a^{5}\left(b\,x^{2}+ax\right)^{\frac{5}{2}}}$	63
orering	$-\frac{2(bx+a)\left(128b^{4}x^{4}+192a\,b^{3}x^{3}+48a^{2}b^{2}x^{2}-8a^{3}bx+3a^{4}\right)}{15a^{5}\left(b\,x^{2}+ax\right)^{\frac{5}{2}}}$	63
pseudoelliptic	$-\frac{2\left(\frac{128}{3}b^{4}x^{4}+64a\,b^{3}x^{3}+16a^{2}b^{2}x^{2}-\frac{8}{3}a^{3}bx+a^{4}\right)}{5\sqrt{x(bx+a)}\,x^{2}(bx+a)a^{5}}$	64
trager	$-\frac{2\left(128b^{4}x^{4}+192a\,b^{3}x^{3}+48a^{2}b^{2}x^{2}-8a^{3}bx+3a^{4}\right)\sqrt{b\,x^{2}+ax}}{15a^{5}x^{3}(bx+a)^{2}}$	68
default	$-\frac{2}{5ax\left(b\,x^{2}+ax\right)^{\frac{3}{2}}}-\frac{8b\left(-\frac{2(2bx+a)}{3a^{2}\left(b\,x^{2}+ax\right)^{\frac{3}{2}}}+\frac{16b(2bx+a)}{3a^{4}\sqrt{b\,x^{2}+ax}}\right)}{5a}$	73
risch	$-\frac{2(bx+a)\left(73b^{2}x^{2}-14abx+3a^{2}\right)}{15a^{5}x^{2}\sqrt{x(bx+a)}}-\frac{2b^{3}(11bx+12a)x}{3\sqrt{x(bx+a)}\,(bx+a)a^{5}}$	76

```
int(1/x/(b*x^2+a*x)^(5/2),x,method=_RETURNVERBOSE)
```

```
-2/15*(b*x+a)*(128*b^4*x^4+192*a*b^3*x^3+48*a^2*b^2*x^2-8*a^3*b*x+3*a^4)/a
^5/(b*x^2+a*x)^(5/2)
```

Fricas [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 83, normalized size of antiderivative = 0.69

$$\int \frac{1}{x\left(ax+bx^2\right)^{5/2}}\,dx = -\frac{2\left(128\,b^4x^4+192\,ab^3x^3+48\,a^2b^2x^2-8\,a^3bx+3\,a^4\right)\sqrt{bx^2+ax}}{15\left(a^5b^2x^5+2\,a^6bx^4+a^7x^3\right)}$$

```
integrate(1/x/(b*x^2+a*x)^(5/2),x, algorithm="fricas")
```

```
-2/15*(128*b^4*x^4 + 192*a*b^3*x^3 + 48*a^2*b^2*x^2 - 8*a^3*b*x + 3*a^4)*s
qrt(b*x^2 + a*x)/(a^5*b^2*x^5 + 2*a^6*b*x^4 + a^7*x^3)
```

Sympy [F]

$$\int \frac{1}{x\left(ax+bx^2\right)^{5/2}}\,dx = \int \frac{1}{x\left(x\left(a+bx\right)\right)^{\frac{5}{2}}}\,dx$$

```
integrate(1/x/(b*x**2+a*x)**(5/2),x)
```

```
Integral(1/(x*(x*(a + b*x))**(5/2)), x)
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 96, normalized size of antiderivative = 0.80

$$\int \frac{1}{x\left(ax+bx^2\right)^{5/2}}\,dx = \frac{32\,b^2x}{15\,(bx^2+ax)^{\frac{3}{2}}a^3} - \frac{256\,b^3x}{15\,\sqrt{bx^2+ax}a^5} + \frac{16\,b}{15\,(bx^2+ax)^{\frac{3}{2}}a^2} - \frac{128\,b^2}{15\,\sqrt{bx^2+ax}a^4} - \frac{2}{5\,(bx^2+ax)^{\frac{3}{2}}ax}$$

```
integrate(1/x/(b*x^2+a*x)^(5/2),x, algorithm="maxima")
```

```
32/15*b^2*x/((b*x^2 + a*x)^(3/2)*a^3) - 256/15*b^3*x/(sqrt(b*x^2 + a*x)*a^
5) + 16/15*b/((b*x^2 + a*x)^(3/2)*a^2) - 128/15*b^2/(sqrt(b*x^2 + a*x)*a^4
) - 2/5/((b*x^2 + a*x)^(3/2)*a*x)
```

Giac [F]

$$\int \frac{1}{x\left(ax+bx^2\right)^{5/2}}\,dx = \int \frac{1}{(bx^2+ax)^{\frac{5}{2}}x}\,dx$$

```
integrate(1/x/(b*x^2+a*x)^(5/2),x, algorithm="giac")
```

```
integrate(1/((b*x^2 + a*x)^(5/2)*x), x)
```

Mupad [B] (verification not implemented)

Time = 10.32 (sec) , antiderivative size = 67, normalized size of antiderivative = 0.56

$$\int \frac{1}{x\left(ax+bx^2\right)^{5/2}}\,dx = -\frac{2\sqrt{b\,x^2+a\,x}\,\left(3\,a^4-8\,a^3\,b\,x+48\,a^2\,b^2\,x^2+192\,a\,b^3\,x^3+128\,b^4\,x^4\right)}{15\,a^5\,x^3\,{\left(a+b\,x\right)}^2}$$

```
int(1/(x*(a*x + b*x^2)^(5/2)),x)
```

```
-(2*(a*x + b*x^2)^(1/2)*(3*a^4 + 128*b^4*x^4 + 192*a*b^3*x^3 + 48*a^2*b^2*
x^2 - 8*a^3*b*x))/(15*a^5*x^3*(a + b*x)^2)
```

Reduce [B] (verification not implemented)

Time = 0.24 (sec) , antiderivative size = 107, normalized size of antiderivative = 0.89

$$\int \frac{1}{x\left(ax+bx^2\right)^{5/2}}\,dx = \frac{\frac{256\sqrt{b}\,\sqrt{bx+a}\,a\,b^2x^3}{15}+\frac{256\sqrt{b}\,\sqrt{bx+a}\,b^3x^4}{15}-\frac{2\sqrt{x}\,a^4}{5}+\frac{16\sqrt{x}\,a^3bx}{15}-\frac{32\sqrt{x}\,a^2b^2x^2}{5}-\frac{128\sqrt{x}\,a\,b^3x^3}{5}-}{\sqrt{bx+a}\,a^5x^3\,(bx+a)}$$

```
int(1/x/(b*x^2+a*x)^(5/2),x)
```

```
(2*(128*sqrt(b)*sqrt(a + b*x)*a*b**2*x**3 + 128*sqrt(b)*sqrt(a + b*x)*b**3
*x**4 - 3*sqrt(x)*a**4 + 8*sqrt(x)*a**3*b*x - 48*sqrt(x)*a**2*b**2*x**2 -
192*sqrt(x)*a*b**3*x**3 - 128*sqrt(x)*b**4*x**4))/(15*sqrt(a + b*x)*a**5*x
**3*(a + b*x))
```

3.64 $\int \frac{1}{x^2(ax+bx^2)^{5/2}}\,dx$

Optimal result

Integrand size = 17, antiderivative size = 146

$$\int \frac{1}{x^2\left(ax+bx^2\right)^{5/2}}\,dx = \frac{2}{3ax^2\left(ax+bx^2\right)^{3/2}}+\frac{20}{3a^2x^3\sqrt{ax+bx^2}} -\frac{160\sqrt{ax+bx^2}}{21a^3x^4}+\frac{64b\sqrt{ax+bx^2}}{7a^4x^3}-\frac{256b^2\sqrt{ax+bx^2}}{21a^5x^2}+\frac{512b^3\sqrt{ax+bx^2}}{21a^6x}$$

```
2/3/a/x^2/(b*x^2+a*x)^(3/2)+20/3/a^2/x^3/(b*x^2+a*x)^(1/2)-160/21*(b*x^2+a
*x)^(1/2)/a^3/x^4+64/7*b*(b*x^2+a*x)^(1/2)/a^4/x^3-256/21*b^2*(b*x^2+a*x)^
(1/2)/a^5/x^2+512/21*b^3*(b*x^2+a*x)^(1/2)/a^6/x
```

Mathematica [A] (verified)

Time = 0.13 (sec) , antiderivative size = 73, normalized size of antiderivative = 0.50

$$\int \frac{1}{x^2\left(ax+bx^2\right)^{5/2}}\,dx = \frac{2(-3a^5+6a^4bx-16a^3b^2x^2+96a^2b^3x^3+384ab^4x^4+256b^5x^5)}{21a^6x^2(x(a+bx))^{3/2}}$$

```
Integrate[1/(x^2*(a*x + b*x^2)^(5/2)),x]
```

```
(2*(-3*a^5 + 6*a^4*b*x - 16*a^3*b^2*x^2 + 96*a^2*b^3*x^3 + 384*a*b^4*x^4 +
 256*b^5*x^5))/(21*a^6*x^2*(x*(a + b*x))^(3/2))
```

Rubi [A] (verified)

Time = 0.40 (sec) , antiderivative size = 118, normalized size of antiderivative = 0.81, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.235$, Rules used = {1129, 1129, 1089, 1088}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{x^2 (ax+bx^2)^{5/2}} \, dx$$

$$\Big\downarrow \ 1129$$

$$-\frac{10b\int \frac{1}{x(bx^2+ax)^{5/2}}dx}{7a} - \frac{2}{7ax^2(ax+bx^2)^{3/2}}$$

$$\Big\downarrow \ 1129$$

$$-\frac{10b\left(-\frac{8b\int \frac{1}{(bx^2+ax)^{5/2}}dx}{5a} - \frac{2}{5ax(ax+bx^2)^{3/2}}\right)}{7a} - \frac{2}{7ax^2(ax+bx^2)^{3/2}}$$

$$\Big\downarrow \ 1089$$

$$-\frac{10b\left(-\frac{8b\left(-\frac{8b\int \frac{1}{(bx^2+ax)^{3/2}}dx}{3a^2} - \frac{2(a+2bx)}{3a^2(ax+bx^2)^{3/2}}\right)}{5a} - \frac{2}{5ax(ax+bx^2)^{3/2}}\right)}{7a} - \frac{2}{7ax^2(ax+bx^2)^{3/2}}$$

$$\Big\downarrow \ 1088$$

$$-\frac{10b\left(-\frac{8b\left(\frac{16b(a+2bx)}{3a^4\sqrt{ax+bx^2}} - \frac{2(a+2bx)}{3a^2(ax+bx^2)^{3/2}}\right)}{5a} - \frac{2}{5ax(ax+bx^2)^{3/2}}\right)}{7a} - \frac{2}{7ax^2(ax+bx^2)^{3/2}}$$

```
Int[1/(x^2*(a*x + b*x^2)^(5/2)),x]
```

```
-2/(7*a*x^2*(a*x + b*x^2)^(3/2)) - (10*b*(-2/(5*a*x*(a*x + b*x^2)^(3/2)) -
 (8*b*((-2*(a + 2*b*x))/(3*a^2*(a*x + b*x^2)^(3/2)) + (16*b*(a + 2*b*x))/(
3*a^4*Sqrt[a*x + b*x^2])))/(5*a)))/(7*a)
```

Defintions of rubi rules used

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-3/2), x_Symbol] :> Simp[-2*((b +
2*c*x)/((b^2 - 4*a*c)*Sqrt[a + b*x + c*x^2])), x] /; FreeQ[{a, b, c}, x] &&
 NeQ[b^2 - 4*a*c, 0]
```

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(b + 2*c*x)
*((a + b*x + c*x^2)^(p + 1)/((p + 1)*(b^2 - 4*a*c))), x] - Simp[2*c*((2*p +
 3)/((p + 1)*(b^2 - 4*a*c)))   Int[(a + b*x + c*x^2)^(p + 1), x], x] /; Fre
eQ[{a, b, c}, x] && LtQ[p, -1] && (IntegerQ[4*p] || IntegerQ[3*p])
```

```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] :> Simp[(-e)*(d + e*x)^m*((a + b*x + c*x^2)^(p + 1)/((m + p + 1)*(2*
c*d - b*e))), x] + Simp[c*(Simplify[m + 2*p + 2]/((m + p + 1)*(2*c*d - b*e)
))   Int[(d + e*x)^(m + 1)*(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d
, e, m, p}, x] && EqQ[c*d^2 - b*d*e + a*e^2, 0] && ILtQ[Simplify[m + 2*p +
2], 0]
```

Maple [A] (verified)

Time = 0.38 (sec) , antiderivative size = 75, normalized size of antiderivative = 0.51

method	result	size
pseudoelliptic	$-\frac{2\left(-\frac{256}{3}b^{5}x^{5}-128a\,b^{4}x^{4}-32a^{2}b^{3}x^{3}+\frac{16}{3}a^{3}b^{2}x^{2}-2a^{4}bx+a^{5}\right)}{7\sqrt{x(bx+a)}\,x^{3}(bx+a)a^{6}}$	75
gosper	$-\frac{2(bx+a)\left(-256b^{5}x^{5}-384a\,b^{4}x^{4}-96a^{2}b^{3}x^{3}+16a^{3}b^{2}x^{2}-6a^{4}bx+3a^{5}\right)}{21x\,a^{6}\left(b\,x^{2}+ax\right)^{\frac{5}{2}}}$	77
orering	$-\frac{2(bx+a)\left(-256b^{5}x^{5}-384a\,b^{4}x^{4}-96a^{2}b^{3}x^{3}+16a^{3}b^{2}x^{2}-6a^{4}bx+3a^{5}\right)}{21x\,a^{6}\left(b\,x^{2}+ax\right)^{\frac{5}{2}}}$	77
trager	$-\frac{2\left(-256b^{5}x^{5}-384a\,b^{4}x^{4}-96a^{2}b^{3}x^{3}+16a^{3}b^{2}x^{2}-6a^{4}bx+3a^{5}\right)\sqrt{b\,x^{2}+ax}}{21a^{6}x^{4}(bx+a)^{2}}$	79
risch	$-\frac{2(bx+a)\left(-158b^{3}x^{3}+37a\,b^{2}x^{2}-12a^{2}bx+3a^{3}\right)}{21a^{6}x^{3}\sqrt{x(bx+a)}}+\frac{2b^{4}(14bx+15a)x}{3\sqrt{x(bx+a)}\,(bx+a)a^{6}}$	87
default	$-\frac{2}{7a\,x^{2}\left(b\,x^{2}+ax\right)^{\frac{3}{2}}}-\frac{10b\left(-\frac{2}{5ax\left(b\,x^{2}+ax\right)^{\frac{3}{2}}}-\frac{8b\left(-\frac{2(2bx+a)}{3a^{2}\left(b\,x^{2}+ax\right)^{\frac{3}{2}}}+\frac{16b(2bx+a)}{3a^{4}\sqrt{b\,x^{2}+ax}}\right)}{5a}\right)}{7a}$	99

```
int(1/x^2/(b*x^2+a*x)^(5/2),x,method=_RETURNVERBOSE)
```

```
-2/7/(x*(b*x+a))^(1/2)*(-256/3*b^5*x^5-128*a*b^4*x^4-32*a^2*b^3*x^3+16/3*a
^3*b^2*x^2-2*a^4*b*x+a^5)/x^3/(b*x+a)/a^6
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 94, normalized size of antiderivative = 0.64

$$\int \frac{1}{x^2\left(ax+bx^2\right)^{5/2}}\,dx=\frac{2\left(256\,b^5x^5+384\,ab^4x^4+96\,a^2b^3x^3-16\,a^3b^2x^2+6\,a^4bx-3\,a^5\right)\sqrt{bx^2+ax}}{21\left(a^6b^2x^6+2\,a^7bx^5+a^8x^4\right)}$$

```
integrate(1/x^2/(b*x^2+a*x)^(5/2),x, algorithm="fricas")
```

```
2/21*(256*b^5*x^5 + 384*a*b^4*x^4 + 96*a^2*b^3*x^3 - 16*a^3*b^2*x^2 + 6*a^
4*b*x - 3*a^5)*sqrt(b*x^2 + a*x)/(a^6*b^2*x^6 + 2*a^7*b*x^5 + a^8*x^4)
```

Sympy [F]

$$\int \frac{1}{x^2 (ax+bx^2)^{5/2}}\,dx = \int \frac{1}{x^2 (x(a+bx))^{\frac{5}{2}}}\,dx$$

```
integrate(1/x**2/(b*x**2+a*x)**(5/2),x)
```

```
Integral(1/(x**2*(x*(a + b*x))**(5/2)), x)
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 118, normalized size of antiderivative = 0.81

$$\int \frac{1}{x^2 (ax+bx^2)^{5/2}}\,dx = -\frac{64\,b^3x}{21\,(bx^2+ax)^{\frac{3}{2}}a^4} + \frac{512\,b^4x}{21\,\sqrt{bx^2+ax}a^6} - \frac{32\,b^2}{21\,(bx^2+ax)^{\frac{3}{2}}a^3} + \frac{256\,b^3}{21\,\sqrt{bx^2+ax}a^5} + \frac{4\,b}{7\,(bx^2+ax)^{\frac{3}{2}}a^2x} - \frac{2}{7\,(bx^2+ax)^{\frac{3}{2}}ax^2}$$

```
integrate(1/x^2/(b*x^2+a*x)^(5/2),x, algorithm="maxima")
```

```
-64/21*b^3*x/((b*x^2 + a*x)^(3/2)*a^4) + 512/21*b^4*x/(sqrt(b*x^2 + a*x)*a
^6) - 32/21*b^2/((b*x^2 + a*x)^(3/2)*a^3) + 256/21*b^3/(sqrt(b*x^2 + a*x)*
a^5) + 4/7*b/((b*x^2 + a*x)^(3/2)*a^2*x) - 2/7/((b*x^2 + a*x)^(3/2)*a*x^2)
```

Giac [F]

$$\int \frac{1}{x^2 (ax+bx^2)^{5/2}}\,dx = \int \frac{1}{(bx^2+ax)^{\frac{5}{2}}x^2}\,dx$$

```
integrate(1/x^2/(b*x^2+a*x)^(5/2),x, algorithm="giac")
```

```
integrate(1/((b*x^2 + a*x)^(5/2)*x^2), x)
```

Mupad [B] (verification not implemented)

Time = 10.09 (sec) , antiderivative size = 121, normalized size of antiderivative = 0.83

$$\int \frac{1}{x^2 (ax+bx^2)^{5/2}}\,dx = \frac{\sqrt{b\,x^2+a\,x}\,\left(\frac{256\,b^3}{21\,a^5}+\frac{512\,b^4\,x}{21\,a^6}\right)}{x\,\left(a+b\,x\right)} - \frac{\sqrt{b\,x^2+a\,x}\,\left(\frac{74\,b^2}{21\,a^3}+\frac{88\,b^3\,x}{21\,a^4}\right)}{x^2\,{\left(a+b\,x\right)}^2} - \frac{2\,\sqrt{b\,x^2+a\,x}}{7\,a^3\,x^4} + \frac{8\,b\,\sqrt{b\,x^2+a\,x}}{7\,a^4\,x^3}$$

```
int(1/(x^2*(a*x + b*x^2)^(5/2)),x)
```

```
((a*x + b*x^2)^(1/2)*((256*b^3)/(21*a^5) + (512*b^4*x)/(21*a^6)))/(x*(a +
b*x)) - ((a*x + b*x^2)^(1/2)*((74*b^2)/(21*a^3) + (88*b^3*x)/(21*a^4)))/(x
^2*(a + b*x)^2) - (2*(a*x + b*x^2)^(1/2))/(7*a^3*x^4) + (8*b*(a*x + b*x^2)
^(1/2))/(7*a^4*x^3)
```

Reduce [B] (verification not implemented)

Time = 0.60 (sec) , antiderivative size = 120, normalized size of antiderivative = 0.82

$$\int \frac{1}{x^2 (ax+bx^2)^{5/2}}\,dx = \frac{-\frac{512\sqrt{b}\,\sqrt{bx+a}\,a\,b^3x^4}{21}-\frac{512\sqrt{b}\,\sqrt{bx+a}\,b^4x^5}{21}-\frac{2\sqrt{x}\,a^5}{7}+\frac{4\sqrt{x}\,a^4bx}{7}-\frac{32\sqrt{x}\,a^3b^2x^2}{21}+\frac{64\sqrt{x}\,a^2b^3x^3}{7} +}{\sqrt{bx+a}\,a^6x^4\,(bx+a)}$$

```
int(1/x^2/(b*x^2+a*x)^(5/2),x)
```

```
(2*( - 256*sqrt(b)*sqrt(a + b*x)*a*b**3*x**4 - 256*sqrt(b)*sqrt(a + b*x)*b
**4*x**5 - 3*sqrt(x)*a**5 + 6*sqrt(x)*a**4*b*x - 16*sqrt(x)*a**3*b**2*x**2
 + 96*sqrt(x)*a**2*b**3*x**3 + 384*sqrt(x)*a*b**4*x**4 + 256*sqrt(x)*b**5*
x**5))/(21*sqrt(a + b*x)*a**6*x**4*(a + b*x))
```

3.65 $\int \frac{1}{x^3(ax+bx^2)^{5/2}}\,dx$

Optimal result

Integrand size = 17, antiderivative size = 170

$$\int \frac{1}{x^3\left(ax+bx^2\right)^{5/2}}\,dx = \frac{2}{3ax^3\left(ax+bx^2\right)^{3/2}}+\frac{8}{a^2x^4\sqrt{ax+bx^2}}-\frac{80\sqrt{ax+bx^2}}{9a^3x^5}+\frac{640b\sqrt{ax+bx^2}}{63a^4x^4}-\frac{256b^2\sqrt{ax+bx^2}}{21a^5x^3}+\frac{1024b^3\sqrt{ax+bx^2}}{63a^6x^2}-\frac{2048b^4\sqrt{ax+bx^2}}{63a^7x}$$

```
2/3/a/x^3/(b*x^2+a*x)^(3/2)+8/a^2/x^4/(b*x^2+a*x)^(1/2)-80/9*(b*x^2+a*x)^(
1/2)/a^3/x^5+640/63*b*(b*x^2+a*x)^(1/2)/a^4/x^4-256/21*b^2*(b*x^2+a*x)^(1/
2)/a^5/x^3+1024/63*b^3*(b*x^2+a*x)^(1/2)/a^6/x^2-2048/63*b^4*(b*x^2+a*x)^(
1/2)/a^7/x
```

Mathematica [A] (verified)

Time = 0.16 (sec) , antiderivative size = 84, normalized size of antiderivative = 0.49

$$\int \frac{1}{x^3\left(ax+bx^2\right)^{5/2}}\,dx = -\frac{2(7a^6-12a^5bx+24a^4b^2x^2-64a^3b^3x^3+384a^2b^4x^4+1536ab^5x^5+1024b^6x^6)}{63a^7x^3(x(a+bx))^{3/2}}$$

```
Integrate[1/(x^3*(a*x + b*x^2)^(5/2)),x]
```

```
(-2*(7*a^6 - 12*a^5*b*x + 24*a^4*b^2*x^2 - 64*a^3*b^3*x^3 + 384*a^2*b^4*x^
4 + 1536*a*b^5*x^5 + 1024*b^6*x^6))/(63*a^7*x^3*(x*(a + b*x))^(3/2))
```

Rubi [A] (verified)

Time = 0.45 (sec) , antiderivative size = 150, normalized size of antiderivative = 0.88, number of steps used = 5, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.294, Rules used = {1129, 1129, 1129, 1089, 1088}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{x^3 (ax+bx^2)^{5/2}} dx$$

$$\Big\downarrow \text{1129}$$

$$-\frac{4b\int \frac{1}{x^2(bx^2+ax)^{5/2}}dx}{3a}-\frac{2}{9ax^3 (ax+bx^2)^{3/2}}$$

$$\Big\downarrow \text{1129}$$

$$-\frac{4b\left(-\frac{10b\int \frac{1}{x\left(bx^2+ax\right)^{5/2}}dx}{7a}-\frac{2}{7ax^2(ax+bx^2)^{3/2}}\right)}{3a}-\frac{2}{9ax^3 (ax+bx^2)^{3/2}}$$

$$\Big\downarrow \text{1129}$$

$$-\frac{4b\left(-\frac{10b\left(-\frac{8b\int \frac{1}{\left(bx^2+ax\right)^{5/2}}dx}{5a}-\frac{2}{5ax\left(ax+bx^2\right)^{3/2}}\right)}{7a}-\frac{2}{7ax^2(ax+bx^2)^{3/2}}\right)}{3a}-\frac{2}{9ax^3 (ax+bx^2)^{3/2}}$$

$$\Big\downarrow \text{1089}$$

$$-\frac{4b\left(-\frac{10b\left(-\frac{8b\left(-\frac{8b\int \frac{1}{\left(bx^2+ax\right)^{3/2}}dx}{3a^2}-\frac{2(a+2bx)}{3a^2\left(ax+bx^2\right)^{3/2}}\right)}{5a}-\frac{2}{5ax\left(ax+bx^2\right)^{3/2}}\right)}{7a}-\frac{2}{7ax^2(ax+bx^2)^{3/2}}\right)}{3a}-\frac{2}{9ax^3\left(ax+bx^2\right)^{3/2}}$$

$$\Big\downarrow \textcolor{blue}{1088}$$

$$-\frac{4b\left(-\frac{10b\left(-\frac{8b\left(\frac{16b(a+2bx)}{3a^4\sqrt{ax+bx^2}}-\frac{2(a+2bx)}{3a^2\left(ax+bx^2\right)^{3/2}}\right)}{5a}-\frac{2}{5ax\left(ax+bx^2\right)^{3/2}}\right)}{7a}-\frac{2}{7ax^2(ax+bx^2)^{3/2}}\right)}{3a}-\frac{2}{9ax^3\left(ax+bx^2\right)^{3/2}}$$

```
Int[1/(x^3*(a*x + b*x^2)^(5/2)),x]
```

```
-2/(9*a*x^3*(a*x + b*x^2)^(3/2)) - (4*b*(-2/(7*a*x^2*(a*x + b*x^2)^(3/2))
- (10*b*(-2/(5*a*x*(a*x + b*x^2)^(3/2)) - (8*b*((-2*(a + 2*b*x))/(3*a^2*(a
*x + b*x^2)^(3/2)) + (16*b*(a + 2*b*x))/(3*a^4*Sqrt[a*x + b*x^2])))/(5*a))
)/(7*a)))/(3*a)
```

Defintions of rubi rules used

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-3/2), x_Symbol] :> Simp[-2*((b +
2*c*x)/((b^2 - 4*a*c)*Sqrt[a + b*x + c*x^2])), x] /; FreeQ[{a, b, c}, x] &&
 NeQ[b^2 - 4*a*c, 0]
```

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(b + 2*c*x)
*((a + b*x + c*x^2)^(p + 1)/((p + 1)*(b^2 - 4*a*c))), x] - Simp[2*c*((2*p +
 3)/((p + 1)*(b^2 - 4*a*c)))   Int[(a + b*x + c*x^2)^(p + 1), x], x] /; Fre
eQ[{a, b, c}, x] && LtQ[p, -1] && (IntegerQ[4*p] || IntegerQ[3*p])
```

```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] :> Simp[(-e)*(d + e*x)^m*((a + b*x + c*x^2)^(p + 1)/((m + p + 1)*(2*
c*d - b*e))), x] + Simp[c*(Simplify[m + 2*p + 2]/((m + p + 1)*(2*c*d - b*e)
))   Int[(d + e*x)^(m + 1)*(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d
, e, m, p}, x] && EqQ[c*d^2 - b*d*e + a*e^2, 0] && ILtQ[Simplify[m + 2*p +
2], 0]
```

Maple [A] (verified)

Time = 0.39 (sec) , antiderivative size = 88, normalized size of antiderivative = 0.52

method	result	si
gosper	$-\frac{2(bx+a)\left(1024b^6x^6+1536a\,b^5x^5+384a^2b^4x^4-64a^3b^3x^3+24a^4b^2x^2-12a^5bx+7a^6\right)}{63x^2a^7\left(b\,x^2+ax\right)^{\frac{5}{2}}}$	88
pseudoelliptic	$\frac{-\frac{2048}{63}b^6x^6-\frac{1024}{21}a\,b^5x^5-\frac{256}{21}a^2b^4x^4+\frac{128}{63}a^3b^3x^3-\frac{16}{21}a^4b^2x^2+\frac{8}{21}a^5bx-\frac{2}{9}a^6}{x^4(bx+a)\sqrt{x(bx+a)}\,a^7}$	88
orering	$-\frac{2(bx+a)\left(1024b^6x^6+1536a\,b^5x^5+384a^2b^4x^4-64a^3b^3x^3+24a^4b^2x^2-12a^5bx+7a^6\right)}{63x^2a^7\left(b\,x^2+ax\right)^{\frac{5}{2}}}$	88
trager	$-\frac{2\left(1024b^6x^6+1536a\,b^5x^5+384a^2b^4x^4-64a^3b^3x^3+24a^4b^2x^2-12a^5bx+7a^6\right)\sqrt{b\,x^2+ax}}{63a^7x^5(bx+a)^2}$	90
risch	$-\frac{2(bx+a)\left(667b^4x^4-176a\,b^3x^3+69a^2b^2x^2-26a^3bx+7a^4\right)}{63a^7x^4\sqrt{x(bx+a)}}-\frac{2b^5(17bx+18a)x}{3\sqrt{x(bx+a)}\,(bx+a)a^7}$	98
default	$-\frac{2}{9a\,x^3\left(b\,x^2+ax\right)^{\frac{3}{2}}}-\frac{4b\left(-\frac{2}{7a\,x^2\left(b\,x^2+ax\right)^{\frac{3}{2}}}-\frac{10b\left(-\frac{2}{5ax\left(b\,x^2+ax\right)^{\frac{3}{2}}}-\frac{8b\left(-\frac{2(2bx+a)}{3a^2\left(b\,x^2+ax\right)^{\frac{3}{2}}}+\frac{16b(2bx+a)}{3a^4\sqrt{b\,x^2+ax}}\right)}{5a}\right)}{7a}\right)}{3a}$	12

```
int(1/x^3/(b*x^2+a*x)^(5/2),x,method=_RETURNVERBOSE)
```

```
-2/63*(b*x+a)*(1024*b^6*x^6+1536*a*b^5*x^5+384*a^2*b^4*x^4-64*a^3*b^3*x^3+
24*a^4*b^2*x^2-12*a^5*b*x+7*a^6)/x^2/a^7/(b*x^2+a*x)^(5/2)
```

Fricas [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 105, normalized size of antiderivative = 0.62

$$\int \frac{1}{x^3\left(ax+bx^2\right)^{5/2}}\,dx = -\frac{2\left(1024\,b^6x^6+1536\,ab^5x^5+384\,a^2b^4x^4-64\,a^3b^3x^3+24\,a^4b^2x^2-12\,a^5bx+7\,a^6\right)\sqrt{bx^2+ax}}{63\left(a^7b^2x^7+2\,a^8bx^6+a^9x^5\right)}$$

```
integrate(1/x^3/(b*x^2+a*x)^(5/2),x, algorithm="fricas")
```

```
-2/63*(1024*b^6*x^6 + 1536*a*b^5*x^5 + 384*a^2*b^4*x^4 - 64*a^3*b^3*x^3 +
24*a^4*b^2*x^2 - 12*a^5*b*x + 7*a^6)*sqrt(b*x^2 + a*x)/(a^7*b^2*x^7 + 2*a^
8*b*x^6 + a^9*x^5)
```

Sympy [F]

$$\int \frac{1}{x^3 (ax + bx^2)^{5/2}}\, dx = \int \frac{1}{x^3 (x (a + bx))^{\frac{5}{2}}}\, dx$$

```
integrate(1/x**3/(b*x**2+a*x)**(5/2),x)
```

```
Integral(1/(x**3*(x*(a + b*x))**(5/2)), x)
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 140, normalized size of antiderivative = 0.82

$$\int \frac{1}{x^3 (ax + bx^2)^{5/2}}\, dx = \frac{256\, b^4 x}{63\, (bx^2 + ax)^{\frac{3}{2}} a^5} - \frac{2048\, b^5 x}{63\, \sqrt{bx^2 + ax} a^7} + \frac{128\, b^3}{63\, (bx^2 + ax)^{\frac{3}{2}} a^4} - \frac{1024\, b^4}{63\, \sqrt{bx^2 + ax} a^6} - \frac{16\, b^2}{21\, (bx^2 + ax)^{\frac{3}{2}} a^3 x} + \frac{8\, b}{21\, (bx^2 + ax)^{\frac{3}{2}} a^2 x^2} - \frac{2}{9\, (bx^2 + ax)^{\frac{3}{2}} a x^3}$$

```
integrate(1/x^3/(b*x^2+a*x)^(5/2),x, algorithm="maxima")
```

```
256/63*b^4*x/((b*x^2 + a*x)^(3/2)*a^5) - 2048/63*b^5*x/(sqrt(b*x^2 + a*x)*
a^7) + 128/63*b^3/((b*x^2 + a*x)^(3/2)*a^4) - 1024/63*b^4/(sqrt(b*x^2 + a*
x)*a^6) - 16/21*b^2/((b*x^2 + a*x)^(3/2)*a^3*x) + 8/21*b/((b*x^2 + a*x)^(3
/2)*a^2*x^2) - 2/9/((b*x^2 + a*x)^(3/2)*a*x^3)
```

Giac [F]

$$\int \frac{1}{x^3 (ax+bx^2)^{5/2}}\,dx = \int \frac{1}{(bx^2+ax)^{\frac{5}{2}} x^3}\,dx$$

```
integrate(1/x^3/(b*x^2+a*x)^(5/2),x, algorithm="giac")
```

```
integrate(1/((b*x^2 + a*x)^(5/2)*x^3), x)
```

Mupad [B] (verification not implemented)

Time = 10.29 (sec) , antiderivative size = 143, normalized size of antiderivative = 0.84

$$\int \frac{1}{x^3 (ax+bx^2)^{5/2}}\,dx = \frac{\sqrt{b\,x^2+a\,x}\,\left(\frac{352\,b^3}{63\,a^4}+\frac{394\,b^4\,x}{63\,a^5}\right)}{x^2\,{\left(a+b\,x\right)}^2} - \frac{46\,b^2\,\sqrt{b\,x^2+a\,x}}{21\,a^5\,x^3} - \frac{2\,\sqrt{b\,x^2+a\,x}}{9\,a^3\,x^5} - \frac{\sqrt{b\,x^2+a\,x}\,\left(\frac{1024\,b^4}{63\,a^6}+\frac{2048\,b^5\,x}{63\,a^7}\right)}{x\,\left(a+b\,x\right)} + \frac{52\,b\,\sqrt{b\,x^2+a\,x}}{63\,a^4\,x^4}$$

```
int(1/(x^3*(a*x + b*x^2)^(5/2)),x)
```

```
((a*x + b*x^2)^(1/2)*((352*b^3)/(63*a^4) + (394*b^4*x)/(63*a^5)))/(x^2*(a
+ b*x)^2) - (46*b^2*(a*x + b*x^2)^(1/2))/(21*a^5*x^3) - (2*(a*x + b*x^2)^(
1/2))/(9*a^3*x^5) - ((a*x + b*x^2)^(1/2)*((1024*b^4)/(63*a^6) + (2048*b^5*
x)/(63*a^7)))/(x*(a + b*x)) + (52*b*(a*x + b*x^2)^(1/2))/(63*a^4*x^4)
```

Reduce [B] (verification not implemented)

Time = 0.63 (sec) , antiderivative size = 133, normalized size of antiderivative = 0.78

$$\int \frac{1}{x^3\left(ax+bx^2\right)^{5/2}}\,dx = \frac{\frac{2048\sqrt{b}\,\sqrt{bx+a}\,a\,b^4x^5}{63}+\frac{2048\sqrt{b}\,\sqrt{bx+a}\,b^5x^6}{63}-\frac{2\sqrt{x}\,a^6}{9}+\frac{8\sqrt{x}\,a^5bx}{21}-\frac{16\sqrt{x}\,a^4b^2x^2}{21}+\frac{128\sqrt{x}\,a^3b^3x^3}{63}}{\sqrt{bx+a}\,a^7x^5\left(bx+a\right)}$$

```
int(1/x^3/(b*x^2+a*x)^(5/2),x)
```

```
(2*(1024*sqrt(b)*sqrt(a + b*x)*a*b**4*x**5 + 1024*sqrt(b)*sqrt(a + b*x)*b*
*5*x**6 - 7*sqrt(x)*a**6 + 12*sqrt(x)*a**5*b*x - 24*sqrt(x)*a**4*b**2*x**2
 + 64*sqrt(x)*a**3*b**3*x**3 - 384*sqrt(x)*a**2*b**4*x**4 - 1536*sqrt(x)*a
*b**5*x**5 - 1024*sqrt(x)*b**6*x**6))/(63*sqrt(a + b*x)*a**7*x**5*(a + b*x
))
```

3.66 $\int \frac{x^6}{(ax+bx^2)^{7/2}}\,dx$

Optimal result

Integrand size = 17, antiderivative size = 100

$$\int \frac{x^6}{(ax+bx^2)^{7/2}}\,dx = -\frac{2a^2x^3}{5b^3(ax+bx^2)^{5/2}} + \frac{22ax^2}{15b^3(ax+bx^2)^{3/2}} - \frac{46x}{15b^3\sqrt{ax+bx^2}} + \frac{2\operatorname{arctanh}\left(\frac{\sqrt{b}x}{\sqrt{ax+bx^2}}\right)}{b^{7/2}}$$

```
-2/5*a^2*x^3/b^3/(b*x^2+a*x)^(5/2)+22/15*a*x^2/b^3/(b*x^2+a*x)^(3/2)-46/15
*x/b^3/(b*x^2+a*x)^(1/2)+2*arctanh(b^(1/2)*x/(b*x^2+a*x)^(1/2))/b^(7/2)
```

Mathematica [A] (verified)

Time = 0.32 (sec) , antiderivative size = 101, normalized size of antiderivative = 1.01

$$\int \frac{x^6}{(ax+bx^2)^{7/2}}\,dx = -\frac{2\left(\sqrt{b}x(15a^2+35abx+23b^2x^2)+30\sqrt{x}(a+bx)^{5/2}\operatorname{arctanh}\left(\frac{\sqrt{b}\sqrt{x}}{\sqrt{a}-\sqrt{a+bx}}\right)\right)}{15b^{7/2}(a+bx)^2\sqrt{x(a+bx)}}$$

```
Integrate[x^6/(a*x + b*x^2)^(7/2),x]
```

```
(-2*(Sqrt[b]*x*(15*a^2 + 35*a*b*x + 23*b^2*x^2) + 30*Sqrt[x]*(a + b*x)^(5/
2)*ArcTanh[(Sqrt[b]*Sqrt[x])/(Sqrt[a] - Sqrt[a + b*x])]))/(15*b^(7/2)*(a +
 b*x)^2*Sqrt[x*(a + b*x)])
```

Rubi [A] (verified)

Time = 0.41 (sec) , antiderivative size = 104, normalized size of antiderivative = 1.04, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.294$, Rules used = {1133, 1133, 1124, 1091, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^6}{(ax+bx^2)^{7/2}}dx$$

$$\Big\downarrow \text{1133}$$

$$\frac{\int \frac{x^4}{(bx^2+ax)^{5/2}}dx}{b}-\frac{2x^5}{5b\left(ax+bx^2\right)^{5/2}}$$

$$\Big\downarrow \text{1133}$$

$$\frac{\frac{\int \frac{x^2}{\left(bx^2+ax\right)^{3/2}}dx}{b}-\frac{2x^3}{3b(ax+bx^2)^{3/2}}}{b}-\frac{2x^5}{5b\left(ax+bx^2\right)^{5/2}}$$

$$\Big\downarrow \text{1124}$$

$$\frac{\frac{\frac{\int \frac{1}{\sqrt{bx^2+ax}}dx}{b}-\frac{2x}{b\sqrt{ax+bx^2}}}{b}-\frac{2x^3}{3b(ax+bx^2)^{3/2}}}{b}-\frac{2x^5}{5b\left(ax+bx^2\right)^{5/2}}$$

$$\Big\downarrow \text{1091}$$

$$\frac{\frac{\frac{2\int \frac{1}{1-\frac{bx^2}{bx^2+ax}}d\frac{x}{\sqrt{bx^2+ax}}}{b}-\frac{2x}{b\sqrt{ax+bx^2}}}{b}-\frac{2x^3}{3b(ax+bx^2)^{3/2}}}{b}-\frac{2x^5}{5b\left(ax+bx^2\right)^{5/2}}$$

$$\Big\downarrow \text{219}$$

$$\frac{\frac{\frac{2\operatorname{arctanh}\left(\frac{\sqrt{b}x}{\sqrt{ax+bx^2}}\right)}{b^{3/2}}-\frac{2x}{b\sqrt{ax+bx^2}}}{b}-\frac{2x^3}{3b(ax+bx^2)^{3/2}}}{b}-\frac{2x^5}{5b\left(ax+bx^2\right)^{5/2}}$$

```
Int[x^6/(a*x + b*x^2)^(7/2),x]
```

```
(-2*x^5)/(5*b*(a*x + b*x^2)^(5/2)) + ((-2*x^3)/(3*b*(a*x + b*x^2)^(3/2)) +
 ((-2*x)/(b*Sqrt[a*x + b*x^2]) + (2*ArcTanh[(Sqrt[b]*x)/Sqrt[a*x + b*x^2]]
)/b^(3/2))/b)/b
```

Defintions of rubi rules used

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

```
Int[1/Sqrt[(b_.)*(x_) + (c_.)*(x_)^2], x_Symbol] :> Simp[2   Subst[Int[1/(1
 - c*x^2), x], x, x/Sqrt[b*x + c*x^2]], x] /; FreeQ[{b, c}, x]
```

```
Int[((d_.) + (e_.)*(x_))^(m_.)/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(3/2), x
_Symbol] :> Simp[-2*e*(2*c*d - b*e)^(m - 2)*((d + e*x)/(c^(m - 1)*Sqrt[a +
b*x + c*x^2])), x] + Simp[e^2/c^(m - 1)   Int[(1/Sqrt[a + b*x + c*x^2])*Exp
andToSum[((2*c*d - b*e)^(m - 1) - c^(m - 1)*(d + e*x)^(m - 1))/(c*d - b*e -
 c*e*x), x], x], x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[c*d^2 - b*d*e + a*e
^2, 0] && IGtQ[m, 0]
```

```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] :> Simp[e*(d + e*x)^(m - 1)*((a + b*x + c*x^2)^(p + 1)/(c*(p + 1))),
 x] - Simp[e^2*((m + p)/(c*(p + 1)))   Int[(d + e*x)^(m - 2)*(a + b*x + c*x
^2)^(p + 1), x], x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[c*d^2 - b*d*e + a*e
^2, 0] && LtQ[p, -1] && GtQ[m, 1] && IntegerQ[2*p]
```

Maple [A] (verified)

Time = 0.44 (sec) , antiderivative size = 83, normalized size of antiderivative = 0.83

method	result
pseudoelliptic	$\frac{2\sqrt{x(bx+a)}\,(bx+a)^2 \operatorname{arctanh}\left(\frac{\sqrt{x(bx+a)}}{x\sqrt{b}}\right)-2a^2x\sqrt{b}-\frac{14b^{\frac{3}{2}}a\,x^2}{3}-\frac{46b^{\frac{5}{2}}x^3}{15}}{b^{\frac{7}{2}}(bx+a)^2\sqrt{x(bx+a)}}$
	$3a\left(-\frac{x^3}{2b\left(b\,x^2+ax\right)^{\frac{5}{2}}}+\frac{a\left(-\frac{x^2}{3b\left(b\,x^2+ax\right)^{\frac{5}{2}}}-\frac{a\left(-\frac{x}{4b\left(b\,x^2+ax\right)^{\frac{5}{2}}}-\frac{3a\left(-\frac{1}{5b\left(b\,x^2+ax\right)\cdots}\right.}{\cdots}\right.}{\cdots}\right.}{\cdots}\right.$

```
int(x^6/(b*x^2+a*x)^(7/2),x,method=_RETURNVERBOSE)
```

```
2*((x*(b*x+a))^(1/2)*(b*x+a)^2*arctanh((x*(b*x+a))^(1/2)/x/b^(1/2))-a^2*x*
b^(1/2)-7/3*b^(3/2)*a*x^2-23/15*b^(5/2)*x^3)/b^(7/2)/(x*(b*x+a))^(1/2)/(b*
x+a)^2
```

Fricas [A] (verification not implemented)

Time = 0.15 (sec) , antiderivative size = 260, normalized size of antiderivative = 2.60

$$\int \frac{x^6}{(ax+bx^2)^{7/2}}\,dx = \left[\frac{15\,(b^3x^3+3\,ab^2x^2+3\,a^2bx+a^3)\sqrt{b}\log\left(2\,bx+a+2\,\sqrt{bx^2+ax}\sqrt{b}\right)-2\,(23\,b^3x^2}{15\,(b^7x^3+3\,ab^6x^2+3\,a^2b^5x+a^3b^4)}\right.$$

$$-\frac{2\left(15\,(b^3x^3+3\,ab^2x^2+3\,a^2bx+a^3)\sqrt{-b}\arctan\left(\frac{\sqrt{bx^2+ax}\sqrt{-b}}{bx+a}\right)+(23\,b^3x^2+35\,ab^2x+15\,a^2b)\sqrt{bx^2+ax}\right.}{15\,(b^7x^3+3\,ab^6x^2+3\,a^2b^5x+a^3b^4)}$$

```
integrate(x^6/(b*x^2+a*x)^(7/2),x, algorithm="fricas")
```

```
[1/15*(15*(b^3*x^3 + 3*a*b^2*x^2 + 3*a^2*b*x + a^3)*sqrt(b)*log(2*b*x + a
+ 2*sqrt(b*x^2 + a*x)*sqrt(b)) - 2*(23*b^3*x^2 + 35*a*b^2*x + 15*a^2*b)*sq
rt(b*x^2 + a*x))/(b^7*x^3 + 3*a*b^6*x^2 + 3*a^2*b^5*x + a^3*b^4), -2/15*(1
5*(b^3*x^3 + 3*a*b^2*x^2 + 3*a^2*b*x + a^3)*sqrt(-b)*arctan(sqrt(b*x^2 + a
*x)*sqrt(-b)/(b*x + a)) + (23*b^3*x^2 + 35*a*b^2*x + 15*a^2*b)*sqrt(b*x^2
+ a*x))/(b^7*x^3 + 3*a*b^6*x^2 + 3*a^2*b^5*x + a^3*b^4)]
```

Sympy [F]

$$\int \frac{x^6}{(ax+bx^2)^{7/2}}\,dx = \int \frac{x^6}{(x\,(a+bx))^{\frac{7}{2}}}\,dx$$

```
integrate(x**6/(b*x**2+a*x)**(7/2),x)
```

```
Integral(x**6/(x*(a + b*x))**(7/2), x)
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 357 vs. 2(82) = 164.

Time = 0.05 (sec) , antiderivative size = 357, normalized size of antiderivative = 3.57

$$\int \frac{x^6}{(ax+bx^2)^{7/2}}\,dx =$$

$$-\frac{1}{40}\,x\Bigg(\frac{40\,x^4}{(bx^2+ax)^{\frac{5}{2}}b}+\frac{30\,ax^3}{(bx^2+ax)^{\frac{5}{2}}b^2}+\frac{5\,a^2x^2}{(bx^2+ax)^{\frac{5}{2}}b^3}-\frac{a^3x}{(bx^2+ax)^{\frac{5}{2}}b^4}+\frac{2\,ax}{(bx^2+ax)^{\frac{3}{2}}b^3}-\frac{16\,x}{\sqrt{bx^2+ax}a}$$

$$-\frac{x\left(\frac{3\,x^2}{(bx^2+ax)^{\frac{3}{2}}b}+\frac{ax}{(bx^2+ax)^{\frac{3}{2}}b^2}-\frac{2\,x}{\sqrt{bx^2+ax}ab}-\frac{1}{\sqrt{bx^2+ax}b^2}\right)}{3\,b}-\frac{3\,ax^2}{4\,(bx^2+ax)^{\frac{3}{2}}b^3}$$

$$-\frac{7\,a^2x}{20\,(bx^2+ax)^{\frac{3}{2}}b^4}-\frac{8\,x}{15\,\sqrt{bx^2+ax}b^3}$$

$$+\frac{\log\left(2\,bx+a+2\,\sqrt{bx^2+ax}\sqrt{b}\right)}{b^{\frac{7}{2}}}+\frac{7\,a}{20\,\sqrt{bx^2+ax}b^4}-\frac{16\,\sqrt{bx^2+ax}}{15\,ab^3}$$

```
integrate(x^6/(b*x^2+a*x)^(7/2),x, algorithm="maxima")
```

```
-1/40*x*(40*x^4/((b*x^2 + a*x)^(5/2)*b) + 30*a*x^3/((b*x^2 + a*x)^(5/2)*b^
2) + 5*a^2*x^2/((b*x^2 + a*x)^(5/2)*b^3) - a^3*x/((b*x^2 + a*x)^(5/2)*b^4)
 + 2*a*x/((b*x^2 + a*x)^(3/2)*b^3) - 16*x/(sqrt(b*x^2 + a*x)*a*b^2) + a^2/
((b*x^2 + a*x)^(3/2)*b^4) - 8/(sqrt(b*x^2 + a*x)*b^3)) - 1/3*x*(3*x^2/((b*
x^2 + a*x)^(3/2)*b) + a*x/((b*x^2 + a*x)^(3/2)*b^2) - 2*x/(sqrt(b*x^2 + a*
x)*a*b) - 1/(sqrt(b*x^2 + a*x)*b^2))/b - 3/4*a*x^2/((b*x^2 + a*x)^(3/2)*b^
3) - 7/20*a^2*x/((b*x^2 + a*x)^(3/2)*b^4) - 8/15*x/(sqrt(b*x^2 + a*x)*b^3)
 + log(2*b*x + a + 2*sqrt(b*x^2 + a*x)*sqrt(b))/b^(7/2) + 7/20*a/(sqrt(b*x
^2 + a*x)*b^4) - 16/15*sqrt(b*x^2 + a*x)/(a*b^3)
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 182 vs. $2(82) = 164$.

Time = 0.17 (sec) , antiderivative size = 182, normalized size of antiderivative = 1.82

$$\int \frac{x^6}{(ax+bx^2)^{7/2}}\,dx = -\frac{\log\left(\left|2\left(\sqrt{b}x-\sqrt{bx^2+ax}\right)\sqrt{b}+a\right|\right)}{b^{\frac{7}{2}}} - \frac{2\left(45\left(\sqrt{b}x-\sqrt{bx^2+ax}\right)^4ab^2+135\left(\sqrt{b}x-\sqrt{bx^2+ax}\right)^3a^2b^{\frac{3}{2}}+170\left(\sqrt{b}x-\sqrt{bx^2+ax}\right)^2a^3b+100\right.}{15\left(\left(\sqrt{b}x-\sqrt{bx^2+ax}\right)\sqrt{b}+a\right)^5b^{\frac{7}{2}}}$$

```
integrate(x^6/(b*x^2+a*x)^(7/2),x, algorithm="giac")
```

```
-log(abs(2*(sqrt(b)*x - sqrt(b*x^2 + a*x))*sqrt(b) + a))/b^(7/2) - 2/15*(4
5*(sqrt(b)*x - sqrt(b*x^2 + a*x))^4*a*b^2 + 135*(sqrt(b)*x - sqrt(b*x^2 +
a*x))^3*a^2*b^(3/2) + 170*(sqrt(b)*x - sqrt(b*x^2 + a*x))^2*a^3*b + 100*(s
qrt(b)*x - sqrt(b*x^2 + a*x))*a^4*sqrt(b) + 23*a^5)/(((sqrt(b)*x - sqrt(b*
x^2 + a*x))*sqrt(b) + a)^5*b^(7/2))
```

Mupad [F(-1)]

Timed out.

$$\int \frac{x^6}{(ax+bx^2)^{7/2}}\,dx = \int \frac{x^6}{(b\,x^2+a\,x)^{7/2}}\,dx$$

```
int(x^6/(a*x + b*x^2)^(7/2),x)
```

```
int(x^6/(a*x + b*x^2)^(7/2), x)
```

Reduce [B] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 197, normalized size of antiderivative = 1.97

$$\int \frac{x^6}{(ax+bx^2)^{7/2}}\,dx = \frac{2\sqrt{b}\,\sqrt{bx+a}\log\left(\frac{\sqrt{bx+a}+\sqrt{x}\,\sqrt{b}}{\sqrt{a}}\right)a^2+4\sqrt{b}\,\sqrt{bx+a}\log\left(\frac{\sqrt{bx+a}+\sqrt{x}\,\sqrt{b}}{\sqrt{a}}\right)abx+2\sqrt{b}\,\sqrt{bx}}{\sqrt{bx}}$$

```
int(x^6/(b*x^2+a*x)^(7/2),x)
```

```
(2*(15*sqrt(b)*sqrt(a + b*x)*log((sqrt(a + b*x) + sqrt(x)*sqrt(b))/sqrt(a)
)*a**2 + 30*sqrt(b)*sqrt(a + b*x)*log((sqrt(a + b*x) + sqrt(x)*sqrt(b))/sq
rt(a))*a*b*x + 15*sqrt(b)*sqrt(a + b*x)*log((sqrt(a + b*x) + sqrt(x)*sqrt(
b))/sqrt(a))*b**2*x**2 + 5*sqrt(b)*sqrt(a + b*x)*a**2 + 10*sqrt(b)*sqrt(a
+ b*x)*a*b*x + 5*sqrt(b)*sqrt(a + b*x)*b**2*x**2 - 15*sqrt(x)*a**2*b - 35*
sqrt(x)*a*b**2*x - 23*sqrt(x)*b**3*x**2))/(15*sqrt(a + b*x)*b**4*(a**2 + 2
*a*b*x + b**2*x**2))
```

3.67 $\int \frac{x^4}{\sqrt{6x-9x^2}}\,dx$

Optimal result

Integrand size = 17, antiderivative size = 111

$$\int \frac{x^4}{\sqrt{6x-9x^2}}\,dx = -\frac{35\sqrt{2-3x}\sqrt{x}}{648\sqrt{3}}-\frac{35\sqrt{2-3x}x^{3/2}}{648\sqrt{3}}-\frac{7\sqrt{2-3x}x^{5/2}}{108\sqrt{3}} -\frac{\sqrt{2-3x}x^{7/2}}{12\sqrt{3}}+\frac{35}{972}\arcsin\left(\sqrt{\frac{3}{2}}\sqrt{x}\right)$$

```
-35/1944*(2-3*x)^(1/2)*x^(1/2)*3^(1/2)-35/1944*(2-3*x)^(1/2)*x^(3/2)*3^(1/
2)-7/324*(2-3*x)^(1/2)*x^(5/2)*3^(1/2)-1/36*(2-3*x)^(1/2)*x^(7/2)*3^(1/2)+
35/972*arcsin(1/2*6^(1/2)*x^(1/2))
```

Mathematica [A] (verified)

Time = 0.09 (sec) , antiderivative size = 83, normalized size of antiderivative = 0.75

$$\int \frac{x^4}{\sqrt{6x-9x^2}}\,dx = \frac{\sqrt{3}x(-70+35x+21x^2+18x^3+162x^4)-70\sqrt{x}\sqrt{-2+3x}\log\left(-\sqrt{3}\sqrt{x}+\sqrt{-2+3x}\right)}{1944\sqrt{-x(-2+3x)}}$$

```
Integrate[x^4/Sqrt[6*x - 9*x^2],x]
```

```
(Sqrt[3]*x*(-70 + 35*x + 21*x^2 + 18*x^3 + 162*x^4) - 70*Sqrt[x]*Sqrt[-2 +
 3*x]*Log[-(Sqrt[3]*Sqrt[x]) + Sqrt[-2 + 3*x]])/(1944*Sqrt[-(x*(-2 + 3*x))
])
```

Rubi [A] (verified)

Time = 0.46 (sec) , antiderivative size = 120, normalized size of antiderivative = 1.08, number of steps used = 8, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}} = 0.412$, Rules used = {1134, 27, 1134, 1134, 1160, 1090, 223}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^4}{\sqrt{6x-9x^2}}dx$$

$$\Big\downarrow \text{1134}$$

$$\frac{7}{12}\int \frac{x^3}{\sqrt{3}\sqrt{2x-3x^2}}dx-\frac{x^3\sqrt{2x-3x^2}}{12\sqrt{3}}$$

$$\Big\downarrow \text{27}$$

$$\frac{7\int \frac{x^3}{\sqrt{2x-3x^2}}dx}{12\sqrt{3}}-\frac{x^3\sqrt{2x-3x^2}}{12\sqrt{3}}$$

$$\Big\downarrow \text{1134}$$

$$\frac{7\left(\frac{5}{9}\int \frac{x^2}{\sqrt{2x-3x^2}}dx-\frac{1}{9}x^2\sqrt{2x-3x^2}\right)}{12\sqrt{3}}-\frac{x^3\sqrt{2x-3x^2}}{12\sqrt{3}}$$

$$\Big\downarrow \text{1134}$$

$$\frac{7\left(\frac{5}{9}\left(\frac{1}{2}\int \frac{x}{\sqrt{2x-3x^2}}dx-\frac{1}{6}x\sqrt{2x-3x^2}\right)-\frac{1}{9}x^2\sqrt{2x-3x^2}\right)}{12\sqrt{3}}-\frac{x^3\sqrt{2x-3x^2}}{12\sqrt{3}}$$

$$\Big\downarrow \text{1160}$$

$$\frac{7\left(\frac{5}{9}\left(\frac{1}{2}\left(\frac{1}{3}\int \frac{1}{\sqrt{2x-3x^2}}dx-\frac{1}{3}\sqrt{2x-3x^2}\right)-\frac{1}{6}x\sqrt{2x-3x^2}\right)-\frac{1}{9}x^2\sqrt{2x-3x^2}\right)}{12\sqrt{3}}-\frac{x^3\sqrt{2x-3x^2}}{12\sqrt{3}}$$

$$\Big\downarrow \text{1090}$$

$$\frac{7\left(\frac{5}{9}\left(\frac{1}{2}\left(-\frac{\int \frac{1}{\sqrt{1-\frac{1}{4}(2-6x)^2}}d(2-6x)}{6\sqrt{3}}-\frac{1}{3}\sqrt{2x-3x^2}\right)-\frac{1}{6}x\sqrt{2x-3x^2}\right)-\frac{1}{9}x^2\sqrt{2x-3x^2}\right)}{12\sqrt{3}}-\frac{x^3\sqrt{2x-3x^2}}{12\sqrt{3}}$$

$$\Big\downarrow \; 223$$

$$\frac{7\left(\frac{5}{9}\left(\frac{1}{2}\left(-\frac{\arcsin\left(\frac{1}{2}(2-6x)\right)}{3\sqrt{3}}-\frac{1}{3}\sqrt{2x-3x^2}\right)-\frac{1}{6}x\sqrt{2x-3x^2}\right)-\frac{1}{9}x^2\sqrt{2x-3x^2}\right)}{12\sqrt{3}}-\frac{x^3\sqrt{2x-3x^2}}{12\sqrt{3}}$$

```
Int[x^4/Sqrt[6*x - 9*x^2],x]
```

```
-1/12*(x^3*Sqrt[2*x - 3*x^2])/Sqrt[3] + (7*(-1/9*(x^2*Sqrt[2*x - 3*x^2]) +
 (5*(-1/6*(x*Sqrt[2*x - 3*x^2]) + (-1/3*Sqrt[2*x - 3*x^2] - ArcSin[(2 - 6*
x)/2]/(3*Sqrt[3]))/2))/9))/(12*Sqrt[3])
```

Defintions of rubi rules used

```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSin[Rt[-b, 2]*(x/Sqrt
[a])]/Rt[-b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && NegQ[b]
```

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[1/(2*c*(-4*
(c/(b^2 - 4*a*c)))^p)   Subst[Int[Simp[1 - x^2/(b^2 - 4*a*c), x]^p, x], x,
b + 2*c*x], x] /; FreeQ[{a, b, c, p}, x] && GtQ[4*a - b^2/c, 0]
```

```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] :> Simp[e*(d + e*x)^(m - 1)*((a + b*x + c*x^2)^(p + 1)/(c*(m + 2*p +
 1))), x] + Simp[(m + p)*((2*c*d - b*e)/(c*(m + 2*p + 1)))   Int[(d + e*x)^
(m - 1)*(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, p}, x] && EqQ[
c*d^2 - b*d*e + a*e^2, 0] && GtQ[m, 1] && NeQ[m + 2*p + 1, 0] && IntegerQ[2
*p]
```

```
Int[((d_.) + (e_.)*(x_))*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol
] :> Simp[e*((a + b*x + c*x^2)^(p + 1)/(2*c*(p + 1))), x] + Simp[(2*c*d - b
*e)/(2*c)   Int[(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, p}, x]
 && NeQ[p, -1]
```

Maple [A] (verified)

Time = 0.36 (sec) , antiderivative size = 46, normalized size of antiderivative = 0.41

method	result
risch	$\frac{(54x^3+42x^2+35x+35)x(-2+3x)\sqrt{3}}{1944\sqrt{-x(-2+3x)}}+\frac{35\arcsin(3x-1)}{1944}$
pseudoelliptic	$-\frac{35\arctan\left(\frac{\sqrt{-3x^2+2x}\,\sqrt{3}}{3x}\right)}{972}+\frac{(-54x^3-42x^2-35x-35)\sqrt{3}\,\sqrt{-3x^2+2x}}{1944}$
meijerg	$-\frac{16i\left(-\frac{i\sqrt{6}\,\sqrt{\pi}\,\sqrt{x}\,(486x^3+378x^2+315x+315)\sqrt{1-\frac{3x}{2}}}{1152}+\frac{35i\sqrt{\pi}\,\arcsin\left(\frac{\sqrt{x}\,\sqrt{2}\,\sqrt{3}}{2}\right)}{64}\right)}{243\sqrt{\pi}}$
trager	$\frac{\left(-\frac{1}{12}x^3-\frac{7}{108}x^2-\frac{35}{648}x-\frac{35}{648}\right)\sqrt{-9x^2+6x}}{3}+\frac{35\,\mathrm{RootOf}\left(\_Z^2+1\right)\ln\left(-3\,\mathrm{RootOf}\left(\_Z^2+1\right)x+\sqrt{-9x^2+6x}+\mathrm{RootOf}\left(\_\right.\right.}{1944}$
default	$-\frac{x^3\sqrt{-9x^2+6x}}{36}-\frac{7x^2\sqrt{-9x^2+6x}}{324}-\frac{35x\sqrt{-9x^2+6x}}{1944}-\frac{35\sqrt{-9x^2+6x}}{1944}+\frac{35\arcsin(3x-1)}{1944}$

```
int(x^4/(-9*x^2+6*x)^(1/2),x,method=_RETURNVERBOSE)
```

```
1/1944*(54*x^3+42*x^2+35*x+35)*x*(-2+3*x)/(-x*(-2+3*x))^(1/2)*3^(1/2)+35/1
944*arcsin(3*x-1)
```

Fricas [A] (verification not implemented)

Time = 0.10 (sec) , antiderivative size = 51, normalized size of antiderivative = 0.46

$$\int \frac{x^4}{\sqrt{6x-9x^2}}\,dx = -\frac{1}{1944}\left(54\,x^3+42\,x^2+35\,x+35\right)\sqrt{-9\,x^2+6\,x} - \frac{35}{972}\arctan\left(\frac{\sqrt{-9\,x^2+6\,x}}{3\,x-2}\right)$$

```
integrate(x^4/(-9*x^2+6*x)^(1/2),x, algorithm="fricas")
```

```
-1/1944*(54*x^3 + 42*x^2 + 35*x + 35)*sqrt(-9*x^2 + 6*x) - 35/972*arctan(s
qrt(-9*x^2 + 6*x)/(3*x - 2))
```

Sympy [A] (verification not implemented)

Time = 0.49 (sec) , antiderivative size = 42, normalized size of antiderivative = 0.38

$$\int \frac{x^4}{\sqrt{6x-9x^2}}\,dx = \sqrt{-9x^2+6x}\left(-\frac{x^3}{36}-\frac{7x^2}{324}-\frac{35x}{1944}-\frac{35}{1944}\right)+\frac{35\operatorname{asin}(3x-1)}{1944}$$

```
integrate(x**4/(-9*x**2+6*x)**(1/2),x)
```

```
sqrt(-9*x**2 + 6*x)*(-x**3/36 - 7*x**2/324 - 35*x/1944 - 35/1944) + 35*asi
n(3*x - 1)/1944
```

Maxima [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 68, normalized size of antiderivative = 0.61

$$\int \frac{x^4}{\sqrt{6x-9x^2}}\,dx = -\frac{1}{36}\sqrt{-9\,x^2+6\,x}x^3 - \frac{7}{324}\sqrt{-9\,x^2+6\,x}x^2 - \frac{35}{1944}\sqrt{-9\,x^2+6\,x}x - \frac{35}{1944}\sqrt{-9\,x^2+6\,x} - \frac{35}{1944}\arcsin\left(-3\,x+1\right)$$

```
integrate(x^4/(-9*x^2+6*x)^(1/2),x, algorithm="maxima")
```

```
-1/36*sqrt(-9*x^2 + 6*x)*x^3 - 7/324*sqrt(-9*x^2 + 6*x)*x^2 - 35/1944*sqrt
(-9*x^2 + 6*x)*x - 35/1944*sqrt(-9*x^2 + 6*x) - 35/1944*arcsin(-3*x + 1)
```

Giac [A] (verification not implemented)

Time = 0.15 (sec) , antiderivative size = 51, normalized size of antiderivative = 0.46

$$\int \frac{x^4}{\sqrt{6x-9x^2}}\,dx = -\frac{1}{1944}\left(\left(6\left(9\sqrt{3}x+7\sqrt{3}\right)x+35\sqrt{3}\right)x+35\sqrt{3}\right)\sqrt{-3x^2+2x} + \frac{35}{1944}\arcsin\left(3x-1\right)$$

```
integrate(x^4/(-9*x^2+6*x)^(1/2),x, algorithm="giac")
```

```
-1/1944*((6*(9*sqrt(3)*x + 7*sqrt(3))*x + 35*sqrt(3))*x + 35*sqrt(3))*sqrt
(-3*x^2 + 2*x) + 35/1944*arcsin(3*x - 1)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{x^4}{\sqrt{6x-9x^2}}\,dx = \int \frac{x^4}{\sqrt{6\,x-9\,x^2}}\,dx$$

```
int(x^4/(6*x - 9*x^2)^(1/2),x)
```

```
int(x^4/(6*x - 9*x^2)^(1/2), x)
```

Reduce [B] (verification not implemented)

Time = 0.35 (sec) , antiderivative size = 78, normalized size of antiderivative = 0.70

$$\int \frac{x^4}{\sqrt{6x-9x^2}}\,dx = -\frac{\sqrt{x}\,\sqrt{-3x+2}\,\sqrt{3}\,x^3}{36} - \frac{7\sqrt{x}\,\sqrt{-3x+2}\,\sqrt{3}\,x^2}{324} - \frac{35\sqrt{x}\,\sqrt{-3x+2}\,\sqrt{3}\,x}{1944} - \frac{35\sqrt{x}\,\sqrt{-3x+2}\,\sqrt{3}}{1944} - \frac{35\log\left(\frac{\sqrt{-3x+2}+\sqrt{x}\,\sqrt{3}\,i}{\sqrt{2}}\right)i}{972}$$

```
int(x^4/(-9*x^2+6*x)^(1/2),x)
```

```
( - 54*sqrt(x)*sqrt( - 3*x + 2)*sqrt(3)*x**3 - 42*sqrt(x)*sqrt( - 3*x + 2)
*sqrt(3)*x**2 - 35*sqrt(x)*sqrt( - 3*x + 2)*sqrt(3)*x - 35*sqrt(x)*sqrt( -
 3*x + 2)*sqrt(3) - 70*log((sqrt( - 3*x + 2) + sqrt(x)*sqrt(3)*i)/sqrt(2))
*i)/1944
```

3.68 $\int \frac{x^3}{\sqrt{6x-9x^2}}\,dx$

Optimal result

Integrand size = 17, antiderivative size = 88

$$\int \frac{x^3}{\sqrt{6x-9x^2}}\,dx=-\frac{5\sqrt{2-3x}\sqrt{x}}{54\sqrt{3}}-\frac{5\sqrt{2-3x}x^{3/2}}{54\sqrt{3}}-\frac{\sqrt{2-3x}x^{5/2}}{9\sqrt{3}}+\frac{5}{81}\arcsin\left(\sqrt{\frac{3}{2}}\sqrt{x}\right)$$

```
-5/162*(2-3*x)^(1/2)*x^(1/2)*3^(1/2)-5/162*(2-3*x)^(1/2)*x^(3/2)*3^(1/2)-1
/27*(2-3*x)^(1/2)*x^(5/2)*3^(1/2)+5/81*arcsin(1/2*6^(1/2)*x^(1/2))
```

Mathematica [A] (verified)

Time = 0.08 (sec) , antiderivative size = 78, normalized size of antiderivative = 0.89

$$\int \frac{x^3}{\sqrt{6x-9x^2}}\,dx = \frac{\sqrt{3}x(-10+5x+3x^2+18x^3)-10\sqrt{x}\sqrt{-2+3x}\log\left(-\sqrt{3}\sqrt{x}+\sqrt{-2+3x}\right)}{162\sqrt{-x(-2+3x)}}$$

```
Integrate[x^3/Sqrt[6*x - 9*x^2],x]
```

```
(Sqrt[3]*x*(-10 + 5*x + 3*x^2 + 18*x^3) - 10*Sqrt[x]*Sqrt[-2 + 3*x]*Log[-(
Sqrt[3]*Sqrt[x]) + Sqrt[-2 + 3*x]])/(162*Sqrt[-(x*(-2 + 3*x))])
```

Rubi [A] (verified)

Time = 0.40 (sec) , antiderivative size = 95, normalized size of antiderivative = 1.08, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.353, Rules used = {1134, 27, 1134, 1160, 1090, 223}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^3}{\sqrt{6x-9x^2}}dx$$

$$\Big\downarrow \text{1134}$$

$$\frac{5}{9}\int \frac{x^2}{\sqrt{3}\sqrt{2x-3x^2}}dx-\frac{x^2\sqrt{2x-3x^2}}{9\sqrt{3}}$$

$$\Big\downarrow \text{27}$$

$$\frac{5\int \frac{x^2}{\sqrt{2x-3x^2}}dx}{9\sqrt{3}}-\frac{x^2\sqrt{2x-3x^2}}{9\sqrt{3}}$$

$$\Big\downarrow \text{1134}$$

$$\frac{5\left(\frac{1}{2}\int \frac{x}{\sqrt{2x-3x^2}}dx-\frac{1}{6}x\sqrt{2x-3x^2}\right)}{9\sqrt{3}}-\frac{x^2\sqrt{2x-3x^2}}{9\sqrt{3}}$$

$$\Big\downarrow \text{1160}$$

$$\frac{5\left(\frac{1}{2}\left(\frac{1}{3}\int \frac{1}{\sqrt{2x-3x^2}}dx-\frac{1}{3}\sqrt{2x-3x^2}\right)-\frac{1}{6}x\sqrt{2x-3x^2}\right)}{9\sqrt{3}}-\frac{x^2\sqrt{2x-3x^2}}{9\sqrt{3}}$$

$$\Big\downarrow \text{1090}$$

$$\frac{5\left(\frac{1}{2}\left(-\frac{\int \frac{1}{\sqrt{1-\frac{1}{4}(2-6x)^2}}d(2-6x)}{6\sqrt{3}}-\frac{1}{3}\sqrt{2x-3x^2}\right)-\frac{1}{6}x\sqrt{2x-3x^2}\right)}{9\sqrt{3}}-\frac{x^2\sqrt{2x-3x^2}}{9\sqrt{3}}$$

$$\Big\downarrow \text{223}$$

$$\frac{5\left(\frac{1}{2}\left(-\frac{\arcsin\left(\frac{1}{2}(2-6x)\right)}{3\sqrt{3}}-\frac{1}{3}\sqrt{2x-3x^2}\right)-\frac{1}{6}x\sqrt{2x-3x^2}\right)}{9\sqrt{3}}-\frac{x^2\sqrt{2x-3x^2}}{9\sqrt{3}}$$

```
Int[x^3/Sqrt[6*x - 9*x^2],x]
```

```
-1/9*(x^2*Sqrt[2*x - 3*x^2])/Sqrt[3] + (5*(-1/6*(x*Sqrt[2*x - 3*x^2]) + (-
1/3*Sqrt[2*x - 3*x^2] - ArcSin[(2 - 6*x)/2]/(3*Sqrt[3]))/2))/(9*Sqrt[3])
```

Defintions of rubi rules used

```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSin[Rt[-b, 2]*(x/Sqrt
[a])]/Rt[-b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && NegQ[b]
```

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[1/(2*c*(-4*
(c/(b^2 - 4*a*c)))^p)   Subst[Int[Simp[1 - x^2/(b^2 - 4*a*c), x]^p, x], x,
b + 2*c*x], x] /; FreeQ[{a, b, c, p}, x] && GtQ[4*a - b^2/c, 0]
```

```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] :> Simp[e*(d + e*x)^(m - 1)*((a + b*x + c*x^2)^(p + 1)/(c*(m + 2*p +
 1))), x] + Simp[(m + p)*((2*c*d - b*e)/(c*(m + 2*p + 1)))   Int[(d + e*x)^
(m - 1)*(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, p}, x] && EqQ[
c*d^2 - b*d*e + a*e^2, 0] && GtQ[m, 1] && NeQ[m + 2*p + 1, 0] && IntegerQ[2
*p]
```

```
Int[((d_.) + (e_.)*(x_))*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol
] :> Simp[e*((a + b*x + c*x^2)^(p + 1)/(2*c*(p + 1))), x] + Simp[(2*c*d - b
*e)/(2*c)   Int[(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, p}, x]
 && NeQ[p, -1]
```

Maple [A] (verified)

Time = 0.33 (sec) , antiderivative size = 41, normalized size of antiderivative = 0.47

method	result
risch	$\frac{(6x^2+5x+5)x(-2+3x)\sqrt{3}}{162\sqrt{-x(-2+3x)}}+\frac{5\arcsin(3x-1)}{162}$
pseudoelliptic	$-\frac{5\arctan\left(\frac{\sqrt{-3x^2+2x}\,\sqrt{3}}{3x}\right)}{81}+\frac{(-6x^2-5x-5)\sqrt{3}\,\sqrt{-3x^2+2x}}{162}$
default	$-\frac{x^2\sqrt{-9x^2+6x}}{27}-\frac{5x\sqrt{-9x^2+6x}}{162}-\frac{5\sqrt{-9x^2+6x}}{162}+\frac{5\arcsin(3x-1)}{162}$
meijerg	$\frac{8i\left(\frac{i\sqrt{6}\,\sqrt{\pi}\,\sqrt{x}\,\left(126x^2+105x+105\right)\sqrt{1-\frac{3x}{2}}}{336}-\frac{5i\sqrt{\pi}\,\arcsin\left(\frac{\sqrt{x}\,\sqrt{2}\,\sqrt{3}}{2}\right)}{8}\right)}{81\sqrt{\pi}}$
trager	$\frac{\left(-\frac{1}{9}x^2-\frac{5}{54}x-\frac{5}{54}\right)\sqrt{-9x^2+6x}}{3}+\frac{5\,\mathrm{RootOf}\left(\_Z^2+1\right)\ln\left(-3\,\mathrm{RootOf}\left(\_Z^2+1\right)x+\sqrt{-9x^2+6x}+\mathrm{RootOf}\left(\_Z^2+1\right)\right)}{162}$

```
int(x^3/(-9*x^2+6*x)^(1/2),x,method=_RETURNVERBOSE)
```

```
1/162*(6*x^2+5*x+5)*x*(-2+3*x)/(-x*(-2+3*x))^(1/2)*3^(1/2)+5/162*arcsin(3*
x-1)
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 46, normalized size of antiderivative = 0.52

$$\int \frac{x^3}{\sqrt{6x-9x^2}}\,dx=-\frac{1}{162}\left(6\,x^2+5\,x+5\right)\sqrt{-9\,x^2+6\,x}-\frac{5}{81}\arctan\left(\frac{\sqrt{-9\,x^2+6\,x}}{3\,x-2}\right)$$

```
integrate(x^3/(-9*x^2+6*x)^(1/2),x, algorithm="fricas")
```

```
-1/162*(6*x^2 + 5*x + 5)*sqrt(-9*x^2 + 6*x) - 5/81*arctan(sqrt(-9*x^2 + 6*
x)/(3*x - 2))
```

Sympy [A] (verification not implemented)

Time = 0.39 (sec) , antiderivative size = 36, normalized size of antiderivative = 0.41

$$\int \frac{x^3}{\sqrt{6x-9x^2}}\,dx = \sqrt{-9x^2+6x}\left(-\frac{x^2}{27}-\frac{5x}{162}-\frac{5}{162}\right)+\frac{5\operatorname{asin}{(3x-1)}}{162}$$

```
integrate(x**3/(-9*x**2+6*x)**(1/2),x)
```

```
sqrt(-9*x**2 + 6*x)*(-x**2/27 - 5*x/162 - 5/162) + 5*asin(3*x - 1)/162
```

Maxima [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 52, normalized size of antiderivative = 0.59

$$\int \frac{x^3}{\sqrt{6x-9x^2}}\,dx = -\frac{1}{27}\sqrt{-9x^2+6x}x^2-\frac{5}{162}\sqrt{-9x^2+6x}x - \frac{5}{162}\sqrt{-9x^2+6x}-\frac{5}{162}\arcsin(-3x+1)$$

```
integrate(x^3/(-9*x^2+6*x)^(1/2),x, algorithm="maxima")
```

```
-1/27*sqrt(-9*x^2 + 6*x)*x^2 - 5/162*sqrt(-9*x^2 + 6*x)*x - 5/162*sqrt(-9*
x^2 + 6*x) - 5/162*arcsin(-3*x + 1)
```

Giac [A] (verification not implemented)

Time = 0.14 (sec) , antiderivative size = 42, normalized size of antiderivative = 0.48

$$\int \frac{x^3}{\sqrt{6x-9x^2}}\,dx = -\frac{1}{162}\left(\left(6\sqrt{3}x+5\sqrt{3}\right)x+5\sqrt{3}\right)\sqrt{-3x^2+2x} + \frac{5}{162}\arcsin(3x-1)$$

```
integrate(x^3/(-9*x^2+6*x)^(1/2),x, algorithm="giac")
```

```
-1/162*((6*sqrt(3)*x + 5*sqrt(3))*x + 5*sqrt(3))*sqrt(-3*x^2 + 2*x) + 5/16
2*arcsin(3*x - 1)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{x^3}{\sqrt{6x-9x^2}}\,dx=\int \frac{x^3}{\sqrt{6\,x-9\,x^2}}\,dx$$

```
int(x^3/(6*x - 9*x^2)^(1/2),x)
```

```
int(x^3/(6*x - 9*x^2)^(1/2), x)
```

Reduce [B] (verification not implemented)

Time = 0.52 (sec) , antiderivative size = 63, normalized size of antiderivative = 0.72

$$\int \frac{x^3}{\sqrt{6x-9x^2}}\,dx=-\frac{\sqrt{x}\,\sqrt{-3x+2}\,\sqrt{3}\,x^2}{27}-\frac{5\sqrt{x}\,\sqrt{-3x+2}\,\sqrt{3}\,x}{162}-\frac{5\sqrt{x}\,\sqrt{-3x+2}\,\sqrt{3}}{162}-\frac{5\log\left(\frac{\sqrt{-3x+2}+\sqrt{x}\,\sqrt{3}\,i}{\sqrt{2}}\right)i}{81}$$

```
int(x^3/(-9*x^2+6*x)^(1/2),x)
```

```
( - 6*sqrt(x)*sqrt( - 3*x + 2)*sqrt(3)*x**2 - 5*sqrt(x)*sqrt( - 3*x + 2)*s
qrt(3)*x - 5*sqrt(x)*sqrt( - 3*x + 2)*sqrt(3) - 10*log((sqrt( - 3*x + 2) +
 sqrt(x)*sqrt(3)*i)/sqrt(2))*i)/162
```

3.69 $\int \frac{x^2}{\sqrt{6x-9x^2}}\,dx$

Optimal result

Integrand size = 17, antiderivative size = 65

$$\int \frac{x^2}{\sqrt{6x-9x^2}}\,dx = -\frac{\sqrt{2-3x}\sqrt{x}}{6\sqrt{3}} - \frac{\sqrt{2-3x}x^{3/2}}{6\sqrt{3}} + \frac{1}{9}\arcsin\left(\sqrt{\frac{3}{2}}\sqrt{x}\right)$$

```
-1/18*(2-3*x)^(1/2)*x^(1/2)*3^(1/2)-1/18*(2-3*x)^(1/2)*x^(3/2)*3^(1/2)+1/9
*arcsin(1/2*6^(1/2)*x^(1/2))
```

Mathematica [A] (verified)

Time = 0.08 (sec) , antiderivative size = 71, normalized size of antiderivative = 1.09

$$\int \frac{x^2}{\sqrt{6x-9x^2}}\,dx = \frac{\sqrt{3}x(-2+x+3x^2) - 2\sqrt{x}\sqrt{-2+3x}\log\left(-\sqrt{3}\sqrt{x}+\sqrt{-2+3x}\right)}{18\sqrt{-x(-2+3x)}}$$

```
Integrate[x^2/Sqrt[6*x - 9*x^2],x]
```

```
(Sqrt[3]*x*(-2 + x + 3*x^2) - 2*Sqrt[x]*Sqrt[-2 + 3*x]*Log[-(Sqrt[3]*Sqrt[
x]) + Sqrt[-2 + 3*x]])/(18*Sqrt[-(x*(-2 + 3*x))])
```

Rubi [A] (verified)

Time = 0.33 (sec) , antiderivative size = 70, normalized size of antiderivative = 1.08, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.294$, Rules used = {1134, 27, 1160, 1090, 223}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^2}{\sqrt{6x-9x^2}}dx$$

$$\Big\downarrow \text{1134}$$

$$\frac{1}{2}\int \frac{x}{\sqrt{3}\sqrt{2x-3x^2}}dx-\frac{x\sqrt{2x-3x^2}}{6\sqrt{3}}$$

$$\Big\downarrow \text{27}$$

$$\frac{\int \frac{x}{\sqrt{2x-3x^2}}dx}{2\sqrt{3}}-\frac{x\sqrt{2x-3x^2}}{6\sqrt{3}}$$

$$\Big\downarrow \text{1160}$$

$$\frac{\frac{1}{3}\int \frac{1}{\sqrt{2x-3x^2}}dx-\frac{1}{3}\sqrt{2x-3x^2}}{2\sqrt{3}}-\frac{x\sqrt{2x-3x^2}}{6\sqrt{3}}$$

$$\Big\downarrow \text{1090}$$

$$\frac{-\frac{\int \frac{1}{\sqrt{1-\frac{1}{4}(2-6x)^2}}d(2-6x)}{6\sqrt{3}}-\frac{1}{3}\sqrt{2x-3x^2}}{2\sqrt{3}}-\frac{x\sqrt{2x-3x^2}}{6\sqrt{3}}$$

$$\Big\downarrow \text{223}$$

$$\frac{-\frac{\arcsin\left(\frac{1}{2}(2-6x)\right)}{3\sqrt{3}}-\frac{1}{3}\sqrt{2x-3x^2}}{2\sqrt{3}}-\frac{x\sqrt{2x-3x^2}}{6\sqrt{3}}$$

```
Int[x^2/Sqrt[6*x - 9*x^2],x]
```

```
-1/6*(x*Sqrt[2*x - 3*x^2])/Sqrt[3] + (-1/3*Sqrt[2*x - 3*x^2] - ArcSin[(2 -
 6*x)/2]/(3*Sqrt[3]))/(2*Sqrt[3])
```

Defintions of rubi rules used

```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSin[Rt[-b, 2]*(x/Sqrt
[a])]/Rt[-b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && NegQ[b]
```

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[1/(2*c*(-4*
(c/(b^2 - 4*a*c)))^p)   Subst[Int[Simp[1 - x^2/(b^2 - 4*a*c), x]^p, x], x,
b + 2*c*x], x] /; FreeQ[{a, b, c, p}, x] && GtQ[4*a - b^2/c, 0]
```

```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] :> Simp[e*(d + e*x)^(m - 1)*((a + b*x + c*x^2)^(p + 1)/(c*(m + 2*p +
 1))), x] + Simp[(m + p)*((2*c*d - b*e)/(c*(m + 2*p + 1)))   Int[(d + e*x)^
(m - 1)*(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, p}, x] && EqQ[
c*d^2 - b*d*e + a*e^2, 0] && GtQ[m, 1] && NeQ[m + 2*p + 1, 0] && IntegerQ[2
*p]
```

```
Int[((d_.) + (e_.)*(x_))*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol
] :> Simp[e*((a + b*x + c*x^2)^(p + 1)/(2*c*(p + 1))), x] + Simp[(2*c*d - b
*e)/(2*c)   Int[(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, p}, x]
 && NeQ[p, -1]
```

Maple [A] (verified)

Time = 0.34 (sec) , antiderivative size = 34, normalized size of antiderivative = 0.52

method	result	size
risch	$\frac{(x+1)x(-2+3x)\sqrt{3}}{18\sqrt{-x(-2+3x)}}+\frac{\arcsin(3x-1)}{18}$	34
default	$-\frac{x\sqrt{-9x^2+6x}}{18}-\frac{\sqrt{-9x^2+6x}}{18}+\frac{\arcsin(3x-1)}{18}$	37
pseudoelliptic	$-\frac{\arctan\left(\frac{\sqrt{-3x^2+2x}\,\sqrt{3}}{3x}\right)}{9}+\frac{(-x-1)\sqrt{3}\,\sqrt{-3x^2+2x}}{18}$	45
meijerg	$-\frac{4i\left(-\frac{i\sqrt{6}\,\sqrt{\pi}\,\sqrt{x}\,(15x+15)\sqrt{1-\frac{3x}{2}}}{40}+\frac{3i\sqrt{\pi}\,\arcsin\left(\frac{\sqrt{x}\,\sqrt{2}\,\sqrt{3}}{2}\right)}{4}\right)}{27\sqrt{\pi}}$	50
trager	$\frac{\left(-\frac{x}{6}-\frac{1}{6}\right)\sqrt{-9x^2+6x}}{3}+\frac{\operatorname{RootOf}\left(\_Z^2+1\right)\ln\left(-3\operatorname{RootOf}\left(\_Z^2+1\right)x+\sqrt{-9x^2+6x}+\operatorname{RootOf}\left(\_Z^2+1\right)\right)}{18}$	56

```
int(x^2/(-9*x^2+6*x)^(1/2),x,method=_RETURNVERBOSE)
```

```
1/18*(x+1)*x*(-2+3*x)/(-x*(-2+3*x))^(1/2)*3^(1/2)+1/18*arcsin(3*x-1)
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 39, normalized size of antiderivative = 0.60

$$\int \frac{x^2}{\sqrt{6x-9x^2}}\,dx = -\frac{1}{18}\,\sqrt{-9\,x^2+6\,x}(x+1)-\frac{1}{9}\,\arctan\left(\frac{\sqrt{-9\,x^2+6\,x}}{3\,x-2}\right)$$

```
integrate(x^2/(-9*x^2+6*x)^(1/2),x, algorithm="fricas")
```

```
-1/18*sqrt(-9*x^2 + 6*x)*(x + 1) - 1/9*arctan(sqrt(-9*x^2 + 6*x)/(3*x - 2)
)
```

Sympy [A] (verification not implemented)

Time = 0.33 (sec) , antiderivative size = 27, normalized size of antiderivative = 0.42

$$\int \frac{x^2}{\sqrt{6x-9x^2}}\,dx = \left(-\frac{x}{18}-\frac{1}{18}\right)\sqrt{-9x^2+6x}+\frac{\operatorname{asin}(3x-1)}{18}$$

```
integrate(x**2/(-9*x**2+6*x)**(1/2),x)
```

```
(-x/18 - 1/18)*sqrt(-9*x**2 + 6*x) + asin(3*x - 1)/18
```

Maxima [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 36, normalized size of antiderivative = 0.55

$$\int \frac{x^2}{\sqrt{6x-9x^2}}\,dx = -\frac{1}{18}\sqrt{-9\,x^2+6\,x}x-\frac{1}{18}\sqrt{-9\,x^2+6\,x}-\frac{1}{18}\arcsin(-3\,x+1)$$

```
integrate(x^2/(-9*x^2+6*x)^(1/2),x, algorithm="maxima")
```

```
-1/18*sqrt(-9*x^2 + 6*x)*x - 1/18*sqrt(-9*x^2 + 6*x) - 1/18*arcsin(-3*x +
1)
```

Giac [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 31, normalized size of antiderivative = 0.48

$$\int \frac{x^2}{\sqrt{6x-9x^2}}\,dx = -\frac{1}{18}\sqrt{-3\,x^2+2\,x}\left(\sqrt{3}x+\sqrt{3}\right)+\frac{1}{18}\arcsin(3\,x-1)$$

```
integrate(x^2/(-9*x^2+6*x)^(1/2),x, algorithm="giac")
```

```
-1/18*sqrt(-3*x^2 + 2*x)*(sqrt(3)*x + sqrt(3)) + 1/18*arcsin(3*x - 1)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{x^2}{\sqrt{6x-9x^2}}\,dx=\int \frac{x^2}{\sqrt{6\,x-9\,x^2}}\,dx$$

```
int(x^2/(6*x - 9*x^2)^(1/2),x)
```

```
int(x^2/(6*x - 9*x^2)^(1/2), x)
```

Reduce [B] (verification not implemented)

Time = 0.33 (sec) , antiderivative size = 48, normalized size of antiderivative = 0.74

$$\int \frac{x^2}{\sqrt{6x-9x^2}}\,dx=-\frac{\sqrt{x}\,\sqrt{-3x+2}\,\sqrt{3}\,x}{18}-\frac{\sqrt{x}\,\sqrt{-3x+2}\,\sqrt{3}}{18}-\frac{\log\left(\frac{\sqrt{-3x+2}+\sqrt{x}\,\sqrt{3}\,i}{\sqrt{2}}\right)i}{9}$$

```
int(x^2/(-9*x^2+6*x)^(1/2),x)
```

```
( - sqrt(x)*sqrt( - 3*x + 2)*sqrt(3)*x - sqrt(x)*sqrt( - 3*x + 2)*sqrt(3)
- 2*log((sqrt( - 3*x + 2) + sqrt(x)*sqrt(3)*i)/sqrt(2))*i)/18
```

3.70 $\int \frac{x}{\sqrt{6x-9x^2}}\,dx$

Optimal result

Integrand size = 15, antiderivative size = 42

$$\int \frac{x}{\sqrt{6x-9x^2}}\,dx=-\frac{\sqrt{2-3x}\sqrt{x}}{3\sqrt{3}}+\frac{2}{9}\arcsin\left(\sqrt{\frac{3}{2}}\sqrt{x}\right)$$

```
-1/9*(2-3*x)^(1/2)*x^(1/2)*3^(1/2)+2/9*arcsin(1/2*6^(1/2)*x^(1/2))
```

Mathematica [A] (verified)

Time = 0.05 (sec) , antiderivative size = 68, normalized size of antiderivative = 1.62

$$\int \frac{x}{\sqrt{6x-9x^2}}\,dx=\frac{\sqrt{3}x(-2+3x)-2\sqrt{x}\sqrt{-2+3x}\log\left(-\sqrt{3}\sqrt{x}+\sqrt{-2+3x}\right)}{9\sqrt{-x(-2+3x)}}$$

```
Integrate[x/Sqrt[6*x - 9*x^2],x]
```

```
(Sqrt[3]*x*(-2 + 3*x) - 2*Sqrt[x]*Sqrt[-2 + 3*x]*Log[-(Sqrt[3]*Sqrt[x]) +
Sqrt[-2 + 3*x]])/(9*Sqrt[-(x*(-2 + 3*x))])
```

Rubi [A] (verified)

Time = 0.28 (sec) , antiderivative size = 37, normalized size of antiderivative = 0.88, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.200, Rules used = {1160, 1090, 223}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x}{\sqrt{6x-9x^2}}dx$$
$$\Big\downarrow \text{ 1160}$$
$$\frac{1}{3}\int \frac{1}{\sqrt{6x-9x^2}}dx-\frac{\sqrt{2x-3x^2}}{3\sqrt{3}}$$
$$\Big\downarrow \text{ 1090}$$
$$-\frac{1}{54}\int \frac{1}{\sqrt{1-\frac{1}{36}(6-18x)^2}}d(6-18x)-\frac{\sqrt{2x-3x^2}}{3\sqrt{3}}$$
$$\Big\downarrow \text{ 223}$$
$$-\frac{1}{9}\arcsin\left(\frac{1}{6}(6-18x)\right)-\frac{\sqrt{2x-3x^2}}{3\sqrt{3}}$$

```
Int[x/Sqrt[6*x - 9*x^2],x]
```

```
-1/3*Sqrt[2*x - 3*x^2]/Sqrt[3] - ArcSin[(6 - 18*x)/6]/9
```

Defintions of rubi rules used

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSin[Rt[-b, 2]*(x/Sqrt
[a])]/Rt[-b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && NegQ[b]
```

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[1/(2*c*(-4*
(c/(b^2 - 4*a*c)))^p)   Subst[Int[Simp[1 - x^2/(b^2 - 4*a*c), x]^p, x], x,
b + 2*c*x], x] /; FreeQ[{a, b, c, p}, x] && GtQ[4*a - b^2/c, 0]
```

```
Int[((d_.) + (e_.)*(x_))*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol
] :> Simp[e*((a + b*x + c*x^2)^(p + 1)/(2*c*(p + 1))), x] + Simp[(2*c*d - b
*e)/(2*c)   Int[(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, p}, x]
 && NeQ[p, -1]
```

Maple [A] (verified)

Time = 0.33 (sec) , antiderivative size = 23, normalized size of antiderivative = 0.55

method	result	size
default	$-\frac{\sqrt{-9x^2+6x}}{9}+\frac{\arcsin(3x-1)}{9}$	23
risch	$\frac{x(-2+3x)\sqrt{3}}{9\sqrt{-x(-2+3x)}}+\frac{\arcsin(3x-1)}{9}$	31
pseudoelliptic	$-\frac{\sqrt{3}\,\sqrt{-3x^2+2x}}{9}-\frac{2\arctan\left(\frac{\sqrt{-3x^2+2x}\,\sqrt{3}}{3x}\right)}{9}$	40
meijerg	$\frac{2i\left(\frac{i\sqrt{6}\,\sqrt{\pi}\,\sqrt{x}\,\sqrt{1-\frac{3x}{2}}}{2}-i\sqrt{\pi}\,\arcsin\left(\frac{\sqrt{x}\,\sqrt{2}\,\sqrt{3}}{2}\right)\right)}{9\sqrt{\pi}}$	45
trager	$-\frac{\sqrt{-9x^2+6x}}{9}+\frac{\operatorname{RootOf}\left(\_Z^2+1\right)\ln\left(-3\operatorname{RootOf}\left(\_Z^2+1\right)x+\sqrt{-9x^2+6x}+\operatorname{RootOf}\left(\_Z^2+1\right)\right)}{9}$	51

```
int(x/(-9*x^2+6*x)^(1/2),x,method=_RETURNVERBOSE)
```

```
-1/9*(-9*x^2+6*x)^(1/2)+1/9*arcsin(3*x-1)
```

Fricas [A] (verification not implemented)

Time = 0.10 (sec) , antiderivative size = 36, normalized size of antiderivative = 0.86

$$\int \frac{x}{\sqrt{6x-9x^2}}\,dx=-\frac{1}{9}\sqrt{-9\,x^2+6\,x}-\frac{2}{9}\arctan\left(\frac{\sqrt{-9\,x^2+6\,x}}{3\,x-2}\right)$$

```
integrate(x/(-9*x^2+6*x)^(1/2),x, algorithm="fricas")
```

```
-1/9*sqrt(-9*x^2 + 6*x) - 2/9*arctan(sqrt(-9*x^2 + 6*x)/(3*x - 2))
```

Sympy [A] (verification not implemented)

Time = 0.29 (sec) , antiderivative size = 20, normalized size of antiderivative = 0.48

$$\int \frac{x}{\sqrt{6x-9x^2}}\,dx = -\frac{\sqrt{-9x^2+6x}}{9} + \frac{\operatorname{asin}(3x-1)}{9}$$

```
integrate(x/(-9*x**2+6*x)**(1/2),x)
```

```
-sqrt(-9*x**2 + 6*x)/9 + asin(3*x - 1)/9
```

Maxima [A] (verification not implemented)

Time = 0.10 (sec) , antiderivative size = 22, normalized size of antiderivative = 0.52

$$\int \frac{x}{\sqrt{6x-9x^2}}\,dx = -\frac{1}{9}\sqrt{-9x^2+6x} - \frac{1}{9}\arcsin(-3x+1)$$

```
integrate(x/(-9*x^2+6*x)^(1/2),x, algorithm="maxima")
```

```
-1/9*sqrt(-9*x^2 + 6*x) - 1/9*arcsin(-3*x + 1)
```

Giac [A] (verification not implemented)

Time = 0.39 (sec) , antiderivative size = 25, normalized size of antiderivative = 0.60

$$\int \frac{x}{\sqrt{6x-9x^2}}\,dx = -\frac{1}{9}\sqrt{3}\sqrt{-3x^2+2x} + \frac{1}{9}\arcsin(3x-1)$$

```
integrate(x/(-9*x^2+6*x)^(1/2),x, algorithm="giac")
```

```
-1/9*sqrt(3)*sqrt(-3*x^2 + 2*x) + 1/9*arcsin(3*x - 1)
```

Mupad [B] (verification not implemented)

Time = 9.32 (sec) , antiderivative size = 37, normalized size of antiderivative = 0.88

$$\int \frac{x}{\sqrt{6x-9x^2}}\,dx = -\frac{\sqrt{6\,x-9\,x^2}}{9}-\frac{\ln\left(x-\frac{1}{3}-\frac{\sqrt{3}\,\sqrt{-x\,(3\,x-2)}\,1\mathrm{i}}{3}\right)\,1\mathrm{i}}{9}$$

```
int(x/(6*x - 9*x^2)^(1/2),x)
```

```
- (log(x - (3^(1/2)*(-x*(3*x - 2))^(1/2)*1i)/3 - 1/3)*1i)/9 - (6*x - 9*x^2
)^(1/2)/9
```

Reduce [B] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 35, normalized size of antiderivative = 0.83

$$\int \frac{x}{\sqrt{6x-9x^2}}\,dx = -\frac{\sqrt{x}\,\sqrt{-3x+2}\,\sqrt{3}}{9}-\frac{2\log\left(\frac{\sqrt{-3x+2}+\sqrt{x}\,\sqrt{3}\,i}{\sqrt{2}}\right)i}{9}$$

```
int(x/(-9*x^2+6*x)^(1/2),x)
```

```
( - sqrt(x)*sqrt( - 3*x + 2)*sqrt(3) - 2*log((sqrt( - 3*x + 2) + sqrt(x)*s
qrt(3)*i)/sqrt(2))*i)/9
```

3.71 $\int \frac{1}{\sqrt{6x-9x^2}} \, dx$

Optimal result

Integrand size = 13, antiderivative size = 18

$$\int \frac{1}{\sqrt{6x-9x^2}} \, dx = \frac{2}{3} \arcsin \left(\sqrt{\frac{3}{2}}\sqrt{x}\right)$$

```
2/3*arcsin(1/2*6^(1/2)*x^(1/2))
```

Mathematica [B] (verified)

Leaf count is larger than twice the leaf count of optimal. 53 vs. 2(18) = 36.

Time = 0.01 (sec) , antiderivative size = 53, normalized size of antiderivative = 2.94

$$\int \frac{1}{\sqrt{6x-9x^2}} \, dx = \frac{2x^{3/2}(-2+3x)^{3/2} \log \left(-\sqrt{3}\sqrt{x}+\sqrt{-2+3x}\right)}{3(-x(-2+3x))^{3/2}}$$

```
Integrate[1/Sqrt[6*x - 9*x^2],x]
```

```
(2*x^(3/2)*(-2 + 3*x)^(3/2)*Log[-(Sqrt[3]*Sqrt[x]) + Sqrt[-2 + 3*x]])/(3*(
-(x*(-2 + 3*x)))^(3/2))
```

Rubi [A] (verified)

Time = 0.25 (sec) , antiderivative size = 14, normalized size of antiderivative = 0.78, number of steps used = 3, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.154, Rules used = {1090, 223}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{\sqrt{6x-9x^2}}dx$$
$$\Big\downarrow \text{1090}$$
$$-\frac{1}{18}\int \frac{1}{\sqrt{1-\frac{1}{36}(6-18x)^2}}d(6-18x)$$
$$\Big\downarrow \text{223}$$
$$-\frac{1}{3}\arcsin\left(\frac{1}{6}(6-18x)\right)$$

```
Int[1/Sqrt[6*x - 9*x^2],x]
```

```
-1/3*ArcSin[(6 - 18*x)/6]
```

Defintions of rubi rules used

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSin[Rt[-b, 2]*(x/Sqrt
[a])]/Rt[-b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && NegQ[b]
```

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[1/(2*c*(-4*
(c/(b^2 - 4*a*c)))^p)   Subst[Int[Simp[1 - x^2/(b^2 - 4*a*c), x]^p, x], x,
b + 2*c*x], x] /; FreeQ[{a, b, c, p}, x] && GtQ[4*a - b^2/c, 0]
```

Maple [A] (verified)

Time = 0.34 (sec) , antiderivative size = 9, normalized size of antiderivative = 0.50

method	result	size
default	$\frac{\arcsin(3x-1)}{3}$	9
meijerg	$\frac{2\arcsin\left(\frac{\sqrt{x}\,\sqrt{2}\,\sqrt{3}}{2}\right)}{3}$	15
pseudoelliptic	$-\frac{2\arctan\left(\frac{\sqrt{-3x^2+2x}\,\sqrt{3}}{3x}\right)}{3}$	23
trager	$\frac{\mathrm{RootOf}\left(\_Z^2+1\right)\ln\left(-3\,\mathrm{RootOf}\left(\_Z^2+1\right)x+\sqrt{-9x^2+6x}+\mathrm{RootOf}\left(\_Z^2+1\right)\right)}{3}$	37

```
int(1/(-9*x^2+6*x)^(1/2),x,method=_RETURNVERBOSE)
```

```
1/3*arcsin(3*x-1)
```

Fricas [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.22

$$\int \frac{1}{\sqrt{6x-9x^2}}\,dx = -\frac{2}{3}\,\arctan\left(\frac{\sqrt{-9\,x^2+6\,x}}{3\,x-2}\right)$$

```
integrate(1/(-9*x^2+6*x)^(1/2),x, algorithm="fricas")
```

```
-2/3*arctan(sqrt(-9*x^2 + 6*x)/(3*x - 2))
```

Sympy [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 7, normalized size of antiderivative = 0.39

$$\int \frac{1}{\sqrt{6x-9x^2}}\,dx = \frac{\operatorname{asin}(3x-1)}{3}$$

```
integrate(1/(-9*x**2+6*x)**(1/2),x)
```

```
asin(3*x - 1)/3
```

Maxima [A] (verification not implemented)

Time = 0.10 (sec) , antiderivative size = 8, normalized size of antiderivative = 0.44

$$\int \frac{1}{\sqrt{6x-9x^2}}\,dx = -\frac{1}{3}\arcsin(-3x+1)$$

```
integrate(1/(-9*x^2+6*x)^(1/2),x, algorithm="maxima")
```

```
-1/3*arcsin(-3*x + 1)
```

Giac [A] (verification not implemented)

Time = 0.20 (sec) , antiderivative size = 8, normalized size of antiderivative = 0.44

$$\int \frac{1}{\sqrt{6x-9x^2}}\,dx = \frac{1}{3}\arcsin(3x-1)$$

```
integrate(1/(-9*x^2+6*x)^(1/2),x, algorithm="giac")
```

```
1/3*arcsin(3*x - 1)
```

Mupad [B] (verification not implemented)

Time = 9.73 (sec) , antiderivative size = 8, normalized size of antiderivative = 0.44

$$\int \frac{1}{\sqrt{6x-9x^2}}\,dx = \frac{\operatorname{asin}(3\,x-1)}{3}$$

```
int(1/(6*x - 9*x^2)^(1/2),x)
```

```
asin(3*x - 1)/3
```

Reduce [B] (verification not implemented)

Time = 0.33 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.22

$$\int \frac{1}{\sqrt{6x-9x^2}}\,dx = -\frac{2\log\left(\frac{\sqrt{-3x+2}+\sqrt{x}\,\sqrt{3}\,i}{\sqrt{2}}\right)i}{3}$$

```
int(1/(-9*x^2+6*x)^(1/2),x)
```

```
( - 2*log((sqrt( - 3*x + 2) + sqrt(x)*sqrt(3)*i)/sqrt(2))*i)/3
```

3.72 $\int \frac{1}{x\sqrt{6x-9x^2}}\,dx$

Optimal result

Integrand size = 17, antiderivative size = 21

$$\int \frac{1}{x\sqrt{6x-9x^2}}\,dx = -\frac{\sqrt{2-3x}}{\sqrt{3}\sqrt{x}}$$

```
-1/3*(2-3*x)^(1/2)*3^(1/2)/x^(1/2)
```

Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 19, normalized size of antiderivative = 0.90

$$\int \frac{1}{x\sqrt{6x-9x^2}}\,dx = \frac{-2+3x}{\sqrt{6x-9x^2}}$$

```
Integrate[1/(x*Sqrt[6*x - 9*x^2]),x]
```

```
(-2 + 3*x)/Sqrt[6*x - 9*x^2]
```

Rubi [A] (verified)

Time = 0.24 (sec) , antiderivative size = 23, normalized size of antiderivative = 1.10, number of steps used = 1, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.059, Rules used = {1123}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{x\sqrt{6x-9x^2}}dx$$
$$\Big\downarrow \text{1123}$$
$$-\frac{\sqrt{2x-3x^2}}{\sqrt{3}x}$$

```
Int[1/(x*Sqrt[6*x - 9*x^2]),x]
```

```
-(Sqrt[2*x - 3*x^2]/(Sqrt[3]*x))
```

Defintions of rubi rules used

```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] :> Simp[e*(d + e*x)^m*((a + b*x + c*x^2)^(p + 1)/((p + 1)*(2*c*d - b
*e))), x] /; FreeQ[{a, b, c, d, e, m, p}, x] && EqQ[c*d^2 - b*d*e + a*e^2,
0] && EqQ[m + 2*p + 2, 0]
```

Maple [A] (verified)

Time = 0.31 (sec) , antiderivative size = 17, normalized size of antiderivative = 0.81

method	result	size
default	$-\frac{\sqrt{-9x^2+6x}}{3x}$	17
trager	$-\frac{\sqrt{-9x^2+6x}}{3x}$	17
gosper	$\frac{-2+3x}{\sqrt{-9x^2+6x}}$	18
orering	$\frac{-2+3x}{\sqrt{-9x^2+6x}}$	18
meijerg	$-\frac{\sqrt{3}\,\sqrt{2}\,\sqrt{1-\frac{3x}{2}}}{3\sqrt{x}}$	19
pseudoelliptic	$-\frac{\sqrt{-3x^2+2x}\,\sqrt{3}}{3x}$	20
risch	$\frac{\sqrt{3}\,(-2+3x)}{3\sqrt{-x(-2+3x)}}$	21

```
int(1/x/(-9*x^2+6*x)^(1/2),x,method=_RETURNVERBOSE)
```

```
-1/3/x*(-9*x^2+6*x)^(1/2)
```

Fricas [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 16, normalized size of antiderivative = 0.76

$$\int \frac{1}{x\sqrt{6x-9x^2}}\,dx = -\frac{\sqrt{-9\,x^2+6\,x}}{3\,x}$$

```
integrate(1/x/(-9*x^2+6*x)^(1/2),x, algorithm="fricas")
```

```
-1/3*sqrt(-9*x^2 + 6*x)/x
```

Sympy [F]

$$\int \frac{1}{x\sqrt{6x-9x^2}}\,dx = \frac{\sqrt{3}\int \frac{1}{x\sqrt{-3x^2+2x}}\,dx}{3}$$

```
integrate(1/x/(-9*x**2+6*x)**(1/2),x)
```

```
sqrt(3)*Integral(1/(x*sqrt(-3*x**2 + 2*x)), x)/3
```

Maxima [A] (verification not implemented)

Time = 0.10 (sec) , antiderivative size = 16, normalized size of antiderivative = 0.76

$$\int \frac{1}{x\sqrt{6x-9x^2}}\,dx = -\frac{\sqrt{-9\,x^2+6\,x}}{3\,x}$$

```
integrate(1/x/(-9*x^2+6*x)^(1/2),x, algorithm="maxima")
```

```
-1/3*sqrt(-9*x^2 + 6*x)/x
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 31 vs. 2(15) = 30.

Time = 0.15 (sec) , antiderivative size = 31, normalized size of antiderivative = 1.48

$$\int \frac{1}{x\sqrt{6x-9x^2}}\,dx = \frac{2}{\frac{\sqrt{3}\sqrt{-3\,x^2+2\,x}-1}{3\,x-1}-1}$$

```
integrate(1/x/(-9*x^2+6*x)^(1/2),x, algorithm="giac")
```

```
2/((sqrt(3)*sqrt(-3*x^2 + 2*x) - 1)/(3*x - 1) - 1)
```

Mupad [B] (verification not implemented)

Time = 9.50 (sec) , antiderivative size = 16, normalized size of antiderivative = 0.76

$$\int \frac{1}{x\sqrt{6x-9x^2}}\,dx = -\frac{\sqrt{6\,x-9\,x^2}}{3\,x}$$

```
int(1/(x*(6*x - 9*x^2)^(1/2)),x)
```

```
-(6*x - 9*x^2)^(1/2)/(3*x)
```

Reduce [B] (verification not implemented)

Time = 0.50 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.05

$$\int \frac{1}{x\sqrt{6x-9x^2}}\,dx = \frac{-\sqrt{x}\,\sqrt{-3x+2}\,\sqrt{3}-3ix}{3x}$$

```
int(1/x/(-9*x^2+6*x)^(1/2),x)
```

```
( - sqrt(x)*sqrt( - 3*x + 2)*sqrt(3) - 3*i*x)/(3*x)
```

3.73 $\int \frac{1}{x^2\sqrt{6x-9x^2}}\,dx$

Optimal result

Integrand size = 17, antiderivative size = 45

$$\int \frac{1}{x^2\sqrt{6x-9x^2}}\,dx = -\frac{\sqrt{2-3x}}{3\sqrt{3}x^{3/2}} - \frac{\sqrt{2-3x}}{\sqrt{3}\sqrt{x}}$$

```
-1/9*(2-3*x)^(1/2)*3^(1/2)/x^(3/2)-1/3*(2-3*x)^(1/2)*3^(1/2)/x^(1/2)
```

Mathematica [A] (verified)

Time = 0.05 (sec) , antiderivative size = 30, normalized size of antiderivative = 0.67

$$\int \frac{1}{x^2\sqrt{6x-9x^2}}\,dx = \frac{(-2+3x)(1+3x)}{3x\sqrt{6x-9x^2}}$$

```
Integrate[1/(x^2*Sqrt[6*x - 9*x^2]),x]
```

```
((-2 + 3*x)*(1 + 3*x))/(3*x*Sqrt[6*x - 9*x^2])
```

Rubi [A] (verified)

Time = 0.29 (sec) , antiderivative size = 49, normalized size of antiderivative = 1.09, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.176$, Rules used = {1129, 27, 1123}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{x^2\sqrt{6x-9x^2}}dx$$

$$\Big\downarrow \textbf{1129}$$

$$\int \frac{1}{\sqrt{3}x\sqrt{2x-3x^2}}dx-\frac{\sqrt{2x-3x^2}}{3\sqrt{3}x^2}$$

$$\Big\downarrow \textbf{27}$$

$$\frac{\int \frac{1}{x\sqrt{2x-3x^2}}dx}{\sqrt{3}}-\frac{\sqrt{2x-3x^2}}{3\sqrt{3}x^2}$$

$$\Big\downarrow \textbf{1123}$$

$$-\frac{\sqrt{2x-3x^2}}{\sqrt{3}x}-\frac{\sqrt{2x-3x^2}}{3\sqrt{3}x^2}$$

```
Int[1/(x^2*Sqrt[6*x - 9*x^2]),x]
```

```
-1/3*Sqrt[2*x - 3*x^2]/(Sqrt[3]*x^2) - Sqrt[2*x - 3*x^2]/(Sqrt[3]*x)
```

Defintions of rubi rules used

```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] :> Simp[e*(d + e*x)^m*((a + b*x + c*x^2)^(p + 1)/((p + 1)*(2*c*d - b
*e))), x] /; FreeQ[{a, b, c, d, e, m, p}, x] && EqQ[c*d^2 - b*d*e + a*e^2,
0] && EqQ[m + 2*p + 2, 0]
```

```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] :> Simp[(-e)*(d + e*x)^m*((a + b*x + c*x^2)^(p + 1)/((m + p + 1)*(2*
c*d - b*e))), x] + Simp[c*(Simplify[m + 2*p + 2]/((m + p + 1)*(2*c*d - b*e)
))   Int[(d + e*x)^(m + 1)*(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d
, e, m, p}, x] && EqQ[c*d^2 - b*d*e + a*e^2, 0] && ILtQ[Simplify[m + 2*p +
2], 0]
```

Maple [A] (verified)

Time = 0.33 (sec) , antiderivative size = 22, normalized size of antiderivative = 0.49

method	result	size
trager	$-\frac{(3x+1)\sqrt{-9x^2+6x}}{9x^2}$	22
meijerg	$-\frac{\sqrt{3}\,\sqrt{2}\,(3x+1)\sqrt{1-\frac{3x}{2}}}{9x^{\frac{3}{2}}}$	24
pseudoelliptic	$-\frac{(3x+1)\sqrt{-3x^2+2x}\,\sqrt{3}}{9x^2}$	25
gosper	$\frac{(-2+3x)(3x+1)}{3x\sqrt{-9x^2+6x}}$	27
orering	$\frac{(-2+3x)(3x+1)}{3x\sqrt{-9x^2+6x}}$	27
risch	$\frac{\sqrt{3}\,(9x^2-3x-2)}{9x\sqrt{-x(-2+3x)}}$	29
default	$-\frac{\sqrt{-9x^2+6x}}{9x^2}-\frac{\sqrt{-9x^2+6x}}{3x}$	34

```
int(1/x^2/(-9*x^2+6*x)^(1/2),x,method=_RETURNVERBOSE)
```

```
-1/9*(3*x+1)/x^2*(-9*x^2+6*x)^(1/2)
```

Fricas [A] (verification not implemented)

Time = 0.10 (sec) , antiderivative size = 21, normalized size of antiderivative = 0.47

$$\int \frac{1}{x^2\sqrt{6x-9x^2}}\,dx = -\frac{\sqrt{-9\,x^2+6\,x}(3\,x+1)}{9\,x^2}$$

```
integrate(1/x^2/(-9*x^2+6*x)^(1/2),x, algorithm="fricas")
```

```
-1/9*sqrt(-9*x^2 + 6*x)*(3*x + 1)/x^2
```

Sympy [F]

$$\int \frac{1}{x^2\sqrt{6x-9x^2}}\,dx = \frac{\sqrt{3}\int \frac{1}{x^2\sqrt{-3x^2+2x}}\,dx}{3}$$

```
integrate(1/x**2/(-9*x**2+6*x)**(1/2),x)
```

```
sqrt(3)*Integral(1/(x**2*sqrt(-3*x**2 + 2*x)), x)/3
```

Maxima [A] (verification not implemented)

Time = 0.10 (sec) , antiderivative size = 33, normalized size of antiderivative = 0.73

$$\int \frac{1}{x^2\sqrt{6x-9x^2}}\,dx = -\frac{\sqrt{-9\,x^2+6\,x}}{3\,x} - \frac{\sqrt{-9\,x^2+6\,x}}{9\,x^2}$$

```
integrate(1/x^2/(-9*x^2+6*x)^(1/2),x, algorithm="maxima")
```

```
-1/3*sqrt(-9*x^2 + 6*x)/x - 1/9*sqrt(-9*x^2 + 6*x)/x^2
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 87 vs. $2(31) = 62$.

Time = 0.14 (sec) , antiderivative size = 87, normalized size of antiderivative = 1.93

$$\int \frac{1}{x^2\sqrt{6x-9x^2}}\,dx = \frac{2\left(\frac{3\left(\sqrt{3}\sqrt{-3\,x^2+2\,x}-1\right)^2}{(3\,x-1)^2} - \frac{3\left(\sqrt{3}\sqrt{-3\,x^2+2\,x}-1\right)}{3\,x-1} + 2\right)}{\left(\frac{\sqrt{3}\sqrt{-3\,x^2+2\,x}-1}{3\,x-1} - 1\right)^3}$$

```
integrate(1/x^2/(-9*x^2+6*x)^(1/2),x, algorithm="giac")
```

```
2*(3*(sqrt(3)*sqrt(-3*x^2 + 2*x) - 1)^2/(3*x - 1)^2 - 3*(sqrt(3)*sqrt(-3*x
^2 + 2*x) - 1)/(3*x - 1) + 2)/((sqrt(3)*sqrt(-3*x^2 + 2*x) - 1)/(3*x - 1)
- 1)^3
```

Mupad [B] (verification not implemented)

Time = 9.27 (sec) , antiderivative size = 21, normalized size of antiderivative = 0.47

$$\int \frac{1}{x^2\sqrt{6x-9x^2}}\,dx = -\frac{(3\,x+1)\,\sqrt{6\,x-9\,x^2}}{9\,x^2}$$

```
int(1/(x^2*(6*x - 9*x^2)^(1/2)),x)
```

```
-((3*x + 1)*(6*x - 9*x^2)^(1/2))/(9*x^2)
```

Reduce [B] (verification not implemented)

Time = 0.36 (sec) , antiderivative size = 37, normalized size of antiderivative = 0.82

$$\int \frac{1}{x^2\sqrt{6x-9x^2}}\,dx = \frac{-3\sqrt{x}\,\sqrt{-3x+2}\,\sqrt{3}\,x-\sqrt{x}\,\sqrt{-3x+2}\,\sqrt{3}+9i\,x^2}{9x^2}$$

```
int(1/x^2/(-9*x^2+6*x)^(1/2),x)
```

```
( - 3*sqrt(x)*sqrt( - 3*x + 2)*sqrt(3)*x - sqrt(x)*sqrt( - 3*x + 2)*sqrt(3
) + 9*i*x**2)/(9*x**2)
```

3.74 $\int \frac{1}{x^3\sqrt{6x-9x^2}}\,dx$

Optimal result

Integrand size = 17, antiderivative size = 70

$$\int \frac{1}{x^3\sqrt{6x-9x^2}}\,dx = -\frac{\sqrt{2-3x}}{5\sqrt{3}x^{5/2}} - \frac{2\sqrt{2-3x}}{5\sqrt{3}x^{3/2}} - \frac{2\sqrt{3}\sqrt{2-3x}}{5\sqrt{x}}$$

```
-1/15*(2-3*x)^(1/2)*3^(1/2)/x^(5/2)-2/15*(2-3*x)^(1/2)*3^(1/2)/x^(3/2)-2/5
*(2-3*x)^(1/2)*3^(1/2)/x^(1/2)
```

Mathematica [A] (verified)

Time = 0.06 (sec) , antiderivative size = 35, normalized size of antiderivative = 0.50

$$\int \frac{1}{x^3\sqrt{6x-9x^2}}\,dx = \frac{(-2+3x)\,(1+2x+6x^2)}{5x^2\sqrt{6x-9x^2}}$$

```
Integrate[1/(x^3*Sqrt[6*x - 9*x^2]),x]
```

```
((-2 + 3*x)*(1 + 2*x + 6*x^2))/(5*x^2*Sqrt[6*x - 9*x^2])
```

Rubi [A] (verified)

Time = 0.33 (sec) , antiderivative size = 74, normalized size of antiderivative = 1.06, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.235, Rules used = {1129, 27, 1129, 1123}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{x^3\sqrt{6x-9x^2}}dx$$

$$\Big\downarrow \text{1129}$$

$$\frac{6}{5}\int \frac{1}{\sqrt{3}x^2\sqrt{2x-3x^2}}dx-\frac{\sqrt{2x-3x^2}}{5\sqrt{3}x^3}$$

$$\Big\downarrow \text{27}$$

$$\frac{2}{5}\sqrt{3}\int \frac{1}{x^2\sqrt{2x-3x^2}}dx-\frac{\sqrt{2x-3x^2}}{5\sqrt{3}x^3}$$

$$\Big\downarrow \text{1129}$$

$$\frac{2}{5}\sqrt{3}\left(\int \frac{1}{x\sqrt{2x-3x^2}}dx-\frac{\sqrt{2x-3x^2}}{3x^2}\right)-\frac{\sqrt{2x-3x^2}}{5\sqrt{3}x^3}$$

$$\Big\downarrow \text{1123}$$

$$\frac{2}{5}\sqrt{3}\left(-\frac{\sqrt{2x-3x^2}}{x}-\frac{\sqrt{2x-3x^2}}{3x^2}\right)-\frac{\sqrt{2x-3x^2}}{5\sqrt{3}x^3}$$

```
Int[1/(x^3*Sqrt[6*x - 9*x^2]),x]
```

```
-1/5*Sqrt[2*x - 3*x^2]/(Sqrt[3]*x^3) + (2*Sqrt[3]*(-1/3*Sqrt[2*x - 3*x^2]/
x^2 - Sqrt[2*x - 3*x^2]/x))/5
```

Defintions of rubi rules used

```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] :> Simp[e*(d + e*x)^m*((a + b*x + c*x^2)^(p + 1)/((p + 1)*(2*c*d - b
*e))), x] /; FreeQ[{a, b, c, d, e, m, p}, x] && EqQ[c*d^2 - b*d*e + a*e^2,
0] && EqQ[m + 2*p + 2, 0]
```

```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] :> Simp[(-e)*(d + e*x)^m*((a + b*x + c*x^2)^(p + 1)/((m + p + 1)*(2*
c*d - b*e))), x] + Simp[c*(Simplify[m + 2*p + 2]/((m + p + 1)*(2*c*d - b*e)
))   Int[(d + e*x)^(m + 1)*(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d
, e, m, p}, x] && EqQ[c*d^2 - b*d*e + a*e^2, 0] && ILtQ[Simplify[m + 2*p +
2], 0]
```

Maple [A] (verified)

Time = 0.38 (sec) , antiderivative size = 27, normalized size of antiderivative = 0.39

method	result	size
trager	$-\frac{\left(6x^{2}+2x+1\right)\sqrt{-9x^{2}+6x}}{15x^{3}}$	27
meijerg	$-\frac{\sqrt{3}\,\sqrt{2}\,\left(6x^{2}+2x+1\right)\sqrt{1-\frac{3x}{2}}}{15x^{\frac{5}{2}}}$	29
pseudoelliptic	$-\frac{\left(6x^{2}+2x+1\right)\sqrt{-3x^{2}+2x}\,\sqrt{3}}{15x^{3}}$	30
gosper	$\frac{\left(-2+3x\right)\left(6x^{2}+2x+1\right)}{5x^{2}\sqrt{-9x^{2}+6x}}$	32
orering	$\frac{\left(-2+3x\right)\left(6x^{2}+2x+1\right)}{5x^{2}\sqrt{-9x^{2}+6x}}$	32
risch	$\frac{\sqrt{3}\,\left(18x^{3}-6x^{2}-x-2\right)}{15x^{2}\sqrt{-x\left(-2+3x\right)}}$	34
default	$-\frac{\sqrt{-9x^{2}+6x}}{15x^{3}}-\frac{2\sqrt{-9x^{2}+6x}}{15x^{2}}-\frac{2\sqrt{-9x^{2}+6x}}{5x}$	50

```
int(1/x^3/(-9*x^2+6*x)^(1/2),x,method=_RETURNVERBOSE)
```

```
-1/15*(6*x^2+2*x+1)/x^3*(-9*x^2+6*x)^(1/2)
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 26, normalized size of antiderivative = 0.37

$$\int \frac{1}{x^3\sqrt{6x-9x^2}}\,dx = -\frac{(6\,x^2+2\,x+1)\sqrt{-9\,x^2+6\,x}}{15\,x^3}$$

```
integrate(1/x^3/(-9*x^2+6*x)^(1/2),x, algorithm="fricas")
```

```
-1/15*(6*x^2 + 2*x + 1)*sqrt(-9*x^2 + 6*x)/x^3
```

Sympy [F]

$$\int \frac{1}{x^3\sqrt{6x-9x^2}}\,dx = \frac{\sqrt{3}\int \frac{1}{x^3\sqrt{-3x^2+2x}}\,dx}{3}$$

```
integrate(1/x**3/(-9*x**2+6*x)**(1/2),x)
```

```
sqrt(3)*Integral(1/(x**3*sqrt(-3*x**2 + 2*x)), x)/3
```

Maxima [A] (verification not implemented)

Time = 0.10 (sec) , antiderivative size = 49, normalized size of antiderivative = 0.70

$$\int \frac{1}{x^3\sqrt{6x-9x^2}}\,dx = -\frac{2\,\sqrt{-9\,x^2+6\,x}}{5\,x} - \frac{2\,\sqrt{-9\,x^2+6\,x}}{15\,x^2} - \frac{\sqrt{-9\,x^2+6\,x}}{15\,x^3}$$

```
integrate(1/x^3/(-9*x^2+6*x)^(1/2),x, algorithm="maxima")
```

```
-2/5*sqrt(-9*x^2 + 6*x)/x - 2/15*sqrt(-9*x^2 + 6*x)/x^2 - 1/15*sqrt(-9*x^2
 + 6*x)/x^3
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 143 vs. $2(46) = 92$.

Time = 0.14 (sec) , antiderivative size = 143, normalized size of antiderivative = 2.04

$$\int \frac{1}{x^3\sqrt{6x-9x^2}}\,dx = \frac{6\left(\frac{15\left(\sqrt{3}\sqrt{-3\,x^2+2\,x}-1\right)^4}{(3\,x-1)^4} - \frac{30\left(\sqrt{3}\sqrt{-3\,x^2+2\,x}-1\right)^3}{(3\,x-1)^3} + \frac{40\left(\sqrt{3}\sqrt{-3\,x^2+2\,x}-1\right)^2}{(3\,x-1)^2} - \frac{20\left(\sqrt{3}\sqrt{-3\,x^2+2\,x}-1\right)}{3\,x-1} + 7\right)}{5\left(\frac{\sqrt{3}\sqrt{-3\,x^2+2\,x}-1}{3\,x-1} - 1\right)^5}$$

```
integrate(1/x^3/(-9*x^2+6*x)^(1/2),x, algorithm="giac")
```

```
6/5*(15*(sqrt(3)*sqrt(-3*x^2 + 2*x) - 1)^4/(3*x - 1)^4 - 30*(sqrt(3)*sqrt(
-3*x^2 + 2*x) - 1)^3/(3*x - 1)^3 + 40*(sqrt(3)*sqrt(-3*x^2 + 2*x) - 1)^2/(
3*x - 1)^2 - 20*(sqrt(3)*sqrt(-3*x^2 + 2*x) - 1)/(3*x - 1) + 7)/((sqrt(3)*
sqrt(-3*x^2 + 2*x) - 1)/(3*x - 1) - 1)^5
```

Mupad [B] (verification not implemented)

Time = 8.85 (sec) , antiderivative size = 26, normalized size of antiderivative = 0.37

$$\int \frac{1}{x^3\sqrt{6x-9x^2}}\,dx = -\frac{\sqrt{6\,x-9\,x^2}\,\left(6\,x^2+2\,x+1\right)}{15\,x^3}$$

```
int(1/(x^3*(6*x - 9*x^2)^(1/2)),x)
```

```
-((6*x - 9*x^2)^(1/2)*(2*x + 6*x^2 + 1))/(15*x^3)
```

Reduce [B] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 52, normalized size of antiderivative = 0.74

$$\int \frac{1}{x^3\sqrt{6x-9x^2}}\,dx = \frac{-6\sqrt{x}\,\sqrt{-3x+2}\,\sqrt{3}\,x^2-2\sqrt{x}\,\sqrt{-3x+2}\,\sqrt{3}\,x-\sqrt{x}\,\sqrt{-3x+2}\,\sqrt{3}+18i\,x^3}{15x^3}$$

```
int(1/x^3/(-9*x^2+6*x)^(1/2),x)
```

```
( - 6*sqrt(x)*sqrt( - 3*x + 2)*sqrt(3)*x**2 - 2*sqrt(x)*sqrt( - 3*x + 2)*s
qrt(3)*x - sqrt(x)*sqrt( - 3*x + 2)*sqrt(3) + 18*i*x**3)/(15*x**3)
```

3.75 $\int \frac{1}{x^4\sqrt{6x-9x^2}}\,dx$

Optimal result

Integrand size = 17, antiderivative size = 93

$$\int \frac{1}{x^4\sqrt{6x-9x^2}}\,dx = -\frac{\sqrt{2-3x}}{7\sqrt{3}x^{7/2}} - \frac{3\sqrt{3}\sqrt{2-3x}}{35x^{5/2}} - \frac{6\sqrt{3}\sqrt{2-3x}}{35x^{3/2}} - \frac{18\sqrt{3}\sqrt{2-3x}}{35\sqrt{x}}$$

```
-1/21*(2-3*x)^(1/2)*3^(1/2)/x^(7/2)-3/35*(2-3*x)^(1/2)*3^(1/2)/x^(5/2)-6/3
5*(2-3*x)^(1/2)*3^(1/2)/x^(3/2)-18/35*(2-3*x)^(1/2)*3^(1/2)/x^(1/2)
```

Mathematica [A] (verified)

Time = 0.07 (sec) , antiderivative size = 40, normalized size of antiderivative = 0.43

$$\int \frac{1}{x^4\sqrt{6x-9x^2}}\,dx = \frac{(-2+3x)\left(5+9x+18x^2+54x^3\right)}{35x^3\sqrt{6x-9x^2}}$$

```
Integrate[1/(x^4*Sqrt[6*x - 9*x^2]),x]
```

```
((-2 + 3*x)*(5 + 9*x + 18*x^2 + 54*x^3))/(35*x^3*Sqrt[6*x - 9*x^2])
```

Rubi [A] (verified)

Time = 0.36 (sec) , antiderivative size = 99, normalized size of antiderivative = 1.06, number of steps used = 5, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.294$, Rules used = {1129, 27, 1129, 1129, 1123}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{x^4\sqrt{6x-9x^2}}dx$$

$$\Big\downarrow \text{1129}$$

$$\frac{9}{7}\int \frac{1}{\sqrt{3}x^3\sqrt{2x-3x^2}}dx-\frac{\sqrt{2x-3x^2}}{7\sqrt{3}x^4}$$

$$\Big\downarrow \text{27}$$

$$\frac{3}{7}\sqrt{3}\int \frac{1}{x^3\sqrt{2x-3x^2}}dx-\frac{\sqrt{2x-3x^2}}{7\sqrt{3}x^4}$$

$$\Big\downarrow \text{1129}$$

$$\frac{3}{7}\sqrt{3}\left(\frac{6}{5}\int \frac{1}{x^2\sqrt{2x-3x^2}}dx-\frac{\sqrt{2x-3x^2}}{5x^3}\right)-\frac{\sqrt{2x-3x^2}}{7\sqrt{3}x^4}$$

$$\Big\downarrow \text{1129}$$

$$\frac{3}{7}\sqrt{3}\left(\frac{6}{5}\left(\int \frac{1}{x\sqrt{2x-3x^2}}dx-\frac{\sqrt{2x-3x^2}}{3x^2}\right)-\frac{\sqrt{2x-3x^2}}{5x^3}\right)-\frac{\sqrt{2x-3x^2}}{7\sqrt{3}x^4}$$

$$\Big\downarrow \text{1123}$$

$$\frac{3}{7}\sqrt{3}\left(\frac{6}{5}\left(-\frac{\sqrt{2x-3x^2}}{x}-\frac{\sqrt{2x-3x^2}}{3x^2}\right)-\frac{\sqrt{2x-3x^2}}{5x^3}\right)-\frac{\sqrt{2x-3x^2}}{7\sqrt{3}x^4}$$

```
Int[1/(x^4*Sqrt[6*x - 9*x^2]),x]
```

```
-1/7*Sqrt[2*x - 3*x^2]/(Sqrt[3]*x^4) + (3*Sqrt[3]*(-1/5*Sqrt[2*x - 3*x^2]/
x^3 + (6*(-1/3*Sqrt[2*x - 3*x^2]/x^2 - Sqrt[2*x - 3*x^2]/x))/5))/7
```

Defintions of rubi rules used

```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] :> Simp[e*(d + e*x)^m*((a + b*x + c*x^2)^(p + 1)/((p + 1)*(2*c*d - b
*e))), x] /; FreeQ[{a, b, c, d, e, m, p}, x] && EqQ[c*d^2 - b*d*e + a*e^2,
0] && EqQ[m + 2*p + 2, 0]
```

```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] :> Simp[(-e)*(d + e*x)^m*((a + b*x + c*x^2)^(p + 1)/((m + p + 1)*(2*
c*d - b*e))), x] + Simp[c*(Simplify[m + 2*p + 2]/((m + p + 1)*(2*c*d - b*e)
))   Int[(d + e*x)^(m + 1)*(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d
, e, m, p}, x] && EqQ[c*d^2 - b*d*e + a*e^2, 0] && ILtQ[Simplify[m + 2*p +
2], 0]
```

Maple [A] (verified)

Time = 0.32 (sec) , antiderivative size = 32, normalized size of antiderivative = 0.34

method	result	size
trager	$-\frac{\left(54 x^{3}+18 x^{2}+9 x +5\right) \sqrt{-9 x^{2}+6 x}}{105 x^{4}}$	32
meijerg	$-\frac{\sqrt{3}\, \sqrt{2}\, \left(\frac{54}{5} x^{3}+\frac{18}{5} x^{2}+\frac{9}{5} x +1\right) \sqrt{1-\frac{3 x}{2}}}{21 x^{\frac{7}{2}}}$	34
pseudoelliptic	$-\frac{\left(54 x^{3}+18 x^{2}+9 x +5\right) \sqrt{-3 x^{2}+2 x}\, \sqrt{3}}{105 x^{4}}$	35
gosper	$\frac{\left(-2+3 x \right) \left(54 x^{3}+18 x^{2}+9 x +5\right)}{35 x^{3} \sqrt{-9 x^{2}+6 x}}$	37
orering	$\frac{\left(-2+3 x \right) \left(54 x^{3}+18 x^{2}+9 x +5\right)}{35 x^{3} \sqrt{-9 x^{2}+6 x}}$	37
risch	$\frac{\sqrt{3}\, \left(162 x^{4}-54 x^{3}-9 x^{2}-3 x -10\right)}{105 x^{3} \sqrt{-x \left(-2+3 x \right)}}$	39
default	$-\frac{\sqrt{-9 x^{2}+6 x}}{21 x^{4}}-\frac{3 \sqrt{-9 x^{2}+6 x}}{35 x^{3}}-\frac{6 \sqrt{-9 x^{2}+6 x}}{35 x^{2}}-\frac{18 \sqrt{-9 x^{2}+6 x}}{35 x}$	66

```
int(1/x^4/(-9*x^2+6*x)^(1/2),x,method=_RETURNVERBOSE)
```

```
-1/105*(54*x^3+18*x^2+9*x+5)/x^4*(-9*x^2+6*x)^(1/2)
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 31, normalized size of antiderivative = 0.33

$$\int \frac{1}{x^4\sqrt{6x-9x^2}}\,dx = -\frac{(54\,x^3+18\,x^2+9\,x+5)\sqrt{-9\,x^2+6\,x}}{105\,x^4}$$

```
integrate(1/x^4/(-9*x^2+6*x)^(1/2),x, algorithm="fricas")
```

```
-1/105*(54*x^3 + 18*x^2 + 9*x + 5)*sqrt(-9*x^2 + 6*x)/x^4
```

Sympy [F]

$$\int \frac{1}{x^4\sqrt{6x-9x^2}}\,dx = \frac{\sqrt{3}\int \frac{1}{x^4\sqrt{-3x^2+2x}}\,dx}{3}$$

```
integrate(1/x**4/(-9*x**2+6*x)**(1/2),x)
```

```
sqrt(3)*Integral(1/(x**4*sqrt(-3*x**2 + 2*x)), x)/3
```

Maxima [A] (verification not implemented)

Time = 0.10 (sec) , antiderivative size = 65, normalized size of antiderivative = 0.70

$$\int \frac{1}{x^4\sqrt{6x-9x^2}}\,dx = -\frac{18\sqrt{-9\,x^2+6\,x}}{35\,x} - \frac{6\sqrt{-9\,x^2+6\,x}}{35\,x^2} - \frac{3\sqrt{-9\,x^2+6\,x}}{35\,x^3} - \frac{\sqrt{-9\,x^2+6\,x}}{21\,x^4}$$

```
integrate(1/x^4/(-9*x^2+6*x)^(1/2),x, algorithm="maxima")
```

```
-18/35*sqrt(-9*x^2 + 6*x)/x - 6/35*sqrt(-9*x^2 + 6*x)/x^2 - 3/35*sqrt(-9*x
^2 + 6*x)/x^3 - 1/21*sqrt(-9*x^2 + 6*x)/x^4
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 199 vs. $2(61) = 122$.

Time = 0.13 (sec) , antiderivative size = 199, normalized size of antiderivative = 2.14

$$\int \frac{1}{x^4\sqrt{6x-9x^2}}\,dx = \frac{54\left(\frac{35\left(\sqrt{3}\sqrt{-3\,x^2+2\,x}-1\right)^6}{(3\,x-1)^6}-\frac{105\left(\sqrt{3}\sqrt{-3\,x^2+2\,x}-1\right)^5}{(3\,x-1)^5}+\frac{210\left(\sqrt{3}\sqrt{-3\,x^2+2\,x}-1\right)^4}{(3\,x-1)^4}-\frac{210\left(\sqrt{3}\sqrt{-3\,x^2+2\,x}-1\right)^3}{(3\,x-1)^3}+\frac{147\left(\sqrt{3}\sqrt{-\right.}{(3\,}\right.}{35\left(\frac{\sqrt{3}\sqrt{-3\,x^2+2\,x}-1}{3\,x-1}-1\right)^7}$$

```
integrate(1/x^4/(-9*x^2+6*x)^(1/2),x, algorithm="giac")
```

```
54/35*(35*(sqrt(3)*sqrt(-3*x^2 + 2*x) - 1)^6/(3*x - 1)^6 - 105*(sqrt(3)*sq
rt(-3*x^2 + 2*x) - 1)^5/(3*x - 1)^5 + 210*(sqrt(3)*sqrt(-3*x^2 + 2*x) - 1)
^4/(3*x - 1)^4 - 210*(sqrt(3)*sqrt(-3*x^2 + 2*x) - 1)^3/(3*x - 1)^3 + 147*
(sqrt(3)*sqrt(-3*x^2 + 2*x) - 1)^2/(3*x - 1)^2 - 49*(sqrt(3)*sqrt(-3*x^2 +
 2*x) - 1)/(3*x - 1) + 12)/((sqrt(3)*sqrt(-3*x^2 + 2*x) - 1)/(3*x - 1) - 1
)^7
```

Mupad [B] (verification not implemented)

Time = 8.93 (sec) , antiderivative size = 65, normalized size of antiderivative = 0.70

$$\int \frac{1}{x^4\sqrt{6x-9x^2}}\,dx = -\frac{18\,\sqrt{6\,x-9\,x^2}}{35\,x}-\frac{6\,\sqrt{6\,x-9\,x^2}}{35\,x^2}-\frac{3\,\sqrt{6\,x-9\,x^2}}{35\,x^3}-\frac{\sqrt{6\,x-9\,x^2}}{21\,x^4}$$

```
int(1/(x^4*(6*x - 9*x^2)^(1/2)),x)
```

```
- (18*(6*x - 9*x^2)^(1/2))/(35*x) - (6*(6*x - 9*x^2)^(1/2))/(35*x^2) - (3*
(6*x - 9*x^2)^(1/2))/(35*x^3) - (6*x - 9*x^2)^(1/2)/(21*x^4)
```

Reduce [B] (verification not implemented)

Time = 0.29 (sec) , antiderivative size = 67, normalized size of antiderivative = 0.72

$$\int \frac{1}{x^4\sqrt{6x-9x^2}}\,dx$$
$$= \frac{-54\sqrt{x}\,\sqrt{-3x+2}\,\sqrt{3}\,x^3-18\sqrt{x}\,\sqrt{-3x+2}\,\sqrt{3}\,x^2-9\sqrt{x}\,\sqrt{-3x+2}\,\sqrt{3}\,x-5\sqrt{x}\,\sqrt{-3x+2}\,\sqrt{3}+162i}{105x^4}$$

```
int(1/x^4/(-9*x^2+6*x)^(1/2),x)
```

```
( - 54*sqrt(x)*sqrt( - 3*x + 2)*sqrt(3)*x**3 - 18*sqrt(x)*sqrt( - 3*x + 2)
*sqrt(3)*x**2 - 9*sqrt(x)*sqrt( - 3*x + 2)*sqrt(3)*x - 5*sqrt(x)*sqrt( - 3
*x + 2)*sqrt(3) + 162*i*x**4)/(105*x**4)
```

3.76 $\int \frac{x}{\sqrt{4x-x^2}}\,dx$

Optimal result

Integrand size = 15, antiderivative size = 29

$$\int \frac{x}{\sqrt{4x-x^2}}\,dx = -\sqrt{4-x}\sqrt{x}+4\arcsin\left(\frac{\sqrt{x}}{2}\right)$$

```
-(4-x)^(1/2)*x^(1/2)+4*arcsin(1/2*x^(1/2))
```

Mathematica [A] (verified)

Time = 0.05 (sec) , antiderivative size = 47, normalized size of antiderivative = 1.62

$$\int \frac{x}{\sqrt{4x-x^2}}\,dx = \frac{(-4+x)x-4\sqrt{-4+x}\sqrt{x}\log\left(\sqrt{-4+x}-\sqrt{x}\right)}{\sqrt{-((-4+x)x)}}$$

```
Integrate[x/Sqrt[4*x - x^2],x]
```

```
((-4 + x)*x - 4*Sqrt[-4 + x]*Sqrt[x]*Log[Sqrt[-4 + x] - Sqrt[x]])/Sqrt[-((
-4 + x)*x)]
```

Rubi [A] (verified)

Time = 0.26 (sec) , antiderivative size = 28, normalized size of antiderivative = 0.97, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.200, Rules used = {1160, 1090, 223}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x}{\sqrt{4x-x^2}}dx$$

$$\Big\downarrow \text{ 1160}$$

$$2\int \frac{1}{\sqrt{4x-x^2}}dx-\sqrt{4x-x^2}$$

$$\Big\downarrow \text{ 1090}$$

$$-\frac{1}{2}\int \frac{1}{\sqrt{1-\frac{1}{16}(4-2x)^2}}d(4-2x)-\sqrt{4x-x^2}$$

$$\Big\downarrow \text{ 223}$$

$$-2\arcsin\left(\frac{1}{4}(4-2x)\right)-\sqrt{4x-x^2}$$

```
Int[x/Sqrt[4*x - x^2],x]
```

```
-Sqrt[4*x - x^2] - 2*ArcSin[(4 - 2*x)/4]
```

Defintions of rubi rules used

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSin[Rt[-b, 2]*(x/Sqrt
[a])]/Rt[-b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && NegQ[b]
```

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[1/(2*c*(-4*
(c/(b^2 - 4*a*c)))^p)   Subst[Int[Simp[1 - x^2/(b^2 - 4*a*c), x]^p, x], x,
b + 2*c*x], x] /; FreeQ[{a, b, c, p}, x] && GtQ[4*a - b^2/c, 0]
```

```
Int[((d_.) + (e_.)*(x_))*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol
] :> Simp[e*((a + b*x + c*x^2)^(p + 1)/(2*c*(p + 1))), x] + Simp[(2*c*d - b
*e)/(2*c)   Int[(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, p}, x]
 && NeQ[p, -1]
```

Maple [A] (verified)

Time = 0.65 (sec) , antiderivative size = 23, normalized size of antiderivative = 0.79

method	result
default	$-\sqrt{-x^{2}+4x}+2\arcsin\left(\frac{x}{2}-1\right)$
risch	$\frac{x(x-4)}{\sqrt{-x(x-4)}}+2\arcsin\left(\frac{x}{2}-1\right)$
pseudoelliptic	$-\sqrt{-x\left(x-4\right)}-4\arctan\left(\frac{\sqrt{-x(x-4)}}{x}\right)$
meijerg	$\frac{4i\left(\frac{i\sqrt{\pi}\,\sqrt{x}\,\sqrt{-\frac{x}{4}+1}}{2}-i\sqrt{\pi}\,\arcsin\left(\frac{\sqrt{x}}{2}\right)\right)}{\sqrt{\pi}}$
trager	$-\sqrt{-x^{2}+4x}+2\,\mathrm{RootOf}\left(\_Z^{2}+1\right)\ln\left(-\mathrm{RootOf}\left(\_Z^{2}+1\right)x+\sqrt{-x^{2}+4x}+2\,\mathrm{Root}\right.$

```
int(x/(-x^2+4*x)^(1/2),x,method=_RETURNVERBOSE)
```

```
-(-x^2+4*x)^(1/2)+2*arcsin(1/2*x-1)
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 34, normalized size of antiderivative = 1.17

$$\int \frac{x}{\sqrt{4x-x^{2}}}\,dx=-\sqrt{-x^{2}+4\,x}-4\,\arctan\left(\frac{\sqrt{-x^{2}+4\,x}}{x-4}\right)$$

```
integrate(x/(-x^2+4*x)^(1/2),x, algorithm="fricas")
```

```
-sqrt(-x^2 + 4*x) - 4*arctan(sqrt(-x^2 + 4*x)/(x - 4))
```

Sympy [A] (verification not implemented)

Time = 0.25 (sec) , antiderivative size = 17, normalized size of antiderivative = 0.59

$$\int \frac{x}{\sqrt{4x-x^2}}\,dx = -\sqrt{-x^2+4x} + 2\operatorname{asin}\left(\frac{x}{2}-1\right)$$

```
integrate(x/(-x**2+4*x)**(1/2),x)
```

```
-sqrt(-x**2 + 4*x) + 2*asin(x/2 - 1)
```

Maxima [A] (verification not implemented)

Time = 0.10 (sec) , antiderivative size = 22, normalized size of antiderivative = 0.76

$$\int \frac{x}{\sqrt{4x-x^2}}\,dx = -\sqrt{-x^2+4\,x} - 2\,\arcsin\left(-\frac{1}{2}\,x+1\right)$$

```
integrate(x/(-x^2+4*x)^(1/2),x, algorithm="maxima")
```

```
-sqrt(-x^2 + 4*x) - 2*arcsin(-1/2*x + 1)
```

Giac [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 22, normalized size of antiderivative = 0.76

$$\int \frac{x}{\sqrt{4x-x^2}}\,dx = -\sqrt{-x^2+4\,x} + 2\,\arcsin\left(\frac{1}{2}\,x-1\right)$$

```
integrate(x/(-x^2+4*x)^(1/2),x, algorithm="giac")
```

```
-sqrt(-x^2 + 4*x) + 2*arcsin(1/2*x - 1)
```

Mupad [B] (verification not implemented)

Time = 0.05 (sec) , antiderivative size = 22, normalized size of antiderivative = 0.76

$$\int \frac{x}{\sqrt{4x-x^2}}\,dx = 2\,\mathrm{asin}\left(\frac{x}{2}-1\right)-\sqrt{4\,x-x^2}$$

```
int(x/(4*x - x^2)^(1/2),x)
```

```
2*asin(x/2 - 1) - (4*x - x^2)^(1/2)
```

Reduce [B] (verification not implemented)

Time = 0.49 (sec) , antiderivative size = 29, normalized size of antiderivative = 1.00

$$\int \frac{x}{\sqrt{4x-x^2}}\,dx = -\sqrt{x}\,\sqrt{-x+4}-4\log\left(\frac{\sqrt{-x+4}}{2}+\frac{\sqrt{x}\,i}{2}\right)i$$

```
int(x/(-x^2+4*x)^(1/2),x)
```

```
 - sqrt(x)*sqrt( - x + 4) - 4*log((sqrt( - x + 4) + sqrt(x)*i)/2)*i
```

3.77 $\int \frac{x}{\sqrt{-4x+x^2}}\,dx$

Optimal result

Integrand size = 13, antiderivative size = 35

$$\int \frac{x}{\sqrt{-4x+x^2}}\,dx = \sqrt{-4x+x^2} + 4\text{arcsinh}\left(\frac{\sqrt{-4x+x^2}}{2\sqrt{x}}\right)$$

```
(x^2-4*x)^(1/2)+4*arcsinh(1/2*(x^2-4*x)^(1/2)/x^(1/2))
```

Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 46, normalized size of antiderivative = 1.31

$$\int \frac{x}{\sqrt{-4x+x^2}}\,dx = \frac{(-4+x)x - 4\sqrt{-4+x}\sqrt{x}\log\left(\sqrt{-4+x}-\sqrt{x}\right)}{\sqrt{(-4+x)x}}$$

```
Integrate[x/Sqrt[-4*x + x^2],x]
```

```
((-4 + x)*x - 4*Sqrt[-4 + x]*Sqrt[x]*Log[Sqrt[-4 + x] - Sqrt[x]])/Sqrt[(-4
 + x)*x]
```

Rubi [A] (verified)

Time = 0.26 (sec) , antiderivative size = 28, normalized size of antiderivative = 0.80, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.231, Rules used = {1160, 1091, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x}{\sqrt{x^2-4x}}dx$$

$$\Big\downarrow \text{ 1160}$$

$$2\int \frac{1}{\sqrt{x^2-4x}}dx+\sqrt{x^2-4x}$$

$$\Big\downarrow \text{ 1091}$$

$$4\int \frac{1}{1-\frac{x^2}{x^2-4x}}d\frac{x}{\sqrt{x^2-4x}}+\sqrt{x^2-4x}$$

$$\Big\downarrow \text{ 219}$$

$$4\text{arctanh}\left(\frac{x}{\sqrt{x^2-4x}}\right)+\sqrt{x^2-4x}$$

```
Int[x/Sqrt[-4*x + x^2],x]
```

```
Sqrt[-4*x + x^2] + 4*ArcTanh[x/Sqrt[-4*x + x^2]]
```

Defintions of rubi rules used

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

```
Int[1/Sqrt[(b_.)*(x_) + (c_.)*(x_)^2], x_Symbol] :> Simp[2   Subst[Int[1/(1
 - c*x^2), x], x, x/Sqrt[b*x + c*x^2]], x] /; FreeQ[{b, c}, x]
```

```
Int[((d_.) + (e_.)*(x_))*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol
] :> Simp[e*((a + b*x + c*x^2)^(p + 1)/(2*c*(p + 1))), x] + Simp[(2*c*d - b
*e)/(2*c)   Int[(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, p}, x]
 && NeQ[p, -1]
```

Maple [A] (verified)

Time = 0.34 (sec) , antiderivative size = 26, normalized size of antiderivative = 0.74

method	result	size
default	$\sqrt{x^{2}-4x}+2\ln\left(x-2+\sqrt{x^{2}-4x}\right)$	26
trager	$\sqrt{x^{2}-4x}+2\ln\left(x-2+\sqrt{x^{2}-4x}\right)$	26
risch	$\frac{x(x-4)}{\sqrt{x(x-4)}}+2\ln\left(x-2+\sqrt{x^{2}-4x}\right)$	29
pseudoelliptic	$\sqrt{x\left(x-4\right)}+2\ln\left(\frac{x+\sqrt{x(x-4)}}{x}\right)-2\ln\left(\frac{\sqrt{x(x-4)}-x}{x}\right)$	43
meijerg	$\frac{4i\sqrt{-\operatorname{signum}(x-4)}\left(\frac{i\sqrt{\pi}\,\sqrt{x}\,\sqrt{-\frac{x}{4}+1}}{2}-i\sqrt{\pi}\,\arcsin\left(\frac{\sqrt{x}}{2}\right)\right)}{\sqrt{\pi}\,\sqrt{\operatorname{signum}(x-4)}}$	50

```
int(x/(x^2-4*x)^(1/2),x,method=_RETURNVERBOSE)
```

```
(x^2-4*x)^(1/2)+2*ln(x-2+(x^2-4*x)^(1/2))
```

Fricas [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 27, normalized size of antiderivative = 0.77

$$\int \frac{x}{\sqrt{-4x+x^2}}\,dx=\sqrt{x^{2}-4\,x}-2\,\log\left(-x+\sqrt{x^{2}-4\,x}+2\right)$$

```
integrate(x/(x^2-4*x)^(1/2),x, algorithm="fricas")
```

```
sqrt(x^2 - 4*x) - 2*log(-x + sqrt(x^2 - 4*x) + 2)
```

Sympy [A] (verification not implemented)

Time = 0.21 (sec) , antiderivative size = 29, normalized size of antiderivative = 0.83

$$\int \frac{x}{\sqrt{-4x+x^2}}\,dx = \sqrt{x^2-4x} + 2\log\left(2x+2\sqrt{x^2-4x}-4\right)$$

```
integrate(x/(x**2-4*x)**(1/2),x)
```

```
sqrt(x**2 - 4*x) + 2*log(2*x + 2*sqrt(x**2 - 4*x) - 4)
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 29, normalized size of antiderivative = 0.83

$$\int \frac{x}{\sqrt{-4x+x^2}}\,dx = \sqrt{x^2-4\,x} + 2\,\log\left(2\,x+2\,\sqrt{x^2-4\,x}-4\right)$$

```
integrate(x/(x^2-4*x)^(1/2),x, algorithm="maxima")
```

```
sqrt(x^2 - 4*x) + 2*log(2*x + 2*sqrt(x^2 - 4*x) - 4)
```

Giac [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 28, normalized size of antiderivative = 0.80

$$\int \frac{x}{\sqrt{-4x+x^2}}\,dx = \sqrt{x^2-4\,x} - 2\,\log\left(\left|-x+\sqrt{x^2-4\,x}+2\right|\right)$$

```
integrate(x/(x^2-4*x)^(1/2),x, algorithm="giac")
```

```
sqrt(x^2 - 4*x) - 2*log(abs(-x + sqrt(x^2 - 4*x) + 2))
```

Mupad [B] (verification not implemented)

Time = 9.03 (sec) , antiderivative size = 23, normalized size of antiderivative = 0.66

$$\int \frac{x}{\sqrt{-4x+x^2}}\,dx = 2\,\ln\left(x+\sqrt{x\,\left(x-4\right)}-2\right)+\sqrt{x^2-4\,x}$$

```
int(x/(x^2 - 4*x)^(1/2),x)
```

```
2*log(x + (x*(x - 4))^(1/2) - 2) + (x^2 - 4*x)^(1/2)
```

Reduce [B] (verification not implemented)

Time = 0.49 (sec) , antiderivative size = 22, normalized size of antiderivative = 0.63

$$\int \frac{x}{\sqrt{-4x+x^2}}\,dx = \sqrt{x}\,\sqrt{x-4}+4\log\left(\frac{\sqrt{x-4}}{2}+\frac{\sqrt{x}}{2}\right)$$

```
int(x/(x^2-4*x)^(1/2),x)
```

```
sqrt(x)*sqrt(x - 4) + 4*log((sqrt(x - 4) + sqrt(x))/2)
```

3.78 $\int \frac{x^2}{\sqrt{2x-x^2}} \, dx$

Optimal result

Integrand size = 17, antiderivative size = 51

$$\int \frac{x^2}{\sqrt{2x-x^2}} \, dx = -\frac{3}{2}\sqrt{2-x}\sqrt{x} - \frac{1}{2}\sqrt{2-x}x^{3/2} + 3\arcsin\left(\frac{\sqrt{x}}{\sqrt{2}}\right)$$

```
-3/2*(2-x)^(1/2)*x^(1/2)-1/2*(2-x)^(1/2)*x^(3/2)+3*arcsin(1/2*2^(1/2)*x^(1
/2))
```

Mathematica [A] (verified)

Time = 0.05 (sec) , antiderivative size = 53, normalized size of antiderivative = 1.04

$$\int \frac{x^2}{\sqrt{2x-x^2}} \, dx = \frac{x(-6+x+x^2) - 6\sqrt{-2+x}\sqrt{x}\log\left(\sqrt{-2+x}-\sqrt{x}\right)}{2\sqrt{-((-2+x)x)}}$$

```
Integrate[x^2/Sqrt[2*x - x^2],x]
```

```
(x*(-6 + x + x^2) - 6*Sqrt[-2 + x]*Sqrt[x]*Log[Sqrt[-2 + x] - Sqrt[x]])/(2
*Sqrt[-((-2 + x)*x)])
```

Rubi [A] (verified)

Time = 0.31 (sec) , antiderivative size = 51, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.235$, Rules used = {1134, 1160, 1090, 223}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^2}{\sqrt{2x-x^2}}dx$$

$$\Big\downarrow \text{1134}$$

$$\frac{3}{2}\int \frac{x}{\sqrt{2x-x^2}}dx-\frac{1}{2}x\sqrt{2x-x^2}$$

$$\Big\downarrow \text{1160}$$

$$\frac{3}{2}\left(\int \frac{1}{\sqrt{2x-x^2}}dx-\sqrt{2x-x^2}\right)-\frac{1}{2}x\sqrt{2x-x^2}$$

$$\Big\downarrow \text{1090}$$

$$\frac{3}{2}\left(-\frac{1}{2}\int \frac{1}{\sqrt{1-\frac{1}{4}(2-2x)^2}}d(2-2x)-\sqrt{2x-x^2}\right)-\frac{1}{2}x\sqrt{2x-x^2}$$

$$\Big\downarrow \text{223}$$

$$\frac{3}{2}\left(-\arcsin\left(\frac{1}{2}(2-2x)\right)-\sqrt{2x-x^2}\right)-\frac{1}{2}x\sqrt{2x-x^2}$$

```
Int[x^2/Sqrt[2*x - x^2],x]
```

```
-1/2*(x*Sqrt[2*x - x^2]) + (3*(-Sqrt[2*x - x^2] - ArcSin[(2 - 2*x)/2]))/2
```

Defintions of rubi rules used

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSin[Rt[-b, 2]*(x/Sqrt
[a])]/Rt[-b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && NegQ[b]
```

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[1/(2*c*(-4*
(c/(b^2 - 4*a*c)))^p)   Subst[Int[Simp[1 - x^2/(b^2 - 4*a*c), x]^p, x], x,
b + 2*c*x], x] /; FreeQ[{a, b, c, p}, x] && GtQ[4*a - b^2/c, 0]
```

```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] :> Simp[e*(d + e*x)^(m - 1)*((a + b*x + c*x^2)^(p + 1)/(c*(m + 2*p +
 1))), x] + Simp[(m + p)*((2*c*d - b*e)/(c*(m + 2*p + 1)))   Int[(d + e*x)^
(m - 1)*(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, p}, x] && EqQ[
c*d^2 - b*d*e + a*e^2, 0] && GtQ[m, 1] && NeQ[m + 2*p + 1, 0] && IntegerQ[2
*p]
```

```
Int[((d_.) + (e_.)*(x_))*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol
] :> Simp[e*((a + b*x + c*x^2)^(p + 1)/(2*c*(p + 1))), x] + Simp[(2*c*d - b
*e)/(2*c)   Int[(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, p}, x]
 && NeQ[p, -1]
```

Maple [A] (verified)

Time = 0.45 (sec) , antiderivative size = 25, normalized size of antiderivative = 0.49

method	result	size
risch	$\frac{(3+x)x(x-2)}{2\sqrt{-x(x-2)}}+\frac{3\arcsin(x-1)}{2}$	25
pseudoelliptic	$-3\arctan\left(\frac{\sqrt{-x(x-2)}}{x}\right)+\frac{(-x-3)\sqrt{-x(x-2)}}{2}$	32
default	$-\frac{x\sqrt{-x^2+2x}}{2}-\frac{3\sqrt{-x^2+2x}}{2}+\frac{3\arcsin(x-1)}{2}$	35
meijerg	$-\frac{4i\left(-\frac{i\sqrt{\pi}\,\sqrt{x}\,\sqrt{2}\,(5x+15)\sqrt{-\frac{x}{2}+1}}{40}+\frac{3i\sqrt{\pi}\,\arcsin\left(\frac{\sqrt{2}\,\sqrt{x}}{2}\right)}{4}\right)}{\sqrt{\pi}}$	47
trager	$\left(-\frac{3}{2}-\frac{x}{2}\right)\sqrt{-x^2+2x}+\frac{3\,\mathrm{RootOf}\left(\_Z^2+1\right)\ln\left(\mathrm{RootOf}\left(\_Z^2+1\right)\sqrt{-x^2+2x}+x-1\right)}{2}$	49

```
int(x^2/(-x^2+2*x)^(1/2),x,method=_RETURNVERBOSE)
```

```
1/2*(3+x)*x*(x-2)/(-x*(x-2))^(1/2)+3/2*arcsin(x-1)
```

Fricas [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 37, normalized size of antiderivative = 0.73

$$\int \frac{x^2}{\sqrt{2x-x^2}}\,dx = -\frac{1}{2}\sqrt{-x^2+2\,x}(x+3) - 3\,\arctan\left(\frac{\sqrt{-x^2+2\,x}}{x-2}\right)$$

```
integrate(x^2/(-x^2+2*x)^(1/2),x, algorithm="fricas")
```

```
-1/2*sqrt(-x^2 + 2*x)*(x + 3) - 3*arctan(sqrt(-x^2 + 2*x)/(x - 2))
```

Sympy [A] (verification not implemented)

Time = 0.23 (sec) , antiderivative size = 26, normalized size of antiderivative = 0.51

$$\int \frac{x^2}{\sqrt{2x-x^2}}\,dx = \left(-\frac{x}{2}-\frac{3}{2}\right)\sqrt{-x^2+2x} + \frac{3\,\mathrm{asin}\,(x-1)}{2}$$

```
integrate(x**2/(-x**2+2*x)**(1/2),x)
```

```
(-x/2 - 3/2)*sqrt(-x**2 + 2*x) + 3*asin(x - 1)/2
```

Maxima [A] (verification not implemented)

Time = 0.10 (sec) , antiderivative size = 36, normalized size of antiderivative = 0.71

$$\int \frac{x^2}{\sqrt{2x-x^2}}\,dx = -\frac{1}{2}\sqrt{-x^2+2\,x}x - \frac{3}{2}\sqrt{-x^2+2\,x} - \frac{3}{2}\arcsin\left(-x+1\right)$$

```
integrate(x^2/(-x^2+2*x)^(1/2),x, algorithm="maxima")
```

```
-1/2*sqrt(-x^2 + 2*x)*x - 3/2*sqrt(-x^2 + 2*x) - 3/2*arcsin(-x + 1)
```

Giac [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 23, normalized size of antiderivative = 0.45

$$\int \frac{x^2}{\sqrt{2x-x^2}}\,dx = -\frac{1}{2}\sqrt{-x^2+2\,x}(x+3) + \frac{3}{2}\arcsin\left(x-1\right)$$

```
integrate(x^2/(-x^2+2*x)^(1/2),x, algorithm="giac")
```

```
-1/2*sqrt(-x^2 + 2*x)*(x + 3) + 3/2*arcsin(x - 1)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{x^2}{\sqrt{2x-x^2}}\,dx = \int \frac{x^2}{\sqrt{2\,x-x^2}}\,dx$$

```
int(x^2/(2*x - x^2)^(1/2),x)
```

```
int(x^2/(2*x - x^2)^(1/2), x)
```

Reduce [B] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 42, normalized size of antiderivative = 0.82

$$\int \frac{x^2}{\sqrt{2x-x^2}}\,dx = -\frac{\sqrt{x}\,\sqrt{-x+2}\,x}{2} - \frac{3\sqrt{x}\,\sqrt{-x+2}}{2} - 3\log\left(\frac{\sqrt{-x+2}+\sqrt{x}\,i}{\sqrt{2}}\right)i$$

```
int(x^2/(-x^2+2*x)^(1/2),x)
```

```
( - sqrt(x)*sqrt( - x + 2)*x - 3*sqrt(x)*sqrt( - x + 2) - 6*log((sqrt( - x
 + 2) + sqrt(x)*i)/sqrt(2))*i)/2
```

3.79 $\int (cx)^{5/2}\sqrt{ax+bx^2}\,dx$

Optimal result

Integrand size = 21, antiderivative size = 128

$$\int (cx)^{5/2}\sqrt{ax+bx^2}\,dx=-\frac{2a^3c^4(ax+bx^2)^{3/2}}{3b^4(cx)^{3/2}}+\frac{6a^2c^5(ax+bx^2)^{5/2}}{5b^4(cx)^{5/2}}-\frac{6ac^6(ax+bx^2)^{7/2}}{7b^4(cx)^{7/2}}+\frac{2c^7(ax+bx^2)^{9/2}}{9b^4(cx)^{9/2}}$$

```
-2/3*a^3*c^4*(b*x^2+a*x)^(3/2)/b^4/(c*x)^(3/2)+6/5*a^2*c^5*(b*x^2+a*x)^(5/
2)/b^4/(c*x)^(5/2)-6/7*a*c^6*(b*x^2+a*x)^(7/2)/b^4/(c*x)^(7/2)+2/9*c^7*(b*
x^2+a*x)^(9/2)/b^4/(c*x)^(9/2)
```

Mathematica [A] (verified)

Time = 0.05 (sec) , antiderivative size = 58, normalized size of antiderivative = 0.45

$$\int (cx)^{5/2}\sqrt{ax+bx^2}\,dx=\frac{2c^4(x(a+bx))^{3/2}\left(-16a^3+24a^2bx-30ab^2x^2+35b^3x^3\right)}{315b^4(cx)^{3/2}}$$

```
Integrate[(c*x)^(5/2)*Sqrt[a*x + b*x^2],x]
```

```
(2*c^4*(x*(a + b*x))^(3/2)*(-16*a^3 + 24*a^2*b*x - 30*a*b^2*x^2 + 35*b^3*x
^3))/(315*b^4*(c*x)^(3/2))
```

Rubi [A] (verified)

Time = 0.43 (sec) , antiderivative size = 136, normalized size of antiderivative = 1.06, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.190$, Rules used = {1128, 1128, 1128, 1122}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (cx)^{5/2}\sqrt{ax+bx^2}\,dx$$

$$\Big\downarrow \text{1128}$$

$$\frac{2c(cx)^{3/2}\left(ax+bx^2\right)^{3/2}}{9b}-\frac{2ac\int (cx)^{3/2}\sqrt{bx^2+ax}dx}{3b}$$

$$\Big\downarrow \text{1128}$$

$$\frac{2c(cx)^{3/2}\left(ax+bx^2\right)^{3/2}}{9b}-\frac{2ac\left(\frac{2c\sqrt{cx}\left(ax+bx^2\right)^{3/2}}{7b}-\frac{4ac\int \sqrt{cx}\sqrt{bx^2+ax}dx}{7b}\right)}{3b}$$

$$\Big\downarrow \text{1128}$$

$$\frac{2c(cx)^{3/2}\left(ax+bx^2\right)^{3/2}}{9b}-\frac{2ac\left(\frac{2c\sqrt{cx}\left(ax+bx^2\right)^{3/2}}{7b}-\frac{4ac\left(\frac{2c\left(ax+bx^2\right)^{3/2}}{5b\sqrt{cx}}-\frac{2ac\int \frac{\sqrt{bx^2+ax}}{\sqrt{cx}}dx}{5b}\right)}{7b}\right)}{3b}$$

$$\Big\downarrow \text{1122}$$

$$\frac{2c(cx)^{3/2}\left(ax+bx^2\right)^{3/2}}{9b}-\frac{2ac\left(\frac{2c\sqrt{cx}\left(ax+bx^2\right)^{3/2}}{7b}-\frac{4ac\left(\frac{2c\left(ax+bx^2\right)^{3/2}}{5b\sqrt{cx}}-\frac{4ac^2\left(ax+bx^2\right)^{3/2}}{15b^2(cx)^{3/2}}\right)}{7b}\right)}{3b}$$

```
Int[(c*x)^(5/2)*Sqrt[a*x + b*x^2],x]
```

```
(2*c*(c*x)^(3/2)*(a*x + b*x^2)^(3/2))/(9*b) - (2*a*c*((2*c*Sqrt[c*x]*(a*x
+ b*x^2)^(3/2))/(7*b) - (4*a*c*((-4*a*c^2*(a*x + b*x^2)^(3/2))/(15*b^2*(c*
x)^(3/2)) + (2*c*(a*x + b*x^2)^(3/2))/(5*b*Sqrt[c*x])))/(7*b)))/(3*b)
```

Defintions of rubi rules used

```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] :> Simp[e*(d + e*x)^(m - 1)*((a + b*x + c*x^2)^(p + 1)/(c*(p + 1))),
 x] /; FreeQ[{a, b, c, d, e, m, p}, x] && EqQ[c*d^2 - b*d*e + a*e^2, 0] &&
EqQ[m + p, 0]
```

```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] :> Simp[e*(d + e*x)^(m - 1)*((a + b*x + c*x^2)^(p + 1)/(c*(m + 2*p +
 1))), x] + Simp[Simplify[m + p]*((2*c*d - b*e)/(c*(m + 2*p + 1)))   Int[(d
 + e*x)^(m - 1)*(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, m, p},
 x] && EqQ[c*d^2 - b*d*e + a*e^2, 0] && IGtQ[Simplify[m + p], 0]
```

Maple [A] (verified)

Time = 0.36 (sec) , antiderivative size = 60, normalized size of antiderivative = 0.47

method	result	size
gosper	$-\frac{2(bx+a)\left(-35b^{3}x^{3}+30a\,b^{2}x^{2}-24a^{2}bx+16a^{3}\right)(cx)^{\frac{5}{2}}\sqrt{b\,x^{2}+ax}}{315b^{4}x^{3}}$	60
orering	$-\frac{2(bx+a)\left(-35b^{3}x^{3}+30a\,b^{2}x^{2}-24a^{2}bx+16a^{3}\right)(cx)^{\frac{5}{2}}\sqrt{b\,x^{2}+ax}}{315b^{4}x^{3}}$	60
default	$-\frac{2c^{2}\sqrt{cx}\,\sqrt{x(bx+a)}\,(bx+a)\left(-35b^{3}x^{3}+30a\,b^{2}x^{2}-24a^{2}bx+16a^{3}\right)}{315x\,b^{4}}$	61
risch	$-\frac{2c^{3}x\left(-35b^{4}x^{4}-5a\,b^{3}x^{3}+6a^{2}b^{2}x^{2}-8a^{3}bx+16a^{4}\right)(bx+a)}{315\sqrt{cx}\,\sqrt{x(bx+a)}\,b^{4}}$	70

```
int((c*x)^(5/2)*(b*x^2+a*x)^(1/2),x,method=_RETURNVERBOSE)
```

```
-2/315*(b*x+a)*(-35*b^3*x^3+30*a*b^2*x^2-24*a^2*b*x+16*a^3)*(c*x)^(5/2)*(b
*x^2+a*x)^(1/2)/b^4/x^3
```

Fricas [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 80, normalized size of antiderivative = 0.62

$$\int (cx)^{5/2}\sqrt{ax+bx^2}\,dx = \frac{2\,(35\,b^4c^2x^4+5\,ab^3c^2x^3-6\,a^2b^2c^2x^2+8\,a^3bc^2x-16\,a^4c^2)\sqrt{bx^2+ax}\sqrt{cx}}{315\,b^4x}$$

```
integrate((c*x)^(5/2)*(b*x^2+a*x)^(1/2),x, algorithm="fricas")
```

```
2/315*(35*b^4*c^2*x^4 + 5*a*b^3*c^2*x^3 - 6*a^2*b^2*c^2*x^2 + 8*a^3*b*c^2*
x - 16*a^4*c^2)*sqrt(b*x^2 + a*x)*sqrt(c*x)/(b^4*x)
```

Sympy [F]

$$\int (cx)^{5/2}\sqrt{ax+bx^2}\,dx = \int (cx)^{\frac{5}{2}}\,\sqrt{x\,(a+bx)}\,dx$$

```
integrate((c*x)**(5/2)*(b*x**2+a*x)**(1/2),x)
```

```
Integral((c*x)**(5/2)*sqrt(x*(a + b*x)), x)
```

Maxima [A] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 68, normalized size of antiderivative = 0.53

$$\int (cx)^{5/2}\sqrt{ax+bx^2}\,dx = \frac{2\left(35\,b^4c^{\frac{5}{2}}x^4+5\,ab^3c^{\frac{5}{2}}x^3-6\,a^2b^2c^{\frac{5}{2}}x^2+8\,a^3bc^{\frac{5}{2}}x-16\,a^4c^{\frac{5}{2}}\right)\sqrt{bx+a}}{315\,b^4}$$

```
integrate((c*x)^(5/2)*(b*x^2+a*x)^(1/2),x, algorithm="maxima")
```

```
2/315*(35*b^4*c^(5/2)*x^4 + 5*a*b^3*c^(5/2)*x^3 - 6*a^2*b^2*c^(5/2)*x^2 +
8*a^3*b*c^(5/2)*x - 16*a^4*c^(5/2))*sqrt(b*x + a)/b^4
```

Giac [A] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 90, normalized size of antiderivative = 0.70

$$\int (cx)^{5/2}\sqrt{ax+bx^2}\,dx = \frac{2}{315}\left(\frac{16\,\sqrt{ac}a^4c}{b^4} - \frac{105\,(bcx+ac)^{\frac{3}{2}}a^3c^3 - 189\,(bcx+ac)^{\frac{5}{2}}a^2c^2 + 135\,(bcx+ac)^{\frac{7}{2}}\right.$$

```
integrate((c*x)^(5/2)*(b*x^2+a*x)^(1/2),x, algorithm="giac")
```

```
2/315*(16*sqrt(a*c)*a^4*c/b^4 - (105*(b*c*x + a*c)^(3/2)*a^3*c^3 - 189*(b*
c*x + a*c)^(5/2)*a^2*c^2 + 135*(b*c*x + a*c)^(7/2)*a*c - 35*(b*c*x + a*c)^
(9/2))/(b^4*c^3))*abs(c)
```

Mupad [B] (verification not implemented)

Time = 9.39 (sec) , antiderivative size = 68, normalized size of antiderivative = 0.53

$$\int (cx)^{5/2}\sqrt{ax+bx^2}\,dx = \frac{2\,c^2\,\sqrt{b\,x^2+a\,x}\,\sqrt{c\,x}\,\left(-16\,a^4+8\,a^3\,b\,x-6\,a^2\,b^2\,x^2+5\,a\,b^3\,x^3+35\,b^4\,x^4\right)}{315\,b^4\,x}$$

```
int((a*x + b*x^2)^(1/2)*(c*x)^(5/2),x)
```

```
(2*c^2*(a*x + b*x^2)^(1/2)*(c*x)^(1/2)*(35*b^4*x^4 - 16*a^4 + 5*a*b^3*x^3
- 6*a^2*b^2*x^2 + 8*a^3*b*x))/(315*b^4*x)
```

Reduce [B] (verification not implemented)

Time = 0.29 (sec) , antiderivative size = 57, normalized size of antiderivative = 0.45

$$\int (cx)^{5/2}\sqrt{ax+bx^2}\,dx = \frac{2\sqrt{c}\,\sqrt{bx+a}\,c^2(35b^4x^4+5a\,b^3x^3-6a^2b^2x^2+8a^3bx-16a^4)}{315b^4}$$

```
int((c*x)^(5/2)*(b*x^2+a*x)^(1/2),x)
```

```
(2*sqrt(c)*sqrt(a + b*x)*c**2*( - 16*a**4 + 8*a**3*b*x - 6*a**2*b**2*x**2
+ 5*a*b**3*x**3 + 35*b**4*x**4))/(315*b**4)
```

3.80 $\int (cx)^{3/2}\sqrt{ax+bx^2}\,dx$

Optimal result

Integrand size = 21, antiderivative size = 95

$$\int (cx)^{3/2}\sqrt{ax+bx^2}\,dx=\frac{2a^2c^3(ax+bx^2)^{3/2}}{3b^3(cx)^{3/2}}-\frac{4ac^4(ax+bx^2)^{5/2}}{5b^3(cx)^{5/2}}+\frac{2c^5(ax+bx^2)^{7/2}}{7b^3(cx)^{7/2}}$$

```
2/3*a^2*c^3*(b*x^2+a*x)^(3/2)/b^3/(c*x)^(3/2)-4/5*a*c^4*(b*x^2+a*x)^(5/2)/
b^3/(c*x)^(5/2)+2/7*c^5*(b*x^2+a*x)^(7/2)/b^3/(c*x)^(7/2)
```

Mathematica [A] (verified)

Time = 0.04 (sec) , antiderivative size = 47, normalized size of antiderivative = 0.49

$$\int (cx)^{3/2}\sqrt{ax+bx^2}\,dx=\frac{2c^3(x(a+bx))^{3/2}\,(8a^2-12abx+15b^2x^2)}{105b^3(cx)^{3/2}}$$

```
Integrate[(c*x)^(3/2)*Sqrt[a*x + b*x^2],x]
```

```
(2*c^3*(x*(a + b*x))^(3/2)*(8*a^2 - 12*a*b*x + 15*b^2*x^2))/(105*b^3*(c*x)
^(3/2))
```

Rubi [A] (verified)

Time = 0.36 (sec) , antiderivative size = 98, normalized size of antiderivative = 1.03, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.143$, Rules used = {1128, 1128, 1122}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (cx)^{3/2}\sqrt{ax+bx^2}\,dx$$

$$\Big\downarrow \text{1128}$$

$$\frac{2c\sqrt{cx}\left(ax+bx^2\right)^{3/2}}{7b}-\frac{4ac\int \sqrt{cx}\sqrt{bx^2+ax}dx}{7b}$$

$$\Big\downarrow \text{1128}$$

$$\frac{2c\sqrt{cx}\left(ax+bx^2\right)^{3/2}}{7b}-\frac{4ac\left(\frac{2c\left(ax+bx^2\right)^{3/2}}{5b\sqrt{cx}}-\frac{2ac\int \frac{\sqrt{bx^2+ax}}{\sqrt{cx}}dx}{5b}\right)}{7b}$$

$$\Big\downarrow \text{1122}$$

$$\frac{2c\sqrt{cx}\left(ax+bx^2\right)^{3/2}}{7b}-\frac{4ac\left(\frac{2c\left(ax+bx^2\right)^{3/2}}{5b\sqrt{cx}}-\frac{4ac^2\left(ax+bx^2\right)^{3/2}}{15b^2(cx)^{3/2}}\right)}{7b}$$

```
Int[(c*x)^(3/2)*Sqrt[a*x + b*x^2],x]
```

```
(2*c*Sqrt[c*x]*(a*x + b*x^2)^(3/2))/(7*b) - (4*a*c*((-4*a*c^2*(a*x + b*x^2
)^(3/2))/(15*b^2*(c*x)^(3/2)) + (2*c*(a*x + b*x^2)^(3/2))/(5*b*Sqrt[c*x]))
)/(7*b)
```

Defintions of rubi rules used

```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] :> Simp[e*(d + e*x)^(m - 1)*((a + b*x + c*x^2)^(p + 1)/(c*(p + 1))),
 x] /; FreeQ[{a, b, c, d, e, m, p}, x] && EqQ[c*d^2 - b*d*e + a*e^2, 0] &&
EqQ[m + p, 0]
```

```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] :> Simp[e*(d + e*x)^(m - 1)*((a + b*x + c*x^2)^(p + 1)/(c*(m + 2*p +
 1))), x] + Simp[Simplify[m + p]*((2*c*d - b*e)/(c*(m + 2*p + 1)))   Int[(d
 + e*x)^(m - 1)*(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, m, p},
 x] && EqQ[c*d^2 - b*d*e + a*e^2, 0] && IGtQ[Simplify[m + p], 0]
```

Maple [A] (verified)

Time = 0.35 (sec) , antiderivative size = 48, normalized size of antiderivative = 0.51

method	result	size
default	$\frac{2c\sqrt{cx}\,\sqrt{x(bx+a)}\,(bx+a)\left(15b^{2}x^{2}-12abx+8a^{2}\right)}{105x\,b^{3}}$	48
gosper	$\frac{2(bx+a)\left(15b^{2}x^{2}-12abx+8a^{2}\right)(cx)^{\frac{3}{2}}\sqrt{b\,x^{2}+ax}}{105b^{3}x^{2}}$	49
orering	$\frac{2(bx+a)\left(15b^{2}x^{2}-12abx+8a^{2}\right)(cx)^{\frac{3}{2}}\sqrt{b\,x^{2}+ax}}{105b^{3}x^{2}}$	49
risch	$\frac{2c^{2}x\left(15b^{3}x^{3}+3a\,b^{2}x^{2}-4a^{2}bx+8a^{3}\right)(bx+a)}{105\sqrt{cx}\,\sqrt{x(bx+a)}\,b^{3}}$	59

```
int((c*x)^(3/2)*(b*x^2+a*x)^(1/2),x,method=_RETURNVERBOSE)
```

```
2/105*c/x*(c*x)^(1/2)*(x*(b*x+a))^(1/2)*(b*x+a)*(15*b^2*x^2-12*a*b*x+8*a^2
)/b^3
```

Fricas [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 58, normalized size of antiderivative = 0.61

$$\int (cx)^{3/2}\sqrt{ax+bx^2}\,dx = \frac{2\,(15\,b^3cx^3+3\,ab^2cx^2-4\,a^2bcx+8\,a^3c)\sqrt{bx^2+ax}\sqrt{cx}}{105\,b^3x}$$

```
integrate((c*x)^(3/2)*(b*x^2+a*x)^(1/2),x, algorithm="fricas")
```

```
2/105*(15*b^3*c*x^3 + 3*a*b^2*c*x^2 - 4*a^2*b*c*x + 8*a^3*c)*sqrt(b*x^2 +
a*x)*sqrt(c*x)/(b^3*x)
```

Sympy [F]

$$\int (cx)^{3/2}\sqrt{ax+bx^2}\,dx = \int (cx)^{\frac{3}{2}}\,\sqrt{x\,(a+bx)}\,dx$$

```
integrate((c*x)**(3/2)*(b*x**2+a*x)**(1/2),x)
```

```
Integral((c*x)**(3/2)*sqrt(x*(a + b*x)), x)
```

Maxima [A] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 54, normalized size of antiderivative = 0.57

$$\int (cx)^{3/2}\sqrt{ax+bx^2}\,dx = \frac{2\left(15\,b^3c^{\frac{3}{2}}x^3+3\,ab^2c^{\frac{3}{2}}x^2-4\,a^2bc^{\frac{3}{2}}x+8\,a^3c^{\frac{3}{2}}\right)\sqrt{bx+a}}{105\,b^3}$$

```
integrate((c*x)^(3/2)*(b*x^2+a*x)^(1/2),x, algorithm="maxima")
```

```
2/105*(15*b^3*c^(3/2)*x^3 + 3*a*b^2*c^(3/2)*x^2 - 4*a^2*b*c^(3/2)*x + 8*a^
3*c^(3/2))*sqrt(b*x + a)/b^3
```

Giac [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 74, normalized size of antiderivative = 0.78

$$\int (cx)^{3/2}\sqrt{ax+bx^2}\,dx = -\frac{2\left(\frac{8\sqrt{ac}a^3c^3}{b^3} - \frac{35\,(bcx+ac)^{\frac{3}{2}}a^2c^2-42\,(bcx+ac)^{\frac{5}{2}}ac+15\,(bcx+ac)^{\frac{7}{2}}}{b^3}\right)|c|}{105\,c^3}$$

```
integrate((c*x)^(3/2)*(b*x^2+a*x)^(1/2),x, algorithm="giac")
```

```
-2/105*(8*sqrt(a*c)*a^3*c^3/b^3 - (35*(b*c*x + a*c)^(3/2)*a^2*c^2 - 42*(b*
c*x + a*c)^(5/2)*a*c + 15*(b*c*x + a*c)^(7/2))/b^3)*abs(c)/c^3
```

Mupad [B] (verification not implemented)

Time = 9.25 (sec) , antiderivative size = 55, normalized size of antiderivative = 0.58

$$\int (cx)^{3/2}\sqrt{ax+bx^2}\,dx = \frac{2\,c\,\sqrt{b\,x^2+a\,x}\,\sqrt{c\,x}\,(8\,a^3-4\,a^2\,b\,x+3\,a\,b^2\,x^2+15\,b^3\,x^3)}{105\,b^3\,x}$$

```
int((a*x + b*x^2)^(1/2)*(c*x)^(3/2),x)
```

```
(2*c*(a*x + b*x^2)^(1/2)*(c*x)^(1/2)*(8*a^3 + 15*b^3*x^3 + 3*a*b^2*x^2 - 4
*a^2*b*x))/(105*b^3*x)
```

Reduce [B] (verification not implemented)

Time = 0.49 (sec) , antiderivative size = 44, normalized size of antiderivative = 0.46

$$\int (cx)^{3/2}\sqrt{ax+bx^2}\,dx = \frac{2\sqrt{c}\,\sqrt{bx+a}\,c(15b^3x^3+3a\,b^2x^2-4a^2bx+8a^3)}{105b^3}$$

```
int((c*x)^(3/2)*(b*x^2+a*x)^(1/2),x)
```

```
(2*sqrt(c)*sqrt(a + b*x)*c*(8*a**3 - 4*a**2*b*x + 3*a*b**2*x**2 + 15*b**3*
x**3))/(105*b**3)
```

3.81 $\int \sqrt{cx}\sqrt{ax+bx^2}\,dx$

Optimal result

Integrand size = 21, antiderivative size = 62

$$\int \sqrt{cx}\sqrt{ax+bx^2}\,dx = -\frac{2ac^2(ax+bx^2)^{3/2}}{3b^2(cx)^{3/2}}+\frac{2c^3(ax+bx^2)^{5/2}}{5b^2(cx)^{5/2}}$$

```
-2/3*a*c^2*(b*x^2+a*x)^(3/2)/b^2/(c*x)^(3/2)+2/5*c^3*(b*x^2+a*x)^(5/2)/b^2
/(c*x)^(5/2)
```

Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 46, normalized size of antiderivative = 0.74

$$\int \sqrt{cx}\sqrt{ax+bx^2}\,dx = \frac{2\sqrt{cx}\sqrt{x(a+bx)}(-2a^2+abx+3b^2x^2)}{15b^2x}$$

```
Integrate[Sqrt[c*x]*Sqrt[a*x + b*x^2],x]
```

```
(2*Sqrt[c*x]*Sqrt[x*(a + b*x)]*(-2*a^2 + a*b*x + 3*b^2*x^2))/(15*b^2*x)
```

Rubi [A] (verified)

Time = 0.30 (sec) , antiderivative size = 60, normalized size of antiderivative = 0.97, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.095$, Rules used = {1128, 1122}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sqrt{cx}\sqrt{ax+bx^2}\,dx$$

$$\Big\downarrow \text{1128}$$

$$\frac{2c\left(ax+bx^2\right)^{3/2}}{5b\sqrt{cx}}-\frac{2ac\int \frac{\sqrt{bx^2+ax}}{\sqrt{cx}}dx}{5b}$$

$$\Big\downarrow \text{1122}$$

$$\frac{2c\left(ax+bx^2\right)^{3/2}}{5b\sqrt{cx}}-\frac{4ac^2\left(ax+bx^2\right)^{3/2}}{15b^2(cx)^{3/2}}$$

```
Int[Sqrt[c*x]*Sqrt[a*x + b*x^2],x]
```

```
(-4*a*c^2*(a*x + b*x^2)^(3/2))/(15*b^2*(c*x)^(3/2)) + (2*c*(a*x + b*x^2)^(
3/2))/(5*b*Sqrt[c*x])
```

Defintions of rubi rules used

```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] :> Simp[e*(d + e*x)^(m - 1)*((a + b*x + c*x^2)^(p + 1)/(c*(p + 1))),
 x] /; FreeQ[{a, b, c, d, e, m, p}, x] && EqQ[c*d^2 - b*d*e + a*e^2, 0] &&
EqQ[m + p, 0]
```

```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] :> Simp[e*(d + e*x)^(m - 1)*((a + b*x + c*x^2)^(p + 1)/(c*(m + 2*p +
 1))), x] + Simp[Simplify[m + p]*((2*c*d - b*e)/(c*(m + 2*p + 1)))   Int[(d
 + e*x)^(m - 1)*(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, m, p},
 x] && EqQ[c*d^2 - b*d*e + a*e^2, 0] && IGtQ[Simplify[m + p], 0]
```

Maple [A] (verified)

Time = 0.36 (sec) , antiderivative size = 36, normalized size of antiderivative = 0.58

method	result	size
default	$-\frac{2\sqrt{cx}\,\sqrt{x(bx+a)}\,(bx+a)(-3bx+2a)}{15x\,b^{2}}$	36
gosper	$-\frac{2(bx+a)(-3bx+2a)\sqrt{cx}\,\sqrt{b\,x^{2}+ax}}{15b^{2}x}$	38
orering	$-\frac{2(bx+a)(-3bx+2a)\sqrt{cx}\,\sqrt{b\,x^{2}+ax}}{15b^{2}x}$	38
risch	$-\frac{2cx\left(-3b^{2}x^{2}-abx+2a^{2}\right)(bx+a)}{15\sqrt{cx}\,\sqrt{x(bx+a)}\,b^{2}}$	46

```
int((c*x)^(1/2)*(b*x^2+a*x)^(1/2),x,method=_RETURNVERBOSE)
```

```
-2/15*(c*x)^(1/2)*(x*(b*x+a))^(1/2)*(b*x+a)*(-3*b*x+2*a)/x/b^2
```

Fricas [A] (verification not implemented)

Time = 0.10 (sec) , antiderivative size = 42, normalized size of antiderivative = 0.68

$$\int \sqrt{cx}\sqrt{ax+bx^2}\,dx = \frac{2\left(3\,b^2x^2+abx-2\,a^2\right)\sqrt{bx^2+ax}\sqrt{cx}}{15\,b^2x}$$

```
integrate((c*x)^(1/2)*(b*x^2+a*x)^(1/2),x, algorithm="fricas")
```

```
2/15*(3*b^2*x^2 + a*b*x - 2*a^2)*sqrt(b*x^2 + a*x)*sqrt(c*x)/(b^2*x)
```

Sympy [F]

$$\int \sqrt{cx}\sqrt{ax+bx^2}\,dx = \int \sqrt{cx}\sqrt{x(a+bx)}\,dx$$

```
integrate((c*x)**(1/2)*(b*x**2+a*x)**(1/2),x)
```

```
Integral(sqrt(c*x)*sqrt(x*(a + b*x)), x)
```

Maxima [A] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 39, normalized size of antiderivative = 0.63

$$\int \sqrt{cx}\sqrt{ax+bx^2}\,dx = \frac{2\left(3\,b^2\sqrt{c}x^2 + ab\sqrt{c}x - 2\,a^2\sqrt{c}\right)\sqrt{bx+a}}{15\,b^2}$$

```
integrate((c*x)^(1/2)*(b*x^2+a*x)^(1/2),x, algorithm="maxima")
```

```
2/15*(3*b^2*sqrt(c)*x^2 + a*b*sqrt(c)*x - 2*a^2*sqrt(c))*sqrt(b*x + a)/b^2
```

Giac [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 56, normalized size of antiderivative = 0.90

$$\int \sqrt{cx}\sqrt{ax+bx^2}\,dx = \frac{2\left(\frac{2\sqrt{ac}a^2c^2}{b^2} - \frac{5\,(bcx+ac)^{\frac{3}{2}}ac - 3\,(bcx+ac)^{\frac{5}{2}}}{b^2}\right)|c|}{15\,c^3}$$

```
integrate((c*x)^(1/2)*(b*x^2+a*x)^(1/2),x, algorithm="giac")
```

```
2/15*(2*sqrt(a*c)*a^2*c^2/b^2 - (5*(b*c*x + a*c)^(3/2)*a*c - 3*(b*c*x + a*
c)^(5/2))/b^2)*abs(c)/c^3
```

Mupad [B] (verification not implemented)

Time = 9.27 (sec) , antiderivative size = 42, normalized size of antiderivative = 0.68

$$\int \sqrt{cx}\sqrt{ax+bx^2}\,dx = \frac{2\sqrt{b\,x^2+a\,x}\,\sqrt{c\,x}\,\left(-2\,a^2+a\,b\,x+3\,b^2\,x^2\right)}{15\,b^2\,x}$$

```
int((a*x + b*x^2)^(1/2)*(c*x)^(1/2),x)
```

```
(2*(a*x + b*x^2)^(1/2)*(c*x)^(1/2)*(3*b^2*x^2 - 2*a^2 + a*b*x))/(15*b^2*x)
```

Reduce [B] (verification not implemented)

Time = 0.44 (sec) , antiderivative size = 31, normalized size of antiderivative = 0.50

$$\int \sqrt{cx}\sqrt{ax+bx^2}\,dx = \frac{2\sqrt{c}\,\sqrt{bx+a}\,(3b^2x^2+abx-2a^2)}{15b^2}$$

```
int((c*x)^(1/2)*(b*x^2+a*x)^(1/2),x)
```

```
(2*sqrt(c)*sqrt(a + b*x)*( - 2*a**2 + a*b*x + 3*b**2*x**2))/(15*b**2)
```

3.82 $\int \frac{\sqrt{ax+bx^2}}{\sqrt{cx}}\,dx$

Optimal result

Integrand size = 21, antiderivative size = 28

$$\int \frac{\sqrt{ax+bx^2}}{\sqrt{cx}}\,dx=\frac{2c(ax+bx^2)^{3/2}}{3b(cx)^{3/2}}$$

```
2/3*c*(b*x^2+a*x)^(3/2)/b/(c*x)^(3/2)
```

Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 26, normalized size of antiderivative = 0.93

$$\int \frac{\sqrt{ax+bx^2}}{\sqrt{cx}}\,dx=\frac{2c(x(a+bx))^{3/2}}{3b(cx)^{3/2}}$$

```
Integrate[Sqrt[a*x + b*x^2]/Sqrt[c*x],x]
```

```
(2*c*(x*(a + b*x))^(3/2))/(3*b*(c*x)^(3/2))
```

Rubi [A] (verified)

Time = 0.25 (sec) , antiderivative size = 28, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}} = 0.048$, Rules used = {1122}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sqrt{ax+bx^2}}{\sqrt{cx}}\,dx$$

$$\Big\downarrow \text{ 1122}$$

$$\frac{2c(ax+bx^2)^{3/2}}{3b(cx)^{3/2}}$$

```
Int[Sqrt[a*x + b*x^2]/Sqrt[c*x],x]
```

```
(2*c*(a*x + b*x^2)^(3/2))/(3*b*(c*x)^(3/2))
```

Defintions of rubi rules used

```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] :> Simp[e*(d + e*x)^(m - 1)*((a + b*x + c*x^2)^(p + 1)/(c*(p + 1))),
 x] /; FreeQ[{a, b, c, d, e, m, p}, x] && EqQ[c*d^2 - b*d*e + a*e^2, 0] &&
EqQ[m + p, 0]
```

Maple [A] (verified)

Time = 0.33 (sec) , antiderivative size = 25, normalized size of antiderivative = 0.89

method	result	size
default	$\frac{2\sqrt{x(bx+a)}\,(bx+a)}{3\sqrt{cx}\,b}$	25
gosper	$\frac{2(bx+a)\sqrt{b\,x^2+ax}}{3b\sqrt{cx}}$	27
orering	$\frac{2(bx+a)\sqrt{b\,x^2+ax}}{3b\sqrt{cx}}$	27
risch	$\frac{2x(bx+a)^2}{3\sqrt{cx}\,\sqrt{x(bx+a)}\,b}$	28

```
int((b*x^2+a*x)^(1/2)/(c*x)^(1/2),x,method=_RETURNVERBOSE)
```

```
2/3*(x*(b*x+a))^(1/2)/(c*x)^(1/2)*(b*x+a)/b
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 32, normalized size of antiderivative = 1.14

$$\int \frac{\sqrt{ax+bx^2}}{\sqrt{cx}}\,dx = \frac{2\sqrt{bx^2+ax}(bx+a)\sqrt{cx}}{3\,bcx}$$

```
integrate((b*x^2+a*x)^(1/2)/(c*x)^(1/2),x, algorithm="fricas")
```

```
2/3*sqrt(b*x^2 + a*x)*(b*x + a)*sqrt(c*x)/(b*c*x)
```

Sympy [F]

$$\int \frac{\sqrt{ax+bx^2}}{\sqrt{cx}}\,dx = \int \frac{\sqrt{x\,(a+bx)}}{\sqrt{cx}}\,dx$$

```
integrate((b*x**2+a*x)**(1/2)/(c*x)**(1/2),x)
```

```
Integral(sqrt(x*(a + b*x))/sqrt(c*x), x)
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 27, normalized size of antiderivative = 0.96

$$\int \frac{\sqrt{ax+bx^2}}{\sqrt{cx}}\,dx = \frac{2\left(b\sqrt{c}x + a\sqrt{c}\right)\sqrt{bx+a}}{3\,bc}$$

```
integrate((b*x^2+a*x)^(1/2)/(c*x)^(1/2),x, algorithm="maxima")
```

```
2/3*(b*sqrt(c)*x + a*sqrt(c))*sqrt(b*x + a)/(b*c)
```

Giac [A] (verification not implemented)

Time = 0.10 (sec) , antiderivative size = 34, normalized size of antiderivative = 1.21

$$\int \frac{\sqrt{ax+bx^2}}{\sqrt{cx}}\,dx = -\frac{2\left(\frac{\sqrt{ac}ac}{b} - \frac{(bcx+ac)^{\frac{3}{2}}}{b}\right)|c|}{3\,c^3}$$

```
integrate((b*x^2+a*x)^(1/2)/(c*x)^(1/2),x, algorithm="giac")
```

```
-2/3*(sqrt(a*c)*a*c/b - (b*c*x + a*c)^(3/2)/b)*abs(c)/c^3
```

Mupad [B] (verification not implemented)

Time = 9.60 (sec) , antiderivative size = 27, normalized size of antiderivative = 0.96

$$\int \frac{\sqrt{ax+bx^2}}{\sqrt{cx}}\,dx = \frac{\sqrt{b\,x^2+a\,x}\,\left(\frac{2\,x}{3}+\frac{2\,a}{3\,b}\right)}{\sqrt{c\,x}}$$

```
int((a*x + b*x^2)^(1/2)/(c*x)^(1/2),x)
```

```
((a*x + b*x^2)^(1/2)*((2*x)/3 + (2*a)/(3*b)))/(c*x)^(1/2)
```

Reduce [B] (verification not implemented)

Time = 0.25 (sec) , antiderivative size = 21, normalized size of antiderivative = 0.75

$$\int \frac{\sqrt{ax+bx^2}}{\sqrt{cx}}\,dx = \frac{2\sqrt{c}\,\sqrt{bx+a}\,(bx+a)}{3bc}$$

```
int((b*x^2+a*x)^(1/2)/(c*x)^(1/2),x)
```

```
(2*sqrt(c)*sqrt(a + b*x)*(a + b*x))/(3*b*c)
```

3.83 $\int \frac{\sqrt{ax+bx^2}}{(cx)^{3/2}}\,dx$

Optimal result

Integrand size = 21, antiderivative size = 70

$$\int \frac{\sqrt{ax+bx^2}}{(cx)^{3/2}}\,dx=\frac{2\sqrt{ax+bx^2}}{c\sqrt{cx}}-\frac{2\sqrt{a}\operatorname{arctanh}\left(\frac{\sqrt{c}\sqrt{ax+bx^2}}{\sqrt{a}\sqrt{cx}}\right)}{c^{3/2}}$$

```
2*(b*x^2+a*x)^(1/2)/c/(c*x)^(1/2)-2*a^(1/2)*arctanh(c^(1/2)*(b*x^2+a*x)^(1
/2)/a^(1/2)/(c*x)^(1/2))/c^(3/2)
```

Mathematica [A] (verified)

Time = 0.06 (sec) , antiderivative size = 62, normalized size of antiderivative = 0.89

$$\int \frac{\sqrt{ax+bx^2}}{(cx)^{3/2}}\,dx=\frac{2(x(a+bx))^{3/2}\left(\sqrt{a+bx}-\sqrt{a}\operatorname{arctanh}\left(\frac{\sqrt{a+bx}}{\sqrt{a}}\right)\right)}{(cx)^{3/2}(a+bx)^{3/2}}$$

```
Integrate[Sqrt[a*x + b*x^2]/(c*x)^(3/2),x]
```

```
(2*(x*(a + b*x))^(3/2)*(Sqrt[a + b*x] - Sqrt[a]*ArcTanh[Sqrt[a + b*x]/Sqrt
[a]]))/((c*x)^(3/2)*(a + b*x)^(3/2))
```

Rubi [A] (verified)

Time = 0.35 (sec) , antiderivative size = 70, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.143$, Rules used = {1131, 1136, 221}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sqrt{ax+bx^2}}{(cx)^{3/2}} \, dx$$

$$\Big\downarrow \text{1131}$$

$$\frac{a\int \frac{1}{\sqrt{cx}\sqrt{bx^2+ax}}dx}{c}+\frac{2\sqrt{ax+bx^2}}{c\sqrt{cx}}$$

$$\Big\downarrow \text{1136}$$

$$2a\int \frac{1}{\frac{c(bx^2+ax)}{x}-ac}d\frac{\sqrt{bx^2+ax}}{\sqrt{cx}}+\frac{2\sqrt{ax+bx^2}}{c\sqrt{cx}}$$

$$\Big\downarrow \text{221}$$

$$\frac{2\sqrt{ax+bx^2}}{c\sqrt{cx}}-\frac{2\sqrt{a}\text{arctanh}\left(\frac{\sqrt{c}\sqrt{ax+bx^2}}{\sqrt{a}\sqrt{cx}}\right)}{c^{3/2}}$$

```
Int[Sqrt[a*x + b*x^2]/(c*x)^(3/2),x]
```

```
(2*Sqrt[a*x + b*x^2])/(c*Sqrt[c*x]) - (2*Sqrt[a]*ArcTanh[(Sqrt[c]*Sqrt[a*x
 + b*x^2])/(Sqrt[a]*Sqrt[c*x])])/c^(3/2)
```

Defintions of rubi rules used

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] :> Simp[(d + e*x)^(m + 1)*((a + b*x + c*x^2)^p/(e*(m + 2*p + 1))), x
] - Simp[p*((2*c*d - b*e)/(e^2*(m + 2*p + 1)))   Int[(d + e*x)^(m + 1)*(a +
 b*x + c*x^2)^(p - 1), x], x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[c*d^2 - b
*d*e + a*e^2, 0] && GtQ[p, 0] && (LeQ[-2, m, 0] || EqQ[m + p + 1, 0]) && Ne
Q[m + 2*p + 1, 0] && IntegerQ[2*p]
```

```
Int[1/(Sqrt[(d_.) + (e_.)*(x_)]*Sqrt[(a_.) + (b_.)*(x_) + (c_.)*(x_)^2]), x
_Symbol] :> Simp[2*e   Subst[Int[1/(2*c*d - b*e + e^2*x^2), x], x, Sqrt[a +
 b*x + c*x^2]/Sqrt[d + e*x]], x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[c*d^2
- b*d*e + a*e^2, 0]
```

Maple [A] (verified)

Time = 0.33 (sec) , antiderivative size = 70, normalized size of antiderivative = 1.00

method	result	size
default	$-\frac{2\sqrt{x(bx+a)}\left(ac\operatorname{arctanh}\left(\frac{\sqrt{c(bx+a)}}{\sqrt{ac}}\right)-\sqrt{c(bx+a)}\sqrt{ac}\right)}{c\sqrt{cx}\sqrt{c(bx+a)}\sqrt{ac}}$	70

```
int((b*x^2+a*x)^(1/2)/(c*x)^(3/2),x,method=_RETURNVERBOSE)
```

```
-2*(x*(b*x+a))^(1/2)*(a*c*arctanh((c*(b*x+a))^(1/2)/(a*c)^(1/2))-(c*(b*x+a
))^(1/2)*(a*c)^(1/2))/c/(c*x)^(1/2)/(c*(b*x+a))^(1/2)/(a*c)^(1/2)
```

Fricas [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 147, normalized size of antiderivative = 2.10

$$\int \frac{\sqrt{ax+bx^2}}{(cx)^{3/2}}\,dx = \left[\frac{cx\sqrt{\frac{a}{c}}\log\left(-\frac{bx^2+2\,ax-2\,\sqrt{bx^2+ax}\sqrt{cx}\sqrt{\frac{a}{c}}}{x^2}\right)+2\,\sqrt{bx^2+ax}\sqrt{cx}}{c^2x}, \frac{2\left(cx\sqrt{-\frac{a}{c}}\arctan\left(\frac{\sqrt{bx^2}}{}\right.\right.}{}\right.$$

```
integrate((b*x^2+a*x)^(1/2)/(c*x)^(3/2),x, algorithm="fricas")
```

```
[(c*x*sqrt(a/c)*log(-(b*x^2 + 2*a*x - 2*sqrt(b*x^2 + a*x)*sqrt(c*x)*sqrt(a
/c))/x^2) + 2*sqrt(b*x^2 + a*x)*sqrt(c*x))/(c^2*x), 2*(c*x*sqrt(-a/c)*arct
an(sqrt(b*x^2 + a*x)*sqrt(c*x)*sqrt(-a/c)/(a*x)) + sqrt(b*x^2 + a*x)*sqrt(
c*x))/(c^2*x)]
```

Sympy [F]

$$\int \frac{\sqrt{ax+bx^2}}{(cx)^{3/2}}\,dx = \int \frac{\sqrt{x\,(a+bx)}}{(cx)^{\frac{3}{2}}}\,dx$$

```
integrate((b*x**2+a*x)**(1/2)/(c*x)**(3/2),x)
```

```
Integral(sqrt(x*(a + b*x))/(c*x)**(3/2), x)
```

Maxima [F]

$$\int \frac{\sqrt{ax+bx^2}}{(cx)^{3/2}}\,dx = \int \frac{\sqrt{bx^2+ax}}{(cx)^{\frac{3}{2}}}\,dx$$

```
integrate((b*x^2+a*x)^(1/2)/(c*x)^(3/2),x, algorithm="maxima")
```

```
integrate(sqrt(b*x^2 + a*x)/(c*x)^(3/2), x)
```

Giac [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 82, normalized size of antiderivative = 1.17

$$\int \frac{\sqrt{ax+bx^2}}{(cx)^{3/2}}\,dx = \frac{2\left(\frac{ac\arctan\left(\frac{\sqrt{bcx+ac}}{\sqrt{-ac}}\right)}{\sqrt{-ac}}+\sqrt{bcx+ac}-\frac{ac\arctan\left(\frac{\sqrt{ac}}{\sqrt{-ac}}\right)+\sqrt{ac}\sqrt{-ac}}{\sqrt{-ac}}\right)|c|}{c^3}$$

```
integrate((b*x^2+a*x)^(1/2)/(c*x)^(3/2),x, algorithm="giac")
```

```
2*(a*c*arctan(sqrt(b*c*x + a*c)/sqrt(-a*c))/sqrt(-a*c) + sqrt(b*c*x + a*c)
 - (a*c*arctan(sqrt(a*c)/sqrt(-a*c)) + sqrt(a*c)*sqrt(-a*c))/sqrt(-a*c))*a
bs(c)/c^3
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\sqrt{ax+bx^2}}{(cx)^{3/2}}\,dx = \int \frac{\sqrt{b\,x^2+a\,x}}{{\left(c\,x\right)}^{3/2}} \,d x$$

```
int((a*x + b*x^2)^(1/2)/(c*x)^(3/2),x)
```

```
int((a*x + b*x^2)^(1/2)/(c*x)^(3/2), x)
```

Reduce [B] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 44, normalized size of antiderivative = 0.63

$$\int \frac{\sqrt{ax+bx^2}}{(cx)^{3/2}}\,dx = \frac{\sqrt{c}\left(2\sqrt{bx+a}+\sqrt{a}\log\left(\sqrt{bx+a}-\sqrt{a}\right)-\sqrt{a}\log\left(\sqrt{bx+a}+\sqrt{a}\right)\right)}{c^2}$$

```
int((b*x^2+a*x)^(1/2)/(c*x)^(3/2),x)
```

```
(sqrt(c)*(2*sqrt(a + b*x) + sqrt(a)*log(sqrt(a + b*x) - sqrt(a)) - sqrt(a)
*log(sqrt(a + b*x) + sqrt(a))))/c**2
```

3.84 $\int \frac{\sqrt{ax+bx^2}}{(cx)^{5/2}} \, dx$

Optimal result

Integrand size = 21, antiderivative size = 71

$$\int \frac{\sqrt{ax+bx^2}}{(cx)^{5/2}} \, dx = -\frac{\sqrt{ax+bx^2}}{c(cx)^{3/2}} - \frac{b\operatorname{arctanh}\left(\frac{\sqrt{c}\sqrt{ax+bx^2}}{\sqrt{a}\sqrt{cx}}\right)}{\sqrt{a}c^{5/2}}$$

```
-(b*x^2+a*x)^(1/2)/c/(c*x)^(3/2)-b*arctanh(c^(1/2)*(b*x^2+a*x)^(1/2)/a^(1/
2)/(c*x)^(1/2))/a^(1/2)/c^(5/2)
```

Mathematica [A] (verified)

Time = 0.08 (sec) , antiderivative size = 69, normalized size of antiderivative = 0.97

$$\int \frac{\sqrt{ax+bx^2}}{(cx)^{5/2}} \, dx = -\frac{(x(a+bx))^{3/2}\left(\sqrt{a}\sqrt{a+bx} + bx\operatorname{arctanh}\left(\frac{\sqrt{a+bx}}{\sqrt{a}}\right)\right)}{\sqrt{a}(cx)^{5/2}(a+bx)^{3/2}}$$

```
Integrate[Sqrt[a*x + b*x^2]/(c*x)^(5/2),x]
```

```
-(((x*(a + b*x))^(3/2)*(Sqrt[a]*Sqrt[a + b*x] + b*x*ArcTanh[Sqrt[a + b*x]/
Sqrt[a]]))/(Sqrt[a]*(c*x)^(5/2)*(a + b*x)^(3/2)))
```

Rubi [A] (verified)

Time = 0.35 (sec) , antiderivative size = 71, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.143, Rules used = {1130, 1136, 221}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sqrt{ax+bx^2}}{(cx)^{5/2}}\,dx$$

$$\Big\downarrow \ 1130$$

$$\frac{b\int \frac{1}{\sqrt{cx}\sqrt{bx^2+ax}}dx}{2c^2}-\frac{\sqrt{ax+bx^2}}{c(cx)^{3/2}}$$

$$\Big\downarrow \ 1136$$

$$\frac{b\int \frac{1}{\frac{c\left(bx^2+ax\right)}{x}-ac}d\frac{\sqrt{bx^2+ax}}{\sqrt{cx}}}{c}-\frac{\sqrt{ax+bx^2}}{c(cx)^{3/2}}$$

$$\Big\downarrow \ 221$$

$$-\frac{b\text{arctanh}\left(\frac{\sqrt{c}\sqrt{ax+bx^2}}{\sqrt{a}\sqrt{cx}}\right)}{\sqrt{a}c^{5/2}}-\frac{\sqrt{ax+bx^2}}{c(cx)^{3/2}}$$

```
Int[Sqrt[a*x + b*x^2]/(c*x)^(5/2),x]
```

```
-(Sqrt[a*x + b*x^2]/(c*(c*x)^(3/2))) - (b*ArcTanh[(Sqrt[c]*Sqrt[a*x + b*x^
2])/(Sqrt[a]*Sqrt[c*x])])/(Sqrt[a]*c^(5/2))
```

Defintions of rubi rules used

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] :> Simp[(d + e*x)^(m + 1)*((a + b*x + c*x^2)^p/(e*(m + p + 1))), x]
- Simp[c*(p/(e^2*(m + p + 1)))   Int[(d + e*x)^(m + 2)*(a + b*x + c*x^2)^(p
 - 1), x], x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[c*d^2 - b*d*e + a*e^2, 0]
 && GtQ[p, 0] && (LtQ[m, -2] || EqQ[m + 2*p + 1, 0]) && NeQ[m + p + 1, 0] &
& IntegerQ[2*p]
```

```
Int[1/(Sqrt[(d_.) + (e_.)*(x_)]*Sqrt[(a_.) + (b_.)*(x_) + (c_.)*(x_)^2]), x
_Symbol] :> Simp[2*e   Subst[Int[1/(2*c*d - b*e + e^2*x^2), x], x, Sqrt[a +
 b*x + c*x^2]/Sqrt[d + e*x]], x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[c*d^2
- b*d*e + a*e^2, 0]
```

Maple [A] (verified)

Time = 0.34 (sec) , antiderivative size = 74, normalized size of antiderivative = 1.04

method	result	size
default	$\frac{\left(-\operatorname{arctanh}\left(\frac{\sqrt{c(bx+a)}}{\sqrt{ac}}\right)cbx-\sqrt{c(bx+a)}\,\sqrt{ac}\right)\sqrt{x(bx+a)}}{c^2x\sqrt{cx}\,\sqrt{c(bx+a)}\,\sqrt{ac}}$	74
risch	$-\frac{bx+a}{c^2\sqrt{cx}\,\sqrt{x(bx+a)}}-\frac{b\operatorname{arctanh}\left(\frac{\sqrt{cbx+ac}}{\sqrt{ac}}\right)\sqrt{c(bx+a)}\,x}{\sqrt{ac}\,c^2\sqrt{cx}\,\sqrt{x(bx+a)}}$	78

```
int((b*x^2+a*x)^(1/2)/(c*x)^(5/2),x,method=_RETURNVERBOSE)
```

```
(-arctanh((c*(b*x+a))^(1/2)/(a*c)^(1/2))*c*b*x-(c*(b*x+a))^(1/2)*(a*c)^(1/
2))*(x*(b*x+a))^(1/2)/c^2/x/(c*x)^(1/2)/(c*(b*x+a))^(1/2)/(a*c)^(1/2)
```

Fricas [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 157, normalized size of antiderivative = 2.21

$$\int \frac{\sqrt{ax+bx^2}}{(cx)^{5/2}}\,dx = \left[\frac{\sqrt{ac}bx^2\log\left(-\frac{bcx^2+2\,acx-2\,\sqrt{bx^2+ax}\sqrt{ac}\sqrt{cx}}{x^2}\right)-2\,\sqrt{bx^2+ax}\sqrt{cx}a}{2\,ac^3x^2}, \frac{\sqrt{-ac}bx^2\arctan\left(\frac{\sqrt{bx}}{}\right.}{}\right.$$

```
integrate((b*x^2+a*x)^(1/2)/(c*x)^(5/2),x, algorithm="fricas")
```

```
[1/2*(sqrt(a*c)*b*x^2*log(-(b*c*x^2 + 2*a*c*x - 2*sqrt(b*x^2 + a*x)*sqrt(a
*c)*sqrt(c*x))/x^2) - 2*sqrt(b*x^2 + a*x)*sqrt(c*x)*a)/(a*c^3*x^2), (sqrt(
-a*c)*b*x^2*arctan(sqrt(b*x^2 + a*x)*sqrt(-a*c)*sqrt(c*x)/(a*c*x)) - sqrt(
b*x^2 + a*x)*sqrt(c*x)*a)/(a*c^3*x^2)]
```

Sympy [F]

$$\int \frac{\sqrt{ax+bx^2}}{(cx)^{5/2}}\,dx = \int \frac{\sqrt{x\,(a+bx)}}{(cx)^{\frac{5}{2}}}\,dx$$

```
integrate((b*x**2+a*x)**(1/2)/(c*x)**(5/2),x)
```

```
Integral(sqrt(x*(a + b*x))/(c*x)**(5/2), x)
```

Maxima [F]

$$\int \frac{\sqrt{ax+bx^2}}{(cx)^{5/2}}\,dx = \int \frac{\sqrt{bx^2+ax}}{(cx)^{\frac{5}{2}}}\,dx$$

```
integrate((b*x^2+a*x)^(1/2)/(c*x)^(5/2),x, algorithm="maxima")
```

```
integrate(sqrt(b*x^2 + a*x)/(c*x)^(5/2), x)
```

Giac [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 54, normalized size of antiderivative = 0.76

$$\int \frac{\sqrt{ax+bx^2}}{(cx)^{5/2}}\,dx = \frac{b\left(\frac{\arctan\left(\frac{\sqrt{bcx+ac}}{\sqrt{-ac}}\right)}{\sqrt{-ac}} - \frac{\sqrt{bcx+ac}}{bcx}\right)|c|}{c^3}$$

```
integrate((b*x^2+a*x)^(1/2)/(c*x)^(5/2),x, algorithm="giac")
```

```
b*(arctan(sqrt(b*c*x + a*c)/sqrt(-a*c))/sqrt(-a*c) - sqrt(b*c*x + a*c)/(b*
c*x))*abs(c)/c^3
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\sqrt{ax+bx^2}}{(cx)^{5/2}}\,dx = \int \frac{\sqrt{b\,x^2+a\,x}}{{(c\,x)}^{5/2}} \,d x$$

```
int((a*x + b*x^2)^(1/2)/(c*x)^(5/2),x)
```

```
int((a*x + b*x^2)^(1/2)/(c*x)^(5/2), x)
```

Reduce [B] (verification not implemented)

Time = 0.48 (sec) , antiderivative size = 56, normalized size of antiderivative = 0.79

$$\int \frac{\sqrt{ax+bx^2}}{(cx)^{5/2}}\,dx = \frac{\sqrt{c}\left(-2\sqrt{bx+a}\,a+\sqrt{a}\log\left(\sqrt{bx+a}-\sqrt{a}\right)bx-\sqrt{a}\log\left(\sqrt{bx+a}+\sqrt{a}\right)bx\right)}{2a\,c^3x}$$

```
int((b*x^2+a*x)^(1/2)/(c*x)^(5/2),x)
```

```
(sqrt(c)*( - 2*sqrt(a + b*x)*a + sqrt(a)*log(sqrt(a + b*x) - sqrt(a))*b*x
- sqrt(a)*log(sqrt(a + b*x) + sqrt(a))*b*x))/(2*a*c**3*x)
```

3.85 $\int \frac{\sqrt{ax+bx^2}}{(cx)^{7/2}}\,dx$

Optimal result

Integrand size = 21, antiderivative size = 108

$$\int \frac{\sqrt{ax+bx^2}}{(cx)^{7/2}}\,dx=-\frac{\sqrt{ax+bx^2}}{2c(cx)^{5/2}}-\frac{b\sqrt{ax+bx^2}}{4ac^2(cx)^{3/2}}+\frac{b^2\text{arctanh}\left(\frac{\sqrt{c}\sqrt{ax+bx^2}}{\sqrt{a}\sqrt{cx}}\right)}{4a^{3/2}c^{7/2}}$$

```
-1/2*(b*x^2+a*x)^(1/2)/c/(c*x)^(5/2)-1/4*b*(b*x^2+a*x)^(1/2)/a/c^2/(c*x)^(
3/2)+1/4*b^2*arctanh(c^(1/2)*(b*x^2+a*x)^(1/2)/a^(1/2)/(c*x)^(1/2))/a^(3/2
)/c^(7/2)
```

Mathematica [A] (verified)

Time = 0.11 (sec) , antiderivative size = 83, normalized size of antiderivative = 0.77

$$\int \frac{\sqrt{ax+bx^2}}{(cx)^{7/2}}\,dx=\frac{(x(a+bx))^{3/2}\left(-\sqrt{a}\sqrt{a+bx}(2a+bx)+b^2x^2\text{arctanh}\left(\frac{\sqrt{a+bx}}{\sqrt{a}}\right)\right)}{4a^{3/2}(cx)^{7/2}(a+bx)^{3/2}}$$

```
Integrate[Sqrt[a*x + b*x^2]/(c*x)^(7/2),x]
```

```
((x*(a + b*x))^(3/2)*(-(Sqrt[a]*Sqrt[a + b*x]*(2*a + b*x)) + b^2*x^2*ArcTa
nh[Sqrt[a + b*x]/Sqrt[a]]))/(4*a^(3/2)*(c*x)^(7/2)*(a + b*x)^(3/2))
```

Rubi [A] (verified)

Time = 0.40 (sec) , antiderivative size = 106, normalized size of antiderivative = 0.98, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.190$, Rules used = {1130, 1135, 1136, 221}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sqrt{ax+bx^2}}{(cx)^{7/2}}dx$$

$$\Big\downarrow \text{1130}$$

$$\frac{b\int \frac{1}{(cx)^{3/2}\sqrt{bx^2+ax}}dx}{4c^2}-\frac{\sqrt{ax+bx^2}}{2c(cx)^{5/2}}$$

$$\Big\downarrow \text{1135}$$

$$\frac{b\left(-\frac{b\int \frac{1}{\sqrt{cx}\sqrt{bx^2+ax}}dx}{2ac}-\frac{\sqrt{ax+bx^2}}{a(cx)^{3/2}}\right)}{4c^2}-\frac{\sqrt{ax+bx^2}}{2c(cx)^{5/2}}$$

$$\Big\downarrow \text{1136}$$

$$\frac{b\left(-\frac{b\int \frac{1}{\frac{c\left(bx^2+ax\right)}{x}-ac}d\frac{\sqrt{bx^2+ax}}{\sqrt{cx}}}{a}-\frac{\sqrt{ax+bx^2}}{a(cx)^{3/2}}\right)}{4c^2}-\frac{\sqrt{ax+bx^2}}{2c(cx)^{5/2}}$$

$$\Big\downarrow \text{221}$$

$$\frac{b\left(\frac{b\,\text{arctanh}\left(\frac{\sqrt{c}\sqrt{ax+bx^2}}{\sqrt{a}\sqrt{cx}}\right)}{a^{3/2}c^{3/2}}-\frac{\sqrt{ax+bx^2}}{a(cx)^{3/2}}\right)}{4c^2}-\frac{\sqrt{ax+bx^2}}{2c(cx)^{5/2}}$$

```
Int[Sqrt[a*x + b*x^2]/(c*x)^(7/2),x]
```

```
-1/2*Sqrt[a*x + b*x^2]/(c*(c*x)^(5/2)) + (b*(-(Sqrt[a*x + b*x^2]/(a*(c*x)^
(3/2))) + (b*ArcTanh[(Sqrt[c]*Sqrt[a*x + b*x^2])/(Sqrt[a]*Sqrt[c*x])])/(a^
(3/2)*c^(3/2))))/(4*c^2)
```

Defintions of rubi rules used

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] :> Simp[(d + e*x)^(m + 1)*((a + b*x + c*x^2)^p/(e*(m + p + 1))), x]
- Simp[c*(p/(e^2*(m + p + 1)))   Int[(d + e*x)^(m + 2)*(a + b*x + c*x^2)^(p
 - 1), x], x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[c*d^2 - b*d*e + a*e^2, 0]
 && GtQ[p, 0] && (LtQ[m, -2] || EqQ[m + 2*p + 1, 0]) && NeQ[m + p + 1, 0] &
& IntegerQ[2*p]
```

```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] :> Simp[(-e)*(d + e*x)^m*((a + b*x + c*x^2)^(p + 1)/((m + p + 1)*(2*
c*d - b*e))), x] + Simp[c*((m + 2*p + 2)/((m + p + 1)*(2*c*d - b*e)))   Int
[(d + e*x)^(m + 1)*(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, p},
 x] && EqQ[c*d^2 - b*d*e + a*e^2, 0] && LtQ[m, 0] && NeQ[m + p + 1, 0] && I
ntegerQ[2*p]
```

```
Int[1/(Sqrt[(d_.) + (e_.)*(x_)]*Sqrt[(a_.) + (b_.)*(x_) + (c_.)*(x_)^2]), x
_Symbol] :> Simp[2*e   Subst[Int[1/(2*c*d - b*e + e^2*x^2), x], x, Sqrt[a +
 b*x + c*x^2]/Sqrt[d + e*x]], x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[c*d^2
- b*d*e + a*e^2, 0]
```

Maple [A] (verified)

Time = 0.35 (sec) , antiderivative size = 96, normalized size of antiderivative = 0.89

method	result	size
risch	$-\frac{(bx+a)(bx+2a)}{4xa\,c^{3}\sqrt{cx}\,\sqrt{x(bx+a)}}+\frac{b^{2}\operatorname{arctanh}\left(\frac{\sqrt{cbx+ac}}{\sqrt{ac}}\right)\sqrt{c(bx+a)}\,x}{4a\sqrt{ac}\,c^{3}\sqrt{cx}\,\sqrt{x(bx+a)}}$	96
default	$\frac{\sqrt{x(bx+a)}\left(\operatorname{arctanh}\left(\frac{\sqrt{c(bx+a)}}{\sqrt{ac}}\right)b^{2}c\,x^{2}-bx\sqrt{c(bx+a)}\,\sqrt{ac}-2\sqrt{c(bx+a)}\,\sqrt{ac}\,a\right)}{4c^{3}x^{2}\sqrt{cx}\,\sqrt{c(bx+a)}\,a\sqrt{ac}}$	100

```
int((b*x^2+a*x)^(1/2)/(c*x)^(7/2),x,method=_RETURNVERBOSE)
```

```
-1/4*(b*x+a)*(b*x+2*a)/x/a/c^3/(c*x)^(1/2)/(x*(b*x+a))^(1/2)+1/4/a*b^2/(a*
c)^(1/2)*arctanh((b*c*x+a*c)^(1/2)/(a*c)^(1/2))/c^3*(c*(b*x+a))^(1/2)/(c*x
)^(1/2)/(x*(b*x+a))^(1/2)*x
```

Fricas [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 179, normalized size of antiderivative = 1.66

$$\int \frac{\sqrt{ax+bx^2}}{(cx)^{7/2}}\,dx = \left[\frac{\sqrt{ac}b^2x^3\log\left(-\frac{bcx^2+2\,acx+2\,\sqrt{bx^2+ax}\sqrt{ac}\sqrt{cx}}{x^2}\right)-2\,(abx+2\,a^2)\sqrt{bx^2+ax}\sqrt{cx}}{8\,a^2c^4x^3},\right.$$
$$\left.-\frac{\sqrt{-ac}b^2x^3\arctan\left(\frac{\sqrt{bx^2+ax}\sqrt{-ac}\sqrt{cx}}{acx}\right)+(abx+2\,a^2)\sqrt{bx^2+ax}\sqrt{cx}}{4\,a^2c^4x^3}\right]$$

```
integrate((b*x^2+a*x)^(1/2)/(c*x)^(7/2),x, algorithm="fricas")
```

```
[1/8*(sqrt(a*c)*b^2*x^3*log(-(b*c*x^2 + 2*a*c*x + 2*sqrt(b*x^2 + a*x)*sqrt
(a*c)*sqrt(c*x))/x^2) - 2*(a*b*x + 2*a^2)*sqrt(b*x^2 + a*x)*sqrt(c*x))/(a^
2*c^4*x^3), -1/4*(sqrt(-a*c)*b^2*x^3*arctan(sqrt(b*x^2 + a*x)*sqrt(-a*c)*s
qrt(c*x)/(a*c*x)) + (a*b*x + 2*a^2)*sqrt(b*x^2 + a*x)*sqrt(c*x))/(a^2*c^4*
x^3)]
```

Sympy [F]

$$\int \frac{\sqrt{ax+bx^2}}{(cx)^{7/2}}\,dx = \int \frac{\sqrt{x\,(a+bx)}}{(cx)^{\frac{7}{2}}}\,dx$$

```
integrate((b*x**2+a*x)**(1/2)/(c*x)**(7/2),x)
```

```
Integral(sqrt(x*(a + b*x))/(c*x)**(7/2), x)
```

Maxima [F]

$$\int \frac{\sqrt{ax+bx^2}}{(cx)^{7/2}}\,dx = \int \frac{\sqrt{bx^2+ax}}{(cx)^{\frac{7}{2}}}\,dx$$

```
integrate((b*x^2+a*x)^(1/2)/(c*x)^(7/2),x, algorithm="maxima")
```

```
integrate(sqrt(b*x^2 + a*x)/(c*x)^(7/2), x)
```

Giac [A] (verification not implemented)

Time = 0.15 (sec) , antiderivative size = 94, normalized size of antiderivative = 0.87

$$\int \frac{\sqrt{ax+bx^2}}{(cx)^{7/2}}\,dx = -\frac{\left(\frac{b^3c^2\arctan\left(\frac{\sqrt{bcx+ac}}{\sqrt{-ac}}\right)}{\sqrt{-ac}a} + \frac{\sqrt{bcx+ac}ab^3c^3+(bcx+ac)^{\frac{3}{2}}b^3c^2}{ab^2c^2x^2}\right)|c|}{4\,bc^6}$$

```
integrate((b*x^2+a*x)^(1/2)/(c*x)^(7/2),x, algorithm="giac")
```

```
-1/4*(b^3*c^2*arctan(sqrt(b*c*x + a*c)/sqrt(-a*c))/(sqrt(-a*c)*a) + (sqrt(
b*c*x + a*c)*a*b^3*c^3 + (b*c*x + a*c)^(3/2)*b^3*c^2)/(a*b^2*c^2*x^2))*abs
(c)/(b*c^6)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\sqrt{ax+bx^2}}{(cx)^{7/2}}\,dx = \int \frac{\sqrt{b\,x^2+a\,x}}{{\left(c\,x\right)}^{7/2}}\,dx$$

```
int((a*x + b*x^2)^(1/2)/(c*x)^(7/2),x)
```

```
int((a*x + b*x^2)^(1/2)/(c*x)^(7/2), x)
```

Reduce [B] (verification not implemented)

Time = 0.56 (sec) , antiderivative size = 77, normalized size of antiderivative = 0.71

$$\int \frac{\sqrt{ax+bx^2}}{(cx)^{7/2}}\,dx = \frac{\sqrt{c}\left(-4\sqrt{bx+a}\,a^2-2\sqrt{bx+a}\,abx-\sqrt{a}\log\left(\sqrt{bx+a}-\sqrt{a}\right)b^2x^2+\sqrt{a}\log\left(\sqrt{bx+a}\right.\right.}{8a^2c^4x^2}$$

```
int((b*x^2+a*x)^(1/2)/(c*x)^(7/2),x)
```

```
(sqrt(c)*( - 4*sqrt(a + b*x)*a**2 - 2*sqrt(a + b*x)*a*b*x - sqrt(a)*log(sq
rt(a + b*x) - sqrt(a))*b**2*x**2 + sqrt(a)*log(sqrt(a + b*x) + sqrt(a))*b*
*2*x**2))/(8*a**2*c**4*x**2)
```

3.86 $\int \frac{\sqrt{ax+bx^2}}{(cx)^{9/2}}\,dx$

Optimal result

Integrand size = 21, antiderivative size = 141

$$\int \frac{\sqrt{ax+bx^2}}{(cx)^{9/2}}\,dx = -\frac{\sqrt{ax+bx^2}}{3c(cx)^{7/2}} - \frac{b\sqrt{ax+bx^2}}{12ac^2(cx)^{5/2}} + \frac{b^2\sqrt{ax+bx^2}}{8a^2c^3(cx)^{3/2}} - \frac{b^3\operatorname{arctanh}\left(\frac{\sqrt{c}\sqrt{ax+bx^2}}{\sqrt{a}\sqrt{cx}}\right)}{8a^{5/2}c^{9/2}}$$

```
-1/3*(b*x^2+a*x)^(1/2)/c/(c*x)^(7/2)-1/12*b*(b*x^2+a*x)^(1/2)/a/c^2/(c*x)^
(5/2)+1/8*b^2*(b*x^2+a*x)^(1/2)/a^2/c^3/(c*x)^(3/2)-1/8*b^3*arctanh(c^(1/2
)*(b*x^2+a*x)^(1/2)/a^(1/2)/(c*x)^(1/2))/a^(5/2)/c^(9/2)
```

Mathematica [A] (verified)

Time = 0.14 (sec) , antiderivative size = 101, normalized size of antiderivative = 0.72

$$\int \frac{\sqrt{ax+bx^2}}{(cx)^{9/2}}\,dx = -\frac{\sqrt{cx}\sqrt{x(a+bx)}\left(\sqrt{a}\sqrt{a+bx}(8a^2+2abx-3b^2x^2)+3b^3x^3\operatorname{arctanh}\left(\frac{\sqrt{a+bx}}{\sqrt{a}}\right)\right)}{24a^{5/2}c^5x^4\sqrt{a+bx}}$$

```
Integrate[Sqrt[a*x + b*x^2]/(c*x)^(9/2),x]
```

```
-1/24*(Sqrt[c*x]*Sqrt[x*(a + b*x)]*(Sqrt[a]*Sqrt[a + b*x]*(8*a^2 + 2*a*b*x
 - 3*b^2*x^2) + 3*b^3*x^3*ArcTanh[Sqrt[a + b*x]/Sqrt[a]]))/(a^(5/2)*c^5*x^
4*Sqrt[a + b*x])
```

Rubi [A] (verified)

Time = 0.46 (sec) , antiderivative size = 145, normalized size of antiderivative = 1.03, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.238$, Rules used = {1130, 1135, 1135, 1136, 221}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sqrt{ax+bx^2}}{(cx)^{9/2}}dx$$

$$\Big\downarrow \; 1130$$

$$\frac{b\int \frac{1}{(cx)^{5/2}\sqrt{bx^2+ax}}dx}{6c^2}-\frac{\sqrt{ax+bx^2}}{3c(cx)^{7/2}}$$

$$\Big\downarrow \; 1135$$

$$\frac{b\left(-\frac{3b\int \frac{1}{(cx)^{3/2}\sqrt{bx^2+ax}}dx}{4ac}-\frac{\sqrt{ax+bx^2}}{2a(cx)^{5/2}}\right)}{6c^2}-\frac{\sqrt{ax+bx^2}}{3c(cx)^{7/2}}$$

$$\Big\downarrow \; 1135$$

$$\frac{b\left(-\frac{3b\left(-\frac{b\int \frac{1}{\sqrt{cx}\sqrt{bx^2+ax}}dx}{2ac}-\frac{\sqrt{ax+bx^2}}{a(cx)^{3/2}}\right)}{4ac}-\frac{\sqrt{ax+bx^2}}{2a(cx)^{5/2}}\right)}{6c^2}-\frac{\sqrt{ax+bx^2}}{3c(cx)^{7/2}}$$

$$\Big\downarrow \; 1136$$

$$\frac{b\left(-\frac{3b\left(-\frac{b\int \frac{1}{\frac{c\left(bx^2+ax\right)}{x}-ac}d\frac{\sqrt{bx^2+ax}}{\sqrt{cx}}}{a}-\frac{\sqrt{ax+bx^2}}{a(cx)^{3/2}}\right)}{4ac}-\frac{\sqrt{ax+bx^2}}{2a(cx)^{5/2}}\right)}{6c^2}-\frac{\sqrt{ax+bx^2}}{3c(cx)^{7/2}}$$

$$\Big\downarrow \text{221}$$

$$\frac{b\left(-\frac{3b\left(\frac{b\,\text{arctanh}\left(\frac{\sqrt{c}\sqrt{ax+bx^2}}{\sqrt{a}\sqrt{cx}}\right)}{a^{3/2}c^{3/2}}-\frac{\sqrt{ax+bx^2}}{a(cx)^{3/2}}\right)}{4ac}-\frac{\sqrt{ax+bx^2}}{2a(cx)^{5/2}}\right)}{6c^2}-\frac{\sqrt{ax+bx^2}}{3c(cx)^{7/2}}$$

```
Int[Sqrt[a*x + b*x^2]/(c*x)^(9/2),x]
```

```
-1/3*Sqrt[a*x + b*x^2]/(c*(c*x)^(7/2)) + (b*(-1/2*Sqrt[a*x + b*x^2]/(a*(c*
x)^(5/2)) - (3*b*(-(Sqrt[a*x + b*x^2]/(a*(c*x)^(3/2))) + (b*ArcTanh[(Sqrt[
c]*Sqrt[a*x + b*x^2])/(Sqrt[a]*Sqrt[c*x])])/(a^(3/2)*c^(3/2))))/(4*a*c)))/
(6*c^2)
```

Defintions of rubi rules used

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] :> Simp[(d + e*x)^(m + 1)*((a + b*x + c*x^2)^p/(e*(m + p + 1))), x]
- Simp[c*(p/(e^2*(m + p + 1)))   Int[(d + e*x)^(m + 2)*(a + b*x + c*x^2)^(p
 - 1), x], x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[c*d^2 - b*d*e + a*e^2, 0]
 && GtQ[p, 0] && (LtQ[m, -2] || EqQ[m + 2*p + 1, 0]) && NeQ[m + p + 1, 0] &
& IntegerQ[2*p]
```

```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] :> Simp[(-e)*(d + e*x)^m*((a + b*x + c*x^2)^(p + 1)/((m + p + 1)*(2*
c*d - b*e))), x] + Simp[c*((m + 2*p + 2)/((m + p + 1)*(2*c*d - b*e)))   Int
[(d + e*x)^(m + 1)*(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, p},
 x] && EqQ[c*d^2 - b*d*e + a*e^2, 0] && LtQ[m, 0] && NeQ[m + p + 1, 0] && I
ntegerQ[2*p]
```

```
Int[1/(Sqrt[(d_.) + (e_.)*(x_)]*Sqrt[(a_.) + (b_.)*(x_) + (c_.)*(x_)^2]), x
_Symbol] :> Simp[2*e   Subst[Int[1/(2*c*d - b*e + e^2*x^2), x], x, Sqrt[a +
 b*x + c*x^2]/Sqrt[d + e*x]], x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[c*d^2
- b*d*e + a*e^2, 0]
```

Maple [A] (verified)

Time = 0.36 (sec) , antiderivative size = 108, normalized size of antiderivative = 0.77

method	result	size
risch	$-\frac{(bx+a)\left(-3b^{2}x^{2}+2abx+8a^{2}\right)}{24x^{2}a^{2}c^{4}\sqrt{cx}\,\sqrt{x(bx+a)}}-\frac{b^{3}\operatorname{arctanh}\left(\frac{\sqrt{cbx+ac}}{\sqrt{ac}}\right)\sqrt{c(bx+a)}\,x}{8a^{2}\sqrt{ac}\,c^{4}\sqrt{cx}\,\sqrt{x(bx+a)}}$	108
default	$-\frac{\sqrt{x(bx+a)}\left(3\operatorname{arctanh}\left(\frac{\sqrt{c(bx+a)}}{\sqrt{ac}}\right)c\,b^{3}x^{3}-3b^{2}x^{2}\sqrt{c(bx+a)}\,\sqrt{ac}+2abx\sqrt{c(bx+a)}\,\sqrt{ac}+8\sqrt{c(bx+a)}\,\sqrt{ac}\,a^{2}\right)}{24c^{4}x^{3}\sqrt{cx}\,\sqrt{c(bx+a)}\,a^{2}\sqrt{ac}}$	126

```
int((b*x^2+a*x)^(1/2)/(c*x)^(9/2),x,method=_RETURNVERBOSE)
```

```
-1/24*(b*x+a)*(-3*b^2*x^2+2*a*b*x+8*a^2)/x^2/a^2/c^4/(c*x)^(1/2)/(x*(b*x+a
))^(1/2)-1/8/a^2*b^3/(a*c)^(1/2)*arctanh((b*c*x+a*c)^(1/2)/(a*c)^(1/2))/c^
4*(c*(b*x+a))^(1/2)/(c*x)^(1/2)/(x*(b*x+a))^(1/2)*x
```

Fricas [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 205, normalized size of antiderivative = 1.45

$$\int \frac{\sqrt{ax+bx^2}}{(cx)^{9/2}}\,dx = \left[\frac{3\sqrt{ac}b^3x^4\log\left(-\frac{bcx^2+2\,acx-2\,\sqrt{bx^2+ax}\sqrt{ac}\sqrt{cx}}{x^2}\right)+2\left(3\,ab^2x^2-2\,a^2bx-8\,a^3\right)\sqrt{bx^2+ax}\sqrt{}}{48\,a^3c^5x^4}\right.$$

```
integrate((b*x^2+a*x)^(1/2)/(c*x)^(9/2),x, algorithm="fricas")
```

```
[1/48*(3*sqrt(a*c)*b^3*x^4*log(-(b*c*x^2 + 2*a*c*x - 2*sqrt(b*x^2 + a*x)*s
qrt(a*c)*sqrt(c*x))/x^2) + 2*(3*a*b^2*x^2 - 2*a^2*b*x - 8*a^3)*sqrt(b*x^2
+ a*x)*sqrt(c*x))/(a^3*c^5*x^4), 1/24*(3*sqrt(-a*c)*b^3*x^4*arctan(sqrt(b*
x^2 + a*x)*sqrt(-a*c)*sqrt(c*x)/(a*c*x)) + (3*a*b^2*x^2 - 2*a^2*b*x - 8*a^
3)*sqrt(b*x^2 + a*x)*sqrt(c*x))/(a^3*c^5*x^4)]
```

Sympy [F]

$$\int \frac{\sqrt{ax+bx^2}}{(cx)^{9/2}}\,dx = \int \frac{\sqrt{x\,(a+bx)}}{(cx)^{\frac{9}{2}}}\,dx$$

```
integrate((b*x**2+a*x)**(1/2)/(c*x)**(9/2),x)
```

```
Integral(sqrt(x*(a + b*x))/(c*x)**(9/2), x)
```

Maxima [F]

$$\int \frac{\sqrt{ax+bx^2}}{(cx)^{9/2}}\,dx = \int \frac{\sqrt{bx^2+ax}}{(cx)^{\frac{9}{2}}}\,dx$$

```
integrate((b*x^2+a*x)^(1/2)/(c*x)^(9/2),x, algorithm="maxima")
```

```
integrate(sqrt(b*x^2 + a*x)/(c*x)^(9/2), x)
```

Giac [A] (verification not implemented)

Time = 0.15 (sec) , antiderivative size = 102, normalized size of antiderivative = 0.72

$$\int \frac{\sqrt{ax+bx^2}}{(cx)^{9/2}}\,dx = \frac{b^3\left(\frac{3\arctan\left(\frac{\sqrt{bcx+ac}}{\sqrt{-ac}}\right)}{\sqrt{-ac}a^2c^2} - \frac{3\sqrt{bcx+ac}a^2c^2+8\,(bcx+ac)^{\frac{3}{2}}ac-3\,(bcx+ac)^{\frac{5}{2}}}{a^2b^3c^5x^3}\right)|c|}{24\,c^3}$$

```
integrate((b*x^2+a*x)^(1/2)/(c*x)^(9/2),x, algorithm="giac")
```

```
1/24*b^3*(3*arctan(sqrt(b*c*x + a*c)/sqrt(-a*c))/(sqrt(-a*c)*a^2*c^2) - (3
*sqrt(b*c*x + a*c)*a^2*c^2 + 8*(b*c*x + a*c)^(3/2)*a*c - 3*(b*c*x + a*c)^(
5/2))/(a^2*b^3*c^5*x^3))*abs(c)/c^3
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\sqrt{ax+bx^2}}{(cx)^{9/2}}\,dx = \int \frac{\sqrt{b\,x^2+a\,x}}{{\left(c\,x\right)}^{9/2}} \,d x$$

```
int((a*x + b*x^2)^(1/2)/(c*x)^(9/2),x)
```

```
int((a*x + b*x^2)^(1/2)/(c*x)^(9/2), x)
```

Reduce [B] (verification not implemented)

Time = 0.24 (sec) , antiderivative size = 95, normalized size of antiderivative = 0.67

$$\int \frac{\sqrt{ax+bx^2}}{(cx)^{9/2}}\,dx = \frac{\sqrt{c}\left(-16\sqrt{bx+a}\,a^3-4\sqrt{bx+a}\,a^2bx+6\sqrt{bx+a}\,a\,b^2x^2+3\sqrt{a}\log\left(\sqrt{bx+a}-\sqrt{a}\right)b^3}{48a^3c^5x^3}$$

```
int((b*x^2+a*x)^(1/2)/(c*x)^(9/2),x)
```

```
(sqrt(c)*( - 16*sqrt(a + b*x)*a**3 - 4*sqrt(a + b*x)*a**2*b*x + 6*sqrt(a +
 b*x)*a*b**2*x**2 + 3*sqrt(a)*log(sqrt(a + b*x) - sqrt(a))*b**3*x**3 - 3*s
qrt(a)*log(sqrt(a + b*x) + sqrt(a))*b**3*x**3))/(48*a**3*c**5*x**3)
```

3.87 $\int \frac{\sqrt{ax+bx^2}}{(cx)^{11/2}}\,dx$

Optimal result

Integrand size = 21, antiderivative size = 174

$$\int \frac{\sqrt{ax+bx^2}}{(cx)^{11/2}}\,dx = -\frac{\sqrt{ax+bx^2}}{4c(cx)^{9/2}} - \frac{b\sqrt{ax+bx^2}}{24ac^2(cx)^{7/2}} + \frac{5b^2\sqrt{ax+bx^2}}{96a^2c^3(cx)^{5/2}} - \frac{5b^3\sqrt{ax+bx^2}}{64a^3c^4(cx)^{3/2}} + \frac{5b^4\text{arctanh}\left(\frac{\sqrt{c}\sqrt{ax+bx^2}}{\sqrt{a}\sqrt{cx}}\right)}{64a^{7/2}c^{11/2}}$$

```
-1/4*(b*x^2+a*x)^(1/2)/c/(c*x)^(9/2)-1/24*b*(b*x^2+a*x)^(1/2)/a/c^2/(c*x)^
(7/2)+5/96*b^2*(b*x^2+a*x)^(1/2)/a^2/c^3/(c*x)^(5/2)-5/64*b^3*(b*x^2+a*x)^
(1/2)/a^3/c^4/(c*x)^(3/2)+5/64*b^4*arctanh(c^(1/2)*(b*x^2+a*x)^(1/2)/a^(1/
2)/(c*x)^(1/2))/a^(7/2)/c^(11/2)
```

Mathematica [A] (verified)

Time = 0.15 (sec) , antiderivative size = 113, normalized size of antiderivative = 0.65

$$\int \frac{\sqrt{ax+bx^2}}{(cx)^{11/2}}\,dx = \frac{\sqrt{cx}\sqrt{x(a+bx)}\left(-\sqrt{a}\sqrt{a+bx}(48a^3+8a^2bx-10ab^2x^2+15b^3x^3)+15b^4x^4\text{arctanh}\left(\right.\right.}{192a^{7/2}c^6x^5\sqrt{a+bx}}$$

```
Integrate[Sqrt[a*x + b*x^2]/(c*x)^(11/2),x]
```

```
(Sqrt[c*x]*Sqrt[x*(a + b*x)]*(-(Sqrt[a]*Sqrt[a + b*x]*(48*a^3 + 8*a^2*b*x
- 10*a*b^2*x^2 + 15*b^3*x^3)) + 15*b^4*x^4*ArcTanh[Sqrt[a + b*x]/Sqrt[a]])
)/(192*a^(7/2)*c^6*x^5*Sqrt[a + b*x])
```

Rubi [A] (verified)

Time = 0.55 (sec) , antiderivative size = 184, normalized size of antiderivative = 1.06, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.286$, Rules used = {1130, 1135, 1135, 1135, 1136, 221}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sqrt{ax+bx^2}}{(cx)^{11/2}}dx$$

$$\Big\downarrow \text{1130}$$

$$\frac{b\int \frac{1}{(cx)^{7/2}\sqrt{bx^2+ax}}dx}{8c^2}-\frac{\sqrt{ax+bx^2}}{4c(cx)^{9/2}}$$

$$\Big\downarrow \text{1135}$$

$$\frac{b\left(-\frac{5b\int \frac{1}{(cx)^{5/2}\sqrt{bx^2+ax}}dx}{6ac}-\frac{\sqrt{ax+bx^2}}{3a(cx)^{7/2}}\right)}{8c^2}-\frac{\sqrt{ax+bx^2}}{4c(cx)^{9/2}}$$

$$\Big\downarrow \text{1135}$$

$$\frac{b\left(-\frac{5b\left(-\frac{3b\int \frac{1}{(cx)^{3/2}\sqrt{bx^2+ax}}dx}{4ac}-\frac{\sqrt{ax+bx^2}}{2a(cx)^{5/2}}\right)}{6ac}-\frac{\sqrt{ax+bx^2}}{3a(cx)^{7/2}}\right)}{8c^2}-\frac{\sqrt{ax+bx^2}}{4c(cx)^{9/2}}$$

$$\Big\downarrow \text{1135}$$

$$\frac{b\left(-\frac{5b\left(-\frac{3b\left(-\frac{b\int \frac{1}{\sqrt{cx}\sqrt{bx^2+ax}}dx}{2ac}-\frac{\sqrt{ax+bx^2}}{a(cx)^{3/2}}\right)}{4ac}-\frac{\sqrt{ax+bx^2}}{2a(cx)^{5/2}}\right)}{6ac}-\frac{\sqrt{ax+bx^2}}{3a(cx)^{7/2}}\right)}{8c^2}-\frac{\sqrt{ax+bx^2}}{4c(cx)^{9/2}}$$

$$\Big\downarrow \ 1136$$

$$\frac{b\left(-\frac{5b\left(-\frac{3b\left(-\frac{b\int \frac{1}{\frac{c\left(bx^2+ax\right)}{x}-ac}d\frac{\sqrt{bx^2+ax}}{\sqrt{cx}}}{a}-\frac{\sqrt{ax+bx^2}}{a(cx)^{3/2}}\right)}{4ac}-\frac{\sqrt{ax+bx^2}}{2a(cx)^{5/2}}\right)}{6ac}-\frac{\sqrt{ax+bx^2}}{3a(cx)^{7/2}}\right)}{8c^2}-\frac{\sqrt{ax+bx^2}}{4c(cx)^{9/2}}$$

$$\Big\downarrow \ 221$$

$$\frac{b\left(-\frac{5b\left(-\frac{3b\left(\frac{b\,\text{arctanh}\left(\frac{\sqrt{c}\sqrt{ax+bx^2}}{\sqrt{a}\sqrt{cx}}\right)}{a^{3/2}c^{3/2}}-\frac{\sqrt{ax+bx^2}}{a(cx)^{3/2}}\right)}{4ac}-\frac{\sqrt{ax+bx^2}}{2a(cx)^{5/2}}\right)}{6ac}-\frac{\sqrt{ax+bx^2}}{3a(cx)^{7/2}}\right)}{8c^2}-\frac{\sqrt{ax+bx^2}}{4c(cx)^{9/2}}$$

```
Int[Sqrt[a*x + b*x^2]/(c*x)^(11/2),x]
```

```
-1/4*Sqrt[a*x + b*x^2]/(c*(c*x)^(9/2)) + (b*(-1/3*Sqrt[a*x + b*x^2]/(a*(c*
x)^(7/2)) - (5*b*(-1/2*Sqrt[a*x + b*x^2]/(a*(c*x)^(5/2)) - (3*b*(-(Sqrt[a*
x + b*x^2]/(a*(c*x)^(3/2))) + (b*ArcTanh[(Sqrt[c]*Sqrt[a*x + b*x^2])/(Sqrt
[a]*Sqrt[c*x])])/(a^(3/2)*c^(3/2))))/(4*a*c)))/(6*a*c)))/(8*c^2)
```

Defintions of rubi rules used

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] :> Simp[(d + e*x)^(m + 1)*((a + b*x + c*x^2)^p/(e*(m + p + 1))), x]
- Simp[c*(p/(e^2*(m + p + 1)))   Int[(d + e*x)^(m + 2)*(a + b*x + c*x^2)^(p
 - 1), x], x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[c*d^2 - b*d*e + a*e^2, 0]
 && GtQ[p, 0] && (LtQ[m, -2] || EqQ[m + 2*p + 1, 0]) && NeQ[m + p + 1, 0] &
& IntegerQ[2*p]
```

```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] :> Simp[(-e)*(d + e*x)^m*((a + b*x + c*x^2)^(p + 1)/((m + p + 1)*(2*
c*d - b*e))), x] + Simp[c*((m + 2*p + 2)/((m + p + 1)*(2*c*d - b*e)))   Int
[(d + e*x)^(m + 1)*(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, p},
 x] && EqQ[c*d^2 - b*d*e + a*e^2, 0] && LtQ[m, 0] && NeQ[m + p + 1, 0] && I
ntegerQ[2*p]
```

```
Int[1/(Sqrt[(d_.) + (e_.)*(x_)]*Sqrt[(a_.) + (b_.)*(x_) + (c_.)*(x_)^2]), x
_Symbol] :> Simp[2*e   Subst[Int[1/(2*c*d - b*e + e^2*x^2), x], x, Sqrt[a +
 b*x + c*x^2]/Sqrt[d + e*x]], x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[c*d^2
- b*d*e + a*e^2, 0]
```

Maple [A] (verified)

Time = 0.40 (sec) , antiderivative size = 119, normalized size of antiderivative = 0.68

method	result
risch	$-\frac{(bx+a)\left(15b^3x^3-10a\,b^2x^2+8a^2bx+48a^3\right)}{192x^3a^3c^5\sqrt{cx}\,\sqrt{x(bx+a)}}+\frac{5b^4\operatorname{arctanh}\left(\frac{\sqrt{cbx+ac}}{\sqrt{ac}}\right)\sqrt{c(bx+a)}\,x}{64a^3\sqrt{ac}\,c^5\sqrt{cx}\,\sqrt{x(bx+a)}}$
default	$\frac{\sqrt{x(bx+a)}\left(15\operatorname{arctanh}\left(\frac{\sqrt{c(bx+a)}}{\sqrt{ac}}\right)c\,b^4x^4-15b^3x^3\sqrt{c(bx+a)}\,\sqrt{ac}+10a\,b^2x^2\sqrt{c(bx+a)}\,\sqrt{ac}-8a^2bx\sqrt{c(bx+a)}\,\sqrt{ac}-48\sqrt{c(bx+a}\right.}{192c^5x^4\sqrt{cx}\,\sqrt{c(bx+a)}\,a^3\sqrt{ac}}$

```
int((b*x^2+a*x)^(1/2)/(c*x)^(11/2),x,method=_RETURNVERBOSE)
```

```
-1/192*(b*x+a)*(15*b^3*x^3-10*a*b^2*x^2+8*a^2*b*x+48*a^3)/x^3/a^3/c^5/(c*x
)^(1/2)/(x*(b*x+a))^(1/2)+5/64*b^4/a^3/(a*c)^(1/2)*arctanh((b*c*x+a*c)^(1/
2)/(a*c)^(1/2))/c^5*(c*(b*x+a))^(1/2)/(c*x)^(1/2)/(x*(b*x+a))^(1/2)*x
```

Fricas [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 227, normalized size of antiderivative = 1.30

$$\int \frac{\sqrt{ax+bx^2}}{(cx)^{11/2}}\,dx = \left[\frac{15\,\sqrt{ac}b^4x^5\log\left(-\frac{bcx^2+2\,acx+2\,\sqrt{bx^2+ax}\sqrt{ac}\sqrt{cx}}{x^2}\right)-2\,(15\,ab^3x^3-10\,a^2b^2x^2+8\,a^3bx+48}{384\,a^4c^6x^5}\right.$$
$$\left.-\frac{15\,\sqrt{-ac}b^4x^5\arctan\left(\frac{\sqrt{bx^2+ax}\sqrt{-ac}\sqrt{cx}}{acx}\right)+(15\,ab^3x^3-10\,a^2b^2x^2+8\,a^3bx+48\,a^4)\sqrt{bx^2+ax}\sqrt{cx}}{192\,a^4c^6x^5}\right]$$

```
integrate((b*x^2+a*x)^(1/2)/(c*x)^(11/2),x, algorithm="fricas")
```

```
[1/384*(15*sqrt(a*c)*b^4*x^5*log(-(b*c*x^2 + 2*a*c*x + 2*sqrt(b*x^2 + a*x)
*sqrt(a*c)*sqrt(c*x))/x^2) - 2*(15*a*b^3*x^3 - 10*a^2*b^2*x^2 + 8*a^3*b*x
+ 48*a^4)*sqrt(b*x^2 + a*x)*sqrt(c*x))/(a^4*c^6*x^5), -1/192*(15*sqrt(-a*c
)*b^4*x^5*arctan(sqrt(b*x^2 + a*x)*sqrt(-a*c)*sqrt(c*x)/(a*c*x)) + (15*a*b
^3*x^3 - 10*a^2*b^2*x^2 + 8*a^3*b*x + 48*a^4)*sqrt(b*x^2 + a*x)*sqrt(c*x))
/(a^4*c^6*x^5)]
```

Sympy [F]

$$\int \frac{\sqrt{ax+bx^2}}{(cx)^{11/2}}\,dx = \int \frac{\sqrt{x\,(a+bx)}}{(cx)^{\frac{11}{2}}}\,dx$$

```
integrate((b*x**2+a*x)**(1/2)/(c*x)**(11/2),x)
```

```
Integral(sqrt(x*(a + b*x))/(c*x)**(11/2), x)
```

Maxima [F]

$$\int \frac{\sqrt{ax+bx^2}}{(cx)^{11/2}}\,dx = \int \frac{\sqrt{bx^2+ax}}{(cx)^{\frac{11}{2}}}\,dx$$

```
integrate((b*x^2+a*x)^(1/2)/(c*x)^(11/2),x, algorithm="maxima")
```

```
integrate(sqrt(b*x^2 + a*x)/(c*x)^(11/2), x)
```

Giac [A] (verification not implemented)

Time = 0.31 (sec) , antiderivative size = 139, normalized size of antiderivative = 0.80

$$\int \frac{\sqrt{ax+bx^2}}{(cx)^{11/2}}\,dx = -\frac{\left(\frac{15\,b^5c^2\arctan\left(\frac{\sqrt{bcx+ac}}{\sqrt{-ac}}\right)}{\sqrt{-ac}a^3} + \frac{15\,\sqrt{bcx+ac}a^3b^5c^5+73\,(bcx+ac)^{\frac{3}{2}}\,a^2b^5c^4-55\,(bcx+ac)^{\frac{5}{2}}\,ab^5c^3+15\,(bcx+ac)^{\frac{7}{2}}\,b^5c^2}{a^3b^4c^4x^4}\right)|c|}{192\,bc^8}$$

```
integrate((b*x^2+a*x)^(1/2)/(c*x)^(11/2),x, algorithm="giac")
```

```
-1/192*(15*b^5*c^2*arctan(sqrt(b*c*x + a*c)/sqrt(-a*c))/(sqrt(-a*c)*a^3) +
 (15*sqrt(b*c*x + a*c)*a^3*b^5*c^5 + 73*(b*c*x + a*c)^(3/2)*a^2*b^5*c^4 -
55*(b*c*x + a*c)^(5/2)*a*b^5*c^3 + 15*(b*c*x + a*c)^(7/2)*b^5*c^2)/(a^3*b^
4*c^4*x^4))*abs(c)/(b*c^8)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\sqrt{ax+bx^2}}{(cx)^{11/2}}\,dx = \int \frac{\sqrt{b\,x^2+a\,x}}{(c\,x)^{11/2}}\,dx$$

```
int((a*x + b*x^2)^(1/2)/(c*x)^(11/2),x)
```

```
int((a*x + b*x^2)^(1/2)/(c*x)^(11/2), x)
```

Reduce [B] (verification not implemented)

Time = 0.30 (sec) , antiderivative size = 112, normalized size of antiderivative = 0.64

$$\int \frac{\sqrt{ax+bx^2}}{(cx)^{11/2}}\,dx = \frac{\sqrt{c}\left(-96\sqrt{bx+a}\,a^4-16\sqrt{bx+a}\,a^3bx+20\sqrt{bx+a}\,a^2b^2x^2-30\sqrt{bx+a}\,ab^3x^3-15\sqrt{}\right.}{384a^4c^6x^4}$$

```
int((b*x^2+a*x)^(1/2)/(c*x)^(11/2),x)
```

```
(sqrt(c)*( - 96*sqrt(a + b*x)*a**4 - 16*sqrt(a + b*x)*a**3*b*x + 20*sqrt(a
 + b*x)*a**2*b**2*x**2 - 30*sqrt(a + b*x)*a*b**3*x**3 - 15*sqrt(a)*log(sqr
t(a + b*x) - sqrt(a))*b**4*x**4 + 15*sqrt(a)*log(sqrt(a + b*x) + sqrt(a))*
b**4*x**4))/(384*a**4*c**6*x**4)
```

3.88 $\int (cx)^{3/2} (ax + bx^2)^{3/2} \, dx$

Optimal result

Integrand size = 21, antiderivative size = 128

$$\int (cx)^{3/2} (ax + bx^2)^{3/2} \, dx = -\frac{2a^3c^4(ax+bx^2)^{5/2}}{5b^4(cx)^{5/2}} + \frac{6a^2c^5(ax+bx^2)^{7/2}}{7b^4(cx)^{7/2}} - \frac{2ac^6(ax+bx^2)^{9/2}}{3b^4(cx)^{9/2}} + \frac{2c^7(ax+bx^2)^{11/2}}{11b^4(cx)^{11/2}}$$

```
-2/5*a^3*c^4*(b*x^2+a*x)^(5/2)/b^4/(c*x)^(5/2)+6/7*a^2*c^5*(b*x^2+a*x)^(7/
2)/b^4/(c*x)^(7/2)-2/3*a*c^6*(b*x^2+a*x)^(9/2)/b^4/(c*x)^(9/2)+2/11*c^7*(b
*x^2+a*x)^(11/2)/b^4/(c*x)^(11/2)
```

Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 65, normalized size of antiderivative = 0.51

$$\int (cx)^{3/2} (ax + bx^2)^{3/2} \, dx = \frac{2c^2(a+bx)^2\sqrt{x(a+bx)}(-16a^3+40a^2bx-70ab^2x^2+105b^3x^3)}{1155b^4\sqrt{cx}}$$

```
Integrate[(c*x)^(3/2)*(a*x + b*x^2)^(3/2),x]
```

```
(2*c^2*(a + b*x)^2*Sqrt[x*(a + b*x)]*(-16*a^3 + 40*a^2*b*x - 70*a*b^2*x^2
+ 105*b^3*x^3))/(1155*b^4*Sqrt[c*x])
```

Rubi [A] (verified)

Time = 0.41 (sec) , antiderivative size = 136, normalized size of antiderivative = 1.06, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.190, Rules used = {1128, 1128, 1128, 1122}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (cx)^{3/2} \left(ax+bx^2\right)^{3/2} dx$$

$$\Big\downarrow \ 1128$$

$$\frac{2c\sqrt{cx}\left(ax+bx^2\right)^{5/2}}{11b}-\frac{6ac\int \sqrt{cx}\left(bx^2+ax\right)^{3/2}dx}{11b}$$

$$\Big\downarrow \ 1128$$

$$\frac{2c\sqrt{cx}\left(ax+bx^2\right)^{5/2}}{11b}-\frac{6ac\left(\frac{2c\left(ax+bx^2\right)^{5/2}}{9b\sqrt{cx}}-\frac{4ac\int \frac{\left(bx^2+ax\right)^{3/2}}{\sqrt{cx}}dx}{9b}\right)}{11b}$$

$$\Big\downarrow \ 1128$$

$$\frac{2c\sqrt{cx}\left(ax+bx^2\right)^{5/2}}{11b}-\frac{6ac\left(\frac{2c\left(ax+bx^2\right)^{5/2}}{9b\sqrt{cx}}-\frac{4ac\left(\frac{2c\left(ax+bx^2\right)^{5/2}}{7b(cx)^{3/2}}-\frac{2ac\int \frac{\left(bx^2+ax\right)^{3/2}}{(cx)^{3/2}}dx}{7b}\right)}{9b}\right)}{11b}$$

$$\Big\downarrow \ 1122$$

$$\frac{2c\sqrt{cx}\left(ax+bx^2\right)^{5/2}}{11b}-\frac{6ac\left(\frac{2c\left(ax+bx^2\right)^{5/2}}{9b\sqrt{cx}}-\frac{4ac\left(\frac{2c\left(ax+bx^2\right)^{5/2}}{7b(cx)^{3/2}}-\frac{4ac^2\left(ax+bx^2\right)^{5/2}}{35b^2(cx)^{5/2}}\right)}{9b}\right)}{11b}$$

```
Int[(c*x)^(3/2)*(a*x + b*x^2)^(3/2),x]
```

```
(2*c*Sqrt[c*x]*(a*x + b*x^2)^(5/2))/(11*b) - (6*a*c*((2*c*(a*x + b*x^2)^(5
/2))/(9*b*Sqrt[c*x]) - (4*a*c*((-4*a*c^2*(a*x + b*x^2)^(5/2))/(35*b^2*(c*x
)^(5/2)) + (2*c*(a*x + b*x^2)^(5/2))/(7*b*(c*x)^(3/2))))/(9*b)))/(11*b)
```

Defintions of rubi rules used

```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] :> Simp[e*(d + e*x)^(m - 1)*((a + b*x + c*x^2)^(p + 1)/(c*(p + 1))),
 x] /; FreeQ[{a, b, c, d, e, m, p}, x] && EqQ[c*d^2 - b*d*e + a*e^2, 0] &&
EqQ[m + p, 0]
```

```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] :> Simp[e*(d + e*x)^(m - 1)*((a + b*x + c*x^2)^(p + 1)/(c*(m + 2*p +
 1))), x] + Simp[Simplify[m + p]*((2*c*d - b*e)/(c*(m + 2*p + 1)))   Int[(d
 + e*x)^(m - 1)*(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, m, p},
 x] && EqQ[c*d^2 - b*d*e + a*e^2, 0] && IGtQ[Simplify[m + p], 0]
```

Maple [A] (verified)

Time = 0.33 (sec) , antiderivative size = 60, normalized size of antiderivative = 0.47

method	result	size
gosper	$-\frac{2(bx+a)\left(-105b^{3}x^{3}+70a\,b^{2}x^{2}-40a^{2}bx+16a^{3}\right)(cx)^{\frac{3}{2}}\left(b\,x^{2}+ax\right)^{\frac{3}{2}}}{1155b^{4}x^{3}}$	60
orering	$-\frac{2(bx+a)\left(-105b^{3}x^{3}+70a\,b^{2}x^{2}-40a^{2}bx+16a^{3}\right)(cx)^{\frac{3}{2}}\left(b\,x^{2}+ax\right)^{\frac{3}{2}}}{1155b^{4}x^{3}}$	60
default	$-\frac{2c\sqrt{cx}\,\sqrt{x(bx+a)}\,(bx+a)^{2}\left(-105b^{3}x^{3}+70a\,b^{2}x^{2}-40a^{2}bx+16a^{3}\right)}{1155x\,b^{4}}$	61
risch	$-\frac{2c^{2}x\left(-105b^{5}x^{5}-140a\,b^{4}x^{4}-5a^{2}b^{3}x^{3}+6a^{3}b^{2}x^{2}-8a^{4}bx+16a^{5}\right)(bx+a)}{1155\sqrt{cx}\,\sqrt{x(bx+a)}\,b^{4}}$	81

```
int((c*x)^(3/2)*(b*x^2+a*x)^(3/2),x,method=_RETURNVERBOSE)
```

```
-2/1155*(b*x+a)*(-105*b^3*x^3+70*a*b^2*x^2-40*a^2*b*x+16*a^3)*(c*x)^(3/2)*
(b*x^2+a*x)^(3/2)/b^4/x^3
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 82, normalized size of antiderivative = 0.64

$$\int (cx)^{3/2} \left(ax+bx^2\right)^{3/2} \, dx = \frac{2 \, (105 \, b^5 c x^5 + 140 \, a b^4 c x^4 + 5 \, a^2 b^3 c x^3 - 6 \, a^3 b^2 c x^2 + 8 \, a^4 b c x - 16 \, a^5 c) \sqrt{b x^2 + a x} \sqrt{c x}}{1155 \, b^4 x}$$

```
integrate((c*x)^(3/2)*(b*x^2+a*x)^(3/2),x, algorithm="fricas")
```

```
2/1155*(105*b^5*c*x^5 + 140*a*b^4*c*x^4 + 5*a^2*b^3*c*x^3 - 6*a^3*b^2*c*x^
2 + 8*a^4*b*c*x - 16*a^5*c)*sqrt(b*x^2 + a*x)*sqrt(c*x)/(b^4*x)
```

Sympy [F]

$$\int (cx)^{3/2} \left(ax+bx^2\right)^{3/2} \, dx = \int (cx)^{\frac{3}{2}} \left(x(a+bx)\right)^{\frac{3}{2}} \, dx$$

```
integrate((c*x)**(3/2)*(b*x**2+a*x)**(3/2),x)
```

```
Integral((c*x)**(3/2)*(x*(a + b*x))**(3/2), x)
```

Maxima [A] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 157, normalized size of antiderivative = 1.23

$$\int (cx)^{3/2} \left(ax+bx^2\right)^{3/2} dx = \frac{2\left(\left(315\,b^5c^{\frac{3}{2}}x^5+35\,ab^4c^{\frac{3}{2}}x^4-40\,a^2b^3c^{\frac{3}{2}}x^3+48\,a^3b^2c^{\frac{3}{2}}x^2-64\,a^4bc^{\frac{3}{2}}x+128\,a^5c^{\frac{3}{2}}\right)x^4+11\,\ldots}{3465\,b^4x^4}$$

```
integrate((c*x)^(3/2)*(b*x^2+a*x)^(3/2),x, algorithm="maxima")
```

```
2/3465*((315*b^5*c^(3/2)*x^5 + 35*a*b^4*c^(3/2)*x^4 - 40*a^2*b^3*c^(3/2)*x
^3 + 48*a^3*b^2*c^(3/2)*x^2 - 64*a^4*b*c^(3/2)*x + 128*a^5*c^(3/2))*x^4 +
11*(35*a*b^4*c^(3/2)*x^5 + 5*a^2*b^3*c^(3/2)*x^4 - 6*a^3*b^2*c^(3/2)*x^3 +
 8*a^4*b*c^(3/2)*x^2 - 16*a^5*c^(3/2)*x)*x^3)*sqrt(b*x + a)/(b^4*x^4)
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 209 vs. 2(104) = 208.

Time = 0.33 (sec) , antiderivative size = 209, normalized size of antiderivative = 1.63

$$\int (cx)^{3/2} \left(ax+bx^2\right)^{3/2} dx = \frac{2}{3465}\,c\left(\frac{11\left(\frac{16\,\sqrt{ac}a^4c}{b^4}-\frac{105\,(bcx+ac)^{\frac{3}{2}}a^3c^3-189\,(bcx+ac)^{\frac{5}{2}}a^2c^2+135\,(bcx+ac)^{\frac{7}{2}}ac-35\,(bcx+ac)^{\frac{9}{2}}}{b^4c^3}\right)a|c|}{c^2}-\frac{\Big(1\ldots}{}\right.$$

```
integrate((c*x)^(3/2)*(b*x^2+a*x)^(3/2),x, algorithm="giac")
```

```
2/3465*c*(11*(16*sqrt(a*c)*a^4*c/b^4 - (105*(b*c*x + a*c)^(3/2)*a^3*c^3 -
189*(b*c*x + a*c)^(5/2)*a^2*c^2 + 135*(b*c*x + a*c)^(7/2)*a*c - 35*(b*c*x
+ a*c)^(9/2))/(b^4*c^3))*a*abs(c)/c^2 - (128*sqrt(a*c)*a^5*c^5/b^5 - (1155
*(b*c*x + a*c)^(3/2)*a^4*c^4 - 2772*(b*c*x + a*c)^(5/2)*a^3*c^3 + 2970*(b*
c*x + a*c)^(7/2)*a^2*c^2 - 1540*(b*c*x + a*c)^(9/2)*a*c + 315*(b*c*x + a*c
)^(11/2))/b^5)*b*abs(c)/c^6)
```

Mupad [B] (verification not implemented)

Time = 9.32 (sec) , antiderivative size = 62, normalized size of antiderivative = 0.48

$$\int (cx)^{3/2} \left(ax+bx^2\right)^{3/2} \, dx = -\frac{2\,c\,\sqrt{b\,x^2+a\,x}\,\sqrt{c\,x}\,{\left(a+b\,x\right)}^2\,\left(16\,a^3-40\,a^2\,b\,x+70\,a\,b^2\,x^2-105\,b^3\,x^3\right)}{1155\,b^4\,x}$$

```
int((a*x + b*x^2)^(3/2)*(c*x)^(3/2),x)
```

```
-(2*c*(a*x + b*x^2)^(1/2)*(c*x)^(1/2)*(a + b*x)^2*(16*a^3 - 105*b^3*x^3 +
70*a*b^2*x^2 - 40*a^2*b*x))/(1155*b^4*x)
```

Reduce [B] (verification not implemented)

Time = 0.45 (sec) , antiderivative size = 66, normalized size of antiderivative = 0.52

$$\int (cx)^{3/2} \left(ax+bx^2\right)^{3/2} \, dx = \frac{2\sqrt{c}\,\sqrt{bx+a}\,c(105b^5x^5+140a\,b^4x^4+5a^2b^3x^3-6a^3b^2x^2+8a^4bx-16a^5)}{1155b^4}$$

```
int((c*x)^(3/2)*(b*x^2+a*x)^(3/2),x)
```

```
(2*sqrt(c)*sqrt(a + b*x)*c*( - 16*a**5 + 8*a**4*b*x - 6*a**3*b**2*x**2 + 5
*a**2*b**3*x**3 + 140*a*b**4*x**4 + 105*b**5*x**5))/(1155*b**4)
```

3.89 $\int \sqrt{cx}(ax+bx^2)^{3/2}\,dx$

Optimal result

Integrand size = 21, antiderivative size = 95

$$\int \sqrt{cx}\left(ax+bx^2\right)^{3/2}\,dx=\frac{2a^2c^3\left(ax+bx^2\right)^{5/2}}{5b^3(cx)^{5/2}}-\frac{4ac^4\left(ax+bx^2\right)^{7/2}}{7b^3(cx)^{7/2}}+\frac{2c^5\left(ax+bx^2\right)^{9/2}}{9b^3(cx)^{9/2}}$$

```
2/5*a^2*c^3*(b*x^2+a*x)^(5/2)/b^3/(c*x)^(5/2)-4/7*a*c^4*(b*x^2+a*x)^(7/2)/
b^3/(c*x)^(7/2)+2/9*c^5*(b*x^2+a*x)^(9/2)/b^3/(c*x)^(9/2)
```

Mathematica [A] (verified)

Time = 0.04 (sec) , antiderivative size = 52, normalized size of antiderivative = 0.55

$$\int \sqrt{cx}\left(ax+bx^2\right)^{3/2}\,dx=\frac{2c(a+bx)^2\sqrt{x(a+bx)}\left(8a^2-20abx+35b^2x^2\right)}{315b^3\sqrt{cx}}$$

```
Integrate[Sqrt[c*x]*(a*x + b*x^2)^(3/2),x]
```

```
(2*c*(a + b*x)^2*Sqrt[x*(a + b*x)]*(8*a^2 - 20*a*b*x + 35*b^2*x^2))/(315*b
^3*Sqrt[c*x])
```

Rubi [A] (verified)

Time = 0.37 (sec) , antiderivative size = 98, normalized size of antiderivative = 1.03, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.143, Rules used = {1128, 1128, 1122}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sqrt{cx}\left(ax+bx^2\right)^{3/2}dx$$

$$\Big\downarrow \text{1128}$$

$$\frac{2c\left(ax+bx^2\right)^{5/2}}{9b\sqrt{cx}}-\frac{4ac\int \frac{\left(bx^2+ax\right)^{3/2}}{\sqrt{cx}}dx}{9b}$$

$$\Big\downarrow \text{1128}$$

$$\frac{2c\left(ax+bx^2\right)^{5/2}}{9b\sqrt{cx}}-\frac{4ac\left(\frac{2c\left(ax+bx^2\right)^{5/2}}{7b(cx)^{3/2}}-\frac{2ac\int \frac{\left(bx^2+ax\right)^{3/2}}{(cx)^{3/2}}dx}{7b}\right)}{9b}$$

$$\Big\downarrow \text{1122}$$

$$\frac{2c\left(ax+bx^2\right)^{5/2}}{9b\sqrt{cx}}-\frac{4ac\left(\frac{2c\left(ax+bx^2\right)^{5/2}}{7b(cx)^{3/2}}-\frac{4ac^2\left(ax+bx^2\right)^{5/2}}{35b^2(cx)^{5/2}}\right)}{9b}$$

```
Int[Sqrt[c*x]*(a*x + b*x^2)^(3/2),x]
```

```
(2*c*(a*x + b*x^2)^(5/2))/(9*b*Sqrt[c*x]) - (4*a*c*((-4*a*c^2*(a*x + b*x^2
)^(5/2))/(35*b^2*(c*x)^(5/2)) + (2*c*(a*x + b*x^2)^(5/2))/(7*b*(c*x)^(3/2)
)))/(9*b)
```

Defintions of rubi rules used

```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] :> Simp[e*(d + e*x)^(m - 1)*((a + b*x + c*x^2)^(p + 1)/(c*(p + 1))),
 x] /; FreeQ[{a, b, c, d, e, m, p}, x] && EqQ[c*d^2 - b*d*e + a*e^2, 0] &&
EqQ[m + p, 0]
```

```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] :> Simp[e*(d + e*x)^(m - 1)*((a + b*x + c*x^2)^(p + 1)/(c*(m + 2*p +
 1))), x] + Simp[Simplify[m + p]*((2*c*d - b*e)/(c*(m + 2*p + 1)))   Int[(d
 + e*x)^(m - 1)*(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, m, p},
 x] && EqQ[c*d^2 - b*d*e + a*e^2, 0] && IGtQ[Simplify[m + p], 0]
```

Maple [A] (verified)

Time = 0.33 (sec) , antiderivative size = 49, normalized size of antiderivative = 0.52

method	result	size
gosper	$\frac{2(bx+a)\left(35b^{2}x^{2}-20abx+8a^{2}\right)\sqrt{cx}\,\left(b\,x^{2}+ax\right)^{\frac{3}{2}}}{315b^{3}x^{2}}$	49
default	$\frac{2\sqrt{cx}\,\sqrt{x(bx+a)}\,(bx+a)^{2}\left(35b^{2}x^{2}-20abx+8a^{2}\right)}{315x\,b^{3}}$	49
orering	$\frac{2(bx+a)\left(35b^{2}x^{2}-20abx+8a^{2}\right)\sqrt{cx}\,\left(b\,x^{2}+ax\right)^{\frac{3}{2}}}{315b^{3}x^{2}}$	49
risch	$\frac{2cx\left(35b^{4}x^{4}+50a\,b^{3}x^{3}+3a^{2}b^{2}x^{2}-4a^{3}bx+8a^{4}\right)(bx+a)}{315\sqrt{cx}\,\sqrt{x(bx+a)}\,b^{3}}$	68

```
int((c*x)^(1/2)*(b*x^2+a*x)^(3/2),x,method=_RETURNVERBOSE)
```

```
2/315*(b*x+a)*(35*b^2*x^2-20*a*b*x+8*a^2)*(c*x)^(1/2)*(b*x^2+a*x)^(3/2)/b^
3/x^2
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 65, normalized size of antiderivative = 0.68

$$\int \sqrt{cx}\,(ax+bx^2)^{3/2}\,dx = \frac{2\,(35\,b^4x^4+50\,ab^3x^3+3\,a^2b^2x^2-4\,a^3bx+8\,a^4)\sqrt{bx^2+ax}\sqrt{cx}}{315\,b^3x}$$

```
integrate((c*x)^(1/2)*(b*x^2+a*x)^(3/2),x, algorithm="fricas")
```

```
2/315*(35*b^4*x^4 + 50*a*b^3*x^3 + 3*a^2*b^2*x^2 - 4*a^3*b*x + 8*a^4)*sqrt
(b*x^2 + a*x)*sqrt(c*x)/(b^3*x)
```

Sympy [F]

$$\int \sqrt{cx}\,(ax+bx^2)^{3/2}\,dx = \int \sqrt{cx}\,(x(a+bx))^{\frac{3}{2}}\,dx$$

```
integrate((c*x)**(1/2)*(b*x**2+a*x)**(3/2),x)
```

```
Integral(sqrt(c*x)*(x*(a + b*x))**(3/2), x)
```

Maxima [A] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 129, normalized size of antiderivative = 1.36

$$\int \sqrt{cx}\,(ax+bx^2)^{3/2}\,dx = \frac{2\left(\left(35\,b^4\sqrt{c}x^4+5\,ab^3\sqrt{c}x^3-6\,a^2b^2\sqrt{c}x^2+8\,a^3b\sqrt{c}x-16\,a^4\sqrt{c}\right)x^3+3\left(15\,ab^3\sqrt{c}x^4+3\,a^2b\cdots\right.\right.}{315\,b^3x^3}$$

```
integrate((c*x)^(1/2)*(b*x^2+a*x)^(3/2),x, algorithm="maxima")
```

```
2/315*((35*b^4*sqrt(c)*x^4 + 5*a*b^3*sqrt(c)*x^3 - 6*a^2*b^2*sqrt(c)*x^2 +
 8*a^3*b*sqrt(c)*x - 16*a^4*sqrt(c))*x^3 + 3*(15*a*b^3*sqrt(c)*x^4 + 3*a^2
*b^2*sqrt(c)*x^3 - 4*a^3*b*sqrt(c)*x^2 + 8*a^4*sqrt(c)*x)*x^2)*sqrt(b*x +
a)/(b^3*x^3)
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 170 vs. $2(77) = 154$.

Time = 0.31 (sec) , antiderivative size = 170, normalized size of antiderivative = 1.79

$$\int \sqrt{cx}(ax+bx^2)^{3/2}\,dx = \frac{2\left(\frac{16\sqrt{ac}a^4c}{b^4} - \frac{105\,(bcx+ac)^{\frac{3}{2}}a^3c^3-189\,(bcx+ac)^{\frac{5}{2}}a^2c^2+135\,(bcx+ac)^{\frac{7}{2}}ac-35\,(bcx+ac)^{\frac{9}{2}}}{b^4c^3}\right)b|c|}{315\,c^2} - \frac{2\left(\frac{8\sqrt{ac}a^3c^3}{b^3} - \frac{35\,(bcx+ac)^{\frac{3}{2}}a^2c^2-42\,(bcx+ac)^{\frac{5}{2}}ac+15\,(bcx+ac)^{\frac{7}{2}}}{b^3}\right)a|c|}{105\,c^4}$$

```
integrate((c*x)^(1/2)*(b*x^2+a*x)^(3/2),x, algorithm="giac")
```

```
2/315*(16*sqrt(a*c)*a^4*c/b^4 - (105*(b*c*x + a*c)^(3/2)*a^3*c^3 - 189*(b*
c*x + a*c)^(5/2)*a^2*c^2 + 135*(b*c*x + a*c)^(7/2)*a*c - 35*(b*c*x + a*c)^
(9/2))/(b^4*c^3))*b*abs(c)/c^2 - 2/105*(8*sqrt(a*c)*a^3*c^3/b^3 - (35*(b*c
*x + a*c)^(3/2)*a^2*c^2 - 42*(b*c*x + a*c)^(5/2)*a*c + 15*(b*c*x + a*c)^(7
/2))/b^3)*a*abs(c)/c^4
```

Mupad [B] (verification not implemented)

Time = 9.13 (sec) , antiderivative size = 50, normalized size of antiderivative = 0.53

$$\int \sqrt{cx}(ax+bx^2)^{3/2}\,dx = \frac{2\sqrt{b\,x^2+a\,x}\,\sqrt{c\,x}\,(a+b\,x)^2\,(8\,a^2-20\,a\,b\,x+35\,b^2\,x^2)}{315\,b^3\,x}$$

```
int((a*x + b*x^2)^(3/2)*(c*x)^(1/2),x)
```

```
(2*(a*x + b*x^2)^(1/2)*(c*x)^(1/2)*(a + b*x)^2*(8*a^2 + 35*b^2*x^2 - 20*a*
b*x))/(315*b^3*x)
```

Reduce [B] (verification not implemented)

Time = 0.45 (sec) , antiderivative size = 54, normalized size of antiderivative = 0.57

$$\int \sqrt{cx}\,(ax+bx^2)^{3/2}\,dx = \frac{2\sqrt{c}\,\sqrt{bx+a}\,(35b^4x^4+50a\,b^3x^3+3a^2b^2x^2-4a^3bx+8a^4)}{315b^3}$$

```
int((c*x)^(1/2)*(b*x^2+a*x)^(3/2),x)
```

```
(2*sqrt(c)*sqrt(a + b*x)*(8*a**4 - 4*a**3*b*x + 3*a**2*b**2*x**2 + 50*a*b*
*3*x**3 + 35*b**4*x**4))/(315*b**3)
```

3.90 $\int \frac{(ax+bx^2)^{3/2}}{\sqrt{cx}}\,dx$

Optimal result

Integrand size = 21, antiderivative size = 62

$$\int \frac{(ax+bx^2)^{3/2}}{\sqrt{cx}}\,dx = -\frac{2ac^2(ax+bx^2)^{5/2}}{5b^2(cx)^{5/2}} + \frac{2c^3(ax+bx^2)^{7/2}}{7b^2(cx)^{7/2}}$$

```
-2/5*a*c^2*(b*x^2+a*x)^(5/2)/b^2/(c*x)^(5/2)+2/7*c^3*(b*x^2+a*x)^(7/2)/b^2
/(c*x)^(7/2)
```

Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 41, normalized size of antiderivative = 0.66

$$\int \frac{(ax+bx^2)^{3/2}}{\sqrt{cx}}\,dx = \frac{2(a+bx)(x(a+bx))^{3/2}(-2a+5bx)}{35b^2x\sqrt{cx}}$$

```
Integrate[(a*x + b*x^2)^(3/2)/Sqrt[c*x],x]
```

```
(2*(a + b*x)*(x*(a + b*x))^(3/2)*(-2*a + 5*b*x))/(35*b^2*x*Sqrt[c*x])
```

Rubi [A] (verified)

Time = 0.29 (sec) , antiderivative size = 60, normalized size of antiderivative = 0.97, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.095$, Rules used = {1128, 1122}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\left(ax+bx^2\right)^{3/2}}{\sqrt{cx}}\,dx$$

$$\Big\downarrow \ 1128$$

$$\frac{2c\left(ax+bx^2\right)^{5/2}}{7b(cx)^{3/2}}-\frac{2ac\int \frac{\left(bx^2+ax\right)^{3/2}}{(cx)^{3/2}}dx}{7b}$$

$$\Big\downarrow \ 1122$$

$$\frac{2c\left(ax+bx^2\right)^{5/2}}{7b(cx)^{3/2}}-\frac{4ac^2\left(ax+bx^2\right)^{5/2}}{35b^2(cx)^{5/2}}$$

```
Int[(a*x + b*x^2)^(3/2)/Sqrt[c*x],x]
```

```
(-4*a*c^2*(a*x + b*x^2)^(5/2))/(35*b^2*(c*x)^(5/2)) + (2*c*(a*x + b*x^2)^(
5/2))/(7*b*(c*x)^(3/2))
```

Defintions of rubi rules used

```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] :> Simp[e*(d + e*x)^(m - 1)*((a + b*x + c*x^2)^(p + 1)/(c*(p + 1))),
 x] /; FreeQ[{a, b, c, d, e, m, p}, x] && EqQ[c*d^2 - b*d*e + a*e^2, 0] &&
EqQ[m + p, 0]
```

```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] :> Simp[e*(d + e*x)^(m - 1)*((a + b*x + c*x^2)^(p + 1)/(c*(m + 2*p +
 1))), x] + Simp[Simplify[m + p]*((2*c*d - b*e)/(c*(m + 2*p + 1)))   Int[(d
 + e*x)^(m - 1)*(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, m, p},
 x] && EqQ[c*d^2 - b*d*e + a*e^2, 0] && IGtQ[Simplify[m + p], 0]
```

Maple [A] (verified)

Time = 0.34 (sec) , antiderivative size = 35, normalized size of antiderivative = 0.56

method	result	size
default	$-\frac{2\sqrt{x(bx+a)}\,(bx+a)^{2}(-5bx+2a)}{35\sqrt{cx}\,b^{2}}$	35
gosper	$-\frac{2(bx+a)(-5bx+2a)\left(b\,x^{2}+ax\right)^{\frac{3}{2}}}{35b^{2}x\sqrt{cx}}$	38
orering	$-\frac{2(bx+a)(-5bx+2a)\left(b\,x^{2}+ax\right)^{\frac{3}{2}}}{35b^{2}x\sqrt{cx}}$	38
risch	$-\frac{2x\left(-5b^{3}x^{3}-8a\,b^{2}x^{2}-a^{2}bx+2a^{3}\right)(bx+a)}{35\sqrt{cx}\,\sqrt{x(bx+a)}\,b^{2}}$	56

```
int((b*x^2+a*x)^(3/2)/(c*x)^(1/2),x,method=_RETURNVERBOSE)
```

```
-2/35*(x*(b*x+a))^(1/2)/(c*x)^(1/2)*(b*x+a)^2*(-5*b*x+2*a)/b^2
```

Fricas [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 56, normalized size of antiderivative = 0.90

$$\int \frac{(ax+bx^2)^{3/2}}{\sqrt{cx}}\,dx = \frac{2\,(5\,b^3x^3+8\,ab^2x^2+a^2bx-2\,a^3)\sqrt{bx^2+ax}\sqrt{cx}}{35\,b^2cx}$$

```
integrate((b*x^2+a*x)^(3/2)/(c*x)^(1/2),x, algorithm="fricas")
```

```
2/35*(5*b^3*x^3 + 8*a*b^2*x^2 + a^2*b*x - 2*a^3)*sqrt(b*x^2 + a*x)*sqrt(c*
x)/(b^2*c*x)
```

Sympy [F]

$$\int \frac{(ax+bx^2)^{3/2}}{\sqrt{cx}}\,dx = \int \frac{(x(a+bx))^{\frac{3}{2}}}{\sqrt{cx}}\,dx$$

```
integrate((b*x**2+a*x)**(3/2)/(c*x)**(1/2),x)
```

```
Integral((x*(a + b*x))**(3/2)/sqrt(c*x), x)
```

Maxima [A] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 80, normalized size of antiderivative = 1.29

$$\int \frac{(ax+bx^2)^{3/2}}{\sqrt{cx}}\,dx = \frac{2\left((15\,b^3x^3+3\,ab^2x^2-4\,a^2bx+8\,a^3)x^2+7\,(3\,ab^2x^3+a^2bx^2-2\,a^3x)x\right)\sqrt{bx+a}}{105\,b^2\sqrt{c}x^2}$$

```
integrate((b*x^2+a*x)^(3/2)/(c*x)^(1/2),x, algorithm="maxima")
```

```
2/105*((15*b^3*x^3 + 3*a*b^2*x^2 - 4*a^2*b*x + 8*a^3)*x^2 + 7*(3*a*b^2*x^3
 + a^2*b*x^2 - 2*a^3*x)*x)*sqrt(b*x + a)/(b^2*sqrt(c)*x^2)
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 138 vs. $2(50) = 100$.

Time = 0.25 (sec) , antiderivative size = 138, normalized size of antiderivative = 2.23

$$\int \frac{(ax+bx^2)^{3/2}}{\sqrt{cx}}\,dx = \frac{2\left(\frac{7\left(\frac{2\sqrt{ac}a^2c^2}{b^2}-\frac{5\,(bcx+ac)^{\frac{3}{2}}ac-3\,(bcx+ac)^{\frac{5}{2}}}{b^2}\right)a|c|}{c^3}-\frac{\left(\frac{8\sqrt{ac}a^3c^3}{b^3}-\frac{35\,(bcx+ac)^{\frac{3}{2}}a^2c^2-42\,(bcx+ac)^{\frac{5}{2}}ac+15\,(bcx+a}{b^3}\right.}{c^4}\right.}{105\,c}$$

```
integrate((b*x^2+a*x)^(3/2)/(c*x)^(1/2),x, algorithm="giac")
```

```
2/105*(7*(2*sqrt(a*c)*a^2*c^2/b^2 - (5*(b*c*x + a*c)^(3/2)*a*c - 3*(b*c*x
+ a*c)^(5/2))/b^2)*a*abs(c)/c^3 - (8*sqrt(a*c)*a^3*c^3/b^3 - (35*(b*c*x +
a*c)^(3/2)*a^2*c^2 - 42*(b*c*x + a*c)^(5/2)*a*c + 15*(b*c*x + a*c)^(7/2))/
b^3)*b*abs(c)/c^4)/c
```

Mupad [B] (verification not implemented)

Time = 9.22 (sec) , antiderivative size = 47, normalized size of antiderivative = 0.76

$$\int \frac{(ax+bx^2)^{3/2}}{\sqrt{cx}}\,dx = \frac{\sqrt{b\,x^2+a\,x}\,\left(\frac{16\,a\,x^2}{35}+\frac{2\,b\,x^3}{7}-\frac{4\,a^3}{35\,b^2}+\frac{2\,a^2\,x}{35\,b}\right)}{\sqrt{c\,x}}$$

```
int((a*x + b*x^2)^(3/2)/(c*x)^(1/2),x)
```

```
((a*x + b*x^2)^(1/2)*((16*a*x^2)/35 + (2*b*x^3)/7 - (4*a^3)/(35*b^2) + (2*
a^2*x)/(35*b)))/(c*x)^(1/2)
```

Reduce [B] (verification not implemented)

Time = 0.23 (sec) , antiderivative size = 45, normalized size of antiderivative = 0.73

$$\int \frac{(ax+bx^2)^{3/2}}{\sqrt{cx}}\,dx = \frac{2\sqrt{c}\,\sqrt{bx+a}\,(5b^3x^3+8a\,b^2x^2+a^2bx-2a^3)}{35b^2c}$$

```
int((b*x^2+a*x)^(3/2)/(c*x)^(1/2),x)
```

```
(2*sqrt(c)*sqrt(a + b*x)*( - 2*a**3 + a**2*b*x + 8*a*b**2*x**2 + 5*b**3*x*
*3))/(35*b**2*c)
```

3.91 $\int \frac{(ax+bx^2)^{3/2}}{(cx)^{3/2}}\,dx$

Optimal result

Integrand size = 21, antiderivative size = 28

$$\int \frac{(ax+bx^2)^{3/2}}{(cx)^{3/2}}\,dx = \frac{2c(ax+bx^2)^{5/2}}{5b(cx)^{5/2}}$$

```
2/5*c*(b*x^2+a*x)^(5/2)/b/(c*x)^(5/2)
```

Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 26, normalized size of antiderivative = 0.93

$$\int \frac{(ax+bx^2)^{3/2}}{(cx)^{3/2}}\,dx = \frac{2c(x(a+bx))^{5/2}}{5b(cx)^{5/2}}$$

```
Integrate[(a*x + b*x^2)^(3/2)/(c*x)^(3/2),x]
```

```
(2*c*(x*(a + b*x))^(5/2))/(5*b*(c*x)^(5/2))
```

Rubi [A] (verified)

Time = 0.24 (sec) , antiderivative size = 28, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}} = 0.048$, Rules used = {1122}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(ax+bx^2)^{3/2}}{(cx)^{3/2}} \, dx$$

$$\Big\downarrow \; 1122$$

$$\frac{2c(ax+bx^2)^{5/2}}{5b(cx)^{5/2}}$$

```
Int[(a*x + b*x^2)^(3/2)/(c*x)^(3/2),x]
```

```
(2*c*(a*x + b*x^2)^(5/2))/(5*b*(c*x)^(5/2))
```

Defintions of rubi rules used

```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] :> Simp[e*(d + e*x)^(m - 1)*((a + b*x + c*x^2)^(p + 1)/(c*(p + 1))),
 x] /; FreeQ[{a, b, c, d, e, m, p}, x] && EqQ[c*d^2 - b*d*e + a*e^2, 0] &&
EqQ[m + p, 0]
```

Maple [A] (verified)

Time = 0.33 (sec) , antiderivative size = 27, normalized size of antiderivative = 0.96

method	result	size
gosper	$\frac{2(bx+a)\left(b\,x^{2}+ax\right)^{\frac{3}{2}}}{5b\left(cx\right)^{\frac{3}{2}}}$	27
orering	$\frac{2(bx+a)\left(b\,x^{2}+ax\right)^{\frac{3}{2}}}{5b\left(cx\right)^{\frac{3}{2}}}$	27
default	$\frac{2\sqrt{x\left(bx+a\right)}\,\left(bx+a\right)^{2}}{5c\sqrt{cx}\,b}$	30
risch	$\frac{2x\left(b^{2}x^{2}+2abx+a^{2}\right)\left(bx+a\right)}{5c\sqrt{cx}\,\sqrt{x\left(bx+a\right)}\,b}$	45

```
int((b*x^2+a*x)^(3/2)/(c*x)^(3/2),x,method=_RETURNVERBOSE)
```

```
2/5*(b*x+a)*(b*x^2+a*x)^(3/2)/b/(c*x)^(3/2)
```

Fricas [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 43, normalized size of antiderivative = 1.54

$$\int \frac{(ax+bx^2)^{3/2}}{(cx)^{3/2}}\,dx = \frac{2\,(b^2x^2+2\,abx+a^2)\sqrt{bx^2+ax}\sqrt{cx}}{5\,bc^2x}$$

```
integrate((b*x^2+a*x)^(3/2)/(c*x)^(3/2),x, algorithm="fricas")
```

```
2/5*(b^2*x^2 + 2*a*b*x + a^2)*sqrt(b*x^2 + a*x)*sqrt(c*x)/(b*c^2*x)
```

Sympy [F]

$$\int \frac{(ax+bx^2)^{3/2}}{(cx)^{3/2}}\,dx = \int \frac{\left(x(a+bx)\right)^{\frac{3}{2}}}{\left(cx\right)^{\frac{3}{2}}}\,dx$$

```
integrate((b*x**2+a*x)**(3/2)/(c*x)**(3/2),x)
```

```
Integral((x*(a + b*x))**(3/2)/(c*x)**(3/2), x)
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 52 vs. $2(22) = 44$.

Time = 0.04 (sec) , antiderivative size = 52, normalized size of antiderivative = 1.86

$$\int \frac{(ax+bx^2)^{3/2}}{(cx)^{3/2}}\,dx = \frac{2\left(5\,abx^2 + 5\,a^2x + (3\,b^2x^2 + abx - 2\,a^2)x\right)\sqrt{bx+a}}{15\,bc^{\frac{3}{2}}x}$$

```
integrate((b*x^2+a*x)^(3/2)/(c*x)^(3/2),x, algorithm="maxima")
```

```
2/15*(5*a*b*x^2 + 5*a^2*x + (3*b^2*x^2 + a*b*x - 2*a^2)*x)*sqrt(b*x + a)/(
b*c^(3/2)*x)
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 98 vs. $2(22) = 44$.

Time = 0.19 (sec) , antiderivative size = 98, normalized size of antiderivative = 3.50

$$\int \frac{(ax+bx^2)^{3/2}}{(cx)^{3/2}}\,dx = -\frac{2\left(\frac{5\left(\frac{\sqrt{ac}ac}{b} - \frac{(bcx+ac)^{\frac{3}{2}}}{b}\right)a|c|}{c^2} - \frac{\left(\frac{2\sqrt{ac}a^2c^2}{b^2} - \frac{5\,(bcx+ac)^{\frac{3}{2}}ac - 3\,(bcx+ac)^{\frac{5}{2}}}{b^2}\right)b|c|}{c^3}\right)}{15\,c^2}$$

```
integrate((b*x^2+a*x)^(3/2)/(c*x)^(3/2),x, algorithm="giac")
```

```
-2/15*(5*(sqrt(a*c)*a*c/b - (b*c*x + a*c)^(3/2)/b)*a*abs(c)/c^2 - (2*sqrt(
a*c)*a^2*c^2/b^2 - (5*(b*c*x + a*c)^(3/2)*a*c - 3*(b*c*x + a*c)^(5/2))/b^2
)*b*abs(c)/c^3)/c^2
```

Mupad [B] (verification not implemented)

Time = 9.38 (sec) , antiderivative size = 45, normalized size of antiderivative = 1.61

$$\int \frac{(ax+bx^2)^{3/2}}{(cx)^{3/2}}\,dx = \frac{\sqrt{b\,x^2+a\,x}\,\left(\frac{2\,b\,x^2}{5\,c}+\frac{2\,a^2}{5\,b\,c}+\frac{4\,a\,x}{5\,c}\right)}{\sqrt{c\,x}}$$

```
int((a*x + b*x^2)^(3/2)/(c*x)^(3/2),x)
```

```
((a*x + b*x^2)^(1/2)*((2*b*x^2)/(5*c) + (2*a^2)/(5*b*c) + (4*a*x)/(5*c)))/
(c*x)^(1/2)
```

Reduce [B] (verification not implemented)

Time = 0.36 (sec) , antiderivative size = 32, normalized size of antiderivative = 1.14

$$\int \frac{(ax+bx^2)^{3/2}}{(cx)^{3/2}}\,dx = \frac{2\sqrt{c}\,\sqrt{bx+a}\,(b^2x^2+2abx+a^2)}{5b\,c^2}$$

```
int((b*x^2+a*x)^(3/2)/(c*x)^(3/2),x)
```

```
(2*sqrt(c)*sqrt(a + b*x)*(a**2 + 2*a*b*x + b**2*x**2))/(5*b*c**2)
```

3.92 $\int \frac{(ax+bx^2)^{3/2}}{(cx)^{5/2}} dx$

Optimal result

Integrand size = 21, antiderivative size = 98

$$\int \frac{(ax+bx^2)^{3/2}}{(cx)^{5/2}}\,dx=\frac{2a\sqrt{ax+bx^2}}{c^2\sqrt{cx}}+\frac{2(ax+bx^2)^{3/2}}{3c(cx)^{3/2}}-\frac{2a^{3/2}\text{arctanh}\left(\frac{\sqrt{c}\sqrt{ax+bx^2}}{\sqrt{a}\sqrt{cx}}\right)}{c^{5/2}}$$

```
2*a*(b*x^2+a*x)^(1/2)/c^2/(c*x)^(1/2)+2/3*(b*x^2+a*x)^(3/2)/c/(c*x)^(3/2)-
2*a^(3/2)*arctanh(c^(1/2)*(b*x^2+a*x)^(1/2)/a^(1/2)/(c*x)^(1/2))/c^(5/2)
```

Mathematica [A] (verified)

Time = 0.04 (sec) , antiderivative size = 75, normalized size of antiderivative = 0.77

$$\int \frac{(ax+bx^2)^{3/2}}{(cx)^{5/2}}\,dx=\frac{2\sqrt{x(a+bx)}\left(\sqrt{a+bx}(4a+bx)-3a^{3/2}\text{arctanh}\left(\frac{\sqrt{a+bx}}{\sqrt{a}}\right)\right)}{3c^2\sqrt{cx}\sqrt{a+bx}}$$

```
Integrate[(a*x + b*x^2)^(3/2)/(c*x)^(5/2),x]
```

```
(2*Sqrt[x*(a + b*x)]*(Sqrt[a + b*x]*(4*a + b*x) - 3*a^(3/2)*ArcTanh[Sqrt[a
 + b*x]/Sqrt[a]]))/(3*c^2*Sqrt[c*x]*Sqrt[a + b*x])
```

Rubi [A] (verified)

Time = 0.38 (sec) , antiderivative size = 103, normalized size of antiderivative = 1.05, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.190$, Rules used = {1131, 1131, 1136, 221}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(ax+bx^2)^{3/2}}{(cx)^{5/2}}dx$$

$$\Big\downarrow \text{ 1131}$$

$$\frac{a\int \frac{\sqrt{bx^2+ax}}{(cx)^{3/2}}dx}{c}+\frac{2(ax+bx^2)^{3/2}}{3c(cx)^{3/2}}$$

$$\Big\downarrow \text{ 1131}$$

$$\frac{a\left(\frac{a\int \frac{1}{\sqrt{cx}\sqrt{bx^2+ax}}dx}{c}+\frac{2\sqrt{ax+bx^2}}{c\sqrt{cx}}\right)}{c}+\frac{2(ax+bx^2)^{3/2}}{3c(cx)^{3/2}}$$

$$\Big\downarrow \text{ 1136}$$

$$\frac{a\left(2a\int \frac{1}{\frac{c(bx^2+ax)}{x}-ac}d\frac{\sqrt{bx^2+ax}}{\sqrt{cx}}+\frac{2\sqrt{ax+bx^2}}{c\sqrt{cx}}\right)}{c}+\frac{2(ax+bx^2)^{3/2}}{3c(cx)^{3/2}}$$

$$\Big\downarrow \text{ 221}$$

$$\frac{a\left(\frac{2\sqrt{ax+bx^2}}{c\sqrt{cx}}-\frac{2\sqrt{a}\text{arctanh}\left(\frac{\sqrt{c}\sqrt{ax+bx^2}}{\sqrt{a}\sqrt{cx}}\right)}{c^{3/2}}\right)}{c}+\frac{2(ax+bx^2)^{3/2}}{3c(cx)^{3/2}}$$

```
Int[(a*x + b*x^2)^(3/2)/(c*x)^(5/2),x]
```

```
(2*(a*x + b*x^2)^(3/2))/(3*c*(c*x)^(3/2)) + (a*((2*Sqrt[a*x + b*x^2])/(c*S
qrt[c*x]) - (2*Sqrt[a]*ArcTanh[(Sqrt[c]*Sqrt[a*x + b*x^2])/(Sqrt[a]*Sqrt[c
*x])])/c^(3/2)))/c
```

Defintions of rubi rules used

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] :> Simp[(d + e*x)^(m + 1)*((a + b*x + c*x^2)^p/(e*(m + 2*p + 1))), x
] - Simp[p*((2*c*d - b*e)/(e^2*(m + 2*p + 1)))   Int[(d + e*x)^(m + 1)*(a +
 b*x + c*x^2)^(p - 1), x], x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[c*d^2 - b
*d*e + a*e^2, 0] && GtQ[p, 0] && (LeQ[-2, m, 0] || EqQ[m + p + 1, 0]) && Ne
Q[m + 2*p + 1, 0] && IntegerQ[2*p]
```

```
Int[1/(Sqrt[(d_.) + (e_.)*(x_)]*Sqrt[(a_.) + (b_.)*(x_) + (c_.)*(x_)^2]), x
_Symbol] :> Simp[2*e   Subst[Int[1/(2*c*d - b*e + e^2*x^2), x], x, Sqrt[a +
 b*x + c*x^2]/Sqrt[d + e*x]], x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[c*d^2
- b*d*e + a*e^2, 0]
```

Maple [A] (verified)

Time = 0.35 (sec) , antiderivative size = 92, normalized size of antiderivative = 0.94

method	result	size
default	$-\frac{2\sqrt{x(bx+a)}\left(3a^{2}c\,\operatorname{arctanh}\left(\frac{\sqrt{c(bx+a)}}{\sqrt{ac}}\right)-bx\sqrt{c(bx+a)}\,\sqrt{ac}-4\sqrt{c(bx+a)}\,\sqrt{ac}\,a\right)}{3c^{2}\sqrt{cx}\,\sqrt{c(bx+a)}\,\sqrt{ac}}$	92

```
int((b*x^2+a*x)^(3/2)/(c*x)^(5/2),x,method=_RETURNVERBOSE)
```

```
-2/3*(x*(b*x+a))^(1/2)/c^2*(3*a^2*c*arctanh((c*(b*x+a))^(1/2)/(a*c)^(1/2))
-b*x*(c*(b*x+a))^(1/2)*(a*c)^(1/2)-4*(c*(b*x+a))^(1/2)*(a*c)^(1/2)*a)/(c*x
)^(1/2)/(c*(b*x+a))^(1/2)/(a*c)^(1/2)
```

Fricas [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 166, normalized size of antiderivative = 1.69

$$\int \frac{(ax+bx^2)^{3/2}}{(cx)^{5/2}}\,dx = \left[\frac{3\,acx\sqrt{\frac{a}{c}}\log\left(-\frac{bx^2+2\,ax-2\,\sqrt{bx^2+ax}\sqrt{cx}\sqrt{\frac{a}{c}}}{x^2}\right)+2\,\sqrt{bx^2+ax}(bx+4\,a)\sqrt{cx}}{3\,c^3x}, \; 2\left(3\,acx\right.\right.$$

```
integrate((b*x^2+a*x)^(3/2)/(c*x)^(5/2),x, algorithm="fricas")
```

```
[1/3*(3*a*c*x*sqrt(a/c)*log(-(b*x^2 + 2*a*x - 2*sqrt(b*x^2 + a*x)*sqrt(c*x
)*sqrt(a/c))/x^2) + 2*sqrt(b*x^2 + a*x)*(b*x + 4*a)*sqrt(c*x))/(c^3*x), 2/
3*(3*a*c*x*sqrt(-a/c)*arctan(sqrt(b*x^2 + a*x)*sqrt(c*x)*sqrt(-a/c)/(a*x))
 + sqrt(b*x^2 + a*x)*(b*x + 4*a)*sqrt(c*x))/(c^3*x)]
```

Sympy [F]

$$\int \frac{(ax+bx^2)^{3/2}}{(cx)^{5/2}}\,dx = \int \frac{(x(a+bx))^{\frac{3}{2}}}{(cx)^{\frac{5}{2}}}\,dx$$

```
integrate((b*x**2+a*x)**(3/2)/(c*x)**(5/2),x)
```

```
Integral((x*(a + b*x))**(3/2)/(c*x)**(5/2), x)
```

Maxima [F]

$$\int \frac{(ax+bx^2)^{3/2}}{(cx)^{5/2}}\,dx = \int \frac{(bx^2+ax)^{\frac{3}{2}}}{(cx)^{\frac{5}{2}}}\,dx$$

```
integrate((b*x^2+a*x)^(3/2)/(c*x)^(5/2),x, algorithm="maxima")
```

```
a*integrate(sqrt(b*x + a)/x, x)/c^(5/2) + 2/3*(b*sqrt(c)*x + a*sqrt(c))*sq
rt(b*x + a)/c^3
```

Giac [A] (verification not implemented)

Time = 0.14 (sec) , antiderivative size = 123, normalized size of antiderivative = 1.26

$$\int \frac{(ax+bx^2)^{3/2}}{(cx)^{5/2}}\,dx = -\frac{2\left(\frac{3\,a^2c|c|\arctan\left(\frac{\sqrt{ac}}{\sqrt{-ac}}\right)+4\,\sqrt{ac}\sqrt{-ac}a|c|}{\sqrt{-ac}c^2}-\frac{\frac{3\,a^2c^2|c|\arctan\left(\frac{\sqrt{bcx+ac}}{\sqrt{-ac}}\right)}{\sqrt{-ac}}+3\,\sqrt{bcx+ac}ac|c|+(bcx+ac)^{\frac{3}{2}}|c|}{c^3}\right)}{3\,c^2}$$

```
integrate((b*x^2+a*x)^(3/2)/(c*x)^(5/2),x, algorithm="giac")
```

```
-2/3*((3*a^2*c*abs(c)*arctan(sqrt(a*c)/sqrt(-a*c)) + 4*sqrt(a*c)*sqrt(-a*c
)*a*abs(c))/(sqrt(-a*c)*c^2) - (3*a^2*c^2*abs(c)*arctan(sqrt(b*c*x + a*c)/
sqrt(-a*c))/sqrt(-a*c) + 3*sqrt(b*c*x + a*c)*a*c*abs(c) + (b*c*x + a*c)^(3
/2)*abs(c))/c^3)/c^2
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(ax+bx^2)^{3/2}}{(cx)^{5/2}}\,dx = \int \frac{{\left(b\,x^2+a\,x\right)}^{3/2}}{{\left(c\,x\right)}^{5/2}} \,d x$$

```
int((a*x + b*x^2)^(3/2)/(c*x)^(5/2),x)
```

```
int((a*x + b*x^2)^(3/2)/(c*x)^(5/2), x)
```

Reduce [B] (verification not implemented)

Time = 0.51 (sec) , antiderivative size = 59, normalized size of antiderivative = 0.60

$$\int \frac{(ax+bx^2)^{3/2}}{(cx)^{5/2}}\,dx = \frac{\sqrt{c}\left(8\sqrt{bx+a}\,a+2\sqrt{bx+a}\,bx+3\sqrt{a}\log\left(\sqrt{bx+a}-\sqrt{a}\right)a-3\sqrt{a}\log\left(\sqrt{bx+a}+\right.\right.}{3c^3}$$

```
int((b*x^2+a*x)^(3/2)/(c*x)^(5/2),x)
```

```
(sqrt(c)*(8*sqrt(a + b*x)*a + 2*sqrt(a + b*x)*b*x + 3*sqrt(a)*log(sqrt(a +
 b*x) - sqrt(a))*a - 3*sqrt(a)*log(sqrt(a + b*x) + sqrt(a))*a))/(3*c**3)
```

3.93 $\int \frac{(ax+bx^2)^{3/2}}{(cx)^{7/2}}\,dx$

Optimal result

Integrand size = 21, antiderivative size = 98

$$\int \frac{(ax+bx^2)^{3/2}}{(cx)^{7/2}}\,dx=-\frac{a\sqrt{ax+bx^2}}{c^2(cx)^{3/2}}+\frac{2b\sqrt{ax+bx^2}}{c^3\sqrt{cx}}-\frac{3\sqrt{a}b\,\text{arctanh}\left(\frac{\sqrt{c}\sqrt{ax+bx^2}}{\sqrt{a}\sqrt{cx}}\right)}{c^{7/2}}$$

```
-a*(b*x^2+a*x)^(1/2)/c^2/(c*x)^(3/2)+2*b*(b*x^2+a*x)^(1/2)/c^3/(c*x)^(1/2)
-3*a^(1/2)*b*arctanh(c^(1/2)*(b*x^2+a*x)^(1/2)/a^(1/2)/(c*x)^(1/2))/c^(7/2
)
```

Mathematica [A] (verified)

Time = 0.08 (sec) , antiderivative size = 74, normalized size of antiderivative = 0.76

$$\int \frac{(ax+bx^2)^{3/2}}{(cx)^{7/2}}\,dx=-\frac{\sqrt{x(a+bx)}\left((a-2bx)\sqrt{a+bx}+3\sqrt{a}bx\,\text{arctanh}\left(\frac{\sqrt{a+bx}}{\sqrt{a}}\right)\right)}{c^2(cx)^{3/2}\sqrt{a+bx}}$$

```
Integrate[(a*x + b*x^2)^(3/2)/(c*x)^(7/2),x]
```

```
-((Sqrt[x*(a + b*x)]*((a - 2*b*x)*Sqrt[a + b*x] + 3*Sqrt[a]*b*x*ArcTanh[Sq
rt[a + b*x]/Sqrt[a]]))/(c^2*(c*x)^(3/2)*Sqrt[a + b*x]))
```

Rubi [A] (verified)

Time = 0.37 (sec) , antiderivative size = 104, normalized size of antiderivative = 1.06, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.190$, Rules used = {1130, 1131, 1136, 221}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\left(ax+bx^2\right)^{3/2}}{(cx)^{7/2}}dx$$

$$\Big\downarrow \text{ 1130}$$

$$\frac{3b\int \frac{\sqrt{bx^2+ax}}{(cx)^{3/2}}dx}{2c^2}-\frac{\left(ax+bx^2\right)^{3/2}}{c(cx)^{5/2}}$$

$$\Big\downarrow \text{ 1131}$$

$$\frac{3b\left(\frac{a\int \frac{1}{\sqrt{cx}\sqrt{bx^2+ax}}dx}{c}+\frac{2\sqrt{ax+bx^2}}{c\sqrt{cx}}\right)}{2c^2}-\frac{\left(ax+bx^2\right)^{3/2}}{c(cx)^{5/2}}$$

$$\Big\downarrow \text{ 1136}$$

$$\frac{3b\left(2a\int \frac{1}{\frac{c\left(bx^2+ax\right)}{x}-ac}d\frac{\sqrt{bx^2+ax}}{\sqrt{cx}}+\frac{2\sqrt{ax+bx^2}}{c\sqrt{cx}}\right)}{2c^2}-\frac{\left(ax+bx^2\right)^{3/2}}{c(cx)^{5/2}}$$

$$\Big\downarrow \text{ 221}$$

$$\frac{3b\left(\frac{2\sqrt{ax+bx^2}}{c\sqrt{cx}}-\frac{2\sqrt{a}\text{arctanh}\left(\frac{\sqrt{c}\sqrt{ax+bx^2}}{\sqrt{a}\sqrt{cx}}\right)}{c^{3/2}}\right)}{2c^2}-\frac{\left(ax+bx^2\right)^{3/2}}{c(cx)^{5/2}}$$

```
Int[(a*x + b*x^2)^(3/2)/(c*x)^(7/2),x]
```

```
-((a*x + b*x^2)^(3/2)/(c*(c*x)^(5/2))) + (3*b*((2*Sqrt[a*x + b*x^2])/(c*Sq
rt[c*x]) - (2*Sqrt[a]*ArcTanh[(Sqrt[c]*Sqrt[a*x + b*x^2])/(Sqrt[a]*Sqrt[c*
x])])/c^(3/2)))/(2*c^2)
```

Defintions of rubi rules used

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] :> Simp[(d + e*x)^(m + 1)*((a + b*x + c*x^2)^p/(e*(m + p + 1))), x]
- Simp[c*(p/(e^2*(m + p + 1)))   Int[(d + e*x)^(m + 2)*(a + b*x + c*x^2)^(p
 - 1), x], x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[c*d^2 - b*d*e + a*e^2, 0]
 && GtQ[p, 0] && (LtQ[m, -2] || EqQ[m + 2*p + 1, 0]) && NeQ[m + p + 1, 0] &
& IntegerQ[2*p]
```

```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] :> Simp[(d + e*x)^(m + 1)*((a + b*x + c*x^2)^p/(e*(m + 2*p + 1))), x
] - Simp[p*((2*c*d - b*e)/(e^2*(m + 2*p + 1)))   Int[(d + e*x)^(m + 1)*(a +
 b*x + c*x^2)^(p - 1), x], x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[c*d^2 - b
*d*e + a*e^2, 0] && GtQ[p, 0] && (LeQ[-2, m, 0] || EqQ[m + p + 1, 0]) && Ne
Q[m + 2*p + 1, 0] && IntegerQ[2*p]
```

```
Int[1/(Sqrt[(d_.) + (e_.)*(x_)]*Sqrt[(a_.) + (b_.)*(x_) + (c_.)*(x_)^2]), x
_Symbol] :> Simp[2*e   Subst[Int[1/(2*c*d - b*e + e^2*x^2), x], x, Sqrt[a +
 b*x + c*x^2]/Sqrt[d + e*x]], x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[c*d^2
- b*d*e + a*e^2, 0]
```

Maple [A] (verified)

Time = 0.37 (sec) , antiderivative size = 94, normalized size of antiderivative = 0.96

method	result	size
default	$\frac{\left(-3 \operatorname{arctanh}\left(\frac{\sqrt{c(bx+a)}}{\sqrt{ac}}\right) abcx+2bx\sqrt{c(bx+a)}\,\sqrt{ac}-\sqrt{c(bx+a)}\,\sqrt{ac}\,a\right)\sqrt{x(bx+a)}}{c^3 x\sqrt{cx}\,\sqrt{c(bx+a)}\,\sqrt{ac}}$	94
risch	$-\frac{a(bx+a)}{c^3\sqrt{cx}\,\sqrt{x(bx+a)}}+\frac{b\left(2\sqrt{cbx+ac}-\frac{3ac \operatorname{arctanh}\left(\frac{\sqrt{cbx+ac}}{\sqrt{ac}}\right)}{\sqrt{ac}}\right)\sqrt{c(bx+a)}\,x}{c^4\sqrt{cx}\,\sqrt{x(bx+a)}}$	95

```
int((b*x^2+a*x)^(3/2)/(c*x)^(7/2),x,method=_RETURNVERBOSE)
```

```
(-3*arctanh((c*(b*x+a))^(1/2)/(a*c)^(1/2))*a*b*c*x+2*b*x*(c*(b*x+a))^(1/2)
*(a*c)^(1/2)-(c*(b*x+a))^(1/2)*(a*c)^(1/2)*a)*(x*(b*x+a))^(1/2)/c^3/x/(c*x
)^(1/2)/(c*(b*x+a))^(1/2)/(a*c)^(1/2)
```

Fricas [A] (verification not implemented)

Time = 0.10 (sec) , antiderivative size = 171, normalized size of antiderivative = 1.74

$$\int \frac{(ax+bx^2)^{3/2}}{(cx)^{7/2}}\,dx = \left[\frac{3\,bcx^2\sqrt{\frac{a}{c}}\log\left(-\frac{bx^2+2\,ax-2\,\sqrt{bx^2+ax}\sqrt{cx}\sqrt{\frac{a}{c}}}{x^2}\right)+2\,\sqrt{bx^2+ax}(2\,bx-a)\sqrt{cx}}{2\,c^4x^2}, \frac{3\,bcx^2\sqrt{-}}{}\right.$$

```
integrate((b*x^2+a*x)^(3/2)/(c*x)^(7/2),x, algorithm="fricas")
```

```
[1/2*(3*b*c*x^2*sqrt(a/c)*log(-(b*x^2 + 2*a*x - 2*sqrt(b*x^2 + a*x)*sqrt(c
*x)*sqrt(a/c))/x^2) + 2*sqrt(b*x^2 + a*x)*(2*b*x - a)*sqrt(c*x))/(c^4*x^2)
, (3*b*c*x^2*sqrt(-a/c)*arctan(sqrt(b*x^2 + a*x)*sqrt(c*x)*sqrt(-a/c)/(a*x
)) + sqrt(b*x^2 + a*x)*(2*b*x - a)*sqrt(c*x))/(c^4*x^2)]
```

Sympy [F]

$$\int \frac{(ax+bx^2)^{3/2}}{(cx)^{7/2}}\,dx = \int \frac{(x(a+bx))^{\frac{3}{2}}}{(cx)^{\frac{7}{2}}}\,dx$$

```
integrate((b*x**2+a*x)**(3/2)/(c*x)**(7/2),x)
```

```
Integral((x*(a + b*x))**(3/2)/(c*x)**(7/2), x)
```

Maxima [F]

$$\int \frac{(ax+bx^2)^{3/2}}{(cx)^{7/2}}\,dx = \int \frac{(bx^2+ax)^{\frac{3}{2}}}{(cx)^{\frac{7}{2}}}\,dx$$

```
integrate((b*x^2+a*x)^(3/2)/(c*x)^(7/2),x, algorithm="maxima")
```

```
integrate((b*x^2 + a*x)^(3/2)/(c*x)^(7/2), x)
```

Giac [A] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 71, normalized size of antiderivative = 0.72

$$\int \frac{(ax+bx^2)^{3/2}}{(cx)^{7/2}}\,dx = \frac{\left(\frac{3\,ac|c|\arctan\left(\frac{\sqrt{bcx+ac}}{\sqrt{-ac}}\right)}{\sqrt{-ac}} + 2\sqrt{bcx+ac}|c| - \frac{\sqrt{bcx+ac}a|c|}{bx}\right)b}{c^5}$$

```
integrate((b*x^2+a*x)^(3/2)/(c*x)^(7/2),x, algorithm="giac")
```

```
(3*a*c*abs(c)*arctan(sqrt(b*c*x + a*c)/sqrt(-a*c))/sqrt(-a*c) + 2*sqrt(b*c
*x + a*c)*abs(c) - sqrt(b*c*x + a*c)*a*abs(c)/(b*x))*b/c^5
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(ax+bx^2)^{3/2}}{(cx)^{7/2}}\,dx=\int \frac{{\left(b\,x^2+a\,x\right)}^{3/2}}{{\left(c\,x\right)}^{7/2}} \,d x$$

```
int((a*x + b*x^2)^(3/2)/(c*x)^(7/2),x)
```

```
int((a*x + b*x^2)^(3/2)/(c*x)^(7/2), x)
```

Reduce [B] (verification not implemented)

Time = 0.46 (sec) , antiderivative size = 64, normalized size of antiderivative = 0.65

$$\int \frac{(ax+bx^2)^{3/2}}{(cx)^{7/2}}\,dx=\frac{\sqrt{c}\,\left(-2\sqrt{bx+a}\,a+4\sqrt{bx+a}\,bx+3\sqrt{a}\,\log\left(\sqrt{bx+a}-\sqrt{a}\right)bx-3\sqrt{a}\,\log\left(\sqrt{bx+a}\right.\right.}{2c^4x}$$

```
int((b*x^2+a*x)^(3/2)/(c*x)^(7/2),x)
```

```
(sqrt(c)*( - 2*sqrt(a + b*x)*a + 4*sqrt(a + b*x)*b*x + 3*sqrt(a)*log(sqrt(
a + b*x) - sqrt(a))*b*x - 3*sqrt(a)*log(sqrt(a + b*x) + sqrt(a))*b*x))/(2*
c**4*x)
```

3.94 $\int \frac{(ax+bx^2)^{3/2}}{(cx)^{9/2}} \, dx$

Optimal result

Integrand size = 21, antiderivative size = 106

$$\int \frac{(ax+bx^2)^{3/2}}{(cx)^{9/2}} \, dx = -\frac{a\sqrt{ax+bx^2}}{2c^2(cx)^{5/2}} - \frac{5b\sqrt{ax+bx^2}}{4c^3(cx)^{3/2}} - \frac{3b^2 \operatorname{arctanh}\left(\frac{\sqrt{c}\sqrt{ax+bx^2}}{\sqrt{a}\sqrt{cx}}\right)}{4\sqrt{a}c^{9/2}}$$

```
-1/2*a*(b*x^2+a*x)^(1/2)/c^2/(c*x)^(5/2)-5/4*b*(b*x^2+a*x)^(1/2)/c^3/(c*x)
^(3/2)-3/4*b^2*arctanh(c^(1/2)*(b*x^2+a*x)^(1/2)/a^(1/2)/(c*x)^(1/2))/a^(1
/2)/c^(9/2)
```

Mathematica [A] (verified)

Time = 0.12 (sec) , antiderivative size = 87, normalized size of antiderivative = 0.82

$$\int \frac{(ax+bx^2)^{3/2}}{(cx)^{9/2}} \, dx = -\frac{\sqrt{x(a+bx)}\left(\sqrt{a}\sqrt{a+bx}(2a+5bx) + 3b^2x^2 \operatorname{arctanh}\left(\frac{\sqrt{a+bx}}{\sqrt{a}}\right)\right)}{4\sqrt{a}c^2(cx)^{5/2}\sqrt{a+bx}}$$

```
Integrate[(a*x + b*x^2)^(3/2)/(c*x)^(9/2),x]
```

```
-1/4*(Sqrt[x*(a + b*x)]*(Sqrt[a]*Sqrt[a + b*x]*(2*a + 5*b*x) + 3*b^2*x^2*A
rcTanh[Sqrt[a + b*x]/Sqrt[a]]))/(Sqrt[a]*c^2*(c*x)^(5/2)*Sqrt[a + b*x])
```

Rubi [A] (verified)

Time = 0.37 (sec) , antiderivative size = 107, normalized size of antiderivative = 1.01, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.190$, Rules used = {1130, 1130, 1136, 221}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\left(ax+bx^2\right)^{3/2}}{(cx)^{9/2}}dx$$

$$\Big\downarrow \text{1130}$$

$$\frac{3b\int \frac{\sqrt{bx^2+ax}}{(cx)^{5/2}}dx}{4c^2}-\frac{\left(ax+bx^2\right)^{3/2}}{2c(cx)^{7/2}}$$

$$\Big\downarrow \text{1130}$$

$$\frac{3b\left(\frac{b\int \frac{1}{\sqrt{cx}\sqrt{bx^2+ax}}dx}{2c^2}-\frac{\sqrt{ax+bx^2}}{c(cx)^{3/2}}\right)}{4c^2}-\frac{\left(ax+bx^2\right)^{3/2}}{2c(cx)^{7/2}}$$

$$\Big\downarrow \text{1136}$$

$$\frac{3b\left(\frac{b\int \frac{1}{\frac{c\left(bx^2+ax\right)}{x}-ac}d\frac{\sqrt{bx^2+ax}}{\sqrt{cx}}}{c}-\frac{\sqrt{ax+bx^2}}{c(cx)^{3/2}}\right)}{4c^2}-\frac{\left(ax+bx^2\right)^{3/2}}{2c(cx)^{7/2}}$$

$$\Big\downarrow \text{221}$$

$$\frac{3b\left(-\frac{b\,\text{arctanh}\left(\frac{\sqrt{c}\sqrt{ax+bx^2}}{\sqrt{a}\sqrt{cx}}\right)}{\sqrt{a}c^{5/2}}-\frac{\sqrt{ax+bx^2}}{c(cx)^{3/2}}\right)}{4c^2}-\frac{\left(ax+bx^2\right)^{3/2}}{2c(cx)^{7/2}}$$

```
Int[(a*x + b*x^2)^(3/2)/(c*x)^(9/2),x]
```

```
-1/2*(a*x + b*x^2)^(3/2)/(c*(c*x)^(7/2)) + (3*b*(-(Sqrt[a*x + b*x^2]/(c*(c
*x)^(3/2))) - (b*ArcTanh[(Sqrt[c]*Sqrt[a*x + b*x^2])/(Sqrt[a]*Sqrt[c*x])])
/(Sqrt[a]*c^(5/2))))/(4*c^2)
```

Defintions of rubi rules used

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] :> Simp[(d + e*x)^(m + 1)*((a + b*x + c*x^2)^p/(e*(m + p + 1))), x]
- Simp[c*(p/(e^2*(m + p + 1)))   Int[(d + e*x)^(m + 2)*(a + b*x + c*x^2)^(p
 - 1), x], x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[c*d^2 - b*d*e + a*e^2, 0]
 && GtQ[p, 0] && (LtQ[m, -2] || EqQ[m + 2*p + 1, 0]) && NeQ[m + p + 1, 0] &
& IntegerQ[2*p]
```

```
Int[1/(Sqrt[(d_.) + (e_.)*(x_)]*Sqrt[(a_.) + (b_.)*(x_) + (c_.)*(x_)^2]), x
_Symbol] :> Simp[2*e   Subst[Int[1/(2*c*d - b*e + e^2*x^2), x], x, Sqrt[a +
 b*x + c*x^2]/Sqrt[d + e*x]], x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[c*d^2
- b*d*e + a*e^2, 0]
```

Maple [A] (verified)

Time = 0.36 (sec) , antiderivative size = 91, normalized size of antiderivative = 0.86

method	result	size
risch	$-\frac{(bx+a)(5bx+2a)}{4x\,c^{4}\sqrt{cx}\,\sqrt{x(bx+a)}}-\frac{3b^{2}\operatorname{arctanh}\left(\frac{\sqrt{cbx+ac}}{\sqrt{ac}}\right)\sqrt{c(bx+a)}\,x}{4\sqrt{ac}\,c^{4}\sqrt{cx}\,\sqrt{x(bx+a)}}$	91
default	$-\frac{\sqrt{x(bx+a)}\,\left(3\operatorname{arctanh}\left(\frac{\sqrt{c(bx+a)}}{\sqrt{ac}}\right)b^{2}c\,x^{2}+5bx\sqrt{c(bx+a)}\,\sqrt{ac}+2\sqrt{c(bx+a)}\,\sqrt{ac}\,a\right)}{4c^{4}x^{2}\sqrt{cx}\,\sqrt{c(bx+a)}\,\sqrt{ac}}$	98

```
int((b*x^2+a*x)^(3/2)/(c*x)^(9/2),x,method=_RETURNVERBOSE)
```

```
-1/4*(b*x+a)*(5*b*x+2*a)/x/c^4/(c*x)^(1/2)/(x*(b*x+a))^(1/2)-3/4*b^2/(a*c)
^(1/2)*arctanh((b*c*x+a*c)^(1/2)/(a*c)^(1/2))/c^4*(c*(b*x+a))^(1/2)/(c*x)^
(1/2)/(x*(b*x+a))^(1/2)*x
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 184, normalized size of antiderivative = 1.74

$$\int \frac{(ax+bx^2)^{3/2}}{(cx)^{9/2}}\,dx = \left[\frac{3\sqrt{ac}b^2x^3\log\left(-\frac{bcx^2+2\,acx-2\,\sqrt{bx^2+ax}\sqrt{ac}\sqrt{cx}}{x^2}\right)-2\,(5\,abx+2\,a^2)\sqrt{bx^2+ax}\sqrt{cx}}{8\,ac^5x^3}, \frac{3\,\sqrt{}\right.$$

```
integrate((b*x^2+a*x)^(3/2)/(c*x)^(9/2),x, algorithm="fricas")
```

```
[1/8*(3*sqrt(a*c)*b^2*x^3*log(-(b*c*x^2 + 2*a*c*x - 2*sqrt(b*x^2 + a*x)*sq
rt(a*c)*sqrt(c*x))/x^2) - 2*(5*a*b*x + 2*a^2)*sqrt(b*x^2 + a*x)*sqrt(c*x))
/(a*c^5*x^3), 1/4*(3*sqrt(-a*c)*b^2*x^3*arctan(sqrt(b*x^2 + a*x)*sqrt(-a*c
)*sqrt(c*x)/(a*c*x)) - (5*a*b*x + 2*a^2)*sqrt(b*x^2 + a*x)*sqrt(c*x))/(a*c
^5*x^3)]
```

Sympy [F]

$$\int \frac{(ax+bx^2)^{3/2}}{(cx)^{9/2}}\,dx = \int \frac{(x(a+bx))^{\frac{3}{2}}}{(cx)^{\frac{9}{2}}}\,dx$$

```
integrate((b*x**2+a*x)**(3/2)/(c*x)**(9/2),x)
```

```
Integral((x*(a + b*x))**(3/2)/(c*x)**(9/2), x)
```

Maxima [F]

$$\int \frac{(ax+bx^2)^{3/2}}{(cx)^{9/2}}\,dx = \int \frac{(bx^2+ax)^{\frac{3}{2}}}{(cx)^{\frac{9}{2}}}\,dx$$

```
integrate((b*x^2+a*x)^(3/2)/(c*x)^(9/2),x, algorithm="maxima")
```

```
integrate((b*x^2 + a*x)^(3/2)/(c*x)^(9/2), x)
```

Giac [A] (verification not implemented)

Time = 0.15 (sec) , antiderivative size = 87, normalized size of antiderivative = 0.82

$$\int \frac{(ax+bx^2)^{3/2}}{(cx)^{9/2}}\,dx = \frac{\frac{3\,b^3|c|\arctan\left(\frac{\sqrt{bcx+ac}}{\sqrt{-ac}}\right)}{\sqrt{-ac}} + \frac{3\,\sqrt{bcx+ac}ab^3c|c| - 5\,(bcx+ac)^{\frac{3}{2}}\,b^3|c|}{b^2c^2x^2}}{4\,bc^5}$$

```
integrate((b*x^2+a*x)^(3/2)/(c*x)^(9/2),x, algorithm="giac")
```

```
1/4*(3*b^3*abs(c)*arctan(sqrt(b*c*x + a*c)/sqrt(-a*c))/sqrt(-a*c) + (3*sqr
t(b*c*x + a*c)*a*b^3*c*abs(c) - 5*(b*c*x + a*c)^(3/2)*b^3*abs(c))/(b^2*c^2
*x^2))/(b*c^5)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(ax+bx^2)^{3/2}}{(cx)^{9/2}}\,dx = \int \frac{{\left(b\,x^2+a\,x\right)}^{3/2}}{{\left(c\,x\right)}^{9/2}}\,dx$$

```
int((a*x + b*x^2)^(3/2)/(c*x)^(9/2),x)
```

```
int((a*x + b*x^2)^(3/2)/(c*x)^(9/2), x)
```

Reduce [B] (verification not implemented)

Time = 0.24 (sec) , antiderivative size = 78, normalized size of antiderivative = 0.74

$$\int \frac{(ax+bx^2)^{3/2}}{(cx)^{9/2}}\,dx = \frac{\sqrt{c}\left(-4\sqrt{bx+a}\,a^2-10\sqrt{bx+a}\,abx+3\sqrt{a}\log\left(\sqrt{bx+a}-\sqrt{a}\right)b^2x^2-3\sqrt{a}\log\left(\sqrt{b\right.\right.}{8a\,c^5x^2}$$

```
int((b*x^2+a*x)^(3/2)/(c*x)^(9/2),x)
```

```
(sqrt(c)*( - 4*sqrt(a + b*x)*a**2 - 10*sqrt(a + b*x)*a*b*x + 3*sqrt(a)*log
(sqrt(a + b*x) - sqrt(a))*b**2*x**2 - 3*sqrt(a)*log(sqrt(a + b*x) + sqrt(a
))*b**2*x**2))/(8*a*c**5*x**2)
```

3.95 $\int \frac{(ax+bx^2)^{3/2}}{(cx)^{11/2}}\,dx$

Optimal result

Integrand size = 21, antiderivative size = 139

$$\int \frac{(ax+bx^2)^{3/2}}{(cx)^{11/2}}\,dx = -\frac{a\sqrt{ax+bx^2}}{3c^2(cx)^{7/2}} - \frac{7b\sqrt{ax+bx^2}}{12c^3(cx)^{5/2}} - \frac{b^2\sqrt{ax+bx^2}}{8ac^4(cx)^{3/2}} + \frac{b^3\operatorname{arctanh}\left(\frac{\sqrt{c}\sqrt{ax+bx^2}}{\sqrt{a}\sqrt{cx}}\right)}{8a^{3/2}c^{11/2}}$$

```
-1/3*a*(b*x^2+a*x)^(1/2)/c^2/(c*x)^(7/2)-7/12*b*(b*x^2+a*x)^(1/2)/c^3/(c*x
)^(5/2)-1/8*b^2*(b*x^2+a*x)^(1/2)/a/c^4/(c*x)^(3/2)+1/8*b^3*arctanh(c^(1/2
)*(b*x^2+a*x)^(1/2)/a^(1/2)/(c*x)^(1/2))/a^(3/2)/c^(11/2)
```

Mathematica [A] (verified)

Time = 0.15 (sec) , antiderivative size = 102, normalized size of antiderivative = 0.73

$$\int \frac{(ax+bx^2)^{3/2}}{(cx)^{11/2}}\,dx = \frac{\sqrt{cx}\sqrt{x(a+bx)}\left(-\sqrt{a}\sqrt{a+bx}(8a^2+14abx+3b^2x^2)+3b^3x^3\operatorname{arctanh}\left(\frac{\sqrt{a+bx}}{\sqrt{a}}\right)\right)}{24a^{3/2}c^6x^4\sqrt{a+bx}}$$

```
Integrate[(a*x + b*x^2)^(3/2)/(c*x)^(11/2),x]
```

```
(Sqrt[c*x]*Sqrt[x*(a + b*x)]*(-(Sqrt[a]*Sqrt[a + b*x]*(8*a^2 + 14*a*b*x +
3*b^2*x^2)) + 3*b^3*x^3*ArcTanh[Sqrt[a + b*x]/Sqrt[a]]))/(24*a^(3/2)*c^6*x
^4*Sqrt[a + b*x])
```

Rubi [A] (verified)

Time = 0.43 (sec) , antiderivative size = 142, normalized size of antiderivative = 1.02, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.238$, Rules used = {1130, 1130, 1135, 1136, 221}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(ax+bx^2)^{3/2}}{(cx)^{11/2}}dx$$

$$\Big\downarrow \text{1130}$$

$$\frac{b\int \frac{\sqrt{bx^2+ax}}{(cx)^{7/2}}dx}{2c^2}-\frac{(ax+bx^2)^{3/2}}{3c(cx)^{9/2}}$$

$$\Big\downarrow \text{1130}$$

$$\frac{b\left(\frac{b\int \frac{1}{(cx)^{3/2}\sqrt{bx^2+ax}}dx}{4c^2}-\frac{\sqrt{ax+bx^2}}{2c(cx)^{5/2}}\right)}{2c^2}-\frac{(ax+bx^2)^{3/2}}{3c(cx)^{9/2}}$$

$$\Big\downarrow \text{1135}$$

$$\frac{b\left(\frac{b\left(-\frac{b\int \frac{1}{\sqrt{cx}\sqrt{bx^2+ax}}dx}{2ac}-\frac{\sqrt{ax+bx^2}}{a(cx)^{3/2}}\right)}{4c^2}-\frac{\sqrt{ax+bx^2}}{2c(cx)^{5/2}}\right)}{2c^2}-\frac{(ax+bx^2)^{3/2}}{3c(cx)^{9/2}}$$

$$\Big\downarrow \text{1136}$$

$$\frac{b\left(\frac{b\left(-\frac{\frac{b\int \frac{1}{\frac{c\left(bx^2+ax\right)}{x}-ac}d\frac{\sqrt{bx^2+ax}}{\sqrt{cx}}}{a}}{}-\frac{\sqrt{ax+bx^2}}{a(cx)^{3/2}}\right)}{4c^2}-\frac{\sqrt{ax+bx^2}}{2c(cx)^{5/2}}\right)}{2c^2}-\frac{\left(ax+bx^2\right)^{3/2}}{3c(cx)^{9/2}}$$

$$\Big\downarrow \ 221$$

$$\frac{b\left(\frac{b\left(\frac{b\,\text{arctanh}\left(\frac{\sqrt{c}\sqrt{ax+bx^2}}{\sqrt{a}\sqrt{cx}}\right)}{a^{3/2}c^{3/2}}-\frac{\sqrt{ax+bx^2}}{a(cx)^{3/2}}\right)}{4c^2}-\frac{\sqrt{ax+bx^2}}{2c(cx)^{5/2}}\right)}{2c^2}-\frac{\left(ax+bx^2\right)^{3/2}}{3c(cx)^{9/2}}$$

```
Int[(a*x + b*x^2)^(3/2)/(c*x)^(11/2),x]
```

```
-1/3*(a*x + b*x^2)^(3/2)/(c*(c*x)^(9/2)) + (b*(-1/2*Sqrt[a*x + b*x^2]/(c*(
c*x)^(5/2)) + (b*(-(Sqrt[a*x + b*x^2]/(a*(c*x)^(3/2))) + (b*ArcTanh[(Sqrt[
c]*Sqrt[a*x + b*x^2])/(Sqrt[a]*Sqrt[c*x])])/(a^(3/2)*c^(3/2))))/(4*c^2)))/
(2*c^2)
```

Defintions of rubi rules used

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] :> Simp[(d + e*x)^(m + 1)*((a + b*x + c*x^2)^p/(e*(m + p + 1))), x]
- Simp[c*(p/(e^2*(m + p + 1)))   Int[(d + e*x)^(m + 2)*(a + b*x + c*x^2)^(p
 - 1), x], x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[c*d^2 - b*d*e + a*e^2, 0]
 && GtQ[p, 0] && (LtQ[m, -2] || EqQ[m + 2*p + 1, 0]) && NeQ[m + p + 1, 0] &
& IntegerQ[2*p]
```

```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] :> Simp[(-e)*(d + e*x)^m*((a + b*x + c*x^2)^(p + 1)/((m + p + 1)*(2*
c*d - b*e))), x] + Simp[c*((m + 2*p + 2)/((m + p + 1)*(2*c*d - b*e)))   Int
[(d + e*x)^(m + 1)*(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, p},
 x] && EqQ[c*d^2 - b*d*e + a*e^2, 0] && LtQ[m, 0] && NeQ[m + p + 1, 0] && I
ntegerQ[2*p]
```

```
Int[1/(Sqrt[(d_.) + (e_.)*(x_)]*Sqrt[(a_.) + (b_.)*(x_) + (c_.)*(x_)^2]), x
_Symbol] :> Simp[2*e   Subst[Int[1/(2*c*d - b*e + e^2*x^2), x], x, Sqrt[a +
 b*x + c*x^2]/Sqrt[d + e*x]], x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[c*d^2
- b*d*e + a*e^2, 0]
```

Maple [A] (verified)

Time = 0.50 (sec) , antiderivative size = 108, normalized size of antiderivative = 0.78

method	result	size
risch	$-\frac{(bx+a)\left(3b^{2}x^{2}+14abx+8a^{2}\right)}{24x^{2}a\,c^{5}\sqrt{cx}\,\sqrt{x(bx+a)}}+\frac{b^{3}\operatorname{arctanh}\left(\frac{\sqrt{cbx+ac}}{\sqrt{ac}}\right)\sqrt{c(bx+a)}\,x}{8a\sqrt{ac}\,c^{5}\sqrt{cx}\,\sqrt{x(bx+a)}}$	108
default	$\frac{\sqrt{x(bx+a)}\left(3\operatorname{arctanh}\left(\frac{\sqrt{c(bx+a)}}{\sqrt{ac}}\right)c\,b^{3}x^{3}-3b^{2}x^{2}\sqrt{c(bx+a)}\,\sqrt{ac}-14abx\sqrt{c(bx+a)}\,\sqrt{ac}-8\sqrt{c(bx+a)}\,\sqrt{ac}\,a^{2}\right)}{24c^{5}x^{3}\sqrt{cx}\,\sqrt{c(bx+a)}\,a\sqrt{ac}}$	126

```
int((b*x^2+a*x)^(3/2)/(c*x)^(11/2),x,method=_RETURNVERBOSE)
```

```
-1/24*(b*x+a)*(3*b^2*x^2+14*a*b*x+8*a^2)/x^2/a/c^5/(c*x)^(1/2)/(x*(b*x+a))
^(1/2)+1/8/a*b^3/(a*c)^(1/2)*arctanh((b*c*x+a*c)^(1/2)/(a*c)^(1/2))/c^5*(c
*(b*x+a))^(1/2)/(c*x)^(1/2)/(x*(b*x+a))^(1/2)*x
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 205, normalized size of antiderivative = 1.47

$$\int \frac{(ax+bx^2)^{3/2}}{(cx)^{11/2}}\,dx = \left[\frac{3\sqrt{ac}b^3x^4\log\left(-\frac{bcx^2+2\,acx+2\sqrt{bx^2+ax}\sqrt{ac}\sqrt{cx}}{x^2}\right)-2\,(3\,ab^2x^2+14\,a^2bx+8\,a^3)\sqrt{bx^2+}}{48\,a^2c^6x^4}\right.$$

```
integrate((b*x^2+a*x)^(3/2)/(c*x)^(11/2),x, algorithm="fricas")
```

```
[1/48*(3*sqrt(a*c)*b^3*x^4*log(-(b*c*x^2 + 2*a*c*x + 2*sqrt(b*x^2 + a*x)*s
qrt(a*c)*sqrt(c*x))/x^2) - 2*(3*a*b^2*x^2 + 14*a^2*b*x + 8*a^3)*sqrt(b*x^2
 + a*x)*sqrt(c*x))/(a^2*c^6*x^4), -1/24*(3*sqrt(-a*c)*b^3*x^4*arctan(sqrt(
b*x^2 + a*x)*sqrt(-a*c)*sqrt(c*x)/(a*c*x)) + (3*a*b^2*x^2 + 14*a^2*b*x + 8
*a^3)*sqrt(b*x^2 + a*x)*sqrt(c*x))/(a^2*c^6*x^4)]
```

Sympy [F]

$$\int \frac{(ax+bx^2)^{3/2}}{(cx)^{11/2}}\,dx = \int \frac{(x(a+bx))^{\frac{3}{2}}}{(cx)^{\frac{11}{2}}}\,dx$$

```
integrate((b*x**2+a*x)**(3/2)/(c*x)**(11/2),x)
```

```
Integral((x*(a + b*x))**(3/2)/(c*x)**(11/2), x)
```

Maxima [F]

$$\int \frac{(ax+bx^2)^{3/2}}{(cx)^{11/2}}\,dx = \int \frac{(bx^2+ax)^{\frac{3}{2}}}{(cx)^{\frac{11}{2}}}\,dx$$

```
integrate((b*x^2+a*x)^(3/2)/(c*x)^(11/2),x, algorithm="maxima")
```

```
integrate((b*x^2 + a*x)^(3/2)/(c*x)^(11/2), x)
```

Giac [A] (verification not implemented)

Time = 0.40 (sec) , antiderivative size = 108, normalized size of antiderivative = 0.78

$$\int \frac{(ax+bx^2)^{3/2}}{(cx)^{11/2}}\,dx = -\frac{b^3\left(\frac{3\,|c|\arctan\left(\frac{\sqrt{bcx+ac}}{\sqrt{-ac}}\right)}{\sqrt{-ac}ac} - \frac{3\,\sqrt{bcx+ac}a^2c^2|c|-8\,(bcx+ac)^{\frac{3}{2}}ac|c|-3\,(bcx+ac)^{\frac{5}{2}}|c|}{ab^3c^4x^3}\right)}{24\,c^5}$$

```
integrate((b*x^2+a*x)^(3/2)/(c*x)^(11/2),x, algorithm="giac")
```

```
-1/24*b^3*(3*abs(c)*arctan(sqrt(b*c*x + a*c)/sqrt(-a*c))/(sqrt(-a*c)*a*c)
- (3*sqrt(b*c*x + a*c)*a^2*c^2*abs(c) - 8*(b*c*x + a*c)^(3/2)*a*c*abs(c) -
 3*(b*c*x + a*c)^(5/2)*abs(c))/(a*b^3*c^4*x^3))/c^5
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(ax+bx^2)^{3/2}}{(cx)^{11/2}}\,dx = \int \frac{{\left(b\,x^2+a\,x\right)}^{3/2}}{{\left(c\,x\right)}^{11/2}} \,dx$$

```
int((a*x + b*x^2)^(3/2)/(c*x)^(11/2),x)
```

```
int((a*x + b*x^2)^(3/2)/(c*x)^(11/2), x)
```

Reduce [B] (verification not implemented)

Time = 0.36 (sec) , antiderivative size = 95, normalized size of antiderivative = 0.68

$$\int \frac{(ax+bx^2)^{3/2}}{(cx)^{11/2}}\,dx = \frac{\sqrt{c}\,\big(-16\sqrt{bx+a}\,a^3-28\sqrt{bx+a}\,a^2bx-6\sqrt{bx+a}\,a\,b^2x^2-3\sqrt{a}\log\big(\sqrt{bx+a}-\sqrt{a}}{48a^2c^6x^3}$$

```
int((b*x^2+a*x)^(3/2)/(c*x)^(11/2),x)
```

```
(sqrt(c)*( - 16*sqrt(a + b*x)*a**3 - 28*sqrt(a + b*x)*a**2*b*x - 6*sqrt(a
+ b*x)*a*b**2*x**2 - 3*sqrt(a)*log(sqrt(a + b*x) - sqrt(a))*b**3*x**3 + 3*
sqrt(a)*log(sqrt(a + b*x) + sqrt(a))*b**3*x**3))/(48*a**2*c**6*x**3)
```

3.96 $\int \frac{(ax+bx^2)^{3/2}}{(cx)^{13/2}}\,dx$

Optimal result

Integrand size = 21, antiderivative size = 172

$$\int \frac{(ax+bx^2)^{3/2}}{(cx)^{13/2}}\,dx = -\frac{a\sqrt{ax+bx^2}}{4c^2(cx)^{9/2}} - \frac{3b\sqrt{ax+bx^2}}{8c^3(cx)^{7/2}} - \frac{b^2\sqrt{ax+bx^2}}{32ac^4(cx)^{5/2}} + \frac{3b^3\sqrt{ax+bx^2}}{64a^2c^5(cx)^{3/2}} - \frac{3b^4\operatorname{arctanh}\left(\frac{\sqrt{c}\sqrt{ax+bx^2}}{\sqrt{a}\sqrt{cx}}\right)}{64a^{5/2}c^{13/2}}$$

```
-1/4*a*(b*x^2+a*x)^(1/2)/c^2/(c*x)^(9/2)-3/8*b*(b*x^2+a*x)^(1/2)/c^3/(c*x)
^(7/2)-1/32*b^2*(b*x^2+a*x)^(1/2)/a/c^4/(c*x)^(5/2)+3/64*b^3*(b*x^2+a*x)^(
1/2)/a^2/c^5/(c*x)^(3/2)-3/64*b^4*arctanh(c^(1/2)*(b*x^2+a*x)^(1/2)/a^(1/2
)/(c*x)^(1/2))/a^(5/2)/c^(13/2)
```

Mathematica [A] (verified)

Time = 0.17 (sec) , antiderivative size = 112, normalized size of antiderivative = 0.65

$$\int \frac{(ax+bx^2)^{3/2}}{(cx)^{13/2}}\,dx = -\frac{\sqrt{cx}\sqrt{x(a+bx)}\left(\sqrt{a}\sqrt{a+bx}(16a^3+24a^2bx+2ab^2x^2-3b^3x^3)+3b^4x^4\operatorname{arctanh}\left(\frac{\sqrt{a+bx}}{\sqrt{a}}\right)\right)}{64a^{5/2}c^7x^5\sqrt{a+bx}}$$

```
Integrate[(a*x + b*x^2)^(3/2)/(c*x)^(13/2),x]
```

```
-1/64*(Sqrt[c*x]*Sqrt[x*(a + b*x)]*(Sqrt[a]*Sqrt[a + b*x]*(16*a^3 + 24*a^2
*b*x + 2*a*b^2*x^2 - 3*b^3*x^3) + 3*b^4*x^4*ArcTanh[Sqrt[a + b*x]/Sqrt[a]]
))/(a^(5/2)*c^7*x^5*Sqrt[a + b*x])
```

Rubi [A] (verified)

Time = 0.50 (sec) , antiderivative size = 181, normalized size of antiderivative = 1.05, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.286$, Rules used = {1130, 1130, 1135, 1135, 1136, 221}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(ax+bx^2)^{3/2}}{(cx)^{13/2}}dx$$

$$\Big\downarrow \text{1130}$$

$$\frac{3b\int \frac{\sqrt{bx^2+ax}}{(cx)^{9/2}}dx}{8c^2}-\frac{(ax+bx^2)^{3/2}}{4c(cx)^{11/2}}$$

$$\Big\downarrow \text{1130}$$

$$\frac{3b\left(\frac{b\int \frac{1}{(cx)^{5/2}\sqrt{bx^2+ax}}dx}{6c^2}-\frac{\sqrt{ax+bx^2}}{3c(cx)^{7/2}}\right)}{8c^2}-\frac{(ax+bx^2)^{3/2}}{4c(cx)^{11/2}}$$

$$\Big\downarrow \text{1135}$$

$$\frac{3b\left(\frac{b\left(-\frac{3b\int \frac{1}{(cx)^{3/2}\sqrt{bx^2+ax}}dx}{4ac}-\frac{\sqrt{ax+bx^2}}{2a(cx)^{5/2}}\right)}{6c^2}-\frac{\sqrt{ax+bx^2}}{3c(cx)^{7/2}}\right)}{8c^2}-\frac{(ax+bx^2)^{3/2}}{4c(cx)^{11/2}}$$

$$\Big\downarrow \text{1135}$$

$$\frac{3b\left(\frac{b\left(-\frac{3b\left(-\frac{b\int \frac{1}{\sqrt{cx}\sqrt{bx^2+ax}}dx}{2ac}-\frac{\sqrt{ax+bx^2}}{a(cx)^{3/2}}\right)}{4ac}-\frac{\sqrt{ax+bx^2}}{2a(cx)^{5/2}}\right)}{6c^2}-\frac{\sqrt{ax+bx^2}}{3c(cx)^{7/2}}\right)}{8c^2}-\frac{(ax+bx^2)^{3/2}}{4c(cx)^{11/2}}$$

$$\Big\downarrow \text{1136}$$

$$\frac{3b\left(\frac{b\left(-\frac{3b\left(-\frac{b\int \frac{1}{\frac{c\left(bx^2+ax\right)}{x}-ac}d\frac{\sqrt{bx^2+ax}}{\sqrt{cx}}}{a}-\frac{\sqrt{ax+bx^2}}{a(cx)^{3/2}}\right)}{4ac}-\frac{\sqrt{ax+bx^2}}{2a(cx)^{5/2}}\right)}{6c^2}-\frac{\sqrt{ax+bx^2}}{3c(cx)^{7/2}}\right)}{8c^2}-\frac{(ax+bx^2)^{3/2}}{4c(cx)^{11/2}}$$

$$\Big\downarrow \text{221}$$

$$\frac{3b\left(\frac{b\left(-\frac{3b\left(\frac{b\,\text{arctanh}\left(\frac{\sqrt{c}\sqrt{ax+bx^2}}{\sqrt{a}\sqrt{cx}}\right)}{a^{3/2}c^{3/2}}-\frac{\sqrt{ax+bx^2}}{a(cx)^{3/2}}\right)}{4ac}-\frac{\sqrt{ax+bx^2}}{2a(cx)^{5/2}}\right)}{6c^2}-\frac{\sqrt{ax+bx^2}}{3c(cx)^{7/2}}\right)}{8c^2}-\frac{(ax+bx^2)^{3/2}}{4c(cx)^{11/2}}$$

```
Int[(a*x + b*x^2)^(3/2)/(c*x)^(13/2),x]
```

```
-1/4*(a*x + b*x^2)^(3/2)/(c*(c*x)^(11/2)) + (3*b*(-1/3*Sqrt[a*x + b*x^2]/(
c*(c*x)^(7/2)) + (b*(-1/2*Sqrt[a*x + b*x^2]/(a*(c*x)^(5/2)) - (3*b*(-(Sqrt
[a*x + b*x^2]/(a*(c*x)^(3/2))) + (b*ArcTanh[(Sqrt[c]*Sqrt[a*x + b*x^2])/(S
qrt[a]*Sqrt[c*x])])/(a^(3/2)*c^(3/2))))/(4*a*c)))/(6*c^2)))/(8*c^2)
```

Defintions of rubi rules used

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] :> Simp[(d + e*x)^(m + 1)*((a + b*x + c*x^2)^p/(e*(m + p + 1))), x]
- Simp[c*(p/(e^2*(m + p + 1)))   Int[(d + e*x)^(m + 2)*(a + b*x + c*x^2)^(p
 - 1), x], x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[c*d^2 - b*d*e + a*e^2, 0]
 && GtQ[p, 0] && (LtQ[m, -2] || EqQ[m + 2*p + 1, 0]) && NeQ[m + p + 1, 0] &
& IntegerQ[2*p]
```

```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] :> Simp[(-e)*(d + e*x)^m*((a + b*x + c*x^2)^(p + 1)/((m + p + 1)*(2*
c*d - b*e))), x] + Simp[c*((m + 2*p + 2)/((m + p + 1)*(2*c*d - b*e)))   Int
[(d + e*x)^(m + 1)*(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, p},
 x] && EqQ[c*d^2 - b*d*e + a*e^2, 0] && LtQ[m, 0] && NeQ[m + p + 1, 0] && I
ntegerQ[2*p]
```

```
Int[1/(Sqrt[(d_.) + (e_.)*(x_)]*Sqrt[(a_.) + (b_.)*(x_) + (c_.)*(x_)^2]), x
_Symbol] :> Simp[2*e   Subst[Int[1/(2*c*d - b*e + e^2*x^2), x], x, Sqrt[a +
 b*x + c*x^2]/Sqrt[d + e*x]], x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[c*d^2
- b*d*e + a*e^2, 0]
```

Maple [A] (verified)

Time = 0.38 (sec) , antiderivative size = 119, normalized size of antiderivative = 0.69

method	result
risch	$-\frac{(bx+a)(-3b^3x^3+2a\,b^2x^2+24a^2bx+16a^3)}{64x^3a^2c^6\sqrt{cx}\,\sqrt{x(bx+a)}}-\frac{3b^4\operatorname{arctanh}\left(\frac{\sqrt{cbx+ac}}{\sqrt{ac}}\right)\sqrt{c(bx+a)}\,x}{64a^2\sqrt{ac}\,c^6\sqrt{cx}\,\sqrt{x(bx+a)}}$
default	$-\frac{\sqrt{x(bx+a)}\left(3\operatorname{arctanh}\left(\frac{\sqrt{c(bx+a)}}{\sqrt{ac}}\right)c\,b^4x^4-3b^3x^3\sqrt{c(bx+a)}\,\sqrt{ac}+2a\,b^2x^2\sqrt{c(bx+a)}\,\sqrt{ac}+24a^2bx\sqrt{c(bx+a)}\,\sqrt{ac}+16\sqrt{c(bx+a}\right.}{64c^6x^4\sqrt{cx}\,\sqrt{c(bx+a)}\,a^2\sqrt{ac}}$

```
int((b*x^2+a*x)^(3/2)/(c*x)^(13/2),x,method=_RETURNVERBOSE)
```

```
-1/64*(b*x+a)*(-3*b^3*x^3+2*a*b^2*x^2+24*a^2*b*x+16*a^3)/x^3/a^2/c^6/(c*x)
^(1/2)/(x*(b*x+a))^(1/2)-3/64/a^2*b^4/(a*c)^(1/2)*arctanh((b*c*x+a*c)^(1/2
)/(a*c)^(1/2))/c^6*(c*(b*x+a))^(1/2)/(c*x)^(1/2)/(x*(b*x+a))^(1/2)*x
```

Fricas [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 227, normalized size of antiderivative = 1.32

$$\int \frac{(ax+bx^2)^{3/2}}{(cx)^{13/2}}\,dx = \left[\frac{3\sqrt{ac}b^4x^5\log\left(-\frac{bcx^2+2\,acx-2\,\sqrt{bx^2+ax}\sqrt{ac}\sqrt{cx}}{x^2}\right)+2\,(3\,ab^3x^3-2\,a^2b^2x^2-24\,a^3bx-16}{128\,a^3c^7x^5}\right.$$

```
integrate((b*x^2+a*x)^(3/2)/(c*x)^(13/2),x, algorithm="fricas")
```

```
[1/128*(3*sqrt(a*c)*b^4*x^5*log(-(b*c*x^2 + 2*a*c*x - 2*sqrt(b*x^2 + a*x)*
sqrt(a*c)*sqrt(c*x))/x^2) + 2*(3*a*b^3*x^3 - 2*a^2*b^2*x^2 - 24*a^3*b*x -
16*a^4)*sqrt(b*x^2 + a*x)*sqrt(c*x))/(a^3*c^7*x^5), 1/64*(3*sqrt(-a*c)*b^4
*x^5*arctan(sqrt(b*x^2 + a*x)*sqrt(-a*c)*sqrt(c*x)/(a*c*x)) + (3*a*b^3*x^3
 - 2*a^2*b^2*x^2 - 24*a^3*b*x - 16*a^4)*sqrt(b*x^2 + a*x)*sqrt(c*x))/(a^3*
c^7*x^5)]
```

Sympy [F(-1)]

Timed out.

$$\int \frac{(ax+bx^2)^{3/2}}{(cx)^{13/2}}\,dx = \text{Timed out}$$

```
integrate((b*x**2+a*x)**(3/2)/(c*x)**(13/2),x)
```

```
Timed out
```

Maxima [F]

$$\int \frac{(ax+bx^2)^{3/2}}{(cx)^{13/2}}\,dx = \int \frac{(bx^2+ax)^{\frac{3}{2}}}{(cx)^{\frac{13}{2}}}\,dx$$

```
integrate((b*x^2+a*x)^(3/2)/(c*x)^(13/2),x, algorithm="maxima")
```

```
integrate((b*x^2 + a*x)^(3/2)/(c*x)^(13/2), x)
```

Giac [A] (verification not implemented)

Time = 0.21 (sec) , antiderivative size = 139, normalized size of antiderivative = 0.81

$$\int \frac{(ax+bx^2)^{3/2}}{(cx)^{13/2}}\,dx = \frac{\frac{3\,b^5|c|\arctan\left(\frac{\sqrt{bcx+ac}}{\sqrt{-ac}}\right)}{\sqrt{-ac}a^2} + \frac{3\,\sqrt{bcx+ac}a^3b^5c^3|c|-11\,(bcx+ac)^{\frac{3}{2}}a^2b^5c^2|c|-11\,(bcx+ac)^{\frac{5}{2}}ab^5c|c|+3\,(bcx+ac)^{\frac{7}{2}}b^5|c}{a^2b^4c^4x^4}}{64\,bc^7}$$

```
integrate((b*x^2+a*x)^(3/2)/(c*x)^(13/2),x, algorithm="giac")
```

```
1/64*(3*b^5*abs(c)*arctan(sqrt(b*c*x + a*c)/sqrt(-a*c))/(sqrt(-a*c)*a^2) +
 (3*sqrt(b*c*x + a*c)*a^3*b^5*c^3*abs(c) - 11*(b*c*x + a*c)^(3/2)*a^2*b^5*
c^2*abs(c) - 11*(b*c*x + a*c)^(5/2)*a*b^5*c*abs(c) + 3*(b*c*x + a*c)^(7/2)
*b^5*abs(c))/(a^2*b^4*c^4*x^4))/(b*c^7)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(ax+bx^2)^{3/2}}{(cx)^{13/2}}\,dx = \int \frac{(b\,x^2+a\,x)^{3/2}}{(c\,x)^{13/2}}\,dx$$

```
int((a*x + b*x^2)^(3/2)/(c*x)^(13/2),x)
```

```
int((a*x + b*x^2)^(3/2)/(c*x)^(13/2), x)
```

Reduce [B] (verification not implemented)

Time = 0.52 (sec) , antiderivative size = 112, normalized size of antiderivative = 0.65

$$\int \frac{(ax+bx^2)^{3/2}}{(cx)^{13/2}}\,dx = \frac{\sqrt{c}\left(-32\sqrt{bx+a}\,a^4-48\sqrt{bx+a}\,a^3bx-4\sqrt{bx+a}\,a^2b^2x^2+6\sqrt{bx+a}\,a\,b^3x^3+3\sqrt{a}\right.}{128a^3c^7x^4}$$

```
int((b*x^2+a*x)^(3/2)/(c*x)^(13/2),x)
```

```
(sqrt(c)*( - 32*sqrt(a + b*x)*a**4 - 48*sqrt(a + b*x)*a**3*b*x - 4*sqrt(a
+ b*x)*a**2*b**2*x**2 + 6*sqrt(a + b*x)*a*b**3*x**3 + 3*sqrt(a)*log(sqrt(a
 + b*x) - sqrt(a))*b**4*x**4 - 3*sqrt(a)*log(sqrt(a + b*x) + sqrt(a))*b**4
*x**4))/(128*a**3*c**7*x**4)
```

3.97 $\int \frac{(cx)^{7/2}}{\sqrt{ax+bx^2}}\,dx$

Optimal result

Integrand size = 21, antiderivative size = 124

$$\int \frac{(cx)^{7/2}}{\sqrt{ax+bx^2}}\,dx = -\frac{2a^3c^4\sqrt{ax+bx^2}}{b^4\sqrt{cx}}+\frac{2a^2c^5(ax+bx^2)^{3/2}}{b^4(cx)^{3/2}} -\frac{6ac^6(ax+bx^2)^{5/2}}{5b^4(cx)^{5/2}}+\frac{2c^7(ax+bx^2)^{7/2}}{7b^4(cx)^{7/2}}$$

```
-2*a^3*c^4*(b*x^2+a*x)^(1/2)/b^4/(c*x)^(1/2)+2*a^2*c^5*(b*x^2+a*x)^(3/2)/b
^4/(c*x)^(3/2)-6/5*a*c^6*(b*x^2+a*x)^(5/2)/b^4/(c*x)^(5/2)+2/7*c^7*(b*x^2+
a*x)^(7/2)/b^4/(c*x)^(7/2)
```

Mathematica [A] (verified)

Time = 0.04 (sec) , antiderivative size = 58, normalized size of antiderivative = 0.47

$$\int \frac{(cx)^{7/2}}{\sqrt{ax+bx^2}}\,dx = \frac{2c^4\sqrt{x(a+bx)}(-16a^3+8a^2bx-6ab^2x^2+5b^3x^3)}{35b^4\sqrt{cx}}$$

```
Integrate[(c*x)^(7/2)/Sqrt[a*x + b*x^2],x]
```

```
(2*c^4*Sqrt[x*(a + b*x)]*(-16*a^3 + 8*a^2*b*x - 6*a*b^2*x^2 + 5*b^3*x^3))/
(35*b^4*Sqrt[c*x])
```

Rubi [A] (verified)

Time = 0.41 (sec) , antiderivative size = 136, normalized size of antiderivative = 1.10, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.190$, Rules used = {1128, 1128, 1128, 1122}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(cx)^{7/2}}{\sqrt{ax+bx^2}}dx$$

$$\Big\downarrow \text{ 1128}$$

$$\frac{2c(cx)^{5/2}\sqrt{ax+bx^2}}{7b}-\frac{6ac\int \frac{(cx)^{5/2}}{\sqrt{bx^2+ax}}dx}{7b}$$

$$\Big\downarrow \text{ 1128}$$

$$\frac{2c(cx)^{5/2}\sqrt{ax+bx^2}}{7b}-\frac{6ac\left(\frac{2c(cx)^{3/2}\sqrt{ax+bx^2}}{5b}-\frac{4ac\int \frac{(cx)^{3/2}}{\sqrt{bx^2+ax}}dx}{5b}\right)}{7b}$$

$$\Big\downarrow \text{ 1128}$$

$$\frac{2c(cx)^{5/2}\sqrt{ax+bx^2}}{7b}-\frac{6ac\left(\frac{2c(cx)^{3/2}\sqrt{ax+bx^2}}{5b}-\frac{4ac\left(\frac{2c\sqrt{cx}\sqrt{ax+bx^2}}{3b}-\frac{2ac\int \frac{\sqrt{cx}}{\sqrt{bx^2+ax}}dx}{3b}\right)}{5b}\right)}{7b}$$

$$\Big\downarrow \text{ 1122}$$

$$\frac{2c(cx)^{5/2}\sqrt{ax+bx^2}}{7b}-\frac{6ac\left(\frac{2c(cx)^{3/2}\sqrt{ax+bx^2}}{5b}-\frac{4ac\left(\frac{2c\sqrt{cx}\sqrt{ax+bx^2}}{3b}-\frac{4ac^2\sqrt{ax+bx^2}}{3b^2\sqrt{cx}}\right)}{5b}\right)}{7b}$$

```
Int[(c*x)^(7/2)/Sqrt[a*x + b*x^2],x]
```

```
(2*c*(c*x)^(5/2)*Sqrt[a*x + b*x^2])/(7*b) - (6*a*c*((2*c*(c*x)^(3/2)*Sqrt[
a*x + b*x^2])/(5*b) - (4*a*c*((-4*a*c^2*Sqrt[a*x + b*x^2])/(3*b^2*Sqrt[c*x
]) + (2*c*Sqrt[c*x]*Sqrt[a*x + b*x^2])/(3*b)))/(5*b)))/(7*b)
```

Defintions of rubi rules used

```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] :> Simp[e*(d + e*x)^(m - 1)*((a + b*x + c*x^2)^(p + 1)/(c*(p + 1))),
 x] /; FreeQ[{a, b, c, d, e, m, p}, x] && EqQ[c*d^2 - b*d*e + a*e^2, 0] &&
EqQ[m + p, 0]
```

```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] :> Simp[e*(d + e*x)^(m - 1)*((a + b*x + c*x^2)^(p + 1)/(c*(m + 2*p +
 1))), x] + Simp[Simplify[m + p]*((2*c*d - b*e)/(c*(m + 2*p + 1)))   Int[(d
 + e*x)^(m - 1)*(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, m, p},
 x] && EqQ[c*d^2 - b*d*e + a*e^2, 0] && IGtQ[Simplify[m + p], 0]
```

Maple [A] (verified)

Time = 0.34 (sec) , antiderivative size = 56, normalized size of antiderivative = 0.45

method	result	size
default	$-\frac{2 c^{3} \sqrt{c x}\, \sqrt{x \left(b x +a \right)}\, \left(-5 b^{3} x^{3}+6 a \,b^{2} x^{2}-8 a^{2} b x +16 a^{3}\right)}{35 x \,b^{4}}$	56
risch	$-\frac{2 c^{4} x \left(-5 b^{3} x^{3}+6 a \,b^{2} x^{2}-8 a^{2} b x +16 a^{3}\right) \left(b x +a \right)}{35 \sqrt{c x}\, \sqrt{x \left(b x +a \right)}\, b^{4}}$	59
gosper	$-\frac{2 \left(b x +a \right) \left(-5 b^{3} x^{3}+6 a \,b^{2} x^{2}-8 a^{2} b x +16 a^{3}\right) \left(c x \right)^{\frac{7}{2}}}{35 b^{4} x^{3} \sqrt{b \,x^{2}+a x}}$	60
orering	$-\frac{2 \left(b x +a \right) \left(-5 b^{3} x^{3}+6 a \,b^{2} x^{2}-8 a^{2} b x +16 a^{3}\right) \left(c x \right)^{\frac{7}{2}}}{35 b^{4} x^{3} \sqrt{b \,x^{2}+a x}}$	60

```
int((c*x)^(7/2)/(b*x^2+a*x)^(1/2),x,method=_RETURNVERBOSE)
```

```
-2/35*c^3/x*(c*x)^(1/2)*(x*(b*x+a))^(1/2)*(-5*b^3*x^3+6*a*b^2*x^2-8*a^2*b*
x+16*a^3)/b^4
```

Fricas [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 66, normalized size of antiderivative = 0.53

$$\int \frac{(cx)^{7/2}}{\sqrt{ax+bx^2}}\,dx = \frac{2\,(5\,b^3c^3x^3 - 6\,ab^2c^3x^2 + 8\,a^2bc^3x - 16\,a^3c^3)\sqrt{bx^2+ax}\sqrt{cx}}{35\,b^4x}$$

```
integrate((c*x)^(7/2)/(b*x^2+a*x)^(1/2),x, algorithm="fricas")
```

```
2/35*(5*b^3*c^3*x^3 - 6*a*b^2*c^3*x^2 + 8*a^2*b*c^3*x - 16*a^3*c^3)*sqrt(b
*x^2 + a*x)*sqrt(c*x)/(b^4*x)
```

Sympy [F]

$$\int \frac{(cx)^{7/2}}{\sqrt{ax+bx^2}}\,dx = \int \frac{(cx)^{\frac{7}{2}}}{\sqrt{x\,(a+bx)}}\,dx$$

```
integrate((c*x)**(7/2)/(b*x**2+a*x)**(1/2),x)
```

```
Integral((c*x)**(7/2)/sqrt(x*(a + b*x)), x)
```

Maxima [A] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 68, normalized size of antiderivative = 0.55

$$\int \frac{(cx)^{7/2}}{\sqrt{ax+bx^2}}\,dx = \frac{2\left(5\,b^4c^{\frac{7}{2}}x^4 - ab^3c^{\frac{7}{2}}x^3 + 2\,a^2b^2c^{\frac{7}{2}}x^2 - 8\,a^3bc^{\frac{7}{2}}x - 16\,a^4c^{\frac{7}{2}}\right)}{35\,\sqrt{bx+a}b^4}$$

```
integrate((c*x)^(7/2)/(b*x^2+a*x)^(1/2),x, algorithm="maxima")
```

```
2/35*(5*b^4*c^(7/2)*x^4 - a*b^3*c^(7/2)*x^3 + 2*a^2*b^2*c^(7/2)*x^2 - 8*a^
3*b*c^(7/2)*x - 16*a^4*c^(7/2))/(sqrt(b*x + a)*b^4)
```

Giac [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 95, normalized size of antiderivative = 0.77

$$\int \frac{(cx)^{7/2}}{\sqrt{ax+bx^2}}\,dx = -\frac{2\left(\frac{35\sqrt{bcx+ac}a^3c^3}{b^4} - \frac{16\sqrt{ac}a^3c^3}{b^4} - \frac{35\,(bcx+ac)^{\frac{3}{2}}a^2c^2 - 21\,(bcx+ac)^{\frac{5}{2}}ac + 5\,(bcx+ac)^{\frac{7}{2}}}{b^4}\right)c}{35\,|c|}$$

```
integrate((c*x)^(7/2)/(b*x^2+a*x)^(1/2),x, algorithm="giac")
```

```
-2/35*(35*sqrt(b*c*x + a*c)*a^3*c^3/b^4 - 16*sqrt(a*c)*a^3*c^3/b^4 - (35*(
b*c*x + a*c)^(3/2)*a^2*c^2 - 21*(b*c*x + a*c)^(5/2)*a*c + 5*(b*c*x + a*c)^
(7/2))/b^4)*c/abs(c)
```

Mupad [B] (verification not implemented)

Time = 9.56 (sec) , antiderivative size = 57, normalized size of antiderivative = 0.46

$$\int \frac{(cx)^{7/2}}{\sqrt{ax+bx^2}}\,dx = -\frac{2\,c^3\,\sqrt{b\,x^2+a\,x}\,\sqrt{c\,x}\,\left(16\,a^3-8\,a^2\,b\,x+6\,a\,b^2\,x^2-5\,b^3\,x^3\right)}{35\,b^4\,x}$$

```
int((c*x)^(7/2)/(a*x + b*x^2)^(1/2),x)
```

```
-(2*c^3*(a*x + b*x^2)^(1/2)*(c*x)^(1/2)*(16*a^3 - 5*b^3*x^3 + 6*a*b^2*x^2
- 8*a^2*b*x))/(35*b^4*x)
```

Reduce [B] (verification not implemented)

Time = 0.39 (sec) , antiderivative size = 46, normalized size of antiderivative = 0.37

$$\int \frac{(cx)^{7/2}}{\sqrt{ax+bx^2}}\,dx = \frac{2\sqrt{c}\,\sqrt{bx+a}\,c^3(5b^3x^3-6a\,b^2x^2+8a^2bx-16a^3)}{35b^4}$$

```
int((c*x)^(7/2)/(b*x^2+a*x)^(1/2),x)
```

```
(2*sqrt(c)*sqrt(a + b*x)*c**3*( - 16*a**3 + 8*a**2*b*x - 6*a*b**2*x**2 + 5
*b**3*x**3))/(35*b**4)
```

3.98 $\int \frac{(cx)^{5/2}}{\sqrt{ax+bx^2}}\,dx$

Optimal result

Integrand size = 21, antiderivative size = 93

$$\int \frac{(cx)^{5/2}}{\sqrt{ax+bx^2}}\,dx = \frac{2a^2c^3\sqrt{ax+bx^2}}{b^3\sqrt{cx}} - \frac{4ac^4(ax+bx^2)^{3/2}}{3b^3(cx)^{3/2}} + \frac{2c^5(ax+bx^2)^{5/2}}{5b^3(cx)^{5/2}}$$

```
2*a^2*c^3*(b*x^2+a*x)^(1/2)/b^3/(c*x)^(1/2)-4/3*a*c^4*(b*x^2+a*x)^(3/2)/b^
3/(c*x)^(3/2)+2/5*c^5*(b*x^2+a*x)^(5/2)/b^3/(c*x)^(5/2)
```

Mathematica [A] (verified)

Time = 0.03 (sec) , antiderivative size = 47, normalized size of antiderivative = 0.51

$$\int \frac{(cx)^{5/2}}{\sqrt{ax+bx^2}}\,dx = \frac{2c^3\sqrt{x(a+bx)}(8a^2-4abx+3b^2x^2)}{15b^3\sqrt{cx}}$$

```
Integrate[(c*x)^(5/2)/Sqrt[a*x + b*x^2],x]
```

```
(2*c^3*Sqrt[x*(a + b*x)]*(8*a^2 - 4*a*b*x + 3*b^2*x^2))/(15*b^3*Sqrt[c*x])
```

Rubi [A] (verified)

Time = 0.36 (sec) , antiderivative size = 98, normalized size of antiderivative = 1.05, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.143, Rules used = {1128, 1128, 1122}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(cx)^{5/2}}{\sqrt{ax+bx^2}}dx$$

$$\Big\downarrow \text{1128}$$

$$\frac{2c(cx)^{3/2}\sqrt{ax+bx^2}}{5b}-\frac{4ac\int \frac{(cx)^{3/2}}{\sqrt{bx^2+ax}}dx}{5b}$$

$$\Big\downarrow \text{1128}$$

$$\frac{2c(cx)^{3/2}\sqrt{ax+bx^2}}{5b}-\frac{4ac\left(\frac{2c\sqrt{cx}\sqrt{ax+bx^2}}{3b}-\frac{2ac\int \frac{\sqrt{cx}}{\sqrt{bx^2+ax}}dx}{3b}\right)}{5b}$$

$$\Big\downarrow \text{1122}$$

$$\frac{2c(cx)^{3/2}\sqrt{ax+bx^2}}{5b}-\frac{4ac\left(\frac{2c\sqrt{cx}\sqrt{ax+bx^2}}{3b}-\frac{4ac^2\sqrt{ax+bx^2}}{3b^2\sqrt{cx}}\right)}{5b}$$

```
Int[(c*x)^(5/2)/Sqrt[a*x + b*x^2],x]
```

```
(2*c*(c*x)^(3/2)*Sqrt[a*x + b*x^2])/(5*b) - (4*a*c*((-4*a*c^2*Sqrt[a*x + b
*x^2])/(3*b^2*Sqrt[c*x]) + (2*c*Sqrt[c*x]*Sqrt[a*x + b*x^2])/(3*b)))/(5*b)
```

Defintions of rubi rules used

```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] :> Simp[e*(d + e*x)^(m - 1)*((a + b*x + c*x^2)^(p + 1)/(c*(p + 1))),
 x] /; FreeQ[{a, b, c, d, e, m, p}, x] && EqQ[c*d^2 - b*d*e + a*e^2, 0] &&
EqQ[m + p, 0]
```

```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] :> Simp[e*(d + e*x)^(m - 1)*((a + b*x + c*x^2)^(p + 1)/(c*(m + 2*p +
 1))), x] + Simp[Simplify[m + p]*((2*c*d - b*e)/(c*(m + 2*p + 1)))   Int[(d
 + e*x)^(m - 1)*(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, m, p},
 x] && EqQ[c*d^2 - b*d*e + a*e^2, 0] && IGtQ[Simplify[m + p], 0]
```

Maple [A] (verified)

Time = 0.40 (sec) , antiderivative size = 45, normalized size of antiderivative = 0.48

method	result	size
default	$\frac{2c^{2}\sqrt{cx}\,\sqrt{x(bx+a)}\,(3b^{2}x^{2}-4abx+8a^{2})}{15x\,b^{3}}$	45
risch	$\frac{2c^{3}x(3b^{2}x^{2}-4abx+8a^{2})(bx+a)}{15\sqrt{cx}\,\sqrt{x(bx+a)}\,b^{3}}$	48
gosper	$\frac{2(bx+a)(3b^{2}x^{2}-4abx+8a^{2})(cx)^{\frac{5}{2}}}{15b^{3}x^{2}\sqrt{b\,x^{2}+ax}}$	49
orering	$\frac{2(bx+a)(3b^{2}x^{2}-4abx+8a^{2})(cx)^{\frac{5}{2}}}{15b^{3}x^{2}\sqrt{b\,x^{2}+ax}}$	49

```
int((c*x)^(5/2)/(b*x^2+a*x)^(1/2),x,method=_RETURNVERBOSE)
```

```
2/15*c^2/x*(c*x)^(1/2)*(x*(b*x+a))^(1/2)*(3*b^2*x^2-4*a*b*x+8*a^2)/b^3
```

Fricas [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 52, normalized size of antiderivative = 0.56

$$\int \frac{(cx)^{5/2}}{\sqrt{ax+bx^2}}\,dx = \frac{2\,(3\,b^2c^2x^2 - 4\,abc^2x + 8\,a^2c^2)\sqrt{bx^2+ax}\sqrt{cx}}{15\,b^3x}$$

```
integrate((c*x)^(5/2)/(b*x^2+a*x)^(1/2),x, algorithm="fricas")
```

```
2/15*(3*b^2*c^2*x^2 - 4*a*b*c^2*x + 8*a^2*c^2)*sqrt(b*x^2 + a*x)*sqrt(c*x)
/(b^3*x)
```

Sympy [F]

$$\int \frac{(cx)^{5/2}}{\sqrt{ax+bx^2}}\,dx = \int \frac{(cx)^{\frac{5}{2}}}{\sqrt{x\,(a+bx)}}\,dx$$

```
integrate((c*x)**(5/2)/(b*x**2+a*x)**(1/2),x)
```

```
Integral((c*x)**(5/2)/sqrt(x*(a + b*x)), x)
```

Maxima [A] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 54, normalized size of antiderivative = 0.58

$$\int \frac{(cx)^{5/2}}{\sqrt{ax+bx^2}}\,dx = \frac{2\left(3\,b^3c^{\frac{5}{2}}x^3 - ab^2c^{\frac{5}{2}}x^2 + 4\,a^2bc^{\frac{5}{2}}x + 8\,a^3c^{\frac{5}{2}}\right)}{15\,\sqrt{bx+a}b^3}$$

```
integrate((c*x)^(5/2)/(b*x^2+a*x)^(1/2),x, algorithm="maxima")
```

```
2/15*(3*b^3*c^(5/2)*x^3 - a*b^2*c^(5/2)*x^2 + 4*a^2*b*c^(5/2)*x + 8*a^3*c^
(5/2))/(sqrt(b*x + a)*b^3)
```

Giac [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 82, normalized size of antiderivative = 0.88

$$\int \frac{(cx)^{5/2}}{\sqrt{ax+bx^2}}\,dx = \frac{2\,c^4\left(\frac{15\,\sqrt{bcx+ac}a^2}{b^3c} - \frac{8\,\sqrt{ac}a^2}{b^3c} - \frac{10\,(bcx+ac)^{\frac{3}{2}}ac-3\,(bcx+ac)^{\frac{5}{2}}}{b^3c^3}\right)}{15\,|c|}$$

```
integrate((c*x)^(5/2)/(b*x^2+a*x)^(1/2),x, algorithm="giac")
```

```
2/15*c^4*(15*sqrt(b*c*x + a*c)*a^2/(b^3*c) - 8*sqrt(a*c)*a^2/(b^3*c) - (10
*(b*c*x + a*c)^(3/2)*a*c - 3*(b*c*x + a*c)^(5/2))/(b^3*c^3))/abs(c)
```

Mupad [B] (verification not implemented)

Time = 9.18 (sec) , antiderivative size = 46, normalized size of antiderivative = 0.49

$$\int \frac{(cx)^{5/2}}{\sqrt{ax+bx^2}}\,dx = \frac{2\,c^2\,\sqrt{b\,x^2+a\,x}\,\sqrt{c\,x}\,(8\,a^2-4\,a\,b\,x+3\,b^2\,x^2)}{15\,b^3\,x}$$

```
int((c*x)^(5/2)/(a*x + b*x^2)^(1/2),x)
```

```
(2*c^2*(a*x + b*x^2)^(1/2)*(c*x)^(1/2)*(8*a^2 + 3*b^2*x^2 - 4*a*b*x))/(15*
b^3*x)
```

Reduce [B] (verification not implemented)

Time = 0.22 (sec) , antiderivative size = 35, normalized size of antiderivative = 0.38

$$\int \frac{(cx)^{5/2}}{\sqrt{ax+bx^2}}\,dx = \frac{2\sqrt{c}\,\sqrt{bx+a}\,c^2(3b^2x^2-4abx+8a^2)}{15b^3}$$

```
int((c*x)^(5/2)/(b*x^2+a*x)^(1/2),x)
```

```
(2*sqrt(c)*sqrt(a + b*x)*c**2*(8*a**2 - 4*a*b*x + 3*b**2*x**2))/(15*b**3)
```

3.99 $\int \frac{(cx)^{3/2}}{\sqrt{ax+bx^2}}\,dx$

Optimal result

Integrand size = 21, antiderivative size = 60

$$\int \frac{(cx)^{3/2}}{\sqrt{ax+bx^2}}\,dx = -\frac{2ac^2\sqrt{ax+bx^2}}{b^2\sqrt{cx}} + \frac{2c^3(ax+bx^2)^{3/2}}{3b^2(cx)^{3/2}}$$

```
-2*a*c^2*(b*x^2+a*x)^(1/2)/b^2/(c*x)^(1/2)+2/3*c^3*(b*x^2+a*x)^(3/2)/b^2/(
c*x)^(3/2)
```

Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 35, normalized size of antiderivative = 0.58

$$\int \frac{(cx)^{3/2}}{\sqrt{ax+bx^2}}\,dx = \frac{2c^2(-2a+bx)\sqrt{x(a+bx)}}{3b^2\sqrt{cx}}$$

```
Integrate[(c*x)^(3/2)/Sqrt[a*x + b*x^2],x]
```

```
(2*c^2*(-2*a + b*x)*Sqrt[x*(a + b*x)])/(3*b^2*Sqrt[c*x])
```

Rubi [A] (verified)

Time = 0.29 (sec) , antiderivative size = 60, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.095$, Rules used = $\{1128, 1122\}$

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(cx)^{3/2}}{\sqrt{ax+bx^2}}dx$$

$$\Big\downarrow \text{1128}$$

$$\frac{2c\sqrt{cx}\sqrt{ax+bx^2}}{3b}-\frac{2ac\int \frac{\sqrt{cx}}{\sqrt{bx^2+ax}}dx}{3b}$$

$$\Big\downarrow \text{1122}$$

$$\frac{2c\sqrt{cx}\sqrt{ax+bx^2}}{3b}-\frac{4ac^2\sqrt{ax+bx^2}}{3b^2\sqrt{cx}}$$

```
Int[(c*x)^(3/2)/Sqrt[a*x + b*x^2],x]
```

```
(-4*a*c^2*Sqrt[a*x + b*x^2])/(3*b^2*Sqrt[c*x]) + (2*c*Sqrt[c*x]*Sqrt[a*x +
 b*x^2])/(3*b)
```

Defintions of rubi rules used

```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] :> Simp[e*(d + e*x)^(m - 1)*((a + b*x + c*x^2)^(p + 1)/(c*(p + 1))),
 x] /; FreeQ[{a, b, c, d, e, m, p}, x] && EqQ[c*d^2 - b*d*e + a*e^2, 0] &&
EqQ[m + p, 0]
```

```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] :> Simp[e*(d + e*x)^(m - 1)*((a + b*x + c*x^2)^(p + 1)/(c*(m + 2*p +
 1))), x] + Simp[Simplify[m + p]*((2*c*d - b*e)/(c*(m + 2*p + 1)))   Int[(d
 + e*x)^(m - 1)*(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, m, p},
 x] && EqQ[c*d^2 - b*d*e + a*e^2, 0] && IGtQ[Simplify[m + p], 0]
```

Maple [A] (verified)

Time = 0.34 (sec) , antiderivative size = 32, normalized size of antiderivative = 0.53

method	result	size
default	$-\frac{2c\sqrt{cx}\,\sqrt{x(bx+a)}\,(-bx+2a)}{3x\,b^{2}}$	32
risch	$-\frac{2c^{2}x(-bx+2a)(bx+a)}{3\sqrt{cx}\,\sqrt{x(bx+a)}\,b^{2}}$	37
gosper	$-\frac{2(bx+a)(-bx+2a)(cx)^{\frac{3}{2}}}{3b^{2}x\sqrt{b\,x^{2}+ax}}$	38
orering	$-\frac{2(bx+a)(-bx+2a)(cx)^{\frac{3}{2}}}{3b^{2}x\sqrt{b\,x^{2}+ax}}$	38

```
int((c*x)^(3/2)/(b*x^2+a*x)^(1/2),x,method=_RETURNVERBOSE)
```

```
-2/3*c/x*(c*x)^(1/2)*(x*(b*x+a))^(1/2)*(-b*x+2*a)/b^2
```

Fricas [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 33, normalized size of antiderivative = 0.55

$$\int \frac{(cx)^{3/2}}{\sqrt{ax+bx^2}}\,dx = \frac{2\,(bcx-2\,ac)\sqrt{bx^2+ax}\sqrt{cx}}{3\,b^2x}$$

```
integrate((c*x)^(3/2)/(b*x^2+a*x)^(1/2),x, algorithm="fricas")
```

```
2/3*(b*c*x - 2*a*c)*sqrt(b*x^2 + a*x)*sqrt(c*x)/(b^2*x)
```

Sympy [F]

$$\int \frac{(cx)^{3/2}}{\sqrt{ax+bx^2}}\,dx = \int \frac{(cx)^{\frac{3}{2}}}{\sqrt{x\,(a+bx)}}\,dx$$

```
integrate((c*x)**(3/2)/(b*x**2+a*x)**(1/2),x)
```

```
Integral((c*x)**(3/2)/sqrt(x*(a + b*x)), x)
```

Maxima [A] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 39, normalized size of antiderivative = 0.65

$$\int \frac{(cx)^{3/2}}{\sqrt{ax+bx^2}}\,dx = \frac{2\left(b^2c^{\frac{3}{2}}x^2 - abc^{\frac{3}{2}}x - 2\,a^2c^{\frac{3}{2}}\right)}{3\,\sqrt{bx+a}b^2}$$

```
integrate((c*x)^(3/2)/(b*x^2+a*x)^(1/2),x, algorithm="maxima")
```

```
2/3*(b^2*c^(3/2)*x^2 - a*b*c^(3/2)*x - 2*a^2*c^(3/2))/(sqrt(b*x + a)*b^2)
```

Giac [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 52, normalized size of antiderivative = 0.87

$$\int \frac{(cx)^{3/2}}{\sqrt{ax+bx^2}}\,dx = -\frac{2\,c\left(\frac{3\,\sqrt{bcx+ac}ac}{b^2} - \frac{2\,\sqrt{ac}ac}{b^2} - \frac{(bcx+ac)^{\frac{3}{2}}}{b^2}\right)}{3\,|c|}$$

```
integrate((c*x)^(3/2)/(b*x^2+a*x)^(1/2),x, algorithm="giac")
```

```
-2/3*c*(3*sqrt(b*c*x + a*c)*a*c/b^2 - 2*sqrt(a*c)*a*c/b^2 - (b*c*x + a*c)^
(3/2)/b^2)/abs(c)
```

Mupad [B] (verification not implemented)

Time = 9.20 (sec) , antiderivative size = 41, normalized size of antiderivative = 0.68

$$\int \frac{(cx)^{3/2}}{\sqrt{ax+bx^2}}\,dx = -\frac{\sqrt{b\,x^2+a\,x}\,\left(\frac{4\,a\,c\,\sqrt{c\,x}}{3\,b^2}-\frac{2\,c\,x\,\sqrt{c\,x}}{3\,b}\right)}{x}$$

```
int((c*x)^(3/2)/(a*x + b*x^2)^(1/2),x)
```

```
-((a*x + b*x^2)^(1/2)*((4*a*c*(c*x)^(1/2))/(3*b^2) - (2*c*x*(c*x)^(1/2))/(
3*b)))/x
```

Reduce [B] (verification not implemented)

Time = 0.34 (sec) , antiderivative size = 21, normalized size of antiderivative = 0.35

$$\int \frac{(cx)^{3/2}}{\sqrt{ax+bx^2}}\,dx = \frac{2\sqrt{c}\,\sqrt{bx+a}\,c(bx-2a)}{3b^2}$$

```
int((c*x)^(3/2)/(b*x^2+a*x)^(1/2),x)
```

```
(2*sqrt(c)*sqrt(a + b*x)*c*( - 2*a + b*x))/(3*b**2)
```

3.100 $\int \frac{\sqrt{cx}}{\sqrt{ax+bx^2}}\,dx$

Optimal result

Integrand size = 21, antiderivative size = 26

$$\int \frac{\sqrt{cx}}{\sqrt{ax+bx^2}}\,dx = \frac{2c\sqrt{ax+bx^2}}{b\sqrt{cx}}$$

```
2*c*(b*x^2+a*x)^(1/2)/b/(c*x)^(1/2)
```

Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 24, normalized size of antiderivative = 0.92

$$\int \frac{\sqrt{cx}}{\sqrt{ax+bx^2}}\,dx = \frac{2c\sqrt{x(a+bx)}}{b\sqrt{cx}}$$

```
Integrate[Sqrt[c*x]/Sqrt[a*x + b*x^2],x]
```

```
(2*c*Sqrt[x*(a + b*x)])/(b*Sqrt[c*x])
```

Rubi [A] (verified)

Time = 0.24 (sec) , antiderivative size = 26, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}} = 0.048$, Rules used = $\{1122\}$

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sqrt{cx}}{\sqrt{ax+bx^2}}\,dx$$
$$\Big\downarrow \ 1122$$
$$\frac{2c\sqrt{ax+bx^2}}{b\sqrt{cx}}$$

```
Int[Sqrt[c*x]/Sqrt[a*x + b*x^2],x]
```

```
(2*c*Sqrt[a*x + b*x^2])/(b*Sqrt[c*x])
```

Defintions of rubi rules used

```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] :> Simp[e*(d + e*x)^(m - 1)*((a + b*x + c*x^2)^(p + 1)/(c*(p + 1))),
 x] /; FreeQ[{a, b, c, d, e, m, p}, x] && EqQ[c*d^2 - b*d*e + a*e^2, 0] &&
EqQ[m + p, 0]
```

Maple [A] (verified)

Time = 0.34 (sec) , antiderivative size = 23, normalized size of antiderivative = 0.88

method	result	size
default	$\frac{2\sqrt{cx}\,\sqrt{x(bx+a)}}{xb}$	23
gosper	$\frac{2(bx+a)\sqrt{cx}}{b\sqrt{b\,x^2+ax}}$	27
risch	$\frac{2cx(bx+a)}{\sqrt{cx}\,\sqrt{x(bx+a)}\,b}$	27
orering	$\frac{2(bx+a)\sqrt{cx}}{b\sqrt{b\,x^2+ax}}$	27

```
int((c*x)^(1/2)/(b*x^2+a*x)^(1/2),x,method=_RETURNVERBOSE)
```

```
2*(c*x)^(1/2)*(x*(b*x+a))^(1/2)/x/b
```

Fricas [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 24, normalized size of antiderivative = 0.92

$$\int \frac{\sqrt{cx}}{\sqrt{ax+bx^2}}\,dx = \frac{2\sqrt{bx^2+ax}\sqrt{cx}}{bx}$$

```
integrate((c*x)^(1/2)/(b*x^2+a*x)^(1/2),x, algorithm="fricas")
```

```
2*sqrt(b*x^2 + a*x)*sqrt(c*x)/(b*x)
```

Sympy [F]

$$\int \frac{\sqrt{cx}}{\sqrt{ax+bx^2}}\,dx = \int \frac{\sqrt{cx}}{\sqrt{x\,(a+bx)}}\,dx$$

```
integrate((c*x)**(1/2)/(b*x**2+a*x)**(1/2),x)
```

```
Integral(sqrt(c*x)/sqrt(x*(a + b*x)), x)
```

Maxima [A] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 24, normalized size of antiderivative = 0.92

$$\int \frac{\sqrt{cx}}{\sqrt{ax+bx^2}}\,dx = \frac{2\left(b\sqrt{c}x + a\sqrt{c}\right)}{\sqrt{bx+a}b}$$

```
integrate((c*x)^(1/2)/(b*x^2+a*x)^(1/2),x, algorithm="maxima")
```

```
2*(b*sqrt(c)*x + a*sqrt(c))/(sqrt(b*x + a)*b)
```

Giac [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 32, normalized size of antiderivative = 1.23

$$\int \frac{\sqrt{cx}}{\sqrt{ax+bx^2}}\,dx = \frac{2\,c\left(\frac{\sqrt{bcx+ac}}{b} - \frac{\sqrt{ac}}{b}\right)}{|c|}$$

```
integrate((c*x)^(1/2)/(b*x^2+a*x)^(1/2),x, algorithm="giac")
```

```
2*c*(sqrt(b*c*x + a*c)/b - sqrt(a*c)/b)/abs(c)
```

Mupad [B] (verification not implemented)

Time = 9.39 (sec) , antiderivative size = 24, normalized size of antiderivative = 0.92

$$\int \frac{\sqrt{cx}}{\sqrt{ax+bx^2}}\,dx = \frac{2\,\sqrt{b\,x^2+a\,x}\,\sqrt{c\,x}}{b\,x}$$

```
int((c*x)^(1/2)/(a*x + b*x^2)^(1/2),x)
```

```
(2*(a*x + b*x^2)^(1/2)*(c*x)^(1/2))/(b*x)
```

Reduce [B] (verification not implemented)

Time = 0.48 (sec) , antiderivative size = 13, normalized size of antiderivative = 0.50

$$\int \frac{\sqrt{cx}}{\sqrt{ax+bx^2}}\,dx=\frac{2\sqrt{c}\,\sqrt{bx+a}}{b}$$

```
int((c*x)^(1/2)/(b*x^2+a*x)^(1/2),x)
```

```
(2*sqrt(c)*sqrt(a + b*x))/b
```

3.101 $\int \frac{1}{\sqrt{cx}\sqrt{ax+bx^2}}\,dx$

Optimal result

Integrand size = 21, antiderivative size = 44

$$\int \frac{1}{\sqrt{cx}\sqrt{ax+bx^2}}\,dx = -\frac{2\operatorname{arctanh}\left(\frac{\sqrt{c}\sqrt{ax+bx^2}}{\sqrt{a}\sqrt{cx}}\right)}{\sqrt{a}\sqrt{c}}$$

```
-2*arctanh(c^(1/2)*(b*x^2+a*x)^(1/2)/a^(1/2)/(c*x)^(1/2))/a^(1/2)/c^(1/2)
```

Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 51, normalized size of antiderivative = 1.16

$$\int \frac{1}{\sqrt{cx}\sqrt{ax+bx^2}}\,dx = -\frac{2x\sqrt{a+bx}\operatorname{arctanh}\left(\frac{\sqrt{a+bx}}{\sqrt{a}}\right)}{\sqrt{a}\sqrt{cx}\sqrt{x(a+bx)}}$$

```
Integrate[1/(Sqrt[c*x]*Sqrt[a*x + b*x^2]),x]
```

```
(-2*x*Sqrt[a + b*x]*ArcTanh[Sqrt[a + b*x]/Sqrt[a]])/(Sqrt[a]*Sqrt[c*x]*Sqr
t[x*(a + b*x)])
```

Rubi [A] (verified)

Time = 0.26 (sec) , antiderivative size = 44, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.095$, Rules used = {1136, 221}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{\sqrt{cx}\sqrt{ax+bx^2}}\,dx$$
$$\Big\downarrow \text{1136}$$
$$2c\int \frac{1}{\frac{c(bx^2+ax)}{x}-ac}d\frac{\sqrt{bx^2+ax}}{\sqrt{cx}}$$
$$\Big\downarrow \text{221}$$
$$-\frac{2\text{arctanh}\left(\frac{\sqrt{c}\sqrt{ax+bx^2}}{\sqrt{a}\sqrt{cx}}\right)}{\sqrt{a}\sqrt{c}}$$

```
Int[1/(Sqrt[c*x]*Sqrt[a*x + b*x^2]),x]
```

```
(-2*ArcTanh[(Sqrt[c]*Sqrt[a*x + b*x^2])/(Sqrt[a]*Sqrt[c*x])])/(Sqrt[a]*Sqr
t[c])
```

Defintions of rubi rules used

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

```
Int[1/(Sqrt[(d_.) + (e_.)*(x_)]*Sqrt[(a_.) + (b_.)*(x_) + (c_.)*(x_)^2]), x
_Symbol] :> Simp[2*e   Subst[Int[1/(2*c*d - b*e + e^2*x^2), x], x, Sqrt[a +
 b*x + c*x^2]/Sqrt[d + e*x]], x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[c*d^2
- b*d*e + a*e^2, 0]
```

Maple [A] (verified)

Time = 0.34 (sec) , antiderivative size = 48, normalized size of antiderivative = 1.09

method	result	size
default	$-\frac{2\sqrt{x(bx+a)}\,c\,\operatorname{arctanh}\left(\frac{\sqrt{c(bx+a)}}{\sqrt{ac}}\right)}{\sqrt{cx}\,\sqrt{c(bx+a)}\,\sqrt{ac}}$	48

```
int(1/(c*x)^(1/2)/(b*x^2+a*x)^(1/2),x,method=_RETURNVERBOSE)
```

```
-2/(c*x)^(1/2)*(x*(b*x+a))^(1/2)*c/(c*(b*x+a))^(1/2)/(a*c)^(1/2)*arctanh((
c*(b*x+a))^(1/2)/(a*c)^(1/2))
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 101, normalized size of antiderivative = 2.30

$$\int \frac{1}{\sqrt{cx}\sqrt{ax+bx^2}}\,dx = \left[\frac{\sqrt{ac}\log\left(-\frac{bcx^2+2\,acx-2\,\sqrt{bx^2+ax}\sqrt{ac}\sqrt{cx}}{x^2}\right)}{ac}, \frac{2\,\sqrt{-ac}\arctan\left(\frac{\sqrt{bx^2+ax}\sqrt{-ac}\sqrt{cx}}{acx}\right)}{ac}\right]$$

```
integrate(1/(c*x)^(1/2)/(b*x^2+a*x)^(1/2),x, algorithm="fricas")
```

```
[sqrt(a*c)*log(-(b*c*x^2 + 2*a*c*x - 2*sqrt(b*x^2 + a*x)*sqrt(a*c)*sqrt(c*
x))/x^2)/(a*c), 2*sqrt(-a*c)*arctan(sqrt(b*x^2 + a*x)*sqrt(-a*c)*sqrt(c*x)
/(a*c*x))/(a*c)]
```

Sympy [F]

$$\int \frac{1}{\sqrt{cx}\sqrt{ax+bx^2}}\,dx = \int \frac{1}{\sqrt{cx}\sqrt{x\,(a+bx)}}\,dx$$

```
integrate(1/(c*x)**(1/2)/(b*x**2+a*x)**(1/2),x)
```

```
Integral(1/(sqrt(c*x)*sqrt(x*(a + b*x))), x)
```

Maxima [F]

$$\int \frac{1}{\sqrt{cx}\sqrt{ax+bx^2}}\,dx = \int \frac{1}{\sqrt{bx^2+ax}\sqrt{cx}}\,dx$$

```
integrate(1/(c*x)^(1/2)/(b*x^2+a*x)^(1/2),x, algorithm="maxima")
```

```
integrate(1/(sqrt(b*x^2 + a*x)*sqrt(c*x)), x)
```

Giac [A] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 54, normalized size of antiderivative = 1.23

$$\int \frac{1}{\sqrt{cx}\sqrt{ax+bx^2}}\,dx = \frac{2\,c\left(\frac{\arctan\left(\frac{\sqrt{bcx+ac}}{\sqrt{-ac}}\right)}{\sqrt{-ac}} - \frac{\arctan\left(\frac{\sqrt{ac}}{\sqrt{-ac}}\right)}{\sqrt{-ac}}\right)}{|c|}$$

```
integrate(1/(c*x)^(1/2)/(b*x^2+a*x)^(1/2),x, algorithm="giac")
```

```
2*c*(arctan(sqrt(b*c*x + a*c)/sqrt(-a*c))/sqrt(-a*c) - arctan(sqrt(a*c)/sq
rt(-a*c))/sqrt(-a*c))/abs(c)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{\sqrt{cx}\sqrt{ax+bx^2}}\,dx = \int \frac{1}{\sqrt{b\,x^2+a\,x}\,\sqrt{c\,x}}\,dx$$

```
int(1/((a*x + b*x^2)^(1/2)*(c*x)^(1/2)),x)
```

```
int(1/((a*x + b*x^2)^(1/2)*(c*x)^(1/2)), x)
```

Reduce [B] (verification not implemented)

Time = 0.52 (sec) , antiderivative size = 36, normalized size of antiderivative = 0.82

$$\int \frac{1}{\sqrt{cx}\sqrt{ax+bx^2}}\,dx = \frac{\sqrt{c}\,\sqrt{a}\,\left(\log\left(\sqrt{bx+a}-\sqrt{a}\right)-\log\left(\sqrt{bx+a}+\sqrt{a}\right)\right)}{ac}$$

```
int(1/(c*x)^(1/2)/(b*x^2+a*x)^(1/2),x)
```

```
(sqrt(c)*sqrt(a)*(log(sqrt(a + b*x) - sqrt(a)) - log(sqrt(a + b*x) + sqrt(
a))))/(a*c)
```

3.102 $\int \frac{1}{(cx)^{3/2}\sqrt{ax+bx^2}}\,dx$

Optimal result

Integrand size = 21, antiderivative size = 70

$$\int \frac{1}{(cx)^{3/2}\sqrt{ax+bx^2}}\,dx = -\frac{\sqrt{ax+bx^2}}{a(cx)^{3/2}}+\frac{b\,\text{arctanh}\left(\frac{\sqrt{c}\sqrt{ax+bx^2}}{\sqrt{a}\sqrt{cx}}\right)}{a^{3/2}c^{3/2}}$$

```
-(b*x^2+a*x)^(1/2)/a/(c*x)^(3/2)+b*arctanh(c^(1/2)*(b*x^2+a*x)^(1/2)/a^(1/
2)/(c*x)^(1/2))/a^(3/2)/c^(3/2)
```

Mathematica [A] (verified)

Time = 0.08 (sec) , antiderivative size = 66, normalized size of antiderivative = 0.94

$$\int \frac{1}{(cx)^{3/2}\sqrt{ax+bx^2}}\,dx = \frac{x\left(-\sqrt{a}(a+bx)+bx\sqrt{a+bx}\,\text{arctanh}\left(\frac{\sqrt{a+bx}}{\sqrt{a}}\right)\right)}{a^{3/2}(cx)^{3/2}\sqrt{x(a+bx)}}$$

```
Integrate[1/((c*x)^(3/2)*Sqrt[a*x + b*x^2]),x]
```

```
(x*(-(Sqrt[a]*(a + b*x)) + b*x*Sqrt[a + b*x]*ArcTanh[Sqrt[a + b*x]/Sqrt[a]
]))/(a^(3/2)*(c*x)^(3/2)*Sqrt[x*(a + b*x)])
```

Rubi [A] (verified)

Time = 0.31 (sec) , antiderivative size = 70, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.143$, Rules used = {1135, 1136, 221}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{(cx)^{3/2}\sqrt{ax+bx^2}}\,dx$$

$$\Big\downarrow \text{1135}$$

$$-\frac{b\int \frac{1}{\sqrt{cx}\sqrt{bx^2+ax}}dx}{2ac}-\frac{\sqrt{ax+bx^2}}{a(cx)^{3/2}}$$

$$\Big\downarrow \text{1136}$$

$$-\frac{b\int \frac{1}{\frac{c\left(bx^2+ax\right)}{x}-ac}d\frac{\sqrt{bx^2+ax}}{\sqrt{cx}}}{a}-\frac{\sqrt{ax+bx^2}}{a(cx)^{3/2}}$$

$$\Big\downarrow \text{221}$$

$$\frac{b\,\text{arctanh}\left(\frac{\sqrt{c}\sqrt{ax+bx^2}}{\sqrt{a}\sqrt{cx}}\right)}{a^{3/2}c^{3/2}}-\frac{\sqrt{ax+bx^2}}{a(cx)^{3/2}}$$

```
Int[1/((c*x)^(3/2)*Sqrt[a*x + b*x^2]),x]
```

```
-(Sqrt[a*x + b*x^2]/(a*(c*x)^(3/2))) + (b*ArcTanh[(Sqrt[c]*Sqrt[a*x + b*x^
2])/(Sqrt[a]*Sqrt[c*x])])/(a^(3/2)*c^(3/2))
```

Defintions of rubi rules used

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] :> Simp[(-e)*(d + e*x)^m*((a + b*x + c*x^2)^(p + 1)/((m + p + 1)*(2*
c*d - b*e))), x] + Simp[c*((m + 2*p + 2)/((m + p + 1)*(2*c*d - b*e)))   Int
[(d + e*x)^(m + 1)*(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, p},
 x] && EqQ[c*d^2 - b*d*e + a*e^2, 0] && LtQ[m, 0] && NeQ[m + p + 1, 0] && I
ntegerQ[2*p]
```

```
Int[1/(Sqrt[(d_.) + (e_.)*(x_)]*Sqrt[(a_.) + (b_.)*(x_) + (c_.)*(x_)^2]), x
_Symbol] :> Simp[2*e   Subst[Int[1/(2*c*d - b*e + e^2*x^2), x], x, Sqrt[a +
 b*x + c*x^2]/Sqrt[d + e*x]], x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[c*d^2
- b*d*e + a*e^2, 0]
```

Maple [A] (verified)

Time = 0.35 (sec) , antiderivative size = 76, normalized size of antiderivative = 1.09

method	result	size
default	$\frac{\sqrt{x(bx+a)}\left(\operatorname{arctanh}\left(\frac{\sqrt{c(bx+a)}}{\sqrt{ac}}\right)cbx-\sqrt{c(bx+a)}\,\sqrt{ac}\right)}{cx\sqrt{cx}\,\sqrt{c(bx+a)}\,a\sqrt{ac}}$	76
risch	$-\frac{bx+a}{ac\sqrt{cx}\,\sqrt{x(bx+a)}}+\frac{b\,\operatorname{arctanh}\left(\frac{\sqrt{cbx+ac}}{\sqrt{ac}}\right)\sqrt{c(bx+a)}\,x}{a\sqrt{ac}\,c\sqrt{cx}\,\sqrt{x(bx+a)}}$	83

```
int(1/(c*x)^(3/2)/(b*x^2+a*x)^(1/2),x,method=_RETURNVERBOSE)
```

```
(x*(b*x+a))^(1/2)*(arctanh((c*(b*x+a))^(1/2)/(a*c)^(1/2))*c*b*x-(c*(b*x+a)
)^(1/2)*(a*c)^(1/2))/c/x/(c*x)^(1/2)/(c*(b*x+a))^(1/2)/a/(a*c)^(1/2)
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 157, normalized size of antiderivative = 2.24

$$\int \frac{1}{(cx)^{3/2}\sqrt{ax+bx^2}}\,dx = \left[\frac{\sqrt{ac}bx^2\log\left(-\frac{bcx^2+2\,acx+2\,\sqrt{bx^2+ax}\sqrt{ac}\sqrt{cx}}{x^2}\right)-2\,\sqrt{bx^2+ax}\sqrt{cx}a}{2\,a^2c^2x^2},\right.$$
$$\left.-\frac{\sqrt{-ac}bx^2\arctan\left(\frac{\sqrt{bx^2+ax}\sqrt{-ac}\sqrt{cx}}{acx}\right)+\sqrt{bx^2+ax}\sqrt{cx}a}{a^2c^2x^2}\right]$$

```
integrate(1/(c*x)^(3/2)/(b*x^2+a*x)^(1/2),x, algorithm="fricas")
```

```
[1/2*(sqrt(a*c)*b*x^2*log(-(b*c*x^2 + 2*a*c*x + 2*sqrt(b*x^2 + a*x)*sqrt(a
*c)*sqrt(c*x))/x^2) - 2*sqrt(b*x^2 + a*x)*sqrt(c*x)*a)/(a^2*c^2*x^2), -(sq
rt(-a*c)*b*x^2*arctan(sqrt(b*x^2 + a*x)*sqrt(-a*c)*sqrt(c*x)/(a*c*x)) + sq
rt(b*x^2 + a*x)*sqrt(c*x)*a)/(a^2*c^2*x^2)]
```

Sympy [F]

$$\int \frac{1}{(cx)^{3/2}\sqrt{ax+bx^2}}\,dx = \int \frac{1}{(cx)^{\frac{3}{2}}\sqrt{x\,(a+bx)}}\,dx$$

```
integrate(1/(c*x)**(3/2)/(b*x**2+a*x)**(1/2),x)
```

```
Integral(1/((c*x)**(3/2)*sqrt(x*(a + b*x))), x)
```

Maxima [F]

$$\int \frac{1}{(cx)^{3/2}\sqrt{ax+bx^2}}\,dx = \int \frac{1}{\sqrt{bx^2+ax}\,(cx)^{\frac{3}{2}}}\,dx$$

```
integrate(1/(c*x)^(3/2)/(b*x^2+a*x)^(1/2),x, algorithm="maxima")
```

```
integrate(1/(sqrt(b*x^2 + a*x)*(c*x)^(3/2)), x)
```

Giac [A] (verification not implemented)

Time = 0.17 (sec) , antiderivative size = 63, normalized size of antiderivative = 0.90

$$\int \frac{1}{(cx)^{3/2}\sqrt{ax+bx^2}}\,dx = -\frac{bc\left(\frac{\arctan\left(\frac{\sqrt{bcx+ac}}{\sqrt{-ac}}\right)}{\sqrt{-ac}ac} + \frac{\sqrt{bcx+ac}}{abc^2x}\right)}{|c|}$$

```
integrate(1/(c*x)^(3/2)/(b*x^2+a*x)^(1/2),x, algorithm="giac")
```

```
-b*c*(arctan(sqrt(b*c*x + a*c)/sqrt(-a*c))/(sqrt(-a*c)*a*c) + sqrt(b*c*x +
 a*c)/(a*b*c^2*x))/abs(c)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{(cx)^{3/2}\sqrt{ax+bx^2}}\,dx = \int \frac{1}{\sqrt{b\,x^2+a\,x}\,(c\,x)^{3/2}}\,dx$$

```
int(1/((a*x + b*x^2)^(1/2)*(c*x)^(3/2)),x)
```

```
int(1/((a*x + b*x^2)^(1/2)*(c*x)^(3/2)), x)
```

Reduce [B] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 56, normalized size of antiderivative = 0.80

$$\int \frac{1}{(cx)^{3/2}\sqrt{ax+bx^2}}\,dx = \frac{\sqrt{c}\left(-2\sqrt{bx+a}\,a-\sqrt{a}\log\left(\sqrt{bx+a}-\sqrt{a}\right)bx+\sqrt{a}\log\left(\sqrt{bx+a}+\sqrt{a}\right)bx\right)}{2a^2c^2x}$$

```
int(1/(c*x)^(3/2)/(b*x^2+a*x)^(1/2),x)
```

```
(sqrt(c)*( - 2*sqrt(a + b*x)*a - sqrt(a)*log(sqrt(a + b*x) - sqrt(a))*b*x
+ sqrt(a)*log(sqrt(a + b*x) + sqrt(a))*b*x))/(2*a**2*c**2*x)
```

3.103 $\int \frac{1}{(cx)^{5/2}\sqrt{ax+bx^2}}\,dx$

Optimal result

Integrand size = 21, antiderivative size = 108

$$\int \frac{1}{(cx)^{5/2}\sqrt{ax+bx^2}}\,dx = -\frac{\sqrt{ax+bx^2}}{2a(cx)^{5/2}}+\frac{3b\sqrt{ax+bx^2}}{4a^2c(cx)^{3/2}}-\frac{3b^2\operatorname{arctanh}\left(\frac{\sqrt{c}\sqrt{ax+bx^2}}{\sqrt{a}\sqrt{cx}}\right)}{4a^{5/2}c^{5/2}}$$

```
-1/2*(b*x^2+a*x)^(1/2)/a/(c*x)^(5/2)+3/4*b*(b*x^2+a*x)^(1/2)/a^2/c/(c*x)^(
3/2)-3/4*b^2*arctanh(c^(1/2)*(b*x^2+a*x)^(1/2)/a^(1/2)/(c*x)^(1/2))/a^(5/2
)/c^(5/2)
```

Mathematica [A] (verified)

Time = 0.09 (sec) , antiderivative size = 86, normalized size of antiderivative = 0.80

$$\int \frac{1}{(cx)^{5/2}\sqrt{ax+bx^2}}\,dx = \frac{x\left(\sqrt{a}(-2a^2+abx+3b^2x^2)-3b^2x^2\sqrt{a+bx}\operatorname{arctanh}\left(\frac{\sqrt{a+bx}}{\sqrt{a}}\right)\right)}{4a^{5/2}(cx)^{5/2}\sqrt{x(a+bx)}}$$

```
Integrate[1/((c*x)^(5/2)*Sqrt[a*x + b*x^2]),x]
```

```
(x*(Sqrt[a]*(-2*a^2 + a*b*x + 3*b^2*x^2) - 3*b^2*x^2*Sqrt[a + b*x]*ArcTanh
[Sqrt[a + b*x]/Sqrt[a]]))/(4*a^(5/2)*(c*x)^(5/2)*Sqrt[x*(a + b*x)])
```

Rubi [A] (verified)

Time = 0.38 (sec) , antiderivative size = 109, normalized size of antiderivative = 1.01, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.190$, Rules used = $\{1135, 1135, 1136, 221\}$

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{(cx)^{5/2}\sqrt{ax+bx^2}}dx$$

$$\Big\downarrow \text{1135}$$

$$-\frac{3b\int \frac{1}{(cx)^{3/2}\sqrt{bx^2+ax}}dx}{4ac}-\frac{\sqrt{ax+bx^2}}{2a(cx)^{5/2}}$$

$$\Big\downarrow \text{1135}$$

$$-\frac{3b\left(-\frac{b\int \frac{1}{\sqrt{cx}\sqrt{bx^2+ax}}dx}{2ac}-\frac{\sqrt{ax+bx^2}}{a(cx)^{3/2}}\right)}{4ac}-\frac{\sqrt{ax+bx^2}}{2a(cx)^{5/2}}$$

$$\Big\downarrow \text{1136}$$

$$-\frac{3b\left(-\frac{b\int \frac{1}{\frac{c(bx^2+ax)}{x}-ac}d\frac{\sqrt{bx^2+ax}}{\sqrt{cx}}}{a}-\frac{\sqrt{ax+bx^2}}{a(cx)^{3/2}}\right)}{4ac}-\frac{\sqrt{ax+bx^2}}{2a(cx)^{5/2}}$$

$$\Big\downarrow \text{221}$$

$$-\frac{3b\left(\frac{b\,\text{arctanh}\left(\frac{\sqrt{c}\sqrt{ax+bx^2}}{\sqrt{a}\sqrt{cx}}\right)}{a^{3/2}c^{3/2}}-\frac{\sqrt{ax+bx^2}}{a(cx)^{3/2}}\right)}{4ac}-\frac{\sqrt{ax+bx^2}}{2a(cx)^{5/2}}$$

```
Int[1/((c*x)^(5/2)*Sqrt[a*x + b*x^2]),x]
```

```
-1/2*Sqrt[a*x + b*x^2]/(a*(c*x)^(5/2)) - (3*b*(-(Sqrt[a*x + b*x^2]/(a*(c*x
)^(3/2))) + (b*ArcTanh[(Sqrt[c]*Sqrt[a*x + b*x^2])/(Sqrt[a]*Sqrt[c*x])])/(
a^(3/2)*c^(3/2))))/(4*a*c)
```

Defintions of rubi rules used

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] :> Simp[(-e)*(d + e*x)^m*((a + b*x + c*x^2)^(p + 1)/((m + p + 1)*(2*
c*d - b*e))), x] + Simp[c*((m + 2*p + 2)/((m + p + 1)*(2*c*d - b*e)))   Int
[(d + e*x)^(m + 1)*(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, p},
 x] && EqQ[c*d^2 - b*d*e + a*e^2, 0] && LtQ[m, 0] && NeQ[m + p + 1, 0] && I
ntegerQ[2*p]
```

```
Int[1/(Sqrt[(d_.) + (e_.)*(x_)]*Sqrt[(a_.) + (b_.)*(x_) + (c_.)*(x_)^2]), x
_Symbol] :> Simp[2*e   Subst[Int[1/(2*c*d - b*e + e^2*x^2), x], x, Sqrt[a +
 b*x + c*x^2]/Sqrt[d + e*x]], x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[c*d^2
- b*d*e + a*e^2, 0]
```

Maple [A] (verified)

Time = 0.72 (sec) , antiderivative size = 97, normalized size of antiderivative = 0.90

method	result	size
risch	$-\frac{(bx+a)(-3bx+2a)}{4a^{2}x\,c^{2}\sqrt{cx}\,\sqrt{x(bx+a)}}-\frac{3b^{2}\operatorname{arctanh}\left(\frac{\sqrt{cbx+ac}}{\sqrt{ac}}\right)\sqrt{c(bx+a)}\,x}{4a^{2}\sqrt{ac}\,c^{2}\sqrt{cx}\,\sqrt{x(bx+a)}}$	97
default	$-\frac{\sqrt{x(bx+a)}\left(3\operatorname{arctanh}\left(\frac{\sqrt{c(bx+a)}}{\sqrt{ac}}\right)b^{2}c\,x^{2}-3bx\sqrt{c(bx+a)}\,\sqrt{ac}+2\sqrt{c(bx+a)}\,\sqrt{ac}\,a\right)}{4c^{2}x^{2}\sqrt{cx}\,\sqrt{c(bx+a)}\,a^{2}\sqrt{ac}}$	101

```
int(1/(c*x)^(5/2)/(b*x^2+a*x)^(1/2),x,method=_RETURNVERBOSE)
```

```
-1/4*(b*x+a)*(-3*b*x+2*a)/a^2/x/c^2/(c*x)^(1/2)/(x*(b*x+a))^(1/2)-3/4*b^2/
a^2/(a*c)^(1/2)*arctanh((b*c*x+a*c)^(1/2)/(a*c)^(1/2))/c^2*(c*(b*x+a))^(1/
2)/(c*x)^(1/2)/(x*(b*x+a))^(1/2)*x
```

Fricas [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 183, normalized size of antiderivative = 1.69

$$\int \frac{1}{(cx)^{5/2}\sqrt{ax+bx^2}}\,dx = \left[\frac{3\,\sqrt{ac}b^2x^3\log\left(-\frac{bcx^2+2\,acx-2\,\sqrt{bx^2+ax}\sqrt{ac}\sqrt{cx}}{x^2}\right)+2\,(3\,abx-2\,a^2)\sqrt{bx^2+ax}\sqrt{cx}}{8\,a^3c^3x^3},\right.$$

```
integrate(1/(c*x)^(5/2)/(b*x^2+a*x)^(1/2),x, algorithm="fricas")
```

```
[1/8*(3*sqrt(a*c)*b^2*x^3*log(-(b*c*x^2 + 2*a*c*x - 2*sqrt(b*x^2 + a*x)*sq
rt(a*c)*sqrt(c*x))/x^2) + 2*(3*a*b*x - 2*a^2)*sqrt(b*x^2 + a*x)*sqrt(c*x))
/(a^3*c^3*x^3), 1/4*(3*sqrt(-a*c)*b^2*x^3*arctan(sqrt(b*x^2 + a*x)*sqrt(-a
*c)*sqrt(c*x)/(a*c*x)) + (3*a*b*x - 2*a^2)*sqrt(b*x^2 + a*x)*sqrt(c*x))/(a
^3*c^3*x^3)]
```

Sympy [F]

$$\int \frac{1}{(cx)^{5/2}\sqrt{ax+bx^2}}\,dx = \int \frac{1}{(cx)^{\frac{5}{2}}\sqrt{x\,(a+bx)}}\,dx$$

```
integrate(1/(c*x)**(5/2)/(b*x**2+a*x)**(1/2),x)
```

```
Integral(1/((c*x)**(5/2)*sqrt(x*(a + b*x))), x)
```

Maxima [F]

$$\int \frac{1}{(cx)^{5/2}\sqrt{ax+bx^2}}\,dx = \int \frac{1}{\sqrt{bx^2+ax}\,(cx)^{\frac{5}{2}}}\,dx$$

```
integrate(1/(c*x)^(5/2)/(b*x^2+a*x)^(1/2),x, algorithm="maxima")
```

```
integrate(1/(sqrt(b*x^2 + a*x)*(c*x)^(5/2)), x)
```

Giac [A] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 93, normalized size of antiderivative = 0.86

$$\int \frac{1}{(cx)^{5/2}\sqrt{ax+bx^2}}\,dx = \frac{c\left(\frac{3\,b^3\arctan\left(\frac{\sqrt{bcx+ac}}{\sqrt{-ac}}\right)}{\sqrt{-ac}a^2c^2} - \frac{5\,\sqrt{bcx+ac}ab^3c-3\,(bcx+ac)^{\frac{3}{2}}b^3}{a^2b^2c^4x^2}\right)}{4\,b|c|}$$

```
integrate(1/(c*x)^(5/2)/(b*x^2+a*x)^(1/2),x, algorithm="giac")
```

```
1/4*c*(3*b^3*arctan(sqrt(b*c*x + a*c)/sqrt(-a*c))/(sqrt(-a*c)*a^2*c^2) - (
5*sqrt(b*c*x + a*c)*a*b^3*c - 3*(b*c*x + a*c)^(3/2)*b^3)/(a^2*b^2*c^4*x^2)
)/(b*abs(c))
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{(cx)^{5/2}\sqrt{ax+bx^2}}\,dx = \int \frac{1}{\sqrt{b\,x^2+a\,x}\,{(c\,x)}^{5/2}}\,dx$$

```
int(1/((a*x + b*x^2)^(1/2)*(c*x)^(5/2)),x)
```

```
int(1/((a*x + b*x^2)^(1/2)*(c*x)^(5/2)), x)
```

Reduce [B] (verification not implemented)

Time = 0.50 (sec) , antiderivative size = 78, normalized size of antiderivative = 0.72

$$\int \frac{1}{(cx)^{5/2}\sqrt{ax+bx^2}}\,dx = \frac{\sqrt{c}\left(-4\sqrt{bx+a}\,a^2+6\sqrt{bx+a}\,abx+3\sqrt{a}\log\left(\sqrt{bx+a}-\sqrt{a}\right)b^2x^2-3\sqrt{a}\log(\right.}{8a^3c^3x^2}$$

```
int(1/(c*x)^(5/2)/(b*x^2+a*x)^(1/2),x)
```

```
(sqrt(c)*( - 4*sqrt(a + b*x)*a**2 + 6*sqrt(a + b*x)*a*b*x + 3*sqrt(a)*log(
sqrt(a + b*x) - sqrt(a))*b**2*x**2 - 3*sqrt(a)*log(sqrt(a + b*x) + sqrt(a)
)*b**2*x**2))/(8*a**3*c**3*x**2)
```

3.104 $\int \frac{1}{(cx)^{7/2}\sqrt{ax+bx^2}}\,dx$

Optimal result

Integrand size = 21, antiderivative size = 141

$$\int \frac{1}{(cx)^{7/2}\sqrt{ax+bx^2}}\,dx = -\frac{\sqrt{ax+bx^2}}{3a(cx)^{7/2}}+\frac{5b\sqrt{ax+bx^2}}{12a^2c(cx)^{5/2}} - \frac{5b^2\sqrt{ax+bx^2}}{8a^3c^2(cx)^{3/2}}+\frac{5b^3\text{arctanh}\left(\frac{\sqrt{c}\sqrt{ax+bx^2}}{\sqrt{a}\sqrt{cx}}\right)}{8a^{7/2}c^{7/2}}$$

```
-1/3*(b*x^2+a*x)^(1/2)/a/(c*x)^(7/2)+5/12*b*(b*x^2+a*x)^(1/2)/a^2/c/(c*x)^
(5/2)-5/8*b^2*(b*x^2+a*x)^(1/2)/a^3/c^2/(c*x)^(3/2)+5/8*b^3*arctanh(c^(1/2
)*(b*x^2+a*x)^(1/2)/a^(1/2)/(c*x)^(1/2))/a^(7/2)/c^(7/2)
```

Mathematica [A] (verified)

Time = 0.10 (sec) , antiderivative size = 99, normalized size of antiderivative = 0.70

$$\int \frac{1}{(cx)^{7/2}\sqrt{ax+bx^2}}\,dx = \frac{x\left(-\sqrt{a}(8a^3-2a^2bx+5ab^2x^2+15b^3x^3)+15b^3x^3\sqrt{a+bx}\,\text{arctanh}\left(\frac{\sqrt{a+bx}}{\sqrt{a}}\right)\right)}{24a^{7/2}(cx)^{7/2}\sqrt{x(a+bx)}}$$

```
Integrate[1/((c*x)^(7/2)*Sqrt[a*x + b*x^2]),x]
```

```
(x*(-(Sqrt[a]*(8*a^3 - 2*a^2*b*x + 5*a*b^2*x^2 + 15*b^3*x^3)) + 15*b^3*x^3
*Sqrt[a + b*x]*ArcTanh[Sqrt[a + b*x]/Sqrt[a]]))/(24*a^(7/2)*(c*x)^(7/2)*Sq
rt[x*(a + b*x)])
```

Rubi [A] (verified)

Time = 0.43 (sec) , antiderivative size = 148, normalized size of antiderivative = 1.05, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.238$, Rules used = {1135, 1135, 1135, 1136, 221}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{(cx)^{7/2}\sqrt{ax+bx^2}}dx$$

$$\Big\downarrow \text{1135}$$

$$-\frac{5b\int \frac{1}{(cx)^{5/2}\sqrt{bx^2+ax}}dx}{6ac}-\frac{\sqrt{ax+bx^2}}{3a(cx)^{7/2}}$$

$$\Big\downarrow \text{1135}$$

$$-\frac{5b\left(-\frac{3b\int \frac{1}{(cx)^{3/2}\sqrt{bx^2+ax}}dx}{4ac}-\frac{\sqrt{ax+bx^2}}{2a(cx)^{5/2}}\right)}{6ac}-\frac{\sqrt{ax+bx^2}}{3a(cx)^{7/2}}$$

$$\Big\downarrow \text{1135}$$

$$-\frac{5b\left(-\frac{3b\left(-\frac{b\int \frac{1}{\sqrt{cx}\sqrt{bx^2+ax}}dx}{2ac}-\frac{\sqrt{ax+bx^2}}{a(cx)^{3/2}}\right)}{4ac}-\frac{\sqrt{ax+bx^2}}{2a(cx)^{5/2}}\right)}{6ac}-\frac{\sqrt{ax+bx^2}}{3a(cx)^{7/2}}$$

$$\Big\downarrow \text{1136}$$

$$-\frac{5b\left(-\frac{3b\left(-\frac{b\int \frac{1}{\frac{c(bx^2+ax)}{x}-ac}d\frac{\sqrt{bx^2+ax}}{\sqrt{cx}}}{a}-\frac{\sqrt{ax+bx^2}}{a(cx)^{3/2}}\right)}{4ac}-\frac{\sqrt{ax+bx^2}}{2a(cx)^{5/2}}\right)}{6ac}-\frac{\sqrt{ax+bx^2}}{3a(cx)^{7/2}}$$

$$\Big\downarrow \text{221}$$

$$-\frac{5b\left(-\frac{3b\left(\frac{b\,\text{arctanh}\left(\frac{\sqrt{c}\sqrt{ax+bx^2}}{\sqrt{a}\sqrt{cx}}\right)}{a^{3/2}c^{3/2}}-\frac{\sqrt{ax+bx^2}}{a(cx)^{3/2}}\right)}{4ac}-\frac{\sqrt{ax+bx^2}}{2a(cx)^{5/2}}\right)}{6ac}-\frac{\sqrt{ax+bx^2}}{3a(cx)^{7/2}}$$

```
Int[1/((c*x)^(7/2)*Sqrt[a*x + b*x^2]),x]
```

```
-1/3*Sqrt[a*x + b*x^2]/(a*(c*x)^(7/2)) - (5*b*(-1/2*Sqrt[a*x + b*x^2]/(a*(
c*x)^(5/2)) - (3*b*(-(Sqrt[a*x + b*x^2]/(a*(c*x)^(3/2))) + (b*ArcTanh[(Sqr
t[c]*Sqrt[a*x + b*x^2])/(Sqrt[a]*Sqrt[c*x])])/(a^(3/2)*c^(3/2))))/(4*a*c))
)/(6*a*c)
```

Defintions of rubi rules used

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] :> Simp[(-e)*(d + e*x)^m*((a + b*x + c*x^2)^(p + 1)/((m + p + 1)*(2*
c*d - b*e))), x] + Simp[c*((m + 2*p + 2)/((m + p + 1)*(2*c*d - b*e)))   Int
[(d + e*x)^(m + 1)*(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, p},
 x] && EqQ[c*d^2 - b*d*e + a*e^2, 0] && LtQ[m, 0] && NeQ[m + p + 1, 0] && I
ntegerQ[2*p]
```

```
Int[1/(Sqrt[(d_.) + (e_.)*(x_)]*Sqrt[(a_.) + (b_.)*(x_) + (c_.)*(x_)^2]), x
_Symbol] :> Simp[2*e   Subst[Int[1/(2*c*d - b*e + e^2*x^2), x], x, Sqrt[a +
 b*x + c*x^2]/Sqrt[d + e*x]], x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[c*d^2
- b*d*e + a*e^2, 0]
```

Maple [A] (verified)

Time = 0.54 (sec) , antiderivative size = 108, normalized size of antiderivative = 0.77

method	result	size
risch	$-\frac{(bx+a)(15b^2x^2-10abx+8a^2)}{24a^3x^2c^3\sqrt{cx}\,\sqrt{x(bx+a)}}+\frac{5b^3\,\mathrm{arctanh}\left(\frac{\sqrt{cbx+ac}}{\sqrt{ac}}\right)\sqrt{c(bx+a)}\,x}{8a^3\sqrt{ac}\,c^3\sqrt{cx}\,\sqrt{x(bx+a)}}$	108
default	$\frac{\sqrt{x(bx+a)}\left(15\,\mathrm{arctanh}\left(\frac{\sqrt{c(bx+a)}}{\sqrt{ac}}\right)c\,b^3x^3-15b^2x^2\sqrt{c(bx+a)}\,\sqrt{ac}+10abx\sqrt{c(bx+a)}\,\sqrt{ac}-8\sqrt{c(bx+a)}\,\sqrt{ac}\,a^2\right)}{24c^3x^3\sqrt{cx}\,\sqrt{c(bx+a)}\,a^3\sqrt{ac}}$	126

```
int(1/(c*x)^(7/2)/(b*x^2+a*x)^(1/2),x,method=_RETURNVERBOSE)
```

```
-1/24*(b*x+a)*(15*b^2*x^2-10*a*b*x+8*a^2)/a^3/x^2/c^3/(c*x)^(1/2)/(x*(b*x+
a))^(1/2)+5/8*b^3/a^3/(a*c)^(1/2)*arctanh((b*c*x+a*c)^(1/2)/(a*c)^(1/2))/c
^3*(c*(b*x+a))^(1/2)/(c*x)^(1/2)/(x*(b*x+a))^(1/2)*x
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 205, normalized size of antiderivative = 1.45

$$\int \frac{1}{(cx)^{7/2}\sqrt{ax+bx^2}}\,dx = \left[\frac{15\,\sqrt{ac}b^3x^4\log\left(-\frac{bcx^2+2\,acx+2\,\sqrt{bx^2+ax}\sqrt{ac}\sqrt{cx}}{x^2}\right)-2\,(15\,ab^2x^2-10\,a^2bx+8\,a^3)\sqrt{}}{48\,a^4c^4x^4}\right.$$
$$\left.-\frac{15\,\sqrt{-ac}b^3x^4\arctan\left(\frac{\sqrt{bx^2+ax}\sqrt{-ac}\sqrt{cx}}{acx}\right)+(15\,ab^2x^2-10\,a^2bx+8\,a^3)\sqrt{bx^2+ax}\sqrt{cx}}{24\,a^4c^4x^4}\right]$$

```
integrate(1/(c*x)^(7/2)/(b*x^2+a*x)^(1/2),x, algorithm="fricas")
```

```
[1/48*(15*sqrt(a*c)*b^3*x^4*log(-(b*c*x^2 + 2*a*c*x + 2*sqrt(b*x^2 + a*x)*
sqrt(a*c)*sqrt(c*x))/x^2) - 2*(15*a*b^2*x^2 - 10*a^2*b*x + 8*a^3)*sqrt(b*x
^2 + a*x)*sqrt(c*x))/(a^4*c^4*x^4), -1/24*(15*sqrt(-a*c)*b^3*x^4*arctan(sq
rt(b*x^2 + a*x)*sqrt(-a*c)*sqrt(c*x)/(a*c*x)) + (15*a*b^2*x^2 - 10*a^2*b*x
 + 8*a^3)*sqrt(b*x^2 + a*x)*sqrt(c*x))/(a^4*c^4*x^4)]
```

Sympy [F]

$$\int \frac{1}{(cx)^{7/2}\sqrt{ax+bx^2}}\,dx = \int \frac{1}{(cx)^{\frac{7}{2}}\sqrt{x\,(a+bx)}}\,dx$$

```
integrate(1/(c*x)**(7/2)/(b*x**2+a*x)**(1/2),x)
```

```
Integral(1/((c*x)**(7/2)*sqrt(x*(a + b*x))), x)
```

Maxima [F]

$$\int \frac{1}{(cx)^{7/2}\sqrt{ax+bx^2}}\,dx = \int \frac{1}{\sqrt{bx^2+ax}\,(cx)^{\frac{7}{2}}}\,dx$$

```
integrate(1/(c*x)^(7/2)/(b*x^2+a*x)^(1/2),x, algorithm="maxima")
```

```
integrate(1/(sqrt(b*x^2 + a*x)*(c*x)^(7/2)), x)
```

Giac [A] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 101, normalized size of antiderivative = 0.72

$$\int \frac{1}{(cx)^{7/2}\sqrt{ax+bx^2}}\,dx = -\frac{b^3c\left(\frac{15\arctan\left(\frac{\sqrt{bcx+ac}}{\sqrt{-ac}}\right)}{\sqrt{-ac}a^3c^3}+\frac{33\sqrt{bcx+ac}a^2c^2-40(bcx+ac)^{\frac{3}{2}}ac+15(bcx+ac)^{\frac{5}{2}}}{a^3b^3c^6x^3}\right)}{24|c|}$$

```
integrate(1/(c*x)^(7/2)/(b*x^2+a*x)^(1/2),x, algorithm="giac")
```

```
-1/24*b^3*c*(15*arctan(sqrt(b*c*x + a*c)/sqrt(-a*c))/(sqrt(-a*c)*a^3*c^3)
+ (33*sqrt(b*c*x + a*c)*a^2*c^2 - 40*(b*c*x + a*c)^(3/2)*a*c + 15*(b*c*x +
 a*c)^(5/2))/(a^3*b^3*c^6*x^3))/abs(c)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{(cx)^{7/2}\sqrt{ax+bx^2}}\,dx = \int \frac{1}{\sqrt{b\,x^2+a\,x}\,(c\,x)^{7/2}}\,dx$$

```
int(1/((a*x + b*x^2)^(1/2)*(c*x)^(7/2)),x)
```

```
int(1/((a*x + b*x^2)^(1/2)*(c*x)^(7/2)), x)
```

Reduce [B] (verification not implemented)

Time = 0.54 (sec) , antiderivative size = 95, normalized size of antiderivative = 0.67

$$\int \frac{1}{(cx)^{7/2}\sqrt{ax+bx^2}}\,dx = \frac{\sqrt{c}\left(-16\sqrt{bx+a}\,a^3+20\sqrt{bx+a}\,a^2bx-30\sqrt{bx+a}\,a\,b^2x^2-15\sqrt{a}\log\left(\sqrt{bx+\right.\right.}{48a^4c^4x^3}$$

```
int(1/(c*x)^(7/2)/(b*x^2+a*x)^(1/2),x)
```

```
(sqrt(c)*( - 16*sqrt(a + b*x)*a**3 + 20*sqrt(a + b*x)*a**2*b*x - 30*sqrt(a
 + b*x)*a*b**2*x**2 - 15*sqrt(a)*log(sqrt(a + b*x) - sqrt(a))*b**3*x**3 +
15*sqrt(a)*log(sqrt(a + b*x) + sqrt(a))*b**3*x**3))/(48*a**4*c**4*x**3)
```

3.105 $\int \frac{(cx)^{9/2}}{(ax+bx^2)^{3/2}}\,dx$

Optimal result

Integrand size = 21, antiderivative size = 122

$$\int \frac{(cx)^{9/2}}{(ax+bx^2)^{3/2}}\,dx = \frac{2a^3c^4\sqrt{cx}}{b^4\sqrt{ax+bx^2}} + \frac{6a^2c^5\sqrt{ax+bx^2}}{b^4\sqrt{cx}} - \frac{2ac^6(ax+bx^2)^{3/2}}{b^4(cx)^{3/2}} + \frac{2c^7(ax+bx^2)^{5/2}}{5b^4(cx)^{5/2}}$$

```
2*a^3*c^4*(c*x)^(1/2)/b^4/(b*x^2+a*x)^(1/2)+6*a^2*c^5*(b*x^2+a*x)^(1/2)/b^
4/(c*x)^(1/2)-2*a*c^6*(b*x^2+a*x)^(3/2)/b^4/(c*x)^(3/2)+2/5*c^7*(b*x^2+a*x
)^(5/2)/b^4/(c*x)^(5/2)
```

Mathematica [A] (verified)

Time = 0.03 (sec) , antiderivative size = 57, normalized size of antiderivative = 0.47

$$\int \frac{(cx)^{9/2}}{(ax+bx^2)^{3/2}}\,dx = \frac{2c^4\sqrt{cx}(16a^3+8a^2bx-2ab^2x^2+b^3x^3)}{5b^4\sqrt{x(a+bx)}}$$

```
Integrate[(c*x)^(9/2)/(a*x + b*x^2)^(3/2),x]
```

```
(2*c^4*Sqrt[c*x]*(16*a^3 + 8*a^2*b*x - 2*a*b^2*x^2 + b^3*x^3))/(5*b^4*Sqrt
[x*(a + b*x)])
```

Rubi [A] (verified)

Time = 0.41 (sec) , antiderivative size = 132, normalized size of antiderivative = 1.08, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.190$, Rules used = {1128, 1128, 1128, 1122}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(cx)^{9/2}}{(ax+bx^2)^{3/2}}dx$$
$$\Big\downarrow \text{1128}$$
$$\frac{2c(cx)^{7/2}}{5b\sqrt{ax+bx^2}}-\frac{6ac\int \frac{(cx)^{7/2}}{(bx^2+ax)^{3/2}}dx}{5b}$$
$$\Big\downarrow \text{1128}$$
$$\frac{2c(cx)^{7/2}}{5b\sqrt{ax+bx^2}}-\frac{6ac\left(\frac{2c(cx)^{5/2}}{3b\sqrt{ax+bx^2}}-\frac{4ac\int \frac{(cx)^{5/2}}{\left(bx^2+ax\right)^{3/2}}dx}{3b}\right)}{5b}$$
$$\Big\downarrow \text{1128}$$
$$\frac{2c(cx)^{7/2}}{5b\sqrt{ax+bx^2}}-\frac{6ac\left(\frac{2c(cx)^{5/2}}{3b\sqrt{ax+bx^2}}-\frac{4ac\left(\frac{2c(cx)^{3/2}}{b\sqrt{ax+bx^2}}-\frac{2ac\int \frac{(cx)^{3/2}}{\left(bx^2+ax\right)^{3/2}}dx}{b}\right)}{3b}\right)}{5b}$$
$$\Big\downarrow \text{1122}$$
$$\frac{2c(cx)^{7/2}}{5b\sqrt{ax+bx^2}}-\frac{6ac\left(\frac{2c(cx)^{5/2}}{3b\sqrt{ax+bx^2}}-\frac{4ac\left(\frac{4ac^2\sqrt{cx}}{b^2\sqrt{ax+bx^2}}+\frac{2c(cx)^{3/2}}{b\sqrt{ax+bx^2}}\right)}{3b}\right)}{5b}$$

```
Int[(c*x)^(9/2)/(a*x + b*x^2)^(3/2),x]
```

```
(2*c*(c*x)^(7/2))/(5*b*Sqrt[a*x + b*x^2]) - (6*a*c*((2*c*(c*x)^(5/2))/(3*b
*Sqrt[a*x + b*x^2]) - (4*a*c*((4*a*c^2*Sqrt[c*x])/(b^2*Sqrt[a*x + b*x^2])
+ (2*c*(c*x)^(3/2))/(b*Sqrt[a*x + b*x^2])))/(3*b)))/(5*b)
```

Defintions of rubi rules used

```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] :> Simp[e*(d + e*x)^(m - 1)*((a + b*x + c*x^2)^(p + 1)/(c*(p + 1))),
 x] /; FreeQ[{a, b, c, d, e, m, p}, x] && EqQ[c*d^2 - b*d*e + a*e^2, 0] &&
EqQ[m + p, 0]
```

```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] :> Simp[e*(d + e*x)^(m - 1)*((a + b*x + c*x^2)^(p + 1)/(c*(m + 2*p +
 1))), x] + Simp[Simplify[m + p]*((2*c*d - b*e)/(c*(m + 2*p + 1)))   Int[(d
 + e*x)^(m - 1)*(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, m, p},
 x] && EqQ[c*d^2 - b*d*e + a*e^2, 0] && IGtQ[Simplify[m + p], 0]
```

Maple [A] (verified)

Time = 0.62 (sec) , antiderivative size = 59, normalized size of antiderivative = 0.48

method	result	size
gosper	$\frac{2(bx+a)\left(b^{3}x^{3}-2a\,b^{2}x^{2}+8a^{2}bx+16a^{3}\right)(cx)^{\frac{9}{2}}}{5b^{4}x^{3}\left(b\,x^{2}+ax\right)^{\frac{3}{2}}}$	59
orering	$\frac{2(bx+a)\left(b^{3}x^{3}-2a\,b^{2}x^{2}+8a^{2}bx+16a^{3}\right)(cx)^{\frac{9}{2}}}{5b^{4}x^{3}\left(b\,x^{2}+ax\right)^{\frac{3}{2}}}$	59
default	$\frac{2c^{4}\sqrt{cx}\,\sqrt{x(bx+a)}\,\left(b^{3}x^{3}-2a\,b^{2}x^{2}+8a^{2}bx+16a^{3}\right)}{5x(bx+a)b^{4}}$	62
risch	$\frac{2\left(b^{2}x^{2}-3abx+11a^{2}\right)(bx+a)c^{5}x}{5b^{4}\sqrt{cx}\,\sqrt{x(bx+a)}}+\frac{2a^{3}c^{5}x}{b^{4}\sqrt{cx}\,\sqrt{x(bx+a)}}$	74

```
int((c*x)^(9/2)/(b*x^2+a*x)^(3/2),x,method=_RETURNVERBOSE)
```

```
2/5*(b*x+a)*(b^3*x^3-2*a*b^2*x^2+8*a^2*b*x+16*a^3)*(c*x)^(9/2)/b^4/x^3/(b*
x^2+a*x)^(3/2)
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 75, normalized size of antiderivative = 0.61

$$\int \frac{(cx)^{9/2}}{(ax+bx^2)^{3/2}}\,dx = \frac{2\,(b^3c^4x^3 - 2\,ab^2c^4x^2 + 8\,a^2bc^4x + 16\,a^3c^4)\sqrt{bx^2+ax}\sqrt{cx}}{5\,(b^5x^2 + ab^4x)}$$

```
integrate((c*x)^(9/2)/(b*x^2+a*x)^(3/2),x, algorithm="fricas")
```

```
2/5*(b^3*c^4*x^3 - 2*a*b^2*c^4*x^2 + 8*a^2*b*c^4*x + 16*a^3*c^4)*sqrt(b*x^
2 + a*x)*sqrt(c*x)/(b^5*x^2 + a*b^4*x)
```

Sympy [F]

$$\int \frac{(cx)^{9/2}}{(ax+bx^2)^{3/2}}\,dx = \int \frac{(cx)^{\frac{9}{2}}}{(x\,(a+bx))^{\frac{3}{2}}}\,dx$$

```
integrate((c*x)**(9/2)/(b*x**2+a*x)**(3/2),x)
```

```
Integral((c*x)**(9/2)/(x*(a + b*x))**(3/2), x)
```

Maxima [F]

$$\int \frac{(cx)^{9/2}}{(ax+bx^2)^{3/2}}\,dx = \int \frac{(cx)^{\frac{9}{2}}}{(bx^2+ax)^{\frac{3}{2}}}\,dx$$

```
integrate((c*x)^(9/2)/(b*x^2+a*x)^(3/2),x, algorithm="maxima")
```

```
2/15*((3*b^4*c^(9/2)*x^3 - a*b^3*c^(9/2)*x^2 + 4*a^2*b^2*c^(9/2)*x + 8*a^3
*b*c^(9/2))*x^3 - 2*(a*b^3*c^(9/2)*x^3 - 2*a^2*b^2*c^(9/2)*x^2 - 7*a^3*b*c
^(9/2)*x - 4*a^4*c^(9/2))*x^2 + 10*(a^2*b^2*c^(9/2)*x^3 + 2*a^3*b*c^(9/2)*
x^2 + a^4*c^(9/2)*x)*x)/((b^5*x^3 + a*b^4*x^2)*sqrt(b*x + a)) - integrate(
2*(a^3*b*c^(9/2)*x + a^4*c^(9/2))*x/((b^5*x^3 + 2*a*b^4*x^2 + a^2*b^3*x)*s
qrt(b*x + a)), x)
```

Giac [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 121, normalized size of antiderivative = 0.99

$$\int \frac{(cx)^{9/2}}{(ax+bx^2)^{3/2}}\,dx = \frac{2}{5}\,c^3\left(\frac{5\,a^3c^3}{\sqrt{bcx+ac}\,b^4|c|} + \frac{15\,\sqrt{bcx+ac}\,a^2b^{16}c^6 - 5\,(bcx+ac)^{\frac{3}{2}}ab^{16}c^5 + (bcx+ac)^{\frac{5}{2}}b^{16}}{b^{20}c^4|c|}\right.$$
$$\left. - \frac{32\,a^3c^6}{5\,\sqrt{ac}\,b^4|c|}\right)$$

```
integrate((c*x)^(9/2)/(b*x^2+a*x)^(3/2),x, algorithm="giac")
```

```
2/5*c^3*(5*a^3*c^3/(sqrt(b*c*x + a*c)*b^4*abs(c)) + (15*sqrt(b*c*x + a*c)*
a^2*b^16*c^6 - 5*(b*c*x + a*c)^(3/2)*a*b^16*c^5 + (b*c*x + a*c)^(5/2)*b^16
*c^4)/(b^20*c^4*abs(c))) - 32/5*a^3*c^6/(sqrt(a*c)*b^4*abs(c))
```

Mupad [B] (verification not implemented)

Time = 9.29 (sec) , antiderivative size = 91, normalized size of antiderivative = 0.75

$$\int \frac{(cx)^{9/2}}{(ax+bx^2)^{3/2}}\,dx = \frac{\sqrt{b\,x^2+a\,x}\,\left(\frac{32\,a^3\,c^4\,\sqrt{c\,x}}{5\,b^5}+\frac{2\,c^4\,x^3\,\sqrt{c\,x}}{5\,b^2}-\frac{4\,a\,c^4\,x^2\,\sqrt{c\,x}}{5\,b^3}+\frac{16\,a^2\,c^4\,x\,\sqrt{c\,x}}{5\,b^4}\right)}{x^2+\frac{a\,x}{b}}$$

```
int((c*x)^(9/2)/(a*x + b*x^2)^(3/2),x)
```

```
((a*x + b*x^2)^(1/2)*((32*a^3*c^4*(c*x)^(1/2))/(5*b^5) + (2*c^4*x^3*(c*x)^
(1/2))/(5*b^2) - (4*a*c^4*x^2*(c*x)^(1/2))/(5*b^3) + (16*a^2*c^4*x*(c*x)^(
1/2))/(5*b^4)))/(x^2 + (a*x)/b)
```

Reduce [B] (verification not implemented)

Time = 0.35 (sec) , antiderivative size = 47, normalized size of antiderivative = 0.39

$$\int \frac{(cx)^{9/2}}{(ax+bx^2)^{3/2}}\,dx = \frac{2\sqrt{c}\,c^4(b^3x^3-2a\,b^2x^2+8a^2bx+16a^3)}{5\sqrt{bx+a}\,b^4}$$

```
int((c*x)^(9/2)/(b*x^2+a*x)^(3/2),x)
```

```
(2*sqrt(c)*c**4*(16*a**3 + 8*a**2*b*x - 2*a*b**2*x**2 + b**3*x**3))/(5*sqr
t(a + b*x)*b**4)
```

3.106 $\int \frac{(cx)^{7/2}}{(ax+bx^2)^{3/2}}\,dx$

Optimal result

Integrand size = 21, antiderivative size = 91

$$\int \frac{(cx)^{7/2}}{(ax+bx^2)^{3/2}}\,dx = -\frac{2a^2c^3\sqrt{cx}}{b^3\sqrt{ax+bx^2}} - \frac{4ac^4\sqrt{ax+bx^2}}{b^3\sqrt{cx}} + \frac{2c^5(ax+bx^2)^{3/2}}{3b^3(cx)^{3/2}}$$

```
-2*a^2*c^3*(c*x)^(1/2)/b^3/(b*x^2+a*x)^(1/2)-4*a*c^4*(b*x^2+a*x)^(1/2)/b^3
/(c*x)^(1/2)+2/3*c^5*(b*x^2+a*x)^(3/2)/b^3/(c*x)^(3/2)
```

Mathematica [A] (verified)

Time = 0.03 (sec) , antiderivative size = 46, normalized size of antiderivative = 0.51

$$\int \frac{(cx)^{7/2}}{(ax+bx^2)^{3/2}}\,dx = \frac{2c^3\sqrt{cx}(-8a^2-4abx+b^2x^2)}{3b^3\sqrt{x(a+bx)}}$$

```
Integrate[(c*x)^(7/2)/(a*x + b*x^2)^(3/2),x]
```

```
(2*c^3*Sqrt[c*x]*(-8*a^2 - 4*a*b*x + b^2*x^2))/(3*b^3*Sqrt[x*(a + b*x)])
```

Rubi [A] (verified)

Time = 0.36 (sec) , antiderivative size = 94, normalized size of antiderivative = 1.03, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.143, Rules used = {1128, 1128, 1122}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(cx)^{7/2}}{(ax+bx^2)^{3/2}}\,dx$$

$$\Big\downarrow \text{ 1128}$$

$$\frac{2c(cx)^{5/2}}{3b\sqrt{ax+bx^2}}-\frac{4ac\int \frac{(cx)^{5/2}}{(bx^2+ax)^{3/2}}dx}{3b}$$

$$\Big\downarrow \text{ 1128}$$

$$\frac{2c(cx)^{5/2}}{3b\sqrt{ax+bx^2}}-\frac{4ac\left(\frac{2c(cx)^{3/2}}{b\sqrt{ax+bx^2}}-\frac{2ac\int \frac{(cx)^{3/2}}{\left(bx^2+ax\right)^{3/2}}dx}{b}\right)}{3b}$$

$$\Big\downarrow \text{ 1122}$$

$$\frac{2c(cx)^{5/2}}{3b\sqrt{ax+bx^2}}-\frac{4ac\left(\frac{4ac^2\sqrt{cx}}{b^2\sqrt{ax+bx^2}}+\frac{2c(cx)^{3/2}}{b\sqrt{ax+bx^2}}\right)}{3b}$$

```
Int[(c*x)^(7/2)/(a*x + b*x^2)^(3/2),x]
```

```
(2*c*(c*x)^(5/2))/(3*b*Sqrt[a*x + b*x^2]) - (4*a*c*((4*a*c^2*Sqrt[c*x])/(b
^2*Sqrt[a*x + b*x^2]) + (2*c*(c*x)^(3/2))/(b*Sqrt[a*x + b*x^2])))/(3*b)
```

Defintions of rubi rules used

```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] :> Simp[e*(d + e*x)^(m - 1)*((a + b*x + c*x^2)^(p + 1)/(c*(p + 1))),
 x] /; FreeQ[{a, b, c, d, e, m, p}, x] && EqQ[c*d^2 - b*d*e + a*e^2, 0] &&
EqQ[m + p, 0]
```

```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] :> Simp[e*(d + e*x)^(m - 1)*((a + b*x + c*x^2)^(p + 1)/(c*(m + 2*p +
 1))), x] + Simp[Simplify[m + p]*((2*c*d - b*e)/(c*(m + 2*p + 1)))   Int[(d
 + e*x)^(m - 1)*(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, m, p},
 x] && EqQ[c*d^2 - b*d*e + a*e^2, 0] && IGtQ[Simplify[m + p], 0]
```

Maple [A] (verified)

Time = 0.35 (sec) , antiderivative size = 49, normalized size of antiderivative = 0.54

method	result	size
gosper	$-\frac{2(bx+a)\left(-b^{2}x^{2}+4abx+8a^{2}\right)(cx)^{\frac{7}{2}}}{3b^{3}x^{2}\left(b\,x^{2}+ax\right)^{\frac{3}{2}}}$	49
orering	$-\frac{2(bx+a)\left(-b^{2}x^{2}+4abx+8a^{2}\right)(cx)^{\frac{7}{2}}}{3b^{3}x^{2}\left(b\,x^{2}+ax\right)^{\frac{3}{2}}}$	49
default	$-\frac{2c^{3}\sqrt{cx}\,\sqrt{x(bx+a)}\,\left(-b^{2}x^{2}+4abx+8a^{2}\right)}{3x(bx+a)b^{3}}$	52
risch	$-\frac{2(-bx+5a)(bx+a)c^{4}x}{3b^{3}\sqrt{cx}\,\sqrt{x(bx+a)}}-\frac{2a^{2}c^{4}x}{b^{3}\sqrt{cx}\,\sqrt{x(bx+a)}}$	64

```
int((c*x)^(7/2)/(b*x^2+a*x)^(3/2),x,method=_RETURNVERBOSE)
```

```
-2/3*(b*x+a)*(-b^2*x^2+4*a*b*x+8*a^2)*(c*x)^(7/2)/b^3/x^2/(b*x^2+a*x)^(3/2
)
```

Fricas [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 61, normalized size of antiderivative = 0.67

$$\int \frac{(cx)^{7/2}}{(ax+bx^2)^{3/2}}\,dx = \frac{2\,(b^2c^3x^2 - 4\,abc^3x - 8\,a^2c^3)\sqrt{bx^2+ax}\sqrt{cx}}{3\,(b^4x^2+ab^3x)}$$

```
integrate((c*x)^(7/2)/(b*x^2+a*x)^(3/2),x, algorithm="fricas")
```

```
2/3*(b^2*c^3*x^2 - 4*a*b*c^3*x - 8*a^2*c^3)*sqrt(b*x^2 + a*x)*sqrt(c*x)/(b
^4*x^2 + a*b^3*x)
```

Sympy [F]

$$\int \frac{(cx)^{7/2}}{(ax+bx^2)^{3/2}}\,dx = \int \frac{(cx)^{\frac{7}{2}}}{(x\,(a+bx))^{\frac{3}{2}}}\,dx$$

```
integrate((c*x)**(7/2)/(b*x**2+a*x)**(3/2),x)
```

```
Integral((c*x)**(7/2)/(x*(a + b*x))**(3/2), x)
```

Maxima [F]

$$\int \frac{(cx)^{7/2}}{(ax+bx^2)^{3/2}}\,dx = \int \frac{(cx)^{\frac{7}{2}}}{(bx^2+ax)^{\frac{3}{2}}}\,dx$$

```
integrate((c*x)^(7/2)/(b*x^2+a*x)^(3/2),x, algorithm="maxima")
```

```
2/3*((b^3*c^(7/2)*x^2 - a*b^2*c^(7/2)*x - 2*a^2*b*c^(7/2))*x^2 - 2*(a*b^2*
c^(7/2)*x^2 + 2*a^2*b*c^(7/2)*x + a^3*c^(7/2))*x)/((b^4*x^2 + a*b^3*x)*sqr
t(b*x + a)) + integrate(2*(a^2*b*c^(7/2)*x + a^3*c^(7/2))*x/((b^4*x^3 + 2*
a*b^3*x^2 + a^2*b^2*x)*sqrt(b*x + a)), x)
```

Giac [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 106, normalized size of antiderivative = 1.16

$$\int \frac{(cx)^{7/2}}{(ax+bx^2)^{3/2}}\,dx = \frac{2}{3}c^3\left(\frac{8\,a^2c^2}{\sqrt{ac}b^3|c|} - \frac{\frac{3\,a^2c^2}{\sqrt{bcx+ac}b|c|} + \frac{6\,\sqrt{bcx+ac}ab^2c^3-(bcx+ac)^{\frac{3}{2}}b^2c^2}{b^3c^2|c|}}{b^2}\right)$$

```
integrate((c*x)^(7/2)/(b*x^2+a*x)^(3/2),x, algorithm="giac")
```

```
2/3*c^3*(8*a^2*c^2/(sqrt(a*c)*b^3*abs(c)) - (3*a^2*c^2/(sqrt(b*c*x + a*c)*
b*abs(c)) + (6*sqrt(b*c*x + a*c)*a*b^2*c^3 - (b*c*x + a*c)^(3/2)*b^2*c^2)/
(b^3*c^2*abs(c)))/b^2)
```

Mupad [B] (verification not implemented)

Time = 9.14 (sec) , antiderivative size = 53, normalized size of antiderivative = 0.58

$$\int \frac{(cx)^{7/2}}{(ax+bx^2)^{3/2}}\,dx = -\frac{2\,c^3\,\sqrt{b\,x^2+a\,x}\,\sqrt{c\,x}\,(8\,a^2+4\,a\,b\,x-b^2\,x^2)}{3\,b^3\,x\,(a+b\,x)}$$

```
int((c*x)^(7/2)/(a*x + b*x^2)^(3/2),x)
```

```
-(2*c^3*(a*x + b*x^2)^(1/2)*(c*x)^(1/2)*(8*a^2 - b^2*x^2 + 4*a*b*x))/(3*b^
3*x*(a + b*x))
```

Reduce [B] (verification not implemented)

Time = 0.23 (sec) , antiderivative size = 36, normalized size of antiderivative = 0.40

$$\int \frac{(cx)^{7/2}}{(ax+bx^2)^{3/2}}\,dx = \frac{2\sqrt{c}\,c^3(b^2x^2-4abx-8a^2)}{3\sqrt{bx+a}\,b^3}$$

```
int((c*x)^(7/2)/(b*x^2+a*x)^(3/2),x)
```

```
(2*sqrt(c)*c**3*( - 8*a**2 - 4*a*b*x + b**2*x**2))/(3*sqrt(a + b*x)*b**3)
```

3.107 $\int \frac{(cx)^{5/2}}{(ax+bx^2)^{3/2}}\,dx$

Optimal result

Integrand size = 21, antiderivative size = 58

$$\int \frac{(cx)^{5/2}}{(ax+bx^2)^{3/2}}\,dx = \frac{2ac^2\sqrt{cx}}{b^2\sqrt{ax+bx^2}} + \frac{2c^3\sqrt{ax+bx^2}}{b^2\sqrt{cx}}$$

```
2*a*c^2*(c*x)^(1/2)/b^2/(b*x^2+a*x)^(1/2)+2*c^3*(b*x^2+a*x)^(1/2)/b^2/(c*x
)^(1/2)
```

Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 33, normalized size of antiderivative = 0.57

$$\int \frac{(cx)^{5/2}}{(ax+bx^2)^{3/2}}\,dx = \frac{2c^2\sqrt{cx}(2a+bx)}{b^2\sqrt{x(a+bx)}}$$

```
Integrate[(c*x)^(5/2)/(a*x + b*x^2)^(3/2),x]
```

```
(2*c^2*Sqrt[c*x]*(2*a + b*x))/(b^2*Sqrt[x*(a + b*x)])
```

Rubi [A] (verified)

Time = 0.28 (sec) , antiderivative size = 56, normalized size of antiderivative = 0.97, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.095, Rules used = {1128, 1122}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(cx)^{5/2}}{(ax+bx^2)^{3/2}}\,dx$$
$$\Big\downarrow \text{ 1128}$$
$$\frac{2c(cx)^{3/2}}{b\sqrt{ax+bx^2}}-\frac{2ac\int \frac{(cx)^{3/2}}{(bx^2+ax)^{3/2}}dx}{b}$$
$$\Big\downarrow \text{ 1122}$$
$$\frac{4ac^2\sqrt{cx}}{b^2\sqrt{ax+bx^2}}+\frac{2c(cx)^{3/2}}{b\sqrt{ax+bx^2}}$$

```
Int[(c*x)^(5/2)/(a*x + b*x^2)^(3/2),x]
```

```
(4*a*c^2*Sqrt[c*x])/(b^2*Sqrt[a*x + b*x^2]) + (2*c*(c*x)^(3/2))/(b*Sqrt[a*
x + b*x^2])
```

Defintions of rubi rules used

```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] :> Simp[e*(d + e*x)^(m - 1)*((a + b*x + c*x^2)^(p + 1)/(c*(p + 1))),
 x] /; FreeQ[{a, b, c, d, e, m, p}, x] && EqQ[c*d^2 - b*d*e + a*e^2, 0] &&
EqQ[m + p, 0]
```

```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] :> Simp[e*(d + e*x)^(m - 1)*((a + b*x + c*x^2)^(p + 1)/(c*(m + 2*p +
 1))), x] + Simp[Simplify[m + p]*((2*c*d - b*e)/(c*(m + 2*p + 1)))   Int[(d
 + e*x)^(m - 1)*(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, m, p},
 x] && EqQ[c*d^2 - b*d*e + a*e^2, 0] && IGtQ[Simplify[m + p], 0]
```

Maple [A] (verified)

Time = 0.54 (sec) , antiderivative size = 37, normalized size of antiderivative = 0.64

method	result	size
gosper	$\frac{2(bx+a)(bx+2a)(cx)^{\frac{5}{2}}}{b^{2}x(b\,x^{2}+ax)^{\frac{3}{2}}}$	37
orering	$\frac{2(bx+a)(bx+2a)(cx)^{\frac{5}{2}}}{b^{2}x(b\,x^{2}+ax)^{\frac{3}{2}}}$	37
default	$\frac{2c^{2}\sqrt{cx}\,\sqrt{x(bx+a)}\,(bx+2a)}{x(bx+a)b^{2}}$	40
risch	$\frac{2(bx+a)c^{3}x}{b^{2}\sqrt{cx}\,\sqrt{x(bx+a)}}+\frac{2a\,c^{3}x}{b^{2}\sqrt{cx}\,\sqrt{x(bx+a)}}$	54

```
int((c*x)^(5/2)/(b*x^2+a*x)^(3/2),x,method=_RETURNVERBOSE)
```

```
2*(b*x+a)*(b*x+2*a)*(c*x)^(5/2)/b^2/x/(b*x^2+a*x)^(3/2)
```

Fricas [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 47, normalized size of antiderivative = 0.81

$$\int \frac{(cx)^{5/2}}{(ax+bx^2)^{3/2}}\,dx=\frac{2\,(bc^2x+2\,ac^2)\sqrt{bx^2+ax}\sqrt{cx}}{b^3x^2+ab^2x}$$

```
integrate((c*x)^(5/2)/(b*x^2+a*x)^(3/2),x, algorithm="fricas")
```

```
2*(b*c^2*x + 2*a*c^2)*sqrt(b*x^2 + a*x)*sqrt(c*x)/(b^3*x^2 + a*b^2*x)
```

Sympy [F]

$$\int \frac{(cx)^{5/2}}{(ax+bx^2)^{3/2}}\,dx = \int \frac{(cx)^{\frac{5}{2}}}{(x(a+bx))^{\frac{3}{2}}}\,dx$$

```
integrate((c*x)**(5/2)/(b*x**2+a*x)**(3/2),x)
```

```
Integral((c*x)**(5/2)/(x*(a + b*x))**(3/2), x)
```

Maxima [F]

$$\int \frac{(cx)^{5/2}}{(ax+bx^2)^{3/2}}\,dx = \int \frac{(cx)^{\frac{5}{2}}}{(bx^2+ax)^{\frac{3}{2}}}\,dx$$

```
integrate((c*x)^(5/2)/(b*x^2+a*x)^(3/2),x, algorithm="maxima")
```

```
2*(b*c^(5/2)*x + a*c^(5/2))*x/((b^2*x + a*b)*sqrt(b*x + a)) - integrate(2*
(a*b*c^(5/2)*x + a^2*c^(5/2))*x/((b^3*x^3 + 2*a*b^2*x^2 + a^2*b*x)*sqrt(b*
x + a)), x)
```

Giac [A] (verification not implemented)

Time = 0.39 (sec) , antiderivative size = 66, normalized size of antiderivative = 1.14

$$\int \frac{(cx)^{5/2}}{(ax+bx^2)^{3/2}}\,dx = \frac{2\,c^3\left(\frac{ac}{\sqrt{bcx+ac}b|c|}+\frac{\sqrt{bcx+ac}}{b|c|}\right)}{b} - \frac{4\,ac^4}{\sqrt{ac}b^2|c|}$$

```
integrate((c*x)^(5/2)/(b*x^2+a*x)^(3/2),x, algorithm="giac")
```

```
2*c^3*(a*c/(sqrt(b*c*x + a*c)*b*abs(c)) + sqrt(b*c*x + a*c)/(b*abs(c)))/b
- 4*a*c^4/(sqrt(a*c)*b^2*abs(c))
```

Mupad [B] (verification not implemented)

Time = 9.01 (sec) , antiderivative size = 53, normalized size of antiderivative = 0.91

$$\int \frac{(cx)^{5/2}}{(ax+bx^2)^{3/2}}\,dx = \frac{\sqrt{b\,x^2+a\,x}\,\left(\frac{4\,a\,c^2\,\sqrt{c\,x}}{b^3}+\frac{2\,c^2\,x\,\sqrt{c\,x}}{b^2}\right)}{x^2+\frac{a\,x}{b}}$$

```
int((c*x)^(5/2)/(a*x + b*x^2)^(3/2),x)
```

```
((a*x + b*x^2)^(1/2)*((4*a*c^2*(c*x)^(1/2))/b^3 + (2*c^2*x*(c*x)^(1/2))/b^
2))/(x^2 + (a*x)/b)
```

Reduce [B] (verification not implemented)

Time = 0.52 (sec) , antiderivative size = 25, normalized size of antiderivative = 0.43

$$\int \frac{(cx)^{5/2}}{(ax+bx^2)^{3/2}}\,dx = \frac{2\sqrt{c}\,c^2(bx+2a)}{\sqrt{bx+a}\,b^2}$$

```
int((c*x)^(5/2)/(b*x^2+a*x)^(3/2),x)
```

```
(2*sqrt(c)*c**2*(2*a + b*x))/(sqrt(a + b*x)*b**2)
```

3.108 $\int \frac{(cx)^{3/2}}{(ax+bx^2)^{3/2}} \, dx$

Optimal result

Integrand size = 21, antiderivative size = 26

$$\int \frac{(cx)^{3/2}}{(ax+bx^2)^{3/2}} \, dx = -\frac{2c\sqrt{cx}}{b\sqrt{ax+bx^2}}$$

```
-2*c*(c*x)^(1/2)/b/(b*x^2+a*x)^(1/2)
```

Mathematica [A] (verified)

Time = 0.00 (sec) , antiderivative size = 24, normalized size of antiderivative = 0.92

$$\int \frac{(cx)^{3/2}}{(ax+bx^2)^{3/2}} \, dx = -\frac{2c\sqrt{cx}}{b\sqrt{x(a+bx)}}$$

```
Integrate[(c*x)^(3/2)/(a*x + b*x^2)^(3/2),x]
```

```
(-2*c*Sqrt[c*x])/(b*Sqrt[x*(a + b*x)])
```

Rubi [A] (verified)

Time = 0.24 (sec) , antiderivative size = 26, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}} = 0.048$, Rules used = $\{1122\}$

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(cx)^{3/2}}{(ax+bx^2)^{3/2}}\,dx$$
$$\Big\downarrow \; 1122$$
$$-\frac{2c\sqrt{cx}}{b\sqrt{ax+bx^2}}$$

```
Int[(c*x)^(3/2)/(a*x + b*x^2)^(3/2),x]
```

```
(-2*c*Sqrt[c*x])/(b*Sqrt[a*x + b*x^2])
```

Defintions of rubi rules used

```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] :> Simp[e*(d + e*x)^(m - 1)*((a + b*x + c*x^2)^(p + 1)/(c*(p + 1))),
 x] /; FreeQ[{a, b, c, d, e, m, p}, x] && EqQ[c*d^2 - b*d*e + a*e^2, 0] &&
EqQ[m + p, 0]
```

Maple [A] (verified)

Time = 0.43 (sec) , antiderivative size = 27, normalized size of antiderivative = 1.04

method	result	size
gosper	$-\frac{2(bx+a)(cx)^{\frac{3}{2}}}{b(b\,x^{2}+ax)^{\frac{3}{2}}}$	27
orering	$-\frac{2(bx+a)(cx)^{\frac{3}{2}}}{b(b\,x^{2}+ax)^{\frac{3}{2}}}$	27
default	$-\frac{2c\sqrt{cx}\,\sqrt{x(bx+a)}}{x(bx+a)b}$	31

```
int((c*x)^(3/2)/(b*x^2+a*x)^(3/2),x,method=_RETURNVERBOSE)
```

```
-2*(b*x+a)*(c*x)^(3/2)/b/(b*x^2+a*x)^(3/2)
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 33, normalized size of antiderivative = 1.27

$$\int \frac{(cx)^{3/2}}{(ax+bx^2)^{3/2}}\,dx = -\frac{2\sqrt{bx^2+ax}\sqrt{cx}c}{b^2x^2+abx}$$

```
integrate((c*x)^(3/2)/(b*x^2+a*x)^(3/2),x, algorithm="fricas")
```

```
-2*sqrt(b*x^2 + a*x)*sqrt(c*x)*c/(b^2*x^2 + a*b*x)
```

Sympy [F]

$$\int \frac{(cx)^{3/2}}{(ax+bx^2)^{3/2}}\,dx = \int \frac{(cx)^{\frac{3}{2}}}{(x\,(a+bx))^{\frac{3}{2}}}\,dx$$

```
integrate((c*x)**(3/2)/(b*x**2+a*x)**(3/2),x)
```

```
Integral((c*x)**(3/2)/(x*(a + b*x))**(3/2), x)
```

Maxima [F]

$$\int \frac{(cx)^{3/2}}{(ax+bx^2)^{3/2}}\,dx = \int \frac{(cx)^{\frac{3}{2}}}{(bx^2+ax)^{\frac{3}{2}}}\,dx$$

```
integrate((c*x)^(3/2)/(b*x^2+a*x)^(3/2),x, algorithm="maxima")
```

```
integrate((c*x)^(3/2)/(b*x^2 + a*x)^(3/2), x)
```

Giac [A] (verification not implemented)

Time = 0.23 (sec) , antiderivative size = 40, normalized size of antiderivative = 1.54

$$\int \frac{(cx)^{3/2}}{(ax+bx^2)^{3/2}}\,dx = -\frac{2\,c^3}{\sqrt{bcx+ac}b|c|} + \frac{2\,c^3}{\sqrt{ac}b|c|}$$

```
integrate((c*x)^(3/2)/(b*x^2+a*x)^(3/2),x, algorithm="giac")
```

```
-2*c^3/(sqrt(b*c*x + a*c)*b*abs(c)) + 2*c^3/(sqrt(a*c)*b*abs(c))
```

Mupad [B] (verification not implemented)

Time = 8.93 (sec) , antiderivative size = 22, normalized size of antiderivative = 0.85

$$\int \frac{(cx)^{3/2}}{(ax+bx^2)^{3/2}}\,dx = -\frac{2\,c\,\sqrt{c\,x}}{b\,\sqrt{b\,x^2+a\,x}}$$

```
int((c*x)^(3/2)/(a*x + b*x^2)^(3/2),x)
```

```
-(2*c*(c*x)^(1/2))/(b*(a*x + b*x^2)^(1/2))
```

Reduce [B] (verification not implemented)

Time = 0.51 (sec) , antiderivative size = 16, normalized size of antiderivative = 0.62

$$\int \frac{(cx)^{3/2}}{(ax+bx^2)^{3/2}}\,dx = -\frac{2\sqrt{c}\,c}{\sqrt{bx+a}\,b}$$

```
int((c*x)^(3/2)/(b*x^2+a*x)^(3/2),x)
```

```
( - 2*sqrt(c)*c)/(sqrt(a + b*x)*b)
```

3.109 $\int \frac{\sqrt{cx}}{(ax+bx^2)^{3/2}}\,dx$

Optimal result

Integrand size = 21, antiderivative size = 70

$$\int \frac{\sqrt{cx}}{(ax+bx^2)^{3/2}}\,dx=\frac{2\sqrt{cx}}{a\sqrt{ax+bx^2}}-\frac{2\sqrt{c}\,\text{arctanh}\left(\frac{\sqrt{c}\sqrt{ax+bx^2}}{\sqrt{a}\sqrt{cx}}\right)}{a^{3/2}}$$

```
2*(c*x)^(1/2)/a/(b*x^2+a*x)^(1/2)-2*c^(1/2)*arctanh(c^(1/2)*(b*x^2+a*x)^(1
/2)/a^(1/2)/(c*x)^(1/2))/a^(3/2)
```

Mathematica [A] (verified)

Time = 0.04 (sec) , antiderivative size = 58, normalized size of antiderivative = 0.83

$$\int \frac{\sqrt{cx}}{(ax+bx^2)^{3/2}}\,dx=\frac{2\sqrt{cx}\left(\sqrt{a}-\sqrt{a+bx}\,\text{arctanh}\left(\frac{\sqrt{a+bx}}{\sqrt{a}}\right)\right)}{a^{3/2}\sqrt{x(a+bx)}}$$

```
Integrate[Sqrt[c*x]/(a*x + b*x^2)^(3/2),x]
```

```
(2*Sqrt[c*x]*(Sqrt[a] - Sqrt[a + b*x]*ArcTanh[Sqrt[a + b*x]/Sqrt[a]]))/(a^
(3/2)*Sqrt[x*(a + b*x)])
```

Rubi [A] (verified)

Time = 0.32 (sec) , antiderivative size = 70, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.143$, Rules used = {1132, 1136, 221}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sqrt{cx}}{(ax+bx^2)^{3/2}}\,dx$$

$$\Big\downarrow \text{1132}$$

$$\frac{c\int \frac{1}{\sqrt{cx}\sqrt{bx^2+ax}}dx}{a}+\frac{2\sqrt{cx}}{a\sqrt{ax+bx^2}}$$

$$\Big\downarrow \text{1136}$$

$$\frac{2c^2\int \frac{1}{\frac{c\left(bx^2+ax\right)}{x}-ac}d\frac{\sqrt{bx^2+ax}}{\sqrt{cx}}}{a}+\frac{2\sqrt{cx}}{a\sqrt{ax+bx^2}}$$

$$\Big\downarrow \text{221}$$

$$\frac{2\sqrt{cx}}{a\sqrt{ax+bx^2}}-\frac{2\sqrt{c}\,\text{arctanh}\left(\frac{\sqrt{c}\sqrt{ax+bx^2}}{\sqrt{a}\sqrt{cx}}\right)}{a^{3/2}}$$

```
Int[Sqrt[c*x]/(a*x + b*x^2)^(3/2),x]
```

```
(2*Sqrt[c*x])/(a*Sqrt[a*x + b*x^2]) - (2*Sqrt[c]*ArcTanh[(Sqrt[c]*Sqrt[a*x
 + b*x^2])/(Sqrt[a]*Sqrt[c*x])])/a^(3/2)
```

Defintions of rubi rules used

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] :> Simp[(2*c*d - b*e)*(d + e*x)^m*((a + b*x + c*x^2)^(p + 1)/(e*(p +
 1)*(b^2 - 4*a*c))), x] - Simp[(2*c*d - b*e)*((m + 2*p + 2)/((p + 1)*(b^2 -
 4*a*c)))   Int[(d + e*x)^(m - 1)*(a + b*x + c*x^2)^(p + 1), x], x] /; Free
Q[{a, b, c, d, e}, x] && EqQ[c*d^2 - b*d*e + a*e^2, 0] && LtQ[p, -1] && LtQ
[0, m, 1] && IntegerQ[2*p]
```

```
Int[1/(Sqrt[(d_.) + (e_.)*(x_)]*Sqrt[(a_.) + (b_.)*(x_) + (c_.)*(x_)^2]), x
_Symbol] :> Simp[2*e   Subst[Int[1/(2*c*d - b*e + e^2*x^2), x], x, Sqrt[a +
 b*x + c*x^2]/Sqrt[d + e*x]], x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[c*d^2
- b*d*e + a*e^2, 0]
```

Maple [A] (verified)

Time = 0.50 (sec) , antiderivative size = 69, normalized size of antiderivative = 0.99

method	result	size
default	$-\frac{2\sqrt{cx}\,\sqrt{x(bx+a)}\left(\operatorname{arctanh}\left(\frac{\sqrt{c(bx+a)}}{\sqrt{ac}}\right)\sqrt{c(bx+a)}-\sqrt{ac}\right)}{x(bx+a)a\sqrt{ac}}$	69

```
int((c*x)^(1/2)/(b*x^2+a*x)^(3/2),x,method=_RETURNVERBOSE)
```

```
-2*(c*x)^(1/2)*(x*(b*x+a))^(1/2)*(arctanh((c*(b*x+a))^(1/2)/(a*c)^(1/2))*(
c*(b*x+a))^(1/2)-(a*c)^(1/2))/x/(b*x+a)/a/(a*c)^(1/2)
```

Fricas [A] (verification not implemented)

Time = 0.10 (sec) , antiderivative size = 188, normalized size of antiderivative = 2.69

$$\int \frac{\sqrt{cx}}{\left(ax+bx^2\right)^{3/2}}\,dx = \left[\frac{\left(bx^2+ax\right)\sqrt{\frac{c}{a}}\log\left(-\frac{bcx^2+2\,acx-2\,\sqrt{bx^2+ax}\sqrt{cx}a\sqrt{\frac{c}{a}}}{x^2}\right)+2\,\sqrt{bx^2+ax}\sqrt{cx}}{abx^2+a^2x}, \frac{2\left(\left(bx^2+a\right.\right.}{}\right.$$

```
integrate((c*x)^(1/2)/(b*x^2+a*x)^(3/2),x, algorithm="fricas")
```

```
[((b*x^2 + a*x)*sqrt(c/a)*log(-(b*c*x^2 + 2*a*c*x - 2*sqrt(b*x^2 + a*x)*sq
rt(c*x)*a*sqrt(c/a))/x^2) + 2*sqrt(b*x^2 + a*x)*sqrt(c*x))/(a*b*x^2 + a^2*
x), 2*((b*x^2 + a*x)*sqrt(-c/a)*arctan(sqrt(b*x^2 + a*x)*sqrt(c*x)*a*sqrt(
-c/a)/(b*c*x^2 + a*c*x)) + sqrt(b*x^2 + a*x)*sqrt(c*x))/(a*b*x^2 + a^2*x)]
```

Sympy [F]

$$\int \frac{\sqrt{cx}}{\left(ax+bx^2\right)^{3/2}}\,dx = \int \frac{\sqrt{cx}}{\left(x\left(a+bx\right)\right)^{\frac{3}{2}}}\,dx$$

```
integrate((c*x)**(1/2)/(b*x**2+a*x)**(3/2),x)
```

```
Integral(sqrt(c*x)/(x*(a + b*x))**(3/2), x)
```

Maxima [F]

$$\int \frac{\sqrt{cx}}{\left(ax+bx^2\right)^{3/2}}\,dx = \int \frac{\sqrt{cx}}{\left(bx^2+ax\right)^{\frac{3}{2}}}\,dx$$

```
integrate((c*x)^(1/2)/(b*x^2+a*x)^(3/2),x, algorithm="maxima")
```

```
integrate(sqrt(c*x)/(b*x^2 + a*x)^(3/2), x)
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 116 vs. $2(54) = 108$.

Time = 0.13 (sec) , antiderivative size = 116, normalized size of antiderivative = 1.66

$$\int \frac{\sqrt{cx}}{(ax+bx^2)^{3/2}}\,dx = 2\,c^3\left(\frac{\arctan\left(\frac{\sqrt{bcx+ac}}{\sqrt{-ac}}\right)}{\sqrt{-ac}ac|c|} + \frac{1}{\sqrt{bcx+ac}ac|c|}\right) - \frac{2\left(\sqrt{ac}c^2\arctan\left(\frac{\sqrt{ac}}{\sqrt{-ac}}\right) + \sqrt{-ac}c^2\right)}{\sqrt{ac}\sqrt{-ac}a|c|}$$

```
integrate((c*x)^(1/2)/(b*x^2+a*x)^(3/2),x, algorithm="giac")
```

```
2*c^3*(arctan(sqrt(b*c*x + a*c)/sqrt(-a*c))/(sqrt(-a*c)*a*c*abs(c)) + 1/(s
qrt(b*c*x + a*c)*a*c*abs(c))) - 2*(sqrt(a*c)*c^2*arctan(sqrt(a*c)/sqrt(-a*
c)) + sqrt(-a*c)*c^2)/(sqrt(a*c)*sqrt(-a*c)*a*abs(c))
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\sqrt{cx}}{(ax+bx^2)^{3/2}}\,dx = \int \frac{\sqrt{c\,x}}{{\left(b\,x^2+a\,x\right)}^{3/2}}\,dx$$

```
int((c*x)^(1/2)/(a*x + b*x^2)^(3/2),x)
```

```
int((c*x)^(1/2)/(a*x + b*x^2)^(3/2), x)
```

Reduce [B] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 59, normalized size of antiderivative = 0.84

$$\int \frac{\sqrt{cx}}{(ax+bx^2)^{3/2}}\,dx = \frac{\sqrt{c}\,\left(\sqrt{a}\,\sqrt{bx+a}\log\left(\sqrt{bx+a}-\sqrt{a}\right)-\sqrt{a}\,\sqrt{bx+a}\log\left(\sqrt{bx+a}+\sqrt{a}\right)+2a\right)}{\sqrt{bx+a}\,a^2}$$

```
int((c*x)^(1/2)/(b*x^2+a*x)^(3/2),x)
```

```
(sqrt(c)*(sqrt(a)*sqrt(a + b*x)*log(sqrt(a + b*x) - sqrt(a)) - sqrt(a)*sqr
t(a + b*x)*log(sqrt(a + b*x) + sqrt(a)) + 2*a))/(sqrt(a + b*x)*a**2)
```

3.110 $\int \frac{1}{\sqrt{cx}\,(ax+bx^2)^{3/2}}\,dx$

Optimal result

Integrand size = 21, antiderivative size = 100

$$\int \frac{1}{\sqrt{cx}\,(ax+bx^2)^{3/2}}\,dx = -\frac{1}{a\sqrt{cx}\sqrt{ax+bx^2}} - \frac{3b\sqrt{cx}}{a^2c\sqrt{ax+bx^2}} + \frac{3b\,\text{arctanh}\left(\frac{\sqrt{c}\sqrt{ax+bx^2}}{\sqrt{a}\sqrt{cx}}\right)}{a^{5/2}\sqrt{c}}$$

```
-1/a/(c*x)^(1/2)/(b*x^2+a*x)^(1/2)-3*b*(c*x)^(1/2)/a^2/c/(b*x^2+a*x)^(1/2)
+3*b*arctanh(c^(1/2)*(b*x^2+a*x)^(1/2)/a^(1/2)/(c*x)^(1/2))/a^(5/2)/c^(1/2
)
```

Mathematica [A] (verified)

Time = 0.07 (sec) , antiderivative size = 67, normalized size of antiderivative = 0.67

$$\int \frac{1}{\sqrt{cx}\,(ax+bx^2)^{3/2}}\,dx = \frac{-\sqrt{a}(a+3bx)+3bx\sqrt{a+bx}\,\text{arctanh}\left(\frac{\sqrt{a+bx}}{\sqrt{a}}\right)}{a^{5/2}\sqrt{cx}\sqrt{x(a+bx)}}$$

```
Integrate[1/(Sqrt[c*x]*(a*x + b*x^2)^(3/2)),x]
```

```
(-(Sqrt[a]*(a + 3*b*x)) + 3*b*x*Sqrt[a + b*x]*ArcTanh[Sqrt[a + b*x]/Sqrt[a
]])/(a^(5/2)*Sqrt[c*x]*Sqrt[x*(a + b*x)])
```

Rubi [A] (verified)

Time = 0.37 (sec) , antiderivative size = 107, normalized size of antiderivative = 1.07, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.190$, Rules used = {1135, 1132, 1136, 221}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{\sqrt{cx}\,(ax+bx^2)^{3/2}}\,dx$$

$$\Big\downarrow \ 1135$$

$$-\frac{3b\int \frac{\sqrt{cx}}{(bx^2+ax)^{3/2}}dx}{2ac}-\frac{1}{a\sqrt{cx}\sqrt{ax+bx^2}}$$

$$\Big\downarrow \ 1132$$

$$-\frac{3b\left(\frac{c\int \frac{1}{\sqrt{cx}\sqrt{bx^2+ax}}dx}{a}+\frac{2\sqrt{cx}}{a\sqrt{ax+bx^2}}\right)}{2ac}-\frac{1}{a\sqrt{cx}\sqrt{ax+bx^2}}$$

$$\Big\downarrow \ 1136$$

$$-\frac{3b\left(\frac{2c^2\int \frac{1}{\frac{c(bx^2+ax)}{x}-ac}d\frac{\sqrt{bx^2+ax}}{\sqrt{cx}}}{a}+\frac{2\sqrt{cx}}{a\sqrt{ax+bx^2}}\right)}{2ac}-\frac{1}{a\sqrt{cx}\sqrt{ax+bx^2}}$$

$$\Big\downarrow \ 221$$

$$-\frac{3b\left(\frac{2\sqrt{cx}}{a\sqrt{ax+bx^2}}-\frac{2\sqrt{c}\,\text{arctanh}\left(\frac{\sqrt{c}\sqrt{ax+bx^2}}{\sqrt{a}\sqrt{cx}}\right)}{a^{3/2}}\right)}{2ac}-\frac{1}{a\sqrt{cx}\sqrt{ax+bx^2}}$$

```
Int[1/(Sqrt[c*x]*(a*x + b*x^2)^(3/2)),x]
```

```
-(1/(a*Sqrt[c*x]*Sqrt[a*x + b*x^2])) - (3*b*((2*Sqrt[c*x])/(a*Sqrt[a*x + b
*x^2]) - (2*Sqrt[c]*ArcTanh[(Sqrt[c]*Sqrt[a*x + b*x^2])/(Sqrt[a]*Sqrt[c*x]
)])/a^(3/2)))/(2*a*c)
```

Defintions of rubi rules used

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] :> Simp[(2*c*d - b*e)*(d + e*x)^m*((a + b*x + c*x^2)^(p + 1)/(e*(p +
 1)*(b^2 - 4*a*c))), x] - Simp[(2*c*d - b*e)*((m + 2*p + 2)/((p + 1)*(b^2 -
 4*a*c)))   Int[(d + e*x)^(m - 1)*(a + b*x + c*x^2)^(p + 1), x], x] /; Free
Q[{a, b, c, d, e}, x] && EqQ[c*d^2 - b*d*e + a*e^2, 0] && LtQ[p, -1] && LtQ
[0, m, 1] && IntegerQ[2*p]
```

```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] :> Simp[(-e)*(d + e*x)^m*((a + b*x + c*x^2)^(p + 1)/((m + p + 1)*(2*
c*d - b*e))), x] + Simp[c*((m + 2*p + 2)/((m + p + 1)*(2*c*d - b*e)))   Int
[(d + e*x)^(m + 1)*(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, p},
 x] && EqQ[c*d^2 - b*d*e + a*e^2, 0] && LtQ[m, 0] && NeQ[m + p + 1, 0] && I
ntegerQ[2*p]
```

```
Int[1/(Sqrt[(d_.) + (e_.)*(x_)]*Sqrt[(a_.) + (b_.)*(x_) + (c_.)*(x_)^2]), x
_Symbol] :> Simp[2*e   Subst[Int[1/(2*c*d - b*e + e^2*x^2), x], x, Sqrt[a +
 b*x + c*x^2]/Sqrt[d + e*x]], x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[c*d^2
- b*d*e + a*e^2, 0]
```

Maple [A] (verified)

Time = 0.37 (sec) , antiderivative size = 81, normalized size of antiderivative = 0.81

method	result	size
default	$\frac{\sqrt{x(bx+a)}\,\left(3\sqrt{c(bx+a)}\,\operatorname{arctanh}\left(\frac{\sqrt{c(bx+a)}}{\sqrt{ac}}\right)bx-3\sqrt{ac}\,bx-\sqrt{ac}\,a\right)}{\sqrt{cx}\,x(bx+a)a^{2}\sqrt{ac}}$	81
risch	$-\frac{bx+a}{a^{2}\sqrt{cx}\,\sqrt{x(bx+a)}}-\frac{b\left(\frac{4}{\sqrt{cbx+ac}}-\frac{6\,\operatorname{arctanh}\left(\frac{\sqrt{cbx+ac}}{\sqrt{ac}}\right)}{\sqrt{ac}}\right)\sqrt{c(bx+a)}\,x}{2a^{2}\sqrt{cx}\,\sqrt{x(bx+a)}}$	93

```
int(1/(c*x)^(1/2)/(b*x^2+a*x)^(3/2),x,method=_RETURNVERBOSE)
```

```
1/(c*x)^(1/2)*(x*(b*x+a))^(1/2)/x*(3*(c*(b*x+a))^(1/2)*arctanh((c*(b*x+a))
^(1/2)/(a*c)^(1/2))*b*x-3*(a*c)^(1/2)*b*x-(a*c)^(1/2)*a)/(b*x+a)/a^2/(a*c)
^(1/2)
```

Fricas [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 217, normalized size of antiderivative = 2.17

$$\int \frac{1}{\sqrt{cx}\,(ax+bx^2)^{3/2}}\,dx = \left[\frac{3\,(b^2x^3+abx^2)\sqrt{ac}\log\left(-\frac{bcx^2+2\,acx+2\,\sqrt{bx^2+ax}\sqrt{ac}\sqrt{cx}}{x^2}\right)-2\,(3\,abx+a^2)\sqrt{bx^2+}}{2\,(a^3bcx^3+a^4cx^2)}\right.$$
$$\left.-\frac{3\,(b^2x^3+abx^2)\sqrt{-ac}\arctan\left(\frac{\sqrt{bx^2+ax}\sqrt{-ac}\sqrt{cx}}{acx}\right)+(3\,abx+a^2)\sqrt{bx^2+ax}\sqrt{cx}}{a^3bcx^3+a^4cx^2}\right]$$

```
integrate(1/(c*x)^(1/2)/(b*x^2+a*x)^(3/2),x, algorithm="fricas")
```

```
[1/2*(3*(b^2*x^3 + a*b*x^2)*sqrt(a*c)*log(-(b*c*x^2 + 2*a*c*x + 2*sqrt(b*x
^2 + a*x)*sqrt(a*c)*sqrt(c*x))/x^2) - 2*(3*a*b*x + a^2)*sqrt(b*x^2 + a*x)*
sqrt(c*x))/(a^3*b*c*x^3 + a^4*c*x^2), -(3*(b^2*x^3 + a*b*x^2)*sqrt(-a*c)*a
rctan(sqrt(b*x^2 + a*x)*sqrt(-a*c)*sqrt(c*x)/(a*c*x)) + (3*a*b*x + a^2)*sq
rt(b*x^2 + a*x)*sqrt(c*x))/(a^3*b*c*x^3 + a^4*c*x^2)]
```

Sympy [F]

$$\int \frac{1}{\sqrt{cx}\,(ax+bx^2)^{3/2}}\,dx = \int \frac{1}{\sqrt{cx}\,(x\,(a+bx))^{\frac{3}{2}}}\,dx$$

```
integrate(1/(c*x)**(1/2)/(b*x**2+a*x)**(3/2),x)
```

```
Integral(1/(sqrt(c*x)*(x*(a + b*x))**(3/2)), x)
```

Maxima [F]

$$\int \frac{1}{\sqrt{cx}\,(ax+bx^2)^{3/2}}\,dx = \int \frac{1}{(bx^2+ax)^{\frac{3}{2}}\sqrt{cx}}\,dx$$

```
integrate(1/(c*x)^(1/2)/(b*x^2+a*x)^(3/2),x, algorithm="maxima")
```

```
integrate(1/((b*x^2 + a*x)^(3/2)*sqrt(c*x)), x)
```

Giac [A] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 99, normalized size of antiderivative = 0.99

$$\int \frac{1}{\sqrt{cx}\,(ax+bx^2)^{3/2}}\,dx = -c^3\left(\frac{3\,b\arctan\left(\frac{\sqrt{bcx+ac}}{\sqrt{-ac}}\right)}{\sqrt{-ac}a^2c^2|c|} + \frac{2\,abc-3\,(bcx+ac)b}{\left(\sqrt{bcx+ac}ac-(bcx+ac)^{\frac{3}{2}}\right)a^2c^2|c|}\right)$$

```
integrate(1/(c*x)^(1/2)/(b*x^2+a*x)^(3/2),x, algorithm="giac")
```

```
-c^3*(3*b*arctan(sqrt(b*c*x + a*c)/sqrt(-a*c))/(sqrt(-a*c)*a^2*c^2*abs(c))
 + (2*a*b*c - 3*(b*c*x + a*c)*b)/((sqrt(b*c*x + a*c)*a*c - (b*c*x + a*c)^(
3/2))*a^2*c^2*abs(c)))
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{\sqrt{cx}\,(ax+bx^2)^{3/2}}\,dx = \int \frac{1}{(b\,x^2+a\,x)^{3/2}\,\sqrt{c\,x}}\,dx$$

```
int(1/((a*x + b*x^2)^(3/2)*(c*x)^(1/2)),x)
```

```
int(1/((a*x + b*x^2)^(3/2)*(c*x)^(1/2)), x)
```

Reduce [B] (verification not implemented)

Time = 0.22 (sec) , antiderivative size = 78, normalized size of antiderivative = 0.78

$$\int \frac{1}{\sqrt{cx}\,(ax+bx^2)^{3/2}}\,dx = \frac{\sqrt{c}\,\left(-3\sqrt{a}\,\sqrt{bx+a}\log\left(\sqrt{bx+a}-\sqrt{a}\right)bx+3\sqrt{a}\,\sqrt{bx+a}\log\left(\sqrt{bx+a}+\sqrt{a}\right)}{2\sqrt{bx+a}\,a^3cx}$$

```
int(1/(c*x)^(1/2)/(b*x^2+a*x)^(3/2),x)
```

```
(sqrt(c)*( - 3*sqrt(a)*sqrt(a + b*x)*log(sqrt(a + b*x) - sqrt(a))*b*x + 3*
sqrt(a)*sqrt(a + b*x)*log(sqrt(a + b*x) + sqrt(a))*b*x - 2*a**2 - 6*a*b*x)
)/(2*sqrt(a + b*x)*a**3*c*x)
```

3.111 $\int \frac{1}{(cx)^{3/2}\left(ax+bx^2\right)^{3/2}}\,dx$

Optimal result

Integrand size = 21, antiderivative size = 141

$$\int \frac{1}{(cx)^{3/2}\left(ax+bx^2\right)^{3/2}}\,dx = -\frac{1}{2a(cx)^{3/2}\sqrt{ax+bx^2}} + \frac{5b}{4a^2c\sqrt{cx}\sqrt{ax+bx^2}} + \frac{15b^2\sqrt{cx}}{4a^3c^2\sqrt{ax+bx^2}} - \frac{15b^2\operatorname{arctanh}\left(\frac{\sqrt{c}\sqrt{ax+bx^2}}{\sqrt{a}\sqrt{cx}}\right)}{4a^{7/2}c^{3/2}}$$

```
-1/2/a/(c*x)^(3/2)/(b*x^2+a*x)^(1/2)+5/4*b/a^2/c/(c*x)^(1/2)/(b*x^2+a*x)^(
1/2)+15/4*b^2*(c*x)^(1/2)/a^3/c^2/(b*x^2+a*x)^(1/2)-15/4*b^2*arctanh(c^(1/
2)*(b*x^2+a*x)^(1/2)/a^(1/2)/(c*x)^(1/2))/a^(7/2)/c^(3/2)
```

Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 86, normalized size of antiderivative = 0.61

$$\int \frac{1}{(cx)^{3/2}\left(ax+bx^2\right)^{3/2}}\,dx = \frac{\sqrt{a}(-2a^2+5abx+15b^2x^2)-15b^2x^2\sqrt{a+bx}\operatorname{arctanh}\left(\frac{\sqrt{a+bx}}{\sqrt{a}}\right)}{4a^{7/2}(cx)^{3/2}\sqrt{x(a+bx)}}$$

```
Integrate[1/((c*x)^(3/2)*(a*x + b*x^2)^(3/2)),x]
```

```
(Sqrt[a]*(-2*a^2 + 5*a*b*x + 15*b^2*x^2) - 15*b^2*x^2*Sqrt[a + b*x]*ArcTan
h[Sqrt[a + b*x]/Sqrt[a]])/(4*a^(7/2)*(c*x)^(3/2)*Sqrt[x*(a + b*x)])
```

Rubi [A] (verified)

Time = 0.45 (sec) , antiderivative size = 146, normalized size of antiderivative = 1.04, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.238$, Rules used = {1135, 1135, 1132, 1136, 221}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{(cx)^{3/2} (ax+bx^2)^{3/2}} \, dx$$

$$\Big\downarrow \text{1135}$$

$$-\frac{5b\int \frac{1}{\sqrt{cx}(bx^2+ax)^{3/2}}dx}{4ac}-\frac{1}{2a(cx)^{3/2}\sqrt{ax+bx^2}}$$

$$\Big\downarrow \text{1135}$$

$$-\frac{5b\left(-\frac{3b\int \frac{\sqrt{cx}}{(bx^2+ax)^{3/2}}dx}{2ac}-\frac{1}{a\sqrt{cx}\sqrt{ax+bx^2}}\right)}{4ac}-\frac{1}{2a(cx)^{3/2}\sqrt{ax+bx^2}}$$

$$\Big\downarrow \text{1132}$$

$$-\frac{5b\left(-\frac{3b\left(\frac{c\int \frac{1}{\sqrt{cx}\sqrt{bx^2+ax}}dx}{a}+\frac{2\sqrt{cx}}{a\sqrt{ax+bx^2}}\right)}{2ac}-\frac{1}{a\sqrt{cx}\sqrt{ax+bx^2}}\right)}{4ac}-\frac{1}{2a(cx)^{3/2}\sqrt{ax+bx^2}}$$

$$\Big\downarrow \text{1136}$$

$$-\frac{5b\left(-\frac{3b\left(\frac{2c^2\int\frac{1}{\frac{c\left(bx^2+ax\right)}{x}-ac}d\frac{\sqrt{bx^2+ax}}{\sqrt{cx}}}{a}+\frac{2\sqrt{cx}}{a\sqrt{ax+bx^2}}\right)}{2ac}-\frac{1}{a\sqrt{cx}\sqrt{ax+bx^2}}\right)}{4ac}-\frac{1}{2a(cx)^{3/2}\sqrt{ax+bx^2}}$$

$$\Big\downarrow \text{221}$$

$$-\frac{5b\left(-\frac{3b\left(\frac{2\sqrt{cx}}{a\sqrt{ax+bx^2}}-\frac{2\sqrt{c}\,\text{arctanh}\left(\frac{\sqrt{c}\sqrt{ax+bx^2}}{\sqrt{a}\sqrt{cx}}\right)}{a^{3/2}}\right)}{2ac}-\frac{1}{a\sqrt{cx}\sqrt{ax+bx^2}}\right)}{4ac}-\frac{1}{2a(cx)^{3/2}\sqrt{ax+bx^2}}$$

```
Int[1/((c*x)^(3/2)*(a*x + b*x^2)^(3/2)),x]
```

```
-1/2*1/(a*(c*x)^(3/2)*Sqrt[a*x + b*x^2]) - (5*b*(-(1/(a*Sqrt[c*x]*Sqrt[a*x
 + b*x^2])) - (3*b*((2*Sqrt[c*x])/(a*Sqrt[a*x + b*x^2]) - (2*Sqrt[c]*ArcTa
nh[(Sqrt[c]*Sqrt[a*x + b*x^2])/(Sqrt[a]*Sqrt[c*x])])/a^(3/2)))/(2*a*c)))/(
4*a*c)
```

Defintions of rubi rules used

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] :> Simp[(2*c*d - b*e)*(d + e*x)^m*((a + b*x + c*x^2)^(p + 1)/(e*(p +
 1)*(b^2 - 4*a*c))), x] - Simp[(2*c*d - b*e)*((m + 2*p + 2)/((p + 1)*(b^2 -
 4*a*c)))   Int[(d + e*x)^(m - 1)*(a + b*x + c*x^2)^(p + 1), x], x] /; Free
Q[{a, b, c, d, e}, x] && EqQ[c*d^2 - b*d*e + a*e^2, 0] && LtQ[p, -1] && LtQ
[0, m, 1] && IntegerQ[2*p]
```

```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] :> Simp[(-e)*(d + e*x)^m*((a + b*x + c*x^2)^(p + 1)/((m + p + 1)*(2*
c*d - b*e))), x] + Simp[c*((m + 2*p + 2)/((m + p + 1)*(2*c*d - b*e)))   Int
[(d + e*x)^(m + 1)*(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, p},
 x] && EqQ[c*d^2 - b*d*e + a*e^2, 0] && LtQ[m, 0] && NeQ[m + p + 1, 0] && I
ntegerQ[2*p]
```

```
Int[1/(Sqrt[(d_.) + (e_.)*(x_)]*Sqrt[(a_.) + (b_.)*(x_) + (c_.)*(x_)^2]), x
_Symbol] :> Simp[2*e   Subst[Int[1/(2*c*d - b*e + e^2*x^2), x], x, Sqrt[a +
 b*x + c*x^2]/Sqrt[d + e*x]], x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[c*d^2
- b*d*e + a*e^2, 0]
```

Maple [A] (verified)

Time = 0.37 (sec) , antiderivative size = 105, normalized size of antiderivative = 0.74

method	result	size
default	$-\frac{\sqrt{x(bx+a)}\left(15\sqrt{c(bx+a)}\,\operatorname{arctanh}\left(\frac{\sqrt{c(bx+a)}}{\sqrt{ac}}\right)b^2x^2-15\sqrt{ac}\,b^2x^2-5\sqrt{ac}\,abx+2\sqrt{ac}\,a^2\right)}{4c\,x^2\sqrt{cx}\,(bx+a)a^3\sqrt{ac}}$	105
risch	$-\frac{(bx+a)(-7bx+2a)}{4a^3xc\sqrt{cx}\,\sqrt{x(bx+a)}}+\frac{b^2\left(\frac{16}{\sqrt{cbx+ac}}-\frac{30\,\operatorname{arctanh}\left(\frac{\sqrt{cbx+ac}}{\sqrt{ac}}\right)}{\sqrt{ac}}\right)\sqrt{c(bx+a)}\,x}{8a^3c\sqrt{cx}\,\sqrt{x(bx+a)}}$	112

```
int(1/(c*x)^(3/2)/(b*x^2+a*x)^(3/2),x,method=_RETURNVERBOSE)
```

```
-1/4/c/x^2/(c*x)^(1/2)*(x*(b*x+a))^(1/2)*(15*(c*(b*x+a))^(1/2)*arctanh((c*
(b*x+a))^(1/2)/(a*c)^(1/2))*b^2*x^2-15*(a*c)^(1/2)*b^2*x^2-5*(a*c)^(1/2)*a
*b*x+2*(a*c)^(1/2)*a^2)/(b*x+a)/a^3/(a*c)^(1/2)
```

Fricas [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 255, normalized size of antiderivative = 1.81

$$\int \frac{1}{(cx)^{3/2}(ax+bx^2)^{3/2}}\,dx = \left[\frac{15\,(b^3x^4+ab^2x^3)\sqrt{ac}\log\left(-\frac{bcx^2+2\,acx-2\,\sqrt{bx^2+ax}\sqrt{ac}\sqrt{cx}}{x^2}\right)+2\,(15\,ab^2x^2+5\,a^2}{8\,(a^4bc^2x^4+a^5c^2x^3)}\right.$$

```
integrate(1/(c*x)^(3/2)/(b*x^2+a*x)^(3/2),x, algorithm="fricas")
```

```
[1/8*(15*(b^3*x^4 + a*b^2*x^3)*sqrt(a*c)*log(-(b*c*x^2 + 2*a*c*x - 2*sqrt(
b*x^2 + a*x)*sqrt(a*c)*sqrt(c*x))/x^2) + 2*(15*a*b^2*x^2 + 5*a^2*b*x - 2*a
^3)*sqrt(b*x^2 + a*x)*sqrt(c*x))/(a^4*b*c^2*x^4 + a^5*c^2*x^3), 1/4*(15*(b
^3*x^4 + a*b^2*x^3)*sqrt(-a*c)*arctan(sqrt(b*x^2 + a*x)*sqrt(-a*c)*sqrt(c*
x)/(a*c*x)) + (15*a*b^2*x^2 + 5*a^2*b*x - 2*a^3)*sqrt(b*x^2 + a*x)*sqrt(c*
x))/(a^4*b*c^2*x^4 + a^5*c^2*x^3)]
```

Sympy [F]

$$\int \frac{1}{(cx)^{3/2}(ax+bx^2)^{3/2}}\,dx = \int \frac{1}{(cx)^{\frac{3}{2}}\,(x\,(a+bx))^{\frac{3}{2}}}\,dx$$

```
integrate(1/(c*x)**(3/2)/(b*x**2+a*x)**(3/2),x)
```

```
Integral(1/((c*x)**(3/2)*(x*(a + b*x))**(3/2)), x)
```

Maxima [F]

$$\int \frac{1}{(cx)^{3/2}(ax+bx^2)^{3/2}}\,dx = \int \frac{1}{(bx^2+ax)^{\frac{3}{2}}\,(cx)^{\frac{3}{2}}}\,dx$$

```
integrate(1/(c*x)^(3/2)/(b*x^2+a*x)^(3/2),x, algorithm="maxima")
```

```
integrate(1/((b*x^2 + a*x)^(3/2)*(c*x)^(3/2)), x)
```

Giac [A] (verification not implemented)

Time = 0.15 (sec) , antiderivative size = 121, normalized size of antiderivative = 0.86

$$\int \frac{1}{(cx)^{3/2}\left(ax+bx^2\right)^{3/2}}\,dx = \frac{1}{4}\,c^3\left(\frac{15\,b^2\arctan\left(\frac{\sqrt{bcx+ac}}{\sqrt{-ac}}\right)}{\sqrt{-ac}a^3c^3|c|}+\frac{8\,b^2}{\sqrt{bcx+ac}a^3c^3|c|}-\frac{9\,\sqrt{bcx+ac}ab^2c-7\,(bc}{a^3b^2c^5x^2|c|}\right.$$

```
integrate(1/(c*x)^(3/2)/(b*x^2+a*x)^(3/2),x, algorithm="giac")
```

```
1/4*c^3*(15*b^2*arctan(sqrt(b*c*x + a*c)/sqrt(-a*c))/(sqrt(-a*c)*a^3*c^3*a
bs(c)) + 8*b^2/(sqrt(b*c*x + a*c)*a^3*c^3*abs(c)) - (9*sqrt(b*c*x + a*c)*a
*b^2*c - 7*(b*c*x + a*c)^(3/2)*b^2)/(a^3*b^2*c^5*x^2*abs(c)))
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{(cx)^{3/2}\left(ax+bx^2\right)^{3/2}}\,dx = \int \frac{1}{\left(b\,x^2+a\,x\right)^{3/2}\,\left(c\,x\right)^{3/2}}\,dx$$

```
int(1/((a*x + b*x^2)^(3/2)*(c*x)^(3/2)),x)
```

```
int(1/((a*x + b*x^2)^(3/2)*(c*x)^(3/2)), x)
```

Reduce [B] (verification not implemented)

Time = 0.33 (sec) , antiderivative size = 97, normalized size of antiderivative = 0.69

$$\int \frac{1}{(cx)^{3/2}\left(ax+bx^2\right)^{3/2}}\,dx = \frac{\sqrt{c}\left(15\sqrt{a}\,\sqrt{bx+a}\log\left(\sqrt{bx+a}-\sqrt{a}\right)b^2x^2-15\sqrt{a}\,\sqrt{bx+a}\log\left(\sqrt{bx+a}+\right.\right.}{8\sqrt{bx+a}\,a^4c^2x^2}$$

```
int(1/(c*x)^(3/2)/(b*x^2+a*x)^(3/2),x)
```

```
(sqrt(c)*(15*sqrt(a)*sqrt(a + b*x)*log(sqrt(a + b*x) - sqrt(a))*b**2*x**2
- 15*sqrt(a)*sqrt(a + b*x)*log(sqrt(a + b*x) + sqrt(a))*b**2*x**2 - 4*a**3
 + 10*a**2*b*x + 30*a*b**2*x**2))/(8*sqrt(a + b*x)*a**4*c**2*x**2)
```

3.112 $\int \frac{1}{(cx)^{5/2}(ax+bx^2)^{3/2}}\,dx$

Optimal result

Integrand size = 21, antiderivative size = 174

$$\int \frac{1}{(cx)^{5/2}(ax+bx^2)^{3/2}}\,dx = -\frac{1}{3a(cx)^{5/2}\sqrt{ax+bx^2}} + \frac{7b}{12a^2c(cx)^{3/2}\sqrt{ax+bx^2}} - \frac{35b^2}{24a^3c^2\sqrt{cx}\sqrt{ax+bx^2}} - \frac{35b^3\sqrt{cx}}{8a^4c^3\sqrt{ax+bx^2}} + \frac{35b^3\operatorname{arctanh}\left(\frac{\sqrt{c}\sqrt{ax+bx^2}}{\sqrt{a}\sqrt{cx}}\right)}{8a^{9/2}c^{5/2}}$$

```
-1/3/a/(c*x)^(5/2)/(b*x^2+a*x)^(1/2)+7/12*b/a^2/c/(c*x)^(3/2)/(b*x^2+a*x)^
(1/2)-35/24*b^2/a^3/c^2/(c*x)^(1/2)/(b*x^2+a*x)^(1/2)-35/8*b^3*(c*x)^(1/2)
/a^4/c^3/(b*x^2+a*x)^(1/2)+35/8*b^3*arctanh(c^(1/2)*(b*x^2+a*x)^(1/2)/a^(1
/2)/(c*x)^(1/2))/a^(9/2)/c^(5/2)
```

Mathematica [A] (verified)

Time = 0.13 (sec) , antiderivative size = 98, normalized size of antiderivative = 0.56

$$\int \frac{1}{(cx)^{5/2}(ax+bx^2)^{3/2}}\,dx = \frac{-\sqrt{a}(8a^3-14a^2bx+35ab^2x^2+105b^3x^3)+105b^3x^3\sqrt{a+bx}\operatorname{arctanh}\left(\frac{\sqrt{a+bx}}{\sqrt{a}}\right)}{24a^{9/2}(cx)^{5/2}\sqrt{x(a+bx)}}$$

```
Integrate[1/((c*x)^(5/2)*(a*x + b*x^2)^(3/2)),x]
```

```
(-(Sqrt[a]*(8*a^3 - 14*a^2*b*x + 35*a*b^2*x^2 + 105*b^3*x^3)) + 105*b^3*x^
3*Sqrt[a + b*x]*ArcTanh[Sqrt[a + b*x]/Sqrt[a]])/(24*a^(9/2)*(c*x)^(5/2)*Sq
rt[x*(a + b*x)])
```

Rubi [A] (verified)

Time = 0.54 (sec) , antiderivative size = 185, normalized size of antiderivative = 1.06, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.286, Rules used = {1135, 1135, 1135, 1132, 1136, 221}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{(cx)^{5/2} \left(ax+bx^2\right)^{3/2}} \, dx$$

$$\Big\downarrow \text{1135}$$

$$-\frac{7b\int \frac{1}{(cx)^{3/2}(bx^2+ax)^{3/2}}dx}{6ac}-\frac{1}{3a(cx)^{5/2}\sqrt{ax+bx^2}}$$

$$\Big\downarrow \text{1135}$$

$$-\frac{7b\left(-\frac{5b\int \frac{1}{\sqrt{cx}\left(bx^2+ax\right)^{3/2}}dx}{4ac}-\frac{1}{2a(cx)^{3/2}\sqrt{ax+bx^2}}\right)}{6ac}-\frac{1}{3a(cx)^{5/2}\sqrt{ax+bx^2}}$$

$$\Big\downarrow \text{1135}$$

$$-\frac{7b\left(-\frac{5b\left(-\frac{3b\int \frac{\sqrt{cx}}{\left(bx^2+ax\right)^{3/2}}dx}{2ac}-\frac{1}{a\sqrt{cx}\sqrt{ax+bx^2}}\right)}{4ac}-\frac{1}{2a(cx)^{3/2}\sqrt{ax+bx^2}}\right)}{6ac}-\frac{1}{3a(cx)^{5/2}\sqrt{ax+bx^2}}$$

$$\Big\downarrow \text{1132}$$

$$-\frac{7b\left(-\frac{5b\left(-\frac{3b\left(\frac{c\int\frac{1}{\sqrt{cx}\sqrt{bx^2+ax}}dx}{a}+\frac{2\sqrt{cx}}{a\sqrt{ax+bx^2}}\right)}{2ac}-\frac{1}{a\sqrt{cx}\sqrt{ax+bx^2}}\right)}{4ac}-\frac{1}{2a(cx)^{3/2}\sqrt{ax+bx^2}}\right)}{6ac}-\frac{1}{3a(cx)^{5/2}\sqrt{ax+bx^2}}$$

$$\Big\downarrow \; 1136$$

$$-\frac{7b\left(-\frac{5b\left(-\frac{3b\left(\frac{2c^2\int\frac{1}{\frac{c(bx^2+ax)}{x}-ac}d\frac{\sqrt{bx^2+ax}}{\sqrt{cx}}}{a}+\frac{2\sqrt{cx}}{a\sqrt{ax+bx^2}}\right)}{2ac}-\frac{1}{a\sqrt{cx}\sqrt{ax+bx^2}}\right)}{4ac}-\frac{1}{2a(cx)^{3/2}\sqrt{ax+bx^2}}\right)}{6ac}-\frac{1}{3a(cx)^{5/2}\sqrt{ax+bx^2}}$$

$$\Big\downarrow \; 221$$

$$-\frac{7b\left(-\frac{5b\left(-\frac{3b\left(\frac{2\sqrt{cx}}{a\sqrt{ax+bx^2}}-\frac{2\sqrt{c}\,\text{arctanh}\left(\frac{\sqrt{c}\sqrt{ax+bx^2}}{\sqrt{a}\sqrt{cx}}\right)}{a^{3/2}}\right)}{2ac}-\frac{1}{a\sqrt{cx}\sqrt{ax+bx^2}}\right)}{4ac}-\frac{1}{2a(cx)^{3/2}\sqrt{ax+bx^2}}\right)}{6ac}-\frac{1}{3a(cx)^{5/2}\sqrt{ax+bx^2}}$$

```
Int[1/((c*x)^(5/2)*(a*x + b*x^2)^(3/2)),x]
```

```
-1/3*1/(a*(c*x)^(5/2)*Sqrt[a*x + b*x^2]) - (7*b*(-1/2*1/(a*(c*x)^(3/2)*Sqr
t[a*x + b*x^2]) - (5*b*(-(1/(a*Sqrt[c*x]*Sqrt[a*x + b*x^2])) - (3*b*((2*Sq
rt[c*x])/(a*Sqrt[a*x + b*x^2]) - (2*Sqrt[c]*ArcTanh[(Sqrt[c]*Sqrt[a*x + b*
x^2])/(Sqrt[a]*Sqrt[c*x])])/a^(3/2)))/(2*a*c)))/(4*a*c)))/(6*a*c)
```

Defintions of rubi rules used

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] :> Simp[(2*c*d - b*e)*(d + e*x)^m*((a + b*x + c*x^2)^(p + 1)/(e*(p +
 1)*(b^2 - 4*a*c))), x] - Simp[(2*c*d - b*e)*((m + 2*p + 2)/((p + 1)*(b^2 -
 4*a*c)))   Int[(d + e*x)^(m - 1)*(a + b*x + c*x^2)^(p + 1), x], x] /; Free
Q[{a, b, c, d, e}, x] && EqQ[c*d^2 - b*d*e + a*e^2, 0] && LtQ[p, -1] && LtQ
[0, m, 1] && IntegerQ[2*p]
```

```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] :> Simp[(-e)*(d + e*x)^m*((a + b*x + c*x^2)^(p + 1)/((m + p + 1)*(2*
c*d - b*e))), x] + Simp[c*((m + 2*p + 2)/((m + p + 1)*(2*c*d - b*e)))   Int
[(d + e*x)^(m + 1)*(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, p},
 x] && EqQ[c*d^2 - b*d*e + a*e^2, 0] && LtQ[m, 0] && NeQ[m + p + 1, 0] && I
ntegerQ[2*p]
```

```
Int[1/(Sqrt[(d_.) + (e_.)*(x_)]*Sqrt[(a_.) + (b_.)*(x_) + (c_.)*(x_)^2]), x
_Symbol] :> Simp[2*e   Subst[Int[1/(2*c*d - b*e + e^2*x^2), x], x, Sqrt[a +
 b*x + c*x^2]/Sqrt[d + e*x]], x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[c*d^2
- b*d*e + a*e^2, 0]
```

Maple [A] (verified)

Time = 0.37 (sec) , antiderivative size = 121, normalized size of antiderivative = 0.70

method	result	size
default	$\frac{\sqrt{x(bx+a)}\left(105\sqrt{c(bx+a)}\,\operatorname{arctanh}\left(\frac{\sqrt{c(bx+a)}}{\sqrt{ac}}\right)b^3x^3-105\sqrt{ac}\,b^3x^3-35\sqrt{ac}\,a\,b^2x^2+14\sqrt{ac}\,a^2bx-8\sqrt{ac}\,a^3\right)}{24c^2x^3\sqrt{cx}\,(bx+a)a^4\sqrt{ac}}$	121
risch	$-\frac{(bx+a)(57b^2x^2-22abx+8a^2)}{24a^4x^2c^2\sqrt{cx}\,\sqrt{x(bx+a)}}-\frac{b^3\left(\frac{32}{\sqrt{cbx+ac}}-\frac{70\,\operatorname{arctanh}\left(\frac{\sqrt{cbx+ac}}{\sqrt{ac}}\right)}{\sqrt{ac}}\right)\sqrt{c(bx+a)}\,x}{16a^4c^2\sqrt{cx}\,\sqrt{x(bx+a)}}$	123

```
int(1/(c*x)^(5/2)/(b*x^2+a*x)^(3/2),x,method=_RETURNVERBOSE)
```

```
1/24/c^2/x^3/(c*x)^(1/2)*(x*(b*x+a))^(1/2)*(105*(c*(b*x+a))^(1/2)*arctanh(
(c*(b*x+a))^(1/2)/(a*c)^(1/2))*b^3*x^3-105*(a*c)^(1/2)*b^3*x^3-35*(a*c)^(1
/2)*a*b^2*x^2+14*(a*c)^(1/2)*a^2*b*x-8*(a*c)^(1/2)*a^3)/(b*x+a)/a^4/(a*c)^
(1/2)
```

Fricas [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 277, normalized size of antiderivative = 1.59

$$\int \frac{1}{(cx)^{5/2}\,(ax+bx^2)^{3/2}}\,dx = \left[\frac{105\,(b^4x^5+ab^3x^4)\sqrt{ac}\log\left(-\frac{bcx^2+2\,acx+2\,\sqrt{bx^2+ax}\sqrt{ac}\sqrt{cx}}{x^2}\right)-2\,(105\,ab^3x^3+3}{48\,(a^5bc^3x^5+a^6c^3x^4)}\right.$$

```
integrate(1/(c*x)^(5/2)/(b*x^2+a*x)^(3/2),x, algorithm="fricas")
```

```
[1/48*(105*(b^4*x^5 + a*b^3*x^4)*sqrt(a*c)*log(-(b*c*x^2 + 2*a*c*x + 2*sqr
t(b*x^2 + a*x)*sqrt(a*c)*sqrt(c*x))/x^2) - 2*(105*a*b^3*x^3 + 35*a^2*b^2*x
^2 - 14*a^3*b*x + 8*a^4)*sqrt(b*x^2 + a*x)*sqrt(c*x))/(a^5*b*c^3*x^5 + a^6
*c^3*x^4), -1/24*(105*(b^4*x^5 + a*b^3*x^4)*sqrt(-a*c)*arctan(sqrt(b*x^2 +
 a*x)*sqrt(-a*c)*sqrt(c*x)/(a*c*x)) + (105*a*b^3*x^3 + 35*a^2*b^2*x^2 - 14
*a^3*b*x + 8*a^4)*sqrt(b*x^2 + a*x)*sqrt(c*x))/(a^5*b*c^3*x^5 + a^6*c^3*x^
4)]
```

Sympy [F]

$$\int \frac{1}{(cx)^{5/2}\left(ax+bx^2\right)^{3/2}}\,dx=\int \frac{1}{(cx)^{\frac{5}{2}}\left(x\left(a+bx\right)\right)^{\frac{3}{2}}}\,dx$$

```
integrate(1/(c*x)**(5/2)/(b*x**2+a*x)**(3/2),x)
```

```
Integral(1/((c*x)**(5/2)*(x*(a + b*x))**(3/2)), x)
```

Maxima [F]

$$\int \frac{1}{(cx)^{5/2}\left(ax+bx^2\right)^{3/2}}\,dx=\int \frac{1}{\left(bx^2+ax\right)^{\frac{3}{2}}(cx)^{\frac{5}{2}}}\,dx$$

```
integrate(1/(c*x)^(5/2)/(b*x^2+a*x)^(3/2),x, algorithm="maxima")
```

```
integrate(1/((b*x^2 + a*x)^(3/2)*(c*x)^(5/2)), x)
```

Giac [A] (verification not implemented)

Time = 0.17 (sec) , antiderivative size = 141, normalized size of antiderivative = 0.81

$$\int \frac{1}{(cx)^{5/2}\left(ax+bx^2\right)^{3/2}}\,dx=$$

$$-\frac{1}{24}\,c^3\left(\frac{105\,b^3\arctan\left(\frac{\sqrt{bcx+ac}}{\sqrt{-ac}}\right)}{\sqrt{-ac}a^4c^4|c|}+\frac{48\,b^3}{\sqrt{bcx+ac}a^4c^4|c|}+\frac{87\,\sqrt{bcx+ac}a^2b^3c^2-136\,(bcx+ac)^{\frac{3}{2}}ab^3c+57\,(b}{a^4b^3c^7x^3|c|}\right.$$

```
integrate(1/(c*x)^(5/2)/(b*x^2+a*x)^(3/2),x, algorithm="giac")
```

```
-1/24*c^3*(105*b^3*arctan(sqrt(b*c*x + a*c)/sqrt(-a*c))/(sqrt(-a*c)*a^4*c^
4*abs(c)) + 48*b^3/(sqrt(b*c*x + a*c)*a^4*c^4*abs(c)) + (87*sqrt(b*c*x + a
*c)*a^2*b^3*c^2 - 136*(b*c*x + a*c)^(3/2)*a*b^3*c + 57*(b*c*x + a*c)^(5/2)
*b^3)/(a^4*b^3*c^7*x^3*abs(c)))
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{(cx)^{5/2}\left(ax+bx^2\right)^{3/2}}\,dx = \int \frac{1}{\left(b\,x^2+a\,x\right)^{3/2}\,(c\,x)^{5/2}}\,dx$$

```
int(1/((a*x + b*x^2)^(3/2)*(c*x)^(5/2)),x)
```

```
int(1/((a*x + b*x^2)^(3/2)*(c*x)^(5/2)), x)
```

Reduce [B] (verification not implemented)

Time = 0.45 (sec) , antiderivative size = 108, normalized size of antiderivative = 0.62

$$\int \frac{1}{(cx)^{5/2}\left(ax+bx^2\right)^{3/2}}\,dx = \frac{\sqrt{c}\left(-105\sqrt{a}\,\sqrt{bx+a}\log\left(\sqrt{bx+a}-\sqrt{a}\right)b^3x^3+105\sqrt{a}\,\sqrt{bx+a}\log\left(\sqrt{bx+\cdots}\right.\right.}{48\sqrt{bx+a}\,a^5c^3x^3}$$

```
int(1/(c*x)^(5/2)/(b*x^2+a*x)^(3/2),x)
```

```
(sqrt(c)*( - 105*sqrt(a)*sqrt(a + b*x)*log(sqrt(a + b*x) - sqrt(a))*b**3*x
**3 + 105*sqrt(a)*sqrt(a + b*x)*log(sqrt(a + b*x) + sqrt(a))*b**3*x**3 - 1
6*a**4 + 28*a**3*b*x - 70*a**2*b**2*x**2 - 210*a*b**3*x**3))/(48*sqrt(a +
b*x)*a**5*c**3*x**3)
```

3.113 $\int \frac{(cx)^{11/2}}{(ax+bx^2)^{5/2}}\,dx$

Optimal result

Integrand size = 21, antiderivative size = 124

$$\int \frac{(cx)^{11/2}}{(ax+bx^2)^{5/2}}\,dx = \frac{2a^3c^4(cx)^{3/2}}{3b^4(ax+bx^2)^{3/2}} - \frac{6a^2c^5\sqrt{cx}}{b^4\sqrt{ax+bx^2}} - \frac{6ac^6\sqrt{ax+bx^2}}{b^4\sqrt{cx}} + \frac{2c^7(ax+bx^2)^{3/2}}{3b^4(cx)^{3/2}}$$

```
2/3*a^3*c^4*(c*x)^(3/2)/b^4/(b*x^2+a*x)^(3/2)-6*a^2*c^5*(c*x)^(1/2)/b^4/(b
*x^2+a*x)^(1/2)-6*a*c^6*(b*x^2+a*x)^(1/2)/b^4/(c*x)^(1/2)+2/3*c^7*(b*x^2+a
*x)^(3/2)/b^4/(c*x)^(3/2)
```

Mathematica [A] (verified)

Time = 0.04 (sec) , antiderivative size = 57, normalized size of antiderivative = 0.46

$$\int \frac{(cx)^{11/2}}{(ax+bx^2)^{5/2}}\,dx = \frac{2c^4(cx)^{3/2}(-16a^3-24a^2bx-6ab^2x^2+b^3x^3)}{3b^4(x(a+bx))^{3/2}}$$

```
Integrate[(c*x)^(11/2)/(a*x + b*x^2)^(5/2),x]
```

```
(2*c^4*(c*x)^(3/2)*(-16*a^3 - 24*a^2*b*x - 6*a*b^2*x^2 + b^3*x^3))/(3*b^4*
(x*(a + b*x))^(3/2))
```

Rubi [A] (verified)

Time = 0.43 (sec) , antiderivative size = 128, normalized size of antiderivative = 1.03, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.190$, Rules used = {1128, 1128, 1128, 1122}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(cx)^{11/2}}{(ax+bx^2)^{5/2}}dx$$

$$\Big\downarrow \text{1128}$$

$$\frac{2c(cx)^{9/2}}{3b(ax+bx^2)^{3/2}}-\frac{2ac\int \frac{(cx)^{9/2}}{(bx^2+ax)^{5/2}}dx}{b}$$

$$\Big\downarrow \text{1128}$$

$$\frac{2c(cx)^{9/2}}{3b(ax+bx^2)^{3/2}}-\frac{2ac\left(\frac{2c(cx)^{7/2}}{b(ax+bx^2)^{3/2}}-\frac{4ac\int \frac{(cx)^{7/2}}{(bx^2+ax)^{5/2}}dx}{b}\right)}{b}$$

$$\Big\downarrow \text{1128}$$

$$\frac{2c(cx)^{9/2}}{3b(ax+bx^2)^{3/2}}-\frac{2ac\left(\frac{2c(cx)^{7/2}}{b(ax+bx^2)^{3/2}}-\frac{4ac\left(\frac{2ac\int \frac{(cx)^{5/2}}{(bx^2+ax)^{5/2}}dx}{b}-\frac{2c(cx)^{5/2}}{b(ax+bx^2)^{3/2}}\right)}{b}\right)}{b}$$

$$\Big\downarrow \text{1122}$$

$$\frac{2c(cx)^{9/2}}{3b\left(ax+bx^2\right)^{3/2}}-\frac{2ac\left(\frac{2c(cx)^{7/2}}{b(ax+bx^2)^{3/2}}-\frac{4ac\left(-\frac{4ac^2(cx)^{3/2}}{3b^2\left(ax+bx^2\right)^{3/2}}-\frac{2c(cx)^{5/2}}{b\left(ax+bx^2\right)^{3/2}}\right)}{b}\right)}{b}$$

```
Int[(c*x)^(11/2)/(a*x + b*x^2)^(5/2),x]
```

```
(2*c*(c*x)^(9/2))/(3*b*(a*x + b*x^2)^(3/2)) - (2*a*c*((2*c*(c*x)^(7/2))/(b
*(a*x + b*x^2)^(3/2)) - (4*a*c*((-4*a*c^2*(c*x)^(3/2))/(3*b^2*(a*x + b*x^2
)^(3/2)) - (2*c*(c*x)^(5/2))/(b*(a*x + b*x^2)^(3/2))))/b))/b
```

Defintions of rubi rules used

```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] :> Simp[e*(d + e*x)^(m - 1)*((a + b*x + c*x^2)^(p + 1)/(c*(p + 1))),
 x] /; FreeQ[{a, b, c, d, e, m, p}, x] && EqQ[c*d^2 - b*d*e + a*e^2, 0] &&
EqQ[m + p, 0]
```

```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] :> Simp[e*(d + e*x)^(m - 1)*((a + b*x + c*x^2)^(p + 1)/(c*(m + 2*p +
 1))), x] + Simp[Simplify[m + p]*((2*c*d - b*e)/(c*(m + 2*p + 1)))   Int[(d
 + e*x)^(m - 1)*(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, m, p},
 x] && EqQ[c*d^2 - b*d*e + a*e^2, 0] && IGtQ[Simplify[m + p], 0]
```

Maple [A] (verified)

Time = 0.37 (sec) , antiderivative size = 60, normalized size of antiderivative = 0.48

method	result	size
gosper	$-\frac{2(bx+a)\left(-b^{3}x^{3}+6a\,b^{2}x^{2}+24a^{2}bx+16a^{3}\right)(cx)^{\frac{11}{2}}}{3b^{4}x^{3}\left(b\,x^{2}+ax\right)^{\frac{5}{2}}}$	60
orering	$-\frac{2(bx+a)\left(-b^{3}x^{3}+6a\,b^{2}x^{2}+24a^{2}bx+16a^{3}\right)(cx)^{\frac{11}{2}}}{3b^{4}x^{3}\left(b\,x^{2}+ax\right)^{\frac{5}{2}}}$	60
default	$-\frac{2c^{5}\sqrt{cx}\,\sqrt{x(bx+a)}\,\left(-b^{3}x^{3}+6a\,b^{2}x^{2}+24a^{2}bx+16a^{3}\right)}{3x(bx+a)^{2}b^{4}}$	63
risch	$-\frac{2(-bx+8a)(bx+a)c^{6}x}{3b^{4}\sqrt{cx}\,\sqrt{x(bx+a)}}-\frac{2a^{2}(9bx+8a)c^{6}x}{3b^{4}(bx+a)\sqrt{cx}\,\sqrt{x(bx+a)}}$	79

```
int((c*x)^(11/2)/(b*x^2+a*x)^(5/2),x,method=_RETURNVERBOSE)
```

```
-2/3*(b*x+a)*(-b^3*x^3+6*a*b^2*x^2+24*a^2*b*x+16*a^3)*(c*x)^(11/2)/b^4/x^3
/(b*x^2+a*x)^(5/2)
```

Fricas [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 86, normalized size of antiderivative = 0.69

$$\int \frac{(cx)^{11/2}}{(ax+bx^2)^{5/2}}\,dx = \frac{2\left(b^3c^5x^3-6\,ab^2c^5x^2-24\,a^2bc^5x-16\,a^3c^5\right)\sqrt{bx^2+ax}\sqrt{cx}}{3\left(b^6x^3+2\,ab^5x^2+a^2b^4x\right)}$$

```
integrate((c*x)^(11/2)/(b*x^2+a*x)^(5/2),x, algorithm="fricas")
```

```
2/3*(b^3*c^5*x^3 - 6*a*b^2*c^5*x^2 - 24*a^2*b*c^5*x - 16*a^3*c^5)*sqrt(b*x
^2 + a*x)*sqrt(c*x)/(b^6*x^3 + 2*a*b^5*x^2 + a^2*b^4*x)
```

Sympy [F]

$$\int \frac{(cx)^{11/2}}{(ax+bx^2)^{5/2}}\,dx = \int \frac{(cx)^{\frac{11}{2}}}{(x\,(a+bx))^{\frac{5}{2}}}\,dx$$

```
integrate((c*x)**(11/2)/(b*x**2+a*x)**(5/2),x)
```

```
Integral((c*x)**(11/2)/(x*(a + b*x))**(5/2), x)
```

Maxima [F]

$$\int \frac{(cx)^{11/2}}{(ax+bx^2)^{5/2}}\,dx = \int \frac{(cx)^{\frac{11}{2}}}{(bx^2+ax)^{\frac{5}{2}}}\,dx$$

```
integrate((c*x)^(11/2)/(b*x^2+a*x)^(5/2),x, algorithm="maxima")
```

```
2/3*((b^3*c^(11/2)*x^2 - a*b^2*c^(11/2)*x - 2*a^2*b*c^(11/2))*x^3 - 4*(a*b
^2*c^(11/2)*x^2 + 2*a^2*b*c^(11/2)*x + a^3*c^(11/2))*x^2)/((b^5*x^3 + 2*a*
b^4*x^2 + a^2*b^3*x)*sqrt(b*x + a)) + integrate(4/3*(5*a^2*b^2*c^(11/2)*x^
2 + 7*a^3*b*c^(11/2)*x + 2*a^4*c^(11/2))*x^2/((b^6*x^5 + 3*a*b^5*x^4 + 3*a
^2*b^4*x^3 + a^3*b^3*x^2)*sqrt(b*x + a)), x)
```

Giac [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 121, normalized size of antiderivative = 0.98

$$\int \frac{(cx)^{11/2}}{(ax+bx^2)^{5/2}}\,dx = \frac{2}{3}\left(c^4\left(\frac{a^3c^3-9\,(bcx+ac)a^2c^2}{(bcx+ac)^{\frac{3}{2}}b^4|c|} - \frac{9\,\sqrt{bcx+ac}ab^8c^3-(bcx+ac)^{\frac{3}{2}}b^8c^2}{b^{12}c^2|c|}\right) + \frac{16\,a^2c^6}{\sqrt{ac}b^4|c}\right.$$

```
integrate((c*x)^(11/2)/(b*x^2+a*x)^(5/2),x, algorithm="giac")
```

```
2/3*(c^4*((a^3*c^3 - 9*(b*c*x + a*c)*a^2*c^2)/((b*c*x + a*c)^(3/2)*b^4*abs
(c)) - (9*sqrt(b*c*x + a*c)*a*b^8*c^3 - (b*c*x + a*c)^(3/2)*b^8*c^2)/(b^12
*c^2*abs(c))) + 16*a^2*c^6/(sqrt(a*c)*b^4*abs(c)))*c
```

Mupad [B] (verification not implemented)

Time = 9.53 (sec) , antiderivative size = 103, normalized size of antiderivative = 0.83

$$\int \frac{(cx)^{11/2}}{(ax+bx^2)^{5/2}}\,dx = -\frac{\sqrt{b\,x^2+a\,x}\,\left(\frac{32\,a^3\,c^5\,\sqrt{c\,x}}{3\,b^6}-\frac{2\,c^5\,x^3\,\sqrt{c\,x}}{3\,b^3}+\frac{4\,a\,c^5\,x^2\,\sqrt{c\,x}}{b^4}+\frac{16\,a^2\,c^5\,x\,\sqrt{c\,x}}{b^5}\right)}{x^3+\frac{2\,a\,x^2}{b}+\frac{a^2\,x}{b^2}}$$

```
int((c*x)^(11/2)/(a*x + b*x^2)^(5/2),x)
```

```
-((a*x + b*x^2)^(1/2)*((32*a^3*c^5*(c*x)^(1/2))/(3*b^6) - (2*c^5*x^3*(c*x)
^(1/2))/(3*b^3) + (4*a*c^5*x^2*(c*x)^(1/2))/b^4 + (16*a^2*c^5*x*(c*x)^(1/2
))/b^5))/(x^3 + (2*a*x^2)/b + (a^2*x)/b^2)
```

Reduce [B] (verification not implemented)

Time = 0.46 (sec) , antiderivative size = 54, normalized size of antiderivative = 0.44

$$\int \frac{(cx)^{11/2}}{(ax+bx^2)^{5/2}}\,dx = \frac{2\sqrt{c}\,c^5(b^3x^3-6a\,b^2x^2-24a^2bx-16a^3)}{3\sqrt{bx+a}\,b^4\,(bx+a)}$$

```
int((c*x)^(11/2)/(b*x^2+a*x)^(5/2),x)
```

```
(2*sqrt(c)*c**5*( - 16*a**3 - 24*a**2*b*x - 6*a*b**2*x**2 + b**3*x**3))/(3
*sqrt(a + b*x)*b**4*(a + b*x))
```

3.114 $\int \frac{(cx)^{9/2}}{(ax+bx^2)^{5/2}}\,dx$

Optimal result

Integrand size = 21, antiderivative size = 91

$$\int \frac{(cx)^{9/2}}{(ax+bx^2)^{5/2}}\,dx = -\frac{2a^2c^3(cx)^{3/2}}{3b^3(ax+bx^2)^{3/2}} + \frac{4ac^4\sqrt{cx}}{b^3\sqrt{ax+bx^2}} + \frac{2c^5\sqrt{ax+bx^2}}{b^3\sqrt{cx}}$$

```
-2/3*a^2*c^3*(c*x)^(3/2)/b^3/(b*x^2+a*x)^(3/2)+4*a*c^4*(c*x)^(1/2)/b^3/(b*
x^2+a*x)^(1/2)+2*c^5*(b*x^2+a*x)^(1/2)/b^3/(c*x)^(1/2)
```

Mathematica [A] (verified)

Time = 0.04 (sec) , antiderivative size = 47, normalized size of antiderivative = 0.52

$$\int \frac{(cx)^{9/2}}{(ax+bx^2)^{5/2}}\,dx = \frac{2c^3(cx)^{3/2}(8a^2+12abx+3b^2x^2)}{3b^3(x(a+bx))^{3/2}}$$

```
Integrate[(c*x)^(9/2)/(a*x + b*x^2)^(5/2),x]
```

```
(2*c^3*(c*x)^(3/2)*(8*a^2 + 12*a*b*x + 3*b^2*x^2))/(3*b^3*(x*(a + b*x))^(3
/2))
```

Rubi [A] (verified)

Time = 0.36 (sec) , antiderivative size = 92, normalized size of antiderivative = 1.01, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.143, Rules used = {1128, 1128, 1122}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(cx)^{9/2}}{(ax+bx^2)^{5/2}}\,dx$$

$$\Big\downarrow \; 1128$$

$$\frac{2c(cx)^{7/2}}{b\,(ax+bx^2)^{3/2}}-\frac{4ac\int \frac{(cx)^{7/2}}{(bx^2+ax)^{5/2}}dx}{b}$$

$$\Big\downarrow \; 1128$$

$$\frac{2c(cx)^{7/2}}{b\,(ax+bx^2)^{3/2}}-\frac{4ac\left(\frac{2ac\int \frac{(cx)^{5/2}}{\left(bx^2+ax\right)^{5/2}}dx}{b}-\frac{2c(cx)^{5/2}}{b(ax+bx^2)^{3/2}}\right)}{b}$$

$$\Big\downarrow \; 1122$$

$$\frac{2c(cx)^{7/2}}{b\,(ax+bx^2)^{3/2}}-\frac{4ac\left(-\frac{4ac^2(cx)^{3/2}}{3b^2(ax+bx^2)^{3/2}}-\frac{2c(cx)^{5/2}}{b(ax+bx^2)^{3/2}}\right)}{b}$$

```
Int[(c*x)^(9/2)/(a*x + b*x^2)^(5/2),x]
```

```
(2*c*(c*x)^(7/2))/(b*(a*x + b*x^2)^(3/2)) - (4*a*c*((-4*a*c^2*(c*x)^(3/2))
/(3*b^2*(a*x + b*x^2)^(3/2)) - (2*c*(c*x)^(5/2))/(b*(a*x + b*x^2)^(3/2))))
/b
```

Defintions of rubi rules used

```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] :> Simp[e*(d + e*x)^(m - 1)*((a + b*x + c*x^2)^(p + 1)/(c*(p + 1))),
 x] /; FreeQ[{a, b, c, d, e, m, p}, x] && EqQ[c*d^2 - b*d*e + a*e^2, 0] &&
EqQ[m + p, 0]
```

```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] :> Simp[e*(d + e*x)^(m - 1)*((a + b*x + c*x^2)^(p + 1)/(c*(m + 2*p +
 1))), x] + Simp[Simplify[m + p]*((2*c*d - b*e)/(c*(m + 2*p + 1)))   Int[(d
 + e*x)^(m - 1)*(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, m, p},
 x] && EqQ[c*d^2 - b*d*e + a*e^2, 0] && IGtQ[Simplify[m + p], 0]
```

Maple [A] (verified)

Time = 0.36 (sec) , antiderivative size = 49, normalized size of antiderivative = 0.54

method	result	size
gosper	$\frac{2(bx+a)\left(3b^{2}x^{2}+12abx+8a^{2}\right)\left(cx\right)^{\frac{9}{2}}}{3b^{3}x^{2}\left(b\,x^{2}+ax\right)^{\frac{5}{2}}}$	49
orering	$\frac{2(bx+a)\left(3b^{2}x^{2}+12abx+8a^{2}\right)\left(cx\right)^{\frac{9}{2}}}{3b^{3}x^{2}\left(b\,x^{2}+ax\right)^{\frac{5}{2}}}$	49
default	$\frac{2c^{4}\sqrt{cx}\,\sqrt{x(bx+a)}\,\left(3b^{2}x^{2}+12abx+8a^{2}\right)}{3x\left(bx+a\right)^{2}b^{3}}$	52
risch	$\frac{2(bx+a)c^{5}x}{b^{3}\sqrt{cx}\,\sqrt{x(bx+a)}}+\frac{2a(6bx+5a)c^{5}x}{3b^{3}(bx+a)\sqrt{cx}\,\sqrt{x(bx+a)}}$	69

```
int((c*x)^(9/2)/(b*x^2+a*x)^(5/2),x,method=_RETURNVERBOSE)
```

```
2/3*(b*x+a)*(3*b^2*x^2+12*a*b*x+8*a^2)*(c*x)^(9/2)/b^3/x^2/(b*x^2+a*x)^(5/
2)
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 73, normalized size of antiderivative = 0.80

$$\int \frac{(cx)^{9/2}}{(ax+bx^2)^{5/2}}\,dx = \frac{2\,(3\,b^2c^4x^2+12\,abc^4x+8\,a^2c^4)\sqrt{bx^2+ax}\sqrt{cx}}{3\,(b^5x^3+2\,ab^4x^2+a^2b^3x)}$$

```
integrate((c*x)^(9/2)/(b*x^2+a*x)^(5/2),x, algorithm="fricas")
```

```
2/3*(3*b^2*c^4*x^2 + 12*a*b*c^4*x + 8*a^2*c^4)*sqrt(b*x^2 + a*x)*sqrt(c*x)
/(b^5*x^3 + 2*a*b^4*x^2 + a^2*b^3*x)
```

Sympy [F]

$$\int \frac{(cx)^{9/2}}{(ax+bx^2)^{5/2}}\,dx = \int \frac{(cx)^{\frac{9}{2}}}{(x\,(a+bx))^{\frac{5}{2}}}\,dx$$

```
integrate((c*x)**(9/2)/(b*x**2+a*x)**(5/2),x)
```

```
Integral((c*x)**(9/2)/(x*(a + b*x))**(5/2), x)
```

Maxima [F]

$$\int \frac{(cx)^{9/2}}{(ax+bx^2)^{5/2}}\,dx = \int \frac{(cx)^{\frac{9}{2}}}{(bx^2+ax)^{\frac{5}{2}}}\,dx$$

```
integrate((c*x)^(9/2)/(b*x^2+a*x)^(5/2),x, algorithm="maxima")
```

```
2*(b*c^(9/2)*x + a*c^(9/2))*x^2/((b^3*x^2 + 2*a*b^2*x + a^2*b)*sqrt(b*x +
a)) - integrate(4*(a*b*c^(9/2)*x + a^2*c^(9/2))*x^2/((b^4*x^4 + 3*a*b^3*x^
3 + 3*a^2*b^2*x^2 + a^3*b*x)*sqrt(b*x + a)), x)
```

Giac [A] (verification not implemented)

Time = 0.21 (sec) , antiderivative size = 88, normalized size of antiderivative = 0.97

$$\int \frac{(cx)^{9/2}}{(ax+bx^2)^{5/2}}\,dx = \frac{2}{3}\,c^4\left(\frac{c\left(\frac{3\,\sqrt{bcx+ac}}{b|c|}-\frac{a^2c^2-6\,(bcx+ac)ac}{(bcx+ac)^{\frac{3}{2}}\,b|c|}\right)}{b^2}-\frac{8\,ac^2}{\sqrt{ac}b^3|c|}\right)$$

```
integrate((c*x)^(9/2)/(b*x^2+a*x)^(5/2),x, algorithm="giac")
```

```
2/3*c^4*(c*(3*sqrt(b*c*x + a*c)/(b*abs(c)) - (a^2*c^2 - 6*(b*c*x + a*c)*a*
c)/((b*c*x + a*c)^(3/2)*b*abs(c)))/b^2 - 8*a*c^2/(sqrt(a*c)*b^3*abs(c)))
```

Mupad [B] (verification not implemented)

Time = 9.91 (sec) , antiderivative size = 53, normalized size of antiderivative = 0.58

$$\int \frac{(cx)^{9/2}}{(ax+bx^2)^{5/2}}\,dx = \frac{2\,c^4\,\sqrt{b\,x^2+a\,x}\,\sqrt{c\,x}\,\left(8\,a^2+12\,a\,b\,x+3\,b^2\,x^2\right)}{3\,b^3\,x\,{\left(a+b\,x\right)}^2}$$

```
int((c*x)^(9/2)/(a*x + b*x^2)^(5/2),x)
```

```
(2*c^4*(a*x + b*x^2)^(1/2)*(c*x)^(1/2)*(8*a^2 + 3*b^2*x^2 + 12*a*b*x))/(3*
b^3*x*(a + b*x)^2)
```

Reduce [B] (verification not implemented)

Time = 0.35 (sec) , antiderivative size = 44, normalized size of antiderivative = 0.48

$$\int \frac{(cx)^{9/2}}{(ax+bx^2)^{5/2}}\,dx = \frac{2\sqrt{c}\,c^4(3b^2x^2+12abx+8a^2)}{3\sqrt{bx+a}\,b^3\,(bx+a)}$$

```
int((c*x)^(9/2)/(b*x^2+a*x)^(5/2),x)
```

```
(2*sqrt(c)*c**4*(8*a**2 + 12*a*b*x + 3*b**2*x**2))/(3*sqrt(a + b*x)*b**3*(
a + b*x))
```

3.115 $\int \frac{(cx)^{7/2}}{(ax+bx^2)^{5/2}}\,dx$

Optimal result

Integrand size = 21, antiderivative size = 60

$$\int \frac{(cx)^{7/2}}{(ax+bx^2)^{5/2}}\,dx = \frac{2ac^2(cx)^{3/2}}{3b^2(ax+bx^2)^{3/2}} - \frac{2c^3\sqrt{cx}}{b^2\sqrt{ax+bx^2}}$$

```
2/3*a*c^2*(c*x)^(3/2)/b^2/(b*x^2+a*x)^(3/2)-2*c^3*(c*x)^(1/2)/b^2/(b*x^2+a
*x)^(1/2)
```

Mathematica [A] (verified)

Time = 0.04 (sec) , antiderivative size = 36, normalized size of antiderivative = 0.60

$$\int \frac{(cx)^{7/2}}{(ax+bx^2)^{5/2}}\,dx = -\frac{2c^2(cx)^{3/2}(2a+3bx)}{3b^2(x(a+bx))^{3/2}}$$

```
Integrate[(c*x)^(7/2)/(a*x + b*x^2)^(5/2),x]
```

```
(-2*c^2*(c*x)^(3/2)*(2*a + 3*b*x))/(3*b^2*(x*(a + b*x))^(3/2))
```

Rubi [A] (verified)

Time = 0.30 (sec) , antiderivative size = 58, normalized size of antiderivative = 0.97, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.095$, Rules used = {1128, 1122}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(cx)^{7/2}}{(ax+bx^2)^{5/2}}\,dx$$
$$\Big\downarrow \text{ 1128}$$
$$\frac{2ac\int \frac{(cx)^{5/2}}{(bx^2+ax)^{5/2}}dx}{b}-\frac{2c(cx)^{5/2}}{b\,(ax+bx^2)^{3/2}}$$
$$\Big\downarrow \text{ 1122}$$
$$-\frac{4ac^2(cx)^{3/2}}{3b^2\,(ax+bx^2)^{3/2}}-\frac{2c(cx)^{5/2}}{b\,(ax+bx^2)^{3/2}}$$

```
Int[(c*x)^(7/2)/(a*x + b*x^2)^(5/2),x]
```

```
(-4*a*c^2*(c*x)^(3/2))/(3*b^2*(a*x + b*x^2)^(3/2)) - (2*c*(c*x)^(5/2))/(b*
(a*x + b*x^2)^(3/2))
```

Defintions of rubi rules used

```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] :> Simp[e*(d + e*x)^(m - 1)*((a + b*x + c*x^2)^(p + 1)/(c*(p + 1))),
 x] /; FreeQ[{a, b, c, d, e, m, p}, x] && EqQ[c*d^2 - b*d*e + a*e^2, 0] &&
EqQ[m + p, 0]
```

```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] :> Simp[e*(d + e*x)^(m - 1)*((a + b*x + c*x^2)^(p + 1)/(c*(m + 2*p +
 1))), x] + Simp[Simplify[m + p]*((2*c*d - b*e)/(c*(m + 2*p + 1)))   Int[(d
 + e*x)^(m - 1)*(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, m, p},
 x] && EqQ[c*d^2 - b*d*e + a*e^2, 0] && IGtQ[Simplify[m + p], 0]
```

Maple [A] (verified)

Time = 0.35 (sec) , antiderivative size = 38, normalized size of antiderivative = 0.63

method	result	size
gosper	$-\frac{2(bx+a)(3bx+2a)(cx)^{\frac{7}{2}}}{3b^{2}x(b\,x^{2}+ax)^{\frac{5}{2}}}$	38
orering	$-\frac{2(bx+a)(3bx+2a)(cx)^{\frac{7}{2}}}{3b^{2}x(b\,x^{2}+ax)^{\frac{5}{2}}}$	38
default	$-\frac{2c^{3}\sqrt{cx}\,\sqrt{x(bx+a)}\,(3bx+2a)}{3x(bx+a)^{2}b^{2}}$	41

```
int((c*x)^(7/2)/(b*x^2+a*x)^(5/2),x,method=_RETURNVERBOSE)
```

```
-2/3*(b*x+a)*(3*b*x+2*a)*(c*x)^(7/2)/b^2/x/(b*x^2+a*x)^(5/2)
```

Fricas [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 59, normalized size of antiderivative = 0.98

$$\int \frac{(cx)^{7/2}}{(ax+bx^2)^{5/2}}\,dx = -\frac{2\,(3\,bc^3x+2\,ac^3)\sqrt{bx^2+ax}\sqrt{cx}}{3\,(b^4x^3+2\,ab^3x^2+a^2b^2x)}$$

```
integrate((c*x)^(7/2)/(b*x^2+a*x)^(5/2),x, algorithm="fricas")
```

```
-2/3*(3*b*c^3*x + 2*a*c^3)*sqrt(b*x^2 + a*x)*sqrt(c*x)/(b^4*x^3 + 2*a*b^3*
x^2 + a^2*b^2*x)
```

Sympy [F]

$$\int \frac{(cx)^{7/2}}{(ax+bx^2)^{5/2}}\,dx = \int \frac{(cx)^{\frac{7}{2}}}{(x\,(a+bx))^{\frac{5}{2}}}\,dx$$

```
integrate((c*x)**(7/2)/(b*x**2+a*x)**(5/2),x)
```

```
Integral((c*x)**(7/2)/(x*(a + b*x))**(5/2), x)
```

Maxima [F]

$$\int \frac{(cx)^{7/2}}{(ax+bx^2)^{5/2}}\,dx = \int \frac{(cx)^{\frac{7}{2}}}{(bx^2+ax)^{\frac{5}{2}}}\,dx$$

```
integrate((c*x)^(7/2)/(b*x^2+a*x)^(5/2),x, algorithm="maxima")
```

```
integrate((c*x)^(7/2)/(b*x^2 + a*x)^(5/2), x)
```

Giac [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 52, normalized size of antiderivative = 0.87

$$\int \frac{(cx)^{7/2}}{(ax+bx^2)^{5/2}}\,dx = -\frac{2}{3}\,c\left(\frac{(3\,bcx+2\,ac)c^4}{(bcx+ac)^{\frac{3}{2}}b^2|c|}-\frac{2\,c^4}{\sqrt{ac}b^2|c|}\right)$$

```
integrate((c*x)^(7/2)/(b*x^2+a*x)^(5/2),x, algorithm="giac")
```

```
-2/3*c*((3*b*c*x + 2*a*c)*c^4/((b*c*x + a*c)^(3/2)*b^2*abs(c)) - 2*c^4/(sq
rt(a*c)*b^2*abs(c)))
```

Mupad [B] (verification not implemented)

Time = 9.51 (sec) , antiderivative size = 65, normalized size of antiderivative = 1.08

$$\int \frac{(cx)^{7/2}}{(ax+bx^2)^{5/2}}\,dx = -\frac{\sqrt{b\,x^2+a\,x}\,\left(\frac{4\,a\,c^3\,\sqrt{c\,x}}{3\,b^4}+\frac{2\,c^3\,x\,\sqrt{c\,x}}{b^3}\right)}{x^3+\frac{2\,a\,x^2}{b}+\frac{a^2\,x}{b^2}}$$

```
int((c*x)^(7/2)/(a*x + b*x^2)^(5/2),x)
```

```
-((a*x + b*x^2)^(1/2)*((4*a*c^3*(c*x)^(1/2))/(3*b^4) + (2*c^3*x*(c*x)^(1/2
))/b^3))/(x^3 + (2*a*x^2)/b + (a^2*x)/b^2)
```

Reduce [B] (verification not implemented)

Time = 0.49 (sec) , antiderivative size = 33, normalized size of antiderivative = 0.55

$$\int \frac{(cx)^{7/2}}{(ax+bx^2)^{5/2}}\,dx = \frac{2\sqrt{c}\,c^3(-3bx-2a)}{3\sqrt{bx+a}\,b^2\,(bx+a)}$$

```
int((c*x)^(7/2)/(b*x^2+a*x)^(5/2),x)
```

```
(2*sqrt(c)*c**3*( - 2*a - 3*b*x))/(3*sqrt(a + b*x)*b**2*(a + b*x))
```

3.116 $\int \frac{(cx)^{5/2}}{(ax+bx^2)^{5/2}} dx$

Optimal result

Integrand size = 21, antiderivative size = 28

$$\int \frac{(cx)^{5/2}}{(ax+bx^2)^{5/2}} dx = -\frac{2c(cx)^{3/2}}{3b\,(ax+bx^2)^{3/2}}$$

```
-2/3*c*(c*x)^(3/2)/b/(b*x^2+a*x)^(3/2)
```

Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 26, normalized size of antiderivative = 0.93

$$\int \frac{(cx)^{5/2}}{(ax+bx^2)^{5/2}} dx = -\frac{2c(cx)^{3/2}}{3b(x(a+bx))^{3/2}}$$

```
Integrate[(c*x)^(5/2)/(a*x + b*x^2)^(5/2),x]
```

```
(-2*c*(c*x)^(3/2))/(3*b*(x*(a + b*x))^(3/2))
```

Rubi [A] (verified)

Time = 0.24 (sec) , antiderivative size = 28, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.048, Rules used = {1122}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(cx)^{5/2}}{(ax+bx^2)^{5/2}}\,dx$$
$$\Big\downarrow \; 1122$$
$$-\frac{2c(cx)^{3/2}}{3b\,(ax+bx^2)^{3/2}}$$

```
Int[(c*x)^(5/2)/(a*x + b*x^2)^(5/2),x]
```

```
(-2*c*(c*x)^(3/2))/(3*b*(a*x + b*x^2)^(3/2))
```

Defintions of rubi rules used

```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] :> Simp[e*(d + e*x)^(m - 1)*((a + b*x + c*x^2)^(p + 1)/(c*(p + 1))),
 x] /; FreeQ[{a, b, c, d, e, m, p}, x] && EqQ[c*d^2 - b*d*e + a*e^2, 0] &&
EqQ[m + p, 0]
```

Maple [A] (verified)

Time = 0.36 (sec) , antiderivative size = 27, normalized size of antiderivative = 0.96

method	result	size
gosper	$-\frac{2(bx+a)(cx)^{\frac{5}{2}}}{3b(b\,x^{2}+ax)^{\frac{5}{2}}}$	27
orering	$-\frac{2(bx+a)(cx)^{\frac{5}{2}}}{3b(b\,x^{2}+ax)^{\frac{5}{2}}}$	27
default	$-\frac{2c^{2}\sqrt{cx}\,\sqrt{x(bx+a)}}{3x(bx+a)^{2}b}$	33

```
int((c*x)^(5/2)/(b*x^2+a*x)^(5/2),x,method=_RETURNVERBOSE)
```

```
-2/3*(b*x+a)*(c*x)^(5/2)/b/(b*x^2+a*x)^(5/2)
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 46 vs. $2(22) = 44$.

Time = 0.09 (sec) , antiderivative size = 46, normalized size of antiderivative = 1.64

$$\int \frac{(cx)^{5/2}}{(ax+bx^2)^{5/2}}\,dx = -\frac{2\sqrt{bx^2+ax}\sqrt{cx}c^2}{3\,(b^3x^3+2\,ab^2x^2+a^2bx)}$$

```
integrate((c*x)^(5/2)/(b*x^2+a*x)^(5/2),x, algorithm="fricas")
```

```
-2/3*sqrt(b*x^2 + a*x)*sqrt(c*x)*c^2/(b^3*x^3 + 2*a*b^2*x^2 + a^2*b*x)
```

Sympy [F]

$$\int \frac{(cx)^{5/2}}{(ax+bx^2)^{5/2}}\,dx = \int \frac{(cx)^{\frac{5}{2}}}{(x\,(a+bx))^{\frac{5}{2}}}\,dx$$

```
integrate((c*x)**(5/2)/(b*x**2+a*x)**(5/2),x)
```

```
Integral((c*x)**(5/2)/(x*(a + b*x))**(5/2), x)
```

Maxima [F]

$$\int \frac{(cx)^{5/2}}{(ax+bx^2)^{5/2}}\,dx = \int \frac{(cx)^{\frac{5}{2}}}{(bx^2+ax)^{\frac{5}{2}}}\,dx$$

```
integrate((c*x)^(5/2)/(b*x^2+a*x)^(5/2),x, algorithm="maxima")
```

```
integrate((c*x)^(5/2)/(b*x^2 + a*x)^(5/2), x)
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 45 vs. $2(22) = 44$.

Time = 0.13 (sec) , antiderivative size = 45, normalized size of antiderivative = 1.61

$$\int \frac{(cx)^{5/2}}{(ax+bx^2)^{5/2}}\,dx = -\frac{2}{3}\,c\left(\frac{c^4}{(bcx+ac)^{\frac{3}{2}}b|c|} - \frac{c^3}{\sqrt{ac}ab|c|}\right)$$

```
integrate((c*x)^(5/2)/(b*x^2+a*x)^(5/2),x, algorithm="giac")
```

```
-2/3*c*(c^4/((b*c*x + a*c)^(3/2)*b*abs(c)) - c^3/(sqrt(a*c)*a*b*abs(c)))
```

Mupad [B] (verification not implemented)

Time = 9.72 (sec) , antiderivative size = 48, normalized size of antiderivative = 1.71

$$\int \frac{(cx)^{5/2}}{(ax+bx^2)^{5/2}}\,dx = -\frac{2\,c^2\,\sqrt{b\,x^2+a\,x}\,\sqrt{c\,x}}{3\,\left(a^2\,b\,x+2\,a\,b^2\,x^2+b^3\,x^3\right)}$$

```
int((c*x)^(5/2)/(a*x + b*x^2)^(5/2),x)
```

```
-(2*c^2*(a*x + b*x^2)^(1/2)*(c*x)^(1/2))/(3*(b^3*x^3 + 2*a*b^2*x^2 + a^2*b
*x))
```

Reduce [B] (verification not implemented)

Time = 0.40 (sec) , antiderivative size = 25, normalized size of antiderivative = 0.89

$$\int \frac{(cx)^{5/2}}{(ax+bx^2)^{5/2}}\,dx = -\frac{2\sqrt{c}\,c^2}{3\sqrt{bx+a}\,b\,(bx+a)}$$

```
int((c*x)^(5/2)/(b*x^2+a*x)^(5/2),x)
```

```
( - 2*sqrt(c)*c**2)/(3*sqrt(a + b*x)*b*(a + b*x))
```

3.117 $\int \frac{(cx)^{3/2}}{(ax+bx^2)^{5/2}} \, dx$

Optimal result

Integrand size = 21, antiderivative size = 98

$$\int \frac{(cx)^{3/2}}{(ax+bx^2)^{5/2}} \, dx = \frac{2(cx)^{3/2}}{3a\,(ax+bx^2)^{3/2}} + \frac{2c\sqrt{cx}}{a^2\sqrt{ax+bx^2}} - \frac{2c^{3/2}\text{arctanh}\left(\frac{\sqrt{c}\sqrt{ax+bx^2}}{\sqrt{a}\sqrt{cx}}\right)}{a^{5/2}}$$

```
2/3*(c*x)^(3/2)/a/(b*x^2+a*x)^(3/2)+2*c*(c*x)^(1/2)/a^2/(b*x^2+a*x)^(1/2)-
2*c^(3/2)*arctanh(c^(1/2)*(b*x^2+a*x)^(1/2)/a^(1/2)/(c*x)^(1/2))/a^(5/2)
```

Mathematica [A] (verified)

Time = 0.07 (sec) , antiderivative size = 69, normalized size of antiderivative = 0.70

$$\int \frac{(cx)^{3/2}}{(ax+bx^2)^{5/2}} \, dx = \frac{2(cx)^{3/2}\left(\sqrt{a}(4a+3bx) - 3(a+bx)^{3/2}\text{arctanh}\left(\frac{\sqrt{a+bx}}{\sqrt{a}}\right)\right)}{3a^{5/2}(x(a+bx))^{3/2}}$$

```
Integrate[(c*x)^(3/2)/(a*x + b*x^2)^(5/2),x]
```

```
(2*(c*x)^(3/2)*(Sqrt[a]*(4*a + 3*b*x) - 3*(a + b*x)^(3/2)*ArcTanh[Sqrt[a +
 b*x]/Sqrt[a]]))/(3*a^(5/2)*(x*(a + b*x))^(3/2))
```

Rubi [A] (verified)

Time = 0.47 (sec) , antiderivative size = 146, normalized size of antiderivative = 1.49, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.238$, Rules used = {1133, 1135, 1132, 1136, 221}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(cx)^{3/2}}{(ax+bx^2)^{5/2}}dx$$

$$\Big\downarrow \ 1133$$

$$-\frac{2c^2\int \frac{1}{\sqrt{cx}(bx^2+ax)^{3/2}}dx}{3b}-\frac{2c\sqrt{cx}}{3b\,(ax+bx^2)^{3/2}}$$

$$\Big\downarrow \ 1135$$

$$-\frac{2c^2\left(-\frac{3b\int \frac{\sqrt{cx}}{\left(bx^2+ax\right)^{3/2}}dx}{2ac}-\frac{1}{a\sqrt{cx}\sqrt{ax+bx^2}}\right)}{3b}-\frac{2c\sqrt{cx}}{3b\,(ax+bx^2)^{3/2}}$$

$$\Big\downarrow \ 1132$$

$$-\frac{2c^2\left(-\frac{3b\left(\frac{c\int \frac{1}{\sqrt{cx}\sqrt{bx^2+ax}}dx}{a}+\frac{2\sqrt{cx}}{a\sqrt{ax+bx^2}}\right)}{2ac}-\frac{1}{a\sqrt{cx}\sqrt{ax+bx^2}}\right)}{3b}-\frac{2c\sqrt{cx}}{3b\,(ax+bx^2)^{3/2}}$$

$$\Big\downarrow \ 1136$$

$$-\frac{2c^2\left(-\frac{3b\left(\frac{2c^2\int \frac{1}{\frac{c\left(bx^2+ax\right)}{x}-ac}d\frac{\sqrt{bx^2+ax}}{\sqrt{cx}}}{a}+\frac{2\sqrt{cx}}{a\sqrt{ax+bx^2}}\right)}{2ac}-\frac{1}{a\sqrt{cx}\sqrt{ax+bx^2}}\right)}{3b}-\frac{2c\sqrt{cx}}{3b\,(ax+bx^2)^{3/2}}$$

$$\Big\downarrow \ 221$$

$$-\frac{2c^2\left(-\frac{3b\left(\frac{2\sqrt{cx}}{a\sqrt{ax+bx^2}}-\frac{2\sqrt{c}\,\text{arctanh}\left(\frac{\sqrt{c}\sqrt{ax+bx^2}}{\sqrt{a}\sqrt{cx}}\right)}{a^{3/2}}\right)}{2ac}-\frac{1}{a\sqrt{cx}\sqrt{ax+bx^2}}\right)}{3b}-\frac{2c\sqrt{cx}}{3b\left(ax+bx^2\right)^{3/2}}$$

```
Int[(c*x)^(3/2)/(a*x + b*x^2)^(5/2),x]
```

```
(-2*c*Sqrt[c*x])/(3*b*(a*x + b*x^2)^(3/2)) - (2*c^2*(-(1/(a*Sqrt[c*x]*Sqrt
[a*x + b*x^2])) - (3*b*((2*Sqrt[c*x])/(a*Sqrt[a*x + b*x^2]) - (2*Sqrt[c]*A
rcTanh[(Sqrt[c]*Sqrt[a*x + b*x^2])/(Sqrt[a]*Sqrt[c*x])])/a^(3/2)))/(2*a*c)
))/(3*b)
```

Defintions of rubi rules used

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] :> Simp[(2*c*d - b*e)*(d + e*x)^m*((a + b*x + c*x^2)^(p + 1)/(e*(p +
 1)*(b^2 - 4*a*c))), x] - Simp[(2*c*d - b*e)*((m + 2*p + 2)/((p + 1)*(b^2 -
 4*a*c)))   Int[(d + e*x)^(m - 1)*(a + b*x + c*x^2)^(p + 1), x], x] /; Free
Q[{a, b, c, d, e}, x] && EqQ[c*d^2 - b*d*e + a*e^2, 0] && LtQ[p, -1] && LtQ
[0, m, 1] && IntegerQ[2*p]
```

```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] :> Simp[e*(d + e*x)^(m - 1)*((a + b*x + c*x^2)^(p + 1)/(c*(p + 1))),
 x] - Simp[e^2*((m + p)/(c*(p + 1)))   Int[(d + e*x)^(m - 2)*(a + b*x + c*x
^2)^(p + 1), x], x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[c*d^2 - b*d*e + a*e
^2, 0] && LtQ[p, -1] && GtQ[m, 1] && IntegerQ[2*p]
```

```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] :> Simp[(-e)*(d + e*x)^m*((a + b*x + c*x^2)^(p + 1)/((m + p + 1)*(2*
c*d - b*e))), x] + Simp[c*((m + 2*p + 2)/((m + p + 1)*(2*c*d - b*e)))   Int
[(d + e*x)^(m + 1)*(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, p},
 x] && EqQ[c*d^2 - b*d*e + a*e^2, 0] && LtQ[m, 0] && NeQ[m + p + 1, 0] && I
ntegerQ[2*p]
```

```
Int[1/(Sqrt[(d_.) + (e_.)*(x_)]*Sqrt[(a_.) + (b_.)*(x_) + (c_.)*(x_)^2]), x
_Symbol] :> Simp[2*e   Subst[Int[1/(2*c*d - b*e + e^2*x^2), x], x, Sqrt[a +
 b*x + c*x^2]/Sqrt[d + e*x]], x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[c*d^2
- b*d*e + a*e^2, 0]
```

Maple [A] (verified)

Time = 0.34 (sec) , antiderivative size = 111, normalized size of antiderivative = 1.13

method	result	size
default	$-\frac{2\sqrt{cx}\,\sqrt{x(bx+a)}\,c\left(3\sqrt{c(bx+a)}\,\operatorname{arctanh}\left(\frac{\sqrt{c(bx+a)}}{\sqrt{ac}}\right)bx+3\,\operatorname{arctanh}\left(\frac{\sqrt{c(bx+a)}}{\sqrt{ac}}\right)a\sqrt{c(bx+a)}-3\sqrt{ac}\,bx-4\sqrt{ac}\,a\right)}{3x(bx+a)^2a^2\sqrt{ac}}$	111

```
int((c*x)^(3/2)/(b*x^2+a*x)^(5/2),x,method=_RETURNVERBOSE)
```

```
-2/3*(c*x)^(1/2)*(x*(b*x+a))^(1/2)*c*(3*(c*(b*x+a))^(1/2)*arctanh((c*(b*x+
a))^(1/2)/(a*c)^(1/2))*b*x+3*arctanh((c*(b*x+a))^(1/2)/(a*c)^(1/2))*a*(c*(
b*x+a))^(1/2)-3*(a*c)^(1/2)*b*x-4*(a*c)^(1/2)*a)/x/(b*x+a)^2/a^2/(a*c)^(1/
2)
```

Fricas [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 265, normalized size of antiderivative = 2.70

$$\int \frac{(cx)^{3/2}}{(ax+bx^2)^{5/2}}\,dx = \left[\frac{3\,(b^2cx^3+2\,abcx^2+a^2cx)\sqrt{\frac{c}{a}}\log\left(-\frac{bcx^2+2\,acx-2\,\sqrt{bx^2+ax}\sqrt{cx}a\sqrt{\frac{c}{a}}}{x^2}\right)+2\,(3\,bcx+4\,ac}{3\,(a^2b^2x^3+2\,a^3bx^2+a^4x)}\right.$$

```
integrate((c*x)^(3/2)/(b*x^2+a*x)^(5/2),x, algorithm="fricas")
```

```
[1/3*(3*(b^2*c*x^3 + 2*a*b*c*x^2 + a^2*c*x)*sqrt(c/a)*log(-(b*c*x^2 + 2*a*
c*x - 2*sqrt(b*x^2 + a*x)*sqrt(c*x)*a*sqrt(c/a))/x^2) + 2*(3*b*c*x + 4*a*c
)*sqrt(b*x^2 + a*x)*sqrt(c*x))/(a^2*b^2*x^3 + 2*a^3*b*x^2 + a^4*x), 2/3*(3
*(b^2*c*x^3 + 2*a*b*c*x^2 + a^2*c*x)*sqrt(-c/a)*arctan(sqrt(b*x^2 + a*x)*s
qrt(c*x)*a*sqrt(-c/a)/(b*c*x^2 + a*c*x)) + (3*b*c*x + 4*a*c)*sqrt(b*x^2 +
a*x)*sqrt(c*x))/(a^2*b^2*x^3 + 2*a^3*b*x^2 + a^4*x)]
```

Sympy [F]

$$\int \frac{(cx)^{3/2}}{(ax+bx^2)^{5/2}}\,dx = \int \frac{(cx)^{\frac{3}{2}}}{(x\,(a+bx))^{\frac{5}{2}}}\,dx$$

```
integrate((c*x)**(3/2)/(b*x**2+a*x)**(5/2),x)
```

```
Integral((c*x)**(3/2)/(x*(a + b*x))**(5/2), x)
```

Maxima [F]

$$\int \frac{(cx)^{3/2}}{(ax+bx^2)^{5/2}}\,dx = \int \frac{(cx)^{\frac{3}{2}}}{(bx^2+ax)^{\frac{5}{2}}}\,dx$$

```
integrate((c*x)^(3/2)/(b*x^2+a*x)^(5/2),x, algorithm="maxima")
```

```
integrate((c*x)^(3/2)/(b*x^2 + a*x)^(5/2), x)
```

Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 131, normalized size of antiderivative = 1.34

$$\int \frac{(cx)^{3/2}}{(ax+bx^2)^{5/2}}\,dx = \frac{2}{3}\left(c^4\left(\frac{3\arctan\left(\frac{\sqrt{bcx+ac}}{\sqrt{-ac}}\right)}{\sqrt{-ac}a^2c^2|c|} + \frac{3\,bcx+4\,ac}{(bcx+ac)^{\frac{3}{2}}a^2c^2|c|}\right) - \frac{3\sqrt{ac}c^2\arctan\left(\frac{\sqrt{ac}}{\sqrt{-ac}}\right)+4\sqrt{}}{\sqrt{ac}\sqrt{-ac}a^2|c|}\right.$$

```
integrate((c*x)^(3/2)/(b*x^2+a*x)^(5/2),x, algorithm="giac")
```

```
2/3*(c^4*(3*arctan(sqrt(b*c*x + a*c)/sqrt(-a*c))/(sqrt(-a*c)*a^2*c^2*abs(c
)) + (3*b*c*x + 4*a*c)/((b*c*x + a*c)^(3/2)*a^2*c^2*abs(c))) - (3*sqrt(a*c
)*c^2*arctan(sqrt(a*c)/sqrt(-a*c)) + 4*sqrt(-a*c)*c^2)/(sqrt(a*c)*sqrt(-a*
c)*a^2*abs(c)))*c
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(cx)^{3/2}}{(ax+bx^2)^{5/2}}\,dx = \int \frac{(c\,x)^{3/2}}{(b\,x^2+a\,x)^{5/2}}\,dx$$

```
int((c*x)^(3/2)/(a*x + b*x^2)^(5/2),x)
```

```
int((c*x)^(3/2)/(a*x + b*x^2)^(5/2), x)
```

Reduce [B] (verification not implemented)

Time = 0.23 (sec) , antiderivative size = 124, normalized size of antiderivative = 1.27

$$\int \frac{(cx)^{3/2}}{(ax+bx^2)^{5/2}}\,dx = \frac{\sqrt{c}\,c\left(3\sqrt{a}\,\sqrt{bx+a}\log\left(\sqrt{bx+a}-\sqrt{a}\right)a+3\sqrt{a}\,\sqrt{bx+a}\log\left(\sqrt{bx+a}-\sqrt{a}\right)bx-3\right.}{3\sqrt{bx+}}$$

```
int((c*x)^(3/2)/(b*x^2+a*x)^(5/2),x)
```

```
(sqrt(c)*c*(3*sqrt(a)*sqrt(a + b*x)*log(sqrt(a + b*x) - sqrt(a))*a + 3*sqr
t(a)*sqrt(a + b*x)*log(sqrt(a + b*x) - sqrt(a))*b*x - 3*sqrt(a)*sqrt(a + b
*x)*log(sqrt(a + b*x) + sqrt(a))*a - 3*sqrt(a)*sqrt(a + b*x)*log(sqrt(a +
b*x) + sqrt(a))*b*x + 8*a**2 + 6*a*b*x))/(3*sqrt(a + b*x)*a**3*(a + b*x))
```

3.118 $\int \frac{\sqrt{cx}}{(ax+bx^2)^{5/2}}\,dx$

Optimal result

Integrand size = 21, antiderivative size = 128

$$\int \frac{\sqrt{cx}}{(ax+bx^2)^{5/2}}\,dx = -\frac{\sqrt{cx}}{a(ax+bx^2)^{3/2}} - \frac{5b(cx)^{3/2}}{3a^2c(ax+bx^2)^{3/2}} - \frac{5b\sqrt{cx}}{a^3\sqrt{ax+bx^2}} + \frac{5b\sqrt{c}\,\text{arctanh}\left(\frac{\sqrt{c}\sqrt{ax+bx^2}}{\sqrt{a}\sqrt{cx}}\right)}{a^{7/2}}$$

```
-(c*x)^(1/2)/a/(b*x^2+a*x)^(3/2)-5/3*b*(c*x)^(3/2)/a^2/c/(b*x^2+a*x)^(3/2)
-5*b*(c*x)^(1/2)/a^3/(b*x^2+a*x)^(1/2)+5*b*c^(1/2)*arctanh(c^(1/2)*(b*x^2+
a*x)^(1/2)/a^(1/2)/(c*x)^(1/2))/a^(7/2)
```

Mathematica [A] (verified)

Time = 0.10 (sec) , antiderivative size = 83, normalized size of antiderivative = 0.65

$$\int \frac{\sqrt{cx}}{(ax+bx^2)^{5/2}}\,dx = \frac{\sqrt{cx}\left(-\sqrt{a}(3a^2+20abx+15b^2x^2)+15bx(a+bx)^{3/2}\text{arctanh}\left(\frac{\sqrt{a+bx}}{\sqrt{a}}\right)\right)}{3a^{7/2}(x(a+bx))^{3/2}}$$

```
Integrate[Sqrt[c*x]/(a*x + b*x^2)^(5/2),x]
```

```
(Sqrt[c*x]*(-(Sqrt[a]*(3*a^2 + 20*a*b*x + 15*b^2*x^2)) + 15*b*x*(a + b*x)^
(3/2)*ArcTanh[Sqrt[a + b*x]/Sqrt[a]]))/(3*a^(7/2)*(x*(a + b*x))^(3/2))
```

Rubi [A] (verified)

Time = 0.45 (sec) , antiderivative size = 143, normalized size of antiderivative = 1.12, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.238, Rules used = {1132, 1135, 1132, 1136, 221}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sqrt{cx}}{(ax+bx^2)^{5/2}}dx$$

$$\Big\downarrow \text{1132}$$

$$\frac{5c\int \frac{1}{\sqrt{cx}(bx^2+ax)^{3/2}}dx}{3a}+\frac{2\sqrt{cx}}{3a(ax+bx^2)^{3/2}}$$

$$\Big\downarrow \text{1135}$$

$$\frac{5c\left(-\frac{3b\int \frac{\sqrt{cx}}{\left(bx^2+ax\right)^{3/2}}dx}{2ac}-\frac{1}{a\sqrt{cx}\sqrt{ax+bx^2}}\right)}{3a}+\frac{2\sqrt{cx}}{3a(ax+bx^2)^{3/2}}$$

$$\Big\downarrow \text{1132}$$

$$\frac{5c\left(-\frac{3b\left(\frac{c\int \frac{1}{\sqrt{cx}\sqrt{bx^2+ax}}dx}{a}+\frac{2\sqrt{cx}}{a\sqrt{ax+bx^2}}\right)}{2ac}-\frac{1}{a\sqrt{cx}\sqrt{ax+bx^2}}\right)}{3a}+\frac{2\sqrt{cx}}{3a(ax+bx^2)^{3/2}}$$

$$\Big\downarrow \text{1136}$$

$$\frac{5c\left(-\frac{3b\left(\frac{2c^2\int\frac{1}{\frac{c(bx^2+ax)}{x}-ac}d\frac{\sqrt{bx^2+ax}}{\sqrt{cx}}}{a}+\frac{2\sqrt{cx}}{a\sqrt{ax+bx^2}}\right)}{2ac}-\frac{1}{a\sqrt{cx}\sqrt{ax+bx^2}}\right)}{3a}+\frac{2\sqrt{cx}}{3a\left(ax+bx^2\right)^{3/2}}$$

$$\Big\downarrow \text{ 221}$$

$$\frac{5c\left(-\frac{3b\left(\frac{2\sqrt{cx}}{a\sqrt{ax+bx^2}}-\frac{2\sqrt{c}\,\text{arctanh}\left(\frac{\sqrt{c}\sqrt{ax+bx^2}}{\sqrt{a}\sqrt{cx}}\right)}{a^{3/2}}\right)}{2ac}-\frac{1}{a\sqrt{cx}\sqrt{ax+bx^2}}\right)}{3a}+\frac{2\sqrt{cx}}{3a\left(ax+bx^2\right)^{3/2}}$$

```
Int[Sqrt[c*x]/(a*x + b*x^2)^(5/2),x]
```

```
(2*Sqrt[c*x])/(3*a*(a*x + b*x^2)^(3/2)) + (5*c*(-(1/(a*Sqrt[c*x]*Sqrt[a*x
+ b*x^2])) - (3*b*((2*Sqrt[c*x])/(a*Sqrt[a*x + b*x^2]) - (2*Sqrt[c]*ArcTan
h[(Sqrt[c]*Sqrt[a*x + b*x^2])/(Sqrt[a]*Sqrt[c*x])])/a^(3/2)))/(2*a*c)))/(3
*a)
```

Defintions of rubi rules used

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] :> Simp[(2*c*d - b*e)*(d + e*x)^m*((a + b*x + c*x^2)^(p + 1)/(e*(p +
 1)*(b^2 - 4*a*c))), x] - Simp[(2*c*d - b*e)*((m + 2*p + 2)/((p + 1)*(b^2 -
 4*a*c)))   Int[(d + e*x)^(m - 1)*(a + b*x + c*x^2)^(p + 1), x], x] /; Free
Q[{a, b, c, d, e}, x] && EqQ[c*d^2 - b*d*e + a*e^2, 0] && LtQ[p, -1] && LtQ
[0, m, 1] && IntegerQ[2*p]
```

```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] :> Simp[(-e)*(d + e*x)^m*((a + b*x + c*x^2)^(p + 1)/((m + p + 1)*(2*
c*d - b*e))), x] + Simp[c*((m + 2*p + 2)/((m + p + 1)*(2*c*d - b*e)))   Int
[(d + e*x)^(m + 1)*(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, p},
 x] && EqQ[c*d^2 - b*d*e + a*e^2, 0] && LtQ[m, 0] && NeQ[m + p + 1, 0] && I
ntegerQ[2*p]
```

```
Int[1/(Sqrt[(d_.) + (e_.)*(x_)]*Sqrt[(a_.) + (b_.)*(x_) + (c_.)*(x_)^2]), x
_Symbol] :> Simp[2*e   Subst[Int[1/(2*c*d - b*e + e^2*x^2), x], x, Sqrt[a +
 b*x + c*x^2]/Sqrt[d + e*x]], x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[c*d^2
- b*d*e + a*e^2, 0]
```

Maple [A] (verified)

Time = 0.37 (sec) , antiderivative size = 109, normalized size of antiderivative = 0.85

method	result
risch	$-\frac{(bx+a)c}{a^3\sqrt{cx}\,\sqrt{x(bx+a)}}-\frac{b\left(\frac{8}{\sqrt{cbx+ac}}+\frac{4ac}{3(cbx+ac)^{\frac{3}{2}}}-\frac{10\,\operatorname{arctanh}\left(\frac{\sqrt{cbx+ac}}{\sqrt{ac}}\right)}{\sqrt{ac}}\right)c\sqrt{c(bx+a)}\,x}{2a^3\sqrt{cx}\,\sqrt{x(bx+a)}}$
default	$\frac{\sqrt{cx}\,\sqrt{x(bx+a)}\left(15\sqrt{c(bx+a)}\,\operatorname{arctanh}\left(\frac{\sqrt{c(bx+a)}}{\sqrt{ac}}\right)b^2x^2+15\,\operatorname{arctanh}\left(\frac{\sqrt{c(bx+a)}}{\sqrt{ac}}\right)abx\sqrt{c(bx+a)}-15\sqrt{ac}\,b^2x^2-20\sqrt{ac}\,abx-3\sqrt{a\cdots}\right.}{3x^2(bx+a)^2a^3\sqrt{ac}}$

```
int((c*x)^(1/2)/(b*x^2+a*x)^(5/2),x,method=_RETURNVERBOSE)
```

```
-1/a^3*(b*x+a)*c/(c*x)^(1/2)/(x*(b*x+a))^(1/2)-1/2/a^3*b*(8/(b*c*x+a*c)^(1
/2)+4/3*a*c/(b*c*x+a*c)^(3/2)-10/(a*c)^(1/2)*arctanh((b*c*x+a*c)^(1/2)/(a*
c)^(1/2)))*c*(c*(b*x+a))^(1/2)/(c*x)^(1/2)/(x*(b*x+a))^(1/2)*x
```

Fricas [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 291, normalized size of antiderivative = 2.27

$$\int \frac{\sqrt{cx}}{(ax+bx^2)^{5/2}}\,dx = \left[\frac{15\,(b^3x^4+2\,ab^2x^3+a^2bx^2)\sqrt{\frac{c}{a}}\log\left(-\frac{bcx^2+2\,acx+2\,\sqrt{bx^2+ax}\sqrt{cx}a\sqrt{\frac{c}{a}}}{x^2}\right)-2\,(15\,b^2x^2+2}{6\,(a^3b^2x^4+2\,a^4bx^3+a^5x^2)}\right.$$

$$\left.-\frac{15\,(b^3x^4+2\,ab^2x^3+a^2bx^2)\sqrt{-\frac{c}{a}}\arctan\left(\frac{\sqrt{bx^2+ax}\sqrt{cx}a\sqrt{-\frac{c}{a}}}{bcx^2+acx}\right)+(15\,b^2x^2+20\,abx+3\,a^2)\sqrt{bx^2+ax}\sqrt{cx}}{3\,(a^3b^2x^4+2\,a^4bx^3+a^5x^2)}\right]$$

```
integrate((c*x)^(1/2)/(b*x^2+a*x)^(5/2),x, algorithm="fricas")
```

```
[1/6*(15*(b^3*x^4 + 2*a*b^2*x^3 + a^2*b*x^2)*sqrt(c/a)*log(-(b*c*x^2 + 2*a
*c*x + 2*sqrt(b*x^2 + a*x)*sqrt(c*x)*a*sqrt(c/a))/x^2) - 2*(15*b^2*x^2 + 2
0*a*b*x + 3*a^2)*sqrt(b*x^2 + a*x)*sqrt(c*x))/(a^3*b^2*x^4 + 2*a^4*b*x^3 +
 a^5*x^2), -1/3*(15*(b^3*x^4 + 2*a*b^2*x^3 + a^2*b*x^2)*sqrt(-c/a)*arctan(
sqrt(b*x^2 + a*x)*sqrt(c*x)*a*sqrt(-c/a)/(b*c*x^2 + a*c*x)) + (15*b^2*x^2
+ 20*a*b*x + 3*a^2)*sqrt(b*x^2 + a*x)*sqrt(c*x))/(a^3*b^2*x^4 + 2*a^4*b*x^
3 + a^5*x^2)]
```

Sympy [F]

$$\int \frac{\sqrt{cx}}{(ax+bx^2)^{5/2}}\,dx = \int \frac{\sqrt{cx}}{(x\,(a+bx))^{\frac{5}{2}}}\,dx$$

```
integrate((c*x)**(1/2)/(b*x**2+a*x)**(5/2),x)
```

```
Integral(sqrt(c*x)/(x*(a + b*x))**(5/2), x)
```

Maxima [F]

$$\int \frac{\sqrt{cx}}{(ax+bx^2)^{5/2}}\,dx = \int \frac{\sqrt{cx}}{(bx^2+ax)^{\frac{5}{2}}}\,dx$$

```
integrate((c*x)^(1/2)/(b*x^2+a*x)^(5/2),x, algorithm="maxima")
```

```
integrate(sqrt(c*x)/(b*x^2 + a*x)^(5/2), x)
```

Giac [A] (verification not implemented)

Time = 0.15 (sec) , antiderivative size = 106, normalized size of antiderivative = 0.83

$$\int \frac{\sqrt{cx}}{(ax+bx^2)^{5/2}}\,dx = -\frac{1}{3}c^5\left(\frac{15\,b\arctan\left(\frac{\sqrt{bcx+ac}}{\sqrt{-ac}}\right)}{\sqrt{-ac}a^3c^3|c|} + \frac{2\,(abc+6\,(bcx+ac)b)}{(bcx+ac)^{\frac{3}{2}}a^3c^3|c|} + \frac{3\,\sqrt{bcx+ac}}{a^3c^4x|c|}\right)$$

```
integrate((c*x)^(1/2)/(b*x^2+a*x)^(5/2),x, algorithm="giac")
```

```
-1/3*c^5*(15*b*arctan(sqrt(b*c*x + a*c)/sqrt(-a*c))/(sqrt(-a*c)*a^3*c^3*ab
s(c)) + 2*(a*b*c + 6*(b*c*x + a*c)*b)/((b*c*x + a*c)^(3/2)*a^3*c^3*abs(c))
 + 3*sqrt(b*c*x + a*c)/(a^3*c^4*x*abs(c)))
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\sqrt{cx}}{(ax+bx^2)^{5/2}}\,dx = \int \frac{\sqrt{c\,x}}{(b\,x^2+a\,x)^{5/2}}\,dx$$

```
int((c*x)^(1/2)/(a*x + b*x^2)^(5/2),x)
```

```
int((c*x)^(1/2)/(a*x + b*x^2)^(5/2), x)
```

Reduce [B] (verification not implemented)

Time = 0.42 (sec) , antiderivative size = 149, normalized size of antiderivative = 1.16

$$\int \frac{\sqrt{cx}}{(ax+bx^2)^{5/2}}\,dx = \frac{\sqrt{c}\left(-15\sqrt{a}\,\sqrt{bx+a}\log\left(\sqrt{bx+a}-\sqrt{a}\right)abx-15\sqrt{a}\,\sqrt{bx+a}\log\left(\sqrt{bx+a}-\sqrt{a}\right)\right.}{}$$

```
int((c*x)^(1/2)/(b*x^2+a*x)^(5/2),x)
```

```
(sqrt(c)*( - 15*sqrt(a)*sqrt(a + b*x)*log(sqrt(a + b*x) - sqrt(a))*a*b*x -
 15*sqrt(a)*sqrt(a + b*x)*log(sqrt(a + b*x) - sqrt(a))*b**2*x**2 + 15*sqrt
(a)*sqrt(a + b*x)*log(sqrt(a + b*x) + sqrt(a))*a*b*x + 15*sqrt(a)*sqrt(a +
 b*x)*log(sqrt(a + b*x) + sqrt(a))*b**2*x**2 - 6*a**3 - 40*a**2*b*x - 30*a
*b**2*x**2))/(6*sqrt(a + b*x)*a**4*x*(a + b*x))
```

3.119 $\int \frac{1}{\sqrt{cx}\left(ax+bx^2\right)^{5/2}}\,dx$

Optimal result

Integrand size = 21, antiderivative size = 174

$$\int \frac{1}{\sqrt{cx}\,(ax+bx^2)^{5/2}}\,dx = -\frac{1}{2a\sqrt{cx}\,(ax+bx^2)^{3/2}} + \frac{7b\sqrt{cx}}{4a^2c\,(ax+bx^2)^{3/2}} + \frac{35b^2(cx)^{3/2}}{12a^3c^2\,(ax+bx^2)^{3/2}} + \frac{35b^2\sqrt{cx}}{4a^4c\sqrt{ax+bx^2}} - \frac{35b^2\text{arctanh}\left(\frac{\sqrt{c}\sqrt{ax+bx^2}}{\sqrt{a}\sqrt{cx}}\right)}{4a^{9/2}\sqrt{c}}$$

```
-1/2/a/(c*x)^(1/2)/(b*x^2+a*x)^(3/2)+7/4*b*(c*x)^(1/2)/a^2/c/(b*x^2+a*x)^(
3/2)+35/12*b^2*(c*x)^(3/2)/a^3/c^2/(b*x^2+a*x)^(3/2)+35/4*b^2*(c*x)^(1/2)/
a^4/c/(b*x^2+a*x)^(1/2)-35/4*b^2*arctanh(c^(1/2)*(b*x^2+a*x)^(1/2)/a^(1/2)
/(c*x)^(1/2))/a^(9/2)/c^(1/2)
```

Mathematica [A] (verified)

Time = 0.12 (sec) , antiderivative size = 97, normalized size of antiderivative = 0.56

$$\int \frac{1}{\sqrt{cx}\,(ax+bx^2)^{5/2}}\,dx = \frac{\sqrt{a}(-6a^3+21a^2bx+140ab^2x^2+105b^3x^3)-105b^2x^2(a+bx)^{3/2}\text{arctanh}\left(\frac{\sqrt{a+b}}{\sqrt{a}}\right.}{12a^{9/2}\sqrt{cx}(x(a+bx))^{3/2}}$$

```
Integrate[1/(Sqrt[c*x]*(a*x + b*x^2)^(5/2)),x]
```

```
(Sqrt[a]*(-6*a^3 + 21*a^2*b*x + 140*a*b^2*x^2 + 105*b^3*x^3) - 105*b^2*x^2
*(a + b*x)^(3/2)*ArcTanh[Sqrt[a + b*x]/Sqrt[a]])/(12*a^(9/2)*Sqrt[c*x]*(x*
(a + b*x))^(3/2))
```

Rubi [A] (verified)

Time = 0.52 (sec) , antiderivative size = 182, normalized size of antiderivative = 1.05, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.286$, Rules used = {1135, 1132, 1135, 1132, 1136, 221}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{\sqrt{cx}\left(ax+bx^2\right)^{5/2}}dx$$

$$\Big\downarrow \text{1135}$$

$$-\frac{7b\int \frac{\sqrt{cx}}{\left(bx^2+ax\right)^{5/2}}dx}{4ac}-\frac{1}{2a\sqrt{cx}\left(ax+bx^2\right)^{3/2}}$$

$$\Big\downarrow \text{1132}$$

$$-\frac{7b\left(\frac{5c\int \frac{1}{\sqrt{cx}\left(bx^2+ax\right)^{3/2}}dx}{3a}+\frac{2\sqrt{cx}}{3a\left(ax+bx^2\right)^{3/2}}\right)}{4ac}-\frac{1}{2a\sqrt{cx}\left(ax+bx^2\right)^{3/2}}$$

$$\Big\downarrow \text{1135}$$

$$-\frac{7b\left(\frac{5c\left(-\frac{3b\int \frac{\sqrt{cx}}{\left(bx^2+ax\right)^{3/2}}dx}{2ac}-\frac{1}{a\sqrt{cx}\sqrt{ax+bx^2}}\right)}{3a}+\frac{2\sqrt{cx}}{3a\left(ax+bx^2\right)^{3/2}}\right)}{4ac}-\frac{1}{2a\sqrt{cx}\left(ax+bx^2\right)^{3/2}}$$

$$\Big\downarrow \text{1132}$$

$$-\frac{7b\left(\frac{5c\left(-\frac{3b\left(\frac{c\int \frac{1}{\sqrt{cx}\sqrt{bx^2+ax}}dx}{a}+\frac{2\sqrt{cx}}{a\sqrt{ax+bx^2}}\right)}{2ac}-\frac{1}{a\sqrt{cx}\sqrt{ax+bx^2}}\right)}{3a}+\frac{2\sqrt{cx}}{3a(ax+bx^2)^{3/2}}\right)}{4ac}-\frac{1}{2a\sqrt{cx}\,(ax+bx^2)^{3/2}}$$

$$\Big\downarrow \; 1136$$

$$-\frac{7b\left(\frac{5c\left(-\frac{3b\left(\frac{2c^2\int \frac{1}{\frac{c(bx^2+ax)}{x}-ac}d\frac{\sqrt{bx^2+ax}}{\sqrt{cx}}}{a}+\frac{2\sqrt{cx}}{a\sqrt{ax+bx^2}}\right)}{2ac}-\frac{1}{a\sqrt{cx}\sqrt{ax+bx^2}}\right)}{3a}+\frac{2\sqrt{cx}}{3a(ax+bx^2)^{3/2}}\right)}{4ac}-\frac{1}{2a\sqrt{cx}\,(ax+bx^2)^{3/2}}$$

$$\Big\downarrow \; 221$$

$$-\frac{7b\left(\frac{5c\left(-\frac{3b\left(\frac{2\sqrt{cx}}{a\sqrt{ax+bx^2}}-\frac{2\sqrt{c}\,\text{arctanh}\left(\frac{\sqrt{c}\sqrt{ax+bx^2}}{\sqrt{a}\sqrt{cx}}\right)}{a^{3/2}}\right)}{2ac}-\frac{1}{a\sqrt{cx}\sqrt{ax+bx^2}}\right)}{3a}+\frac{2\sqrt{cx}}{3a(ax+bx^2)^{3/2}}\right)}{4ac}-\frac{1}{2a\sqrt{cx}\,(ax+bx^2)^{3/2}}$$

```
Int[1/(Sqrt[c*x]*(a*x + b*x^2)^(5/2)),x]
```

```
-1/2*1/(a*Sqrt[c*x]*(a*x + b*x^2)^(3/2)) - (7*b*((2*Sqrt[c*x])/(3*a*(a*x +
 b*x^2)^(3/2)) + (5*c*(-(1/(a*Sqrt[c*x]*Sqrt[a*x + b*x^2])) - (3*b*((2*Sqr
t[c*x])/(a*Sqrt[a*x + b*x^2]) - (2*Sqrt[c]*ArcTanh[(Sqrt[c]*Sqrt[a*x + b*x
^2])/(Sqrt[a]*Sqrt[c*x])])/a^(3/2)))/(2*a*c)))/(3*a)))/(4*a*c)
```

Defintions of rubi rules used

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] :> Simp[(2*c*d - b*e)*(d + e*x)^m*((a + b*x + c*x^2)^(p + 1)/(e*(p +
 1)*(b^2 - 4*a*c))), x] - Simp[(2*c*d - b*e)*((m + 2*p + 2)/((p + 1)*(b^2 -
 4*a*c)))   Int[(d + e*x)^(m - 1)*(a + b*x + c*x^2)^(p + 1), x], x] /; Free
Q[{a, b, c, d, e}, x] && EqQ[c*d^2 - b*d*e + a*e^2, 0] && LtQ[p, -1] && LtQ
[0, m, 1] && IntegerQ[2*p]
```

```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] :> Simp[(-e)*(d + e*x)^m*((a + b*x + c*x^2)^(p + 1)/((m + p + 1)*(2*
c*d - b*e))), x] + Simp[c*((m + 2*p + 2)/((m + p + 1)*(2*c*d - b*e)))   Int
[(d + e*x)^(m + 1)*(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, p},
 x] && EqQ[c*d^2 - b*d*e + a*e^2, 0] && LtQ[m, 0] && NeQ[m + p + 1, 0] && I
ntegerQ[2*p]
```

```
Int[1/(Sqrt[(d_.) + (e_.)*(x_)]*Sqrt[(a_.) + (b_.)*(x_) + (c_.)*(x_)^2]), x
_Symbol] :> Simp[2*e   Subst[Int[1/(2*c*d - b*e + e^2*x^2), x], x, Sqrt[a +
 b*x + c*x^2]/Sqrt[d + e*x]], x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[c*d^2
- b*d*e + a*e^2, 0]
```

Maple [A] (verified)

Time = 0.36 (sec) , antiderivative size = 120, normalized size of antiderivative = 0.69

method	result
risch	$-\frac{(bx+a)(-11bx+2a)}{4a^4x\sqrt{cx}\,\sqrt{x(bx+a)}}+\frac{b^2\left(\frac{48}{\sqrt{cbx+ac}}+\frac{16ac}{3(cbx+ac)^{\frac{3}{2}}}-\frac{70\,\operatorname{arctanh}\left(\frac{\sqrt{cbx+ac}}{\sqrt{ac}}\right)}{\sqrt{ac}}\right)\sqrt{c(bx+a)}\,x}{8a^4\sqrt{cx}\,\sqrt{x(bx+a)}}$
default	$-\frac{\sqrt{x(bx+a)}\left(105\sqrt{c(bx+a)}\,\operatorname{arctanh}\left(\frac{\sqrt{c(bx+a)}}{\sqrt{ac}}\right)b^3x^3+105\,\operatorname{arctanh}\left(\frac{\sqrt{c(bx+a)}}{\sqrt{ac}}\right)a\,b^2x^2\sqrt{c(bx+a)}-105\sqrt{ac}\,b^3x^3-140\sqrt{ac}\,a\,b^2x\right.}{12\sqrt{cx}\,x^2(bx+a)^2a^4\sqrt{ac}}$

```
int(1/(c*x)^(1/2)/(b*x^2+a*x)^(5/2),x,method=_RETURNVERBOSE)
```

```
-1/4*(b*x+a)*(-11*b*x+2*a)/a^4/x/(c*x)^(1/2)/(x*(b*x+a))^(1/2)+1/8*b^2/a^4
*(48/(b*c*x+a*c)^(1/2)+16/3*a*c/(b*c*x+a*c)^(3/2)-70/(a*c)^(1/2)*arctanh((
b*c*x+a*c)^(1/2)/(a*c)^(1/2)))*(c*(b*x+a))^(1/2)/(c*x)^(1/2)/(x*(b*x+a))^(
1/2)*x
```

Fricas [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 315, normalized size of antiderivative = 1.81

$$\int \frac{1}{\sqrt{cx}\,(ax+bx^2)^{5/2}}\,dx = \left[\frac{105\,(b^4x^5+2\,ab^3x^4+a^2b^2x^3)\sqrt{ac}\log\left(-\frac{bcx^2+2\,acx-2\,\sqrt{bx^2+ax}\sqrt{ac}\sqrt{cx}}{x^2}\right)+2\,(105\,a}{24\,(a^5b^2cx^5+2\,a^6bcx^4+a^7cx}\right.$$

```
integrate(1/(c*x)^(1/2)/(b*x^2+a*x)^(5/2),x, algorithm="fricas")
```

```
[1/24*(105*(b^4*x^5 + 2*a*b^3*x^4 + a^2*b^2*x^3)*sqrt(a*c)*log(-(b*c*x^2 +
 2*a*c*x - 2*sqrt(b*x^2 + a*x)*sqrt(a*c)*sqrt(c*x))/x^2) + 2*(105*a*b^3*x^
3 + 140*a^2*b^2*x^2 + 21*a^3*b*x - 6*a^4)*sqrt(b*x^2 + a*x)*sqrt(c*x))/(a^
5*b^2*c*x^5 + 2*a^6*b*c*x^4 + a^7*c*x^3), 1/12*(105*(b^4*x^5 + 2*a*b^3*x^4
 + a^2*b^2*x^3)*sqrt(-a*c)*arctan(sqrt(b*x^2 + a*x)*sqrt(-a*c)*sqrt(c*x)/(
a*c*x)) + (105*a*b^3*x^3 + 140*a^2*b^2*x^2 + 21*a^3*b*x - 6*a^4)*sqrt(b*x^
2 + a*x)*sqrt(c*x))/(a^5*b^2*c*x^5 + 2*a^6*b*c*x^4 + a^7*c*x^3)]
```

Sympy [F]

$$\int \frac{1}{\sqrt{cx}\,(ax+bx^2)^{5/2}}\,dx = \int \frac{1}{\sqrt{cx}\,(x\,(a+bx))^{\frac{5}{2}}}\,dx$$

```
integrate(1/(c*x)**(1/2)/(b*x**2+a*x)**(5/2),x)
```

```
Integral(1/(sqrt(c*x)*(x*(a + b*x))**(5/2)), x)
```

Maxima [F]

$$\int \frac{1}{\sqrt{cx}\,(ax+bx^2)^{5/2}}\,dx = \int \frac{1}{(bx^2+ax)^{\frac{5}{2}}\sqrt{cx}}\,dx$$

```
integrate(1/(c*x)^(1/2)/(b*x^2+a*x)^(5/2),x, algorithm="maxima")
```

```
integrate(1/((b*x^2 + a*x)^(5/2)*sqrt(c*x)), x)
```

Giac [A] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 138, normalized size of antiderivative = 0.79

$$\int \frac{1}{\sqrt{cx}\,(ax+bx^2)^{5/2}}\,dx = \frac{1}{12}c^5\left(\frac{105\,b^2\arctan\left(\frac{\sqrt{bcx+ac}}{\sqrt{-ac}}\right)}{\sqrt{-ac}a^4c^4|c|} + \frac{8\,(ab^2c+9\,(bcx+ac)b^2)}{(bcx+ac)^{\frac{3}{2}}a^4c^4|c|} - \frac{3\left(13\sqrt{bcx+ac}\right.}{a}\right.$$

```
integrate(1/(c*x)^(1/2)/(b*x^2+a*x)^(5/2),x, algorithm="giac")
```

```
1/12*c^5*(105*b^2*arctan(sqrt(b*c*x + a*c)/sqrt(-a*c))/(sqrt(-a*c)*a^4*c^4
*abs(c)) + 8*(a*b^2*c + 9*(b*c*x + a*c)*b^2)/((b*c*x + a*c)^(3/2)*a^4*c^4*
abs(c)) - 3*(13*sqrt(b*c*x + a*c)*a*b^2*c - 11*(b*c*x + a*c)^(3/2)*b^2)/(a
^4*b^2*c^6*x^2*abs(c)))
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{\sqrt{cx}\,(ax+bx^2)^{5/2}}\,dx = \int \frac{1}{(b\,x^2+a\,x)^{5/2}\,\sqrt{c\,x}}\,dx$$

```
int(1/((a*x + b*x^2)^(5/2)*(c*x)^(1/2)),x)
```

```
int(1/((a*x + b*x^2)^(5/2)*(c*x)^(1/2)), x)
```

Reduce [B] (verification not implemented)

Time = 0.54 (sec) , antiderivative size = 171, normalized size of antiderivative = 0.98

$$\int \frac{1}{\sqrt{cx}\,(ax+bx^2)^{5/2}}\,dx = \frac{\sqrt{c}\,\big(105\sqrt{a}\,\sqrt{bx+a}\,\log\big(\sqrt{bx+a}-\sqrt{a}\big)\,a\,b^2x^2+105\sqrt{a}\,\sqrt{bx+a}\,\log\big(\sqrt{bx+a}$$

```
int(1/(c*x)^(1/2)/(b*x^2+a*x)^(5/2),x)
```

```
(sqrt(c)*(105*sqrt(a)*sqrt(a + b*x)*log(sqrt(a + b*x) - sqrt(a))*a*b**2*x*
*2 + 105*sqrt(a)*sqrt(a + b*x)*log(sqrt(a + b*x) - sqrt(a))*b**3*x**3 - 10
5*sqrt(a)*sqrt(a + b*x)*log(sqrt(a + b*x) + sqrt(a))*a*b**2*x**2 - 105*sqr
t(a)*sqrt(a + b*x)*log(sqrt(a + b*x) + sqrt(a))*b**3*x**3 - 12*a**4 + 42*a
**3*b*x + 280*a**2*b**2*x**2 + 210*a*b**3*x**3))/(24*sqrt(a + b*x)*a**5*c*
x**2*(a + b*x))
```

3.120 $\int \frac{1}{(cx)^{3/2}(ax+bx^2)^{5/2}}\,dx$

Optimal result

Integrand size = 21, antiderivative size = 207

$$
\begin{aligned}
\int \frac{1}{(cx)^{3/2}\left(ax+bx^2\right)^{5/2}}\,dx &= -\frac{1}{3a(cx)^{3/2}\left(ax+bx^2\right)^{3/2}} \\
&+ \frac{3b}{4a^2c\sqrt{cx}\left(ax+bx^2\right)^{3/2}} - \frac{21b^2\sqrt{cx}}{8a^3c^2\left(ax+bx^2\right)^{3/2}} - \frac{35b^3(cx)^{3/2}}{8a^4c^3\left(ax+bx^2\right)^{3/2}} \\
&- \frac{105b^3\sqrt{cx}}{8a^5c^2\sqrt{ax+bx^2}} + \frac{105b^3\operatorname{arctanh}\left(\frac{\sqrt{c}\sqrt{ax+bx^2}}{\sqrt{a}\sqrt{cx}}\right)}{8a^{11/2}c^{3/2}}
\end{aligned}
$$

```
-1/3/a/(c*x)^(3/2)/(b*x^2+a*x)^(3/2)+3/4*b/a^2/c/(c*x)^(1/2)/(b*x^2+a*x)^(
3/2)-21/8*b^2*(c*x)^(1/2)/a^3/c^2/(b*x^2+a*x)^(3/2)-35/8*b^3*(c*x)^(3/2)/a
^4/c^3/(b*x^2+a*x)^(3/2)-105/8*b^3*(c*x)^(1/2)/a^5/c^2/(b*x^2+a*x)^(1/2)+1
05/8*b^3*arctanh(c^(1/2)*(b*x^2+a*x)^(1/2)/a^(1/2)/(c*x)^(1/2))/a^(11/2)/c
^(3/2)
```

Mathematica [A] (verified)

Time = 0.14 (sec) , antiderivative size = 109, normalized size of antiderivative = 0.53

$$\int \frac{1}{(cx)^{3/2} (ax+bx^2)^{5/2}} \, dx = \frac{-\sqrt{a}(8a^4 - 18a^3bx + 63a^2b^2x^2 + 420ab^3x^3 + 315b^4x^4) + 315b^3x^3(a+bx)^{3/2}}{24a^{11/2}(cx)^{3/2}(x(a+bx))^{3/2}}$$

```
Integrate[1/((c*x)^(3/2)*(a*x + b*x^2)^(5/2)),x]
```

```
(-(Sqrt[a]*(8*a^4 - 18*a^3*b*x + 63*a^2*b^2*x^2 + 420*a*b^3*x^3 + 315*b^4*
x^4)) + 315*b^3*x^3*(a + b*x)^(3/2)*ArcTanh[Sqrt[a + b*x]/Sqrt[a]])/(24*a^
(11/2)*(c*x)^(3/2)*(x*(a + b*x))^(3/2))
```

Rubi [A] (verified)

Time = 0.60 (sec) , antiderivative size = 221, normalized size of antiderivative = 1.07, number of steps used = 8, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}} = 0.333$, Rules used = {1135, 1135, 1132, 1135, 1132, 1136, 221}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{(cx)^{3/2} (ax+bx^2)^{5/2}} dx$$

$$\Big\downarrow \text{ 1135}$$

$$-\frac{3b \int \frac{1}{\sqrt{cx}(bx^2+ax)^{5/2}}dx}{2ac} - \frac{1}{3a(cx)^{3/2} (ax+bx^2)^{3/2}}$$

$$\Big\downarrow \text{ 1135}$$

$$-\frac{3b \left(-\frac{7b \int \frac{\sqrt{cx}}{(bx^2+ax)^{5/2}}dx}{4ac} - \frac{1}{2a\sqrt{cx}(ax+bx^2)^{3/2}}\right)}{2ac} - \frac{1}{3a(cx)^{3/2} (ax+bx^2)^{3/2}}$$

$$\Big\downarrow \text{ 1132}$$

$$-\frac{3b\left(-\frac{7b\left(\frac{5c\int \frac{1}{\sqrt{cx}\left(bx^2+ax\right)^{3/2}}dx}{3a}+\frac{2\sqrt{cx}}{3a\left(ax+bx^2\right)^{3/2}}\right)}{4ac}-\frac{1}{2a\sqrt{cx}(ax+bx^2)^{3/2}}\right)}{2ac}-\frac{1}{3a(cx)^{3/2}\left(ax+bx^2\right)^{3/2}}$$

$$\Big\downarrow \; 1135$$

$$-\frac{3b\left(-\frac{7b\left(\frac{5c\left(-\frac{3b\int \frac{\sqrt{cx}}{\left(bx^2+ax\right)^{3/2}}dx}{2ac}-\frac{1}{a\sqrt{cx}\sqrt{ax+bx^2}}\right)}{3a}+\frac{2\sqrt{cx}}{3a\left(ax+bx^2\right)^{3/2}}\right)}{4ac}-\frac{1}{2a\sqrt{cx}(ax+bx^2)^{3/2}}\right)}{2ac}-\frac{1}{3a(cx)^{3/2}\left(ax+bx^2\right)^{3/2}}$$

$$\Big\downarrow \; 1132$$

$$-\frac{3b\left(-\frac{7b\left(\frac{5c\left(-\frac{3b\left(\frac{c\int \frac{1}{\sqrt{cx}\sqrt{bx^2+ax}}dx}{a}+\frac{2\sqrt{cx}}{a\sqrt{ax+bx^2}}\right)}{2ac}-\frac{1}{a\sqrt{cx}\sqrt{ax+bx^2}}\right)}{3a}+\frac{2\sqrt{cx}}{3a\left(ax+bx^2\right)^{3/2}}\right)}{4ac}-\frac{1}{2a\sqrt{cx}(ax+bx^2)^{3/2}}\right)}{2ac}-\frac{1}{3a(cx)^{3/2}\left(ax+bx^2\right)^{3/2}}$$

$$\Big\downarrow \text{1136}$$

$$-\frac{3b\left(-\frac{7b\left(\frac{5c\left(-\frac{3b\left(\frac{2c^2\int \frac{1}{\frac{c\left(bx^2+ax\right)}{x}-ac}d\frac{\sqrt{bx^2+ax}}{\sqrt{cx}}}{a}+\frac{2\sqrt{cx}}{a\sqrt{ax+bx^2}}\right)}{2ac}-\frac{1}{a\sqrt{cx}\sqrt{ax+bx^2}}\right)}{3a}+\frac{2\sqrt{cx}}{3a\left(ax+bx^2\right)^{3/2}}\right)}{4ac}-\frac{1}{2a\sqrt{cx}(ax+bx^2)^{3/2}}\right)}{2ac}-\frac{1}{3a(cx)^{3/2}\left(ax+bx^2\right)^{3/2}}$$

$$\Big\downarrow \text{221}$$

$$-\frac{3b\left(-\frac{7b\left(\frac{5c\left(-\frac{3b\left(\frac{2\sqrt{cx}}{a\sqrt{ax+bx^2}}-\frac{2\sqrt{c}\,\text{arctanh}\left(\frac{\sqrt{c}\sqrt{ax+bx^2}}{\sqrt{a}\sqrt{cx}}\right)}{a^{3/2}}\right)}{2ac}-\frac{1}{a\sqrt{cx}\sqrt{ax+bx^2}}\right)}{3a}+\frac{2\sqrt{cx}}{3a\left(ax+bx^2\right)^{3/2}}\right)}{4ac}-\frac{1}{2a\sqrt{cx}(ax+bx^2)^{3/2}}\right)}{2ac}-\frac{1}{3a(cx)^{3/2}\left(ax+bx^2\right)^{3/2}}$$

```
Int[1/((c*x)^(3/2)*(a*x + b*x^2)^(5/2)),x]
```

```
-1/3*1/(a*(c*x)^(3/2)*(a*x + b*x^2)^(3/2)) - (3*b*(-1/2*1/(a*Sqrt[c*x]*(a*
x + b*x^2)^(3/2)) - (7*b*((2*Sqrt[c*x])/(3*a*(a*x + b*x^2)^(3/2)) + (5*c*(
-(1/(a*Sqrt[c*x]*Sqrt[a*x + b*x^2])) - (3*b*((2*Sqrt[c*x])/(a*Sqrt[a*x + b
*x^2]) - (2*Sqrt[c]*ArcTanh[(Sqrt[c]*Sqrt[a*x + b*x^2])/(Sqrt[a]*Sqrt[c*x]
)])/a^(3/2)))/(2*a*c)))/(3*a)))/(4*a*c)))/(2*a*c)
```

Defintions of rubi rules used

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] :> Simp[(2*c*d - b*e)*(d + e*x)^m*((a + b*x + c*x^2)^(p + 1)/(e*(p +
 1)*(b^2 - 4*a*c))), x] - Simp[(2*c*d - b*e)*((m + 2*p + 2)/((p + 1)*(b^2 -
 4*a*c)))   Int[(d + e*x)^(m - 1)*(a + b*x + c*x^2)^(p + 1), x], x] /; Free
Q[{a, b, c, d, e}, x] && EqQ[c*d^2 - b*d*e + a*e^2, 0] && LtQ[p, -1] && LtQ
[0, m, 1] && IntegerQ[2*p]
```

```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] :> Simp[(-e)*(d + e*x)^m*((a + b*x + c*x^2)^(p + 1)/((m + p + 1)*(2*
c*d - b*e))), x] + Simp[c*((m + 2*p + 2)/((m + p + 1)*(2*c*d - b*e)))   Int
[(d + e*x)^(m + 1)*(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, p},
 x] && EqQ[c*d^2 - b*d*e + a*e^2, 0] && LtQ[m, 0] && NeQ[m + p + 1, 0] && I
ntegerQ[2*p]
```

```
Int[1/(Sqrt[(d_.) + (e_.)*(x_)]*Sqrt[(a_.) + (b_.)*(x_) + (c_.)*(x_)^2]), x
_Symbol] :> Simp[2*e   Subst[Int[1/(2*c*d - b*e + e^2*x^2), x], x, Sqrt[a +
 b*x + c*x^2]/Sqrt[d + e*x]], x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[c*d^2
- b*d*e + a*e^2, 0]
```

Maple [A] (verified)

Time = 0.36 (sec) , antiderivative size = 137, normalized size of antiderivative = 0.66

method	result
risch	$-\frac{(bx+a)\left(123b^2x^2-34abx+8a^2\right)}{24a^5x^2c\sqrt{cx}\,\sqrt{x(bx+a)}}-\frac{b^3\left(\frac{128}{\sqrt{cbx+ac}}+\frac{32ac}{3(cbx+ac)^{\frac{3}{2}}}-\frac{210\,\mathrm{arctanh}\left(\frac{\sqrt{cbx+ac}}{\sqrt{ac}}\right)}{\sqrt{ac}}\right)\sqrt{c(bx+a)}\,x}{16a^5c\sqrt{cx}\,\sqrt{x(bx+a)}}$
default	$\frac{\sqrt{x(bx+a)}\left(315\,\mathrm{arctanh}\left(\frac{\sqrt{c(bx+a)}}{\sqrt{ac}}\right)b^4x^4\sqrt{c(bx+a)}+315\,\mathrm{arctanh}\left(\frac{\sqrt{c(bx+a)}}{\sqrt{ac}}\right)a\,b^3x^3\sqrt{c(bx+a)}-315\sqrt{ac}\,b^4x^4-420\sqrt{ac}\,a\,b^3x^3-\right.}{24c\,x^3\sqrt{cx}\,(bx+a)^2a^5\sqrt{ac}}$

```
int(1/(c*x)^(3/2)/(b*x^2+a*x)^(5/2),x,method=_RETURNVERBOSE)
```

```
-1/24*(b*x+a)*(123*b^2*x^2-34*a*b*x+8*a^2)/a^5/x^2/c/(c*x)^(1/2)/(x*(b*x+a
))^(1/2)-1/16*b^3/a^5*(128/(b*c*x+a*c)^(1/2)+32/3*a*c/(b*c*x+a*c)^(3/2)-21
0/(a*c)^(1/2)*arctanh((b*c*x+a*c)^(1/2)/(a*c)^(1/2)))/c*(c*(b*x+a))^(1/2)/
(c*x)^(1/2)/(x*(b*x+a))^(1/2)*x
```

Fricas [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 349, normalized size of antiderivative = 1.69

$$\int \frac{1}{(cx)^{3/2}\left(ax+bx^2\right)^{5/2}}\,dx = \left[\frac{315\left(b^5x^6+2\,ab^4x^5+a^2b^3x^4\right)\sqrt{ac}\log\left(-\frac{bcx^2+2\,acx+2\,\sqrt{bx^2+ax}\sqrt{ac}\sqrt{cx}}{x^2}\right)-2\,(31}{48\left(a^6b^2c^2x^6+2\,a^7bc^2}\right.$$

```
integrate(1/(c*x)^(3/2)/(b*x^2+a*x)^(5/2),x, algorithm="fricas")
```

```
[1/48*(315*(b^5*x^6 + 2*a*b^4*x^5 + a^2*b^3*x^4)*sqrt(a*c)*log(-(b*c*x^2 +
 2*a*c*x + 2*sqrt(b*x^2 + a*x)*sqrt(a*c)*sqrt(c*x))/x^2) - 2*(315*a*b^4*x^
4 + 420*a^2*b^3*x^3 + 63*a^3*b^2*x^2 - 18*a^4*b*x + 8*a^5)*sqrt(b*x^2 + a*
x)*sqrt(c*x))/(a^6*b^2*c^2*x^6 + 2*a^7*b*c^2*x^5 + a^8*c^2*x^4), -1/24*(31
5*(b^5*x^6 + 2*a*b^4*x^5 + a^2*b^3*x^4)*sqrt(-a*c)*arctan(sqrt(b*x^2 + a*x
)*sqrt(-a*c)*sqrt(c*x)/(a*c*x)) + (315*a*b^4*x^4 + 420*a^2*b^3*x^3 + 63*a^
3*b^2*x^2 - 18*a^4*b*x + 8*a^5)*sqrt(b*x^2 + a*x)*sqrt(c*x))/(a^6*b^2*c^2*
x^6 + 2*a^7*b*c^2*x^5 + a^8*c^2*x^4)]
```

Sympy [F]

$$\int \frac{1}{(cx)^{3/2}\left(ax+bx^2\right)^{5/2}}\,dx = \int \frac{1}{(cx)^{\frac{3}{2}}\left(x\left(a+bx\right)\right)^{\frac{5}{2}}}\,dx$$

```
integrate(1/(c*x)**(3/2)/(b*x**2+a*x)**(5/2),x)
```

```
Integral(1/((c*x)**(3/2)*(x*(a + b*x))**(5/2)), x)
```

Maxima [F]

$$\int \frac{1}{(cx)^{3/2}\,(ax+bx^2)^{5/2}}\,dx = \int \frac{1}{(bx^2+ax)^{\frac{5}{2}}\,(cx)^{\frac{3}{2}}}\,dx$$

```
integrate(1/(c*x)^(3/2)/(b*x^2+a*x)^(5/2),x, algorithm="maxima")
```

```
integrate(1/((b*x^2 + a*x)^(5/2)*(c*x)^(3/2)), x)
```

Giac [A] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 168, normalized size of antiderivative = 0.81

$$\int \frac{1}{(cx)^{3/2}\,(ax+bx^2)^{5/2}}\,dx = -\frac{1}{24}\,c^5\left(\frac{315\,b^3\arctan\left(\frac{\sqrt{bcx+ac}}{\sqrt{-ac}}\right)}{\sqrt{-ac}a^5c^5|c|} + \frac{16\,a^4b^3c^4+144\,(bcx+ac)a^3b^3c^3-693\,(bcx+ac)^2a^2b^3c^2+840\,(bcx+}{\left(\sqrt{bcx+ac}ac-(bcx+ac)^{\frac{3}{2}}\right)^3a^5c^5|c|}\right.$$

```
integrate(1/(c*x)^(3/2)/(b*x^2+a*x)^(5/2),x, algorithm="giac")
```

```
-1/24*c^5*(315*b^3*arctan(sqrt(b*c*x + a*c)/sqrt(-a*c))/(sqrt(-a*c)*a^5*c^
5*abs(c)) + (16*a^4*b^3*c^4 + 144*(b*c*x + a*c)*a^3*b^3*c^3 - 693*(b*c*x +
 a*c)^2*a^2*b^3*c^2 + 840*(b*c*x + a*c)^3*a*b^3*c - 315*(b*c*x + a*c)^4*b^
3)/((sqrt(b*c*x + a*c)*a*c - (b*c*x + a*c)^(3/2))^3*a^5*c^5*abs(c)))
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{(cx)^{3/2}\,(ax+bx^2)^{5/2}}\,dx = \int \frac{1}{\left(b\,x^2+a\,x\right)^{5/2}\,\left(c\,x\right)^{3/2}}\,dx$$

```
int(1/((a*x + b*x^2)^(5/2)*(c*x)^(3/2)),x)
```

```
int(1/((a*x + b*x^2)^(5/2)*(c*x)^(3/2)), x)
```

Reduce [B] (verification not implemented)

Time = 0.40 (sec) , antiderivative size = 182, normalized size of antiderivative = 0.88

$$\int \frac{1}{(cx)^{3/2}\,(ax+bx^2)^{5/2}}\,dx = \frac{\sqrt{c}\left(-315\sqrt{a}\,\sqrt{bx+a}\,\log\left(\sqrt{bx+a}-\sqrt{a}\right)a\,b^3x^3-315\sqrt{a}\,\sqrt{bx+a}\,\log\left(\sqrt{bx\right.\right.}{}$$

```
int(1/(c*x)^(3/2)/(b*x^2+a*x)^(5/2),x)
```

```
(sqrt(c)*( - 315*sqrt(a)*sqrt(a + b*x)*log(sqrt(a + b*x) - sqrt(a))*a*b**3
*x**3 - 315*sqrt(a)*sqrt(a + b*x)*log(sqrt(a + b*x) - sqrt(a))*b**4*x**4 +
 315*sqrt(a)*sqrt(a + b*x)*log(sqrt(a + b*x) + sqrt(a))*a*b**3*x**3 + 315*
sqrt(a)*sqrt(a + b*x)*log(sqrt(a + b*x) + sqrt(a))*b**4*x**4 - 16*a**5 + 3
6*a**4*b*x - 126*a**3*b**2*x**2 - 840*a**2*b**3*x**3 - 630*a*b**4*x**4))/(
48*sqrt(a + b*x)*a**6*c**2*x**3*(a + b*x))
```

3.121 $\int \frac{(cx)^{7/3}}{\sqrt[3]{ax+bx^2}}\,dx$

Optimal result

Integrand size = 21, antiderivative size = 95

$$\int \frac{(cx)^{7/3}}{\sqrt[3]{ax+bx^2}}\,dx = \frac{3a^2c^3(ax+bx^2)^{2/3}}{2b^3(cx)^{2/3}} - \frac{6ac^4(ax+bx^2)^{5/3}}{5b^3(cx)^{5/3}} + \frac{3c^5(ax+bx^2)^{8/3}}{8b^3(cx)^{8/3}}$$

```
3/2*a^2*c^3*(b*x^2+a*x)^(2/3)/b^3/(c*x)^(2/3)-6/5*a*c^4*(b*x^2+a*x)^(5/3)/
b^3/(c*x)^(5/3)+3/8*c^5*(b*x^2+a*x)^(8/3)/b^3/(c*x)^(8/3)
```

Mathematica [A] (verified)

Time = 0.03 (sec) , antiderivative size = 47, normalized size of antiderivative = 0.49

$$\int \frac{(cx)^{7/3}}{\sqrt[3]{ax+bx^2}}\,dx = \frac{3c^3(x(a+bx))^{2/3}\,(9a^2-6abx+5b^2x^2)}{40b^3(cx)^{2/3}}$$

```
Integrate[(c*x)^(7/3)/(a*x + b*x^2)^(1/3),x]
```

```
(3*c^3*(x*(a + b*x))^(2/3)*(9*a^2 - 6*a*b*x + 5*b^2*x^2))/(40*b^3*(c*x)^(2
/3))
```

Rubi [A] (verified)

Time = 0.35 (sec) , antiderivative size = 98, normalized size of antiderivative = 1.03, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.143, Rules used = {1128, 1128, 1122}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(cx)^{7/3}}{\sqrt[3]{ax+bx^2}}dx$$

$$\Big\downarrow \text{1128}$$

$$\frac{3c(cx)^{4/3}\left(ax+bx^2\right)^{2/3}}{8b}-\frac{3ac\int \frac{(cx)^{4/3}}{\sqrt[3]{bx^2+ax}}dx}{4b}$$

$$\Big\downarrow \text{1128}$$

$$\frac{3c(cx)^{4/3}\left(ax+bx^2\right)^{2/3}}{8b}-\frac{3ac\left(\frac{3c\sqrt[3]{cx}(ax+bx^2)^{2/3}}{5b}-\frac{3ac\int \frac{\sqrt[3]{cx}}{\sqrt[3]{bx^2+ax}}dx}{5b}\right)}{4b}$$

$$\Big\downarrow \text{1122}$$

$$\frac{3c(cx)^{4/3}\left(ax+bx^2\right)^{2/3}}{8b}-\frac{3ac\left(\frac{3c\sqrt[3]{cx}(ax+bx^2)^{2/3}}{5b}-\frac{9ac^2(ax+bx^2)^{2/3}}{10b^2(cx)^{2/3}}\right)}{4b}$$

```
Int[(c*x)^(7/3)/(a*x + b*x^2)^(1/3),x]
```

```
(3*c*(c*x)^(4/3)*(a*x + b*x^2)^(2/3))/(8*b) - (3*a*c*((-9*a*c^2*(a*x + b*x
^2)^(2/3))/(10*b^2*(c*x)^(2/3)) + (3*c*(c*x)^(1/3)*(a*x + b*x^2)^(2/3))/(5
*b)))/(4*b)
```

Defintions of rubi rules used

```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] :> Simp[e*(d + e*x)^(m - 1)*((a + b*x + c*x^2)^(p + 1)/(c*(p + 1))),
 x] /; FreeQ[{a, b, c, d, e, m, p}, x] && EqQ[c*d^2 - b*d*e + a*e^2, 0] &&
EqQ[m + p, 0]
```

```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] :> Simp[e*(d + e*x)^(m - 1)*((a + b*x + c*x^2)^(p + 1)/(c*(m + 2*p +
 1))), x] + Simp[Simplify[m + p]*((2*c*d - b*e)/(c*(m + 2*p + 1)))   Int[(d
 + e*x)^(m - 1)*(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, m, p},
 x] && EqQ[c*d^2 - b*d*e + a*e^2, 0] && IGtQ[Simplify[m + p], 0]
```

Maple [A] (verified)

Time = 0.33 (sec) , antiderivative size = 47, normalized size of antiderivative = 0.49

method	result	size
risch	$\frac{3c^{2}\left(cx\right)^{\frac{1}{3}}\left(5b^{2}x^{2}-6abx+9a^{2}\right)\left(bx+a\right)}{40\left(x\left(bx+a\right)\right)^{\frac{1}{3}}b^{3}}$	47
gosper	$\frac{3\left(bx+a\right)\left(5b^{2}x^{2}-6abx+9a^{2}\right)\left(cx\right)^{\frac{7}{3}}}{40b^{3}x^{2}\left(b\,x^{2}+ax\right)^{\frac{1}{3}}}$	49
orering	$\frac{3\left(bx+a\right)\left(5b^{2}x^{2}-6abx+9a^{2}\right)\left(cx\right)^{\frac{7}{3}}}{40b^{3}x^{2}\left(b\,x^{2}+ax\right)^{\frac{1}{3}}}$	49

```
int((c*x)^(7/3)/(b*x^2+a*x)^(1/3),x,method=_RETURNVERBOSE)
```

```
3/40*c^2*(c*x)^(1/3)/(x*(b*x+a))^(1/3)*(5*b^2*x^2-6*a*b*x+9*a^2)*(b*x+a)/b
^3
```

Fricas [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 52, normalized size of antiderivative = 0.55

$$\int \frac{(cx)^{7/3}}{\sqrt[3]{ax+bx^2}}\,dx = \frac{3\,(5\,b^2c^2x^2-6\,abc^2x+9\,a^2c^2)(bx^2+ax)^{\frac{2}{3}}(cx)^{\frac{1}{3}}}{40\,b^3x}$$

```
integrate((c*x)^(7/3)/(b*x^2+a*x)^(1/3),x, algorithm="fricas")
```

```
3/40*(5*b^2*c^2*x^2 - 6*a*b*c^2*x + 9*a^2*c^2)*(b*x^2 + a*x)^(2/3)*(c*x)^(
1/3)/(b^3*x)
```

Sympy [F]

$$\int \frac{(cx)^{7/3}}{\sqrt[3]{ax+bx^2}}\,dx = \int \frac{(cx)^{\frac{7}{3}}}{\sqrt[3]{x\,(a+bx)}}\,dx$$

```
integrate((c*x)**(7/3)/(b*x**2+a*x)**(1/3),x)
```

```
Integral((c*x)**(7/3)/(x*(a + b*x))**(1/3), x)
```

Maxima [A] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 54, normalized size of antiderivative = 0.57

$$\int \frac{(cx)^{7/3}}{\sqrt[3]{ax+bx^2}}\,dx = \frac{3\left(5\,b^3c^{\frac{7}{3}}x^3-ab^2c^{\frac{7}{3}}x^2+3\,a^2bc^{\frac{7}{3}}x+9\,a^3c^{\frac{7}{3}}\right)}{40\,(bx+a)^{\frac{1}{3}}b^3}$$

```
integrate((c*x)^(7/3)/(b*x^2+a*x)^(1/3),x, algorithm="maxima")
```

```
3/40*(5*b^3*c^(7/3)*x^3 - a*b^2*c^(7/3)*x^2 + 3*a^2*b*c^(7/3)*x + 9*a^3*c^
(7/3))/((b*x + a)^(1/3)*b^3)
```

Giac [F]

$$\int \frac{(cx)^{7/3}}{\sqrt[3]{ax+bx^2}}\,dx = \int \frac{(cx)^{\frac{7}{3}}}{(bx^2+ax)^{\frac{1}{3}}}\,dx$$

```
integrate((c*x)^(7/3)/(b*x^2+a*x)^(1/3),x, algorithm="giac")
```

```
integrate((c*x)^(7/3)/(b*x^2 + a*x)^(1/3), x)
```

Mupad [B] (verification not implemented)

Time = 10.24 (sec) , antiderivative size = 46, normalized size of antiderivative = 0.48

$$\int \frac{(cx)^{7/3}}{\sqrt[3]{ax+bx^2}}\,dx = \frac{3\,c^2\,(b\,x^2+a\,x)^{2/3}\,(c\,x)^{1/3}\,(9\,a^2-6\,a\,b\,x+5\,b^2\,x^2)}{40\,b^3\,x}$$

```
int((c*x)^(7/3)/(a*x + b*x^2)^(1/3),x)
```

```
(3*c^2*(a*x + b*x^2)^(2/3)*(c*x)^(1/3)*(9*a^2 + 5*b^2*x^2 - 6*a*b*x))/(40*
b^3*x)
```

Reduce [B] (verification not implemented)

Time = 0.34 (sec) , antiderivative size = 34, normalized size of antiderivative = 0.36

$$\int \frac{(cx)^{7/3}}{\sqrt[3]{ax+bx^2}}\,dx = \frac{3c^{\frac{7}{3}}(bx+a)^{\frac{2}{3}}\,(5b^2x^2-6abx+9a^2)}{40b^3}$$

```
int((c*x)^(7/3)/(b*x^2+a*x)^(1/3),x)
```

```
(3*c**(1/3)*(a + b*x)**(2/3)*c**2*(9*a**2 - 6*a*b*x + 5*b**2*x**2))/(40*b*
*3)
```

3.122 $\int \frac{(cx)^{4/3}}{\sqrt[3]{ax+bx^2}}\,dx$

Optimal result

Integrand size = 21, antiderivative size = 62

$$\int \frac{(cx)^{4/3}}{\sqrt[3]{ax+bx^2}}\,dx=-\frac{3ac^2(ax+bx^2)^{2/3}}{2b^2(cx)^{2/3}}+\frac{3c^3(ax+bx^2)^{5/3}}{5b^2(cx)^{5/3}}$$

```
-3/2*a*c^2*(b*x^2+a*x)^(2/3)/b^2/(c*x)^(2/3)+3/5*c^3*(b*x^2+a*x)^(5/3)/b^2
/(c*x)^(5/3)
```

Mathematica [A] (verified)

Time = 0.03 (sec) , antiderivative size = 36, normalized size of antiderivative = 0.58

$$\int \frac{(cx)^{4/3}}{\sqrt[3]{ax+bx^2}}\,dx=\frac{3c^2(x(a+bx))^{2/3}(-3a+2bx)}{10b^2(cx)^{2/3}}$$

```
Integrate[(c*x)^(4/3)/(a*x + b*x^2)^(1/3),x]
```

```
(3*c^2*(x*(a + b*x))^(2/3)*(-3*a + 2*b*x))/(10*b^2*(c*x)^(2/3))
```

Rubi [A] (verified)

Time = 0.29 (sec) , antiderivative size = 60, normalized size of antiderivative = 0.97, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.095, Rules used = {1128, 1122}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(cx)^{4/3}}{\sqrt[3]{ax+bx^2}}dx$$
$$\Big\downarrow \text{1128}$$
$$\frac{3c\sqrt[3]{cx}\left(ax+bx^2\right)^{2/3}}{5b}-\frac{3ac\int \frac{\sqrt[3]{cx}}{\sqrt[3]{bx^2+ax}}dx}{5b}$$
$$\Big\downarrow \text{1122}$$
$$\frac{3c\sqrt[3]{cx}\left(ax+bx^2\right)^{2/3}}{5b}-\frac{9ac^2\left(ax+bx^2\right)^{2/3}}{10b^2(cx)^{2/3}}$$

```
Int[(c*x)^(4/3)/(a*x + b*x^2)^(1/3),x]
```

```
(-9*a*c^2*(a*x + b*x^2)^(2/3))/(10*b^2*(c*x)^(2/3)) + (3*c*(c*x)^(1/3)*(a*
x + b*x^2)^(2/3))/(5*b)
```

Defintions of rubi rules used

```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] :> Simp[e*(d + e*x)^(m - 1)*((a + b*x + c*x^2)^(p + 1)/(c*(p + 1))),
 x] /; FreeQ[{a, b, c, d, e, m, p}, x] && EqQ[c*d^2 - b*d*e + a*e^2, 0] &&
EqQ[m + p, 0]
```

```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] :> Simp[e*(d + e*x)^(m - 1)*((a + b*x + c*x^2)^(p + 1)/(c*(m + 2*p +
 1))), x] + Simp[Simplify[m + p]*((2*c*d - b*e)/(c*(m + 2*p + 1)))   Int[(d
 + e*x)^(m - 1)*(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, m, p},
 x] && EqQ[c*d^2 - b*d*e + a*e^2, 0] && IGtQ[Simplify[m + p], 0]
```

Maple [A] (verified)

Time = 0.33 (sec) , antiderivative size = 34, normalized size of antiderivative = 0.55

method	result	size
risch	$-\frac{3c(cx)^{\frac{1}{3}}(-2bx+3a)(bx+a)}{10(x(bx+a))^{\frac{1}{3}}b^{2}}$	34
gosper	$-\frac{3(bx+a)(-2bx+3a)(cx)^{\frac{4}{3}}}{10b^{2}x(b\,x^{2}+ax)^{\frac{1}{3}}}$	38
orering	$-\frac{3(bx+a)(-2bx+3a)(cx)^{\frac{4}{3}}}{10b^{2}x(b\,x^{2}+ax)^{\frac{1}{3}}}$	38

```
int((c*x)^(4/3)/(b*x^2+a*x)^(1/3),x,method=_RETURNVERBOSE)
```

```
-3/10*c*(c*x)^(1/3)/(x*(b*x+a))^(1/3)*(-2*b*x+3*a)*(b*x+a)/b^2
```

Fricas [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 34, normalized size of antiderivative = 0.55

$$\int \frac{(cx)^{4/3}}{\sqrt[3]{ax+bx^2}}\,dx = \frac{3\,(2\,bcx-3\,ac)(bx^2+ax)^{\frac{2}{3}}(cx)^{\frac{1}{3}}}{10\,b^2x}$$

```
integrate((c*x)^(4/3)/(b*x^2+a*x)^(1/3),x, algorithm="fricas")
```

```
3/10*(2*b*c*x - 3*a*c)*(b*x^2 + a*x)^(2/3)*(c*x)^(1/3)/(b^2*x)
```

Sympy [F]

$$\int \frac{(cx)^{4/3}}{\sqrt[3]{ax+bx^2}}\,dx = \int \frac{(cx)^{\frac{4}{3}}}{\sqrt[3]{x\,(a+bx)}}\,dx$$

```
integrate((c*x)**(4/3)/(b*x**2+a*x)**(1/3),x)
```

```
Integral((c*x)**(4/3)/(x*(a + b*x))**(1/3), x)
```

Maxima [A] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 40, normalized size of antiderivative = 0.65

$$\int \frac{(cx)^{4/3}}{\sqrt[3]{ax+bx^2}}\,dx = \frac{3\left(2\,b^2c^{\frac{4}{3}}x^2 - abc^{\frac{4}{3}}x - 3\,a^2c^{\frac{4}{3}}\right)}{10\,(bx+a)^{\frac{1}{3}}b^2}$$

```
integrate((c*x)^(4/3)/(b*x^2+a*x)^(1/3),x, algorithm="maxima")
```

```
3/10*(2*b^2*c^(4/3)*x^2 - a*b*c^(4/3)*x - 3*a^2*c^(4/3))/((b*x + a)^(1/3)*
b^2)
```

Giac [F]

$$\int \frac{(cx)^{4/3}}{\sqrt[3]{ax+bx^2}}\,dx = \int \frac{(cx)^{\frac{4}{3}}}{(bx^2+ax)^{\frac{1}{3}}}\,dx$$

```
integrate((c*x)^(4/3)/(b*x^2+a*x)^(1/3),x, algorithm="giac")
```

```
integrate((c*x)^(4/3)/(b*x^2 + a*x)^(1/3), x)
```

Mupad [B] (verification not implemented)

Time = 10.13 (sec) , antiderivative size = 33, normalized size of antiderivative = 0.53

$$\int \frac{(cx)^{4/3}}{\sqrt[3]{ax+bx^2}}\,dx = -\frac{3\,c\,(b\,x^2+a\,x)^{2/3}\,(c\,x)^{1/3}\,(3\,a-2\,b\,x)}{10\,b^2\,x}$$

```
int((c*x)^(4/3)/(a*x + b*x^2)^(1/3),x)
```

```
-(3*c*(a*x + b*x^2)^(2/3)*(c*x)^(1/3)*(3*a - 2*b*x))/(10*b^2*x)
```

Reduce [B] (verification not implemented)

Time = 0.50 (sec) , antiderivative size = 23, normalized size of antiderivative = 0.37

$$\int \frac{(cx)^{4/3}}{\sqrt[3]{ax+bx^2}}\,dx = \frac{3c^{\frac{4}{3}}(bx+a)^{\frac{2}{3}}\,(2bx-3a)}{10b^2}$$

```
int((c*x)^(4/3)/(b*x^2+a*x)^(1/3),x)
```

```
(3*c**(1/3)*(a + b*x)**(2/3)*c*( - 3*a + 2*b*x))/(10*b**2)
```

3.123 $\int \frac{\sqrt[3]{cx}}{\sqrt[3]{ax+bx^2}}\,dx$

Optimal result

Integrand size = 21, antiderivative size = 28

$$\int \frac{\sqrt[3]{cx}}{\sqrt[3]{ax+bx^2}}\,dx = \frac{3c\left(ax+bx^2\right)^{2/3}}{2b(cx)^{2/3}}$$

```
3/2*c*(b*x^2+a*x)^(2/3)/b/(c*x)^(2/3)
```

Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 26, normalized size of antiderivative = 0.93

$$\int \frac{\sqrt[3]{cx}}{\sqrt[3]{ax+bx^2}}\,dx = \frac{3c(x(a+bx))^{2/3}}{2b(cx)^{2/3}}$$

```
Integrate[(c*x)^(1/3)/(a*x + b*x^2)^(1/3),x]
```

```
(3*c*(x*(a + b*x))^(2/3))/(2*b*(c*x)^(2/3))
```

Rubi [A] (verified)

Time = 0.24 (sec) , antiderivative size = 28, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}} = 0.048$, Rules used = $\{1122\}$

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sqrt[3]{cx}}{\sqrt[3]{ax+bx^2}}\,dx$$

$$\Big\downarrow \text{1122}$$

$$\frac{3c(ax+bx^2)^{2/3}}{2b(cx)^{2/3}}$$

```
Int[(c*x)^(1/3)/(a*x + b*x^2)^(1/3),x]
```

```
(3*c*(a*x + b*x^2)^(2/3))/(2*b*(c*x)^(2/3))
```

Defintions of rubi rules used

```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] :> Simp[e*(d + e*x)^(m - 1)*((a + b*x + c*x^2)^(p + 1)/(c*(p + 1))),
 x] /; FreeQ[{a, b, c, d, e, m, p}, x] && EqQ[c*d^2 - b*d*e + a*e^2, 0] &&
EqQ[m + p, 0]
```

Maple [A] (verified)

Time = 0.33 (sec) , antiderivative size = 25, normalized size of antiderivative = 0.89

method	result	size
risch	$\frac{3(cx)^{\frac{1}{3}}(bx+a)}{2(x(bx+a))^{\frac{1}{3}}b}$	25
gosper	$\frac{3(bx+a)(cx)^{\frac{1}{3}}}{2b(b\,x^{2}+ax)^{\frac{1}{3}}}$	27
orering	$\frac{3(bx+a)(cx)^{\frac{1}{3}}}{2b(b\,x^{2}+ax)^{\frac{1}{3}}}$	27

```
int((c*x)^(1/3)/(b*x^2+a*x)^(1/3),x,method=_RETURNVERBOSE)
```

```
3/2*(c*x)^(1/3)/(x*(b*x+a))^(1/3)/b*(b*x+a)
```

Fricas [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 24, normalized size of antiderivative = 0.86

$$\int \frac{\sqrt[3]{cx}}{\sqrt[3]{ax+bx^2}}\,dx = \frac{3\,(bx^2+ax)^{\frac{2}{3}}(cx)^{\frac{1}{3}}}{2\,bx}$$

```
integrate((c*x)^(1/3)/(b*x^2+a*x)^(1/3),x, algorithm="fricas")
```

```
3/2*(b*x^2 + a*x)^(2/3)*(c*x)^(1/3)/(b*x)
```

Sympy [F]

$$\int \frac{\sqrt[3]{cx}}{\sqrt[3]{ax+bx^2}}\,dx = \int \frac{\sqrt[3]{cx}}{\sqrt[3]{x\,(a+bx)}}\,dx$$

```
integrate((c*x)**(1/3)/(b*x**2+a*x)**(1/3),x)
```

```
Integral((c*x)**(1/3)/(x*(a + b*x))**(1/3), x)
```

Maxima [A] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 24, normalized size of antiderivative = 0.86

$$\int \frac{\sqrt[3]{cx}}{\sqrt[3]{ax+bx^2}}\,dx = \frac{3\left(bc^{\frac{1}{3}}x+ac^{\frac{1}{3}}\right)}{2\,(bx+a)^{\frac{1}{3}}b}$$

```
integrate((c*x)^(1/3)/(b*x^2+a*x)^(1/3),x, algorithm="maxima")
```

```
3/2*(b*c^(1/3)*x + a*c^(1/3))/((b*x + a)^(1/3)*b)
```

Giac [F]

$$\int \frac{\sqrt[3]{cx}}{\sqrt[3]{ax+bx^2}}\,dx = \int \frac{(cx)^{\frac{1}{3}}}{(bx^2+ax)^{\frac{1}{3}}}\,dx$$

```
integrate((c*x)^(1/3)/(b*x^2+a*x)^(1/3),x, algorithm="giac")
```

```
integrate((c*x)^(1/3)/(b*x^2 + a*x)^(1/3), x)
```

Mupad [B] (verification not implemented)

Time = 10.00 (sec) , antiderivative size = 24, normalized size of antiderivative = 0.86

$$\int \frac{\sqrt[3]{cx}}{\sqrt[3]{ax+bx^2}}\,dx = \frac{3\,(b\,x^2+a\,x)^{2/3}\,(c\,x)^{1/3}}{2\,b\,x}$$

```
int((c*x)^(1/3)/(a*x + b*x^2)^(1/3),x)
```

```
(3*(a*x + b*x^2)^(2/3)*(c*x)^(1/3))/(2*b*x)
```

Reduce [B] (verification not implemented)

Time = 0.53 (sec) , antiderivative size = 15, normalized size of antiderivative = 0.54

$$\int \frac{\sqrt[3]{cx}}{\sqrt[3]{ax+bx^2}}\,dx = \frac{3c^{\frac{1}{3}}(bx+a)^{\frac{2}{3}}}{2b}$$

```
int((c*x)^(1/3)/(b*x^2+a*x)^(1/3),x)
```

```
(3*c**(1/3)*(a + b*x)**(2/3))/(2*b)
```

3.124 $\int \frac{1}{(cx)^{4/3} \sqrt[3]{ax+bx^2}}\,dx$

Optimal result

Integrand size = 21, antiderivative size = 27

$$\int \frac{1}{(cx)^{4/3} \sqrt[3]{ax+bx^2}}\,dx = -\frac{3(ax+bx^2)^{2/3}}{2a(cx)^{4/3}}$$

```
-3/2*(b*x^2+a*x)^(2/3)/a/(c*x)^(4/3)
```

Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 25, normalized size of antiderivative = 0.93

$$\int \frac{1}{(cx)^{4/3} \sqrt[3]{ax+bx^2}}\,dx = -\frac{3(x(a+bx))^{2/3}}{2a(cx)^{4/3}}$$

```
Integrate[1/((c*x)^(4/3)*(a*x + b*x^2)^(1/3)),x]
```

```
(-3*(x*(a + b*x))^(2/3))/(2*a*(c*x)^(4/3))
```

Rubi [A] (verified)

Time = 0.23 (sec) , antiderivative size = 27, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.048, Rules used = {1123}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{(cx)^{4/3}\sqrt[3]{ax+bx^2}}\,dx$$

$$\Big\downarrow \, 1123$$

$$-\frac{3(ax+bx^2)^{2/3}}{2a(cx)^{4/3}}$$

```
Int[1/((c*x)^(4/3)*(a*x + b*x^2)^(1/3)),x]
```

```
(-3*(a*x + b*x^2)^(2/3))/(2*a*(c*x)^(4/3))
```

Defintions of rubi rules used

```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] :> Simp[e*(d + e*x)^m*((a + b*x + c*x^2)^(p + 1)/((p + 1)*(2*c*d - b
*e))), x] /; FreeQ[{a, b, c, d, e, m, p}, x] && EqQ[c*d^2 - b*d*e + a*e^2,
0] && EqQ[m + 2*p + 2, 0]
```

Maple [A] (verified)

Time = 0.34 (sec) , antiderivative size = 28, normalized size of antiderivative = 1.04

method	result	size
gosper	$-\frac{3x(bx+a)}{2a(cx)^{\frac{4}{3}}(b\,x^2+ax)^{\frac{1}{3}}}$	28
risch	$-\frac{3(bx+a)}{2c(cx)^{\frac{1}{3}}(x(bx+a))^{\frac{1}{3}}a}$	28
orering	$-\frac{3x(bx+a)}{2a(cx)^{\frac{4}{3}}(b\,x^2+ax)^{\frac{1}{3}}}$	28

```
int(1/(c*x)^(4/3)/(b*x^2+a*x)^(1/3),x,method=_RETURNVERBOSE)
```

```
-3/2*x*(b*x+a)/a/(c*x)^(4/3)/(b*x^2+a*x)^(1/3)
```

Fricas [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 27, normalized size of antiderivative = 1.00

$$\int \frac{1}{(cx)^{4/3}\sqrt[3]{ax+bx^2}}\,dx = -\frac{3\,(bx^2+ax)^{\frac{2}{3}}(cx)^{\frac{2}{3}}}{2\,ac^2x^2}$$

```
integrate(1/(c*x)^(4/3)/(b*x^2+a*x)^(1/3),x, algorithm="fricas")
```

```
-3/2*(b*x^2 + a*x)^(2/3)*(c*x)^(2/3)/(a*c^2*x^2)
```

Sympy [F]

$$\int \frac{1}{(cx)^{4/3}\sqrt[3]{ax+bx^2}}\,dx = \int \frac{1}{(cx)^{\frac{4}{3}}\sqrt[3]{x\,(a+bx)}}\,dx$$

```
integrate(1/(c*x)**(4/3)/(b*x**2+a*x)**(1/3),x)
```

```
Integral(1/((c*x)**(4/3)*(x*(a + b*x))**(1/3)), x)
```

Maxima [F]

$$\int \frac{1}{(cx)^{4/3}\sqrt[3]{ax+bx^2}}\,dx = \int \frac{1}{(bx^2+ax)^{\frac{1}{3}}(cx)^{\frac{4}{3}}}\,dx$$

```
integrate(1/(c*x)^(4/3)/(b*x^2+a*x)^(1/3),x, algorithm="maxima")
```

```
integrate(1/((b*x^2 + a*x)^(1/3)*(c*x)^(4/3)), x)
```

Giac [A] (verification not implemented)

Time = 0.14 (sec) , antiderivative size = 20, normalized size of antiderivative = 0.74

$$\int \frac{1}{(cx)^{4/3}\sqrt[3]{ax+bx^2}}\,dx = -\frac{3\left(bc+\frac{ac}{x}\right)^{\frac{2}{3}}}{2\,ac^2}$$

```
integrate(1/(c*x)^(4/3)/(b*x^2+a*x)^(1/3),x, algorithm="giac")
```

```
-3/2*(b*c + a*c/x)^(2/3)/(a*c^2)
```

Mupad [B] (verification not implemented)

Time = 10.68 (sec) , antiderivative size = 27, normalized size of antiderivative = 1.00

$$\int \frac{1}{(cx)^{4/3}\sqrt[3]{ax+bx^2}}\,dx = -\frac{3\left(b\,x^2+a\,x\right)^{2/3}}{2\,a\,c\,x\,\left(c\,x\right)^{1/3}}$$

```
int(1/((a*x + b*x^2)^(1/3)*(c*x)^(4/3)),x)
```

```
-(3*(a*x + b*x^2)^(2/3))/(2*a*c*x*(c*x)^(1/3))
```

Reduce [F]

$$\int \frac{1}{(cx)^{4/3}\sqrt[3]{ax+bx^2}}\,dx=\frac{\int \frac{1}{x^{\frac{5}{3}}(bx+a)^{\frac{1}{3}}}dx}{c^{\frac{4}{3}}}$$

```
int(1/(c*x)^(4/3)/(b*x^2+a*x)^(1/3),x)
```

```
int(1/(x**(2/3)*(a + b*x)**(1/3)*x),x)/(c**(1/3)*c)
```

3.125 $\int \frac{1}{(cx)^{7/3}\sqrt[3]{ax+bx^2}}\,dx$

Optimal result

Integrand size = 21, antiderivative size = 59

$$\int \frac{1}{(cx)^{7/3}\sqrt[3]{ax+bx^2}}\,dx=-\frac{3(ax+bx^2)^{2/3}}{5a(cx)^{7/3}}+\frac{9b(ax+bx^2)^{2/3}}{10a^2c(cx)^{4/3}}$$

```
-3/5*(b*x^2+a*x)^(2/3)/a/(c*x)^(7/3)+9/10*b*(b*x^2+a*x)^(2/3)/a^2/c/(c*x)^
(4/3)
```

Mathematica [A] (verified)

Time = 0.15 (sec) , antiderivative size = 33, normalized size of antiderivative = 0.56

$$\int \frac{1}{(cx)^{7/3}\sqrt[3]{ax+bx^2}}\,dx=\frac{3(x(a+bx))^{2/3}(-2a+3bx)}{10a^2(cx)^{7/3}}$$

```
Integrate[1/((c*x)^(7/3)*(a*x + b*x^2)^(1/3)),x]
```

```
(3*(x*(a + b*x))^(2/3)*(-2*a + 3*b*x))/(10*a^2*(c*x)^(7/3))
```

Rubi [A] (verified)

Time = 0.29 (sec) , antiderivative size = 59, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.095$, Rules used = {1129, 1123}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{(cx)^{7/3}\sqrt[3]{ax+bx^2}}\,dx$$

$$\Big\downarrow \text{1129}$$

$$-\frac{3b\int \frac{1}{(cx)^{4/3}\sqrt[3]{bx^2+ax}}dx}{5ac}-\frac{3(ax+bx^2)^{2/3}}{5a(cx)^{7/3}}$$

$$\Big\downarrow \text{1123}$$

$$\frac{9b(ax+bx^2)^{2/3}}{10a^2c(cx)^{4/3}}-\frac{3(ax+bx^2)^{2/3}}{5a(cx)^{7/3}}$$

```
Int[1/((c*x)^(7/3)*(a*x + b*x^2)^(1/3)),x]
```

```
(-3*(a*x + b*x^2)^(2/3))/(5*a*(c*x)^(7/3)) + (9*b*(a*x + b*x^2)^(2/3))/(10
*a^2*c*(c*x)^(4/3))
```

Defintions of rubi rules used

```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] :> Simp[e*(d + e*x)^m*((a + b*x + c*x^2)^(p + 1)/((p + 1)*(2*c*d - b
*e))), x] /; FreeQ[{a, b, c, d, e, m, p}, x] && EqQ[c*d^2 - b*d*e + a*e^2,
0] && EqQ[m + 2*p + 2, 0]
```

```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] :> Simp[(-e)*(d + e*x)^m*((a + b*x + c*x^2)^(p + 1)/((m + p + 1)*(2*
c*d - b*e))), x] + Simp[c*(Simplify[m + 2*p + 2]/((m + p + 1)*(2*c*d - b*e)
))   Int[(d + e*x)^(m + 1)*(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d
, e, m, p}, x] && EqQ[c*d^2 - b*d*e + a*e^2, 0] && ILtQ[Simplify[m + 2*p +
2], 0]
```

Maple [A] (verified)

Time = 0.35 (sec) , antiderivative size = 36, normalized size of antiderivative = 0.61

method	result	size
gosper	$-\frac{3x(bx+a)(-3bx+2a)}{10a^2(cx)^{\frac{7}{3}}(b\,x^2+ax)^{\frac{1}{3}}}$	36
orering	$-\frac{3x(bx+a)(-3bx+2a)}{10a^2(cx)^{\frac{7}{3}}(b\,x^2+ax)^{\frac{1}{3}}}$	36
risch	$-\frac{3(bx+a)(-3bx+2a)}{10c^2(cx)^{\frac{1}{3}}(x(bx+a))^{\frac{1}{3}}a^2x}$	39

```
int(1/(c*x)^(7/3)/(b*x^2+a*x)^(1/3),x,method=_RETURNVERBOSE)
```

```
-3/10*x*(b*x+a)*(-3*b*x+2*a)/a^2/(c*x)^(7/3)/(b*x^2+a*x)^(1/3)
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 35, normalized size of antiderivative = 0.59

$$\int \frac{1}{(cx)^{7/3}\sqrt[3]{ax+bx^2}}\,dx = \frac{3\,(bx^2+ax)^{\frac{2}{3}}(3\,bx-2\,a)(cx)^{\frac{2}{3}}}{10\,a^2c^3x^3}$$

```
integrate(1/(c*x)^(7/3)/(b*x^2+a*x)^(1/3),x, algorithm="fricas")
```

```
3/10*(b*x^2 + a*x)^(2/3)*(3*b*x - 2*a)*(c*x)^(2/3)/(a^2*c^3*x^3)
```

Sympy [F]

$$\int \frac{1}{(cx)^{7/3}\sqrt[3]{ax+bx^2}}\,dx = \int \frac{1}{(cx)^{\frac{7}{3}}\sqrt[3]{x(a+bx)}}\,dx$$

```
integrate(1/(c*x)**(7/3)/(b*x**2+a*x)**(1/3),x)
```

```
Integral(1/((c*x)**(7/3)*(x*(a + b*x))**(1/3)), x)
```

Maxima [F]

$$\int \frac{1}{(cx)^{7/3}\sqrt[3]{ax+bx^2}}\,dx = \int \frac{1}{(bx^2+ax)^{\frac{1}{3}}(cx)^{\frac{7}{3}}}\,dx$$

```
integrate(1/(c*x)^(7/3)/(b*x^2+a*x)^(1/3),x, algorithm="maxima")
```

```
integrate(1/((b*x^2 + a*x)^(1/3)*(c*x)^(7/3)), x)
```

Giac [A] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 47, normalized size of antiderivative = 0.80

$$\int \frac{1}{(cx)^{7/3}\sqrt[3]{ax+bx^2}}\,dx = \frac{3\left(\frac{5\left(bc+\frac{ac}{x}\right)^{\frac{2}{3}}b}{a^2c^2} - \frac{2\left(bc+\frac{ac}{x}\right)^{\frac{5}{3}}}{a^2c^3}\right)}{10\,c}$$

```
integrate(1/(c*x)^(7/3)/(b*x^2+a*x)^(1/3),x, algorithm="giac")
```

```
3/10*(5*(b*c + a*c/x)^(2/3)*b/(a^2*c^2) - 2*(b*c + a*c/x)^(5/3)/(a^2*c^3))
/c
```

Mupad [B] (verification not implemented)

Time = 10.33 (sec) , antiderivative size = 40, normalized size of antiderivative = 0.68

$$\int \frac{1}{(cx)^{7/3}\sqrt[3]{ax+bx^2}}\,dx=-\frac{{\left(b\,x^2+a\,x\right)}^{2/3}\,\left(\frac{3}{5\,a\,c^2}-\frac{9\,b\,x}{10\,a^2\,c^2}\right)}{x^2\,{\left(c\,x\right)}^{1/3}}$$

```
int(1/((a*x + b*x^2)^(1/3)*(c*x)^(7/3)),x)
```

```
-((a*x + b*x^2)^(2/3)*(3/(5*a*c^2) - (9*b*x)/(10*a^2*c^2)))/(x^2*(c*x)^(1/
3))
```

Reduce [F]

$$\int \frac{1}{(cx)^{7/3}\sqrt[3]{ax+bx^2}}\,dx=\frac{\int \frac{1}{x^{\frac{8}{3}}\,(bx+a)^{\frac{1}{3}}}dx}{c^{\frac{7}{3}}}$$

```
int(1/(c*x)^(7/3)/(b*x^2+a*x)^(1/3),x)
```

```
int(1/(x**(2/3)*(a + b*x)**(1/3)*x**2),x)/(c**(1/3)*c**2)
```

3.126 $\int \frac{1}{(cx)^{10/3}\sqrt[3]{ax+bx^2}}\,dx$

Optimal result

Integrand size = 21, antiderivative size = 92

$$\int \frac{1}{(cx)^{10/3}\sqrt[3]{ax+bx^2}}\,dx = -\frac{3(ax+bx^2)^{2/3}}{8a(cx)^{10/3}}+\frac{9b(ax+bx^2)^{2/3}}{20a^2c(cx)^{7/3}}-\frac{27b^2(ax+bx^2)^{2/3}}{40a^3c^2(cx)^{4/3}}$$

```
-3/8*(b*x^2+a*x)^(2/3)/a/(c*x)^(10/3)+9/20*b*(b*x^2+a*x)^(2/3)/a^2/c/(c*x)
^(7/3)-27/40*b^2*(b*x^2+a*x)^(2/3)/a^3/c^2/(c*x)^(4/3)
```

Mathematica [A] (verified)

Time = 0.17 (sec) , antiderivative size = 44, normalized size of antiderivative = 0.48

$$\int \frac{1}{(cx)^{10/3}\sqrt[3]{ax+bx^2}}\,dx = -\frac{3(x(a+bx))^{2/3}\left(5a^2-6abx+9b^2x^2\right)}{40a^3(cx)^{10/3}}$$

```
Integrate[1/((c*x)^(10/3)*(a*x + b*x^2)^(1/3)),x]
```

```
(-3*(x*(a + b*x))^(2/3)*(5*a^2 - 6*a*b*x + 9*b^2*x^2))/(40*a^3*(c*x)^(10/3
))
```

Rubi [A] (verified)

Time = 0.35 (sec) , antiderivative size = 98, normalized size of antiderivative = 1.07, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.143$, Rules used = {1129, 1129, 1123}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{(cx)^{10/3}\sqrt[3]{ax+bx^2}}dx$$

$$\Big\downarrow \text{ 1129}$$

$$-\frac{3b\int \frac{1}{(cx)^{7/3}\sqrt[3]{bx^2+ax}}dx}{4ac}-\frac{3(ax+bx^2)^{2/3}}{8a(cx)^{10/3}}$$

$$\Big\downarrow \text{ 1129}$$

$$-\frac{3b\left(-\frac{3b\int \frac{1}{(cx)^{4/3}\sqrt[3]{bx^2+ax}}dx}{5ac}-\frac{3(ax+bx^2)^{2/3}}{5a(cx)^{7/3}}\right)}{4ac}-\frac{3(ax+bx^2)^{2/3}}{8a(cx)^{10/3}}$$

$$\Big\downarrow \text{ 1123}$$

$$-\frac{3b\left(\frac{9b(ax+bx^2)^{2/3}}{10a^2c(cx)^{4/3}}-\frac{3(ax+bx^2)^{2/3}}{5a(cx)^{7/3}}\right)}{4ac}-\frac{3(ax+bx^2)^{2/3}}{8a(cx)^{10/3}}$$

```
Int[1/((c*x)^(10/3)*(a*x + b*x^2)^(1/3)),x]
```

```
(-3*(a*x + b*x^2)^(2/3))/(8*a*(c*x)^(10/3)) - (3*b*((-3*(a*x + b*x^2)^(2/3
))/(5*a*(c*x)^(7/3)) + (9*b*(a*x + b*x^2)^(2/3))/(10*a^2*c*(c*x)^(4/3))))/
(4*a*c)
```

Defintions of rubi rules used

```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] :> Simp[e*(d + e*x)^m*((a + b*x + c*x^2)^(p + 1)/((p + 1)*(2*c*d - b
*e))), x] /; FreeQ[{a, b, c, d, e, m, p}, x] && EqQ[c*d^2 - b*d*e + a*e^2,
0] && EqQ[m + 2*p + 2, 0]
```

```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] :> Simp[(-e)*(d + e*x)^m*((a + b*x + c*x^2)^(p + 1)/((m + p + 1)*(2*
c*d - b*e))), x] + Simp[c*(Simplify[m + 2*p + 2]/((m + p + 1)*(2*c*d - b*e)
))   Int[(d + e*x)^(m + 1)*(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d
, e, m, p}, x] && EqQ[c*d^2 - b*d*e + a*e^2, 0] && ILtQ[Simplify[m + 2*p +
2], 0]
```

Maple [A] (verified)

Time = 0.34 (sec) , antiderivative size = 47, normalized size of antiderivative = 0.51

method	result	size
gosper	$-\frac{3x(bx+a)(9b^2x^2-6abx+5a^2)}{40a^3(cx)^{\frac{10}{3}}(b\,x^2+ax)^{\frac{1}{3}}}$	47
orering	$-\frac{3x(bx+a)(9b^2x^2-6abx+5a^2)}{40a^3(cx)^{\frac{10}{3}}(b\,x^2+ax)^{\frac{1}{3}}}$	47
risch	$-\frac{3(bx+a)(9b^2x^2-6abx+5a^2)}{40c^3(cx)^{\frac{1}{3}}(x(bx+a))^{\frac{1}{3}}a^3x^2}$	50

```
int(1/(c*x)^(10/3)/(b*x^2+a*x)^(1/3),x,method=_RETURNVERBOSE)
```

```
-3/40*x*(b*x+a)*(9*b^2*x^2-6*a*b*x+5*a^2)/a^3/(c*x)^(10/3)/(b*x^2+a*x)^(1/
3)
```

Fricas [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 46, normalized size of antiderivative = 0.50

$$\int \frac{1}{(cx)^{10/3}\sqrt[3]{ax+bx^2}}\,dx = -\frac{3\,(9\,b^2x^2-6\,abx+5\,a^2)(bx^2+ax)^{\frac{2}{3}}(cx)^{\frac{2}{3}}}{40\,a^3c^4x^4}$$

```
integrate(1/(c*x)^(10/3)/(b*x^2+a*x)^(1/3),x, algorithm="fricas")
```

```
-3/40*(9*b^2*x^2 - 6*a*b*x + 5*a^2)*(b*x^2 + a*x)^(2/3)*(c*x)^(2/3)/(a^3*c
^4*x^4)
```

Sympy [F]

$$\int \frac{1}{(cx)^{10/3}\sqrt[3]{ax+bx^2}}\,dx = \int \frac{1}{(cx)^{\frac{10}{3}}\,\sqrt[3]{x\,(a+bx)}}\,dx$$

```
integrate(1/(c*x)**(10/3)/(b*x**2+a*x)**(1/3),x)
```

```
Integral(1/((c*x)**(10/3)*(x*(a + b*x))**(1/3)), x)
```

Maxima [F]

$$\int \frac{1}{(cx)^{10/3}\sqrt[3]{ax+bx^2}}\,dx = \int \frac{1}{(bx^2+ax)^{\frac{1}{3}}\,(cx)^{\frac{10}{3}}}\,dx$$

```
integrate(1/(c*x)^(10/3)/(b*x^2+a*x)^(1/3),x, algorithm="maxima")
```

```
integrate(1/((b*x^2 + a*x)^(1/3)*(c*x)^(10/3)), x)
```

Giac [A] (verification not implemented)

Time = 0.20 (sec) , antiderivative size = 65, normalized size of antiderivative = 0.71

$$\int \frac{1}{(cx)^{10/3}\sqrt[3]{ax+bx^2}}\,dx = -\frac{3\left(\frac{20\left(bc+\frac{ac}{x}\right)^{\frac{2}{3}}b^2}{a^3} - \frac{16\left(bc+\frac{ac}{x}\right)^{\frac{5}{3}}bc-5\left(bc+\frac{ac}{x}\right)^{\frac{8}{3}}}{a^3c^2}\right)}{40\,c^4}$$

```
integrate(1/(c*x)^(10/3)/(b*x^2+a*x)^(1/3),x, algorithm="giac")
```

```
-3/40*(20*(b*c + a*c/x)^(2/3)*b^2/a^3 - (16*(b*c + a*c/x)^(5/3)*b*c - 5*(b
*c + a*c/x)^(8/3))/(a^3*c^2))/c^4
```

Mupad [B] (verification not implemented)

Time = 9.90 (sec) , antiderivative size = 54, normalized size of antiderivative = 0.59

$$\int \frac{1}{(cx)^{10/3}\sqrt[3]{ax+bx^2}}\,dx = -\frac{\left(b\,x^2+a\,x\right)^{2/3}\left(\frac{3}{8\,a\,c^3}+\frac{27\,b^2\,x^2}{40\,a^3\,c^3}-\frac{9\,b\,x}{20\,a^2\,c^3}\right)}{x^3\,\left(c\,x\right)^{1/3}}$$

```
int(1/((a*x + b*x^2)^(1/3)*(c*x)^(10/3)),x)
```

```
-((a*x + b*x^2)^(2/3)*(3/(8*a*c^3) + (27*b^2*x^2)/(40*a^3*c^3) - (9*b*x)/(
20*a^2*c^3)))/(x^3*(c*x)^(1/3))
```

Reduce [F]

$$\int \frac{1}{(cx)^{10/3}\sqrt[3]{ax+bx^2}}\,dx = \frac{\int \frac{1}{x^{\frac{11}{3}}(bx+a)^{\frac{1}{3}}}dx}{c^{\frac{10}{3}}}$$

```
int(1/(c*x)^(10/3)/(b*x^2+a*x)^(1/3),x)
```

```
int(1/(x**(2/3)*(a + b*x)**(1/3)*x**3),x)/(c**(1/3)*c**3)
```

3.127 $\int \frac{(cx)^{5/3}}{\sqrt[3]{ax+bx^2}}\,dx$

Optimal result

Integrand size = 21, antiderivative size = 278

$$\begin{aligned}\int \frac{(cx)^{5/3}}{\sqrt[3]{ax+bx^2}}\,dx &= -\frac{2ac^2(ax+bx^2)^{2/3}}{3b^2\sqrt[3]{cx}}+\frac{c(cx)^{2/3}\,(ax+bx^2)^{2/3}}{2b}\\ &\quad-\frac{2a^2c^{4/3}\sqrt[3]{cx}\sqrt[3]{a+bx}\arctan\left(\frac{1}{\sqrt{3}}+\frac{2\sqrt[3]{c}\sqrt[3]{a+bx}}{\sqrt{3}\sqrt[3]{b}\sqrt[3]{cx}}\right)}{3\sqrt{3}b^{7/3}\sqrt[3]{ax+bx^2}}\\ &\quad-\frac{a^2c^{4/3}\sqrt[3]{cx}\sqrt[3]{a+bx}\log(cx)}{9b^{7/3}\sqrt[3]{ax+bx^2}}-\frac{a^2c^{4/3}\sqrt[3]{cx}\sqrt[3]{a+bx}\log\left(1-\frac{\sqrt[3]{c}\sqrt[3]{a+bx}}{\sqrt[3]{b}\sqrt[3]{cx}}\right)}{3b^{7/3}\sqrt[3]{ax+bx^2}}\end{aligned}$$

```
-2/3*a*c^2*(b*x^2+a*x)^(2/3)/b^2/(c*x)^(1/3)+1/2*c*(c*x)^(2/3)*(b*x^2+a*x)
^(2/3)/b-2/9*a^2*c^(4/3)*(c*x)^(1/3)*(b*x+a)^(1/3)*arctan(1/3*3^(1/2)+2/3*
c^(1/3)*(b*x+a)^(1/3)*3^(1/2)/b^(1/3)/(c*x)^(1/3))*3^(1/2)/b^(7/3)/(b*x^2+
a*x)^(1/3)-1/9*a^2*c^(4/3)*(c*x)^(1/3)*(b*x+a)^(1/3)*ln(c*x)/b^(7/3)/(b*x^
2+a*x)^(1/3)-1/3*a^2*c^(4/3)*(c*x)^(1/3)*(b*x+a)^(1/3)*ln(1-c^(1/3)*(b*x+a
)^(1/3)/b^(1/3)/(c*x)^(1/3))/b^(7/3)/(b*x^2+a*x)^(1/3)
```

Mathematica [A] (verified)

Time = 0.40 (sec) , antiderivative size = 227, normalized size of antiderivative = 0.82

$$\int \frac{(cx)^{5/3}}{\sqrt[3]{ax+bx^2}}\,dx = \frac{(cx)^{5/3}\left(-12a^2\sqrt[3]{b}\sqrt[3]{x}-3ab^{4/3}x^{4/3}+9b^{7/3}x^{7/3}+4\sqrt{3}a^2\sqrt[3]{a+bx}\arctan\left(\frac{\sqrt{3}\sqrt[3]{b}\sqrt[3]{}}{\sqrt[3]{b}\sqrt[3]{x}+2\sqrt[3]{}}\right.\right.}{}$$

```
Integrate[(c*x)^(5/3)/(a*x + b*x^2)^(1/3),x]
```

```
((c*x)^(5/3)*(-12*a^2*b^(1/3)*x^(1/3) - 3*a*b^(4/3)*x^(4/3) + 9*b^(7/3)*x^
(7/3) + 4*Sqrt[3]*a^2*(a + b*x)^(1/3)*ArcTan[(Sqrt[3]*b^(1/3)*x^(1/3))/(b^
(1/3)*x^(1/3) + 2*(a + b*x)^(1/3))] - 4*a^2*(a + b*x)^(1/3)*Log[-(b^(1/3)*
x^(1/3)) + (a + b*x)^(1/3)] + 2*a^2*(a + b*x)^(1/3)*Log[b^(2/3)*x^(2/3) +
b^(1/3)*x^(1/3)*(a + b*x)^(1/3) + (a + b*x)^(2/3)]))/(18*b^(7/3)*x^(4/3)*(
x*(a + b*x))^(1/3))
```

Rubi [A] (verified)

Time = 0.39 (sec) , antiderivative size = 181, normalized size of antiderivative = 0.65, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.190$, Rules used = {1137, 60, 60, 71}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(cx)^{5/3}}{\sqrt[3]{ax+bx^2}}\,dx$$

$$\Big\downarrow \; 1137$$

$$\frac{(cx)^{5/3}\sqrt[3]{a+bx}\int \frac{x^{4/3}}{\sqrt[3]{a+bx}}dx}{x^{4/3}\sqrt[3]{ax+bx^2}}$$

$$\Big\downarrow \; 60$$

$$\frac{(cx)^{5/3}\sqrt[3]{a+bx}\left(\frac{x^{4/3}(a+bx)^{2/3}}{2b}-\frac{2a\int\frac{\sqrt[3]{x}}{\sqrt[3]{a+bx}}dx}{3b}\right)}{x^{4/3}\sqrt[3]{ax+bx^2}}$$

$$\Big\downarrow \text{60}$$

$$\frac{(cx)^{5/3}\sqrt[3]{a+bx}\left(\frac{x^{4/3}(a+bx)^{2/3}}{2b}-\frac{2a\left(\frac{\sqrt[3]{x}(a+bx)^{2/3}}{b}-\frac{a\int\frac{1}{x^{2/3}\sqrt[3]{a+bx}}dx}{3b}\right)}{3b}\right)}{x^{4/3}\sqrt[3]{ax+bx^2}}$$

$$\Big\downarrow \text{71}$$

$$\frac{(cx)^{5/3}\sqrt[3]{a+bx}\left(\frac{x^{4/3}(a+bx)^{2/3}}{2b}-\frac{2a\left(\frac{\sqrt[3]{x}(a+bx)^{2/3}}{b}-\frac{a\left(-\frac{\sqrt{3}\arctan\left(\frac{2\sqrt[3]{a+bx}}{\sqrt{3}\sqrt[3]{b}\sqrt[3]{x}}+\frac{1}{\sqrt{3}}\right)}{\sqrt[3]{b}}-\frac{3\log\left(\frac{\sqrt[3]{a+bx}}{\sqrt[3]{b}\sqrt[3]{x}}-1\right)}{2\sqrt[3]{b}}-\frac{\log(x)}{2\sqrt[3]{b}}\right)}{3b}\right)}{3b}\right)}{x^{4/3}\sqrt[3]{ax+bx^2}}$$

```
Int[(c*x)^(5/3)/(a*x + b*x^2)^(1/3),x]
```

```
((c*x)^(5/3)*(a + b*x)^(1/3)*((x^(4/3)*(a + b*x)^(2/3))/(2*b) - (2*a*((x^(
1/3)*(a + b*x)^(2/3))/b - (a*(-((Sqrt[3]*ArcTan[1/Sqrt[3] + (2*(a + b*x)^(
1/3))/(Sqrt[3]*b^(1/3)*x^(1/3))])/b^(1/3)) - Log[x]/(2*b^(1/3)) - (3*Log[-
1 + (a + b*x)^(1/3)/(b^(1/3)*x^(1/3))])/(2*b^(1/3))))/(3*b)))/(3*b)))/(x^(
4/3)*(a*x + b*x^2)^(1/3))
```

Defintions of rubi rules used

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^n/(b*(m + n + 1))), x] + Simp[n*((b*c - a*d)/(
b*(m + n + 1)))   Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
 c, d}, x] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ[m, 0] && ( !Integer
Q[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n + 2, 0] && IntLinear
Q[a, b, c, d, m, n, x]
```

```
Int[1/(((a_.) + (b_.)*(x_))^(1/3)*((c_.) + (d_.)*(x_))^(2/3)), x_Symbol] :>
 With[{q = Rt[d/b, 3]}, Simp[(-Sqrt[3])*(q/d)*ArcTan[2*q*((a + b*x)^(1/3)/(
Sqrt[3]*(c + d*x)^(1/3))) + 1/Sqrt[3]], x] + (-Simp[3*(q/(2*d))*Log[q*((a +
 b*x)^(1/3)/(c + d*x)^(1/3)) - 1], x] - Simp[(q/(2*d))*Log[c + d*x], x])] /
; FreeQ[{a, b, c, d}, x] && PosQ[d/b]
```

```
Int[((e_.)*(x_))^(m_)*((b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(
e*x)^m*((b*x + c*x^2)^p/(x^(m + p)*(b + c*x)^p))   Int[x^(m + p)*(b + c*x)^
p, x], x] /; FreeQ[{b, c, e, m}, x]
```

Maple [F]

$$\int \frac{(cx)^{\frac{5}{3}}}{(b\,x^2+ax)^{\frac{1}{3}}}dx$$

```
int((c*x)^(5/3)/(b*x^2+a*x)^(1/3),x)
```

```
int((c*x)^(5/3)/(b*x^2+a*x)^(1/3),x)
```

Fricas [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 295, normalized size of antiderivative = 1.06

$$\int \frac{(cx)^{5/3}}{\sqrt[3]{ax+bx^2}}\,dx = -\frac{4\sqrt{3}a^2c\left(-\frac{c^2}{b}\right)^{\frac{1}{3}} x \arctan\left(\frac{2\sqrt{3}(bx^2+ax)^{\frac{2}{3}}(cx)^{\frac{2}{3}}b\left(-\frac{c^2}{b}\right)^{\frac{2}{3}}+\sqrt{3}(bc^2x^2+ac^2x)}{3\,(bc^2x^2+ac^2x)}\right) + 2\,a^2c\left(-\frac{c^2}{b}\right)^{\frac{1}{3}} x\log\left(\frac{(bx^2+ax)^{\frac{1}{3}}(cx)^{\frac{1}{3}}\cdots}{\cdots}\right.\cdots}{\cdots}$$

```
integrate((c*x)^(5/3)/(b*x^2+a*x)^(1/3),x, algorithm="fricas")
```

```
-1/18*(4*sqrt(3)*a^2*c*(-c^2/b)^(1/3)*x*arctan(1/3*(2*sqrt(3)*(b*x^2 + a*x
)^(2/3)*(c*x)^(2/3)*b*(-c^2/b)^(2/3) + sqrt(3)*(b*c^2*x^2 + a*c^2*x))/(b*c
^2*x^2 + a*c^2*x)) + 2*a^2*c*(-c^2/b)^(1/3)*x*log(((b*x^2 + a*x)^(1/3)*(c*
x)^(1/3)*c*x - (b*x^2 + a*x)^(2/3)*(c*x)^(2/3)*(-c^2/b)^(1/3) + (b*x^2 + a
*x)*(-c^2/b)^(2/3))/(b*x^2 + a*x)) - 4*a^2*c*(-c^2/b)^(1/3)*x*log(((b*x^2
+ a*x)^(2/3)*(c*x)^(2/3) + (b*x^2 + a*x)*(-c^2/b)^(1/3))/(b*x^2 + a*x)) -
3*(3*b*c*x - 4*a*c)*(b*x^2 + a*x)^(2/3)*(c*x)^(2/3))/(b^2*x)
```

Sympy [F]

$$\int \frac{(cx)^{5/3}}{\sqrt[3]{ax+bx^2}}\,dx = \int \frac{(cx)^{\frac{5}{3}}}{\sqrt[3]{x\,(a+bx)}}\,dx$$

```
integrate((c*x)**(5/3)/(b*x**2+a*x)**(1/3),x)
```

```
Integral((c*x)**(5/3)/(x*(a + b*x))**(1/3), x)
```

Maxima [F]

$$\int \frac{(cx)^{5/3}}{\sqrt[3]{ax+bx^2}}\,dx = \int \frac{(cx)^{\frac{5}{3}}}{(bx^2+ax)^{\frac{1}{3}}}\,dx$$

```
integrate((c*x)^(5/3)/(b*x^2+a*x)^(1/3),x, algorithm="maxima")
```

```
integrate((c*x)^(5/3)/(b*x^2 + a*x)^(1/3), x)
```

Giac [A] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 206, normalized size of antiderivative = 0.74

$$\int \frac{(cx)^{5/3}}{\sqrt[3]{ax+bx^2}}\,dx = -\frac{\frac{4\sqrt{3}(bc)^{\frac{2}{3}}a^3c^3\arctan\left(\frac{\sqrt{3}\left(2\left(bc+\frac{ac}{x}\right)^{\frac{1}{3}}+(bc)^{\frac{1}{3}}\right)}{3\,(bc)^{\frac{1}{3}}}\right)}{b^3} - \frac{2\,(bc)^{\frac{2}{3}}a^3c^3\log\left(\left(bc+\frac{ac}{x}\right)^{\frac{2}{3}}+\left(bc+\frac{ac}{x}\right)^{\frac{1}{3}}(bc)^{\frac{1}{3}}+(bc)^{\frac{2}{3}}\right)}{b^3} + \frac{4\,(bc)^{\frac{2}{3}}a^3c^3\log\left(\left|\left(bc+\frac{ac}{x}\right)^{\frac{1}{3}}\right.\right.}{b^3}}{18\,ac^2}$$

```
integrate((c*x)^(5/3)/(b*x^2+a*x)^(1/3),x, algorithm="giac")
```

```
-1/18*(4*sqrt(3)*(b*c)^(2/3)*a^3*c^3*arctan(1/3*sqrt(3)*(2*(b*c + a*c/x)^(
1/3) + (b*c)^(1/3))/(b*c)^(1/3))/b^3 - 2*(b*c)^(2/3)*a^3*c^3*log((b*c + a*
c/x)^(2/3) + (b*c + a*c/x)^(1/3)*(b*c)^(1/3) + (b*c)^(2/3))/b^3 + 4*(b*c)^
(2/3)*a^3*c^3*log(abs((b*c + a*c/x)^(1/3) - (b*c)^(1/3)))/b^3 - 3*(7*(b*c
+ a*c/x)^(2/3)*a^3*b*c^5 - 4*(b*c + a*c/x)^(5/3)*a^3*c^4)*x^2/(a^2*b^2*c^2
))/(a*c^2)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(cx)^{5/3}}{\sqrt[3]{ax+bx^2}}\,dx=\int \frac{(c\,x)^{5/3}}{(b\,x^2+a\,x)^{1/3}}\,dx$$

```
int((c*x)^(5/3)/(a*x + b*x^2)^(1/3),x)
```

```
int((c*x)^(5/3)/(a*x + b*x^2)^(1/3), x)
```

Reduce [F]

$$\int \frac{(cx)^{5/3}}{\sqrt[3]{ax+bx^2}}\,dx=c^{\frac{5}{3}}\left(\int \frac{x^{\frac{4}{3}}}{(bx+a)^{\frac{1}{3}}}dx\right)$$

```
int((c*x)^(5/3)/(b*x^2+a*x)^(1/3),x)
```

```
c**(2/3)*int((x**(1/3)*x)/(a + b*x)**(1/3),x)*c
```

3.128 $\int \frac{(cx)^{2/3}}{\sqrt[3]{ax+bx^2}}\,dx$

Optimal result

Integrand size = 21, antiderivative size = 235

$$\begin{aligned}\int \frac{(cx)^{2/3}}{\sqrt[3]{ax+bx^2}}\,dx &= \frac{c(ax+bx^2)^{2/3}}{b\sqrt[3]{cx}}\\
&+\frac{a\sqrt[3]{c}\sqrt[3]{cx}\sqrt[3]{a+bx}\arctan\left(\frac{1}{\sqrt{3}}+\frac{2\sqrt[3]{c}\sqrt[3]{a+bx}}{\sqrt{3}\sqrt[3]{b}\sqrt[3]{cx}}\right)}{\sqrt{3}b^{4/3}\sqrt[3]{ax+bx^2}}\\
&+\frac{a\sqrt[3]{c}\sqrt[3]{cx}\sqrt[3]{a+bx}\log(cx)}{6b^{4/3}\sqrt[3]{ax+bx^2}}+\frac{a\sqrt[3]{c}\sqrt[3]{cx}\sqrt[3]{a+bx}\log\left(1-\frac{\sqrt[3]{c}\sqrt[3]{a+bx}}{\sqrt[3]{b}\sqrt[3]{cx}}\right)}{2b^{4/3}\sqrt[3]{ax+bx^2}}\end{aligned}$$

```
c*(b*x^2+a*x)^(2/3)/b/(c*x)^(1/3)+1/3*a*c^(1/3)*(c*x)^(1/3)*(b*x+a)^(1/3)*
arctan(1/3*3^(1/2)+2/3*c^(1/3)*(b*x+a)^(1/3)*3^(1/2)/b^(1/3)/(c*x)^(1/3))*
3^(1/2)/b^(4/3)/(b*x^2+a*x)^(1/3)+1/6*a*c^(1/3)*(c*x)^(1/3)*(b*x+a)^(1/3)*
ln(c*x)/b^(4/3)/(b*x^2+a*x)^(1/3)+1/2*a*c^(1/3)*(c*x)^(1/3)*(b*x+a)^(1/3)*
ln(1-c^(1/3)*(b*x+a)^(1/3)/b^(1/3)/(c*x)^(1/3))/b^(4/3)/(b*x^2+a*x)^(1/3)
```

Mathematica [A] (verified)

Time = 0.33 (sec) , antiderivative size = 206, normalized size of antiderivative = 0.88

$$\int \frac{(cx)^{2/3}}{\sqrt[3]{ax+bx^2}}\,dx = \frac{(cx)^{2/3}\left(6a\sqrt[3]{b}\sqrt[3]{x}+6b^{4/3}x^{4/3}-2\sqrt{3}a\sqrt[3]{a+bx}\arctan\left(\frac{\sqrt{3}\sqrt[3]{b}\sqrt[3]{x}}{\sqrt[3]{b}\sqrt[3]{x}+2\sqrt[3]{a+bx}}\right)+2a\sqrt[3]{a-}\right.}{6b^{4/3}\sqrt[3]{x}}$$

```
Integrate[(c*x)^(2/3)/(a*x + b*x^2)^(1/3),x]
```

```
((c*x)^(2/3)*(6*a*b^(1/3)*x^(1/3) + 6*b^(4/3)*x^(4/3) - 2*Sqrt[3]*a*(a + b
*x)^(1/3)*ArcTan[(Sqrt[3]*b^(1/3)*x^(1/3))/(b^(1/3)*x^(1/3) + 2*(a + b*x)^
(1/3))] + 2*a*(a + b*x)^(1/3)*Log[-(b^(1/3)*x^(1/3)) + (a + b*x)^(1/3)] -
a*(a + b*x)^(1/3)*Log[b^(2/3)*x^(2/3) + b^(1/3)*x^(1/3)*(a + b*x)^(1/3) +
(a + b*x)^(2/3)]))/(6*b^(4/3)*x^(1/3)*(x*(a + b*x))^(1/3))
```

Rubi [A] (verified)

Time = 0.36 (sec) , antiderivative size = 151, normalized size of antiderivative = 0.64, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.143$, Rules used = {1137, 60, 71}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(cx)^{2/3}}{\sqrt[3]{ax+bx^2}}dx$$

$$\Big\downarrow \; 1137$$

$$\frac{(cx)^{2/3}\sqrt[3]{a+bx}\int \frac{\sqrt[3]{x}}{\sqrt[3]{a+bx}}dx}{\sqrt[3]{x}\sqrt[3]{ax+bx^2}}$$

$$\Big\downarrow \; 60$$

$$\frac{(cx)^{2/3}\sqrt[3]{a+bx}\left(\frac{\sqrt[3]{x}(a+bx)^{2/3}}{b}-\frac{a\int \frac{1}{x^{2/3}\sqrt[3]{a+bx}}dx}{3b}\right)}{\sqrt[3]{x}\sqrt[3]{ax+bx^2}}$$

$$\Big\downarrow \text{71}$$

$$\frac{(cx)^{2/3}\sqrt[3]{a+bx}\left(\frac{\sqrt[3]{x}(a+bx)^{2/3}}{b}-\frac{a\left(-\frac{\sqrt{3}\arctan\left(\frac{2\sqrt[3]{a+bx}}{\sqrt{3}\sqrt[3]{b}\sqrt[3]{x}}+\frac{1}{\sqrt{3}}\right)}{\sqrt[3]{b}}-\frac{3\log\left(\frac{\sqrt[3]{a+bx}}{\sqrt[3]{b}\sqrt[3]{x}}-1\right)}{2\sqrt[3]{b}}-\frac{\log(x)}{2\sqrt[3]{b}}\right)}{3b}\right)}{\sqrt[3]{x}\sqrt[3]{ax+bx^2}}$$

```
Int[(c*x)^(2/3)/(a*x + b*x^2)^(1/3),x]
```

```
((c*x)^(2/3)*(a + b*x)^(1/3)*((x^(1/3)*(a + b*x)^(2/3))/b - (a*(-((Sqrt[3]
*ArcTan[1/Sqrt[3] + (2*(a + b*x)^(1/3))/(Sqrt[3]*b^(1/3)*x^(1/3))])/b^(1/3
)) - Log[x]/(2*b^(1/3)) - (3*Log[-1 + (a + b*x)^(1/3)/(b^(1/3)*x^(1/3))])/
(2*b^(1/3))))/(3*b)))/(x^(1/3)*(a*x + b*x^2)^(1/3))
```

Defintions of rubi rules used

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^n/(b*(m + n + 1))), x] + Simp[n*((b*c - a*d)/(
b*(m + n + 1)))   Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
 c, d}, x] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ[m, 0] && ( !Integer
Q[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n + 2, 0] && IntLinear
Q[a, b, c, d, m, n, x]
```

```
Int[1/(((a_.) + (b_.)*(x_))^(1/3)*((c_.) + (d_.)*(x_))^(2/3)), x_Symbol] :>
 With[{q = Rt[d/b, 3]}, Simp[(-Sqrt[3])*(q/d)*ArcTan[2*q*((a + b*x)^(1/3)/(
Sqrt[3]*(c + d*x)^(1/3))) + 1/Sqrt[3]], x] + (-Simp[3*(q/(2*d))*Log[q*((a +
 b*x)^(1/3)/(c + d*x)^(1/3)) - 1], x] - Simp[(q/(2*d))*Log[c + d*x], x])] /
; FreeQ[{a, b, c, d}, x] && PosQ[d/b]
```

```
Int[((e_.)*(x_))^(m_)*((b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(
e*x)^m*((b*x + c*x^2)^p/(x^(m + p)*(b + c*x)^p))   Int[x^(m + p)*(b + c*x)^
p, x], x] /; FreeQ[{b, c, e, m}, x]
```

Maple [F]

$$\int \frac{(cx)^{\frac{2}{3}}}{(b\,x^{2}+ax)^{\frac{1}{3}}}dx$$

```
int((c*x)^(2/3)/(b*x^2+a*x)^(1/3),x)
```

```
int((c*x)^(2/3)/(b*x^2+a*x)^(1/3),x)
```

Fricas [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 268, normalized size of antiderivative = 1.14

$$\int \frac{(cx)^{2/3}}{\sqrt[3]{ax+bx^2}}\,dx =$$

$$-\frac{2\sqrt{3}a\left(\frac{c^2}{b}\right)^{\frac{1}{3}}x\arctan\left(\frac{2\sqrt{3}(bx^2+ax)^{\frac{2}{3}}(cx)^{\frac{2}{3}}b\left(\frac{c^2}{b}\right)^{\frac{2}{3}}+\sqrt{3}(bc^2x^2+ac^2x)}{3\,(bc^2x^2+ac^2x)}\right)+a\left(\frac{c^2}{b}\right)^{\frac{1}{3}}x\log\left(\frac{(bx^2+ax)^{\frac{1}{3}}(cx)^{\frac{1}{3}}cx+(bx^2+ax)}{bx^2}\right.}{6\,bx}$$

```
integrate((c*x)^(2/3)/(b*x^2+a*x)^(1/3),x, algorithm="fricas")
```

```
-1/6*(2*sqrt(3)*a*(c^2/b)^(1/3)*x*arctan(1/3*(2*sqrt(3)*(b*x^2 + a*x)^(2/3
)*(c*x)^(2/3)*b*(c^2/b)^(2/3) + sqrt(3)*(b*c^2*x^2 + a*c^2*x))/(b*c^2*x^2
+ a*c^2*x)) + a*(c^2/b)^(1/3)*x*log(((b*x^2 + a*x)^(1/3)*(c*x)^(1/3)*c*x +
 (b*x^2 + a*x)^(2/3)*(c*x)^(2/3)*(c^2/b)^(1/3) + (b*x^2 + a*x)*(c^2/b)^(2/
3))/(b*x^2 + a*x)) - 2*a*(c^2/b)^(1/3)*x*log(((b*x^2 + a*x)^(2/3)*(c*x)^(2
/3) - (b*x^2 + a*x)*(c^2/b)^(1/3))/(b*x^2 + a*x)) - 6*(b*x^2 + a*x)^(2/3)*
(c*x)^(2/3))/(b*x)
```

Sympy [F]

$$\int \frac{(cx)^{2/3}}{\sqrt[3]{ax+bx^2}}\,dx = \int \frac{(cx)^{\frac{2}{3}}}{\sqrt[3]{x\,(a+bx)}}\,dx$$

```
integrate((c*x)**(2/3)/(b*x**2+a*x)**(1/3),x)
```

```
Integral((c*x)**(2/3)/(x*(a + b*x))**(1/3), x)
```

Maxima [F]

$$\int \frac{(cx)^{2/3}}{\sqrt[3]{ax+bx^2}}\,dx = \int \frac{(cx)^{\frac{2}{3}}}{(bx^2+ax)^{\frac{1}{3}}}\,dx$$

```
integrate((c*x)^(2/3)/(b*x^2+a*x)^(1/3),x, algorithm="maxima")
```

```
integrate((c*x)^(2/3)/(b*x^2 + a*x)^(1/3), x)
```

Giac [A] (verification not implemented)

Time = 0.19 (sec) , antiderivative size = 163, normalized size of antiderivative = 0.69

$$\int \frac{(cx)^{2/3}}{\sqrt[3]{ax+bx^2}}\,dx = \frac{1}{6}\,ac^2\left(\frac{2\sqrt{3}(bc)^{\frac{2}{3}}\arctan\left(\frac{\sqrt{3}\left(2\left(bc+\frac{ac}{x}\right)^{\frac{1}{3}}+(bc)^{\frac{1}{3}}\right)}{3\,(bc)^{\frac{1}{3}}}\right)}{b^2c^2} + \frac{6\left(bc+\frac{ac}{x}\right)^{\frac{2}{3}}x}{abc^2} - \frac{(bc)^{\frac{2}{3}}\log\left(\left(bc+\frac{a}{x}\right.\right.}{}\right.$$

```
integrate((c*x)^(2/3)/(b*x^2+a*x)^(1/3),x, algorithm="giac")
```

```
1/6*a*c^2*(2*sqrt(3)*(b*c)^(2/3)*arctan(1/3*sqrt(3)*(2*(b*c + a*c/x)^(1/3)
 + (b*c)^(1/3))/(b*c)^(1/3))/(b^2*c^2) + 6*(b*c + a*c/x)^(2/3)*x/(a*b*c^2)
 - (b*c)^(2/3)*log((b*c + a*c/x)^(2/3) + (b*c + a*c/x)^(1/3)*(b*c)^(1/3) +
 (b*c)^(2/3))/(b^2*c^2) + 2*(b*c)^(2/3)*log(abs((b*c + a*c/x)^(1/3) - (b*c
)^(1/3)))/(b^2*c^2))
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(cx)^{2/3}}{\sqrt[3]{ax+bx^2}}\,dx = \int \frac{(c\,x)^{2/3}}{(b\,x^2+a\,x)^{1/3}}\,dx$$

```
int((c*x)^(2/3)/(a*x + b*x^2)^(1/3),x)
```

```
int((c*x)^(2/3)/(a*x + b*x^2)^(1/3), x)
```

Reduce [F]

$$\int \frac{(cx)^{2/3}}{\sqrt[3]{ax+bx^2}}\,dx = c^{\frac{2}{3}}\left(\int \frac{x^{\frac{1}{3}}}{(bx+a)^{\frac{1}{3}}}dx\right)$$

```
int((c*x)^(2/3)/(b*x^2+a*x)^(1/3),x)
```

```
c**(2/3)*int(x**(1/3)/(a + b*x)**(1/3),x)
```

3.129 $\int \frac{1}{\sqrt[3]{cx}\sqrt[3]{ax+bx^2}}\,dx$

Optimal result

Integrand size = 21, antiderivative size = 208

$$\int \frac{1}{\sqrt[3]{cx}\sqrt[3]{ax+bx^2}}\,dx = -\frac{\sqrt{3}\sqrt[3]{cx}\sqrt[3]{a+bx}\arctan\left(\frac{1}{\sqrt{3}}+\frac{2\sqrt[3]{c}\sqrt[3]{a+bx}}{\sqrt{3}\sqrt[3]{b}\sqrt[3]{cx}}\right)}{\sqrt[3]{b}c^{2/3}\sqrt[3]{ax+bx^2}} - \frac{\sqrt[3]{cx}\sqrt[3]{a+bx}\log(cx)}{2\sqrt[3]{b}c^{2/3}\sqrt[3]{ax+bx^2}} - \frac{3\sqrt[3]{cx}\sqrt[3]{a+bx}\log\left(1-\frac{\sqrt[3]{c}\sqrt[3]{a+bx}}{\sqrt[3]{b}\sqrt[3]{cx}}\right)}{2\sqrt[3]{b}c^{2/3}\sqrt[3]{ax+bx^2}}$$

```
-3^(1/2)*(c*x)^(1/3)*(b*x+a)^(1/3)*arctan(1/3*3^(1/2)+2/3*c^(1/3)*(b*x+a)^
(1/3)*3^(1/2)/b^(1/3)/(c*x)^(1/3))/b^(1/3)/c^(2/3)/(b*x^2+a*x)^(1/3)-1/2*(
c*x)^(1/3)*(b*x+a)^(1/3)*ln(c*x)/b^(1/3)/c^(2/3)/(b*x^2+a*x)^(1/3)-3/2*(c*
x)^(1/3)*(b*x+a)^(1/3)*ln(1-c^(1/3)*(b*x+a)^(1/3)/b^(1/3)/(c*x)^(1/3))/b^(
1/3)/c^(2/3)/(b*x^2+a*x)^(1/3)
```

Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 158, normalized size of antiderivative = 0.76

$$\int \frac{1}{\sqrt[3]{cx}\sqrt[3]{ax+bx^2}}\,dx$$
$$= \frac{x^{2/3}\sqrt[3]{a+bx}\Bigl(2\sqrt{3}\arctan\left(\frac{\sqrt{3}\sqrt[3]{b}\sqrt[3]{x}}{\sqrt[3]{b}\sqrt[3]{x}+2\sqrt[3]{a+bx}}\right)-2\log\left(-\sqrt[3]{b}\sqrt[3]{x}+\sqrt[3]{a+bx}\right)+\log\bigl(b^{2/3}x^{2/3}+\sqrt[3]{b}\sqrt[3]{x}}{2\sqrt[3]{b}\sqrt[3]{cx}\sqrt[3]{x(a+bx)}}$$

```
Integrate[1/((c*x)^(1/3)*(a*x + b*x^2)^(1/3)),x]
```

```
(x^(2/3)*(a + b*x)^(1/3)*(2*Sqrt[3]*ArcTan[(Sqrt[3]*b^(1/3)*x^(1/3))/(b^(1
/3)*x^(1/3) + 2*(a + b*x)^(1/3))] - 2*Log[-(b^(1/3)*x^(1/3)) + (a + b*x)^(
1/3)] + Log[b^(2/3)*x^(2/3) + b^(1/3)*x^(1/3)*(a + b*x)^(1/3) + (a + b*x)^
(2/3)]))/(2*b^(1/3)*(c*x)^(1/3)*(x*(a + b*x))^(1/3))
```

Rubi [A] (verified)

Time = 0.32 (sec) , antiderivative size = 124, normalized size of antiderivative = 0.60, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.095, Rules used = {1137, 71}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{\sqrt[3]{cx}\sqrt[3]{ax+bx^2}}\,dx$$
$$\Big\downarrow \text{ 1137}$$
$$\frac{x^{2/3}\sqrt[3]{a+bx}\int \frac{1}{x^{2/3}\sqrt[3]{a+bx}}dx}{\sqrt[3]{cx}\sqrt[3]{ax+bx^2}}$$
$$\Big\downarrow \text{ 71}$$

$$\frac{x^{2/3}\sqrt[3]{a+bx}\left(-\frac{\sqrt{3}\arctan\left(\frac{2\sqrt[3]{a+bx}}{\sqrt{3}\sqrt[3]{b}\sqrt[3]{x}}+\frac{1}{\sqrt{3}}\right)}{\sqrt[3]{b}}-\frac{3\log\left(\frac{\sqrt[3]{a+bx}}{\sqrt[3]{b}\sqrt[3]{x}}-1\right)}{2\sqrt[3]{b}}-\frac{\log(x)}{2\sqrt[3]{b}}\right)}{\sqrt[3]{cx}\sqrt[3]{ax+bx^2}}$$

```
Int[1/((c*x)^(1/3)*(a*x + b*x^2)^(1/3)),x]
```

```
(x^(2/3)*(a + b*x)^(1/3)*(-((Sqrt[3]*ArcTan[1/Sqrt[3] + (2*(a + b*x)^(1/3)
)/(Sqrt[3]*b^(1/3)*x^(1/3))])/b^(1/3)) - Log[x]/(2*b^(1/3)) - (3*Log[-1 +
(a + b*x)^(1/3)/(b^(1/3)*x^(1/3))])/(2*b^(1/3))))/((c*x)^(1/3)*(a*x + b*x^
2)^(1/3))
```

Defintions of rubi rules used

```
Int[1/(((a_.) + (b_.)*(x_))^(1/3)*((c_.) + (d_.)*(x_))^(2/3)), x_Symbol] :>
 With[{q = Rt[d/b, 3]}, Simp[(-Sqrt[3])*(q/d)*ArcTan[2*q*((a + b*x)^(1/3)/(
Sqrt[3]*(c + d*x)^(1/3))) + 1/Sqrt[3]], x] + (-Simp[3*(q/(2*d))*Log[q*((a +
 b*x)^(1/3)/(c + d*x)^(1/3)) - 1], x] - Simp[(q/(2*d))*Log[c + d*x], x])] /
; FreeQ[{a, b, c, d}, x] && PosQ[d/b]
```

```
Int[((e_.)*(x_))^(m_)*((b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(
e*x)^m*((b*x + c*x^2)^p/(x^(m + p)*(b + c*x)^p))   Int[x^(m + p)*(b + c*x)^
p, x], x] /; FreeQ[{b, c, e, m}, x]
```

Maple [F]

$$\int \frac{1}{(cx)^{\frac{1}{3}}\left(b\,x^{2}+ax\right)^{\frac{1}{3}}}dx$$

```
int(1/(c*x)^(1/3)/(b*x^2+a*x)^(1/3),x)
```

```
int(1/(c*x)^(1/3)/(b*x^2+a*x)^(1/3),x)
```

Fricas [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 547, normalized size of antiderivative = 2.63

$$\int \frac{1}{\sqrt[3]{cx}\sqrt[3]{ax+bx^2}}\,dx = \text{Too large to display}$$

```
integrate(1/(c*x)^(1/3)/(b*x^2+a*x)^(1/3),x, algorithm="fricas")
```

```
[1/2*(sqrt(3)*b*c*sqrt((-b*c)^(1/3)/(b*c))*log(-(3*b*c*x^2 + a*c*x + sqrt(
3)*(2*(b*x^2 + a*x)^(1/3)*(c*x)^(1/3)*b*c*x - (b*x^2 + a*x)^(2/3)*(-b*c)^(
2/3)*(c*x)^(2/3) + (b*c*x^2 + a*c*x)*(-b*c)^(1/3))*sqrt((-b*c)^(1/3)/(b*c)
) + 3*(b*x^2 + a*x)^(2/3)*(-b*c)^(1/3)*(c*x)^(2/3))/x) + (-b*c)^(2/3)*log(
((b*x^2 + a*x)^(1/3)*(c*x)^(1/3)*b*c*x + (b*x^2 + a*x)^(2/3)*(-b*c)^(2/3)*
(c*x)^(2/3) - (b*c*x^2 + a*c*x)*(-b*c)^(1/3))/(b*x^2 + a*x)) - 2*(-b*c)^(2
/3)*log(((b*x^2 + a*x)^(2/3)*(c*x)^(2/3)*b - (b*x^2 + a*x)*(-b*c)^(2/3))/(
b*x^2 + a*x)))/(b*c), 1/2*(2*sqrt(3)*b*c*sqrt(-(-b*c)^(1/3)/(b*c))*arctan(
1/3*sqrt(3)*(2*(b*x^2 + a*x)^(2/3)*(-b*c)^(2/3)*(c*x)^(2/3) - (b*c*x^2 + a
*c*x)*(-b*c)^(1/3))*sqrt(-(-b*c)^(1/3)/(b*c))/(b*c*x^2 + a*c*x)) + (-b*c)^
(2/3)*log(((b*x^2 + a*x)^(1/3)*(c*x)^(1/3)*b*c*x + (b*x^2 + a*x)^(2/3)*(-b
*c)^(2/3)*(c*x)^(2/3) - (b*c*x^2 + a*c*x)*(-b*c)^(1/3))/(b*x^2 + a*x)) - 2
*(-b*c)^(2/3)*log(((b*x^2 + a*x)^(2/3)*(c*x)^(2/3)*b - (b*x^2 + a*x)*(-b*c
)^(2/3))/(b*x^2 + a*x)))/(b*c)]
```

Sympy [F]

$$\int \frac{1}{\sqrt[3]{cx}\sqrt[3]{ax+bx^2}}\,dx = \int \frac{1}{\sqrt[3]{cx}\sqrt[3]{x(a+bx)}}\,dx$$

```
integrate(1/(c*x)**(1/3)/(b*x**2+a*x)**(1/3),x)
```

```
Integral(1/((c*x)**(1/3)*(x*(a + b*x))**(1/3)), x)
```

Maxima [F]

$$\int \frac{1}{\sqrt[3]{cx}\sqrt[3]{ax+bx^2}}\,dx = \int \frac{1}{(bx^2+ax)^{\frac{1}{3}}(cx)^{\frac{1}{3}}}\,dx$$

```
integrate(1/(c*x)^(1/3)/(b*x^2+a*x)^(1/3),x, algorithm="maxima")
```

```
integrate(1/((b*x^2 + a*x)^(1/3)*(c*x)^(1/3)), x)
```

Giac [A] (verification not implemented)

Time = 0.17 (sec) , antiderivative size = 133, normalized size of antiderivative = 0.64

$$\int \frac{1}{\sqrt[3]{cx}\sqrt[3]{ax+bx^2}}\,dx = -\frac{\sqrt{3}(bc)^{\frac{2}{3}}\arctan\left(\frac{\sqrt{3}\left(2\left(bc+\frac{ac}{x}\right)^{\frac{1}{3}}+(bc)^{\frac{1}{3}}\right)}{3(bc)^{\frac{1}{3}}}\right)}{bc} + \frac{(bc)^{\frac{2}{3}}\log\left(\left(bc+\frac{ac}{x}\right)^{\frac{2}{3}}+\left(bc+\frac{ac}{x}\right)^{\frac{1}{3}}(bc)^{\frac{1}{3}}+(bc)^{\frac{2}{3}}\right)}{2\,bc} - \frac{(bc)^{\frac{2}{3}}\log\left(\left|\left(bc+\frac{ac}{x}\right)^{\frac{1}{3}}-(bc)^{\frac{1}{3}}\right|\right)}{bc}$$

```
integrate(1/(c*x)^(1/3)/(b*x^2+a*x)^(1/3),x, algorithm="giac")
```

```
-sqrt(3)*(b*c)^(2/3)*arctan(1/3*sqrt(3)*(2*(b*c + a*c/x)^(1/3) + (b*c)^(1/
3))/(b*c)^(1/3))/(b*c) + 1/2*(b*c)^(2/3)*log((b*c + a*c/x)^(2/3) + (b*c +
a*c/x)^(1/3)*(b*c)^(1/3) + (b*c)^(2/3))/(b*c) - (b*c)^(2/3)*log(abs((b*c +
 a*c/x)^(1/3) - (b*c)^(1/3)))/(b*c)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{\sqrt[3]{cx}\sqrt[3]{ax+bx^2}}\,dx = \int \frac{1}{{\left(b\,x^2+a\,x\right)}^{1/3}\,{\left(c\,x\right)}^{1/3}}\,dx$$

```
int(1/((a*x + b*x^2)^(1/3)*(c*x)^(1/3)),x)
```

```
int(1/((a*x + b*x^2)^(1/3)*(c*x)^(1/3)), x)
```

Reduce [F]

$$\int \frac{1}{\sqrt[3]{cx}\sqrt[3]{ax+bx^2}}\,dx = \frac{\int \frac{1}{x^{\frac{2}{3}}(bx+a)^{\frac{1}{3}}}dx}{c^{\frac{1}{3}}}$$

```
int(1/(c*x)^(1/3)/(b*x^2+a*x)^(1/3),x)
```

```
int(1/(x**(2/3)*(a + b*x)**(1/3)),x)/c**(1/3)
```

3.130 $\int \frac{1}{(cx)^{2/3}\sqrt[3]{ax+bx^2}}\,dx$

Optimal result

Integrand size = 21, antiderivative size = 175

$$\int \frac{1}{(cx)^{2/3}\sqrt[3]{ax+bx^2}}\,dx = \frac{\sqrt{3}\sqrt[3]{cx}\sqrt[3]{a+bx}\arctan\left(\frac{\sqrt[3]{a}+2\sqrt[3]{a+bx}}{\sqrt{3}\sqrt[3]{a}}\right)}{\sqrt[3]{a}c\sqrt[3]{ax+bx^2}} - \frac{\sqrt[3]{cx}\sqrt[3]{a+bx}\log(x)}{2\sqrt[3]{a}c\sqrt[3]{ax+bx^2}} + \frac{3\sqrt[3]{cx}\sqrt[3]{a+bx}\log\left(\sqrt[3]{a}-\sqrt[3]{a+bx}\right)}{2\sqrt[3]{a}c\sqrt[3]{ax+bx^2}}$$

```
3^(1/2)*(c*x)^(1/3)*(b*x+a)^(1/3)*arctan(1/3*(a^(1/3)+2*(b*x+a)^(1/3))*3^(
1/2)/a^(1/3))/a^(1/3)/c/(b*x^2+a*x)^(1/3)-1/2*(c*x)^(1/3)*(b*x+a)^(1/3)*ln
(x)/a^(1/3)/c/(b*x^2+a*x)^(1/3)+3/2*(c*x)^(1/3)*(b*x+a)^(1/3)*ln(a^(1/3)-(
b*x+a)^(1/3))/a^(1/3)/c/(b*x^2+a*x)^(1/3)
```

Mathematica [A] (verified)

Time = 0.08 (sec) , antiderivative size = 122, normalized size of antiderivative = 0.70

$$\int \frac{1}{(cx)^{2/3}\sqrt[3]{ax+bx^2}}\,dx = \frac{(x(a+bx))^{2/3}\left(2\sqrt{3}\arctan\left(\frac{1+\frac{2\sqrt[3]{a+bx}}{\sqrt[3]{a}}}{\sqrt{3}}\right)+2\log\left(\sqrt[3]{a}-\sqrt[3]{a+bx}\right)-\log\left(\right.\right.}{2\sqrt[3]{a}(cx)^{2/3}(a+bx)^{2/3}}$$

```
Integrate[1/((c*x)^(2/3)*(a*x + b*x^2)^(1/3)),x]
```

```
((x*(a + b*x))^(2/3)*(2*Sqrt[3]*ArcTan[(1 + (2*(a + b*x)^(1/3))/a^(1/3))/S
qrt[3]] + 2*Log[a^(1/3) - (a + b*x)^(1/3)] - Log[a^(2/3) + a^(1/3)*(a + b*
x)^(1/3) + (a + b*x)^(2/3)]))/(2*a^(1/3)*(c*x)^(2/3)*(a + b*x)^(2/3))
```

Rubi [A] (verified)

Time = 0.37 (sec) , antiderivative size = 106, normalized size of antiderivative = 0.61, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.238$, Rules used = {1137, 67, 16, 1082, 217}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{(cx)^{2/3}\sqrt[3]{ax+bx^2}}dx$$

$$\Big\downarrow \ 1137$$

$$\frac{x\sqrt[3]{a+bx}\int \frac{1}{x\sqrt[3]{a+bx}}dx}{(cx)^{2/3}\sqrt[3]{ax+bx^2}}$$

$$\Big\downarrow \ 67$$

$$\frac{x\sqrt[3]{a+bx}\left(\frac{3}{2}\int \frac{1}{a^{2/3}+\sqrt[3]{a+bx}\sqrt[3]{a}+(a+bx)^{2/3}}d\sqrt[3]{a+bx}-\frac{3\int \frac{1}{\sqrt[3]{a}-\sqrt[3]{a+bx}}d\sqrt[3]{a+bx}}{2\sqrt[3]{a}}-\frac{\log(x)}{2\sqrt[3]{a}}\right)}{(cx)^{2/3}\sqrt[3]{ax+bx^2}}$$

$$\Big\downarrow \; 16$$

$$\frac{x\sqrt[3]{a+bx}\left(\frac{3}{2}\int \frac{1}{a^{2/3}+\sqrt[3]{a+bx}\sqrt[3]{a}+(a+bx)^{2/3}}d\sqrt[3]{a+bx}+\frac{3\log\left(\sqrt[3]{a}-\sqrt[3]{a+bx}\right)}{2\sqrt[3]{a}}-\frac{\log(x)}{2\sqrt[3]{a}}\right)}{(cx)^{2/3}\sqrt[3]{ax+bx^2}}$$

$$\Big\downarrow \; 1082$$

$$\frac{x\sqrt[3]{a+bx}\left(-\frac{3\int \frac{1}{-(a+bx)^{2/3}-3}d\left(\frac{2\sqrt[3]{a+bx}}{\sqrt[3]{a}}+1\right)}{\sqrt[3]{a}}+\frac{3\log\left(\sqrt[3]{a}-\sqrt[3]{a+bx}\right)}{2\sqrt[3]{a}}-\frac{\log(x)}{2\sqrt[3]{a}}\right)}{(cx)^{2/3}\sqrt[3]{ax+bx^2}}$$

$$\Big\downarrow \; 217$$

$$\frac{x\sqrt[3]{a+bx}\left(\frac{\sqrt{3}\arctan\left(\frac{\frac{2\sqrt[3]{a+bx}}{\sqrt[3]{a}}+1}{\sqrt{3}}\right)}{\sqrt[3]{a}}+\frac{3\log\left(\sqrt[3]{a}-\sqrt[3]{a+bx}\right)}{2\sqrt[3]{a}}-\frac{\log(x)}{2\sqrt[3]{a}}\right)}{(cx)^{2/3}\sqrt[3]{ax+bx^2}}$$

```
Int[1/((c*x)^(2/3)*(a*x + b*x^2)^(1/3)),x]
```

```
(x*(a + b*x)^(1/3)*((Sqrt[3]*ArcTan[(1 + (2*(a + b*x)^(1/3))/a^(1/3))/Sqrt
[3]])/a^(1/3) - Log[x]/(2*a^(1/3)) + (3*Log[a^(1/3) - (a + b*x)^(1/3)])/(2
*a^(1/3))))/((c*x)^(2/3)*(a*x + b*x^2)^(1/3))
```

Defintions of rubi rules used

```
Int[(c_.)/((a_.) + (b_.)*(x_)), x_Symbol] :> Simp[c*(Log[RemoveContent[a +
b*x, x]]/b), x] /; FreeQ[{a, b, c}, x]
```

```
Int[1/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(1/3)), x_Symbol] :> With[
{q = Rt[(b*c - a*d)/b, 3]}, Simp[-Log[RemoveContent[a + b*x, x]]/(2*b*q), x
] + (Simp[3/(2*b)   Subst[Int[1/(q^2 + q*x + x^2), x], x, (c + d*x)^(1/3)],
 x] - Simp[3/(2*b*q)   Subst[Int[1/(q - x), x], x, (c + d*x)^(1/3)], x])] /
; FreeQ[{a, b, c, d}, x] && PosQ[(b*c - a*d)/b]
```

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Simp[-2/b   Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b
)], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Fre
eQ[{a, b, c}, x]
```

```
Int[((e_.)*(x_))^(m_)*((b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(
e*x)^m*((b*x + c*x^2)^p/(x^(m + p)*(b + c*x)^p))   Int[x^(m + p)*(b + c*x)^
p, x], x] /; FreeQ[{b, c, e, m}, x]
```

Maple [F]

$$\int \frac{1}{(cx)^{\frac{2}{3}} \left(b\,x^{2}+ax\right)^{\frac{1}{3}}}dx$$

```
int(1/(c*x)^(2/3)/(b*x^2+a*x)^(1/3),x)
```


```
int(1/(c*x)^(2/3)/(b*x^2+a*x)^(1/3),x)
```

Fricas [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 562, normalized size of antiderivative = 3.21

$$\int \frac{1}{(cx)^{2/3}\sqrt[3]{ax+bx^2}}\,dx = \text{Too large to display}$$

```
integrate(1/(c*x)^(2/3)/(b*x^2+a*x)^(1/3),x, algorithm="fricas")
```

```
[1/2*(sqrt(3)*a*c*sqrt(-(a*c^2)^(1/3)/a)*log(-(b*c^2*x^2 + 3*a*c^2*x - 3*(
a*c^2)^(1/3)*(b*x^2 + a*x)^(2/3)*(c*x)^(1/3)*c - sqrt(3)*(2*(b*x^2 + a*x)^
(1/3)*(c*x)^(2/3)*a*c - (a*c^2)^(2/3)*(b*x^2 + a*x)^(2/3)*(c*x)^(1/3) - (b
*c*x^2 + a*c*x)*(a*c^2)^(1/3))*sqrt(-(a*c^2)^(1/3)/a))/x^2) + 2*(a*c^2)^(2
/3)*log(((b*x^2 + a*x)^(2/3)*(c*x)^(1/3)*a*c - (a*c^2)^(2/3)*(b*x^2 + a*x)
)/(b*x^2 + a*x)) - (a*c^2)^(2/3)*log(((b*x^2 + a*x)^(1/3)*(c*x)^(2/3)*a*c
+ (a*c^2)^(2/3)*(b*x^2 + a*x)^(2/3)*(c*x)^(1/3) + (b*c*x^2 + a*c*x)*(a*c^2
)^(1/3))/(b*x^2 + a*x)))/(a*c^2), -1/2*(2*sqrt(3)*a*c*sqrt((a*c^2)^(1/3)/a
)*arctan(1/3*sqrt(3)*(2*(a*c^2)^(2/3)*(b*x^2 + a*x)^(2/3)*(c*x)^(1/3) + (b
*c*x^2 + a*c*x)*(a*c^2)^(1/3))*sqrt((a*c^2)^(1/3)/a)/(b*c^2*x^2 + a*c^2*x)
) - 2*(a*c^2)^(2/3)*log(((b*x^2 + a*x)^(2/3)*(c*x)^(1/3)*a*c - (a*c^2)^(2/
3)*(b*x^2 + a*x))/(b*x^2 + a*x)) + (a*c^2)^(2/3)*log(((b*x^2 + a*x)^(1/3)*
(c*x)^(2/3)*a*c + (a*c^2)^(2/3)*(b*x^2 + a*x)^(2/3)*(c*x)^(1/3) + (b*c*x^2
 + a*c*x)*(a*c^2)^(1/3))/(b*x^2 + a*x)))/(a*c^2)]
```

Sympy [F]

$$\int \frac{1}{(cx)^{2/3}\sqrt[3]{ax+bx^2}}\,dx = \int \frac{1}{(cx)^{\frac{2}{3}}\sqrt[3]{x(a+bx)}}\,dx$$

```
integrate(1/(c*x)**(2/3)/(b*x**2+a*x)**(1/3),x)
```

```
Integral(1/((c*x)**(2/3)*(x*(a + b*x))**(1/3)), x)
```

Maxima [F]

$$\int \frac{1}{(cx)^{2/3}\sqrt[3]{ax+bx^2}}\,dx = \int \frac{1}{(bx^2+ax)^{\frac{1}{3}}\,(cx)^{\frac{2}{3}}}\,dx$$

```
integrate(1/(c*x)^(2/3)/(b*x^2+a*x)^(1/3),x, algorithm="maxima")
```

```
integrate(1/((b*x^2 + a*x)^(1/3)*(c*x)^(2/3)), x)
```

Giac [F]

$$\int \frac{1}{(cx)^{2/3}\sqrt[3]{ax+bx^2}}\,dx = \int \frac{1}{(bx^2+ax)^{\frac{1}{3}}\,(cx)^{\frac{2}{3}}}\,dx$$

```
integrate(1/(c*x)^(2/3)/(b*x^2+a*x)^(1/3),x, algorithm="giac")
```

```
integrate(1/((b*x^2 + a*x)^(1/3)*(c*x)^(2/3)), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{(cx)^{2/3}\sqrt[3]{ax+bx^2}}\,dx = \int \frac{1}{(b\,x^2+a\,x)^{1/3}\,(c\,x)^{2/3}}\,dx$$

```
int(1/((a*x + b*x^2)^(1/3)*(c*x)^(2/3)),x)
```

```
int(1/((a*x + b*x^2)^(1/3)*(c*x)^(2/3)), x)
```

Reduce [B] (verification not implemented)

Time = 0.48 (sec) , antiderivative size = 144, normalized size of antiderivative = 0.82

$$\int \frac{1}{(cx)^{2/3}\sqrt[3]{ax+bx^2}}\,dx = \frac{-2\sqrt{3}\,atan\left(\frac{2(bx+a)^{\frac{1}{6}}+a^{\frac{1}{6}}}{a^{\frac{1}{6}}\sqrt{3}}\right)+2\sqrt{3}\,atan\left(\frac{2(bx+a)^{\frac{1}{6}}-a^{\frac{1}{6}}}{a^{\frac{1}{6}}\sqrt{3}}\right)+2\log\left((bx+a)^{\frac{1}{6}}+a^{\frac{1}{6}}\right)}{}$$

```
int(1/(c*x)^(2/3)/(b*x^2+a*x)^(1/3),x)
```

```
( - 2*sqrt(3)*atan((2*(a + b*x)**(1/6) + a**(1/6))/(a**(1/6)*sqrt(3))) + 2
*sqrt(3)*atan((2*(a + b*x)**(1/6) - a**(1/6))/(a**(1/6)*sqrt(3))) + 2*log(
(a + b*x)**(1/6) + a**(1/6)) + 2*log((a + b*x)**(1/6) - a**(1/6)) - log( -
 a**(1/6)*(a + b*x)**(1/6) + (a + b*x)**(1/3) + a**(1/3)) - log(a**(1/6)*(
a + b*x)**(1/6) + (a + b*x)**(1/3) + a**(1/3)))/(2*c**(2/3)*a**(1/3))
```

3.131 $\int \frac{1}{(cx)^{5/3}\sqrt[3]{ax+bx^2}}\,dx$

Optimal result

Integrand size = 21, antiderivative size = 204

$$\begin{aligned}\int \frac{1}{(cx)^{5/3}\sqrt[3]{ax+bx^2}}\,dx &= -\frac{(ax+bx^2)^{2/3}}{a(cx)^{5/3}} \\ &\quad - \frac{b\sqrt[3]{cx}\sqrt[3]{a+bx}\arctan\left(\frac{\sqrt[3]{a}+2\sqrt[3]{a+bx}}{\sqrt{3}\sqrt[3]{a}}\right)}{\sqrt{3}a^{4/3}c^2\sqrt[3]{ax+bx^2}} \\ &\quad + \frac{b\sqrt[3]{cx}\sqrt[3]{a+bx}\log(x)}{6a^{4/3}c^2\sqrt[3]{ax+bx^2}} - \frac{b\sqrt[3]{cx}\sqrt[3]{a+bx}\log\left(\sqrt[3]{a}-\sqrt[3]{a+bx}\right)}{2a^{4/3}c^2\sqrt[3]{ax+bx^2}}\end{aligned}$$

```
-(b*x^2+a*x)^(2/3)/a/(c*x)^(5/3)-1/3*b*(c*x)^(1/3)*(b*x+a)^(1/3)*arctan(1/
3*(a^(1/3)+2*(b*x+a)^(1/3))*3^(1/2)/a^(1/3))*3^(1/2)/a^(4/3)/c^2/(b*x^2+a*
x)^(1/3)+1/6*b*(c*x)^(1/3)*(b*x+a)^(1/3)*ln(x)/a^(4/3)/c^2/(b*x^2+a*x)^(1/
3)-1/2*b*(c*x)^(1/3)*(b*x+a)^(1/3)*ln(a^(1/3)-(b*x+a)^(1/3))/a^(4/3)/c^2/(
b*x^2+a*x)^(1/3)
```

Mathematica [A] (verified)

Time = 0.17 (sec) , antiderivative size = 163, normalized size of antiderivative = 0.80

$$\int \frac{1}{(cx)^{5/3}\sqrt[3]{ax+bx^2}}\,dx =$$
$$-\frac{x\left(6a^{4/3}+6\sqrt[3]{a}bx+2\sqrt{3}bx\sqrt[3]{a+bx}\arctan\left(\frac{1+\frac{2\sqrt[3]{a+bx}}{\sqrt[3]{a}}}{\sqrt{3}}\right)+2bx\sqrt[3]{a+bx}\log\left(\sqrt[3]{a}-\sqrt[3]{a+bx}\right)-bx\right.}{6a^{4/3}(cx)^{5/3}\sqrt[3]{x(a+bx)}}$$

```
Integrate[1/((c*x)^(5/3)*(a*x + b*x^2)^(1/3)),x]
```

```
-1/6*(x*(6*a^(4/3) + 6*a^(1/3)*b*x + 2*Sqrt[3]*b*x*(a + b*x)^(1/3)*ArcTan[
(1 + (2*(a + b*x)^(1/3))/a^(1/3))/Sqrt[3]] + 2*b*x*(a + b*x)^(1/3)*Log[a^(
1/3) - (a + b*x)^(1/3)] - b*x*(a + b*x)^(1/3)*Log[a^(2/3) + a^(1/3)*(a + b
*x)^(1/3) + (a + b*x)^(2/3)]))/(a^(4/3)*(c*x)^(5/3)*(x*(a + b*x))^(1/3))
```

Rubi [A] (verified)

Time = 0.40 (sec) , antiderivative size = 134, normalized size of antiderivative = 0.66, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.286$, Rules used = {1137, 52, 67, 16, 1082, 217}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{(cx)^{5/3}\sqrt[3]{ax+bx^2}}dx$$
$$\Big\downarrow \text{1137}$$
$$\frac{x^2\sqrt[3]{a+bx}\int \frac{1}{x^2\sqrt[3]{a+bx}}dx}{(cx)^{5/3}\sqrt[3]{ax+bx^2}}$$
$$\Big\downarrow \text{52}$$

$$\frac{x^2\sqrt[3]{a+bx}\left(-\frac{b\int \frac{1}{x\sqrt[3]{a+bx}}dx}{3a}-\frac{(a+bx)^{2/3}}{ax}\right)}{(cx)^{5/3}\sqrt[3]{ax+bx^2}}$$

$$\Big\downarrow \text{67}$$

$$\frac{x^2\sqrt[3]{a+bx}\left(-\frac{b\left(\frac{3}{2}\int \frac{1}{a^{2/3}+\sqrt[3]{a+bx}\sqrt[3]{a}+(a+bx)^{2/3}}d\sqrt[3]{a+bx}-\frac{3\int \frac{1}{\sqrt[3]{a}-\sqrt[3]{a+bx}}d\sqrt[3]{a+bx}}{2\sqrt[3]{a}}-\frac{\log(x)}{2\sqrt[3]{a}}\right)}{3a}-\frac{(a+bx)^{2/3}}{ax}\right)}{(cx)^{5/3}\sqrt[3]{ax+bx^2}}$$

$$\Big\downarrow \text{16}$$

$$\frac{x^2\sqrt[3]{a+bx}\left(-\frac{b\left(\frac{3}{2}\int \frac{1}{a^{2/3}+\sqrt[3]{a+bx}\sqrt[3]{a}+(a+bx)^{2/3}}d\sqrt[3]{a+bx}+\frac{3\log\left(\sqrt[3]{a}-\sqrt[3]{a+bx}\right)}{2\sqrt[3]{a}}-\frac{\log(x)}{2\sqrt[3]{a}}\right)}{3a}-\frac{(a+bx)^{2/3}}{ax}\right)}{(cx)^{5/3}\sqrt[3]{ax+bx^2}}$$

$$\Big\downarrow \text{1082}$$

$$\frac{x^2\sqrt[3]{a+bx}\left(-\frac{b\left(-\frac{3\int \frac{1}{-(a+bx)^{2/3}-3}d\left(\frac{2\sqrt[3]{a+bx}}{\sqrt[3]{a}}+1\right)}{\sqrt[3]{a}}+\frac{3\log\left(\sqrt[3]{a}-\sqrt[3]{a+bx}\right)}{2\sqrt[3]{a}}-\frac{\log(x)}{2\sqrt[3]{a}}\right)}{3a}-\frac{(a+bx)^{2/3}}{ax}\right)}{(cx)^{5/3}\sqrt[3]{ax+bx^2}}$$

$$\Big\downarrow \text{217}$$

$$\frac{x^2\sqrt[3]{a+bx}\left(-\frac{b\left(\frac{\sqrt{3}\arctan\left(\frac{\frac{2\sqrt[3]{a+bx}}{\sqrt[3]{a}}+1}{\sqrt{3}}\right)}{\sqrt[3]{a}}+\frac{3\log\left(\sqrt[3]{a}-\sqrt[3]{a+bx}\right)}{2\sqrt[3]{a}}-\frac{\log(x)}{2\sqrt[3]{a}}\right)}{3a}-\frac{(a+bx)^{2/3}}{ax}\right)}{(cx)^{5/3}\sqrt[3]{ax+bx^2}}$$

```
Int[1/((c*x)^(5/3)*(a*x + b*x^2)^(1/3)),x]
```

```
(x^2*(a + b*x)^(1/3)*(-((a + b*x)^(2/3)/(a*x)) - (b*((Sqrt[3]*ArcTan[(1 +
(2*(a + b*x)^(1/3))/a^(1/3))/Sqrt[3]])/a^(1/3) - Log[x]/(2*a^(1/3)) + (3*L
og[a^(1/3) - (a + b*x)^(1/3)])/(2*a^(1/3))))/(3*a)))/((c*x)^(5/3)*(a*x + b
*x^2)^(1/3))
```

Defintions of rubi rules used

```
Int[(c_.)/((a_.) + (b_.)*(x_)), x_Symbol] :> Simp[c*(Log[RemoveContent[a +
b*x, x]]/b), x] /; FreeQ[{a, b, c}, x]
```

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] - Simp[d*((
m + n + 2)/((b*c - a*d)*(m + 1)))   Int[(a + b*x)^(m + 1)*(c + d*x)^n, x],
x] /; FreeQ[{a, b, c, d, n}, x] && ILtQ[m, -1] && FractionQ[n] && LtQ[n, 0]
```

```
Int[1/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(1/3)), x_Symbol] :> With[
{q = Rt[(b*c - a*d)/b, 3]}, Simp[-Log[RemoveContent[a + b*x, x]]/(2*b*q), x
] + (Simp[3/(2*b)   Subst[Int[1/(q^2 + q*x + x^2), x], x, (c + d*x)^(1/3)],
 x] - Simp[3/(2*b*q)   Subst[Int[1/(q - x), x], x, (c + d*x)^(1/3)], x])] /
; FreeQ[{a, b, c, d}, x] && PosQ[(b*c - a*d)/b]
```

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Simp[-2/b   Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b
)], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Fre
eQ[{a, b, c}, x]
```

```
Int[((e_.)*(x_))^(m_)*((b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(
e*x)^m*((b*x + c*x^2)^p/(x^(m + p)*(b + c*x)^p))   Int[x^(m + p)*(b + c*x)^
p, x], x] /; FreeQ[{b, c, e, m}, x]
```

Maple [A] (verified)

Time = 0.37 (sec) , antiderivative size = 194, normalized size of antiderivative = 0.95

method	result
risch	$-\frac{bx+a}{ac(cx)^{\frac{2}{3}}(x(bx+a))^{\frac{1}{3}}}-\frac{b\left(\frac{\ln\left(\left(c^{2}bx+a\,c^{2}\right)^{\frac{1}{3}}-\left(a\,c^{2}\right)^{\frac{1}{3}}\right)}{\left(a\,c^{2}\right)^{\frac{1}{3}}}-\frac{\ln\left(\left(c^{2}bx+a\,c^{2}\right)^{\frac{2}{3}}+\left(a\,c^{2}\right)^{\frac{1}{3}}\left(c^{2}bx+a\,c^{2}\right)^{\frac{1}{3}}+\left(a\,c^{2}\right)^{\frac{2}{3}}\right)}{2\left(a\,c^{2}\right)^{\frac{1}{3}}}+\frac{\sqrt{3}\arctan\left(\frac{\sqrt{3}\left(\ldots\right.}{\ldots}\right.}{\left(\ldots\right.}\right.}{3ac(cx)^{\frac{2}{3}}(x(bx+a))^{\frac{1}{3}}}$

```
int(1/(c*x)^(5/3)/(b*x^2+a*x)^(1/3),x,method=_RETURNVERBOSE)
```

```
-1/a*(b*x+a)/c/(c*x)^(2/3)/(x*(b*x+a))^(1/3)-1/3*b/a*(1/(a*c^2)^(1/3)*ln((
b*c^2*x+a*c^2)^(1/3)-(a*c^2)^(1/3))-1/2/(a*c^2)^(1/3)*ln((b*c^2*x+a*c^2)^(
2/3)+(a*c^2)^(1/3)*(b*c^2*x+a*c^2)^(1/3)+(a*c^2)^(2/3))+3^(1/2)/(a*c^2)^(1
/3)*arctan(1/3*3^(1/2)*(2/(a*c^2)^(1/3)*(b*c^2*x+a*c^2)^(1/3)+1)))/c/(c*x)
^(2/3)*(c^2*(b*x+a))^(1/3)/(x*(b*x+a))^(1/3)*x
```

Fricas [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 652, normalized size of antiderivative = 3.20

$$\int \frac{1}{(cx)^{5/3}\sqrt[3]{ax+bx^2}}\,dx = \text{Too large to display}$$

```
integrate(1/(c*x)^(5/3)/(b*x^2+a*x)^(1/3),x, algorithm="fricas")
```

```
[1/6*(3*sqrt(1/3)*a*b*c*x^2*sqrt((-a*c^2)^(1/3)/a)*log(-(b*c^2*x^2 + 3*a*c
^2*x + 3*(-a*c^2)^(1/3)*(b*x^2 + a*x)^(2/3)*(c*x)^(1/3)*c + 3*sqrt(1/3)*(2
*(b*x^2 + a*x)^(1/3)*(c*x)^(2/3)*a*c - (-a*c^2)^(2/3)*(b*x^2 + a*x)^(2/3)*
(c*x)^(1/3) + (b*c*x^2 + a*c*x)*(-a*c^2)^(1/3))*sqrt((-a*c^2)^(1/3)/a))/x^
2) - 2*(-a*c^2)^(2/3)*b*x^2*log(((b*x^2 + a*x)^(2/3)*(c*x)^(1/3)*a*c - (-a
*c^2)^(2/3)*(b*x^2 + a*x))/(b*x^2 + a*x)) + (-a*c^2)^(2/3)*b*x^2*log(((b*x
^2 + a*x)^(1/3)*(c*x)^(2/3)*a*c + (-a*c^2)^(2/3)*(b*x^2 + a*x)^(2/3)*(c*x)
^(1/3) - (b*c*x^2 + a*c*x)*(-a*c^2)^(1/3))/(b*x^2 + a*x)) - 6*(b*x^2 + a*x
)^(2/3)*(c*x)^(1/3)*a*c)/(a^2*c^3*x^2), 1/6*(6*sqrt(1/3)*a*b*c*x^2*sqrt(-(
-a*c^2)^(1/3)/a)*arctan(sqrt(1/3)*(2*(-a*c^2)^(2/3)*(b*x^2 + a*x)^(2/3)*(c
*x)^(1/3) - (b*c*x^2 + a*c*x)*(-a*c^2)^(1/3))*sqrt(-(-a*c^2)^(1/3)/a)/(b*c
^2*x^2 + a*c^2*x)) - 2*(-a*c^2)^(2/3)*b*x^2*log(((b*x^2 + a*x)^(2/3)*(c*x)
^(1/3)*a*c - (-a*c^2)^(2/3)*(b*x^2 + a*x))/(b*x^2 + a*x)) + (-a*c^2)^(2/3)
*b*x^2*log(((b*x^2 + a*x)^(1/3)*(c*x)^(2/3)*a*c + (-a*c^2)^(2/3)*(b*x^2 +
a*x)^(2/3)*(c*x)^(1/3) - (b*c*x^2 + a*c*x)*(-a*c^2)^(1/3))/(b*x^2 + a*x))
- 6*(b*x^2 + a*x)^(2/3)*(c*x)^(1/3)*a*c)/(a^2*c^3*x^2)]
```

Sympy [F]

$$\int \frac{1}{(cx)^{5/3}\sqrt[3]{ax+bx^2}}\,dx = \int \frac{1}{(cx)^{\frac{5}{3}}\sqrt[3]{x\,(a+bx)}}\,dx$$

```
integrate(1/(c*x)**(5/3)/(b*x**2+a*x)**(1/3),x)
```

```
Integral(1/((c*x)**(5/3)*(x*(a + b*x))**(1/3)), x)
```

Maxima [F]

$$\int \frac{1}{(cx)^{5/3}\sqrt[3]{ax+bx^2}}\,dx = \int \frac{1}{(bx^2+ax)^{\frac{1}{3}}\,(cx)^{\frac{5}{3}}}\,dx$$

```
integrate(1/(c*x)^(5/3)/(b*x^2+a*x)^(1/3),x, algorithm="maxima")
```

```
integrate(1/((b*x^2 + a*x)^(1/3)*(c*x)^(5/3)), x)
```

Giac [F]

$$\int \frac{1}{(cx)^{5/3}\sqrt[3]{ax+bx^2}}\,dx = \int \frac{1}{(bx^2+ax)^{\frac{1}{3}}\,(cx)^{\frac{5}{3}}}\,dx$$

```
integrate(1/(c*x)^(5/3)/(b*x^2+a*x)^(1/3),x, algorithm="giac")
```

```
integrate(1/((b*x^2 + a*x)^(1/3)*(c*x)^(5/3)), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{(cx)^{5/3}\sqrt[3]{ax+bx^2}}\,dx = \int \frac{1}{\left(b\,x^2+a\,x\right)^{1/3}\,\left(c\,x\right)^{5/3}}\,dx$$

```
int(1/((a*x + b*x^2)^(1/3)*(c*x)^(5/3)),x)
```

```
int(1/((a*x + b*x^2)^(1/3)*(c*x)^(5/3)), x)
```

Reduce [B] (verification not implemented)

Time = 0.58 (sec) , antiderivative size = 169, normalized size of antiderivative = 0.83

$$\int \frac{1}{(cx)^{5/3}\sqrt[3]{ax+bx^2}}\,dx = \frac{2\sqrt{3}\,atan\left(\frac{2(bx+a)^{\frac{1}{6}}+a^{\frac{1}{6}}}{a^{\frac{1}{6}}\sqrt{3}}\right)bx-2\sqrt{3}\,atan\left(\frac{2(bx+a)^{\frac{1}{6}}-a^{\frac{1}{6}}}{a^{\frac{1}{6}}\sqrt{3}}\right)bx-6a^{\frac{1}{3}}(bx+a)^{\frac{2}{3}}-2\,\mathrm{lo}}{}$$

```
int(1/(c*x)^(5/3)/(b*x^2+a*x)^(1/3),x)
```

```
(2*sqrt(3)*atan((2*(a + b*x)**(1/6) + a**(1/6))/(a**(1/6)*sqrt(3)))*b*x -
2*sqrt(3)*atan((2*(a + b*x)**(1/6) - a**(1/6))/(a**(1/6)*sqrt(3)))*b*x - 6
*a**(1/3)*(a + b*x)**(2/3) - 2*log((a + b*x)**(1/6) + a**(1/6))*b*x - 2*lo
g((a + b*x)**(1/6) - a**(1/6))*b*x + log( - a**(1/6)*(a + b*x)**(1/6) + (a
 + b*x)**(1/3) + a**(1/3))*b*x + log(a**(1/6)*(a + b*x)**(1/6) + (a + b*x)
**(1/3) + a**(1/3))*b*x)/(6*c**(2/3)*a**(1/3)*a*c*x)
```

3.132 $\int \frac{1}{(cx)^{8/3}\sqrt[3]{ax+bx^2}}\,dx$

Optimal result

Integrand size = 21, antiderivative size = 245

$$\int \frac{1}{(cx)^{8/3}\sqrt[3]{ax+bx^2}}\,dx = -\frac{(ax+bx^2)^{2/3}}{2a(cx)^{8/3}} + \frac{2b(ax+bx^2)^{2/3}}{3a^2c(cx)^{5/3}} + \frac{2b^2\sqrt[3]{cx}\sqrt[3]{a+bx}\arctan\left(\frac{\sqrt[3]{a}+2\sqrt[3]{a+bx}}{\sqrt{3}\sqrt[3]{a}}\right)}{3\sqrt{3}a^{7/3}c^3\sqrt[3]{ax+bx^2}} - \frac{b^2\sqrt[3]{cx}\sqrt[3]{a+bx}\log(x)}{9a^{7/3}c^3\sqrt[3]{ax+bx^2}} + \frac{b^2\sqrt[3]{cx}\sqrt[3]{a+bx}\log\left(\sqrt[3]{a}-\sqrt[3]{a+bx}\right)}{3a^{7/3}c^3\sqrt[3]{ax+bx^2}}$$

```
-1/2*(b*x^2+a*x)^(2/3)/a/(c*x)^(8/3)+2/3*b*(b*x^2+a*x)^(2/3)/a^2/c/(c*x)^(
5/3)+2/9*b^2*(c*x)^(1/3)*(b*x+a)^(1/3)*arctan(1/3*(a^(1/3)+2*(b*x+a)^(1/3)
)*3^(1/2)/a^(1/3))*3^(1/2)/a^(7/3)/c^3/(b*x^2+a*x)^(1/3)-1/9*b^2*(c*x)^(1/
3)*(b*x+a)^(1/3)*ln(x)/a^(7/3)/c^3/(b*x^2+a*x)^(1/3)+1/3*b^2*(c*x)^(1/3)*(
b*x+a)^(1/3)*ln(a^(1/3)-(b*x+a)^(1/3))/a^(7/3)/c^3/(b*x^2+a*x)^(1/3)
```

Mathematica [A] (verified)

Time = 0.20 (sec) , antiderivative size = 188, normalized size of antiderivative = 0.77

$$\int \frac{1}{(cx)^{8/3}\sqrt[3]{ax+bx^2}}\,dx = \frac{x\left(-9a^{7/3}+3a^{4/3}bx+12\sqrt[3]{a}b^2x^2+4\sqrt{3}b^2x^2\sqrt[3]{a+bx}\arctan\left(\frac{1+\frac{2\sqrt[3]{a+bx}}{\sqrt[3]{a}}}{\sqrt{3}}\right)\right.}{18a^7}$$

```
Integrate[1/((c*x)^(8/3)*(a*x + b*x^2)^(1/3)),x]
```

```
(x*(-9*a^(7/3) + 3*a^(4/3)*b*x + 12*a^(1/3)*b^2*x^2 + 4*Sqrt[3]*b^2*x^2*(a
 + b*x)^(1/3)*ArcTan[(1 + (2*(a + b*x)^(1/3))/a^(1/3))/Sqrt[3]] + 4*b^2*x^
2*(a + b*x)^(1/3)*Log[a^(1/3) - (a + b*x)^(1/3)] - 2*b^2*x^2*(a + b*x)^(1/
3)*Log[a^(2/3) + a^(1/3)*(a + b*x)^(1/3) + (a + b*x)^(2/3)]))/(18*a^(7/3)*
(c*x)^(8/3)*(x*(a + b*x))^(1/3))
```

Rubi [A] (verified)

Time = 0.42 (sec) , antiderivative size = 162, normalized size of antiderivative = 0.66, number of steps used = 8, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}} = 0.333$, Rules used = {1137, 52, 52, 67, 16, 1082, 217}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{(cx)^{8/3}\sqrt[3]{ax+bx^2}}dx$$

$$\Big\downarrow \text{1137}$$

$$\frac{x^3\sqrt[3]{a+bx}\int \frac{1}{x^3\sqrt[3]{a+bx}}dx}{(cx)^{8/3}\sqrt[3]{ax+bx^2}}$$

$$\Big\downarrow \text{52}$$

$$\frac{x^3\sqrt[3]{a+bx}\left(-\frac{2b\int \frac{1}{x^2\sqrt[3]{a+bx}}dx}{3a}-\frac{(a+bx)^{2/3}}{2ax^2}\right)}{(cx)^{8/3}\sqrt[3]{ax+bx^2}}$$

$$\Big\downarrow\ 52$$

$$\frac{x^3\sqrt[3]{a+bx}\left(-\frac{2b\left(-\frac{b\int\frac{1}{x\sqrt[3]{a+bx}}dx}{3a}-\frac{(a+bx)^{2/3}}{ax}\right)}{3a}-\frac{(a+bx)^{2/3}}{2ax^2}\right)}{(cx)^{8/3}\sqrt[3]{ax+bx^2}}$$

$$\Big\downarrow\ 67$$

$$\frac{x^3\sqrt[3]{a+bx}\left(-\frac{2b\left(-\frac{b\left(\frac{3}{2}\int\frac{1}{a^{2/3}+\sqrt[3]{a+bx}\sqrt[3]{a}+(a+bx)^{2/3}}d\sqrt[3]{a+bx}-\frac{3\int\frac{1}{\sqrt[3]{a}-\sqrt[3]{a+bx}}d\sqrt[3]{a+bx}}{2\sqrt[3]{a}}-\frac{\log(x)}{2\sqrt[3]{a}}\right)}{3a}-\frac{(a+bx)^{2/3}}{ax}\right)}{3a}-(a\right)}{(cx)^{8/3}\sqrt[3]{ax+bx^2}}$$

$$\Big\downarrow\ 16$$

$$\frac{x^3\sqrt[3]{a+bx}\left(-\frac{2b\left(-\frac{b\left(\frac{3}{2}\int\frac{1}{a^{2/3}+\sqrt[3]{a+bx}\sqrt[3]{a}+(a+bx)^{2/3}}d\sqrt[3]{a+bx}+\frac{3\log\left(\sqrt[3]{a}-\sqrt[3]{a+bx}\right)}{2\sqrt[3]{a}}-\frac{\log(x)}{2\sqrt[3]{a}}\right)}{3a}-\frac{(a+bx)^{2/3}}{ax}\right)}{3a}-\frac{(a+bx)^{2/3}}{2ax^2}\right)}{(cx)^{8/3}\sqrt[3]{ax+bx^2}}$$

$$\Big\downarrow \text{1082}$$

$$\frac{x^3\sqrt[3]{a+bx}\left(-\frac{2b\left(-\frac{b\left(-\frac{3\int\frac{1}{-(a+bx)^{2/3}-3}d\left(\frac{2\sqrt[3]{a+bx}}{\sqrt[3]{a}}+1\right)}{\sqrt[3]{a}}+\frac{3\log\left(\sqrt[3]{a}-\sqrt[3]{a+bx}\right)}{2\sqrt[3]{a}}-\frac{\log(x)}{2\sqrt[3]{a}}\right)}{3a}-\frac{(a+bx)^{2/3}}{ax}\right)}{3a}-\frac{(a+bx)^{2/3}}{2ax^2}\right)}{(cx)^{8/3}\sqrt[3]{ax+bx^2}}$$

$$\Big\downarrow \text{217}$$

$$\frac{x^3\sqrt[3]{a+bx}\left(-\frac{2b\left(-\frac{b\left(\frac{\sqrt{3}\arctan\left(\frac{\frac{2\sqrt[3]{a+bx}}{\sqrt[3]{a}}+1}{\sqrt{3}}\right)}{\sqrt[3]{a}}+\frac{3\log\left(\sqrt[3]{a}-\sqrt[3]{a+bx}\right)}{2\sqrt[3]{a}}-\frac{\log(x)}{2\sqrt[3]{a}}\right)}{3a}-\frac{(a+bx)^{2/3}}{ax}\right)}{3a}-\frac{(a+bx)^{2/3}}{2ax^2}\right)}{(cx)^{8/3}\sqrt[3]{ax+bx^2}}$$

```
Int[1/((c*x)^(8/3)*(a*x + b*x^2)^(1/3)),x]
```

```
(x^3*(a + b*x)^(1/3)*(-1/2*(a + b*x)^(2/3)/(a*x^2) - (2*b*(-((a + b*x)^(2/
3)/(a*x)) - (b*((Sqrt[3]*ArcTan[(1 + (2*(a + b*x)^(1/3))/a^(1/3))/Sqrt[3]]
)/a^(1/3) - Log[x]/(2*a^(1/3)) + (3*Log[a^(1/3) - (a + b*x)^(1/3)])/(2*a^(
1/3))))/(3*a)))/(3*a)))/((c*x)^(8/3)*(a*x + b*x^2)^(1/3))
```

Defintions of rubi rules used

```
Int[(c_.)/((a_.) + (b_.)*(x_)), x_Symbol] :> Simp[c*(Log[RemoveContent[a +
b*x, x]]/b), x] /; FreeQ[{a, b, c}, x]
```

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] - Simp[d*((
m + n + 2)/((b*c - a*d)*(m + 1)))   Int[(a + b*x)^(m + 1)*(c + d*x)^n, x],
x] /; FreeQ[{a, b, c, d, n}, x] && ILtQ[m, -1] && FractionQ[n] && LtQ[n, 0]
```

```
Int[1/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(1/3)), x_Symbol] :> With[
{q = Rt[(b*c - a*d)/b, 3]}, Simp[-Log[RemoveContent[a + b*x, x]]/(2*b*q), x
] + (Simp[3/(2*b)   Subst[Int[1/(q^2 + q*x + x^2), x], x, (c + d*x)^(1/3)],
 x] - Simp[3/(2*b*q)   Subst[Int[1/(q - x), x], x, (c + d*x)^(1/3)], x])] /
; FreeQ[{a, b, c, d}, x] && PosQ[(b*c - a*d)/b]
```

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Simp[-2/b   Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b
)], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Fre
eQ[{a, b, c}, x]
```

```
Int[((e_.)*(x_))^(m_)*((b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(
e*x)^m*((b*x + c*x^2)^p/(x^(m + p)*(b + c*x)^p))   Int[x^(m + p)*(b + c*x)^
p, x], x] /; FreeQ[{b, c, e, m}, x]
```

Maple [A] (verified)

Time = 0.37 (sec) , antiderivative size = 207, normalized size of antiderivative = 0.84

method	result
risch	$-\frac{(bx+a)(-4bx+3a)}{6a^2 x\, c^2 (cx)^{\frac{2}{3}} (x(bx+a))^{\frac{1}{3}}}+\frac{2b^2\left(\frac{\ln\left(\left(c^2bx+a\,c^2\right)^{\frac{1}{3}}-\left(a\,c^2\right)^{\frac{1}{3}}\right)}{\left(a\,c^2\right)^{\frac{1}{3}}}-\frac{\ln\left(\left(c^2bx+a\,c^2\right)^{\frac{2}{3}}+\left(a\,c^2\right)^{\frac{1}{3}}\left(c^2bx+a\,c^2\right)^{\frac{1}{3}}+\left(a\,c^2\right)^{\frac{2}{3}}\right)}{2\left(a\,c^2\right)^{\frac{1}{3}}}+\frac{\sqrt{3}\,\arctan}{}\right.}{9a^2c^2(cx)^{\frac{2}{3}}(x(bx+a))^{\frac{1}{3}}}$

```
int(1/(c*x)^(8/3)/(b*x^2+a*x)^(1/3),x,method=_RETURNVERBOSE)
```

```
-1/6*(b*x+a)*(-4*b*x+3*a)/a^2/x/c^2/(c*x)^(2/3)/(x*(b*x+a))^(1/3)+2/9*b^2/
a^2*(1/(a*c^2)^(1/3)*ln((b*c^2*x+a*c^2)^(1/3)-(a*c^2)^(1/3))-1/2/(a*c^2)^(
1/3)*ln((b*c^2*x+a*c^2)^(2/3)+(a*c^2)^(1/3)*(b*c^2*x+a*c^2)^(1/3)+(a*c^2)^
(2/3))+3^(1/2)/(a*c^2)^(1/3)*arctan(1/3*3^(1/2)*(2/(a*c^2)^(1/3)*(b*c^2*x+
a*c^2)^(1/3)+1)))/c^2/(c*x)^(2/3)*(c^2*(b*x+a))^(1/3)/(x*(b*x+a))^(1/3)*x
```

Fricas [A] (verification not implemented)

Time = 0.10 (sec) , antiderivative size = 667, normalized size of antiderivative = 2.72

$$\int \frac{1}{(cx)^{8/3}\sqrt[3]{ax+bx^2}}\,dx = \text{Too large to display}$$

```
integrate(1/(c*x)^(8/3)/(b*x^2+a*x)^(1/3),x, algorithm="fricas")
```

```
[1/18*(6*sqrt(1/3)*a*b^2*c*x^3*sqrt(-(a*c^2)^(1/3)/a)*log(-(b*c^2*x^2 + 3*
a*c^2*x - 3*(a*c^2)^(1/3)*(b*x^2 + a*x)^(2/3)*(c*x)^(1/3)*c - 3*sqrt(1/3)*
(2*(b*x^2 + a*x)^(1/3)*(c*x)^(2/3)*a*c - (a*c^2)^(2/3)*(b*x^2 + a*x)^(2/3)
*(c*x)^(1/3) - (b*c*x^2 + a*c*x)*(a*c^2)^(1/3))*sqrt(-(a*c^2)^(1/3)/a))/x^
2) + 4*(a*c^2)^(2/3)*b^2*x^3*log(((b*x^2 + a*x)^(2/3)*(c*x)^(1/3)*a*c - (a
*c^2)^(2/3)*(b*x^2 + a*x))/(b*x^2 + a*x)) - 2*(a*c^2)^(2/3)*b^2*x^3*log(((
b*x^2 + a*x)^(1/3)*(c*x)^(2/3)*a*c + (a*c^2)^(2/3)*(b*x^2 + a*x)^(2/3)*(c*
x)^(1/3) + (b*c*x^2 + a*c*x)*(a*c^2)^(1/3))/(b*x^2 + a*x)) + 3*(4*a*b*c*x
- 3*a^2*c)*(b*x^2 + a*x)^(2/3)*(c*x)^(1/3))/(a^3*c^4*x^3), -1/18*(12*sqrt(
1/3)*a*b^2*c*x^3*sqrt((a*c^2)^(1/3)/a)*arctan(sqrt(1/3)*(2*(a*c^2)^(2/3)*(
b*x^2 + a*x)^(2/3)*(c*x)^(1/3) + (b*c*x^2 + a*c*x)*(a*c^2)^(1/3))*sqrt((a*
c^2)^(1/3)/a)/(b*c^2*x^2 + a*c^2*x)) - 4*(a*c^2)^(2/3)*b^2*x^3*log(((b*x^2
 + a*x)^(2/3)*(c*x)^(1/3)*a*c - (a*c^2)^(2/3)*(b*x^2 + a*x))/(b*x^2 + a*x)
) + 2*(a*c^2)^(2/3)*b^2*x^3*log(((b*x^2 + a*x)^(1/3)*(c*x)^(2/3)*a*c + (a*
c^2)^(2/3)*(b*x^2 + a*x)^(2/3)*(c*x)^(1/3) + (b*c*x^2 + a*c*x)*(a*c^2)^(1/
3))/(b*x^2 + a*x)) - 3*(4*a*b*c*x - 3*a^2*c)*(b*x^2 + a*x)^(2/3)*(c*x)^(1/
3))/(a^3*c^4*x^3)]
```

Sympy [F]

$$\int \frac{1}{(cx)^{8/3}\sqrt[3]{ax+bx^2}}\,dx = \int \frac{1}{(cx)^{\frac{8}{3}}\sqrt[3]{x\,(a+bx)}}\,dx$$

```
integrate(1/(c*x)**(8/3)/(b*x**2+a*x)**(1/3),x)
```

```
Integral(1/((c*x)**(8/3)*(x*(a + b*x))**(1/3)), x)
```

Maxima [F]

$$\int \frac{1}{(cx)^{8/3}\sqrt[3]{ax+bx^2}}\,dx = \int \frac{1}{(bx^2+ax)^{\frac{1}{3}}\,(cx)^{\frac{8}{3}}}\,dx$$

```
integrate(1/(c*x)^(8/3)/(b*x^2+a*x)^(1/3),x, algorithm="maxima")
```

```
integrate(1/((b*x^2 + a*x)^(1/3)*(c*x)^(8/3)), x)
```

Giac [F]

$$\int \frac{1}{(cx)^{8/3}\sqrt[3]{ax+bx^2}}\,dx = \int \frac{1}{(bx^2+ax)^{\frac{1}{3}}\,(cx)^{\frac{8}{3}}}\,dx$$

```
integrate(1/(c*x)^(8/3)/(b*x^2+a*x)^(1/3),x, algorithm="giac")
```

```
integrate(1/((b*x^2 + a*x)^(1/3)*(c*x)^(8/3)), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{(cx)^{8/3}\sqrt[3]{ax+bx^2}}\,dx = \int \frac{1}{(b\,x^2+a\,x)^{1/3}\,(c\,x)^{8/3}}\,dx$$

```
int(1/((a*x + b*x^2)^(1/3)*(c*x)^(8/3)),x)
```

```
int(1/((a*x + b*x^2)^(1/3)*(c*x)^(8/3)), x)
```

Reduce [B] (verification not implemented)

Time = 0.33 (sec) , antiderivative size = 209, normalized size of antiderivative = 0.85

$$\int \frac{1}{(cx)^{8/3}\sqrt[3]{ax+bx^2}}\,dx = \frac{-4\sqrt{3}\,atan\left(\frac{2(bx+a)^{\frac{1}{6}}+a^{\frac{1}{6}}}{a^{\frac{1}{6}}\sqrt{3}}\right)b^2x^2+4\sqrt{3}\,atan\left(\frac{2(bx+a)^{\frac{1}{6}}-a^{\frac{1}{6}}}{a^{\frac{1}{6}}\sqrt{3}}\right)b^2x^2-9a^{\frac{4}{3}}(bx+a)^{\frac{2}{3}}}{}$$

```
int(1/(c*x)^(8/3)/(b*x^2+a*x)^(1/3),x)
```

```
( - 4*sqrt(3)*atan((2*(a + b*x)**(1/6) + a**(1/6))/(a**(1/6)*sqrt(3)))*b**
2*x**2 + 4*sqrt(3)*atan((2*(a + b*x)**(1/6) - a**(1/6))/(a**(1/6)*sqrt(3))
)*b**2*x**2 - 9*a**(1/3)*(a + b*x)**(2/3)*a + 12*a**(1/3)*(a + b*x)**(2/3)
*b*x + 4*log((a + b*x)**(1/6) + a**(1/6))*b**2*x**2 + 4*log((a + b*x)**(1/
6) - a**(1/6))*b**2*x**2 - 2*log( - a**(1/6)*(a + b*x)**(1/6) + (a + b*x)*
*(1/3) + a**(1/3))*b**2*x**2 - 2*log(a**(1/6)*(a + b*x)**(1/6) + (a + b*x)
**(1/3) + a**(1/3))*b**2*x**2)/(18*c**(2/3)*a**(1/3)*a**2*c**2*x**2)
```

3.133 $\int \frac{(cx)^{8/3}}{(ax+bx^2)^{2/3}} dx$

Optimal result

Integrand size = 21, antiderivative size = 93

$$\int \frac{(cx)^{8/3}}{(ax+bx^2)^{2/3}}\,dx=\frac{3a^2c^3\sqrt[3]{ax+bx^2}}{b^3\sqrt[3]{cx}}-\frac{3ac^4(ax+bx^2)^{4/3}}{2b^3(cx)^{4/3}}+\frac{3c^5(ax+bx^2)^{7/3}}{7b^3(cx)^{7/3}}$$

```
3*a^2*c^3*(b*x^2+a*x)^(1/3)/b^3/(c*x)^(1/3)-3/2*a*c^4*(b*x^2+a*x)^(4/3)/b^
3/(c*x)^(4/3)+3/7*c^5*(b*x^2+a*x)^(7/3)/b^3/(c*x)^(7/3)
```

Mathematica [A] (verified)

Time = 0.04 (sec) , antiderivative size = 47, normalized size of antiderivative = 0.51

$$\int \frac{(cx)^{8/3}}{(ax+bx^2)^{2/3}}\,dx=\frac{3c^3\sqrt[3]{x(a+bx)}(9a^2-3abx+2b^2x^2)}{14b^3\sqrt[3]{cx}}$$

```
Integrate[(c*x)^(8/3)/(a*x + b*x^2)^(2/3),x]
```

```
(3*c^3*(x*(a + b*x))^(1/3)*(9*a^2 - 3*a*b*x + 2*b^2*x^2))/(14*b^3*(c*x)^(1
/3))
```

Rubi [A] (verified)

Time = 0.37 (sec) , antiderivative size = 98, normalized size of antiderivative = 1.05, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.143$, Rules used = {1128, 1128, 1122}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(cx)^{8/3}}{(ax+bx^2)^{2/3}}dx$$

$$\Big\downarrow \text{1128}$$

$$\frac{3c(cx)^{5/3}\sqrt[3]{ax+bx^2}}{7b}-\frac{6ac\int \frac{(cx)^{5/3}}{(bx^2+ax)^{2/3}}dx}{7b}$$

$$\Big\downarrow \text{1128}$$

$$\frac{3c(cx)^{5/3}\sqrt[3]{ax+bx^2}}{7b}-\frac{6ac\left(\frac{3c(cx)^{2/3}\sqrt[3]{ax+bx^2}}{4b}-\frac{3ac\int \frac{(cx)^{2/3}}{\left(bx^2+ax\right)^{2/3}}dx}{4b}\right)}{7b}$$

$$\Big\downarrow \text{1122}$$

$$\frac{3c(cx)^{5/3}\sqrt[3]{ax+bx^2}}{7b}-\frac{6ac\left(\frac{3c(cx)^{2/3}\sqrt[3]{ax+bx^2}}{4b}-\frac{9ac^2\sqrt[3]{ax+bx^2}}{4b^2\sqrt[3]{cx}}\right)}{7b}$$

```
Int[(c*x)^(8/3)/(a*x + b*x^2)^(2/3),x]
```

```
(3*c*(c*x)^(5/3)*(a*x + b*x^2)^(1/3))/(7*b) - (6*a*c*((-9*a*c^2*(a*x + b*x
^2)^(1/3))/(4*b^2*(c*x)^(1/3)) + (3*c*(c*x)^(2/3)*(a*x + b*x^2)^(1/3))/(4*
b)))/(7*b)
```

Defintions of rubi rules used

```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] :> Simp[e*(d + e*x)^(m - 1)*((a + b*x + c*x^2)^(p + 1)/(c*(p + 1))),
 x] /; FreeQ[{a, b, c, d, e, m, p}, x] && EqQ[c*d^2 - b*d*e + a*e^2, 0] &&
EqQ[m + p, 0]
```

```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] :> Simp[e*(d + e*x)^(m - 1)*((a + b*x + c*x^2)^(p + 1)/(c*(m + 2*p +
 1))), x] + Simp[Simplify[m + p]*((2*c*d - b*e)/(c*(m + 2*p + 1)))   Int[(d
 + e*x)^(m - 1)*(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, m, p},
 x] && EqQ[c*d^2 - b*d*e + a*e^2, 0] && IGtQ[Simplify[m + p], 0]
```

Maple [A] (verified)

Time = 0.60 (sec) , antiderivative size = 48, normalized size of antiderivative = 0.52

method	result	size
risch	$\frac{3c^{3}x\left(2b^{2}x^{2}-3abx+9a^{2}\right)\left(bx+a\right)}{14\left(x\left(bx+a\right)\right)^{\frac{2}{3}}\left(cx\right)^{\frac{1}{3}}b^{3}}$	48
gosper	$\frac{3\left(bx+a\right)\left(2b^{2}x^{2}-3abx+9a^{2}\right)\left(cx\right)^{\frac{8}{3}}}{14b^{3}x^{2}\left(b\,x^{2}+ax\right)^{\frac{2}{3}}}$	49
orering	$\frac{3\left(bx+a\right)\left(2b^{2}x^{2}-3abx+9a^{2}\right)\left(cx\right)^{\frac{8}{3}}}{14b^{3}x^{2}\left(b\,x^{2}+ax\right)^{\frac{2}{3}}}$	49

```
int((c*x)^(8/3)/(b*x^2+a*x)^(2/3),x,method=_RETURNVERBOSE)
```

```
3/14*c^3/(x*(b*x+a))^(2/3)/(c*x)^(1/3)*x*(2*b^2*x^2-3*a*b*x+9*a^2)*(b*x+a)
/b^3
```

Fricas [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 52, normalized size of antiderivative = 0.56

$$\int \frac{(cx)^{8/3}}{(ax+bx^2)^{2/3}}\,dx = \frac{3\,(2\,b^2c^2x^2-3\,abc^2x+9\,a^2c^2)(bx^2+ax)^{\frac{1}{3}}(cx)^{\frac{2}{3}}}{14\,b^3x}$$

```
integrate((c*x)^(8/3)/(b*x^2+a*x)^(2/3),x, algorithm="fricas")
```

```
3/14*(2*b^2*c^2*x^2 - 3*a*b*c^2*x + 9*a^2*c^2)*(b*x^2 + a*x)^(1/3)*(c*x)^(
2/3)/(b^3*x)
```

Sympy [F]

$$\int \frac{(cx)^{8/3}}{(ax+bx^2)^{2/3}}\,dx = \int \frac{(cx)^{\frac{8}{3}}}{(x\,(a+bx))^{\frac{2}{3}}}\,dx$$

```
integrate((c*x)**(8/3)/(b*x**2+a*x)**(2/3),x)
```

```
Integral((c*x)**(8/3)/(x*(a + b*x))**(2/3), x)
```

Maxima [A] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 54, normalized size of antiderivative = 0.58

$$\int \frac{(cx)^{8/3}}{(ax+bx^2)^{2/3}}\,dx = \frac{3\left(2\,b^3c^{\frac{8}{3}}x^3-ab^2c^{\frac{8}{3}}x^2+6\,a^2bc^{\frac{8}{3}}x+9\,a^3c^{\frac{8}{3}}\right)}{14\,(bx+a)^{\frac{2}{3}}b^3}$$

```
integrate((c*x)^(8/3)/(b*x^2+a*x)^(2/3),x, algorithm="maxima")
```

```
3/14*(2*b^3*c^(8/3)*x^3 - a*b^2*c^(8/3)*x^2 + 6*a^2*b*c^(8/3)*x + 9*a^3*c^
(8/3))/((b*x + a)^(2/3)*b^3)
```

Giac [F]

$$\int \frac{(cx)^{8/3}}{(ax+bx^2)^{2/3}}\,dx = \int \frac{(cx)^{\frac{8}{3}}}{(bx^2+ax)^{\frac{2}{3}}}\,dx$$

```
integrate((c*x)^(8/3)/(b*x^2+a*x)^(2/3),x, algorithm="giac")
```

```
integrate((c*x)^(8/3)/(b*x^2 + a*x)^(2/3), x)
```

Mupad [B] (verification not implemented)

Time = 9.57 (sec) , antiderivative size = 46, normalized size of antiderivative = 0.49

$$\int \frac{(cx)^{8/3}}{(ax+bx^2)^{2/3}}\,dx = \frac{3\,c^2\,{\left(b\,x^2+a\,x\right)}^{1/3}\,{\left(c\,x\right)}^{2/3}\,\left(9\,a^2-3\,a\,b\,x+2\,b^2\,x^2\right)}{14\,b^3\,x}$$

```
int((c*x)^(8/3)/(a*x + b*x^2)^(2/3),x)
```

```
(3*c^2*(a*x + b*x^2)^(1/3)*(c*x)^(2/3)*(9*a^2 + 2*b^2*x^2 - 3*a*b*x))/(14*
b^3*x)
```

Reduce [B] (verification not implemented)

Time = 0.29 (sec) , antiderivative size = 34, normalized size of antiderivative = 0.37

$$\int \frac{(cx)^{8/3}}{(ax+bx^2)^{2/3}}\,dx = \frac{3c^{\frac{8}{3}}(bx+a)^{\frac{1}{3}}\,(2b^2x^2-3abx+9a^2)}{14b^3}$$

```
int((c*x)^(8/3)/(b*x^2+a*x)^(2/3),x)
```

```
(3*c**(2/3)*(a + b*x)**(1/3)*c**2*(9*a**2 - 3*a*b*x + 2*b**2*x**2))/(14*b*
*3)
```

3.134 $\int \frac{(cx)^{5/3}}{(ax+bx^2)^{2/3}}\,dx$

Optimal result

Integrand size = 21, antiderivative size = 60

$$\int \frac{(cx)^{5/3}}{(ax+bx^2)^{2/3}}\,dx=-\frac{3ac^2\sqrt[3]{ax+bx^2}}{b^2\sqrt[3]{cx}}+\frac{3c^3(ax+bx^2)^{4/3}}{4b^2(cx)^{4/3}}$$

```
-3*a*c^2*(b*x^2+a*x)^(1/3)/b^2/(c*x)^(1/3)+3/4*c^3*(b*x^2+a*x)^(4/3)/b^2/(
c*x)^(4/3)
```

Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 35, normalized size of antiderivative = 0.58

$$\int \frac{(cx)^{5/3}}{(ax+bx^2)^{2/3}}\,dx=\frac{3c^2(-3a+bx)\sqrt[3]{x(a+bx)}}{4b^2\sqrt[3]{cx}}$$

```
Integrate[(c*x)^(5/3)/(a*x + b*x^2)^(2/3),x]
```

```
(3*c^2*(-3*a + b*x)*(x*(a + b*x))^(1/3))/(4*b^2*(c*x)^(1/3))
```

Rubi [A] (verified)

Time = 0.30 (sec) , antiderivative size = 60, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.095$, Rules used = {1128, 1122}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(cx)^{5/3}}{(ax+bx^2)^{2/3}}\,dx$$

$$\Big\downarrow \ 1128$$

$$\frac{3c(cx)^{2/3}\sqrt[3]{ax+bx^2}}{4b}-\frac{3ac\int \frac{(cx)^{2/3}}{(bx^2+ax)^{2/3}}dx}{4b}$$

$$\Big\downarrow \ 1122$$

$$\frac{3c(cx)^{2/3}\sqrt[3]{ax+bx^2}}{4b}-\frac{9ac^2\sqrt[3]{ax+bx^2}}{4b^2\sqrt[3]{cx}}$$

```
Int[(c*x)^(5/3)/(a*x + b*x^2)^(2/3),x]
```

```
(-9*a*c^2*(a*x + b*x^2)^(1/3))/(4*b^2*(c*x)^(1/3)) + (3*c*(c*x)^(2/3)*(a*x
 + b*x^2)^(1/3))/(4*b)
```

Defintions of rubi rules used

```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] :> Simp[e*(d + e*x)^(m - 1)*((a + b*x + c*x^2)^(p + 1)/(c*(p + 1))),
 x] /; FreeQ[{a, b, c, d, e, m, p}, x] && EqQ[c*d^2 - b*d*e + a*e^2, 0] &&
EqQ[m + p, 0]
```

```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] :> Simp[e*(d + e*x)^(m - 1)*((a + b*x + c*x^2)^(p + 1)/(c*(m + 2*p +
 1))), x] + Simp[Simplify[m + p]*((2*c*d - b*e)/(c*(m + 2*p + 1)))   Int[(d
 + e*x)^(m - 1)*(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, m, p},
 x] && EqQ[c*d^2 - b*d*e + a*e^2, 0] && IGtQ[Simplify[m + p], 0]
```

Maple [A] (verified)

Time = 0.44 (sec) , antiderivative size = 37, normalized size of antiderivative = 0.62

method	result	size
risch	$-\frac{3c^{2}x(-bx+3a)(bx+a)}{4(x(bx+a))^{\frac{2}{3}}(cx)^{\frac{1}{3}}b^{2}}$	37
gosper	$-\frac{3(bx+a)(-bx+3a)(cx)^{\frac{5}{3}}}{4b^{2}x(b\,x^{2}+ax)^{\frac{2}{3}}}$	38
orering	$-\frac{3(bx+a)(-bx+3a)(cx)^{\frac{5}{3}}}{4b^{2}x(b\,x^{2}+ax)^{\frac{2}{3}}}$	38

```
int((c*x)^(5/3)/(b*x^2+a*x)^(2/3),x,method=_RETURNVERBOSE)
```

```
-3/4*c^2/(x*(b*x+a))^(2/3)/(c*x)^(1/3)*x*(-b*x+3*a)*(b*x+a)/b^2
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 33, normalized size of antiderivative = 0.55

$$\int \frac{(cx)^{5/3}}{(ax+bx^2)^{2/3}}\,dx = \frac{3\,(bcx-3\,ac)(bx^2+ax)^{\frac{1}{3}}(cx)^{\frac{2}{3}}}{4\,b^2x}$$

```
integrate((c*x)^(5/3)/(b*x^2+a*x)^(2/3),x, algorithm="fricas")
```

```
3/4*(b*c*x - 3*a*c)*(b*x^2 + a*x)^(1/3)*(c*x)^(2/3)/(b^2*x)
```

Sympy [F]

$$\int \frac{(cx)^{5/3}}{(ax+bx^2)^{2/3}}\,dx = \int \frac{(cx)^{\frac{5}{3}}}{(x(a+bx))^{\frac{2}{3}}}\,dx$$

```
integrate((c*x)**(5/3)/(b*x**2+a*x)**(2/3),x)
```

```
Integral((c*x)**(5/3)/(x*(a + b*x))**(2/3), x)
```

Maxima [A] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 39, normalized size of antiderivative = 0.65

$$\int \frac{(cx)^{5/3}}{(ax+bx^2)^{2/3}}\,dx = \frac{3\left(b^2c^{\frac{5}{3}}x^2 - 2\,abc^{\frac{5}{3}}x - 3\,a^2c^{\frac{5}{3}}\right)}{4\,(bx+a)^{\frac{2}{3}}b^2}$$

```
integrate((c*x)^(5/3)/(b*x^2+a*x)^(2/3),x, algorithm="maxima")
```

```
3/4*(b^2*c^(5/3)*x^2 - 2*a*b*c^(5/3)*x - 3*a^2*c^(5/3))/((b*x + a)^(2/3)*b
^2)
```

Giac [F]

$$\int \frac{(cx)^{5/3}}{(ax+bx^2)^{2/3}}\,dx = \int \frac{(cx)^{\frac{5}{3}}}{(bx^2+ax)^{\frac{2}{3}}}\,dx$$

```
integrate((c*x)^(5/3)/(b*x^2+a*x)^(2/3),x, algorithm="giac")
```

```
integrate((c*x)^(5/3)/(b*x^2 + a*x)^(2/3), x)
```

Mupad [B] (verification not implemented)

Time = 9.36 (sec) , antiderivative size = 41, normalized size of antiderivative = 0.68

$$\int \frac{(cx)^{5/3}}{(ax+bx^2)^{2/3}}\,dx = -\frac{(b\,x^2+a\,x)^{1/3}\,\left(\frac{9\,a\,c\,(c\,x)^{2/3}}{4\,b^2}-\frac{3\,c\,x\,(c\,x)^{2/3}}{4\,b}\right)}{x}$$

```
int((c*x)^(5/3)/(a*x + b*x^2)^(2/3),x)
```

```
-((a*x + b*x^2)^(1/3)*((9*a*c*(c*x)^(2/3))/(4*b^2) - (3*c*x*(c*x)^(2/3))/(
4*b)))/x
```

Reduce [B] (verification not implemented)

Time = 0.52 (sec) , antiderivative size = 22, normalized size of antiderivative = 0.37

$$\int \frac{(cx)^{5/3}}{(ax+bx^2)^{2/3}}\,dx = \frac{3c^{\frac{5}{3}}(bx+a)^{\frac{1}{3}}\,(bx-3a)}{4b^2}$$

```
int((c*x)^(5/3)/(b*x^2+a*x)^(2/3),x)
```

```
(3*c**(2/3)*(a + b*x)**(1/3)*c*( - 3*a + b*x))/(4*b**2)
```

3.135 $\int \frac{(cx)^{2/3}}{(ax+bx^2)^{2/3}}\,dx$

Optimal result

Integrand size = 21, antiderivative size = 26

$$\int \frac{(cx)^{2/3}}{(ax+bx^2)^{2/3}}\,dx=\frac{3c\sqrt[3]{ax+bx^2}}{b\sqrt[3]{cx}}$$

```
3*c*(b*x^2+a*x)^(1/3)/b/(c*x)^(1/3)
```

Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 24, normalized size of antiderivative = 0.92

$$\int \frac{(cx)^{2/3}}{(ax+bx^2)^{2/3}}\,dx=\frac{3c\sqrt[3]{x(a+bx)}}{b\sqrt[3]{cx}}$$

```
Integrate[(c*x)^(2/3)/(a*x + b*x^2)^(2/3),x]
```

```
(3*c*(x*(a + b*x))^(1/3))/(b*(c*x)^(1/3))
```

Rubi [A] (verified)

Time = 0.24 (sec) , antiderivative size = 26, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.048, Rules used = {1122}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(cx)^{2/3}}{(ax+bx^2)^{2/3}}\,dx$$
$$\Big\downarrow \text{1122}$$
$$\frac{3c\sqrt[3]{ax+bx^2}}{b\sqrt[3]{cx}}$$

```
Int[(c*x)^(2/3)/(a*x + b*x^2)^(2/3),x]
```

```
(3*c*(a*x + b*x^2)^(1/3))/(b*(c*x)^(1/3))
```

Defintions of rubi rules used

```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] :> Simp[e*(d + e*x)^(m - 1)*((a + b*x + c*x^2)^(p + 1)/(c*(p + 1))),
 x] /; FreeQ[{a, b, c, d, e, m, p}, x] && EqQ[c*d^2 - b*d*e + a*e^2, 0] &&
EqQ[m + p, 0]
```

Maple [A] (verified)

Time = 0.35 (sec) , antiderivative size = 27, normalized size of antiderivative = 1.04

method	result	size
gosper	$\frac{3(bx+a)(cx)^{\frac{2}{3}}}{b(b\,x^{2}+ax)^{\frac{2}{3}}}$	27
risch	$\frac{3cx(bx+a)}{(x(bx+a))^{\frac{2}{3}}(cx)^{\frac{1}{3}}b}$	27
orering	$\frac{3(bx+a)(cx)^{\frac{2}{3}}}{b(b\,x^{2}+ax)^{\frac{2}{3}}}$	27

```
int((c*x)^(2/3)/(b*x^2+a*x)^(2/3),x,method=_RETURNVERBOSE)
```

```
3*(b*x+a)*(c*x)^(2/3)/b/(b*x^2+a*x)^(2/3)
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 24, normalized size of antiderivative = 0.92

$$\int \frac{(cx)^{2/3}}{(ax+bx^2)^{2/3}}\,dx = \frac{3\left(bx^2+ax\right)^{\frac{1}{3}}(cx)^{\frac{2}{3}}}{bx}$$

```
integrate((c*x)^(2/3)/(b*x^2+a*x)^(2/3),x, algorithm="fricas")
```

```
3*(b*x^2 + a*x)^(1/3)*(c*x)^(2/3)/(b*x)
```

Sympy [F]

$$\int \frac{(cx)^{2/3}}{(ax+bx^2)^{2/3}}\,dx = \int \frac{(cx)^{\frac{2}{3}}}{(x\,(a+bx))^{\frac{2}{3}}}\,dx$$

```
integrate((c*x)**(2/3)/(b*x**2+a*x)**(2/3),x)
```

```
Integral((c*x)**(2/3)/(x*(a + b*x))**(2/3), x)
```

Maxima [A] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 24, normalized size of antiderivative = 0.92

$$\int \frac{(cx)^{2/3}}{(ax+bx^2)^{2/3}}\,dx = \frac{3\left(bc^{\frac{2}{3}}x + ac^{\frac{2}{3}}\right)}{(bx+a)^{\frac{2}{3}}b}$$

```
integrate((c*x)^(2/3)/(b*x^2+a*x)^(2/3),x, algorithm="maxima")
```

```
3*(b*c^(2/3)*x + a*c^(2/3))/((b*x + a)^(2/3)*b)
```

Giac [F]

$$\int \frac{(cx)^{2/3}}{(ax+bx^2)^{2/3}}\,dx = \int \frac{(cx)^{\frac{2}{3}}}{(bx^2+ax)^{\frac{2}{3}}}\,dx$$

```
integrate((c*x)^(2/3)/(b*x^2+a*x)^(2/3),x, algorithm="giac")
```

```
integrate((c*x)^(2/3)/(b*x^2 + a*x)^(2/3), x)
```

Mupad [B] (verification not implemented)

Time = 8.96 (sec) , antiderivative size = 24, normalized size of antiderivative = 0.92

$$\int \frac{(cx)^{2/3}}{(ax+bx^2)^{2/3}}\,dx = \frac{3\,(b\,x^2+a\,x)^{1/3}\,(c\,x)^{2/3}}{b\,x}$$

```
int((c*x)^(2/3)/(a*x + b*x^2)^(2/3),x)
```

```
(3*(a*x + b*x^2)^(1/3)*(c*x)^(2/3))/(b*x)
```

Reduce [B] (verification not implemented)

Time = 0.47 (sec) , antiderivative size = 15, normalized size of antiderivative = 0.58

$$\int \frac{(cx)^{2/3}}{(ax+bx^2)^{2/3}}\,dx = \frac{3c^{\frac{2}{3}}(bx+a)^{\frac{1}{3}}}{b}$$

```
int((c*x)^(2/3)/(b*x^2+a*x)^(2/3),x)
```

```
(3*c**(2/3)*(a + b*x)**(1/3))/b
```

3.136 $\int \frac{1}{(cx)^{2/3}(ax+bx^2)^{2/3}}\,dx$

Optimal result

Integrand size = 21, antiderivative size = 25

$$\int \frac{1}{(cx)^{2/3}\,(ax+bx^2)^{2/3}}\,dx=-\frac{3\sqrt[3]{ax+bx^2}}{a(cx)^{2/3}}$$

```
-3*(b*x^2+a*x)^(1/3)/a/(c*x)^(2/3)
```

Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 23, normalized size of antiderivative = 0.92

$$\int \frac{1}{(cx)^{2/3}\,(ax+bx^2)^{2/3}}\,dx=-\frac{3\sqrt[3]{x(a+bx)}}{a(cx)^{2/3}}$$

```
Integrate[1/((c*x)^(2/3)*(a*x + b*x^2)^(2/3)),x]
```

```
(-3*(x*(a + b*x))^(1/3))/(a*(c*x)^(2/3))
```

Rubi [A] (verified)

Time = 0.24 (sec) , antiderivative size = 25, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.048, Rules used = {1123}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{(cx)^{2/3}\left(ax+bx^2\right)^{2/3}}dx$$
$$\Big\downarrow \text{ 1123}$$
$$-\frac{3\sqrt[3]{ax+bx^2}}{a(cx)^{2/3}}$$

```
Int[1/((c*x)^(2/3)*(a*x + b*x^2)^(2/3)),x]
```

```
(-3*(a*x + b*x^2)^(1/3))/(a*(c*x)^(2/3))
```

Defintions of rubi rules used

```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] :> Simp[e*(d + e*x)^m*((a + b*x + c*x^2)^(p + 1)/((p + 1)*(2*c*d - b
*e))), x] /; FreeQ[{a, b, c, d, e, m, p}, x] && EqQ[c*d^2 - b*d*e + a*e^2,
0] && EqQ[m + 2*p + 2, 0]
```

Maple [A] (verified)

Time = 0.39 (sec) , antiderivative size = 26, normalized size of antiderivative = 1.04

method	result	size
risch	$-\frac{3x(bx+a)}{(cx)^{\frac{2}{3}}(x(bx+a))^{\frac{2}{3}}a}$	26
gosper	$-\frac{3x(bx+a)}{a(cx)^{\frac{2}{3}}(bx^2+ax)^{\frac{2}{3}}}$	28
orering	$-\frac{3x(bx+a)}{a(cx)^{\frac{2}{3}}(bx^2+ax)^{\frac{2}{3}}}$	28

```
int(1/(c*x)^(2/3)/(b*x^2+a*x)^(2/3),x,method=_RETURNVERBOSE)
```

```
-3/(c*x)^(2/3)/(x*(b*x+a))^(2/3)*x/a*(b*x+a)
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 27, normalized size of antiderivative = 1.08

$$\int \frac{1}{(cx)^{2/3}\,(ax+bx^2)^{2/3}}\,dx = -\frac{3\,(bx^2+ax)^{\frac{1}{3}}(cx)^{\frac{1}{3}}}{acx}$$

```
integrate(1/(c*x)^(2/3)/(b*x^2+a*x)^(2/3),x, algorithm="fricas")
```

```
-3*(b*x^2 + a*x)^(1/3)*(c*x)^(1/3)/(a*c*x)
```

Sympy [F]

$$\int \frac{1}{(cx)^{2/3}\,(ax+bx^2)^{2/3}}\,dx = \int \frac{1}{(cx)^{\frac{2}{3}}\,(x\,(a+bx))^{\frac{2}{3}}}\,dx$$

```
integrate(1/(c*x)**(2/3)/(b*x**2+a*x)**(2/3),x)
```

```
Integral(1/((c*x)**(2/3)*(x*(a + b*x))**(2/3)), x)
```

Maxima [F]

$$\int \frac{1}{(cx)^{2/3}(ax+bx^2)^{2/3}}\,dx = \int \frac{1}{(bx^2+ax)^{\frac{2}{3}}(cx)^{\frac{2}{3}}}\,dx$$

```
integrate(1/(c*x)^(2/3)/(b*x^2+a*x)^(2/3),x, algorithm="maxima")
```

```
integrate(1/((b*x^2 + a*x)^(2/3)*(c*x)^(2/3)), x)
```

Giac [A] (verification not implemented)

Time = 0.15 (sec) , antiderivative size = 20, normalized size of antiderivative = 0.80

$$\int \frac{1}{(cx)^{2/3}(ax+bx^2)^{2/3}}\,dx = -\frac{3\left(bc+\frac{ac}{x}\right)^{\frac{1}{3}}}{ac}$$

```
integrate(1/(c*x)^(2/3)/(b*x^2+a*x)^(2/3),x, algorithm="giac")
```

```
-3*(b*c + a*c/x)^(1/3)/(a*c)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{(cx)^{2/3}(ax+bx^2)^{2/3}}\,dx = \int \frac{1}{(b\,x^2+a\,x)^{2/3}\,(c\,x)^{2/3}}\,dx$$

```
int(1/((a*x + b*x^2)^(2/3)*(c*x)^(2/3)),x)
```

```
int(1/((a*x + b*x^2)^(2/3)*(c*x)^(2/3)), x)
```

Reduce [F]

$$\int \frac{1}{(cx)^{2/3}\left(ax+bx^2\right)^{2/3}}\,dx = \frac{\int \frac{1}{x^{\frac{4}{3}}(bx+a)^{\frac{2}{3}}}\,dx}{c^{\frac{2}{3}}}$$

```
int(1/(c*x)^(2/3)/(b*x^2+a*x)^(2/3),x)
```

```
int(1/(x**(1/3)*(a + b*x)**(2/3)*x),x)/c**(2/3)
```

3.137 $\int \frac{1}{(cx)^{5/3}(ax+bx^2)^{2/3}}\,dx$

Optimal result

Integrand size = 21, antiderivative size = 59

$$\int \frac{1}{(cx)^{5/3}\,(ax+bx^2)^{2/3}}\,dx=-\frac{3\sqrt[3]{ax+bx^2}}{4a(cx)^{5/3}}+\frac{9b\sqrt[3]{ax+bx^2}}{4a^2c(cx)^{2/3}}$$

```
-3/4*(b*x^2+a*x)^(1/3)/a/(c*x)^(5/3)+9/4*b*(b*x^2+a*x)^(1/3)/a^2/c/(c*x)^(
2/3)
```

Mathematica [A] (verified)

Time = 0.18 (sec) , antiderivative size = 31, normalized size of antiderivative = 0.53

$$\int \frac{1}{(cx)^{5/3}\,(ax+bx^2)^{2/3}}\,dx=-\frac{3(a-3bx)\sqrt[3]{x(a+bx)}}{4a^2(cx)^{5/3}}$$

```
Integrate[1/((c*x)^(5/3)*(a*x + b*x^2)^(2/3)),x]
```

```
(-3*(a - 3*b*x)*(x*(a + b*x))^(1/3))/(4*a^2*(c*x)^(5/3))
```

Rubi [A] (verified)

Time = 0.29 (sec) , antiderivative size = 59, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.095, Rules used = {1129, 1123}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{(cx)^{5/3}\left(ax+bx^2\right)^{2/3}}dx$$
$$\Big\downarrow \text{1129}$$
$$-\frac{3b\int \frac{1}{(cx)^{2/3}(bx^2+ax)^{2/3}}dx}{4ac}-\frac{3\sqrt[3]{ax+bx^2}}{4a(cx)^{5/3}}$$
$$\Big\downarrow \text{1123}$$
$$\frac{9b\sqrt[3]{ax+bx^2}}{4a^2c(cx)^{2/3}}-\frac{3\sqrt[3]{ax+bx^2}}{4a(cx)^{5/3}}$$

```
Int[1/((c*x)^(5/3)*(a*x + b*x^2)^(2/3)),x]
```

```
(-3*(a*x + b*x^2)^(1/3))/(4*a*(c*x)^(5/3)) + (9*b*(a*x + b*x^2)^(1/3))/(4*
a^2*c*(c*x)^(2/3))
```

Defintions of rubi rules used

```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] :> Simp[e*(d + e*x)^m*((a + b*x + c*x^2)^(p + 1)/((p + 1)*(2*c*d - b
*e))), x] /; FreeQ[{a, b, c, d, e, m, p}, x] && EqQ[c*d^2 - b*d*e + a*e^2,
0] && EqQ[m + 2*p + 2, 0]
```

```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] :> Simp[(-e)*(d + e*x)^m*((a + b*x + c*x^2)^(p + 1)/((m + p + 1)*(2*
c*d - b*e))), x] + Simp[c*(Simplify[m + 2*p + 2]/((m + p + 1)*(2*c*d - b*e)
))   Int[(d + e*x)^(m + 1)*(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d
, e, m, p}, x] && EqQ[c*d^2 - b*d*e + a*e^2, 0] && ILtQ[Simplify[m + 2*p +
2], 0]
```

Maple [A] (verified)

Time = 0.36 (sec) , antiderivative size = 34, normalized size of antiderivative = 0.58

method	result	size
gosper	$-\frac{3x(bx+a)(-3bx+a)}{4a^{2}(cx)^{\frac{5}{3}}(b\,x^{2}+ax)^{\frac{2}{3}}}$	34
risch	$-\frac{3(bx+a)(-3bx+a)}{4c(cx)^{\frac{2}{3}}(x(bx+a))^{\frac{2}{3}}a^{2}}$	34
orering	$-\frac{3x(bx+a)(-3bx+a)}{4a^{2}(cx)^{\frac{5}{3}}(b\,x^{2}+ax)^{\frac{2}{3}}}$	34

```
int(1/(c*x)^(5/3)/(b*x^2+a*x)^(2/3),x,method=_RETURNVERBOSE)
```

```
-3/4*x*(b*x+a)*(-3*b*x+a)/a^2/(c*x)^(5/3)/(b*x^2+a*x)^(2/3)
```

Fricas [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 35, normalized size of antiderivative = 0.59

$$\int \frac{1}{(cx)^{5/3}\,(ax+bx^2)^{2/3}}\,dx = \frac{3\,(bx^2+ax)^{\frac{1}{3}}(3\,bx-a)(cx)^{\frac{1}{3}}}{4\,a^2c^2x^2}$$

```
integrate(1/(c*x)^(5/3)/(b*x^2+a*x)^(2/3),x, algorithm="fricas")
```

```
3/4*(b*x^2 + a*x)^(1/3)*(3*b*x - a)*(c*x)^(1/3)/(a^2*c^2*x^2)
```

Sympy [F]

$$\int \frac{1}{(cx)^{5/3}\left(ax+bx^2\right)^{2/3}}\,dx=\int \frac{1}{(cx)^{\frac{5}{3}}\left(x\left(a+bx\right)\right)^{\frac{2}{3}}}\,dx$$

```
integrate(1/(c*x)**(5/3)/(b*x**2+a*x)**(2/3),x)
```

```
Integral(1/((c*x)**(5/3)*(x*(a + b*x))**(2/3)), x)
```

Maxima [F]

$$\int \frac{1}{(cx)^{5/3}\left(ax+bx^2\right)^{2/3}}\,dx=\int \frac{1}{\left(bx^2+ax\right)^{\frac{2}{3}}(cx)^{\frac{5}{3}}}\,dx$$

```
integrate(1/(c*x)^(5/3)/(b*x^2+a*x)^(2/3),x, algorithm="maxima")
```

```
integrate(1/((b*x^2 + a*x)^(2/3)*(c*x)^(5/3)), x)
```

Giac [A] (verification not implemented)

Time = 0.35 (sec) , antiderivative size = 47, normalized size of antiderivative = 0.80

$$\int \frac{1}{(cx)^{5/3}\left(ax+bx^2\right)^{2/3}}\,dx=\frac{3\left(\frac{4\left(bc+\frac{ac}{x}\right)^{\frac{1}{3}}b}{a^2c}-\frac{\left(bc+\frac{ac}{x}\right)^{\frac{4}{3}}}{a^2c^2}\right)}{4\,c}$$

```
integrate(1/(c*x)^(5/3)/(b*x^2+a*x)^(2/3),x, algorithm="giac")
```

```
3/4*(4*(b*c + a*c/x)^(1/3)*b/(a^2*c) - (b*c + a*c/x)^(4/3)/(a^2*c^2))/c
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{(cx)^{5/3}\left(ax+bx^2\right)^{2/3}}\,dx=\int \frac{1}{{\left(b\,x^2+a\,x\right)}^{2/3}\,{\left(c\,x\right)}^{5/3}} \,d x$$

```
int(1/((a*x + b*x^2)^(2/3)*(c*x)^(5/3)),x)
```

```
int(1/((a*x + b*x^2)^(2/3)*(c*x)^(5/3)), x)
```

Reduce [F]

$$\int \frac{1}{(cx)^{5/3}\left(ax+bx^2\right)^{2/3}}\,dx=\frac{\int \frac{1}{x^{\frac{7}{3}} (b x +a)^{\frac{2}{3}}}d x}{c^{\frac{5}{3}}}$$

```
int(1/(c*x)^(5/3)/(b*x^2+a*x)^(2/3),x)
```

```
int(1/(x**(1/3)*(a + b*x)**(2/3)*x**2),x)/(c**(2/3)*c)
```

3.138 $\int \frac{1}{(cx)^{8/3}(ax+bx^2)^{2/3}}\,dx$

Optimal result

Integrand size = 21, antiderivative size = 92

$$\int \frac{1}{(cx)^{8/3}\,(ax+bx^2)^{2/3}}\,dx = -\frac{3\sqrt[3]{ax+bx^2}}{7a(cx)^{8/3}} + \frac{9b\sqrt[3]{ax+bx^2}}{14a^2c(cx)^{5/3}} - \frac{27b^2\sqrt[3]{ax+bx^2}}{14a^3c^2(cx)^{2/3}}$$

```
-3/7*(b*x^2+a*x)^(1/3)/a/(c*x)^(8/3)+9/14*b*(b*x^2+a*x)^(1/3)/a^2/c/(c*x)^
(5/3)-27/14*b^2*(b*x^2+a*x)^(1/3)/a^3/c^2/(c*x)^(2/3)
```

Mathematica [A] (verified)

Time = 0.22 (sec) , antiderivative size = 44, normalized size of antiderivative = 0.48

$$\int \frac{1}{(cx)^{8/3}\,(ax+bx^2)^{2/3}}\,dx = -\frac{3\sqrt[3]{x(a+bx)}(2a^2-3abx+9b^2x^2)}{14a^3(cx)^{8/3}}$$

```
Integrate[1/((c*x)^(8/3)*(a*x + b*x^2)^(2/3)),x]
```

```
(-3*(x*(a + b*x))^(1/3)*(2*a^2 - 3*a*b*x + 9*b^2*x^2))/(14*a^3*(c*x)^(8/3)
)
```

Rubi [A] (verified)

Time = 0.35 (sec) , antiderivative size = 98, normalized size of antiderivative = 1.07, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.143, Rules used = {1129, 1129, 1123}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{(cx)^{8/3} (ax+bx^2)^{2/3}} \, dx$$

$$\Big\downarrow \ 1129$$

$$-\frac{6b\int \frac{1}{(cx)^{5/3}(bx^2+ax)^{2/3}}dx}{7ac}-\frac{3\sqrt[3]{ax+bx^2}}{7a(cx)^{8/3}}$$

$$\Big\downarrow \ 1129$$

$$-\frac{6b\left(-\frac{3b\int \frac{1}{(cx)^{2/3}\left(bx^2+ax\right)^{2/3}}dx}{4ac}-\frac{3\sqrt[3]{ax+bx^2}}{4a(cx)^{5/3}}\right)}{7ac}-\frac{3\sqrt[3]{ax+bx^2}}{7a(cx)^{8/3}}$$

$$\Big\downarrow \ 1123$$

$$-\frac{6b\left(\frac{9b\sqrt[3]{ax+bx^2}}{4a^2c(cx)^{2/3}}-\frac{3\sqrt[3]{ax+bx^2}}{4a(cx)^{5/3}}\right)}{7ac}-\frac{3\sqrt[3]{ax+bx^2}}{7a(cx)^{8/3}}$$

```
Int[1/((c*x)^(8/3)*(a*x + b*x^2)^(2/3)),x]
```

```
(-3*(a*x + b*x^2)^(1/3))/(7*a*(c*x)^(8/3)) - (6*b*((-3*(a*x + b*x^2)^(1/3)
)/(4*a*(c*x)^(5/3)) + (9*b*(a*x + b*x^2)^(1/3))/(4*a^2*c*(c*x)^(2/3))))/(7
*a*c)
```

Defintions of rubi rules used

```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] :> Simp[e*(d + e*x)^m*((a + b*x + c*x^2)^(p + 1)/((p + 1)*(2*c*d - b
*e))), x] /; FreeQ[{a, b, c, d, e, m, p}, x] && EqQ[c*d^2 - b*d*e + a*e^2,
0] && EqQ[m + 2*p + 2, 0]
```

```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] :> Simp[(-e)*(d + e*x)^m*((a + b*x + c*x^2)^(p + 1)/((m + p + 1)*(2*
c*d - b*e))), x] + Simp[c*(Simplify[m + 2*p + 2]/((m + p + 1)*(2*c*d - b*e)
))   Int[(d + e*x)^(m + 1)*(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d
, e, m, p}, x] && EqQ[c*d^2 - b*d*e + a*e^2, 0] && ILtQ[Simplify[m + 2*p +
2], 0]
```

Maple [A] (verified)

Time = 0.40 (sec) , antiderivative size = 47, normalized size of antiderivative = 0.51

method	result	size
gosper	$-\frac{3x(bx+a)(9b^2x^2-3abx+2a^2)}{14a^3(cx)^{\frac{8}{3}}(b\,x^2+ax)^{\frac{2}{3}}}$	47
orering	$-\frac{3x(bx+a)(9b^2x^2-3abx+2a^2)}{14a^3(cx)^{\frac{8}{3}}(b\,x^2+ax)^{\frac{2}{3}}}$	47
risch	$-\frac{3(bx+a)(9b^2x^2-3abx+2a^2)}{14c^2(cx)^{\frac{2}{3}}(x(bx+a))^{\frac{2}{3}}x\,a^3}$	50

```
int(1/(c*x)^(8/3)/(b*x^2+a*x)^(2/3),x,method=_RETURNVERBOSE)
```

```
-3/14*x*(b*x+a)*(9*b^2*x^2-3*a*b*x+2*a^2)/a^3/(c*x)^(8/3)/(b*x^2+a*x)^(2/3
)
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 46, normalized size of antiderivative = 0.50

$$\int \frac{1}{(cx)^{8/3}\,(ax+bx^2)^{2/3}}\,dx = -\frac{3\,(9\,b^2x^2-3\,abx+2\,a^2)(bx^2+ax)^{\frac{1}{3}}(cx)^{\frac{1}{3}}}{14\,a^3c^3x^3}$$

```
integrate(1/(c*x)^(8/3)/(b*x^2+a*x)^(2/3),x, algorithm="fricas")
```

```
-3/14*(9*b^2*x^2 - 3*a*b*x + 2*a^2)*(b*x^2 + a*x)^(1/3)*(c*x)^(1/3)/(a^3*c
^3*x^3)
```

Sympy [F]

$$\int \frac{1}{(cx)^{8/3}\,(ax+bx^2)^{2/3}}\,dx = \int \frac{1}{(cx)^{\frac{8}{3}}\,(x\,(a+bx))^{\frac{2}{3}}}\,dx$$

```
integrate(1/(c*x)**(8/3)/(b*x**2+a*x)**(2/3),x)
```

```
Integral(1/((c*x)**(8/3)*(x*(a + b*x))**(2/3)), x)
```

Maxima [F]

$$\int \frac{1}{(cx)^{8/3}\,(ax+bx^2)^{2/3}}\,dx = \int \frac{1}{(bx^2+ax)^{\frac{2}{3}}\,(cx)^{\frac{8}{3}}}\,dx$$

```
integrate(1/(c*x)^(8/3)/(b*x^2+a*x)^(2/3),x, algorithm="maxima")
```

```
integrate(1/((b*x^2 + a*x)^(2/3)*(c*x)^(8/3)), x)
```

Giac [A] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 68, normalized size of antiderivative = 0.74

$$\int \frac{1}{(cx)^{8/3} (ax + bx^2)^{2/3}} dx = -\frac{3 \left(\frac{14 \left(bc+\frac{ac}{x}\right)^{\frac{1}{3}} b^2}{a^3 c^2} - \frac{7 \left(bc+\frac{ac}{x}\right)^{\frac{4}{3}} bc - 2 \left(bc+\frac{ac}{x}\right)^{\frac{7}{3}}}{a^3 c^4} \right)}{14 c}$$

```
integrate(1/(c*x)^(8/3)/(b*x^2+a*x)^(2/3),x, algorithm="giac")
```

```
-3/14*(14*(b*c + a*c/x)^(1/3)*b^2/(a^3*c^2) - (7*(b*c + a*c/x)^(4/3)*b*c -
 2*(b*c + a*c/x)^(7/3))/(a^3*c^4))/c
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{(cx)^{8/3} (ax + bx^2)^{2/3}} dx = \int \frac{1}{(b\,x^2 + a\,x)^{2/3}\,(c\,x)^{8/3}} dx$$

```
int(1/((a*x + b*x^2)^(2/3)*(c*x)^(8/3)),x)
```

```
int(1/((a*x + b*x^2)^(2/3)*(c*x)^(8/3)), x)
```

Reduce [F]

$$\int \frac{1}{(cx)^{8/3} (ax + bx^2)^{2/3}} dx = \frac{\int \frac{1}{x^{\frac{10}{3}} (bx+a)^{\frac{2}{3}}} dx}{c^{\frac{8}{3}}}$$

```
int(1/(c*x)^(8/3)/(b*x^2+a*x)^(2/3),x)
```

```
int(1/(x**(1/3)*(a + b*x)**(2/3)*x**3),x)/(c**(2/3)*c**2)
```

3.139 $\int \frac{(cx)^{7/3}}{(ax+bx^2)^{2/3}}\,dx$

Optimal result

Integrand size = 21, antiderivative size = 280

$$\int \frac{(cx)^{7/3}}{(ax+bx^2)^{2/3}}\,dx = -\frac{5ac^2\sqrt[3]{cx}\sqrt[3]{ax+bx^2}}{6b^2}+\frac{c(cx)^{4/3}\sqrt[3]{ax+bx^2}}{2b} - \frac{5a^2c^{5/3}(cx)^{2/3}(a+bx)^{2/3}\arctan\left(\frac{1}{\sqrt{3}}+\frac{2\sqrt[3]{b}\sqrt[3]{cx}}{\sqrt{3}\sqrt[3]{c}\sqrt[3]{a+bx}}\right)}{3\sqrt{3}b^{8/3}\left(ax+bx^2\right)^{2/3}} - \frac{5a^2c^{5/3}(cx)^{2/3}(a+bx)^{2/3}\log(a+bx)}{18b^{8/3}\left(ax+bx^2\right)^{2/3}} - \frac{5a^2c^{5/3}(cx)^{2/3}(a+bx)^{2/3}\log\left(1-\frac{\sqrt[3]{b}\sqrt[3]{cx}}{\sqrt[3]{c}\sqrt[3]{a+bx}}\right)}{6b^{8/3}\left(ax+bx^2\right)^{2/3}}$$

```
-5/6*a*c^2*(c*x)^(1/3)*(b*x^2+a*x)^(1/3)/b^2+1/2*c*(c*x)^(4/3)*(b*x^2+a*x)
^(1/3)/b-5/9*a^2*c^(5/3)*(c*x)^(2/3)*(b*x+a)^(2/3)*arctan(1/3*3^(1/2)+2/3*
b^(1/3)*(c*x)^(1/3)*3^(1/2)/c^(1/3)/(b*x+a)^(1/3))*3^(1/2)/b^(8/3)/(b*x^2+
a*x)^(2/3)-5/18*a^2*c^(5/3)*(c*x)^(2/3)*(b*x+a)^(2/3)*ln(b*x+a)/b^(8/3)/(b
*x^2+a*x)^(2/3)-5/6*a^2*c^(5/3)*(c*x)^(2/3)*(b*x+a)^(2/3)*ln(1-b^(1/3)*(c*
x)^(1/3)/c^(1/3)/(b*x+a)^(1/3))/b^(8/3)/(b*x^2+a*x)^(2/3)
```

Mathematica [A] (verified)

Time = 0.44 (sec) , antiderivative size = 227, normalized size of antiderivative = 0.81

$$\int \frac{(cx)^{7/3}}{(ax+bx^2)^{2/3}}\,dx = \frac{(cx)^{7/3}\left(-15a^2b^{2/3}x^{2/3}-6ab^{5/3}x^{5/3}+9b^{8/3}x^{8/3}-10\sqrt{3}a^2(a+bx)^{2/3}\arctan\left(\frac{}{\sqrt[3]{b}\sqrt[3]{}}\right.\right.}{}$$

```
Integrate[(c*x)^(7/3)/(a*x + b*x^2)^(2/3),x]
```

```
((c*x)^(7/3)*(-15*a^2*b^(2/3)*x^(2/3) - 6*a*b^(5/3)*x^(5/3) + 9*b^(8/3)*x^
(8/3) - 10*Sqrt[3]*a^2*(a + b*x)^(2/3)*ArcTan[(Sqrt[3]*b^(1/3)*x^(1/3))/(b
^(1/3)*x^(1/3) + 2*(a + b*x)^(1/3))] - 10*a^2*(a + b*x)^(2/3)*Log[-(b^(1/3
)*x^(1/3)) + (a + b*x)^(1/3)] + 5*a^2*(a + b*x)^(2/3)*Log[b^(2/3)*x^(2/3)
+ b^(1/3)*x^(1/3)*(a + b*x)^(1/3) + (a + b*x)^(2/3)]))/(18*b^(8/3)*x^(5/3)
*(x*(a + b*x))^(2/3))
```

Rubi [A] (verified)

Time = 0.39 (sec) , antiderivative size = 185, normalized size of antiderivative = 0.66, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.190, Rules used = {1137, 60, 60, 71}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(cx)^{7/3}}{(ax+bx^2)^{2/3}}\,dx$$

$$\Big\downarrow \ 1137$$

$$\frac{(cx)^{7/3}(a+bx)^{2/3}\int \frac{x^{5/3}}{(a+bx)^{2/3}}dx}{x^{5/3}\left(ax+bx^2\right)^{2/3}}$$

$$\Big\downarrow \ 60$$

$$\frac{(cx)^{7/3}(a+bx)^{2/3}\left(\frac{x^{5/3}\sqrt[3]{a+bx}}{2b}-\frac{5a\int\frac{x^{2/3}}{(a+bx)^{2/3}}dx}{6b}\right)}{x^{5/3}\left(ax+bx^2\right)^{2/3}}$$

$$\Big\downarrow \; 60$$

$$\frac{(cx)^{7/3}(a+bx)^{2/3}\left(\frac{x^{5/3}\sqrt[3]{a+bx}}{2b}-\frac{5a\left(\frac{x^{2/3}\sqrt[3]{a+bx}}{b}-\frac{2a\int\frac{1}{\sqrt[3]{x}(a+bx)^{2/3}}dx}{3b}\right)}{6b}\right)}{x^{5/3}\left(ax+bx^2\right)^{2/3}}$$

$$\Big\downarrow \; 71$$

$$\frac{(cx)^{7/3}(a+bx)^{2/3}\left(\frac{x^{5/3}\sqrt[3]{a+bx}}{2b}-\frac{5a\left(\frac{x^{2/3}\sqrt[3]{a+bx}}{b}-\frac{2a\left(-\frac{\sqrt{3}\arctan\left(\frac{2\sqrt[3]{b}\sqrt[3]{x}}{\sqrt{3}\sqrt[3]{a+bx}}+\frac{1}{\sqrt{3}}\right)}{b^{2/3}}-\frac{\log(a+bx)}{2b^{2/3}}-\frac{3\log\left(\frac{\sqrt[3]{b}\sqrt[3]{x}}{\sqrt[3]{a+bx}}-1\right)}{2b^{2/3}}\right)}{3b}\right)}{6b}\right)}{x^{5/3}\left(ax+bx^2\right)^{2/3}}$$

```
Int[(c*x)^(7/3)/(a*x + b*x^2)^(2/3),x]
```

```
((c*x)^(7/3)*(a + b*x)^(2/3)*((x^(5/3)*(a + b*x)^(1/3))/(2*b) - (5*a*((x^(
2/3)*(a + b*x)^(1/3))/b - (2*a*(-((Sqrt[3]*ArcTan[1/Sqrt[3] + (2*b^(1/3)*x
^(1/3))/(Sqrt[3]*(a + b*x)^(1/3))])/b^(2/3)) - Log[a + b*x]/(2*b^(2/3)) -
(3*Log[-1 + (b^(1/3)*x^(1/3))/(a + b*x)^(1/3)])/(2*b^(2/3))))/(3*b)))/(6*b
)))/(x^(5/3)*(a*x + b*x^2)^(2/3))
```

Defintions of rubi rules used

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^n/(b*(m + n + 1))), x] + Simp[n*((b*c - a*d)/(
b*(m + n + 1)))   Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
 c, d}, x] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ[m, 0] && ( !Integer
Q[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n + 2, 0] && IntLinear
Q[a, b, c, d, m, n, x]
```

```
Int[1/(((a_.) + (b_.)*(x_))^(1/3)*((c_.) + (d_.)*(x_))^(2/3)), x_Symbol] :>
 With[{q = Rt[d/b, 3]}, Simp[(-Sqrt[3])*(q/d)*ArcTan[2*q*((a + b*x)^(1/3)/(
Sqrt[3]*(c + d*x)^(1/3))) + 1/Sqrt[3]], x] + (-Simp[3*(q/(2*d))*Log[q*((a +
 b*x)^(1/3)/(c + d*x)^(1/3)) - 1], x] - Simp[(q/(2*d))*Log[c + d*x], x])] /
; FreeQ[{a, b, c, d}, x] && PosQ[d/b]
```

```
Int[((e_.)*(x_))^(m_)*((b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(
e*x)^m*((b*x + c*x^2)^p/(x^(m + p)*(b + c*x)^p))   Int[x^(m + p)*(b + c*x)^
p, x], x] /; FreeQ[{b, c, e, m}, x]
```

Maple [F]

$$\int \frac{(cx)^{\frac{7}{3}}}{(b\,x^{2}+ax)^{\frac{2}{3}}}dx$$

```
int((c*x)^(7/3)/(b*x^2+a*x)^(2/3),x)
```

```
int((c*x)^(7/3)/(b*x^2+a*x)^(2/3),x)
```

Fricas [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 235, normalized size of antiderivative = 0.84

$$\int \frac{(cx)^{7/3}}{(ax+bx^2)^{2/3}}\,dx =$$

$$-\frac{10\sqrt{3}a^2c^2\left(-\frac{c}{b^2}\right)^{\frac{1}{3}}\arctan\left(\frac{2\sqrt{3}(bx^2+ax)^{\frac{1}{3}}(cx)^{\frac{1}{3}}b\left(-\frac{c}{b^2}\right)^{\frac{2}{3}}+\sqrt{3}cx}{3\,cx}\right)-10\,a^2c^2\left(-\frac{c}{b^2}\right)^{\frac{1}{3}}\log\left(\frac{bx\left(-\frac{c}{b^2}\right)^{\frac{1}{3}}+(bx^2+ax)^{\frac{1}{3}}(cx)^{\frac{1}{3}}}{x}\right.}{}$$

```
integrate((c*x)^(7/3)/(b*x^2+a*x)^(2/3),x, algorithm="fricas")
```

```
-1/18*(10*sqrt(3)*a^2*c^2*(-c/b^2)^(1/3)*arctan(1/3*(2*sqrt(3)*(b*x^2 + a*
x)^(1/3)*(c*x)^(1/3)*b*(-c/b^2)^(2/3) + sqrt(3)*c*x)/(c*x)) - 10*a^2*c^2*(
-c/b^2)^(1/3)*log((b*x*(-c/b^2)^(1/3) + (b*x^2 + a*x)^(1/3)*(c*x)^(1/3))/x
) + 5*a^2*c^2*(-c/b^2)^(1/3)*log((b^2*x^2*(-c/b^2)^(2/3) - (b*x^2 + a*x)^(
1/3)*(c*x)^(1/3)*b*x*(-c/b^2)^(1/3) + (b*x^2 + a*x)^(2/3)*(c*x)^(2/3))/x^2
) - 3*(3*b*c^2*x - 5*a*c^2)*(b*x^2 + a*x)^(1/3)*(c*x)^(1/3))/b^2
```

Sympy [F]

$$\int \frac{(cx)^{7/3}}{(ax+bx^2)^{2/3}}\,dx = \int \frac{(cx)^{\frac{7}{3}}}{(x\,(a+bx))^{\frac{2}{3}}}\,dx$$

```
integrate((c*x)**(7/3)/(b*x**2+a*x)**(2/3),x)
```

```
Integral((c*x)**(7/3)/(x*(a + b*x))**(2/3), x)
```

Maxima [F]

$$\int \frac{(cx)^{7/3}}{(ax+bx^2)^{2/3}}\,dx = \int \frac{(cx)^{\frac{7}{3}}}{(bx^2+ax)^{\frac{2}{3}}}\,dx$$

```
integrate((c*x)^(7/3)/(b*x^2+a*x)^(2/3),x, algorithm="maxima")
```

```
integrate((c*x)^(7/3)/(b*x^2 + a*x)^(2/3), x)
```

Giac [A] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 206, normalized size of antiderivative = 0.74

$$\int \frac{(cx)^{7/3}}{(ax+bx^2)^{2/3}}\,dx = \frac{\frac{10\sqrt{3}(bc)^{\frac{1}{3}}a^3c^3\arctan\left(\frac{\sqrt{3}\left(2\left(bc+\frac{ac}{x}\right)^{\frac{1}{3}}+(bc)^{\frac{1}{3}}\right)}{3(bc)^{\frac{1}{3}}}\right)}{b^3}+\frac{5(bc)^{\frac{1}{3}}a^3c^3\log\left(\left(bc+\frac{ac}{x}\right)^{\frac{2}{3}}+\left(bc+\frac{ac}{x}\right)^{\frac{1}{3}}(bc)^{\frac{1}{3}}+(bc)^{\frac{2}{3}}\right)}{b^3}-}{18\,ac}$$

```
integrate((c*x)^(7/3)/(b*x^2+a*x)^(2/3),x, algorithm="giac")
```

```
1/18*(10*sqrt(3)*(b*c)^(1/3)*a^3*c^3*arctan(1/3*sqrt(3)*(2*(b*c + a*c/x)^(
1/3) + (b*c)^(1/3))/(b*c)^(1/3))/b^3 + 5*(b*c)^(1/3)*a^3*c^3*log((b*c + a*
c/x)^(2/3) + (b*c + a*c/x)^(1/3)*(b*c)^(1/3) + (b*c)^(2/3))/b^3 - 10*(b*c)
^(1/3)*a^3*c^3*log(abs((b*c + a*c/x)^(1/3) - (b*c)^(1/3)))/b^3 + 3*(8*(b*c
 + a*c/x)^(1/3)*a^3*b*c^5 - 5*(b*c + a*c/x)^(4/3)*a^3*c^4)*x^2/(a^2*b^2*c^
2))/(a*c)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(cx)^{7/3}}{(ax+bx^2)^{2/3}}\,dx=\int \frac{(c\,x)^{7/3}}{(b\,x^2+a\,x)^{2/3}}\,dx$$

```
int((c*x)^(7/3)/(a*x + b*x^2)^(2/3),x)
```

```
int((c*x)^(7/3)/(a*x + b*x^2)^(2/3), x)
```

Reduce [F]

$$\int \frac{(cx)^{7/3}}{(ax+bx^2)^{2/3}}\,dx=c^{\frac{7}{3}}\left(\int \frac{x^{\frac{5}{3}}}{(bx+a)^{\frac{2}{3}}}dx\right)$$

```
int((c*x)^(7/3)/(b*x^2+a*x)^(2/3),x)
```

```
c**(1/3)*int(x**2/(x**(1/3)*(a + b*x)**(2/3)),x)*c**2
```

3.140 $\int \frac{(cx)^{4/3}}{(ax+bx^2)^{2/3}}\,dx$

Optimal result

Integrand size = 21, antiderivative size = 235

$$\begin{aligned}\int \frac{(cx)^{4/3}}{(ax+bx^2)^{2/3}}\,dx &= \frac{c\sqrt[3]{cx}\sqrt[3]{ax+bx^2}}{b}\\ &+ \frac{2ac^{2/3}(cx)^{2/3}(a+bx)^{2/3}\arctan\left(\frac{1}{\sqrt{3}}+\frac{2\sqrt[3]{b}\sqrt[3]{cx}}{\sqrt{3}\sqrt[3]{c}\sqrt[3]{a+bx}}\right)}{\sqrt{3}b^{5/3}\left(ax+bx^2\right)^{2/3}}\\ &+ \frac{ac^{2/3}(cx)^{2/3}(a+bx)^{2/3}\log(a+bx)}{3b^{5/3}\left(ax+bx^2\right)^{2/3}}\\ &+ \frac{ac^{2/3}(cx)^{2/3}(a+bx)^{2/3}\log\left(1-\frac{\sqrt[3]{b}\sqrt[3]{cx}}{\sqrt[3]{c}\sqrt[3]{a+bx}}\right)}{b^{5/3}\left(ax+bx^2\right)^{2/3}}\end{aligned}$$

```
c*(c*x)^(1/3)*(b*x^2+a*x)^(1/3)/b+2/3*a*c^(2/3)*(c*x)^(2/3)*(b*x+a)^(2/3)*
arctan(1/3*3^(1/2)+2/3*b^(1/3)*(c*x)^(1/3)*3^(1/2)/c^(1/3)/(b*x+a)^(1/3))*
3^(1/2)/b^(5/3)/(b*x^2+a*x)^(2/3)+1/3*a*c^(2/3)*(c*x)^(2/3)*(b*x+a)^(2/3)*
ln(b*x+a)/b^(5/3)/(b*x^2+a*x)^(2/3)+a*c^(2/3)*(c*x)^(2/3)*(b*x+a)^(2/3)*ln
(1-b^(1/3)*(c*x)^(1/3)/c^(1/3)/(b*x+a)^(1/3))/b^(5/3)/(b*x^2+a*x)^(2/3)
```

Mathematica [A] (verified)

Time = 0.37 (sec) , antiderivative size = 207, normalized size of antiderivative = 0.88

$$\int \frac{(cx)^{4/3}}{(ax+bx^2)^{2/3}}\,dx = \frac{c\sqrt[3]{x}\sqrt[3]{cx}\left(3ab^{2/3}x^{2/3}+3b^{5/3}x^{5/3}+2\sqrt{3}a(a+bx)^{2/3}\arctan\left(\frac{\sqrt{3}\sqrt[3]{b}\sqrt[3]{x}}{\sqrt[3]{b}\sqrt[3]{x}+2\sqrt[3]{a+bx}}\right)+\right.}{}$$

```
Integrate[(c*x)^(4/3)/(a*x + b*x^2)^(2/3),x]
```

```
(c*x^(1/3)*(c*x)^(1/3)*(3*a*b^(2/3)*x^(2/3) + 3*b^(5/3)*x^(5/3) + 2*Sqrt[3
]*a*(a + b*x)^(2/3)*ArcTan[(Sqrt[3]*b^(1/3)*x^(1/3))/(b^(1/3)*x^(1/3) + 2*
(a + b*x)^(1/3))] + 2*a*(a + b*x)^(2/3)*Log[-(b^(1/3)*x^(1/3)) + (a + b*x)
^(1/3)] - a*(a + b*x)^(2/3)*Log[b^(2/3)*x^(2/3) + b^(1/3)*x^(1/3)*(a + b*x
)^(1/3) + (a + b*x)^(2/3)]))/(3*b^(5/3)*(x*(a + b*x))^(2/3))
```

Rubi [A] (verified)

Time = 0.36 (sec) , antiderivative size = 155, normalized size of antiderivative = 0.66, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.143$, Rules used = {1137, 60, 71}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(cx)^{4/3}}{(ax+bx^2)^{2/3}}\,dx$$

$$\Big\downarrow \text{1137}$$

$$\frac{(cx)^{4/3}(a+bx)^{2/3}\int \frac{x^{2/3}}{(a+bx)^{2/3}}dx}{x^{2/3}\left(ax+bx^2\right)^{2/3}}$$

$$\Big\downarrow \text{60}$$

$$\frac{(cx)^{4/3}(a+bx)^{2/3}\left(\frac{x^{2/3}\sqrt[3]{a+bx}}{b}-\frac{2a\int \frac{1}{\sqrt[3]{x}(a+bx)^{2/3}}dx}{3b}\right)}{x^{2/3}\left(ax+bx^2\right)^{2/3}}$$

$$\Big\downarrow \text{71}$$

$$\frac{(cx)^{4/3}(a+bx)^{2/3}\left(\frac{x^{2/3}\sqrt[3]{a+bx}}{b}-\frac{2a\left(-\frac{\sqrt{3}\arctan\left(\frac{2\sqrt[3]{b}\sqrt[3]{x}}{\sqrt{3}\sqrt[3]{a+bx}}+\frac{1}{\sqrt{3}}\right)}{b^{2/3}}-\frac{\log(a+bx)}{2b^{2/3}}-\frac{3\log\left(\frac{\sqrt[3]{b}\sqrt[3]{x}}{\sqrt[3]{a+bx}}-1\right)}{2b^{2/3}}\right)}{3b}\right)}{x^{2/3}\left(ax+bx^2\right)^{2/3}}$$

```
Int[(c*x)^(4/3)/(a*x + b*x^2)^(2/3),x]
```

```
((c*x)^(4/3)*(a + b*x)^(2/3)*((x^(2/3)*(a + b*x)^(1/3))/b - (2*a*(-((Sqrt[
3]*ArcTan[1/Sqrt[3] + (2*b^(1/3)*x^(1/3))/(Sqrt[3]*(a + b*x)^(1/3))])/b^(2
/3)) - Log[a + b*x]/(2*b^(2/3)) - (3*Log[-1 + (b^(1/3)*x^(1/3))/(a + b*x)^
(1/3)])/(2*b^(2/3))))/(3*b)))/(x^(2/3)*(a*x + b*x^2)^(2/3))
```

Defintions of rubi rules used

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^n/(b*(m + n + 1))), x] + Simp[n*((b*c - a*d)/(
b*(m + n + 1)))   Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
 c, d}, x] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ[m, 0] && ( !Integer
Q[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n + 2, 0] && IntLinear
Q[a, b, c, d, m, n, x]
```

```
Int[1/(((a_.) + (b_.)*(x_))^(1/3)*((c_.) + (d_.)*(x_))^(2/3)), x_Symbol] :>
 With[{q = Rt[d/b, 3]}, Simp[(-Sqrt[3])*(q/d)*ArcTan[2*q*((a + b*x)^(1/3)/(
Sqrt[3]*(c + d*x)^(1/3))) + 1/Sqrt[3]], x] + (-Simp[3*(q/(2*d))*Log[q*((a +
 b*x)^(1/3)/(c + d*x)^(1/3)) - 1], x] - Simp[(q/(2*d))*Log[c + d*x], x])] /
; FreeQ[{a, b, c, d}, x] && PosQ[d/b]
```

```
Int[((e_.)*(x_))^(m_)*((b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(
e*x)^m*((b*x + c*x^2)^p/(x^(m + p)*(b + c*x)^p))   Int[x^(m + p)*(b + c*x)^
p, x], x] /; FreeQ[{b, c, e, m}, x]
```

Maple [F]

$$\int \frac{(cx)^{\frac{4}{3}}}{(b\,x^2+ax)^{\frac{2}{3}}}dx$$

```
int((c*x)^(4/3)/(b*x^2+a*x)^(2/3),x)
```

```
int((c*x)^(4/3)/(b*x^2+a*x)^(2/3),x)
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 203, normalized size of antiderivative = 0.86

$$\int \frac{(cx)^{4/3}}{(ax+bx^2)^{2/3}}\,dx = -\frac{2\sqrt{3}ac\left(\frac{c}{b^2}\right)^{\frac{1}{3}}\arctan\left(\frac{2\sqrt{3}(bx^2+ax)^{\frac{1}{3}}(cx)^{\frac{1}{3}}b\left(\frac{c}{b^2}\right)^{\frac{2}{3}}+\sqrt{3}cx}{3\,cx}\right)-2\,ac\left(\frac{c}{b^2}\right)^{\frac{1}{3}}\log\left(-\frac{bx\left(\frac{c}{b^2}\right)^{\frac{1}{3}}-(bx^2+ax)^{\frac{1}{3}}(cx)^{\frac{1}{3}}}{x}\right)+ac\left(\frac{c}{b^2}\right.}{3\,b}$$

```
integrate((c*x)^(4/3)/(b*x^2+a*x)^(2/3),x, algorithm="fricas")
```

```
-1/3*(2*sqrt(3)*a*c*(c/b^2)^(1/3)*arctan(1/3*(2*sqrt(3)*(b*x^2 + a*x)^(1/3
)*(c*x)^(1/3)*b*(c/b^2)^(2/3) + sqrt(3)*c*x)/(c*x)) - 2*a*c*(c/b^2)^(1/3)*
log(-(b*x*(c/b^2)^(1/3) - (b*x^2 + a*x)^(1/3)*(c*x)^(1/3))/x) + a*c*(c/b^2
)^(1/3)*log((b^2*x^2*(c/b^2)^(2/3) + (b*x^2 + a*x)^(1/3)*(c*x)^(1/3)*b*x*(
c/b^2)^(1/3) + (b*x^2 + a*x)^(2/3)*(c*x)^(2/3))/x^2) - 3*(b*x^2 + a*x)^(1/
3)*(c*x)^(1/3)*c)/b
```

Sympy [F]

$$\int \frac{(cx)^{4/3}}{(ax+bx^2)^{2/3}}\,dx = \int \frac{(cx)^{\frac{4}{3}}}{(x\,(a+bx))^{\frac{2}{3}}}\,dx$$

```
integrate((c*x)**(4/3)/(b*x**2+a*x)**(2/3),x)
```

```
Integral((c*x)**(4/3)/(x*(a + b*x))**(2/3), x)
```

Maxima [F]

$$\int \frac{(cx)^{4/3}}{(ax+bx^2)^{2/3}}\,dx = \int \frac{(cx)^{\frac{4}{3}}}{(bx^2+ax)^{\frac{2}{3}}}\,dx$$

```
integrate((c*x)^(4/3)/(b*x^2+a*x)^(2/3),x, algorithm="maxima")
```

```
integrate((c*x)^(4/3)/(b*x^2 + a*x)^(2/3), x)
```

Giac [A] (verification not implemented)

Time = 0.18 (sec) , antiderivative size = 162, normalized size of antiderivative = 0.69

$$\int \frac{(cx)^{4/3}}{(ax+bx^2)^{2/3}}\,dx =$$

$$-\frac{1}{3}\,ac^3\left(\frac{2\sqrt{3}(bc)^{\frac{1}{3}}\arctan\left(\frac{\sqrt{3}\left(2\left(bc+\frac{ac}{x}\right)^{\frac{1}{3}}+(bc)^{\frac{1}{3}}\right)}{3\,(bc)^{\frac{1}{3}}}\right)}{b^2c^2} - \frac{3\left(bc+\frac{ac}{x}\right)^{\frac{1}{3}}x}{abc^2} + \frac{(bc)^{\frac{1}{3}}\log\left(\left(bc+\frac{ac}{x}\right)^{\frac{2}{3}}+\left(bc+\frac{ac}{x}\right)^{\frac{1}{3}}(\right.}{b^2c^2}\right.$$

```
integrate((c*x)^(4/3)/(b*x^2+a*x)^(2/3),x, algorithm="giac")
```

```
-1/3*a*c^3*(2*sqrt(3)*(b*c)^(1/3)*arctan(1/3*sqrt(3)*(2*(b*c + a*c/x)^(1/3
) + (b*c)^(1/3))/(b*c)^(1/3))/(b^2*c^2) - 3*(b*c + a*c/x)^(1/3)*x/(a*b*c^2
) + (b*c)^(1/3)*log((b*c + a*c/x)^(2/3) + (b*c + a*c/x)^(1/3)*(b*c)^(1/3)
+ (b*c)^(2/3))/(b^2*c^2) - 2*(b*c)^(1/3)*log(abs((b*c + a*c/x)^(1/3) - (b*
c)^(1/3)))/(b^2*c^2))
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(cx)^{4/3}}{(ax+bx^2)^{2/3}}\,dx = \int \frac{(c\,x)^{4/3}}{(b\,x^2+a\,x)^{2/3}}\,dx$$

```
int((c*x)^(4/3)/(a*x + b*x^2)^(2/3),x)
```

```
int((c*x)^(4/3)/(a*x + b*x^2)^(2/3), x)
```

Reduce [F]

$$\int \frac{(cx)^{4/3}}{(ax+bx^2)^{2/3}}\,dx = c^{\frac{4}{3}}\left(\int \frac{x^{\frac{2}{3}}}{(bx+a)^{\frac{2}{3}}}dx\right)$$

```
int((c*x)^(4/3)/(b*x^2+a*x)^(2/3),x)
```

```
c**(1/3)*int(x/(x**(1/3)*(a + b*x)**(2/3)),x)*c
```

3.141 $\int \frac{\sqrt[3]{cx}}{(ax+bx^2)^{2/3}} \, dx$

Optimal result

Integrand size = 21, antiderivative size = 210

$$\int \frac{\sqrt[3]{cx}}{(ax+bx^2)^{2/3}} \, dx = -\frac{\sqrt{3}(cx)^{2/3}(a+bx)^{2/3}\arctan\left(\frac{1}{\sqrt{3}}+\frac{2\sqrt[3]{b}\sqrt[3]{cx}}{\sqrt{3}\sqrt[3]{c}\sqrt[3]{a+bx}}\right)}{b^{2/3}\sqrt[3]{c}\,(ax+bx^2)^{2/3}} - \frac{(cx)^{2/3}(a+bx)^{2/3}\log(a+bx)}{2b^{2/3}\sqrt[3]{c}\,(ax+bx^2)^{2/3}} - \frac{3(cx)^{2/3}(a+bx)^{2/3}\log\left(1-\frac{\sqrt[3]{b}\sqrt[3]{cx}}{\sqrt[3]{c}\sqrt[3]{a+bx}}\right)}{2b^{2/3}\sqrt[3]{c}\,(ax+bx^2)^{2/3}}$$

```
-3^(1/2)*(c*x)^(2/3)*(b*x+a)^(2/3)*arctan(1/3*3^(1/2)+2/3*b^(1/3)*(c*x)^(1
/3)*3^(1/2)/c^(1/3)/(b*x+a)^(1/3))/b^(2/3)/c^(1/3)/(b*x^2+a*x)^(2/3)-1/2*(
c*x)^(2/3)*(b*x+a)^(2/3)*ln(b*x+a)/b^(2/3)/c^(1/3)/(b*x^2+a*x)^(2/3)-3/2*(
c*x)^(2/3)*(b*x+a)^(2/3)*ln(1-b^(1/3)*(c*x)^(1/3)/c^(1/3)/(b*x+a)^(1/3))/b
^(2/3)/c^(1/3)/(b*x^2+a*x)^(2/3)
```

Mathematica [A] (verified)

Time = 0.24 (sec) , antiderivative size = 158, normalized size of antiderivative = 0.75

$$\int \frac{\sqrt[3]{cx}}{(ax+bx^2)^{2/3}}\,dx = \frac{\sqrt[3]{x}\sqrt[3]{cx}(a+bx)^{2/3}\left(-2\sqrt{3}\arctan\left(\frac{\sqrt{3}\sqrt[3]{b}\sqrt[3]{x}}{\sqrt[3]{b}\sqrt[3]{x}+2\sqrt[3]{a+bx}}\right)-2\log\left(-\sqrt[3]{b}\sqrt[3]{x}+\sqrt[3]{a+\cdots}\right.\right.}{2b^{2/3}(x(a+bx))^{2/3}}$$

```
Integrate[(c*x)^(1/3)/(a*x + b*x^2)^(2/3),x]
```

```
(x^(1/3)*(c*x)^(1/3)*(a + b*x)^(2/3)*(-2*Sqrt[3]*ArcTan[(Sqrt[3]*b^(1/3)*x
^(1/3))/(b^(1/3)*x^(1/3) + 2*(a + b*x)^(1/3))] - 2*Log[-(b^(1/3)*x^(1/3))
+ (a + b*x)^(1/3)] + Log[b^(2/3)*x^(2/3) + b^(1/3)*x^(1/3)*(a + b*x)^(1/3)
 + (a + b*x)^(2/3)]))/(2*b^(2/3)*(x*(a + b*x))^(2/3))
```

Rubi [A] (verified)

Time = 0.33 (sec) , antiderivative size = 128, normalized size of antiderivative = 0.61, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.095$, Rules used = {1137, 71}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sqrt[3]{cx}}{(ax+bx^2)^{2/3}}\,dx$$

$$\Big\downarrow \; 1137$$

$$\frac{\sqrt[3]{x}\sqrt[3]{cx}(a+bx)^{2/3}\int \frac{1}{\sqrt[3]{x}(a+bx)^{2/3}}dx}{(ax+bx^2)^{2/3}}$$

$$\Big\downarrow \; 71$$

$$\frac{\sqrt[3]{x}\sqrt[3]{cx}(a+bx)^{2/3}\left(-\frac{\sqrt{3}\arctan\left(\frac{2\sqrt[3]{b}\sqrt[3]{x}}{\sqrt{3}\sqrt[3]{a+bx}}+\frac{1}{\sqrt{3}}\right)}{b^{2/3}}-\frac{\log(a+bx)}{2b^{2/3}}-\frac{3\log\left(\frac{\sqrt[3]{b}\sqrt[3]{x}}{\sqrt[3]{a+bx}}-1\right)}{2b^{2/3}}\right)}{(ax+bx^2)^{2/3}}$$

```
Int[(c*x)^(1/3)/(a*x + b*x^2)^(2/3),x]
```

```
(x^(1/3)*(c*x)^(1/3)*(a + b*x)^(2/3)*(-((Sqrt[3]*ArcTan[1/Sqrt[3] + (2*b^(
1/3)*x^(1/3))/(Sqrt[3]*(a + b*x)^(1/3))])/b^(2/3)) - Log[a + b*x]/(2*b^(2/
3)) - (3*Log[-1 + (b^(1/3)*x^(1/3))/(a + b*x)^(1/3)])/(2*b^(2/3))))/(a*x +
 b*x^2)^(2/3)
```

Defintions of rubi rules used

```
Int[1/(((a_.) + (b_.)*(x_))^(1/3)*((c_.) + (d_.)*(x_))^(2/3)), x_Symbol] :>
 With[{q = Rt[d/b, 3]}, Simp[(-Sqrt[3])*(q/d)*ArcTan[2*q*((a + b*x)^(1/3)/(
Sqrt[3]*(c + d*x)^(1/3))) + 1/Sqrt[3]], x] + (-Simp[3*(q/(2*d))*Log[q*((a +
 b*x)^(1/3)/(c + d*x)^(1/3)) - 1], x] - Simp[(q/(2*d))*Log[c + d*x], x])] /
; FreeQ[{a, b, c, d}, x] && PosQ[d/b]
```

```
Int[((e_.)*(x_))^(m_)*((b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(
e*x)^m*((b*x + c*x^2)^p/(x^(m + p)*(b + c*x)^p))   Int[x^(m + p)*(b + c*x)^
p, x], x] /; FreeQ[{b, c, e, m}, x]
```

Maple [F]

$$\int \frac{(cx)^{\frac{1}{3}}}{(b\,x^2 + ax)^{\frac{2}{3}}}dx$$

```
int((c*x)^(1/3)/(b*x^2+a*x)^(2/3),x)
```

```
int((c*x)^(1/3)/(b*x^2+a*x)^(2/3),x)
```

Fricas [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 179, normalized size of antiderivative = 0.85

$$\int \frac{\sqrt[3]{cx}}{(ax+bx^2)^{2/3}}\,dx = -\sqrt{3}\left(-\frac{c}{b^2}\right)^{\frac{1}{3}}\arctan\left(\frac{2\sqrt{3}(bx^2+ax)^{\frac{1}{3}}(cx)^{\frac{1}{3}}\,b\left(-\frac{c}{b^2}\right)^{\frac{2}{3}}+\sqrt{3}cx}{3\,cx}\right) + \left(-\frac{c}{b^2}\right)^{\frac{1}{3}}\log\left(\frac{bx\left(-\frac{c}{b^2}\right)^{\frac{1}{3}}+(bx^2+ax)^{\frac{1}{3}}(cx)^{\frac{1}{3}}}{x}\right) - \frac{1}{2}\left(-\frac{c}{b^2}\right)^{\frac{1}{3}}\log\left(\frac{b^2x^2\left(-\frac{c}{b^2}\right)^{\frac{2}{3}}-(bx^2+ax)^{\frac{1}{3}}(cx)^{\frac{1}{3}}\,bx\left(-\frac{c}{b^2}\right)^{\frac{1}{3}}+(bx^2+ax)^{\frac{2}{3}}(cx)^{\frac{2}{3}}}{x^2}\right)$$

```
integrate((c*x)^(1/3)/(b*x^2+a*x)^(2/3),x, algorithm="fricas")
```

```
-sqrt(3)*(-c/b^2)^(1/3)*arctan(1/3*(2*sqrt(3)*(b*x^2 + a*x)^(1/3)*(c*x)^(1
/3)*b*(-c/b^2)^(2/3) + sqrt(3)*c*x)/(c*x)) + (-c/b^2)^(1/3)*log((b*x*(-c/b
^2)^(1/3) + (b*x^2 + a*x)^(1/3)*(c*x)^(1/3))/x) - 1/2*(-c/b^2)^(1/3)*log((
b^2*x^2*(-c/b^2)^(2/3) - (b*x^2 + a*x)^(1/3)*(c*x)^(1/3)*b*x*(-c/b^2)^(1/3
) + (b*x^2 + a*x)^(2/3)*(c*x)^(2/3))/x^2)
```

Sympy [F]

$$\int \frac{\sqrt[3]{cx}}{(ax+bx^2)^{2/3}}\,dx = \int \frac{\sqrt[3]{cx}}{(x\,(a+bx))^{\frac{2}{3}}}\,dx$$

```
integrate((c*x)**(1/3)/(b*x**2+a*x)**(2/3),x)
```

```
Integral((c*x)**(1/3)/(x*(a + b*x))**(2/3), x)
```

Maxima [F]

$$\int \frac{\sqrt[3]{cx}}{(ax+bx^2)^{2/3}}\,dx = \int \frac{(cx)^{\frac{1}{3}}}{(bx^2+ax)^{\frac{2}{3}}}\,dx$$

```
integrate((c*x)^(1/3)/(b*x^2+a*x)^(2/3),x, algorithm="maxima")
```

```
integrate((c*x)^(1/3)/(b*x^2 + a*x)^(2/3), x)
```

Giac [A] (verification not implemented)

Time = 0.17 (sec) , antiderivative size = 135, normalized size of antiderivative = 0.64

$$\int \frac{\sqrt[3]{cx}}{(ax+bx^2)^{2/3}}\,dx = \frac{1}{2}c\left(\frac{2\sqrt{3}(bc)^{\frac{1}{3}}\arctan\left(\frac{\sqrt{3}\left(2\left(bc+\frac{ac}{x}\right)^{\frac{1}{3}}+(bc)^{\frac{1}{3}}\right)}{3(bc)^{\frac{1}{3}}}\right)}{bc} + \frac{(bc)^{\frac{1}{3}}\log\left(\left(bc+\frac{ac}{x}\right)^{\frac{2}{3}}+\left(bc+\frac{ac}{x}\right)^{\frac{1}{3}}}{bc}\right.$$

```
integrate((c*x)^(1/3)/(b*x^2+a*x)^(2/3),x, algorithm="giac")
```

```
1/2*c*(2*sqrt(3)*(b*c)^(1/3)*arctan(1/3*sqrt(3)*(2*(b*c + a*c/x)^(1/3) + (
b*c)^(1/3))/(b*c)^(1/3))/(b*c) + (b*c)^(1/3)*log((b*c + a*c/x)^(2/3) + (b*
c + a*c/x)^(1/3)*(b*c)^(1/3) + (b*c)^(2/3))/(b*c) - 2*(b*c)^(1/3)*log(abs(
(b*c + a*c/x)^(1/3) - (b*c)^(1/3)))/(b*c))
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\sqrt[3]{cx}}{(ax+bx^2)^{2/3}}\,dx = \int \frac{(c\,x)^{1/3}}{(b\,x^2+a\,x)^{2/3}}\,dx$$

```
int((c*x)^(1/3)/(a*x + b*x^2)^(2/3),x)
```

```
int((c*x)^(1/3)/(a*x + b*x^2)^(2/3), x)
```

Reduce [F]

$$\int \frac{\sqrt[3]{cx}}{(ax+bx^2)^{2/3}}\,dx = c^{\frac{1}{3}}\left(\int \frac{1}{x^{\frac{1}{3}}\,(bx+a)^{\frac{2}{3}}}dx\right)$$

```
int((c*x)^(1/3)/(b*x^2+a*x)^(2/3),x)
```

```
c**(1/3)*int(1/(x**(1/3)*(a + b*x)**(2/3)),x)
```

3.142 $\int \frac{1}{\sqrt[3]{cx}\,(ax+bx^2)^{2/3}}\,dx$

Optimal result

Integrand size = 21, antiderivative size = 176

$$\int \frac{1}{\sqrt[3]{cx}\,(ax+bx^2)^{2/3}}\,dx = -\frac{\sqrt{3}(cx)^{2/3}(a+bx)^{2/3}\arctan\left(\frac{\sqrt[3]{a}+2\sqrt[3]{a+bx}}{\sqrt{3}\sqrt[3]{a}}\right)}{a^{2/3}c\,(ax+bx^2)^{2/3}} - \frac{(cx)^{2/3}(a+bx)^{2/3}\log(x)}{2a^{2/3}c\,(ax+bx^2)^{2/3}} + \frac{3(cx)^{2/3}(a+bx)^{2/3}\log\left(\sqrt[3]{a}-\sqrt[3]{a+bx}\right)}{2a^{2/3}c\,(ax+bx^2)^{2/3}}$$

```
-3^(1/2)*(c*x)^(2/3)*(b*x+a)^(2/3)*arctan(1/3*(a^(1/3)+2*(b*x+a)^(1/3))*3^
(1/2)/a^(1/3))/a^(2/3)/c/(b*x^2+a*x)^(2/3)-1/2*(c*x)^(2/3)*(b*x+a)^(2/3)*l
n(x)/a^(2/3)/c/(b*x^2+a*x)^(2/3)+3/2*(c*x)^(2/3)*(b*x+a)^(2/3)*ln(a^(1/3)-
(b*x+a)^(1/3))/a^(2/3)/c/(b*x^2+a*x)^(2/3)
```

Mathematica [A] (verified)

Time = 0.10 (sec) , antiderivative size = 120, normalized size of antiderivative = 0.68

$$\int \frac{1}{\sqrt[3]{cx}\,(ax+bx^2)^{2/3}}\,dx =$$

$$-\frac{\sqrt[3]{x(a+bx)}\left(2\sqrt{3}\arctan\left(\frac{1+\frac{2\sqrt[3]{a+bx}}{\sqrt[3]{a}}}{\sqrt{3}}\right)-2\log\left(\sqrt[3]{a}-\sqrt[3]{a+bx}\right)+\log\left(a^{2/3}+\sqrt[3]{a}\sqrt[3]{a+bx}+(a+b\right.\right.}{2a^{2/3}\sqrt[3]{cx}\sqrt[3]{a+bx}}$$

```
Integrate[1/((c*x)^(1/3)*(a*x + b*x^2)^(2/3)),x]
```

```
-1/2*((x*(a + b*x))^(1/3)*(2*Sqrt[3]*ArcTan[(1 + (2*(a + b*x)^(1/3))/a^(1/
3))/Sqrt[3]] - 2*Log[a^(1/3) - (a + b*x)^(1/3)] + Log[a^(2/3) + a^(1/3)*(a
 + b*x)^(1/3) + (a + b*x)^(2/3)]))/(a^(2/3)*(c*x)^(1/3)*(a + b*x)^(1/3))
```

Rubi [A] (verified)

Time = 0.36 (sec) , antiderivative size = 107, normalized size of antiderivative = 0.61, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.238$, Rules used = {1137, 69, 16, 1082, 217}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{\sqrt[3]{cx}\,(ax+bx^2)^{2/3}}\,dx$$

$$\Big\downarrow \; 1137$$

$$\frac{x(a+bx)^{2/3}\int \frac{1}{x(a+bx)^{2/3}}dx}{\sqrt[3]{cx}\,(ax+bx^2)^{2/3}}$$

$$\Big\downarrow \; 69$$

$$\frac{x(a+bx)^{2/3}\left(-\frac{3\int \frac{1}{\sqrt[3]{a}-\sqrt[3]{a+bx}}d\sqrt[3]{a+bx}}{2a^{2/3}}-\frac{3\int \frac{1}{a^{2/3}+\sqrt[3]{a+bx}\sqrt[3]{a}+(a+bx)^{2/3}}d\sqrt[3]{a+bx}}{2\sqrt[3]{a}}-\frac{\log(x)}{2a^{2/3}}\right)}{\sqrt[3]{cx}\,(ax+bx^2)^{2/3}}$$

$$\Big\downarrow \; 16$$

$$\frac{x(a+bx)^{2/3}\left(-\frac{3\int \frac{1}{a^{2/3}+\sqrt[3]{a+bx}\sqrt[3]{a}+(a+bx)^{2/3}}d\sqrt[3]{a+bx}}{2\sqrt[3]{a}}+\frac{3\log\left(\sqrt[3]{a}-\sqrt[3]{a+bx}\right)}{2a^{2/3}}-\frac{\log(x)}{2a^{2/3}}\right)}{\sqrt[3]{cx}\,(ax+bx^2)^{2/3}}$$

$$\Big\downarrow \; 1082$$

$$\frac{x(a+bx)^{2/3}\left(\frac{3\int \frac{1}{-(a+bx)^{2/3}-3}d\left(\frac{2\sqrt[3]{a+bx}}{\sqrt[3]{a}}+1\right)}{a^{2/3}}+\frac{3\log\left(\sqrt[3]{a}-\sqrt[3]{a+bx}\right)}{2a^{2/3}}-\frac{\log(x)}{2a^{2/3}}\right)}{\sqrt[3]{cx}\,(ax+bx^2)^{2/3}}$$

$$\Big\downarrow \; 217$$

$$\frac{x(a+bx)^{2/3}\left(-\frac{\sqrt{3}\arctan\left(\frac{\frac{2\sqrt[3]{a+bx}}{\sqrt[3]{a}}+1}{\sqrt{3}}\right)}{a^{2/3}}+\frac{3\log\left(\sqrt[3]{a}-\sqrt[3]{a+bx}\right)}{2a^{2/3}}-\frac{\log(x)}{2a^{2/3}}\right)}{\sqrt[3]{cx}\,(ax+bx^2)^{2/3}}$$

```
Int[1/((c*x)^(1/3)*(a*x + b*x^2)^(2/3)),x]
```

```
(x*(a + b*x)^(2/3)*(-((Sqrt[3]*ArcTan[(1 + (2*(a + b*x)^(1/3))/a^(1/3))/Sq
rt[3]])/a^(2/3)) - Log[x]/(2*a^(2/3)) + (3*Log[a^(1/3) - (a + b*x)^(1/3)])
/(2*a^(2/3))))/((c*x)^(1/3)*(a*x + b*x^2)^(2/3))
```

Defintions of rubi rules used

```
Int[(c_.)/((a_.) + (b_.)*(x_)), x_Symbol] :> Simp[c*(Log[RemoveContent[a +
b*x, x]]/b), x] /; FreeQ[{a, b, c}, x]
```

```
Int[1/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(2/3)), x_Symbol] :> With[
{q = Rt[(b*c - a*d)/b, 3]}, Simp[-Log[RemoveContent[a + b*x, x]]/(2*b*q^2),
 x] + (-Simp[3/(2*b*q)   Subst[Int[1/(q^2 + q*x + x^2), x], x, (c + d*x)^(1
/3)], x] - Simp[3/(2*b*q^2)   Subst[Int[1/(q - x), x], x, (c + d*x)^(1/3)],
 x])] /; FreeQ[{a, b, c, d}, x] && PosQ[(b*c - a*d)/b]
```

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Simp[-2/b   Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b
)], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Fre
eQ[{a, b, c}, x]
```

```
Int[((e_.)*(x_))^(m_)*((b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(
e*x)^m*((b*x + c*x^2)^p/(x^(m + p)*(b + c*x)^p))   Int[x^(m + p)*(b + c*x)^
p, x], x] /; FreeQ[{b, c, e, m}, x]
```

Maple [F]

$$\int \frac{1}{(cx)^{\frac{1}{3}} (b\,x^2+ax)^{\frac{2}{3}}}dx$$

```
int(1/(c*x)^(1/3)/(b*x^2+a*x)^(2/3),x)
```


```
int(1/(c*x)^(1/3)/(b*x^2+a*x)^(2/3),x)
```

Fricas [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 468, normalized size of antiderivative = 2.66

$$\int \frac{1}{\sqrt[3]{cx}\left(ax+bx^2\right)^{2/3}}\,dx = \left[\frac{\sqrt{3}ac\sqrt{-\frac{(a^2c)^{\frac{1}{3}}}{c}}\log\left(\frac{2\,abcx^2+3\,a^2cx-3\,(a^2c)^{\frac{1}{3}}(bx^2+ax)^{\frac{1}{3}}(cx)^{\frac{2}{3}}a+\sqrt{3}\left((a^2c)^{\frac{1}{3}}acx-2\,(bx^2+ax)\right.}{x^2}\right.}{}\right.$$

$$-\frac{2\sqrt{3}ac\sqrt{\frac{(a^2c)^{\frac{1}{3}}}{c}}\arctan\left(\frac{\sqrt{3}\left((a^2c)^{\frac{1}{3}}acx+2\,(a^2c)^{\frac{2}{3}}(bx^2+ax)^{\frac{1}{3}}(cx)^{\frac{2}{3}}\right)\sqrt{\frac{(a^2c)^{\frac{1}{3}}}{c}}}{3\,a^2cx}\right)+(a^2c)^{\frac{2}{3}}\log\left(\frac{(a^2c)^{\frac{1}{3}}acx+(bx^2+ax)^{\frac{2}{3}}(cx)}{}\right.}{2\,a^2c}$$

```
integrate(1/(c*x)^(1/3)/(b*x^2+a*x)^(2/3),x, algorithm="fricas")
```

```
[1/2*(sqrt(3)*a*c*sqrt(-(a^2*c)^(1/3)/c)*log((2*a*b*c*x^2 + 3*a^2*c*x - 3*
(a^2*c)^(1/3)*(b*x^2 + a*x)^(1/3)*(c*x)^(2/3)*a + sqrt(3)*((a^2*c)^(1/3)*a
*c*x - 2*(b*x^2 + a*x)^(2/3)*(c*x)^(1/3)*a*c + (a^2*c)^(2/3)*(b*x^2 + a*x)
^(1/3)*(c*x)^(2/3))*sqrt(-(a^2*c)^(1/3)/c))/x^2) - (a^2*c)^(2/3)*log(((a^2
*c)^(1/3)*a*c*x + (b*x^2 + a*x)^(2/3)*(c*x)^(1/3)*a*c + (a^2*c)^(2/3)*(b*x
^2 + a*x)^(1/3)*(c*x)^(2/3))/x) + 2*(a^2*c)^(2/3)*log(((b*x^2 + a*x)^(1/3)
*(c*x)^(2/3)*a - (a^2*c)^(2/3)*x)/x))/(a^2*c), -1/2*(2*sqrt(3)*a*c*sqrt((a
^2*c)^(1/3)/c)*arctan(1/3*sqrt(3)*((a^2*c)^(1/3)*a*c*x + 2*(a^2*c)^(2/3)*(
b*x^2 + a*x)^(1/3)*(c*x)^(2/3))*sqrt((a^2*c)^(1/3)/c)/(a^2*c*x)) + (a^2*c)
^(2/3)*log(((a^2*c)^(1/3)*a*c*x + (b*x^2 + a*x)^(2/3)*(c*x)^(1/3)*a*c + (a
^2*c)^(2/3)*(b*x^2 + a*x)^(1/3)*(c*x)^(2/3))/x) - 2*(a^2*c)^(2/3)*log(((b*
x^2 + a*x)^(1/3)*(c*x)^(2/3)*a - (a^2*c)^(2/3)*x)/x))/(a^2*c)]
```

Sympy [F]

$$\int \frac{1}{\sqrt[3]{cx}\,(ax+bx^2)^{2/3}}\,dx = \int \frac{1}{\sqrt[3]{cx}\,(x\,(a+bx))^{\frac{2}{3}}}\,dx$$

```
integrate(1/(c*x)**(1/3)/(b*x**2+a*x)**(2/3),x)
```

```
Integral(1/((c*x)**(1/3)*(x*(a + b*x))**(2/3)), x)
```

Maxima [F]

$$\int \frac{1}{\sqrt[3]{cx}\,(ax+bx^2)^{2/3}}\,dx = \int \frac{1}{(bx^2+ax)^{\frac{2}{3}}\,(cx)^{\frac{1}{3}}}\,dx$$

```
integrate(1/(c*x)^(1/3)/(b*x^2+a*x)^(2/3),x, algorithm="maxima")
```

```
integrate(1/((b*x^2 + a*x)^(2/3)*(c*x)^(1/3)), x)
```

Giac [F]

$$\int \frac{1}{\sqrt[3]{cx}\,(ax+bx^2)^{2/3}}\,dx = \int \frac{1}{(bx^2+ax)^{\frac{2}{3}}\,(cx)^{\frac{1}{3}}}\,dx$$

```
integrate(1/(c*x)^(1/3)/(b*x^2+a*x)^(2/3),x, algorithm="giac")
```

```
integrate(1/((b*x^2 + a*x)^(2/3)*(c*x)^(1/3)), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{\sqrt[3]{cx}\,(ax+bx^2)^{2/3}}\,dx = \int \frac{1}{(b\,x^2+a\,x)^{2/3}\,(c\,x)^{1/3}}\,dx$$

```
int(1/((a*x + b*x^2)^(2/3)*(c*x)^(1/3)),x)
```

```
int(1/((a*x + b*x^2)^(2/3)*(c*x)^(1/3)), x)
```

Reduce [B] (verification not implemented)

Time = 0.54 (sec) , antiderivative size = 144, normalized size of antiderivative = 0.82

$$\int \frac{1}{\sqrt[3]{cx}\,(ax+bx^2)^{2/3}}\,dx = \frac{2\sqrt{3}\,atan\left(\frac{2(bx+a)^{\frac{1}{6}}+a^{\frac{1}{6}}}{a^{\frac{1}{6}}\sqrt{3}}\right)-2\sqrt{3}\,atan\left(\frac{2(bx+a)^{\frac{1}{6}}-a^{\frac{1}{6}}}{a^{\frac{1}{6}}\sqrt{3}}\right)+2\log\left((bx+a)^{\frac{1}{6}}+a^{\frac{1}{6}}\right)+}{}$$

```
int(1/(c*x)^(1/3)/(b*x^2+a*x)^(2/3),x)
```

```
(2*sqrt(3)*atan((2*(a + b*x)**(1/6) + a**(1/6))/(a**(1/6)*sqrt(3))) - 2*sq
rt(3)*atan((2*(a + b*x)**(1/6) - a**(1/6))/(a**(1/6)*sqrt(3))) + 2*log((a
+ b*x)**(1/6) + a**(1/6)) + 2*log((a + b*x)**(1/6) - a**(1/6)) - log( - a*
*(1/6)*(a + b*x)**(1/6) + (a + b*x)**(1/3) + a**(1/3)) - log(a**(1/6)*(a +
 b*x)**(1/6) + (a + b*x)**(1/3) + a**(1/3)))/(2*c**(1/3)*a**(2/3))
```

3.143 $\int \frac{1}{(cx)^{4/3}\left(ax+bx^2\right)^{2/3}}\,dx$

Optimal result

Integrand size = 21, antiderivative size = 202

$$\begin{aligned}\int \frac{1}{(cx)^{4/3}\left(ax+bx^2\right)^{2/3}}\,dx &= -\frac{\sqrt[3]{ax+bx^2}}{a(cx)^{4/3}} \\ &+ \frac{2b(cx)^{2/3}(a+bx)^{2/3}\arctan\left(\frac{\sqrt[3]{a}+2\sqrt[3]{a+bx}}{\sqrt{3}\sqrt[3]{a}}\right)}{\sqrt{3}a^{5/3}c^2\left(ax+bx^2\right)^{2/3}} \\ &+ \frac{b(cx)^{2/3}(a+bx)^{2/3}\log(x)}{3a^{5/3}c^2\left(ax+bx^2\right)^{2/3}} - \frac{b(cx)^{2/3}(a+bx)^{2/3}\log\left(\sqrt[3]{a}-\sqrt[3]{a+bx}\right)}{a^{5/3}c^2\left(ax+bx^2\right)^{2/3}}\end{aligned}$$

```
-(b*x^2+a*x)^(1/3)/a/(c*x)^(4/3)+2/3*b*(c*x)^(2/3)*(b*x+a)^(2/3)*arctan(1/
3*(a^(1/3)+2*(b*x+a)^(1/3))*3^(1/2)/a^(1/3))*3^(1/2)/a^(5/3)/c^2/(b*x^2+a*
x)^(2/3)+1/3*b*(c*x)^(2/3)*(b*x+a)^(2/3)*ln(x)/a^(5/3)/c^2/(b*x^2+a*x)^(2/
3)-b*(c*x)^(2/3)*(b*x+a)^(2/3)*ln(a^(1/3)-(b*x+a)^(1/3))/a^(5/3)/c^2/(b*x^
2+a*x)^(2/3)
```

Mathematica [A] (verified)

Time = 0.17 (sec) , antiderivative size = 162, normalized size of antiderivative = 0.80

$$\int \frac{1}{(cx)^{4/3} \left(ax+bx^2\right)^{2/3}} \, dx = \frac{x \left(-3a^{5/3} - 3a^{2/3}bx + 2\sqrt{3}bx(a+bx)^{2/3} \arctan\left(\frac{1+\frac{2\sqrt[3]{a+bx}}{\sqrt[3]{a}}}{\sqrt{3}}\right) - 2bx(a+ \right.}{3a^{5/3}(cx}$$

```
Integrate[1/((c*x)^(4/3)*(a*x + b*x^2)^(2/3)),x]
```

```
(x*(-3*a^(5/3) - 3*a^(2/3)*b*x + 2*Sqrt[3]*b*x*(a + b*x)^(2/3)*ArcTan[(1 +
 (2*(a + b*x)^(1/3))/a^(1/3))/Sqrt[3]] - 2*b*x*(a + b*x)^(2/3)*Log[a^(1/3)
 - (a + b*x)^(1/3)] + b*x*(a + b*x)^(2/3)*Log[a^(2/3) + a^(1/3)*(a + b*x)^
(1/3) + (a + b*x)^(2/3)]))/(3*a^(5/3)*(c*x)^(4/3)*(x*(a + b*x))^(2/3))
```

Rubi [A] (verified)

Time = 0.41 (sec) , antiderivative size = 135, normalized size of antiderivative = 0.67, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.286, Rules used = {1137, 52, 69, 16, 1082, 217}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{(cx)^{4/3} \left(ax+bx^2\right)^{2/3}} \, dx$$

$$\Big\downarrow \text{ 1137}$$

$$\frac{x^2(a+bx)^{2/3} \int \frac{1}{x^2(a+bx)^{2/3}}dx}{(cx)^{4/3} \left(ax+bx^2\right)^{2/3}}$$

$$\Big\downarrow \text{ 52}$$

$$\frac{x^2(a+bx)^{2/3} \left(-\frac{2b\int \frac{1}{x(a+bx)^{2/3}}dx}{3a} - \frac{\sqrt[3]{a+bx}}{ax}\right)}{(cx)^{4/3} \left(ax+bx^2\right)^{2/3}}$$

$$\Big\downarrow \; 69$$

$$\frac{x^2(a+bx)^{2/3}\left(-\frac{2b\left(-\frac{3\int \frac{1}{\sqrt[3]{a}-\sqrt[3]{a+bx}}d\sqrt[3]{a+bx}}{2a^{2/3}}-\frac{3\int \frac{1}{a^{2/3}+\sqrt[3]{a+bx}\sqrt[3]{a}+(a+bx)^{2/3}}d\sqrt[3]{a+bx}}{2\sqrt[3]{a}}-\frac{\log(x)}{2a^{2/3}}\right)}{3a}-\frac{\sqrt[3]{a+bx}}{ax}\right)}{(cx)^{4/3}\left(ax+bx^2\right)^{2/3}}$$

$$\Big\downarrow \; 16$$

$$\frac{x^2(a+bx)^{2/3}\left(-\frac{2b\left(-\frac{3\int \frac{1}{a^{2/3}+\sqrt[3]{a+bx}\sqrt[3]{a}+(a+bx)^{2/3}}d\sqrt[3]{a+bx}}{2\sqrt[3]{a}}+\frac{3\log\left(\sqrt[3]{a}-\sqrt[3]{a+bx}\right)}{2a^{2/3}}-\frac{\log(x)}{2a^{2/3}}\right)}{3a}-\frac{\sqrt[3]{a+bx}}{ax}\right)}{(cx)^{4/3}\left(ax+bx^2\right)^{2/3}}$$

$$\Big\downarrow \; 1082$$

$$\frac{x^2(a+bx)^{2/3}\left(-\frac{2b\left(\frac{3\int \frac{1}{-(a+bx)^{2/3}-3}d\left(\frac{2\sqrt[3]{a+bx}}{\sqrt[3]{a}}+1\right)}{a^{2/3}}+\frac{3\log\left(\sqrt[3]{a}-\sqrt[3]{a+bx}\right)}{2a^{2/3}}-\frac{\log(x)}{2a^{2/3}}\right)}{3a}-\frac{\sqrt[3]{a+bx}}{ax}\right)}{(cx)^{4/3}\left(ax+bx^2\right)^{2/3}}$$

$$\Big\downarrow \; 217$$

$$\frac{x^2(a+bx)^{2/3}\left(-\frac{2b\left(-\frac{\sqrt{3}\arctan\left(\frac{\frac{2\sqrt[3]{a+bx}}{\sqrt[3]{a}}+1}{\sqrt{3}}\right)}{a^{2/3}}+\frac{3\log\left(\sqrt[3]{a}-\sqrt[3]{a+bx}\right)}{2a^{2/3}}-\frac{\log(x)}{2a^{2/3}}\right)}{3a}-\frac{\sqrt[3]{a+bx}}{ax}\right)}{(cx)^{4/3}\left(ax+bx^2\right)^{2/3}}$$

```
Int[1/((c*x)^(4/3)*(a*x + b*x^2)^(2/3)),x]
```

```
(x^2*(a + b*x)^(2/3)*(-((a + b*x)^(1/3)/(a*x)) - (2*b*(-((Sqrt[3]*ArcTan[(
1 + (2*(a + b*x)^(1/3))/a^(1/3))/Sqrt[3]])/a^(2/3)) - Log[x]/(2*a^(2/3)) +
 (3*Log[a^(1/3) - (a + b*x)^(1/3)])/(2*a^(2/3))))/(3*a)))/((c*x)^(4/3)*(a*
x + b*x^2)^(2/3))
```

Defintions of rubi rules used

```
Int[(c_.)/((a_.) + (b_.)*(x_)), x_Symbol] :> Simp[c*(Log[RemoveContent[a +
b*x, x]]/b), x] /; FreeQ[{a, b, c}, x]
```

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] - Simp[d*((
m + n + 2)/((b*c - a*d)*(m + 1)))   Int[(a + b*x)^(m + 1)*(c + d*x)^n, x],
x] /; FreeQ[{a, b, c, d, n}, x] && ILtQ[m, -1] && FractionQ[n] && LtQ[n, 0]
```

```
Int[1/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(2/3)), x_Symbol] :> With[
{q = Rt[(b*c - a*d)/b, 3]}, Simp[-Log[RemoveContent[a + b*x, x]]/(2*b*q^2),
 x] + (-Simp[3/(2*b*q)   Subst[Int[1/(q^2 + q*x + x^2), x], x, (c + d*x)^(1
/3)], x] - Simp[3/(2*b*q^2)   Subst[Int[1/(q - x), x], x, (c + d*x)^(1/3)],
 x])] /; FreeQ[{a, b, c, d}, x] && PosQ[(b*c - a*d)/b]
```

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Simp[-2/b   Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b
)], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Fre
eQ[{a, b, c}, x]
```

```
Int[((e_.)*(x_))^(m_)*((b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(
e*x)^m*((b*x + c*x^2)^p/(x^(m + p)*(b + c*x)^p))   Int[x^(m + p)*(b + c*x)^
p, x], x] /; FreeQ[{b, c, e, m}, x]
```

Maple [F]

$$\int \frac{1}{(cx)^{\frac{4}{3}} (b\,x^{2}+ax)^{\frac{2}{3}}}dx$$

```
int(1/(c*x)^(4/3)/(b*x^2+a*x)^(2/3),x)
```

```
int(1/(c*x)^(4/3)/(b*x^2+a*x)^(2/3),x)
```

Fricas [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 567, normalized size of antiderivative = 2.81

$$\int \frac{1}{(cx)^{4/3}\left(ax+bx^2\right)^{2/3}}\,dx = \text{Too large to display}$$

```
integrate(1/(c*x)^(4/3)/(b*x^2+a*x)^(2/3),x, algorithm="fricas")
```

```
[1/3*(3*sqrt(1/3)*a*b*c*x^2*sqrt((-a^2*c)^(1/3)/c)*log((2*a*b*c*x^2 + 3*a^
2*c*x + 3*(-a^2*c)^(1/3)*(b*x^2 + a*x)^(1/3)*(c*x)^(2/3)*a + 3*sqrt(1/3)*(
(-a^2*c)^(1/3)*a*c*x + 2*(b*x^2 + a*x)^(2/3)*(c*x)^(1/3)*a*c - (-a^2*c)^(2
/3)*(b*x^2 + a*x)^(1/3)*(c*x)^(2/3))*sqrt((-a^2*c)^(1/3)/c))/x^2) + (-a^2*
c)^(2/3)*b*x^2*log(-((-a^2*c)^(1/3)*a*c*x - (b*x^2 + a*x)^(2/3)*(c*x)^(1/3
)*a*c - (-a^2*c)^(2/3)*(b*x^2 + a*x)^(1/3)*(c*x)^(2/3))/x) - 2*(-a^2*c)^(2
/3)*b*x^2*log(((b*x^2 + a*x)^(1/3)*(c*x)^(2/3)*a - (-a^2*c)^(2/3)*x)/x) -
3*(b*x^2 + a*x)^(1/3)*(c*x)^(2/3)*a^2)/(a^3*c^2*x^2), 1/3*(6*sqrt(1/3)*a*b
*c*x^2*sqrt(-(-a^2*c)^(1/3)/c)*arctan(-sqrt(1/3)*((-a^2*c)^(1/3)*a*c*x - 2
*(-a^2*c)^(2/3)*(b*x^2 + a*x)^(1/3)*(c*x)^(2/3))*sqrt(-(-a^2*c)^(1/3)/c)/(
a^2*c*x)) + (-a^2*c)^(2/3)*b*x^2*log(-((-a^2*c)^(1/3)*a*c*x - (b*x^2 + a*x
)^(2/3)*(c*x)^(1/3)*a*c - (-a^2*c)^(2/3)*(b*x^2 + a*x)^(1/3)*(c*x)^(2/3))/
x) - 2*(-a^2*c)^(2/3)*b*x^2*log(((b*x^2 + a*x)^(1/3)*(c*x)^(2/3)*a - (-a^2
*c)^(2/3)*x)/x) - 3*(b*x^2 + a*x)^(1/3)*(c*x)^(2/3)*a^2)/(a^3*c^2*x^2)]
```

Sympy [F]

$$\int \frac{1}{(cx)^{4/3}\left(ax+bx^2\right)^{2/3}}\,dx = \int \frac{1}{(cx)^{\frac{4}{3}}\,(x\,(a+bx))^{\frac{2}{3}}}\,dx$$

```
integrate(1/(c*x)**(4/3)/(b*x**2+a*x)**(2/3),x)
```

```
Integral(1/((c*x)**(4/3)*(x*(a + b*x))**(2/3)), x)
```

Maxima [F]

$$\int \frac{1}{(cx)^{4/3}\left(ax+bx^2\right)^{2/3}}\,dx=\int \frac{1}{\left(bx^2+ax\right)^{\frac{2}{3}}(cx)^{\frac{4}{3}}}\,dx$$

```
integrate(1/(c*x)^(4/3)/(b*x^2+a*x)^(2/3),x, algorithm="maxima")
```

```
integrate(1/((b*x^2 + a*x)^(2/3)*(c*x)^(4/3)), x)
```

Giac [F]

$$\int \frac{1}{(cx)^{4/3}\left(ax+bx^2\right)^{2/3}}\,dx=\int \frac{1}{\left(bx^2+ax\right)^{\frac{2}{3}}(cx)^{\frac{4}{3}}}\,dx$$

```
integrate(1/(c*x)^(4/3)/(b*x^2+a*x)^(2/3),x, algorithm="giac")
```

```
integrate(1/((b*x^2 + a*x)^(2/3)*(c*x)^(4/3)), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{(cx)^{4/3}\left(ax+bx^2\right)^{2/3}}\,dx=\int \frac{1}{\left(b\,x^2+a\,x\right)^{2/3}\,(c\,x)^{4/3}}\,dx$$

```
int(1/((a*x + b*x^2)^(2/3)*(c*x)^(4/3)),x)
```

```
int(1/((a*x + b*x^2)^(2/3)*(c*x)^(4/3)), x)
```

Reduce [B] (verification not implemented)

Time = 0.48 (sec) , antiderivative size = 169, normalized size of antiderivative = 0.84

$$\int \frac{1}{(cx)^{4/3}\left(ax+bx^2\right)^{2/3}}\,dx = \frac{-2\sqrt{3}\,atan\left(\frac{2(bx+a)^{\frac{1}{6}}+a^{\frac{1}{6}}}{a^{\frac{1}{6}}\sqrt{3}}\right)bx+2\sqrt{3}\,atan\left(\frac{2(bx+a)^{\frac{1}{6}}-a^{\frac{1}{6}}}{a^{\frac{1}{6}}\sqrt{3}}\right)bx-3a^{\frac{2}{3}}(bx+a)^{\frac{1}{3}}$$

```
int(1/(c*x)^(4/3)/(b*x^2+a*x)^(2/3),x)
```

```
( - 2*sqrt(3)*atan((2*(a + b*x)**(1/6) + a**(1/6))/(a**(1/6)*sqrt(3)))*b*x
 + 2*sqrt(3)*atan((2*(a + b*x)**(1/6) - a**(1/6))/(a**(1/6)*sqrt(3)))*b*x
- 3*a**(2/3)*(a + b*x)**(1/3) - 2*log((a + b*x)**(1/6) + a**(1/6))*b*x - 2
*log((a + b*x)**(1/6) - a**(1/6))*b*x + log( - a**(1/6)*(a + b*x)**(1/6) +
 (a + b*x)**(1/3) + a**(1/3))*b*x + log(a**(1/6)*(a + b*x)**(1/6) + (a + b
*x)**(1/3) + a**(1/3))*b*x)/(3*c**(1/3)*a**(2/3)*a*c*x)
```

3.144 $\int \frac{1}{(cx)^{7/3}(ax+bx^2)^{2/3}}\,dx$

Optimal result

Integrand size = 21, antiderivative size = 245

$$\begin{aligned}\int \frac{1}{(cx)^{7/3}\left(ax+bx^2\right)^{2/3}}\,dx &= -\frac{\sqrt[3]{ax+bx^2}}{2a(cx)^{7/3}}+\frac{5b\sqrt[3]{ax+bx^2}}{6a^2c(cx)^{4/3}} \\ &\quad -\frac{5b^2(cx)^{2/3}(a+bx)^{2/3}\arctan\left(\frac{\sqrt[3]{a}+2\sqrt[3]{a+bx}}{\sqrt{3}\sqrt[3]{a}}\right)}{3\sqrt{3}a^{8/3}c^3\left(ax+bx^2\right)^{2/3}} \\ &\quad -\frac{5b^2(cx)^{2/3}(a+bx)^{2/3}\log(x)}{18a^{8/3}c^3\left(ax+bx^2\right)^{2/3}}+\frac{5b^2(cx)^{2/3}(a+bx)^{2/3}\log\left(\sqrt[3]{a}-\sqrt[3]{a+bx}\right)}{6a^{8/3}c^3\left(ax+bx^2\right)^{2/3}}\end{aligned}$$

```
-1/2*(b*x^2+a*x)^(1/3)/a/(c*x)^(7/3)+5/6*b*(b*x^2+a*x)^(1/3)/a^2/c/(c*x)^(
4/3)-5/9*b^2*(c*x)^(2/3)*(b*x+a)^(2/3)*arctan(1/3*(a^(1/3)+2*(b*x+a)^(1/3)
)*3^(1/2)/a^(1/3))*3^(1/2)/a^(8/3)/c^3/(b*x^2+a*x)^(2/3)-5/18*b^2*(c*x)^(2
/3)*(b*x+a)^(2/3)*ln(x)/a^(8/3)/c^3/(b*x^2+a*x)^(2/3)+5/6*b^2*(c*x)^(2/3)*
(b*x+a)^(2/3)*ln(a^(1/3)-(b*x+a)^(1/3))/a^(8/3)/c^3/(b*x^2+a*x)^(2/3)
```

Mathematica [A] (verified)

Time = 0.18 (sec) , antiderivative size = 188, normalized size of antiderivative = 0.77

$$\int \frac{1}{(cx)^{7/3}\left(ax+bx^2\right)^{2/3}}\,dx = \frac{x\left(-9a^{8/3}+6a^{5/3}bx+15a^{2/3}b^2x^2-10\sqrt{3}b^2x^2(a+bx)^{2/3}\arctan\left(\frac{1+\frac{2\sqrt[3]{a+bx}}{\sqrt[3]{a}}}{\sqrt{3}}\right)\right.}{}$$

```
Integrate[1/((c*x)^(7/3)*(a*x + b*x^2)^(2/3)),x]
```

```
(x*(-9*a^(8/3) + 6*a^(5/3)*b*x + 15*a^(2/3)*b^2*x^2 - 10*Sqrt[3]*b^2*x^2*(
a + b*x)^(2/3)*ArcTan[(1 + (2*(a + b*x)^(1/3))/a^(1/3))/Sqrt[3]] + 10*b^2*
x^2*(a + b*x)^(2/3)*Log[a^(1/3) - (a + b*x)^(1/3)] - 5*b^2*x^2*(a + b*x)^(
2/3)*Log[a^(2/3) + a^(1/3)*(a + b*x)^(1/3) + (a + b*x)^(2/3)]))/(18*a^(8/3
)*(c*x)^(7/3)*(x*(a + b*x))^(2/3))
```

Rubi [A] (verified)

Time = 0.43 (sec) , antiderivative size = 163, normalized size of antiderivative = 0.67, number of steps used = 8, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}} = 0.333$, Rules used = {1137, 52, 52, 69, 16, 1082, 217}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{(cx)^{7/3}\left(ax+bx^2\right)^{2/3}}dx$$

$$\Big\downarrow \text{1137}$$

$$\frac{x^3(a+bx)^{2/3}\int \frac{1}{x^3(a+bx)^{2/3}}dx}{(cx)^{7/3}\left(ax+bx^2\right)^{2/3}}$$

$$\Big\downarrow \text{52}$$

$$\frac{x^3(a+bx)^{2/3}\left(-\frac{5b\int \frac{1}{x^2(a+bx)^{2/3}}dx}{6a}-\frac{\sqrt[3]{a+bx}}{2ax^2}\right)}{(cx)^{7/3}\left(ax+bx^2\right)^{2/3}}$$

$$\Big\downarrow \textcolor{blue}{52}$$

$$\frac{x^3(a+bx)^{2/3}\left(-\frac{5b\left(-\frac{2b\int \frac{1}{x(a+bx)^{2/3}}dx}{3a}-\frac{\sqrt[3]{a+bx}}{ax}\right)}{6a}-\frac{\sqrt[3]{a+bx}}{2ax^2}\right)}{(cx)^{7/3}\left(ax+bx^2\right)^{2/3}}$$

$$\Big\downarrow \textcolor{blue}{69}$$

$$\frac{x^3(a+bx)^{2/3}\left(-\frac{5b\left(-\frac{2b\left(-\frac{3\int \frac{1}{\sqrt[3]{a}-\sqrt[3]{a+bx}}d\sqrt[3]{a+bx}}{2a^{2/3}}-\frac{3\int \frac{1}{a^{2/3}+\sqrt[3]{a+bx}\sqrt[3]{a}+(a+bx)^{2/3}}d\sqrt[3]{a+bx}}{2\sqrt[3]{a}}-\frac{\log(x)}{2a^{2/3}}\right)}{3a}-\frac{\sqrt[3]{a+bx}}{ax}\right)}{6a}\right)}{(cx)^{7/3}\left(ax+bx^2\right)^{2/3}}$$

$$\Big\downarrow \textcolor{blue}{16}$$

$$\frac{x^3(a+bx)^{2/3}\left(-\frac{5b\left(-\frac{2b\left(-\frac{3\int \frac{1}{a^{2/3}+\sqrt[3]{a+bx}\sqrt[3]{a}+(a+bx)^{2/3}}d\sqrt[3]{a+bx}}{2\sqrt[3]{a}}+\frac{3\log\left(\sqrt[3]{a}-\sqrt[3]{a+bx}\right)}{2a^{2/3}}-\frac{\log(x)}{2a^{2/3}}\right)}{3a}-\frac{\sqrt[3]{a+bx}}{ax}\right)}{6a}-\sqrt[3]{\cdots}\right)}{(cx)^{7/3}\left(ax+bx^2\right)^{2/3}}$$

$$\Big\downarrow \textcolor{blue}{1082}$$

$$\frac{x^3(a+bx)^{2/3}\left(-\frac{5b\left(-\frac{2b\left(\frac{3\int\frac{1}{-(a+bx)^{2/3}-3}d\left(\frac{2\sqrt[3]{a+bx}}{\sqrt[3]{a}}+1\right)}{a^{2/3}}+\frac{3\log\left(\sqrt[3]{a}-\sqrt[3]{a+bx}\right)}{2a^{2/3}}-\frac{\log(x)}{2a^{2/3}}\right)}{3a}-\frac{\sqrt[3]{a+bx}}{ax}\right)}{6a}-\frac{\sqrt[3]{a+bx}}{2ax^2}\right)}{(cx)^{7/3}\left(ax+bx^2\right)^{2/3}}$$

$$\Big\downarrow \text{217}$$

$$\frac{x^3(a+bx)^{2/3}\left(-\frac{5b\left(-\frac{2b\left(-\frac{\sqrt{3}\arctan\left(\frac{\frac{2\sqrt[3]{a+bx}}{\sqrt[3]{a}}+1}{\sqrt{3}}\right)}{a^{2/3}}+\frac{3\log\left(\sqrt[3]{a}-\sqrt[3]{a+bx}\right)}{2a^{2/3}}-\frac{\log(x)}{2a^{2/3}}\right)}{3a}-\frac{\sqrt[3]{a+bx}}{ax}\right)}{6a}-\frac{\sqrt[3]{a+bx}}{2ax^2}\right)}{(cx)^{7/3}\left(ax+bx^2\right)^{2/3}}$$

```
Int[1/((c*x)^(7/3)*(a*x + b*x^2)^(2/3)),x]
```

```
(x^3*(a + b*x)^(2/3)*(-1/2*(a + b*x)^(1/3)/(a*x^2) - (5*b*(-((a + b*x)^(1/
3)/(a*x)) - (2*b*(-((Sqrt[3]*ArcTan[(1 + (2*(a + b*x)^(1/3))/a^(1/3))/Sqrt
[3]])/a^(2/3)) - Log[x]/(2*a^(2/3)) + (3*Log[a^(1/3) - (a + b*x)^(1/3)])/(
2*a^(2/3))))/(3*a)))/(6*a)))/((c*x)^(7/3)*(a*x + b*x^2)^(2/3))
```

Defintions of rubi rules used

```
Int[(c_.)/((a_.) + (b_.)*(x_)), x_Symbol] :> Simp[c*(Log[RemoveContent[a +
b*x, x]]/b), x] /; FreeQ[{a, b, c}, x]
```

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] - Simp[d*((
m + n + 2)/((b*c - a*d)*(m + 1)))   Int[(a + b*x)^(m + 1)*(c + d*x)^n, x],
x] /; FreeQ[{a, b, c, d, n}, x] && ILtQ[m, -1] && FractionQ[n] && LtQ[n, 0]
```

```
Int[1/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(2/3)), x_Symbol] :> With[
{q = Rt[(b*c - a*d)/b, 3]}, Simp[-Log[RemoveContent[a + b*x, x]]/(2*b*q^2),
 x] + (-Simp[3/(2*b*q)   Subst[Int[1/(q^2 + q*x + x^2), x], x, (c + d*x)^(1
/3)], x] - Simp[3/(2*b*q^2)   Subst[Int[1/(q - x), x], x, (c + d*x)^(1/3)],
 x])] /; FreeQ[{a, b, c, d}, x] && PosQ[(b*c - a*d)/b]
```

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Simp[-2/b   Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b
)], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Fre
eQ[{a, b, c}, x]
```

```
Int[((e_.)*(x_))^(m_)*((b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(
e*x)^m*((b*x + c*x^2)^p/(x^(m + p)*(b + c*x)^p))   Int[x^(m + p)*(b + c*x)^
p, x], x] /; FreeQ[{b, c, e, m}, x]
```

Maple [F]

$$\int \frac{1}{(cx)^{\frac{7}{3}} (b\,x^2+ax)^{\frac{2}{3}}}dx$$

```
int(1/(c*x)^(7/3)/(b*x^2+a*x)^(2/3),x)
```

```
int(1/(c*x)^(7/3)/(b*x^2+a*x)^(2/3),x)
```

Fricas [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 574, normalized size of antiderivative = 2.34

$$\int \frac{1}{(cx)^{7/3} (ax+bx^2)^{2/3}}\,dx = \text{Too large to display}$$

```
integrate(1/(c*x)^(7/3)/(b*x^2+a*x)^(2/3),x, algorithm="fricas")
```

```
[1/18*(15*sqrt(1/3)*a*b^2*c*x^3*sqrt(-(a^2*c)^(1/3)/c)*log((2*a*b*c*x^2 +
3*a^2*c*x - 3*(a^2*c)^(1/3)*(b*x^2 + a*x)^(1/3)*(c*x)^(2/3)*a + 3*sqrt(1/3
)*((a^2*c)^(1/3)*a*c*x - 2*(b*x^2 + a*x)^(2/3)*(c*x)^(1/3)*a*c + (a^2*c)^(
2/3)*(b*x^2 + a*x)^(1/3)*(c*x)^(2/3))*sqrt(-(a^2*c)^(1/3)/c))/x^2) - 5*(a^
2*c)^(2/3)*b^2*x^3*log(((a^2*c)^(1/3)*a*c*x + (b*x^2 + a*x)^(2/3)*(c*x)^(1
/3)*a*c + (a^2*c)^(2/3)*(b*x^2 + a*x)^(1/3)*(c*x)^(2/3))/x) + 10*(a^2*c)^(
2/3)*b^2*x^3*log(((b*x^2 + a*x)^(1/3)*(c*x)^(2/3)*a - (a^2*c)^(2/3)*x)/x)
+ 3*(5*a^2*b*x - 3*a^3)*(b*x^2 + a*x)^(1/3)*(c*x)^(2/3))/(a^4*c^3*x^3), -1
/18*(30*sqrt(1/3)*a*b^2*c*x^3*sqrt((a^2*c)^(1/3)/c)*arctan(sqrt(1/3)*((a^2
*c)^(1/3)*a*c*x + 2*(a^2*c)^(2/3)*(b*x^2 + a*x)^(1/3)*(c*x)^(2/3))*sqrt((a
^2*c)^(1/3)/c)/(a^2*c*x)) + 5*(a^2*c)^(2/3)*b^2*x^3*log(((a^2*c)^(1/3)*a*c
*x + (b*x^2 + a*x)^(2/3)*(c*x)^(1/3)*a*c + (a^2*c)^(2/3)*(b*x^2 + a*x)^(1/
3)*(c*x)^(2/3))/x) - 10*(a^2*c)^(2/3)*b^2*x^3*log(((b*x^2 + a*x)^(1/3)*(c*
x)^(2/3)*a - (a^2*c)^(2/3)*x)/x) - 3*(5*a^2*b*x - 3*a^3)*(b*x^2 + a*x)^(1/
3)*(c*x)^(2/3))/(a^4*c^3*x^3)]
```

Sympy [F]

$$\int \frac{1}{(cx)^{7/3}\left(ax+bx^2\right)^{2/3}}\,dx=\int \frac{1}{(cx)^{\frac{7}{3}}\left(x\left(a+bx\right)\right)^{\frac{2}{3}}}\,dx$$

```
integrate(1/(c*x)**(7/3)/(b*x**2+a*x)**(2/3),x)
```

```
Integral(1/((c*x)**(7/3)*(x*(a + b*x))**(2/3)), x)
```

Maxima [F]

$$\int \frac{1}{(cx)^{7/3}\left(ax+bx^2\right)^{2/3}}\,dx=\int \frac{1}{\left(bx^2+ax\right)^{\frac{2}{3}}\left(cx\right)^{\frac{7}{3}}}\,dx$$

```
integrate(1/(c*x)^(7/3)/(b*x^2+a*x)^(2/3),x, algorithm="maxima")
```

```
integrate(1/((b*x^2 + a*x)^(2/3)*(c*x)^(7/3)), x)
```

Giac [F]

$$\int \frac{1}{(cx)^{7/3}\left(ax+bx^2\right)^{2/3}}\,dx=\int \frac{1}{\left(bx^2+ax\right)^{\frac{2}{3}}\left(cx\right)^{\frac{7}{3}}}\,dx$$

```
integrate(1/(c*x)^(7/3)/(b*x^2+a*x)^(2/3),x, algorithm="giac")
```

```
integrate(1/((b*x^2 + a*x)^(2/3)*(c*x)^(7/3)), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{(cx)^{7/3}(ax+bx^2)^{2/3}}\,dx = \int \frac{1}{(b\,x^2+a\,x)^{2/3}\,(c\,x)^{7/3}}\,dx$$

```
int(1/((a*x + b*x^2)^(2/3)*(c*x)^(7/3)),x)
```

```
int(1/((a*x + b*x^2)^(2/3)*(c*x)^(7/3)), x)
```

Reduce [B] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 209, normalized size of antiderivative = 0.85

$$\int \frac{1}{(cx)^{7/3}(ax+bx^2)^{2/3}}\,dx = \frac{10\sqrt{3}\,atan\left(\frac{2(bx+a)^{\frac{1}{6}}+a^{\frac{1}{6}}}{a^{\frac{1}{6}}\sqrt{3}}\right)b^2x^2 - 10\sqrt{3}\,atan\left(\frac{2(bx+a)^{\frac{1}{6}}-a^{\frac{1}{6}}}{a^{\frac{1}{6}}\sqrt{3}}\right)b^2x^2 - 9a^{\frac{5}{3}}(bx+$$

```
int(1/(c*x)^(7/3)/(b*x^2+a*x)^(2/3),x)
```

```
(10*sqrt(3)*atan((2*(a + b*x)**(1/6) + a**(1/6))/(a**(1/6)*sqrt(3)))*b**2*
x**2 - 10*sqrt(3)*atan((2*(a + b*x)**(1/6) - a**(1/6))/(a**(1/6)*sqrt(3)))
*b**2*x**2 - 9*a**(2/3)*(a + b*x)**(1/3)*a + 15*a**(2/3)*(a + b*x)**(1/3)*
b*x + 10*log((a + b*x)**(1/6) + a**(1/6))*b**2*x**2 + 10*log((a + b*x)**(1
/6) - a**(1/6))*b**2*x**2 - 5*log( - a**(1/6)*(a + b*x)**(1/6) + (a + b*x)
**(1/3) + a**(1/3))*b**2*x**2 - 5*log(a**(1/6)*(a + b*x)**(1/6) + (a + b*x
)**(1/3) + a**(1/3))*b**2*x**2)/(18*c**(1/3)*a**(2/3)*a**2*c**2*x**2)
```

3.145 $\int x^3 \sqrt[4]{ax+bx^2}\,dx$

Optimal result

Integrand size = 17, antiderivative size = 191

$$\begin{aligned}
&\int x^3 \sqrt[4]{ax+bx^2}\,dx \\
&= -\frac{13a^4\sqrt[4]{ax+bx^2}}{168b^4}+\frac{13a^3x\sqrt[4]{ax+bx^2}}{420b^3} \\
&\quad -\frac{13a^2x^2\sqrt[4]{ax+bx^2}}{630b^2}+\frac{ax^3\sqrt[4]{ax+bx^2}}{63b}+\frac{2}{9}x^4\sqrt[4]{ax+bx^2} \\
&\quad -\frac{13a^{9/2}\left(\frac{bx}{a+bx}\right)^{3/4}\sqrt{a+bx}\sqrt[4]{ax+bx^2}\,\text{EllipticF}\left(\frac{1}{2}\arcsin\left(\frac{\sqrt{a}}{\sqrt{a+bx}}\right),2\right)}{168b^5x}
\end{aligned}$$

```
-13/168*a^4*(b*x^2+a*x)^(1/4)/b^4+13/420*a^3*x*(b*x^2+a*x)^(1/4)/b^3-13/63
0*a^2*x^2*(b*x^2+a*x)^(1/4)/b^2+1/63*a*x^3*(b*x^2+a*x)^(1/4)/b+2/9*x^4*(b*
x^2+a*x)^(1/4)-13/168*a^(9/2)*(b*x/(b*x+a))^(3/4)*(b*x+a)^(1/2)*(b*x^2+a*x
)^(1/4)*InverseJacobiAM(1/2*arcsin(a^(1/2)/(b*x+a)^(1/2)),2^(1/2))/b^5/x
```

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 10.01 (sec) , antiderivative size = 47, normalized size of antiderivative = 0.25

$$\int x^3 \sqrt[4]{ax+bx^2}\,dx = \frac{4x^4\sqrt[4]{x(a+bx)}\,\text{Hypergeometric2F1}\left(-\frac{1}{4},\frac{17}{4},\frac{21}{4},-\frac{bx}{a}\right)}{17\sqrt[4]{1+\frac{bx}{a}}}$$

```
Integrate[x^3*(a*x + b*x^2)^(1/4),x]
```

```
(4*x^4*(x*(a + b*x))^(1/4)*Hypergeometric2F1[-1/4, 17/4, 21/4, -((b*x)/a)]
)/(17*(1 + (b*x)/a)^(1/4))
```

Rubi [A] (warning: unable to verify)

Time = 0.56 (sec) , antiderivative size = 225, normalized size of antiderivative = 1.18, number of steps used = 12, number of rules used = 11, $\frac{\text{number of rules}}{\text{integrand size}} = 0.647$, Rules used = {1137, 60, 60, 60, 60, 60, 73, 768, 858, 807, 229}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^3 \sqrt[4]{ax+bx^2}\,dx$$

$$\Big\downarrow \text{1137}$$

$$\frac{\sqrt[4]{ax+bx^2}\int x^{13/4}\sqrt[4]{a+bx}dx}{\sqrt[4]{x}\sqrt[4]{a+bx}}$$

$$\Big\downarrow \text{60}$$

$$\frac{\sqrt[4]{ax+bx^2}\left(\frac{1}{18}a\int \frac{x^{13/4}}{(a+bx)^{3/4}}dx+\frac{2}{9}x^{17/4}\sqrt[4]{a+bx}\right)}{\sqrt[4]{x}\sqrt[4]{a+bx}}$$

$$\Big\downarrow \text{60}$$

$$\frac{\sqrt[4]{ax+bx^2}\left(\frac{1}{18}a\left(\frac{2x^{13/4}\sqrt[4]{a+bx}}{7b}-\frac{13a\int\frac{x^{9/4}}{(a+bx)^{3/4}}dx}{14b}\right)+\frac{2}{9}x^{17/4}\sqrt[4]{a+bx}\right)}{\sqrt[4]{x}\sqrt[4]{a+bx}}$$

$$\Big\downarrow \text{60}$$

$$\frac{\sqrt[4]{ax+bx^2}\left(\frac{1}{18}a\left(\frac{2x^{13/4}\sqrt[4]{a+bx}}{7b}-\frac{13a\left(\frac{2x^{9/4}\sqrt[4]{a+bx}}{5b}-\frac{9a\int\frac{x^{5/4}}{(a+bx)^{3/4}}dx}{10b}\right)}{14b}\right)+\frac{2}{9}x^{17/4}\sqrt[4]{a+bx}\right)}{\sqrt[4]{x}\sqrt[4]{a+bx}}$$

$$\Big\downarrow \text{60}$$

$$\frac{\sqrt[4]{ax+bx^2}\left(\frac{1}{18}a\left(\frac{2x^{13/4}\sqrt[4]{a+bx}}{7b}-\frac{13a\left(\frac{2x^{9/4}\sqrt[4]{a+bx}}{5b}-\frac{9a\left(\frac{2x^{5/4}\sqrt[4]{a+bx}}{3b}-\frac{5a\int\frac{\sqrt[4]{x}}{(a+bx)^{3/4}}dx}{6b}\right)}{10b}\right)}{14b}\right)+\frac{2}{9}x^{17/4}\sqrt[4]{a+bx}\right)}{\sqrt[4]{x}\sqrt[4]{a+bx}}$$

$$\Big\downarrow \text{60}$$

$$\frac{\sqrt[4]{ax+bx^2}\left(\frac{1}{18}a\left(\frac{2x^{13/4}\sqrt[4]{a+bx}}{7b}-\frac{13a\left(\frac{2x^{9/4}\sqrt[4]{a+bx}}{5b}-\frac{9a\left(\frac{2x^{5/4}\sqrt[4]{a+bx}}{3b}-\frac{5a\left(\frac{2\sqrt[4]{x}\sqrt[4]{a+bx}}{b}-\frac{a\int\frac{1}{x^{3/4}(a+bx)^{3/4}}dx}{2b}\right)}{6b}\right)}{10b}\right)}{14b}\right)\right)}{\sqrt[4]{x}\sqrt[4]{a+bx}}$$

$\Big\downarrow$ 73

$$\frac{\sqrt[4]{ax+bx^2}\left(\frac{1}{18}a\left(\frac{2x^{13/4}\sqrt[4]{a+bx}}{7b}-\frac{13a\left(\frac{2x^{9/4}\sqrt[4]{a+bx}}{5b}-\frac{9a\left(\frac{2x^{5/4}\sqrt[4]{a+bx}}{3b}-\frac{5a\left(\frac{2\sqrt[4]{x}\sqrt[4]{a+bx}}{b}-\frac{2a\int\frac{1}{(a+bx)^{3/4}}d\sqrt[4]{x}}{b}\right)}{6b}\right)}{10b}\right)}{14b}\right)\right)}{\sqrt[4]{x}\sqrt[4]{a+bx}}$$

$\Big\downarrow$ 768

$$\frac{\sqrt[4]{ax+bx^2}\left(\frac{1}{18}a\left(\frac{2x^{13/4}\sqrt[4]{a+bx}}{7b}-\frac{13a\left(\frac{2x^{9/4}\sqrt[4]{a+bx}}{5b}-\frac{9a\left(\frac{2x^{5/4}\sqrt[4]{a+bx}}{3b}-\frac{5a\left(\frac{2\sqrt[4]{x}\sqrt[4]{a+bx}}{b}-\frac{2ax^{3/4}\left(\frac{a}{bx}+1\right)^{3/4}\int \frac{}{\left(\frac{a}{bx}+\right.}}{b(a+bx)^{3/4}}\right.}{6b}\right.}{10b}\right.}{14b}\right.\right.}{\sqrt[4]{x}\sqrt[4]{a+bx}}$$

$\downarrow$ 858

$$\frac{\sqrt[4]{ax+bx^2}\left(\frac{1}{18}a\left(\frac{2x^{13/4}\sqrt[4]{a+bx}}{7b}-\frac{13a\left(\frac{2x^{9/4}\sqrt[4]{a+bx}}{5b}-\frac{9a\left(\frac{2x^{5/4}\sqrt[4]{a+bx}}{3b}-\frac{5a\left(\frac{2ax^{3/4}\left(\frac{a}{bx}+1\right)^{3/4}\int\frac{1}{\sqrt[4]{x}\left(\frac{ax}{b}+1\right)^{3/4}}d\frac{1}{\sqrt[4]{x}}}{b(a+bx)^{3/4}}+\cdots\right.}{6b}\cdots\right.}{10b}\cdots\right.}{14b}\cdots\right.\right.}{\sqrt[4]{x}\sqrt[4]{a+bx}\cdots}$$

$\downarrow$ 807

$$\frac{\sqrt[4]{ax+bx^2}\left(\frac{1}{18}a\left(\frac{2x^{13/4}\sqrt[4]{a+bx}}{7b}-\frac{13a\left(\frac{2x^{9/4}\sqrt[4]{a+bx}}{5b}-\frac{9a\left(\frac{2x^{5/4}\sqrt[4]{a+bx}}{3b}-\frac{5a\left(\frac{ax^{3/4}\left(\frac{a}{bx}+1\right)^{3/4}\int\frac{1}{\left(\frac{\sqrt{x}a}{b}+1\right)^{3/4}}d\sqrt{x}}{b(a+bx)^{3/4}}+\frac{2\sqrt[4]{x}\cdots}{\cdots}\right.}{6b}\right.}{10b}\right.}{14b}\right.\right.}{\sqrt[4]{x}\sqrt[4]{a+bx}}$$

$\downarrow$ 229

$$\frac{\sqrt[4]{ax+bx^2}\left(\frac{1}{18}a\left(\frac{2x^{13/4}\sqrt[4]{a+bx}}{7b}-\frac{13a\left(\frac{2x^{9/4}\sqrt[4]{a+bx}}{5b}-\frac{9a\left(\frac{2x^{5/4}\sqrt[4]{a+bx}}{3b}-\frac{5a\left(\frac{2\sqrt{a}x^{3/4}\left(\frac{a}{bx}+1\right)^{3/4}\text{EllipticF}\left(\frac{1}{2}\arctan\left(\frac{\sqrt{a}\sqrt{x}}{\sqrt{b}}\right.\right.}{\sqrt{b}(a+bx)^{3/4}}\right.}{6b}\right.}{10b}\right.}{14b}\right.\right.}{\sqrt[4]{x}\sqrt[4]{a+bx}}$$

```
Int[x^3*(a*x + b*x^2)^(1/4),x]
```

```
((a*x + b*x^2)^(1/4)*((2*x^(17/4)*(a + b*x)^(1/4))/9 + (a*((2*x^(13/4)*(a
+ b*x)^(1/4))/(7*b) - (13*a*((2*x^(9/4)*(a + b*x)^(1/4))/(5*b) - (9*a*((2*
x^(5/4)*(a + b*x)^(1/4))/(3*b) - (5*a*((2*x^(1/4)*(a + b*x)^(1/4))/b + (2*
Sqrt[a]*(1 + a/(b*x))^(3/4)*x^(3/4)*EllipticF[ArcTan[(Sqrt[a]*Sqrt[x])/Sqr
t[b]]/2, 2])/(Sqrt[b]*(a + b*x)^(3/4))))/(6*b)))/(10*b)))/(14*b)))/18))/(x
^(1/4)*(a + b*x)^(1/4))
```

Defintions of rubi rules used

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^n/(b*(m + n + 1))), x] + Simp[n*((b*c - a*d)/(
b*(m + n + 1)))   Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
 c, d}, x] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ[m, 0] && ( !Integer
Q[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n + 2, 0] && IntLinear
Q[a, b, c, d, m, n, x]
```

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Simp[p/b   Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
 d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && Lt
Q[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntL
inearQ[a, b, c, d, m, n, x]
```

```
Int[((a_) + (b_.)*(x_)^2)^(-3/4), x_Symbol] :> Simp[(2/(a^(3/4)*Rt[b/a, 2])
)*EllipticF[(1/2)*ArcTan[Rt[b/a, 2]*x], 2], x] /; FreeQ[{a, b}, x] && GtQ[a
, 0] && PosQ[b/a]
```

```
Int[((a_) + (b_.)*(x_)^4)^(-3/4), x_Symbol] :> Simp[x^3*((1 + a/(b*x^4))^(3
/4)/(a + b*x^4)^(3/4))   Int[1/(x^3*(1 + a/(b*x^4))^(3/4)), x], x] /; FreeQ
[{a, b}, x]
```

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k = GCD[m
+ 1, n]}, Simp[1/k   Subst[Int[x^((m + 1)/k - 1)*(a + b*x^(n/k))^p, x], x,
x^k], x] /; k != 1] /; FreeQ[{a, b, p}, x] && IGtQ[n, 0] && IntegerQ[m]
```

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> -Subst[Int[(a +
b/x^n)^p/x^(m + 2), x], x, 1/x] /; FreeQ[{a, b, p}, x] && ILtQ[n, 0] && Int
egerQ[m]
```

```
Int[((e_.)*(x_))^(m_)*((b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(
e*x)^m*((b*x + c*x^2)^p/(x^(m + p)*(b + c*x)^p))   Int[x^(m + p)*(b + c*x)^
p, x], x] /; FreeQ[{b, c, e, m}, x]
```

Maple [F]

$$\int x^{3} \left(b \,x^{2}+a x \right)^{\frac{1}{4}}d x$$

```
int(x^3*(b*x^2+a*x)^(1/4),x)
```

```
int(x^3*(b*x^2+a*x)^(1/4),x)
```

Fricas [F]

$$\int x^3 \sqrt[4]{ax+bx^2}\,dx = \int \left(bx^2+ax\right)^{\frac{1}{4}} x^3\,dx$$

```
integrate(x^3*(b*x^2+a*x)^(1/4),x, algorithm="fricas")
```

```
integral((b*x^2 + a*x)^(1/4)*x^3, x)
```

Sympy [F]

$$\int x^3 \sqrt[4]{ax+bx^2}\,dx = \int x^3 \sqrt[4]{x\,(a+bx)}\,dx$$

```
integrate(x**3*(b*x**2+a*x)**(1/4),x)
```

```
Integral(x**3*(x*(a + b*x))**(1/4), x)
```

Maxima [F]

$$\int x^3 \sqrt[4]{ax+bx^2}\,dx = \int \left(bx^2+ax\right)^{\frac{1}{4}} x^3\,dx$$

```
integrate(x^3*(b*x^2+a*x)^(1/4),x, algorithm="maxima")
```

```
integrate((b*x^2 + a*x)^(1/4)*x^3, x)
```

Giac [F]

$$\int x^3 \sqrt[4]{ax+bx^2}\,dx = \int \left(bx^2+ax\right)^{\frac{1}{4}} x^3\,dx$$

```
integrate(x^3*(b*x^2+a*x)^(1/4),x, algorithm="giac")
```

```
integrate((b*x^2 + a*x)^(1/4)*x^3, x)
```

Mupad [F(-1)]

Timed out.

$$\int x^3 \sqrt[4]{ax+bx^2}\,dx = \int x^3\,\left(b\,x^2+a\,x\right)^{1/4}\,dx$$

```
int(x^3*(a*x + b*x^2)^(1/4),x)
```

```
int(x^3*(a*x + b*x^2)^(1/4), x)
```

Reduce [F]

$$\int x^3 \sqrt[4]{ax+bx^2}\,dx$$
$$= \frac{-780x^{\frac{1}{4}}(bx+a)^{\frac{1}{4}}a^4+312x^{\frac{5}{4}}(bx+a)^{\frac{1}{4}}a^3b-208x^{\frac{9}{4}}(bx+a)^{\frac{1}{4}}a^2b^2+160x^{\frac{13}{4}}(bx+a)^{\frac{1}{4}}a\,b^3+2240x^{\frac{17}{4}}(bx}{10080b^4}$$

```
int(x^3*(b*x^2+a*x)^(1/4),x)
```

```
( - 780*x**(1/4)*(a + b*x)**(1/4)*a**4 + 312*x**(1/4)*(a + b*x)**(1/4)*a**
3*b*x - 208*x**(1/4)*(a + b*x)**(1/4)*a**2*b**2*x**2 + 160*x**(1/4)*(a + b
*x)**(1/4)*a*b**3*x**3 + 2240*x**(1/4)*(a + b*x)**(1/4)*b**4*x**4 + 195*in
t((a + b*x)**(1/4)/(x**(3/4)*a + x**(3/4)*b*x),x)*a**5)/(10080*b**4)
```

3.146 $\int x^2 \sqrt[4]{ax+bx^2}\,dx$

Optimal result

Integrand size = 17, antiderivative size = 165

$$\int x^2 \sqrt[4]{ax+bx^2}\,dx = \frac{3a^3\sqrt[4]{ax+bx^2}}{28b^3} - \frac{3a^2x\sqrt[4]{ax+bx^2}}{70b^2} + \frac{ax^2\sqrt[4]{ax+bx^2}}{35b} + \frac{2}{7}x^3\sqrt[4]{ax+bx^2} + \frac{3a^{7/2}\left(\frac{bx}{a+bx}\right)^{3/4}\sqrt{a+bx}\sqrt[4]{ax+bx^2}\,\text{EllipticF}\left(\frac{1}{2}\arcsin\left(\frac{\sqrt{a}}{\sqrt{a+bx}}\right),2\right)}{28b^4x}$$

```
3/28*a^3*(b*x^2+a*x)^(1/4)/b^3-3/70*a^2*x*(b*x^2+a*x)^(1/4)/b^2+1/35*a*x^2
*(b*x^2+a*x)^(1/4)/b+2/7*x^3*(b*x^2+a*x)^(1/4)+3/28*a^(7/2)*(b*x/(b*x+a))^
(3/4)*(b*x+a)^(1/2)*(b*x^2+a*x)^(1/4)*InverseJacobiAM(1/2*arcsin(a^(1/2)/(
b*x+a)^(1/2)),2^(1/2))/b^4/x
```

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 10.01 (sec) , antiderivative size = 47, normalized size of antiderivative = 0.28

$$\int x^2\sqrt[4]{ax+bx^2}\,dx=\frac{4x^3\sqrt[4]{x(a+bx)}\,\text{Hypergeometric2F1}\left(-\frac{1}{4},\frac{13}{4},\frac{17}{4},-\frac{bx}{a}\right)}{13\sqrt[4]{1+\frac{bx}{a}}}$$

```
Integrate[x^2*(a*x + b*x^2)^(1/4),x]
```

```
(4*x^3*(x*(a + b*x))^(1/4)*Hypergeometric2F1[-1/4, 13/4, 17/4, -((b*x)/a)]
)/(13*(1 + (b*x)/a)^(1/4))
```

Rubi [A] (warning: unable to verify)

Time = 0.56 (sec) , antiderivative size = 195, normalized size of antiderivative = 1.18, number of steps used = 11, number of rules used = 10, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.588, Rules used = {1137, 60, 60, 60, 60, 73, 768, 858, 807, 229}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^2\sqrt[4]{ax+bx^2}\,dx$$

$$\Big\downarrow \text{1137}$$

$$\frac{\sqrt[4]{ax+bx^2}\int x^{9/4}\sqrt[4]{a+bx}dx}{\sqrt[4]{x}\sqrt[4]{a+bx}}$$

$$\Big\downarrow \text{60}$$

$$\frac{\sqrt[4]{ax+bx^2}\left(\frac{1}{14}a\int\frac{x^{9/4}}{(a+bx)^{3/4}}dx+\frac{2}{7}x^{13/4}\sqrt[4]{a+bx}\right)}{\sqrt[4]{x}\sqrt[4]{a+bx}}$$

$$\Big\downarrow \text{60}$$

$$\frac{\sqrt[4]{ax+bx^2}\left(\frac{1}{14}a\left(\frac{2x^{9/4}\sqrt[4]{a+bx}}{5b}-\frac{9a\int\frac{x^{5/4}}{(a+bx)^{3/4}}dx}{10b}\right)+\frac{2}{7}x^{13/4}\sqrt[4]{a+bx}\right)}{\sqrt[4]{x}\sqrt[4]{a+bx}}$$

$$\Big\downarrow 60$$

$$\frac{\sqrt[4]{ax+bx^2}\left(\frac{1}{14}a\left(\frac{2x^{9/4}\sqrt[4]{a+bx}}{5b}-\frac{9a\left(\frac{2x^{5/4}\sqrt[4]{a+bx}}{3b}-\frac{5a\int\frac{\sqrt[4]{x}}{(a+bx)^{3/4}}dx}{6b}\right)}{10b}\right)+\frac{2}{7}x^{13/4}\sqrt[4]{a+bx}\right)}{\sqrt[4]{x}\sqrt[4]{a+bx}}$$

$$\Big\downarrow 60$$

$$\frac{\sqrt[4]{ax+bx^2}\left(\frac{1}{14}a\left(\frac{2x^{9/4}\sqrt[4]{a+bx}}{5b}-\frac{9a\left(\frac{2x^{5/4}\sqrt[4]{a+bx}}{3b}-\frac{5a\left(\frac{2\sqrt[4]{x}\sqrt[4]{a+bx}}{b}-\frac{a\int\frac{1}{x^{3/4}(a+bx)^{3/4}}dx}{2b}\right)}{6b}\right)}{10b}\right)+\frac{2}{7}x^{13/4}\sqrt[4]{a+bx}\right)}{\sqrt[4]{x}\sqrt[4]{a+bx}}$$

$$\Big\downarrow 73$$

$$\frac{\sqrt[4]{ax+bx^2}\left(\frac{1}{14}a\left(\frac{2x^{9/4}\sqrt[4]{a+bx}}{5b}-\frac{9a\left(\frac{2x^{5/4}\sqrt[4]{a+bx}}{3b}-\frac{5a\left(\frac{2\sqrt[4]{x}\sqrt[4]{a+bx}}{b}-\frac{2a\int\frac{1}{(a+bx)^{3/4}}d\sqrt[4]{x}}{b}\right)}{6b}\right)}{10b}\right)+\frac{2}{7}x^{13/4}\sqrt[4]{a+bx}\right)}{\sqrt[4]{x}\sqrt[4]{a+bx}}$$

$$\Big\downarrow 768$$

$$\frac{\sqrt[4]{ax+bx^2}\left(\frac{1}{14}a\left(\frac{2x^{9/4}\sqrt[4]{a+bx}}{5b}-\frac{9a\left(\frac{2x^{5/4}\sqrt[4]{a+bx}}{3b}-\frac{5a\left(\frac{2\sqrt[4]{x}\sqrt[4]{a+bx}}{b}-\frac{2ax^{3/4}\left(\frac{a}{bx}+1\right)^{3/4}\int \frac{1}{\left(\frac{a}{bx}+1\right)^{3/4}x^{3/4}}d\sqrt[4]{x}}{b(a+bx)^{3/4}}\right)}{6b}\right)}{10b}\right)+$$

$$\sqrt[4]{x}\sqrt[4]{a+bx}$$

$\downarrow$ 858

$$\frac{\sqrt[4]{ax+bx^2}\left(\frac{1}{14}a\left(\frac{2x^{9/4}\sqrt[4]{a+bx}}{5b}-\frac{9a\left(\frac{2x^{5/4}\sqrt[4]{a+bx}}{3b}-\frac{5a\left(\frac{2ax^{3/4}\left(\frac{a}{bx}+1\right)^{3/4}\int \frac{1}{\sqrt[4]{x}\left(\frac{ax}{b}+1\right)^{3/4}}d\frac{1}{\sqrt[4]{x}}}{b(a+bx)^{3/4}}+\frac{2\sqrt[4]{x}\sqrt[4]{a+bx}}{b}\right)}{6b}\right)}{10b}\right)}{\sqrt[4]{x}\sqrt[4]{a+bx}}$$

$\downarrow$ 807

$$\frac{\sqrt[4]{ax+bx^2}\left(\frac{1}{14}a\left(\frac{2x^{9/4}\sqrt[4]{a+bx}}{5b}-\frac{9a\left(\frac{2x^{5/4}\sqrt[4]{a+bx}}{3b}-\frac{5a\left(\frac{ax^{3/4}\left(\frac{a}{bx}+1\right)^{3/4}\int\frac{1}{\left(\frac{\sqrt{x}a}{b}+1\right)^{3/4}}d\sqrt{x}}{b(a+bx)^{3/4}}+\frac{2\sqrt[4]{x}\sqrt[4]{a+bx}}{b}\right)}{6b}\right)}{10b}\right)+\frac{2}{7}x}{\sqrt[4]{x}\sqrt[4]{a+bx}}$$

$\downarrow$ 229

$$\frac{\sqrt[4]{ax+bx^2}\left(\frac{1}{14}a\left(\frac{2x^{9/4}\sqrt[4]{a+bx}}{5b}-\frac{9a\left(\frac{2x^{5/4}\sqrt[4]{a+bx}}{3b}-\frac{5a\left(\frac{2\sqrt{a}x^{3/4}\left(\frac{a}{bx}+1\right)^{3/4}\text{EllipticF}\left(\frac{1}{2}\arctan\left(\frac{\sqrt{a}\sqrt{x}}{\sqrt{b}}\right),2\right)}{\sqrt{b}(a+bx)^{3/4}}+\frac{2\sqrt[4]{x}\sqrt[4]{a+bx}}{b}\right)}{6b}}{10b}}{\sqrt[4]{x}\sqrt[4]{a+bx}}$$

```
Int[x^2*(a*x + b*x^2)^(1/4),x]
```

```
((a*x + b*x^2)^(1/4)*((2*x^(13/4)*(a + b*x)^(1/4))/7 + (a*((2*x^(9/4)*(a +
 b*x)^(1/4))/(5*b) - (9*a*((2*x^(5/4)*(a + b*x)^(1/4))/(3*b) - (5*a*((2*x^
(1/4)*(a + b*x)^(1/4))/b + (2*Sqrt[a]*(1 + a/(b*x))^(3/4)*x^(3/4)*Elliptic
F[ArcTan[(Sqrt[a]*Sqrt[x])/Sqrt[b]]/2, 2])/(Sqrt[b]*(a + b*x)^(3/4))))/(6*
b)))/(10*b)))/14))/(x^(1/4)*(a + b*x)^(1/4))
```

Defintions of rubi rules used

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^n/(b*(m + n + 1))), x] + Simp[n*((b*c - a*d)/(
b*(m + n + 1)))   Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
 c, d}, x] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ[m, 0] && ( !Integer
Q[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n + 2, 0] && IntLinear
Q[a, b, c, d, m, n, x]
```

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Simp[p/b   Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
 d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && Lt
Q[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntL
inearQ[a, b, c, d, m, n, x]
```

```
Int[((a_) + (b_.)*(x_)^2)^(-3/4), x_Symbol] :> Simp[(2/(a^(3/4)*Rt[b/a, 2])
)*EllipticF[(1/2)*ArcTan[Rt[b/a, 2]*x], 2], x] /; FreeQ[{a, b}, x] && GtQ[a
, 0] && PosQ[b/a]
```

```
Int[((a_) + (b_.)*(x_)^4)^(-3/4), x_Symbol] :> Simp[x^3*((1 + a/(b*x^4))^(3
/4)/(a + b*x^4)^(3/4))   Int[1/(x^3*(1 + a/(b*x^4))^(3/4)), x], x] /; FreeQ
[{a, b}, x]
```

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k = GCD[m
+ 1, n]}, Simp[1/k   Subst[Int[x^((m + 1)/k - 1)*(a + b*x^(n/k))^p, x], x,
x^k], x] /; k != 1] /; FreeQ[{a, b, p}, x] && IGtQ[n, 0] && IntegerQ[m]
```

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> -Subst[Int[(a +
b/x^n)^p/x^(m + 2), x], x, 1/x] /; FreeQ[{a, b, p}, x] && ILtQ[n, 0] && Int
egerQ[m]
```

```
Int[((e_.)*(x_))^(m_)*((b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(
e*x)^m*((b*x + c*x^2)^p/(x^(m + p)*(b + c*x)^p))   Int[x^(m + p)*(b + c*x)^
p, x], x] /; FreeQ[{b, c, e, m}, x]
```

Maple [F]

$$\int x^{2} \left(b \,x^{2}+a x \right)^{\frac{1}{4}}d x$$

```
int(x^2*(b*x^2+a*x)^(1/4),x)
```

```
int(x^2*(b*x^2+a*x)^(1/4),x)
```

Fricas [F]

$$\int x^2 \sqrt[4]{ax + bx^2}\, dx = \int (bx^2 + ax)^{\frac{1}{4}} x^2\, dx$$

```
integrate(x^2*(b*x^2+a*x)^(1/4),x, algorithm="fricas")
```

```
integral((b*x^2 + a*x)^(1/4)*x^2, x)
```

Sympy [F]

$$\int x^2 \sqrt[4]{ax + bx^2}\, dx = \int x^{2} \sqrt[4]{x (a + bx)}\, dx$$

```
integrate(x**2*(b*x**2+a*x)**(1/4),x)
```

```
Integral(x**2*(x*(a + b*x))**(1/4), x)
```

Maxima [F]

$$\int x^2 \sqrt[4]{ax+bx^2}\,dx = \int (bx^2+ax)^{\frac{1}{4}} x^2\,dx$$

```
integrate(x^2*(b*x^2+a*x)^(1/4),x, algorithm="maxima")
```

```
integrate((b*x^2 + a*x)^(1/4)*x^2, x)
```

Giac [F]

$$\int x^2 \sqrt[4]{ax+bx^2}\,dx = \int (bx^2+ax)^{\frac{1}{4}} x^2\,dx$$

```
integrate(x^2*(b*x^2+a*x)^(1/4),x, algorithm="giac")
```

```
integrate((b*x^2 + a*x)^(1/4)*x^2, x)
```

Mupad [F(-1)]

Timed out.

$$\int x^2 \sqrt[4]{ax+bx^2}\,dx = \int x^2\,\left(b\,x^2+a\,x\right)^{1/4}\,dx$$

```
int(x^2*(a*x + b*x^2)^(1/4),x)
```

```
int(x^2*(a*x + b*x^2)^(1/4), x)
```

Reduce [F]

$$\int x^2 \sqrt[4]{ax+bx^2}\,dx$$
$$= \frac{60x^{\frac{1}{4}}(bx+a)^{\frac{1}{4}}a^3 - 24x^{\frac{5}{4}}(bx+a)^{\frac{1}{4}}a^2b + 16x^{\frac{9}{4}}(bx+a)^{\frac{1}{4}}a\,b^2 + 160x^{\frac{13}{4}}(bx+a)^{\frac{1}{4}}b^3 - 15\left(\int \frac{(bx+a)^{\frac{1}{4}}}{x^{\frac{3}{4}}a+x^{\frac{7}{4}}b}dx\right)}{560b^3}$$

```
int(x^2*(b*x^2+a*x)^(1/4),x)
```

```
(60*x**(1/4)*(a + b*x)**(1/4)*a**3 - 24*x**(1/4)*(a + b*x)**(1/4)*a**2*b*x
 + 16*x**(1/4)*(a + b*x)**(1/4)*a*b**2*x**2 + 160*x**(1/4)*(a + b*x)**(1/4
)*b**3*x**3 - 15*int((a + b*x)**(1/4)/(x**(3/4)*a + x**(3/4)*b*x),x)*a**4)
/(560*b**3)
```

3.147 $\int x\sqrt[4]{ax+bx^2}\,dx$

Optimal result

Integrand size = 15, antiderivative size = 139

$$\int x\sqrt[4]{ax+bx^2}\,dx = -\frac{a^2\sqrt[4]{ax+bx^2}}{6b^2}+\frac{ax\sqrt[4]{ax+bx^2}}{15b}+\frac{2}{5}x^2\sqrt[4]{ax+bx^2} - \frac{a^{5/2}\left(\frac{bx}{a+bx}\right)^{3/4}\sqrt{a+bx}\sqrt[4]{ax+bx^2}\,\text{EllipticF}\left(\frac{1}{2}\arcsin\left(\frac{\sqrt{a}}{\sqrt{a+bx}}\right),2\right)}{6b^3x}$$

```
-1/6*a^2*(b*x^2+a*x)^(1/4)/b^2+1/15*a*x*(b*x^2+a*x)^(1/4)/b+2/5*x^2*(b*x^2
+a*x)^(1/4)-1/6*a^(5/2)*(b*x/(b*x+a))^(3/4)*(b*x+a)^(1/2)*(b*x^2+a*x)^(1/4
)*InverseJacobiAM(1/2*arcsin(a^(1/2)/(b*x+a)^(1/2)),2^(1/2))/b^3/x
```

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 10.01 (sec) , antiderivative size = 47, normalized size of antiderivative = 0.34

$$\int x\sqrt[4]{ax+bx^2}\,dx = \frac{4x^2\sqrt[4]{x(a+bx)}\,\text{Hypergeometric2F1}\left(-\frac{1}{4},\frac{9}{4},\frac{13}{4},-\frac{bx}{a}\right)}{9\sqrt[4]{1+\frac{bx}{a}}}$$

```
Integrate[x*(a*x + b*x^2)^(1/4),x]
```

```
(4*x^2*(x*(a + b*x))^(1/4)*Hypergeometric2F1[-1/4, 9/4, 13/4, -((b*x)/a)])
/(9*(1 + (b*x)/a)^(1/4))
```

Rubi [A] (verified)

Time = 0.45 (sec) , antiderivative size = 131, normalized size of antiderivative = 0.94, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.333, Rules used = {1160, 1087, 1093, 1090, 230}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x\sqrt[4]{ax+bx^2}\,dx$$

$$\Big\downarrow \; 1160$$

$$\frac{2\left(ax+bx^2\right)^{5/4}}{5b}-\frac{a\int \sqrt[4]{bx^2+ax}dx}{2b}$$

$$\Big\downarrow \; 1087$$

$$\frac{2\left(ax+bx^2\right)^{5/4}}{5b}-\frac{a\left(\frac{(a+2bx)\sqrt[4]{ax+bx^2}}{3b}-\frac{a^2\int \frac{1}{\left(bx^2+ax\right)^{3/4}}dx}{12b}\right)}{2b}$$

$$\Big\downarrow \; 1093$$

$$\frac{2\left(ax+bx^2\right)^{5/4}}{5b}-\frac{a\left(\frac{(a+2bx)\sqrt[4]{ax+bx^2}}{3b}-\frac{a^2\left(-\frac{b\left(ax+bx^2\right)}{a^2}\right)^{3/4}\int \frac{1}{\left(-\frac{b^2x^2}{a^2}-\frac{bx}{a}\right)^{3/4}}dx}{12b(ax+bx^2)^{3/4}}\right)}{2b}$$

$$\Big\downarrow \; 1090$$

$$\frac{2\left(ax+bx^2\right)^{5/4}}{5b}-\frac{a\left(\frac{a^4\left(-\frac{b\left(ax+bx^2\right)}{a^2}\right)^{3/4}\int \frac{1}{\left(1-\frac{a^2\left(-\frac{2xb^2}{a^2}-\frac{b}{a}\right)^2}{b^2}\right)^{3/4}}d\left(-\frac{2xb^2}{a^2}-\frac{b}{a}\right)}{6\sqrt{2}b^3(ax+bx^2)^{3/4}}+\frac{(a+2bx)\sqrt[4]{ax+bx^2}}{3b}\right)}{2b}$$

$$\Big\downarrow \text{230}$$

$$\frac{2\left(ax+bx^2\right)^{5/4}}{5b}-\frac{a\left(\frac{a^3\left(-\frac{b\left(ax+bx^2\right)}{a^2}\right)^{3/4}\text{EllipticF}\left(\frac{1}{2}\arcsin\left(\frac{a\left(-\frac{2xb^2}{a^2}-\frac{b}{a}\right)}{b}\right),2\right)}{3\sqrt{2}b^2(ax+bx^2)^{3/4}}+\frac{(a+2bx)\sqrt[4]{ax+bx^2}}{3b}\right)}{2b}$$

```
Int[x*(a*x + b*x^2)^(1/4),x]
```

```
(2*(a*x + b*x^2)^(5/4))/(5*b) - (a*(((a + 2*b*x)*(a*x + b*x^2)^(1/4))/(3*b
) + (a^3*(-((b*(a*x + b*x^2))/a^2))^(3/4)*EllipticF[ArcSin[(a*(-(b/a) - (2
*b^2*x)/a^2))/b]/2, 2])/(3*Sqrt[2]*b^2*(a*x + b*x^2)^(3/4))))/(2*b)
```

Defintions of rubi rules used

```
Int[((a_) + (b_.)*(x_)^2)^(-3/4), x_Symbol] :> Simp[(2/(a^(3/4)*Rt[-b/a, 2]
))*EllipticF[(1/2)*ArcSin[Rt[-b/a, 2]*x], 2], x] /; FreeQ[{a, b}, x] && GtQ
[a, 0] && NegQ[b/a]
```

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(b + 2*c*x)
*((a + b*x + c*x^2)^p/(2*c*(2*p + 1))), x] - Simp[p*((b^2 - 4*a*c)/(2*c*(2*
p + 1)))   Int[(a + b*x + c*x^2)^(p - 1), x], x] /; FreeQ[{a, b, c}, x] &&
GtQ[p, 0] && (IntegerQ[4*p] || IntegerQ[3*p])
```

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[1/(2*c*(-4*
(c/(b^2 - 4*a*c)))^p)   Subst[Int[Simp[1 - x^2/(b^2 - 4*a*c), x]^p, x], x,
b + 2*c*x], x] /; FreeQ[{a, b, c, p}, x] && GtQ[4*a - b^2/c, 0]
```

```
Int[((b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(b*x + c*x^2)^p/((-
c)*((b*x + c*x^2)/b^2))^p   Int[((-c)*(x/b) - c^2*(x^2/b^2))^p, x], x] /; F
reeQ[{b, c}, x] && (IntegerQ[4*p] || IntegerQ[3*p])
```

```
Int[((d_.) + (e_.)*(x_))*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol
] :> Simp[e*((a + b*x + c*x^2)^(p + 1)/(2*c*(p + 1))), x] + Simp[(2*c*d - b
*e)/(2*c)   Int[(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, p}, x]
 && NeQ[p, -1]
```

Maple [F]

$$\int x\left(b\,x^{2}+ax\right)^{\frac{1}{4}}dx$$

```
int(x*(b*x^2+a*x)^(1/4),x)
```

```
int(x*(b*x^2+a*x)^(1/4),x)
```

Fricas [F]

$$\int x\sqrt[4]{ax+bx^2}\,dx=\int\left(bx^2+ax\right)^{\frac{1}{4}}x\,dx$$

```
integrate(x*(b*x^2+a*x)^(1/4),x, algorithm="fricas")
```

```
integral((b*x^2 + a*x)^(1/4)*x, x)
```

Sympy [F]

$$\int x\sqrt[4]{ax+bx^2}\,dx = \int x\sqrt[4]{x(a+bx)}\,dx$$

```
integrate(x*(b*x**2+a*x)**(1/4),x)
```

```
Integral(x*(x*(a + b*x))**(1/4), x)
```

Maxima [F]

$$\int x\sqrt[4]{ax+bx^2}\,dx = \int (bx^2+ax)^{\frac{1}{4}}x\,dx$$

```
integrate(x*(b*x^2+a*x)^(1/4),x, algorithm="maxima")
```

```
integrate((b*x^2 + a*x)^(1/4)*x, x)
```

Giac [F]

$$\int x\sqrt[4]{ax+bx^2}\,dx = \int (bx^2+ax)^{\frac{1}{4}}x\,dx$$

```
integrate(x*(b*x^2+a*x)^(1/4),x, algorithm="giac")
```

```
integrate((b*x^2 + a*x)^(1/4)*x, x)
```

Mupad [F(-1)]

Timed out.

$$\int x\sqrt[4]{ax+bx^2}\,dx = \int x\,\left(b\,x^2+a\,x\right)^{1/4}\,dx$$

```
int(x*(a*x + b*x^2)^(1/4),x)
```

```
int(x*(a*x + b*x^2)^(1/4), x)
```

Reduce [F]

$$\int x\sqrt[4]{ax+bx^2}\,dx = \frac{-20x^{\frac{1}{4}}(bx+a)^{\frac{1}{4}}a^2+8x^{\frac{5}{4}}(bx+a)^{\frac{1}{4}}ab+48x^{\frac{9}{4}}(bx+a)^{\frac{1}{4}}b^2+5\left(\int \frac{(bx+a)^{\frac{1}{4}}}{x^{\frac{3}{4}}a+x^{\frac{7}{4}}b}dx\right)a^3}{120b^2}$$

```
int(x*(b*x^2+a*x)^(1/4),x)
```

```
( - 20*x**(1/4)*(a + b*x)**(1/4)*a**2 + 8*x**(1/4)*(a + b*x)**(1/4)*a*b*x
+ 48*x**(1/4)*(a + b*x)**(1/4)*b**2*x**2 + 5*int((a + b*x)**(1/4)/(x**(3/4
)*a + x**(3/4)*b*x),x)*a**3)/(120*b**2)
```

3.148 $\int \sqrt[4]{ax+bx^2}\,dx$

Optimal result

Integrand size = 13, antiderivative size = 113

$$\int \sqrt[4]{ax+bx^2}\,dx=\frac{a\sqrt[4]{ax+bx^2}}{3b}+\frac{2}{3}x\sqrt[4]{ax+bx^2}+\frac{a^{3/2}\left(\frac{bx}{a+bx}\right)^{3/4}\sqrt{a+bx}\sqrt[4]{ax+bx^2}\,\text{EllipticF}\left(\frac{1}{2}\arcsin\left(\frac{\sqrt{a}}{\sqrt{a+bx}}\right),2\right)}{3b^2x}$$

```
1/3*a*(b*x^2+a*x)^(1/4)/b+2/3*x*(b*x^2+a*x)^(1/4)+1/3*a^(3/2)*(b*x/(b*x+a)
)^(3/4)*(b*x+a)^(1/2)*(b*x^2+a*x)^(1/4)*InverseJacobiAM(1/2*arcsin(a^(1/2)
/(b*x+a)^(1/2)),2^(1/2))/b^2/x
```

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 10.01 (sec) , antiderivative size = 45, normalized size of antiderivative = 0.40

$$\int \sqrt[4]{ax+bx^2}\,dx=\frac{4x\sqrt[4]{x(a+bx)}\,\text{Hypergeometric2F1}\left(-\frac{1}{4},\frac{5}{4},\frac{9}{4},-\frac{bx}{a}\right)}{5\sqrt[4]{1+\frac{bx}{a}}}$$

```
Integrate[(a*x + b*x^2)^(1/4),x]
```

```
(4*x*(x*(a + b*x))^(1/4)*Hypergeometric2F1[-1/4, 5/4, 9/4, -((b*x)/a)])/(5
*(1 + (b*x)/a)^(1/4))
```

Rubi [A] (verified)

Time = 0.40 (sec) , antiderivative size = 102, normalized size of antiderivative = 0.90, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.308$, Rules used = {1087, 1093, 1090, 230}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sqrt[4]{ax+bx^2}\,dx$$

$$\Big\downarrow \text{1087}$$

$$\frac{(a+2bx)\sqrt[4]{ax+bx^2}}{3b}-\frac{a^2\int \frac{1}{(bx^2+ax)^{3/4}}dx}{12b}$$

$$\Big\downarrow \text{1093}$$

$$\frac{(a+2bx)\sqrt[4]{ax+bx^2}}{3b}-\frac{a^2\left(-\frac{b(ax+bx^2)}{a^2}\right)^{3/4}\int \frac{1}{\left(-\frac{b^2x^2}{a^2}-\frac{bx}{a}\right)^{3/4}}dx}{12b\,(ax+bx^2)^{3/4}}$$

$$\Big\downarrow \text{1090}$$

$$\frac{a^4\left(-\frac{b(ax+bx^2)}{a^2}\right)^{3/4}\int \frac{1}{\left(1-\frac{a^2\left(-\frac{2xb^2}{a^2}-\frac{b}{a}\right)^2}{b^2}\right)^{3/4}}d\left(-\frac{2xb^2}{a^2}-\frac{b}{a}\right)}{6\sqrt{2}b^3\,(ax+bx^2)^{3/4}}+\frac{(a+2bx)\sqrt[4]{ax+bx^2}}{3b}$$

$$\Big\downarrow \text{230}$$

$$\frac{a^3\left(-\frac{b(ax+bx^2)}{a^2}\right)^{3/4}\text{EllipticF}\left(\frac{1}{2}\arcsin\left(\frac{a\left(-\frac{2xb^2}{a^2}-\frac{b}{a}\right)}{b}\right),2\right)}{3\sqrt{2}b^2\,(ax+bx^2)^{3/4}}+\frac{(a+2bx)\sqrt[4]{ax+bx^2}}{3b}$$

```
Int[(a*x + b*x^2)^(1/4),x]
```

```
((a + 2*b*x)*(a*x + b*x^2)^(1/4))/(3*b) + (a^3*(-((b*(a*x + b*x^2))/a^2))^
(3/4)*EllipticF[ArcSin[(a*(-(b/a) - (2*b^2*x)/a^2))/b]/2, 2])/(3*Sqrt[2]*b
^2*(a*x + b*x^2)^(3/4))
```

Defintions of rubi rules used

```
Int[((a_) + (b_.)*(x_)^2)^(-3/4), x_Symbol] :> Simp[(2/(a^(3/4)*Rt[-b/a, 2]
))*EllipticF[(1/2)*ArcSin[Rt[-b/a, 2]*x], 2], x] /; FreeQ[{a, b}, x] && GtQ
[a, 0] && NegQ[b/a]
```

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(b + 2*c*x)
*((a + b*x + c*x^2)^p/(2*c*(2*p + 1))), x] - Simp[p*((b^2 - 4*a*c)/(2*c*(2*
p + 1)))   Int[(a + b*x + c*x^2)^(p - 1), x], x] /; FreeQ[{a, b, c}, x] &&
GtQ[p, 0] && (IntegerQ[4*p] || IntegerQ[3*p])
```

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[1/(2*c*(-4*
(c/(b^2 - 4*a*c)))^p)   Subst[Int[Simp[1 - x^2/(b^2 - 4*a*c), x]^p, x], x,
b + 2*c*x], x] /; FreeQ[{a, b, c, p}, x] && GtQ[4*a - b^2/c, 0]
```

```
Int[((b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(b*x + c*x^2)^p/((-
c)*((b*x + c*x^2)/b^2))^p   Int[((-c)*(x/b) - c^2*(x^2/b^2))^p, x], x] /; F
reeQ[{b, c}, x] && (IntegerQ[4*p] || IntegerQ[3*p])
```

Maple [F]

$$\int \left(b\,x^{2}+a x\right)^{\frac{1}{4}}d x$$

```
int((b*x^2+a*x)^(1/4),x)
```

```
int((b*x^2+a*x)^(1/4),x)
```

Fricas [F]

$$\int \sqrt[4]{ax+bx^2}\,dx = \int \left(bx^2+ax\right)^{\frac{1}{4}}\,dx$$

```
integrate((b*x^2+a*x)^(1/4),x, algorithm="fricas")
```

```
integral((b*x^2 + a*x)^(1/4), x)
```

Sympy [F]

$$\int \sqrt[4]{ax+bx^2}\,dx = \int \sqrt[4]{ax+bx^2}\,dx$$

```
integrate((b*x**2+a*x)**(1/4),x)
```

```
Integral((a*x + b*x**2)**(1/4), x)
```

Maxima [F]

$$\int \sqrt[4]{ax+bx^2}\,dx = \int \left(bx^2+ax\right)^{\frac{1}{4}}\,dx$$

```
integrate((b*x^2+a*x)^(1/4),x, algorithm="maxima")
```

```
integrate((b*x^2 + a*x)^(1/4), x)
```

Giac [F]

$$\int \sqrt[4]{ax+bx^2}\,dx = \int \left(bx^2+ax\right)^{\frac{1}{4}}\,dx$$

```
integrate((b*x^2+a*x)^(1/4),x, algorithm="giac")
```

```
integrate((b*x^2 + a*x)^(1/4), x)
```

Mupad [B] (verification not implemented)

Time = 9.20 (sec) , antiderivative size = 36, normalized size of antiderivative = 0.32

$$\int \sqrt[4]{ax+bx^2}\,dx = \frac{4\,x\,\left(b\,x^2+a\,x\right)^{1/4}\,{}_2\mathrm{F}_1\left(-\frac{1}{4},\frac{5}{4};\ \frac{9}{4};\ -\frac{b\,x}{a}\right)}{5\,\left(\frac{b\,x}{a}+1\right)^{1/4}}$$

```
int((a*x + b*x^2)^(1/4),x)
```

```
(4*x*(a*x + b*x^2)^(1/4)*hypergeom([-1/4, 5/4], 9/4, -(b*x)/a))/(5*((b*x)/
a + 1)^(1/4))
```

Reduce [F]

$$\int \sqrt[4]{ax+bx^2}\,dx = \frac{4x^{\frac{1}{4}}(bx+a)^{\frac{1}{4}}\,a+8x^{\frac{5}{4}}(bx+a)^{\frac{1}{4}}\,b-\left(\int \frac{(bx+a)^{\frac{1}{4}}}{x^{\frac{3}{4}}a+x^{\frac{7}{4}}b}dx\right)a^2}{12b}$$

```
int((b*x^2+a*x)^(1/4),x)
```

```
(4*x**(1/4)*(a + b*x)**(1/4)*a + 8*x**(1/4)*(a + b*x)**(1/4)*b*x - int((a
+ b*x)**(1/4)/(x**(3/4)*a + x**(3/4)*b*x),x)*a**2)/(12*b)
```

3.149 $\int \frac{\sqrt[4]{ax+bx^2}}{x}\,dx$

Optimal result

Integrand size = 17, antiderivative size = 87

$$\int \frac{\sqrt[4]{ax+bx^2}}{x}\,dx = 2\sqrt[4]{ax+bx^2} - \frac{2\sqrt{a}\left(\frac{bx}{a+bx}\right)^{3/4}\sqrt{a+bx}\sqrt[4]{ax+bx^2}\,\text{EllipticF}\left(\frac{1}{2}\arcsin\left(\frac{\sqrt{a}}{\sqrt{a+bx}}\right),2\right)}{bx}$$

```
2*(b*x^2+a*x)^(1/4)-2*a^(1/2)*(b*x/(b*x+a))^(3/4)*(b*x+a)^(1/2)*(b*x^2+a*x
)^(1/4)*InverseJacobiAM(1/2*arcsin(a^(1/2)/(b*x+a)^(1/2)),2^(1/2))/b/x
```

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 10.01 (sec) , antiderivative size = 42, normalized size of antiderivative = 0.48

$$\int \frac{\sqrt[4]{ax+bx^2}}{x}\,dx = \frac{4\sqrt[4]{x(a+bx)}\,\text{Hypergeometric2F1}\left(-\frac{1}{4},\frac{1}{4},\frac{5}{4},-\frac{bx}{a}\right)}{\sqrt[4]{1+\frac{bx}{a}}}$$

```
Integrate[(a*x + b*x^2)^(1/4)/x,x]
```

```
(4*(x*(a + b*x))^(1/4)*Hypergeometric2F1[-1/4, 1/4, 5/4, -((b*x)/a)])/(1 +
 (b*x)/a)^(1/4)
```

Rubi [A] (warning: unable to verify)

Time = 0.44 (sec) , antiderivative size = 108, normalized size of antiderivative = 1.24, number of steps used = 8, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}} = 0.412$, Rules used = {1137, 60, 73, 768, 858, 807, 229}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sqrt[4]{ax+bx^2}}{x}dx$$

$$\Big\downarrow \text{1137}$$

$$\frac{\sqrt[4]{ax+bx^2}\int \frac{\sqrt[4]{a+bx}}{x^{3/4}}dx}{\sqrt[4]{x}\sqrt[4]{a+bx}}$$

$$\Big\downarrow \text{60}$$

$$\frac{\sqrt[4]{ax+bx^2}\left(\frac{1}{2}a\int \frac{1}{x^{3/4}(a+bx)^{3/4}}dx+2\sqrt[4]{x}\sqrt[4]{a+bx}\right)}{\sqrt[4]{x}\sqrt[4]{a+bx}}$$

$$\Big\downarrow \text{73}$$

$$\frac{\sqrt[4]{ax+bx^2}\left(2a\int \frac{1}{(a+bx)^{3/4}}d\sqrt[4]{x}+2\sqrt[4]{x}\sqrt[4]{a+bx}\right)}{\sqrt[4]{x}\sqrt[4]{a+bx}}$$

$$\Big\downarrow \text{768}$$

$$\frac{\sqrt[4]{ax+bx^2}\left(\frac{2ax^{3/4}\left(\frac{a}{bx}+1\right)^{3/4}\int \frac{1}{\left(\frac{a}{bx}+1\right)^{3/4}x^{3/4}}d\sqrt[4]{x}}{(a+bx)^{3/4}}+2\sqrt[4]{x}\sqrt[4]{a+bx}\right)}{\sqrt[4]{x}\sqrt[4]{a+bx}}$$

$$\Big\downarrow \text{858}$$

$$\frac{\sqrt[4]{ax+bx^2}\left(2\sqrt[4]{x}\sqrt[4]{a+bx}-\frac{2ax^{3/4}\left(\frac{a}{bx}+1\right)^{3/4}\int \frac{1}{\sqrt[4]{x}\left(\frac{ax}{b}+1\right)^{3/4}}d\frac{1}{\sqrt[4]{x}}}{(a+bx)^{3/4}}\right)}{\sqrt[4]{x}\sqrt[4]{a+bx}}$$

$$\Big\downarrow \text{807}$$

$$\frac{\sqrt[4]{ax+bx^2}\left(2\sqrt[4]{x}\sqrt[4]{a+bx}-\frac{ax^{3/4}\left(\frac{a}{bx}+1\right)^{3/4}\int \frac{1}{\left(\frac{\sqrt{x}a}{b}+1\right)^{3/4}}d\sqrt{x}}{(a+bx)^{3/4}}\right)}{\sqrt[4]{x}\sqrt[4]{a+bx}}$$

$$\Big\downarrow \text{229}$$

$$\frac{\sqrt[4]{ax+bx^2}\left(2\sqrt[4]{x}\sqrt[4]{a+bx}-\frac{2\sqrt{a}\sqrt{b}x^{3/4}\left(\frac{a}{bx}+1\right)^{3/4}\text{EllipticF}\left(\frac{1}{2}\arctan\left(\frac{\sqrt{a}\sqrt{x}}{\sqrt{b}}\right),2\right)}{(a+bx)^{3/4}}\right)}{\sqrt[4]{x}\sqrt[4]{a+bx}}$$

```
Int[(a*x + b*x^2)^(1/4)/x,x]
```

```
((a*x + b*x^2)^(1/4)*(2*x^(1/4)*(a + b*x)^(1/4) - (2*Sqrt[a]*Sqrt[b]*(1 +
a/(b*x))^(3/4)*x^(3/4)*EllipticF[ArcTan[(Sqrt[a]*Sqrt[x])/Sqrt[b]]/2, 2])/
(a + b*x)^(3/4)))/(x^(1/4)*(a + b*x)^(1/4))
```

Defintions of rubi rules used

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^n/(b*(m + n + 1))), x] + Simp[n*((b*c - a*d)/(
b*(m + n + 1)))   Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
 c, d}, x] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ[m, 0] && ( !Integer
Q[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n + 2, 0] && IntLinear
Q[a, b, c, d, m, n, x]
```

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Simp[p/b   Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
 d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && Lt
Q[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntL
inearQ[a, b, c, d, m, n, x]
```

```
Int[((a_) + (b_.)*(x_)^2)^(-3/4), x_Symbol] :> Simp[(2/(a^(3/4)*Rt[b/a, 2])
)*EllipticF[(1/2)*ArcTan[Rt[b/a, 2]*x], 2], x] /; FreeQ[{a, b}, x] && GtQ[a
, 0] && PosQ[b/a]
```

```
Int[((a_) + (b_.)*(x_)^4)^(-3/4), x_Symbol] :> Simp[x^3*((1 + a/(b*x^4))^(3
/4)/(a + b*x^4)^(3/4))   Int[1/(x^3*(1 + a/(b*x^4))^(3/4)), x], x] /; FreeQ
[{a, b}, x]
```

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k = GCD[m
+ 1, n]}, Simp[1/k   Subst[Int[x^((m + 1)/k - 1)*(a + b*x^(n/k))^p, x], x,
x^k], x] /; k != 1] /; FreeQ[{a, b, p}, x] && IGtQ[n, 0] && IntegerQ[m]
```

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> -Subst[Int[(a +
b/x^n)^p/x^(m + 2), x], x, 1/x] /; FreeQ[{a, b, p}, x] && ILtQ[n, 0] && Int
egerQ[m]
```

```
Int[((e_.)*(x_))^(m_)*((b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(
e*x)^m*((b*x + c*x^2)^p/(x^(m + p)*(b + c*x)^p))   Int[x^(m + p)*(b + c*x)^
p, x], x] /; FreeQ[{b, c, e, m}, x]
```

Maple [F]

$$\int \frac{(b\,x^2 + ax)^{\frac{1}{4}}}{x}dx$$

```
int((b*x^2+a*x)^(1/4)/x,x)
```


```
int((b*x^2+a*x)^(1/4)/x,x)
```

Fricas [F]

$$\int \frac{\sqrt[4]{ax+bx^2}}{x}\,dx = \int \frac{(bx^2+ax)^{\frac{1}{4}}}{x}\,dx$$

```
integrate((b*x^2+a*x)^(1/4)/x,x, algorithm="fricas")
```

```
integral((b*x^2 + a*x)^(1/4)/x, x)
```

Sympy [F]

$$\int \frac{\sqrt[4]{ax+bx^2}}{x}\,dx = \int \frac{\sqrt[4]{x\,(a+bx)}}{x}\,dx$$

```
integrate((b*x**2+a*x)**(1/4)/x,x)
```

```
Integral((x*(a + b*x))**(1/4)/x, x)
```

Maxima [F]

$$\int \frac{\sqrt[4]{ax+bx^2}}{x}\,dx = \int \frac{(bx^2+ax)^{\frac{1}{4}}}{x}\,dx$$

```
integrate((b*x^2+a*x)^(1/4)/x,x, algorithm="maxima")
```

```
integrate((b*x^2 + a*x)^(1/4)/x, x)
```

Giac [F]

$$\int \frac{\sqrt[4]{ax+bx^2}}{x}\,dx = \int \frac{(bx^2+ax)^{\frac{1}{4}}}{x}\,dx$$

```
integrate((b*x^2+a*x)^(1/4)/x,x, algorithm="giac")
```

```
integrate((b*x^2 + a*x)^(1/4)/x, x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\sqrt[4]{ax+bx^2}}{x}\,dx = \int \frac{(b\,x^2+a\,x)^{1/4}}{x}\,dx$$

```
int((a*x + b*x^2)^(1/4)/x,x)
```

```
int((a*x + b*x^2)^(1/4)/x, x)
```

Reduce [F]

$$\int \frac{\sqrt[4]{ax+bx^2}}{x}\,dx = 2x^{\frac{1}{4}}(bx+a)^{\frac{1}{4}} + \frac{\left(\int \frac{(bx+a)^{\frac{1}{4}}}{x^{\frac{3}{4}}a+x^{\frac{7}{4}}b}dx\right)a}{2}$$

```
int((b*x^2+a*x)^(1/4)/x,x)
```

```
(4*x**(1/4)*(a + b*x)**(1/4) + int((a + b*x)**(1/4)/(x**(3/4)*a + x**(3/4)
*b*x),x)*a)/2
```

3.150 $\int \frac{\sqrt[4]{ax+bx^2}}{x^2}\,dx$

Optimal result

Integrand size = 17, antiderivative size = 91

$$\int \frac{\sqrt[4]{ax+bx^2}}{x^2}\,dx=-\frac{4\sqrt[4]{ax+bx^2}}{3x}-\frac{4\left(\frac{bx}{a+bx}\right)^{3/4}\sqrt{a+bx}\sqrt[4]{ax+bx^2}\,\text{EllipticF}\left(\frac{1}{2}\arcsin\left(\frac{\sqrt{a}}{\sqrt{a+bx}}\right),2\right)}{3\sqrt{a}x}$$

```
-4/3*(b*x^2+a*x)^(1/4)/x-4/3*(b*x/(b*x+a))^(3/4)*(b*x+a)^(1/2)*(b*x^2+a*x)
^(1/4)*InverseJacobiAM(1/2*arcsin(a^(1/2)/(b*x+a)^(1/2)),2^(1/2))/a^(1/2)/
x
```

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 10.01 (sec) , antiderivative size = 47, normalized size of antiderivative = 0.52

$$\int \frac{\sqrt[4]{ax+bx^2}}{x^2}\,dx=-\frac{4\sqrt[4]{x(a+bx)}\,\text{Hypergeometric2F1}\left(-\frac{3}{4},-\frac{1}{4},\frac{1}{4},-\frac{bx}{a}\right)}{3x\sqrt[4]{1+\frac{bx}{a}}}$$

```
Integrate[(a*x + b*x^2)^(1/4)/x^2,x]
```

```
(-4*(x*(a + b*x))^(1/4)*Hypergeometric2F1[-3/4, -1/4, 1/4, -((b*x)/a)])/(3
*x*(1 + (b*x)/a)^(1/4))
```

Rubi [A] (warning: unable to verify)

Time = 0.43 (sec) , antiderivative size = 112, normalized size of antiderivative = 1.23, number of steps used = 8, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.412, Rules used = {1137, 57, 73, 768, 858, 807, 229}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sqrt[4]{ax+bx^2}}{x^2}dx$$

$$\Big\downarrow \text{ 1137}$$

$$\frac{\sqrt[4]{ax+bx^2}\int \frac{\sqrt[4]{a+bx}}{x^{7/4}}dx}{\sqrt[4]{x}\sqrt[4]{a+bx}}$$

$$\Big\downarrow \text{ 57}$$

$$\frac{\sqrt[4]{ax+bx^2}\left(\frac{1}{3}b\int \frac{1}{x^{3/4}(a+bx)^{3/4}}dx-\frac{4\sqrt[4]{a+bx}}{3x^{3/4}}\right)}{\sqrt[4]{x}\sqrt[4]{a+bx}}$$

$$\Big\downarrow \text{ 73}$$

$$\frac{\sqrt[4]{ax+bx^2}\left(\frac{4}{3}b\int \frac{1}{(a+bx)^{3/4}}d\sqrt[4]{x}-\frac{4\sqrt[4]{a+bx}}{3x^{3/4}}\right)}{\sqrt[4]{x}\sqrt[4]{a+bx}}$$

$$\Big\downarrow \text{ 768}$$

$$\frac{\sqrt[4]{ax+bx^2}\left(\frac{4bx^{3/4}\left(\frac{a}{bx}+1\right)^{3/4}\int \frac{1}{\left(\frac{a}{bx}+1\right)^{3/4}x^{3/4}}d\sqrt[4]{x}}{3(a+bx)^{3/4}}-\frac{4\sqrt[4]{a+bx}}{3x^{3/4}}\right)}{\sqrt[4]{x}\sqrt[4]{a+bx}}$$

$$\Big\downarrow \text{ 858}$$

$$\frac{\sqrt[4]{ax+bx^2}\left(-\frac{4bx^{3/4}\left(\frac{a}{bx}+1\right)^{3/4}\int\frac{1}{\sqrt[4]{x}\left(\frac{ax}{b}+1\right)^{3/4}}d\frac{1}{\sqrt[4]{x}}}{3(a+bx)^{3/4}}-\frac{4\sqrt[4]{a+bx}}{3x^{3/4}}\right)}{\sqrt[4]{x}\sqrt[4]{a+bx}}$$

$$\Big\downarrow \text{807}$$

$$\frac{\sqrt[4]{ax+bx^2}\left(-\frac{2bx^{3/4}\left(\frac{a}{bx}+1\right)^{3/4}\int\frac{1}{\left(\frac{\sqrt{x}a}{b}+1\right)^{3/4}}d\sqrt{x}}{3(a+bx)^{3/4}}-\frac{4\sqrt[4]{a+bx}}{3x^{3/4}}\right)}{\sqrt[4]{x}\sqrt[4]{a+bx}}$$

$$\Big\downarrow \text{229}$$

$$\frac{\sqrt[4]{ax+bx^2}\left(-\frac{4b^{3/2}x^{3/4}\left(\frac{a}{bx}+1\right)^{3/4}\text{EllipticF}\left(\frac{1}{2}\arctan\left(\frac{\sqrt{a}\sqrt{x}}{\sqrt{b}}\right),2\right)}{3\sqrt{a}(a+bx)^{3/4}}-\frac{4\sqrt[4]{a+bx}}{3x^{3/4}}\right)}{\sqrt[4]{x}\sqrt[4]{a+bx}}$$

```
Int[(a*x + b*x^2)^(1/4)/x^2,x]
```

```
((a*x + b*x^2)^(1/4)*((-4*(a + b*x)^(1/4))/(3*x^(3/4)) - (4*b^(3/2)*(1 + a
/(b*x))^(3/4)*x^(3/4)*EllipticF[ArcTan[(Sqrt[a]*Sqrt[x])/Sqrt[b]]/2, 2])/(
3*Sqrt[a]*(a + b*x)^(3/4))))/(x^(1/4)*(a + b*x)^(1/4))
```

Defintions of rubi rules used

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^n/(b*(m + 1))), x] - Simp[d*(n/(b*(m + 1)))
Int[(a + b*x)^(m + 1)*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b, c, d}, x] &
& GtQ[n, 0] && LtQ[m, -1] &&  !(IntegerQ[n] &&  !IntegerQ[m]) &&  !(ILeQ[m
+ n + 2, 0] && (FractionQ[m] || GeQ[2*n + m + 1, 0])) && IntLinearQ[a, b, c
, d, m, n, x]
```

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Simp[p/b   Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
 d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && Lt
Q[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntL
inearQ[a, b, c, d, m, n, x]
```

```
Int[((a_) + (b_.)*(x_)^2)^(-3/4), x_Symbol] :> Simp[(2/(a^(3/4)*Rt[b/a, 2])
)*EllipticF[(1/2)*ArcTan[Rt[b/a, 2]*x], 2], x] /; FreeQ[{a, b}, x] && GtQ[a
, 0] && PosQ[b/a]
```

```
Int[((a_) + (b_.)*(x_)^4)^(-3/4), x_Symbol] :> Simp[x^3*((1 + a/(b*x^4))^(3
/4)/(a + b*x^4)^(3/4))   Int[1/(x^3*(1 + a/(b*x^4))^(3/4)), x], x] /; FreeQ
[{a, b}, x]
```

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k = GCD[m
+ 1, n]}, Simp[1/k   Subst[Int[x^((m + 1)/k - 1)*(a + b*x^(n/k))^p, x], x,
x^k], x] /; k != 1] /; FreeQ[{a, b, p}, x] && IGtQ[n, 0] && IntegerQ[m]
```

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> -Subst[Int[(a +
b/x^n)^p/x^(m + 2), x], x, 1/x] /; FreeQ[{a, b, p}, x] && ILtQ[n, 0] && Int
egerQ[m]
```

```
Int[((e_.)*(x_))^(m_)*((b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(
e*x)^m*((b*x + c*x^2)^p/(x^(m + p)*(b + c*x)^p))   Int[x^(m + p)*(b + c*x)^
p, x], x] /; FreeQ[{b, c, e, m}, x]
```

Maple [F]

$$\int \frac{(b\,x^2+ax)^{\frac{1}{4}}}{x^2}dx$$

```
int((b*x^2+a*x)^(1/4)/x^2,x)
```

```
int((b*x^2+a*x)^(1/4)/x^2,x)
```

Fricas [F]

$$\int \frac{\sqrt[4]{ax+bx^2}}{x^2}\,dx = \int \frac{(bx^2+ax)^{\frac{1}{4}}}{x^2}\,dx$$

```
integrate((b*x^2+a*x)^(1/4)/x^2,x, algorithm="fricas")
```

```
integral((b*x^2 + a*x)^(1/4)/x^2, x)
```

Sympy [F]

$$\int \frac{\sqrt[4]{ax+bx^2}}{x^2}\,dx = \int \frac{\sqrt[4]{x\,(a+bx)}}{x^2}\,dx$$

```
integrate((b*x**2+a*x)**(1/4)/x**2,x)
```

```
Integral((x*(a + b*x))**(1/4)/x**2, x)
```

Maxima [F]

$$\int \frac{\sqrt[4]{ax+bx^2}}{x^2}\,dx = \int \frac{(bx^2+ax)^{\frac{1}{4}}}{x^2}\,dx$$

```
integrate((b*x^2+a*x)^(1/4)/x^2,x, algorithm="maxima")
```

```
integrate((b*x^2 + a*x)^(1/4)/x^2, x)
```

Giac [F]

$$\int \frac{\sqrt[4]{ax+bx^2}}{x^2}\,dx=\int \frac{(bx^2+ax)^{\frac{1}{4}}}{x^2}\,dx$$

```
integrate((b*x^2+a*x)^(1/4)/x^2,x, algorithm="giac")
```

```
integrate((b*x^2 + a*x)^(1/4)/x^2, x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\sqrt[4]{ax+bx^2}}{x^2}\,dx=\int \frac{(b\,x^2+a\,x)^{1/4}}{x^2}\,dx$$

```
int((a*x + b*x^2)^(1/4)/x^2,x)
```

```
int((a*x + b*x^2)^(1/4)/x^2, x)
```

Reduce [F]

$$\int \frac{\sqrt[4]{ax+bx^2}}{x^2}\,dx=\frac{-4(bx+a)^{\frac{1}{4}}-x^{\frac{3}{4}}\left(\int \frac{(bx+a)^{\frac{1}{4}}}{x^{\frac{7}{4}}a+x^{\frac{11}{4}}b}dx\right)a}{2x^{\frac{3}{4}}}$$

```
int((b*x^2+a*x)^(1/4)/x^2,x)
```

```
( - 4*(a + b*x)**(1/4) - x**(3/4)*int((a + b*x)**(1/4)/(x**(3/4)*a*x + x**
(3/4)*b*x**2),x)*a)/(2*x**(3/4))
```

3.151 $\int \frac{\sqrt[4]{ax+bx^2}}{x^3}\,dx$

Optimal result

Integrand size = 17, antiderivative size = 116

$$\int \frac{\sqrt[4]{ax+bx^2}}{x^3}\,dx = -\frac{4\sqrt[4]{ax+bx^2}}{7x^2} - \frac{4b\sqrt[4]{ax+bx^2}}{21ax} + \frac{8b\left(\frac{bx}{a+bx}\right)^{3/4}\sqrt{a+bx}\sqrt[4]{ax+bx^2}\,\text{EllipticF}\left(\frac{1}{2}\arcsin\left(\frac{\sqrt{a}}{\sqrt{a+bx}}\right),2\right)}{21a^{3/2}x}$$

```
-4/7*(b*x^2+a*x)^(1/4)/x^2-4/21*b*(b*x^2+a*x)^(1/4)/a/x+8/21*b*(b*x/(b*x+a
))^(3/4)*(b*x+a)^(1/2)*(b*x^2+a*x)^(1/4)*InverseJacobiAM(1/2*arcsin(a^(1/2
)/(b*x+a)^(1/2)),2^(1/2))/a^(3/2)/x
```

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 10.01 (sec) , antiderivative size = 47, normalized size of antiderivative = 0.41

$$\int \frac{\sqrt[4]{ax+bx^2}}{x^3}\,dx = -\frac{4\sqrt[4]{x(a+bx)}\,\text{Hypergeometric2F1}\left(-\frac{7}{4},-\frac{1}{4},-\frac{3}{4},-\frac{bx}{a}\right)}{7x^2\sqrt[4]{1+\frac{bx}{a}}}$$

```
Integrate[(a*x + b*x^2)^(1/4)/x^3,x]
```

```
(-4*(x*(a + b*x))^(1/4)*Hypergeometric2F1[-7/4, -1/4, -3/4, -((b*x)/a)])/(
7*x^2*(1 + (b*x)/a)^(1/4))
```

Rubi [A] (warning: unable to verify)

Time = 0.47 (sec) , antiderivative size = 139, normalized size of antiderivative = 1.20, number of steps used = 9, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}} = 0.471$, Rules used = {1137, 57, 61, 73, 768, 858, 807, 229}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sqrt[4]{ax+bx^2}}{x^3}dx$$

$$\Big\downarrow \text{1137}$$

$$\frac{\sqrt[4]{ax+bx^2}\int \frac{\sqrt[4]{a+bx}}{x^{11/4}}dx}{\sqrt[4]{x}\sqrt[4]{a+bx}}$$

$$\Big\downarrow \text{57}$$

$$\frac{\sqrt[4]{ax+bx^2}\left(\frac{1}{7}b\int \frac{1}{x^{7/4}(a+bx)^{3/4}}dx-\frac{4\sqrt[4]{a+bx}}{7x^{7/4}}\right)}{\sqrt[4]{x}\sqrt[4]{a+bx}}$$

$$\Big\downarrow \text{61}$$

$$\frac{\sqrt[4]{ax+bx^2}\left(\frac{1}{7}b\left(-\frac{2b\int \frac{1}{x^{3/4}(a+bx)^{3/4}}dx}{3a}-\frac{4\sqrt[4]{a+bx}}{3ax^{3/4}}\right)-\frac{4\sqrt[4]{a+bx}}{7x^{7/4}}\right)}{\sqrt[4]{x}\sqrt[4]{a+bx}}$$

$$\Big\downarrow \text{73}$$

$$\frac{\sqrt[4]{ax+bx^2}\left(\frac{1}{7}b\left(-\frac{8b\int \frac{1}{(a+bx)^{3/4}}d\sqrt[4]{x}}{3a}-\frac{4\sqrt[4]{a+bx}}{3ax^{3/4}}\right)-\frac{4\sqrt[4]{a+bx}}{7x^{7/4}}\right)}{\sqrt[4]{x}\sqrt[4]{a+bx}}$$

$$\Big\downarrow \text{768}$$

$$\frac{\sqrt[4]{ax+bx^2}\left(\frac{1}{7}b\left(-\frac{8bx^{3/4}\left(\frac{a}{bx}+1\right)^{3/4}\int\frac{1}{\left(\frac{a}{bx}+1\right)^{3/4}x^{3/4}}d\sqrt[4]{x}}{3a(a+bx)^{3/4}}-\frac{4\sqrt[4]{a+bx}}{3ax^{3/4}}\right)-\frac{4\sqrt[4]{a+bx}}{7x^{7/4}}\right)}{\sqrt[4]{x}\sqrt[4]{a+bx}}$$

$$\Big\downarrow \text{858}$$

$$\frac{\sqrt[4]{ax+bx^2}\left(\frac{1}{7}b\left(\frac{8bx^{3/4}\left(\frac{a}{bx}+1\right)^{3/4}\int\frac{1}{\sqrt[4]{x}\left(\frac{ax}{b}+1\right)^{3/4}}d\frac{1}{\sqrt[4]{x}}}{3a(a+bx)^{3/4}}-\frac{4\sqrt[4]{a+bx}}{3ax^{3/4}}\right)-\frac{4\sqrt[4]{a+bx}}{7x^{7/4}}\right)}{\sqrt[4]{x}\sqrt[4]{a+bx}}$$

$$\Big\downarrow \text{807}$$

$$\frac{\sqrt[4]{ax+bx^2}\left(\frac{1}{7}b\left(\frac{4bx^{3/4}\left(\frac{a}{bx}+1\right)^{3/4}\int\frac{1}{\left(\frac{\sqrt{x}a}{b}+1\right)^{3/4}}d\sqrt{x}}{3a(a+bx)^{3/4}}-\frac{4\sqrt[4]{a+bx}}{3ax^{3/4}}\right)-\frac{4\sqrt[4]{a+bx}}{7x^{7/4}}\right)}{\sqrt[4]{x}\sqrt[4]{a+bx}}$$

$$\Big\downarrow \text{229}$$

$$\frac{\sqrt[4]{ax+bx^2}\left(\frac{1}{7}b\left(\frac{8b^{3/2}x^{3/4}\left(\frac{a}{bx}+1\right)^{3/4}\text{EllipticF}\left(\frac{1}{2}\arctan\left(\frac{\sqrt{a}\sqrt{x}}{\sqrt{b}}\right),2\right)}{3a^{3/2}(a+bx)^{3/4}}-\frac{4\sqrt[4]{a+bx}}{3ax^{3/4}}\right)-\frac{4\sqrt[4]{a+bx}}{7x^{7/4}}\right)}{\sqrt[4]{x}\sqrt[4]{a+bx}}$$

```
Int[(a*x + b*x^2)^(1/4)/x^3,x]
```

```
((a*x + b*x^2)^(1/4)*((-4*(a + b*x)^(1/4))/(7*x^(7/4)) + (b*((-4*(a + b*x)
^(1/4))/(3*a*x^(3/4)) + (8*b^(3/2)*(1 + a/(b*x))^(3/4)*x^(3/4)*EllipticF[A
rcTan[(Sqrt[a]*Sqrt[x])/Sqrt[b]]/2, 2])/(3*a^(3/2)*(a + b*x)^(3/4))))/7))/
(x^(1/4)*(a + b*x)^(1/4))
```

Defintions of rubi rules used

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^n/(b*(m + 1))), x] - Simp[d*(n/(b*(m + 1)))
Int[(a + b*x)^(m + 1)*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b, c, d}, x] &
& GtQ[n, 0] && LtQ[m, -1] && !(IntegerQ[n] && !IntegerQ[m]) && !(ILeQ[m
+ n + 2, 0] && (FractionQ[m] || GeQ[2*n + m + 1, 0])) && IntLinearQ[a, b, c
, d, m, n, x]
```

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] - Simp[d*((
m + n + 2)/((b*c - a*d)*(m + 1)))   Int[(a + b*x)^(m + 1)*(c + d*x)^n, x],
x] /; FreeQ[{a, b, c, d, n}, x] && LtQ[m, -1] &&  !(LtQ[n, -1] && (EqQ[a, 0
] || (NeQ[c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && IntLinearQ[a, b, c, d
, m, n, x]
```

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Simp[p/b   Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
 d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && Lt
Q[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntL
inearQ[a, b, c, d, m, n, x]
```

```
Int[((a_) + (b_.)*(x_)^2)^(-3/4), x_Symbol] :> Simp[(2/(a^(3/4)*Rt[b/a, 2])
)*EllipticF[(1/2)*ArcTan[Rt[b/a, 2]*x], 2], x] /; FreeQ[{a, b}, x] && GtQ[a
, 0] && PosQ[b/a]
```

```
Int[((a_) + (b_.)*(x_)^4)^(-3/4), x_Symbol] :> Simp[x^3*((1 + a/(b*x^4))^(3
/4)/(a + b*x^4)^(3/4))   Int[1/(x^3*(1 + a/(b*x^4))^(3/4)), x], x] /; FreeQ
[{a, b}, x]
```

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k = GCD[m
+ 1, n]}, Simp[1/k   Subst[Int[x^((m + 1)/k - 1)*(a + b*x^(n/k))^p, x], x,
x^k], x] /; k != 1] /; FreeQ[{a, b, p}, x] && IGtQ[n, 0] && IntegerQ[m]
```

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> -Subst[Int[(a +
b/x^n)^p/x^(m + 2), x], x, 1/x] /; FreeQ[{a, b, p}, x] && ILtQ[n, 0] && Int
egerQ[m]
```

```
Int[((e_.)*(x_))^(m_)*((b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(
e*x)^m*((b*x + c*x^2)^p/(x^(m + p)*(b + c*x)^p))   Int[x^(m + p)*(b + c*x)^
p, x], x] /; FreeQ[{b, c, e, m}, x]
```

Maple [F]

$$\int \frac{(b\,x^2+ax)^{\frac{1}{4}}}{x^3}dx$$

```
int((b*x^2+a*x)^(1/4)/x^3,x)
```

```
int((b*x^2+a*x)^(1/4)/x^3,x)
```

Fricas [F]

$$\int \frac{\sqrt[4]{ax+bx^2}}{x^3}\,dx = \int \frac{(bx^2+ax)^{\frac{1}{4}}}{x^3}\,dx$$

```
integrate((b*x^2+a*x)^(1/4)/x^3,x, algorithm="fricas")
```

```
integral((b*x^2 + a*x)^(1/4)/x^3, x)
```

Sympy [F]

$$\int \frac{\sqrt[4]{ax+bx^2}}{x^3}\,dx = \int \frac{\sqrt[4]{x\,(a+bx)}}{x^3}\,dx$$

```
integrate((b*x**2+a*x)**(1/4)/x**3,x)
```

```
Integral((x*(a + b*x))**(1/4)/x**3, x)
```

Maxima [F]

$$\int \frac{\sqrt[4]{ax+bx^2}}{x^3}\,dx = \int \frac{(bx^2+ax)^{\frac{1}{4}}}{x^3}\,dx$$

```
integrate((b*x^2+a*x)^(1/4)/x^3,x, algorithm="maxima")
```

```
integrate((b*x^2 + a*x)^(1/4)/x^3, x)
```

Giac [F]

$$\int \frac{\sqrt[4]{ax+bx^2}}{x^3}\,dx = \int \frac{(bx^2+ax)^{\frac{1}{4}}}{x^3}\,dx$$

```
integrate((b*x^2+a*x)^(1/4)/x^3,x, algorithm="giac")
```

```
integrate((b*x^2 + a*x)^(1/4)/x^3, x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\sqrt[4]{ax+bx^2}}{x^3}\,dx = \int \frac{\left(b\,x^2+a\,x\right)^{1/4}}{x^3}\,dx$$

```
int((a*x + b*x^2)^(1/4)/x^3,x)
```

```
int((a*x + b*x^2)^(1/4)/x^3, x)
```

Reduce [F]

$$\int \frac{\sqrt[4]{ax+bx^2}}{x^3}\,dx = \frac{-4(bx+a)^{\frac{1}{4}} - x^{\frac{7}{4}}\left(\int \frac{(bx+a)^{\frac{1}{4}}}{x^{\frac{11}{4}}a+x^{\frac{15}{4}}b}dx\right)a}{6x^{\frac{7}{4}}}$$

```
int((b*x^2+a*x)^(1/4)/x^3,x)
```

```
( - 4*(a + b*x)**(1/4) - x**(3/4)*int((a + b*x)**(1/4)/(x**(3/4)*a*x**2 +
x**(3/4)*b*x**3),x)*a*x)/(6*x**(3/4)*x)
```

3.152 $\int \frac{\sqrt[4]{ax+bx^2}}{x^4}\,dx$

Optimal result

Integrand size = 17, antiderivative size = 144

$$\int \frac{\sqrt[4]{ax+bx^2}}{x^4}\,dx=-\frac{4\sqrt[4]{ax+bx^2}}{11x^3}-\frac{4b\sqrt[4]{ax+bx^2}}{77ax^2}+\frac{8b^2\sqrt[4]{ax+bx^2}}{77a^2x}-\frac{16b^2\left(\frac{bx}{a+bx}\right)^{3/4}\sqrt{a+bx}\sqrt[4]{ax+bx^2}\,\text{EllipticF}\left(\frac{1}{2}\arcsin\left(\frac{\sqrt{a}}{\sqrt{a+bx}}\right),2\right)}{77a^{5/2}x}$$

```
-4/11*(b*x^2+a*x)^(1/4)/x^3-4/77*b*(b*x^2+a*x)^(1/4)/a/x^2+8/77*b^2*(b*x^2
+a*x)^(1/4)/a^2/x-16/77*b^2*(b*x/(b*x+a))^(3/4)*(b*x+a)^(1/2)*(b*x^2+a*x)^
(1/4)*InverseJacobiAM(1/2*arcsin(a^(1/2)/(b*x+a)^(1/2)),2^(1/2))/a^(5/2)/x
```

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 10.01 (sec) , antiderivative size = 47, normalized size of antiderivative = 0.33

$$\int \frac{\sqrt[4]{ax+bx^2}}{x^4}\,dx=-\frac{4\sqrt[4]{x(a+bx)}\,\text{Hypergeometric2F1}\left(-\frac{11}{4},-\frac{1}{4},-\frac{7}{4},-\frac{bx}{a}\right)}{11x^3\sqrt[4]{1+\frac{bx}{a}}}$$

```
Integrate[(a*x + b*x^2)^(1/4)/x^4,x]
```

```
(-4*(x*(a + b*x))^(1/4)*Hypergeometric2F1[-11/4, -1/4, -7/4, -((b*x)/a)])/
(11*x^3*(1 + (b*x)/a)^(1/4))
```

Rubi [A] (warning: unable to verify)

Time = 0.50 (sec) , antiderivative size = 169, normalized size of antiderivative = 1.17, number of steps used = 10, number of rules used = 9, $\frac{\text{number of rules}}{\text{integrand size}} = 0.529$, Rules used = {1137, 57, 61, 61, 73, 768, 858, 807, 229}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sqrt[4]{ax+bx^2}}{x^4}dx$$

$$\Big\downarrow \; 1137$$

$$\frac{\sqrt[4]{ax+bx^2}\int \frac{\sqrt[4]{a+bx}}{x^{15/4}}dx}{\sqrt[4]{x}\sqrt[4]{a+bx}}$$

$$\Big\downarrow \; 57$$

$$\frac{\sqrt[4]{ax+bx^2}\left(\frac{1}{11}b\int \frac{1}{x^{11/4}(a+bx)^{3/4}}dx-\frac{4\sqrt[4]{a+bx}}{11x^{11/4}}\right)}{\sqrt[4]{x}\sqrt[4]{a+bx}}$$

$$\Big\downarrow \; 61$$

$$\frac{\sqrt[4]{ax+bx^2}\left(\frac{1}{11}b\left(-\frac{6b\int \frac{1}{x^{7/4}(a+bx)^{3/4}}dx}{7a}-\frac{4\sqrt[4]{a+bx}}{7ax^{7/4}}\right)-\frac{4\sqrt[4]{a+bx}}{11x^{11/4}}\right)}{\sqrt[4]{x}\sqrt[4]{a+bx}}$$

$$\Big\downarrow \; 61$$

$$\frac{\sqrt[4]{ax+bx^2}\left(\frac{1}{11}b\left(-\frac{6b\left(-\frac{2b\int \frac{1}{x^{3/4}(a+bx)^{3/4}}dx}{3a}-\frac{4\sqrt[4]{a+bx}}{3ax^{3/4}}\right)}{7a}-\frac{4\sqrt[4]{a+bx}}{7ax^{7/4}}\right)-\frac{4\sqrt[4]{a+bx}}{11x^{11/4}}\right)}{\sqrt[4]{x}\sqrt[4]{a+bx}}$$

$$\Big\downarrow \text{73}$$

$$\frac{\sqrt[4]{ax+bx^2}\left(\frac{1}{11}b\left(-\frac{6b\left(-\frac{8b\int\frac{1}{(a+bx)^{3/4}}d\sqrt[4]{x}}{3a}-\frac{4\sqrt[4]{a+bx}}{3ax^{3/4}}\right)}{7a}-\frac{4\sqrt[4]{a+bx}}{7ax^{7/4}}\right)-\frac{4\sqrt[4]{a+bx}}{11x^{11/4}}\right)}{\sqrt[4]{x}\sqrt[4]{a+bx}}$$

$$\Big\downarrow \text{768}$$

$$\frac{\sqrt[4]{ax+bx^2}\left(\frac{1}{11}b\left(-\frac{6b\left(-\frac{8bx^{3/4}\left(\frac{a}{bx}+1\right)^{3/4}\int\frac{1}{\left(\frac{a}{bx}+1\right)^{3/4}x^{3/4}}d\sqrt[4]{x}}{3a(a+bx)^{3/4}}-\frac{4\sqrt[4]{a+bx}}{3ax^{3/4}}\right)}{7a}-\frac{4\sqrt[4]{a+bx}}{7ax^{7/4}}\right)-\frac{4\sqrt[4]{a+bx}}{11x^{11/4}}\right)}{\sqrt[4]{x}\sqrt[4]{a+bx}}$$

$$\Big\downarrow \text{858}$$

$$\frac{\sqrt[4]{ax+bx^2}\left(\frac{1}{11}b\left(-\frac{6b\left(\frac{8bx^{3/4}\left(\frac{a}{bx}+1\right)^{3/4}\int\frac{1}{\sqrt[4]{x}\left(\frac{ax}{b}+1\right)^{3/4}}d\frac{1}{\sqrt[4]{x}}}{3a(a+bx)^{3/4}}-\frac{4\sqrt[4]{a+bx}}{3ax^{3/4}}\right)}{7a}-\frac{4\sqrt[4]{a+bx}}{7ax^{7/4}}\right)-\frac{4\sqrt[4]{a+bx}}{11x^{11/4}}\right)}{\sqrt[4]{x}\sqrt[4]{a+bx}}$$

$$\Big\downarrow \text{807}$$

$$\frac{\sqrt[4]{ax+bx^2}\left(\frac{1}{11}b\left(-\frac{6b\left(\frac{4bx^{3/4}\left(\frac{a}{bx}+1\right)^{3/4}\int\frac{1}{\left(\frac{\sqrt{x}a}{b}+1\right)^{3/4}}d\sqrt{x}}{3a(a+bx)^{3/4}}-\frac{4\sqrt[4]{a+bx}}{3ax^{3/4}}\right)}{7a}-\frac{4\sqrt[4]{a+bx}}{7ax^{7/4}}\right)-\frac{4\sqrt[4]{a+bx}}{11x^{11/4}}\right)}{\sqrt[4]{x}\sqrt[4]{a+bx}}$$

$$\Big\downarrow \text{229}$$

$$\frac{\sqrt[4]{ax+bx^2}\left(\frac{1}{11}b\left(-\frac{6b\left(\frac{8b^{3/2}x^{3/4}\left(\frac{a}{bx}+1\right)^{3/4}\text{EllipticF}\left(\frac{1}{2}\arctan\left(\frac{\sqrt{a}\sqrt{x}}{\sqrt{b}}\right),2\right)}{3a^{3/2}(a+bx)^{3/4}}-\frac{4\sqrt[4]{a+bx}}{3ax^{3/4}}\right)}{7a}-\frac{4\sqrt[4]{a+bx}}{7ax^{7/4}}\right)-\frac{4\sqrt[4]{a+bx}}{11x^{11/4}}\right)}{\sqrt[4]{x}\sqrt[4]{a+bx}}$$

```
Int[(a*x + b*x^2)^(1/4)/x^4,x]
```

```
((a*x + b*x^2)^(1/4)*((-4*(a + b*x)^(1/4))/(11*x^(11/4)) + (b*((-4*(a + b*
x)^(1/4))/(7*a*x^(7/4)) - (6*b*((-4*(a + b*x)^(1/4))/(3*a*x^(3/4)) + (8*b^
(3/2)*(1 + a/(b*x))^(3/4)*x^(3/4)*EllipticF[ArcTan[(Sqrt[a]*Sqrt[x])/Sqrt[
b]]/2, 2])/(3*a^(3/2)*(a + b*x)^(3/4))))/(7*a)))/11))/(x^(1/4)*(a + b*x)^(
1/4))
```

Defintions of rubi rules used

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^n/(b*(m + 1))), x] - Simp[d*(n/(b*(m + 1)))
Int[(a + b*x)^(m + 1)*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b, c, d}, x] &
& GtQ[n, 0] && LtQ[m, -1] &&  !(IntegerQ[n] &&  !IntegerQ[m]) &&  !(ILeQ[m
+ n + 2, 0] && (FractionQ[m] || GeQ[2*n + m + 1, 0])) && IntLinearQ[a, b, c
, d, m, n, x]
```

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] - Simp[d*((
m + n + 2)/((b*c - a*d)*(m + 1)))   Int[(a + b*x)^(m + 1)*(c + d*x)^n, x],
x] /; FreeQ[{a, b, c, d, n}, x] && LtQ[m, -1] &&  !(LtQ[n, -1] && (EqQ[a, 0
] || (NeQ[c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && IntLinearQ[a, b, c, d
, m, n, x]
```

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Simp[p/b   Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
 d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && Lt
Q[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntL
inearQ[a, b, c, d, m, n, x]
```

```
Int[((a_) + (b_.)*(x_)^2)^(-3/4), x_Symbol] :> Simp[(2/(a^(3/4)*Rt[b/a, 2])
)*EllipticF[(1/2)*ArcTan[Rt[b/a, 2]*x], 2], x] /; FreeQ[{a, b}, x] && GtQ[a
, 0] && PosQ[b/a]
```

```
Int[((a_) + (b_.)*(x_)^4)^(-3/4), x_Symbol] :> Simp[x^3*((1 + a/(b*x^4))^(3
/4)/(a + b*x^4)^(3/4))   Int[1/(x^3*(1 + a/(b*x^4))^(3/4)), x], x] /; FreeQ
[{a, b}, x]
```

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k = GCD[m
+ 1, n]}, Simp[1/k   Subst[Int[x^((m + 1)/k - 1)*(a + b*x^(n/k))^p, x], x,
x^k], x] /; k != 1] /; FreeQ[{a, b, p}, x] && IGtQ[n, 0] && IntegerQ[m]
```

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> -Subst[Int[(a +
b/x^n)^p/x^(m + 2), x], x, 1/x] /; FreeQ[{a, b, p}, x] && ILtQ[n, 0] && Int
egerQ[m]
```

```
Int[((e_.)*(x_))^(m_)*((b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(
e*x)^m*((b*x + c*x^2)^p/(x^(m + p)*(b + c*x)^p))   Int[x^(m + p)*(b + c*x)^
p, x], x] /; FreeQ[{b, c, e, m}, x]
```

Maple [F]

$$\int \frac{(b\,x^2 + ax)^{\frac{1}{4}}}{x^4}dx$$

```
int((b*x^2+a*x)^(1/4)/x^4,x)
```

```
int((b*x^2+a*x)^(1/4)/x^4,x)
```

Fricas [F]

$$\int \frac{\sqrt[4]{ax+bx^2}}{x^4}\,dx=\int \frac{(bx^2+ax)^{\frac{1}{4}}}{x^4}\,dx$$

```
integrate((b*x^2+a*x)^(1/4)/x^4,x, algorithm="fricas")
```

```
integral((b*x^2 + a*x)^(1/4)/x^4, x)
```

Sympy [F]

$$\int \frac{\sqrt[4]{ax+bx^2}}{x^4}\,dx=\int \frac{\sqrt[4]{x\,(a+bx)}}{x^4}\,dx$$

```
integrate((b*x**2+a*x)**(1/4)/x**4,x)
```

```
Integral((x*(a + b*x))**(1/4)/x**4, x)
```

Maxima [F]

$$\int \frac{\sqrt[4]{ax+bx^2}}{x^4}\,dx=\int \frac{(bx^2+ax)^{\frac{1}{4}}}{x^4}\,dx$$

```
integrate((b*x^2+a*x)^(1/4)/x^4,x, algorithm="maxima")
```

```
integrate((b*x^2 + a*x)^(1/4)/x^4, x)
```

Giac [F]

$$\int \frac{\sqrt[4]{ax+bx^2}}{x^4}\,dx = \int \frac{(bx^2+ax)^{\frac{1}{4}}}{x^4}\,dx$$

```
integrate((b*x^2+a*x)^(1/4)/x^4,x, algorithm="giac")
```

```
integrate((b*x^2 + a*x)^(1/4)/x^4, x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\sqrt[4]{ax+bx^2}}{x^4}\,dx = \int \frac{(b\,x^2+a\,x)^{1/4}}{x^4}\,dx$$

```
int((a*x + b*x^2)^(1/4)/x^4,x)
```

```
int((a*x + b*x^2)^(1/4)/x^4, x)
```

Reduce [F]

$$\int \frac{\sqrt[4]{ax+bx^2}}{x^4}\,dx = \frac{-4(bx+a)^{\frac{1}{4}} - x^{\frac{11}{4}}\left(\int \frac{(bx+a)^{\frac{1}{4}}}{x^{\frac{15}{4}}a+x^{\frac{19}{4}}b}dx\right)a}{10x^{\frac{11}{4}}}$$

```
int((b*x^2+a*x)^(1/4)/x^4,x)
```

```
( - 4*(a + b*x)**(1/4) - x**(3/4)*int((a + b*x)**(1/4)/(x**(3/4)*a*x**3 +
x**(3/4)*b*x**4),x)*a*x**2)/(10*x**(3/4)*x**2)
```

3.153 $\int \frac{\sqrt[4]{ax+bx^2}}{x^5}\,dx$

Optimal result

Integrand size = 17, antiderivative size = 170

$$\int \frac{\sqrt[4]{ax+bx^2}}{x^5}\,dx = -\frac{4\sqrt[4]{ax+bx^2}}{15x^4}-\frac{4b\sqrt[4]{ax+bx^2}}{165ax^3}+\frac{8b^2\sqrt[4]{ax+bx^2}}{231a^2x^2}-\frac{16b^3\sqrt[4]{ax+bx^2}}{231a^3x}+\frac{32b^3\left(\frac{bx}{a+bx}\right)^{3/4}\sqrt{a+bx}\sqrt[4]{ax+bx^2}\,\mathrm{EllipticF}\left(\frac{1}{2}\arcsin\left(\frac{\sqrt{a}}{\sqrt{a+bx}}\right),2\right)}{231a^{7/2}x}$$

```
-4/15*(b*x^2+a*x)^(1/4)/x^4-4/165*b*(b*x^2+a*x)^(1/4)/a/x^3+8/231*b^2*(b*x
^2+a*x)^(1/4)/a^2/x^2-16/231*b^3*(b*x^2+a*x)^(1/4)/a^3/x+32/231*b^3*(b*x/(
b*x+a))^(3/4)*(b*x+a)^(1/2)*(b*x^2+a*x)^(1/4)*InverseJacobiAM(1/2*arcsin(a
^(1/2)/(b*x+a)^(1/2)),2^(1/2))/a^(7/2)/x
```

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 10.01 (sec) , antiderivative size = 47, normalized size of antiderivative = 0.28

$$\int \frac{\sqrt[4]{ax+bx^2}}{x^5}\,dx = -\frac{4\sqrt[4]{x(a+bx)}\,\mathrm{Hypergeometric2F1}\left(-\frac{15}{4},-\frac{1}{4},-\frac{11}{4},-\frac{bx}{a}\right)}{15x^4\sqrt[4]{1+\frac{bx}{a}}}$$

```
Integrate[(a*x + b*x^2)^(1/4)/x^5,x]
```

```
(-4*(x*(a + b*x))^(1/4)*Hypergeometric2F1[-15/4, -1/4, -11/4, -((b*x)/a)])
/(15*x^4*(1 + (b*x)/a)^(1/4))
```

Rubi [A] (warning: unable to verify)

Time = 0.52 (sec) , antiderivative size = 199, normalized size of antiderivative = 1.17, number of steps used = 11, number of rules used = 10, $\frac{\text{number of rules}}{\text{integrand size}} = 0.588$, Rules used = {1137, 57, 61, 61, 61, 73, 768, 858, 807, 229}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sqrt[4]{ax+bx^2}}{x^5}\,dx$$

$$\Big\downarrow \text{1137}$$

$$\frac{\sqrt[4]{ax+bx^2}\int \frac{\sqrt[4]{a+bx}}{x^{19/4}}dx}{\sqrt[4]{x}\sqrt[4]{a+bx}}$$

$$\Big\downarrow \text{57}$$

$$\frac{\sqrt[4]{ax+bx^2}\left(\frac{1}{15}b\int \frac{1}{x^{15/4}(a+bx)^{3/4}}dx-\frac{4\sqrt[4]{a+bx}}{15x^{15/4}}\right)}{\sqrt[4]{x}\sqrt[4]{a+bx}}$$

$$\Big\downarrow \text{61}$$

$$\frac{\sqrt[4]{ax+bx^2}\left(\frac{1}{15}b\left(-\frac{10b\int \frac{1}{x^{11/4}(a+bx)^{3/4}}dx}{11a}-\frac{4\sqrt[4]{a+bx}}{11ax^{11/4}}\right)-\frac{4\sqrt[4]{a+bx}}{15x^{15/4}}\right)}{\sqrt[4]{x}\sqrt[4]{a+bx}}$$

$$\Big\downarrow \text{61}$$

$$\frac{\sqrt[4]{ax+bx^2}\left(\frac{1}{15}b\left(-\frac{10b\left(-\frac{6b\int \frac{1}{x^{7/4}(a+bx)^{3/4}}dx}{7a}-\frac{4\sqrt[4]{a+bx}}{7ax^{7/4}}\right)}{11a}-\frac{4\sqrt[4]{a+bx}}{11ax^{11/4}}\right)-\frac{4\sqrt[4]{a+bx}}{15x^{15/4}}\right)}{\sqrt[4]{x}\sqrt[4]{a+bx}}$$

$$\Big\downarrow \text{61}$$

$$\frac{\sqrt[4]{ax+bx^2}\left(\frac{1}{15}b\left(-\frac{10b\left(-\frac{6b\left(-\frac{2b\int \frac{1}{x^{3/4}(a+bx)^{3/4}}dx}{3a}-\frac{4\sqrt[4]{a+bx}}{3ax^{3/4}}\right)}{7a}-\frac{4\sqrt[4]{a+bx}}{7ax^{7/4}}\right)}{11a}-\frac{4\sqrt[4]{a+bx}}{11ax^{11/4}}\right)-\frac{4\sqrt[4]{a+bx}}{15x^{15/4}}\right)}{\sqrt[4]{x}\sqrt[4]{a+bx}}$$

$$\Big\downarrow \text{73}$$

$$\frac{\sqrt[4]{ax+bx^2}\left(\frac{1}{15}b\left(-\frac{10b\left(-\frac{6b\left(-\frac{8b\int \frac{1}{(a+bx)^{3/4}}d\sqrt[4]{x}}{3a}-\frac{4\sqrt[4]{a+bx}}{3ax^{3/4}}\right)}{7a}-\frac{4\sqrt[4]{a+bx}}{7ax^{7/4}}\right)}{11a}-\frac{4\sqrt[4]{a+bx}}{11ax^{11/4}}\right)-\frac{4\sqrt[4]{a+bx}}{15x^{15/4}}\right)}{\sqrt[4]{x}\sqrt[4]{a+bx}}$$

$$\Big\downarrow \text{768}$$

$$\frac{\sqrt[4]{ax+bx^2}\left(\frac{1}{15}b\left(-\frac{10b\left(-\frac{6b\left(-\frac{8bx^{3/4}\left(\frac{a}{bx}+1\right)^{3/4}\int \frac{1}{\left(\frac{a}{bx}+1\right)^{3/4}x^{3/4}}d\sqrt[4]{x}}{3a(a+bx)^{3/4}}-\frac{4\sqrt[4]{a+bx}}{3ax^{3/4}}\right)}{7a}-\frac{4\sqrt[4]{a+bx}}{7ax^{7/4}}\right)}{11a}-\frac{4\sqrt[4]{a+bx}}{11ax^{11/4}}\right)-\frac{4\sqrt[4]{a}}{15x}\right)}{\sqrt[4]{x}\sqrt[4]{a+bx}}$$

$$\Big\downarrow \text{858}$$

$$\frac{\sqrt[4]{ax+bx^2}\left(\frac{1}{15}b\left(-\frac{10b\left(-\frac{6b\left(\frac{8bx^{3/4}\left(\frac{a}{bx}+1\right)^{3/4}\int \frac{1}{\sqrt[4]{x}\left(\frac{ax}{b}+1\right)^{3/4}}d\frac{1}{\sqrt[4]{x}}}{3a(a+bx)^{3/4}}-\frac{4\sqrt[4]{a+bx}}{3ax^{3/4}}\right)}{7a}-\frac{4\sqrt[4]{a+bx}}{7ax^{7/4}}\right)}{11a}-\frac{4\sqrt[4]{a+bx}}{11ax^{11/4}}\right)-\frac{4\sqrt[4]{a}}{15x}\right.}{\sqrt[4]{x}\sqrt[4]{a+bx}}$$

$\downarrow$ 807

$$\frac{\sqrt[4]{ax+bx^2}\left(\frac{1}{15}b\left(-\frac{10b\left(-\frac{6b\left(\frac{4bx^{3/4}\left(\frac{a}{bx}+1\right)^{3/4}\int \frac{1}{\left(\frac{\sqrt{x}a}{b}+1\right)^{3/4}}d\sqrt{x}}{3a(a+bx)^{3/4}}-\frac{4\sqrt[4]{a+bx}}{3ax^{3/4}}\right)}{7a}-\frac{4\sqrt[4]{a+bx}}{7ax^{7/4}}\right)}{11a}-\frac{4\sqrt[4]{a+bx}}{11ax^{11/4}}\right)-\frac{4\sqrt[4]{a+b}}{15x^{15/4}}\right.}{\sqrt[4]{x}\sqrt[4]{a+bx}}$$

$\downarrow$ 229

$$\frac{\sqrt[4]{ax+bx^2}\left(\frac{1}{15}b\left(-\frac{10b\left(-\frac{6b\left(\frac{8b^{3/2}x^{3/4}\left(\frac{a}{bx}+1\right)^{3/4}\text{EllipticF}\left(\frac{1}{2}\arctan\left(\frac{\sqrt{a}\sqrt{x}}{\sqrt{b}}\right),2\right)}{3a^{3/2}(a+bx)^{3/4}}-\frac{4\sqrt[4]{a+bx}}{3ax^{3/4}}\right)}{7a}-\frac{4\sqrt[4]{a+bx}}{7ax^{7/4}}\right)}{11a}-\frac{4\sqrt[4]{a+bx}}{11ax^{11/4}}\right)-\right.}{\sqrt[4]{x}\sqrt[4]{a+bx}}$$

```
Int[(a*x + b*x^2)^(1/4)/x^5,x]
```

```
((a*x + b*x^2)^(1/4)*((-4*(a + b*x)^(1/4))/(15*x^(15/4)) + (b*((-4*(a + b*
x)^(1/4))/(11*a*x^(11/4)) - (10*b*((-4*(a + b*x)^(1/4))/(7*a*x^(7/4)) - (6
*b*((-4*(a + b*x)^(1/4))/(3*a*x^(3/4)) + (8*b^(3/2)*(1 + a/(b*x))^(3/4)*x^
(3/4)*EllipticF[ArcTan[(Sqrt[a]*Sqrt[x])/Sqrt[b]]/2, 2])/(3*a^(3/2)*(a + b
*x)^(3/4))))/(7*a)))/(11*a)))/15))/(x^(1/4)*(a + b*x)^(1/4))
```

Defintions of rubi rules used

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^n/(b*(m + 1))), x] - Simp[d*(n/(b*(m + 1)))
Int[(a + b*x)^(m + 1)*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b, c, d}, x] &
& GtQ[n, 0] && LtQ[m, -1] &&  !(IntegerQ[n] &&  !IntegerQ[m]) &&  !(ILeQ[m
+ n + 2, 0] && (FractionQ[m] || GeQ[2*n + m + 1, 0])) && IntLinearQ[a, b, c
, d, m, n, x]
```

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] - Simp[d*((
m + n + 2)/((b*c - a*d)*(m + 1)))   Int[(a + b*x)^(m + 1)*(c + d*x)^n, x],
x] /; FreeQ[{a, b, c, d, n}, x] && LtQ[m, -1] &&  !(LtQ[n, -1] && (EqQ[a, 0
] || (NeQ[c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && IntLinearQ[a, b, c, d
, m, n, x]
```

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Simp[p/b   Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
 d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && Lt
Q[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntL
inearQ[a, b, c, d, m, n, x]
```

```
Int[((a_) + (b_.)*(x_)^2)^(-3/4), x_Symbol] :> Simp[(2/(a^(3/4)*Rt[b/a, 2])
)*EllipticF[(1/2)*ArcTan[Rt[b/a, 2]*x], 2], x] /; FreeQ[{a, b}, x] && GtQ[a
, 0] && PosQ[b/a]
```

```
Int[((a_) + (b_.)*(x_)^4)^(-3/4), x_Symbol] :> Simp[x^3*((1 + a/(b*x^4))^(3
/4)/(a + b*x^4)^(3/4))   Int[1/(x^3*(1 + a/(b*x^4))^(3/4)), x], x] /; FreeQ
[{a, b}, x]
```

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k = GCD[m
+ 1, n]}, Simp[1/k   Subst[Int[x^((m + 1)/k - 1)*(a + b*x^(n/k))^p, x], x,
x^k], x] /; k != 1] /; FreeQ[{a, b, p}, x] && IGtQ[n, 0] && IntegerQ[m]
```

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> -Subst[Int[(a +
b/x^n)^p/x^(m + 2), x], x, 1/x] /; FreeQ[{a, b, p}, x] && ILtQ[n, 0] && Int
egerQ[m]
```

```
Int[((e_.)*(x_))^(m_)*((b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(
e*x)^m*((b*x + c*x^2)^p/(x^(m + p)*(b + c*x)^p))   Int[x^(m + p)*(b + c*x)^
p, x], x] /; FreeQ[{b, c, e, m}, x]
```

Maple [F]

$$\int \frac{(b\,x^{2}+ax)^{\frac{1}{4}}}{x^{5}}dx$$

```
int((b*x^2+a*x)^(1/4)/x^5,x)
```

```
int((b*x^2+a*x)^(1/4)/x^5,x)
```

Fricas [F]

$$\int \frac{\sqrt[4]{ax+bx^2}}{x^5}\,dx = \int \frac{(bx^2+ax)^{\frac{1}{4}}}{x^5}\,dx$$

```
integrate((b*x^2+a*x)^(1/4)/x^5,x, algorithm="fricas")
```

```
integral((b*x^2 + a*x)^(1/4)/x^5, x)
```

Sympy [F]

$$\int \frac{\sqrt[4]{ax+bx^2}}{x^5}\,dx = \int \frac{\sqrt[4]{x\,(a+bx)}}{x^5}\,dx$$

```
integrate((b*x**2+a*x)**(1/4)/x**5,x)
```

```
Integral((x*(a + b*x))**(1/4)/x**5, x)
```

Maxima [F]

$$\int \frac{\sqrt[4]{ax+bx^2}}{x^5}\,dx = \int \frac{(bx^2+ax)^{\frac{1}{4}}}{x^5}\,dx$$

```
integrate((b*x^2+a*x)^(1/4)/x^5,x, algorithm="maxima")
```

```
integrate((b*x^2 + a*x)^(1/4)/x^5, x)
```

Giac [F]

$$\int \frac{\sqrt[4]{ax+bx^2}}{x^5}\,dx = \int \frac{(bx^2+ax)^{\frac{1}{4}}}{x^5}\,dx$$

```
integrate((b*x^2+a*x)^(1/4)/x^5,x, algorithm="giac")
```

```
integrate((b*x^2 + a*x)^(1/4)/x^5, x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\sqrt[4]{ax+bx^2}}{x^5}\,dx = \int \frac{{\left(b\,x^2+a\,x\right)}^{1/4}}{x^5}\,dx$$

```
int((a*x + b*x^2)^(1/4)/x^5,x)
```

```
int((a*x + b*x^2)^(1/4)/x^5, x)
```

Reduce [F]

$$\int \frac{\sqrt[4]{ax+bx^2}}{x^5}\,dx = \frac{-4(bx+a)^{\frac{1}{4}} - x^{\frac{15}{4}}\left(\int \frac{(bx+a)^{\frac{1}{4}}}{x^{\frac{19}{4}}a+x^{\frac{23}{4}}b}dx\right)a}{14x^{\frac{15}{4}}}$$

```
int((b*x^2+a*x)^(1/4)/x^5,x)
```

```
( - 4*(a + b*x)**(1/4) - x**(3/4)*int((a + b*x)**(1/4)/(x**(3/4)*a*x**4 +
x**(3/4)*b*x**5),x)*a*x**3)/(14*x**(3/4)*x**3)
```

3.154 $\int x^3(ax+bx^2)^{3/4}\,dx$

Optimal result

Integrand size = 17, antiderivative size = 185

$$\int x^3\left(ax+bx^2\right)^{3/4}\,dx=-\frac{a^4(ax+bx^2)^{3/4}}{24b^4}+\frac{a^3x(ax+bx^2)^{3/4}}{28b^3}-\frac{5a^2x^2(ax+bx^2)^{3/4}}{154b^2}+\frac{ax^3(ax+bx^2)^{3/4}}{33b}+\frac{2}{11}x^4\left(ax+bx^2\right)^{3/4}+\frac{a^6\sqrt[4]{-\frac{bx}{a}-\frac{b^2x^2}{a^2}}E\left(\frac{1}{2}\arcsin\left(1+\frac{2bx}{a}\right)\middle|2\right)}{16\sqrt{2}b^5\sqrt[4]{ax+bx^2}}$$

```
-1/24*a^4*(b*x^2+a*x)^(3/4)/b^4+1/28*a^3*x*(b*x^2+a*x)^(3/4)/b^3-5/154*a^2
*x^2*(b*x^2+a*x)^(3/4)/b^2+1/33*a*x^3*(b*x^2+a*x)^(3/4)/b+2/11*x^4*(b*x^2+
a*x)^(3/4)+1/32*a^6*(-b*x/a-b^2*x^2/a^2)^(1/4)*EllipticE(sin(1/2*arcsin(1+
2*b*x/a)),2^(1/2))*2^(1/2)/b^5/(b*x^2+a*x)^(1/4)
```

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 10.01 (sec) , antiderivative size = 47, normalized size of antiderivative = 0.25

$$\int x^3\left(ax+bx^2\right)^{3/4}\,dx=\frac{4x^4(x(a+bx))^{3/4}\,\text{Hypergeometric2F1}\left(-\frac{3}{4},\frac{19}{4},\frac{23}{4},-\frac{bx}{a}\right)}{19\left(1+\frac{bx}{a}\right)^{3/4}}$$

```
Integrate[x^3*(a*x + b*x^2)^(3/4),x]
```

```
(4*x^4*(x*(a + b*x))^(3/4)*Hypergeometric2F1[-3/4, 19/4, 23/4, -((b*x)/a)]
)/(19*(1 + (b*x)/a)^(3/4))
```

Rubi [A] (warning: unable to verify)

Time = 0.63 (sec) , antiderivative size = 254, normalized size of antiderivative = 1.37, number of steps used = 13, number of rules used = 12, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.706, Rules used = {1137, 60, 60, 60, 60, 60, 73, 839, 813, 858, 807, 212}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^3 \left(ax+bx^2\right)^{3/4} \, dx$$

$$\Big\downarrow \text{1137}$$

$$\frac{\left(ax+bx^2\right)^{3/4} \int x^{15/4}(a+bx)^{3/4}dx}{x^{3/4}(a+bx)^{3/4}}$$

$$\Big\downarrow \text{60}$$

$$\frac{\left(ax+bx^2\right)^{3/4} \left(\frac{3}{22}a\int \frac{x^{15/4}}{\sqrt[4]{a+bx}}dx+\frac{2}{11}x^{19/4}(a+bx)^{3/4}\right)}{x^{3/4}(a+bx)^{3/4}}$$

$$\Big\downarrow \text{60}$$

$$\frac{\left(ax+bx^2\right)^{3/4} \left(\frac{3}{22}a\left(\frac{2x^{15/4}(a+bx)^{3/4}}{9b}-\frac{5a\int \frac{x^{11/4}}{\sqrt[4]{a+bx}}dx}{6b}\right)+\frac{2}{11}x^{19/4}(a+bx)^{3/4}\right)}{x^{3/4}(a+bx)^{3/4}}$$

$$\Big\downarrow \text{60}$$

$$\frac{\left(ax+bx^2\right)^{3/4}\left(\frac{3}{22}a\left(\frac{2x^{15/4}(a+bx)^{3/4}}{9b}-\frac{5a\left(\frac{2x^{11/4}(a+bx)^{3/4}}{7b}-\frac{11a\int\frac{x^{7/4}}{\sqrt[4]{a+bx}}dx}{14b}\right)}{6b}\right)+\frac{2}{11}x^{19/4}(a+bx)^{3/4}\right)}{x^{3/4}(a+bx)^{3/4}}$$

$$\Big\downarrow \text{60}$$

$$\frac{\left(ax+bx^2\right)^{3/4}\left(\frac{3}{22}a\left(\frac{2x^{15/4}(a+bx)^{3/4}}{9b}-\frac{5a\left(\frac{2x^{11/4}(a+bx)^{3/4}}{7b}-\frac{11a\left(\frac{2x^{7/4}(a+bx)^{3/4}}{5b}-\frac{7a\int\frac{x^{3/4}}{\sqrt[4]{a+bx}}dx}{10b}\right)}{14b}\right)}{6b}\right)+\frac{2}{11}x^{19/4}(a+bx\right)}{x^{3/4}(a+bx)^{3/4}}$$

$$\Big\downarrow \text{60}$$

$$\frac{\left(ax+bx^2\right)^{3/4}\left(\frac{3}{22}a\left(\frac{2x^{15/4}(a+bx)^{3/4}}{9b}-\frac{5a\left(\frac{2x^{11/4}(a+bx)^{3/4}}{7b}-\frac{11a\left(\frac{2x^{7/4}(a+bx)^{3/4}}{5b}-\frac{7a\left(\frac{2x^{3/4}(a+bx)^{3/4}}{3b}-\frac{a\int\frac{1}{\sqrt[4]{x}\sqrt[4]{a+bx}}dx}{2b}\right)}{10b}\right)}{14b}\right)}{6b}\right.\right.}{x^{3/4}(a+bx)^{3/4}}$$

$\downarrow$ 73

$$\frac{\left(ax+bx^2\right)^{3/4}\left(\frac{3}{22}a\left(\frac{2x^{15/4}(a+bx)^{3/4}}{9b}-\frac{5a\left(\frac{2x^{11/4}(a+bx)^{3/4}}{7b}-\frac{11a\left(\frac{2x^{7/4}(a+bx)^{3/4}}{5b}-\frac{7a\left(\frac{2x^{3/4}(a+bx)^{3/4}}{3b}-\frac{2a\int\frac{\sqrt{x}}{\sqrt[4]{a+bx}}d\sqrt[4]{x}}{b}\right)}{10b}\right)}{14b}\right.}{6b}\right.\right.}{x^{3/4}(a+bx)^{3/4}}$$

$\downarrow$ 839

$$\frac{\left(ax+bx^2\right)^{3/4}\left(\frac{3}{22}a\left(\frac{2x^{15/4}(a+bx)^{3/4}}{9b}-\frac{5a\left(\frac{2x^{11/4}(a+bx)^{3/4}}{7b}-\frac{11a\left(\frac{2x^{7/4}(a+bx)^{3/4}}{5b}-\frac{7a\left(\frac{2x^{3/4}(a+bx)^{3/4}}{3b}-\frac{2a\left(\frac{x^{3/4}}{2\sqrt[4]{a+bx}}-\frac{1}{2}a\int\frac{}{(a+\cdots}\right.}{b}\right.}{10b}\right.}{14b}\right.}{6b}\right.\right.}{x^{3/4}(a+bx)^{3/4}}$$

$\downarrow$ 813

$$2a\left(\frac{x^{3/4}}{2\sqrt[4]{a+bx}}-\frac{a\sqrt[4]{x}\,\sqrt[4]{\ }}{}\right.$$

$$7a\left(\frac{2x^{3/4}(a+bx)^{3/4}}{3b}-\right.$$

$$11a\left(\frac{2x^{7/4}(a+bx)^{3/4}}{5b}-\frac{}{10b}\right.$$

$$5a\left(\frac{2x^{11/4}(a+bx)^{3/4}}{7b}-\frac{}{14b}\right.$$

$$\left(ax+bx^2\right)^{3/4}\left(\frac{3}{22}a\left(\frac{2x^{15/4}(a+bx)^{3/4}}{9b}-\frac{}{6b}\right.\right.$$

↓ 858

$$\left(ax+bx^2\right)^{3/4}\left(\frac{3}{22}a\left(\frac{2x^{15/4}(a+bx)^{3/4}}{9b}-\frac{5a\left(\frac{2x^{11/4}(a+bx)^{3/4}}{7b}-\frac{11a\left(\frac{2x^{7/4}(a+bx)^{3/4}}{5b}-\frac{7a\left(\frac{2x^{3/4}(a+bx)^{3/4}}{3b}-\frac{2a\left(\frac{a\sqrt[4]{x}\sqrt[4]{\frac{a}{bx}+1}\int \frac{}{\sqrt[4]{}}}{2b\sqrt[4]{a}}\right.}{}\right.}{10b}\right.}{14b}\right.}{6b}\right.\right.$$

$\downarrow$ 807

$$(ax+bx^2)^{3/4}\left(\frac{3}{22}a\left(\frac{2x^{15/4}(a+bx)^{3/4}}{9b}-\frac{5a\left(\frac{2x^{11/4}(a+bx)^{3/4}}{7b}-\frac{11a\left(\frac{2x^{7/4}(a+bx)^{3/4}}{5b}-\frac{7a\left(\frac{2x^{3/4}(a+bx)^{3/4}}{3b}-\frac{2a\left(\frac{a\sqrt[4]{x}\sqrt[4]{\frac{a}{bx}+1}\int}{4b\sqrt[4]{a+}}\right.}{}\right.}{10b}\right.}{14b}\right.}{6b}\right.\right.$$

$\downarrow$ 212

$$2a\left(\frac{\sqrt{a}\,\sqrt[4]{x}\,\sqrt[4]{\frac{a}{bx}+1}\,E\Big(}{2\sqrt{b}\,\sqrt[4]{a}}\right.$$

$$7a\left(\frac{2x^{3/4}(a+bx)^{3/4}}{3b}-\frac{\quad}{10b}\right.$$

$$11a\left(\frac{2x^{7/4}(a+bx)^{3/4}}{5b}-\frac{\quad}{14b}\right.$$

$$5a\left(\frac{2x^{11/4}(a+bx)^{3/4}}{7b}-\frac{\quad}{6b}\right.$$

$$\left(ax+bx^2\right)^{3/4}\left(\frac{3}{22}a\left(\frac{2x^{15/4}(a+bx)^{3/4}}{9b}-\frac{\quad}{6b}\right.\right.$$

```
Int[x^3*(a*x + b*x^2)^(3/4),x]
```

```
((a*x + b*x^2)^(3/4)*((2*x^(19/4)*(a + b*x)^(3/4))/11 + (3*a*((2*x^(15/4)*
(a + b*x)^(3/4))/(9*b) - (5*a*((2*x^(11/4)*(a + b*x)^(3/4))/(7*b) - (11*a*
((2*x^(7/4)*(a + b*x)^(3/4))/(5*b) - (7*a*((2*x^(3/4)*(a + b*x)^(3/4))/(3*
b) - (2*a*(x^(3/4)/(2*(a + b*x)^(1/4)) + (Sqrt[a]*(1 + a/(b*x))^(1/4)*x^(1
/4)*EllipticE[ArcTan[(Sqrt[a]*Sqrt[x])/Sqrt[b]]/2, 2])/(2*Sqrt[b]*(a + b*x
)^(1/4))))/b))/(10*b)))/(14*b)))/(6*b)))/22))/(x^(3/4)*(a + b*x)^(3/4))
```

Defintions of rubi rules used

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^n/(b*(m + n + 1))), x] + Simp[n*((b*c - a*d)/(
b*(m + n + 1)))   Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
 c, d}, x] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ[m, 0] && ( !Integer
Q[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n + 2, 0] && IntLinear
Q[a, b, c, d, m, n, x]
```

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Simp[p/b   Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
 d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && Lt
Q[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntL
inearQ[a, b, c, d, m, n, x]
```

```
Int[((a_) + (b_.)*(x_)^2)^(-5/4), x_Symbol] :> Simp[(2/(a^(5/4)*Rt[b/a, 2])
)*EllipticE[(1/2)*ArcTan[Rt[b/a, 2]*x], 2], x] /; FreeQ[{a, b}, x] && GtQ[a
, 0] && PosQ[b/a]
```

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k = GCD[m
+ 1, n]}, Simp[1/k   Subst[Int[x^((m + 1)/k - 1)*(a + b*x^(n/k))^p, x], x,
x^k], x] /; k != 1] /; FreeQ[{a, b, p}, x] && IGtQ[n, 0] && IntegerQ[m]
```

```
Int[(x_)^2/((a_) + (b_.)*(x_)^4)^(5/4), x_Symbol] :> Simp[x*((1 + a/(b*x^4)
)^(1/4)/(b*(a + b*x^4)^(1/4)))   Int[1/(x^3*(1 + a/(b*x^4))^(5/4)), x], x]
/; FreeQ[{a, b}, x] && PosQ[b/a]
```

```
Int[(x_)^2/((a_) + (b_.)*(x_)^4)^(1/4), x_Symbol] :> Simp[x^3/(2*(a + b*x^4
)^(1/4)), x] - Simp[a/2   Int[x^2/(a + b*x^4)^(5/4), x], x] /; FreeQ[{a, b}
, x] && PosQ[b/a]
```

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> -Subst[Int[(a +
b/x^n)^p/x^(m + 2), x], x, 1/x] /; FreeQ[{a, b, p}, x] && ILtQ[n, 0] && Int
egerQ[m]
```

```
Int[((e_.)*(x_))^(m_)*((b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(
e*x)^m*((b*x + c*x^2)^p/(x^(m + p)*(b + c*x)^p))   Int[x^(m + p)*(b + c*x)^
p, x], x] /; FreeQ[{b, c, e, m}, x]
```

Maple [F]

$$\int x^{3} \left(b\,x^{2}+a x \right)^{\frac{3}{4}}d x$$

```
int(x^3*(b*x^2+a*x)^(3/4),x)
```

```
int(x^3*(b*x^2+a*x)^(3/4),x)
```

Fricas [F]

$$\int x^3 \left(ax+bx^2\right)^{3/4}\,dx=\int \left(bx^2+ax\right)^{\frac{3}{4}}x^3\,dx$$

```
integrate(x^3*(b*x^2+a*x)^(3/4),x, algorithm="fricas")
```

```
integral((b*x^2 + a*x)^(3/4)*x^3, x)
```

Sympy [F]

$$\int x^3 \left(ax + bx^2\right)^{3/4} \, dx = \int x^3 (x(a + bx))^{\frac{3}{4}} \, dx$$

```
integrate(x**3*(b*x**2+a*x)**(3/4),x)
```

```
Integral(x**3*(x*(a + b*x))**(3/4), x)
```

Maxima [F]

$$\int x^3 \left(ax + bx^2\right)^{3/4} \, dx = \int \left(bx^2 + ax\right)^{\frac{3}{4}} x^3 \, dx$$

```
integrate(x^3*(b*x^2+a*x)^(3/4),x, algorithm="maxima")
```

```
integrate((b*x^2 + a*x)^(3/4)*x^3, x)
```

Giac [F]

$$\int x^3 \left(ax + bx^2\right)^{3/4} \, dx = \int \left(bx^2 + ax\right)^{\frac{3}{4}} x^3 \, dx$$

```
integrate(x^3*(b*x^2+a*x)^(3/4),x, algorithm="giac")
```

```
integrate((b*x^2 + a*x)^(3/4)*x^3, x)
```

Mupad [F(-1)]

Timed out.

$$\int x^3 (ax + bx^2)^{3/4} \, dx = \int x^3 \left(b\,x^2 + a\,x\right)^{3/4} dx$$

```
int(x^3*(a*x + b*x^2)^(3/4),x)
```

```
int(x^3*(a*x + b*x^2)^(3/4), x)
```

Reduce [F]

$$\int x^3 (ax + bx^2)^{3/4} \, dx = \frac{-308x^{\frac{3}{4}}(bx+a)^{\frac{3}{4}} a^4 + 264x^{\frac{7}{4}}(bx+a)^{\frac{3}{4}} a^3 b - 240x^{\frac{11}{4}}(bx+a)^{\frac{3}{4}} a^2 b^2 + 224x^{\frac{15}{4}}(bx+a)^{\frac{3}{4}} a\,b^3 \cdots}{7392b^4}$$

```
int(x^3*(b*x^2+a*x)^(3/4),x)
```

```
( - 308*x**(3/4)*(a + b*x)**(3/4)*a**4 + 264*x**(3/4)*(a + b*x)**(3/4)*a**
3*b*x - 240*x**(3/4)*(a + b*x)**(3/4)*a**2*b**2*x**2 + 224*x**(3/4)*(a + b
*x)**(3/4)*a*b**3*x**3 + 1344*x**(3/4)*(a + b*x)**(3/4)*b**4*x**4 + 231*in
t((a + b*x)**(3/4)/(x**(1/4)*a + x**(1/4)*b*x),x)*a**5)/(7392*b**4)
```

3.155 $\int x^2(ax+bx^2)^{3/4}\,dx$

Optimal result

Integrand size = 17, antiderivative size = 159

$$\int x^2\left(ax+bx^2\right)^{3/4}\,dx=\frac{11a^3(ax+bx^2)^{3/4}}{180b^3}-\frac{11a^2x(ax+bx^2)^{3/4}}{210b^2}+\frac{ax^2(ax+bx^2)^{3/4}}{21b}+\frac{2}{9}x^3\left(ax+bx^2\right)^{3/4}-\frac{11a^5\sqrt[4]{-\frac{bx}{a}-\frac{b^2x^2}{a^2}}\,E\left(\frac{1}{2}\arcsin\left(1+\frac{2bx}{a}\right)\middle|2\right)}{120\sqrt{2}b^4\sqrt[4]{ax+bx^2}}$$

```
11/180*a^3*(b*x^2+a*x)^(3/4)/b^3-11/210*a^2*x*(b*x^2+a*x)^(3/4)/b^2+1/21*a
*x^2*(b*x^2+a*x)^(3/4)/b+2/9*x^3*(b*x^2+a*x)^(3/4)-11/240*a^5*(-b*x/a-b^2*
x^2/a^2)^(1/4)*EllipticE(sin(1/2*arcsin(1+2*b*x/a)),2^(1/2))*2^(1/2)/b^4/(
b*x^2+a*x)^(1/4)
```

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 10.01 (sec) , antiderivative size = 47, normalized size of antiderivative = 0.30

$$\int x^2\left(ax+bx^2\right)^{3/4}\,dx=\frac{4x^3(x(a+bx))^{3/4}\,\text{Hypergeometric2F1}\left(-\frac{3}{4},\frac{15}{4},\frac{19}{4},-\frac{bx}{a}\right)}{15\left(1+\frac{bx}{a}\right)^{3/4}}$$

```
Integrate[x^2*(a*x + b*x^2)^(3/4),x]
```

```
(4*x^3*(x*(a + b*x))^(3/4)*Hypergeometric2F1[-3/4, 15/4, 19/4, -((b*x)/a)]
)/(15*(1 + (b*x)/a)^(3/4))
```

Rubi [A] (warning: unable to verify)

Time = 0.60 (sec) , antiderivative size = 224, normalized size of antiderivative = 1.41, number of steps used = 12, number of rules used = 11, $\frac{\text{number of rules}}{\text{integrand size}} = 0.647$, Rules used = {1137, 60, 60, 60, 60, 73, 839, 813, 858, 807, 212}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^2 \left(ax+bx^2\right)^{3/4} \, dx$$

$$\Big\downarrow \text{1137}$$

$$\frac{\left(ax+bx^2\right)^{3/4} \int x^{11/4}(a+bx)^{3/4}dx}{x^{3/4}(a+bx)^{3/4}}$$

$$\Big\downarrow \text{60}$$

$$\frac{\left(ax+bx^2\right)^{3/4} \left(\frac{1}{6}a\int \frac{x^{11/4}}{\sqrt[4]{a+bx}}dx+\frac{2}{9}x^{15/4}(a+bx)^{3/4}\right)}{x^{3/4}(a+bx)^{3/4}}$$

$$\Big\downarrow \text{60}$$

$$\frac{\left(ax+bx^2\right)^{3/4} \left(\frac{1}{6}a\left(\frac{2x^{11/4}(a+bx)^{3/4}}{7b}-\frac{11a\int \frac{x^{7/4}}{\sqrt[4]{a+bx}}dx}{14b}\right)+\frac{2}{9}x^{15/4}(a+bx)^{3/4}\right)}{x^{3/4}(a+bx)^{3/4}}$$

$$\Big\downarrow \text{60}$$

$$\frac{(ax+bx^2)^{3/4}\left(\frac{1}{6}a\left(\frac{2x^{11/4}(a+bx)^{3/4}}{7b}-\frac{11a\left(\frac{2x^{7/4}(a+bx)^{3/4}}{5b}-\frac{7a\int\frac{x^{3/4}}{\sqrt[4]{a+bx}}dx}{10b}\right)}{14b}\right)+\frac{2}{9}x^{15/4}(a+bx)^{3/4}\right)}{x^{3/4}(a+bx)^{3/4}}$$

$$\Big\downarrow \text{ 60}$$

$$\frac{(ax+bx^2)^{3/4}\left(\frac{1}{6}a\left(\frac{2x^{11/4}(a+bx)^{3/4}}{7b}-\frac{11a\left(\frac{2x^{7/4}(a+bx)^{3/4}}{5b}-\frac{7a\left(\frac{2x^{3/4}(a+bx)^{3/4}}{3b}-\frac{a\int\frac{1}{\sqrt[4]{x}\sqrt[4]{a+bx}}dx}{2b}\right)}{10b}\right)}{14b}\right)+\frac{2}{9}x^{15/4}(a+bx}{x^{3/4}(a+bx)^{3/4}}$$

$$\Big\downarrow \text{ 73}$$

$$\frac{(ax+bx^2)^{3/4}\left(\frac{1}{6}a\left(\frac{2x^{11/4}(a+bx)^{3/4}}{7b}-\frac{11a\left(\frac{2x^{7/4}(a+bx)^{3/4}}{5b}-\frac{7a\left(\frac{2x^{3/4}(a+bx)^{3/4}}{3b}-\frac{2a\int\frac{\sqrt{x}}{\sqrt[4]{a+bx}}d\sqrt[4]{x}}{b}\right)}{10b}\right)}{14b}\right)+\frac{2}{9}x^{15/4}(a+bx}{x^{3/4}(a+bx)^{3/4}}$$

$$\Big\downarrow \text{ 839}$$

$$\frac{\left(ax+bx^2\right)^{3/4}\left(\frac{1}{6}a\left(\frac{2x^{11/4}(a+bx)^{3/4}}{7b}-\frac{11a\left(\frac{2x^{7/4}(a+bx)^{3/4}}{5b}-\frac{7a\left(\frac{2x^{3/4}(a+bx)^{3/4}}{3b}-\frac{2a\left(\frac{x^{3/4}}{2\sqrt[4]{a+bx}}-\frac{1}{2}a\int\frac{\sqrt{x}}{(a+bx)^{5/4}}d\sqrt[4]{x}\right)}{b}\right)}{10b}\right)}{14b}\right)+\right.}{x^{3/4}(a+bx)^{3/4}}$$

$\downarrow$ 813

$$\frac{\left(ax+bx^2\right)^{3/4}\left(\frac{1}{6}a\left(\frac{2x^{11/4}(a+bx)^{3/4}}{7b}-\frac{11a\left(\frac{2x^{7/4}(a+bx)^{3/4}}{5b}-\frac{7a\left(\frac{2x^{3/4}(a+bx)^{3/4}}{3b}-\frac{2a\left(\frac{x^{3/4}}{2\sqrt[4]{a+bx}}-\frac{a\sqrt[4]{x}\sqrt[4]{\frac{a}{bx}+1}\int\frac{1}{\left(\frac{a}{bx}+1\right)^{5/4}\cdots}\cdots}{2b\sqrt[4]{a+bx}}\cdots\right)}{b}\cdots\right)}{10b}\cdots\right)}{14b}\cdots\right)\cdots\right)}{x^{3/4}(a+bx)^{3/4}\cdots}$$

$\downarrow$ 858

$$\frac{\left(ax+bx^2\right)^{3/4}\left(\frac{1}{6}a\left(\frac{2x^{11/4}(a+bx)^{3/4}}{7b}-\frac{11a\left(\frac{2x^{7/4}(a+bx)^{3/4}}{5b}-\frac{7a\left(\frac{2x^{3/4}(a+bx)^{3/4}}{3b}-\frac{2a\left(\frac{a\sqrt[4]{x}\sqrt[4]{\frac{a}{bx}+1}\int\frac{1}{\sqrt[4]{x}\left(\frac{ax}{b}+1\right)^{5/4}}d\frac{1}{\sqrt[4]{x}}}{2b\sqrt[4]{a+bx}}+\cdots\right)}{b}\cdots\right)}{10b}\cdots\right)}{14b}\cdots\right)\cdots\right)}{x^{3/4}(a+bx)^{3/4}\cdots}$$

$\downarrow$ 807

$$\frac{\left(ax+bx^2\right)^{3/4}\left(\frac{1}{6}a\left(\frac{2x^{11/4}(a+bx)^{3/4}}{7b}-\frac{11a\left(\frac{2x^{7/4}(a+bx)^{3/4}}{5b}-\frac{7a\left(\frac{2x^{3/4}(a+bx)^{3/4}}{3b}-\frac{2a\left(\frac{a\sqrt[4]{x}\sqrt[4]{\frac{a}{bx}+1}\int\frac{1}{\left(\frac{\sqrt{x}a}{b}+1\right)^{5/4}}d\sqrt{x}}{4b\sqrt[4]{a+bx}}+\frac{x^3}{2\sqrt[4]{a}}\right.}{b}\right.}{10b}\right.}{14b}\right.\right.}{x^{3/4}(a+bx)^{3/4}}$$

$\downarrow$ 212

$$\frac{\left(ax+bx^2\right)^{3/4}\left(\frac{1}{6}a\left(\frac{2x^{11/4}(a+bx)^{3/4}}{7b}-\frac{11a\left(\frac{2x^{7/4}(a+bx)^{3/4}}{5b}-\frac{7a\left(\frac{2x^{3/4}(a+bx)^{3/4}}{3b}-\frac{2a\left(\frac{\sqrt{a}\sqrt[4]{x}\sqrt[4]{\frac{a}{bx}+1}E\left(\frac{1}{2}\arctan\left(\frac{\sqrt{a}\sqrt{x}}{\sqrt{b}}\right)\middle|2\right)}{2\sqrt{b}\sqrt[4]{a+bx}}+\cdots\right.}{b}\right.}{10b}\right.}{14b}\right.\right.}{x^{3/4}(a+bx)^{3/4}}$$

```
Int[x^2*(a*x + b*x^2)^(3/4),x]
```

```
((a*x + b*x^2)^(3/4)*((2*x^(15/4)*(a + b*x)^(3/4))/9 + (a*((2*x^(11/4)*(a
+ b*x)^(3/4))/(7*b) - (11*a*((2*x^(7/4)*(a + b*x)^(3/4))/(5*b) - (7*a*((2*
x^(3/4)*(a + b*x)^(3/4))/(3*b) - (2*a*(x^(3/4)/(2*(a + b*x)^(1/4)) + (Sqrt
[a]*(1 + a/(b*x))^(1/4)*x^(1/4)*EllipticE[ArcTan[(Sqrt[a]*Sqrt[x])/Sqrt[b]
]/2, 2])/(2*Sqrt[b]*(a + b*x)^(1/4))))/b))/(10*b)))/(14*b)))/6))/(x^(3/4)*
(a + b*x)^(3/4))
```

Defintions of rubi rules used

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^n/(b*(m + n + 1))), x] + Simp[n*((b*c - a*d)/(
b*(m + n + 1)))   Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
 c, d}, x] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ[m, 0] && ( !Integer
Q[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n + 2, 0] && IntLinear
Q[a, b, c, d, m, n, x]
```

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Simp[p/b   Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
 d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && Lt
Q[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntL
inearQ[a, b, c, d, m, n, x]
```

```
Int[((a_) + (b_.)*(x_)^2)^(-5/4), x_Symbol] :> Simp[(2/(a^(5/4)*Rt[b/a, 2])
)*EllipticE[(1/2)*ArcTan[Rt[b/a, 2]*x], 2], x] /; FreeQ[{a, b}, x] && GtQ[a
, 0] && PosQ[b/a]
```

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k = GCD[m
+ 1, n]}, Simp[1/k   Subst[Int[x^((m + 1)/k - 1)*(a + b*x^(n/k))^p, x], x,
x^k], x] /; k != 1] /; FreeQ[{a, b, p}, x] && IGtQ[n, 0] && IntegerQ[m]
```

```
Int[(x_)^2/((a_) + (b_.)*(x_)^4)^(5/4), x_Symbol] :> Simp[x*((1 + a/(b*x^4)
)^(1/4)/(b*(a + b*x^4)^(1/4)))   Int[1/(x^3*(1 + a/(b*x^4))^(5/4)), x], x]
/; FreeQ[{a, b}, x] && PosQ[b/a]
```

```
Int[(x_)^2/((a_) + (b_.)*(x_)^4)^(1/4), x_Symbol] :> Simp[x^3/(2*(a + b*x^4
)^(1/4)), x] - Simp[a/2   Int[x^2/(a + b*x^4)^(5/4), x], x] /; FreeQ[{a, b}
, x] && PosQ[b/a]
```

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> -Subst[Int[(a +
b/x^n)^p/x^(m + 2), x], x, 1/x] /; FreeQ[{a, b, p}, x] && ILtQ[n, 0] && Int
egerQ[m]
```

```
Int[((e_.)*(x_))^(m_)*((b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(
e*x)^m*((b*x + c*x^2)^p/(x^(m + p)*(b + c*x)^p))   Int[x^(m + p)*(b + c*x)^
p, x], x] /; FreeQ[{b, c, e, m}, x]
```

Maple [F]

$$\int x^{2} \left(b\,x^{2}+a x \right)^{\frac{3}{4}}d x$$

```
int(x^2*(b*x^2+a*x)^(3/4),x)
```

```
int(x^2*(b*x^2+a*x)^(3/4),x)
```

Fricas [F]

$$\int x^2 \left(ax+bx^2\right)^{3/4} \, dx = \int \left(bx^{2}+ax\right)^{\frac{3}{4}}x^{2}\,dx$$

```
integrate(x^2*(b*x^2+a*x)^(3/4),x, algorithm="fricas")
```

```
integral((b*x^2 + a*x)^(3/4)*x^2, x)
```

Sympy [F]

$$\int x^2 \left(ax+bx^2\right)^{3/4} \, dx = \int x^{2}(x(a+bx))^{\frac{3}{4}}\,dx$$

```
integrate(x**2*(b*x**2+a*x)**(3/4),x)
```

```
Integral(x**2*(x*(a + b*x))**(3/4), x)
```

Maxima [F]

$$\int x^2 \left(ax+bx^2\right)^{3/4} \, dx = \int \left(bx^2+ax\right)^{\frac{3}{4}} x^2 \, dx$$

```
integrate(x^2*(b*x^2+a*x)^(3/4),x, algorithm="maxima")
```

```
integrate((b*x^2 + a*x)^(3/4)*x^2, x)
```

Giac [F]

$$\int x^2 \left(ax+bx^2\right)^{3/4} \, dx = \int \left(bx^2+ax\right)^{\frac{3}{4}} x^2 \, dx$$

```
integrate(x^2*(b*x^2+a*x)^(3/4),x, algorithm="giac")
```

```
integrate((b*x^2 + a*x)^(3/4)*x^2, x)
```

Mupad [F(-1)]

Timed out.

$$\int x^2 \left(ax+bx^2\right)^{3/4} \, dx = \int x^2 \left(b\,x^2+a\,x\right)^{3/4} \,dx$$

```
int(x^2*(a*x + b*x^2)^(3/4),x)
```

```
int(x^2*(a*x + b*x^2)^(3/4), x)
```

Reduce [F]

$$\int x^2(ax+bx^2)^{3/4}\,dx = \frac{308x^{\frac{3}{4}}(bx+a)^{\frac{3}{4}}a^3 - 264x^{\frac{7}{4}}(bx+a)^{\frac{3}{4}}a^2b + 240x^{\frac{11}{4}}(bx+a)^{\frac{3}{4}}a\,b^2 + 1120x^{\frac{15}{4}}(bx+a)^{\frac{3}{4}}b^3 - 2}{5040b^3}$$

```
int(x^2*(b*x^2+a*x)^(3/4),x)
```

```
(308*x**(3/4)*(a + b*x)**(3/4)*a**3 - 264*x**(3/4)*(a + b*x)**(3/4)*a**2*b
*x + 240*x**(3/4)*(a + b*x)**(3/4)*a*b**2*x**2 + 1120*x**(3/4)*(a + b*x)**
(3/4)*b**3*x**3 - 231*int((a + b*x)**(3/4)/(x**(1/4)*a + x**(1/4)*b*x),x)*
a**4)/(5040*b**3)
```

3.156 $\int x(ax+bx^2)^{3/4}\,dx$

Optimal result

Integrand size = 15, antiderivative size = 133

$$\int x(ax+bx^2)^{3/4}\,dx=-\frac{a^2(ax+bx^2)^{3/4}}{10b^2}+\frac{3ax(ax+bx^2)^{3/4}}{35b}+\frac{2}{7}x^2(ax+bx^2)^{3/4}+\frac{3a^4\sqrt[4]{-\frac{bx}{a}-\frac{b^2x^2}{a^2}}\,E\left(\frac{1}{2}\arcsin\left(1+\frac{2bx}{a}\right)\middle|2\right)}{20\sqrt{2}b^3\sqrt[4]{ax+bx^2}}$$

```
-1/10*a^2*(b*x^2+a*x)^(3/4)/b^2+3/35*a*x*(b*x^2+a*x)^(3/4)/b+2/7*x^2*(b*x^
2+a*x)^(3/4)+3/40*a^4*(-b*x/a-b^2*x^2/a^2)^(1/4)*EllipticE(sin(1/2*arcsin(
1+2*b*x/a)),2^(1/2))*2^(1/2)/b^3/(b*x^2+a*x)^(1/4)
```

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 10.01 (sec) , antiderivative size = 47, normalized size of antiderivative = 0.35

$$\int x(ax+bx^2)^{3/4}\,dx=\frac{4x^2(x(a+bx))^{3/4}\,\text{Hypergeometric2F1}\left(-\frac{3}{4},\frac{11}{4},\frac{15}{4},-\frac{bx}{a}\right)}{11\left(1+\frac{bx}{a}\right)^{3/4}}$$

```
Integrate[x*(a*x + b*x^2)^(3/4),x]
```

```
(4*x^2*(x*(a + b*x))^(3/4)*Hypergeometric2F1[-3/4, 11/4, 15/4, -((b*x)/a)]
)/(11*(1 + (b*x)/a)^(3/4))
```

Rubi [A] (verified)

Time = 0.44 (sec) , antiderivative size = 131, normalized size of antiderivative = 0.98, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.333$, Rules used = {1160, 1087, 1093, 1090, 226}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x\left(ax+bx^2\right)^{3/4}dx$$

$$\Big\downarrow \; 1160$$

$$\frac{2\left(ax+bx^2\right)^{7/4}}{7b}-\frac{a\int \left(bx^2+ax\right)^{3/4}dx}{2b}$$

$$\Big\downarrow \; 1087$$

$$\frac{2\left(ax+bx^2\right)^{7/4}}{7b}-\frac{a\left(\frac{(a+2bx)\left(ax+bx^2\right)^{3/4}}{5b}-\frac{3a^2\int \frac{1}{\sqrt[4]{bx^2+ax}}dx}{20b}\right)}{2b}$$

$$\Big\downarrow \; 1093$$

$$\frac{2\left(ax+bx^2\right)^{7/4}}{7b}-\frac{a\left(\frac{(a+2bx)\left(ax+bx^2\right)^{3/4}}{5b}-\frac{3a^2\sqrt[4]{-\frac{b\left(ax+bx^2\right)}{a^2}}\int \frac{1}{\sqrt[4]{-\frac{b^2x^2}{a^2}-\frac{bx}{a}}}dx}{20b\sqrt[4]{ax+bx^2}}\right)}{2b}$$

$$\Big\downarrow \; 1090$$

$$\frac{2\left(ax+bx^2\right)^{7/4}}{7b}-\frac{a\left(\frac{3a^4\sqrt[4]{-\frac{b\left(ax+bx^2\right)}{a^2}}\int \frac{1}{\sqrt[4]{1-\frac{a^2\left(-\frac{2xb^2}{a^2}-\frac{b}{a}\right)^2}{b^2}}}d\left(-\frac{2xb^2}{a^2}-\frac{b}{a}\right)}{20\sqrt{2}b^3\sqrt[4]{ax+bx^2}}+\frac{(a+2bx)\left(ax+bx^2\right)^{3/4}}{5b}\right)}{2b}$$

$$\Big\downarrow \; 226$$

$$\frac{2\left(ax+bx^2\right)^{7/4}}{7b}-\frac{a\left(\frac{3a^3\sqrt[4]{-\frac{b\left(ax+bx^2\right)}{a^2}}E\left(\frac{1}{2}\arcsin\left(\frac{a\left(-\frac{2xb^2}{a^2}-\frac{b}{a}\right)}{b}\right)\middle|2\right)}{10\sqrt{2}b^2\sqrt[4]{ax+bx^2}}+\frac{(a+2bx)\left(ax+bx^2\right)^{3/4}}{5b}\right)}{2b}$$

```
Int[x*(a*x + b*x^2)^(3/4),x]
```

```
(2*(a*x + b*x^2)^(7/4))/(7*b) - (a*(((a + 2*b*x)*(a*x + b*x^2)^(3/4))/(5*b
) + (3*a^3*(-((b*(a*x + b*x^2))/a^2))^(1/4)*EllipticE[ArcSin[(a*(-(b/a) -
(2*b^2*x)/a^2))/b]/2, 2])/(10*Sqrt[2]*b^2*(a*x + b*x^2)^(1/4))))/(2*b)
```

Defintions of rubi rules used

```
Int[((a_) + (b_.)*(x_)^2)^(-1/4), x_Symbol] :> Simp[(2/(a^(1/4)*Rt[-b/a, 2]
))*EllipticE[(1/2)*ArcSin[Rt[-b/a, 2]*x], 2], x] /; FreeQ[{a, b}, x] && GtQ
[a, 0] && NegQ[b/a]
```

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(b + 2*c*x)
*((a + b*x + c*x^2)^p/(2*c*(2*p + 1))), x] - Simp[p*((b^2 - 4*a*c)/(2*c*(2*
p + 1)))   Int[(a + b*x + c*x^2)^(p - 1), x], x] /; FreeQ[{a, b, c}, x] &&
GtQ[p, 0] && (IntegerQ[4*p] || IntegerQ[3*p])
```

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[1/(2*c*(-4*
(c/(b^2 - 4*a*c)))^p)   Subst[Int[Simp[1 - x^2/(b^2 - 4*a*c), x]^p, x], x,
b + 2*c*x], x] /; FreeQ[{a, b, c, p}, x] && GtQ[4*a - b^2/c, 0]
```

```
Int[((b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(b*x + c*x^2)^p/((-
c)*((b*x + c*x^2)/b^2))^p   Int[((-c)*(x/b) - c^2*(x^2/b^2))^p, x], x] /; F
reeQ[{b, c}, x] && (IntegerQ[4*p] || IntegerQ[3*p])
```

```
Int[((d_.) + (e_.)*(x_))*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol
] :> Simp[e*((a + b*x + c*x^2)^(p + 1)/(2*c*(p + 1))), x] + Simp[(2*c*d - b
*e)/(2*c)   Int[(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, p}, x]
 && NeQ[p, -1]
```

Maple [F]

$$\int x \left(b \,x^{2}+a x \right)^{\frac{3}{4}}d x$$

```
int(x*(b*x^2+a*x)^(3/4),x)
```

```
int(x*(b*x^2+a*x)^(3/4),x)
```

Fricas [F]

$$\int x \left(ax+bx^2\right)^{3/4} \, dx=\int \left(bx^{2}+ax\right)^{\frac{3}{4}}x\,dx$$

```
integrate(x*(b*x^2+a*x)^(3/4),x, algorithm="fricas")
```

```
integral((b*x^2 + a*x)^(3/4)*x, x)
```

Sympy [F]

$$\int x(ax + bx^2)^{3/4} \, dx = \int x(x(a + bx))^{\frac{3}{4}} \, dx$$

```
integrate(x*(b*x**2+a*x)**(3/4),x)
```

```
Integral(x*(x*(a + b*x))**(3/4), x)
```

Maxima [F]

$$\int x(ax + bx^2)^{3/4} \, dx = \int (bx^2 + ax)^{\frac{3}{4}} x \, dx$$

```
integrate(x*(b*x^2+a*x)^(3/4),x, algorithm="maxima")
```

```
integrate((b*x^2 + a*x)^(3/4)*x, x)
```

Giac [F]

$$\int x(ax + bx^2)^{3/4} \, dx = \int (bx^2 + ax)^{\frac{3}{4}} x \, dx$$

```
integrate(x*(b*x^2+a*x)^(3/4),x, algorithm="giac")
```

```
integrate((b*x^2 + a*x)^(3/4)*x, x)
```

Mupad [F(-1)]

Timed out.

$$\int x(ax+bx^2)^{3/4}\,dx=\int x\,(b\,x^2+a\,x)^{3/4}\,dx$$

```
int(x*(a*x + b*x^2)^(3/4),x)
```

```
int(x*(a*x + b*x^2)^(3/4), x)
```

Reduce [F]

$$\int x(ax+bx^2)^{3/4}\,dx=\frac{-28x^{\frac{3}{4}}(bx+a)^{\frac{3}{4}}a^2+24x^{\frac{7}{4}}(bx+a)^{\frac{3}{4}}ab+80x^{\frac{11}{4}}(bx+a)^{\frac{3}{4}}b^2+21\left(\int\frac{(bx+a)^{\frac{3}{4}}}{x^{\frac{1}{4}}a+x^{\frac{5}{4}}b}dx\right)a^3}{280b^2}$$

```
int(x*(b*x^2+a*x)^(3/4),x)
```

```
( - 28*x**(3/4)*(a + b*x)**(3/4)*a**2 + 24*x**(3/4)*(a + b*x)**(3/4)*a*b*x
 + 80*x**(3/4)*(a + b*x)**(3/4)*b**2*x**2 + 21*int((a + b*x)**(3/4)/(x**(1
/4)*a + x**(1/4)*b*x),x)*a**3)/(280*b**2)
```

3.157 $\int (ax+bx^2)^{3/4}\,dx$

Optimal result

Integrand size = 13, antiderivative size = 107

$$\int (ax+bx^2)^{3/4}\,dx = \frac{a(ax+bx^2)^{3/4}}{5b} + \frac{2}{5}x(ax+bx^2)^{3/4} - \frac{3a^3\sqrt[4]{-\frac{bx}{a}-\frac{b^2x^2}{a^2}}\,E\left(\frac{1}{2}\arcsin\left(1+\frac{2bx}{a}\right)\middle|\,2\right)}{10\sqrt{2}b^2\sqrt[4]{ax+bx^2}}$$

```
1/5*a*(b*x^2+a*x)^(3/4)/b+2/5*x*(b*x^2+a*x)^(3/4)-3/20*a^3*(-b*x/a-b^2*x^2
/a^2)^(1/4)*EllipticE(sin(1/2*arcsin(1+2*b*x/a)),2^(1/2))*2^(1/2)/b^2/(b*x
^2+a*x)^(1/4)
```

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 10.01 (sec) , antiderivative size = 45, normalized size of antiderivative = 0.42

$$\int (ax+bx^2)^{3/4}\,dx = \frac{4x(x(a+bx))^{3/4}\,\text{Hypergeometric2F1}\left(-\frac{3}{4},\frac{7}{4},\frac{11}{4},-\frac{bx}{a}\right)}{7\left(1+\frac{bx}{a}\right)^{3/4}}$$

```
Integrate[(a*x + b*x^2)^(3/4),x]
```

```
(4*x*(x*(a + b*x))^(3/4)*Hypergeometric2F1[-3/4, 7/4, 11/4, -((b*x)/a)])/(
7*(1 + (b*x)/a)^(3/4))
```

Rubi [A] (verified)

Time = 0.39 (sec) , antiderivative size = 102, normalized size of antiderivative = 0.95, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.308, Rules used = {1087, 1093, 1090, 226}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \left(ax+bx^2\right)^{3/4}dx$$

$$\Big\downarrow \text{ 1087}$$

$$\frac{(a+2bx)\left(ax+bx^2\right)^{3/4}}{5b}-\frac{3a^2\int \frac{1}{\sqrt[4]{bx^2+ax}}dx}{20b}$$

$$\Big\downarrow \text{ 1093}$$

$$\frac{(a+2bx)\left(ax+bx^2\right)^{3/4}}{5b}-\frac{3a^2\sqrt[4]{-\frac{b\left(ax+bx^2\right)}{a^2}}\int \frac{1}{\sqrt[4]{-\frac{b^2x^2}{a^2}-\frac{bx}{a}}}dx}{20b\sqrt[4]{ax+bx^2}}$$

$$\Big\downarrow \text{ 1090}$$

$$\frac{3a^4\sqrt[4]{-\frac{b\left(ax+bx^2\right)}{a^2}}\int \frac{1}{\sqrt[4]{1-\frac{a^2\left(-\frac{2xb^2}{a^2}-\frac{b}{a}\right)^2}{b^2}}}d\left(-\frac{2xb^2}{a^2}-\frac{b}{a}\right)}{20\sqrt{2}b^3\sqrt[4]{ax+bx^2}}+\frac{(a+2bx)\left(ax+bx^2\right)^{3/4}}{5b}$$

$$\Big\downarrow \text{ 226}$$

$$\frac{3a^3\sqrt[4]{-\frac{b\left(ax+bx^2\right)}{a^2}}E\left(\frac{1}{2}\arcsin\left(\frac{a\left(-\frac{2xb^2}{a^2}-\frac{b}{a}\right)}{b}\right)\middle|2\right)}{10\sqrt{2}b^2\sqrt[4]{ax+bx^2}}+\frac{(a+2bx)\left(ax+bx^2\right)^{3/4}}{5b}$$

```
Int[(a*x + b*x^2)^(3/4),x]
```

```
((a + 2*b*x)*(a*x + b*x^2)^(3/4))/(5*b) + (3*a^3*(-((b*(a*x + b*x^2))/a^2)
)^(1/4)*EllipticE[ArcSin[(a*(-(b/a) - (2*b^2*x)/a^2))/b]/2, 2])/(10*Sqrt[2
]*b^2*(a*x + b*x^2)^(1/4))
```

Defintions of rubi rules used

```
Int[((a_) + (b_.)*(x_)^2)^(-1/4), x_Symbol] :> Simp[(2/(a^(1/4)*Rt[-b/a, 2]
))*EllipticE[(1/2)*ArcSin[Rt[-b/a, 2]*x], 2], x] /; FreeQ[{a, b}, x] && GtQ
[a, 0] && NegQ[b/a]
```

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(b + 2*c*x)
*((a + b*x + c*x^2)^p/(2*c*(2*p + 1))), x] - Simp[p*((b^2 - 4*a*c)/(2*c*(2*
p + 1)))   Int[(a + b*x + c*x^2)^(p - 1), x], x] /; FreeQ[{a, b, c}, x] &&
GtQ[p, 0] && (IntegerQ[4*p] || IntegerQ[3*p])
```

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[1/(2*c*(-4*
(c/(b^2 - 4*a*c)))^p)   Subst[Int[Simp[1 - x^2/(b^2 - 4*a*c), x]^p, x], x,
b + 2*c*x], x] /; FreeQ[{a, b, c, p}, x] && GtQ[4*a - b^2/c, 0]
```

```
Int[((b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(b*x + c*x^2)^p/((-
c)*((b*x + c*x^2)/b^2))^p   Int[((-c)*(x/b) - c^2*(x^2/b^2))^p, x], x] /; F
reeQ[{b, c}, x] && (IntegerQ[4*p] || IntegerQ[3*p])
```

Maple [F]

$$\int \left(b\,x^2+ax\right)^{\frac{3}{4}}dx$$

```
int((b*x^2+a*x)^(3/4),x)
```

```
int((b*x^2+a*x)^(3/4),x)
```

Fricas [F]

$$\int \left(ax + bx^2\right)^{3/4} dx = \int \left(bx^2 + ax\right)^{\frac{3}{4}} dx$$

```
integrate((b*x^2+a*x)^(3/4),x, algorithm="fricas")
```

```
integral((b*x^2 + a*x)^(3/4), x)
```

Sympy [F]

$$\int \left(ax + bx^2\right)^{3/4} dx = \int \left(ax + bx^2\right)^{\frac{3}{4}} dx$$

```
integrate((b*x**2+a*x)**(3/4),x)
```

```
Integral((a*x + b*x**2)**(3/4), x)
```

Maxima [F]

$$\int \left(ax + bx^2\right)^{3/4} dx = \int \left(bx^2 + ax\right)^{\frac{3}{4}} dx$$

```
integrate((b*x^2+a*x)^(3/4),x, algorithm="maxima")
```

```
integrate((b*x^2 + a*x)^(3/4), x)
```

Giac [F]

$$\int \left(ax+bx^2\right)^{3/4}\,dx=\int \left(bx^2+ax\right)^{\frac{3}{4}}\,dx$$

```
integrate((b*x^2+a*x)^(3/4),x, algorithm="giac")
```

```
integrate((b*x^2 + a*x)^(3/4), x)
```

Mupad [B] (verification not implemented)

Time = 9.35 (sec) , antiderivative size = 36, normalized size of antiderivative = 0.34

$$\int \left(ax+bx^2\right)^{3/4}\,dx=\frac{4\,x\,\left(b\,x^2+a\,x\right)^{3/4}\,{}_2\mathrm{F}_1\left(-\frac{3}{4},\frac{7}{4};\ \frac{11}{4};\ -\frac{b\,x}{a}\right)}{7\,\left(\frac{b\,x}{a}+1\right)^{3/4}}$$

```
int((a*x + b*x^2)^(3/4),x)
```

```
(4*x*(a*x + b*x^2)^(3/4)*hypergeom([-3/4, 7/4], 11/4, -(b*x)/a))/(7*((b*x)
/a + 1)^(3/4))
```

Reduce [F]

$$\int \left(ax+bx^2\right)^{3/4}\,dx=\frac{4x^{\frac{3}{4}}(bx+a)^{\frac{3}{4}}a+8x^{\frac{7}{4}}(bx+a)^{\frac{3}{4}}b-3\left(\int \frac{(bx+a)^{\frac{3}{4}}}{x^{\frac{1}{4}}a+x^{\frac{5}{4}}b}dx\right)a^2}{20b}$$

```
int((b*x^2+a*x)^(3/4),x)
```

```
(4*x**(3/4)*(a + b*x)**(3/4)*a + 8*x**(3/4)*(a + b*x)**(3/4)*b*x - 3*int((
a + b*x)**(3/4)/(x**(1/4)*a + x**(1/4)*b*x),x)*a**2)/(20*b)
```

3.158 $\int \frac{(ax+bx^2)^{3/4}}{x}\,dx$

Optimal result

Integrand size = 17, antiderivative size = 82

$$\int \frac{(ax+bx^2)^{3/4}}{x}\,dx = \frac{2}{3}\left(ax+bx^2\right)^{3/4} + \frac{a^2\sqrt[4]{-\frac{bx}{a}-\frac{b^2x^2}{a^2}}\,E\left(\frac{1}{2}\arcsin\left(1+\frac{2bx}{a}\right)\middle|2\right)}{\sqrt{2}b\sqrt[4]{ax+bx^2}}$$

```
2/3*(b*x^2+a*x)^(3/4)+1/2*a^2*(-b*x/a-b^2*x^2/a^2)^(1/4)*EllipticE(sin(1/2
*arcsin(1+2*b*x/a)),2^(1/2))*2^(1/2)/b/(b*x^2+a*x)^(1/4)
```

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 10.02 (sec) , antiderivative size = 44, normalized size of antiderivative = 0.54

$$\int \frac{(ax+bx^2)^{3/4}}{x}\,dx = \frac{4(x(a+bx))^{3/4}\,\text{Hypergeometric2F1}\left(-\frac{3}{4},\frac{3}{4},\frac{7}{4},-\frac{bx}{a}\right)}{3\left(1+\frac{bx}{a}\right)^{3/4}}$$

```
Integrate[(a*x + b*x^2)^(3/4)/x,x]
```

```
(4*(x*(a + b*x))^(3/4)*Hypergeometric2F1[-3/4, 3/4, 7/4, -((b*x)/a)])/(3*(
1 + (b*x)/a)^(3/4))
```

Rubi [A] (warning: unable to verify)

Time = 0.47 (sec) , antiderivative size = 134, normalized size of antiderivative = 1.63, number of steps used = 9, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.471, Rules used = {1137, 60, 73, 839, 813, 858, 807, 212}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\left(ax+bx^2\right)^{3/4}}{x}dx$$

$$\Big\downarrow \text{1137}$$

$$\frac{\left(ax+bx^2\right)^{3/4}\int \frac{(a+bx)^{3/4}}{\sqrt[4]{x}}dx}{x^{3/4}(a+bx)^{3/4}}$$

$$\Big\downarrow \text{60}$$

$$\frac{\left(ax+bx^2\right)^{3/4}\left(\frac{1}{2}a\int \frac{1}{\sqrt[4]{x}\sqrt[4]{a+bx}}dx+\frac{2}{3}x^{3/4}(a+bx)^{3/4}\right)}{x^{3/4}(a+bx)^{3/4}}$$

$$\Big\downarrow \text{73}$$

$$\frac{\left(ax+bx^2\right)^{3/4}\left(2a\int \frac{\sqrt{x}}{\sqrt[4]{a+bx}}d\sqrt[4]{x}+\frac{2}{3}x^{3/4}(a+bx)^{3/4}\right)}{x^{3/4}(a+bx)^{3/4}}$$

$$\Big\downarrow \text{839}$$

$$\frac{\left(ax+bx^2\right)^{3/4}\left(2a\left(\frac{x^{3/4}}{2\sqrt[4]{a+bx}}-\frac{1}{2}a\int \frac{\sqrt{x}}{(a+bx)^{5/4}}d\sqrt[4]{x}\right)+\frac{2}{3}x^{3/4}(a+bx)^{3/4}\right)}{x^{3/4}(a+bx)^{3/4}}$$

$$\Big\downarrow \text{813}$$

$$\frac{\left(ax+bx^2\right)^{3/4}\left(2a\left(\frac{x^{3/4}}{2\sqrt[4]{a+bx}}-\frac{a\sqrt[4]{x}\sqrt[4]{\frac{a}{bx}+1}\int\frac{1}{\left(\frac{a}{bx}+1\right)^{5/4}x^{3/4}}d\sqrt[4]{x}}{2b\sqrt[4]{a+bx}}\right)+\frac{2}{3}x^{3/4}(a+bx)^{3/4}\right)}{x^{3/4}(a+bx)^{3/4}}$$

$$\Big\downarrow \text{858}$$

$$\frac{\left(ax+bx^2\right)^{3/4}\left(2a\left(\frac{a\sqrt[4]{x}\sqrt[4]{\frac{a}{bx}+1}\int\frac{1}{\sqrt[4]{x}\left(\frac{ax}{b}+1\right)^{5/4}}d\frac{1}{\sqrt[4]{x}}}{2b\sqrt[4]{a+bx}}+\frac{x^{3/4}}{2\sqrt[4]{a+bx}}\right)+\frac{2}{3}x^{3/4}(a+bx)^{3/4}\right)}{x^{3/4}(a+bx)^{3/4}}$$

$$\Big\downarrow \text{807}$$

$$\frac{\left(ax+bx^2\right)^{3/4}\left(2a\left(\frac{a\sqrt[4]{x}\sqrt[4]{\frac{a}{bx}+1}\int\frac{1}{\left(\frac{\sqrt{x}a}{b}+1\right)^{5/4}}d\sqrt{x}}{4b\sqrt[4]{a+bx}}+\frac{x^{3/4}}{2\sqrt[4]{a+bx}}\right)+\frac{2}{3}x^{3/4}(a+bx)^{3/4}\right)}{x^{3/4}(a+bx)^{3/4}}$$

$$\Big\downarrow \text{212}$$

$$\frac{\left(ax+bx^2\right)^{3/4}\left(2a\left(\frac{\sqrt{a}\sqrt[4]{x}\sqrt[4]{\frac{a}{bx}+1}E\left(\frac{1}{2}\arctan\left(\frac{\sqrt{a}\sqrt{x}}{\sqrt{b}}\right)\middle|2\right)}{2\sqrt{b}\sqrt[4]{a+bx}}+\frac{x^{3/4}}{2\sqrt[4]{a+bx}}\right)+\frac{2}{3}x^{3/4}(a+bx)^{3/4}\right)}{x^{3/4}(a+bx)^{3/4}}$$

```
Int[(a*x + b*x^2)^(3/4)/x,x]
```

```
((a*x + b*x^2)^(3/4)*((2*x^(3/4)*(a + b*x)^(3/4))/3 + 2*a*(x^(3/4)/(2*(a +
 b*x)^(1/4)) + (Sqrt[a]*(1 + a/(b*x))^(1/4)*x^(1/4)*EllipticE[ArcTan[(Sqrt
[a]*Sqrt[x])/Sqrt[b]]/2, 2])/(2*Sqrt[b]*(a + b*x)^(1/4)))))/(x^(3/4)*(a +
b*x)^(3/4))
```

Defintions of rubi rules used

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^n/(b*(m + n + 1))), x] + Simp[n*((b*c - a*d)/(
b*(m + n + 1)))   Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
 c, d}, x] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ[m, 0] && ( !Integer
Q[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n + 2, 0] && IntLinear
Q[a, b, c, d, m, n, x]
```

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Simp[p/b   Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
 d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && Lt
Q[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntL
inearQ[a, b, c, d, m, n, x]
```

```
Int[((a_) + (b_.)*(x_)^2)^(-5/4), x_Symbol] :> Simp[(2/(a^(5/4)*Rt[b/a, 2])
)*EllipticE[(1/2)*ArcTan[Rt[b/a, 2]*x], 2], x] /; FreeQ[{a, b}, x] && GtQ[a
, 0] && PosQ[b/a]
```

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k = GCD[m
+ 1, n]}, Simp[1/k   Subst[Int[x^((m + 1)/k - 1)*(a + b*x^(n/k))^p, x], x,
x^k], x] /; k != 1] /; FreeQ[{a, b, p}, x] && IGtQ[n, 0] && IntegerQ[m]
```

```
Int[(x_)^2/((a_) + (b_.)*(x_)^4)^(5/4), x_Symbol] :> Simp[x*((1 + a/(b*x^4)
)^(1/4)/(b*(a + b*x^4)^(1/4)))   Int[1/(x^3*(1 + a/(b*x^4))^(5/4)), x], x]
/; FreeQ[{a, b}, x] && PosQ[b/a]
```

```
Int[(x_)^2/((a_) + (b_.)*(x_)^4)^(1/4), x_Symbol] :> Simp[x^3/(2*(a + b*x^4
)^(1/4)), x] - Simp[a/2   Int[x^2/(a + b*x^4)^(5/4), x], x] /; FreeQ[{a, b}
, x] && PosQ[b/a]
```

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> -Subst[Int[(a +
b/x^n)^p/x^(m + 2), x], x, 1/x] /; FreeQ[{a, b, p}, x] && ILtQ[n, 0] && Int
egerQ[m]
```

```
Int[((e_.)*(x_))^(m_)*((b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(
e*x)^m*((b*x + c*x^2)^p/(x^(m + p)*(b + c*x)^p))   Int[x^(m + p)*(b + c*x)^
p, x], x] /; FreeQ[{b, c, e, m}, x]
```

Maple [F]

$$\int \frac{(b\,x^2 + ax)^{\frac{3}{4}}}{x}dx$$

```
int((b*x^2+a*x)^(3/4)/x,x)
```

```
int((b*x^2+a*x)^(3/4)/x,x)
```

Fricas [F]

$$\int \frac{(ax + bx^2)^{3/4}}{x}\,dx = \int \frac{(bx^2 + ax)^{\frac{3}{4}}}{x}\,dx$$

```
integrate((b*x^2+a*x)^(3/4)/x,x, algorithm="fricas")
```

```
integral((b*x^2 + a*x)^(3/4)/x, x)
```

Sympy [F]

$$\int \frac{(ax + bx^2)^{3/4}}{x}\,dx = \int \frac{(x(a + bx))^{\frac{3}{4}}}{x}\,dx$$

```
integrate((b*x**2+a*x)**(3/4)/x,x)
```

```
Integral((x*(a + b*x))**(3/4)/x, x)
```

Maxima [F]

$$\int \frac{(ax+bx^2)^{3/4}}{x}\,dx = \int \frac{(bx^2+ax)^{\frac{3}{4}}}{x}\,dx$$

```
integrate((b*x^2+a*x)^(3/4)/x,x, algorithm="maxima")
```

```
integrate((b*x^2 + a*x)^(3/4)/x, x)
```

Giac [F]

$$\int \frac{(ax+bx^2)^{3/4}}{x}\,dx = \int \frac{(bx^2+ax)^{\frac{3}{4}}}{x}\,dx$$

```
integrate((b*x^2+a*x)^(3/4)/x,x, algorithm="giac")
```

```
integrate((b*x^2 + a*x)^(3/4)/x, x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(ax+bx^2)^{3/4}}{x}\,dx = \int \frac{(b\,x^2+a\,x)^{3/4}}{x}\,dx$$

```
int((a*x + b*x^2)^(3/4)/x,x)
```

```
int((a*x + b*x^2)^(3/4)/x, x)
```

Reduce [F]

$$\int \frac{(ax+bx^2)^{3/4}}{x}\,dx = \frac{2x^{\frac{3}{4}}(bx+a)^{\frac{3}{4}}}{3} + \frac{\left(\int \frac{(bx+a)^{\frac{3}{4}}}{x^{\frac{1}{4}}a+x^{\frac{5}{4}}b}dx\right)a}{2}$$

```
int((b*x^2+a*x)^(3/4)/x,x)
```

```
(4*x**(3/4)*(a + b*x)**(3/4) + 3*int((a + b*x)**(3/4)/(x**(1/4)*a + x**(1/
4)*b*x),x)*a)/6
```

3.159 $\int \frac{(ax+bx^2)^{3/4}}{x^2} dx$

Optimal result

Integrand size = 17, antiderivative size = 79

$$\int \frac{(ax+bx^2)^{3/4}}{x^2}\,dx = -\frac{4(ax+bx^2)^{3/4}}{x} + \frac{3\sqrt{2}a\sqrt[4]{-\frac{bx}{a}-\frac{b^2x^2}{a^2}}E\left(\frac{1}{2}\arcsin\left(1+\frac{2bx}{a}\right)\middle|2\right)}{\sqrt[4]{ax+bx^2}}$$

```
-4*(b*x^2+a*x)^(3/4)/x+3*2^(1/2)*a*(-b*x/a-b^2*x^2/a^2)^(1/4)*EllipticE(si
n(1/2*arcsin(1+2*b*x/a)),2^(1/2))/(b*x^2+a*x)^(1/4)
```

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 10.01 (sec) , antiderivative size = 45, normalized size of antiderivative = 0.57

$$\int \frac{(ax+bx^2)^{3/4}}{x^2}\,dx = -\frac{4(x(a+bx))^{3/4}\,\text{Hypergeometric2F1}\left(-\frac{3}{4},-\frac{1}{4},\frac{3}{4},-\frac{bx}{a}\right)}{x\left(1+\frac{bx}{a}\right)^{3/4}}$$

```
Integrate[(a*x + b*x^2)^(3/4)/x^2,x]
```

```
(-4*(x*(a + b*x))^(3/4)*Hypergeometric2F1[-3/4, -1/4, 3/4, -((b*x)/a)])/(x
*(1 + (b*x)/a)^(3/4))
```

Rubi [A] (warning: unable to verify)

Time = 0.47 (sec) , antiderivative size = 132, normalized size of antiderivative = 1.67, number of steps used = 9, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.471, Rules used = {1137, 57, 73, 839, 813, 858, 807, 212}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\left(ax+bx^2\right)^{3/4}}{x^2}dx$$

$$\Big\downarrow \text{1137}$$

$$\frac{\left(ax+bx^2\right)^{3/4}\int \frac{(a+bx)^{3/4}}{x^{5/4}}dx}{x^{3/4}(a+bx)^{3/4}}$$

$$\Big\downarrow \text{57}$$

$$\frac{\left(ax+bx^2\right)^{3/4}\left(3b\int \frac{1}{\sqrt[4]{x}\sqrt[4]{a+bx}}dx-\frac{4(a+bx)^{3/4}}{\sqrt[4]{x}}\right)}{x^{3/4}(a+bx)^{3/4}}$$

$$\Big\downarrow \text{73}$$

$$\frac{\left(ax+bx^2\right)^{3/4}\left(12b\int \frac{\sqrt{x}}{\sqrt[4]{a+bx}}d\sqrt[4]{x}-\frac{4(a+bx)^{3/4}}{\sqrt[4]{x}}\right)}{x^{3/4}(a+bx)^{3/4}}$$

$$\Big\downarrow \text{839}$$

$$\frac{\left(ax+bx^2\right)^{3/4}\left(12b\left(\frac{x^{3/4}}{2\sqrt[4]{a+bx}}-\frac{1}{2}a\int \frac{\sqrt{x}}{(a+bx)^{5/4}}d\sqrt[4]{x}\right)-\frac{4(a+bx)^{3/4}}{\sqrt[4]{x}}\right)}{x^{3/4}(a+bx)^{3/4}}$$

$$\Big\downarrow \text{813}$$

$$\frac{\left(ax+bx^2\right)^{3/4}\left(12b\left(\frac{x^{3/4}}{2\sqrt[4]{a+bx}}-\frac{a\sqrt[4]{x}\sqrt[4]{\frac{a}{bx}+1}\int\frac{1}{\left(\frac{a}{bx}+1\right)^{5/4}x^{3/4}}d\sqrt[4]{x}}{2b\sqrt[4]{a+bx}}\right)-\frac{4(a+bx)^{3/4}}{\sqrt[4]{x}}\right)}{x^{3/4}(a+bx)^{3/4}}$$

$$\Big\downarrow \text{858}$$

$$\frac{\left(ax+bx^2\right)^{3/4}\left(12b\left(\frac{a\sqrt[4]{x}\sqrt[4]{\frac{a}{bx}+1}\int\frac{1}{\sqrt[4]{x}\left(\frac{ax}{b}+1\right)^{5/4}}d\frac{1}{\sqrt[4]{x}}}{2b\sqrt[4]{a+bx}}+\frac{x^{3/4}}{2\sqrt[4]{a+bx}}\right)-\frac{4(a+bx)^{3/4}}{\sqrt[4]{x}}\right)}{x^{3/4}(a+bx)^{3/4}}$$

$$\Big\downarrow \text{807}$$

$$\frac{\left(ax+bx^2\right)^{3/4}\left(12b\left(\frac{a\sqrt[4]{x}\sqrt[4]{\frac{a}{bx}+1}\int\frac{1}{\left(\frac{\sqrt{x}a}{b}+1\right)^{5/4}}d\sqrt{x}}{4b\sqrt[4]{a+bx}}+\frac{x^{3/4}}{2\sqrt[4]{a+bx}}\right)-\frac{4(a+bx)^{3/4}}{\sqrt[4]{x}}\right)}{x^{3/4}(a+bx)^{3/4}}$$

$$\Big\downarrow \text{212}$$

$$\frac{\left(ax+bx^2\right)^{3/4}\left(12b\left(\frac{\sqrt{a}\sqrt[4]{x}\sqrt[4]{\frac{a}{bx}+1}E\left(\frac{1}{2}\arctan\left(\frac{\sqrt{a}\sqrt{x}}{\sqrt{b}}\right)\middle|2\right)}{2\sqrt{b}\sqrt[4]{a+bx}}+\frac{x^{3/4}}{2\sqrt[4]{a+bx}}\right)-\frac{4(a+bx)^{3/4}}{\sqrt[4]{x}}\right)}{x^{3/4}(a+bx)^{3/4}}$$

```
Int[(a*x + b*x^2)^(3/4)/x^2,x]
```

```
((a*x + b*x^2)^(3/4)*((-4*(a + b*x)^(3/4))/x^(1/4) + 12*b*(x^(3/4)/(2*(a +
 b*x)^(1/4)) + (Sqrt[a]*(1 + a/(b*x))^(1/4)*x^(1/4)*EllipticE[ArcTan[(Sqrt
[a]*Sqrt[x])/Sqrt[b]]/2, 2])/(2*Sqrt[b]*(a + b*x)^(1/4)))))/(x^(3/4)*(a +
b*x)^(3/4))
```

Defintions of rubi rules used

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^n/(b*(m + 1))), x] - Simp[d*(n/(b*(m + 1)))
Int[(a + b*x)^(m + 1)*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b, c, d}, x] &
& GtQ[n, 0] && LtQ[m, -1] &&  !(IntegerQ[n] &&  !IntegerQ[m]) &&  !(ILeQ[m
+ n + 2, 0] && (FractionQ[m] || GeQ[2*n + m + 1, 0])) && IntLinearQ[a, b, c
, d, m, n, x]
```

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Simp[p/b   Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
 d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && Lt
Q[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntL
inearQ[a, b, c, d, m, n, x]
```

```
Int[((a_) + (b_.)*(x_)^2)^(-5/4), x_Symbol] :> Simp[(2/(a^(5/4)*Rt[b/a, 2])
)*EllipticE[(1/2)*ArcTan[Rt[b/a, 2]*x], 2], x] /; FreeQ[{a, b}, x] && GtQ[a
, 0] && PosQ[b/a]
```

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k = GCD[m
+ 1, n]}, Simp[1/k   Subst[Int[x^((m + 1)/k - 1)*(a + b*x^(n/k))^p, x], x,
x^k], x] /; k != 1] /; FreeQ[{a, b, p}, x] && IGtQ[n, 0] && IntegerQ[m]
```

```
Int[(x_)^2/((a_) + (b_.)*(x_)^4)^(5/4), x_Symbol] :> Simp[x*((1 + a/(b*x^4)
)^(1/4)/(b*(a + b*x^4)^(1/4)))   Int[1/(x^3*(1 + a/(b*x^4))^(5/4)), x], x]
/; FreeQ[{a, b}, x] && PosQ[b/a]
```

```
Int[(x_)^2/((a_) + (b_.)*(x_)^4)^(1/4), x_Symbol] :> Simp[x^3/(2*(a + b*x^4
)^(1/4)), x] - Simp[a/2   Int[x^2/(a + b*x^4)^(5/4), x], x] /; FreeQ[{a, b}
, x] && PosQ[b/a]
```

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> -Subst[Int[(a +
b/x^n)^p/x^(m + 2), x], x, 1/x] /; FreeQ[{a, b, p}, x] && ILtQ[n, 0] && Int
egerQ[m]
```

```
Int[((e_.)*(x_))^(m_)*((b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(
e*x)^m*((b*x + c*x^2)^p/(x^(m + p)*(b + c*x)^p))   Int[x^(m + p)*(b + c*x)^
p, x], x] /; FreeQ[{b, c, e, m}, x]
```

Maple [F]

$$\int \frac{(b\,x^2+ax)^{\frac{3}{4}}}{x^2}dx$$

```
int((b*x^2+a*x)^(3/4)/x^2,x)
```

```
int((b*x^2+a*x)^(3/4)/x^2,x)
```

Fricas [F]

$$\int \frac{(ax+bx^2)^{3/4}}{x^2}\,dx = \int \frac{(bx^2+ax)^{\frac{3}{4}}}{x^2}\,dx$$

```
integrate((b*x^2+a*x)^(3/4)/x^2,x, algorithm="fricas")
```

```
integral((b*x^2 + a*x)^(3/4)/x^2, x)
```

Sympy [F]

$$\int \frac{(ax+bx^2)^{3/4}}{x^2}\,dx = \int \frac{(x(a+bx))^{\frac{3}{4}}}{x^2}\,dx$$

```
integrate((b*x**2+a*x)**(3/4)/x**2,x)
```

```
Integral((x*(a + b*x))**(3/4)/x**2, x)
```

Maxima [F]

$$\int \frac{(ax+bx^2)^{3/4}}{x^2}\,dx = \int \frac{(bx^2+ax)^{\frac{3}{4}}}{x^2}\,dx$$

```
integrate((b*x^2+a*x)^(3/4)/x^2,x, algorithm="maxima")
```

```
integrate((b*x^2 + a*x)^(3/4)/x^2, x)
```

Giac [F]

$$\int \frac{(ax+bx^2)^{3/4}}{x^2}\,dx = \int \frac{(bx^2+ax)^{\frac{3}{4}}}{x^2}\,dx$$

```
integrate((b*x^2+a*x)^(3/4)/x^2,x, algorithm="giac")
```

```
integrate((b*x^2 + a*x)^(3/4)/x^2, x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(ax+bx^2)^{3/4}}{x^2}\,dx = \int \frac{(b\,x^2+a\,x)^{3/4}}{x^2}\,dx$$

```
int((a*x + b*x^2)^(3/4)/x^2,x)
```

```
int((a*x + b*x^2)^(3/4)/x^2, x)
```

Reduce [F]

$$\int \frac{(ax+bx^2)^{3/4}}{x^2}\,dx = \frac{4(bx+a)^{\frac{3}{4}}+3x^{\frac{1}{4}}\left(\int \frac{(bx+a)^{\frac{3}{4}}}{x^{\frac{5}{4}}a+x^{\frac{9}{4}}b}dx\right)a}{2x^{\frac{1}{4}}}$$

```
int((b*x^2+a*x)^(3/4)/x^2,x)
```

```
(4*(a + b*x)**(3/4) + 3*x**(1/4)*int((a + b*x)**(3/4)/(x**(1/4)*a*x + x**(
1/4)*b*x**2),x)*a)/(2*x**(1/4))
```

3.160 $\int \frac{(ax+bx^2)^{3/4}}{x^3}\,dx$

Optimal result

Integrand size = 17, antiderivative size = 110

$$\int \frac{(ax+bx^2)^{3/4}}{x^3}\,dx = -\frac{12b}{5\sqrt[4]{ax+bx^2}}-\frac{4(ax+bx^2)^{3/4}}{5x^2}+\frac{12b\sqrt[4]{\frac{bx}{a+bx}}(ax+bx^2)^{3/4}E\left(\frac{1}{2}\arcsin\left(\frac{\sqrt{a}}{\sqrt{a+bx}}\right)\middle|2\right)}{5\sqrt{a}x\sqrt{a+bx}}$$

```
-12/5*b/(b*x^2+a*x)^(1/4)-4/5*(b*x^2+a*x)^(3/4)/x^2+12/5*b*(b*x/(b*x+a))^(
1/4)*(b*x^2+a*x)^(3/4)*EllipticE(sin(1/2*arcsin(a^(1/2)/(b*x+a)^(1/2))),2^
(1/2))/a^(1/2)/x/(b*x+a)^(1/2)
```

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 10.01 (sec) , antiderivative size = 47, normalized size of antiderivative = 0.43

$$\int \frac{(ax+bx^2)^{3/4}}{x^3}\,dx = -\frac{4(x(a+bx))^{3/4}\,\text{Hypergeometric2F1}\left(-\frac{5}{4},-\frac{3}{4},-\frac{1}{4},-\frac{bx}{a}\right)}{5x^2\left(1+\frac{bx}{a}\right)^{3/4}}$$

```
Integrate[(a*x + b*x^2)^(3/4)/x^3,x]
```

```
(-4*(x*(a + b*x))^(3/4)*Hypergeometric2F1[-5/4, -3/4, -1/4, -((b*x)/a)])/(
5*x^2*(1 + (b*x)/a)^(3/4))
```

Rubi [A] (warning: unable to verify)

Time = 0.51 (sec) , antiderivative size = 162, normalized size of antiderivative = 1.47, number of steps used = 10, number of rules used = 9, $\frac{\text{number of rules}}{\text{integrand size}} = 0.529$, Rules used = {1137, 57, 61, 73, 839, 813, 858, 807, 212}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\left(ax+bx^2\right)^{3/4}}{x^3}dx$$

$$\Big\downarrow \text{1137}$$

$$\frac{\left(ax+bx^2\right)^{3/4}\int \frac{(a+bx)^{3/4}}{x^{9/4}}dx}{x^{3/4}(a+bx)^{3/4}}$$

$$\Big\downarrow \text{57}$$

$$\frac{\left(ax+bx^2\right)^{3/4}\left(\frac{3}{5}b\int \frac{1}{x^{5/4}\sqrt[4]{a+bx}}dx-\frac{4(a+bx)^{3/4}}{5x^{5/4}}\right)}{x^{3/4}(a+bx)^{3/4}}$$

$$\Big\downarrow \text{61}$$

$$\frac{\left(ax+bx^2\right)^{3/4}\left(\frac{3}{5}b\left(\frac{2b\int \frac{1}{\sqrt[4]{x}\sqrt[4]{a+bx}}dx}{a}-\frac{4(a+bx)^{3/4}}{a\sqrt[4]{x}}\right)-\frac{4(a+bx)^{3/4}}{5x^{5/4}}\right)}{x^{3/4}(a+bx)^{3/4}}$$

$$\Big\downarrow \text{73}$$

$$\frac{\left(ax+bx^2\right)^{3/4}\left(\frac{3}{5}b\left(\frac{8b\int \frac{\sqrt{x}}{\sqrt[4]{a+bx}}d\sqrt[4]{x}}{a}-\frac{4(a+bx)^{3/4}}{a\sqrt[4]{x}}\right)-\frac{4(a+bx)^{3/4}}{5x^{5/4}}\right)}{x^{3/4}(a+bx)^{3/4}}$$

$$\Big\downarrow \text{839}$$

$$\frac{\left(ax+bx^2\right)^{3/4}\left(\frac{3}{5}b\left(\frac{8b\left(\frac{x^{3/4}}{2\sqrt[4]{a+bx}}-\frac{1}{2}a\int\frac{\sqrt{x}}{(a+bx)^{5/4}}d\sqrt[4]{x}\right)}{a}-\frac{4(a+bx)^{3/4}}{a\sqrt[4]{x}}\right)-\frac{4(a+bx)^{3/4}}{5x^{5/4}}\right)}{x^{3/4}(a+bx)^{3/4}}$$

$$\Big\downarrow \text{ 813}$$

$$\frac{\left(ax+bx^2\right)^{3/4}\left(\frac{3}{5}b\left(\frac{8b\left(\frac{x^{3/4}}{2\sqrt[4]{a+bx}}-\frac{a\sqrt[4]{x}\sqrt[4]{\frac{a}{bx}+1}\int\frac{1}{\left(\frac{a}{bx}+1\right)^{5/4}x^{3/4}}d\sqrt[4]{x}}{2b\sqrt[4]{a+bx}}\right)}{a}-\frac{4(a+bx)^{3/4}}{a\sqrt[4]{x}}\right)-\frac{4(a+bx)^{3/4}}{5x^{5/4}}\right)}{x^{3/4}(a+bx)^{3/4}}$$

$$\Big\downarrow \text{ 858}$$

$$\frac{\left(ax+bx^2\right)^{3/4}\left(\frac{3}{5}b\left(\frac{8b\left(\frac{a\sqrt[4]{x}\sqrt[4]{\frac{a}{bx}+1}\int\frac{1}{\sqrt[4]{x}\left(\frac{ax}{b}+1\right)^{5/4}}d\frac{1}{\sqrt[4]{x}}}{2b\sqrt[4]{a+bx}}+\frac{x^{3/4}}{2\sqrt[4]{a+bx}}\right)}{a}-\frac{4(a+bx)^{3/4}}{a\sqrt[4]{x}}\right)-\frac{4(a+bx)^{3/4}}{5x^{5/4}}\right)}{x^{3/4}(a+bx)^{3/4}}$$

$$\Big\downarrow \text{ 807}$$

$$\frac{\left(ax+bx^2\right)^{3/4}\left(\frac{3}{5}b\left(\frac{8b\left(\frac{a\sqrt[4]{x}\sqrt[4]{\frac{a}{bx}+1}\int\frac{1}{\left(\frac{\sqrt{x}a}{b}+1\right)^{5/4}}d\sqrt{x}}{4b\sqrt[4]{a+bx}}+\frac{x^{3/4}}{2\sqrt[4]{a+bx}}\right)}{a}-\frac{4(a+bx)^{3/4}}{a\sqrt[4]{x}}\right)-\frac{4(a+bx)^{3/4}}{5x^{5/4}}\right)}{x^{3/4}(a+bx)^{3/4}}$$

$$\Big\downarrow \ 212$$

$$\frac{(ax+bx^2)^{3/4}\left(\frac{3}{5}b\left(\frac{8b\left(\frac{\sqrt{a}\sqrt[4]{x}\sqrt[4]{\frac{a}{bx}+1}E\left(\frac{1}{2}\arctan\left(\frac{\sqrt{a}\sqrt{x}}{\sqrt{b}}\right)\middle|2\right)}{2\sqrt{b}\sqrt[4]{a+bx}}+\frac{x^{3/4}}{2\sqrt[4]{a+bx}}\right)}{a}-\frac{4(a+bx)^{3/4}}{a\sqrt[4]{x}}\right)-\frac{4(a+bx)^{3/4}}{5x^{5/4}}\right)}{x^{3/4}(a+bx)^{3/4}}$$

```
Int[(a*x + b*x^2)^(3/4)/x^3,x]
```

```
((a*x + b*x^2)^(3/4)*((-4*(a + b*x)^(3/4))/(5*x^(5/4)) + (3*b*((-4*(a + b*
x)^(3/4))/(a*x^(1/4)) + (8*b*(x^(3/4)/(2*(a + b*x)^(1/4)) + (Sqrt[a]*(1 +
a/(b*x))^(1/4)*x^(1/4)*EllipticE[ArcTan[(Sqrt[a]*Sqrt[x])/Sqrt[b]]/2, 2])/
(2*Sqrt[b]*(a + b*x)^(1/4))))/a))/5))/(x^(3/4)*(a + b*x)^(3/4))
```

Defintions of rubi rules used

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^n/(b*(m + 1))), x] - Simp[d*(n/(b*(m + 1)))
Int[(a + b*x)^(m + 1)*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b, c, d}, x] &
& GtQ[n, 0] && LtQ[m, -1] && !(IntegerQ[n] && !IntegerQ[m]) && !(ILeQ[m
+ n + 2, 0] && (FractionQ[m] || GeQ[2*n + m + 1, 0])) && IntLinearQ[a, b, c
, d, m, n, x]
```

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] - Simp[d*((
m + n + 2)/((b*c - a*d)*(m + 1)))   Int[(a + b*x)^(m + 1)*(c + d*x)^n, x],
x] /; FreeQ[{a, b, c, d, n}, x] && LtQ[m, -1] && !(LtQ[n, -1] && (EqQ[a, 0
] || (NeQ[c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && IntLinearQ[a, b, c, d
, m, n, x]
```

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Simp[p/b   Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
 d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && Lt
Q[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntL
inearQ[a, b, c, d, m, n, x]
```

```
Int[((a_) + (b_.)*(x_)^2)^(-5/4), x_Symbol] :> Simp[(2/(a^(5/4)*Rt[b/a, 2])
)*EllipticE[(1/2)*ArcTan[Rt[b/a, 2]*x], 2], x] /; FreeQ[{a, b}, x] && GtQ[a
, 0] && PosQ[b/a]
```

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k = GCD[m
+ 1, n]}, Simp[1/k   Subst[Int[x^((m + 1)/k - 1)*(a + b*x^(n/k))^p, x], x,
x^k], x] /; k != 1] /; FreeQ[{a, b, p}, x] && IGtQ[n, 0] && IntegerQ[m]
```

```
Int[(x_)^2/((a_) + (b_.)*(x_)^4)^(5/4), x_Symbol] :> Simp[x*((1 + a/(b*x^4)
)^(1/4)/(b*(a + b*x^4)^(1/4)))   Int[1/(x^3*(1 + a/(b*x^4))^(5/4)), x], x]
/; FreeQ[{a, b}, x] && PosQ[b/a]
```

```
Int[(x_)^2/((a_) + (b_.)*(x_)^4)^(1/4), x_Symbol] :> Simp[x^3/(2*(a + b*x^4
)^(1/4)), x] - Simp[a/2   Int[x^2/(a + b*x^4)^(5/4), x], x] /; FreeQ[{a, b}
, x] && PosQ[b/a]
```

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> -Subst[Int[(a +
b/x^n)^p/x^(m + 2), x], x, 1/x] /; FreeQ[{a, b, p}, x] && ILtQ[n, 0] && Int
egerQ[m]
```

```
Int[((e_.)*(x_))^(m_)*((b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(
e*x)^m*((b*x + c*x^2)^p/(x^(m + p)*(b + c*x)^p))   Int[x^(m + p)*(b + c*x)^
p, x], x] /; FreeQ[{b, c, e, m}, x]
```

Maple [F]

$$\int \frac{(b\,x^2 + ax)^{\frac{3}{4}}}{x^3} dx$$

```
int((b*x^2+a*x)^(3/4)/x^3,x)
```

```
int((b*x^2+a*x)^(3/4)/x^3,x)
```

Fricas [F]

$$\int \frac{(ax + bx^2)^{3/4}}{x^3} dx = \int \frac{(bx^2 + ax)^{\frac{3}{4}}}{x^3} dx$$

```
integrate((b*x^2+a*x)^(3/4)/x^3,x, algorithm="fricas")
```

```
integral((b*x^2 + a*x)^(3/4)/x^3, x)
```

Sympy [F]

$$\int \frac{(ax + bx^2)^{3/4}}{x^3} dx = \int \frac{(x(a + bx))^{\frac{3}{4}}}{x^3} dx$$

```
integrate((b*x**2+a*x)**(3/4)/x**3,x)
```

```
Integral((x*(a + b*x))**(3/4)/x**3, x)
```

Maxima [F]

$$\int \frac{(ax+bx^2)^{3/4}}{x^3}\,dx=\int \frac{(bx^2+ax)^{\frac{3}{4}}}{x^3}\,dx$$

```
integrate((b*x^2+a*x)^(3/4)/x^3,x, algorithm="maxima")
```

```
integrate((b*x^2 + a*x)^(3/4)/x^3, x)
```

Giac [F]

$$\int \frac{(ax+bx^2)^{3/4}}{x^3}\,dx=\int \frac{(bx^2+ax)^{\frac{3}{4}}}{x^3}\,dx$$

```
integrate((b*x^2+a*x)^(3/4)/x^3,x, algorithm="giac")
```

```
integrate((b*x^2 + a*x)^(3/4)/x^3, x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(ax+bx^2)^{3/4}}{x^3}\,dx=\int \frac{(b\,x^2+a\,x)^{3/4}}{x^3}\,dx$$

```
int((a*x + b*x^2)^(3/4)/x^3,x)
```

```
int((a*x + b*x^2)^(3/4)/x^3, x)
```

Reduce [F]

$$\int \frac{(ax+bx^2)^{3/4}}{x^3}\,dx = \frac{-4(bx+a)^{\frac{3}{4}} - 3x^{\frac{5}{4}}\left(\int \frac{(bx+a)^{\frac{3}{4}}}{x^{\frac{9}{4}}a+x^{\frac{13}{4}}b}dx\right)a}{2x^{\frac{5}{4}}}$$

```
int((b*x^2+a*x)^(3/4)/x^3,x)
```

```
( - 4*(a + b*x)**(3/4) - 3*x**(1/4)*int((a + b*x)**(3/4)/(x**(1/4)*a*x**2
+ x**(1/4)*b*x**3),x)*a*x)/(2*x**(1/4)*x)
```

3.161 $\int \frac{(ax+bx^2)^{3/4}}{x^4}\,dx$

Optimal result

Integrand size = 17, antiderivative size = 141

$$\int \frac{(ax+bx^2)^{3/4}}{x^4}\,dx = \frac{8b^2}{15a\sqrt[4]{ax+bx^2}} - \frac{4(ax+bx^2)^{3/4}}{9x^3} - \frac{4b(ax+bx^2)^{3/4}}{15ax^2} - \frac{8b^2\sqrt[4]{\frac{bx}{a+bx}}(ax+bx^2)^{3/4}\,E\left(\frac{1}{2}\arcsin\left(\frac{\sqrt{a}}{\sqrt{a+bx}}\right)\middle|2\right)}{15a^{3/2}x\sqrt{a+bx}}$$

```
8/15*b^2/a/(b*x^2+a*x)^(1/4)-4/9*(b*x^2+a*x)^(3/4)/x^3-4/15*b*(b*x^2+a*x)^
(3/4)/a/x^2-8/15*b^2*(b*x/(b*x+a))^(1/4)*(b*x^2+a*x)^(3/4)*EllipticE(sin(1
/2*arcsin(a^(1/2)/(b*x+a)^(1/2))),2^(1/2))/a^(3/2)/x/(b*x+a)^(1/2)
```

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 10.01 (sec) , antiderivative size = 47, normalized size of antiderivative = 0.33

$$\int \frac{(ax+bx^2)^{3/4}}{x^4}\,dx = -\frac{4(x(a+bx))^{3/4}\,\text{Hypergeometric2F1}\left(-\frac{9}{4},-\frac{3}{4},-\frac{5}{4},-\frac{bx}{a}\right)}{9x^3\left(1+\frac{bx}{a}\right)^{3/4}}$$

```
Integrate[(a*x + b*x^2)^(3/4)/x^4,x]
```

```
(-4*(x*(a + b*x))^(3/4)*Hypergeometric2F1[-9/4, -3/4, -5/4, -((b*x)/a)])/(
9*x^3*(1 + (b*x)/a)^(3/4))
```

Rubi [A] (warning: unable to verify)

Time = 0.53 (sec) , antiderivative size = 192, normalized size of antiderivative = 1.36, number of steps used = 11, number of rules used = 10, $\frac{\text{number of rules}}{\text{integrand size}} = 0.588$, Rules used = {1137, 57, 61, 61, 73, 839, 813, 858, 807, 212}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\left(ax+bx^2\right)^{3/4}}{x^4} \, dx$$

$$\Big\downarrow \text{ 1137}$$

$$\frac{\left(ax+bx^2\right)^{3/4} \int \frac{(a+bx)^{3/4}}{x^{13/4}}dx}{x^{3/4}(a+bx)^{3/4}}$$

$$\Big\downarrow \text{ 57}$$

$$\frac{\left(ax+bx^2\right)^{3/4} \left(\frac{1}{3}b\int \frac{1}{x^{9/4}\sqrt[4]{a+bx}}dx-\frac{4(a+bx)^{3/4}}{9x^{9/4}}\right)}{x^{3/4}(a+bx)^{3/4}}$$

$$\Big\downarrow \text{ 61}$$

$$\frac{\left(ax+bx^2\right)^{3/4} \left(\frac{1}{3}b\left(-\frac{2b\int \frac{1}{x^{5/4}\sqrt[4]{a+bx}}dx}{5a}-\frac{4(a+bx)^{3/4}}{5ax^{5/4}}\right)-\frac{4(a+bx)^{3/4}}{9x^{9/4}}\right)}{x^{3/4}(a+bx)^{3/4}}$$

$$\Big\downarrow \text{ 61}$$

$$\frac{\left(ax+bx^2\right)^{3/4}\left(\frac{1}{3}b\left(-\frac{2b\left(\frac{2b\int \frac{1}{\sqrt[4]{x}\sqrt[4]{a+bx}}dx}{a}-\frac{4(a+bx)^{3/4}}{a\sqrt[4]{x}}\right)}{5a}-\frac{4(a+bx)^{3/4}}{5ax^{5/4}}\right)-\frac{4(a+bx)^{3/4}}{9x^{9/4}}\right)}{x^{3/4}(a+bx)^{3/4}}$$

$$\Big\downarrow \; 73$$

$$\frac{\left(ax+bx^2\right)^{3/4}\left(\frac{1}{3}b\left(-\frac{2b\left(\frac{8b\int \frac{\sqrt{x}}{\sqrt[4]{a+bx}}d\sqrt[4]{x}}{a}-\frac{4(a+bx)^{3/4}}{a\sqrt[4]{x}}\right)}{5a}-\frac{4(a+bx)^{3/4}}{5ax^{5/4}}\right)-\frac{4(a+bx)^{3/4}}{9x^{9/4}}\right)}{x^{3/4}(a+bx)^{3/4}}$$

$$\Big\downarrow \; 839$$

$$\frac{\left(ax+bx^2\right)^{3/4}\left(\frac{1}{3}b\left(-\frac{2b\left(\frac{8b\left(\frac{x^{3/4}}{2\sqrt[4]{a+bx}}-\frac{1}{2}a\int \frac{\sqrt{x}}{(a+bx)^{5/4}}d\sqrt[4]{x}\right)}{a}-\frac{4(a+bx)^{3/4}}{a\sqrt[4]{x}}\right)}{5a}-\frac{4(a+bx)^{3/4}}{5ax^{5/4}}\right)-\frac{4(a+bx)^{3/4}}{9x^{9/4}}\right)}{x^{3/4}(a+bx)^{3/4}}$$

$$\Big\downarrow \; 813$$

$$\frac{\left(ax+bx^2\right)^{3/4}\left(\frac{1}{3}b\left(-\frac{2b\frac{8b\left(\frac{x^{3/4}}{2\sqrt[4]{a+bx}}-\frac{a\sqrt[4]{x}\sqrt[4]{\frac{a}{bx}+1}\int\frac{1}{\left(\frac{a}{bx}+1\right)^{5/4}x^{3/4}}d\sqrt[4]{x}}{2b\sqrt[4]{a+bx}}\right)}{a}-\frac{4(a+bx)^{3/4}}{a\sqrt[4]{x}}}{5a}-\frac{4(a+bx)^{3/4}}{5ax^{5/4}}\right)-\frac{4(a+bx)^{3}}{9x^{9/4}}\right.}{x^{3/4}(a+bx)^{3/4}}$$

$$\Big\downarrow \ 858$$

$$\frac{\left(ax+bx^2\right)^{3/4}\left(\frac{1}{3}b\left(-\frac{2b\frac{8b\left(\frac{a\sqrt[4]{x}\sqrt[4]{\frac{a}{bx}+1}\int\frac{1}{\sqrt[4]{x}\left(\frac{ax}{b}+1\right)^{5/4}}d\frac{1}{\sqrt[4]{x}}}{2b\sqrt[4]{a+bx}}+\frac{x^{3/4}}{2\sqrt[4]{a+bx}}\right)}{a}-\frac{4(a+bx)^{3/4}}{a\sqrt[4]{x}}}{5a}-\frac{4(a+bx)^{3/4}}{5ax^{5/4}}\right)-\frac{4(a+bx)}{9x^{9/4}}\right.}{x^{3/4}(a+bx)^{3/4}}$$

$$\Big\downarrow \ 807$$

$$\frac{\left(ax+bx^2\right)^{3/4}\left(\frac{1}{3}b\left(-\frac{2b\left(\frac{8b\left(\frac{a\sqrt[4]{x}\sqrt[4]{\frac{a}{bx}+1}\int \frac{1}{\left(\frac{\sqrt{x}a}{b}+1\right)^{5/4}}d\sqrt{x}}{4b\sqrt[4]{a+bx}}+\frac{x^{3/4}}{2\sqrt[4]{a+bx}}\right)}{a}-\frac{4(a+bx)^{3/4}}{a\sqrt[4]{x}}\right)}{5a}-\frac{4(a+bx)^{3/4}}{5ax^{5/4}}\right)-\frac{4(a+bx)^{3/4}}{9x^{9/4}}\right)}{x^{3/4}(a+bx)^{3/4}}$$

$$\Big\downarrow \; 212$$

$$\frac{\left(ax+bx^2\right)^{3/4}\left(\frac{1}{3}b\left(-\frac{2b\left(\frac{8b\left(\frac{\sqrt{a}\sqrt[4]{x}\sqrt[4]{\frac{a}{bx}+1}E\left(\frac{1}{2}\arctan\left(\frac{\sqrt{a}\sqrt{x}}{\sqrt{b}}\right)\middle|2\right)}{2\sqrt{b}\sqrt[4]{a+bx}}+\frac{x^{3/4}}{2\sqrt[4]{a+bx}}\right)}{a}-\frac{4(a+bx)^{3/4}}{a\sqrt[4]{x}}\right)}{5a}-\frac{4(a+bx)^{3/4}}{5ax^{5/4}}\right)-\frac{4(a+bx)^{3/4}}{9x^{9/4}}\right)}{x^{3/4}(a+bx)^{3/4}}$$

```
Int[(a*x + b*x^2)^(3/4)/x^4,x]
```

```
((a*x + b*x^2)^(3/4)*((-4*(a + b*x)^(3/4))/(9*x^(9/4)) + (b*((-4*(a + b*x)
^(3/4))/(5*a*x^(5/4)) - (2*b*((-4*(a + b*x)^(3/4))/(a*x^(1/4)) + (8*b*(x^(
3/4)/(2*(a + b*x)^(1/4)) + (Sqrt[a]*(1 + a/(b*x))^(1/4)*x^(1/4)*EllipticE[
ArcTan[(Sqrt[a]*Sqrt[x])/Sqrt[b]]/2, 2])/(2*Sqrt[b]*(a + b*x)^(1/4))))/a))
/(5*a)))/3))/(x^(3/4)*(a + b*x)^(3/4))
```

Defintions of rubi rules used

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^n/(b*(m + 1))), x] - Simp[d*(n/(b*(m + 1)))
Int[(a + b*x)^(m + 1)*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b, c, d}, x] &
& GtQ[n, 0] && LtQ[m, -1] &&  !(IntegerQ[n] &&  !IntegerQ[m]) &&  !(ILeQ[m
+ n + 2, 0] && (FractionQ[m] || GeQ[2*n + m + 1, 0])) && IntLinearQ[a, b, c
, d, m, n, x]
```

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] - Simp[d*((
m + n + 2)/((b*c - a*d)*(m + 1)))   Int[(a + b*x)^(m + 1)*(c + d*x)^n, x],
x] /; FreeQ[{a, b, c, d, n}, x] && LtQ[m, -1] &&  !(LtQ[n, -1] && (EqQ[a, 0
] || (NeQ[c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && IntLinearQ[a, b, c, d
, m, n, x]
```

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Simp[p/b   Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
 d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && Lt
Q[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntL
inearQ[a, b, c, d, m, n, x]
```

```
Int[((a_) + (b_.)*(x_)^2)^(-5/4), x_Symbol] :> Simp[(2/(a^(5/4)*Rt[b/a, 2])
)*EllipticE[(1/2)*ArcTan[Rt[b/a, 2]*x], 2], x] /; FreeQ[{a, b}, x] && GtQ[a
, 0] && PosQ[b/a]
```

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k = GCD[m
+ 1, n]}, Simp[1/k   Subst[Int[x^((m + 1)/k - 1)*(a + b*x^(n/k))^p, x], x,
x^k], x] /; k != 1] /; FreeQ[{a, b, p}, x] && IGtQ[n, 0] && IntegerQ[m]
```

```
Int[(x_)^2/((a_) + (b_.)*(x_)^4)^(5/4), x_Symbol] :> Simp[x*((1 + a/(b*x^4)
)^(1/4)/(b*(a + b*x^4)^(1/4)))   Int[1/(x^3*(1 + a/(b*x^4))^(5/4)), x], x]
/; FreeQ[{a, b}, x] && PosQ[b/a]
```

```
Int[(x_)^2/((a_) + (b_.)*(x_)^4)^(1/4), x_Symbol] :> Simp[x^3/(2*(a + b*x^4
)^(1/4)), x] - Simp[a/2   Int[x^2/(a + b*x^4)^(5/4), x], x] /; FreeQ[{a, b}
, x] && PosQ[b/a]
```

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> -Subst[Int[(a +
b/x^n)^p/x^(m + 2), x], x, 1/x] /; FreeQ[{a, b, p}, x] && ILtQ[n, 0] && Int
egerQ[m]
```

```
Int[((e_.)*(x_))^(m_)*((b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(
e*x)^m*((b*x + c*x^2)^p/(x^(m + p)*(b + c*x)^p))   Int[x^(m + p)*(b + c*x)^
p, x], x] /; FreeQ[{b, c, e, m}, x]
```

Maple [F]

$$\int \frac{(b\,x^2 + ax)^{\frac{3}{4}}}{x^4}dx$$

```
int((b*x^2+a*x)^(3/4)/x^4,x)
```

```
int((b*x^2+a*x)^(3/4)/x^4,x)
```

Fricas [F]

$$\int \frac{(ax + bx^2)^{3/4}}{x^4}\,dx = \int \frac{(bx^2 + ax)^{\frac{3}{4}}}{x^4}\,dx$$

```
integrate((b*x^2+a*x)^(3/4)/x^4,x, algorithm="fricas")
```

```
integral((b*x^2 + a*x)^(3/4)/x^4, x)
```

Sympy [F]

$$\int \frac{(ax + bx^2)^{3/4}}{x^4}\,dx = \int \frac{(x(a + bx))^{\frac{3}{4}}}{x^4}\,dx$$

```
integrate((b*x**2+a*x)**(3/4)/x**4,x)
```

```
Integral((x*(a + b*x))**(3/4)/x**4, x)
```

Maxima [F]

$$\int \frac{(ax + bx^2)^{3/4}}{x^4}\,dx = \int \frac{(bx^2 + ax)^{\frac{3}{4}}}{x^4}\,dx$$

```
integrate((b*x^2+a*x)^(3/4)/x^4,x, algorithm="maxima")
```

```
integrate((b*x^2 + a*x)^(3/4)/x^4, x)
```

Giac [F]

$$\int \frac{(ax+bx^2)^{3/4}}{x^4}\,dx = \int \frac{(bx^2+ax)^{\frac{3}{4}}}{x^4}\,dx$$

```
integrate((b*x^2+a*x)^(3/4)/x^4,x, algorithm="giac")
```

```
integrate((b*x^2 + a*x)^(3/4)/x^4, x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(ax+bx^2)^{3/4}}{x^4}\,dx = \int \frac{(b\,x^2+a\,x)^{3/4}}{x^4}\,dx$$

```
int((a*x + b*x^2)^(3/4)/x^4,x)
```

```
int((a*x + b*x^2)^(3/4)/x^4, x)
```

Reduce [F]

$$\int \frac{(ax+bx^2)^{3/4}}{x^4}\,dx = \frac{-4(bx+a)^{\frac{3}{4}} - 3x^{\frac{9}{4}}\left(\int \frac{(bx+a)^{\frac{3}{4}}}{x^{\frac{13}{4}}a+x^{\frac{17}{4}}b}dx\right)a}{6x^{\frac{9}{4}}}$$

```
int((b*x^2+a*x)^(3/4)/x^4,x)
```

```
( - 4*(a + b*x)**(3/4) - 3*x**(1/4)*int((a + b*x)**(3/4)/(x**(1/4)*a*x**3
+ x**(1/4)*b*x**4),x)*a*x**2)/(6*x**(1/4)*x**2)
```

3.162 $\int \frac{(ax+bx^2)^{3/4}}{x^5} dx$

Optimal result

Integrand size = 17, antiderivative size = 167

$$\int \frac{(ax+bx^2)^{3/4}}{x^5} dx = -\frac{16b^3}{65a^2\sqrt[4]{ax+bx^2}} - \frac{4(ax+bx^2)^{3/4}}{13x^4} - \frac{4b(ax+bx^2)^{3/4}}{39ax^3} + \frac{8b^2(ax+bx^2)^{3/4}}{65a^2x^2} + \frac{16b^3\sqrt[4]{\frac{bx}{a+bx}}(ax+bx^2)^{3/4} E\left(\frac{1}{2}\arcsin\left(\frac{\sqrt{a}}{\sqrt{a+bx}}\right)\middle|2\right)}{65a^{5/2}x\sqrt{a+bx}}$$

```
-16/65*b^3/a^2/(b*x^2+a*x)^(1/4)-4/13*(b*x^2+a*x)^(3/4)/x^4-4/39*b*(b*x^2+
a*x)^(3/4)/a/x^3+8/65*b^2*(b*x^2+a*x)^(3/4)/a^2/x^2+16/65*b^3*(b*x/(b*x+a)
)^(1/4)*(b*x^2+a*x)^(3/4)*EllipticE(sin(1/2*arcsin(a^(1/2)/(b*x+a)^(1/2)))
,2^(1/2))/a^(5/2)/x/(b*x+a)^(1/2)
```

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 10.01 (sec) , antiderivative size = 47, normalized size of antiderivative = 0.28

$$\int \frac{(ax+bx^2)^{3/4}}{x^5} dx = -\frac{4(x(a+bx))^{3/4}\,\text{Hypergeometric2F1}\left(-\frac{13}{4},-\frac{3}{4},-\frac{9}{4},-\frac{bx}{a}\right)}{13x^4\left(1+\frac{bx}{a}\right)^{3/4}}$$

```
Integrate[(a*x + b*x^2)^(3/4)/x^5,x]
```

```
(-4*(x*(a + b*x))^(3/4)*Hypergeometric2F1[-13/4, -3/4, -9/4, -((b*x)/a)])/
(13*x^4*(1 + (b*x)/a)^(3/4))
```

Rubi [A] (warning: unable to verify)

Time = 0.58 (sec) , antiderivative size = 222, normalized size of antiderivative = 1.33, number of steps used = 12, number of rules used = 11, $\frac{\text{number of rules}}{\text{integrand size}} = 0.647$, Rules used = {1137, 57, 61, 61, 61, 73, 839, 813, 858, 807, 212}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(ax+bx^2)^{3/4}}{x^5}\,dx$$

$$\Big\downarrow \text{1137}$$

$$\frac{(ax+bx^2)^{3/4}\int \frac{(a+bx)^{3/4}}{x^{17/4}}dx}{x^{3/4}(a+bx)^{3/4}}$$

$$\Big\downarrow \text{57}$$

$$\frac{(ax+bx^2)^{3/4}\left(\frac{3}{13}b\int \frac{1}{x^{13/4}\sqrt[4]{a+bx}}dx-\frac{4(a+bx)^{3/4}}{13x^{13/4}}\right)}{x^{3/4}(a+bx)^{3/4}}$$

$$\Big\downarrow \text{61}$$

$$\frac{(ax+bx^2)^{3/4}\left(\frac{3}{13}b\left(-\frac{2b\int \frac{1}{x^{9/4}\sqrt[4]{a+bx}}dx}{3a}-\frac{4(a+bx)^{3/4}}{9ax^{9/4}}\right)-\frac{4(a+bx)^{3/4}}{13x^{13/4}}\right)}{x^{3/4}(a+bx)^{3/4}}$$

$$\Big\downarrow \text{61}$$

$$\frac{(ax+bx^2)^{3/4}\left(\frac{3}{13}b\left(-\frac{2b\left(-\frac{2b\int \frac{1}{x^{5/4}\sqrt[4]{a+bx}}dx}{5a}-\frac{4(a+bx)^{3/4}}{5ax^{5/4}}\right)}{3a}-\frac{4(a+bx)^{3/4}}{9ax^{9/4}}\right)-\frac{4(a+bx)^{3/4}}{13x^{13/4}}\right)}{x^{3/4}(a+bx)^{3/4}}$$

$$\Big\downarrow \text{61}$$

$$\frac{\left(ax+bx^2\right)^{3/4}\left(\frac{3}{13}b\left(-\frac{2b\left(-\frac{2b\left(\frac{2b\int\frac{1}{\sqrt[4]{x}\sqrt[4]{a+bx}}dx}{a}-\frac{4(a+bx)^{3/4}}{a\sqrt[4]{x}}\right)}{5a}-\frac{4(a+bx)^{3/4}}{5ax^{5/4}}\right)}{3a}-\frac{4(a+bx)^{3/4}}{9ax^{9/4}}\right)-\frac{4(a+bx)^{3/4}}{13x^{13/4}}\right)}{x^{3/4}(a+bx)^{3/4}}$$

$$\Big\downarrow \text{73}$$

$$\frac{\left(ax+bx^2\right)^{3/4}\left(\frac{3}{13}b\left(-\frac{2b\left(-\frac{2b\left(\frac{8b\int\frac{\sqrt{x}}{\sqrt[4]{a+bx}}d\sqrt[4]{x}}{a}-\frac{4(a+bx)^{3/4}}{a\sqrt[4]{x}}\right)}{5a}-\frac{4(a+bx)^{3/4}}{5ax^{5/4}}\right)}{3a}-\frac{4(a+bx)^{3/4}}{9ax^{9/4}}\right)-\frac{4(a+bx)^{3/4}}{13x^{13/4}}\right)}{x^{3/4}(a+bx)^{3/4}}$$

$$\Big\downarrow \text{839}$$

$$\frac{\left(ax+bx^2\right)^{3/4}\left(\frac{3}{13}b\left(-\frac{2b\left(-\frac{2b\left(\frac{8b\left(\frac{x^{3/4}}{2\sqrt[4]{a+bx}}-\frac{1}{2}a\int\frac{\sqrt{x}}{(a+bx)^{5/4}}d\sqrt[4]{x}\right)}{a}-\frac{4(a+bx)^{3/4}}{a\sqrt[4]{x}}\right)}{5a}-\frac{4(a+bx)^{3/4}}{5ax^{5/4}}\right)}{3a}-\frac{4(a+bx)^{3/4}}{9ax^{9/4}}\right)-\frac{4(a+bx)^{3}}{13x^{13/4}}\right)}{x^{3/4}(a+bx)^{3/4}}$$

$\big\downarrow$ 813

$$\frac{\left(ax+bx^2\right)^{3/4}\left(\frac{3}{13}b\left(-\frac{2b\left(-\frac{2b\left(\frac{8b\left(\frac{x^{3/4}}{2\sqrt[4]{a+bx}}-\frac{a\sqrt[4]{x}\sqrt[4]{\frac{a}{bx}+1}\int\frac{1}{\left(\frac{a}{bx}+1\right)^{5/4}x^{3/4}}d\sqrt[4]{x}}{2b\sqrt[4]{a+bx}}\right)}{a}-\frac{4(a+bx)^{3/4}}{a\sqrt[4]{x}}\right)}{5a}-\frac{4(a+bx)^{3/4}}{5ax^{5/4}}\right)}{3a}-\frac{4(a+b}{9ax}\right.\right.}{x^{3/4}(a+bx)^{3/4}}$$

$\downarrow$ 858

$$\frac{\left(ax+bx^2\right)^{3/4}\left(\frac{3}{13}b\left(-\frac{2b\left(-\frac{2b\left(\frac{8b\left(\frac{a\sqrt[4]{x}\sqrt[4]{\frac{a}{bx}+1}\int\frac{1}{\sqrt[4]{x}\left(\frac{ax}{b}+1\right)^{5/4}}d\frac{1}{\sqrt[4]{x}}}{2b\sqrt[4]{a+bx}}+\frac{x^{3/4}}{2\sqrt[4]{a+bx}}\right)}{a}-\frac{4(a+bx)^{3/4}}{a\sqrt[4]{x}}\right)}{5a}-\frac{4(a+bx)^{3/4}}{5ax^{5/4}}\right)}{3a}-\frac{4(a+}{9a}\right.\right.}{x^{3/4}(a+bx)^{3/4}}$$

$\Big\downarrow$ 807

$$\frac{\left(ax+bx^2\right)^{3/4}\left(\frac{3}{13}b\left(-\frac{2b\left(-\frac{2b\left(\frac{8b\left(\frac{a\sqrt[4]{x}\sqrt[4]{\frac{a}{bx}+1}\int\frac{1}{\left(\frac{\sqrt{x}a}{b}+1\right)^{5/4}}d\sqrt{x}}{4b\sqrt[4]{a+bx}}+\frac{x^{3/4}}{2\sqrt[4]{a+bx}}\right)}{a}-\frac{4(a+bx)^{3/4}}{a\sqrt[4]{x}}\right)}{5a}-\frac{4(a+bx)^{3/4}}{5ax^{5/4}}\right)}{3a}-\frac{4(a+bx)^{3/4}}{9ax^{9/4}}}{x^{3/4}(a+bx)^{3/4}}$$

↓ 212

$$\frac{\left(ax+bx^2\right)^{3/4}\left(\frac{3}{13}b\left(-\frac{2b\left(-\frac{2b\left(\frac{8b\left(\frac{\sqrt{a}\sqrt[4]{x}\sqrt[4]{\frac{a}{bx}+1}E\left(\frac{1}{2}\arctan\left(\frac{\sqrt{a}\sqrt{x}}{\sqrt{b}}\right)\middle|2\right)}{2\sqrt{b}\sqrt[4]{a+bx}}+\frac{x^{3/4}}{2\sqrt[4]{a+bx}}\right)}{a}-\frac{4(a+bx)^{3/4}}{a\sqrt[4]{x}}\right)}{5a}-\frac{4(a+bx)^{3/4}}{5ax^{5/4}}\right)}{3a}-\frac{4(a+\cdots}{9a\cdots}\right.\right.}{x^{3/4}(a+bx)^{3/4}}$$

```
Int[(a*x + b*x^2)^(3/4)/x^5,x]
```

```
((a*x + b*x^2)^(3/4)*((-4*(a + b*x)^(3/4))/(13*x^(13/4)) + (3*b*((-4*(a +
b*x)^(3/4))/(9*a*x^(9/4)) - (2*b*((-4*(a + b*x)^(3/4))/(5*a*x^(5/4)) - (2*
b*((-4*(a + b*x)^(3/4))/(a*x^(1/4)) + (8*b*(x^(3/4)/(2*(a + b*x)^(1/4)) +
(Sqrt[a]*(1 + a/(b*x))^(1/4)*x^(1/4)*EllipticE[ArcTan[(Sqrt[a]*Sqrt[x]])/Sq
rt[b]]/2, 2])/(2*Sqrt[b]*(a + b*x)^(1/4))))/a))/(5*a)))/(3*a)))/13))/(x^(3
/4)*(a + b*x)^(3/4))
```

Defintions of rubi rules used

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^n/(b*(m + 1))), x] - Simp[d*(n/(b*(m + 1)))
Int[(a + b*x)^(m + 1)*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b, c, d}, x] &
& GtQ[n, 0] && LtQ[m, -1] &&  !(IntegerQ[n] &&  !IntegerQ[m]) &&  !(ILeQ[m
+ n + 2, 0] && (FractionQ[m] || GeQ[2*n + m + 1, 0])) && IntLinearQ[a, b, c
, d, m, n, x]
```

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] - Simp[d*((
m + n + 2)/((b*c - a*d)*(m + 1)))   Int[(a + b*x)^(m + 1)*(c + d*x)^n, x],
x] /; FreeQ[{a, b, c, d, n}, x] && LtQ[m, -1] &&  !(LtQ[n, -1] && (EqQ[a, 0
] || (NeQ[c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && IntLinearQ[a, b, c, d
, m, n, x]
```

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Simp[p/b   Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
 d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && Lt
Q[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntL
inearQ[a, b, c, d, m, n, x]
```

```
Int[((a_) + (b_.)*(x_)^2)^(-5/4), x_Symbol] :> Simp[(2/(a^(5/4)*Rt[b/a, 2])
)*EllipticE[(1/2)*ArcTan[Rt[b/a, 2]*x], 2], x] /; FreeQ[{a, b}, x] && GtQ[a
, 0] && PosQ[b/a]
```

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k = GCD[m
+ 1, n]}, Simp[1/k   Subst[Int[x^((m + 1)/k - 1)*(a + b*x^(n/k))^p, x], x,
x^k], x] /; k != 1] /; FreeQ[{a, b, p}, x] && IGtQ[n, 0] && IntegerQ[m]
```

```
Int[(x_)^2/((a_) + (b_.)*(x_)^4)^(5/4), x_Symbol] :> Simp[x*((1 + a/(b*x^4)
)^(1/4)/(b*(a + b*x^4)^(1/4)))   Int[1/(x^3*(1 + a/(b*x^4))^(5/4)), x], x]
/; FreeQ[{a, b}, x] && PosQ[b/a]
```

```
Int[(x_)^2/((a_) + (b_.)*(x_)^4)^(1/4), x_Symbol] :> Simp[x^3/(2*(a + b*x^4
)^(1/4)), x] - Simp[a/2   Int[x^2/(a + b*x^4)^(5/4), x], x] /; FreeQ[{a, b}
, x] && PosQ[b/a]
```

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> -Subst[Int[(a +
b/x^n)^p/x^(m + 2), x], x, 1/x] /; FreeQ[{a, b, p}, x] && ILtQ[n, 0] && Int
egerQ[m]
```

```
Int[((e_.)*(x_))^(m_)*((b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(
e*x)^m*((b*x + c*x^2)^p/(x^(m + p)*(b + c*x)^p))   Int[x^(m + p)*(b + c*x)^
p, x], x] /; FreeQ[{b, c, e, m}, x]
```

Maple [F]

$$\int \frac{\left(b\,x^{2}+a x \right)^{\frac{3}{4}}}{x^{5}}d x$$

```
int((b*x^2+a*x)^(3/4)/x^5,x)
```

```
int((b*x^2+a*x)^(3/4)/x^5,x)
```

Fricas [F]

$$\int \frac{\left(ax+bx^2\right)^{3/4}}{x^5}\,dx=\int \frac{\left(bx^{2}+ax\right)^{\frac{3}{4}}}{x^{5}}\,dx$$

```
integrate((b*x^2+a*x)^(3/4)/x^5,x, algorithm="fricas")
```

```
integral((b*x^2 + a*x)^(3/4)/x^5, x)
```

Sympy [F]

$$\int \frac{(ax+bx^2)^{3/4}}{x^5}\,dx = \int \frac{(x(a+bx))^{\frac{3}{4}}}{x^5}\,dx$$

```
integrate((b*x**2+a*x)**(3/4)/x**5,x)
```

```
Integral((x*(a + b*x))**(3/4)/x**5, x)
```

Maxima [F]

$$\int \frac{(ax+bx^2)^{3/4}}{x^5}\,dx = \int \frac{(bx^2+ax)^{\frac{3}{4}}}{x^5}\,dx$$

```
integrate((b*x^2+a*x)^(3/4)/x^5,x, algorithm="maxima")
```

```
integrate((b*x^2 + a*x)^(3/4)/x^5, x)
```

Giac [F]

$$\int \frac{(ax+bx^2)^{3/4}}{x^5}\,dx = \int \frac{(bx^2+ax)^{\frac{3}{4}}}{x^5}\,dx$$

```
integrate((b*x^2+a*x)^(3/4)/x^5,x, algorithm="giac")
```

```
integrate((b*x^2 + a*x)^(3/4)/x^5, x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(ax+bx^2)^{3/4}}{x^5}\,dx = \int \frac{(b\,x^2+a\,x)^{3/4}}{x^5}\,dx$$

```
int((a*x + b*x^2)^(3/4)/x^5,x)
```

```
int((a*x + b*x^2)^(3/4)/x^5, x)
```

Reduce [F]

$$\int \frac{(ax+bx^2)^{3/4}}{x^5}\,dx = \frac{-4(bx+a)^{\frac{3}{4}}-3x^{\frac{13}{4}}\left(\int \frac{(bx+a)^{\frac{3}{4}}}{x^{\frac{17}{4}}a+x^{\frac{21}{4}}b}dx\right)a}{10x^{\frac{13}{4}}}$$

```
int((b*x^2+a*x)^(3/4)/x^5,x)
```

```
( - 4*(a + b*x)**(3/4) - 3*x**(1/4)*int((a + b*x)**(3/4)/(x**(1/4)*a*x**4
+ x**(1/4)*b*x**5),x)*a*x**3)/(10*x**(1/4)*x**3)
```

3.163 $\int x^2(ax+bx^2)^{5/4}\,dx$

Optimal result

Integrand size = 17, antiderivative size = 214

$$\int x^2\left(ax+bx^2\right)^{5/4}\,dx=-\frac{65a^5\sqrt[4]{ax+bx^2}}{3696b^4}+\frac{13a^4x\sqrt[4]{ax+bx^2}}{1848b^3}-\frac{13a^3x^2\sqrt[4]{ax+bx^2}}{2772b^2}+\frac{5a^2x^3\sqrt[4]{ax+bx^2}}{1386b}+\frac{5}{99}ax^4\sqrt[4]{ax+bx^2}+\frac{2}{11}x^3\left(ax+bx^2\right)^{5/4}-\frac{65a^{11/2}\left(\frac{bx}{a+bx}\right)^{3/4}\sqrt{a+bx}\sqrt[4]{ax+bx^2}\,\text{EllipticF}\left(\frac{1}{2}\arcsin\left(\frac{\sqrt{a}}{\sqrt{a+bx}}\right),2\right)}{3696b^5x}$$

```
-65/3696*a^5*(b*x^2+a*x)^(1/4)/b^4+13/1848*a^4*x*(b*x^2+a*x)^(1/4)/b^3-13/
2772*a^3*x^2*(b*x^2+a*x)^(1/4)/b^2+5/1386*a^2*x^3*(b*x^2+a*x)^(1/4)/b+5/99
*a*x^4*(b*x^2+a*x)^(1/4)+2/11*x^3*(b*x^2+a*x)^(5/4)-65/3696*a^(11/2)*(b*x/
(b*x+a))^(3/4)*(b*x+a)^(1/2)*(b*x^2+a*x)^(1/4)*InverseJacobiAM(1/2*arcsin(
a^(1/2)/(b*x+a)^(1/2)),2^(1/2))/b^5/x
```

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 10.01 (sec) , antiderivative size = 48, normalized size of antiderivative = 0.22

$$\int x^2 (ax+bx^2)^{5/4}\,dx = \frac{4ax^4\sqrt[4]{x(a+bx)}\,\text{Hypergeometric2F1}\left(-\frac{5}{4},\frac{17}{4},\frac{21}{4},-\frac{bx}{a}\right)}{17\sqrt[4]{1+\frac{bx}{a}}}$$

```
Integrate[x^2*(a*x + b*x^2)^(5/4),x]
```

```
(4*a*x^4*(x*(a + b*x))^(1/4)*Hypergeometric2F1[-5/4, 17/4, 21/4, -((b*x)/a
)])/(17*(1 + (b*x)/a)^(1/4))
```

Rubi [A] (warning: unable to verify)

Time = 0.62 (sec) , antiderivative size = 249, normalized size of antiderivative = 1.16, number of steps used = 13, number of rules used = 12, $\frac{\text{number of rules}}{\text{integrand size}} = 0.706$, Rules used = {1137, 60, 60, 60, 60, 60, 60, 73, 768, 858, 807, 229}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^2 (ax+bx^2)^{5/4}\,dx$$

$$\Big\downarrow \text{1137}$$

$$\frac{(ax+bx^2)^{5/4}\int x^{13/4}(a+bx)^{5/4}dx}{x^{5/4}(a+bx)^{5/4}}$$

$$\Big\downarrow \text{60}$$

$$\frac{(ax+bx^2)^{5/4}\left(\frac{5}{22}a\int x^{13/4}\sqrt[4]{a+bx}dx+\frac{2}{11}x^{17/4}(a+bx)^{5/4}\right)}{x^{5/4}(a+bx)^{5/4}}$$

$$\Big\downarrow \text{60}$$

$$\frac{(ax+bx^2)^{5/4}\left(\frac{5}{22}a\left(\frac{1}{18}a\int\frac{x^{13/4}}{(a+bx)^{3/4}}dx+\frac{2}{9}x^{17/4}\sqrt[4]{a+bx}\right)+\frac{2}{11}x^{17/4}(a+bx)^{5/4}\right)}{x^{5/4}(a+bx)^{5/4}}$$

$$\Big\downarrow \text{60}$$

$$\frac{(ax+bx^2)^{5/4}\left(\frac{5}{22}a\left(\frac{1}{18}a\left(\frac{2x^{13/4}\sqrt[4]{a+bx}}{7b}-\frac{13a\int\frac{x^{9/4}}{(a+bx)^{3/4}}dx}{14b}\right)+\frac{2}{9}x^{17/4}\sqrt[4]{a+bx}\right)+\frac{2}{11}x^{17/4}(a+bx)^{5/4}\right)}{x^{5/4}(a+bx)^{5/4}}$$

$$\Big\downarrow \text{60}$$

$$\frac{(ax+bx^2)^{5/4}\left(\frac{5}{22}a\left(\frac{1}{18}a\left(\frac{2x^{13/4}\sqrt[4]{a+bx}}{7b}-\frac{13a\left(\frac{2x^{9/4}\sqrt[4]{a+bx}}{5b}-\frac{9a\int\frac{x^{5/4}}{(a+bx)^{3/4}}dx}{10b}\right)}{14b}\right)+\frac{2}{9}x^{17/4}\sqrt[4]{a+bx}\right)+\frac{2}{11}x^{17/4}\right.}{x^{5/4}(a+bx)^{5/4}}$$

$$\Big\downarrow \text{60}$$

$$\frac{(ax+bx^2)^{5/4}\left(\frac{5}{22}a\left(\frac{1}{18}a\left(\frac{2x^{13/4}\sqrt[4]{a+bx}}{7b}-\frac{13a\left(\frac{2x^{9/4}\sqrt[4]{a+bx}}{5b}-\frac{9a\left(\frac{2x^{5/4}\sqrt[4]{a+bx}}{3b}-\frac{5a\int\frac{\sqrt[4]{x}}{(a+bx)^{3/4}}dx}{6b}\right)}{10b}\right)}{14b}\right)+\frac{2}{9}x^{17/4}\sqrt[4]{\right.\right.}{x^{5/4}(a+bx)^{5/4}}$$

$$\Big\downarrow \text{60}$$

$$\frac{\left(ax+bx^2\right)^{5/4}\Bigg(\frac{5}{22}a\Bigg(\frac{1}{18}a\Bigg(\frac{2x^{13/4}\sqrt[4]{a+bx}}{7b}-\frac{13a\Bigg(\frac{2x^{9/4}\sqrt[4]{a+bx}}{5b}-\frac{9a\Bigg(\frac{2x^{5/4}\sqrt[4]{a+bx}}{3b}-\frac{5a\Bigg(\frac{2\sqrt[4]{x}\sqrt[4]{a+bx}}{b}-\frac{a\int\frac{1}{x^{3/4}(a+bx)}}{2b}}{6b}}{10b}}{14b}}{x^{5/4}(a+bx)^{5/4}}$$

$\downarrow$ 73

$$\frac{\left(ax+bx^2\right)^{5/4}\Bigg(\frac{5}{22}a\Bigg(\frac{1}{18}a\Bigg(\frac{2x^{13/4}\sqrt[4]{a+bx}}{7b}-\frac{13a\Bigg(\frac{2x^{9/4}\sqrt[4]{a+bx}}{5b}-\frac{9a\Bigg(\frac{2x^{5/4}\sqrt[4]{a+bx}}{3b}-\frac{5a\Bigg(\frac{2\sqrt[4]{x}\sqrt[4]{a+bx}}{b}-\frac{2a\int\frac{1}{(a+bx)^{3/4}}}{b}}{6b}}{10b}}{14b}}{x^{5/4}(a+bx)^{5/4}}$$

$\downarrow$ 768

$$\frac{\left(ax+bx^2\right)^{5/4}\left(\frac{5}{22}a\left(\frac{1}{18}a\left(\frac{2x^{13/4}\sqrt[4]{a+bx}}{7b}-\frac{13a\left(\frac{2x^{9/4}\sqrt[4]{a+bx}}{5b}-\frac{9a\left(\frac{2x^{5/4}\sqrt[4]{a+bx}}{3b}-\frac{5a\left(\frac{2\sqrt[4]{x}\sqrt[4]{a+bx}}{b}-\frac{2ax^{3/4}\left(\frac{a}{bx}+1\right)}{}\right.}{6b}\right.}{10b}\right.}{14b}\right.\right.\right.}{x^{5/4}(a+bx)^{5/4}}$$

$\downarrow$ 858

$$\frac{\left(ax+bx^2\right)^{5/4}\left(\frac{5}{22}a\left(\frac{1}{18}a\left(\frac{2x^{13/4}\sqrt[4]{a+bx}}{7b}-\frac{13a\left(\frac{2x^{9/4}\sqrt[4]{a+bx}}{5b}-\frac{9a\left(\frac{2x^{5/4}\sqrt[4]{a+bx}}{3b}-\frac{5a\left(\frac{2ax^{3/4}\left(\frac{a}{bx}+1\right)^{3/4}\int\frac{1}{\sqrt[4]{x}\left(\frac{ax}{b}+1\right)^{3/4}}}{b(a+bx)^{3/4}}\right.}{6b}\right.}{10b}\right.}{14b}\right.\right.\right.}{x^{5/4}(a+bx)^{5/4}}$$

$$\Big\downarrow \; 807$$

$$\frac{\left(ax+bx^2\right)^{5/4}\left(\frac{5}{22}a\left(\frac{1}{18}a\left(\frac{2x^{13/4}\sqrt[4]{a+bx}}{7b}-\frac{13a\left(\frac{2x^{9/4}\sqrt[4]{a+bx}}{5b}-\frac{9a\left(\frac{2x^{5/4}\sqrt[4]{a+bx}}{3b}-\frac{5a\left(\frac{ax^{3/4}\left(\frac{a}{bx}+1\right)^{3/4}\int\frac{1}{\left(\frac{\sqrt{x}a}{b}+1\right)^{3/4}}d}{b(a+bx)^{3/4}}\right.}{6b}\right.}{10b}\right.}{14b}\right.\right.\right.}{x^{5/4}(a+bx)^{5/4}}$$

$\downarrow$ 229

$$\frac{(ax+bx^2)^{5/4}\left(\frac{5}{22}a\left(\frac{1}{18}a\left(\frac{2x^{13/4}\sqrt[4]{a+bx}}{7b}-\frac{13a\left(\frac{2x^{9/4}\sqrt[4]{a+bx}}{5b}-\frac{9a\left(\frac{2x^{5/4}\sqrt[4]{a+bx}}{3b}-\frac{5a\left(\frac{2\sqrt{a}x^{3/4}\left(\frac{a}{bx}+1\right)^{3/4}\text{EllipticF}\left(\frac{1}{2}\text{arc}\right.}{\sqrt{b}(a+bx)^{3/4}}\right.}{6b}\right.}{10b}\right.}{14b}\right.\right.\right.}{x^{5/4}(a+bx)^{5/4}}$$

```
Int[x^2*(a*x + b*x^2)^(5/4),x]
```

```
((a*x + b*x^2)^(5/4)*((2*x^(17/4)*(a + b*x)^(5/4))/11 + (5*a*((2*x^(17/4)*
(a + b*x)^(1/4))/9 + (a*((2*x^(13/4)*(a + b*x)^(1/4))/(7*b) - (13*a*((2*x^
(9/4)*(a + b*x)^(1/4))/(5*b) - (9*a*((2*x^(5/4)*(a + b*x)^(1/4))/(3*b) - (
5*a*((2*x^(1/4)*(a + b*x)^(1/4))/b + (2*Sqrt[a]*(1 + a/(b*x))^(3/4)*x^(3/4
)*EllipticF[ArcTan[(Sqrt[a]*Sqrt[x])/Sqrt[b]]/2, 2])/(Sqrt[b]*(a + b*x)^(3
/4))))/(6*b)))/(10*b)))/(14*b)))/18))/22))/(x^(5/4)*(a + b*x)^(5/4))
```

Defintions of rubi rules used

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^n/(b*(m + n + 1))), x] + Simp[n*((b*c - a*d)/(
b*(m + n + 1)))   Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
 c, d}, x] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ[m, 0] && ( !Integer
Q[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n + 2, 0] && IntLinear
Q[a, b, c, d, m, n, x]
```

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Simp[p/b   Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
 d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && Lt
Q[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntL
inearQ[a, b, c, d, m, n, x]
```

```
Int[((a_) + (b_.)*(x_)^2)^(-3/4), x_Symbol] :> Simp[(2/(a^(3/4)*Rt[b/a, 2])
)*EllipticF[(1/2)*ArcTan[Rt[b/a, 2]*x], 2], x] /; FreeQ[{a, b}, x] && GtQ[a
, 0] && PosQ[b/a]
```

```
Int[((a_) + (b_.)*(x_)^4)^(-3/4), x_Symbol] :> Simp[x^3*((1 + a/(b*x^4))^(3
/4)/(a + b*x^4)^(3/4))   Int[1/(x^3*(1 + a/(b*x^4))^(3/4)), x], x] /; FreeQ
[{a, b}, x]
```

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k = GCD[m
+ 1, n]}, Simp[1/k   Subst[Int[x^((m + 1)/k - 1)*(a + b*x^(n/k))^p, x], x,
x^k], x] /; k != 1] /; FreeQ[{a, b, p}, x] && IGtQ[n, 0] && IntegerQ[m]
```

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> -Subst[Int[(a +
b/x^n)^p/x^(m + 2), x], x, 1/x] /; FreeQ[{a, b, p}, x] && ILtQ[n, 0] && Int
egerQ[m]
```

```
Int[((e_.)*(x_))^(m_)*((b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(
e*x)^m*((b*x + c*x^2)^p/(x^(m + p)*(b + c*x)^p))   Int[x^(m + p)*(b + c*x)^
p, x], x] /; FreeQ[{b, c, e, m}, x]
```

Maple [F]

$$\int x^{2} \left(b\,x^{2}+a x \right)^{\frac{5}{4}}d x$$

```
int(x^2*(b*x^2+a*x)^(5/4),x)
```

```
int(x^2*(b*x^2+a*x)^(5/4),x)
```

Fricas [F]

$$\int x^2 (ax + bx^2)^{5/4} \, dx = \int (bx^2 + ax)^{\frac{5}{4}} x^2 \, dx$$

```
integrate(x^2*(b*x^2+a*x)^(5/4),x, algorithm="fricas")
```

```
integral((b*x^4 + a*x^3)*(b*x^2 + a*x)^(1/4), x)
```

Sympy [F]

$$\int x^2 (ax + bx^2)^{5/4} \, dx = \int x^2 (x(a + bx))^{\frac{5}{4}} \, dx$$

```
integrate(x**2*(b*x**2+a*x)**(5/4),x)
```

```
Integral(x**2*(x*(a + b*x))**(5/4), x)
```

Maxima [F]

$$\int x^2 (ax + bx^2)^{5/4} \, dx = \int (bx^2 + ax)^{\frac{5}{4}} x^2 \, dx$$

```
integrate(x^2*(b*x^2+a*x)^(5/4),x, algorithm="maxima")
```

```
integrate((b*x^2 + a*x)^(5/4)*x^2, x)
```

Giac [F]

$$\int x^2 (ax + bx^2)^{5/4} \, dx = \int (bx^2 + ax)^{\frac{5}{4}} x^2 \, dx$$

```
integrate(x^2*(b*x^2+a*x)^(5/4),x, algorithm="giac")
```

```
integrate((b*x^2 + a*x)^(5/4)*x^2, x)
```

Mupad [F(-1)]

Timed out.

$$\int x^2 (ax + bx^2)^{5/4} \, dx = \int x^2 \left(b\,x^2 + a\,x\right)^{5/4} dx$$

```
int(x^2*(a*x + b*x^2)^(5/4),x)
```

```
int(x^2*(a*x + b*x^2)^(5/4), x)
```

Reduce [F]

$$\int x^2 (ax + bx^2)^{5/4} \, dx = \frac{-780x^{\frac{1}{4}}(bx+a)^{\frac{1}{4}} a^5 + 312x^{\frac{5}{4}}(bx+a)^{\frac{1}{4}} a^4 b - 208x^{\frac{9}{4}}(bx+a)^{\frac{1}{4}} a^3 b^2 + 160x^{\frac{13}{4}}(bx+a)^{\frac{1}{4}} a^2 b^3 -}{44352b^4}$$

```
int(x^2*(b*x^2+a*x)^(5/4),x)
```

```
( - 780*x**(1/4)*(a + b*x)**(1/4)*a**5 + 312*x**(1/4)*(a + b*x)**(1/4)*a**
4*b*x - 208*x**(1/4)*(a + b*x)**(1/4)*a**3*b**2*x**2 + 160*x**(1/4)*(a + b
*x)**(1/4)*a**2*b**3*x**3 + 10304*x**(1/4)*(a + b*x)**(1/4)*a*b**4*x**4 +
8064*x**(1/4)*(a + b*x)**(1/4)*b**5*x**5 + 195*int((a + b*x)**(1/4)/(x**(3
/4)*a + x**(3/4)*b*x),x)*a**6)/(44352*b**4)
```

3.164 $\int x(ax+bx^2)^{5/4}\,dx$

Optimal result

Integrand size = 15, antiderivative size = 188

$$\int x\left(ax+bx^2\right)^{5/4}\,dx=\frac{5a^4\sqrt[4]{ax+bx^2}}{168b^3}-\frac{a^3x\sqrt[4]{ax+bx^2}}{84b^2}+\frac{a^2x^2\sqrt[4]{ax+bx^2}}{126b}+\frac{5}{63}ax^3\sqrt[4]{ax+bx^2}+\frac{2}{9}x^2\left(ax+bx^2\right)^{5/4}+\frac{5a^{9/2}\left(\frac{bx}{a+bx}\right)^{3/4}\sqrt{a+bx}\sqrt[4]{ax+bx^2}\,\text{EllipticF}\left(\frac{1}{2}\arcsin\left(\frac{\sqrt{a}}{\sqrt{a+bx}}\right),2\right)}{168b^4x}$$

```
5/168*a^4*(b*x^2+a*x)^(1/4)/b^3-1/84*a^3*x*(b*x^2+a*x)^(1/4)/b^2+1/126*a^2
*x^2*(b*x^2+a*x)^(1/4)/b+5/63*a*x^3*(b*x^2+a*x)^(1/4)+2/9*x^2*(b*x^2+a*x)^
(5/4)+5/168*a^(9/2)*(b*x/(b*x+a))^(3/4)*(b*x+a)^(1/2)*(b*x^2+a*x)^(1/4)*In
verseJacobiAM(1/2*arcsin(a^(1/2)/(b*x+a)^(1/2)),2^(1/2))/b^4/x
```

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 10.01 (sec) , antiderivative size = 48, normalized size of antiderivative = 0.26

$$\int x(ax+bx^2)^{5/4}\,dx=\frac{4ax^3\sqrt[4]{x(a+bx)}\,\text{Hypergeometric2F1}\left(-\frac{5}{4},\frac{13}{4},\frac{17}{4},-\frac{bx}{a}\right)}{13\sqrt[4]{1+\frac{bx}{a}}}$$

```
Integrate[x*(a*x + b*x^2)^(5/4),x]
```

```
(4*a*x^3*(x*(a + b*x))^(1/4)*Hypergeometric2F1[-5/4, 13/4, 17/4, -((b*x)/a
)])/(13*(1 + (b*x)/a)^(1/4))
```

Rubi [A] (verified)

Time = 0.52 (sec) , antiderivative size = 168, normalized size of antiderivative = 0.89, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.400, Rules used = {1160, 1087, 1087, 1093, 1090, 230}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x(ax+bx^2)^{5/4}\,dx$$

$$\Big\downarrow \text{1160}$$

$$\frac{2(ax+bx^2)^{9/4}}{9b}-\frac{a\int (bx^2+ax)^{5/4}dx}{2b}$$

$$\Big\downarrow \text{1087}$$

$$\frac{2(ax+bx^2)^{9/4}}{9b}-\frac{a\left(\frac{(a+2bx)(ax+bx^2)^{5/4}}{7b}-\frac{5a^2\int \sqrt[4]{bx^2+ax}dx}{28b}\right)}{2b}$$

$$\Big\downarrow \text{1087}$$

$$\frac{2\left(ax+bx^2\right)^{9/4}}{9b}-\frac{a\left(\frac{(a+2bx)\left(ax+bx^2\right)^{5/4}}{7b}-\frac{5a^2\left(\frac{(a+2bx)\sqrt[4]{ax+bx^2}}{3b}-\frac{a^2\int\frac{1}{\left(bx^2+ax\right)^{3/4}}dx}{12b}\right)}{28b}\right)}{2b}$$

$$\Big\downarrow \; 1093$$

$$\frac{2\left(ax+bx^2\right)^{9/4}}{9b}-\frac{a\left(\frac{(a+2bx)\left(ax+bx^2\right)^{5/4}}{7b}-\frac{5a^2\left(\frac{(a+2bx)\sqrt[4]{ax+bx^2}}{3b}-\frac{a^2\left(-\frac{b\left(ax+bx^2\right)}{a^2}\right)^{3/4}\int\frac{1}{\left(-\frac{b^2x^2}{a^2}-\frac{bx}{a}\right)^{3/4}}dx}{12b\left(ax+bx^2\right)^{3/4}}\right)}{28b}\right)}{2b}$$

$$\Big\downarrow \; 1090$$

$$\frac{2\left(ax+bx^2\right)^{9/4}}{9b}-\frac{a\left(\frac{(a+2bx)\left(ax+bx^2\right)^{5/4}}{7b}-\frac{5a^2\left(\frac{a^4\left(-\frac{b\left(ax+bx^2\right)}{a^2}\right)^{3/4}\int\frac{1}{\left(1-\frac{a^2\left(-\frac{2xb^2}{a^2}-\frac{b}{a}\right)^2}{b^2}\right)^{3/4}}d\left(-\frac{2xb^2}{a^2}-\frac{b}{a}\right)}{6\sqrt{2}b^3\left(ax+bx^2\right)^{3/4}}+\frac{(a+2bx)\sqrt[4]{ax+bx^2}}{3b}\right)}{28b}\right)}{2b}$$

$$\Big\downarrow \; 230$$

$$\frac{2\left(ax+bx^2\right)^{9/4}}{9b} - \frac{a\left(\frac{(a+2bx)\left(ax+bx^2\right)^{5/4}}{7b} - \frac{5a^2\left(\frac{a^3\left(-\frac{b\left(ax+bx^2\right)}{a^2}\right)^{3/4}\operatorname{EllipticF}\left(\frac{1}{2}\arcsin\left(\frac{a\left(-\frac{2xb^2}{a^2}-\frac{b}{a}\right)}{b}\right),2\right)}{3\sqrt{2}b^2\left(ax+bx^2\right)^{3/4}} + \frac{(a+2bx)\sqrt[4]{ax+bx^2}}{3b}\right)}{28b}\right)}{2b}$$

```
Int[x*(a*x + b*x^2)^(5/4),x]
```

```
(2*(a*x + b*x^2)^(9/4))/(9*b) - (a*(((a + 2*b*x)*(a*x + b*x^2)^(5/4))/(7*b
) - (5*a^2*(((a + 2*b*x)*(a*x + b*x^2)^(1/4))/(3*b) + (a^3*(-((b*(a*x + b*
x^2))/a^2))^(3/4)*EllipticF[ArcSin[(a*(-(b/a) - (2*b^2*x)/a^2))/b]/2, 2])/
(3*Sqrt[2]*b^2*(a*x + b*x^2)^(3/4))))/(28*b)))/(2*b)
```

Defintions of rubi rules used

```
Int[((a_) + (b_.)*(x_)^2)^(-3/4), x_Symbol] :> Simp[(2/(a^(3/4)*Rt[-b/a, 2]
))*EllipticF[(1/2)*ArcSin[Rt[-b/a, 2]*x], 2], x] /; FreeQ[{a, b}, x] && GtQ
[a, 0] && NegQ[b/a]
```

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(b + 2*c*x)
*((a + b*x + c*x^2)^p/(2*c*(2*p + 1))), x] - Simp[p*((b^2 - 4*a*c)/(2*c*(2*
p + 1)))   Int[(a + b*x + c*x^2)^(p - 1), x], x] /; FreeQ[{a, b, c}, x] &&
GtQ[p, 0] && (IntegerQ[4*p] || IntegerQ[3*p])
```

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[1/(2*c*(-4*
(c/(b^2 - 4*a*c)))^p)   Subst[Int[Simp[1 - x^2/(b^2 - 4*a*c), x]^p, x], x,
b + 2*c*x], x] /; FreeQ[{a, b, c, p}, x] && GtQ[4*a - b^2/c, 0]
```

```
Int[((b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(b*x + c*x^2)^p/((-
c)*((b*x + c*x^2)/b^2))^p   Int[((-c)*(x/b) - c^2*(x^2/b^2))^p, x], x] /; F
reeQ[{b, c}, x] && (IntegerQ[4*p] || IntegerQ[3*p])
```

```
Int[((d_.) + (e_.)*(x_))*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol
] :> Simp[e*((a + b*x + c*x^2)^(p + 1)/(2*c*(p + 1))), x] + Simp[(2*c*d - b
*e)/(2*c)   Int[(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, p}, x]
 && NeQ[p, -1]
```

Maple [F]

$$\int x\left(b\,x^{2}+ax\right)^{\frac{5}{4}}dx$$

```
int(x*(b*x^2+a*x)^(5/4),x)
```

```
int(x*(b*x^2+a*x)^(5/4),x)
```

Fricas [F]

$$\int x\left(ax+bx^2\right)^{5/4}\,dx=\int\left(bx^2+ax\right)^{\frac{5}{4}}x\,dx$$

```
integrate(x*(b*x^2+a*x)^(5/4),x, algorithm="fricas")
```

```
integral((b*x^3 + a*x^2)*(b*x^2 + a*x)^(1/4), x)
```

Sympy [F]

$$\int x\left(ax+bx^2\right)^{5/4}\,dx=\int x(x(a+bx))^{\frac{5}{4}}\,dx$$

```
integrate(x*(b*x**2+a*x)**(5/4),x)
```

```
Integral(x*(x*(a + b*x))**(5/4), x)
```

Maxima [F]

$$\int x\left(ax+bx^2\right)^{5/4}\,dx=\int \left(bx^2+ax\right)^{\frac{5}{4}}x\,dx$$

```
integrate(x*(b*x^2+a*x)^(5/4),x, algorithm="maxima")
```

```
integrate((b*x^2 + a*x)^(5/4)*x, x)
```

Giac [F]

$$\int x\left(ax+bx^2\right)^{5/4}\,dx=\int \left(bx^2+ax\right)^{\frac{5}{4}}x\,dx$$

```
integrate(x*(b*x^2+a*x)^(5/4),x, algorithm="giac")
```

```
integrate((b*x^2 + a*x)^(5/4)*x, x)
```

Mupad [F(-1)]

Timed out.

$$\int x(ax+bx^2)^{5/4}\,dx=\int x\,\left(b\,x^2+a\,x\right)^{5/4}\,dx$$

```
int(x*(a*x + b*x^2)^(5/4),x)
```

```
int(x*(a*x + b*x^2)^(5/4), x)
```

Reduce [F]

$$\int x(ax+bx^2)^{5/4}\,dx=\frac{60x^{\frac{1}{4}}(bx+a)^{\frac{1}{4}}a^4-24x^{\frac{5}{4}}(bx+a)^{\frac{1}{4}}a^3b+16x^{\frac{9}{4}}(bx+a)^{\frac{1}{4}}a^2b^2+608x^{\frac{13}{4}}(bx+a)^{\frac{1}{4}}a\,b^3+448x}{2016b^3}$$

```
int(x*(b*x^2+a*x)^(5/4),x)
```

```
(60*x**(1/4)*(a + b*x)**(1/4)*a**4 - 24*x**(1/4)*(a + b*x)**(1/4)*a**3*b*x
 + 16*x**(1/4)*(a + b*x)**(1/4)*a**2*b**2*x**2 + 608*x**(1/4)*(a + b*x)**(
1/4)*a*b**3*x**3 + 448*x**(1/4)*(a + b*x)**(1/4)*b**4*x**4 - 15*int((a + b
*x)**(1/4)/(x**(3/4)*a + x**(3/4)*b*x),x)*a**5)/(2016*b**3)
```

3.165 $\int (ax + bx^2)^{5/4}\, dx$

Optimal result

Integrand size = 13, antiderivative size = 160

$$\int (ax + bx^2)^{5/4}\, dx = -\frac{5a^3\sqrt[4]{ax+bx^2}}{84b^2} + \frac{a^2x\sqrt[4]{ax+bx^2}}{42b} + \frac{1}{7}ax^2\sqrt[4]{ax+bx^2} + \frac{2}{7}x(ax+bx^2)^{5/4} - \frac{5a^{7/2}\left(\frac{bx}{a+bx}\right)^{3/4}\sqrt{a+bx}\sqrt[4]{ax+bx^2}\,\text{EllipticF}\left(\frac{1}{2}\arcsin\left(\frac{\sqrt{a}}{\sqrt{a+bx}}\right),2\right)}{84b^3x}$$

```
-5/84*a^3*(b*x^2+a*x)^(1/4)/b^2+1/42*a^2*x*(b*x^2+a*x)^(1/4)/b+1/7*a*x^2*(
b*x^2+a*x)^(1/4)+2/7*x*(b*x^2+a*x)^(5/4)-5/84*a^(7/2)*(b*x/(b*x+a))^(3/4)*
(b*x+a)^(1/2)*(b*x^2+a*x)^(1/4)*InverseJacobiAM(1/2*arcsin(a^(1/2)/(b*x+a)
^(1/2)),2^(1/2))/b^3/x
```

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 10.01 (sec) , antiderivative size = 48, normalized size of antiderivative = 0.30

$$\int \left(ax+bx^2\right)^{5/4} dx = \frac{4ax^2\sqrt[4]{x(a+bx)}\,\text{Hypergeometric2F1}\left(-\frac{5}{4},\frac{9}{4},\frac{13}{4},-\frac{bx}{a}\right)}{9\sqrt[4]{1+\frac{bx}{a}}}$$

```
Integrate[(a*x + b*x^2)^(5/4),x]
```

```
(4*a*x^2*(x*(a + b*x))^(1/4)*Hypergeometric2F1[-5/4, 9/4, 13/4, -((b*x)/a)
])/(9*(1 + (b*x)/a)^(1/4))
```

Rubi [A] (verified)

Time = 0.46 (sec) , antiderivative size = 139, normalized size of antiderivative = 0.87, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.385$, Rules used = {1087, 1087, 1093, 1090, 230}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \left(ax+bx^2\right)^{5/4} dx$$

$$\Big\downarrow \text{1087}$$

$$\frac{(a+2bx)\left(ax+bx^2\right)^{5/4}}{7b}-\frac{5a^2\int \sqrt[4]{bx^2+ax}dx}{28b}$$

$$\Big\downarrow \text{1087}$$

$$\frac{(a+2bx)\left(ax+bx^2\right)^{5/4}}{7b}-\frac{5a^2\left(\frac{(a+2bx)\sqrt[4]{ax+bx^2}}{3b}-\frac{a^2\int \frac{1}{\left(bx^2+ax\right)^{3/4}}dx}{12b}\right)}{28b}$$

$$\Big\downarrow \text{1093}$$

$$\frac{(a+2bx)\left(ax+bx^2\right)^{5/4}}{7b}-\frac{5a^2\left(\frac{(a+2bx)\sqrt[4]{ax+bx^2}}{3b}-\frac{a^2\left(-\frac{b\left(ax+bx^2\right)}{a^2}\right)^{3/4}\int\frac{1}{\left(-\frac{b^2x^2}{a^2}-\frac{bx}{a}\right)^{3/4}}dx}{12b\left(ax+bx^2\right)^{3/4}}\right)}{28b}$$

$$\Big\downarrow \; 1090$$

$$\frac{(a+2bx)\left(ax+bx^2\right)^{5/4}}{7b}-\frac{5a^2\left(\frac{a^4\left(-\frac{b\left(ax+bx^2\right)}{a^2}\right)^{3/4}\int\frac{1}{\left(1-\frac{a^2\left(-\frac{2xb^2}{a^2}-\frac{b}{a}\right)^2}{b^2}\right)^{3/4}}d\left(-\frac{2xb^2}{a^2}-\frac{b}{a}\right)}{6\sqrt{2}b^3\left(ax+bx^2\right)^{3/4}}+\frac{(a+2bx)\sqrt[4]{ax+bx^2}}{3b}\right)}{28b}$$

$$\Big\downarrow \; 230$$

$$\frac{(a+2bx)\left(ax+bx^2\right)^{5/4}}{7b}-\frac{5a^2\left(\frac{a^3\left(-\frac{b\left(ax+bx^2\right)}{a^2}\right)^{3/4}\text{EllipticF}\left(\frac{1}{2}\arcsin\left(\frac{a\left(-\frac{2xb^2}{a^2}-\frac{b}{a}\right)}{b}\right),2\right)}{3\sqrt{2}b^2\left(ax+bx^2\right)^{3/4}}+\frac{(a+2bx)\sqrt[4]{ax+bx^2}}{3b}\right)}{28b}$$

```
Int[(a*x + b*x^2)^(5/4),x]
```

```
((a + 2*b*x)*(a*x + b*x^2)^(5/4))/(7*b) - (5*a^2*(((a + 2*b*x)*(a*x + b*x^
2)^(1/4))/(3*b) + (a^3*(-((b*(a*x + b*x^2))/a^2))^(3/4)*EllipticF[ArcSin[(
a*(-(b/a) - (2*b^2*x)/a^2))/b]/2, 2])/(3*Sqrt[2]*b^2*(a*x + b*x^2)^(3/4)))
)/(28*b)
```

Defintions of rubi rules used

```
Int[((a_) + (b_.)*(x_)^2)^(-3/4), x_Symbol] :> Simp[(2/(a^(3/4)*Rt[-b/a, 2]
))*EllipticF[(1/2)*ArcSin[Rt[-b/a, 2]*x], 2], x] /; FreeQ[{a, b}, x] && GtQ
[a, 0] && NegQ[b/a]
```

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(b + 2*c*x)
*((a + b*x + c*x^2)^p/(2*c*(2*p + 1))), x] - Simp[p*((b^2 - 4*a*c)/(2*c*(2*
p + 1)))   Int[(a + b*x + c*x^2)^(p - 1), x], x] /; FreeQ[{a, b, c}, x] &&
GtQ[p, 0] && (IntegerQ[4*p] || IntegerQ[3*p])
```

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[1/(2*c*(-4*
(c/(b^2 - 4*a*c)))^p)   Subst[Int[Simp[1 - x^2/(b^2 - 4*a*c), x]^p, x], x,
b + 2*c*x], x] /; FreeQ[{a, b, c, p}, x] && GtQ[4*a - b^2/c, 0]
```

```
Int[((b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(b*x + c*x^2)^p/((-
c)*((b*x + c*x^2)/b^2))^p   Int[((-c)*(x/b) - c^2*(x^2/b^2))^p, x], x] /; F
reeQ[{b, c}, x] && (IntegerQ[4*p] || IntegerQ[3*p])
```

Maple [F]

$$\int \left(b\,x^{2}+ax\right)^{\frac{5}{4}}dx$$

```
int((b*x^2+a*x)^(5/4),x)
```

```
int((b*x^2+a*x)^(5/4),x)
```

Fricas [F]

$$\int \left(ax+bx^2\right)^{5/4} dx = \int \left(bx^2+ax\right)^{\frac{5}{4}} dx$$

```
integrate((b*x^2+a*x)^(5/4),x, algorithm="fricas")
```

```
integral((b*x^2 + a*x)^(5/4), x)
```

Sympy [F]

$$\int \left(ax+bx^2\right)^{5/4} dx = \int \left(ax+bx^2\right)^{\frac{5}{4}} dx$$

```
integrate((b*x**2+a*x)**(5/4),x)
```

```
Integral((a*x + b*x**2)**(5/4), x)
```

Maxima [F]

$$\int \left(ax+bx^2\right)^{5/4} dx = \int \left(bx^2+ax\right)^{\frac{5}{4}} dx$$

```
integrate((b*x^2+a*x)^(5/4),x, algorithm="maxima")
```

```
integrate((b*x^2 + a*x)^(5/4), x)
```

Giac [F]

$$\int \left(ax+bx^2\right)^{5/4}\,dx=\int \left(bx^2+ax\right)^{\frac{5}{4}}\,dx$$

```
integrate((b*x^2+a*x)^(5/4),x, algorithm="giac")
```

```
integrate((b*x^2 + a*x)^(5/4), x)
```

Mupad [B] (verification not implemented)

Time = 9.21 (sec) , antiderivative size = 36, normalized size of antiderivative = 0.22

$$\int \left(ax+bx^2\right)^{5/4}\,dx=\frac{4\,x\,{\left(b\,x^2+a\,x\right)}^{5/4}\,{{}}_2{\mathrm{F}}_1\left(-\frac{5}{4},\frac{9}{4};\ \frac{13}{4};\ -\frac{b\,x}{a}\right)}{9\,{\left(\frac{b\,x}{a}+1\right)}^{5/4}}$$

```
int((a*x + b*x^2)^(5/4),x)
```

```
(4*x*(a*x + b*x^2)^(5/4)*hypergeom([-5/4, 9/4], 13/4, -(b*x)/a))/(9*((b*x)
/a + 1)^(5/4))
```

Reduce [F]

$$\int (ax+bx^2)^{5/4}\,dx=\frac{-20x^{\frac{1}{4}}(bx+a)^{\frac{1}{4}}a^3+8x^{\frac{5}{4}}(bx+a)^{\frac{1}{4}}a^2b+144x^{\frac{9}{4}}(bx+a)^{\frac{1}{4}}a\,b^2+96x^{\frac{13}{4}}(bx+a)^{\frac{1}{4}}b^3+5\left(\int \cdots\right.}{336b^2}$$

```
int((b*x^2+a*x)^(5/4),x)
```

```
( - 20*x**(1/4)*(a + b*x)**(1/4)*a**3 + 8*x**(1/4)*(a + b*x)**(1/4)*a**2*b
*x + 144*x**(1/4)*(a + b*x)**(1/4)*a*b**2*x**2 + 96*x**(1/4)*(a + b*x)**(1
/4)*b**3*x**3 + 5*int((a + b*x)**(1/4)/(x**(3/4)*a + x**(3/4)*b*x),x)*a**4
)/(336*b**2)
```

3.166 $\int \frac{(ax+bx^2)^{5/4}}{x} dx$

Optimal result

Integrand size = 17, antiderivative size = 133

$$\int \frac{(ax+bx^2)^{5/4}}{x} dx = \frac{a^2\sqrt[4]{ax+bx^2}}{6b} + \frac{1}{3}ax\sqrt[4]{ax+bx^2} + \frac{2}{5}(ax+bx^2)^{5/4} + \frac{a^{5/2}\left(\frac{bx}{a+bx}\right)^{3/4}\sqrt{a+bx}\sqrt[4]{ax+bx^2}\,\text{EllipticF}\left(\frac{1}{2}\arcsin\left(\frac{\sqrt{a}}{\sqrt{a+bx}}\right),2\right)}{6b^2x}$$

```
1/6*a^2*(b*x^2+a*x)^(1/4)/b+1/3*a*x*(b*x^2+a*x)^(1/4)+2/5*(b*x^2+a*x)^(5/4
)+1/6*a^(5/2)*(b*x/(b*x+a))^(3/4)*(b*x+a)^(1/2)*(b*x^2+a*x)^(1/4)*InverseJ
acobiAM(1/2*arcsin(a^(1/2)/(b*x+a)^(1/2)),2^(1/2))/b^2/x
```

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 10.01 (sec) , antiderivative size = 46, normalized size of antiderivative = 0.35

$$\int \frac{(ax+bx^2)^{5/4}}{x} dx = \frac{4ax\sqrt[4]{x(a+bx)}\,\text{Hypergeometric2F1}\left(-\frac{5}{4},\frac{5}{4},\frac{9}{4},-\frac{bx}{a}\right)}{5\sqrt[4]{1+\frac{bx}{a}}}$$

```
Integrate[(a*x + b*x^2)^(5/4)/x,x]
```

```
(4*a*x*(x*(a + b*x))^(1/4)*Hypergeometric2F1[-5/4, 5/4, 9/4, -((b*x)/a)])/
(5*(1 + (b*x)/a)^(1/4))
```

Rubi [A] (warning: unable to verify)

Time = 0.49 (sec) , antiderivative size = 159, normalized size of antiderivative = 1.20, number of steps used = 10, number of rules used = 9, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.529, Rules used = {1137, 60, 60, 60, 73, 768, 858, 807, 229}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(ax+bx^2)^{5/4}}{x}\,dx$$

$$\Big\downarrow \text{1137}$$

$$\frac{(ax+bx^2)^{5/4}\int \sqrt[4]{x}(a+bx)^{5/4}dx}{x^{5/4}(a+bx)^{5/4}}$$

$$\Big\downarrow \text{60}$$

$$\frac{(ax+bx^2)^{5/4}\left(\frac{1}{2}a\int \sqrt[4]{x}\sqrt[4]{a+bx}dx+\frac{2}{5}x^{5/4}(a+bx)^{5/4}\right)}{x^{5/4}(a+bx)^{5/4}}$$

$$\Big\downarrow \text{60}$$

$$\frac{(ax+bx^2)^{5/4}\left(\frac{1}{2}a\left(\frac{1}{6}a\int \frac{\sqrt[4]{x}}{(a+bx)^{3/4}}dx+\frac{2}{3}x^{5/4}\sqrt[4]{a+bx}\right)+\frac{2}{5}x^{5/4}(a+bx)^{5/4}\right)}{x^{5/4}(a+bx)^{5/4}}$$

$$\Big\downarrow \text{60}$$

$$\frac{(ax+bx^2)^{5/4}\left(\frac{1}{2}a\left(\frac{1}{6}a\left(\frac{2\sqrt[4]{x}\sqrt[4]{a+bx}}{b}-\frac{a\int \frac{1}{x^{3/4}(a+bx)^{3/4}}dx}{2b}\right)+\frac{2}{3}x^{5/4}\sqrt[4]{a+bx}\right)+\frac{2}{5}x^{5/4}(a+bx)^{5/4}\right)}{x^{5/4}(a+bx)^{5/4}}$$

$$\Big\downarrow \text{73}$$

$$\frac{(ax+bx^2)^{5/4}\left(\frac{1}{2}a\left(\frac{1}{6}a\left(\frac{2\sqrt[4]{x}\sqrt[4]{a+bx}}{b}-\frac{2a\int\frac{1}{(a+bx)^{3/4}}d\sqrt[4]{x}}{b}\right)+\frac{2}{3}x^{5/4}\sqrt[4]{a+bx}\right)+\frac{2}{5}x^{5/4}(a+bx)^{5/4}\right)}{x^{5/4}(a+bx)^{5/4}}$$

$$\Big\downarrow \; 768$$

$$\frac{(ax+bx^2)^{5/4}\Bigg(\frac{1}{2}a\Bigg(\frac{1}{6}a\Bigg(\frac{2\sqrt[4]{x}\sqrt[4]{a+bx}}{b}-\frac{2ax^{3/4}\left(\frac{a}{bx}+1\right)^{3/4}\int\frac{1}{\left(\frac{a}{bx}+1\right)^{3/4}x^{3/4}}d\sqrt[4]{x}}{b(a+bx)^{3/4}}\Bigg)+\frac{2}{3}x^{5/4}\sqrt[4]{a+bx}\Bigg)+\frac{2}{5}x^{5/4}(a+b}{x^{5/4}(a+bx)^{5/4}}$$

$$\Big\downarrow \; 858$$

$$\frac{(ax+bx^2)^{5/4}\Bigg(\frac{1}{2}a\Bigg(\frac{1}{6}a\Bigg(\frac{2ax^{3/4}\left(\frac{a}{bx}+1\right)^{3/4}\int\frac{1}{\sqrt[4]{x}\left(\frac{ax}{b}+1\right)^{3/4}}d\frac{1}{\sqrt[4]{x}}}{b(a+bx)^{3/4}}+\frac{2\sqrt[4]{x}\sqrt[4]{a+bx}}{b}\Bigg)+\frac{2}{3}x^{5/4}\sqrt[4]{a+bx}\Bigg)+\frac{2}{5}x^{5/4}(a+}{x^{5/4}(a+bx)^{5/4}}$$

$$\Big\downarrow \; 807$$

$$\frac{(ax+bx^2)^{5/4}\Bigg(\frac{1}{2}a\Bigg(\frac{1}{6}a\Bigg(\frac{ax^{3/4}\left(\frac{a}{bx}+1\right)^{3/4}\int\frac{1}{\left(\frac{\sqrt{x}a}{b}+1\right)^{3/4}}d\sqrt{x}}{b(a+bx)^{3/4}}+\frac{2\sqrt[4]{x}\sqrt[4]{a+bx}}{b}\Bigg)+\frac{2}{3}x^{5/4}\sqrt[4]{a+bx}\Bigg)+\frac{2}{5}x^{5/4}(a+bx)^{5/}{x^{5/4}(a+bx)^{5/4}}$$

$$\Big\downarrow \; 229$$

$$\frac{(ax+bx^2)^{5/4}\Bigg(\frac{1}{2}a\Bigg(\frac{1}{6}a\Bigg(\frac{2\sqrt{a}x^{3/4}\left(\frac{a}{bx}+1\right)^{3/4}\operatorname{EllipticF}\left(\frac{1}{2}\arctan\left(\frac{\sqrt{a}\sqrt{x}}{\sqrt{b}}\right),2\right)}{\sqrt{b}(a+bx)^{3/4}}+\frac{2\sqrt[4]{x}\sqrt[4]{a+bx}}{b}\Bigg)+\frac{2}{3}x^{5/4}\sqrt[4]{a+bx}\Bigg)+\frac{2}{5}x^{5/4}}{x^{5/4}(a+bx)^{5/4}}$$

```
Int[(a*x + b*x^2)^(5/4)/x,x]
```

```
((a*x + b*x^2)^(5/4)*((2*x^(5/4)*(a + b*x)^(5/4))/5 + (a*((2*x^(5/4)*(a +
b*x)^(1/4))/3 + (a*((2*x^(1/4)*(a + b*x)^(1/4))/b + (2*Sqrt[a]*(1 + a/(b*x
))^(3/4)*x^(3/4)*EllipticF[ArcTan[(Sqrt[a]*Sqrt[x])/Sqrt[b]]/2, 2])/(Sqrt[
b]*(a + b*x)^(3/4))))/6))/2))/(x^(5/4)*(a + b*x)^(5/4))
```

Defintions of rubi rules used

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^n/(b*(m + n + 1))), x] + Simp[n*((b*c - a*d)/(
b*(m + n + 1)))   Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
 c, d}, x] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ[m, 0] && ( !Integer
Q[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n + 2, 0] && IntLinear
Q[a, b, c, d, m, n, x]
```

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Simp[p/b   Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
 d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && Lt
Q[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntL
inearQ[a, b, c, d, m, n, x]
```

```
Int[((a_) + (b_.)*(x_)^2)^(-3/4), x_Symbol] :> Simp[(2/(a^(3/4)*Rt[b/a, 2])
)*EllipticF[(1/2)*ArcTan[Rt[b/a, 2]*x], 2], x] /; FreeQ[{a, b}, x] && GtQ[a
, 0] && PosQ[b/a]
```

```
Int[((a_) + (b_.)*(x_)^4)^(-3/4), x_Symbol] :> Simp[x^3*((1 + a/(b*x^4))^(3
/4)/(a + b*x^4)^(3/4))   Int[1/(x^3*(1 + a/(b*x^4))^(3/4)), x], x] /; FreeQ
[{a, b}, x]
```

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k = GCD[m
+ 1, n]}, Simp[1/k   Subst[Int[x^((m + 1)/k - 1)*(a + b*x^(n/k))^p, x], x,
x^k], x] /; k != 1] /; FreeQ[{a, b, p}, x] && IGtQ[n, 0] && IntegerQ[m]
```

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> -Subst[Int[(a +
b/x^n)^p/x^(m + 2), x], x, 1/x] /; FreeQ[{a, b, p}, x] && ILtQ[n, 0] && Int
egerQ[m]
```

```
Int[((e_.)*(x_))^(m_)*((b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(
e*x)^m*((b*x + c*x^2)^p/(x^(m + p)*(b + c*x)^p))   Int[x^(m + p)*(b + c*x)^
p, x], x] /; FreeQ[{b, c, e, m}, x]
```

Maple [F]

$$\int \frac{\left(b\,x^{2}+ax\right)^{\frac{5}{4}}}{x}dx$$

```
int((b*x^2+a*x)^(5/4)/x,x)
```

```
int((b*x^2+a*x)^(5/4)/x,x)
```

Fricas [F]

$$\int \frac{(ax+bx^2)^{5/4}}{x}\,dx = \int \frac{(bx^2+ax)^{\frac{5}{4}}}{x}\,dx$$

```
integrate((b*x^2+a*x)^(5/4)/x,x, algorithm="fricas")
```

```
integral((b*x^2 + a*x)^(1/4)*(b*x + a), x)
```

Sympy [F]

$$\int \frac{(ax+bx^2)^{5/4}}{x}\,dx = \int \frac{(x(a+bx))^{\frac{5}{4}}}{x}\,dx$$

```
integrate((b*x**2+a*x)**(5/4)/x,x)
```

```
Integral((x*(a + b*x))**(5/4)/x, x)
```

Maxima [F]

$$\int \frac{\left(ax+bx^2\right)^{5/4}}{x}\,dx=\int \frac{\left(bx^2+ax\right)^{\frac{5}{4}}}{x}\,dx$$

```
integrate((b*x^2+a*x)^(5/4)/x,x, algorithm="maxima")
```

```
integrate((b*x^2 + a*x)^(5/4)/x, x)
```

Giac [F]

$$\int \frac{\left(ax+bx^2\right)^{5/4}}{x}\,dx=\int \frac{\left(bx^2+ax\right)^{\frac{5}{4}}}{x}\,dx$$

```
integrate((b*x^2+a*x)^(5/4)/x,x, algorithm="giac")
```

```
integrate((b*x^2 + a*x)^(5/4)/x, x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\left(ax+bx^2\right)^{5/4}}{x}\,dx=\int \frac{\left(b\,x^2+a\,x\right)^{5/4}}{x}\,dx$$

```
int((a*x + b*x^2)^(5/4)/x,x)
```

```
int((a*x + b*x^2)^(5/4)/x, x)
```

Reduce [F]

$$\int \frac{(ax+bx^2)^{5/4}}{x}\,dx = \frac{20x^{\frac{1}{4}}(bx+a)^{\frac{1}{4}}\,a^2 + 88x^{\frac{5}{4}}(bx+a)^{\frac{1}{4}}\,ab + 48x^{\frac{9}{4}}(bx+a)^{\frac{1}{4}}\,b^2 - 5\left(\int \frac{(bx+a)^{\frac{1}{4}}}{x^{\frac{3}{4}}a+x^{\frac{7}{4}}b}dx\right)a^3}{120b}$$

```
int((b*x^2+a*x)^(5/4)/x,x)
```

```
(20*x**(1/4)*(a + b*x)**(1/4)*a**2 + 88*x**(1/4)*(a + b*x)**(1/4)*a*b*x +
48*x**(1/4)*(a + b*x)**(1/4)*b**2*x**2 - 5*int((a + b*x)**(1/4)/(x**(3/4)*
a + x**(3/4)*b*x),x)*a**3)/(120*b)
```

3.167 $\int \frac{(ax+bx^2)^{5/4}}{x^2}\,dx$

Optimal result

Integrand size = 17, antiderivative size = 112

$$\int \frac{(ax+bx^2)^{5/4}}{x^2}\,dx = \frac{5}{3}a\sqrt[4]{ax+bx^2}+\frac{2(ax+bx^2)^{5/4}}{3x} - \frac{5a^{3/2}\left(\frac{bx}{a+bx}\right)^{3/4}\sqrt{a+bx}\sqrt[4]{ax+bx^2}\,\text{EllipticF}\left(\frac{1}{2}\arcsin\left(\frac{\sqrt{a}}{\sqrt{a+bx}}\right),2\right)}{3bx}$$

```
5/3*a*(b*x^2+a*x)^(1/4)+2/3*(b*x^2+a*x)^(5/4)/x-5/3*a^(3/2)*(b*x/(b*x+a))^
(3/4)*(b*x+a)^(1/2)*(b*x^2+a*x)^(1/4)*InverseJacobiAM(1/2*arcsin(a^(1/2)/(
b*x+a)^(1/2)),2^(1/2))/b/x
```

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 10.01 (sec) , antiderivative size = 43, normalized size of antiderivative = 0.38

$$\int \frac{(ax+bx^2)^{5/4}}{x^2}\,dx = \frac{4a\sqrt[4]{x(a+bx)}\,\text{Hypergeometric2F1}\left(-\frac{5}{4},\frac{1}{4},\frac{5}{4},-\frac{bx}{a}\right)}{\sqrt[4]{1+\frac{bx}{a}}}$$

```
Integrate[(a*x + b*x^2)^(5/4)/x^2,x]
```

```
(4*a*(x*(a + b*x))^(1/4)*Hypergeometric2F1[-5/4, 1/4, 5/4, -((b*x)/a)])/(1
 + (b*x)/a)^(1/4)
```

Rubi [A] (warning: unable to verify)

Time = 0.46 (sec) , antiderivative size = 132, normalized size of antiderivative = 1.18, number of steps used = 9, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}} = 0.471$, Rules used = {1137, 60, 60, 73, 768, 858, 807, 229}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(ax+bx^2)^{5/4}}{x^2}\,dx$$

$$\Big\downarrow \; 1137$$

$$\frac{(ax+bx^2)^{5/4}\int \frac{(a+bx)^{5/4}}{x^{3/4}}dx}{x^{5/4}(a+bx)^{5/4}}$$

$$\Big\downarrow \; 60$$

$$\frac{(ax+bx^2)^{5/4}\left(\frac{5}{6}a\int \frac{\sqrt[4]{a+bx}}{x^{3/4}}dx+\frac{2}{3}\sqrt[4]{x}(a+bx)^{5/4}\right)}{x^{5/4}(a+bx)^{5/4}}$$

$$\Big\downarrow \; 60$$

$$\frac{(ax+bx^2)^{5/4}\left(\frac{5}{6}a\left(\frac{1}{2}a\int \frac{1}{x^{3/4}(a+bx)^{3/4}}dx+2\sqrt[4]{x}\sqrt[4]{a+bx}\right)+\frac{2}{3}\sqrt[4]{x}(a+bx)^{5/4}\right)}{x^{5/4}(a+bx)^{5/4}}$$

$$\Big\downarrow \; 73$$

$$\frac{(ax+bx^2)^{5/4}\left(\frac{5}{6}a\left(2a\int \frac{1}{(a+bx)^{3/4}}d\sqrt[4]{x}+2\sqrt[4]{x}\sqrt[4]{a+bx}\right)+\frac{2}{3}\sqrt[4]{x}(a+bx)^{5/4}\right)}{x^{5/4}(a+bx)^{5/4}}$$

$$\Big\downarrow \; 768$$

$$\frac{\left(ax+bx^2\right)^{5/4}\left(\frac{5}{6}a\left(\frac{2ax^{3/4}\left(\frac{a}{bx}+1\right)^{3/4}\int\frac{1}{\left(\frac{a}{bx}+1\right)^{3/4}x^{3/4}}d\sqrt[4]{x}}{(a+bx)^{3/4}}+2\sqrt[4]{x}\sqrt[4]{a+bx}\right)+\frac{2}{3}\sqrt[4]{x}(a+bx)^{5/4}\right)}{x^{5/4}(a+bx)^{5/4}}$$

$$\Big\downarrow \text{858}$$

$$\frac{\left(ax+bx^2\right)^{5/4}\left(\frac{5}{6}a\left(2\sqrt[4]{x}\sqrt[4]{a+bx}-\frac{2ax^{3/4}\left(\frac{a}{bx}+1\right)^{3/4}\int\frac{1}{\sqrt[4]{x}\left(\frac{ax}{b}+1\right)^{3/4}}d\frac{1}{\sqrt[4]{x}}}{(a+bx)^{3/4}}\right)+\frac{2}{3}\sqrt[4]{x}(a+bx)^{5/4}\right)}{x^{5/4}(a+bx)^{5/4}}$$

$$\Big\downarrow \text{807}$$

$$\frac{\left(ax+bx^2\right)^{5/4}\left(\frac{5}{6}a\left(2\sqrt[4]{x}\sqrt[4]{a+bx}-\frac{ax^{3/4}\left(\frac{a}{bx}+1\right)^{3/4}\int\frac{1}{\left(\frac{\sqrt{x}a}{b}+1\right)^{3/4}}d\sqrt{x}}{(a+bx)^{3/4}}\right)+\frac{2}{3}\sqrt[4]{x}(a+bx)^{5/4}\right)}{x^{5/4}(a+bx)^{5/4}}$$

$$\Big\downarrow \text{229}$$

$$\frac{\left(ax+bx^2\right)^{5/4}\left(\frac{5}{6}a\left(2\sqrt[4]{x}\sqrt[4]{a+bx}-\frac{2\sqrt{a}\sqrt{b}x^{3/4}\left(\frac{a}{bx}+1\right)^{3/4}\text{EllipticF}\left(\frac{1}{2}\arctan\left(\frac{\sqrt{a}\sqrt{x}}{\sqrt{b}}\right),2\right)}{(a+bx)^{3/4}}\right)+\frac{2}{3}\sqrt[4]{x}(a+bx)^{5/4}\right)}{x^{5/4}(a+bx)^{5/4}}$$

```
Int[(a*x + b*x^2)^(5/4)/x^2,x]
```

```
((a*x + b*x^2)^(5/4)*((2*x^(1/4)*(a + b*x)^(5/4))/3 + (5*a*(2*x^(1/4)*(a +
 b*x)^(1/4) - (2*Sqrt[a]*Sqrt[b]*(1 + a/(b*x))^(3/4)*x^(3/4)*EllipticF[Arc
Tan[(Sqrt[a]*Sqrt[x])/Sqrt[b]]/2, 2])/(a + b*x)^(3/4)))/6))/(x^(5/4)*(a +
b*x)^(5/4))
```

Defintions of rubi rules used

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^n/(b*(m + n + 1))), x] + Simp[n*((b*c - a*d)/(
b*(m + n + 1)))   Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
 c, d}, x] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ[m, 0] && ( !Integer
Q[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n + 2, 0] && IntLinear
Q[a, b, c, d, m, n, x]
```

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Simp[p/b   Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
 d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && Lt
Q[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntL
inearQ[a, b, c, d, m, n, x]
```

```
Int[((a_) + (b_.)*(x_)^2)^(-3/4), x_Symbol] :> Simp[(2/(a^(3/4)*Rt[b/a, 2])
)*EllipticF[(1/2)*ArcTan[Rt[b/a, 2]*x], 2], x] /; FreeQ[{a, b}, x] && GtQ[a
, 0] && PosQ[b/a]
```

```
Int[((a_) + (b_.)*(x_)^4)^(-3/4), x_Symbol] :> Simp[x^3*((1 + a/(b*x^4))^(3
/4)/(a + b*x^4)^(3/4))   Int[1/(x^3*(1 + a/(b*x^4))^(3/4)), x], x] /; FreeQ
[{a, b}, x]
```

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k = GCD[m
+ 1, n]}, Simp[1/k   Subst[Int[x^((m + 1)/k - 1)*(a + b*x^(n/k))^p, x], x,
x^k], x] /; k != 1] /; FreeQ[{a, b, p}, x] && IGtQ[n, 0] && IntegerQ[m]
```

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> -Subst[Int[(a +
b/x^n)^p/x^(m + 2), x], x, 1/x] /; FreeQ[{a, b, p}, x] && ILtQ[n, 0] && Int
egerQ[m]
```

```
Int[((e_.)*(x_))^(m_)*((b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(
e*x)^m*((b*x + c*x^2)^p/(x^(m + p)*(b + c*x)^p))   Int[x^(m + p)*(b + c*x)^
p, x], x] /; FreeQ[{b, c, e, m}, x]
```

Maple [F]

$$\int \frac{\left(b\,x^{2}+a x \right)^{\frac{5}{4}}}{x^{2}}d x$$

```
int((b*x^2+a*x)^(5/4)/x^2,x)
```

```
int((b*x^2+a*x)^(5/4)/x^2,x)
```

Fricas [F]

$$\int \frac{\left(ax+bx^2\right)^{5/4}}{x^2}\,dx=\int \frac{{\left(bx^{2}+ax\right)}^{\frac{5}{4}}}{x^{2}}\,dx$$

```
integrate((b*x^2+a*x)^(5/4)/x^2,x, algorithm="fricas")
```

```
integral((b*x^2 + a*x)^(1/4)*(b*x + a)/x, x)
```

Sympy [F]

$$\int \frac{\left(ax+bx^2\right)^{5/4}}{x^2}\,dx=\int \frac{\left(x\left(a+bx\right)\right)^{\frac{5}{4}}}{x^{2}}\,dx$$

```
integrate((b*x**2+a*x)**(5/4)/x**2,x)
```

```
Integral((x*(a + b*x))**(5/4)/x**2, x)
```

Maxima [F]

$$\int \frac{(ax+bx^2)^{5/4}}{x^2}\,dx = \int \frac{(bx^2+ax)^{\frac{5}{4}}}{x^2}\,dx$$

```
integrate((b*x^2+a*x)^(5/4)/x^2,x, algorithm="maxima")
```

```
integrate((b*x^2 + a*x)^(5/4)/x^2, x)
```

Giac [F]

$$\int \frac{(ax+bx^2)^{5/4}}{x^2}\,dx = \int \frac{(bx^2+ax)^{\frac{5}{4}}}{x^2}\,dx$$

```
integrate((b*x^2+a*x)^(5/4)/x^2,x, algorithm="giac")
```

```
integrate((b*x^2 + a*x)^(5/4)/x^2, x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(ax+bx^2)^{5/4}}{x^2}\,dx = \int \frac{(b\,x^2+a\,x)^{5/4}}{x^2}\,dx$$

```
int((a*x + b*x^2)^(5/4)/x^2,x)
```

```
int((a*x + b*x^2)^(5/4)/x^2, x)
```

Reduce [F]

$$\int \frac{(ax+bx^2)^{5/4}}{x^2}\,dx = \frac{7x^{\frac{1}{4}}(bx+a)^{\frac{1}{4}}\,a}{3} + \frac{2x^{\frac{5}{4}}(bx+a)^{\frac{1}{4}}\,b}{3} + \frac{5\left(\int \frac{(bx+a)^{\frac{1}{4}}}{x^{\frac{3}{4}}a+x^{\frac{7}{4}}b}dx\right)a^2}{12}$$

```
int((b*x^2+a*x)^(5/4)/x^2,x)
```

```
(28*x**(1/4)*(a + b*x)**(1/4)*a + 8*x**(1/4)*(a + b*x)**(1/4)*b*x + 5*int(
(a + b*x)**(1/4)/(x**(3/4)*a + x**(3/4)*b*x),x)*a**2)/12
```

3.168 $\int \frac{(ax+bx^2)^{5/4}}{x^3} dx$

Optimal result

Integrand size = 17, antiderivative size = 108

$$\int \frac{(ax+bx^2)^{5/4}}{x^3} dx = 2b\sqrt[4]{ax+bx^2} - \frac{4a\sqrt[4]{ax+bx^2}}{3x} - \frac{10\sqrt{a}\left(\frac{bx}{a+bx}\right)^{3/4}\sqrt{a+bx}\sqrt[4]{ax+bx^2}\,\text{EllipticF}\left(\frac{1}{2}\arcsin\left(\frac{\sqrt{a}}{\sqrt{a+bx}}\right),2\right)}{3x}$$

```
2*b*(b*x^2+a*x)^(1/4)-4/3*a*(b*x^2+a*x)^(1/4)/x-10/3*a^(1/2)*(b*x/(b*x+a))
^(3/4)*(b*x+a)^(1/2)*(b*x^2+a*x)^(1/4)*InverseJacobiAM(1/2*arcsin(a^(1/2)/
(b*x+a)^(1/2)),2^(1/2))/x
```

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 10.01 (sec) , antiderivative size = 48, normalized size of antiderivative = 0.44

$$\int \frac{(ax+bx^2)^{5/4}}{x^3} dx = -\frac{4a\sqrt[4]{x(a+bx)}\,\text{Hypergeometric2F1}\left(-\frac{5}{4},-\frac{3}{4},\frac{1}{4},-\frac{bx}{a}\right)}{3x\sqrt[4]{1+\frac{bx}{a}}}$$

```
Integrate[(a*x + b*x^2)^(5/4)/x^3,x]
```

```
(-4*a*(x*(a + b*x))^(1/4)*Hypergeometric2F1[-5/4, -3/4, 1/4, -((b*x)/a)])/
(3*x*(1 + (b*x)/a)^(1/4))
```

Rubi [A] (warning: unable to verify)

Time = 0.47 (sec) , antiderivative size = 132, normalized size of antiderivative = 1.22, number of steps used = 9, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.471, Rules used = {1137, 57, 60, 73, 768, 858, 807, 229}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\left(ax+bx^2\right)^{5/4}}{x^3}dx$$

$$\Big\downarrow \text{ 1137}$$

$$\frac{\left(ax+bx^2\right)^{5/4}\int \frac{(a+bx)^{5/4}}{x^{7/4}}dx}{x^{5/4}(a+bx)^{5/4}}$$

$$\Big\downarrow \text{ 57}$$

$$\frac{\left(ax+bx^2\right)^{5/4}\left(\frac{5}{3}b\int \frac{\sqrt[4]{a+bx}}{x^{3/4}}dx-\frac{4(a+bx)^{5/4}}{3x^{3/4}}\right)}{x^{5/4}(a+bx)^{5/4}}$$

$$\Big\downarrow \text{ 60}$$

$$\frac{\left(ax+bx^2\right)^{5/4}\left(\frac{5}{3}b\left(\frac{1}{2}a\int \frac{1}{x^{3/4}(a+bx)^{3/4}}dx+2\sqrt[4]{x}\sqrt[4]{a+bx}\right)-\frac{4(a+bx)^{5/4}}{3x^{3/4}}\right)}{x^{5/4}(a+bx)^{5/4}}$$

$$\Big\downarrow \text{ 73}$$

$$\frac{\left(ax+bx^2\right)^{5/4}\left(\frac{5}{3}b\left(2a\int \frac{1}{(a+bx)^{3/4}}d\sqrt[4]{x}+2\sqrt[4]{x}\sqrt[4]{a+bx}\right)-\frac{4(a+bx)^{5/4}}{3x^{3/4}}\right)}{x^{5/4}(a+bx)^{5/4}}$$

$$\Big\downarrow \text{ 768}$$

$$\frac{\left(ax+bx^2\right)^{5/4}\left(\frac{5}{3}b\left(\frac{2ax^{3/4}\left(\frac{a}{bx}+1\right)^{3/4}\int\frac{1}{\left(\frac{a}{bx}+1\right)^{3/4}x^{3/4}}d\sqrt[4]{x}}{(a+bx)^{3/4}}+2\sqrt[4]{x}\sqrt[4]{a+bx}\right)-\frac{4(a+bx)^{5/4}}{3x^{3/4}}\right)}{x^{5/4}(a+bx)^{5/4}}$$

$$\Big\downarrow \text{858}$$

$$\frac{\left(ax+bx^2\right)^{5/4}\left(\frac{5}{3}b\left(2\sqrt[4]{x}\sqrt[4]{a+bx}-\frac{2ax^{3/4}\left(\frac{a}{bx}+1\right)^{3/4}\int\frac{1}{\sqrt[4]{x}\left(\frac{ax}{b}+1\right)^{3/4}}d\frac{1}{\sqrt[4]{x}}}{(a+bx)^{3/4}}\right)-\frac{4(a+bx)^{5/4}}{3x^{3/4}}\right)}{x^{5/4}(a+bx)^{5/4}}$$

$$\Big\downarrow \text{807}$$

$$\frac{\left(ax+bx^2\right)^{5/4}\left(\frac{5}{3}b\left(2\sqrt[4]{x}\sqrt[4]{a+bx}-\frac{ax^{3/4}\left(\frac{a}{bx}+1\right)^{3/4}\int\frac{1}{\left(\frac{\sqrt{x}a}{b}+1\right)^{3/4}}d\sqrt{x}}{(a+bx)^{3/4}}\right)-\frac{4(a+bx)^{5/4}}{3x^{3/4}}\right)}{x^{5/4}(a+bx)^{5/4}}$$

$$\Big\downarrow \text{229}$$

$$\frac{\left(ax+bx^2\right)^{5/4}\left(\frac{5}{3}b\left(2\sqrt[4]{x}\sqrt[4]{a+bx}-\frac{2\sqrt{a}\sqrt{b}x^{3/4}\left(\frac{a}{bx}+1\right)^{3/4}\text{EllipticF}\left(\frac{1}{2}\arctan\left(\frac{\sqrt{a}\sqrt{x}}{\sqrt{b}}\right),2\right)}{(a+bx)^{3/4}}\right)-\frac{4(a+bx)^{5/4}}{3x^{3/4}}\right)}{x^{5/4}(a+bx)^{5/4}}$$

```
Int[(a*x + b*x^2)^(5/4)/x^3,x]
```

```
((a*x + b*x^2)^(5/4)*((-4*(a + b*x)^(5/4))/(3*x^(3/4)) + (5*b*(2*x^(1/4)*(
a + b*x)^(1/4) - (2*Sqrt[a]*Sqrt[b]*(1 + a/(b*x))^(3/4)*x^(3/4)*EllipticF[
ArcTan[(Sqrt[a]*Sqrt[x])/Sqrt[b]]/2, 2])/(a + b*x)^(3/4)))/3))/(x^(5/4)*(a
 + b*x)^(5/4))
```

Defintions of rubi rules used

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^n/(b*(m + 1))), x] - Simp[d*(n/(b*(m + 1)))
Int[(a + b*x)^(m + 1)*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b, c, d}, x] &
& GtQ[n, 0] && LtQ[m, -1] && !(IntegerQ[n] && !IntegerQ[m]) && !(ILeQ[m
+ n + 2, 0] && (FractionQ[m] || GeQ[2*n + m + 1, 0])) && IntLinearQ[a, b, c
, d, m, n, x]
```

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^n/(b*(m + n + 1))), x] + Simp[n*((b*c - a*d)/(
b*(m + n + 1)))   Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
 c, d}, x] && GtQ[n, 0] && NeQ[m + n + 1, 0] && !(IGtQ[m, 0] && ( !Integer
Q[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) && !ILtQ[m + n + 2, 0] && IntLinear
Q[a, b, c, d, m, n, x]
```

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Simp[p/b   Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
 d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && Lt
Q[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntL
inearQ[a, b, c, d, m, n, x]
```

```
Int[((a_) + (b_.)*(x_)^2)^(-3/4), x_Symbol] :> Simp[(2/(a^(3/4)*Rt[b/a, 2])
)*EllipticF[(1/2)*ArcTan[Rt[b/a, 2]*x], 2], x] /; FreeQ[{a, b}, x] && GtQ[a
, 0] && PosQ[b/a]
```

```
Int[((a_) + (b_.)*(x_)^4)^(-3/4), x_Symbol] :> Simp[x^3*((1 + a/(b*x^4))^(3
/4)/(a + b*x^4)^(3/4))   Int[1/(x^3*(1 + a/(b*x^4))^(3/4)), x], x] /; FreeQ
[{a, b}, x]
```

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k = GCD[m
+ 1, n]}, Simp[1/k   Subst[Int[x^((m + 1)/k - 1)*(a + b*x^(n/k))^p, x], x,
x^k], x] /; k != 1] /; FreeQ[{a, b, p}, x] && IGtQ[n, 0] && IntegerQ[m]
```

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> -Subst[Int[(a +
b/x^n)^p/x^(m + 2), x], x, 1/x] /; FreeQ[{a, b, p}, x] && ILtQ[n, 0] && Int
egerQ[m]
```

```
Int[((e_.)*(x_))^(m_)*((b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(
e*x)^m*((b*x + c*x^2)^p/(x^(m + p)*(b + c*x)^p))   Int[x^(m + p)*(b + c*x)^
p, x], x] /; FreeQ[{b, c, e, m}, x]
```

Maple [F]

$$\int \frac{(b\,x^2+ax)^{\frac{5}{4}}}{x^3}dx$$

```
int((b*x^2+a*x)^(5/4)/x^3,x)
```

```
int((b*x^2+a*x)^(5/4)/x^3,x)
```

Fricas [F]

$$\int \frac{(ax+bx^2)^{5/4}}{x^3}\,dx = \int \frac{(bx^2+ax)^{\frac{5}{4}}}{x^3}\,dx$$

```
integrate((b*x^2+a*x)^(5/4)/x^3,x, algorithm="fricas")
```

```
integral((b*x^2 + a*x)^(1/4)*(b*x + a)/x^2, x)
```

Sympy [F]

$$\int \frac{(ax+bx^2)^{5/4}}{x^3}\,dx = \int \frac{(x(a+bx))^{\frac{5}{4}}}{x^3}\,dx$$

```
integrate((b*x**2+a*x)**(5/4)/x**3,x)
```

```
Integral((x*(a + b*x))**(5/4)/x**3, x)
```

Maxima [F]

$$\int \frac{(ax+bx^2)^{5/4}}{x^3}\,dx = \int \frac{(bx^2+ax)^{\frac{5}{4}}}{x^3}\,dx$$

```
integrate((b*x^2+a*x)^(5/4)/x^3,x, algorithm="maxima")
```

```
integrate((b*x^2 + a*x)^(5/4)/x^3, x)
```

Giac [F]

$$\int \frac{(ax+bx^2)^{5/4}}{x^3}\,dx = \int \frac{(bx^2+ax)^{\frac{5}{4}}}{x^3}\,dx$$

```
integrate((b*x^2+a*x)^(5/4)/x^3,x, algorithm="giac")
```

```
integrate((b*x^2 + a*x)^(5/4)/x^3, x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(ax+bx^2)^{5/4}}{x^3}\,dx = \int \frac{(b\,x^2+a\,x)^{5/4}}{x^3}\,dx$$

```
int((a*x + b*x^2)^(5/4)/x^3,x)
```

```
int((a*x + b*x^2)^(5/4)/x^3, x)
```

Reduce [F]

$$\int \frac{(ax+bx^2)^{5/4}}{x^3}\,dx = \frac{-12(bx+a)^{\frac{1}{4}}\,a+8(bx+a)^{\frac{1}{4}}\,bx-5x^{\frac{3}{4}}\left(\int \frac{(bx+a)^{\frac{1}{4}}}{x^{\frac{7}{4}}a+x^{\frac{11}{4}}b}dx\right)a^2}{4x^{\frac{3}{4}}}$$

```
int((b*x^2+a*x)^(5/4)/x^3,x)
```

```
( - 12*(a + b*x)**(1/4)*a + 8*(a + b*x)**(1/4)*b*x - 5*x**(3/4)*int((a + b
*x)**(1/4)/(x**(3/4)*a*x + x**(3/4)*b*x**2),x)*a**2)/(4*x**(3/4))
```

3.169 $\int \frac{(ax+bx^2)^{5/4}}{x^4} \, dx$

Optimal result

Integrand size = 17, antiderivative size = 114

$$\int \frac{(ax+bx^2)^{5/4}}{x^4} \, dx = -\frac{4a\sqrt[4]{ax+bx^2}}{7x^2} - \frac{32b\sqrt[4]{ax+bx^2}}{21x} - \frac{20b\left(\frac{bx}{a+bx}\right)^{3/4}\sqrt{a+bx}\sqrt[4]{ax+bx^2}\,\text{EllipticF}\left(\frac{1}{2}\arcsin\left(\frac{\sqrt{a}}{\sqrt{a+bx}}\right),2\right)}{21\sqrt{a}x}$$

```
-4/7*a*(b*x^2+a*x)^(1/4)/x^2-32/21*b*(b*x^2+a*x)^(1/4)/x-20/21*b*(b*x/(b*x
+a))^(3/4)*(b*x+a)^(1/2)*(b*x^2+a*x)^(1/4)*InverseJacobiAM(1/2*arcsin(a^(1
/2)/(b*x+a)^(1/2)),2^(1/2))/a^(1/2)/x
```

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 10.02 (sec) , antiderivative size = 48, normalized size of antiderivative = 0.42

$$\int \frac{(ax+bx^2)^{5/4}}{x^4} \, dx = -\frac{4a\sqrt[4]{x(a+bx)}\,\text{Hypergeometric2F1}\left(-\frac{7}{4},-\frac{5}{4},-\frac{3}{4},-\frac{bx}{a}\right)}{7x^2\sqrt[4]{1+\frac{bx}{a}}}$$

```
Integrate[(a*x + b*x^2)^(5/4)/x^4,x]
```

```
(-4*a*(x*(a + b*x))^(1/4)*Hypergeometric2F1[-7/4, -5/4, -3/4, -((b*x)/a)])
/(7*x^2*(1 + (b*x)/a)^(1/4))
```

Rubi [A] (warning: unable to verify)

Time = 0.46 (sec) , antiderivative size = 136, normalized size of antiderivative = 1.19, number of steps used = 9, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}} = 0.471$, Rules used = {1137, 57, 57, 73, 768, 858, 807, 229}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(ax+bx^2)^{5/4}}{x^4} \, dx$$

$$\Big\downarrow \text{1137}$$

$$\frac{(ax+bx^2)^{5/4} \int \frac{(a+bx)^{5/4}}{x^{11/4}}dx}{x^{5/4}(a+bx)^{5/4}}$$

$$\Big\downarrow \text{57}$$

$$\frac{(ax+bx^2)^{5/4} \left(\frac{5}{7}b\int \frac{\sqrt[4]{a+bx}}{x^{7/4}}dx - \frac{4(a+bx)^{5/4}}{7x^{7/4}}\right)}{x^{5/4}(a+bx)^{5/4}}$$

$$\Big\downarrow \text{57}$$

$$\frac{(ax+bx^2)^{5/4} \left(\frac{5}{7}b\left(\frac{1}{3}b\int \frac{1}{x^{3/4}(a+bx)^{3/4}}dx - \frac{4\sqrt[4]{a+bx}}{3x^{3/4}}\right) - \frac{4(a+bx)^{5/4}}{7x^{7/4}}\right)}{x^{5/4}(a+bx)^{5/4}}$$

$$\Big\downarrow \text{73}$$

$$\frac{(ax+bx^2)^{5/4} \left(\frac{5}{7}b\left(\frac{4}{3}b\int \frac{1}{(a+bx)^{3/4}}d\sqrt[4]{x} - \frac{4\sqrt[4]{a+bx}}{3x^{3/4}}\right) - \frac{4(a+bx)^{5/4}}{7x^{7/4}}\right)}{x^{5/4}(a+bx)^{5/4}}$$

$$\Big\downarrow \text{768}$$

$$\frac{\left(ax+bx^2\right)^{5/4}\left(\frac{5}{7}b\left(\frac{4bx^{3/4}\left(\frac{a}{bx}+1\right)^{3/4}\int\frac{1}{\left(\frac{a}{bx}+1\right)^{3/4}x^{3/4}}d\sqrt[4]{x}}{3(a+bx)^{3/4}}-\frac{4\sqrt[4]{a+bx}}{3x^{3/4}}\right)-\frac{4(a+bx)^{5/4}}{7x^{7/4}}\right)}{x^{5/4}(a+bx)^{5/4}}$$

$$\Big\downarrow \text{858}$$

$$\frac{\left(ax+bx^2\right)^{5/4}\left(\frac{5}{7}b\left(-\frac{4bx^{3/4}\left(\frac{a}{bx}+1\right)^{3/4}\int\frac{1}{\sqrt[4]{x}\left(\frac{ax}{b}+1\right)^{3/4}}d\frac{1}{\sqrt[4]{x}}}{3(a+bx)^{3/4}}-\frac{4\sqrt[4]{a+bx}}{3x^{3/4}}\right)-\frac{4(a+bx)^{5/4}}{7x^{7/4}}\right)}{x^{5/4}(a+bx)^{5/4}}$$

$$\Big\downarrow \text{807}$$

$$\frac{\left(ax+bx^2\right)^{5/4}\left(\frac{5}{7}b\left(-\frac{2bx^{3/4}\left(\frac{a}{bx}+1\right)^{3/4}\int\frac{1}{\left(\frac{\sqrt{x}a}{b}+1\right)^{3/4}}d\sqrt{x}}{3(a+bx)^{3/4}}-\frac{4\sqrt[4]{a+bx}}{3x^{3/4}}\right)-\frac{4(a+bx)^{5/4}}{7x^{7/4}}\right)}{x^{5/4}(a+bx)^{5/4}}$$

$$\Big\downarrow \text{229}$$

$$\frac{\left(ax+bx^2\right)^{5/4}\left(\frac{5}{7}b\left(-\frac{4b^{3/2}x^{3/4}\left(\frac{a}{bx}+1\right)^{3/4}\text{EllipticF}\left(\frac{1}{2}\arctan\left(\frac{\sqrt{a}\sqrt{x}}{\sqrt{b}}\right),2\right)}{3\sqrt{a}(a+bx)^{3/4}}-\frac{4\sqrt[4]{a+bx}}{3x^{3/4}}\right)-\frac{4(a+bx)^{5/4}}{7x^{7/4}}\right)}{x^{5/4}(a+bx)^{5/4}}$$

```
Int[(a*x + b*x^2)^(5/4)/x^4,x]
```

```
((a*x + b*x^2)^(5/4)*((-4*(a + b*x)^(5/4))/(7*x^(7/4)) + (5*b*((-4*(a + b*
x)^(1/4))/(3*x^(3/4)) - (4*b^(3/2)*(1 + a/(b*x))^(3/4)*x^(3/4)*EllipticF[A
rcTan[(Sqrt[a]*Sqrt[x])/Sqrt[b]]/2, 2])/(3*Sqrt[a]*(a + b*x)^(3/4))))/7))/
(x^(5/4)*(a + b*x)^(5/4))
```

Defintions of rubi rules used

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^n/(b*(m + 1))), x] - Simp[d*(n/(b*(m + 1)))
Int[(a + b*x)^(m + 1)*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b, c, d}, x] &
& GtQ[n, 0] && LtQ[m, -1] && !(IntegerQ[n] && !IntegerQ[m]) && !(ILeQ[m
+ n + 2, 0] && (FractionQ[m] || GeQ[2*n + m + 1, 0])) && IntLinearQ[a, b, c
, d, m, n, x]
```

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Simp[p/b   Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
 d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && Lt
Q[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntL
inearQ[a, b, c, d, m, n, x]
```

```
Int[((a_) + (b_.)*(x_)^2)^(-3/4), x_Symbol] :> Simp[(2/(a^(3/4)*Rt[b/a, 2])
)*EllipticF[(1/2)*ArcTan[Rt[b/a, 2]*x], 2], x] /; FreeQ[{a, b}, x] && GtQ[a
, 0] && PosQ[b/a]
```

```
Int[((a_) + (b_.)*(x_)^4)^(-3/4), x_Symbol] :> Simp[x^3*((1 + a/(b*x^4))^(3
/4)/(a + b*x^4)^(3/4))   Int[1/(x^3*(1 + a/(b*x^4))^(3/4)), x], x] /; FreeQ
[{a, b}, x]
```

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k = GCD[m
+ 1, n]}, Simp[1/k   Subst[Int[x^((m + 1)/k - 1)*(a + b*x^(n/k))^p, x], x,
x^k], x] /; k != 1] /; FreeQ[{a, b, p}, x] && IGtQ[n, 0] && IntegerQ[m]
```

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> -Subst[Int[(a +
b/x^n)^p/x^(m + 2), x], x, 1/x] /; FreeQ[{a, b, p}, x] && ILtQ[n, 0] && Int
egerQ[m]
```

```
Int[((e_.)*(x_))^(m_)*((b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(
e*x)^m*((b*x + c*x^2)^p/(x^(m + p)*(b + c*x)^p))   Int[x^(m + p)*(b + c*x)^
p, x], x] /; FreeQ[{b, c, e, m}, x]
```

Maple [F]

$$\int \frac{(b\,x^2 + ax)^{\frac{5}{4}}}{x^4}dx$$

```
int((b*x^2+a*x)^(5/4)/x^4,x)
```

```
int((b*x^2+a*x)^(5/4)/x^4,x)
```

Fricas [F]

$$\int \frac{(ax+bx^2)^{5/4}}{x^4}\,dx=\int \frac{(bx^2+ax)^{\frac{5}{4}}}{x^4}\,dx$$

```
integrate((b*x^2+a*x)^(5/4)/x^4,x, algorithm="fricas")
```

```
integral((b*x^2 + a*x)^(1/4)*(b*x + a)/x^3, x)
```

Sympy [F]

$$\int \frac{(ax+bx^2)^{5/4}}{x^4}\,dx=\int \frac{(x(a+bx))^{\frac{5}{4}}}{x^4}\,dx$$

```
integrate((b*x**2+a*x)**(5/4)/x**4,x)
```

```
Integral((x*(a + b*x))**(5/4)/x**4, x)
```

Maxima [F]

$$\int \frac{(ax+bx^2)^{5/4}}{x^4}\,dx=\int \frac{(bx^2+ax)^{\frac{5}{4}}}{x^4}\,dx$$

```
integrate((b*x^2+a*x)^(5/4)/x^4,x, algorithm="maxima")
```

```
integrate((b*x^2 + a*x)^(5/4)/x^4, x)
```

Giac [F]

$$\int \frac{(ax+bx^2)^{5/4}}{x^4}\,dx = \int \frac{(bx^2+ax)^{\frac{5}{4}}}{x^4}\,dx$$

```
integrate((b*x^2+a*x)^(5/4)/x^4,x, algorithm="giac")
```

```
integrate((b*x^2 + a*x)^(5/4)/x^4, x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(ax+bx^2)^{5/4}}{x^4}\,dx = \int \frac{(b\,x^2+a\,x)^{5/4}}{x^4}\,dx$$

```
int((a*x + b*x^2)^(5/4)/x^4,x)
```

```
int((a*x + b*x^2)^(5/4)/x^4, x)
```

Reduce [F]

$$\int \frac{(ax+bx^2)^{5/4}}{x^4}\,dx = \frac{-4(bx+a)^{\frac{1}{4}}a-24(bx+a)^{\frac{1}{4}}bx+5x^{\frac{7}{4}}\left(\int \frac{(bx+a)^{\frac{1}{4}}}{x^{\frac{11}{4}}a+x^{\frac{15}{4}}b}dx\right)a^2}{12x^{\frac{7}{4}}}$$

```
int((b*x^2+a*x)^(5/4)/x^4,x)
```

```
( - 4*(a + b*x)**(1/4)*a - 24*(a + b*x)**(1/4)*b*x + 5*x**(3/4)*int((a + b
*x)**(1/4)/(x**(3/4)*a*x**2 + x**(3/4)*b*x**3),x)*a**2*x)/(12*x**(3/4)*x)
```

3.170 $\int \frac{(ax+bx^2)^{5/4}}{x^5}\,dx$

Optimal result

Integrand size = 17, antiderivative size = 142

$$\int \frac{(ax+bx^2)^{5/4}}{x^5}\,dx = -\frac{4a\sqrt[4]{ax+bx^2}}{11x^3}-\frac{48b\sqrt[4]{ax+bx^2}}{77x^2}-\frac{20b^2\sqrt[4]{ax+bx^2}}{231ax}+\frac{40b^2\left(\frac{bx}{a+bx}\right)^{3/4}\sqrt{a+bx}\sqrt[4]{ax+bx^2}\,\text{EllipticF}\left(\frac{1}{2}\arcsin\left(\frac{\sqrt{a}}{\sqrt{a+bx}}\right),2\right)}{231a^{3/2}x}$$

```
-4/11*a*(b*x^2+a*x)^(1/4)/x^3-48/77*b*(b*x^2+a*x)^(1/4)/x^2-20/231*b^2*(b*
x^2+a*x)^(1/4)/a/x+40/231*b^2*(b*x/(b*x+a))^(3/4)*(b*x+a)^(1/2)*(b*x^2+a*x
)^(1/4)*InverseJacobiAM(1/2*arcsin(a^(1/2)/(b*x+a)^(1/2)),2^(1/2))/a^(3/2)
/x
```

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 10.01 (sec) , antiderivative size = 48, normalized size of antiderivative = 0.34

$$\int \frac{(ax+bx^2)^{5/4}}{x^5}\,dx = -\frac{4a\sqrt[4]{x(a+bx)}\,\text{Hypergeometric2F1}\left(-\frac{11}{4},-\frac{5}{4},-\frac{7}{4},-\frac{bx}{a}\right)}{11x^3\sqrt[4]{1+\frac{bx}{a}}}$$

```
Integrate[(a*x + b*x^2)^(5/4)/x^5,x]
```

```
(-4*a*(x*(a + b*x))^(1/4)*Hypergeometric2F1[-11/4, -5/4, -7/4, -((b*x)/a)]
)/(11*x^3*(1 + (b*x)/a)^(1/4))
```

Rubi [A] (warning: unable to verify)

Time = 0.49 (sec) , antiderivative size = 163, normalized size of antiderivative = 1.15, number of steps used = 10, number of rules used = 9, $\frac{\text{number of rules}}{\text{integrand size}} = 0.529$, Rules used = {1137, 57, 57, 61, 73, 768, 858, 807, 229}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(ax+bx^2)^{5/4}}{x^5} \, dx$$

$$\Big\downarrow \text{ 1137}$$

$$\frac{(ax+bx^2)^{5/4} \int \frac{(a+bx)^{5/4}}{x^{15/4}}dx}{x^{5/4}(a+bx)^{5/4}}$$

$$\Big\downarrow \text{ 57}$$

$$\frac{(ax+bx^2)^{5/4} \left(\frac{5}{11}b\int \frac{\sqrt[4]{a+bx}}{x^{11/4}}dx-\frac{4(a+bx)^{5/4}}{11x^{11/4}}\right)}{x^{5/4}(a+bx)^{5/4}}$$

$$\Big\downarrow \text{ 57}$$

$$\frac{(ax+bx^2)^{5/4} \left(\frac{5}{11}b\left(\frac{1}{7}b\int \frac{1}{x^{7/4}(a+bx)^{3/4}}dx-\frac{4\sqrt[4]{a+bx}}{7x^{7/4}}\right)-\frac{4(a+bx)^{5/4}}{11x^{11/4}}\right)}{x^{5/4}(a+bx)^{5/4}}$$

$$\Big\downarrow \text{ 61}$$

$$\frac{(ax+bx^2)^{5/4} \left(\frac{5}{11}b\left(\frac{1}{7}b\left(-\frac{2b\int \frac{1}{x^{3/4}(a+bx)^{3/4}}dx}{3a}-\frac{4\sqrt[4]{a+bx}}{3ax^{3/4}}\right)-\frac{4\sqrt[4]{a+bx}}{7x^{7/4}}\right)-\frac{4(a+bx)^{5/4}}{11x^{11/4}}\right)}{x^{5/4}(a+bx)^{5/4}}$$

$$\Big\downarrow \text{ 73}$$

$$\frac{\left(ax+bx^2\right)^{5/4}\left(\frac{5}{11}b\left(\frac{1}{7}b\left(-\frac{8b\int\frac{1}{(a+bx)^{3/4}}d\sqrt[4]{x}}{3a}-\frac{4\sqrt[4]{a+bx}}{3ax^{3/4}}\right)-\frac{4\sqrt[4]{a+bx}}{7x^{7/4}}\right)-\frac{4(a+bx)^{5/4}}{11x^{11/4}}\right)}{x^{5/4}(a+bx)^{5/4}}$$

$$\Big\downarrow \textbf{768}$$

$$\frac{\left(ax+bx^2\right)^{5/4}\left(\frac{5}{11}b\left(\frac{1}{7}b\left(-\frac{8bx^{3/4}\left(\frac{a}{bx}+1\right)^{3/4}\int\frac{1}{\left(\frac{a}{bx}+1\right)^{3/4}x^{3/4}}d\sqrt[4]{x}}{3a(a+bx)^{3/4}}-\frac{4\sqrt[4]{a+bx}}{3ax^{3/4}}\right)-\frac{4\sqrt[4]{a+bx}}{7x^{7/4}}\right)-\frac{4(a+bx)^{5/4}}{11x^{11/4}}\right)}{x^{5/4}(a+bx)^{5/4}}$$

$$\Big\downarrow \textbf{858}$$

$$\frac{\left(ax+bx^2\right)^{5/4}\left(\frac{5}{11}b\left(\frac{1}{7}b\left(\frac{8bx^{3/4}\left(\frac{a}{bx}+1\right)^{3/4}\int\frac{1}{\sqrt[4]{x}\left(\frac{ax}{b}+1\right)^{3/4}}d\frac{1}{\sqrt[4]{x}}}{3a(a+bx)^{3/4}}-\frac{4\sqrt[4]{a+bx}}{3ax^{3/4}}\right)-\frac{4\sqrt[4]{a+bx}}{7x^{7/4}}\right)-\frac{4(a+bx)^{5/4}}{11x^{11/4}}\right)}{x^{5/4}(a+bx)^{5/4}}$$

$$\Big\downarrow \textbf{807}$$

$$\frac{\left(ax+bx^2\right)^{5/4}\left(\frac{5}{11}b\left(\frac{1}{7}b\left(\frac{4bx^{3/4}\left(\frac{a}{bx}+1\right)^{3/4}\int\frac{1}{\left(\frac{\sqrt{x}a}{b}+1\right)^{3/4}}d\sqrt{x}}{3a(a+bx)^{3/4}}-\frac{4\sqrt[4]{a+bx}}{3ax^{3/4}}\right)-\frac{4\sqrt[4]{a+bx}}{7x^{7/4}}\right)-\frac{4(a+bx)^{5/4}}{11x^{11/4}}\right)}{x^{5/4}(a+bx)^{5/4}}$$

$$\Big\downarrow \textbf{229}$$

$$\frac{\left(ax+bx^2\right)^{5/4}\left(\frac{5}{11}b\left(\frac{1}{7}b\left(\frac{8b^{3/2}x^{3/4}\left(\frac{a}{bx}+1\right)^{3/4}\text{EllipticF}\left(\frac{1}{2}\arctan\left(\frac{\sqrt{a}\sqrt{x}}{\sqrt{b}}\right),2\right)}{3a^{3/2}(a+bx)^{3/4}}-\frac{4\sqrt[4]{a+bx}}{3ax^{3/4}}\right)-\frac{4\sqrt[4]{a+bx}}{7x^{7/4}}\right)-\frac{4(a+bx)^{5/4}}{11x^{11/4}}\right)}{x^{5/4}(a+bx)^{5/4}}$$

```
Int[(a*x + b*x^2)^(5/4)/x^5,x]
```

```
((a*x + b*x^2)^(5/4)*((-4*(a + b*x)^(5/4))/(11*x^(11/4)) + (5*b*((-4*(a +
b*x)^(1/4))/(7*x^(7/4)) + (b*((-4*(a + b*x)^(1/4))/(3*a*x^(3/4)) + (8*b^(3
/2)*(1 + a/(b*x))^(3/4)*x^(3/4)*EllipticF[ArcTan[(Sqrt[a]*Sqrt[x])/Sqrt[b]
]/2, 2])/(3*a^(3/2)*(a + b*x)^(3/4))))/7))/11))/(x^(5/4)*(a + b*x)^(5/4))
```

Defintions of rubi rules used

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^n/(b*(m + 1))), x] - Simp[d*(n/(b*(m + 1)))
Int[(a + b*x)^(m + 1)*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b, c, d}, x] &
& GtQ[n, 0] && LtQ[m, -1] && !(IntegerQ[n] && !IntegerQ[m]) && !(ILeQ[m
+ n + 2, 0] && (FractionQ[m] || GeQ[2*n + m + 1, 0])) && IntLinearQ[a, b, c
, d, m, n, x]
```

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] - Simp[d*((
m + n + 2)/((b*c - a*d)*(m + 1)))   Int[(a + b*x)^(m + 1)*(c + d*x)^n, x],
x] /; FreeQ[{a, b, c, d, n}, x] && LtQ[m, -1] && !(LtQ[n, -1] && (EqQ[a, 0
] || (NeQ[c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && IntLinearQ[a, b, c, d
, m, n, x]
```

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Simp[p/b   Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
 d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && Lt
Q[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntL
inearQ[a, b, c, d, m, n, x]
```

```
Int[((a_) + (b_.)*(x_)^2)^(-3/4), x_Symbol] :> Simp[(2/(a^(3/4)*Rt[b/a, 2])
)*EllipticF[(1/2)*ArcTan[Rt[b/a, 2]*x], 2], x] /; FreeQ[{a, b}, x] && GtQ[a
, 0] && PosQ[b/a]
```

```
Int[((a_) + (b_.)*(x_)^4)^(-3/4), x_Symbol] :> Simp[x^3*((1 + a/(b*x^4))^(3
/4)/(a + b*x^4)^(3/4))   Int[1/(x^3*(1 + a/(b*x^4))^(3/4)), x], x] /; FreeQ
[{a, b}, x]
```

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k = GCD[m
+ 1, n]}, Simp[1/k   Subst[Int[x^((m + 1)/k - 1)*(a + b*x^(n/k))^p, x], x,
x^k], x] /; k != 1] /; FreeQ[{a, b, p}, x] && IGtQ[n, 0] && IntegerQ[m]
```

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> -Subst[Int[(a +
b/x^n)^p/x^(m + 2), x], x, 1/x] /; FreeQ[{a, b, p}, x] && ILtQ[n, 0] && Int
egerQ[m]
```

```
Int[((e_.)*(x_))^(m_)*((b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(
e*x)^m*((b*x + c*x^2)^p/(x^(m + p)*(b + c*x)^p))   Int[x^(m + p)*(b + c*x)^
p, x], x] /; FreeQ[{b, c, e, m}, x]
```

Maple [F]

$$\int \frac{(b\,x^2+ax)^{\frac{5}{4}}}{x^5}dx$$

```
int((b*x^2+a*x)^(5/4)/x^5,x)
```

```
int((b*x^2+a*x)^(5/4)/x^5,x)
```

Fricas [F]

$$\int \frac{(ax+bx^2)^{5/4}}{x^5}\,dx = \int \frac{(bx^2+ax)^{\frac{5}{4}}}{x^5}\,dx$$

```
integrate((b*x^2+a*x)^(5/4)/x^5,x, algorithm="fricas")
```

```
integral((b*x^2 + a*x)^(1/4)*(b*x + a)/x^4, x)
```

Sympy [F]

$$\int \frac{(ax+bx^2)^{5/4}}{x^5}\,dx = \int \frac{(x(a+bx))^{\frac{5}{4}}}{x^5}\,dx$$

```
integrate((b*x**2+a*x)**(5/4)/x**5,x)
```

```
Integral((x*(a + b*x))**(5/4)/x**5, x)
```

Maxima [F]

$$\int \frac{(ax+bx^2)^{5/4}}{x^5}\,dx = \int \frac{(bx^2+ax)^{\frac{5}{4}}}{x^5}\,dx$$

```
integrate((b*x^2+a*x)^(5/4)/x^5,x, algorithm="maxima")
```

```
integrate((b*x^2 + a*x)^(5/4)/x^5, x)
```

Giac [F]

$$\int \frac{(ax+bx^2)^{5/4}}{x^5}\,dx = \int \frac{(bx^2+ax)^{\frac{5}{4}}}{x^5}\,dx$$

```
integrate((b*x^2+a*x)^(5/4)/x^5,x, algorithm="giac")
```

```
integrate((b*x^2 + a*x)^(5/4)/x^5, x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(ax+bx^2)^{5/4}}{x^5}\,dx = \int \frac{\left(b\,x^2+a\,x\right)^{5/4}}{x^5}\,dx$$

```
int((a*x + b*x^2)^(5/4)/x^5,x)
```

```
int((a*x + b*x^2)^(5/4)/x^5, x)
```

Reduce [F]

$$\int \frac{(ax+bx^2)^{5/4}}{x^5}\,dx = \frac{-4(bx+a)^{\frac{1}{4}}a-8(bx+a)^{\frac{1}{4}}bx+x^{\frac{11}{4}}\left(\int \frac{(bx+a)^{\frac{1}{4}}}{x^{\frac{15}{4}}a+x^{\frac{19}{4}}b}dx\right)a^2}{12x^{\frac{11}{4}}}$$

```
int((b*x^2+a*x)^(5/4)/x^5,x)
```

```
( - 4*(a + b*x)**(1/4)*a - 8*(a + b*x)**(1/4)*b*x + x**(3/4)*int((a + b*x)
**(1/4)/(x**(3/4)*a*x**3 + x**(3/4)*b*x**4),x)*a**2*x**2)/(12*x**(3/4)*x**
2)
```

3.171 $\int \frac{(ax+bx^2)^{5/4}}{x^6}\,dx$

Optimal result

Integrand size = 17, antiderivative size = 167

$$\int \frac{(ax+bx^2)^{5/4}}{x^6}\,dx = -\frac{4b\sqrt[4]{ax+bx^2}}{33x^3} - \frac{4b^2\sqrt[4]{ax+bx^2}}{231ax^2} + \frac{8b^3\sqrt[4]{ax+bx^2}}{231a^2x} - \frac{4(ax+bx^2)^{5/4}}{15x^5} - \frac{16b^3\left(\frac{bx}{a+bx}\right)^{3/4}\sqrt{a+bx}\sqrt[4]{ax+bx^2}\,\text{EllipticF}\left(\frac{1}{2}\arcsin\left(\frac{\sqrt{a}}{\sqrt{a+bx}}\right),2\right)}{231a^{5/2}x}$$

```
-4/33*b*(b*x^2+a*x)^(1/4)/x^3-4/231*b^2*(b*x^2+a*x)^(1/4)/a/x^2+8/231*b^3*
(b*x^2+a*x)^(1/4)/a^2/x-4/15*(b*x^2+a*x)^(5/4)/x^5-16/231*b^3*(b*x/(b*x+a)
)^(3/4)*(b*x+a)^(1/2)*(b*x^2+a*x)^(1/4)*InverseJacobiAM(1/2*arcsin(a^(1/2)
/(b*x+a)^(1/2)),2^(1/2))/a^(5/2)/x
```

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 10.01 (sec) , antiderivative size = 48, normalized size of antiderivative = 0.29

$$\int \frac{(ax+bx^2)^{5/4}}{x^6}\,dx = -\frac{4a\sqrt[4]{x(a+bx)}\,\text{Hypergeometric2F1}\left(-\frac{15}{4},-\frac{5}{4},-\frac{11}{4},-\frac{bx}{a}\right)}{15x^4\sqrt[4]{1+\frac{bx}{a}}}$$

```
Integrate[(a*x + b*x^2)^(5/4)/x^6,x]
```

```
(-4*a*(x*(a + b*x))^(1/4)*Hypergeometric2F1[-15/4, -5/4, -11/4, -((b*x)/a)
])/(15*x^4*(1 + (b*x)/a)^(1/4))
```

Rubi [A] (warning: unable to verify)

Time = 0.52 (sec) , antiderivative size = 193, normalized size of antiderivative = 1.16, number of steps used = 11, number of rules used = 10, $\frac{\text{number of rules}}{\text{integrand size}} = 0.588$, Rules used = {1137, 57, 57, 61, 61, 73, 768, 858, 807, 229}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(ax+bx^2)^{5/4}}{x^6}\,dx$$

$$\Big\downarrow \text{1137}$$

$$\frac{(ax+bx^2)^{5/4}\int \frac{(a+bx)^{5/4}}{x^{19/4}}dx}{x^{5/4}(a+bx)^{5/4}}$$

$$\Big\downarrow \text{57}$$

$$\frac{(ax+bx^2)^{5/4}\left(\frac{1}{3}b\int \frac{\sqrt[4]{a+bx}}{x^{15/4}}dx-\frac{4(a+bx)^{5/4}}{15x^{15/4}}\right)}{x^{5/4}(a+bx)^{5/4}}$$

$$\Big\downarrow \text{57}$$

$$\frac{\left(ax+bx^2\right)^{5/4}\left(\frac{1}{3}b\left(\frac{1}{11}b\int\frac{1}{x^{11/4}(a+bx)^{3/4}}dx-\frac{4\sqrt[4]{a+bx}}{11x^{11/4}}\right)-\frac{4(a+bx)^{5/4}}{15x^{15/4}}\right)}{x^{5/4}(a+bx)^{5/4}}$$

$$\Big\downarrow \ 61$$

$$\frac{\left(ax+bx^2\right)^{5/4}\left(\frac{1}{3}b\left(\frac{1}{11}b\left(-\frac{6b\int\frac{1}{x^{7/4}(a+bx)^{3/4}}dx}{7a}-\frac{4\sqrt[4]{a+bx}}{7ax^{7/4}}\right)-\frac{4\sqrt[4]{a+bx}}{11x^{11/4}}\right)-\frac{4(a+bx)^{5/4}}{15x^{15/4}}\right)}{x^{5/4}(a+bx)^{5/4}}$$

$$\Big\downarrow \ 61$$

$$\frac{\left(ax+bx^2\right)^{5/4}\left(\frac{1}{3}b\left(\frac{1}{11}b\left(-\frac{6b\left(-\frac{2b\int\frac{1}{x^{3/4}(a+bx)^{3/4}}dx}{3a}-\frac{4\sqrt[4]{a+bx}}{3ax^{3/4}}\right)}{7a}-\frac{4\sqrt[4]{a+bx}}{7ax^{7/4}}\right)-\frac{4\sqrt[4]{a+bx}}{11x^{11/4}}\right)-\frac{4(a+bx)^{5/4}}{15x^{15/4}}\right)}{x^{5/4}(a+bx)^{5/4}}$$

$$\Big\downarrow \ 73$$

$$\frac{\left(ax+bx^2\right)^{5/4}\left(\frac{1}{3}b\left(\frac{1}{11}b\left(-\frac{6b\left(-\frac{8b\int\frac{1}{(a+bx)^{3/4}}d\sqrt[4]{x}}{3a}-\frac{4\sqrt[4]{a+bx}}{3ax^{3/4}}\right)}{7a}-\frac{4\sqrt[4]{a+bx}}{7ax^{7/4}}\right)-\frac{4\sqrt[4]{a+bx}}{11x^{11/4}}\right)-\frac{4(a+bx)^{5/4}}{15x^{15/4}}\right)}{x^{5/4}(a+bx)^{5/4}}$$

$$\Big\downarrow \ 768$$

$$\frac{\left(ax+bx^2\right)^{5/4}\left(\frac{1}{3}b\left(\frac{1}{11}b\left(-\frac{6b\left(-\frac{8bx^{3/4}\left(\frac{a}{bx}+1\right)^{3/4}\int\frac{1}{\left(\frac{a}{bx}+1\right)^{3/4}x^{3/4}}d\sqrt[4]{x}}{3a(a+bx)^{3/4}}-\frac{4\sqrt[4]{a+bx}}{3ax^{3/4}}\right)}{7a}-\frac{4\sqrt[4]{a+bx}}{7ax^{7/4}}\right)-\frac{4\sqrt[4]{a+bx}}{11x^{11/4}}\right)-\right.}{x^{5/4}(a+bx)^{5/4}}$$

$$\Big\downarrow \ 858$$

$$\frac{\left(ax+bx^2\right)^{5/4}\left(\frac{1}{3}b\left(\frac{1}{11}b\left(-\frac{6b\left(\frac{8bx^{3/4}\left(\frac{a}{bx}+1\right)^{3/4}\int\frac{1}{\sqrt[4]{x}\left(\frac{ax}{b}+1\right)^{3/4}}d\frac{1}{\sqrt[4]{x}}}{3a(a+bx)^{3/4}}-\frac{4\sqrt[4]{a+bx}}{3ax^{3/4}}\right)}{7a}-\frac{4\sqrt[4]{a+bx}}{7ax^{7/4}}\right)-\frac{4\sqrt[4]{a+bx}}{11x^{11/4}}\right)-\right.}{x^{5/4}(a+bx)^{5/4}}$$

$$\Big\downarrow \; 807$$

$$\frac{\left(ax+bx^2\right)^{5/4}\left(\frac{1}{3}b\left(\frac{1}{11}b\left(-\frac{6b\left(\frac{4bx^{3/4}\left(\frac{a}{bx}+1\right)^{3/4}\int\frac{1}{\left(\frac{\sqrt{x}a}{b}+1\right)^{3/4}}d\sqrt{x}}{3a(a+bx)^{3/4}}-\frac{4\sqrt[4]{a+bx}}{3ax^{3/4}}\right)}{7a}-\frac{4\sqrt[4]{a+bx}}{7ax^{7/4}}\right)-\frac{4\sqrt[4]{a+bx}}{11x^{11/4}}\right)-\frac{4(a+b}{15x}\right.}{x^{5/4}(a+bx)^{5/4}}$$

$$\Big\downarrow \; 229$$

$$\frac{\left(ax+bx^2\right)^{5/4}\left(\frac{1}{3}b\left(\frac{1}{11}b\left(-\frac{6b\left(\frac{8b^{3/2}x^{3/4}\left(\frac{a}{bx}+1\right)^{3/4}\text{EllipticF}\left(\frac{1}{2}\arctan\left(\frac{\sqrt{a}\sqrt{x}}{\sqrt{b}}\right),2\right)}{3a^{3/2}(a+bx)^{3/4}}-\frac{4\sqrt[4]{a+bx}}{3ax^{3/4}}\right)}{7a}-\frac{4\sqrt[4]{a+bx}}{7ax^{7/4}}\right)-\frac{4\sqrt[4]{a+bx}}{11x^{11/4}}\right.\right.}{x^{5/4}(a+bx)^{5/4}}$$

```
Int[(a*x + b*x^2)^(5/4)/x^6,x]
```

```
((a*x + b*x^2)^(5/4)*((-4*(a + b*x)^(5/4))/(15*x^(15/4)) + (b*((-4*(a + b*
x)^(1/4))/(11*x^(11/4)) + (b*((-4*(a + b*x)^(1/4))/(7*a*x^(7/4)) - (6*b*((
-4*(a + b*x)^(1/4))/(3*a*x^(3/4)) + (8*b^(3/2)*(1 + a/(b*x))^(3/4)*x^(3/4)
*EllipticF[ArcTan[(Sqrt[a]*Sqrt[x])/Sqrt[b]]/2, 2])/(3*a^(3/2)*(a + b*x)^(
3/4))))/(7*a)))/11))/3))/(x^(5/4)*(a + b*x)^(5/4))
```

Defintions of rubi rules used

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^n/(b*(m + 1))), x] - Simp[d*(n/(b*(m + 1)))
Int[(a + b*x)^(m + 1)*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b, c, d}, x] &
& GtQ[n, 0] && LtQ[m, -1] &&  !(IntegerQ[n] &&  !IntegerQ[m]) &&  !(ILeQ[m
+ n + 2, 0] && (FractionQ[m] || GeQ[2*n + m + 1, 0])) && IntLinearQ[a, b, c
, d, m, n, x]
```

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] - Simp[d*((
m + n + 2)/((b*c - a*d)*(m + 1)))   Int[(a + b*x)^(m + 1)*(c + d*x)^n, x],
x] /; FreeQ[{a, b, c, d, n}, x] && LtQ[m, -1] &&  !(LtQ[n, -1] && (EqQ[a, 0
] || (NeQ[c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && IntLinearQ[a, b, c, d
, m, n, x]
```

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Simp[p/b   Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
 d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && Lt
Q[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntL
inearQ[a, b, c, d, m, n, x]
```

```
Int[((a_) + (b_.)*(x_)^2)^(-3/4), x_Symbol] :> Simp[(2/(a^(3/4)*Rt[b/a, 2])
)*EllipticF[(1/2)*ArcTan[Rt[b/a, 2]*x], 2], x] /; FreeQ[{a, b}, x] && GtQ[a
, 0] && PosQ[b/a]
```

```
Int[((a_) + (b_.)*(x_)^4)^(-3/4), x_Symbol] :> Simp[x^3*((1 + a/(b*x^4))^(3
/4)/(a + b*x^4)^(3/4))   Int[1/(x^3*(1 + a/(b*x^4))^(3/4)), x], x] /; FreeQ
[{a, b}, x]
```

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k = GCD[m
+ 1, n]}, Simp[1/k   Subst[Int[x^((m + 1)/k - 1)*(a + b*x^(n/k))^p, x], x,
x^k], x] /; k != 1] /; FreeQ[{a, b, p}, x] && IGtQ[n, 0] && IntegerQ[m]
```

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> -Subst[Int[(a +
b/x^n)^p/x^(m + 2), x], x, 1/x] /; FreeQ[{a, b, p}, x] && ILtQ[n, 0] && Int
egerQ[m]
```

```
Int[((e_.)*(x_))^(m_)*((b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(
e*x)^m*((b*x + c*x^2)^p/(x^(m + p)*(b + c*x)^p))   Int[x^(m + p)*(b + c*x)^
p, x], x] /; FreeQ[{b, c, e, m}, x]
```

Maple [F]

$$\int \frac{(b\,x^{2}+ax)^{\frac{5}{4}}}{x^{6}}dx$$

```
int((b*x^2+a*x)^(5/4)/x^6,x)
```

```
int((b*x^2+a*x)^(5/4)/x^6,x)
```

Fricas [F]

$$\int \frac{(ax+bx^2)^{5/4}}{x^6}\,dx = \int \frac{(bx^2+ax)^{\frac{5}{4}}}{x^6}\,dx$$

```
integrate((b*x^2+a*x)^(5/4)/x^6,x, algorithm="fricas")
```

```
integral((b*x^2 + a*x)^(1/4)*(b*x + a)/x^5, x)
```

Sympy [F]

$$\int \frac{(ax+bx^2)^{5/4}}{x^6}\,dx = \int \frac{(x(a+bx))^{\frac{5}{4}}}{x^6}\,dx$$

```
integrate((b*x**2+a*x)**(5/4)/x**6,x)
```

```
Integral((x*(a + b*x))**(5/4)/x**6, x)
```

Maxima [F]

$$\int \frac{(ax+bx^2)^{5/4}}{x^6}\,dx = \int \frac{(bx^2+ax)^{\frac{5}{4}}}{x^6}\,dx$$

```
integrate((b*x^2+a*x)^(5/4)/x^6,x, algorithm="maxima")
```

```
integrate((b*x^2 + a*x)^(5/4)/x^6, x)
```

Giac [F]

$$\int \frac{(ax+bx^2)^{5/4}}{x^6}\,dx = \int \frac{(bx^2+ax)^{\frac{5}{4}}}{x^6}\,dx$$

```
integrate((b*x^2+a*x)^(5/4)/x^6,x, algorithm="giac")
```

```
integrate((b*x^2 + a*x)^(5/4)/x^6, x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(ax+bx^2)^{5/4}}{x^6}\,dx = \int \frac{(b\,x^2+a\,x)^{5/4}}{x^6}\,dx$$

```
int((a*x + b*x^2)^(5/4)/x^6,x)
```

```
int((a*x + b*x^2)^(5/4)/x^6, x)
```

Reduce [F]

$$\int \frac{(ax+bx^2)^{5/4}}{x^6}\,dx = \frac{-36(bx+a)^{\frac{1}{4}}a-56(bx+a)^{\frac{1}{4}}bx+5x^{\frac{15}{4}}\left(\int \frac{(bx+a)^{\frac{1}{4}}}{x^{\frac{19}{4}}a+x^{\frac{23}{4}}b}dx\right)a^2}{140x^{\frac{15}{4}}}$$

```
int((b*x^2+a*x)^(5/4)/x^6,x)
```

```
( - 36*(a + b*x)**(1/4)*a - 56*(a + b*x)**(1/4)*b*x + 5*x**(3/4)*int((a +
b*x)**(1/4)/(x**(3/4)*a*x**4 + x**(3/4)*b*x**5),x)*a**2*x**3)/(140*x**(3/4
)*x**3)
```

3.172 $\int \frac{(ax+bx^2)^{5/4}}{x^7} dx$

Optimal result

Integrand size = 17, antiderivative size = 193

$$\begin{aligned}\int \frac{(ax+bx^2)^{5/4}}{x^7}\,dx &= -\frac{4b\sqrt[4]{ax+bx^2}}{57x^4}-\frac{4b^2\sqrt[4]{ax+bx^2}}{627ax^3}\\ &+\frac{40b^3\sqrt[4]{ax+bx^2}}{4389a^2x^2}-\frac{80b^4\sqrt[4]{ax+bx^2}}{4389a^3x}-\frac{4(ax+bx^2)^{5/4}}{19x^6}\\ &+\frac{160b^4\left(\frac{bx}{a+bx}\right)^{3/4}\sqrt{a+bx}\sqrt[4]{ax+bx^2}\,\text{EllipticF}\left(\frac{1}{2}\arcsin\left(\frac{\sqrt{a}}{\sqrt{a+bx}}\right),2\right)}{4389a^{7/2}x}\end{aligned}$$

```
-4/57*b*(b*x^2+a*x)^(1/4)/x^4-4/627*b^2*(b*x^2+a*x)^(1/4)/a/x^3+40/4389*b^
3*(b*x^2+a*x)^(1/4)/a^2/x^2-80/4389*b^4*(b*x^2+a*x)^(1/4)/a^3/x-4/19*(b*x^
2+a*x)^(5/4)/x^6+160/4389*b^4*(b*x/(b*x+a))^(3/4)*(b*x+a)^(1/2)*(b*x^2+a*x
)^(1/4)*InverseJacobiAM(1/2*arcsin(a^(1/2)/(b*x+a)^(1/2)),2^(1/2))/a^(7/2)
/x
```

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 10.01 (sec) , antiderivative size = 48, normalized size of antiderivative = 0.25

$$\int \frac{(ax+bx^2)^{5/4}}{x^7}\,dx = -\frac{4a\sqrt[4]{x(a+bx)}\,\text{Hypergeometric2F1}\left(-\frac{19}{4},-\frac{5}{4},-\frac{15}{4},-\frac{bx}{a}\right)}{19x^5\sqrt[4]{1+\frac{bx}{a}}}$$

```
Integrate[(a*x + b*x^2)^(5/4)/x^7,x]
```

```
(-4*a*(x*(a + b*x))^(1/4)*Hypergeometric2F1[-19/4, -5/4, -15/4, -((b*x)/a)
])/(19*x^5*(1 + (b*x)/a)^(1/4))
```

Rubi [A] (warning: unable to verify)

Time = 0.59 (sec) , antiderivative size = 223, normalized size of antiderivative = 1.16, number of steps used = 12, number of rules used = 11, $\frac{\text{number of rules}}{\text{integrand size}} = 0.647$, Rules used = {1137, 57, 57, 61, 61, 61, 73, 768, 858, 807, 229}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(ax+bx^2)^{5/4}}{x^7}\,dx$$

$$\Big\downarrow \; 1137$$

$$\frac{(ax+bx^2)^{5/4}\int \frac{(a+bx)^{5/4}}{x^{23/4}}dx}{x^{5/4}(a+bx)^{5/4}}$$

$$\Big\downarrow \; 57$$

$$\frac{(ax+bx^2)^{5/4}\left(\frac{5}{19}b\int \frac{\sqrt[4]{a+bx}}{x^{19/4}}dx-\frac{4(a+bx)^{5/4}}{19x^{19/4}}\right)}{x^{5/4}(a+bx)^{5/4}}$$

$$\Big\downarrow \; 57$$

$$\frac{\left(ax+bx^2\right)^{5/4}\left(\frac{5}{19}b\left(\frac{1}{15}b\int \frac{1}{x^{15/4}(a+bx)^{3/4}}dx-\frac{4\sqrt[4]{a+bx}}{15x^{15/4}}\right)-\frac{4(a+bx)^{5/4}}{19x^{19/4}}\right)}{x^{5/4}(a+bx)^{5/4}}$$

$$\Big\downarrow \; 61$$

$$\frac{\left(ax+bx^2\right)^{5/4}\left(\frac{5}{19}b\left(\frac{1}{15}b\left(-\frac{10b\int \frac{1}{x^{11/4}(a+bx)^{3/4}}dx}{11a}-\frac{4\sqrt[4]{a+bx}}{11ax^{11/4}}\right)-\frac{4\sqrt[4]{a+bx}}{15x^{15/4}}\right)-\frac{4(a+bx)^{5/4}}{19x^{19/4}}\right)}{x^{5/4}(a+bx)^{5/4}}$$

$$\Big\downarrow \; 61$$

$$\frac{\left(ax+bx^2\right)^{5/4}\left(\frac{5}{19}b\left(\frac{1}{15}b\left(-\frac{10b\left(-\frac{6b\int \frac{1}{x^{7/4}(a+bx)^{3/4}}dx}{7a}-\frac{4\sqrt[4]{a+bx}}{7ax^{7/4}}\right)}{11a}-\frac{4\sqrt[4]{a+bx}}{11ax^{11/4}}\right)-\frac{4\sqrt[4]{a+bx}}{15x^{15/4}}\right)-\frac{4(a+bx)^{5/4}}{19x^{19/4}}\right)}{x^{5/4}(a+bx)^{5/4}}$$

$$\Big\downarrow \; 61$$

$$\frac{\left(ax+bx^2\right)^{5/4}\left(\frac{5}{19}b\left(\frac{1}{15}b\left(-\frac{10b\left(-\frac{6b\left(-\frac{2b\int \frac{1}{x^{3/4}(a+bx)^{3/4}}dx}{3a}-\frac{4\sqrt[4]{a+bx}}{3ax^{3/4}}\right)}{7a}-\frac{4\sqrt[4]{a+bx}}{7ax^{7/4}}\right)}{11a}-\frac{4\sqrt[4]{a+bx}}{11ax^{11/4}}\right)-\frac{4\sqrt[4]{a+bx}}{15x^{15/4}}\right)\right.}{x^{5/4}(a+bx)^{5/4}}$$

$$\Big\downarrow \; 73$$

$$\frac{\left(ax+bx^2\right)^{5/4}\left(\frac{5}{19}b\left(\frac{1}{15}b\left(-\frac{10b\left(-\frac{6b\left(-\frac{8b\int \frac{1}{(a+bx)^{3/4}}d\sqrt[4]{x}}{3a}-\frac{4\sqrt[4]{a+bx}}{3ax^{3/4}}\right)}{7a}-\frac{4\sqrt[4]{a+bx}}{7ax^{7/4}}\right)}{11a}-\frac{4\sqrt[4]{a+bx}}{11ax^{11/4}}\right)-\frac{4\sqrt[4]{a+bx}}{15x^{15/4}}\right)\right.}{x^{5/4}(a+bx)^{5/4}}$$

$$\Big\downarrow \; 768$$

$$\frac{\left(ax+bx^2\right)^{5/4}\left(\frac{5}{19}b\left(\frac{1}{15}b\left(-\frac{10b\left(-\frac{6b\left(-\frac{8bx^{3/4}\left(\frac{a}{bx}+1\right)^{3/4}\int\frac{1}{\left(\frac{a}{bx}+1\right)^{3/4}x^{3/4}}d\sqrt[4]{x}}{3a(a+bx)^{3/4}}-\frac{4\sqrt[4]{a+bx}}{3ax^{3/4}}\right)}{7a}-\frac{4\sqrt[4]{a+bx}}{7ax^{7/4}}\right)}{11a}-\frac{4\sqrt[4]{a+bx}}{11ax^{11/4}}\right)\right)\right)}{x^{5/4}(a+bx)^{5/4}}$$

$$\Big\downarrow \text{ 858}$$

$$\frac{\left(ax+bx^2\right)^{5/4}\left(\frac{5}{19}b\left(\frac{1}{15}b\left(-\frac{10b\left(-\frac{6b\left(\frac{8bx^{3/4}\left(\frac{a}{bx}+1\right)^{3/4}\int\frac{1}{\sqrt[4]{x}\left(\frac{ax}{b}+1\right)^{3/4}}d\frac{1}{\sqrt[4]{x}}}{3a(a+bx)^{3/4}}-\frac{4\sqrt[4]{a+bx}}{3ax^{3/4}}\right)}{7a}-\frac{4\sqrt[4]{a+bx}}{7ax^{7/4}}\right)}{11a}-\frac{4\sqrt[4]{a+bx}}{11ax^{11/4}}\right)\right)\right)}{x^{5/4}(a+bx)^{5/4}}$$

$$\Big\downarrow \text{ 807}$$

$$\frac{\left(ax+bx^2\right)^{5/4}\left(\frac{5}{19}b\left(\frac{1}{15}b\left(-\frac{10b\left(-\frac{6b\left(\frac{4bx^{3/4}\left(\frac{a}{bx}+1\right)^{3/4}\int \frac{1}{\left(\frac{\sqrt{x}a}{b}+1\right)^{3/4}}d\sqrt{x}}{3a(a+bx)^{3/4}}-\frac{4\sqrt[4]{a+bx}}{3ax^{3/4}}\right)}{7a}-\frac{4\sqrt[4]{a+bx}}{7ax^{7/4}}\right)}{11a}-\frac{4\sqrt[4]{a+bx}}{11ax^{11/4}}\right)-\cdots\right.\right.}{x^{5/4}(a+bx)^{5/4}}$$

$$\Big\downarrow \ 229$$

$$\frac{\left(ax+bx^2\right)^{5/4}\left(\frac{5}{19}b\left(\frac{1}{15}b\left(-\frac{10b\left(-\frac{6b\left(\frac{8b^{3/2}x^{3/4}\left(\frac{a}{bx}+1\right)^{3/4}\text{EllipticF}\left(\frac{1}{2}\arctan\left(\frac{\sqrt{a}\sqrt{x}}{\sqrt{b}}\right),2\right)}{3a^{3/2}(a+bx)^{3/4}}-\frac{4\sqrt[4]{a+bx}}{3ax^{3/4}}\right)}{7a}-\frac{4\sqrt[4]{a+bx}}{7ax^{7/4}}\right)}{11a}-\frac{4\sqrt[4]{a\cdots}}{11ax^{\cdots}}\cdots\right.\right.\right.}{x^{5/4}(a+bx)^{5/4}}$$

```
Int[(a*x + b*x^2)^(5/4)/x^7,x]
```

```
((a*x + b*x^2)^(5/4)*((-4*(a + b*x)^(5/4))/(19*x^(19/4)) + (5*b*((-4*(a +
b*x)^(1/4))/(15*x^(15/4)) + (b*((-4*(a + b*x)^(1/4))/(11*a*x^(11/4)) - (10
*b*((-4*(a + b*x)^(1/4))/(7*a*x^(7/4)) - (6*b*((-4*(a + b*x)^(1/4))/(3*a*x
^(3/4)) + (8*b^(3/2)*(1 + a/(b*x))^(3/4)*x^(3/4)*EllipticF[ArcTan[(Sqrt[a]
*Sqrt[x])/Sqrt[b]]/2, 2])/(3*a^(3/2)*(a + b*x)^(3/4))))/(7*a)))/(11*a)))/1
5))/19))/(x^(5/4)*(a + b*x)^(5/4))
```

Defintions of rubi rules used

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^n/(b*(m + 1))), x] - Simp[d*(n/(b*(m + 1)))
Int[(a + b*x)^(m + 1)*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b, c, d}, x] &
& GtQ[n, 0] && LtQ[m, -1] &&  !(IntegerQ[n] &&  !IntegerQ[m]) &&  !(ILeQ[m
+ n + 2, 0] && (FractionQ[m] || GeQ[2*n + m + 1, 0])) && IntLinearQ[a, b, c
, d, m, n, x]
```

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] - Simp[d*((
m + n + 2)/((b*c - a*d)*(m + 1)))   Int[(a + b*x)^(m + 1)*(c + d*x)^n, x],
x] /; FreeQ[{a, b, c, d, n}, x] && LtQ[m, -1] &&  !(LtQ[n, -1] && (EqQ[a, 0
] || (NeQ[c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && IntLinearQ[a, b, c, d
, m, n, x]
```

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Simp[p/b   Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
 d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && Lt
Q[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntL
inearQ[a, b, c, d, m, n, x]
```

```
Int[((a_) + (b_.)*(x_)^2)^(-3/4), x_Symbol] :> Simp[(2/(a^(3/4)*Rt[b/a, 2])
)*EllipticF[(1/2)*ArcTan[Rt[b/a, 2]*x], 2], x] /; FreeQ[{a, b}, x] && GtQ[a
, 0] && PosQ[b/a]
```

```
Int[((a_) + (b_.)*(x_)^4)^(-3/4), x_Symbol] :> Simp[x^3*((1 + a/(b*x^4))^(3
/4)/(a + b*x^4)^(3/4))   Int[1/(x^3*(1 + a/(b*x^4))^(3/4)), x], x] /; FreeQ
[{a, b}, x]
```

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k = GCD[m
+ 1, n]}, Simp[1/k   Subst[Int[x^((m + 1)/k - 1)*(a + b*x^(n/k))^p, x], x,
x^k], x] /; k != 1] /; FreeQ[{a, b, p}, x] && IGtQ[n, 0] && IntegerQ[m]
```

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> -Subst[Int[(a +
b/x^n)^p/x^(m + 2), x], x, 1/x] /; FreeQ[{a, b, p}, x] && ILtQ[n, 0] && Int
egerQ[m]
```

```
Int[((e_.)*(x_))^(m_)*((b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(
e*x)^m*((b*x + c*x^2)^p/(x^(m + p)*(b + c*x)^p))   Int[x^(m + p)*(b + c*x)^
p, x], x] /; FreeQ[{b, c, e, m}, x]
```

Maple [F]

$$\int \frac{(b\,x^{2}+ax)^{\frac{5}{4}}}{x^{7}}dx$$

```
int((b*x^2+a*x)^(5/4)/x^7,x)
```

```
int((b*x^2+a*x)^(5/4)/x^7,x)
```

Fricas [F]

$$\int \frac{(ax+bx^2)^{5/4}}{x^7}\,dx = \int \frac{(bx^2+ax)^{\frac{5}{4}}}{x^7}\,dx$$

```
integrate((b*x^2+a*x)^(5/4)/x^7,x, algorithm="fricas")
```

```
integral((b*x^2 + a*x)^(1/4)*(b*x + a)/x^6, x)
```

Sympy [F]

$$\int \frac{(ax+bx^2)^{5/4}}{x^7}\,dx = \int \frac{(x(a+bx))^{\frac{5}{4}}}{x^7}\,dx$$

```
integrate((b*x**2+a*x)**(5/4)/x**7,x)
```

```
Integral((x*(a + b*x))**(5/4)/x**7, x)
```

Maxima [F]

$$\int \frac{(ax+bx^2)^{5/4}}{x^7}\,dx = \int \frac{(bx^2+ax)^{\frac{5}{4}}}{x^7}\,dx$$

```
integrate((b*x^2+a*x)^(5/4)/x^7,x, algorithm="maxima")
```

```
integrate((b*x^2 + a*x)^(5/4)/x^7, x)
```

Giac [F]

$$\int \frac{(ax+bx^2)^{5/4}}{x^7}\,dx = \int \frac{(bx^2+ax)^{\frac{5}{4}}}{x^7}\,dx$$

```
integrate((b*x^2+a*x)^(5/4)/x^7,x, algorithm="giac")
```

```
integrate((b*x^2 + a*x)^(5/4)/x^7, x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(ax+bx^2)^{5/4}}{x^7}\,dx = \int \frac{(b\,x^2+a\,x)^{5/4}}{x^7}\,dx$$

```
int((a*x + b*x^2)^(5/4)/x^7,x)
```

```
int((a*x + b*x^2)^(5/4)/x^7, x)
```

Reduce [F]

$$\int \frac{(ax+bx^2)^{5/4}}{x^7}\,dx = \frac{-52(bx+a)^{\frac{1}{4}}a-72(bx+a)^{\frac{1}{4}}bx+5x^{\frac{19}{4}}\left(\int \frac{(bx+a)^{\frac{1}{4}}}{x^{\frac{23}{4}}a+x^{\frac{27}{4}}b}dx\right)a^2}{252x^{\frac{19}{4}}}$$

```
int((b*x^2+a*x)^(5/4)/x^7,x)
```

```
( - 52*(a + b*x)**(1/4)*a - 72*(a + b*x)**(1/4)*b*x + 5*x**(3/4)*int((a +
b*x)**(1/4)/(x**(3/4)*a*x**5 + x**(3/4)*b*x**6),x)*a**2*x**4)/(252*x**(3/4
)*x**4)
```

3.173 $\int \frac{x^4}{\sqrt[4]{ax+bx^2}}\,dx$

Optimal result

Integrand size = 17, antiderivative size = 162

$$\int \frac{x^4}{\sqrt[4]{ax+bx^2}}\,dx = -\frac{11a^3(ax+bx^2)^{3/4}}{36b^4}+\frac{11a^2x(ax+bx^2)^{3/4}}{42b^3}-\frac{5ax^2(ax+bx^2)^{3/4}}{21b^2}+\frac{2x^3(ax+bx^2)^{3/4}}{9b}+\frac{11a^5\sqrt[4]{-\frac{bx}{a}-\frac{b^2x^2}{a^2}}\,E\left(\frac{1}{2}\arcsin\left(1+\frac{2bx}{a}\right)\middle|2\right)}{24\sqrt{2}b^5\sqrt[4]{ax+bx^2}}$$

```
-11/36*a^3*(b*x^2+a*x)^(3/4)/b^4+11/42*a^2*x*(b*x^2+a*x)^(3/4)/b^3-5/21*a*
x^2*(b*x^2+a*x)^(3/4)/b^2+2/9*x^3*(b*x^2+a*x)^(3/4)/b+11/48*a^5*(-b*x/a-b^
2*x^2/a^2)^(1/4)*EllipticE(sin(1/2*arcsin(1+2*b*x/a)),2^(1/2))*2^(1/2)/b^5
/(b*x^2+a*x)^(1/4)
```

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 10.01 (sec) , antiderivative size = 47, normalized size of antiderivative = 0.29

$$\int \frac{x^4}{\sqrt[4]{ax+bx^2}}\,dx = \frac{4x^5\sqrt[4]{1+\frac{bx}{a}}\,\text{Hypergeometric2F1}\left(\frac{1}{4},\frac{19}{4},\frac{23}{4},-\frac{bx}{a}\right)}{19\sqrt[4]{x(a+bx)}}$$

```
Integrate[x^4/(a*x + b*x^2)^(1/4),x]
```

```
(4*x^5*(1 + (b*x)/a)^(1/4)*Hypergeometric2F1[1/4, 19/4, 23/4, -((b*x)/a)])
/(19*(x*(a + b*x))^(1/4))
```

Rubi [A] (warning: unable to verify)

Time = 0.59 (sec) , antiderivative size = 230, normalized size of antiderivative = 1.42, number of steps used = 12, number of rules used = 11, $\frac{\text{number of rules}}{\text{integrand size}} = 0.647$, Rules used = {1137, 60, 60, 60, 60, 73, 839, 813, 858, 807, 212}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^4}{\sqrt[4]{ax+bx^2}}dx$$

$$\Big\downarrow \text{1137}$$

$$\frac{\sqrt[4]{x}\sqrt[4]{a+bx}\int \frac{x^{15/4}}{\sqrt[4]{a+bx}}dx}{\sqrt[4]{ax+bx^2}}$$

$$\Big\downarrow \text{60}$$

$$\frac{\sqrt[4]{x}\sqrt[4]{a+bx}\left(\frac{2x^{15/4}(a+bx)^{3/4}}{9b}-\frac{5a\int \frac{x^{11/4}}{\sqrt[4]{a+bx}}dx}{6b}\right)}{\sqrt[4]{ax+bx^2}}$$

$$\Big\downarrow \text{60}$$

$$\frac{\sqrt[4]{x}\sqrt[4]{a+bx}\left(\frac{2x^{15/4}(a+bx)^{3/4}}{9b}-\frac{5a\left(\frac{2x^{11/4}(a+bx)^{3/4}}{7b}-\frac{11a\int \frac{x^{7/4}}{\sqrt[4]{a+bx}}dx}{14b}\right)}{6b}\right)}{\sqrt[4]{ax+bx^2}}$$

$$\Big\downarrow \text{60}$$

$$\frac{\sqrt[4]{x}\sqrt[4]{a+bx}\left(\frac{2x^{15/4}(a+bx)^{3/4}}{9b}-\frac{5a\left(\frac{2x^{11/4}(a+bx)^{3/4}}{7b}-\frac{11a\left(\frac{2x^{7/4}(a+bx)^{3/4}}{5b}-\frac{7a\int\frac{x^{3/4}}{\sqrt[4]{a+bx}}dx}{10b}\right)}{14b}\right)}{6b}\right)}{\sqrt[4]{ax+bx^2}}$$

$\Big\downarrow$ 60

$$\frac{\sqrt[4]{x}\sqrt[4]{a+bx}\left(\frac{2x^{15/4}(a+bx)^{3/4}}{9b}-\frac{5a\left(\frac{2x^{11/4}(a+bx)^{3/4}}{7b}-\frac{11a\left(\frac{2x^{7/4}(a+bx)^{3/4}}{5b}-\frac{7a\left(\frac{2x^{3/4}(a+bx)^{3/4}}{3b}-\frac{a\int\frac{1}{\sqrt[4]{x}\sqrt[4]{a+bx}}dx}{2b}\right)}{10b}\right)}{14b}\right)}{6b}\right)}{\sqrt[4]{ax+bx^2}}$$

$\Big\downarrow$ 73

$$\frac{\sqrt[4]{x}\sqrt[4]{a+bx}\left(\frac{2x^{15/4}(a+bx)^{3/4}}{9b}-\frac{5a\left(\frac{2x^{11/4}(a+bx)^{3/4}}{7b}-\frac{11a\left(\frac{2x^{7/4}(a+bx)^{3/4}}{5b}-\frac{7a\left(\frac{2x^{3/4}(a+bx)^{3/4}}{3b}-\frac{2a\int\frac{\sqrt{x}}{\sqrt[4]{a+bx}}d\sqrt[4]{x}}{b}\right)}{10b}\right)}{14b}\right)}{6b}\right)}{\sqrt[4]{ax+bx^2}}$$

$$\Big\downarrow \text{ 839}$$

$$\frac{\sqrt[4]{x}\sqrt[4]{a+bx}\left(\frac{2x^{15/4}(a+bx)^{3/4}}{9b}-\frac{5a\left(\frac{2x^{11/4}(a+bx)^{3/4}}{7b}-\frac{11a\left(\frac{2x^{7/4}(a+bx)^{3/4}}{5b}-\frac{7a\left(\frac{2x^{3/4}(a+bx)^{3/4}}{3b}-\frac{2a\left(\frac{x^{3/4}}{2\sqrt[4]{a+bx}}-\frac{1}{2}a\int\frac{\sqrt{x}}{(a+bx)^{5/4}}d\sqrt[4]{\right.}{b}\right.}{10b}\right.}{14b}\right.}{6b}\right.}{\sqrt[4]{ax+bx^2}}$$

$\downarrow$ 813

$$\sqrt[4]{x}\sqrt[4]{a+bx}\left(\frac{2x^{15/4}(a+bx)^{3/4}}{9b}-\frac{5a\left(\frac{2x^{11/4}(a+bx)^{3/4}}{7b}-\frac{11a\left(\frac{2x^{7/4}(a+bx)^{3/4}}{5b}-\frac{7a\left(\frac{2x^{3/4}(a+bx)^{3/4}}{3b}-\frac{2a\left(\frac{x^{3/4}}{2\sqrt[4]{a+bx}}-\frac{a\sqrt[4]{x}\sqrt[4]{\frac{a}{bx}+1}}{2b\sqrt[4]{\cdots}}\right.}{b}\right.}{10b}\right.}{14b}\right.}{6b}\right.$$

↓ 858

$$\sqrt[4]{x}\sqrt[4]{a+bx}\left(\frac{2x^{15/4}(a+bx)^{3/4}}{9b}-\frac{5a\left(\frac{2x^{11/4}(a+bx)^{3/4}}{7b}-\frac{11a\left(\frac{2x^{7/4}(a+bx)^{3/4}}{5b}-\frac{7a\left(\frac{2x^{3/4}(a+bx)^{3/4}}{3b}-\frac{2a\left(\frac{a\sqrt[4]{x}\sqrt[4]{\frac{a}{bx}+1}\int\frac{1}{\sqrt[4]{x}\left(\frac{ax}{b}+1\right)}}{2b\sqrt[4]{a+bx}}\right.}{b}\right.}{10b}\right.}{14b}\right.}{6b}\right.$$

↓ 807

$$\sqrt[4]{x}\,\sqrt[4]{a+bx}\left(\frac{2x^{15/4}(a+bx)^{3/4}}{9b}-\frac{5a\left(\frac{2x^{11/4}(a+bx)^{3/4}}{7b}-\frac{11a\left(\frac{2x^{7/4}(a+bx)^{3/4}}{5b}-\frac{7a\left(\frac{2x^{3/4}(a+bx)^{3/4}}{3b}-\frac{2a\left(\frac{a\sqrt[4]{x}\sqrt[4]{\frac{a}{bx}+1}\int \frac{1}{\left(\frac{\sqrt{x}a}{b}+1\right)^{5/4}\cdots}\cdots}{4b\sqrt[4]{a+bx}}\cdots\right)}{b}\cdots\right)}{10b}\cdots\right)}{14b}\cdots\right)}{6b}\cdots\right.$$

$\downarrow$ 212

$$\sqrt[4]{x}\sqrt[4]{a+bx}\left(\frac{2x^{15/4}(a+bx)^{3/4}}{9b}-\frac{5a\left(\frac{2x^{11/4}(a+bx)^{3/4}}{7b}-\frac{11a\left(\frac{2x^{7/4}(a+bx)^{3/4}}{5b}-\frac{7a\left(\frac{2x^{3/4}(a+bx)^{3/4}}{3b}-\frac{2a\left(\frac{\sqrt{a}\sqrt[4]{x}\sqrt[4]{\frac{a}{bx}+1}E\left(\frac{1}{2}\arctan\left(\right.\right.}{2\sqrt{b}\sqrt[4]{a+bx}}\right.}{b}\right.}{10b}\right.}{14b}\right.}{6b}\right.$$

```
Int[x^4/(a*x + b*x^2)^(1/4),x]
```

```
(x^(1/4)*(a + b*x)^(1/4)*((2*x^(15/4)*(a + b*x)^(3/4))/(9*b) - (5*a*((2*x^
(11/4)*(a + b*x)^(3/4))/(7*b) - (11*a*((2*x^(7/4)*(a + b*x)^(3/4))/(5*b) -
 (7*a*((2*x^(3/4)*(a + b*x)^(3/4))/(3*b) - (2*a*(x^(3/4)/(2*(a + b*x)^(1/4
)) + (Sqrt[a]*(1 + a/(b*x))^(1/4)*x^(1/4)*EllipticE[ArcTan[(Sqrt[a]*Sqrt[x
])/Sqrt[b]]/2, 2])/(2*Sqrt[b]*(a + b*x)^(1/4))))/b))/(10*b)))/(14*b)))/(6*
b)))/(a*x + b*x^2)^(1/4)
```

Defintions of rubi rules used

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^n/(b*(m + n + 1))), x] + Simp[n*((b*c - a*d)/(
b*(m + n + 1)))   Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
 c, d}, x] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ[m, 0] && ( !Integer
Q[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n + 2, 0] && IntLinear
Q[a, b, c, d, m, n, x]
```

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Simp[p/b   Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
 d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && Lt
Q[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntL
inearQ[a, b, c, d, m, n, x]
```

```
Int[((a_) + (b_.)*(x_)^2)^(-5/4), x_Symbol] :> Simp[(2/(a^(5/4)*Rt[b/a, 2])
)*EllipticE[(1/2)*ArcTan[Rt[b/a, 2]*x], 2], x] /; FreeQ[{a, b}, x] && GtQ[a
, 0] && PosQ[b/a]
```

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k = GCD[m
+ 1, n]}, Simp[1/k   Subst[Int[x^((m + 1)/k - 1)*(a + b*x^(n/k))^p, x], x,
x^k], x] /; k != 1] /; FreeQ[{a, b, p}, x] && IGtQ[n, 0] && IntegerQ[m]
```

```
Int[(x_)^2/((a_) + (b_.)*(x_)^4)^(5/4), x_Symbol] :> Simp[x*((1 + a/(b*x^4)
)^(1/4)/(b*(a + b*x^4)^(1/4)))   Int[1/(x^3*(1 + a/(b*x^4))^(5/4)), x], x]
/; FreeQ[{a, b}, x] && PosQ[b/a]
```

```
Int[(x_)^2/((a_) + (b_.)*(x_)^4)^(1/4), x_Symbol] :> Simp[x^3/(2*(a + b*x^4
)^(1/4)), x] - Simp[a/2   Int[x^2/(a + b*x^4)^(5/4), x], x] /; FreeQ[{a, b}
, x] && PosQ[b/a]
```

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> -Subst[Int[(a +
b/x^n)^p/x^(m + 2), x], x, 1/x] /; FreeQ[{a, b, p}, x] && ILtQ[n, 0] && Int
egerQ[m]
```

```
Int[((e_.)*(x_))^(m_)*((b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(
e*x)^m*((b*x + c*x^2)^p/(x^(m + p)*(b + c*x)^p))   Int[x^(m + p)*(b + c*x)^
p, x], x] /; FreeQ[{b, c, e, m}, x]
```

Maple [F]

$$\int \frac{x^4}{\left(b\,x^2+ax\right)^{\frac{1}{4}}}dx$$

```
int(x^4/(b*x^2+a*x)^(1/4),x)
```

```
int(x^4/(b*x^2+a*x)^(1/4),x)
```

Fricas [F]

$$\int \frac{x^4}{\sqrt[4]{ax+bx^2}}\,dx=\int \frac{x^4}{\left(bx^2+ax\right)^{\frac{1}{4}}}\,dx$$

```
integrate(x^4/(b*x^2+a*x)^(1/4),x, algorithm="fricas")
```

```
integral((b*x^2 + a*x)^(3/4)*x^3/(b*x + a), x)
```

Sympy [F]

$$\int \frac{x^4}{\sqrt[4]{ax + bx^2}}\, dx = \int \frac{x^4}{\sqrt[4]{x\,(a + bx)}}\, dx$$

```
integrate(x**4/(b*x**2+a*x)**(1/4),x)
```

```
Integral(x**4/(x*(a + b*x))**(1/4), x)
```

Maxima [F]

$$\int \frac{x^4}{\sqrt[4]{ax + bx^2}}\, dx = \int \frac{x^4}{(bx^2 + ax)^{\frac{1}{4}}}\, dx$$

```
integrate(x^4/(b*x^2+a*x)^(1/4),x, algorithm="maxima")
```

```
integrate(x^4/(b*x^2 + a*x)^(1/4), x)
```

Giac [F]

$$\int \frac{x^4}{\sqrt[4]{ax + bx^2}}\, dx = \int \frac{x^4}{(bx^2 + ax)^{\frac{1}{4}}}\, dx$$

```
integrate(x^4/(b*x^2+a*x)^(1/4),x, algorithm="giac")
```

```
integrate(x^4/(b*x^2 + a*x)^(1/4), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{x^4}{\sqrt[4]{ax+bx^2}}\,dx = \int \frac{x^4}{(b\,x^2+a\,x)^{1/4}}\,dx$$

```
int(x^4/(a*x + b*x^2)^(1/4),x)
```

```
int(x^4/(a*x + b*x^2)^(1/4), x)
```

Reduce [F]

$$\int \frac{x^4}{\sqrt[4]{ax+bx^2}}\,dx = \int \frac{x^{\frac{15}{4}}}{(bx+a)^{\frac{1}{4}}}dx$$

```
int(x^4/(b*x^2+a*x)^(1/4),x)
```

```
int(x**4/(x**(1/4)*(a + b*x)**(1/4)),x)
```

3.174 $\int \frac{x^3}{\sqrt[4]{ax+bx^2}}\,dx$

Optimal result

Integrand size = 17, antiderivative size = 136

$$\int \frac{x^3}{\sqrt[4]{ax+bx^2}}\,dx = \frac{11a^2(ax+bx^2)^{3/4}}{30b^3} - \frac{11ax(ax+bx^2)^{3/4}}{35b^2} + \frac{2x^2(ax+bx^2)^{3/4}}{7b} - \frac{11a^4\sqrt[4]{-\frac{bx}{a}-\frac{b^2x^2}{a^2}}\,E\left(\frac{1}{2}\arcsin\left(1+\frac{2bx}{a}\right)\middle|\,2\right)}{20\sqrt{2}b^4\sqrt[4]{ax+bx^2}}$$

```
11/30*a^2*(b*x^2+a*x)^(3/4)/b^3-11/35*a*x*(b*x^2+a*x)^(3/4)/b^2+2/7*x^2*(b
*x^2+a*x)^(3/4)/b-11/40*a^4*(-b*x/a-b^2*x^2/a^2)^(1/4)*EllipticE(sin(1/2*a
rcsin(1+2*b*x/a)),2^(1/2))*2^(1/2)/b^4/(b*x^2+a*x)^(1/4)
```

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 10.01 (sec) , antiderivative size = 47, normalized size of antiderivative = 0.35

$$\int \frac{x^3}{\sqrt[4]{ax+bx^2}}\,dx = \frac{4x^4\sqrt[4]{1+\frac{bx}{a}}\,\text{Hypergeometric2F1}\left(\frac{1}{4},\frac{15}{4},\frac{19}{4},-\frac{bx}{a}\right)}{15\sqrt[4]{x(a+bx)}}$$

```
Integrate[x^3/(a*x + b*x^2)^(1/4),x]
```

```
(4*x^4*(1 + (b*x)/a)^(1/4)*Hypergeometric2F1[1/4, 15/4, 19/4, -((b*x)/a)])
/(15*(x*(a + b*x))^(1/4))
```

Rubi [A] (warning: unable to verify)

Time = 0.56 (sec) , antiderivative size = 200, normalized size of antiderivative = 1.47, number of steps used = 11, number of rules used = 10, $\frac{\text{number of rules}}{\text{integrand size}} = 0.588$, Rules used = {1137, 60, 60, 60, 73, 839, 813, 858, 807, 212}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^3}{\sqrt[4]{ax+bx^2}}dx$$

$$\Big\downarrow \text{1137}$$

$$\frac{\sqrt[4]{x}\sqrt[4]{a+bx}\int \frac{x^{11/4}}{\sqrt[4]{a+bx}}dx}{\sqrt[4]{ax+bx^2}}$$

$$\Big\downarrow \text{60}$$

$$\frac{\sqrt[4]{x}\sqrt[4]{a+bx}\left(\frac{2x^{11/4}(a+bx)^{3/4}}{7b}-\frac{11a\int \frac{x^{7/4}}{\sqrt[4]{a+bx}}dx}{14b}\right)}{\sqrt[4]{ax+bx^2}}$$

$$\Big\downarrow \text{60}$$

$$\frac{\sqrt[4]{x}\sqrt[4]{a+bx}\left(\frac{2x^{11/4}(a+bx)^{3/4}}{7b}-\frac{11a\left(\frac{2x^{7/4}(a+bx)^{3/4}}{5b}-\frac{7a\int \frac{x^{3/4}}{\sqrt[4]{a+bx}}dx}{10b}\right)}{14b}\right)}{\sqrt[4]{ax+bx^2}}$$

$$\Big\downarrow \text{60}$$

$$\frac{\sqrt[4]{x}\sqrt[4]{a+bx}\left(\frac{2x^{11/4}(a+bx)^{3/4}}{7b}-\frac{11a\left(\frac{2x^{7/4}(a+bx)^{3/4}}{5b}-\frac{7a\left(\frac{2x^{3/4}(a+bx)^{3/4}}{3b}-\frac{a\int\frac{1}{\sqrt[4]{x}\sqrt[4]{a+bx}}dx}{2b}\right)}{10b}\right)}{14b}\right)}{\sqrt[4]{ax+bx^2}}$$

$$\Big\downarrow \; 73$$

$$\frac{\sqrt[4]{x}\sqrt[4]{a+bx}\left(\frac{2x^{11/4}(a+bx)^{3/4}}{7b}-\frac{11a\left(\frac{2x^{7/4}(a+bx)^{3/4}}{5b}-\frac{7a\left(\frac{2x^{3/4}(a+bx)^{3/4}}{3b}-\frac{2a\int\frac{\sqrt{x}}{\sqrt[4]{a+bx}}d\sqrt[4]{x}}{b}\right)}{10b}\right)}{14b}\right)}{\sqrt[4]{ax+bx^2}}$$

$$\Big\downarrow \; 839$$

$$\frac{\sqrt[4]{x}\sqrt[4]{a+bx}\left(\frac{2x^{11/4}(a+bx)^{3/4}}{7b}-\frac{11a\left(\frac{2x^{7/4}(a+bx)^{3/4}}{5b}-\frac{7a\left(\frac{2x^{3/4}(a+bx)^{3/4}}{3b}-\frac{2a\left(\frac{x^{3/4}}{2\sqrt[4]{a+bx}}-\frac{1}{2}a\int\frac{\sqrt{x}}{(a+bx)^{5/4}}d\sqrt[4]{x}\right)}{b}\right)}{10b}\right)}{14b}\right)}{\sqrt[4]{ax+bx^2}}$$

$$\Big\downarrow 813$$

$$\frac{\sqrt[4]{x}\sqrt[4]{a+bx}\left(\frac{2x^{11/4}(a+bx)^{3/4}}{7b}-\frac{11a\left(\frac{2x^{7/4}(a+bx)^{3/4}}{5b}-\frac{7a\left(\frac{2x^{3/4}(a+bx)^{3/4}}{3b}-\frac{2a\left(\frac{x^{3/4}}{2\sqrt[4]{a+bx}}-\frac{a\sqrt[4]{x}\sqrt[4]{\frac{a}{bx}+1}\int\frac{1}{\left(\frac{a}{bx}+1\right)^{5/4}x^{3/4}}d\sqrt[4]{x}}{2b\sqrt[4]{a+bx}}\right)}{b}\right)}{10b}\right)}{14b}\right)}{\sqrt[4]{ax+bx^2}}$$

$$\Big\downarrow 858$$

$$\frac{\sqrt[4]{x}\sqrt[4]{a+bx}\left(\frac{2x^{11/4}(a+bx)^{3/4}}{7b}-\frac{11a\left(\frac{2x^{7/4}(a+bx)^{3/4}}{5b}-\frac{7a\left(\frac{2x^{3/4}(a+bx)^{3/4}}{3b}-\frac{2a\left(\frac{a\sqrt[4]{x}\sqrt[4]{\frac{a}{bx}+1}\int\frac{1}{\sqrt[4]{x}\left(\frac{ax}{b}+1\right)^{5/4}}d\frac{1}{\sqrt[4]{x}}}{2b\sqrt[4]{a+bx}}+\frac{x^{3/4}}{2\sqrt[4]{a+b}}\right.}{b}\right.}{10b}\right.}{14b}\right.}{\sqrt[4]{ax+bx^2}}$$

$\downarrow$ 807

$$\frac{\sqrt[4]{x}\sqrt[4]{a+bx}\left(\frac{2x^{11/4}(a+bx)^{3/4}}{7b}-\frac{11a\left(\frac{2x^{7/4}(a+bx)^{3/4}}{5b}-\frac{7a\left(\frac{2x^{3/4}(a+bx)^{3/4}}{3b}-\frac{2a\left(\frac{a\sqrt[4]{x}\sqrt[4]{\frac{a}{bx}+1}\int \frac{1}{\left(\frac{\sqrt{x}a}{b}+1\right)^{5/4}}d\sqrt{x}}{4b\sqrt[4]{a+bx}}+\frac{x^{3/4}}{2\sqrt[4]{a+bx}}\right)}{b}\right)}{10b}\right)}{14b}\right)}{\sqrt[4]{ax+bx^2}}$$

$$\downarrow \ 212$$

$$\frac{\sqrt[4]{x}\sqrt[4]{a+bx}\left(\frac{2x^{11/4}(a+bx)^{3/4}}{7b}-\frac{11a\left(\frac{2x^{7/4}(a+bx)^{3/4}}{5b}-\frac{7a\left(\frac{2x^{3/4}(a+bx)^{3/4}}{3b}-\frac{2a\left(\frac{\sqrt{a}\sqrt[4]{x}\sqrt[4]{\frac{a}{bx}+1}\,E\left(\frac{1}{2}\arctan\left(\frac{\sqrt{a}\sqrt{x}}{\sqrt{b}}\right)\middle|2\right)}{2\sqrt{b}\sqrt[4]{a+bx}}+\frac{x^{3/4}}{2\sqrt[4]{a+bx}}\right)}{b}\right)}{10b}\right)}{14b}\right)}{\sqrt[4]{ax+bx^2}}$$

```
Int[x^3/(a*x + b*x^2)^(1/4),x]
```

```
(x^(1/4)*(a + b*x)^(1/4)*((2*x^(11/4)*(a + b*x)^(3/4))/(7*b) - (11*a*((2*x
^(7/4)*(a + b*x)^(3/4))/(5*b) - (7*a*((2*x^(3/4)*(a + b*x)^(3/4))/(3*b) -
(2*a*(x^(3/4)/(2*(a + b*x)^(1/4)) + (Sqrt[a]*(1 + a/(b*x))^(1/4)*x^(1/4)*E
llipticE[ArcTan[(Sqrt[a]*Sqrt[x])/Sqrt[b]]/2, 2])/(2*Sqrt[b]*(a + b*x)^(1/
4))))/b))/(10*b)))/(14*b)))/(a*x + b*x^2)^(1/4)
```

Defintions of rubi rules used

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^n/(b*(m + n + 1))), x] + Simp[n*((b*c - a*d)/(
b*(m + n + 1)))   Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
 c, d}, x] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ[m, 0] && ( !Integer
Q[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n + 2, 0] && IntLinear
Q[a, b, c, d, m, n, x]
```

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Simp[p/b   Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
 d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && Lt
Q[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntL
inearQ[a, b, c, d, m, n, x]
```

```
Int[((a_) + (b_.)*(x_)^2)^(-5/4), x_Symbol] :> Simp[(2/(a^(5/4)*Rt[b/a, 2])
)*EllipticE[(1/2)*ArcTan[Rt[b/a, 2]*x], 2], x] /; FreeQ[{a, b}, x] && GtQ[a
, 0] && PosQ[b/a]
```

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k = GCD[m
+ 1, n]}, Simp[1/k   Subst[Int[x^((m + 1)/k - 1)*(a + b*x^(n/k))^p, x], x,
x^k], x] /; k != 1] /; FreeQ[{a, b, p}, x] && IGtQ[n, 0] && IntegerQ[m]
```

```
Int[(x_)^2/((a_) + (b_.)*(x_)^4)^(5/4), x_Symbol] :> Simp[x*((1 + a/(b*x^4)
)^(1/4)/(b*(a + b*x^4)^(1/4)))   Int[1/(x^3*(1 + a/(b*x^4))^(5/4)), x], x]
/; FreeQ[{a, b}, x] && PosQ[b/a]
```

```
Int[(x_)^2/((a_) + (b_.)*(x_)^4)^(1/4), x_Symbol] :> Simp[x^3/(2*(a + b*x^4
)^(1/4)), x] - Simp[a/2   Int[x^2/(a + b*x^4)^(5/4), x], x] /; FreeQ[{a, b}
, x] && PosQ[b/a]
```

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> -Subst[Int[(a +
b/x^n)^p/x^(m + 2), x], x, 1/x] /; FreeQ[{a, b, p}, x] && ILtQ[n, 0] && Int
egerQ[m]
```

```
Int[((e_.)*(x_))^(m_)*((b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(
e*x)^m*((b*x + c*x^2)^p/(x^(m + p)*(b + c*x)^p))   Int[x^(m + p)*(b + c*x)^
p, x], x] /; FreeQ[{b, c, e, m}, x]
```

Maple [F]

$$\int \frac{x^3}{(b\,x^2+ax)^{\frac{1}{4}}}dx$$

```
int(x^3/(b*x^2+a*x)^(1/4),x)
```

```
int(x^3/(b*x^2+a*x)^(1/4),x)
```

Fricas [F]

$$\int \frac{x^3}{\sqrt[4]{ax+bx^2}}\,dx = \int \frac{x^3}{(bx^2+ax)^{\frac{1}{4}}}\,dx$$

```
integrate(x^3/(b*x^2+a*x)^(1/4),x, algorithm="fricas")
```

```
integral((b*x^2 + a*x)^(3/4)*x^2/(b*x + a), x)
```

Sympy [F]

$$\int \frac{x^3}{\sqrt[4]{ax+bx^2}}\,dx = \int \frac{x^3}{\sqrt[4]{x\,(a+bx)}}\,dx$$

```
integrate(x**3/(b*x**2+a*x)**(1/4),x)
```

```
Integral(x**3/(x*(a + b*x))**(1/4), x)
```

Maxima [F]

$$\int \frac{x^3}{\sqrt[4]{ax+bx^2}}\,dx = \int \frac{x^3}{(bx^2+ax)^{\frac{1}{4}}}\,dx$$

```
integrate(x^3/(b*x^2+a*x)^(1/4),x, algorithm="maxima")
```

```
integrate(x^3/(b*x^2 + a*x)^(1/4), x)
```

Giac [F]

$$\int \frac{x^3}{\sqrt[4]{ax+bx^2}}\,dx = \int \frac{x^3}{(bx^2+ax)^{\frac{1}{4}}}\,dx$$

```
integrate(x^3/(b*x^2+a*x)^(1/4),x, algorithm="giac")
```

```
integrate(x^3/(b*x^2 + a*x)^(1/4), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{x^3}{\sqrt[4]{ax+bx^2}}\,dx = \int \frac{x^3}{(b\,x^2+a\,x)^{1/4}}\,dx$$

```
int(x^3/(a*x + b*x^2)^(1/4),x)
```

```
int(x^3/(a*x + b*x^2)^(1/4), x)
```

Reduce [F]

$$\int \frac{x^3}{\sqrt[4]{ax + bx^2}}\,dx = \int \frac{x^{\frac{11}{4}}}{(bx + a)^{\frac{1}{4}}}dx$$

```
int(x^3/(b*x^2+a*x)^(1/4),x)
```

```
int(x**3/(x**(1/4)*(a + b*x)**(1/4)),x)
```

3.175 $\int \frac{x^2}{\sqrt[4]{ax+bx^2}}\,dx$

Optimal result

Integrand size = 17, antiderivative size = 110

$$\int \frac{x^2}{\sqrt[4]{ax+bx^2}}\,dx = -\frac{7a(ax+bx^2)^{3/4}}{15b^2}+\frac{2x(ax+bx^2)^{3/4}}{5b}+\frac{7a^3\sqrt[4]{-\frac{bx}{a}-\frac{b^2x^2}{a^2}}\,E\left(\frac{1}{2}\arcsin\left(1+\frac{2bx}{a}\right)\middle|2\right)}{10\sqrt{2}b^3\sqrt[4]{ax+bx^2}}$$

```
-7/15*a*(b*x^2+a*x)^(3/4)/b^2+2/5*x*(b*x^2+a*x)^(3/4)/b+7/20*a^3*(-b*x/a-b
^2*x^2/a^2)^(1/4)*EllipticE(sin(1/2*arcsin(1+2*b*x/a)),2^(1/2))*2^(1/2)/b^
3/(b*x^2+a*x)^(1/4)
```

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 10.01 (sec) , antiderivative size = 47, normalized size of antiderivative = 0.43

$$\int \frac{x^2}{\sqrt[4]{ax+bx^2}}\,dx = \frac{4x^3\sqrt[4]{1+\frac{bx}{a}}\,\text{Hypergeometric2F1}\left(\frac{1}{4},\frac{11}{4},\frac{15}{4},-\frac{bx}{a}\right)}{11\sqrt[4]{x(a+bx)}}$$

```
Integrate[x^2/(a*x + b*x^2)^(1/4),x]
```

```
(4*x^3*(1 + (b*x)/a)^(1/4)*Hypergeometric2F1[1/4, 11/4, 15/4, -((b*x)/a)])
/(11*(x*(a + b*x))^(1/4))
```

Rubi [A] (warning: unable to verify)

Time = 0.53 (sec) , antiderivative size = 170, normalized size of antiderivative = 1.55, number of steps used = 10, number of rules used = 9, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.529, Rules used = {1137, 60, 60, 73, 839, 813, 858, 807, 212}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^2}{\sqrt[4]{ax+bx^2}}dx$$

$$\Big\downarrow \ 1137$$

$$\frac{\sqrt[4]{x}\sqrt[4]{a+bx}\int \frac{x^{7/4}}{\sqrt[4]{a+bx}}dx}{\sqrt[4]{ax+bx^2}}$$

$$\Big\downarrow \ 60$$

$$\frac{\sqrt[4]{x}\sqrt[4]{a+bx}\left(\frac{2x^{7/4}(a+bx)^{3/4}}{5b}-\frac{7a\int \frac{x^{3/4}}{\sqrt[4]{a+bx}}dx}{10b}\right)}{\sqrt[4]{ax+bx^2}}$$

$$\Big\downarrow \ 60$$

$$\frac{\sqrt[4]{x}\sqrt[4]{a+bx}\left(\frac{2x^{7/4}(a+bx)^{3/4}}{5b}-\frac{7a\left(\frac{2x^{3/4}(a+bx)^{3/4}}{3b}-\frac{a\int \frac{1}{\sqrt[4]{x}\sqrt[4]{a+bx}}dx}{2b}\right)}{10b}\right)}{\sqrt[4]{ax+bx^2}}$$

$$\Big\downarrow \ 73$$

$$\frac{\sqrt[4]{x}\sqrt[4]{a+bx}\left(\frac{2x^{7/4}(a+bx)^{3/4}}{5b}-\frac{7a\left(\frac{2x^{3/4}(a+bx)^{3/4}}{3b}-\frac{2a\int\frac{\sqrt{x}}{\sqrt[4]{a+bx}}d\sqrt[4]{x}}{b}\right)}{10b}\right)}{\sqrt[4]{ax+bx^2}}$$

$\downarrow$ 839

$$\frac{\sqrt[4]{x}\sqrt[4]{a+bx}\left(\frac{2x^{7/4}(a+bx)^{3/4}}{5b}-\frac{7a\left(\frac{2x^{3/4}(a+bx)^{3/4}}{3b}-\frac{2a\left(\frac{x^{3/4}}{2\sqrt[4]{a+bx}}-\frac{1}{2}a\int\frac{\sqrt{x}}{(a+bx)^{5/4}}d\sqrt[4]{x}\right)}{b}\right)}{10b}\right)}{\sqrt[4]{ax+bx^2}}$$

$\downarrow$ 813

$$\frac{\sqrt[4]{x}\sqrt[4]{a+bx}\left(\frac{2x^{7/4}(a+bx)^{3/4}}{5b}-\frac{7a\left(\frac{2x^{3/4}(a+bx)^{3/4}}{3b}-\frac{2a\left(\frac{x^{3/4}}{2\sqrt[4]{a+bx}}-\frac{a\sqrt[4]{x}\sqrt[4]{\frac{a}{bx}+1}\int\frac{1}{\left(\frac{a}{bx}+1\right)^{5/4}x^{3/4}}d\sqrt[4]{x}}{2b\sqrt[4]{a+bx}}\right)}{b}\right)}{10b}\right)}{\sqrt[4]{ax+bx^2}}$$

$\downarrow$ 858

$$\frac{\sqrt[4]{x}\sqrt[4]{a+bx}\left(\frac{2x^{7/4}(a+bx)^{3/4}}{5b}-\frac{7a\left(\frac{2x^{3/4}(a+bx)^{3/4}}{3b}-\frac{2a\left(\frac{a\sqrt[4]{x}\sqrt[4]{\frac{a}{bx}+1}\int\frac{1}{\sqrt[4]{x}\left(\frac{ax}{b}+1\right)^{5/4}}d\frac{1}{\sqrt[4]{x}}}{2b\sqrt[4]{a+bx}}+\frac{x^{3/4}}{2\sqrt[4]{a+bx}}\right)}{b}\right)}{10b}\right)}{\sqrt[4]{ax+bx^2}}$$

$$\Big\downarrow \; 807$$

$$\frac{\sqrt[4]{x}\sqrt[4]{a+bx}\left(\frac{2x^{7/4}(a+bx)^{3/4}}{5b}-\frac{7a\left(\frac{2x^{3/4}(a+bx)^{3/4}}{3b}-\frac{2a\left(\frac{a\sqrt[4]{x}\sqrt[4]{\frac{a}{bx}+1}\int\frac{1}{\left(\frac{\sqrt{x}a}{b}+1\right)^{5/4}}d\sqrt{x}}{4b\sqrt[4]{a+bx}}+\frac{x^{3/4}}{2\sqrt[4]{a+bx}}\right)}{b}\right)}{10b}\right)}{\sqrt[4]{ax+bx^2}}$$

$$\Big\downarrow \; 212$$

$$\frac{\sqrt[4]{x}\sqrt[4]{a+bx}\left(\frac{2x^{7/4}(a+bx)^{3/4}}{5b}-\frac{7a\left(\frac{2x^{3/4}(a+bx)^{3/4}}{3b}-\frac{2a\left(\frac{\sqrt{a}\sqrt[4]{x}\sqrt[4]{\frac{a}{bx}+1}E\left(\frac{1}{2}\arctan\left(\frac{\sqrt{a}\sqrt{x}}{\sqrt{b}}\right)\middle|2\right)}{2\sqrt{b}\sqrt[4]{a+bx}}+\frac{x^{3/4}}{2\sqrt[4]{a+bx}}\right)}{b}\right)}{10b}\right)}{\sqrt[4]{ax+bx^2}}$$

```
Int[x^2/(a*x + b*x^2)^(1/4),x]
```

```
(x^(1/4)*(a + b*x)^(1/4)*((2*x^(7/4)*(a + b*x)^(3/4))/(5*b) - (7*a*((2*x^(
3/4)*(a + b*x)^(3/4))/(3*b) - (2*a*(x^(3/4)/(2*(a + b*x)^(1/4)) + (Sqrt[a]
*(1 + a/(b*x))^(1/4)*x^(1/4)*EllipticE[ArcTan[(Sqrt[a]*Sqrt[x])/Sqrt[b]]/2
, 2])/(2*Sqrt[b]*(a + b*x)^(1/4))))/b))/(10*b)))/(a*x + b*x^2)^(1/4)
```

Defintions of rubi rules used

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^n/(b*(m + n + 1))), x] + Simp[n*((b*c - a*d)/(
b*(m + n + 1)))   Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
 c, d}, x] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ[m, 0] && ( !Integer
Q[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n + 2, 0] && IntLinear
Q[a, b, c, d, m, n, x]
```

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Simp[p/b   Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
 d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && Lt
Q[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntL
inearQ[a, b, c, d, m, n, x]
```

```
Int[((a_) + (b_.)*(x_)^2)^(-5/4), x_Symbol] :> Simp[(2/(a^(5/4)*Rt[b/a, 2])
)*EllipticE[(1/2)*ArcTan[Rt[b/a, 2]*x], 2], x] /; FreeQ[{a, b}, x] && GtQ[a
, 0] && PosQ[b/a]
```

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k = GCD[m
+ 1, n]}, Simp[1/k   Subst[Int[x^((m + 1)/k - 1)*(a + b*x^(n/k))^p, x], x,
x^k], x] /; k != 1] /; FreeQ[{a, b, p}, x] && IGtQ[n, 0] && IntegerQ[m]
```

```
Int[(x_)^2/((a_) + (b_.)*(x_)^4)^(5/4), x_Symbol] :> Simp[x*((1 + a/(b*x^4)
)^(1/4)/(b*(a + b*x^4)^(1/4)))   Int[1/(x^3*(1 + a/(b*x^4))^(5/4)), x], x]
/; FreeQ[{a, b}, x] && PosQ[b/a]
```

```
Int[(x_)^2/((a_) + (b_.)*(x_)^4)^(1/4), x_Symbol] :> Simp[x^3/(2*(a + b*x^4
)^(1/4)), x] - Simp[a/2   Int[x^2/(a + b*x^4)^(5/4), x], x] /; FreeQ[{a, b}
, x] && PosQ[b/a]
```

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> -Subst[Int[(a +
b/x^n)^p/x^(m + 2), x], x, 1/x] /; FreeQ[{a, b, p}, x] && ILtQ[n, 0] && Int
egerQ[m]
```

```
Int[((e_.)*(x_))^(m_)*((b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(
e*x)^m*((b*x + c*x^2)^p/(x^(m + p)*(b + c*x)^p))   Int[x^(m + p)*(b + c*x)^
p, x], x] /; FreeQ[{b, c, e, m}, x]
```

Maple [F]

$$\int \frac{x^2}{\left(b\,x^2+ax\right)^{\frac{1}{4}}}dx$$

```
int(x^2/(b*x^2+a*x)^(1/4),x)
```

```
int(x^2/(b*x^2+a*x)^(1/4),x)
```

Fricas [F]

$$\int \frac{x^2}{\sqrt[4]{ax+bx^2}}\,dx = \int \frac{x^2}{\left(bx^2+ax\right)^{\frac{1}{4}}}\,dx$$

```
integrate(x^2/(b*x^2+a*x)^(1/4),x, algorithm="fricas")
```

```
integral((b*x^2 + a*x)^(3/4)*x/(b*x + a), x)
```

Sympy [F]

$$\int \frac{x^2}{\sqrt[4]{ax+bx^2}}\,dx = \int \frac{x^2}{\sqrt[4]{x\,(a+bx)}}\,dx$$

```
integrate(x**2/(b*x**2+a*x)**(1/4),x)
```

```
Integral(x**2/(x*(a + b*x))**(1/4), x)
```

Maxima [F]

$$\int \frac{x^2}{\sqrt[4]{ax+bx^2}}\,dx = \int \frac{x^2}{(bx^2+ax)^{\frac{1}{4}}}\,dx$$

```
integrate(x^2/(b*x^2+a*x)^(1/4),x, algorithm="maxima")
```

```
integrate(x^2/(b*x^2 + a*x)^(1/4), x)
```

Giac [F]

$$\int \frac{x^2}{\sqrt[4]{ax+bx^2}}\,dx = \int \frac{x^2}{(bx^2+ax)^{\frac{1}{4}}}\,dx$$

```
integrate(x^2/(b*x^2+a*x)^(1/4),x, algorithm="giac")
```

```
integrate(x^2/(b*x^2 + a*x)^(1/4), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{x^2}{\sqrt[4]{ax+bx^2}}\,dx = \int \frac{x^2}{{\left(b\,x^2+a\,x\right)}^{1/4}}\,dx$$

```
int(x^2/(a*x + b*x^2)^(1/4),x)
```

```
int(x^2/(a*x + b*x^2)^(1/4), x)
```

Reduce [F]

$$\int \frac{x^2}{\sqrt[4]{ax+bx^2}}\,dx = \int \frac{x^{\frac{7}{4}}}{(bx+a)^{\frac{1}{4}}}dx$$

```
int(x^2/(b*x^2+a*x)^(1/4),x)
```

```
int(x**2/(x**(1/4)*(a + b*x)**(1/4)),x)
```

3.176 $\int \frac{x}{\sqrt[4]{ax+bx^2}}\,dx$

Optimal result

Integrand size = 15, antiderivative size = 86

$$\int \frac{x}{\sqrt[4]{ax+bx^2}}\,dx=\frac{2(ax+bx^2)^{3/4}}{3b}-\frac{a^2\sqrt[4]{-\frac{bx}{a}-\frac{b^2x^2}{a^2}}E\left(\frac{1}{2}\arcsin\left(1+\frac{2bx}{a}\right)\middle|2\right)}{\sqrt{2}b^2\sqrt[4]{ax+bx^2}}$$

```
2/3*(b*x^2+a*x)^(3/4)/b-1/2*a^2*(-b*x/a-b^2*x^2/a^2)^(1/4)*EllipticE(sin(1
/2*arcsin(1+2*b*x/a)),2^(1/2))*2^(1/2)/b^2/(b*x^2+a*x)^(1/4)
```

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 10.01 (sec) , antiderivative size = 47, normalized size of antiderivative = 0.55

$$\int \frac{x}{\sqrt[4]{ax+bx^2}}\,dx=\frac{4x^2\sqrt[4]{1+\frac{bx}{a}}\,\text{Hypergeometric2F1}\left(\frac{1}{4},\frac{7}{4},\frac{11}{4},-\frac{bx}{a}\right)}{7\sqrt[4]{x(a+bx)}}$$

```
Integrate[x/(a*x + b*x^2)^(1/4),x]
```

```
(4*x^2*(1 + (b*x)/a)^(1/4)*Hypergeometric2F1[1/4, 7/4, 11/4, -((b*x)/a)])/
(7*(x*(a + b*x))^(1/4))
```

Rubi [A] (verified)

Time = 0.41 (sec) , antiderivative size = 93, normalized size of antiderivative = 1.08, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.267, Rules used = {1160, 1093, 1090, 226}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x}{\sqrt[4]{ax+bx^2}}dx$$

$$\Big\downarrow \text{1160}$$

$$\frac{2\left(ax+bx^2\right)^{3/4}}{3b}-\frac{a\int \frac{1}{\sqrt[4]{bx^2+ax}}dx}{2b}$$

$$\Big\downarrow \text{1093}$$

$$\frac{2\left(ax+bx^2\right)^{3/4}}{3b}-\frac{a\sqrt[4]{-\frac{b\left(ax+bx^2\right)}{a^2}}\int \frac{1}{\sqrt[4]{-\frac{b^2x^2}{a^2}-\frac{bx}{a}}}dx}{2b\sqrt[4]{ax+bx^2}}$$

$$\Big\downarrow \text{1090}$$

$$\frac{a^3\sqrt[4]{-\frac{b\left(ax+bx^2\right)}{a^2}}\int \frac{1}{\sqrt[4]{1-\frac{a^2\left(-\frac{2xb^2}{a^2}-\frac{b}{a}\right)^2}{b^2}}}d\left(-\frac{2xb^2}{a^2}-\frac{b}{a}\right)}{2\sqrt{2}b^3\sqrt[4]{ax+bx^2}}+\frac{2\left(ax+bx^2\right)^{3/4}}{3b}$$

$$\Big\downarrow \text{226}$$

$$\frac{a^2\sqrt[4]{-\frac{b\left(ax+bx^2\right)}{a^2}}E\left(\frac{1}{2}\arcsin\left(\frac{a\left(-\frac{2xb^2}{a^2}-\frac{b}{a}\right)}{b}\right)\middle|2\right)}{\sqrt{2}b^2\sqrt[4]{ax+bx^2}}+\frac{2\left(ax+bx^2\right)^{3/4}}{3b}$$

```
Int[x/(a*x + b*x^2)^(1/4),x]
```

```
(2*(a*x + b*x^2)^(3/4))/(3*b) + (a^2*(-((b*(a*x + b*x^2))/a^2))^(1/4)*Elli
pticE[ArcSin[(a*(-(b/a) - (2*b^2*x)/a^2))/b]/2, 2])/(Sqrt[2]*b^2*(a*x + b*
x^2)^(1/4))
```

Defintions of rubi rules used

```
Int[((a_) + (b_.)*(x_)^2)^(-1/4), x_Symbol] :> Simp[(2/(a^(1/4)*Rt[-b/a, 2]
))*EllipticE[(1/2)*ArcSin[Rt[-b/a, 2]*x], 2], x] /; FreeQ[{a, b}, x] && GtQ
[a, 0] && NegQ[b/a]
```

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[1/(2*c*(-4*
(c/(b^2 - 4*a*c)))^p)   Subst[Int[Simp[1 - x^2/(b^2 - 4*a*c), x]^p, x], x,
b + 2*c*x], x] /; FreeQ[{a, b, c, p}, x] && GtQ[4*a - b^2/c, 0]
```

```
Int[((b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(b*x + c*x^2)^p/((-
c)*((b*x + c*x^2)/b^2))^p   Int[((-c)*(x/b) - c^2*(x^2/b^2))^p, x], x] /; F
reeQ[{b, c}, x] && (IntegerQ[4*p] || IntegerQ[3*p])
```

```
Int[((d_.) + (e_.)*(x_))*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol
] :> Simp[e*((a + b*x + c*x^2)^(p + 1)/(2*c*(p + 1))), x] + Simp[(2*c*d - b
*e)/(2*c)   Int[(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, p}, x]
 && NeQ[p, -1]
```

Maple [F]

$$\int \frac{x}{\left(b\,x^{2}+ax\right)^{\frac{1}{4}}}dx$$

```
int(x/(b*x^2+a*x)^(1/4),x)
```

```
int(x/(b*x^2+a*x)^(1/4),x)
```

Fricas [F]

$$\int \frac{x}{\sqrt[4]{ax+bx^2}}\,dx = \int \frac{x}{(bx^2+ax)^{\frac{1}{4}}}\,dx$$

```
integrate(x/(b*x^2+a*x)^(1/4),x, algorithm="fricas")
```

```
integral((b*x^2 + a*x)^(3/4)/(b*x + a), x)
```

Sympy [F]

$$\int \frac{x}{\sqrt[4]{ax+bx^2}}\,dx = \int \frac{x}{\sqrt[4]{x\,(a+bx)}}\,dx$$

```
integrate(x/(b*x**2+a*x)**(1/4),x)
```

```
Integral(x/(x*(a + b*x))**(1/4), x)
```

Maxima [F]

$$\int \frac{x}{\sqrt[4]{ax+bx^2}}\,dx = \int \frac{x}{(bx^2+ax)^{\frac{1}{4}}}\,dx$$

```
integrate(x/(b*x^2+a*x)^(1/4),x, algorithm="maxima")
```

```
integrate(x/(b*x^2 + a*x)^(1/4), x)
```

Giac [F]

$$\int \frac{x}{\sqrt[4]{ax+bx^2}}\,dx = \int \frac{x}{(bx^2+ax)^{\frac{1}{4}}}\,dx$$

```
integrate(x/(b*x^2+a*x)^(1/4),x, algorithm="giac")
```

```
integrate(x/(b*x^2 + a*x)^(1/4), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{x}{\sqrt[4]{ax+bx^2}}\,dx = \int \frac{x}{(b\,x^2+a\,x)^{1/4}}\,dx$$

```
int(x/(a*x + b*x^2)^(1/4),x)
```

```
int(x/(a*x + b*x^2)^(1/4), x)
```

Reduce [F]

$$\int \frac{x}{\sqrt[4]{ax+bx^2}}\,dx = \int \frac{x^{\frac{3}{4}}}{(bx+a)^{\frac{1}{4}}}dx$$

```
int(x/(b*x^2+a*x)^(1/4),x)
```

```
int(x/(x**(1/4)*(a + b*x)**(1/4)),x)
```

3.177 $\int \frac{1}{\sqrt[4]{ax+bx^2}}\,dx$

Optimal result

Integrand size = 13, antiderivative size = 62

$$\int \frac{1}{\sqrt[4]{ax+bx^2}}\,dx=\frac{\sqrt{2}a\sqrt[4]{-\frac{bx}{a}-\frac{b^2x^2}{a^2}}\,E\left(\frac{1}{2}\arcsin\left(1+\frac{2bx}{a}\right)\middle|2\right)}{b\sqrt[4]{ax+bx^2}}$$

```
2^(1/2)*a*(-b*x/a-b^2*x^2/a^2)^(1/4)*EllipticE(sin(1/2*arcsin(1+2*b*x/a)),
2^(1/2))/b/(b*x^2+a*x)^(1/4)
```

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 10.01 (sec) , antiderivative size = 45, normalized size of antiderivative = 0.73

$$\int \frac{1}{\sqrt[4]{ax+bx^2}}\,dx=\frac{4x\sqrt[4]{1+\frac{bx}{a}}\,\text{Hypergeometric2F1}\left(\frac{1}{4},\frac{3}{4},\frac{7}{4},-\frac{bx}{a}\right)}{3\sqrt[4]{x(a+bx)}}$$

```
Integrate[(a*x + b*x^2)^(-1/4),x]
```

```
(4*x*(1 + (b*x)/a)^(1/4)*Hypergeometric2F1[1/4, 3/4, 7/4, -((b*x)/a)])/(3*
(x*(a + b*x))^(1/4))
```

Rubi [A] (verified)

Time = 0.34 (sec) , antiderivative size = 71, normalized size of antiderivative = 1.15, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.231, Rules used = {1093, 1090, 226}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{\sqrt[4]{ax+bx^2}}dx$$

$$\Big\downarrow \text{1093}$$

$$\frac{\sqrt[4]{-\frac{b(ax+bx^2)}{a^2}}\int \frac{1}{\sqrt[4]{-\frac{b^2x^2}{a^2}-\frac{bx}{a}}}dx}{\sqrt[4]{ax+bx^2}}$$

$$\Big\downarrow \text{1090}$$

$$-\frac{a^2\sqrt[4]{-\frac{b(ax+bx^2)}{a^2}}\int \frac{1}{\sqrt[4]{1-\frac{a^2\left(-\frac{2xb^2}{a^2}-\frac{b}{a}\right)^2}{b^2}}}d\left(-\frac{2xb^2}{a^2}-\frac{b}{a}\right)}{\sqrt{2}b^2\sqrt[4]{ax+bx^2}}$$

$$\Big\downarrow \text{226}$$

$$-\frac{\sqrt{2}a\sqrt[4]{-\frac{b(ax+bx^2)}{a^2}}E\left(\frac{1}{2}\arcsin\left(\frac{a\left(-\frac{2xb^2}{a^2}-\frac{b}{a}\right)}{b}\right)\middle|2\right)}{b\sqrt[4]{ax+bx^2}}$$

```
Int[(a*x + b*x^2)^(-1/4),x]
```

```
-((Sqrt[2]*a*(-((b*(a*x + b*x^2))/a^2))^(1/4)*EllipticE[ArcSin[(a*(-(b/a)
- (2*b^2*x)/a^2))/b]/2, 2])/(b*(a*x + b*x^2)^(1/4)))
```

Defintions of rubi rules used

```
Int[((a_) + (b_.)*(x_)^2)^(-1/4), x_Symbol] :> Simp[(2/(a^(1/4)*Rt[-b/a, 2]
))*EllipticE[(1/2)*ArcSin[Rt[-b/a, 2]*x], 2], x] /; FreeQ[{a, b}, x] && GtQ
[a, 0] && NegQ[b/a]
```

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[1/(2*c*(-4*
(c/(b^2 - 4*a*c)))^p)   Subst[Int[Simp[1 - x^2/(b^2 - 4*a*c), x]^p, x], x,
b + 2*c*x], x] /; FreeQ[{a, b, c, p}, x] && GtQ[4*a - b^2/c, 0]
```

```
Int[((b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(b*x + c*x^2)^p/((-
c)*((b*x + c*x^2)/b^2))^p   Int[((-c)*(x/b) - c^2*(x^2/b^2))^p, x], x] /; F
reeQ[{b, c}, x] && (IntegerQ[4*p] || IntegerQ[3*p])
```

Maple [F]

$$\int \frac{1}{(b\,x^2 + ax)^{\frac{1}{4}}}dx$$

```
int(1/(b*x^2+a*x)^(1/4),x)
```


```
int(1/(b*x^2+a*x)^(1/4),x)
```

Fricas [F]

$$\int \frac{1}{\sqrt[4]{ax+bx^2}}\,dx = \int \frac{1}{(bx^2+ax)^{\frac{1}{4}}}\,dx$$

```
integrate(1/(b*x^2+a*x)^(1/4),x, algorithm="fricas")
```

```
integral((b*x^2 + a*x)^(-1/4), x)
```

Sympy [F]

$$\int \frac{1}{\sqrt[4]{ax+bx^2}}\,dx = \int \frac{1}{\sqrt[4]{ax+bx^2}}\,dx$$

```
integrate(1/(b*x**2+a*x)**(1/4),x)
```

```
Integral((a*x + b*x**2)**(-1/4), x)
```

Maxima [F]

$$\int \frac{1}{\sqrt[4]{ax+bx^2}}\,dx = \int \frac{1}{(bx^2+ax)^{\frac{1}{4}}}\,dx$$

```
integrate(1/(b*x^2+a*x)^(1/4),x, algorithm="maxima")
```

```
integrate((b*x^2 + a*x)^(-1/4), x)
```

Giac [F]

$$\int \frac{1}{\sqrt[4]{ax+bx^2}}\,dx=\int \frac{1}{(bx^2+ax)^{\frac{1}{4}}}\,dx$$

```
integrate(1/(b*x^2+a*x)^(1/4),x, algorithm="giac")
```

```
integrate((b*x^2 + a*x)^(-1/4), x)
```

Mupad [B] (verification not implemented)

Time = 9.58 (sec) , antiderivative size = 36, normalized size of antiderivative = 0.58

$$\int \frac{1}{\sqrt[4]{ax+bx^2}}\,dx=\frac{4\,x\,\left(\frac{b\,x}{a}+1\right)^{1/4}\,{}_2\mathrm{F}_1\left(\frac{1}{4},\frac{3}{4};\ \frac{7}{4};\ -\frac{b\,x}{a}\right)}{3\,\left(b\,x^2+a\,x\right)^{1/4}}$$

```
int(1/(a*x + b*x^2)^(1/4),x)
```

```
(4*x*((b*x)/a + 1)^(1/4)*hypergeom([1/4, 3/4], 7/4, -(b*x)/a))/(3*(a*x + b
*x^2)^(1/4))
```

Reduce [F]

$$\int \frac{1}{\sqrt[4]{ax+bx^2}}\,dx=\int \frac{1}{x^{\frac{1}{4}}\,(bx+a)^{\frac{1}{4}}}dx$$

```
int(1/(b*x^2+a*x)^(1/4),x)
```

```
int(1/(x**(1/4)*(a + b*x)**(1/4)),x)
```

3.178 $\int \frac{1}{x\sqrt[4]{ax+bx^2}}\,dx$

Optimal result

Integrand size = 17, antiderivative size = 81

$$\int \frac{1}{x\sqrt[4]{ax+bx^2}}\,dx=-\frac{4}{\sqrt[4]{ax+bx^2}}+\frac{4\sqrt[4]{\frac{bx}{a+bx}}\sqrt{a+bx}\,E\left(\frac{1}{2}\arcsin\left(\frac{\sqrt{a}}{\sqrt{a+bx}}\right)\middle|2\right)}{\sqrt{a}\sqrt[4]{ax+bx^2}}$$

```
-4/(b*x^2+a*x)^(1/4)+4*(b*x/(b*x+a))^(1/4)*(b*x+a)^(1/2)*EllipticE(sin(1/2
*arcsin(a^(1/2)/(b*x+a)^(1/2))),2^(1/2))/a^(1/2)/(b*x^2+a*x)^(1/4)
```

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 10.02 (sec) , antiderivative size = 42, normalized size of antiderivative = 0.52

$$\int \frac{1}{x\sqrt[4]{ax+bx^2}}\,dx=-\frac{4\sqrt[4]{1+\frac{bx}{a}}\,\text{Hypergeometric2F1}\left(-\frac{1}{4},\frac{1}{4},\frac{3}{4},-\frac{bx}{a}\right)}{\sqrt[4]{x(a+bx)}}$$

```
Integrate[1/(x*(a*x + b*x^2)^(1/4)),x]
```

```
(-4*(1 + (b*x)/a)^(1/4)*Hypergeometric2F1[-1/4, 1/4, 3/4, -((b*x)/a)])/(x*
(a + b*x))^(1/4)
```

Rubi [A] (warning: unable to verify)

Time = 0.48 (sec) , antiderivative size = 138, normalized size of antiderivative = 1.70, number of steps used = 9, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.471, Rules used = {1137, 61, 73, 839, 813, 858, 807, 212}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{x\sqrt[4]{ax+bx^2}}dx$$

$$\Big\downarrow \text{1137}$$

$$\frac{\sqrt[4]{x}\sqrt[4]{a+bx}\int \frac{1}{x^{5/4}\sqrt[4]{a+bx}}dx}{\sqrt[4]{ax+bx^2}}$$

$$\Big\downarrow \text{61}$$

$$\frac{\sqrt[4]{x}\sqrt[4]{a+bx}\left(\frac{2b\int \frac{1}{\sqrt[4]{x}\sqrt[4]{a+bx}}dx}{a}-\frac{4(a+bx)^{3/4}}{a\sqrt[4]{x}}\right)}{\sqrt[4]{ax+bx^2}}$$

$$\Big\downarrow \text{73}$$

$$\frac{\sqrt[4]{x}\sqrt[4]{a+bx}\left(\frac{8b\int \frac{\sqrt{x}}{\sqrt[4]{a+bx}}d\sqrt[4]{x}}{a}-\frac{4(a+bx)^{3/4}}{a\sqrt[4]{x}}\right)}{\sqrt[4]{ax+bx^2}}$$

$$\Big\downarrow \text{839}$$

$$\frac{\sqrt[4]{x}\sqrt[4]{a+bx}\left(\frac{8b\left(\frac{x^{3/4}}{2\sqrt[4]{a+bx}}-\frac{1}{2}a\int \frac{\sqrt{x}}{(a+bx)^{5/4}}d\sqrt[4]{x}\right)}{a}-\frac{4(a+bx)^{3/4}}{a\sqrt[4]{x}}\right)}{\sqrt[4]{ax+bx^2}}$$

$$\Big\downarrow \text{813}$$

$$\frac{\sqrt[4]{x}\sqrt[4]{a+bx}\left(\frac{8b\left(\frac{x^{3/4}}{2\sqrt[4]{a+bx}}-\frac{a\sqrt[4]{x}\sqrt[4]{\frac{a}{bx}+1}\int \frac{1}{\left(\frac{a}{bx}+1\right)^{5/4}x^{3/4}}d\sqrt[4]{x}}{2b\sqrt[4]{a+bx}}\right)}{a}-\frac{4(a+bx)^{3/4}}{a\sqrt[4]{x}}\right)}{\sqrt[4]{ax+bx^2}}$$

$$\Big\downarrow \; 858$$

$$\frac{\sqrt[4]{x}\sqrt[4]{a+bx}\left(\frac{8b\left(\frac{a\sqrt[4]{x}\sqrt[4]{\frac{a}{bx}+1}\int \frac{1}{\sqrt[4]{x}\left(\frac{ax}{b}+1\right)^{5/4}}d\frac{1}{\sqrt[4]{x}}}{2b\sqrt[4]{a+bx}}+\frac{x^{3/4}}{2\sqrt[4]{a+bx}}\right)}{a}-\frac{4(a+bx)^{3/4}}{a\sqrt[4]{x}}\right)}{\sqrt[4]{ax+bx^2}}$$

$$\Big\downarrow \; 807$$

$$\frac{\sqrt[4]{x}\sqrt[4]{a+bx}\left(\frac{8b\left(\frac{a\sqrt[4]{x}\sqrt[4]{\frac{a}{bx}+1}\int \frac{1}{\left(\frac{\sqrt{x}a}{b}+1\right)^{5/4}}d\sqrt{x}}{4b\sqrt[4]{a+bx}}+\frac{x^{3/4}}{2\sqrt[4]{a+bx}}\right)}{a}-\frac{4(a+bx)^{3/4}}{a\sqrt[4]{x}}\right)}{\sqrt[4]{ax+bx^2}}$$

$$\Big\downarrow \; 212$$

$$\frac{\sqrt[4]{x}\sqrt[4]{a+bx}\left(\frac{8b\left(\frac{\sqrt{a}\sqrt[4]{x}\sqrt[4]{\frac{a}{bx}+1}E\left(\frac{1}{2}\arctan\left(\frac{\sqrt{a}\sqrt{x}}{\sqrt{b}}\right)\middle|2\right)}{2\sqrt{b}\sqrt[4]{a+bx}}+\frac{x^{3/4}}{2\sqrt[4]{a+bx}}\right)}{a}-\frac{4(a+bx)^{3/4}}{a\sqrt[4]{x}}\right)}{\sqrt[4]{ax+bx^2}}$$

```
Int[1/(x*(a*x + b*x^2)^(1/4)),x]
```

```
(x^(1/4)*(a + b*x)^(1/4)*((-4*(a + b*x)^(3/4))/(a*x^(1/4)) + (8*b*(x^(3/4)
/(2*(a + b*x)^(1/4)) + (Sqrt[a]*(1 + a/(b*x))^(1/4)*x^(1/4)*EllipticE[ArcT
an[(Sqrt[a]*Sqrt[x])/Sqrt[b]]/2, 2])/(2*Sqrt[b]*(a + b*x)^(1/4))))/a))/(a*
x + b*x^2)^(1/4)
```

Defintions of rubi rules used

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] - Simp[d*((
m + n + 2)/((b*c - a*d)*(m + 1)))   Int[(a + b*x)^(m + 1)*(c + d*x)^n, x],
x] /; FreeQ[{a, b, c, d, n}, x] && LtQ[m, -1] &&  !(LtQ[n, -1] && (EqQ[a, 0
] || (NeQ[c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && IntLinearQ[a, b, c, d
, m, n, x]
```

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Simp[p/b   Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
 d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && Lt
Q[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntL
inearQ[a, b, c, d, m, n, x]
```

```
Int[((a_) + (b_.)*(x_)^2)^(-5/4), x_Symbol] :> Simp[(2/(a^(5/4)*Rt[b/a, 2])
)*EllipticE[(1/2)*ArcTan[Rt[b/a, 2]*x], 2], x] /; FreeQ[{a, b}, x] && GtQ[a
, 0] && PosQ[b/a]
```

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k = GCD[m
+ 1, n]}, Simp[1/k   Subst[Int[x^((m + 1)/k - 1)*(a + b*x^(n/k))^p, x], x,
x^k], x] /; k != 1] /; FreeQ[{a, b, p}, x] && IGtQ[n, 0] && IntegerQ[m]
```

```
Int[(x_)^2/((a_) + (b_.)*(x_)^4)^(5/4), x_Symbol] :> Simp[x*((1 + a/(b*x^4)
)^(1/4)/(b*(a + b*x^4)^(1/4)))   Int[1/(x^3*(1 + a/(b*x^4))^(5/4)), x], x]
/; FreeQ[{a, b}, x] && PosQ[b/a]
```

```
Int[(x_)^2/((a_) + (b_.)*(x_)^4)^(1/4), x_Symbol] :> Simp[x^3/(2*(a + b*x^4
)^(1/4)), x] - Simp[a/2   Int[x^2/(a + b*x^4)^(5/4), x], x] /; FreeQ[{a, b}
, x] && PosQ[b/a]
```

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> -Subst[Int[(a +
b/x^n)^p/x^(m + 2), x], x, 1/x] /; FreeQ[{a, b, p}, x] && ILtQ[n, 0] && Int
egerQ[m]
```

```
Int[((e_.)*(x_))^(m_)*((b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(
e*x)^m*((b*x + c*x^2)^p/(x^(m + p)*(b + c*x)^p))   Int[x^(m + p)*(b + c*x)^
p, x], x] /; FreeQ[{b, c, e, m}, x]
```

Maple [F]

$$\int \frac{1}{x\left(b\,x^{2}+ax\right)^{\frac{1}{4}}}dx$$

```
int(1/x/(b*x^2+a*x)^(1/4),x)
```

```
int(1/x/(b*x^2+a*x)^(1/4),x)
```

Fricas [F]

$$\int \frac{1}{x\sqrt[4]{ax+bx^2}}\,dx=\int \frac{1}{(bx^2+ax)^{\frac{1}{4}}x}\,dx$$

```
integrate(1/x/(b*x^2+a*x)^(1/4),x, algorithm="fricas")
```

```
integral((b*x^2 + a*x)^(3/4)/(b*x^3 + a*x^2), x)
```

Sympy [F]

$$\int \frac{1}{x\sqrt[4]{ax+bx^2}}\,dx=\int \frac{1}{x\sqrt[4]{x(a+bx)}}\,dx$$

```
integrate(1/x/(b*x**2+a*x)**(1/4),x)
```

```
Integral(1/(x*(x*(a + b*x))**(1/4)), x)
```

Maxima [F]

$$\int \frac{1}{x\sqrt[4]{ax+bx^2}}\,dx=\int \frac{1}{(bx^2+ax)^{\frac{1}{4}}x}\,dx$$

```
integrate(1/x/(b*x^2+a*x)^(1/4),x, algorithm="maxima")
```

```
integrate(1/((b*x^2 + a*x)^(1/4)*x), x)
```

Giac [F]

$$\int \frac{1}{x\sqrt[4]{ax+bx^2}}\,dx = \int \frac{1}{(bx^2+ax)^{\frac{1}{4}}x}\,dx$$

```
integrate(1/x/(b*x^2+a*x)^(1/4),x, algorithm="giac")
```

```
integrate(1/((b*x^2 + a*x)^(1/4)*x), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{x\sqrt[4]{ax+bx^2}}\,dx = \int \frac{1}{x\,(b\,x^2+a\,x)^{1/4}}\,dx$$

```
int(1/(x*(a*x + b*x^2)^(1/4)),x)
```

```
int(1/(x*(a*x + b*x^2)^(1/4)), x)
```

Reduce [F]

$$\int \frac{1}{x\sqrt[4]{ax+bx^2}}\,dx = \int \frac{1}{x^{\frac{5}{4}}\,(bx+a)^{\frac{1}{4}}}dx$$

```
int(1/x/(b*x^2+a*x)^(1/4),x)
```

```
int(1/(x**(1/4)*(a + b*x)**(1/4)*x),x)
```

3.179 $\int \frac{1}{x^2 \sqrt[4]{ax+bx^2}}\,dx$

Optimal result

Integrand size = 17, antiderivative size = 113

$$\int \frac{1}{x^2 \sqrt[4]{ax+bx^2}}\,dx = \frac{8b}{5a\sqrt[4]{ax+bx^2}} - \frac{4(ax+bx^2)^{3/4}}{5ax^2} - \frac{8b\sqrt[4]{\frac{bx}{a+bx}}\sqrt{a+bx}\,E\left(\frac{1}{2}\arcsin\left(\frac{\sqrt{a}}{\sqrt{a+bx}}\right)\middle|2\right)}{5a^{3/2}\sqrt[4]{ax+bx^2}}$$

```
8/5*b/a/(b*x^2+a*x)^(1/4)-4/5*(b*x^2+a*x)^(3/4)/a/x^2-8/5*b*(b*x/(b*x+a))^
(1/4)*(b*x+a)^(1/2)*EllipticE(sin(1/2*arcsin(a^(1/2)/(b*x+a)^(1/2))),2^(1/
2))/a^(3/2)/(b*x^2+a*x)^(1/4)
```

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 10.01 (sec) , antiderivative size = 47, normalized size of antiderivative = 0.42

$$\int \frac{1}{x^2 \sqrt[4]{ax+bx^2}}\,dx = -\frac{4\sqrt[4]{1+\frac{bx}{a}}\,\text{Hypergeometric2F1}\left(-\frac{5}{4},\frac{1}{4},-\frac{1}{4},-\frac{bx}{a}\right)}{5x\sqrt[4]{x(a+bx)}}$$

```
Integrate[1/(x^2*(a*x + b*x^2)^(1/4)),x]
```

```
(-4*(1 + (b*x)/a)^(1/4)*Hypergeometric2F1[-5/4, 1/4, -1/4, -((b*x)/a)])/(5
*x*(x*(a + b*x))^(1/4))
```

Rubi [A] (warning: unable to verify)

Time = 0.54 (sec) , antiderivative size = 168, normalized size of antiderivative = 1.49, number of steps used = 10, number of rules used = 9, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.529, Rules used = {1137, 61, 61, 73, 839, 813, 858, 807, 212}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{x^2 \sqrt[4]{ax+bx^2}}dx$$

$$\Big\downarrow \; 1137$$

$$\frac{\sqrt[4]{x}\sqrt[4]{a+bx}\int \frac{1}{x^{9/4}\sqrt[4]{a+bx}}dx}{\sqrt[4]{ax+bx^2}}$$

$$\Big\downarrow \; 61$$

$$\frac{\sqrt[4]{x}\sqrt[4]{a+bx}\left(-\frac{2b\int \frac{1}{x^{5/4}\sqrt[4]{a+bx}}dx}{5a}-\frac{4(a+bx)^{3/4}}{5ax^{5/4}}\right)}{\sqrt[4]{ax+bx^2}}$$

$$\Big\downarrow \; 61$$

$$\frac{\sqrt[4]{x}\sqrt[4]{a+bx}\left(-\frac{2b\left(\frac{2b\int \frac{1}{\sqrt[4]{x}\sqrt[4]{a+bx}}dx}{a}-\frac{4(a+bx)^{3/4}}{a\sqrt[4]{x}}\right)}{5a}-\frac{4(a+bx)^{3/4}}{5ax^{5/4}}\right)}{\sqrt[4]{ax+bx^2}}$$

$$\Big\downarrow \; 73$$

$$\frac{\sqrt[4]{x}\sqrt[4]{a+bx}\left(-\frac{2b\left(\frac{8b\int \frac{\sqrt{x}}{\sqrt[4]{a+bx}}d\sqrt[4]{x}}{a}-\frac{4(a+bx)^{3/4}}{a\sqrt[4]{x}}\right)}{5a}-\frac{4(a+bx)^{3/4}}{5ax^{5/4}}\right)}{\sqrt[4]{ax+bx^2}}$$

$$\Big\downarrow \text{839}$$

$$\frac{\sqrt[4]{x}\sqrt[4]{a+bx}\left(-\frac{2b\left(\frac{8b\left(\frac{x^{3/4}}{2\sqrt[4]{a+bx}}-\frac{1}{2}a\int \frac{\sqrt{x}}{(a+bx)^{5/4}}d\sqrt[4]{x}\right)}{a}-\frac{4(a+bx)^{3/4}}{a\sqrt[4]{x}}\right)}{5a}-\frac{4(a+bx)^{3/4}}{5ax^{5/4}}\right)}{\sqrt[4]{ax+bx^2}}$$

$$\Big\downarrow \text{813}$$

$$\frac{\sqrt[4]{x}\sqrt[4]{a+bx}\left(-\frac{2b\left(\frac{8b\left(\frac{x^{3/4}}{2\sqrt[4]{a+bx}}-\frac{a\sqrt[4]{x}\sqrt[4]{\frac{a}{bx}+1}\int \frac{1}{\left(\frac{a}{bx}+1\right)^{5/4}x^{3/4}}d\sqrt[4]{x}}{2b\sqrt[4]{a+bx}}\right)}{a}-\frac{4(a+bx)^{3/4}}{a\sqrt[4]{x}}\right)}{5a}-\frac{4(a+bx)^{3/4}}{5ax^{5/4}}\right)}{\sqrt[4]{ax+bx^2}}$$

$$\Big\downarrow \text{858}$$

$$\frac{\sqrt[4]{x}\sqrt[4]{a+bx}\left(-\frac{2b\left(\frac{8b\left(\frac{a\sqrt[4]{x}\sqrt[4]{\frac{a}{bx}+1}\int \frac{1}{\sqrt[4]{x}\left(\frac{ax}{b}+1\right)^{5/4}}d\frac{1}{\sqrt[4]{x}}}{2b\sqrt[4]{a+bx}}+\frac{x^{3/4}}{2\sqrt[4]{a+bx}}\right)}{a}-\frac{4(a+bx)^{3/4}}{a\sqrt[4]{x}}\right)}{5a}-\frac{4(a+bx)^{3/4}}{5ax^{5/4}}\right)}{\sqrt[4]{ax+bx^2}}$$

$$\Big\downarrow \; 807$$

$$\frac{\sqrt[4]{x}\sqrt[4]{a+bx}\left(-\frac{2b\left(\frac{8b\left(\frac{a\sqrt[4]{x}\sqrt[4]{\frac{a}{bx}+1}\int \frac{1}{\left(\frac{\sqrt{x}a}{b}+1\right)^{5/4}}d\sqrt{x}}{4b\sqrt[4]{a+bx}}+\frac{x^{3/4}}{2\sqrt[4]{a+bx}}\right)}{a}-\frac{4(a+bx)^{3/4}}{a\sqrt[4]{x}}\right)}{5a}-\frac{4(a+bx)^{3/4}}{5ax^{5/4}}\right)}{\sqrt[4]{ax+bx^2}}$$

$$\Big\downarrow \; 212$$

$$\frac{\sqrt[4]{x}\sqrt[4]{a+bx}\left(-\frac{2b\left(\frac{8b\left(\frac{\sqrt{a}\sqrt[4]{x}\sqrt[4]{\frac{a}{bx}+1}E\left(\frac{1}{2}\arctan\left(\frac{\sqrt{a}\sqrt{x}}{\sqrt{b}}\right)\middle|2\right)}{2\sqrt{b}\sqrt[4]{a+bx}}+\frac{x^{3/4}}{2\sqrt[4]{a+bx}}\right)}{a}-\frac{4(a+bx)^{3/4}}{a\sqrt[4]{x}}\right)}{5a}-\frac{4(a+bx)^{3/4}}{5ax^{5/4}}\right)}{\sqrt[4]{ax+bx^2}}$$

```
Int[1/(x^2*(a*x + b*x^2)^(1/4)),x]
```

```
(x^(1/4)*(a + b*x)^(1/4)*((-4*(a + b*x)^(3/4))/(5*a*x^(5/4)) - (2*b*((-4*(
a + b*x)^(3/4))/(a*x^(1/4)) + (8*b*(x^(3/4)/(2*(a + b*x)^(1/4)) + (Sqrt[a]
*(1 + a/(b*x))^(1/4)*x^(1/4)*EllipticE[ArcTan[(Sqrt[a]*Sqrt[x])/Sqrt[b]]/2
, 2])/(2*Sqrt[b]*(a + b*x)^(1/4))))/a))/(5*a)))/(a*x + b*x^2)^(1/4)
```

Defintions of rubi rules used

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] - Simp[d*((
m + n + 2)/((b*c - a*d)*(m + 1)))   Int[(a + b*x)^(m + 1)*(c + d*x)^n, x],
x] /; FreeQ[{a, b, c, d, n}, x] && LtQ[m, -1] &&  !(LtQ[n, -1] && (EqQ[a, 0
] || (NeQ[c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && IntLinearQ[a, b, c, d
, m, n, x]
```

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Simp[p/b   Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
 d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && Lt
Q[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntL
inearQ[a, b, c, d, m, n, x]
```

```
Int[((a_) + (b_.)*(x_)^2)^(-5/4), x_Symbol] :> Simp[(2/(a^(5/4)*Rt[b/a, 2])
)*EllipticE[(1/2)*ArcTan[Rt[b/a, 2]*x], 2], x] /; FreeQ[{a, b}, x] && GtQ[a
, 0] && PosQ[b/a]
```

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k = GCD[m
+ 1, n]}, Simp[1/k   Subst[Int[x^((m + 1)/k - 1)*(a + b*x^(n/k))^p, x], x,
x^k], x] /; k != 1] /; FreeQ[{a, b, p}, x] && IGtQ[n, 0] && IntegerQ[m]
```

```
Int[(x_)^2/((a_) + (b_.)*(x_)^4)^(5/4), x_Symbol] :> Simp[x*((1 + a/(b*x^4)
)^(1/4)/(b*(a + b*x^4)^(1/4)))   Int[1/(x^3*(1 + a/(b*x^4))^(5/4)), x], x]
/; FreeQ[{a, b}, x] && PosQ[b/a]
```

```
Int[(x_)^2/((a_) + (b_.)*(x_)^4)^(1/4), x_Symbol] :> Simp[x^3/(2*(a + b*x^4
)^(1/4)), x] - Simp[a/2   Int[x^2/(a + b*x^4)^(5/4), x], x] /; FreeQ[{a, b}
, x] && PosQ[b/a]
```

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> -Subst[Int[(a +
b/x^n)^p/x^(m + 2), x], x, 1/x] /; FreeQ[{a, b, p}, x] && ILtQ[n, 0] && Int
egerQ[m]
```

```
Int[((e_.)*(x_))^(m_)*((b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(
e*x)^m*((b*x + c*x^2)^p/(x^(m + p)*(b + c*x)^p))   Int[x^(m + p)*(b + c*x)^
p, x], x] /; FreeQ[{b, c, e, m}, x]
```

Maple [F]

$$\int \frac{1}{x^{2} \left(b\,x^{2}+a x \right)^{\frac{1}{4}}}d x$$

```
int(1/x^2/(b*x^2+a*x)^(1/4),x)
```

```
int(1/x^2/(b*x^2+a*x)^(1/4),x)
```

Fricas [F]

$$\int \frac{1}{x^2 \sqrt[4]{ax+bx^2}}\,dx = \int \frac{1}{(bx^2+ax)^{\frac{1}{4}}x^2}\,dx$$

```
integrate(1/x^2/(b*x^2+a*x)^(1/4),x, algorithm="fricas")
```

```
integral((b*x^2 + a*x)^(3/4)/(b*x^4 + a*x^3), x)
```

Sympy [F]

$$\int \frac{1}{x^2 \sqrt[4]{ax+bx^2}}\,dx = \int \frac{1}{x^2 \sqrt[4]{x\,(a+bx)}}\,dx$$

```
integrate(1/x**2/(b*x**2+a*x)**(1/4),x)
```

```
Integral(1/(x**2*(x*(a + b*x))**(1/4)), x)
```

Maxima [F]

$$\int \frac{1}{x^2 \sqrt[4]{ax+bx^2}}\,dx = \int \frac{1}{(bx^2+ax)^{\frac{1}{4}}x^2}\,dx$$

```
integrate(1/x^2/(b*x^2+a*x)^(1/4),x, algorithm="maxima")
```

```
integrate(1/((b*x^2 + a*x)^(1/4)*x^2), x)
```

Giac [F]

$$\int \frac{1}{x^2 \sqrt[4]{ax+bx^2}}\,dx = \int \frac{1}{(bx^2+ax)^{\frac{1}{4}}x^2}\,dx$$

```
integrate(1/x^2/(b*x^2+a*x)^(1/4),x, algorithm="giac")
```

```
integrate(1/((b*x^2 + a*x)^(1/4)*x^2), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{x^2 \sqrt[4]{ax+bx^2}}\,dx = \int \frac{1}{x^2\,(b\,x^2+a\,x)^{1/4}}\,dx$$

```
int(1/(x^2*(a*x + b*x^2)^(1/4)),x)
```

```
int(1/(x^2*(a*x + b*x^2)^(1/4)), x)
```

Reduce [B] (verification not implemented)

Time = 0.57 (sec) , antiderivative size = 19, normalized size of antiderivative = 0.17

$$\int \frac{1}{x^2 \sqrt[4]{ax+bx^2}}\,dx = -\frac{4(bx+a)^{\frac{3}{4}}}{3x^{\frac{3}{4}}\sqrt{x}\,a}$$

```
int(1/x^2/(b*x^2+a*x)^(1/4),x)
```

```
( - 4*x**(1/4)*(a + b*x)**(3/4))/(3*sqrt(x)*a*x)
```

3.180 $\int \frac{1}{x^3 \sqrt[4]{ax+bx^2}}\,dx$

Optimal result

Integrand size = 17, antiderivative size = 141

$$\int \frac{1}{x^3 \sqrt[4]{ax+bx^2}}\,dx = -\frac{16b^2}{15a^2\sqrt[4]{ax+bx^2}} - \frac{4(ax+bx^2)^{3/4}}{9ax^3} + \frac{8b(ax+bx^2)^{3/4}}{15a^2x^2} + \frac{16b^2\sqrt[4]{\frac{bx}{a+bx}}\sqrt{a+bx}\,E\left(\frac{1}{2}\arcsin\left(\frac{\sqrt{a}}{\sqrt{a+bx}}\right)\middle|2\right)}{15a^{5/2}\sqrt[4]{ax+bx^2}}$$

```
-16/15*b^2/a^2/(b*x^2+a*x)^(1/4)-4/9*(b*x^2+a*x)^(3/4)/a/x^3+8/15*b*(b*x^2
+a*x)^(3/4)/a^2/x^2+16/15*b^2*(b*x/(b*x+a))^(1/4)*(b*x+a)^(1/2)*EllipticE(
sin(1/2*arcsin(a^(1/2)/(b*x+a)^(1/2))),2^(1/2))/a^(5/2)/(b*x^2+a*x)^(1/4)
```

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 10.01 (sec) , antiderivative size = 47, normalized size of antiderivative = 0.33

$$\int \frac{1}{x^3 \sqrt[4]{ax+bx^2}}\,dx = -\frac{4\sqrt[4]{1+\frac{bx}{a}}\,\text{Hypergeometric2F1}\left(-\frac{9}{4},\frac{1}{4},-\frac{5}{4},-\frac{bx}{a}\right)}{9x^2\sqrt[4]{x(a+bx)}}$$

```
Integrate[1/(x^3*(a*x + b*x^2)^(1/4)),x]
```

```
(-4*(1 + (b*x)/a)^(1/4)*Hypergeometric2F1[-9/4, 1/4, -5/4, -((b*x)/a)])/(9
*x^2*(x*(a + b*x))^(1/4))
```

Rubi [A] (warning: unable to verify)

Time = 0.57 (sec) , antiderivative size = 198, normalized size of antiderivative = 1.40, number of steps used = 11, number of rules used = 10, $\frac{\text{number of rules}}{\text{integrand size}} = 0.588$, Rules used = {1137, 61, 61, 61, 73, 839, 813, 858, 807, 212}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{x^3 \sqrt[4]{ax+bx^2}}\,dx$$

$$\Big\downarrow \text{1137}$$

$$\frac{\sqrt[4]{x}\sqrt[4]{a+bx}\int \frac{1}{x^{13/4}\sqrt[4]{a+bx}}dx}{\sqrt[4]{ax+bx^2}}$$

$$\Big\downarrow \text{61}$$

$$\frac{\sqrt[4]{x}\sqrt[4]{a+bx}\left(-\frac{2b\int \frac{1}{x^{9/4}\sqrt[4]{a+bx}}dx}{3a}-\frac{4(a+bx)^{3/4}}{9ax^{9/4}}\right)}{\sqrt[4]{ax+bx^2}}$$

$$\Big\downarrow \text{61}$$

$$\frac{\sqrt[4]{x}\sqrt[4]{a+bx}\left(-\frac{2b\left(-\frac{2b\int \frac{1}{x^{5/4}\sqrt[4]{a+bx}}dx}{5a}-\frac{4(a+bx)^{3/4}}{5ax^{5/4}}\right)}{3a}-\frac{4(a+bx)^{3/4}}{9ax^{9/4}}\right)}{\sqrt[4]{ax+bx^2}}$$

$$\Big\downarrow \text{61}$$

$$\frac{\sqrt[4]{x}\sqrt[4]{a+bx}\left(-\frac{2b\left(-\frac{2b\left(\frac{2b\int\frac{1}{\sqrt[4]{x}\sqrt[4]{a+bx}}dx}{a}-\frac{4(a+bx)^{3/4}}{a\sqrt[4]{x}}\right)}{5a}-\frac{4(a+bx)^{3/4}}{5ax^{5/4}}\right)}{3a}-\frac{4(a+bx)^{3/4}}{9ax^{9/4}}\right)}{\sqrt[4]{ax+bx^2}}$$

$$\Big\downarrow \text{73}$$

$$\frac{\sqrt[4]{x}\sqrt[4]{a+bx}\left(-\frac{2b\left(-\frac{2b\left(\frac{8b\int\frac{\sqrt{x}}{\sqrt[4]{a+bx}}d\sqrt[4]{x}}{a}-\frac{4(a+bx)^{3/4}}{a\sqrt[4]{x}}\right)}{5a}-\frac{4(a+bx)^{3/4}}{5ax^{5/4}}\right)}{3a}-\frac{4(a+bx)^{3/4}}{9ax^{9/4}}\right)}{\sqrt[4]{ax+bx^2}}$$

$$\Big\downarrow \text{839}$$

$$\frac{\sqrt[4]{x}\sqrt[4]{a+bx}\left(-\frac{2b\left(-\frac{2b\left(\frac{8b\left(\frac{x^{3/4}}{2\sqrt[4]{a+bx}}-\frac{1}{2}a\int\frac{\sqrt{x}}{(a+bx)^{5/4}}d\sqrt[4]{x}\right)}{a}-\frac{4(a+bx)^{3/4}}{a\sqrt[4]{x}}\right)}{5a}-\frac{4(a+bx)^{3/4}}{5ax^{5/4}}\right)}{3a}-\frac{4(a+bx)^{3/4}}{9ax^{9/4}}\right)}{\sqrt[4]{ax+bx^2}}$$

$$\Big\downarrow \text{813}$$

$$\frac{\sqrt[4]{x}\sqrt[4]{a+bx}\left(-\frac{2b\left(-\frac{2b\left(\frac{8b\left(\frac{x^{3/4}}{2\sqrt[4]{a+bx}}-\frac{a\sqrt[4]{x}\sqrt[4]{\frac{a}{bx}+1}\int\frac{1}{\left(\frac{a}{bx}+1\right)^{5/4}x^{3/4}}d\sqrt[4]{x}}{2b\sqrt[4]{a+bx}}\right)}{a}-\frac{4(a+bx)^{3/4}}{a\sqrt[4]{x}}\right)}{5a}-\frac{4(a+bx)^{3/4}}{5ax^{5/4}}\right)}{3a}-\frac{4(a+bx)^{3/4}}{9ax^{9/4}}\right)}{\sqrt[4]{ax+bx^2}}$$

$\downarrow$ 858

$$\frac{\sqrt[4]{x}\sqrt[4]{a+bx}\left(-\frac{2b\left(-\frac{2b\left(\frac{8b\left(\frac{a\sqrt[4]{x}\sqrt[4]{\frac{a}{bx}+1}\int\frac{1}{\sqrt[4]{x}\left(\frac{ax}{b}+1\right)^{5/4}}d\frac{1}{\sqrt[4]{x}}}{2b\sqrt[4]{a+bx}}+\frac{x^{3/4}}{2\sqrt[4]{a+bx}}\right)}{a}-\frac{4(a+bx)^{3/4}}{a\sqrt[4]{x}}\right)}{5a}-\frac{4(a+bx)^{3/4}}{5ax^{5/4}}\right)}{3a}-\frac{4(a+bx)^{3/4}}{9ax^{9/4}}\right)}{\sqrt[4]{ax+bx^2}}$$

$\downarrow$ 807

$$\frac{\sqrt[4]{x}\sqrt[4]{a+bx}\left(-\frac{2b\left(-\frac{2b\left(\frac{2b\left(\frac{8b\left(\frac{a\sqrt[4]{x}\sqrt[4]{\frac{a}{bx}+1}\int\frac{1}{\left(\frac{\sqrt{x}a}{b}+1\right)^{5/4}}d\sqrt{x}}{4b\sqrt[4]{a+bx}}+\frac{x^{3/4}}{2\sqrt[4]{a+bx}}\right)}{a}-\frac{4(a+bx)^{3/4}}{a\sqrt[4]{x}}\right)}{5a}-\frac{4(a+bx)^{3/4}}{5ax^{5/4}}\right)}{3a}-\frac{4(a+bx)^{3/4}}{9ax^{9/4}}\right)}{\sqrt[4]{ax+bx^2}}$$

$\downarrow$ 212

$$\frac{\sqrt[4]{x}\sqrt[4]{a+bx}\left(-\frac{2b\left(-\frac{2b\left(\frac{8b\left(\frac{\sqrt{a}\sqrt[4]{x}\sqrt[4]{\frac{a}{bx}+1}E\left(\frac{1}{2}\arctan\left(\frac{\sqrt{a}\sqrt{x}}{\sqrt{b}}\right)\middle|2\right)}{2\sqrt{b}\sqrt[4]{a+bx}}+\frac{x^{3/4}}{2\sqrt[4]{a+bx}}\right)}{a}-\frac{4(a+bx)^{3/4}}{a\sqrt[4]{x}}\right)}{5a}-\frac{4(a+bx)^{3/4}}{5ax^{5/4}}\right)}{3a}-\frac{4(a+bx)^{3/4}}{9ax^{9/4}}\right)}{\sqrt[4]{ax+bx^2}}$$

```
Int[1/(x^3*(a*x + b*x^2)^(1/4)),x]
```

```
(x^(1/4)*(a + b*x)^(1/4)*((-4*(a + b*x)^(3/4))/(9*a*x^(9/4)) - (2*b*((-4*(
a + b*x)^(3/4))/(5*a*x^(5/4)) - (2*b*((-4*(a + b*x)^(3/4))/(a*x^(1/4)) + (
8*b*(x^(3/4)/(2*(a + b*x)^(1/4)) + (Sqrt[a]*(1 + a/(b*x))^(1/4)*x^(1/4)*El
lipticE[ArcTan[(Sqrt[a]*Sqrt[x])/Sqrt[b]]/2, 2])/(2*Sqrt[b]*(a + b*x)^(1/4
))))/a))/(5*a)))/(3*a)))/(a*x + b*x^2)^(1/4)
```

Defintions of rubi rules used

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] - Simp[d*((
m + n + 2)/((b*c - a*d)*(m + 1)))   Int[(a + b*x)^(m + 1)*(c + d*x)^n, x],
x] /; FreeQ[{a, b, c, d, n}, x] && LtQ[m, -1] &&  !(LtQ[n, -1] && (EqQ[a, 0
] || (NeQ[c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && IntLinearQ[a, b, c, d
, m, n, x]
```

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Simp[p/b   Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
 d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && Lt
Q[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntL
inearQ[a, b, c, d, m, n, x]
```

```
Int[((a_) + (b_.)*(x_)^2)^(-5/4), x_Symbol] :> Simp[(2/(a^(5/4)*Rt[b/a, 2])
)*EllipticE[(1/2)*ArcTan[Rt[b/a, 2]*x], 2], x] /; FreeQ[{a, b}, x] && GtQ[a
, 0] && PosQ[b/a]
```

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k = GCD[m
+ 1, n]}, Simp[1/k   Subst[Int[x^((m + 1)/k - 1)*(a + b*x^(n/k))^p, x], x,
x^k], x] /; k != 1] /; FreeQ[{a, b, p}, x] && IGtQ[n, 0] && IntegerQ[m]
```

```
Int[(x_)^2/((a_) + (b_.)*(x_)^4)^(5/4), x_Symbol] :> Simp[x*((1 + a/(b*x^4)
)^(1/4)/(b*(a + b*x^4)^(1/4)))   Int[1/(x^3*(1 + a/(b*x^4))^(5/4)), x], x]
/; FreeQ[{a, b}, x] && PosQ[b/a]
```

```
Int[(x_)^2/((a_) + (b_.)*(x_)^4)^(1/4), x_Symbol] :> Simp[x^3/(2*(a + b*x^4
)^(1/4)), x] - Simp[a/2   Int[x^2/(a + b*x^4)^(5/4), x], x] /; FreeQ[{a, b}
, x] && PosQ[b/a]
```

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> -Subst[Int[(a +
b/x^n)^p/x^(m + 2), x], x, 1/x] /; FreeQ[{a, b, p}, x] && ILtQ[n, 0] && Int
egerQ[m]
```

```
Int[((e_.)*(x_))^(m_)*((b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(
e*x)^m*((b*x + c*x^2)^p/(x^(m + p)*(b + c*x)^p))   Int[x^(m + p)*(b + c*x)^
p, x], x] /; FreeQ[{b, c, e, m}, x]
```

Maple [F]

$$\int \frac{1}{x^{3} \left(b\,x^{2}+a x \right)^{\frac{1}{4}}}d x$$

```
int(1/x^3/(b*x^2+a*x)^(1/4),x)
```

```
int(1/x^3/(b*x^2+a*x)^(1/4),x)
```

Fricas [F]

$$\int \frac{1}{x^3 \sqrt[4]{ax+bx^2}}\,dx = \int \frac{1}{\left(bx^{2}+ax\right)^{\frac{1}{4}}x^{3}}\,dx$$

```
integrate(1/x^3/(b*x^2+a*x)^(1/4),x, algorithm="fricas")
```

```
integral((b*x^2 + a*x)^(3/4)/(b*x^5 + a*x^4), x)
```

Sympy [F]

$$\int \frac{1}{x^3 \sqrt[4]{ax+bx^2}}\,dx = \int \frac{1}{x^{3}\sqrt[4]{x\left(a+bx\right)}}\,dx$$

```
integrate(1/x**3/(b*x**2+a*x)**(1/4),x)
```

```
Integral(1/(x**3*(x*(a + b*x))**(1/4)), x)
```

Maxima [F]

$$\int \frac{1}{x^3 \sqrt[4]{ax+bx^2}}\,dx = \int \frac{1}{(bx^2+ax)^{\frac{1}{4}}x^3}\,dx$$

```
integrate(1/x^3/(b*x^2+a*x)^(1/4),x, algorithm="maxima")
```

```
integrate(1/((b*x^2 + a*x)^(1/4)*x^3), x)
```

Giac [F]

$$\int \frac{1}{x^3 \sqrt[4]{ax+bx^2}}\,dx = \int \frac{1}{(bx^2+ax)^{\frac{1}{4}}x^3}\,dx$$

```
integrate(1/x^3/(b*x^2+a*x)^(1/4),x, algorithm="giac")
```

```
integrate(1/((b*x^2 + a*x)^(1/4)*x^3), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{x^3 \sqrt[4]{ax+bx^2}}\,dx = \int \frac{1}{x^3\,\left(b\,x^2+a\,x\right)^{1/4}}\,dx$$

```
int(1/(x^3*(a*x + b*x^2)^(1/4)),x)
```

```
int(1/(x^3*(a*x + b*x^2)^(1/4)), x)
```

Reduce [B] (verification not implemented)

Time = 0.39 (sec) , antiderivative size = 27, normalized size of antiderivative = 0.19

$$\int \frac{1}{x^3 \sqrt[4]{ax+bx^2}}\,dx = \frac{4(bx+a)^{\frac{3}{4}}\,(4bx-3a)}{21x^{\frac{7}{4}}\sqrt{x}\,a^2}$$

```
int(1/x^3/(b*x^2+a*x)^(1/4),x)
```

```
(4*x**(1/4)*(a + b*x)**(3/4)*( - 3*a + 4*b*x))/(21*sqrt(x)*a**2*x**2)
```

3.181 $\int \frac{x^4}{(ax+bx^2)^{3/4}}\,dx$

Optimal result

Integrand size = 17, antiderivative size = 165

$$\int \frac{x^4}{(ax+bx^2)^{3/4}}\,dx = -\frac{39a^3\sqrt[4]{ax+bx^2}}{28b^4}+\frac{39a^2x\sqrt[4]{ax+bx^2}}{70b^3}-\frac{13ax^2\sqrt[4]{ax+bx^2}}{35b^2}+\frac{2x^3\sqrt[4]{ax+bx^2}}{7b}-\frac{39a^{7/2}\left(\frac{bx}{a+bx}\right)^{3/4}(a+bx)^{3/2}\operatorname{EllipticF}\left(\frac{1}{2}\arcsin\left(\frac{\sqrt{a}}{\sqrt{a+bx}}\right),2\right)}{28b^5(ax+bx^2)^{3/4}}$$

```
-39/28*a^3*(b*x^2+a*x)^(1/4)/b^4+39/70*a^2*x*(b*x^2+a*x)^(1/4)/b^3-13/35*a
*x^2*(b*x^2+a*x)^(1/4)/b^2+2/7*x^3*(b*x^2+a*x)^(1/4)/b-39/28*a^(7/2)*(b*x/
(b*x+a))^(3/4)*(b*x+a)^(3/2)*InverseJacobiAM(1/2*arcsin(a^(1/2)/(b*x+a)^(1
/2)),2^(1/2))/b^5/(b*x^2+a*x)^(3/4)
```

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 10.01 (sec) , antiderivative size = 47, normalized size of antiderivative = 0.28

$$\int \frac{x^4}{(ax+bx^2)^{3/4}}\,dx = \frac{4x^5\left(1+\frac{bx}{a}\right)^{3/4}\text{Hypergeometric2F1}\left(\frac{3}{4},\frac{17}{4},\frac{21}{4},-\frac{bx}{a}\right)}{17(x(a+bx))^{3/4}}$$

```
Integrate[x^4/(a*x + b*x^2)^(3/4),x]
```

```
(4*x^5*(1 + (b*x)/a)^(3/4)*Hypergeometric2F1[3/4, 17/4, 21/4, -((b*x)/a)])
/(17*(x*(a + b*x))^(3/4))
```

Rubi [A] (warning: unable to verify)

Time = 0.54 (sec) , antiderivative size = 201, normalized size of antiderivative = 1.22, number of steps used = 11, number of rules used = 10, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.588, Rules used = {1137, 60, 60, 60, 60, 73, 768, 858, 807, 229}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^4}{(ax+bx^2)^{3/4}}\,dx$$

$$\Big\downarrow \; 1137$$

$$\frac{x^{3/4}(a+bx)^{3/4}\int \frac{x^{13/4}}{(a+bx)^{3/4}}dx}{(ax+bx^2)^{3/4}}$$

$$\Big\downarrow \; 60$$

$$\frac{x^{3/4}(a+bx)^{3/4}\left(\frac{2x^{13/4}\sqrt[4]{a+bx}}{7b}-\frac{13a\int \frac{x^{9/4}}{(a+bx)^{3/4}}dx}{14b}\right)}{(ax+bx^2)^{3/4}}$$

$$\Big\downarrow \; 60$$

$$\frac{x^{3/4}(a+bx)^{3/4}\left(\frac{2x^{13/4}\sqrt[4]{a+bx}}{7b}-\frac{13a\left(\frac{2x^{9/4}\sqrt[4]{a+bx}}{5b}-\frac{9a\int \frac{x^{5/4}}{(a+bx)^{3/4}}dx}{10b}\right)}{14b}\right)}{(ax+bx^2)^{3/4}}$$

$$\Big\downarrow \; 60$$

$$\frac{x^{3/4}(a+bx)^{3/4}\left(\frac{2x^{13/4}\sqrt[4]{a+bx}}{7b}-\frac{13a\left(\frac{2x^{9/4}\sqrt[4]{a+bx}}{5b}-\frac{9a\left(\frac{2x^{5/4}\sqrt[4]{a+bx}}{3b}-\frac{5a\int\frac{\sqrt[4]{x}}{(a+bx)^{3/4}}dx}{6b}\right)}{10b}\right)}{14b}\right)}{(ax+bx^2)^{3/4}}$$

$$\Big\downarrow \; 60$$

$$\frac{x^{3/4}(a+bx)^{3/4}\left(\frac{2x^{13/4}\sqrt[4]{a+bx}}{7b}-\frac{13a\left(\frac{2x^{9/4}\sqrt[4]{a+bx}}{5b}-\frac{9a\left(\frac{2x^{5/4}\sqrt[4]{a+bx}}{3b}-\frac{5a\left(\frac{2\sqrt[4]{x}\sqrt[4]{a+bx}}{b}-\frac{a\int\frac{1}{x^{3/4}(a+bx)^{3/4}}dx}{2b}\right)}{6b}\right)}{10b}\right)}{14b}\right)}{(ax+bx^2)^{3/4}}$$

$$\Big\downarrow \; 73$$

$$\frac{x^{3/4}(a+bx)^{3/4}\left(\frac{2x^{13/4}\sqrt[4]{a+bx}}{7b}-\frac{13a\left(\frac{2x^{9/4}\sqrt[4]{a+bx}}{5b}-\frac{9a\left(\frac{2x^{5/4}\sqrt[4]{a+bx}}{3b}-\frac{5a\left(\frac{2\sqrt[4]{x}\sqrt[4]{a+bx}}{b}-\frac{2a\int\frac{1}{(a+bx)^{3/4}}d\sqrt[4]{x}}{b}\right)}{6b}\right)}{10b}\right)}{14b}\right)}{(ax+bx^2)^{3/4}}$$

$\downarrow$ 768

$$\frac{x^{3/4}(a+bx)^{3/4}\left(\frac{2x^{13/4}\sqrt[4]{a+bx}}{7b}-\frac{13a\left(\frac{2x^{9/4}\sqrt[4]{a+bx}}{5b}-\frac{9a\left(\frac{2x^{5/4}\sqrt[4]{a+bx}}{3b}-\frac{5a\left(\frac{2\sqrt[4]{x}\sqrt[4]{a+bx}}{b}-\frac{2ax^{3/4}\left(\frac{a}{bx}+1\right)^{3/4}\int \frac{\cdots}{\left(\frac{a}{bx}+1\right)\cdots}}{b(a+bx)^{3/4}}\cdots\right)}{6b}\cdots\right)}{10b}\cdots\right)}{14b}\cdots\right)}{(ax+bx^2)^{3/4}}\cdots$$

$\downarrow$ 858

$$\frac{x^{3/4}(a+bx)^{3/4}\Bigg(\frac{2x^{13/4}\sqrt[4]{a+bx}}{7b}-\frac{13a\bigg(\frac{2x^{9/4}\sqrt[4]{a+bx}}{5b}-\frac{9a\Big(\frac{2x^{5/4}\sqrt[4]{a+bx}}{3b}-\frac{5a\Big(\frac{2ax^{3/4}\left(\frac{a}{bx}+1\right)^{3/4}\int\frac{1}{\sqrt[4]{x}\left(\frac{ax}{b}+1\right)^{3/4}}d\frac{1}{\sqrt[4]{x}}}{b(a+bx)^{3/4}}+2}{6b}}{10b}}{14b}}{\left(ax+bx^2\right)^{3/4}}$$

$\downarrow$ 807

$$\frac{x^{3/4}(a+bx)^{3/4}\left(\frac{2x^{13/4}\sqrt[4]{a+bx}}{7b}-\frac{13a\left(\frac{2x^{9/4}\sqrt[4]{a+bx}}{5b}-\frac{9a\left(\frac{2x^{5/4}\sqrt[4]{a+bx}}{3b}-\frac{5a\left(\frac{ax^{3/4}\left(\frac{a}{bx}+1\right)^{3/4}\int\frac{1}{\left(\frac{\sqrt{x}a}{b}+1\right)^{3/4}}d\sqrt{x}}{b(a+bx)^{3/4}}+\frac{2\sqrt[4]{x}\sqrt[4]{\cdots}}{\cdots}\right.}{6b}\cdots\right.}{10b}\cdots\right.}{14b}\cdots\right.}{(ax+bx^2)^{3/4}}\cdots$$

$\downarrow$ 229

$$\frac{x^{3/4}(a+bx)^{3/4}\left(\frac{2x^{13/4}\sqrt[4]{a+bx}}{7b}-\frac{13a\left(\frac{2x^{9/4}\sqrt[4]{a+bx}}{5b}-\frac{9a\left(\frac{2x^{5/4}\sqrt[4]{a+bx}}{3b}-\frac{5a\left(\frac{2\sqrt{a}x^{3/4}\left(\frac{a}{bx}+1\right)^{3/4}\text{EllipticF}\left(\frac{1}{2}\arctan\left(\frac{\sqrt{a}\sqrt{x}}{\sqrt{b}}\right),\right.}{\sqrt{b}(a+bx)^{3/4}}\right.}{6b}\right.}{10b}\right.}{14b}\right.}{(ax+bx^2)^{3/4}}$$

```
Int[x^4/(a*x + b*x^2)^(3/4),x]
```

```
(x^(3/4)*(a + b*x)^(3/4)*((2*x^(13/4)*(a + b*x)^(1/4))/(7*b) - (13*a*((2*x
^(9/4)*(a + b*x)^(1/4))/(5*b) - (9*a*((2*x^(5/4)*(a + b*x)^(1/4))/(3*b) -
(5*a*((2*x^(1/4)*(a + b*x)^(1/4))/b + (2*Sqrt[a]*(1 + a/(b*x))^(3/4)*x^(3/
4)*EllipticF[ArcTan[(Sqrt[a]*Sqrt[x])/Sqrt[b]]/2, 2])/(Sqrt[b]*(a + b*x)^(
3/4))))/(6*b)))/(10*b)))/(14*b)))/(a*x + b*x^2)^(3/4)
```

Defintions of rubi rules used

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^n/(b*(m + n + 1))), x] + Simp[n*((b*c - a*d)/(
b*(m + n + 1)))   Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
 c, d}, x] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ[m, 0] && ( !Integer
Q[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n + 2, 0] && IntLinear
Q[a, b, c, d, m, n, x]
```

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Simp[p/b   Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
 d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && Lt
Q[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntL
inearQ[a, b, c, d, m, n, x]
```

```
Int[((a_) + (b_.)*(x_)^2)^(-3/4), x_Symbol] :> Simp[(2/(a^(3/4)*Rt[b/a, 2])
)*EllipticF[(1/2)*ArcTan[Rt[b/a, 2]*x], 2], x] /; FreeQ[{a, b}, x] && GtQ[a
, 0] && PosQ[b/a]
```

```
Int[((a_) + (b_.)*(x_)^4)^(-3/4), x_Symbol] :> Simp[x^3*((1 + a/(b*x^4))^(3
/4)/(a + b*x^4)^(3/4))   Int[1/(x^3*(1 + a/(b*x^4))^(3/4)), x], x] /; FreeQ
[{a, b}, x]
```

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k = GCD[m
+ 1, n]}, Simp[1/k   Subst[Int[x^((m + 1)/k - 1)*(a + b*x^(n/k))^p, x], x,
x^k], x] /; k != 1] /; FreeQ[{a, b, p}, x] && IGtQ[n, 0] && IntegerQ[m]
```

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> -Subst[Int[(a +
b/x^n)^p/x^(m + 2), x], x, 1/x] /; FreeQ[{a, b, p}, x] && ILtQ[n, 0] && Int
egerQ[m]
```

```
Int[((e_.)*(x_))^(m_)*((b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(
e*x)^m*((b*x + c*x^2)^p/(x^(m + p)*(b + c*x)^p))   Int[x^(m + p)*(b + c*x)^
p, x], x] /; FreeQ[{b, c, e, m}, x]
```

Maple [F]

$$\int \frac{x^4}{(b\,x^2 + ax)^{\frac{3}{4}}}dx$$

```
int(x^4/(b*x^2+a*x)^(3/4),x)
```

```
int(x^4/(b*x^2+a*x)^(3/4),x)
```

Fricas [F]

$$\int \frac{x^4}{(ax+bx^2)^{3/4}}\,dx = \int \frac{x^4}{(bx^2+ax)^{\frac{3}{4}}}\,dx$$

```
integrate(x^4/(b*x^2+a*x)^(3/4),x, algorithm="fricas")
```

```
integral((b*x^2 + a*x)^(1/4)*x^3/(b*x + a), x)
```

Sympy [F]

$$\int \frac{x^4}{(ax+bx^2)^{3/4}}\,dx = \int \frac{x^4}{(x(a+bx))^{\frac{3}{4}}}\,dx$$

```
integrate(x**4/(b*x**2+a*x)**(3/4),x)
```

```
Integral(x**4/(x*(a + b*x))**(3/4), x)
```

Maxima [F]

$$\int \frac{x^4}{(ax+bx^2)^{3/4}}\,dx = \int \frac{x^4}{(bx^2+ax)^{\frac{3}{4}}}\,dx$$

```
integrate(x^4/(b*x^2+a*x)^(3/4),x, algorithm="maxima")
```

```
integrate(x^4/(b*x^2 + a*x)^(3/4), x)
```

Giac [F]

$$\int \frac{x^4}{(ax+bx^2)^{3/4}}\,dx = \int \frac{x^4}{(bx^2+ax)^{\frac{3}{4}}}\,dx$$

```
integrate(x^4/(b*x^2+a*x)^(3/4),x, algorithm="giac")
```

```
integrate(x^4/(b*x^2 + a*x)^(3/4), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{x^4}{(ax+bx^2)^{3/4}}\,dx = \int \frac{x^4}{(b\,x^2+a\,x)^{3/4}}\,dx$$

```
int(x^4/(a*x + b*x^2)^(3/4),x)
```

```
int(x^4/(a*x + b*x^2)^(3/4), x)
```

Reduce [F]

$$\int \frac{x^4}{(ax+bx^2)^{3/4}}\,dx = \int \frac{x^{\frac{13}{4}}}{(bx+a)^{\frac{3}{4}}}dx$$

```
int(x^4/(b*x^2+a*x)^(3/4),x)
```

```
int(x**4/(x**(3/4)*(a + b*x)**(3/4)),x)
```

3.182 $\int \frac{x^3}{(ax+bx^2)^{3/4}}\,dx$

Optimal result

Integrand size = 17, antiderivative size = 139

$$\int \frac{x^3}{(ax+bx^2)^{3/4}}\,dx = \frac{3a^2\sqrt[4]{ax+bx^2}}{2b^3}-\frac{3ax\sqrt[4]{ax+bx^2}}{5b^2}+\frac{2x^2\sqrt[4]{ax+bx^2}}{5b} + \frac{3a^{5/2}\left(\frac{bx}{a+bx}\right)^{3/4}(a+bx)^{3/2}\,\mathrm{EllipticF}\left(\frac{1}{2}\arcsin\left(\frac{\sqrt{a}}{\sqrt{a+bx}}\right),2\right)}{2b^4\,(ax+bx^2)^{3/4}}$$

```
3/2*a^2*(b*x^2+a*x)^(1/4)/b^3-3/5*a*x*(b*x^2+a*x)^(1/4)/b^2+2/5*x^2*(b*x^2
+a*x)^(1/4)/b+3/2*a^(5/2)*(b*x/(b*x+a))^(3/4)*(b*x+a)^(3/2)*InverseJacobiA
M(1/2*arcsin(a^(1/2)/(b*x+a)^(1/2)),2^(1/2))/b^4/(b*x^2+a*x)^(3/4)
```

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 10.01 (sec) , antiderivative size = 47, normalized size of antiderivative = 0.34

$$\int \frac{x^3}{(ax+bx^2)^{3/4}}\,dx = \frac{4x^4\left(1+\frac{bx}{a}\right)^{3/4}\mathrm{Hypergeometric2F1}\left(\frac{3}{4},\frac{13}{4},\frac{17}{4},-\frac{bx}{a}\right)}{13(x(a+bx))^{3/4}}$$

```
Integrate[x^3/(a*x + b*x^2)^(3/4),x]
```

```
(4*x^4*(1 + (b*x)/a)^(3/4)*Hypergeometric2F1[3/4, 13/4, 17/4, -((b*x)/a)])
/(13*(x*(a + b*x))^(3/4))
```

Rubi [A] (warning: unable to verify)

Time = 0.49 (sec) , antiderivative size = 171, normalized size of antiderivative = 1.23, number of steps used = 10, number of rules used = 9, $\frac{\text{number of rules}}{\text{integrand size}} = 0.529$, Rules used = {1137, 60, 60, 60, 73, 768, 858, 807, 229}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^3}{(ax+bx^2)^{3/4}}\,dx$$

$$\Big\downarrow \; 1137$$

$$\frac{x^{3/4}(a+bx)^{3/4}\int \frac{x^{9/4}}{(a+bx)^{3/4}}dx}{(ax+bx^2)^{3/4}}$$

$$\Big\downarrow \; 60$$

$$\frac{x^{3/4}(a+bx)^{3/4}\left(\frac{2x^{9/4}\sqrt[4]{a+bx}}{5b}-\frac{9a\int \frac{x^{5/4}}{(a+bx)^{3/4}}dx}{10b}\right)}{(ax+bx^2)^{3/4}}$$

$$\Big\downarrow \; 60$$

$$\frac{x^{3/4}(a+bx)^{3/4}\left(\frac{2x^{9/4}\sqrt[4]{a+bx}}{5b}-\frac{9a\left(\frac{2x^{5/4}\sqrt[4]{a+bx}}{3b}-\frac{5a\int \frac{\sqrt[4]{x}}{(a+bx)^{3/4}}dx}{6b}\right)}{10b}\right)}{(ax+bx^2)^{3/4}}$$

$$\Big\downarrow \; 60$$

$$\frac{x^{3/4}(a+bx)^{3/4}\left(\frac{2x^{9/4}\sqrt[4]{a+bx}}{5b}-\frac{9a\left(\frac{2x^{5/4}\sqrt[4]{a+bx}}{3b}-\frac{5a\left(\frac{2\sqrt[4]{x}\sqrt[4]{a+bx}}{b}-\frac{a\int\frac{1}{x^{3/4}(a+bx)^{3/4}}dx}{2b}\right)}{6b}\right)}{10b}\right)}{(ax+bx^2)^{3/4}}$$

$$\Big\downarrow \ 73$$

$$\frac{x^{3/4}(a+bx)^{3/4}\left(\frac{2x^{9/4}\sqrt[4]{a+bx}}{5b}-\frac{9a\left(\frac{2x^{5/4}\sqrt[4]{a+bx}}{3b}-\frac{5a\left(\frac{2\sqrt[4]{x}\sqrt[4]{a+bx}}{b}-\frac{2a\int\frac{1}{(a+bx)^{3/4}}d\sqrt[4]{x}}{b}\right)}{6b}\right)}{10b}\right)}{(ax+bx^2)^{3/4}}$$

$$\Big\downarrow \ 768$$

$$\frac{x^{3/4}(a+bx)^{3/4}\left(\frac{2x^{9/4}\sqrt[4]{a+bx}}{5b}-\frac{9a\left(\frac{2x^{5/4}\sqrt[4]{a+bx}}{3b}-\frac{5a\left(\frac{2\sqrt[4]{x}\sqrt[4]{a+bx}}{b}-\frac{2ax^{3/4}\left(\frac{a}{bx}+1\right)^{3/4}\int\frac{1}{\left(\frac{a}{bx}+1\right)^{3/4}x^{3/4}}d\sqrt[4]{x}}{b(a+bx)^{3/4}}\right)}{6b}\right)}{10b}\right)}{(ax+bx^2)^{3/4}}$$

$$\Big\downarrow \ 858$$

$$\frac{x^{3/4}(a+bx)^{3/4}\left(\frac{2x^{9/4}\sqrt[4]{a+bx}}{5b}-\frac{9a\left(\frac{2x^{5/4}\sqrt[4]{a+bx}}{3b}-\frac{5a\left(\frac{2ax^{3/4}\left(\frac{a}{bx}+1\right)^{3/4}\int\frac{1}{\sqrt[4]{x}\left(\frac{ax}{b}+1\right)^{3/4}}d\frac{1}{\sqrt[4]{x}}}{b(a+bx)^{3/4}}+\frac{2\sqrt[4]{x}\sqrt[4]{a+bx}}{b}\right)}{6b}\right)}{10b}\right)}{(ax+bx^2)^{3/4}}$$

$$\Big\downarrow \text{807}$$

$$\frac{x^{3/4}(a+bx)^{3/4}\left(\frac{2x^{9/4}\sqrt[4]{a+bx}}{5b}-\frac{9a\left(\frac{2x^{5/4}\sqrt[4]{a+bx}}{3b}-\frac{5a\left(\frac{ax^{3/4}\left(\frac{a}{bx}+1\right)^{3/4}\int\frac{1}{\left(\frac{\sqrt{x}a}{b}+1\right)^{3/4}}d\sqrt{x}}{b(a+bx)^{3/4}}+\frac{2\sqrt[4]{x}\sqrt[4]{a+bx}}{b}\right)}{6b}\right)}{10b}\right)}{(ax+bx^2)^{3/4}}$$

$$\Big\downarrow \text{229}$$

$$\frac{x^{3/4}(a+bx)^{3/4}\left(\frac{2x^{9/4}\sqrt[4]{a+bx}}{5b}-\frac{9a\left(\frac{2x^{5/4}\sqrt[4]{a+bx}}{3b}-\frac{5a\left(\frac{2\sqrt{a}x^{3/4}\left(\frac{a}{bx}+1\right)^{3/4}\operatorname{EllipticF}\left(\frac{1}{2}\arctan\left(\frac{\sqrt{a}\sqrt{x}}{\sqrt{b}}\right),2\right)}{\sqrt{b}(a+bx)^{3/4}}+\frac{2\sqrt[4]{x}\sqrt[4]{a+bx}}{b}\right)}{6b}\right)}{10b}\right)}{(ax+bx^2)^{3/4}}$$

```
Int[x^3/(a*x + b*x^2)^(3/4),x]
```

```
(x^(3/4)*(a + b*x)^(3/4)*((2*x^(9/4)*(a + b*x)^(1/4))/(5*b) - (9*a*((2*x^(
5/4)*(a + b*x)^(1/4))/(3*b) - (5*a*((2*x^(1/4)*(a + b*x)^(1/4))/b + (2*Sqr
t[a]*(1 + a/(b*x))^(3/4)*x^(3/4)*EllipticF[ArcTan[(Sqrt[a]*Sqrt[x])/Sqrt[b
]]/2, 2])/(Sqrt[b]*(a + b*x)^(3/4))))/(6*b)))/(10*b)))/(a*x + b*x^2)^(3/4)
```

Defintions of rubi rules used

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^n/(b*(m + n + 1))), x] + Simp[n*((b*c - a*d)/(
b*(m + n + 1)))   Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
 c, d}, x] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ[m, 0] && ( !Integer
Q[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n + 2, 0] && IntLinear
Q[a, b, c, d, m, n, x]
```

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Simp[p/b   Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
 d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && Lt
Q[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntL
inearQ[a, b, c, d, m, n, x]
```

```
Int[((a_) + (b_.)*(x_)^2)^(-3/4), x_Symbol] :> Simp[(2/(a^(3/4)*Rt[b/a, 2])
)*EllipticF[(1/2)*ArcTan[Rt[b/a, 2]*x], 2], x] /; FreeQ[{a, b}, x] && GtQ[a
, 0] && PosQ[b/a]
```

```
Int[((a_) + (b_.)*(x_)^4)^(-3/4), x_Symbol] :> Simp[x^3*((1 + a/(b*x^4))^(3
/4)/(a + b*x^4)^(3/4))   Int[1/(x^3*(1 + a/(b*x^4))^(3/4)), x], x] /; FreeQ
[{a, b}, x]
```

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k = GCD[m
+ 1, n]}, Simp[1/k   Subst[Int[x^((m + 1)/k - 1)*(a + b*x^(n/k))^p, x], x,
x^k], x] /; k != 1] /; FreeQ[{a, b, p}, x] && IGtQ[n, 0] && IntegerQ[m]
```

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> -Subst[Int[(a +
b/x^n)^p/x^(m + 2), x], x, 1/x] /; FreeQ[{a, b, p}, x] && ILtQ[n, 0] && Int
egerQ[m]
```

```
Int[((e_.)*(x_))^(m_)*((b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(
e*x)^m*((b*x + c*x^2)^p/(x^(m + p)*(b + c*x)^p))   Int[x^(m + p)*(b + c*x)^
p, x], x] /; FreeQ[{b, c, e, m}, x]
```

Maple [F]

$$\int \frac{x^3}{(b\,x^2+ax)^{\frac{3}{4}}}dx$$

```
int(x^3/(b*x^2+a*x)^(3/4),x)
```

```
int(x^3/(b*x^2+a*x)^(3/4),x)
```

Fricas [F]

$$\int \frac{x^3}{(ax+bx^2)^{3/4}}\,dx = \int \frac{x^3}{(bx^2+ax)^{\frac{3}{4}}}\,dx$$

```
integrate(x^3/(b*x^2+a*x)^(3/4),x, algorithm="fricas")
```

```
integral((b*x^2 + a*x)^(1/4)*x^2/(b*x + a), x)
```

Sympy [F]

$$\int \frac{x^3}{(ax+bx^2)^{3/4}}\,dx = \int \frac{x^3}{(x(a+bx))^{\frac{3}{4}}}\,dx$$

```
integrate(x**3/(b*x**2+a*x)**(3/4),x)
```

```
Integral(x**3/(x*(a + b*x))**(3/4), x)
```

Maxima [F]

$$\int \frac{x^3}{(ax+bx^2)^{3/4}}\,dx = \int \frac{x^3}{(bx^2+ax)^{\frac{3}{4}}}\,dx$$

```
integrate(x^3/(b*x^2+a*x)^(3/4),x, algorithm="maxima")
```

```
integrate(x^3/(b*x^2 + a*x)^(3/4), x)
```

Giac [F]

$$\int \frac{x^3}{(ax+bx^2)^{3/4}}\,dx = \int \frac{x^3}{(bx^2+ax)^{\frac{3}{4}}}\,dx$$

```
integrate(x^3/(b*x^2+a*x)^(3/4),x, algorithm="giac")
```

```
integrate(x^3/(b*x^2 + a*x)^(3/4), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{x^3}{(ax+bx^2)^{3/4}}\,dx = \int \frac{x^3}{(b\,x^2+a\,x)^{3/4}}\,dx$$

```
int(x^3/(a*x + b*x^2)^(3/4),x)
```

```
int(x^3/(a*x + b*x^2)^(3/4), x)
```

Reduce [F]

$$\int \frac{x^3}{(ax+bx^2)^{3/4}}\,dx = \int \frac{x^{\frac{9}{4}}}{(bx+a)^{\frac{3}{4}}}dx$$

```
int(x^3/(b*x^2+a*x)^(3/4),x)
```

```
int(x**3/(x**(3/4)*(a + b*x)**(3/4)),x)
```

3.183 $\int \frac{x^2}{(ax+bx^2)^{3/4}}\,dx$

Optimal result

Integrand size = 17, antiderivative size = 113

$$\int \frac{x^2}{(ax+bx^2)^{3/4}}\,dx = -\frac{5a\sqrt[4]{ax+bx^2}}{3b^2}+\frac{2x\sqrt[4]{ax+bx^2}}{3b} - \frac{5a^{3/2}\left(\frac{bx}{a+bx}\right)^{3/4}(a+bx)^{3/2}\,\mathrm{EllipticF}\left(\frac{1}{2}\arcsin\left(\frac{\sqrt{a}}{\sqrt{a+bx}}\right),2\right)}{3b^3\,(ax+bx^2)^{3/4}}$$

```
-5/3*a*(b*x^2+a*x)^(1/4)/b^2+2/3*x*(b*x^2+a*x)^(1/4)/b-5/3*a^(3/2)*(b*x/(b
*x+a))^(3/4)*(b*x+a)^(3/2)*InverseJacobiAM(1/2*arcsin(a^(1/2)/(b*x+a)^(1/2
)),2^(1/2))/b^3/(b*x^2+a*x)^(3/4)
```

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 10.01 (sec) , antiderivative size = 47, normalized size of antiderivative = 0.42

$$\int \frac{x^2}{(ax+bx^2)^{3/4}}\,dx = \frac{4x^3\left(1+\frac{bx}{a}\right)^{3/4}\mathrm{Hypergeometric2F1}\left(\frac{3}{4},\frac{9}{4},\frac{13}{4},-\frac{bx}{a}\right)}{9(x(a+bx))^{3/4}}$$

```
Integrate[x^2/(a*x + b*x^2)^(3/4),x]
```

```
(4*x^3*(1 + (b*x)/a)^(3/4)*Hypergeometric2F1[3/4, 9/4, 13/4, -((b*x)/a)])/
(9*(x*(a + b*x))^(3/4))
```

Rubi [A] (warning: unable to verify)

Time = 0.44 (sec) , antiderivative size = 141, normalized size of antiderivative = 1.25, number of steps used = 9, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}} = 0.471$, Rules used = {1137, 60, 60, 73, 768, 858, 807, 229}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^2}{(ax+bx^2)^{3/4}}dx$$

$$\Big\downarrow \text{1137}$$

$$\frac{x^{3/4}(a+bx)^{3/4}\int \frac{x^{5/4}}{(a+bx)^{3/4}}dx}{(ax+bx^2)^{3/4}}$$

$$\Big\downarrow \text{60}$$

$$\frac{x^{3/4}(a+bx)^{3/4}\left(\frac{2x^{5/4}\sqrt[4]{a+bx}}{3b}-\frac{5a\int \frac{\sqrt[4]{x}}{(a+bx)^{3/4}}dx}{6b}\right)}{(ax+bx^2)^{3/4}}$$

$$\Big\downarrow \text{60}$$

$$\frac{x^{3/4}(a+bx)^{3/4}\left(\frac{2x^{5/4}\sqrt[4]{a+bx}}{3b}-\frac{5a\left(\frac{2\sqrt[4]{x}\sqrt[4]{a+bx}}{b}-\frac{a\int \frac{1}{x^{3/4}(a+bx)^{3/4}}dx}{2b}\right)}{6b}\right)}{(ax+bx^2)^{3/4}}$$

$$\Big\downarrow \text{73}$$

$$\frac{x^{3/4}(a+bx)^{3/4}\left(\frac{2x^{5/4}\sqrt[4]{a+bx}}{3b}-\frac{5a\left(\frac{2\sqrt[4]{x}\sqrt[4]{a+bx}}{b}-\frac{2a\int\frac{1}{(a+bx)^{3/4}}d\sqrt[4]{x}}{b}\right)}{6b}\right)}{(ax+bx^2)^{3/4}}$$

$$\Big\downarrow \text{768}$$

$$\frac{x^{3/4}(a+bx)^{3/4}\left(\frac{2x^{5/4}\sqrt[4]{a+bx}}{3b}-\frac{5a\left(\frac{2\sqrt[4]{x}\sqrt[4]{a+bx}}{b}-\frac{2ax^{3/4}\left(\frac{a}{bx}+1\right)^{3/4}\int\frac{1}{\left(\frac{a}{bx}+1\right)^{3/4}x^{3/4}}d\sqrt[4]{x}}{b(a+bx)^{3/4}}\right)}{6b}\right)}{(ax+bx^2)^{3/4}}$$

$$\Big\downarrow \text{858}$$

$$\frac{x^{3/4}(a+bx)^{3/4}\left(\frac{2x^{5/4}\sqrt[4]{a+bx}}{3b}-\frac{5a\left(\frac{2ax^{3/4}\left(\frac{a}{bx}+1\right)^{3/4}\int\frac{1}{\sqrt[4]{x}\left(\frac{ax}{b}+1\right)^{3/4}}d\frac{1}{\sqrt[4]{x}}}{b(a+bx)^{3/4}}+\frac{2\sqrt[4]{x}\sqrt[4]{a+bx}}{b}\right)}{6b}\right)}{(ax+bx^2)^{3/4}}$$

$$\Big\downarrow \text{807}$$

$$\frac{x^{3/4}(a+bx)^{3/4}\left(\frac{2x^{5/4}\sqrt[4]{a+bx}}{3b}-\frac{5a\left(\frac{ax^{3/4}\left(\frac{a}{bx}+1\right)^{3/4}\int\frac{1}{\left(\frac{\sqrt{x}a}{b}+1\right)^{3/4}}d\sqrt{x}}{b(a+bx)^{3/4}}+\frac{2\sqrt[4]{x}\sqrt[4]{a+bx}}{b}\right)}{6b}\right)}{(ax+bx^2)^{3/4}}$$

$$\Big\downarrow \text{229}$$

$$\frac{x^{3/4}(a+bx)^{3/4}\left(\frac{2x^{5/4}\sqrt[4]{a+bx}}{3b}-\frac{5a\left(\frac{2\sqrt{a}x^{3/4}\left(\frac{a}{bx}+1\right)^{3/4}\operatorname{EllipticF}\left(\frac{1}{2}\arctan\left(\frac{\sqrt{a}\sqrt{x}}{\sqrt{b}}\right),2\right)}{\sqrt{b}(a+bx)^{3/4}}+\frac{2\sqrt[4]{x}\sqrt[4]{a+bx}}{b}\right)}{6b}\right)}{(ax+bx^2)^{3/4}}$$

```
Int[x^2/(a*x + b*x^2)^(3/4),x]
```

```
(x^(3/4)*(a + b*x)^(3/4)*((2*x^(5/4)*(a + b*x)^(1/4))/(3*b) - (5*a*((2*x^(
1/4)*(a + b*x)^(1/4))/b + (2*Sqrt[a]*(1 + a/(b*x))^(3/4)*x^(3/4)*EllipticF
[ArcTan[(Sqrt[a]*Sqrt[x])/Sqrt[b]]/2, 2])/(Sqrt[b]*(a + b*x)^(3/4))))/(6*b
)))/(a*x + b*x^2)^(3/4)
```

Defintions of rubi rules used

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^n/(b*(m + n + 1))), x] + Simp[n*((b*c - a*d)/(
b*(m + n + 1)))   Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
 c, d}, x] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ[m, 0] && ( !Integer
Q[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n + 2, 0] && IntLinear
Q[a, b, c, d, m, n, x]
```

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Simp[p/b   Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
 d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && Lt
Q[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntL
inearQ[a, b, c, d, m, n, x]
```

```
Int[((a_) + (b_.)*(x_)^2)^(-3/4), x_Symbol] :> Simp[(2/(a^(3/4)*Rt[b/a, 2])
)*EllipticF[(1/2)*ArcTan[Rt[b/a, 2]*x], 2], x] /; FreeQ[{a, b}, x] && GtQ[a
, 0] && PosQ[b/a]
```

```
Int[((a_) + (b_.)*(x_)^4)^(-3/4), x_Symbol] :> Simp[x^3*((1 + a/(b*x^4))^(3
/4)/(a + b*x^4)^(3/4))   Int[1/(x^3*(1 + a/(b*x^4))^(3/4)), x], x] /; FreeQ
[{a, b}, x]
```

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k = GCD[m
+ 1, n]}, Simp[1/k   Subst[Int[x^((m + 1)/k - 1)*(a + b*x^(n/k))^p, x], x,
x^k], x] /; k != 1] /; FreeQ[{a, b, p}, x] && IGtQ[n, 0] && IntegerQ[m]
```

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> -Subst[Int[(a +
b/x^n)^p/x^(m + 2), x], x, 1/x] /; FreeQ[{a, b, p}, x] && ILtQ[n, 0] && Int
egerQ[m]
```

```
Int[((e_.)*(x_))^(m_)*((b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(
e*x)^m*((b*x + c*x^2)^p/(x^(m + p)*(b + c*x)^p))   Int[x^(m + p)*(b + c*x)^
p, x], x] /; FreeQ[{b, c, e, m}, x]
```

Maple [F]

$$\int \frac{x^2}{\left(b\,x^2+ax\right)^{\frac{3}{4}}}dx$$

```
int(x^2/(b*x^2+a*x)^(3/4),x)
```

```
int(x^2/(b*x^2+a*x)^(3/4),x)
```

Fricas [F]

$$\int \frac{x^2}{\left(ax+bx^2\right)^{3/4}}\,dx=\int \frac{x^2}{\left(bx^2+ax\right)^{\frac{3}{4}}}\,dx$$

```
integrate(x^2/(b*x^2+a*x)^(3/4),x, algorithm="fricas")
```

```
integral((b*x^2 + a*x)^(1/4)*x/(b*x + a), x)
```

Sympy [F]

$$\int \frac{x^2}{(ax+bx^2)^{3/4}}\,dx = \int \frac{x^2}{(x\,(a+bx))^{\frac{3}{4}}}\,dx$$

```
integrate(x**2/(b*x**2+a*x)**(3/4),x)
```

```
Integral(x**2/(x*(a + b*x))**(3/4), x)
```

Maxima [F]

$$\int \frac{x^2}{(ax+bx^2)^{3/4}}\,dx = \int \frac{x^2}{(bx^2+ax)^{\frac{3}{4}}}\,dx$$

```
integrate(x^2/(b*x^2+a*x)^(3/4),x, algorithm="maxima")
```

```
integrate(x^2/(b*x^2 + a*x)^(3/4), x)
```

Giac [F]

$$\int \frac{x^2}{(ax+bx^2)^{3/4}}\,dx = \int \frac{x^2}{(bx^2+ax)^{\frac{3}{4}}}\,dx$$

```
integrate(x^2/(b*x^2+a*x)^(3/4),x, algorithm="giac")
```

```
integrate(x^2/(b*x^2 + a*x)^(3/4), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{x^2}{(ax+bx^2)^{3/4}}\,dx = \int \frac{x^2}{(b\,x^2+a\,x)^{3/4}}\,dx$$

```
int(x^2/(a*x + b*x^2)^(3/4),x)
```

```
int(x^2/(a*x + b*x^2)^(3/4), x)
```

Reduce [F]

$$\int \frac{x^2}{(ax+bx^2)^{3/4}}\,dx = \int \frac{x^{\frac{5}{4}}}{(bx+a)^{\frac{3}{4}}}dx$$

```
int(x^2/(b*x^2+a*x)^(3/4),x)
```

```
int(x**2/(x**(3/4)*(a + b*x)**(3/4)),x)
```

3.184 $\int \frac{x}{(ax+bx^2)^{3/4}} dx$

Optimal result

Integrand size = 15, antiderivative size = 87

$$\int \frac{x}{(ax+bx^2)^{3/4}} dx = \frac{2\sqrt[4]{ax+bx^2}}{b} + \frac{2\sqrt{a}\left(\frac{bx}{a+bx}\right)^{3/4}(a+bx)^{3/2}\,\text{EllipticF}\left(\frac{1}{2}\arcsin\left(\frac{\sqrt{a}}{\sqrt{a+bx}}\right),2\right)}{b^2(ax+bx^2)^{3/4}}$$

```
2*(b*x^2+a*x)^(1/4)/b+2*a^(1/2)*(b*x/(b*x+a))^(3/4)*(b*x+a)^(3/2)*InverseJ
acobiAM(1/2*arcsin(a^(1/2)/(b*x+a)^(1/2)),2^(1/2))/b^2/(b*x^2+a*x)^(3/4)
```

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 10.02 (sec) , antiderivative size = 47, normalized size of antiderivative = 0.54

$$\int \frac{x}{(ax+bx^2)^{3/4}} dx = \frac{4x^2\left(1+\frac{bx}{a}\right)^{3/4}\text{Hypergeometric2F1}\left(\frac{3}{4},\frac{5}{4},\frac{9}{4},-\frac{bx}{a}\right)}{5(x(a+bx))^{3/4}}$$

```
Integrate[x/(a*x + b*x^2)^(3/4),x]
```

```
(4*x^2*(1 + (b*x)/a)^(3/4)*Hypergeometric2F1[3/4, 5/4, 9/4, -((b*x)/a)])/(
5*(x*(a + b*x))^(3/4))
```

Rubi [A] (verified)

Time = 0.38 (sec) , antiderivative size = 91, normalized size of antiderivative = 1.05, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.267, Rules used = {1160, 1093, 1090, 230}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x}{(ax+bx^2)^{3/4}}dx$$

$$\Big\downarrow \text{1160}$$

$$\frac{2\sqrt[4]{ax+bx^2}}{b}-\frac{a\int \frac{1}{(bx^2+ax)^{3/4}}dx}{2b}$$

$$\Big\downarrow \text{1093}$$

$$\frac{2\sqrt[4]{ax+bx^2}}{b}-\frac{a\left(-\frac{b(ax+bx^2)}{a^2}\right)^{3/4}\int \frac{1}{\left(-\frac{b^2x^2}{a^2}-\frac{bx}{a}\right)^{3/4}}dx}{2b\left(ax+bx^2\right)^{3/4}}$$

$$\Big\downarrow \text{1090}$$

$$\frac{a^3\left(-\frac{b(ax+bx^2)}{a^2}\right)^{3/4}\int \frac{1}{\left(1-\frac{a^2\left(-\frac{2xb^2}{a^2}-\frac{b}{a}\right)^2}{b^2}\right)^{3/4}}d\left(-\frac{2xb^2}{a^2}-\frac{b}{a}\right)}{\sqrt{2}b^3\left(ax+bx^2\right)^{3/4}}+\frac{2\sqrt[4]{ax+bx^2}}{b}$$

$$\Big\downarrow \text{230}$$

$$\frac{\sqrt{2}a^2\left(-\frac{b(ax+bx^2)}{a^2}\right)^{3/4}\text{EllipticF}\left(\frac{1}{2}\arcsin\left(\frac{a\left(-\frac{2xb^2}{a^2}-\frac{b}{a}\right)}{b}\right),2\right)}{b^2\left(ax+bx^2\right)^{3/4}}+\frac{2\sqrt[4]{ax+bx^2}}{b}$$

```
Int[x/(a*x + b*x^2)^(3/4),x]
```

```
(2*(a*x + b*x^2)^(1/4))/b + (Sqrt[2]*a^2*(-((b*(a*x + b*x^2))/a^2))^(3/4)*
EllipticF[ArcSin[(a*(-(b/a) - (2*b^2*x)/a^2))/b]/2, 2])/(b^2*(a*x + b*x^2)
^(3/4))
```

Defintions of rubi rules used

```
Int[((a_) + (b_.)*(x_)^2)^(-3/4), x_Symbol] :> Simp[(2/(a^(3/4)*Rt[-b/a, 2]
))*EllipticF[(1/2)*ArcSin[Rt[-b/a, 2]*x], 2], x] /; FreeQ[{a, b}, x] && GtQ
[a, 0] && NegQ[b/a]
```

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[1/(2*c*(-4*
(c/(b^2 - 4*a*c)))^p)   Subst[Int[Simp[1 - x^2/(b^2 - 4*a*c), x]^p, x], x,
b + 2*c*x], x] /; FreeQ[{a, b, c, p}, x] && GtQ[4*a - b^2/c, 0]
```

```
Int[((b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(b*x + c*x^2)^p/((-
c)*((b*x + c*x^2)/b^2))^p   Int[((-c)*(x/b) - c^2*(x^2/b^2))^p, x], x] /; F
reeQ[{b, c}, x] && (IntegerQ[4*p] || IntegerQ[3*p])
```

```
Int[((d_.) + (e_.)*(x_))*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol
] :> Simp[e*((a + b*x + c*x^2)^(p + 1)/(2*c*(p + 1))), x] + Simp[(2*c*d - b
*e)/(2*c)   Int[(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, p}, x]
 && NeQ[p, -1]
```

Maple [F]

$$\int \frac{x}{(b\,x^2+ax)^{\frac{3}{4}}}dx$$

```
int(x/(b*x^2+a*x)^(3/4),x)
```

```
int(x/(b*x^2+a*x)^(3/4),x)
```

Fricas [F]

$$\int \frac{x}{(ax+bx^2)^{3/4}}\,dx = \int \frac{x}{(bx^2+ax)^{\frac{3}{4}}}\,dx$$

```
integrate(x/(b*x^2+a*x)^(3/4),x, algorithm="fricas")
```

```
integral((b*x^2 + a*x)^(1/4)/(b*x + a), x)
```

Sympy [F]

$$\int \frac{x}{(ax+bx^2)^{3/4}}\,dx = \int \frac{x}{(x\,(a+bx))^{\frac{3}{4}}}\,dx$$

```
integrate(x/(b*x**2+a*x)**(3/4),x)
```

```
Integral(x/(x*(a + b*x))**(3/4), x)
```

Maxima [F]

$$\int \frac{x}{(ax+bx^2)^{3/4}}\,dx = \int \frac{x}{(bx^2+ax)^{\frac{3}{4}}}\,dx$$

```
integrate(x/(b*x^2+a*x)^(3/4),x, algorithm="maxima")
```

```
integrate(x/(b*x^2 + a*x)^(3/4), x)
```

Giac [F]

$$\int \frac{x}{(ax+bx^2)^{3/4}}\,dx = \int \frac{x}{(bx^2+ax)^{\frac{3}{4}}}\,dx$$

```
integrate(x/(b*x^2+a*x)^(3/4),x, algorithm="giac")
```

```
integrate(x/(b*x^2 + a*x)^(3/4), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{x}{(ax+bx^2)^{3/4}}\,dx = \int \frac{x}{(b\,x^2+a\,x)^{3/4}}\,dx$$

```
int(x/(a*x + b*x^2)^(3/4),x)
```

```
int(x/(a*x + b*x^2)^(3/4), x)
```

Reduce [F]

$$\int \frac{x}{(ax+bx^2)^{3/4}}\,dx = \int \frac{x^{\frac{1}{4}}}{(bx+a)^{\frac{3}{4}}}dx$$

```
int(x/(b*x^2+a*x)^(3/4),x)
```

```
int(x/(x**(3/4)*(a + b*x)**(3/4)),x)
```

3.185 $\int \frac{1}{(ax+bx^2)^{3/4}}\,dx$

Optimal result

Integrand size = 13, antiderivative size = 68

$$\int \frac{1}{(ax+bx^2)^{3/4}}\,dx = -\frac{4\left(\frac{bx}{a+bx}\right)^{3/4}(a+bx)^{3/2}\operatorname{EllipticF}\left(\frac{1}{2}\arcsin\left(\frac{\sqrt{a}}{\sqrt{a+bx}}\right),2\right)}{\sqrt{a}b\,(ax+bx^2)^{3/4}}$$

```
-4*(b*x/(b*x+a))^(3/4)*(b*x+a)^(3/2)*InverseJacobiAM(1/2*arcsin(a^(1/2)/(b
*x+a)^(1/2)),2^(1/2))/a^(1/2)/b/(b*x^2+a*x)^(3/4)
```

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 10.01 (sec) , antiderivative size = 43, normalized size of antiderivative = 0.63

$$\int \frac{1}{(ax+bx^2)^{3/4}}\,dx = \frac{4x\left(1+\frac{bx}{a}\right)^{3/4}\operatorname{Hypergeometric2F1}\left(\frac{1}{4},\frac{3}{4},\frac{5}{4},-\frac{bx}{a}\right)}{(x(a+bx))^{3/4}}$$

```
Integrate[(a*x + b*x^2)^(-3/4),x]
```

```
(4*x*(1 + (b*x)/a)^(3/4)*Hypergeometric2F1[1/4, 3/4, 5/4, -((b*x)/a)])/(x*
(a + b*x))^(3/4)
```

Rubi [A] (verified)

Time = 0.33 (sec) , antiderivative size = 71, normalized size of antiderivative = 1.04, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.231, Rules used = {1093, 1090, 230}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{(ax+bx^2)^{3/4}}dx$$

$$\Big\downarrow \text{1093}$$

$$\frac{\left(-\frac{b(ax+bx^2)}{a^2}\right)^{3/4}\int \frac{1}{\left(-\frac{b^2x^2}{a^2}-\frac{bx}{a}\right)^{3/4}}dx}{(ax+bx^2)^{3/4}}$$

$$\Big\downarrow \text{1090}$$

$$-\frac{\sqrt{2}a^2\left(-\frac{b(ax+bx^2)}{a^2}\right)^{3/4}\int \frac{1}{\left(1-\frac{a^2\left(-\frac{2xb^2}{a^2}-\frac{b}{a}\right)^2}{b^2}\right)^{3/4}}d\left(-\frac{2xb^2}{a^2}-\frac{b}{a}\right)}{b^2\left(ax+bx^2\right)^{3/4}}$$

$$\Big\downarrow \text{230}$$

$$-\frac{2\sqrt{2}a\left(-\frac{b(ax+bx^2)}{a^2}\right)^{3/4}\text{EllipticF}\left(\frac{1}{2}\arcsin\left(\frac{a\left(-\frac{2xb^2}{a^2}-\frac{b}{a}\right)}{b}\right),2\right)}{b\left(ax+bx^2\right)^{3/4}}$$

```
Int[(a*x + b*x^2)^(-3/4),x]
```

```
(-2*Sqrt[2]*a*(-((b*(a*x + b*x^2))/a^2))^(3/4)*EllipticF[ArcSin[(a*(-(b/a)
 - (2*b^2*x)/a^2))/b]/2, 2])/(b*(a*x + b*x^2)^(3/4))
```

Defintions of rubi rules used

```
Int[((a_) + (b_.)*(x_)^2)^(-3/4), x_Symbol] :> Simp[(2/(a^(3/4)*Rt[-b/a, 2]
))*EllipticF[(1/2)*ArcSin[Rt[-b/a, 2]*x], 2], x] /; FreeQ[{a, b}, x] && GtQ
[a, 0] && NegQ[b/a]
```

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[1/(2*c*(-4*
(c/(b^2 - 4*a*c)))^p)   Subst[Int[Simp[1 - x^2/(b^2 - 4*a*c), x]^p, x], x,
b + 2*c*x], x] /; FreeQ[{a, b, c, p}, x] && GtQ[4*a - b^2/c, 0]
```

```
Int[((b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(b*x + c*x^2)^p/((-
c)*((b*x + c*x^2)/b^2))^p   Int[((-c)*(x/b) - c^2*(x^2/b^2))^p, x], x] /; F
reeQ[{b, c}, x] && (IntegerQ[4*p] || IntegerQ[3*p])
```

Maple [F]

$$\int \frac{1}{\left(b\,x^{2}+a x \right)^{\frac{3}{4}}}d x$$

```
int(1/(b*x^2+a*x)^(3/4),x)
```

```
int(1/(b*x^2+a*x)^(3/4),x)
```

Fricas [F]

$$\int \frac{1}{(ax+bx^2)^{3/4}}\,dx = \int \frac{1}{\left(bx^{2}+ax\right)^{\frac{3}{4}}}\,dx$$

```
integrate(1/(b*x^2+a*x)^(3/4),x, algorithm="fricas")
```

```
integral((b*x^2 + a*x)^(-3/4), x)
```

Sympy [F]

$$\int \frac{1}{(ax+bx^2)^{3/4}}\,dx = \int \frac{1}{(ax+bx^2)^{\frac{3}{4}}}\,dx$$

```
integrate(1/(b*x**2+a*x)**(3/4),x)
```

```
Integral((a*x + b*x**2)**(-3/4), x)
```

Maxima [F]

$$\int \frac{1}{(ax+bx^2)^{3/4}}\,dx = \int \frac{1}{(bx^2+ax)^{\frac{3}{4}}}\,dx$$

```
integrate(1/(b*x^2+a*x)^(3/4),x, algorithm="maxima")
```

```
integrate((b*x^2 + a*x)^(-3/4), x)
```

Giac [F]

$$\int \frac{1}{(ax+bx^2)^{3/4}}\,dx = \int \frac{1}{(bx^2+ax)^{\frac{3}{4}}}\,dx$$

```
integrate(1/(b*x^2+a*x)^(3/4),x, algorithm="giac")
```

```
integrate((b*x^2 + a*x)^(-3/4), x)
```

Mupad [B] (verification not implemented)

Time = 8.73 (sec) , antiderivative size = 36, normalized size of antiderivative = 0.53

$$\int \frac{1}{(ax+bx^2)^{3/4}}\,dx = \frac{4\,x\,\left(\frac{b\,x}{a}+1\right)^{3/4}{}_2\mathrm{F}_1\left(\frac{1}{4},\frac{3}{4};\ \frac{5}{4};\ -\frac{b\,x}{a}\right)}{\left(b\,x^2+a\,x\right)^{3/4}}$$

```
int(1/(a*x + b*x^2)^(3/4),x)
```

```
(4*x*((b*x)/a + 1)^(3/4)*hypergeom([1/4, 3/4], 5/4, -(b*x)/a))/(a*x + b*x^
2)^(3/4)
```

Reduce [F]

$$\int \frac{1}{(ax+bx^2)^{3/4}}\,dx = \int \frac{1}{x^{\frac{3}{4}}\,(bx+a)^{\frac{3}{4}}}dx$$

```
int(1/(b*x^2+a*x)^(3/4),x)
```

```
int(1/(x**(3/4)*(a + b*x)**(3/4)),x)
```

3.186 $\int \frac{1}{x(ax+bx^2)^{3/4}} \, dx$

Optimal result

Integrand size = 17, antiderivative size = 91

$$\int \frac{1}{x(ax+bx^2)^{3/4}} \, dx = -\frac{4\sqrt[4]{ax+bx^2}}{3ax} + \frac{8\left(\frac{bx}{a+bx}\right)^{3/4}(a+bx)^{3/2}\operatorname{EllipticF}\left(\frac{1}{2}\arcsin\left(\frac{\sqrt{a}}{\sqrt{a+bx}}\right),2\right)}{3a^{3/2}(ax+bx^2)^{3/4}}$$

```
-4/3*(b*x^2+a*x)^(1/4)/a/x+8/3*(b*x/(b*x+a))^(3/4)*(b*x+a)^(3/2)*InverseJa
cobiAM(1/2*arcsin(a^(1/2)/(b*x+a)^(1/2)),2^(1/2))/a^(3/2)/(b*x^2+a*x)^(3/4
)
```

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 10.01 (sec) , antiderivative size = 44, normalized size of antiderivative = 0.48

$$\int \frac{1}{x(ax+bx^2)^{3/4}} \, dx = -\frac{4\left(1+\frac{bx}{a}\right)^{3/4}\text{Hypergeometric2F1}\left(-\frac{3}{4},\frac{3}{4},\frac{1}{4},-\frac{bx}{a}\right)}{3(x(a+bx))^{3/4}}$$

```
Integrate[1/(x*(a*x + b*x^2)^(3/4)),x]
```

```
(-4*(1 + (b*x)/a)^(3/4)*Hypergeometric2F1[-3/4, 3/4, 1/4, -((b*x)/a)])/(3*
(x*(a + b*x))^(3/4))
```

Rubi [A] (warning: unable to verify)

Time = 0.42 (sec) , antiderivative size = 115, normalized size of antiderivative = 1.26, number of steps used = 8, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}} = 0.412$, Rules used = {1137, 61, 73, 768, 858, 807, 229}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{x\left(ax+bx^2\right)^{3/4}}dx$$

$$\Big\downarrow \text{ 1137}$$

$$\frac{x^{3/4}(a+bx)^{3/4}\int \frac{1}{x^{7/4}(a+bx)^{3/4}}dx}{\left(ax+bx^2\right)^{3/4}}$$

$$\Big\downarrow \text{ 61}$$

$$\frac{x^{3/4}(a+bx)^{3/4}\left(-\frac{2b\int \frac{1}{x^{3/4}(a+bx)^{3/4}}dx}{3a}-\frac{4\sqrt[4]{a+bx}}{3ax^{3/4}}\right)}{\left(ax+bx^2\right)^{3/4}}$$

$$\Big\downarrow \text{ 73}$$

$$\frac{x^{3/4}(a+bx)^{3/4}\left(-\frac{8b\int \frac{1}{(a+bx)^{3/4}}d\sqrt[4]{x}}{3a}-\frac{4\sqrt[4]{a+bx}}{3ax^{3/4}}\right)}{\left(ax+bx^2\right)^{3/4}}$$

$$\Big\downarrow \text{ 768}$$

$$\frac{x^{3/4}(a+bx)^{3/4}\left(-\frac{8bx^{3/4}\left(\frac{a}{bx}+1\right)^{3/4}\int \frac{1}{\left(\frac{a}{bx}+1\right)^{3/4}x^{3/4}}d\sqrt[4]{x}}{3a(a+bx)^{3/4}}-\frac{4\sqrt[4]{a+bx}}{3ax^{3/4}}\right)}{\left(ax+bx^2\right)^{3/4}}$$

$$\Big\downarrow \text{ 858}$$

$$\frac{x^{3/4}(a+bx)^{3/4}\left(\frac{8bx^{3/4}\left(\frac{a}{bx}+1\right)^{3/4}\int \frac{1}{\sqrt[4]{x}\left(\frac{ax}{b}+1\right)^{3/4}}d\frac{1}{\sqrt[4]{x}}}{3a(a+bx)^{3/4}}-\frac{4\sqrt[4]{a+bx}}{3ax^{3/4}}\right)}{(ax+bx^2)^{3/4}}$$

$$\Big\downarrow \text{807}$$

$$\frac{x^{3/4}(a+bx)^{3/4}\left(\frac{4bx^{3/4}\left(\frac{a}{bx}+1\right)^{3/4}\int \frac{1}{\left(\frac{\sqrt{x}a}{b}+1\right)^{3/4}}d\sqrt{x}}{3a(a+bx)^{3/4}}-\frac{4\sqrt[4]{a+bx}}{3ax^{3/4}}\right)}{(ax+bx^2)^{3/4}}$$

$$\Big\downarrow \text{229}$$

$$\frac{x^{3/4}(a+bx)^{3/4}\left(\frac{8b^{3/2}x^{3/4}\left(\frac{a}{bx}+1\right)^{3/4}\text{EllipticF}\left(\frac{1}{2}\arctan\left(\frac{\sqrt{a}\sqrt{x}}{\sqrt{b}}\right),2\right)}{3a^{3/2}(a+bx)^{3/4}}-\frac{4\sqrt[4]{a+bx}}{3ax^{3/4}}\right)}{(ax+bx^2)^{3/4}}$$

```
Int[1/(x*(a*x + b*x^2)^(3/4)),x]
```

```
(x^(3/4)*(a + b*x)^(3/4)*((-4*(a + b*x)^(1/4))/(3*a*x^(3/4)) + (8*b^(3/2)*
(1 + a/(b*x))^(3/4)*x^(3/4)*EllipticF[ArcTan[(Sqrt[a]*Sqrt[x])/Sqrt[b]]/2,
 2])/(3*a^(3/2)*(a + b*x)^(3/4))))/(a*x + b*x^2)^(3/4)
```

Defintions of rubi rules used

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] - Simp[d*((
m + n + 2)/((b*c - a*d)*(m + 1)))   Int[(a + b*x)^(m + 1)*(c + d*x)^n, x],
x] /; FreeQ[{a, b, c, d, n}, x] && LtQ[m, -1] &&  !(LtQ[n, -1] && (EqQ[a, 0
] || (NeQ[c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && IntLinearQ[a, b, c, d
, m, n, x]
```

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Simp[p/b   Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
 d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && Lt
Q[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntL
inearQ[a, b, c, d, m, n, x]
```

```
Int[((a_) + (b_.)*(x_)^2)^(-3/4), x_Symbol] :> Simp[(2/(a^(3/4)*Rt[b/a, 2])
)*EllipticF[(1/2)*ArcTan[Rt[b/a, 2]*x], 2], x] /; FreeQ[{a, b}, x] && GtQ[a
, 0] && PosQ[b/a]
```

```
Int[((a_) + (b_.)*(x_)^4)^(-3/4), x_Symbol] :> Simp[x^3*((1 + a/(b*x^4))^(3
/4)/(a + b*x^4)^(3/4))   Int[1/(x^3*(1 + a/(b*x^4))^(3/4)), x], x] /; FreeQ
[{a, b}, x]
```

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k = GCD[m
+ 1, n]}, Simp[1/k   Subst[Int[x^((m + 1)/k - 1)*(a + b*x^(n/k))^p, x], x,
x^k], x] /; k != 1] /; FreeQ[{a, b, p}, x] && IGtQ[n, 0] && IntegerQ[m]
```

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> -Subst[Int[(a +
b/x^n)^p/x^(m + 2), x], x, 1/x] /; FreeQ[{a, b, p}, x] && ILtQ[n, 0] && Int
egerQ[m]
```

```
Int[((e_.)*(x_))^(m_)*((b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(
e*x)^m*((b*x + c*x^2)^p/(x^(m + p)*(b + c*x)^p))   Int[x^(m + p)*(b + c*x)^
p, x], x] /; FreeQ[{b, c, e, m}, x]
```

Maple [F]

$$\int \frac{1}{x\left(b\,x^{2}+a x\right)^{\frac{3}{4}}}d x$$

```
int(1/x/(b*x^2+a*x)^(3/4),x)
```

```
int(1/x/(b*x^2+a*x)^(3/4),x)
```

Fricas [F]

$$\int \frac{1}{x\left(ax+bx^2\right)^{3/4}}\,dx=\int \frac{1}{\left(bx^2+ax\right)^{\frac{3}{4}}x}\,dx$$

```
integrate(1/x/(b*x^2+a*x)^(3/4),x, algorithm="fricas")
```

```
integral((b*x^2 + a*x)^(1/4)/(b*x^3 + a*x^2), x)
```

Sympy [F]

$$\int \frac{1}{x\left(ax+bx^2\right)^{3/4}}\,dx=\int \frac{1}{x\left(x\left(a+bx\right)\right)^{\frac{3}{4}}}\,dx$$

```
integrate(1/x/(b*x**2+a*x)**(3/4),x)
```

```
Integral(1/(x*(x*(a + b*x))**(3/4)), x)
```

Maxima [F]

$$\int \frac{1}{x\left(ax+bx^2\right)^{3/4}}\,dx=\int \frac{1}{\left(bx^2+ax\right)^{\frac{3}{4}}x}\,dx$$

```
integrate(1/x/(b*x^2+a*x)^(3/4),x, algorithm="maxima")
```

```
integrate(1/((b*x^2 + a*x)^(3/4)*x), x)
```

Giac [F]

$$\int \frac{1}{x\left(ax+bx^2\right)^{3/4}}\,dx=\int \frac{1}{\left(bx^2+ax\right)^{\frac{3}{4}}x}\,dx$$

```
integrate(1/x/(b*x^2+a*x)^(3/4),x, algorithm="giac")
```

```
integrate(1/((b*x^2 + a*x)^(3/4)*x), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{x\left(ax+bx^2\right)^{3/4}}\,dx=\int \frac{1}{x\left(b\,x^2+a\,x\right)^{3/4}}\,dx$$

```
int(1/(x*(a*x + b*x^2)^(3/4)),x)
```

```
int(1/(x*(a*x + b*x^2)^(3/4)), x)
```

Reduce [B] (verification not implemented)

Time = 0.54 (sec) , antiderivative size = 19, normalized size of antiderivative = 0.21

$$\int \frac{1}{x\left(ax+bx^2\right)^{3/4}}\,dx=-\frac{4(bx+a)^{\frac{1}{4}}}{x^{\frac{1}{4}}\sqrt{x}\,a}$$

```
int(1/x/(b*x^2+a*x)^(3/4),x)
```

```
( - 4*x**(3/4)*(a + b*x)**(1/4))/(sqrt(x)*a*x)
```

3.187 $\int \frac{1}{x^2(ax+bx^2)^{3/4}}\,dx$

Optimal result

Integrand size = 17, antiderivative size = 116

$$\int \frac{1}{x^2(ax+bx^2)^{3/4}}\,dx = -\frac{4\sqrt[4]{ax+bx^2}}{7ax^2}+\frac{8b\sqrt[4]{ax+bx^2}}{7a^2x} - \frac{16b\left(\frac{bx}{a+bx}\right)^{3/4}(a+bx)^{3/2}\operatorname{EllipticF}\left(\frac{1}{2}\arcsin\left(\frac{\sqrt{a}}{\sqrt{a+bx}}\right),2\right)}{7a^{5/2}(ax+bx^2)^{3/4}}$$

```
-4/7*(b*x^2+a*x)^(1/4)/a/x^2+8/7*b*(b*x^2+a*x)^(1/4)/a^2/x-16/7*b*(b*x/(b*
x+a))^(3/4)*(b*x+a)^(3/2)*InverseJacobiAM(1/2*arcsin(a^(1/2)/(b*x+a)^(1/2)
),2^(1/2))/a^(5/2)/(b*x^2+a*x)^(3/4)
```

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 10.01 (sec) , antiderivative size = 47, normalized size of antiderivative = 0.41

$$\int \frac{1}{x^2(ax+bx^2)^{3/4}}\,dx = -\frac{4\left(1+\frac{bx}{a}\right)^{3/4}\text{Hypergeometric2F1}\left(-\frac{7}{4},\frac{3}{4},-\frac{3}{4},-\frac{bx}{a}\right)}{7x(x(a+bx))^{3/4}}$$

```
Integrate[1/(x^2*(a*x + b*x^2)^(3/4)),x]
```

```
(-4*(1 + (b*x)/a)^(3/4)*Hypergeometric2F1[-7/4, 3/4, -3/4, -((b*x)/a)])/(7
*x*(x*(a + b*x))^(3/4))
```

Rubi [A] (warning: unable to verify)

Time = 0.47 (sec) , antiderivative size = 145, normalized size of antiderivative = 1.25, number of steps used = 9, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}} = 0.471$, Rules used = {1137, 61, 61, 73, 768, 858, 807, 229}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{x^2 (ax+bx^2)^{3/4}} \, dx$$

$$\Big\downarrow \ \color{blue}{1137}$$

$$\frac{x^{3/4}(a+bx)^{3/4} \int \frac{1}{x^{11/4}(a+bx)^{3/4}}dx}{(ax+bx^2)^{3/4}}$$

$$\Big\downarrow \ \color{blue}{61}$$

$$\frac{x^{3/4}(a+bx)^{3/4} \left(-\frac{6b\int \frac{1}{x^{7/4}(a+bx)^{3/4}}dx}{7a}-\frac{4\sqrt[4]{a+bx}}{7ax^{7/4}}\right)}{(ax+bx^2)^{3/4}}$$

$$\Big\downarrow \ \color{blue}{61}$$

$$\frac{x^{3/4}(a+bx)^{3/4} \left(-\frac{6b\left(-\frac{2b\int \frac{1}{x^{3/4}(a+bx)^{3/4}}dx}{3a}-\frac{4\sqrt[4]{a+bx}}{3ax^{3/4}}\right)}{7a}-\frac{4\sqrt[4]{a+bx}}{7ax^{7/4}}\right)}{(ax+bx^2)^{3/4}}$$

$$\Big\downarrow \ \color{blue}{73}$$

$$\frac{x^{3/4}(a+bx)^{3/4} \left(-\frac{6b\left(-\frac{8b\int \frac{1}{(a+bx)^{3/4}}d\sqrt[4]{x}}{3a}-\frac{4\sqrt[4]{a+bx}}{3ax^{3/4}}\right)}{7a}-\frac{4\sqrt[4]{a+bx}}{7ax^{7/4}}\right)}{(ax+bx^2)^{3/4}}$$

$$\Big\downarrow \text{768}$$

$$\frac{x^{3/4}(a+bx)^{3/4}\left(-\frac{6b\left(-\frac{8bx^{3/4}\left(\frac{a}{bx}+1\right)^{3/4}\int \frac{1}{\left(\frac{a}{bx}+1\right)^{3/4}x^{3/4}}d\sqrt[4]{x}}{3a(a+bx)^{3/4}}-\frac{4\sqrt[4]{a+bx}}{3ax^{3/4}}\right)}{7a}-\frac{4\sqrt[4]{a+bx}}{7ax^{7/4}}\right)}{(ax+bx^2)^{3/4}}$$

$$\Big\downarrow \text{858}$$

$$\frac{x^{3/4}(a+bx)^{3/4}\left(-\frac{6b\left(\frac{8bx^{3/4}\left(\frac{a}{bx}+1\right)^{3/4}\int \frac{1}{\sqrt[4]{x}\left(\frac{ax}{b}+1\right)^{3/4}}d\frac{1}{\sqrt[4]{x}}}{3a(a+bx)^{3/4}}-\frac{4\sqrt[4]{a+bx}}{3ax^{3/4}}\right)}{7a}-\frac{4\sqrt[4]{a+bx}}{7ax^{7/4}}\right)}{(ax+bx^2)^{3/4}}$$

$$\Big\downarrow \text{807}$$

$$\frac{x^{3/4}(a+bx)^{3/4}\left(-\frac{6b\left(\frac{4bx^{3/4}\left(\frac{a}{bx}+1\right)^{3/4}\int \frac{1}{\left(\frac{\sqrt{x}a}{b}+1\right)^{3/4}}d\sqrt{x}}{3a(a+bx)^{3/4}}-\frac{4\sqrt[4]{a+bx}}{3ax^{3/4}}\right)}{7a}-\frac{4\sqrt[4]{a+bx}}{7ax^{7/4}}\right)}{(ax+bx^2)^{3/4}}$$

$$\Big\downarrow \text{229}$$

$$\frac{x^{3/4}(a+bx)^{3/4}\left(-\frac{6b\left(\frac{8b^{3/2}x^{3/4}\left(\frac{a}{bx}+1\right)^{3/4}\text{EllipticF}\left(\frac{1}{2}\arctan\left(\frac{\sqrt{a}\sqrt{x}}{\sqrt{b}}\right),2\right)}{3a^{3/2}(a+bx)^{3/4}}-\frac{4\sqrt[4]{a+bx}}{3ax^{3/4}}\right)}{7a}-\frac{4\sqrt[4]{a+bx}}{7ax^{7/4}}\right)}{(ax+bx^2)^{3/4}}$$

```
Int[1/(x^2*(a*x + b*x^2)^(3/4)),x]
```

```
(x^(3/4)*(a + b*x)^(3/4)*((-4*(a + b*x)^(1/4))/(7*a*x^(7/4)) - (6*b*((-4*(
a + b*x)^(1/4))/(3*a*x^(3/4)) + (8*b^(3/2)*(1 + a/(b*x))^(3/4)*x^(3/4)*Ell
ipticF[ArcTan[(Sqrt[a]*Sqrt[x])/Sqrt[b]]/2, 2])/(3*a^(3/2)*(a + b*x)^(3/4)
)))/(7*a)))/(a*x + b*x^2)^(3/4)
```

Defintions of rubi rules used

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] - Simp[d*((
m + n + 2)/((b*c - a*d)*(m + 1)))   Int[(a + b*x)^(m + 1)*(c + d*x)^n, x],
x] /; FreeQ[{a, b, c, d, n}, x] && LtQ[m, -1] &&  !(LtQ[n, -1] && (EqQ[a, 0
] || (NeQ[c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && IntLinearQ[a, b, c, d
, m, n, x]
```

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Simp[p/b   Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
 d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && Lt
Q[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntL
inearQ[a, b, c, d, m, n, x]
```

```
Int[((a_) + (b_.)*(x_)^2)^(-3/4), x_Symbol] :> Simp[(2/(a^(3/4)*Rt[b/a, 2])
)*EllipticF[(1/2)*ArcTan[Rt[b/a, 2]*x], 2], x] /; FreeQ[{a, b}, x] && GtQ[a
, 0] && PosQ[b/a]
```

```
Int[((a_) + (b_.)*(x_)^4)^(-3/4), x_Symbol] :> Simp[x^3*((1 + a/(b*x^4))^(3
/4)/(a + b*x^4)^(3/4))   Int[1/(x^3*(1 + a/(b*x^4))^(3/4)), x], x] /; FreeQ
[{a, b}, x]
```

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k = GCD[m
+ 1, n]}, Simp[1/k   Subst[Int[x^((m + 1)/k - 1)*(a + b*x^(n/k))^p, x], x,
x^k], x] /; k != 1] /; FreeQ[{a, b, p}, x] && IGtQ[n, 0] && IntegerQ[m]
```

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> -Subst[Int[(a +
b/x^n)^p/x^(m + 2), x], x, 1/x] /; FreeQ[{a, b, p}, x] && ILtQ[n, 0] && Int
egerQ[m]
```

```
Int[((e_.)*(x_))^(m_)*((b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(
e*x)^m*((b*x + c*x^2)^p/(x^(m + p)*(b + c*x)^p))   Int[x^(m + p)*(b + c*x)^
p, x], x] /; FreeQ[{b, c, e, m}, x]
```

Maple [F]

$$\int \frac{1}{x^{2} \left(b\,x^{2}+a x \right)^{\frac{3}{4}}}d x$$

```
int(1/x^2/(b*x^2+a*x)^(3/4),x)
```

```
int(1/x^2/(b*x^2+a*x)^(3/4),x)
```

Fricas [F]

$$\int \frac{1}{x^2 \left(ax+bx^2\right)^{3/4}}\,dx=\int \frac{1}{\left(bx^{2}+ax\right)^{\frac{3}{4}}x^{2}}\,dx$$

```
integrate(1/x^2/(b*x^2+a*x)^(3/4),x, algorithm="fricas")
```

```
integral((b*x^2 + a*x)^(1/4)/(b*x^4 + a*x^3), x)
```

Sympy [F]

$$\int \frac{1}{x^2 (ax + bx^2)^{3/4}} \, dx = \int \frac{1}{x^2 (x (a + bx))^{\frac{3}{4}}} \, dx$$

```
integrate(1/x**2/(b*x**2+a*x)**(3/4),x)
```

```
Integral(1/(x**2*(x*(a + b*x))**(3/4)), x)
```

Maxima [F]

$$\int \frac{1}{x^2 (ax + bx^2)^{3/4}} \, dx = \int \frac{1}{(bx^2 + ax)^{\frac{3}{4}} x^2} \, dx$$

```
integrate(1/x^2/(b*x^2+a*x)^(3/4),x, algorithm="maxima")
```

```
integrate(1/((b*x^2 + a*x)^(3/4)*x^2), x)
```

Giac [F]

$$\int \frac{1}{x^2 (ax + bx^2)^{3/4}} \, dx = \int \frac{1}{(bx^2 + ax)^{\frac{3}{4}} x^2} \, dx$$

```
integrate(1/x^2/(b*x^2+a*x)^(3/4),x, algorithm="giac")
```

```
integrate(1/((b*x^2 + a*x)^(3/4)*x^2), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{x^2\left(ax+bx^2\right)^{3/4}}\,dx=\int \frac{1}{x^2\,{\left(b\,x^2+a\,x\right)}^{3/4}} \,dx$$

```
int(1/(x^2*(a*x + b*x^2)^(3/4)),x)
```

```
int(1/(x^2*(a*x + b*x^2)^(3/4)), x)
```

Reduce [B] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 27, normalized size of antiderivative = 0.23

$$\int \frac{1}{x^2\left(ax+bx^2\right)^{3/4}}\,dx=\frac{4(bx+a)^{\frac{1}{4}}\left(4bx-a\right)}{5x^{\frac{5}{4}}\sqrt{x}\,a^{2}}$$

```
int(1/x^2/(b*x^2+a*x)^(3/4),x)
```

```
(4*x**(3/4)*(a + b*x)**(1/4)*( - a + 4*b*x))/(5*sqrt(x)*a**2*x**2)
```

3.188 $\int \frac{1}{x^3\left(ax+bx^2\right)^{3/4}}\,dx$

Optimal result

Integrand size = 17, antiderivative size = 144

$$\int \frac{1}{x^3\left(ax+bx^2\right)^{3/4}}\,dx = -\frac{4\sqrt[4]{ax+bx^2}}{11ax^3}+\frac{40b\sqrt[4]{ax+bx^2}}{77a^2x^2}-\frac{80b^2\sqrt[4]{ax+bx^2}}{77a^3x}+\frac{160b^2\left(\frac{bx}{a+bx}\right)^{3/4}(a+bx)^{3/2}\,\text{EllipticF}\left(\frac{1}{2}\arcsin\left(\frac{\sqrt{a}}{\sqrt{a+bx}}\right),2\right)}{77a^{7/2}\left(ax+bx^2\right)^{3/4}}$$

```
-4/11*(b*x^2+a*x)^(1/4)/a/x^3+40/77*b*(b*x^2+a*x)^(1/4)/a^2/x^2-80/77*b^2*
(b*x^2+a*x)^(1/4)/a^3/x+160/77*b^2*(b*x/(b*x+a))^(3/4)*(b*x+a)^(3/2)*Inver
seJacobiAM(1/2*arcsin(a^(1/2)/(b*x+a)^(1/2)),2^(1/2))/a^(7/2)/(b*x^2+a*x)^
(3/4)
```

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 10.01 (sec) , antiderivative size = 47, normalized size of antiderivative = 0.33

$$\int \frac{1}{x^3\left(ax+bx^2\right)^{3/4}}\,dx = -\frac{4\left(1+\frac{bx}{a}\right)^{3/4}\text{Hypergeometric2F1}\left(-\frac{11}{4},\frac{3}{4},-\frac{7}{4},-\frac{bx}{a}\right)}{11x^2(x(a+bx))^{3/4}}$$

```
Integrate[1/(x^3*(a*x + b*x^2)^(3/4)),x]
```

```
(-4*(1 + (b*x)/a)^(3/4)*Hypergeometric2F1[-11/4, 3/4, -7/4, -((b*x)/a)])/(
11*x^2*(x*(a + b*x))^(3/4))
```

Rubi [A] (warning: unable to verify)

Time = 0.49 (sec) , antiderivative size = 175, normalized size of antiderivative = 1.22, number of steps used = 10, number of rules used = 9, $\frac{\text{number of rules}}{\text{integrand size}} = 0.529$, Rules used = {1137, 61, 61, 61, 73, 768, 858, 807, 229}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{x^3 (ax+bx^2)^{3/4}} dx$$

$$\Big\downarrow \text{1137}$$

$$\frac{x^{3/4}(a+bx)^{3/4} \int \frac{1}{x^{15/4}(a+bx)^{3/4}}dx}{(ax+bx^2)^{3/4}}$$

$$\Big\downarrow \text{61}$$

$$\frac{x^{3/4}(a+bx)^{3/4} \left(-\frac{10b \int \frac{1}{x^{11/4}(a+bx)^{3/4}}dx}{11a}-\frac{4\sqrt[4]{a+bx}}{11ax^{11/4}}\right)}{(ax+bx^2)^{3/4}}$$

$$\Big\downarrow \text{61}$$

$$\frac{x^{3/4}(a+bx)^{3/4} \left(-\frac{10b\left(-\frac{6b \int \frac{1}{x^{7/4}(a+bx)^{3/4}}dx}{7a}-\frac{4\sqrt[4]{a+bx}}{7ax^{7/4}}\right)}{11a}-\frac{4\sqrt[4]{a+bx}}{11ax^{11/4}}\right)}{(ax+bx^2)^{3/4}}$$

$$\Big\downarrow \text{61}$$

$$\frac{x^{3/4}(a+bx)^{3/4}\left(-\frac{10b\left(-\frac{6b\left(-\frac{2b\int \frac{1}{x^{3/4}(a+bx)^{3/4}}dx}{3a}-\frac{4\sqrt[4]{a+bx}}{3ax^{3/4}}\right)}{7a}-\frac{4\sqrt[4]{a+bx}}{7ax^{7/4}}\right)}{11a}-\frac{4\sqrt[4]{a+bx}}{11ax^{11/4}}\right)}{(ax+bx^2)^{3/4}}$$

$$\Big\downarrow \ 73$$

$$\frac{x^{3/4}(a+bx)^{3/4}\left(-\frac{10b\left(-\frac{6b\left(-\frac{8b\int \frac{1}{(a+bx)^{3/4}}d\sqrt[4]{x}}{3a}-\frac{4\sqrt[4]{a+bx}}{3ax^{3/4}}\right)}{7a}-\frac{4\sqrt[4]{a+bx}}{7ax^{7/4}}\right)}{11a}-\frac{4\sqrt[4]{a+bx}}{11ax^{11/4}}\right)}{(ax+bx^2)^{3/4}}$$

$$\Big\downarrow \ 768$$

$$\frac{x^{3/4}(a+bx)^{3/4}\left(-\frac{10b\left(-\frac{6b\left(-\frac{8bx^{3/4}\left(\frac{a}{bx}+1\right)^{3/4}\int \frac{1}{\left(\frac{a}{bx}+1\right)^{3/4}x^{3/4}}d\sqrt[4]{x}}{3a(a+bx)^{3/4}}-\frac{4\sqrt[4]{a+bx}}{3ax^{3/4}}\right)}{7a}-\frac{4\sqrt[4]{a+bx}}{7ax^{7/4}}\right)}{11a}-\frac{4\sqrt[4]{a+bx}}{11ax^{11/4}}\right)}{(ax+bx^2)^{3/4}}$$

$$\Big\downarrow \ 858$$

$$\frac{x^{3/4}(a+bx)^{3/4}\left(-\frac{10b\left(-\frac{6b\left(\frac{8bx^{3/4}\left(\frac{a}{bx}+1\right)^{3/4}\int \frac{1}{\sqrt[4]{x}\left(\frac{ax}{b}+1\right)^{3/4}}d\frac{1}{\sqrt[4]{x}}}{3a(a+bx)^{3/4}}-\frac{4\sqrt[4]{a+bx}}{3ax^{3/4}}\right)}{7a}-\frac{4\sqrt[4]{a+bx}}{7ax^{7/4}}\right)}{11a}-\frac{4\sqrt[4]{a+bx}}{11ax^{11/4}}\right)}{(ax+bx^2)^{3/4}}$$

$$\Big\downarrow \text{807}$$

$$\frac{x^{3/4}(a+bx)^{3/4}\left(-\frac{10b\left(-\frac{6b\left(\frac{4bx^{3/4}\left(\frac{a}{bx}+1\right)^{3/4}\int \frac{1}{\left(\frac{\sqrt{x}a}{b}+1\right)^{3/4}}d\sqrt{x}}{3a(a+bx)^{3/4}}-\frac{4\sqrt[4]{a+bx}}{3ax^{3/4}}\right)}{7a}-\frac{4\sqrt[4]{a+bx}}{7ax^{7/4}}\right)}{11a}-\frac{4\sqrt[4]{a+bx}}{11ax^{11/4}}\right)}{(ax+bx^2)^{3/4}}$$

$$\Big\downarrow \text{229}$$

$$\frac{x^{3/4}(a+bx)^{3/4}\left(-\frac{10b\left(-\frac{6b\left(\frac{8b^{3/2}x^{3/4}\left(\frac{a}{bx}+1\right)^{3/4}\text{EllipticF}\left(\frac{1}{2}\arctan\left(\frac{\sqrt{a}\sqrt{x}}{\sqrt{b}}\right),2\right)}{3a^{3/2}(a+bx)^{3/4}}-\frac{4\sqrt[4]{a+bx}}{3ax^{3/4}}\right)}{7a}-\frac{4\sqrt[4]{a+bx}}{7ax^{7/4}}\right)}{11a}-\frac{4\sqrt[4]{a+bx}}{11ax^{11/4}}\right)}{(ax+bx^2)^{3/4}}$$

```
Int[1/(x^3*(a*x + b*x^2)^(3/4)),x]
```

```
(x^(3/4)*(a + b*x)^(3/4)*((-4*(a + b*x)^(1/4))/(11*a*x^(11/4)) - (10*b*((-
4*(a + b*x)^(1/4))/(7*a*x^(7/4)) - (6*b*((-4*(a + b*x)^(1/4))/(3*a*x^(3/4)
) + (8*b^(3/2)*(1 + a/(b*x))^(3/4)*x^(3/4)*EllipticF[ArcTan[(Sqrt[a]*Sqrt[
x])/Sqrt[b]]/2, 2])/(3*a^(3/2)*(a + b*x)^(3/4))))/(7*a)))/(11*a)))/(a*x +
b*x^2)^(3/4)
```

Defintions of rubi rules used

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] - Simp[d*((
m + n + 2)/((b*c - a*d)*(m + 1)))   Int[(a + b*x)^(m + 1)*(c + d*x)^n, x],
x] /; FreeQ[{a, b, c, d, n}, x] && LtQ[m, -1] &&  !(LtQ[n, -1] && (EqQ[a, 0
] || (NeQ[c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && IntLinearQ[a, b, c, d
, m, n, x]
```

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Simp[p/b   Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
 d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && Lt
Q[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntL
inearQ[a, b, c, d, m, n, x]
```

```
Int[((a_) + (b_.)*(x_)^2)^(-3/4), x_Symbol] :> Simp[(2/(a^(3/4)*Rt[b/a, 2])
)*EllipticF[(1/2)*ArcTan[Rt[b/a, 2]*x], 2], x] /; FreeQ[{a, b}, x] && GtQ[a
, 0] && PosQ[b/a]
```

```
Int[((a_) + (b_.)*(x_)^4)^(-3/4), x_Symbol] :> Simp[x^3*((1 + a/(b*x^4))^(3
/4)/(a + b*x^4)^(3/4))   Int[1/(x^3*(1 + a/(b*x^4))^(3/4)), x], x] /; FreeQ
[{a, b}, x]
```

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k = GCD[m
+ 1, n]}, Simp[1/k   Subst[Int[x^((m + 1)/k - 1)*(a + b*x^(n/k))^p, x], x,
x^k], x] /; k != 1] /; FreeQ[{a, b, p}, x] && IGtQ[n, 0] && IntegerQ[m]
```

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> -Subst[Int[(a +
b/x^n)^p/x^(m + 2), x], x, 1/x] /; FreeQ[{a, b, p}, x] && ILtQ[n, 0] && Int
egerQ[m]
```

```
Int[((e_.)*(x_))^(m_)*((b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(
e*x)^m*((b*x + c*x^2)^p/(x^(m + p)*(b + c*x)^p))   Int[x^(m + p)*(b + c*x)^
p, x], x] /; FreeQ[{b, c, e, m}, x]
```

Maple [F]

$$\int \frac{1}{x^3\left(b\,x^2+ax\right)^{\frac{3}{4}}}dx$$

```
int(1/x^3/(b*x^2+a*x)^(3/4),x)
```

```
int(1/x^3/(b*x^2+a*x)^(3/4),x)
```

Fricas [F]

$$\int \frac{1}{x^3 (ax + bx^2)^{3/4}} \, dx = \int \frac{1}{(bx^2 + ax)^{\frac{3}{4}} x^3} \, dx$$

```
integrate(1/x^3/(b*x^2+a*x)^(3/4),x, algorithm="fricas")
```

```
integral((b*x^2 + a*x)^(1/4)/(b*x^5 + a*x^4), x)
```

Sympy [F]

$$\int \frac{1}{x^3 (ax + bx^2)^{3/4}} \, dx = \int \frac{1}{x^3 (x (a + bx))^{\frac{3}{4}}} \, dx$$

```
integrate(1/x**3/(b*x**2+a*x)**(3/4),x)
```

```
Integral(1/(x**3*(x*(a + b*x))**(3/4)), x)
```

Maxima [F]

$$\int \frac{1}{x^3 (ax + bx^2)^{3/4}} \, dx = \int \frac{1}{(bx^2 + ax)^{\frac{3}{4}} x^3} \, dx$$

```
integrate(1/x^3/(b*x^2+a*x)^(3/4),x, algorithm="maxima")
```

```
integrate(1/((b*x^2 + a*x)^(3/4)*x^3), x)
```

Giac [F]

$$\int \frac{1}{x^3 (ax + bx^2)^{3/4}} dx = \int \frac{1}{(bx^2 + ax)^{\frac{3}{4}} x^3} dx$$

```
integrate(1/x^3/(b*x^2+a*x)^(3/4),x, algorithm="giac")
```

```
integrate(1/((b*x^2 + a*x)^(3/4)*x^3), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{x^3 (ax + bx^2)^{3/4}} dx = \int \frac{1}{x^3 (b x^2 + a x)^{3/4}} dx$$

```
int(1/(x^3*(a*x + b*x^2)^(3/4)),x)
```

```
int(1/(x^3*(a*x + b*x^2)^(3/4)), x)
```

Reduce [B] (verification not implemented)

Time = 0.43 (sec) , antiderivative size = 38, normalized size of antiderivative = 0.26

$$\int \frac{1}{x^3 (ax + bx^2)^{3/4}} dx = \frac{4(bx + a)^{\frac{1}{4}} (-32b^2x^2 + 8abx - 5a^2)}{45x^{\frac{9}{4}} \sqrt{x}\, a^3}$$

```
int(1/x^3/(b*x^2+a*x)^(3/4),x)
```

```
(4*x**(3/4)*(a + b*x)**(1/4)*( - 5*a**2 + 8*a*b*x - 32*b**2*x**2))/(45*sqr
t(x)*a**3*x**3)
```

3.189 $\int \frac{x^5}{\left(ax+bx^2\right)^{5/4}} \, dx$

Optimal result

Integrand size = 17, antiderivative size = 160

$$\int \frac{x^5}{(ax+bx^2)^{5/4}} \, dx = -\frac{4a^2x^2}{b^3\sqrt[4]{ax+bx^2}} + \frac{11a^2(ax+bx^2)^{3/4}}{2b^4} - \frac{5ax(ax+bx^2)^{3/4}}{7b^3} + \frac{2x^2(ax+bx^2)^{3/4}}{7b^2} - \frac{33a^4\sqrt[4]{-\frac{bx}{a}-\frac{b^2x^2}{a^2}}\,E\left(\frac{1}{2}\arcsin\left(1+\frac{2bx}{a}\right)\middle|2\right)}{4\sqrt{2}b^5\sqrt[4]{ax+bx^2}}$$

```
-4*a^2*x^2/b^3/(b*x^2+a*x)^(1/4)+11/2*a^2*(b*x^2+a*x)^(3/4)/b^4-5/7*a*x*(b
*x^2+a*x)^(3/4)/b^3+2/7*x^2*(b*x^2+a*x)^(3/4)/b^2-33/8*a^4*(-b*x/a-b^2*x^2
/a^2)^(1/4)*EllipticE(sin(1/2*arcsin(1+2*b*x/a)),2^(1/2))*2^(1/2)/b^5/(b*x
^2+a*x)^(1/4)
```

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 10.02 (sec) , antiderivative size = 50, normalized size of antiderivative = 0.31

$$\int \frac{x^5}{(ax+bx^2)^{5/4}} \, dx = \frac{4x^5\sqrt[4]{1+\frac{bx}{a}}\,\text{Hypergeometric2F1}\left(\frac{5}{4},\frac{19}{4},\frac{23}{4},-\frac{bx}{a}\right)}{19a\sqrt[4]{x(a+bx)}}$$

```
Integrate[x^5/(a*x + b*x^2)^(5/4),x]
```

```
(4*x^5*(1 + (b*x)/a)^(1/4)*Hypergeometric2F1[5/4, 19/4, 23/4, -((b*x)/a)])
/(19*a*(x*(a + b*x))^(1/4))
```

Rubi [A] (warning: unable to verify)

Time = 0.59 (sec) , antiderivative size = 225, normalized size of antiderivative = 1.41, number of steps used = 12, number of rules used = 11, $\frac{\text{number of rules}}{\text{integrand size}} = 0.647$, Rules used = {1137, 57, 60, 60, 60, 73, 839, 813, 858, 807, 212}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^5}{(ax+bx^2)^{5/4}}dx$$

$$\Big\downarrow \text{1137}$$

$$\frac{x^{5/4}(a+bx)^{5/4}\int \frac{x^{15/4}}{(a+bx)^{5/4}}dx}{(ax+bx^2)^{5/4}}$$

$$\Big\downarrow \text{57}$$

$$\frac{x^{5/4}(a+bx)^{5/4}\left(\frac{15\int \frac{x^{11/4}}{\sqrt[4]{a+bx}}dx}{b}-\frac{4x^{15/4}}{b\sqrt[4]{a+bx}}\right)}{(ax+bx^2)^{5/4}}$$

$$\Big\downarrow \text{60}$$

$$\frac{x^{5/4}(a+bx)^{5/4}\left(\frac{15\left(\frac{2x^{11/4}(a+bx)^{3/4}}{7b}-\frac{11a\int \frac{x^{7/4}}{\sqrt[4]{a+bx}}dx}{14b}\right)}{b}-\frac{4x^{15/4}}{b\sqrt[4]{a+bx}}\right)}{(ax+bx^2)^{5/4}}$$

$$\Big\downarrow \text{60}$$

$$\frac{x^{5/4}(a+bx)^{5/4}\left(\frac{15\left(\frac{2x^{11/4}(a+bx)^{3/4}}{7b}-\frac{11a\left(\frac{2x^{7/4}(a+bx)^{3/4}}{5b}-\frac{7a\int \frac{x^{3/4}}{\sqrt[4]{a+bx}}dx}{10b}\right)}{14b}\right)}{b}-\frac{4x^{15/4}}{b\sqrt[4]{a+bx}}\right)}{(ax+bx^2)^{5/4}}$$

$$\Big\downarrow \; 60$$

$$\frac{x^{5/4}(a+bx)^{5/4}\left(\frac{15\left(\frac{2x^{11/4}(a+bx)^{3/4}}{7b}-\frac{11a\left(\frac{2x^{7/4}(a+bx)^{3/4}}{5b}-\frac{7a\left(\frac{2x^{3/4}(a+bx)^{3/4}}{3b}-\frac{a\int \frac{1}{\sqrt[4]{x}\sqrt[4]{a+bx}}dx}{2b}\right)}{10b}\right)}{14b}\right)}{b}-\frac{4x^{15/4}}{b\sqrt[4]{a+bx}}\right)}{(ax+bx^2)^{5/4}}$$

$$\Big\downarrow \; 73$$

$$\frac{x^{5/4}(a+bx)^{5/4}\left(\frac{15\left(\frac{2x^{11/4}(a+bx)^{3/4}}{7b}-\frac{11a\left(\frac{2x^{7/4}(a+bx)^{3/4}}{5b}-\frac{7a\left(\frac{2x^{3/4}(a+bx)^{3/4}}{3b}-\frac{2a\int\frac{\sqrt{x}}{\sqrt[4]{a+bx}}d\sqrt[4]{x}}{b}\right)}{10b}\right)}{14b}\right)}{b}-\frac{4x^{15/4}}{b\sqrt[4]{a+bx}}\right)}{(ax+bx^2)^{5/4}}$$

$\Big\downarrow$ 839

$$\frac{x^{5/4}(a+bx)^{5/4}\left(\frac{15\left(\frac{2x^{11/4}(a+bx)^{3/4}}{7b}-\frac{11a\left(\frac{2x^{7/4}(a+bx)^{3/4}}{5b}-\frac{7a\left(\frac{2x^{3/4}(a+bx)^{3/4}}{3b}-\frac{2a\left(\frac{x^{3/4}}{2\sqrt[4]{a+bx}}-\frac{1}{2}a\int\frac{\sqrt{x}}{(a+bx)^{5/4}}d\sqrt[4]{x}\right)}{b}\right)}{10b}\right)}{14b}\right)}{b}-\frac{}{b\sqrt[4]{}}\right.}{(ax+bx^2)^{5/4}}$$

$\big\downarrow$ 813

$$x^{5/4}(a+bx)^{5/4}\left(\frac{15\left(\frac{2x^{11/4}(a+bx)^{3/4}}{7b}-\frac{11a\left(\frac{2x^{7/4}(a+bx)^{3/4}}{5b}-\frac{7a\left(\frac{2x^{3/4}(a+bx)^{3/4}}{3b}-\frac{2a\left(\frac{x^{3/4}}{2\sqrt[4]{a+bx}}-\frac{a\sqrt[4]{x}\sqrt[4]{\frac{a}{bx}+1}\int\frac{1}{\left(\frac{a}{bx}+1\right)^{5/4}x^{3/4}}}{2b\sqrt[4]{a+bx}}\right.}{b}\right.}{10b}\right.}{14b}\right.}{b}\right.$$

↓ 858

$$x^{5/4}(a+bx)^{5/4}\left(\frac{15\left(\frac{2x^{11/4}(a+bx)^{3/4}}{7b}-\frac{11a\left(\frac{2x^{7/4}(a+bx)^{3/4}}{5b}-\frac{7a\left(\frac{2x^{3/4}(a+bx)^{3/4}}{3b}-\frac{2a\left(\frac{a\sqrt[4]{x}\sqrt[4]{\frac{a}{bx}+1}\int\frac{1}{\sqrt[4]{x}\left(\frac{ax}{b}+1\right)^{5/4}}d\frac{1}{\sqrt[4]{x}}}{2b\sqrt[4]{a+bx}}+\frac{}{2\sqrt[4]{}}\right.}{b}\right.}{10b}\right.}{14b}\right.}{b}\right.$$

↓ 807

$$x^{5/4}(a+bx)^{5/4}\left(\frac{15\left(\frac{2x^{11/4}(a+bx)^{3/4}}{7b}-\frac{11a\left(\frac{2x^{7/4}(a+bx)^{3/4}}{5b}-\frac{7a\left(\frac{2x^{3/4}(a+bx)^{3/4}}{3b}-\frac{2a\left(\frac{a\sqrt[4]{x}\sqrt[4]{\frac{a}{bx}+1}\int\frac{1}{\left(\frac{\sqrt{x}a}{b}+1\right)^{5/4}}d\sqrt{x}}{4b\sqrt[4]{a+bx}}+\frac{x^{3/4}}{2\sqrt[4]{a+}}\right.}{b}\right.}{10b}\right.}{14b}\right.}{b}\right.$$

$\downarrow$ 212

$$x^{5/4}(a+bx)^{5/4}\left(\frac{15\left(\frac{2x^{11/4}(a+bx)^{3/4}}{7b}-\frac{11a\left(\frac{2x^{7/4}(a+bx)^{3/4}}{5b}-\frac{7a\left(\frac{2x^{3/4}(a+bx)^{3/4}}{3b}-\frac{2a\left(\frac{\sqrt{a}\,\sqrt[4]{x}\,\sqrt[4]{\frac{a}{bx}+1}\,E\left(\frac{1}{2}\arctan\left(\frac{\sqrt{a}\sqrt{x}}{\sqrt{b}}\right)\middle|2\right)}{2\sqrt{b}\,\sqrt[4]{a+bx}}+\frac{}{2\sqrt[4]{}}\right.}{b}\right.}{10b}\right.}{14b}\right.}{b}\right.$$

```
Int[x^5/(a*x + b*x^2)^(5/4),x]
```

```
(x^(5/4)*(a + b*x)^(5/4)*((-4*x^(15/4))/(b*(a + b*x)^(1/4)) + (15*((2*x^(1
1/4)*(a + b*x)^(3/4))/(7*b) - (11*a*((2*x^(7/4)*(a + b*x)^(3/4))/(5*b) - (
7*a*((2*x^(3/4)*(a + b*x)^(3/4))/(3*b) - (2*a*(x^(3/4)/(2*(a + b*x)^(1/4))
 + (Sqrt[a]*(1 + a/(b*x))^(1/4)*x^(1/4)*EllipticE[ArcTan[(Sqrt[a]*Sqrt[x])
/Sqrt[b]]/2, 2])/(2*Sqrt[b]*(a + b*x)^(1/4))))/b))/(10*b)))/(14*b)))/b))/(
a*x + b*x^2)^(5/4)
```

Defintions of rubi rules used

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^n/(b*(m + 1))), x] - Simp[d*(n/(b*(m + 1)))
Int[(a + b*x)^(m + 1)*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b, c, d}, x] &
& GtQ[n, 0] && LtQ[m, -1] &&  !(IntegerQ[n] &&  !IntegerQ[m]) &&  !(ILeQ[m
+ n + 2, 0] && (FractionQ[m] || GeQ[2*n + m + 1, 0])) && IntLinearQ[a, b, c
, d, m, n, x]
```

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^n/(b*(m + n + 1))), x] + Simp[n*((b*c - a*d)/(
b*(m + n + 1)))   Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
 c, d}, x] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ[m, 0] && ( !Integer
Q[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n + 2, 0] && IntLinear
Q[a, b, c, d, m, n, x]
```

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Simp[p/b   Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
 d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && Lt
Q[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntL
inearQ[a, b, c, d, m, n, x]
```

```
Int[((a_) + (b_.)*(x_)^2)^(-5/4), x_Symbol] :> Simp[(2/(a^(5/4)*Rt[b/a, 2])
)*EllipticE[(1/2)*ArcTan[Rt[b/a, 2]*x], 2], x] /; FreeQ[{a, b}, x] && GtQ[a
, 0] && PosQ[b/a]
```

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k = GCD[m
+ 1, n]}, Simp[1/k   Subst[Int[x^((m + 1)/k - 1)*(a + b*x^(n/k))^p, x], x,
x^k], x] /; k != 1] /; FreeQ[{a, b, p}, x] && IGtQ[n, 0] && IntegerQ[m]
```

```
Int[(x_)^2/((a_) + (b_.)*(x_)^4)^(5/4), x_Symbol] :> Simp[x*((1 + a/(b*x^4)
)^(1/4)/(b*(a + b*x^4)^(1/4)))   Int[1/(x^3*(1 + a/(b*x^4))^(5/4)), x], x]
/; FreeQ[{a, b}, x] && PosQ[b/a]
```

```
Int[(x_)^2/((a_) + (b_.)*(x_)^4)^(1/4), x_Symbol] :> Simp[x^3/(2*(a + b*x^4
)^(1/4)), x] - Simp[a/2   Int[x^2/(a + b*x^4)^(5/4), x], x] /; FreeQ[{a, b}
, x] && PosQ[b/a]
```

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> -Subst[Int[(a +
b/x^n)^p/x^(m + 2), x], x, 1/x] /; FreeQ[{a, b, p}, x] && ILtQ[n, 0] && Int
egerQ[m]
```

```
Int[((e_.)*(x_))^(m_)*((b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(
e*x)^m*((b*x + c*x^2)^p/(x^(m + p)*(b + c*x)^p))   Int[x^(m + p)*(b + c*x)^
p, x], x] /; FreeQ[{b, c, e, m}, x]
```

Maple [F]

$$\int \frac{x^5}{(b\,x^2+ax)^{\frac{5}{4}}}dx$$

```
int(x^5/(b*x^2+a*x)^(5/4),x)
```

```
int(x^5/(b*x^2+a*x)^(5/4),x)
```

Fricas [F]

$$\int \frac{x^5}{(ax+bx^2)^{5/4}}\,dx = \int \frac{x^5}{(bx^2+ax)^{\frac{5}{4}}}\,dx$$

```
integrate(x^5/(b*x^2+a*x)^(5/4),x, algorithm="fricas")
```

```
integral((b*x^2 + a*x)^(3/4)*x^3/(b^2*x^2 + 2*a*b*x + a^2), x)
```

Sympy [F]

$$\int \frac{x^5}{(ax+bx^2)^{5/4}}\,dx = \int \frac{x^5}{(x\,(a+bx))^{\frac{5}{4}}}\,dx$$

```
integrate(x**5/(b*x**2+a*x)**(5/4),x)
```

```
Integral(x**5/(x*(a + b*x))**(5/4), x)
```

Maxima [F]

$$\int \frac{x^5}{(ax+bx^2)^{5/4}}\,dx = \int \frac{x^5}{(bx^2+ax)^{\frac{5}{4}}}\,dx$$

```
integrate(x^5/(b*x^2+a*x)^(5/4),x, algorithm="maxima")
```

```
integrate(x^5/(b*x^2 + a*x)^(5/4), x)
```

Giac [F]

$$\int \frac{x^5}{(ax+bx^2)^{5/4}}\,dx = \int \frac{x^5}{(bx^2+ax)^{\frac{5}{4}}}\,dx$$

```
integrate(x^5/(b*x^2+a*x)^(5/4),x, algorithm="giac")
```

```
integrate(x^5/(b*x^2 + a*x)^(5/4), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{x^5}{(ax+bx^2)^{5/4}}\,dx = \int \frac{x^5}{(b\,x^2+a\,x)^{5/4}}\,dx$$

```
int(x^5/(a*x + b*x^2)^(5/4),x)
```

```
int(x^5/(a*x + b*x^2)^(5/4), x)
```

Reduce [F]

$$\int \frac{x^5}{(ax+bx^2)^{5/4}}\,dx = \int \frac{x^4}{x^{\frac{1}{4}}\,(bx+a)^{\frac{1}{4}}\,a+x^{\frac{5}{4}}\,(bx+a)^{\frac{1}{4}}\,b}dx$$

```
int(x^5/(b*x^2+a*x)^(5/4),x)
```

```
int(x**4/(x**(1/4)*(a + b*x)**(1/4)*a + x**(1/4)*(a + b*x)**(1/4)*b*x),x)
```

3.190 $\int \frac{x^4}{(ax+bx^2)^{5/4}}\,dx$

Optimal result

Integrand size = 17, antiderivative size = 132

$$\int \frac{x^4}{(ax+bx^2)^{5/4}}\,dx = -\frac{4a^2x}{b^3\sqrt[4]{ax+bx^2}} - \frac{17a(ax+bx^2)^{3/4}}{15b^3} + \frac{2x(ax+bx^2)^{3/4}}{5b^2} + \frac{77a^3\sqrt[4]{-\frac{bx}{a}-\frac{b^2x^2}{a^2}}\,E\left(\frac{1}{2}\arcsin\left(1+\frac{2bx}{a}\right)\middle|2\right)}{10\sqrt{2}b^4\sqrt[4]{ax+bx^2}}$$

```
-4*a^2*x/b^3/(b*x^2+a*x)^(1/4)-17/15*a*(b*x^2+a*x)^(3/4)/b^3+2/5*x*(b*x^2+
a*x)^(3/4)/b^2+77/20*a^3*(-b*x/a-b^2*x^2/a^2)^(1/4)*EllipticE(sin(1/2*arcs
in(1+2*b*x/a)),2^(1/2))*2^(1/2)/b^4/(b*x^2+a*x)^(1/4)
```

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 10.01 (sec) , antiderivative size = 50, normalized size of antiderivative = 0.38

$$\int \frac{x^4}{(ax+bx^2)^{5/4}}\,dx = \frac{4x^4\sqrt[4]{1+\frac{bx}{a}}\,\text{Hypergeometric2F1}\left(\frac{5}{4},\frac{15}{4},\frac{19}{4},-\frac{bx}{a}\right)}{15a\sqrt[4]{x(a+bx)}}$$

```
Integrate[x^4/(a*x + b*x^2)^(5/4),x]
```

```
(4*x^4*(1 + (b*x)/a)^(1/4)*Hypergeometric2F1[5/4, 15/4, 19/4, -((b*x)/a)])
/(15*a*(x*(a + b*x))^(1/4))
```

Rubi [A] (warning: unable to verify)

Time = 0.57 (sec) , antiderivative size = 195, normalized size of antiderivative = 1.48, number of steps used = 11, number of rules used = 10, $\frac{\text{number of rules}}{\text{integrand size}} = 0.588$, Rules used = {1137, 57, 60, 60, 73, 839, 813, 858, 807, 212}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^4}{(ax+bx^2)^{5/4}}\,dx$$

$$\Big\downarrow \text{1137}$$

$$\frac{x^{5/4}(a+bx)^{5/4}\int \frac{x^{11/4}}{(a+bx)^{5/4}}dx}{(ax+bx^2)^{5/4}}$$

$$\Big\downarrow \text{57}$$

$$\frac{x^{5/4}(a+bx)^{5/4}\left(\frac{11\int \frac{x^{7/4}}{\sqrt[4]{a+bx}}dx}{b}-\frac{4x^{11/4}}{b\sqrt[4]{a+bx}}\right)}{(ax+bx^2)^{5/4}}$$

$$\Big\downarrow \text{60}$$

$$\frac{x^{5/4}(a+bx)^{5/4}\left(\frac{11\left(\frac{2x^{7/4}(a+bx)^{3/4}}{5b}-\frac{7a\int \frac{x^{3/4}}{\sqrt[4]{a+bx}}dx}{10b}\right)}{b}-\frac{4x^{11/4}}{b\sqrt[4]{a+bx}}\right)}{(ax+bx^2)^{5/4}}$$

$$\Big\downarrow \text{60}$$

$$\frac{x^{5/4}(a+bx)^{5/4}\left(\frac{11\left(\frac{2x^{7/4}(a+bx)^{3/4}}{5b}-\frac{7a\left(\frac{2x^{3/4}(a+bx)^{3/4}}{3b}-\frac{a\int\frac{1}{\sqrt[4]{x}\sqrt[4]{a+bx}}dx}{2b}\right)}{10b}\right)}{b}-\frac{4x^{11/4}}{b\sqrt[4]{a+bx}}\right)}{(ax+bx^2)^{5/4}}$$

$$\Big\downarrow \text{73}$$

$$\frac{x^{5/4}(a+bx)^{5/4}\left(\frac{11\left(\frac{2x^{7/4}(a+bx)^{3/4}}{5b}-\frac{7a\left(\frac{2x^{3/4}(a+bx)^{3/4}}{3b}-\frac{2a\int\frac{\sqrt{x}}{\sqrt[4]{a+bx}}d\sqrt[4]{x}}{b}\right)}{10b}\right)}{b}-\frac{4x^{11/4}}{b\sqrt[4]{a+bx}}\right)}{(ax+bx^2)^{5/4}}$$

$$\Big\downarrow \text{839}$$

$$\frac{x^{5/4}(a+bx)^{5/4}\left(\frac{11\left(\frac{2x^{7/4}(a+bx)^{3/4}}{5b}-\frac{7a\left(\frac{2x^{3/4}(a+bx)^{3/4}}{3b}-\frac{2a\left(\frac{x^{3/4}}{2\sqrt[4]{a+bx}}-\frac{1}{2}a\int\frac{\sqrt{x}}{(a+bx)^{5/4}}d\sqrt[4]{x}\right)}{b}\right)}{10b}\right)}{b}-\frac{4x^{11/4}}{b\sqrt[4]{a+bx}}\right)}{(ax+bx^2)^{5/4}}$$

$\Big\downarrow$ 813

$$\frac{x^{5/4}(a+bx)^{5/4}\left(\frac{11\left(\frac{2x^{7/4}(a+bx)^{3/4}}{5b}-\frac{7a\left(\frac{2x^{3/4}(a+bx)^{3/4}}{3b}-\frac{2a\left(\frac{x^{3/4}}{2\sqrt[4]{a+bx}}-\frac{a\sqrt[4]{x}\sqrt[4]{\frac{a}{bx}+1}\int\frac{1}{\left(\frac{a}{bx}+1\right)^{5/4}x^{3/4}}d\sqrt[4]{x}}{2b\sqrt[4]{a+bx}}\right)}{b}\right)}{10b}\right)}{b}-\frac{4x^{1}}{b\sqrt[4]{a}}\right.}{(ax+bx^2)^{5/4}}$$

$\Big\downarrow$ 858

$$\frac{x^{5/4}(a+bx)^{5/4}\left(\frac{11\left(\frac{2x^{7/4}(a+bx)^{3/4}}{5b}-\frac{7a\left(\frac{2x^{3/4}(a+bx)^{3/4}}{3b}-\frac{2a\left(\frac{a\sqrt[4]{x}\sqrt[4]{\frac{a}{bx}+1}\int\frac{1}{\sqrt[4]{x}\left(\frac{ax}{b}+1\right)^{5/4}}d\frac{1}{\sqrt[4]{x}}}{2b\sqrt[4]{a+bx}}+\frac{x^{3/4}}{2\sqrt[4]{a+bx}}\right)}{b}\right)}{10b}\right)}{b}-\frac{4x}{b\sqrt[4]{a}}\cdots\right)}{(ax+bx^2)^{5/4}}$$

$\downarrow$ 807

$$\frac{x^{5/4}(a+bx)^{5/4}\left(\frac{11\left(\frac{2x^{7/4}(a+bx)^{3/4}}{5b}-\frac{7a\left(\frac{2x^{3/4}(a+bx)^{3/4}}{3b}-\frac{2a\left(\frac{a\sqrt[4]{x}\sqrt[4]{\frac{a}{bx}+1}\int\frac{1}{\left(\frac{\sqrt{x}a}{b}+1\right)^{5/4}}d\sqrt{x}}{4b\sqrt[4]{a+bx}}+\frac{x^{3/4}}{2\sqrt[4]{a+bx}}\right)}{b}\right)}{10b}\right)}{b}-\frac{4x^{11/4}}{b\sqrt[4]{a+b}\cdots}\right.}{(ax+bx^2)^{5/4}}$$

$\downarrow$ 212

$$\frac{x^{5/4}(a+bx)^{5/4}\left(\frac{11\left(\frac{2x^{7/4}(a+bx)^{3/4}}{5b}-\frac{7a\left(\frac{2x^{3/4}(a+bx)^{3/4}}{3b}-\frac{2a\left(\frac{\sqrt{a}\sqrt[4]{x}\sqrt[4]{\frac{a}{bx}+1}E\left(\frac{1}{2}\arctan\left(\frac{\sqrt{a}\sqrt{x}}{\sqrt{b}}\right)\middle|2\right)}{2\sqrt{b}\sqrt[4]{a+bx}}+\frac{x^{3/4}}{2\sqrt[4]{a+bx}}\right)}{b}\right)}{10b}\right)}{b}-\frac{4x}{b\sqrt[4]{a}}\right.}{(ax+bx^2)^{5/4}}$$

```
Int[x^4/(a*x + b*x^2)^(5/4),x]
```

```
(x^(5/4)*(a + b*x)^(5/4)*((-4*x^(11/4))/(b*(a + b*x)^(1/4)) + (11*((2*x^(7
/4)*(a + b*x)^(3/4))/(5*b) - (7*a*((2*x^(3/4)*(a + b*x)^(3/4))/(3*b) - (2*
a*(x^(3/4)/(2*(a + b*x)^(1/4)) + (Sqrt[a]*(1 + a/(b*x))^(1/4)*x^(1/4)*Elli
pticE[ArcTan[(Sqrt[a]*Sqrt[x])/Sqrt[b]]/2, 2])/(2*Sqrt[b]*(a + b*x)^(1/4))
))/b))/(10*b)))/b))/(a*x + b*x^2)^(5/4)
```

Defintions of rubi rules used

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^n/(b*(m + 1))), x] - Simp[d*(n/(b*(m + 1)))
Int[(a + b*x)^(m + 1)*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b, c, d}, x] &
& GtQ[n, 0] && LtQ[m, -1] &&  !(IntegerQ[n] &&  !IntegerQ[m]) &&  !(ILeQ[m
+ n + 2, 0] && (FractionQ[m] || GeQ[2*n + m + 1, 0])) && IntLinearQ[a, b, c
, d, m, n, x]
```

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^n/(b*(m + n + 1))), x] + Simp[n*((b*c - a*d)/(
b*(m + n + 1)))   Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
 c, d}, x] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ[m, 0] && ( !Integer
Q[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n + 2, 0] && IntLinear
Q[a, b, c, d, m, n, x]
```

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Simp[p/b   Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
 d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && Lt
Q[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntL
inearQ[a, b, c, d, m, n, x]
```

```
Int[((a_) + (b_.)*(x_)^2)^(-5/4), x_Symbol] :> Simp[(2/(a^(5/4)*Rt[b/a, 2])
)*EllipticE[(1/2)*ArcTan[Rt[b/a, 2]*x], 2], x] /; FreeQ[{a, b}, x] && GtQ[a
, 0] && PosQ[b/a]
```

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k = GCD[m
+ 1, n]}, Simp[1/k   Subst[Int[x^((m + 1)/k - 1)*(a + b*x^(n/k))^p, x], x,
x^k], x] /; k != 1] /; FreeQ[{a, b, p}, x] && IGtQ[n, 0] && IntegerQ[m]
```

```
Int[(x_)^2/((a_) + (b_.)*(x_)^4)^(5/4), x_Symbol] :> Simp[x*((1 + a/(b*x^4)
)^(1/4)/(b*(a + b*x^4)^(1/4)))   Int[1/(x^3*(1 + a/(b*x^4))^(5/4)), x], x]
/; FreeQ[{a, b}, x] && PosQ[b/a]
```

```
Int[(x_)^2/((a_) + (b_.)*(x_)^4)^(1/4), x_Symbol] :> Simp[x^3/(2*(a + b*x^4
)^(1/4)), x] - Simp[a/2   Int[x^2/(a + b*x^4)^(5/4), x], x] /; FreeQ[{a, b}
, x] && PosQ[b/a]
```

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> -Subst[Int[(a +
b/x^n)^p/x^(m + 2), x], x, 1/x] /; FreeQ[{a, b, p}, x] && ILtQ[n, 0] && Int
egerQ[m]
```

```
Int[((e_.)*(x_))^(m_)*((b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(
e*x)^m*((b*x + c*x^2)^p/(x^(m + p)*(b + c*x)^p))   Int[x^(m + p)*(b + c*x)^
p, x], x] /; FreeQ[{b, c, e, m}, x]
```

Maple [F]

$$\int \frac{x^4}{(b\,x^2+ax)^{\frac{5}{4}}}dx$$

```
int(x^4/(b*x^2+a*x)^(5/4),x)
```

```
int(x^4/(b*x^2+a*x)^(5/4),x)
```

Fricas [F]

$$\int \frac{x^4}{(ax+bx^2)^{5/4}}\,dx = \int \frac{x^4}{(bx^2+ax)^{\frac{5}{4}}}\,dx$$

```
integrate(x^4/(b*x^2+a*x)^(5/4),x, algorithm="fricas")
```

```
integral((b*x^2 + a*x)^(3/4)*x^2/(b^2*x^2 + 2*a*b*x + a^2), x)
```

Sympy [F]

$$\int \frac{x^4}{(ax+bx^2)^{5/4}}\,dx = \int \frac{x^4}{(x(a+bx))^{\frac{5}{4}}}\,dx$$

```
integrate(x**4/(b*x**2+a*x)**(5/4),x)
```

```
Integral(x**4/(x*(a + b*x))**(5/4), x)
```

Maxima [F]

$$\int \frac{x^4}{(ax+bx^2)^{5/4}}\,dx = \int \frac{x^4}{(bx^2+ax)^{\frac{5}{4}}}\,dx$$

```
integrate(x^4/(b*x^2+a*x)^(5/4),x, algorithm="maxima")
```

```
integrate(x^4/(b*x^2 + a*x)^(5/4), x)
```

Giac [F]

$$\int \frac{x^4}{(ax+bx^2)^{5/4}}\,dx = \int \frac{x^4}{(bx^2+ax)^{\frac{5}{4}}}\,dx$$

```
integrate(x^4/(b*x^2+a*x)^(5/4),x, algorithm="giac")
```

```
integrate(x^4/(b*x^2 + a*x)^(5/4), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{x^4}{(ax+bx^2)^{5/4}}\,dx = \int \frac{x^4}{{\left(b\,x^2+a\,x\right)}^{5/4}} \,d x$$

```
int(x^4/(a*x + b*x^2)^(5/4),x)
```

```
int(x^4/(a*x + b*x^2)^(5/4), x)
```

Reduce [F]

$$\int \frac{x^4}{(ax+bx^2)^{5/4}}\,dx = \int \frac{x^3}{x^{\frac{1}{4}} (bx+a)^{\frac{1}{4}} a+x^{\frac{5}{4}} (bx+a)^{\frac{1}{4}} b}d x$$

```
int(x^4/(b*x^2+a*x)^(5/4),x)
```

```
int(x**3/(x**(1/4)*(a + b*x)**(1/4)*a + x**(1/4)*(a + b*x)**(1/4)*b*x),x)
```

3.191 $\int \frac{x^3}{(ax+bx^2)^{5/4}}\,dx$

Optimal result

Integrand size = 17, antiderivative size = 106

$$\int \frac{x^3}{(ax+bx^2)^{5/4}}\,dx = \frac{4ax}{b^2\sqrt[4]{ax+bx^2}}+\frac{2(ax+bx^2)^{3/4}}{3b^2}-\frac{7a^2\sqrt[4]{-\frac{bx}{a}-\frac{b^2x^2}{a^2}}\,E\left(\frac{1}{2}\arcsin\left(1+\frac{2bx}{a}\right)\middle|2\right)}{\sqrt{2}b^3\sqrt[4]{ax+bx^2}}$$

```
4*a*x/b^2/(b*x^2+a*x)^(1/4)+2/3*(b*x^2+a*x)^(3/4)/b^2-7/2*a^2*(-b*x/a-b^2*
x^2/a^2)^(1/4)*EllipticE(sin(1/2*arcsin(1+2*b*x/a)),2^(1/2))*2^(1/2)/b^3/(
b*x^2+a*x)^(1/4)
```

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 10.01 (sec) , antiderivative size = 50, normalized size of antiderivative = 0.47

$$\int \frac{x^3}{(ax+bx^2)^{5/4}}\,dx = \frac{4x^3\sqrt[4]{1+\frac{bx}{a}}\,\text{Hypergeometric2F1}\left(\frac{5}{4},\frac{11}{4},\frac{15}{4},-\frac{bx}{a}\right)}{11a\sqrt[4]{x(a+bx)}}$$

```
Integrate[x^3/(a*x + b*x^2)^(5/4),x]
```

```
(4*x^3*(1 + (b*x)/a)^(1/4)*Hypergeometric2F1[5/4, 11/4, 15/4, -((b*x)/a)])
/(11*a*(x*(a + b*x))^(1/4))
```

Rubi [A] (warning: unable to verify)

Time = 0.53 (sec) , antiderivative size = 165, normalized size of antiderivative = 1.56, number of steps used = 10, number of rules used = 9, $\frac{\text{number of rules}}{\text{integrand size}} = 0.529$, Rules used = {1137, 57, 60, 73, 839, 813, 858, 807, 212}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^3}{(ax+bx^2)^{5/4}}dx$$

$$\Big\downarrow \text{1137}$$

$$\frac{x^{5/4}(a+bx)^{5/4}\int \frac{x^{7/4}}{(a+bx)^{5/4}}dx}{(ax+bx^2)^{5/4}}$$

$$\Big\downarrow \text{57}$$

$$\frac{x^{5/4}(a+bx)^{5/4}\left(\frac{7\int \frac{x^{3/4}}{\sqrt[4]{a+bx}}dx}{b}-\frac{4x^{7/4}}{b\sqrt[4]{a+bx}}\right)}{(ax+bx^2)^{5/4}}$$

$$\Big\downarrow \text{60}$$

$$\frac{x^{5/4}(a+bx)^{5/4}\left(\frac{7\left(\frac{2x^{3/4}(a+bx)^{3/4}}{3b}-\frac{a\int \frac{1}{\sqrt[4]{x}\sqrt[4]{a+bx}}dx}{2b}\right)}{b}-\frac{4x^{7/4}}{b\sqrt[4]{a+bx}}\right)}{(ax+bx^2)^{5/4}}$$

$$\Big\downarrow \text{73}$$

$$\frac{x^{5/4}(a+bx)^{5/4}\left(\frac{7\left(\frac{2x^{3/4}(a+bx)^{3/4}}{3b}-\frac{2a\int\frac{\sqrt{x}}{\sqrt[4]{a+bx}}d\sqrt[4]{x}}{b}\right)}{b}-\frac{4x^{7/4}}{b\sqrt[4]{a+bx}}\right)}{(ax+bx^2)^{5/4}}$$

$\Big\downarrow$ 839

$$\frac{x^{5/4}(a+bx)^{5/4}\left(\frac{7\left(\frac{2x^{3/4}(a+bx)^{3/4}}{3b}-\frac{2a\left(\frac{x^{3/4}}{2\sqrt[4]{a+bx}}-\frac{1}{2}a\int\frac{\sqrt{x}}{(a+bx)^{5/4}}d\sqrt[4]{x}\right)}{b}\right)}{b}-\frac{4x^{7/4}}{b\sqrt[4]{a+bx}}\right)}{(ax+bx^2)^{5/4}}$$

$\Big\downarrow$ 813

$$\frac{x^{5/4}(a+bx)^{5/4}\left(\frac{7\left(\frac{2x^{3/4}(a+bx)^{3/4}}{3b}-\frac{2a\left(\frac{x^{3/4}}{2\sqrt[4]{a+bx}}-\frac{a\sqrt[4]{x}\sqrt[4]{\frac{a}{bx}+1}\int\frac{1}{\left(\frac{a}{bx}+1\right)^{5/4}x^{3/4}}d\sqrt[4]{x}}{2b\sqrt[4]{a+bx}}\right)}{b}\right)}{b}-\frac{4x^{7/4}}{b\sqrt[4]{a+bx}}\right)}{(ax+bx^2)^{5/4}}$$

$\Big\downarrow$ 858

$$\frac{x^{5/4}(a+bx)^{5/4}\left(\frac{7\left(\frac{2x^{3/4}(a+bx)^{3/4}}{3b}-\frac{2a\left(\frac{a\sqrt[4]{x}\sqrt[4]{\frac{a}{bx}+1}\int \frac{1}{\sqrt[4]{x}\left(\frac{ax}{b}+1\right)^{5/4}}d\frac{1}{\sqrt[4]{x}}}{2b\sqrt[4]{a+bx}}+\frac{x^{3/4}}{2\sqrt[4]{a+bx}}\right)}{b}\right)}{b}-\frac{4x^{7/4}}{b\sqrt[4]{a+bx}}\right)}{(ax+bx^2)^{5/4}}$$

$$\Big\downarrow \text{807}$$

$$\frac{x^{5/4}(a+bx)^{5/4}\left(\frac{7\left(\frac{2x^{3/4}(a+bx)^{3/4}}{3b}-\frac{2a\left(\frac{a\sqrt[4]{x}\sqrt[4]{\frac{a}{bx}+1}\int \frac{1}{\left(\frac{\sqrt{x}a}{b}+1\right)^{5/4}}d\sqrt{x}}{4b\sqrt[4]{a+bx}}+\frac{x^{3/4}}{2\sqrt[4]{a+bx}}\right)}{b}\right)}{b}-\frac{4x^{7/4}}{b\sqrt[4]{a+bx}}\right)}{(ax+bx^2)^{5/4}}$$

$$\Big\downarrow \text{212}$$

$$\frac{x^{5/4}(a+bx)^{5/4}\left(\frac{7\left(\frac{2x^{3/4}(a+bx)^{3/4}}{3b}-\frac{2a\left(\frac{\sqrt{a}\sqrt[4]{x}\sqrt[4]{\frac{a}{bx}+1}E\left(\frac{1}{2}\arctan\left(\frac{\sqrt{a}\sqrt{x}}{\sqrt{b}}\right)\middle|2\right)}{2\sqrt{b}\sqrt[4]{a+bx}}+\frac{x^{3/4}}{2\sqrt[4]{a+bx}}\right)}{b}\right)}{b}-\frac{4x^{7/4}}{b\sqrt[4]{a+bx}}\right)}{(ax+bx^2)^{5/4}}$$

```
Int[x^3/(a*x + b*x^2)^(5/4),x]
```

```
(x^(5/4)*(a + b*x)^(5/4)*((-4*x^(7/4))/(b*(a + b*x)^(1/4)) + (7*((2*x^(3/4
)*(a + b*x)^(3/4))/(3*b) - (2*a*(x^(3/4)/(2*(a + b*x)^(1/4)) + (Sqrt[a]*(1
 + a/(b*x))^(1/4)*x^(1/4)*EllipticE[ArcTan[(Sqrt[a]*Sqrt[x])/Sqrt[b]]/2, 2
])/(2*Sqrt[b]*(a + b*x)^(1/4))))/b))/b))/(a*x + b*x^2)^(5/4)
```

Defintions of rubi rules used

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^n/(b*(m + 1))), x] - Simp[d*(n/(b*(m + 1)))
Int[(a + b*x)^(m + 1)*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b, c, d}, x] &
& GtQ[n, 0] && LtQ[m, -1] &&  !(IntegerQ[n] &&  !IntegerQ[m]) &&  !(ILeQ[m
+ n + 2, 0] && (FractionQ[m] || GeQ[2*n + m + 1, 0])) && IntLinearQ[a, b, c
, d, m, n, x]
```

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^n/(b*(m + n + 1))), x] + Simp[n*((b*c - a*d)/(
b*(m + n + 1)))   Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
 c, d}, x] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ[m, 0] && ( !Integer
Q[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n + 2, 0] && IntLinear
Q[a, b, c, d, m, n, x]
```

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Simp[p/b   Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
 d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && Lt
Q[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntL
inearQ[a, b, c, d, m, n, x]
```

```
Int[((a_) + (b_.)*(x_)^2)^(-5/4), x_Symbol] :> Simp[(2/(a^(5/4)*Rt[b/a, 2])
)*EllipticE[(1/2)*ArcTan[Rt[b/a, 2]*x], 2], x] /; FreeQ[{a, b}, x] && GtQ[a
, 0] && PosQ[b/a]
```

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k = GCD[m
+ 1, n]}, Simp[1/k   Subst[Int[x^((m + 1)/k - 1)*(a + b*x^(n/k))^p, x], x,
x^k], x] /; k != 1] /; FreeQ[{a, b, p}, x] && IGtQ[n, 0] && IntegerQ[m]
```

```
Int[(x_)^2/((a_) + (b_.)*(x_)^4)^(5/4), x_Symbol] :> Simp[x*((1 + a/(b*x^4)
)^(1/4)/(b*(a + b*x^4)^(1/4)))   Int[1/(x^3*(1 + a/(b*x^4))^(5/4)), x], x]
/; FreeQ[{a, b}, x] && PosQ[b/a]
```

```
Int[(x_)^2/((a_) + (b_.)*(x_)^4)^(1/4), x_Symbol] :> Simp[x^3/(2*(a + b*x^4
)^(1/4)), x] - Simp[a/2   Int[x^2/(a + b*x^4)^(5/4), x], x] /; FreeQ[{a, b}
, x] && PosQ[b/a]
```

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> -Subst[Int[(a +
b/x^n)^p/x^(m + 2), x], x, 1/x] /; FreeQ[{a, b, p}, x] && ILtQ[n, 0] && Int
egerQ[m]
```

```
Int[((e_.)*(x_))^(m_)*((b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(
e*x)^m*((b*x + c*x^2)^p/(x^(m + p)*(b + c*x)^p))   Int[x^(m + p)*(b + c*x)^
p, x], x] /; FreeQ[{b, c, e, m}, x]
```

Maple [F]

$$\int \frac{x^3}{(b\,x^2+ax)^{\frac{5}{4}}}dx$$

```
int(x^3/(b*x^2+a*x)^(5/4),x)
```

```
int(x^3/(b*x^2+a*x)^(5/4),x)
```

Fricas [F]

$$\int \frac{x^3}{(ax+bx^2)^{5/4}}\,dx = \int \frac{x^3}{(bx^2+ax)^{\frac{5}{4}}}\,dx$$

```
integrate(x^3/(b*x^2+a*x)^(5/4),x, algorithm="fricas")
```

```
integral((b*x^2 + a*x)^(3/4)*x/(b^2*x^2 + 2*a*b*x + a^2), x)
```

Sympy [F]

$$\int \frac{x^3}{(ax+bx^2)^{5/4}}\,dx = \int \frac{x^3}{(x\,(a+bx))^{\frac{5}{4}}}\,dx$$

```
integrate(x**3/(b*x**2+a*x)**(5/4),x)
```

```
Integral(x**3/(x*(a + b*x))**(5/4), x)
```

Maxima [F]

$$\int \frac{x^3}{(ax+bx^2)^{5/4}}\,dx = \int \frac{x^3}{(bx^2+ax)^{\frac{5}{4}}}\,dx$$

```
integrate(x^3/(b*x^2+a*x)^(5/4),x, algorithm="maxima")
```

```
integrate(x^3/(b*x^2 + a*x)^(5/4), x)
```

Giac [F]

$$\int \frac{x^3}{(ax+bx^2)^{5/4}}\,dx = \int \frac{x^3}{(bx^2+ax)^{\frac{5}{4}}}\,dx$$

```
integrate(x^3/(b*x^2+a*x)^(5/4),x, algorithm="giac")
```

```
integrate(x^3/(b*x^2 + a*x)^(5/4), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{x^3}{(ax+bx^2)^{5/4}}\,dx = \int \frac{x^3}{(b\,x^2+a\,x)^{5/4}}\,dx$$

```
int(x^3/(a*x + b*x^2)^(5/4),x)
```

```
int(x^3/(a*x + b*x^2)^(5/4), x)
```

Reduce [F]

$$\int \frac{x^3}{(ax+bx^2)^{5/4}}\,dx = \int \frac{x^2}{x^{\frac{1}{4}}\,(bx+a)^{\frac{1}{4}}\,a + x^{\frac{5}{4}}\,(bx+a)^{\frac{1}{4}}\,b}dx$$

```
int(x^3/(b*x^2+a*x)^(5/4),x)
```

```
int(x**2/(x**(1/4)*(a + b*x)**(1/4)*a + x**(1/4)*(a + b*x)**(1/4)*b*x),x)
```

3.192 $\int \frac{x^2}{(ax+bx^2)^{5/4}}\,dx$

Optimal result

Integrand size = 17, antiderivative size = 83

$$\int \frac{x^2}{(ax+bx^2)^{5/4}}\,dx = -\frac{4x}{b\sqrt[4]{ax+bx^2}}+\frac{3\sqrt{2}a\sqrt[4]{-\frac{bx}{a}-\frac{b^2x^2}{a^2}}E\left(\frac{1}{2}\arcsin\left(1+\frac{2bx}{a}\right)\middle|2\right)}{b^2\sqrt[4]{ax+bx^2}}$$

```
-4*x/b/(b*x^2+a*x)^(1/4)+3*2^(1/2)*a*(-b*x/a-b^2*x^2/a^2)^(1/4)*EllipticE(
sin(1/2*arcsin(1+2*b*x/a)),2^(1/2))/b^2/(b*x^2+a*x)^(1/4)
```

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 10.01 (sec) , antiderivative size = 50, normalized size of antiderivative = 0.60

$$\int \frac{x^2}{(ax+bx^2)^{5/4}}\,dx = \frac{4x^2\sqrt[4]{1+\frac{bx}{a}}\,\text{Hypergeometric2F1}\left(\frac{5}{4},\frac{7}{4},\frac{11}{4},-\frac{bx}{a}\right)}{7a\sqrt[4]{x(a+bx)}}$$

```
Integrate[x^2/(a*x + b*x^2)^(5/4),x]
```

```
(4*x^2*(1 + (b*x)/a)^(1/4)*Hypergeometric2F1[5/4, 7/4, 11/4, -((b*x)/a)])/
(7*a*(x*(a + b*x))^(1/4))
```

Rubi [A] (verified)

Time = 0.41 (sec) , antiderivative size = 91, normalized size of antiderivative = 1.10, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.235, Rules used = {1126, 1093, 1090, 226}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^2}{(ax+bx^2)^{5/4}}dx$$

$$\Big\downarrow \text{1126}$$

$$\frac{3\int \frac{1}{\sqrt[4]{bx^2+ax}}dx}{b}-\frac{4x}{b\sqrt[4]{ax+bx^2}}$$

$$\Big\downarrow \text{1093}$$

$$\frac{3\sqrt[4]{-\frac{b(ax+bx^2)}{a^2}}\int \frac{1}{\sqrt[4]{-\frac{b^2x^2}{a^2}-\frac{bx}{a}}}dx}{b\sqrt[4]{ax+bx^2}}-\frac{4x}{b\sqrt[4]{ax+bx^2}}$$

$$\Big\downarrow \text{1090}$$

$$-\frac{3a^2\sqrt[4]{-\frac{b(ax+bx^2)}{a^2}}\int \frac{1}{\sqrt[4]{1-\frac{a^2\left(-\frac{2xb^2}{a^2}-\frac{b}{a}\right)^2}{b^2}}}d\left(-\frac{2xb^2}{a^2}-\frac{b}{a}\right)}{\sqrt{2}b^3\sqrt[4]{ax+bx^2}}-\frac{4x}{b\sqrt[4]{ax+bx^2}}$$

$$\Big\downarrow \text{226}$$

$$-\frac{3\sqrt{2}a\sqrt[4]{-\frac{b(ax+bx^2)}{a^2}}E\left(\frac{1}{2}\arcsin\left(\frac{a\left(-\frac{2xb^2}{a^2}-\frac{b}{a}\right)}{b}\right)\middle|2\right)}{b^2\sqrt[4]{ax+bx^2}}-\frac{4x}{b\sqrt[4]{ax+bx^2}}$$

```
Int[x^2/(a*x + b*x^2)^(5/4),x]
```

```
(-4*x)/(b*(a*x + b*x^2)^(1/4)) - (3*Sqrt[2]*a*(-((b*(a*x + b*x^2))/a^2))^(
1/4)*EllipticE[ArcSin[(a*(-(b/a) - (2*b^2*x)/a^2))/b]/2, 2])/(b^2*(a*x + b
*x^2)^(1/4))
```

Defintions of rubi rules used

```
Int[((a_) + (b_.)*(x_)^2)^(-1/4), x_Symbol] :> Simp[(2/(a^(1/4)*Rt[-b/a, 2]
))*EllipticE[(1/2)*ArcSin[Rt[-b/a, 2]*x], 2], x] /; FreeQ[{a, b}, x] && GtQ
[a, 0] && NegQ[b/a]
```

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[1/(2*c*(-4*
(c/(b^2 - 4*a*c)))^p)   Subst[Int[Simp[1 - x^2/(b^2 - 4*a*c), x]^p, x], x,
b + 2*c*x], x] /; FreeQ[{a, b, c, p}, x] && GtQ[4*a - b^2/c, 0]
```

```
Int[((b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(b*x + c*x^2)^p/((-
c)*((b*x + c*x^2)/b^2))^p   Int[((-c)*(x/b) - c^2*(x^2/b^2))^p, x], x] /; F
reeQ[{b, c}, x] && (IntegerQ[4*p] || IntegerQ[3*p])
```

```
Int[((d_.) + (e_.)*(x_))^2*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symb
ol] :> Simp[e*(d + e*x)*((a + b*x + c*x^2)^(p + 1)/(c*(p + 1))), x] - Simp[
e^2*((p + 2)/(c*(p + 1)))   Int[(a + b*x + c*x^2)^(p + 1), x], x] /; FreeQ[
{a, b, c, d, e, p}, x] && EqQ[c*d^2 - b*d*e + a*e^2, 0] && LtQ[p, -1]
```

Maple [F]

$$\int \frac{x^2}{(b\,x^2+ax)^{\frac{5}{4}}}dx$$

```
int(x^2/(b*x^2+a*x)^(5/4),x)
```

```
int(x^2/(b*x^2+a*x)^(5/4),x)
```

Fricas [F]

$$\int \frac{x^2}{(ax+bx^2)^{5/4}}\,dx = \int \frac{x^2}{(bx^2+ax)^{\frac{5}{4}}}\,dx$$

```
integrate(x^2/(b*x^2+a*x)^(5/4),x, algorithm="fricas")
```

```
integral((b*x^2 + a*x)^(3/4)/(b^2*x^2 + 2*a*b*x + a^2), x)
```

Sympy [F]

$$\int \frac{x^2}{(ax+bx^2)^{5/4}}\,dx = \int \frac{x^2}{(x\,(a+bx))^{\frac{5}{4}}}\,dx$$

```
integrate(x**2/(b*x**2+a*x)**(5/4),x)
```

```
Integral(x**2/(x*(a + b*x))**(5/4), x)
```

Maxima [F]

$$\int \frac{x^2}{(ax+bx^2)^{5/4}}\,dx = \int \frac{x^2}{(bx^2+ax)^{\frac{5}{4}}}\,dx$$

```
integrate(x^2/(b*x^2+a*x)^(5/4),x, algorithm="maxima")
```

```
integrate(x^2/(b*x^2 + a*x)^(5/4), x)
```

Giac [F]

$$\int \frac{x^2}{(ax+bx^2)^{5/4}}\,dx=\int \frac{x^2}{(bx^2+ax)^{\frac{5}{4}}}\,dx$$

```
integrate(x^2/(b*x^2+a*x)^(5/4),x, algorithm="giac")
```

```
integrate(x^2/(b*x^2 + a*x)^(5/4), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{x^2}{(ax+bx^2)^{5/4}}\,dx=\int \frac{x^2}{(b\,x^2+a\,x)^{5/4}}\,dx$$

```
int(x^2/(a*x + b*x^2)^(5/4),x)
```

```
int(x^2/(a*x + b*x^2)^(5/4), x)
```

Reduce [F]

$$\int \frac{x^2}{(ax+bx^2)^{5/4}}\,dx=\int \frac{x}{x^{\frac{1}{4}}\,(bx+a)^{\frac{1}{4}}\,a+x^{\frac{5}{4}}\,(bx+a)^{\frac{1}{4}}\,b}dx$$

```
int(x^2/(b*x^2+a*x)^(5/4),x)
```

```
int(x/(x**(1/4)*(a + b*x)**(1/4)*a + x**(1/4)*(a + b*x)**(1/4)*b*x),x)
```

3.193 $\int \frac{x}{(ax+bx^2)^{5/4}}\,dx$

Optimal result

Integrand size = 15, antiderivative size = 69

$$\int \frac{x}{(ax+bx^2)^{5/4}}\,dx = -\frac{4\sqrt{x}\sqrt[4]{\frac{a+bx}{bx}}E\left(\frac{1}{2}\arctan\left(\frac{\sqrt{a}}{\sqrt{b}\sqrt{x}}\right)\middle|2\right)}{\sqrt{a}\sqrt{b}\sqrt[4]{ax+bx^2}}$$

```
-4*x^(1/2)*((b*x+a)/b/x)^(1/4)*EllipticE(sin(1/2*arctan(1/b^(1/2)/x^(1/2)*
a^(1/2))),2^(1/2))/a^(1/2)/b^(1/2)/(b*x^2+a*x)^(1/4)
```

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 10.02 (sec) , antiderivative size = 48, normalized size of antiderivative = 0.70

$$\int \frac{x}{(ax+bx^2)^{5/4}}\,dx = \frac{4x\sqrt[4]{1+\frac{bx}{a}}\,\text{Hypergeometric2F1}\left(\frac{3}{4},\frac{5}{4},\frac{7}{4},-\frac{bx}{a}\right)}{3a\sqrt[4]{x(a+bx)}}$$

```
Integrate[x/(a*x + b*x^2)^(5/4),x]
```

```
(4*x*(1 + (b*x)/a)^(1/4)*Hypergeometric2F1[3/4, 5/4, 7/4, -((b*x)/a)])/(3*
a*(x*(a + b*x))^(1/4))
```

Rubi [A] (verified)

Time = 0.39 (sec) , antiderivative size = 90, normalized size of antiderivative = 1.30, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.267, Rules used = {1159, 1093, 1090, 226}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x}{(ax+bx^2)^{5/4}}dx$$

$$\Big\downarrow \text{1159}$$

$$\frac{4x}{a\sqrt[4]{ax+bx^2}}-\frac{2\int \frac{1}{\sqrt[4]{bx^2+ax}}dx}{a}$$

$$\Big\downarrow \text{1093}$$

$$\frac{4x}{a\sqrt[4]{ax+bx^2}}-\frac{2\sqrt[4]{-\frac{b(ax+bx^2)}{a^2}}\int \frac{1}{\sqrt[4]{-\frac{b^2x^2}{a^2}-\frac{bx}{a}}}dx}{a\sqrt[4]{ax+bx^2}}$$

$$\Big\downarrow \text{1090}$$

$$\frac{\sqrt{2}a\sqrt[4]{-\frac{b(ax+bx^2)}{a^2}}\int \frac{1}{\sqrt[4]{1-\frac{a^2\left(-\frac{2xb^2}{a^2}-\frac{b}{a}\right)^2}{b^2}}}d\left(-\frac{2xb^2}{a^2}-\frac{b}{a}\right)}{b^2\sqrt[4]{ax+bx^2}}+\frac{4x}{a\sqrt[4]{ax+bx^2}}$$

$$\Big\downarrow \text{226}$$

$$\frac{2\sqrt{2}\sqrt[4]{-\frac{b(ax+bx^2)}{a^2}}E\left(\frac{1}{2}\arcsin\left(\frac{a\left(-\frac{2xb^2}{a^2}-\frac{b}{a}\right)}{b}\right)\middle|2\right)}{b\sqrt[4]{ax+bx^2}}+\frac{4x}{a\sqrt[4]{ax+bx^2}}$$

```
Int[x/(a*x + b*x^2)^(5/4),x]
```

```
(4*x)/(a*(a*x + b*x^2)^(1/4)) + (2*Sqrt[2]*(-((b*(a*x + b*x^2))/a^2))^(1/4
)*EllipticE[ArcSin[(a*(-(b/a) - (2*b^2*x)/a^2))/b]/2, 2])/(b*(a*x + b*x^2)
^(1/4))
```

Defintions of rubi rules used

```
Int[((a_) + (b_.)*(x_)^2)^(-1/4), x_Symbol] :> Simp[(2/(a^(1/4)*Rt[-b/a, 2]
))*EllipticE[(1/2)*ArcSin[Rt[-b/a, 2]*x], 2], x] /; FreeQ[{a, b}, x] && GtQ
[a, 0] && NegQ[b/a]
```

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[1/(2*c*(-4*
(c/(b^2 - 4*a*c)))^p)   Subst[Int[Simp[1 - x^2/(b^2 - 4*a*c), x]^p, x], x,
b + 2*c*x], x] /; FreeQ[{a, b, c, p}, x] && GtQ[4*a - b^2/c, 0]
```

```
Int[((b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(b*x + c*x^2)^p/((-
c)*((b*x + c*x^2)/b^2))^p   Int[((-c)*(x/b) - c^2*(x^2/b^2))^p, x], x] /; F
reeQ[{b, c}, x] && (IntegerQ[4*p] || IntegerQ[3*p])
```

```
Int[((d_.) + (e_.)*(x_))*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol
] :> Simp[((b*d - 2*a*e + (2*c*d - b*e)*x)/((p + 1)*(b^2 - 4*a*c)))*(a + b*
x + c*x^2)^(p + 1), x] - Simp[(2*p + 3)*((2*c*d - b*e)/((p + 1)*(b^2 - 4*a*
c)))   Int[(a + b*x + c*x^2)^(p + 1), x], x] /; FreeQ[{a, b, c, d, e}, x] &
& LtQ[p, -1] && NeQ[p, -3/2]
```

Maple [F]

$$\int \frac{x}{\left(b\,x^{2}+ax\right)^{\frac{5}{4}}}dx$$

```
int(x/(b*x^2+a*x)^(5/4),x)
```

```
int(x/(b*x^2+a*x)^(5/4),x)
```

Fricas [F]

$$\int \frac{x}{(ax+bx^2)^{5/4}}\,dx = \int \frac{x}{(bx^2+ax)^{\frac{5}{4}}}\,dx$$

```
integrate(x/(b*x^2+a*x)^(5/4),x, algorithm="fricas")
```

```
integral((b*x^2 + a*x)^(3/4)/(b^2*x^3 + 2*a*b*x^2 + a^2*x), x)
```

Sympy [F]

$$\int \frac{x}{(ax+bx^2)^{5/4}}\,dx = \int \frac{x}{(x\,(a+bx))^{\frac{5}{4}}}\,dx$$

```
integrate(x/(b*x**2+a*x)**(5/4),x)
```

```
Integral(x/(x*(a + b*x))**(5/4), x)
```

Maxima [F]

$$\int \frac{x}{(ax+bx^2)^{5/4}}\,dx = \int \frac{x}{(bx^2+ax)^{\frac{5}{4}}}\,dx$$

```
integrate(x/(b*x^2+a*x)^(5/4),x, algorithm="maxima")
```

```
integrate(x/(b*x^2 + a*x)^(5/4), x)
```

Giac [F]

$$\int \frac{x}{(ax+bx^2)^{5/4}}\,dx = \int \frac{x}{(bx^2+ax)^{\frac{5}{4}}}\,dx$$

```
integrate(x/(b*x^2+a*x)^(5/4),x, algorithm="giac")
```

```
integrate(x/(b*x^2 + a*x)^(5/4), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{x}{(ax+bx^2)^{5/4}}\,dx = \int \frac{x}{(b\,x^2+a\,x)^{5/4}}\,dx$$

```
int(x/(a*x + b*x^2)^(5/4),x)
```

```
int(x/(a*x + b*x^2)^(5/4), x)
```

Reduce [F]

$$\int \frac{x}{(ax+bx^2)^{5/4}}\,dx = \int \frac{1}{x^{\frac{1}{4}}\,(bx+a)^{\frac{1}{4}}\,a + x^{\frac{5}{4}}\,(bx+a)^{\frac{1}{4}}\,b}dx$$

```
int(x/(b*x^2+a*x)^(5/4),x)
```

```
int(1/(x**(1/4)*(a + b*x)**(1/4)*a + x**(1/4)*(a + b*x)**(1/4)*b*x),x)
```

3.194 $\int \frac{1}{(ax+bx^2)^{5/4}}\,dx$

Optimal result

Integrand size = 13, antiderivative size = 84

$$\int \frac{1}{(ax+bx^2)^{5/4}}\,dx = -\frac{4}{a\sqrt[4]{ax+bx^2}}+\frac{8\sqrt[4]{\frac{bx}{a+bx}}\sqrt{a+bx}\,E\left(\frac{1}{2}\arcsin\left(\frac{\sqrt{a}}{\sqrt{a+bx}}\right)\middle|2\right)}{a^{3/2}\sqrt[4]{ax+bx^2}}$$

```
-4/a/(b*x^2+a*x)^(1/4)+8*(b*x/(b*x+a))^(1/4)*(b*x+a)^(1/2)*EllipticE(sin(1
/2*arcsin(a^(1/2)/(b*x+a)^(1/2))),2^(1/2))/a^(3/2)/(b*x^2+a*x)^(1/4)
```

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 10.01 (sec) , antiderivative size = 45, normalized size of antiderivative = 0.54

$$\int \frac{1}{(ax+bx^2)^{5/4}}\,dx = -\frac{4\sqrt[4]{1+\frac{bx}{a}}\,\text{Hypergeometric2F1}\left(-\frac{1}{4},\frac{5}{4},\frac{3}{4},-\frac{bx}{a}\right)}{a\sqrt[4]{x(a+bx)}}$$

```
Integrate[(a*x + b*x^2)^(-5/4),x]
```

```
(-4*(1 + (b*x)/a)^(1/4)*Hypergeometric2F1[-1/4, 5/4, 3/4, -((b*x)/a)])/(a*
(x*(a + b*x))^(1/4))
```

Rubi [A] (verified)

Time = 0.39 (sec) , antiderivative size = 95, normalized size of antiderivative = 1.13, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.308, Rules used = {1089, 1093, 1090, 226}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{(ax+bx^2)^{5/4}}dx$$

$$\Big\downarrow \text{1089}$$

$$\frac{4b\int \frac{1}{\sqrt[4]{bx^2+ax}}dx}{a^2}-\frac{4(a+2bx)}{a^2\sqrt[4]{ax+bx^2}}$$

$$\Big\downarrow \text{1093}$$

$$\frac{4b\sqrt[4]{-\frac{b(ax+bx^2)}{a^2}}\int \frac{1}{\sqrt[4]{-\frac{b^2x^2}{a^2}-\frac{bx}{a}}}dx}{a^2\sqrt[4]{ax+bx^2}}-\frac{4(a+2bx)}{a^2\sqrt[4]{ax+bx^2}}$$

$$\Big\downarrow \text{1090}$$

$$-\frac{2\sqrt{2}\sqrt[4]{-\frac{b(ax+bx^2)}{a^2}}\int \frac{1}{\sqrt[4]{1-\frac{a^2\left(-\frac{2xb^2}{a^2}-\frac{b}{a}\right)^2}{b^2}}}d\left(-\frac{2xb^2}{a^2}-\frac{b}{a}\right)}{b\sqrt[4]{ax+bx^2}}-\frac{4(a+2bx)}{a^2\sqrt[4]{ax+bx^2}}$$

$$\Big\downarrow \text{226}$$

$$-\frac{4\sqrt{2}\sqrt[4]{-\frac{b(ax+bx^2)}{a^2}}E\left(\frac{1}{2}\arcsin\left(\frac{a\left(-\frac{2xb^2}{a^2}-\frac{b}{a}\right)}{b}\right)\middle|2\right)}{a\sqrt[4]{ax+bx^2}}-\frac{4(a+2bx)}{a^2\sqrt[4]{ax+bx^2}}$$

```
Int[(a*x + b*x^2)^(-5/4),x]
```

```
(-4*(a + 2*b*x))/(a^2*(a*x + b*x^2)^(1/4)) - (4*Sqrt[2]*(-((b*(a*x + b*x^2
))/a^2))^(1/4)*EllipticE[ArcSin[(a*(-(b/a) - (2*b^2*x)/a^2))/b]/2, 2])/(a*
(a*x + b*x^2)^(1/4))
```

Defintions of rubi rules used

```
Int[((a_) + (b_.)*(x_)^2)^(-1/4), x_Symbol] :> Simp[(2/(a^(1/4)*Rt[-b/a, 2]
))*EllipticE[(1/2)*ArcSin[Rt[-b/a, 2]*x], 2], x] /; FreeQ[{a, b}, x] && GtQ
[a, 0] && NegQ[b/a]
```

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(b + 2*c*x)
*((a + b*x + c*x^2)^(p + 1)/((p + 1)*(b^2 - 4*a*c))), x] - Simp[2*c*((2*p +
 3)/((p + 1)*(b^2 - 4*a*c)))   Int[(a + b*x + c*x^2)^(p + 1), x], x] /; Fre
eQ[{a, b, c}, x] && LtQ[p, -1] && (IntegerQ[4*p] || IntegerQ[3*p])
```

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[1/(2*c*(-4*
(c/(b^2 - 4*a*c)))^p)   Subst[Int[Simp[1 - x^2/(b^2 - 4*a*c), x]^p, x], x,
b + 2*c*x], x] /; FreeQ[{a, b, c, p}, x] && GtQ[4*a - b^2/c, 0]
```

```
Int[((b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(b*x + c*x^2)^p/((-
c)*((b*x + c*x^2)/b^2))^p   Int[((-c)*(x/b) - c^2*(x^2/b^2))^p, x], x] /; F
reeQ[{b, c}, x] && (IntegerQ[4*p] || IntegerQ[3*p])
```

Maple [F]

$$\int \frac{1}{(b\,x^2+ax)^{\frac{5}{4}}}dx$$

```
int(1/(b*x^2+a*x)^(5/4),x)
```

```
int(1/(b*x^2+a*x)^(5/4),x)
```

Fricas [F]

$$\int \frac{1}{(ax+bx^2)^{5/4}}\,dx = \int \frac{1}{(bx^2+ax)^{\frac{5}{4}}}\,dx$$

```
integrate(1/(b*x^2+a*x)^(5/4),x, algorithm="fricas")
```

```
integral((b*x^2 + a*x)^(3/4)/(b^2*x^4 + 2*a*b*x^3 + a^2*x^2), x)
```

Sympy [F]

$$\int \frac{1}{(ax+bx^2)^{5/4}}\,dx = \int \frac{1}{(ax+bx^2)^{\frac{5}{4}}}\,dx$$

```
integrate(1/(b*x**2+a*x)**(5/4),x)
```

```
Integral((a*x + b*x**2)**(-5/4), x)
```

Maxima [F]

$$\int \frac{1}{(ax+bx^2)^{5/4}}\,dx = \int \frac{1}{(bx^2+ax)^{\frac{5}{4}}}\,dx$$

```
integrate(1/(b*x^2+a*x)^(5/4),x, algorithm="maxima")
```

```
integrate((b*x^2 + a*x)^(-5/4), x)
```

Giac [F]

$$\int \frac{1}{(ax+bx^2)^{5/4}}\,dx = \int \frac{1}{(bx^2+ax)^{\frac{5}{4}}}\,dx$$

```
integrate(1/(b*x^2+a*x)^(5/4),x, algorithm="giac")
```

```
integrate((b*x^2 + a*x)^(-5/4), x)
```

Mupad [B] (verification not implemented)

Time = 8.75 (sec) , antiderivative size = 36, normalized size of antiderivative = 0.43

$$\int \frac{1}{(ax+bx^2)^{5/4}}\,dx = -\frac{4\,x\,\left(\frac{b\,x}{a}+1\right)^{5/4}\,{}_2\mathrm{F}_1\left(-\frac{1}{4},\frac{5}{4};\ \frac{3}{4};\ -\frac{b\,x}{a}\right)}{\left(b\,x^2+a\,x\right)^{5/4}}$$

```
int(1/(a*x + b*x^2)^(5/4),x)
```

```
-(4*x*((b*x)/a + 1)^(5/4)*hypergeom([-1/4, 5/4], 3/4, -(b*x)/a))/(a*x + b*
x^2)^(5/4)
```

Reduce [B] (verification not implemented)

Time = 0.51 (sec) , antiderivative size = 27, normalized size of antiderivative = 0.32

$$\int \frac{1}{(ax+bx^2)^{5/4}}\,dx = \frac{4x^{\frac{1}{4}}(bx+a)^{\frac{1}{4}}}{\sqrt{x}\,\sqrt{bx+a}\,a}$$

```
int(1/(b*x^2+a*x)^(5/4),x)
```

```
(4*x**(1/4)*(a + b*x)**(1/4))/(sqrt(x)*sqrt(a + b*x)*a)
```

3.195 $\int \frac{1}{x(ax+bx^2)^{5/4}}\,dx$

Optimal result

Integrand size = 17, antiderivative size = 134

$$\begin{aligned}\int \frac{1}{x\left(ax+bx^2\right)^{5/4}}\,dx &= \frac{48b}{5a^2\sqrt[4]{ax+bx^2}}+\frac{4}{ax\sqrt[4]{ax+bx^2}}\\&\quad-\frac{24(ax+bx^2)^{3/4}}{5a^2x^2}-\frac{48b\sqrt[4]{\frac{bx}{a+bx}}\sqrt{a+bx}\,E\left(\frac{1}{2}\arcsin\left(\frac{\sqrt{a}}{\sqrt{a+bx}}\right)\middle|2\right)}{5a^{5/2}\sqrt[4]{ax+bx^2}}\end{aligned}$$

```
48/5*b/a^2/(b*x^2+a*x)^(1/4)+4/a/x/(b*x^2+a*x)^(1/4)-24/5*(b*x^2+a*x)^(3/4
)/a^2/x^2-48/5*b*(b*x/(b*x+a))^(1/4)*(b*x+a)^(1/2)*EllipticE(sin(1/2*arcsi
n(a^(1/2)/(b*x+a)^(1/2))),2^(1/2))/a^(5/2)/(b*x^2+a*x)^(1/4)
```

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 10.02 (sec) , antiderivative size = 52, normalized size of antiderivative = 0.39

$$\int \frac{1}{x\left(ax+bx^2\right)^{5/4}}\,dx = -\frac{4(a+bx)\sqrt[4]{1+\frac{bx}{a}}\,\text{Hypergeometric2F1}\left(-\frac{5}{4},\frac{5}{4},-\frac{1}{4},-\frac{bx}{a}\right)}{5a(x(a+bx))^{5/4}}$$

```
Integrate[1/(x*(a*x + b*x^2)^(5/4)),x]
```

```
(-4*(a + b*x)*(1 + (b*x)/a)^(1/4)*Hypergeometric2F1[-5/4, 5/4, -1/4, -((b*
x)/a)])/(5*a*(x*(a + b*x))^(5/4))
```

Rubi [A] (warning: unable to verify)

Time = 0.56 (sec) , antiderivative size = 193, normalized size of antiderivative = 1.44, number of steps used = 11, number of rules used = 10, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.588, Rules used = {1137, 61, 61, 61, 73, 839, 813, 858, 807, 212}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{x\left(ax+bx^2\right)^{5/4}}dx$$

$$\Big\downarrow \; 1137$$

$$\frac{x^{5/4}(a+bx)^{5/4}\int \frac{1}{x^{9/4}(a+bx)^{5/4}}dx}{\left(ax+bx^2\right)^{5/4}}$$

$$\Big\downarrow \; 61$$

$$\frac{x^{5/4}(a+bx)^{5/4}\left(\frac{6\int \frac{1}{x^{9/4}\sqrt[4]{a+bx}}dx}{a}+\frac{4}{ax^{5/4}\sqrt[4]{a+bx}}\right)}{\left(ax+bx^2\right)^{5/4}}$$

$$\Big\downarrow \; 61$$

$$\frac{x^{5/4}(a+bx)^{5/4}\left(\frac{6\left(-\frac{2b\int \frac{1}{x^{5/4}\sqrt[4]{a+bx}}dx}{5a}-\frac{4(a+bx)^{3/4}}{5ax^{5/4}}\right)}{a}+\frac{4}{ax^{5/4}\sqrt[4]{a+bx}}\right)}{\left(ax+bx^2\right)^{5/4}}$$

$$\Big\downarrow \; 61$$

$$\frac{x^{5/4}(a+bx)^{5/4}\left(\frac{6\left(-\frac{2b\left(\frac{2b\int \frac{1}{\sqrt[4]{x}\sqrt[4]{a+bx}}dx}{a}-\frac{4(a+bx)^{3/4}}{a\sqrt[4]{x}}\right)}{5a}-\frac{4(a+bx)^{3/4}}{5ax^{5/4}}\right)}{a}+\frac{4}{ax^{5/4}\sqrt[4]{a+bx}}\right)}{(ax+bx^2)^{5/4}}$$

$$\Big\downarrow \; 73$$

$$\frac{x^{5/4}(a+bx)^{5/4}\left(\frac{6\left(-\frac{2b\left(\frac{8b\int \frac{\sqrt{x}}{\sqrt[4]{a+bx}}d\sqrt[4]{x}}{a}-\frac{4(a+bx)^{3/4}}{a\sqrt[4]{x}}\right)}{5a}-\frac{4(a+bx)^{3/4}}{5ax^{5/4}}\right)}{a}+\frac{4}{ax^{5/4}\sqrt[4]{a+bx}}\right)}{(ax+bx^2)^{5/4}}$$

$$\Big\downarrow \; 839$$

$$\frac{x^{5/4}(a+bx)^{5/4}\left(\frac{6\left(-\frac{2b\left(\frac{8b\left(\frac{x^{3/4}}{2\sqrt[4]{a+bx}}-\frac{1}{2}a\int \frac{\sqrt{x}}{(a+bx)^{5/4}}d\sqrt[4]{x}\right)}{a}-\frac{4(a+bx)^{3/4}}{a\sqrt[4]{x}}\right)}{5a}-\frac{4(a+bx)^{3/4}}{5ax^{5/4}}\right)}{a}+\frac{4}{ax^{5/4}\sqrt[4]{a+bx}}\right)}{(ax+bx^2)^{5/4}}$$

$$\Big\downarrow \text{813}$$

$$\frac{x^{5/4}(a+bx)^{5/4}\left(\frac{6\left(-\frac{2b\left(\frac{8b\left(\frac{x^{3/4}}{2\sqrt[4]{a+bx}}-\frac{a\sqrt[4]{x}\sqrt[4]{\frac{a}{bx}+1}\int\frac{1}{\left(\frac{a}{bx}+1\right)^{5/4}x^{3/4}}d\sqrt[4]{x}}{2b\sqrt[4]{a+bx}}\right)}{a}-\frac{4(a+bx)^{3/4}}{a\sqrt[4]{x}}\right)}{5a}-\frac{4(a+bx)^{3/4}}{5ax^{5/4}}\right)}{a}+\frac{4}{ax^{5/4}\sqrt[4]{a+b}}\right.}{(ax+bx^2)^{5/4}}$$

$$\Big\downarrow \text{858}$$

$$\frac{x^{5/4}(a+bx)^{5/4}\left(\frac{6\left(-\frac{2b\left(\frac{8b\left(\frac{a\sqrt[4]{x}\sqrt[4]{\frac{a}{bx}+1}\int \frac{1}{\sqrt[4]{x}\left(\frac{ax}{b}+1\right)^{5/4}}d\frac{1}{\sqrt[4]{x}}}{2b\sqrt[4]{a+bx}}+\frac{x^{3/4}}{2\sqrt[4]{a+bx}}\right)}{a}-\frac{4(a+bx)^{3/4}}{a\sqrt[4]{x}}\right)}{5a}-\frac{4(a+bx)^{3/4}}{5ax^{5/4}}\right)}{a}+\frac{4}{ax^{5/4}\sqrt[4]{a+}}\right.}{(ax+bx^2)^{5/4}}$$

$\Big\downarrow$ 807

$$\frac{x^{5/4}(a+bx)^{5/4}\left(\frac{6\left(-\frac{2b\left(\frac{8b\left(\frac{a\sqrt[4]{x}\sqrt[4]{\frac{a}{bx}+1}\int \frac{1}{\left(\frac{\sqrt{x}a}{b}+1\right)^{5/4}}d\sqrt{x}}{4b\sqrt[4]{a+bx}}+\frac{x^{3/4}}{2\sqrt[4]{a+bx}}\right)}{a}-\frac{4(a+bx)^{3/4}}{a\sqrt[4]{x}}\right)}{5a}-\frac{4(a+bx)^{3/4}}{5ax^{5/4}}\right)}{a}+\frac{4}{ax^{5/4}\sqrt[4]{a+bx}}\right)}{(ax+bx^2)^{5/4}}$$

$\downarrow$ 212

$$\frac{x^{5/4}(a+bx)^{5/4}\left(\frac{6\left(-\frac{2b\left(\frac{8b\left(\frac{\sqrt{a}\sqrt[4]{x}\sqrt[4]{\frac{a}{bx}+1}E\left(\frac{1}{2}\arctan\left(\frac{\sqrt{a}\sqrt{x}}{\sqrt{b}}\right)\middle|2\right)}{2\sqrt{b}\sqrt[4]{a+bx}}+\frac{x^{3/4}}{2\sqrt[4]{a+bx}}\right)}{a}-\frac{4(a+bx)^{3/4}}{a\sqrt[4]{x}}\right)}{5a}-\frac{4(a+bx)^{3/4}}{5ax^{5/4}}\right)}{a}+\frac{4}{ax^{5/4}\sqrt[4]{a+bx}}\right)}{(ax+bx^2)^{5/4}}$$

```
Int[1/(x*(a*x + b*x^2)^(5/4)),x]
```

```
(x^(5/4)*(a + b*x)^(5/4)*(4/(a*x^(5/4)*(a + b*x)^(1/4)) + (6*((-4*(a + b*x
)^(3/4))/(5*a*x^(5/4)) - (2*b*((-4*(a + b*x)^(3/4))/(a*x^(1/4)) + (8*b*(x^
(3/4)/(2*(a + b*x)^(1/4)) + (Sqrt[a]*(1 + a/(b*x))^(1/4)*x^(1/4)*EllipticE
[ArcTan[(Sqrt[a]*Sqrt[x])/Sqrt[b]]/2, 2])/(2*Sqrt[b]*(a + b*x)^(1/4))))/a)
)/(5*a)))/a))/(a*x + b*x^2)^(5/4)
```

Defintions of rubi rules used

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] - Simp[d*((
m + n + 2)/((b*c - a*d)*(m + 1)))   Int[(a + b*x)^(m + 1)*(c + d*x)^n, x],
x] /; FreeQ[{a, b, c, d, n}, x] && LtQ[m, -1] &&  !(LtQ[n, -1] && (EqQ[a, 0
] || (NeQ[c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && IntLinearQ[a, b, c, d
, m, n, x]
```

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Simp[p/b   Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
 d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && Lt
Q[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntL
inearQ[a, b, c, d, m, n, x]
```

```
Int[((a_) + (b_.)*(x_)^2)^(-5/4), x_Symbol] :> Simp[(2/(a^(5/4)*Rt[b/a, 2])
)*EllipticE[(1/2)*ArcTan[Rt[b/a, 2]*x], 2], x] /; FreeQ[{a, b}, x] && GtQ[a
, 0] && PosQ[b/a]
```

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k = GCD[m
+ 1, n]}, Simp[1/k   Subst[Int[x^((m + 1)/k - 1)*(a + b*x^(n/k))^p, x], x,
x^k], x] /; k != 1] /; FreeQ[{a, b, p}, x] && IGtQ[n, 0] && IntegerQ[m]
```

```
Int[(x_)^2/((a_) + (b_.)*(x_)^4)^(5/4), x_Symbol] :> Simp[x*((1 + a/(b*x^4)
)^(1/4)/(b*(a + b*x^4)^(1/4)))   Int[1/(x^3*(1 + a/(b*x^4))^(5/4)), x], x]
/; FreeQ[{a, b}, x] && PosQ[b/a]
```

```
Int[(x_)^2/((a_) + (b_.)*(x_)^4)^(1/4), x_Symbol] :> Simp[x^3/(2*(a + b*x^4
)^(1/4)), x] - Simp[a/2   Int[x^2/(a + b*x^4)^(5/4), x], x] /; FreeQ[{a, b}
, x] && PosQ[b/a]
```

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> -Subst[Int[(a +
b/x^n)^p/x^(m + 2), x], x, 1/x] /; FreeQ[{a, b, p}, x] && ILtQ[n, 0] && Int
egerQ[m]
```

```
Int[((e_.)*(x_))^(m_)*((b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(
e*x)^m*((b*x + c*x^2)^p/(x^(m + p)*(b + c*x)^p))   Int[x^(m + p)*(b + c*x)^
p, x], x] /; FreeQ[{b, c, e, m}, x]
```

Maple [F]

$$\int \frac{1}{x\left(b\,x^{2}+ax\right)^{\frac{5}{4}}}dx$$

```
int(1/x/(b*x^2+a*x)^(5/4),x)
```

```
int(1/x/(b*x^2+a*x)^(5/4),x)
```

Fricas [F]

$$\int \frac{1}{x\left(ax+bx^2\right)^{5/4}}\,dx=\int \frac{1}{\left(bx^2+ax\right)^{\frac{5}{4}}x}\,dx$$

```
integrate(1/x/(b*x^2+a*x)^(5/4),x, algorithm="fricas")
```

```
integral((b*x^2 + a*x)^(3/4)/(b^2*x^5 + 2*a*b*x^4 + a^2*x^3), x)
```

Sympy [F]

$$\int \frac{1}{x\left(ax+bx^2\right)^{5/4}}\,dx=\int \frac{1}{x\left(x\left(a+bx\right)\right)^{\frac{5}{4}}}\,dx$$

```
integrate(1/x/(b*x**2+a*x)**(5/4),x)
```

```
Integral(1/(x*(x*(a + b*x))**(5/4)), x)
```

Maxima [F]

$$\int \frac{1}{x\,(ax+bx^2)^{5/4}}\,dx = \int \frac{1}{(bx^2+ax)^{\frac{5}{4}}x}\,dx$$

```
integrate(1/x/(b*x^2+a*x)^(5/4),x, algorithm="maxima")
```

```
integrate(1/((b*x^2 + a*x)^(5/4)*x), x)
```

Giac [F]

$$\int \frac{1}{x\,(ax+bx^2)^{5/4}}\,dx = \int \frac{1}{(bx^2+ax)^{\frac{5}{4}}x}\,dx$$

```
integrate(1/x/(b*x^2+a*x)^(5/4),x, algorithm="giac")
```

```
integrate(1/((b*x^2 + a*x)^(5/4)*x), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{x\,(ax+bx^2)^{5/4}}\,dx = \int \frac{1}{x\,(b\,x^2+a\,x)^{5/4}}\,dx$$

```
int(1/(x*(a*x + b*x^2)^(5/4)),x)
```

```
int(1/(x*(a*x + b*x^2)^(5/4)), x)
```

Reduce [B] (verification not implemented)

Time = 0.61 (sec) , antiderivative size = 35, normalized size of antiderivative = 0.26

$$\int \frac{1}{x\left(ax+bx^2\right)^{5/4}}\,dx = \frac{4(bx+a)^{\frac{1}{4}}\left(-4bx-a\right)}{3x^{\frac{3}{4}}\sqrt{x}\,\sqrt{bx+a}\,a^2}$$

```
int(1/x/(b*x^2+a*x)^(5/4),x)
```

```
(4*x**(1/4)*(a + b*x)**(1/4)*( - a - 4*b*x))/(3*sqrt(x)*sqrt(a + b*x)*a**2
*x)
```

3.196 $\int \frac{1}{x^2(ax+bx^2)^{5/4}}\,dx$

Optimal result

Integrand size = 17, antiderivative size = 162

$$\int \frac{1}{x^2(ax+bx^2)^{5/4}}\,dx=-\frac{32b^2}{3a^3\sqrt[4]{ax+bx^2}}+\frac{4}{ax^2\sqrt[4]{ax+bx^2}}-\frac{40(ax+bx^2)^{3/4}}{9a^2x^3}+\frac{16b(ax+bx^2)^{3/4}}{3a^3x^2}+\frac{32b^2\sqrt[4]{\frac{bx}{a+bx}}\sqrt{a+bx}\,E\left(\frac{1}{2}\arcsin\left(\frac{\sqrt{a}}{\sqrt{a+bx}}\right)\middle|2\right)}{3a^{7/2}\sqrt[4]{ax+bx^2}}$$

```
-32/3*b^2/a^3/(b*x^2+a*x)^(1/4)+4/a/x^2/(b*x^2+a*x)^(1/4)-40/9*(b*x^2+a*x)
^(3/4)/a^2/x^3+16/3*b*(b*x^2+a*x)^(3/4)/a^3/x^2+32/3*b^2*(b*x/(b*x+a))^(1/
4)*(b*x+a)^(1/2)*EllipticE(sin(1/2*arcsin(a^(1/2)/(b*x+a)^(1/2))),2^(1/2))
/a^(7/2)/(b*x^2+a*x)^(1/4)
```

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 10.01 (sec) , antiderivative size = 50, normalized size of antiderivative = 0.31

$$\int \frac{1}{x^2(ax+bx^2)^{5/4}}\,dx=-\frac{4\sqrt[4]{1+\frac{bx}{a}}\,\text{Hypergeometric2F1}\left(-\frac{9}{4},\frac{5}{4},-\frac{5}{4},-\frac{bx}{a}\right)}{9ax^2\sqrt[4]{x(a+bx)}}$$

```
Integrate[1/(x^2*(a*x + b*x^2)^(5/4)),x]
```

```
(-4*(1 + (b*x)/a)^(1/4)*Hypergeometric2F1[-9/4, 5/4, -5/4, -((b*x)/a)])/(9
*a*x^2*(x*(a + b*x))^(1/4))
```

Rubi [A] (warning: unable to verify)

Time = 0.61 (sec) , antiderivative size = 223, normalized size of antiderivative = 1.38, number of steps used = 12, number of rules used = 11, $\frac{\text{number of rules}}{\text{integrand size}} = 0.647$, Rules used = {1137, 61, 61, 61, 61, 73, 839, 813, 858, 807, 212}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{x^2 (ax+bx^2)^{5/4}} \, dx$$

$$\Big\downarrow \text{ 1137}$$

$$\frac{x^{5/4}(a+bx)^{5/4}\int \frac{1}{x^{13/4}(a+bx)^{5/4}}dx}{(ax+bx^2)^{5/4}}$$

$$\Big\downarrow \text{ 61}$$

$$\frac{x^{5/4}(a+bx)^{5/4}\left(\frac{10\int \frac{1}{x^{13/4}\sqrt[4]{a+bx}}dx}{a}+\frac{4}{ax^{9/4}\sqrt[4]{a+bx}}\right)}{(ax+bx^2)^{5/4}}$$

$$\Big\downarrow \text{ 61}$$

$$\frac{x^{5/4}(a+bx)^{5/4}\left(\frac{10\left(-\frac{2b\int \frac{1}{x^{9/4}\sqrt[4]{a+bx}}dx}{3a}-\frac{4(a+bx)^{3/4}}{9ax^{9/4}}\right)}{a}+\frac{4}{ax^{9/4}\sqrt[4]{a+bx}}\right)}{(ax+bx^2)^{5/4}}$$

$$\Big\downarrow \text{ 61}$$

$$\frac{x^{5/4}(a+bx)^{5/4}\left(\frac{10\left(-\frac{2b\left(-\frac{2b\int\frac{1}{x^{5/4}\sqrt[4]{a+bx}}dx}{5a}-\frac{4(a+bx)^{3/4}}{5ax^{5/4}}\right)}{3a}-\frac{4(a+bx)^{3/4}}{9ax^{9/4}}\right)}{a}+\frac{4}{ax^{9/4}\sqrt[4]{a+bx}}\right)}{(ax+bx^2)^{5/4}}$$

$$\Big\downarrow \; 61$$

$$\frac{x^{5/4}(a+bx)^{5/4}\left(\frac{10\left(-\frac{2b\left(-\frac{2b\left(\frac{2b\int\frac{1}{\sqrt[4]{x}\sqrt[4]{a+bx}}dx}{a}-\frac{4(a+bx)^{3/4}}{a\sqrt[4]{x}}\right)}{5a}-\frac{4(a+bx)^{3/4}}{5ax^{5/4}}\right)}{3a}-\frac{4(a+bx)^{3/4}}{9ax^{9/4}}\right)}{a}+\frac{4}{ax^{9/4}\sqrt[4]{a+bx}}\right)}{(ax+bx^2)^{5/4}}$$

$$\Big\downarrow \; 73$$

$$\frac{x^{5/4}(a+bx)^{5/4}\left(\frac{10\left(-\frac{2b\left(-\frac{2b\left(\frac{8b\int\frac{\sqrt{x}}{\sqrt[4]{a+bx}}d\sqrt[4]{x}}{a}-\frac{4(a+bx)^{3/4}}{a\sqrt[4]{x}}\right)}{5a}-\frac{4(a+bx)^{3/4}}{5ax^{5/4}}\right)}{3a}-\frac{4(a+bx)^{3/4}}{9ax^{9/4}}\right)}{a}+\frac{4}{ax^{9/4}\sqrt[4]{a+bx}}\right)}{(ax+bx^2)^{5/4}}$$

$\Big\downarrow$ 839

$$\frac{x^{5/4}(a+bx)^{5/4}\left(\frac{10\left(-\frac{2b\left(-\frac{2b\left(\frac{8b\left(\frac{x^{3/4}}{2\sqrt[4]{a+bx}}-\frac{1}{2}a\int \frac{\sqrt{x}}{(a+bx)^{5/4}}d\sqrt[4]{x}\right)}{a}-\frac{4(a+bx)^{3/4}}{a\sqrt[4]{x}}\right)}{5a}-\frac{4(a+bx)^{3/4}}{5ax^{5/4}}\right)}{3a}-\frac{4(a+bx)^{3/4}}{9ax^{9/4}}\right)}{a}+\frac{4}{ax^{9/4}\sqrt[4]{a+b}}\right.}{(ax+bx^2)^{5/4}}$$

$\Big\downarrow$ 813

$$x^{5/4}(a+bx)^{5/4}\left(\frac{10\left(-\frac{2b\left(-\frac{2b\left(\frac{8b\left(\frac{x^{3/4}}{2\sqrt[4]{a+bx}}-\frac{a\sqrt[4]{x}\sqrt[4]{\frac{a}{bx}+1}\int\frac{1}{\left(\frac{a}{bx}+1\right)^{5/4}x^{3/4}}\,d\sqrt[4]{x}}{2b\sqrt[4]{a+bx}}\right)}{a}-\frac{4(a+bx)^{3/4}}{a\sqrt[4]{x}}\right)}{5a}-\frac{4(a+bx)^{3/4}}{5ax^{5/4}}\right)}{3a}-\frac{4(a+bx)^{3/4}}{9ax^{9/4}}\right.}{a}\right.$$

↓ 858

$$x^{5/4}(a+bx)^{5/4}\left(\frac{10\left(-\frac{2b\left(-\frac{2b\left(\frac{8b\left(\frac{a\sqrt[4]{x}\sqrt[4]{\frac{a}{bx}+1}\int\frac{1}{\sqrt[4]{x}\left(\frac{ax}{b}+1\right)^{5/4}}d\frac{1}{\sqrt[4]{x}}}{2b\sqrt[4]{a+bx}}+\frac{x^{3/4}}{2\sqrt[4]{a+bx}}\right)}{a}-\frac{4(a+bx)^{3/4}}{a\sqrt[4]{x}}\right)}{5a}-\frac{4(a+bx)^{3/4}}{5ax^{5/4}}\right)}{3a}-\frac{4(a+bx)^{3/4}}{9ax^{9/4}}\right.}{a}\right.$$

$\downarrow$ 807

$$x^{5/4}(a+bx)^{5/4}\left(\frac{10\left(-\frac{2b\left(-\frac{2b\left(\frac{8b\left(\frac{a\sqrt[4]{x}\sqrt[4]{\frac{a}{bx}+1}\int\frac{1}{\left(\frac{\sqrt{x}a}{b}+1\right)^{5/4}}\,d\sqrt{x}}{4b\sqrt[4]{a+bx}}+\frac{x^{3/4}}{2\sqrt[4]{a+bx}}\right)}{a}-\frac{4(a+bx)^{3/4}}{a\sqrt[4]{x}}\right)}{5a}-\frac{4(a+bx)^{3/4}}{5ax^{5/4}}\right)}{3a}-\frac{4(a+bx)^{3/4}}{9ax^{9/4}}\right)}{a}\right..$$

$\downarrow$ 212

$$x^{5/4}(a+bx)^{5/4}\left(\frac{10\left(-\frac{2b\left(-\frac{2b\left(\frac{8b\left(\frac{\sqrt{a}\sqrt[4]{x}\sqrt[4]{\frac{a}{bx}+1}E\left(\frac{1}{2}\arctan\left(\frac{\sqrt{a}\sqrt{x}}{\sqrt{b}}\right)\middle|2\right)}{2\sqrt{b}\sqrt[4]{a+bx}}+\frac{x^{3/4}}{2\sqrt[4]{a+bx}}\right)}{a}-\frac{4(a+bx)^{3/4}}{a\sqrt[4]{x}}\right)}{5a}-\frac{4(a+bx)^{3/4}}{5ax^{5/4}}\right)}{3a}-\frac{4(a+bx)^{3/}}{9ax^{9/4}}\right.}{a}\right.$$

```
Int[1/(x^2*(a*x + b*x^2)^(5/4)),x]
```

```
(x^(5/4)*(a + b*x)^(5/4)*(4/(a*x^(9/4)*(a + b*x)^(1/4)) + (10*((-4*(a + b*
x)^(3/4))/(9*a*x^(9/4)) - (2*b*((-4*(a + b*x)^(3/4))/(5*a*x^(5/4)) - (2*b*
((-4*(a + b*x)^(3/4))/(a*x^(1/4)) + (8*b*(x^(3/4)/(2*(a + b*x)^(1/4)) + (S
qrt[a]*(1 + a/(b*x))^(1/4)*x^(1/4)*EllipticE[ArcTan[(Sqrt[a]*Sqrt[x])/Sqrt
[b]]/2, 2])/(2*Sqrt[b]*(a + b*x)^(1/4))))/a))/(5*a)))/(3*a)))/a))/(a*x + b
*x^2)^(5/4)
```

Defintions of rubi rules used

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] - Simp[d*((
m + n + 2)/((b*c - a*d)*(m + 1)))   Int[(a + b*x)^(m + 1)*(c + d*x)^n, x],
x] /; FreeQ[{a, b, c, d, n}, x] && LtQ[m, -1] &&  !(LtQ[n, -1] && (EqQ[a, 0
] || (NeQ[c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && IntLinearQ[a, b, c, d
, m, n, x]
```

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Simp[p/b   Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
 d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && Lt
Q[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntL
inearQ[a, b, c, d, m, n, x]
```

```
Int[((a_) + (b_.)*(x_)^2)^(-5/4), x_Symbol] :> Simp[(2/(a^(5/4)*Rt[b/a, 2])
)*EllipticE[(1/2)*ArcTan[Rt[b/a, 2]*x], 2], x] /; FreeQ[{a, b}, x] && GtQ[a
, 0] && PosQ[b/a]
```

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k = GCD[m
+ 1, n]}, Simp[1/k   Subst[Int[x^((m + 1)/k - 1)*(a + b*x^(n/k))^p, x], x,
x^k], x] /; k != 1] /; FreeQ[{a, b, p}, x] && IGtQ[n, 0] && IntegerQ[m]
```

```
Int[(x_)^2/((a_) + (b_.)*(x_)^4)^(5/4), x_Symbol] :> Simp[x*((1 + a/(b*x^4)
)^(1/4)/(b*(a + b*x^4)^(1/4)))   Int[1/(x^3*(1 + a/(b*x^4))^(5/4)), x], x]
/; FreeQ[{a, b}, x] && PosQ[b/a]
```

```
Int[(x_)^2/((a_) + (b_.)*(x_)^4)^(1/4), x_Symbol] :> Simp[x^3/(2*(a + b*x^4
)^(1/4)), x] - Simp[a/2   Int[x^2/(a + b*x^4)^(5/4), x], x] /; FreeQ[{a, b}
, x] && PosQ[b/a]
```

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> -Subst[Int[(a +
b/x^n)^p/x^(m + 2), x], x, 1/x] /; FreeQ[{a, b, p}, x] && ILtQ[n, 0] && Int
egerQ[m]
```

```
Int[((e_.)*(x_))^(m_)*((b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(
e*x)^m*((b*x + c*x^2)^p/(x^(m + p)*(b + c*x)^p))   Int[x^(m + p)*(b + c*x)^
p, x], x] /; FreeQ[{b, c, e, m}, x]
```

Maple [F]

$$\int \frac{1}{x^{2} \left(b\,x^{2}+a x \right)^{\frac{5}{4}}}d x$$

```
int(1/x^2/(b*x^2+a*x)^(5/4),x)
```

```
int(1/x^2/(b*x^2+a*x)^(5/4),x)
```

Fricas [F]

$$\int \frac{1}{x^2\left(ax+bx^2\right)^{5/4}}\,dx=\int \frac{1}{{\left(b x^{2} + a x\right)}^{\frac{5}{4}} x^{2}}\,{d x}$$

```
integrate(1/x^2/(b*x^2+a*x)^(5/4),x, algorithm="fricas")
```

```
integral((b*x^2 + a*x)^(3/4)/(b^2*x^6 + 2*a*b*x^5 + a^2*x^4), x)
```

Sympy [F]

$$\int \frac{1}{x^2 (ax + bx^2)^{5/4}} \, dx = \int \frac{1}{x^2 (x (a + bx))^{\frac{5}{4}}} \, dx$$

```
integrate(1/x**2/(b*x**2+a*x)**(5/4),x)
```

```
Integral(1/(x**2*(x*(a + b*x))**(5/4)), x)
```

Maxima [F]

$$\int \frac{1}{x^2 (ax + bx^2)^{5/4}} \, dx = \int \frac{1}{(bx^2 + ax)^{\frac{5}{4}} x^2} \, dx$$

```
integrate(1/x^2/(b*x^2+a*x)^(5/4),x, algorithm="maxima")
```

```
integrate(1/((b*x^2 + a*x)^(5/4)*x^2), x)
```

Giac [F]

$$\int \frac{1}{x^2 (ax + bx^2)^{5/4}} \, dx = \int \frac{1}{(bx^2 + ax)^{\frac{5}{4}} x^2} \, dx$$

```
integrate(1/x^2/(b*x^2+a*x)^(5/4),x, algorithm="giac")
```

```
integrate(1/((b*x^2 + a*x)^(5/4)*x^2), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{x^2\left(ax+bx^2\right)^{5/4}}\,dx = \int \frac{1}{x^2\left(b\,x^2+a\,x\right)^{5/4}}\,dx$$

```
int(1/(x^2*(a*x + b*x^2)^(5/4)),x)
```

```
int(1/(x^2*(a*x + b*x^2)^(5/4)), x)
```

Reduce [B] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 46, normalized size of antiderivative = 0.28

$$\int \frac{1}{x^2\left(ax+bx^2\right)^{5/4}}\,dx = \frac{4(bx+a)^{\frac{1}{4}}\left(32b^2x^2+8abx-3a^2\right)}{21x^{\frac{7}{4}}\sqrt{x}\,\sqrt{bx+a}\,a^3}$$

```
int(1/x^2/(b*x^2+a*x)^(5/4),x)
```

```
(4*x**(1/4)*(a + b*x)**(1/4)*( - 3*a**2 + 8*a*b*x + 32*b**2*x**2))/(21*sqr
t(x)*sqrt(a + b*x)*a**3*x**2)
```

3.197 $\int \frac{1}{x^3(ax+bx^2)^{5/4}}\,dx$

Optimal result

Integrand size = 17, antiderivative size = 188

$$\begin{aligned}\int \frac{1}{x^3(ax+bx^2)^{5/4}}\,dx &= \frac{448b^3}{39a^4\sqrt[4]{ax+bx^2}} + \frac{4}{ax^3\sqrt[4]{ax+bx^2}} \\ &\quad - \frac{56(ax+bx^2)^{3/4}}{13a^2x^4} + \frac{560b(ax+bx^2)^{3/4}}{117a^3x^3} - \frac{224b^2(ax+bx^2)^{3/4}}{39a^4x^2} \\ &\quad - \frac{448b^3\sqrt[4]{\frac{bx}{a+bx}}\sqrt{a+bx}\,E\left(\frac{1}{2}\arcsin\left(\frac{\sqrt{a}}{\sqrt{a+bx}}\right)\middle|2\right)}{39a^{9/2}\sqrt[4]{ax+bx^2}}\end{aligned}$$

```
448/39*b^3/a^4/(b*x^2+a*x)^(1/4)+4/a/x^3/(b*x^2+a*x)^(1/4)-56/13*(b*x^2+a*
x)^(3/4)/a^2/x^4+560/117*b*(b*x^2+a*x)^(3/4)/a^3/x^3-224/39*b^2*(b*x^2+a*x
)^(3/4)/a^4/x^2-448/39*b^3*(b*x/(b*x+a))^(1/4)*(b*x+a)^(1/2)*EllipticE(sin
(1/2*arcsin(a^(1/2)/(b*x+a)^(1/2)),2^(1/2))/a^(9/2)/(b*x^2+a*x)^(1/4)
```

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 10.01 (sec) , antiderivative size = 50, normalized size of antiderivative = 0.27

$$\int \frac{1}{x^3\left(ax+bx^2\right)^{5/4}}\,dx = -\frac{4\sqrt[4]{1+\frac{bx}{a}}\,\text{Hypergeometric2F1}\left(-\frac{13}{4},\frac{5}{4},-\frac{9}{4},-\frac{bx}{a}\right)}{13ax^3\sqrt[4]{x(a+bx)}}$$

```
Integrate[1/(x^3*(a*x + b*x^2)^(5/4)),x]
```

```
(-4*(1 + (b*x)/a)^(1/4)*Hypergeometric2F1[-13/4, 5/4, -9/4, -((b*x)/a)])/(
13*a*x^3*(x*(a + b*x))^(1/4))
```

Rubi [A] (warning: unable to verify)

Time = 0.66 (sec) , antiderivative size = 253, normalized size of antiderivative = 1.35, number of steps used = 13, number of rules used = 12, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.706, Rules used = {1137, 61, 61, 61, 61, 61, 73, 839, 813, 858, 807, 212}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{x^3\left(ax+bx^2\right)^{5/4}}dx$$

$$\Big\downarrow \text{1137}$$

$$\frac{x^{5/4}(a+bx)^{5/4}\int \frac{1}{x^{17/4}(a+bx)^{5/4}}dx}{\left(ax+bx^2\right)^{5/4}}$$

$$\Big\downarrow \text{61}$$

$$\frac{x^{5/4}(a+bx)^{5/4}\left(\frac{14\int \frac{1}{x^{17/4}\sqrt[4]{a+bx}}dx}{a}+\frac{4}{ax^{13/4}\sqrt[4]{a+bx}}\right)}{\left(ax+bx^2\right)^{5/4}}$$

$$\Big\downarrow \text{61}$$

$$\frac{x^{5/4}(a+bx)^{5/4}\left(\frac{14\left(-\frac{10b\int\frac{1}{x^{13/4}\sqrt[4]{a+bx}}dx}{13a}-\frac{4(a+bx)^{3/4}}{13ax^{13/4}}\right)}{a}+\frac{4}{ax^{13/4}\sqrt[4]{a+bx}}\right)}{(ax+bx^2)^{5/4}}$$

$\Big\downarrow$ 61

$$\frac{x^{5/4}(a+bx)^{5/4}\left(\frac{14\left(-\frac{10b\left(-\frac{2b\int\frac{1}{x^{9/4}\sqrt[4]{a+bx}}dx}{3a}-\frac{4(a+bx)^{3/4}}{9ax^{9/4}}\right)}{13a}-\frac{4(a+bx)^{3/4}}{13ax^{13/4}}\right)}{a}+\frac{4}{ax^{13/4}\sqrt[4]{a+bx}}\right)}{(ax+bx^2)^{5/4}}$$

$\Big\downarrow$ 61

$$\frac{x^{5/4}(a+bx)^{5/4}\left(\frac{14\left(-\frac{10b\left(-\frac{2b\left(-\frac{2b\int\frac{1}{x^{5/4}\sqrt[4]{a+bx}}dx}{5a}-\frac{4(a+bx)^{3/4}}{5ax^{5/4}}\right)}{3a}-\frac{4(a+bx)^{3/4}}{9ax^{9/4}}\right)}{13a}-\frac{4(a+bx)^{3/4}}{13ax^{13/4}}\right)}{a}+\frac{4}{ax^{13/4}\sqrt[4]{a+bx}}\right)}{(ax+bx^2)^{5/4}}$$

$\Big\downarrow$ 61

$$\frac{x^{5/4}(a+bx)^{5/4}\left(\frac{14\left(-\frac{10b\left(-\frac{2b\left(-\frac{2b\left(\frac{2b\int\frac{1}{\sqrt[4]{x}\sqrt[4]{a+bx}}dx}{a}-\frac{4(a+bx)^{3/4}}{a\sqrt[4]{x}}\right)}{5a}-\frac{4(a+bx)^{3/4}}{5ax^{5/4}}\right)}{3a}-\frac{4(a+bx)^{3/4}}{9ax^{9/4}}\right)}{13a}-\frac{4(a+bx)^{3/4}}{13ax^{13/4}}\right)}{a}+\frac{4}{ax^{13/4}\sqrt[4]{a}}\right.}{(ax+bx^2)^{5/4}}$$

$\downarrow$ 73

$$\frac{x^{5/4}(a+bx)^{5/4}\left(\frac{14\left(-\frac{10b\left(-\frac{2b\left(-\frac{2b\left(\frac{8b\int\frac{\sqrt{x}}{\sqrt[4]{a+bx}}d\sqrt[4]{x}}{a}-\frac{4(a+bx)^{3/4}}{a\sqrt[4]{x}}\right)}{5a}-\frac{4(a+bx)^{3/4}}{5ax^{5/4}}\right)}{3a}-\frac{4(a+bx)^{3/4}}{9ax^{9/4}}\right)}{13a}-\frac{4(a+bx)^{3/4}}{13ax^{13/4}}\right)}{a}+\frac{4}{ax^{13/4}\sqrt[4]{a\cdots}}\right.}{(ax+bx^2)^{5/4}}$$

$\downarrow$ 839

$$x^{5/4}(a+bx)^{5/4}\left(\frac{14\left(-\frac{10b\left(-\frac{2b\left(-\frac{2b\left(\frac{8b\left(\frac{x^{3/4}}{2\sqrt[4]{a+bx}}-\frac{1}{2}a\int\frac{\sqrt{x}}{(a+bx)^{5/4}}\,d\sqrt[4]{x}\right)}{a}-\frac{4(a+bx)^{3/4}}{a\sqrt[4]{x}}\right)}{5a}-\frac{4(a+bx)^{3/4}}{5ax^{5/4}}\right)}{3a}-\frac{4(a+bx)^{3/4}}{9ax^{9/4}}\right)}{13a}-\frac{4(a+bx)^{3/4}}{13ax^{13/4}}\right.}{a}\right.$$

↓ 813

$$\left(14\left(-\frac{10b\left(-\frac{2b\left(-\frac{2b\left(\frac{8b\left(\frac{x^{3/4}}{2\sqrt[4]{a+bx}}-\frac{a\sqrt[4]{x}\sqrt[4]{\frac{a}{bx}+1}\int\frac{1}{\left(\frac{a}{bx}+1\right)^{5/4}x^{3/4}}d\sqrt[4]{x}}{2b\sqrt[4]{a+bx}}\right)}{a}-\frac{4(a+bx)^{3/4}}{a\sqrt[4]{x}}\right)}{5a}-\frac{4(a+bx)^{3/4}}{5ax^{5/4}}\right)}{3a}-\frac{4(a+}{9a}\cdots\right)}{13a}\cdots\right.\right.$$

↓ 858

$$14\left(-\frac{10b\left(-\frac{2b\left(-\frac{2b\left(\frac{8b\left(\frac{a\sqrt[4]{x}\sqrt[4]{\frac{a}{bx}+1}\int\frac{1}{\sqrt[4]{x}\left(\frac{ax}{b}+1\right)^{5/4}}d\frac{1}{\sqrt[4]{x}}}{2b\sqrt[4]{a+bx}}+\frac{x^{3/4}}{2\sqrt[4]{a+bx}}\right)}{a}-\frac{4(a+bx)^{3/4}}{a\sqrt[4]{x}}\right)}{5a}-\frac{4(a+bx)^{3/4}}{5ax^{5/4}}\right)}{3a}-\frac{4(a}{9}}{13a}\right.$$

↓ 807

$$\left(14\left(-\frac{10b\left(-\frac{2b\left(-\frac{2b\left(\frac{8b\left(\frac{a\sqrt[4]{x}\sqrt[4]{\frac{a}{bx}+1}\int\frac{1}{\left(\frac{\sqrt{x}a}{b}+1\right)^{5/4}}d\sqrt{x}}{4b\sqrt[4]{a+bx}}+\frac{x^{3/4}}{2\sqrt[4]{a+bx}}\right)}{a}-\frac{4(a+bx)^{3/4}}{a\sqrt[4]{x}}\right)}{5a}-\frac{4(a+bx)^{3/4}}{5ax^{5/4}}\right)}{3a}-\frac{4(a+bx)^{3}}{9ax^{9/4}}}{13a}\right.\right.$$

$\downarrow$ 212

$$\Bigg(14\left(-\frac{10b\left(-\frac{2b\left(-\frac{2b\left(\frac{8b\left(\frac{\sqrt{a}\sqrt[4]{x}\sqrt[4]{\frac{a}{bx}+1}\,E\left(\frac{1}{2}\arctan\left(\frac{\sqrt{a}\sqrt{x}}{\sqrt{b}}\right)\middle|2\right)}{2\sqrt{b}\sqrt[4]{a+bx}}+\frac{x^{3/4}}{2\sqrt[4]{a+bx}}\right)}{a}-\frac{4(a+bx)^{3/4}}{a\sqrt[4]{x}}\right)}{5a}-\frac{4(a+bx)^{3/4}}{5ax^{5/4}}\right)}{3a}-\frac{4(a}{9}\cdots\right)}{13a}\cdots\right.\right.$$

```
Int[1/(x^3*(a*x + b*x^2)^(5/4)),x]
```

```
(x^(5/4)*(a + b*x)^(5/4)*(4/(a*x^(13/4)*(a + b*x)^(1/4)) + (14*((-4*(a + b
*x)^(3/4))/(13*a*x^(13/4)) - (10*b*((-4*(a + b*x)^(3/4))/(9*a*x^(9/4)) - (
2*b*((-4*(a + b*x)^(3/4))/(5*a*x^(5/4)) - (2*b*((-4*(a + b*x)^(3/4))/(a*x^
(1/4)) + (8*b*(x^(3/4)/(2*(a + b*x)^(1/4)) + (Sqrt[a]*(1 + a/(b*x))^(1/4)*
x^(1/4)*EllipticE[ArcTan[(Sqrt[a]*Sqrt[x])/Sqrt[b]]/2, 2])/(2*Sqrt[b]*(a +
 b*x)^(1/4))))/a))/(5*a)))/(3*a)))/(13*a)))/a))/(a*x + b*x^2)^(5/4)
```

Defintions of rubi rules used

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] - Simp[d*((
m + n + 2)/((b*c - a*d)*(m + 1)))   Int[(a + b*x)^(m + 1)*(c + d*x)^n, x],
x] /; FreeQ[{a, b, c, d, n}, x] && LtQ[m, -1] &&  !(LtQ[n, -1] && (EqQ[a, 0
] || (NeQ[c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && IntLinearQ[a, b, c, d
, m, n, x]
```

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Simp[p/b   Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
 d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && Lt
Q[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntL
inearQ[a, b, c, d, m, n, x]
```

```
Int[((a_) + (b_.)*(x_)^2)^(-5/4), x_Symbol] :> Simp[(2/(a^(5/4)*Rt[b/a, 2])
)*EllipticE[(1/2)*ArcTan[Rt[b/a, 2]*x], 2], x] /; FreeQ[{a, b}, x] && GtQ[a
, 0] && PosQ[b/a]
```

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k = GCD[m
+ 1, n]}, Simp[1/k   Subst[Int[x^((m + 1)/k - 1)*(a + b*x^(n/k))^p, x], x,
x^k], x] /; k != 1] /; FreeQ[{a, b, p}, x] && IGtQ[n, 0] && IntegerQ[m]
```

```
Int[(x_)^2/((a_) + (b_.)*(x_)^4)^(5/4), x_Symbol] :> Simp[x*((1 + a/(b*x^4)
)^(1/4)/(b*(a + b*x^4)^(1/4)))   Int[1/(x^3*(1 + a/(b*x^4))^(5/4)), x], x]
/; FreeQ[{a, b}, x] && PosQ[b/a]
```

```
Int[(x_)^2/((a_) + (b_.)*(x_)^4)^(1/4), x_Symbol] :> Simp[x^3/(2*(a + b*x^4
)^(1/4)), x] - Simp[a/2   Int[x^2/(a + b*x^4)^(5/4), x], x] /; FreeQ[{a, b}
, x] && PosQ[b/a]
```

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> -Subst[Int[(a +
b/x^n)^p/x^(m + 2), x], x, 1/x] /; FreeQ[{a, b, p}, x] && ILtQ[n, 0] && Int
egerQ[m]
```

```
Int[((e_.)*(x_))^(m_)*((b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(
e*x)^m*((b*x + c*x^2)^p/(x^(m + p)*(b + c*x)^p))   Int[x^(m + p)*(b + c*x)^
p, x], x] /; FreeQ[{b, c, e, m}, x]
```

Maple [F]

$$\int \frac{1}{x^{3} \left(b\,x^{2}+a x \right)^{\frac{5}{4}}}dx$$

```
int(1/x^3/(b*x^2+a*x)^(5/4),x)
```

```
int(1/x^3/(b*x^2+a*x)^(5/4),x)
```

Fricas [F]

$$\int \frac{1}{x^3\left(ax+bx^2\right)^{5/4}}\,dx=\int \frac{1}{\left(bx^{2}+ax\right)^{\frac{5}{4}}x^{3}}\,dx$$

```
integrate(1/x^3/(b*x^2+a*x)^(5/4),x, algorithm="fricas")
```

```
integral((b*x^2 + a*x)^(3/4)/(b^2*x^7 + 2*a*b*x^6 + a^2*x^5), x)
```

Sympy [F]

$$\int \frac{1}{x^3 (ax+bx^2)^{5/4}}\,dx = \int \frac{1}{x^3 (x(a+bx))^{\frac{5}{4}}}\,dx$$

```
integrate(1/x**3/(b*x**2+a*x)**(5/4),x)
```

```
Integral(1/(x**3*(x*(a + b*x))**(5/4)), x)
```

Maxima [F]

$$\int \frac{1}{x^3 (ax+bx^2)^{5/4}}\,dx = \int \frac{1}{(bx^2+ax)^{\frac{5}{4}}x^3}\,dx$$

```
integrate(1/x^3/(b*x^2+a*x)^(5/4),x, algorithm="maxima")
```

```
integrate(1/((b*x^2 + a*x)^(5/4)*x^3), x)
```

Giac [F]

$$\int \frac{1}{x^3 (ax+bx^2)^{5/4}}\,dx = \int \frac{1}{(bx^2+ax)^{\frac{5}{4}}x^3}\,dx$$

```
integrate(1/x^3/(b*x^2+a*x)^(5/4),x, algorithm="giac")
```

```
integrate(1/((b*x^2 + a*x)^(5/4)*x^3), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{x^3\left(ax+bx^2\right)^{5/4}}\,dx=\int \frac{1}{x^3\,{\left(b\,x^2+a\,x\right)}^{5/4}} \,dx$$

```
int(1/(x^3*(a*x + b*x^2)^(5/4)),x)
```

```
int(1/(x^3*(a*x + b*x^2)^(5/4)), x)
```

Reduce [B] (verification not implemented)

Time = 0.42 (sec) , antiderivative size = 57, normalized size of antiderivative = 0.30

$$\int \frac{1}{x^3\left(ax+bx^2\right)^{5/4}}\,dx=\frac{4(bx+a)^{\frac{1}{4}}\left(-128b^3x^3-32a\,b^2x^2+12a^2bx-7a^3\right)}{77x^{\frac{11}{4}}\sqrt{x}\,\sqrt{bx+a}\,a^4}$$

```
int(1/x^3/(b*x^2+a*x)^(5/4),x)
```

```
(4*x**(1/4)*(a + b*x)**(1/4)*( - 7*a**3 + 12*a**2*b*x - 32*a*b**2*x**2 - 1
28*b**3*x**3))/(77*sqrt(x)*sqrt(a + b*x)*a**4*x**3)
```

3.198 $\int \frac{1}{x\sqrt[4]{ax-bx^2}}\,dx$

Optimal result

Integrand size = 18, antiderivative size = 120

$$\int \frac{1}{x\sqrt[4]{ax-bx^2}}\,dx = -\frac{4\sqrt[4]{-\frac{bx}{a-bx}}}{\sqrt[4]{ax-bx^2}\sqrt[4]{1-\frac{a}{a-bx}}} + \frac{4\sqrt[4]{-\frac{bx}{a-bx}}\sqrt{a-bx}\,E\left(\frac{1}{2}\arcsin\left(\frac{\sqrt{a}}{\sqrt{a-bx}}\right)\middle|2\right)}{\sqrt{a}\sqrt[4]{ax-bx^2}}$$

```
-4*(-b*x/(-b*x+a))^(1/4)/(-b*x^2+a*x)^(1/4)/(1-a/(-b*x+a))^(1/4)+4*(-b*x/(
-b*x+a))^(1/4)*(-b*x+a)^(1/2)*EllipticE(sin(1/2*arcsin(a^(1/2)/(-b*x+a)^(1
/2))),2^(1/2))/a^(1/2)/(-b*x^2+a*x)^(1/4)
```

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 10.02 (sec) , antiderivative size = 43, normalized size of antiderivative = 0.36

$$\int \frac{1}{x\sqrt[4]{ax-bx^2}}\,dx = -\frac{4\sqrt[4]{1-\frac{bx}{a}}\,\text{Hypergeometric2F1}\left(-\frac{1}{4},\frac{1}{4},\frac{3}{4},\frac{bx}{a}\right)}{\sqrt[4]{x(a-bx)}}$$

```
Integrate[1/(x*(a*x - b*x^2)^(1/4)),x]
```

```
(-4*(1 - (b*x)/a)^(1/4)*Hypergeometric2F1[-1/4, 1/4, 3/4, (b*x)/a])/(x*(a
- b*x))^(1/4)
```

Rubi [A] (warning: unable to verify)

Time = 0.51 (sec) , antiderivative size = 147, normalized size of antiderivative = 1.22, number of steps used = 9, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}} = 0.444$, Rules used = {1137, 61, 73, 840, 842, 858, 807, 226}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{x\sqrt[4]{ax-bx^2}}dx$$

$$\Big\downarrow \text{ 1137}$$

$$\frac{\sqrt[4]{x}\sqrt[4]{a-bx}\int \frac{1}{x^{5/4}\sqrt[4]{a-bx}}dx}{\sqrt[4]{ax-bx^2}}$$

$$\Big\downarrow \text{ 61}$$

$$\frac{\sqrt[4]{x}\sqrt[4]{a-bx}\left(-\frac{2b\int \frac{1}{\sqrt[4]{x}\sqrt[4]{a-bx}}dx}{a}-\frac{4(a-bx)^{3/4}}{a\sqrt[4]{x}}\right)}{\sqrt[4]{ax-bx^2}}$$

$$\Big\downarrow \text{ 73}$$

$$\frac{\sqrt[4]{x}\sqrt[4]{a-bx}\left(-\frac{8b\int \frac{\sqrt{x}}{\sqrt[4]{a-bx}}d\sqrt[4]{x}}{a}-\frac{4(a-bx)^{3/4}}{a\sqrt[4]{x}}\right)}{\sqrt[4]{ax-bx^2}}$$

$$\Big\downarrow \; 840$$

$$\frac{\sqrt[4]{x}\sqrt[4]{a-bx}\left(-\frac{8b\left(-\frac{a\int \frac{1}{\sqrt{x}\sqrt[4]{a-bx}}d\sqrt[4]{x}}{2b}-\frac{(a-bx)^{3/4}}{2b\sqrt[4]{x}}\right)}{a}-\frac{4(a-bx)^{3/4}}{a\sqrt[4]{x}}\right)}{\sqrt[4]{ax-bx^2}}$$

$$\Big\downarrow \; 842$$

$$\frac{\sqrt[4]{x}\sqrt[4]{a-bx}\left(-\frac{8b\left(-\frac{a\sqrt[4]{x}\sqrt[4]{1-\frac{a}{bx}}\int \frac{1}{\sqrt[4]{1-\frac{a}{bx}}x^{3/4}}d\sqrt[4]{x}}{2b\sqrt[4]{a-bx}}-\frac{(a-bx)^{3/4}}{2b\sqrt[4]{x}}\right)}{a}-\frac{4(a-bx)^{3/4}}{a\sqrt[4]{x}}\right)}{\sqrt[4]{ax-bx^2}}$$

$$\Big\downarrow \; 858$$

$$\frac{\sqrt[4]{x}\sqrt[4]{a-bx}\left(-\frac{8b\left(\frac{a\sqrt[4]{x}\sqrt[4]{1-\frac{a}{bx}}\int \frac{1}{\sqrt[4]{x}\sqrt[4]{1-\frac{ax}{b}}}d\frac{1}{\sqrt[4]{x}}}{2b\sqrt[4]{a-bx}}-\frac{(a-bx)^{3/4}}{2b\sqrt[4]{x}}\right)}{a}-\frac{4(a-bx)^{3/4}}{a\sqrt[4]{x}}\right)}{\sqrt[4]{ax-bx^2}}$$

$$\Big\downarrow \textcolor{blue}{807}$$

$$\frac{\sqrt[4]{x}\sqrt[4]{a-bx}\left(-\frac{8b\left(\frac{a\sqrt[4]{x}\sqrt[4]{1-\frac{a}{bx}}\int \frac{1}{\sqrt[4]{1-\frac{a\sqrt{x}}{b}}}d\sqrt{x}}{4b\sqrt[4]{a-bx}}-\frac{(a-bx)^{3/4}}{2b\sqrt[4]{x}}\right)}{a}-\frac{4(a-bx)^{3/4}}{a\sqrt[4]{x}}\right)}{\sqrt[4]{ax-bx^2}}$$

$$\Big\downarrow \textcolor{blue}{226}$$

$$\frac{\sqrt[4]{x}\sqrt[4]{a-bx}\left(-\frac{8b\left(\frac{\sqrt{a}\sqrt[4]{x}\sqrt[4]{1-\frac{a}{bx}}E\left(\frac{1}{2}\arcsin\left(\frac{\sqrt{a}\sqrt{x}}{\sqrt{b}}\right)\middle|2\right)}{2\sqrt{b}\sqrt[4]{a-bx}}-\frac{(a-bx)^{3/4}}{2b\sqrt[4]{x}}\right)}{a}-\frac{4(a-bx)^{3/4}}{a\sqrt[4]{x}}\right)}{\sqrt[4]{ax-bx^2}}$$

```
Int[1/(x*(a*x - b*x^2)^(1/4)),x]
```

```
(x^(1/4)*(a - b*x)^(1/4)*((-4*(a - b*x)^(3/4))/(a*x^(1/4)) - (8*b*(-1/2*(a
 - b*x)^(3/4)/(b*x^(1/4)) + (Sqrt[a]*(1 - a/(b*x))^(1/4)*x^(1/4)*EllipticE
[ArcSin[(Sqrt[a]*Sqrt[x])/Sqrt[b]]/2, 2])/(2*Sqrt[b]*(a - b*x)^(1/4))))/a)
)/(a*x - b*x^2)^(1/4)
```

Defintions of rubi rules used

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] - Simp[d*((
m + n + 2)/((b*c - a*d)*(m + 1)))   Int[(a + b*x)^(m + 1)*(c + d*x)^n, x],
x] /; FreeQ[{a, b, c, d, n}, x] && LtQ[m, -1] &&  !(LtQ[n, -1] && (EqQ[a, 0
] || (NeQ[c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && IntLinearQ[a, b, c, d
, m, n, x]
```

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Simp[p/b   Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
 d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && Lt
Q[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntL
inearQ[a, b, c, d, m, n, x]
```

```
Int[((a_) + (b_.)*(x_)^2)^(-1/4), x_Symbol] :> Simp[(2/(a^(1/4)*Rt[-b/a, 2]
))*EllipticE[(1/2)*ArcSin[Rt[-b/a, 2]*x], 2], x] /; FreeQ[{a, b}, x] && GtQ
[a, 0] && NegQ[b/a]
```

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k = GCD[m
+ 1, n]}, Simp[1/k   Subst[Int[x^((m + 1)/k - 1)*(a + b*x^(n/k))^p, x], x,
x^k], x] /; k != 1] /; FreeQ[{a, b, p}, x] && IGtQ[n, 0] && IntegerQ[m]
```

```
Int[(x_)^2/((a_) + (b_.)*(x_)^4)^(1/4), x_Symbol] :> Simp[(a + b*x^4)^(3/4)
/(2*b*x), x] + Simp[a/(2*b)   Int[1/(x^2*(a + b*x^4)^(1/4)), x], x] /; Free
Q[{a, b}, x] && NegQ[b/a]
```

```
Int[1/((x_)^2*((a_) + (b_.)*(x_)^4)^(1/4)), x_Symbol] :> Simp[x*((1 + a/(b*
x^4))^(1/4)/(a + b*x^4)^(1/4))   Int[1/(x^3*(1 + a/(b*x^4))^(1/4)), x], x]
/; FreeQ[{a, b}, x] && NegQ[b/a]
```

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> -Subst[Int[(a +
b/x^n)^p/x^(m + 2), x], x, 1/x] /; FreeQ[{a, b, p}, x] && ILtQ[n, 0] && Int
egerQ[m]
```

```
Int[((e_.)*(x_))^(m_)*((b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(
e*x)^m*((b*x + c*x^2)^p/(x^(m + p)*(b + c*x)^p))   Int[x^(m + p)*(b + c*x)^
p, x], x] /; FreeQ[{b, c, e, m}, x]
```

Maple [F]

$$\int \frac{1}{x\left(-b\,x^{2}+a x \right)^{\frac{1}{4}}}dx$$

```
int(1/x/(-b*x^2+a*x)^(1/4),x)
```

```
int(1/x/(-b*x^2+a*x)^(1/4),x)
```

Fricas [F]

$$\int \frac{1}{x\sqrt[4]{ax-bx^2}}\,dx = \int \frac{1}{(-bx^2+ax)^{\frac{1}{4}}x}\,dx$$

```
integrate(1/x/(-b*x^2+a*x)^(1/4),x, algorithm="fricas")
```

```
integral(-(-b*x^2 + a*x)^(3/4)/(b*x^3 - a*x^2), x)
```

Sympy [F]

$$\int \frac{1}{x\sqrt[4]{ax-bx^2}}\,dx = \int \frac{1}{x\sqrt[4]{-x\,(-a+bx)}}\,dx$$

```
integrate(1/x/(-b*x**2+a*x)**(1/4),x)
```

```
Integral(1/(x*(-x*(-a + b*x))**(1/4)), x)
```

Maxima [F]

$$\int \frac{1}{x\sqrt[4]{ax-bx^2}}\,dx = \int \frac{1}{(-bx^2+ax)^{\frac{1}{4}}x}\,dx$$

```
integrate(1/x/(-b*x^2+a*x)^(1/4),x, algorithm="maxima")
```

```
integrate(1/((-b*x^2 + a*x)^(1/4)*x), x)
```

Giac [F]

$$\int \frac{1}{x\sqrt[4]{ax-bx^2}}\,dx = \int \frac{1}{(-bx^2+ax)^{\frac{1}{4}}x}\,dx$$

```
integrate(1/x/(-b*x^2+a*x)^(1/4),x, algorithm="giac")
```

```
integrate(1/((-b*x^2 + a*x)^(1/4)*x), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{x\sqrt[4]{ax-bx^2}}\,dx = \int \frac{1}{x\,(a\,x-b\,x^2)^{1/4}}\,dx$$

```
int(1/(x*(a*x - b*x^2)^(1/4)),x)
```

```
int(1/(x*(a*x - b*x^2)^(1/4)), x)
```

Reduce [F]

$$\int \frac{1}{x\sqrt[4]{ax-bx^2}}\,dx = \int \frac{1}{x^{\frac{5}{4}}\,(-bx+a)^{\frac{1}{4}}}dx$$

```
int(1/x/(-b*x^2+a*x)^(1/4),x)
```

```
int(1/(x**(1/4)*(a - b*x)**(1/4)*x),x)
```

3.199 $\int \frac{1}{x\sqrt[4]{-ax+bx^2}}\,dx$

Optimal result

Integrand size = 18, antiderivative size = 72

$$\int \frac{1}{x\sqrt[4]{-ax+bx^2}}\,dx = -\frac{4\sqrt[4]{-\frac{bx}{a-bx}}\sqrt{-a+bx}\,E\left(\frac{1}{2}\arctan\left(\frac{\sqrt{a}}{\sqrt{-a+bx}}\right)\middle|2\right)}{\sqrt{a}\sqrt[4]{-ax+bx^2}}$$

```
-4*(-b*x/(-b*x+a))^(1/4)*(b*x-a)^(1/2)*EllipticE(sin(1/2*arctan(a^(1/2)/(b
*x-a)^(1/2))),2^(1/2))/a^(1/2)/(b*x^2-a*x)^(1/4)
```

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 10.01 (sec) , antiderivative size = 44, normalized size of antiderivative = 0.61

$$\int \frac{1}{x\sqrt[4]{-ax+bx^2}}\,dx = -\frac{4\sqrt[4]{1-\frac{bx}{a}}\,\text{Hypergeometric2F1}\left(-\frac{1}{4},\frac{1}{4},\frac{3}{4},\frac{bx}{a}\right)}{\sqrt[4]{x(-a+bx)}}$$

```
Integrate[1/(x*(-(a*x) + b*x^2)^(1/4)),x]
```

```
(-4*(1 - (b*x)/a)^(1/4)*Hypergeometric2F1[-1/4, 1/4, 3/4, (b*x)/a])/(x*(-a
 + b*x))^(1/4)
```

Rubi [B] (warning: unable to verify)

Leaf count is larger than twice the leaf count of optimal. 151 vs. $2(72) = 144$.

Time = 0.52 (sec) , antiderivative size = 151, normalized size of antiderivative = 2.10, number of steps used = 9, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}} = 0.444$, Rules used = {1137, 61, 73, 840, 842, 858, 807, 226}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{x\sqrt[4]{bx^2-ax}}dx$$

$$\Big\downarrow \text{1137}$$

$$\frac{\sqrt[4]{x}\sqrt[4]{bx-a}\int \frac{1}{x^{5/4}\sqrt[4]{bx-a}}dx}{\sqrt[4]{bx^2-ax}}$$

$$\Big\downarrow \text{61}$$

$$\frac{\sqrt[4]{x}\sqrt[4]{bx-a}\left(\frac{4(bx-a)^{3/4}}{a\sqrt[4]{x}}-\frac{2b\int \frac{1}{\sqrt[4]{x}\sqrt[4]{bx-a}}dx}{a}\right)}{\sqrt[4]{bx^2-ax}}$$

$$\Big\downarrow \text{73}$$

$$\frac{\sqrt[4]{x}\sqrt[4]{bx-a}\left(\frac{4(bx-a)^{3/4}}{a\sqrt[4]{x}}-\frac{8b\int \frac{\sqrt{x}}{\sqrt[4]{bx-a}}d\sqrt[4]{x}}{a}\right)}{\sqrt[4]{bx^2-ax}}$$

$$\Big\downarrow \text{840}$$

$$\frac{\sqrt[4]{x}\sqrt[4]{bx-a}\left(\frac{4(bx-a)^{3/4}}{a\sqrt[4]{x}}-\frac{8b\left(\frac{(bx-a)^{3/4}}{2b\sqrt[4]{x}}-\frac{a\int\frac{1}{\sqrt{x}\sqrt[4]{bx-a}}d\sqrt[4]{x}}{2b}\right)}{a}\right)}{\sqrt[4]{bx^2-ax}}$$

$$\Big\downarrow \text{ 842}$$

$$\frac{\sqrt[4]{x}\sqrt[4]{bx-a}\left(\frac{4(bx-a)^{3/4}}{a\sqrt[4]{x}}-\frac{8b\left(\frac{(bx-a)^{3/4}}{2b\sqrt[4]{x}}-\frac{a\sqrt[4]{x}\sqrt[4]{1-\frac{a}{bx}}\int\frac{1}{\sqrt[4]{1-\frac{a}{bx}}x^{3/4}}d\sqrt[4]{x}}{2b\sqrt[4]{bx-a}}\right)}{a}\right)}{\sqrt[4]{bx^2-ax}}$$

$$\Big\downarrow \text{ 858}$$

$$\frac{\sqrt[4]{x}\sqrt[4]{bx-a}\left(\frac{4(bx-a)^{3/4}}{a\sqrt[4]{x}}-\frac{8b\left(\frac{a\sqrt[4]{x}\sqrt[4]{1-\frac{a}{bx}}\int\frac{1}{\sqrt[4]{x}\sqrt[4]{1-\frac{ax}{b}}}d\frac{1}{\sqrt[4]{x}}}{2b\sqrt[4]{bx-a}}+\frac{(bx-a)^{3/4}}{2b\sqrt[4]{x}}\right)}{a}\right)}{\sqrt[4]{bx^2-ax}}$$

$$\Big\downarrow \text{ 807}$$

$$\frac{\sqrt[4]{x}\sqrt[4]{bx-a}\left(\frac{4(bx-a)^{3/4}}{a\sqrt[4]{x}}-\frac{8b\left(\frac{a\sqrt[4]{x}\sqrt[4]{1-\frac{a}{bx}}\int\frac{1}{\sqrt[4]{1-\frac{a\sqrt{x}}{b}}}d\sqrt{x}}{4b\sqrt[4]{bx-a}}+\frac{(bx-a)^{3/4}}{2b\sqrt[4]{x}}\right)}{a}\right)}{\sqrt[4]{bx^2-ax}}$$

$$\Big\downarrow \textbf{226}$$

$$\frac{\sqrt[4]{x}\sqrt[4]{bx-a}\left(\frac{4(bx-a)^{3/4}}{a\sqrt[4]{x}}-\frac{8b\left(\frac{\sqrt{a}\sqrt[4]{x}\sqrt[4]{1-\frac{a}{bx}}E\left(\frac{1}{2}\arcsin\left(\frac{\sqrt{a}\sqrt{x}}{\sqrt{b}}\right)\middle|2\right)}{2\sqrt{b}\sqrt[4]{bx-a}}+\frac{(bx-a)^{3/4}}{2b\sqrt[4]{x}}\right)}{a}\right)}{\sqrt[4]{bx^2-ax}}$$

```
Int[1/(x*(-(a*x) + b*x^2)^(1/4)),x]
```

```
(x^(1/4)*(-a + b*x)^(1/4)*((4*(-a + b*x)^(3/4))/(a*x^(1/4)) - (8*b*((-a +
b*x)^(3/4)/(2*b*x^(1/4)) + (Sqrt[a]*(1 - a/(b*x))^(1/4)*x^(1/4)*EllipticE[
ArcSin[(Sqrt[a]*Sqrt[x])/Sqrt[b]]/2, 2])/(2*Sqrt[b]*(-a + b*x)^(1/4))))/a)
)/(-(a*x) + b*x^2)^(1/4)
```

Defintions of rubi rules used

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] - Simp[d*((
m + n + 2)/((b*c - a*d)*(m + 1)))   Int[(a + b*x)^(m + 1)*(c + d*x)^n, x],
x] /; FreeQ[{a, b, c, d, n}, x] && LtQ[m, -1] &&  !(LtQ[n, -1] && (EqQ[a, 0
] || (NeQ[c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && IntLinearQ[a, b, c, d
, m, n, x]
```

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Simp[p/b   Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
 d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && Lt
Q[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntL
inearQ[a, b, c, d, m, n, x]
```

```
Int[((a_) + (b_.)*(x_)^2)^(-1/4), x_Symbol] :> Simp[(2/(a^(1/4)*Rt[-b/a, 2]
))*EllipticE[(1/2)*ArcSin[Rt[-b/a, 2]*x], 2], x] /; FreeQ[{a, b}, x] && GtQ
[a, 0] && NegQ[b/a]
```

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k = GCD[m
+ 1, n]}, Simp[1/k   Subst[Int[x^((m + 1)/k - 1)*(a + b*x^(n/k))^p, x], x,
x^k], x] /; k != 1] /; FreeQ[{a, b, p}, x] && IGtQ[n, 0] && IntegerQ[m]
```

```
Int[(x_)^2/((a_) + (b_.)*(x_)^4)^(1/4), x_Symbol] :> Simp[(a + b*x^4)^(3/4)
/(2*b*x), x] + Simp[a/(2*b)   Int[1/(x^2*(a + b*x^4)^(1/4)), x], x] /; Free
Q[{a, b}, x] && NegQ[b/a]
```

```
Int[1/((x_)^2*((a_) + (b_.)*(x_)^4)^(1/4)), x_Symbol] :> Simp[x*((1 + a/(b*
x^4))^(1/4)/(a + b*x^4)^(1/4))   Int[1/(x^3*(1 + a/(b*x^4))^(1/4)), x], x]
/; FreeQ[{a, b}, x] && NegQ[b/a]
```

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> -Subst[Int[(a +
b/x^n)^p/x^(m + 2), x], x, 1/x] /; FreeQ[{a, b, p}, x] && ILtQ[n, 0] && Int
egerQ[m]
```

```
Int[((e_.)*(x_))^(m_)*((b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(
e*x)^m*((b*x + c*x^2)^p/(x^(m + p)*(b + c*x)^p))   Int[x^(m + p)*(b + c*x)^
p, x], x] /; FreeQ[{b, c, e, m}, x]
```

Maple [F]

$$\int \frac{1}{x\left(b\,x^{2}-ax\right)^{\frac{1}{4}}}dx$$

```
int(1/x/(b*x^2-a*x)^(1/4),x)
```

```
int(1/x/(b*x^2-a*x)^(1/4),x)
```

Fricas [F]

$$\int \frac{1}{x\sqrt[4]{-ax+bx^2}}\,dx=\int \frac{1}{\left(bx^{2}-ax\right)^{\frac{1}{4}}x}\,dx$$

```
integrate(1/x/(b*x^2-a*x)^(1/4),x, algorithm="fricas")
```

```
integral((b*x^2 - a*x)^(3/4)/(b*x^3 - a*x^2), x)
```

Sympy [F]

$$\int \frac{1}{x\sqrt[4]{-ax+bx^2}}\,dx=\int \frac{1}{x\sqrt[4]{x\left(-a+bx\right)}}\,dx$$

```
integrate(1/x/(b*x**2-a*x)**(1/4),x)
```

```
Integral(1/(x*(x*(-a + b*x))**(1/4)), x)
```

Maxima [F]

$$\int \frac{1}{x\sqrt[4]{-ax+bx^2}}\,dx = \int \frac{1}{(bx^2-ax)^{\frac{1}{4}}x}\,dx$$

```
integrate(1/x/(b*x^2-a*x)^(1/4),x, algorithm="maxima")
```

```
integrate(1/((b*x^2 - a*x)^(1/4)*x), x)
```

Giac [F]

$$\int \frac{1}{x\sqrt[4]{-ax+bx^2}}\,dx = \int \frac{1}{(bx^2-ax)^{\frac{1}{4}}x}\,dx$$

```
integrate(1/x/(b*x^2-a*x)^(1/4),x, algorithm="giac")
```

```
integrate(1/((b*x^2 - a*x)^(1/4)*x), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{x\sqrt[4]{-ax+bx^2}}\,dx = \int \frac{1}{x\,(b\,x^2-a\,x)^{1/4}}\,dx$$

```
int(1/(x*(b*x^2 - a*x)^(1/4)),x)
```

```
int(1/(x*(b*x^2 - a*x)^(1/4)), x)
```

Reduce [F]

$$\int \frac{1}{x\sqrt[4]{-ax+bx^2}}\,dx = \int \frac{1}{x^{\frac{5}{4}}\,(bx-a)^{\frac{1}{4}}}dx$$

```
int(1/x/(b*x^2-a*x)^(1/4),x)
```

```
int(1/(x**(1/4)*( - a + b*x)**(1/4)*x),x)
```

3.200 $\int \frac{1}{x\sqrt[4]{ax+bx^2}}\,dx$

Optimal result

Integrand size = 17, antiderivative size = 81

$$\int \frac{1}{x\sqrt[4]{ax+bx^2}}\,dx = -\frac{4}{\sqrt[4]{ax+bx^2}} + \frac{4\sqrt[4]{\frac{bx}{a+bx}}\sqrt{a+bx}E\left(\frac{1}{2}\arcsin\left(\frac{\sqrt{a}}{\sqrt{a+bx}}\right)\middle|2\right)}{\sqrt{a}\sqrt[4]{ax+bx^2}}$$

```
-4/(b*x^2+a*x)^(1/4)+4*(b*x/(b*x+a))^(1/4)*(b*x+a)^(1/2)*EllipticE(sin(1/2
*arcsin(a^(1/2)/(b*x+a)^(1/2))),2^(1/2))/a^(1/2)/(b*x^2+a*x)^(1/4)
```

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 0.00 (sec) , antiderivative size = 42, normalized size of antiderivative = 0.52

$$\int \frac{1}{x\sqrt[4]{ax+bx^2}}\,dx = -\frac{4\sqrt[4]{1+\frac{bx}{a}}\,\text{Hypergeometric2F1}\left(-\frac{1}{4},\frac{1}{4},\frac{3}{4},-\frac{bx}{a}\right)}{\sqrt[4]{x(a+bx)}}$$

```
Integrate[1/(x*(a*x + b*x^2)^(1/4)),x]
```

```
(-4*(1 + (b*x)/a)^(1/4)*Hypergeometric2F1[-1/4, 1/4, 3/4, -((b*x)/a)])/(x*
(a + b*x))^(1/4)
```

Rubi [A] (warning: unable to verify)

Time = 0.49 (sec) , antiderivative size = 138, normalized size of antiderivative = 1.70, number of steps used = 9, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}} = 0.471$, Rules used = {1137, 61, 73, 839, 813, 858, 807, 212}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{x\sqrt[4]{ax+bx^2}}dx$$

$$\Big\downarrow \text{1137}$$

$$\frac{\sqrt[4]{x}\sqrt[4]{a+bx}\int \frac{1}{x^{5/4}\sqrt[4]{a+bx}}dx}{\sqrt[4]{ax+bx^2}}$$

$$\Big\downarrow \text{61}$$

$$\frac{\sqrt[4]{x}\sqrt[4]{a+bx}\left(\frac{2b\int \frac{1}{\sqrt[4]{x}\sqrt[4]{a+bx}}dx}{a}-\frac{4(a+bx)^{3/4}}{a\sqrt[4]{x}}\right)}{\sqrt[4]{ax+bx^2}}$$

$$\Big\downarrow \text{73}$$

$$\frac{\sqrt[4]{x}\sqrt[4]{a+bx}\left(\frac{8b\int \frac{\sqrt{x}}{\sqrt[4]{a+bx}}d\sqrt[4]{x}}{a}-\frac{4(a+bx)^{3/4}}{a\sqrt[4]{x}}\right)}{\sqrt[4]{ax+bx^2}}$$

$$\Big\downarrow \text{839}$$

$$\frac{\sqrt[4]{x}\sqrt[4]{a+bx}\left(\frac{8b\left(\frac{x^{3/4}}{2\sqrt[4]{a+bx}}-\frac{1}{2}a\int \frac{\sqrt{x}}{(a+bx)^{5/4}}d\sqrt[4]{x}\right)}{a}-\frac{4(a+bx)^{3/4}}{a\sqrt[4]{x}}\right)}{\sqrt[4]{ax+bx^2}}$$

$$\Big\downarrow \text{813}$$

$$\frac{\sqrt[4]{x}\sqrt[4]{a+bx}\left(\frac{8b\left(\frac{x^{3/4}}{2\sqrt[4]{a+bx}}-\frac{a\sqrt[4]{x}\sqrt[4]{\frac{a}{bx}+1}\int\frac{1}{\left(\frac{a}{bx}+1\right)^{5/4}x^{3/4}}d\sqrt[4]{x}}{2b\sqrt[4]{a+bx}}\right)}{a}-\frac{4(a+bx)^{3/4}}{a\sqrt[4]{x}}\right)}{\sqrt[4]{ax+bx^2}}$$

$$\Big\downarrow \; 858$$

$$\frac{\sqrt[4]{x}\sqrt[4]{a+bx}\left(\frac{8b\left(\frac{a\sqrt[4]{x}\sqrt[4]{\frac{a}{bx}+1}\int\frac{1}{\sqrt[4]{x}\left(\frac{ax}{b}+1\right)^{5/4}}d\frac{1}{\sqrt[4]{x}}}{2b\sqrt[4]{a+bx}}+\frac{x^{3/4}}{2\sqrt[4]{a+bx}}\right)}{a}-\frac{4(a+bx)^{3/4}}{a\sqrt[4]{x}}\right)}{\sqrt[4]{ax+bx^2}}$$

$$\Big\downarrow \; 807$$

$$\frac{\sqrt[4]{x}\sqrt[4]{a+bx}\left(\frac{8b\left(\frac{a\sqrt[4]{x}\sqrt[4]{\frac{a}{bx}+1}\int\frac{1}{\left(\frac{\sqrt{x}a}{b}+1\right)^{5/4}}d\sqrt{x}}{4b\sqrt[4]{a+bx}}+\frac{x^{3/4}}{2\sqrt[4]{a+bx}}\right)}{a}-\frac{4(a+bx)^{3/4}}{a\sqrt[4]{x}}\right)}{\sqrt[4]{ax+bx^2}}$$

$$\Big\downarrow \; 212$$

$$\frac{\sqrt[4]{x}\sqrt[4]{a+bx}\left(\frac{8b\left(\frac{\sqrt{a}\sqrt[4]{x}\sqrt[4]{\frac{a}{bx}+1}E\left(\frac{1}{2}\arctan\left(\frac{\sqrt{a}\sqrt{x}}{\sqrt{b}}\right)\middle|2\right)}{2\sqrt{b}\sqrt[4]{a+bx}}+\frac{x^{3/4}}{2\sqrt[4]{a+bx}}\right)}{a}-\frac{4(a+bx)^{3/4}}{a\sqrt[4]{x}}\right)}{\sqrt[4]{ax+bx^2}}$$

```
Int[1/(x*(a*x + b*x^2)^(1/4)),x]
```

```
(x^(1/4)*(a + b*x)^(1/4)*((-4*(a + b*x)^(3/4))/(a*x^(1/4)) + (8*b*(x^(3/4)
/(2*(a + b*x)^(1/4)) + (Sqrt[a]*(1 + a/(b*x))^(1/4)*x^(1/4)*EllipticE[ArcT
an[(Sqrt[a]*Sqrt[x])/Sqrt[b]]/2, 2])/(2*Sqrt[b]*(a + b*x)^(1/4))))/a))/(a*
x + b*x^2)^(1/4)
```

Defintions of rubi rules used

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] - Simp[d*((
m + n + 2)/((b*c - a*d)*(m + 1)))   Int[(a + b*x)^(m + 1)*(c + d*x)^n, x],
x] /; FreeQ[{a, b, c, d, n}, x] && LtQ[m, -1] &&  !(LtQ[n, -1] && (EqQ[a, 0
] || (NeQ[c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && IntLinearQ[a, b, c, d
, m, n, x]
```

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Simp[p/b   Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
 d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && Lt
Q[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntL
inearQ[a, b, c, d, m, n, x]
```

```
Int[((a_) + (b_.)*(x_)^2)^(-5/4), x_Symbol] :> Simp[(2/(a^(5/4)*Rt[b/a, 2])
)*EllipticE[(1/2)*ArcTan[Rt[b/a, 2]*x], 2], x] /; FreeQ[{a, b}, x] && GtQ[a
, 0] && PosQ[b/a]
```

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k = GCD[m
+ 1, n]}, Simp[1/k   Subst[Int[x^((m + 1)/k - 1)*(a + b*x^(n/k))^p, x], x,
x^k], x] /; k != 1] /; FreeQ[{a, b, p}, x] && IGtQ[n, 0] && IntegerQ[m]
```

```
Int[(x_)^2/((a_) + (b_.)*(x_)^4)^(5/4), x_Symbol] :> Simp[x*((1 + a/(b*x^4)
)^(1/4)/(b*(a + b*x^4)^(1/4)))   Int[1/(x^3*(1 + a/(b*x^4))^(5/4)), x], x]
/; FreeQ[{a, b}, x] && PosQ[b/a]
```

```
Int[(x_)^2/((a_) + (b_.)*(x_)^4)^(1/4), x_Symbol] :> Simp[x^3/(2*(a + b*x^4
)^(1/4)), x] - Simp[a/2   Int[x^2/(a + b*x^4)^(5/4), x], x] /; FreeQ[{a, b}
, x] && PosQ[b/a]
```

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> -Subst[Int[(a +
b/x^n)^p/x^(m + 2), x], x, 1/x] /; FreeQ[{a, b, p}, x] && ILtQ[n, 0] && Int
egerQ[m]
```

```
Int[((e_.)*(x_))^(m_)*((b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(
e*x)^m*((b*x + c*x^2)^p/(x^(m + p)*(b + c*x)^p))   Int[x^(m + p)*(b + c*x)^
p, x], x] /; FreeQ[{b, c, e, m}, x]
```

Maple [F]

$$\int \frac{1}{x\,(b\,x^{2}+ax)^{\frac{1}{4}}}dx$$

```
int(1/x/(b*x^2+a*x)^(1/4),x)
```

```
int(1/x/(b*x^2+a*x)^(1/4),x)
```

Fricas [F]

$$\int \frac{1}{x\sqrt[4]{ax+bx^2}}\,dx = \int \frac{1}{(bx^2+ax)^{\frac{1}{4}}x}\,dx$$

```
integrate(1/x/(b*x^2+a*x)^(1/4),x, algorithm="fricas")
```

```
integral((b*x^2 + a*x)^(3/4)/(b*x^3 + a*x^2), x)
```

Sympy [F]

$$\int \frac{1}{x\sqrt[4]{ax+bx^2}}\,dx = \int \frac{1}{x\sqrt[4]{x\,(a+bx)}}\,dx$$

```
integrate(1/x/(b*x**2+a*x)**(1/4),x)
```

```
Integral(1/(x*(x*(a + b*x))**(1/4)), x)
```

Maxima [F]

$$\int \frac{1}{x\sqrt[4]{ax+bx^2}}\,dx = \int \frac{1}{(bx^2+ax)^{\frac{1}{4}}x}\,dx$$

```
integrate(1/x/(b*x^2+a*x)^(1/4),x, algorithm="maxima")
```

```
integrate(1/((b*x^2 + a*x)^(1/4)*x), x)
```

Giac [F]

$$\int \frac{1}{x\sqrt[4]{ax+bx^2}}\,dx = \int \frac{1}{(bx^2+ax)^{\frac{1}{4}}x}\,dx$$

```
integrate(1/x/(b*x^2+a*x)^(1/4),x, algorithm="giac")
```

```
integrate(1/((b*x^2 + a*x)^(1/4)*x), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{x\sqrt[4]{ax+bx^2}}\,dx = \int \frac{1}{x\,(b\,x^2+a\,x)^{1/4}}\,dx$$

```
int(1/(x*(a*x + b*x^2)^(1/4)),x)
```

```
int(1/(x*(a*x + b*x^2)^(1/4)), x)
```

Reduce [F]

$$\int \frac{1}{x\sqrt[4]{ax+bx^2}}\,dx = \int \frac{1}{x^{\frac{5}{4}}\,(bx+a)^{\frac{1}{4}}}dx$$

```
int(1/x/(b*x^2+a*x)^(1/4),x)
```

```
int(1/(x**(1/4)*(a + b*x)**(1/4)*x),x)
```

3.201 $\int \frac{1}{x\sqrt[4]{-ax-bx^2}}\,dx$

Optimal result

Integrand size = 19, antiderivative size = 73

$$\int \frac{1}{x\sqrt[4]{-ax-bx^2}}\,dx=-\frac{4\sqrt{-a-bx}\sqrt[4]{\frac{bx}{a+bx}}E\left(\frac{1}{2}\arctan\left(\frac{\sqrt{a}}{\sqrt{-a-bx}}\right)\middle|2\right)}{\sqrt{a}\sqrt[4]{-ax-bx^2}}$$

```
-4*(-b*x-a)^(1/2)*(b*x/(b*x+a))^(1/4)*EllipticE(sin(1/2*arctan(a^(1/2)/(-b
*x-a)^(1/2))),2^(1/2))/a^(1/2)/(-b*x^2-a*x)^(1/4)
```

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 10.01 (sec) , antiderivative size = 43, normalized size of antiderivative = 0.59

$$\int \frac{1}{x\sqrt[4]{-ax-bx^2}}\,dx=-\frac{4\sqrt[4]{1+\frac{bx}{a}}\,\text{Hypergeometric2F1}\left(-\frac{1}{4},\frac{1}{4},\frac{3}{4},-\frac{bx}{a}\right)}{\sqrt[4]{-x(a+bx)}}$$

```
Integrate[1/(x*(-(a*x) - b*x^2)^(1/4)),x]
```

```
(-4*(1 + (b*x)/a)^(1/4)*Hypergeometric2F1[-1/4, 1/4, 3/4, -((b*x)/a)])/(-(
x*(a + b*x)))^(1/4)
```

Rubi [B] (warning: unable to verify)

Leaf count is larger than twice the leaf count of optimal. 152 vs. $2(73) = 146$.

Time = 0.51 (sec) , antiderivative size = 152, normalized size of antiderivative = 2.08, number of steps used = 9, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}} = 0.421$, Rules used = {1137, 61, 73, 839, 813, 858, 807, 212}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{x\sqrt[4]{-ax-bx^2}}dx$$

$$\Big\downarrow \text{1137}$$

$$\frac{\sqrt[4]{x}\sqrt[4]{-a-bx}\int \frac{1}{x^{5/4}\sqrt[4]{-a-bx}}dx}{\sqrt[4]{-ax-bx^2}}$$

$$\Big\downarrow \text{61}$$

$$\frac{\sqrt[4]{x}\sqrt[4]{-a-bx}\left(\frac{2b\int \frac{1}{\sqrt[4]{x}\sqrt[4]{-a-bx}}dx}{a}+\frac{4(-a-bx)^{3/4}}{a\sqrt[4]{x}}\right)}{\sqrt[4]{-ax-bx^2}}$$

$$\Big\downarrow \text{73}$$

$$\frac{\sqrt[4]{x}\sqrt[4]{-a-bx}\left(\frac{8b\int \frac{\sqrt{x}}{\sqrt[4]{-a-bx}}d\sqrt[4]{x}}{a}+\frac{4(-a-bx)^{3/4}}{a\sqrt[4]{x}}\right)}{\sqrt[4]{-ax-bx^2}}$$

$$\Big\downarrow \text{839}$$

$$\frac{\sqrt[4]{x}\sqrt[4]{-a-bx}\left(\frac{8b\left(\frac{1}{2}a\int \frac{\sqrt{x}}{(-a-bx)^{5/4}}d\sqrt[4]{x}+\frac{x^{3/4}}{2\sqrt[4]{-a-bx}}\right)}{a}+\frac{4(-a-bx)^{3/4}}{a\sqrt[4]{x}}\right)}{\sqrt[4]{-ax-bx^2}}$$

$$\Big\downarrow \text{813}$$

$$\frac{\sqrt[4]{x}\sqrt[4]{-a-bx}\left(\frac{8b\left(\frac{x^{3/4}}{2\sqrt[4]{-a-bx}}-\frac{a\sqrt[4]{x}\sqrt[4]{\frac{a}{bx}+1}\int \frac{1}{\left(\frac{a}{bx}+1\right)^{5/4}x^{3/4}}d\sqrt[4]{x}}{2b\sqrt[4]{-a-bx}}\right)}{a}+\frac{4(-a-bx)^{3/4}}{a\sqrt[4]{x}}\right)}{\sqrt[4]{-ax-bx^2}}$$

$$\Big\downarrow \text{858}$$

$$\frac{\sqrt[4]{x}\sqrt[4]{-a-bx}\left(\frac{8b\left(\frac{a\sqrt[4]{x}\sqrt[4]{\frac{a}{bx}+1}\int \frac{1}{\sqrt[4]{x}\left(\frac{ax}{b}+1\right)^{5/4}}d\frac{1}{\sqrt[4]{x}}}{2b\sqrt[4]{-a-bx}}+\frac{x^{3/4}}{2\sqrt[4]{-a-bx}}\right)}{a}+\frac{4(-a-bx)^{3/4}}{a\sqrt[4]{x}}\right)}{\sqrt[4]{-ax-bx^2}}$$

$$\Big\downarrow \text{807}$$

$$\frac{\sqrt[4]{x}\sqrt[4]{-a-bx}\left(\frac{8b\left(\frac{a\sqrt[4]{x}\sqrt[4]{\frac{a}{bx}+1}\int \frac{1}{\left(\frac{\sqrt{x}a}{b}+1\right)^{5/4}}d\sqrt{x}}{4b\sqrt[4]{-a-bx}}+\frac{x^{3/4}}{2\sqrt[4]{-a-bx}}\right)}{a}+\frac{4(-a-bx)^{3/4}}{a\sqrt[4]{x}}\right)}{\sqrt[4]{-ax-bx^2}}$$

$$\Big\downarrow \text{212}$$

$$\frac{\sqrt[4]{x}\sqrt[4]{-a-bx}\left(\frac{8b\left(\frac{\sqrt{a}\sqrt[4]{x}\sqrt[4]{\frac{a}{bx}+1}E\left(\frac{1}{2}\arctan\left(\frac{\sqrt{a}\sqrt{x}}{\sqrt{b}}\right)\middle|2\right)}{2\sqrt{b}\sqrt[4]{-a-bx}}+\frac{x^{3/4}}{2\sqrt[4]{-a-bx}}\right)}{a}+\frac{4(-a-bx)^{3/4}}{a\sqrt[4]{x}}\right)}{\sqrt[4]{-ax-bx^2}}$$

```
Int[1/(x*(-(a*x) - b*x^2)^(1/4)),x]
```

```
(x^(1/4)*(-a - b*x)^(1/4)*((4*(-a - b*x)^(3/4))/(a*x^(1/4)) + (8*b*(x^(3/4
)/(2*(-a - b*x)^(1/4)) + (Sqrt[a]*(1 + a/(b*x))^(1/4)*x^(1/4)*EllipticE[Ar
cTan[(Sqrt[a]*Sqrt[x])/Sqrt[b]]/2, 2])/(2*Sqrt[b]*(-a - b*x)^(1/4))))/a))/
(-(a*x) - b*x^2)^(1/4)
```

Defintions of rubi rules used

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] - Simp[d*((
m + n + 2)/((b*c - a*d)*(m + 1)))   Int[(a + b*x)^(m + 1)*(c + d*x)^n, x],
x] /; FreeQ[{a, b, c, d, n}, x] && LtQ[m, -1] &&  !(LtQ[n, -1] && (EqQ[a, 0
] || (NeQ[c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && IntLinearQ[a, b, c, d
, m, n, x]
```

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Simp[p/b   Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
 d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && Lt
Q[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntL
inearQ[a, b, c, d, m, n, x]
```

```
Int[((a_) + (b_.)*(x_)^2)^(-5/4), x_Symbol] :> Simp[(2/(a^(5/4)*Rt[b/a, 2])
)*EllipticE[(1/2)*ArcTan[Rt[b/a, 2]*x], 2], x] /; FreeQ[{a, b}, x] && GtQ[a
, 0] && PosQ[b/a]
```

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k = GCD[m
+ 1, n]}, Simp[1/k   Subst[Int[x^((m + 1)/k - 1)*(a + b*x^(n/k))^p, x], x,
x^k], x] /; k != 1] /; FreeQ[{a, b, p}, x] && IGtQ[n, 0] && IntegerQ[m]
```

```
Int[(x_)^2/((a_) + (b_.)*(x_)^4)^(5/4), x_Symbol] :> Simp[x*((1 + a/(b*x^4)
)^(1/4)/(b*(a + b*x^4)^(1/4)))   Int[1/(x^3*(1 + a/(b*x^4))^(5/4)), x], x]
/; FreeQ[{a, b}, x] && PosQ[b/a]
```

```
Int[(x_)^2/((a_) + (b_.)*(x_)^4)^(1/4), x_Symbol] :> Simp[x^3/(2*(a + b*x^4
)^(1/4)), x] - Simp[a/2   Int[x^2/(a + b*x^4)^(5/4), x], x] /; FreeQ[{a, b}
, x] && PosQ[b/a]
```

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> -Subst[Int[(a +
b/x^n)^p/x^(m + 2), x], x, 1/x] /; FreeQ[{a, b, p}, x] && ILtQ[n, 0] && Int
egerQ[m]
```

```
Int[((e_.)*(x_))^(m_)*((b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(
e*x)^m*((b*x + c*x^2)^p/(x^(m + p)*(b + c*x)^p))   Int[x^(m + p)*(b + c*x)^
p, x], x] /; FreeQ[{b, c, e, m}, x]
```

Maple [F]

$$\int \frac{1}{x\left(-b\,x^{2}-ax\right)^{\frac{1}{4}}}dx$$

```
int(1/x/(-b*x^2-a*x)^(1/4),x)
```

```
int(1/x/(-b*x^2-a*x)^(1/4),x)
```

Fricas [F]

$$\int \frac{1}{x\sqrt[4]{-ax-bx^2}}\,dx = \int \frac{1}{(-bx^2-ax)^{\frac{1}{4}}x}\,dx$$

```
integrate(1/x/(-b*x^2-a*x)^(1/4),x, algorithm="fricas")
```

```
integral(-(-b*x^2 - a*x)^(3/4)/(b*x^3 + a*x^2), x)
```

Sympy [F]

$$\int \frac{1}{x\sqrt[4]{-ax-bx^2}}\,dx = \int \frac{1}{x\sqrt[4]{-x(a+bx)}}\,dx$$

```
integrate(1/x/(-b*x**2-a*x)**(1/4),x)
```

```
Integral(1/(x*(-x*(a + b*x))**(1/4)), x)
```

Maxima [F]

$$\int \frac{1}{x\sqrt[4]{-ax-bx^2}}\,dx = \int \frac{1}{(-bx^2-ax)^{\frac{1}{4}}x}\,dx$$

```
integrate(1/x/(-b*x^2-a*x)^(1/4),x, algorithm="maxima")
```

```
integrate(1/((-b*x^2 - a*x)^(1/4)*x), x)
```

Giac [F]

$$\int \frac{1}{x\sqrt[4]{-ax-bx^2}}\,dx=\int \frac{1}{(-bx^2-ax)^{\frac{1}{4}}x}\,dx$$

```
integrate(1/x/(-b*x^2-a*x)^(1/4),x, algorithm="giac")
```

```
integrate(1/((-b*x^2 - a*x)^(1/4)*x), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{x\sqrt[4]{-ax-bx^2}}\,dx=\int \frac{1}{x\,{\left(-b\,x^2-a\,x\right)}^{1/4}}\,dx$$

```
int(1/(x*(- a*x - b*x^2)^(1/4)),x)
```

```
int(1/(x*(- a*x - b*x^2)^(1/4)), x)
```

Reduce [F]

$$\int \frac{1}{x\sqrt[4]{-ax-bx^2}}\,dx=-\left(\int \frac{1}{x^{\frac{5}{4}}\,(bx+a)^{\frac{1}{4}}}dx\right)(-1)^{\frac{3}{4}}$$

```
int(1/x/(-b*x^2-a*x)^(1/4),x)
```

```
int(1/(x**(1/4)*(a + b*x)**(1/4)*x),x)/( - 1)**(1/4)
```

3.202 $\int \frac{1}{x \sqrt[4]{2x+3x^2}} \, dx$

Optimal result

Integrand size = 17, antiderivative size = 77

$$\int \frac{1}{x \sqrt[4]{2x+3x^2}} \, dx = -\frac{4}{\sqrt[4]{2x+3x^2}} + \frac{2\sqrt{2}\sqrt[4]{3}\sqrt[4]{x}\sqrt[4]{2+3x}E\left(\frac{1}{2}\arcsin\left(\frac{\sqrt{2}}{\sqrt{2+3x}}\right)\middle|2\right)}{\sqrt[4]{2x+3x^2}}$$

```
-4/(3*x^2+2*x)^(1/4)+2*2^(1/2)*3^(1/4)*x^(1/4)*(2+3*x)^(1/4)*EllipticE(sin
(1/2*arcsin(2^(1/2)/(2+3*x)^(1/2))),2^(1/2))/(3*x^2+2*x)^(1/4)
```

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 10.01 (sec) , antiderivative size = 42, normalized size of antiderivative = 0.55

$$\int \frac{1}{x \sqrt[4]{2x+3x^2}} \, dx = -\frac{2\ 2^{3/4}\sqrt[4]{2+3x}\,\text{Hypergeometric2F1}\left(-\frac{1}{4},\frac{1}{4},\frac{3}{4},-\frac{3x}{2}\right)}{\sqrt[4]{x(2+3x)}}$$

```
Integrate[1/(x*(2*x + 3*x^2)^(1/4)),x]
```

```
(-2*2^(3/4)*(2 + 3*x)^(1/4)*Hypergeometric2F1[-1/4, 1/4, 3/4, (-3*x)/2])/(
x*(2 + 3*x))^(1/4)
```

Rubi [A] (warning: unable to verify)

Time = 0.46 (sec) , antiderivative size = 122, normalized size of antiderivative = 1.58, number of steps used = 10, number of rules used = 9, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.529, Rules used = {1137, 61, 73, 839, 813, 27, 858, 807, 212}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{x\sqrt[4]{3x^2+2x}}dx$$

$$\Big\downarrow \text{ 1137}$$

$$\frac{\sqrt[4]{x}\sqrt[4]{3x+2}\int \frac{1}{x^{5/4}\sqrt[4]{3x+2}}dx}{\sqrt[4]{3x^2+2x}}$$

$$\Big\downarrow \text{ 61}$$

$$\frac{\sqrt[4]{x}\sqrt[4]{3x+2}\left(3\int \frac{1}{\sqrt[4]{x}\sqrt[4]{3x+2}}dx-\frac{2(3x+2)^{3/4}}{\sqrt[4]{x}}\right)}{\sqrt[4]{3x^2+2x}}$$

$$\Big\downarrow \text{ 73}$$

$$\frac{\sqrt[4]{x}\sqrt[4]{3x+2}\left(12\int \frac{\sqrt{x}}{\sqrt[4]{3x+2}}d\sqrt[4]{x}-\frac{2(3x+2)^{3/4}}{\sqrt[4]{x}}\right)}{\sqrt[4]{3x^2+2x}}$$

$$\Big\downarrow \text{ 839}$$

$$\frac{\sqrt[4]{x}\sqrt[4]{3x+2}\left(12\left(\frac{x^{3/4}}{2\sqrt[4]{3x+2}}-\int \frac{\sqrt{x}}{(3x+2)^{5/4}}d\sqrt[4]{x}\right)-\frac{2(3x+2)^{3/4}}{\sqrt[4]{x}}\right)}{\sqrt[4]{3x^2+2x}}$$

$$\Big\downarrow \text{ 813}$$

$$\frac{\sqrt[4]{x}\sqrt[4]{3x+2}\left(12\left(\frac{x^{3/4}}{2\sqrt[4]{3x+2}}-\frac{\sqrt[4]{\frac{2}{x}+3}\sqrt[4]{x}\int\frac{3\sqrt[4]{3}}{\left(3+\frac{2}{x}\right)^{5/4}x^{3/4}}d\sqrt[4]{x}}{3\sqrt[4]{3}\sqrt[4]{3x+2}}\right)-\frac{2(3x+2)^{3/4}}{\sqrt[4]{x}}\right)}{\sqrt[4]{3x^2+2x}}$$

$$\Big\downarrow \ 27$$

$$\frac{\sqrt[4]{x}\sqrt[4]{3x+2}\left(12\left(\frac{x^{3/4}}{2\sqrt[4]{3x+2}}-\frac{\sqrt[4]{\frac{2}{x}+3}\sqrt[4]{x}\int\frac{1}{\left(3+\frac{2}{x}\right)^{5/4}x^{3/4}}d\sqrt[4]{x}}{\sqrt[4]{3x+2}}\right)-\frac{2(3x+2)^{3/4}}{\sqrt[4]{x}}\right)}{\sqrt[4]{3x^2+2x}}$$

$$\Big\downarrow \ 858$$

$$\frac{\sqrt[4]{x}\sqrt[4]{3x+2}\left(12\left(\frac{\sqrt[4]{\frac{2}{x}+3}\sqrt[4]{x}\int\frac{1}{\sqrt[4]{x}(2x+3)^{5/4}}d\frac{1}{\sqrt[4]{x}}}{\sqrt[4]{3x+2}}+\frac{x^{3/4}}{2\sqrt[4]{3x+2}}\right)-\frac{2(3x+2)^{3/4}}{\sqrt[4]{x}}\right)}{\sqrt[4]{3x^2+2x}}$$

$$\Big\downarrow \ 807$$

$$\frac{\sqrt[4]{x}\sqrt[4]{3x+2}\left(12\left(\frac{\sqrt[4]{\frac{2}{x}+3}\sqrt[4]{x}\int\frac{1}{(2\sqrt{x}+3)^{5/4}}d\sqrt{x}}{2\sqrt[4]{3x+2}}+\frac{x^{3/4}}{2\sqrt[4]{3x+2}}\right)-\frac{2(3x+2)^{3/4}}{\sqrt[4]{x}}\right)}{\sqrt[4]{3x^2+2x}}$$

$$\Big\downarrow \ 212$$

$$\frac{\sqrt[4]{x}\sqrt[4]{3x+2}\left(12\left(\frac{\sqrt[4]{\frac{2}{x}+3}\sqrt[4]{x}E\left(\frac{1}{2}\arctan\left(\sqrt{\frac{2}{3}}\sqrt{x}\right)\middle|2\right)}{\sqrt{2}3^{3/4}\sqrt[4]{3x+2}}+\frac{x^{3/4}}{2\sqrt[4]{3x+2}}\right)-\frac{2(3x+2)^{3/4}}{\sqrt[4]{x}}\right)}{\sqrt[4]{3x^2+2x}}$$

```
Int[1/(x*(2*x + 3*x^2)^(1/4)),x]
```

```
(x^(1/4)*(2 + 3*x)^(1/4)*((-2*(2 + 3*x)^(3/4))/x^(1/4) + 12*(x^(3/4)/(2*(2
 + 3*x)^(1/4)) + ((3 + 2/x)^(1/4)*x^(1/4)*EllipticE[ArcTan[Sqrt[2/3]*Sqrt[
x]]/2, 2])/(Sqrt[2]*3^(3/4)*(2 + 3*x)^(1/4)))))/(2*x + 3*x^2)^(1/4)
```

Defintions of rubi rules used

```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] - Simp[d*((
m + n + 2)/((b*c - a*d)*(m + 1)))   Int[(a + b*x)^(m + 1)*(c + d*x)^n, x],
x] /; FreeQ[{a, b, c, d, n}, x] && LtQ[m, -1] &&  !(LtQ[n, -1] && (EqQ[a, 0
] || (NeQ[c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && IntLinearQ[a, b, c, d
, m, n, x]
```

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Simp[p/b   Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
 d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && Lt
Q[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntL
inearQ[a, b, c, d, m, n, x]
```

```
Int[((a_) + (b_.)*(x_)^2)^(-5/4), x_Symbol] :> Simp[(2/(a^(5/4)*Rt[b/a, 2])
)*EllipticE[(1/2)*ArcTan[Rt[b/a, 2]*x], 2], x] /; FreeQ[{a, b}, x] && GtQ[a
, 0] && PosQ[b/a]
```

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k = GCD[m
+ 1, n]}, Simp[1/k   Subst[Int[x^((m + 1)/k - 1)*(a + b*x^(n/k))^p, x], x,
x^k], x] /; k != 1] /; FreeQ[{a, b, p}, x] && IGtQ[n, 0] && IntegerQ[m]
```

```
Int[(x_)^2/((a_) + (b_.)*(x_)^4)^(5/4), x_Symbol] :> Simp[x*((1 + a/(b*x^4)
)^(1/4)/(b*(a + b*x^4)^(1/4)))   Int[1/(x^3*(1 + a/(b*x^4))^(5/4)), x], x]
/; FreeQ[{a, b}, x] && PosQ[b/a]
```

```
Int[(x_)^2/((a_) + (b_.)*(x_)^4)^(1/4), x_Symbol] :> Simp[x^3/(2*(a + b*x^4
)^(1/4)), x] - Simp[a/2   Int[x^2/(a + b*x^4)^(5/4), x], x] /; FreeQ[{a, b}
, x] && PosQ[b/a]
```

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> -Subst[Int[(a +
b/x^n)^p/x^(m + 2), x], x, 1/x] /; FreeQ[{a, b, p}, x] && ILtQ[n, 0] && Int
egerQ[m]
```

```
Int[((e_.)*(x_))^(m_)*((b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(
e*x)^m*((b*x + c*x^2)^p/(x^(m + p)*(b + c*x)^p))   Int[x^(m + p)*(b + c*x)^
p, x], x] /; FreeQ[{b, c, e, m}, x]
```

Maple [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4.

Time = 0.38 (sec) , antiderivative size = 18, normalized size of antiderivative = 0.23

method	result	size
meijerg	$-\frac{2\,2^{\frac{3}{4}} \operatorname{hypergeom}\left(\left[-\frac{1}{4},\frac{1}{4}\right],\left[\frac{3}{4}\right],-\frac{3x}{2}\right)}{x^{\frac{1}{4}}}$	18
risch	$-\frac{2(3x+2)}{\left(x(3x+2)\right)^{\frac{1}{4}}}+2\,2^{\frac{3}{4}} x^{\frac{3}{4}} \operatorname{hypergeom}\left(\left[\frac{1}{4},\frac{3}{4}\right],\left[\frac{7}{4}\right],-\frac{3x}{2}\right)$	35

```
int(1/x/(3*x^2+2*x)^(1/4),x,method=_RETURNVERBOSE)
```

```
-2*2^(3/4)/x^(1/4)*hypergeom([-1/4,1/4],[3/4],-3/2*x)
```

Fricas [F]

$$\int \frac{1}{x\sqrt[4]{2x+3x^2}}\,dx=\int \frac{1}{\left(3\,x^2+2\,x\right)^{\frac{1}{4}}x}\,dx$$

```
integrate(1/x/(3*x^2+2*x)^(1/4),x, algorithm="fricas")
```

```
integral((3*x^2 + 2*x)^(3/4)/(3*x^3 + 2*x^2), x)
```

Sympy [F]

$$\int \frac{1}{x\sqrt[4]{2x+3x^2}}\,dx = \int \frac{1}{x\sqrt[4]{x\,(3x+2)}}\,dx$$

```
integrate(1/x/(3*x**2+2*x)**(1/4),x)
```

```
Integral(1/(x*(x*(3*x + 2))**(1/4)), x)
```

Maxima [F]

$$\int \frac{1}{x\sqrt[4]{2x+3x^2}}\,dx = \int \frac{1}{(3\,x^2+2\,x)^{\frac{1}{4}}x}\,dx$$

```
integrate(1/x/(3*x^2+2*x)^(1/4),x, algorithm="maxima")
```

```
integrate(1/((3*x^2 + 2*x)^(1/4)*x), x)
```

Giac [F]

$$\int \frac{1}{x\sqrt[4]{2x+3x^2}}\,dx = \int \frac{1}{(3\,x^2+2\,x)^{\frac{1}{4}}x}\,dx$$

```
integrate(1/x/(3*x^2+2*x)^(1/4),x, algorithm="giac")
```

```
integrate(1/((3*x^2 + 2*x)^(1/4)*x), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{x\sqrt[4]{2x+3x^2}}\,dx = \int \frac{1}{x\,\left(3\,x^2+2\,x\right)^{1/4}}\,dx$$

```
int(1/(x*(2*x + 3*x^2)^(1/4)),x)
```

```
int(1/(x*(2*x + 3*x^2)^(1/4)), x)
```

Reduce [F]

$$\int \frac{1}{x\sqrt[4]{2x+3x^2}}\,dx = \int \frac{1}{x^{\frac{5}{4}}\,(3x+2)^{\frac{1}{4}}}dx$$

```
int(1/x/(3*x^2+2*x)^(1/4),x)
```

```
int(1/(x**(1/4)*(3*x + 2)**(1/4)*x),x)
```

3.203 $\int \frac{1}{x\sqrt[4]{-2x+3x^2}}\,dx$

Optimal result

Integrand size = 17, antiderivative size = 59

$$\int \frac{1}{x\sqrt[4]{-2x+3x^2}}\,dx=-\frac{2\sqrt{2}\sqrt[4]{3}\sqrt[4]{x}\sqrt[4]{-2+3x}E\left(\frac{1}{2}\arcsin\left(\frac{\sqrt{\frac{2}{3}}}{\sqrt{x}}\right)\middle|2\right)}{\sqrt[4]{-2x+3x^2}}$$

```
-2*2^(1/2)*3^(1/4)*x^(1/4)*(-2+3*x)^(1/4)*EllipticE(sin(1/2*arcsin(1/3*6^(
1/2)/x^(1/2))),2^(1/2))/(3*x^2-2*x)^(1/4)
```

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 10.01 (sec) , antiderivative size = 41, normalized size of antiderivative = 0.69

$$\int \frac{1}{x\sqrt[4]{-2x+3x^2}}\,dx=\frac{\left(\frac{2}{3}\right)^{3/4}(x(-2+3x))^{3/4}\,\text{Hypergeometric2F1}\left(\frac{3}{4},\frac{5}{4},\frac{7}{4},1-\frac{3x}{2}\right)}{x^{3/4}}$$

```
Integrate[1/(x*(-2*x + 3*x^2)^(1/4)),x]
```

```
((2/3)^(3/4)*(x*(-2 + 3*x))^(3/4)*Hypergeometric2F1[3/4, 5/4, 7/4, 1 - (3*
x)/2])/x^(3/4)
```

Rubi [B] (warning: unable to verify)

Leaf count is larger than twice the leaf count of optimal. 122 vs. $2(59) = 118$.

Time = 0.47 (sec) , antiderivative size = 122, normalized size of antiderivative = 2.07, number of steps used = 10, number of rules used = 9, $\frac{\text{number of rules}}{\text{integrand size}} = 0.529$, Rules used = {1137, 61, 73, 840, 842, 27, 858, 807, 226}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{x\sqrt[4]{3x^2-2x}}dx$$

$$\Big\downarrow \text{1137}$$

$$\frac{\sqrt[4]{x}\sqrt[4]{3x-2}\int \frac{1}{x^{5/4}\sqrt[4]{3x-2}}dx}{\sqrt[4]{3x^2-2x}}$$

$$\Big\downarrow \text{61}$$

$$\frac{\sqrt[4]{x}\sqrt[4]{3x-2}\left(\frac{2(3x-2)^{3/4}}{\sqrt[4]{x}}-3\int \frac{1}{\sqrt[4]{x}\sqrt[4]{3x-2}}dx\right)}{\sqrt[4]{3x^2-2x}}$$

$$\Big\downarrow \text{73}$$

$$\frac{\sqrt[4]{x}\sqrt[4]{3x-2}\left(\frac{2(3x-2)^{3/4}}{\sqrt[4]{x}}-12\int \frac{\sqrt{x}}{\sqrt[4]{3x-2}}d\sqrt[4]{x}\right)}{\sqrt[4]{3x^2-2x}}$$

$$\Big\downarrow \text{840}$$

$$\frac{\sqrt[4]{x}\sqrt[4]{3x-2}\left(\frac{2(3x-2)^{3/4}}{\sqrt[4]{x}}-12\left(\frac{(3x-2)^{3/4}}{6\sqrt[4]{x}}-\frac{1}{3}\int \frac{1}{\sqrt{x}\sqrt[4]{3x-2}}d\sqrt[4]{x}\right)\right)}{\sqrt[4]{3x^2-2x}}$$

$$\Big\downarrow \text{842}$$

$$\frac{\sqrt[4]{x}\sqrt[4]{3x-2}\left(\frac{2(3x-2)^{3/4}}{\sqrt[4]{x}}-12\left(\frac{(3x-2)^{3/4}}{6\sqrt[4]{x}}-\frac{\sqrt[4]{3-\frac{2}{x}}\sqrt[4]{x}\int\frac{\sqrt[4]{3}}{\sqrt[4]{3-\frac{2}{x}}x^{3/4}}d\sqrt[4]{x}}{3\sqrt[4]{3}\sqrt[4]{3x-2}}\right)\right)}{\sqrt[4]{3x^2-2x}}$$

$$\Big\downarrow \; 27$$

$$\frac{\sqrt[4]{x}\sqrt[4]{3x-2}\left(\frac{2(3x-2)^{3/4}}{\sqrt[4]{x}}-12\left(\frac{(3x-2)^{3/4}}{6\sqrt[4]{x}}-\frac{\sqrt[4]{3-\frac{2}{x}}\sqrt[4]{x}\int\frac{1}{\sqrt[4]{3-\frac{2}{x}}x^{3/4}}d\sqrt[4]{x}}{3\sqrt[4]{3x-2}}\right)\right)}{\sqrt[4]{3x^2-2x}}$$

$$\Big\downarrow \; 858$$

$$\frac{\sqrt[4]{x}\sqrt[4]{3x-2}\left(\frac{2(3x-2)^{3/4}}{\sqrt[4]{x}}-12\left(\frac{\sqrt[4]{3-\frac{2}{x}}\sqrt[4]{x}\int\frac{1}{\sqrt[4]{3-2x}\sqrt[4]{x}}d\frac{1}{\sqrt[4]{x}}}{3\sqrt[4]{3x-2}}+\frac{(3x-2)^{3/4}}{6\sqrt[4]{x}}\right)\right)}{\sqrt[4]{3x^2-2x}}$$

$$\Big\downarrow \; 807$$

$$\frac{\sqrt[4]{x}\sqrt[4]{3x-2}\left(\frac{2(3x-2)^{3/4}}{\sqrt[4]{x}}-12\left(\frac{\sqrt[4]{3-\frac{2}{x}}\sqrt[4]{x}\int\frac{1}{\sqrt[4]{3-2\sqrt{x}}}d\sqrt{x}}{6\sqrt[4]{3x-2}}+\frac{(3x-2)^{3/4}}{6\sqrt[4]{x}}\right)\right)}{\sqrt[4]{3x^2-2x}}$$

$$\Big\downarrow \; 226$$

$$\frac{\sqrt[4]{x}\sqrt[4]{3x-2}\left(\frac{2(3x-2)^{3/4}}{\sqrt[4]{x}}-12\left(\frac{\sqrt[4]{3-\frac{2}{x}}\sqrt[4]{x}E\left(\frac{1}{2}\arcsin\left(\sqrt{\frac{2}{3}}\sqrt{x}\right)\middle|2\right)}{\sqrt{2}3^{3/4}\sqrt[4]{3x-2}}+\frac{(3x-2)^{3/4}}{6\sqrt[4]{x}}\right)\right)}{\sqrt[4]{3x^2-2x}}$$

```
Int[1/(x*(-2*x + 3*x^2)^(1/4)),x]
```

```
(x^(1/4)*(-2 + 3*x)^(1/4)*((2*(-2 + 3*x)^(3/4))/x^(1/4) - 12*((-2 + 3*x)^(
3/4)/(6*x^(1/4)) + ((3 - 2/x)^(1/4)*x^(1/4)*EllipticE[ArcSin[Sqrt[2/3]*Sqr
t[x]]/2, 2])/(Sqrt[2]*3^(3/4)*(-2 + 3*x)^(1/4)))))/(-2*x + 3*x^2)^(1/4)
```

Defintions of rubi rules used

```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] - Simp[d*((
m + n + 2)/((b*c - a*d)*(m + 1)))   Int[(a + b*x)^(m + 1)*(c + d*x)^n, x],
x] /; FreeQ[{a, b, c, d, n}, x] && LtQ[m, -1] &&  !(LtQ[n, -1] && (EqQ[a, 0
] || (NeQ[c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && IntLinearQ[a, b, c, d
, m, n, x]
```

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Simp[p/b   Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
 d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && Lt
Q[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntL
inearQ[a, b, c, d, m, n, x]
```

```
Int[((a_) + (b_.)*(x_)^2)^(-1/4), x_Symbol] :> Simp[(2/(a^(1/4)*Rt[-b/a, 2]
))*EllipticE[(1/2)*ArcSin[Rt[-b/a, 2]*x], 2], x] /; FreeQ[{a, b}, x] && GtQ
[a, 0] && NegQ[b/a]
```

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k = GCD[m
+ 1, n]}, Simp[1/k   Subst[Int[x^((m + 1)/k - 1)*(a + b*x^(n/k))^p, x], x,
x^k], x] /; k != 1] /; FreeQ[{a, b, p}, x] && IGtQ[n, 0] && IntegerQ[m]
```

```
Int[(x_)^2/((a_) + (b_.)*(x_)^4)^(1/4), x_Symbol] :> Simp[(a + b*x^4)^(3/4)
/(2*b*x), x] + Simp[a/(2*b)   Int[1/(x^2*(a + b*x^4)^(1/4)), x], x] /; Free
Q[{a, b}, x] && NegQ[b/a]
```

```
Int[1/((x_)^2*((a_) + (b_.)*(x_)^4)^(1/4)), x_Symbol] :> Simp[x*((1 + a/(b*
x^4))^(1/4)/(a + b*x^4)^(1/4))   Int[1/(x^3*(1 + a/(b*x^4))^(1/4)), x], x]
/; FreeQ[{a, b}, x] && NegQ[b/a]
```

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> -Subst[Int[(a +
b/x^n)^p/x^(m + 2), x], x, 1/x] /; FreeQ[{a, b, p}, x] && ILtQ[n, 0] && Int
egerQ[m]
```

```
Int[((e_.)*(x_))^(m_)*((b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(
e*x)^m*((b*x + c*x^2)^p/(x^(m + p)*(b + c*x)^p))   Int[x^(m + p)*(b + c*x)^
p, x], x] /; FreeQ[{b, c, e, m}, x]
```

Maple [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 0.40 (sec) , antiderivative size = 32, normalized size of antiderivative = 0.54

method	result	size
meijerg	$-\frac{2\,2^{\frac{3}{4}} \left(-\operatorname{signum}\left(x-\frac{2}{3}\right)\right)^{\frac{1}{4}} \operatorname{hypergeom}\left(\left[-\frac{1}{4},\frac{1}{4}\right],\left[\frac{3}{4}\right],\frac{3x}{2}\right)}{\operatorname{signum}\left(x-\frac{2}{3}\right)^{\frac{1}{4}} x^{\frac{1}{4}}}$	32
risch	$\frac{-4+6x}{\left(x(-2+3x)\right)^{\frac{1}{4}}}-\frac{2\,2^{\frac{3}{4}} \left(-\operatorname{signum}\left(x-\frac{2}{3}\right)\right)^{\frac{1}{4}} x^{\frac{3}{4}} \operatorname{hypergeom}\left(\left[\frac{1}{4},\frac{3}{4}\right],\left[\frac{7}{4}\right],\frac{3x}{2}\right)}{\operatorname{signum}\left(x-\frac{2}{3}\right)^{\frac{1}{4}}}$	49

```
int(1/x/(3*x^2-2*x)^(1/4),x,method=_RETURNVERBOSE)
```

```
-2*2^(3/4)/signum(x-2/3)^(1/4)*(-signum(x-2/3))^(1/4)/x^(1/4)*hypergeom([-
1/4,1/4],[3/4],3/2*x)
```

Fricas [F]

$$\int \frac{1}{x\sqrt[4]{-2x+3x^2}}\,dx = \int \frac{1}{(3\,x^2-2\,x)^{\frac{1}{4}}x}\,dx$$

```
integrate(1/x/(3*x^2-2*x)^(1/4),x, algorithm="fricas")
```

```
integral((3*x^2 - 2*x)^(3/4)/(3*x^3 - 2*x^2), x)
```

Sympy [F]

$$\int \frac{1}{x\sqrt[4]{-2x+3x^2}}\,dx = \int \frac{1}{x\sqrt[4]{x\,(3x-2)}}\,dx$$

```
integrate(1/x/(3*x**2-2*x)**(1/4),x)
```

```
Integral(1/(x*(x*(3*x - 2))**(1/4)), x)
```

Maxima [F]

$$\int \frac{1}{x\sqrt[4]{-2x+3x^2}}\,dx = \int \frac{1}{(3\,x^2-2\,x)^{\frac{1}{4}}x}\,dx$$

```
integrate(1/x/(3*x^2-2*x)^(1/4),x, algorithm="maxima")
```

```
integrate(1/((3*x^2 - 2*x)^(1/4)*x), x)
```

Giac [F]

$$\int \frac{1}{x\sqrt[4]{-2x+3x^2}}\,dx = \int \frac{1}{(3\,x^2-2\,x)^{\frac{1}{4}}x}\,dx$$

```
integrate(1/x/(3*x^2-2*x)^(1/4),x, algorithm="giac")
```

```
integrate(1/((3*x^2 - 2*x)^(1/4)*x), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{x\sqrt[4]{-2x+3x^2}}\,dx = \int \frac{1}{x\,(3\,x^2-2\,x)^{1/4}}\,dx$$

```
int(1/(x*(3*x^2 - 2*x)^(1/4)),x)
```

```
int(1/(x*(3*x^2 - 2*x)^(1/4)), x)
```

Reduce [F]

$$\int \frac{1}{x\sqrt[4]{-2x+3x^2}}\,dx = \int \frac{1}{x^{\frac{5}{4}}\,(3x-2)^{\frac{1}{4}}}dx$$

```
int(1/x/(3*x^2-2*x)^(1/4),x)
```

```
int(1/(x**(1/4)*(3*x - 2)**(1/4)*x),x)
```

3.204 $\int \frac{1}{x\sqrt[4]{ax+3x^2}}\,dx$

Optimal result

Integrand size = 17, antiderivative size = 85

$$\int \frac{1}{x\sqrt[4]{ax+3x^2}}\,dx = -\frac{4}{\sqrt[4]{ax+3x^2}} + \frac{4\sqrt[4]{3}\sqrt[4]{\frac{x}{a+3x}}\sqrt{a+3x}\,E\left(\frac{1}{2}\arcsin\left(\frac{\sqrt{a}}{\sqrt{a+3x}}\right)\middle|2\right)}{\sqrt{a}\sqrt[4]{ax+3x^2}}$$

```
-4/(a*x+3*x^2)^(1/4)+4*3^(1/4)*(x/(a+3*x))^(1/4)*(a+3*x)^(1/2)*EllipticE(s
in(1/2*arcsin(a^(1/2)/(a+3*x)^(1/2))),2^(1/2))/a^(1/2)/(a*x+3*x^2)^(1/4)
```

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 10.01 (sec) , antiderivative size = 41, normalized size of antiderivative = 0.48

$$\int \frac{1}{x\sqrt[4]{ax+3x^2}}\,dx = -\frac{4\sqrt[4]{1+\frac{3x}{a}}\,\text{Hypergeometric2F1}\left(-\frac{1}{4},\frac{1}{4},\frac{3}{4},-\frac{3x}{a}\right)}{\sqrt[4]{x(a+3x)}}$$

```
Integrate[1/(x*(a*x + 3*x^2)^(1/4)),x]
```

```
(-4*(1 + (3*x)/a)^(1/4)*Hypergeometric2F1[-1/4, 1/4, 3/4, (-3*x)/a])/(x*(a
 + 3*x))^(1/4)
```

Rubi [A] (warning: unable to verify)

Time = 0.50 (sec) , antiderivative size = 134, normalized size of antiderivative = 1.58, number of steps used = 10, number of rules used = 9, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.529, Rules used = {1137, 61, 73, 839, 813, 27, 858, 807, 212}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{x\sqrt[4]{ax+3x^2}}\,dx$$

$$\Big\downarrow \text{1137}$$

$$\frac{\sqrt[4]{x}\sqrt[4]{a+3x}\int \frac{1}{x^{5/4}\sqrt[4]{a+3x}}dx}{\sqrt[4]{ax+3x^2}}$$

$$\Big\downarrow \text{61}$$

$$\frac{\sqrt[4]{x}\sqrt[4]{a+3x}\left(\frac{6\int \frac{1}{\sqrt[4]{x}\sqrt[4]{a+3x}}dx}{a}-\frac{4(a+3x)^{3/4}}{a\sqrt[4]{x}}\right)}{\sqrt[4]{ax+3x^2}}$$

$$\Big\downarrow \text{73}$$

$$\frac{\sqrt[4]{x}\sqrt[4]{a+3x}\left(\frac{24\int \frac{\sqrt{x}}{\sqrt[4]{a+3x}}d\sqrt[4]{x}}{a}-\frac{4(a+3x)^{3/4}}{a\sqrt[4]{x}}\right)}{\sqrt[4]{ax+3x^2}}$$

$$\Big\downarrow \text{839}$$

$$\frac{\sqrt[4]{x}\sqrt[4]{a+3x}\left(\frac{24\left(\frac{x^{3/4}}{2\sqrt[4]{a+3x}}-\frac{1}{2}a\int \frac{\sqrt{x}}{(a+3x)^{5/4}}d\sqrt[4]{x}\right)}{a}-\frac{4(a+3x)^{3/4}}{a\sqrt[4]{x}}\right)}{\sqrt[4]{ax+3x^2}}$$

$$\Big\downarrow \text{813}$$

$$\frac{\sqrt[4]{x}\sqrt[4]{a+3x}\left(\frac{24\left(\frac{x^{3/4}}{2\sqrt[4]{a+3x}}-\frac{a\sqrt[4]{x}\sqrt[4]{\frac{a}{x}+3}\int \frac{3\sqrt[4]{3}}{\left(\frac{a}{x}+3\right)^{5/4}x^{3/4}}d\sqrt[4]{x}}{6\sqrt[4]{3}\sqrt[4]{a+3x}}\right)}{a}-\frac{4(a+3x)^{3/4}}{a\sqrt[4]{x}}\right)}{\sqrt[4]{ax+3x^2}}$$

$$\Big\downarrow \text{27}$$

$$\frac{\sqrt[4]{x}\sqrt[4]{a+3x}\left(\frac{24\left(\frac{x^{3/4}}{2\sqrt[4]{a+3x}}-\frac{a\sqrt[4]{x}\sqrt[4]{\frac{a}{x}+3}\int \frac{1}{\left(\frac{a}{x}+3\right)^{5/4}x^{3/4}}d\sqrt[4]{x}}{2\sqrt[4]{a+3x}}\right)}{a}-\frac{4(a+3x)^{3/4}}{a\sqrt[4]{x}}\right)}{\sqrt[4]{ax+3x^2}}$$

$$\Big\downarrow \text{858}$$

$$\frac{\sqrt[4]{x}\sqrt[4]{a+3x}\left(\frac{24\left(\frac{a\sqrt[4]{x}\sqrt[4]{\frac{a}{x}+3}\int \frac{1}{\sqrt[4]{x}(ax+3)^{5/4}}d\frac{1}{\sqrt[4]{x}}}{2\sqrt[4]{a+3x}}+\frac{x^{3/4}}{2\sqrt[4]{a+3x}}\right)}{a}-\frac{4(a+3x)^{3/4}}{a\sqrt[4]{x}}\right)}{\sqrt[4]{ax+3x^2}}$$

$$\Big\downarrow \text{807}$$

$$\frac{\sqrt[4]{x}\sqrt[4]{a+3x}\left(\frac{24\left(\frac{a\sqrt[4]{x}\sqrt[4]{\frac{a}{x}+3}\int \frac{1}{(\sqrt{x}a+3)^{5/4}}d\sqrt{x}}{4\sqrt[4]{a+3x}}+\frac{x^{3/4}}{2\sqrt[4]{a+3x}}\right)}{a}-\frac{4(a+3x)^{3/4}}{a\sqrt[4]{x}}\right)}{\sqrt[4]{ax+3x^2}}$$

$$\Big\downarrow \text{212}$$

$$\frac{\sqrt[4]{x}\sqrt[4]{a+3x}\left(\frac{24\left(\frac{\sqrt{a}\sqrt[4]{x}\sqrt[4]{\frac{a}{x}+3}E\left(\frac{1}{2}\arctan\left(\frac{\sqrt{a}\sqrt{x}}{\sqrt{3}}\right)\middle|2\right)}{2\,3^{3/4}\sqrt[4]{a+3x}}+\frac{x^{3/4}}{2\sqrt[4]{a+3x}}\right)}{a}-\frac{4(a+3x)^{3/4}}{a\sqrt[4]{x}}\right)}{\sqrt[4]{ax+3x^2}}$$

```
Int[1/(x*(a*x + 3*x^2)^(1/4)),x]
```

```
(x^(1/4)*(a + 3*x)^(1/4)*((-4*(a + 3*x)^(3/4))/(a*x^(1/4)) + (24*(x^(3/4)/
(2*(a + 3*x)^(1/4)) + (Sqrt[a]*(3 + a/x)^(1/4)*x^(1/4)*EllipticE[ArcTan[(S
qrt[a]*Sqrt[x])/Sqrt[3]]/2, 2])/(2*3^(3/4)*(a + 3*x)^(1/4))))/a))/(a*x + 3
*x^2)^(1/4)
```

Defintions of rubi rules used

```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] - Simp[d*((
m + n + 2)/((b*c - a*d)*(m + 1)))   Int[(a + b*x)^(m + 1)*(c + d*x)^n, x],
x] /; FreeQ[{a, b, c, d, n}, x] && LtQ[m, -1] &&  !(LtQ[n, -1] && (EqQ[a, 0
] || (NeQ[c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && IntLinearQ[a, b, c, d
, m, n, x]
```

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Simp[p/b   Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
 d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && Lt
Q[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntL
inearQ[a, b, c, d, m, n, x]
```

```
Int[((a_) + (b_.)*(x_)^2)^(-5/4), x_Symbol] :> Simp[(2/(a^(5/4)*Rt[b/a, 2])
)*EllipticE[(1/2)*ArcTan[Rt[b/a, 2]*x], 2], x] /; FreeQ[{a, b}, x] && GtQ[a
, 0] && PosQ[b/a]
```

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k = GCD[m
+ 1, n]}, Simp[1/k   Subst[Int[x^((m + 1)/k - 1)*(a + b*x^(n/k))^p, x], x,
x^k], x] /; k != 1] /; FreeQ[{a, b, p}, x] && IGtQ[n, 0] && IntegerQ[m]
```

```
Int[(x_)^2/((a_) + (b_.)*(x_)^4)^(5/4), x_Symbol] :> Simp[x*((1 + a/(b*x^4)
)^(1/4)/(b*(a + b*x^4)^(1/4)))   Int[1/(x^3*(1 + a/(b*x^4))^(5/4)), x], x]
/; FreeQ[{a, b}, x] && PosQ[b/a]
```

```
Int[(x_)^2/((a_) + (b_.)*(x_)^4)^(1/4), x_Symbol] :> Simp[x^3/(2*(a + b*x^4
)^(1/4)), x] - Simp[a/2   Int[x^2/(a + b*x^4)^(5/4), x], x] /; FreeQ[{a, b}
, x] && PosQ[b/a]
```

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> -Subst[Int[(a +
b/x^n)^p/x^(m + 2), x], x, 1/x] /; FreeQ[{a, b, p}, x] && ILtQ[n, 0] && Int
egerQ[m]
```

```
Int[((e_.)*(x_))^(m_)*((b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(
e*x)^m*((b*x + c*x^2)^p/(x^(m + p)*(b + c*x)^p))   Int[x^(m + p)*(b + c*x)^
p, x], x] /; FreeQ[{b, c, e, m}, x]
```

Maple [F]

$$\int \frac{1}{x\,(ax+3x^2)^{\frac{1}{4}}}dx$$

```
int(1/x/(a*x+3*x^2)^(1/4),x)
```

```
int(1/x/(a*x+3*x^2)^(1/4),x)
```

Fricas [F]

$$\int \frac{1}{x\sqrt[4]{ax+3x^2}}\,dx = \int \frac{1}{\left(ax+3\,x^2\right)^{\frac{1}{4}}x}\,dx$$

```
integrate(1/x/(a*x+3*x^2)^(1/4),x, algorithm="fricas")
```

```
integral((a*x + 3*x^2)^(3/4)/(a*x^2 + 3*x^3), x)
```

Sympy [F]

$$\int \frac{1}{x\sqrt[4]{ax+3x^2}}\,dx = \int \frac{1}{x\sqrt[4]{x\left(a+3x\right)}}\,dx$$

```
integrate(1/x/(a*x+3*x**2)**(1/4),x)
```

```
Integral(1/(x*(x*(a + 3*x))**(1/4)), x)
```

Maxima [F]

$$\int \frac{1}{x\sqrt[4]{ax+3x^2}}\,dx = \int \frac{1}{\left(ax+3\,x^2\right)^{\frac{1}{4}}x}\,dx$$

```
integrate(1/x/(a*x+3*x^2)^(1/4),x, algorithm="maxima")
```

```
integrate(1/((a*x + 3*x^2)^(1/4)*x), x)
```

Giac [F]

$$\int \frac{1}{x\sqrt[4]{ax+3x^2}}\,dx = \int \frac{1}{(ax+3\,x^2)^{\frac{1}{4}}x}\,dx$$

```
integrate(1/x/(a*x+3*x^2)^(1/4),x, algorithm="giac")
```

```
integrate(1/((a*x + 3*x^2)^(1/4)*x), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{x\sqrt[4]{ax+3x^2}}\,dx = \int \frac{1}{x\,(3\,x^2+a\,x)^{1/4}}\,dx$$

```
int(1/(x*(a*x + 3*x^2)^(1/4)),x)
```

```
int(1/(x*(a*x + 3*x^2)^(1/4)), x)
```

Reduce [F]

$$\int \frac{1}{x\sqrt[4]{ax+3x^2}}\,dx = \int \frac{1}{x^{\frac{5}{4}}\,(a+3x)^{\frac{1}{4}}}dx$$

```
int(1/x/(a*x+3*x^2)^(1/4),x)
```

```
int(1/(x**(1/4)*(a + 3*x)**(1/4)*x),x)
```

3.205 $\int \frac{1}{x\sqrt[4]{2x-3x^2}}\,dx$

Optimal result

Integrand size = 17, antiderivative size = 36

$$\int \frac{1}{x\sqrt[4]{2x-3x^2}}\,dx = -\frac{2(2-3x)^{3/4}}{\sqrt[4]{x}} + 2\sqrt[4]{3}E\left(\frac{1}{2}\arcsin(1-3x)\middle|2\right)$$

```
-2*(2-3*x)^(3/4)/x^(1/4)-2*EllipticE(sin(1/2*arcsin(-1+3*x)),2^(1/2))*3^(1
/4)
```

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 10.01 (sec) , antiderivative size = 43, normalized size of antiderivative = 1.19

$$\int \frac{1}{x\sqrt[4]{2x-3x^2}}\,dx = -\frac{\left(\frac{2}{3}\right)^{3/4}(-x(-2+3x))^{3/4}\,\text{Hypergeometric2F1}\left(\frac{3}{4},\frac{5}{4},\frac{7}{4},1-\frac{3x}{2}\right)}{x^{3/4}}$$

```
Integrate[1/(x*(2*x - 3*x^2)^(1/4)),x]
```

```
-(((2/3)^(3/4)*(-(x*(-2 + 3*x)))^(3/4)*Hypergeometric2F1[3/4, 5/4, 7/4, 1
- (3*x)/2])/x^(3/4))
```

Rubi [B] (warning: unable to verify)

Leaf count is larger than twice the leaf count of optimal. 138 vs. $2(36) = 72$.

Time = 0.48 (sec) , antiderivative size = 138, normalized size of antiderivative = 3.83, number of steps used = 10, number of rules used = 9, $\frac{\text{number of rules}}{\text{integrand size}} = 0.529$, Rules used = {1137, 61, 73, 27, 840, 842, 858, 807, 226}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{x\sqrt[4]{2x-3x^2}}dx$$

$$\Big\downarrow \text{1137}$$

$$\frac{\sqrt[4]{2-3x}\sqrt[4]{x}\int \frac{1}{\sqrt[4]{2-3x}x^{5/4}}dx}{\sqrt[4]{2x-3x^2}}$$

$$\Big\downarrow \text{61}$$

$$\frac{\sqrt[4]{2-3x}\sqrt[4]{x}\left(-3\int \frac{1}{\sqrt[4]{2-3x}\sqrt[4]{x}}dx-\frac{2(2-3x)^{3/4}}{\sqrt[4]{x}}\right)}{\sqrt[4]{2x-3x^2}}$$

$$\Big\downarrow \text{73}$$

$$\frac{\sqrt[4]{2-3x}\sqrt[4]{x}\left(4\int \frac{\sqrt{2-3x}}{\sqrt[4]{x}}d\sqrt[4]{2-3x}-\frac{2(2-3x)^{3/4}}{\sqrt[4]{x}}\right)}{\sqrt[4]{2x-3x^2}}$$

$$\Big\downarrow \text{27}$$

$$\frac{\sqrt[4]{2-3x}\sqrt[4]{x}\left(4\sqrt[4]{3}\int \frac{\sqrt{2-3x}}{\sqrt[4]{3}\sqrt[4]{x}}d\sqrt[4]{2-3x}-\frac{2(2-3x)^{3/4}}{\sqrt[4]{x}}\right)}{\sqrt[4]{2x-3x^2}}$$

$$\Big\downarrow \text{840}$$

$$\frac{\sqrt[4]{2-3x}\sqrt[4]{x}\left(4\sqrt[4]{3}\left(-\int \frac{1}{\sqrt[4]{3}\sqrt{2-3x}\sqrt[4]{x}}d\sqrt[4]{2-3x}-\frac{3^{3/4}x^{3/4}}{2\sqrt[4]{2-3x}}\right)-\frac{2(2-3x)^{3/4}}{\sqrt[4]{x}}\right)}{\sqrt[4]{2x-3x^2}}$$

$$\Big\downarrow \text{842}$$

$$\frac{\sqrt[4]{2-3x}\sqrt[4]{x}\left(4\sqrt[4]{3}\left(-\frac{\sqrt[4]{1-\frac{2}{2-3x}}\sqrt[4]{2-3x}\int\frac{1}{\sqrt[4]{1-\frac{2}{2-3x}}(2-3x)^{3/4}}d\sqrt[4]{2-3x}}{\sqrt[4]{3}\sqrt[4]{x}}-\frac{3^{3/4}x^{3/4}}{2\sqrt[4]{2-3x}}\right)-\frac{2(2-3x)^{3/4}}{\sqrt[4]{x}}\right)}{\sqrt[4]{2x-3x^2}}$$

$$\Big\downarrow \; 858$$

$$\frac{\sqrt[4]{2-3x}\sqrt[4]{x}\left(4\sqrt[4]{3}\left(\frac{\sqrt[4]{1-\frac{2}{2-3x}}\sqrt[4]{2-3x}\int\frac{1}{\sqrt[4]{1-2(2-3x)}\sqrt[4]{2-3x}}d\frac{1}{\sqrt[4]{2-3x}}}{\sqrt[4]{3}\sqrt[4]{x}}-\frac{3^{3/4}x^{3/4}}{2\sqrt[4]{2-3x}}\right)-\frac{2(2-3x)^{3/4}}{\sqrt[4]{x}}\right)}{\sqrt[4]{2x-3x^2}}$$

$$\Big\downarrow \; 807$$

$$\frac{\sqrt[4]{2-3x}\sqrt[4]{x}\left(4\sqrt[4]{3}\left(\frac{\sqrt[4]{1-\frac{2}{2-3x}}\sqrt[4]{2-3x}\int\frac{1}{\sqrt[4]{1-2\sqrt{2-3x}}}d\sqrt{2-3x}}{2\sqrt[4]{3}\sqrt[4]{x}}-\frac{3^{3/4}x^{3/4}}{2\sqrt[4]{2-3x}}\right)-\frac{2(2-3x)^{3/4}}{\sqrt[4]{x}}\right)}{\sqrt[4]{2x-3x^2}}$$

$$\Big\downarrow \; 226$$

$$\frac{\sqrt[4]{2-3x}\sqrt[4]{x}\left(4\sqrt[4]{3}\left(\frac{\sqrt[4]{1-\frac{2}{2-3x}}\sqrt[4]{2-3x}E\left(\frac{1}{2}\arcsin\left(\sqrt{2}\sqrt{2-3x}\right)\middle|2\right)}{\sqrt{2}\sqrt[4]{3}\sqrt[4]{x}}-\frac{3^{3/4}x^{3/4}}{2\sqrt[4]{2-3x}}\right)-\frac{2(2-3x)^{3/4}}{\sqrt[4]{x}}\right)}{\sqrt[4]{2x-3x^2}}$$

```
Int[1/(x*(2*x - 3*x^2)^(1/4)),x]
```

```
((2 - 3*x)^(1/4)*x^(1/4)*((-2*(2 - 3*x)^(3/4))/x^(1/4) + 4*3^(1/4)*(-1/2*(
3^(3/4)*x^(3/4))/(2 - 3*x)^(1/4) + ((1 - 2/(2 - 3*x))^(1/4)*(2 - 3*x)^(1/4
)*EllipticE[ArcSin[Sqrt[2]*Sqrt[2 - 3*x]]/2, 2])/(Sqrt[2]*3^(1/4)*x^(1/4))
)))/(2*x - 3*x^2)^(1/4)
```

Defintions of rubi rules used

```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] - Simp[d*((
m + n + 2)/((b*c - a*d)*(m + 1)))   Int[(a + b*x)^(m + 1)*(c + d*x)^n, x],
x] /; FreeQ[{a, b, c, d, n}, x] && LtQ[m, -1] &&  !(LtQ[n, -1] && (EqQ[a, 0
] || (NeQ[c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && IntLinearQ[a, b, c, d
, m, n, x]
```

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Simp[p/b   Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
 d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && Lt
Q[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntL
inearQ[a, b, c, d, m, n, x]
```

```
Int[((a_) + (b_.)*(x_)^2)^(-1/4), x_Symbol] :> Simp[(2/(a^(1/4)*Rt[-b/a, 2]
))*EllipticE[(1/2)*ArcSin[Rt[-b/a, 2]*x], 2], x] /; FreeQ[{a, b}, x] && GtQ
[a, 0] && NegQ[b/a]
```

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k = GCD[m
+ 1, n]}, Simp[1/k   Subst[Int[x^((m + 1)/k - 1)*(a + b*x^(n/k))^p, x], x,
x^k], x] /; k != 1] /; FreeQ[{a, b, p}, x] && IGtQ[n, 0] && IntegerQ[m]
```

```
Int[(x_)^2/((a_) + (b_.)*(x_)^4)^(1/4), x_Symbol] :> Simp[(a + b*x^4)^(3/4)
/(2*b*x), x] + Simp[a/(2*b)   Int[1/(x^2*(a + b*x^4)^(1/4)), x], x] /; Free
Q[{a, b}, x] && NegQ[b/a]
```

```
Int[1/((x_)^2*((a_) + (b_.)*(x_)^4)^(1/4)), x_Symbol] :> Simp[x*((1 + a/(b*
x^4))^(1/4)/(a + b*x^4)^(1/4))   Int[1/(x^3*(1 + a/(b*x^4))^(1/4)), x], x]
/; FreeQ[{a, b}, x] && NegQ[b/a]
```

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> -Subst[Int[(a +
b/x^n)^p/x^(m + 2), x], x, 1/x] /; FreeQ[{a, b, p}, x] && ILtQ[n, 0] && Int
egerQ[m]
```

```
Int[((e_.)*(x_))^(m_)*((b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(
e*x)^m*((b*x + c*x^2)^p/(x^(m + p)*(b + c*x)^p))   Int[x^(m + p)*(b + c*x)^
p, x], x] /; FreeQ[{b, c, e, m}, x]
```

Maple [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4.

Time = 0.38 (sec) , antiderivative size = 18, normalized size of antiderivative = 0.50

method	result	size
meijerg	$-\frac{2\,2^{\frac{3}{4}} \operatorname{hypergeom}\left(\left[-\frac{1}{4},\frac{1}{4}\right],\left[\frac{3}{4}\right],\frac{3x}{2}\right)}{x^{\frac{1}{4}}}$	18
risch	$\frac{-4+6x}{\left(-x(-2+3x)\right)^{\frac{1}{4}}}-2\,2^{\frac{3}{4}} x^{\frac{3}{4}} \operatorname{hypergeom}\left(\left[\frac{1}{4},\frac{3}{4}\right],\left[\frac{7}{4}\right],\frac{3x}{2}\right)$	36

```
int(1/x/(-3*x^2+2*x)^(1/4),x,method=_RETURNVERBOSE)
```

```
-2*2^(3/4)/x^(1/4)*hypergeom([-1/4,1/4],[3/4],3/2*x)
```

Fricas [F]

$$\int \frac{1}{x\sqrt[4]{2x-3x^2}}\,dx=\int \frac{1}{\left(-3\,x^{2}+2\,x\right)^{\frac{1}{4}}x}\,dx$$

```
integrate(1/x/(-3*x^2+2*x)^(1/4),x, algorithm="fricas")
```

```
integral(-(-3*x^2 + 2*x)^(3/4)/(3*x^3 - 2*x^2), x)
```

Sympy [F]

$$\int \frac{1}{x\sqrt[4]{2x-3x^2}}\,dx=\int \frac{1}{x\sqrt[4]{-x\,(3x-2)}}\,dx$$

```
integrate(1/x/(-3*x**2+2*x)**(1/4),x)
```

```
Integral(1/(x*(-x*(3*x - 2))**(1/4)), x)
```

Maxima [F]

$$\int \frac{1}{x\sqrt[4]{2x-3x^2}}\,dx=\int \frac{1}{\left(-3\,x^{2}+2\,x\right)^{\frac{1}{4}}x}\,dx$$

```
integrate(1/x/(-3*x^2+2*x)^(1/4),x, algorithm="maxima")
```

```
integrate(1/((-3*x^2 + 2*x)^(1/4)*x), x)
```

Giac [F]

$$\int \frac{1}{x\sqrt[4]{2x-3x^2}}\,dx = \int \frac{1}{\left(-3\,x^2+2\,x\right)^{\frac{1}{4}}x}\,dx$$

```
integrate(1/x/(-3*x^2+2*x)^(1/4),x, algorithm="giac")
```

```
integrate(1/((-3*x^2 + 2*x)^(1/4)*x), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{x\sqrt[4]{2x-3x^2}}\,dx = \int \frac{1}{x\,\left(2\,x-3\,x^2\right)^{1/4}}\,dx$$

```
int(1/(x*(2*x - 3*x^2)^(1/4)),x)
```

```
int(1/(x*(2*x - 3*x^2)^(1/4)), x)
```

Reduce [F]

$$\int \frac{1}{x\sqrt[4]{2x-3x^2}}\,dx = \int \frac{1}{x^{\frac{5}{4}}\left(-3x+2\right)^{\frac{1}{4}}}dx$$

```
int(1/x/(-3*x^2+2*x)^(1/4),x)
```

```
int(1/(x**(1/4)*( - 3*x + 2)**(1/4)*x),x)
```

3.206 $\int \frac{1}{x\sqrt[4]{-2x-3x^2}}\,dx$

Optimal result

Integrand size = 17, antiderivative size = 38

$$\int \frac{1}{x\sqrt[4]{-2x-3x^2}}\,dx=\frac{2(-2x-3x^2)^{3/4}}{x}+2\sqrt[4]{3}E\left(\frac{1}{2}\arcsin(1+3x)\middle|2\right)$$

```
2*(-3*x^2-2*x)^(3/4)/x+2*EllipticE(sin(1/2*arcsin(1+3*x)),2^(1/2))*3^(1/4)
```

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 10.01 (sec) , antiderivative size = 43, normalized size of antiderivative = 1.13

$$\int \frac{1}{x\sqrt[4]{-2x-3x^2}}\,dx=-\frac{2\,2^{3/4}\sqrt[4]{2+3x}\,\text{Hypergeometric2F1}\left(-\frac{1}{4},\frac{1}{4},\frac{3}{4},-\frac{3x}{2}\right)}{\sqrt[4]{-x(2+3x)}}$$

```
Integrate[1/(x*(-2*x - 3*x^2)^(1/4)),x]
```

```
(-2*2^(3/4)*(2 + 3*x)^(1/4)*Hypergeometric2F1[-1/4, 1/4, 3/4, (-3*x)/2])/(
-(x*(2 + 3*x)))^(1/4)
```

Rubi [B] (warning: unable to verify)

Leaf count is larger than twice the leaf count of optimal. 138 vs. $2(38) = 76$.

Time = 0.47 (sec) , antiderivative size = 138, normalized size of antiderivative = 3.63, number of steps used = 10, number of rules used = 9, $\frac{\text{number of rules}}{\text{integrand size}} = 0.529$, Rules used = {1137, 61, 73, 27, 839, 813, 858, 807, 212}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{x\sqrt[4]{-3x^2-2x}}dx$$

$$\Big\downarrow \text{1137}$$

$$\frac{\sqrt[4]{-3x-2}\sqrt[4]{x}\int \frac{1}{\sqrt[4]{-3x-2}x^{5/4}}dx}{\sqrt[4]{-3x^2-2x}}$$

$$\Big\downarrow \text{61}$$

$$\frac{\sqrt[4]{-3x-2}\sqrt[4]{x}\left(3\int \frac{1}{\sqrt[4]{-3x-2}\sqrt[4]{x}}dx+\frac{2(-3x-2)^{3/4}}{\sqrt[4]{x}}\right)}{\sqrt[4]{-3x^2-2x}}$$

$$\Big\downarrow \text{73}$$

$$\frac{\sqrt[4]{-3x-2}\sqrt[4]{x}\left(\frac{2(-3x-2)^{3/4}}{\sqrt[4]{x}}-4\int \frac{\sqrt{-3x-2}}{\sqrt[4]{x}}d\sqrt[4]{-3x-2}\right)}{\sqrt[4]{-3x^2-2x}}$$

$$\Big\downarrow \text{27}$$

$$\frac{\sqrt[4]{-3x-2}\sqrt[4]{x}\left(\frac{2(-3x-2)^{3/4}}{\sqrt[4]{x}}-4\sqrt[4]{3}\int \frac{\sqrt{-3x-2}}{\sqrt[4]{3}\sqrt[4]{x}}d\sqrt[4]{-3x-2}\right)}{\sqrt[4]{-3x^2-2x}}$$

$$\Big\downarrow \text{839}$$

$$\frac{\sqrt[4]{-3x-2}\sqrt[4]{x}\left(\frac{2(-3x-2)^{3/4}}{\sqrt[4]{x}}-4\sqrt[4]{3}\left(\int \frac{\sqrt{-3x-2}}{3\sqrt[4]{3}x^{5/4}}d\sqrt[4]{-3x-2}+\frac{(-3x-2)^{3/4}}{2\sqrt[4]{3}\sqrt[4]{x}}\right)\right)}{\sqrt[4]{-3x^2-2x}}$$

$$\Big\downarrow \text{813}$$

$$\frac{\sqrt[4]{-3x-2}\sqrt[4]{x}\left(\frac{2(-3x-2)^{3/4}}{\sqrt[4]{x}}-4\sqrt[4]{3}\left(\frac{(-3x-2)^{3/4}}{2\sqrt[4]{3}\sqrt[4]{x}}-\frac{\sqrt[4]{\frac{2}{-3x-2}+1}\sqrt[4]{-3x-2}\int \frac{1}{\left(1+\frac{2}{-3x-2}\right)^{5/4}(-3x-2)^{3/4}}d\sqrt[4]{-3x-2}}{\sqrt[4]{3}\sqrt[4]{x}}\right)\right)}{\sqrt[4]{-3x^2-2x}}$$

$$\Big\downarrow \; 858$$

$$\frac{\sqrt[4]{-3x-2}\sqrt[4]{x}\left(\frac{2(-3x-2)^{3/4}}{\sqrt[4]{x}}-4\sqrt[4]{3}\left(\frac{\sqrt[4]{\frac{2}{-3x-2}+1}\sqrt[4]{-3x-2}\int \frac{1}{(2(-3x-2)+1)^{5/4}\sqrt[4]{-3x-2}}d\frac{1}{\sqrt[4]{-3x-2}}}{\sqrt[4]{3}\sqrt[4]{x}}+\frac{(-3x-2)^{3/4}}{2\sqrt[4]{3}\sqrt[4]{x}}\right)\right)}{\sqrt[4]{-3x^2-2x}}$$

$$\Big\downarrow \; 807$$

$$\frac{\sqrt[4]{-3x-2}\sqrt[4]{x}\left(\frac{2(-3x-2)^{3/4}}{\sqrt[4]{x}}-4\sqrt[4]{3}\left(\frac{\sqrt[4]{\frac{2}{-3x-2}+1}\sqrt[4]{-3x-2}\int \frac{1}{\left(2\sqrt{-3x-2}+1\right)^{5/4}}d\sqrt{-3x-2}}{2\sqrt[4]{3}\sqrt[4]{x}}+\frac{(-3x-2)^{3/4}}{2\sqrt[4]{3}\sqrt[4]{x}}\right)\right)}{\sqrt[4]{-3x^2-2x}}$$

$$\Big\downarrow \; 212$$

$$\frac{\sqrt[4]{-3x-2}\sqrt[4]{x}\left(\frac{2(-3x-2)^{3/4}}{\sqrt[4]{x}}-4\sqrt[4]{3}\left(\frac{\sqrt[4]{\frac{2}{-3x-2}+1}\sqrt[4]{-3x-2}E\left(\frac{1}{2}\arctan\left(\sqrt{2}\sqrt{-3x-2}\right)\middle|2\right)}{\sqrt{2}\sqrt[4]{3}\sqrt[4]{x}}+\frac{(-3x-2)^{3/4}}{2\sqrt[4]{3}\sqrt[4]{x}}\right)\right)}{\sqrt[4]{-3x^2-2x}}$$

```
Int[1/(x*(-2*x - 3*x^2)^(1/4)),x]
```

```
((-2 - 3*x)^(1/4)*x^(1/4)*((2*(-2 - 3*x)^(3/4))/x^(1/4) - 4*3^(1/4)*((-2 -
 3*x)^(3/4)/(2*3^(1/4)*x^(1/4)) + ((1 + 2/(-2 - 3*x))^(1/4)*(-2 - 3*x)^(1/
4)*EllipticE[ArcTan[Sqrt[2]*Sqrt[-2 - 3*x]]/2, 2])/(Sqrt[2]*3^(1/4)*x^(1/4
)))))/(-2*x - 3*x^2)^(1/4)
```

Defintions of rubi rules used

```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] - Simp[d*((
m + n + 2)/((b*c - a*d)*(m + 1)))   Int[(a + b*x)^(m + 1)*(c + d*x)^n, x],
x] /; FreeQ[{a, b, c, d, n}, x] && LtQ[m, -1] &&  !(LtQ[n, -1] && (EqQ[a, 0
] || (NeQ[c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && IntLinearQ[a, b, c, d
, m, n, x]
```

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Simp[p/b   Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
 d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && Lt
Q[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntL
inearQ[a, b, c, d, m, n, x]
```

```
Int[((a_) + (b_.)*(x_)^2)^(-5/4), x_Symbol] :> Simp[(2/(a^(5/4)*Rt[b/a, 2])
)*EllipticE[(1/2)*ArcTan[Rt[b/a, 2]*x], 2], x] /; FreeQ[{a, b}, x] && GtQ[a
, 0] && PosQ[b/a]
```

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k = GCD[m
+ 1, n]}, Simp[1/k   Subst[Int[x^((m + 1)/k - 1)*(a + b*x^(n/k))^p, x], x,
x^k], x] /; k != 1] /; FreeQ[{a, b, p}, x] && IGtQ[n, 0] && IntegerQ[m]
```

```
Int[(x_)^2/((a_) + (b_.)*(x_)^4)^(5/4), x_Symbol] :> Simp[x*((1 + a/(b*x^4)
)^(1/4)/(b*(a + b*x^4)^(1/4)))   Int[1/(x^3*(1 + a/(b*x^4))^(5/4)), x], x]
/; FreeQ[{a, b}, x] && PosQ[b/a]
```

```
Int[(x_)^2/((a_) + (b_.)*(x_)^4)^(1/4), x_Symbol] :> Simp[x^3/(2*(a + b*x^4
)^(1/4)), x] - Simp[a/2   Int[x^2/(a + b*x^4)^(5/4), x], x] /; FreeQ[{a, b}
, x] && PosQ[b/a]
```

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> -Subst[Int[(a +
b/x^n)^p/x^(m + 2), x], x, 1/x] /; FreeQ[{a, b, p}, x] && ILtQ[n, 0] && Int
egerQ[m]
```

```
Int[((e_.)*(x_))^(m_)*((b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(
e*x)^m*((b*x + c*x^2)^p/(x^(m + p)*(b + c*x)^p))   Int[x^(m + p)*(b + c*x)^
p, x], x] /; FreeQ[{b, c, e, m}, x]
```

Maple [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4.

Time = 0.38 (sec) , antiderivative size = 21, normalized size of antiderivative = 0.55

method	result	size
meijerg	$\frac{2(-1)^{\frac{3}{4}} 2^{\frac{3}{4}} \operatorname{hypergeom}\left(\left[-\frac{1}{4},\frac{1}{4}\right],\left[\frac{3}{4}\right],-\frac{3x}{2}\right)}{x^{\frac{1}{4}}}$	21
risch	$-\frac{2(3x+2)}{(-x(3x+2))^{\frac{1}{4}}}-2(-1)^{\frac{3}{4}} 2^{\frac{3}{4}} x^{\frac{3}{4}} \operatorname{hypergeom}\left(\left[\frac{1}{4},\frac{3}{4}\right],\left[\frac{7}{4}\right],-\frac{3x}{2}\right)$	39

```
int(1/x/(-3*x^2-2*x)^(1/4),x,method=_RETURNVERBOSE)
```

```
2*(-1)^(3/4)*2^(3/4)/x^(1/4)*hypergeom([-1/4,1/4],[3/4],-3/2*x)
```

Fricas [F]

$$\int \frac{1}{x\sqrt[4]{-2x-3x^2}}\,dx=\int \frac{1}{\left(-3\,x^{2}-2\,x\right)^{\frac{1}{4}}x}\,dx$$

```
integrate(1/x/(-3*x^2-2*x)^(1/4),x, algorithm="fricas")
```

```
integral(-(-3*x^2 - 2*x)^(3/4)/(3*x^3 + 2*x^2), x)
```

Sympy [F]

$$\int \frac{1}{x\sqrt[4]{-2x-3x^2}}\,dx = \int \frac{1}{x\sqrt[4]{-x\,(3x+2)}}\,dx$$

```
integrate(1/x/(-3*x**2-2*x)**(1/4),x)
```

```
Integral(1/(x*(-x*(3*x + 2))**(1/4)), x)
```

Maxima [F]

$$\int \frac{1}{x\sqrt[4]{-2x-3x^2}}\,dx = \int \frac{1}{(-3\,x^2-2\,x)^{\frac{1}{4}}x}\,dx$$

```
integrate(1/x/(-3*x^2-2*x)^(1/4),x, algorithm="maxima")
```

```
integrate(1/((-3*x^2 - 2*x)^(1/4)*x), x)
```

Giac [F]

$$\int \frac{1}{x\sqrt[4]{-2x-3x^2}}\,dx = \int \frac{1}{(-3\,x^2-2\,x)^{\frac{1}{4}}x}\,dx$$

```
integrate(1/x/(-3*x^2-2*x)^(1/4),x, algorithm="giac")
```

```
integrate(1/((-3*x^2 - 2*x)^(1/4)*x), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{x\sqrt[4]{-2x-3x^2}}\,dx = \int \frac{1}{x\left(-3\,x^2-2\,x\right)^{1/4}}\,dx$$

```
int(1/(x*(- 2*x - 3*x^2)^(1/4)),x)
```

```
int(1/(x*(- 2*x - 3*x^2)^(1/4)), x)
```

Reduce [F]

$$\int \frac{1}{x\sqrt[4]{-2x-3x^2}}\,dx = \int \frac{1}{x^{\frac{5}{4}}\left(-3x-2\right)^{\frac{1}{4}}}dx$$

```
int(1/x/(-3*x^2-2*x)^(1/4),x)
```

```
int(1/(x**(1/4)*( - 3*x - 2)**(1/4)*x),x)
```

3.207 $\int \frac{1}{x\sqrt[4]{ax-3x^2}}\,dx$

Optimal result

Integrand size = 17, antiderivative size = 80

$$\int \frac{1}{x\sqrt[4]{ax-3x^2}}\,dx=-\frac{4(ax-3x^2)^{3/4}}{ax}+\frac{2\sqrt{2}\sqrt[4]{3}\sqrt[4]{\frac{x}{a}-\frac{3x^2}{a^2}}\,E\left(\frac{1}{2}\arcsin\left(1-\frac{6x}{a}\right)\middle|2\right)}{\sqrt[4]{ax-3x^2}}$$

```
-4*(a*x-3*x^2)^(3/4)/a/x+2*2^(1/2)*3^(1/4)*(x/a-3*x^2/a^2)^(1/4)*EllipticE
(sin(1/2*arcsin(1-6*x/a)),2^(1/2))/(a*x-3*x^2)^(1/4)
```

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 10.02 (sec) , antiderivative size = 41, normalized size of antiderivative = 0.51

$$\int \frac{1}{x\sqrt[4]{ax-3x^2}}\,dx=-\frac{4\sqrt[4]{1-\frac{3x}{a}}\,\text{Hypergeometric2F1}\left(-\frac{1}{4},\frac{1}{4},\frac{3}{4},\frac{3x}{a}\right)}{\sqrt[4]{(a-3x)x}}$$

```
Integrate[1/(x*(a*x - 3*x^2)^(1/4)),x]
```

```
(-4*(1 - (3*x)/a)^(1/4)*Hypergeometric2F1[-1/4, 1/4, 3/4, (3*x)/a])/((a -
3*x)*x)^(1/4)
```

Rubi [A] (warning: unable to verify)

Time = 0.49 (sec) , antiderivative size = 148, normalized size of antiderivative = 1.85, number of steps used = 10, number of rules used = 9, $\frac{\text{number of rules}}{\text{integrand size}} = 0.529$, Rules used = {1137, 61, 73, 27, 840, 842, 858, 807, 226}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{x\sqrt[4]{ax-3x^2}}dx$$

$$\Big\downarrow \text{1137}$$

$$\frac{\sqrt[4]{x}\sqrt[4]{a-3x}\int \frac{1}{\sqrt[4]{a-3x}x^{5/4}}dx}{\sqrt[4]{ax-3x^2}}$$

$$\Big\downarrow \text{61}$$

$$\frac{\sqrt[4]{x}\sqrt[4]{a-3x}\left(-\frac{6\int \frac{1}{\sqrt[4]{a-3x}\sqrt[4]{x}}dx}{a}-\frac{4(a-3x)^{3/4}}{a\sqrt[4]{x}}\right)}{\sqrt[4]{ax-3x^2}}$$

$$\Big\downarrow \text{73}$$

$$\frac{\sqrt[4]{x}\sqrt[4]{a-3x}\left(\frac{8\int \frac{\sqrt{a-3x}}{\sqrt[4]{x}}d\sqrt[4]{a-3x}}{a}-\frac{4(a-3x)^{3/4}}{a\sqrt[4]{x}}\right)}{\sqrt[4]{ax-3x^2}}$$

$$\Big\downarrow \text{27}$$

$$\frac{\sqrt[4]{x}\sqrt[4]{a-3x}\left(\frac{8\sqrt[4]{3}\int \frac{\sqrt{a-3x}}{\sqrt[4]{3}\sqrt[4]{x}}d\sqrt[4]{a-3x}}{a}-\frac{4(a-3x)^{3/4}}{a\sqrt[4]{x}}\right)}{\sqrt[4]{ax-3x^2}}$$

$$\Big\downarrow \text{840}$$

$$\frac{\sqrt[4]{x}\sqrt[4]{a-3x}\left(\frac{8\sqrt[4]{3}\left(-\frac{1}{2}a\int\frac{1}{\sqrt[4]{3}\sqrt{a-3x}\sqrt[4]{x}}d\sqrt[4]{a-3x}-\frac{3^{3/4}x^{3/4}}{2\sqrt[4]{a-3x}}\right)}{a}-\frac{4(a-3x)^{3/4}}{a\sqrt[4]{x}}\right)}{\sqrt[4]{ax-3x^2}}$$

$$\Big\downarrow \; 842$$

$$\frac{\sqrt[4]{x}\sqrt[4]{a-3x}\left(\frac{8\sqrt[4]{3}\left(-\frac{a\sqrt[4]{1-\frac{a}{a-3x}}\sqrt[4]{a-3x}\int\frac{1}{\sqrt[4]{1-\frac{a}{a-3x}}(a-3x)^{3/4}}d\sqrt[4]{a-3x}}{2\sqrt[4]{3}\sqrt[4]{x}}-\frac{3^{3/4}x^{3/4}}{2\sqrt[4]{a-3x}}\right)}{a}-\frac{4(a-3x)^{3/4}}{a\sqrt[4]{x}}\right)}{\sqrt[4]{ax-3x^2}}$$

$$\Big\downarrow \; 858$$

$$\frac{\sqrt[4]{x}\sqrt[4]{a-3x}\left(\frac{8\sqrt[4]{3}\left(\frac{a\sqrt[4]{1-\frac{a}{a-3x}}\sqrt[4]{a-3x}\int\frac{1}{\sqrt[4]{1-a(a-3x)}\sqrt[4]{a-3x}}d\frac{1}{\sqrt[4]{a-3x}}}{2\sqrt[4]{3}\sqrt[4]{x}}-\frac{3^{3/4}x^{3/4}}{2\sqrt[4]{a-3x}}\right)}{a}-\frac{4(a-3x)^{3/4}}{a\sqrt[4]{x}}\right)}{\sqrt[4]{ax-3x^2}}$$

$$\Big\downarrow \; 807$$

$$\frac{\sqrt[4]{x}\sqrt[4]{a-3x}\left(\frac{8\sqrt[4]{3}\left(\frac{a\sqrt[4]{1-\frac{a}{a-3x}}\sqrt[4]{a-3x}\int\frac{1}{\sqrt[4]{1-a\sqrt{a-3x}}}d\sqrt{a-3x}}{4\sqrt[4]{3}\sqrt[4]{x}}-\frac{3^{3/4}x^{3/4}}{2\sqrt[4]{a-3x}}\right)}{a}-\frac{4(a-3x)^{3/4}}{a\sqrt[4]{x}}\right)}{\sqrt[4]{ax-3x^2}}$$

$$\Big\downarrow \text{226}$$

$$\frac{\sqrt[4]{x}\sqrt[4]{a-3x}\left(\frac{8\sqrt[4]{3}\left(\frac{\sqrt{a}\sqrt[4]{1-\frac{a}{a-3x}}\sqrt[4]{a-3x}E\left(\frac{1}{2}\arcsin\left(\sqrt{a}\sqrt{a-3x}\right)\middle|2\right)}{2\sqrt[4]{3}\sqrt[4]{x}}-\frac{3^{3/4}x^{3/4}}{2\sqrt[4]{a-3x}}\right)}{a}-\frac{4(a-3x)^{3/4}}{a\sqrt[4]{x}}\right)}{\sqrt[4]{ax-3x^2}}$$

```
Int[1/(x*(a*x - 3*x^2)^(1/4)),x]
```

```
((a - 3*x)^(1/4)*x^(1/4)*((-4*(a - 3*x)^(3/4))/(a*x^(1/4)) + (8*3^(1/4)*(-
1/2*(3^(3/4)*x^(3/4))/(a - 3*x)^(1/4) + (Sqrt[a]*(1 - a/(a - 3*x))^(1/4)*(
a - 3*x)^(1/4)*EllipticE[ArcSin[Sqrt[a]*Sqrt[a - 3*x]]/2, 2])/(2*3^(1/4)*x
^(1/4))))/a))/(a*x - 3*x^2)^(1/4)
```

Defintions of rubi rules used

```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] - Simp[d*((
m + n + 2)/((b*c - a*d)*(m + 1)))   Int[(a + b*x)^(m + 1)*(c + d*x)^n, x],
x] /; FreeQ[{a, b, c, d, n}, x] && LtQ[m, -1] &&  !(LtQ[n, -1] && (EqQ[a, 0
] || (NeQ[c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && IntLinearQ[a, b, c, d
, m, n, x]
```

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Simp[p/b   Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
 d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && Lt
Q[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntL
inearQ[a, b, c, d, m, n, x]
```

```
Int[((a_) + (b_.)*(x_)^2)^(-1/4), x_Symbol] :> Simp[(2/(a^(1/4)*Rt[-b/a, 2]
))*EllipticE[(1/2)*ArcSin[Rt[-b/a, 2]*x], 2], x] /; FreeQ[{a, b}, x] && GtQ
[a, 0] && NegQ[b/a]
```

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k = GCD[m
+ 1, n]}, Simp[1/k   Subst[Int[x^((m + 1)/k - 1)*(a + b*x^(n/k))^p, x], x,
x^k], x] /; k != 1] /; FreeQ[{a, b, p}, x] && IGtQ[n, 0] && IntegerQ[m]
```

```
Int[(x_)^2/((a_) + (b_.)*(x_)^4)^(1/4), x_Symbol] :> Simp[(a + b*x^4)^(3/4)
/(2*b*x), x] + Simp[a/(2*b)   Int[1/(x^2*(a + b*x^4)^(1/4)), x], x] /; Free
Q[{a, b}, x] && NegQ[b/a]
```

```
Int[1/((x_)^2*((a_) + (b_.)*(x_)^4)^(1/4)), x_Symbol] :> Simp[x*((1 + a/(b*
x^4))^(1/4)/(a + b*x^4)^(1/4))   Int[1/(x^3*(1 + a/(b*x^4))^(1/4)), x], x]
/; FreeQ[{a, b}, x] && NegQ[b/a]
```

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> -Subst[Int[(a +
b/x^n)^p/x^(m + 2), x], x, 1/x] /; FreeQ[{a, b, p}, x] && ILtQ[n, 0] && Int
egerQ[m]
```

```
Int[((e_.)*(x_))^(m_)*((b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(
e*x)^m*((b*x + c*x^2)^p/(x^(m + p)*(b + c*x)^p))   Int[x^(m + p)*(b + c*x)^
p, x], x] /; FreeQ[{b, c, e, m}, x]
```

Maple [F]

$$\int \frac{1}{x\left(ax-3x^{2}\right)^{\frac{1}{4}}}dx$$

```
int(1/x/(a*x-3*x^2)^(1/4),x)
```

```
int(1/x/(a*x-3*x^2)^(1/4),x)
```

Fricas [F]

$$\int \frac{1}{x\sqrt[4]{ax-3x^2}}\,dx = \int \frac{1}{\left(ax-3\,x^{2}\right)^{\frac{1}{4}}x}\,dx$$

```
integrate(1/x/(a*x-3*x^2)^(1/4),x, algorithm="fricas")
```

```
integral((a*x - 3*x^2)^(3/4)/(a*x^2 - 3*x^3), x)
```

Sympy [F]

$$\int \frac{1}{x\sqrt[4]{ax-3x^2}}\,dx = \int \frac{1}{x\sqrt[4]{-x\left(-a+3x\right)}}\,dx$$

```
integrate(1/x/(a*x-3*x**2)**(1/4),x)
```

```
Integral(1/(x*(-x*(-a + 3*x))**(1/4)), x)
```

Maxima [F]

$$\int \frac{1}{x\sqrt[4]{ax-3x^2}}\,dx = \int \frac{1}{(ax-3\,x^2)^{\frac{1}{4}}x}\,dx$$

```
integrate(1/x/(a*x-3*x^2)^(1/4),x, algorithm="maxima")
```

```
integrate(1/((a*x - 3*x^2)^(1/4)*x), x)
```

Giac [F]

$$\int \frac{1}{x\sqrt[4]{ax-3x^2}}\,dx = \int \frac{1}{(ax-3\,x^2)^{\frac{1}{4}}x}\,dx$$

```
integrate(1/x/(a*x-3*x^2)^(1/4),x, algorithm="giac")
```

```
integrate(1/((a*x - 3*x^2)^(1/4)*x), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{x\sqrt[4]{ax-3x^2}}\,dx = \int \frac{1}{x\,(a\,x-3\,x^2)^{1/4}}\,dx$$

```
int(1/(x*(a*x - 3*x^2)^(1/4)),x)
```

```
int(1/(x*(a*x - 3*x^2)^(1/4)), x)
```

Reduce [F]

$$\int \frac{1}{x\sqrt[4]{ax-3x^2}}\,dx = \int \frac{1}{x^{\frac{5}{4}}\,(a-3x)^{\frac{1}{4}}}dx$$

```
int(1/x/(a*x-3*x^2)^(1/4),x)
```

```
int(1/(x**(1/4)*(a - 3*x)**(1/4)*x),x)
```

3.208 $\int \frac{x}{(2x+3x^2)^{5/4}}\,dx$

Optimal result

Integrand size = 15, antiderivative size = 61

$$\int \frac{x}{(2x+3x^2)^{5/4}}\,dx = -\frac{2\sqrt{2}\sqrt[4]{x}\sqrt[4]{2+3x}E\left(\frac{1}{2}\arcsin\left(\frac{\sqrt{2}}{\sqrt{2+3x}}\right)\middle|2\right)}{3^{3/4}\sqrt[4]{2x+3x^2}}$$

```
-2/3*2^(1/2)*3^(1/4)*x^(1/4)*(2+3*x)^(1/4)*EllipticE(sin(1/2*arcsin(2^(1/2
)/(2+3*x)^(1/2))),2^(1/2))/(3*x^2+2*x)^(1/4)
```

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 10.01 (sec) , antiderivative size = 41, normalized size of antiderivative = 0.67

$$\int \frac{x}{(2x+3x^2)^{5/4}}\,dx = \frac{(x(2+3x))^{3/4}\,\text{Hypergeometric2F1}\left(\frac{3}{4},\frac{5}{4},\frac{7}{4},-\frac{3x}{2}\right)}{3\left(1+\frac{3x}{2}\right)^{3/4}}$$

```
Integrate[x/(2*x + 3*x^2)^(5/4),x]
```

```
((x*(2 + 3*x))^(3/4)*Hypergeometric2F1[3/4, 5/4, 7/4, (-3*x)/2])/(3*(1 + (
3*x)/2)^(3/4))
```

Rubi [A] (verified)

Time = 0.33 (sec) , antiderivative size = 70, normalized size of antiderivative = 1.15, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.267$, Rules used = {1159, 1093, 1090, 226}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x}{(3x^2+2x)^{5/4}}\,dx$$

$$\Big\downarrow \text{1159}$$

$$\frac{2x}{\sqrt[4]{3x^2+2x}}-\int \frac{1}{\sqrt[4]{3x^2+2x}}dx$$

$$\Big\downarrow \text{1093}$$

$$\frac{2x}{\sqrt[4]{3x^2+2x}}-\frac{\sqrt[4]{3}\sqrt[4]{-3x^2-2x}\int \frac{1}{\sqrt[4]{-\frac{9x^2}{4}-\frac{3x}{2}}}dx}{\sqrt{2}\sqrt[4]{3x^2+2x}}$$

$$\Big\downarrow \text{1090}$$

$$\frac{2\sqrt[4]{-3x^2-2x}\int \frac{1}{\sqrt[4]{1-\frac{4}{9}\left(-\frac{9x}{2}-\frac{3}{2}\right)^2}}d\left(-\frac{9x}{2}-\frac{3}{2}\right)}{3\ 3^{3/4}\sqrt[4]{3x^2+2x}}+\frac{2x}{\sqrt[4]{3x^2+2x}}$$

$$\Big\downarrow \text{226}$$

$$\frac{2\sqrt[4]{-3x^2-2x}E\left(\frac{1}{2}\arcsin\left(\frac{2}{3}\left(-\frac{9x}{2}-\frac{3}{2}\right)\right)\middle|2\right)}{3^{3/4}\sqrt[4]{3x^2+2x}}+\frac{2x}{\sqrt[4]{3x^2+2x}}$$

```
Int[x/(2*x + 3*x^2)^(5/4),x]
```

```
(2*x)/(2*x + 3*x^2)^(1/4) + (2*(-2*x - 3*x^2)^(1/4)*EllipticE[ArcSin[(2*(-
3/2 - (9*x)/2))/3]/2, 2])/(3^(3/4)*(2*x + 3*x^2)^(1/4))
```

Defintions of rubi rules used

```
Int[((a_) + (b_.)*(x_)^2)^(-1/4), x_Symbol] :> Simp[(2/(a^(1/4)*Rt[-b/a, 2]
))*EllipticE[(1/2)*ArcSin[Rt[-b/a, 2]*x], 2], x] /; FreeQ[{a, b}, x] && GtQ
[a, 0] && NegQ[b/a]
```

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[1/(2*c*(-4*
(c/(b^2 - 4*a*c)))^p)   Subst[Int[Simp[1 - x^2/(b^2 - 4*a*c), x]^p, x], x,
b + 2*c*x], x] /; FreeQ[{a, b, c, p}, x] && GtQ[4*a - b^2/c, 0]
```

```
Int[((b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(b*x + c*x^2)^p/((-
c)*((b*x + c*x^2)/b^2))^p   Int[((-c)*(x/b) - c^2*(x^2/b^2))^p, x], x] /; F
reeQ[{b, c}, x] && (IntegerQ[4*p] || IntegerQ[3*p])
```

```
Int[((d_.) + (e_.)*(x_))*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol
] :> Simp[((b*d - 2*a*e + (2*c*d - b*e)*x)/((p + 1)*(b^2 - 4*a*c)))*(a + b*
x + c*x^2)^(p + 1), x] - Simp[(2*p + 3)*((2*c*d - b*e)/((p + 1)*(b^2 - 4*a*
c)))   Int[(a + b*x + c*x^2)^(p + 1), x], x] /; FreeQ[{a, b, c, d, e}, x] &
& LtQ[p, -1] && NeQ[p, -3/2]
```

Maple [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4.

Time = 0.32 (sec) , antiderivative size = 18, normalized size of antiderivative = 0.30

method	result	size
meijerg	$\frac{2^{\frac{3}{4}} x^{\frac{3}{4}} \operatorname{hypergeom}\left(\left[\frac{3}{4}, \frac{5}{4}\right], \left[\frac{7}{4}\right], -\frac{3x}{2}\right)}{3}$	18

```
int(x/(3*x^2+2*x)^(5/4),x,method=_RETURNVERBOSE)
```

```
1/3*2^(3/4)*x^(3/4)*hypergeom([3/4,5/4],[7/4],-3/2*x)
```

Fricas [F]

$$\int \frac{x}{(2x+3x^2)^{5/4}}\,dx=\int \frac{x}{(3\,x^2+2\,x)^{\frac{5}{4}}}\,dx$$

```
integrate(x/(3*x^2+2*x)^(5/4),x, algorithm="fricas")
```

```
integral((3*x^2 + 2*x)^(3/4)/(9*x^3 + 12*x^2 + 4*x), x)
```

Sympy [F]

$$\int \frac{x}{(2x+3x^2)^{5/4}}\,dx=\int \frac{x}{(x\,(3x+2))^{\frac{5}{4}}}\,dx$$

```
integrate(x/(3*x**2+2*x)**(5/4),x)
```

```
Integral(x/(x*(3*x + 2))**(5/4), x)
```

Maxima [F]

$$\int \frac{x}{(2x+3x^2)^{5/4}}\,dx=\int \frac{x}{(3\,x^2+2\,x)^{\frac{5}{4}}}\,dx$$

```
integrate(x/(3*x^2+2*x)^(5/4),x, algorithm="maxima")
```

```
integrate(x/(3*x^2 + 2*x)^(5/4), x)
```

Giac [F]

$$\int \frac{x}{(2x+3x^2)^{5/4}}\,dx = \int \frac{x}{(3\,x^2+2\,x)^{\frac{5}{4}}}\,dx$$

```
integrate(x/(3*x^2+2*x)^(5/4),x, algorithm="giac")
```

```
integrate(x/(3*x^2 + 2*x)^(5/4), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{x}{(2x+3x^2)^{5/4}}\,dx = \int \frac{x}{(3\,x^2+2\,x)^{5/4}}\,dx$$

```
int(x/(2*x + 3*x^2)^(5/4),x)
```

```
int(x/(2*x + 3*x^2)^(5/4), x)
```

Reduce [F]

$$\int \frac{x}{(2x+3x^2)^{5/4}}\,dx = \int \frac{1}{3x^{\frac{5}{4}}\,(3x+2)^{\frac{1}{4}}+2x^{\frac{1}{4}}\,(3x+2)^{\frac{1}{4}}}dx$$

```
int(x/(3*x^2+2*x)^(5/4),x)
```

```
int(1/(3*x**(1/4)*(3*x + 2)**(1/4)*x + 2*x**(1/4)*(3*x + 2)**(1/4)),x)
```

3.209 $\int \frac{x}{(-2x+3x^2)^{5/4}}\,dx$

Optimal result

Integrand size = 15, antiderivative size = 77

$$\int \frac{x}{(-2x+3x^2)^{5/4}}\,dx=-\frac{4}{3\sqrt[4]{-2x+3x^2}}+\frac{2\sqrt{2}\sqrt[4]{x}\sqrt[4]{-2+3x}E\left(\frac{1}{2}\arcsin\left(\frac{\sqrt{\frac{2}{3}}}{\sqrt{x}}\right)\middle|2\right)}{3^{3/4}\sqrt[4]{-2x+3x^2}}$$

```
-4/3/(3*x^2-2*x)^(1/4)+2/3*2^(1/2)*3^(1/4)*x^(1/4)*(-2+3*x)^(1/4)*Elliptic
E(sin(1/2*arcsin(1/3*6^(1/2)/x^(1/2))),2^(1/2))/(3*x^2-2*x)^(1/4)
```

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 10.01 (sec) , antiderivative size = 42, normalized size of antiderivative = 0.55

$$\int \frac{x}{(-2x+3x^2)^{5/4}}\,dx=-\frac{2\left(\frac{2}{3}\right)^{3/4}\sqrt[4]{x}\,\text{Hypergeometric2F1}\left(-\frac{1}{4},\frac{1}{4},\frac{3}{4},1-\frac{3x}{2}\right)}{\sqrt[4]{x(-2+3x)}}$$

```
Integrate[x/(-2*x + 3*x^2)^(5/4),x]
```

```
(-2*(2/3)^(3/4)*x^(1/4)*Hypergeometric2F1[-1/4, 1/4, 3/4, 1 - (3*x)/2])/(x
*(-2 + 3*x))^(1/4)
```

Rubi [A] (verified)

Time = 0.34 (sec) , antiderivative size = 70, normalized size of antiderivative = 0.91, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.267$, Rules used = {1159, 1093, 1090, 226}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x}{(3x^2-2x)^{5/4}}dx$$

$$\Big\downarrow \text{1159}$$

$$\int \frac{1}{\sqrt[4]{3x^2-2x}}dx - \frac{2x}{\sqrt[4]{3x^2-2x}}$$

$$\Big\downarrow \text{1093}$$

$$\frac{\sqrt[4]{3}\sqrt[4]{2x-3x^2}\int \frac{1}{\sqrt[4]{\frac{3x}{2}-\frac{9x^2}{4}}}dx}{\sqrt{2}\sqrt[4]{3x^2-2x}} - \frac{2x}{\sqrt[4]{3x^2-2x}}$$

$$\Big\downarrow \text{1090}$$

$$-\frac{2\sqrt[4]{2x-3x^2}\int \frac{1}{\sqrt[4]{1-\frac{4}{9}\left(\frac{3}{2}-\frac{9x}{2}\right)^2}}d\left(\frac{3}{2}-\frac{9x}{2}\right)}{3\ 3^{3/4}\sqrt[4]{3x^2-2x}} - \frac{2x}{\sqrt[4]{3x^2-2x}}$$

$$\Big\downarrow \text{226}$$

$$-\frac{2\sqrt[4]{2x-3x^2}E\left(\frac{1}{2}\arcsin\left(\frac{2}{3}\left(\frac{3}{2}-\frac{9x}{2}\right)\right)\middle|2\right)}{3^{3/4}\sqrt[4]{3x^2-2x}} - \frac{2x}{\sqrt[4]{3x^2-2x}}$$

```
Int[x/(-2*x + 3*x^2)^(5/4),x]
```

```
(-2*x)/(-2*x + 3*x^2)^(1/4) - (2*(2*x - 3*x^2)^(1/4)*EllipticE[ArcSin[(2*(
3/2 - (9*x)/2))/3]/2, 2])/(3^(3/4)*(-2*x + 3*x^2)^(1/4))
```

Defintions of rubi rules used

```
Int[((a_) + (b_.)*(x_)^2)^(-1/4), x_Symbol] :> Simp[(2/(a^(1/4)*Rt[-b/a, 2]
))*EllipticE[(1/2)*ArcSin[Rt[-b/a, 2]*x], 2], x] /; FreeQ[{a, b}, x] && GtQ
[a, 0] && NegQ[b/a]
```

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[1/(2*c*(-4*
(c/(b^2 - 4*a*c)))^p)   Subst[Int[Simp[1 - x^2/(b^2 - 4*a*c), x]^p, x], x,
b + 2*c*x], x] /; FreeQ[{a, b, c, p}, x] && GtQ[4*a - b^2/c, 0]
```

```
Int[((b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(b*x + c*x^2)^p/((-
c)*((b*x + c*x^2)/b^2))^p   Int[((-c)*(x/b) - c^2*(x^2/b^2))^p, x], x] /; F
reeQ[{b, c}, x] && (IntegerQ[4*p] || IntegerQ[3*p])
```

```
Int[((d_.) + (e_.)*(x_))*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol
] :> Simp[((b*d - 2*a*e + (2*c*d - b*e)*x)/((p + 1)*(b^2 - 4*a*c)))*(a + b*
x + c*x^2)^(p + 1), x] - Simp[(2*p + 3)*((2*c*d - b*e)/((p + 1)*(b^2 - 4*a*
c)))   Int[(a + b*x + c*x^2)^(p + 1), x], x] /; FreeQ[{a, b, c, d, e}, x] &
& LtQ[p, -1] && NeQ[p, -3/2]
```

Maple [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 0.32 (sec) , antiderivative size = 32, normalized size of antiderivative = 0.42

method	result	size
meijerg	$\frac{2^{\frac{3}{4}} \left(-\operatorname{signum}\left(x-\frac{2}{3}\right)\right)^{\frac{5}{4}} x^{\frac{3}{4}} \operatorname{hypergeom}\left(\left[\frac{3}{4}, \frac{5}{4}\right], \left[\frac{7}{4}\right], \frac{3x}{2}\right)}{3 \operatorname{signum}\left(x-\frac{2}{3}\right)^{\frac{5}{4}}}$	32

```
int(x/(3*x^2-2*x)^(5/4),x,method=_RETURNVERBOSE)
```

```
1/3*2^(3/4)/signum(x-2/3)^(5/4)*(-signum(x-2/3))^(5/4)*x^(3/4)*hypergeom([
3/4,5/4],[7/4],3/2*x)
```

Fricas [F]

$$\int \frac{x}{(-2x+3x^2)^{5/4}}\,dx = \int \frac{x}{(3\,x^2-2\,x)^{\frac{5}{4}}}\,dx$$

```
integrate(x/(3*x^2-2*x)^(5/4),x, algorithm="fricas")
```

```
integral((3*x^2 - 2*x)^(3/4)/(9*x^3 - 12*x^2 + 4*x), x)
```

Sympy [F]

$$\int \frac{x}{(-2x+3x^2)^{5/4}}\,dx = \int \frac{x}{(x\,(3x-2))^{\frac{5}{4}}}\,dx$$

```
integrate(x/(3*x**2-2*x)**(5/4),x)
```

```
Integral(x/(x*(3*x - 2))**(5/4), x)
```

Maxima [F]

$$\int \frac{x}{(-2x+3x^2)^{5/4}}\,dx = \int \frac{x}{(3\,x^2-2\,x)^{\frac{5}{4}}}\,dx$$

```
integrate(x/(3*x^2-2*x)^(5/4),x, algorithm="maxima")
```

```
integrate(x/(3*x^2 - 2*x)^(5/4), x)
```

Giac [F]

$$\int \frac{x}{(-2x+3x^2)^{5/4}}\,dx = \int \frac{x}{(3\,x^2-2\,x)^{\frac{5}{4}}}\,dx$$

```
integrate(x/(3*x^2-2*x)^(5/4),x, algorithm="giac")
```

```
integrate(x/(3*x^2 - 2*x)^(5/4), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{x}{(-2x+3x^2)^{5/4}}\,dx = \int \frac{x}{(3\,x^2-2\,x)^{5/4}}\,dx$$

```
int(x/(3*x^2 - 2*x)^(5/4),x)
```

```
int(x/(3*x^2 - 2*x)^(5/4), x)
```

Reduce [F]

$$\int \frac{x}{(-2x+3x^2)^{5/4}}\,dx = \int \frac{1}{3x^{\frac{5}{4}}\,(3x-2)^{\frac{1}{4}}-2x^{\frac{1}{4}}\,(3x-2)^{\frac{1}{4}}}dx$$

```
int(x/(3*x^2-2*x)^(5/4),x)
```

```
int(1/(3*x**(1/4)*(3*x - 2)**(1/4)*x - 2*x**(1/4)*(3*x - 2)**(1/4)),x)
```

3.210 $\int \frac{x}{\left(ax+3x^2\right)^{5/4}}\,dx$

Optimal result

Integrand size = 15, antiderivative size = 66

$$\int \frac{x}{(ax+3x^2)^{5/4}}\,dx=-\frac{4\sqrt{x}\sqrt[4]{\frac{a+3x}{x}}E\left(\frac{1}{2}\arctan\left(\frac{\sqrt{a}}{\sqrt{3}\sqrt{x}}\right)\middle|2\right)}{3^{3/4}\sqrt{a}\sqrt[4]{ax+3x^2}}$$

```
-4/3*x^(1/2)*((a+3*x)/x)^(1/4)*EllipticE(sin(1/2*arctan(1/3*a^(1/2)*3^(1/2
)/x^(1/2))),2^(1/2))*3^(1/4)/a^(1/2)/(a*x+3*x^2)^(1/4)
```

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 10.01 (sec) , antiderivative size = 47, normalized size of antiderivative = 0.71

$$\int \frac{x}{(ax+3x^2)^{5/4}}\,dx=\frac{4x\sqrt[4]{1+\frac{3x}{a}}\,\text{Hypergeometric2F1}\left(\frac{3}{4},\frac{5}{4},\frac{7}{4},-\frac{3x}{a}\right)}{3a\sqrt[4]{x(a+3x)}}$$

```
Integrate[x/(a*x + 3*x^2)^(5/4),x]
```

```
(4*x*(1 + (3*x)/a)^(1/4)*Hypergeometric2F1[3/4, 5/4, 7/4, (-3*x)/a])/(3*a*
(x*(a + 3*x))^(1/4))
```

Rubi [A] (verified)

Time = 0.38 (sec) , antiderivative size = 87, normalized size of antiderivative = 1.32, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.267, Rules used = {1159, 1093, 1090, 226}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x}{\left(ax+3x^2\right)^{5/4}}dx$$

$$\Big\downarrow \text{1159}$$

$$\frac{4x}{a\sqrt[4]{ax+3x^2}}-\frac{2\int \frac{1}{\sqrt[4]{3x^2+ax}}dx}{a}$$

$$\Big\downarrow \text{1093}$$

$$\frac{4x}{a\sqrt[4]{ax+3x^2}}-\frac{2\sqrt[4]{3}\sqrt[4]{-\frac{ax+3x^2}{a^2}}\int \frac{1}{\sqrt[4]{-\frac{9x^2}{a^2}-\frac{3x}{a}}}dx}{a\sqrt[4]{ax+3x^2}}$$

$$\Big\downarrow \text{1090}$$

$$\frac{\sqrt{2}a\sqrt[4]{-\frac{ax+3x^2}{a^2}}\int \frac{1}{\sqrt[4]{1-\frac{1}{9}a^2\left(-\frac{18x}{a^2}-\frac{3}{a}\right)^2}}d\left(-\frac{18x}{a^2}-\frac{3}{a}\right)}{3\ 3^{3/4}\sqrt[4]{ax+3x^2}}+\frac{4x}{a\sqrt[4]{ax+3x^2}}$$

$$\Big\downarrow \text{226}$$

$$\frac{2\sqrt{2}\sqrt[4]{-\frac{ax+3x^2}{a^2}}E\left(\frac{1}{2}\arcsin\left(\frac{1}{3}a\left(-\frac{18x}{a^2}-\frac{3}{a}\right)\right)\middle|2\right)}{3^{3/4}\sqrt[4]{ax+3x^2}}+\frac{4x}{a\sqrt[4]{ax+3x^2}}$$

```
Int[x/(a*x + 3*x^2)^(5/4),x]
```

```
(4*x)/(a*(a*x + 3*x^2)^(1/4)) + (2*Sqrt[2]*(-((a*x + 3*x^2)/a^2))^(1/4)*El
lipticE[ArcSin[(a*(-3/a - (18*x)/a^2))/3]/2, 2])/(3^(3/4)*(a*x + 3*x^2)^(1
/4))
```

Defintions of rubi rules used

```
Int[((a_) + (b_.)*(x_)^2)^(-1/4), x_Symbol] :> Simp[(2/(a^(1/4)*Rt[-b/a, 2]
))*EllipticE[(1/2)*ArcSin[Rt[-b/a, 2]*x], 2], x] /; FreeQ[{a, b}, x] && GtQ
[a, 0] && NegQ[b/a]
```

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[1/(2*c*(-4*
(c/(b^2 - 4*a*c)))^p)   Subst[Int[Simp[1 - x^2/(b^2 - 4*a*c), x]^p, x], x,
b + 2*c*x], x] /; FreeQ[{a, b, c, p}, x] && GtQ[4*a - b^2/c, 0]
```

```
Int[((b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(b*x + c*x^2)^p/((-
c)*((b*x + c*x^2)/b^2))^p   Int[((-c)*(x/b) - c^2*(x^2/b^2))^p, x], x] /; F
reeQ[{b, c}, x] && (IntegerQ[4*p] || IntegerQ[3*p])
```

```
Int[((d_.) + (e_.)*(x_))*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol
] :> Simp[((b*d - 2*a*e + (2*c*d - b*e)*x)/((p + 1)*(b^2 - 4*a*c)))*(a + b*
x + c*x^2)^(p + 1), x] - Simp[(2*p + 3)*((2*c*d - b*e)/((p + 1)*(b^2 - 4*a*
c)))   Int[(a + b*x + c*x^2)^(p + 1), x], x] /; FreeQ[{a, b, c, d, e}, x] &
& LtQ[p, -1] && NeQ[p, -3/2]
```

Maple [F]

$$\int \frac{x}{(ax+3x^2)^{\frac{5}{4}}}dx$$

```
int(x/(a*x+3*x^2)^(5/4),x)
```

```
int(x/(a*x+3*x^2)^(5/4),x)
```

Fricas [F]

$$\int \frac{x}{(ax+3x^2)^{5/4}}\,dx = \int \frac{x}{(ax+3\,x^2)^{\frac{5}{4}}}\,dx$$

```
integrate(x/(a*x+3*x^2)^(5/4),x, algorithm="fricas")
```

```
integral((a*x + 3*x^2)^(3/4)/(a^2*x + 6*a*x^2 + 9*x^3), x)
```

Sympy [F]

$$\int \frac{x}{(ax+3x^2)^{5/4}}\,dx = \int \frac{x}{(x\,(a+3x))^{\frac{5}{4}}}\,dx$$

```
integrate(x/(a*x+3*x**2)**(5/4),x)
```

```
Integral(x/(x*(a + 3*x))**(5/4), x)
```

Maxima [F]

$$\int \frac{x}{(ax+3x^2)^{5/4}}\,dx = \int \frac{x}{(ax+3\,x^2)^{\frac{5}{4}}}\,dx$$

```
integrate(x/(a*x+3*x^2)^(5/4),x, algorithm="maxima")
```

```
integrate(x/(a*x + 3*x^2)^(5/4), x)
```

Giac [F]

$$\int \frac{x}{(ax+3x^2)^{5/4}}\,dx = \int \frac{x}{(ax+3\,x^2)^{\frac{5}{4}}}\,dx$$

```
integrate(x/(a*x+3*x^2)^(5/4),x, algorithm="giac")
```

```
integrate(x/(a*x + 3*x^2)^(5/4), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{x}{(ax+3x^2)^{5/4}}\,dx = \int \frac{x}{(3\,x^2+a\,x)^{5/4}}\,dx$$

```
int(x/(a*x + 3*x^2)^(5/4),x)
```

```
int(x/(a*x + 3*x^2)^(5/4), x)
```

Reduce [F]

$$\int \frac{x}{(ax+3x^2)^{5/4}}\,dx = \int \frac{1}{x^{\frac{1}{4}}\,(a+3x)^{\frac{1}{4}}\,a+3x^{\frac{5}{4}}\,(a+3x)^{\frac{1}{4}}}dx$$

```
int(x/(a*x+3*x^2)^(5/4),x)
```

```
int(1/(x**(1/4)*(a + 3*x)**(1/4)*a + 3*x**(1/4)*(a + 3*x)**(1/4)*x),x)
```

3.211 $\int \frac{x}{(2x-3x^2)^{5/4}}\,dx$

Optimal result

Integrand size = 15, antiderivative size = 36

$$\int \frac{x}{(2x-3x^2)^{5/4}}\,dx=\frac{2x^{3/4}}{\sqrt[4]{2-3x}}+\frac{2E\left(\frac{1}{2}\arcsin(1-3x)\middle|2\right)}{3^{3/4}}$$

```
2*x^(3/4)/(2-3*x)^(1/4)-2/3*EllipticE(sin(1/2*arcsin(-1+3*x)),2^(1/2))*3^(
1/4)
```

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 10.01 (sec) , antiderivative size = 43, normalized size of antiderivative = 1.19

$$\int \frac{x}{(2x-3x^2)^{5/4}}\,dx=\frac{2\left(\frac{2}{3}\right)^{3/4}\sqrt[4]{x}\,\text{Hypergeometric2F1}\left(-\frac{1}{4},\frac{1}{4},\frac{3}{4},1-\frac{3x}{2}\right)}{\sqrt[4]{-x(-2+3x)}}$$

```
Integrate[x/(2*x - 3*x^2)^(5/4),x]
```

```
(2*(2/3)^(3/4)*x^(1/4)*Hypergeometric2F1[-1/4, 1/4, 3/4, 1 - (3*x)/2])/(-(
x*(-2 + 3*x)))^(1/4)
```

Rubi [A] (verified)

Time = 0.29 (sec) , antiderivative size = 40, normalized size of antiderivative = 1.11, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.200, Rules used = {1159, 1090, 226}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x}{(2x-3x^2)^{5/4}}\,dx$$

$$\Big\downarrow \; 1159$$

$$\frac{2x}{\sqrt[4]{2x-3x^2}}-\int \frac{1}{\sqrt[4]{2x-3x^2}}dx$$

$$\Big\downarrow \; 1090$$

$$\frac{\int \frac{1}{\sqrt[4]{1-\frac{1}{4}(2-6x)^2}}d(2-6x)}{2\ 3^{3/4}}+\frac{2x}{\sqrt[4]{2x-3x^2}}$$

$$\Big\downarrow \; 226$$

$$\frac{2E\left(\frac{1}{2}\arcsin\left(\frac{1}{2}(2-6x)\right)\middle|2\right)}{3^{3/4}}+\frac{2x}{\sqrt[4]{2x-3x^2}}$$

```
Int[x/(2*x - 3*x^2)^(5/4),x]
```

```
(2*x)/(2*x - 3*x^2)^(1/4) + (2*EllipticE[ArcSin[(2 - 6*x)/2]/2, 2])/3^(3/4
)
```

Defintions of rubi rules used

```
Int[((a_) + (b_.)*(x_)^2)^(-1/4), x_Symbol] :> Simp[(2/(a^(1/4)*Rt[-b/a, 2]
))*EllipticE[(1/2)*ArcSin[Rt[-b/a, 2]*x], 2], x] /; FreeQ[{a, b}, x] && GtQ
[a, 0] && NegQ[b/a]
```

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[1/(2*c*(-4*
(c/(b^2 - 4*a*c)))^p)   Subst[Int[Simp[1 - x^2/(b^2 - 4*a*c), x]^p, x], x,
b + 2*c*x], x] /; FreeQ[{a, b, c, p}, x] && GtQ[4*a - b^2/c, 0]
```

```
Int[((d_.) + (e_.)*(x_))*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol
] :> Simp[((b*d - 2*a*e + (2*c*d - b*e)*x)/((p + 1)*(b^2 - 4*a*c)))*(a + b*
x + c*x^2)^(p + 1), x] - Simp[(2*p + 3)*((2*c*d - b*e)/((p + 1)*(b^2 - 4*a*
c)))   Int[(a + b*x + c*x^2)^(p + 1), x], x] /; FreeQ[{a, b, c, d, e}, x] &
& LtQ[p, -1] && NeQ[p, -3/2]
```

Maple [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4.

Time = 0.32 (sec) , antiderivative size = 18, normalized size of antiderivative = 0.50

method	result	size
meijerg	$\frac{2^{\frac{3}{4}} x^{\frac{3}{4}} \operatorname{hypergeom}\left(\left[\frac{3}{4}, \frac{5}{4}\right], \left[\frac{7}{4}\right], \frac{3x}{2}\right)}{3}$	18

```
int(x/(-3*x^2+2*x)^(5/4),x,method=_RETURNVERBOSE)
```

```
1/3*2^(3/4)*x^(3/4)*hypergeom([3/4,5/4],[7/4],3/2*x)
```

Fricas [F]

$$\int \frac{x}{(2x-3x^2)^{5/4}}\,dx = \int \frac{x}{(-3\,x^2+2\,x)^{\frac{5}{4}}}\,dx$$

```
integrate(x/(-3*x^2+2*x)^(5/4),x, algorithm="fricas")
```

```
integral((-3*x^2 + 2*x)^(3/4)/(9*x^3 - 12*x^2 + 4*x), x)
```

Sympy [F]

$$\int \frac{x}{(2x-3x^2)^{5/4}}\,dx = \int \frac{x}{(-x\,(3x-2))^{\frac{5}{4}}}\,dx$$

```
integrate(x/(-3*x**2+2*x)**(5/4),x)
```

```
Integral(x/(-x*(3*x - 2))**(5/4), x)
```

Maxima [F]

$$\int \frac{x}{(2x-3x^2)^{5/4}}\,dx = \int \frac{x}{(-3\,x^2+2\,x)^{\frac{5}{4}}}\,dx$$

```
integrate(x/(-3*x^2+2*x)^(5/4),x, algorithm="maxima")
```

```
integrate(x/(-3*x^2 + 2*x)^(5/4), x)
```

Giac [F]

$$\int \frac{x}{(2x-3x^2)^{5/4}}\,dx = \int \frac{x}{(-3\,x^2+2\,x)^{\frac{5}{4}}}\,dx$$

```
integrate(x/(-3*x^2+2*x)^(5/4),x, algorithm="giac")
```

```
integrate(x/(-3*x^2 + 2*x)^(5/4), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{x}{(2x-3x^2)^{5/4}}\,dx = \int \frac{x}{(2\,x-3\,x^2)^{5/4}}\,dx$$

```
int(x/(2*x - 3*x^2)^(5/4),x)
```

```
int(x/(2*x - 3*x^2)^(5/4), x)
```

Reduce [F]

$$\int \frac{x}{(2x-3x^2)^{5/4}}\,dx = -\left(\int \frac{1}{3x^{\frac{5}{4}}\,(-3x+2)^{\frac{1}{4}}-2x^{\frac{1}{4}}\,(-3x+2)^{\frac{1}{4}}}dx\right)$$

```
int(x/(-3*x^2+2*x)^(5/4),x)
```

```
 - int(1/(3*x**(1/4)*( - 3*x + 2)**(1/4)*x - 2*x**(1/4)*( - 3*x + 2)**(1/4
)),x)
```

3.212 $\int \frac{x}{(-2x-3x^2)^{5/4}}\,dx$

Optimal result

Integrand size = 15, antiderivative size = 36

$$\int \frac{x}{(-2x-3x^2)^{5/4}}\,dx = -\frac{2x}{\sqrt[4]{-2x-3x^2}} + \frac{2E\left(\frac{1}{2}\arcsin(1+3x)\middle|\,2\right)}{3^{3/4}}$$

```
-2*x/(-3*x^2-2*x)^(1/4)+2/3*EllipticE(sin(1/2*arcsin(1+3*x)),2^(1/2))*3^(1
/4)
```

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 10.01 (sec) , antiderivative size = 42, normalized size of antiderivative = 1.17

$$\int \frac{x}{(-2x-3x^2)^{5/4}}\,dx = \frac{(-x(2+3x))^{3/4}\,\text{Hypergeometric2F1}\left(\frac{3}{4},\frac{5}{4},\frac{7}{4},-\frac{3x}{2}\right)}{3\left(1+\frac{3x}{2}\right)^{3/4}}$$

```
Integrate[x/(-2*x - 3*x^2)^(5/4),x]
```

```
((-(x*(2 + 3*x)))^(3/4)*Hypergeometric2F1[3/4, 5/4, 7/4, (-3*x)/2])/(3*(1
+ (3*x)/2)^(3/4))
```

Rubi [A] (verified)

Time = 0.29 (sec) , antiderivative size = 40, normalized size of antiderivative = 1.11, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.200$, Rules used = {1159, 1090, 226}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x}{(-3x^2-2x)^{5/4}}dx$$

$$\Big\downarrow \text{1159}$$

$$\int \frac{1}{\sqrt[4]{-3x^2-2x}}dx - \frac{2x}{\sqrt[4]{-3x^2-2x}}$$

$$\Big\downarrow \text{1090}$$

$$-\frac{\int \frac{1}{\sqrt[4]{1-\frac{1}{4}(-6x-2)^2}}d(-6x-2)}{2\ 3^{3/4}} - \frac{2x}{\sqrt[4]{-3x^2-2x}}$$

$$\Big\downarrow \text{226}$$

$$-\frac{2E\left(\frac{1}{2}\arcsin\left(\frac{1}{2}(-6x-2)\right)\middle|2\right)}{3^{3/4}} - \frac{2x}{\sqrt[4]{-3x^2-2x}}$$

```
Int[x/(-2*x - 3*x^2)^(5/4),x]
```

```
(-2*x)/(-2*x - 3*x^2)^(1/4) - (2*EllipticE[ArcSin[(-2 - 6*x)/2]/2, 2])/3^(
3/4)
```

Defintions of rubi rules used

```
Int[((a_) + (b_.)*(x_)^2)^(-1/4), x_Symbol] :> Simp[(2/(a^(1/4)*Rt[-b/a, 2]
))*EllipticE[(1/2)*ArcSin[Rt[-b/a, 2]*x], 2], x] /; FreeQ[{a, b}, x] && GtQ
[a, 0] && NegQ[b/a]
```

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[1/(2*c*(-4*
(c/(b^2 - 4*a*c)))^p)   Subst[Int[Simp[1 - x^2/(b^2 - 4*a*c), x]^p, x], x,
b + 2*c*x], x] /; FreeQ[{a, b, c, p}, x] && GtQ[4*a - b^2/c, 0]
```

```
Int[((d_.) + (e_.)*(x_))*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol
] :> Simp[((b*d - 2*a*e + (2*c*d - b*e)*x)/((p + 1)*(b^2 - 4*a*c)))*(a + b*
x + c*x^2)^(p + 1), x] - Simp[(2*p + 3)*((2*c*d - b*e)/((p + 1)*(b^2 - 4*a*
c)))   Int[(a + b*x + c*x^2)^(p + 1), x], x] /; FreeQ[{a, b, c, d, e}, x] &
& LtQ[p, -1] && NeQ[p, -3/2]
```

Maple [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4.

Time = 0.36 (sec) , antiderivative size = 21, normalized size of antiderivative = 0.58

method	result	size
meijerg	$\frac{(-1)^{\frac{3}{4}} 2^{\frac{3}{4}} x^{\frac{3}{4}} \text{hypergeom}\left(\left[\frac{3}{4},\frac{5}{4}\right],\left[\frac{7}{4}\right],-\frac{3x}{2}\right)}{3}$	21

```
int(x/(-3*x^2-2*x)^(5/4),x,method=_RETURNVERBOSE)
```

```
1/3*(-1)^(3/4)*2^(3/4)*x^(3/4)*hypergeom([3/4,5/4],[7/4],-3/2*x)
```

Fricas [F]

$$\int \frac{x}{(-2x-3x^2)^{5/4}}\,dx = \int \frac{x}{\left(-3\,x^2-2\,x\right)^{\frac{5}{4}}}\,dx$$

```
integrate(x/(-3*x^2-2*x)^(5/4),x, algorithm="fricas")
```

```
integral((-3*x^2 - 2*x)^(3/4)/(9*x^3 + 12*x^2 + 4*x), x)
```

Sympy [F]

$$\int \frac{x}{(-2x-3x^2)^{5/4}}\,dx = \int \frac{x}{\left(-x\,(3x+2)\right)^{\frac{5}{4}}}\,dx$$

```
integrate(x/(-3*x**2-2*x)**(5/4),x)
```

```
Integral(x/(-x*(3*x + 2))**(5/4), x)
```

Maxima [F]

$$\int \frac{x}{(-2x-3x^2)^{5/4}}\,dx = \int \frac{x}{\left(-3\,x^2-2\,x\right)^{\frac{5}{4}}}\,dx$$

```
integrate(x/(-3*x^2-2*x)^(5/4),x, algorithm="maxima")
```

```
integrate(x/(-3*x^2 - 2*x)^(5/4), x)
```

Giac [F]

$$\int \frac{x}{(-2x-3x^2)^{5/4}}\,dx = \int \frac{x}{(-3\,x^2-2\,x)^{\frac{5}{4}}}\,dx$$

```
integrate(x/(-3*x^2-2*x)^(5/4),x, algorithm="giac")
```

```
integrate(x/(-3*x^2 - 2*x)^(5/4), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{x}{(-2x-3x^2)^{5/4}}\,dx = \int \frac{x}{(-3\,x^2-2\,x)^{5/4}}\,dx$$

```
int(x/(- 2*x - 3*x^2)^(5/4),x)
```

```
int(x/(- 2*x - 3*x^2)^(5/4), x)
```

Reduce [F]

$$\int \frac{x}{(-2x-3x^2)^{5/4}}\,dx = -\left(\int \frac{1}{3x^{\frac{5}{4}}(-3x-2)^{\frac{1}{4}}+2x^{\frac{1}{4}}(-3x-2)^{\frac{1}{4}}}dx\right)$$

```
int(x/(-3*x^2-2*x)^(5/4),x)
```

```
 - int(1/(3*x**(1/4)*( - 3*x - 2)**(1/4)*x + 2*x**(1/4)*( - 3*x - 2)**(1/4
)),x)
```

3.213 $\int \frac{x}{(ax-3x^2)^{5/4}}\,dx$

Optimal result

Integrand size = 15, antiderivative size = 78

$$\int \frac{x}{(ax-3x^2)^{5/4}}\,dx = \frac{4x}{a\sqrt[4]{ax-3x^2}} + \frac{2\sqrt{2}\sqrt[4]{\frac{x}{a}-\frac{3x^2}{a^2}}\,E\left(\frac{1}{2}\arcsin\left(1-\frac{6x}{a}\right)\middle|2\right)}{3^{3/4}\sqrt[4]{ax-3x^2}}$$

```
4*x/a/(a*x-3*x^2)^(1/4)+2/3*2^(1/2)*3^(1/4)*(x/a-3*x^2/a^2)^(1/4)*Elliptic
E(sin(1/2*arcsin(1-6*x/a)),2^(1/2))/(a*x-3*x^2)^(1/4)
```

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 10.01 (sec) , antiderivative size = 47, normalized size of antiderivative = 0.60

$$\int \frac{x}{(ax-3x^2)^{5/4}}\,dx = \frac{4x\sqrt[4]{1-\frac{3x}{a}}\,\text{Hypergeometric2F1}\left(\frac{3}{4},\frac{5}{4},\frac{7}{4},\frac{3x}{a}\right)}{3a\sqrt[4]{(a-3x)x}}$$

```
Integrate[x/(a*x - 3*x^2)^(5/4),x]
```

```
(4*x*(1 - (3*x)/a)^(1/4)*Hypergeometric2F1[3/4, 5/4, 7/4, (3*x)/a])/(3*a*(
(a - 3*x)*x)^(1/4))
```

Rubi [A] (verified)

Time = 0.37 (sec) , antiderivative size = 86, normalized size of antiderivative = 1.10, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.267$, Rules used = {1159, 1093, 1090, 226}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x}{(ax-3x^2)^{5/4}}dx$$

$$\Big\downarrow \ 1159$$

$$\frac{4x}{a\sqrt[4]{ax-3x^2}}-\frac{2\int \frac{1}{\sqrt[4]{ax-3x^2}}dx}{a}$$

$$\Big\downarrow \ 1093$$

$$\frac{4x}{a\sqrt[4]{ax-3x^2}}-\frac{2\sqrt[4]{3}\sqrt[4]{\frac{ax-3x^2}{a^2}}\int \frac{1}{\sqrt[4]{\frac{3x}{a}-\frac{9x^2}{a^2}}}dx}{a\sqrt[4]{ax-3x^2}}$$

$$\Big\downarrow \ 1090$$

$$\frac{\sqrt{2}a\sqrt[4]{\frac{ax-3x^2}{a^2}}\int \frac{1}{\sqrt[4]{1-\frac{1}{9}a^2\left(\frac{3}{a}-\frac{18x}{a^2}\right)^2}}d\left(\frac{3}{a}-\frac{18x}{a^2}\right)}{3\ 3^{3/4}\sqrt[4]{ax-3x^2}}+\frac{4x}{a\sqrt[4]{ax-3x^2}}$$

$$\Big\downarrow \ 226$$

$$\frac{2\sqrt{2}\sqrt[4]{\frac{ax-3x^2}{a^2}}E\left(\frac{1}{2}\arcsin\left(\frac{1}{3}a\left(\frac{3}{a}-\frac{18x}{a^2}\right)\right)\middle|2\right)}{3^{3/4}\sqrt[4]{ax-3x^2}}+\frac{4x}{a\sqrt[4]{ax-3x^2}}$$

```
Int[x/(a*x - 3*x^2)^(5/4),x]
```

```
(4*x)/(a*(a*x - 3*x^2)^(1/4)) + (2*Sqrt[2]*((a*x - 3*x^2)/a^2)^(1/4)*Ellip
ticE[ArcSin[(a*(3/a - (18*x)/a^2))/3]/2, 2])/(3^(3/4)*(a*x - 3*x^2)^(1/4))
```

Defintions of rubi rules used

```
Int[((a_) + (b_.)*(x_)^2)^(-1/4), x_Symbol] :> Simp[(2/(a^(1/4)*Rt[-b/a, 2]
))*EllipticE[(1/2)*ArcSin[Rt[-b/a, 2]*x], 2], x] /; FreeQ[{a, b}, x] && GtQ
[a, 0] && NegQ[b/a]
```

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[1/(2*c*(-4*
(c/(b^2 - 4*a*c)))^p)   Subst[Int[Simp[1 - x^2/(b^2 - 4*a*c), x]^p, x], x,
b + 2*c*x], x] /; FreeQ[{a, b, c, p}, x] && GtQ[4*a - b^2/c, 0]
```

```
Int[((b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(b*x + c*x^2)^p/((-
c)*((b*x + c*x^2)/b^2))^p   Int[((-c)*(x/b) - c^2*(x^2/b^2))^p, x], x] /; F
reeQ[{b, c}, x] && (IntegerQ[4*p] || IntegerQ[3*p])
```

```
Int[((d_.) + (e_.)*(x_))*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol
] :> Simp[((b*d - 2*a*e + (2*c*d - b*e)*x)/((p + 1)*(b^2 - 4*a*c)))*(a + b*
x + c*x^2)^(p + 1), x] - Simp[(2*p + 3)*((2*c*d - b*e)/((p + 1)*(b^2 - 4*a*
c)))   Int[(a + b*x + c*x^2)^(p + 1), x], x] /; FreeQ[{a, b, c, d, e}, x] &
& LtQ[p, -1] && NeQ[p, -3/2]
```

Maple [F]

$$\int \frac{x}{(ax-3x^2)^{\frac{5}{4}}}dx$$

```
int(x/(a*x-3*x^2)^(5/4),x)
```

```
int(x/(a*x-3*x^2)^(5/4),x)
```

Fricas [F]

$$\int \frac{x}{(ax-3x^2)^{5/4}}\,dx = \int \frac{x}{(ax-3\,x^2)^{\frac{5}{4}}}\,dx$$

```
integrate(x/(a*x-3*x^2)^(5/4),x, algorithm="fricas")
```

```
integral((a*x - 3*x^2)^(3/4)/(a^2*x - 6*a*x^2 + 9*x^3), x)
```

Sympy [F]

$$\int \frac{x}{(ax-3x^2)^{5/4}}\,dx = \int \frac{x}{(-x\,(-a+3x))^{\frac{5}{4}}}\,dx$$

```
integrate(x/(a*x-3*x**2)**(5/4),x)
```

```
Integral(x/(-x*(-a + 3*x))**(5/4), x)
```

Maxima [F]

$$\int \frac{x}{(ax-3x^2)^{5/4}}\,dx = \int \frac{x}{(ax-3\,x^2)^{\frac{5}{4}}}\,dx$$

```
integrate(x/(a*x-3*x^2)^(5/4),x, algorithm="maxima")
```

```
integrate(x/(a*x - 3*x^2)^(5/4), x)
```

Giac [F]

$$\int \frac{x}{(ax-3x^2)^{5/4}}\,dx = \int \frac{x}{(ax-3\,x^2)^{\frac{5}{4}}}\,dx$$

```
integrate(x/(a*x-3*x^2)^(5/4),x, algorithm="giac")
```

```
integrate(x/(a*x - 3*x^2)^(5/4), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{x}{(ax-3x^2)^{5/4}}\,dx = \int \frac{x}{(a\,x-3\,x^2)^{5/4}}\,dx$$

```
int(x/(a*x - 3*x^2)^(5/4),x)
```

```
int(x/(a*x - 3*x^2)^(5/4), x)
```

Reduce [F]

$$\int \frac{x}{(ax-3x^2)^{5/4}}\,dx = \int \frac{1}{x^{\frac{1}{4}}\,(a-3x)^{\frac{1}{4}}\,a-3x^{\frac{5}{4}}\,(a-3x)^{\frac{1}{4}}}dx$$

```
int(x/(a*x-3*x^2)^(5/4),x)
```

```
int(1/(x**(1/4)*(a - 3*x)**(1/4)*a - 3*x**(1/4)*(a - 3*x)**(1/4)*x),x)
```

3.214 $\int \frac{1}{x\sqrt[4]{-x+x^2}}\,dx$

Optimal result

Integrand size = 15, antiderivative size = 37

$$\int \frac{1}{x\sqrt[4]{-x+x^2}}\,dx=-\frac{4\sqrt[4]{-1+x}\sqrt[4]{x}E\left(\frac{1}{2}\arcsin\left(\frac{1}{\sqrt{x}}\right)\middle|2\right)}{\sqrt[4]{-x+x^2}}$$

```
-4*(-1+x)^(1/4)*x^(1/4)*EllipticE(sin(1/2*arcsin(1/x^(1/2))),2^(1/2))/(x^2
-x)^(1/4)
```

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 10.01 (sec) , antiderivative size = 33, normalized size of antiderivative = 0.89

$$\int \frac{1}{x\sqrt[4]{-x+x^2}}\,dx=\frac{4((-1+x)x)^{3/4}\,\text{Hypergeometric2F1}\left(\frac{3}{4},\frac{5}{4},\frac{7}{4},1-x\right)}{3x^{3/4}}$$

```
Integrate[1/(x*(-x + x^2)^(1/4)),x]
```

```
(4*((-1 + x)*x)^(3/4)*Hypergeometric2F1[3/4, 5/4, 7/4, 1 - x])/(3*x^(3/4))
```

Rubi [B] (warning: unable to verify)

Leaf count is larger than twice the leaf count of optimal. 99 vs. $2(37) = 74$.

Time = 0.43 (sec) , antiderivative size = 99, normalized size of antiderivative = 2.68, number of steps used = 9, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}} = 0.533$, Rules used = {1137, 61, 73, 839, 813, 858, 807, 212}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{x\sqrt[4]{x^2-x}}dx$$

$$\Big\downarrow \text{1137}$$

$$\frac{\sqrt[4]{x-1}\sqrt[4]{x}\int \frac{1}{\sqrt[4]{x-1}x^{5/4}}dx}{\sqrt[4]{x^2-x}}$$

$$\Big\downarrow \text{61}$$

$$\frac{\sqrt[4]{x-1}\sqrt[4]{x}\left(\frac{4(x-1)^{3/4}}{\sqrt[4]{x}}-2\int \frac{1}{\sqrt[4]{x-1}\sqrt[4]{x}}dx\right)}{\sqrt[4]{x^2-x}}$$

$$\Big\downarrow \text{73}$$

$$\frac{\sqrt[4]{x-1}\sqrt[4]{x}\left(\frac{4(x-1)^{3/4}}{\sqrt[4]{x}}-8\int \frac{\sqrt{x-1}}{\sqrt[4]{x}}d\sqrt[4]{x-1}\right)}{\sqrt[4]{x^2-x}}$$

$$\Big\downarrow \text{839}$$

$$\frac{\sqrt[4]{x-1}\sqrt[4]{x}\left(\frac{4(x-1)^{3/4}}{\sqrt[4]{x}}-8\left(\frac{(x-1)^{3/4}}{2\sqrt[4]{x}}-\frac{1}{2}\int \frac{\sqrt{x-1}}{x^{5/4}}d\sqrt[4]{x-1}\right)\right)}{\sqrt[4]{x^2-x}}$$

$$\Big\downarrow \text{813}$$

$$\frac{\sqrt[4]{x-1}\sqrt[4]{x}\left(\frac{4(x-1)^{3/4}}{\sqrt[4]{x}}-8\left(\frac{(x-1)^{3/4}}{2\sqrt[4]{x}}-\frac{\sqrt[4]{\frac{1}{x-1}+1}\sqrt[4]{x-1}\int \frac{1}{\left(1+\frac{1}{x-1}\right)^{5/4}(x-1)^{3/4}}d\sqrt[4]{x-1}}{2\sqrt[4]{x}}\right)\right)}{\sqrt[4]{x^2-x}}$$

$$\Big\downarrow \text{858}$$

$$\frac{\sqrt[4]{x-1}\sqrt[4]{x}\left(\frac{4(x-1)^{3/4}}{\sqrt[4]{x}}-8\left(\frac{\sqrt[4]{\frac{1}{x-1}+1}\sqrt[4]{x-1}\int\frac{1}{\sqrt[4]{x-1}x^{5/4}}d\frac{1}{\sqrt[4]{x-1}}}{2\sqrt[4]{x}}+\frac{(x-1)^{3/4}}{2\sqrt[4]{x}}\right)\right)}{\sqrt[4]{x^2-x}}$$

$$\Big\downarrow \textbf{807}$$

$$\frac{\sqrt[4]{x-1}\sqrt[4]{x}\left(\frac{4(x-1)^{3/4}}{\sqrt[4]{x}}-8\left(\frac{\sqrt[4]{\frac{1}{x-1}+1}\sqrt[4]{x-1}\int\frac{1}{\left(\sqrt{x-1}+1\right)^{5/4}}d\sqrt{x-1}}{4\sqrt[4]{x}}+\frac{(x-1)^{3/4}}{2\sqrt[4]{x}}\right)\right)}{\sqrt[4]{x^2-x}}$$

$$\Big\downarrow \textbf{212}$$

$$\frac{\sqrt[4]{x-1}\sqrt[4]{x}\left(\frac{4(x-1)^{3/4}}{\sqrt[4]{x}}-8\left(\frac{\sqrt[4]{\frac{1}{x-1}+1}\sqrt[4]{x-1}E\left(\frac{1}{2}\arctan\left(\sqrt{x-1}\right)|2\right)}{2\sqrt[4]{x}}+\frac{(x-1)^{3/4}}{2\sqrt[4]{x}}\right)\right)}{\sqrt[4]{x^2-x}}$$

```
Int[1/(x*(-x + x^2)^(1/4)),x]
```

```
((-1 + x)^(1/4)*x^(1/4)*((4*(-1 + x)^(3/4))/x^(1/4) - 8*((-1 + x)^(3/4)/(2
*x^(1/4)) + ((1 + (-1 + x)^(-1))^(1/4)*(-1 + x)^(1/4)*EllipticE[ArcTan[Sqr
t[-1 + x]]/2, 2])/(2*x^(1/4)))))/(-x + x^2)^(1/4)
```

Defintions of rubi rules used

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] - Simp[d*((
m + n + 2)/((b*c - a*d)*(m + 1)))   Int[(a + b*x)^(m + 1)*(c + d*x)^n, x],
x] /; FreeQ[{a, b, c, d, n}, x] && LtQ[m, -1] &&  !(LtQ[n, -1] && (EqQ[a, 0
] || (NeQ[c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && IntLinearQ[a, b, c, d
, m, n, x]
```

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Simp[p/b   Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
 d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && Lt
Q[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntL
inearQ[a, b, c, d, m, n, x]
```

```
Int[((a_) + (b_.)*(x_)^2)^(-5/4), x_Symbol] :> Simp[(2/(a^(5/4)*Rt[b/a, 2])
)*EllipticE[(1/2)*ArcTan[Rt[b/a, 2]*x], 2], x] /; FreeQ[{a, b}, x] && GtQ[a
, 0] && PosQ[b/a]
```

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k = GCD[m
+ 1, n]}, Simp[1/k   Subst[Int[x^((m + 1)/k - 1)*(a + b*x^(n/k))^p, x], x,
x^k], x] /; k != 1] /; FreeQ[{a, b, p}, x] && IGtQ[n, 0] && IntegerQ[m]
```

```
Int[(x_)^2/((a_) + (b_.)*(x_)^4)^(5/4), x_Symbol] :> Simp[x*((1 + a/(b*x^4)
)^(1/4)/(b*(a + b*x^4)^(1/4)))   Int[1/(x^3*(1 + a/(b*x^4))^(5/4)), x], x]
/; FreeQ[{a, b}, x] && PosQ[b/a]
```

```
Int[(x_)^2/((a_) + (b_.)*(x_)^4)^(1/4), x_Symbol] :> Simp[x^3/(2*(a + b*x^4
)^(1/4)), x] - Simp[a/2   Int[x^2/(a + b*x^4)^(5/4), x], x] /; FreeQ[{a, b}
, x] && PosQ[b/a]
```

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> -Subst[Int[(a +
b/x^n)^p/x^(m + 2), x], x, 1/x] /; FreeQ[{a, b, p}, x] && ILtQ[n, 0] && Int
egerQ[m]
```

```
Int[((e_.)*(x_))^(m_)*((b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(
e*x)^m*((b*x + c*x^2)^p/(x^(m + p)*(b + c*x)^p))   Int[x^(m + p)*(b + c*x)^
p, x], x] /; FreeQ[{b, c, e, m}, x]
```

Maple [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 0.40 (sec) , antiderivative size = 27, normalized size of antiderivative = 0.73

method	result	size
meijerg	$-\frac{4(-\operatorname{signum}(x-1))^{\frac{1}{4}} \operatorname{hypergeom}\left(\left[-\frac{1}{4},\frac{1}{4}\right],\left[\frac{3}{4}\right],x\right)}{\operatorname{signum}(x-1)^{\frac{1}{4}} x^{\frac{1}{4}}}$	27
risch	$\frac{4x-4}{(x(x-1))^{\frac{1}{4}}}-\frac{8(-\operatorname{signum}(x-1))^{\frac{1}{4}} x^{\frac{3}{4}} \operatorname{hypergeom}\left(\left[\frac{1}{4},\frac{3}{4}\right],\left[\frac{7}{4}\right],x\right)}{3 \operatorname{signum}(x-1)^{\frac{1}{4}}}$	40

```
int(1/x/(x^2-x)^(1/4),x,method=_RETURNVERBOSE)
```

```
-4/signum(x-1)^(1/4)*(-signum(x-1))^(1/4)/x^(1/4)*hypergeom([-1/4,1/4],[3/
4],x)
```

Fricas [F]

$$\int \frac{1}{x\sqrt[4]{-x+x^2}}\,dx=\int \frac{1}{(x^2-x)^{\frac{1}{4}}x}\,dx$$

```
integrate(1/x/(x^2-x)^(1/4),x, algorithm="fricas")
```

```
integral((x^2 - x)^(3/4)/(x^3 - x^2), x)
```

Sympy [F]

$$\int \frac{1}{x\sqrt[4]{-x+x^2}}\,dx=\int \frac{1}{x\sqrt[4]{x(x-1)}}\,dx$$

```
integrate(1/x/(x**2-x)**(1/4),x)
```

```
Integral(1/(x*(x*(x - 1))**(1/4)), x)
```

Maxima [F]

$$\int \frac{1}{x\sqrt[4]{-x+x^2}}\,dx = \int \frac{1}{(x^2-x)^{\frac{1}{4}}x}\,dx$$

```
integrate(1/x/(x^2-x)^(1/4),x, algorithm="maxima")
```

```
integrate(1/((x^2 - x)^(1/4)*x), x)
```

Giac [F]

$$\int \frac{1}{x\sqrt[4]{-x+x^2}}\,dx = \int \frac{1}{(x^2-x)^{\frac{1}{4}}x}\,dx$$

```
integrate(1/x/(x^2-x)^(1/4),x, algorithm="giac")
```

```
integrate(1/((x^2 - x)^(1/4)*x), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{x\sqrt[4]{-x+x^2}}\,dx = \int \frac{1}{x\,(x^2-x)^{1/4}}\,dx$$

```
int(1/(x*(x^2 - x)^(1/4)),x)
```

```
int(1/(x*(x^2 - x)^(1/4)), x)
```

Reduce [F]

$$\int \frac{1}{x\sqrt[4]{-x+x^2}}\,dx = \int \frac{1}{x^{\frac{5}{4}}\,(x-1)^{\frac{1}{4}}}dx$$

```
int(1/x/(x^2-x)^(1/4),x)
```

```
int(1/(x**(1/4)*(x - 1)**(1/4)*x),x)
```

3.215 $\int \frac{1}{\sqrt[4]{3-2x}x^{3/4}}\,dx$

Optimal result

Integrand size = 15, antiderivative size = 112

$$\int \frac{1}{\sqrt[4]{3-2x}x^{3/4}}\,dx = -\sqrt[4]{2}\arctan\left(1-\frac{2^{3/4}\sqrt[4]{x}}{\sqrt[4]{3-2x}}\right) + \sqrt[4]{2}\arctan\left(1+\frac{2^{3/4}\sqrt[4]{x}}{\sqrt[4]{3-2x}}\right) + \sqrt[4]{2}\text{arctanh}\left(\frac{2^{3/4}\sqrt[4]{x}}{\left(1+\frac{\sqrt{2}\sqrt{x}}{\sqrt{3-2x}}\right)\sqrt[4]{3-2x}}\right)$$

```
2^(1/4)*arctan(-1+2^(3/4)*x^(1/4)/(3-2*x)^(1/4))+2^(1/4)*arctan(1+2^(3/4)*
x^(1/4)/(3-2*x)^(1/4))+2^(1/4)*arctanh(2^(3/4)*x^(1/4)/(1+2^(1/2)*x^(1/2)/
(3-2*x)^(1/2))/(3-2*x)^(1/4))
```

Mathematica [A] (verified)

Time = 0.34 (sec) , antiderivative size = 88, normalized size of antiderivative = 0.79

$$\int \frac{1}{\sqrt[4]{3-2x}x^{3/4}}\,dx = \sqrt[4]{2}\left(\arctan\left(\frac{2^{3/4}\sqrt[4]{3-2x}\sqrt[4]{x}}{\sqrt{3-2x}-\sqrt{2}\sqrt{x}}\right)+\text{arctanh}\left(\frac{2\sqrt[4]{6-4x}\sqrt[4]{x}}{\sqrt{6-4x}+2\sqrt{x}}\right)\right)$$

```
Integrate[1/((3 - 2*x)^(1/4)*x^(3/4)),x]
```

```
2^(1/4)*(ArcTan[(2^(3/4)*(3 - 2*x)^(1/4)*x^(1/4))/(Sqrt[3 - 2*x] - Sqrt[2]
*Sqrt[x])] + ArcTanh[(2*(6 - 4*x)^(1/4)*x^(1/4))/(Sqrt[6 - 4*x] + 2*Sqrt[x
])])
```

Rubi [A] (warning: unable to verify)

Time = 0.54 (sec) , antiderivative size = 161, normalized size of antiderivative = 1.44, number of steps used = 12, number of rules used = 11, $\frac{\text{number of rules}}{\text{integrand size}} = 0.733$, Rules used = {73, 27, 854, 826, 1476, 1082, 217, 1479, 25, 27, 1103}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{\sqrt[4]{3-2x}x^{3/4}}dx$$

$$\Big\downarrow \text{73}$$

$$-2\int \frac{\sqrt{3-2x}}{x^{3/4}}d\sqrt[4]{3-2x}$$

$$\Big\downarrow \text{27}$$

$$-22^{3/4}\int \frac{\sqrt{3-2x}}{2^{3/4}x^{3/4}}d\sqrt[4]{3-2x}$$

$$\Big\downarrow \text{854}$$

$$-22^{3/4}\int \frac{\sqrt{3-2x}}{4-2x}d\frac{\sqrt[4]{3-2x}}{\sqrt[4]{2}\sqrt[4]{x}}$$

$$\Big\downarrow \text{826}$$

$$-22^{3/4}\left(\frac{1}{2}\int \frac{\sqrt{3-2x}+1}{4-2x}d\frac{\sqrt[4]{3-2x}}{\sqrt[4]{2}\sqrt[4]{x}}-\frac{1}{2}\int \frac{1-\sqrt{3-2x}}{4-2x}d\frac{\sqrt[4]{3-2x}}{\sqrt[4]{2}\sqrt[4]{x}}\right)$$

$$\Big\downarrow \text{1476}$$

$$-22^{3/4}\left(\frac{1}{2}\left(\frac{1}{2}\int \frac{1}{\sqrt{3-2x}-\frac{\sqrt[4]{2}\sqrt[4]{3-2x}}{\sqrt[4]{x}}+1}d\frac{\sqrt[4]{3-2x}}{\sqrt[4]{2}\sqrt[4]{x}}+\frac{1}{2}\int \frac{1}{\sqrt{3-2x}+\frac{\sqrt[4]{2}\sqrt[4]{3-2x}}{\sqrt[4]{x}}+1}d\frac{\sqrt[4]{3-2x}}{\sqrt[4]{2}\sqrt[4]{x}}\right)-\frac{1}{2}\int$$

$$\Big\downarrow \text{1082}$$

$$-22^{3/4}\left(\frac{1}{2}\left(\frac{\int \frac{1}{-\sqrt{3-2x}-1}d\left(1-\frac{\sqrt[4]{2}\sqrt[4]{3-2x}}{\sqrt[4]{x}}\right)}{\sqrt{2}}-\frac{\int \frac{1}{-\sqrt{3-2x}-1}d\left(\frac{\sqrt[4]{2}\sqrt[4]{3-2x}}{\sqrt[4]{x}}+1\right)}{\sqrt{2}}\right)-\frac{1}{2}\int \frac{1-\sqrt{3-2x}}{4-2x}d\frac{\sqrt[4]{3}}{\sqrt[4]{2}}\right.$$

$$\Big\downarrow \text{217}$$

$$-22^{3/4}\left(\frac{1}{2}\left(\frac{\arctan\left(\frac{\sqrt[4]{2}\sqrt[4]{3-2x}}{\sqrt[4]{x}}+1\right)}{\sqrt{2}}-\frac{\arctan\left(1-\frac{\sqrt[4]{2}\sqrt[4]{3-2x}}{\sqrt[4]{x}}\right)}{\sqrt{2}}\right)-\frac{1}{2}\int \frac{1-\sqrt{3-2x}}{4-2x}d\frac{\sqrt[4]{3-2x}}{\sqrt[4]{2}\sqrt[4]{x}}\right)$$

$$\Big\downarrow \text{1479}$$

$$-22^{3/4}\left(\frac{1}{2}\left(\frac{\int -\frac{\sqrt{2}\left(\frac{\sqrt[4]{2}\sqrt[4]{3-2x}}{\sqrt[4]{x}}+1\right)}{\sqrt{3-2x}+\frac{\sqrt[4]{2}\sqrt[4]{3-2x}}{\sqrt[4]{x}}+1}d\frac{\sqrt[4]{3-2x}}{\sqrt[4]{2}\sqrt[4]{x}}}{2\sqrt{2}}+\frac{\int -\frac{\sqrt{2}-\frac{2^{3/4}\sqrt[4]{3-2x}}{\sqrt[4]{x}}}{\sqrt{3-2x}-\frac{\sqrt[4]{2}\sqrt[4]{3-2x}}{\sqrt[4]{x}}+1}d\frac{\sqrt[4]{3-2x}}{\sqrt[4]{2}\sqrt[4]{x}}}{2\sqrt{2}}\right)+\frac{1}{2}\left(\frac{\arctan\left(\frac{\sqrt[4]{2}}{}\right.}{}\right.\right.$$

$$\Big\downarrow \text{25}$$

$$-22^{3/4}\left(\frac{1}{2}\left(-\frac{\int \frac{\sqrt{2}\left(\frac{\sqrt[4]{2}\sqrt[4]{3-2x}}{\sqrt[4]{x}}+1\right)}{\sqrt{3-2x}+\frac{\sqrt[4]{2}\sqrt[4]{3-2x}}{\sqrt[4]{x}}+1}d\frac{\sqrt[4]{3-2x}}{\sqrt[4]{2}\sqrt[4]{x}}}{2\sqrt{2}}-\frac{\int \frac{\sqrt{2}-\frac{2^{3/4}\sqrt[4]{3-2x}}{\sqrt[4]{x}}}{\sqrt{3-2x}-\frac{\sqrt[4]{2}\sqrt[4]{3-2x}}{\sqrt[4]{x}}+1}d\frac{\sqrt[4]{3-2x}}{\sqrt[4]{2}\sqrt[4]{x}}}{2\sqrt{2}}\right)+\frac{1}{2}\left(\frac{\arctan\left(\frac{\sqrt[4]{2}\sqrt[4]{}}{}\right.}{\sqrt{}}\right.\right.$$

$$\Big\downarrow \text{27}$$

$$-22^{3/4}\left(\frac{1}{2}\left(-\frac{1}{2}\int \frac{\frac{\sqrt[4]{2}\sqrt[4]{3-2x}}{\sqrt[4]{x}}+1}{\sqrt{3-2x}+\frac{\sqrt[4]{2}\sqrt[4]{3-2x}}{\sqrt[4]{x}}+1}d\frac{\sqrt[4]{3-2x}}{\sqrt[4]{2}\sqrt[4]{x}}-\frac{\int \frac{\sqrt{2}-\frac{2^{3/4}\sqrt[4]{3-2x}}{\sqrt[4]{x}}}{\sqrt{3-2x}-\frac{\sqrt[4]{2}\sqrt[4]{3-2x}}{\sqrt[4]{x}}+1}d\frac{\sqrt[4]{3-2x}}{\sqrt[4]{2}\sqrt[4]{x}}}{2\sqrt{2}}\right)+\frac{1}{2}\left(\frac{\text{arctan}}{}\right.\right.$$

$$\Big\downarrow \text{1103}$$

$$-22^{3/4}\left(\frac{1}{2}\left(\frac{\arctan\left(\frac{\sqrt[4]{2}\sqrt[4]{3-2x}}{\sqrt[4]{x}}+1\right)}{\sqrt{2}}-\frac{\arctan\left(1-\frac{\sqrt[4]{2}\sqrt[4]{3-2x}}{\sqrt[4]{x}}\right)}{\sqrt{2}}\right)+\frac{1}{2}\left(\frac{\log\left(\sqrt{3-2x}-\frac{\sqrt[4]{2}\sqrt[4]{3-2x}}{\sqrt[4]{x}}+\right.}{2\sqrt{2}}\right.\right.$$

```
Int[1/((3 - 2*x)^(1/4)*x^(3/4)),x]
```

```
-2*2^(3/4)*((-(ArcTan[1 - (2^(1/4)*(3 - 2*x)^(1/4))/x^(1/4)]/Sqrt[2]) + Ar
cTan[1 + (2^(1/4)*(3 - 2*x)^(1/4))/x^(1/4)]/Sqrt[2])/2 + (Log[1 + Sqrt[3 -
 2*x] - (2^(1/4)*(3 - 2*x)^(1/4))/x^(1/4)]/(2*Sqrt[2]) - Log[1 + Sqrt[3 -
2*x] + (2^(1/4)*(3 - 2*x)^(1/4))/x^(1/4)]/(2*Sqrt[2]))/2)
```

Defintions of rubi rules used

```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Simp[p/b   Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
 d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && Lt
Q[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntL
inearQ[a, b, c, d, m, n, x]
```

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

```
Int[(x_)^2/((a_) + (b_.)*(x_)^4), x_Symbol] :> With[{r = Numerator[Rt[a/b,
2]], s = Denominator[Rt[a/b, 2]]}, Simp[1/(2*s)   Int[(r + s*x^2)/(a + b*x^
4), x], x] - Simp[1/(2*s)   Int[(r - s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{
a, b}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]]
 && AtomQ[SplitProduct[SumBaseQ, b]]))
```

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[a^(p + (m +
 1)/n)   Subst[Int[x^m/(1 - b*x^n)^(p + (m + 1)/n + 1), x], x, x/(a + b*x^n
)^(1/n)], x] /; FreeQ[{a, b}, x] && IGtQ[n, 0] && LtQ[-1, p, 0] && NeQ[p, -
2^(-1)] && IntegersQ[m, p + (m + 1)/n]
```

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Simp[-2/b   Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b
)], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Fre
eQ[{a, b, c}, x]
```

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
2*(d/e), 2]}, Simp[e/(2*c)   Int[1/Simp[d/e + q*x + x^2, x], x], x] + Simp[
e/(2*c)   Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x]
 && EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
-2*(d/e), 2]}, Simp[e/(2*c*q)   Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
 x] + Simp[e/(2*c*q)   Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; F
reeQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Maple [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 3.

Time = 0.33 (sec) , antiderivative size = 18, normalized size of antiderivative = 0.16

method	result	size
meijerg	$\frac{4x^{\frac{1}{4}}3^{\frac{3}{4}}\,\mathrm{hypergeom}\left(\left[\frac{1}{4},\frac{1}{4}\right],\left[\frac{5}{4}\right],\frac{2x}{3}\right)}{3}$	18

```
int(1/(3-2*x)^(1/4)/x^(3/4),x,method=_RETURNVERBOSE)
```

```
4/3*x^(1/4)*3^(3/4)*hypergeom([1/4,1/4],[5/4],2/3*x)
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 170 vs. 2(80) = 160.

Time = 0.10 (sec) , antiderivative size = 170, normalized size of antiderivative = 1.52

$$\begin{aligned}\int \frac{1}{\sqrt[4]{3-2x}x^{3/4}}\,dx =& -\frac{1}{4}\cdot 8^{\frac{3}{4}}\arctan\left(\frac{8^{\frac{1}{4}}x^{\frac{1}{4}}(-2\,x+3)^{\frac{3}{4}}+2\,x-3}{2\,x-3}\right)\\&-\frac{1}{4}\cdot 8^{\frac{3}{4}}\arctan\left(\frac{8^{\frac{1}{4}}x^{\frac{1}{4}}(-2\,x+3)^{\frac{3}{4}}-2\,x+3}{2\,x-3}\right)-\frac{1}{8}\\&\cdot 8^{\frac{3}{4}}\log\left(\frac{8^{\frac{3}{4}}x^{\frac{1}{4}}(-2\,x+3)^{\frac{3}{4}}+2\,\sqrt{2}(2\,x-3)-4\,\sqrt{x}\sqrt{-2\,x+3}}{2\,x-3}\right)+\frac{1}{8}\\&\cdot 8^{\frac{3}{4}}\log\left(-\frac{8^{\frac{3}{4}}x^{\frac{1}{4}}(-2\,x+3)^{\frac{3}{4}}-2\,\sqrt{2}(2\,x-3)+4\,\sqrt{x}\sqrt{-2\,x+3}}{2\,x-3}\right)\end{aligned}$$

```
integrate(1/(3-2*x)^(1/4)/x^(3/4),x, algorithm="fricas")
```

```
-1/4*8^(3/4)*arctan((8^(1/4)*x^(1/4)*(-2*x + 3)^(3/4) + 2*x - 3)/(2*x - 3)
) - 1/4*8^(3/4)*arctan((8^(1/4)*x^(1/4)*(-2*x + 3)^(3/4) - 2*x + 3)/(2*x -
 3)) - 1/8*8^(3/4)*log((8^(3/4)*x^(1/4)*(-2*x + 3)^(3/4) + 2*sqrt(2)*(2*x
- 3) - 4*sqrt(x)*sqrt(-2*x + 3))/(2*x - 3)) + 1/8*8^(3/4)*log(-(8^(3/4)*x^
(1/4)*(-2*x + 3)^(3/4) - 2*sqrt(2)*(2*x - 3) + 4*sqrt(x)*sqrt(-2*x + 3))/(
2*x - 3))
```

Sympy [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.75 (sec) , antiderivative size = 39, normalized size of antiderivative = 0.35

$$\int \frac{1}{\sqrt[4]{3-2x}x^{3/4}}\,dx = \frac{3^{\frac{3}{4}}\sqrt[4]{x}\Gamma\left(\frac{1}{4}\right){}_{2}F_{1}\left(\left.\begin{matrix}\frac{1}{4},\frac{1}{4}\\ \frac{5}{4}\end{matrix}\right|\frac{2xe^{2i\pi}}{3}\right)}{3\Gamma\left(\frac{5}{4}\right)}$$

```
integrate(1/(3-2*x)**(1/4)/x**(3/4),x)
```

```
3**(3/4)*x**(1/4)*gamma(1/4)*hyper((1/4, 1/4), (5/4,), 2*x*exp_polar(2*I*p
i)/3)/(3*gamma(5/4))
```

Maxima [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 126, normalized size of antiderivative = 1.12

$$\begin{aligned}\int \frac{1}{\sqrt[4]{3-2x}x^{3/4}}\,dx &= -2^{\frac{1}{4}}\arctan\left(\frac{1}{2}\cdot 2^{\frac{1}{4}}\left(2^{\frac{3}{4}}+\frac{2\,(-2\,x+3)^{\frac{1}{4}}}{x^{\frac{1}{4}}}\right)\right)\\ &\quad - 2^{\frac{1}{4}}\arctan\left(-\frac{1}{2}\cdot 2^{\frac{1}{4}}\left(2^{\frac{3}{4}}-\frac{2\,(-2\,x+3)^{\frac{1}{4}}}{x^{\frac{1}{4}}}\right)\right)+\frac{1}{2}\\ &\quad \cdot 2^{\frac{1}{4}}\log\left(\frac{2^{\frac{3}{4}}(-2\,x+3)^{\frac{1}{4}}}{x^{\frac{1}{4}}}+\sqrt{2}+\frac{\sqrt{-2\,x+3}}{\sqrt{x}}\right)-\frac{1}{2}\\ &\quad \cdot 2^{\frac{1}{4}}\log\left(-\frac{2^{\frac{3}{4}}(-2\,x+3)^{\frac{1}{4}}}{x^{\frac{1}{4}}}+\sqrt{2}+\frac{\sqrt{-2\,x+3}}{\sqrt{x}}\right)\end{aligned}$$

```
integrate(1/(3-2*x)^(1/4)/x^(3/4),x, algorithm="maxima")
```

```
-2^(1/4)*arctan(1/2*2^(1/4)*(2^(3/4) + 2*(-2*x + 3)^(1/4)/x^(1/4))) - 2^(1
/4)*arctan(-1/2*2^(1/4)*(2^(3/4) - 2*(-2*x + 3)^(1/4)/x^(1/4))) + 1/2*2^(1
/4)*log(2^(3/4)*(-2*x + 3)^(1/4)/x^(1/4) + sqrt(2) + sqrt(-2*x + 3)/sqrt(x
)) - 1/2*2^(1/4)*log(-2^(3/4)*(-2*x + 3)^(1/4)/x^(1/4) + sqrt(2) + sqrt(-2
*x + 3)/sqrt(x))
```

Giac [F]

$$\int \frac{1}{\sqrt[4]{3-2x}x^{3/4}}\,dx = \int \frac{1}{x^{\frac{3}{4}}(-2\,x+3)^{\frac{1}{4}}}\,dx$$

```
integrate(1/(3-2*x)^(1/4)/x^(3/4),x, algorithm="giac")
```

```
integrate(1/(x^(3/4)*(-2*x + 3)^(1/4)), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{\sqrt[4]{3-2x}x^{3/4}}\,dx = \int \frac{1}{x^{3/4}\,(3-2\,x)^{1/4}}\,dx$$

```
int(1/(x^(3/4)*(3 - 2*x)^(1/4)),x)
```

```
int(1/(x^(3/4)*(3 - 2*x)^(1/4)), x)
```

Reduce [F]

$$\int \frac{1}{\sqrt[4]{3-2x}x^{3/4}}\,dx = \int \frac{1}{x^{\frac{3}{4}}\left(-2x+3\right)^{\frac{1}{4}}}dx$$

```
int(1/(3-2*x)^(1/4)/x^(3/4),x)
```

```
int(1/(x**(3/4)*( - 2*x + 3)**(1/4)),x)
```

3.216 $\int \frac{1}{\sqrt{x}\sqrt[4]{3x-2x^2}}\,dx$

Optimal result

Integrand size = 19, antiderivative size = 112

$$\int \frac{1}{\sqrt{x}\sqrt[4]{3x-2x^2}}\,dx = -\sqrt[4]{2}\arctan\left(1-\frac{2^{3/4}\sqrt[4]{x}}{\sqrt[4]{3-2x}}\right)+\sqrt[4]{2}\arctan\left(1+\frac{2^{3/4}\sqrt[4]{x}}{\sqrt[4]{3-2x}}\right)+\sqrt[4]{2}\operatorname{arctanh}\left(\frac{2^{3/4}\sqrt[4]{x}}{\left(1+\frac{\sqrt{2}\sqrt{x}}{\sqrt{3-2x}}\right)\sqrt[4]{3-2x}}\right)$$

```
2^(1/4)*arctan(-1+2^(3/4)*x^(1/4)/(3-2*x)^(1/4))+2^(1/4)*arctan(1+2^(3/4)*
x^(1/4)/(3-2*x)^(1/4))+2^(1/4)*arctanh(2^(3/4)*x^(1/4)/(1+2^(1/2)*x^(1/2)/
(3-2*x)^(1/2))/(3-2*x)^(1/4))
```

Mathematica [A] (verified)

Time = 0.21 (sec) , antiderivative size = 75, normalized size of antiderivative = 0.67

$$\int \frac{1}{\sqrt{x}\sqrt[4]{3x-2x^2}}\,dx = \frac{2^{3/4}\sqrt[4]{x}\sqrt[4]{-3+2x}\left(\arctan\left(\frac{\sqrt[4]{2}\sqrt[4]{x}}{\sqrt[4]{-3+2x}}\right)+\operatorname{arctanh}\left(\frac{\sqrt[4]{2}\sqrt[4]{x}}{\sqrt[4]{-3+2x}}\right)\right)}{\sqrt[4]{-x(-3+2x)}}$$

```
Integrate[1/(Sqrt[x]*(3*x - 2*x^2)^(1/4)),x]
```

```
(2^(3/4)*x^(1/4)*(-3 + 2*x)^(1/4)*(ArcTan[(2^(1/4)*x^(1/4))/(-3 + 2*x)^(1/
4)] + ArcTanh[(2^(1/4)*x^(1/4))/(-3 + 2*x)^(1/4)]))/(-(x*(-3 + 2*x)))^(1/4
)
```

Rubi [A] (warning: unable to verify)

Time = 0.61 (sec) , antiderivative size = 188, normalized size of antiderivative = 1.68, number of steps used = 13, number of rules used = 12, $\frac{\text{number of rules}}{\text{integrand size}} = 0.632$, Rules used = {1137, 73, 27, 854, 826, 1476, 1082, 217, 1479, 25, 27, 1103}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{\sqrt{x}\sqrt[4]{3x-2x^2}}dx$$

$$\Big\downarrow \text{1137}$$

$$\frac{\sqrt[4]{3-2x}\sqrt[4]{x}\int \frac{1}{\sqrt[4]{3-2x}x^{3/4}}dx}{\sqrt[4]{3x-2x^2}}$$

$$\Big\downarrow \text{73}$$

$$-\frac{2\sqrt[4]{3-2x}\sqrt[4]{x}\int \frac{\sqrt{3-2x}}{x^{3/4}}d\sqrt[4]{3-2x}}{\sqrt[4]{3x-2x^2}}$$

$$\Big\downarrow \text{27}$$

$$-\frac{2\ 2^{3/4}\sqrt[4]{3-2x}\sqrt[4]{x}\int \frac{\sqrt{3-2x}}{2^{3/4}x^{3/4}}d\sqrt[4]{3-2x}}{\sqrt[4]{3x-2x^2}}$$

$$\Big\downarrow \text{854}$$

$$-\frac{2\ 2^{3/4}\sqrt[4]{3-2x}\sqrt[4]{x}\int \frac{\sqrt{3-2x}}{4-2x}d\frac{\sqrt[4]{3-2x}}{\sqrt[4]{2}\sqrt[4]{x}}}{\sqrt[4]{3x-2x^2}}$$

$$\Big\downarrow \text{826}$$

$$-\frac{2\,2^{3/4}\sqrt[4]{3-2x}\sqrt[4]{x}\left(\frac{1}{2}\int\frac{\sqrt{3-2x}+1}{4-2x}d\frac{\sqrt[4]{3-2x}}{\sqrt[4]{2}\sqrt[4]{x}}-\frac{1}{2}\int\frac{1-\sqrt{3-2x}}{4-2x}d\frac{\sqrt[4]{3-2x}}{\sqrt[4]{2}\sqrt[4]{x}}\right)}{\sqrt[4]{3x-2x^2}}$$

$$\Big\downarrow \text{1476}$$

$$-\frac{2\,2^{3/4}\sqrt[4]{3-2x}\sqrt[4]{x}\left(\frac{1}{2}\left(\frac{1}{2}\int\frac{1}{\sqrt{3-2x}-\frac{\sqrt[4]{2}\sqrt[4]{3-2x}}{\sqrt[4]{x}}+1}d\frac{\sqrt[4]{3-2x}}{\sqrt[4]{2}\sqrt[4]{x}}+\frac{1}{2}\int\frac{1}{\sqrt{3-2x}+\frac{\sqrt[4]{2}\sqrt[4]{3-2x}}{\sqrt[4]{x}}+1}d\frac{\sqrt[4]{3-2x}}{\sqrt[4]{2}\sqrt[4]{x}}\right)-\frac{1}{2}\int\right.}{\sqrt[4]{3x-2x^2}}$$

$$\Big\downarrow \text{1082}$$

$$-\frac{2\,2^{3/4}\sqrt[4]{3-2x}\sqrt[4]{x}\left(\frac{1}{2}\left(\frac{\int\frac{1}{-\sqrt{3-2x}-1}d\left(1-\frac{\sqrt[4]{2}\sqrt[4]{3-2x}}{\sqrt[4]{x}}\right)}{\sqrt{2}}-\frac{\int\frac{1}{-\sqrt{3-2x}-1}d\left(\frac{\sqrt[4]{2}\sqrt[4]{3-2x}}{\sqrt[4]{x}}+1\right)}{\sqrt{2}}\right)-\frac{1}{2}\int\frac{1-\sqrt{3-2x}}{4-2x}d\frac{\sqrt[4]{3-}}{\sqrt[4]{2}}\right.}{\sqrt[4]{3x-2x^2}}$$

$$\Big\downarrow \text{217}$$

$$-\frac{2\,2^{3/4}\sqrt[4]{3-2x}\sqrt[4]{x}\left(\frac{1}{2}\left(\frac{\arctan\left(\frac{\sqrt[4]{2}\sqrt[4]{3-2x}}{\sqrt[4]{x}}+1\right)}{\sqrt{2}}-\frac{\arctan\left(1-\frac{\sqrt[4]{2}\sqrt[4]{3-2x}}{\sqrt[4]{x}}\right)}{\sqrt{2}}\right)-\frac{1}{2}\int\frac{1-\sqrt{3-2x}}{4-2x}d\frac{\sqrt[4]{3-2x}}{\sqrt[4]{2}\sqrt[4]{x}}\right)}{\sqrt[4]{3x-2x^2}}$$

$$\Big\downarrow \text{1479}$$

$$-\frac{2\,2^{3/4}\sqrt[4]{3-2x}\sqrt[4]{x}\left(\frac{1}{2}\left(\frac{\int-\frac{\sqrt{2}\left(\frac{\sqrt[4]{2}\sqrt[4]{3-2x}}{\sqrt[4]{x}}+1\right)}{\sqrt{3-2x}+\frac{\sqrt[4]{2}\sqrt[4]{3-2x}}{\sqrt[4]{x}}+1}d\frac{\sqrt[4]{3-2x}}{\sqrt[4]{2}\sqrt[4]{x}}}{2\sqrt{2}}+\frac{\int-\frac{\sqrt{2}-\frac{2^{3/4}\sqrt[4]{3-2x}}{\sqrt[4]{x}}}{\sqrt{3-2x}-\frac{\sqrt[4]{2}\sqrt[4]{3-2x}}{\sqrt[4]{x}}+1}d\frac{\sqrt[4]{3-2x}}{\sqrt[4]{2}\sqrt[4]{x}}}{2\sqrt{2}}\right)+\frac{1}{2}\left(\frac{\text{arcta}}{}\right.\right.}{\sqrt[4]{3x-2x^2}}$$

$$\Big\downarrow \text{25}$$

$$-\frac{2\,2^{3/4}\sqrt[4]{3-2x}\sqrt[4]{x}\left(\frac{1}{2}\left(-\frac{\int \frac{\sqrt{2}\left(\frac{\sqrt[4]{2}\sqrt[4]{3-2x}}{\sqrt[4]{x}}+1\right)}{\sqrt{3-2x}+\frac{\sqrt[4]{2}\sqrt[4]{3-2x}}{\sqrt[4]{x}}+1}d\frac{\sqrt[4]{3-2x}}{\sqrt[4]{2}\sqrt[4]{x}}}{2\sqrt{2}}-\frac{\int \frac{\sqrt{2}-\frac{2^{3/4}\sqrt[4]{3-2x}}{\sqrt[4]{x}}}{\sqrt{3-2x}-\frac{\sqrt[4]{2}\sqrt[4]{3-2x}}{\sqrt[4]{x}}+1}d\frac{\sqrt[4]{3-2x}}{\sqrt[4]{2}\sqrt[4]{x}}}{2\sqrt{2}}\right)+\frac{1}{2}\left(\frac{\arctan\cdots}{\cdots}\right.\right.}{\sqrt[4]{3x-2x^2}}$$

$$\Big\downarrow \; 27$$

$$-\frac{2\,2^{3/4}\sqrt[4]{3-2x}\sqrt[4]{x}\left(\frac{1}{2}\left(-\frac{1}{2}\int \frac{\frac{\sqrt[4]{2}\sqrt[4]{3-2x}}{\sqrt[4]{x}}+1}{\sqrt{3-2x}+\frac{\sqrt[4]{2}\sqrt[4]{3-2x}}{\sqrt[4]{x}}+1}d\frac{\sqrt[4]{3-2x}}{\sqrt[4]{2}\sqrt[4]{x}}-\frac{\int \frac{\sqrt{2}-\frac{2^{3/4}\sqrt[4]{3-2x}}{\sqrt[4]{x}}}{\sqrt{3-2x}-\frac{\sqrt[4]{2}\sqrt[4]{3-2x}}{\sqrt[4]{x}}+1}d\frac{\sqrt[4]{3-2x}}{\sqrt[4]{2}\sqrt[4]{x}}}{2\sqrt{2}}\right)+\frac{1}{2}\left(\frac{\mathrm{ar}\cdots}{\cdots}\right.\right.}{\sqrt[4]{3x-2x^2}}$$

$$\Big\downarrow \; 1103$$

$$-\frac{2\,2^{3/4}\sqrt[4]{3-2x}\sqrt[4]{x}\left(\frac{1}{2}\left(\frac{\arctan\left(\frac{\sqrt[4]{2}\sqrt[4]{3-2x}}{\sqrt[4]{x}}+1\right)}{\sqrt{2}}-\frac{\arctan\left(1-\frac{\sqrt[4]{2}\sqrt[4]{3-2x}}{\sqrt[4]{x}}\right)}{\sqrt{2}}\right)+\frac{1}{2}\left(\frac{\log\left(\sqrt{3-2x}-\frac{\sqrt[4]{2}\sqrt[4]{3-2x}}{\sqrt[4]{x}}+1\right)}{2\sqrt{2}}\cdots\right.\right.}{\sqrt[4]{3x-2x^2}}$$

```
Int[1/(Sqrt[x]*(3*x - 2*x^2)^(1/4)),x]
```

```
(-2*2^(3/4)*(3 - 2*x)^(1/4)*x^(1/4)*((-(ArcTan[1 - (2^(1/4)*(3 - 2*x)^(1/4
))/x^(1/4)]/Sqrt[2]) + ArcTan[1 + (2^(1/4)*(3 - 2*x)^(1/4))/x^(1/4)]/Sqrt[
2])/2 + (Log[1 + Sqrt[3 - 2*x] - (2^(1/4)*(3 - 2*x)^(1/4))/x^(1/4)]/(2*Sqr
t[2]) - Log[1 + Sqrt[3 - 2*x] + (2^(1/4)*(3 - 2*x)^(1/4))/x^(1/4)]/(2*Sqrt
[2]))/2))/(3*x - 2*x^2)^(1/4)
```

Defintions of rubi rules used

```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Simp[p/b   Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
 d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && Lt
Q[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntL
inearQ[a, b, c, d, m, n, x]
```

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

```
Int[(x_)^2/((a_) + (b_.)*(x_)^4), x_Symbol] :> With[{r = Numerator[Rt[a/b,
2]], s = Denominator[Rt[a/b, 2]]}, Simp[1/(2*s)   Int[(r + s*x^2)/(a + b*x^
4), x], x] - Simp[1/(2*s)   Int[(r - s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{
a, b}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]]
 && AtomQ[SplitProduct[SumBaseQ, b]]))
```

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[a^(p + (m +
 1)/n)   Subst[Int[x^m/(1 - b*x^n)^(p + (m + 1)/n + 1), x], x, x/(a + b*x^n
)^(1/n)], x] /; FreeQ[{a, b}, x] && IGtQ[n, 0] && LtQ[-1, p, 0] && NeQ[p, -
2^(-1)] && IntegersQ[m, p + (m + 1)/n]
```

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Simp[-2/b   Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b
)], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Fre
eQ[{a, b, c}, x]
```

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

```
Int[((e_.)*(x_))^(m_)*((b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(
e*x)^m*((b*x + c*x^2)^p/(x^(m + p)*(b + c*x)^p))   Int[x^(m + p)*(b + c*x)^
p, x], x] /; FreeQ[{b, c, e, m}, x]
```

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
2*(d/e), 2]}, Simp[e/(2*c)   Int[1/Simp[d/e + q*x + x^2, x], x], x] + Simp[
e/(2*c)   Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x]
 && EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
-2*(d/e), 2]}, Simp[e/(2*c*q)   Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
 x] + Simp[e/(2*c*q)   Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; F
reeQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Maple [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 3.

Time = 0.33 (sec) , antiderivative size = 18, normalized size of antiderivative = 0.16

method	result	size
meijerg	$\frac{4x^{\frac{1}{4}}3^{\frac{3}{4}}\operatorname{hypergeom}\left(\left[\frac{1}{4},\frac{1}{4}\right],\left[\frac{5}{4}\right],\frac{2x}{3}\right)}{3}$	18

```
int(1/x^(1/2)/(-2*x^2+3*x)^(1/4),x,method=_RETURNVERBOSE)
```

```
4/3*x^(1/4)*3^(3/4)*hypergeom([1/4,1/4],[5/4],2/3*x)
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 224 vs. $2(80) = 160$.

Time = 0.10 (sec) , antiderivative size = 224, normalized size of antiderivative = 2.00

$$\int \frac{1}{\sqrt{x}\sqrt[4]{3x-2x^2}}\,dx$$
$$= -\frac{1}{4}\cdot 8^{\frac{3}{4}}\arctan\left(\frac{2\,x^2+8^{\frac{1}{4}}(-2\,x^2+3\,x)^{\frac{3}{4}}\sqrt{x}-3\,x}{2\,x^2-3\,x}\right)-\frac{1}{4}$$
$$\cdot 8^{\frac{3}{4}}\arctan\left(-\frac{2\,x^2-8^{\frac{1}{4}}(-2\,x^2+3\,x)^{\frac{3}{4}}\sqrt{x}-3\,x}{2\,x^2-3\,x}\right)-\frac{1}{8}$$
$$\cdot 8^{\frac{3}{4}}\log\left(\frac{8^{\frac{3}{4}}(-2\,x^2+3\,x)^{\frac{3}{4}}\sqrt{x}+2\,\sqrt{2}(2\,x^2-3\,x)-4\,\sqrt{-2\,x^2+3\,x}x}{2\,x^2-3\,x}\right)+\frac{1}{8}$$
$$\cdot 8^{\frac{3}{4}}\log\left(-\frac{8^{\frac{3}{4}}(-2\,x^2+3\,x)^{\frac{3}{4}}\sqrt{x}-2\,\sqrt{2}(2\,x^2-3\,x)+4\,\sqrt{-2\,x^2+3\,x}x}{2\,x^2-3\,x}\right)$$

```
integrate(1/x^(1/2)/(-2*x^2+3*x)^(1/4),x, algorithm="fricas")
```

```
-1/4*8^(3/4)*arctan((2*x^2 + 8^(1/4)*(-2*x^2 + 3*x)^(3/4)*sqrt(x) - 3*x)/(
2*x^2 - 3*x)) - 1/4*8^(3/4)*arctan(-(2*x^2 - 8^(1/4)*(-2*x^2 + 3*x)^(3/4)*
sqrt(x) - 3*x)/(2*x^2 - 3*x)) - 1/8*8^(3/4)*log((8^(3/4)*(-2*x^2 + 3*x)^(3
/4)*sqrt(x) + 2*sqrt(2)*(2*x^2 - 3*x) - 4*sqrt(-2*x^2 + 3*x)*x)/(2*x^2 - 3
*x)) + 1/8*8^(3/4)*log(-(8^(3/4)*(-2*x^2 + 3*x)^(3/4)*sqrt(x) - 2*sqrt(2)*
(2*x^2 - 3*x) + 4*sqrt(-2*x^2 + 3*x)*x)/(2*x^2 - 3*x))
```

Sympy [F]

$$\int \frac{1}{\sqrt{x}\sqrt[4]{3x-2x^2}}\,dx = \int \frac{1}{\sqrt{x}\,\sqrt[4]{-x\,(2x-3)}}\,dx$$

```
integrate(1/x**(1/2)/(-2*x**2+3*x)**(1/4),x)
```

```
Integral(1/(sqrt(x)*(-x*(2*x - 3))**(1/4)), x)
```

Maxima [F]

$$\int \frac{1}{\sqrt{x}\sqrt[4]{3x-2x^2}}\,dx = \int \frac{1}{\left(-2\,x^2+3\,x\right)^{\frac{1}{4}}\sqrt{x}}\,dx$$

```
integrate(1/x^(1/2)/(-2*x^2+3*x)^(1/4),x, algorithm="maxima")
```

```
integrate(1/((-2*x^2 + 3*x)^(1/4)*sqrt(x)), x)
```

Giac [A] (verification not implemented)

Time = 0.20 (sec) , antiderivative size = 118, normalized size of antiderivative = 1.05

$$\begin{aligned}\int \frac{1}{\sqrt{x}\sqrt[4]{3x-2x^2}}\,dx = & -2^{\frac{1}{4}}\arctan\left(\frac{1}{2}\cdot 2^{\frac{1}{4}}\left(2^{\frac{3}{4}}+2\left(\frac{3}{x}-2\right)^{\frac{1}{4}}\right)\right) \\ & -2^{\frac{1}{4}}\arctan\left(-\frac{1}{2}\cdot 2^{\frac{1}{4}}\left(2^{\frac{3}{4}}-2\left(\frac{3}{x}-2\right)^{\frac{1}{4}}\right)\right) \\ & +\frac{1}{2}\cdot 2^{\frac{1}{4}}\log\left(2^{\frac{3}{4}}\left(\frac{3}{x}-2\right)^{\frac{1}{4}}+\sqrt{2}+\sqrt{\frac{3}{x}-2}\right) \\ & -\frac{1}{2}\cdot 2^{\frac{1}{4}}\log\left(-2^{\frac{3}{4}}\left(\frac{3}{x}-2\right)^{\frac{1}{4}}+\sqrt{2}+\sqrt{\frac{3}{x}-2}\right)\end{aligned}$$

```
integrate(1/x^(1/2)/(-2*x^2+3*x)^(1/4),x, algorithm="giac")
```

```
-2^(1/4)*arctan(1/2*2^(1/4)*(2^(3/4) + 2*(3/x - 2)^(1/4))) - 2^(1/4)*arcta
n(-1/2*2^(1/4)*(2^(3/4) - 2*(3/x - 2)^(1/4))) + 1/2*2^(1/4)*log(2^(3/4)*(3
/x - 2)^(1/4) + sqrt(2) + sqrt(3/x - 2)) - 1/2*2^(1/4)*log(-2^(3/4)*(3/x -
 2)^(1/4) + sqrt(2) + sqrt(3/x - 2))
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{\sqrt{x}\sqrt[4]{3x-2x^2}}\,dx = \int \frac{1}{\sqrt{x}\,\left(3\,x-2\,x^2\right)^{1/4}}\,dx$$

```
int(1/(x^(1/2)*(3*x - 2*x^2)^(1/4)),x)
```

```
int(1/(x^(1/2)*(3*x - 2*x^2)^(1/4)), x)
```

Reduce [B] (verification not implemented)

Time = 0.43 (sec) , antiderivative size = 32, normalized size of antiderivative = 0.29

$$\int \frac{1}{\sqrt{x}\sqrt[4]{3x-2x^2}}\,dx = \frac{2x^{\frac{1}{4}}(-2x+3)^{\frac{1}{4}}\,(4x-6)}{\sqrt{-2x+3}\,(2x-3)}$$

```
int(1/x^(1/2)/(-2*x^2+3*x)^(1/4),x)
```

```
(2*x**(1/4)*( - 2*x + 3)**(1/4)*( - ( - 2*x + 3) + 2*x - 3))/(sqrt( - 2*x
+ 3)*(2*x - 3))
```

3.217 $\int (cx)^m (ax+bx^2)^3 \, dx$

Optimal result

Integrand size = 17, antiderivative size = 81

$$\int (cx)^m (ax+bx^2)^3 \, dx = \frac{a^3(cx)^{4+m}}{c^4(4+m)} + \frac{3a^2b(cx)^{5+m}}{c^5(5+m)} + \frac{3ab^2(cx)^{6+m}}{c^6(6+m)} + \frac{b^3(cx)^{7+m}}{c^7(7+m)}$$

```
a^3*(c*x)^(4+m)/c^4/(4+m)+3*a^2*b*(c*x)^(5+m)/c^5/(5+m)+3*a*b^2*(c*x)^(6+m
)/c^6/(6+m)+b^3*(c*x)^(7+m)/c^7/(7+m)
```

Mathematica [A] (verified)

Time = 0.07 (sec) , antiderivative size = 57, normalized size of antiderivative = 0.70

$$\int (cx)^m (ax+bx^2)^3 \, dx = x^4(cx)^m \left(\frac{a^3}{4+m} + \frac{3a^2bx}{5+m} + \frac{3ab^2x^2}{6+m} + \frac{b^3x^3}{7+m}\right)$$

```
Integrate[(c*x)^m*(a*x + b*x^2)^3,x]
```

```
x^4*(c*x)^m*(a^3/(4 + m) + (3*a^2*b*x)/(5 + m) + (3*a*b^2*x^2)/(6 + m) + (
b^3*x^3)/(7 + m))
```

Rubi [A] (verified)

Time = 0.39 (sec) , antiderivative size = 85, normalized size of antiderivative = 1.05, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.176, Rules used = {9, 53, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (ax+bx^2)^3 (cx)^m\,dx$$

$$\Big\downarrow \text{9}$$

$$\frac{\int (cx)^{m+3}(a+bx)^3dx}{c^3}$$

$$\Big\downarrow \text{53}$$

$$\frac{\int \left(a^3(cx)^{m+3}+\frac{3a^2b(cx)^{m+4}}{c}+\frac{3ab^2(cx)^{m+5}}{c^2}+\frac{b^3(cx)^{m+6}}{c^3}\right)dx}{c^3}$$

$$\Big\downarrow \text{2009}$$

$$\frac{\frac{a^3(cx)^{m+4}}{c(m+4)}+\frac{3a^2b(cx)^{m+5}}{c^2(m+5)}+\frac{3ab^2(cx)^{m+6}}{c^3(m+6)}+\frac{b^3(cx)^{m+7}}{c^4(m+7)}}{c^3}$$

```
Int[(c*x)^m*(a*x + b*x^2)^3,x]
```

```
((a^3*(c*x)^(4 + m))/(c*(4 + m)) + (3*a^2*b*(c*x)^(5 + m))/(c^2*(5 + m)) +
 (3*a*b^2*(c*x)^(6 + m))/(c^3*(6 + m)) + (b^3*(c*x)^(7 + m))/(c^4*(7 + m))
)/c^3
```

Defintions of rubi rules used

```
Int[(u_.)*(Px_)^(p_.)*((e_.)*(x_))^(m_.), x_Symbol] :> With[{r = Expon[Px,
x, Min]}, Simp[1/e^(p*r)   Int[u*(e*x)^(m + p*r)*ExpandToSum[Px/x^r, x]^p,
x], x] /; IGtQ[r, 0]] /; FreeQ[{e, m}, x] && PolyQ[Px, x] && IntegerQ[p] &&
  !MonomialQ[Px, x]
```

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && LeQ[7*m + 4*n + 4, 0])
|| LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 0.30 (sec) , antiderivative size = 82, normalized size of antiderivative = 1.01

method	result
norman	$\frac{a^3 x^4 e^{m\ln(cx)}}{4+m}+\frac{b^3 x^7 e^{m\ln(cx)}}{7+m}+\frac{3a\,b^2 x^6 e^{m\ln(cx)}}{6+m}+\frac{3a^2 b\,x^5 e^{m\ln(cx)}}{5+m}$
gosper	$\frac{(cx)^m\,(b^3m^3x^3+3a\,b^2m^3x^2+15b^3m^2x^3+3a^2b\,m^3x+48a\,b^2m^2x^2+74m\,x^3b^3+a^3m^3+51a^2b\,m^2x+249a\,b^2m\,x^2+120b^3x^3+1}{(7+m)(6+m)(5+m)(4+m)}$
risch	$\frac{(cx)^m\,(b^3m^3x^3+3a\,b^2m^3x^2+15b^3m^2x^3+3a^2b\,m^3x+48a\,b^2m^2x^2+74m\,x^3b^3+a^3m^3+51a^2b\,m^2x+249a\,b^2m\,x^2+120b^3x^3+1}{(7+m)(6+m)(5+m)(4+m)}$
orering	$\frac{(b^3m^3x^3+3a\,b^2m^3x^2+15b^3m^2x^3+3a^2b\,m^3x+48a\,b^2m^2x^2+74m\,x^3b^3+a^3m^3+51a^2b\,m^2x+249a\,b^2m\,x^2+120b^3x^3+18a^3m^2}{(7+m)(6+m)(5+m)(4+m)(bx+a)^3}$
parallelrisch	$\frac{x^7(cx)^m b^3m^3+15x^7(cx)^m b^3m^2+3x^6(cx)^m a\,b^2m^3+74x^7(cx)^m b^3m+48x^6(cx)^m a\,b^2m^2+3x^5(cx)^m a^2b\,m^3+120x^7(cx)^m b^3+}{}$

```
int((c*x)^m*(b*x^2+a*x)^3,x,method=_RETURNVERBOSE)
```

```
a^3/(4+m)*x^4*exp(m*ln(c*x))+b^3/(7+m)*x^7*exp(m*ln(c*x))+3*a*b^2/(6+m)*x^
6*exp(m*ln(c*x))+3*a^2*b/(5+m)*x^5*exp(m*ln(c*x))
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 161, normalized size of antiderivative = 1.99

$$\int (cx)^m \left(ax+bx^2\right)^3 \, dx$$
$$= \frac{((b^3m^3 + 15\,b^3m^2 + 74\,b^3m + 120\,b^3)x^7 + 3\,(ab^2m^3 + 16\,ab^2m^2 + 83\,ab^2m + 140\,ab^2)x^6 + 3\,(a^2bm^3 + 17}{m^4 + 22\,m^3 + 179\,m^2 + 638\,m}$$

```
integrate((c*x)^m*(b*x^2+a*x)^3,x, algorithm="fricas")
```

```
((b^3*m^3 + 15*b^3*m^2 + 74*b^3*m + 120*b^3)*x^7 + 3*(a*b^2*m^3 + 16*a*b^2
*m^2 + 83*a*b^2*m + 140*a*b^2)*x^6 + 3*(a^2*b*m^3 + 17*a^2*b*m^2 + 94*a^2*
b*m + 168*a^2*b)*x^5 + (a^3*m^3 + 18*a^3*m^2 + 107*a^3*m + 210*a^3)*x^4)*(
c*x)^m/(m^4 + 22*m^3 + 179*m^2 + 638*m + 840)
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 711 vs. 2(73) = 146.

Time = 0.39 (sec) , antiderivative size = 711, normalized size of antiderivative = 8.78

$$\int (cx)^m \left(ax+bx^2\right)^3 \, dx = \text{Too large to display}$$

```
integrate((c*x)**m*(b*x**2+a*x)**3,x)
```

```
Piecewise(((-a**3/(3*x**3) - 3*a**2*b/(2*x**2) - 3*a*b**2/x + b**3*log(x))
/c**7, Eq(m, -7)), ((-a**3/(2*x**2) - 3*a**2*b/x + 3*a*b**2*log(x) + b**3*
x)/c**6, Eq(m, -6)), ((-a**3/x + 3*a**2*b*log(x) + 3*a*b**2*x + b**3*x**2/
2)/c**5, Eq(m, -5)), ((a**3*log(x) + 3*a**2*b*x + 3*a*b**2*x**2/2 + b**3*x
**3/3)/c**4, Eq(m, -4)), (a**3*m**3*x**4*(c*x)**m/(m**4 + 22*m**3 + 179*m*
*2 + 638*m + 840) + 18*a**3*m**2*x**4*(c*x)**m/(m**4 + 22*m**3 + 179*m**2
+ 638*m + 840) + 107*a**3*m*x**4*(c*x)**m/(m**4 + 22*m**3 + 179*m**2 + 638
*m + 840) + 210*a**3*x**4*(c*x)**m/(m**4 + 22*m**3 + 179*m**2 + 638*m + 84
0) + 3*a**2*b*m**3*x**5*(c*x)**m/(m**4 + 22*m**3 + 179*m**2 + 638*m + 840)
 + 51*a**2*b*m**2*x**5*(c*x)**m/(m**4 + 22*m**3 + 179*m**2 + 638*m + 840)
+ 282*a**2*b*m*x**5*(c*x)**m/(m**4 + 22*m**3 + 179*m**2 + 638*m + 840) + 5
04*a**2*b*x**5*(c*x)**m/(m**4 + 22*m**3 + 179*m**2 + 638*m + 840) + 3*a*b*
*2*m**3*x**6*(c*x)**m/(m**4 + 22*m**3 + 179*m**2 + 638*m + 840) + 48*a*b**
2*m**2*x**6*(c*x)**m/(m**4 + 22*m**3 + 179*m**2 + 638*m + 840) + 249*a*b**
2*m*x**6*(c*x)**m/(m**4 + 22*m**3 + 179*m**2 + 638*m + 840) + 420*a*b**2*x
**6*(c*x)**m/(m**4 + 22*m**3 + 179*m**2 + 638*m + 840) + b**3*m**3*x**7*(c
*x)**m/(m**4 + 22*m**3 + 179*m**2 + 638*m + 840) + 15*b**3*m**2*x**7*(c*x)
**m/(m**4 + 22*m**3 + 179*m**2 + 638*m + 840) + 74*b**3*m*x**7*(c*x)**m/(m
**4 + 22*m**3 + 179*m**2 + 638*m + 840) + 120*b**3*x**7*(c*x)**m/(m**4 + 2
2*m**3 + 179*m**2 + 638*m + 840), True))
```

Maxima [A] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 77, normalized size of antiderivative = 0.95

$$\int (cx)^m \left(ax+bx^2\right)^3 \, dx = \frac{b^3 c^m x^7 x^m}{m+7} + \frac{3\,ab^2 c^m x^6 x^m}{m+6} + \frac{3\,a^2 b c^m x^5 x^m}{m+5} + \frac{a^3 c^m x^4 x^m}{m+4}$$

```
integrate((c*x)^m*(b*x^2+a*x)^3,x, algorithm="maxima")
```

```
b^3*c^m*x^7*x^m/(m + 7) + 3*a*b^2*c^m*x^6*x^m/(m + 6) + 3*a^2*b*c^m*x^5*x^
m/(m + 5) + a^3*c^m*x^4*x^m/(m + 4)
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 264 vs. $2(81) = 162$.

Time = 0.23 (sec) , antiderivative size = 264, normalized size of antiderivative = 3.26

$$\int (cx)^m \left(ax+bx^2\right)^3 \, dx$$
$$= \frac{(cx)^m b^3 m^3 x^7 + 3\,(cx)^m ab^2 m^3 x^6 + 15\,(cx)^m b^3 m^2 x^7 + 3\,(cx)^m a^2 b m^3 x^5 + 48\,(cx)^m ab^2 m^2 x^6 + 74\,(cx)^m}{}$$

```
integrate((c*x)^m*(b*x^2+a*x)^3,x, algorithm="giac")
```

```
((c*x)^m*b^3*m^3*x^7 + 3*(c*x)^m*a*b^2*m^3*x^6 + 15*(c*x)^m*b^3*m^2*x^7 +
3*(c*x)^m*a^2*b*m^3*x^5 + 48*(c*x)^m*a*b^2*m^2*x^6 + 74*(c*x)^m*b^3*m*x^7
+ (c*x)^m*a^3*m^3*x^4 + 51*(c*x)^m*a^2*b*m^2*x^5 + 249*(c*x)^m*a*b^2*m*x^6
 + 120*(c*x)^m*b^3*x^7 + 18*(c*x)^m*a^3*m^2*x^4 + 282*(c*x)^m*a^2*b*m*x^5
+ 420*(c*x)^m*a*b^2*x^6 + 107*(c*x)^m*a^3*m*x^4 + 504*(c*x)^m*a^2*b*x^5 +
210*(c*x)^m*a^3*x^4)/(m^4 + 22*m^3 + 179*m^2 + 638*m + 840)
```

Mupad [B] (verification not implemented)

Time = 9.00 (sec) , antiderivative size = 171, normalized size of antiderivative = 2.11

$$\int (cx)^m \left(ax+bx^2\right)^3 \, dx = (c\,x)^m \left(\frac{a^3\,x^4\,\left(m^3+18\,m^2+107\,m+210\right)}{m^4+22\,m^3+179\,m^2+638\,m+840} + \frac{b^3\,x^7\,\left(m^3+15\,m^2+74\,m+120\right)}{m^4+22\,m^3+179\,m^2+638\,m+840} + \frac{3\,a\,b^2\,x^6\,\left(m^3+16\,m^2+83\,m+140\right)}{m^4+22\,m^3+179\,m^2+638\,m+840} + \frac{3\,a^2\,b\,x^5\,\left(m^3+17\,m^2+94\,m+168\right)}{m^4+22\,m^3+179\,m^2+638\,m+840}\right)$$

```
int((a*x + b*x^2)^3*(c*x)^m,x)
```

```
(c*x)^m*((a^3*x^4*(107*m + 18*m^2 + m^3 + 210))/(638*m + 179*m^2 + 22*m^3
+ m^4 + 840) + (b^3*x^7*(74*m + 15*m^2 + m^3 + 120))/(638*m + 179*m^2 + 22
*m^3 + m^4 + 840) + (3*a*b^2*x^6*(83*m + 16*m^2 + m^3 + 140))/(638*m + 179
*m^2 + 22*m^3 + m^4 + 840) + (3*a^2*b*x^5*(94*m + 17*m^2 + m^3 + 168))/(63
8*m + 179*m^2 + 22*m^3 + m^4 + 840))
```

Reduce [B] (verification not implemented)

Time = 0.24 (sec) , antiderivative size = 173, normalized size of antiderivative = 2.14

$$\int (cx)^m \left(ax+bx^2\right)^3 \, dx = \frac{x^m c^m x^4 (b^3 m^3 x^3 + 3a\,b^2 m^3 x^2 + 15 b^3 m^2 x^3 + 3a^2 b\,m^3 x + 48a\,b^2 m^2 x^2 + 74 b^3 m\,x^3 + a^3 m^3 + 51 a^2 b\,m^2 x +}{m^4 + 22m^3 + 179m^2 + 6}$$

```
int((c*x)^m*(b*x^2+a*x)^3,x)
```

```
(x**m*c**m*x**4*(a**3*m**3 + 18*a**3*m**2 + 107*a**3*m + 210*a**3 + 3*a**2
*b*m**3*x + 51*a**2*b*m**2*x + 282*a**2*b*m*x + 504*a**2*b*x + 3*a*b**2*m*
*3*x**2 + 48*a*b**2*m**2*x**2 + 249*a*b**2*m*x**2 + 420*a*b**2*x**2 + b**3
*m**3*x**3 + 15*b**3*m**2*x**3 + 74*b**3*m*x**3 + 120*b**3*x**3))/(m**4 +
22*m**3 + 179*m**2 + 638*m + 840)
```

3.218 $\int (cx)^m (ax+bx^2)^2 \, dx$

Optimal result

Integrand size = 17, antiderivative size = 58

$$\int (cx)^m (ax+bx^2)^2 \, dx = \frac{a^2(cx)^{3+m}}{c^3(3+m)} + \frac{2ab(cx)^{4+m}}{c^4(4+m)} + \frac{b^2(cx)^{5+m}}{c^5(5+m)}$$

```
a^2*(c*x)^(3+m)/c^3/(3+m)+2*a*b*(c*x)^(4+m)/c^4/(4+m)+b^2*(c*x)^(5+m)/c^5/
(5+m)
```

Mathematica [A] (verified)

Time = 0.05 (sec) , antiderivative size = 41, normalized size of antiderivative = 0.71

$$\int (cx)^m (ax+bx^2)^2 \, dx = x^3(cx)^m \left(\frac{a^2}{3+m} + \frac{2abx}{4+m} + \frac{b^2x^2}{5+m}\right)$$

```
Integrate[(c*x)^m*(a*x + b*x^2)^2,x]
```

```
x^3*(c*x)^m*(a^2/(3 + m) + (2*a*b*x)/(4 + m) + (b^2*x^2)/(5 + m))
```

Rubi [A] (verified)

Time = 0.34 (sec) , antiderivative size = 62, normalized size of antiderivative = 1.07, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.176, Rules used = {9, 53, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (ax+bx^2)^2 (cx)^m \, dx$$

$$\Big\downarrow \; 9$$

$$\frac{\int (cx)^{m+2}(a+bx)^2 dx}{c^2}$$

$$\Big\downarrow \; 53$$

$$\frac{\int \left(a^2(cx)^{m+2}+\frac{2ab(cx)^{m+3}}{c}+\frac{b^2(cx)^{m+4}}{c^2}\right)dx}{c^2}$$

$$\Big\downarrow \; 2009$$

$$\frac{\frac{a^2(cx)^{m+3}}{c(m+3)}+\frac{2ab(cx)^{m+4}}{c^2(m+4)}+\frac{b^2(cx)^{m+5}}{c^3(m+5)}}{c^2}$$

```
Int[(c*x)^m*(a*x + b*x^2)^2,x]
```

```
((a^2*(c*x)^(3 + m))/(c*(3 + m)) + (2*a*b*(c*x)^(4 + m))/(c^2*(4 + m)) + (
b^2*(c*x)^(5 + m))/(c^3*(5 + m)))/c^2
```

Defintions of rubi rules used

```
Int[(u_.)*(Px_)^(p_.)*((e_.)*(x_))^(m_.), x_Symbol] :> With[{r = Expon[Px,
x, Min]}, Simp[1/e^(p*r)   Int[u*(e*x)^(m + p*r)*ExpandToSum[Px/x^r, x]^p,
x], x] /; IGtQ[r, 0]] /; FreeQ[{e, m}, x] && PolyQ[Px, x] && IntegerQ[p] &&
 !MonomialQ[Px, x]
```

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && LeQ[7*m + 4*n + 4, 0])
|| LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 0.27 (sec) , antiderivative size = 59, normalized size of antiderivative = 1.02

method	result
norman	$\frac{a^{2} x^{3} {\mathrm e}^{m \ln \left(c x \right)}}{3+m}+\frac{b^{2} x^{5} {\mathrm e}^{m \ln \left(c x \right)}}{5+m}+\frac{2 a b \,x^{4} {\mathrm e}^{m \ln \left(c x \right)}}{4+m}$
gosper	$\frac{\left(c x \right)^{m} \left(b^{2} m^{2} x^{2}+2 a b \,m^{2} x +7 m \,x^{2} b^{2}+a^{2} m^{2}+16 m x a b +12 b^{2} x^{2}+9 a^{2} m +30 a b x +20 a^{2}\right) x^{3}}{\left(5+m \right) \left(4+m \right) \left(3+m \right)}$
risch	$\frac{\left(c x \right)^{m} \left(b^{2} m^{2} x^{2}+2 a b \,m^{2} x +7 m \,x^{2} b^{2}+a^{2} m^{2}+16 m x a b +12 b^{2} x^{2}+9 a^{2} m +30 a b x +20 a^{2}\right) x^{3}}{\left(5+m \right) \left(4+m \right) \left(3+m \right)}$
orering	$\frac{\left(b^{2} m^{2} x^{2}+2 a b \,m^{2} x +7 m \,x^{2} b^{2}+a^{2} m^{2}+16 m x a b +12 b^{2} x^{2}+9 a^{2} m +30 a b x +20 a^{2}\right) x \left(c x \right)^{m} \left(b \,x^{2}+a x \right)^{2}}{\left(5+m \right) \left(4+m \right) \left(3+m \right) \left(b x +a \right)^{2}}$
parallelrisch	$\frac{x^{5} \left(c x \right)^{m} b^{2} m^{2}+7 x^{5} \left(c x \right)^{m} b^{2} m +2 x^{4} \left(c x \right)^{m} a b \,m^{2}+12 x^{5} \left(c x \right)^{m} b^{2}+16 x^{4} \left(c x \right)^{m} a b m +x^{3} \left(c x \right)^{m} a^{2} m^{2}+30 x^{4} \left(c x \right)^{m} a b +9 x^{3} \left(c x \right)^{m} a}{\left(5+m \right) \left(4+m \right) \left(3+m \right)}$

```
int((c*x)^m*(b*x^2+a*x)^2,x,method=_RETURNVERBOSE)
```

```
a^2/(3+m)*x^3*exp(m*ln(c*x))+b^2/(5+m)*x^5*exp(m*ln(c*x))+2*a*b/(4+m)*x^4*
exp(m*ln(c*x))
```

Fricas [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 89, normalized size of antiderivative = 1.53

$$\int (cx)^m \left(ax+bx^2\right)^2 \, dx$$
$$= \frac{((b^2m^2 + 7\,b^2m + 12\,b^2)x^5 + 2\,(abm^2 + 8\,abm + 15\,ab)x^4 + (a^2m^2 + 9\,a^2m + 20\,a^2)x^3)(cx)^m}{m^3 + 12\,m^2 + 47\,m + 60}$$

```
integrate((c*x)^m*(b*x^2+a*x)^2,x, algorithm="fricas")
```

```
((b^2*m^2 + 7*b^2*m + 12*b^2)*x^5 + 2*(a*b*m^2 + 8*a*b*m + 15*a*b)*x^4 + (
a^2*m^2 + 9*a^2*m + 20*a^2)*x^3)*(c*x)^m/(m^3 + 12*m^2 + 47*m + 60)
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 330 vs. $2(51) = 102$.

Time = 0.29 (sec) , antiderivative size = 330, normalized size of antiderivative = 5.69

$$\int (cx)^m \left(ax + bx^2\right)^2 \, dx$$

$$= \begin{cases} \frac{-\frac{a^2}{2x^2} - \frac{2ab}{x} + b^2 \log(x)}{c^5} \\ \frac{-\frac{a^2}{x} + 2ab \log(x) + b^2 x}{c^4} \\ \frac{a^2 \log(x) + 2abx + \frac{b^2x^2}{2}}{c^3} \\ \frac{a^2m^2x^3(cx)^m}{m^3+12m^2+47m+60} + \frac{9a^2mx^3(cx)^m}{m^3+12m^2+47m+60} + \frac{20a^2x^3(cx)^m}{m^3+12m^2+47m+60} + \frac{2abm^2x^4(cx)^m}{m^3+12m^2+47m+60} + \frac{16abmx^4(cx)^m}{m^3+12m^2+47m+60} + \frac{30abx^4}{m^3+12m^2} \end{cases}$$

```
integrate((c*x)**m*(b*x**2+a*x)**2,x)
```

```
Piecewise(((-a**2/(2*x**2) - 2*a*b/x + b**2*log(x))/c**5, Eq(m, -5)), ((-a
**2/x + 2*a*b*log(x) + b**2*x)/c**4, Eq(m, -4)), ((a**2*log(x) + 2*a*b*x +
 b**2*x**2/2)/c**3, Eq(m, -3)), (a**2*m**2*x**3*(c*x)**m/(m**3 + 12*m**2 +
 47*m + 60) + 9*a**2*m*x**3*(c*x)**m/(m**3 + 12*m**2 + 47*m + 60) + 20*a**
2*x**3*(c*x)**m/(m**3 + 12*m**2 + 47*m + 60) + 2*a*b*m**2*x**4*(c*x)**m/(m
**3 + 12*m**2 + 47*m + 60) + 16*a*b*m*x**4*(c*x)**m/(m**3 + 12*m**2 + 47*m
 + 60) + 30*a*b*x**4*(c*x)**m/(m**3 + 12*m**2 + 47*m + 60) + b**2*m**2*x**
5*(c*x)**m/(m**3 + 12*m**2 + 47*m + 60) + 7*b**2*m*x**5*(c*x)**m/(m**3 + 1
2*m**2 + 47*m + 60) + 12*b**2*x**5*(c*x)**m/(m**3 + 12*m**2 + 47*m + 60),
True))
```

Maxima [A] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 55, normalized size of antiderivative = 0.95

$$\int (cx)^m \left(ax+bx^2\right)^2 \, dx = \frac{b^2c^mx^5x^m}{m+5} + \frac{2\,abc^mx^4x^m}{m+4} + \frac{a^2c^mx^3x^m}{m+3}$$

```
integrate((c*x)^m*(b*x^2+a*x)^2,x, algorithm="maxima")
```

```
b^2*c^m*x^5*x^m/(m + 5) + 2*a*b*c^m*x^4*x^m/(m + 4) + a^2*c^m*x^3*x^m/(m +
 3)
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 141 vs. $2(58) = 116$.

Time = 0.13 (sec) , antiderivative size = 141, normalized size of antiderivative = 2.43

$$\int (cx)^m \left(ax+bx^2\right)^2 \, dx$$
$$= \frac{(cx)^m\,b^2m^2x^5 + 2\,(cx)^m\,abm^2x^4 + 7\,(cx)^m\,b^2mx^5 + (cx)^m\,a^2m^2x^3 + 16\,(cx)^m\,abmx^4 + 12\,(cx)^m\,b^2x^5 + 9}{m^3 + 12\,m^2 + 47\,m + 60}$$

```
integrate((c*x)^m*(b*x^2+a*x)^2,x, algorithm="giac")
```

```
((c*x)^m*b^2*m^2*x^5 + 2*(c*x)^m*a*b*m^2*x^4 + 7*(c*x)^m*b^2*m*x^5 + (c*x)
^m*a^2*m^2*x^3 + 16*(c*x)^m*a*b*m*x^4 + 12*(c*x)^m*b^2*x^5 + 9*(c*x)^m*a^2
*m*x^3 + 30*(c*x)^m*a*b*x^4 + 20*(c*x)^m*a^2*x^3)/(m^3 + 12*m^2 + 47*m + 6
0)
```

Mupad [B] (verification not implemented)

Time = 9.20 (sec) , antiderivative size = 97, normalized size of antiderivative = 1.67

$$\int (cx)^m \left(ax+bx^2\right)^2 \, dx = (c\,x)^m \left(\frac{a^2\,x^3\,(m^2+9\,m+20)}{m^3+12\,m^2+47\,m+60} + \frac{b^2\,x^5\,(m^2+7\,m+12)}{m^3+12\,m^2+47\,m+60} + \frac{2\,a\,b\,x^4\,(m^2+8\,m+15)}{m^3+12\,m^2+47\,m+60}\right)$$

```
int((a*x + b*x^2)^2*(c*x)^m,x)
```

```
(c*x)^m*((a^2*x^3*(9*m + m^2 + 20))/(47*m + 12*m^2 + m^3 + 60) + (b^2*x^5*
(7*m + m^2 + 12))/(47*m + 12*m^2 + m^3 + 60) + (2*a*b*x^4*(8*m + m^2 + 15)
)/(47*m + 12*m^2 + m^3 + 60))
```

Reduce [B] (verification not implemented)

Time = 0.52 (sec) , antiderivative size = 90, normalized size of antiderivative = 1.55

$$\int (cx)^m \left(ax+bx^2\right)^2 \, dx = \frac{x^m c^m x^3 (b^2 m^2 x^2 + 2ab\,m^2 x + 7b^2 m\,x^2 + a^2 m^2 + 16abmx + 12b^2 x^2 + 9a^2 m + 30abx + 20a^2)}{m^3 + 12m^2 + 47m + 60}$$

```
int((c*x)^m*(b*x^2+a*x)^2,x)
```

```
(x**m*c**m*x**3*(a**2*m**2 + 9*a**2*m + 20*a**2 + 2*a*b*m**2*x + 16*a*b*m*
x + 30*a*b*x + b**2*m**2*x**2 + 7*b**2*m*x**2 + 12*b**2*x**2))/(m**3 + 12*
m**2 + 47*m + 60)
```

3.219 $\int (cx)^m (ax+bx^2)\ dx$

Optimal result

Integrand size = 15, antiderivative size = 35

$$\int (cx)^m (ax+bx^2)\ dx = \frac{a(cx)^{2+m}}{c^2(2+m)} + \frac{b(cx)^{3+m}}{c^3(3+m)}$$

```
a*(c*x)^(2+m)/c^2/(2+m)+b*(c*x)^(3+m)/c^3/(3+m)
```

Mathematica [A] (verified)

Time = 0.03 (sec) , antiderivative size = 25, normalized size of antiderivative = 0.71

$$\int (cx)^m (ax+bx^2)\ dx = x^2(cx)^m\left(\frac{a}{2+m} + \frac{bx}{3+m}\right)$$

```
Integrate[(c*x)^m*(a*x + b*x^2),x]
```

```
x^2*(c*x)^m*(a/(2 + m) + (b*x)/(3 + m))
```

Rubi [A] (verified)

Time = 0.30 (sec) , antiderivative size = 39, normalized size of antiderivative = 1.11, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.200$, Rules used = {9, 53, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \left(ax+bx^2\right)(cx)^m\,dx$$

$$\Big\downarrow \text{ 9}$$

$$\frac{\int (cx)^{m+1}(a+bx)dx}{c}$$

$$\Big\downarrow \text{ 53}$$

$$\frac{\int \left(a(cx)^{m+1}+\frac{b(cx)^{m+2}}{c}\right)dx}{c}$$

$$\Big\downarrow \text{ 2009}$$

$$\frac{\frac{a(cx)^{m+2}}{c(m+2)}+\frac{b(cx)^{m+3}}{c^2(m+3)}}{c}$$

```
Int[(c*x)^m*(a*x + b*x^2),x]
```

```
((a*(c*x)^(2 + m))/(c*(2 + m)) + (b*(c*x)^(3 + m))/(c^2*(3 + m)))/c
```

Defintions of rubi rules used

```
Int[(u_.)*(Px_)^(p_.)*((e_.)*(x_))^(m_.), x_Symbol] :> With[{r = Expon[Px,
x, Min]}, Simp[1/e^(p*r)   Int[u*(e*x)^(m + p*r)*ExpandToSum[Px/x^r, x]^p,
x], x] /; IGtQ[r, 0]] /; FreeQ[{e, m}, x] && PolyQ[Px, x] && IntegerQ[p] &&
  !MonomialQ[Px, x]
```

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && LeQ[7*m + 4*n + 4, 0])
|| LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 0.04 (sec) , antiderivative size = 35, normalized size of antiderivative = 1.00

method	result	size
gosper	$\frac{(cx)^m (bmx+am+2bx+3a)x^2}{(3+m)(2+m)}$	35
risch	$\frac{(cx)^m (bmx+am+2bx+3a)x^2}{(3+m)(2+m)}$	35
norman	$\frac{a\,x^2 e^{m\ln(cx)}}{2+m}+\frac{b\,x^3 e^{m\ln(cx)}}{3+m}$	36
orering	$\frac{(bmx+am+2bx+3a)x(cx)^m (b\,x^2+ax)}{(3+m)(2+m)(bx+a)}$	49
parallelrisch	$\frac{x^3(cx)^m bm+2x^3(cx)^m b+x^2(cx)^m am+3x^2(cx)^m a}{(3+m)(2+m)}$	57

```
int((c*x)^m*(b*x^2+a*x),x,method=_RETURNVERBOSE)
```

```
(c*x)^m*(b*m*x+a*m+2*b*x+3*a)*x^2/(3+m)/(2+m)
```

Fricas [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 39, normalized size of antiderivative = 1.11

$$\int (cx)^m \left(ax+bx^2\right)\,dx=\frac{((bm+2\,b)x^3+(am+3\,a)x^2)(cx)^m}{m^2+5\,m+6}$$

```
integrate((c*x)^m*(b*x^2+a*x),x, algorithm="fricas")
```

```
((b*m + 2*b)*x^3 + (a*m + 3*a)*x^2)*(c*x)^m/(m^2 + 5*m + 6)
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 105 vs. 2(29) = 58.

Time = 0.20 (sec) , antiderivative size = 105, normalized size of antiderivative = 3.00

$$\int (cx)^m \left(ax+bx^2\right)\,dx = \begin{cases} \frac{-\frac{a}{x}+b\log{(x)}}{c^3} & \text{for}\ m=-3 \\ \frac{a\log{(x)}+bx}{c^2} & \text{for}\ m=-2 \\ \frac{amx^2(cx)^m}{m^2+5m+6}+\frac{3ax^2(cx)^m}{m^2+5m+6}+\frac{bmx^3(cx)^m}{m^2+5m+6}+\frac{2bx^3(cx)^m}{m^2+5m+6} & \text{otherwise} \end{cases}$$

```
integrate((c*x)**m*(b*x**2+a*x),x)
```

```
Piecewise(((-a/x + b*log(x))/c**3, Eq(m, -3)), ((a*log(x) + b*x)/c**2, Eq(
m, -2)), (a*m*x**2*(c*x)**m/(m**2 + 5*m + 6) + 3*a*x**2*(c*x)**m/(m**2 + 5
*m + 6) + b*m*x**3*(c*x)**m/(m**2 + 5*m + 6) + 2*b*x**3*(c*x)**m/(m**2 + 5
*m + 6), True))
```

Maxima [A] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 33, normalized size of antiderivative = 0.94

$$\int (cx)^m \left(ax+bx^2\right)\,dx = \frac{bc^m x^3 x^m}{m+3}+\frac{ac^m x^2 x^m}{m+2}$$

```
integrate((c*x)^m*(b*x^2+a*x),x, algorithm="maxima")
```

```
b*c^m*x^3*x^m/(m + 3) + a*c^m*x^2*x^m/(m + 2)
```

Giac [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 56, normalized size of antiderivative = 1.60

$$\int (cx)^m \left(ax+bx^2\right) \, dx = \frac{(cx)^m bmx^3 + (cx)^m amx^2 + 2\,(cx)^m bx^3 + 3\,(cx)^m ax^2}{m^2 + 5\,m + 6}$$

```
integrate((c*x)^m*(b*x^2+a*x),x, algorithm="giac")
```

```
((c*x)^m*b*m*x^3 + (c*x)^m*a*m*x^2 + 2*(c*x)^m*b*x^3 + 3*(c*x)^m*a*x^2)/(m
^2 + 5*m + 6)
```

Mupad [B] (verification not implemented)

Time = 8.83 (sec) , antiderivative size = 34, normalized size of antiderivative = 0.97

$$\int (cx)^m \left(ax+bx^2\right) \, dx = \frac{x^2\,(c\,x)^m\,(3\,a + a\,m + 2\,b\,x + b\,m\,x)}{m^2 + 5\,m + 6}$$

```
int((a*x + b*x^2)*(c*x)^m,x)
```

```
(x^2*(c*x)^m*(3*a + a*m + 2*b*x + b*m*x))/(5*m + m^2 + 6)
```

Reduce [B] (verification not implemented)

Time = 0.45 (sec) , antiderivative size = 35, normalized size of antiderivative = 1.00

$$\int (cx)^m \left(ax+bx^2\right) \, dx = \frac{x^m c^m x^2(bmx + am + 2bx + 3a)}{m^2 + 5m + 6}$$

```
int((c*x)^m*(b*x^2+a*x),x)
```

```
(x**m*c**m*x**2*(a*m + 3*a + b*m*x + 2*b*x))/(m**2 + 5*m + 6)
```

3.220 $\int \frac{(cx)^m}{ax+bx^2} dx$

Optimal result

Integrand size = 17, antiderivative size = 25

$$\int \frac{(cx)^m}{ax+bx^2} dx = \frac{(cx)^m \text{Hypergeometric2F1}\left(1,m,1+m,-\frac{bx}{a}\right)}{am}$$

```
(c*x)^m*hypergeom([1, m],[1+m],-b*x/a)/a/m
```

Mathematica [A] (verified)

Time = 0.03 (sec) , antiderivative size = 25, normalized size of antiderivative = 1.00

$$\int \frac{(cx)^m}{ax+bx^2} dx = \frac{(cx)^m \text{Hypergeometric2F1}\left(1,m,1+m,-\frac{bx}{a}\right)}{am}$$

```
Integrate[(c*x)^m/(a*x + b*x^2),x]
```

```
((c*x)^m*Hypergeometric2F1[1, m, 1 + m, -((b*x)/a)])/(a*m)
```

Rubi [A] (verified)

Time = 0.25 (sec) , antiderivative size = 25, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.118, Rules used = {9, 74}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(cx)^m}{ax+bx^2}\,dx$$
$$\Big\downarrow \text{9}$$
$$c\int \frac{(cx)^{m-1}}{a+bx}dx$$
$$\Big\downarrow \text{74}$$
$$\frac{(cx)^m \text{Hypergeometric2F1}\left(1,m,m+1,-\frac{bx}{a}\right)}{am}$$

```
Int[(c*x)^m/(a*x + b*x^2),x]
```

```
((c*x)^m*Hypergeometric2F1[1, m, 1 + m, -((b*x)/a)])/(a*m)
```

Defintions of rubi rules used

```
Int[(u_.)*(Px_)^(p_.)*((e_.)*(x_))^(m_.), x_Symbol] :> With[{r = Expon[Px,
x, Min]}, Simp[1/e^(p*r)   Int[u*(e*x)^(m + p*r)*ExpandToSum[Px/x^r, x]^p,
x], x] /; IGtQ[r, 0]] /; FreeQ[{e, m}, x] && PolyQ[Px, x] && IntegerQ[p] &&
  !MonomialQ[Px, x]
```

```
Int[((b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[c^n*((b*x
)^(m + 1)/(b*(m + 1)))*Hypergeometric2F1[-n, m + 1, m + 2, (-d)*(x/c)], x]
/; FreeQ[{b, c, d, m, n}, x] &&  !IntegerQ[m] && (IntegerQ[n] || (GtQ[c, 0]
 &&  !(EqQ[n, -2^(-1)] && EqQ[c^2 - d^2, 0] && GtQ[-d/(b*c), 0])))
```

Maple [F]

$$\int \frac{(cx)^m}{b\,x^2 + ax} dx$$

```
int((c*x)^m/(b*x^2+a*x),x)
```

```
int((c*x)^m/(b*x^2+a*x),x)
```

Fricas [F]

$$\int \frac{(cx)^m}{ax + bx^2}\, dx = \int \frac{(cx)^m}{bx^2 + ax}\, dx$$

```
integrate((c*x)^m/(b*x^2+a*x),x, algorithm="fricas")
```

```
integral((c*x)^m/(b*x^2 + a*x), x)
```

Sympy [F]

$$\int \frac{(cx)^m}{ax + bx^2}\, dx = \int \frac{(cx)^m}{x\,(a + bx)}\, dx$$

```
integrate((c*x)**m/(b*x**2+a*x),x)
```

```
Integral((c*x)**m/(x*(a + b*x)), x)
```

Maxima [F]

$$\int \frac{(cx)^m}{ax+bx^2}\,dx = \int \frac{(cx)^m}{bx^2+ax}\,dx$$

```
integrate((c*x)^m/(b*x^2+a*x),x, algorithm="maxima")
```

```
integrate((c*x)^m/(b*x^2 + a*x), x)
```

Giac [F]

$$\int \frac{(cx)^m}{ax+bx^2}\,dx = \int \frac{(cx)^m}{bx^2+ax}\,dx$$

```
integrate((c*x)^m/(b*x^2+a*x),x, algorithm="giac")
```

```
integrate((c*x)^m/(b*x^2 + a*x), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(cx)^m}{ax+bx^2}\,dx = \int \frac{(c\,x)^m}{b\,x^2+a\,x}\,dx$$

```
int((c*x)^m/(a*x + b*x^2),x)
```

```
int((c*x)^m/(a*x + b*x^2), x)
```

Reduce [F]

$$\int \frac{(cx)^m}{ax+bx^2}\,dx = c^m\left(\int \frac{x^m}{b\,x^2+ax}dx\right)$$

```
int((c*x)^m/(b*x^2+a*x),x)
```

```
c**m*int(x**m/(a*x + b*x**2),x)
```

3.221 $\int \frac{(cx)^m}{(ax+bx^2)^2}\,dx$

Optimal result

Integrand size = 17, antiderivative size = 33

$$\int \frac{(cx)^m}{(ax+bx^2)^2}\,dx = -\frac{c(cx)^{-1+m}\,\text{Hypergeometric2F1}\left(2,-1+m,m,-\frac{bx}{a}\right)}{a^2(1-m)}$$

```
-c*(c*x)^(-1+m)*hypergeom([2, -1+m],[m],-b*x/a)/a^2/(1-m)
```

Mathematica [A] (verified)

Time = 0.03 (sec) , antiderivative size = 30, normalized size of antiderivative = 0.91

$$\int \frac{(cx)^m}{(ax+bx^2)^2}\,dx = \frac{(cx)^m\,\text{Hypergeometric2F1}\left(2,-1+m,m,-\frac{bx}{a}\right)}{a^2(-1+m)x}$$

```
Integrate[(c*x)^m/(a*x + b*x^2)^2,x]
```

```
((c*x)^m*Hypergeometric2F1[2, -1 + m, m, -((b*x)/a)])/(a^2*(-1 + m)*x)
```

Rubi [A] (verified)

Time = 0.26 (sec) , antiderivative size = 33, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.118, Rules used = {9, 74}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(cx)^m}{(ax+bx^2)^2}dx$$
$$\Big\downarrow 9$$
$$c^2\int \frac{(cx)^{m-2}}{(a+bx)^2}dx$$
$$\Big\downarrow 74$$
$$-\frac{c(cx)^{m-1}\text{Hypergeometric2F1}\left(2,m-1,m,-\frac{bx}{a}\right)}{a^2(1-m)}$$

```
Int[(c*x)^m/(a*x + b*x^2)^2,x]
```

```
-((c*(c*x)^(-1 + m)*Hypergeometric2F1[2, -1 + m, m, -((b*x)/a)])/(a^2*(1 -
 m)))
```

Defintions of rubi rules used

```
Int[(u_.)*(Px_)^(p_.)*((e_.)*(x_))^(m_.), x_Symbol] :> With[{r = Expon[Px,
x, Min]}, Simp[1/e^(p*r)   Int[u*(e*x)^(m + p*r)*ExpandToSum[Px/x^r, x]^p,
x], x] /; IGtQ[r, 0]] /; FreeQ[{e, m}, x] && PolyQ[Px, x] && IntegerQ[p] &&
  !MonomialQ[Px, x]
```

```
Int[((b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[c^n*((b*x
)^(m + 1)/(b*(m + 1)))*Hypergeometric2F1[-n, m + 1, m + 2, (-d)*(x/c)], x]
/; FreeQ[{b, c, d, m, n}, x] &&  !IntegerQ[m] && (IntegerQ[n] || (GtQ[c, 0]
 &&  !(EqQ[n, -2^(-1)] && EqQ[c^2 - d^2, 0] && GtQ[-d/(b*c), 0])))
```

Maple [F]

$$\int \frac{(cx)^m}{(b\,x^2 + ax)^2}dx$$

```
int((c*x)^m/(b*x^2+a*x)^2,x)
```

```
int((c*x)^m/(b*x^2+a*x)^2,x)
```

Fricas [F]

$$\int \frac{(cx)^m}{(ax + bx^2)^2}\,dx = \int \frac{(cx)^m}{(bx^2 + ax)^2}\,dx$$

```
integrate((c*x)^m/(b*x^2+a*x)^2,x, algorithm="fricas")
```

```
integral((c*x)^m/(b^2*x^4 + 2*a*b*x^3 + a^2*x^2), x)
```

Sympy [F]

$$\int \frac{(cx)^m}{(ax + bx^2)^2}\,dx = \int \frac{(cx)^m}{x^2\,(a + bx)^2}\,dx$$

```
integrate((c*x)**m/(b*x**2+a*x)**2,x)
```

```
Integral((c*x)**m/(x**2*(a + b*x)**2), x)
```

Maxima [F]

$$\int \frac{(cx)^m}{(ax+bx^2)^2}\,dx = \int \frac{(cx)^m}{(bx^2+ax)^2}\,dx$$

```
integrate((c*x)^m/(b*x^2+a*x)^2,x, algorithm="maxima")
```

```
integrate((c*x)^m/(b*x^2 + a*x)^2, x)
```

Giac [F]

$$\int \frac{(cx)^m}{(ax+bx^2)^2}\,dx = \int \frac{(cx)^m}{(bx^2+ax)^2}\,dx$$

```
integrate((c*x)^m/(b*x^2+a*x)^2,x, algorithm="giac")
```

```
integrate((c*x)^m/(b*x^2 + a*x)^2, x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(cx)^m}{(ax+bx^2)^2}\,dx = \int \frac{(c\,x)^m}{(b\,x^2+a\,x)^2}\,dx$$

```
int((c*x)^m/(a*x + b*x^2)^2,x)
```

```
int((c*x)^m/(a*x + b*x^2)^2, x)
```

Reduce [F]

$$\int \frac{(cx)^m}{(ax+bx^2)^2}\,dx = c^m\left(\int \frac{x^m}{b^2x^4+2ab\,x^3+a^2x^2}dx\right)$$

```
int((c*x)^m/(b*x^2+a*x)^2,x)
```

```
c**m*int(x**m/(a**2*x**2 + 2*a*b*x**3 + b**2*x**4),x)
```

3.222 $\int \frac{(cx)^m}{(ax+bx^2)^3} dx$

Optimal result

Integrand size = 17, antiderivative size = 37

$$\int \frac{(cx)^m}{(ax+bx^2)^3} dx = -\frac{c^2(cx)^{-2+m} \text{Hypergeometric2F1}\left(3,-2+m,-1+m,-\frac{bx}{a}\right)}{a^3(2-m)}$$

```
-c^2*(c*x)^(-2+m)*hypergeom([3, -2+m],[-1+m],-b*x/a)/a^3/(2-m)
```

Mathematica [A] (verified)

Time = 0.03 (sec) , antiderivative size = 32, normalized size of antiderivative = 0.86

$$\int \frac{(cx)^m}{(ax+bx^2)^3} dx = \frac{(cx)^m \text{Hypergeometric2F1}\left(3,-2+m,-1+m,-\frac{bx}{a}\right)}{a^3(-2+m)x^2}$$

```
Integrate[(c*x)^m/(a*x + b*x^2)^3,x]
```

```
((c*x)^m*Hypergeometric2F1[3, -2 + m, -1 + m, -((b*x)/a)])/(a^3*(-2 + m)*x
^2)
```

Rubi [A] (verified)

Time = 0.26 (sec) , antiderivative size = 37, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.118, Rules used = {9, 74}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(cx)^m}{(ax+bx^2)^3} \, dx$$
$$\Big\downarrow \, 9$$
$$c^3 \int \frac{(cx)^{m-3}}{(a+bx)^3} dx$$
$$\Big\downarrow \, 74$$
$$-\frac{c^2 (cx)^{m-2} \text{Hypergeometric2F1}\left(3, m-2, m-1, -\frac{bx}{a}\right)}{a^3 (2-m)}$$

```
Int[(c*x)^m/(a*x + b*x^2)^3,x]
```

```
-((c^2*(c*x)^(-2 + m)*Hypergeometric2F1[3, -2 + m, -1 + m, -((b*x)/a)])/(a
^3*(2 - m)))
```

Defintions of rubi rules used

```
Int[(u_.)*(Px_)^(p_.)*((e_.)*(x_))^(m_.), x_Symbol] :> With[{r = Expon[Px,
x, Min]}, Simp[1/e^(p*r)   Int[u*(e*x)^(m + p*r)*ExpandToSum[Px/x^r, x]^p,
x], x] /; IGtQ[r, 0]] /; FreeQ[{e, m}, x] && PolyQ[Px, x] && IntegerQ[p] &&
  !MonomialQ[Px, x]
```

```
Int[((b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[c^n*((b*x
)^(m + 1)/(b*(m + 1)))*Hypergeometric2F1[-n, m + 1, m + 2, (-d)*(x/c)], x]
/; FreeQ[{b, c, d, m, n}, x] &&  !IntegerQ[m] && (IntegerQ[n] || (GtQ[c, 0]
 &&  !(EqQ[n, -2^(-1)] && EqQ[c^2 - d^2, 0] && GtQ[-d/(b*c), 0])))
```

Maple [F]

$$\int \frac{(cx)^m}{(b\,x^2+ax)^3}dx$$

```
int((c*x)^m/(b*x^2+a*x)^3,x)
```

```
int((c*x)^m/(b*x^2+a*x)^3,x)
```

Fricas [F]

$$\int \frac{(cx)^m}{(ax+bx^2)^3}\,dx = \int \frac{(cx)^m}{(bx^2+ax)^3}\,dx$$

```
integrate((c*x)^m/(b*x^2+a*x)^3,x, algorithm="fricas")
```

```
integral((c*x)^m/(b^3*x^6 + 3*a*b^2*x^5 + 3*a^2*b*x^4 + a^3*x^3), x)
```

Sympy [F]

$$\int \frac{(cx)^m}{(ax+bx^2)^3}\,dx = \int \frac{(cx)^m}{x^3\,(a+bx)^3}\,dx$$

```
integrate((c*x)**m/(b*x**2+a*x)**3,x)
```

```
Integral((c*x)**m/(x**3*(a + b*x)**3), x)
```

Maxima [F]

$$\int \frac{(cx)^m}{(ax+bx^2)^3}\,dx = \int \frac{(cx)^m}{(bx^2+ax)^3}\,dx$$

```
integrate((c*x)^m/(b*x^2+a*x)^3,x, algorithm="maxima")
```

```
integrate((c*x)^m/(b*x^2 + a*x)^3, x)
```

Giac [F]

$$\int \frac{(cx)^m}{(ax+bx^2)^3}\,dx = \int \frac{(cx)^m}{(bx^2+ax)^3}\,dx$$

```
integrate((c*x)^m/(b*x^2+a*x)^3,x, algorithm="giac")
```

```
integrate((c*x)^m/(b*x^2 + a*x)^3, x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(cx)^m}{(ax+bx^2)^3}\,dx = \int \frac{(c\,x)^m}{(b\,x^2+a\,x)^3}\,dx$$

```
int((c*x)^m/(a*x + b*x^2)^3,x)
```

```
int((c*x)^m/(a*x + b*x^2)^3, x)
```

Reduce [F]

$$\int \frac{(cx)^m}{(ax+bx^2)^3}\,dx = c^m\left(\int \frac{x^m}{b^3x^6+3a\,b^2x^5+3a^2b\,x^4+a^3x^3}dx\right)$$

```
int((c*x)^m/(b*x^2+a*x)^3,x)
```

```
c**m*int(x**m/(a**3*x**3 + 3*a**2*b*x**4 + 3*a*b**2*x**5 + b**3*x**6),x)
```

3.223 $\int (cx)^m \left(ax+bx^2\right)^{3/2} \, dx$

Optimal result

Integrand size = 19, antiderivative size = 65

$$\int (cx)^m \left(ax+bx^2\right)^{3/2} \, dx=\frac{2c\left(-\frac{bx}{a}\right)^{-\frac{3}{2}-m} (cx)^{-1+m} \left(ax+bx^2\right)^{5/2} \text{Hypergeometric2F1}\left(\frac{5}{2},-\frac{3}{2}-m,\frac{7}{2},1+\frac{bx}{a}\right)}{5b}$$

```
2/5*c*(-b*x/a)^(-3/2-m)*(c*x)^(-1+m)*(b*x^2+a*x)^(5/2)*hypergeom([5/2, -3/
2-m],[7/2],1+b*x/a)/b
```

Mathematica [A] (verified)

Time = 0.18 (sec) , antiderivative size = 60, normalized size of antiderivative = 0.92

$$\int (cx)^m \left(ax+bx^2\right)^{3/2} \, dx = -\frac{2\left(-\frac{bx}{a}\right)^{-\frac{5}{2}-m} (cx)^m (x(a+bx))^{5/2} \text{Hypergeometric2F1}\left(\frac{5}{2},-\frac{3}{2}-m,\frac{7}{2},1+\frac{bx}{a}\right)}{5a}$$

```
Integrate[(c*x)^m*(a*x + b*x^2)^(3/2),x]
```

```
(-2*(-((b*x)/a))^(-5/2 - m)*(c*x)^m*(x*(a + b*x))^(5/2)*Hypergeometric2F1[
5/2, -3/2 - m, 7/2, 1 + (b*x)/a])/(5*a)
```

Rubi [A] (verified)

Time = 0.32 (sec) , antiderivative size = 71, normalized size of antiderivative = 1.09, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.158, Rules used = {1137, 77, 75}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \left(ax+bx^2\right)^{3/2} (cx)^m \, dx$$

$$\Big\downarrow \text{1137}$$

$$\frac{x^{-m-\frac{3}{2}} \left(ax+bx^2\right)^{3/2} (cx)^m \int x^{m+\frac{3}{2}} (a+bx)^{3/2} dx}{(a+bx)^{3/2}}$$

$$\Big\downarrow \text{77}$$

$$-\frac{a \left(ax+bx^2\right)^{3/2} (cx)^m \left(-\frac{bx}{a}\right)^{-m-\frac{1}{2}} \int \left(-\frac{bx}{a}\right)^{m+\frac{3}{2}} (a+bx)^{3/2} dx}{bx (a+bx)^{3/2}}$$

$$\Big\downarrow \text{75}$$

$$-\frac{2a(a+bx) \left(ax+bx^2\right)^{3/2} (cx)^m \left(-\frac{bx}{a}\right)^{-m-\frac{1}{2}} \text{Hypergeometric2F1}\left(\frac{5}{2},-m-\frac{3}{2},\frac{7}{2},\frac{bx}{a}+1\right)}{5b^2 x}$$

```
Int[(c*x)^m*(a*x + b*x^2)^(3/2),x]
```

```
(-2*a*(-((b*x)/a))^(-1/2 - m)*(c*x)^m*(a + b*x)*(a*x + b*x^2)^(3/2)*Hyperg
eometric2F1[5/2, -3/2 - m, 7/2, 1 + (b*x)/a])/(5*b^2*x)
```

Defintions of rubi rules used

```
Int[((b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[((c + d*x
)^(n + 1)/(d*(n + 1)*(-d/(b*c))^m))*Hypergeometric2F1[-m, n + 1, n + 2, 1 +
 d*(x/c)], x] /; FreeQ[{b, c, d, m, n}, x] &&  !IntegerQ[n] && (IntegerQ[m]
 || GtQ[-d/(b*c), 0])
```

```
Int[((b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[((-b)*(c/
d))^IntPart[m]*((b*x)^FracPart[m]/((-d)*(x/c))^FracPart[m])   Int[((-d)*(x/
c))^m*(c + d*x)^n, x], x] /; FreeQ[{b, c, d, m, n}, x] &&  !IntegerQ[m] &&
 !IntegerQ[n] &&  !GtQ[c, 0] &&  !GtQ[-d/(b*c), 0]
```

```
Int[((e_.)*(x_))^(m_)*((b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(
e*x)^m*((b*x + c*x^2)^p/(x^(m + p)*(b + c*x)^p))   Int[x^(m + p)*(b + c*x)^
p, x], x] /; FreeQ[{b, c, e, m}, x]
```

Maple [F]

$$\int (cx)^m \left(b\,x^2+ax\right)^{\frac{3}{2}}dx$$

```
int((c*x)^m*(b*x^2+a*x)^(3/2),x)
```

```
int((c*x)^m*(b*x^2+a*x)^(3/2),x)
```

Fricas [F]

$$\int (cx)^m \left(ax+bx^2\right)^{3/2}\,dx=\int \left(bx^2+ax\right)^{\frac{3}{2}}(cx)^m\,dx$$

```
integrate((c*x)^m*(b*x^2+a*x)^(3/2),x, algorithm="fricas")
```

```
integral((b*x^2 + a*x)^(3/2)*(c*x)^m, x)
```

Sympy [F]

$$\int (cx)^m \left(ax + bx^2\right)^{3/2} dx = \int (cx)^m (x(a + bx))^{\frac{3}{2}} dx$$

```
integrate((c*x)**m*(b*x**2+a*x)**(3/2),x)
```

```
Integral((c*x)**m*(x*(a + b*x))**(3/2), x)
```

Maxima [F]

$$\int (cx)^m \left(ax + bx^2\right)^{3/2} dx = \int \left(bx^2 + ax\right)^{\frac{3}{2}} (cx)^m dx$$

```
integrate((c*x)^m*(b*x^2+a*x)^(3/2),x, algorithm="maxima")
```

```
integrate((b*x^2 + a*x)^(3/2)*(c*x)^m, x)
```

Giac [F]

$$\int (cx)^m \left(ax + bx^2\right)^{3/2} dx = \int \left(bx^2 + ax\right)^{\frac{3}{2}} (cx)^m dx$$

```
integrate((c*x)^m*(b*x^2+a*x)^(3/2),x, algorithm="giac")
```

```
integrate((b*x^2 + a*x)^(3/2)*(c*x)^m, x)
```

Mupad [F(-1)]

Timed out.

$$\int (cx)^m \left(ax+bx^2\right)^{3/2} dx = \int \left(b\,x^2+a\,x\right)^{3/2} (c\,x)^m\,dx$$

```
int((a*x + b*x^2)^(3/2)*(c*x)^m,x)
```

```
int((a*x + b*x^2)^(3/2)*(c*x)^m, x)
```

Reduce [F]

$$\int (cx)^m \left(ax+bx^2\right)^{3/2} dx = c^m\left(\left(\int x^{m+\frac{1}{2}}\sqrt{bx+a}\,x^2dx\right)b+\left(\int x^{m+\frac{1}{2}}\sqrt{bx+a}\,xdx\right)a\right)$$

```
int((c*x)^m*(b*x^2+a*x)^(3/2),x)
```

```
c**m*(int(x**((2*m + 1)/2)*sqrt(a + b*x)*x**2,x)*b + int(x**((2*m + 1)/2)*
sqrt(a + b*x)*x,x)*a)
```

3.224 $\int (cx)^m \sqrt{ax+bx^2}\,dx$

Optimal result

Integrand size = 19, antiderivative size = 65

$$\int (cx)^m \sqrt{ax+bx^2}\,dx = \frac{2c\left(-\frac{bx}{a}\right)^{-\frac{1}{2}-m}(cx)^{-1+m}\left(ax+bx^2\right)^{3/2}\text{Hypergeometric2F1}\left(\frac{3}{2},-\frac{1}{2}-m,\frac{5}{2},1+\frac{bx}{a}\right)}{3b}$$

```
2/3*c*(-b*x/a)^(-1/2-m)*(c*x)^(-1+m)*(b*x^2+a*x)^(3/2)*hypergeom([3/2, -1/
2-m],[5/2],1+b*x/a)/b
```

Mathematica [A] (verified)

Time = 0.11 (sec) , antiderivative size = 60, normalized size of antiderivative = 0.92

$$\int (cx)^m \sqrt{ax+bx^2}\,dx = -\frac{2\left(-\frac{bx}{a}\right)^{-\frac{3}{2}-m}(cx)^m(x(a+bx))^{3/2}\text{Hypergeometric2F1}\left(\frac{3}{2},-\frac{1}{2}-m,\frac{5}{2},1+\frac{bx}{a}\right)}{3a}$$

```
Integrate[(c*x)^m*Sqrt[a*x + b*x^2],x]
```

```
(-2*(-((b*x)/a))^(-3/2 - m)*(c*x)^m*(x*(a + b*x))^(3/2)*Hypergeometric2F1[
3/2, -1/2 - m, 5/2, 1 + (b*x)/a])/(3*a)
```

Rubi [A] (verified)

Time = 0.30 (sec) , antiderivative size = 67, normalized size of antiderivative = 1.03, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.158, Rules used = {1137, 77, 75}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sqrt{ax+bx^2}(cx)^m\,dx$$

$$\Big\downarrow \text{1137}$$

$$\frac{x^{-m-\frac{1}{2}}\sqrt{ax+bx^2}(cx)^m\int x^{m+\frac{1}{2}}\sqrt{a+bx}dx}{\sqrt{a+bx}}$$

$$\Big\downarrow \text{77}$$

$$\frac{\sqrt{ax+bx^2}(cx)^m\left(-\frac{bx}{a}\right)^{-m-\frac{1}{2}}\int\left(-\frac{bx}{a}\right)^{m+\frac{1}{2}}\sqrt{a+bx}dx}{\sqrt{a+bx}}$$

$$\Big\downarrow \text{75}$$

$$\frac{2(a+bx)\sqrt{ax+bx^2}(cx)^m\left(-\frac{bx}{a}\right)^{-m-\frac{1}{2}}\text{Hypergeometric2F1}\left(\frac{3}{2},-m-\frac{1}{2},\frac{5}{2},\frac{bx}{a}+1\right)}{3b}$$

```
Int[(c*x)^m*Sqrt[a*x + b*x^2],x]
```

```
(2*(-((b*x)/a))^(-1/2 - m)*(c*x)^m*(a + b*x)*Sqrt[a*x + b*x^2]*Hypergeomet
ric2F1[3/2, -1/2 - m, 5/2, 1 + (b*x)/a])/(3*b)
```

Defintions of rubi rules used

```
Int[((b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[((c + d*x
)^(n + 1)/(d*(n + 1)*(-d/(b*c))^m))*Hypergeometric2F1[-m, n + 1, n + 2, 1 +
 d*(x/c)], x] /; FreeQ[{b, c, d, m, n}, x] && !IntegerQ[n] && (IntegerQ[m]
 || GtQ[-d/(b*c), 0])
```

```
Int[((b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[((-b)*(c/
d))^IntPart[m]*((b*x)^FracPart[m]/((-d)*(x/c))^FracPart[m])   Int[((-d)*(x/
c))^m*(c + d*x)^n, x], x] /; FreeQ[{b, c, d, m, n}, x] && !IntegerQ[m] &&
 !IntegerQ[n] && !GtQ[c, 0] && !GtQ[-d/(b*c), 0]
```

```
Int[((e_.)*(x_))^(m_)*((b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(
e*x)^m*((b*x + c*x^2)^p/(x^(m + p)*(b + c*x)^p))   Int[x^(m + p)*(b + c*x)^
p, x], x] /; FreeQ[{b, c, e, m}, x]
```

Maple [F]

$$\int (cx)^m \sqrt{b\,x^2+ax}dx$$

```
int((c*x)^m*(b*x^2+a*x)^(1/2),x)
```

```
int((c*x)^m*(b*x^2+a*x)^(1/2),x)
```

Fricas [F]

$$\int (cx)^m\sqrt{ax+bx^2}\,dx = \int \sqrt{bx^2+ax}(cx)^m\,dx$$

```
integrate((c*x)^m*(b*x^2+a*x)^(1/2),x, algorithm="fricas")
```

```
integral(sqrt(b*x^2 + a*x)*(c*x)^m, x)
```

Sympy [F]

$$\int (cx)^m \sqrt{ax+bx^2}\, dx = \int (cx)^m \sqrt{x(a+bx)}\, dx$$

```
integrate((c*x)**m*(b*x**2+a*x)**(1/2),x)
```

```
Integral((c*x)**m*sqrt(x*(a + b*x)), x)
```

Maxima [F]

$$\int (cx)^m \sqrt{ax+bx^2}\, dx = \int \sqrt{bx^2+ax}(cx)^m \, dx$$

```
integrate((c*x)^m*(b*x^2+a*x)^(1/2),x, algorithm="maxima")
```

```
integrate(sqrt(b*x^2 + a*x)*(c*x)^m, x)
```

Giac [F]

$$\int (cx)^m \sqrt{ax+bx^2}\, dx = \int \sqrt{bx^2+ax}(cx)^m \, dx$$

```
integrate((c*x)^m*(b*x^2+a*x)^(1/2),x, algorithm="giac")
```

```
integrate(sqrt(b*x^2 + a*x)*(c*x)^m, x)
```

Mupad [F(-1)]

Timed out.

$$\int (cx)^m \sqrt{ax+bx^2}\,dx = \int \sqrt{b\,x^2+a\,x}\,(c\,x)^m\,dx$$

```
int((a*x + b*x^2)^(1/2)*(c*x)^m,x)
```

```
int((a*x + b*x^2)^(1/2)*(c*x)^m, x)
```

Reduce [F]

$$\int (cx)^m \sqrt{ax+bx^2}\,dx = c^m \left(\int x^{m+\frac{1}{2}} \sqrt{bx+a}dx \right)$$

```
int((c*x)^m*(b*x^2+a*x)^(1/2),x)
```

```
c**m*int(x**((2*m + 1)/2)*sqrt(a + b*x),x)
```

3.225 $\int \frac{(cx)^m}{\sqrt{ax+bx^2}}\,dx$

Optimal result

Integrand size = 19, antiderivative size = 63

$$\int \frac{(cx)^m}{\sqrt{ax+bx^2}}\,dx = \frac{2c\left(-\frac{bx}{a}\right)^{\frac{1}{2}-m}(cx)^{-1+m}\sqrt{ax+bx^2}\,\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{1}{2}-m,\frac{3}{2},1+\frac{bx}{a}\right)}{b}$$

```
2*c*(-b*x/a)^(1/2-m)*(c*x)^(-1+m)*(b*x^2+a*x)^(1/2)*hypergeom([1/2, 1/2-m]
,[3/2],1+b*x/a)/b
```

Mathematica [A] (verified)

Time = 0.10 (sec) , antiderivative size = 58, normalized size of antiderivative = 0.92

$$\int \frac{(cx)^m}{\sqrt{ax+bx^2}}\,dx = -\frac{2\left(-\frac{bx}{a}\right)^{-\frac{1}{2}-m}(cx)^m\sqrt{x(a+bx)}\,\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{1}{2}-m,\frac{3}{2},1+\frac{bx}{a}\right)}{a}$$

```
Integrate[(c*x)^m/Sqrt[a*x + b*x^2],x]
```

```
(-2*(-((b*x)/a))^(-1/2 - m)*(c*x)^m*Sqrt[x*(a + b*x)]*Hypergeometric2F1[1/
2, 1/2 - m, 3/2, 1 + (b*x)/a])/a
```

Rubi [A] (verified)

Time = 0.30 (sec) , antiderivative size = 65, normalized size of antiderivative = 1.03, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.158, Rules used = {1137, 77, 75}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(cx)^m}{\sqrt{ax+bx^2}}\,dx$$
$$\Big\downarrow \text{1137}$$
$$\frac{x^{\frac{1}{2}-m}\sqrt{a+bx}(cx)^m\int \frac{x^{m-\frac{1}{2}}}{\sqrt{a+bx}}dx}{\sqrt{ax+bx^2}}$$
$$\Big\downarrow \text{77}$$
$$\frac{\sqrt{a+bx}(cx)^m\left(-\frac{bx}{a}\right)^{\frac{1}{2}-m}\int \frac{\left(-\frac{bx}{a}\right)^{m-\frac{1}{2}}}{\sqrt{a+bx}}dx}{\sqrt{ax+bx^2}}$$
$$\Big\downarrow \text{75}$$
$$\frac{2(a+bx)(cx)^m\left(-\frac{bx}{a}\right)^{\frac{1}{2}-m}\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{1}{2}-m,\frac{3}{2},\frac{bx}{a}+1\right)}{b\sqrt{ax+bx^2}}$$

```
Int[(c*x)^m/Sqrt[a*x + b*x^2],x]
```

```
(2*(-((b*x)/a))^(1/2 - m)*(c*x)^m*(a + b*x)*Hypergeometric2F1[1/2, 1/2 - m
, 3/2, 1 + (b*x)/a])/(b*Sqrt[a*x + b*x^2])
```

Defintions of rubi rules used

```
Int[((b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[((c + d*x
)^(n + 1)/(d*(n + 1)*(-d/(b*c))^m))*Hypergeometric2F1[-m, n + 1, n + 2, 1 +
 d*(x/c)], x] /; FreeQ[{b, c, d, m, n}, x] &&  !IntegerQ[n] && (IntegerQ[m]
 || GtQ[-d/(b*c), 0])
```

```
Int[((b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[((-b)*(c/
d))^IntPart[m]*((b*x)^FracPart[m]/((-d)*(x/c))^FracPart[m])   Int[((-d)*(x/
c))^m*(c + d*x)^n, x], x] /; FreeQ[{b, c, d, m, n}, x] &&  !IntegerQ[m] &&
 !IntegerQ[n] &&  !GtQ[c, 0] &&  !GtQ[-d/(b*c), 0]
```

```
Int[((e_.)*(x_))^(m_)*((b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(
e*x)^m*((b*x + c*x^2)^p/(x^(m + p)*(b + c*x)^p))   Int[x^(m + p)*(b + c*x)^
p, x], x] /; FreeQ[{b, c, e, m}, x]
```

Maple [F]

$$\int \frac{(cx)^m}{\sqrt{b\,x^2+ax}}dx$$

```
int((c*x)^m/(b*x^2+a*x)^(1/2),x)
```

```
int((c*x)^m/(b*x^2+a*x)^(1/2),x)
```

Fricas [F]

$$\int \frac{(cx)^m}{\sqrt{ax+bx^2}}\,dx = \int \frac{(cx)^m}{\sqrt{bx^2+ax}}\,dx$$

```
integrate((c*x)^m/(b*x^2+a*x)^(1/2),x, algorithm="fricas")
```

```
integral((c*x)^m/sqrt(b*x^2 + a*x), x)
```

Sympy [F]

$$\int \frac{(cx)^m}{\sqrt{ax+bx^2}}\,dx = \int \frac{(cx)^m}{\sqrt{x\,(a+bx)}}\,dx$$

```
integrate((c*x)**m/(b*x**2+a*x)**(1/2),x)
```

```
Integral((c*x)**m/sqrt(x*(a + b*x)), x)
```

Maxima [F]

$$\int \frac{(cx)^m}{\sqrt{ax+bx^2}}\,dx = \int \frac{(cx)^m}{\sqrt{bx^2+ax}}\,dx$$

```
integrate((c*x)^m/(b*x^2+a*x)^(1/2),x, algorithm="maxima")
```

```
integrate((c*x)^m/sqrt(b*x^2 + a*x), x)
```

Giac [F]

$$\int \frac{(cx)^m}{\sqrt{ax+bx^2}}\,dx = \int \frac{(cx)^m}{\sqrt{bx^2+ax}}\,dx$$

```
integrate((c*x)^m/(b*x^2+a*x)^(1/2),x, algorithm="giac")
```

```
integrate((c*x)^m/sqrt(b*x^2 + a*x), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(cx)^m}{\sqrt{ax+bx^2}}\,dx = \int \frac{(c\,x)^m}{\sqrt{b\,x^2+a\,x}}\,dx$$

```
int((c*x)^m/(a*x + b*x^2)^(1/2),x)
```

```
int((c*x)^m/(a*x + b*x^2)^(1/2), x)
```

Reduce [F]

$$\int \frac{(cx)^m}{\sqrt{ax+bx^2}}\,dx = c^m \left(\int \frac{x^m}{\sqrt{x}\,\sqrt{bx+a}}dx\right)$$

```
int((c*x)^m/(b*x^2+a*x)^(1/2),x)
```

```
c**m*int(x**m/(sqrt(x)*sqrt(a + b*x)),x)
```

3.226 $\int \frac{(cx)^m}{(ax+bx^2)^{3/2}}\,dx$

Optimal result

Integrand size = 19, antiderivative size = 63

$$\int \frac{(cx)^m}{(ax+bx^2)^{3/2}}\,dx=-\frac{2c\left(-\frac{bx}{a}\right)^{\frac{3}{2}-m}(cx)^{-1+m}\,\text{Hypergeometric2F1}\left(-\frac{1}{2},\frac{3}{2}-m,\frac{1}{2},1+\frac{bx}{a}\right)}{b\sqrt{ax+bx^2}}$$

```
-2*c*(-b*x/a)^(3/2-m)*(c*x)^(-1+m)*hypergeom([-1/2, 3/2-m],[1/2],1+b*x/a)/
b/(b*x^2+a*x)^(1/2)
```

Mathematica [A] (verified)

Time = 0.16 (sec) , antiderivative size = 58, normalized size of antiderivative = 0.92

$$\int \frac{(cx)^m}{(ax+bx^2)^{3/2}}\,dx=\frac{2\left(-\frac{bx}{a}\right)^{\frac{1}{2}-m}(cx)^m\,\text{Hypergeometric2F1}\left(-\frac{1}{2},\frac{3}{2}-m,\frac{1}{2},1+\frac{bx}{a}\right)}{a\sqrt{x(a+bx)}}$$

```
Integrate[(c*x)^m/(a*x + b*x^2)^(3/2),x]
```

```
(2*(-((b*x)/a))^(1/2 - m)*(c*x)^m*Hypergeometric2F1[-1/2, 3/2 - m, 1/2, 1
+ (b*x)/a])/(a*Sqrt[x*(a + b*x)])
```

Rubi [A] (verified)

Time = 0.30 (sec) , antiderivative size = 66, normalized size of antiderivative = 1.05, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.158, Rules used = {1137, 77, 75}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(cx)^m}{(ax+bx^2)^{3/2}}\,dx$$

$$\Big\downarrow \ 1137$$

$$\frac{x^{\frac{3}{2}-m}(a+bx)^{3/2}(cx)^m\int \frac{x^{m-\frac{3}{2}}}{(a+bx)^{3/2}}dx}{(ax+bx^2)^{3/2}}$$

$$\Big\downarrow \ 77$$

$$-\frac{bx(a+bx)^{3/2}(cx)^m\left(-\frac{bx}{a}\right)^{\frac{1}{2}-m}\int \frac{\left(-\frac{bx}{a}\right)^{m-\frac{3}{2}}}{(a+bx)^{3/2}}dx}{a\left(ax+bx^2\right)^{3/2}}$$

$$\Big\downarrow \ 75$$

$$\frac{2x(a+bx)(cx)^m\left(-\frac{bx}{a}\right)^{\frac{1}{2}-m}\text{Hypergeometric2F1}\left(-\frac{1}{2},\frac{3}{2}-m,\frac{1}{2},\frac{bx}{a}+1\right)}{a\left(ax+bx^2\right)^{3/2}}$$

```
Int[(c*x)^m/(a*x + b*x^2)^(3/2),x]
```

```
(2*x*(-((b*x)/a))^(1/2 - m)*(c*x)^m*(a + b*x)*Hypergeometric2F1[-1/2, 3/2
- m, 1/2, 1 + (b*x)/a])/(a*(a*x + b*x^2)^(3/2))
```

Defintions of rubi rules used

```
Int[((b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[((c + d*x
)^(n + 1)/(d*(n + 1)*(-d/(b*c))^m))*Hypergeometric2F1[-m, n + 1, n + 2, 1 +
 d*(x/c)], x] /; FreeQ[{b, c, d, m, n}, x] &&  !IntegerQ[n] && (IntegerQ[m]
 || GtQ[-d/(b*c), 0])
```

```
Int[((b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[((-b)*(c/
d))^IntPart[m]*((b*x)^FracPart[m]/((-d)*(x/c))^FracPart[m])   Int[((-d)*(x/
c))^m*(c + d*x)^n, x], x] /; FreeQ[{b, c, d, m, n}, x] &&  !IntegerQ[m] &&
 !IntegerQ[n] &&  !GtQ[c, 0] &&  !GtQ[-d/(b*c), 0]
```

```
Int[((e_.)*(x_))^(m_)*((b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(
e*x)^m*((b*x + c*x^2)^p/(x^(m + p)*(b + c*x)^p))   Int[x^(m + p)*(b + c*x)^
p, x], x] /; FreeQ[{b, c, e, m}, x]
```

Maple [F]

$$\int \frac{(cx)^{m}}{\left(b\,x^{2}+ax\right)^{\frac{3}{2}}}dx$$

```
int((c*x)^m/(b*x^2+a*x)^(3/2),x)
```

```
int((c*x)^m/(b*x^2+a*x)^(3/2),x)
```

Fricas [F]

$$\int \frac{(cx)^m}{(ax+bx^2)^{3/2}}\,dx=\int \frac{(cx)^{m}}{(bx^{2}+ax)^{\frac{3}{2}}}\,dx$$

```
integrate((c*x)^m/(b*x^2+a*x)^(3/2),x, algorithm="fricas")
```

```
integral(sqrt(b*x^2 + a*x)*(c*x)^m/(b^2*x^4 + 2*a*b*x^3 + a^2*x^2), x)
```

Sympy [F]

$$\int \frac{(cx)^m}{(ax+bx^2)^{3/2}}\,dx = \int \frac{(cx)^m}{(x\,(a+bx))^{\frac{3}{2}}}\,dx$$

```
integrate((c*x)**m/(b*x**2+a*x)**(3/2),x)
```

```
Integral((c*x)**m/(x*(a + b*x))**(3/2), x)
```

Maxima [F]

$$\int \frac{(cx)^m}{(ax+bx^2)^{3/2}}\,dx = \int \frac{(cx)^m}{(bx^2+ax)^{\frac{3}{2}}}\,dx$$

```
integrate((c*x)^m/(b*x^2+a*x)^(3/2),x, algorithm="maxima")
```

```
integrate((c*x)^m/(b*x^2 + a*x)^(3/2), x)
```

Giac [F]

$$\int \frac{(cx)^m}{(ax+bx^2)^{3/2}}\,dx = \int \frac{(cx)^m}{(bx^2+ax)^{\frac{3}{2}}}\,dx$$

```
integrate((c*x)^m/(b*x^2+a*x)^(3/2),x, algorithm="giac")
```

```
integrate((c*x)^m/(b*x^2 + a*x)^(3/2), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(cx)^m}{(ax+bx^2)^{3/2}}\,dx = \int \frac{(c\,x)^m}{(b\,x^2+a\,x)^{3/2}}\,dx$$

```
int((c*x)^m/(a*x + b*x^2)^(3/2),x)
```

```
int((c*x)^m/(a*x + b*x^2)^(3/2), x)
```

Reduce [F]

$$\int \frac{(cx)^m}{(ax+bx^2)^{3/2}}\,dx = c^m\left(\int \frac{x^m}{\sqrt{x}\,\sqrt{bx+a}\,ax+\sqrt{x}\,\sqrt{bx+a}\,b\,x^2}dx\right)$$

```
int((c*x)^m/(b*x^2+a*x)^(3/2),x)
```

```
c**m*int(x**m/(sqrt(x)*sqrt(a + b*x)*a*x + sqrt(x)*sqrt(a + b*x)*b*x**2),x
)
```

3.227 $\int \frac{(cx)^m}{(ax+bx^2)^{5/2}}\,dx$

Optimal result

Integrand size = 19, antiderivative size = 65

$$\int \frac{(cx)^m}{(ax+bx^2)^{5/2}}\,dx=-\frac{2c\left(-\frac{bx}{a}\right)^{\frac{5}{2}-m}(cx)^{-1+m}\,\text{Hypergeometric2F1}\left(-\frac{3}{2},\frac{5}{2}-m,-\frac{1}{2},1+\frac{bx}{a}\right)}{3b\,(ax+bx^2)^{3/2}}$$

```
-2/3*c*(-b*x/a)^(5/2-m)*(c*x)^(-1+m)*hypergeom([-3/2, 5/2-m],[-1/2],1+b*x/
a)/b/(b*x^2+a*x)^(3/2)
```

Mathematica [A] (verified)

Time = 0.16 (sec) , antiderivative size = 60, normalized size of antiderivative = 0.92

$$\int \frac{(cx)^m}{(ax+bx^2)^{5/2}}\,dx=\frac{2\left(-\frac{bx}{a}\right)^{\frac{3}{2}-m}(cx)^m\,\text{Hypergeometric2F1}\left(-\frac{3}{2},\frac{5}{2}-m,-\frac{1}{2},1+\frac{bx}{a}\right)}{3a(x(a+bx))^{3/2}}$$

```
Integrate[(c*x)^m/(a*x + b*x^2)^(5/2),x]
```

```
(2*(-((b*x)/a))^(3/2 - m)*(c*x)^m*Hypergeometric2F1[-3/2, 5/2 - m, -1/2, 1
 + (b*x)/a])/(3*a*(x*(a + b*x))^(3/2))
```

Rubi [A] (verified)

Time = 0.30 (sec) , antiderivative size = 71, normalized size of antiderivative = 1.09, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.158, Rules used = {1137, 77, 75}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(cx)^m}{(ax+bx^2)^{5/2}}\,dx$$

$$\Big\downarrow \text{ 1137}$$

$$\frac{x^{\frac{5}{2}-m}(a+bx)^{5/2}(cx)^m\int \frac{x^{m-\frac{5}{2}}}{(a+bx)^{5/2}}dx}{(ax+bx^2)^{5/2}}$$

$$\Big\downarrow \text{ 77}$$

$$\frac{b^2x^2(a+bx)^{5/2}(cx)^m\left(-\frac{bx}{a}\right)^{\frac{1}{2}-m}\int \frac{\left(-\frac{bx}{a}\right)^{m-\frac{5}{2}}}{(a+bx)^{5/2}}dx}{a^2\left(ax+bx^2\right)^{5/2}}$$

$$\Big\downarrow \text{ 75}$$

$$-\frac{2bx^2(a+bx)(cx)^m\left(-\frac{bx}{a}\right)^{\frac{1}{2}-m}\text{Hypergeometric2F1}\left(-\frac{3}{2},\frac{5}{2}-m,-\frac{1}{2},\frac{bx}{a}+1\right)}{3a^2\left(ax+bx^2\right)^{5/2}}$$

```
Int[(c*x)^m/(a*x + b*x^2)^(5/2),x]
```

```
(-2*b*x^2*(-((b*x)/a))^(1/2 - m)*(c*x)^m*(a + b*x)*Hypergeometric2F1[-3/2,
 5/2 - m, -1/2, 1 + (b*x)/a])/(3*a^2*(a*x + b*x^2)^(5/2))
```

Defintions of rubi rules used

```
Int[((b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[((c + d*x
)^(n + 1)/(d*(n + 1)*(-d/(b*c))^m))*Hypergeometric2F1[-m, n + 1, n + 2, 1 +
 d*(x/c)], x] /; FreeQ[{b, c, d, m, n}, x] &&  !IntegerQ[n] && (IntegerQ[m]
 || GtQ[-d/(b*c), 0])
```

```
Int[((b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[((-b)*(c/
d))^IntPart[m]*((b*x)^FracPart[m]/((-d)*(x/c))^FracPart[m])   Int[((-d)*(x/
c))^m*(c + d*x)^n, x], x] /; FreeQ[{b, c, d, m, n}, x] &&  !IntegerQ[m] &&
 !IntegerQ[n] &&  !GtQ[c, 0] &&  !GtQ[-d/(b*c), 0]
```

```
Int[((e_.)*(x_))^(m_)*((b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(
e*x)^m*((b*x + c*x^2)^p/(x^(m + p)*(b + c*x)^p))   Int[x^(m + p)*(b + c*x)^
p, x], x] /; FreeQ[{b, c, e, m}, x]
```

Maple [F]

$$\int \frac{(cx)^m}{(b\,x^2+ax)^{\frac{5}{2}}}dx$$

```
int((c*x)^m/(b*x^2+a*x)^(5/2),x)
```

```
int((c*x)^m/(b*x^2+a*x)^(5/2),x)
```

Fricas [F]

$$\int \frac{(cx)^m}{(ax+bx^2)^{5/2}}\,dx = \int \frac{(cx)^m}{(bx^2+ax)^{\frac{5}{2}}}\,dx$$

```
integrate((c*x)^m/(b*x^2+a*x)^(5/2),x, algorithm="fricas")
```

```
integral(sqrt(b*x^2 + a*x)*(c*x)^m/(b^3*x^6 + 3*a*b^2*x^5 + 3*a^2*b*x^4 +
a^3*x^3), x)
```

Sympy [F]

$$\int \frac{(cx)^m}{(ax+bx^2)^{5/2}}\,dx = \int \frac{(cx)^m}{(x\,(a+bx))^{\frac{5}{2}}}\,dx$$

```
integrate((c*x)**m/(b*x**2+a*x)**(5/2),x)
```

```
Integral((c*x)**m/(x*(a + b*x))**(5/2), x)
```

Maxima [F]

$$\int \frac{(cx)^m}{(ax+bx^2)^{5/2}}\,dx = \int \frac{(cx)^m}{(bx^2+ax)^{\frac{5}{2}}}\,dx$$

```
integrate((c*x)^m/(b*x^2+a*x)^(5/2),x, algorithm="maxima")
```

```
integrate((c*x)^m/(b*x^2 + a*x)^(5/2), x)
```

Giac [F]

$$\int \frac{(cx)^m}{(ax+bx^2)^{5/2}}\,dx = \int \frac{(cx)^m}{(bx^2+ax)^{\frac{5}{2}}}\,dx$$

```
integrate((c*x)^m/(b*x^2+a*x)^(5/2),x, algorithm="giac")
```

```
integrate((c*x)^m/(b*x^2 + a*x)^(5/2), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(cx)^m}{(ax+bx^2)^{5/2}}\,dx=\int \frac{(c\,x)^m}{(b\,x^2+a\,x)^{5/2}}\,dx$$

```
int((c*x)^m/(a*x + b*x^2)^(5/2),x)
```

```
int((c*x)^m/(a*x + b*x^2)^(5/2), x)
```

Reduce [F]

$$\int \frac{(cx)^m}{(ax+bx^2)^{5/2}}\,dx=c^m\left(\int \frac{x^m}{\sqrt{x}\,\sqrt{bx+a}\,a^2x^2+2\sqrt{x}\,\sqrt{bx+a}\,ab\,x^3+\sqrt{x}\,\sqrt{bx+a}\,b^2x^4}dx\right)$$

```
int((c*x)^m/(b*x^2+a*x)^(5/2),x)
```

```
c**m*int(x**m/(sqrt(x)*sqrt(a + b*x)*a**2*x**2 + 2*sqrt(x)*sqrt(a + b*x)*a
*b*x**3 + sqrt(x)*sqrt(a + b*x)*b**2*x**4),x)
```

3.228 $\int x^2 (ax + bx^2)^p \, dx$

Optimal result

Integrand size = 15, antiderivative size = 42

$$\int x^2 \left(ax + bx^2\right)^p \, dx = \frac{x^2 (ax + bx^2)^{1+p} \text{Hypergeometric2F1}\left(1, 2(2+p), 4+p, -\frac{bx}{a}\right)}{a(3+p)}$$

```
x^2*(b*x^2+a*x)^(p+1)*hypergeom([1, 4+2*p],[4+p],-b*x/a)/a/(3+p)
```

Mathematica [A] (verified)

Time = 0.03 (sec) , antiderivative size = 47, normalized size of antiderivative = 1.12

$$\int x^2 \left(ax + bx^2\right)^p \, dx = \frac{x^3 (x(a+bx))^p \left(1 + \frac{bx}{a}\right)^{-p} \text{Hypergeometric2F1}\left(-p, 3+p, 4+p, -\frac{bx}{a}\right)}{3+p}$$

```
Integrate[x^2*(a*x + b*x^2)^p,x]
```

```
(x^3*(x*(a + b*x))^p*Hypergeometric2F1[-p, 3 + p, 4 + p, -((b*x)/a)])/((3
+ p)*(1 + (b*x)/a)^p)
```

Rubi [A] (verified)

Time = 0.28 (sec) , antiderivative size = 49, normalized size of antiderivative = 1.17, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.200$, Rules used = {1137, 76, 74}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^2 (ax + bx^2)^p \, dx$$

$$\Big\downarrow \text{ 1137}$$

$$x^{-p}(a+bx)^{-p} \left(ax+bx^2\right)^p \int x^{p+2}(a+bx)^p dx$$

$$\Big\downarrow \text{ 76}$$

$$x^{-p}\left(\frac{bx}{a}+1\right)^{-p} \left(ax+bx^2\right)^p \int x^{p+2}\left(\frac{bx}{a}+1\right)^p dx$$

$$\Big\downarrow \text{ 74}$$

$$\frac{x^3 \left(\frac{bx}{a}+1\right)^{-p} (ax+bx^2)^p \text{Hypergeometric2F1}\left(-p,p+3,p+4,-\frac{bx}{a}\right)}{p+3}$$

```
Int[x^2*(a*x + b*x^2)^p,x]
```

```
(x^3*(a*x + b*x^2)^p*Hypergeometric2F1[-p, 3 + p, 4 + p, -((b*x)/a)])/((3
+ p)*(1 + (b*x)/a)^p)
```

Defintions of rubi rules used

```
Int[((b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[c^n*((b*x
)^(m + 1)/(b*(m + 1)))*Hypergeometric2F1[-n, m + 1, m + 2, (-d)*(x/c)], x]
/; FreeQ[{b, c, d, m, n}, x] && !IntegerQ[m] && (IntegerQ[n] || (GtQ[c, 0]
 && !(EqQ[n, -2^(-1)] && EqQ[c^2 - d^2, 0] && GtQ[-d/(b*c), 0])))
```

```
Int[((b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[c^IntPart
[n]*((c + d*x)^FracPart[n]/(1 + d*(x/c))^FracPart[n])   Int[(b*x)^m*(1 + d*
(x/c))^n, x], x] /; FreeQ[{b, c, d, m, n}, x] &&  !IntegerQ[m] &&  !Integer
Q[n] &&  !GtQ[c, 0] &&  !GtQ[-d/(b*c), 0] && ((RationalQ[m] &&  !(EqQ[n, -2
^(-1)] && EqQ[c^2 - d^2, 0])) ||  !RationalQ[n])
```

```
Int[((e_.)*(x_))^(m_)*((b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(
e*x)^m*((b*x + c*x^2)^p/(x^(m + p)*(b + c*x)^p))   Int[x^(m + p)*(b + c*x)^
p, x], x] /; FreeQ[{b, c, e, m}, x]
```

Maple [F]

$$\int x^{2} \left(b \,x^{2}+a x \right)^{p}d x$$

```
int(x^2*(b*x^2+a*x)^p,x)
```

```
int(x^2*(b*x^2+a*x)^p,x)
```

Fricas [F]

$$\int x^2 (ax + bx^2)^p \, dx = \int (bx^2 + ax)^p x^2 \, dx$$

```
integrate(x^2*(b*x^2+a*x)^p,x, algorithm="fricas")
```

```
integral((b*x^2 + a*x)^p*x^2, x)
```

Sympy [F]

$$\int x^2 (ax+bx^2)^p \, dx = \int x^2 (x(a+bx))^p \, dx$$

```
integrate(x**2*(b*x**2+a*x)**p,x)
```

```
Integral(x**2*(x*(a + b*x))**p, x)
```

Maxima [F]

$$\int x^2 (ax+bx^2)^p \, dx = \int (bx^2+ax)^p x^2 \, dx$$

```
integrate(x^2*(b*x^2+a*x)^p,x, algorithm="maxima")
```

```
integrate((b*x^2 + a*x)^p*x^2, x)
```

Giac [F]

$$\int x^2 (ax+bx^2)^p \, dx = \int (bx^2+ax)^p x^2 \, dx$$

```
integrate(x^2*(b*x^2+a*x)^p,x, algorithm="giac")
```

```
integrate((b*x^2 + a*x)^p*x^2, x)
```

Mupad [F(-1)]

Timed out.

$$\int x^2(ax+bx^2)^p\,dx=\int x^2\,\left(b\,x^2+a\,x\right)^p\,dx$$

```
int(x^2*(a*x + b*x^2)^p,x)
```

```
int(x^2*(a*x + b*x^2)^p, x)
```

Reduce [F]

$$\int x^2(ax+bx^2)^p\,dx$$
$$=\frac{(b\,x^2+ax)^p\,a^3p^2+3(b\,x^2+ax)^p\,a^3p+2(b\,x^2+ax)^p\,a^3-2(b\,x^2+ax)^p\,a^2b\,p^2x-4(b\,x^2+ax)^p\,a^2bpx+}{}$$

```
int(x^2*(b*x^2+a*x)^p,x)
```

```
((a*x + b*x**2)**p*a**3*p**2 + 3*(a*x + b*x**2)**p*a**3*p + 2*(a*x + b*x**
2)**p*a**3 - 2*(a*x + b*x**2)**p*a**2*b*p**2*x - 4*(a*x + b*x**2)**p*a**2*
b*p*x + 4*(a*x + b*x**2)**p*a*b**2*p**2*x**2 + 2*(a*x + b*x**2)**p*a*b**2*
p*x**2 + 8*(a*x + b*x**2)**p*b**3*p**2*x**3 + 12*(a*x + b*x**2)**p*b**3*p*
x**3 + 4*(a*x + b*x**2)**p*b**3*x**3 - 4*int((a*x + b*x**2)**p/(4*a*p**2*x
 + 8*a*p*x + 3*a*x + 4*b*p**2*x**2 + 8*b*p*x**2 + 3*b*x**2),x)*a**4*p**5 -
 20*int((a*x + b*x**2)**p/(4*a*p**2*x + 8*a*p*x + 3*a*x + 4*b*p**2*x**2 +
8*b*p*x**2 + 3*b*x**2),x)*a**4*p**4 - 35*int((a*x + b*x**2)**p/(4*a*p**2*x
 + 8*a*p*x + 3*a*x + 4*b*p**2*x**2 + 8*b*p*x**2 + 3*b*x**2),x)*a**4*p**3 -
 25*int((a*x + b*x**2)**p/(4*a*p**2*x + 8*a*p*x + 3*a*x + 4*b*p**2*x**2 +
8*b*p*x**2 + 3*b*x**2),x)*a**4*p**2 - 6*int((a*x + b*x**2)**p/(4*a*p**2*x
+ 8*a*p*x + 3*a*x + 4*b*p**2*x**2 + 8*b*p*x**2 + 3*b*x**2),x)*a**4*p)/(4*b
**3*(4*p**3 + 12*p**2 + 11*p + 3))
```

3.229 $\int x(ax+bx^2)^p \, dx$

Optimal result

Integrand size = 13, antiderivative size = 40

$$\int x(ax+bx^2)^p \, dx = \frac{x(ax+bx^2)^{1+p}\,\text{Hypergeometric2F1}\left(1,3+2p,3+p,-\frac{bx}{a}\right)}{a(2+p)}$$

```
x*(b*x^2+a*x)^(p+1)*hypergeom([1, 3+2*p],[3+p],-b*x/a)/a/(2+p)
```

Mathematica [A] (verified)

Time = 0.03 (sec) , antiderivative size = 47, normalized size of antiderivative = 1.18

$$\int x(ax+bx^2)^p \, dx = \frac{x^2(x(a+bx))^p\left(1+\frac{bx}{a}\right)^{-p}\text{Hypergeometric2F1}\left(-p,2+p,3+p,-\frac{bx}{a}\right)}{2+p}$$

```
Integrate[x*(a*x + b*x^2)^p,x]
```

```
(x^2*(x*(a + b*x))^p*Hypergeometric2F1[-p, 2 + p, 3 + p, -((b*x)/a)])/((2
+ p)*(1 + (b*x)/a)^p)
```

Rubi [B] (verified)

Leaf count is larger than twice the leaf count of optimal. 83 vs. $2(40) = 80$.

Time = 0.33 (sec) , antiderivative size = 83, normalized size of antiderivative = 2.08, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.154$, Rules used = {1160, 1096}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x\left(ax+bx^2\right)^p\,dx$$

$$\Big\downarrow \; 1160$$

$$\frac{\left(ax+bx^2\right)^{p+1}}{2b(p+1)}-\frac{a\int \left(bx^2+ax\right)^p\,dx}{2b}$$

$$\Big\downarrow \; 1096$$

$$\frac{\left(ax+bx^2\right)^{p+1}\left(-\frac{bx}{a}\right)^{-p-1}\text{Hypergeometric2F1}\left(-p,p+1,p+2,\frac{a+bx}{a}\right)}{2b(p+1)}+\frac{\left(ax+bx^2\right)^{p+1}}{2b(p+1)}$$

```
Int[x*(a*x + b*x^2)^p,x]
```

```
(a*x + b*x^2)^(1 + p)/(2*b*(1 + p)) + ((-((b*x)/a))^(-1 - p)*(a*x + b*x^2)
^(1 + p)*Hypergeometric2F1[-p, 1 + p, 2 + p, (a + b*x)/a])/(2*b*(1 + p))
```

Defintions of rubi rules used

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> With[{q = Rt[b^2
 - 4*a*c, 2]}, Simp[(-(a + b*x + c*x^2)^(p + 1)/(q*(p + 1)*((q - b - 2*c*x)
/(2*q))^(p + 1)))*Hypergeometric2F1[-p, p + 1, p + 2, (b + q + 2*c*x)/(2*q)
], x]] /; FreeQ[{a, b, c, p}, x] && !IntegerQ[4*p] && !IntegerQ[3*p]
```

```
Int[((d_.) + (e_.)*(x_))*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol
] :> Simp[e*((a + b*x + c*x^2)^(p + 1)/(2*c*(p + 1))), x] + Simp[(2*c*d - b
*e)/(2*c)   Int[(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, p}, x]
 && NeQ[p, -1]
```

Maple [F]

$$\int x\left(b\,x^{2}+ax\right)^{p}dx$$

```
int(x*(b*x^2+a*x)^p,x)
```

```
int(x*(b*x^2+a*x)^p,x)
```

Fricas [F]

$$\int x(ax+bx^2)^p\;dx=\int\left(bx^2+ax\right)^p x\,dx$$

```
integrate(x*(b*x^2+a*x)^p,x, algorithm="fricas")
```

```
integral((b*x^2 + a*x)^p*x, x)
```

Sympy [F]

$$\int x(ax+bx^2)^p\;dx=\int x(x(a+bx))^p\;dx$$

```
integrate(x*(b*x**2+a*x)**p,x)
```

```
Integral(x*(x*(a + b*x))**p, x)
```

Maxima [F]

$$\int x(ax+bx^2)^p \, dx = \int (bx^2+ax)^p x \, dx$$

```
integrate(x*(b*x^2+a*x)^p,x, algorithm="maxima")
```

```
integrate((b*x^2 + a*x)^p*x, x)
```

Giac [F]

$$\int x(ax+bx^2)^p \, dx = \int (bx^2+ax)^p x \, dx$$

```
integrate(x*(b*x^2+a*x)^p,x, algorithm="giac")
```

```
integrate((b*x^2 + a*x)^p*x, x)
```

Mupad [F(-1)]

Timed out.

$$\int x(ax+bx^2)^p \, dx = \int x \left(b\,x^2+a\,x\right)^p \, dx$$

```
int(x*(a*x + b*x^2)^p,x)
```

```
int(x*(a*x + b*x^2)^p, x)
```

Reduce [F]

$$\int x(ax+bx^2)^p\,dx$$
$$= \frac{-(b\,x^2+ax)^p\,a^2p-(b\,x^2+ax)^p\,a^2+2(b\,x^2+ax)^p\,abpx+4(b\,x^2+ax)^p\,b^2p\,x^2+2(b\,x^2+ax)^p\,b^2x^2+2}{4b^2\,(2p^2+3p}$$

```
int(x*(b*x^2+a*x)^p,x)
```

```
( - (a*x + b*x**2)**p*a**2*p - (a*x + b*x**2)**p*a**2 + 2*(a*x + b*x**2)**
p*a*b*p*x + 4*(a*x + b*x**2)**p*b**2*p*x**2 + 2*(a*x + b*x**2)**p*b**2*x**
2 + 2*int((a*x + b*x**2)**p/(2*a*p*x + a*x + 2*b*p*x**2 + b*x**2),x)*a**3*
p**3 + 3*int((a*x + b*x**2)**p/(2*a*p*x + a*x + 2*b*p*x**2 + b*x**2),x)*a*
*3*p**2 + int((a*x + b*x**2)**p/(2*a*p*x + a*x + 2*b*p*x**2 + b*x**2),x)*a
**3*p)/(4*b**2*(2*p**2 + 3*p + 1))
```

3.230 $\int (ax+bx^2)^p \, dx$

Optimal result

Integrand size = 11, antiderivative size = 39

$$\int (ax+bx^2)^p \, dx = \frac{(ax+bx^2)^{1+p} \operatorname{Hypergeometric2F1}\left(1,2(1+p),2+p,-\frac{bx}{a}\right)}{a(1+p)}$$

```
(b*x^2+a*x)^(p+1)*hypergeom([1, 2*p+2],[2+p],-b*x/a)/a/(p+1)
```

Mathematica [A] (verified)

Time = 0.03 (sec) , antiderivative size = 45, normalized size of antiderivative = 1.15

$$\int (ax+bx^2)^p \, dx = \frac{x(x(a+bx))^p \left(1+\frac{bx}{a}\right)^{-p} \operatorname{Hypergeometric2F1}\left(-p,1+p,2+p,-\frac{bx}{a}\right)}{1+p}$$

```
Integrate[(a*x + b*x^2)^p,x]
```

```
(x*(x*(a + b*x))^p*Hypergeometric2F1[-p, 1 + p, 2 + p, -((b*x)/a)])/((1 +
p)*(1 + (b*x)/a)^p)
```

Rubi [A] (verified)

Time = 0.27 (sec) , antiderivative size = 55, normalized size of antiderivative = 1.41, number of steps used = 1, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.091, Rules used = {1096}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \left(ax+bx^2\right)^p\,dx$$

$$\Big\downarrow \ 1096$$

$$-\frac{\left(-\frac{bx}{a}\right)^{-p-1}\left(ax+bx^2\right)^{p+1}\text{Hypergeometric2F1}\left(-p,p+1,p+2,\frac{a+bx}{a}\right)}{a(p+1)}$$

```
Int[(a*x + b*x^2)^p,x]
```

```
-(((-((b*x)/a))^(-1 - p)*(a*x + b*x^2)^(1 + p)*Hypergeometric2F1[-p, 1 + p
, 2 + p, (a + b*x)/a])/(a*(1 + p)))
```

Defintions of rubi rules used

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> With[{q = Rt[b^2
 - 4*a*c, 2]}, Simp[(-(a + b*x + c*x^2)^(p + 1)/(q*(p + 1)*((q - b - 2*c*x)
/(2*q))^(p + 1)))*Hypergeometric2F1[-p, p + 1, p + 2, (b + q + 2*c*x)/(2*q)
], x]] /; FreeQ[{a, b, c, p}, x] &&  !IntegerQ[4*p] &&  !IntegerQ[3*p]
```

Maple [F]

$$\int \left(b\,x^2+ax\right)^p dx$$

```
int((b*x^2+a*x)^p,x)
```

```
int((b*x^2+a*x)^p,x)
```

Fricas [F]

$$\int \left(ax + bx^2\right)^p \, dx = \int \left(bx^2 + ax\right)^p \, dx$$

```
integrate((b*x^2+a*x)^p,x, algorithm="fricas")
```

```
integral((b*x^2 + a*x)^p, x)
```

Sympy [F]

$$\int \left(ax + bx^2\right)^p \, dx = \int \left(ax + bx^2\right)^p \, dx$$

```
integrate((b*x**2+a*x)**p,x)
```

```
Integral((a*x + b*x**2)**p, x)
```

Maxima [F]

$$\int \left(ax + bx^2\right)^p \, dx = \int \left(bx^2 + ax\right)^p \, dx$$

```
integrate((b*x^2+a*x)^p,x, algorithm="maxima")
```

```
integrate((b*x^2 + a*x)^p, x)
```

Giac [F]

$$\int \left(ax+bx^2\right)^p\,dx=\int \left(bx^2+ax\right)^p\,dx$$

```
integrate((b*x^2+a*x)^p,x, algorithm="giac")
```

```
integrate((b*x^2 + a*x)^p, x)
```

Mupad [B] (verification not implemented)

Time = 9.14 (sec) , antiderivative size = 48, normalized size of antiderivative = 1.23

$$\int \left(ax+bx^2\right)^p\,dx=\frac{x\,\left(b\,x^2+a\,x\right)^p\,{}_2\mathrm{F}_1\left(-p,p+1;\ p+2;\ -\frac{b\,x}{a}\right)}{\left(\frac{b\,x}{a}+1\right)^p\,\left(p+1\right)}$$

```
int((a*x + b*x^2)^p,x)
```

```
(x*(a*x + b*x^2)^p*hypergeom([-p, p + 1], p + 2, -(b*x)/a))/(((b*x)/a + 1)
^p*(p + 1))
```

Reduce [F]

$$\int \left(ax+bx^2\right)^p\,dx$$
$$=\frac{(b\,x^2+ax)^p\,a+2(b\,x^2+ax)^p\,bx-2\left(\int\frac{(b\,x^2+ax)^p}{2bp\,x^2+2apx+b\,x^2+ax}dx\right)a^2p^2-\left(\int\frac{(b\,x^2+ax)^p}{2bp\,x^2+2apx+b\,x^2+ax}dx\right)a^2p}{2b\,(2p+1)}$$

```
int((b*x^2+a*x)^p,x)
```

```
((a*x + b*x**2)**p*a + 2*(a*x + b*x**2)**p*b*x - 2*int((a*x + b*x**2)**p/(
2*a*p*x + a*x + 2*b*p*x**2 + b*x**2),x)*a**2*p**2 - int((a*x + b*x**2)**p/
(2*a*p*x + a*x + 2*b*p*x**2 + b*x**2),x)*a**2*p)/(2*b*(2*p + 1))
```

3.231 $\int \frac{(ax+bx^2)^p}{x}\,dx$

Optimal result

Integrand size = 15, antiderivative size = 40

$$\int \frac{(ax+bx^2)^p}{x}\,dx = \frac{(ax+bx^2)^{1+p}\,\text{Hypergeometric2F1}\left(1,1+2p,1+p,-\frac{bx}{a}\right)}{apx}$$

```
(b*x^2+a*x)^(p+1)*hypergeom([1, 1+2*p],[p+1],-b*x/a)/a/p/x
```

Mathematica [A] (verified)

Time = 0.04 (sec) , antiderivative size = 40, normalized size of antiderivative = 1.00

$$\int \frac{(ax+bx^2)^p}{x}\,dx = \frac{(x(a+bx))^p\left(1+\frac{bx}{a}\right)^{-p}\text{Hypergeometric2F1}\left(-p,p,1+p,-\frac{bx}{a}\right)}{p}$$

```
Integrate[(a*x + b*x^2)^p/x,x]
```

```
((x*(a + b*x))^p*Hypergeometric2F1[-p, p, 1 + p, -((b*x)/a)])/(p*(1 + (b*x
)/a)^p)
```

Rubi [A] (verified)

Time = 0.27 (sec) , antiderivative size = 42, normalized size of antiderivative = 1.05, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.200$, Rules used = {1137, 76, 74}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(ax+bx^2)^p}{x} \, dx$$

$$\Big\downarrow \text{1137}$$

$$x^{-p}(a+bx)^{-p}\left(ax+bx^2\right)^p \int x^{p-1}(a+bx)^p dx$$

$$\Big\downarrow \text{76}$$

$$x^{-p}\left(\frac{bx}{a}+1\right)^{-p}\left(ax+bx^2\right)^p \int x^{p-1}\left(\frac{bx}{a}+1\right)^p dx$$

$$\Big\downarrow \text{74}$$

$$\frac{\left(\frac{bx}{a}+1\right)^{-p}\left(ax+bx^2\right)^p \text{Hypergeometric2F1}\left(-p,p,p+1,-\frac{bx}{a}\right)}{p}$$

```
Int[(a*x + b*x^2)^p/x,x]
```

```
((a*x + b*x^2)^p*Hypergeometric2F1[-p, p, 1 + p, -((b*x)/a)])/(p*(1 + (b*x
)/a)^p)
```

Defintions of rubi rules used

```
Int[((b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[c^n*((b*x
)^(m + 1)/(b*(m + 1)))*Hypergeometric2F1[-n, m + 1, m + 2, (-d)*(x/c)], x]
/; FreeQ[{b, c, d, m, n}, x] &&  !IntegerQ[m] && (IntegerQ[n] || (GtQ[c, 0]
 &&  !(EqQ[n, -2^(-1)] && EqQ[c^2 - d^2, 0] && GtQ[-d/(b*c), 0])))
```

```
Int[((b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[c^IntPart
[n]*((c + d*x)^FracPart[n]/(1 + d*(x/c))^FracPart[n])   Int[(b*x)^m*(1 + d*
(x/c))^n, x], x] /; FreeQ[{b, c, d, m, n}, x] &&  !IntegerQ[m] &&  !Integer
Q[n] &&  !GtQ[c, 0] &&  !GtQ[-d/(b*c), 0] && ((RationalQ[m] &&  !(EqQ[n, -2
^(-1)] && EqQ[c^2 - d^2, 0])) ||  !RationalQ[n])
```

```
Int[((e_.)*(x_))^(m_)*((b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(
e*x)^m*((b*x + c*x^2)^p/(x^(m + p)*(b + c*x)^p))   Int[x^(m + p)*(b + c*x)^
p, x], x] /; FreeQ[{b, c, e, m}, x]
```

Maple [F]

$$\int \frac{(b\,x^2 + ax)^p}{x}dx$$

```
int((b*x^2+a*x)^p/x,x)
```

```
int((b*x^2+a*x)^p/x,x)
```

Fricas [F]

$$\int \frac{(ax + bx^2)^p}{x}\,dx = \int \frac{(bx^2 + ax)^p}{x}\,dx$$

```
integrate((b*x^2+a*x)^p/x,x, algorithm="fricas")
```

```
integral((b*x^2 + a*x)^p/x, x)
```

Sympy [F]

$$\int \frac{(ax + bx^2)^p}{x} \, dx = \int \frac{(x(a + bx))^p}{x} \, dx$$

```
integrate((b*x**2+a*x)**p/x,x)
```

```
Integral((x*(a + b*x))**p/x, x)
```

Maxima [F]

$$\int \frac{(ax + bx^2)^p}{x} \, dx = \int \frac{(bx^2 + ax)^p}{x} \, dx$$

```
integrate((b*x^2+a*x)^p/x,x, algorithm="maxima")
```

```
integrate((b*x^2 + a*x)^p/x, x)
```

Giac [F]

$$\int \frac{(ax + bx^2)^p}{x} \, dx = \int \frac{(bx^2 + ax)^p}{x} \, dx$$

```
integrate((b*x^2+a*x)^p/x,x, algorithm="giac")
```

```
integrate((b*x^2 + a*x)^p/x, x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(ax+bx^2)^p}{x}\,dx=\int \frac{{\left(b\,x^2+a\,x\right)}^p}{x}\,dx$$

```
int((a*x + b*x^2)^p/x,x)
```

```
int((a*x + b*x^2)^p/x, x)
```

Reduce [F]

$$\int \frac{(ax+bx^2)^p}{x}\,dx=\frac{(b\,x^2+ax)^p+\left(\int \frac{(b\,x^2+ax)^p}{b\,x^2+ax}dx\right)ap}{2p}$$

```
int((b*x^2+a*x)^p/x,x)
```

```
((a*x + b*x**2)**p + int((a*x + b*x**2)**p/(a*x + b*x**2),x)*a*p)/(2*p)
```

3.232 $\int \frac{(ax+bx^2)^p}{x^2} dx$

Optimal result

Integrand size = 15, antiderivative size = 41

$$\int \frac{(ax+bx^2)^p}{x^2} dx = -\frac{(ax+bx^2)^{1+p} \text{Hypergeometric2F1}\left(1,2p,p,-\frac{bx}{a}\right)}{a(1-p)x^2}$$

```
-(b*x^2+a*x)^(p+1)*hypergeom([1, 2*p],[p],-b*x/a)/a/(1-p)/x^2
```

Mathematica [A] (verified)

Time = 0.03 (sec) , antiderivative size = 45, normalized size of antiderivative = 1.10

$$\int \frac{(ax+bx^2)^p}{x^2} dx = \frac{(x(a+bx))^p \left(1+\frac{bx}{a}\right)^{-p} \text{Hypergeometric2F1}\left(-1+p,-p,p,-\frac{bx}{a}\right)}{(-1+p)x}$$

```
Integrate[(a*x + b*x^2)^p/x^2,x]
```

```
((x*(a + b*x))^p*Hypergeometric2F1[-1 + p, -p, p, -((b*x)/a)])/((-1 + p)*x
*(1 + (b*x)/a)^p)
```

Rubi [A] (verified)

Time = 0.29 (sec) , antiderivative size = 50, normalized size of antiderivative = 1.22, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.200$, Rules used = {1137, 76, 74}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(ax+bx^2)^p}{x^2} dx$$

$$\Big\downarrow \ 1137$$

$$x^{-p}(a+bx)^{-p} \left(ax+bx^2\right)^p \int x^{p-2}(a+bx)^p dx$$

$$\Big\downarrow \ 76$$

$$x^{-p}\left(\frac{bx}{a}+1\right)^{-p} \left(ax+bx^2\right)^p \int x^{p-2}\left(\frac{bx}{a}+1\right)^p dx$$

$$\Big\downarrow \ 74$$

$$-\frac{\left(\frac{bx}{a}+1\right)^{-p} \left(ax+bx^2\right)^p \text{Hypergeometric2F1}\left(p-1,-p,p,-\frac{bx}{a}\right)}{(1-p)x}$$

```
Int[(a*x + b*x^2)^p/x^2,x]
```

```
-(((a*x + b*x^2)^p*Hypergeometric2F1[-1 + p, -p, p, -((b*x)/a)])/((1 - p)*
x*(1 + (b*x)/a)^p))
```

Defintions of rubi rules used

```
Int[((b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[c^n*((b*x
)^(m + 1)/(b*(m + 1)))*Hypergeometric2F1[-n, m + 1, m + 2, (-d)*(x/c)], x]
/; FreeQ[{b, c, d, m, n}, x] &&  !IntegerQ[m] && (IntegerQ[n] || (GtQ[c, 0]
 &&  !(EqQ[n, -2^(-1)] && EqQ[c^2 - d^2, 0] && GtQ[-d/(b*c), 0])))
```

```
Int[((b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[c^IntPart
[n]*((c + d*x)^FracPart[n]/(1 + d*(x/c))^FracPart[n])   Int[(b*x)^m*(1 + d*
(x/c))^n, x], x] /; FreeQ[{b, c, d, m, n}, x] &&  !IntegerQ[m] &&  !Integer
Q[n] &&  !GtQ[c, 0] &&  !GtQ[-d/(b*c), 0] && ((RationalQ[m] &&  !(EqQ[n, -2
^(-1)] && EqQ[c^2 - d^2, 0])) ||  !RationalQ[n])
```

```
Int[((e_.)*(x_))^(m_)*((b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(
e*x)^m*((b*x + c*x^2)^p/(x^(m + p)*(b + c*x)^p))   Int[x^(m + p)*(b + c*x)^
p, x], x] /; FreeQ[{b, c, e, m}, x]
```

Maple [F]

$$\int \frac{(b\,x^{2}+ax)^{p}}{x^{2}}dx$$

```
int((b*x^2+a*x)^p/x^2,x)
```

```
int((b*x^2+a*x)^p/x^2,x)
```

Fricas [F]

$$\int \frac{(ax+bx^2)^p}{x^2}\,dx = \int \frac{(bx^2+ax)^p}{x^2}\,dx$$

```
integrate((b*x^2+a*x)^p/x^2,x, algorithm="fricas")
```

```
integral((b*x^2 + a*x)^p/x^2, x)
```

Sympy [F]

$$\int \frac{(ax + bx^2)^p}{x^2} dx = \int \frac{(x(a + bx))^p}{x^2} dx$$

```
integrate((b*x**2+a*x)**p/x**2,x)
```

```
Integral((x*(a + b*x))**p/x**2, x)
```

Maxima [F]

$$\int \frac{(ax + bx^2)^p}{x^2} dx = \int \frac{(bx^2 + ax)^p}{x^2} dx$$

```
integrate((b*x^2+a*x)^p/x^2,x, algorithm="maxima")
```

```
integrate((b*x^2 + a*x)^p/x^2, x)
```

Giac [F]

$$\int \frac{(ax + bx^2)^p}{x^2} dx = \int \frac{(bx^2 + ax)^p}{x^2} dx$$

```
integrate((b*x^2+a*x)^p/x^2,x, algorithm="giac")
```

```
integrate((b*x^2 + a*x)^p/x^2, x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(ax+bx^2)^p}{x^2}\,dx = \int \frac{(b\,x^2+a\,x)^p}{x^2}\,dx$$

```
int((a*x + b*x^2)^p/x^2,x)
```

```
int((a*x + b*x^2)^p/x^2, x)
```

Reduce [F]

$$\int \frac{(ax+bx^2)^p}{x^2}\,dx = \frac{(b\,x^2+ax)^p+2\left(\int \frac{(b\,x^2+ax)^p}{2bp\,x^3+2ap\,x^2-b\,x^3-a\,x^2}dx\right)a\,p^2x-\left(\int \frac{(b\,x^2+ax)^p}{2bp\,x^3+2ap\,x^2-b\,x^3-a\,x^2}dx\right)apx}{x\,(2p-1)}$$

```
int((b*x^2+a*x)^p/x^2,x)
```

```
((a*x + b*x**2)**p + 2*int((a*x + b*x**2)**p/(2*a*p*x**2 - a*x**2 + 2*b*p*
x**3 - b*x**3),x)*a*p**2*x - int((a*x + b*x**2)**p/(2*a*p*x**2 - a*x**2 +
2*b*p*x**3 - b*x**3),x)*a*p*x)/(x*(2*p - 1))
```

3.233 $\int \frac{(ax+bx^2)^p}{x^3}\,dx$

Optimal result

Integrand size = 15, antiderivative size = 45

$$\int \frac{(ax+bx^2)^p}{x^3}\,dx = -\frac{(ax+bx^2)^{1+p}\,\text{Hypergeometric2F1}\left(1,-1+2p,-1+p,-\frac{bx}{a}\right)}{a(2-p)x^3}$$

```
-(b*x^2+a*x)^(p+1)*hypergeom([1, -1+2*p],[-1+p],-b*x/a)/a/(2-p)/x^3
```

Mathematica [A] (verified)

Time = 0.03 (sec) , antiderivative size = 47, normalized size of antiderivative = 1.04

$$\int \frac{(ax+bx^2)^p}{x^3}\,dx = \frac{(x(a+bx))^p\left(1+\frac{bx}{a}\right)^{-p}\text{Hypergeometric2F1}\left(-2+p,-p,-1+p,-\frac{bx}{a}\right)}{(-2+p)x^2}$$

```
Integrate[(a*x + b*x^2)^p/x^3,x]
```

```
((x*(a + b*x))^p*Hypergeometric2F1[-2 + p, -p, -1 + p, -((b*x)/a)])/((-2 +
 p)*x^2*(1 + (b*x)/a)^p)
```

Rubi [A] (verified)

Time = 0.29 (sec) , antiderivative size = 52, normalized size of antiderivative = 1.16, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.200$, Rules used = {1137, 76, 74}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(ax+bx^2)^p}{x^3}dx$$
$$\Big\downarrow \text{ 1137}$$
$$x^{-p}(a+bx)^{-p}\left(ax+bx^2\right)^p\int x^{p-3}(a+bx)^pdx$$
$$\Big\downarrow \text{ 76}$$
$$x^{-p}\left(\frac{bx}{a}+1\right)^{-p}\left(ax+bx^2\right)^p\int x^{p-3}\left(\frac{bx}{a}+1\right)^pdx$$
$$\Big\downarrow \text{ 74}$$
$$-\frac{\left(\frac{bx}{a}+1\right)^{-p}\left(ax+bx^2\right)^p\text{Hypergeometric2F1}\left(p-2,-p,p-1,-\frac{bx}{a}\right)}{(2-p)x^2}$$

```
Int[(a*x + b*x^2)^p/x^3,x]
```

```
-(((a*x + b*x^2)^p*Hypergeometric2F1[-2 + p, -p, -1 + p, -((b*x)/a)])/((2
- p)*x^2*(1 + (b*x)/a)^p))
```

Defintions of rubi rules used

```
Int[((b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[c^n*((b*x
)^(m + 1)/(b*(m + 1)))*Hypergeometric2F1[-n, m + 1, m + 2, (-d)*(x/c)], x]
/; FreeQ[{b, c, d, m, n}, x] &&  !IntegerQ[m] && (IntegerQ[n] || (GtQ[c, 0]
 &&  !(EqQ[n, -2^(-1)] && EqQ[c^2 - d^2, 0] && GtQ[-d/(b*c), 0])))
```

```
Int[((b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[c^IntPart
[n]*((c + d*x)^FracPart[n]/(1 + d*(x/c))^FracPart[n])   Int[(b*x)^m*(1 + d*
(x/c))^n, x], x] /; FreeQ[{b, c, d, m, n}, x] &&  !IntegerQ[m] &&  !Integer
Q[n] &&  !GtQ[c, 0] &&  !GtQ[-d/(b*c), 0] && ((RationalQ[m] &&  !(EqQ[n, -2
^(-1)] && EqQ[c^2 - d^2, 0])) ||  !RationalQ[n])
```

```
Int[((e_.)*(x_))^(m_)*((b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(
e*x)^m*((b*x + c*x^2)^p/(x^(m + p)*(b + c*x)^p))   Int[x^(m + p)*(b + c*x)^
p, x], x] /; FreeQ[{b, c, e, m}, x]
```

Maple [F]

$$\int \frac{(b\,x^2+ax)^p}{x^3}dx$$

```
int((b*x^2+a*x)^p/x^3,x)
```

```
int((b*x^2+a*x)^p/x^3,x)
```

Fricas [F]

$$\int \frac{(ax+bx^2)^p}{x^3}\,dx = \int \frac{(bx^2+ax)^p}{x^3}\,dx$$

```
integrate((b*x^2+a*x)^p/x^3,x, algorithm="fricas")
```

```
integral((b*x^2 + a*x)^p/x^3, x)
```

Sympy [F]

$$\int \frac{(ax + bx^2)^p}{x^3} \, dx = \int \frac{(x(a + bx))^p}{x^3} \, dx$$

```
integrate((b*x**2+a*x)**p/x**3,x)
```

```
Integral((x*(a + b*x))**p/x**3, x)
```

Maxima [F]

$$\int \frac{(ax + bx^2)^p}{x^3} \, dx = \int \frac{(bx^2 + ax)^p}{x^3} \, dx$$

```
integrate((b*x^2+a*x)^p/x^3,x, algorithm="maxima")
```

```
integrate((b*x^2 + a*x)^p/x^3, x)
```

Giac [F]

$$\int \frac{(ax + bx^2)^p}{x^3} \, dx = \int \frac{(bx^2 + ax)^p}{x^3} \, dx$$

```
integrate((b*x^2+a*x)^p/x^3,x, algorithm="giac")
```

```
integrate((b*x^2 + a*x)^p/x^3, x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(ax+bx^2)^p}{x^3}\,dx = \int \frac{(b\,x^2+a\,x)^p}{x^3}\,dx$$

```
int((a*x + b*x^2)^p/x^3,x)
```

```
int((a*x + b*x^2)^p/x^3, x)
```

Reduce [F]

$$\int \frac{(ax+bx^2)^p}{x^3}\,dx = \frac{(b\,x^2+ax)^p + \left(\int \frac{(b\,x^2+ax)^p}{bp\,x^4+ap\,x^3-b\,x^4-a\,x^3}dx\right) a\,p^2x^2 - \left(\int \frac{(b\,x^2+ax)^p}{bp\,x^4+ap\,x^3-b\,x^4-a\,x^3}dx\right) ap\,x^2}{2x^2\,(p-1)}$$

```
int((b*x^2+a*x)^p/x^3,x)
```

```
((a*x + b*x**2)**p + int((a*x + b*x**2)**p/(a*p*x**3 - a*x**3 + b*p*x**4 -
 b*x**4),x)*a*p**2*x**2 - int((a*x + b*x**2)**p/(a*p*x**3 - a*x**3 + b*p*x
**4 - b*x**4),x)*a*p*x**2)/(2*x**2*(p - 1))
```

3.234 $\int x^2(2x-3x^2)^p\,dx$

Optimal result

Integrand size = 15, antiderivative size = 29

$$\int x^2\left(2x-3x^2\right)^p\,dx=\frac{2^px^{3+p}\,\text{Hypergeometric2F1}\left(-p,3+p,4+p,\frac{3x}{2}\right)}{3+p}$$

```
2^p*x^(3+p)*hypergeom([-p, 3+p],[4+p],3/2*x)/(3+p)
```

Mathematica [A] (verified)

Time = 0.03 (sec) , antiderivative size = 45, normalized size of antiderivative = 1.55

$$\int x^2\left(2x-3x^2\right)^p\,dx\\=\frac{2^p(2-3x)^{-p}x^3((2-3x)x)^p\,\text{Hypergeometric2F1}\left(-p,3+p,4+p,\frac{3x}{2}\right)}{3+p}$$

```
Integrate[x^2*(2*x - 3*x^2)^p,x]
```

```
(2^p*x^3*((2 - 3*x)*x)^p*Hypergeometric2F1[-p, 3 + p, 4 + p, (3*x)/2])/((3
 + p)*(2 - 3*x)^p)
```

Rubi [A] (verified)

Time = 0.28 (sec) , antiderivative size = 47, normalized size of antiderivative = 1.62, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.133$, Rules used = {1137, 74}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^2 (2x - 3x^2)^p \, dx$$

$$\Big\downarrow \ 1137$$

$$(2-3x)^{-p}x^{-p}(2x-3x^2)^p \int (2-3x)^p x^{p+2} dx$$

$$\Big\downarrow \ 74$$

$$\frac{2^p x^3 (2-3x)^{-p} (2x-3x^2)^p \text{Hypergeometric2F1}\left(-p, p+3, p+4, \frac{3x}{2}\right)}{p+3}$$

```
Int[x^2*(2*x - 3*x^2)^p,x]
```

```
(2^p*x^3*(2*x - 3*x^2)^p*Hypergeometric2F1[-p, 3 + p, 4 + p, (3*x)/2])/((3
 + p)*(2 - 3*x)^p)
```

Defintions of rubi rules used

```
Int[((b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[c^n*((b*x
)^(m + 1)/(b*(m + 1)))*Hypergeometric2F1[-n, m + 1, m + 2, (-d)*(x/c)], x]
/; FreeQ[{b, c, d, m, n}, x] &&  !IntegerQ[m] && (IntegerQ[n] || (GtQ[c, 0]
 &&  !(EqQ[n, -2^(-1)] && EqQ[c^2 - d^2, 0] && GtQ[-d/(b*c), 0])))
```

```
Int[((e_.)*(x_))^(m_)*((b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(
e*x)^m*((b*x + c*x^2)^p/(x^(m + p)*(b + c*x)^p))   Int[x^(m + p)*(b + c*x)^
p, x], x] /; FreeQ[{b, c, e, m}, x]
```

Maple [A] (verified)

Time = 0.46 (sec) , antiderivative size = 30, normalized size of antiderivative = 1.03

method	result	size
meijerg	$\frac{2^{p} x^{3+p} \operatorname{hypergeom}\left(\left[-p,3+p\right],\left[4+p\right],\frac{3x}{2}\right)}{3+p}$	30

```
int(x^2*(-3*x^2+2*x)^p,x,method=_RETURNVERBOSE)
```

```
2^p*x^(3+p)*hypergeom([-p,3+p],[4+p],3/2*x)/(3+p)
```

Fricas [F]

$$\int x^2 (2x-3x^2)^p \,dx = \int (-3\,x^2+2\,x)^p x^2 \,dx$$

```
integrate(x^2*(-3*x^2+2*x)^p,x, algorithm="fricas")
```

```
integral((-3*x^2 + 2*x)^p*x^2, x)
```

Sympy [F]

$$\int x^2 (2x-3x^2)^p \,dx = \int x^2 (-x(3x-2))^p \,dx$$

```
integrate(x**2*(-3*x**2+2*x)**p,x)
```

```
Integral(x**2*(-x*(3*x - 2))**p, x)
```

Maxima [F]

$$\int x^2 (2x - 3x^2)^p \, dx = \int (-3\,x^2 + 2\,x)^p x^2 \, dx$$

```
integrate(x^2*(-3*x^2+2*x)^p,x, algorithm="maxima")
```

```
integrate((-3*x^2 + 2*x)^p*x^2, x)
```

Giac [F]

$$\int x^2 (2x - 3x^2)^p \, dx = \int (-3\,x^2 + 2\,x)^p x^2 \, dx$$

```
integrate(x^2*(-3*x^2+2*x)^p,x, algorithm="giac")
```

```
integrate((-3*x^2 + 2*x)^p*x^2, x)
```

Mupad [F(-1)]

Timed out.

$$\int x^2 (2x - 3x^2)^p \, dx = \int x^2 \left(2\,x - 3\,x^2\right)^p \, dx$$

```
int(x^2*(2*x - 3*x^2)^p,x)
```

```
int(x^2*(2*x - 3*x^2)^p, x)
```

Reduce [F]

$$\int x^2 (2x - 3x^2)^p \, dx$$
$$= \frac{54(-3x^2+2x)^p p^2 x^3 - 18(-3x^2+2x)^p p^2 x^2 - 6(-3x^2+2x)^p p^2 x - 2(-3x^2+2x)^p p^2 + 81(-3x^2+2x)}{}$$

```
int(x^2*(-3*x^2+2*x)^p,x)
```

```
(54*( - 3*x**2 + 2*x)**p*p**2*x**3 - 18*( - 3*x**2 + 2*x)**p*p**2*x**2 - 6
*( - 3*x**2 + 2*x)**p*p**2*x - 2*( - 3*x**2 + 2*x)**p*p**2 + 81*( - 3*x**2
 + 2*x)**p*p*x**3 - 9*( - 3*x**2 + 2*x)**p*p*x**2 - 12*( - 3*x**2 + 2*x)**
p*p*x - 6*( - 3*x**2 + 2*x)**p*p + 27*( - 3*x**2 + 2*x)**p*x**3 - 4*( - 3*
x**2 + 2*x)**p - 16*int(( - 3*x**2 + 2*x)**p/(12*p**2*x**2 - 8*p**2*x + 24
*p*x**2 - 16*p*x + 9*x**2 - 6*x),x)*p**5 - 80*int(( - 3*x**2 + 2*x)**p/(12
*p**2*x**2 - 8*p**2*x + 24*p*x**2 - 16*p*x + 9*x**2 - 6*x),x)*p**4 - 140*i
nt(( - 3*x**2 + 2*x)**p/(12*p**2*x**2 - 8*p**2*x + 24*p*x**2 - 16*p*x + 9*
x**2 - 6*x),x)*p**3 - 100*int(( - 3*x**2 + 2*x)**p/(12*p**2*x**2 - 8*p**2*
x + 24*p*x**2 - 16*p*x + 9*x**2 - 6*x),x)*p**2 - 24*int(( - 3*x**2 + 2*x)*
*p/(12*p**2*x**2 - 8*p**2*x + 24*p*x**2 - 16*p*x + 9*x**2 - 6*x),x)*p)/(27
*(4*p**3 + 12*p**2 + 11*p + 3))
```

3.235 $\int x(2x-3x^2)^p\,dx$

Optimal result

Integrand size = 13, antiderivative size = 29

$$\int x\left(2x-3x^2\right)^p\,dx=\frac{2^p x^{2+p}\,\text{Hypergeometric2F1}\left(-p,2+p,3+p,\frac{3x}{2}\right)}{2+p}$$

```
2^p*x^(2+p)*hypergeom([-p, 2+p],[3+p],3/2*x)/(2+p)
```

Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 45, normalized size of antiderivative = 1.55

$$\int x\left(2x-3x^2\right)^p\,dx
=\frac{2^p(2-3x)^{-p}x^2((2-3x)x)^p\,\text{Hypergeometric2F1}\left(-p,2+p,3+p,\frac{3x}{2}\right)}{2+p}$$

```
Integrate[x*(2*x - 3*x^2)^p,x]
```

```
(2^p*x^2*((2 - 3*x)*x)^p*Hypergeometric2F1[-p, 2 + p, 3 + p, (3*x)/2])/((2
 + p)*(2 - 3*x)^p)
```

Rubi [B] (verified)

Leaf count is larger than twice the leaf count of optimal. 60 vs. $2(29) = 58$.

Time = 0.30 (sec) , antiderivative size = 60, normalized size of antiderivative = 2.07, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.231$, Rules used = $\{1160, 1090, 237\}$

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x\left(2x-3x^2\right)^p\,dx$$

$$\Big\downarrow \text{1160}$$

$$\frac{1}{3}\int \left(2x-3x^2\right)^p\,dx-\frac{\left(2x-3x^2\right)^{p+1}}{6(p+1)}$$

$$\Big\downarrow \text{1090}$$

$$-\frac{1}{2}3^{-p-2}\int \left(1-\frac{1}{4}(2-6x)^2\right)^p d(2-6x)-\frac{\left(2x-3x^2\right)^{p+1}}{6(p+1)}$$

$$\Big\downarrow \text{237}$$

$$-\frac{1}{2}3^{-p-2}(2-6x)\,\text{Hypergeometric2F1}\left(\frac{1}{2},-p,\frac{3}{2},\frac{1}{4}(2-6x)^2\right)-\frac{\left(2x-3x^2\right)^{p+1}}{6(p+1)}$$

```
Int[x*(2*x - 3*x^2)^p,x]
```

```
-1/6*(2*x - 3*x^2)^(1 + p)/(1 + p) - (3^(-2 - p)*(2 - 6*x)*Hypergeometric2
F1[1/2, -p, 3/2, (2 - 6*x)^2/4])/2
```

Defintions of rubi rules used

```
Int[((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[a^p*x*Hypergeometric2F1[-
p, 1/2, 1/2 + 1, (-b)*(x^2/a)], x] /; FreeQ[{a, b, p}, x] &&  !IntegerQ[2*p
] && GtQ[a, 0]
```

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[1/(2*c*(-4*
(c/(b^2 - 4*a*c)))^p)   Subst[Int[Simp[1 - x^2/(b^2 - 4*a*c), x]^p, x], x,
b + 2*c*x], x] /; FreeQ[{a, b, c, p}, x] && GtQ[4*a - b^2/c, 0]
```

```
Int[((d_.) + (e_.)*(x_))*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol
] :> Simp[e*((a + b*x + c*x^2)^(p + 1)/(2*c*(p + 1))), x] + Simp[(2*c*d - b
*e)/(2*c)   Int[(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, p}, x]
 && NeQ[p, -1]
```

Maple [A] (verified)

Time = 0.44 (sec) , antiderivative size = 30, normalized size of antiderivative = 1.03

method	result	size
meijerg	$\frac{2^{p} x^{2+p} \operatorname{hypergeom}\left(\left[-p,2+p\right],\left[3+p\right],\frac{3x}{2}\right)}{2+p}$	30

```
int(x*(-3*x^2+2*x)^p,x,method=_RETURNVERBOSE)
```

```
2^p*x^(2+p)*hypergeom([-p,2+p],[3+p],3/2*x)/(2+p)
```

Fricas [F]

$$\int x(2x-3x^2)^p\,dx = \int (-3\,x^2+2\,x)^p x\,dx$$

```
integrate(x*(-3*x^2+2*x)^p,x, algorithm="fricas")
```

```
integral((-3*x^2 + 2*x)^p*x, x)
```

Sympy [F]

$$\int x(2x-3x^2)^p\,dx = \int x(-x(3x-2))^p\,dx$$

```
integrate(x*(-3*x**2+2*x)**p,x)
```

```
Integral(x*(-x*(3*x - 2))**p, x)
```

Maxima [F]

$$\int x(2x-3x^2)^p\,dx = \int (-3\,x^2+2\,x)^p x\,dx$$

```
integrate(x*(-3*x^2+2*x)^p,x, algorithm="maxima")
```

```
integrate((-3*x^2 + 2*x)^p*x, x)
```

Giac [F]

$$\int x(2x - 3x^2)^p \, dx = \int (-3x^2 + 2x)^p x \, dx$$

```
integrate(x*(-3*x^2+2*x)^p,x, algorithm="giac")
```

```
integrate((-3*x^2 + 2*x)^p*x, x)
```

Mupad [F(-1)]

Timed out.

$$\int x(2x - 3x^2)^p \, dx = \int x (2x - 3x^2)^p \, dx$$

```
int(x*(2*x - 3*x^2)^p,x)
```

```
int(x*(2*x - 3*x^2)^p, x)
```

Reduce [F]

$$\int x(2x - 3x^2)^p \, dx$$
$$= \frac{18(-3x^2 + 2x)^p p x^2 - 6(-3x^2 + 2x)^p px - 2(-3x^2 + 2x)^p p + 9(-3x^2 + 2x)^p x^2 - 2(-3x^2 + 2x)^p - 8(}{36p^2 + 54p + 18}$$

```
int(x*(-3*x^2+2*x)^p,x)
```

```
(18*( - 3*x**2 + 2*x)**p*p*x**2 - 6*( - 3*x**2 + 2*x)**p*p*x - 2*( - 3*x**
2 + 2*x)**p*p + 9*( - 3*x**2 + 2*x)**p*x**2 - 2*( - 3*x**2 + 2*x)**p - 8*i
nt(( - 3*x**2 + 2*x)**p/(6*p*x**2 - 4*p*x + 3*x**2 - 2*x),x)*p**3 - 12*int
(( - 3*x**2 + 2*x)**p/(6*p*x**2 - 4*p*x + 3*x**2 - 2*x),x)*p**2 - 4*int((
- 3*x**2 + 2*x)**p/(6*p*x**2 - 4*p*x + 3*x**2 - 2*x),x)*p)/(18*(2*p**2 + 3
*p + 1))
```

3.236 $\int (2x - 3x^2)^p \, dx$

Optimal result

Integrand size = 11, antiderivative size = 29

$$\int (2x - 3x^2)^p \, dx = \frac{2^p x^{1+p} \text{Hypergeometric2F1}\left(-p, 1+p, 2+p, \frac{3x}{2}\right)}{1+p}$$

```
2^p*x^(p+1)*hypergeom([-p, p+1],[2+p],3/2*x)/(p+1)
```

Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 43, normalized size of antiderivative = 1.48

$$\int (2x - 3x^2)^p \, dx = \frac{2^p (2-3x)^{-p} x ((2-3x)x)^p \text{Hypergeometric2F1}\left(-p, 1+p, 2+p, \frac{3x}{2}\right)}{1+p}$$

```
Integrate[(2*x - 3*x^2)^p,x]
```

```
(2^p*x*((2 - 3*x)*x)^p*Hypergeometric2F1[-p, 1 + p, 2 + p, (3*x)/2])/((1 +
 p)*(2 - 3*x)^p)
```

Rubi [A] (verified)

Time = 0.25 (sec) , antiderivative size = 37, normalized size of antiderivative = 1.28, number of steps used = 3, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.182, Rules used = {1090, 237}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (2x-3x^2)^p \, dx$$

$$\Big\downarrow \text{ 1090}$$

$$-\frac{1}{2}3^{-p-1}\int \left(1-\frac{1}{4}(2-6x)^2\right)^p d(2-6x)$$

$$\Big\downarrow \text{ 237}$$

$$-\frac{1}{2}3^{-p-1}(2-6x)\,\text{Hypergeometric2F1}\left(\frac{1}{2},-p,\frac{3}{2},\frac{1}{4}(2-6x)^2\right)$$

```
Int[(2*x - 3*x^2)^p,x]
```

```
-1/2*(3^(-1 - p)*(2 - 6*x)*Hypergeometric2F1[1/2, -p, 3/2, (2 - 6*x)^2/4])
```

Defintions of rubi rules used

```
Int[((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[a^p*x*Hypergeometric2F1[-
p, 1/2, 1/2 + 1, (-b)*(x^2/a)], x] /; FreeQ[{a, b, p}, x] &&  !IntegerQ[2*p
] && GtQ[a, 0]
```

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[1/(2*c*(-4*
(c/(b^2 - 4*a*c)))^p)   Subst[Int[Simp[1 - x^2/(b^2 - 4*a*c), x]^p, x], x,
b + 2*c*x], x] /; FreeQ[{a, b, c, p}, x] && GtQ[4*a - b^2/c, 0]
```

Maple [A] (verified)

Time = 0.34 (sec) , antiderivative size = 30, normalized size of antiderivative = 1.03

method	result	size
meijerg	$\frac{2^{p} x^{p+1} \operatorname{hypergeom}\left(\left[-p,p+1\right],\left[2+p\right],\frac{3x}{2}\right)}{p+1}$	30

```
int((-3*x^2+2*x)^p,x,method=_RETURNVERBOSE)
```

```
2^p*x^(p+1)*hypergeom([-p,p+1],[2+p],3/2*x)/(p+1)
```

Fricas [F]

$$\int \left(2x-3x^2\right)^p\,dx=\int \left(-3\,x^2+2\,x\right)^p\,dx$$

```
integrate((-3*x^2+2*x)^p,x, algorithm="fricas")
```

```
integral((-3*x^2 + 2*x)^p, x)
```

Sympy [F]

$$\int \left(2x-3x^2\right)^p\,dx=\int \left(-3x^2+2x\right)^p\,dx$$

```
integrate((-3*x**2+2*x)**p,x)
```

```
Integral((-3*x**2 + 2*x)**p, x)
```

Maxima [F]

$$\int \left(2x-3x^2\right)^p\,dx=\int \left(-3\,x^2+2\,x\right)^p\,dx$$

```
integrate((-3*x^2+2*x)^p,x, algorithm="maxima")
```

```
integrate((-3*x^2 + 2*x)^p, x)
```

Giac [F]

$$\int \left(2x-3x^2\right)^p\,dx=\int \left(-3\,x^2+2\,x\right)^p\,dx$$

```
integrate((-3*x^2+2*x)^p,x, algorithm="giac")
```

```
integrate((-3*x^2 + 2*x)^p, x)
```

Mupad [B] (verification not implemented)

Time = 5.82 (sec) , antiderivative size = 41, normalized size of antiderivative = 1.41

$$\int \left(2x-3x^2\right)^p\,dx=\frac{x\,\left(2\,x-3\,x^2\right)^p\,{}_2\mathrm{F}_1\left(-p,p+1;\ p+2;\ \frac{3\,x}{2}\right)}{\left(1-\frac{3\,x}{2}\right)^p\,\left(p+1\right)}$$

```
int((2*x - 3*x^2)^p,x)
```

```
(x*(2*x - 3*x^2)^p*hypergeom([-p, p + 1], p + 2, (3*x)/2))/((1 - (3*x)/2)^
p*(p + 1))
```

Reduce [F]

$$\int \left(2x-3x^2\right)^p\,dx$$
$$=\frac{3(-3x^2+2x)^p\,x-(-3x^2+2x)^p-4\left(\int\frac{(-3x^2+2x)^p}{6p\,x^2-4px+3x^2-2x}dx\right)p^2-2\left(\int\frac{(-3x^2+2x)^p}{6p\,x^2-4px+3x^2-2x}dx\right)p}{6p+3}$$

```
int((-3*x^2+2*x)^p,x)
```

```
(3*( - 3*x**2 + 2*x)**p*x - ( - 3*x**2 + 2*x)**p - 4*int(( - 3*x**2 + 2*x)
**p/(6*p*x**2 - 4*p*x + 3*x**2 - 2*x),x)*p**2 - 2*int(( - 3*x**2 + 2*x)**p
/(6*p*x**2 - 4*p*x + 3*x**2 - 2*x),x)*p)/(3*(2*p + 1))
```

3.237 $\int \frac{(2x-3x^2)^p}{x} dx$

Optimal result

Integrand size = 15, antiderivative size = 23

$$\int \frac{(2x-3x^2)^p}{x} dx = \frac{2^p x^p \text{Hypergeometric2F1}\left(-p, p, 1+p, \frac{3x}{2}\right)}{p}$$

```
2^p*x^p*hypergeom([p, -p],[p+1],3/2*x)/p
```

Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 38, normalized size of antiderivative = 1.65

$$\int \frac{(2x-3x^2)^p}{x} dx = \frac{2^p(2-3x)^{-p}((2-3x)x)^p \text{Hypergeometric2F1}\left(-p, p, 1+p, \frac{3x}{2}\right)}{p}$$

```
Integrate[(2*x - 3*x^2)^p/x,x]
```

```
(2^p*((2 - 3*x)*x)^p*Hypergeometric2F1[-p, p, 1 + p, (3*x)/2])/(p*(2 - 3*x
)^p)
```

Rubi [A] (verified)

Time = 0.26 (sec) , antiderivative size = 40, normalized size of antiderivative = 1.74, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.133, Rules used = {1137, 74}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(2x-3x^2)^p}{x} \, dx$$

$$\Big\downarrow \text{1137}$$

$$(2-3x)^{-p}x^{-p}(2x-3x^2)^p \int (2-3x)^p x^{p-1}dx$$

$$\Big\downarrow \text{74}$$

$$\frac{2^p(2-3x)^{-p}\,(2x-3x^2)^p \,\text{Hypergeometric2F1}\left(-p,p,p+1,\frac{3x}{2}\right)}{p}$$

```
Int[(2*x - 3*x^2)^p/x,x]
```

```
(2^p*(2*x - 3*x^2)^p*Hypergeometric2F1[-p, p, 1 + p, (3*x)/2])/(p*(2 - 3*x
)^p)
```

Defintions of rubi rules used

```
Int[((b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[c^n*((b*x
)^(m + 1)/(b*(m + 1)))*Hypergeometric2F1[-n, m + 1, m + 2, (-d)*(x/c)], x]
/; FreeQ[{b, c, d, m, n}, x] &&  !IntegerQ[m] && (IntegerQ[n] || (GtQ[c, 0]
 &&  !(EqQ[n, -2^(-1)] && EqQ[c^2 - d^2, 0] && GtQ[-d/(b*c), 0])))
```

```
Int[((e_.)*(x_))^(m_)*((b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(
e*x)^m*((b*x + c*x^2)^p/(x^(m + p)*(b + c*x)^p))   Int[x^(m + p)*(b + c*x)^
p, x], x] /; FreeQ[{b, c, e, m}, x]
```

Maple [A] (verified)

Time = 0.35 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.04

method	result	size
meijerg	$\frac{2^{p} x^{p} \operatorname{hypergeom}\left(\left[p,-p\right],\left[p+1\right],\frac{3x}{2}\right)}{p}$	24

```
int((-3*x^2+2*x)^p/x,x,method=_RETURNVERBOSE)
```

```
2^p*x^p*hypergeom([p,-p],[p+1],3/2*x)/p
```

Fricas [F]

$$\int \frac{(2x-3x^2)^p}{x}\,dx=\int \frac{(-3\,x^2+2\,x)^p}{x}\,dx$$

```
integrate((-3*x^2+2*x)^p/x,x, algorithm="fricas")
```

```
integral((-3*x^2 + 2*x)^p/x, x)
```

Sympy [F]

$$\int \frac{(2x-3x^2)^p}{x}\,dx=\int \frac{(-x(3x-2))^p}{x}\,dx$$

```
integrate((-3*x**2+2*x)**p/x,x)
```

```
Integral((-x*(3*x - 2))**p/x, x)
```

Maxima [F]

$$\int \frac{(2x - 3x^2)^p}{x} \, dx = \int \frac{(-3\,x^2 + 2\,x)^p}{x} \, dx$$

```
integrate((-3*x^2+2*x)^p/x,x, algorithm="maxima")
```

```
integrate((-3*x^2 + 2*x)^p/x, x)
```

Giac [F]

$$\int \frac{(2x - 3x^2)^p}{x} \, dx = \int \frac{(-3\,x^2 + 2\,x)^p}{x} \, dx$$

```
integrate((-3*x^2+2*x)^p/x,x, algorithm="giac")
```

```
integrate((-3*x^2 + 2*x)^p/x, x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(2x - 3x^2)^p}{x} \, dx = \int \frac{(2\,x - 3\,x^2)^p}{x} \, dx$$

```
int((2*x - 3*x^2)^p/x,x)
```

```
int((2*x - 3*x^2)^p/x, x)
```

Reduce [F]

$$\int \frac{(2x-3x^2)^p}{x}\,dx = \frac{(-3x^2+2x)^p - 2\left(\int \frac{(-3x^2+2x)^p}{3x^2-2x}dx\right)p}{2p}$$

```
int((-3*x^2+2*x)^p/x,x)
```

```
(( - 3*x**2 + 2*x)**p - 2*int(( - 3*x**2 + 2*x)**p/(3*x**2 - 2*x),x)*p)/(2
*p)
```

3.238 $\int \frac{(2x-3x^2)^p}{x^2} \, dx$

Optimal result

Integrand size = 15, antiderivative size = 30

$$\int \frac{(2x-3x^2)^p}{x^2} \, dx = -\frac{2^p x^{-1+p} \text{Hypergeometric2F1}\left(-1+p,-p,p,\frac{3x}{2}\right)}{1-p}$$

```
-2^p*x^(-1+p)*hypergeom([-p, -1+p],[p],3/2*x)/(1-p)
```

Mathematica [A] (verified)

Time = 0.03 (sec) , antiderivative size = 43, normalized size of antiderivative = 1.43

$$\int \frac{(2x-3x^2)^p}{x^2} \, dx = \frac{2^p(2-3x)^{-p}((2-3x)x)^p \text{Hypergeometric2F1}\left(-1+p,-p,p,\frac{3x}{2}\right)}{(-1+p)x}$$

```
Integrate[(2*x - 3*x^2)^p/x^2,x]
```

```
(2^p*((2 - 3*x)*x)^p*Hypergeometric2F1[-1 + p, -p, p, (3*x)/2])/((-1 + p)*
(2 - 3*x)^p*x)
```

Rubi [A] (verified)

Time = 0.27 (sec) , antiderivative size = 48, normalized size of antiderivative = 1.60, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.133$, Rules used = {1137, 74}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(2x-3x^2)^p}{x^2} \, dx$$
$$\Big\downarrow \text{1137}$$
$$(2-3x)^{-p}x^{-p}(2x-3x^2)^p \int (2-3x)^p x^{p-2} dx$$
$$\Big\downarrow \text{74}$$
$$-\frac{2^p(2-3x)^{-p}\left(2x-3x^2\right)^p \text{Hypergeometric2F1}\left(p-1,-p,p,\frac{3x}{2}\right)}{(1-p)x}$$

```
Int[(2*x - 3*x^2)^p/x^2,x]
```

```
-((2^p*(2*x - 3*x^2)^p*Hypergeometric2F1[-1 + p, -p, p, (3*x)/2])/((1 - p)
*(2 - 3*x)^p*x))
```

Defintions of rubi rules used

```
Int[((b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[c^n*((b*x
)^(m + 1)/(b*(m + 1)))*Hypergeometric2F1[-n, m + 1, m + 2, (-d)*(x/c)], x]
/; FreeQ[{b, c, d, m, n}, x] && !IntegerQ[m] && (IntegerQ[n] || (GtQ[c, 0]
 && !(EqQ[n, -2^(-1)] && EqQ[c^2 - d^2, 0] && GtQ[-d/(b*c), 0])))
```

```
Int[((e_.)*(x_))^(m_)*((b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(
e*x)^m*((b*x + c*x^2)^p/(x^(m + p)*(b + c*x)^p))   Int[x^(m + p)*(b + c*x)^
p, x], x] /; FreeQ[{b, c, e, m}, x]
```

Maple [A] (verified)

Time = 0.35 (sec) , antiderivative size = 28, normalized size of antiderivative = 0.93

method	result	size
meijerg	$\frac{2^{p} x^{p-1} \operatorname{hypergeom}\left(\left[-p,p-1\right],\left[p\right],\frac{3x}{2}\right)}{p-1}$	28

```
int((-3*x^2+2*x)^p/x^2,x,method=_RETURNVERBOSE)
```

```
2^p/(p-1)*x^(p-1)*hypergeom([-p,p-1],[p],3/2*x)
```

Fricas [F]

$$\int \frac{(2x-3x^2)^p}{x^2}\,dx = \int \frac{(-3\,x^2+2\,x)^p}{x^2}\,dx$$

```
integrate((-3*x^2+2*x)^p/x^2,x, algorithm="fricas")
```

```
integral((-3*x^2 + 2*x)^p/x^2, x)
```

Sympy [F]

$$\int \frac{(2x-3x^2)^p}{x^2}\,dx = \int \frac{(-x(3x-2))^p}{x^2}\,dx$$

```
integrate((-3*x**2+2*x)**p/x**2,x)
```

```
Integral((-x*(3*x - 2))**p/x**2, x)
```

Maxima [F]

$$\int \frac{(2x-3x^2)^p}{x^2}\,dx = \int \frac{(-3\,x^2+2\,x)^p}{x^2}\,dx$$

```
integrate((-3*x^2+2*x)^p/x^2,x, algorithm="maxima")
```

```
integrate((-3*x^2 + 2*x)^p/x^2, x)
```

Giac [F]

$$\int \frac{(2x-3x^2)^p}{x^2}\,dx = \int \frac{(-3\,x^2+2\,x)^p}{x^2}\,dx$$

```
integrate((-3*x^2+2*x)^p/x^2,x, algorithm="giac")
```

```
integrate((-3*x^2 + 2*x)^p/x^2, x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(2x-3x^2)^p}{x^2}\,dx = \int \frac{(2\,x-3\,x^2)^p}{x^2}\,dx$$

```
int((2*x - 3*x^2)^p/x^2,x)
```

```
int((2*x - 3*x^2)^p/x^2, x)
```

Reduce [F]

$$\int \frac{(2x-3x^2)^p}{x^2}\,dx = \frac{(-3x^2+2x)^p - 4\left(\int \frac{(-3x^2+2x)^p}{6p\,x^3-4p\,x^2-3x^3+2x^2}dx\right)p^2x + 2\left(\int \frac{(-3x^2+2x)^p}{6p\,x^3-4p\,x^2-3x^3+2x^2}dx\right)px}{x\,(2p-1)}$$

```
int((-3*x^2+2*x)^p/x^2,x)
```

```
(( - 3*x**2 + 2*x)**p - 4*int(( - 3*x**2 + 2*x)**p/(6*p*x**3 - 4*p*x**2 -
3*x**3 + 2*x**2),x)*p**2*x + 2*int(( - 3*x**2 + 2*x)**p/(6*p*x**3 - 4*p*x*
*2 - 3*x**3 + 2*x**2),x)*p*x)/(x*(2*p - 1))
```

3.239 $\int \frac{(2x-3x^2)^p}{x^3} dx$

Optimal result

Integrand size = 15, antiderivative size = 32

$$\int \frac{(2x-3x^2)^p}{x^3} dx = -\frac{2^p x^{-2+p} \text{Hypergeometric2F1}\left(-2+p,-p,-1+p,\frac{3x}{2}\right)}{2-p}$$

```
-2^p*x^(-2+p)*hypergeom([-p, -2+p],[-1+p],3/2*x)/(2-p)
```

Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 45, normalized size of antiderivative = 1.41

$$\int \frac{(2x-3x^2)^p}{x^3} dx = \frac{2^p(2-3x)^{-p}((2-3x)x)^p \text{Hypergeometric2F1}\left(-2+p,-p,-1+p,\frac{3x}{2}\right)}{(-2+p)x^2}$$

```
Integrate[(2*x - 3*x^2)^p/x^3,x]
```

```
(2^p*((2 - 3*x)*x)^p*Hypergeometric2F1[-2 + p, -p, -1 + p, (3*x)/2])/((-2
+ p)*(2 - 3*x)^p*x^2)
```

Rubi [A] (verified)

Time = 0.27 (sec) , antiderivative size = 50, normalized size of antiderivative = 1.56, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.133, Rules used = {1137, 74}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(2x-3x^2)^p}{x^3} \, dx$$

$$\Big\downarrow \text{1137}$$

$$(2-3x)^{-p}x^{-p}(2x-3x^2)^p \int (2-3x)^p x^{p-3} dx$$

$$\Big\downarrow \text{74}$$

$$-\frac{2^p(2-3x)^{-p}\left(2x-3x^2\right)^p \text{Hypergeometric2F1}\left(p-2,-p,p-1,\frac{3x}{2}\right)}{(2-p)x^2}$$

```
Int[(2*x - 3*x^2)^p/x^3,x]
```

```
-((2^p*(2*x - 3*x^2)^p*Hypergeometric2F1[-2 + p, -p, -1 + p, (3*x)/2])/((2
 - p)*(2 - 3*x)^p*x^2))
```

Defintions of rubi rules used

```
Int[((b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[c^n*((b*x
)^(m + 1)/(b*(m + 1)))*Hypergeometric2F1[-n, m + 1, m + 2, (-d)*(x/c)], x]
/; FreeQ[{b, c, d, m, n}, x] &&  !IntegerQ[m] && (IntegerQ[n] || (GtQ[c, 0]
 &&  !(EqQ[n, -2^(-1)] && EqQ[c^2 - d^2, 0] && GtQ[-d/(b*c), 0])))
```

```
Int[((e_.)*(x_))^(m_)*((b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(
e*x)^m*((b*x + c*x^2)^p/(x^(m + p)*(b + c*x)^p))   Int[x^(m + p)*(b + c*x)^
p, x], x] /; FreeQ[{b, c, e, m}, x]
```

Maple [A] (verified)

Time = 0.41 (sec) , antiderivative size = 30, normalized size of antiderivative = 0.94

method	result	size
meijerg	$\frac{2^{p} x^{-2+p} \operatorname{hypergeom}\left(\left[-p,-2+p\right],\left[p-1\right],\frac{3x}{2}\right)}{-2+p}$	30

```
int((-3*x^2+2*x)^p/x^3,x,method=_RETURNVERBOSE)
```

```
2^p/(-2+p)*x^(-2+p)*hypergeom([-p,-2+p],[p-1],3/2*x)
```

Fricas [F]

$$\int \frac{(2x-3x^2)^p}{x^3}\,dx = \int \frac{(-3\,x^2+2\,x)^p}{x^3}\,dx$$

```
integrate((-3*x^2+2*x)^p/x^3,x, algorithm="fricas")
```

```
integral((-3*x^2 + 2*x)^p/x^3, x)
```

Sympy [F]

$$\int \frac{(2x-3x^2)^p}{x^3}\,dx = \int \frac{(-x(3x-2))^p}{x^3}\,dx$$

```
integrate((-3*x**2+2*x)**p/x**3,x)
```

```
Integral((-x*(3*x - 2))**p/x**3, x)
```

Maxima [F]

$$\int \frac{(2x-3x^2)^p}{x^3}\,dx = \int \frac{(-3\,x^2+2\,x)^p}{x^3}\,dx$$

```
integrate((-3*x^2+2*x)^p/x^3,x, algorithm="maxima")
```

```
integrate((-3*x^2 + 2*x)^p/x^3, x)
```

Giac [F]

$$\int \frac{(2x-3x^2)^p}{x^3}\,dx = \int \frac{(-3\,x^2+2\,x)^p}{x^3}\,dx$$

```
integrate((-3*x^2+2*x)^p/x^3,x, algorithm="giac")
```

```
integrate((-3*x^2 + 2*x)^p/x^3, x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(2x-3x^2)^p}{x^3}\,dx = \int \frac{(2\,x-3\,x^2)^p}{x^3}\,dx$$

```
int((2*x - 3*x^2)^p/x^3,x)
```

```
int((2*x - 3*x^2)^p/x^3, x)
```

Reduce [F]

$$\int \frac{(2x - 3x^2)^p}{x^3} \, dx$$
$$= \frac{(-3x^2 + 2x)^p - 2\left(\int \frac{(-3x^2+2x)^p}{3p\,x^4-2p\,x^3-3x^4+2x^3} dx\right) p^2x^2 + 2\left(\int \frac{(-3x^2+2x)^p}{3p\,x^4-2p\,x^3-3x^4+2x^3} dx\right) p\,x^2}{2x^2\,(p-1)}$$

```
int((-3*x^2+2*x)^p/x^3,x)
```

```
(( - 3*x**2 + 2*x)**p - 2*int(( - 3*x**2 + 2*x)**p/(3*p*x**4 - 2*p*x**3 -
3*x**4 + 2*x**3),x)*p**2*x**2 + 2*int(( - 3*x**2 + 2*x)**p/(3*p*x**4 - 2*p
*x**3 - 3*x**4 + 2*x**3),x)*p*x**2)/(2*x**2*(p - 1))
```

3.240 $\int x^2(2x-x^2)^p\,dx$

Optimal result

Integrand size = 15, antiderivative size = 29

$$\int x^2(2x-x^2)^p\,dx=\frac{2^p x^{3+p}\,\text{Hypergeometric2F1}\left(-p,3+p,4+p,\frac{x}{2}\right)}{3+p}$$

```
2^p*x^(3+p)*hypergeom([-p, 3+p],[4+p],1/2*x)/(3+p)
```

Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 46, normalized size of antiderivative = 1.59

$$\int x^2(2x-x^2)^p\,dx = \frac{2^{2+p}(-2+x)x^{-p}(-((-2+x)x))^p\,\text{Hypergeometric2F1}\left(-2-p,1+p,2+p,1-\frac{x}{2}\right)}{1+p}$$

```
Integrate[x^2*(2*x - x^2)^p,x]
```

```
(2^(2 + p)*(-2 + x)*(-((-2 + x)*x))^p*Hypergeometric2F1[-2 - p, 1 + p, 2 +
 p, 1 - x/2])/((1 + p)*x^p)
```

Rubi [A] (verified)

Time = 0.28 (sec) , antiderivative size = 47, normalized size of antiderivative = 1.62, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.133$, Rules used = $\{1137, 74\}$

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^2 \left(2x - x^2\right)^p \, dx$$

$$\Big\downarrow \text{ 1137}$$

$$(2-x)^{-p} x^{-p} \left(2x - x^2\right)^p \int (2-x)^p x^{p+2} dx$$

$$\Big\downarrow \text{ 74}$$

$$\frac{2^p x^3 (2-x)^{-p} \left(2x - x^2\right)^p \text{Hypergeometric2F1}\left(-p, p+3, p+4, \frac{x}{2}\right)}{p+3}$$

```
Int[x^2*(2*x - x^2)^p,x]
```

```
(2^p*x^3*(2*x - x^2)^p*Hypergeometric2F1[-p, 3 + p, 4 + p, x/2])/((3 + p)*
(2 - x)^p)
```

Defintions of rubi rules used

```
Int[((b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[c^n*((b*x
)^(m + 1)/(b*(m + 1)))*Hypergeometric2F1[-n, m + 1, m + 2, (-d)*(x/c)], x]
/; FreeQ[{b, c, d, m, n}, x] &&  !IntegerQ[m] && (IntegerQ[n] || (GtQ[c, 0]
 &&  !(EqQ[n, -2^(-1)] && EqQ[c^2 - d^2, 0] && GtQ[-d/(b*c), 0])))
```

```
Int[((e_.)*(x_))^(m_)*((b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(
e*x)^m*((b*x + c*x^2)^p/(x^(m + p)*(b + c*x)^p))   Int[x^(m + p)*(b + c*x)^
p, x], x] /; FreeQ[{b, c, e, m}, x]
```

Maple [A] (verified)

Time = 0.46 (sec) , antiderivative size = 30, normalized size of antiderivative = 1.03

method	result	size
meijerg	$\frac{2^{p} x^{3+p} \operatorname{hypergeom}\left(\left[-p,3+p\right],\left[4+p\right],\frac{x}{2}\right)}{3+p}$	30

```
int(x^2*(-x^2+2*x)^p,x,method=_RETURNVERBOSE)
```

```
2^p*x^(3+p)*hypergeom([-p,3+p],[4+p],1/2*x)/(3+p)
```

Fricas [F]

$$\int x^2 (2x - x^2)^p \, dx = \int (-x^2 + 2\,x)^p x^2 \, dx$$

```
integrate(x^2*(-x^2+2*x)^p,x, algorithm="fricas")
```

```
integral((-x^2 + 2*x)^p*x^2, x)
```

Sympy [F]

$$\int x^2 (2x - x^2)^p \, dx = \int x^2 (-x(x-2))^p \, dx$$

```
integrate(x**2*(-x**2+2*x)**p,x)
```

```
Integral(x**2*(-x*(x - 2))**p, x)
```

Maxima [F]

$$\int x^2 (2x - x^2)^p \, dx = \int (-x^2 + 2x)^p x^2 \, dx$$

```
integrate(x^2*(-x^2+2*x)^p,x, algorithm="maxima")
```

```
integrate((-x^2 + 2*x)^p*x^2, x)
```

Giac [F]

$$\int x^2 (2x - x^2)^p \, dx = \int (-x^2 + 2x)^p x^2 \, dx$$

```
integrate(x^2*(-x^2+2*x)^p,x, algorithm="giac")
```

```
integrate((-x^2 + 2*x)^p*x^2, x)
```

Mupad [F(-1)]

Timed out.

$$\int x^2 (2x - x^2)^p \, dx = \int x^2 (2x - x^2)^p \, dx$$

```
int(x^2*(2*x - x^2)^p,x)
```

```
int(x^2*(2*x - x^2)^p, x)
```

Reduce [F]

$$\int x^2 (2x - x^2)^p \, dx$$

$$= \frac{2(-x^2+2x)^p p^2 x^3 - 2(-x^2+2x)^p p^2 x^2 - 2(-x^2+2x)^p p^2 x - 2(-x^2+2x)^p p^2 + 3(-x^2+2x)^p p x^3 - (}{}$$

```
int(x^2*(-x^2+2*x)^p,x)
```

```
(2*( - x**2 + 2*x)**p*p**2*x**3 - 2*( - x**2 + 2*x)**p*p**2*x**2 - 2*( - x
**2 + 2*x)**p*p**2*x - 2*( - x**2 + 2*x)**p*p**2 + 3*( - x**2 + 2*x)**p*p*
x**3 - ( - x**2 + 2*x)**p*p*x**2 - 4*( - x**2 + 2*x)**p*p*x - 6*( - x**2 +
 2*x)**p*p + ( - x**2 + 2*x)**p*x**3 - 4*( - x**2 + 2*x)**p - 16*int(( - x
**2 + 2*x)**p/(4*p**2*x**2 - 8*p**2*x + 8*p*x**2 - 16*p*x + 3*x**2 - 6*x),
x)*p**5 - 80*int(( - x**2 + 2*x)**p/(4*p**2*x**2 - 8*p**2*x + 8*p*x**2 - 1
6*p*x + 3*x**2 - 6*x),x)*p**4 - 140*int(( - x**2 + 2*x)**p/(4*p**2*x**2 -
8*p**2*x + 8*p*x**2 - 16*p*x + 3*x**2 - 6*x),x)*p**3 - 100*int(( - x**2 +
2*x)**p/(4*p**2*x**2 - 8*p**2*x + 8*p*x**2 - 16*p*x + 3*x**2 - 6*x),x)*p**
2 - 24*int(( - x**2 + 2*x)**p/(4*p**2*x**2 - 8*p**2*x + 8*p*x**2 - 16*p*x
+ 3*x**2 - 6*x),x)*p)/(4*p**3 + 12*p**2 + 11*p + 3)
```

3.241 $\int x^2(2dx-d^2x^2)^p\,dx$

Optimal result

Integrand size = 19, antiderivative size = 35

$$\int x^2\left(2dx-d^2x^2\right)^p\,dx=\frac{2^p(dx)^{3+p}\,\text{Hypergeometric2F1}\left(-p,3+p,4+p,\frac{dx}{2}\right)}{d^3(3+p)}$$

```
2^p*(d*x)^(3+p)*hypergeom([-p, 3+p],[4+p],1/2*d*x)/d^3/(3+p)
```

Mathematica [A] (verified)

Time = 0.03 (sec) , antiderivative size = 49, normalized size of antiderivative = 1.40

$$\int x^2\left(2dx-d^2x^2\right)^p\,dx=\frac{2^px^3(2-dx)^{-p}(dx(2-dx))^p\,\text{Hypergeometric2F1}\left(-p,3+p,4+p,\frac{dx}{2}\right)}{3+p}$$

```
Integrate[x^2*(2*d*x - d^2*x^2)^p,x]
```

```
(2^p*x^3*(d*x*(2 - d*x))^p*Hypergeometric2F1[-p, 3 + p, 4 + p, (d*x)/2])/(
(3 + p)*(2 - d*x)^p)
```

Rubi [A] (verified)

Time = 0.29 (sec) , antiderivative size = 52, normalized size of antiderivative = 1.49, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.158, Rules used = {1137, 76, 74}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^2 \left(2dx-d^2x^2\right)^p \, dx$$

$$\Big\downarrow \text{1137}$$

$$x^{-p}\left(2d-d^2x\right)^{-p}\left(2dx-d^2x^2\right)^p\int x^{p+2}\left(2d-d^2x\right)^p dx$$

$$\Big\downarrow \text{76}$$

$$x^{-p}\left(1-\frac{dx}{2}\right)^{-p}\left(2dx-d^2x^2\right)^p\int x^{p+2}\left(1-\frac{dx}{2}\right)^p dx$$

$$\Big\downarrow \text{74}$$

$$\frac{x^3\left(1-\frac{dx}{2}\right)^{-p}\left(2dx-d^2x^2\right)^p \text{Hypergeometric2F1}\left(-p,p+3,p+4,\frac{dx}{2}\right)}{p+3}$$

```
Int[x^2*(2*d*x - d^2*x^2)^p,x]
```

```
(x^3*(2*d*x - d^2*x^2)^p*Hypergeometric2F1[-p, 3 + p, 4 + p, (d*x)/2])/((3
 + p)*(1 - (d*x)/2)^p)
```

Defintions of rubi rules used

```
Int[((b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[c^n*((b*x
)^(m + 1)/(b*(m + 1)))*Hypergeometric2F1[-n, m + 1, m + 2, (-d)*(x/c)], x]
/; FreeQ[{b, c, d, m, n}, x] && !IntegerQ[m] && (IntegerQ[n] || (GtQ[c, 0]
 && !(EqQ[n, -2^(-1)] && EqQ[c^2 - d^2, 0] && GtQ[-d/(b*c), 0])))
```

```
Int[((b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[c^IntPart
[n]*((c + d*x)^FracPart[n]/(1 + d*(x/c))^FracPart[n])   Int[(b*x)^m*(1 + d*
(x/c))^n, x], x] /; FreeQ[{b, c, d, m, n}, x] &&  !IntegerQ[m] &&  !Integer
Q[n] &&  !GtQ[c, 0] &&  !GtQ[-d/(b*c), 0] && ((RationalQ[m] &&  !(EqQ[n, -2
^(-1)] && EqQ[c^2 - d^2, 0])) ||  !RationalQ[n])
```

```
Int[((e_.)*(x_))^(m_)*((b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(
e*x)^m*((b*x + c*x^2)^p/(x^(m + p)*(b + c*x)^p))   Int[x^(m + p)*(b + c*x)^
p, x], x] /; FreeQ[{b, c, e, m}, x]
```

Maple [F]

$$\int x^2 \left(-d^2 x^2 + 2dx\right)^p dx$$

```
int(x^2*(-d^2*x^2+2*d*x)^p,x)
```

```
int(x^2*(-d^2*x^2+2*d*x)^p,x)
```

Fricas [F]

$$\int x^2 \left(2dx - d^2x^2\right)^p \, dx = \int \left(-d^2x^2 + 2\,dx\right)^p x^2 \, dx$$

```
integrate(x^2*(-d^2*x^2+2*d*x)^p,x, algorithm="fricas")
```

```
integral((-d^2*x^2 + 2*d*x)^p*x^2, x)
```

Sympy [F]

$$\int x^2 (2dx - d^2x^2)^p \, dx = \int x^2(-dx(dx-2))^p \, dx$$

```
integrate(x**2*(-d**2*x**2+2*d*x)**p,x)
```

```
Integral(x**2*(-d*x*(d*x - 2))**p, x)
```

Maxima [F]

$$\int x^2 (2dx - d^2x^2)^p \, dx = \int (-d^2x^2 + 2\,dx)^p x^2 \, dx$$

```
integrate(x^2*(-d^2*x^2+2*d*x)^p,x, algorithm="maxima")
```

```
integrate((-d^2*x^2 + 2*d*x)^p*x^2, x)
```

Giac [F]

$$\int x^2 (2dx - d^2x^2)^p \, dx = \int (-d^2x^2 + 2\,dx)^p x^2 \, dx$$

```
integrate(x^2*(-d^2*x^2+2*d*x)^p,x, algorithm="giac")
```

```
integrate((-d^2*x^2 + 2*d*x)^p*x^2, x)
```

Mupad [F(-1)]

Timed out.

$$\int x^2 (2dx - d^2x^2)^p \, dx = \int x^2 \left(2\,d\,x - d^2\,x^2\right)^p \, dx$$

```
int(x^2*(2*d*x - d^2*x^2)^p,x)
```

```
int(x^2*(2*d*x - d^2*x^2)^p, x)
```

Reduce [F]

$$\int x^2 (2dx - d^2x^2)^p \, dx$$
$$= \frac{2(-d^2x^2 + 2dx)^p\, d^3p^2x^3 + 3(-d^2x^2 + 2dx)^p\, d^3p\,x^3 + (-d^2x^2 + 2dx)^p\, d^3x^3 - 2(-d^2x^2 + 2dx)^p\, d^2p^2x^2 -}{}$$

```
int(x^2*(-d^2*x^2+2*d*x)^p,x)
```

```
(2*( - d**2*x**2 + 2*d*x)**p*d**3*p**2*x**3 + 3*( - d**2*x**2 + 2*d*x)**p*
d**3*p*x**3 + ( - d**2*x**2 + 2*d*x)**p*d**3*x**3 - 2*( - d**2*x**2 + 2*d*
x)**p*d**2*p**2*x**2 - ( - d**2*x**2 + 2*d*x)**p*d**2*p*x**2 - 2*( - d**2*
x**2 + 2*d*x)**p*d*p**2*x - 4*( - d**2*x**2 + 2*d*x)**p*d*p*x - 2*( - d**2
*x**2 + 2*d*x)**p*p**2 - 6*( - d**2*x**2 + 2*d*x)**p*p - 4*( - d**2*x**2 +
 2*d*x)**p - 16*int(( - d**2*x**2 + 2*d*x)**p/(4*d*p**2*x**2 + 8*d*p*x**2
+ 3*d*x**2 - 8*p**2*x - 16*p*x - 6*x),x)*p**5 - 80*int(( - d**2*x**2 + 2*d
*x)**p/(4*d*p**2*x**2 + 8*d*p*x**2 + 3*d*x**2 - 8*p**2*x - 16*p*x - 6*x),x
)*p**4 - 140*int(( - d**2*x**2 + 2*d*x)**p/(4*d*p**2*x**2 + 8*d*p*x**2 + 3
*d*x**2 - 8*p**2*x - 16*p*x - 6*x),x)*p**3 - 100*int(( - d**2*x**2 + 2*d*x
)**p/(4*d*p**2*x**2 + 8*d*p*x**2 + 3*d*x**2 - 8*p**2*x - 16*p*x - 6*x),x)*
p**2 - 24*int(( - d**2*x**2 + 2*d*x)**p/(4*d*p**2*x**2 + 8*d*p*x**2 + 3*d*
x**2 - 8*p**2*x - 16*p*x - 6*x),x)*p)/(d**3*(4*p**3 + 12*p**2 + 11*p + 3))
```

3.242 $\int x(3dx - 2d^2x^2)^p \, dx$

Optimal result

Integrand size = 17, antiderivative size = 35

$$\int x(3dx - 2d^2x^2)^p \, dx = \frac{3^p (dx)^{2+p} \operatorname{Hypergeometric2F1}\left(-p, 2+p, 3+p, \frac{2dx}{3}\right)}{d^2(2+p)}$$

```
3^p*(d*x)^(2+p)*hypergeom([-p, 2+p],[3+p],2/3*d*x)/d^2/(2+p)
```

Mathematica [A] (verified)

Time = 0.03 (sec) , antiderivative size = 49, normalized size of antiderivative = 1.40

$$\int x(3dx - 2d^2x^2)^p \, dx = \frac{3^p x^2 (3-2dx)^{-p} (dx(3-2dx))^p \operatorname{Hypergeometric2F1}\left(-p, 2+p, 3+p, \frac{2dx}{3}\right)}{2+p}$$

```
Integrate[x*(3*d*x - 2*d^2*x^2)^p,x]
```

```
(3^p*x^2*(d*x*(3 - 2*d*x))^p*Hypergeometric2F1[-p, 2 + p, 3 + p, (2*d*x)/3
])/((2 + p)*(3 - 2*d*x)^p)
```

Rubi [B] (verified)

Leaf count is larger than twice the leaf count of optimal. 88 vs. $2(35) = 70$.

Time = 0.34 (sec) , antiderivative size = 88, normalized size of antiderivative = 2.51, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.176$, Rules used = {1160, 1090, 237}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x\left(3dx-2d^2x^2\right)^p\,dx$$

$$\Big\downarrow \text{1160}$$

$$\frac{3\int \left(3dx-2d^2x^2\right)^p\,dx}{4d}-\frac{\left(3dx-2d^2x^2\right)^{p+1}}{4d^2(p+1)}$$

$$\Big\downarrow \text{1090}$$

$$-\frac{2^{-3p-4}3^{2p+1}\int \left(1-\frac{\left(3d-4d^2x\right)^2}{9d^2}\right)^p d\left(3d-4d^2x\right)}{d^3}-\frac{\left(3dx-2d^2x^2\right)^{p+1}}{4d^2(p+1)}$$

$$\Big\downarrow \text{237}$$

$$-\frac{\left(3dx-2d^2x^2\right)^{p+1}}{4d^2(p+1)}-\frac{2^{-3p-4}3^{2p+1}\left(3d-4d^2x\right)\text{Hypergeometric2F1}\left(\frac{1}{2},-p,\frac{3}{2},\frac{\left(3d-4d^2x\right)^2}{9d^2}\right)}{d^3}$$

```
Int[x*(3*d*x - 2*d^2*x^2)^p,x]
```

```
-1/4*(3*d*x - 2*d^2*x^2)^(1 + p)/(d^2*(1 + p)) - (2^(-4 - 3*p)*3^(1 + 2*p)
*(3*d - 4*d^2*x)*Hypergeometric2F1[1/2, -p, 3/2, (3*d - 4*d^2*x)^2/(9*d^2)
])/d^3
```

Defintions of rubi rules used

```
Int[((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[a^p*x*Hypergeometric2F1[-
p, 1/2, 1/2 + 1, (-b)*(x^2/a)], x] /; FreeQ[{a, b, p}, x] &&  !IntegerQ[2*p
] && GtQ[a, 0]
```

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[1/(2*c*(-4*
(c/(b^2 - 4*a*c)))^p)   Subst[Int[Simp[1 - x^2/(b^2 - 4*a*c), x]^p, x], x,
b + 2*c*x], x] /; FreeQ[{a, b, c, p}, x] && GtQ[4*a - b^2/c, 0]
```

```
Int[((d_.) + (e_.)*(x_))*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol
] :> Simp[e*((a + b*x + c*x^2)^(p + 1)/(2*c*(p + 1))), x] + Simp[(2*c*d - b
*e)/(2*c)   Int[(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, p}, x]
 && NeQ[p, -1]
```

Maple [F]

$$\int x\left(-2d^2x^2+3dx\right)^p dx$$

```
int(x*(-2*d^2*x^2+3*d*x)^p,x)
```

```
int(x*(-2*d^2*x^2+3*d*x)^p,x)
```

Fricas [F]

$$\int x\left(3dx-2d^2x^2\right)^p\,dx=\int\left(-2\,d^2x^2+3\,dx\right)^p x\,dx$$

```
integrate(x*(-2*d^2*x^2+3*d*x)^p,x, algorithm="fricas")
```

```
integral((-2*d^2*x^2 + 3*d*x)^p*x, x)
```

Sympy [F]

$$\int x(3dx - 2d^2x^2)^p \, dx = \int x(-dx(2dx - 3))^p \, dx$$

```
integrate(x*(-2*d**2*x**2+3*d*x)**p,x)
```

```
Integral(x*(-d*x*(2*d*x - 3))**p, x)
```

Maxima [F]

$$\int x(3dx - 2d^2x^2)^p \, dx = \int (-2\,d^2x^2 + 3\,dx)^p x \, dx$$

```
integrate(x*(-2*d^2*x^2+3*d*x)^p,x, algorithm="maxima")
```

```
integrate((-2*d^2*x^2 + 3*d*x)^p*x, x)
```

Giac [F]

$$\int x(3dx - 2d^2x^2)^p \, dx = \int (-2\,d^2x^2 + 3\,dx)^p x \, dx$$

```
integrate(x*(-2*d^2*x^2+3*d*x)^p,x, algorithm="giac")
```

```
integrate((-2*d^2*x^2 + 3*d*x)^p*x, x)
```

Mupad [F(-1)]

Timed out.

$$\int x(3dx-2d^2x^2)^p\,dx=\int x\left(3\,d\,x-2\,d^2\,x^2\right)^p\,dx$$

```
int(x*(3*d*x - 2*d^2*x^2)^p,x)
```

```
int(x*(3*d*x - 2*d^2*x^2)^p, x)
```

Reduce [F]

$$\int x(3dx-2d^2x^2)^p\,dx$$
$$=\frac{16(-2d^2x^2+3dx)^p\,d^2p\,x^2+8(-2d^2x^2+3dx)^p\,d^2x^2-12(-2d^2x^2+3dx)^p\,dpx-9(-2d^2x^2+3dx)^p\,p-}{16d}$$

```
int(x*(-2*d^2*x^2+3*d*x)^p,x)
```

```
(16*( - 2*d**2*x**2 + 3*d*x)**p*d**2*p*x**2 + 8*( - 2*d**2*x**2 + 3*d*x)**
p*d**2*x**2 - 12*( - 2*d**2*x**2 + 3*d*x)**p*d*p*x - 9*( - 2*d**2*x**2 + 3
*d*x)**p*p - 9*( - 2*d**2*x**2 + 3*d*x)**p - 54*int(( - 2*d**2*x**2 + 3*d*
x)**p/(4*d*p*x**2 + 2*d*x**2 - 6*p*x - 3*x),x)*p**3 - 81*int(( - 2*d**2*x*
*2 + 3*d*x)**p/(4*d*p*x**2 + 2*d*x**2 - 6*p*x - 3*x),x)*p**2 - 27*int(( -
2*d**2*x**2 + 3*d*x)**p/(4*d*p*x**2 + 2*d*x**2 - 6*p*x - 3*x),x)*p)/(16*d*
*2*(2*p**2 + 3*p + 1))
```

3.243 $\int (3dx - 2d^2x^2)^p \, dx$

Optimal result

Integrand size = 15, antiderivative size = 35

$$\int (3dx - 2d^2x^2)^p \, dx = \frac{3^p(dx)^{1+p}\,\text{Hypergeometric2F1}\left(-p, 1+p, 2+p, \frac{2dx}{3}\right)}{d(1+p)}$$

```
3^p*(d*x)^(p+1)*hypergeom([-p, p+1],[2+p],2/3*d*x)/d/(p+1)
```

Mathematica [A] (verified)

Time = 0.03 (sec) , antiderivative size = 47, normalized size of antiderivative = 1.34

$$\int (3dx - 2d^2x^2)^p \, dx = \frac{3^p x(3-2dx)^{-p}(dx(3-2dx))^p\,\text{Hypergeometric2F1}\left(-p, 1+p, 2+p, \frac{2dx}{3}\right)}{1+p}$$

```
Integrate[(3*d*x - 2*d^2*x^2)^p,x]
```

```
(3^p*x*(d*x*(3 - 2*d*x))^p*Hypergeometric2F1[-p, 1 + p, 2 + p, (2*d*x)/3])
/((1 + p)*(3 - 2*d*x)^p)
```

Rubi [A] (verified)

Time = 0.27 (sec) , antiderivative size = 54, normalized size of antiderivative = 1.54, number of steps used = 3, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.133, Rules used = {1090, 237}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \left(3dx-2d^2x^2\right)^p\,dx$$
$$\Big\downarrow \text{1090}$$
$$-\frac{2^{-3p-2}9^p\int \left(1-\frac{\left(3d-4d^2x\right)^2}{9d^2}\right)^p d\left(3d-4d^2x\right)}{d^2}$$
$$\Big\downarrow \text{237}$$
$$-\frac{2^{-3p-2}9^p\left(3d-4d^2x\right)\text{Hypergeometric2F1}\left(\frac{1}{2},-p,\frac{3}{2},\frac{\left(3d-4d^2x\right)^2}{9d^2}\right)}{d^2}$$

```
Int[(3*d*x - 2*d^2*x^2)^p,x]
```

```
-((2^(-2 - 3*p)*9^p*(3*d - 4*d^2*x)*Hypergeometric2F1[1/2, -p, 3/2, (3*d -
 4*d^2*x)^2/(9*d^2)])/d^2)
```

Defintions of rubi rules used

```
Int[((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[a^p*x*Hypergeometric2F1[-
p, 1/2, 1/2 + 1, (-b)*(x^2/a)], x] /; FreeQ[{a, b, p}, x] &&  !IntegerQ[2*p
] && GtQ[a, 0]
```

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[1/(2*c*(-4*
(c/(b^2 - 4*a*c)))^p)   Subst[Int[Simp[1 - x^2/(b^2 - 4*a*c), x]^p, x], x,
b + 2*c*x], x] /; FreeQ[{a, b, c, p}, x] && GtQ[4*a - b^2/c, 0]
```

Maple [F]

$$\int \left(-2d^2x^2+3dx\right)^p dx$$

```
int((-2*d^2*x^2+3*d*x)^p,x)
```

```
int((-2*d^2*x^2+3*d*x)^p,x)
```

Fricas [F]

$$\int \left(3dx-2d^2x^2\right)^p \, dx = \int \left(-2\,d^2x^2+3\,dx\right)^p dx$$

```
integrate((-2*d^2*x^2+3*d*x)^p,x, algorithm="fricas")
```

```
integral((-2*d^2*x^2 + 3*d*x)^p, x)
```

Sympy [F]

$$\int \left(3dx-2d^2x^2\right)^p \, dx = \int \left(-2d^2x^2+3dx\right)^p \, dx$$

```
integrate((-2*d**2*x**2+3*d*x)**p,x)
```

```
Integral((-2*d**2*x**2 + 3*d*x)**p, x)
```

Maxima [F]

$$\int \left(3dx-2d^2x^2\right)^p\,dx=\int \left(-2\,d^2x^2+3\,dx\right)^p\,dx$$

```
integrate((-2*d^2*x^2+3*d*x)^p,x, algorithm="maxima")
```

```
integrate((-2*d^2*x^2 + 3*d*x)^p, x)
```

Giac [F]

$$\int \left(3dx-2d^2x^2\right)^p\,dx=\int \left(-2\,d^2x^2+3\,dx\right)^p\,dx$$

```
integrate((-2*d^2*x^2+3*d*x)^p,x, algorithm="giac")
```

```
integrate((-2*d^2*x^2 + 3*d*x)^p, x)
```

Mupad [B] (verification not implemented)

Time = 9.20 (sec) , antiderivative size = 47, normalized size of antiderivative = 1.34

$$\int \left(3dx-2d^2x^2\right)^p\,dx=\frac{x\,\left(3\,d\,x-2\,d^2\,x^2\right)^p\,{}_2\mathrm{F}_1\left(-p,p+1;\ p+2;\ \frac{2\,d\,x}{3}\right)}{\left(1-\frac{2\,d\,x}{3}\right)^p\,\left(p+1\right)}$$

```
int((3*d*x - 2*d^2*x^2)^p,x)
```

```
(x*(3*d*x - 2*d^2*x^2)^p*hypergeom([-p, p + 1], p + 2, (2*d*x)/3))/((1 - (
2*d*x)/3)^p*(p + 1))
```

Reduce [F]

$$\int \left(3dx - 2d^2x^2\right)^p \, dx$$
$$= \frac{4(-2d^2x^2 + 3dx)^p\, dx - 3(-2d^2x^2 + 3dx)^p - 18\left(\int \frac{(-2d^2x^2+3dx)^p}{4dp\,x^2+2d\,x^2-6px-3x}dx\right) p^2 - 9\left(\int \frac{(-2d^2x^2+3dx)^p}{4dp\,x^2+2d\,x^2-6px-3x}dx\right)}{4d\,(2p+1)}$$

```
int((-2*d^2*x^2+3*d*x)^p,x)
```

```
(4*( - 2*d**2*x**2 + 3*d*x)**p*d*x - 3*( - 2*d**2*x**2 + 3*d*x)**p - 18*in
t(( - 2*d**2*x**2 + 3*d*x)**p/(4*d*p*x**2 + 2*d*x**2 - 6*p*x - 3*x),x)*p**
2 - 9*int(( - 2*d**2*x**2 + 3*d*x)**p/(4*d*p*x**2 + 2*d*x**2 - 6*p*x - 3*x
),x)*p)/(4*d*(2*p + 1))
```

3.244 $\int \frac{(3dx-2d^2x^2)^p}{x}\,dx$

Optimal result

Integrand size = 19, antiderivative size = 26

$$\int \frac{(3dx-2d^2x^2)^p}{x}\,dx=\frac{3^p(dx)^p\,\text{Hypergeometric2F1}\left(-p,p,1+p,\frac{2dx}{3}\right)}{p}$$

```
3^p*(d*x)^p*hypergeom([p, -p],[p+1],2/3*d*x)/p
```

Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 42, normalized size of antiderivative = 1.62

$$\int \frac{(3dx-2d^2x^2)^p}{x}\,dx =\frac{3^p(3-2dx)^{-p}(dx(3-2dx))^p\,\text{Hypergeometric2F1}\left(-p,p,1+p,\frac{2dx}{3}\right)}{p}$$

```
Integrate[(3*d*x - 2*d^2*x^2)^p/x,x]
```

```
(3^p*(d*x*(3 - 2*d*x))^p*Hypergeometric2F1[-p, p, 1 + p, (2*d*x)/3])/(p*(3
 - 2*d*x)^p)
```

Rubi [A] (verified)

Time = 0.27 (sec) , antiderivative size = 45, normalized size of antiderivative = 1.73, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.158$, Rules used = {1137, 76, 74}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(3dx-2d^2x^2)^p}{x}\,dx$$

$$\Big\downarrow \text{1137}$$

$$x^{-p}(3d-2d^2x)^{-p}(3dx-2d^2x^2)^p\int x^{p-1}(3d-2d^2x)^p\,dx$$

$$\Big\downarrow \text{76}$$

$$x^{-p}\left(1-\frac{2dx}{3}\right)^{-p}(3dx-2d^2x^2)^p\int x^{p-1}\left(1-\frac{2dx}{3}\right)^p\,dx$$

$$\Big\downarrow \text{74}$$

$$\frac{\left(1-\frac{2dx}{3}\right)^{-p}(3dx-2d^2x^2)^p\,\text{Hypergeometric2F1}\left(-p,p,p+1,\frac{2dx}{3}\right)}{p}$$

```
Int[(3*d*x - 2*d^2*x^2)^p/x,x]
```

```
((3*d*x - 2*d^2*x^2)^p*Hypergeometric2F1[-p, p, 1 + p, (2*d*x)/3])/(p*(1 -
 (2*d*x)/3)^p)
```

Defintions of rubi rules used

```
Int[((b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[c^n*((b*x
)^(m + 1)/(b*(m + 1)))*Hypergeometric2F1[-n, m + 1, m + 2, (-d)*(x/c)], x]
/; FreeQ[{b, c, d, m, n}, x] &&  !IntegerQ[m] && (IntegerQ[n] || (GtQ[c, 0]
 &&  !(EqQ[n, -2^(-1)] && EqQ[c^2 - d^2, 0] && GtQ[-d/(b*c), 0])))
```

```
Int[((b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[c^IntPart
[n]*((c + d*x)^FracPart[n]/(1 + d*(x/c))^FracPart[n])   Int[(b*x)^m*(1 + d*
(x/c))^n, x], x] /; FreeQ[{b, c, d, m, n}, x] &&  !IntegerQ[m] &&  !Integer
Q[n] &&  !GtQ[c, 0] &&  !GtQ[-d/(b*c), 0] && ((RationalQ[m] &&  !(EqQ[n, -2
^(-1)] && EqQ[c^2 - d^2, 0])) ||  !RationalQ[n])
```

```
Int[((e_.)*(x_))^(m_)*((b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(
e*x)^m*((b*x + c*x^2)^p/(x^(m + p)*(b + c*x)^p))   Int[x^(m + p)*(b + c*x)^
p, x], x] /; FreeQ[{b, c, e, m}, x]
```

Maple [F]

$$\int \frac{(-2d^2x^2 + 3dx)^p}{x} dx$$

```
int((-2*d^2*x^2+3*d*x)^p/x,x)
```

```
int((-2*d^2*x^2+3*d*x)^p/x,x)
```

Fricas [F]

$$\int \frac{(3dx - 2d^2x^2)^p}{x} dx = \int \frac{(-2\,d^2x^2 + 3\,dx)^p}{x} dx$$

```
integrate((-2*d^2*x^2+3*d*x)^p/x,x, algorithm="fricas")
```

```
integral((-2*d^2*x^2 + 3*d*x)^p/x, x)
```

Sympy [F]

$$\int \frac{(3dx-2d^2x^2)^p}{x}\,dx=\int \frac{(-dx(2dx-3))^p}{x}\,dx$$

```
integrate((-2*d**2*x**2+3*d*x)**p/x,x)
```

```
Integral((-d*x*(2*d*x - 3))**p/x, x)
```

Maxima [F]

$$\int \frac{(3dx-2d^2x^2)^p}{x}\,dx=\int \frac{(-2\,d^2x^2+3\,dx)^p}{x}\,dx$$

```
integrate((-2*d^2*x^2+3*d*x)^p/x,x, algorithm="maxima")
```

```
integrate((-2*d^2*x^2 + 3*d*x)^p/x, x)
```

Giac [F]

$$\int \frac{(3dx-2d^2x^2)^p}{x}\,dx=\int \frac{(-2\,d^2x^2+3\,dx)^p}{x}\,dx$$

```
integrate((-2*d^2*x^2+3*d*x)^p/x,x, algorithm="giac")
```

```
integrate((-2*d^2*x^2 + 3*d*x)^p/x, x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(3dx - 2d^2x^2)^p}{x}\,dx = \int \frac{(3\,d\,x - 2\,d^2\,x^2)^p}{x}\,dx$$

```
int((3*d*x - 2*d^2*x^2)^p/x,x)
```

```
int((3*d*x - 2*d^2*x^2)^p/x, x)
```

Reduce [F]

$$\int \frac{(3dx - 2d^2x^2)^p}{x}\,dx = \frac{(-2d^2x^2 + 3dx)^p - 3\left(\int \frac{(-2d^2x^2+3dx)^p}{2d\,x^2-3x}dx\right)p}{2p}$$

```
int((-2*d^2*x^2+3*d*x)^p/x,x)
```

```
(( - 2*d**2*x**2 + 3*d*x)**p - 3*int(( - 2*d**2*x**2 + 3*d*x)**p/(2*d*x**2
 - 3*x),x)*p)/(2*p)
```

3.245 $\int \frac{(3dx-2d^2x^2)^p}{x^2}\,dx$

Optimal result

Integrand size = 19, antiderivative size = 34

$$\int \frac{(3dx-2d^2x^2)^p}{x^2}\,dx = -\frac{3^p d(dx)^{-1+p}\,\text{Hypergeometric2F1}\left(-1+p,-p,p,\frac{2dx}{3}\right)}{1-p}$$

```
-3^p*d*(d*x)^(-1+p)*hypergeom([-p, -1+p],[p],2/3*d*x)/(1-p)
```

Mathematica [A] (verified)

Time = 0.03 (sec) , antiderivative size = 47, normalized size of antiderivative = 1.38

$$\int \frac{(3dx-2d^2x^2)^p}{x^2}\,dx = \frac{3^p(3-2dx)^{-p}(dx(3-2dx))^p\,\text{Hypergeometric2F1}\left(-1+p,-p,p,\frac{2dx}{3}\right)}{(-1+p)x}$$

```
Integrate[(3*d*x - 2*d^2*x^2)^p/x^2,x]
```

```
(3^p*(d*x*(3 - 2*d*x))^p*Hypergeometric2F1[-1 + p, -p, p, (2*d*x)/3])/((-1
 + p)*x*(3 - 2*d*x)^p)
```

Rubi [A] (verified)

Time = 0.29 (sec) , antiderivative size = 53, normalized size of antiderivative = 1.56, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.158, Rules used = {1137, 76, 74}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(3dx-2d^2x^2)^p}{x^2} \, dx$$

$$\Big\downarrow \text{1137}$$

$$x^{-p}(3d-2d^2x)^{-p}(3dx-2d^2x^2)^p \int x^{p-2}(3d-2d^2x)^p dx$$

$$\Big\downarrow \text{76}$$

$$x^{-p}\left(1-\frac{2dx}{3}\right)^{-p}(3dx-2d^2x^2)^p \int x^{p-2}\left(1-\frac{2dx}{3}\right)^p dx$$

$$\Big\downarrow \text{74}$$

$$-\frac{\left(1-\frac{2dx}{3}\right)^{-p}(3dx-2d^2x^2)^p \text{Hypergeometric2F1}\left(p-1,-p,p,\frac{2dx}{3}\right)}{(1-p)x}$$

```
Int[(3*d*x - 2*d^2*x^2)^p/x^2,x]
```

```
-(((3*d*x - 2*d^2*x^2)^p*Hypergeometric2F1[-1 + p, -p, p, (2*d*x)/3])/((1
- p)*x*(1 - (2*d*x)/3)^p))
```

Defintions of rubi rules used

```
Int[((b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[c^n*((b*x
)^(m + 1)/(b*(m + 1)))*Hypergeometric2F1[-n, m + 1, m + 2, (-d)*(x/c)], x]
/; FreeQ[{b, c, d, m, n}, x] &&  !IntegerQ[m] && (IntegerQ[n] || (GtQ[c, 0]
 &&  !(EqQ[n, -2^(-1)] && EqQ[c^2 - d^2, 0] && GtQ[-d/(b*c), 0])))
```

```
Int[((b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[c^IntPart
[n]*((c + d*x)^FracPart[n]/(1 + d*(x/c))^FracPart[n])   Int[(b*x)^m*(1 + d*
(x/c))^n, x], x] /; FreeQ[{b, c, d, m, n}, x] &&  !IntegerQ[m] &&  !Integer
Q[n] &&  !GtQ[c, 0] &&  !GtQ[-d/(b*c), 0] && ((RationalQ[m] &&  !(EqQ[n, -2
^(-1)] && EqQ[c^2 - d^2, 0])) ||  !RationalQ[n])
```

```
Int[((e_.)*(x_))^(m_)*((b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(
e*x)^m*((b*x + c*x^2)^p/(x^(m + p)*(b + c*x)^p))   Int[x^(m + p)*(b + c*x)^
p, x], x] /; FreeQ[{b, c, e, m}, x]
```

Maple [F]

$$\int \frac{(-2d^2x^2 + 3dx)^p}{x^2} dx$$

```
int((-2*d^2*x^2+3*d*x)^p/x^2,x)
```

```
int((-2*d^2*x^2+3*d*x)^p/x^2,x)
```

Fricas [F]

$$\int \frac{(3dx - 2d^2x^2)^p}{x^2} dx = \int \frac{(-2\,d^2x^2 + 3\,dx)^p}{x^2} dx$$

```
integrate((-2*d^2*x^2+3*d*x)^p/x^2,x, algorithm="fricas")
```

```
integral((-2*d^2*x^2 + 3*d*x)^p/x^2, x)
```

Sympy [F]

$$\int \frac{(3dx-2d^2x^2)^p}{x^2}\,dx = \int \frac{(-dx(2dx-3))^p}{x^2}\,dx$$

```
integrate((-2*d**2*x**2+3*d*x)**p/x**2,x)
```

```
Integral((-d*x*(2*d*x - 3))**p/x**2, x)
```

Maxima [F]

$$\int \frac{(3dx-2d^2x^2)^p}{x^2}\,dx = \int \frac{(-2\,d^2x^2+3\,dx)^p}{x^2}\,dx$$

```
integrate((-2*d^2*x^2+3*d*x)^p/x^2,x, algorithm="maxima")
```

```
integrate((-2*d^2*x^2 + 3*d*x)^p/x^2, x)
```

Giac [F]

$$\int \frac{(3dx-2d^2x^2)^p}{x^2}\,dx = \int \frac{(-2\,d^2x^2+3\,dx)^p}{x^2}\,dx$$

```
integrate((-2*d^2*x^2+3*d*x)^p/x^2,x, algorithm="giac")
```

```
integrate((-2*d^2*x^2 + 3*d*x)^p/x^2, x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(3dx-2d^2x^2)^p}{x^2}\,dx = \int \frac{(3\,d\,x-2\,d^2\,x^2)^p}{x^2}\,dx$$

```
int((3*d*x - 2*d^2*x^2)^p/x^2,x)
```

```
int((3*d*x - 2*d^2*x^2)^p/x^2, x)
```

Reduce [F]

$$\int \frac{(3dx-2d^2x^2)^p}{x^2}\,dx$$
$$= \frac{(-2d^2x^2+3dx)^p - 6\left(\int \frac{(-2d^2x^2+3dx)^p}{4dp\,x^3-2d\,x^3-6p\,x^2+3x^2}dx\right)p^2x + 3\left(\int \frac{(-2d^2x^2+3dx)^p}{4dp\,x^3-2d\,x^3-6p\,x^2+3x^2}dx\right)px}{x\,(2p-1)}$$

```
int((-2*d^2*x^2+3*d*x)^p/x^2,x)
```

```
(( - 2*d**2*x**2 + 3*d*x)**p - 6*int(( - 2*d**2*x**2 + 3*d*x)**p/(4*d*p*x*
*3 - 2*d*x**3 - 6*p*x**2 + 3*x**2),x)*p**2*x + 3*int(( - 2*d**2*x**2 + 3*d
*x)**p/(4*d*p*x**3 - 2*d*x**3 - 6*p*x**2 + 3*x**2),x)*p*x)/(x*(2*p - 1))
```

3.246 $\int \frac{(3dx-2d^2x^2)^p}{x^3}\,dx$

Optimal result

Integrand size = 19, antiderivative size = 38

$$\int \frac{(3dx-2d^2x^2)^p}{x^3}\,dx = -\frac{3^p d^2 (dx)^{-2+p}\,\text{Hypergeometric2F1}\left(-2+p,-p,-1+p,\frac{2dx}{3}\right)}{2-p}$$

```
-3^p*d^2*(d*x)^(-2+p)*hypergeom([-p, -2+p],[-1+p],2/3*d*x)/(2-p)
```

Mathematica [A] (verified)

Time = 0.03 (sec) , antiderivative size = 49, normalized size of antiderivative = 1.29

$$\int \frac{(3dx-2d^2x^2)^p}{x^3}\,dx = \frac{3^p(3-2dx)^{-p}(dx(3-2dx))^p\,\text{Hypergeometric2F1}\left(-2+p,-p,-1+p,\frac{2dx}{3}\right)}{(-2+p)x^2}$$

```
Integrate[(3*d*x - 2*d^2*x^2)^p/x^3,x]
```

```
(3^p*(d*x*(3 - 2*d*x))^p*Hypergeometric2F1[-2 + p, -p, -1 + p, (2*d*x)/3])
/((-2 + p)*x^2*(3 - 2*d*x)^p)
```

Rubi [A] (verified)

Time = 0.29 (sec) , antiderivative size = 55, normalized size of antiderivative = 1.45, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.158$, Rules used = {1137, 76, 74}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(3dx-2d^2x^2)^p}{x^3}dx$$
$$\Big\downarrow \text{1137}$$
$$x^{-p}(3d-2d^2x)^{-p}(3dx-2d^2x^2)^p\int x^{p-3}(3d-2d^2x)^pdx$$
$$\Big\downarrow \text{76}$$
$$x^{-p}\left(1-\frac{2dx}{3}\right)^{-p}(3dx-2d^2x^2)^p\int x^{p-3}\left(1-\frac{2dx}{3}\right)^pdx$$
$$\Big\downarrow \text{74}$$
$$-\frac{\left(1-\frac{2dx}{3}\right)^{-p}(3dx-2d^2x^2)^p\text{Hypergeometric2F1}\left(p-2,-p,p-1,\frac{2dx}{3}\right)}{(2-p)x^2}$$

```
Int[(3*d*x - 2*d^2*x^2)^p/x^3,x]
```

```
-(((3*d*x - 2*d^2*x^2)^p*Hypergeometric2F1[-2 + p, -p, -1 + p, (2*d*x)/3])
/((2 - p)*x^2*(1 - (2*d*x)/3)^p))
```

Defintions of rubi rules used

```
Int[((b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[c^n*((b*x
)^(m + 1)/(b*(m + 1)))*Hypergeometric2F1[-n, m + 1, m + 2, (-d)*(x/c)], x]
/; FreeQ[{b, c, d, m, n}, x] &&  !IntegerQ[m] && (IntegerQ[n] || (GtQ[c, 0]
 &&  !(EqQ[n, -2^(-1)] && EqQ[c^2 - d^2, 0] && GtQ[-d/(b*c), 0])))
```

```
Int[((b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[c^IntPart
[n]*((c + d*x)^FracPart[n]/(1 + d*(x/c))^FracPart[n])   Int[(b*x)^m*(1 + d*
(x/c))^n, x], x] /; FreeQ[{b, c, d, m, n}, x] &&  !IntegerQ[m] &&  !Integer
Q[n] &&  !GtQ[c, 0] &&  !GtQ[-d/(b*c), 0] && ((RationalQ[m] &&  !(EqQ[n, -2
^(-1)] && EqQ[c^2 - d^2, 0])) ||  !RationalQ[n])
```

```
Int[((e_.)*(x_))^(m_)*((b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(
e*x)^m*((b*x + c*x^2)^p/(x^(m + p)*(b + c*x)^p))   Int[x^(m + p)*(b + c*x)^
p, x], x] /; FreeQ[{b, c, e, m}, x]
```

Maple [F]

$$\int \frac{(-2d^2x^2 + 3dx)^p}{x^3}dx$$

```
int((-2*d^2*x^2+3*d*x)^p/x^3,x)
```

```
int((-2*d^2*x^2+3*d*x)^p/x^3,x)
```

Fricas [F]

$$\int \frac{(3dx - 2d^2x^2)^p}{x^3}\,dx = \int \frac{(-2\,d^2x^2 + 3\,dx)^p}{x^3}\,dx$$

```
integrate((-2*d^2*x^2+3*d*x)^p/x^3,x, algorithm="fricas")
```

```
integral((-2*d^2*x^2 + 3*d*x)^p/x^3, x)
```

Sympy [F]

$$\int \frac{(3dx - 2d^2x^2)^p}{x^3} \, dx = \int \frac{(-dx(2dx - 3))^p}{x^3} \, dx$$

```
integrate((-2*d**2*x**2+3*d*x)**p/x**3,x)
```

```
Integral((-d*x*(2*d*x - 3))**p/x**3, x)
```

Maxima [F]

$$\int \frac{(3dx - 2d^2x^2)^p}{x^3} \, dx = \int \frac{(-2\,d^2x^2 + 3\,dx)^p}{x^3} \, dx$$

```
integrate((-2*d^2*x^2+3*d*x)^p/x^3,x, algorithm="maxima")
```

```
integrate((-2*d^2*x^2 + 3*d*x)^p/x^3, x)
```

Giac [F]

$$\int \frac{(3dx - 2d^2x^2)^p}{x^3} \, dx = \int \frac{(-2\,d^2x^2 + 3\,dx)^p}{x^3} \, dx$$

```
integrate((-2*d^2*x^2+3*d*x)^p/x^3,x, algorithm="giac")
```

```
integrate((-2*d^2*x^2 + 3*d*x)^p/x^3, x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(3dx-2d^2x^2)^p}{x^3}\,dx = \int \frac{(3\,d\,x-2\,d^2\,x^2)^p}{x^3}\,dx$$

```
int((3*d*x - 2*d^2*x^2)^p/x^3,x)
```

```
int((3*d*x - 2*d^2*x^2)^p/x^3, x)
```

Reduce [F]

$$\int \frac{(3dx-2d^2x^2)^p}{x^3}\,dx = \frac{(-2d^2x^2+3dx)^p-3\left(\int \frac{(-2d^2x^2+3dx)^p}{2dp\,x^4-2d\,x^4-3p\,x^3+3x^3}dx\right)p^2x^2+3\left(\int \frac{(-2d^2x^2+3dx)^p}{2dp\,x^4-2d\,x^4-3p\,x^3+3x^3}dx\right)p\,x^2}{2x^2\,(p-1)}$$

```
int((-2*d^2*x^2+3*d*x)^p/x^3,x)
```

```
(( - 2*d**2*x**2 + 3*d*x)**p - 3*int(( - 2*d**2*x**2 + 3*d*x)**p/(2*d*p*x*
*4 - 2*d*x**4 - 3*p*x**3 + 3*x**3),x)*p**2*x**2 + 3*int(( - 2*d**2*x**2 +
3*d*x)**p/(2*d*p*x**4 - 2*d*x**4 - 3*p*x**3 + 3*x**3),x)*p*x**2)/(2*x**2*(
p - 1))
```

3.247 $\int (cx)^{3/2} (ax+bx^2)^p \, dx$

Optimal result

Integrand size = 19, antiderivative size = 53

$$\int (cx)^{3/2} (ax+bx^2)^p \, dx = \frac{2(cx)^{3/2} (ax+bx^2)^{1+p} \text{Hypergeometric2F1}\left(1, \frac{7}{2}+2p, \frac{7}{2}+p, -\frac{bx}{a}\right)}{a(5+2p)}$$

```
2*(c*x)^(3/2)*(b*x^2+a*x)^(p+1)*hypergeom([1, 7/2+2*p],[7/2+p],-b*x/a)/a/(
5+2*p)
```

Mathematica [A] (verified)

Time = 0.04 (sec) , antiderivative size = 58, normalized size of antiderivative = 1.09

$$\int (cx)^{3/2} (ax+bx^2)^p \, dx = \frac{x(cx)^{3/2}(x(a+bx))^p \left(1+\frac{bx}{a}\right)^{-p} \text{Hypergeometric2F1}\left(-p, \frac{5}{2}+p, \frac{7}{2}+p, -\frac{bx}{a}\right)}{\frac{5}{2}+p}$$

```
Integrate[(c*x)^(3/2)*(a*x + b*x^2)^p,x]
```

```
(x*(c*x)^(3/2)*(x*(a + b*x))^p*Hypergeometric2F1[-p, 5/2 + p, 7/2 + p, -((
b*x)/a)])/((5/2 + p)*(1 + (b*x)/a)^p)
```

Rubi [A] (verified)

Time = 0.30 (sec) , antiderivative size = 61, normalized size of antiderivative = 1.15, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.158$, Rules used = $\{1137, 76, 74\}$

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (cx)^{3/2} \left(ax+bx^2\right)^p \, dx$$

$$\Big\downarrow \; 1137$$

$$(cx)^{3/2} x^{-p-\frac{3}{2}} (a+bx)^{-p} \left(ax+bx^2\right)^p \int x^{p+\frac{3}{2}} (a+bx)^p dx$$

$$\Big\downarrow \; 76$$

$$(cx)^{3/2} x^{-p-\frac{3}{2}} \left(\frac{bx}{a}+1\right)^{-p} \left(ax+bx^2\right)^p \int x^{p+\frac{3}{2}} \left(\frac{bx}{a}+1\right)^p dx$$

$$\Big\downarrow \; 74$$

$$\frac{2x(cx)^{3/2} \left(\frac{bx}{a}+1\right)^{-p} \left(ax+bx^2\right)^p \text{Hypergeometric2F1}\left(-p,p+\frac{5}{2},p+\frac{7}{2},-\frac{bx}{a}\right)}{2p+5}$$

```
Int[(c*x)^(3/2)*(a*x + b*x^2)^p,x]
```

```
(2*x*(c*x)^(3/2)*(a*x + b*x^2)^p*Hypergeometric2F1[-p, 5/2 + p, 7/2 + p, -
((b*x)/a)])/((5 + 2*p)*(1 + (b*x)/a)^p)
```

Defintions of rubi rules used

```
Int[((b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[c^n*((b*x
)^(m + 1)/(b*(m + 1)))*Hypergeometric2F1[-n, m + 1, m + 2, (-d)*(x/c)], x]
/; FreeQ[{b, c, d, m, n}, x] &&  !IntegerQ[m] && (IntegerQ[n] || (GtQ[c, 0]
 &&  !(EqQ[n, -2^(-1)] && EqQ[c^2 - d^2, 0] && GtQ[-d/(b*c), 0])))
```

```
Int[((b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[c^IntPart
[n]*((c + d*x)^FracPart[n]/(1 + d*(x/c))^FracPart[n])   Int[(b*x)^m*(1 + d*
(x/c))^n, x], x] /; FreeQ[{b, c, d, m, n}, x] &&  !IntegerQ[m] &&  !Integer
Q[n] &&  !GtQ[c, 0] &&  !GtQ[-d/(b*c), 0] && ((RationalQ[m] &&  !(EqQ[n, -2
^(-1)] && EqQ[c^2 - d^2, 0])) ||  !RationalQ[n])
```

```
Int[((e_.)*(x_))^(m_)*((b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(
e*x)^m*((b*x + c*x^2)^p/(x^(m + p)*(b + c*x)^p))   Int[x^(m + p)*(b + c*x)^
p, x], x] /; FreeQ[{b, c, e, m}, x]
```

Maple [F]

$$\int (cx)^{\frac{3}{2}} \left(b\,x^{2}+ax\right)^{p}dx$$

```
int((c*x)^(3/2)*(b*x^2+a*x)^p,x)
```

```
int((c*x)^(3/2)*(b*x^2+a*x)^p,x)
```

Fricas [F]

$$\int (cx)^{3/2} \left(ax+bx^2\right)^p \, dx = \int (cx)^{\frac{3}{2}} \left(bx^{2}+ax\right)^{p} dx$$

```
integrate((c*x)^(3/2)*(b*x^2+a*x)^p,x, algorithm="fricas")
```

```
integral(sqrt(c*x)*(b*x^2 + a*x)^p*c*x, x)
```

Sympy [F]

$$\int (cx)^{3/2} \left(ax+bx^2\right)^p \, dx = \int (cx)^{\frac{3}{2}} \left(x(a+bx)\right)^p \, dx$$

```
integrate((c*x)**(3/2)*(b*x**2+a*x)**p,x)
```

```
Integral((c*x)**(3/2)*(x*(a + b*x))**p, x)
```

Maxima [F]

$$\int (cx)^{3/2} \left(ax+bx^2\right)^p \, dx = \int (cx)^{\frac{3}{2}} \left(bx^2+ax\right)^p \, dx$$

```
integrate((c*x)^(3/2)*(b*x^2+a*x)^p,x, algorithm="maxima")
```

```
integrate((c*x)^(3/2)*(b*x^2 + a*x)^p, x)
```

Giac [F]

$$\int (cx)^{3/2} \left(ax+bx^2\right)^p \, dx = \int (cx)^{\frac{3}{2}} \left(bx^2+ax\right)^p \, dx$$

```
integrate((c*x)^(3/2)*(b*x^2+a*x)^p,x, algorithm="giac")
```

```
integrate((c*x)^(3/2)*(b*x^2 + a*x)^p, x)
```

Mupad [F(-1)]

Timed out.

$$\int (cx)^{3/2} \left(ax+bx^2\right)^p \,dx = \int {\left(b\,x^2+a\,x\right)}^p\,{\left(c\,x\right)}^{3/2} \,dx$$

```
int((a*x + b*x^2)^p*(c*x)^(3/2),x)
```

```
int((a*x + b*x^2)^p*(c*x)^(3/2), x)
```

Reduce [F]

$$\int (cx)^{3/2} \left(ax+bx^2\right)^p \,dx = \frac{2\sqrt{c}\,c\Big(-4\sqrt{x}\,(b\,x^2+ax)^p\,a^2p^2-6\sqrt{x}\,(b\,x^2+ax)^p\,a^2p+8\sqrt{x}\,(b\,x^2+ax)^p\,ab\,p^2x+2\sqrt{x}\,(b\,x^2}{}$$

```
int((c*x)^(3/2)*(b*x^2+a*x)^p,x)
```

```
(2*sqrt(c)*c*( - 4*sqrt(x)*(a*x + b*x**2)**p*a**2*p**2 - 6*sqrt(x)*(a*x +
b*x**2)**p*a**2*p + 8*sqrt(x)*(a*x + b*x**2)**p*a*b*p**2*x + 2*sqrt(x)*(a*
x + b*x**2)**p*a*b*p*x + 16*sqrt(x)*(a*x + b*x**2)**p*b**2*p**2*x**2 + 16*
sqrt(x)*(a*x + b*x**2)**p*b**2*p*x**2 + 3*sqrt(x)*(a*x + b*x**2)**p*b**2*x
**2 + 256*int((sqrt(x)*(a*x + b*x**2)**p)/(64*a*p**3*x + 144*a*p**2*x + 92
*a*p*x + 15*a*x + 64*b*p**3*x**2 + 144*b*p**2*x**2 + 92*b*p*x**2 + 15*b*x*
*2),x)*a**3*p**6 + 1088*int((sqrt(x)*(a*x + b*x**2)**p)/(64*a*p**3*x + 144
*a*p**2*x + 92*a*p*x + 15*a*x + 64*b*p**3*x**2 + 144*b*p**2*x**2 + 92*b*p*
x**2 + 15*b*x**2),x)*a**3*p**5 + 1712*int((sqrt(x)*(a*x + b*x**2)**p)/(64*
a*p**3*x + 144*a*p**2*x + 92*a*p*x + 15*a*x + 64*b*p**3*x**2 + 144*b*p**2*
x**2 + 92*b*p*x**2 + 15*b*x**2),x)*a**3*p**4 + 1228*int((sqrt(x)*(a*x + b*
x**2)**p)/(64*a*p**3*x + 144*a*p**2*x + 92*a*p*x + 15*a*x + 64*b*p**3*x**2
 + 144*b*p**2*x**2 + 92*b*p*x**2 + 15*b*x**2),x)*a**3*p**3 + 396*int((sqrt
(x)*(a*x + b*x**2)**p)/(64*a*p**3*x + 144*a*p**2*x + 92*a*p*x + 15*a*x + 6
4*b*p**3*x**2 + 144*b*p**2*x**2 + 92*b*p*x**2 + 15*b*x**2),x)*a**3*p**2 +
45*int((sqrt(x)*(a*x + b*x**2)**p)/(64*a*p**3*x + 144*a*p**2*x + 92*a*p*x
+ 15*a*x + 64*b*p**3*x**2 + 144*b*p**2*x**2 + 92*b*p*x**2 + 15*b*x**2),x)*
a**3*p))/(b**2*(64*p**3 + 144*p**2 + 92*p + 15))
```

3.248 $\int \sqrt{cx}(ax+bx^2)^p \, dx$

Optimal result

Integrand size = 19, antiderivative size = 53

$$\int \sqrt{cx}\left(ax+bx^2\right)^p \, dx = \frac{2\sqrt{cx}(ax+bx^2)^{1+p}\,\text{Hypergeometric2F1}\left(1,\frac{5}{2}+2p,\frac{5}{2}+p,-\frac{bx}{a}\right)}{a(3+2p)}$$

```
2*(c*x)^(1/2)*(b*x^2+a*x)^(p+1)*hypergeom([1, 5/2+2*p],[5/2+p],-b*x/a)/a/(
3+2*p)
```

Mathematica [A] (verified)

Time = 0.04 (sec) , antiderivative size = 58, normalized size of antiderivative = 1.09

$$\int \sqrt{cx}\left(ax+bx^2\right)^p \, dx = \frac{x\sqrt{cx}(x(a+bx))^p\left(1+\frac{bx}{a}\right)^{-p}\text{Hypergeometric2F1}\left(-p,\frac{3}{2}+p,\frac{5}{2}+p,-\frac{bx}{a}\right)}{\frac{3}{2}+p}$$

```
Integrate[Sqrt[c*x]*(a*x + b*x^2)^p,x]
```

```
(x*Sqrt[c*x]*(x*(a + b*x))^p*Hypergeometric2F1[-p, 3/2 + p, 5/2 + p, -((b*
x)/a)])/((3/2 + p)*(1 + (b*x)/a)^p)
```

Rubi [A] (verified)

Time = 0.30 (sec) , antiderivative size = 61, normalized size of antiderivative = 1.15, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.158, Rules used = {1137, 76, 74}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sqrt{cx}\left(ax+bx^2\right)^p\,dx$$

$$\Big\downarrow \text{1137}$$

$$\sqrt{cx}x^{-p-\frac{1}{2}}(a+bx)^{-p}\left(ax+bx^2\right)^p\int x^{p+\frac{1}{2}}(a+bx)^pdx$$

$$\Big\downarrow \text{76}$$

$$\sqrt{cx}x^{-p-\frac{1}{2}}\left(\frac{bx}{a}+1\right)^{-p}\left(ax+bx^2\right)^p\int x^{p+\frac{1}{2}}\left(\frac{bx}{a}+1\right)^p dx$$

$$\Big\downarrow \text{74}$$

$$\frac{2x\sqrt{cx}\left(\frac{bx}{a}+1\right)^{-p}\left(ax+bx^2\right)^p\text{Hypergeometric2F1}\left(-p,p+\frac{3}{2},p+\frac{5}{2},-\frac{bx}{a}\right)}{2p+3}$$

```
Int[Sqrt[c*x]*(a*x + b*x^2)^p,x]
```

```
(2*x*Sqrt[c*x]*(a*x + b*x^2)^p*Hypergeometric2F1[-p, 3/2 + p, 5/2 + p, -((
b*x)/a)])/((3 + 2*p)*(1 + (b*x)/a)^p)
```

Defintions of rubi rules used

```
Int[((b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[c^n*((b*x
)^(m + 1)/(b*(m + 1)))*Hypergeometric2F1[-n, m + 1, m + 2, (-d)*(x/c)], x]
/; FreeQ[{b, c, d, m, n}, x] && !IntegerQ[m] && (IntegerQ[n] || (GtQ[c, 0]
 && !(EqQ[n, -2^(-1)] && EqQ[c^2 - d^2, 0] && GtQ[-d/(b*c), 0])))
```

```
Int[((b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[c^IntPart
[n]*((c + d*x)^FracPart[n]/(1 + d*(x/c))^FracPart[n])   Int[(b*x)^m*(1 + d*
(x/c))^n, x], x] /; FreeQ[{b, c, d, m, n}, x] &&  !IntegerQ[m] &&  !Integer
Q[n] &&  !GtQ[c, 0] &&  !GtQ[-d/(b*c), 0] && ((RationalQ[m] &&  !(EqQ[n, -2
^(-1)] && EqQ[c^2 - d^2, 0])) ||  !RationalQ[n])
```

```
Int[((e_.)*(x_))^(m_)*((b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(
e*x)^m*((b*x + c*x^2)^p/(x^(m + p)*(b + c*x)^p))   Int[x^(m + p)*(b + c*x)^
p, x], x] /; FreeQ[{b, c, e, m}, x]
```

Maple [F]

$$\int \sqrt{cx}\,\left(b\,x^{2}+ax\right)^{p}dx$$

```
int((c*x)^(1/2)*(b*x^2+a*x)^p,x)
```

```
int((c*x)^(1/2)*(b*x^2+a*x)^p,x)
```

Fricas [F]

$$\int \sqrt{cx}(ax+bx^2)^p\,dx=\int \sqrt{cx}\left(bx^2+ax\right)^p dx$$

```
integrate((c*x)^(1/2)*(b*x^2+a*x)^p,x, algorithm="fricas")
```

```
integral(sqrt(c*x)*(b*x^2 + a*x)^p, x)
```

Sympy [F]

$$\int \sqrt{cx}\left(ax+bx^2\right)^p\,dx=\int \sqrt{cx}(x(a+bx))^p\,dx$$

```
integrate((c*x)**(1/2)*(b*x**2+a*x)**p,x)
```

```
Integral(sqrt(c*x)*(x*(a + b*x))**p, x)
```

Maxima [F]

$$\int \sqrt{cx}\left(ax+bx^2\right)^p\,dx=\int \sqrt{cx}\left(bx^2+ax\right)^p\,dx$$

```
integrate((c*x)^(1/2)*(b*x^2+a*x)^p,x, algorithm="maxima")
```

```
integrate(sqrt(c*x)*(b*x^2 + a*x)^p, x)
```

Giac [F]

$$\int \sqrt{cx}\left(ax+bx^2\right)^p\,dx=\int \sqrt{cx}\left(bx^2+ax\right)^p\,dx$$

```
integrate((c*x)^(1/2)*(b*x^2+a*x)^p,x, algorithm="giac")
```

```
integrate(sqrt(c*x)*(b*x^2 + a*x)^p, x)
```

Mupad [F(-1)]

Timed out.

$$\int \sqrt{cx}\left(ax+bx^2\right)^p\,dx=\int \left(b\,x^2+a\,x\right)^p\,\sqrt{c\,x}\,dx$$

```
int((a*x + b*x^2)^p*(c*x)^(1/2),x)
```

```
int((a*x + b*x^2)^p*(c*x)^(1/2), x)
```

Reduce [F]

$$\int \sqrt{cx}\left(ax+bx^2\right)^p\,dx$$
$$=\frac{2\sqrt{c}\left(2\sqrt{x}\,(b\,x^2+ax)^p\,ap+4\sqrt{x}\,(b\,x^2+ax)^p\,bpx+\sqrt{x}\,(b\,x^2+ax)^p\,bx-32\left(\int\frac{\sqrt{x}\,(b\,x^2+ax)^p}{16b\,p^2x^2+16a\,p^2x+16bp\,x^2+16a}\right.\right.}{}$$

```
int((c*x)^(1/2)*(b*x^2+a*x)^p,x)
```

```
(2*sqrt(c)*(2*sqrt(x)*(a*x + b*x**2)**p*a*p + 4*sqrt(x)*(a*x + b*x**2)**p*
b*p*x + sqrt(x)*(a*x + b*x**2)**p*b*x - 32*int((sqrt(x)*(a*x + b*x**2)**p)
/(16*a*p**2*x + 16*a*p*x + 3*a*x + 16*b*p**2*x**2 + 16*b*p*x**2 + 3*b*x**2
),x)*a**2*p**4 - 48*int((sqrt(x)*(a*x + b*x**2)**p)/(16*a*p**2*x + 16*a*p*
x + 3*a*x + 16*b*p**2*x**2 + 16*b*p*x**2 + 3*b*x**2),x)*a**2*p**3 - 22*int
((sqrt(x)*(a*x + b*x**2)**p)/(16*a*p**2*x + 16*a*p*x + 3*a*x + 16*b*p**2*x
**2 + 16*b*p*x**2 + 3*b*x**2),x)*a**2*p**2 - 3*int((sqrt(x)*(a*x + b*x**2)
**p)/(16*a*p**2*x + 16*a*p*x + 3*a*x + 16*b*p**2*x**2 + 16*b*p*x**2 + 3*b*
x**2),x)*a**2*p))/(b*(16*p**2 + 16*p + 3))
```

3.249 $\int \frac{(ax+bx^2)^p}{\sqrt{cx}}\,dx$

Optimal result

Integrand size = 19, antiderivative size = 53

$$\int \frac{(ax+bx^2)^p}{\sqrt{cx}}\,dx = \frac{2(ax+bx^2)^{1+p}\,\text{Hypergeometric2F1}\left(1,\frac{3}{2}+2p,\frac{3}{2}+p,-\frac{bx}{a}\right)}{a(1+2p)\sqrt{cx}}$$

```
2*(b*x^2+a*x)^(p+1)*hypergeom([1, 3/2+2*p],[3/2+p],-b*x/a)/a/(1+2*p)/(c*x)
^(1/2)
```

Mathematica [A] (verified)

Time = 0.03 (sec) , antiderivative size = 58, normalized size of antiderivative = 1.09

$$\int \frac{(ax+bx^2)^p}{\sqrt{cx}}\,dx = \frac{x(x(a+bx))^p\left(1+\frac{bx}{a}\right)^{-p}\text{Hypergeometric2F1}\left(-p,\frac{1}{2}+p,\frac{3}{2}+p,-\frac{bx}{a}\right)}{\left(\frac{1}{2}+p\right)\sqrt{cx}}$$

```
Integrate[(a*x + b*x^2)^p/Sqrt[c*x],x]
```

```
(x*(x*(a + b*x))^p*Hypergeometric2F1[-p, 1/2 + p, 3/2 + p, -((b*x)/a)])/((
1/2 + p)*Sqrt[c*x]*(1 + (b*x)/a)^p)
```

Rubi [A] (verified)

Time = 0.30 (sec) , antiderivative size = 61, normalized size of antiderivative = 1.15, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.158, Rules used = {1137, 76, 74}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(ax+bx^2)^p}{\sqrt{cx}}dx$$

$$\Big\downarrow \text{ 1137}$$

$$\frac{x^{\frac{1}{2}-p}(a+bx)^{-p}(ax+bx^2)^p\int x^{p-\frac{1}{2}}(a+bx)^pdx}{\sqrt{cx}}$$

$$\Big\downarrow \text{ 76}$$

$$\frac{x^{\frac{1}{2}-p}\left(\frac{bx}{a}+1\right)^{-p}(ax+bx^2)^p\int x^{p-\frac{1}{2}}\left(\frac{bx}{a}+1\right)^pdx}{\sqrt{cx}}$$

$$\Big\downarrow \text{ 74}$$

$$\frac{2x\left(\frac{bx}{a}+1\right)^{-p}(ax+bx^2)^p\text{Hypergeometric2F1}\left(-p,p+\frac{1}{2},p+\frac{3}{2},-\frac{bx}{a}\right)}{(2p+1)\sqrt{cx}}$$

```
Int[(a*x + b*x^2)^p/Sqrt[c*x],x]
```

```
(2*x*(a*x + b*x^2)^p*Hypergeometric2F1[-p, 1/2 + p, 3/2 + p, -((b*x)/a)])/
((1 + 2*p)*Sqrt[c*x]*(1 + (b*x)/a)^p)
```

Defintions of rubi rules used

```
Int[((b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[c^n*((b*x
)^(m + 1)/(b*(m + 1)))*Hypergeometric2F1[-n, m + 1, m + 2, (-d)*(x/c)], x]
/; FreeQ[{b, c, d, m, n}, x] &&  !IntegerQ[m] && (IntegerQ[n] || (GtQ[c, 0]
 &&  !(EqQ[n, -2^(-1)] && EqQ[c^2 - d^2, 0] && GtQ[-d/(b*c), 0])))
```

```
Int[((b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[c^IntPart
[n]*((c + d*x)^FracPart[n]/(1 + d*(x/c))^FracPart[n])   Int[(b*x)^m*(1 + d*
(x/c))^n, x], x] /; FreeQ[{b, c, d, m, n}, x] &&  !IntegerQ[m] &&  !Integer
Q[n] &&  !GtQ[c, 0] &&  !GtQ[-d/(b*c), 0] && ((RationalQ[m] &&  !(EqQ[n, -2
^(-1)] && EqQ[c^2 - d^2, 0])) ||  !RationalQ[n])
```

```
Int[((e_.)*(x_))^(m_)*((b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(
e*x)^m*((b*x + c*x^2)^p/(x^(m + p)*(b + c*x)^p))   Int[x^(m + p)*(b + c*x)^
p, x], x] /; FreeQ[{b, c, e, m}, x]
```

Maple [F]

$$\int \frac{(b\,x^2+ax)^p}{\sqrt{cx}}dx$$

```
int((b*x^2+a*x)^p/(c*x)^(1/2),x)
```

```
int((b*x^2+a*x)^p/(c*x)^(1/2),x)
```

Fricas [F]

$$\int \frac{(ax+bx^2)^p}{\sqrt{cx}}\,dx = \int \frac{(bx^2+ax)^p}{\sqrt{cx}}\,dx$$

```
integrate((b*x^2+a*x)^p/(c*x)^(1/2),x, algorithm="fricas")
```

```
integral(sqrt(c*x)*(b*x^2 + a*x)^p/(c*x), x)
```

Sympy [F]

$$\int \frac{(ax+bx^2)^p}{\sqrt{cx}}\,dx = \int \frac{(x(a+bx))^p}{\sqrt{cx}}\,dx$$

```
integrate((b*x**2+a*x)**p/(c*x)**(1/2),x)
```

```
Integral((x*(a + b*x))**p/sqrt(c*x), x)
```

Maxima [F]

$$\int \frac{(ax+bx^2)^p}{\sqrt{cx}}\,dx = \int \frac{(bx^2+ax)^p}{\sqrt{cx}}\,dx$$

```
integrate((b*x^2+a*x)^p/(c*x)^(1/2),x, algorithm="maxima")
```

```
integrate((b*x^2 + a*x)^p/sqrt(c*x), x)
```

Giac [F]

$$\int \frac{(ax+bx^2)^p}{\sqrt{cx}}\,dx = \int \frac{(bx^2+ax)^p}{\sqrt{cx}}\,dx$$

```
integrate((b*x^2+a*x)^p/(c*x)^(1/2),x, algorithm="giac")
```

```
integrate((b*x^2 + a*x)^p/sqrt(c*x), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(ax+bx^2)^p}{\sqrt{cx}}\,dx=\int \frac{(b\,x^2+a\,x)^p}{\sqrt{c\,x}}\,dx$$

```
int((a*x + b*x^2)^p/(c*x)^(1/2),x)
```

```
int((a*x + b*x^2)^p/(c*x)^(1/2), x)
```

Reduce [F]

$$\int \frac{(ax+bx^2)^p}{\sqrt{cx}}\,dx$$
$$=\frac{2\sqrt{c}\left(\sqrt{x}\,(b\,x^2+ax)^p+4\left(\int \frac{\sqrt{x}\,(b\,x^2+ax)^p}{4bp\,x^2+4apx+b\,x^2+ax}dx\right)a\,p^2+\left(\int \frac{\sqrt{x}\,(b\,x^2+ax)^p}{4bp\,x^2+4apx+b\,x^2+ax}dx\right)ap\right)}{c\,(4p+1)}$$

```
int((b*x^2+a*x)^p/(c*x)^(1/2),x)
```

```
(2*sqrt(c)*(sqrt(x)*(a*x + b*x**2)**p + 4*int((sqrt(x)*(a*x + b*x**2)**p)/
(4*a*p*x + a*x + 4*b*p*x**2 + b*x**2),x)*a*p**2 + int((sqrt(x)*(a*x + b*x*
*2)**p)/(4*a*p*x + a*x + 4*b*p*x**2 + b*x**2),x)*a*p))/(c*(4*p + 1))
```

3.250 $\int \frac{(ax+bx^2)^p}{(cx)^{3/2}}\,dx$

Optimal result

Integrand size = 19, antiderivative size = 53

$$\int \frac{(ax+bx^2)^p}{(cx)^{3/2}}\,dx = -\frac{2(ax+bx^2)^{1+p}\,\text{Hypergeometric2F1}\left(1,\frac{1}{2}+2p,\frac{1}{2}+p,-\frac{bx}{a}\right)}{a(1-2p)(cx)^{3/2}}$$

```
-2*(b*x^2+a*x)^(p+1)*hypergeom([1, 1/2+2*p],[1/2+p],-b*x/a)/a/(1-2*p)/(c*x
)^(3/2)
```

Mathematica [A] (verified)

Time = 0.03 (sec) , antiderivative size = 58, normalized size of antiderivative = 1.09

$$\int \frac{(ax+bx^2)^p}{(cx)^{3/2}}\,dx = \frac{x(x(a+bx))^p\left(1+\frac{bx}{a}\right)^{-p}\text{Hypergeometric2F1}\left(-\frac{1}{2}+p,-p,\frac{1}{2}+p,-\frac{bx}{a}\right)}{\left(-\frac{1}{2}+p\right)(cx)^{3/2}}$$

```
Integrate[(a*x + b*x^2)^p/(c*x)^(3/2),x]
```

```
(x*(x*(a + b*x))^p*Hypergeometric2F1[-1/2 + p, -p, 1/2 + p, -((b*x)/a)])/(
(-1/2 + p)*(c*x)^(3/2)*(1 + (b*x)/a)^p)
```

Rubi [A] (verified)

Time = 0.31 (sec) , antiderivative size = 61, normalized size of antiderivative = 1.15, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.158, Rules used = {1137, 76, 74}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(ax+bx^2)^p}{(cx)^{3/2}}\,dx$$

$$\Big\downarrow \text{ 1137}$$

$$\frac{x^{\frac{3}{2}-p}(a+bx)^{-p}\left(ax+bx^2\right)^p\int x^{p-\frac{3}{2}}(a+bx)^pdx}{(cx)^{3/2}}$$

$$\Big\downarrow \text{ 76}$$

$$\frac{x^{\frac{3}{2}-p}\left(\frac{bx}{a}+1\right)^{-p}\left(ax+bx^2\right)^p\int x^{p-\frac{3}{2}}\left(\frac{bx}{a}+1\right)^pdx}{(cx)^{3/2}}$$

$$\Big\downarrow \text{ 74}$$

$$-\frac{2x\left(\frac{bx}{a}+1\right)^{-p}\left(ax+bx^2\right)^p\text{Hypergeometric2F1}\left(p-\frac{1}{2},-p,p+\frac{1}{2},-\frac{bx}{a}\right)}{(1-2p)(cx)^{3/2}}$$

```
Int[(a*x + b*x^2)^p/(c*x)^(3/2),x]
```

```
(-2*x*(a*x + b*x^2)^p*Hypergeometric2F1[-1/2 + p, -p, 1/2 + p, -((b*x)/a)]
)/((1 - 2*p)*(c*x)^(3/2)*(1 + (b*x)/a)^p)
```

Defintions of rubi rules used

```
Int[((b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[c^n*((b*x
)^(m + 1)/(b*(m + 1)))*Hypergeometric2F1[-n, m + 1, m + 2, (-d)*(x/c)], x]
/; FreeQ[{b, c, d, m, n}, x] &&  !IntegerQ[m] && (IntegerQ[n] || (GtQ[c, 0]
 &&  !(EqQ[n, -2^(-1)] && EqQ[c^2 - d^2, 0] && GtQ[-d/(b*c), 0])))
```

```
Int[((b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[c^IntPart
[n]*((c + d*x)^FracPart[n]/(1 + d*(x/c))^FracPart[n])   Int[(b*x)^m*(1 + d*
(x/c))^n, x], x] /; FreeQ[{b, c, d, m, n}, x] &&  !IntegerQ[m] &&  !Integer
Q[n] &&  !GtQ[c, 0] &&  !GtQ[-d/(b*c), 0] && ((RationalQ[m] &&  !(EqQ[n, -2
^(-1)] && EqQ[c^2 - d^2, 0])) ||  !RationalQ[n])
```

```
Int[((e_.)*(x_))^(m_)*((b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(
e*x)^m*((b*x + c*x^2)^p/(x^(m + p)*(b + c*x)^p))   Int[x^(m + p)*(b + c*x)^
p, x], x] /; FreeQ[{b, c, e, m}, x]
```

Maple [F]

$$\int \frac{\left(b\,x^{2}+a x \right)^{p}}{\left(c x \right)^{\frac{3}{2}}}d x$$

```
int((b*x^2+a*x)^p/(c*x)^(3/2),x)
```

```
int((b*x^2+a*x)^p/(c*x)^(3/2),x)
```

Fricas [F]

$$\int \frac{(ax+bx^2)^p}{(cx)^{3/2}}\,dx = \int \frac{(bx^2+ax)^p}{(cx)^{\frac{3}{2}}}\,dx$$

```
integrate((b*x^2+a*x)^p/(c*x)^(3/2),x, algorithm="fricas")
```

```
integral(sqrt(c*x)*(b*x^2 + a*x)^p/(c^2*x^2), x)
```

Sympy [F]

$$\int \frac{(ax+bx^2)^p}{(cx)^{3/2}}\,dx = \int \frac{(x(a+bx))^p}{(cx)^{\frac{3}{2}}}\,dx$$

```
integrate((b*x**2+a*x)**p/(c*x)**(3/2),x)
```

```
Integral((x*(a + b*x))**p/(c*x)**(3/2), x)
```

Maxima [F]

$$\int \frac{(ax+bx^2)^p}{(cx)^{3/2}}\,dx = \int \frac{(bx^2+ax)^p}{(cx)^{\frac{3}{2}}}\,dx$$

```
integrate((b*x^2+a*x)^p/(c*x)^(3/2),x, algorithm="maxima")
```

```
integrate((b*x^2 + a*x)^p/(c*x)^(3/2), x)
```

Giac [F]

$$\int \frac{(ax+bx^2)^p}{(cx)^{3/2}}\,dx = \int \frac{(bx^2+ax)^p}{(cx)^{\frac{3}{2}}}\,dx$$

```
integrate((b*x^2+a*x)^p/(c*x)^(3/2),x, algorithm="giac")
```

```
integrate((b*x^2 + a*x)^p/(c*x)^(3/2), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(ax+bx^2)^p}{(cx)^{3/2}}\,dx = \int \frac{(b\,x^2+a\,x)^p}{(c\,x)^{3/2}}\,dx$$

```
int((a*x + b*x^2)^p/(c*x)^(3/2),x)
```

```
int((a*x + b*x^2)^p/(c*x)^(3/2), x)
```

Reduce [F]

$$\int \frac{(ax+bx^2)^p}{(cx)^{3/2}}\,dx = \frac{2\sqrt{c}\left((b\,x^2+ax)^p + 4\sqrt{x}\left(\int \frac{\sqrt{x}\,(b\,x^2+ax)^p}{4bp\,x^3+4ap\,x^2-b\,x^3-a\,x^2}dx\right)a\,p^2 - \sqrt{x}\left(\int \frac{\sqrt{x}\,(b\,x^2+ax)^p}{4bp\,x^3+4ap\,x^2-b\,x^3-a\,x}\right.\right.}{\sqrt{x}\,c^2\,(4p-1)}$$

```
int((b*x^2+a*x)^p/(c*x)^(3/2),x)
```

```
(2*sqrt(c)*((a*x + b*x**2)**p + 4*sqrt(x)*int((sqrt(x)*(a*x + b*x**2)**p)/
(4*a*p*x**2 - a*x**2 + 4*b*p*x**3 - b*x**3),x)*a*p**2 - sqrt(x)*int((sqrt(
x)*(a*x + b*x**2)**p)/(4*a*p*x**2 - a*x**2 + 4*b*p*x**3 - b*x**3),x)*a*p))
/(sqrt(x)*c**2*(4*p - 1))
```

3.251 $\int (cx)^{3/2} (2x-3x^2)^p \, dx$

Optimal result

Integrand size = 19, antiderivative size = 45

$$\int (cx)^{3/2} (2x-3x^2)^p \, dx = \frac{2^{1+p} c x^{2+p} \sqrt{cx} \operatorname{Hypergeometric2F1}\left(-p, \frac{5}{2}+p, \frac{7}{2}+p, \frac{3x}{2}\right)}{5+2p}$$

```
2^(p+1)*c*x^(2+p)*(c*x)^(1/2)*hypergeom([-p, 5/2+p],[7/2+p],3/2*x)/(5+2*p)
```

Mathematica [A] (verified)

Time = 0.03 (sec) , antiderivative size = 56, normalized size of antiderivative = 1.24

$$\int (cx)^{3/2} (2x-3x^2)^p \, dx = \frac{2^p (2-3x)^{-p} x (cx)^{3/2} ((2-3x)x)^p \operatorname{Hypergeometric2F1}\left(-p, \frac{5}{2}+p, \frac{7}{2}+p, \frac{3x}{2}\right)}{\frac{5}{2}+p}$$

```
Integrate[(c*x)^(3/2)*(2*x - 3*x^2)^p,x]
```

```
(2^p*x*(c*x)^(3/2)*((2 - 3*x)*x)^p*Hypergeometric2F1[-p, 5/2 + p, 7/2 + p,
 (3*x)/2])/((5/2 + p)*(2 - 3*x)^p)
```

Rubi [A] (verified)

Time = 0.29 (sec) , antiderivative size = 60, normalized size of antiderivative = 1.33, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.105$, Rules used = {1137, 74}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (cx)^{3/2} \left(2x-3x^2\right)^p \, dx$$

$$\Big\downarrow \ 1137$$

$$(cx)^{3/2}(2-3x)^{-p}x^{-p-\frac{3}{2}}\left(2x-3x^2\right)^p \int (2-3x)^p x^{p+\frac{3}{2}}dx$$

$$\Big\downarrow \ 74$$

$$\frac{2^{p+1}x(cx)^{3/2}(2-3x)^{-p}\left(2x-3x^2\right)^p \text{Hypergeometric2F1}\left(-p,p+\frac{5}{2},p+\frac{7}{2},\frac{3x}{2}\right)}{2p+5}$$

```
Int[(c*x)^(3/2)*(2*x - 3*x^2)^p,x]
```

```
(2^(1 + p)*x*(c*x)^(3/2)*(2*x - 3*x^2)^p*Hypergeometric2F1[-p, 5/2 + p, 7/
2 + p, (3*x)/2])/((5 + 2*p)*(2 - 3*x)^p)
```

Defintions of rubi rules used

```
Int[((b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[c^n*((b*x
)^(m + 1)/(b*(m + 1)))*Hypergeometric2F1[-n, m + 1, m + 2, (-d)*(x/c)], x]
/; FreeQ[{b, c, d, m, n}, x] &&  !IntegerQ[m] && (IntegerQ[n] || (GtQ[c, 0]
 &&  !(EqQ[n, -2^(-1)] && EqQ[c^2 - d^2, 0] && GtQ[-d/(b*c), 0])))
```

```
Int[((e_.)*(x_))^(m_)*((b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(
e*x)^m*((b*x + c*x^2)^p/(x^(m + p)*(b + c*x)^p))   Int[x^(m + p)*(b + c*x)^
p, x], x] /; FreeQ[{b, c, e, m}, x]
```

Maple [A] (verified)

Time = 0.31 (sec) , antiderivative size = 39, normalized size of antiderivative = 0.87

method	result	size
meijerg	$\frac{2^{p+1}(cx)^{\frac{3}{2}} x^{p+1} \operatorname{hypergeom}\left(\left[-p,\frac{5}{2}+p\right],\left[\frac{7}{2}+p\right],\frac{3x}{2}\right)}{5+2p}$	39

```
int((c*x)^(3/2)*(-3*x^2+2*x)^p,x,method=_RETURNVERBOSE)
```

```
2^(p+1)*(c*x)^(3/2)*x^(p+1)/(5+2*p)*hypergeom([-p,5/2+p],[7/2+p],3/2*x)
```

Fricas [F]

$$\int (cx)^{3/2} \left(2x-3x^2\right)^p \, dx = \int (cx)^{\frac{3}{2}} \left(-3\,x^2+2\,x\right)^p \, dx$$

```
integrate((c*x)^(3/2)*(-3*x^2+2*x)^p,x, algorithm="fricas")
```

```
integral(sqrt(c*x)*(-3*x^2 + 2*x)^p*c*x, x)
```

Sympy [F]

$$\int (cx)^{3/2} \left(2x-3x^2\right)^p \, dx = \int (cx)^{\frac{3}{2}} \left(-x(3x-2)\right)^p \, dx$$

```
integrate((c*x)**(3/2)*(-3*x**2+2*x)**p,x)
```

```
Integral((c*x)**(3/2)*(-x*(3*x - 2))**p, x)
```

Maxima [F]

$$\int (cx)^{3/2} \left(2x-3x^2\right)^p \, dx = \int (cx)^{\frac{3}{2}} \left(-3\,x^2+2\,x\right)^p \, dx$$

```
integrate((c*x)^(3/2)*(-3*x^2+2*x)^p,x, algorithm="maxima")
```

```
integrate((c*x)^(3/2)*(-3*x^2 + 2*x)^p, x)
```

Giac [F]

$$\int (cx)^{3/2} \left(2x-3x^2\right)^p \, dx = \int (cx)^{\frac{3}{2}} \left(-3\,x^2+2\,x\right)^p \, dx$$

```
integrate((c*x)^(3/2)*(-3*x^2+2*x)^p,x, algorithm="giac")
```

```
integrate((c*x)^(3/2)*(-3*x^2 + 2*x)^p, x)
```

Mupad [F(-1)]

Timed out.

$$\int (cx)^{3/2} \left(2x-3x^2\right)^p \, dx = \int (c\,x)^{3/2} \left(2\,x-3\,x^2\right)^p \, dx$$

```
int((c*x)^(3/2)*(2*x - 3*x^2)^p,x)
```

```
int((c*x)^(3/2)*(2*x - 3*x^2)^p, x)
```

Reduce [F]

$$\int (cx)^{3/2} \left(2x-3x^2\right)^p dx = \frac{2\sqrt{c}\, c\Big(144\sqrt{x}\left(-3x^2+2x\right)^p p^2x^2 - 48\sqrt{x}\left(-3x^2+2x\right)^p p^2x - 16\sqrt{x}\left(-3x^2+2x\right)^p p^2 + 144\sqrt{}$$

```
int((c*x)^(3/2)*(-3*x^2+2*x)^p,x)
```

```
(2*sqrt(c)*c*(144*sqrt(x)*( - 3*x**2 + 2*x)**p*p**2*x**2 - 48*sqrt(x)*( -
3*x**2 + 2*x)**p*p**2*x - 16*sqrt(x)*( - 3*x**2 + 2*x)**p*p**2 + 144*sqrt(
x)*( - 3*x**2 + 2*x)**p*p*x**2 - 12*sqrt(x)*( - 3*x**2 + 2*x)**p*p*x - 24*
sqrt(x)*( - 3*x**2 + 2*x)**p*p + 27*sqrt(x)*( - 3*x**2 + 2*x)**p*x**2 - 20
48*int((sqrt(x)*( - 3*x**2 + 2*x)**p)/(192*p**3*x**2 - 128*p**3*x + 432*p*
*2*x**2 - 288*p**2*x + 276*p*x**2 - 184*p*x + 45*x**2 - 30*x),x)*p**6 - 87
04*int((sqrt(x)*( - 3*x**2 + 2*x)**p)/(192*p**3*x**2 - 128*p**3*x + 432*p*
*2*x**2 - 288*p**2*x + 276*p*x**2 - 184*p*x + 45*x**2 - 30*x),x)*p**5 - 13
696*int((sqrt(x)*( - 3*x**2 + 2*x)**p)/(192*p**3*x**2 - 128*p**3*x + 432*p
**2*x**2 - 288*p**2*x + 276*p*x**2 - 184*p*x + 45*x**2 - 30*x),x)*p**4 - 9
824*int((sqrt(x)*( - 3*x**2 + 2*x)**p)/(192*p**3*x**2 - 128*p**3*x + 432*p
**2*x**2 - 288*p**2*x + 276*p*x**2 - 184*p*x + 45*x**2 - 30*x),x)*p**3 - 3
168*int((sqrt(x)*( - 3*x**2 + 2*x)**p)/(192*p**3*x**2 - 128*p**3*x + 432*p
**2*x**2 - 288*p**2*x + 276*p*x**2 - 184*p*x + 45*x**2 - 30*x),x)*p**2 - 3
60*int((sqrt(x)*( - 3*x**2 + 2*x)**p)/(192*p**3*x**2 - 128*p**3*x + 432*p*
*2*x**2 - 288*p**2*x + 276*p*x**2 - 184*p*x + 45*x**2 - 30*x),x)*p))/(9*(6
4*p**3 + 144*p**2 + 92*p + 15))
```

3.252 $\int \sqrt{cx}(2x-3x^2)^p\,dx$

Optimal result

Integrand size = 19, antiderivative size = 44

$$\int \sqrt{cx}\left(2x-3x^2\right)^p\,dx=\frac{2^{1+p}x^{1+p}\sqrt{cx}\,\text{Hypergeometric2F1}\left(-p,\frac{3}{2}+p,\frac{5}{2}+p,\frac{3x}{2}\right)}{3+2p}$$

```
2^(p+1)*x^(p+1)*(c*x)^(1/2)*hypergeom([-p, 3/2+p],[5/2+p],3/2*x)/(3+2*p)
```

Mathematica [A] (verified)

Time = 0.03 (sec) , antiderivative size = 56, normalized size of antiderivative = 1.27

$$\int \sqrt{cx}\left(2x-3x^2\right)^p\,dx=\frac{2^p(2-3x)^{-p}x\sqrt{cx}((2-3x)x)^p\,\text{Hypergeometric2F1}\left(-p,\frac{3}{2}+p,\frac{5}{2}+p,\frac{3x}{2}\right)}{\frac{3}{2}+p}$$

```
Integrate[Sqrt[c*x]*(2*x - 3*x^2)^p,x]
```

```
(2^p*x*Sqrt[c*x]*((2 - 3*x)*x)^p*Hypergeometric2F1[-p, 3/2 + p, 5/2 + p, (
3*x)/2])/((3/2 + p)*(2 - 3*x)^p)
```

Rubi [A] (verified)

Time = 0.29 (sec) , antiderivative size = 60, normalized size of antiderivative = 1.36, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.105$, Rules used = {1137, 74}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sqrt{cx}\left(2x-3x^2\right)^p\,dx$$

$$\Big\downarrow \ 1137$$

$$\sqrt{cx}(2-3x)^{-p}x^{-p-\frac{1}{2}}\left(2x-3x^2\right)^p\int (2-3x)^p x^{p+\frac{1}{2}}dx$$

$$\Big\downarrow \ 74$$

$$\frac{2^{p+1}x\sqrt{cx}(2-3x)^{-p}\left(2x-3x^2\right)^p \text{Hypergeometric2F1}\left(-p,p+\frac{3}{2},p+\frac{5}{2},\frac{3x}{2}\right)}{2p+3}$$

```
Int[Sqrt[c*x]*(2*x - 3*x^2)^p,x]
```

```
(2^(1 + p)*x*Sqrt[c*x]*(2*x - 3*x^2)^p*Hypergeometric2F1[-p, 3/2 + p, 5/2
+ p, (3*x)/2])/((3 + 2*p)*(2 - 3*x)^p)
```

Defintions of rubi rules used

```
Int[((b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[c^n*((b*x
)^(m + 1)/(b*(m + 1)))*Hypergeometric2F1[-n, m + 1, m + 2, (-d)*(x/c)], x]
/; FreeQ[{b, c, d, m, n}, x] &&  !IntegerQ[m] && (IntegerQ[n] || (GtQ[c, 0]
 &&  !(EqQ[n, -2^(-1)] && EqQ[c^2 - d^2, 0] && GtQ[-d/(b*c), 0])))
```

```
Int[((e_.)*(x_))^(m_)*((b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(
e*x)^m*((b*x + c*x^2)^p/(x^(m + p)*(b + c*x)^p))   Int[x^(m + p)*(b + c*x)^
p, x], x] /; FreeQ[{b, c, e, m}, x]
```

Maple [A] (verified)

Time = 0.30 (sec) , antiderivative size = 39, normalized size of antiderivative = 0.89

method	result	size
meijerg	$\frac{2^{p+1}x^{p+1}\sqrt{cx}\ \mathrm{hypergeom}\left(\left[-p,\frac{3}{2}+p\right],\left[\frac{5}{2}+p\right],\frac{3x}{2}\right)}{3+2p}$	39

```
int((c*x)^(1/2)*(-3*x^2+2*x)^p,x,method=_RETURNVERBOSE)
```

```
2^(p+1)*x^(p+1)*(c*x)^(1/2)*hypergeom([-p,3/2+p],[5/2+p],3/2*x)/(3+2*p)
```

Fricas [F]

$$\int \sqrt{cx}\left(2x-3x^2\right)^p\,dx=\int \sqrt{cx}\left(-3\,x^2+2\,x\right)^p\,dx$$

```
integrate((c*x)^(1/2)*(-3*x^2+2*x)^p,x, algorithm="fricas")
```

```
integral(sqrt(c*x)*(-3*x^2 + 2*x)^p, x)
```

Sympy [F]

$$\int \sqrt{cx}\left(2x-3x^2\right)^p\,dx=\int \sqrt{cx}(-x(3x-2))^p\,dx$$

```
integrate((c*x)**(1/2)*(-3*x**2+2*x)**p,x)
```

```
Integral(sqrt(c*x)*(-x*(3*x - 2))**p, x)
```

Maxima [F]

$$\int \sqrt{cx}\,(2x-3x^2)^p\,dx = \int \sqrt{cx}\,(-3\,x^2+2\,x)^p\,dx$$

```
integrate((c*x)^(1/2)*(-3*x^2+2*x)^p,x, algorithm="maxima")
```

```
integrate(sqrt(c*x)*(-3*x^2 + 2*x)^p, x)
```

Giac [F]

$$\int \sqrt{cx}\,(2x-3x^2)^p\,dx = \int \sqrt{cx}\,(-3\,x^2+2\,x)^p\,dx$$

```
integrate((c*x)^(1/2)*(-3*x^2+2*x)^p,x, algorithm="giac")
```

```
integrate(sqrt(c*x)*(-3*x^2 + 2*x)^p, x)
```

Mupad [F(-1)]

Timed out.

$$\int \sqrt{cx}\,(2x-3x^2)^p\,dx = \int \sqrt{c\,x}\,\left(2\,x-3\,x^2\right)^p\,dx$$

```
int((c*x)^(1/2)*(2*x - 3*x^2)^p,x)
```

```
int((c*x)^(1/2)*(2*x - 3*x^2)^p, x)
```

Reduce [F]

$$\int \sqrt{cx}\,(2x-3x^2)^p\,dx$$
$$= \frac{2\sqrt{c}\left(12\sqrt{x}\,(-3x^2+2x)^p\,px-4\sqrt{x}\,(-3x^2+2x)^p\,p+3\sqrt{x}\,(-3x^2+2x)^p\,x-128\left(\int \frac{\sqrt{x}\,(-3x^2+2x}{48p^2x^2-32p^2x+48p\,x^2-3}\right.\right.}{}$$

```
int((c*x)^(1/2)*(-3*x^2+2*x)^p,x)
```

```
(2*sqrt(c)*(12*sqrt(x)*( - 3*x**2 + 2*x)**p*p*x - 4*sqrt(x)*( - 3*x**2 + 2
*x)**p*p + 3*sqrt(x)*( - 3*x**2 + 2*x)**p*x - 128*int((sqrt(x)*( - 3*x**2
+ 2*x)**p)/(48*p**2*x**2 - 32*p**2*x + 48*p*x**2 - 32*p*x + 9*x**2 - 6*x),
x)*p**4 - 192*int((sqrt(x)*( - 3*x**2 + 2*x)**p)/(48*p**2*x**2 - 32*p**2*x
 + 48*p*x**2 - 32*p*x + 9*x**2 - 6*x),x)*p**3 - 88*int((sqrt(x)*( - 3*x**2
 + 2*x)**p)/(48*p**2*x**2 - 32*p**2*x + 48*p*x**2 - 32*p*x + 9*x**2 - 6*x)
,x)*p**2 - 12*int((sqrt(x)*( - 3*x**2 + 2*x)**p)/(48*p**2*x**2 - 32*p**2*x
 + 48*p*x**2 - 32*p*x + 9*x**2 - 6*x),x)*p))/(3*(16*p**2 + 16*p + 3))
```

3.253 $\int \frac{(2x-3x^2)^p}{\sqrt{cx}}\,dx$

Optimal result

Integrand size = 19, antiderivative size = 44

$$\int \frac{(2x-3x^2)^p}{\sqrt{cx}}\,dx = \frac{2^{1+p}x^{1+p}\,\text{Hypergeometric2F1}\left(-p,\frac{1}{2}+p,\frac{3}{2}+p,\frac{3x}{2}\right)}{(1+2p)\sqrt{cx}}$$

```
2^(p+1)*x^(p+1)*hypergeom([-p, 1/2+p],[3/2+p],3/2*x)/(1+2*p)/(c*x)^(1/2)
```

Mathematica [A] (verified)

Time = 0.03 (sec) , antiderivative size = 56, normalized size of antiderivative = 1.27

$$\int \frac{(2x-3x^2)^p}{\sqrt{cx}}\,dx = \frac{2^p(2-3x)^{-p}x((2-3x)x)^p\,\text{Hypergeometric2F1}\left(-p,\frac{1}{2}+p,\frac{3}{2}+p,\frac{3x}{2}\right)}{\left(\frac{1}{2}+p\right)\sqrt{cx}}$$

```
Integrate[(2*x - 3*x^2)^p/Sqrt[c*x],x]
```

```
(2^p*x*((2 - 3*x)*x)^p*Hypergeometric2F1[-p, 1/2 + p, 3/2 + p, (3*x)/2])/(
(1/2 + p)*(2 - 3*x)^p*Sqrt[c*x])
```

Rubi [A] (verified)

Time = 0.28 (sec) , antiderivative size = 60, normalized size of antiderivative = 1.36, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.105, Rules used = {1137, 74}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(2x-3x^2)^p}{\sqrt{cx}}\,dx$$

$$\Big\downarrow \text{1137}$$

$$\frac{(2-3x)^{-p}x^{\frac{1}{2}-p}(2x-3x^2)^p\int (2-3x)^p x^{p-\frac{1}{2}}dx}{\sqrt{cx}}$$

$$\Big\downarrow \text{74}$$

$$\frac{2^{p+1}x(2-3x)^{-p}\left(2x-3x^2\right)^p\text{Hypergeometric2F1}\left(-p,p+\frac{1}{2},p+\frac{3}{2},\frac{3x}{2}\right)}{(2p+1)\sqrt{cx}}$$

```
Int[(2*x - 3*x^2)^p/Sqrt[c*x],x]
```

```
(2^(1 + p)*x*(2*x - 3*x^2)^p*Hypergeometric2F1[-p, 1/2 + p, 3/2 + p, (3*x)
/2])/((1 + 2*p)*(2 - 3*x)^p*Sqrt[c*x])
```

Defintions of rubi rules used

```
Int[((b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[c^n*((b*x
)^(m + 1)/(b*(m + 1)))*Hypergeometric2F1[-n, m + 1, m + 2, (-d)*(x/c)], x]
/; FreeQ[{b, c, d, m, n}, x] && !IntegerQ[m] && (IntegerQ[n] || (GtQ[c, 0]
 && !(EqQ[n, -2^(-1)] && EqQ[c^2 - d^2, 0] && GtQ[-d/(b*c), 0])))
```

```
Int[((e_.)*(x_))^(m_)*((b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(
e*x)^m*((b*x + c*x^2)^p/(x^(m + p)*(b + c*x)^p))   Int[x^(m + p)*(b + c*x)^
p, x], x] /; FreeQ[{b, c, e, m}, x]
```

Maple [A] (verified)

Time = 0.31 (sec) , antiderivative size = 39, normalized size of antiderivative = 0.89

method	result	size
meijerg	$\frac{2^{p+1} x^{p+1} \operatorname{hypergeom}\left(\left[-p, \frac{1}{2}+p\right], \left[\frac{3}{2}+p\right], \frac{3x}{2}\right)}{(2p+1)\sqrt{cx}}$	39

```
int((-3*x^2+2*x)^p/(c*x)^(1/2),x,method=_RETURNVERBOSE)
```

```
2^(p+1)*x^(p+1)*hypergeom([-p,1/2+p],[3/2+p],3/2*x)/(2*p+1)/(c*x)^(1/2)
```

Fricas [F]

$$\int \frac{(2x-3x^2)^p}{\sqrt{cx}}\,dx = \int \frac{(-3\,x^2+2\,x)^p}{\sqrt{cx}}\,dx$$

```
integrate((-3*x^2+2*x)^p/(c*x)^(1/2),x, algorithm="fricas")
```

```
integral(sqrt(c*x)*(-3*x^2 + 2*x)^p/(c*x), x)
```

Sympy [F]

$$\int \frac{(2x-3x^2)^p}{\sqrt{cx}}\,dx = \int \frac{(-x(3x-2))^p}{\sqrt{cx}}\,dx$$

```
integrate((-3*x**2+2*x)**p/(c*x)**(1/2),x)
```

```
Integral((-x*(3*x - 2))**p/sqrt(c*x), x)
```

Maxima [F]

$$\int \frac{(2x-3x^2)^p}{\sqrt{cx}}\,dx = \int \frac{(-3\,x^2+2\,x)^p}{\sqrt{cx}}\,dx$$

```
integrate((-3*x^2+2*x)^p/(c*x)^(1/2),x, algorithm="maxima")
```

```
integrate((-3*x^2 + 2*x)^p/sqrt(c*x), x)
```

Giac [F]

$$\int \frac{(2x-3x^2)^p}{\sqrt{cx}}\,dx = \int \frac{(-3\,x^2+2\,x)^p}{\sqrt{cx}}\,dx$$

```
integrate((-3*x^2+2*x)^p/(c*x)^(1/2),x, algorithm="giac")
```

```
integrate((-3*x^2 + 2*x)^p/sqrt(c*x), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(2x-3x^2)^p}{\sqrt{cx}}\,dx = \int \frac{(2\,x-3\,x^2)^p}{\sqrt{c\,x}}\,dx$$

```
int((2*x - 3*x^2)^p/(c*x)^(1/2),x)
```

```
int((2*x - 3*x^2)^p/(c*x)^(1/2), x)
```

Reduce [F]

$$\int \frac{(2x-3x^2)^p}{\sqrt{cx}}\,dx$$
$$=\frac{2\sqrt{c}\left(\sqrt{x}\,(-3x^2+2x)^p-8\left(\int \frac{\sqrt{x}\,(-3x^2+2x)^p}{12p\,x^2-8px+3x^2-2x}dx\right)p^2-2\left(\int \frac{\sqrt{x}\,(-3x^2+2x)^p}{12p\,x^2-8px+3x^2-2x}dx\right)p\right)}{c\,(4p+1)}$$

```
int((-3*x^2+2*x)^p/(c*x)^(1/2),x)
```

```
(2*sqrt(c)*(sqrt(x)*( - 3*x**2 + 2*x)**p - 8*int((sqrt(x)*( - 3*x**2 + 2*x
)**p)/(12*p*x**2 - 8*p*x + 3*x**2 - 2*x),x)*p**2 - 2*int((sqrt(x)*( - 3*x*
*2 + 2*x)**p)/(12*p*x**2 - 8*p*x + 3*x**2 - 2*x),x)*p))/(c*(4*p + 1))
```

3.254 $\int \frac{(2x-3x^2)^p}{(cx)^{3/2}}\,dx$

Optimal result

Integrand size = 19, antiderivative size = 46

$$\int \frac{(2x-3x^2)^p}{(cx)^{3/2}}\,dx = -\frac{2^{1+p}x^p\,\text{Hypergeometric2F1}\left(-\frac{1}{2}+p,-p,\frac{1}{2}+p,\frac{3x}{2}\right)}{c(1-2p)\sqrt{cx}}$$

```
-2^(p+1)*x^p*hypergeom([-p, -1/2+p],[1/2+p],3/2*x)/c/(1-2*p)/(c*x)^(1/2)
```

Mathematica [A] (verified)

Time = 0.03 (sec) , antiderivative size = 56, normalized size of antiderivative = 1.22

$$\int \frac{(2x-3x^2)^p}{(cx)^{3/2}}\,dx = \frac{2^p(2-3x)^{-p}x((2-3x)x)^p\,\text{Hypergeometric2F1}\left(-\frac{1}{2}+p,-p,\frac{1}{2}+p,\frac{3x}{2}\right)}{\left(-\frac{1}{2}+p\right)(cx)^{3/2}}$$

```
Integrate[(2*x - 3*x^2)^p/(c*x)^(3/2),x]
```

```
(2^p*x*((2 - 3*x)*x)^p*Hypergeometric2F1[-1/2 + p, -p, 1/2 + p, (3*x)/2])/
((-1/2 + p)*(2 - 3*x)^p*(c*x)^(3/2))
```

Rubi [A] (verified)

Time = 0.28 (sec) , antiderivative size = 61, normalized size of antiderivative = 1.33, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.105, Rules used = {1137, 74}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(2x-3x^2)^p}{(cx)^{3/2}} \, dx$$

$$\Big\downarrow \; 1137$$

$$\frac{(2-3x)^{-p}x^{\frac{3}{2}-p}(2x-3x^2)^p \int (2-3x)^p x^{p-\frac{3}{2}}dx}{(cx)^{3/2}}$$

$$\Big\downarrow \; 74$$

$$-\frac{2^{p+1}x(2-3x)^{-p}\left(2x-3x^2\right)^p \text{Hypergeometric2F1}\left(p-\frac{1}{2},-p,p+\frac{1}{2},\frac{3x}{2}\right)}{(1-2p)(cx)^{3/2}}$$

```
Int[(2*x - 3*x^2)^p/(c*x)^(3/2),x]
```

```
-((2^(1 + p)*x*(2*x - 3*x^2)^p*Hypergeometric2F1[-1/2 + p, -p, 1/2 + p, (3
*x)/2])/((1 - 2*p)*(2 - 3*x)^p*(c*x)^(3/2)))
```

Defintions of rubi rules used

```
Int[((b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[c^n*((b*x
)^(m + 1)/(b*(m + 1)))*Hypergeometric2F1[-n, m + 1, m + 2, (-d)*(x/c)], x]
/; FreeQ[{b, c, d, m, n}, x] &&  !IntegerQ[m] && (IntegerQ[n] || (GtQ[c, 0]
 &&  !(EqQ[n, -2^(-1)] && EqQ[c^2 - d^2, 0] && GtQ[-d/(b*c), 0])))
```

```
Int[((e_.)*(x_))^(m_)*((b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(
e*x)^m*((b*x + c*x^2)^p/(x^(m + p)*(b + c*x)^p))   Int[x^(m + p)*(b + c*x)^
p, x], x] /; FreeQ[{b, c, e, m}, x]
```

Maple [A] (verified)

Time = 0.32 (sec) , antiderivative size = 39, normalized size of antiderivative = 0.85

method	result	size
meijerg	$\frac{2^{p+1} x^{p+1} \operatorname{hypergeom}\left(\left[-p,-\frac{1}{2}+p\right],\left[\frac{1}{2}+p\right],\frac{3x}{2}\right)}{(cx)^{\frac{3}{2}}(-1+2p)}$	39

```
int((-3*x^2+2*x)^p/(c*x)^(3/2),x,method=_RETURNVERBOSE)
```

```
2^(p+1)/(c*x)^(3/2)*x^(p+1)/(-1+2*p)*hypergeom([-p,-1/2+p],[1/2+p],3/2*x)
```

Fricas [F]

$$\int \frac{(2x-3x^2)^p}{(cx)^{3/2}}\,dx = \int \frac{(-3\,x^2+2\,x)^p}{(cx)^{\frac{3}{2}}}\,dx$$

```
integrate((-3*x^2+2*x)^p/(c*x)^(3/2),x, algorithm="fricas")
```

```
integral(sqrt(c*x)*(-3*x^2 + 2*x)^p/(c^2*x^2), x)
```

Sympy [F]

$$\int \frac{(2x-3x^2)^p}{(cx)^{3/2}}\,dx = \int \frac{(-x(3x-2))^p}{(cx)^{\frac{3}{2}}}\,dx$$

```
integrate((-3*x**2+2*x)**p/(c*x)**(3/2),x)
```

```
Integral((-x*(3*x - 2))**p/(c*x)**(3/2), x)
```

Maxima [F]

$$\int \frac{(2x-3x^2)^p}{(cx)^{3/2}}\,dx = \int \frac{(-3\,x^2+2\,x)^p}{(cx)^{\frac{3}{2}}}\,dx$$

```
integrate((-3*x^2+2*x)^p/(c*x)^(3/2),x, algorithm="maxima")
```

```
integrate((-3*x^2 + 2*x)^p/(c*x)^(3/2), x)
```

Giac [F]

$$\int \frac{(2x-3x^2)^p}{(cx)^{3/2}}\,dx = \int \frac{(-3\,x^2+2\,x)^p}{(cx)^{\frac{3}{2}}}\,dx$$

```
integrate((-3*x^2+2*x)^p/(c*x)^(3/2),x, algorithm="giac")
```

```
integrate((-3*x^2 + 2*x)^p/(c*x)^(3/2), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(2x-3x^2)^p}{(cx)^{3/2}}\,dx = \int \frac{(2\,x-3\,x^2)^p}{(c\,x)^{3/2}}\,dx$$

```
int((2*x - 3*x^2)^p/(c*x)^(3/2),x)
```

```
int((2*x - 3*x^2)^p/(c*x)^(3/2), x)
```

Reduce [F]

$$\int \frac{(2x-3x^2)^p}{(cx)^{3/2}}\,dx = \frac{2\sqrt{c}\left((-3x^2+2x)^p - 8\sqrt{x}\left(\int \frac{\sqrt{x}\,(-3x^2+2x)^p}{12p\,x^3-8p\,x^2-3x^3+2x^2}dx\right)p^2 + 2\sqrt{x}\left(\int \frac{\sqrt{x}\,(-3x^2+2x)^p}{12p\,x^3-8p\,x^2-3x^3+2x^2}\right.\right.}{\sqrt{x}\,c^2\,(4p-1)}$$

```
int((-3*x^2+2*x)^p/(c*x)^(3/2),x)
```

```
(2*sqrt(c)*(( - 3*x**2 + 2*x)**p - 8*sqrt(x)*int((sqrt(x)*( - 3*x**2 + 2*x
)**p)/(12*p*x**3 - 8*p*x**2 - 3*x**3 + 2*x**2),x)*p**2 + 2*sqrt(x)*int((sq
rt(x)*( - 3*x**2 + 2*x)**p)/(12*p*x**3 - 8*p*x**2 - 3*x**3 + 2*x**2),x)*p)
)/(sqrt(x)*c**2*(4*p - 1))
```

3.255 $\int (cx)^m (ax+bx^2)^p \, dx$

Optimal result

Integrand size = 17, antiderivative size = 47

$$\int (cx)^m (ax+bx^2)^p \, dx = \frac{(cx)^m (ax+bx^2)^{1+p} \text{Hypergeometric2F1}\left(1, 2+m+2p, 2+m+p, -\frac{bx}{a}\right)}{a(1+m+p)}$$

```
(c*x)^m*(b*x^2+a*x)^(p+1)*hypergeom([1, 2+m+2*p],[2+m+p],-b*x/a)/a/(1+m+p)
```

Mathematica [A] (verified)

Time = 0.03 (sec) , antiderivative size = 53, normalized size of antiderivative = 1.13

$$\int (cx)^m (ax+bx^2)^p \, dx = \frac{x(cx)^m (x(a+bx))^p \left(1+\frac{bx}{a}\right)^{-p} \text{Hypergeometric2F1}\left(-p, 1+m+p, 2+m+p, -\frac{bx}{a}\right)}{1+m+p}$$

```
Integrate[(c*x)^m*(a*x + b*x^2)^p,x]
```

```
(x*(c*x)^m*(x*(a + b*x))^p*Hypergeometric2F1[-p, 1 + m + p, 2 + m + p, -((
b*x)/a)])/((1 + m + p)*(1 + (b*x)/a)^p)
```

Rubi [A] (verified)

Time = 0.29 (sec) , antiderivative size = 55, normalized size of antiderivative = 1.17, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.176, Rules used = {1137, 76, 74}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (cx)^m \left(ax+bx^2\right)^p \, dx$$

$$\Big\downarrow \ 1137$$

$$(cx)^m x^{-m-p} (a+bx)^{-p} \left(ax+bx^2\right)^p \int x^{m+p} (a+bx)^p dx$$

$$\Big\downarrow \ 76$$

$$(cx)^m x^{-m-p} \left(\frac{bx}{a}+1\right)^{-p} \left(ax+bx^2\right)^p \int x^{m+p} \left(\frac{bx}{a}+1\right)^p dx$$

$$\Big\downarrow \ 74$$

$$\frac{x(cx)^m \left(\frac{bx}{a}+1\right)^{-p} \left(ax+bx^2\right)^p \text{Hypergeometric2F1}\left(-p,m+p+1,m+p+2,-\frac{bx}{a}\right)}{m+p+1}$$

```
Int[(c*x)^m*(a*x + b*x^2)^p,x]
```

```
(x*(c*x)^m*(a*x + b*x^2)^p*Hypergeometric2F1[-p, 1 + m + p, 2 + m + p, -((
b*x)/a)])/((1 + m + p)*(1 + (b*x)/a)^p)
```

Defintions of rubi rules used

```
Int[((b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[c^n*((b*x
)^(m + 1)/(b*(m + 1)))*Hypergeometric2F1[-n, m + 1, m + 2, (-d)*(x/c)], x]
/; FreeQ[{b, c, d, m, n}, x] &&  !IntegerQ[m] && (IntegerQ[n] || (GtQ[c, 0]
 &&  !(EqQ[n, -2^(-1)] && EqQ[c^2 - d^2, 0] && GtQ[-d/(b*c), 0])))
```

```
Int[((b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[c^IntPart
[n]*((c + d*x)^FracPart[n]/(1 + d*(x/c))^FracPart[n])   Int[(b*x)^m*(1 + d*
(x/c))^n, x], x] /; FreeQ[{b, c, d, m, n}, x] &&  !IntegerQ[m] &&  !Integer
Q[n] &&  !GtQ[c, 0] &&  !GtQ[-d/(b*c), 0] && ((RationalQ[m] &&  !(EqQ[n, -2
^(-1)] && EqQ[c^2 - d^2, 0])) ||  !RationalQ[n])
```

```
Int[((e_.)*(x_))^(m_)*((b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(
e*x)^m*((b*x + c*x^2)^p/(x^(m + p)*(b + c*x)^p))   Int[x^(m + p)*(b + c*x)^
p, x], x] /; FreeQ[{b, c, e, m}, x]
```

Maple [F]

$$\int (cx)^m \left(b\,x^2+ax\right)^p dx$$

```
int((c*x)^m*(b*x^2+a*x)^p,x)
```

```
int((c*x)^m*(b*x^2+a*x)^p,x)
```

Fricas [F]

$$\int (cx)^m \left(ax+bx^2\right)^p \, dx = \int \left(bx^2+ax\right)^p (cx)^m \, dx$$

```
integrate((c*x)^m*(b*x^2+a*x)^p,x, algorithm="fricas")
```

```
integral((b*x^2 + a*x)^p*(c*x)^m, x)
```

Sympy [F]

$$\int (cx)^m \left(ax+bx^2\right)^p \, dx = \int (cx)^m \left(x(a+bx)\right)^p \, dx$$

```
integrate((c*x)**m*(b*x**2+a*x)**p,x)
```

```
Integral((c*x)**m*(x*(a + b*x))**p, x)
```

Maxima [F]

$$\int (cx)^m \left(ax+bx^2\right)^p \, dx = \int \left(bx^2+ax\right)^p (cx)^m \, dx$$

```
integrate((c*x)^m*(b*x^2+a*x)^p,x, algorithm="maxima")
```

```
integrate((b*x^2 + a*x)^p*(c*x)^m, x)
```

Giac [F]

$$\int (cx)^m \left(ax+bx^2\right)^p \, dx = \int \left(bx^2+ax\right)^p (cx)^m \, dx$$

```
integrate((c*x)^m*(b*x^2+a*x)^p,x, algorithm="giac")
```

```
integrate((b*x^2 + a*x)^p*(c*x)^m, x)
```

Mupad [F(-1)]

Timed out.

$$\int (cx)^m \left(ax+bx^2\right)^p \, dx = \int \left(b\,x^2+a\,x\right)^p\,(c\,x)^m \,dx$$

```
int((a*x + b*x^2)^p*(c*x)^m,x)
```

```
int((a*x + b*x^2)^p*(c*x)^m, x)
```

Reduce [F]

$$\int (cx)^m \left(ax+bx^2\right)^p \, dx = \frac{c^m \Big(x^m(b\,x^2+ax)^p\,ap + x^m(b\,x^2+ax)^p\,bmx + 2x^m(b\,x^2+ax)^p\,bpx - \Big(\int \frac{x^m\,(b\,x^2}{b\,m^2x^2+4bmp\,x^2+4b\,p^2x^2+a\,m^2x+4am}}{}$$

```
int((c*x)^m*(b*x^2+a*x)^p,x)
```

```
(c**m*(x**m*(a*x + b*x**2)**p*a*p + x**m*(a*x + b*x**2)**p*b*m*x + 2*x**m*
(a*x + b*x**2)**p*b*p*x - int((x**m*(a*x + b*x**2)**p)/(a*m**2*x + 4*a*m*p
*x + a*m*x + 4*a*p**2*x + 2*a*p*x + b*m**2*x**2 + 4*b*m*p*x**2 + b*m*x**2
+ 4*b*p**2*x**2 + 2*b*p*x**2),x)*a**2*m**3*p - 5*int((x**m*(a*x + b*x**2)*
*p)/(a*m**2*x + 4*a*m*p*x + a*m*x + 4*a*p**2*x + 2*a*p*x + b*m**2*x**2 + 4
*b*m*p*x**2 + b*m*x**2 + 4*b*p**2*x**2 + 2*b*p*x**2),x)*a**2*m**2*p**2 - i
nt((x**m*(a*x + b*x**2)**p)/(a*m**2*x + 4*a*m*p*x + a*m*x + 4*a*p**2*x + 2
*a*p*x + b*m**2*x**2 + 4*b*m*p*x**2 + b*m*x**2 + 4*b*p**2*x**2 + 2*b*p*x**
2),x)*a**2*m**2*p - 8*int((x**m*(a*x + b*x**2)**p)/(a*m**2*x + 4*a*m*p*x +
 a*m*x + 4*a*p**2*x + 2*a*p*x + b*m**2*x**2 + 4*b*m*p*x**2 + b*m*x**2 + 4*
b*p**2*x**2 + 2*b*p*x**2),x)*a**2*m*p**3 - 3*int((x**m*(a*x + b*x**2)**p)/
(a*m**2*x + 4*a*m*p*x + a*m*x + 4*a*p**2*x + 2*a*p*x + b*m**2*x**2 + 4*b*m
*p*x**2 + b*m*x**2 + 4*b*p**2*x**2 + 2*b*p*x**2),x)*a**2*m*p**2 - 4*int((x
**m*(a*x + b*x**2)**p)/(a*m**2*x + 4*a*m*p*x + a*m*x + 4*a*p**2*x + 2*a*p*
x + b*m**2*x**2 + 4*b*m*p*x**2 + b*m*x**2 + 4*b*p**2*x**2 + 2*b*p*x**2),x)
*a**2*p**4 - 2*int((x**m*(a*x + b*x**2)**p)/(a*m**2*x + 4*a*m*p*x + a*m*x
+ 4*a*p**2*x + 2*a*p*x + b*m**2*x**2 + 4*b*m*p*x**2 + b*m*x**2 + 4*b*p**2*
x**2 + 2*b*p*x**2),x)*a**2*p**3))/(b*(m**2 + 4*m*p + m + 4*p**2 + 2*p))
```

3.256 $\int (cx)^{-5-2p} (ax+bx^2)^p \, dx$

Optimal result

Integrand size = 21, antiderivative size = 175

$$
\begin{aligned}
\int (cx)^{-5-2p} (ax+bx^2)^p \, dx = & -\frac{(cx)^{-5-2p} (ax+bx^2)^{1+p}}{a(4+p)} - \frac{6b^2 (cx)^{-3-2p} (ax+bx^2)^{1+p}}{a^3c^2(2+p)(3+p)(4+p)} \\
& + \frac{6b^3 (cx)^{-2(1+p)} (ax+bx^2)^{1+p}}{a^4c^3(1+p)(2+p)(3+p)(4+p)} \\
& + \frac{3b (cx)^{-2(2+p)} (ax+bx^2)^{1+p}}{a^2c(3+p)(4+p)}
\end{aligned}
$$

```
-(c*x)^(-5-2*p)*(b*x^2+a*x)^(p+1)/a/(4+p)-6*b^2*(c*x)^(-3-2*p)*(b*x^2+a*x)
^(p+1)/a^3/c^2/(2+p)/(3+p)/(4+p)+6*b^3*(b*x^2+a*x)^(p+1)/a^4/c^3/(p+1)/(2+
p)/(3+p)/(4+p)/((c*x)^(2*p+2))+3*b*(b*x^2+a*x)^(p+1)/a^2/c/(3+p)/(4+p)/((c
*x)^(4+2*p))
```

Mathematica [A] (verified)

Time = 0.06 (sec) , antiderivative size = 102, normalized size of antiderivative = 0.58

$$\int (cx)^{-5-2p} \left(ax+bx^2\right)^p \, dx = -\frac{(cx)^{-2p}(x(a+bx))^{1+p} \left(a^3(6+11p+6p^2+p^3)-3a^2b(2+3p+p^2)\,x+6ab^2(1+p)x^2-6b^3x^3\right)}{a^4c^5(1+p)(2+p)(3+p)(4+p)x^5}$$

```
Integrate[(c*x)^(-5 - 2*p)*(a*x + b*x^2)^p,x]
```

```
-(((x*(a + b*x))^(1 + p)*(a^3*(6 + 11*p + 6*p^2 + p^3) - 3*a^2*b*(2 + 3*p
+ p^2)*x + 6*a*b^2*(1 + p)*x^2 - 6*b^3*x^3))/(a^4*c^5*(1 + p)*(2 + p)*(3 +
 p)*(4 + p)*x^5*(c*x)^(2*p)))
```

Rubi [A] (verified)

Time = 0.55 (sec) , antiderivative size = 167, normalized size of antiderivative = 0.95, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.190$, Rules used = {1129, 1129, 1129, 1123}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (cx)^{-2p-5} \left(ax+bx^2\right)^p \, dx$$

$$\Big\downarrow \ 1129$$

$$-\frac{3b\int (cx)^{-2(p+2)} \left(bx^2+ax\right)^p dx}{ac(p+4)}-\frac{(cx)^{-2p-5} \left(ax+bx^2\right)^{p+1}}{a(p+4)}$$

$$\Big\downarrow \ 1129$$

$$-\frac{3b\left(-\frac{2b\int (cx)^{-2p-3} \left(bx^2+ax\right)^p dx}{ac(p+3)}-\frac{(cx)^{-2(p+2)} \left(ax+bx^2\right)^{p+1}}{a(p+3)}\right)}{ac(p+4)}-\frac{(cx)^{-2p-5} \left(ax+bx^2\right)^{p+1}}{a(p+4)}$$

$$\Big\downarrow \ 1129$$

$$-\frac{3b\left(-\frac{2b\left(-\frac{b\int (cx)^{-2(p+1)}\left(bx^2+ax\right)^p dx}{ac(p+2)}-\frac{(cx)^{-2p-3}\left(ax+bx^2\right)^{p+1}}{a(p+2)}\right)}{ac(p+3)}-\frac{(cx)^{-2(p+2)}\left(ax+bx^2\right)^{p+1}}{a(p+3)}\right)}{ac(p+4)}-\frac{(cx)^{-2p-5}\left(ax+bx^2\right)^{p+1}}{a(p+4)}$$

$$\Big\downarrow \text{ 1123}$$

$$-\frac{3b\left(-\frac{2b\left(\frac{b(cx)^{-2(p+1)}\left(ax+bx^2\right)^{p+1}}{a^2c(p+1)(p+2)}-\frac{(cx)^{-2p-3}\left(ax+bx^2\right)^{p+1}}{a(p+2)}\right)}{ac(p+3)}-\frac{(cx)^{-2(p+2)}\left(ax+bx^2\right)^{p+1}}{a(p+3)}\right)}{ac(p+4)}-\frac{(cx)^{-2p-5}\left(ax+bx^2\right)^{p+1}}{a(p+4)}$$

```
Int[(c*x)^(-5 - 2*p)*(a*x + b*x^2)^p,x]
```

```
-(((c*x)^(-5 - 2*p)*(a*x + b*x^2)^(1 + p))/(a*(4 + p))) - (3*b*(-((a*x + b
*x^2)^(1 + p)/(a*(3 + p)*(c*x)^(2*(2 + p)))) - (2*b*(-(((c*x)^(-3 - 2*p)*(
a*x + b*x^2)^(1 + p))/(a*(2 + p))) + (b*(a*x + b*x^2)^(1 + p))/(a^2*c*(1 +
 p)*(2 + p)*(c*x)^(2*(1 + p)))))/(a*c*(3 + p))))/(a*c*(4 + p))
```

Defintions of rubi rules used

```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] :> Simp[e*(d + e*x)^m*((a + b*x + c*x^2)^(p + 1)/((p + 1)*(2*c*d - b
*e))), x] /; FreeQ[{a, b, c, d, e, m, p}, x] && EqQ[c*d^2 - b*d*e + a*e^2,
0] && EqQ[m + 2*p + 2, 0]
```

```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] :> Simp[(-e)*(d + e*x)^m*((a + b*x + c*x^2)^(p + 1)/((m + p + 1)*(2*
c*d - b*e))), x] + Simp[c*(Simplify[m + 2*p + 2]/((m + p + 1)*(2*c*d - b*e)
))   Int[(d + e*x)^(m + 1)*(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d
, e, m, p}, x] && EqQ[c*d^2 - b*d*e + a*e^2, 0] && ILtQ[Simplify[m + 2*p +
2], 0]
```

Maple [A] (verified)

Time = 0.48 (sec) , antiderivative size = 131, normalized size of antiderivative = 0.75

method	result	size
gosper	$-\frac{\left(b\,x^{2}+a x \right)^{p} \left(c x \right)^{-5-2 p} \left(a^{3} p^{3}-3 a^{2} b\,p^{2} x +6 a\,b^{2} p\,x^{2}-6 b^{3} x^{3}+6 a^{3} p^{2}-9 a^{2} b p x +6 a\,b^{2} x^{2}+11 a^{3} p -6 a^{2} b x +6 a^{3}\right) x \left(b x +a \right)}{\left(4+p \right) \left(3+p \right) \left(2+p \right) \left(p +1\right) a^{4}}$	131
orering	$-\frac{\left(b\,x^{2}+a x \right)^{p} \left(c x \right)^{-5-2 p} \left(a^{3} p^{3}-3 a^{2} b\,p^{2} x +6 a\,b^{2} p\,x^{2}-6 b^{3} x^{3}+6 a^{3} p^{2}-9 a^{2} b p x +6 a\,b^{2} x^{2}+11 a^{3} p -6 a^{2} b x +6 a^{3}\right) x \left(b x +a \right)}{\left(4+p \right) \left(3+p \right) \left(2+p \right) \left(p +1\right) a^{4}}$	131

```
int((c*x)^(-5-2*p)*(b*x^2+a*x)^p,x,method=_RETURNVERBOSE)
```

```
-(b*x^2+a*x)^p*(c*x)^(-5-2*p)*(a^3*p^3-3*a^2*b*p^2*x+6*a*b^2*p*x^2-6*b^3*x
^3+6*a^3*p^2-9*a^2*b*p*x+6*a*b^2*x^2+11*a^3*p-6*a^2*b*x+6*a^3)*x*(b*x+a)/(
4+p)/(3+p)/(2+p)/(p+1)/a^4
```

Fricas [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 160, normalized size of antiderivative = 0.91

$$\int (cx)^{-5-2p} \left(ax+bx^2\right)^p \, dx = -\frac{\left(6\,ab^{3}px^{4}-6\,b^{4}x^{5}-3\,(a^{2}b^{2}p^{2}+a^{2}b^{2}p)x^{3}+(a^{3}bp^{3}+3\,a^{3}bp^{2}+2\,a^{3}bp)x^{2}+(a^{4}p^{3}+6\,a^{4}p^{2}+11\,a^{4}p+\right.}{a^{4}p^{4}+10\,a^{4}p^{3}+35\,a^{4}p^{2}+50\,a^{4}p+24\,a^{4}}$$

```
integrate((c*x)^(-5-2*p)*(b*x^2+a*x)^p,x, algorithm="fricas")
```

```
-(6*a*b^3*p*x^4 - 6*b^4*x^5 - 3*(a^2*b^2*p^2 + a^2*b^2*p)*x^3 + (a^3*b*p^3
 + 3*a^3*b*p^2 + 2*a^3*b*p)*x^2 + (a^4*p^3 + 6*a^4*p^2 + 11*a^4*p + 6*a^4)
*x)*(b*x^2 + a*x)^p*(c*x)^(-2*p - 5)/(a^4*p^4 + 10*a^4*p^3 + 35*a^4*p^2 +
50*a^4*p + 24*a^4)
```

Sympy [F]

$$\int (cx)^{-5-2p} \left(ax+bx^2\right)^p \, dx = \int (cx)^{-2p-5} \left(x(a+bx)\right)^p \, dx$$

```
integrate((c*x)**(-5-2*p)*(b*x**2+a*x)**p,x)
```

```
Integral((c*x)**(-2*p - 5)*(x*(a + b*x))**p, x)
```

Maxima [F]

$$\int (cx)^{-5-2p} \left(ax+bx^2\right)^p \, dx = \int \left(bx^2+ax\right)^p (cx)^{-2\,p-5} \, dx$$

```
integrate((c*x)^(-5-2*p)*(b*x^2+a*x)^p,x, algorithm="maxima")
```

```
integrate((b*x^2 + a*x)^p*(c*x)^(-2*p - 5), x)
```

Giac [F]

$$\int (cx)^{-5-2p} \left(ax+bx^2\right)^p \, dx = \int \left(bx^2+ax\right)^p (cx)^{-2\,p-5} \, dx$$

```
integrate((c*x)^(-5-2*p)*(b*x^2+a*x)^p,x, algorithm="giac")
```

```
integrate((b*x^2 + a*x)^p*(c*x)^(-2*p - 5), x)
```

Mupad [B] (verification not implemented)

Time = 9.01 (sec) , antiderivative size = 239, normalized size of antiderivative = 1.37

$$\int (cx)^{-5-2p}\left(ax+bx^2\right)^p\,dx=-\left(b\,x^2+a\,x\right)^p\left(\frac{x\,\left(p^3+6\,p^2+11\,p+6\right)}{{\left(c\,x\right)}^{2\,p+5}\,\left(p^4+10\,p^3+35\,p^2+50\,p+24\right)}-\frac{6\,b^4\,x^5}{a^4\,{\left(c\,x\right)}^{2\,p+5}\,\left(p^4+10\,p^3+35\,p^2+50\,p+24\right)}+\frac{6\,b^3\,p\,x^4}{a^3\,{\left(c\,x\right)}^{2\,p+5}\,\left(p^4+10\,p^3+35\,p^2+50\,p+24\right)}+\frac{b\,p\,x^2\,\left(p^2+3\,p+2\right)}{a\,{\left(c\,x\right)}^{2\,p+5}\,\left(p^4+10\,p^3+35\,p^2+50\,p+24\right)}-\frac{3\,b^2\,p\,x^3\,\left(p+1\right)}{a^2\,{\left(c\,x\right)}^{2\,p+5}\,\left(p^4+10\,p^3+35\,p^2+50\,p+24\right)}\right)$$

```
int((a*x + b*x^2)^p/(c*x)^(2*p + 5),x)
```

```
-(a*x + b*x^2)^p*((x*(11*p + 6*p^2 + p^3 + 6))/((c*x)^(2*p + 5)*(50*p + 35
*p^2 + 10*p^3 + p^4 + 24)) - (6*b^4*x^5)/(a^4*(c*x)^(2*p + 5)*(50*p + 35*p
^2 + 10*p^3 + p^4 + 24)) + (6*b^3*p*x^4)/(a^3*(c*x)^(2*p + 5)*(50*p + 35*p
^2 + 10*p^3 + p^4 + 24)) + (b*p*x^2*(3*p + p^2 + 2))/(a*(c*x)^(2*p + 5)*(5
0*p + 35*p^2 + 10*p^3 + p^4 + 24)) - (3*b^2*p*x^3*(p + 1))/(a^2*(c*x)^(2*p
 + 5)*(50*p + 35*p^2 + 10*p^3 + p^4 + 24)))
```

Reduce [B] (verification not implemented)

Time = 0.55 (sec) , antiderivative size = 155, normalized size of antiderivative = 0.89

$$\int (cx)^{-5-2p}\left(ax+bx^2\right)^p\,dx$$
$$=\frac{\left(b\,x^2+ax\right)^p\left(-a^3b\,p^3x+3a^2b^2p^2x^2-6a\,b^3p\,x^3+6b^4x^4-a^4p^3-3a^3b\,p^2x+3a^2b^2p\,x^2-6a^4p^2-2a^3bpx\right.}{x^{2p}c^{2p}a^4c^5x^4\left(p^4+10p^3+35p^2+50p+24\right)}$$

```
int((c*x)^(-5-2*p)*(b*x^2+a*x)^p,x)
```

```
((a*x + b*x**2)**p*( - a**4*p**3 - 6*a**4*p**2 - 11*a**4*p - 6*a**4 - a**3
*b*p**3*x - 3*a**3*b*p**2*x - 2*a**3*b*p*x + 3*a**2*b**2*p**2*x**2 + 3*a**
2*b**2*p*x**2 - 6*a*b**3*p*x**3 + 6*b**4*x**4))/(x**(2*p)*c**(2*p)*a**4*c*
*5*x**4*(p**4 + 10*p**3 + 35*p**2 + 50*p + 24))
```

3.257 $\int (cx)^{-4-2p} (ax+bx^2)^p \, dx$

Optimal result

Integrand size = 21, antiderivative size = 122

$$\int (cx)^{-4-2p} (ax+bx^2)^p \, dx = \frac{2b(cx)^{-3-2p} (ax+bx^2)^{1+p}}{a^2c(2+p)(3+p)} - \frac{2b^2(cx)^{-2(1+p)} (ax+bx^2)^{1+p}}{a^3c^2(1+p)(2+p)(3+p)} - \frac{(cx)^{-2(2+p)} (ax+bx^2)^{1+p}}{a(3+p)}$$

```
2*b*(c*x)^(-3-2*p)*(b*x^2+a*x)^(p+1)/a^2/c/(2+p)/(3+p)-2*b^2*(b*x^2+a*x)^(
p+1)/a^3/c^2/(p+1)/(2+p)/(3+p)/((c*x)^(2*p+2))-(b*x^2+a*x)^(p+1)/a/(3+p)/(
(c*x)^(4+2*p))
```

Mathematica [A] (verified)

Time = 0.05 (sec) , antiderivative size = 73, normalized size of antiderivative = 0.60

$$\int (cx)^{-4-2p} (ax+bx^2)^p \, dx = -\frac{(cx)^{-2p}(x(a+bx))^{1+p} (a^2(2+3p+p^2) - 2ab(1+p)x + 2b^2x^2)}{a^3c^4(1+p)(2+p)(3+p)x^4}$$

```
Integrate[(c*x)^(-4 - 2*p)*(a*x + b*x^2)^p,x]
```

```
-(((x*(a + b*x))^(1 + p)*(a^2*(2 + 3*p + p^2) - 2*a*b*(1 + p)*x + 2*b^2*x^
2))/(a^3*c^4*(1 + p)*(2 + p)*(3 + p)*x^4*(c*x)^(2*p)))
```

Rubi [A] (verified)

Time = 0.43 (sec) , antiderivative size = 120, normalized size of antiderivative = 0.98, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.143$, Rules used = {1129, 1129, 1123}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (cx)^{-2p-4} \left(ax+bx^2\right)^p \, dx$$

$$\Big\downarrow \text{ 1129}$$

$$-\frac{2b\int (cx)^{-2p-3} \left(bx^2+ax\right)^p dx}{ac(p+3)}-\frac{(cx)^{-2(p+2)} \left(ax+bx^2\right)^{p+1}}{a(p+3)}$$

$$\Big\downarrow \text{ 1129}$$

$$-\frac{2b\left(-\frac{b\int (cx)^{-2(p+1)}\left(bx^2+ax\right)^p dx}{ac(p+2)}-\frac{(cx)^{-2p-3}\left(ax+bx^2\right)^{p+1}}{a(p+2)}\right)}{ac(p+3)}-\frac{(cx)^{-2(p+2)} \left(ax+bx^2\right)^{p+1}}{a(p+3)}$$

$$\Big\downarrow \text{ 1123}$$

$$-\frac{2b\left(\frac{b(cx)^{-2(p+1)}\left(ax+bx^2\right)^{p+1}}{a^2c(p+1)(p+2)}-\frac{(cx)^{-2p-3}\left(ax+bx^2\right)^{p+1}}{a(p+2)}\right)}{ac(p+3)}-\frac{(cx)^{-2(p+2)} \left(ax+bx^2\right)^{p+1}}{a(p+3)}$$

```
Int[(c*x)^(-4 - 2*p)*(a*x + b*x^2)^p,x]
```

```
-((a*x + b*x^2)^(1 + p)/(a*(3 + p)*(c*x)^(2*(2 + p)))) - (2*b*(-(((c*x)^(-
3 - 2*p)*(a*x + b*x^2)^(1 + p))/(a*(2 + p))) + (b*(a*x + b*x^2)^(1 + p))/(
a^2*c*(1 + p)*(2 + p)*(c*x)^(2*(1 + p)))))/(a*c*(3 + p))
```

Defintions of rubi rules used

```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] :> Simp[e*(d + e*x)^m*((a + b*x + c*x^2)^(p + 1)/((p + 1)*(2*c*d - b
*e))), x] /; FreeQ[{a, b, c, d, e, m, p}, x] && EqQ[c*d^2 - b*d*e + a*e^2,
0] && EqQ[m + 2*p + 2, 0]
```

```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] :> Simp[(-e)*(d + e*x)^m*((a + b*x + c*x^2)^(p + 1)/((m + p + 1)*(2*
c*d - b*e))), x] + Simp[c*(Simplify[m + 2*p + 2]/((m + p + 1)*(2*c*d - b*e)
))   Int[(d + e*x)^(m + 1)*(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d
, e, m, p}, x] && EqQ[c*d^2 - b*d*e + a*e^2, 0] && ILtQ[Simplify[m + 2*p +
2], 0]
```

Maple [A] (verified)

Time = 0.47 (sec) , antiderivative size = 85, normalized size of antiderivative = 0.70

method	result	size
gosper	$-\frac{(bx+a)x\left(a^{2}p^{2}-2abpx+2b^{2}x^{2}+3a^{2}p-2abx+2a^{2}\right)(cx)^{-4-2p}\left(b\,x^{2}+ax\right)^{p}}{(3+p)(2+p)(p+1)a^{3}}$	85
orering	$-\frac{(bx+a)x\left(a^{2}p^{2}-2abpx+2b^{2}x^{2}+3a^{2}p-2abx+2a^{2}\right)(cx)^{-4-2p}\left(b\,x^{2}+ax\right)^{p}}{(3+p)(2+p)(p+1)a^{3}}$	85

```
int((c*x)^(-4-2*p)*(b*x^2+a*x)^p,x,method=_RETURNVERBOSE)
```

```
-(b*x+a)*x*(a^2*p^2-2*a*b*p*x+2*b^2*x^2+3*a^2*p-2*a*b*x+2*a^2)*(c*x)^(-4-2
*p)*(b*x^2+a*x)^p/(3+p)/(2+p)/(p+1)/a^3
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 111, normalized size of antiderivative = 0.91

$$\int (cx)^{-4-2p} \left(ax+bx^2\right)^p \, dx = \frac{\left(2\, ab^2px^3 - 2\, b^3x^4 - \left(a^2bp^2 + a^2bp\right)x^2 - \left(a^3p^2 + 3\, a^3p + 2\, a^3\right)x\right)\left(bx^2+ax\right)^p (cx)^{-2\,p-4}}{a^3p^3 + 6\, a^3p^2 + 11\, a^3p + 6\, a^3}$$

```
integrate((c*x)^(-4-2*p)*(b*x^2+a*x)^p,x, algorithm="fricas")
```

```
(2*a*b^2*p*x^3 - 2*b^3*x^4 - (a^2*b*p^2 + a^2*b*p)*x^2 - (a^3*p^2 + 3*a^3*
p + 2*a^3)*x)*(b*x^2 + a*x)^p*(c*x)^(-2*p - 4)/(a^3*p^3 + 6*a^3*p^2 + 11*a
^3*p + 6*a^3)
```

Sympy [F]

$$\int (cx)^{-4-2p} \left(ax+bx^2\right)^p \, dx = \int (cx)^{-2p-4} \left(x(a+bx)\right)^p \, dx$$

```
integrate((c*x)**(-4-2*p)*(b*x**2+a*x)**p,x)
```

```
Integral((c*x)**(-2*p - 4)*(x*(a + b*x))**p, x)
```

Maxima [F]

$$\int (cx)^{-4-2p} \left(ax+bx^2\right)^p \, dx = \int \left(bx^2+ax\right)^p (cx)^{-2\,p-4} \, dx$$

```
integrate((c*x)^(-4-2*p)*(b*x^2+a*x)^p,x, algorithm="maxima")
```

```
integrate((b*x^2 + a*x)^p*(c*x)^(-2*p - 4), x)
```

Giac [F]

$$\int (cx)^{-4-2p} \left(ax+bx^2\right)^p \, dx = \int \left(bx^2+ax\right)^p (cx)^{-2\,p-4} \, dx$$

```
integrate((c*x)^(-4-2*p)*(b*x^2+a*x)^p,x, algorithm="giac")
```

```
integrate((b*x^2 + a*x)^p*(c*x)^(-2*p - 4), x)
```

Mupad [B] (verification not implemented)

Time = 8.84 (sec) , antiderivative size = 163, normalized size of antiderivative = 1.34

$$\int (cx)^{-4-2p} \left(ax+bx^2\right)^p \, dx = -\left(b\,x^2+a\,x\right)^p \left(\frac{x\,\left(p^2+3\,p+2\right)}{{\left(c\,x\right)}^{2\,p+4}\,\left(p^3+6\,p^2+11\,p+6\right)} + \frac{2\,b^3\,x^4}{a^3\,{\left(c\,x\right)}^{2\,p+4}\,\left(p^3+6\,p^2+11\,p+6\right)} - \frac{2\,b^2\,p\,x^3}{a^2\,{\left(c\,x\right)}^{2\,p+4}\,\left(p^3+6\,p^2+11\,p+6\right)} + \frac{b\,p\,x^2\,\left(p+1\right)}{a\,{\left(c\,x\right)}^{2\,p+4}\,\left(p^3+6\,p^2+11\,p+6\right)}\right)$$

```
int((a*x + b*x^2)^p/(c*x)^(2*p + 4),x)
```

```
-(a*x + b*x^2)^p*((x*(3*p + p^2 + 2))/((c*x)^(2*p + 4)*(11*p + 6*p^2 + p^3
 + 6)) + (2*b^3*x^4)/(a^3*(c*x)^(2*p + 4)*(11*p + 6*p^2 + p^3 + 6)) - (2*b
^2*p*x^3)/(a^2*(c*x)^(2*p + 4)*(11*p + 6*p^2 + p^3 + 6)) + (b*p*x^2*(p + 1
))/(a*(c*x)^(2*p + 4)*(11*p + 6*p^2 + p^3 + 6)))
```

Reduce [B] (verification not implemented)

Time = 0.57 (sec) , antiderivative size = 106, normalized size of antiderivative = 0.87

$$\int (cx)^{-4-2p} \left(ax+bx^2\right)^p \, dx = \frac{(b\,x^2+ax)^p\,(-a^2b\,p^2x+2a\,b^2p\,x^2-2b^3x^3-a^3p^2-a^2bpx-3a^3p-2a^3)}{x^{2p}c^{2p}a^3c^4x^3\,(p^3+6p^2+11p+6)}$$

```
int((c*x)^(-4-2*p)*(b*x^2+a*x)^p,x)
```

```
((a*x + b*x**2)**p*( - a**3*p**2 - 3*a**3*p - 2*a**3 - a**2*b*p**2*x - a**
2*b*p*x + 2*a*b**2*p*x**2 - 2*b**3*x**3))/(x**(2*p)*c**(2*p)*a**3*c**4*x**
3*(p**3 + 6*p**2 + 11*p + 6))
```

3.258 $\int (cx)^{-3-2p} (ax+bx^2)^p \, dx$

Optimal result

Integrand size = 21, antiderivative size = 73

$$\int (cx)^{-3-2p} (ax+bx^2)^p \, dx = -\frac{(cx)^{-3-2p} (ax+bx^2)^{1+p}}{a(2+p)} + \frac{b(cx)^{-2(1+p)} (ax+bx^2)^{1+p}}{a^2 c(1+p)(2+p)}$$

```
-(c*x)^(-3-2*p)*(b*x^2+a*x)^(p+1)/a/(2+p)+b*(b*x^2+a*x)^(p+1)/a^2/c/(p+1)/
(2+p)/((c*x)^(2*p+2))
```

Mathematica [A] (verified)

Time = 0.04 (sec) , antiderivative size = 48, normalized size of antiderivative = 0.66

$$\int (cx)^{-3-2p} (ax+bx^2)^p \, dx = -\frac{(cx)^{-2p}(a+ap-bx)(x(a+bx))^{1+p}}{a^2c^3(1+p)(2+p)x^3}$$

```
Integrate[(c*x)^(-3 - 2*p)*(a*x + b*x^2)^p,x]
```

```
-(((a + a*p - b*x)*(x*(a + b*x))^(1 + p))/(a^2*c^3*(1 + p)*(2 + p)*x^3*(c*
x)^(2*p)))
```

Rubi [A] (verified)

Time = 0.32 (sec) , antiderivative size = 73, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.095$, Rules used = {1129, 1123}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (cx)^{-2p-3} \left(ax+bx^2\right)^p \, dx$$

$$\Big\downarrow \ \color{blue}{1129}$$

$$-\frac{b \int (cx)^{-2(p+1)} \left(bx^2+ax\right)^p dx}{ac(p+2)} - \frac{(cx)^{-2p-3} \left(ax+bx^2\right)^{p+1}}{a(p+2)}$$

$$\Big\downarrow \ \color{blue}{1123}$$

$$\frac{b(cx)^{-2(p+1)} \left(ax+bx^2\right)^{p+1}}{a^2c(p+1)(p+2)} - \frac{(cx)^{-2p-3} \left(ax+bx^2\right)^{p+1}}{a(p+2)}$$

```
Int[(c*x)^(-3 - 2*p)*(a*x + b*x^2)^p,x]
```

```
-(((c*x)^(-3 - 2*p)*(a*x + b*x^2)^(1 + p))/(a*(2 + p))) + (b*(a*x + b*x^2)
^(1 + p))/(a^2*c*(1 + p)*(2 + p)*(c*x)^(2*(1 + p)))
```

Defintions of rubi rules used

```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] :> Simp[e*(d + e*x)^m*((a + b*x + c*x^2)^(p + 1)/((p + 1)*(2*c*d - b
*e))), x] /; FreeQ[{a, b, c, d, e, m, p}, x] && EqQ[c*d^2 - b*d*e + a*e^2,
0] && EqQ[m + 2*p + 2, 0]
```

```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] :> Simp[(-e)*(d + e*x)^m*((a + b*x + c*x^2)^(p + 1)/((m + p + 1)*(2*
c*d - b*e))), x] + Simp[c*(Simplify[m + 2*p + 2]/((m + p + 1)*(2*c*d - b*e)
))   Int[(d + e*x)^(m + 1)*(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d
, e, m, p}, x] && EqQ[c*d^2 - b*d*e + a*e^2, 0] && ILtQ[Simplify[m + 2*p +
2], 0]
```

Maple [A] (verified)

Time = 0.47 (sec) , antiderivative size = 51, normalized size of antiderivative = 0.70

method	result	size
gosper	$-\frac{(b\,x^2+ax)^p (cx)^{-3-2p} (ap-bx+a) x (bx+a)}{(2+p)(p+1)a^2}$	51
orering	$-\frac{(b\,x^2+ax)^p (cx)^{-3-2p} (ap-bx+a) x (bx+a)}{(2+p)(p+1)a^2}$	51

```
int((c*x)^(-3-2*p)*(b*x^2+a*x)^p,x,method=_RETURNVERBOSE)
```

```
-(b*x^2+a*x)^p*(c*x)^(-3-2*p)*(a*p-b*x+a)*x*(b*x+a)/(2+p)/(p+1)/a^2
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 70, normalized size of antiderivative = 0.96

$$\int (cx)^{-3-2p} \left(ax+bx^2\right)^p \, dx = -\frac{(abpx^2 - b^2x^3 + (a^2p + a^2)x)(bx^2 + ax)^p (cx)^{-2\,p-3}}{a^2p^2 + 3\,a^2p + 2\,a^2}$$

```
integrate((c*x)^(-3-2*p)*(b*x^2+a*x)^p,x, algorithm="fricas")
```

```
-(a*b*p*x^2 - b^2*x^3 + (a^2*p + a^2)*x)*(b*x^2 + a*x)^p*(c*x)^(-2*p - 3)/
(a^2*p^2 + 3*a^2*p + 2*a^2)
```

Sympy [F]

$$\int (cx)^{-3-2p}\left(ax+bx^2\right)^p\,dx=\int (cx)^{-2p-3}\left(x(a+bx)\right)^p\,dx$$

```
integrate((c*x)**(-3-2*p)*(b*x**2+a*x)**p,x)
```

```
Integral((c*x)**(-2*p - 3)*(x*(a + b*x))**p, x)
```

Maxima [F]

$$\int (cx)^{-3-2p}\left(ax+bx^2\right)^p\,dx=\int \left(bx^2+ax\right)^p(cx)^{-2\,p-3}\,dx$$

```
integrate((c*x)^(-3-2*p)*(b*x^2+a*x)^p,x, algorithm="maxima")
```

```
integrate((b*x^2 + a*x)^p*(c*x)^(-2*p - 3), x)
```

Giac [F]

$$\int (cx)^{-3-2p}\left(ax+bx^2\right)^p\,dx=\int \left(bx^2+ax\right)^p(cx)^{-2\,p-3}\,dx$$

```
integrate((c*x)^(-3-2*p)*(b*x^2+a*x)^p,x, algorithm="giac")
```

```
integrate((b*x^2 + a*x)^p*(c*x)^(-2*p - 3), x)
```

Mupad [B] (verification not implemented)

Time = 8.93 (sec) , antiderivative size = 102, normalized size of antiderivative = 1.40

$$\int (cx)^{-3-2p}\left(ax+bx^2\right)^p\,dx = -\left(b\,x^2+a\,x\right)^p\left(\frac{x\,(p+1)}{(c\,x)^{2\,p+3}\,(p^2+3\,p+2)} - \frac{b^2\,x^3}{a^2\,(c\,x)^{2\,p+3}\,(p^2+3\,p+2)} + \frac{b\,p\,x^2}{a\,(c\,x)^{2\,p+3}\,(p^2+3\,p+2)}\right)$$

```
int((a*x + b*x^2)^p/(c*x)^(2*p + 3),x)
```

```
-(a*x + b*x^2)^p*((x*(p + 1))/((c*x)^(2*p + 3)*(3*p + p^2 + 2)) - (b^2*x^3
)/(a^2*(c*x)^(2*p + 3)*(3*p + p^2 + 2)) + (b*p*x^2)/(a*(c*x)^(2*p + 3)*(3*
p + p^2 + 2)))
```

Reduce [B] (verification not implemented)

Time = 0.24 (sec) , antiderivative size = 70, normalized size of antiderivative = 0.96

$$\int (cx)^{-3-2p}\left(ax+bx^2\right)^p\,dx = \frac{(b\,x^2+ax)^p\,(-abpx+b^2x^2-a^2p-a^2)}{x^{2p}c^{2p}a^2c^3x^2\,(p^2+3p+2)}$$

```
int((c*x)^(-3-2*p)*(b*x^2+a*x)^p,x)
```

```
((a*x + b*x**2)**p*( - a**2*p - a**2 - a*b*p*x + b**2*x**2))/(x**(2*p)*c**
(2*p)*a**2*c**3*x**2*(p**2 + 3*p + 2))
```

3.259 $\int (cx)^{-2-2p} \left(ax+bx^2\right)^p \, dx$

Optimal result

Integrand size = 21, antiderivative size = 32

$$\int (cx)^{-2-2p} \left(ax+bx^2\right)^p \, dx = -\frac{(cx)^{-2(1+p)} \left(ax+bx^2\right)^{1+p}}{a(1+p)}$$

```
-(b*x^2+a*x)^(p+1)/a/(p+1)/((c*x)^(2*p+2))
```

Mathematica [A] (verified)

Time = 0.04 (sec) , antiderivative size = 30, normalized size of antiderivative = 0.94

$$\int (cx)^{-2-2p} \left(ax+bx^2\right)^p \, dx = -\frac{(cx)^{-2(1+p)}(x(a+bx))^{1+p}}{a(1+p)}$$

```
Integrate[(c*x)^(-2 - 2*p)*(a*x + b*x^2)^p,x]
```

```
-((x*(a + b*x))^(1 + p)/(a*(1 + p)*(c*x)^(2*(1 + p))))
```

Rubi [A] (verified)

Time = 0.25 (sec) , antiderivative size = 32, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}} = 0.048$, Rules used = $\{1123\}$

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (cx)^{-2p-2}\left(ax+bx^2\right)^p\,dx$$

$$\Big\downarrow \text{1123}$$

$$-\frac{(cx)^{-2(p+1)}\left(ax+bx^2\right)^{p+1}}{a(p+1)}$$

```
Int[(c*x)^(-2 - 2*p)*(a*x + b*x^2)^p,x]
```

```
-((a*x + b*x^2)^(1 + p)/(a*(1 + p)*(c*x)^(2*(1 + p))))
```

Defintions of rubi rules used

```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] :> Simp[e*(d + e*x)^m*((a + b*x + c*x^2)^(p + 1)/((p + 1)*(2*c*d - b
*e))), x] /; FreeQ[{a, b, c, d, e, m, p}, x] && EqQ[c*d^2 - b*d*e + a*e^2,
0] && EqQ[m + 2*p + 2, 0]
```

Maple [A] (verified)

Time = 0.43 (sec) , antiderivative size = 37, normalized size of antiderivative = 1.16

method	result	size
gosper	$-\frac{x(bx+a)(cx)^{-2p-2}(b\,x^{2}+ax)^{p}}{a(p+1)}$	37
orering	$-\frac{x(bx+a)(cx)^{-2p-2}(b\,x^{2}+ax)^{p}}{a(p+1)}$	37
parallelrisch	$-\frac{x^{2}(cx)^{-2p-2}(x(bx+a))^{p}b+x(cx)^{-2p-2}(x(bx+a))^{p}a}{a(p+1)}$	56

```
int((c*x)^(-2*p-2)*(b*x^2+a*x)^p,x,method=_RETURNVERBOSE)
```

```
-x*(b*x+a)/a/(p+1)*(c*x)^(-2*p-2)*(b*x^2+a*x)^p
```

Fricas [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 38, normalized size of antiderivative = 1.19

$$\int (cx)^{-2-2p}\left(ax+bx^2\right)^p\,dx=-\frac{(bx^2+ax)(bx^2+ax)^p(cx)^{-2\,p-2}}{ap+a}$$

```
integrate((c*x)^(-2-2*p)*(b*x^2+a*x)^p,x, algorithm="fricas")
```

```
-(b*x^2 + a*x)*(b*x^2 + a*x)^p*(c*x)^(-2*p - 2)/(a*p + a)
```

Sympy [F]

$$\int (cx)^{-2-2p}\left(ax+bx^2\right)^p\,dx=\int (cx)^{-2p-2}\left(x(a+bx)\right)^p\,dx$$

```
integrate((c*x)**(-2-2*p)*(b*x**2+a*x)**p,x)
```

```
Integral((c*x)**(-2*p - 2)*(x*(a + b*x))**p, x)
```

Maxima [F]

$$\int (cx)^{-2-2p} \left(ax + bx^2\right)^p \, dx = \int (bx^2 + ax)^p (cx)^{-2\,p-2} \, dx$$

```
integrate((c*x)^(-2-2*p)*(b*x^2+a*x)^p,x, algorithm="maxima")
```

```
integrate((b*x^2 + a*x)^p*(c*x)^(-2*p - 2), x)
```

Giac [F]

$$\int (cx)^{-2-2p} \left(ax + bx^2\right)^p \, dx = \int (bx^2 + ax)^p (cx)^{-2\,p-2} \, dx$$

```
integrate((c*x)^(-2-2*p)*(b*x^2+a*x)^p,x, algorithm="giac")
```

```
integrate((b*x^2 + a*x)^p*(c*x)^(-2*p - 2), x)
```

Mupad [B] (verification not implemented)

Time = 9.20 (sec) , antiderivative size = 41, normalized size of antiderivative = 1.28

$$\int (cx)^{-2-2p} \left(ax + bx^2\right)^p \, dx = -\frac{(b\,x^2 + a\,x)^p\,(a + b\,x)}{a\,c^2\,x\,(c\,x)^{2\,p}\,(p + 1)}$$

```
int((a*x + b*x^2)^p/(c*x)^(2*p + 2),x)
```

```
-((a*x + b*x^2)^p*(a + b*x))/(a*c^2*x*(c*x)^(2*p)*(p + 1))
```

Reduce [B] (verification not implemented)

Time = 0.34 (sec) , antiderivative size = 46, normalized size of antiderivative = 1.44

$$\int (cx)^{-2-2p}\left(ax+bx^2\right)^p\,dx=-\frac{\left(b\,x^2+ax\right)^p(bx+a)}{x^{2p}c^{2p}a\,c^2x\,(p+1)}$$

```
int((c*x)^(-2-2*p)*(b*x^2+a*x)^p,x)
```

```
( - (a*x + b*x**2)**p*(a + b*x))/(x**(2*p)*c**(2*p)*a*c**2*x*(p + 1))
```

3.260 $\int (cx)^{-1-2p}\left(ax+bx^2\right)^p\,dx$

Optimal result

Integrand size = 21, antiderivative size = 45

$$\int (cx)^{-1-2p}\left(ax+bx^2\right)^p\,dx = -\frac{(cx)^{-1-2p}\left(ax+bx^2\right)^{1+p}\,\text{Hypergeometric2F1}\left(1,1,1-p,-\frac{bx}{a}\right)}{ap}$$

```
-(c*x)^(-1-2*p)*(b*x^2+a*x)^(p+1)*hypergeom([1, 1],[1-p],-b*x/a)/a/p
```

Mathematica [A] (verified)

Time = 0.03 (sec) , antiderivative size = 55, normalized size of antiderivative = 1.22

$$\int (cx)^{-1-2p}\left(ax+bx^2\right)^p\,dx = -\frac{x(cx)^{-1-2p}(x(a+bx))^p\left(1+\frac{bx}{a}\right)^{-p}\text{Hypergeometric2F1}\left(-p,-p,1-p,-\frac{bx}{a}\right)}{p}$$

```
Integrate[(c*x)^(-1 - 2*p)*(a*x + b*x^2)^p,x]
```

```
-((x*(c*x)^(-1 - 2*p)*(x*(a + b*x))^p*Hypergeometric2F1[-p, -p, 1 - p, -((
b*x)/a)])/(p*(1 + (b*x)/a)^p))
```

Rubi [A] (verified)

Time = 0.29 (sec) , antiderivative size = 57, normalized size of antiderivative = 1.27, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.143, Rules used = {1137, 76, 74}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (cx)^{-2p-1} \left(ax+bx^2\right)^p \, dx$$

$$\Big\downarrow \text{ 1137}$$

$$x^{p+1}(cx)^{-2p-1}(a+bx)^{-p} \left(ax+bx^2\right)^p \int x^{-p-1}(a+bx)^p dx$$

$$\Big\downarrow \text{ 76}$$

$$x^{p+1}(cx)^{-2p-1} \left(\frac{bx}{a}+1\right)^{-p} \left(ax+bx^2\right)^p \int x^{-p-1} \left(\frac{bx}{a}+1\right)^p dx$$

$$\Big\downarrow \text{ 74}$$

$$-\frac{x(cx)^{-2p-1} \left(\frac{bx}{a}+1\right)^{-p} \left(ax+bx^2\right)^p \text{Hypergeometric2F1}\left(-p,-p,1-p,-\frac{bx}{a}\right)}{p}$$

```
Int[(c*x)^(-1 - 2*p)*(a*x + b*x^2)^p,x]
```

```
-((x*(c*x)^(-1 - 2*p)*(a*x + b*x^2)^p*Hypergeometric2F1[-p, -p, 1 - p, -((
b*x)/a)])/(p*(1 + (b*x)/a)^p))
```

Defintions of rubi rules used

```
Int[((b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[c^n*((b*x
)^(m + 1)/(b*(m + 1)))*Hypergeometric2F1[-n, m + 1, m + 2, (-d)*(x/c)], x]
/; FreeQ[{b, c, d, m, n}, x] &&  !IntegerQ[m] && (IntegerQ[n] || (GtQ[c, 0]
 &&  !(EqQ[n, -2^(-1)] && EqQ[c^2 - d^2, 0] && GtQ[-d/(b*c), 0])))
```

```
Int[((b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[c^IntPart
[n]*((c + d*x)^FracPart[n]/(1 + d*(x/c))^FracPart[n])   Int[(b*x)^m*(1 + d*
(x/c))^n, x], x] /; FreeQ[{b, c, d, m, n}, x] &&  !IntegerQ[m] &&  !Integer
Q[n] &&  !GtQ[c, 0] &&  !GtQ[-d/(b*c), 0] && ((RationalQ[m] &&  !(EqQ[n, -2
^(-1)] && EqQ[c^2 - d^2, 0])) ||  !RationalQ[n])
```

```
Int[((e_.)*(x_))^(m_)*((b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(
e*x)^m*((b*x + c*x^2)^p/(x^(m + p)*(b + c*x)^p))   Int[x^(m + p)*(b + c*x)^
p, x], x] /; FreeQ[{b, c, e, m}, x]
```

Maple [F]

$$\int \left(c x \right)^{-1-2 p} \left(b \,x^{2}+a x \right)^{p}d x$$

```
int((c*x)^(-1-2*p)*(b*x^2+a*x)^p,x)
```

```
int((c*x)^(-1-2*p)*(b*x^2+a*x)^p,x)
```

Fricas [F]

$$\int (cx)^{-1-2p} \left(ax+bx^2\right)^p \, dx = \int { {\left(b x^{2} + a x\right)}^{p} \left(c x\right)^{-2 \, p - 1} \,d x }$$

```
integrate((c*x)^(-1-2*p)*(b*x^2+a*x)^p,x, algorithm="fricas")
```

```
integral((b*x^2 + a*x)^p*(c*x)^(-2*p - 1), x)
```

Sympy [F]

$$\int (cx)^{-1-2p} \left(ax+bx^2\right)^p \, dx = \int (cx)^{-2p-1} \left(x(a+bx)\right)^p \, dx$$

```
integrate((c*x)**(-1-2*p)*(b*x**2+a*x)**p,x)
```

```
Integral((c*x)**(-2*p - 1)*(x*(a + b*x))**p, x)
```

Maxima [F]

$$\int (cx)^{-1-2p} \left(ax+bx^2\right)^p \, dx = \int \left(bx^2+ax\right)^p (cx)^{-2p-1} \, dx$$

```
integrate((c*x)^(-1-2*p)*(b*x^2+a*x)^p,x, algorithm="maxima")
```

```
integrate((b*x^2 + a*x)^p*(c*x)^(-2*p - 1), x)
```

Giac [F]

$$\int (cx)^{-1-2p} \left(ax+bx^2\right)^p \, dx = \int \left(bx^2+ax\right)^p (cx)^{-2p-1} \, dx$$

```
integrate((c*x)^(-1-2*p)*(b*x^2+a*x)^p,x, algorithm="giac")
```

```
integrate((b*x^2 + a*x)^p*(c*x)^(-2*p - 1), x)
```

Mupad [F(-1)]

Timed out.

$$\int (cx)^{-1-2p} \left(ax+bx^2\right)^p \, dx = \int \frac{\left(b\,x^2+a\,x\right)^p}{\left(c\,x\right)^{2\,p+1}} \,dx$$

```
int((a*x + b*x^2)^p/(c*x)^(2*p + 1),x)
```

```
int((a*x + b*x^2)^p/(c*x)^(2*p + 1), x)
```

Reduce [F]

$$\int (cx)^{-1-2p} \left(ax+bx^2\right)^p \, dx = \frac{\int \frac{\left(b\,x^2+ax\right)^p}{x^{2p}x}dx}{c^{2p}c}$$

```
int((c*x)^(-1-2*p)*(b*x^2+a*x)^p,x)
```

```
int((a*x + b*x**2)**p/(x**(2*p)*x),x)/(c**(2*p)*c)
```

3.261 $\int (cx)^{-2p} \left(ax+bx^2\right)^p \, dx$

Optimal result

Integrand size = 19, antiderivative size = 46

$$\int (cx)^{-2p} \left(ax+bx^2\right)^p \, dx = \frac{(cx)^{-2p} \left(ax+bx^2\right)^{1+p} \text{Hypergeometric2F1}\left(1,2,2-p,-\frac{bx}{a}\right)}{a(1-p)}$$

```
(b*x^2+a*x)^(p+1)*hypergeom([1, 2],[2-p],-b*x/a)/a/(1-p)/((c*x)^(2*p))
```

Mathematica [A] (verified)

Time = 0.03 (sec) , antiderivative size = 58, normalized size of antiderivative = 1.26

$$\int (cx)^{-2p} \left(ax+bx^2\right)^p \, dx = \frac{x(cx)^{-2p}(x(a+bx))^p \left(1+\frac{bx}{a}\right)^{-p} \text{Hypergeometric2F1}\left(1-p,-p,2-p,-\frac{bx}{a}\right)}{1-p}$$

```
Integrate[(a*x + b*x^2)^p/(c*x)^(2*p),x]
```

```
(x*(x*(a + b*x))^p*Hypergeometric2F1[1 - p, -p, 2 - p, -((b*x)/a)])/((1 -
p)*(c*x)^(2*p)*(1 + (b*x)/a)^p)
```

Rubi [A] (verified)

Time = 0.30 (sec) , antiderivative size = 60, normalized size of antiderivative = 1.30, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.158$, Rules used = {1137, 76, 74}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (cx)^{-2p} \left(ax+bx^2\right)^p \, dx$$

$$\Big\downarrow \text{ 1137}$$

$$x^p (cx)^{-2p} (a+bx)^{-p} \left(ax+bx^2\right)^p \int x^{-p} (a+bx)^p dx$$

$$\Big\downarrow \text{ 76}$$

$$x^p (cx)^{-2p} \left(\frac{bx}{a}+1\right)^{-p} \left(ax+bx^2\right)^p \int x^{-p} \left(\frac{bx}{a}+1\right)^p dx$$

$$\Big\downarrow \text{ 74}$$

$$\frac{x (cx)^{-2p} \left(\frac{bx}{a}+1\right)^{-p} \left(ax+bx^2\right)^p \text{Hypergeometric2F1}\left(1-p,-p,2-p,-\frac{bx}{a}\right)}{1-p}$$

```
Int[(a*x + b*x^2)^p/(c*x)^(2*p),x]
```

```
(x*(a*x + b*x^2)^p*Hypergeometric2F1[1 - p, -p, 2 - p, -((b*x)/a)])/((1 -
p)*(c*x)^(2*p)*(1 + (b*x)/a)^p)
```

Defintions of rubi rules used

```
Int[((b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[c^n*((b*x
)^(m + 1)/(b*(m + 1)))*Hypergeometric2F1[-n, m + 1, m + 2, (-d)*(x/c)], x]
/; FreeQ[{b, c, d, m, n}, x] &&  !IntegerQ[m] && (IntegerQ[n] || (GtQ[c, 0]
 &&  !(EqQ[n, -2^(-1)] && EqQ[c^2 - d^2, 0] && GtQ[-d/(b*c), 0])))
```

```
Int[((b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[c^IntPart
[n]*((c + d*x)^FracPart[n]/(1 + d*(x/c))^FracPart[n])   Int[(b*x)^m*(1 + d*
(x/c))^n, x], x] /; FreeQ[{b, c, d, m, n}, x] &&  !IntegerQ[m] &&  !Integer
Q[n] &&  !GtQ[c, 0] &&  !GtQ[-d/(b*c), 0] && ((RationalQ[m] &&  !(EqQ[n, -2
^(-1)] && EqQ[c^2 - d^2, 0])) ||  !RationalQ[n])
```

```
Int[((e_.)*(x_))^(m_)*((b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(
e*x)^m*((b*x + c*x^2)^p/(x^(m + p)*(b + c*x)^p))   Int[x^(m + p)*(b + c*x)^
p, x], x] /; FreeQ[{b, c, e, m}, x]
```

Maple [F]

$$\int \left(b\,x^{2}+a x \right)^{p} \left(c x \right)^{-2 p}d x$$

```
int((b*x^2+a*x)^p/((c*x)^(2*p)),x)
```

```
int((b*x^2+a*x)^p/((c*x)^(2*p)),x)
```

Fricas [F]

$$\int (cx)^{-2p}\left(ax+bx^2\right)^p\,dx=\int\frac{(bx^2+ax)^p}{(cx)^{2p}}\,dx$$

```
integrate((b*x^2+a*x)^p/((c*x)^(2*p)),x, algorithm="fricas")
```

```
integral((b*x^2 + a*x)^p/(c*x)^(2*p), x)
```

Sympy [F]

$$\int (cx)^{-2p} \left(ax + bx^2\right)^p \, dx = \int (cx)^{-2p} \left(x(a + bx)\right)^p \, dx$$

```
integrate((b*x**2+a*x)**p/((c*x)**(2*p)),x)
```

```
Integral((x*(a + b*x))**p/(c*x)**(2*p), x)
```

Maxima [F]

$$\int (cx)^{-2p} \left(ax + bx^2\right)^p \, dx = \int \frac{\left(bx^2 + ax\right)^p}{(cx)^{2p}} \, dx$$

```
integrate((b*x^2+a*x)^p/((c*x)^(2*p)),x, algorithm="maxima")
```

```
integrate((b*x^2 + a*x)^p/(c*x)^(2*p), x)
```

Giac [F]

$$\int (cx)^{-2p} \left(ax + bx^2\right)^p \, dx = \int \frac{\left(bx^2 + ax\right)^p}{(cx)^{2p}} \, dx$$

```
integrate((b*x^2+a*x)^p/((c*x)^(2*p)),x, algorithm="giac")
```

```
integrate((b*x^2 + a*x)^p/(c*x)^(2*p), x)
```

Mupad [F(-1)]

Timed out.

$$\int (cx)^{-2p} \left(ax+bx^2\right)^p \, dx = \int \frac{\left(b\,x^2+a\,x\right)^p}{\left(c\,x\right)^{2\,p}} \,dx$$

```
int((a*x + b*x^2)^p/(c*x)^(2*p),x)
```

```
int((a*x + b*x^2)^p/(c*x)^(2*p), x)
```

Reduce [F]

$$\int (cx)^{-2p} \left(ax+bx^2\right)^p \, dx = \frac{\left(b\,x^2+ax\right)^p x+x^{2p}\left(\int \frac{\left(b\,x^2+ax\right)^p}{x^{2p}a+x^{2p}bx}dx\right)ap}{x^{2p}c^{2p}}$$

```
int((b*x^2+a*x)^p/((c*x)^(2*p)),x)
```

```
((a*x + b*x**2)**p*x + x**(2*p)*int((a*x + b*x**2)**p/(x**(2*p)*a + x**(2*
p)*b*x),x)*a*p)/(x**(2*p)*c**(2*p))
```

3.262 $\int (cx)^{1-2p} \left(ax+bx^2\right)^p \, dx$

Optimal result

Integrand size = 21, antiderivative size = 48

$$\int (cx)^{1-2p} \left(ax+bx^2\right)^p \, dx = \frac{(cx)^{1-2p} \left(ax+bx^2\right)^{1+p} \text{Hypergeometric2F1}\left(1,3,3-p,-\frac{bx}{a}\right)}{a(2-p)}$$

```
(c*x)^(1-2*p)*(b*x^2+a*x)^(p+1)*hypergeom([1, 3],[3-p],-b*x/a)/a/(2-p)
```

Mathematica [A] (verified)

Time = 0.03 (sec) , antiderivative size = 61, normalized size of antiderivative = 1.27

$$\int (cx)^{1-2p} \left(ax+bx^2\right)^p \, dx = \frac{cx^2(cx)^{-2p}(x(a+bx))^p \left(1+\frac{bx}{a}\right)^{-p} \text{Hypergeometric2F1}\left(2-p,-p,3-p,-\frac{bx}{a}\right)}{2-p}$$

```
Integrate[(c*x)^(1 - 2*p)*(a*x + b*x^2)^p,x]
```

```
(c*x^2*(x*(a + b*x))^p*Hypergeometric2F1[2 - p, -p, 3 - p, -((b*x)/a)])/((
2 - p)*(c*x)^(2*p)*(1 + (b*x)/a)^p)
```

Rubi [A] (verified)

Time = 0.31 (sec) , antiderivative size = 62, normalized size of antiderivative = 1.29, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.143$, Rules used = {1137, 76, 74}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (cx)^{1-2p} \left(ax+bx^2\right)^p \, dx$$

$$\Big\downarrow \ 1137$$

$$x^{p-1}(cx)^{1-2p}(a+bx)^{-p} \left(ax+bx^2\right)^p \int x^{1-p}(a+bx)^p dx$$

$$\Big\downarrow \ 76$$

$$x^{p-1}(cx)^{1-2p} \left(\frac{bx}{a}+1\right)^{-p} \left(ax+bx^2\right)^p \int x^{1-p} \left(\frac{bx}{a}+1\right)^p dx$$

$$\Big\downarrow \ 74$$

$$\frac{x(cx)^{1-2p} \left(\frac{bx}{a}+1\right)^{-p} \left(ax+bx^2\right)^p \text{Hypergeometric2F1}\left(2-p,-p,3-p,-\frac{bx}{a}\right)}{2-p}$$

```
Int[(c*x)^(1 - 2*p)*(a*x + b*x^2)^p,x]
```

```
(x*(c*x)^(1 - 2*p)*(a*x + b*x^2)^p*Hypergeometric2F1[2 - p, -p, 3 - p, -((
b*x)/a)])/((2 - p)*(1 + (b*x)/a)^p)
```

Defintions of rubi rules used

```
Int[((b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[c^n*((b*x
)^(m + 1)/(b*(m + 1)))*Hypergeometric2F1[-n, m + 1, m + 2, (-d)*(x/c)], x]
/; FreeQ[{b, c, d, m, n}, x] && !IntegerQ[m] && (IntegerQ[n] || (GtQ[c, 0]
 && !(EqQ[n, -2^(-1)] && EqQ[c^2 - d^2, 0] && GtQ[-d/(b*c), 0])))
```

```
Int[((b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[c^IntPart
[n]*((c + d*x)^FracPart[n]/(1 + d*(x/c))^FracPart[n])   Int[(b*x)^m*(1 + d*
(x/c))^n, x], x] /; FreeQ[{b, c, d, m, n}, x] &&  !IntegerQ[m] &&  !Integer
Q[n] &&  !GtQ[c, 0] &&  !GtQ[-d/(b*c), 0] && ((RationalQ[m] &&  !(EqQ[n, -2
^(-1)] && EqQ[c^2 - d^2, 0])) ||  !RationalQ[n])
```

```
Int[((e_.)*(x_))^(m_)*((b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(
e*x)^m*((b*x + c*x^2)^p/(x^(m + p)*(b + c*x)^p))   Int[x^(m + p)*(b + c*x)^
p, x], x] /; FreeQ[{b, c, e, m}, x]
```

Maple [F]

$$\int (cx)^{1-2p} \left(b\,x^{2}+ax\right)^{p} dx$$

```
int((c*x)^(1-2*p)*(b*x^2+a*x)^p,x)
```

```
int((c*x)^(1-2*p)*(b*x^2+a*x)^p,x)
```

Fricas [F]

$$\int (cx)^{1-2p} \left(ax+bx^2\right)^p \, dx = \int \left(bx^2+ax\right)^p (cx)^{-2\,p+1} \, dx$$

```
integrate((c*x)^(1-2*p)*(b*x^2+a*x)^p,x, algorithm="fricas")
```

```
integral((b*x^2 + a*x)^p*(c*x)^(-2*p + 1), x)
```

Sympy [F]

$$\int (cx)^{1-2p} \left(ax+bx^2\right)^p \, dx = \int (cx)^{1-2p} \left(x(a+bx)\right)^p \, dx$$

```
integrate((c*x)**(1-2*p)*(b*x**2+a*x)**p,x)
```

```
Integral((c*x)**(1 - 2*p)*(x*(a + b*x))**p, x)
```

Maxima [F]

$$\int (cx)^{1-2p} \left(ax+bx^2\right)^p \, dx = \int \left(bx^2+ax\right)^p (cx)^{-2\,p+1} \, dx$$

```
integrate((c*x)^(1-2*p)*(b*x^2+a*x)^p,x, algorithm="maxima")
```

```
integrate((b*x^2 + a*x)^p*(c*x)^(-2*p + 1), x)
```

Giac [F]

$$\int (cx)^{1-2p} \left(ax+bx^2\right)^p \, dx = \int \left(bx^2+ax\right)^p (cx)^{-2\,p+1} \, dx$$

```
integrate((c*x)^(1-2*p)*(b*x^2+a*x)^p,x, algorithm="giac")
```

```
integrate((b*x^2 + a*x)^p*(c*x)^(-2*p + 1), x)
```

Mupad [F(-1)]

Timed out.

$$\int (cx)^{1-2p}\left(ax+bx^2\right)^p\,dx=\int \left(b\,x^2+a\,x\right)^p\,(c\,x)^{1-2\,p}\,dx$$

```
int((a*x + b*x^2)^p*(c*x)^(1 - 2*p),x)
```

```
int((a*x + b*x^2)^p*(c*x)^(1 - 2*p), x)
```

Reduce [F]

$$\int (cx)^{1-2p}\left(ax+bx^2\right)^p\,dx$$
$$=\frac{c\left(\left(b\,x^2+ax\right)^p apx+\left(b\,x^2+ax\right)^p b\,x^2+x^{2p}\left(\int\frac{\left(b\,x^2+ax\right)^p}{x^{2p}a+x^{2p}bx}dx\right)a^2p^2-x^{2p}\left(\int\frac{\left(b\,x^2+ax\right)^p}{x^{2p}a+x^{2p}bx}dx\right)a^2p\right)}{2x^{2p}c^{2p}b}$$

```
int((c*x)^(1-2*p)*(b*x^2+a*x)^p,x)
```

```
(c*((a*x + b*x**2)**p*a*p*x + (a*x + b*x**2)**p*b*x**2 + x**(2*p)*int((a*x
 + b*x**2)**p/(x**(2*p)*a + x**(2*p)*b*x),x)*a**2*p**2 - x**(2*p)*int((a*x
 + b*x**2)**p/(x**(2*p)*a + x**(2*p)*b*x),x)*a**2*p))/(2*x**(2*p)*c**(2*p)
*b)
```

3.263 $\int (2-3x)^p x^{m+p}\,dx$

Optimal result

Integrand size = 13, antiderivative size = 33

$$\int (2-3x)^p x^{m+p}\,dx=\frac{2^p x^{1+m+p}\,\text{Hypergeometric2F1}\left(-p,1+m+p,2+m+p,\frac{3x}{2}\right)}{1+m+p}$$

```
2^p*x^(1+m+p)*hypergeom([-p, 1+m+p],[2+m+p],3/2*x)/(1+m+p)
```

Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 33, normalized size of antiderivative = 1.00

$$\int (2-3x)^p x^{m+p}\,dx=\frac{2^p x^{1+m+p}\,\text{Hypergeometric2F1}\left(-p,1+m+p,2+m+p,\frac{3x}{2}\right)}{1+m+p}$$

```
Integrate[(2 - 3*x)^p*x^(m + p),x]
```

```
(2^p*x^(1 + m + p)*Hypergeometric2F1[-p, 1 + m + p, 2 + m + p, (3*x)/2])/(
1 + m + p)
```

Rubi [A] (verified)

Time = 0.24 (sec) , antiderivative size = 33, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.077, Rules used = $\{74\}$

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (2-3x)^p x^{m+p} \, dx$$

$$\Big\downarrow \; 74$$

$$\frac{2^p x^{m+p+1} \text{Hypergeometric2F1}\left(-p,m+p+1,m+p+2,\frac{3x}{2}\right)}{m+p+1}$$

```
Int[(2 - 3*x)^p*x^(m + p),x]
```

```
(2^p*x^(1 + m + p)*Hypergeometric2F1[-p, 1 + m + p, 2 + m + p, (3*x)/2])/(
1 + m + p)
```

Defintions of rubi rules used

```
Int[((b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[c^n*((b*x
)^(m + 1)/(b*(m + 1)))*Hypergeometric2F1[-n, m + 1, m + 2, (-d)*(x/c)], x]
/; FreeQ[{b, c, d, m, n}, x] &&  !IntegerQ[m] && (IntegerQ[n] || (GtQ[c, 0]
 &&  !(EqQ[n, -2^(-1)] && EqQ[c^2 - d^2, 0] && GtQ[-d/(b*c), 0])))
```

Maple [A] (verified)

Time = 0.39 (sec) , antiderivative size = 34, normalized size of antiderivative = 1.03

method	result	size
meijerg	$\frac{2^{p} x^{1+m+p} \operatorname{hypergeom}\left(\left[-p,1+m+p\right],\left[2+m+p\right],\frac{3x}{2}\right)}{1+m+p}$	34

```
int((2-3*x)^p*x^(m+p),x,method=_RETURNVERBOSE)
```

```
2^p*x^(1+m+p)*hypergeom([-p,1+m+p],[2+m+p],3/2*x)/(1+m+p)
```

Fricas [F]

$$\int (2-3x)^p x^{m+p}\,dx = \int x^{m+p}(-3\,x+2)^p\,dx$$

```
integrate((2-3*x)^p*x^(m+p),x, algorithm="fricas")
```

```
integral(x^(m + p)*(-3*x + 2)^p, x)
```

Sympy [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 1.23 (sec) , antiderivative size = 44, normalized size of antiderivative = 1.33

$$\int (2-3x)^p x^{m+p}\,dx = \frac{2^p x^{m+p+1}\Gamma(m+p+1)\,{}_2F_1\left(\begin{matrix}-p, m+p+1\\ m+p+2\end{matrix}\middle|\frac{3xe^{2i\pi}}{2}\right)}{\Gamma\left(m+p+2\right)}$$

```
integrate((2-3*x)**p*x**(m+p),x)
```

```
2**p*x**(m + p + 1)*gamma(m + p + 1)*hyper((-p, m + p + 1), (m + p + 2,),
3*x*exp_polar(2*I*pi)/2)/gamma(m + p + 2)
```

Maxima [F]

$$\int (2-3x)^p x^{m+p}\,dx = \int x^{m+p}(-3\,x+2)^p\,dx$$

```
integrate((2-3*x)^p*x^(m+p),x, algorithm="maxima")
```

```
integrate(x^(m + p)*(-3*x + 2)^p, x)
```

Giac [F]

$$\int (2-3x)^p x^{m+p}\,dx = \int x^{m+p}(-3\,x+2)^p\,dx$$

```
integrate((2-3*x)^p*x^(m+p),x, algorithm="giac")
```

```
integrate(x^(m + p)*(-3*x + 2)^p, x)
```

Mupad [F(-1)]

Timed out.

$$\int (2-3x)^p x^{m+p}\,dx = \int x^{m+p}\,(2-3\,x)^p\,dx$$

```
int(x^(m + p)*(2 - 3*x)^p,x)
```

```
int(x^(m + p)*(2 - 3*x)^p, x)
```

Reduce [F]

$$\int (2-3x)^p x^{m+p}\,dx$$
$$= \frac{3x^{m+p}(-3x+2)^p\,mx + 6x^{m+p}(-3x+2)^p\,px - 2x^{m+p}(-3x+2)^p\,p - 4\Big(\int \frac{x^{m+p}(-3}{3m^2x^2+12mp\,x^2+12p^2x^2-2m^2x-8mp}}{}$$

```
int((2-3*x)^p*x^(m+p),x)
```

```
(3*x**(m + p)*( - 3*x + 2)**p*m*x + 6*x**(m + p)*( - 3*x + 2)**p*p*x - 2*x
**(m + p)*( - 3*x + 2)**p*p - 4*int((x**(m + p)*( - 3*x + 2)**p)/(3*m**2*x
**2 - 2*m**2*x + 12*m*p*x**2 - 8*m*p*x + 3*m*x**2 - 2*m*x + 12*p**2*x**2 -
 8*p**2*x + 6*p*x**2 - 4*p*x),x)*m**3*p - 20*int((x**(m + p)*( - 3*x + 2)*
*p)/(3*m**2*x**2 - 2*m**2*x + 12*m*p*x**2 - 8*m*p*x + 3*m*x**2 - 2*m*x + 1
2*p**2*x**2 - 8*p**2*x + 6*p*x**2 - 4*p*x),x)*m**2*p**2 - 4*int((x**(m + p
)*( - 3*x + 2)**p)/(3*m**2*x**2 - 2*m**2*x + 12*m*p*x**2 - 8*m*p*x + 3*m*x
**2 - 2*m*x + 12*p**2*x**2 - 8*p**2*x + 6*p*x**2 - 4*p*x),x)*m**2*p - 32*i
nt((x**(m + p)*( - 3*x + 2)**p)/(3*m**2*x**2 - 2*m**2*x + 12*m*p*x**2 - 8*
m*p*x + 3*m*x**2 - 2*m*x + 12*p**2*x**2 - 8*p**2*x + 6*p*x**2 - 4*p*x),x)*
m*p**3 - 12*int((x**(m + p)*( - 3*x + 2)**p)/(3*m**2*x**2 - 2*m**2*x + 12*
m*p*x**2 - 8*m*p*x + 3*m*x**2 - 2*m*x + 12*p**2*x**2 - 8*p**2*x + 6*p*x**2
 - 4*p*x),x)*m*p**2 - 16*int((x**(m + p)*( - 3*x + 2)**p)/(3*m**2*x**2 - 2
*m**2*x + 12*m*p*x**2 - 8*m*p*x + 3*m*x**2 - 2*m*x + 12*p**2*x**2 - 8*p**2
*x + 6*p*x**2 - 4*p*x),x)*p**4 - 8*int((x**(m + p)*( - 3*x + 2)**p)/(3*m**
2*x**2 - 2*m**2*x + 12*m*p*x**2 - 8*m*p*x + 3*m*x**2 - 2*m*x + 12*p**2*x**
2 - 8*p**2*x + 6*p*x**2 - 4*p*x),x)*p**3)/(3*(m**2 + 4*m*p + m + 4*p**2 +
2*p))
```

3.264 $\int x^m(2x-3x^2)^p\,dx$

Optimal result

Integrand size = 15, antiderivative size = 33

$$\int x^m\left(2x-3x^2\right)^p\,dx=\frac{2^px^{1+m+p}\operatorname{Hypergeometric2F1}\left(-p,1+m+p,2+m+p,\frac{3x}{2}\right)}{1+m+p}$$

```
2^p*x^(1+m+p)*hypergeom([-p, 1+m+p],[2+m+p],3/2*x)/(1+m+p)
```

Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 50, normalized size of antiderivative = 1.52

$$\int x^m\left(2x-3x^2\right)^p\,dx=\frac{2^p(2-3x)^{-p}x^{1+m}((2-3x)x)^p\operatorname{Hypergeometric2F1}\left(-p,1+m+p,2+m+p,\frac{3x}{2}\right)}{1+m+p}$$

```
Integrate[x^m*(2*x - 3*x^2)^p,x]
```

```
(2^p*x^(1 + m)*((2 - 3*x)*x)^p*Hypergeometric2F1[-p, 1 + m + p, 2 + m + p,
 (3*x)/2])/((1 + m + p)*(2 - 3*x)^p)
```

Rubi [A] (verified)

Time = 0.27 (sec) , antiderivative size = 52, normalized size of antiderivative = 1.58, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.133, Rules used = {1137, 74}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^m (2x-3x^2)^p \, dx$$

$$\Big\downarrow \text{ 1137}$$

$$(2-3x)^{-p}x^{-p}(2x-3x^2)^p \int (2-3x)^p x^{m+p} dx$$

$$\Big\downarrow \text{ 74}$$

$$\frac{2^p x^{m+1}(2-3x)^{-p}\left(2x-3x^2\right)^p \text{Hypergeometric2F1}\left(-p,m+p+1,m+p+2,\frac{3x}{2}\right)}{m+p+1}$$

```
Int[x^m*(2*x - 3*x^2)^p,x]
```

```
(2^p*x^(1 + m)*(2*x - 3*x^2)^p*Hypergeometric2F1[-p, 1 + m + p, 2 + m + p,
 (3*x)/2])/((1 + m + p)*(2 - 3*x)^p)
```

Defintions of rubi rules used

```
Int[((b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[c^n*((b*x
)^(m + 1)/(b*(m + 1)))*Hypergeometric2F1[-n, m + 1, m + 2, (-d)*(x/c)], x]
/; FreeQ[{b, c, d, m, n}, x] &&  !IntegerQ[m] && (IntegerQ[n] || (GtQ[c, 0]
 &&  !(EqQ[n, -2^(-1)] && EqQ[c^2 - d^2, 0] && GtQ[-d/(b*c), 0])))
```

```
Int[((e_.)*(x_))^(m_)*((b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(
e*x)^m*((b*x + c*x^2)^p/(x^(m + p)*(b + c*x)^p))   Int[x^(m + p)*(b + c*x)^
p, x], x] /; FreeQ[{b, c, e, m}, x]
```

Maple [A] (verified)

Time = 0.39 (sec) , antiderivative size = 34, normalized size of antiderivative = 1.03

method	result	size
meijerg	$\frac{2^{p} x^{1+m+p} \operatorname{hypergeom}\left(\left[-p,1+m+p\right],\left[2+m+p\right],\frac{3 x}{2}\right)}{1+m+p}$	34

```
int(x^m*(-3*x^2+2*x)^p,x,method=_RETURNVERBOSE)
```

```
2^p*x^(1+m+p)*hypergeom([-p,1+m+p],[2+m+p],3/2*x)/(1+m+p)
```

Fricas [F]

$$\int x^{m}\left(2x-3x^{2}\right)^{p}\,dx=\int \left(-3\,x^{2}+2\,x\right)^{p}x^{m}\,dx$$

```
integrate(x^m*(-3*x^2+2*x)^p,x, algorithm="fricas")
```

```
integral((-3*x^2 + 2*x)^p*x^m, x)
```

Sympy [F]

$$\int x^{m}\left(2x-3x^{2}\right)^{p}\,dx=\int x^{m}(-x(3x-2))^{p}\,dx$$

```
integrate(x**m*(-3*x**2+2*x)**p,x)
```

```
Integral(x**m*(-x*(3*x - 2))**p, x)
```

Maxima [F]

$$\int x^m (2x - 3x^2)^p \, dx = \int (-3\,x^2 + 2\,x)^p x^m \, dx$$

```
integrate(x^m*(-3*x^2+2*x)^p,x, algorithm="maxima")
```

```
integrate((-3*x^2 + 2*x)^p*x^m, x)
```

Giac [F]

$$\int x^m (2x - 3x^2)^p \, dx = \int (-3\,x^2 + 2\,x)^p x^m \, dx$$

```
integrate(x^m*(-3*x^2+2*x)^p,x, algorithm="giac")
```

```
integrate((-3*x^2 + 2*x)^p*x^m, x)
```

Mupad [F(-1)]

Timed out.

$$\int x^m (2x - 3x^2)^p \, dx = \int x^m \,(2\,x - 3\,x^2)^p \, dx$$

```
int(x^m*(2*x - 3*x^2)^p,x)
```

```
int(x^m*(2*x - 3*x^2)^p, x)
```

Reduce [F]

$$\int x^m \left(2x-3x^2\right)^p \, dx$$
$$= \frac{3x^m(-3x^2+2x)^p mx + 6x^m(-3x^2+2x)^p px - 2x^m(-3x^2+2x)^p p - 4\left(\int \frac{x^m(-3x^2}{3m^2x^2+12mpx^2+12p^2x^2-2m^2x-8m}\right.}{}$$

```
int(x^m*(-3*x^2+2*x)^p,x)
```

```
(3*x**m*( - 3*x**2 + 2*x)**p*m*x + 6*x**m*( - 3*x**2 + 2*x)**p*p*x - 2*x**
m*( - 3*x**2 + 2*x)**p*p - 4*int((x**m*( - 3*x**2 + 2*x)**p)/(3*m**2*x**2
- 2*m**2*x + 12*m*p*x**2 - 8*m*p*x + 3*m*x**2 - 2*m*x + 12*p**2*x**2 - 8*p
**2*x + 6*p*x**2 - 4*p*x),x)*m**3*p - 20*int((x**m*( - 3*x**2 + 2*x)**p)/(
3*m**2*x**2 - 2*m**2*x + 12*m*p*x**2 - 8*m*p*x + 3*m*x**2 - 2*m*x + 12*p**
2*x**2 - 8*p**2*x + 6*p*x**2 - 4*p*x),x)*m**2*p**2 - 4*int((x**m*( - 3*x**
2 + 2*x)**p)/(3*m**2*x**2 - 2*m**2*x + 12*m*p*x**2 - 8*m*p*x + 3*m*x**2 -
2*m*x + 12*p**2*x**2 - 8*p**2*x + 6*p*x**2 - 4*p*x),x)*m**2*p - 32*int((x*
*m*( - 3*x**2 + 2*x)**p)/(3*m**2*x**2 - 2*m**2*x + 12*m*p*x**2 - 8*m*p*x +
 3*m*x**2 - 2*m*x + 12*p**2*x**2 - 8*p**2*x + 6*p*x**2 - 4*p*x),x)*m*p**3
- 12*int((x**m*( - 3*x**2 + 2*x)**p)/(3*m**2*x**2 - 2*m**2*x + 12*m*p*x**2
 - 8*m*p*x + 3*m*x**2 - 2*m*x + 12*p**2*x**2 - 8*p**2*x + 6*p*x**2 - 4*p*x
),x)*m*p**2 - 16*int((x**m*( - 3*x**2 + 2*x)**p)/(3*m**2*x**2 - 2*m**2*x +
 12*m*p*x**2 - 8*m*p*x + 3*m*x**2 - 2*m*x + 12*p**2*x**2 - 8*p**2*x + 6*p*
x**2 - 4*p*x),x)*p**4 - 8*int((x**m*( - 3*x**2 + 2*x)**p)/(3*m**2*x**2 - 2
*m**2*x + 12*m*p*x**2 - 8*m*p*x + 3*m*x**2 - 2*m*x + 12*p**2*x**2 - 8*p**2
*x + 6*p*x**2 - 4*p*x),x)*p**3)/(3*(m**2 + 4*m*p + m + 4*p**2 + 2*p))
```

3.265 $\int x^3\sqrt{ax^2+bx^3}\,dx$

Optimal result

Integrand size = 19, antiderivative size = 136

$$\int x^3\sqrt{ax^2+bx^3}\,dx=\frac{2a^4(ax^2+bx^3)^{3/2}}{3b^5x^3}-\frac{8a^3(ax^2+bx^3)^{5/2}}{5b^5x^5}+\frac{12a^2(ax^2+bx^3)^{7/2}}{7b^5x^7}-\frac{8a(ax^2+bx^3)^{9/2}}{9b^5x^9}+\frac{2(ax^2+bx^3)^{11/2}}{11b^5x^{11}}$$

```
2/3*a^4*(b*x^3+a*x^2)^(3/2)/b^5/x^3-8/5*a^3*(b*x^3+a*x^2)^(5/2)/b^5/x^5+12
/7*a^2*(b*x^3+a*x^2)^(7/2)/b^5/x^7-8/9*a*(b*x^3+a*x^2)^(9/2)/b^5/x^9+2/11*
(b*x^3+a*x^2)^(11/2)/b^5/x^11
```

Mathematica [A] (verified)

Time = 0.03 (sec) , antiderivative size = 64, normalized size of antiderivative = 0.47

$$\int x^3\sqrt{ax^2+bx^3}\,dx=\frac{2(x^2(a+bx))^{3/2}\left(128a^4-192a^3bx+240a^2b^2x^2-280ab^3x^3+315b^4x^4\right)}{3465b^5x^3}$$

```
Integrate[x^3*Sqrt[a*x^2 + b*x^3],x]
```

```
(2*(x^2*(a + b*x))^(3/2)*(128*a^4 - 192*a^3*b*x + 240*a^2*b^2*x^2 - 280*a*
b^3*x^3 + 315*b^4*x^4))/(3465*b^5*x^3)
```

Rubi [A] (verified)

Time = 0.54 (sec) , antiderivative size = 149, normalized size of antiderivative = 1.10, number of steps used = 5, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.263$, Rules used = {1922, 1922, 1922, 1908, 1920}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^3\sqrt{ax^2+bx^3}\,dx$$

$$\Big\downarrow \text{1922}$$

$$\frac{2x\left(ax^2+bx^3\right)^{3/2}}{11b}-\frac{8a\int x^2\sqrt{bx^3+ax^2}dx}{11b}$$

$$\Big\downarrow \text{1922}$$

$$\frac{2x\left(ax^2+bx^3\right)^{3/2}}{11b}-\frac{8a\left(\frac{2\left(ax^2+bx^3\right)^{3/2}}{9b}-\frac{2a\int x\sqrt{bx^3+ax^2}dx}{3b}\right)}{11b}$$

$$\Big\downarrow \text{1922}$$

$$\frac{2x\left(ax^2+bx^3\right)^{3/2}}{11b}-\frac{8a\left(\frac{2\left(ax^2+bx^3\right)^{3/2}}{9b}-\frac{2a\left(\frac{2\left(ax^2+bx^3\right)^{3/2}}{7bx}-\frac{4a\int\sqrt{bx^3+ax^2}dx}{7b}\right)}{3b}\right)}{11b}$$

$$\Big\downarrow \text{1908}$$

$$\frac{2x\left(ax^2+bx^3\right)^{3/2}}{11b}-\frac{8a\left(\frac{2\left(ax^2+bx^3\right)^{3/2}}{9b}-\frac{2a\left(\frac{2\left(ax^2+bx^3\right)^{3/2}}{7bx}-\frac{4a\left(\frac{2\left(ax^2+bx^3\right)^{3/2}}{5bx^2}-\frac{2a\int\frac{\sqrt{bx^3+ax^2}}{x}dx}{5b}\right)}{7b}\right)}{3b}\right)}{11b}$$

$$\downarrow \text{1920}$$

$$\frac{2x\left(ax^2+bx^3\right)^{3/2}}{11b}-\frac{8a\left(\frac{2\left(ax^2+bx^3\right)^{3/2}}{9b}-\frac{2a\left(\frac{2\left(ax^2+bx^3\right)^{3/2}}{7bx}-\frac{4a\left(\frac{2\left(ax^2+bx^3\right)^{3/2}}{5bx^2}-\frac{4a\left(ax^2+bx^3\right)^{3/2}}{15b^2x^3}\right)}{7b}\right)}{3b}\right)}{11b}$$

```
Int[x^3*Sqrt[a*x^2 + b*x^3],x]
```

```
(2*x*(a*x^2 + b*x^3)^(3/2))/(11*b) - (8*a*((2*(a*x^2 + b*x^3)^(3/2))/(9*b)
 - (2*a*((2*(a*x^2 + b*x^3)^(3/2))/(7*b*x) - (4*a*((-4*a*(a*x^2 + b*x^3)^(
3/2))/(15*b^2*x^3) + (2*(a*x^2 + b*x^3)^(3/2))/(5*b*x^2)))/(7*b)))/(3*b)))
/(11*b)
```

Defintions of rubi rules used

```
Int[((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol] :> Simp[(a*x^j +
b*x^n)^(p + 1)/(a*(j*p + 1)*x^(j - 1)), x] - Simp[b*((n*p + n - j + 1)/(a*(
j*p + 1)))   Int[x^(n - j)*(a*x^j + b*x^n)^p, x], x] /; FreeQ[{a, b, j, n,
p}, x] &&  !IntegerQ[p] && NeQ[n, j] && ILtQ[Simplify[(n*p + n - j + 1)/(n
- j)], 0] && NeQ[j*p + 1, 0]
```

```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> Simp[(-c^(j - 1))*(c*x)^(m - j + 1)*((a*x^j + b*x^n)^(p + 1)/(a*(n - j
)*(p + 1))), x] /; FreeQ[{a, b, c, j, m, n, p}, x] &&  !IntegerQ[p] && NeQ[
n, j] && EqQ[m + n*p + n - j + 1, 0] && (IntegerQ[j] || GtQ[c, 0])
```

```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> Simp[c^(j - 1)*(c*x)^(m - j + 1)*((a*x^j + b*x^n)^(p + 1)/(a*(m + j*p
+ 1))), x] - Simp[b*((m + n*p + n - j + 1)/(a*c^(n - j)*(m + j*p + 1)))   I
nt[(c*x)^(m + n - j)*(a*x^j + b*x^n)^p, x], x] /; FreeQ[{a, b, c, j, m, n,
p}, x] &&  !IntegerQ[p] && NeQ[n, j] && ILtQ[Simplify[(m + n*p + n - j + 1)
/(n - j)], 0] && NeQ[m + j*p + 1, 0] && (IntegersQ[j, n] || GtQ[c, 0])
```

Maple [A] (verified)

Time = 0.44 (sec) , antiderivative size = 43, normalized size of antiderivative = 0.32

method	result	size
pseudoelliptic	$-\frac{2(bx+a)^{\frac{3}{2}}\left(-35b^{3}x^{3}+30a\,b^{2}x^{2}-24a^{2}bx+16a^{3}\right)}{315b^{4}}$	43
gosper	$\frac{2(bx+a)\left(315b^{4}x^{4}-280a\,b^{3}x^{3}+240a^{2}b^{2}x^{2}-192a^{3}bx+128a^{4}\right)\sqrt{b\,x^{3}+a\,x^{2}}}{3465b^{5}x}$	68
default	$\frac{2(bx+a)\left(315b^{4}x^{4}-280a\,b^{3}x^{3}+240a^{2}b^{2}x^{2}-192a^{3}bx+128a^{4}\right)\sqrt{b\,x^{3}+a\,x^{2}}}{3465b^{5}x}$	68
orering	$\frac{2(bx+a)\left(315b^{4}x^{4}-280a\,b^{3}x^{3}+240a^{2}b^{2}x^{2}-192a^{3}bx+128a^{4}\right)\sqrt{b\,x^{3}+a\,x^{2}}}{3465b^{5}x}$	68
risch	$\frac{2\sqrt{x^{2}(bx+a)}\,\left(315b^{5}x^{5}+35a\,b^{4}x^{4}-40a^{2}b^{3}x^{3}+48a^{3}b^{2}x^{2}-64a^{4}bx+128a^{5}\right)}{3465x\,b^{5}}$	72
trager	$\frac{2\left(315b^{5}x^{5}+35a\,b^{4}x^{4}-40a^{2}b^{3}x^{3}+48a^{3}b^{2}x^{2}-64a^{4}bx+128a^{5}\right)\sqrt{b\,x^{3}+a\,x^{2}}}{3465b^{5}x}$	74

```
int(x^3*(b*x^3+a*x^2)^(1/2),x,method=_RETURNVERBOSE)
```

```
-2/315*(b*x+a)^(3/2)*(-35*b^3*x^3+30*a*b^2*x^2-24*a^2*b*x+16*a^3)/b^4
```

Fricas [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 73, normalized size of antiderivative = 0.54

$$\int x^3\sqrt{ax^2+bx^3}\,dx = \frac{2\,(315\,b^5x^5+35\,ab^4x^4-40\,a^2b^3x^3+48\,a^3b^2x^2-64\,a^4bx+128\,a^5)\sqrt{bx^3+ax^2}}{3465\,b^5x}$$

```
integrate(x^3*(b*x^3+a*x^2)^(1/2),x, algorithm="fricas")
```

```
2/3465*(315*b^5*x^5 + 35*a*b^4*x^4 - 40*a^2*b^3*x^3 + 48*a^3*b^2*x^2 - 64*
a^4*b*x + 128*a^5)*sqrt(b*x^3 + a*x^2)/(b^5*x)
```

Sympy [F]

$$\int x^3\sqrt{ax^2+bx^3}\,dx=\int x^3\sqrt{x^2\left(a+bx\right)}\,dx$$

```
integrate(x**3*(b*x**3+a*x**2)**(1/2),x)
```

```
Integral(x**3*sqrt(x**2*(a + b*x)), x)
```

Maxima [A] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 64, normalized size of antiderivative = 0.47

$$\int x^3\sqrt{ax^2+bx^3}\,dx=\frac{2\left(315\,b^5x^5+35\,ab^4x^4-40\,a^2b^3x^3+48\,a^3b^2x^2-64\,a^4bx+128\,a^5\right)\sqrt{bx+a}}{3465\,b^5}$$

```
integrate(x^3*(b*x^3+a*x^2)^(1/2),x, algorithm="maxima")
```

```
2/3465*(315*b^5*x^5 + 35*a*b^4*x^4 - 40*a^2*b^3*x^3 + 48*a^3*b^2*x^2 - 64*
a^4*b*x + 128*a^5)*sqrt(b*x + a)/b^5
```

Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 156, normalized size of antiderivative = 1.15

$$\int x^3\sqrt{ax^2+bx^3}\,dx = -\frac{256\,a^{\frac{11}{2}}\operatorname{sgn}(x)}{3465\,b^5} + \frac{2\left(\frac{11\left(35\,(bx+a)^{\frac{9}{2}}-180\,(bx+a)^{\frac{7}{2}}a+378\,(bx+a)^{\frac{5}{2}}a^2-420\,(bx+a)^{\frac{3}{2}}a^3+315\,\sqrt{bx+a}a^4\right)a\operatorname{sgn}(x)}{b^4} + \frac{5\left(63\,(bx+a)^{\frac{11}{2}}-385\,(bx+a)^{\frac{9}{2}}a+990\,(\cdots\right.}{\cdots}\right.}{3465\,b}$$

```
integrate(x^3*(b*x^3+a*x^2)^(1/2),x, algorithm="giac")
```

```
-256/3465*a^(11/2)*sgn(x)/b^5 + 2/3465*(11*(35*(b*x + a)^(9/2) - 180*(b*x
+ a)^(7/2)*a + 378*(b*x + a)^(5/2)*a^2 - 420*(b*x + a)^(3/2)*a^3 + 315*sqr
t(b*x + a)*a^4)*a*sgn(x)/b^4 + 5*(63*(b*x + a)^(11/2) - 385*(b*x + a)^(9/2
)*a + 990*(b*x + a)^(7/2)*a^2 - 1386*(b*x + a)^(5/2)*a^3 + 1155*(b*x + a)^
(3/2)*a^4 - 693*sqrt(b*x + a)*a^5)*sgn(x)/b^4)/b
```

Mupad [B] (verification not implemented)

Time = 9.20 (sec) , antiderivative size = 73, normalized size of antiderivative = 0.54

$$\int x^3\sqrt{ax^2+bx^3}\,dx = \frac{2\,\sqrt{b\,x^3+a\,x^2}\,\left(128\,a^5-64\,a^4\,b\,x+48\,a^3\,b^2\,x^2-40\,a^2\,b^3\,x^3+35\,a\,b^4\,x^4+315\,b^5\,x^5\right)}{3465\,b^5\,x}$$

```
int(x^3*(a*x^2 + b*x^3)^(1/2),x)
```

```
(2*(a*x^2 + b*x^3)^(1/2)*(128*a^5 + 315*b^5*x^5 + 35*a*b^4*x^4 + 48*a^3*b^
2*x^2 - 40*a^2*b^3*x^3 - 64*a^4*b*x))/(3465*b^5*x)
```

Reduce [B] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 63, normalized size of antiderivative = 0.46

$$\int x^3\sqrt{ax^2+bx^3}\,dx = \frac{2\sqrt{bx+a}\,(315b^5x^5+35a\,b^4x^4-40a^2b^3x^3+48a^3b^2x^2-64a^4bx+128a^5)}{3465b^5}$$

```
int(x^3*(b*x^3+a*x^2)^(1/2),x)
```

```
(2*sqrt(a + b*x)*(128*a**5 - 64*a**4*b*x + 48*a**3*b**2*x**2 - 40*a**2*b**
3*x**3 + 35*a*b**4*x**4 + 315*b**5*x**5))/(3465*b**5)
```

3.266 $\int x^2\sqrt{ax^2+bx^3}\,dx$

Optimal result

Integrand size = 19, antiderivative size = 108

$$\int x^2\sqrt{ax^2+bx^3}\,dx=-\frac{2a^3(ax^2+bx^3)^{3/2}}{3b^4x^3}+\frac{6a^2(ax^2+bx^3)^{5/2}}{5b^4x^5}-\frac{6a(ax^2+bx^3)^{7/2}}{7b^4x^7}+\frac{2(ax^2+bx^3)^{9/2}}{9b^4x^9}$$

```
-2/3*a^3*(b*x^3+a*x^2)^(3/2)/b^4/x^3+6/5*a^2*(b*x^3+a*x^2)^(5/2)/b^4/x^5-6
/7*a*(b*x^3+a*x^2)^(7/2)/b^4/x^7+2/9*(b*x^3+a*x^2)^(9/2)/b^4/x^9
```

Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 53, normalized size of antiderivative = 0.49

$$\int x^2\sqrt{ax^2+bx^3}\,dx=\frac{2(x^2(a+bx))^{3/2}(-16a^3+24a^2bx-30ab^2x^2+35b^3x^3)}{315b^4x^3}$$

```
Integrate[x^2*Sqrt[a*x^2 + b*x^3],x]
```

```
(2*(x^2*(a + b*x))^(3/2)*(-16*a^3 + 24*a^2*b*x - 30*a*b^2*x^2 + 35*b^3*x^3
))/(315*b^4*x^3)
```

Rubi [A] (verified)

Time = 0.46 (sec) , antiderivative size = 117, normalized size of antiderivative = 1.08, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.211$, Rules used = {1922, 1922, 1908, 1920}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^2\sqrt{ax^2+bx^3}\,dx$$

$$\Big\downarrow \text{1922}$$

$$\frac{2\left(ax^2+bx^3\right)^{3/2}}{9b}-\frac{2a\int x\sqrt{bx^3+ax^2}dx}{3b}$$

$$\Big\downarrow \text{1922}$$

$$\frac{2\left(ax^2+bx^3\right)^{3/2}}{9b}-\frac{2a\left(\frac{2\left(ax^2+bx^3\right)^{3/2}}{7bx}-\frac{4a\int\sqrt{bx^3+ax^2}dx}{7b}\right)}{3b}$$

$$\Big\downarrow \text{1908}$$

$$\frac{2\left(ax^2+bx^3\right)^{3/2}}{9b}-\frac{2a\left(\frac{2\left(ax^2+bx^3\right)^{3/2}}{7bx}-\frac{4a\left(\frac{2\left(ax^2+bx^3\right)^{3/2}}{5bx^2}-\frac{2a\int\frac{\sqrt{bx^3+ax^2}}{x}dx}{5b}\right)}{7b}\right)}{3b}$$

$$\Big\downarrow \text{1920}$$

$$\frac{2\left(ax^2+bx^3\right)^{3/2}}{9b}-\frac{2a\left(\frac{2\left(ax^2+bx^3\right)^{3/2}}{7bx}-\frac{4a\left(\frac{2\left(ax^2+bx^3\right)^{3/2}}{5bx^2}-\frac{4a\left(ax^2+bx^3\right)^{3/2}}{15b^2x^3}\right)}{7b}\right)}{3b}$$

```
Int[x^2*Sqrt[a*x^2 + b*x^3],x]
```

```
(2*(a*x^2 + b*x^3)^(3/2))/(9*b) - (2*a*((2*(a*x^2 + b*x^3)^(3/2))/(7*b*x)
- (4*a*((-4*a*(a*x^2 + b*x^3)^(3/2))/(15*b^2*x^3) + (2*(a*x^2 + b*x^3)^(3/
2))/(5*b*x^2)))/(7*b)))/(3*b)
```

Defintions of rubi rules used

```
Int[((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol] :> Simp[(a*x^j +
b*x^n)^(p + 1)/(a*(j*p + 1)*x^(j - 1)), x] - Simp[b*((n*p + n - j + 1)/(a*(
j*p + 1)))   Int[x^(n - j)*(a*x^j + b*x^n)^p, x], x] /; FreeQ[{a, b, j, n,
p}, x] &&  !IntegerQ[p] && NeQ[n, j] && ILtQ[Simplify[(n*p + n - j + 1)/(n
- j)], 0] && NeQ[j*p + 1, 0]
```

```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> Simp[(-c^(j - 1))*(c*x)^(m - j + 1)*((a*x^j + b*x^n)^(p + 1)/(a*(n - j
)*(p + 1))), x] /; FreeQ[{a, b, c, j, m, n, p}, x] &&  !IntegerQ[p] && NeQ[
n, j] && EqQ[m + n*p + n - j + 1, 0] && (IntegerQ[j] || GtQ[c, 0])
```

```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> Simp[c^(j - 1)*(c*x)^(m - j + 1)*((a*x^j + b*x^n)^(p + 1)/(a*(m + j*p
+ 1))), x] - Simp[b*((m + n*p + n - j + 1)/(a*c^(n - j)*(m + j*p + 1)))   I
nt[(c*x)^(m + n - j)*(a*x^j + b*x^n)^p, x], x] /; FreeQ[{a, b, c, j, m, n,
p}, x] &&  !IntegerQ[p] && NeQ[n, j] && ILtQ[Simplify[(m + n*p + n - j + 1)
/(n - j)], 0] && NeQ[m + j*p + 1, 0] && (IntegersQ[j, n] || GtQ[c, 0])
```

Maple [A] (verified)

Time = 0.41 (sec) , antiderivative size = 32, normalized size of antiderivative = 0.30

method	result	size
pseudoelliptic	$\frac{2(bx+a)^{\frac{3}{2}}(15b^2x^2-12abx+8a^2)}{105b^3}$	32
gosper	$-\frac{2(bx+a)(-35b^3x^3+30a\,b^2x^2-24a^2bx+16a^3)\sqrt{b\,x^3+a\,x^2}}{315b^4x}$	57
default	$-\frac{2(bx+a)(-35b^3x^3+30a\,b^2x^2-24a^2bx+16a^3)\sqrt{b\,x^3+a\,x^2}}{315b^4x}$	57
orering	$-\frac{2(bx+a)(-35b^3x^3+30a\,b^2x^2-24a^2bx+16a^3)\sqrt{b\,x^3+a\,x^2}}{315b^4x}$	57
risch	$-\frac{2\sqrt{x^2(bx+a)}\,(-35b^4x^4-5a\,b^3x^3+6a^2b^2x^2-8a^3bx+16a^4)}{315x\,b^4}$	61
trager	$-\frac{2(-35b^4x^4-5a\,b^3x^3+6a^2b^2x^2-8a^3bx+16a^4)\sqrt{b\,x^3+a\,x^2}}{315b^4x}$	63

```
int(x^2*(b*x^3+a*x^2)^(1/2),x,method=_RETURNVERBOSE)
```

```
2/105*(b*x+a)^(3/2)*(15*b^2*x^2-12*a*b*x+8*a^2)/b^3
```

Fricas [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 62, normalized size of antiderivative = 0.57

$$\int x^2\sqrt{ax^2+bx^3}\,dx = \frac{2\,(35\,b^4x^4+5\,ab^3x^3-6\,a^2b^2x^2+8\,a^3bx-16\,a^4)\sqrt{bx^3+ax^2}}{315\,b^4x}$$

```
integrate(x^2*(b*x^3+a*x^2)^(1/2),x, algorithm="fricas")
```

```
2/315*(35*b^4*x^4 + 5*a*b^3*x^3 - 6*a^2*b^2*x^2 + 8*a^3*b*x - 16*a^4)*sqrt
(b*x^3 + a*x^2)/(b^4*x)
```

Sympy [F]

$$\int x^2\sqrt{ax^2+bx^3}\,dx = \int x^2\sqrt{x^2\,(a+bx)}\,dx$$

```
integrate(x**2*(b*x**3+a*x**2)**(1/2),x)
```

```
Integral(x**2*sqrt(x**2*(a + b*x)), x)
```

Maxima [A] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 53, normalized size of antiderivative = 0.49

$$\int x^2\sqrt{ax^2+bx^3}\,dx = \frac{2\,(35\,b^4x^4+5\,ab^3x^3-6\,a^2b^2x^2+8\,a^3bx-16\,a^4)\sqrt{bx+a}}{315\,b^4}$$

```
integrate(x^2*(b*x^3+a*x^2)^(1/2),x, algorithm="maxima")
```

```
2/315*(35*b^4*x^4 + 5*a*b^3*x^3 - 6*a^2*b^2*x^2 + 8*a^3*b*x - 16*a^4)*sqrt
(b*x + a)/b^4
```

Giac [A] (verification not implemented)

Time = 0.14 (sec) , antiderivative size = 131, normalized size of antiderivative = 1.21

$$\int x^2\sqrt{ax^2+bx^3}\,dx = \frac{32\,a^{\frac{9}{2}}\mathrm{sgn}(x)}{315\,b^4} + \frac{2\left(\frac{9\left(5\,(bx+a)^{\frac{7}{2}}-21\,(bx+a)^{\frac{5}{2}}a+35\,(bx+a)^{\frac{3}{2}}a^2-35\,\sqrt{bx+a}a^3\right)a\mathrm{sgn}(x)}{b^3}+\frac{\left(35\,(bx+a)^{\frac{9}{2}}-180\,(bx+a)^{\frac{7}{2}}a+378\,(bx+a)^{\frac{5}{2}}a^2-420\,(bx+a)^{\frac{3}{2}}a\right.}{b^3}\right.}{315\,b}$$

```
integrate(x^2*(b*x^3+a*x^2)^(1/2),x, algorithm="giac")
```

```
32/315*a^(9/2)*sgn(x)/b^4 + 2/315*(9*(5*(b*x + a)^(7/2) - 21*(b*x + a)^(5/
2)*a + 35*(b*x + a)^(3/2)*a^2 - 35*sqrt(b*x + a)*a^3)*a*sgn(x)/b^3 + (35*(
b*x + a)^(9/2) - 180*(b*x + a)^(7/2)*a + 378*(b*x + a)^(5/2)*a^2 - 420*(b*
x + a)^(3/2)*a^3 + 315*sqrt(b*x + a)*a^4)*sgn(x)/b^3)/b
```

Mupad [B] (verification not implemented)

Time = 8.99 (sec) , antiderivative size = 62, normalized size of antiderivative = 0.57

$$\int x^2\sqrt{ax^2+bx^3}\,dx = \frac{2\sqrt{b\,x^3+a\,x^2}\,\left(-16\,a^4+8\,a^3\,b\,x-6\,a^2\,b^2\,x^2+5\,a\,b^3\,x^3+35\,b^4\,x^4\right)}{315\,b^4\,x}$$

```
int(x^2*(a*x^2 + b*x^3)^(1/2),x)
```

```
(2*(a*x^2 + b*x^3)^(1/2)*(35*b^4*x^4 - 16*a^4 + 5*a*b^3*x^3 - 6*a^2*b^2*x^
2 + 8*a^3*b*x))/(315*b^4*x)
```

Reduce [B] (verification not implemented)

Time = 0.38 (sec) , antiderivative size = 52, normalized size of antiderivative = 0.48

$$\int x^2\sqrt{ax^2+bx^3}\,dx = \frac{2\sqrt{bx+a}\,(35b^4x^4+5a\,b^3x^3-6a^2b^2x^2+8a^3bx-16a^4)}{315b^4}$$

```
int(x^2*(b*x^3+a*x^2)^(1/2),x)
```

```
(2*sqrt(a + b*x)*( - 16*a**4 + 8*a**3*b*x - 6*a**2*b**2*x**2 + 5*a*b**3*x*
*3 + 35*b**4*x**4))/(315*b**4)
```

3.267 $\int x\sqrt{ax^2+bx^3}\,dx$

Optimal result

Integrand size = 17, antiderivative size = 80

$$\int x\sqrt{ax^2+bx^3}\,dx=\frac{2a^2(ax^2+bx^3)^{3/2}}{3b^3x^3}-\frac{4a(ax^2+bx^3)^{5/2}}{5b^3x^5}+\frac{2(ax^2+bx^3)^{7/2}}{7b^3x^7}$$

```
2/3*a^2*(b*x^3+a*x^2)^(3/2)/b^3/x^3-4/5*a*(b*x^3+a*x^2)^(5/2)/b^3/x^5+2/7*
(b*x^3+a*x^2)^(7/2)/b^3/x^7
```

Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 42, normalized size of antiderivative = 0.52

$$\int x\sqrt{ax^2+bx^3}\,dx=\frac{2(x^2(a+bx))^{3/2}\,(8a^2-12abx+15b^2x^2)}{105b^3x^3}$$

```
Integrate[x*Sqrt[a*x^2 + b*x^3],x]
```

```
(2*(x^2*(a + b*x))^(3/2)*(8*a^2 - 12*a*b*x + 15*b^2*x^2))/(105*b^3*x^3)
```

Rubi [A] (verified)

Time = 0.37 (sec) , antiderivative size = 86, normalized size of antiderivative = 1.08, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.176$, Rules used = {1922, 1908, 1920}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x\sqrt{ax^2+bx^3}\,dx$$

$$\Big\downarrow \text{ 1922}$$

$$\frac{2(ax^2+bx^3)^{3/2}}{7bx}-\frac{4a\int\sqrt{bx^3+ax^2}dx}{7b}$$

$$\Big\downarrow \text{ 1908}$$

$$\frac{2(ax^2+bx^3)^{3/2}}{7bx}-\frac{4a\left(\frac{2(ax^2+bx^3)^{3/2}}{5bx^2}-\frac{2a\int\frac{\sqrt{bx^3+ax^2}}{x}dx}{5b}\right)}{7b}$$

$$\Big\downarrow \text{ 1920}$$

$$\frac{2(ax^2+bx^3)^{3/2}}{7bx}-\frac{4a\left(\frac{2(ax^2+bx^3)^{3/2}}{5bx^2}-\frac{4a(ax^2+bx^3)^{3/2}}{15b^2x^3}\right)}{7b}$$

```
Int[x*Sqrt[a*x^2 + b*x^3],x]
```

```
(2*(a*x^2 + b*x^3)^(3/2))/(7*b*x) - (4*a*((-4*a*(a*x^2 + b*x^3)^(3/2))/(15
*b^2*x^3) + (2*(a*x^2 + b*x^3)^(3/2))/(5*b*x^2)))/(7*b)
```

Defintions of rubi rules used

```
Int[((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol] :> Simp[(a*x^j +
b*x^n)^(p + 1)/(a*(j*p + 1)*x^(j - 1)), x] - Simp[b*((n*p + n - j + 1)/(a*(
j*p + 1)))   Int[x^(n - j)*(a*x^j + b*x^n)^p, x], x] /; FreeQ[{a, b, j, n,
p}, x] &&  !IntegerQ[p] && NeQ[n, j] && ILtQ[Simplify[(n*p + n - j + 1)/(n
- j)], 0] && NeQ[j*p + 1, 0]
```

```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> Simp[(-c^(j - 1))*(c*x)^(m - j + 1)*((a*x^j + b*x^n)^(p + 1)/(a*(n - j
)*(p + 1))), x] /; FreeQ[{a, b, c, j, m, n, p}, x] &&  !IntegerQ[p] && NeQ[
n, j] && EqQ[m + n*p + n - j + 1, 0] && (IntegerQ[j] || GtQ[c, 0])
```

```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> Simp[c^(j - 1)*(c*x)^(m - j + 1)*((a*x^j + b*x^n)^(p + 1)/(a*(m + j*p
+ 1))), x] - Simp[b*((m + n*p + n - j + 1)/(a*c^(n - j)*(m + j*p + 1)))   I
nt[(c*x)^(m + n - j)*(a*x^j + b*x^n)^p, x], x] /; FreeQ[{a, b, c, j, m, n,
p}, x] &&  !IntegerQ[p] && NeQ[n, j] && ILtQ[Simplify[(m + n*p + n - j + 1)
/(n - j)], 0] && NeQ[m + j*p + 1, 0] && (IntegersQ[j, n] || GtQ[c, 0])
```

Maple [A] (verified)

Time = 0.41 (sec) , antiderivative size = 21, normalized size of antiderivative = 0.26

method	result	size
pseudoelliptic	$-\frac{2(bx+a)^{\frac{3}{2}}(-3bx+2a)}{15b^{2}}$	21
gosper	$\frac{2(bx+a)(15b^{2}x^{2}-12abx+8a^{2})\sqrt{b\,x^{3}+a\,x^{2}}}{105b^{3}x}$	46
default	$\frac{2(bx+a)(15b^{2}x^{2}-12abx+8a^{2})\sqrt{b\,x^{3}+a\,x^{2}}}{105b^{3}x}$	46
orering	$\frac{2(bx+a)(15b^{2}x^{2}-12abx+8a^{2})\sqrt{b\,x^{3}+a\,x^{2}}}{105b^{3}x}$	46
risch	$\frac{2\sqrt{x^{2}(bx+a)}\,(15b^{3}x^{3}+3a\,b^{2}x^{2}-4a^{2}bx+8a^{3})}{105x\,b^{3}}$	50
trager	$\frac{2(15b^{3}x^{3}+3a\,b^{2}x^{2}-4a^{2}bx+8a^{3})\sqrt{b\,x^{3}+a\,x^{2}}}{105b^{3}x}$	52

```
int(x*(b*x^3+a*x^2)^(1/2),x,method=_RETURNVERBOSE)
```

```
-2/15*(b*x+a)^(3/2)*(-3*b*x+2*a)/b^2
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 51, normalized size of antiderivative = 0.64

$$\int x\sqrt{ax^2+bx^3}\,dx = \frac{2\,(15\,b^3x^3+3\,ab^2x^2-4\,a^2bx+8\,a^3)\sqrt{bx^3+ax^2}}{105\,b^3x}$$

```
integrate(x*(b*x^3+a*x^2)^(1/2),x, algorithm="fricas")
```

```
2/105*(15*b^3*x^3 + 3*a*b^2*x^2 - 4*a^2*b*x + 8*a^3)*sqrt(b*x^3 + a*x^2)/(
b^3*x)
```

Sympy [F]

$$\int x\sqrt{ax^2+bx^3}\,dx = \int x\sqrt{x^2\,(a+bx)}\,dx$$

```
integrate(x*(b*x**3+a*x**2)**(1/2),x)
```

```
Integral(x*sqrt(x**2*(a + b*x)), x)
```

Maxima [A] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 42, normalized size of antiderivative = 0.52

$$\int x\sqrt{ax^2+bx^3}\,dx = \frac{2\,(15\,b^3x^3+3\,ab^2x^2-4\,a^2bx+8\,a^3)\sqrt{bx+a}}{105\,b^3}$$

```
integrate(x*(b*x^3+a*x^2)^(1/2),x, algorithm="maxima")
```

```
2/105*(15*b^3*x^3 + 3*a*b^2*x^2 - 4*a^2*b*x + 8*a^3)*sqrt(b*x + a)/b^3
```

Giac [A] (verification not implemented)

Time = 0.14 (sec) , antiderivative size = 108, normalized size of antiderivative = 1.35

$$\int x\sqrt{ax^2+bx^3}\,dx = -\frac{16\,a^{\frac{7}{2}}\mathrm{sgn}(x)}{105\,b^3} + \frac{2\left(\frac{7\left(3\,(bx+a)^{\frac{5}{2}}-10\,(bx+a)^{\frac{3}{2}}a+15\,\sqrt{bx+a}a^2\right)a\mathrm{sgn}(x)}{b^2}+\frac{3\left(5\,(bx+a)^{\frac{7}{2}}-21\,(bx+a)^{\frac{5}{2}}a+35\,(bx+a)^{\frac{3}{2}}a^2-35\,\sqrt{bx+a}a^3\right)\mathrm{sgn}(x)}{b^2}\right)}{105\,b}$$

```
integrate(x*(b*x^3+a*x^2)^(1/2),x, algorithm="giac")
```

```
-16/105*a^(7/2)*sgn(x)/b^3 + 2/105*(7*(3*(b*x + a)^(5/2) - 10*(b*x + a)^(3
/2)*a + 15*sqrt(b*x + a)*a^2)*a*sgn(x)/b^2 + 3*(5*(b*x + a)^(7/2) - 21*(b*
x + a)^(5/2)*a + 35*(b*x + a)^(3/2)*a^2 - 35*sqrt(b*x + a)*a^3)*sgn(x)/b^2
)/b
```

Mupad [B] (verification not implemented)

Time = 9.09 (sec) , antiderivative size = 51, normalized size of antiderivative = 0.64

$$\int x\sqrt{ax^2+bx^3}\,dx = \frac{2\,\sqrt{b\,x^3+a\,x^2}\,\left(8\,a^3-4\,a^2\,b\,x+3\,a\,b^2\,x^2+15\,b^3\,x^3\right)}{105\,b^3\,x}$$

```
int(x*(a*x^2 + b*x^3)^(1/2),x)
```

```
(2*(a*x^2 + b*x^3)^(1/2)*(8*a^3 + 15*b^3*x^3 + 3*a*b^2*x^2 - 4*a^2*b*x))/(
105*b^3*x)
```

Reduce [B] (verification not implemented)

Time = 0.56 (sec) , antiderivative size = 41, normalized size of antiderivative = 0.51

$$\int x\sqrt{ax^2+bx^3}\,dx = \frac{2\sqrt{bx+a}\,(15b^3x^3+3a\,b^2x^2-4a^2bx+8a^3)}{105b^3}$$

```
int(x*(b*x^3+a*x^2)^(1/2),x)
```

```
(2*sqrt(a + b*x)*(8*a**3 - 4*a**2*b*x + 3*a*b**2*x**2 + 15*b**3*x**3))/(10
5*b**3)
```

3.268 $\int \sqrt{ax^2+bx^3}\,dx$

Optimal result

Integrand size = 15, antiderivative size = 52

$$\int \sqrt{ax^2+bx^3}\,dx=-\frac{2a(ax^2+bx^3)^{3/2}}{3b^2x^3}+\frac{2(ax^2+bx^3)^{5/2}}{5b^2x^5}$$

```
-2/3*a*(b*x^3+a*x^2)^(3/2)/b^2/x^3+2/5*(b*x^3+a*x^2)^(5/2)/b^2/x^5
```

Mathematica [A] (verified)

Time = 0.00 (sec) , antiderivative size = 41, normalized size of antiderivative = 0.79

$$\int \sqrt{ax^2+bx^3}\,dx=\frac{2\sqrt{x^2(a+bx)}(-2a^2+abx+3b^2x^2)}{15b^2x}$$

```
Integrate[Sqrt[a*x^2 + b*x^3],x]
```

```
(2*Sqrt[x^2*(a + b*x)]*(-2*a^2 + a*b*x + 3*b^2*x^2))/(15*b^2*x)
```

Rubi [A] (verified)

Time = 0.30 (sec) , antiderivative size = 52, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.133, Rules used = {1908, 1920}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sqrt{ax^2+bx^3}\,dx$$

$$\Big\downarrow \text{ 1908}$$

$$\frac{2\left(ax^2+bx^3\right)^{3/2}}{5bx^2}-\frac{2a\int \frac{\sqrt{bx^3+ax^2}}{x}dx}{5b}$$

$$\Big\downarrow \text{ 1920}$$

$$\frac{2\left(ax^2+bx^3\right)^{3/2}}{5bx^2}-\frac{4a\left(ax^2+bx^3\right)^{3/2}}{15b^2x^3}$$

```
Int[Sqrt[a*x^2 + b*x^3],x]
```

```
(-4*a*(a*x^2 + b*x^3)^(3/2))/(15*b^2*x^3) + (2*(a*x^2 + b*x^3)^(3/2))/(5*b
*x^2)
```

Defintions of rubi rules used

```
Int[((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol] :> Simp[(a*x^j +
b*x^n)^(p + 1)/(a*(j*p + 1)*x^(j - 1)), x] - Simp[b*((n*p + n - j + 1)/(a*(
j*p + 1)))   Int[x^(n - j)*(a*x^j + b*x^n)^p, x], x] /; FreeQ[{a, b, j, n,
p}, x] && !IntegerQ[p] && NeQ[n, j] && ILtQ[Simplify[(n*p + n - j + 1)/(n
- j)], 0] && NeQ[j*p + 1, 0]
```

```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> Simp[(-c^(j - 1))*(c*x)^(m - j + 1)*((a*x^j + b*x^n)^(p + 1)/(a*(n - j
)*(p + 1))), x] /; FreeQ[{a, b, c, j, m, n, p}, x] &&  !IntegerQ[p] && NeQ[
n, j] && EqQ[m + n*p + n - j + 1, 0] && (IntegerQ[j] || GtQ[c, 0])
```

Maple [A] (verified)

Time = 0.38 (sec) , antiderivative size = 13, normalized size of antiderivative = 0.25

method	result	size
pseudoelliptic	$\frac{2(bx+a)^{\frac{3}{2}}}{3b}$	13
gosper	$-\frac{2(bx+a)(-3bx+2a)\sqrt{b\,x^{3}+a\,x^{2}}}{15b^{2}x}$	35
default	$-\frac{2(bx+a)(-3bx+2a)\sqrt{b\,x^{3}+a\,x^{2}}}{15b^{2}x}$	35
orering	$-\frac{2(bx+a)(-3bx+2a)\sqrt{b\,x^{3}+a\,x^{2}}}{15b^{2}x}$	35
risch	$-\frac{2\sqrt{x^{2}(bx+a)}\,\left(-3b^{2}x^{2}-abx+2a^{2}\right)}{15x\,b^{2}}$	39
trager	$-\frac{2\left(-3b^{2}x^{2}-abx+2a^{2}\right)\sqrt{b\,x^{3}+a\,x^{2}}}{15b^{2}x}$	41

```
int((b*x^3+a*x^2)^(1/2),x,method=_RETURNVERBOSE)
```

```
2/3*(b*x+a)^(3/2)/b
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 39, normalized size of antiderivative = 0.75

$$\int \sqrt{ax^2+bx^3}\,dx = \frac{2\,(3\,b^2x^2+abx-2\,a^2)\sqrt{bx^3+ax^2}}{15\,b^2x}$$

```
integrate((b*x^3+a*x^2)^(1/2),x, algorithm="fricas")
```

```
2/15*(3*b^2*x^2 + a*b*x - 2*a^2)*sqrt(b*x^3 + a*x^2)/(b^2*x)
```

Sympy [F]

$$\int \sqrt{ax^2+bx^3}\,dx=\int \sqrt{ax^2+bx^3}\,dx$$

```
integrate((b*x**3+a*x**2)**(1/2),x)
```

```
Integral(sqrt(a*x**2 + b*x**3), x)
```

Maxima [A] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 30, normalized size of antiderivative = 0.58

$$\int \sqrt{ax^2+bx^3}\,dx=\frac{2\,(3\,b^2x^2+abx-2\,a^2)\sqrt{bx+a}}{15\,b^2}$$

```
integrate((b*x^3+a*x^2)^(1/2),x, algorithm="maxima")
```

```
2/15*(3*b^2*x^2 + a*b*x - 2*a^2)*sqrt(b*x + a)/b^2
```

Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 81, normalized size of antiderivative = 1.56

$$\int \sqrt{ax^2+bx^3}\,dx$$
$$=\frac{4\,a^{\frac{5}{2}}\mathrm{sgn}\left(x\right)}{15\,b^2}+\frac{2\left(\frac{5\left((bx+a)^{\frac{3}{2}}-3\,\sqrt{bx+a}a\right)a\mathrm{sgn}\left(x\right)}{b}+\frac{\left(3\,(bx+a)^{\frac{5}{2}}-10\,(bx+a)^{\frac{3}{2}}a+15\,\sqrt{bx+a}a^2\right)\mathrm{sgn}\left(x\right)}{b}\right)}{15\,b}$$

```
integrate((b*x^3+a*x^2)^(1/2),x, algorithm="giac")
```

```
4/15*a^(5/2)*sgn(x)/b^2 + 2/15*(5*((b*x + a)^(3/2) - 3*sqrt(b*x + a)*a)*a*
sgn(x)/b + (3*(b*x + a)^(5/2) - 10*(b*x + a)^(3/2)*a + 15*sqrt(b*x + a)*a^
2)*sgn(x)/b)/b
```

Mupad [B] (verification not implemented)

Time = 9.11 (sec) , antiderivative size = 39, normalized size of antiderivative = 0.75

$$\int \sqrt{ax^2+bx^3}\,dx=\frac{2\sqrt{b\,x^3+a\,x^2}\,\left(-2\,a^2+a\,b\,x+3\,b^2\,x^2\right)}{15\,b^2\,x}$$

```
int((a*x^2 + b*x^3)^(1/2),x)
```

```
(2*(a*x^2 + b*x^3)^(1/2)*(3*b^2*x^2 - 2*a^2 + a*b*x))/(15*b^2*x)
```

Reduce [B] (verification not implemented)

Time = 0.43 (sec) , antiderivative size = 29, normalized size of antiderivative = 0.56

$$\int \sqrt{ax^2+bx^3}\,dx=\frac{2\sqrt{bx+a}\,(3b^2x^2+abx-2a^2)}{15b^2}$$

```
int((b*x^3+a*x^2)^(1/2),x)
```

```
(2*sqrt(a + b*x)*( - 2*a**2 + a*b*x + 3*b**2*x**2))/(15*b**2)
```

3.269 $\int \frac{\sqrt{ax^2+bx^3}}{x}\,dx$

Optimal result

Integrand size = 19, antiderivative size = 25

$$\int \frac{\sqrt{ax^2+bx^3}}{x}\,dx = \frac{2(ax^2+bx^3)^{3/2}}{3bx^3}$$

```
2/3*(b*x^3+a*x^2)^(3/2)/b/x^3
```

Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 23, normalized size of antiderivative = 0.92

$$\int \frac{\sqrt{ax^2+bx^3}}{x}\,dx = \frac{2(x^2(a+bx))^{3/2}}{3bx^3}$$

```
Integrate[Sqrt[a*x^2 + b*x^3]/x,x]
```

```
(2*(x^2*(a + b*x))^(3/2))/(3*b*x^3)
```

Rubi [A] (verified)

Time = 0.25 (sec) , antiderivative size = 25, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.053, Rules used = {1920}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sqrt{ax^2+bx^3}}{x}\,dx$$
$$\Big\downarrow \text{1920}$$
$$\frac{2(ax^2+bx^3)^{3/2}}{3bx^3}$$

```
Int[Sqrt[a*x^2 + b*x^3]/x,x]
```

```
(2*(a*x^2 + b*x^3)^(3/2))/(3*b*x^3)
```

Defintions of rubi rules used

```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> Simp[(-c^(j - 1))*(c*x)^(m - j + 1)*((a*x^j + b*x^n)^(p + 1)/(a*(n - j
)*(p + 1))), x] /; FreeQ[{a, b, c, j, m, n, p}, x] && !IntegerQ[p] && NeQ[
n, j] && EqQ[m + n*p + n - j + 1, 0] && (IntegerQ[j] || GtQ[c, 0])
```

Maple [A] (verified)

Time = 0.39 (sec) , antiderivative size = 25, normalized size of antiderivative = 1.00

method	result	size
risch	$\frac{2\sqrt{x^2(bx+a)}\,(bx+a)}{3xb}$	25
gosper	$\frac{2(bx+a)\sqrt{b\,x^3+a\,x^2}}{3bx}$	27
default	$\frac{2(bx+a)\sqrt{b\,x^3+a\,x^2}}{3bx}$	27
trager	$\frac{2(bx+a)\sqrt{b\,x^3+a\,x^2}}{3bx}$	27
orering	$\frac{2(bx+a)\sqrt{b\,x^3+a\,x^2}}{3bx}$	27
pseudoelliptic	$2\sqrt{bx+a}-2\sqrt{a}\,\operatorname{arctanh}\left(\frac{\sqrt{bx+a}}{\sqrt{a}}\right)$	28

```
int((b*x^3+a*x^2)^(1/2)/x,x,method=_RETURNVERBOSE)
```

```
2/3*(x^2*(b*x+a))^(1/2)/x*(b*x+a)/b
```

Fricas [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 26, normalized size of antiderivative = 1.04

$$\int \frac{\sqrt{ax^2+bx^3}}{x}\,dx = \frac{2\sqrt{bx^3+ax^2}(bx+a)}{3\,bx}$$

```
integrate((b*x^3+a*x^2)^(1/2)/x,x, algorithm="fricas")
```

```
2/3*sqrt(b*x^3 + a*x^2)*(b*x + a)/(b*x)
```

Sympy [F]

$$\int \frac{\sqrt{ax^2+bx^3}}{x}\,dx = \int \frac{\sqrt{x^2\,(a+bx)}}{x}\,dx$$

```
integrate((b*x**3+a*x**2)**(1/2)/x,x)
```

```
Integral(sqrt(x**2*(a + b*x))/x, x)
```

Maxima [A] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 12, normalized size of antiderivative = 0.48

$$\int \frac{\sqrt{ax^2+bx^3}}{x}\,dx=\frac{2\,(bx+a)^{\frac{3}{2}}}{3\,b}$$

```
integrate((b*x^3+a*x^2)^(1/2)/x,x, algorithm="maxima")
```

```
2/3*(b*x + a)^(3/2)/b
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 50 vs. 2(21) = 42.

Time = 0.14 (sec) , antiderivative size = 50, normalized size of antiderivative = 2.00

$$\int \frac{\sqrt{ax^2+bx^3}}{x}\,dx=-\frac{2\,a^{\frac{3}{2}}\mathrm{sgn}(x)}{3\,b}+\frac{2\left(3\,\sqrt{bx+a}a\mathrm{sgn}(x)+\left((bx+a)^{\frac{3}{2}}-3\,\sqrt{bx+a}a\right)\mathrm{sgn}(x)\right)}{3\,b}$$

```
integrate((b*x^3+a*x^2)^(1/2)/x,x, algorithm="giac")
```

```
-2/3*a^(3/2)*sgn(x)/b + 2/3*(3*sqrt(b*x + a)*a*sgn(x) + ((b*x + a)^(3/2) -
 3*sqrt(b*x + a)*a)*sgn(x))/b
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\sqrt{ax^2+bx^3}}{x}\,dx=\int \frac{\sqrt{b\,x^3+a\,x^2}}{x}\,dx$$

```
int((a*x^2 + b*x^3)^(1/2)/x,x)
```

```
int((a*x^2 + b*x^3)^(1/2)/x, x)
```

Reduce [B] (verification not implemented)

Time = 0.52 (sec) , antiderivative size = 16, normalized size of antiderivative = 0.64

$$\int \frac{\sqrt{ax^2+bx^3}}{x}\,dx=\frac{2\sqrt{bx+a}\,(bx+a)}{3b}$$

```
int((b*x^3+a*x^2)^(1/2)/x,x)
```

```
(2*sqrt(a + b*x)*(a + b*x))/(3*b)
```

3.270 $\int \frac{\sqrt{ax^2+bx^3}}{x^2}\,dx$

Optimal result

Integrand size = 19, antiderivative size = 53

$$\int \frac{\sqrt{ax^2+bx^3}}{x^2}\,dx=\frac{2\sqrt{ax^2+bx^3}}{x}-2\sqrt{a}\operatorname{arctanh}\left(\frac{\sqrt{ax^2+bx^3}}{\sqrt{a}x}\right)$$

```
2*(b*x^3+a*x^2)^(1/2)/x-2*a^(1/2)*arctanh((b*x^3+a*x^2)^(1/2)/a^(1/2)/x)
```

Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 53, normalized size of antiderivative = 1.00

$$\int \frac{\sqrt{ax^2+bx^3}}{x^2}\,dx=\frac{2x\left(a+bx-\sqrt{a}\sqrt{a+bx}\operatorname{arctanh}\left(\frac{\sqrt{a+bx}}{\sqrt{a}}\right)\right)}{\sqrt{x^2(a+bx)}}$$

```
Integrate[Sqrt[a*x^2 + b*x^3]/x^2,x]
```

```
(2*x*(a + b*x - Sqrt[a]*Sqrt[a + b*x]*ArcTanh[Sqrt[a + b*x]/Sqrt[a]]))/Sqr
t[x^2*(a + b*x)]
```

Rubi [A] (verified)

Time = 0.32 (sec) , antiderivative size = 51, normalized size of antiderivative = 0.96, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.158$, Rules used = {1927, 1914, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sqrt{ax^2+bx^3}}{x^2}\,dx$$

$$\Big\downarrow \; 1927$$

$$a\int \frac{1}{\sqrt{bx^3+ax^2}}dx+\frac{2\sqrt{ax^2+bx^3}}{x}$$

$$\Big\downarrow \; 1914$$

$$\frac{2\sqrt{ax^2+bx^3}}{x}-2a\int \frac{1}{1-\frac{ax^2}{bx^3+ax^2}}d\frac{x}{\sqrt{bx^3+ax^2}}$$

$$\Big\downarrow \; 219$$

$$\frac{2\sqrt{ax^2+bx^3}}{x}-2\sqrt{a}\text{arctanh}\left(\frac{\sqrt{a}x}{\sqrt{ax^2+bx^3}}\right)$$

```
Int[Sqrt[a*x^2 + b*x^3]/x^2,x]
```

```
(2*Sqrt[a*x^2 + b*x^3])/x - 2*Sqrt[a]*ArcTanh[(Sqrt[a]*x)/Sqrt[a*x^2 + b*x
^3]]
```

Defintions of rubi rules used

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

```
Int[1/Sqrt[(a_.)*(x_)^2 + (b_.)*(x_)^(n_.)], x_Symbol] :> Simp[2/(2 - n)
Subst[Int[1/(1 - a*x^2), x], x, x/Sqrt[a*x^2 + b*x^n]], x] /; FreeQ[{a, b,
n}, x] && NeQ[n, 2]
```

```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> Simp[(c*x)^(m + 1)*((a*x^j + b*x^n)^p/(c*(m + n*p + 1))), x] + Simp[a*
(n - j)*(p/(c^j*(m + n*p + 1)))   Int[(c*x)^(m + j)*(a*x^j + b*x^n)^(p - 1)
, x], x] /; FreeQ[{a, b, c, m}, x] &&  !IntegerQ[p] && LtQ[0, j, n] && (Int
egersQ[j, n] || GtQ[c, 0]) && GtQ[p, 0] && NeQ[m + n*p + 1, 0]
```

Maple [A] (verified)

Time = 0.42 (sec) , antiderivative size = 36, normalized size of antiderivative = 0.68

method	result	size
pseudoelliptic	$b\left(-\frac{\sqrt{bx+a}}{bx}-\frac{\operatorname{arctanh}\left(\frac{\sqrt{bx+a}}{\sqrt{a}}\right)}{\sqrt{a}}\right)$	36
default	$\frac{2\sqrt{b\,x^{3}+a\,x^{2}}\,\left(\sqrt{bx+a}-\sqrt{a}\,\operatorname{arctanh}\left(\frac{\sqrt{bx+a}}{\sqrt{a}}\right)\right)}{x\sqrt{bx+a}}$	51

```
int((b*x^3+a*x^2)^(1/2)/x^2,x,method=_RETURNVERBOSE)
```

```
b*(-(b*x+a)^(1/2)/b/x-1/a^(1/2)*arctanh((b*x+a)^(1/2)/a^(1/2)))
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 116, normalized size of antiderivative = 2.19

$$\int \frac{\sqrt{ax^2+bx^3}}{x^2}\,dx$$
$$=\left[\frac{\sqrt{a}x\log\left(\frac{bx^2+2\,ax-2\,\sqrt{bx^3+ax^2}\sqrt{a}}{x^2}\right)+2\,\sqrt{bx^3+ax^2}}{x},\frac{2\left(\sqrt{-a}x\arctan\left(\frac{\sqrt{bx^3+ax^2}\sqrt{-a}}{bx^2+ax}\right)+\sqrt{bx^3+ax^2}\right)}{x}\right]$$

```
integrate((b*x^3+a*x^2)^(1/2)/x^2,x, algorithm="fricas")
```

```
[(sqrt(a)*x*log((b*x^2 + 2*a*x - 2*sqrt(b*x^3 + a*x^2)*sqrt(a))/x^2) + 2*s
qrt(b*x^3 + a*x^2))/x, 2*(sqrt(-a)*x*arctan(sqrt(b*x^3 + a*x^2)*sqrt(-a)/(
b*x^2 + a*x)) + sqrt(b*x^3 + a*x^2))/x]
```

Sympy [F]

$$\int \frac{\sqrt{ax^2+bx^3}}{x^2}\,dx=\int \frac{\sqrt{x^2\,(a+bx)}}{x^2}\,dx$$

```
integrate((b*x**3+a*x**2)**(1/2)/x**2,x)
```

```
Integral(sqrt(x**2*(a + b*x))/x**2, x)
```

Maxima [F]

$$\int \frac{\sqrt{ax^2+bx^3}}{x^2}\,dx=\int \frac{\sqrt{bx^3+ax^2}}{x^2}\,dx$$

```
integrate((b*x^3+a*x^2)^(1/2)/x^2,x, algorithm="maxima")
```

```
integrate(sqrt(b*x^3 + a*x^2)/x^2, x)
```

Giac [A] (verification not implemented)

Time = 0.15 (sec) , antiderivative size = 67, normalized size of antiderivative = 1.26

$$\int \frac{\sqrt{ax^2+bx^3}}{x^2}\,dx=\frac{2\,a\arctan\left(\frac{\sqrt{bx+a}}{\sqrt{-a}}\right)\operatorname{sgn}(x)}{\sqrt{-a}}+2\,\sqrt{bx+a}\operatorname{sgn}(x)-\frac{2\left(a\arctan\left(\frac{\sqrt{a}}{\sqrt{-a}}\right)+\sqrt{-a}\sqrt{a}\right)\operatorname{sgn}(x)}{\sqrt{-a}}$$

```
integrate((b*x^3+a*x^2)^(1/2)/x^2,x, algorithm="giac")
```

```
2*a*arctan(sqrt(b*x + a)/sqrt(-a))*sgn(x)/sqrt(-a) + 2*sqrt(b*x + a)*sgn(x
) - 2*(a*arctan(sqrt(a)/sqrt(-a)) + sqrt(-a)*sqrt(a))*sgn(x)/sqrt(-a)
```

Mupad [B] (verification not implemented)

Time = 9.25 (sec) , antiderivative size = 73, normalized size of antiderivative = 1.38

$$\int \frac{\sqrt{ax^2+bx^3}}{x^2}\,dx = \frac{2\sqrt{b\,x^3+a\,x^2}}{x} + \frac{\sqrt{a}\,\mathrm{asin}\left(\frac{\sqrt{a}\,\sqrt{\frac{1}{x}}\,1\mathrm{i}}{\sqrt{b}}\right)\,\sqrt{b\,x^3+a\,x^2}\,\left(\frac{1}{x}\right)^{3/2}\,2\mathrm{i}}{\sqrt{b}\,\sqrt{\frac{a}{b\,x}+1}}$$

```
int((a*x^2 + b*x^3)^(1/2)/x^2,x)
```

```
(2*(a*x^2 + b*x^3)^(1/2))/x + (a^(1/2)*asin((a^(1/2)*(1/x)^(1/2)*1i)/b^(1/
2))*(a*x^2 + b*x^3)^(1/2)*(1/x)^(3/2)*2i)/(b^(1/2)*(a/(b*x) + 1)^(1/2))
```

Reduce [B] (verification not implemented)

Time = 0.54 (sec) , antiderivative size = 38, normalized size of antiderivative = 0.72

$$\int \frac{\sqrt{ax^2+bx^3}}{x^2}\,dx = 2\sqrt{bx+a} + \sqrt{a}\log\left(\sqrt{bx+a}-\sqrt{a}\right) - \sqrt{a}\log\left(\sqrt{bx+a}+\sqrt{a}\right)$$

```
int((b*x^3+a*x^2)^(1/2)/x^2,x)
```

```
2*sqrt(a + b*x) + sqrt(a)*log(sqrt(a + b*x) - sqrt(a)) - sqrt(a)*log(sqrt(
a + b*x) + sqrt(a))
```

3.271 $\int \frac{\sqrt{ax^2+bx^3}}{x^3}\,dx$

Optimal result

Integrand size = 19, antiderivative size = 54

$$\int \frac{\sqrt{ax^2+bx^3}}{x^3}\,dx = -\frac{\sqrt{ax^2+bx^3}}{x^2} - \frac{b\operatorname{arctanh}\left(\frac{\sqrt{ax^2+bx^3}}{\sqrt{a}x}\right)}{\sqrt{a}}$$

```
-(b*x^3+a*x^2)^(1/2)/x^2-b*arctanh((b*x^3+a*x^2)^(1/2)/a^(1/2)/x)/a^(1/2)
```

Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 64, normalized size of antiderivative = 1.19

$$\int \frac{\sqrt{ax^2+bx^3}}{x^3}\,dx = -\frac{\sqrt{a+bx}\left(\sqrt{a}\sqrt{a+bx} + bx\operatorname{arctanh}\left(\frac{\sqrt{a+bx}}{\sqrt{a}}\right)\right)}{\sqrt{a}\sqrt{x^2(a+bx)}}$$

```
Integrate[Sqrt[a*x^2 + b*x^3]/x^3,x]
```

```
-((Sqrt[a + b*x]*(Sqrt[a]*Sqrt[a + b*x] + b*x*ArcTanh[Sqrt[a + b*x]/Sqrt[a
]]))/(Sqrt[a]*Sqrt[x^2*(a + b*x)]))
```

Rubi [A] (verified)

Time = 0.32 (sec) , antiderivative size = 52, normalized size of antiderivative = 0.96, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.158, Rules used = {1926, 1914, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sqrt{ax^2+bx^3}}{x^3} dx$$

$$\Big\downarrow \text{1926}$$

$$\frac{1}{2}b\int \frac{1}{\sqrt{bx^3+ax^2}}dx-\frac{\sqrt{ax^2+bx^3}}{x^2}$$

$$\Big\downarrow \text{1914}$$

$$-b\int \frac{1}{1-\frac{ax^2}{bx^3+ax^2}}d\frac{x}{\sqrt{bx^3+ax^2}}-\frac{\sqrt{ax^2+bx^3}}{x^2}$$

$$\Big\downarrow \text{219}$$

$$-\frac{b\text{arctanh}\left(\frac{\sqrt{a}x}{\sqrt{ax^2+bx^3}}\right)}{\sqrt{a}}-\frac{\sqrt{ax^2+bx^3}}{x^2}$$

```
Int[Sqrt[a*x^2 + b*x^3]/x^3,x]
```

```
-(Sqrt[a*x^2 + b*x^3]/x^2) - (b*ArcTanh[(Sqrt[a]*x)/Sqrt[a*x^2 + b*x^3]])/
Sqrt[a]
```

Defintions of rubi rules used

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

```
Int[1/Sqrt[(a_.)*(x_)^2 + (b_.)*(x_)^(n_.)], x_Symbol] :> Simp[2/(2 - n)
Subst[Int[1/(1 - a*x^2), x], x, x/Sqrt[a*x^2 + b*x^n]], x] /; FreeQ[{a, b,
n}, x] && NeQ[n, 2]
```

```
Int[((c_.)*(x_))^(m_)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol]
 :> Simp[(c*x)^(m + 1)*((a*x^j + b*x^n)^p/(c*(m + j*p + 1))), x] - Simp[b*p
*((n - j)/(c^n*(m + j*p + 1)))   Int[(c*x)^(m + n)*(a*x^j + b*x^n)^(p - 1),
 x], x] /; FreeQ[{a, b, c}, x] && !IntegerQ[p] && LtQ[0, j, n] && (Integer
sQ[j, n] || GtQ[c, 0]) && GtQ[p, 0] && LtQ[m + j*p + 1, 0]
```

Maple [A] (verified)

Time = 0.45 (sec) , antiderivative size = 50, normalized size of antiderivative = 0.93

method	result	size
pseudoelliptic	$\frac{\operatorname{arctanh}\left(\frac{\sqrt{bx+a}}{\sqrt{a}}\right) b^{2} x^{2}-\left(2 a^{\frac{3}{2}}+b x \sqrt{a}\right) \sqrt{bx+a}}{4 a^{\frac{3}{2}} x^{2}}$	50
default	$-\frac{\sqrt{b\,x^{3}+a\,x^{2}}\,\left(\operatorname{arctanh}\left(\frac{\sqrt{bx+a}}{\sqrt{a}}\right) bx+\sqrt{bx+a}\,\sqrt{a}\right)}{x^{2} \sqrt{bx+a}\,\sqrt{a}}$	56
risch	$-\frac{\sqrt{x^{2}(bx+a)}}{x^{2}}-\frac{b \operatorname{arctanh}\left(\frac{\sqrt{bx+a}}{\sqrt{a}}\right) \sqrt{x^{2}(bx+a)}}{\sqrt{a}\,x \sqrt{bx+a}}$	57

```
int((b*x^3+a*x^2)^(1/2)/x^3,x,method=_RETURNVERBOSE)
```

```
1/4*(arctanh((b*x+a)^(1/2)/a^(1/2))*b^2*x^2-(2*a^(3/2)+b*x*a^(1/2))*(b*x+a
)^(1/2))/a^(3/2)/x^2
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 132, normalized size of antiderivative = 2.44

$$\int \frac{\sqrt{ax^2+bx^3}}{x^3}\,dx$$
$$=\left[\frac{\sqrt{a}bx^{2}\log\left(\frac{bx^{2}+2\,ax-2\,\sqrt{bx^{3}+ax^{2}}\sqrt{a}}{x^{2}}\right)-2\,\sqrt{bx^{3}+ax^{2}}a}{2\,ax^{2}},\ \frac{\sqrt{-a}bx^{2}\arctan\left(\frac{\sqrt{bx^{3}+ax^{2}}\sqrt{-a}}{bx^{2}+ax}\right)-\sqrt{bx^{3}+ax^{2}}a}{ax^{2}}\right]$$

```
integrate((b*x^3+a*x^2)^(1/2)/x^3,x, algorithm="fricas")
```

```
[1/2*(sqrt(a)*b*x^2*log((b*x^2 + 2*a*x - 2*sqrt(b*x^3 + a*x^2)*sqrt(a))/x^
2) - 2*sqrt(b*x^3 + a*x^2)*a)/(a*x^2), (sqrt(-a)*b*x^2*arctan(sqrt(b*x^3 +
 a*x^2)*sqrt(-a)/(b*x^2 + a*x)) - sqrt(b*x^3 + a*x^2)*a)/(a*x^2)]
```

Sympy [F]

$$\int \frac{\sqrt{ax^2 + bx^3}}{x^3}\,dx = \int \frac{\sqrt{x^2 (a + bx)}}{x^3}\,dx$$

```
integrate((b*x**3+a*x**2)**(1/2)/x**3,x)
```

```
Integral(sqrt(x**2*(a + b*x))/x**3, x)
```

Maxima [F]

$$\int \frac{\sqrt{ax^2 + bx^3}}{x^3}\,dx = \int \frac{\sqrt{bx^3 + ax^2}}{x^3}\,dx$$

```
integrate((b*x^3+a*x^2)^(1/2)/x^3,x, algorithm="maxima")
```

```
integrate(sqrt(b*x^3 + a*x^2)/x^3, x)
```

Giac [A] (verification not implemented)

Time = 0.21 (sec) , antiderivative size = 42, normalized size of antiderivative = 0.78

$$\int \frac{\sqrt{ax^2+bx^3}}{x^3}\,dx = \left(\frac{\arctan\left(\frac{\sqrt{bx+a}}{\sqrt{-a}}\right)\operatorname{sgn}(x)}{\sqrt{-a}} - \frac{\sqrt{bx+a}\operatorname{sgn}(x)}{bx}\right)b$$

```
integrate((b*x^3+a*x^2)^(1/2)/x^3,x, algorithm="giac")
```

```
(arctan(sqrt(b*x + a)/sqrt(-a))*sgn(x)/sqrt(-a) - sqrt(b*x + a)*sgn(x)/(b*
x))*b
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\sqrt{ax^2+bx^3}}{x^3}\,dx = \int \frac{\sqrt{b\,x^3+a\,x^2}}{x^3}\,dx$$

```
int((a*x^2 + b*x^3)^(1/2)/x^3,x)
```

```
int((a*x^2 + b*x^3)^(1/2)/x^3, x)
```

Reduce [B] (verification not implemented)

Time = 0.30 (sec) , antiderivative size = 51, normalized size of antiderivative = 0.94

$$\int \frac{\sqrt{ax^2+bx^3}}{x^3}\,dx = \frac{-2\sqrt{bx+a}\,a+\sqrt{a}\log\left(\sqrt{bx+a}-\sqrt{a}\right)bx-\sqrt{a}\log\left(\sqrt{bx+a}+\sqrt{a}\right)bx}{2ax}$$

```
int((b*x^3+a*x^2)^(1/2)/x^3,x)
```

```
( - 2*sqrt(a + b*x)*a + sqrt(a)*log(sqrt(a + b*x) - sqrt(a))*b*x - sqrt(a)
*log(sqrt(a + b*x) + sqrt(a))*b*x)/(2*a*x)
```

3.272 $\int \frac{\sqrt{ax^2+bx^3}}{x^4} dx$

Optimal result

Integrand size = 19, antiderivative size = 86

$$\int \frac{\sqrt{ax^2+bx^3}}{x^4} dx = -\frac{\sqrt{ax^2+bx^3}}{2x^3} - \frac{b\sqrt{ax^2+bx^3}}{4ax^2} + \frac{b^2 \operatorname{arctanh}\left(\frac{\sqrt{ax^2+bx^3}}{\sqrt{a}x}\right)}{4a^{3/2}}$$

```
-1/2*(b*x^3+a*x^2)^(1/2)/x^3-1/4*b*(b*x^3+a*x^2)^(1/2)/a/x^2+1/4*b^2*arcta
nh((b*x^3+a*x^2)^(1/2)/a^(1/2)/x)/a^(3/2)
```

Mathematica [A] (verified)

Time = 0.03 (sec) , antiderivative size = 81, normalized size of antiderivative = 0.94

$$\int \frac{\sqrt{ax^2+bx^3}}{x^4} dx = \frac{\sqrt{x^2(a+bx)}\left(-\sqrt{a}\sqrt{a+bx}(2a+bx)+b^2x^2 \operatorname{arctanh}\left(\frac{\sqrt{a+bx}}{\sqrt{a}}\right)\right)}{4a^{3/2}x^3\sqrt{a+bx}}$$

```
Integrate[Sqrt[a*x^2 + b*x^3]/x^4,x]
```

```
(Sqrt[x^2*(a + b*x)]*(-(Sqrt[a]*Sqrt[a + b*x]*(2*a + b*x)) + b^2*x^2*ArcTa
nh[Sqrt[a + b*x]/Sqrt[a]]))/(4*a^(3/2)*x^3*Sqrt[a + b*x])
```

Rubi [A] (verified)

Time = 0.38 (sec) , antiderivative size = 82, normalized size of antiderivative = 0.95, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.211, Rules used = {1926, 1931, 1914, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sqrt{ax^2+bx^3}}{x^4}dx$$

$$\Big\downarrow \text{1926}$$

$$\frac{1}{4}b\int \frac{1}{x\sqrt{bx^3+ax^2}}dx-\frac{\sqrt{ax^2+bx^3}}{2x^3}$$

$$\Big\downarrow \text{1931}$$

$$\frac{1}{4}b\left(-\frac{b\int \frac{1}{\sqrt{bx^3+ax^2}}dx}{2a}-\frac{\sqrt{ax^2+bx^3}}{ax^2}\right)-\frac{\sqrt{ax^2+bx^3}}{2x^3}$$

$$\Big\downarrow \text{1914}$$

$$\frac{1}{4}b\left(\frac{b\int \frac{1}{1-\frac{ax^2}{bx^3+ax^2}}d\frac{x}{\sqrt{bx^3+ax^2}}}{a}-\frac{\sqrt{ax^2+bx^3}}{ax^2}\right)-\frac{\sqrt{ax^2+bx^3}}{2x^3}$$

$$\Big\downarrow \text{219}$$

$$\frac{1}{4}b\left(\frac{b\operatorname{arctanh}\left(\frac{\sqrt{a}x}{\sqrt{ax^2+bx^3}}\right)}{a^{3/2}}-\frac{\sqrt{ax^2+bx^3}}{ax^2}\right)-\frac{\sqrt{ax^2+bx^3}}{2x^3}$$

```
Int[Sqrt[a*x^2 + b*x^3]/x^4,x]
```

```
-1/2*Sqrt[a*x^2 + b*x^3]/x^3 + (b*(-(Sqrt[a*x^2 + b*x^3]/(a*x^2)) + (b*Arc
Tanh[(Sqrt[a]*x)/Sqrt[a*x^2 + b*x^3]])/a^(3/2)))/4
```

Defintions of rubi rules used

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

```
Int[1/Sqrt[(a_.)*(x_)^2 + (b_.)*(x_)^(n_.)], x_Symbol] :> Simp[2/(2 - n)
Subst[Int[1/(1 - a*x^2), x], x, x/Sqrt[a*x^2 + b*x^n]], x] /; FreeQ[{a, b,
n}, x] && NeQ[n, 2]
```

```
Int[((c_.)*(x_))^(m_)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol]
 :> Simp[(c*x)^(m + 1)*((a*x^j + b*x^n)^p/(c*(m + j*p + 1))), x] - Simp[b*p
*((n - j)/(c^n*(m + j*p + 1)))   Int[(c*x)^(m + n)*(a*x^j + b*x^n)^(p - 1),
 x], x] /; FreeQ[{a, b, c}, x] &&  !IntegerQ[p] && LtQ[0, j, n] && (Integer
sQ[j, n] || GtQ[c, 0]) && GtQ[p, 0] && LtQ[m + j*p + 1, 0]
```

```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> Simp[c^(j - 1)*(c*x)^(m - j + 1)*((a*x^j + b*x^n)^(p + 1)/(a*(m + j*p
+ 1))), x] - Simp[b*((m + n*p + n - j + 1)/(a*c^(n - j)*(m + j*p + 1)))   I
nt[(c*x)^(m + n - j)*(a*x^j + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, p}, x]
 &&  !IntegerQ[p] && LtQ[0, j, n] && (IntegersQ[j, n] || GtQ[c, 0]) && LtQ[
m + j*p + 1, 0]
```

Maple [A] (verified)

Time = 0.46 (sec) , antiderivative size = 61, normalized size of antiderivative = 0.71

method	result	size
pseudoelliptic	$-\frac{\operatorname{arctanh}\left(\frac{\sqrt{bx+a}}{\sqrt{a}}\right) b^{3} x^{3}-\sqrt{bx+a}\,\left(\sqrt{a}\, b^{2} x^{2}-\frac{2 a^{\frac{3}{2}} b x}{3}-\frac{8 a^{\frac{5}{2}}}{3}\right)}{8 a^{\frac{5}{2}} x^{3}}$	61
risch	$-\frac{(bx+2a)\sqrt{x^{2}(bx+a)}}{4x^{3}a}+\frac{b^{2}\operatorname{arctanh}\left(\frac{\sqrt{bx+a}}{\sqrt{a}}\right)\sqrt{x^{2}(bx+a)}}{4a^{\frac{3}{2}}x\sqrt{bx+a}}$	69
default	$-\frac{\sqrt{b\,x^{3}+a\,x^{2}}\,\left((bx+a)^{\frac{3}{2}} a^{\frac{3}{2}}-\operatorname{arctanh}\left(\frac{\sqrt{bx+a}}{\sqrt{a}}\right) a\, b^{2} x^{2}+\sqrt{bx+a}\, a^{\frac{5}{2}}\right)}{4x^{3}\sqrt{bx+a}\, a^{\frac{5}{2}}}$	73

```
int((b*x^3+a*x^2)^(1/2)/x^4,x,method=_RETURNVERBOSE)
```

```
-1/8/a^(5/2)*(arctanh((b*x+a)^(1/2)/a^(1/2))*b^3*x^3-(b*x+a)^(1/2)*(a^(1/2
)*b^2*x^2-2/3*a^(3/2)*b*x-8/3*a^(5/2)))/x^3
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 154, normalized size of antiderivative = 1.79

$$\int \frac{\sqrt{ax^2+bx^3}}{x^4}\,dx = \left[\frac{\sqrt{a}b^2x^3\log\left(\frac{bx^2+2\,ax+2\,\sqrt{bx^3+ax^2}\sqrt{a}}{x^2}\right)-2\,\sqrt{bx^3+ax^2}(abx+2\,a^2)}{8\,a^2x^3},\ -\frac{\sqrt{-a}b^2x^3\arctan\left(\frac{\sqrt{bx^3+ax^2}\sqrt{-a}}{bx^2+ax}\right)+\sqrt{bx^3+ax^2}(abx+2\,a^2)}{4\,a^2x^3}\right]$$

```
integrate((b*x^3+a*x^2)^(1/2)/x^4,x, algorithm="fricas")
```

```
[1/8*(sqrt(a)*b^2*x^3*log((b*x^2 + 2*a*x + 2*sqrt(b*x^3 + a*x^2)*sqrt(a))/
x^2) - 2*sqrt(b*x^3 + a*x^2)*(a*b*x + 2*a^2))/(a^2*x^3), -1/4*(sqrt(-a)*b^
2*x^3*arctan(sqrt(b*x^3 + a*x^2)*sqrt(-a)/(b*x^2 + a*x)) + sqrt(b*x^3 + a*
x^2)*(a*b*x + 2*a^2))/(a^2*x^3)]
```

Sympy [F]

$$\int \frac{\sqrt{ax^2+bx^3}}{x^4}\,dx = \int \frac{\sqrt{x^2\,(a+bx)}}{x^4}\,dx$$

```
integrate((b*x**3+a*x**2)**(1/2)/x**4,x)
```

```
Integral(sqrt(x**2*(a + b*x))/x**4, x)
```

Maxima [F]

$$\int \frac{\sqrt{ax^2+bx^3}}{x^4}\,dx=\int \frac{\sqrt{bx^3+ax^2}}{x^4}\,dx$$

```
integrate((b*x^3+a*x^2)^(1/2)/x^4,x, algorithm="maxima")
```

```
integrate(sqrt(b*x^3 + a*x^2)/x^4, x)
```

Giac [A] (verification not implemented)

Time = 0.21 (sec) , antiderivative size = 72, normalized size of antiderivative = 0.84

$$\int \frac{\sqrt{ax^2+bx^3}}{x^4}\,dx=-\frac{\frac{b^3\arctan\left(\frac{\sqrt{bx+a}}{\sqrt{-a}}\right)\operatorname{sgn}(x)}{\sqrt{-a}a}+\frac{(bx+a)^{\frac{3}{2}}b^3\operatorname{sgn}(x)+\sqrt{bx+a}ab^3\operatorname{sgn}(x)}{ab^2x^2}}{4\,b}$$

```
integrate((b*x^3+a*x^2)^(1/2)/x^4,x, algorithm="giac")
```

```
-1/4*(b^3*arctan(sqrt(b*x + a)/sqrt(-a))*sgn(x)/(sqrt(-a)*a) + ((b*x + a)^
(3/2)*b^3*sgn(x) + sqrt(b*x + a)*a*b^3*sgn(x))/(a*b^2*x^2))/b
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\sqrt{ax^2+bx^3}}{x^4}\,dx=\int \frac{\sqrt{b\,x^3+a\,x^2}}{x^4}\,dx$$

```
int((a*x^2 + b*x^3)^(1/2)/x^4,x)
```

```
int((a*x^2 + b*x^3)^(1/2)/x^4, x)
```

Reduce [B] (verification not implemented)

Time = 0.54 (sec) , antiderivative size = 72, normalized size of antiderivative = 0.84

$$\int \frac{\sqrt{a x^2 + b x^3}}{x^4} \, dx$$
$$= \frac{-4\sqrt{bx+a}\,a^2 - 2\sqrt{bx+a}\,abx - \sqrt{a}\log\left(\sqrt{bx+a} - \sqrt{a}\right) b^2x^2 + \sqrt{a}\log\left(\sqrt{bx+a} + \sqrt{a}\right) b^2x^2}{8a^2x^2}$$

```
int((b*x^3+a*x^2)^(1/2)/x^4,x)
```

```
( - 4*sqrt(a + b*x)*a**2 - 2*sqrt(a + b*x)*a*b*x - sqrt(a)*log(sqrt(a + b*
x) - sqrt(a))*b**2*x**2 + sqrt(a)*log(sqrt(a + b*x) + sqrt(a))*b**2*x**2)/
(8*a**2*x**2)
```

3.273 $\int \frac{\sqrt{ax^2+bx^3}}{x^5}\,dx$

Optimal result

Integrand size = 19, antiderivative size = 114

$$\int \frac{\sqrt{ax^2+bx^3}}{x^5}\,dx=-\frac{\sqrt{ax^2+bx^3}}{3x^4}-\frac{b\sqrt{ax^2+bx^3}}{12ax^3}+\frac{b^2\sqrt{ax^2+bx^3}}{8a^2x^2}-\frac{b^3\operatorname{arctanh}\left(\frac{\sqrt{ax^2+bx^3}}{\sqrt{a}x}\right)}{8a^{5/2}}$$

```
-1/3*(b*x^3+a*x^2)^(1/2)/x^4-1/12*b*(b*x^3+a*x^2)^(1/2)/a/x^3+1/8*b^2*(b*x
^3+a*x^2)^(1/2)/a^2/x^2-1/8*b^3*arctanh((b*x^3+a*x^2)^(1/2)/a^(1/2)/x)/a^(
5/2)
```

Mathematica [A] (verified)

Time = 0.03 (sec) , antiderivative size = 93, normalized size of antiderivative = 0.82

$$\int \frac{\sqrt{ax^2+bx^3}}{x^5}\,dx = -\frac{\sqrt{x^2(a+bx)}\left(\sqrt{a}\sqrt{a+bx}(8a^2+2abx-3b^2x^2)+3b^3x^3\operatorname{arctanh}\left(\frac{\sqrt{a+bx}}{\sqrt{a}}\right)\right)}{24a^{5/2}x^4\sqrt{a+bx}}$$

```
Integrate[Sqrt[a*x^2 + b*x^3]/x^5,x]
```

```
-1/24*(Sqrt[x^2*(a + b*x)]*(Sqrt[a]*Sqrt[a + b*x]*(8*a^2 + 2*a*b*x - 3*b^2
*x^2) + 3*b^3*x^3*ArcTanh[Sqrt[a + b*x]/Sqrt[a]]))/(a^(5/2)*x^4*Sqrt[a + b
*x])
```

Rubi [A] (verified)

Time = 0.48 (sec) , antiderivative size = 116, normalized size of antiderivative = 1.02, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.263$, Rules used = {1926, 1931, 1931, 1914, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sqrt{ax^2+bx^3}}{x^5}dx$$

$$\Big\downarrow \text{1926}$$

$$\frac{1}{6}b\int \frac{1}{x^2\sqrt{bx^3+ax^2}}dx-\frac{\sqrt{ax^2+bx^3}}{3x^4}$$

$$\Big\downarrow \text{1931}$$

$$\frac{1}{6}b\left(-\frac{3b\int \frac{1}{x\sqrt{bx^3+ax^2}}dx}{4a}-\frac{\sqrt{ax^2+bx^3}}{2ax^3}\right)-\frac{\sqrt{ax^2+bx^3}}{3x^4}$$

$$\Big\downarrow \text{1931}$$

$$\frac{1}{6}b\left(-\frac{3b\left(-\frac{b\int \frac{1}{\sqrt{bx^3+ax^2}}dx}{2a}-\frac{\sqrt{ax^2+bx^3}}{ax^2}\right)}{4a}-\frac{\sqrt{ax^2+bx^3}}{2ax^3}\right)-\frac{\sqrt{ax^2+bx^3}}{3x^4}$$

$$\Big\downarrow \text{1914}$$

$$\frac{1}{6}b\left(-\frac{3b\left(\frac{b\int \frac{1}{1-\frac{ax^2}{bx^3+ax^2}}d\frac{x}{\sqrt{bx^3+ax^2}}}{a}-\frac{\sqrt{ax^2+bx^3}}{ax^2}\right)}{4a}-\frac{\sqrt{ax^2+bx^3}}{2ax^3}\right)-\frac{\sqrt{ax^2+bx^3}}{3x^4}$$

$$\Big\downarrow \text{219}$$

$$\frac{1}{6}b\left(-\frac{3b\left(\frac{b\operatorname{arctanh}\left(\frac{\sqrt{a}x}{\sqrt{ax^2+bx^3}}\right)}{a^{3/2}}-\frac{\sqrt{ax^2+bx^3}}{ax^2}\right)}{4a}-\frac{\sqrt{ax^2+bx^3}}{2ax^3}\right)-\frac{\sqrt{ax^2+bx^3}}{3x^4}$$

```
Int[Sqrt[a*x^2 + b*x^3]/x^5,x]
```

```
-1/3*Sqrt[a*x^2 + b*x^3]/x^4 + (b*(-1/2*Sqrt[a*x^2 + b*x^3]/(a*x^3) - (3*b
*(-(Sqrt[a*x^2 + b*x^3]/(a*x^2)) + (b*ArcTanh[(Sqrt[a]*x)/Sqrt[a*x^2 + b*x
^3]])/a^(3/2)))/(4*a)))/6
```

Defintions of rubi rules used

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

```
Int[1/Sqrt[(a_.)*(x_)^2 + (b_.)*(x_)^(n_.)], x_Symbol] :> Simp[2/(2 - n)
Subst[Int[1/(1 - a*x^2), x], x, x/Sqrt[a*x^2 + b*x^n]], x] /; FreeQ[{a, b,
n}, x] && NeQ[n, 2]
```

```
Int[((c_.)*(x_))^(m_)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol]
 :> Simp[(c*x)^(m + 1)*((a*x^j + b*x^n)^p/(c*(m + j*p + 1))), x] - Simp[b*p
*((n - j)/(c^n*(m + j*p + 1)))   Int[(c*x)^(m + n)*(a*x^j + b*x^n)^(p - 1),
 x], x] /; FreeQ[{a, b, c}, x] &&  !IntegerQ[p] && LtQ[0, j, n] && (Integer
sQ[j, n] || GtQ[c, 0]) && GtQ[p, 0] && LtQ[m + j*p + 1, 0]
```

```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> Simp[c^(j - 1)*(c*x)^(m - j + 1)*((a*x^j + b*x^n)^(p + 1)/(a*(m + j*p
+ 1))), x] - Simp[b*((m + n*p + n - j + 1)/(a*c^(n - j)*(m + j*p + 1)))   I
nt[(c*x)^(m + n - j)*(a*x^j + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, p}, x]
 &&  !IntegerQ[p] && LtQ[0, j, n] && (IntegersQ[j, n] || GtQ[c, 0]) && LtQ[
m + j*p + 1, 0]
```

Maple [A] (verified)

Time = 0.49 (sec) , antiderivative size = 72, normalized size of antiderivative = 0.63

method	result	size
pseudoelliptic	$-\frac{5\left(-\operatorname{arctanh}\left(\frac{\sqrt{bx+a}}{\sqrt{a}}\right)b^4x^4+\sqrt{bx+a}\left(\sqrt{a}\,b^3x^3-\frac{2a^{\frac{3}{2}}b^2x^2}{3}+\frac{8a^{\frac{5}{2}}bx}{15}+\frac{16a^{\frac{7}{2}}}{5}\right)\right)}{64a^{\frac{7}{2}}x^4}$	72
risch	$-\frac{\left(-3b^2x^2+2abx+8a^2\right)\sqrt{x^2(bx+a)}}{24x^4a^2}-\frac{b^3\operatorname{arctanh}\left(\frac{\sqrt{bx+a}}{\sqrt{a}}\right)\sqrt{x^2(bx+a)}}{8a^{\frac{5}{2}}x\sqrt{bx+a}}$	81
default	$\frac{\sqrt{b\,x^3+a\,x^2}\left(3(bx+a)^{\frac{5}{2}}a^{\frac{5}{2}}-8(bx+a)^{\frac{3}{2}}a^{\frac{7}{2}}-3\operatorname{arctanh}\left(\frac{\sqrt{bx+a}}{\sqrt{a}}\right)a^2b^3x^3-3\sqrt{bx+a}\,a^{\frac{9}{2}}\right)}{24x^4\sqrt{bx+a}\,a^{\frac{9}{2}}}$	89

```
int((b*x^3+a*x^2)^(1/2)/x^5,x,method=_RETURNVERBOSE)
```

```
-5/64/a^(7/2)*(-arctanh((b*x+a)^(1/2)/a^(1/2))*b^4*x^4+(b*x+a)^(1/2)*(a^(1
/2)*b^3*x^3-2/3*a^(3/2)*b^2*x^2+8/15*a^(5/2)*b*x+16/5*a^(7/2)))/x^4
```

Fricas [A] (verification not implemented)

Time = 0.10 (sec) , antiderivative size = 180, normalized size of antiderivative = 1.58

$$\int \frac{\sqrt{ax^2+bx^3}}{x^5}\,dx$$
$$=\left[\frac{3\sqrt{a}b^3x^4\log\left(\frac{bx^2+2\,ax-2\,\sqrt{bx^3+ax^2}\sqrt{a}}{x^2}\right)+2\left(3\,ab^2x^2-2\,a^2bx-8\,a^3\right)\sqrt{bx^3+ax^2}}{48\,a^3x^4},\frac{3\sqrt{-a}b^3x^4\arctan\left(\frac{\sqrt{bx^3}}{b}\right.}{}\right.$$

```
integrate((b*x^3+a*x^2)^(1/2)/x^5,x, algorithm="fricas")
```

```
[1/48*(3*sqrt(a)*b^3*x^4*log((b*x^2 + 2*a*x - 2*sqrt(b*x^3 + a*x^2)*sqrt(a
))/x^2) + 2*(3*a*b^2*x^2 - 2*a^2*b*x - 8*a^3)*sqrt(b*x^3 + a*x^2))/(a^3*x^
4), 1/24*(3*sqrt(-a)*b^3*x^4*arctan(sqrt(b*x^3 + a*x^2)*sqrt(-a)/(b*x^2 +
a*x)) + (3*a*b^2*x^2 - 2*a^2*b*x - 8*a^3)*sqrt(b*x^3 + a*x^2))/(a^3*x^4)]
```

Sympy [F]

$$\int \frac{\sqrt{ax^2+bx^3}}{x^5}\,dx = \int \frac{\sqrt{x^2\,(a+bx)}}{x^5}\,dx$$

```
integrate((b*x**3+a*x**2)**(1/2)/x**5,x)
```

```
Integral(sqrt(x**2*(a + b*x))/x**5, x)
```

Maxima [F]

$$\int \frac{\sqrt{ax^2+bx^3}}{x^5}\,dx = \int \frac{\sqrt{bx^3+ax^2}}{x^5}\,dx$$

```
integrate((b*x^3+a*x^2)^(1/2)/x^5,x, algorithm="maxima")
```

```
integrate(sqrt(b*x^3 + a*x^2)/x^5, x)
```

Giac [A] (verification not implemented)

Time = 0.23 (sec) , antiderivative size = 80, normalized size of antiderivative = 0.70

$$\int \frac{\sqrt{ax^2+bx^3}}{x^5}\,dx$$
$$= \frac{1}{24}\,b^3\left(\frac{3\,\arctan\left(\frac{\sqrt{bx+a}}{\sqrt{-a}}\right)\mathrm{sgn}(x)}{\sqrt{-a}a^2} + \frac{3\,(bx+a)^{\frac{5}{2}}\mathrm{sgn}(x) - 8\,(bx+a)^{\frac{3}{2}}a\mathrm{sgn}(x) - 3\,\sqrt{bx+a}a^2\mathrm{sgn}(x)}{a^2b^3x^3}\right)$$

```
integrate((b*x^3+a*x^2)^(1/2)/x^5,x, algorithm="giac")
```

```
1/24*b^3*(3*arctan(sqrt(b*x + a)/sqrt(-a))*sgn(x)/(sqrt(-a)*a^2) + (3*(b*x
 + a)^(5/2)*sgn(x) - 8*(b*x + a)^(3/2)*a*sgn(x) - 3*sqrt(b*x + a)*a^2*sgn(
x))/(a^2*b^3*x^3))
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\sqrt{ax^2 + bx^3}}{x^5} \, dx = \int \frac{\sqrt{b\,x^3 + a\,x^2}}{x^5} \, dx$$

```
int((a*x^2 + b*x^3)^(1/2)/x^5,x)
```

```
int((a*x^2 + b*x^3)^(1/2)/x^5, x)
```

Reduce [B] (verification not implemented)

Time = 0.52 (sec) , antiderivative size = 90, normalized size of antiderivative = 0.79

$$\int \frac{\sqrt{ax^2 + bx^3}}{x^5} \, dx$$
$$= \frac{-16\sqrt{bx+a}\,a^3 - 4\sqrt{bx+a}\,a^2bx + 6\sqrt{bx+a}\,a\,b^2x^2 + 3\sqrt{a}\log\left(\sqrt{bx+a} - \sqrt{a}\right)b^3x^3 - 3\sqrt{a}\log\left(\sqrt{bx+\right.}{48a^3x^3}$$

```
int((b*x^3+a*x^2)^(1/2)/x^5,x)
```

```
( - 16*sqrt(a + b*x)*a**3 - 4*sqrt(a + b*x)*a**2*b*x + 6*sqrt(a + b*x)*a*b
**2*x**2 + 3*sqrt(a)*log(sqrt(a + b*x) - sqrt(a))*b**3*x**3 - 3*sqrt(a)*lo
g(sqrt(a + b*x) + sqrt(a))*b**3*x**3)/(48*a**3*x**3)
```

3.274 $\int \frac{\sqrt{ax^2+bx^3}}{x^6}\,dx$

Optimal result

Integrand size = 19, antiderivative size = 142

$$\int \frac{\sqrt{ax^2+bx^3}}{x^6}\,dx=-\frac{\sqrt{ax^2+bx^3}}{4x^5}-\frac{b\sqrt{ax^2+bx^3}}{24ax^4}+\frac{5b^2\sqrt{ax^2+bx^3}}{96a^2x^3}-\frac{5b^3\sqrt{ax^2+bx^3}}{64a^3x^2}+\frac{5b^4\text{arctanh}\left(\frac{\sqrt{ax^2+bx^3}}{\sqrt{a}x}\right)}{64a^{7/2}}$$

```
-1/4*(b*x^3+a*x^2)^(1/2)/x^5-1/24*b*(b*x^3+a*x^2)^(1/2)/a/x^4+5/96*b^2*(b*
x^3+a*x^2)^(1/2)/a^2/x^3-5/64*b^3*(b*x^3+a*x^2)^(1/2)/a^3/x^2+5/64*b^4*arc
tanh((b*x^3+a*x^2)^(1/2)/a^(1/2)/x)/a^(7/2)
```

Mathematica [A] (verified)

Time = 0.05 (sec) , antiderivative size = 105, normalized size of antiderivative = 0.74

$$\int \frac{\sqrt{ax^2+bx^3}}{x^6}\,dx=\frac{\sqrt{x^2(a+bx)}\left(-\sqrt{a}\sqrt{a+bx}(48a^3+8a^2bx-10ab^2x^2+15b^3x^3)+15b^4x^4\text{arctanh}\left(\frac{\sqrt{a+bx}}{\sqrt{a}}\right)\right)}{192a^{7/2}x^5\sqrt{a+bx}}$$

```
Integrate[Sqrt[a*x^2 + b*x^3]/x^6,x]
```

```
(Sqrt[x^2*(a + b*x)]*(-(Sqrt[a]*Sqrt[a + b*x]*(48*a^3 + 8*a^2*b*x - 10*a*b
^2*x^2 + 15*b^3*x^3)) + 15*b^4*x^4*ArcTanh[Sqrt[a + b*x]/Sqrt[a]]))/(192*a
^(7/2)*x^5*Sqrt[a + b*x])
```

Rubi [A] (verified)

Time = 0.55 (sec) , antiderivative size = 150, normalized size of antiderivative = 1.06, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.316$, Rules used = {1926, 1931, 1931, 1931, 1914, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sqrt{ax^2+bx^3}}{x^6}dx$$

$$\Big\downarrow \text{1926}$$

$$\frac{1}{8}b\int \frac{1}{x^3\sqrt{bx^3+ax^2}}dx-\frac{\sqrt{ax^2+bx^3}}{4x^5}$$

$$\Big\downarrow \text{1931}$$

$$\frac{1}{8}b\left(-\frac{5b\int \frac{1}{x^2\sqrt{bx^3+ax^2}}dx}{6a}-\frac{\sqrt{ax^2+bx^3}}{3ax^4}\right)-\frac{\sqrt{ax^2+bx^3}}{4x^5}$$

$$\Big\downarrow \text{1931}$$

$$\frac{1}{8}b\left(-\frac{5b\left(-\frac{3b\int \frac{1}{x\sqrt{bx^3+ax^2}}dx}{4a}-\frac{\sqrt{ax^2+bx^3}}{2ax^3}\right)}{6a}-\frac{\sqrt{ax^2+bx^3}}{3ax^4}\right)-\frac{\sqrt{ax^2+bx^3}}{4x^5}$$

$$\Big\downarrow \text{1931}$$

$$\frac{1}{8}b\left(-\frac{5b\left(-\frac{3b\left(-\frac{b\int \frac{1}{\sqrt{bx^3+ax^2}}dx}{2a}-\frac{\sqrt{ax^2+bx^3}}{ax^2}\right)}{4a}-\frac{\sqrt{ax^2+bx^3}}{2ax^3}\right)}{6a}-\frac{\sqrt{ax^2+bx^3}}{3ax^4}\right)-\frac{\sqrt{ax^2+bx^3}}{4x^5}$$

$$\Big\downarrow \textcolor{blue}{1914}$$

$$\frac{1}{8}b\left(-\frac{5b\left(-\frac{3b\left(\frac{b\int \frac{1}{1-\frac{ax^2}{bx^3+ax^2}}d\frac{x}{\sqrt{bx^3+ax^2}}}{a}-\frac{\sqrt{ax^2+bx^3}}{ax^2}\right)}{4a}-\frac{\sqrt{ax^2+bx^3}}{2ax^3}\right)}{6a}-\frac{\sqrt{ax^2+bx^3}}{3ax^4}\right)-\frac{\sqrt{ax^2+bx^3}}{4x^5}$$

$$\Big\downarrow \textcolor{blue}{219}$$

$$\frac{1}{8}b\left(-\frac{5b\left(-\frac{3b\left(\frac{b\,\text{arctanh}\left(\frac{\sqrt{a}x}{\sqrt{ax^2+bx^3}}\right)}{a^{3/2}}-\frac{\sqrt{ax^2+bx^3}}{ax^2}\right)}{4a}-\frac{\sqrt{ax^2+bx^3}}{2ax^3}\right)}{6a}-\frac{\sqrt{ax^2+bx^3}}{3ax^4}\right)-\frac{\sqrt{ax^2+bx^3}}{4x^5}$$

```
Int[Sqrt[a*x^2 + b*x^3]/x^6,x]
```

```
-1/4*Sqrt[a*x^2 + b*x^3]/x^5 + (b*(-1/3*Sqrt[a*x^2 + b*x^3]/(a*x^4) - (5*b
*(-1/2*Sqrt[a*x^2 + b*x^3]/(a*x^3) - (3*b*(-(Sqrt[a*x^2 + b*x^3]/(a*x^2))
+ (b*ArcTanh[(Sqrt[a]*x)/Sqrt[a*x^2 + b*x^3]])/a^(3/2)))/(4*a)))/(6*a)))/8
```

Defintions of rubi rules used

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

```
Int[1/Sqrt[(a_.)*(x_)^2 + (b_.)*(x_)^(n_.)], x_Symbol] :> Simp[2/(2 - n)
Subst[Int[1/(1 - a*x^2), x], x, x/Sqrt[a*x^2 + b*x^n]], x] /; FreeQ[{a, b,
n}, x] && NeQ[n, 2]
```

```
Int[((c_.)*(x_))^(m_)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol]
 :> Simp[(c*x)^(m + 1)*((a*x^j + b*x^n)^p/(c*(m + j*p + 1))), x] - Simp[b*p
*((n - j)/(c^n*(m + j*p + 1)))   Int[(c*x)^(m + n)*(a*x^j + b*x^n)^(p - 1),
 x], x] /; FreeQ[{a, b, c}, x] &&  !IntegerQ[p] && LtQ[0, j, n] && (Integer
sQ[j, n] || GtQ[c, 0]) && GtQ[p, 0] && LtQ[m + j*p + 1, 0]
```

```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> Simp[c^(j - 1)*(c*x)^(m - j + 1)*((a*x^j + b*x^n)^(p + 1)/(a*(m + j*p
+ 1))), x] - Simp[b*((m + n*p + n - j + 1)/(a*c^(n - j)*(m + j*p + 1)))   I
nt[(c*x)^(m + n - j)*(a*x^j + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, p}, x]
 &&  !IntegerQ[p] && LtQ[0, j, n] && (IntegersQ[j, n] || GtQ[c, 0]) && LtQ[
m + j*p + 1, 0]
```

Maple [A] (verified)

Time = 0.50 (sec) , antiderivative size = 82, normalized size of antiderivative = 0.58

method	result	size
pseudoelliptic	$-\frac{\frac{35\,\operatorname{arctanh}\left(\frac{\sqrt{bx+a}}{\sqrt{a}}\right) b^{5} x^{5}}{128}+\sqrt{bx+a}\,\left(-\frac{35\sqrt{a}\,b^{4} x^{4}}{128}+\frac{35 a^{\frac{3}{2}} b^{3} x^{3}}{192}-\frac{7 a^{\frac{5}{2}} b^{2} x^{2}}{48}+\frac{a^{\frac{7}{2}} b x}{8}+a^{\frac{9}{2}}\right)}{5 a^{\frac{9}{2}} x^{5}}$	82
risch	$-\frac{\left(15 b^{3} x^{3}-10 a\,b^{2} x^{2}+8 a^{2} b x+48 a^{3}\right)\sqrt{x^{2}\left(bx+a\right)}}{192 x^{5} a^{3}}+\frac{5 b^{4}\operatorname{arctanh}\left(\frac{\sqrt{bx+a}}{\sqrt{a}}\right)\sqrt{x^{2}\left(bx+a\right)}}{64 a^{\frac{7}{2}} x\sqrt{bx+a}}$	92
default	$-\frac{\sqrt{b\,x^{3}+a\,x^{2}}\,\left(15\left(bx+a\right)^{\frac{7}{2}} a^{\frac{7}{2}}-55\left(bx+a\right)^{\frac{5}{2}} a^{\frac{9}{2}}+73\left(bx+a\right)^{\frac{3}{2}} a^{\frac{11}{2}}-15\,\operatorname{arctanh}\left(\frac{\sqrt{bx+a}}{\sqrt{a}}\right) b^{4} x^{4} a^{3}+15\sqrt{bx+a}\,a^{\frac{13}{2}}\right)}{192 x^{5}\sqrt{bx+a}\,a^{\frac{13}{2}}}$	101

```
int((b*x^3+a*x^2)^(1/2)/x^6,x,method=_RETURNVERBOSE)
```

```
-1/5/a^(9/2)*(35/128*arctanh((b*x+a)^(1/2)/a^(1/2))*b^5*x^5+(b*x+a)^(1/2)*
(-35/128*a^(1/2)*b^4*x^4+35/192*a^(3/2)*b^3*x^3-7/48*a^(5/2)*b^2*x^2+1/8*a
^(7/2)*b*x+a^(9/2)))/x^5
```

Fricas [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 202, normalized size of antiderivative = 1.42

$$\int \frac{\sqrt{ax^2+bx^3}}{x^6}\,dx = \left[\frac{15\,\sqrt{a}b^4x^5\log\left(\frac{bx^2+2\,ax+2\,\sqrt{bx^3+ax^2}\sqrt{a}}{x^2}\right)-2\,(15\,ab^3x^3-10\,a^2b^2x^2+8\,a^3bx+48\,a^4)\sqrt{bx^3+ax^2}}{384\,a^4x^5},\right.$$
$$\left.-\frac{15\,\sqrt{-a}b^4x^5\arctan\left(\frac{\sqrt{bx^3+ax^2}\sqrt{-a}}{bx^2+ax}\right)+(15\,ab^3x^3-10\,a^2b^2x^2+8\,a^3bx+48\,a^4)\sqrt{bx^3+ax^2}}{192\,a^4x^5}\right]$$

```
integrate((b*x^3+a*x^2)^(1/2)/x^6,x, algorithm="fricas")
```

```
[1/384*(15*sqrt(a)*b^4*x^5*log((b*x^2 + 2*a*x + 2*sqrt(b*x^3 + a*x^2)*sqrt
(a))/x^2) - 2*(15*a*b^3*x^3 - 10*a^2*b^2*x^2 + 8*a^3*b*x + 48*a^4)*sqrt(b*
x^3 + a*x^2))/(a^4*x^5), -1/192*(15*sqrt(-a)*b^4*x^5*arctan(sqrt(b*x^3 + a
*x^2)*sqrt(-a)/(b*x^2 + a*x)) + (15*a*b^3*x^3 - 10*a^2*b^2*x^2 + 8*a^3*b*x
 + 48*a^4)*sqrt(b*x^3 + a*x^2))/(a^4*x^5)]
```

Sympy [F]

$$\int \frac{\sqrt{ax^2+bx^3}}{x^6}\,dx = \int \frac{\sqrt{x^2(a+bx)}}{x^6}\,dx$$

```
integrate((b*x**3+a*x**2)**(1/2)/x**6,x)
```

```
Integral(sqrt(x**2*(a + b*x))/x**6, x)
```

Maxima [F]

$$\int \frac{\sqrt{ax^2+bx^3}}{x^6}\,dx = \int \frac{\sqrt{bx^3+ax^2}}{x^6}\,dx$$

```
integrate((b*x^3+a*x^2)^(1/2)/x^6,x, algorithm="maxima")
```

```
integrate(sqrt(b*x^3 + a*x^2)/x^6, x)
```

Giac [A] (verification not implemented)

Time = 0.24 (sec) , antiderivative size = 109, normalized size of antiderivative = 0.77

$$\int \frac{\sqrt{ax^2+bx^3}}{x^6}\,dx = -\frac{\frac{15\,b^5\arctan\left(\frac{\sqrt{bx+a}}{\sqrt{-a}}\right)\mathrm{sgn}(x)}{\sqrt{-a}a^3}+\frac{15\,(bx+a)^{\frac{7}{2}}b^5\mathrm{sgn}(x)-55\,(bx+a)^{\frac{5}{2}}ab^5\mathrm{sgn}(x)+73\,(bx+a)^{\frac{3}{2}}a^2b^5\mathrm{sgn}(x)+15\,\sqrt{bx+a}a^3b^5\mathrm{sgn}(x)}{a^3b^4x^4}}{192\,b}$$

```
integrate((b*x^3+a*x^2)^(1/2)/x^6,x, algorithm="giac")
```

```
-1/192*(15*b^5*arctan(sqrt(b*x + a)/sqrt(-a))*sgn(x)/(sqrt(-a)*a^3) + (15*
(b*x + a)^(7/2)*b^5*sgn(x) - 55*(b*x + a)^(5/2)*a*b^5*sgn(x) + 73*(b*x + a
)^(3/2)*a^2*b^5*sgn(x) + 15*sqrt(b*x + a)*a^3*b^5*sgn(x))/(a^3*b^4*x^4))/b
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\sqrt{ax^2+bx^3}}{x^6}\,dx = \int \frac{\sqrt{b\,x^3+a\,x^2}}{x^6}\,dx$$

```
int((a*x^2 + b*x^3)^(1/2)/x^6,x)
```

```
int((a*x^2 + b*x^3)^(1/2)/x^6, x)
```

Reduce [B] (verification not implemented)

Time = 0.24 (sec) , antiderivative size = 107, normalized size of antiderivative = 0.75

$$\int \frac{\sqrt{ax^2+bx^3}}{x^6}\,dx = \frac{-96\sqrt{bx+a}\,a^4-16\sqrt{bx+a}\,a^3bx+20\sqrt{bx+a}\,a^2b^2x^2-30\sqrt{bx+a}\,a\,b^3x^3-15\sqrt{a}\log\left(\sqrt{bx+a}-\sqrt{a}\right)}{384a^4x^4}$$

```
int((b*x^3+a*x^2)^(1/2)/x^6,x)
```

```
( - 96*sqrt(a + b*x)*a**4 - 16*sqrt(a + b*x)*a**3*b*x + 20*sqrt(a + b*x)*a
**2*b**2*x**2 - 30*sqrt(a + b*x)*a*b**3*x**3 - 15*sqrt(a)*log(sqrt(a + b*x
) - sqrt(a))*b**4*x**4 + 15*sqrt(a)*log(sqrt(a + b*x) + sqrt(a))*b**4*x**4
)/(384*a**4*x**4)
```

3.275 $\int x^2(ax^2+bx^3)^{3/2}\,dx$

Optimal result

Integrand size = 19, antiderivative size = 164

$$\int x^2\left(ax^2+bx^3\right)^{3/2}\,dx=-\frac{2a^5(ax^2+bx^3)^{5/2}}{5b^6x^5}+\frac{10a^4(ax^2+bx^3)^{7/2}}{7b^6x^7}-\frac{20a^3(ax^2+bx^3)^{9/2}}{9b^6x^9}+\frac{20a^2(ax^2+bx^3)^{11/2}}{11b^6x^{11}}-\frac{10a(ax^2+bx^3)^{13/2}}{13b^6x^{13}}+\frac{2(ax^2+bx^3)^{15/2}}{15b^6x^{15}}$$

```
-2/5*a^5*(b*x^3+a*x^2)^(5/2)/b^6/x^5+10/7*a^4*(b*x^3+a*x^2)^(7/2)/b^6/x^7-
20/9*a^3*(b*x^3+a*x^2)^(9/2)/b^6/x^9+20/11*a^2*(b*x^3+a*x^2)^(11/2)/b^6/x^
11-10/13*a*(b*x^3+a*x^2)^(13/2)/b^6/x^13+2/15*(b*x^3+a*x^2)^(15/2)/b^6/x^1
5
```

Mathematica [A] (verified)

Time = 0.04 (sec) , antiderivative size = 80, normalized size of antiderivative = 0.49

$$\int x^2\left(ax^2+bx^3\right)^{3/2}\,dx=\frac{2x(a+bx)^3\left(-256a^5+640a^4bx-1120a^3b^2x^2+1680a^2b^3x^3-2310ab^4x^4+3003b^5x^5\right)}{45045b^6\sqrt{x^2(a+bx)}}$$

```
Integrate[x^2*(a*x^2 + b*x^3)^(3/2),x]
```

```
(2*x*(a + b*x)^3*(-256*a^5 + 640*a^4*b*x - 1120*a^3*b^2*x^2 + 1680*a^2*b^3
*x^3 - 2310*a*b^4*x^4 + 3003*b^5*x^5))/(45045*b^6*Sqrt[x^2*(a + b*x)])
```

Rubi [A] (verified)

Time = 0.61 (sec) , antiderivative size = 185, normalized size of antiderivative = 1.13, number of steps used = 6, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.316, Rules used = {1922, 1922, 1908, 1922, 1922, 1920}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^2 \left(ax^2+bx^3\right)^{3/2} dx$$

$$\Big\downarrow \text{1922}$$

$$\frac{2\left(ax^2+bx^3\right)^{5/2}}{15b}-\frac{2a\int x\left(bx^3+ax^2\right)^{3/2}dx}{3b}$$

$$\Big\downarrow \text{1922}$$

$$\frac{2\left(ax^2+bx^3\right)^{5/2}}{15b}-\frac{2a\left(\frac{2\left(ax^2+bx^3\right)^{5/2}}{13bx}-\frac{8a\int \left(bx^3+ax^2\right)^{3/2}dx}{13b}\right)}{3b}$$

$$\Big\downarrow \text{1908}$$

$$\frac{2\left(ax^2+bx^3\right)^{5/2}}{15b}-\frac{2a\left(\frac{2\left(ax^2+bx^3\right)^{5/2}}{13bx}-\frac{8a\left(\frac{2\left(ax^2+bx^3\right)^{5/2}}{11bx^2}-\frac{6a\int \frac{\left(bx^3+ax^2\right)^{3/2}}{x}dx}{11b}\right)}{13b}\right)}{3b}$$

$$\Big\downarrow \text{1922}$$

$$\frac{2\left(ax^2+bx^3\right)^{5/2}}{15b}-\frac{2a\left(\frac{2\left(ax^2+bx^3\right)^{5/2}}{13bx}-\frac{8a\left(\frac{2\left(ax^2+bx^3\right)^{5/2}}{11bx^2}-\frac{6a\left(\frac{2\left(ax^2+bx^3\right)^{5/2}}{9bx^3}-\frac{4a\int\frac{\left(bx^3+ax^2\right)^{3/2}}{x^2}dx}{9b}\right)}{11b}\right)}{13b}\right)}{3b}$$

$$\Big\downarrow \; 1922$$

$$\frac{2\left(ax^2+bx^3\right)^{5/2}}{15b}-\frac{2a\left(\frac{2\left(ax^2+bx^3\right)^{5/2}}{13bx}-\frac{8a\left(\frac{2\left(ax^2+bx^3\right)^{5/2}}{11bx^2}-\frac{6a\left(\frac{2\left(ax^2+bx^3\right)^{5/2}}{9bx^3}-\frac{4a\left(\frac{2\left(ax^2+bx^3\right)^{5/2}}{7bx^4}-\frac{2a\int\frac{\left(bx^3+ax^2\right)^{3/2}}{x^3}dx}{7b}\right)}{9b}\right)}{11b}\right)}{13b}\right)}{3b}$$

$$\Big\downarrow \; 1920$$

$$\frac{2(ax^2+bx^3)^{5/2}}{15b} - \frac{2a\left(\frac{2(ax^2+bx^3)^{5/2}}{13bx} - \frac{8a\left(\frac{2(ax^2+bx^3)^{5/2}}{11bx^2} - \frac{6a\left(\frac{2(ax^2+bx^3)^{5/2}}{9bx^3} - \frac{4a\left(\frac{2(ax^2+bx^3)^{5/2}}{7bx^4} - \frac{4a(ax^2+bx^3)^{5/2}}{35b^2x^5}\right)}{9b}\right)}{11b}\right)}{13b}\right)}{3b}$$

```
Int[x^2*(a*x^2 + b*x^3)^(3/2),x]
```

```
(2*(a*x^2 + b*x^3)^(5/2))/(15*b) - (2*a*((2*(a*x^2 + b*x^3)^(5/2))/(13*b*x
) - (8*a*((2*(a*x^2 + b*x^3)^(5/2))/(11*b*x^2) - (6*a*((2*(a*x^2 + b*x^3)^
(5/2))/(9*b*x^3) - (4*a*((-4*a*(a*x^2 + b*x^3)^(5/2))/(35*b^2*x^5) + (2*(a
*x^2 + b*x^3)^(5/2))/(7*b*x^4)))/(9*b)))/(11*b)))/(13*b)))/(3*b)
```

Defintions of rubi rules used

```
Int[((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol] :> Simp[(a*x^j +
b*x^n)^(p + 1)/(a*(j*p + 1)*x^(j - 1)), x] - Simp[b*((n*p + n - j + 1)/(a*(
j*p + 1)))   Int[x^(n - j)*(a*x^j + b*x^n)^p, x], x] /; FreeQ[{a, b, j, n,
p}, x] &&  !IntegerQ[p] && NeQ[n, j] && ILtQ[Simplify[(n*p + n - j + 1)/(n
- j)], 0] && NeQ[j*p + 1, 0]
```

```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> Simp[(-c^(j - 1))*(c*x)^(m - j + 1)*((a*x^j + b*x^n)^(p + 1)/(a*(n - j
)*(p + 1))), x] /; FreeQ[{a, b, c, j, m, n, p}, x] &&  !IntegerQ[p] && NeQ[
n, j] && EqQ[m + n*p + n - j + 1, 0] && (IntegerQ[j] || GtQ[c, 0])
```

```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> Simp[c^(j - 1)*(c*x)^(m - j + 1)*((a*x^j + b*x^n)^(p + 1)/(a*(m + j*p
+ 1))), x] - Simp[b*((m + n*p + n - j + 1)/(a*c^(n - j)*(m + j*p + 1)))   I
nt[(c*x)^(m + n - j)*(a*x^j + b*x^n)^p, x], x] /; FreeQ[{a, b, c, j, m, n,
p}, x] &&  !IntegerQ[p] && NeQ[n, j] && ILtQ[Simplify[(m + n*p + n - j + 1)
/(n - j)], 0] && NeQ[m + j*p + 1, 0] && (IntegersQ[j, n] || GtQ[c, 0])
```

Maple [A] (verified)

Time = 0.42 (sec) , antiderivative size = 32, normalized size of antiderivative = 0.20

method	result	size
pseudoelliptic	$\frac{2(bx+a)^{\frac{5}{2}}\left(35b^2x^2-20abx+8a^2\right)}{315b^3}$	32
gosper	$-\frac{2(bx+a)\left(-3003b^5x^5+2310a\,b^4x^4-1680a^2b^3x^3+1120a^3b^2x^2-640a^4bx+256a^5\right)\left(b\,x^3+a\,x^2\right)^{\frac{3}{2}}}{45045b^6x^3}$	79
default	$-\frac{2(bx+a)\left(-3003b^5x^5+2310a\,b^4x^4-1680a^2b^3x^3+1120a^3b^2x^2-640a^4bx+256a^5\right)\left(b\,x^3+a\,x^2\right)^{\frac{3}{2}}}{45045b^6x^3}$	79
orering	$-\frac{2(bx+a)\left(-3003b^5x^5+2310a\,b^4x^4-1680a^2b^3x^3+1120a^3b^2x^2-640a^4bx+256a^5\right)\left(b\,x^3+a\,x^2\right)^{\frac{3}{2}}}{45045b^6x^3}$	79
risch	$-\frac{2\sqrt{x^2(bx+a)}\left(-3003x^7b^7-3696b^6a\,x^6-63a^2x^5b^5+70b^4x^4a^3-80b^3x^3a^4+96x^2b^2a^5-128xb\,a^6+256a^7\right)}{45045x\,b^6}$	94
trager	$-\frac{2\left(-3003x^7b^7-3696b^6a\,x^6-63a^2x^5b^5+70b^4x^4a^3-80b^3x^3a^4+96x^2b^2a^5-128xb\,a^6+256a^7\right)\sqrt{b\,x^3+a\,x^2}}{45045b^6x}$	96

```
int(x^2*(b*x^3+a*x^2)^(3/2),x,method=_RETURNVERBOSE)
```

```
2/315*(b*x+a)^(5/2)*(35*b^2*x^2-20*a*b*x+8*a^2)/b^3
```

Fricas [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 95, normalized size of antiderivative = 0.58

$$\int x^2\left(ax^2+bx^3\right)^{3/2}\,dx=\frac{2\left(3003\,b^7x^7+3696\,ab^6x^6+63\,a^2b^5x^5-70\,a^3b^4x^4+80\,a^4b^3x^3-96\,a^5b^2x^2+128\,a^6bx-256\right.}{45045\,b^6x}$$

```
integrate(x^2*(b*x^3+a*x^2)^(3/2),x, algorithm="fricas")
```

```
2/45045*(3003*b^7*x^7 + 3696*a*b^6*x^6 + 63*a^2*b^5*x^5 - 70*a^3*b^4*x^4 +
 80*a^4*b^3*x^3 - 96*a^5*b^2*x^2 + 128*a^6*b*x - 256*a^7)*sqrt(b*x^3 + a*x
^2)/(b^6*x)
```

Sympy [F]

$$\int x^2 \left(ax^2 + bx^3\right)^{3/2} \, dx = \int x^2 \left(x^2 (a + bx)\right)^{\frac{3}{2}} \, dx$$

```
integrate(x**2*(b*x**3+a*x**2)**(3/2),x)
```

```
Integral(x**2*(x**2*(a + b*x))**(3/2), x)
```

Maxima [A] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 86, normalized size of antiderivative = 0.52

$$\int x^2 \left(ax^2 + bx^3\right)^{3/2} \, dx = \frac{2 \left(3003 \, b^7 x^7 + 3696 \, ab^6 x^6 + 63 \, a^2 b^5 x^5 - 70 \, a^3 b^4 x^4 + 80 \, a^4 b^3 x^3 - 96 \, a^5 b^2 x^2 + 128 \, a^6 bx - 256}{45045 \, b^6}$$

```
integrate(x^2*(b*x^3+a*x^2)^(3/2),x, algorithm="maxima")
```

```
2/45045*(3003*b^7*x^7 + 3696*a*b^6*x^6 + 63*a^2*b^5*x^5 - 70*a^3*b^4*x^4 +
 80*a^4*b^3*x^3 - 96*a^5*b^2*x^2 + 128*a^6*b*x - 256*a^7)*sqrt(b*x + a)/b^
6
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 282 vs. $2(140) = 280$.

Time = 0.14 (sec) , antiderivative size = 282, normalized size of antiderivative = 1.72

$$\int x^2 \left(ax^2+bx^3\right)^{3/2} dx = \frac{512 a^{\frac{15}{2}} \operatorname{sgn}(x)}{45045 b^6} + \frac{2 \left(\frac{65 \left(63 (bx+a)^{\frac{11}{2}} - 385 (bx+a)^{\frac{9}{2}} a + 990 (bx+a)^{\frac{7}{2}} a^2 - 1386 (bx+a)^{\frac{5}{2}} a^3 + 1155 (bx+a)^{\frac{3}{2}} a^4 - 693 \sqrt{bx+a} a^5\right) a^2 \operatorname{sgn}(x)}{b^5} + \frac{30 \left(231 (bx+a)^{\frac{13}{2}} - 1\right.}{}\right.}{}$$

```
integrate(x^2*(b*x^3+a*x^2)^(3/2),x, algorithm="giac")
```

```
512/45045*a^(15/2)*sgn(x)/b^6 + 2/45045*(65*(63*(b*x + a)^(11/2) - 385*(b*
x + a)^(9/2)*a + 990*(b*x + a)^(7/2)*a^2 - 1386*(b*x + a)^(5/2)*a^3 + 1155
*(b*x + a)^(3/2)*a^4 - 693*sqrt(b*x + a)*a^5)*a^2*sgn(x)/b^5 + 30*(231*(b*
x + a)^(13/2) - 1638*(b*x + a)^(11/2)*a + 5005*(b*x + a)^(9/2)*a^2 - 8580*
(b*x + a)^(7/2)*a^3 + 9009*(b*x + a)^(5/2)*a^4 - 6006*(b*x + a)^(3/2)*a^5
+ 3003*sqrt(b*x + a)*a^6)*a*sgn(x)/b^5 + 7*(429*(b*x + a)^(15/2) - 3465*(b
*x + a)^(13/2)*a + 12285*(b*x + a)^(11/2)*a^2 - 25025*(b*x + a)^(9/2)*a^3
+ 32175*(b*x + a)^(7/2)*a^4 - 27027*(b*x + a)^(5/2)*a^5 + 15015*(b*x + a)^
(3/2)*a^6 - 6435*sqrt(b*x + a)*a^7)*sgn(x)/b^5)/b
```

Mupad [B] (verification not implemented)

Time = 9.32 (sec) , antiderivative size = 80, normalized size of antiderivative = 0.49

$$\int x^2 \left(ax^2+bx^3\right)^{3/2} dx = -\frac{2 \sqrt{b x^3 + a x^2} (a + b x)^2 (256 a^5 - 640 a^4 b x + 1120 a^3 b^2 x^2 - 1680 a^2 b^3 x^3 + 2310 a b^4 x^4 - 3003 b^5 x^5)}{45045 b^6 x}$$

```
int(x^2*(a*x^2 + b*x^3)^(3/2),x)
```

```
-(2*(a*x^2 + b*x^3)^(1/2)*(a + b*x)^2*(256*a^5 - 3003*b^5*x^5 + 2310*a*b^4
*x^4 + 1120*a^3*b^2*x^2 - 1680*a^2*b^3*x^3 - 640*a^4*b*x))/(45045*b^6*x)
```

Reduce [B] (verification not implemented)

Time = 0.48 (sec) , antiderivative size = 85, normalized size of antiderivative = 0.52

$$\int x^2(ax^2+bx^3)^{3/2}\,dx = \frac{2\sqrt{bx+a}\,(3003b^7x^7+3696a\,b^6x^6+63a^2b^5x^5-70a^3b^4x^4+80a^4b^3x^3-96a^5b^2x^2+128a^6bx}{45045b^6}$$

```
int(x^2*(b*x^3+a*x^2)^(3/2),x)
```

```
(2*sqrt(a + b*x)*( - 256*a**7 + 128*a**6*b*x - 96*a**5*b**2*x**2 + 80*a**4
*b**3*x**3 - 70*a**3*b**4*x**4 + 63*a**2*b**5*x**5 + 3696*a*b**6*x**6 + 30
03*b**7*x**7))/(45045*b**6)
```

3.276 $\int x(ax^2+bx^3)^{3/2}\,dx$

Optimal result

Integrand size = 17, antiderivative size = 136

$$\int x\left(ax^2+bx^3\right)^{3/2}\,dx=\frac{2a^4(ax^2+bx^3)^{5/2}}{5b^5x^5}-\frac{8a^3(ax^2+bx^3)^{7/2}}{7b^5x^7}+\frac{4a^2(ax^2+bx^3)^{9/2}}{3b^5x^9}-\frac{8a(ax^2+bx^3)^{11/2}}{11b^5x^{11}}+\frac{2(ax^2+bx^3)^{13/2}}{13b^5x^{13}}$$

```
2/5*a^4*(b*x^3+a*x^2)^(5/2)/b^5/x^5-8/7*a^3*(b*x^3+a*x^2)^(7/2)/b^5/x^7+4/
3*a^2*(b*x^3+a*x^2)^(9/2)/b^5/x^9-8/11*a*(b*x^3+a*x^2)^(11/2)/b^5/x^11+2/1
3*(b*x^3+a*x^2)^(13/2)/b^5/x^13
```

Mathematica [A] (verified)

Time = 0.03 (sec) , antiderivative size = 69, normalized size of antiderivative = 0.51

$$\int x\left(ax^2+bx^3\right)^{3/2}\,dx=\frac{2x(a+bx)^3\left(128a^4-320a^3bx+560a^2b^2x^2-840ab^3x^3+1155b^4x^4\right)}{15015b^5\sqrt{x^2(a+bx)}}$$

```
Integrate[x*(a*x^2 + b*x^3)^(3/2),x]
```

```
(2*x*(a + b*x)^3*(128*a^4 - 320*a^3*b*x + 560*a^2*b^2*x^2 - 840*a*b^3*x^3
+ 1155*b^4*x^4))/(15015*b^5*Sqrt[x^2*(a + b*x)])
```

Rubi [A] (verified)

Time = 0.56 (sec) , antiderivative size = 154, normalized size of antiderivative = 1.13, number of steps used = 5, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.294, Rules used = {1922, 1908, 1922, 1922, 1920}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x\left(ax^2+bx^3\right)^{3/2}dx$$

$$\Big\downarrow \text{1922}$$

$$\frac{2\left(ax^2+bx^3\right)^{5/2}}{13bx}-\frac{8a\int \left(bx^3+ax^2\right)^{3/2}dx}{13b}$$

$$\Big\downarrow \text{1908}$$

$$\frac{2\left(ax^2+bx^3\right)^{5/2}}{13bx}-\frac{8a\left(\frac{2\left(ax^2+bx^3\right)^{5/2}}{11bx^2}-\frac{6a\int \frac{\left(bx^3+ax^2\right)^{3/2}}{x}dx}{11b}\right)}{13b}$$

$$\Big\downarrow \text{1922}$$

$$\frac{2\left(ax^2+bx^3\right)^{5/2}}{13bx}-\frac{8a\left(\frac{2\left(ax^2+bx^3\right)^{5/2}}{11bx^2}-\frac{6a\left(\frac{2\left(ax^2+bx^3\right)^{5/2}}{9bx^3}-\frac{4a\int \frac{\left(bx^3+ax^2\right)^{3/2}}{x^2}dx}{9b}\right)}{11b}\right)}{13b}$$

$$\Big\downarrow \text{1922}$$

$$\frac{2\left(ax^2+bx^3\right)^{5/2}}{13bx}-\frac{8a\left(\frac{2\left(ax^2+bx^3\right)^{5/2}}{11bx^2}-\frac{6a\left(\frac{2\left(ax^2+bx^3\right)^{5/2}}{9bx^3}-\frac{4a\left(\frac{2\left(ax^2+bx^3\right)^{5/2}}{7bx^4}-\frac{2a\int\frac{\left(bx^3+ax^2\right)^{3/2}}{x^3}dx}{7b}\right)}{9b}\right)}{11b}\right)}{13b}$$

$$\Big\downarrow \ 1920$$

$$\frac{2\left(ax^2+bx^3\right)^{5/2}}{13bx}-\frac{8a\left(\frac{2\left(ax^2+bx^3\right)^{5/2}}{11bx^2}-\frac{6a\left(\frac{2\left(ax^2+bx^3\right)^{5/2}}{9bx^3}-\frac{4a\left(\frac{2\left(ax^2+bx^3\right)^{5/2}}{7bx^4}-\frac{4a\left(ax^2+bx^3\right)^{5/2}}{35b^2x^5}\right)}{9b}\right)}{11b}\right)}{13b}$$

```
Int[x*(a*x^2 + b*x^3)^(3/2),x]
```

```
(2*(a*x^2 + b*x^3)^(5/2))/(13*b*x) - (8*a*((2*(a*x^2 + b*x^3)^(5/2))/(11*b
*x^2) - (6*a*((2*(a*x^2 + b*x^3)^(5/2))/(9*b*x^3) - (4*a*((-4*a*(a*x^2 + b
*x^3)^(5/2))/(35*b^2*x^5) + (2*(a*x^2 + b*x^3)^(5/2))/(7*b*x^4)))/(9*b)))/
(11*b)))/(13*b)
```

Defintions of rubi rules used

```
Int[((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol] :> Simp[(a*x^j +
b*x^n)^(p + 1)/(a*(j*p + 1)*x^(j - 1)), x] - Simp[b*((n*p + n - j + 1)/(a*(
j*p + 1)))   Int[x^(n - j)*(a*x^j + b*x^n)^p, x], x] /; FreeQ[{a, b, j, n,
p}, x] &&  !IntegerQ[p] && NeQ[n, j] && ILtQ[Simplify[(n*p + n - j + 1)/(n
- j)], 0] && NeQ[j*p + 1, 0]
```

```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> Simp[(-c^(j - 1))*(c*x)^(m - j + 1)*((a*x^j + b*x^n)^(p + 1)/(a*(n - j
)*(p + 1))), x] /; FreeQ[{a, b, c, j, m, n, p}, x] &&  !IntegerQ[p] && NeQ[
n, j] && EqQ[m + n*p + n - j + 1, 0] && (IntegerQ[j] || GtQ[c, 0])
```

```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> Simp[c^(j - 1)*(c*x)^(m - j + 1)*((a*x^j + b*x^n)^(p + 1)/(a*(m + j*p
+ 1))), x] - Simp[b*((m + n*p + n - j + 1)/(a*c^(n - j)*(m + j*p + 1)))   I
nt[(c*x)^(m + n - j)*(a*x^j + b*x^n)^p, x], x] /; FreeQ[{a, b, c, j, m, n,
p}, x] &&  !IntegerQ[p] && NeQ[n, j] && ILtQ[Simplify[(m + n*p + n - j + 1)
/(n - j)], 0] && NeQ[m + j*p + 1, 0] && (IntegersQ[j, n] || GtQ[c, 0])
```

Maple [A] (verified)

Time = 0.40 (sec) , antiderivative size = 21, normalized size of antiderivative = 0.15

method	result	size
pseudoelliptic	$-\frac{2(bx+a)^{\frac{5}{2}}(-5bx+2a)}{35b^{2}}$	21
gosper	$\frac{2(bx+a)\left(1155b^{4}x^{4}-840a\,b^{3}x^{3}+560a^{2}b^{2}x^{2}-320a^{3}bx+128a^{4}\right)\left(b\,x^{3}+a\,x^{2}\right)^{\frac{3}{2}}}{15015b^{5}x^{3}}$	68
default	$\frac{2(bx+a)\left(1155b^{4}x^{4}-840a\,b^{3}x^{3}+560a^{2}b^{2}x^{2}-320a^{3}bx+128a^{4}\right)\left(b\,x^{3}+a\,x^{2}\right)^{\frac{3}{2}}}{15015b^{5}x^{3}}$	68
orering	$\frac{2(bx+a)\left(1155b^{4}x^{4}-840a\,b^{3}x^{3}+560a^{2}b^{2}x^{2}-320a^{3}bx+128a^{4}\right)\left(b\,x^{3}+a\,x^{2}\right)^{\frac{3}{2}}}{15015b^{5}x^{3}}$	68
risch	$\frac{2\sqrt{x^{2}(bx+a)}\,\left(1155b^{6}x^{6}+1470a\,b^{5}x^{5}+35a^{2}b^{4}x^{4}-40a^{3}b^{3}x^{3}+48a^{4}b^{2}x^{2}-64a^{5}bx+128a^{6}\right)}{15015x\,b^{5}}$	83
trager	$\frac{2\left(1155b^{6}x^{6}+1470a\,b^{5}x^{5}+35a^{2}b^{4}x^{4}-40a^{3}b^{3}x^{3}+48a^{4}b^{2}x^{2}-64a^{5}bx+128a^{6}\right)\sqrt{b\,x^{3}+a\,x^{2}}}{15015b^{5}x}$	85

```
int(x*(b*x^3+a*x^2)^(3/2),x,method=_RETURNVERBOSE)
```

```
-2/35*(b*x+a)^(5/2)*(-5*b*x+2*a)/b^2
```

Fricas [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 84, normalized size of antiderivative = 0.62

$$\int x(ax^2 + bx^3)^{3/2} dx = \frac{2 (1155 b^6x^6 + 1470 ab^5x^5 + 35 a^2b^4x^4 - 40 a^3b^3x^3 + 48 a^4b^2x^2 - 64 a^5bx + 128 a^6)\sqrt{bx^3 + a}}{15015 b^5x}$$

```
integrate(x*(b*x^3+a*x^2)^(3/2),x, algorithm="fricas")
```

```
2/15015*(1155*b^6*x^6 + 1470*a*b^5*x^5 + 35*a^2*b^4*x^4 - 40*a^3*b^3*x^3 +
 48*a^4*b^2*x^2 - 64*a^5*b*x + 128*a^6)*sqrt(b*x^3 + a*x^2)/(b^5*x)
```

Sympy [F]

$$\int x(ax^2 + bx^3)^{3/2} dx = \int x(x^2(a + bx))^{\frac{3}{2}} dx$$

```
integrate(x*(b*x**3+a*x**2)**(3/2),x)
```

```
Integral(x*(x**2*(a + b*x))**(3/2), x)
```

Maxima [A] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 75, normalized size of antiderivative = 0.55

$$\int x(ax^2 + bx^3)^{3/2} dx = \frac{2 (1155 b^6x^6 + 1470 ab^5x^5 + 35 a^2b^4x^4 - 40 a^3b^3x^3 + 48 a^4b^2x^2 - 64 a^5bx + 128 a^6)\sqrt{bx + a}}{15015 b^5}$$

```
integrate(x*(b*x^3+a*x^2)^(3/2),x, algorithm="maxima")
```

```
2/15015*(1155*b^6*x^6 + 1470*a*b^5*x^5 + 35*a^2*b^4*x^4 - 40*a^3*b^3*x^3 +
 48*a^4*b^2*x^2 - 64*a^5*b*x + 128*a^6)*sqrt(b*x + a)/b^5
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 246 vs. $2(116) = 232$.

Time = 0.12 (sec) , antiderivative size = 246, normalized size of antiderivative = 1.81

$$\int x(ax^2+bx^3)^{3/2}\,dx = -\frac{256\,a^{\frac{13}{2}}\operatorname{sgn}(x)}{15015\,b^5} + \frac{2\left(\frac{143\left(35\,(bx+a)^{\frac{9}{2}}-180\,(bx+a)^{\frac{7}{2}}a+378\,(bx+a)^{\frac{5}{2}}a^2-420\,(bx+a)^{\frac{3}{2}}a^3+315\,\sqrt{bx+a}a^4\right)a^2\operatorname{sgn}(x)}{b^4} + \frac{130\left(63\,(bx+a)^{\frac{11}{2}}-385\,(bx+a)^{\frac{9}{2}}a+990\,(\right.}{}\right.}{}$$

```
integrate(x*(b*x^3+a*x^2)^(3/2),x, algorithm="giac")
```

```
-256/15015*a^(13/2)*sgn(x)/b^5 + 2/45045*(143*(35*(b*x + a)^(9/2) - 180*(b
*x + a)^(7/2)*a + 378*(b*x + a)^(5/2)*a^2 - 420*(b*x + a)^(3/2)*a^3 + 315*
sqrt(b*x + a)*a^4)*a^2*sgn(x)/b^4 + 130*(63*(b*x + a)^(11/2) - 385*(b*x +
a)^(9/2)*a + 990*(b*x + a)^(7/2)*a^2 - 1386*(b*x + a)^(5/2)*a^3 + 1155*(b*
x + a)^(3/2)*a^4 - 693*sqrt(b*x + a)*a^5)*a*sgn(x)/b^4 + 15*(231*(b*x + a)
^(13/2) - 1638*(b*x + a)^(11/2)*a + 5005*(b*x + a)^(9/2)*a^2 - 8580*(b*x +
 a)^(7/2)*a^3 + 9009*(b*x + a)^(5/2)*a^4 - 6006*(b*x + a)^(3/2)*a^5 + 3003
*sqrt(b*x + a)*a^6)*sgn(x)/b^4)/b
```

Mupad [B] (verification not implemented)

Time = 9.06 (sec) , antiderivative size = 69, normalized size of antiderivative = 0.51

$$\int x(ax^2+bx^3)^{3/2}\,dx = \frac{2\,\sqrt{b\,x^3+a\,x^2}\,(a+b\,x)^2\,(128\,a^4-320\,a^3\,b\,x+560\,a^2\,b^2\,x^2-840\,a\,b^3\,x^3+1155\,b^4\,x^4)}{15015\,b^5\,x}$$

```
int(x*(a*x^2 + b*x^3)^(3/2),x)
```

```
(2*(a*x^2 + b*x^3)^(1/2)*(a + b*x)^2*(128*a^4 + 1155*b^4*x^4 - 840*a*b^3*x
^3 + 560*a^2*b^2*x^2 - 320*a^3*b*x))/(15015*b^5*x)
```

Reduce [B] (verification not implemented)

Time = 0.48 (sec) , antiderivative size = 74, normalized size of antiderivative = 0.54

$$\int x(ax^2+bx^3)^{3/2}\,dx = \frac{2\sqrt{bx+a}\,(1155b^6x^6+1470a\,b^5x^5+35a^2b^4x^4-40a^3b^3x^3+48a^4b^2x^2-64a^5bx+128a^6)}{15015b^5}$$

```
int(x*(b*x^3+a*x^2)^(3/2),x)
```

```
(2*sqrt(a + b*x)*(128*a**6 - 64*a**5*b*x + 48*a**4*b**2*x**2 - 40*a**3*b**
3*x**3 + 35*a**2*b**4*x**4 + 1470*a*b**5*x**5 + 1155*b**6*x**6))/(15015*b*
*5)
```

3.277 $\int (ax^2 + bx^3)^{3/2} \, dx$

Optimal result

Integrand size = 15, antiderivative size = 108

$$\int (ax^2+bx^3)^{3/2} \, dx = -\frac{2a^3(ax^2+bx^3)^{5/2}}{5b^4x^5} + \frac{6a^2(ax^2+bx^3)^{7/2}}{7b^4x^7} - \frac{2a(ax^2+bx^3)^{9/2}}{3b^4x^9} + \frac{2(ax^2+bx^3)^{11/2}}{11b^4x^{11}}$$

```
-2/5*a^3*(b*x^3+a*x^2)^(5/2)/b^4/x^5+6/7*a^2*(b*x^3+a*x^2)^(7/2)/b^4/x^7-2
/3*a*(b*x^3+a*x^2)^(9/2)/b^4/x^9+2/11*(b*x^3+a*x^2)^(11/2)/b^4/x^11
```

Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 58, normalized size of antiderivative = 0.54

$$\int (ax^2+bx^3)^{3/2} \, dx = \frac{2x(a+bx)^3 (-16a^3 + 40a^2bx - 70ab^2x^2 + 105b^3x^3)}{1155b^4\sqrt{x^2(a+bx)}}$$

```
Integrate[(a*x^2 + b*x^3)^(3/2),x]
```

```
(2*x*(a + b*x)^3*(-16*a^3 + 40*a^2*b*x - 70*a*b^2*x^2 + 105*b^3*x^3))/(115
5*b^4*Sqrt[x^2*(a + b*x)])
```

Rubi [A] (verified)

Time = 0.45 (sec) , antiderivative size = 120, normalized size of antiderivative = 1.11, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.267, Rules used = {1908, 1922, 1922, 1920}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \left(ax^2+bx^3\right)^{3/2}\,dx$$

$$\Big\downarrow \text{1908}$$

$$\frac{2\left(ax^2+bx^3\right)^{5/2}}{11bx^2}-\frac{6a\int \frac{\left(bx^3+ax^2\right)^{3/2}}{x}dx}{11b}$$

$$\Big\downarrow \text{1922}$$

$$\frac{2\left(ax^2+bx^3\right)^{5/2}}{11bx^2}-\frac{6a\left(\frac{2\left(ax^2+bx^3\right)^{5/2}}{9bx^3}-\frac{4a\int \frac{\left(bx^3+ax^2\right)^{3/2}}{x^2}dx}{9b}\right)}{11b}$$

$$\Big\downarrow \text{1922}$$

$$\frac{2\left(ax^2+bx^3\right)^{5/2}}{11bx^2}-\frac{6a\left(\frac{2\left(ax^2+bx^3\right)^{5/2}}{9bx^3}-\frac{4a\left(\frac{2\left(ax^2+bx^3\right)^{5/2}}{7bx^4}-\frac{2a\int \frac{\left(bx^3+ax^2\right)^{3/2}}{x^3}dx}{7b}\right)}{9b}\right)}{11b}$$

$$\Big\downarrow \text{1920}$$

$$\frac{2\left(ax^2+bx^3\right)^{5/2}}{11bx^2}-\frac{6a\left(\frac{2\left(ax^2+bx^3\right)^{5/2}}{9bx^3}-\frac{4a\left(\frac{2\left(ax^2+bx^3\right)^{5/2}}{7bx^4}-\frac{4a\left(ax^2+bx^3\right)^{5/2}}{35b^2x^5}\right)}{9b}\right)}{11b}$$

```
Int[(a*x^2 + b*x^3)^(3/2),x]
```

```
(2*(a*x^2 + b*x^3)^(5/2))/(11*b*x^2) - (6*a*((2*(a*x^2 + b*x^3)^(5/2))/(9*
b*x^3) - (4*a*((-4*a*(a*x^2 + b*x^3)^(5/2))/(35*b^2*x^5) + (2*(a*x^2 + b*x
^3)^(5/2))/(7*b*x^4)))/(9*b)))/(11*b)
```

Defintions of rubi rules used

```
Int[((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol] :> Simp[(a*x^j +
b*x^n)^(p + 1)/(a*(j*p + 1)*x^(j - 1)), x] - Simp[b*((n*p + n - j + 1)/(a*(
j*p + 1)))   Int[x^(n - j)*(a*x^j + b*x^n)^p, x], x] /; FreeQ[{a, b, j, n,
p}, x] &&  !IntegerQ[p] && NeQ[n, j] && ILtQ[Simplify[(n*p + n - j + 1)/(n
- j)], 0] && NeQ[j*p + 1, 0]
```

```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> Simp[(-c^(j - 1))*(c*x)^(m - j + 1)*((a*x^j + b*x^n)^(p + 1)/(a*(n - j
)*(p + 1))), x] /; FreeQ[{a, b, c, j, m, n, p}, x] &&  !IntegerQ[p] && NeQ[
n, j] && EqQ[m + n*p + n - j + 1, 0] && (IntegerQ[j] || GtQ[c, 0])
```

```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> Simp[c^(j - 1)*(c*x)^(m - j + 1)*((a*x^j + b*x^n)^(p + 1)/(a*(m + j*p
+ 1))), x] - Simp[b*((m + n*p + n - j + 1)/(a*c^(n - j)*(m + j*p + 1)))   I
nt[(c*x)^(m + n - j)*(a*x^j + b*x^n)^p, x], x] /; FreeQ[{a, b, c, j, m, n,
p}, x] &&  !IntegerQ[p] && NeQ[n, j] && ILtQ[Simplify[(m + n*p + n - j + 1)
/(n - j)], 0] && NeQ[m + j*p + 1, 0] && (IntegersQ[j, n] || GtQ[c, 0])
```

Maple [A] (verified)

Time = 0.37 (sec) , antiderivative size = 13, normalized size of antiderivative = 0.12

method	result	size
pseudoelliptic	$\frac{2(bx+a)^{\frac{5}{2}}}{5b}$	13
gosper	$-\frac{2(bx+a)\left(-105b^{3}x^{3}+70a\,b^{2}x^{2}-40a^{2}bx+16a^{3}\right)\left(b\,x^{3}+a\,x^{2}\right)^{\frac{3}{2}}}{1155b^{4}x^{3}}$	57
default	$-\frac{2(bx+a)\left(-105b^{3}x^{3}+70a\,b^{2}x^{2}-40a^{2}bx+16a^{3}\right)\left(b\,x^{3}+a\,x^{2}\right)^{\frac{3}{2}}}{1155b^{4}x^{3}}$	57
orering	$-\frac{2(bx+a)\left(-105b^{3}x^{3}+70a\,b^{2}x^{2}-40a^{2}bx+16a^{3}\right)\left(b\,x^{3}+a\,x^{2}\right)^{\frac{3}{2}}}{1155b^{4}x^{3}}$	57
risch	$-\frac{2\sqrt{x^{2}(bx+a)}\,\left(-105b^{5}x^{5}-140a\,b^{4}x^{4}-5a^{2}b^{3}x^{3}+6a^{3}b^{2}x^{2}-8a^{4}bx+16a^{5}\right)}{1155x\,b^{4}}$	72
trager	$-\frac{2\left(-105b^{5}x^{5}-140a\,b^{4}x^{4}-5a^{2}b^{3}x^{3}+6a^{3}b^{2}x^{2}-8a^{4}bx+16a^{5}\right)\sqrt{b\,x^{3}+a\,x^{2}}}{1155b^{4}x}$	74

```
int((b*x^3+a*x^2)^(3/2),x,method=_RETURNVERBOSE)
```

```
2/5*(b*x+a)^(5/2)/b
```

Fricas [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 73, normalized size of antiderivative = 0.68

$$\int \left(ax^2+bx^3\right)^{3/2}\,dx=\frac{2\left(105\,b^5x^5+140\,ab^4x^4+5\,a^2b^3x^3-6\,a^3b^2x^2+8\,a^4bx-16\,a^5\right)\sqrt{bx^3+ax^2}}{1155\,b^4x}$$

```
integrate((b*x^3+a*x^2)^(3/2),x, algorithm="fricas")
```

```
2/1155*(105*b^5*x^5 + 140*a*b^4*x^4 + 5*a^2*b^3*x^3 - 6*a^3*b^2*x^2 + 8*a^
4*b*x - 16*a^5)*sqrt(b*x^3 + a*x^2)/(b^4*x)
```

Sympy [F]

$$\int \left(ax^2+bx^3\right)^{3/2}\,dx=\int \left(ax^2+bx^3\right)^{\frac{3}{2}}\,dx$$

```
integrate((b*x**3+a*x**2)**(3/2),x)
```

```
Integral((a*x**2 + b*x**3)**(3/2), x)
```

Maxima [A] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 64, normalized size of antiderivative = 0.59

$$\int \left(ax^2+bx^3\right)^{3/2}\,dx=\frac{2\,(105\,b^5x^5+140\,ab^4x^4+5\,a^2b^3x^3-6\,a^3b^2x^2+8\,a^4bx-16\,a^5)\sqrt{bx+a}}{1155\,b^4}$$

```
integrate((b*x^3+a*x^2)^(3/2),x, algorithm="maxima")
```

```
2/1155*(105*b^5*x^5 + 140*a*b^4*x^4 + 5*a^2*b^3*x^3 - 6*a^3*b^2*x^2 + 8*a^
4*b*x - 16*a^5)*sqrt(b*x + a)/b^4
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 210 vs. $2(92) = 184$.

Time = 0.12 (sec) , antiderivative size = 210, normalized size of antiderivative = 1.94

$$\int \left(ax^2+bx^3\right)^{3/2}\,dx=\frac{32\,a^{\frac{11}{2}}\,\mathrm{sgn}(x)}{1155\,b^4}+\frac{2\left(\frac{99\left(5\,(bx+a)^{\frac{7}{2}}-21\,(bx+a)^{\frac{5}{2}}a+35\,(bx+a)^{\frac{3}{2}}a^2-35\,\sqrt{bx+a}a^3\right)a^2\mathrm{sgn}(x)}{b^3}+\frac{22\left(35\,(bx+a)^{\frac{9}{2}}-180\,(bx+a)^{\frac{7}{2}}a+378\,(bx+a)^{\frac{5}{2}}a^2-420\,(bx+a)^{\frac{3}{2}}\right.}{b^3}\right.}{346}$$

```
integrate((b*x^3+a*x^2)^(3/2),x, algorithm="giac")
```

```
32/1155*a^(11/2)*sgn(x)/b^4 + 2/3465*(99*(5*(b*x + a)^(7/2) - 21*(b*x + a)
^(5/2)*a + 35*(b*x + a)^(3/2)*a^2 - 35*sqrt(b*x + a)*a^3)*a^2*sgn(x)/b^3 +
 22*(35*(b*x + a)^(9/2) - 180*(b*x + a)^(7/2)*a + 378*(b*x + a)^(5/2)*a^2
- 420*(b*x + a)^(3/2)*a^3 + 315*sqrt(b*x + a)*a^4)*a*sgn(x)/b^3 + 5*(63*(b
*x + a)^(11/2) - 385*(b*x + a)^(9/2)*a + 990*(b*x + a)^(7/2)*a^2 - 1386*(b
*x + a)^(5/2)*a^3 + 1155*(b*x + a)^(3/2)*a^4 - 693*sqrt(b*x + a)*a^5)*sgn(
x)/b^3)/b
```

Mupad [B] (verification not implemented)

Time = 8.70 (sec) , antiderivative size = 58, normalized size of antiderivative = 0.54

$$\int \left(ax^2+bx^3\right)^{3/2}\,dx=-\frac{2\,\sqrt{b\,x^3+a\,x^2}\,{\left(a+b\,x\right)}^2\,\left(16\,a^3-40\,a^2\,b\,x+70\,a\,b^2\,x^2-105\,b^3\,x^3\right)}{1155\,b^4\,x}$$

```
int((a*x^2 + b*x^3)^(3/2),x)
```

```
-(2*(a*x^2 + b*x^3)^(1/2)*(a + b*x)^2*(16*a^3 - 105*b^3*x^3 + 70*a*b^2*x^2
 - 40*a^2*b*x))/(1155*b^4*x)
```

Reduce [B] (verification not implemented)

Time = 0.38 (sec) , antiderivative size = 63, normalized size of antiderivative = 0.58

$$\int \left(ax^2+bx^3\right)^{3/2}\,dx=\frac{2\sqrt{bx+a}\,(105b^5x^5+140a\,b^4x^4+5a^2b^3x^3-6a^3b^2x^2+8a^4bx-16a^5)}{1155b^4}$$

```
int((b*x^3+a*x^2)^(3/2),x)
```

```
(2*sqrt(a + b*x)*( - 16*a**5 + 8*a**4*b*x - 6*a**3*b**2*x**2 + 5*a**2*b**3
*x**3 + 140*a*b**4*x**4 + 105*b**5*x**5))/(1155*b**4)
```

3.278 $\int \frac{(ax^2+bx^3)^{3/2}}{x} dx$

Optimal result

Integrand size = 19, antiderivative size = 80

$$\int \frac{(ax^2+bx^3)^{3/2}}{x} dx = \frac{2a^2(ax^2+bx^3)^{5/2}}{5b^3x^5} - \frac{4a(ax^2+bx^3)^{7/2}}{7b^3x^7} + \frac{2(ax^2+bx^3)^{9/2}}{9b^3x^9}$$

```
2/5*a^2*(b*x^3+a*x^2)^(5/2)/b^3/x^5-4/7*a*(b*x^3+a*x^2)^(7/2)/b^3/x^7+2/9*
(b*x^3+a*x^2)^(9/2)/b^3/x^9
```

Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 47, normalized size of antiderivative = 0.59

$$\int \frac{(ax^2+bx^3)^{3/2}}{x} dx = \frac{2x(a+bx)^3(8a^2-20abx+35b^2x^2)}{315b^3\sqrt{x^2(a+bx)}}$$

```
Integrate[(a*x^2 + b*x^3)^(3/2)/x,x]
```

```
(2*x*(a + b*x)^3*(8*a^2 - 20*a*b*x + 35*b^2*x^2))/(315*b^3*Sqrt[x^2*(a + b
*x)])
```

Rubi [A] (verified)

Time = 0.39 (sec) , antiderivative size = 86, normalized size of antiderivative = 1.08, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.158, Rules used = {1922, 1922, 1920}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(ax^2+bx^3)^{3/2}}{x}dx$$

$$\Big\downarrow \text{1922}$$

$$\frac{2(ax^2+bx^3)^{5/2}}{9bx^3}-\frac{4a\int \frac{(bx^3+ax^2)^{3/2}}{x^2}dx}{9b}$$

$$\Big\downarrow \text{1922}$$

$$\frac{2(ax^2+bx^3)^{5/2}}{9bx^3}-\frac{4a\left(\frac{2(ax^2+bx^3)^{5/2}}{7bx^4}-\frac{2a\int \frac{(bx^3+ax^2)^{3/2}}{x^3}dx}{7b}\right)}{9b}$$

$$\Big\downarrow \text{1920}$$

$$\frac{2(ax^2+bx^3)^{5/2}}{9bx^3}-\frac{4a\left(\frac{2(ax^2+bx^3)^{5/2}}{7bx^4}-\frac{4a(ax^2+bx^3)^{5/2}}{35b^2x^5}\right)}{9b}$$

```
Int[(a*x^2 + b*x^3)^(3/2)/x,x]
```

```
(2*(a*x^2 + b*x^3)^(5/2))/(9*b*x^3) - (4*a*((-4*a*(a*x^2 + b*x^3)^(5/2))/(
35*b^2*x^5) + (2*(a*x^2 + b*x^3)^(5/2))/(7*b*x^4)))/(9*b)
```

Defintions of rubi rules used

```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> Simp[(-c^(j - 1))*(c*x)^(m - j + 1)*((a*x^j + b*x^n)^(p + 1)/(a*(n - j
)*(p + 1))), x] /; FreeQ[{a, b, c, j, m, n, p}, x] && !IntegerQ[p] && NeQ[
n, j] && EqQ[m + n*p + n - j + 1, 0] && (IntegerQ[j] || GtQ[c, 0])
```

```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> Simp[c^(j - 1)*(c*x)^(m - j + 1)*((a*x^j + b*x^n)^(p + 1)/(a*(m + j*p
+ 1))), x] - Simp[b*((m + n*p + n - j + 1)/(a*c^(n - j)*(m + j*p + 1)))   I
nt[(c*x)^(m + n - j)*(a*x^j + b*x^n)^p, x], x] /; FreeQ[{a, b, c, j, m, n,
p}, x] && !IntegerQ[p] && NeQ[n, j] && ILtQ[Simplify[(m + n*p + n - j + 1)
/(n - j)], 0] && NeQ[m + j*p + 1, 0] && (IntegersQ[j, n] || GtQ[c, 0])
```

Maple [C] (verified)

Result contains higher order function than in optimal. Order 3 vs. order 2.

Time = 0.38 (sec) , antiderivative size = 35, normalized size of antiderivative = 0.44

method	result	size
pseudoelliptic	$-2a^{\frac{3}{2}} \operatorname{arctanh}\left(\frac{\sqrt{bx+a}}{\sqrt{a}}\right)+\frac{2\sqrt{bx+a}\,(bx+4a)}{3}$	35
gosper	$\frac{2(bx+a)\left(35b^{2}x^{2}-20abx+8a^{2}\right)\left(b\,x^{3}+a\,x^{2}\right)^{\frac{3}{2}}}{315b^{3}x^{3}}$	46
default	$\frac{2(bx+a)\left(35b^{2}x^{2}-20abx+8a^{2}\right)\left(b\,x^{3}+a\,x^{2}\right)^{\frac{3}{2}}}{315b^{3}x^{3}}$	46
orering	$\frac{2(bx+a)\left(35b^{2}x^{2}-20abx+8a^{2}\right)\left(b\,x^{3}+a\,x^{2}\right)^{\frac{3}{2}}}{315b^{3}x^{3}}$	46
risch	$\frac{2\sqrt{x^{2}(bx+a)}\,\left(35b^{4}x^{4}+50a\,b^{3}x^{3}+3a^{2}b^{2}x^{2}-4a^{3}bx+8a^{4}\right)}{315x\,b^{3}}$	61
trager	$\frac{2\left(35b^{4}x^{4}+50a\,b^{3}x^{3}+3a^{2}b^{2}x^{2}-4a^{3}bx+8a^{4}\right)\sqrt{b\,x^{3}+a\,x^{2}}}{315b^{3}x}$	63

```
int((b*x^3+a*x^2)^(3/2)/x,x,method=_RETURNVERBOSE)
```

```
-2*a^(3/2)*arctanh((b*x+a)^(1/2)/a^(1/2))+2/3*(b*x+a)^(1/2)*(b*x+4*a)
```

Fricas [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 62, normalized size of antiderivative = 0.78

$$\int \frac{(ax^2+bx^3)^{3/2}}{x}\,dx = \frac{2\,(35\,b^4x^4+50\,ab^3x^3+3\,a^2b^2x^2-4\,a^3bx+8\,a^4)\sqrt{bx^3+ax^2}}{315\,b^3x}$$

```
integrate((b*x^3+a*x^2)^(3/2)/x,x, algorithm="fricas")
```

```
2/315*(35*b^4*x^4 + 50*a*b^3*x^3 + 3*a^2*b^2*x^2 - 4*a^3*b*x + 8*a^4)*sqrt
(b*x^3 + a*x^2)/(b^3*x)
```

Sympy [F]

$$\int \frac{(ax^2+bx^3)^{3/2}}{x}\,dx = \int \frac{(x^2(a+bx))^{\frac{3}{2}}}{x}\,dx$$

```
integrate((b*x**3+a*x**2)**(3/2)/x,x)
```

```
Integral((x**2*(a + b*x))**(3/2)/x, x)
```

Maxima [A] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 53, normalized size of antiderivative = 0.66

$$\int \frac{(ax^2+bx^3)^{3/2}}{x}\,dx = \frac{2\,(35\,b^4x^4+50\,ab^3x^3+3\,a^2b^2x^2-4\,a^3bx+8\,a^4)\sqrt{bx+a}}{315\,b^3}$$

```
integrate((b*x^3+a*x^2)^(3/2)/x,x, algorithm="maxima")
```

```
2/315*(35*b^4*x^4 + 50*a*b^3*x^3 + 3*a^2*b^2*x^2 - 4*a^3*b*x + 8*a^4)*sqrt
(b*x + a)/b^3
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 173 vs. $2(68) = 136$.

Time = 0.27 (sec) , antiderivative size = 173, normalized size of antiderivative = 2.16

$$\int \frac{(ax^2+bx^3)^{3/2}}{x}\,dx = -\frac{16\,a^{\frac{9}{2}}\mathrm{sgn}(x)}{315\,b^3} + \frac{2\left(\frac{21\left(3\,(bx+a)^{\frac{5}{2}}-10\,(bx+a)^{\frac{3}{2}}a+15\,\sqrt{bx+a}a^2\right)a^2\mathrm{sgn}(x)}{b^2}+\frac{18\left(5\,(bx+a)^{\frac{7}{2}}-21\,(bx+a)^{\frac{5}{2}}a+35\,(bx+a)^{\frac{3}{2}}a^2-35\,\sqrt{bx+a}a^3\right)a\mathrm{sgn}(x)}{b^2}+\frac{(35\,(b}{}\right.}{315\,b}$$

```
integrate((b*x^3+a*x^2)^(3/2)/x,x, algorithm="giac")
```

```
-16/315*a^(9/2)*sgn(x)/b^3 + 2/315*(21*(3*(b*x + a)^(5/2) - 10*(b*x + a)^(
3/2)*a + 15*sqrt(b*x + a)*a^2)*a^2*sgn(x)/b^2 + 18*(5*(b*x + a)^(7/2) - 21
*(b*x + a)^(5/2)*a + 35*(b*x + a)^(3/2)*a^2 - 35*sqrt(b*x + a)*a^3)*a*sgn(
x)/b^2 + (35*(b*x + a)^(9/2) - 180*(b*x + a)^(7/2)*a + 378*(b*x + a)^(5/2)
*a^2 - 420*(b*x + a)^(3/2)*a^3 + 315*sqrt(b*x + a)*a^4)*sgn(x)/b^2)/b
```

Mupad [B] (verification not implemented)

Time = 8.54 (sec) , antiderivative size = 47, normalized size of antiderivative = 0.59

$$\int \frac{(ax^2+bx^3)^{3/2}}{x}\,dx = \frac{2\,\sqrt{b\,x^3+a\,x^2}\,(a+b\,x)^2\,(8\,a^2-20\,a\,b\,x+35\,b^2\,x^2)}{315\,b^3\,x}$$

```
int((a*x^2 + b*x^3)^(3/2)/x,x)
```

```
(2*(a*x^2 + b*x^3)^(1/2)*(a + b*x)^2*(8*a^2 + 35*b^2*x^2 - 20*a*b*x))/(315
*b^3*x)
```

Reduce [B] (verification not implemented)

Time = 0.53 (sec) , antiderivative size = 52, normalized size of antiderivative = 0.65

$$\int \frac{(ax^2+bx^3)^{3/2}}{x}\,dx = \frac{2\sqrt{bx+a}\,(35b^4x^4+50a\,b^3x^3+3a^2b^2x^2-4a^3bx+8a^4)}{315b^3}$$

```
int((b*x^3+a*x^2)^(3/2)/x,x)
```

```
(2*sqrt(a + b*x)*(8*a**4 - 4*a**3*b*x + 3*a**2*b**2*x**2 + 50*a*b**3*x**3
+ 35*b**4*x**4))/(315*b**3)
```

3.279 $\int \frac{(ax^2+bx^3)^{3/2}}{x^2} \, dx$

Optimal result

Integrand size = 19, antiderivative size = 52

$$\int \frac{(ax^2+bx^3)^{3/2}}{x^2} \, dx = -\frac{2a(ax^2+bx^3)^{5/2}}{5b^2x^5} + \frac{2(ax^2+bx^3)^{7/2}}{7b^2x^7}$$

```
-2/5*a*(b*x^3+a*x^2)^(5/2)/b^2/x^5+2/7*(b*x^3+a*x^2)^(7/2)/b^2/x^7
```

Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 31, normalized size of antiderivative = 0.60

$$\int \frac{(ax^2+bx^3)^{3/2}}{x^2} \, dx = \frac{2(x^2(a+bx))^{5/2}(-2a+5bx)}{35b^2x^5}$$

```
Integrate[(a*x^2 + b*x^3)^(3/2)/x^2,x]
```

```
(2*(x^2*(a + b*x))^(5/2)*(-2*a + 5*b*x))/(35*b^2*x^5)
```

Rubi [A] (verified)

Time = 0.32 (sec) , antiderivative size = 52, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.105, Rules used = {1922, 1920}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(ax^2+bx^3)^{3/2}}{x^2}\,dx$$

$$\Big\downarrow \text{1922}$$

$$\frac{2(ax^2+bx^3)^{5/2}}{7bx^4}-\frac{2a\int \frac{(bx^3+ax^2)^{3/2}}{x^3}dx}{7b}$$

$$\Big\downarrow \text{1920}$$

$$\frac{2(ax^2+bx^3)^{5/2}}{7bx^4}-\frac{4a(ax^2+bx^3)^{5/2}}{35b^2x^5}$$

```
Int[(a*x^2 + b*x^3)^(3/2)/x^2,x]
```

```
(-4*a*(a*x^2 + b*x^3)^(5/2))/(35*b^2*x^5) + (2*(a*x^2 + b*x^3)^(5/2))/(7*b
*x^4)
```

Defintions of rubi rules used

```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> Simp[(-c^(j - 1))*(c*x)^(m - j + 1)*((a*x^j + b*x^n)^(p + 1)/(a*(n - j
)*(p + 1))), x] /; FreeQ[{a, b, c, j, m, n, p}, x] &&  !IntegerQ[p] && NeQ[
n, j] && EqQ[m + n*p + n - j + 1, 0] && (IntegerQ[j] || GtQ[c, 0])
```

```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> Simp[c^(j - 1)*(c*x)^(m - j + 1)*((a*x^j + b*x^n)^(p + 1)/(a*(m + j*p
+ 1))), x] - Simp[b*((m + n*p + n - j + 1)/(a*c^(n - j)*(m + j*p + 1)))   I
nt[(c*x)^(m + n - j)*(a*x^j + b*x^n)^p, x], x] /; FreeQ[{a, b, c, j, m, n,
p}, x] &&  !IntegerQ[p] && NeQ[n, j] && ILtQ[Simplify[(m + n*p + n - j + 1)
/(n - j)], 0] && NeQ[m + j*p + 1, 0] && (IntegersQ[j, n] || GtQ[c, 0])
```

Maple [A] (verified)

Time = 0.42 (sec) , antiderivative size = 35, normalized size of antiderivative = 0.67

method	result	size
gosper	$-\frac{2(bx+a)(-5bx+2a)\left(b\,x^{3}+a\,x^{2}\right)^{\frac{3}{2}}}{35b^{2}x^{3}}$	35
default	$-\frac{2(bx+a)(-5bx+2a)\left(b\,x^{3}+a\,x^{2}\right)^{\frac{3}{2}}}{35b^{2}x^{3}}$	35
orering	$-\frac{2(bx+a)(-5bx+2a)\left(b\,x^{3}+a\,x^{2}\right)^{\frac{3}{2}}}{35b^{2}x^{3}}$	35
pseudoelliptic	$-\frac{3\,\operatorname{arctanh}\left(\frac{\sqrt{bx+a}}{\sqrt{a}}\right)abx+\sqrt{bx+a}\,(-2bx+a)\sqrt{a}}{\sqrt{a}\,x}$	44
risch	$-\frac{2\sqrt{x^{2}(bx+a)}\,\left(-5b^{3}x^{3}-8a\,b^{2}x^{2}-a^{2}bx+2a^{3}\right)}{35x\,b^{2}}$	50
trager	$-\frac{2\left(-5b^{3}x^{3}-8a\,b^{2}x^{2}-a^{2}bx+2a^{3}\right)\sqrt{b\,x^{3}+a\,x^{2}}}{35b^{2}x}$	52

```
int((b*x^3+a*x^2)^(3/2)/x^2,x,method=_RETURNVERBOSE)
```

```
-2/35*(b*x+a)*(-5*b*x+2*a)*(b*x^3+a*x^2)^(3/2)/b^2/x^3
```

Fricas [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 50, normalized size of antiderivative = 0.96

$$\int \frac{(ax^2+bx^3)^{3/2}}{x^2}\,dx = \frac{2\,(5\,b^3x^3+8\,ab^2x^2+a^2bx-2\,a^3)\sqrt{bx^3+ax^2}}{35\,b^2x}$$

```
integrate((b*x^3+a*x^2)^(3/2)/x^2,x, algorithm="fricas")
```

```
2/35*(5*b^3*x^3 + 8*a*b^2*x^2 + a^2*b*x - 2*a^3)*sqrt(b*x^3 + a*x^2)/(b^2*
x)
```

Sympy [F]

$$\int \frac{(ax^2+bx^3)^{3/2}}{x^2}\,dx = \int \frac{(x^2(a+bx))^{\frac{3}{2}}}{x^2}\,dx$$

```
integrate((b*x**3+a*x**2)**(3/2)/x**2,x)
```

```
Integral((x**2*(a + b*x))**(3/2)/x**2, x)
```

Maxima [A] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 41, normalized size of antiderivative = 0.79

$$\int \frac{(ax^2+bx^3)^{3/2}}{x^2}\,dx = \frac{2\,(5\,b^3x^3+8\,ab^2x^2+a^2bx-2\,a^3)\sqrt{bx+a}}{35\,b^2}$$

```
integrate((b*x^3+a*x^2)^(3/2)/x^2,x, algorithm="maxima")
```

```
2/35*(5*b^3*x^3 + 8*a*b^2*x^2 + a^2*b*x - 2*a^3)*sqrt(b*x + a)/b^2
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 136 vs. 2(44) = 88.

Time = 0.13 (sec) , antiderivative size = 136, normalized size of antiderivative = 2.62

$$\int \frac{(ax^2+bx^3)^{3/2}}{x^2}\,dx = \frac{4\,a^{\frac{7}{2}}\mathrm{sgn}(x)}{35\,b^2} + \frac{2\left(\frac{35\left((bx+a)^{\frac{3}{2}}-3\sqrt{bx+a}a\right)a^2\mathrm{sgn}(x)}{b} + \frac{14\left(3\,(bx+a)^{\frac{5}{2}}-10\,(bx+a)^{\frac{3}{2}}a+15\sqrt{bx+a}a^2\right)a\mathrm{sgn}(x)}{b} + \frac{3\left(5\,(bx+a)^{\frac{7}{2}}-21\,(bx+a)^{\frac{5}{2}}a+35\,(bx+a)\right.}{b}\right.}{105\,b}$$

```
integrate((b*x^3+a*x^2)^(3/2)/x^2,x, algorithm="giac")
```

```
4/35*a^(7/2)*sgn(x)/b^2 + 2/105*(35*((b*x + a)^(3/2) - 3*sqrt(b*x + a)*a)*
a^2*sgn(x)/b + 14*(3*(b*x + a)^(5/2) - 10*(b*x + a)^(3/2)*a + 15*sqrt(b*x
+ a)*a^2)*a*sgn(x)/b + 3*(5*(b*x + a)^(7/2) - 21*(b*x + a)^(5/2)*a + 35*(b
*x + a)^(3/2)*a^2 - 35*sqrt(b*x + a)*a^3)*sgn(x)/b)/b
```

Mupad [B] (verification not implemented)

Time = 8.71 (sec) , antiderivative size = 36, normalized size of antiderivative = 0.69

$$\int \frac{(ax^2+bx^3)^{3/2}}{x^2}\,dx = -\frac{2\,(2\,a-5\,b\,x)\,\sqrt{b\,x^3+a\,x^2}\,{(a+b\,x)}^2}{35\,b^2\,x}$$

```
int((a*x^2 + b*x^3)^(3/2)/x^2,x)
```

```
-(2*(2*a - 5*b*x)*(a*x^2 + b*x^3)^(1/2)*(a + b*x)^2)/(35*b^2*x)
```

Reduce [B] (verification not implemented)

Time = 0.50 (sec) , antiderivative size = 40, normalized size of antiderivative = 0.77

$$\int \frac{(ax^2+bx^3)^{3/2}}{x^2}\,dx = \frac{2\sqrt{bx+a}\,(5b^3x^3+8a\,b^2x^2+a^2bx-2a^3)}{35b^2}$$

```
int((b*x^3+a*x^2)^(3/2)/x^2,x)
```

```
(2*sqrt(a + b*x)*( - 2*a**3 + a**2*b*x + 8*a*b**2*x**2 + 5*b**3*x**3))/(35
*b**2)
```

3.280 $\int \frac{(ax^2+bx^3)^{3/2}}{x^3}\,dx$

Optimal result

Integrand size = 19, antiderivative size = 25

$$\int \frac{(ax^2+bx^3)^{3/2}}{x^3}\,dx = \frac{2(ax^2+bx^3)^{5/2}}{5bx^5}$$

```
2/5*(b*x^3+a*x^2)^(5/2)/b/x^5
```

Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 23, normalized size of antiderivative = 0.92

$$\int \frac{(ax^2+bx^3)^{3/2}}{x^3}\,dx = \frac{2(x^2(a+bx))^{5/2}}{5bx^5}$$

```
Integrate[(a*x^2 + b*x^3)^(3/2)/x^3,x]
```

```
(2*(x^2*(a + b*x))^(5/2))/(5*b*x^5)
```

Rubi [A] (verified)

Time = 0.25 (sec) , antiderivative size = 25, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.053, Rules used = {1920}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(ax^2+bx^3)^{3/2}}{x^3}\,dx$$
$$\Big\downarrow \color{blue}{1920}$$
$$\frac{2(ax^2+bx^3)^{5/2}}{5bx^5}$$

```
Int[(a*x^2 + b*x^3)^(3/2)/x^3,x]
```

```
(2*(a*x^2 + b*x^3)^(5/2))/(5*b*x^5)
```

Defintions of rubi rules used

```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> Simp[(-c^(j - 1))*(c*x)^(m - j + 1)*((a*x^j + b*x^n)^(p + 1)/(a*(n - j
)*(p + 1))), x] /; FreeQ[{a, b, c, j, m, n, p}, x] &&  !IntegerQ[p] && NeQ[
n, j] && EqQ[m + n*p + n - j + 1, 0] && (IntegerQ[j] || GtQ[c, 0])
```

Maple [A] (verified)

Time = 0.41 (sec) , antiderivative size = 27, normalized size of antiderivative = 1.08

method	result	size
gosper	$\frac{2(bx+a)\left(b\,x^{3}+a\,x^{2}\right)^{\frac{3}{2}}}{5b\,x^{3}}$	27
default	$\frac{2(bx+a)\left(b\,x^{3}+a\,x^{2}\right)^{\frac{3}{2}}}{5b\,x^{3}}$	27
orering	$\frac{2(bx+a)\left(b\,x^{3}+a\,x^{2}\right)^{\frac{3}{2}}}{5b\,x^{3}}$	27
risch	$\frac{2\sqrt{x^{2}(bx+a)}\,\left(b^{2}x^{2}+2abx+a^{2}\right)}{5xb}$	36
trager	$\frac{2\left(b^{2}x^{2}+2abx+a^{2}\right)\sqrt{b\,x^{3}+a\,x^{2}}}{5bx}$	38
pseudoelliptic	$\frac{-3\,\operatorname{arctanh}\left(\frac{\sqrt{bx+a}}{\sqrt{a}}\right)b^{2}x^{2}-5bx\sqrt{bx+a}\,\sqrt{a}-2a^{\frac{3}{2}}\sqrt{bx+a}}{4x^{2}\sqrt{a}}$	56

```
int((b*x^3+a*x^2)^(3/2)/x^3,x,method=_RETURNVERBOSE)
```

```
2/5*(b*x+a)/b*(b*x^3+a*x^2)^(3/2)/x^3
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 37, normalized size of antiderivative = 1.48

$$\int \frac{(ax^2+bx^3)^{3/2}}{x^3}\,dx = \frac{2\,(b^2x^2+2\,abx+a^2)\sqrt{bx^3+ax^2}}{5\,bx}$$

```
integrate((b*x^3+a*x^2)^(3/2)/x^3,x, algorithm="fricas")
```

```
2/5*(b^2*x^2 + 2*a*b*x + a^2)*sqrt(b*x^3 + a*x^2)/(b*x)
```

Sympy [F]

$$\int \frac{(ax^2+bx^3)^{3/2}}{x^3}\,dx=\int \frac{\left(x^2(a+bx)\right)^{\frac{3}{2}}}{x^3}\,dx$$

```
integrate((b*x**3+a*x**2)**(3/2)/x**3,x)
```

```
Integral((x**2*(a + b*x))**(3/2)/x**3, x)
```

Maxima [A] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 28, normalized size of antiderivative = 1.12

$$\int \frac{(ax^2+bx^3)^{3/2}}{x^3}\,dx=\frac{2\,(b^2x^2+2\,abx+a^2)\sqrt{bx+a}}{5\,b}$$

```
integrate((b*x^3+a*x^2)^(3/2)/x^3,x, algorithm="maxima")
```

```
2/5*(b^2*x^2 + 2*a*b*x + a^2)*sqrt(b*x + a)/b
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 89 vs. 2(21) = 42.

Time = 0.12 (sec) , antiderivative size = 89, normalized size of antiderivative = 3.56

$$\int \frac{(ax^2+bx^3)^{3/2}}{x^3}\,dx=-\frac{2\,a^{\frac{5}{2}}\mathrm{sgn}(x)}{5\,b} + \frac{2\left(15\,\sqrt{bx+a}a^2\mathrm{sgn}(x)+10\left((bx+a)^{\frac{3}{2}}-3\,\sqrt{bx+a}a\right)a\mathrm{sgn}(x)+\left(3\,(bx+a)^{\frac{5}{2}}-10\,(bx+a)^{\frac{3}{2}}a+15\,\sqrt{b}\right.}{15\,b}$$

```
integrate((b*x^3+a*x^2)^(3/2)/x^3,x, algorithm="giac")
```

```
-2/5*a^(5/2)*sgn(x)/b + 2/15*(15*sqrt(b*x + a)*a^2*sgn(x) + 10*((b*x + a)^
(3/2) - 3*sqrt(b*x + a)*a)*a*sgn(x) + (3*(b*x + a)^(5/2) - 10*(b*x + a)^(3
/2)*a + 15*sqrt(b*x + a)*a^2)*sgn(x))/b
```

Mupad [B] (verification not implemented)

Time = 9.13 (sec) , antiderivative size = 28, normalized size of antiderivative = 1.12

$$\int \frac{(ax^2+bx^3)^{3/2}}{x^3}\,dx = \frac{2\,\sqrt{b\,x^3+a\,x^2}\,{\left(a+b\,x\right)}^2}{5\,b\,x}$$

```
int((a*x^2 + b*x^3)^(3/2)/x^3,x)
```

```
(2*(a*x^2 + b*x^3)^(1/2)*(a + b*x)^2)/(5*b*x)
```

Reduce [B] (verification not implemented)

Time = 0.30 (sec) , antiderivative size = 27, normalized size of antiderivative = 1.08

$$\int \frac{(ax^2+bx^3)^{3/2}}{x^3}\,dx = \frac{2\sqrt{bx+a}\,(b^2x^2+2abx+a^2)}{5b}$$

```
int((b*x^3+a*x^2)^(3/2)/x^3,x)
```

```
(2*sqrt(a + b*x)*(a**2 + 2*a*b*x + b**2*x**2))/(5*b)
```

3.281 $\int \frac{(ax^2+bx^3)^{3/2}}{x^4}\,dx$

Optimal result

Integrand size = 19, antiderivative size = 76

$$\int \frac{(ax^2+bx^3)^{3/2}}{x^4}\,dx = \frac{2a\sqrt{ax^2+bx^3}}{x} + \frac{2(ax^2+bx^3)^{3/2}}{3x^3} - 2a^{3/2}\text{arctanh}\left(\frac{\sqrt{ax^2+bx^3}}{\sqrt{a}x}\right)$$

```
2*a*(b*x^3+a*x^2)^(1/2)/x+2/3*(b*x^3+a*x^2)^(3/2)/x^3-2*a^(3/2)*arctanh((b
*x^3+a*x^2)^(1/2)/a^(1/2)/x)
```

Mathematica [A] (verified)

Time = 0.05 (sec) , antiderivative size = 68, normalized size of antiderivative = 0.89

$$\int \frac{(ax^2+bx^3)^{3/2}}{x^4}\,dx = \frac{2x\sqrt{a+bx}\left(\sqrt{a+bx}(4a+bx) - 3a^{3/2}\text{arctanh}\left(\frac{\sqrt{a+bx}}{\sqrt{a}}\right)\right)}{3\sqrt{x^2(a+bx)}}$$

```
Integrate[(a*x^2 + b*x^3)^(3/2)/x^4,x]
```

```
(2*x*Sqrt[a + b*x]*(Sqrt[a + b*x]*(4*a + b*x) - 3*a^(3/2)*ArcTanh[Sqrt[a +
 b*x]/Sqrt[a]]))/(3*Sqrt[x^2*(a + b*x)])
```

Rubi [A] (verified)

Time = 0.39 (sec) , antiderivative size = 76, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.211, Rules used = {1927, 1927, 1914, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(ax^2+bx^3)^{3/2}}{x^4}dx$$

$$\Big\downarrow \ 1927$$

$$a\int \frac{\sqrt{bx^3+ax^2}}{x^2}dx+\frac{2(ax^2+bx^3)^{3/2}}{3x^3}$$

$$\Big\downarrow \ 1927$$

$$a\left(a\int \frac{1}{\sqrt{bx^3+ax^2}}dx+\frac{2\sqrt{ax^2+bx^3}}{x}\right)+\frac{2(ax^2+bx^3)^{3/2}}{3x^3}$$

$$\Big\downarrow \ 1914$$

$$a\left(\frac{2\sqrt{ax^2+bx^3}}{x}-2a\int \frac{1}{1-\frac{ax^2}{bx^3+ax^2}}d\frac{x}{\sqrt{bx^3+ax^2}}\right)+\frac{2(ax^2+bx^3)^{3/2}}{3x^3}$$

$$\Big\downarrow \ 219$$

$$a\left(\frac{2\sqrt{ax^2+bx^3}}{x}-2\sqrt{a}\text{arctanh}\left(\frac{\sqrt{a}x}{\sqrt{ax^2+bx^3}}\right)\right)+\frac{2(ax^2+bx^3)^{3/2}}{3x^3}$$

```
Int[(a*x^2 + b*x^3)^(3/2)/x^4,x]
```

```
(2*(a*x^2 + b*x^3)^(3/2))/(3*x^3) + a*((2*Sqrt[a*x^2 + b*x^3])/x - 2*Sqrt[
a]*ArcTanh[(Sqrt[a]*x)/Sqrt[a*x^2 + b*x^3]])
```

Defintions of rubi rules used

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

```
Int[1/Sqrt[(a_.)*(x_)^2 + (b_.)*(x_)^(n_.)], x_Symbol] :> Simp[2/(2 - n)
Subst[Int[1/(1 - a*x^2), x], x, x/Sqrt[a*x^2 + b*x^n]], x] /; FreeQ[{a, b,
n}, x] && NeQ[n, 2]
```

```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> Simp[(c*x)^(m + 1)*((a*x^j + b*x^n)^p/(c*(m + n*p + 1))), x] + Simp[a*
(n - j)*(p/(c^j*(m + n*p + 1)))   Int[(c*x)^(m + j)*(a*x^j + b*x^n)^(p - 1)
, x], x] /; FreeQ[{a, b, c, m}, x] &&  !IntegerQ[p] && LtQ[0, j, n] && (Int
egersQ[j, n] || GtQ[c, 0]) && GtQ[p, 0] && NeQ[m + n*p + 1, 0]
```

Maple [A] (verified)

Time = 0.43 (sec) , antiderivative size = 61, normalized size of antiderivative = 0.80

method	result	size
default	$\frac{2\left(b\,x^{3}+a\,x^{2}\right)^{\frac{3}{2}}\left(-3a^{\frac{3}{2}}\,\mathrm{arctanh}\left(\frac{\sqrt{bx+a}}{\sqrt{a}}\right)+\left(bx+a\right)^{\frac{3}{2}}+3a\sqrt{bx+a}\right)}{3x^{3}\left(bx+a\right)^{\frac{3}{2}}}$	61
pseudoelliptic	$\frac{\mathrm{arctanh}\left(\frac{\sqrt{bx+a}}{\sqrt{a}}\right)b^{3}x^{3}-\left(\sqrt{a}\,b^{2}x^{2}+\frac{14a^{\frac{3}{2}}\,bx}{3}+\frac{8a^{\frac{5}{2}}}{3}\right)\sqrt{bx+a}}{8a^{\frac{3}{2}}\,x^{3}}$	61

```
int((b*x^3+a*x^2)^(3/2)/x^4,x,method=_RETURNVERBOSE)
```

```
2/3*(b*x^3+a*x^2)^(3/2)*(-3*a^(3/2)*arctanh((b*x+a)^(1/2)/a^(1/2))+(b*x+a)
^(3/2)+3*a*(b*x+a)^(1/2))/x^3/(b*x+a)^(3/2)
```

Fricas [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 135, normalized size of antiderivative = 1.78

$$\int \frac{(ax^2+bx^3)^{3/2}}{x^4}\,dx = \left[\frac{3\,a^{\frac{3}{2}}x\log\left(\frac{bx^2+2\,ax-2\,\sqrt{bx^3+ax^2}\sqrt{a}}{x^2}\right)+2\,\sqrt{bx^3+ax^2}(bx+4\,a)}{3\,x}, \frac{2\left(3\,\sqrt{-a}ax\arctan\left(\right.\right.}{}\right.$$

```
integrate((b*x^3+a*x^2)^(3/2)/x^4,x, algorithm="fricas")
```

```
[1/3*(3*a^(3/2)*x*log((b*x^2 + 2*a*x - 2*sqrt(b*x^3 + a*x^2)*sqrt(a))/x^2)
 + 2*sqrt(b*x^3 + a*x^2)*(b*x + 4*a))/x, 2/3*(3*sqrt(-a)*a*x*arctan(sqrt(b
*x^3 + a*x^2)*sqrt(-a)/(b*x^2 + a*x)) + sqrt(b*x^3 + a*x^2)*(b*x + 4*a))/x
]
```

Sympy [F]

$$\int \frac{(ax^2+bx^3)^{3/2}}{x^4}\,dx = \int \frac{(x^2(a+bx))^{\frac{3}{2}}}{x^4}\,dx$$

```
integrate((b*x**3+a*x**2)**(3/2)/x**4,x)
```

```
Integral((x**2*(a + b*x))**(3/2)/x**4, x)
```

Maxima [F]

$$\int \frac{(ax^2+bx^3)^{3/2}}{x^4}\,dx = \int \frac{(bx^3+ax^2)^{\frac{3}{2}}}{x^4}\,dx$$

```
integrate((b*x^3+a*x^2)^(3/2)/x^4,x, algorithm="maxima")
```

```
integrate((b*x^3 + a*x^2)^(3/2)/x^4, x)
```

Giac [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 85, normalized size of antiderivative = 1.12

$$\int \frac{(ax^2+bx^3)^{3/2}}{x^4}\,dx = \frac{2\,a^2\arctan\left(\frac{\sqrt{bx+a}}{\sqrt{-a}}\right)\operatorname{sgn}(x)}{\sqrt{-a}} + \frac{2}{3}\,(bx+a)^{\frac{3}{2}}\operatorname{sgn}(x) + 2\,\sqrt{bx+a}\,a\operatorname{sgn}(x) - \frac{2\left(3\,a^2\arctan\left(\frac{\sqrt{a}}{\sqrt{-a}}\right)+4\,\sqrt{-a}a^{\frac{3}{2}}\right)\operatorname{sgn}(x)}{3\,\sqrt{-a}}$$

```
integrate((b*x^3+a*x^2)^(3/2)/x^4,x, algorithm="giac")
```

```
2*a^2*arctan(sqrt(b*x + a)/sqrt(-a))*sgn(x)/sqrt(-a) + 2/3*(b*x + a)^(3/2)
*sgn(x) + 2*sqrt(b*x + a)*a*sgn(x) - 2/3*(3*a^2*arctan(sqrt(a)/sqrt(-a)) +
 4*sqrt(-a)*a^(3/2))*sgn(x)/sqrt(-a)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(ax^2+bx^3)^{3/2}}{x^4}\,dx = \int \frac{{\left(b\,x^3+a\,x^2\right)}^{3/2}}{x^4}\,dx$$

```
int((a*x^2 + b*x^3)^(3/2)/x^4,x)
```

```
int((a*x^2 + b*x^3)^(3/2)/x^4, x)
```

Reduce [B] (verification not implemented)

Time = 0.25 (sec) , antiderivative size = 51, normalized size of antiderivative = 0.67

$$\int \frac{(ax^2+bx^3)^{3/2}}{x^4}\,dx = \frac{8\sqrt{bx+a}\,a}{3} + \frac{2\sqrt{bx+a}\,bx}{3} + \sqrt{a}\log\left(\sqrt{bx+a}-\sqrt{a}\right)a - \sqrt{a}\log\left(\sqrt{bx+a}+\sqrt{a}\right)a$$

```
int((b*x^3+a*x^2)^(3/2)/x^4,x)
```

```
(8*sqrt(a + b*x)*a + 2*sqrt(a + b*x)*b*x + 3*sqrt(a)*log(sqrt(a + b*x) - s
qrt(a))*a - 3*sqrt(a)*log(sqrt(a + b*x) + sqrt(a))*a)/3
```

3.282 $\int \frac{(ax^2+bx^3)^{3/2}}{x^5}\,dx$

Optimal result

Integrand size = 19, antiderivative size = 76

$$\int \frac{(ax^2+bx^3)^{3/2}}{x^5}\,dx = -\frac{a\sqrt{ax^2+bx^3}}{x^2}+\frac{2b\sqrt{ax^2+bx^3}}{x}-3\sqrt{a}b\,\text{arctanh}\left(\frac{\sqrt{ax^2+bx^3}}{\sqrt{a}\,x}\right)$$

```
-a*(b*x^3+a*x^2)^(1/2)/x^2+2*b*(b*x^3+a*x^2)^(1/2)/x-3*a^(1/2)*b*arctanh((
b*x^3+a*x^2)^(1/2)/a^(1/2)/x)
```

Mathematica [A] (verified)

Time = 0.03 (sec) , antiderivative size = 66, normalized size of antiderivative = 0.87

$$\int \frac{(ax^2+bx^3)^{3/2}}{x^5}\,dx = -\frac{\sqrt{a+bx}\left((a-2bx)\sqrt{a+bx}+3\sqrt{a}bx\,\text{arctanh}\left(\frac{\sqrt{a+bx}}{\sqrt{a}}\right)\right)}{\sqrt{x^2(a+bx)}}$$

```
Integrate[(a*x^2 + b*x^3)^(3/2)/x^5,x]
```

```
-((Sqrt[a + b*x]*((a - 2*b*x)*Sqrt[a + b*x] + 3*Sqrt[a]*b*x*ArcTanh[Sqrt[a
 + b*x]/Sqrt[a]]))/Sqrt[x^2*(a + b*x)])
```

Rubi [A] (verified)

Time = 0.39 (sec) , antiderivative size = 77, normalized size of antiderivative = 1.01, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.211$, Rules used = {1926, 1927, 1914, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(ax^2+bx^3)^{3/2}}{x^5}dx$$
$$\Big\downarrow \text{1926}$$
$$\frac{3}{2}b\int \frac{\sqrt{bx^3+ax^2}}{x^2}dx-\frac{(ax^2+bx^3)^{3/2}}{x^4}$$
$$\Big\downarrow \text{1927}$$
$$\frac{3}{2}b\left(a\int \frac{1}{\sqrt{bx^3+ax^2}}dx+\frac{2\sqrt{ax^2+bx^3}}{x}\right)-\frac{(ax^2+bx^3)^{3/2}}{x^4}$$
$$\Big\downarrow \text{1914}$$
$$\frac{3}{2}b\left(\frac{2\sqrt{ax^2+bx^3}}{x}-2a\int \frac{1}{1-\frac{ax^2}{bx^3+ax^2}}d\frac{x}{\sqrt{bx^3+ax^2}}\right)-\frac{(ax^2+bx^3)^{3/2}}{x^4}$$
$$\Big\downarrow \text{219}$$
$$\frac{3}{2}b\left(\frac{2\sqrt{ax^2+bx^3}}{x}-2\sqrt{a}\text{arctanh}\left(\frac{\sqrt{a}x}{\sqrt{ax^2+bx^3}}\right)\right)-\frac{(ax^2+bx^3)^{3/2}}{x^4}$$

```
Int[(a*x^2 + b*x^3)^(3/2)/x^5,x]
```

```
-((a*x^2 + b*x^3)^(3/2)/x^4) + (3*b*((2*Sqrt[a*x^2 + b*x^3])/x - 2*Sqrt[a]
*ArcTanh[(Sqrt[a]*x)/Sqrt[a*x^2 + b*x^3]]))/2
```

Defintions of rubi rules used

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

```
Int[1/Sqrt[(a_.)*(x_)^2 + (b_.)*(x_)^(n_.)], x_Symbol] :> Simp[2/(2 - n)
Subst[Int[1/(1 - a*x^2), x], x, x/Sqrt[a*x^2 + b*x^n]], x] /; FreeQ[{a, b,
n}, x] && NeQ[n, 2]
```

```
Int[((c_.)*(x_))^(m_)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol]
 :> Simp[(c*x)^(m + 1)*((a*x^j + b*x^n)^p/(c*(m + j*p + 1))), x] - Simp[b*p
*((n - j)/(c^n*(m + j*p + 1)))   Int[(c*x)^(m + n)*(a*x^j + b*x^n)^(p - 1),
 x], x] /; FreeQ[{a, b, c}, x] &&  !IntegerQ[p] && LtQ[0, j, n] && (Integer
sQ[j, n] || GtQ[c, 0]) && GtQ[p, 0] && LtQ[m + j*p + 1, 0]
```

```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> Simp[(c*x)^(m + 1)*((a*x^j + b*x^n)^p/(c*(m + n*p + 1))), x] + Simp[a*
(n - j)*(p/(c^j*(m + n*p + 1)))   Int[(c*x)^(m + j)*(a*x^j + b*x^n)^(p - 1)
, x], x] /; FreeQ[{a, b, c, m}, x] &&  !IntegerQ[p] && LtQ[0, j, n] && (Int
egersQ[j, n] || GtQ[c, 0]) && GtQ[p, 0] && NeQ[m + n*p + 1, 0]
```

Maple [A] (verified)

Time = 0.47 (sec) , antiderivative size = 70, normalized size of antiderivative = 0.92

method	result	size
risch	$-\frac{a\sqrt{x^{2}(bx+a)}}{x^{2}}+\frac{b\left(4\sqrt{bx+a}-6\sqrt{a}\,\operatorname{arctanh}\left(\frac{\sqrt{bx+a}}{\sqrt{a}}\right)\right)\sqrt{x^{2}(bx+a)}}{2x\sqrt{bx+a}}$	70
default	$-\frac{\left(b\,x^{3}+a\,x^{2}\right)^{\frac{3}{2}}\left(-2bx\sqrt{bx+a}\,\sqrt{a}+3\,\operatorname{arctanh}\left(\frac{\sqrt{bx+a}}{\sqrt{a}}\right)abx+a^{\frac{3}{2}}\sqrt{bx+a}\right)}{x^{4}(bx+a)^{\frac{3}{2}}\sqrt{a}}$	72
pseudoelliptic	$\frac{-\frac{3\,\operatorname{arctanh}\left(\frac{\sqrt{bx+a}}{\sqrt{a}}\right)b^{4}x^{4}}{64}+\frac{3\sqrt{bx+a}\,\left(\sqrt{a}\,b^{3}x^{3}-\frac{2a^{\frac{3}{2}}b^{2}x^{2}}{3}-8a^{\frac{5}{2}}bx-\frac{16a^{\frac{7}{2}}}{3}\right)}{64}}{a^{\frac{5}{2}}x^{4}}$	72

```
int((b*x^3+a*x^2)^(3/2)/x^5,x,method=_RETURNVERBOSE)
```

```
-a/x^2*(x^2*(b*x+a))^(1/2)+1/2*b*(4*(b*x+a)^(1/2)-6*a^(1/2)*arctanh((b*x+a
)^(1/2)/a^(1/2)))*(x^2*(b*x+a))^(1/2)/x/(b*x+a)^(1/2)
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 141, normalized size of antiderivative = 1.86

$$\int \frac{(ax^2+bx^3)^{3/2}}{x^5}\,dx = \left[\frac{3\sqrt{a}bx^2\log\left(\frac{bx^2+2\,ax-2\,\sqrt{bx^3+ax^2}\sqrt{a}}{x^2}\right)+2\,\sqrt{bx^3+ax^2}(2\,bx-a)}{2\,x^2}, \frac{3\sqrt{-a}bx^2\arctan\left(\right.}{}\right.$$

```
integrate((b*x^3+a*x^2)^(3/2)/x^5,x, algorithm="fricas")
```

```
[1/2*(3*sqrt(a)*b*x^2*log((b*x^2 + 2*a*x - 2*sqrt(b*x^3 + a*x^2)*sqrt(a))/
x^2) + 2*sqrt(b*x^3 + a*x^2)*(2*b*x - a))/x^2, (3*sqrt(-a)*b*x^2*arctan(sq
rt(b*x^3 + a*x^2)*sqrt(-a)/(b*x^2 + a*x)) + sqrt(b*x^3 + a*x^2)*(2*b*x - a
))/x^2]
```

Sympy [F]

$$\int \frac{(ax^2+bx^3)^{3/2}}{x^5}\,dx = \int \frac{(x^2(a+bx))^{\frac{3}{2}}}{x^5}\,dx$$

```
integrate((b*x**3+a*x**2)**(3/2)/x**5,x)
```

```
Integral((x**2*(a + b*x))**(3/2)/x**5, x)
```

Maxima [F]

$$\int \frac{(ax^2+bx^3)^{3/2}}{x^5}\,dx = \int \frac{(bx^3+ax^2)^{\frac{3}{2}}}{x^5}\,dx$$

```
integrate((b*x^3+a*x^2)^(3/2)/x^5,x, algorithm="maxima")
```

```
integrate((b*x^3 + a*x^2)^(3/2)/x^5, x)
```

Giac [A] (verification not implemented)

Time = 0.15 (sec) , antiderivative size = 56, normalized size of antiderivative = 0.74

$$\int \frac{(ax^2+bx^3)^{3/2}}{x^5}\,dx = \left(\frac{3\,a\arctan\left(\frac{\sqrt{bx+a}}{\sqrt{-a}}\right)\operatorname{sgn}(x)}{\sqrt{-a}} + 2\sqrt{bx+a}\operatorname{sgn}(x) - \frac{\sqrt{bx+a}\,a\operatorname{sgn}(x)}{bx}\right)b$$

```
integrate((b*x^3+a*x^2)^(3/2)/x^5,x, algorithm="giac")
```

```
(3*a*arctan(sqrt(b*x + a)/sqrt(-a))*sgn(x)/sqrt(-a) + 2*sqrt(b*x + a)*sgn(
x) - sqrt(b*x + a)*a*sgn(x)/(b*x))*b
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(ax^2+bx^3)^{3/2}}{x^5}\,dx = \int \frac{(b\,x^3+a\,x^2)^{3/2}}{x^5}\,dx$$

```
int((a*x^2 + b*x^3)^(3/2)/x^5,x)
```

```
int((a*x^2 + b*x^3)^(3/2)/x^5, x)
```

Reduce [B] (verification not implemented)

Time = 0.53 (sec) , antiderivative size = 59, normalized size of antiderivative = 0.78

$$\int \frac{(ax^2+bx^3)^{3/2}}{x^5}\,dx = \frac{-2\sqrt{bx+a}\,a+4\sqrt{bx+a}\,bx+3\sqrt{a}\log\left(\sqrt{bx+a}-\sqrt{a}\right)bx-3\sqrt{a}\log\left(\sqrt{bx+a}+\right.}{2x}$$

```
int((b*x^3+a*x^2)^(3/2)/x^5,x)
```

```
( - 2*sqrt(a + b*x)*a + 4*sqrt(a + b*x)*b*x + 3*sqrt(a)*log(sqrt(a + b*x)
- sqrt(a))*b*x - 3*sqrt(a)*log(sqrt(a + b*x) + sqrt(a))*b*x)/(2*x)
```

3.283 $\int \frac{(ax^2+bx^3)^{3/2}}{x^6} dx$

Optimal result

Integrand size = 19, antiderivative size = 84

$$\int \frac{(ax^2+bx^3)^{3/2}}{x^6} dx = -\frac{a\sqrt{ax^2+bx^3}}{2x^3} - \frac{5b\sqrt{ax^2+bx^3}}{4x^2} - \frac{3b^2\text{arctanh}\left(\frac{\sqrt{ax^2+bx^3}}{\sqrt{a}x}\right)}{4\sqrt{a}}$$

```
-1/2*a*(b*x^3+a*x^2)^(1/2)/x^3-5/4*b*(b*x^3+a*x^2)^(1/2)/x^2-3/4*b^2*arcta
nh((b*x^3+a*x^2)^(1/2)/a^(1/2)/x)/a^(1/2)
```

Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 82, normalized size of antiderivative = 0.98

$$\int \frac{(ax^2+bx^3)^{3/2}}{x^6} dx = -\frac{\sqrt{x^2(a+bx)}\left(\sqrt{a}\sqrt{a+bx}(2a+5bx) + 3b^2x^2\text{arctanh}\left(\frac{\sqrt{a+bx}}{\sqrt{a}}\right)\right)}{4\sqrt{a}x^3\sqrt{a+bx}}$$

```
Integrate[(a*x^2 + b*x^3)^(3/2)/x^6,x]
```

```
-1/4*(Sqrt[x^2*(a + b*x)]*(Sqrt[a]*Sqrt[a + b*x]*(2*a + 5*b*x) + 3*b^2*x^2
*ArcTanh[Sqrt[a + b*x]/Sqrt[a]]))/(Sqrt[a]*x^3*Sqrt[a + b*x])
```

Rubi [A] (verified)

Time = 0.37 (sec) , antiderivative size = 80, normalized size of antiderivative = 0.95, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.211, Rules used = {1926, 1926, 1914, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(ax^2+bx^3)^{3/2}}{x^6}dx$$

$$\Big\downarrow \text{1926}$$

$$\frac{3}{4}b\int \frac{\sqrt{bx^3+ax^2}}{x^3}dx-\frac{(ax^2+bx^3)^{3/2}}{2x^5}$$

$$\Big\downarrow \text{1926}$$

$$\frac{3}{4}b\left(\frac{1}{2}b\int \frac{1}{\sqrt{bx^3+ax^2}}dx-\frac{\sqrt{ax^2+bx^3}}{x^2}\right)-\frac{(ax^2+bx^3)^{3/2}}{2x^5}$$

$$\Big\downarrow \text{1914}$$

$$\frac{3}{4}b\left(-b\int \frac{1}{1-\frac{ax^2}{bx^3+ax^2}}d\frac{x}{\sqrt{bx^3+ax^2}}-\frac{\sqrt{ax^2+bx^3}}{x^2}\right)-\frac{(ax^2+bx^3)^{3/2}}{2x^5}$$

$$\Big\downarrow \text{219}$$

$$\frac{3}{4}b\left(-\frac{b\,\text{arctanh}\left(\frac{\sqrt{a}x}{\sqrt{ax^2+bx^3}}\right)}{\sqrt{a}}-\frac{\sqrt{ax^2+bx^3}}{x^2}\right)-\frac{(ax^2+bx^3)^{3/2}}{2x^5}$$

```
Int[(a*x^2 + b*x^3)^(3/2)/x^6,x]
```

```
-1/2*(a*x^2 + b*x^3)^(3/2)/x^5 + (3*b*(-(Sqrt[a*x^2 + b*x^3]/x^2) - (b*Arc
Tanh[(Sqrt[a]*x)/Sqrt[a*x^2 + b*x^3]])/Sqrt[a]))/4
```

Defintions of rubi rules used

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

```
Int[1/Sqrt[(a_.)*(x_)^2 + (b_.)*(x_)^(n_.)], x_Symbol] :> Simp[2/(2 - n)
Subst[Int[1/(1 - a*x^2), x], x, x/Sqrt[a*x^2 + b*x^n]], x] /; FreeQ[{a, b,
n}, x] && NeQ[n, 2]
```

```
Int[((c_.)*(x_))^(m_)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol]
 :> Simp[(c*x)^(m + 1)*((a*x^j + b*x^n)^p/(c*(m + j*p + 1))), x] - Simp[b*p
*((n - j)/(c^n*(m + j*p + 1)))   Int[(c*x)^(m + n)*(a*x^j + b*x^n)^(p - 1),
 x], x] /; FreeQ[{a, b, c}, x] &&  !IntegerQ[p] && LtQ[0, j, n] && (Integer
sQ[j, n] || GtQ[c, 0]) && GtQ[p, 0] && LtQ[m + j*p + 1, 0]
```

Maple [A] (verified)

Time = 0.50 (sec) , antiderivative size = 67, normalized size of antiderivative = 0.80

method	result	size
risch	$-\frac{(5bx+2a)\sqrt{x^{2}(bx+a)}}{4x^{3}}-\frac{3b^{2}\operatorname{arctanh}\left(\frac{\sqrt{bx+a}}{\sqrt{a}}\right)\sqrt{x^{2}(bx+a)}}{4\sqrt{a}\,x\sqrt{bx+a}}$	67
default	$-\frac{(b\,x^{3}+a\,x^{2})^{\frac{3}{2}}\left(3\operatorname{arctanh}\left(\frac{\sqrt{bx+a}}{\sqrt{a}}\right)b^{2}x^{2}+5(bx+a)^{\frac{3}{2}}\sqrt{a}-3a^{\frac{3}{2}}\sqrt{bx+a}\right)}{4x^{5}(bx+a)^{\frac{3}{2}}\sqrt{a}}$	74
pseudoelliptic	$-\frac{-\frac{15\operatorname{arctanh}\left(\frac{\sqrt{bx+a}}{\sqrt{a}}\right)b^{5}x^{5}}{128}+\sqrt{bx+a}\left(\frac{15\sqrt{a}\,b^{4}x^{4}}{128}-\frac{5a^{\frac{3}{2}}b^{3}x^{3}}{64}+\frac{a^{\frac{5}{2}}b^{2}x^{2}}{16}+\frac{11a^{\frac{7}{2}}bx}{8}+a^{\frac{9}{2}}\right)}{5a^{\frac{7}{2}}x^{5}}$	82

```
int((b*x^3+a*x^2)^(3/2)/x^6,x,method=_RETURNVERBOSE)
```

```
-1/4*(5*b*x+2*a)/x^3*(x^2*(b*x+a))^(1/2)-3/4*b^2/a^(1/2)*arctanh((b*x+a)^(
1/2)/a^(1/2))*(x^2*(b*x+a))^(1/2)/x/(b*x+a)^(1/2)
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 159, normalized size of antiderivative = 1.89

$$\int \frac{(ax^2+bx^3)^{3/2}}{x^6}\,dx = \left[\frac{3\sqrt{a}b^2x^3\log\left(\frac{bx^2+2\,ax-2\,\sqrt{bx^3+ax^2}\sqrt{a}}{x^2}\right)-2\,\sqrt{bx^3+ax^2}(5\,abx+2\,a^2)}{8\,ax^3}, \frac{3\,\sqrt{-a}b^2x^3\,\mathrm{ar}}{}\right.$$

```
integrate((b*x^3+a*x^2)^(3/2)/x^6,x, algorithm="fricas")
```

```
[1/8*(3*sqrt(a)*b^2*x^3*log((b*x^2 + 2*a*x - 2*sqrt(b*x^3 + a*x^2)*sqrt(a)
)/x^2) - 2*sqrt(b*x^3 + a*x^2)*(5*a*b*x + 2*a^2))/(a*x^3), 1/4*(3*sqrt(-a)
*b^2*x^3*arctan(sqrt(b*x^3 + a*x^2)*sqrt(-a)/(b*x^2 + a*x)) - sqrt(b*x^3 +
 a*x^2)*(5*a*b*x + 2*a^2))/(a*x^3)]
```

Sympy [F]

$$\int \frac{(ax^2+bx^3)^{3/2}}{x^6}\,dx = \int \frac{(x^2(a+bx))^{\frac{3}{2}}}{x^6}\,dx$$

```
integrate((b*x**3+a*x**2)**(3/2)/x**6,x)
```

```
Integral((x**2*(a + b*x))**(3/2)/x**6, x)
```

Maxima [F]

$$\int \frac{(ax^2+bx^3)^{3/2}}{x^6}\,dx = \int \frac{(bx^3+ax^2)^{\frac{3}{2}}}{x^6}\,dx$$

```
integrate((b*x^3+a*x^2)^(3/2)/x^6,x, algorithm="maxima")
```

```
integrate((b*x^3 + a*x^2)^(3/2)/x^6, x)
```

Giac [A] (verification not implemented)

Time = 0.15 (sec) , antiderivative size = 70, normalized size of antiderivative = 0.83

$$\int \frac{(ax^2+bx^3)^{3/2}}{x^6}\,dx = \frac{\frac{3\,b^3\arctan\left(\frac{\sqrt{bx+a}}{\sqrt{-a}}\right)\mathrm{sgn}(x)}{\sqrt{-a}} - \frac{5\,(bx+a)^{\frac{3}{2}}b^3\mathrm{sgn}(x)-3\,\sqrt{bx+a}ab^3\mathrm{sgn}(x)}{b^2x^2}}{4\,b}$$

```
integrate((b*x^3+a*x^2)^(3/2)/x^6,x, algorithm="giac")
```

```
1/4*(3*b^3*arctan(sqrt(b*x + a)/sqrt(-a))*sgn(x)/sqrt(-a) - (5*(b*x + a)^(
3/2)*b^3*sgn(x) - 3*sqrt(b*x + a)*a*b^3*sgn(x))/(b^2*x^2))/b
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(ax^2+bx^3)^{3/2}}{x^6}\,dx = \int \frac{(b\,x^3+a\,x^2)^{3/2}}{x^6}\,dx$$

```
int((a*x^2 + b*x^3)^(3/2)/x^6,x)
```

```
int((a*x^2 + b*x^3)^(3/2)/x^6, x)
```

Reduce [B] (verification not implemented)

Time = 0.51 (sec) , antiderivative size = 73, normalized size of antiderivative = 0.87

$$\int \frac{(ax^2+bx^3)^{3/2}}{x^6}\,dx = \frac{-4\sqrt{bx+a}\,a^2-10\sqrt{bx+a}\,abx+3\sqrt{a}\log\left(\sqrt{bx+a}-\sqrt{a}\right)b^2x^2-3\sqrt{a}\log\left(\sqrt{bx+\right.}{8a\,x^2}$$

```
int((b*x^3+a*x^2)^(3/2)/x^6,x)
```

```
( - 4*sqrt(a + b*x)*a**2 - 10*sqrt(a + b*x)*a*b*x + 3*sqrt(a)*log(sqrt(a +
 b*x) - sqrt(a))*b**2*x**2 - 3*sqrt(a)*log(sqrt(a + b*x) + sqrt(a))*b**2*x
**2)/(8*a*x**2)
```

3.284 $\int \frac{(ax^2+bx^3)^{3/2}}{x^7} dx$

Optimal result

Integrand size = 19, antiderivative size = 112

$$\int \frac{(ax^2+bx^3)^{3/2}}{x^7} dx = -\frac{a\sqrt{ax^2+bx^3}}{3x^4} - \frac{7b\sqrt{ax^2+bx^3}}{12x^3} - \frac{b^2\sqrt{ax^2+bx^3}}{8ax^2} + \frac{b^3 \operatorname{arctanh}\left(\frac{\sqrt{ax^2+bx^3}}{\sqrt{a}x}\right)}{8a^{3/2}}$$

```
-1/3*a*(b*x^3+a*x^2)^(1/2)/x^4-7/12*b*(b*x^3+a*x^2)^(1/2)/x^3-1/8*b^2*(b*x
^3+a*x^2)^(1/2)/a/x^2+1/8*b^3*arctanh((b*x^3+a*x^2)^(1/2)/a^(1/2)/x)/a^(3/
2)
```

Mathematica [A] (verified)

Time = 0.03 (sec) , antiderivative size = 94, normalized size of antiderivative = 0.84

$$\int \frac{(ax^2+bx^3)^{3/2}}{x^7} dx = \frac{\sqrt{x^2(a+bx)}\left(-\sqrt{a}\sqrt{a+bx}(8a^2+14abx+3b^2x^2)+3b^3x^3 \operatorname{arctanh}\left(\frac{\sqrt{a+bx}}{\sqrt{a}}\right)\right)}{24a^{3/2}x^4\sqrt{a+bx}}$$

```
Integrate[(a*x^2 + b*x^3)^(3/2)/x^7,x]
```

```
(Sqrt[x^2*(a + b*x)]*(-(Sqrt[a]*Sqrt[a + b*x]*(8*a^2 + 14*a*b*x + 3*b^2*x^
2)) + 3*b^3*x^3*ArcTanh[Sqrt[a + b*x]/Sqrt[a]]))/(24*a^(3/2)*x^4*Sqrt[a +
b*x])
```

Rubi [A] (verified)

Time = 0.45 (sec) , antiderivative size = 110, normalized size of antiderivative = 0.98, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.263, Rules used = {1926, 1926, 1931, 1914, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\left(ax^2+bx^3\right)^{3/2}}{x^7}dx$$

$$\Big\downarrow \text{1926}$$

$$\frac{1}{2}b\int \frac{\sqrt{bx^3+ax^2}}{x^4}dx-\frac{\left(ax^2+bx^3\right)^{3/2}}{3x^6}$$

$$\Big\downarrow \text{1926}$$

$$\frac{1}{2}b\left(\frac{1}{4}b\int \frac{1}{x\sqrt{bx^3+ax^2}}dx-\frac{\sqrt{ax^2+bx^3}}{2x^3}\right)-\frac{\left(ax^2+bx^3\right)^{3/2}}{3x^6}$$

$$\Big\downarrow \text{1931}$$

$$\frac{1}{2}b\left(\frac{1}{4}b\left(-\frac{b\int \frac{1}{\sqrt{bx^3+ax^2}}dx}{2a}-\frac{\sqrt{ax^2+bx^3}}{ax^2}\right)-\frac{\sqrt{ax^2+bx^3}}{2x^3}\right)-\frac{\left(ax^2+bx^3\right)^{3/2}}{3x^6}$$

$$\Big\downarrow \text{1914}$$

$$\frac{1}{2}b\left(\frac{1}{4}b\left(\frac{b\int \frac{1}{1-\frac{ax^2}{bx^3+ax^2}}d\frac{x}{\sqrt{bx^3+ax^2}}}{a}-\frac{\sqrt{ax^2+bx^3}}{ax^2}\right)-\frac{\sqrt{ax^2+bx^3}}{2x^3}\right)-\frac{\left(ax^2+bx^3\right)^{3/2}}{3x^6}$$

$$\Big\downarrow \text{219}$$

$$\frac{1}{2}b\left(\frac{1}{4}b\left(\frac{b\,\text{arctanh}\left(\frac{\sqrt{a}x}{\sqrt{ax^2+bx^3}}\right)}{a^{3/2}}-\frac{\sqrt{ax^2+bx^3}}{ax^2}\right)-\frac{\sqrt{ax^2+bx^3}}{2x^3}\right)-\frac{\left(ax^2+bx^3\right)^{3/2}}{3x^6}$$

```
Int[(a*x^2 + b*x^3)^(3/2)/x^7,x]
```

```
-1/3*(a*x^2 + b*x^3)^(3/2)/x^6 + (b*(-1/2*Sqrt[a*x^2 + b*x^3]/x^3 + (b*(-(
Sqrt[a*x^2 + b*x^3]/(a*x^2)) + (b*ArcTanh[(Sqrt[a]*x)/Sqrt[a*x^2 + b*x^3]]
)/a^(3/2)))/4))/2
```

Defintions of rubi rules used

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

```
Int[1/Sqrt[(a_.)*(x_)^2 + (b_.)*(x_)^(n_.)], x_Symbol] :> Simp[2/(2 - n)
Subst[Int[1/(1 - a*x^2), x], x, x/Sqrt[a*x^2 + b*x^n]], x] /; FreeQ[{a, b,
n}, x] && NeQ[n, 2]
```

```
Int[((c_.)*(x_))^(m_)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol]
 :> Simp[(c*x)^(m + 1)*((a*x^j + b*x^n)^p/(c*(m + j*p + 1))), x] - Simp[b*p
*((n - j)/(c^n*(m + j*p + 1)))   Int[(c*x)^(m + n)*(a*x^j + b*x^n)^(p - 1),
 x], x] /; FreeQ[{a, b, c}, x] &&  !IntegerQ[p] && LtQ[0, j, n] && (Integer
sQ[j, n] || GtQ[c, 0]) && GtQ[p, 0] && LtQ[m + j*p + 1, 0]
```

```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> Simp[c^(j - 1)*(c*x)^(m - j + 1)*((a*x^j + b*x^n)^(p + 1)/(a*(m + j*p
+ 1))), x] - Simp[b*((m + n*p + n - j + 1)/(a*c^(n - j)*(m + j*p + 1)))   I
nt[(c*x)^(m + n - j)*(a*x^j + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, p}, x]
 &&  !IntegerQ[p] && LtQ[0, j, n] && (IntegersQ[j, n] || GtQ[c, 0]) && LtQ[
m + j*p + 1, 0]
```

Maple [A] (verified)

Time = 0.52 (sec) , antiderivative size = 81, normalized size of antiderivative = 0.72

method	result	size
risch	$-\frac{\left(3 b^{2} x^{2}+14 a b x+8 a^{2}\right) \sqrt{x^{2} \left(b x+a\right)}}{24 x^{4} a}+\frac{b^{3} \operatorname{arctanh}\left(\frac{\sqrt{b x+a}}{\sqrt{a}}\right) \sqrt{x^{2} \left(b x+a\right)}}{8 a^{\frac{3}{2}} x \sqrt{b x+a}}$	81
default	$-\frac{\left(b \,x^{3}+a \,x^{2}\right)^{\frac{3}{2}} \left(3 \left(b x+a\right)^{\frac{5}{2}} a^{\frac{3}{2}}-3 \,\operatorname{arctanh}\left(\frac{\sqrt{b x+a}}{\sqrt{a}}\right) a \,b^{3} x^{3}+8 \left(b x+a\right)^{\frac{3}{2}} a^{\frac{5}{2}}-3 a^{\frac{7}{2}} \sqrt{b x+a}\right)}{24 x^{6} \left(b x+a\right)^{\frac{3}{2}} a^{\frac{5}{2}}}$	87
pseudoelliptic	$-\frac{13 \left(\frac{105 \,\operatorname{arctanh}\left(\frac{\sqrt{b x+a}}{\sqrt{a}}\right) b^{6} x^{6}}{1664}+\sqrt{b x+a}\, \left(-\frac{105 \sqrt{a}\, b^{5} x^{5}}{1664}+\frac{35 a^{\frac{3}{2}} b^{4} x^{4}}{832}-\frac{7 a^{\frac{5}{2}} b^{3} x^{3}}{208}+\frac{3 a^{\frac{7}{2}} b^{2} x^{2}}{104}+a^{\frac{9}{2}} b x +\frac{10 a^{\frac{11}{2}}}{13}\right)\right)}{60 a^{\frac{9}{2}} x^{6}}$	94

```
int((b*x^3+a*x^2)^(3/2)/x^7,x,method=_RETURNVERBOSE)
```

```
-1/24*(3*b^2*x^2+14*a*b*x+8*a^2)/x^4/a*(x^2*(b*x+a))^(1/2)+1/8/a^(3/2)*b^3
*arctanh((b*x+a)^(1/2)/a^(1/2))*(x^2*(b*x+a))^(1/2)/x/(b*x+a)^(1/2)
```

Fricas [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 180, normalized size of antiderivative = 1.61

$$\int \frac{(ax^2+bx^3)^{3/2}}{x^7}\,dx = \left[\frac{3\sqrt{a}b^3x^4\log\left(\frac{bx^2+2\,ax+2\,\sqrt{bx^3+ax^2}\sqrt{a}}{x^2}\right)-2\,(3\,ab^2x^2+14\,a^2bx+8\,a^3)\sqrt{bx^3+ax^2}}{48\,a^2x^4}, -\frac{3\,\sqrt{-a}b^3x^4\arctan\left(\frac{\sqrt{bx^3+ax^2}\sqrt{-a}}{bx^2+ax}\right)+(3\,ab^2x^2+14\,a^2bx+8\,a^3)\sqrt{bx^3+ax^2}}{24\,a^2x^4}\right]$$

```
integrate((b*x^3+a*x^2)^(3/2)/x^7,x, algorithm="fricas")
```

```
[1/48*(3*sqrt(a)*b^3*x^4*log((b*x^2 + 2*a*x + 2*sqrt(b*x^3 + a*x^2)*sqrt(a
))/x^2) - 2*(3*a*b^2*x^2 + 14*a^2*b*x + 8*a^3)*sqrt(b*x^3 + a*x^2))/(a^2*x
^4), -1/24*(3*sqrt(-a)*b^3*x^4*arctan(sqrt(b*x^3 + a*x^2)*sqrt(-a)/(b*x^2
+ a*x)) + (3*a*b^2*x^2 + 14*a^2*b*x + 8*a^3)*sqrt(b*x^3 + a*x^2))/(a^2*x^4
)]
```

Sympy [F]

$$\int \frac{(ax^2+bx^3)^{3/2}}{x^7}\,dx=\int \frac{(x^2(a+bx))^{\frac{3}{2}}}{x^7}\,dx$$

```
integrate((b*x**3+a*x**2)**(3/2)/x**7,x)
```

```
Integral((x**2*(a + b*x))**(3/2)/x**7, x)
```

Maxima [F]

$$\int \frac{(ax^2+bx^3)^{3/2}}{x^7}\,dx=\int \frac{(bx^3+ax^2)^{\frac{3}{2}}}{x^7}\,dx$$

```
integrate((b*x^3+a*x^2)^(3/2)/x^7,x, algorithm="maxima")
```

```
integrate((b*x^3 + a*x^2)^(3/2)/x^7, x)
```

Giac [A] (verification not implemented)

Time = 0.14 (sec) , antiderivative size = 80, normalized size of antiderivative = 0.71

$$\int \frac{(ax^2+bx^3)^{3/2}}{x^7}\,dx=$$
$$-\frac{1}{24}b^3\left(\frac{3\arctan\left(\frac{\sqrt{bx+a}}{\sqrt{-a}}\right)\mathrm{sgn}(x)}{\sqrt{-a}a}+\frac{3(bx+a)^{\frac{5}{2}}\mathrm{sgn}(x)+8(bx+a)^{\frac{3}{2}}a\mathrm{sgn}(x)-3\sqrt{bx+a}a^2\mathrm{sgn}(x)}{ab^3x^3}\right)$$

```
integrate((b*x^3+a*x^2)^(3/2)/x^7,x, algorithm="giac")
```

```
-1/24*b^3*(3*arctan(sqrt(b*x + a)/sqrt(-a))*sgn(x)/(sqrt(-a)*a) + (3*(b*x
+ a)^(5/2)*sgn(x) + 8*(b*x + a)^(3/2)*a*sgn(x) - 3*sqrt(b*x + a)*a^2*sgn(x
))/(a*b^3*x^3))
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(ax^2+bx^3)^{3/2}}{x^7}\,dx=\int \frac{{\left(b\,x^3+a\,x^2\right)}^{3/2}}{x^7}\,dx$$

```
int((a*x^2 + b*x^3)^(3/2)/x^7,x)
```

```
int((a*x^2 + b*x^3)^(3/2)/x^7, x)
```

Reduce [B] (verification not implemented)

Time = 0.25 (sec) , antiderivative size = 90, normalized size of antiderivative = 0.80

$$\int \frac{(ax^2+bx^3)^{3/2}}{x^7}\,dx=\frac{-16\sqrt{bx+a}\,a^3-28\sqrt{bx+a}\,a^2bx-6\sqrt{bx+a}\,a\,b^2x^2-3\sqrt{a}\log\left(\sqrt{bx+a}-\sqrt{a}\right)b^3}{48a^2x^3}$$

```
int((b*x^3+a*x^2)^(3/2)/x^7,x)
```

```
( - 16*sqrt(a + b*x)*a**3 - 28*sqrt(a + b*x)*a**2*b*x - 6*sqrt(a + b*x)*a*
b**2*x**2 - 3*sqrt(a)*log(sqrt(a + b*x) - sqrt(a))*b**3*x**3 + 3*sqrt(a)*l
og(sqrt(a + b*x) + sqrt(a))*b**3*x**3)/(48*a**2*x**3)
```

3.285 $\int \frac{(ax^2+bx^3)^{3/2}}{x^8}\,dx$

Optimal result

Integrand size = 19, antiderivative size = 140

$$\int \frac{(ax^2+bx^3)^{3/2}}{x^8}\,dx = -\frac{a\sqrt{ax^2+bx^3}}{4x^5}-\frac{3b\sqrt{ax^2+bx^3}}{8x^4}-\frac{b^2\sqrt{ax^2+bx^3}}{32ax^3}+\frac{3b^3\sqrt{ax^2+bx^3}}{64a^2x^2}-\frac{3b^4\operatorname{arctanh}\left(\frac{\sqrt{ax^2+bx^3}}{\sqrt{a}x}\right)}{64a^{5/2}}$$

```
-1/4*a*(b*x^3+a*x^2)^(1/2)/x^5-3/8*b*(b*x^3+a*x^2)^(1/2)/x^4-1/32*b^2*(b*x
^3+a*x^2)^(1/2)/a/x^3+3/64*b^3*(b*x^3+a*x^2)^(1/2)/a^2/x^2-3/64*b^4*arctan
h((b*x^3+a*x^2)^(1/2)/a^(1/2)/x)/a^(5/2)
```

Mathematica [A] (verified)

Time = 0.04 (sec) , antiderivative size = 104, normalized size of antiderivative = 0.74

$$\int \frac{(ax^2+bx^3)^{3/2}}{x^8}\,dx = -\frac{\sqrt{x^2(a+bx)}\left(\sqrt{a}\sqrt{a+bx}(16a^3+24a^2bx+2ab^2x^2-3b^3x^3)+3b^4x^4\operatorname{arctanh}\left(\frac{\sqrt{a+bx}}{\sqrt{a}}\right)\right)}{64a^{5/2}x^5\sqrt{a+bx}}$$

```
Integrate[(a*x^2 + b*x^3)^(3/2)/x^8,x]
```

```
-1/64*(Sqrt[x^2*(a + b*x)]*(Sqrt[a]*Sqrt[a + b*x]*(16*a^3 + 24*a^2*b*x + 2
*a*b^2*x^2 - 3*b^3*x^3) + 3*b^4*x^4*ArcTanh[Sqrt[a + b*x]/Sqrt[a]]))/(a^(5
/2)*x^5*Sqrt[a + b*x])
```

Rubi [A] (verified)

Time = 0.54 (sec) , antiderivative size = 144, normalized size of antiderivative = 1.03, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.316$, Rules used = $\{1926, 1926, 1931, 1931, 1914, 219\}$

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\left(ax^2+bx^3\right)^{3/2}}{x^8}dx$$

$$\Big\downarrow \text{1926}$$

$$\frac{3}{8}b\int \frac{\sqrt{bx^3+ax^2}}{x^5}dx-\frac{\left(ax^2+bx^3\right)^{3/2}}{4x^7}$$

$$\Big\downarrow \text{1926}$$

$$\frac{3}{8}b\left(\frac{1}{6}b\int \frac{1}{x^2\sqrt{bx^3+ax^2}}dx-\frac{\sqrt{ax^2+bx^3}}{3x^4}\right)-\frac{\left(ax^2+bx^3\right)^{3/2}}{4x^7}$$

$$\Big\downarrow \text{1931}$$

$$\frac{3}{8}b\left(\frac{1}{6}b\left(-\frac{3b\int \frac{1}{x\sqrt{bx^3+ax^2}}dx}{4a}-\frac{\sqrt{ax^2+bx^3}}{2ax^3}\right)-\frac{\sqrt{ax^2+bx^3}}{3x^4}\right)-\frac{\left(ax^2+bx^3\right)^{3/2}}{4x^7}$$

$$\Big\downarrow \text{1931}$$

$$\frac{3}{8}b\left(\frac{1}{6}b\left(-\frac{3b\left(-\frac{b\int \frac{1}{\sqrt{bx^3+ax^2}}dx}{2a}-\frac{\sqrt{ax^2+bx^3}}{ax^2}\right)}{4a}-\frac{\sqrt{ax^2+bx^3}}{2ax^3}\right)-\frac{\sqrt{ax^2+bx^3}}{3x^4}\right)-\frac{\left(ax^2+bx^3\right)^{3/2}}{4x^7}$$

$$\Big\downarrow \text{1914}$$

$$\frac{3}{8}b\left(\frac{1}{6}b\left(-\frac{3b\left(\frac{b\int \frac{1}{1-\frac{ax^2}{bx^3+ax^2}}d\frac{x}{\sqrt{bx^3+ax^2}}}{a}-\frac{\sqrt{ax^2+bx^3}}{ax^2}\right)}{4a}-\frac{\sqrt{ax^2+bx^3}}{2ax^3}\right)-\frac{\sqrt{ax^2+bx^3}}{3x^4}\right)-\frac{\left(ax^2+bx^3\right)^{3/2}}{4x^7}$$

$$\Big\downarrow \text{219}$$

$$\frac{3}{8}b\left(\frac{1}{6}b\left(-\frac{3b\left(\frac{b\text{arctanh}\left(\frac{\sqrt{a}x}{\sqrt{ax^2+bx^3}}\right)}{a^{3/2}}-\frac{\sqrt{ax^2+bx^3}}{ax^2}\right)}{4a}-\frac{\sqrt{ax^2+bx^3}}{2ax^3}\right)-\frac{\sqrt{ax^2+bx^3}}{3x^4}\right)-\frac{\left(ax^2+bx^3\right)^{3/2}}{4x^7}$$

```
Int[(a*x^2 + b*x^3)^(3/2)/x^8,x]
```

```
-1/4*(a*x^2 + b*x^3)^(3/2)/x^7 + (3*b*(-1/3*Sqrt[a*x^2 + b*x^3]/x^4 + (b*(
-1/2*Sqrt[a*x^2 + b*x^3]/(a*x^3) - (3*b*(-(Sqrt[a*x^2 + b*x^3]/(a*x^2)) +
(b*ArcTanh[(Sqrt[a]*x)/Sqrt[a*x^2 + b*x^3]])/a^(3/2)))/(4*a)))/6))/8
```

Defintions of rubi rules used

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

```
Int[1/Sqrt[(a_.)*(x_)^2 + (b_.)*(x_)^(n_.)], x_Symbol] :> Simp[2/(2 - n)
Subst[Int[1/(1 - a*x^2), x], x, x/Sqrt[a*x^2 + b*x^n]], x] /; FreeQ[{a, b,
n}, x] && NeQ[n, 2]
```

```
Int[((c_.)*(x_))^(m_)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol]
 :> Simp[(c*x)^(m + 1)*((a*x^j + b*x^n)^p/(c*(m + j*p + 1))), x] - Simp[b*p
*((n - j)/(c^n*(m + j*p + 1)))   Int[(c*x)^(m + n)*(a*x^j + b*x^n)^(p - 1),
 x], x] /; FreeQ[{a, b, c}, x] &&  !IntegerQ[p] && LtQ[0, j, n] && (Integer
sQ[j, n] || GtQ[c, 0]) && GtQ[p, 0] && LtQ[m + j*p + 1, 0]
```

```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> Simp[c^(j - 1)*(c*x)^(m - j + 1)*((a*x^j + b*x^n)^(p + 1)/(a*(m + j*p
+ 1))), x] - Simp[b*((m + n*p + n - j + 1)/(a*c^(n - j)*(m + j*p + 1)))   I
nt[(c*x)^(m + n - j)*(a*x^j + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, p}, x]
 &&  !IntegerQ[p] && LtQ[0, j, n] && (IntegersQ[j, n] || GtQ[c, 0]) && LtQ[
m + j*p + 1, 0]
```

Maple [A] (verified)

Time = 0.56 (sec) , antiderivative size = 92, normalized size of antiderivative = 0.66

method	result
risch	$-\frac{\left(-3 b^{3} x^{3}+2 a \,b^{2} x^{2}+24 a^{2} b x +16 a^{3}\right) \sqrt{x^{2} \left(b x +a \right)}}{64 x^{5} a^{2}}-\frac{3 b^{4} \operatorname{arctanh}\left(\frac{\sqrt{b x +a}}{\sqrt{a}}\right) \sqrt{x^{2} \left(b x +a \right)}}{64 a^{\frac{5}{2}} x \sqrt{b x +a}}$
default	$\frac{\left(b \,x^{3}+a \,x^{2}\right)^{\frac{3}{2}} \left(3 \left(b x +a \right)^{\frac{7}{2}} a^{\frac{5}{2}}-11 \left(b x +a \right)^{\frac{5}{2}} a^{\frac{7}{2}}-3 \,\operatorname{arctanh}\left(\frac{\sqrt{b x +a}}{\sqrt{a}}\right) a^{2} b^{4} x^{4}-11 \left(b x +a \right)^{\frac{3}{2}} a^{\frac{9}{2}}+3 a^{\frac{11}{2}} \sqrt{b x +a}\right)}{64 x^{7} \left(b x +a \right)^{\frac{3}{2}} a^{\frac{9}{2}}}$
pseudoelliptic	$-\frac{5 \left(-\frac{63 \,\operatorname{arctanh}\left(\frac{\sqrt{b x +a}}{\sqrt{a}}\right) x^{7} b^{7}}{1280}+\sqrt{b x +a}\, \left(\frac{63 \sqrt{a}\, b^{6} x^{6}}{1280}-\frac{21 a^{\frac{3}{2}} b^{5} x^{5}}{640}+\frac{21 a^{\frac{5}{2}} b^{4} x^{4}}{800}-\frac{9 a^{\frac{7}{2}} b^{3} x^{3}}{400}+\frac{a^{\frac{9}{2}} b^{2} x^{2}}{50}+a^{\frac{11}{2}} b x +\frac{4 a^{\frac{13}{2}}}{5}\right)\right)}{28 a^{\frac{11}{2}} x^{7}}$

```
int((b*x^3+a*x^2)^(3/2)/x^8,x,method=_RETURNVERBOSE)
```

```
-1/64*(-3*b^3*x^3+2*a*b^2*x^2+24*a^2*b*x+16*a^3)/x^5/a^2*(x^2*(b*x+a))^(1/
2)-3/64/a^(5/2)*b^4*arctanh((b*x+a)^(1/2)/a^(1/2))*(x^2*(b*x+a))^(1/2)/x/(
b*x+a)^(1/2)
```

Fricas [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 202, normalized size of antiderivative = 1.44

$$\int \frac{(ax^2+bx^3)^{3/2}}{x^8}\,dx = \left[\frac{3\sqrt{a}b^4x^5\log\left(\frac{bx^2+2\,ax-2\,\sqrt{bx^3+ax^2}\sqrt{a}}{x^2}\right)+2\,(3\,ab^3x^3-2\,a^2b^2x^2-24\,a^3bx-16\,a^4)\sqrt{b}}{128\,a^3x^5}\right.$$

```
integrate((b*x^3+a*x^2)^(3/2)/x^8,x, algorithm="fricas")
```

```
[1/128*(3*sqrt(a)*b^4*x^5*log((b*x^2 + 2*a*x - 2*sqrt(b*x^3 + a*x^2)*sqrt(
a))/x^2) + 2*(3*a*b^3*x^3 - 2*a^2*b^2*x^2 - 24*a^3*b*x - 16*a^4)*sqrt(b*x^
3 + a*x^2))/(a^3*x^5), 1/64*(3*sqrt(-a)*b^4*x^5*arctan(sqrt(b*x^3 + a*x^2)
*sqrt(-a)/(b*x^2 + a*x)) + (3*a*b^3*x^3 - 2*a^2*b^2*x^2 - 24*a^3*b*x - 16*
a^4)*sqrt(b*x^3 + a*x^2))/(a^3*x^5)]
```

Sympy [F]

$$\int \frac{(ax^2+bx^3)^{3/2}}{x^8}\,dx = \int \frac{(x^2(a+bx))^{\frac{3}{2}}}{x^8}\,dx$$

```
integrate((b*x**3+a*x**2)**(3/2)/x**8,x)
```

```
Integral((x**2*(a + b*x))**(3/2)/x**8, x)
```

Maxima [F]

$$\int \frac{(ax^2+bx^3)^{3/2}}{x^8}\,dx = \int \frac{(bx^3+ax^2)^{\frac{3}{2}}}{x^8}\,dx$$

```
integrate((b*x^3+a*x^2)^(3/2)/x^8,x, algorithm="maxima")
```

```
integrate((b*x^3 + a*x^2)^(3/2)/x^8, x)
```

Giac [A] (verification not implemented)

Time = 0.15 (sec) , antiderivative size = 109, normalized size of antiderivative = 0.78

$$\int \frac{(ax^2+bx^3)^{3/2}}{x^8}\,dx = \frac{\frac{3\,b^5\arctan\left(\frac{\sqrt{bx+a}}{\sqrt{-a}}\right)\mathrm{sgn}(x)}{\sqrt{-a}a^2} + \frac{3\,(bx+a)^{\frac{7}{2}}b^5\mathrm{sgn}(x)-11\,(bx+a)^{\frac{5}{2}}ab^5\mathrm{sgn}(x)-11\,(bx+a)^{\frac{3}{2}}a^2b^5\mathrm{sgn}(x)+3\,\sqrt{bx+a}a^3b}{a^2b^4x^4}}{64\,b}$$

```
integrate((b*x^3+a*x^2)^(3/2)/x^8,x, algorithm="giac")
```

```
1/64*(3*b^5*arctan(sqrt(b*x + a)/sqrt(-a))*sgn(x)/(sqrt(-a)*a^2) + (3*(b*x
 + a)^(7/2)*b^5*sgn(x) - 11*(b*x + a)^(5/2)*a*b^5*sgn(x) - 11*(b*x + a)^(3
/2)*a^2*b^5*sgn(x) + 3*sqrt(b*x + a)*a^3*b^5*sgn(x))/(a^2*b^4*x^4))/b
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(ax^2+bx^3)^{3/2}}{x^8}\,dx = \int \frac{(b\,x^3+a\,x^2)^{3/2}}{x^8}\,dx$$

```
int((a*x^2 + b*x^3)^(3/2)/x^8,x)
```

```
int((a*x^2 + b*x^3)^(3/2)/x^8, x)
```

Reduce [B] (verification not implemented)

Time = 0.50 (sec) , antiderivative size = 107, normalized size of antiderivative = 0.76

$$\int \frac{(ax^2+bx^3)^{3/2}}{x^8}\,dx = \frac{-32\sqrt{bx+a}\,a^4-48\sqrt{bx+a}\,a^3bx-4\sqrt{bx+a}\,a^2b^2x^2+6\sqrt{bx+a}\,a\,b^3x^3+3\sqrt{a}\log}{128a^3x^4}$$

```
int((b*x^3+a*x^2)^(3/2)/x^8,x)
```

```
( - 32*sqrt(a + b*x)*a**4 - 48*sqrt(a + b*x)*a**3*b*x - 4*sqrt(a + b*x)*a*
*2*b**2*x**2 + 6*sqrt(a + b*x)*a*b**3*x**3 + 3*sqrt(a)*log(sqrt(a + b*x) -
 sqrt(a))*b**4*x**4 - 3*sqrt(a)*log(sqrt(a + b*x) + sqrt(a))*b**4*x**4)/(1
28*a**3*x**4)
```

3.286 $\int x(ax^2+bx^3)^{5/2}\,dx$

Optimal result

Integrand size = 17, antiderivative size = 190

$$\begin{aligned}\int x\left(ax^2+bx^3\right)^{5/2}\,dx=&\frac{2a^6(ax^2+bx^3)^{7/2}}{7b^7x^7}-\frac{4a^5(ax^2+bx^3)^{9/2}}{3b^7x^9}\\&+\frac{30a^4(ax^2+bx^3)^{11/2}}{11b^7x^{11}}-\frac{40a^3(ax^2+bx^3)^{13/2}}{13b^7x^{13}}\\&+\frac{2a^2(ax^2+bx^3)^{15/2}}{b^7x^{15}}-\frac{12a(ax^2+bx^3)^{17/2}}{17b^7x^{17}}+\frac{2(ax^2+bx^3)^{19/2}}{19b^7x^{19}}\end{aligned}$$

```
2/7*a^6*(b*x^3+a*x^2)^(7/2)/b^7/x^7-4/3*a^5*(b*x^3+a*x^2)^(9/2)/b^7/x^9+30
/11*a^4*(b*x^3+a*x^2)^(11/2)/b^7/x^11-40/13*a^3*(b*x^3+a*x^2)^(13/2)/b^7/x
^13+2*a^2*(b*x^3+a*x^2)^(15/2)/b^7/x^15-12/17*a*(b*x^3+a*x^2)^(17/2)/b^7/x
^17+2/19*(b*x^3+a*x^2)^(19/2)/b^7/x^19
```

Mathematica [A] (verified)

Time = 0.04 (sec) , antiderivative size = 91, normalized size of antiderivative = 0.48

$$\int x\left(ax^2+bx^3\right)^{5/2}\,dx=\frac{2x(a+bx)^4\left(1024a^6-3584a^5bx+8064a^4b^2x^2-14784a^3b^3x^3+24024a^2b^4x^4-36036ab^5x^5\right.}{969969b^7\sqrt{x^2(a+bx)}}$$

```
Integrate[x*(a*x^2 + b*x^3)^(5/2),x]
```

```
(2*x*(a + b*x)^4*(1024*a^6 - 3584*a^5*b*x + 8064*a^4*b^2*x^2 - 14784*a^3*b
^3*x^3 + 24024*a^2*b^4*x^4 - 36036*a*b^5*x^5 + 51051*b^6*x^6))/(969969*b^7
*Sqrt[x^2*(a + b*x)])
```

Rubi [A] (verified)

Time = 0.74 (sec) , antiderivative size = 222, normalized size of antiderivative = 1.17, number of steps used = 7, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.412, Rules used = {1922, 1908, 1922, 1922, 1922, 1922, 1920}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x\left(ax^2+bx^3\right)^{5/2}\,dx$$

$$\Big\downarrow \text{1922}$$

$$\frac{2\left(ax^2+bx^3\right)^{7/2}}{19bx}-\frac{12a\int \left(bx^3+ax^2\right)^{5/2}dx}{19b}$$

$$\Big\downarrow \text{1908}$$

$$\frac{2\left(ax^2+bx^3\right)^{7/2}}{19bx}-\frac{12a\left(\frac{2\left(ax^2+bx^3\right)^{7/2}}{17bx^2}-\frac{10a\int \frac{\left(bx^3+ax^2\right)^{5/2}}{x}dx}{17b}\right)}{19b}$$

$$\Big\downarrow \text{1922}$$

$$\frac{2\left(ax^2+bx^3\right)^{7/2}}{19bx}-\frac{12a\left(\frac{2\left(ax^2+bx^3\right)^{7/2}}{17bx^2}-\frac{10a\left(\frac{2\left(ax^2+bx^3\right)^{7/2}}{15bx^3}-\frac{8a\int\frac{\left(bx^3+ax^2\right)^{5/2}}{x^2}dx}{15b}\right)}{17b}\right)}{19b}$$

$$\Big\downarrow \; 1922$$

$$\frac{2\left(ax^2+bx^3\right)^{7/2}}{19bx}-\frac{12a\left(\frac{2\left(ax^2+bx^3\right)^{7/2}}{17bx^2}-\frac{10a\left(\frac{2\left(ax^2+bx^3\right)^{7/2}}{15bx^3}-\frac{8a\left(\frac{2\left(ax^2+bx^3\right)^{7/2}}{13bx^4}-\frac{6a\int\frac{\left(bx^3+ax^2\right)^{5/2}}{x^3}dx}{13b}\right)}{15b}\right)}{17b}\right)}{19b}$$

$$\Big\downarrow \; 1922$$

$$\frac{\frac{2\left(ax^2+bx^3\right)^{7/2}}{19bx}-\frac{12a\left(\frac{2\left(ax^2+bx^3\right)^{7/2}}{17bx^2}-\frac{10a\left(\frac{2\left(ax^2+bx^3\right)^{7/2}}{15bx^3}-\frac{8a\left(\frac{2\left(ax^2+bx^3\right)^{7/2}}{13bx^4}-\frac{6a\left(\frac{2\left(ax^2+bx^3\right)^{7/2}}{11bx^5}-\frac{4a\int\frac{\left(bx^3+ax^2\right)^{5/2}}{x^4}dx}{11b}\right)}{13b}\right)}{15b}\right)}{17b}\right)}{19b}}{}$$

$$\downarrow 1922$$

$$\frac{\frac{2\left(ax^2+bx^3\right)^{7/2}}{19bx}-\frac{12a\left(\frac{2\left(ax^2+bx^3\right)^{7/2}}{17bx^2}-\frac{10a\left(\frac{2\left(ax^2+bx^3\right)^{7/2}}{15bx^3}-\frac{8a\left(\frac{2\left(ax^2+bx^3\right)^{7/2}}{13bx^4}-\frac{6a\left(\frac{2\left(ax^2+bx^3\right)^{7/2}}{11bx^5}-\frac{4a\left(\frac{2\left(ax^2+bx^3\right)^{7/2}}{9bx^6}-\frac{2a\int\frac{\left(bx^3+ax^2\right)^{5/2}}{x^5}dx}{9b}\right)}{11b}\right)}{13b}\right)}{15b}\right)}{17b}\right)}{19b}$$

$$\Big\downarrow \text{1920}$$

$$\frac{2(ax^2+bx^3)^{7/2}}{19bx}-\frac{12a\left(\frac{2(ax^2+bx^3)^{7/2}}{17bx^2}-\frac{10a\left(\frac{2(ax^2+bx^3)^{7/2}}{15bx^3}-\frac{8a\left(\frac{2(ax^2+bx^3)^{7/2}}{13bx^4}-\frac{6a\left(\frac{2(ax^2+bx^3)^{7/2}}{11bx^5}-\frac{4a\left(\frac{2(ax^2+bx^3)^{7/2}}{9bx^6}-\frac{4a(ax^2+bx^3)^{7/2}}{63b^2x^7}\right)}{11b}\right)}{13b}\right)}{15b}\right)}{17b}\right)}{19b}$$

```
Int[x*(a*x^2 + b*x^3)^(5/2),x]
```

```
(2*(a*x^2 + b*x^3)^(7/2))/(19*b*x) - (12*a*((2*(a*x^2 + b*x^3)^(7/2))/(17*
b*x^2) - (10*a*((2*(a*x^2 + b*x^3)^(7/2))/(15*b*x^3) - (8*a*((2*(a*x^2 + b
*x^3)^(7/2))/(13*b*x^4) - (6*a*((2*(a*x^2 + b*x^3)^(7/2))/(11*b*x^5) - (4*
a*((-4*a*(a*x^2 + b*x^3)^(7/2))/(63*b^2*x^7) + (2*(a*x^2 + b*x^3)^(7/2))/(
9*b*x^6)))/(11*b)))/(13*b)))/(15*b)))/(17*b)))/(19*b)
```

Defintions of rubi rules used

```
Int[((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol] :> Simp[(a*x^j +
b*x^n)^(p + 1)/(a*(j*p + 1)*x^(j - 1)), x] - Simp[b*((n*p + n - j + 1)/(a*(
j*p + 1)))   Int[x^(n - j)*(a*x^j + b*x^n)^p, x], x] /; FreeQ[{a, b, j, n,
p}, x] &&  !IntegerQ[p] && NeQ[n, j] && ILtQ[Simplify[(n*p + n - j + 1)/(n
- j)], 0] && NeQ[j*p + 1, 0]
```

```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> Simp[(-c^(j - 1))*(c*x)^(m - j + 1)*((a*x^j + b*x^n)^(p + 1)/(a*(n - j
)*(p + 1))), x] /; FreeQ[{a, b, c, j, m, n, p}, x] &&  !IntegerQ[p] && NeQ[
n, j] && EqQ[m + n*p + n - j + 1, 0] && (IntegerQ[j] || GtQ[c, 0])
```

```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> Simp[c^(j - 1)*(c*x)^(m - j + 1)*((a*x^j + b*x^n)^(p + 1)/(a*(m + j*p
+ 1))), x] - Simp[b*((m + n*p + n - j + 1)/(a*c^(n - j)*(m + j*p + 1)))   I
nt[(c*x)^(m + n - j)*(a*x^j + b*x^n)^p, x], x] /; FreeQ[{a, b, c, j, m, n,
p}, x] &&  !IntegerQ[p] && NeQ[n, j] && ILtQ[Simplify[(m + n*p + n - j + 1)
/(n - j)], 0] && NeQ[m + j*p + 1, 0] && (IntegersQ[j, n] || GtQ[c, 0])
```

Maple [A] (verified)

Time = 0.41 (sec) , antiderivative size = 21, normalized size of antiderivative = 0.11

method	result
pseudoelliptic	$-\frac{2(bx+a)^{\frac{7}{2}}(-7bx+2a)}{63b^2}$
gosper	$\frac{2(bx+a)(51051b^6x^6-36036a\,b^5x^5+24024a^2b^4x^4-14784a^3b^3x^3+8064a^4b^2x^2-3584a^5bx+1024a^6)(b\,x^3+a\,x^2)^{\frac{5}{2}}}{969969b^7x^5}$
default	$\frac{2(bx+a)(51051b^6x^6-36036a\,b^5x^5+24024a^2b^4x^4-14784a^3b^3x^3+8064a^4b^2x^2-3584a^5bx+1024a^6)(b\,x^3+a\,x^2)^{\frac{5}{2}}}{969969b^7x^5}$
orering	$\frac{2(bx+a)(51051b^6x^6-36036a\,b^5x^5+24024a^2b^4x^4-14784a^3b^3x^3+8064a^4b^2x^2-3584a^5bx+1024a^6)(b\,x^3+a\,x^2)^{\frac{5}{2}}}{969969b^7x^5}$
risch	$\frac{2\sqrt{x^2(bx+a)}\,(51051b^9x^9+117117a\,b^8x^8+69069a^2b^7x^7+231a^3b^6x^6-252a^4b^5x^5+280a^5x^4b^4-320a^6x^3b^3+384a^7x^2b^2-51}{969969x\,b^7}$
trager	$\frac{2(51051b^9x^9+117117a\,b^8x^8+69069a^2b^7x^7+231a^3b^6x^6-252a^4b^5x^5+280a^5x^4b^4-320a^6x^3b^3+384a^7x^2b^2-512a^8bx+1024}{969969b^7x}$

```
int(x*(b*x^3+a*x^2)^(5/2),x,method=_RETURNVERBOSE)
```

```
-2/63*(b*x+a)^(7/2)*(-7*b*x+2*a)/b^2
```

Fricas [A] (verification not implemented)

Time = 0.10 (sec) , antiderivative size = 117, normalized size of antiderivative = 0.62

$$\int x\left(ax^2+bx^3\right)^{5/2}\,dx = \frac{2\left(51051\,b^9x^9+117117\,ab^8x^8+69069\,a^2b^7x^7+231\,a^3b^6x^6-252\,a^4b^5x^5+280\,a^5b^4x^4-320\right.}{969969\,b^7x}$$

```
integrate(x*(b*x^3+a*x^2)^(5/2),x, algorithm="fricas")
```

```
2/969969*(51051*b^9*x^9 + 117117*a*b^8*x^8 + 69069*a^2*b^7*x^7 + 231*a^3*b
^6*x^6 - 252*a^4*b^5*x^5 + 280*a^5*b^4*x^4 - 320*a^6*b^3*x^3 + 384*a^7*b^2
*x^2 - 512*a^8*b*x + 1024*a^9)*sqrt(b*x^3 + a*x^2)/(b^7*x)
```

Sympy [F]

$$\int x(ax^2+bx^3)^{5/2}\,dx = \int x\left(x^2(a+bx)\right)^{\frac{5}{2}}\,dx$$

```
integrate(x*(b*x**3+a*x**2)**(5/2),x)
```

```
Integral(x*(x**2*(a + b*x))**(5/2), x)
```

Maxima [A] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 108, normalized size of antiderivative = 0.57

$$\int x(ax^2+bx^3)^{5/2}\,dx = \frac{2\,(51051\,b^9x^9+117117\,ab^8x^8+69069\,a^2b^7x^7+231\,a^3b^6x^6-252\,a^4b^5x^5+280\,a^5b^4x^4-320}{969969\,b^7}$$

```
integrate(x*(b*x^3+a*x^2)^(5/2),x, algorithm="maxima")
```

```
2/969969*(51051*b^9*x^9 + 117117*a*b^8*x^8 + 69069*a^2*b^7*x^7 + 231*a^3*b
^6*x^6 - 252*a^4*b^5*x^5 + 280*a^5*b^4*x^4 - 320*a^6*b^3*x^3 + 384*a^7*b^2
*x^2 - 512*a^8*b*x + 1024*a^9)*sqrt(b*x + a)/b^7
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 444 vs. 2(164) = 328.

Time = 0.13 (sec) , antiderivative size = 444, normalized size of antiderivative = 2.34

$$\int x(ax^2+bx^3)^{5/2}\,dx = \text{Too large to display}$$

```
integrate(x*(b*x^3+a*x^2)^(5/2),x, algorithm="giac")
```

```
-2048/969969*a^(19/2)*sgn(x)/b^7 + 2/4849845*(1615*(231*(b*x + a)^(13/2) -
 1638*(b*x + a)^(11/2)*a + 5005*(b*x + a)^(9/2)*a^2 - 8580*(b*x + a)^(7/2)
*a^3 + 9009*(b*x + a)^(5/2)*a^4 - 6006*(b*x + a)^(3/2)*a^5 + 3003*sqrt(b*x
 + a)*a^6)*a^3*sgn(x)/b^6 + 2261*(429*(b*x + a)^(15/2) - 3465*(b*x + a)^(1
3/2)*a + 12285*(b*x + a)^(11/2)*a^2 - 25025*(b*x + a)^(9/2)*a^3 + 32175*(b
*x + a)^(7/2)*a^4 - 27027*(b*x + a)^(5/2)*a^5 + 15015*(b*x + a)^(3/2)*a^6
- 6435*sqrt(b*x + a)*a^7)*a^2*sgn(x)/b^6 + 133*(6435*(b*x + a)^(17/2) - 58
344*(b*x + a)^(15/2)*a + 235620*(b*x + a)^(13/2)*a^2 - 556920*(b*x + a)^(1
1/2)*a^3 + 850850*(b*x + a)^(9/2)*a^4 - 875160*(b*x + a)^(7/2)*a^5 + 61261
2*(b*x + a)^(5/2)*a^6 - 291720*(b*x + a)^(3/2)*a^7 + 109395*sqrt(b*x + a)*
a^8)*a*sgn(x)/b^6 + 21*(12155*(b*x + a)^(19/2) - 122265*(b*x + a)^(17/2)*a
 + 554268*(b*x + a)^(15/2)*a^2 - 1492260*(b*x + a)^(13/2)*a^3 + 2645370*(b
*x + a)^(11/2)*a^4 - 3233230*(b*x + a)^(9/2)*a^5 + 2771340*(b*x + a)^(7/2)
*a^6 - 1662804*(b*x + a)^(5/2)*a^7 + 692835*(b*x + a)^(3/2)*a^8 - 230945*s
qrt(b*x + a)*a^9)*sgn(x)/b^6)/b
```

Mupad [B] (verification not implemented)

Time = 8.78 (sec) , antiderivative size = 91, normalized size of antiderivative = 0.48

$$\int x\left(ax^2+bx^3\right)^{5/2}\,dx=\frac{2\,\sqrt{b\,x^3+a\,x^2}\,\left(a+b\,x\right)^3\,\left(1024\,a^6-3584\,a^5\,b\,x+8064\,a^4\,b^2\,x^2-14784\,a^3\,b^3\,x^3+24024\,a^2\,b^4\cdots\right)}{969969\,b^7\,x}$$

```
int(x*(a*x^2 + b*x^3)^(5/2),x)
```

```
(2*(a*x^2 + b*x^3)^(1/2)*(a + b*x)^3*(1024*a^6 + 51051*b^6*x^6 - 36036*a*b
^5*x^5 + 8064*a^4*b^2*x^2 - 14784*a^3*b^3*x^3 + 24024*a^2*b^4*x^4 - 3584*a
^5*b*x))/(969969*b^7*x)
```

Reduce [B] (verification not implemented)

Time = 0.57 (sec) , antiderivative size = 107, normalized size of antiderivative = 0.56

$$\int x(ax^2+bx^3)^{5/2}\,dx = \frac{2\sqrt{bx+a}\,(51051b^9x^9+117117a\,b^8x^8+69069a^2b^7x^7+231a^3b^6x^6-252a^4b^5x^5+280a^5b^4x^4}{969969b^7}$$

```
int(x*(b*x^3+a*x^2)^(5/2),x)
```

```
(2*sqrt(a + b*x)*(1024*a**9 - 512*a**8*b*x + 384*a**7*b**2*x**2 - 320*a**6
*b**3*x**3 + 280*a**5*b**4*x**4 - 252*a**4*b**5*x**5 + 231*a**3*b**6*x**6
+ 69069*a**2*b**7*x**7 + 117117*a*b**8*x**8 + 51051*b**9*x**9))/(969969*b*
*7)
```

3.287 $\int (ax^2+bx^3)^{5/2}\,dx$

Optimal result

Integrand size = 15, antiderivative size = 164

$$\int (ax^2+bx^3)^{5/2}\,dx = -\frac{2a^5(ax^2+bx^3)^{7/2}}{7b^6x^7}+\frac{10a^4(ax^2+bx^3)^{9/2}}{9b^6x^9} -\frac{20a^3(ax^2+bx^3)^{11/2}}{11b^6x^{11}}+\frac{20a^2(ax^2+bx^3)^{13/2}}{13b^6x^{13}}-\frac{2a(ax^2+bx^3)^{15/2}}{3b^6x^{15}}+\frac{2(ax^2+bx^3)^{17/2}}{17b^6x^{17}}$$

```
-2/7*a^5*(b*x^3+a*x^2)^(7/2)/b^6/x^7+10/9*a^4*(b*x^3+a*x^2)^(9/2)/b^6/x^9-
20/11*a^3*(b*x^3+a*x^2)^(11/2)/b^6/x^11+20/13*a^2*(b*x^3+a*x^2)^(13/2)/b^6
/x^13-2/3*a*(b*x^3+a*x^2)^(15/2)/b^6/x^15+2/17*(b*x^3+a*x^2)^(17/2)/b^6/x^
17
```

Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 80, normalized size of antiderivative = 0.49

$$\int (ax^2+bx^3)^{5/2}\,dx = \frac{2x(a+bx)^4(-256a^5+896a^4bx-2016a^3b^2x^2+3696a^2b^3x^3-6006ab^4x^4+9009b^5x^5)}{153153b^6\sqrt{x^2(a+bx)}}$$

```
Integrate[(a*x^2 + b*x^3)^(5/2),x]
```

```
(2*x*(a + b*x)^4*(-256*a^5 + 896*a^4*b*x - 2016*a^3*b^2*x^2 + 3696*a^2*b^3
*x^3 - 6006*a*b^4*x^4 + 9009*b^5*x^5))/(153153*b^6*Sqrt[x^2*(a + b*x)])
```

Rubi [A] (verified)

Time = 0.66 (sec) , antiderivative size = 188, normalized size of antiderivative = 1.15, number of steps used = 6, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.400, Rules used = {1908, 1922, 1922, 1922, 1922, 1920}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \left(ax^2+bx^3\right)^{5/2} dx$$

$$\Big\downarrow \text{1908}$$

$$\frac{2\left(ax^2+bx^3\right)^{7/2}}{17bx^2}-\frac{10a\int \frac{\left(bx^3+ax^2\right)^{5/2}}{x}dx}{17b}$$

$$\Big\downarrow \text{1922}$$

$$\frac{2\left(ax^2+bx^3\right)^{7/2}}{17bx^2}-\frac{10a\left(\frac{2\left(ax^2+bx^3\right)^{7/2}}{15bx^3}-\frac{8a\int \frac{\left(bx^3+ax^2\right)^{5/2}}{x^2}dx}{15b}\right)}{17b}$$

$$\Big\downarrow \text{1922}$$

$$\frac{2\left(ax^2+bx^3\right)^{7/2}}{17bx^2}-\frac{10a\left(\frac{2\left(ax^2+bx^3\right)^{7/2}}{15bx^3}-\frac{8a\left(\frac{2\left(ax^2+bx^3\right)^{7/2}}{13bx^4}-\frac{6a\int \frac{\left(bx^3+ax^2\right)^{5/2}}{x^3}dx}{13b}\right)}{15b}\right)}{17b}$$

$$\Big\downarrow \text{1922}$$

$$\frac{2\left(ax^2+bx^3\right)^{7/2}}{17bx^2}-\frac{10a\left(\frac{2\left(ax^2+bx^3\right)^{7/2}}{15bx^3}-\frac{8a\left(\frac{2\left(ax^2+bx^3\right)^{7/2}}{13bx^4}-\frac{6a\left(\frac{2\left(ax^2+bx^3\right)^{7/2}}{11bx^5}-\frac{4a\int\frac{\left(bx^3+ax^2\right)^{5/2}}{x^4}dx}{11b}\right)}{13b}\right)}{15b}\right)}{17b}$$

$$\Big\downarrow \ 1922$$

$$\frac{2\left(ax^2+bx^3\right)^{7/2}}{17bx^2}-\frac{10a\left(\frac{2\left(ax^2+bx^3\right)^{7/2}}{15bx^3}-\frac{8a\left(\frac{2\left(ax^2+bx^3\right)^{7/2}}{13bx^4}-\frac{6a\left(\frac{2\left(ax^2+bx^3\right)^{7/2}}{11bx^5}-\frac{4a\left(\frac{2\left(ax^2+bx^3\right)^{7/2}}{9bx^6}-\frac{2a\int\frac{\left(bx^3+ax^2\right)^{5/2}}{x^5}dx}{9b}\right)}{11b}\right)}{13b}\right)}{15b}\right)}{17b}$$

$$\Big\downarrow \ 1920$$

$$\frac{2\left(ax^2+bx^3\right)^{7/2}}{17bx^2}-\frac{10a\left(\frac{2\left(ax^2+bx^3\right)^{7/2}}{15bx^3}-\frac{8a\left(\frac{2\left(ax^2+bx^3\right)^{7/2}}{13bx^4}-\frac{6a\left(\frac{2\left(ax^2+bx^3\right)^{7/2}}{11bx^5}-\frac{4a\left(\frac{2\left(ax^2+bx^3\right)^{7/2}}{9bx^6}-\frac{4a\left(ax^2+bx^3\right)^{7/2}}{63b^2x^7}\right)}{11b}\right)}{13b}\right)}{15b}\right)}{17b}$$

```
Int[(a*x^2 + b*x^3)^(5/2),x]
```

```
(2*(a*x^2 + b*x^3)^(7/2))/(17*b*x^2) - (10*a*((2*(a*x^2 + b*x^3)^(7/2))/(1
5*b*x^3) - (8*a*((2*(a*x^2 + b*x^3)^(7/2))/(13*b*x^4) - (6*a*((2*(a*x^2 +
b*x^3)^(7/2))/(11*b*x^5) - (4*a*((-4*a*(a*x^2 + b*x^3)^(7/2))/(63*b^2*x^7)
 + (2*(a*x^2 + b*x^3)^(7/2))/(9*b*x^6)))/(11*b)))/(13*b)))/(15*b)))/(17*b)
```

Defintions of rubi rules used

```
Int[((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol] :> Simp[(a*x^j +
b*x^n)^(p + 1)/(a*(j*p + 1)*x^(j - 1)), x] - Simp[b*((n*p + n - j + 1)/(a*(
j*p + 1)))   Int[x^(n - j)*(a*x^j + b*x^n)^p, x], x] /; FreeQ[{a, b, j, n,
p}, x] &&  !IntegerQ[p] && NeQ[n, j] && ILtQ[Simplify[(n*p + n - j + 1)/(n
- j)], 0] && NeQ[j*p + 1, 0]
```

```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> Simp[(-c^(j - 1))*(c*x)^(m - j + 1)*((a*x^j + b*x^n)^(p + 1)/(a*(n - j
)*(p + 1))), x] /; FreeQ[{a, b, c, j, m, n, p}, x] &&  !IntegerQ[p] && NeQ[
n, j] && EqQ[m + n*p + n - j + 1, 0] && (IntegerQ[j] || GtQ[c, 0])
```

```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> Simp[c^(j - 1)*(c*x)^(m - j + 1)*((a*x^j + b*x^n)^(p + 1)/(a*(m + j*p
+ 1))), x] - Simp[b*((m + n*p + n - j + 1)/(a*c^(n - j)*(m + j*p + 1)))   I
nt[(c*x)^(m + n - j)*(a*x^j + b*x^n)^p, x], x] /; FreeQ[{a, b, c, j, m, n,
p}, x] &&  !IntegerQ[p] && NeQ[n, j] && ILtQ[Simplify[(m + n*p + n - j + 1)
/(n - j)], 0] && NeQ[m + j*p + 1, 0] && (IntegersQ[j, n] || GtQ[c, 0])
```

Maple [A] (verified)

Time = 0.37 (sec) , antiderivative size = 13, normalized size of antiderivative = 0.08

method	result
pseudoelliptic	$\frac{2(bx+a)^{\frac{7}{2}}}{7b}$
gosper	$-\frac{2(bx+a)\left(-9009b^5x^5+6006a\,b^4x^4-3696a^2b^3x^3+2016a^3b^2x^2-896a^4bx+256a^5\right)\left(b\,x^3+a\,x^2\right)^{\frac{5}{2}}}{153153b^6x^5}$
default	$-\frac{2(bx+a)\left(-9009b^5x^5+6006a\,b^4x^4-3696a^2b^3x^3+2016a^3b^2x^2-896a^4bx+256a^5\right)\left(b\,x^3+a\,x^2\right)^{\frac{5}{2}}}{153153b^6x^5}$
orering	$-\frac{2(bx+a)\left(-9009b^5x^5+6006a\,b^4x^4-3696a^2b^3x^3+2016a^3b^2x^2-896a^4bx+256a^5\right)\left(b\,x^3+a\,x^2\right)^{\frac{5}{2}}}{153153b^6x^5}$
risch	$-\frac{2\sqrt{x^2(bx+a)}\left(-9009b^8x^8-21021a\,b^7x^7-12705a^2b^6x^6-63a^3b^5x^5+70a^4b^4x^4-80a^5b^3x^3+96a^6b^2x^2-128a^7bx+256a^8\right)}{153153x\,b^6}$
trager	$-\frac{2\left(-9009b^8x^8-21021a\,b^7x^7-12705a^2b^6x^6-63a^3b^5x^5+70a^4b^4x^4-80a^5b^3x^3+96a^6b^2x^2-128a^7bx+256a^8\right)\sqrt{b\,x^3+a\,x^2}}{153153b^6x}$

```
int((b*x^3+a*x^2)^(5/2),x,method=_RETURNVERBOSE)
```

```
2/7*(b*x+a)^(7/2)/b
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 106, normalized size of antiderivative = 0.65

$$\int \left(ax^2+bx^3\right)^{5/2}\,dx=\frac{2\left(9009\,b^8x^8+21021\,ab^7x^7+12705\,a^2b^6x^6+63\,a^3b^5x^5-70\,a^4b^4x^4+80\,a^5b^3x^3-96\,a^6b^2x^2\right.}{153153\,b^6x}$$

```
integrate((b*x^3+a*x^2)^(5/2),x, algorithm="fricas")
```

```
2/153153*(9009*b^8*x^8 + 21021*a*b^7*x^7 + 12705*a^2*b^6*x^6 + 63*a^3*b^5*
x^5 - 70*a^4*b^4*x^4 + 80*a^5*b^3*x^3 - 96*a^6*b^2*x^2 + 128*a^7*b*x - 256
*a^8)*sqrt(b*x^3 + a*x^2)/(b^6*x)
```

Sympy [F]

$$\int \left(ax^2 + bx^3\right)^{5/2}\,dx = \int \left(ax^2 + bx^3\right)^{\frac{5}{2}}\,dx$$

```
integrate((b*x**3+a*x**2)**(5/2),x)
```

```
Integral((a*x**2 + b*x**3)**(5/2), x)
```

Maxima [A] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 97, normalized size of antiderivative = 0.59

$$\int \left(ax^2 + bx^3\right)^{5/2}\,dx = \frac{2\left(9009\,b^8x^8 + 21021\,ab^7x^7 + 12705\,a^2b^6x^6 + 63\,a^3b^5x^5 - 70\,a^4b^4x^4 + 80\,a^5b^3x^3 - 96\,a^6b^2x^2\right.}{153153\,b^6}$$

```
integrate((b*x^3+a*x^2)^(5/2),x, algorithm="maxima")
```

```
2/153153*(9009*b^8*x^8 + 21021*a*b^7*x^7 + 12705*a^2*b^6*x^6 + 63*a^3*b^5*
x^5 - 70*a^4*b^4*x^4 + 80*a^5*b^3*x^3 - 96*a^6*b^2*x^2 + 128*a^7*b*x - 256
*a^8)*sqrt(b*x + a)/b^6
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 396 vs. $2(140) = 280$.

Time = 0.12 (sec) , antiderivative size = 396, normalized size of antiderivative = 2.41

$$\int \left(ax^2+bx^3\right)^{5/2}\,dx=\frac{512\,a^{\frac{17}{2}}\operatorname{sgn}(x)}{153153\,b^6}+\frac{2\left(\frac{1105\left(63\,(bx+a)^{\frac{11}{2}}-385\,(bx+a)^{\frac{9}{2}}a+990\,(bx+a)^{\frac{7}{2}}a^2-1386\,(bx+a)^{\frac{5}{2}}a^3+1155\,(bx+a)^{\frac{3}{2}}a^4-693\,\sqrt{bx+a}a^5\right)a^3\operatorname{sgn}(x)}{b^5}+\frac{765\left(231\,(bx+a)^{\frac{13}{2}}\right.}{}\right.}{}$$

```
integrate((b*x^3+a*x^2)^(5/2),x, algorithm="giac")
```

```
512/153153*a^(17/2)*sgn(x)/b^6 + 2/765765*(1105*(63*(b*x + a)^(11/2) - 385
*(b*x + a)^(9/2)*a + 990*(b*x + a)^(7/2)*a^2 - 1386*(b*x + a)^(5/2)*a^3 +
1155*(b*x + a)^(3/2)*a^4 - 693*sqrt(b*x + a)*a^5)*a^3*sgn(x)/b^5 + 765*(23
1*(b*x + a)^(13/2) - 1638*(b*x + a)^(11/2)*a + 5005*(b*x + a)^(9/2)*a^2 -
8580*(b*x + a)^(7/2)*a^3 + 9009*(b*x + a)^(5/2)*a^4 - 6006*(b*x + a)^(3/2)
*a^5 + 3003*sqrt(b*x + a)*a^6)*a^2*sgn(x)/b^5 + 357*(429*(b*x + a)^(15/2)
- 3465*(b*x + a)^(13/2)*a + 12285*(b*x + a)^(11/2)*a^2 - 25025*(b*x + a)^(
9/2)*a^3 + 32175*(b*x + a)^(7/2)*a^4 - 27027*(b*x + a)^(5/2)*a^5 + 15015*(
b*x + a)^(3/2)*a^6 - 6435*sqrt(b*x + a)*a^7)*a*sgn(x)/b^5 + 7*(6435*(b*x +
 a)^(17/2) - 58344*(b*x + a)^(15/2)*a + 235620*(b*x + a)^(13/2)*a^2 - 5569
20*(b*x + a)^(11/2)*a^3 + 850850*(b*x + a)^(9/2)*a^4 - 875160*(b*x + a)^(7
/2)*a^5 + 612612*(b*x + a)^(5/2)*a^6 - 291720*(b*x + a)^(3/2)*a^7 + 109395
*sqrt(b*x + a)*a^8)*sgn(x)/b^5)/b
```

Mupad [B] (verification not implemented)

Time = 8.82 (sec) , antiderivative size = 80, normalized size of antiderivative = 0.49

$$\int \left(ax^2+bx^3\right)^{5/2}\,dx=-\frac{2\sqrt{b\,x^3+a\,x^2}\,(a+b\,x)^3\,(256\,a^5-896\,a^4\,b\,x+2016\,a^3\,b^2\,x^2-3696\,a^2\,b^3\,x^3+6006\,a\,b^4\,x^4-9009\,b^5\,x^5)}{153153\,b^6\,x}$$

```
int((a*x^2 + b*x^3)^(5/2),x)
```

```
-(2*(a*x^2 + b*x^3)^(1/2)*(a + b*x)^3*(256*a^5 - 9009*b^5*x^5 + 6006*a*b^4
*x^4 + 2016*a^3*b^2*x^2 - 3696*a^2*b^3*x^3 - 896*a^4*b*x))/(153153*b^6*x)
```

Reduce [B] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 96, normalized size of antiderivative = 0.59

$$\int (ax^2 + bx^3)^{5/2}\,dx = \frac{2\sqrt{bx+a}\,(9009b^8x^8 + 21021a\,b^7x^7 + 12705a^2b^6x^6 + 63a^3b^5x^5 - 70a^4b^4x^4 + 80a^5b^3x^3 - 96a}{153153b^6}$$

```
int((b*x^3+a*x^2)^(5/2),x)
```

```
(2*sqrt(a + b*x)*( - 256*a**8 + 128*a**7*b*x - 96*a**6*b**2*x**2 + 80*a**5
*b**3*x**3 - 70*a**4*b**4*x**4 + 63*a**3*b**5*x**5 + 12705*a**2*b**6*x**6
+ 21021*a*b**7*x**7 + 9009*b**8*x**8))/(153153*b**6)
```

3.288 $\int \frac{\left(ax^2+bx^3\right)^{5/2}}{x}\,dx$

Optimal result

Integrand size = 19, antiderivative size = 136

$$\int \frac{\left(ax^2+bx^3\right)^{5/2}}{x}\,dx = \frac{2a^4\left(ax^2+bx^3\right)^{7/2}}{7b^5x^7} - \frac{8a^3\left(ax^2+bx^3\right)^{9/2}}{9b^5x^9} + \frac{12a^2\left(ax^2+bx^3\right)^{11/2}}{11b^5x^{11}} - \frac{8a\left(ax^2+bx^3\right)^{13/2}}{13b^5x^{13}} + \frac{2\left(ax^2+bx^3\right)^{15/2}}{15b^5x^{15}}$$

```
2/7*a^4*(b*x^3+a*x^2)^(7/2)/b^5/x^7-8/9*a^3*(b*x^3+a*x^2)^(9/2)/b^5/x^9+12
/11*a^2*(b*x^3+a*x^2)^(11/2)/b^5/x^11-8/13*a*(b*x^3+a*x^2)^(13/2)/b^5/x^13
+2/15*(b*x^3+a*x^2)^(15/2)/b^5/x^15
```

Mathematica [A] (verified)

Time = 0.03 (sec) , antiderivative size = 69, normalized size of antiderivative = 0.51

$$\int \frac{\left(ax^2+bx^3\right)^{5/2}}{x}\,dx = \frac{2x(a+bx)^4\left(128a^4-448a^3bx+1008a^2b^2x^2-1848ab^3x^3+3003b^4x^4\right)}{45045b^5\sqrt{x^2(a+bx)}}$$

```
Integrate[(a*x^2 + b*x^3)^(5/2)/x,x]
```

```
(2*x*(a + b*x)^4*(128*a^4 - 448*a^3*b*x + 1008*a^2*b^2*x^2 - 1848*a*b^3*x^
3 + 3003*b^4*x^4))/(45045*b^5*Sqrt[x^2*(a + b*x)])
```

Rubi [A] (verified)

Time = 0.57 (sec) , antiderivative size = 154, normalized size of antiderivative = 1.13, number of steps used = 5, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.263, Rules used = {1922, 1922, 1922, 1922, 1920}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(ax^2+bx^3)^{5/2}}{x}\,dx$$

$$\Big\downarrow \text{1922}$$

$$\frac{2(ax^2+bx^3)^{7/2}}{15bx^3}-\frac{8a\int \frac{(bx^3+ax^2)^{5/2}}{x^2}dx}{15b}$$

$$\Big\downarrow \text{1922}$$

$$\frac{2(ax^2+bx^3)^{7/2}}{15bx^3}-\frac{8a\left(\frac{2(ax^2+bx^3)^{7/2}}{13bx^4}-\frac{6a\int \frac{(bx^3+ax^2)^{5/2}}{x^3}dx}{13b}\right)}{15b}$$

$$\Big\downarrow \text{1922}$$

$$\frac{2(ax^2+bx^3)^{7/2}}{15bx^3}-\frac{8a\left(\frac{2(ax^2+bx^3)^{7/2}}{13bx^4}-\frac{6a\left(\frac{2(ax^2+bx^3)^{7/2}}{11bx^5}-\frac{4a\int \frac{(bx^3+ax^2)^{5/2}}{x^4}dx}{11b}\right)}{13b}\right)}{15b}$$

$$\Big\downarrow \text{1922}$$

$$\frac{2\left(ax^2+bx^3\right)^{7/2}}{15bx^3}-\frac{8a\left(\frac{2\left(ax^2+bx^3\right)^{7/2}}{13bx^4}-\frac{6a\left(\frac{2\left(ax^2+bx^3\right)^{7/2}}{11bx^5}-\frac{4a\left(\frac{2\left(ax^2+bx^3\right)^{7/2}}{9bx^6}-\frac{2a\int\frac{\left(bx^3+ax^2\right)^{5/2}}{x^5}dx}{9b}\right)}{11b}\right)}{13b}\right)}{15b}$$

$$\Big\downarrow \; 1920$$

$$\frac{2\left(ax^2+bx^3\right)^{7/2}}{15bx^3}-\frac{8a\left(\frac{2\left(ax^2+bx^3\right)^{7/2}}{13bx^4}-\frac{6a\left(\frac{2\left(ax^2+bx^3\right)^{7/2}}{11bx^5}-\frac{4a\left(\frac{2\left(ax^2+bx^3\right)^{7/2}}{9bx^6}-\frac{4a\left(ax^2+bx^3\right)^{7/2}}{63b^2x^7}\right)}{11b}\right)}{13b}\right)}{15b}$$

```
Int[(a*x^2 + b*x^3)^(5/2)/x,x]
```

```
(2*(a*x^2 + b*x^3)^(7/2))/(15*b*x^3) - (8*a*((2*(a*x^2 + b*x^3)^(7/2))/(13
*b*x^4) - (6*a*((2*(a*x^2 + b*x^3)^(7/2))/(11*b*x^5) - (4*a*((-4*a*(a*x^2
+ b*x^3)^(7/2))/(63*b^2*x^7) + (2*(a*x^2 + b*x^3)^(7/2))/(9*b*x^6)))/(11*b
)))/(13*b)))/(15*b)
```

Defintions of rubi rules used

```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> Simp[(-c^(j - 1))*(c*x)^(m - j + 1)*((a*x^j + b*x^n)^(p + 1)/(a*(n - j
)*(p + 1))), x] /; FreeQ[{a, b, c, j, m, n, p}, x] &&  !IntegerQ[p] && NeQ[
n, j] && EqQ[m + n*p + n - j + 1, 0] && (IntegerQ[j] || GtQ[c, 0])
```

```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> Simp[c^(j - 1)*(c*x)^(m - j + 1)*((a*x^j + b*x^n)^(p + 1)/(a*(m + j*p
+ 1))), x] - Simp[b*((m + n*p + n - j + 1)/(a*c^(n - j)*(m + j*p + 1)))   I
nt[(c*x)^(m + n - j)*(a*x^j + b*x^n)^p, x], x] /; FreeQ[{a, b, c, j, m, n,
p}, x] &&  !IntegerQ[p] && NeQ[n, j] && ILtQ[Simplify[(m + n*p + n - j + 1)
/(n - j)], 0] && NeQ[m + j*p + 1, 0] && (IntegersQ[j, n] || GtQ[c, 0])
```

Maple [C] (verified)

Result contains higher order function than in optimal. Order 3 vs. order 2.

Time = 0.39 (sec) , antiderivative size = 47, normalized size of antiderivative = 0.35

method	result	size
pseudoelliptic	$-2a^{\frac{5}{2}} \operatorname{arctanh}\left(\frac{\sqrt{bx+a}}{\sqrt{a}}\right)+\frac{2\sqrt{bx+a}\,(3b^2x^2+11abx+23a^2)}{15}$	47
gosper	$\frac{2(bx+a)(3003b^4x^4-1848a\,b^3x^3+1008a^2b^2x^2-448a^3bx+128a^4)(b\,x^3+a\,x^2)^{\frac{5}{2}}}{45045b^5x^5}$	68
default	$\frac{2(bx+a)(3003b^4x^4-1848a\,b^3x^3+1008a^2b^2x^2-448a^3bx+128a^4)(b\,x^3+a\,x^2)^{\frac{5}{2}}}{45045b^5x^5}$	68
orering	$\frac{2(bx+a)(3003b^4x^4-1848a\,b^3x^3+1008a^2b^2x^2-448a^3bx+128a^4)(b\,x^3+a\,x^2)^{\frac{5}{2}}}{45045b^5x^5}$	68
risch	$\frac{2\sqrt{x^2(bx+a)}\,(3003x^7b^7+7161b^6a\,x^6+4473a^2x^5b^5+35b^4x^4a^3-40b^3x^3a^4+48x^2b^2a^5-64xb\,a^6+128a^7)}{45045x\,b^5}$	94
trager	$\frac{2(3003x^7b^7+7161b^6a\,x^6+4473a^2x^5b^5+35b^4x^4a^3-40b^3x^3a^4+48x^2b^2a^5-64xb\,a^6+128a^7)\sqrt{b\,x^3+a\,x^2}}{45045b^5x}$	96

```
int((b*x^3+a*x^2)^(5/2)/x,x,method=_RETURNVERBOSE)
```

```
-2*a^(5/2)*arctanh((b*x+a)^(1/2)/a^(1/2))+2/15*(b*x+a)^(1/2)*(3*b^2*x^2+11
*a*b*x+23*a^2)
```

Fricas [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 95, normalized size of antiderivative = 0.70

$$\int \frac{(ax^2+bx^3)^{5/2}}{x}\,dx = \frac{2\,(3003\,b^7x^7+7161\,ab^6x^6+4473\,a^2b^5x^5+35\,a^3b^4x^4-40\,a^4b^3x^3+48\,a^5b^2x^2-64\,a}{45045\,b^5x}$$

```
integrate((b*x^3+a*x^2)^(5/2)/x,x, algorithm="fricas")
```

```
2/45045*(3003*b^7*x^7 + 7161*a*b^6*x^6 + 4473*a^2*b^5*x^5 + 35*a^3*b^4*x^4
 - 40*a^4*b^3*x^3 + 48*a^5*b^2*x^2 - 64*a^6*b*x + 128*a^7)*sqrt(b*x^3 + a*
x^2)/(b^5*x)
```

Sympy [F]

$$\int \frac{(ax^2+bx^3)^{5/2}}{x}\,dx = \int \frac{(x^2(a+bx))^{\frac{5}{2}}}{x}\,dx$$

```
integrate((b*x**3+a*x**2)**(5/2)/x,x)
```

```
Integral((x**2*(a + b*x))**(5/2)/x, x)
```

Maxima [A] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 86, normalized size of antiderivative = 0.63

$$\int \frac{(ax^2+bx^3)^{5/2}}{x}\,dx = \frac{2\,(3003\,b^7x^7+7161\,ab^6x^6+4473\,a^2b^5x^5+35\,a^3b^4x^4-40\,a^4b^3x^3+48\,a^5b^2x^2-64\,a}{45045\,b^5}$$

```
integrate((b*x^3+a*x^2)^(5/2)/x,x, algorithm="maxima")
```

```
2/45045*(3003*b^7*x^7 + 7161*a*b^6*x^6 + 4473*a^2*b^5*x^5 + 35*a^3*b^4*x^4
 - 40*a^4*b^3*x^3 + 48*a^5*b^2*x^2 - 64*a^6*b*x + 128*a^7)*sqrt(b*x + a)/b
^5
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 348 vs. $2(116) = 232$.

Time = 0.13 (sec) , antiderivative size = 348, normalized size of antiderivative = 2.56

$$\int \frac{(ax^2+bx^3)^{5/2}}{x}\,dx = -\frac{256\,a^{\frac{15}{2}}\operatorname{sgn}(x)}{45045\,b^5} + \frac{2\left(\frac{143\left(35\,(bx+a)^{\frac{9}{2}}-180\,(bx+a)^{\frac{7}{2}}a+378\,(bx+a)^{\frac{5}{2}}a^2-420\,(bx+a)^{\frac{3}{2}}a^3+315\,\sqrt{bx+a}a^4\right)a^3\operatorname{sgn}(x)}{b^4} + \frac{195\left(63\,(bx+a)^{\frac{11}{2}}-385\,(bx+a)^{\frac{9}{2}}a+990\,(\right.}{}\right.}{}$$

```
integrate((b*x^3+a*x^2)^(5/2)/x,x, algorithm="giac")
```

```
-256/45045*a^(15/2)*sgn(x)/b^5 + 2/45045*(143*(35*(b*x + a)^(9/2) - 180*(b
*x + a)^(7/2)*a + 378*(b*x + a)^(5/2)*a^2 - 420*(b*x + a)^(3/2)*a^3 + 315*
sqrt(b*x + a)*a^4)*a^3*sgn(x)/b^4 + 195*(63*(b*x + a)^(11/2) - 385*(b*x +
a)^(9/2)*a + 990*(b*x + a)^(7/2)*a^2 - 1386*(b*x + a)^(5/2)*a^3 + 1155*(b*
x + a)^(3/2)*a^4 - 693*sqrt(b*x + a)*a^5)*a^2*sgn(x)/b^4 + 45*(231*(b*x +
a)^(13/2) - 1638*(b*x + a)^(11/2)*a + 5005*(b*x + a)^(9/2)*a^2 - 8580*(b*x
 + a)^(7/2)*a^3 + 9009*(b*x + a)^(5/2)*a^4 - 6006*(b*x + a)^(3/2)*a^5 + 30
03*sqrt(b*x + a)*a^6)*a*sgn(x)/b^4 + 7*(429*(b*x + a)^(15/2) - 3465*(b*x +
 a)^(13/2)*a + 12285*(b*x + a)^(11/2)*a^2 - 25025*(b*x + a)^(9/2)*a^3 + 32
175*(b*x + a)^(7/2)*a^4 - 27027*(b*x + a)^(5/2)*a^5 + 15015*(b*x + a)^(3/2
)*a^6 - 6435*sqrt(b*x + a)*a^7)*sgn(x)/b^4)/b
```

Mupad [B] (verification not implemented)

Time = 9.00 (sec) , antiderivative size = 69, normalized size of antiderivative = 0.51

$$\int \frac{(ax^2+bx^3)^{5/2}}{x}\,dx = \frac{2\sqrt{b\,x^3+a\,x^2}\,(a+b\,x)^3\,(128\,a^4-448\,a^3\,b\,x+1008\,a^2\,b^2\,x^2-1848\,a\,b^3\,x^3+3003\,b}{45045\,b^5\,x}$$

```
int((a*x^2 + b*x^3)^(5/2)/x,x)
```

```
(2*(a*x^2 + b*x^3)^(1/2)*(a + b*x)^3*(128*a^4 + 3003*b^4*x^4 - 1848*a*b^3*
x^3 + 1008*a^2*b^2*x^2 - 448*a^3*b*x))/(45045*b^5*x)
```

Reduce [B] (verification not implemented)

Time = 0.34 (sec) , antiderivative size = 85, normalized size of antiderivative = 0.62

$$\int \frac{(ax^2+bx^3)^{5/2}}{x}\,dx = \frac{2\sqrt{bx+a}\,(3003b^7x^7+7161a\,b^6x^6+4473a^2b^5x^5+35a^3b^4x^4-40a^4b^3x^3+48a^5b^2x^2}{45045b^5}$$

```
int((b*x^3+a*x^2)^(5/2)/x,x)
```

```
(2*sqrt(a + b*x)*(128*a**7 - 64*a**6*b*x + 48*a**5*b**2*x**2 - 40*a**4*b**
3*x**3 + 35*a**3*b**4*x**4 + 4473*a**2*b**5*x**5 + 7161*a*b**6*x**6 + 3003
*b**7*x**7))/(45045*b**5)
```

3.289 $\int \frac{(ax^2+bx^3)^{5/2}}{x^2}\,dx$

Optimal result

Integrand size = 19, antiderivative size = 108

$$\int \frac{(ax^2+bx^3)^{5/2}}{x^2}\,dx = -\frac{2a^3(ax^2+bx^3)^{7/2}}{7b^4x^7} + \frac{2a^2(ax^2+bx^3)^{9/2}}{3b^4x^9} - \frac{6a(ax^2+bx^3)^{11/2}}{11b^4x^{11}} + \frac{2(ax^2+bx^3)^{13/2}}{13b^4x^{13}}$$

```
-2/7*a^3*(b*x^3+a*x^2)^(7/2)/b^4/x^7+2/3*a^2*(b*x^3+a*x^2)^(9/2)/b^4/x^9-6
/11*a*(b*x^3+a*x^2)^(11/2)/b^4/x^11+2/13*(b*x^3+a*x^2)^(13/2)/b^4/x^13
```

Mathematica [A] (verified)

Time = 0.03 (sec) , antiderivative size = 58, normalized size of antiderivative = 0.54

$$\int \frac{(ax^2+bx^3)^{5/2}}{x^2}\,dx = \frac{2x(a+bx)^4\left(-16a^3+56a^2bx-126ab^2x^2+231b^3x^3\right)}{3003b^4\sqrt{x^2(a+bx)}}$$

```
Integrate[(a*x^2 + b*x^3)^(5/2)/x^2,x]
```

```
(2*x*(a + b*x)^4*(-16*a^3 + 56*a^2*b*x - 126*a*b^2*x^2 + 231*b^3*x^3))/(30
03*b^4*Sqrt[x^2*(a + b*x)])
```

Rubi [A] (verified)

Time = 0.48 (sec) , antiderivative size = 120, normalized size of antiderivative = 1.11, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.211, Rules used = {1922, 1922, 1922, 1920}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\left(ax^2+bx^3\right)^{5/2}}{x^2}dx$$

$$\Big\downarrow \text{1922}$$

$$\frac{2\left(ax^2+bx^3\right)^{7/2}}{13bx^4}-\frac{6a\int \frac{\left(bx^3+ax^2\right)^{5/2}}{x^3}dx}{13b}$$

$$\Big\downarrow \text{1922}$$

$$\frac{2\left(ax^2+bx^3\right)^{7/2}}{13bx^4}-\frac{6a\left(\frac{2\left(ax^2+bx^3\right)^{7/2}}{11bx^5}-\frac{4a\int \frac{\left(bx^3+ax^2\right)^{5/2}}{x^4}dx}{11b}\right)}{13b}$$

$$\Big\downarrow \text{1922}$$

$$\frac{2\left(ax^2+bx^3\right)^{7/2}}{13bx^4}-\frac{6a\left(\frac{2\left(ax^2+bx^3\right)^{7/2}}{11bx^5}-\frac{4a\left(\frac{2\left(ax^2+bx^3\right)^{7/2}}{9bx^6}-\frac{2a\int \frac{\left(bx^3+ax^2\right)^{5/2}}{x^5}dx}{9b}\right)}{11b}\right)}{13b}$$

$$\Big\downarrow \text{1920}$$

$$\frac{2\left(ax^2+bx^3\right)^{7/2}}{13bx^4}-\frac{6a\left(\frac{2\left(ax^2+bx^3\right)^{7/2}}{11bx^5}-\frac{4a\left(\frac{2\left(ax^2+bx^3\right)^{7/2}}{9bx^6}-\frac{4a\left(ax^2+bx^3\right)^{7/2}}{63b^2x^7}\right)}{11b}\right)}{13b}$$

```
Int[(a*x^2 + b*x^3)^(5/2)/x^2,x]
```

```
(2*(a*x^2 + b*x^3)^(7/2))/(13*b*x^4) - (6*a*((2*(a*x^2 + b*x^3)^(7/2))/(11
*b*x^5) - (4*a*((-4*a*(a*x^2 + b*x^3)^(7/2))/(63*b^2*x^7) + (2*(a*x^2 + b*
x^3)^(7/2))/(9*b*x^6)))/(11*b)))/(13*b)
```

Defintions of rubi rules used

```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> Simp[(-c^(j - 1))*(c*x)^(m - j + 1)*((a*x^j + b*x^n)^(p + 1)/(a*(n - j
)*(p + 1))), x] /; FreeQ[{a, b, c, j, m, n, p}, x] &&  !IntegerQ[p] && NeQ[
n, j] && EqQ[m + n*p + n - j + 1, 0] && (IntegerQ[j] || GtQ[c, 0])
```

```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> Simp[c^(j - 1)*(c*x)^(m - j + 1)*((a*x^j + b*x^n)^(p + 1)/(a*(m + j*p
+ 1))), x] - Simp[b*((m + n*p + n - j + 1)/(a*c^(n - j)*(m + j*p + 1)))   I
nt[(c*x)^(m + n - j)*(a*x^j + b*x^n)^p, x], x] /; FreeQ[{a, b, c, j, m, n,
p}, x] &&  !IntegerQ[p] && NeQ[n, j] && ILtQ[Simplify[(m + n*p + n - j + 1)
/(n - j)], 0] && NeQ[m + j*p + 1, 0] && (IntegersQ[j, n] || GtQ[c, 0])
```

Maple [A] (verified)

Time = 0.42 (sec) , antiderivative size = 57, normalized size of antiderivative = 0.53

method	result	size
gosper	$-\frac{2(bx+a)\left(-231b^{3}x^{3}+126a\,b^{2}x^{2}-56a^{2}bx+16a^{3}\right)\left(b\,x^{3}+a\,x^{2}\right)^{\frac{5}{2}}}{3003b^{4}x^{5}}$	57
default	$-\frac{2(bx+a)\left(-231b^{3}x^{3}+126a\,b^{2}x^{2}-56a^{2}bx+16a^{3}\right)\left(b\,x^{3}+a\,x^{2}\right)^{\frac{5}{2}}}{3003b^{4}x^{5}}$	57
orering	$-\frac{2(bx+a)\left(-231b^{3}x^{3}+126a\,b^{2}x^{2}-56a^{2}bx+16a^{3}\right)\left(b\,x^{3}+a\,x^{2}\right)^{\frac{5}{2}}}{3003b^{4}x^{5}}$	57
pseudoelliptic	$-\frac{5\,\operatorname{arctanh}\left(\frac{\sqrt{bx+a}}{\sqrt{a}}\right)a^{2}bx+\sqrt{bx+a}\,\left(-\frac{2\sqrt{a}\,b^{2}x^{2}}{3}+a^{\frac{3}{2}}\left(a-\frac{14bx}{3}\right)\right)}{\sqrt{a}\,x}$	59
risch	$-\frac{2\sqrt{x^{2}(bx+a)}\,\left(-231b^{6}x^{6}-567a\,b^{5}x^{5}-371a^{2}b^{4}x^{4}-5a^{3}b^{3}x^{3}+6a^{4}b^{2}x^{2}-8a^{5}bx+16a^{6}\right)}{3003x\,b^{4}}$	83
trager	$-\frac{2\left(-231b^{6}x^{6}-567a\,b^{5}x^{5}-371a^{2}b^{4}x^{4}-5a^{3}b^{3}x^{3}+6a^{4}b^{2}x^{2}-8a^{5}bx+16a^{6}\right)\sqrt{b\,x^{3}+a\,x^{2}}}{3003b^{4}x}$	85

```
int((b*x^3+a*x^2)^(5/2)/x^2,x,method=_RETURNVERBOSE)
```

```
-2/3003*(b*x+a)*(-231*b^3*x^3+126*a*b^2*x^2-56*a^2*b*x+16*a^3)*(b*x^3+a*x^
2)^(5/2)/b^4/x^5
```

Fricas [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 84, normalized size of antiderivative = 0.78

$$\int \frac{(ax^2+bx^3)^{5/2}}{x^2}\,dx = \frac{2\,(231\,b^6x^6+567\,ab^5x^5+371\,a^2b^4x^4+5\,a^3b^3x^3-6\,a^4b^2x^2+8\,a^5bx-16\,a^6)\sqrt{bx^3}}{3003\,b^4x}$$

```
integrate((b*x^3+a*x^2)^(5/2)/x^2,x, algorithm="fricas")
```

```
2/3003*(231*b^6*x^6 + 567*a*b^5*x^5 + 371*a^2*b^4*x^4 + 5*a^3*b^3*x^3 - 6*
a^4*b^2*x^2 + 8*a^5*b*x - 16*a^6)*sqrt(b*x^3 + a*x^2)/(b^4*x)
```

Sympy [F]

$$\int \frac{(ax^2+bx^3)^{5/2}}{x^2}\,dx = \int \frac{\left(x^2(a+bx)\right)^{\frac{5}{2}}}{x^2}\,dx$$

```
integrate((b*x**3+a*x**2)**(5/2)/x**2,x)
```

```
Integral((x**2*(a + b*x))**(5/2)/x**2, x)
```

Maxima [A] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 75, normalized size of antiderivative = 0.69

$$\int \frac{(ax^2+bx^3)^{5/2}}{x^2}\,dx = \frac{2\,(231\,b^6x^6+567\,ab^5x^5+371\,a^2b^4x^4+5\,a^3b^3x^3-6\,a^4b^2x^2+8\,a^5bx-16\,a^6)\sqrt{bx+}}{3003\,b^4}$$

```
integrate((b*x^3+a*x^2)^(5/2)/x^2,x, algorithm="maxima")
```

```
2/3003*(231*b^6*x^6 + 567*a*b^5*x^5 + 371*a^2*b^4*x^4 + 5*a^3*b^3*x^3 - 6*
a^4*b^2*x^2 + 8*a^5*b*x - 16*a^6)*sqrt(b*x + a)/b^4
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 300 vs. 2(92) = 184.

Time = 0.12 (sec) , antiderivative size = 300, normalized size of antiderivative = 2.78

$$\int \frac{(ax^2+bx^3)^{5/2}}{x^2}\,dx = \frac{32\,a^{\frac{13}{2}}\,\mathrm{sgn}(x)}{3003\,b^4} + \frac{2\left(\frac{429\left(5\,(bx+a)^{\frac{7}{2}}-21\,(bx+a)^{\frac{5}{2}}a+35\,(bx+a)^{\frac{3}{2}}a^2-35\,\sqrt{bx+a}a^3\right)a^3\mathrm{sgn}(x)}{b^3}+\frac{143\left(35\,(bx+a)^{\frac{9}{2}}-180\,(bx+a)^{\frac{7}{2}}a+378\,(bx+a)^{\frac{5}{2}}a^2-420\,(bx+a\right.}{b^3}\right.}{}$$

```
integrate((b*x^3+a*x^2)^(5/2)/x^2,x, algorithm="giac")
```

```
32/3003*a^(13/2)*sgn(x)/b^4 + 2/15015*(429*(5*(b*x + a)^(7/2) - 21*(b*x +
a)^(5/2)*a + 35*(b*x + a)^(3/2)*a^2 - 35*sqrt(b*x + a)*a^3)*a^3*sgn(x)/b^3
 + 143*(35*(b*x + a)^(9/2) - 180*(b*x + a)^(7/2)*a + 378*(b*x + a)^(5/2)*a
^2 - 420*(b*x + a)^(3/2)*a^3 + 315*sqrt(b*x + a)*a^4)*a^2*sgn(x)/b^3 + 65*
(63*(b*x + a)^(11/2) - 385*(b*x + a)^(9/2)*a + 990*(b*x + a)^(7/2)*a^2 - 1
386*(b*x + a)^(5/2)*a^3 + 1155*(b*x + a)^(3/2)*a^4 - 693*sqrt(b*x + a)*a^5
)*a*sgn(x)/b^3 + 5*(231*(b*x + a)^(13/2) - 1638*(b*x + a)^(11/2)*a + 5005*
(b*x + a)^(9/2)*a^2 - 8580*(b*x + a)^(7/2)*a^3 + 9009*(b*x + a)^(5/2)*a^4
- 6006*(b*x + a)^(3/2)*a^5 + 3003*sqrt(b*x + a)*a^6)*sgn(x)/b^3)/b
```

Mupad [B] (verification not implemented)

Time = 8.90 (sec) , antiderivative size = 58, normalized size of antiderivative = 0.54

$$\int \frac{(ax^2+bx^3)^{5/2}}{x^2}\,dx = -\frac{2\sqrt{b\,x^3+a\,x^2}\,(a+b\,x)^3\,(16\,a^3-56\,a^2\,b\,x+126\,a\,b^2\,x^2-231\,b^3\,x^3)}{3003\,b^4\,x}$$

```
int((a*x^2 + b*x^3)^(5/2)/x^2,x)
```

```
-(2*(a*x^2 + b*x^3)^(1/2)*(a + b*x)^3*(16*a^3 - 231*b^3*x^3 + 126*a*b^2*x^
2 - 56*a^2*b*x))/(3003*b^4*x)
```

Reduce [B] (verification not implemented)

Time = 0.59 (sec) , antiderivative size = 74, normalized size of antiderivative = 0.69

$$\int \frac{(ax^2+bx^3)^{5/2}}{x^2}\,dx = \frac{2\sqrt{bx+a}\,(231b^6x^6+567a\,b^5x^5+371a^2b^4x^4+5a^3b^3x^3-6a^4b^2x^2+8a^5bx-16a^6)}{3003b^4}$$

```
int((b*x^3+a*x^2)^(5/2)/x^2,x)
```

```
(2*sqrt(a + b*x)*( - 16*a**6 + 8*a**5*b*x - 6*a**4*b**2*x**2 + 5*a**3*b**3
*x**3 + 371*a**2*b**4*x**4 + 567*a*b**5*x**5 + 231*b**6*x**6))/(3003*b**4)
```

3.290 $\int \frac{(ax^2+bx^3)^{5/2}}{x^3}\,dx$

Optimal result

Integrand size = 19, antiderivative size = 80

$$\int \frac{(ax^2+bx^3)^{5/2}}{x^3}\,dx = \frac{2a^2(ax^2+bx^3)^{7/2}}{7b^3x^7} - \frac{4a(ax^2+bx^3)^{9/2}}{9b^3x^9} + \frac{2(ax^2+bx^3)^{11/2}}{11b^3x^{11}}$$

```
2/7*a^2*(b*x^3+a*x^2)^(7/2)/b^3/x^7-4/9*a*(b*x^3+a*x^2)^(9/2)/b^3/x^9+2/11
*(b*x^3+a*x^2)^(11/2)/b^3/x^11
```

Mathematica [A] (verified)

Time = 0.04 (sec) , antiderivative size = 47, normalized size of antiderivative = 0.59

$$\int \frac{(ax^2+bx^3)^{5/2}}{x^3}\,dx = \frac{2x(a+bx)^4\,(8a^2-28abx+63b^2x^2)}{693b^3\sqrt{x^2(a+bx)}}$$

```
Integrate[(a*x^2 + b*x^3)^(5/2)/x^3,x]
```

```
(2*x*(a + b*x)^4*(8*a^2 - 28*a*b*x + 63*b^2*x^2))/(693*b^3*Sqrt[x^2*(a + b
*x)])
```

Rubi [A] (verified)

Time = 0.40 (sec) , antiderivative size = 86, normalized size of antiderivative = 1.08, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.158, Rules used = {1922, 1922, 1920}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(ax^2+bx^3)^{5/2}}{x^3} \, dx$$
$$\Big\downarrow \text{ 1922}$$
$$\frac{2(ax^2+bx^3)^{7/2}}{11bx^5}-\frac{4a\int \frac{(bx^3+ax^2)^{5/2}}{x^4}dx}{11b}$$
$$\Big\downarrow \text{ 1922}$$
$$\frac{2(ax^2+bx^3)^{7/2}}{11bx^5}-\frac{4a\left(\frac{2(ax^2+bx^3)^{7/2}}{9bx^6}-\frac{2a\int \frac{(bx^3+ax^2)^{5/2}}{x^5}dx}{9b}\right)}{11b}$$
$$\Big\downarrow \text{ 1920}$$
$$\frac{2(ax^2+bx^3)^{7/2}}{11bx^5}-\frac{4a\left(\frac{2(ax^2+bx^3)^{7/2}}{9bx^6}-\frac{4a(ax^2+bx^3)^{7/2}}{63b^2x^7}\right)}{11b}$$

```
Int[(a*x^2 + b*x^3)^(5/2)/x^3,x]
```

```
(2*(a*x^2 + b*x^3)^(7/2))/(11*b*x^5) - (4*a*((-4*a*(a*x^2 + b*x^3)^(7/2))/
(63*b^2*x^7) + (2*(a*x^2 + b*x^3)^(7/2))/(9*b*x^6)))/(11*b)
```

Defintions of rubi rules used

```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> Simp[(-c^(j - 1))*(c*x)^(m - j + 1)*((a*x^j + b*x^n)^(p + 1)/(a*(n - j
)*(p + 1))), x] /; FreeQ[{a, b, c, j, m, n, p}, x] &&  !IntegerQ[p] && NeQ[
n, j] && EqQ[m + n*p + n - j + 1, 0] && (IntegerQ[j] || GtQ[c, 0])
```

```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> Simp[c^(j - 1)*(c*x)^(m - j + 1)*((a*x^j + b*x^n)^(p + 1)/(a*(m + j*p
+ 1))), x] - Simp[b*((m + n*p + n - j + 1)/(a*c^(n - j)*(m + j*p + 1)))   I
nt[(c*x)^(m + n - j)*(a*x^j + b*x^n)^p, x], x] /; FreeQ[{a, b, c, j, m, n,
p}, x] &&  !IntegerQ[p] && NeQ[n, j] && ILtQ[Simplify[(m + n*p + n - j + 1)
/(n - j)], 0] && NeQ[m + j*p + 1, 0] && (IntegersQ[j, n] || GtQ[c, 0])
```

Maple [A] (verified)

Time = 0.43 (sec) , antiderivative size = 46, normalized size of antiderivative = 0.58

method	result	size
gosper	$\frac{2(bx+a)\left(63b^{2}x^{2}-28abx+8a^{2}\right)\left(b\,x^{3}+a\,x^{2}\right)^{\frac{5}{2}}}{693b^{3}x^{5}}$	46
default	$\frac{2(bx+a)\left(63b^{2}x^{2}-28abx+8a^{2}\right)\left(b\,x^{3}+a\,x^{2}\right)^{\frac{5}{2}}}{693b^{3}x^{5}}$	46
orering	$\frac{2(bx+a)\left(63b^{2}x^{2}-28abx+8a^{2}\right)\left(b\,x^{3}+a\,x^{2}\right)^{\frac{5}{2}}}{693b^{3}x^{5}}$	46
pseudoelliptic	$-\frac{\frac{15\,\operatorname{arctanh}\left(\frac{\sqrt{bx+a}}{\sqrt{a}}\right)a\,b^{2}x^{2}}{2}+\sqrt{bx+a}\,\sqrt{a}\,\left(a^{2}+\frac{9}{2}abx-4b^{2}x^{2}\right)}{2\sqrt{a}\,x^{2}}$	59
risch	$\frac{2\sqrt{x^{2}(bx+a)}\,\left(63b^{5}x^{5}+161a\,b^{4}x^{4}+113a^{2}b^{3}x^{3}+3a^{3}b^{2}x^{2}-4a^{4}bx+8a^{5}\right)}{693x\,b^{3}}$	72
trager	$\frac{2\left(63b^{5}x^{5}+161a\,b^{4}x^{4}+113a^{2}b^{3}x^{3}+3a^{3}b^{2}x^{2}-4a^{4}bx+8a^{5}\right)\sqrt{b\,x^{3}+a\,x^{2}}}{693b^{3}x}$	74

```
int((b*x^3+a*x^2)^(5/2)/x^3,x,method=_RETURNVERBOSE)
```

```
2/693*(b*x+a)*(63*b^2*x^2-28*a*b*x+8*a^2)*(b*x^3+a*x^2)^(5/2)/b^3/x^5
```

Fricas [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 73, normalized size of antiderivative = 0.91

$$\int \frac{(ax^2 + bx^3)^{5/2}}{x^3}\,dx = \frac{2\,(63\,b^5x^5 + 161\,ab^4x^4 + 113\,a^2b^3x^3 + 3\,a^3b^2x^2 - 4\,a^4bx + 8\,a^5)\sqrt{bx^3 + ax^2}}{693\,b^3x}$$

```
integrate((b*x^3+a*x^2)^(5/2)/x^3,x, algorithm="fricas")
```

```
2/693*(63*b^5*x^5 + 161*a*b^4*x^4 + 113*a^2*b^3*x^3 + 3*a^3*b^2*x^2 - 4*a^
4*b*x + 8*a^5)*sqrt(b*x^3 + a*x^2)/(b^3*x)
```

Sympy [F]

$$\int \frac{(ax^2 + bx^3)^{5/2}}{x^3}\,dx = \int \frac{(x^2(a + bx))^{\frac{5}{2}}}{x^3}\,dx$$

```
integrate((b*x**3+a*x**2)**(5/2)/x**3,x)
```

```
Integral((x**2*(a + b*x))**(5/2)/x**3, x)
```

Maxima [A] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 64, normalized size of antiderivative = 0.80

$$\int \frac{(ax^2 + bx^3)^{5/2}}{x^3}\,dx = \frac{2\,(63\,b^5x^5 + 161\,ab^4x^4 + 113\,a^2b^3x^3 + 3\,a^3b^2x^2 - 4\,a^4bx + 8\,a^5)\sqrt{bx + a}}{693\,b^3}$$

```
integrate((b*x^3+a*x^2)^(5/2)/x^3,x, algorithm="maxima")
```

```
2/693*(63*b^5*x^5 + 161*a*b^4*x^4 + 113*a^2*b^3*x^3 + 3*a^3*b^2*x^2 - 4*a^
4*b*x + 8*a^5)*sqrt(b*x + a)/b^3
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 252 vs. $2(68) = 136$.

Time = 0.20 (sec) , antiderivative size = 252, normalized size of antiderivative = 3.15

$$\int \frac{(ax^2+bx^3)^{5/2}}{x^3}\,dx = -\frac{16\,a^{\frac{11}{2}}\operatorname{sgn}(x)}{693\,b^3} + \frac{2\left(\frac{231\left(3\,(bx+a)^{\frac{5}{2}}-10\,(bx+a)^{\frac{3}{2}}a+15\,\sqrt{bx+a}\,a^2\right)a^3\operatorname{sgn}(x)}{b^2} + \frac{297\left(5\,(bx+a)^{\frac{7}{2}}-21\,(bx+a)^{\frac{5}{2}}a+35\,(bx+a)^{\frac{3}{2}}a^2-35\,\sqrt{bx+a}\,a^3\right)a^2\operatorname{sgn}(x)}{b^2} + \frac{33\cdots}{\cdots}\right.}{\cdots}$$

```
integrate((b*x^3+a*x^2)^(5/2)/x^3,x, algorithm="giac")
```

```
-16/693*a^(11/2)*sgn(x)/b^3 + 2/3465*(231*(3*(b*x + a)^(5/2) - 10*(b*x + a
)^(3/2)*a + 15*sqrt(b*x + a)*a^2)*a^3*sgn(x)/b^2 + 297*(5*(b*x + a)^(7/2)
- 21*(b*x + a)^(5/2)*a + 35*(b*x + a)^(3/2)*a^2 - 35*sqrt(b*x + a)*a^3)*a^
2*sgn(x)/b^2 + 33*(35*(b*x + a)^(9/2) - 180*(b*x + a)^(7/2)*a + 378*(b*x +
 a)^(5/2)*a^2 - 420*(b*x + a)^(3/2)*a^3 + 315*sqrt(b*x + a)*a^4)*a*sgn(x)/
b^2 + 5*(63*(b*x + a)^(11/2) - 385*(b*x + a)^(9/2)*a + 990*(b*x + a)^(7/2)
*a^2 - 1386*(b*x + a)^(5/2)*a^3 + 1155*(b*x + a)^(3/2)*a^4 - 693*sqrt(b*x
+ a)*a^5)*sgn(x)/b^2)/b
```

Mupad [B] (verification not implemented)

Time = 9.05 (sec) , antiderivative size = 47, normalized size of antiderivative = 0.59

$$\int \frac{(ax^2+bx^3)^{5/2}}{x^3}\,dx = \frac{2\,\sqrt{b\,x^3+a\,x^2}\,\left(a+b\,x\right)^3\,\left(8\,a^2-28\,a\,b\,x+63\,b^2\,x^2\right)}{693\,b^3\,x}$$

```
int((a*x^2 + b*x^3)^(1/2)... 
```

Reduce [B] (verification not implemented)

Time = 0.35 (sec) , antiderivative size = 63, normalized size of antiderivative = 0.79

$$\int \frac{(ax^2+bx^3)^{5/2}}{x^3}\,dx = \frac{2\sqrt{bx+a}\,(63b^5x^5+161a\,b^4x^4+113a^2b^3x^3+3a^3b^2x^2-4a^4bx+8a^5)}{693b^3}$$

```
int((b*x^3+a*x^2)^(5/2)/x^3,x)
```

```
(2*sqrt(a + b*x)*(8*a**5 - 4*a**4*b*x + 3*a**3*b**2*x**2 + 113*a**2*b**3*x
**3 + 161*a*b**4*x**4 + 63*b**5*x**5))/(693*b**3)
```

3.291 $\int \frac{(ax^2+bx^3)^{5/2}}{x^4}\,dx$

Optimal result

Integrand size = 19, antiderivative size = 52

$$\int \frac{(ax^2+bx^3)^{5/2}}{x^4}\,dx = -\frac{2a(ax^2+bx^3)^{7/2}}{7b^2x^7}+\frac{2(ax^2+bx^3)^{9/2}}{9b^2x^9}$$

```
-2/7*a*(b*x^3+a*x^2)^(7/2)/b^2/x^7+2/9*(b*x^3+a*x^2)^(9/2)/b^2/x^9
```

Mathematica [A] (verified)

Time = 0.03 (sec) , antiderivative size = 31, normalized size of antiderivative = 0.60

$$\int \frac{(ax^2+bx^3)^{5/2}}{x^4}\,dx = \frac{2(x^2(a+bx))^{7/2}\,(-2a+7bx)}{63b^2x^7}$$

```
Integrate[(a*x^2 + b*x^3)^(5/2)/x^4,x]
```

```
(2*(x^2*(a + b*x))^(7/2)*(-2*a + 7*b*x))/(63*b^2*x^7)
```

Rubi [A] (verified)

Time = 0.32 (sec) , antiderivative size = 52, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.105, Rules used = {1922, 1920}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(ax^2+bx^3)^{5/2}}{x^4} \, dx$$
$$\Big\downarrow \text{ 1922}$$
$$\frac{2(ax^2+bx^3)^{7/2}}{9bx^6}-\frac{2a\int \frac{(bx^3+ax^2)^{5/2}}{x^5}dx}{9b}$$
$$\Big\downarrow \text{ 1920}$$
$$\frac{2(ax^2+bx^3)^{7/2}}{9bx^6}-\frac{4a(ax^2+bx^3)^{7/2}}{63b^2x^7}$$

```
Int[(a*x^2 + b*x^3)^(5/2)/x^4,x]
```

```
(-4*a*(a*x^2 + b*x^3)^(7/2))/(63*b^2*x^7) + (2*(a*x^2 + b*x^3)^(7/2))/(9*b
*x^6)
```

Defintions of rubi rules used

```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> Simp[(-c^(j - 1))*(c*x)^(m - j + 1)*((a*x^j + b*x^n)^(p + 1)/(a*(n - j
)*(p + 1))), x] /; FreeQ[{a, b, c, j, m, n, p}, x] &&  !IntegerQ[p] && NeQ[
n, j] && EqQ[m + n*p + n - j + 1, 0] && (IntegerQ[j] || GtQ[c, 0])
```

```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> Simp[c^(j - 1)*(c*x)^(m - j + 1)*((a*x^j + b*x^n)^(p + 1)/(a*(m + j*p
+ 1))), x] - Simp[b*((m + n*p + n - j + 1)/(a*c^(n - j)*(m + j*p + 1)))   I
nt[(c*x)^(m + n - j)*(a*x^j + b*x^n)^p, x], x] /; FreeQ[{a, b, c, j, m, n,
p}, x] &&  !IntegerQ[p] && NeQ[n, j] && ILtQ[Simplify[(m + n*p + n - j + 1)
/(n - j)], 0] && NeQ[m + j*p + 1, 0] && (IntegersQ[j, n] || GtQ[c, 0])
```

Maple [A] (verified)

Time = 0.43 (sec) , antiderivative size = 35, normalized size of antiderivative = 0.67

method	result	size
gosper	$-\frac{2(bx+a)(-7bx+2a)\left(b\,x^{3}+a\,x^{2}\right)^{\frac{5}{2}}}{63b^{2}x^{5}}$	35
default	$-\frac{2(bx+a)(-7bx+2a)\left(b\,x^{3}+a\,x^{2}\right)^{\frac{5}{2}}}{63b^{2}x^{5}}$	35
orering	$-\frac{2(bx+a)(-7bx+2a)\left(b\,x^{3}+a\,x^{2}\right)^{\frac{5}{2}}}{63b^{2}x^{5}}$	35
risch	$-\frac{2\sqrt{x^{2}(bx+a)}\,\left(-7b^{4}x^{4}-19a\,b^{3}x^{3}-15a^{2}b^{2}x^{2}-a^{3}bx+2a^{4}\right)}{63x\,b^{2}}$	61
trager	$-\frac{2\left(-7b^{4}x^{4}-19a\,b^{3}x^{3}-15a^{2}b^{2}x^{2}-a^{3}bx+2a^{4}\right)\sqrt{b\,x^{3}+a\,x^{2}}}{63b^{2}x}$	63
pseudoelliptic	$\frac{-15\,\operatorname{arctanh}\left(\frac{\sqrt{bx+a}}{\sqrt{a}}\right)b^{3}x^{3}-33b^{2}x^{2}\sqrt{a}\,\sqrt{bx+a}-26a^{\frac{3}{2}}\,bx\sqrt{bx+a}-8\sqrt{bx+a}\,a^{\frac{5}{2}}}{24x^{3}\sqrt{a}}$	74

```
int((b*x^3+a*x^2)^(5/2)/x^4,x,method=_RETURNVERBOSE)
```

```
-2/63*(b*x+a)*(-7*b*x+2*a)*(b*x^3+a*x^2)^(5/2)/b^2/x^5
```

Fricas [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 61, normalized size of antiderivative = 1.17

$$\int \frac{\left(ax^2+bx^3\right)^{5/2}}{x^4}\,dx = \frac{2\,\left(7\,b^4x^4+19\,ab^3x^3+15\,a^2b^2x^2+a^3bx-2\,a^4\right)\sqrt{bx^3+ax^2}}{63\,b^2x}$$

```
integrate((b*x^3+a*x^2)^(5/2)/x^4,x, algorithm="fricas")
```

```
2/63*(7*b^4*x^4 + 19*a*b^3*x^3 + 15*a^2*b^2*x^2 + a^3*b*x - 2*a^4)*sqrt(b*
x^3 + a*x^2)/(b^2*x)
```

Sympy [F]

$$\int \frac{(ax^2+bx^3)^{5/2}}{x^4}\,dx = \int \frac{(x^2(a+bx))^{\frac{5}{2}}}{x^4}\,dx$$

```
integrate((b*x**3+a*x**2)**(5/2)/x**4,x)
```

```
Integral((x**2*(a + b*x))**(5/2)/x**4, x)
```

Maxima [A] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 52, normalized size of antiderivative = 1.00

$$\int \frac{(ax^2+bx^3)^{5/2}}{x^4}\,dx = \frac{2\,(7\,b^4x^4+19\,ab^3x^3+15\,a^2b^2x^2+a^3bx-2\,a^4)\sqrt{bx+a}}{63\,b^2}$$

```
integrate((b*x^3+a*x^2)^(5/2)/x^4,x, algorithm="maxima")
```

```
2/63*(7*b^4*x^4 + 19*a*b^3*x^3 + 15*a^2*b^2*x^2 + a^3*b*x - 2*a^4)*sqrt(b*
x + a)/b^2
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 201 vs. 2(44) = 88.

Time = 0.12 (sec) , antiderivative size = 201, normalized size of antiderivative = 3.87

$$\int \frac{(ax^2+bx^3)^{5/2}}{x^4}\,dx = \frac{4\,a^{\frac{9}{2}}\mathrm{sgn}(x)}{63\,b^2} + \frac{2\left(\frac{105\left((bx+a)^{\frac{3}{2}}-3\sqrt{bx+a}a\right)a^3\mathrm{sgn}(x)}{b}+\frac{63\left(3\,(bx+a)^{\frac{5}{2}}-10\,(bx+a)^{\frac{3}{2}}a+15\sqrt{bx+a}a^2\right)a^2\mathrm{sgn}(x)}{b}+\frac{27\left(5\,(bx+a)^{\frac{7}{2}}-21\,(bx+a)^{\frac{5}{2}}a+35\,(bx-\right.}{b}\right.}{315\,b}$$

```
integrate((b*x^3+a*x^2)^(5/2)/x^4,x, algorithm="giac")
```

```
4/63*a^(9/2)*sgn(x)/b^2 + 2/315*(105*((b*x + a)^(3/2) - 3*sqrt(b*x + a)*a)
*a^3*sgn(x)/b + 63*(3*(b*x + a)^(5/2) - 10*(b*x + a)^(3/2)*a + 15*sqrt(b*x
 + a)*a^2)*a^2*sgn(x)/b + 27*(5*(b*x + a)^(7/2) - 21*(b*x + a)^(5/2)*a + 3
5*(b*x + a)^(3/2)*a^2 - 35*sqrt(b*x + a)*a^3)*a*sgn(x)/b + (35*(b*x + a)^(
9/2) - 180*(b*x + a)^(7/2)*a + 378*(b*x + a)^(5/2)*a^2 - 420*(b*x + a)^(3/
2)*a^3 + 315*sqrt(b*x + a)*a^4)*sgn(x)/b)/b
```

Mupad [B] (verification not implemented)

Time = 8.88 (sec) , antiderivative size = 36, normalized size of antiderivative = 0.69

$$\int \frac{(ax^2+bx^3)^{5/2}}{x^4}\,dx = -\frac{2\,(2\,a-7\,b\,x)\,\sqrt{b\,x^3+a\,x^2}\,(a+b\,x)^3}{63\,b^2\,x}$$

```
int((a*x^2 + b*x^3)^(5/2)/x^4,x)
```

```
-(2*(2*a - 7*b*x)*(a*x^2 + b*x^3)^(1/2)*(a + b*x)^3)/(63*b^2*x)
```

Reduce [B] (verification not implemented)

Time = 0.21 (sec) , antiderivative size = 51, normalized size of antiderivative = 0.98

$$\int \frac{(ax^2+bx^3)^{5/2}}{x^4}\,dx = \frac{2\sqrt{bx+a}\,(7b^4x^4+19a\,b^3x^3+15a^2b^2x^2+a^3bx-2a^4)}{63b^2}$$

```
int((b*x^3+a*x^2)^(5/2)/x^4,x)
```

```
(2*sqrt(a + b*x)*( - 2*a**4 + a**3*b*x + 15*a**2*b**2*x**2 + 19*a*b**3*x**
3 + 7*b**4*x**4))/(63*b**2)
```

3.292 $\int \frac{(ax^2+bx^3)^{5/2}}{x^5}\,dx$

Optimal result

Integrand size = 19, antiderivative size = 25

$$\int \frac{(ax^2+bx^3)^{5/2}}{x^5}\,dx = \frac{2(ax^2+bx^3)^{7/2}}{7bx^7}$$

```
2/7*(b*x^3+a*x^2)^(7/2)/b/x^7
```

Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 23, normalized size of antiderivative = 0.92

$$\int \frac{(ax^2+bx^3)^{5/2}}{x^5}\,dx = \frac{2(x^2(a+bx))^{7/2}}{7bx^7}$$

```
Integrate[(a*x^2 + b*x^3)^(5/2)/x^5,x]
```

```
(2*(x^2*(a + b*x))^(7/2))/(7*b*x^7)
```

Rubi [A] (verified)

Time = 0.26 (sec) , antiderivative size = 25, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.053, Rules used = {1920}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(ax^2+bx^3)^{5/2}}{x^5} dx$$
$$\Big\downarrow \text{ 1920}$$
$$\frac{2(ax^2+bx^3)^{7/2}}{7bx^7}$$

```
Int[(a*x^2 + b*x^3)^(5/2)/x^5,x]
```

```
(2*(a*x^2 + b*x^3)^(7/2))/(7*b*x^7)
```

Defintions of rubi rules used

```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> Simp[(-c^(j - 1))*(c*x)^(m - j + 1)*((a*x^j + b*x^n)^(p + 1)/(a*(n - j
)*(p + 1))), x] /; FreeQ[{a, b, c, j, m, n, p}, x] &&  !IntegerQ[p] && NeQ[
n, j] && EqQ[m + n*p + n - j + 1, 0] && (IntegerQ[j] || GtQ[c, 0])
```

Maple [A] (verified)

Time = 0.46 (sec) , antiderivative size = 27, normalized size of antiderivative = 1.08

method	result	size
gosper	$\frac{2(bx+a)\left(b\,x^{3}+a\,x^{2}\right)^{\frac{5}{2}}}{7b\,x^{5}}$	27
default	$\frac{2(bx+a)\left(b\,x^{3}+a\,x^{2}\right)^{\frac{5}{2}}}{7b\,x^{5}}$	27
orering	$\frac{2(bx+a)\left(b\,x^{3}+a\,x^{2}\right)^{\frac{5}{2}}}{7b\,x^{5}}$	27
risch	$\frac{2\sqrt{x^{2}(bx+a)}\,\left(b^{3}x^{3}+3a\,b^{2}x^{2}+3a^{2}bx+a^{3}\right)}{7xb}$	47
trager	$\frac{2\left(b^{3}x^{3}+3a\,b^{2}x^{2}+3a^{2}bx+a^{3}\right)\sqrt{b\,x^{3}+a\,x^{2}}}{7bx}$	49
pseudoelliptic	$-\frac{5\left(-\operatorname{arctanh}\left(\frac{\sqrt{bx+a}}{\sqrt{a}}\right)b^{4}x^{4}+\sqrt{bx+a}\,\left(\sqrt{a}\,b^{3}x^{3}+\frac{118a^{\frac{3}{2}}b^{2}x^{2}}{15}+\frac{136a^{\frac{5}{2}}bx}{15}+\frac{16a^{\frac{7}{2}}}{5}\right)\right)}{64a^{\frac{3}{2}}x^{4}}$	72

```
int((b*x^3+a*x^2)^(5/2)/x^5,x,method=_RETURNVERBOSE)
```

```
2/7*(b*x+a)/b*(b*x^3+a*x^2)^(5/2)/x^5
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 48 vs. $2(21) = 42$.

Time = 0.07 (sec) , antiderivative size = 48, normalized size of antiderivative = 1.92

$$\int \frac{(ax^2+bx^3)^{5/2}}{x^5}\,dx = \frac{2\,(b^3x^3+3\,ab^2x^2+3\,a^2bx+a^3)\sqrt{bx^3+ax^2}}{7\,bx}$$

```
integrate((b*x^3+a*x^2)^(5/2)/x^5,x, algorithm="fricas")
```

```
2/7*(b^3*x^3 + 3*a*b^2*x^2 + 3*a^2*b*x + a^3)*sqrt(b*x^3 + a*x^2)/(b*x)
```

Sympy [F]

$$\int \frac{(ax^2 + bx^3)^{5/2}}{x^5} \, dx = \int \frac{(x^2(a + bx))^{\frac{5}{2}}}{x^5} \, dx$$

```
integrate((b*x**3+a*x**2)**(5/2)/x**5,x)
```

```
Integral((x**2*(a + b*x))**(5/2)/x**5, x)
```

Maxima [A] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 39, normalized size of antiderivative = 1.56

$$\int \frac{(ax^2 + bx^3)^{5/2}}{x^5} \, dx = \frac{2\,(b^3x^3 + 3\,ab^2x^2 + 3\,a^2bx + a^3)\sqrt{bx + a}}{7\,b}$$

```
integrate((b*x^3+a*x^2)^(5/2)/x^5,x, algorithm="maxima")
```

```
2/7*(b^3*x^3 + 3*a*b^2*x^2 + 3*a^2*b*x + a^3)*sqrt(b*x + a)/b
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 140 vs. 2(21) = 42.

Time = 0.12 (sec) , antiderivative size = 140, normalized size of antiderivative = 5.60

$$\int \frac{(ax^2 + bx^3)^{5/2}}{x^5} \, dx = -\frac{2\,a^{\frac{7}{2}}\operatorname{sgn}(x)}{7\,b} + \frac{2\left(35\,\sqrt{bx + a}a^3\operatorname{sgn}(x) + 35\left((bx + a)^{\frac{3}{2}} - 3\,\sqrt{bx + a}a\right)a^2\operatorname{sgn}(x) + 7\left(3\,(bx + a)^{\frac{5}{2}} - 10\,(bx + a)^{\frac{3}{2}}a + 15\right.\right.}{35\,b}$$

```
integrate((b*x^3+a*x^2)^(5/2)/x^5,x, algorithm="giac")
```

```
-2/7*a^(7/2)*sgn(x)/b + 2/35*(35*sqrt(b*x + a)*a^3*sgn(x) + 35*((b*x + a)^
(3/2) - 3*sqrt(b*x + a)*a)*a^2*sgn(x) + 7*(3*(b*x + a)^(5/2) - 10*(b*x + a
)^(3/2)*a + 15*sqrt(b*x + a)*a^2)*a*sgn(x) + (5*(b*x + a)^(7/2) - 21*(b*x
+ a)^(5/2)*a + 35*(b*x + a)^(3/2)*a^2 - 35*sqrt(b*x + a)*a^3)*sgn(x))/b
```

Mupad [B] (verification not implemented)

Time = 9.60 (sec) , antiderivative size = 82, normalized size of antiderivative = 3.28

$$\int \frac{(ax^2+bx^3)^{5/2}}{x^5}\,dx = \frac{6\,a^2\,\sqrt{b\,x^3+a\,x^2}}{7}+\frac{2\,b^2\,x^2\,\sqrt{b\,x^3+a\,x^2}}{7}+\frac{6\,a\,b\,x\,\sqrt{b\,x^3+a\,x^2}}{7}+\frac{2\,a^3\,\sqrt{b\,x^3+a\,x^2}}{7\,b\,x}$$

```
int((a*x^2 + b*x^3)^(5/2)/x^5,x)
```

```
(6*a^2*(a*x^2 + b*x^3)^(1/2))/7 + (2*b^2*x^2*(a*x^2 + b*x^3)^(1/2))/7 + (6
*a*b*x*(a*x^2 + b*x^3)^(1/2))/7 + (2*a^3*(a*x^2 + b*x^3)^(1/2))/(7*b*x)
```

Reduce [B] (verification not implemented)

Time = 0.54 (sec) , antiderivative size = 38, normalized size of antiderivative = 1.52

$$\int \frac{(ax^2+bx^3)^{5/2}}{x^5}\,dx = \frac{2\sqrt{bx+a}\,(b^3x^3+3a\,b^2x^2+3a^2bx+a^3)}{7b}$$

```
int((b*x^3+a*x^2)^(5/2)/x^5,x)
```

```
(2*sqrt(a + b*x)*(a**3 + 3*a**2*b*x + 3*a*b**2*x**2 + b**3*x**3))/(7*b)
```

3.293 $\int \frac{(ax^2+bx^3)^{5/2}}{x^6}\,dx$

Optimal result

Integrand size = 19, antiderivative size = 101

$$\int \frac{(ax^2+bx^3)^{5/2}}{x^6}\,dx = \frac{2a^2\sqrt{ax^2+bx^3}}{x} + \frac{2a(ax^2+bx^3)^{3/2}}{3x^3} + \frac{2(ax^2+bx^3)^{5/2}}{5x^5} - 2a^{5/2}\text{arctanh}\left(\frac{\sqrt{ax^2+bx^3}}{\sqrt{a}x}\right)$$

```
2*a^2*(b*x^3+a*x^2)^(1/2)/x+2/3*a*(b*x^3+a*x^2)^(3/2)/x^3+2/5*(b*x^3+a*x^2
)^(5/2)/x^5-2*a^(5/2)*arctanh((b*x^3+a*x^2)^(1/2)/a^(1/2)/x)
```

Mathematica [A] (verified)

Time = 0.06 (sec) , antiderivative size = 80, normalized size of antiderivative = 0.79

$$\int \frac{(ax^2+bx^3)^{5/2}}{x^6}\,dx = \frac{2x\sqrt{a+bx}\left(\sqrt{a+bx}(23a^2+11abx+3b^2x^2) - 15a^{5/2}\text{arctanh}\left(\frac{\sqrt{a+bx}}{\sqrt{a}}\right)\right)}{15\sqrt{x^2(a+bx)}}$$

```
Integrate[(a*x^2 + b*x^3)^(5/2)/x^6,x]
```

```
(2*x*Sqrt[a + b*x]*(Sqrt[a + b*x]*(23*a^2 + 11*a*b*x + 3*b^2*x^2) - 15*a^(
5/2)*ArcTanh[Sqrt[a + b*x]/Sqrt[a]]))/(15*Sqrt[x^2*(a + b*x)])
```

Rubi [A] (verified)

Time = 0.47 (sec) , antiderivative size = 101, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.263$, Rules used = {1927, 1927, 1927, 1914, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\left(ax^2+bx^3\right)^{5/2}}{x^6}dx$$

$$\Big\downarrow \text{1927}$$

$$a\int \frac{\left(bx^3+ax^2\right)^{3/2}}{x^4}dx+\frac{2\left(ax^2+bx^3\right)^{5/2}}{5x^5}$$

$$\Big\downarrow \text{1927}$$

$$a\left(a\int \frac{\sqrt{bx^3+ax^2}}{x^2}dx+\frac{2\left(ax^2+bx^3\right)^{3/2}}{3x^3}\right)+\frac{2\left(ax^2+bx^3\right)^{5/2}}{5x^5}$$

$$\Big\downarrow \text{1927}$$

$$a\left(a\left(a\int \frac{1}{\sqrt{bx^3+ax^2}}dx+\frac{2\sqrt{ax^2+bx^3}}{x}\right)+\frac{2\left(ax^2+bx^3\right)^{3/2}}{3x^3}\right)+\frac{2\left(ax^2+bx^3\right)^{5/2}}{5x^5}$$

$$\Big\downarrow \text{1914}$$

$$a\left(a\left(\frac{2\sqrt{ax^2+bx^3}}{x}-2a\int \frac{1}{1-\frac{ax^2}{bx^3+ax^2}}d\frac{x}{\sqrt{bx^3+ax^2}}\right)+\frac{2\left(ax^2+bx^3\right)^{3/2}}{3x^3}\right)+\frac{2\left(ax^2+bx^3\right)^{5/2}}{5x^5}$$

$$\Big\downarrow \text{219}$$

$$a\left(a\left(\frac{2\sqrt{ax^2+bx^3}}{x}-2\sqrt{a}\text{arctanh}\left(\frac{\sqrt{a}x}{\sqrt{ax^2+bx^3}}\right)\right)+\frac{2\left(ax^2+bx^3\right)^{3/2}}{3x^3}\right)+\frac{2\left(ax^2+bx^3\right)^{5/2}}{5x^5}$$

```
Int[(a*x^2 + b*x^3)^(5/2)/x^6,x]
```

```
(2*(a*x^2 + b*x^3)^(5/2))/(5*x^5) + a*((2*(a*x^2 + b*x^3)^(3/2))/(3*x^3) +
 a*((2*Sqrt[a*x^2 + b*x^3])/x - 2*Sqrt[a]*ArcTanh[(Sqrt[a]*x)/Sqrt[a*x^2 +
 b*x^3]]))
```

Defintions of rubi rules used

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

```
Int[1/Sqrt[(a_.)*(x_)^2 + (b_.)*(x_)^(n_.)], x_Symbol] :> Simp[2/(2 - n)
Subst[Int[1/(1 - a*x^2), x], x, x/Sqrt[a*x^2 + b*x^n]], x] /; FreeQ[{a, b,
n}, x] && NeQ[n, 2]
```

```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> Simp[(c*x)^(m + 1)*((a*x^j + b*x^n)^p/(c*(m + n*p + 1))), x] + Simp[a*
(n - j)*(p/(c^j*(m + n*p + 1)))   Int[(c*x)^(m + j)*(a*x^j + b*x^n)^(p - 1)
, x], x] /; FreeQ[{a, b, c, m}, x] &&  !IntegerQ[p] && LtQ[0, j, n] && (Int
egersQ[j, n] || GtQ[c, 0]) && GtQ[p, 0] && NeQ[m + n*p + 1, 0]
```

Maple [A] (verified)

Time = 0.50 (sec) , antiderivative size = 75, normalized size of antiderivative = 0.74

method	result	size
default	$\frac{2\left(b\,x^{3}+a\,x^{2}\right)^{\frac{5}{2}} \left(-15 a^{\frac{5}{2}} \operatorname{arctanh}\left(\frac{\sqrt{b x +a}}{\sqrt{a}}\right)+3 \left(b x +a \right)^{\frac{5}{2}}+5 \left(b x +a \right)^{\frac{3}{2}} a +15 a^{2} \sqrt{b x +a}\right)}{15 x^{5} \left(b x +a \right)^{\frac{5}{2}}}$	75
pseudoelliptic	$-\frac{\frac{15 \operatorname{arctanh}\left(\frac{\sqrt{b x +a}}{\sqrt{a}}\right) b^{5} x^{5}}{128}+\sqrt{b x +a}\, \left(-\frac{15 \sqrt{a}\, b^{4} x^{4}}{128}+\frac{5 a^{\frac{3}{2}} b^{3} x^{3}}{64}+\frac{31 a^{\frac{5}{2}} b^{2} x^{2}}{16}+\frac{21 a^{\frac{7}{2}} b x}{8}+a^{\frac{9}{2}}\right)}{5 a^{\frac{5}{2}} x^{5}}$	82

```
int((b*x^3+a*x^2)^(5/2)/x^6,x,method=_RETURNVERBOSE)
```

```
2/15*(b*x^3+a*x^2)^(5/2)*(-15*a^(5/2)*arctanh((b*x+a)^(1/2)/a^(1/2))+3*(b*
x+a)^(5/2)+5*(b*x+a)^(3/2)*a+15*a^2*(b*x+a)^(1/2))/x^5/(b*x+a)^(5/2)
```

Fricas [A] (verification not implemented)

Time = 0.10 (sec) , antiderivative size = 161, normalized size of antiderivative = 1.59

$$\int \frac{(ax^2+bx^3)^{5/2}}{x^6}\,dx = \left[\frac{15\,a^{\frac{5}{2}}x\log\left(\frac{bx^2+2\,ax-2\,\sqrt{bx^3+ax^2}\sqrt{a}}{x^2}\right)+2\,(3\,b^2x^2+11\,abx+23\,a^2)\sqrt{bx^3+ax^2}}{15\,x}, \frac{2\left(15\right.}{}\right.$$

```
integrate((b*x^3+a*x^2)^(5/2)/x^6,x, algorithm="fricas")
```

```
[1/15*(15*a^(5/2)*x*log((b*x^2 + 2*a*x - 2*sqrt(b*x^3 + a*x^2)*sqrt(a))/x^
2) + 2*(3*b^2*x^2 + 11*a*b*x + 23*a^2)*sqrt(b*x^3 + a*x^2))/x, 2/15*(15*sq
rt(-a)*a^2*x*arctan(sqrt(b*x^3 + a*x^2)*sqrt(-a)/(b*x^2 + a*x)) + (3*b^2*x
^2 + 11*a*b*x + 23*a^2)*sqrt(b*x^3 + a*x^2))/x]
```

Sympy [F]

$$\int \frac{(ax^2+bx^3)^{5/2}}{x^6}\,dx = \int \frac{(x^2(a+bx))^{\frac{5}{2}}}{x^6}\,dx$$

```
integrate((b*x**3+a*x**2)**(5/2)/x**6,x)
```

```
Integral((x**2*(a + b*x))**(5/2)/x**6, x)
```

Maxima [F]

$$\int \frac{(ax^2+bx^3)^{5/2}}{x^6}\,dx = \int \frac{(bx^3+ax^2)^{\frac{5}{2}}}{x^6}\,dx$$

```
integrate((b*x^3+a*x^2)^(5/2)/x^6,x, algorithm="maxima")
```

```
integrate((b*x^3 + a*x^2)^(5/2)/x^6, x)
```

Giac [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 99, normalized size of antiderivative = 0.98

$$\int \frac{(ax^2+bx^3)^{5/2}}{x^6}\,dx = \frac{2\,a^3\arctan\left(\frac{\sqrt{bx+a}}{\sqrt{-a}}\right)\mathrm{sgn}(x)}{\sqrt{-a}} + \frac{2}{5}\,(bx+a)^{\frac{5}{2}}\mathrm{sgn}(x) + \frac{2}{3}\,(bx+a)^{\frac{3}{2}}a\mathrm{sgn}(x) + 2\,\sqrt{bx+a}a^2\mathrm{sgn}(x) - \frac{2\left(15\,a^3\arctan\left(\frac{\sqrt{a}}{\sqrt{-a}}\right) + 23\,\sqrt{-a}a^{\frac{5}{2}}\right)\mathrm{sgn}(x)}{15\,\sqrt{-a}}$$

```
integrate((b*x^3+a*x^2)^(5/2)/x^6,x, algorithm="giac")
```

```
2*a^3*arctan(sqrt(b*x + a)/sqrt(-a))*sgn(x)/sqrt(-a) + 2/5*(b*x + a)^(5/2)
*sgn(x) + 2/3*(b*x + a)^(3/2)*a*sgn(x) + 2*sqrt(b*x + a)*a^2*sgn(x) - 2/15
*(15*a^3*arctan(sqrt(a)/sqrt(-a)) + 23*sqrt(-a)*a^(5/2))*sgn(x)/sqrt(-a)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(ax^2+bx^3)^{5/2}}{x^6}\,dx = \int \frac{(b\,x^3+a\,x^2)^{5/2}}{x^6}\,dx$$

```
int((a*x^2 + b*x^3)^(5/2)/x^6,x)
```

```
int((a*x^2 + b*x^3)^(5/2)/x^6, x)
```

Reduce [B] (verification not implemented)

Time = 0.44 (sec) , antiderivative size = 72, normalized size of antiderivative = 0.71

$$\int \frac{(ax^2+bx^3)^{5/2}}{x^6}\,dx = \frac{46\sqrt{bx+a}\,a^2}{15}+\frac{22\sqrt{bx+a}\,abx}{15}+\frac{2\sqrt{bx+a}\,b^2x^2}{5} + \sqrt{a}\log\left(\sqrt{bx+a}-\sqrt{a}\right)a^2-\sqrt{a}\log\left(\sqrt{bx+a}+\sqrt{a}\right)a^2$$

```
int((b*x^3+a*x^2)^(5/2)/x^6,x)
```

```
(46*sqrt(a + b*x)*a**2 + 22*sqrt(a + b*x)*a*b*x + 6*sqrt(a + b*x)*b**2*x**
2 + 15*sqrt(a)*log(sqrt(a + b*x) - sqrt(a))*a**2 - 15*sqrt(a)*log(sqrt(a +
 b*x) + sqrt(a))*a**2)/15
```

3.294 $\int \frac{(ax^2+bx^3)^{5/2}}{x^7}\,dx$

Optimal result

Integrand size = 19, antiderivative size = 102

$$\int \frac{(ax^2+bx^3)^{5/2}}{x^7}\,dx = -\frac{a^2\sqrt{ax^2+bx^3}}{x^2}+\frac{4ab\sqrt{ax^2+bx^3}}{x} + \frac{2b(ax^2+bx^3)^{3/2}}{3x^3}-5a^{3/2}b\,\text{arctanh}\left(\frac{\sqrt{ax^2+bx^3}}{\sqrt{a}x}\right)$$

```
-a^2*(b*x^3+a*x^2)^(1/2)/x^2+4*a*b*(b*x^3+a*x^2)^(1/2)/x+2/3*b*(b*x^3+a*x^
2)^(3/2)/x^3-5*a^(3/2)*b*arctanh((b*x^3+a*x^2)^(1/2)/a^(1/2)/x)
```

Mathematica [A] (verified)

Time = 0.10 (sec) , antiderivative size = 81, normalized size of antiderivative = 0.79

$$\int \frac{(ax^2+bx^3)^{5/2}}{x^7}\,dx = \frac{\sqrt{a+bx}\left(\sqrt{a+bx}(-3a^2+14abx+2b^2x^2)-15a^{3/2}bx\,\text{arctanh}\left(\frac{\sqrt{a+bx}}{\sqrt{a}}\right)\right)}{3\sqrt{x^2(a+bx)}}$$

```
Integrate[(a*x^2 + b*x^3)^(5/2)/x^7,x]
```

```
(Sqrt[a + b*x]*(Sqrt[a + b*x]*(-3*a^2 + 14*a*b*x + 2*b^2*x^2) - 15*a^(3/2)
*b*x*ArcTanh[Sqrt[a + b*x]/Sqrt[a]]))/(3*Sqrt[x^2*(a + b*x)])
```

Rubi [A] (verified)

Time = 0.45 (sec) , antiderivative size = 102, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.263, Rules used = {1926, 1927, 1927, 1914, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\left(ax^2+bx^3\right)^{5/2}}{x^7}dx$$

$$\Big\downarrow \text{1926}$$

$$\frac{5}{2}b\int \frac{\left(bx^3+ax^2\right)^{3/2}}{x^4}dx-\frac{\left(ax^2+bx^3\right)^{5/2}}{x^6}$$

$$\Big\downarrow \text{1927}$$

$$\frac{5}{2}b\left(a\int \frac{\sqrt{bx^3+ax^2}}{x^2}dx+\frac{2\left(ax^2+bx^3\right)^{3/2}}{3x^3}\right)-\frac{\left(ax^2+bx^3\right)^{5/2}}{x^6}$$

$$\Big\downarrow \text{1927}$$

$$\frac{5}{2}b\left(a\left(a\int \frac{1}{\sqrt{bx^3+ax^2}}dx+\frac{2\sqrt{ax^2+bx^3}}{x}\right)+\frac{2\left(ax^2+bx^3\right)^{3/2}}{3x^3}\right)-\frac{\left(ax^2+bx^3\right)^{5/2}}{x^6}$$

$$\Big\downarrow \text{1914}$$

$$\frac{5}{2}b\left(a\left(\frac{2\sqrt{ax^2+bx^3}}{x}-2a\int \frac{1}{1-\frac{ax^2}{bx^3+ax^2}}d\frac{x}{\sqrt{bx^3+ax^2}}\right)+\frac{2\left(ax^2+bx^3\right)^{3/2}}{3x^3}\right)-\frac{\left(ax^2+bx^3\right)^{5/2}}{x^6}$$

$$\Big\downarrow \text{219}$$

$$\frac{5}{2}b\left(a\left(\frac{2\sqrt{ax^2+bx^3}}{x}-2\sqrt{a}\text{arctanh}\left(\frac{\sqrt{a}x}{\sqrt{ax^2+bx^3}}\right)\right)+\frac{2\left(ax^2+bx^3\right)^{3/2}}{3x^3}\right)-\frac{\left(ax^2+bx^3\right)^{5/2}}{x^6}$$

```
Int[(a*x^2 + b*x^3)^(5/2)/x^7,x]
```

```
-((a*x^2 + b*x^3)^(5/2)/x^6) + (5*b*((2*(a*x^2 + b*x^3)^(3/2))/(3*x^3) + a
*((2*Sqrt[a*x^2 + b*x^3])/x - 2*Sqrt[a]*ArcTanh[(Sqrt[a]*x)/Sqrt[a*x^2 + b
*x^3]])))/2
```

Defintions of rubi rules used

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

```
Int[1/Sqrt[(a_.)*(x_)^2 + (b_.)*(x_)^(n_.)], x_Symbol] :> Simp[2/(2 - n)
Subst[Int[1/(1 - a*x^2), x], x, x/Sqrt[a*x^2 + b*x^n]], x] /; FreeQ[{a, b,
n}, x] && NeQ[n, 2]
```

```
Int[((c_.)*(x_))^(m_)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol]
 :> Simp[(c*x)^(m + 1)*((a*x^j + b*x^n)^p/(c*(m + j*p + 1))), x] - Simp[b*p
*((n - j)/(c^n*(m + j*p + 1)))   Int[(c*x)^(m + n)*(a*x^j + b*x^n)^(p - 1),
 x], x] /; FreeQ[{a, b, c}, x] &&  !IntegerQ[p] && LtQ[0, j, n] && (Integer
sQ[j, n] || GtQ[c, 0]) && GtQ[p, 0] && LtQ[m + j*p + 1, 0]
```

```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> Simp[(c*x)^(m + 1)*((a*x^j + b*x^n)^p/(c*(m + n*p + 1))), x] + Simp[a*
(n - j)*(p/(c^j*(m + n*p + 1)))   Int[(c*x)^(m + j)*(a*x^j + b*x^n)^(p - 1)
, x], x] /; FreeQ[{a, b, c, m}, x] &&  !IntegerQ[p] && LtQ[0, j, n] && (Int
egersQ[j, n] || GtQ[c, 0]) && GtQ[p, 0] && NeQ[m + n*p + 1, 0]
```

Maple [A] (verified)

Time = 0.53 (sec) , antiderivative size = 82, normalized size of antiderivative = 0.80

method	result	size
risch	$-\frac{a^{2} \sqrt{x^{2} (bx+a)}}{x^{2}}+\frac{b \left(\frac{4 (bx+a)^{\frac{3}{2}}}{3}+8 a \sqrt{bx+a}-10 a^{\frac{3}{2}} \operatorname{arctanh}\left(\frac{\sqrt{bx+a}}{\sqrt{a}}\right)\right) \sqrt{x^{2} (bx+a)}}{2 x \sqrt{bx+a}}$	82
default	$\frac{\left(b\,x^{3}+a\,x^{2}\right)^{\frac{5}{2}} \left(2 (bx+a)^{\frac{3}{2}} b x \sqrt{a}+12 a^{\frac{3}{2}} b x \sqrt{bx+a}-15 \operatorname{arctanh}\left(\frac{\sqrt{bx+a}}{\sqrt{a}}\right) a^{2} b x-3 \sqrt{bx+a}\, a^{\frac{5}{2}}\right)}{3 x^{6} (bx+a)^{\frac{5}{2}} \sqrt{a}}$	89
pseudoelliptic	$-\frac{5 \left(-\frac{3 \operatorname{arctanh}\left(\frac{\sqrt{bx+a}}{\sqrt{a}}\right) b^{6} x^{6}}{128}+\sqrt{bx+a}\left(\frac{3 \sqrt{a}\, b^{5} x^{5}}{128}-\frac{a^{\frac{3}{2}} b^{4} x^{4}}{64}+\frac{a^{\frac{5}{2}} b^{3} x^{3}}{80}+\frac{27 a^{\frac{7}{2}} b^{2} x^{2}}{40}+a^{\frac{9}{2}} b x+\frac{2 a^{\frac{11}{2}}}{5}\right)\right)}{12 a^{\frac{7}{2}} x^{6}}$	94

```
int((b*x^3+a*x^2)^(5/2)/x^7,x,method=_RETURNVERBOSE)
```

```
-a^2/x^2*(x^2*(b*x+a))^(1/2)+1/2*b*(4/3*(b*x+a)^(3/2)+8*a*(b*x+a)^(1/2)-10
*a^(3/2)*arctanh((b*x+a)^(1/2)/a^(1/2)))*(x^2*(b*x+a))^(1/2)/x/(b*x+a)^(1/
2)
```

Fricas [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 165, normalized size of antiderivative = 1.62

$$\int \frac{(ax^2+bx^3)^{5/2}}{x^7}\,dx = \left[\frac{15\,a^{\frac{3}{2}}bx^2\log\left(\frac{bx^2+2\,ax-2\,\sqrt{bx^3+ax^2}\sqrt{a}}{x^2}\right)+2\,(2\,b^2x^2+14\,abx-3\,a^2)\sqrt{bx^3+ax^2}}{6\,x^2}, \frac{15\,\sqrt{}}{}\right.$$

```
integrate((b*x^3+a*x^2)^(5/2)/x^7,x, algorithm="fricas")
```

```
[1/6*(15*a^(3/2)*b*x^2*log((b*x^2 + 2*a*x - 2*sqrt(b*x^3 + a*x^2)*sqrt(a))
/x^2) + 2*(2*b^2*x^2 + 14*a*b*x - 3*a^2)*sqrt(b*x^3 + a*x^2))/x^2, 1/3*(15
*sqrt(-a)*a*b*x^2*arctan(sqrt(b*x^3 + a*x^2)*sqrt(-a)/(b*x^2 + a*x)) + (2*
b^2*x^2 + 14*a*b*x - 3*a^2)*sqrt(b*x^3 + a*x^2))/x^2]
```

Sympy [F]

$$\int \frac{(ax^2+bx^3)^{5/2}}{x^7}\,dx = \int \frac{(x^2(a+bx))^{\frac{5}{2}}}{x^7}\,dx$$

```
integrate((b*x**3+a*x**2)**(5/2)/x**7,x)
```

```
Integral((x**2*(a + b*x))**(5/2)/x**7, x)
```

Maxima [F]

$$\int \frac{(ax^2+bx^3)^{5/2}}{x^7}\,dx = \int \frac{(bx^3+ax^2)^{\frac{5}{2}}}{x^7}\,dx$$

```
integrate((b*x^3+a*x^2)^(5/2)/x^7,x, algorithm="maxima")
```

```
integrate((b*x^3 + a*x^2)^(5/2)/x^7, x)
```

Giac [A] (verification not implemented)

Time = 0.23 (sec) , antiderivative size = 73, normalized size of antiderivative = 0.72

$$\int \frac{(ax^2+bx^3)^{5/2}}{x^7}\,dx = \frac{1}{3}\left(\frac{15\,a^2\arctan\left(\frac{\sqrt{bx+a}}{\sqrt{-a}}\right)\operatorname{sgn}(x)}{\sqrt{-a}} + 2\,(bx+a)^{\frac{3}{2}}\operatorname{sgn}(x) + 12\,\sqrt{bx+a}\,a\operatorname{sgn}(x) - \frac{3\,\sqrt{b}$$

```
integrate((b*x^3+a*x^2)^(5/2)/x^7,x, algorithm="giac")
```

```
1/3*(15*a^2*arctan(sqrt(b*x + a)/sqrt(-a))*sgn(x)/sqrt(-a) + 2*(b*x + a)^(
3/2)*sgn(x) + 12*sqrt(b*x + a)*a*sgn(x) - 3*sqrt(b*x + a)*a^2*sgn(x)/(b*x)
)*b
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(ax^2 + bx^3)^{5/2}}{x^7} \, dx = \int \frac{(b\,x^3 + a\,x^2)^{5/2}}{x^7} \, dx$$

```
int((a*x^2 + b*x^3)^(5/2)/x^7,x)
```

```
int((a*x^2 + b*x^3)^(5/2)/x^7, x)
```

Reduce [B] (verification not implemented)

Time = 0.30 (sec) , antiderivative size = 78, normalized size of antiderivative = 0.76

$$\int \frac{(ax^2 + bx^3)^{5/2}}{x^7} \, dx = \frac{-6\sqrt{bx+a}\,a^2 + 28\sqrt{bx+a}\,abx + 4\sqrt{bx+a}\,b^2x^2 + 15\sqrt{a}\log\left(\sqrt{bx+a} - \sqrt{a}\right)abx}{6x}$$

```
int((b*x^3+a*x^2)^(5/2)/x^7,x)
```

```
( - 6*sqrt(a + b*x)*a**2 + 28*sqrt(a + b*x)*a*b*x + 4*sqrt(a + b*x)*b**2*x
**2 + 15*sqrt(a)*log(sqrt(a + b*x) - sqrt(a))*a*b*x - 15*sqrt(a)*log(sqrt(
a + b*x) + sqrt(a))*a*b*x)/(6*x)
```

3.295 $\int \frac{(ax^2+bx^3)^{5/2}}{x^8}\,dx$

Optimal result

Integrand size = 19, antiderivative size = 110

$$\int \frac{(ax^2+bx^3)^{5/2}}{x^8}\,dx = -\frac{a^2\sqrt{ax^2+bx^3}}{2x^3} - \frac{9ab\sqrt{ax^2+bx^3}}{4x^2} + \frac{2b^2\sqrt{ax^2+bx^3}}{x} - \frac{15}{4}\sqrt{a}b^2\operatorname{arctanh}\left(\frac{\sqrt{ax^2+bx^3}}{\sqrt{a}x}\right)$$

```
-1/2*a^2*(b*x^3+a*x^2)^(1/2)/x^3-9/4*a*b*(b*x^3+a*x^2)^(1/2)/x^2+2*b^2*(b*
x^3+a*x^2)^(1/2)/x-15/4*a^(1/2)*b^2*arctanh((b*x^3+a*x^2)^(1/2)/a^(1/2)/x)
```

Mathematica [A] (verified)

Time = 0.12 (sec) , antiderivative size = 88, normalized size of antiderivative = 0.80

$$\int \frac{(ax^2+bx^3)^{5/2}}{x^8}\,dx = -\frac{\sqrt{x^2(a+bx)}\left(\sqrt{a+bx}(2a^2+9abx-8b^2x^2)+15\sqrt{a}b^2x^2\operatorname{arctanh}\left(\frac{\sqrt{a+bx}}{\sqrt{a}}\right)\right)}{4x^3\sqrt{a+bx}}$$

```
Integrate[(a*x^2 + b*x^3)^(5/2)/x^8,x]
```

```
-1/4*(Sqrt[x^2*(a + b*x)]*(Sqrt[a + b*x]*(2*a^2 + 9*a*b*x - 8*b^2*x^2) + 1
5*Sqrt[a]*b^2*x^2*ArcTanh[Sqrt[a + b*x]/Sqrt[a]]))/(x^3*Sqrt[a + b*x])
```

Rubi [A] (verified)

Time = 0.46 (sec) , antiderivative size = 105, normalized size of antiderivative = 0.95, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.263, Rules used = {1926, 1926, 1927, 1914, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(ax^2+bx^3)^{5/2}}{x^8}dx$$

$$\Big\downarrow \text{1926}$$

$$\frac{5}{4}b\int \frac{(bx^3+ax^2)^{3/2}}{x^5}dx-\frac{(ax^2+bx^3)^{5/2}}{2x^7}$$

$$\Big\downarrow \text{1926}$$

$$\frac{5}{4}b\left(\frac{3}{2}b\int \frac{\sqrt{bx^3+ax^2}}{x^2}dx-\frac{(ax^2+bx^3)^{3/2}}{x^4}\right)-\frac{(ax^2+bx^3)^{5/2}}{2x^7}$$

$$\Big\downarrow \text{1927}$$

$$\frac{5}{4}b\left(\frac{3}{2}b\left(a\int \frac{1}{\sqrt{bx^3+ax^2}}dx+\frac{2\sqrt{ax^2+bx^3}}{x}\right)-\frac{(ax^2+bx^3)^{3/2}}{x^4}\right)-\frac{(ax^2+bx^3)^{5/2}}{2x^7}$$

$$\Big\downarrow \text{1914}$$

$$\frac{5}{4}b\left(\frac{3}{2}b\left(\frac{2\sqrt{ax^2+bx^3}}{x}-2a\int \frac{1}{1-\frac{ax^2}{bx^3+ax^2}}d\frac{x}{\sqrt{bx^3+ax^2}}\right)-\frac{(ax^2+bx^3)^{3/2}}{x^4}\right)-\frac{(ax^2+bx^3)^{5/2}}{2x^7}$$

$$\Big\downarrow \text{219}$$

$$\frac{5}{4}b\left(\frac{3}{2}b\left(\frac{2\sqrt{ax^2+bx^3}}{x}-2\sqrt{a}\text{arctanh}\left(\frac{\sqrt{a}x}{\sqrt{ax^2+bx^3}}\right)\right)-\frac{(ax^2+bx^3)^{3/2}}{x^4}\right)-\frac{(ax^2+bx^3)^{5/2}}{2x^7}$$

```
Int[(a*x^2 + b*x^3)^(5/2)/x^8,x]
```

```
-1/2*(a*x^2 + b*x^3)^(5/2)/x^7 + (5*b*(-((a*x^2 + b*x^3)^(3/2)/x^4) + (3*b
*((2*Sqrt[a*x^2 + b*x^3])/x - 2*Sqrt[a]*ArcTanh[(Sqrt[a]*x)/Sqrt[a*x^2 + b
*x^3]]))/2))/4
```

Defintions of rubi rules used

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

```
Int[1/Sqrt[(a_.)*(x_)^2 + (b_.)*(x_)^(n_.)], x_Symbol] :> Simp[2/(2 - n)
Subst[Int[1/(1 - a*x^2), x], x, x/Sqrt[a*x^2 + b*x^n]], x] /; FreeQ[{a, b,
n}, x] && NeQ[n, 2]
```

```
Int[((c_.)*(x_))^(m_)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol]
 :> Simp[(c*x)^(m + 1)*((a*x^j + b*x^n)^p/(c*(m + j*p + 1))), x] - Simp[b*p
*((n - j)/(c^n*(m + j*p + 1)))   Int[(c*x)^(m + n)*(a*x^j + b*x^n)^(p - 1),
 x], x] /; FreeQ[{a, b, c}, x] &&  !IntegerQ[p] && LtQ[0, j, n] && (Integer
sQ[j, n] || GtQ[c, 0]) && GtQ[p, 0] && LtQ[m + j*p + 1, 0]
```

```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> Simp[(c*x)^(m + 1)*((a*x^j + b*x^n)^p/(c*(m + n*p + 1))), x] + Simp[a*
(n - j)*(p/(c^j*(m + n*p + 1)))   Int[(c*x)^(m + j)*(a*x^j + b*x^n)^(p - 1)
, x], x] /; FreeQ[{a, b, c, m}, x] &&  !IntegerQ[p] && LtQ[0, j, n] && (Int
egersQ[j, n] || GtQ[c, 0]) && GtQ[p, 0] && NeQ[m + n*p + 1, 0]
```

Maple [A] (verified)

Time = 0.58 (sec) , antiderivative size = 80, normalized size of antiderivative = 0.73

method	result
risch	$-\frac{a(9bx+2a)\sqrt{x^2(bx+a)}}{4x^3}+\frac{b^2\left(16\sqrt{bx+a}-30\sqrt{a}\,\operatorname{arctanh}\left(\frac{\sqrt{bx+a}}{\sqrt{a}}\right)\right)\sqrt{x^2(bx+a)}}{8x\sqrt{bx+a}}$
default	$-\frac{\left(b\,x^3+a\,x^2\right)^{\frac{5}{2}}\left(-8b^2x^2\sqrt{a}\,\sqrt{bx+a}+15\,\operatorname{arctanh}\left(\frac{\sqrt{bx+a}}{\sqrt{a}}\right)a\,b^2x^2+9(bx+a)^{\frac{3}{2}}a^{\frac{3}{2}}-7\sqrt{bx+a}\,a^{\frac{5}{2}}\right)}{4x^7(bx+a)^{\frac{5}{2}}\sqrt{a}}$
pseudoelliptic	$-\frac{37\left(\frac{105\,\operatorname{arctanh}\left(\frac{\sqrt{bx+a}}{\sqrt{a}}\right)x^7b^7}{4736}+\sqrt{bx+a}\left(-\frac{105\sqrt{a}\,b^6x^6}{4736}+\frac{35a^{\frac{3}{2}}b^5x^5}{2368}-\frac{7a^{\frac{5}{2}}b^4x^4}{592}+\frac{3a^{\frac{7}{2}}b^3x^3}{296}+a^{\frac{9}{2}}b^2x^2+\frac{58a^{\frac{11}{2}}bx}{37}+\frac{24a^{\frac{13}{2}}}{37}\right)\right.}{168a^{\frac{9}{2}}x^7}$

```
int((b*x^3+a*x^2)^(5/2)/x^8,x,method=_RETURNVERBOSE)
```

```
-1/4*a*(9*b*x+2*a)/x^3*(x^2*(b*x+a))^(1/2)+1/8*b^2*(16*(b*x+a)^(1/2)-30*a^
(1/2)*arctanh((b*x+a)^(1/2)/a^(1/2)))*(x^2*(b*x+a))^(1/2)/x/(b*x+a)^(1/2)
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 168, normalized size of antiderivative = 1.53

$$\int \frac{\left(ax^2+bx^3\right)^{5/2}}{x^8}\,dx=\left[\frac{15\,\sqrt{a}b^2x^3\log\left(\frac{bx^2+2\,ax-2\,\sqrt{bx^3+ax^2}\sqrt{a}}{x^2}\right)+2\,(8\,b^2x^2-9\,abx-2\,a^2)\sqrt{bx^3+ax^2}}{8\,x^3},\ 15\,\sqrt{}\right.$$

```
integrate((b*x^3+a*x^2)^(5/2)/x^8,x, algorithm="fricas")
```

```
[1/8*(15*sqrt(a)*b^2*x^3*log((b*x^2 + 2*a*x - 2*sqrt(b*x^3 + a*x^2)*sqrt(a
))/x^2) + 2*(8*b^2*x^2 - 9*a*b*x - 2*a^2)*sqrt(b*x^3 + a*x^2))/x^3, 1/4*(1
5*sqrt(-a)*b^2*x^3*arctan(sqrt(b*x^3 + a*x^2)*sqrt(-a)/(b*x^2 + a*x)) + (8
*b^2*x^2 - 9*a*b*x - 2*a^2)*sqrt(b*x^3 + a*x^2))/x^3]
```

Sympy [F]

$$\int \frac{(ax^2+bx^3)^{5/2}}{x^8}\,dx = \int \frac{(x^2(a+bx))^{\frac{5}{2}}}{x^8}\,dx$$

```
integrate((b*x**3+a*x**2)**(5/2)/x**8,x)
```

```
Integral((x**2*(a + b*x))**(5/2)/x**8, x)
```

Maxima [F]

$$\int \frac{(ax^2+bx^3)^{5/2}}{x^8}\,dx = \int \frac{(bx^3+ax^2)^{\frac{5}{2}}}{x^8}\,dx$$

```
integrate((b*x^3+a*x^2)^(5/2)/x^8,x, algorithm="maxima")
```

```
integrate((b*x^3 + a*x^2)^(5/2)/x^8, x)
```

Giac [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 88, normalized size of antiderivative = 0.80

$$\int \frac{(ax^2+bx^3)^{5/2}}{x^8}\,dx = \frac{\frac{15\,ab^3\arctan\left(\frac{\sqrt{bx+a}}{\sqrt{-a}}\right)\mathrm{sgn}(x)}{\sqrt{-a}} + 8\,\sqrt{bx+a}b^3\mathrm{sgn}(x) - \frac{9\,(bx+a)^{\frac{3}{2}}ab^3\mathrm{sgn}(x)-7\,\sqrt{bx+a}a^2b^3\mathrm{sgn}(x)}{b^2x^2}}{4\,b}$$

```
integrate((b*x^3+a*x^2)^(5/2)/x^8,x, algorithm="giac")
```

```
1/4*(15*a*b^3*arctan(sqrt(b*x + a)/sqrt(-a))*sgn(x)/sqrt(-a) + 8*sqrt(b*x
+ a)*b^3*sgn(x) - (9*(b*x + a)^(3/2)*a*b^3*sgn(x) - 7*sqrt(b*x + a)*a^2*b^
3*sgn(x))/(b^2*x^2))/b
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(ax^2+bx^3)^{5/2}}{x^8}\,dx=\int \frac{{\left(b\,x^3+a\,x^2\right)}^{5/2}}{x^8}\,dx$$

```
int((a*x^2 + b*x^3)^(5/2)/x^8,x)
```

```
int((a*x^2 + b*x^3)^(5/2)/x^8, x)
```

Reduce [B] (verification not implemented)

Time = 0.32 (sec) , antiderivative size = 84, normalized size of antiderivative = 0.76

$$\int \frac{(ax^2+bx^3)^{5/2}}{x^8}\,dx=\frac{-4\sqrt{bx+a}\,a^2-18\sqrt{bx+a}\,abx+16\sqrt{bx+a}\,b^2x^2+15\sqrt{a}\log\left(\sqrt{bx+a}-\sqrt{a}\right)b^2x}{8x^2}$$

```
int((b*x^3+a*x^2)^(5/2)/x^8,x)
```

```
( - 4*sqrt(a + b*x)*a**2 - 18*sqrt(a + b*x)*a*b*x + 16*sqrt(a + b*x)*b**2*
x**2 + 15*sqrt(a)*log(sqrt(a + b*x) - sqrt(a))*b**2*x**2 - 15*sqrt(a)*log(
sqrt(a + b*x) + sqrt(a))*b**2*x**2)/(8*x**2)
```

3.296 $\int \frac{(ax^2+bx^3)^{5/2}}{x^9} dx$

Optimal result

Integrand size = 19, antiderivative size = 112

$$\int \frac{(ax^2+bx^3)^{5/2}}{x^9} dx = -\frac{a^2\sqrt{ax^2+bx^3}}{3x^4} - \frac{13ab\sqrt{ax^2+bx^3}}{12x^3} - \frac{11b^2\sqrt{ax^2+bx^3}}{8x^2} - \frac{5b^3\text{arctanh}\left(\frac{\sqrt{ax^2+bx^3}}{\sqrt{a}x}\right)}{8\sqrt{a}}$$

```
-1/3*a^2*(b*x^3+a*x^2)^(1/2)/x^4-13/12*a*b*(b*x^3+a*x^2)^(1/2)/x^3-11/8*b^
2*(b*x^3+a*x^2)^(1/2)/x^2-5/8*b^3*arctanh((b*x^3+a*x^2)^(1/2)/a^(1/2)/x)/a
^(1/2)
```

Mathematica [A] (verified)

Time = 0.16 (sec) , antiderivative size = 93, normalized size of antiderivative = 0.83

$$\int \frac{(ax^2+bx^3)^{5/2}}{x^9} dx = -\frac{\sqrt{x^2(a+bx)}\left(\sqrt{a}\sqrt{a+bx}(8a^2+26abx+33b^2x^2)+15b^3x^3\text{arctanh}\left(\frac{\sqrt{a+bx}}{\sqrt{a}}\right)\right)}{24\sqrt{a}x^4\sqrt{a+bx}}$$

```
Integrate[(a*x^2 + b*x^3)^(5/2)/x^9,x]
```

```
-1/24*(Sqrt[x^2*(a + b*x)]*(Sqrt[a]*Sqrt[a + b*x]*(8*a^2 + 26*a*b*x + 33*b
^2*x^2) + 15*b^3*x^3*ArcTanh[Sqrt[a + b*x]/Sqrt[a]]))/(Sqrt[a]*x^4*Sqrt[a
+ b*x])
```

Rubi [A] (verified)

Time = 0.45 (sec) , antiderivative size = 108, normalized size of antiderivative = 0.96, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.263$, Rules used = {1926, 1926, 1926, 1914, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\left(ax^2+bx^3\right)^{5/2}}{x^9}dx$$

$$\Big\downarrow \text{1926}$$

$$\frac{5}{6}b\int \frac{\left(bx^3+ax^2\right)^{3/2}}{x^6}dx-\frac{\left(ax^2+bx^3\right)^{5/2}}{3x^8}$$

$$\Big\downarrow \text{1926}$$

$$\frac{5}{6}b\left(\frac{3}{4}b\int \frac{\sqrt{bx^3+ax^2}}{x^3}dx-\frac{\left(ax^2+bx^3\right)^{3/2}}{2x^5}\right)-\frac{\left(ax^2+bx^3\right)^{5/2}}{3x^8}$$

$$\Big\downarrow \text{1926}$$

$$\frac{5}{6}b\left(\frac{3}{4}b\left(\frac{1}{2}b\int \frac{1}{\sqrt{bx^3+ax^2}}dx-\frac{\sqrt{ax^2+bx^3}}{x^2}\right)-\frac{\left(ax^2+bx^3\right)^{3/2}}{2x^5}\right)-\frac{\left(ax^2+bx^3\right)^{5/2}}{3x^8}$$

$$\Big\downarrow \text{1914}$$

$$\frac{5}{6}b\left(\frac{3}{4}b\left(-b\int \frac{1}{1-\frac{ax^2}{bx^3+ax^2}}d\frac{x}{\sqrt{bx^3+ax^2}}-\frac{\sqrt{ax^2+bx^3}}{x^2}\right)-\frac{\left(ax^2+bx^3\right)^{3/2}}{2x^5}\right)-\frac{\left(ax^2+bx^3\right)^{5/2}}{3x^8}$$

$$\Big\downarrow \text{219}$$

$$\frac{5}{6}b\left(\frac{3}{4}b\left(-\frac{b\operatorname{arctanh}\left(\frac{\sqrt{a}x}{\sqrt{ax^2+bx^3}}\right)}{\sqrt{a}}-\frac{\sqrt{ax^2+bx^3}}{x^2}\right)-\frac{(ax^2+bx^3)^{3/2}}{2x^5}\right)-\frac{(ax^2+bx^3)^{5/2}}{3x^8}$$

```
Int[(a*x^2 + b*x^3)^(5/2)/x^9,x]
```

```
-1/3*(a*x^2 + b*x^3)^(5/2)/x^8 + (5*b*(-1/2*(a*x^2 + b*x^3)^(3/2)/x^5 + (3
*b*(-(Sqrt[a*x^2 + b*x^3]/x^2) - (b*ArcTanh[(Sqrt[a]*x)/Sqrt[a*x^2 + b*x^3
]])/Sqrt[a]))/4))/6
```

Defintions of rubi rules used

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

```
Int[1/Sqrt[(a_.)*(x_)^2 + (b_.)*(x_)^(n_.)], x_Symbol] :> Simp[2/(2 - n)
Subst[Int[1/(1 - a*x^2), x], x, x/Sqrt[a*x^2 + b*x^n]], x] /; FreeQ[{a, b,
n}, x] && NeQ[n, 2]
```

```
Int[((c_.)*(x_))^(m_)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol]
 :> Simp[(c*x)^(m + 1)*((a*x^j + b*x^n)^p/(c*(m + j*p + 1))), x] - Simp[b*p
*((n - j)/(c^n*(m + j*p + 1)))   Int[(c*x)^(m + n)*(a*x^j + b*x^n)^(p - 1),
 x], x] /; FreeQ[{a, b, c}, x] &&  !IntegerQ[p] && LtQ[0, j, n] && (Integer
sQ[j, n] || GtQ[c, 0]) && GtQ[p, 0] && LtQ[m + j*p + 1, 0]
```

Maple [A] (verified)

Time = 0.62 (sec) , antiderivative size = 78, normalized size of antiderivative = 0.70

method	result
risch	$-\frac{(33b^2x^2+26abx+8a^2)\sqrt{x^2(bx+a)}}{24x^4}-\frac{5b^3\operatorname{arctanh}\left(\frac{\sqrt{bx+a}}{\sqrt{a}}\right)\sqrt{x^2(bx+a)}}{8\sqrt{a}\,x\sqrt{bx+a}}$
default	$-\frac{(b\,x^3+a\,x^2)^{\frac{5}{2}}\left(15\operatorname{arctanh}\left(\frac{\sqrt{bx+a}}{\sqrt{a}}\right)b^3x^3+33(bx+a)^{\frac{5}{2}}\sqrt{a}-40(bx+a)^{\frac{3}{2}}a^{\frac{3}{2}}+15\sqrt{bx+a}\,a^{\frac{5}{2}}\right)}{24x^8(bx+a)^{\frac{5}{2}}\sqrt{a}}$
pseudoelliptic	$-\frac{81\left(-\frac{35\operatorname{arctanh}\left(\frac{\sqrt{bx+a}}{\sqrt{a}}\right)b^8x^8}{2304}+\sqrt{bx+a}\left(\frac{35\sqrt{a}\,b^7x^7}{2304}-\frac{35a^{\frac{3}{2}}b^6x^6}{3456}+\frac{7a^{\frac{5}{2}}b^5x^5}{864}-\frac{a^{\frac{7}{2}}b^4x^4}{144}+\frac{a^{\frac{9}{2}}b^3x^3}{162}+a^{\frac{11}{2}}b^2x^2+\frac{44a^{\frac{13}{2}}bx}{27}+\right.\right.}{448a^{\frac{11}{2}}x^8}$

```
int((b*x^3+a*x^2)^(5/2)/x^9,x,method=_RETURNVERBOSE)
```

```
-1/24*(33*b^2*x^2+26*a*b*x+8*a^2)/x^4*(x^2*(b*x+a))^(1/2)-5/8*b^3/a^(1/2)*
arctanh((b*x+a)^(1/2)/a^(1/2))*(x^2*(b*x+a))^(1/2)/x/(b*x+a)^(1/2)
```

Fricas [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 181, normalized size of antiderivative = 1.62

$$\int \frac{(ax^2+bx^3)^{5/2}}{x^9}\,dx = \left[\frac{15\sqrt{a}b^3x^4\log\left(\frac{bx^2+2\,ax-2\,\sqrt{bx^3+ax^2}\sqrt{a}}{x^2}\right)-2\,(33\,ab^2x^2+26\,a^2bx+8\,a^3)\sqrt{bx^3+ax^2}}{48\,ax^4},\right.$$

```
integrate((b*x^3+a*x^2)^(5/2)/x^9,x, algorithm="fricas")
```

```
[1/48*(15*sqrt(a)*b^3*x^4*log((b*x^2 + 2*a*x - 2*sqrt(b*x^3 + a*x^2)*sqrt(
a))/x^2) - 2*(33*a*b^2*x^2 + 26*a^2*b*x + 8*a^3)*sqrt(b*x^3 + a*x^2))/(a*x
^4), 1/24*(15*sqrt(-a)*b^3*x^4*arctan(sqrt(b*x^3 + a*x^2)*sqrt(-a)/(b*x^2
+ a*x)) - (33*a*b^2*x^2 + 26*a^2*b*x + 8*a^3)*sqrt(b*x^3 + a*x^2))/(a*x^4)
]
```

Sympy [F]

$$\int \frac{(ax^2 + bx^3)^{5/2}}{x^9} \, dx = \int \frac{(x^2(a + bx))^{\frac{5}{2}}}{x^9} \, dx$$

```
integrate((b*x**3+a*x**2)**(5/2)/x**9,x)
```

```
Integral((x**2*(a + b*x))**(5/2)/x**9, x)
```

Maxima [F]

$$\int \frac{(ax^2 + bx^3)^{5/2}}{x^9} \, dx = \int \frac{(bx^3 + ax^2)^{\frac{5}{2}}}{x^9} \, dx$$

```
integrate((b*x^3+a*x^2)^(5/2)/x^9,x, algorithm="maxima")
```

```
integrate((b*x^3 + a*x^2)^(5/2)/x^9, x)
```

Giac [A] (verification not implemented)

Time = 0.15 (sec) , antiderivative size = 75, normalized size of antiderivative = 0.67

$$\int \frac{(ax^2 + bx^3)^{5/2}}{x^9} \, dx = \frac{1}{24} \left(\frac{15 \arctan\left(\frac{\sqrt{bx+a}}{\sqrt{-a}}\right) \operatorname{sgn}(x)}{\sqrt{-a}} - \frac{33 (bx + a)^{\frac{5}{2}} \operatorname{sgn}(x) - 40 (bx + a)^{\frac{3}{2}} a \operatorname{sgn}(x) + 15}{b^3 x^3} \right.$$

```
integrate((b*x^3+a*x^2)^(5/2)/x^9,x, algorithm="giac")
```

```
1/24*(15*arctan(sqrt(b*x + a)/sqrt(-a))*sgn(x)/sqrt(-a) - (33*(b*x + a)^(5
/2)*sgn(x) - 40*(b*x + a)^(3/2)*a*sgn(x) + 15*sqrt(b*x + a)*a^2*sgn(x))/(b
^3*x^3))*b^3
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(ax^2 + bx^3)^{5/2}}{x^9}\, dx = \int \frac{(b\,x^3 + a\,x^2)^{5/2}}{x^9}\, dx$$

```
int((a*x^2 + b*x^3)^(5/2)/x^9,x)
```

```
int((a*x^2 + b*x^3)^(5/2)/x^9, x)
```

Reduce [B] (verification not implemented)

Time = 0.52 (sec) , antiderivative size = 90, normalized size of antiderivative = 0.80

$$\int \frac{(ax^2 + bx^3)^{5/2}}{x^9}\, dx = \frac{-16\sqrt{bx+a}\,a^3 - 52\sqrt{bx+a}\,a^2bx - 66\sqrt{bx+a}\,a\,b^2x^2 + 15\sqrt{a}\log\left(\sqrt{bx+a} - \sqrt{a}\right)}{48a\,x^3}$$

```
int((b*x^3+a*x^2)^(5/2)/x^9,x)
```

```
( - 16*sqrt(a + b*x)*a**3 - 52*sqrt(a + b*x)*a**2*b*x - 66*sqrt(a + b*x)*a
*b**2*x**2 + 15*sqrt(a)*log(sqrt(a + b*x) - sqrt(a))*b**3*x**3 - 15*sqrt(a
)*log(sqrt(a + b*x) + sqrt(a))*b**3*x**3)/(48*a*x**3)
```

3.297 $\int \frac{(ax^2+bx^3)^{5/2}}{x^{10}}\,dx$

Optimal result

Integrand size = 19, antiderivative size = 140

$$\int \frac{(ax^2+bx^3)^{5/2}}{x^{10}}\,dx = -\frac{a^2\sqrt{ax^2+bx^3}}{4x^5}-\frac{17ab\sqrt{ax^2+bx^3}}{24x^4} -\frac{59b^2\sqrt{ax^2+bx^3}}{96x^3}-\frac{5b^3\sqrt{ax^2+bx^3}}{64ax^2}+\frac{5b^4\text{arctanh}\left(\frac{\sqrt{ax^2+bx^3}}{\sqrt{a}x}\right)}{64a^{3/2}}$$

```
-1/4*a^2*(b*x^3+a*x^2)^(1/2)/x^5-17/24*a*b*(b*x^3+a*x^2)^(1/2)/x^4-59/96*b
^2*(b*x^3+a*x^2)^(1/2)/x^3-5/64*b^3*(b*x^3+a*x^2)^(1/2)/a/x^2+5/64*b^4*arc
tanh((b*x^3+a*x^2)^(1/2)/a^(1/2)/x)/a^(3/2)
```

Mathematica [A] (verified)

Time = 0.18 (sec) , antiderivative size = 105, normalized size of antiderivative = 0.75

$$\int \frac{(ax^2+bx^3)^{5/2}}{x^{10}}\,dx = \frac{\sqrt{x^2(a+bx)}\left(-\sqrt{a}\sqrt{a+bx}(48a^3+136a^2bx+118ab^2x^2+15b^3x^3)+15b^4x^4\text{arctan}\right.}{192a^{3/2}x^5\sqrt{a+bx}}$$

```
Integrate[(a*x^2 + b*x^3)^(5/2)/x^10,x]
```

```
(Sqrt[x^2*(a + b*x)]*(-(Sqrt[a]*Sqrt[a + b*x]*(48*a^3 + 136*a^2*b*x + 118*
a*b^2*x^2 + 15*b^3*x^3)) + 15*b^4*x^4*ArcTanh[Sqrt[a + b*x]/Sqrt[a]]))/(19
2*a^(3/2)*x^5*Sqrt[a + b*x])
```

Rubi [A] (verified)

Time = 0.54 (sec) , antiderivative size = 138, normalized size of antiderivative = 0.99, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.316$, Rules used = {1926, 1926, 1926, 1931, 1914, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\left(ax^2+bx^3\right)^{5/2}}{x^{10}}dx$$

$$\Big\downarrow \text{1926}$$

$$\frac{5}{8}b\int \frac{\left(bx^3+ax^2\right)^{3/2}}{x^7}dx-\frac{\left(ax^2+bx^3\right)^{5/2}}{4x^9}$$

$$\Big\downarrow \text{1926}$$

$$\frac{5}{8}b\left(\frac{1}{2}b\int \frac{\sqrt{bx^3+ax^2}}{x^4}dx-\frac{\left(ax^2+bx^3\right)^{3/2}}{3x^6}\right)-\frac{\left(ax^2+bx^3\right)^{5/2}}{4x^9}$$

$$\Big\downarrow \text{1926}$$

$$\frac{5}{8}b\left(\frac{1}{2}b\left(\frac{1}{4}b\int \frac{1}{x\sqrt{bx^3+ax^2}}dx-\frac{\sqrt{ax^2+bx^3}}{2x^3}\right)-\frac{\left(ax^2+bx^3\right)^{3/2}}{3x^6}\right)-\frac{\left(ax^2+bx^3\right)^{5/2}}{4x^9}$$

$$\Big\downarrow \text{1931}$$

$$\frac{5}{8}b\left(\frac{1}{2}b\left(\frac{1}{4}b\left(-\frac{b\int \frac{1}{\sqrt{bx^3+ax^2}}dx}{2a}-\frac{\sqrt{ax^2+bx^3}}{ax^2}\right)-\frac{\sqrt{ax^2+bx^3}}{2x^3}\right)-\frac{\left(ax^2+bx^3\right)^{3/2}}{3x^6}\right)-\frac{\left(ax^2+bx^3\right)^{5/2}}{4x^9}$$

$$\Big\downarrow \text{1914}$$

$$\frac{5}{8}b\left(\frac{1}{2}b\left(\frac{1}{4}b\left(\frac{b\int \frac{1}{1-\frac{ax^2}{bx^3+ax^2}}d\frac{x}{\sqrt{bx^3+ax^2}}}{a}-\frac{\sqrt{ax^2+bx^3}}{ax^2}\right)-\frac{\sqrt{ax^2+bx^3}}{2x^3}\right)-\frac{(ax^2+bx^3)^{3/2}}{3x^6}\right)-\frac{(ax^2+bx^3)^{5/2}}{4x^9}$$

$$\Big\downarrow \; 219$$

$$\frac{5}{8}b\left(\frac{1}{2}b\left(\frac{1}{4}b\left(\frac{b\,\text{arctanh}\left(\frac{\sqrt{a}x}{\sqrt{ax^2+bx^3}}\right)}{a^{3/2}}-\frac{\sqrt{ax^2+bx^3}}{ax^2}\right)-\frac{\sqrt{ax^2+bx^3}}{2x^3}\right)-\frac{(ax^2+bx^3)^{3/2}}{3x^6}\right)-\frac{(ax^2+bx^3)^{5/2}}{4x^9}$$

```
Int[(a*x^2 + b*x^3)^(5/2)/x^10,x]
```

```
-1/4*(a*x^2 + b*x^3)^(5/2)/x^9 + (5*b*(-1/3*(a*x^2 + b*x^3)^(3/2)/x^6 + (b
*(-1/2*Sqrt[a*x^2 + b*x^3]/x^3 + (b*(-(Sqrt[a*x^2 + b*x^3]/(a*x^2)) + (b*A
rcTanh[(Sqrt[a]*x)/Sqrt[a*x^2 + b*x^3]])/a^(3/2)))/4))/2))/8
```

Defintions of rubi rules used

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

```
Int[1/Sqrt[(a_.)*(x_)^2 + (b_.)*(x_)^(n_.)], x_Symbol] :> Simp[2/(2 - n)
Subst[Int[1/(1 - a*x^2), x], x, x/Sqrt[a*x^2 + b*x^n]], x] /; FreeQ[{a, b,
n}, x] && NeQ[n, 2]
```

```
Int[((c_.)*(x_))^(m_)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol]
 :> Simp[(c*x)^(m + 1)*((a*x^j + b*x^n)^p/(c*(m + j*p + 1))), x] - Simp[b*p
*((n - j)/(c^n*(m + j*p + 1)))   Int[(c*x)^(m + n)*(a*x^j + b*x^n)^(p - 1),
 x], x] /; FreeQ[{a, b, c}, x] &&  !IntegerQ[p] && LtQ[0, j, n] && (Integer
sQ[j, n] || GtQ[c, 0]) && GtQ[p, 0] && LtQ[m + j*p + 1, 0]
```

```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> Simp[c^(j - 1)*(c*x)^(m - j + 1)*((a*x^j + b*x^n)^(p + 1)/(a*(m + j*p
+ 1))), x] - Simp[b*((m + n*p + n - j + 1)/(a*c^(n - j)*(m + j*p + 1)))   I
nt[(c*x)^(m + n - j)*(a*x^j + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, p}, x]
 &&  !IntegerQ[p] && LtQ[0, j, n] && (IntegersQ[j, n] || GtQ[c, 0]) && LtQ[
m + j*p + 1, 0]
```

Maple [A] (verified)

Time = 0.66 (sec) , antiderivative size = 92, normalized size of antiderivative = 0.66

method	result
risch	$-\frac{\left(15 b^{3} x^{3}+118 a \,b^{2} x^{2}+136 a^{2} b x +48 a^{3}\right) \sqrt{x^{2} \left(b x +a \right)}}{192 x^{5} a}+\frac{5 b^{4} \operatorname{arctanh}\left(\frac{\sqrt{b x +a}}{\sqrt{a}}\right) \sqrt{x^{2} \left(b x +a \right)}}{64 a^{\frac{3}{2}} x \sqrt{b x +a}}$
default	$-\frac{\left(b \,x^{3}+a \,x^{2}\right)^{\frac{5}{2}} \left(15 \left(b x +a \right)^{\frac{7}{2}} a^{\frac{3}{2}}-15 \operatorname{arctanh}\left(\frac{\sqrt{b x +a}}{\sqrt{a}}\right) a \,b^{4} x^{4}+73 \left(b x +a \right)^{\frac{5}{2}} a^{\frac{5}{2}}-55 \left(b x +a \right)^{\frac{3}{2}} a^{\frac{7}{2}}+15 \sqrt{b x +a}\, a^{\frac{9}{2}}\right)}{192 x^{9} \left(b x +a \right)^{\frac{5}{2}} a^{\frac{5}{2}}}$
pseudoelliptic	$-\frac{5 \left(\frac{693 \operatorname{arctanh}\left(\frac{\sqrt{b x +a}}{\sqrt{a}}\right) b^{9} x^{9}}{256}+\sqrt{b x +a}\, \left(-\frac{693 \sqrt{a}\, b^{8} x^{8}}{256}+\frac{231 a^{\frac{3}{2}} b^{7} x^{7}}{128}-\frac{231 a^{\frac{5}{2}} b^{6} x^{6}}{160}+\frac{99 a^{\frac{7}{2}} b^{5} x^{5}}{80}-\frac{11 a^{\frac{9}{2}} b^{4} x^{4}}{10}+a^{\frac{11}{2}} b^{3} x^{3}+123\right.\right.}{8064 a^{\frac{13}{2}} x^{9}}$

```
int((b*x^3+a*x^2)^(5/2)/x^10,x,method=_RETURNVERBOSE)
```

```
-1/192*(15*b^3*x^3+118*a*b^2*x^2+136*a^2*b*x+48*a^3)/x^5/a*(x^2*(b*x+a))^(
1/2)+5/64*b^4/a^(3/2)*arctanh((b*x+a)^(1/2)/a^(1/2))*(x^2*(b*x+a))^(1/2)/x
/(b*x+a)^(1/2)
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 202, normalized size of antiderivative = 1.44

$$\int \frac{(ax^2+bx^3)^{5/2}}{x^{10}}\,dx = \left[\frac{15\,\sqrt{a}b^4x^5\log\left(\frac{bx^2+2\,ax+2\,\sqrt{bx^3+ax^2}\sqrt{a}}{x^2}\right)-2\,(15\,ab^3x^3+118\,a^2b^2x^2+136\,a^3bx+48}{384\,a^2x^5}\right.$$

$$\left.-\frac{15\,\sqrt{-a}b^4x^5\arctan\left(\frac{\sqrt{bx^3+ax^2}\sqrt{-a}}{bx^2+ax}\right)+(15\,ab^3x^3+118\,a^2b^2x^2+136\,a^3bx+48\,a^4)\sqrt{bx^3+ax^2}}{192\,a^2x^5}\right]$$

```
integrate((b*x^3+a*x^2)^(5/2)/x^10,x, algorithm="fricas")
```

```
[1/384*(15*sqrt(a)*b^4*x^5*log((b*x^2 + 2*a*x + 2*sqrt(b*x^3 + a*x^2)*sqrt
(a))/x^2) - 2*(15*a*b^3*x^3 + 118*a^2*b^2*x^2 + 136*a^3*b*x + 48*a^4)*sqrt
(b*x^3 + a*x^2))/(a^2*x^5), -1/192*(15*sqrt(-a)*b^4*x^5*arctan(sqrt(b*x^3
+ a*x^2)*sqrt(-a)/(b*x^2 + a*x)) + (15*a*b^3*x^3 + 118*a^2*b^2*x^2 + 136*a
^3*b*x + 48*a^4)*sqrt(b*x^3 + a*x^2))/(a^2*x^5)]
```

Sympy [F]

$$\int \frac{(ax^2 + bx^3)^{5/2}}{x^{10}}\,dx = \int \frac{(x^2(a + bx))^{\frac{5}{2}}}{x^{10}}\,dx$$

```
integrate((b*x**3+a*x**2)**(5/2)/x**10,x)
```

```
Integral((x**2*(a + b*x))**(5/2)/x**10, x)
```

Maxima [F]

$$\int \frac{(ax^2 + bx^3)^{5/2}}{x^{10}}\,dx = \int \frac{(bx^3 + ax^2)^{\frac{5}{2}}}{x^{10}}\,dx$$

```
integrate((b*x^3+a*x^2)^(5/2)/x^10,x, algorithm="maxima")
```

```
integrate((b*x^3 + a*x^2)^(5/2)/x^10, x)
```

Giac [A] (verification not implemented)

Time = 0.15 (sec) , antiderivative size = 109, normalized size of antiderivative = 0.78

$$\int \frac{(ax^2+bx^3)^{5/2}}{x^{10}}\,dx = -\frac{\frac{15\,b^5\arctan\left(\frac{\sqrt{bx+a}}{\sqrt{-a}}\right)\mathrm{sgn}(x)}{\sqrt{-a}a}+\frac{15\,(bx+a)^{\frac{7}{2}}b^5\mathrm{sgn}(x)+73\,(bx+a)^{\frac{5}{2}}ab^5\mathrm{sgn}(x)-55\,(bx+a)^{\frac{3}{2}}a^2b^5\mathrm{sgn}(x)+15\,\sqrt{bx+a}a^3b^5\mathrm{sgn}(x)}{ab^4x^4}}{192\,b}$$

```
integrate((b*x^3+a*x^2)^(5/2)/x^10,x, algorithm="giac")
```

```
-1/192*(15*b^5*arctan(sqrt(b*x + a)/sqrt(-a))*sgn(x)/(sqrt(-a)*a) + (15*(b
*x + a)^(7/2)*b^5*sgn(x) + 73*(b*x + a)^(5/2)*a*b^5*sgn(x) - 55*(b*x + a)^
(3/2)*a^2*b^5*sgn(x) + 15*sqrt(b*x + a)*a^3*b^5*sgn(x))/(a*b^4*x^4))/b
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(ax^2+bx^3)^{5/2}}{x^{10}}\,dx = \int \frac{(b\,x^3+a\,x^2)^{5/2}}{x^{10}}\,dx$$

```
int((a*x^2 + b*x^3)^(5/2)/x^10,x)
```

```
int((a*x^2 + b*x^3)^(5/2)/x^10, x)
```

Reduce [B] (verification not implemented)

Time = 0.49 (sec) , antiderivative size = 107, normalized size of antiderivative = 0.76

$$\int \frac{(ax^2+bx^3)^{5/2}}{x^{10}}\,dx = \frac{-96\sqrt{bx+a}\,a^4-272\sqrt{bx+a}\,a^3bx-236\sqrt{bx+a}\,a^2b^2x^2-30\sqrt{bx+a}\,a\,b^3x^3-15}{384a^2x^4}$$

```
int((b*x^3+a*x^2)^(5/2)/x^10,x)
```

```
( - 96*sqrt(a + b*x)*a**4 - 272*sqrt(a + b*x)*a**3*b*x - 236*sqrt(a + b*x)
*a**2*b**2*x**2 - 30*sqrt(a + b*x)*a*b**3*x**3 - 15*sqrt(a)*log(sqrt(a + b
*x) - sqrt(a))*b**4*x**4 + 15*sqrt(a)*log(sqrt(a + b*x) + sqrt(a))*b**4*x*
*4)/(384*a**2*x**4)
```

3.298 $\int \frac{(ax^2+bx^3)^{5/2}}{x^{11}}\,dx$

Optimal result

Integrand size = 19, antiderivative size = 168

$$\int \frac{(ax^2+bx^3)^{5/2}}{x^{11}}\,dx = -\frac{a^2\sqrt{ax^2+bx^3}}{5x^6}-\frac{21ab\sqrt{ax^2+bx^3}}{40x^5}-\frac{31b^2\sqrt{ax^2+bx^3}}{80x^4}-\frac{b^3\sqrt{ax^2+bx^3}}{64ax^3}+\frac{3b^4\sqrt{ax^2+bx^3}}{128a^2x^2}-\frac{3b^5\operatorname{arctanh}\left(\frac{\sqrt{ax^2+bx^3}}{\sqrt{a}x}\right)}{128a^{5/2}}$$

```
-1/5*a^2*(b*x^3+a*x^2)^(1/2)/x^6-21/40*a*b*(b*x^3+a*x^2)^(1/2)/x^5-31/80*b
^2*(b*x^3+a*x^2)^(1/2)/x^4-1/64*b^3*(b*x^3+a*x^2)^(1/2)/a/x^3+3/128*b^4*(b
*x^3+a*x^2)^(1/2)/a^2/x^2-3/128*b^5*arctanh((b*x^3+a*x^2)^(1/2)/a^(1/2)/x)
/a^(5/2)
```

Mathematica [A] (verified)

Time = 0.19 (sec) , antiderivative size = 115, normalized size of antiderivative = 0.68

$$\int \frac{(ax^2+bx^3)^{5/2}}{x^{11}}\,dx = -\frac{\sqrt{x^2(a+bx)}\left(\sqrt{a}\sqrt{a+bx}(128a^4+336a^3bx+248a^2b^2x^2+10ab^3x^3-15b^4x^4)+15b^5x^5\operatorname{arctanh}\left(\frac{\sqrt{a+bx}}{\sqrt{a}}\right)\right)}{640a^{5/2}x^6\sqrt{a+bx}}$$

```
Integrate[(a*x^2 + b*x^3)^(5/2)/x^11,x]
```

```
-1/640*(Sqrt[x^2*(a + b*x)]*(Sqrt[a]*Sqrt[a + b*x]*(128*a^4 + 336*a^3*b*x
+ 248*a^2*b^2*x^2 + 10*a*b^3*x^3 - 15*b^4*x^4) + 15*b^5*x^5*ArcTanh[Sqrt[a
 + b*x]/Sqrt[a]]))/(a^(5/2)*x^6*Sqrt[a + b*x])
```

Rubi [A] (verified)

Time = 0.65 (sec) , antiderivative size = 172, normalized size of antiderivative = 1.02, number of steps used = 8, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.368, Rules used = {1926, 1926, 1926, 1931, 1931, 1914, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\left(ax^2+bx^3\right)^{5/2}}{x^{11}}dx$$

$$\Big\downarrow \text{1926}$$

$$\frac{1}{2}b\int \frac{\left(bx^3+ax^2\right)^{3/2}}{x^8}dx-\frac{\left(ax^2+bx^3\right)^{5/2}}{5x^{10}}$$

$$\Big\downarrow \text{1926}$$

$$\frac{1}{2}b\left(\frac{3}{8}b\int \frac{\sqrt{bx^3+ax^2}}{x^5}dx-\frac{\left(ax^2+bx^3\right)^{3/2}}{4x^7}\right)-\frac{\left(ax^2+bx^3\right)^{5/2}}{5x^{10}}$$

$$\Big\downarrow \text{1926}$$

$$\frac{1}{2}b\left(\frac{3}{8}b\left(\frac{1}{6}b\int \frac{1}{x^2\sqrt{bx^3+ax^2}}dx-\frac{\sqrt{ax^2+bx^3}}{3x^4}\right)-\frac{\left(ax^2+bx^3\right)^{3/2}}{4x^7}\right)-\frac{\left(ax^2+bx^3\right)^{5/2}}{5x^{10}}$$

$$\Big\downarrow \text{1931}$$

$$\frac{1}{2}b\left(\frac{3}{8}b\left(\frac{1}{6}b\left(-\frac{3b\int \frac{1}{x\sqrt{bx^3+ax^2}}dx}{4a}-\frac{\sqrt{ax^2+bx^3}}{2ax^3}\right)-\frac{\sqrt{ax^2+bx^3}}{3x^4}\right)-\frac{\left(ax^2+bx^3\right)^{3/2}}{4x^7}\right)-\frac{\left(ax^2+bx^3\right)^{5/2}}{5x^{10}}$$

$$\Big\downarrow \text{1931}$$

$$\frac{1}{2}b\left(\frac{3}{8}b\left(\frac{1}{6}b\left(-\frac{3b\left(-\frac{b\int\frac{1}{\sqrt{bx^3+ax^2}}dx}{2a}-\frac{\sqrt{ax^2+bx^3}}{ax^2}\right)}{4a}-\frac{\sqrt{ax^2+bx^3}}{2ax^3}\right)-\frac{\sqrt{ax^2+bx^3}}{3x^4}\right)-\frac{(ax^2+bx^3)^{3/2}}{4x^7}\right)-\frac{(ax^2+bx^3)^{5/2}}{5x^{10}}$$

$$\Big\downarrow \text{1914}$$

$$\frac{1}{2}b\left(\frac{3}{8}b\left(\frac{1}{6}b\left(-\frac{3b\left(\frac{b\int\frac{1}{1-\frac{ax^2}{bx^3+ax^2}}d\frac{x}{\sqrt{bx^3+ax^2}}}{a}-\frac{\sqrt{ax^2+bx^3}}{ax^2}\right)}{4a}-\frac{\sqrt{ax^2+bx^3}}{2ax^3}\right)-\frac{\sqrt{ax^2+bx^3}}{3x^4}\right)-\frac{(ax^2+bx^3)^{3/2}}{4x^7}\right)-\frac{(ax^2+bx^3)^{5/2}}{5x^{10}}$$

$$\Big\downarrow \text{219}$$

$$\frac{1}{2}b\left(\frac{3}{8}b\left(\frac{1}{6}b\left(-\frac{3b\left(\frac{b\,\text{arctanh}\left(\frac{\sqrt{a}x}{\sqrt{ax^2+bx^3}}\right)}{a^{3/2}}-\frac{\sqrt{ax^2+bx^3}}{ax^2}\right)}{4a}-\frac{\sqrt{ax^2+bx^3}}{2ax^3}\right)-\frac{\sqrt{ax^2+bx^3}}{3x^4}\right)-\frac{(ax^2+bx^3)^{3/2}}{4x^7}\right)-\frac{(ax^2+bx^3)^{5/2}}{5x^{10}}$$

```
Int[(a*x^2 + b*x^3)^(5/2)/x^11,x]
```

```
-1/5*(a*x^2 + b*x^3)^(5/2)/x^10 + (b*(-1/4*(a*x^2 + b*x^3)^(3/2)/x^7 + (3*
b*(-1/3*Sqrt[a*x^2 + b*x^3]/x^4 + (b*(-1/2*Sqrt[a*x^2 + b*x^3]/(a*x^3) - (
3*b*(-(Sqrt[a*x^2 + b*x^3]/(a*x^2)) + (b*ArcTanh[(Sqrt[a]*x)/Sqrt[a*x^2 +
b*x^3]])/a^(3/2)))/(4*a)))/6))/8))/2
```

Defintions of rubi rules used

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

```
Int[1/Sqrt[(a_.)*(x_)^2 + (b_.)*(x_)^(n_.)], x_Symbol] :> Simp[2/(2 - n)
Subst[Int[1/(1 - a*x^2), x], x, x/Sqrt[a*x^2 + b*x^n]], x] /; FreeQ[{a, b,
n}, x] && NeQ[n, 2]
```

```
Int[((c_.)*(x_))^(m_)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol]
 :> Simp[(c*x)^(m + 1)*((a*x^j + b*x^n)^p/(c*(m + j*p + 1))), x] - Simp[b*p
*((n - j)/(c^n*(m + j*p + 1)))   Int[(c*x)^(m + n)*(a*x^j + b*x^n)^(p - 1),
 x], x] /; FreeQ[{a, b, c}, x] &&  !IntegerQ[p] && LtQ[0, j, n] && (Integer
sQ[j, n] || GtQ[c, 0]) && GtQ[p, 0] && LtQ[m + j*p + 1, 0]
```

```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> Simp[c^(j - 1)*(c*x)^(m - j + 1)*((a*x^j + b*x^n)^(p + 1)/(a*(m + j*p
+ 1))), x] - Simp[b*((m + n*p + n - j + 1)/(a*c^(n - j)*(m + j*p + 1)))   I
nt[(c*x)^(m + n - j)*(a*x^j + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, p}, x]
 &&  !IntegerQ[p] && LtQ[0, j, n] && (IntegersQ[j, n] || GtQ[c, 0]) && LtQ[
m + j*p + 1, 0]
```

Maple [A] (verified)

Time = 0.83 (sec) , antiderivative size = 103, normalized size of antiderivative = 0.61

method	result
risch	$-\frac{\left(-15 b^{4} x^{4}+10 a\, b^{3} x^{3}+248 a^{2} b^{2} x^{2}+336 a^{3} b x +128 a^{4}\right) \sqrt{x^{2} \left(b x +a \right)}}{640 x^{6} a^{2}}-\frac{3 b^{5} \operatorname{arctanh}\left(\frac{\sqrt{b x +a}}{\sqrt{a}}\right) \sqrt{x^{2} \left(b x +a \right)}}{128 a^{\frac{5}{2}} x \sqrt{b x +a}}$
default	$\frac{\left(b\, x^{3}+a\, x^{2}\right)^{\frac{5}{2}} \left(15 \left(b x +a \right)^{\frac{9}{2}} a^{\frac{5}{2}}-70 \left(b x +a \right)^{\frac{7}{2}} a^{\frac{7}{2}}-15 \operatorname{arctanh}\left(\frac{\sqrt{b x +a}}{\sqrt{a}}\right) a^{2} x^{5} b^{5}-128 \left(b x +a \right)^{\frac{5}{2}} a^{\frac{9}{2}}+70 \left(b x +a \right)^{\frac{3}{2}} a^{\frac{11}{2}}-15 \sqrt{b x +}\right.}{640 x^{10} \left(b x +a \right)^{\frac{5}{2}} a^{\frac{9}{2}}}$
pseudoelliptic	$\frac{\frac{143 \operatorname{arctanh}\left(\frac{\sqrt{b x +a}}{\sqrt{a}}\right) b^{10} x^{10}}{131072}+\frac{13 \sqrt{b x +a}\, \left(-\frac{693 \sqrt{a}\, b^{9} x^{9}}{256}+\frac{231 a^{\frac{3}{2}} b^{8} x^{8}}{128}-\frac{231 a^{\frac{5}{2}} b^{7} x^{7}}{160}+\frac{99 a^{\frac{7}{2}} b^{6} x^{6}}{80}-\frac{11 a^{\frac{9}{2}} b^{5} x^{5}}{10}+a^{\frac{11}{2}} b^{4} x^{4}-\frac{12 a^{\frac{13}{2}} b^{3}}{13}\right.}{32256}}{a^{\frac{15}{2}} x^{10}}$

```
int((b*x^3+a*x^2)^(5/2)/x^11,x,method=_RETURNVERBOSE)
```

```
-1/640*(-15*b^4*x^4+10*a*b^3*x^3+248*a^2*b^2*x^2+336*a^3*b*x+128*a^4)/x^6/
a^2*(x^2*(b*x+a))^(1/2)-3/128*b^5/a^(5/2)*arctanh((b*x+a)^(1/2)/a^(1/2))*(
x^2*(b*x+a))^(1/2)/x/(b*x+a)^(1/2)
```

Fricas [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 224, normalized size of antiderivative = 1.33

$$\int \frac{(ax^2+bx^3)^{5/2}}{x^{11}}\,dx = \left[\frac{15\sqrt{a}b^5x^6\log\left(\frac{bx^2+2\,ax-2\,\sqrt{bx^3+ax^2}\sqrt{a}}{x^2}\right)+2\,(15\,ab^4x^4-10\,a^2b^3x^3-248\,a^3b^2x^2-33}{1280\,a^3x^6}\right.$$

```
integrate((b*x^3+a*x^2)^(5/2)/x^11,x, algorithm="fricas")
```

```
[1/1280*(15*sqrt(a)*b^5*x^6*log((b*x^2 + 2*a*x - 2*sqrt(b*x^3 + a*x^2)*sqr
t(a))/x^2) + 2*(15*a*b^4*x^4 - 10*a^2*b^3*x^3 - 248*a^3*b^2*x^2 - 336*a^4*
b*x - 128*a^5)*sqrt(b*x^3 + a*x^2))/(a^3*x^6), 1/640*(15*sqrt(-a)*b^5*x^6*
arctan(sqrt(b*x^3 + a*x^2)*sqrt(-a)/(b*x^2 + a*x)) + (15*a*b^4*x^4 - 10*a^
2*b^3*x^3 - 248*a^3*b^2*x^2 - 336*a^4*b*x - 128*a^5)*sqrt(b*x^3 + a*x^2))/
(a^3*x^6)]
```

Sympy [F]

$$\int \frac{(ax^2+bx^3)^{5/2}}{x^{11}}\,dx = \int \frac{(x^2(a+bx))^{\frac{5}{2}}}{x^{11}}\,dx$$

```
integrate((b*x**3+a*x**2)**(5/2)/x**11,x)
```

```
Integral((x**2*(a + b*x))**(5/2)/x**11, x)
```

Maxima [F]

$$\int \frac{(ax^2+bx^3)^{5/2}}{x^{11}}\,dx = \int \frac{(bx^3+ax^2)^{\frac{5}{2}}}{x^{11}}\,dx$$

```
integrate((b*x^3+a*x^2)^(5/2)/x^11,x, algorithm="maxima")
```

```
integrate((b*x^3 + a*x^2)^(5/2)/x^11, x)
```

Giac [A] (verification not implemented)

Time = 0.18 (sec) , antiderivative size = 108, normalized size of antiderivative = 0.64

$$\int \frac{(ax^2+bx^3)^{5/2}}{x^{11}}\,dx = \frac{1}{640}b^5\left(\frac{15\arctan\left(\frac{\sqrt{bx+a}}{\sqrt{-a}}\right)\operatorname{sgn}(x)}{\sqrt{-a}a^2} + \frac{15\,(bx+a)^{\frac{9}{2}}\operatorname{sgn}(x) - 70\,(bx+a)^{\frac{7}{2}}a\operatorname{sgn}(x) -}{}\right.$$

```
integrate((b*x^3+a*x^2)^(5/2)/x^11,x, algorithm="giac")
```

```
1/640*b^5*(15*arctan(sqrt(b*x + a)/sqrt(-a))*sgn(x)/(sqrt(-a)*a^2) + (15*(
b*x + a)^(9/2)*sgn(x) - 70*(b*x + a)^(7/2)*a*sgn(x) - 128*(b*x + a)^(5/2)*
a^2*sgn(x) + 70*(b*x + a)^(3/2)*a^3*sgn(x) - 15*sqrt(b*x + a)*a^4*sgn(x))/
(a^2*b^5*x^5))
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(ax^2+bx^3)^{5/2}}{x^{11}}\,dx = \int \frac{(b\,x^3+a\,x^2)^{5/2}}{x^{11}}\,dx$$

```
int((a*x^2 + b*x^3)^(5/2)/x^11,x)
```

```
int((a*x^2 + b*x^3)^(5/2)/x^11, x)
```

Reduce [B] (verification not implemented)

Time = 0.23 (sec) , antiderivative size = 124, normalized size of antiderivative = 0.74

$$\int \frac{(ax^2+bx^3)^{5/2}}{x^{11}}\,dx = \frac{-256\sqrt{bx+a}\,a^5-672\sqrt{bx+a}\,a^4bx-496\sqrt{bx+a}\,a^3b^2x^2-20\sqrt{bx+a}\,a^2b^3x^3+3}{1280}$$

```
int((b*x^3+a*x^2)^(5/2)/x^11,x)
```

```
( - 256*sqrt(a + b*x)*a**5 - 672*sqrt(a + b*x)*a**4*b*x - 496*sqrt(a + b*x
)*a**3*b**2*x**2 - 20*sqrt(a + b*x)*a**2*b**3*x**3 + 30*sqrt(a + b*x)*a*b*
*4*x**4 + 15*sqrt(a)*log(sqrt(a + b*x) - sqrt(a))*b**5*x**5 - 15*sqrt(a)*l
og(sqrt(a + b*x) + sqrt(a))*b**5*x**5)/(1280*a**3*x**5)
```

3.299 $\int \frac{(ax^2+bx^3)^{5/2}}{x^{12}}\,dx$

Optimal result

Integrand size = 19, antiderivative size = 196

$$\int \frac{(ax^2+bx^3)^{5/2}}{x^{12}}\,dx = -\frac{a^2\sqrt{ax^2+bx^3}}{6x^7}-\frac{5ab\sqrt{ax^2+bx^3}}{12x^6}-\frac{9b^2\sqrt{ax^2+bx^3}}{32x^5} -\frac{b^3\sqrt{ax^2+bx^3}}{192ax^4}+\frac{5b^4\sqrt{ax^2+bx^3}}{768a^2x^3}-\frac{5b^5\sqrt{ax^2+bx^3}}{512a^3x^2}+\frac{5b^6\operatorname{arctanh}\left(\frac{\sqrt{ax^2+bx^3}}{\sqrt{a}x}\right)}{512a^{7/2}}$$

```
-1/6*a^2*(b*x^3+a*x^2)^(1/2)/x^7-5/12*a*b*(b*x^3+a*x^2)^(1/2)/x^6-9/32*b^2
*(b*x^3+a*x^2)^(1/2)/x^5-1/192*b^3*(b*x^3+a*x^2)^(1/2)/a/x^4+5/768*b^4*(b*
x^3+a*x^2)^(1/2)/a^2/x^3-5/512*b^5*(b*x^3+a*x^2)^(1/2)/a^3/x^2+5/512*b^6*a
rctanh((b*x^3+a*x^2)^(1/2)/a^(1/2)/x)/a^(7/2)
```

Mathematica [A] (verified)

Time = 0.24 (sec) , antiderivative size = 127, normalized size of antiderivative = 0.65

$$\int \frac{(ax^2+bx^3)^{5/2}}{x^{12}}\,dx = \frac{\sqrt{x^2(a+bx)}\left(-\sqrt{a}\sqrt{a+bx}(256a^5+640a^4bx+432a^3b^2x^2+8a^2b^3x^3-10ab^4x^4+\right.}{1536a^{7/2}x^7\sqrt{a+bx}}$$

```
Integrate[(a*x^2 + b*x^3)^(5/2)/x^12,x]
```

```
(Sqrt[x^2*(a + b*x)]*(-(Sqrt[a]*Sqrt[a + b*x]*(256*a^5 + 640*a^4*b*x + 432
*a^3*b^2*x^2 + 8*a^2*b^3*x^3 - 10*a*b^4*x^4 + 15*b^5*x^5)) + 15*b^6*x^6*Ar
cTanh[Sqrt[a + b*x]/Sqrt[a]]))/(1536*a^(7/2)*x^7*Sqrt[a + b*x])
```

Rubi [A] (verified)

Time = 0.74 (sec) , antiderivative size = 206, normalized size of antiderivative = 1.05, number of steps used = 9, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.421, Rules used = {1926, 1926, 1926, 1931, 1931, 1931, 1914, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(ax^2+bx^3)^{5/2}}{x^{12}}\,dx$$

$$\Big\downarrow \text{1926}$$

$$\frac{5}{12}b\int \frac{(bx^3+ax^2)^{3/2}}{x^9}dx-\frac{(ax^2+bx^3)^{5/2}}{6x^{11}}$$

$$\Big\downarrow \text{1926}$$

$$\frac{5}{12}b\left(\frac{3}{10}b\int \frac{\sqrt{bx^3+ax^2}}{x^6}dx-\frac{(ax^2+bx^3)^{3/2}}{5x^8}\right)-\frac{(ax^2+bx^3)^{5/2}}{6x^{11}}$$

$$\Big\downarrow \text{1926}$$

$$\frac{5}{12}b\left(\frac{3}{10}b\left(\frac{1}{8}b\int \frac{1}{x^3\sqrt{bx^3+ax^2}}dx-\frac{\sqrt{ax^2+bx^3}}{4x^5}\right)-\frac{(ax^2+bx^3)^{3/2}}{5x^8}\right)-\frac{(ax^2+bx^3)^{5/2}}{6x^{11}}$$

$$\Big\downarrow \text{1931}$$

$$\frac{5}{12}b\left(\frac{3}{10}b\left(\frac{1}{8}b\left(-\frac{5b\int \frac{1}{x^2\sqrt{bx^3+ax^2}}dx}{6a}-\frac{\sqrt{ax^2+bx^3}}{3ax^4}\right)-\frac{\sqrt{ax^2+bx^3}}{4x^5}\right)-\frac{(ax^2+bx^3)^{3/2}}{5x^8}\right)-\frac{(ax^2+bx^3)^{5/2}}{6x^{11}}$$

$$\Big\downarrow \text{1931}$$

$$\frac{5}{12}b\left(\frac{3}{10}b\left(\frac{1}{8}b\left(-\frac{5b\left(-\frac{3b\int\frac{1}{x\sqrt{bx^3+ax^2}}dx}{4a}-\frac{\sqrt{ax^2+bx^3}}{2ax^3}\right)}{6a}-\frac{\sqrt{ax^2+bx^3}}{3ax^4}\right)-\frac{\sqrt{ax^2+bx^3}}{4x^5}\right)-\frac{\left(ax^2+bx^3\right)^{3/2}}{5x^8}\right)-\frac{\left(ax^2+bx^3\right)^{5/2}}{6x^{11}}$$

$$\Big\downarrow \text{ 1931}$$

$$\frac{5}{12}b\left(\frac{3}{10}b\left(\frac{1}{8}b\left(-\frac{5b\left(-\frac{3b\left(-\frac{b\int\frac{1}{\sqrt{bx^3+ax^2}}dx}{2a}-\frac{\sqrt{ax^2+bx^3}}{ax^2}\right)}{4a}-\frac{\sqrt{ax^2+bx^3}}{2ax^3}\right)}{6a}-\frac{\sqrt{ax^2+bx^3}}{3ax^4}\right)-\frac{\sqrt{ax^2+bx^3}}{4x^5}\right)-\frac{\left(ax^2+bx^3\right)^{3/2}}{5x^8}\right)-\frac{\left(ax^2+bx^3\right)^{5/2}}{6x^{11}}$$

$$\Big\downarrow \text{ 1914}$$

$$\frac{5}{12}b\left(\frac{3}{10}b\left(\frac{1}{8}b\left(-\frac{5b\left(-\frac{3b\left(\frac{b\int\frac{1}{1-\frac{ax^2}{bx^3+ax^2}}d\frac{x}{\sqrt{bx^3+ax^2}}}{a}-\frac{\sqrt{ax^2+bx^3}}{ax^2}\right)}{4a}-\frac{\sqrt{ax^2+bx^3}}{2ax^3}\right)}{6a}-\frac{\sqrt{ax^2+bx^3}}{3ax^4}\right)-\frac{\sqrt{ax^2+bx^3}}{4x^5}\right)-\frac{\left(ax^2+bx^3\right)^{3/2}}{5x^8}\right)-\frac{\left(ax^2+bx^3\right)^{5/2}}{6x^{11}}$$

$$\Big\downarrow \text{ 219}$$

$$\frac{5}{12}b\left(\frac{3}{10}b\left(\frac{1}{8}b\left(-\frac{5b\left(-\frac{3b\left(\frac{b\operatorname{arctanh}\left(\frac{\sqrt{a}x}{\sqrt{ax^2+bx^3}}\right)}{a^{3/2}}-\frac{\sqrt{ax^2+bx^3}}{ax^2}\right)}{4a}-\frac{\sqrt{ax^2+bx^3}}{2ax^3}\right)}{6a}-\frac{\sqrt{ax^2+bx^3}}{3ax^4}\right)-\frac{\sqrt{ax^2+bx^3}}{4x^5}\right)-\left(\right.\right.$$

$$\frac{\left(ax^2+bx^3\right)^{5/2}}{6x^{11}}$$

```
Int[(a*x^2 + b*x^3)^(5/2)/x^12,x]
```

```
-1/6*(a*x^2 + b*x^3)^(5/2)/x^11 + (5*b*(-1/5*(a*x^2 + b*x^3)^(3/2)/x^8 + (
3*b*(-1/4*Sqrt[a*x^2 + b*x^3]/x^5 + (b*(-1/3*Sqrt[a*x^2 + b*x^3]/(a*x^4) -
 (5*b*(-1/2*Sqrt[a*x^2 + b*x^3]/(a*x^3) - (3*b*(-(Sqrt[a*x^2 + b*x^3]/(a*x
^2)) + (b*ArcTanh[(Sqrt[a]*x)/Sqrt[a*x^2 + b*x^3]])/a^(3/2)))/(4*a)))/(6*a
)))/8))/10))/12
```

Defintions of rubi rules used

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

```
Int[1/Sqrt[(a_.)*(x_)^2 + (b_.)*(x_)^(n_.)], x_Symbol] :> Simp[2/(2 - n)
Subst[Int[1/(1 - a*x^2), x], x, x/Sqrt[a*x^2 + b*x^n]], x] /; FreeQ[{a, b,
n}, x] && NeQ[n, 2]
```

```
Int[((c_.)*(x_))^(m_)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol]
 :> Simp[(c*x)^(m + 1)*((a*x^j + b*x^n)^p/(c*(m + j*p + 1))), x] - Simp[b*p
*((n - j)/(c^n*(m + j*p + 1)))   Int[(c*x)^(m + n)*(a*x^j + b*x^n)^(p - 1),
 x], x] /; FreeQ[{a, b, c}, x] &&  !IntegerQ[p] && LtQ[0, j, n] && (Integer
sQ[j, n] || GtQ[c, 0]) && GtQ[p, 0] && LtQ[m + j*p + 1, 0]
```

```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> Simp[c^(j - 1)*(c*x)^(m - j + 1)*((a*x^j + b*x^n)^(p + 1)/(a*(m + j*p
+ 1))), x] - Simp[b*((m + n*p + n - j + 1)/(a*c^(n - j)*(m + j*p + 1)))   I
nt[(c*x)^(m + n - j)*(a*x^j + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, p}, x]
 &&  !IntegerQ[p] && LtQ[0, j, n] && (IntegersQ[j, n] || GtQ[c, 0]) && LtQ[
m + j*p + 1, 0]
```

Maple [A] (verified)

Time = 0.84 (sec) , antiderivative size = 114, normalized size of antiderivative = 0.58

method	result
risch	$-\frac{\left(15 b^{5} x^{5}-10 a \,b^{4} x^{4}+8 a^{2} b^{3} x^{3}+432 a^{3} b^{2} x^{2}+640 a^{4} b x +256 a^{5}\right) \sqrt{x^{2} \left(b x +a \right)}}{1536 x^{7} a^{3}}+\frac{5 b^{6} \operatorname{arctanh}\left(\frac{\sqrt{b x +a}}{\sqrt{a}}\right) \sqrt{x^{2} \left(b x +a \right)}}{512 a^{\frac{7}{2}} x \sqrt{b x +a}}$
default	$-\frac{\left(b \,x^{3}+a \,x^{2}\right)^{\frac{5}{2}} \left(15 \left(b x +a \right)^{\frac{11}{2}} a^{\frac{7}{2}}-85 \left(b x +a \right)^{\frac{9}{2}} a^{\frac{9}{2}}+198 \left(b x +a \right)^{\frac{7}{2}} a^{\frac{11}{2}}-15 \operatorname{arctanh}\left(\frac{\sqrt{b x +a}}{\sqrt{a}}\right) a^{3} b^{6} x^{6}+198 \left(b x +a \right)^{\frac{5}{2}} a^{\frac{13}{2}}-85 (}{1536 x^{11} \left(b x +a \right)^{\frac{5}{2}} a^{\frac{13}{2}}}$
pseudoelliptic	$-\frac{65 \left(\frac{693 \operatorname{arctanh}\left(\frac{\sqrt{b x +a}}{\sqrt{a}}\right) b^{11} x^{11}}{256}+\sqrt{b x +a}\, \left(-\frac{693 \sqrt{a}\, b^{10} x^{10}}{256}+\frac{231 a^{\frac{3}{2}} b^{9} x^{9}}{128}-\frac{231 a^{\frac{5}{2}} b^{8} x^{8}}{160}+\frac{99 a^{\frac{7}{2}} b^{7} x^{7}}{80}-\frac{11 a^{\frac{9}{2}} b^{6} x^{6}}{10}+a^{\frac{11}{2}} b^{5} x^{5}\right.\right.}{236544 a^{\frac{17}{2}} x^{11}}$

```
int((b*x^3+a*x^2)^(5/2)/x^12,x,method=_RETURNVERBOSE)
```

```
-1/1536*(15*b^5*x^5-10*a*b^4*x^4+8*a^2*b^3*x^3+432*a^3*b^2*x^2+640*a^4*b*x
+256*a^5)/x^7/a^3*(x^2*(b*x+a))^(1/2)+5/512*b^6/a^(7/2)*arctanh((b*x+a)^(1
/2)/a^(1/2))*(x^2*(b*x+a))^(1/2)/x/(b*x+a)^(1/2)
```

Fricas [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 246, normalized size of antiderivative = 1.26

$$\int \frac{(ax^2+bx^3)^{5/2}}{x^{12}}\,dx = \left[\frac{15\,\sqrt{a}b^6x^7\log\left(\frac{bx^2+2\,ax+2\,\sqrt{bx^3+ax^2}\sqrt{a}}{x^2}\right)-2\,(15\,ab^5x^5-10\,a^2b^4x^4+8\,a^3b^3x^3+432\,a}{3072\,a^4x^7}\right.$$

$$-\frac{15\,\sqrt{-a}b^6x^7\arctan\left(\frac{\sqrt{bx^3+ax^2}\sqrt{-a}}{bx^2+ax}\right)+(15\,ab^5x^5-10\,a^2b^4x^4+8\,a^3b^3x^3+432\,a^4b^2x^2+640\,a^5bx+256\,a^6)}{1536\,a^4x^7}$$

```
integrate((b*x^3+a*x^2)^(5/2)/x^12,x, algorithm="fricas")
```

```
[1/3072*(15*sqrt(a)*b^6*x^7*log((b*x^2 + 2*a*x + 2*sqrt(b*x^3 + a*x^2)*sqr
t(a))/x^2) - 2*(15*a*b^5*x^5 - 10*a^2*b^4*x^4 + 8*a^3*b^3*x^3 + 432*a^4*b^
2*x^2 + 640*a^5*b*x + 256*a^6)*sqrt(b*x^3 + a*x^2))/(a^4*x^7), -1/1536*(15
*sqrt(-a)*b^6*x^7*arctan(sqrt(b*x^3 + a*x^2)*sqrt(-a)/(b*x^2 + a*x)) + (15
*a*b^5*x^5 - 10*a^2*b^4*x^4 + 8*a^3*b^3*x^3 + 432*a^4*b^2*x^2 + 640*a^5*b*
x + 256*a^6)*sqrt(b*x^3 + a*x^2))/(a^4*x^7)]
```

Sympy [F]

$$\int \frac{(ax^2 + bx^3)^{5/2}}{x^{12}}\,dx = \int \frac{(x^2(a + bx))^{\frac{5}{2}}}{x^{12}}\,dx$$

```
integrate((b*x**3+a*x**2)**(5/2)/x**12,x)
```

```
Integral((x**2*(a + b*x))**(5/2)/x**12, x)
```

Maxima [F]

$$\int \frac{(ax^2 + bx^3)^{5/2}}{x^{12}}\,dx = \int \frac{(bx^3 + ax^2)^{\frac{5}{2}}}{x^{12}}\,dx$$

```
integrate((b*x^3+a*x^2)^(5/2)/x^12,x, algorithm="maxima")
```

```
integrate((b*x^3 + a*x^2)^(5/2)/x^12, x)
```

Giac [A] (verification not implemented)

Time = 0.22 (sec) , antiderivative size = 143, normalized size of antiderivative = 0.73

$$\int \frac{\left(ax^2+bx^3\right)^{5/2}}{x^{12}}\,dx =$$

$$-\frac{\frac{15\,b^7\arctan\left(\frac{\sqrt{bx+a}}{\sqrt{-a}}\right)\mathrm{sgn}(x)}{\sqrt{-a}a^3}+\frac{15\,(bx+a)^{\frac{11}{2}}b^7\mathrm{sgn}(x)-85\,(bx+a)^{\frac{9}{2}}ab^7\mathrm{sgn}(x)+198\,(bx+a)^{\frac{7}{2}}a^2b^7\mathrm{sgn}(x)+198\,(bx+a)^{\frac{5}{2}}a^3b^7\mathrm{sgn}(x)-85\,(bx+a)^{\frac{3}{2}}}{a^3b^6x^6}}{1536\,b}$$

```
integrate((b*x^3+a*x^2)^(5/2)/x^12,x, algorithm="giac")
```

```
-1/1536*(15*b^7*arctan(sqrt(b*x + a)/sqrt(-a))*sgn(x)/(sqrt(-a)*a^3) + (15
*(b*x + a)^(11/2)*b^7*sgn(x) - 85*(b*x + a)^(9/2)*a*b^7*sgn(x) + 198*(b*x
+ a)^(7/2)*a^2*b^7*sgn(x) + 198*(b*x + a)^(5/2)*a^3*b^7*sgn(x) - 85*(b*x +
 a)^(3/2)*a^4*b^7*sgn(x) + 15*sqrt(b*x + a)*a^5*b^7*sgn(x))/(a^3*b^6*x^6))
/b
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\left(ax^2+bx^3\right)^{5/2}}{x^{12}}\,dx = \int \frac{{\left(b\,x^3+a\,x^2\right)}^{5/2}}{x^{12}}\,dx$$

```
int((a*x^2 + b*x^3)^(5/2)/x^12,x)
```

```
int((a*x^2 + b*x^3)^(5/2)/x^12, x)
```

Reduce [B] (verification not implemented)

Time = 0.44 (sec) , antiderivative size = 141, normalized size of antiderivative = 0.72

$$\int \frac{(ax^2+bx^3)^{5/2}}{x^{12}}\,dx = \frac{-512\sqrt{bx+a}\,a^6-1280\sqrt{bx+a}\,a^5bx-864\sqrt{bx+a}\,a^4b^2x^2-16\sqrt{bx+a}\,a^3b^3x^3+}{}$$

```
int((b*x^3+a*x^2)^(5/2)/x^12,x)
```

```
( - 512*sqrt(a + b*x)*a**6 - 1280*sqrt(a + b*x)*a**5*b*x - 864*sqrt(a + b*
x)*a**4*b**2*x**2 - 16*sqrt(a + b*x)*a**3*b**3*x**3 + 20*sqrt(a + b*x)*a**
2*b**4*x**4 - 30*sqrt(a + b*x)*a*b**5*x**5 - 15*sqrt(a)*log(sqrt(a + b*x)
- sqrt(a))*b**6*x**6 + 15*sqrt(a)*log(sqrt(a + b*x) + sqrt(a))*b**6*x**6)/
(3072*a**4*x**6)
```

3.300 $\int \frac{x^4}{\sqrt{ax^2+bx^3}}\,dx$

Optimal result

Integrand size = 19, antiderivative size = 104

$$\int \frac{x^4}{\sqrt{ax^2+bx^3}}\,dx = -\frac{2a^3\sqrt{ax^2+bx^3}}{b^4x} + \frac{2a^2(ax^2+bx^3)^{3/2}}{b^4x^3} - \frac{6a(ax^2+bx^3)^{5/2}}{5b^4x^5} + \frac{2(ax^2+bx^3)^{7/2}}{7b^4x^7}$$

```
-2*a^3*(b*x^3+a*x^2)^(1/2)/b^4/x+2*a^2*(b*x^3+a*x^2)^(3/2)/b^4/x^3-6/5*a*(
b*x^3+a*x^2)^(5/2)/b^4/x^5+2/7*(b*x^3+a*x^2)^(7/2)/b^4/x^7
```

Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 53, normalized size of antiderivative = 0.51

$$\int \frac{x^4}{\sqrt{ax^2+bx^3}}\,dx = \frac{2\sqrt{x^2(a+bx)}(-16a^3+8a^2bx-6ab^2x^2+5b^3x^3)}{35b^4x}$$

```
Integrate[x^4/Sqrt[a*x^2 + b*x^3],x]
```

```
(2*Sqrt[x^2*(a + b*x)]*(-16*a^3 + 8*a^2*b*x - 6*a*b^2*x^2 + 5*b^3*x^3))/(3
5*b^4*x)
```

Rubi [A] (verified)

Time = 0.48 (sec) , antiderivative size = 115, normalized size of antiderivative = 1.11, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.211, Rules used = {1922, 1922, 1922, 1920}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^4}{\sqrt{ax^2+bx^3}}dx$$

$$\Big\downarrow \text{1922}$$

$$\frac{2x^2\sqrt{ax^2+bx^3}}{7b}-\frac{6a\int \frac{x^3}{\sqrt{bx^3+ax^2}}dx}{7b}$$

$$\Big\downarrow \text{1922}$$

$$\frac{2x^2\sqrt{ax^2+bx^3}}{7b}-\frac{6a\left(\frac{2x\sqrt{ax^2+bx^3}}{5b}-\frac{4a\int \frac{x^2}{\sqrt{bx^3+ax^2}}dx}{5b}\right)}{7b}$$

$$\Big\downarrow \text{1922}$$

$$\frac{2x^2\sqrt{ax^2+bx^3}}{7b}-\frac{6a\left(\frac{2x\sqrt{ax^2+bx^3}}{5b}-\frac{4a\left(\frac{2\sqrt{ax^2+bx^3}}{3b}-\frac{2a\int \frac{x}{\sqrt{bx^3+ax^2}}dx}{3b}\right)}{5b}\right)}{7b}$$

$$\Big\downarrow \text{1920}$$

$$\frac{2x^2\sqrt{ax^2+bx^3}}{7b}-\frac{6a\left(\frac{2x\sqrt{ax^2+bx^3}}{5b}-\frac{4a\left(\frac{2\sqrt{ax^2+bx^3}}{3b}-\frac{4a\sqrt{ax^2+bx^3}}{3b^2x}\right)}{5b}\right)}{7b}$$

```
Int[x^4/Sqrt[a*x^2 + b*x^3],x]
```

```
(2*x^2*Sqrt[a*x^2 + b*x^3])/(7*b) - (6*a*((2*x*Sqrt[a*x^2 + b*x^3])/(5*b)
- (4*a*((2*Sqrt[a*x^2 + b*x^3])/(3*b) - (4*a*Sqrt[a*x^2 + b*x^3])/(3*b^2*x
)))/(5*b)))/(7*b)
```

Defintions of rubi rules used

```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> Simp[(-c^(j - 1))*(c*x)^(m - j + 1)*((a*x^j + b*x^n)^(p + 1)/(a*(n - j
)*(p + 1))), x] /; FreeQ[{a, b, c, j, m, n, p}, x] &&  !IntegerQ[p] && NeQ[
n, j] && EqQ[m + n*p + n - j + 1, 0] && (IntegerQ[j] || GtQ[c, 0])
```

```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> Simp[c^(j - 1)*(c*x)^(m - j + 1)*((a*x^j + b*x^n)^(p + 1)/(a*(m + j*p
+ 1))), x] - Simp[b*((m + n*p + n - j + 1)/(a*c^(n - j)*(m + j*p + 1)))   I
nt[(c*x)^(m + n - j)*(a*x^j + b*x^n)^p, x], x] /; FreeQ[{a, b, c, j, m, n,
p}, x] &&  !IntegerQ[p] && NeQ[n, j] && ILtQ[Simplify[(m + n*p + n - j + 1)
/(n - j)], 0] && NeQ[m + j*p + 1, 0] && (IntegersQ[j, n] || GtQ[c, 0])
```

Maple [A] (verified)

Time = 0.43 (sec) , antiderivative size = 52, normalized size of antiderivative = 0.50

method	result	size
trager	$-\frac{2\left(-5 b^{3} x^{3}+6 a \,b^{2} x^{2}-8 a^{2} b x +16 a^{3}\right) \sqrt{b \,x^{3}+a \,x^{2}}}{35 b^{4} x}$	52
risch	$-\frac{2 x \left(b x +a \right) \left(-5 b^{3} x^{3}+6 a \,b^{2} x^{2}-8 a^{2} b x +16 a^{3}\right)}{35 \sqrt{x^{2} \left(b x +a \right)}\, b^{4}}$	53
pseudoelliptic	$\frac{2 \sqrt{b x +a}\, \left(35 b^{4} x^{4}-40 a \,b^{3} x^{3}+48 a^{2} b^{2} x^{2}-64 a^{3} b x +128 a^{4}\right)}{315 b^{5}}$	54
gosper	$-\frac{2 \left(b x +a \right) \left(-5 b^{3} x^{3}+6 a \,b^{2} x^{2}-8 a^{2} b x +16 a^{3}\right) x}{35 b^{4} \sqrt{b \,x^{3}+a \,x^{2}}}$	55
default	$-\frac{2 \left(b x +a \right) \left(-5 b^{3} x^{3}+6 a \,b^{2} x^{2}-8 a^{2} b x +16 a^{3}\right) x}{35 b^{4} \sqrt{b \,x^{3}+a \,x^{2}}}$	55
orering	$-\frac{2 \left(b x +a \right) \left(-5 b^{3} x^{3}+6 a \,b^{2} x^{2}-8 a^{2} b x +16 a^{3}\right) x}{35 b^{4} \sqrt{b \,x^{3}+a \,x^{2}}}$	55

```
int(x^4/(b*x^3+a*x^2)^(1/2),x,method=_RETURNVERBOSE)
```

```
-2/35*(-5*b^3*x^3+6*a*b^2*x^2-8*a^2*b*x+16*a^3)/b^4/x*(b*x^3+a*x^2)^(1/2)
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 51, normalized size of antiderivative = 0.49

$$\int \frac{x^4}{\sqrt{ax^2+bx^3}}\,dx = \frac{2\,(5\,b^3x^3-6\,ab^2x^2+8\,a^2bx-16\,a^3)\sqrt{bx^3+ax^2}}{35\,b^4x}$$

```
integrate(x^4/(b*x^3+a*x^2)^(1/2),x, algorithm="fricas")
```

```
2/35*(5*b^3*x^3 - 6*a*b^2*x^2 + 8*a^2*b*x - 16*a^3)*sqrt(b*x^3 + a*x^2)/(b
^4*x)
```

Sympy [F]

$$\int \frac{x^4}{\sqrt{ax^2+bx^3}}\,dx = \int \frac{x^4}{\sqrt{x^2\,(a+bx)}}\,dx$$

```
integrate(x**4/(b*x**3+a*x**2)**(1/2),x)
```

```
Integral(x**4/sqrt(x**2*(a + b*x)), x)
```

Maxima [A] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 53, normalized size of antiderivative = 0.51

$$\int \frac{x^4}{\sqrt{ax^2+bx^3}}\,dx = \frac{2\,(5\,b^4x^4-ab^3x^3+2\,a^2b^2x^2-8\,a^3bx-16\,a^4)}{35\,\sqrt{bx+a}b^4}$$

```
integrate(x^4/(b*x^3+a*x^2)^(1/2),x, algorithm="maxima")
```

```
2/35*(5*b^4*x^4 - a*b^3*x^3 + 2*a^2*b^2*x^2 - 8*a^3*b*x - 16*a^4)/(sqrt(b*
x + a)*b^4)
```

Giac [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 64, normalized size of antiderivative = 0.62

$$\int \frac{x^4}{\sqrt{ax^2+bx^3}}\,dx$$
$$= \frac{32\,a^{\frac{7}{2}}\mathrm{sgn}(x)}{35\,b^4} + \frac{2\left(5\,(bx+a)^{\frac{7}{2}} - 21\,(bx+a)^{\frac{5}{2}}a + 35\,(bx+a)^{\frac{3}{2}}a^2 - 35\,\sqrt{bx+a}a^3\right)}{35\,b^4\mathrm{sgn}\,(x)}$$

```
integrate(x^4/(b*x^3+a*x^2)^(1/2),x, algorithm="giac")
```

```
32/35*a^(7/2)*sgn(x)/b^4 + 2/35*(5*(b*x + a)^(7/2) - 21*(b*x + a)^(5/2)*a
+ 35*(b*x + a)^(3/2)*a^2 - 35*sqrt(b*x + a)*a^3)/(b^4*sgn(x))
```

Mupad [B] (verification not implemented)

Time = 8.76 (sec) , antiderivative size = 51, normalized size of antiderivative = 0.49

$$\int \frac{x^4}{\sqrt{ax^2+bx^3}}\,dx = -\frac{2\,\sqrt{b\,x^3+a\,x^2}\,\left(16\,a^3-8\,a^2\,b\,x+6\,a\,b^2\,x^2-5\,b^3\,x^3\right)}{35\,b^4\,x}$$

```
int(x^4/(a*x^2 + b*x^3)^(1/2),x)
```

```
-(2*(a*x^2 + b*x^3)^(1/2)*(16*a^3 - 5*b^3*x^3 + 6*a*b^2*x^2 - 8*a^2*b*x))/
(35*b^4*x)
```

Reduce [B] (verification not implemented)

Time = 0.53 (sec) , antiderivative size = 41, normalized size of antiderivative = 0.39

$$\int \frac{x^4}{\sqrt{ax^2+bx^3}}\,dx = \frac{2\sqrt{bx+a}\,(5b^3x^3-6a\,b^2x^2+8a^2bx-16a^3)}{35b^4}$$

```
int(x^4/(b*x^3+a*x^2)^(1/2),x)
```

```
(2*sqrt(a + b*x)*( - 16*a**3 + 8*a**2*b*x - 6*a*b**2*x**2 + 5*b**3*x**3))/
(35*b**4)
```

3.301 $\int \frac{x^3}{\sqrt{ax^2+bx^3}}\,dx$

Optimal result

Integrand size = 19, antiderivative size = 78

$$\int \frac{x^3}{\sqrt{ax^2+bx^3}}\,dx = \frac{2a^2\sqrt{ax^2+bx^3}}{b^3x} - \frac{4a(ax^2+bx^3)^{3/2}}{3b^3x^3} + \frac{2(ax^2+bx^3)^{5/2}}{5b^3x^5}$$

```
2*a^2*(b*x^3+a*x^2)^(1/2)/b^3/x-4/3*a*(b*x^3+a*x^2)^(3/2)/b^3/x^3+2/5*(b*x
^3+a*x^2)^(5/2)/b^3/x^5
```

Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 42, normalized size of antiderivative = 0.54

$$\int \frac{x^3}{\sqrt{ax^2+bx^3}}\,dx = \frac{2\sqrt{x^2(a+bx)}(8a^2-4abx+3b^2x^2)}{15b^3x}$$

```
Integrate[x^3/Sqrt[a*x^2 + b*x^3],x]
```

```
(2*Sqrt[x^2*(a + b*x)]*(8*a^2 - 4*a*b*x + 3*b^2*x^2))/(15*b^3*x)
```

Rubi [A] (verified)

Time = 0.38 (sec) , antiderivative size = 81, normalized size of antiderivative = 1.04, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.158, Rules used = {1922, 1922, 1920}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^3}{\sqrt{ax^2+bx^3}}dx$$

$$\Big\downarrow \; 1922$$

$$\frac{2x\sqrt{ax^2+bx^3}}{5b}-\frac{4a\int \frac{x^2}{\sqrt{bx^3+ax^2}}dx}{5b}$$

$$\Big\downarrow \; 1922$$

$$\frac{2x\sqrt{ax^2+bx^3}}{5b}-\frac{4a\left(\frac{2\sqrt{ax^2+bx^3}}{3b}-\frac{2a\int \frac{x}{\sqrt{bx^3+ax^2}}dx}{3b}\right)}{5b}$$

$$\Big\downarrow \; 1920$$

$$\frac{2x\sqrt{ax^2+bx^3}}{5b}-\frac{4a\left(\frac{2\sqrt{ax^2+bx^3}}{3b}-\frac{4a\sqrt{ax^2+bx^3}}{3b^2x}\right)}{5b}$$

```
Int[x^3/Sqrt[a*x^2 + b*x^3],x]
```

```
(2*x*Sqrt[a*x^2 + b*x^3])/(5*b) - (4*a*((2*Sqrt[a*x^2 + b*x^3])/(3*b) - (4
*a*Sqrt[a*x^2 + b*x^3])/(3*b^2*x)))/(5*b)
```

Defintions of rubi rules used

```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> Simp[(-c^(j - 1))*(c*x)^(m - j + 1)*((a*x^j + b*x^n)^(p + 1)/(a*(n - j
)*(p + 1))), x] /; FreeQ[{a, b, c, j, m, n, p}, x] &&  !IntegerQ[p] && NeQ[
n, j] && EqQ[m + n*p + n - j + 1, 0] && (IntegerQ[j] || GtQ[c, 0])
```

```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> Simp[c^(j - 1)*(c*x)^(m - j + 1)*((a*x^j + b*x^n)^(p + 1)/(a*(m + j*p
+ 1))), x] - Simp[b*((m + n*p + n - j + 1)/(a*c^(n - j)*(m + j*p + 1)))   I
nt[(c*x)^(m + n - j)*(a*x^j + b*x^n)^p, x], x] /; FreeQ[{a, b, c, j, m, n,
p}, x] &&  !IntegerQ[p] && NeQ[n, j] && ILtQ[Simplify[(m + n*p + n - j + 1)
/(n - j)], 0] && NeQ[m + j*p + 1, 0] && (IntegersQ[j, n] || GtQ[c, 0])
```

Maple [A] (verified)

Time = 0.41 (sec) , antiderivative size = 41, normalized size of antiderivative = 0.53

method	result	size
trager	$\frac{2\left(3b^{2}x^{2}-4abx+8a^{2}\right)\sqrt{b\,x^{3}+a\,x^{2}}}{15b^{3}x}$	41
risch	$\frac{2x\left(bx+a\right)\left(3b^{2}x^{2}-4abx+8a^{2}\right)}{15\sqrt{x^{2}\left(bx+a\right)}\,b^{3}}$	42
pseudoelliptic	$-\frac{2\sqrt{bx+a}\,\left(-5b^{3}x^{3}+6a\,b^{2}x^{2}-8a^{2}bx+16a^{3}\right)}{35b^{4}}$	43
gosper	$\frac{2\left(bx+a\right)\left(3b^{2}x^{2}-4abx+8a^{2}\right)x}{15b^{3}\sqrt{b\,x^{3}+a\,x^{2}}}$	44
default	$\frac{2\left(bx+a\right)\left(3b^{2}x^{2}-4abx+8a^{2}\right)x}{15b^{3}\sqrt{b\,x^{3}+a\,x^{2}}}$	44
orering	$\frac{2\left(bx+a\right)\left(3b^{2}x^{2}-4abx+8a^{2}\right)x}{15b^{3}\sqrt{b\,x^{3}+a\,x^{2}}}$	44

```
int(x^3/(b*x^3+a*x^2)^(1/2),x,method=_RETURNVERBOSE)
```

```
2/15*(3*b^2*x^2-4*a*b*x+8*a^2)/b^3/x*(b*x^3+a*x^2)^(1/2)
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 40, normalized size of antiderivative = 0.51

$$\int \frac{x^3}{\sqrt{ax^2+bx^3}}\,dx = \frac{2\,(3\,b^2x^2-4\,abx+8\,a^2)\sqrt{bx^3+ax^2}}{15\,b^3x}$$

```
integrate(x^3/(b*x^3+a*x^2)^(1/2),x, algorithm="fricas")
```

```
2/15*(3*b^2*x^2 - 4*a*b*x + 8*a^2)*sqrt(b*x^3 + a*x^2)/(b^3*x)
```

Sympy [F]

$$\int \frac{x^3}{\sqrt{ax^2+bx^3}}\,dx = \int \frac{x^3}{\sqrt{x^2\,(a+bx)}}\,dx$$

```
integrate(x**3/(b*x**3+a*x**2)**(1/2),x)
```

```
Integral(x**3/sqrt(x**2*(a + b*x)), x)
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 42, normalized size of antiderivative = 0.54

$$\int \frac{x^3}{\sqrt{ax^2+bx^3}}\,dx = \frac{2\,(3\,b^3x^3-ab^2x^2+4\,a^2bx+8\,a^3)}{15\,\sqrt{bx+a}b^3}$$

```
integrate(x^3/(b*x^3+a*x^2)^(1/2),x, algorithm="maxima")
```

```
2/15*(3*b^3*x^3 - a*b^2*x^2 + 4*a^2*b*x + 8*a^3)/(sqrt(b*x + a)*b^3)
```

Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 52, normalized size of antiderivative = 0.67

$$\int \frac{x^3}{\sqrt{ax^2+bx^3}}\,dx = -\frac{16\,a^{\frac{5}{2}}\mathrm{sgn}(x)}{15\,b^3} + \frac{2\left(3\,(bx+a)^{\frac{5}{2}} - 10\,(bx+a)^{\frac{3}{2}}a + 15\,\sqrt{bx+a}a^2\right)}{15\,b^3\mathrm{sgn}\,(x)}$$

```
integrate(x^3/(b*x^3+a*x^2)^(1/2),x, algorithm="giac")
```

```
-16/15*a^(5/2)*sgn(x)/b^3 + 2/15*(3*(b*x + a)^(5/2) - 10*(b*x + a)^(3/2)*a
 + 15*sqrt(b*x + a)*a^2)/(b^3*sgn(x))
```

Mupad [B] (verification not implemented)

Time = 8.70 (sec) , antiderivative size = 40, normalized size of antiderivative = 0.51

$$\int \frac{x^3}{\sqrt{ax^2+bx^3}}\,dx = \frac{2\sqrt{b\,x^3+a\,x^2}\,(8\,a^2-4\,a\,b\,x+3\,b^2\,x^2)}{15\,b^3\,x}$$

```
int(x^3/(a*x^2 + b*x^3)^(1/2),x)
```

```
(2*(a*x^2 + b*x^3)^(1/2)*(8*a^2 + 3*b^2*x^2 - 4*a*b*x))/(15*b^3*x)
```

Reduce [B] (verification not implemented)

Time = 0.35 (sec) , antiderivative size = 30, normalized size of antiderivative = 0.38

$$\int \frac{x^3}{\sqrt{ax^2+bx^3}}\,dx = \frac{2\sqrt{bx+a}\,(3b^2x^2-4abx+8a^2)}{15b^3}$$

```
int(x^3/(b*x^3+a*x^2)^(1/2),x)
```

```
(2*sqrt(a + b*x)*(8*a**2 - 4*a*b*x + 3*b**2*x**2))/(15*b**3)
```

3.302 $\int \frac{x^2}{\sqrt{ax^2+bx^3}}\,dx$

Optimal result

Integrand size = 19, antiderivative size = 50

$$\int \frac{x^2}{\sqrt{ax^2+bx^3}}\,dx = -\frac{2a\sqrt{ax^2+bx^3}}{b^2x}+\frac{2(ax^2+bx^3)^{3/2}}{3b^2x^3}$$

```
-2*a*(b*x^3+a*x^2)^(1/2)/b^2/x+2/3*(b*x^3+a*x^2)^(3/2)/b^2/x^3
```

Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 30, normalized size of antiderivative = 0.60

$$\int \frac{x^2}{\sqrt{ax^2+bx^3}}\,dx = \frac{2(-2a+bx)\sqrt{x^2(a+bx)}}{3b^2x}$$

```
Integrate[x^2/Sqrt[a*x^2 + b*x^3],x]
```

```
(2*(-2*a + b*x)*Sqrt[x^2*(a + b*x)])/(3*b^2*x)
```

Rubi [A] (verified)

Time = 0.30 (sec) , antiderivative size = 49, normalized size of antiderivative = 0.98, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.105$, Rules used = {1922, 1920}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^2}{\sqrt{ax^2+bx^3}}\,dx$$
$$\Big\downarrow \ 1922$$
$$\frac{2\sqrt{ax^2+bx^3}}{3b}-\frac{2a\int \frac{x}{\sqrt{bx^3+ax^2}}dx}{3b}$$
$$\Big\downarrow \ 1920$$
$$\frac{2\sqrt{ax^2+bx^3}}{3b}-\frac{4a\sqrt{ax^2+bx^3}}{3b^2x}$$

```
Int[x^2/Sqrt[a*x^2 + b*x^3],x]
```

```
(2*Sqrt[a*x^2 + b*x^3])/(3*b) - (4*a*Sqrt[a*x^2 + b*x^3])/(3*b^2*x)
```

Defintions of rubi rules used

```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> Simp[(-c^(j - 1))*(c*x)^(m - j + 1)*((a*x^j + b*x^n)^(p + 1)/(a*(n - j
)*(p + 1))), x] /; FreeQ[{a, b, c, j, m, n, p}, x] &&  !IntegerQ[p] && NeQ[
n, j] && EqQ[m + n*p + n - j + 1, 0] && (IntegerQ[j] || GtQ[c, 0])
```

```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> Simp[c^(j - 1)*(c*x)^(m - j + 1)*((a*x^j + b*x^n)^(p + 1)/(a*(m + j*p
+ 1))), x] - Simp[b*((m + n*p + n - j + 1)/(a*c^(n - j)*(m + j*p + 1)))   I
nt[(c*x)^(m + n - j)*(a*x^j + b*x^n)^p, x], x] /; FreeQ[{a, b, c, j, m, n,
p}, x] &&  !IntegerQ[p] && NeQ[n, j] && ILtQ[Simplify[(m + n*p + n - j + 1)
/(n - j)], 0] && NeQ[m + j*p + 1, 0] && (IntegersQ[j, n] || GtQ[c, 0])
```

Maple [A] (verified)

Time = 0.39 (sec) , antiderivative size = 30, normalized size of antiderivative = 0.60

method	result	size
trager	$-\frac{2(-bx+2a)\sqrt{b\,x^{3}+a\,x^{2}}}{3b^{2}x}$	30
risch	$-\frac{2x(bx+a)(-bx+2a)}{3\sqrt{x^{2}(bx+a)}\,b^{2}}$	31
pseudoelliptic	$\frac{2\sqrt{bx+a}\,(3b^{2}x^{2}-4abx+8a^{2})}{15b^{3}}$	32
gosper	$-\frac{2(bx+a)(-bx+2a)x}{3b^{2}\sqrt{b\,x^{3}+a\,x^{2}}}$	33
default	$-\frac{2(bx+a)(-bx+2a)x}{3b^{2}\sqrt{b\,x^{3}+a\,x^{2}}}$	33
orering	$-\frac{2(bx+a)(-bx+2a)x}{3b^{2}\sqrt{b\,x^{3}+a\,x^{2}}}$	33

```
int(x^2/(b*x^3+a*x^2)^(1/2),x,method=_RETURNVERBOSE)
```

```
-2/3*(-b*x+2*a)/b^2/x*(b*x^3+a*x^2)^(1/2)
```

Fricas [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 28, normalized size of antiderivative = 0.56

$$\int \frac{x^2}{\sqrt{ax^2+bx^3}}\,dx = \frac{2\,\sqrt{bx^3+ax^2}(bx-2\,a)}{3\,b^2x}$$

```
integrate(x^2/(b*x^3+a*x^2)^(1/2),x, algorithm="fricas")
```

```
2/3*sqrt(b*x^3 + a*x^2)*(b*x - 2*a)/(b^2*x)
```

Sympy [F]

$$\int \frac{x^2}{\sqrt{ax^2+bx^3}}\,dx = \int \frac{x^2}{\sqrt{x^2\,(a+bx)}}\,dx$$

```
integrate(x**2/(b*x**3+a*x**2)**(1/2),x)
```

```
Integral(x**2/sqrt(x**2*(a + b*x)), x)
```

Maxima [A] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 30, normalized size of antiderivative = 0.60

$$\int \frac{x^2}{\sqrt{ax^2+bx^3}}\,dx = \frac{2\,(b^2x^2-abx-2\,a^2)}{3\,\sqrt{bx+a}b^2}$$

```
integrate(x^2/(b*x^3+a*x^2)^(1/2),x, algorithm="maxima")
```

```
2/3*(b^2*x^2 - a*b*x - 2*a^2)/(sqrt(b*x + a)*b^2)
```

Giac [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 38, normalized size of antiderivative = 0.76

$$\int \frac{x^2}{\sqrt{ax^2+bx^3}}\,dx = \frac{4\,a^{\frac{3}{2}}\mathrm{sgn}(x)}{3\,b^2} + \frac{2\left((bx+a)^{\frac{3}{2}}-3\,\sqrt{bx+a}a\right)}{3\,b^2\mathrm{sgn}\,(x)}$$

```
integrate(x^2/(b*x^3+a*x^2)^(1/2),x, algorithm="giac")
```

```
4/3*a^(3/2)*sgn(x)/b^2 + 2/3*((b*x + a)^(3/2) - 3*sqrt(b*x + a)*a)/(b^2*sg
n(x))
```

Mupad [B] (verification not implemented)

Time = 8.59 (sec) , antiderivative size = 31, normalized size of antiderivative = 0.62

$$\int \frac{x^2}{\sqrt{ax^2+bx^3}}\,dx = -\frac{\left(\frac{4\,a}{3\,b^2}-\frac{2\,x}{3\,b}\right)\,\sqrt{b\,x^3+a\,x^2}}{x}$$

```
int(x^2/(a*x^2 + b*x^3)^(1/2),x)
```

```
-(((4*a)/(3*b^2) - (2*x)/(3*b))*(a*x^2 + b*x^3)^(1/2))/x
```

Reduce [B] (verification not implemented)

Time = 0.23 (sec) , antiderivative size = 18, normalized size of antiderivative = 0.36

$$\int \frac{x^2}{\sqrt{ax^2+bx^3}}\,dx = \frac{2\sqrt{bx+a}\,(bx-2a)}{3b^2}$$

```
int(x^2/(b*x^3+a*x^2)^(1/2),x)
```

```
(2*sqrt(a + b*x)*( - 2*a + b*x))/(3*b**2)
```

3.303 $\int \frac{x}{\sqrt{ax^2+bx^3}}\,dx$

Optimal result

Integrand size = 17, antiderivative size = 23

$$\int \frac{x}{\sqrt{ax^2+bx^3}}\,dx=\frac{2\sqrt{ax^2+bx^3}}{bx}$$

```
2*(b*x^3+a*x^2)^(1/2)/b/x
```

Mathematica [A] (verified)

Time = 0.00 (sec) , antiderivative size = 21, normalized size of antiderivative = 0.91

$$\int \frac{x}{\sqrt{ax^2+bx^3}}\,dx=\frac{2\sqrt{x^2(a+bx)}}{bx}$$

```
Integrate[x/Sqrt[a*x^2 + b*x^3],x]
```

```
(2*Sqrt[x^2*(a + b*x)])/(b*x)
```

Rubi [A] (verified)

Time = 0.24 (sec) , antiderivative size = 23, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}} = 0.059$, Rules used = {1920}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x}{\sqrt{ax^2+bx^3}}\,dx$$

$$\Big\downarrow \; 1920$$

$$\frac{2\sqrt{ax^2+bx^3}}{bx}$$

```
Int[x/Sqrt[a*x^2 + b*x^3],x]
```

```
(2*Sqrt[a*x^2 + b*x^3])/(b*x)
```

Defintions of rubi rules used

```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> Simp[(-c^(j - 1))*(c*x)^(m - j + 1)*((a*x^j + b*x^n)^(p + 1)/(a*(n - j
)*(p + 1))), x] /; FreeQ[{a, b, c, j, m, n, p}, x] &&  !IntegerQ[p] && NeQ[
n, j] && EqQ[m + n*p + n - j + 1, 0] && (IntegerQ[j] || GtQ[c, 0])
```

Maple [A] (verified)

Time = 0.38 (sec) , antiderivative size = 21, normalized size of antiderivative = 0.91

method	result	size
pseudoelliptic	$-\frac{2\sqrt{bx+a}\,(-bx+2a)}{3b^{2}}$	21
trager	$\frac{2\sqrt{b\,x^{3}+a\,x^{2}}}{bx}$	22
risch	$\frac{2x(bx+a)}{\sqrt{x^{2}(bx+a)}\,b}$	23
gosper	$\frac{2x(bx+a)}{b\sqrt{b\,x^{3}+a\,x^{2}}}$	25
default	$\frac{2x(bx+a)}{b\sqrt{b\,x^{3}+a\,x^{2}}}$	25
orering	$\frac{2x(bx+a)}{b\sqrt{b\,x^{3}+a\,x^{2}}}$	25

```
int(x/(b*x^3+a*x^2)^(1/2),x,method=_RETURNVERBOSE)
```

```
-2/3*(b*x+a)^(1/2)*(-b*x+2*a)/b^2
```

Fricas [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 21, normalized size of antiderivative = 0.91

$$\int \frac{x}{\sqrt{ax^2+bx^3}}\,dx = \frac{2\,\sqrt{bx^3+ax^2}}{bx}$$

```
integrate(x/(b*x^3+a*x^2)^(1/2),x, algorithm="fricas")
```

```
2*sqrt(b*x^3 + a*x^2)/(b*x)
```

Sympy [F]

$$\int \frac{x}{\sqrt{ax^2+bx^3}}\,dx = \int \frac{x}{\sqrt{x^2\,(a+bx)}}\,dx$$

```
integrate(x/(b*x**3+a*x**2)**(1/2),x)
```

```
Integral(x/sqrt(x**2*(a + b*x)), x)
```

Maxima [A] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 12, normalized size of antiderivative = 0.52

$$\int \frac{x}{\sqrt{ax^2 + bx^3}}\,dx = \frac{2\sqrt{bx+a}}{b}$$

```
integrate(x/(b*x^3+a*x^2)^(1/2),x, algorithm="maxima")
```

```
2*sqrt(b*x + a)/b
```

Giac [A] (verification not implemented)

Time = 0.14 (sec) , antiderivative size = 27, normalized size of antiderivative = 1.17

$$\int \frac{x}{\sqrt{ax^2 + bx^3}}\,dx = -\frac{2\sqrt{a}\mathrm{sgn}(x)}{b} + \frac{2\sqrt{bx+a}}{b\mathrm{sgn}\,(x)}$$

```
integrate(x/(b*x^3+a*x^2)^(1/2),x, algorithm="giac")
```

```
-2*sqrt(a)*sgn(x)/b + 2*sqrt(b*x + a)/(b*sgn(x))
```

Mupad [B] (verification not implemented)

Time = 8.55 (sec) , antiderivative size = 17, normalized size of antiderivative = 0.74

$$\int \frac{x}{\sqrt{ax^2 + bx^3}}\,dx = \frac{2\,|x|\,\sqrt{a+b\,x}}{b\,x}$$

```
int(x/(a*x^2 + b*x^3)^(1/2),x)
```

```
(2*abs(x)*(a + b*x)^(1/2))/(b*x)
```

Reduce [B] (verification not implemented)

Time = 0.41 (sec) , antiderivative size = 11, normalized size of antiderivative = 0.48

$$\int \frac{x}{\sqrt{ax^2 + bx^3}}\,dx = \frac{2\sqrt{bx + a}}{b}$$

```
int(x/(b*x^3+a*x^2)^(1/2),x)
```

```
(2*sqrt(a + b*x))/b
```

3.304 $\int \frac{1}{\sqrt{ax^2+bx^3}}\,dx$

Optimal result

Integrand size = 15, antiderivative size = 32

$$\int \frac{1}{\sqrt{ax^2+bx^3}}\,dx = -\frac{2\operatorname{arctanh}\left(\frac{\sqrt{ax^2+bx^3}}{\sqrt{a}x}\right)}{\sqrt{a}}$$

```
-2*arctanh((b*x^3+a*x^2)^(1/2)/a^(1/2)/x)/a^(1/2)
```

Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 46, normalized size of antiderivative = 1.44

$$\int \frac{1}{\sqrt{ax^2+bx^3}}\,dx = -\frac{2x\sqrt{a+bx}\operatorname{arctanh}\left(\frac{\sqrt{a+bx}}{\sqrt{a}}\right)}{\sqrt{a}\sqrt{x^2(a+bx)}}$$

```
Integrate[1/Sqrt[a*x^2 + b*x^3],x]
```

```
(-2*x*Sqrt[a + b*x]*ArcTanh[Sqrt[a + b*x]/Sqrt[a]])/(Sqrt[a]*Sqrt[x^2*(a +
 b*x)])
```

Rubi [A] (verified)

Time = 0.26 (sec) , antiderivative size = 30, normalized size of antiderivative = 0.94, number of steps used = 3, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.133$, Rules used = {1914, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{\sqrt{ax^2+bx^3}}dx$$
$$\Big\downarrow \text{1914}$$
$$-2\int \frac{1}{1-\frac{ax^2}{bx^3+ax^2}}d\frac{x}{\sqrt{bx^3+ax^2}}$$
$$\Big\downarrow \text{219}$$
$$-\frac{2\text{arctanh}\left(\frac{\sqrt{a}x}{\sqrt{ax^2+bx^3}}\right)}{\sqrt{a}}$$

```
Int[1/Sqrt[a*x^2 + b*x^3],x]
```

```
(-2*ArcTanh[(Sqrt[a]*x)/Sqrt[a*x^2 + b*x^3]])/Sqrt[a]
```

Defintions of rubi rules used

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

```
Int[1/Sqrt[(a_.)*(x_)^2 + (b_.)*(x_)^(n_.)], x_Symbol] :> Simp[2/(2 - n)
Subst[Int[1/(1 - a*x^2), x], x, x/Sqrt[a*x^2 + b*x^n]], x] /; FreeQ[{a, b,
n}, x] && NeQ[n, 2]
```

Maple [A] (verified)

Time = 0.37 (sec) , antiderivative size = 13, normalized size of antiderivative = 0.41

method	result	size
pseudoelliptic	$\frac{2\sqrt{bx+a}}{b}$	13
default	$-\frac{2x\sqrt{bx+a}\,\operatorname{arctanh}\left(\frac{\sqrt{bx+a}}{\sqrt{a}}\right)}{\sqrt{b\,x^{3}+a\,x^{2}}\,\sqrt{a}}$	39

```
int(1/(b*x^3+a*x^2)^(1/2),x,method=_RETURNVERBOSE)
```

```
2*(b*x+a)^(1/2)/b
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 79, normalized size of antiderivative = 2.47

$$\int \frac{1}{\sqrt{ax^2+bx^3}}\,dx = \left[\frac{\log\left(\frac{bx^2+2\,ax-2\,\sqrt{bx^3+ax^2}\sqrt{a}}{x^2}\right)}{\sqrt{a}}, \frac{2\,\sqrt{-a}\arctan\left(\frac{\sqrt{bx^3+ax^2}\sqrt{-a}}{bx^2+ax}\right)}{a}\right]$$

```
integrate(1/(b*x^3+a*x^2)^(1/2),x, algorithm="fricas")
```

```
[log((b*x^2 + 2*a*x - 2*sqrt(b*x^3 + a*x^2)*sqrt(a))/x^2)/sqrt(a), 2*sqrt(
-a)*arctan(sqrt(b*x^3 + a*x^2)*sqrt(-a)/(b*x^2 + a*x))/a]
```

Sympy [F]

$$\int \frac{1}{\sqrt{ax^2+bx^3}}\,dx = \int \frac{1}{\sqrt{ax^2+bx^3}}\,dx$$

```
integrate(1/(b*x**3+a*x**2)**(1/2),x)
```

```
Integral(1/sqrt(a*x**2 + b*x**3), x)
```

Maxima [F]

$$\int \frac{1}{\sqrt{ax^2+bx^3}}\,dx = \int \frac{1}{\sqrt{bx^3+ax^2}}\,dx$$

```
integrate(1/(b*x^3+a*x^2)^(1/2),x, algorithm="maxima")
```

```
integrate(1/sqrt(b*x^3 + a*x^2), x)
```

Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 45, normalized size of antiderivative = 1.41

$$\int \frac{1}{\sqrt{ax^2+bx^3}}\,dx = -\frac{2\arctan\left(\frac{\sqrt{a}}{\sqrt{-a}}\right)\mathrm{sgn}(x)}{\sqrt{-a}} + \frac{2\arctan\left(\frac{\sqrt{bx+a}}{\sqrt{-a}}\right)}{\sqrt{-a}\mathrm{sgn}\left(x\right)}$$

```
integrate(1/(b*x^3+a*x^2)^(1/2),x, algorithm="giac")
```

```
-2*arctan(sqrt(a)/sqrt(-a))*sgn(x)/sqrt(-a) + 2*arctan(sqrt(b*x + a)/sqrt(
-a))/(sqrt(-a)*sgn(x))
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{\sqrt{ax^2+bx^3}}\,dx = \int \frac{1}{\sqrt{b\,x^3+a\,x^2}}\,dx$$

```
int(1/(a*x^2 + b*x^3)^(1/2),x)
```

```
int(1/(a*x^2 + b*x^3)^(1/2), x)
```

Reduce [B] (verification not implemented)

Time = 0.44 (sec) , antiderivative size = 31, normalized size of antiderivative = 0.97

$$\int \frac{1}{\sqrt{ax^2+bx^3}}\,dx=\frac{\sqrt{a}\,\left(\log\left(\sqrt{bx+a}-\sqrt{a}\right)-\log\left(\sqrt{bx+a}+\sqrt{a}\right)\right)}{a}$$

```
int(1/(b*x^3+a*x^2)^(1/2),x)
```

```
(sqrt(a)*(log(sqrt(a + b*x) - sqrt(a)) - log(sqrt(a + b*x) + sqrt(a))))/a
```

3.305 $\int \frac{1}{x\sqrt{ax^2+bx^3}}\,dx$

Optimal result

Integrand size = 19, antiderivative size = 56

$$\int \frac{1}{x\sqrt{ax^2+bx^3}}\,dx = -\frac{\sqrt{ax^2+bx^3}}{ax^2}+\frac{b\operatorname{arctanh}\left(\frac{\sqrt{ax^2+bx^3}}{\sqrt{a}x}\right)}{a^{3/2}}$$

```
-(b*x^3+a*x^2)^(1/2)/a/x^2+b*arctanh((b*x^3+a*x^2)^(1/2)/a^(1/2)/x)/a^(3/2
)
```

Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 60, normalized size of antiderivative = 1.07

$$\int \frac{1}{x\sqrt{ax^2+bx^3}}\,dx = \frac{-\sqrt{a}(a+bx)+bx\sqrt{a+bx}\operatorname{arctanh}\left(\frac{\sqrt{a+bx}}{\sqrt{a}}\right)}{a^{3/2}\sqrt{x^2(a+bx)}}$$

```
Integrate[1/(x*Sqrt[a*x^2 + b*x^3]),x]
```

```
(-(Sqrt[a]*(a + b*x)) + b*x*Sqrt[a + b*x]*ArcTanh[Sqrt[a + b*x]/Sqrt[a]])/
(a^(3/2)*Sqrt[x^2*(a + b*x)])
```

Rubi [A] (verified)

Time = 0.32 (sec) , antiderivative size = 54, normalized size of antiderivative = 0.96, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.158, Rules used = {1931, 1914, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{x\sqrt{ax^2+bx^3}}dx$$
$$\Big\downarrow \text{1931}$$
$$-\frac{b\int \frac{1}{\sqrt{bx^3+ax^2}}dx}{2a}-\frac{\sqrt{ax^2+bx^3}}{ax^2}$$
$$\Big\downarrow \text{1914}$$
$$\frac{b\int \frac{1}{1-\frac{ax^2}{bx^3+ax^2}}d\frac{x}{\sqrt{bx^3+ax^2}}}{a}-\frac{\sqrt{ax^2+bx^3}}{ax^2}$$
$$\Big\downarrow \text{219}$$
$$\frac{b\text{arctanh}\left(\frac{\sqrt{a}x}{\sqrt{ax^2+bx^3}}\right)}{a^{3/2}}-\frac{\sqrt{ax^2+bx^3}}{ax^2}$$

```
Int[1/(x*Sqrt[a*x^2 + b*x^3]),x]
```

```
-(Sqrt[a*x^2 + b*x^3]/(a*x^2)) + (b*ArcTanh[(Sqrt[a]*x)/Sqrt[a*x^2 + b*x^3
]])/a^(3/2)
```

Defintions of rubi rules used

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

```
Int[1/Sqrt[(a_.)*(x_)^2 + (b_.)*(x_)^(n_.)], x_Symbol] :> Simp[2/(2 - n)
Subst[Int[1/(1 - a*x^2), x], x, x/Sqrt[a*x^2 + b*x^n]], x] /; FreeQ[{a, b,
n}, x] && NeQ[n, 2]
```

```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> Simp[c^(j - 1)*(c*x)^(m - j + 1)*((a*x^j + b*x^n)^(p + 1)/(a*(m + j*p
+ 1))), x] - Simp[b*((m + n*p + n - j + 1)/(a*c^(n - j)*(m + j*p + 1)))   I
nt[(c*x)^(m + n - j)*(a*x^j + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, p}, x]
 &&  !IntegerQ[p] && LtQ[0, j, n] && (IntegersQ[j, n] || GtQ[c, 0]) && LtQ[
m + j*p + 1, 0]
```

Maple [A] (verified)

Time = 0.39 (sec) , antiderivative size = 18, normalized size of antiderivative = 0.32

method	result	size
pseudoelliptic	$-\frac{2 \operatorname{arctanh}\left(\frac{\sqrt{bx+a}}{\sqrt{a}}\right)}{\sqrt{a}}$	18
default	$-\frac{\sqrt{bx+a}\left(a^{\frac{3}{2}}\sqrt{bx+a}-\operatorname{arctanh}\left(\frac{\sqrt{bx+a}}{\sqrt{a}}\right)abx\right)}{\sqrt{b\,x^{3}+a\,x^{2}}\,a^{\frac{5}{2}}}$	55
risch	$-\frac{bx+a}{a\sqrt{x^{2}(bx+a)}}+\frac{b \operatorname{arctanh}\left(\frac{\sqrt{bx+a}}{\sqrt{a}}\right)\sqrt{bx+a}\,x}{a^{\frac{3}{2}}\sqrt{x^{2}(bx+a)}}$	59

```
int(1/x/(b*x^3+a*x^2)^(1/2),x,method=_RETURNVERBOSE)
```

```
-2/a^(1/2)*arctanh((b*x+a)^(1/2)/a^(1/2))
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 132, normalized size of antiderivative = 2.36

$$\int \frac{1}{x\sqrt{ax^2+bx^3}}\,dx = \left[\frac{\sqrt{a}bx^2\log\left(\frac{bx^2+2\,ax+2\,\sqrt{bx^3+ax^2}\sqrt{a}}{x^2}\right)-2\,\sqrt{bx^3+ax^2}a}{2\,a^2x^2},\; -\frac{\sqrt{-a}bx^2\arctan\left(\frac{\sqrt{bx^3+ax^2}\sqrt{-a}}{bx^2+ax}\right)+\sqrt{bx^3+ax^2}a}{a^2x^2}\right]$$

```
integrate(1/x/(b*x^3+a*x^2)^(1/2),x, algorithm="fricas")
```

```
[1/2*(sqrt(a)*b*x^2*log((b*x^2 + 2*a*x + 2*sqrt(b*x^3 + a*x^2)*sqrt(a))/x^
2) - 2*sqrt(b*x^3 + a*x^2)*a)/(a^2*x^2), -(sqrt(-a)*b*x^2*arctan(sqrt(b*x^
3 + a*x^2)*sqrt(-a)/(b*x^2 + a*x)) + sqrt(b*x^3 + a*x^2)*a)/(a^2*x^2)]
```

Sympy [F]

$$\int \frac{1}{x\sqrt{ax^2+bx^3}}\,dx = \int \frac{1}{x\sqrt{x^2\,(a+bx)}}\,dx$$

```
integrate(1/x/(b*x**3+a*x**2)**(1/2),x)
```

```
Integral(1/(x*sqrt(x**2*(a + b*x))), x)
```

Maxima [F]

$$\int \frac{1}{x\sqrt{ax^2+bx^3}}\,dx = \int \frac{1}{\sqrt{bx^3+ax^2}x}\,dx$$

```
integrate(1/x/(b*x^3+a*x^2)^(1/2),x, algorithm="maxima")
```

```
integrate(1/(sqrt(b*x^3 + a*x^2)*x), x)
```

Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 48, normalized size of antiderivative = 0.86

$$\int \frac{1}{x\sqrt{ax^2+bx^3}}\,dx = -\frac{b\left(\frac{\arctan\left(\frac{\sqrt{bx+a}}{\sqrt{-a}}\right)}{\sqrt{-a}a}+\frac{\sqrt{bx+a}}{abx}\right)}{\mathrm{sgn}\left(x\right)}$$

```
integrate(1/x/(b*x^3+a*x^2)^(1/2),x, algorithm="giac")
```

```
-b*(arctan(sqrt(b*x + a)/sqrt(-a))/(sqrt(-a)*a) + sqrt(b*x + a)/(a*b*x))/s
gn(x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{x\sqrt{ax^2+bx^3}}\,dx = \int \frac{1}{x\,\sqrt{b\,x^3+a\,x^2}}\,dx$$

```
int(1/(x*(a*x^2 + b*x^3)^(1/2)),x)
```

```
int(1/(x*(a*x^2 + b*x^3)^(1/2)), x)
```

Reduce [B] (verification not implemented)

Time = 0.43 (sec) , antiderivative size = 51, normalized size of antiderivative = 0.91

$$\int \frac{1}{x\sqrt{ax^2+bx^3}}\,dx = \frac{-2\sqrt{bx+a}\,a-\sqrt{a}\log\left(\sqrt{bx+a}-\sqrt{a}\right)bx+\sqrt{a}\log\left(\sqrt{bx+a}+\sqrt{a}\right)bx}{2a^2x}$$

```
int(1/x/(b*x^3+a*x^2)^(1/2),x)
```

```
( - 2*sqrt(a + b*x)*a - sqrt(a)*log(sqrt(a + b*x) - sqrt(a))*b*x + sqrt(a)
*log(sqrt(a + b*x) + sqrt(a))*b*x)/(2*a**2*x)
```

3.306 $\int \frac{1}{x^2\sqrt{ax^2+bx^3}}\,dx$

Optimal result

Integrand size = 19, antiderivative size = 89

$$\int \frac{1}{x^2\sqrt{ax^2+bx^3}}\,dx = -\frac{\sqrt{ax^2+bx^3}}{2ax^3}+\frac{3b\sqrt{ax^2+bx^3}}{4a^2x^2}-\frac{3b^2\operatorname{arctanh}\left(\frac{\sqrt{ax^2+bx^3}}{\sqrt{a}x}\right)}{4a^{5/2}}$$

```
-1/2*(b*x^3+a*x^2)^(1/2)/a/x^3+3/4*b*(b*x^3+a*x^2)^(1/2)/a^2/x^2-3/4*b^2*a
rctanh((b*x^3+a*x^2)^(1/2)/a^(1/2)/x)/a^(5/2)
```

Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 83, normalized size of antiderivative = 0.93

$$\int \frac{1}{x^2\sqrt{ax^2+bx^3}}\,dx = \frac{\sqrt{a}(-2a^2+abx+3b^2x^2)-3b^2x^2\sqrt{a+bx}\operatorname{arctanh}\left(\frac{\sqrt{a+bx}}{\sqrt{a}}\right)}{4a^{5/2}x\sqrt{x^2(a+bx)}}$$

```
Integrate[1/(x^2*Sqrt[a*x^2 + b*x^3]),x]
```

```
(Sqrt[a]*(-2*a^2 + a*b*x + 3*b^2*x^2) - 3*b^2*x^2*Sqrt[a + b*x]*ArcTanh[Sq
rt[a + b*x]/Sqrt[a]])/(4*a^(5/2)*x*Sqrt[x^2*(a + b*x)])
```

Rubi [A] (verified)

Time = 0.42 (sec) , antiderivative size = 88, normalized size of antiderivative = 0.99, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.211, Rules used = {1931, 1931, 1914, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{x^2\sqrt{ax^2+bx^3}}\,dx$$
$$\Big\downarrow \text{1931}$$
$$-\frac{3b\int \frac{1}{x\sqrt{bx^3+ax^2}}dx}{4a}-\frac{\sqrt{ax^2+bx^3}}{2ax^3}$$
$$\Big\downarrow \text{1931}$$
$$-\frac{3b\left(-\frac{b\int \frac{1}{\sqrt{bx^3+ax^2}}dx}{2a}-\frac{\sqrt{ax^2+bx^3}}{ax^2}\right)}{4a}-\frac{\sqrt{ax^2+bx^3}}{2ax^3}$$
$$\Big\downarrow \text{1914}$$
$$-\frac{3b\left(\frac{b\int \frac{1}{1-\frac{ax^2}{bx^3+ax^2}}d\frac{x}{\sqrt{bx^3+ax^2}}}{a}-\frac{\sqrt{ax^2+bx^3}}{ax^2}\right)}{4a}-\frac{\sqrt{ax^2+bx^3}}{2ax^3}$$
$$\Big\downarrow \text{219}$$
$$-\frac{3b\left(\frac{b\,\text{arctanh}\left(\frac{\sqrt{a}x}{\sqrt{ax^2+bx^3}}\right)}{a^{3/2}}-\frac{\sqrt{ax^2+bx^3}}{ax^2}\right)}{4a}-\frac{\sqrt{ax^2+bx^3}}{2ax^3}$$

```
Int[1/(x^2*Sqrt[a*x^2 + b*x^3]),x]
```

```
-1/2*Sqrt[a*x^2 + b*x^3]/(a*x^3) - (3*b*(-(Sqrt[a*x^2 + b*x^3]/(a*x^2)) +
(b*ArcTanh[(Sqrt[a]*x)/Sqrt[a*x^2 + b*x^3]])/a^(3/2)))/(4*a)
```

Defintions of rubi rules used

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

```
Int[1/Sqrt[(a_.)*(x_)^2 + (b_.)*(x_)^(n_.)], x_Symbol] :> Simp[2/(2 - n)
Subst[Int[1/(1 - a*x^2), x], x, x/Sqrt[a*x^2 + b*x^n]], x] /; FreeQ[{a, b,
n}, x] && NeQ[n, 2]
```

```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> Simp[c^(j - 1)*(c*x)^(m - j + 1)*((a*x^j + b*x^n)^(p + 1)/(a*(m + j*p
+ 1))), x] - Simp[b*((m + n*p + n - j + 1)/(a*c^(n - j)*(m + j*p + 1)))   I
nt[(c*x)^(m + n - j)*(a*x^j + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, p}, x]
 &&  !IntegerQ[p] && LtQ[0, j, n] && (IntegersQ[j, n] || GtQ[c, 0]) && LtQ[
m + j*p + 1, 0]
```

Maple [A] (verified)

Time = 0.42 (sec) , antiderivative size = 36, normalized size of antiderivative = 0.40

method	result	size
pseudoelliptic	$\frac{\operatorname{arctanh}\left(\frac{\sqrt{bx+a}}{\sqrt{a}}\right) bx-\sqrt{bx+a}\,\sqrt{a}}{a^{\frac{3}{2}} x}$	36
risch	$-\frac{(bx+a)(-3bx+2a)}{4a^{2} x\sqrt{x^{2}(bx+a)}}-\frac{3b^{2} \operatorname{arctanh}\left(\frac{\sqrt{bx+a}}{\sqrt{a}}\right) \sqrt{bx+a}\, x}{4a^{\frac{5}{2}} \sqrt{x^{2}(bx+a)}}$	73
default	$-\frac{\sqrt{bx+a}\,\left(-3a^{\frac{3}{2}} bx\sqrt{bx+a}+3 \operatorname{arctanh}\left(\frac{\sqrt{bx+a}}{\sqrt{a}}\right) a\, b^{2} x^{2}+2\sqrt{bx+a}\, a^{\frac{5}{2}}\right)}{4x\sqrt{b\, x^{3}+a\, x^{2}}\, a^{\frac{7}{2}}}$	77

```
int(1/x^2/(b*x^3+a*x^2)^(1/2),x,method=_RETURNVERBOSE)
```

```
(arctanh((b*x+a)^(1/2)/a^(1/2))*b*x-(b*x+a)^(1/2)*a^(1/2))/a^(3/2)/x
```

Fricas [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 158, normalized size of antiderivative = 1.78

$$\int \frac{1}{x^2\sqrt{ax^2+bx^3}}\,dx$$
$$= \left[\frac{3\,\sqrt{a}b^2x^3\log\left(\frac{bx^2+2\,ax-2\,\sqrt{bx^3+ax^2}\sqrt{a}}{x^2}\right)+2\,\sqrt{bx^3+ax^2}(3\,abx-2\,a^2)}{8\,a^3x^3}, \frac{3\,\sqrt{-a}b^2x^3\arctan\left(\frac{\sqrt{bx^3+ax^2}\sqrt{-a}}{bx^2+ax}\right) -}{4\,a^3x^3}\right.$$

```
integrate(1/x^2/(b*x^3+a*x^2)^(1/2),x, algorithm="fricas")
```

```
[1/8*(3*sqrt(a)*b^2*x^3*log((b*x^2 + 2*a*x - 2*sqrt(b*x^3 + a*x^2)*sqrt(a)
)/x^2) + 2*sqrt(b*x^3 + a*x^2)*(3*a*b*x - 2*a^2))/(a^3*x^3), 1/4*(3*sqrt(-
a)*b^2*x^3*arctan(sqrt(b*x^3 + a*x^2)*sqrt(-a)/(b*x^2 + a*x)) + sqrt(b*x^3
 + a*x^2)*(3*a*b*x - 2*a^2))/(a^3*x^3)]
```

Sympy [F]

$$\int \frac{1}{x^2\sqrt{ax^2+bx^3}}\,dx = \int \frac{1}{x^2\sqrt{x^2\,(a+bx)}}\,dx$$

```
integrate(1/x**2/(b*x**3+a*x**2)**(1/2),x)
```

```
Integral(1/(x**2*sqrt(x**2*(a + b*x))), x)
```

Maxima [F]

$$\int \frac{1}{x^2\sqrt{ax^2+bx^3}}\,dx = \int \frac{1}{\sqrt{bx^3+ax^2}x^2}\,dx$$

```
integrate(1/x^2/(b*x^3+a*x^2)^(1/2),x, algorithm="maxima")
```

```
integrate(1/(sqrt(b*x^3 + a*x^2)*x^2), x)
```

Giac [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 73, normalized size of antiderivative = 0.82

$$\int \frac{1}{x^2\sqrt{ax^2+bx^3}}\,dx = \frac{\frac{3\,b^3\arctan\left(\frac{\sqrt{bx+a}}{\sqrt{-a}}\right)}{\sqrt{-a}a^2} + \frac{3\,(bx+a)^{\frac{3}{2}}b^3 - 5\,\sqrt{bx+a}ab^3}{a^2b^2x^2}}{4\,b\mathrm{sgn}\left(x\right)}$$

```
integrate(1/x^2/(b*x^3+a*x^2)^(1/2),x, algorithm="giac")
```

```
1/4*(3*b^3*arctan(sqrt(b*x + a)/sqrt(-a))/(sqrt(-a)*a^2) + (3*(b*x + a)^(3
/2)*b^3 - 5*sqrt(b*x + a)*a*b^3)/(a^2*b^2*x^2))/(b*sgn(x))
```

Mupad [B] (verification not implemented)

Time = 9.06 (sec) , antiderivative size = 44, normalized size of antiderivative = 0.49

$$\int \frac{1}{x^2\sqrt{ax^2+bx^3}}\,dx = -\frac{2\,\sqrt{\frac{a}{b\,x}+1}\,{}_2\mathrm{F}_1\left(\frac{1}{2},\frac{5}{2};\ \frac{7}{2};\ -\frac{a}{b\,x}\right)}{5\,x\,\sqrt{b\,x^3+a\,x^2}}$$

```
int(1/(x^2*(a*x^2 + b*x^3)^(1/2)),x)
```

```
-(2*(a/(b*x) + 1)^(1/2)*hypergeom([1/2, 5/2], 7/2, -a/(b*x)))/(5*x*(a*x^2
+ b*x^3)^(1/2))
```

Reduce [B] (verification not implemented)

Time = 0.25 (sec) , antiderivative size = 73, normalized size of antiderivative = 0.82

$$\int \frac{1}{x^2\sqrt{ax^2+bx^3}}\,dx = \frac{-4\sqrt{bx+a}\,a^2+6\sqrt{bx+a}\,abx+3\sqrt{a}\log\left(\sqrt{bx+a}-\sqrt{a}\right)b^2x^2-3\sqrt{a}\log\left(\sqrt{bx+a}+\sqrt{a}\right)b^2x^2}{8a^3x^2}$$

```
int(1/x^2/(b*x^3+a*x^2)^(1/2),x)
```

```
( - 4*sqrt(a + b*x)*a**2 + 6*sqrt(a + b*x)*a*b*x + 3*sqrt(a)*log(sqrt(a +
b*x) - sqrt(a))*b**2*x**2 - 3*sqrt(a)*log(sqrt(a + b*x) + sqrt(a))*b**2*x*
*2)/(8*a**3*x**2)
```

3.307 $\int \frac{1}{x^3\sqrt{ax^2+bx^3}}\,dx$

Optimal result

Integrand size = 19, antiderivative size = 117

$$\int \frac{1}{x^3\sqrt{ax^2+bx^3}}\,dx = -\frac{\sqrt{ax^2+bx^3}}{3ax^4}+\frac{5b\sqrt{ax^2+bx^3}}{12a^2x^3}-\frac{5b^2\sqrt{ax^2+bx^3}}{8a^3x^2}+\frac{5b^3\operatorname{arctanh}\left(\frac{\sqrt{ax^2+bx^3}}{\sqrt{a}x}\right)}{8a^{7/2}}$$

```
-1/3*(b*x^3+a*x^2)^(1/2)/a/x^4+5/12*b*(b*x^3+a*x^2)^(1/2)/a^2/x^3-5/8*b^2*
(b*x^3+a*x^2)^(1/2)/a^3/x^2+5/8*b^3*arctanh((b*x^3+a*x^2)^(1/2)/a^(1/2)/x)
/a^(7/2)
```

Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 96, normalized size of antiderivative = 0.82

$$\int \frac{1}{x^3\sqrt{ax^2+bx^3}}\,dx = \frac{-\sqrt{a}(8a^3-2a^2bx+5ab^2x^2+15b^3x^3)+15b^3x^3\sqrt{a+bx}\operatorname{arctanh}\left(\frac{\sqrt{a+bx}}{\sqrt{a}}\right)}{24a^{7/2}x^2\sqrt{x^2(a+bx)}}$$

```
Integrate[1/(x^3*Sqrt[a*x^2 + b*x^3]),x]
```

```
(-(Sqrt[a]*(8*a^3 - 2*a^2*b*x + 5*a*b^2*x^2 + 15*b^3*x^3)) + 15*b^3*x^3*Sq
rt[a + b*x]*ArcTanh[Sqrt[a + b*x]/Sqrt[a]])/(24*a^(7/2)*x^2*Sqrt[x^2*(a +
b*x)])
```

Rubi [A] (verified)

Time = 0.48 (sec) , antiderivative size = 122, normalized size of antiderivative = 1.04, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.263, Rules used = {1931, 1931, 1931, 1914, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{x^3\sqrt{ax^2+bx^3}}dx$$

$$\Big\downarrow \text{1931}$$

$$-\frac{5b\int \frac{1}{x^2\sqrt{bx^3+ax^2}}dx}{6a}-\frac{\sqrt{ax^2+bx^3}}{3ax^4}$$

$$\Big\downarrow \text{1931}$$

$$-\frac{5b\left(-\frac{3b\int \frac{1}{x\sqrt{bx^3+ax^2}}dx}{4a}-\frac{\sqrt{ax^2+bx^3}}{2ax^3}\right)}{6a}-\frac{\sqrt{ax^2+bx^3}}{3ax^4}$$

$$\Big\downarrow \text{1931}$$

$$-\frac{5b\left(-\frac{3b\left(-\frac{b\int \frac{1}{\sqrt{bx^3+ax^2}}dx}{2a}-\frac{\sqrt{ax^2+bx^3}}{ax^2}\right)}{4a}-\frac{\sqrt{ax^2+bx^3}}{2ax^3}\right)}{6a}-\frac{\sqrt{ax^2+bx^3}}{3ax^4}$$

$$\Big\downarrow \text{1914}$$

$$-\frac{5b\left(-\frac{3b\left(\frac{b\int \frac{1}{1-\frac{ax^2}{bx^3+ax^2}}d\frac{x}{\sqrt{bx^3+ax^2}}}{a}-\frac{\sqrt{ax^2+bx^3}}{ax^2}\right)}{4a}-\frac{\sqrt{ax^2+bx^3}}{2ax^3}\right)}{6a}-\frac{\sqrt{ax^2+bx^3}}{3ax^4}$$

$$\Big\downarrow \text{219}$$

$$-\frac{5b\left(-\frac{3b\left(\frac{b\,\text{arctanh}\left(\frac{\sqrt{a}x}{\sqrt{ax^2+bx^3}}\right)}{a^{3/2}}-\frac{\sqrt{ax^2+bx^3}}{ax^2}\right)}{4a}-\frac{\sqrt{ax^2+bx^3}}{2ax^3}\right)}{6a}-\frac{\sqrt{ax^2+bx^3}}{3ax^4}$$

```
Int[1/(x^3*Sqrt[a*x^2 + b*x^3]),x]
```

```
-1/3*Sqrt[a*x^2 + b*x^3]/(a*x^4) - (5*b*(-1/2*Sqrt[a*x^2 + b*x^3]/(a*x^3)
- (3*b*(-(Sqrt[a*x^2 + b*x^3]/(a*x^2)) + (b*ArcTanh[(Sqrt[a]*x)/Sqrt[a*x^2
 + b*x^3]])/a^(3/2)))/(4*a)))/(6*a)
```

Defintions of rubi rules used

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

```
Int[1/Sqrt[(a_.)*(x_)^2 + (b_.)*(x_)^(n_.)], x_Symbol] :> Simp[2/(2 - n)
Subst[Int[1/(1 - a*x^2), x], x, x/Sqrt[a*x^2 + b*x^n]], x] /; FreeQ[{a, b,
n}, x] && NeQ[n, 2]
```

```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> Simp[c^(j - 1)*(c*x)^(m - j + 1)*((a*x^j + b*x^n)^(p + 1)/(a*(m + j*p
+ 1))), x] - Simp[b*((m + n*p + n - j + 1)/(a*c^(n - j)*(m + j*p + 1)))   I
nt[(c*x)^(m + n - j)*(a*x^j + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, p}, x]
 &&  !IntegerQ[p] && LtQ[0, j, n] && (IntegersQ[j, n] || GtQ[c, 0]) && LtQ[
m + j*p + 1, 0]
```

Maple [A] (verified)

Time = 0.42 (sec) , antiderivative size = 56, normalized size of antiderivative = 0.48

method	result	size
pseudoelliptic	$\frac{-3 \operatorname{arctanh}\left(\frac{\sqrt{bx+a}}{\sqrt{a}}\right) b^{2} x^{2}+3 b x \sqrt{bx+a}\, \sqrt{a}-2 a^{\frac{3}{2}} \sqrt{bx+a}}{4 a^{\frac{5}{2}} x^{2}}$	56
risch	$-\frac{(bx+a)\left(15 b^{2} x^{2}-10 a b x+8 a^{2}\right)}{24 a^{3} x^{2} \sqrt{x^{2}(bx+a)}}+\frac{5 b^{3} \operatorname{arctanh}\left(\frac{\sqrt{bx+a}}{\sqrt{a}}\right) \sqrt{bx+a}\, x}{8 a^{\frac{7}{2}} \sqrt{x^{2}(bx+a)}}$	84
default	$-\frac{\sqrt{bx+a}\,\left(15 a^{\frac{3}{2}} b^{2} x^{2} \sqrt{bx+a}-15 \operatorname{arctanh}\left(\frac{\sqrt{bx+a}}{\sqrt{a}}\right) a\, b^{3} x^{3}-10 a^{\frac{5}{2}} b x \sqrt{bx+a}+8 a^{\frac{7}{2}} \sqrt{bx+a}\right)}{24 x^{2} \sqrt{b\, x^{3}+a\, x^{2}}\, a^{\frac{9}{2}}}$	95

```
int(1/x^3/(b*x^3+a*x^2)^(1/2),x,method=_RETURNVERBOSE)
```

```
1/4*(-3*arctanh((b*x+a)^(1/2)/a^(1/2))*b^2*x^2+3*b*x*(b*x+a)^(1/2)*a^(1/2)
-2*a^(3/2)*(b*x+a)^(1/2))/a^(5/2)/x^2
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 180, normalized size of antiderivative = 1.54

$$\int \frac{1}{x^3\sqrt{ax^2+bx^3}}\,dx = \left[\frac{15\,\sqrt{a}b^3x^4\log\left(\frac{bx^2+2\,ax+2\,\sqrt{bx^3+ax^2}\sqrt{a}}{x^2}\right)-2\,(15\,ab^2x^2-10\,a^2bx+8\,a^3)\sqrt{bx^3+ax^2}}{48\,a^4x^4},\right.$$
$$\left.-\frac{15\,\sqrt{-a}b^3x^4\arctan\left(\frac{\sqrt{bx^3+ax^2}\sqrt{-a}}{bx^2+ax}\right)+(15\,ab^2x^2-10\,a^2bx+8\,a^3)\sqrt{bx^3+ax^2}}{24\,a^4x^4}\right]$$

```
integrate(1/x^3/(b*x^3+a*x^2)^(1/2),x, algorithm="fricas")
```

```
[1/48*(15*sqrt(a)*b^3*x^4*log((b*x^2 + 2*a*x + 2*sqrt(b*x^3 + a*x^2)*sqrt(
a))/x^2) - 2*(15*a*b^2*x^2 - 10*a^2*b*x + 8*a^3)*sqrt(b*x^3 + a*x^2))/(a^4
*x^4), -1/24*(15*sqrt(-a)*b^3*x^4*arctan(sqrt(b*x^3 + a*x^2)*sqrt(-a)/(b*x
^2 + a*x)) + (15*a*b^2*x^2 - 10*a^2*b*x + 8*a^3)*sqrt(b*x^3 + a*x^2))/(a^4
*x^4)]
```

Sympy [F]

$$\int \frac{1}{x^3\sqrt{ax^2+bx^3}}\,dx = \int \frac{1}{x^3\sqrt{x^2\left(a+bx\right)}}\,dx$$

```
integrate(1/x**3/(b*x**3+a*x**2)**(1/2),x)
```

```
Integral(1/(x**3*sqrt(x**2*(a + b*x))), x)
```

Maxima [F]

$$\int \frac{1}{x^3\sqrt{ax^2+bx^3}}\,dx = \int \frac{1}{\sqrt{bx^3+ax^2}x^3}\,dx$$

```
integrate(1/x^3/(b*x^3+a*x^2)^(1/2),x, algorithm="maxima")
```

```
integrate(1/(sqrt(b*x^3 + a*x^2)*x^3), x)
```

Giac [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 76, normalized size of antiderivative = 0.65

$$\int \frac{1}{x^3\sqrt{ax^2+bx^3}}\,dx = -\frac{b^3\left(\frac{15\arctan\left(\frac{\sqrt{bx+a}}{\sqrt{-a}}\right)}{\sqrt{-a}a^3}+\frac{15\,(bx+a)^{\frac{5}{2}}-40\,(bx+a)^{\frac{3}{2}}a+33\,\sqrt{bx+a}a^2}{a^3b^3x^3}\right)}{24\,\mathrm{sgn}\,(x)}$$

```
integrate(1/x^3/(b*x^3+a*x^2)^(1/2),x, algorithm="giac")
```

```
-1/24*b^3*(15*arctan(sqrt(b*x + a)/sqrt(-a))/(sqrt(-a)*a^3) + (15*(b*x + a
)^(5/2) - 40*(b*x + a)^(3/2)*a + 33*sqrt(b*x + a)*a^2)/(a^3*b^3*x^3))/sgn(
x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{x^3\sqrt{ax^2+bx^3}}\,dx = \int \frac{1}{x^3\,\sqrt{b\,x^3+a\,x^2}}\,dx$$

```
int(1/(x^3*(a*x^2 + b*x^3)^(1/2)),x)
```

```
int(1/(x^3*(a*x^2 + b*x^3)^(1/2)), x)
```

Reduce [B] (verification not implemented)

Time = 0.47 (sec) , antiderivative size = 90, normalized size of antiderivative = 0.77

$$\int \frac{1}{x^3\sqrt{ax^2+bx^3}}\,dx$$
$$= \frac{-16\sqrt{bx+a}\,a^3+20\sqrt{bx+a}\,a^2bx-30\sqrt{bx+a}\,a\,b^2x^2-15\sqrt{a}\log\left(\sqrt{bx+a}-\sqrt{a}\right)b^3x^3+15\sqrt{a}\log\left(\sqrt{b\right.}{48a^4x^3}$$

```
int(1/x^3/(b*x^3+a*x^2)^(1/2),x)
```

```
( - 16*sqrt(a + b*x)*a**3 + 20*sqrt(a + b*x)*a**2*b*x - 30*sqrt(a + b*x)*a
*b**2*x**2 - 15*sqrt(a)*log(sqrt(a + b*x) - sqrt(a))*b**3*x**3 + 15*sqrt(a
)*log(sqrt(a + b*x) + sqrt(a))*b**3*x**3)/(48*a**4*x**3)
```

3.308 $\int \frac{x^6}{\left(ax^2+bx^3\right)^{3/2}}\,dx$

Optimal result

Integrand size = 19, antiderivative size = 100

$$\int \frac{x^6}{\left(ax^2+bx^3\right)^{3/2}}\,dx=\frac{2a^3x}{b^4\sqrt{ax^2+bx^3}}+\frac{6a^2\sqrt{ax^2+bx^3}}{b^4x}-\frac{2a\left(ax^2+bx^3\right)^{3/2}}{b^4x^3}+\frac{2\left(ax^2+bx^3\right)^{5/2}}{5b^4x^5}$$

```
2*a^3*x/b^4/(b*x^3+a*x^2)^(1/2)+6*a^2*(b*x^3+a*x^2)^(1/2)/b^4/x-2*a*(b*x^3
+a*x^2)^(3/2)/b^4/x^3+2/5*(b*x^3+a*x^2)^(5/2)/b^4/x^5
```

Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 50, normalized size of antiderivative = 0.50

$$\int \frac{x^6}{\left(ax^2+bx^3\right)^{3/2}}\,dx=\frac{2x(16a^3+8a^2bx-2ab^2x^2+b^3x^3)}{5b^4\sqrt{x^2(a+bx)}}$$

```
Integrate[x^6/(a*x^2 + b*x^3)^(3/2),x]
```

```
(2*x*(16*a^3 + 8*a^2*b*x - 2*a*b^2*x^2 + b^3*x^3))/(5*b^4*Sqrt[x^2*(a + b*
x)])
```

Rubi [A] (verified)

Time = 0.45 (sec) , antiderivative size = 110, normalized size of antiderivative = 1.10, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.211, Rules used = {1921, 1922, 1922, 1920}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^6}{\left(ax^2+bx^3\right)^{3/2}}\,dx$$

$$\Big\downarrow \text{1921}$$

$$\frac{6\int \frac{x^3}{\sqrt{bx^3+ax^2}}dx}{b}-\frac{2x^4}{b\sqrt{ax^2+bx^3}}$$

$$\Big\downarrow \text{1922}$$

$$\frac{6\left(\frac{2x\sqrt{ax^2+bx^3}}{5b}-\frac{4a\int \frac{x^2}{\sqrt{bx^3+ax^2}}dx}{5b}\right)}{b}-\frac{2x^4}{b\sqrt{ax^2+bx^3}}$$

$$\Big\downarrow \text{1922}$$

$$\frac{6\left(\frac{2x\sqrt{ax^2+bx^3}}{5b}-\frac{4a\left(\frac{2\sqrt{ax^2+bx^3}}{3b}-\frac{2a\int \frac{x}{\sqrt{bx^3+ax^2}}dx}{3b}\right)}{5b}\right)}{b}-\frac{2x^4}{b\sqrt{ax^2+bx^3}}$$

$$\Big\downarrow \text{1920}$$

$$\frac{6\left(\frac{2x\sqrt{ax^2+bx^3}}{5b}-\frac{4a\left(\frac{2\sqrt{ax^2+bx^3}}{3b}-\frac{4a\sqrt{ax^2+bx^3}}{3b^2x}\right)}{5b}\right)}{b}-\frac{2x^4}{b\sqrt{ax^2+bx^3}}$$

```
Int[x^6/(a*x^2 + b*x^3)^(3/2),x]
```

```
(-2*x^4)/(b*Sqrt[a*x^2 + b*x^3]) + (6*((2*x*Sqrt[a*x^2 + b*x^3])/(5*b) - (
4*a*((2*Sqrt[a*x^2 + b*x^3])/(3*b) - (4*a*Sqrt[a*x^2 + b*x^3])/(3*b^2*x)))
/(5*b)))/b
```

Defintions of rubi rules used

```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> Simp[(-c^(j - 1))*(c*x)^(m - j + 1)*((a*x^j + b*x^n)^(p + 1)/(a*(n - j
)*(p + 1))), x] /; FreeQ[{a, b, c, j, m, n, p}, x] && !IntegerQ[p] && NeQ[
n, j] && EqQ[m + n*p + n - j + 1, 0] && (IntegerQ[j] || GtQ[c, 0])
```

```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> Simp[(-c^(j - 1))*(c*x)^(m - j + 1)*((a*x^j + b*x^n)^(p + 1)/(a*(n - j
)*(p + 1))), x] + Simp[c^j*((m + n*p + n - j + 1)/(a*(n - j)*(p + 1)))   In
t[(c*x)^(m - j)*(a*x^j + b*x^n)^(p + 1), x], x] /; FreeQ[{a, b, c, j, m, n}
, x] && !IntegerQ[p] && NeQ[n, j] && ILtQ[Simplify[(m + n*p + n - j + 1)/(
n - j)], 0] && LtQ[p, -1] && (IntegerQ[j] || GtQ[c, 0])
```

```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> Simp[c^(j - 1)*(c*x)^(m - j + 1)*((a*x^j + b*x^n)^(p + 1)/(a*(m + j*p
+ 1))), x] - Simp[b*((m + n*p + n - j + 1)/(a*c^(n - j)*(m + j*p + 1)))   I
nt[(c*x)^(m + n - j)*(a*x^j + b*x^n)^p, x], x] /; FreeQ[{a, b, c, j, m, n,
p}, x] && !IntegerQ[p] && NeQ[n, j] && ILtQ[Simplify[(m + n*p + n - j + 1)
/(n - j)], 0] && NeQ[m + j*p + 1, 0] && (IntegersQ[j, n] || GtQ[c, 0])
```

Maple [A] (verified)

Time = 0.47 (sec) , antiderivative size = 56, normalized size of antiderivative = 0.56

method	result	size
gosper	$\frac{2(bx+a)(b^3x^3-2a\,b^2x^2+8a^2bx+16a^3)x^3}{5b^4(b\,x^3+a\,x^2)^{\frac{3}{2}}}$	56
default	$\frac{2(bx+a)(b^3x^3-2a\,b^2x^2+8a^2bx+16a^3)x^3}{5b^4(b\,x^3+a\,x^2)^{\frac{3}{2}}}$	56
orering	$\frac{2(bx+a)(b^3x^3-2a\,b^2x^2+8a^2bx+16a^3)x^3}{5b^4(b\,x^3+a\,x^2)^{\frac{3}{2}}}$	56
trager	$\frac{2(b^3x^3-2a\,b^2x^2+8a^2bx+16a^3)\sqrt{b\,x^3+a\,x^2}}{5(bx+a)b^4x}$	58
risch	$\frac{2(b^2x^2-3abx+11a^2)(bx+a)x}{5b^4\sqrt{x^2(bx+a)}}+\frac{2a^3x}{b^4\sqrt{x^2(bx+a)}}$	62
pseudoelliptic	$\frac{\frac{2}{11}b^6x^6-\frac{8}{33}a\,b^5x^5+\frac{80}{231}a^2b^4x^4-\frac{128}{231}a^3b^3x^3+\frac{256}{231}a^4b^2x^2-\frac{1024}{231}a^5bx-\frac{2048}{231}a^6}{b^7\sqrt{bx+a}}$	76

```
int(x^6/(b*x^3+a*x^2)^(3/2),x,method=_RETURNVERBOSE)
```

```
2/5*(b*x+a)*(b^3*x^3-2*a*b^2*x^2+8*a^2*b*x+16*a^3)*x^3/b^4/(b*x^3+a*x^2)^(
3/2)
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 60, normalized size of antiderivative = 0.60

$$\int \frac{x^6}{(ax^2+bx^3)^{3/2}}\,dx = \frac{2\,(b^3x^3-2\,ab^2x^2+8\,a^2bx+16\,a^3)\sqrt{bx^3+ax^2}}{5\,(b^5x^2+ab^4x)}$$

```
integrate(x^6/(b*x^3+a*x^2)^(3/2),x, algorithm="fricas")
```

```
2/5*(b^3*x^3 - 2*a*b^2*x^2 + 8*a^2*b*x + 16*a^3)*sqrt(b*x^3 + a*x^2)/(b^5*
x^2 + a*b^4*x)
```

Sympy [F]

$$\int \frac{x^6}{(ax^2+bx^3)^{3/2}}\,dx = \int \frac{x^6}{\left(x^2\,(a+bx)\right)^{\frac{3}{2}}}\,dx$$

```
integrate(x**6/(b*x**3+a*x**2)**(3/2),x)
```

```
Integral(x**6/(x**2*(a + b*x))**(3/2), x)
```

Maxima [A] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 41, normalized size of antiderivative = 0.41

$$\int \frac{x^6}{(ax^2+bx^3)^{3/2}}\,dx = \frac{2\,(b^3x^3 - 2\,ab^2x^2 + 8\,a^2bx + 16\,a^3)}{5\,\sqrt{bx+a}b^4}$$

```
integrate(x^6/(b*x^3+a*x^2)^(3/2),x, algorithm="maxima")
```

```
2/5*(b^3*x^3 - 2*a*b^2*x^2 + 8*a^2*b*x + 16*a^3)/(sqrt(b*x + a)*b^4)
```

Giac [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 79, normalized size of antiderivative = 0.79

$$\int \frac{x^6}{(ax^2+bx^3)^{3/2}}\,dx = -\frac{32\,a^{\frac{5}{2}}\mathrm{sgn}(x)}{5\,b^4} + \frac{2\,a^3}{\sqrt{bx+a}b^4\mathrm{sgn}\,(x)} + \frac{2\left((bx+a)^{\frac{5}{2}}b^{16} - 5\,(bx+a)^{\frac{3}{2}}ab^{16} + 15\,\sqrt{bx+a}a^2b^{16}\right)}{5\,b^{20}\mathrm{sgn}\,(x)}$$

```
integrate(x^6/(b*x^3+a*x^2)^(3/2),x, algorithm="giac")
```

```
-32/5*a^(5/2)*sgn(x)/b^4 + 2*a^3/(sqrt(b*x + a)*b^4*sgn(x)) + 2/5*((b*x +
a)^(5/2)*b^16 - 5*(b*x + a)^(3/2)*a*b^16 + 15*sqrt(b*x + a)*a^2*b^16)/(b^2
0*sgn(x))
```

Mupad [B] (verification not implemented)

Time = 8.91 (sec) , antiderivative size = 57, normalized size of antiderivative = 0.57

$$\int \frac{x^6}{(ax^2+bx^3)^{3/2}}\,dx=\frac{2\,\sqrt{b\,x^3+a\,x^2}\,\left(16\,a^3+8\,a^2\,b\,x-2\,a\,b^2\,x^2+b^3\,x^3\right)}{5\,b^4\,x\,\left(a+b\,x\right)}$$

```
int(x^6/(a*x^2 + b*x^3)^(3/2),x)
```

```
(2*(a*x^2 + b*x^3)^(1/2)*(16*a^3 + b^3*x^3 - 2*a*b^2*x^2 + 8*a^2*b*x))/(5*
b^4*x*(a + b*x))
```

Reduce [B] (verification not implemented)

Time = 0.52 (sec) , antiderivative size = 42, normalized size of antiderivative = 0.42

$$\int \frac{x^6}{(ax^2+bx^3)^{3/2}}\,dx=\frac{\frac{2}{5}b^3x^3-\frac{4}{5}a\,b^2x^2+\frac{16}{5}a^2bx+\frac{32}{5}a^3}{\sqrt{bx+a}\,b^4}$$

```
int(x^6/(b*x^3+a*x^2)^(3/2),x)
```

```
(2*(16*a**3 + 8*a**2*b*x - 2*a*b**2*x**2 + b**3*x**3))/(5*sqrt(a + b*x)*b*
*4)
```

3.309 $\int \frac{x^5}{\left(ax^2+bx^3\right)^{3/2}}\,dx$

Optimal result

Integrand size = 19, antiderivative size = 74

$$\int \frac{x^5}{\left(ax^2+bx^3\right)^{3/2}}\,dx = -\frac{2a^2x}{b^3\sqrt{ax^2+bx^3}} - \frac{4a\sqrt{ax^2+bx^3}}{b^3x} + \frac{2\left(ax^2+bx^3\right)^{3/2}}{3b^3x^3}$$

```
-2*a^2*x/b^3/(b*x^3+a*x^2)^(1/2)-4*a*(b*x^3+a*x^2)^(1/2)/b^3/x+2/3*(b*x^3+
a*x^2)^(3/2)/b^3/x^3
```

Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 39, normalized size of antiderivative = 0.53

$$\int \frac{x^5}{\left(ax^2+bx^3\right)^{3/2}}\,dx = \frac{2x(-8a^2-4abx+b^2x^2)}{3b^3\sqrt{x^2(a+bx)}}$$

```
Integrate[x^5/(a*x^2 + b*x^3)^(3/2),x]
```

```
(2*x*(-8*a^2 - 4*a*b*x + b^2*x^2))/(3*b^3*Sqrt[x^2*(a + b*x)])
```

Rubi [A] (verified)

Time = 0.38 (sec) , antiderivative size = 78, normalized size of antiderivative = 1.05, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.158$, Rules used = {1921, 1922, 1920}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^5}{(ax^2+bx^3)^{3/2}}\,dx$$

$$\Big\downarrow \text{1921}$$

$$\frac{4\int \frac{x^2}{\sqrt{bx^3+ax^2}}dx}{b}-\frac{2x^3}{b\sqrt{ax^2+bx^3}}$$

$$\Big\downarrow \text{1922}$$

$$\frac{4\left(\frac{2\sqrt{ax^2+bx^3}}{3b}-\frac{2a\int \frac{x}{\sqrt{bx^3+ax^2}}dx}{3b}\right)}{b}-\frac{2x^3}{b\sqrt{ax^2+bx^3}}$$

$$\Big\downarrow \text{1920}$$

$$\frac{4\left(\frac{2\sqrt{ax^2+bx^3}}{3b}-\frac{4a\sqrt{ax^2+bx^3}}{3b^2x}\right)}{b}-\frac{2x^3}{b\sqrt{ax^2+bx^3}}$$

```
Int[x^5/(a*x^2 + b*x^3)^(3/2),x]
```

```
(-2*x^3)/(b*Sqrt[a*x^2 + b*x^3]) + (4*((2*Sqrt[a*x^2 + b*x^3])/(3*b) - (4*
a*Sqrt[a*x^2 + b*x^3])/(3*b^2*x)))/b
```

Defintions of rubi rules used

```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> Simp[(-c^(j - 1))*(c*x)^(m - j + 1)*((a*x^j + b*x^n)^(p + 1)/(a*(n - j
)*(p + 1))), x] /; FreeQ[{a, b, c, j, m, n, p}, x] &&  !IntegerQ[p] && NeQ[
n, j] && EqQ[m + n*p + n - j + 1, 0] && (IntegerQ[j] || GtQ[c, 0])
```

```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> Simp[(-c^(j - 1))*(c*x)^(m - j + 1)*((a*x^j + b*x^n)^(p + 1)/(a*(n - j
)*(p + 1))), x] + Simp[c^j*((m + n*p + n - j + 1)/(a*(n - j)*(p + 1)))   In
t[(c*x)^(m - j)*(a*x^j + b*x^n)^(p + 1), x], x] /; FreeQ[{a, b, c, j, m, n}
, x] &&  !IntegerQ[p] && NeQ[n, j] && ILtQ[Simplify[(m + n*p + n - j + 1)/(
n - j)], 0] && LtQ[p, -1] && (IntegerQ[j] || GtQ[c, 0])
```

```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> Simp[c^(j - 1)*(c*x)^(m - j + 1)*((a*x^j + b*x^n)^(p + 1)/(a*(m + j*p
+ 1))), x] - Simp[b*((m + n*p + n - j + 1)/(a*c^(n - j)*(m + j*p + 1)))   I
nt[(c*x)^(m + n - j)*(a*x^j + b*x^n)^p, x], x] /; FreeQ[{a, b, c, j, m, n,
p}, x] &&  !IntegerQ[p] && NeQ[n, j] && ILtQ[Simplify[(m + n*p + n - j + 1)
/(n - j)], 0] && NeQ[m + j*p + 1, 0] && (IntegersQ[j, n] || GtQ[c, 0])
```

Maple [A] (verified)

Time = 0.44 (sec) , antiderivative size = 46, normalized size of antiderivative = 0.62

method	result	size
gosper	$-\frac{2(bx+a)\left(-b^{2}x^{2}+4abx+8a^{2}\right)x^{3}}{3b^{3}\left(b\,x^{3}+a\,x^{2}\right)^{\frac{3}{2}}}$	46
default	$-\frac{2(bx+a)\left(-b^{2}x^{2}+4abx+8a^{2}\right)x^{3}}{3b^{3}\left(b\,x^{3}+a\,x^{2}\right)^{\frac{3}{2}}}$	46
orering	$-\frac{2(bx+a)\left(-b^{2}x^{2}+4abx+8a^{2}\right)x^{3}}{3b^{3}\left(b\,x^{3}+a\,x^{2}\right)^{\frac{3}{2}}}$	46
trager	$-\frac{2\left(-b^{2}x^{2}+4abx+8a^{2}\right)\sqrt{b\,x^{3}+a\,x^{2}}}{3(bx+a)\,b^{3}x}$	48
risch	$-\frac{2(-bx+5a)(bx+a)x}{3b^{3}\sqrt{x^{2}(bx+a)}}-\frac{2a^{2}x}{b^{3}\sqrt{x^{2}(bx+a)}}$	52
pseudoelliptic	$\frac{\frac{2}{9}b^{5}x^{5}-\frac{20}{63}a\,b^{4}x^{4}+\frac{32}{63}a^{2}b^{3}x^{3}-\frac{64}{63}a^{3}b^{2}x^{2}+\frac{256}{63}a^{4}bx+\frac{512}{63}a^{5}}{b^{6}\sqrt{bx+a}}$	65

```
int(x^5/(b*x^3+a*x^2)^(3/2),x,method=_RETURNVERBOSE)
```

```
-2/3*(b*x+a)*(-b^2*x^2+4*a*b*x+8*a^2)*x^3/b^3/(b*x^3+a*x^2)^(3/2)
```

Fricas [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 49, normalized size of antiderivative = 0.66

$$\int \frac{x^5}{(ax^2+bx^3)^{3/2}}\,dx = \frac{2\,(b^2x^2-4\,abx-8\,a^2)\sqrt{bx^3+ax^2}}{3\,(b^4x^2+ab^3x)}$$

```
integrate(x^5/(b*x^3+a*x^2)^(3/2),x, algorithm="fricas")
```

```
2/3*(b^2*x^2 - 4*a*b*x - 8*a^2)*sqrt(b*x^3 + a*x^2)/(b^4*x^2 + a*b^3*x)
```

Sympy [F]

$$\int \frac{x^5}{(ax^2+bx^3)^{3/2}}\,dx = \int \frac{x^5}{(x^2\,(a+bx))^{\frac{3}{2}}}\,dx$$

```
integrate(x**5/(b*x**3+a*x**2)**(3/2),x)
```

```
Integral(x**5/(x**2*(a + b*x))**(3/2), x)
```

Maxima [A] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 30, normalized size of antiderivative = 0.41

$$\int \frac{x^5}{\left(ax^2+bx^3\right)^{3/2}}\,dx = \frac{2\left(b^2x^2-4\,abx-8\,a^2\right)}{3\,\sqrt{bx+a}b^3}$$

```
integrate(x^5/(b*x^3+a*x^2)^(3/2),x, algorithm="maxima")
```

```
2/3*(b^2*x^2 - 4*a*b*x - 8*a^2)/(sqrt(b*x + a)*b^3)
```

Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 70, normalized size of antiderivative = 0.95

$$\int \frac{x^5}{\left(ax^2+bx^3\right)^{3/2}}\,dx = \frac{16\,a^{\frac{3}{2}}\mathrm{sgn}(x)}{3\,b^3} - \frac{2\left(\frac{3\,a^2}{\sqrt{bx+a}b\mathrm{sgn}(x)} - \frac{(bx+a)^{\frac{3}{2}}b^2-6\,\sqrt{bx+a}ab^2}{b^3\mathrm{sgn}(x)}\right)}{3\,b^2}$$

```
integrate(x^5/(b*x^3+a*x^2)^(3/2),x, algorithm="giac")
```

```
16/3*a^(3/2)*sgn(x)/b^3 - 2/3*(3*a^2/(sqrt(b*x + a)*b*sgn(x)) - ((b*x + a)
^(3/2)*b^2 - 6*sqrt(b*x + a)*a*b^2)/(b^3*sgn(x)))/b^2
```

Mupad [B] (verification not implemented)

Time = 9.47 (sec) , antiderivative size = 47, normalized size of antiderivative = 0.64

$$\int \frac{x^5}{\left(ax^2+bx^3\right)^{3/2}}\,dx = -\frac{2\,\sqrt{b\,x^3+a\,x^2}\,\left(8\,a^2+4\,a\,b\,x-b^2\,x^2\right)}{3\,b^3\,x\,\left(a+b\,x\right)}$$

```
int(x^5/(a*x^2 + b*x^3)^(3/2),x)
```

```
-(2*(a*x^2 + b*x^3)^(1/2)*(8*a^2 - b^2*x^2 + 4*a*b*x))/(3*b^3*x*(a + b*x))
```

Reduce [B] (verification not implemented)

Time = 0.31 (sec) , antiderivative size = 31, normalized size of antiderivative = 0.42

$$\int \frac{x^5}{(ax^2+bx^3)^{3/2}}\,dx = \frac{\frac{2}{3}b^2x^2 - \frac{8}{3}abx - \frac{16}{3}a^2}{\sqrt{bx+a}\,b^3}$$

```
int(x^5/(b*x^3+a*x^2)^(3/2),x)
```

```
(2*( - 8*a**2 - 4*a*b*x + b**2*x**2))/(3*sqrt(a + b*x)*b**3)
```

3.310 $\int \frac{x^4}{\left(ax^2+bx^3\right)^{3/2}}\,dx$

Optimal result

Integrand size = 19, antiderivative size = 46

$$\int \frac{x^4}{\left(ax^2+bx^3\right)^{3/2}}\,dx = \frac{2ax}{b^2\sqrt{ax^2+bx^3}} + \frac{2\sqrt{ax^2+bx^3}}{b^2x}$$

```
2*a*x/b^2/(b*x^3+a*x^2)^(1/2)+2*(b*x^3+a*x^2)^(1/2)/b^2/x
```

Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 26, normalized size of antiderivative = 0.57

$$\int \frac{x^4}{\left(ax^2+bx^3\right)^{3/2}}\,dx = \frac{2x(2a+bx)}{b^2\sqrt{x^2(a+bx)}}$$

```
Integrate[x^4/(a*x^2 + b*x^3)^(3/2),x]
```

```
(2*x*(2*a + b*x))/(b^2*Sqrt[x^2*(a + b*x)])
```

Rubi [A] (verified)

Time = 0.32 (sec) , antiderivative size = 47, normalized size of antiderivative = 1.02, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.105, Rules used = {1921, 1920}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^4}{(ax^2+bx^3)^{3/2}} \, dx$$

$$\Big\downarrow \text{ 1921}$$

$$\frac{2\int \frac{x}{\sqrt{bx^3+ax^2}}dx}{b}-\frac{2x^2}{b\sqrt{ax^2+bx^3}}$$

$$\Big\downarrow \text{ 1920}$$

$$\frac{4\sqrt{ax^2+bx^3}}{b^2x}-\frac{2x^2}{b\sqrt{ax^2+bx^3}}$$

```
Int[x^4/(a*x^2 + b*x^3)^(3/2),x]
```

```
(-2*x^2)/(b*Sqrt[a*x^2 + b*x^3]) + (4*Sqrt[a*x^2 + b*x^3])/(b^2*x)
```

Defintions of rubi rules used

```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> Simp[(-c^(j - 1))*(c*x)^(m - j + 1)*((a*x^j + b*x^n)^(p + 1)/(a*(n - j
)*(p + 1))), x] /; FreeQ[{a, b, c, j, m, n, p}, x] &&  !IntegerQ[p] && NeQ[
n, j] && EqQ[m + n*p + n - j + 1, 0] && (IntegerQ[j] || GtQ[c, 0])
```

```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> Simp[(-c^(j - 1))*(c*x)^(m - j + 1)*((a*x^j + b*x^n)^(p + 1)/(a*(n - j
)*(p + 1))), x] + Simp[c^j*((m + n*p + n - j + 1)/(a*(n - j)*(p + 1)))   In
t[(c*x)^(m - j)*(a*x^j + b*x^n)^(p + 1), x], x] /; FreeQ[{a, b, c, j, m, n}
, x] &&  !IntegerQ[p] && NeQ[n, j] && ILtQ[Simplify[(m + n*p + n - j + 1)/(
n - j)], 0] && LtQ[p, -1] && (IntegerQ[j] || GtQ[c, 0])
```

Maple [A] (verified)

Time = 0.44 (sec) , antiderivative size = 34, normalized size of antiderivative = 0.74

method	result	size
gosper	$\frac{2(bx+a)(bx+2a)x^3}{b^2(b\,x^3+a\,x^2)^{\frac{3}{2}}}$	34
default	$\frac{2(bx+a)(bx+2a)x^3}{b^2(b\,x^3+a\,x^2)^{\frac{3}{2}}}$	34
orering	$\frac{2(bx+a)(bx+2a)x^3}{b^2(b\,x^3+a\,x^2)^{\frac{3}{2}}}$	34
trager	$\frac{2(bx+2a)\sqrt{b\,x^3+a\,x^2}}{(bx+a)b^2x}$	36
risch	$\frac{2(bx+a)x}{b^2\sqrt{x^2(bx+a)}}+\frac{2ax}{b^2\sqrt{x^2(bx+a)}}$	42
pseudoelliptic	$\frac{\frac{2}{7}b^4x^4-\frac{16}{35}a\,b^3x^3+\frac{32}{35}a^2b^2x^2-\frac{128}{35}a^3bx-\frac{256}{35}a^4}{b^5\sqrt{bx+a}}$	54

```
int(x^4/(b*x^3+a*x^2)^(3/2),x,method=_RETURNVERBOSE)
```

```
2*(b*x+a)*(b*x+2*a)*x^3/b^2/(b*x^3+a*x^2)^(3/2)
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 38, normalized size of antiderivative = 0.83

$$\int \frac{x^4}{\left(ax^2+bx^3\right)^{3/2}}\,dx=\frac{2\,\sqrt{bx^3+ax^2}(bx+2\,a)}{b^3x^2+ab^2x}$$

```
integrate(x^4/(b*x^3+a*x^2)^(3/2),x, algorithm="fricas")
```

```
2*sqrt(b*x^3 + a*x^2)*(b*x + 2*a)/(b^3*x^2 + a*b^2*x)
```

Sympy [F]

$$\int \frac{x^4}{\left(ax^2+bx^3\right)^{3/2}}\,dx=\int \frac{x^4}{\left(x^2\left(a+bx\right)\right)^{\frac{3}{2}}}\,dx$$

```
integrate(x**4/(b*x**3+a*x**2)**(3/2),x)
```

```
Integral(x**4/(x**2*(a + b*x))**(3/2), x)
```

Maxima [A] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 19, normalized size of antiderivative = 0.41

$$\int \frac{x^4}{\left(ax^2+bx^3\right)^{3/2}}\,dx=\frac{2\,(bx+2\,a)}{\sqrt{bx+a}b^2}$$

```
integrate(x^4/(b*x^3+a*x^2)^(3/2),x, algorithm="maxima")
```

```
2*(b*x + 2*a)/(sqrt(b*x + a)*b^2)
```

Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 48, normalized size of antiderivative = 1.04

$$\int \frac{x^4}{(ax^2+bx^3)^{3/2}}\,dx = \frac{2\left(\frac{\sqrt{bx+a}}{b\mathrm{sgn}(x)}+\frac{a}{\sqrt{bx+a}b\mathrm{sgn}(x)}\right)}{b}-\frac{4\sqrt{a}\mathrm{sgn}(x)}{b^2}$$

```
integrate(x^4/(b*x^3+a*x^2)^(3/2),x, algorithm="giac")
```

```
2*(sqrt(b*x + a)/(b*sgn(x)) + a/(sqrt(b*x + a)*b*sgn(x)))/b - 4*sqrt(a)*sg
n(x)/b^2
```

Mupad [B] (verification not implemented)

Time = 9.01 (sec) , antiderivative size = 35, normalized size of antiderivative = 0.76

$$\int \frac{x^4}{(ax^2+bx^3)^{3/2}}\,dx = \frac{2\,(2\,a+b\,x)\,\sqrt{b\,x^3+a\,x^2}}{b^2\,x\,(a+b\,x)}$$

```
int(x^4/(a*x^2 + b*x^3)^(3/2),x)
```

```
(2*(2*a + b*x)*(a*x^2 + b*x^3)^(1/2))/(b^2*x*(a + b*x))
```

Reduce [B] (verification not implemented)

Time = 0.49 (sec) , antiderivative size = 20, normalized size of antiderivative = 0.43

$$\int \frac{x^4}{(ax^2+bx^3)^{3/2}}\,dx = \frac{2bx+4a}{\sqrt{bx+a}\,b^2}$$

```
int(x^4/(b*x^3+a*x^2)^(3/2),x)
```

```
(2*(2*a + b*x))/(sqrt(a + b*x)*b**2)
```

3.311 $\int \frac{x^3}{\left(ax^2+bx^3\right)^{3/2}} \, dx$

Optimal result

Integrand size = 19, antiderivative size = 21

$$\int \frac{x^3}{\left(ax^2+bx^3\right)^{3/2}} \, dx = -\frac{2x}{b\sqrt{ax^2+bx^3}}$$

```
-2*x/b/(b*x^3+a*x^2)^(1/2)
```

Mathematica [A] (verified)

Time = 0.00 (sec) , antiderivative size = 19, normalized size of antiderivative = 0.90

$$\int \frac{x^3}{\left(ax^2+bx^3\right)^{3/2}} \, dx = -\frac{2x}{b\sqrt{x^2(a+bx)}}$$

```
Integrate[x^3/(a*x^2 + b*x^3)^(3/2),x]
```

```
(-2*x)/(b*Sqrt[x^2*(a + b*x)])
```

Rubi [A] (verified)

Time = 0.25 (sec) , antiderivative size = 21, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}} = 0.053$, Rules used = $\{1920\}$

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^3}{(ax^2+bx^3)^{3/2}}\,dx$$
$$\Big\downarrow \; 1920$$
$$-\frac{2x}{b\sqrt{ax^2+bx^3}}$$

```
Int[x^3/(a*x^2 + b*x^3)^(3/2),x]
```

```
(-2*x)/(b*Sqrt[a*x^2 + b*x^3])
```

Defintions of rubi rules used

```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> Simp[(-c^(j - 1))*(c*x)^(m - j + 1)*((a*x^j + b*x^n)^(p + 1)/(a*(n - j
)*(p + 1))), x] /; FreeQ[{a, b, c, j, m, n, p}, x] &&  !IntegerQ[p] && NeQ[
n, j] && EqQ[m + n*p + n - j + 1, 0] && (IntegerQ[j] || GtQ[c, 0])
```

Maple [A] (verified)

Time = 0.42 (sec) , antiderivative size = 27, normalized size of antiderivative = 1.29

method	result	size
gosper	$-\frac{2(bx+a)x^3}{b(b\,x^3+a\,x^2)^{\frac{3}{2}}}$	27
default	$-\frac{2(bx+a)x^3}{b(b\,x^3+a\,x^2)^{\frac{3}{2}}}$	27
orering	$-\frac{2(bx+a)x^3}{b(b\,x^3+a\,x^2)^{\frac{3}{2}}}$	27
trager	$-\frac{2\sqrt{b\,x^3+a\,x^2}}{(bx+a)bx}$	29
pseudoelliptic	$\frac{\frac{2}{5}b^3x^3-\frac{4}{5}a\,b^2x^2+\frac{16}{5}a^2bx+\frac{32}{5}a^3}{b^4\sqrt{bx+a}}$	42

```
int(x^3/(b*x^3+a*x^2)^(3/2),x,method=_RETURNVERBOSE)
```

```
-2*(b*x+a)/b*x^3/(b*x^3+a*x^2)^(3/2)
```

Fricas [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 29, normalized size of antiderivative = 1.38

$$\int \frac{x^3}{(ax^2+bx^3)^{3/2}}\,dx = -\frac{2\sqrt{bx^3+ax^2}}{b^2x^2+abx}$$

```
integrate(x^3/(b*x^3+a*x^2)^(3/2),x, algorithm="fricas")
```

```
-2*sqrt(b*x^3 + a*x^2)/(b^2*x^2 + a*b*x)
```

Sympy [F]

$$\int \frac{x^3}{(ax^2+bx^3)^{3/2}}\,dx = \int \frac{x^3}{(x^2\,(a+bx))^{\frac{3}{2}}}\,dx$$

```
integrate(x**3/(b*x**3+a*x**2)**(3/2),x)
```

```
Integral(x**3/(x**2*(a + b*x))**(3/2), x)
```

Maxima [A] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 12, normalized size of antiderivative = 0.57

$$\int \frac{x^3}{(ax^2+bx^3)^{3/2}}\,dx = -\frac{2}{\sqrt{bx+a}b}$$

```
integrate(x^3/(b*x^3+a*x^2)^(3/2),x, algorithm="maxima")
```

```
-2/(sqrt(b*x + a)*b)
```

Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 27, normalized size of antiderivative = 1.29

$$\int \frac{x^3}{(ax^2+bx^3)^{3/2}}\,dx = \frac{2\,\mathrm{sgn}(x)}{\sqrt{a}b} - \frac{2}{\sqrt{bx+a}b\,\mathrm{sgn}\,(x)}$$

```
integrate(x^3/(b*x^3+a*x^2)^(3/2),x, algorithm="giac")
```

```
2*sgn(x)/(sqrt(a)*b) - 2/(sqrt(b*x + a)*b*sgn(x))
```

Mupad [B] (verification not implemented)

Time = 8.76 (sec) , antiderivative size = 28, normalized size of antiderivative = 1.33

$$\int \frac{x^3}{(ax^2+bx^3)^{3/2}}\,dx = -\frac{2\sqrt{b\,x^3+a\,x^2}}{b\,x\,\left(a+b\,x\right)}$$

```
int(x^3/(a*x^2 + b*x^3)^(3/2),x)
```

```
-(2*(a*x^2 + b*x^3)^(1/2))/(b*x*(a + b*x))
```

Reduce [B] (verification not implemented)

Time = 0.61 (sec) , antiderivative size = 13, normalized size of antiderivative = 0.62

$$\int \frac{x^3}{(ax^2 + bx^3)^{3/2}}\,dx = -\frac{2}{\sqrt{bx + a}\,b}$$

```
int(x^3/(b*x^3+a*x^2)^(3/2),x)
```

```
( - 2)/(sqrt(a + b*x)*b)
```

3.312 $\int \frac{x^2}{\left(ax^2+bx^3\right)^{3/2}}\,dx$

Optimal result

Integrand size = 19, antiderivative size = 54

$$\int \frac{x^2}{\left(ax^2+bx^3\right)^{3/2}}\,dx=\frac{2x}{a\sqrt{ax^2+bx^3}}-\frac{2\text{arctanh}\left(\frac{\sqrt{ax^2+bx^3}}{\sqrt{a}x}\right)}{a^{3/2}}$$

```
2*x/a/(b*x^3+a*x^2)^(1/2)-2*arctanh((b*x^3+a*x^2)^(1/2)/a^(1/2)/x)/a^(3/2)
```

Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 54, normalized size of antiderivative = 1.00

$$\int \frac{x^2}{\left(ax^2+bx^3\right)^{3/2}}\,dx=\frac{2x\left(\sqrt{a}-\sqrt{a+bx}\text{arctanh}\left(\frac{\sqrt{a+bx}}{\sqrt{a}}\right)\right)}{a^{3/2}\sqrt{x^2(a+bx)}}$$

```
Integrate[x^2/(a*x^2 + b*x^3)^(3/2),x]
```

```
(2*x*(Sqrt[a] - Sqrt[a + b*x]*ArcTanh[Sqrt[a + b*x]/Sqrt[a]]))/(a^(3/2)*Sq
rt[x^2*(a + b*x)])
```

Rubi [A] (verified)

Time = 0.32 (sec) , antiderivative size = 52, normalized size of antiderivative = 0.96, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.158, Rules used = {1929, 1914, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^2}{(ax^2+bx^3)^{3/2}} \, dx$$

$$\Big\downarrow \text{ 1929}$$

$$\frac{\int \frac{1}{\sqrt{bx^3+ax^2}}dx}{a}+\frac{2x}{a\sqrt{ax^2+bx^3}}$$

$$\Big\downarrow \text{ 1914}$$

$$\frac{2x}{a\sqrt{ax^2+bx^3}}-\frac{2\int \frac{1}{1-\frac{ax^2}{bx^3+ax^2}}d\frac{x}{\sqrt{bx^3+ax^2}}}{a}$$

$$\Big\downarrow \text{ 219}$$

$$\frac{2x}{a\sqrt{ax^2+bx^3}}-\frac{2\text{arctanh}\left(\frac{\sqrt{a}x}{\sqrt{ax^2+bx^3}}\right)}{a^{3/2}}$$

```
Int[x^2/(a*x^2 + b*x^3)^(3/2),x]
```

```
(2*x)/(a*Sqrt[a*x^2 + b*x^3]) - (2*ArcTanh[(Sqrt[a]*x)/Sqrt[a*x^2 + b*x^3]
])/a^(3/2)
```

Defintions of rubi rules used

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

```
Int[1/Sqrt[(a_.)*(x_)^2 + (b_.)*(x_)^(n_.)], x_Symbol] :> Simp[2/(2 - n)
Subst[Int[1/(1 - a*x^2), x], x, x/Sqrt[a*x^2 + b*x^n]], x] /; FreeQ[{a, b,
n}, x] && NeQ[n, 2]
```

```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> Simp[(-c^(j - 1))*(c*x)^(m - j + 1)*((a*x^j + b*x^n)^(p + 1)/(a*(n - j
)*(p + 1))), x] + Simp[c^j*((m + n*p + n - j + 1)/(a*(n - j)*(p + 1)))   In
t[(c*x)^(m - j)*(a*x^j + b*x^n)^(p + 1), x], x] /; FreeQ[{a, b, c, m}, x] &
&  !IntegerQ[p] && LtQ[0, j, n] && (IntegersQ[j, n] || GtQ[c, 0]) && LtQ[p,
 -1]
```

Maple [A] (verified)

Time = 0.40 (sec) , antiderivative size = 31, normalized size of antiderivative = 0.57

method	result	size
pseudoelliptic	$\frac{\frac{2}{3}b^{2}x^{2}-\frac{8}{3}abx-\frac{16}{3}a^{2}}{b^{3}\sqrt{bx+a}}$	31
default	$-\frac{2x^{3}\left(bx+a\right)\left(\operatorname{arctanh}\left(\frac{\sqrt{bx+a}}{\sqrt{a}}\right)a\sqrt{bx+a}-a^{\frac{3}{2}}\right)}{\left(b\,x^{3}+a\,x^{2}\right)^{\frac{3}{2}}a^{\frac{5}{2}}}$	54

```
int(x^2/(b*x^3+a*x^2)^(3/2),x,method=_RETURNVERBOSE)
```

```
2/3*(b^2*x^2-4*a*b*x-8*a^2)/b^3/(b*x+a)^(1/2)
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 161, normalized size of antiderivative = 2.98

$$\int \frac{x^2}{(ax^2+bx^3)^{3/2}}\,dx = \left[\frac{(bx^2+ax)\sqrt{a}\log\left(\frac{bx^2+2\,ax-2\,\sqrt{bx^3+ax^2}\sqrt{a}}{x^2}\right)+2\,\sqrt{bx^3+ax^2}a}{a^2bx^2+a^3x}, \frac{2\left((bx^2+ax)\sqrt{-a}\,\mathrm{ar}\right.}{}\right.$$

```
integrate(x^2/(b*x^3+a*x^2)^(3/2),x, algorithm="fricas")
```

```
[((b*x^2 + a*x)*sqrt(a)*log((b*x^2 + 2*a*x - 2*sqrt(b*x^3 + a*x^2)*sqrt(a)
)/x^2) + 2*sqrt(b*x^3 + a*x^2)*a)/(a^2*b*x^2 + a^3*x), 2*((b*x^2 + a*x)*sq
rt(-a)*arctan(sqrt(b*x^3 + a*x^2)*sqrt(-a)/(b*x^2 + a*x)) + sqrt(b*x^3 + a
*x^2)*a)/(a^2*b*x^2 + a^3*x)]
```

Sympy [F]

$$\int \frac{x^2}{(ax^2+bx^3)^{3/2}}\,dx = \int \frac{x^2}{\left(x^2\,(a+bx)\right)^{\frac{3}{2}}}\,dx$$

```
integrate(x**2/(b*x**3+a*x**2)**(3/2),x)
```

```
Integral(x**2/(x**2*(a + b*x))**(3/2), x)
```

Maxima [F]

$$\int \frac{x^2}{(ax^2+bx^3)^{3/2}}\,dx = \int \frac{x^2}{(bx^3+ax^2)^{\frac{3}{2}}}\,dx$$

```
integrate(x^2/(b*x^3+a*x^2)^(3/2),x, algorithm="maxima")
```

```
integrate(x^2/(b*x^3 + a*x^2)^(3/2), x)
```

Giac [A] (verification not implemented)

Time = 0.17 (sec) , antiderivative size = 77, normalized size of antiderivative = 1.43

$$\int \frac{x^2}{(ax^2+bx^3)^{3/2}}\,dx = -\frac{2\left(\sqrt{a}\arctan\left(\frac{\sqrt{a}}{\sqrt{-a}}\right)+\sqrt{-a}\right)\mathrm{sgn}(x)}{\sqrt{-a}a^{\frac{3}{2}}} + \frac{2\arctan\left(\frac{\sqrt{bx+a}}{\sqrt{-a}}\right)}{\sqrt{-a}a\mathrm{sgn}\left(x\right)} + \frac{2}{\sqrt{bx+a}a\mathrm{sgn}\left(x\right)}$$

```
integrate(x^2/(b*x^3+a*x^2)^(3/2),x, algorithm="giac")
```

```
-2*(sqrt(a)*arctan(sqrt(a)/sqrt(-a)) + sqrt(-a))*sgn(x)/(sqrt(-a)*a^(3/2))
 + 2*arctan(sqrt(b*x + a)/sqrt(-a))/(sqrt(-a)*a*sgn(x)) + 2/(sqrt(b*x + a)
*a*sgn(x))
```

Mupad [F(-1)]

Timed out.

$$\int \frac{x^2}{(ax^2+bx^3)^{3/2}}\,dx = \int \frac{x^2}{\left(b\,x^3+a\,x^2\right)^{3/2}}\,dx$$

```
int(x^2/(a*x^2 + b*x^3)^(3/2),x)
```

```
int(x^2/(a*x^2 + b*x^3)^(3/2), x)
```

Reduce [B] (verification not implemented)

Time = 0.30 (sec) , antiderivative size = 57, normalized size of antiderivative = 1.06

$$\int \frac{x^2}{(ax^2+bx^3)^{3/2}}\,dx = \frac{\sqrt{a}\,\sqrt{bx+a}\log\left(\sqrt{bx+a}-\sqrt{a}\right)-\sqrt{a}\,\sqrt{bx+a}\log\left(\sqrt{bx+a}+\sqrt{a}\right)+2a}{\sqrt{bx+a}\,a^2}$$

```
int(x^2/(b*x^3+a*x^2)^(3/2),x)
```

```
(sqrt(a)*sqrt(a + b*x)*log(sqrt(a + b*x) - sqrt(a)) - sqrt(a)*sqrt(a + b*x
)*log(sqrt(a + b*x) + sqrt(a)) + 2*a)/(sqrt(a + b*x)*a**2)
```

3.313 $\int \frac{x}{(ax^2+bx^3)^{3/2}}\,dx$

Optimal result

Integrand size = 17, antiderivative size = 76

$$\int \frac{x}{(ax^2+bx^3)^{3/2}}\,dx = -\frac{1}{a\sqrt{ax^2+bx^3}} - \frac{3bx}{a^2\sqrt{ax^2+bx^3}} + \frac{3b\operatorname{arctanh}\left(\frac{\sqrt{ax^2+bx^3}}{\sqrt{a}x}\right)}{a^{5/2}}$$

```
-1/a/(b*x^3+a*x^2)^(1/2)-3*b*x/a^2/(b*x^3+a*x^2)^(1/2)+3*b*arctanh((b*x^3+
a*x^2)^(1/2)/a^(1/2)/x)/a^(5/2)
```

Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 62, normalized size of antiderivative = 0.82

$$\int \frac{x}{(ax^2+bx^3)^{3/2}}\,dx = \frac{-\sqrt{a}(a+3bx)+3bx\sqrt{a+bx}\operatorname{arctanh}\left(\frac{\sqrt{a+bx}}{\sqrt{a}}\right)}{a^{5/2}\sqrt{x^2(a+bx)}}$$

```
Integrate[x/(a*x^2 + b*x^3)^(3/2),x]
```

```
(-(Sqrt[a]*(a + 3*b*x)) + 3*b*x*Sqrt[a + b*x]*ArcTanh[Sqrt[a + b*x]/Sqrt[a
]])/(a^(5/2)*Sqrt[x^2*(a + b*x)])
```

Rubi [A] (verified)

Time = 0.39 (sec) , antiderivative size = 80, normalized size of antiderivative = 1.05, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.235$, Rules used = {1929, 1931, 1914, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x}{(ax^2+bx^3)^{3/2}}\,dx$$

$$\Big\downarrow \ 1929$$

$$\frac{3\int \frac{1}{x\sqrt{bx^3+ax^2}}dx}{a}+\frac{2}{a\sqrt{ax^2+bx^3}}$$

$$\Big\downarrow \ 1931$$

$$\frac{3\left(-\frac{b\int \frac{1}{\sqrt{bx^3+ax^2}}dx}{2a}-\frac{\sqrt{ax^2+bx^3}}{ax^2}\right)}{a}+\frac{2}{a\sqrt{ax^2+bx^3}}$$

$$\Big\downarrow \ 1914$$

$$\frac{3\left(\frac{b\int \frac{1}{1-\frac{ax^2}{bx^3+ax^2}}d\frac{x}{\sqrt{bx^3+ax^2}}}{a}-\frac{\sqrt{ax^2+bx^3}}{ax^2}\right)}{a}+\frac{2}{a\sqrt{ax^2+bx^3}}$$

$$\Big\downarrow \ 219$$

$$\frac{3\left(\frac{b\,\text{arctanh}\left(\frac{\sqrt{a}x}{\sqrt{ax^2+bx^3}}\right)}{a^{3/2}}-\frac{\sqrt{ax^2+bx^3}}{ax^2}\right)}{a}+\frac{2}{a\sqrt{ax^2+bx^3}}$$

```
Int[x/(a*x^2 + b*x^3)^(3/2),x]
```

```
2/(a*Sqrt[a*x^2 + b*x^3]) + (3*(-(Sqrt[a*x^2 + b*x^3]/(a*x^2)) + (b*ArcTan
h[(Sqrt[a]*x)/Sqrt[a*x^2 + b*x^3]])/a^(3/2)))/a
```

Defintions of rubi rules used

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

```
Int[1/Sqrt[(a_.)*(x_)^2 + (b_.)*(x_)^(n_.)], x_Symbol] :> Simp[2/(2 - n)
Subst[Int[1/(1 - a*x^2), x], x, x/Sqrt[a*x^2 + b*x^n]], x] /; FreeQ[{a, b,
n}, x] && NeQ[n, 2]
```

```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> Simp[(-c^(j - 1))*(c*x)^(m - j + 1)*((a*x^j + b*x^n)^(p + 1)/(a*(n - j
)*(p + 1))), x] + Simp[c^j*((m + n*p + n - j + 1)/(a*(n - j)*(p + 1)))   In
t[(c*x)^(m - j)*(a*x^j + b*x^n)^(p + 1), x], x] /; FreeQ[{a, b, c, m}, x] &
&  !IntegerQ[p] && LtQ[0, j, n] && (IntegersQ[j, n] || GtQ[c, 0]) && LtQ[p,
 -1]
```

```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> Simp[c^(j - 1)*(c*x)^(m - j + 1)*((a*x^j + b*x^n)^(p + 1)/(a*(m + j*p
+ 1))), x] - Simp[b*((m + n*p + n - j + 1)/(a*c^(n - j)*(m + j*p + 1)))   I
nt[(c*x)^(m + n - j)*(a*x^j + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, p}, x]
 &&  !IntegerQ[p] && LtQ[0, j, n] && (IntegersQ[j, n] || GtQ[c, 0]) && LtQ[
m + j*p + 1, 0]
```

Maple [A] (verified)

Time = 0.41 (sec) , antiderivative size = 20, normalized size of antiderivative = 0.26

method	result	size
pseudoelliptic	$\frac{2bx+4a}{b^2\sqrt{bx+a}}$	20
default	$\frac{x^2(bx+a)\left(3\sqrt{bx+a}\,\operatorname{arctanh}\left(\frac{\sqrt{bx+a}}{\sqrt{a}}\right)bx-3bx\sqrt{a}-a^{\frac{3}{2}}\right)}{\left(b\,x^3+a\,x^2\right)^{\frac{3}{2}}a^{\frac{5}{2}}}$	62
risch	$-\frac{bx+a}{a^2\sqrt{x^2(bx+a)}}-\frac{b\left(-\frac{6\,\operatorname{arctanh}\left(\frac{\sqrt{bx+a}}{\sqrt{a}}\right)}{\sqrt{a}}+\frac{4}{\sqrt{bx+a}}\right)\sqrt{bx+a}\,x}{2a^2\sqrt{x^2(bx+a)}}$	75

```
int(x/(b*x^3+a*x^2)^(3/2),x,method=_RETURNVERBOSE)
```

```
(2*b*x+4*a)/b^2/(b*x+a)^(1/2)
```

Fricas [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 194, normalized size of antiderivative = 2.55

$$\int \frac{x}{(ax^2+bx^3)^{3/2}}\,dx = \left[\frac{3\,(b^2x^3+abx^2)\sqrt{a}\log\left(\frac{bx^2+2\,ax+2\,\sqrt{bx^3+ax^2}\sqrt{a}}{x^2}\right)-2\,\sqrt{bx^3+ax^2}(3\,abx+a^2)}{2\,(a^3bx^3+a^4x^2)},\right.$$
$$\left.-\frac{3\,(b^2x^3+abx^2)\sqrt{-a}\arctan\left(\frac{\sqrt{bx^3+ax^2}\sqrt{-a}}{bx^2+ax}\right)+\sqrt{bx^3+ax^2}(3\,abx+a^2)}{a^3bx^3+a^4x^2}\right]$$

```
integrate(x/(b*x^3+a*x^2)^(3/2),x, algorithm="fricas")
```

```
[1/2*(3*(b^2*x^3 + a*b*x^2)*sqrt(a)*log((b*x^2 + 2*a*x + 2*sqrt(b*x^3 + a*
x^2)*sqrt(a))/x^2) - 2*sqrt(b*x^3 + a*x^2)*(3*a*b*x + a^2))/(a^3*b*x^3 + a
^4*x^2), -(3*(b^2*x^3 + a*b*x^2)*sqrt(-a)*arctan(sqrt(b*x^3 + a*x^2)*sqrt(
-a)/(b*x^2 + a*x)) + sqrt(b*x^3 + a*x^2)*(3*a*b*x + a^2))/(a^3*b*x^3 + a^4
*x^2)]
```

Sympy [F]

$$\int \frac{x}{(ax^2+bx^3)^{3/2}}\,dx = \int \frac{x}{(x^2\,(a+bx))^{\frac{3}{2}}}\,dx$$

```
integrate(x/(b*x**3+a*x**2)**(3/2),x)
```

```
Integral(x/(x**2*(a + b*x))**(3/2), x)
```

Maxima [F]

$$\int \frac{x}{(ax^2+bx^3)^{3/2}}\,dx=\int \frac{x}{(bx^3+ax^2)^{\frac{3}{2}}}\,dx$$

```
integrate(x/(b*x^3+a*x^2)^(3/2),x, algorithm="maxima")
```

```
integrate(x/(b*x^3 + a*x^2)^(3/2), x)
```

Giac [A] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 72, normalized size of antiderivative = 0.95

$$\int \frac{x}{(ax^2+bx^3)^{3/2}}\,dx=-\frac{3\,b\arctan\left(\frac{\sqrt{bx+a}}{\sqrt{-a}}\right)}{\sqrt{-a}a^2\mathrm{sgn}\left(x\right)}-\frac{3\,(bx+a)b-2\,ab}{\left((bx+a)^{\frac{3}{2}}-\sqrt{bx+a}a\right)a^2\mathrm{sgn}\left(x\right)}$$

```
integrate(x/(b*x^3+a*x^2)^(3/2),x, algorithm="giac")
```

```
-3*b*arctan(sqrt(b*x + a)/sqrt(-a))/(sqrt(-a)*a^2*sgn(x)) - (3*(b*x + a)*b
 - 2*a*b)/(((b*x + a)^(3/2) - sqrt(b*x + a)*a)*a^2*sgn(x))
```

Mupad [F(-1)]

Timed out.

$$\int \frac{x}{(ax^2+bx^3)^{3/2}}\,dx=\int \frac{x}{{\left(b\,x^3+a\,x^2\right)}^{3/2}}\,dx$$

```
int(x/(a*x^2 + b*x^3)^(3/2),x)
```

```
int(x/(a*x^2 + b*x^3)^(3/2), x)
```

Reduce [B] (verification not implemented)

Time = 0.39 (sec) , antiderivative size = 73, normalized size of antiderivative = 0.96

$$\int \frac{x}{\left(ax^2+bx^3\right)^{3/2}}\,dx = \frac{-3\sqrt{a}\,\sqrt{bx+a}\,\log\left(\sqrt{bx+a}-\sqrt{a}\right)bx+3\sqrt{a}\,\sqrt{bx+a}\,\log\left(\sqrt{bx+a}+\sqrt{a}\right)bx-2a}{2\sqrt{bx+a}\,a^3x}$$

```
int(x/(b*x^3+a*x^2)^(3/2),x)
```

```
( - 3*sqrt(a)*sqrt(a + b*x)*log(sqrt(a + b*x) - sqrt(a))*b*x + 3*sqrt(a)*s
qrt(a + b*x)*log(sqrt(a + b*x) + sqrt(a))*b*x - 2*a**2 - 6*a*b*x)/(2*sqrt(
a + b*x)*a**3*x)
```

3.314 $\int \frac{1}{(ax^2+bx^3)^{3/2}}\,dx$

Optimal result

Integrand size = 15, antiderivative size = 112

$$\int \frac{1}{(ax^2+bx^3)^{3/2}}\,dx=\frac{5b}{4a^2\sqrt{ax^2+bx^3}}-\frac{1}{2ax\sqrt{ax^2+bx^3}}+\frac{15b^2x}{4a^3\sqrt{ax^2+bx^3}}-\frac{15b^2\operatorname{arctanh}\left(\frac{\sqrt{ax^2+bx^3}}{\sqrt{a}x}\right)}{4a^{7/2}}$$

```
5/4*b/a^2/(b*x^3+a*x^2)^(1/2)-1/2/a/x/(b*x^3+a*x^2)^(1/2)+15/4*b^2*x/a^3/(
b*x^3+a*x^2)^(1/2)-15/4*b^2*arctanh((b*x^3+a*x^2)^(1/2)/a^(1/2)/x)/a^(7/2)
```

Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 84, normalized size of antiderivative = 0.75

$$\int \frac{1}{(ax^2+bx^3)^{3/2}}\,dx=\frac{\sqrt{a}(-2a^2+5abx+15b^2x^2)-15b^2x^2\sqrt{a+bx}\operatorname{arctanh}\left(\frac{\sqrt{a+bx}}{\sqrt{a}}\right)}{4a^{7/2}x\sqrt{x^2(a+bx)}}$$

```
Integrate[(a*x^2 + b*x^3)^(-3/2),x]
```

```
(Sqrt[a]*(-2*a^2 + 5*a*b*x + 15*b^2*x^2) - 15*b^2*x^2*Sqrt[a + b*x]*ArcTan
h[Sqrt[a + b*x]/Sqrt[a]])/(4*a^(7/2)*x*Sqrt[x^2*(a + b*x)])
```

Rubi [A] (verified)

Time = 0.46 (sec) , antiderivative size = 117, normalized size of antiderivative = 1.04, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.333, Rules used = {1912, 1931, 1931, 1914, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{(ax^2+bx^3)^{3/2}}dx$$

$$\Big\downarrow \text{1912}$$

$$\frac{5\int \frac{1}{x^2\sqrt{bx^3+ax^2}}dx}{a}+\frac{2}{ax\sqrt{ax^2+bx^3}}$$

$$\Big\downarrow \text{1931}$$

$$\frac{5\left(-\frac{3b\int \frac{1}{x\sqrt{bx^3+ax^2}}dx}{4a}-\frac{\sqrt{ax^2+bx^3}}{2ax^3}\right)}{a}+\frac{2}{ax\sqrt{ax^2+bx^3}}$$

$$\Big\downarrow \text{1931}$$

$$\frac{5\left(-\frac{3b\left(-\frac{b\int \frac{1}{\sqrt{bx^3+ax^2}}dx}{2a}-\frac{\sqrt{ax^2+bx^3}}{ax^2}\right)}{4a}-\frac{\sqrt{ax^2+bx^3}}{2ax^3}\right)}{a}+\frac{2}{ax\sqrt{ax^2+bx^3}}$$

$$\Big\downarrow \text{1914}$$

$$\frac{5\left(-\frac{3b\left(\frac{b\int \frac{1}{1-\frac{ax^2}{bx^3+ax^2}}d\frac{x}{\sqrt{bx^3+ax^2}}}{a}-\frac{\sqrt{ax^2+bx^3}}{ax^2}\right)}{4a}-\frac{\sqrt{ax^2+bx^3}}{2ax^3}\right)}{a}+\frac{2}{ax\sqrt{ax^2+bx^3}}$$

$$\Big\downarrow \text{219}$$

$$\frac{5\left(-\frac{3b\left(\frac{b\,\text{arctanh}\left(\frac{\sqrt{a}x}{\sqrt{ax^2+bx^3}}\right)}{a^{3/2}}-\frac{\sqrt{ax^2+bx^3}}{ax^2}\right)}{4a}-\frac{\sqrt{ax^2+bx^3}}{2ax^3}\right)}{a}+\frac{2}{ax\sqrt{ax^2+bx^3}}$$

```
Int[(a*x^2 + b*x^3)^(-3/2),x]
```

```
2/(a*x*Sqrt[a*x^2 + b*x^3]) + (5*(-1/2*Sqrt[a*x^2 + b*x^3]/(a*x^3) - (3*b*
(-(Sqrt[a*x^2 + b*x^3]/(a*x^2)) + (b*ArcTanh[(Sqrt[a]*x)/Sqrt[a*x^2 + b*x^
3]])/a^(3/2)))/(4*a)))/a
```

Defintions of rubi rules used

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

```
Int[((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol] :> Simp[-(a*x^j +
 b*x^n)^(p + 1)/(a*(n - j)*(p + 1)*x^(j - 1)), x] + Simp[(n*p + n - j + 1)/
(a*(n - j)*(p + 1))   Int[(a*x^j + b*x^n)^(p + 1)/x^j, x], x] /; FreeQ[{a,
b}, x] &&  !IntegerQ[p] && LtQ[0, j, n] && LtQ[p, -1]
```

```
Int[1/Sqrt[(a_.)*(x_)^2 + (b_.)*(x_)^(n_.)], x_Symbol] :> Simp[2/(2 - n)
Subst[Int[1/(1 - a*x^2), x], x, x/Sqrt[a*x^2 + b*x^n]], x] /; FreeQ[{a, b,
n}, x] && NeQ[n, 2]
```

```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> Simp[c^(j - 1)*(c*x)^(m - j + 1)*((a*x^j + b*x^n)^(p + 1)/(a*(m + j*p
+ 1))), x] - Simp[b*((m + n*p + n - j + 1)/(a*c^(n - j)*(m + j*p + 1)))   I
nt[(c*x)^(m + n - j)*(a*x^j + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, p}, x]
 &&  !IntegerQ[p] && LtQ[0, j, n] && (IntegersQ[j, n] || GtQ[c, 0]) && LtQ[
m + j*p + 1, 0]
```

Maple [A] (verified)

Time = 0.37 (sec) , antiderivative size = 13, normalized size of antiderivative = 0.12

method	result	size
pseudoelliptic	$-\frac{2}{\sqrt{bx+a}\,b}$	13
default	$-\frac{x(bx+a)\left(15\sqrt{bx+a}\,\operatorname{arctanh}\left(\frac{\sqrt{bx+a}}{\sqrt{a}}\right)b^2x^2-5a^{\frac{3}{2}}bx-15\sqrt{a}\,b^2x^2+2a^{\frac{5}{2}}\right)}{4\left(b\,x^3+a\,x^2\right)^{\frac{3}{2}}a^{\frac{7}{2}}}$	76
risch	$-\frac{(bx+a)(-7bx+2a)}{4a^3x\sqrt{x^2(bx+a)}}+\frac{b^2\left(-\frac{30\,\operatorname{arctanh}\left(\frac{\sqrt{bx+a}}{\sqrt{a}}\right)}{\sqrt{a}}+\frac{16}{\sqrt{bx+a}}\right)\sqrt{bx+a}\,x}{8a^3\sqrt{x^2(bx+a)}}$	88

```
int(1/(b*x^3+a*x^2)^(3/2),x,method=_RETURNVERBOSE)
```

```
-2/(b*x+a)^(1/2)/b
```

Fricas [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 224, normalized size of antiderivative = 2.00

$$\int \frac{1}{\left(ax^2+bx^3\right)^{3/2}}\,dx = \left[\frac{15\left(b^3x^4+ab^2x^3\right)\sqrt{a}\log\left(\frac{bx^2+2\,ax-2\,\sqrt{bx^3+ax^2}\sqrt{a}}{x^2}\right)+2\left(15\,ab^2x^2+5\,a^2bx-2\,a^3\right)\sqrt{b}}{8\left(a^4bx^4+a^5x^3\right)}\right.$$

```
integrate(1/(b*x^3+a*x^2)^(3/2),x, algorithm="fricas")
```

```
[1/8*(15*(b^3*x^4 + a*b^2*x^3)*sqrt(a)*log((b*x^2 + 2*a*x - 2*sqrt(b*x^3 +
 a*x^2)*sqrt(a))/x^2) + 2*(15*a*b^2*x^2 + 5*a^2*b*x - 2*a^3)*sqrt(b*x^3 +
a*x^2))/(a^4*b*x^4 + a^5*x^3), 1/4*(15*(b^3*x^4 + a*b^2*x^3)*sqrt(-a)*arct
an(sqrt(b*x^3 + a*x^2)*sqrt(-a)/(b*x^2 + a*x)) + (15*a*b^2*x^2 + 5*a^2*b*x
 - 2*a^3)*sqrt(b*x^3 + a*x^2))/(a^4*b*x^4 + a^5*x^3)]
```

Sympy [F]

$$\int \frac{1}{(ax^2 + bx^3)^{3/2}} \, dx = \int \frac{1}{(ax^2 + bx^3)^{\frac{3}{2}}} \, dx$$

```
integrate(1/(b*x**3+a*x**2)**(3/2),x)
```

```
Integral((a*x**2 + b*x**3)**(-3/2), x)
```

Maxima [F]

$$\int \frac{1}{(ax^2 + bx^3)^{3/2}} \, dx = \int \frac{1}{(bx^3 + ax^2)^{\frac{3}{2}}} \, dx$$

```
integrate(1/(b*x^3+a*x^2)^(3/2),x, algorithm="maxima")
```

```
integrate((b*x^3 + a*x^2)^(-3/2), x)
```

Giac [A] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 92, normalized size of antiderivative = 0.82

$$\int \frac{1}{(ax^2+bx^3)^{3/2}}\,dx = \frac{15\,b^2\arctan\left(\frac{\sqrt{bx+a}}{\sqrt{-a}}\right)}{4\,\sqrt{-a}a^3\mathrm{sgn}\left(x\right)} + \frac{2\,b^2}{\sqrt{bx+a}a^3\mathrm{sgn}\left(x\right)} + \frac{7\,(bx+a)^{\frac{3}{2}}b^2 - 9\,\sqrt{bx+a}ab^2}{4\,a^3b^2x^2\mathrm{sgn}\left(x\right)}$$

```
integrate(1/(b*x^3+a*x^2)^(3/2),x, algorithm="giac")
```

```
15/4*b^2*arctan(sqrt(b*x + a)/sqrt(-a))/(sqrt(-a)*a^3*sgn(x)) + 2*b^2/(sqr
t(b*x + a)*a^3*sgn(x)) + 1/4*(7*(b*x + a)^(3/2)*b^2 - 9*sqrt(b*x + a)*a*b^
2)/(a^3*b^2*x^2*sgn(x))
```

Mupad [B] (verification not implemented)

Time = 9.00 (sec) , antiderivative size = 42, normalized size of antiderivative = 0.38

$$\int \frac{1}{(ax^2+bx^3)^{3/2}}\,dx = -\frac{2\,x\,\left(\frac{a}{b\,x}+1\right)^{3/2}\,{}_2\mathrm{F}_1\left(\frac{3}{2},\frac{7}{2};\ \frac{9}{2};\ -\frac{a}{b\,x}\right)}{7\,\left(b\,x^3+a\,x^2\right)^{3/2}}$$

```
int(1/(a*x^2 + b*x^3)^(3/2),x)
```

```
-(2*x*(a/(b*x) + 1)^(3/2)*hypergeom([3/2, 7/2], 9/2, -a/(b*x)))/(7*(a*x^2
+ b*x^3)^(3/2))
```

Reduce [B] (verification not implemented)

Time = 0.54 (sec) , antiderivative size = 92, normalized size of antiderivative = 0.82

$$\int \frac{1}{\left(ax^2+bx^3\right)^{3/2}}\,dx = \frac{15\sqrt{a}\,\sqrt{bx+a}\,\log\left(\sqrt{bx+a}-\sqrt{a}\right)b^2x^2-15\sqrt{a}\,\sqrt{bx+a}\,\log\left(\sqrt{bx+a}+\sqrt{a}\right)b^2x^2}{8\sqrt{bx+a}\,a^4x^2}$$

```
int(1/(b*x^3+a*x^2)^(3/2),x)
```

```
(15*sqrt(a)*sqrt(a + b*x)*log(sqrt(a + b*x) - sqrt(a))*b**2*x**2 - 15*sqrt
(a)*sqrt(a + b*x)*log(sqrt(a + b*x) + sqrt(a))*b**2*x**2 - 4*a**3 + 10*a**
2*b*x + 30*a*b**2*x**2)/(8*sqrt(a + b*x)*a**4*x**2)
```

3.315 $\int \frac{1}{x(ax^2+bx^3)^{3/2}}\,dx$

Optimal result

Integrand size = 19, antiderivative size = 140

$$\int \frac{1}{x\left(ax^2+bx^3\right)^{3/2}}\,dx=-\frac{35b^2}{24a^3\sqrt{ax^2+bx^3}}-\frac{1}{3ax^2\sqrt{ax^2+bx^3}}+\frac{7b}{12a^2x\sqrt{ax^2+bx^3}}-\frac{35b^3x}{8a^4\sqrt{ax^2+bx^3}}+\frac{35b^3\operatorname{arctanh}\left(\frac{\sqrt{ax^2+bx^3}}{\sqrt{a}x}\right)}{8a^{9/2}}$$

```
-35/24*b^2/a^3/(b*x^3+a*x^2)^(1/2)-1/3/a/x^2/(b*x^3+a*x^2)^(1/2)+7/12*b/a^
2/x/(b*x^3+a*x^2)^(1/2)-35/8*b^3*x/a^4/(b*x^3+a*x^2)^(1/2)+35/8*b^3*arctan
h((b*x^3+a*x^2)^(1/2)/a^(1/2)/x)/a^(9/2)
```

Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 96, normalized size of antiderivative = 0.69

$$\int \frac{1}{x\left(ax^2+bx^3\right)^{3/2}}\,dx=\frac{-\sqrt{a}(8a^3-14a^2bx+35ab^2x^2+105b^3x^3)+105b^3x^3\sqrt{a+bx}\operatorname{arctanh}\left(\frac{\sqrt{a+bx}}{\sqrt{a}}\right)}{24a^{9/2}x^2\sqrt{x^2(a+bx)}}$$

```
Integrate[1/(x*(a*x^2 + b*x^3)^(3/2)),x]
```

```
(-(Sqrt[a]*(8*a^3 - 14*a^2*b*x + 35*a*b^2*x^2 + 105*b^3*x^3)) + 105*b^3*x^
3*Sqrt[a + b*x]*ArcTanh[Sqrt[a + b*x]/Sqrt[a]])/(24*a^(9/2)*x^2*Sqrt[x^2*(
a + b*x)])
```

Rubi [A] (verified)

Time = 0.57 (sec) , antiderivative size = 151, normalized size of antiderivative = 1.08, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.316, Rules used = {1929, 1931, 1931, 1931, 1914, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{x\left(ax^2+bx^3\right)^{3/2}}dx$$

$$\Big\downarrow \text{1929}$$

$$\frac{7\int \frac{1}{x^3\sqrt{bx^3+ax^2}}dx}{a}+\frac{2}{ax^2\sqrt{ax^2+bx^3}}$$

$$\Big\downarrow \text{1931}$$

$$\frac{7\left(-\frac{5b\int \frac{1}{x^2\sqrt{bx^3+ax^2}}dx}{6a}-\frac{\sqrt{ax^2+bx^3}}{3ax^4}\right)}{a}+\frac{2}{ax^2\sqrt{ax^2+bx^3}}$$

$$\Big\downarrow \text{1931}$$

$$\frac{7\left(-\frac{5b\left(-\frac{3b\int \frac{1}{x\sqrt{bx^3+ax^2}}dx}{4a}-\frac{\sqrt{ax^2+bx^3}}{2ax^3}\right)}{6a}-\frac{\sqrt{ax^2+bx^3}}{3ax^4}\right)}{a}+\frac{2}{ax^2\sqrt{ax^2+bx^3}}$$

$$\Big\downarrow \text{1931}$$

$$\frac{7\left(-\frac{5b\left(-\frac{3b\left(-\frac{b\int \frac{1}{\sqrt{bx^3+ax^2}}dx}{2a}-\frac{\sqrt{ax^2+bx^3}}{ax^2}\right)}{4a}-\frac{\sqrt{ax^2+bx^3}}{2ax^3}\right)}{6a}-\frac{\sqrt{ax^2+bx^3}}{3ax^4}\right)}{a}+\frac{2}{ax^2\sqrt{ax^2+bx^3}}$$

$$\Big\downarrow \; 1914$$

$$\frac{7\left(-\frac{5b\left(-\frac{3b\left(\frac{b\int \frac{1}{1-\frac{ax^2}{bx^3+ax^2}}d\frac{x}{\sqrt{bx^3+ax^2}}}{a}-\frac{\sqrt{ax^2+bx^3}}{ax^2}\right)}{4a}-\frac{\sqrt{ax^2+bx^3}}{2ax^3}\right)}{6a}-\frac{\sqrt{ax^2+bx^3}}{3ax^4}\right)}{a}+\frac{2}{ax^2\sqrt{ax^2+bx^3}}$$

$$\Big\downarrow \; 219$$

$$\frac{7\left(-\frac{5b\left(-\frac{3b\left(\frac{b\,\text{arctanh}\left(\frac{\sqrt{a}x}{\sqrt{ax^2+bx^3}}\right)}{a^{3/2}}-\frac{\sqrt{ax^2+bx^3}}{ax^2}\right)}{4a}-\frac{\sqrt{ax^2+bx^3}}{2ax^3}\right)}{6a}-\frac{\sqrt{ax^2+bx^3}}{3ax^4}\right)}{a}+\frac{2}{ax^2\sqrt{ax^2+bx^3}}$$

```
Int[1/(x*(a*x^2 + b*x^3)^(3/2)),x]
```

```
2/(a*x^2*Sqrt[a*x^2 + b*x^3]) + (7*(-1/3*Sqrt[a*x^2 + b*x^3]/(a*x^4) - (5*
b*(-1/2*Sqrt[a*x^2 + b*x^3]/(a*x^3) - (3*b*(-(Sqrt[a*x^2 + b*x^3]/(a*x^2))
 + (b*ArcTanh[(Sqrt[a]*x)/Sqrt[a*x^2 + b*x^3]])/a^(3/2)))/(4*a)))/(6*a)))/
a
```

Defintions of rubi rules used

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

```
Int[1/Sqrt[(a_.)*(x_)^2 + (b_.)*(x_)^(n_.)], x_Symbol] :> Simp[2/(2 - n)
Subst[Int[1/(1 - a*x^2), x], x, x/Sqrt[a*x^2 + b*x^n]], x] /; FreeQ[{a, b,
n}, x] && NeQ[n, 2]
```

```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> Simp[(-c^(j - 1))*(c*x)^(m - j + 1)*((a*x^j + b*x^n)^(p + 1)/(a*(n - j
)*(p + 1))), x] + Simp[c^j*((m + n*p + n - j + 1)/(a*(n - j)*(p + 1)))   In
t[(c*x)^(m - j)*(a*x^j + b*x^n)^(p + 1), x], x] /; FreeQ[{a, b, c, m}, x] &
&  !IntegerQ[p] && LtQ[0, j, n] && (IntegersQ[j, n] || GtQ[c, 0]) && LtQ[p,
 -1]
```

```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> Simp[c^(j - 1)*(c*x)^(m - j + 1)*((a*x^j + b*x^n)^(p + 1)/(a*(m + j*p
+ 1))), x] - Simp[b*((m + n*p + n - j + 1)/(a*c^(n - j)*(m + j*p + 1)))   I
nt[(c*x)^(m + n - j)*(a*x^j + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, p}, x]
 &&  !IntegerQ[p] && LtQ[0, j, n] && (IntegersQ[j, n] || GtQ[c, 0]) && LtQ[
m + j*p + 1, 0]
```

Maple [A] (verified)

Time = 0.41 (sec) , antiderivative size = 31, normalized size of antiderivative = 0.22

method	result	size
pseudoelliptic	$\frac{2}{a\sqrt{bx+a}}-\frac{2\,\operatorname{arctanh}\left(\frac{\sqrt{bx+a}}{\sqrt{a}}\right)}{a^{\frac{3}{2}}}$	31
default	$\frac{(bx+a)\left(105\sqrt{bx+a}\,\operatorname{arctanh}\left(\frac{\sqrt{bx+a}}{\sqrt{a}}\right)b^{3}x^{3}+14a^{\frac{5}{2}}bx-35a^{\frac{3}{2}}b^{2}x^{2}-105\sqrt{a}\,b^{3}x^{3}-8a^{\frac{7}{2}}\right)}{24(b\,x^{3}+a\,x^{2})^{\frac{3}{2}}a^{\frac{9}{2}}}$	86
risch	$-\frac{(bx+a)(57b^{2}x^{2}-22abx+8a^{2})}{24a^{4}x^{2}\sqrt{x^{2}(bx+a)}}-\frac{b^{3}\left(-\frac{70\,\operatorname{arctanh}\left(\frac{\sqrt{bx+a}}{\sqrt{a}}\right)}{\sqrt{a}}+\frac{32}{\sqrt{bx+a}}\right)\sqrt{bx+a}\,x}{16a^{4}\sqrt{x^{2}(bx+a)}}$	99

```
int(1/x/(b*x^3+a*x^2)^(3/2),x,method=_RETURNVERBOSE)
```

```
2/a/(b*x+a)^(1/2)-2/a^(3/2)*arctanh((b*x+a)^(1/2)/a^(1/2))
```

Fricas [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 246, normalized size of antiderivative = 1.76

$$\int \frac{1}{x\left(ax^2+bx^3\right)^{3/2}}\,dx = \left[\frac{105\left(b^4x^5+ab^3x^4\right)\sqrt{a}\log\left(\frac{bx^2+2\,ax+2\,\sqrt{bx^3+ax^2}\sqrt{a}}{x^2}\right)-2\left(105\,ab^3x^3+35\,a^2b^2x^2-1\right.}{48\left(a^5bx^5+a^6x^4\right)}\right.$$
$$\left.-\frac{105\left(b^4x^5+ab^3x^4\right)\sqrt{-a}\arctan\left(\frac{\sqrt{bx^3+ax^2}\sqrt{-a}}{bx^2+ax}\right)+\left(105\,ab^3x^3+35\,a^2b^2x^2-14\,a^3bx+8\,a^4\right)\sqrt{bx^3+ax^2}}{24\left(a^5bx^5+a^6x^4\right)}\right]$$

```
integrate(1/x/(b*x^3+a*x^2)^(3/2),x, algorithm="fricas")
```

```
[1/48*(105*(b^4*x^5 + a*b^3*x^4)*sqrt(a)*log((b*x^2 + 2*a*x + 2*sqrt(b*x^3
 + a*x^2)*sqrt(a))/x^2) - 2*(105*a*b^3*x^3 + 35*a^2*b^2*x^2 - 14*a^3*b*x +
 8*a^4)*sqrt(b*x^3 + a*x^2))/(a^5*b*x^5 + a^6*x^4), -1/24*(105*(b^4*x^5 +
a*b^3*x^4)*sqrt(-a)*arctan(sqrt(b*x^3 + a*x^2)*sqrt(-a)/(b*x^2 + a*x)) + (
105*a*b^3*x^3 + 35*a^2*b^2*x^2 - 14*a^3*b*x + 8*a^4)*sqrt(b*x^3 + a*x^2))/
(a^5*b*x^5 + a^6*x^4)]
```

Sympy [F]

$$\int \frac{1}{x\left(ax^2+bx^3\right)^{3/2}}\,dx = \int \frac{1}{x\left(x^2\left(a+bx\right)\right)^{\frac{3}{2}}}\,dx$$

```
integrate(1/x/(b*x**3+a*x**2)**(3/2),x)
```

```
Integral(1/(x*(x**2*(a + b*x))**(3/2)), x)
```

Maxima [F]

$$\int \frac{1}{x\left(ax^2+bx^3\right)^{3/2}}\,dx=\int \frac{1}{\left(bx^3+ax^2\right)^{\frac{3}{2}}x}\,dx$$

```
integrate(1/x/(b*x^3+a*x^2)^(3/2),x, algorithm="maxima")
```

```
integrate(1/((b*x^3 + a*x^2)^(3/2)*x), x)
```

Giac [A] (verification not implemented)

Time = 0.14 (sec) , antiderivative size = 107, normalized size of antiderivative = 0.76

$$\int \frac{1}{x\left(ax^2+bx^3\right)^{3/2}}\,dx=-\frac{35\,b^3\arctan\left(\frac{\sqrt{bx+a}}{\sqrt{-a}}\right)}{8\,\sqrt{-a}a^4\mathrm{sgn}\left(x\right)}-\frac{2\,b^3}{\sqrt{bx+a}a^4\mathrm{sgn}\left(x\right)}-\frac{57\,(bx+a)^{\frac{5}{2}}b^3-136\,(bx+a)^{\frac{3}{2}}ab^3+87\,\sqrt{bx+a}a^2b^3}{24\,a^4b^3x^3\mathrm{sgn}\left(x\right)}$$

```
integrate(1/x/(b*x^3+a*x^2)^(3/2),x, algorithm="giac")
```

```
-35/8*b^3*arctan(sqrt(b*x + a)/sqrt(-a))/(sqrt(-a)*a^4*sgn(x)) - 2*b^3/(sq
rt(b*x + a)*a^4*sgn(x)) - 1/24*(57*(b*x + a)^(5/2)*b^3 - 136*(b*x + a)^(3/
2)*a*b^3 + 87*sqrt(b*x + a)*a^2*b^3)/(a^4*b^3*x^3*sgn(x))
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{x\left(ax^2+bx^3\right)^{3/2}}\,dx=\int \frac{1}{x\,{\left(b\,x^3+a\,x^2\right)}^{3/2}}\,dx$$

```
int(1/(x*(a*x^2 + b*x^3)^(3/2)),x)
```

```
int(1/(x*(a*x^2 + b*x^3)^(3/2)), x)
```

Reduce [B] (verification not implemented)

Time = 0.49 (sec) , antiderivative size = 103, normalized size of antiderivative = 0.74

$$\int \frac{1}{x\,(ax^2+bx^3)^{3/2}}\,dx = \frac{-105\sqrt{a}\,\sqrt{bx+a}\log\left(\sqrt{bx+a}-\sqrt{a}\right)b^3x^3+105\sqrt{a}\,\sqrt{bx+a}\log\left(\sqrt{bx+a}+\sqrt{a}\right)}{48\sqrt{bx+a}\,a^5x^3}$$

```
int(1/x/(b*x^3+a*x^2)^(3/2),x)
```

```
( - 105*sqrt(a)*sqrt(a + b*x)*log(sqrt(a + b*x) - sqrt(a))*b**3*x**3 + 105
*sqrt(a)*sqrt(a + b*x)*log(sqrt(a + b*x) + sqrt(a))*b**3*x**3 - 16*a**4 +
28*a**3*b*x - 70*a**2*b**2*x**2 - 210*a*b**3*x**3)/(48*sqrt(a + b*x)*a**5*
x**3)
```

3.316 $\int \frac{1}{x^2\left(ax^2+bx^3\right)^{3/2}}\,dx$

Optimal result

Integrand size = 19, antiderivative size = 168

$$\int \frac{1}{x^2\left(ax^2+bx^3\right)^{3/2}}\,dx=\frac{105b^3}{64a^4\sqrt{ax^2+bx^3}}-\frac{1}{4ax^3\sqrt{ax^2+bx^3}}+\frac{3b}{8a^2x^2\sqrt{ax^2+bx^3}}-\frac{21b^2}{32a^3x\sqrt{ax^2+bx^3}}+\frac{315b^4x}{64a^5\sqrt{ax^2+bx^3}}-\frac{315b^4\text{arctanh}\left(\frac{\sqrt{ax^2+bx^3}}{\sqrt{a}x}\right)}{64a^{11/2}}$$

```
105/64*b^3/a^4/(b*x^3+a*x^2)^(1/2)-1/4/a/x^3/(b*x^3+a*x^2)^(1/2)+3/8*b/a^2
/x^2/(b*x^3+a*x^2)^(1/2)-21/32*b^2/a^3/x/(b*x^3+a*x^2)^(1/2)+315/64*b^4*x/
a^5/(b*x^3+a*x^2)^(1/2)-315/64*b^4*arctanh((b*x^3+a*x^2)^(1/2)/a^(1/2)/x)/
a^(11/2)
```

Mathematica [A] (verified)

Time = 0.16 (sec) , antiderivative size = 106, normalized size of antiderivative = 0.63

$$\int \frac{1}{x^2\left(ax^2+bx^3\right)^{3/2}}\,dx=\frac{\sqrt{a}(-16a^4+24a^3bx-42a^2b^2x^2+105ab^3x^3+315b^4x^4)-315b^4x^4\sqrt{a+bx}\text{arcta}}{64a^{11/2}x^3\sqrt{x^2(a+bx)}}$$

```
Integrate[1/(x^2*(a*x^2 + b*x^3)^(3/2)),x]
```

```
(Sqrt[a]*(-16*a^4 + 24*a^3*b*x - 42*a^2*b^2*x^2 + 105*a*b^3*x^3 + 315*b^4*
x^4) - 315*b^4*x^4*Sqrt[a + b*x]*ArcTanh[Sqrt[a + b*x]/Sqrt[a]])/(64*a^(11
/2)*x^3*Sqrt[x^2*(a + b*x)])
```

Rubi [A] (verified)

Time = 0.68 (sec) , antiderivative size = 185, normalized size of antiderivative = 1.10, number of steps used = 8, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}} = 0.368$, Rules used = {1929, 1931, 1931, 1931, 1931, 1914, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{x^2 (ax^2+bx^3)^{3/2}} \, dx$$

$$\Big\downarrow \text{1929}$$

$$\frac{9 \int \frac{1}{x^4\sqrt{bx^3+ax^2}}dx}{a}+\frac{2}{ax^3\sqrt{ax^2+bx^3}}$$

$$\Big\downarrow \text{1931}$$

$$\frac{9 \left(-\frac{7b \int \frac{1}{x^3\sqrt{bx^3+ax^2}}dx}{8a}-\frac{\sqrt{ax^2+bx^3}}{4ax^5}\right)}{a}+\frac{2}{ax^3\sqrt{ax^2+bx^3}}$$

$$\Big\downarrow \text{1931}$$

$$\frac{9 \left(-\frac{7b \left(-\frac{5b \int \frac{1}{x^2\sqrt{bx^3+ax^2}}dx}{6a}-\frac{\sqrt{ax^2+bx^3}}{3ax^4}\right)}{8a}-\frac{\sqrt{ax^2+bx^3}}{4ax^5}\right)}{a}+\frac{2}{ax^3\sqrt{ax^2+bx^3}}$$

$$\Big\downarrow \text{1931}$$

$$\frac{9\left(-\frac{7b\left(-\frac{5b\left(-\frac{3b\int\frac{1}{x\sqrt{bx^3+ax^2}}dx}{4a}-\frac{\sqrt{ax^2+bx^3}}{2ax^3}\right)}{6a}-\frac{\sqrt{ax^2+bx^3}}{3ax^4}\right)}{8a}-\frac{\sqrt{ax^2+bx^3}}{4ax^5}\right)}{a}+\frac{2}{ax^3\sqrt{ax^2+bx^3}}$$

$$\Big\downarrow \text{1931}$$

$$\frac{9\left(-\frac{7b\left(-\frac{5b\left(-\frac{3b\left(-\frac{b\int\frac{1}{\sqrt{bx^3+ax^2}}dx}{2a}-\frac{\sqrt{ax^2+bx^3}}{ax^2}\right)}{4a}-\frac{\sqrt{ax^2+bx^3}}{2ax^3}\right)}{6a}-\frac{\sqrt{ax^2+bx^3}}{3ax^4}\right)}{8a}-\frac{\sqrt{ax^2+bx^3}}{4ax^5}\right)}{a}+\frac{2}{ax^3\sqrt{ax^2+bx^3}}$$

$$\Big\downarrow \text{1914}$$

$$\frac{9\left(-\frac{7b\left(-\frac{5b\left(-\frac{3b\left(\frac{b\int \frac{1}{1-\frac{ax^2}{bx^3+ax^2}}d\frac{x}{\sqrt{bx^3+ax^2}}}{a}-\frac{\sqrt{ax^2+bx^3}}{ax^2}\right)}{4a}-\frac{\sqrt{ax^2+bx^3}}{2ax^3}\right)}{6a}-\frac{\sqrt{ax^2+bx^3}}{3ax^4}\right)}{8a}-\frac{\sqrt{ax^2+bx^3}}{4ax^5}\right)}{a}+$$

$$\frac{2}{ax^3\sqrt{ax^2+bx^3}}$$

$$\Big\downarrow \; 219$$

$$\frac{9\left(-\frac{7b\left(-\frac{5b\left(-\frac{3b\left(\frac{b\operatorname{arctanh}\left(\frac{\sqrt{a}x}{\sqrt{ax^2+bx^3}}\right)}{a^{3/2}}-\frac{\sqrt{ax^2+bx^3}}{ax^2}\right)}{4a}-\frac{\sqrt{ax^2+bx^3}}{2ax^3}\right)}{6a}-\frac{\sqrt{ax^2+bx^3}}{3ax^4}\right)}{8a}-\frac{\sqrt{ax^2+bx^3}}{4ax^5}\right)}{a}+\frac{2}{ax^3\sqrt{ax^2+bx^3}}$$

```
Int[1/(x^2*(a*x^2 + b*x^3)^(3/2)),x]
```

```
2/(a*x^3*Sqrt[a*x^2 + b*x^3]) + (9*(-1/4*Sqrt[a*x^2 + b*x^3]/(a*x^5) - (7*
b*(-1/3*Sqrt[a*x^2 + b*x^3]/(a*x^4) - (5*b*(-1/2*Sqrt[a*x^2 + b*x^3]/(a*x^
3) - (3*b*(-(Sqrt[a*x^2 + b*x^3]/(a*x^2)) + (b*ArcTanh[(Sqrt[a]*x)/Sqrt[a*
x^2 + b*x^3]])/a^(3/2)))/(4*a)))/(6*a)))/(8*a)))/a
```

Defintions of rubi rules used

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

```
Int[1/Sqrt[(a_.)*(x_)^2 + (b_.)*(x_)^(n_.)], x_Symbol] :> Simp[2/(2 - n)
Subst[Int[1/(1 - a*x^2), x], x, x/Sqrt[a*x^2 + b*x^n]], x] /; FreeQ[{a, b,
n}, x] && NegQ[n, 2]
```

```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> Simp[(-c^(j - 1))*(c*x)^(m - j + 1)*((a*x^j + b*x^n)^(p + 1)/(a*(n - j
)*(p + 1))), x] + Simp[c^j*((m + n*p + n - j + 1)/(a*(n - j)*(p + 1)))   In
t[(c*x)^(m - j)*(a*x^j + b*x^n)^(p + 1), x], x] /; FreeQ[{a, b, c, m}, x] &
&  !IntegerQ[p] && LtQ[0, j, n] && (IntegersQ[j, n] || GtQ[c, 0]) && LtQ[p,
 -1]
```

```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> Simp[c^(j - 1)*(c*x)^(m - j + 1)*((a*x^j + b*x^n)^(p + 1)/(a*(m + j*p
+ 1))), x] - Simp[b*((m + n*p + n - j + 1)/(a*c^(n - j)*(m + j*p + 1)))   I
nt[(c*x)^(m + n - j)*(a*x^j + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, p}, x]
 &&  !IntegerQ[p] && LtQ[0, j, n] && (IntegersQ[j, n] || GtQ[c, 0]) && LtQ[
m + j*p + 1, 0]
```

Maple [A] (verified)

Time = 0.42 (sec) , antiderivative size = 50, normalized size of antiderivative = 0.30

method	result	size
pseudoelliptic	$-\frac{-3\sqrt{bx+a}\,\operatorname{arctanh}\left(\frac{\sqrt{bx+a}}{\sqrt{a}}\right)bx+\sqrt{a}\,(3bx+a)}{\sqrt{bx+a}\,x\,a^{\frac{5}{2}}}$	50
default	$-\frac{(bx+a)\left(315\sqrt{bx+a}\,\operatorname{arctanh}\left(\frac{\sqrt{bx+a}}{\sqrt{a}}\right)b^4x^4-24a^{\frac{7}{2}}bx+42a^{\frac{5}{2}}b^2x^2-105a^{\frac{3}{2}}b^3x^3-315\sqrt{a}\,b^4x^4+16a^{\frac{9}{2}}\right)}{64x(b\,x^3+a\,x^2)^{\frac{3}{2}}a^{\frac{11}{2}}}$	100
risch	$-\frac{(bx+a)(-187b^3x^3+82a\,b^2x^2-40a^2bx+16a^3)}{64a^5x^3\sqrt{x^2(bx+a)}}+\frac{b^4\left(-\frac{630\,\operatorname{arctanh}\left(\frac{\sqrt{bx+a}}{\sqrt{a}}\right)}{\sqrt{a}}+\frac{256}{\sqrt{bx+a}}\right)\sqrt{bx+a}\,x}{128a^5\sqrt{x^2(bx+a)}}$	110

```
int(1/x^2/(b*x^3+a*x^2)^(3/2),x,method=_RETURNVERBOSE)
```

```
-1/(b*x+a)^(1/2)*(-3*(b*x+a)^(1/2)*arctanh((b*x+a)^(1/2)/a^(1/2))*b*x+a^(1
/2)*(3*b*x+a))/x/a^(5/2)
```

Fricas [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 268, normalized size of antiderivative = 1.60

$$\int \frac{1}{x^2 (ax^2 + bx^3)^{3/2}} \, dx = \left[\frac{315 (b^5x^6 + ab^4x^5)\sqrt{a} \log \left(\frac{bx^2 + 2\,ax - 2\,\sqrt{bx^3 + ax^2}\sqrt{a}}{x^2} \right) + 2 \left(315\,ab^4x^4 + 105\,a^2b^3x^3 - \right.}{128 \left(a^6bx^6 + a^7x^5\right)} \right.$$

```
integrate(1/x^2/(b*x^3+a*x^2)^(3/2),x, algorithm="fricas")
```

```
[1/128*(315*(b^5*x^6 + a*b^4*x^5)*sqrt(a)*log((b*x^2 + 2*a*x - 2*sqrt(b*x^
3 + a*x^2)*sqrt(a))/x^2) + 2*(315*a*b^4*x^4 + 105*a^2*b^3*x^3 - 42*a^3*b^2
*x^2 + 24*a^4*b*x - 16*a^5)*sqrt(b*x^3 + a*x^2))/(a^6*b*x^6 + a^7*x^5), 1/
64*(315*(b^5*x^6 + a*b^4*x^5)*sqrt(-a)*arctan(sqrt(b*x^3 + a*x^2)*sqrt(-a)
/(b*x^2 + a*x)) + (315*a*b^4*x^4 + 105*a^2*b^3*x^3 - 42*a^3*b^2*x^2 + 24*a
^4*b*x - 16*a^5)*sqrt(b*x^3 + a*x^2))/(a^6*b*x^6 + a^7*x^5)]
```

Sympy [F]

$$\int \frac{1}{x^2 (ax^2 + bx^3)^{3/2}} \, dx = \int \frac{1}{x^2 \left(x^2 (a + bx)\right)^{\frac{3}{2}}} \, dx$$

```
integrate(1/x**2/(b*x**3+a*x**2)**(3/2),x)
```

```
Integral(1/(x**2*(x**2*(a + b*x))**(3/2)), x)
```

Maxima [F]

$$\int \frac{1}{x^2 (ax^2 + bx^3)^{3/2}} \, dx = \int \frac{1}{(bx^3 + ax^2)^{\frac{3}{2}} x^2} \, dx$$

```
integrate(1/x^2/(b*x^3+a*x^2)^(3/2),x, algorithm="maxima")
```

```
integrate(1/((b*x^3 + a*x^2)^(3/2)*x^2), x)
```

Giac [A] (verification not implemented)

Time = 0.15 (sec) , antiderivative size = 122, normalized size of antiderivative = 0.73

$$\int \frac{1}{x^2 (ax^2+bx^3)^{3/2}}\,dx = \frac{315\,b^4 \arctan\left(\frac{\sqrt{bx+a}}{\sqrt{-a}}\right)}{64\,\sqrt{-a}a^5 \mathrm{sgn}\left(x\right)} + \frac{2\,b^4}{\sqrt{bx+a}a^5 \mathrm{sgn}\left(x\right)} + \frac{187\,(bx+a)^{\frac{7}{2}}b^4 - 643\,(bx+a)^{\frac{5}{2}}ab^4 + 765\,(bx+a)^{\frac{3}{2}}a^2b^4 - 325\,\sqrt{bx+a}a^3b^4}{64\,a^5b^4x^4 \mathrm{sgn}\left(x\right)}$$

```
integrate(1/x^2/(b*x^3+a*x^2)^(3/2),x, algorithm="giac")
```

```
315/64*b^4*arctan(sqrt(b*x + a)/sqrt(-a))/(sqrt(-a)*a^5*sgn(x)) + 2*b^4/(s
qrt(b*x + a)*a^5*sgn(x)) + 1/64*(187*(b*x + a)^(7/2)*b^4 - 643*(b*x + a)^(
5/2)*a*b^4 + 765*(b*x + a)^(3/2)*a^2*b^4 - 325*sqrt(b*x + a)*a^3*b^4)/(a^5
*b^4*x^4*sgn(x))
```

Mupad [B] (verification not implemented)

Time = 9.34 (sec) , antiderivative size = 44, normalized size of antiderivative = 0.26

$$\int \frac{1}{x^2 (ax^2+bx^3)^{3/2}}\,dx = -\frac{2\left(\frac{a}{b\,x}+1\right)^{3/2}{}_2\mathrm{F}_1\left(\frac{3}{2},\frac{11}{2};\ \frac{13}{2};\ -\frac{a}{b\,x}\right)}{11\,x\,\left(b\,x^3+a\,x^2\right)^{3/2}}$$

```
int(1/(x^2*(a*x^2 + b*x^3)^(3/2)),x)
```

```
-(2*(a/(b*x) + 1)^(3/2)*hypergeom([3/2, 11/2], 13/2, -a/(b*x)))/(11*x*(a*x
^2 + b*x^3)^(3/2))
```

Reduce [B] (verification not implemented)

Time = 0.42 (sec) , antiderivative size = 114, normalized size of antiderivative = 0.68

$$\int \frac{1}{x^2 (ax^2+bx^3)^{3/2}}\,dx = \frac{315\sqrt{a}\,\sqrt{bx+a}\log\left(\sqrt{bx+a}-\sqrt{a}\right)b^4x^4 - 315\sqrt{a}\,\sqrt{bx+a}\log\left(\sqrt{bx+a}+\sqrt{a}\right)}{128\sqrt{bx+a}\,a^6x^4}$$

```
int(1/x^2/(b*x^3+a*x^2)^(3/2),x)
```

```
(315*sqrt(a)*sqrt(a + b*x)*log(sqrt(a + b*x) - sqrt(a))*b**4*x**4 - 315*sq
rt(a)*sqrt(a + b*x)*log(sqrt(a + b*x) + sqrt(a))*b**4*x**4 - 32*a**5 + 48*
a**4*b*x - 84*a**3*b**2*x**2 + 210*a**2*b**3*x**3 + 630*a*b**4*x**4)/(128*
sqrt(a + b*x)*a**6*x**4)
```

3.317 $\int \frac{x^8}{(ax^2+bx^3)^{5/2}} \, dx$

Optimal result

Integrand size = 19, antiderivative size = 102

$$\int \frac{x^8}{(ax^2+bx^3)^{5/2}} \, dx = \frac{2a^3x^3}{3b^4(ax^2+bx^3)^{3/2}} - \frac{6a^2x}{b^4\sqrt{ax^2+bx^3}} - \frac{6a\sqrt{ax^2+bx^3}}{b^4x} + \frac{2(ax^2+bx^3)^{3/2}}{3b^4x^3}$$

```
2/3*a^3*x^3/b^4/(b*x^3+a*x^2)^(3/2)-6*a^2*x/b^4/(b*x^3+a*x^2)^(1/2)-6*a*(b
*x^3+a*x^2)^(1/2)/b^4/x+2/3*(b*x^3+a*x^2)^(3/2)/b^4/x^3
```

Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 52, normalized size of antiderivative = 0.51

$$\int \frac{x^8}{(ax^2+bx^3)^{5/2}} \, dx = \frac{2x^3(-16a^3-24a^2bx-6ab^2x^2+b^3x^3)}{3b^4(x^2(a+bx))^{3/2}}$$

```
Integrate[x^8/(a*x^2 + b*x^3)^(5/2),x]
```

```
(2*x^3*(-16*a^3 - 24*a^2*b*x - 6*a*b^2*x^2 + b^3*x^3))/(3*b^4*(x^2*(a + b*
x))^(3/2))
```

Rubi [A] (verified)

Time = 0.47 (sec) , antiderivative size = 109, normalized size of antiderivative = 1.07, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.211, Rules used = {1921, 1921, 1922, 1920}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^8}{(ax^2+bx^3)^{5/2}}dx$$

$$\Big\downarrow \text{1921}$$

$$\frac{2\int \frac{x^5}{(bx^3+ax^2)^{3/2}}dx}{b}-\frac{2x^6}{3b\left(ax^2+bx^3\right)^{3/2}}$$

$$\Big\downarrow \text{1921}$$

$$\frac{2\left(\frac{4\int \frac{x^2}{\sqrt{bx^3+ax^2}}dx}{b}-\frac{2x^3}{b\sqrt{ax^2+bx^3}}\right)}{b}-\frac{2x^6}{3b\left(ax^2+bx^3\right)^{3/2}}$$

$$\Big\downarrow \text{1922}$$

$$\frac{2\left(\frac{4\left(\frac{2\sqrt{ax^2+bx^3}}{3b}-\frac{2a\int \frac{x}{\sqrt{bx^3+ax^2}}dx}{3b}\right)}{b}-\frac{2x^3}{b\sqrt{ax^2+bx^3}}\right)}{b}-\frac{2x^6}{3b\left(ax^2+bx^3\right)^{3/2}}$$

$$\Big\downarrow \text{1920}$$

$$\frac{2\left(\frac{4\left(\frac{2\sqrt{ax^2+bx^3}}{3b}-\frac{4a\sqrt{ax^2+bx^3}}{3b^2x}\right)}{b}-\frac{2x^3}{b\sqrt{ax^2+bx^3}}\right)}{b}-\frac{2x^6}{3b\left(ax^2+bx^3\right)^{3/2}}$$

```
Int[x^8/(a*x^2 + b*x^3)^(5/2),x]
```

```
(-2*x^6)/(3*b*(a*x^2 + b*x^3)^(3/2)) + (2*((-2*x^3)/(b*Sqrt[a*x^2 + b*x^3]
) + (4*((2*Sqrt[a*x^2 + b*x^3])/(3*b) - (4*a*Sqrt[a*x^2 + b*x^3])/(3*b^2*x
)))/b))/b
```

Defintions of rubi rules used

```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> Simp[(-c^(j - 1))*(c*x)^(m - j + 1)*((a*x^j + b*x^n)^(p + 1)/(a*(n - j
)*(p + 1))), x] /; FreeQ[{a, b, c, j, m, n, p}, x] &&  !IntegerQ[p] && NeQ[
n, j] && EqQ[m + n*p + n - j + 1, 0] && (IntegerQ[j] || GtQ[c, 0])
```

```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> Simp[(-c^(j - 1))*(c*x)^(m - j + 1)*((a*x^j + b*x^n)^(p + 1)/(a*(n - j
)*(p + 1))), x] + Simp[c^j*((m + n*p + n - j + 1)/(a*(n - j)*(p + 1)))   In
t[(c*x)^(m - j)*(a*x^j + b*x^n)^(p + 1), x], x] /; FreeQ[{a, b, c, j, m, n}
, x] &&  !IntegerQ[p] && NeQ[n, j] && ILtQ[Simplify[(m + n*p + n - j + 1)/(
n - j)], 0] && LtQ[p, -1] && (IntegerQ[j] || GtQ[c, 0])
```

```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> Simp[c^(j - 1)*(c*x)^(m - j + 1)*((a*x^j + b*x^n)^(p + 1)/(a*(m + j*p
+ 1))), x] - Simp[b*((m + n*p + n - j + 1)/(a*c^(n - j)*(m + j*p + 1)))   I
nt[(c*x)^(m + n - j)*(a*x^j + b*x^n)^p, x], x] /; FreeQ[{a, b, c, j, m, n,
p}, x] &&  !IntegerQ[p] && NeQ[n, j] && ILtQ[Simplify[(m + n*p + n - j + 1)
/(n - j)], 0] && NeQ[m + j*p + 1, 0] && (IntegersQ[j, n] || GtQ[c, 0])
```

Maple [A] (verified)

Time = 0.50 (sec) , antiderivative size = 57, normalized size of antiderivative = 0.56

method	result	size
gosper	$-\frac{2(bx+a)\left(-b^{3} x^{3}+6 a \,b^{2} x^{2}+24 a^{2} b x +16 a^{3}\right) x^{5}}{3 b^{4} \left(b \,x^{3}+a \,x^{2}\right)^{\frac{5}{2}}}$	57
default	$-\frac{2(bx+a)\left(-b^{3} x^{3}+6 a \,b^{2} x^{2}+24 a^{2} b x +16 a^{3}\right) x^{5}}{3 b^{4} \left(b \,x^{3}+a \,x^{2}\right)^{\frac{5}{2}}}$	57
orering	$-\frac{2(bx+a)\left(-b^{3} x^{3}+6 a \,b^{2} x^{2}+24 a^{2} b x +16 a^{3}\right) x^{5}}{3 b^{4} \left(b \,x^{3}+a \,x^{2}\right)^{\frac{5}{2}}}$	57
trager	$-\frac{2\left(-b^{3} x^{3}+6 a \,b^{2} x^{2}+24 a^{2} b x +16 a^{3}\right) \sqrt{b \,x^{3}+a \,x^{2}}}{3 x \,b^{4} \left(b x +a \right)^{2}}$	59
risch	$-\frac{2\left(-b x +8 a \right) \left(b x +a \right) x}{3 b^{4} \sqrt{x^{2} \left(b x +a \right)}}-\frac{2 a^{2} \left(9 b x +8 a \right) x}{3 b^{4} \left(b x +a \right) \sqrt{x^{2} \left(b x +a \right)}}$	67
pseudoelliptic	$\frac{\frac{2}{13} b^{8} x^{8}-\frac{32}{143} a \,b^{7} x^{7}+\frac{448}{1287} a^{2} b^{6} x^{6}-\frac{256}{429} a^{3} b^{5} x^{5}+\frac{512}{429} a^{4} b^{4} x^{4}-\frac{4096}{1287} a^{5} b^{3} x^{3}+\frac{8192}{429} a^{6} b^{2} x^{2}+\frac{32768}{429} a^{7} b x +\frac{65536}{1287} a^{8}}{b^{9} \left(b x +a \right)^{\frac{3}{2}}}$	98

```
int(x^8/(b*x^3+a*x^2)^(5/2),x,method=_RETURNVERBOSE)
```

```
-2/3*(b*x+a)*(-b^3*x^3+6*a*b^2*x^2+24*a^2*b*x+16*a^3)*x^5/b^4/(b*x^3+a*x^2
)^(5/2)
```

Fricas [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 71, normalized size of antiderivative = 0.70

$$\int \frac{x^8}{(ax^2+bx^3)^{5/2}}\,dx = \frac{2\,(b^3x^3-6\,ab^2x^2-24\,a^2bx-16\,a^3)\sqrt{bx^3+ax^2}}{3\,(b^6x^3+2\,ab^5x^2+a^2b^4x)}$$

```
integrate(x^8/(b*x^3+a*x^2)^(5/2),x, algorithm="fricas")
```

```
2/3*(b^3*x^3 - 6*a*b^2*x^2 - 24*a^2*b*x - 16*a^3)*sqrt(b*x^3 + a*x^2)/(b^6
*x^3 + 2*a*b^5*x^2 + a^2*b^4*x)
```

Sympy [F]

$$\int \frac{x^8}{(ax^2+bx^3)^{5/2}}\,dx = \int \frac{x^8}{(x^2(a+bx))^{\frac{5}{2}}}\,dx$$

```
integrate(x**8/(b*x**3+a*x**2)**(5/2),x)
```

```
Integral(x**8/(x**2*(a + b*x))**(5/2), x)
```

Maxima [A] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 51, normalized size of antiderivative = 0.50

$$\int \frac{x^8}{(ax^2+bx^3)^{5/2}}\,dx = \frac{2\,(b^3x^3-6\,ab^2x^2-24\,a^2bx-16\,a^3)}{3\,(b^5x+ab^4)\sqrt{bx+a}}$$

```
integrate(x^8/(b*x^3+a*x^2)^(5/2),x, algorithm="maxima")
```

```
2/3*(b^3*x^3 - 6*a*b^2*x^2 - 24*a^2*b*x - 16*a^3)/((b^5*x + a*b^4)*sqrt(b*
x + a))
```

Giac [A] (verification not implemented)

Time = 0.24 (sec) , antiderivative size = 77, normalized size of antiderivative = 0.75

$$\int \frac{x^8}{(ax^2+bx^3)^{5/2}}\,dx = \frac{32\,a^{\frac{3}{2}}\mathrm{sgn}(x)}{3\,b^4} - \frac{2\,(9\,(bx+a)a^2-a^3)}{3\,(bx+a)^{\frac{3}{2}}b^4\mathrm{sgn}\,(x)} + \frac{2\left((bx+a)^{\frac{3}{2}}b^8-9\,\sqrt{bx+a}ab^8\right)}{3\,b^{12}\mathrm{sgn}\,(x)}$$

```
integrate(x^8/(b*x^3+a*x^2)^(5/2),x, algorithm="giac")
```

```
32/3*a^(3/2)*sgn(x)/b^4 - 2/3*(9*(b*x + a)*a^2 - a^3)/((b*x + a)^(3/2)*b^4
*sgn(x)) + 2/3*((b*x + a)^(3/2)*b^8 - 9*sqrt(b*x + a)*a*b^8)/(b^12*sgn(x))
```

Mupad [B] (verification not implemented)

Time = 9.19 (sec) , antiderivative size = 58, normalized size of antiderivative = 0.57

$$\int \frac{x^8}{(ax^2+bx^3)^{5/2}}\,dx = -\frac{2\sqrt{b\,x^3+a\,x^2}\,\left(16\,a^3+24\,a^2\,b\,x+6\,a\,b^2\,x^2-b^3\,x^3\right)}{3\,b^4\,x\,{\left(a+b\,x\right)}^2}$$

```
int(x^8/(a*x^2 + b*x^3)^(5/2),x)
```

```
-(2*(a*x^2 + b*x^3)^(1/2)*(16*a^3 - b^3*x^3 + 6*a*b^2*x^2 + 24*a^2*b*x))/(
3*b^4*x*(a + b*x)^2)
```

Reduce [B] (verification not implemented)

Time = 0.49 (sec) , antiderivative size = 49, normalized size of antiderivative = 0.48

$$\int \frac{x^8}{(ax^2+bx^3)^{5/2}}\,dx = \frac{\frac{2}{3}b^3x^3-4a\,b^2x^2-16a^2bx-\frac{32}{3}a^3}{\sqrt{bx+a}\,b^4\,(bx+a)}$$

```
int(x^8/(b*x^3+a*x^2)^(5/2),x)
```

```
(2*( - 16*a**3 - 24*a**2*b*x - 6*a*b**2*x**2 + b**3*x**3))/(3*sqrt(a + b*x
)*b**4*(a + b*x))
```

3.318 $\int \frac{x^7}{\left(ax^2+bx^3\right)^{5/2}}\,dx$

Optimal result

Integrand size = 19, antiderivative size = 74

$$\int \frac{x^7}{(ax^2+bx^3)^{5/2}}\,dx=-\frac{2a^2x^3}{3b^3\,(ax^2+bx^3)^{3/2}}+\frac{4ax}{b^3\sqrt{ax^2+bx^3}}+\frac{2\sqrt{ax^2+bx^3}}{b^3x}$$

```
-2/3*a^2*x^3/b^3/(b*x^3+a*x^2)^(3/2)+4*a*x/b^3/(b*x^3+a*x^2)^(1/2)+2*(b*x^
3+a*x^2)^(1/2)/b^3/x
```

Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 42, normalized size of antiderivative = 0.57

$$\int \frac{x^7}{(ax^2+bx^3)^{5/2}}\,dx=\frac{2x^3(8a^2+12abx+3b^2x^2)}{3b^3\,(x^2(a+bx))^{3/2}}$$

```
Integrate[x^7/(a*x^2 + b*x^3)^(5/2),x]
```

```
(2*x^3*(8*a^2 + 12*a*b*x + 3*b^2*x^2))/(3*b^3*(x^2*(a + b*x))^(3/2))
```

Rubi [A] (verified)

Time = 0.39 (sec) , antiderivative size = 80, normalized size of antiderivative = 1.08, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.158, Rules used = {1921, 1921, 1920}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^7}{(ax^2+bx^3)^{5/2}}\,dx$$

$$\Big\downarrow \text{1921}$$

$$\frac{4\int \frac{x^4}{(bx^3+ax^2)^{3/2}}dx}{3b}-\frac{2x^5}{3b\,(ax^2+bx^3)^{3/2}}$$

$$\Big\downarrow \text{1921}$$

$$\frac{4\left(\frac{2\int \frac{x}{\sqrt{bx^3+ax^2}}dx}{b}-\frac{2x^2}{b\sqrt{ax^2+bx^3}}\right)}{3b}-\frac{2x^5}{3b\,(ax^2+bx^3)^{3/2}}$$

$$\Big\downarrow \text{1920}$$

$$\frac{4\left(\frac{4\sqrt{ax^2+bx^3}}{b^2x}-\frac{2x^2}{b\sqrt{ax^2+bx^3}}\right)}{3b}-\frac{2x^5}{3b\,(ax^2+bx^3)^{3/2}}$$

```
Int[x^7/(a*x^2 + b*x^3)^(5/2),x]
```

```
(-2*x^5)/(3*b*(a*x^2 + b*x^3)^(3/2)) + (4*((-2*x^2)/(b*Sqrt[a*x^2 + b*x^3]
) + (4*Sqrt[a*x^2 + b*x^3])/(b^2*x)))/(3*b)
```

Defintions of rubi rules used

```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> Simp[(-c^(j - 1))*(c*x)^(m - j + 1)*((a*x^j + b*x^n)^(p + 1)/(a*(n - j
)*(p + 1))), x] /; FreeQ[{a, b, c, j, m, n, p}, x] && !IntegerQ[p] && NeQ[
n, j] && EqQ[m + n*p + n - j + 1, 0] && (IntegerQ[j] || GtQ[c, 0])
```

```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> Simp[(-c^(j - 1))*(c*x)^(m - j + 1)*((a*x^j + b*x^n)^(p + 1)/(a*(n - j
)*(p + 1))), x] + Simp[c^j*((m + n*p + n - j + 1)/(a*(n - j)*(p + 1)))   In
t[(c*x)^(m - j)*(a*x^j + b*x^n)^(p + 1), x], x] /; FreeQ[{a, b, c, j, m, n}
, x] && !IntegerQ[p] && NeQ[n, j] && ILtQ[Simplify[(m + n*p + n - j + 1)/(
n - j)], 0] && LtQ[p, -1] && (IntegerQ[j] || GtQ[c, 0])
```

Maple [A] (verified)

Time = 0.47 (sec) , antiderivative size = 46, normalized size of antiderivative = 0.62

method	result	size
gosper	$\frac{2(bx+a)\left(3b^{2}x^{2}+12abx+8a^{2}\right)x^{5}}{3b^{3}\left(b\,x^{3}+a\,x^{2}\right)^{\frac{5}{2}}}$	46
default	$\frac{2(bx+a)\left(3b^{2}x^{2}+12abx+8a^{2}\right)x^{5}}{3b^{3}\left(b\,x^{3}+a\,x^{2}\right)^{\frac{5}{2}}}$	46
orering	$\frac{2(bx+a)\left(3b^{2}x^{2}+12abx+8a^{2}\right)x^{5}}{3b^{3}\left(b\,x^{3}+a\,x^{2}\right)^{\frac{5}{2}}}$	46
trager	$\frac{2\left(3b^{2}x^{2}+12abx+8a^{2}\right)\sqrt{b\,x^{3}+a\,x^{2}}}{3x\,b^{3}(bx+a)^{2}}$	48
risch	$\frac{2(bx+a)x}{b^{3}\sqrt{x^{2}(bx+a)}}+\frac{2a(6bx+5a)x}{3b^{3}(bx+a)\sqrt{x^{2}(bx+a)}}$	57
pseudoelliptic	$\frac{\frac{2}{11}x^{7}b^{7}-\frac{28}{99}b^{6}a\,x^{6}+\frac{16}{33}a^{2}x^{5}b^{5}-\frac{32}{33}b^{4}x^{4}a^{3}+\frac{256}{99}b^{3}x^{3}a^{4}-\frac{512}{33}x^{2}b^{2}a^{5}-\frac{2048}{33}xb\,a^{6}-\frac{4096}{99}a^{7}}{b^{8}(bx+a)^{\frac{3}{2}}}$	87

```
int(x^7/(b*x^3+a*x^2)^(5/2),x,method=_RETURNVERBOSE)
```

```
2/3*(b*x+a)*(3*b^2*x^2+12*a*b*x+8*a^2)*x^5/b^3/(b*x^3+a*x^2)^(5/2)
```

Fricas [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 61, normalized size of antiderivative = 0.82

$$\int \frac{x^7}{(ax^2+bx^3)^{5/2}}\,dx = \frac{2\,(3\,b^2x^2+12\,abx+8\,a^2)\sqrt{bx^3+ax^2}}{3\,(b^5x^3+2\,ab^4x^2+a^2b^3x)}$$

```
integrate(x^7/(b*x^3+a*x^2)^(5/2),x, algorithm="fricas")
```

```
2/3*(3*b^2*x^2 + 12*a*b*x + 8*a^2)*sqrt(b*x^3 + a*x^2)/(b^5*x^3 + 2*a*b^4*
x^2 + a^2*b^3*x)
```

Sympy [F]

$$\int \frac{x^7}{(ax^2+bx^3)^{5/2}}\,dx = \int \frac{x^7}{(x^2\,(a+bx))^{\frac{5}{2}}}\,dx$$

```
integrate(x**7/(b*x**3+a*x**2)**(5/2),x)
```

```
Integral(x**7/(x**2*(a + b*x))**(5/2), x)
```

Maxima [A] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 41, normalized size of antiderivative = 0.55

$$\int \frac{x^7}{(ax^2+bx^3)^{5/2}}\,dx = \frac{2\,(3\,b^2x^2+12\,abx+8\,a^2)}{3\,(b^4x+ab^3)\sqrt{bx+a}}$$

```
integrate(x^7/(b*x^3+a*x^2)^(5/2),x, algorithm="maxima")
```

```
2/3*(3*b^2*x^2 + 12*a*b*x + 8*a^2)/((b^4*x + a*b^3)*sqrt(b*x + a))
```

Giac [A] (verification not implemented)

Time = 0.20 (sec) , antiderivative size = 62, normalized size of antiderivative = 0.84

$$\int \frac{x^7}{\left(ax^2+bx^3\right)^{5/2}}\,dx = \frac{2\left(\frac{3\sqrt{bx+a}}{b\mathrm{sgn}(x)} + \frac{6\,(bx+a)a-a^2}{(bx+a)^{\frac{3}{2}}b\mathrm{sgn}(x)}\right)}{3\,b^2} - \frac{16\,\sqrt{a}\mathrm{sgn}(x)}{3\,b^3}$$

```
integrate(x^7/(b*x^3+a*x^2)^(5/2),x, algorithm="giac")
```

```
2/3*(3*sqrt(b*x + a)/(b*sgn(x)) + (6*(b*x + a)*a - a^2)/((b*x + a)^(3/2)*b
*sgn(x)))/b^2 - 16/3*sqrt(a)*sgn(x)/b^3
```

Mupad [B] (verification not implemented)

Time = 8.78 (sec) , antiderivative size = 82, normalized size of antiderivative = 1.11

$$\int \frac{x^7}{\left(ax^2+bx^3\right)^{5/2}}\,dx = \frac{2\,\sqrt{b\,x^3+a\,x^2}}{b^3\,x} - \frac{2\,a^2\,\sqrt{b\,x^3+a\,x^2}}{3\,b^3\,x\,{\left(a+b\,x\right)}^2} + \frac{4\,a\,\sqrt{b\,x^3+a\,x^2}}{b^3\,x\,\left(a+b\,x\right)}$$

```
int(x^7/(a*x^2 + b*x^3)^(5/2),x)
```

```
(2*(a*x^2 + b*x^3)^(1/2))/(b^3*x) - (2*a^2*(a*x^2 + b*x^3)^(1/2))/(3*b^3*x
*(a + b*x)^2) + (4*a*(a*x^2 + b*x^3)^(1/2))/(b^3*x*(a + b*x))
```

Reduce [B] (verification not implemented)

Time = 0.43 (sec) , antiderivative size = 39, normalized size of antiderivative = 0.53

$$\int \frac{x^7}{\left(ax^2+bx^3\right)^{5/2}}\,dx = \frac{2b^2x^2+8abx+\frac{16}{3}a^2}{\sqrt{bx+a}\,b^3\,(bx+a)}$$

```
int(x^7/(b*x^3+a*x^2)^(5/2),x)
```

```
(2*(8*a**2 + 12*a*b*x + 3*b**2*x**2))/(3*sqrt(a + b*x)*b**3*(a + b*x))
```

3.319 $\int \frac{x^6}{(ax^2+bx^3)^{5/2}}\,dx$

Optimal result

Integrand size = 19, antiderivative size = 48

$$\int \frac{x^6}{(ax^2+bx^3)^{5/2}}\,dx = \frac{2ax^3}{3b^2(ax^2+bx^3)^{3/2}} - \frac{2x}{b^2\sqrt{ax^2+bx^3}}$$

```
2/3*a*x^3/b^2/(b*x^3+a*x^2)^(3/2)-2*x/b^2/(b*x^3+a*x^2)^(1/2)
```

Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 31, normalized size of antiderivative = 0.65

$$\int \frac{x^6}{(ax^2+bx^3)^{5/2}}\,dx = -\frac{2x^3(2a+3bx)}{3b^2(x^2(a+bx))^{3/2}}$$

```
Integrate[x^6/(a*x^2 + b*x^3)^(5/2),x]
```

```
(-2*x^3*(2*a + 3*b*x))/(3*b^2*(x^2*(a + b*x))^(3/2))
```

Rubi [A] (verified)

Time = 0.31 (sec) , antiderivative size = 49, normalized size of antiderivative = 1.02, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.105, Rules used = {1921, 1920}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^6}{(ax^2+bx^3)^{5/2}}\,dx$$

$$\Big\downarrow \text{ 1921}$$

$$\frac{2\int \frac{x^3}{(bx^3+ax^2)^{3/2}}dx}{3b}-\frac{2x^4}{3b\,(ax^2+bx^3)^{3/2}}$$

$$\Big\downarrow \text{ 1920}$$

$$-\frac{4x}{3b^2\sqrt{ax^2+bx^3}}-\frac{2x^4}{3b\,(ax^2+bx^3)^{3/2}}$$

```
Int[x^6/(a*x^2 + b*x^3)^(5/2),x]
```

```
(-2*x^4)/(3*b*(a*x^2 + b*x^3)^(3/2)) - (4*x)/(3*b^2*Sqrt[a*x^2 + b*x^3])
```

Defintions of rubi rules used

```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> Simp[(-c^(j - 1))*(c*x)^(m - j + 1)*((a*x^j + b*x^n)^(p + 1)/(a*(n - j
)*(p + 1))), x] /; FreeQ[{a, b, c, j, m, n, p}, x] &&  !IntegerQ[p] && NeQ[
n, j] && EqQ[m + n*p + n - j + 1, 0] && (IntegerQ[j] || GtQ[c, 0])
```

```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> Simp[(-c^(j - 1))*(c*x)^(m - j + 1)*((a*x^j + b*x^n)^(p + 1)/(a*(n - j
)*(p + 1))), x] + Simp[c^j*((m + n*p + n - j + 1)/(a*(n - j)*(p + 1)))   In
t[(c*x)^(m - j)*(a*x^j + b*x^n)^(p + 1), x], x] /; FreeQ[{a, b, c, j, m, n}
, x] &&  !IntegerQ[p] && NeQ[n, j] && ILtQ[Simplify[(m + n*p + n - j + 1)/(
n - j)], 0] && LtQ[p, -1] && (IntegerQ[j] || GtQ[c, 0])
```

Maple [A] (verified)

Time = 0.44 (sec) , antiderivative size = 35, normalized size of antiderivative = 0.73

method	result	size
gosper	$-\frac{2(bx+a)(3bx+2a)x^{5}}{3b^{2}\left(b\,x^{3}+a\,x^{2}\right)^{\frac{5}{2}}}$	35
default	$-\frac{2(bx+a)(3bx+2a)x^{5}}{3b^{2}\left(b\,x^{3}+a\,x^{2}\right)^{\frac{5}{2}}}$	35
orering	$-\frac{2(bx+a)(3bx+2a)x^{5}}{3b^{2}\left(b\,x^{3}+a\,x^{2}\right)^{\frac{5}{2}}}$	35
trager	$-\frac{2(3bx+2a)\sqrt{b\,x^{3}+a\,x^{2}}}{3x\,b^{2}\left(bx+a\right)^{2}}$	37
pseudoelliptic	$\frac{\frac{2}{9}b^{6}x^{6}-\frac{8}{21}a\,b^{5}x^{5}+\frac{16}{21}a^{2}b^{4}x^{4}-\frac{128}{63}a^{3}b^{3}x^{3}+\frac{256}{21}a^{4}b^{2}x^{2}+\frac{1024}{21}a^{5}bx+\frac{2048}{63}a^{6}}{b^{7}\left(bx+a\right)^{\frac{3}{2}}}$	76

```
int(x^6/(b*x^3+a*x^2)^(5/2),x,method=_RETURNVERBOSE)
```

```
-2/3*(b*x+a)*(3*b*x+2*a)*x^5/b^2/(b*x^3+a*x^2)^(5/2)
```

Fricas [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 50, normalized size of antiderivative = 1.04

$$\int \frac{x^6}{(ax^2+bx^3)^{5/2}}\,dx = -\frac{2\,\sqrt{bx^3+ax^2}(3\,bx+2\,a)}{3\,(b^4x^3+2\,ab^3x^2+a^2b^2x)}$$

```
integrate(x^6/(b*x^3+a*x^2)^(5/2),x, algorithm="fricas")
```

```
-2/3*sqrt(b*x^3 + a*x^2)*(3*b*x + 2*a)/(b^4*x^3 + 2*a*b^3*x^2 + a^2*b^2*x)
```

Sympy [F]

$$\int \frac{x^6}{(ax^2+bx^3)^{5/2}}\,dx = \int \frac{x^6}{(x^2\,(a+bx))^{\frac{5}{2}}}\,dx$$

```
integrate(x**6/(b*x**3+a*x**2)**(5/2),x)
```

```
Integral(x**6/(x**2*(a + b*x))**(5/2), x)
```

Maxima [A] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 30, normalized size of antiderivative = 0.62

$$\int \frac{x^6}{(ax^2+bx^3)^{5/2}}\,dx = -\frac{2\,(3\,bx+2\,a)}{3\,(b^3x+ab^2)\sqrt{bx+a}}$$

```
integrate(x^6/(b*x^3+a*x^2)^(5/2),x, algorithm="maxima")
```

```
-2/3*(3*b*x + 2*a)/((b^3*x + a*b^2)*sqrt(b*x + a))
```

Giac [A] (verification not implemented)

Time = 0.20 (sec) , antiderivative size = 35, normalized size of antiderivative = 0.73

$$\int \frac{x^6}{(ax^2+bx^3)^{5/2}}\,dx = \frac{4\,\mathrm{sgn}(x)}{3\,\sqrt{a}b^2} - \frac{2\,(3\,bx+2\,a)}{3\,(bx+a)^{\frac{3}{2}}b^2\mathrm{sgn}\,(x)}$$

```
integrate(x^6/(b*x^3+a*x^2)^(5/2),x, algorithm="giac")
```

```
4/3*sgn(x)/(sqrt(a)*b^2) - 2/3*(3*b*x + 2*a)/((b*x + a)^(3/2)*b^2*sgn(x))
```

Mupad [B] (verification not implemented)

Time = 8.68 (sec) , antiderivative size = 36, normalized size of antiderivative = 0.75

$$\int \frac{x^6}{(ax^2+bx^3)^{5/2}}\,dx = -\frac{2\,(2\,a+3\,b\,x)\,\sqrt{b\,x^3+a\,x^2}}{3\,b^2\,x\,(a+b\,x)^2}$$

```
int(x^6/(a*x^2 + b*x^3)^(5/2),x)
```

```
-(2*(2*a + 3*b*x)*(a*x^2 + b*x^3)^(1/2))/(3*b^2*x*(a + b*x)^2)
```

Reduce [B] (verification not implemented)

Time = 0.24 (sec) , antiderivative size = 28, normalized size of antiderivative = 0.58

$$\int \frac{x^6}{(ax^2+bx^3)^{5/2}}\,dx = \frac{-2bx-\frac{4a}{3}}{\sqrt{bx+a}\,b^2\,(bx+a)}$$

```
int(x^6/(b*x^3+a*x^2)^(5/2),x)
```

```
(2*( - 2*a - 3*b*x))/(3*sqrt(a + b*x)*b**2*(a + b*x))
```

3.320 $\int \frac{x^5}{\left(ax^2+bx^3\right)^{5/2}}\,dx$

Optimal result

Integrand size = 19, antiderivative size = 25

$$\int \frac{x^5}{\left(ax^2+bx^3\right)^{5/2}}\,dx=-\frac{2x^3}{3b\left(ax^2+bx^3\right)^{3/2}}$$

```
-2/3*x^3/b/(b*x^3+a*x^2)^(3/2)
```

Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 23, normalized size of antiderivative = 0.92

$$\int \frac{x^5}{\left(ax^2+bx^3\right)^{5/2}}\,dx=-\frac{2x^3}{3b\left(x^2(a+bx)\right)^{3/2}}$$

```
Integrate[x^5/(a*x^2 + b*x^3)^(5/2),x]
```

```
(-2*x^3)/(3*b*(x^2*(a + b*x))^(3/2))
```

Rubi [A] (verified)

Time = 0.25 (sec) , antiderivative size = 25, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.053, Rules used = {1920}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^5}{(ax^2+bx^3)^{5/2}} \, dx$$
$$\Big\downarrow \text{ 1920}$$
$$-\frac{2x^3}{3b\,(ax^2+bx^3)^{3/2}}$$

```
Int[x^5/(a*x^2 + b*x^3)^(5/2),x]
```

```
(-2*x^3)/(3*b*(a*x^2 + b*x^3)^(3/2))
```

Defintions of rubi rules used

```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> Simp[(-c^(j - 1))*(c*x)^(m - j + 1)*((a*x^j + b*x^n)^(p + 1)/(a*(n - j
)*(p + 1))), x] /; FreeQ[{a, b, c, j, m, n, p}, x] &&  !IntegerQ[p] && NeQ[
n, j] && EqQ[m + n*p + n - j + 1, 0] && (IntegerQ[j] || GtQ[c, 0])
```

Maple [A] (verified)

Time = 0.43 (sec) , antiderivative size = 27, normalized size of antiderivative = 1.08

method	result	size
gosper	$-\frac{2(bx+a)x^{5}}{3b(b\,x^{3}+a\,x^{2})^{\frac{5}{2}}}$	27
default	$-\frac{2(bx+a)x^{5}}{3b(b\,x^{3}+a\,x^{2})^{\frac{5}{2}}}$	27
orering	$-\frac{2(bx+a)x^{5}}{3b(b\,x^{3}+a\,x^{2})^{\frac{5}{2}}}$	27
trager	$-\frac{2\sqrt{b\,x^{3}+a\,x^{2}}}{3x(bx+a)^{2}b}$	29
pseudoelliptic	$\frac{\frac{2}{7}b^{5}x^{5}-\frac{4}{7}a\,b^{4}x^{4}+\frac{32}{21}a^{2}b^{3}x^{3}-\frac{64}{7}a^{3}b^{2}x^{2}-\frac{256}{7}a^{4}bx-\frac{512}{21}a^{5}}{b^{6}(bx+a)^{\frac{3}{2}}}$	65

```
int(x^5/(b*x^3+a*x^2)^(5/2),x,method=_RETURNVERBOSE)
```

```
-2/3*(b*x+a)/b*x^5/(b*x^3+a*x^2)^(5/2)
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 40, normalized size of antiderivative = 1.60

$$\int \frac{x^5}{(ax^2+bx^3)^{5/2}}\,dx = -\frac{2\sqrt{bx^3+ax^2}}{3\,(b^3x^3+2\,ab^2x^2+a^2bx)}$$

```
integrate(x^5/(b*x^3+a*x^2)^(5/2),x, algorithm="fricas")
```

```
-2/3*sqrt(b*x^3 + a*x^2)/(b^3*x^3 + 2*a*b^2*x^2 + a^2*b*x)
```

Sympy [F]

$$\int \frac{x^5}{(ax^2+bx^3)^{5/2}}\,dx = \int \frac{x^5}{(x^2\,(a+bx))^{\frac{5}{2}}}\,dx$$

```
integrate(x**5/(b*x**3+a*x**2)**(5/2),x)
```

```
Integral(x**5/(x**2*(a + b*x))**(5/2), x)
```

Maxima [A] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 20, normalized size of antiderivative = 0.80

$$\int \frac{x^5}{(ax^2+bx^3)^{5/2}}\,dx = -\frac{2}{3\,(b^2x+ab)\sqrt{bx+a}}$$

```
integrate(x^5/(b*x^3+a*x^2)^(5/2),x, algorithm="maxima")
```

```
-2/3/((b^2*x + a*b)*sqrt(b*x + a))
```

Giac [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 27, normalized size of antiderivative = 1.08

$$\int \frac{x^5}{(ax^2+bx^3)^{5/2}}\,dx = \frac{2\,\mathrm{sgn}(x)}{3\,a^{\frac{3}{2}}b} - \frac{2}{3\,(bx+a)^{\frac{3}{2}}b\mathrm{sgn}\,(x)}$$

```
integrate(x^5/(b*x^3+a*x^2)^(5/2),x, algorithm="giac")
```

```
2/3*sgn(x)/(a^(3/2)*b) - 2/3/((b*x + a)^(3/2)*b*sgn(x))
```

Mupad [B] (verification not implemented)

Time = 8.64 (sec) , antiderivative size = 28, normalized size of antiderivative = 1.12

$$\int \frac{x^5}{(ax^2+bx^3)^{5/2}}\,dx = -\frac{2\sqrt{b\,x^3+a\,x^2}}{3\,b\,x\,(a+b\,x)^2}$$

```
int(x^5/(a*x^2 + b*x^3)^(5/2),x)
```

```
-(2*(a*x^2 + b*x^3)^(1/2))/(3*b*x*(a + b*x)^2)
```

Reduce [B] (verification not implemented)

Time = 0.48 (sec) , antiderivative size = 20, normalized size of antiderivative = 0.80

$$\int \frac{x^5}{(ax^2+bx^3)^{5/2}}\,dx = -\frac{2}{3\sqrt{bx+a}\,b\,(bx+a)}$$

```
int(x^5/(b*x^3+a*x^2)^(5/2),x)
```

```
( - 2)/(3*sqrt(a + b*x)*b*(a + b*x))
```

3.321 $\int \frac{x^4}{(ax^2+bx^3)^{5/2}}\,dx$

Optimal result

Integrand size = 19, antiderivative size = 79

$$\int \frac{x^4}{(ax^2+bx^3)^{5/2}}\,dx = \frac{2x^3}{3a\left(ax^2+bx^3\right)^{3/2}} + \frac{2x}{a^2\sqrt{ax^2+bx^3}} - \frac{2\,\text{arctanh}\left(\frac{\sqrt{ax^2+bx^3}}{\sqrt{a}x}\right)}{a^{5/2}}$$

```
2/3*x^3/a/(b*x^3+a*x^2)^(3/2)+2*x/a^2/(b*x^3+a*x^2)^(1/2)-2*arctanh((b*x^3
+a*x^2)^(1/2)/a^(1/2)/x)/a^(5/2)
```

Mathematica [A] (verified)

Time = 0.03 (sec) , antiderivative size = 67, normalized size of antiderivative = 0.85

$$\int \frac{x^4}{(ax^2+bx^3)^{5/2}}\,dx = \frac{2x^3\left(\sqrt{a}(4a+3bx)-3(a+bx)^{3/2}\text{arctanh}\left(\frac{\sqrt{a+bx}}{\sqrt{a}}\right)\right)}{3a^{5/2}\left(x^2(a+bx)\right)^{3/2}}$$

```
Integrate[x^4/(a*x^2 + b*x^3)^(5/2),x]
```

```
(2*x^3*(Sqrt[a]*(4*a + 3*b*x) - 3*(a + b*x)^(3/2)*ArcTanh[Sqrt[a + b*x]/Sq
rt[a]]))/(3*a^(5/2)*(x^2*(a + b*x))^(3/2))
```

Rubi [A] (verified)

Time = 0.39 (sec) , antiderivative size = 82, normalized size of antiderivative = 1.04, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.211, Rules used = {1929, 1929, 1914, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^4}{(ax^2+bx^3)^{5/2}}dx$$

$$\Big\downarrow \text{1929}$$

$$\frac{\int \frac{x^2}{(bx^3+ax^2)^{3/2}}dx}{a}+\frac{2x^3}{3a\,(ax^2+bx^3)^{3/2}}$$

$$\Big\downarrow \text{1929}$$

$$\frac{\frac{\int \frac{1}{\sqrt{bx^3+ax^2}}dx}{a}+\frac{2x}{a\sqrt{ax^2+bx^3}}}{a}+\frac{2x^3}{3a\,(ax^2+bx^3)^{3/2}}$$

$$\Big\downarrow \text{1914}$$

$$\frac{\frac{2x}{a\sqrt{ax^2+bx^3}}-\frac{2\int \frac{1}{1-\frac{ax^2}{bx^3+ax^2}}d\frac{x}{\sqrt{bx^3+ax^2}}}{a}}{a}+\frac{2x^3}{3a\,(ax^2+bx^3)^{3/2}}$$

$$\Big\downarrow \text{219}$$

$$\frac{\frac{2x}{a\sqrt{ax^2+bx^3}}-\frac{2\text{arctanh}\left(\frac{\sqrt{a}x}{\sqrt{ax^2+bx^3}}\right)}{a^{3/2}}}{a}+\frac{2x^3}{3a\,(ax^2+bx^3)^{3/2}}$$

```
Int[x^4/(a*x^2 + b*x^3)^(5/2),x]
```

```
(2*x^3)/(3*a*(a*x^2 + b*x^3)^(3/2)) + ((2*x)/(a*Sqrt[a*x^2 + b*x^3]) - (2*
ArcTanh[(Sqrt[a]*x)/Sqrt[a*x^2 + b*x^3]])/a^(3/2))/a
```

Defintions of rubi rules used

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

```
Int[1/Sqrt[(a_.)*(x_)^2 + (b_.)*(x_)^(n_.)], x_Symbol] :> Simp[2/(2 - n)
Subst[Int[1/(1 - a*x^2), x], x, x/Sqrt[a*x^2 + b*x^n]], x] /; FreeQ[{a, b,
n}, x] && NeQ[n, 2]
```

```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> Simp[(-c^(j - 1))*(c*x)^(m - j + 1)*((a*x^j + b*x^n)^(p + 1)/(a*(n - j
)*(p + 1))), x] + Simp[c^j*((m + n*p + n - j + 1)/(a*(n - j)*(p + 1)))   In
t[(c*x)^(m - j)*(a*x^j + b*x^n)^(p + 1), x], x] /; FreeQ[{a, b, c, m}, x] &
&  !IntegerQ[p] && LtQ[0, j, n] && (IntegersQ[j, n] || GtQ[c, 0]) && LtQ[p,
 -1]
```

Maple [A] (verified)

Time = 0.43 (sec) , antiderivative size = 54, normalized size of antiderivative = 0.68

method	result	size
pseudoelliptic	$\frac{\frac{2}{5} b^{4} x^{4}-\frac{16}{15} a \,b^{3} x^{3}+\frac{32}{5} a^{2} b^{2} x^{2}+\frac{128}{5} a^{3} b x +\frac{256}{15} a^{4}}{b^{5} \left(b x +a \right)^{\frac{3}{2}}}$	54
default	$-\frac{2 x^{5} \left(b x +a \right) \left(-4 a^{\frac{7}{2}}-3 a^{\frac{5}{2}} b x +3 \,\operatorname{arctanh}\left(\frac{\sqrt{b x +a}}{\sqrt{a}}\right) a^{2} \left(b x +a \right)^{\frac{3}{2}}\right)}{3 \left(b \,x^{3}+a \,x^{2}\right)^{\frac{5}{2}} a^{\frac{9}{2}}}$	64

```
int(x^4/(b*x^3+a*x^2)^(5/2),x,method=_RETURNVERBOSE)
```

```
2/15*(3*b^4*x^4-8*a*b^3*x^3+48*a^2*b^2*x^2+192*a^3*b*x+128*a^4)/b^5/(b*x+a
)^(3/2)
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 228, normalized size of antiderivative = 2.89

$$\int \frac{x^4}{(ax^2+bx^3)^{5/2}}\,dx = \left[\frac{3\,(b^2x^3+2\,abx^2+a^2x)\sqrt{a}\log\left(\frac{bx^2+2\,ax-2\,\sqrt{bx^3+ax^2}\sqrt{a}}{x^2}\right)+2\,\sqrt{bx^3+ax^2}(3\,abx+4\,a}{3\,(a^3b^2x^3+2\,a^4bx^2+a^5x)}\right.$$

```
integrate(x^4/(b*x^3+a*x^2)^(5/2),x, algorithm="fricas")
```

```
[1/3*(3*(b^2*x^3 + 2*a*b*x^2 + a^2*x)*sqrt(a)*log((b*x^2 + 2*a*x - 2*sqrt(
b*x^3 + a*x^2)*sqrt(a))/x^2) + 2*sqrt(b*x^3 + a*x^2)*(3*a*b*x + 4*a^2))/(a
^3*b^2*x^3 + 2*a^4*b*x^2 + a^5*x), 2/3*(3*(b^2*x^3 + 2*a*b*x^2 + a^2*x)*sq
rt(-a)*arctan(sqrt(b*x^3 + a*x^2)*sqrt(-a)/(b*x^2 + a*x)) + sqrt(b*x^3 + a
*x^2)*(3*a*b*x + 4*a^2))/(a^3*b^2*x^3 + 2*a^4*b*x^2 + a^5*x)]
```

Sympy [F]

$$\int \frac{x^4}{(ax^2+bx^3)^{5/2}}\,dx = \int \frac{x^4}{(x^2\,(a+bx))^{\frac{5}{2}}}\,dx$$

```
integrate(x**4/(b*x**3+a*x**2)**(5/2),x)
```

```
Integral(x**4/(x**2*(a + b*x))**(5/2), x)
```

Maxima [F]

$$\int \frac{x^4}{(ax^2+bx^3)^{5/2}}\,dx = \int \frac{x^4}{(bx^3+ax^2)^{\frac{5}{2}}}\,dx$$

```
integrate(x^4/(b*x^3+a*x^2)^(5/2),x, algorithm="maxima")
```

```
integrate(x^4/(b*x^3 + a*x^2)^(5/2), x)
```

Giac [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 88, normalized size of antiderivative = 1.11

$$\int \frac{x^4}{(ax^2+bx^3)^{5/2}}\,dx = -\frac{2\left(3\sqrt{a}\arctan\left(\frac{\sqrt{a}}{\sqrt{-a}}\right)+4\sqrt{-a}\right)\mathrm{sgn}(x)}{3\sqrt{-a}a^{\frac{5}{2}}} + \frac{2\arctan\left(\frac{\sqrt{bx+a}}{\sqrt{-a}}\right)}{\sqrt{-a}a^2\mathrm{sgn}\left(x\right)} + \frac{2\left(3\,bx+4\,a\right)}{3\left(bx+a\right)^{\frac{3}{2}}a^2\mathrm{sgn}\left(x\right)}$$

```
integrate(x^4/(b*x^3+a*x^2)^(5/2),x, algorithm="giac")
```

```
-2/3*(3*sqrt(a)*arctan(sqrt(a)/sqrt(-a)) + 4*sqrt(-a))*sgn(x)/(sqrt(-a)*a^
(5/2)) + 2*arctan(sqrt(b*x + a)/sqrt(-a))/(sqrt(-a)*a^2*sgn(x)) + 2/3*(3*b
*x + 4*a)/((b*x + a)^(3/2)*a^2*sgn(x))
```

Mupad [F(-1)]

Timed out.

$$\int \frac{x^4}{(ax^2+bx^3)^{5/2}}\,dx = \int \frac{x^4}{\left(b\,x^3+a\,x^2\right)^{5/2}}\,dx$$

```
int(x^4/(a*x^2 + b*x^3)^(5/2),x)
```

```
int(x^4/(a*x^2 + b*x^3)^(5/2), x)
```

Reduce [B] (verification not implemented)

Time = 0.49 (sec) , antiderivative size = 121, normalized size of antiderivative = 1.53

$$\int \frac{x^4}{(ax^2+bx^3)^{5/2}}\,dx = \frac{3\sqrt{a}\,\sqrt{bx+a}\log\left(\sqrt{bx+a}-\sqrt{a}\right)a+3\sqrt{a}\,\sqrt{bx+a}\log\left(\sqrt{bx+a}-\sqrt{a}\right)bx-3\sqrt{a}}{3\sqrt{bx+a}}$$

```
int(x^4/(b*x^3+a*x^2)^(5/2),x)
```

```
(3*sqrt(a)*sqrt(a + b*x)*log(sqrt(a + b*x) - sqrt(a))*a + 3*sqrt(a)*sqrt(a
 + b*x)*log(sqrt(a + b*x) - sqrt(a))*b*x - 3*sqrt(a)*sqrt(a + b*x)*log(sqr
t(a + b*x) + sqrt(a))*a - 3*sqrt(a)*sqrt(a + b*x)*log(sqrt(a + b*x) + sqrt
(a))*b*x + 8*a**2 + 6*a*b*x)/(3*sqrt(a + b*x)*a**3*(a + b*x))
```

3.322 $\int \frac{x^3}{(ax^2+bx^3)^{5/2}}\,dx$

Optimal result

Integrand size = 19, antiderivative size = 105

$$\int \frac{x^3}{(ax^2+bx^3)^{5/2}}\,dx = -\frac{x^2}{a\,(ax^2+bx^3)^{3/2}} - \frac{5bx^3}{3a^2\,(ax^2+bx^3)^{3/2}} - \frac{5bx}{a^3\sqrt{ax^2+bx^3}} + \frac{5b\,\text{arctanh}\left(\frac{\sqrt{ax^2+bx^3}}{\sqrt{a}x}\right)}{a^{7/2}}$$

```
-x^2/a/(b*x^3+a*x^2)^(3/2)-5/3*b*x^3/a^2/(b*x^3+a*x^2)^(3/2)-5*b*x/a^3/(b*
x^3+a*x^2)^(1/2)+5*b*arctanh((b*x^3+a*x^2)^(1/2)/a^(1/2)/x)/a^(7/2)
```

Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 81, normalized size of antiderivative = 0.77

$$\int \frac{x^3}{(ax^2+bx^3)^{5/2}}\,dx = \frac{x^2\left(-\sqrt{a}(3a^2+20abx+15b^2x^2)+15bx(a+bx)^{3/2}\text{arctanh}\left(\frac{\sqrt{a+bx}}{\sqrt{a}}\right)\right)}{3a^{7/2}\,(x^2(a+bx))^{3/2}}$$

```
Integrate[x^3/(a*x^2 + b*x^3)^(5/2),x]
```

```
(x^2*(-(Sqrt[a]*(3*a^2 + 20*a*b*x + 15*b^2*x^2)) + 15*b*x*(a + b*x)^(3/2)*
ArcTanh[Sqrt[a + b*x]/Sqrt[a]]))/(3*a^(7/2)*(x^2*(a + b*x))^(3/2))
```

Rubi [A] (verified)

Time = 0.46 (sec) , antiderivative size = 113, normalized size of antiderivative = 1.08, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.263, Rules used = {1929, 1929, 1931, 1914, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^3}{(ax^2+bx^3)^{5/2}}dx$$

$$\Big\downarrow \text{1929}$$

$$\frac{5\int \frac{x}{(bx^3+ax^2)^{3/2}}dx}{3a}+\frac{2x^2}{3a(ax^2+bx^3)^{3/2}}$$

$$\Big\downarrow \text{1929}$$

$$\frac{5\left(\frac{3\int \frac{1}{x\sqrt{bx^3+ax^2}}dx}{a}+\frac{2}{a\sqrt{ax^2+bx^3}}\right)}{3a}+\frac{2x^2}{3a(ax^2+bx^3)^{3/2}}$$

$$\Big\downarrow \text{1931}$$

$$\frac{5\left(\frac{3\left(-\frac{b\int \frac{1}{\sqrt{bx^3+ax^2}}dx}{2a}-\frac{\sqrt{ax^2+bx^3}}{ax^2}\right)}{a}+\frac{2}{a\sqrt{ax^2+bx^3}}\right)}{3a}+\frac{2x^2}{3a(ax^2+bx^3)^{3/2}}$$

$$\Big\downarrow \text{1914}$$

$$\frac{5\left(\frac{3\left(\frac{b\int \frac{1}{1-\frac{ax^2}{bx^3+ax^2}}d\frac{x}{\sqrt{bx^3+ax^2}}}{a}-\frac{\sqrt{ax^2+bx^3}}{ax^2}\right)}{a}+\frac{2}{a\sqrt{ax^2+bx^3}}\right)}{3a}+\frac{2x^2}{3a(ax^2+bx^3)^{3/2}}$$

$$\Big\downarrow \text{219}$$

$$\frac{5\left(\frac{3\left(\frac{b\,\text{arctanh}\left(\frac{\sqrt{a}x}{\sqrt{ax^2+bx^3}}\right)}{a^{3/2}}-\frac{\sqrt{ax^2+bx^3}}{ax^2}\right)}{a}+\frac{2}{a\sqrt{ax^2+bx^3}}\right)}{3a}+\frac{2x^2}{3a\left(ax^2+bx^3\right)^{3/2}}$$

```
Int[x^3/(a*x^2 + b*x^3)^(5/2),x]
```

```
(2*x^2)/(3*a*(a*x^2 + b*x^3)^(3/2)) + (5*(2/(a*Sqrt[a*x^2 + b*x^3]) + (3*(
-(Sqrt[a*x^2 + b*x^3]/(a*x^2)) + (b*ArcTanh[(Sqrt[a]*x)/Sqrt[a*x^2 + b*x^3
]])/a^(3/2)))/a))/(3*a)
```

Defintions of rubi rules used

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

```
Int[1/Sqrt[(a_.)*(x_)^2 + (b_.)*(x_)^(n_.)], x_Symbol] :> Simp[2/(2 - n)
Subst[Int[1/(1 - a*x^2), x], x, x/Sqrt[a*x^2 + b*x^n]], x] /; FreeQ[{a, b,
n}, x] && NeQ[n, 2]
```

```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> Simp[(-c^(j - 1))*(c*x)^(m - j + 1)*((a*x^j + b*x^n)^(p + 1)/(a*(n - j
)*(p + 1))), x] + Simp[c^j*((m + n*p + n - j + 1)/(a*(n - j)*(p + 1)))   In
t[(c*x)^(m - j)*(a*x^j + b*x^n)^(p + 1), x], x] /; FreeQ[{a, b, c, m}, x] &
&  !IntegerQ[p] && LtQ[0, j, n] && (IntegersQ[j, n] || GtQ[c, 0]) && LtQ[p,
 -1]
```

```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> Simp[c^(j - 1)*(c*x)^(m - j + 1)*((a*x^j + b*x^n)^(p + 1)/(a*(m + j*p
+ 1))), x] - Simp[b*((m + n*p + n - j + 1)/(a*c^(n - j)*(m + j*p + 1)))   I
nt[(c*x)^(m + n - j)*(a*x^j + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, p}, x]
 &&  !IntegerQ[p] && LtQ[0, j, n] && (IntegersQ[j, n] || GtQ[c, 0]) && LtQ[
m + j*p + 1, 0]
```

Maple [A] (verified)

Time = 0.42 (sec) , antiderivative size = 42, normalized size of antiderivative = 0.40

method	result	size
pseudoelliptic	$\frac{\frac{2}{3}b^{3}x^{3}-4a\,b^{2}x^{2}-16a^{2}bx-\frac{32}{3}a^{3}}{b^{4}\left(bx+a\right)^{\frac{3}{2}}}$	42
default	$\frac{x^{4}\left(bx+a\right)\left(15\left(bx+a\right)^{\frac{3}{2}}\operatorname{arctanh}\left(\frac{\sqrt{bx+a}}{\sqrt{a}}\right)bx-20a^{\frac{3}{2}}bx-15\sqrt{a}\,b^{2}x^{2}-3a^{\frac{5}{2}}\right)}{3\left(b\,x^{3}+a\,x^{2}\right)^{\frac{5}{2}}a^{\frac{7}{2}}}$	74
risch	$-\frac{bx+a}{a^{3}\sqrt{x^{2}\left(bx+a\right)}}-\frac{b\left(-\frac{10\,\operatorname{arctanh}\left(\frac{\sqrt{bx+a}}{\sqrt{a}}\right)}{\sqrt{a}}+\frac{8}{\sqrt{bx+a}}+\frac{4a}{3\left(bx+a\right)^{\frac{3}{2}}}\right)\sqrt{bx+a}\,x}{2a^{3}\sqrt{x^{2}\left(bx+a\right)}}$	85

```
int(x^3/(b*x^3+a*x^2)^(5/2),x,method=_RETURNVERBOSE)
```

```
2/3*(b^3*x^3-6*a*b^2*x^2-24*a^2*b*x-16*a^3)/b^4/(b*x+a)^(3/2)
```

Fricas [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 264, normalized size of antiderivative = 2.51

$$\int \frac{x^3}{\left(ax^2+bx^3\right)^{5/2}}\,dx = \left[\frac{15\left(b^3x^4+2\,ab^2x^3+a^2bx^2\right)\sqrt{a}\log\left(\frac{bx^2+2\,ax+2\,\sqrt{bx^3+ax^2}\sqrt{a}}{x^2}\right)-2\left(15\,ab^2x^2+20\,a^2bx\right.}{6\left(a^4b^2x^4+2\,a^5bx^3+a^6x^2\right)}\right.$$
$$\left.-\frac{15\left(b^3x^4+2\,ab^2x^3+a^2bx^2\right)\sqrt{-a}\arctan\left(\frac{\sqrt{bx^3+ax^2}\sqrt{-a}}{bx^2+ax}\right)+\left(15\,ab^2x^2+20\,a^2bx+3\,a^3\right)\sqrt{bx^3+ax^2}}{3\left(a^4b^2x^4+2\,a^5bx^3+a^6x^2\right)}\right]$$

```
integrate(x^3/(b*x^3+a*x^2)^(5/2),x, algorithm="fricas")
```

```
[1/6*(15*(b^3*x^4 + 2*a*b^2*x^3 + a^2*b*x^2)*sqrt(a)*log((b*x^2 + 2*a*x +
2*sqrt(b*x^3 + a*x^2)*sqrt(a))/x^2) - 2*(15*a*b^2*x^2 + 20*a^2*b*x + 3*a^3
)*sqrt(b*x^3 + a*x^2))/(a^4*b^2*x^4 + 2*a^5*b*x^3 + a^6*x^2), -1/3*(15*(b^
3*x^4 + 2*a*b^2*x^3 + a^2*b*x^2)*sqrt(-a)*arctan(sqrt(b*x^3 + a*x^2)*sqrt(
-a)/(b*x^2 + a*x)) + (15*a*b^2*x^2 + 20*a^2*b*x + 3*a^3)*sqrt(b*x^3 + a*x^
2))/(a^4*b^2*x^4 + 2*a^5*b*x^3 + a^6*x^2)]
```

Sympy [F]

$$\int \frac{x^3}{(ax^2+bx^3)^{5/2}}\,dx = \int \frac{x^3}{(x^2(a+bx))^{\frac{5}{2}}}\,dx$$

```
integrate(x**3/(b*x**3+a*x**2)**(5/2),x)
```

```
Integral(x**3/(x**2*(a + b*x))**(5/2), x)
```

Maxima [F]

$$\int \frac{x^3}{(ax^2+bx^3)^{5/2}}\,dx = \int \frac{x^3}{(bx^3+ax^2)^{\frac{5}{2}}}\,dx$$

```
integrate(x^3/(b*x^3+a*x^2)^(5/2),x, algorithm="maxima")
```

```
integrate(x^3/(b*x^3 + a*x^2)^(5/2), x)
```

Giac [A] (verification not implemented)

Time = 0.14 (sec) , antiderivative size = 77, normalized size of antiderivative = 0.73

$$\int \frac{x^3}{(ax^2+bx^3)^{5/2}}\,dx = -\frac{5\,b\arctan\left(\frac{\sqrt{bx+a}}{\sqrt{-a}}\right)}{\sqrt{-a}a^3\mathrm{sgn}\left(x\right)} - \frac{2\left(6\left(bx+a\right)b+ab\right)}{3\left(bx+a\right)^{\frac{3}{2}}a^3\mathrm{sgn}\left(x\right)} - \frac{\sqrt{bx+a}}{a^3x\mathrm{sgn}\left(x\right)}$$

```
integrate(x^3/(b*x^3+a*x^2)^(5/2),x, algorithm="giac")
```

```
-5*b*arctan(sqrt(b*x + a)/sqrt(-a))/(sqrt(-a)*a^3*sgn(x)) - 2/3*(6*(b*x +
a)*b + a*b)/((b*x + a)^(3/2)*a^3*sgn(x)) - sqrt(b*x + a)/(a^3*x*sgn(x))
```

Mupad [F(-1)]

Timed out.

$$\int \frac{x^3}{(ax^2+bx^3)^{5/2}}\,dx = \int \frac{x^3}{\left(b\,x^3+a\,x^2\right)^{5/2}}\,dx$$

```
int(x^3/(a*x^2 + b*x^3)^(5/2),x)
```

```
int(x^3/(a*x^2 + b*x^3)^(5/2), x)
```

Reduce [B] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 147, normalized size of antiderivative = 1.40

$$\int \frac{x^3}{(ax^2+bx^3)^{5/2}}\,dx = \frac{-15\sqrt{a}\,\sqrt{bx+a}\log\left(\sqrt{bx+a}-\sqrt{a}\right)abx-15\sqrt{a}\,\sqrt{bx+a}\log\left(\sqrt{bx+a}-\sqrt{a}\right)b^2x^2}{}$$

```
int(x^3/(b*x^3+a*x^2)^(5/2),x)
```

```
( - 15*sqrt(a)*sqrt(a + b*x)*log(sqrt(a + b*x) - sqrt(a))*a*b*x - 15*sqrt(
a)*sqrt(a + b*x)*log(sqrt(a + b*x) - sqrt(a))*b**2*x**2 + 15*sqrt(a)*sqrt(
a + b*x)*log(sqrt(a + b*x) + sqrt(a))*a*b*x + 15*sqrt(a)*sqrt(a + b*x)*log
(sqrt(a + b*x) + sqrt(a))*b**2*x**2 - 6*a**3 - 40*a**2*b*x - 30*a*b**2*x**
2)/(6*sqrt(a + b*x)*a**4*x*(a + b*x))
```

3.323 $\int \frac{x^2}{(ax^2+bx^3)^{5/2}}\,dx$

Optimal result

Integrand size = 19, antiderivative size = 141

$$\int \frac{x^2}{(ax^2+bx^3)^{5/2}}\,dx = -\frac{x}{2a\,(ax^2+bx^3)^{3/2}} + \frac{7bx^2}{4a^2\,(ax^2+bx^3)^{3/2}} + \frac{35b^2x^3}{12a^3\,(ax^2+bx^3)^{3/2}} + \frac{35b^2x}{4a^4\sqrt{ax^2+bx^3}} - \frac{35b^2\operatorname{arctanh}\left(\frac{\sqrt{ax^2+bx^3}}{\sqrt{a}x}\right)}{4a^{9/2}}$$

```
-1/2*x/a/(b*x^3+a*x^2)^(3/2)+7/4*b*x^2/a^2/(b*x^3+a*x^2)^(3/2)+35/12*b^2*x
^3/a^3/(b*x^3+a*x^2)^(3/2)+35/4*b^2*x/a^4/(b*x^3+a*x^2)^(1/2)-35/4*b^2*arc
tanh((b*x^3+a*x^2)^(1/2)/a^(1/2)/x)/a^(9/2)
```

Mathematica [A] (verified)

Time = 0.05 (sec) , antiderivative size = 93, normalized size of antiderivative = 0.66

$$\int \frac{x^2}{(ax^2+bx^3)^{5/2}}\,dx = \frac{x\left(\sqrt{a}(-6a^3+21a^2bx+140ab^2x^2+105b^3x^3) - 105b^2x^2(a+bx)^{3/2}\operatorname{arctanh}\left(\frac{\sqrt{a+bx}}{\sqrt{a}}\right.\right.}{12a^{9/2}\,(x^2(a+bx))^{3/2}}$$

```
Integrate[x^2/(a*x^2 + b*x^3)^(5/2),x]
```

```
(x*(Sqrt[a]*(-6*a^3 + 21*a^2*b*x + 140*a*b^2*x^2 + 105*b^3*x^3) - 105*b^2*
x^2*(a + b*x)^(3/2)*ArcTanh[Sqrt[a + b*x]/Sqrt[a]]))/(12*a^(9/2)*(x^2*(a +
 b*x))^(3/2))
```

Rubi [A] (verified)

Time = 0.55 (sec) , antiderivative size = 148, normalized size of antiderivative = 1.05, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.316, Rules used = {1929, 1912, 1931, 1931, 1914, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^2}{(ax^2+bx^3)^{5/2}}dx$$

$$\Big\downarrow \text{1929}$$

$$\frac{7\int \frac{1}{(bx^3+ax^2)^{3/2}}dx}{3a}+\frac{2x}{3a(ax^2+bx^3)^{3/2}}$$

$$\Big\downarrow \text{1912}$$

$$\frac{7\left(\frac{5\int \frac{1}{x^2\sqrt{bx^3+ax^2}}dx}{a}+\frac{2}{ax\sqrt{ax^2+bx^3}}\right)}{3a}+\frac{2x}{3a(ax^2+bx^3)^{3/2}}$$

$$\Big\downarrow \text{1931}$$

$$\frac{7\left(\frac{5\left(-\frac{3b\int \frac{1}{x\sqrt{bx^3+ax^2}}dx}{4a}-\frac{\sqrt{ax^2+bx^3}}{2ax^3}\right)}{a}+\frac{2}{ax\sqrt{ax^2+bx^3}}\right)}{3a}+\frac{2x}{3a(ax^2+bx^3)^{3/2}}$$

$$\Big\downarrow \text{1931}$$

$$\frac{7\left(\frac{5\left(-\frac{3b\left(-\frac{b\int \frac{1}{\sqrt{bx^3+ax^2}}dx}{2a}-\frac{\sqrt{ax^2+bx^3}}{ax^2}\right)}{4a}-\frac{\sqrt{ax^2+bx^3}}{2ax^3}\right)}{a}+\frac{2}{ax\sqrt{ax^2+bx^3}}\right)}{3a}+\frac{2x}{3a\left(ax^2+bx^3\right)^{3/2}}$$

$$\Big\downarrow \text{ 1914}$$

$$\frac{7\left(\frac{5\left(-\frac{3b\left(\frac{b\int \frac{1}{1-\frac{ax^2}{bx^3+ax^2}}d\frac{x}{\sqrt{bx^3+ax^2}}}{a}-\frac{\sqrt{ax^2+bx^3}}{ax^2}\right)}{4a}-\frac{\sqrt{ax^2+bx^3}}{2ax^3}\right)}{a}+\frac{2}{ax\sqrt{ax^2+bx^3}}\right)}{3a}+\frac{2x}{3a\left(ax^2+bx^3\right)^{3/2}}$$

$$\Big\downarrow \text{ 219}$$

$$\frac{7\left(\frac{5\left(-\frac{3b\left(\frac{b\,\text{arctanh}\left(\frac{\sqrt{a}x}{\sqrt{ax^2+bx^3}}\right)}{a^{3/2}}-\frac{\sqrt{ax^2+bx^3}}{ax^2}\right)}{4a}-\frac{\sqrt{ax^2+bx^3}}{2ax^3}\right)}{a}+\frac{2}{ax\sqrt{ax^2+bx^3}}\right)}{3a}+\frac{2x}{3a\left(ax^2+bx^3\right)^{3/2}}$$

```
Int[x^2/(a*x^2 + b*x^3)^(5/2),x]
```

```
(2*x)/(3*a*(a*x^2 + b*x^3)^(3/2)) + (7*(2/(a*x*Sqrt[a*x^2 + b*x^3]) + (5*(
-1/2*Sqrt[a*x^2 + b*x^3]/(a*x^3) - (3*b*(-(Sqrt[a*x^2 + b*x^3]/(a*x^2)) +
(b*ArcTanh[(Sqrt[a]*x)/Sqrt[a*x^2 + b*x^3]])/a^(3/2)))/(4*a)))/a))/(3*a)
```

Defintions of rubi rules used

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

```
Int[((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol] :> Simp[-(a*x^j +
 b*x^n)^(p + 1)/(a*(n - j)*(p + 1)*x^(j - 1)), x] + Simp[(n*p + n - j + 1)/
(a*(n - j)*(p + 1))   Int[(a*x^j + b*x^n)^(p + 1)/x^j, x], x] /; FreeQ[{a,
b}, x] &&  !IntegerQ[p] && LtQ[0, j, n] && LtQ[p, -1]
```

```
Int[1/Sqrt[(a_.)*(x_)^2 + (b_.)*(x_)^(n_.)], x_Symbol] :> Simp[2/(2 - n)
Subst[Int[1/(1 - a*x^2), x], x, x/Sqrt[a*x^2 + b*x^n]], x] /; FreeQ[{a, b,
n}, x] && NeQ[n, 2]
```

```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> Simp[(-c^(j - 1))*(c*x)^(m - j + 1)*((a*x^j + b*x^n)^(p + 1)/(a*(n - j
)*(p + 1))), x] + Simp[c^j*((m + n*p + n - j + 1)/(a*(n - j)*(p + 1)))   In
t[(c*x)^(m - j)*(a*x^j + b*x^n)^(p + 1), x], x] /; FreeQ[{a, b, c, m}, x] &
&  !IntegerQ[p] && LtQ[0, j, n] && (IntegersQ[j, n] || GtQ[c, 0]) && LtQ[p,
 -1]
```

```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> Simp[c^(j - 1)*(c*x)^(m - j + 1)*((a*x^j + b*x^n)^(p + 1)/(a*(m + j*p
+ 1))), x] - Simp[b*((m + n*p + n - j + 1)/(a*c^(n - j)*(m + j*p + 1)))   I
nt[(c*x)^(m + n - j)*(a*x^j + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, p}, x]
 &&  !IntegerQ[p] && LtQ[0, j, n] && (IntegersQ[j, n] || GtQ[c, 0]) && LtQ[
m + j*p + 1, 0]
```

Maple [A] (verified)

Time = 0.41 (sec) , antiderivative size = 32, normalized size of antiderivative = 0.23

method	result	size
pseudoelliptic	$\frac{2b^2x^2+8abx+\frac{16}{3}a^2}{b^3(bx+a)^{\frac{3}{2}}}$	32
default	$-\frac{x^3(bx+a)\left(105(bx+a)^{\frac{3}{2}}\operatorname{arctanh}\left(\frac{\sqrt{bx+a}}{\sqrt{a}}\right)b^2x^2-140a^{\frac{3}{2}}b^2x^2-105\sqrt{a}\,b^3x^3-21a^{\frac{5}{2}}bx+6a^{\frac{7}{2}}\right)}{12(b\,x^3+a\,x^2)^{\frac{5}{2}}a^{\frac{9}{2}}}$	89
risch	$-\frac{(bx+a)(-11bx+2a)}{4a^4x\sqrt{x^2(bx+a)}}+\frac{b^2\left(-\frac{70\operatorname{arctanh}\left(\frac{\sqrt{bx+a}}{\sqrt{a}}\right)}{\sqrt{a}}+\frac{48}{\sqrt{bx+a}}+\frac{16a}{3(bx+a)^{\frac{3}{2}}}\right)\sqrt{bx+a}\,x}{8a^4\sqrt{x^2(bx+a)}}$	98

```
int(x^2/(b*x^3+a*x^2)^(5/2),x,method=_RETURNVERBOSE)
```

```
2/3*(3*b^2*x^2+12*a*b*x+8*a^2)/b^3/(b*x+a)^(3/2)
```

Fricas [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 290, normalized size of antiderivative = 2.06

$$\int \frac{x^2}{(ax^2+bx^3)^{5/2}}\,dx = \left[\frac{105\left(b^4x^5+2\,ab^3x^4+a^2b^2x^3\right)\sqrt{a}\log\left(\frac{bx^2+2\,ax-2\,\sqrt{bx^3+ax^2}\sqrt{a}}{x^2}\right)+2\left(105\,ab^3x^3+140\right.}{24\left(a^5b^2x^5+2\,a^6bx^4+a^7x^3\right)}\right.$$

```
integrate(x^2/(b*x^3+a*x^2)^(5/2),x, algorithm="fricas")
```

```
[1/24*(105*(b^4*x^5 + 2*a*b^3*x^4 + a^2*b^2*x^3)*sqrt(a)*log((b*x^2 + 2*a*
x - 2*sqrt(b*x^3 + a*x^2)*sqrt(a))/x^2) + 2*(105*a*b^3*x^3 + 140*a^2*b^2*x
^2 + 21*a^3*b*x - 6*a^4)*sqrt(b*x^3 + a*x^2))/(a^5*b^2*x^5 + 2*a^6*b*x^4 +
 a^7*x^3), 1/12*(105*(b^4*x^5 + 2*a*b^3*x^4 + a^2*b^2*x^3)*sqrt(-a)*arctan
(sqrt(b*x^3 + a*x^2)*sqrt(-a)/(b*x^2 + a*x)) + (105*a*b^3*x^3 + 140*a^2*b^
2*x^2 + 21*a^3*b*x - 6*a^4)*sqrt(b*x^3 + a*x^2))/(a^5*b^2*x^5 + 2*a^6*b*x^
4 + a^7*x^3)]
```

Sympy [F]

$$\int \frac{x^2}{(ax^2+bx^3)^{5/2}}\,dx = \int \frac{x^2}{(x^2(a+bx))^{\frac{5}{2}}}\,dx$$

```
integrate(x**2/(b*x**3+a*x**2)**(5/2),x)
```

```
Integral(x**2/(x**2*(a + b*x))**(5/2), x)
```

Maxima [F]

$$\int \frac{x^2}{(ax^2+bx^3)^{5/2}}\,dx = \int \frac{x^2}{(bx^3+ax^2)^{\frac{5}{2}}}\,dx$$

```
integrate(x^2/(b*x^3+a*x^2)^(5/2),x, algorithm="maxima")
```

```
integrate(x^2/(b*x^3 + a*x^2)^(5/2), x)
```

Giac [A] (verification not implemented)

Time = 0.38 (sec) , antiderivative size = 105, normalized size of antiderivative = 0.74

$$\int \frac{x^2}{(ax^2+bx^3)^{5/2}}\,dx = \frac{35\,b^2\arctan\left(\frac{\sqrt{bx+a}}{\sqrt{-a}}\right)}{4\,\sqrt{-a}a^4\mathrm{sgn}\left(x\right)} + \frac{2\,(9\,(bx+a)b^2+ab^2)}{3\,(bx+a)^{\frac{3}{2}}a^4\mathrm{sgn}\left(x\right)} + \frac{11\,(bx+a)^{\frac{3}{2}}b^2-13\,\sqrt{bx+a}ab^2}{4\,a^4b^2x^2\mathrm{sgn}\left(x\right)}$$

```
integrate(x^2/(b*x^3+a*x^2)^(5/2),x, algorithm="giac")
```

```
35/4*b^2*arctan(sqrt(b*x + a)/sqrt(-a))/(sqrt(-a)*a^4*sgn(x)) + 2/3*(9*(b*
x + a)*b^2 + a*b^2)/((b*x + a)^(3/2)*a^4*sgn(x)) + 1/4*(11*(b*x + a)^(3/2)
*b^2 - 13*sqrt(b*x + a)*a*b^2)/(a^4*b^2*x^2*sgn(x))
```

Mupad [F(-1)]

Timed out.

$$\int \frac{x^2}{(ax^2+bx^3)^{5/2}}\,dx = \int \frac{x^2}{(b\,x^3+a\,x^2)^{5/2}}\,dx$$

```
int(x^2/(a*x^2 + b*x^3)^(5/2),x)
```

```
int(x^2/(a*x^2 + b*x^3)^(5/2), x)
```

Reduce [B] (verification not implemented)

Time = 0.31 (sec) , antiderivative size = 166, normalized size of antiderivative = 1.18

$$\int \frac{x^2}{(ax^2+bx^3)^{5/2}}\,dx = \frac{105\sqrt{a}\,\sqrt{bx+a}\log\left(\sqrt{bx+a}-\sqrt{a}\right)a\,b^2x^2+105\sqrt{a}\,\sqrt{bx+a}\log\left(\sqrt{bx+a}-\sqrt{a}\right)b$$

```
int(x^2/(b*x^3+a*x^2)^(5/2),x)
```

```
(105*sqrt(a)*sqrt(a + b*x)*log(sqrt(a + b*x) - sqrt(a))*a*b**2*x**2 + 105*
sqrt(a)*sqrt(a + b*x)*log(sqrt(a + b*x) - sqrt(a))*b**3*x**3 - 105*sqrt(a)
*sqrt(a + b*x)*log(sqrt(a + b*x) + sqrt(a))*a*b**2*x**2 - 105*sqrt(a)*sqrt
(a + b*x)*log(sqrt(a + b*x) + sqrt(a))*b**3*x**3 - 12*a**4 + 42*a**3*b*x +
 280*a**2*b**2*x**2 + 210*a*b**3*x**3)/(24*sqrt(a + b*x)*a**5*x**2*(a + b*
x))
```

3.324 $\int \frac{x}{(ax^2+bx^3)^{5/2}}\,dx$

Optimal result

Integrand size = 17, antiderivative size = 166

$$\int \frac{x}{(ax^2+bx^3)^{5/2}}\,dx = -\frac{1}{3a(ax^2+bx^3)^{3/2}} + \frac{3bx}{4a^2(ax^2+bx^3)^{3/2}} - \frac{21b^2x^2}{8a^3(ax^2+bx^3)^{3/2}} - \frac{35b^3x^3}{8a^4(ax^2+bx^3)^{3/2}} - \frac{105b^3x}{8a^5\sqrt{ax^2+bx^3}} + \frac{105b^3\operatorname{arctanh}\left(\frac{\sqrt{ax^2+bx^3}}{\sqrt{a}x}\right)}{8a^{11/2}}$$

```
-1/3/a/(b*x^3+a*x^2)^(3/2)+3/4*b*x/a^2/(b*x^3+a*x^2)^(3/2)-21/8*b^2*x^2/a^
3/(b*x^3+a*x^2)^(3/2)-35/8*b^3*x^3/a^4/(b*x^3+a*x^2)^(3/2)-105/8*b^3*x/a^5
/(b*x^3+a*x^2)^(1/2)+105/8*b^3*arctanh((b*x^3+a*x^2)^(1/2)/a^(1/2)/x)/a^(1
1/2)
```

Mathematica [A] (verified)

Time = 0.04 (sec) , antiderivative size = 104, normalized size of antiderivative = 0.63

$$\int \frac{x}{(ax^2+bx^3)^{5/2}}\,dx = \frac{-\sqrt{a}(8a^4-18a^3bx+63a^2b^2x^2+420ab^3x^3+315b^4x^4)+315b^3x^3(a+bx)^{3/2}\text{arctan}}{24a^{11/2}\left(x^2(a+bx)\right)^{3/2}}$$

```
Integrate[x/(a*x^2 + b*x^3)^(5/2),x]
```

```
(-(Sqrt[a]*(8*a^4 - 18*a^3*b*x + 63*a^2*b^2*x^2 + 420*a*b^3*x^3 + 315*b^4*
x^4)) + 315*b^3*x^3*(a + b*x)^(3/2)*ArcTanh[Sqrt[a + b*x]/Sqrt[a]])/(24*a^
(11/2)*(x^2*(a + b*x))^(3/2))
```

Rubi [A] (verified)

Time = 0.64 (sec) , antiderivative size = 179, normalized size of antiderivative = 1.08, number of steps used = 8, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}} = 0.412$, Rules used = {1929, 1929, 1931, 1931, 1931, 1914, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x}{(ax^2+bx^3)^{5/2}}dx$$

$$\Big\downarrow \text{1929}$$

$$\frac{3\int \frac{1}{x(bx^3+ax^2)^{3/2}}dx}{a}+\frac{2}{3a\left(ax^2+bx^3\right)^{3/2}}$$

$$\Big\downarrow \text{1929}$$

$$\frac{3\left(\frac{7\int \frac{1}{x^3\sqrt{bx^3+ax^2}}dx}{a}+\frac{2}{ax^2\sqrt{ax^2+bx^3}}\right)}{a}+\frac{2}{3a\left(ax^2+bx^3\right)^{3/2}}$$

$$\Big\downarrow \text{1931}$$

$$\frac{3\left(\frac{7\left(-\frac{5b\int \frac{1}{x^2\sqrt{bx^3+ax^2}}dx}{6a}-\frac{\sqrt{ax^2+bx^3}}{3ax^4}\right)}{a}+\frac{2}{ax^2\sqrt{ax^2+bx^3}}\right)}{a}+\frac{2}{3a\left(ax^2+bx^3\right)^{3/2}}$$

$$\Big\downarrow \text{1931}$$

$$\frac{3\left(\frac{7\left(-\frac{5b\left(-\frac{3b\int \frac{1}{x\sqrt{bx^3+ax^2}}dx}{4a}-\frac{\sqrt{ax^2+bx^3}}{2ax^3}\right)}{6a}-\frac{\sqrt{ax^2+bx^3}}{3ax^4}\right)}{a}+\frac{2}{ax^2\sqrt{ax^2+bx^3}}\right)}{a}+\frac{2}{3a\left(ax^2+bx^3\right)^{3/2}}$$

$$\Big\downarrow \text{1931}$$

$$\frac{3\left(\frac{7\left(-\frac{5b\left(-\frac{3b\left(-\frac{b\int \frac{1}{\sqrt{bx^3+ax^2}}dx}{2a}-\frac{\sqrt{ax^2+bx^3}}{ax^2}\right)}{4a}-\frac{\sqrt{ax^2+bx^3}}{2ax^3}\right)}{6a}-\frac{\sqrt{ax^2+bx^3}}{3ax^4}\right)}{a}+\frac{2}{ax^2\sqrt{ax^2+bx^3}}\right)}{a}+\frac{2}{3a\left(ax^2+bx^3\right)^{3/2}}$$

$$\Big\downarrow \text{1914}$$

$$\frac{3\left(\frac{7\left(-\frac{5b\left(-\frac{3b\left(\frac{b\int \frac{1}{1-\frac{ax^2}{bx^3+ax^2}}d\frac{x}{\sqrt{bx^3+ax^2}}}{a}-\frac{\sqrt{ax^2+bx^3}}{ax^2}\right)}{4a}-\frac{\sqrt{ax^2+bx^3}}{2ax^3}\right)}{6a}-\frac{\sqrt{ax^2+bx^3}}{3ax^4}\right)}{a}+\frac{2}{ax^2\sqrt{ax^2+bx^3}}\right)}{a}+$$

$$\frac{2}{3a\left(ax^2+bx^3\right)^{3/2}}$$

$$\Big\downarrow \text{219}$$

$$\frac{3\left(\frac{7\left(-\frac{5b\left(-\frac{3b\left(\frac{b\operatorname{arctanh}\left(\frac{\sqrt{a}x}{\sqrt{ax^2+bx^3}}\right)}{a^{3/2}}-\frac{\sqrt{ax^2+bx^3}}{ax^2}\right)}{4a}-\frac{\sqrt{ax^2+bx^3}}{2ax^3}\right)}{6a}-\frac{\sqrt{ax^2+bx^3}}{3ax^4}\right)}{a}+\frac{2}{ax^2\sqrt{ax^2+bx^3}}\right)}{a}+\frac{2}{3a\left(ax^2+bx^3\right)^{3/2}}$$

```
Int[x/(a*x^2 + b*x^3)^(5/2),x]
```

```
2/(3*a*(a*x^2 + b*x^3)^(3/2)) + (3*(2/(a*x^2*Sqrt[a*x^2 + b*x^3]) + (7*(-1
/3*Sqrt[a*x^2 + b*x^3]/(a*x^4) - (5*b*(-1/2*Sqrt[a*x^2 + b*x^3]/(a*x^3) -
(3*b*(-(Sqrt[a*x^2 + b*x^3]/(a*x^2)) + (b*ArcTanh[(Sqrt[a]*x)/Sqrt[a*x^2 +
 b*x^3]])/a^(3/2)))/(4*a)))/(6*a)))/a))/a
```

Defintions of rubi rules used

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

```
Int[1/Sqrt[(a_.)*(x_)^2 + (b_.)*(x_)^(n_.)], x_Symbol] :> Simp[2/(2 - n)
Subst[Int[1/(1 - a*x^2), x], x, x/Sqrt[a*x^2 + b*x^n]], x] /; FreeQ[{a, b,
n}, x] && NeQ[n, 2]
```

```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> Simp[(-c^(j - 1))*(c*x)^(m - j + 1)*((a*x^j + b*x^n)^(p + 1)/(a*(n - j
)*(p + 1))), x] + Simp[c^j*((m + n*p + n - j + 1)/(a*(n - j)*(p + 1)))   In
t[(c*x)^(m - j)*(a*x^j + b*x^n)^(p + 1), x], x] /; FreeQ[{a, b, c, m}, x] &
&  !IntegerQ[p] && LtQ[0, j, n] && (IntegersQ[j, n] || GtQ[c, 0]) && LtQ[p,
 -1]
```

```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> Simp[c^(j - 1)*(c*x)^(m - j + 1)*((a*x^j + b*x^n)^(p + 1)/(a*(m + j*p
+ 1))), x] - Simp[b*((m + n*p + n - j + 1)/(a*c^(n - j)*(m + j*p + 1)))   I
nt[(c*x)^(m + n - j)*(a*x^j + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, p}, x]
 &&  !IntegerQ[p] && LtQ[0, j, n] && (IntegersQ[j, n] || GtQ[c, 0]) && LtQ[
m + j*p + 1, 0]
```

Maple [A] (verified)

Time = 0.40 (sec) , antiderivative size = 21, normalized size of antiderivative = 0.13

method	result	size
pseudoelliptic	$\frac{-6bx-4a}{3b^{2}(bx+a)^{\frac{3}{2}}}$	21
default	$\frac{x^{2}(bx+a)\left(315(bx+a)^{\frac{3}{2}}\operatorname{arctanh}\left(\frac{\sqrt{bx+a}}{\sqrt{a}}\right)b^{3}x^{3}-63a^{\frac{5}{2}}b^{2}x^{2}-420a^{\frac{3}{2}}b^{3}x^{3}-315\sqrt{a}\,b^{4}x^{4}+18a^{\frac{7}{2}}bx-8a^{\frac{9}{2}}\right)}{24(b\,x^{3}+a\,x^{2})^{\frac{5}{2}}a^{\frac{11}{2}}}$	100
risch	$-\frac{(bx+a)(123b^{2}x^{2}-34abx+8a^{2})}{24a^{5}x^{2}\sqrt{x^{2}(bx+a)}}-\frac{b^{3}\left(-\frac{210\operatorname{arctanh}\left(\frac{\sqrt{bx+a}}{\sqrt{a}}\right)}{\sqrt{a}}+\frac{128}{\sqrt{bx+a}}+\frac{32a}{3(bx+a)^{\frac{3}{2}}}\right)\sqrt{bx+a}\,x}{16a^{5}\sqrt{x^{2}(bx+a)}}$	109

```
int(x/(b*x^3+a*x^2)^(5/2),x,method=_RETURNVERBOSE)
```

```
1/3*(-6*b*x-4*a)/b^2/(b*x+a)^(3/2)
```

Fricas [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 312, normalized size of antiderivative = 1.88

$$\int \frac{x}{(ax^2+bx^3)^{5/2}}\,dx = \left[\frac{315\,(b^5x^6+2\,ab^4x^5+a^2b^3x^4)\sqrt{a}\log\left(\frac{bx^2+2\,ax+2\,\sqrt{bx^3+ax^2}\sqrt{a}}{x^2}\right)-2\,(315\,ab^4x^4+420}{48\,(a^6b^2x^6+2\,a^7bx^5+a^8x^4)}\right.$$

$$-\frac{315\,(b^5x^6+2\,ab^4x^5+a^2b^3x^4)\sqrt{-a}\arctan\left(\frac{\sqrt{bx^3+ax^2}\sqrt{-a}}{bx^2+ax}\right)+(315\,ab^4x^4+420\,a^2b^3x^3+63\,a^3b^2x^2-18\,a^4b}{24\,(a^6b^2x^6+2\,a^7bx^5+a^8x^4)}$$

```
integrate(x/(b*x^3+a*x^2)^(5/2),x, algorithm="fricas")
```

```
[1/48*(315*(b^5*x^6 + 2*a*b^4*x^5 + a^2*b^3*x^4)*sqrt(a)*log((b*x^2 + 2*a*
x + 2*sqrt(b*x^3 + a*x^2)*sqrt(a))/x^2) - 2*(315*a*b^4*x^4 + 420*a^2*b^3*x
^3 + 63*a^3*b^2*x^2 - 18*a^4*b*x + 8*a^5)*sqrt(b*x^3 + a*x^2))/(a^6*b^2*x^
6 + 2*a^7*b*x^5 + a^8*x^4), -1/24*(315*(b^5*x^6 + 2*a*b^4*x^5 + a^2*b^3*x^
4)*sqrt(-a)*arctan(sqrt(b*x^3 + a*x^2)*sqrt(-a)/(b*x^2 + a*x)) + (315*a*b^
4*x^4 + 420*a^2*b^3*x^3 + 63*a^3*b^2*x^2 - 18*a^4*b*x + 8*a^5)*sqrt(b*x^3
+ a*x^2))/(a^6*b^2*x^6 + 2*a^7*b*x^5 + a^8*x^4)]
```

Sympy [F]

$$\int \frac{x}{(ax^2+bx^3)^{5/2}}\,dx = \int \frac{x}{\left(x^2\,(a+bx)\right)^{\frac{5}{2}}}\,dx$$

```
integrate(x/(b*x**3+a*x**2)**(5/2),x)
```

```
Integral(x/(x**2*(a + b*x))**(5/2), x)
```

Maxima [F]

$$\int \frac{x}{\left(ax^2+bx^3\right)^{5/2}}\,dx = \int \frac{x}{\left(bx^3+ax^2\right)^{\frac{5}{2}}}\,dx$$

```
integrate(x/(b*x^3+a*x^2)^(5/2),x, algorithm="maxima")
```

```
integrate(x/(b*x^3 + a*x^2)^(5/2), x)
```

Giac [A] (verification not implemented)

Time = 0.30 (sec) , antiderivative size = 123, normalized size of antiderivative = 0.74

$$\int \frac{x}{\left(ax^2+bx^3\right)^{5/2}}\,dx = -\frac{105\,b^3\arctan\left(\frac{\sqrt{bx+a}}{\sqrt{-a}}\right)}{8\,\sqrt{-a}a^5\mathrm{sgn}\left(x\right)} - \frac{315\,(bx+a)^4b^3-840\,(bx+a)^3ab^3+693\,(bx+a)^2a^2b^3-144\,(bx+a)a^3b^3-16\,a^4b^3}{24\,\left((bx+a)^{\frac{3}{2}}-\sqrt{bx+a}a\right)^3a^5\mathrm{sgn}\left(x\right)}$$

```
integrate(x/(b*x^3+a*x^2)^(5/2),x, algorithm="giac")
```

```
-105/8*b^3*arctan(sqrt(b*x + a)/sqrt(-a))/(sqrt(-a)*a^5*sgn(x)) - 1/24*(31
5*(b*x + a)^4*b^3 - 840*(b*x + a)^3*a*b^3 + 693*(b*x + a)^2*a^2*b^3 - 144*
(b*x + a)*a^3*b^3 - 16*a^4*b^3)/(((b*x + a)^(3/2) - sqrt(b*x + a)*a)^3*a^5
*sgn(x))
```

Mupad [F(-1)]

Timed out.

$$\int \frac{x}{(ax^2+bx^3)^{5/2}}\,dx=\int \frac{x}{(b\,x^3+a\,x^2)^{5/2}}\,dx$$

```
int(x/(a*x^2 + b*x^3)^(5/2),x)
```

```
int(x/(a*x^2 + b*x^3)^(5/2), x)
```

Reduce [B] (verification not implemented)

Time = 0.50 (sec) , antiderivative size = 177, normalized size of antiderivative = 1.07

$$\int \frac{x}{(ax^2+bx^3)^{5/2}}\,dx=\frac{-315\sqrt{a}\,\sqrt{bx+a}\log\left(\sqrt{bx+a}-\sqrt{a}\right)a\,b^3x^3-315\sqrt{a}\,\sqrt{bx+a}\log\left(\sqrt{bx+a}-\sqrt{a}\right)}{}$$

```
int(x/(b*x^3+a*x^2)^(5/2),x)
```

```
( - 315*sqrt(a)*sqrt(a + b*x)*log(sqrt(a + b*x) - sqrt(a))*a*b**3*x**3 - 3
15*sqrt(a)*sqrt(a + b*x)*log(sqrt(a + b*x) - sqrt(a))*b**4*x**4 + 315*sqrt
(a)*sqrt(a + b*x)*log(sqrt(a + b*x) + sqrt(a))*a*b**3*x**3 + 315*sqrt(a)*s
qrt(a + b*x)*log(sqrt(a + b*x) + sqrt(a))*b**4*x**4 - 16*a**5 + 36*a**4*b*
x - 126*a**3*b**2*x**2 - 840*a**2*b**3*x**3 - 630*a*b**4*x**4)/(48*sqrt(a
+ b*x)*a**6*x**3*(a + b*x))
```

3.325 $\int (cx)^{5/2}\sqrt{ax^2+bx^3}\,dx$

Optimal result

Integrand size = 23, antiderivative size = 214

$$
\begin{aligned}
\int (cx)^{5/2}\sqrt{ax^2+bx^3}\,dx = & -\frac{7a^4c^3\sqrt{ax^2+bx^3}}{128b^4\sqrt{cx}}+\frac{7a^3c^2\sqrt{cx}\sqrt{ax^2+bx^3}}{192b^3} \\
& -\frac{7a^2c(cx)^{3/2}\sqrt{ax^2+bx^3}}{240b^2}+\frac{a(cx)^{5/2}\sqrt{ax^2+bx^3}}{40b} \\
& +\frac{(cx)^{7/2}\sqrt{ax^2+bx^3}}{5c}+\frac{7a^5c^{5/2}\operatorname{arctanh}\left(\frac{\sqrt{b}(cx)^{3/2}}{c^{3/2}\sqrt{ax^2+bx^3}}\right)}{128b^{9/2}}
\end{aligned}
$$

```
-7/128*a^4*c^3*(b*x^3+a*x^2)^(1/2)/b^4/(c*x)^(1/2)+7/192*a^3*c^2*(c*x)^(1/
2)*(b*x^3+a*x^2)^(1/2)/b^3-7/240*a^2*c*(c*x)^(3/2)*(b*x^3+a*x^2)^(1/2)/b^2
+1/40*a*(c*x)^(5/2)*(b*x^3+a*x^2)^(1/2)/b+1/5*(c*x)^(7/2)*(b*x^3+a*x^2)^(1
/2)/c+7/128*a^5*c^(5/2)*arctanh(b^(1/2)*(c*x)^(3/2)/c^(3/2)/(b*x^3+a*x^2)^
(1/2))/b^(9/2)
```

Mathematica [A] (verified)

Time = 0.07 (sec) , antiderivative size = 141, normalized size of antiderivative = 0.66

$$\int (cx)^{5/2}\sqrt{ax^2+bx^3}\,dx = \frac{(cx)^{5/2}\sqrt{x^2(a+bx)}\left(\sqrt{b}\sqrt{x}\sqrt{a+bx}(-105a^4+70a^3bx-56a^2b^2x^2+48ab^3x^3+\right.}{1920b^{9/2}x^{7/2}\sqrt{a+bx}}$$

```
Integrate[(c*x)^(5/2)*Sqrt[a*x^2 + b*x^3],x]
```

```
((c*x)^(5/2)*Sqrt[x^2*(a + b*x)]*(Sqrt[b]*Sqrt[x]*Sqrt[a + b*x]*(-105*a^4
+ 70*a^3*b*x - 56*a^2*b^2*x^2 + 48*a*b^3*x^3 + 384*b^4*x^4) + 210*a^5*ArcT
anh[(Sqrt[b]*Sqrt[x])/(-Sqrt[a] + Sqrt[a + b*x])]))/(1920*b^(9/2)*x^(7/2)*
Sqrt[a + b*x])
```

Rubi [A] (verified)

Time = 0.85 (sec) , antiderivative size = 242, normalized size of antiderivative = 1.13, number of steps used = 9, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.348, Rules used = {1927, 1930, 1930, 1930, 1930, 1937, 1935, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (cx)^{5/2}\sqrt{ax^2+bx^3}\,dx$$

$$\Big\downarrow \text{1927}$$

$$\frac{a\int \frac{(cx)^{9/2}}{\sqrt{bx^3+ax^2}}dx}{10c^2}+\frac{(cx)^{7/2}\sqrt{ax^2+bx^3}}{5c}$$

$$\Big\downarrow \text{1930}$$

$$\frac{a\left(\frac{c^2(cx)^{5/2}\sqrt{ax^2+bx^3}}{4b}-\frac{7ac\int \frac{(cx)^{7/2}}{\sqrt{bx^3+ax^2}}dx}{8b}\right)}{10c^2}+\frac{(cx)^{7/2}\sqrt{ax^2+bx^3}}{5c}$$

$$\Big\downarrow \text{1930}$$

$$\frac{a\left(\frac{c^2(cx)^{5/2}\sqrt{ax^2+bx^3}}{4b}-\frac{7ac\left(\frac{c^2(cx)^{3/2}\sqrt{ax^2+bx^3}}{3b}-\frac{5ac\int\frac{(cx)^{5/2}}{\sqrt{bx^3+ax^2}}dx}{6b}\right)}{8b}\right)}{10c^2}+\frac{(cx)^{7/2}\sqrt{ax^2+bx^3}}{5c}$$

$$\Big\downarrow \text{1930}$$

$$\frac{a\left(\frac{c^2(cx)^{5/2}\sqrt{ax^2+bx^3}}{4b}-\frac{7ac\left(\frac{c^2(cx)^{3/2}\sqrt{ax^2+bx^3}}{3b}-\frac{5ac\left(\frac{c^2\sqrt{cx}\sqrt{ax^2+bx^3}}{2b}-\frac{3ac\int\frac{(cx)^{3/2}}{\sqrt{bx^3+ax^2}}dx}{4b}\right)}{6b}\right)}{8b}\right)}{10c^2}+\frac{(cx)^{7/2}\sqrt{ax^2+bx^3}}{5c}$$

$$\Big\downarrow \text{1930}$$

$$\frac{a\left(\frac{c^2(cx)^{5/2}\sqrt{ax^2+bx^3}}{4b}-\frac{7ac\left(\frac{c^2(cx)^{3/2}\sqrt{ax^2+bx^3}}{3b}-\frac{5ac\left(\frac{c^2\sqrt{cx}\sqrt{ax^2+bx^3}}{2b}-\frac{3ac\left(\frac{c^2\sqrt{ax^2+bx^3}}{b\sqrt{cx}}-\frac{ac\int\frac{\sqrt{cx}}{\sqrt{bx^3+ax^2}}dx}{2b}\right)}{4b}\right)}{6b}\right)}{8b}\right)}{10c^2}+\frac{(cx)^{7/2}\sqrt{ax^2+bx^3}}{5c}$$

$$\Big\downarrow \text{1937}$$

$$\frac{a\left(\frac{c^2(cx)^{5/2}\sqrt{ax^2+bx^3}}{4b}-\frac{7ac\left(\frac{c^2(cx)^{3/2}\sqrt{ax^2+bx^3}}{3b}-\frac{5ac\left(\frac{c^2\sqrt{cx}\sqrt{ax^2+bx^3}}{2b}-\frac{3ac\left(\frac{c^2\sqrt{ax^2+bx^3}}{b\sqrt{cx}}-\frac{ac\sqrt{cx}\int\frac{\sqrt{x}}{\sqrt{bx^3+ax^2}}dx}{2b\sqrt{x}}\right)}{4b}\right)}{6b}\right)}{8b}\right)}{10c^2}+\frac{(cx)^{7/2}\sqrt{ax^2+bx^3}}{5c}$$

$$\Big\downarrow \text{1935}$$

$$\frac{a\left(\frac{c^2(cx)^{5/2}\sqrt{ax^2+bx^3}}{4b}-\frac{7ac\left(\frac{c^2(cx)^{3/2}\sqrt{ax^2+bx^3}}{3b}-\frac{5ac\left(\frac{c^2\sqrt{cx}\sqrt{ax^2+bx^3}}{2b}-\frac{3ac\left(\frac{c^2\sqrt{ax^2+bx^3}}{b\sqrt{cx}}-\frac{ac\sqrt{cx}\int\frac{1}{1-\frac{bx^3}{bx^3+ax^2}}d\frac{x^{3/2}}{\sqrt{bx^3+ax^2}}}{b\sqrt{x}}\right)}{4b}\right)}{6b}\right)}{8b}\right)}{10c^2}+\frac{(cx)^{7/2}\sqrt{ax^2+bx^3}}{5c}$$

$$\Big\downarrow \; 219$$

$$\frac{a\left(\frac{c^2(cx)^{5/2}\sqrt{ax^2+bx^3}}{4b}-\frac{7ac\left(\frac{c^2(cx)^{3/2}\sqrt{ax^2+bx^3}}{3b}-\frac{5ac\left(\frac{c^2\sqrt{cx}\sqrt{ax^2+bx^3}}{2b}-\frac{3ac\left(\frac{c^2\sqrt{ax^2+bx^3}}{b\sqrt{cx}}-\frac{ac\sqrt{cx}\operatorname{arctanh}\left(\frac{\sqrt{b}x^{3/2}}{\sqrt{ax^2+bx^3}}\right)}{b^{3/2}\sqrt{x}}\right)}{4b}\right)}{6b}\right)}{8b}\right)}{10c^2}+\frac{(cx)^{7/2}\sqrt{ax^2+bx^3}}{5c}$$

```
Int[(c*x)^(5/2)*Sqrt[a*x^2 + b*x^3],x]
```

```
((c*x)^(7/2)*Sqrt[a*x^2 + b*x^3])/(5*c) + (a*((c^2*(c*x)^(5/2)*Sqrt[a*x^2
+ b*x^3])/(4*b) - (7*a*c*((c^2*(c*x)^(3/2)*Sqrt[a*x^2 + b*x^3])/(3*b) - (5
*a*c*((c^2*Sqrt[c*x]*Sqrt[a*x^2 + b*x^3])/(2*b) - (3*a*c*((c^2*Sqrt[a*x^2
+ b*x^3])/(b*Sqrt[c*x]) - (a*c*Sqrt[c*x]*ArcTanh[(Sqrt[b]*x^(3/2))/Sqrt[a*
x^2 + b*x^3]])/(b^(3/2)*Sqrt[x])))/(4*b)))/(6*b)))/(8*b)))/(10*c^2)
```

Defintions of rubi rules used

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> Simp[(c*x)^(m + 1)*((a*x^j + b*x^n)^p/(c*(m + n*p + 1))), x] + Simp[a*
(n - j)*(p/(c^j*(m + n*p + 1)))   Int[(c*x)^(m + j)*(a*x^j + b*x^n)^(p - 1)
, x], x] /; FreeQ[{a, b, c, m}, x] &&  !IntegerQ[p] && LtQ[0, j, n] && (Int
egersQ[j, n] || GtQ[c, 0]) && GtQ[p, 0] && NeQ[m + n*p + 1, 0]
```

```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> Simp[c^(n - 1)*(c*x)^(m - n + 1)*((a*x^j + b*x^n)^(p + 1)/(b*(m + n*p
+ 1))), x] - Simp[a*c^(n - j)*((m + j*p - n + j + 1)/(b*(m + n*p + 1)))   I
nt[(c*x)^(m - (n - j))*(a*x^j + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, p},
x] &&  !IntegerQ[p] && LtQ[0, j, n] && (IntegersQ[j, n] || GtQ[c, 0]) && Gt
Q[m + j*p - n + j + 1, 0] && NeQ[m + n*p + 1, 0]
```

```
Int[(x_)^(m_.)/Sqrt[(a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.)], x_Symbol] :> Simp
[-2/(n - j)   Subst[Int[1/(1 - a*x^2), x], x, x^(j/2)/Sqrt[a*x^j + b*x^n]],
 x] /; FreeQ[{a, b, j, n}, x] && EqQ[m, j/2 - 1] && NeQ[n, j]
```

```
Int[((c_)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_))^(p_), x_Symbol]
:> Simp[c^IntPart[m]*((c*x)^FracPart[m]/x^FracPart[m])   Int[x^m*(a*x^j + b
*x^n)^p, x], x] /; FreeQ[{a, b, c, j, m, n, p}, x] && IntegerQ[p + 1/2] &&
NeQ[n, j] && EqQ[Simplify[m + j*p + 1], 0]
```

Maple [A] (verified)

Time = 0.37 (sec) , antiderivative size = 149, normalized size of antiderivative = 0.70

method	result
risch	$-\frac{\left(-384 b^{4} x^{4}-48 a \,b^{3} x^{3}+56 a^{2} b^{2} x^{2}-70 a^{3} b x +105 a^{4}\right) c^{3} \sqrt{x^{2} \left(b x +a \right)}}{1920 b^{4} \sqrt{c x}}+\frac{7 a^{5} \ln \left(\frac{\frac{1}{2} a c +c b x}{\sqrt{b c}}+\sqrt{b c \,x^{2}+a c x}\right) c^{3} \sqrt{x^{2} \left(b x +a \right)}\, \sqrt{c x \left(b x +a\right.}}{256 b^{4} \sqrt{b c}\, x \left(b x +a \right) \sqrt{c x}}$
default	$-\frac{\sqrt{b \,x^{3}+a \,x^{2}}\, c^{2} \sqrt{c x}\, \left(-768 b^{4} x^{4} \sqrt{b c}\, \sqrt{c x \left(b x +a \right)}-96 a \,b^{3} x^{3} \sqrt{b c}\, \sqrt{c x \left(b x +a \right)}+112 a^{2} b^{2} x^{2} \sqrt{b c}\, \sqrt{c x \left(b x +a \right)}-105 \ln \left(\frac{2 c b x +2 \sqrt{c x \left(b x +\right.}}{2 \sqrt{b c}}\right.\right.}{3840 x \sqrt{c x \left(b x +a \right)}\, b^{4} \sqrt{b c}}$

```
int((c*x)^(5/2)*(b*x^3+a*x^2)^(1/2),x,method=_RETURNVERBOSE)
```

```
-1/1920*(-384*b^4*x^4-48*a*b^3*x^3+56*a^2*b^2*x^2-70*a^3*b*x+105*a^4)/b^4*
c^3*(x^2*(b*x+a))^(1/2)/(c*x)^(1/2)+7/256*a^5/b^4*ln((1/2*a*c+c*b*x)/(b*c)
^(1/2)+(b*c*x^2+a*c*x)^(1/2))/(b*c)^(1/2)*c^3*(x^2*(b*x+a))^(1/2)/x/(b*x+a
)*(c*x*(b*x+a))^(1/2)/(c*x)^(1/2)
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 291, normalized size of antiderivative = 1.36

$$\int (cx)^{5/2}\sqrt{ax^2+bx^3}\,dx = \left[\frac{105\,a^5c^2x\sqrt{\frac{c}{b}}\log\left(\frac{2\,bcx^2+acx+2\,\sqrt{bx^3+ax^2}\sqrt{cx}b\sqrt{\frac{c}{b}}}{x}\right)+2\,(384\,b^4c^2x^4+48\,ab^3c^2x^3-}{3840\,b^4x}\right.$$

$$-\frac{105\,a^5c^2x\sqrt{-\frac{c}{b}}\arctan\left(\frac{\sqrt{bx^3+ax^2}\sqrt{cx}b\sqrt{-\frac{c}{b}}}{bcx^2+acx}\right)-(384\,b^4c^2x^4+48\,ab^3c^2x^3-56\,a^2b^2c^2x^2+70\,a^3bc^2x-105\,a}{1920\,b^4x}$$

```
integrate((c*x)^(5/2)*(b*x^3+a*x^2)^(1/2),x, algorithm="fricas")
```

```
[1/3840*(105*a^5*c^2*x*sqrt(c/b)*log((2*b*c*x^2 + a*c*x + 2*sqrt(b*x^3 + a
*x^2)*sqrt(c*x)*b*sqrt(c/b))/x) + 2*(384*b^4*c^2*x^4 + 48*a*b^3*c^2*x^3 -
56*a^2*b^2*c^2*x^2 + 70*a^3*b*c^2*x - 105*a^4*c^2)*sqrt(b*x^3 + a*x^2)*sqr
t(c*x))/(b^4*x), -1/1920*(105*a^5*c^2*x*sqrt(-c/b)*arctan(sqrt(b*x^3 + a*x
^2)*sqrt(c*x)*b*sqrt(-c/b)/(b*c*x^2 + a*c*x)) - (384*b^4*c^2*x^4 + 48*a*b^
3*c^2*x^3 - 56*a^2*b^2*c^2*x^2 + 70*a^3*b*c^2*x - 105*a^4*c^2)*sqrt(b*x^3
+ a*x^2)*sqrt(c*x))/(b^4*x)]
```

Sympy [F]

$$\int (cx)^{5/2}\sqrt{ax^2+bx^3}\,dx = \int (cx)^{\frac{5}{2}}\sqrt{x^2(a+bx)}\,dx$$

```
integrate((c*x)**(5/2)*(b*x**3+a*x**2)**(1/2),x)
```

```
Integral((c*x)**(5/2)*sqrt(x**2*(a + b*x)), x)
```

Maxima [F]

$$\int (cx)^{5/2}\sqrt{ax^2+bx^3}\,dx = \int \sqrt{bx^3+ax^2}(cx)^{\frac{5}{2}}\,dx$$

```
integrate((c*x)^(5/2)*(b*x^3+a*x^2)^(1/2),x, algorithm="maxima")
```

```
integrate(sqrt(b*x^3 + a*x^2)*(c*x)^(5/2), x)
```

Giac [A] (verification not implemented)

Time = 0.31 (sec) , antiderivative size = 162, normalized size of antiderivative = 0.76

$$\int (cx)^{5/2}\sqrt{ax^2+bx^3}\,dx = \frac{7\,a^5c^2|c|\log\left(c^2|a|\right)\operatorname{sgn}(x)}{256\,\sqrt{bc}b^4} - \frac{\left(\frac{105\,a^5c^6\log\left(\left|-\sqrt{bc}\sqrt{cx}+\sqrt{bc^2x+ac^2}\right|\right)}{\sqrt{bc}b^4} + \left(\frac{105\,a^4c^4}{b^4} - 2\left(\frac{35\,a^3c^3}{b^3} + 4\left(6\left(8\,cx + \frac{ac}{b}\right)cx - \frac{7\,a^2c^2}{b^2}\right)cx\right)cx\right)\sqrt{bc^2x+ac^2}\right.}{1920\,c^4}$$

```
integrate((c*x)^(5/2)*(b*x^3+a*x^2)^(1/2),x, algorithm="giac")
```

```
7/256*a^5*c^2*abs(c)*log(c^2*abs(a))*sgn(x)/(sqrt(b*c)*b^4) - 1/1920*(105*
a^5*c^6*log(abs(-sqrt(b*c)*sqrt(c*x) + sqrt(b*c^2*x + a*c^2)))/(sqrt(b*c)*
b^4) + (105*a^4*c^4/b^4 - 2*(35*a^3*c^3/b^3 + 4*(6*(8*c*x + a*c/b)*c*x - 7
*a^2*c^2/b^2)*c*x)*c*x)*sqrt(b*c^2*x + a*c^2)*sqrt(c*x))*abs(c)*sgn(x)/c^4
```

Mupad [F(-1)]

Timed out.

$$\int (cx)^{5/2}\sqrt{ax^2+bx^3}\,dx = \int (c\,x)^{5/2}\,\sqrt{b\,x^3+a\,x^2}\,dx$$

```
int((c*x)^(5/2)*(a*x^2 + b*x^3)^(1/2),x)
```

```
int((c*x)^(5/2)*(a*x^2 + b*x^3)^(1/2), x)
```

Reduce [B] (verification not implemented)

Time = 0.48 (sec) , antiderivative size = 119, normalized size of antiderivative = 0.56

$$\int (cx)^{5/2}\sqrt{ax^2+bx^3}\,dx = \frac{\sqrt{c}\,c^2\left(-105\sqrt{x}\,\sqrt{bx+a}\,a^4b + 70\sqrt{x}\,\sqrt{bx+a}\,a^3b^2x - 56\sqrt{x}\,\sqrt{bx+a}\,a^2b^3x^2 + \right.}{1920}$$

```
int((c*x)^(5/2)*(b*x^3+a*x^2)^(1/2),x)
```

```
(sqrt(c)*c**2*( - 105*sqrt(x)*sqrt(a + b*x)*a**4*b + 70*sqrt(x)*sqrt(a + b
*x)*a**3*b**2*x - 56*sqrt(x)*sqrt(a + b*x)*a**2*b**3*x**2 + 48*sqrt(x)*sqr
t(a + b*x)*a*b**4*x**3 + 384*sqrt(x)*sqrt(a + b*x)*b**5*x**4 + 105*sqrt(b)
*log((sqrt(a + b*x) + sqrt(x)*sqrt(b))/sqrt(a))*a**5))/(1920*b**5)
```

3.326 $\int (cx)^{3/2}\sqrt{ax^2+bx^3}\,dx$

Optimal result

Integrand size = 23, antiderivative size = 179

$$\int (cx)^{3/2}\sqrt{ax^2+bx^3}\,dx=\frac{5a^3c^2\sqrt{ax^2+bx^3}}{64b^3\sqrt{cx}}-\frac{5a^2c\sqrt{cx}\sqrt{ax^2+bx^3}}{96b^2}+\frac{a(cx)^{3/2}\sqrt{ax^2+bx^3}}{24b}+\frac{(cx)^{5/2}\sqrt{ax^2+bx^3}}{4c}-\frac{5a^4c^{3/2}\operatorname{arctanh}\left(\frac{\sqrt{b}(cx)^{3/2}}{c^{3/2}\sqrt{ax^2+bx^3}}\right)}{64b^{7/2}}$$

```
5/64*a^3*c^2*(b*x^3+a*x^2)^(1/2)/b^3/(c*x)^(1/2)-5/96*a^2*c*(c*x)^(1/2)*(b
*x^3+a*x^2)^(1/2)/b^2+1/24*a*(c*x)^(3/2)*(b*x^3+a*x^2)^(1/2)/b+1/4*(c*x)^(
5/2)*(b*x^3+a*x^2)^(1/2)/c-5/64*a^4*c^(3/2)*arctanh(b^(1/2)*(c*x)^(3/2)/c^
(3/2)/(b*x^3+a*x^2)^(1/2))/b^(7/2)
```

Mathematica [A] (verified)

Time = 0.07 (sec) , antiderivative size = 130, normalized size of antiderivative = 0.73

$$\int (cx)^{3/2}\sqrt{ax^2+bx^3}\,dx=\frac{(cx)^{3/2}\sqrt{x^2(a+bx)}\left(\sqrt{b}\sqrt{x}\sqrt{a+bx}(15a^3-10a^2bx+8ab^2x^2+48b^3x^3)+30a^4\right.}{192b^{7/2}x^{5/2}\sqrt{a+bx}}$$

```
Integrate[(c*x)^(3/2)*Sqrt[a*x^2 + b*x^3],x]
```

```
((c*x)^(3/2)*Sqrt[x^2*(a + b*x)]*(Sqrt[b]*Sqrt[x]*Sqrt[a + b*x]*(15*a^3 -
10*a^2*b*x + 8*a*b^2*x^2 + 48*b^3*x^3) + 30*a^4*ArcTanh[(Sqrt[b]*Sqrt[x])/
(Sqrt[a] - Sqrt[a + b*x])]))/(192*b^(7/2)*x^(5/2)*Sqrt[a + b*x])
```

Rubi [A] (verified)

Time = 0.70 (sec) , antiderivative size = 200, normalized size of antiderivative = 1.12, number of steps used = 8, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}} = 0.304$, Rules used = {1927, 1930, 1930, 1930, 1937, 1935, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (cx)^{3/2}\sqrt{ax^2+bx^3}\,dx$$

$$\Big\downarrow \text{1927}$$

$$\frac{a\int \frac{(cx)^{7/2}}{\sqrt{bx^3+ax^2}}dx}{8c^2}+\frac{(cx)^{5/2}\sqrt{ax^2+bx^3}}{4c}$$

$$\Big\downarrow \text{1930}$$

$$\frac{a\left(\frac{c^2(cx)^{3/2}\sqrt{ax^2+bx^3}}{3b}-\frac{5ac\int \frac{(cx)^{5/2}}{\sqrt{bx^3+ax^2}}dx}{6b}\right)}{8c^2}+\frac{(cx)^{5/2}\sqrt{ax^2+bx^3}}{4c}$$

$$\Big\downarrow \text{1930}$$

$$\frac{a\left(\frac{c^2(cx)^{3/2}\sqrt{ax^2+bx^3}}{3b}-\frac{5ac\left(\frac{c^2\sqrt{cx}\sqrt{ax^2+bx^3}}{2b}-\frac{3ac\int \frac{(cx)^{3/2}}{\sqrt{bx^3+ax^2}}dx}{4b}\right)}{6b}\right)}{8c^2}+\frac{(cx)^{5/2}\sqrt{ax^2+bx^3}}{4c}$$

$$\Big\downarrow \text{1930}$$

$$\frac{a\left(\frac{c^2(cx)^{3/2}\sqrt{ax^2+bx^3}}{3b}-\frac{5ac\left(\frac{c^2\sqrt{cx}\sqrt{ax^2+bx^3}}{2b}-\frac{3ac\left(\frac{c^2\sqrt{ax^2+bx^3}}{b\sqrt{cx}}-\frac{ac\int\frac{\sqrt{cx}}{\sqrt{bx^3+ax^2}}dx}{2b}\right)}{4b}\right)}{6b}\right)}{8c^2}+\frac{(cx)^{5/2}\sqrt{ax^2+bx^3}}{4c}$$

$$\Big\downarrow \; 1937$$

$$\frac{a\left(\frac{c^2(cx)^{3/2}\sqrt{ax^2+bx^3}}{3b}-\frac{5ac\left(\frac{c^2\sqrt{cx}\sqrt{ax^2+bx^3}}{2b}-\frac{3ac\left(\frac{c^2\sqrt{ax^2+bx^3}}{b\sqrt{cx}}-\frac{ac\sqrt{cx}\int\frac{\sqrt{x}}{\sqrt{bx^3+ax^2}}dx}{2b\sqrt{x}}\right)}{4b}\right)}{6b}\right)}{8c^2}+\frac{(cx)^{5/2}\sqrt{ax^2+bx^3}}{4c}$$

$$\Big\downarrow \; 1935$$

$$\frac{a\left(\frac{c^2(cx)^{3/2}\sqrt{ax^2+bx^3}}{3b}-\frac{5ac\left(\frac{c^2\sqrt{cx}\sqrt{ax^2+bx^3}}{2b}-\frac{3ac\left(\frac{c^2\sqrt{ax^2+bx^3}}{b\sqrt{cx}}-\frac{ac\sqrt{cx}\int\frac{1}{1-\frac{bx^3}{bx^3+ax^2}}d\frac{x^{3/2}}{\sqrt{bx^3+ax^2}}}{b\sqrt{x}}\right)}{4b}\right)}{6b}\right)}{8c^2}+\frac{(cx)^{5/2}\sqrt{ax^2+bx^3}}{4c}$$

$$\Big\downarrow \; 219$$

$$\frac{a\left(\frac{c^2(cx)^{3/2}\sqrt{ax^2+bx^3}}{3b}-\frac{5ac\left(\frac{c^2\sqrt{cx}\sqrt{ax^2+bx^3}}{2b}-\frac{3ac\left(\frac{c^2\sqrt{ax^2+bx^3}}{b\sqrt{cx}}-\frac{ac\sqrt{cx}\text{arctanh}\left(\frac{\sqrt{b}x^{3/2}}{\sqrt{ax^2+bx^3}}\right)}{b^{3/2}\sqrt{x}}\right)}{4b}\right)}{6b}\right)}{8c^2}+\frac{(cx)^{5/2}\sqrt{ax^2+bx^3}}{4c}$$

```
Int[(c*x)^(3/2)*Sqrt[a*x^2 + b*x^3],x]
```

```
((c*x)^(5/2)*Sqrt[a*x^2 + b*x^3])/(4*c) + (a*((c^2*(c*x)^(3/2)*Sqrt[a*x^2
+ b*x^3])/(3*b) - (5*a*c*((c^2*Sqrt[c*x]*Sqrt[a*x^2 + b*x^3])/(2*b) - (3*a
*c*((c^2*Sqrt[a*x^2 + b*x^3])/(b*Sqrt[c*x]) - (a*c*Sqrt[c*x]*ArcTanh[(Sqrt
[b]*x^(3/2))/Sqrt[a*x^2 + b*x^3]])/(b^(3/2)*Sqrt[x])))/(4*b)))/(6*b)))/(8*
c^2)
```

Defintions of rubi rules used

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> Simp[(c*x)^(m + 1)*((a*x^j + b*x^n)^p/(c*(m + n*p + 1))), x] + Simp[a*
(n - j)*(p/(c^j*(m + n*p + 1)))   Int[(c*x)^(m + j)*(a*x^j + b*x^n)^(p - 1)
, x], x] /; FreeQ[{a, b, c, m}, x] &&  !IntegerQ[p] && LtQ[0, j, n] && (Int
egersQ[j, n] || GtQ[c, 0]) && GtQ[p, 0] && NeQ[m + n*p + 1, 0]
```

```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> Simp[c^(n - 1)*(c*x)^(m - n + 1)*((a*x^j + b*x^n)^(p + 1)/(b*(m + n*p
+ 1))), x] - Simp[a*c^(n - j)*((m + j*p - n + j + 1)/(b*(m + n*p + 1)))   I
nt[(c*x)^(m - (n - j))*(a*x^j + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, p},
x] &&  !IntegerQ[p] && LtQ[0, j, n] && (IntegersQ[j, n] || GtQ[c, 0]) && Gt
Q[m + j*p - n + j + 1, 0] && NeQ[m + n*p + 1, 0]
```

```
Int[(x_)^(m_.)/Sqrt[(a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.)], x_Symbol] :> Simp
[-2/(n - j)   Subst[Int[1/(1 - a*x^2), x], x, x^(j/2)/Sqrt[a*x^j + b*x^n]],
 x] /; FreeQ[{a, b, j, n}, x] && EqQ[m, j/2 - 1] && NeQ[n, j]
```

```
Int[((c_)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_))^(p_), x_Symbol]
:> Simp[c^IntPart[m]*((c*x)^FracPart[m]/x^FracPart[m])   Int[x^m*(a*x^j + b
*x^n)^p, x], x] /; FreeQ[{a, b, c, j, m, n, p}, x] && IntegerQ[p + 1/2] &&
NeQ[n, j] && EqQ[Simplify[m + j*p + 1], 0]
```

Maple [A] (verified)

Time = 0.37 (sec) , antiderivative size = 138, normalized size of antiderivative = 0.77

method	result
risch	$\frac{\left(48 b^{3} x^{3}+8 a \,b^{2} x^{2}-10 a^{2} b x +15 a^{3}\right) c^{2} \sqrt{x^{2} \left(b x +a \right)}}{192 b^{3} \sqrt{c x}}-\frac{5 a^{4} \ln \left(\frac{\frac{1}{2} a c +c b x}{\sqrt{b c}}+\sqrt{b c \,x^{2}+a c x}\right) c^{2} \sqrt{x^{2} \left(b x +a \right)}\, \sqrt{c x \left(b x +a \right)}}{128 b^{3} \sqrt{b c}\, x \left(b x +a \right) \sqrt{c x}}$
default	$\frac{\sqrt{b \,x^{3}+a \,x^{2}}\, \sqrt{c x}\, c \left(96 b^{3} x^{3} \sqrt{c x \left(b x +a \right)}\, \sqrt{b c}+16 a \,b^{2} x^{2} \sqrt{c x \left(b x +a \right)}\, \sqrt{b c}-15 \ln \left(\frac{2 c b x +2 \sqrt{c x \left(b x +a \right)}\, \sqrt{b c}+a c}{2 \sqrt{b c}}\right) a^{4} c -20 \sqrt{b c}\, \sqrt{c x \left(b x +a \right)}\, a \cdots}{384 x \sqrt{c x \left(b x +a \right)}\, b^{3} \sqrt{b c}}$

```
int((c*x)^(3/2)*(b*x^3+a*x^2)^(1/2),x,method=_RETURNVERBOSE)
```

```
1/192*(48*b^3*x^3+8*a*b^2*x^2-10*a^2*b*x+15*a^3)/b^3*c^2*(x^2*(b*x+a))^(1/
2)/(c*x)^(1/2)-5/128*a^4/b^3*ln((1/2*a*c+c*b*x)/(b*c)^(1/2)+(b*c*x^2+a*c*x
)^(1/2))/(b*c)^(1/2)*c^2*(x^2*(b*x+a))^(1/2)/x/(b*x+a)*(c*x*(b*x+a))^(1/2)
/(c*x)^(1/2)
```

Fricas [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 242, normalized size of antiderivative = 1.35

$$\int (cx)^{3/2}\sqrt{ax^2+bx^3}\,dx = \left[\frac{15\,a^4cx\sqrt{\frac{c}{b}}\log\left(\frac{2\,bcx^2+acx-2\,\sqrt{bx^3+ax^2}\sqrt{cx}b\sqrt{\frac{c}{b}}}{x}\right)+2\,(48\,b^3cx^3+8\,ab^2cx^2-10\,a^2b}{384\,b^3x}\right.$$

```
integrate((c*x)^(3/2)*(b*x^3+a*x^2)^(1/2),x, algorithm="fricas")
```

```
[1/384*(15*a^4*c*x*sqrt(c/b)*log((2*b*c*x^2 + a*c*x - 2*sqrt(b*x^3 + a*x^2
)*sqrt(c*x)*b*sqrt(c/b))/x) + 2*(48*b^3*c*x^3 + 8*a*b^2*c*x^2 - 10*a^2*b*c
*x + 15*a^3*c)*sqrt(b*x^3 + a*x^2)*sqrt(c*x))/(b^3*x), 1/192*(15*a^4*c*x*s
qrt(-c/b)*arctan(sqrt(b*x^3 + a*x^2)*sqrt(c*x)*b*sqrt(-c/b)/(b*c*x^2 + a*c
*x)) + (48*b^3*c*x^3 + 8*a*b^2*c*x^2 - 10*a^2*b*c*x + 15*a^3*c)*sqrt(b*x^3
 + a*x^2)*sqrt(c*x))/(b^3*x)]
```

Sympy [F]

$$\int (cx)^{3/2}\sqrt{ax^2+bx^3}\,dx = \int (cx)^{\frac{3}{2}}\sqrt{x^2\,(a+bx)}\,dx$$

```
integrate((c*x)**(3/2)*(b*x**3+a*x**2)**(1/2),x)
```

```
Integral((c*x)**(3/2)*sqrt(x**2*(a + b*x)), x)
```

Maxima [F]

$$\int (cx)^{3/2}\sqrt{ax^2+bx^3}\,dx = \int \sqrt{bx^3+ax^2}(cx)^{\frac{3}{2}}\,dx$$

```
integrate((c*x)^(3/2)*(b*x^3+a*x^2)^(1/2),x, algorithm="maxima")
```

```
integrate(sqrt(b*x^3 + a*x^2)*(c*x)^(3/2), x)
```

Giac [A] (verification not implemented)

Time = 0.14 (sec) , antiderivative size = 147, normalized size of antiderivative = 0.82

$$\int (cx)^{3/2}\sqrt{ax^2+bx^3}\,dx =$$

$$-\frac{1}{384}\left(\frac{15\,a^4|c|\log\left(c^2|a|\right)\operatorname{sgn}(x)}{\sqrt{bc}b^3} - \frac{2\left(\frac{15\,a^4c^2\log\left(\left|-\sqrt{bc}\sqrt{cx}+\sqrt{bc^2x+ac^2}\right|\right)}{\sqrt{bc}b^3} + \sqrt{bc^2x+ac^2}\left(2\left(4\,cx\left(\frac{6\,x}{c^2}+\frac{a}{bc^2}\right) - \right.\right.\right.}{c^2}\right.$$

```
integrate((c*x)^(3/2)*(b*x^3+a*x^2)^(1/2),x, algorithm="giac")
```

```
-1/384*(15*a^4*abs(c)*log(c^2*abs(a))*sgn(x)/(sqrt(b*c)*b^3) - 2*(15*a^4*c
^2*log(abs(-sqrt(b*c)*sqrt(c*x) + sqrt(b*c^2*x + a*c^2)))/(sqrt(b*c)*b^3)
+ sqrt(b*c^2*x + a*c^2)*(2*(4*c*x*(6*x/c^2 + a/(b*c^2)) - 5*a^2/(b^2*c))*c
*x + 15*a^3/b^3)*sqrt(c*x))*abs(c)*sgn(x)/c^2)*c
```

Mupad [F(-1)]

Timed out.

$$\int (cx)^{3/2}\sqrt{ax^2+bx^3}\,dx = \int (c\,x)^{3/2}\,\sqrt{b\,x^3+a\,x^2}\,dx$$

```
int((c*x)^(3/2)*(a*x^2 + b*x^3)^(1/2),x)
```

```
int((c*x)^(3/2)*(a*x^2 + b*x^3)^(1/2), x)
```

Reduce [B] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 98, normalized size of antiderivative = 0.55

$$\int (cx)^{3/2}\sqrt{ax^2+bx^3}\,dx = \frac{\sqrt{c}\,c\Big(15\sqrt{x}\,\sqrt{bx+a}\,a^3b-10\sqrt{x}\,\sqrt{bx+a}\,a^2b^2x+8\sqrt{x}\,\sqrt{bx+a}\,a\,b^3x^2+48\sqrt{x}}{192b^4}$$

```
int((c*x)^(3/2)*(b*x^3+a*x^2)^(1/2),x)
```

```
(sqrt(c)*c*(15*sqrt(x)*sqrt(a + b*x)*a**3*b - 10*sqrt(x)*sqrt(a + b*x)*a**
2*b**2*x + 8*sqrt(x)*sqrt(a + b*x)*a*b**3*x**2 + 48*sqrt(x)*sqrt(a + b*x)*
b**4*x**3 - 15*sqrt(b)*log((sqrt(a + b*x) + sqrt(x)*sqrt(b))/sqrt(a))*a**4
))/(192*b**4)
```

3.327 $\int \sqrt{cx}\sqrt{ax^2+bx^3}\,dx$

Optimal result

Integrand size = 23, antiderivative size = 144

$$\int \sqrt{cx}\sqrt{ax^2+bx^3}\,dx = -\frac{a^2c\sqrt{ax^2+bx^3}}{8b^2\sqrt{cx}}+\frac{a\sqrt{cx}\sqrt{ax^2+bx^3}}{12b}+\frac{(cx)^{3/2}\sqrt{ax^2+bx^3}}{3c}+\frac{a^3\sqrt{c}\,\text{arctanh}\left(\frac{\sqrt{b}(cx)^{3/2}}{c^{3/2}\sqrt{ax^2+bx^3}}\right)}{8b^{5/2}}$$

```
-1/8*a^2*c*(b*x^3+a*x^2)^(1/2)/b^2/(c*x)^(1/2)+1/12*a*(c*x)^(1/2)*(b*x^3+a
*x^2)^(1/2)/b+1/3*(c*x)^(3/2)*(b*x^3+a*x^2)^(1/2)/c+1/8*a^3*c^(1/2)*arctan
h(b^(1/2)*(c*x)^(3/2)/c^(3/2)/(b*x^3+a*x^2)^(1/2))/b^(5/2)
```

Mathematica [A] (verified)

Time = 0.08 (sec) , antiderivative size = 115, normalized size of antiderivative = 0.80

$$\int \sqrt{cx}\sqrt{ax^2+bx^3}\,dx = \frac{\sqrt{x}\sqrt{cx}\left(\sqrt{b}\sqrt{x}(a+bx)\left(-3a^2+2abx+8b^2x^2\right)+6a^3\sqrt{a+bx}\,\text{arctanh}\left(\frac{\sqrt{b}\sqrt{x}}{-\sqrt{a}+\sqrt{a+bx}}\right)\right)}{24b^{5/2}\sqrt{x^2(a+bx)}}$$

```
Integrate[Sqrt[c*x]*Sqrt[a*x^2 + b*x^3],x]
```

```
(Sqrt[x]*Sqrt[c*x]*(Sqrt[b]*Sqrt[x]*(a + b*x)*(-3*a^2 + 2*a*b*x + 8*b^2*x^
2) + 6*a^3*Sqrt[a + b*x]*ArcTanh[(Sqrt[b]*Sqrt[x])/(-Sqrt[a] + Sqrt[a + b*
x])]))/(24*b^(5/2)*Sqrt[x^2*(a + b*x)])
```

Rubi [A] (verified)

Time = 0.57 (sec) , antiderivative size = 158, normalized size of antiderivative = 1.10, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.261$, Rules used = {1927, 1930, 1930, 1937, 1935, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sqrt{cx}\sqrt{ax^2+bx^3}\,dx$$

$$\Big\downarrow \text{1927}$$

$$\frac{a\int \frac{(cx)^{5/2}}{\sqrt{bx^3+ax^2}}dx}{6c^2}+\frac{(cx)^{3/2}\sqrt{ax^2+bx^3}}{3c}$$

$$\Big\downarrow \text{1930}$$

$$\frac{a\left(\frac{c^2\sqrt{cx}\sqrt{ax^2+bx^3}}{2b}-\frac{3ac\int \frac{(cx)^{3/2}}{\sqrt{bx^3+ax^2}}dx}{4b}\right)}{6c^2}+\frac{(cx)^{3/2}\sqrt{ax^2+bx^3}}{3c}$$

$$\Big\downarrow \text{1930}$$

$$\frac{a\left(\frac{c^2\sqrt{cx}\sqrt{ax^2+bx^3}}{2b}-\frac{3ac\left(\frac{c^2\sqrt{ax^2+bx^3}}{b\sqrt{cx}}-\frac{ac\int \frac{\sqrt{cx}}{\sqrt{bx^3+ax^2}}dx}{2b}\right)}{4b}\right)}{6c^2}+\frac{(cx)^{3/2}\sqrt{ax^2+bx^3}}{3c}$$

$$\Big\downarrow \text{1937}$$

$$\frac{a\left(\frac{c^2\sqrt{cx}\sqrt{ax^2+bx^3}}{2b}-\frac{3ac\left(\frac{c^2\sqrt{ax^2+bx^3}}{b\sqrt{cx}}-\frac{ac\sqrt{cx}\int \frac{\sqrt{x}}{\sqrt{bx^3+ax^2}}dx}{2b\sqrt{x}}\right)}{4b}\right)}{6c^2}+\frac{(cx)^{3/2}\sqrt{ax^2+bx^3}}{3c}$$

$$\Big\downarrow 1935$$

$$\frac{a\left(\frac{c^2\sqrt{cx}\sqrt{ax^2+bx^3}}{2b}-\frac{3ac\left(\frac{c^2\sqrt{ax^2+bx^3}}{b\sqrt{cx}}-\frac{ac\sqrt{cx}\int\frac{1}{1-\frac{bx^3}{bx^3+ax^2}}d\frac{x^{3/2}}{\sqrt{bx^3+ax^2}}}{b\sqrt{x}}\right)}{4b}\right)}{6c^2}+\frac{(cx)^{3/2}\sqrt{ax^2+bx^3}}{3c}$$

$$\Big\downarrow 219$$

$$\frac{a\left(\frac{c^2\sqrt{cx}\sqrt{ax^2+bx^3}}{2b}-\frac{3ac\left(\frac{c^2\sqrt{ax^2+bx^3}}{b\sqrt{cx}}-\frac{ac\sqrt{cx}\,\text{arctanh}\left(\frac{\sqrt{b}x^{3/2}}{\sqrt{ax^2+bx^3}}\right)}{b^{3/2}\sqrt{x}}\right)}{4b}\right)}{6c^2}+\frac{(cx)^{3/2}\sqrt{ax^2+bx^3}}{3c}$$

```
Int[Sqrt[c*x]*Sqrt[a*x^2 + b*x^3],x]
```

```
((c*x)^(3/2)*Sqrt[a*x^2 + b*x^3])/(3*c) + (a*((c^2*Sqrt[c*x]*Sqrt[a*x^2 +
b*x^3])/(2*b) - (3*a*c*((c^2*Sqrt[a*x^2 + b*x^3])/(b*Sqrt[c*x]) - (a*c*Sqr
t[c*x]*ArcTanh[(Sqrt[b]*x^(3/2))/Sqrt[a*x^2 + b*x^3]])/(b^(3/2)*Sqrt[x])))
/(4*b)))/(6*c^2)
```

Defintions of rubi rules used

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> Simp[(c*x)^(m + 1)*((a*x^j + b*x^n)^p/(c*(m + n*p + 1))), x] + Simp[a*
(n - j)*(p/(c^j*(m + n*p + 1)))   Int[(c*x)^(m + j)*(a*x^j + b*x^n)^(p - 1)
, x], x] /; FreeQ[{a, b, c, m}, x] && !IntegerQ[p] && LtQ[0, j, n] && (Int
egersQ[j, n] || GtQ[c, 0]) && GtQ[p, 0] && NeQ[m + n*p + 1, 0]
```

```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> Simp[c^(n - 1)*(c*x)^(m - n + 1)*((a*x^j + b*x^n)^(p + 1)/(b*(m + n*p
+ 1))), x] - Simp[a*c^(n - j)*((m + j*p - n + j + 1)/(b*(m + n*p + 1)))   I
nt[(c*x)^(m - (n - j))*(a*x^j + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, p},
x] &&  !IntegerQ[p] && LtQ[0, j, n] && (IntegersQ[j, n] || GtQ[c, 0]) && Gt
Q[m + j*p - n + j + 1, 0] && NeQ[m + n*p + 1, 0]
```

```
Int[(x_)^(m_.)/Sqrt[(a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.)], x_Symbol] :> Simp
[-2/(n - j)   Subst[Int[1/(1 - a*x^2), x], x, x^(j/2)/Sqrt[a*x^j + b*x^n]],
 x] /; FreeQ[{a, b, j, n}, x] && EqQ[m, j/2 - 1] && NeQ[n, j]
```

```
Int[((c_)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_))^(p_), x_Symbol]
:> Simp[c^IntPart[m]*((c*x)^FracPart[m]/x^FracPart[m])   Int[x^m*(a*x^j + b
*x^n)^p, x], x] /; FreeQ[{a, b, c, j, m, n, p}, x] && IntegerQ[p + 1/2] &&
NeQ[n, j] && EqQ[Simplify[m + j*p + 1], 0]
```

Maple [A] (verified)

Time = 0.36 (sec) , antiderivative size = 123, normalized size of antiderivative = 0.85

method	result
risch	$-\frac{\left(-8b^{2}x^{2}-2abx+3a^{2}\right)c\sqrt{x^{2}\left(bx+a\right)}}{24b^{2}\sqrt{cx}}+\frac{a^{3}\ln\left(\frac{\frac{1}{2}ac+cbx}{\sqrt{bc}}+\sqrt{bc\,x^{2}+acx}\right)c\sqrt{x^{2}\left(bx+a\right)}\,\sqrt{cx\left(bx+a\right)}}{16b^{2}\sqrt{bc}\,x\left(bx+a\right)\sqrt{cx}}$
default	$-\frac{\sqrt{b\,x^{3}+a\,x^{2}}\,\sqrt{cx}\,\left(-16b^{2}x^{2}\sqrt{cx\left(bx+a\right)}\,\sqrt{bc}-3\ln\left(\frac{2cbx+2\sqrt{cx\left(bx+a\right)}\,\sqrt{bc}+ac}{2\sqrt{bc}}\right)a^{3}c-4\sqrt{bc}\,\sqrt{cx\left(bx+a\right)}\,abx+6\sqrt{bc}\,\sqrt{cx\left(bx+a\right)}\,a^{2}\right)}{48x\sqrt{cx\left(bx+a\right)}\,b^{2}\sqrt{bc}}$

```
int((c*x)^(1/2)*(b*x^3+a*x^2)^(1/2),x,method=_RETURNVERBOSE)
```

```
-1/24*(-8*b^2*x^2-2*a*b*x+3*a^2)/b^2*c*(x^2*(b*x+a))^(1/2)/(c*x)^(1/2)+1/1
6*a^3/b^2*ln((1/2*a*c+c*b*x)/(b*c)^(1/2)+(b*c*x^2+a*c*x)^(1/2))/(b*c)^(1/2
)*c*(x^2*(b*x+a))^(1/2)/x/(b*x+a)*(c*x*(b*x+a))^(1/2)/(c*x)^(1/2)
```

Fricas [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 211, normalized size of antiderivative = 1.47

$$\int \sqrt{cx}\sqrt{ax^2+bx^3}\,dx$$
$$= \left[\frac{3\,a^3x\sqrt{\frac{c}{b}}\log\left(\frac{2\,bcx^2+acx+2\,\sqrt{bx^3+ax^2}\sqrt{cx}b\sqrt{\frac{c}{b}}}{x}\right)+2\,(8\,b^2x^2+2\,abx-3\,a^2)\sqrt{bx^3+ax^2}\sqrt{cx}}{48\,b^2x},\right.$$
$$\left.-\frac{3\,a^3x\sqrt{-\frac{c}{b}}\arctan\left(\frac{\sqrt{bx^3+ax^2}\sqrt{cx}b\sqrt{-\frac{c}{b}}}{bcx^2+acx}\right)-(8\,b^2x^2+2\,abx-3\,a^2)\sqrt{bx^3+ax^2}\sqrt{cx}}{24\,b^2x}\right]$$

```
integrate((c*x)^(1/2)*(b*x^3+a*x^2)^(1/2),x, algorithm="fricas")
```

```
[1/48*(3*a^3*x*sqrt(c/b)*log((2*b*c*x^2 + a*c*x + 2*sqrt(b*x^3 + a*x^2)*sq
rt(c*x)*b*sqrt(c/b))/x) + 2*(8*b^2*x^2 + 2*a*b*x - 3*a^2)*sqrt(b*x^3 + a*x
^2)*sqrt(c*x))/(b^2*x), -1/24*(3*a^3*x*sqrt(-c/b)*arctan(sqrt(b*x^3 + a*x^
2)*sqrt(c*x)*b*sqrt(-c/b)/(b*c*x^2 + a*c*x)) - (8*b^2*x^2 + 2*a*b*x - 3*a^
2)*sqrt(b*x^3 + a*x^2)*sqrt(c*x))/(b^2*x)]
```

Sympy [F]

$$\int \sqrt{cx}\sqrt{ax^2+bx^3}\,dx = \int \sqrt{cx}\sqrt{x^2\,(a+bx)}\,dx$$

```
integrate((c*x)**(1/2)*(b*x**3+a*x**2)**(1/2),x)
```

```
Integral(sqrt(c*x)*sqrt(x**2*(a + b*x)), x)
```

Maxima [F]

$$\int \sqrt{cx}\sqrt{ax^2+bx^3}\,dx=\int \sqrt{bx^3+ax^2}\sqrt{cx}\,dx$$

```
integrate((c*x)^(1/2)*(b*x^3+a*x^2)^(1/2),x, algorithm="maxima")
```

```
integrate(sqrt(b*x^3 + a*x^2)*sqrt(c*x), x)
```

Giac [A] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 128, normalized size of antiderivative = 0.89

$$\int \sqrt{cx}\sqrt{ax^2+bx^3}\,dx=\frac{a^3|c|\log\left(c^2|a|\right)\mathrm{sgn}(x)}{16\sqrt{bc}b^2}-\frac{\left(\frac{3\,a^3c^4\log\left(\left|-\sqrt{bc}\sqrt{cx}+\sqrt{bc^2x+ac^2}\right|\right)}{\sqrt{bc}b^2}-\sqrt{bc^2x+ac^2}\left(2\left(4\,cx+\frac{ac}{b}\right)cx-\frac{3\,a^2c^2}{b^2}\right)\sqrt{cx}\right)|c|\mathrm{sgn}(x)}{24\,c^4}$$

```
integrate((c*x)^(1/2)*(b*x^3+a*x^2)^(1/2),x, algorithm="giac")
```

```
1/16*a^3*abs(c)*log(c^2*abs(a))*sgn(x)/(sqrt(b*c)*b^2) - 1/24*(3*a^3*c^4*l
og(abs(-sqrt(b*c)*sqrt(c*x) + sqrt(b*c^2*x + a*c^2)))/(sqrt(b*c)*b^2) - sq
rt(b*c^2*x + a*c^2)*(2*(4*c*x + a*c/b)*c*x - 3*a^2*c^2/b^2)*sqrt(c*x))*abs
(c)*sgn(x)/c^4
```

Mupad [F(-1)]

Timed out.

$$\int \sqrt{cx}\sqrt{ax^2+bx^3}\,dx=\int \sqrt{c\,x}\,\sqrt{b\,x^3+a\,x^2}\,dx$$

```
int((c*x)^(1/2)*(a*x^2 + b*x^3)^(1/2),x)
```

```
int((c*x)^(1/2)*(a*x^2 + b*x^3)^(1/2), x)
```

Reduce [B] (verification not implemented)

Time = 0.35 (sec) , antiderivative size = 78, normalized size of antiderivative = 0.54

$$\int \sqrt{cx}\sqrt{ax^2+bx^3}\,dx$$
$$=\frac{\sqrt{c}\left(-3\sqrt{x}\,\sqrt{bx+a}\,a^2b+2\sqrt{x}\,\sqrt{bx+a}\,a\,b^2x+8\sqrt{x}\,\sqrt{bx+a}\,b^3x^2+3\sqrt{b}\log\left(\frac{\sqrt{bx+a}+\sqrt{x}\,\sqrt{b}}{\sqrt{a}}\right)a^3\right)}{24b^3}$$

```
int((c*x)^(1/2)*(b*x^3+a*x^2)^(1/2),x)
```

```
(sqrt(c)*( - 3*sqrt(x)*sqrt(a + b*x)*a**2*b + 2*sqrt(x)*sqrt(a + b*x)*a*b*
*2*x + 8*sqrt(x)*sqrt(a + b*x)*b**3*x**2 + 3*sqrt(b)*log((sqrt(a + b*x) +
sqrt(x)*sqrt(b))/sqrt(a))*a**3))/(24*b**3)
```

3.328 $\int \frac{\sqrt{ax^2+bx^3}}{\sqrt{cx}}\,dx$

Optimal result

Integrand size = 23, antiderivative size = 111

$$\int \frac{\sqrt{ax^2+bx^3}}{\sqrt{cx}}\,dx=\frac{a\sqrt{ax^2+bx^3}}{4b\sqrt{cx}}+\frac{\sqrt{cx}\sqrt{ax^2+bx^3}}{2c}-\frac{a^2\text{arctanh}\left(\frac{\sqrt{b}(cx)^{3/2}}{c^{3/2}\sqrt{ax^2+bx^3}}\right)}{4b^{3/2}\sqrt{c}}$$

```
1/4*a*(b*x^3+a*x^2)^(1/2)/b/(c*x)^(1/2)+1/2*(c*x)^(1/2)*(b*x^3+a*x^2)^(1/2
)/c-1/4*a^2*arctanh(b^(1/2)*(c*x)^(3/2)/c^(3/2)/(b*x^3+a*x^2)^(1/2))/b^(3/
2)/c^(1/2)
```

Mathematica [A] (verified)

Time = 0.07 (sec) , antiderivative size = 106, normalized size of antiderivative = 0.95

$$\int \frac{\sqrt{ax^2+bx^3}}{\sqrt{cx}}\,dx=\frac{x^{3/2}\sqrt{a+bx}\left(\sqrt{b}\sqrt{x}\sqrt{a+bx}(a+2bx)+2a^2\text{arctanh}\left(\frac{\sqrt{b}\sqrt{x}}{\sqrt{a}-\sqrt{a+bx}}\right)\right)}{4b^{3/2}\sqrt{cx}\sqrt{x^2(a+bx)}}$$

```
Integrate[Sqrt[a*x^2 + b*x^3]/Sqrt[c*x],x]
```

```
(x^(3/2)*Sqrt[a + b*x]*(Sqrt[b]*Sqrt[x]*Sqrt[a + b*x]*(a + 2*b*x) + 2*a^2*
ArcTanh[(Sqrt[b]*Sqrt[x])/(Sqrt[a] - Sqrt[a + b*x])]))/(4*b^(3/2)*Sqrt[c*x
]*Sqrt[x^2*(a + b*x)])
```

Rubi [A] (verified)

Time = 0.47 (sec) , antiderivative size = 116, normalized size of antiderivative = 1.05, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.217$, Rules used = {1927, 1930, 1937, 1935, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sqrt{ax^2+bx^3}}{\sqrt{cx}}dx$$

$$\Big\downarrow \text{1927}$$

$$\frac{a\int \frac{(cx)^{3/2}}{\sqrt{bx^3+ax^2}}dx}{4c^2}+\frac{\sqrt{cx}\sqrt{ax^2+bx^3}}{2c}$$

$$\Big\downarrow \text{1930}$$

$$\frac{a\left(\frac{c^2\sqrt{ax^2+bx^3}}{b\sqrt{cx}}-\frac{ac\int \frac{\sqrt{cx}}{\sqrt{bx^3+ax^2}}dx}{2b}\right)}{4c^2}+\frac{\sqrt{cx}\sqrt{ax^2+bx^3}}{2c}$$

$$\Big\downarrow \text{1937}$$

$$\frac{a\left(\frac{c^2\sqrt{ax^2+bx^3}}{b\sqrt{cx}}-\frac{ac\sqrt{cx}\int \frac{\sqrt{x}}{\sqrt{bx^3+ax^2}}dx}{2b\sqrt{x}}\right)}{4c^2}+\frac{\sqrt{cx}\sqrt{ax^2+bx^3}}{2c}$$

$$\Big\downarrow \text{1935}$$

$$\frac{a\left(\frac{c^2\sqrt{ax^2+bx^3}}{b\sqrt{cx}}-\frac{ac\sqrt{cx}\int \frac{1}{1-\frac{bx^3}{bx^3+ax^2}}d\frac{x^{3/2}}{\sqrt{bx^3+ax^2}}}{b\sqrt{x}}\right)}{4c^2}+\frac{\sqrt{cx}\sqrt{ax^2+bx^3}}{2c}$$

$$\Big\downarrow \text{219}$$

$$\frac{a\left(\frac{c^2\sqrt{ax^2+bx^3}}{b\sqrt{cx}}-\frac{ac\sqrt{cx}\,\text{arctanh}\left(\frac{\sqrt{b}x^{3/2}}{\sqrt{ax^2+bx^3}}\right)}{b^{3/2}\sqrt{x}}\right)}{4c^2}+\frac{\sqrt{cx}\sqrt{ax^2+bx^3}}{2c}$$

```
Int[Sqrt[a*x^2 + b*x^3]/Sqrt[c*x],x]
```

```
(Sqrt[c*x]*Sqrt[a*x^2 + b*x^3])/(2*c) + (a*((c^2*Sqrt[a*x^2 + b*x^3])/(b*S
qrt[c*x]) - (a*c*Sqrt[c*x]*ArcTanh[(Sqrt[b]*x^(3/2))/Sqrt[a*x^2 + b*x^3]])
/(b^(3/2)*Sqrt[x])))/(4*c^2)
```

Defintions of rubi rules used

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> Simp[(c*x)^(m + 1)*((a*x^j + b*x^n)^p/(c*(m + n*p + 1))), x] + Simp[a*
(n - j)*(p/(c^j*(m + n*p + 1)))   Int[(c*x)^(m + j)*(a*x^j + b*x^n)^(p - 1)
, x], x] /; FreeQ[{a, b, c, m}, x] &&  !IntegerQ[p] && LtQ[0, j, n] && (Int
egersQ[j, n] || GtQ[c, 0]) && GtQ[p, 0] && NeQ[m + n*p + 1, 0]
```

```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> Simp[c^(n - 1)*(c*x)^(m - n + 1)*((a*x^j + b*x^n)^(p + 1)/(b*(m + n*p
+ 1))), x] - Simp[a*c^(n - j)*((m + j*p - n + j + 1)/(b*(m + n*p + 1)))   I
nt[(c*x)^(m - (n - j))*(a*x^j + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, p},
x] &&  !IntegerQ[p] && LtQ[0, j, n] && (IntegersQ[j, n] || GtQ[c, 0]) && Gt
Q[m + j*p - n + j + 1, 0] && NeQ[m + n*p + 1, 0]
```

```
Int[(x_)^(m_.)/Sqrt[(a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.)], x_Symbol] :> Simp
[-2/(n - j)   Subst[Int[1/(1 - a*x^2), x], x, x^(j/2)/Sqrt[a*x^j + b*x^n]],
 x] /; FreeQ[{a, b, j, n}, x] && EqQ[m, j/2 - 1] && NeQ[n, j]
```

```
Int[((c_)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_))^(p_), x_Symbol]
:> Simp[c^IntPart[m]*((c*x)^FracPart[m]/x^FracPart[m])   Int[x^m*(a*x^j + b
*x^n)^p, x], x] /; FreeQ[{a, b, c, j, m, n, p}, x] && IntegerQ[p + 1/2] &&
NeQ[n, j] && EqQ[Simplify[m + j*p + 1], 0]
```

Maple [A] (verified)

Time = 0.36 (sec) , antiderivative size = 108, normalized size of antiderivative = 0.97

method	result	size
risch	$\frac{(2bx+a)\sqrt{x^2(bx+a)}}{4b\sqrt{cx}}-\frac{a^2\ln\left(\frac{\frac{1}{2}ac+cbx}{\sqrt{bc}}+\sqrt{bc\,x^2+acx}\right)\sqrt{x^2(bx+a)}\,\sqrt{cx(bx+a)}}{8b\sqrt{bc}\,x(bx+a)\sqrt{cx}}$	108
default	$\frac{\sqrt{b\,x^3+a\,x^2}\left(4\sqrt{bc}\,\sqrt{cx(bx+a)}\,bx-a^2c\ln\left(\frac{2cbx+2\sqrt{cx(bx+a)}\,\sqrt{bc}+ac}{2\sqrt{bc}}\right)+2\sqrt{bc}\,\sqrt{cx(bx+a)}\,a\right)}{8\sqrt{cx}\,\sqrt{cx(bx+a)}\,b\sqrt{bc}}$	117

```
int((b*x^3+a*x^2)^(1/2)/(c*x)^(1/2),x,method=_RETURNVERBOSE)
```

```
1/4*(2*b*x+a)/b*(x^2*(b*x+a))^(1/2)/(c*x)^(1/2)-1/8*a^2/b*ln((1/2*a*c+c*b*
x)/(b*c)^(1/2)+(b*c*x^2+a*c*x)^(1/2))/(b*c)^(1/2)*(x^2*(b*x+a))^(1/2)/x/(b
*x+a)*(c*x*(b*x+a))^(1/2)/(c*x)^(1/2)
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 186, normalized size of antiderivative = 1.68

$$\int \frac{\sqrt{ax^2+bx^3}}{\sqrt{cx}}\,dx$$
$$=\left[\frac{\sqrt{bc}a^2x\log\left(\frac{2\,bcx^2+acx-2\,\sqrt{bx^3+ax^2}\sqrt{bc}\sqrt{cx}}{x}\right)+2\,\sqrt{bx^3+ax^2}(2\,b^2x+ab)\sqrt{cx}}{8\,b^2cx},\frac{\sqrt{-bc}a^2x\arctan\left(\frac{\sqrt{bx^3+ax^2}\sqrt{-}}{bcx^2+ac}\right.}{}\right.$$

```
integrate((b*x^3+a*x^2)^(1/2)/(c*x)^(1/2),x, algorithm="fricas")
```

```
[1/8*(sqrt(b*c)*a^2*x*log((2*b*c*x^2 + a*c*x - 2*sqrt(b*x^3 + a*x^2)*sqrt(
b*c)*sqrt(c*x))/x) + 2*sqrt(b*x^3 + a*x^2)*(2*b^2*x + a*b)*sqrt(c*x))/(b^2
*c*x), 1/4*(sqrt(-b*c)*a^2*x*arctan(sqrt(b*x^3 + a*x^2)*sqrt(-b*c)*sqrt(c*
x)/(b*c*x^2 + a*c*x)) + sqrt(b*x^3 + a*x^2)*(2*b^2*x + a*b)*sqrt(c*x))/(b^
2*c*x)]
```

Sympy [F]

$$\int \frac{\sqrt{ax^2 + bx^3}}{\sqrt{cx}}\,dx = \int \frac{\sqrt{x^2\,(a + bx)}}{\sqrt{cx}}\,dx$$

```
integrate((b*x**3+a*x**2)**(1/2)/(c*x)**(1/2),x)
```

```
Integral(sqrt(x**2*(a + b*x))/sqrt(c*x), x)
```

Maxima [F]

$$\int \frac{\sqrt{ax^2 + bx^3}}{\sqrt{cx}}\,dx = \int \frac{\sqrt{bx^3 + ax^2}}{\sqrt{cx}}\,dx$$

```
integrate((b*x^3+a*x^2)^(1/2)/(c*x)^(1/2),x, algorithm="maxima")
```

```
integrate(sqrt(b*x^3 + a*x^2)/sqrt(c*x), x)
```

Giac [A] (verification not implemented)

Time = 0.17 (sec) , antiderivative size = 133, normalized size of antiderivative = 1.20

$$\int \frac{\sqrt{ax^2+bx^3}}{\sqrt{cx}}\,dx = -\frac{a^2\log\left(\sqrt{bc}\sqrt{a}\right)\operatorname{sgn}(x)}{4\sqrt{bc}|b|} + \frac{\left(\frac{a^2\log\left(\left|-\sqrt{bc}\sqrt{bx+a}+\sqrt{(bx+a)bc-abc}\right|\right)\operatorname{sgn}(x)}{\sqrt{bc}b} + \sqrt{(bx+a)bc-abc}\sqrt{bx+a}\left(\frac{2(bx+a)\operatorname{sgn}(x)}{b^2c} - \frac{a\operatorname{sgn}(x)}{b^2c}\right)\right)b}{4|b|}$$

```
integrate((b*x^3+a*x^2)^(1/2)/(c*x)^(1/2),x, algorithm="giac")
```

```
-1/4*a^2*log(sqrt(b*c)*sqrt(a))*sgn(x)/(sqrt(b*c)*abs(b)) + 1/4*(a^2*log(a
bs(-sqrt(b*c)*sqrt(b*x + a) + sqrt((b*x + a)*b*c - a*b*c)))*sgn(x)/(sqrt(b
*c)*b) + sqrt((b*x + a)*b*c - a*b*c)*sqrt(b*x + a)*(2*(b*x + a)*sgn(x)/(b^
2*c) - a*sgn(x)/(b^2*c)))*b/abs(b)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\sqrt{ax^2+bx^3}}{\sqrt{cx}}\,dx = \int \frac{\sqrt{b\,x^3+a\,x^2}}{\sqrt{c\,x}}\,dx$$

```
int((a*x^2 + b*x^3)^(1/2)/(c*x)^(1/2),x)
```

```
int((a*x^2 + b*x^3)^(1/2)/(c*x)^(1/2), x)
```

Reduce [B] (verification not implemented)

Time = 0.56 (sec) , antiderivative size = 61, normalized size of antiderivative = 0.55

$$\int \frac{\sqrt{ax^2+bx^3}}{\sqrt{cx}}\,dx = \frac{\sqrt{c}\left(\sqrt{x}\,\sqrt{bx+a}\,ab+2\sqrt{x}\,\sqrt{bx+a}\,b^2x-\sqrt{b}\log\left(\frac{\sqrt{bx+a}+\sqrt{x}\,\sqrt{b}}{\sqrt{a}}\right)a^2\right)}{4b^2c}$$

```
int((b*x^3+a*x^2)^(1/2)/(c*x)^(1/2),x)
```

```
(sqrt(c)*(sqrt(x)*sqrt(a + b*x)*a*b + 2*sqrt(x)*sqrt(a + b*x)*b**2*x - sqr
t(b)*log((sqrt(a + b*x) + sqrt(x)*sqrt(b))/sqrt(a))*a**2))/(4*b**2*c)
```

3.329 $\int \frac{\sqrt{ax^2+bx^3}}{(cx)^{3/2}}\,dx$

Optimal result

Integrand size = 23, antiderivative size = 73

$$\int \frac{\sqrt{ax^2+bx^3}}{(cx)^{3/2}}\,dx=\frac{\sqrt{ax^2+bx^3}}{c\sqrt{cx}}+\frac{a\operatorname{arctanh}\left(\frac{\sqrt{b}(cx)^{3/2}}{c^{3/2}\sqrt{ax^2+bx^3}}\right)}{\sqrt{b}c^{3/2}}$$

```
(b*x^3+a*x^2)^(1/2)/c/(c*x)^(1/2)+a*arctanh(b^(1/2)*(c*x)^(3/2)/c^(3/2)/(b
*x^3+a*x^2)^(1/2))/b^(1/2)/c^(3/2)
```

Mathematica [A] (verified)

Time = 0.03 (sec) , antiderivative size = 72, normalized size of antiderivative = 0.99

$$\int \frac{\sqrt{ax^2+bx^3}}{(cx)^{3/2}}\,dx=\frac{\sqrt{x}\sqrt{x^2(a+bx)}\left(\sqrt{x}-\frac{a\log\left(-\sqrt{b}\sqrt{x}+\sqrt{a+bx}\right)}{\sqrt{b}\sqrt{a+bx}}\right)}{(cx)^{3/2}}$$

```
Integrate[Sqrt[a*x^2 + b*x^3]/(c*x)^(3/2),x]
```

```
(Sqrt[x]*Sqrt[x^2*(a + b*x)]*(Sqrt[x] - (a*Log[-(Sqrt[b]*Sqrt[x]) + Sqrt[a
 + b*x]])/(Sqrt[b]*Sqrt[a + b*x])))/(c*x)^(3/2)
```

Rubi [A] (verified)

Time = 0.38 (sec) , antiderivative size = 76, normalized size of antiderivative = 1.04, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.174$, Rules used = {1927, 1937, 1935, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sqrt{ax^2+bx^3}}{(cx)^{3/2}}\,dx$$

$$\Big\downarrow \text{ 1927}$$

$$\frac{a\int \frac{\sqrt{cx}}{\sqrt{bx^3+ax^2}}dx}{2c^2}+\frac{\sqrt{ax^2+bx^3}}{c\sqrt{cx}}$$

$$\Big\downarrow \text{ 1937}$$

$$\frac{a\sqrt{cx}\int \frac{\sqrt{x}}{\sqrt{bx^3+ax^2}}dx}{2c^2\sqrt{x}}+\frac{\sqrt{ax^2+bx^3}}{c\sqrt{cx}}$$

$$\Big\downarrow \text{ 1935}$$

$$\frac{a\sqrt{cx}\int \frac{1}{1-\frac{bx^3}{bx^3+ax^2}}d\frac{x^{3/2}}{\sqrt{bx^3+ax^2}}}{c^2\sqrt{x}}+\frac{\sqrt{ax^2+bx^3}}{c\sqrt{cx}}$$

$$\Big\downarrow \text{ 219}$$

$$\frac{a\sqrt{cx}\text{arctanh}\left(\frac{\sqrt{b}x^{3/2}}{\sqrt{ax^2+bx^3}}\right)}{\sqrt{b}c^2\sqrt{x}}+\frac{\sqrt{ax^2+bx^3}}{c\sqrt{cx}}$$

```
Int[Sqrt[a*x^2 + b*x^3]/(c*x)^(3/2),x]
```

```
Sqrt[a*x^2 + b*x^3]/(c*Sqrt[c*x]) + (a*Sqrt[c*x]*ArcTanh[(Sqrt[b]*x^(3/2))
/Sqrt[a*x^2 + b*x^3]])/(Sqrt[b]*c^2*Sqrt[x])
```

Defintions of rubi rules used

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> Simp[(c*x)^(m + 1)*((a*x^j + b*x^n)^p/(c*(m + n*p + 1))), x] + Simp[a*
(n - j)*(p/(c^j*(m + n*p + 1)))   Int[(c*x)^(m + j)*(a*x^j + b*x^n)^(p - 1)
, x], x] /; FreeQ[{a, b, c, m}, x] &&  !IntegerQ[p] && LtQ[0, j, n] && (Int
egersQ[j, n] || GtQ[c, 0]) && GtQ[p, 0] && NeQ[m + n*p + 1, 0]
```

```
Int[(x_)^(m_.)/Sqrt[(a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.)], x_Symbol] :> Simp
[-2/(n - j)   Subst[Int[1/(1 - a*x^2), x], x, x^(j/2)/Sqrt[a*x^j + b*x^n]],
 x] /; FreeQ[{a, b, j, n}, x] && EqQ[m, j/2 - 1] && NeQ[n, j]
```

```
Int[((c_)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_))^(p_), x_Symbol]
:> Simp[c^IntPart[m]*((c*x)^FracPart[m]/x^FracPart[m])   Int[x^m*(a*x^j + b
*x^n)^p, x], x] /; FreeQ[{a, b, c, j, m, n, p}, x] && IntegerQ[p + 1/2] &&
NeQ[n, j] && EqQ[Simplify[m + j*p + 1], 0]
```

Maple [A] (verified)

Time = 0.36 (sec) , antiderivative size = 94, normalized size of antiderivative = 1.29

method	result	size
default	$\frac{\sqrt{b\,x^{3}+a\,x^{2}}\,\left(a c \ln\left(\frac{2 c b x +2 \sqrt{c x \left(b x +a \right)}\,\sqrt{b c}+a c}{2 \sqrt{b c}}\right)+2 \sqrt{c x \left(b x +a \right)}\,\sqrt{b c}\right)}{2 c \sqrt{c x}\,\sqrt{c x \left(b x +a \right)}\,\sqrt{b c}}$	94
risch	$\frac{\sqrt{x^{2} \left(b x +a \right)}}{c \sqrt{c x}}+\frac{a \ln\left(\frac{\frac{1}{2} a c +c b x}{\sqrt{b c}}+\sqrt{b c\,x^{2}+a c x}\right) \sqrt{x^{2} \left(b x +a \right)}\,\sqrt{c x \left(b x +a \right)}}{2 \sqrt{b c}\,c x \left(b x +a \right) \sqrt{c x}}$	99

```
int((b*x^3+a*x^2)^(1/2)/(c*x)^(3/2),x,method=_RETURNVERBOSE)
```

```
1/2*(b*x^3+a*x^2)^(1/2)*(a*c*ln(1/2*(2*c*b*x+2*(c*x*(b*x+a))^(1/2)*(b*c)^(
1/2)+a*c)/(b*c)^(1/2))+2*(c*x*(b*x+a))^(1/2)*(b*c)^(1/2))/c/(c*x)^(1/2)/(c
*x*(b*x+a))^(1/2)/(b*c)^(1/2)
```

Fricas [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 165, normalized size of antiderivative = 2.26

$$\int \frac{\sqrt{ax^2+bx^3}}{(cx)^{3/2}}\,dx = \left[\frac{\sqrt{bc}ax\log\left(\frac{2\,bcx^2+acx+2\,\sqrt{bx^3+ax^2}\sqrt{bc}\sqrt{cx}}{x}\right)+2\,\sqrt{bx^3+ax^2}\sqrt{cx}b}{2\,bc^2x}, -\frac{\sqrt{-bc}ax\arctan\left(\frac{\sqrt{bx^3+ax^2}\sqrt{-bc}\sqrt{cx}}{bcx^2+acx}\right)-\sqrt{bx^3+ax^2}\sqrt{cx}b}{bc^2x}\right]$$

```
integrate((b*x^3+a*x^2)^(1/2)/(c*x)^(3/2),x, algorithm="fricas")
```

```
[1/2*(sqrt(b*c)*a*x*log((2*b*c*x^2 + a*c*x + 2*sqrt(b*x^3 + a*x^2)*sqrt(b*
c)*sqrt(c*x))/x) + 2*sqrt(b*x^3 + a*x^2)*sqrt(c*x)*b)/(b*c^2*x), -(sqrt(-b
*c)*a*x*arctan(sqrt(b*x^3 + a*x^2)*sqrt(-b*c)*sqrt(c*x)/(b*c*x^2 + a*c*x))
 - sqrt(b*x^3 + a*x^2)*sqrt(c*x)*b)/(b*c^2*x)]
```

Sympy [F]

$$\int \frac{\sqrt{ax^2+bx^3}}{(cx)^{3/2}}\,dx = \int \frac{\sqrt{x^2\,(a+bx)}}{(cx)^{\frac{3}{2}}}\,dx$$

```
integrate((b*x**3+a*x**2)**(1/2)/(c*x)**(3/2),x)
```

```
Integral(sqrt(x**2*(a + b*x))/(c*x)**(3/2), x)
```

Maxima [F]

$$\int \frac{\sqrt{ax^2+bx^3}}{(cx)^{3/2}}\,dx = \int \frac{\sqrt{bx^3+ax^2}}{(cx)^{\frac{3}{2}}}\,dx$$

```
integrate((b*x^3+a*x^2)^(1/2)/(c*x)^(3/2),x, algorithm="maxima")
```

```
integrate(sqrt(b*x^3 + a*x^2)/(c*x)^(3/2), x)
```

Giac [A] (verification not implemented)

Time = 0.43 (sec) , antiderivative size = 112, normalized size of antiderivative = 1.53

$$\int \frac{\sqrt{ax^2+bx^3}}{(cx)^{3/2}}\,dx = \frac{\frac{ab\log\left(\sqrt{bc}\sqrt{a}\right)\mathrm{sgn}(x)}{\sqrt{bc}|b|} - \frac{\left(\frac{a\log\left(\left|-\sqrt{bc}\sqrt{bx+a}+\sqrt{(bx+a)bc-abc}\right|\right)\mathrm{sgn}(x)}{\sqrt{bc}} - \frac{\sqrt{(bx+a)bc-abc}\sqrt{bx+a}\,\mathrm{sgn}(x)}{bc}\right)b}{|b|}}{c}$$

```
integrate((b*x^3+a*x^2)^(1/2)/(c*x)^(3/2),x, algorithm="giac")
```

```
(a*b*log(sqrt(b*c)*sqrt(a))*sgn(x)/(sqrt(b*c)*abs(b)) - (a*log(abs(-sqrt(b
*c)*sqrt(b*x + a) + sqrt((b*x + a)*b*c - a*b*c)))*sgn(x)/sqrt(b*c) - sqrt(
(b*x + a)*b*c - a*b*c)*sqrt(b*x + a)*sgn(x)/(b*c))*b/abs(b))/c
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\sqrt{ax^2+bx^3}}{(cx)^{3/2}}\,dx = \int \frac{\sqrt{b\,x^3+a\,x^2}}{{\left(c\,x\right)}^{3/2}}\,dx$$

```
int((a*x^2 + b*x^3)^(1/2)/(c*x)^(3/2),x)
```

```
int((a*x^2 + b*x^3)^(1/2)/(c*x)^(3/2), x)
```

Reduce [B] (verification not implemented)

Time = 0.47 (sec) , antiderivative size = 42, normalized size of antiderivative = 0.58

$$\int \frac{\sqrt{a x^2+b x^3}}{(c x)^{3/2}}\,dx=\frac{\sqrt{c}\left(\sqrt{x}\,\sqrt{b x+a}\,b+\sqrt{b}\log\left(\frac{\sqrt{b x+a}+\sqrt{x}\,\sqrt{b}}{\sqrt{a}}\right)a\right)}{b\,c^2}$$

```
int((b*x^3+a*x^2)^(1/2)/(c*x)^(3/2),x)
```

```
(sqrt(c)*(sqrt(x)*sqrt(a + b*x)*b + sqrt(b)*log((sqrt(a + b*x) + sqrt(x)*s
qrt(b))/sqrt(a))*a))/(b*c**2)
```

3.330 $\int \frac{\sqrt{ax^2+bx^3}}{(cx)^{5/2}} \, dx$

Optimal result

Integrand size = 23, antiderivative size = 74

$$\int \frac{\sqrt{ax^2+bx^3}}{(cx)^{5/2}} \, dx = -\frac{2\sqrt{ax^2+bx^3}}{c(cx)^{3/2}} + \frac{2\sqrt{b}\operatorname{arctanh}\left(\frac{\sqrt{b}(cx)^{3/2}}{c^{3/2}\sqrt{ax^2+bx^3}}\right)}{c^{5/2}}$$

```
-2*(b*x^3+a*x^2)^(1/2)/c/(c*x)^(3/2)+2*b^(1/2)*arctanh(b^(1/2)*(c*x)^(3/2)
/c^(3/2)/(b*x^3+a*x^2)^(1/2))/c^(5/2)
```

Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 85, normalized size of antiderivative = 1.15

$$\int \frac{\sqrt{ax^2+bx^3}}{(cx)^{5/2}} \, dx = -\frac{2x\sqrt{x^2(a+bx)}\left(\sqrt{a+bx} + 2\sqrt{b}\sqrt{x}\operatorname{arctanh}\left(\frac{\sqrt{b}\sqrt{x}}{\sqrt{a}-\sqrt{a+bx}}\right)\right)}{(cx)^{5/2}\sqrt{a+bx}}$$

```
Integrate[Sqrt[a*x^2 + b*x^3]/(c*x)^(5/2),x]
```

```
(-2*x*Sqrt[x^2*(a + b*x)]*(Sqrt[a + b*x] + 2*Sqrt[b]*Sqrt[x]*ArcTanh[(Sqrt
[b]*Sqrt[x])/(Sqrt[a] - Sqrt[a + b*x])]))/((c*x)^(5/2)*Sqrt[a + b*x])
```

Rubi [A] (verified)

Time = 0.39 (sec) , antiderivative size = 77, normalized size of antiderivative = 1.04, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.174, Rules used = {1926, 1937, 1935, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sqrt{ax^2+bx^3}}{(cx)^{5/2}}dx$$

$$\Big\downarrow \; 1926$$

$$\frac{b\int \frac{\sqrt{cx}}{\sqrt{bx^3+ax^2}}dx}{c^3}-\frac{2\sqrt{ax^2+bx^3}}{c(cx)^{3/2}}$$

$$\Big\downarrow \; 1937$$

$$\frac{b\sqrt{cx}\int \frac{\sqrt{x}}{\sqrt{bx^3+ax^2}}dx}{c^3\sqrt{x}}-\frac{2\sqrt{ax^2+bx^3}}{c(cx)^{3/2}}$$

$$\Big\downarrow \; 1935$$

$$\frac{2b\sqrt{cx}\int \frac{1}{1-\frac{bx^3}{bx^3+ax^2}}d\frac{x^{3/2}}{\sqrt{bx^3+ax^2}}}{c^3\sqrt{x}}-\frac{2\sqrt{ax^2+bx^3}}{c(cx)^{3/2}}$$

$$\Big\downarrow \; 219$$

$$\frac{2\sqrt{b}\sqrt{cx}\text{arctanh}\left(\frac{\sqrt{b}x^{3/2}}{\sqrt{ax^2+bx^3}}\right)}{c^3\sqrt{x}}-\frac{2\sqrt{ax^2+bx^3}}{c(cx)^{3/2}}$$

```
Int[Sqrt[a*x^2 + b*x^3]/(c*x)^(5/2),x]
```

```
(-2*Sqrt[a*x^2 + b*x^3])/(c*(c*x)^(3/2)) + (2*Sqrt[b]*Sqrt[c*x]*ArcTanh[(S
qrt[b]*x^(3/2))/Sqrt[a*x^2 + b*x^3]])/(c^3*Sqrt[x])
```

Defintions of rubi rules used

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

```
Int[((c_.)*(x_))^(m_)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol]
 :> Simp[(c*x)^(m + 1)*((a*x^j + b*x^n)^p/(c*(m + j*p + 1))), x] - Simp[b*p
*((n - j)/(c^n*(m + j*p + 1)))   Int[(c*x)^(m + n)*(a*x^j + b*x^n)^(p - 1),
 x], x] /; FreeQ[{a, b, c}, x] &&  !IntegerQ[p] && LtQ[0, j, n] && (Integer
sQ[j, n] || GtQ[c, 0]) && GtQ[p, 0] && LtQ[m + j*p + 1, 0]
```

```
Int[(x_)^(m_.)/Sqrt[(a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.)], x_Symbol] :> Simp
[-2/(n - j)   Subst[Int[1/(1 - a*x^2), x], x, x^(j/2)/Sqrt[a*x^j + b*x^n]],
 x] /; FreeQ[{a, b, j, n}, x] && EqQ[m, j/2 - 1] && NeQ[n, j]
```

```
Int[((c_)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_))^(p_), x_Symbol]
:> Simp[c^IntPart[m]*((c*x)^FracPart[m]/x^FracPart[m])   Int[x^m*(a*x^j + b
*x^n)^p, x], x] /; FreeQ[{a, b, c, j, m, n, p}, x] && IntegerQ[p + 1/2] &&
NeQ[n, j] && EqQ[Simplify[m + j*p + 1], 0]
```

Maple [A] (verified)

Time = 0.36 (sec) , antiderivative size = 99, normalized size of antiderivative = 1.34

method	result	size
default	$-\frac{\sqrt{b\,x^{3}+a\,x^{2}}\,\left(-\ln\left(\frac{2cbx+2\sqrt{cx(bx+a)}\,\sqrt{bc}+ac}{2\sqrt{bc}}\right)bcx+2\sqrt{cx(bx+a)}\,\sqrt{bc}\right)}{x\,c^{2}\sqrt{cx}\,\sqrt{cx(bx+a)}\,\sqrt{bc}}$	99
risch	$-\frac{2\sqrt{x^{2}(bx+a)}}{c^{2}x\sqrt{cx}}+\frac{b\ln\left(\frac{\frac{1}{2}ac+cbx}{\sqrt{bc}}+\sqrt{bc\,x^{2}+acx}\right)\sqrt{x^{2}(bx+a)}\,\sqrt{cx(bx+a)}}{\sqrt{bc}\,c^{2}x(bx+a)\sqrt{cx}}$	102

```
int((b*x^3+a*x^2)^(1/2)/(c*x)^(5/2),x,method=_RETURNVERBOSE)
```

```
-(b*x^3+a*x^2)^(1/2)*(-ln(1/2*(2*c*b*x+2*(c*x*(b*x+a))^(1/2)*(b*c)^(1/2)+a
*c)/(b*c)^(1/2))*b*c*x+2*(c*x*(b*x+a))^(1/2)*(b*c)^(1/2))/x/c^2/(c*x)^(1/2
)/(c*x*(b*x+a))^(1/2)/(b*c)^(1/2)
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 163, normalized size of antiderivative = 2.20

$$\int \frac{\sqrt{ax^2+bx^3}}{(cx)^{5/2}}\,dx = \left[\frac{cx^2\sqrt{\frac{b}{c}}\log\left(\frac{2\,bx^2+ax+2\,\sqrt{bx^3+ax^2}\sqrt{cx}\sqrt{\frac{b}{c}}}{x}\right)-2\,\sqrt{bx^3+ax^2}\sqrt{cx}}{c^3x^2},\right.$$
$$\left.-\frac{2\left(cx^2\sqrt{-\frac{b}{c}}\arctan\left(\frac{\sqrt{bx^3+ax^2}\sqrt{cx}\sqrt{-\frac{b}{c}}}{bx^2+ax}\right)+\sqrt{bx^3+ax^2}\sqrt{cx}\right)}{c^3x^2}\right]$$

```
integrate((b*x^3+a*x^2)^(1/2)/(c*x)^(5/2),x, algorithm="fricas")
```

```
[(c*x^2*sqrt(b/c)*log((2*b*x^2 + a*x + 2*sqrt(b*x^3 + a*x^2)*sqrt(c*x)*sqr
t(b/c))/x) - 2*sqrt(b*x^3 + a*x^2)*sqrt(c*x))/(c^3*x^2), -2*(c*x^2*sqrt(-b
/c)*arctan(sqrt(b*x^3 + a*x^2)*sqrt(c*x)*sqrt(-b/c)/(b*x^2 + a*x)) + sqrt(
b*x^3 + a*x^2)*sqrt(c*x))/(c^3*x^2)]
```

Sympy [F]

$$\int \frac{\sqrt{ax^2+bx^3}}{(cx)^{5/2}}\,dx = \int \frac{\sqrt{x^2\,(a+bx)}}{(cx)^{\frac{5}{2}}}\,dx$$

```
integrate((b*x**3+a*x**2)**(1/2)/(c*x)**(5/2),x)
```

```
Integral(sqrt(x**2*(a + b*x))/(c*x)**(5/2), x)
```

Maxima [F]

$$\int \frac{\sqrt{ax^2 + bx^3}}{(cx)^{5/2}}\,dx = \int \frac{\sqrt{bx^3 + ax^2}}{(cx)^{\frac{5}{2}}}\,dx$$

```
integrate((b*x^3+a*x^2)^(1/2)/(c*x)^(5/2),x, algorithm="maxima")
```

```
integrate(sqrt(b*x^3 + a*x^2)/(c*x)^(5/2), x)
```

Giac [A] (verification not implemented)

Time = 0.30 (sec) , antiderivative size = 80, normalized size of antiderivative = 1.08

$$\int \frac{\sqrt{ax^2 + bx^3}}{(cx)^{5/2}}\,dx = -\frac{2\left(\frac{\log\left(\left|-\sqrt{bc}\sqrt{bx+a}+\sqrt{(bx+a)bc-abc}\right|\right)\operatorname{sgn}(x)}{\sqrt{bc}} + \frac{\sqrt{bx+a}\operatorname{sgn}(x)}{\sqrt{(bx+a)bc-abc}}\right)b^2}{c^2|b|}$$

```
integrate((b*x^3+a*x^2)^(1/2)/(c*x)^(5/2),x, algorithm="giac")
```

```
-2*(log(abs(-sqrt(b*c)*sqrt(b*x + a) + sqrt((b*x + a)*b*c - a*b*c)))*sgn(x
)/sqrt(b*c) + sqrt(b*x + a)*sgn(x)/sqrt((b*x + a)*b*c - a*b*c))*b^2/(c^2*a
bs(b))
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\sqrt{ax^2 + bx^3}}{(cx)^{5/2}}\,dx = \int \frac{\sqrt{b\,x^3 + a\,x^2}}{{(c\,x)}^{5/2}}\,dx$$

```
int((a*x^2 + b*x^3)^(1/2)/(c*x)^(5/2),x)
```

```
int((a*x^2 + b*x^3)^(1/2)/(c*x)^(5/2), x)
```

Reduce [B] (verification not implemented)

Time = 0.33 (sec) , antiderivative size = 48, normalized size of antiderivative = 0.65

$$\int \frac{\sqrt{ax^2+bx^3}}{(cx)^{5/2}}\,dx = \frac{2\sqrt{c}\left(-\sqrt{x}\,\sqrt{bx+a}+\sqrt{b}\log\left(\frac{\sqrt{bx+a}+\sqrt{x}\,\sqrt{b}}{\sqrt{a}}\right)x-\sqrt{b}\,x\right)}{c^3x}$$

```
int((b*x^3+a*x^2)^(1/2)/(c*x)^(5/2),x)
```

```
(2*sqrt(c)*( - sqrt(x)*sqrt(a + b*x) + sqrt(b)*log((sqrt(a + b*x) + sqrt(x
)*sqrt(b))/sqrt(a))*x - sqrt(b)*x))/(c**3*x)
```

3.331 $\int \frac{\sqrt{ax^2+bx^3}}{(cx)^{7/2}}\,dx$

Optimal result

Integrand size = 23, antiderivative size = 30

$$\int \frac{\sqrt{ax^2+bx^3}}{(cx)^{7/2}}\,dx=-\frac{2c(ax^2+bx^3)^{3/2}}{3a(cx)^{9/2}}$$

```
-2/3*c*(b*x^3+a*x^2)^(3/2)/a/(c*x)^(9/2)
```

Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 28, normalized size of antiderivative = 0.93

$$\int \frac{\sqrt{ax^2+bx^3}}{(cx)^{7/2}}\,dx=-\frac{2c(x^2(a+bx))^{3/2}}{3a(cx)^{9/2}}$$

```
Integrate[Sqrt[a*x^2 + b*x^3]/(c*x)^(7/2),x]
```

```
(-2*c*(x^2*(a + b*x))^(3/2))/(3*a*(c*x)^(9/2))
```

Rubi [A] (verified)

Time = 0.25 (sec) , antiderivative size = 30, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.043, Rules used = {1920}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sqrt{ax^2+bx^3}}{(cx)^{7/2}} \, dx$$
$$\Big\downarrow \ 1920$$
$$-\frac{2c(ax^2+bx^3)^{3/2}}{3a(cx)^{9/2}}$$

```
Int[Sqrt[a*x^2 + b*x^3]/(c*x)^(7/2),x]
```

```
(-2*c*(a*x^2 + b*x^3)^(3/2))/(3*a*(c*x)^(9/2))
```

Defintions of rubi rules used

```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> Simp[(-c^(j - 1))*(c*x)^(m - j + 1)*((a*x^j + b*x^n)^(p + 1)/(a*(n - j
)*(p + 1))), x] /; FreeQ[{a, b, c, j, m, n, p}, x] &&  !IntegerQ[p] && NeQ[
n, j] && EqQ[m + n*p + n - j + 1, 0] && (IntegerQ[j] || GtQ[c, 0])
```

Maple [A] (verified)

Time = 0.35 (sec) , antiderivative size = 30, normalized size of antiderivative = 1.00

method	result	size
gosper	$-\frac{2x(bx+a)\sqrt{b\,x^3+a\,x^2}}{3a(cx)^{\frac{7}{2}}}$	30
orering	$-\frac{2x(bx+a)\sqrt{b\,x^3+a\,x^2}}{3a(cx)^{\frac{7}{2}}}$	30
risch	$-\frac{2\sqrt{x^2(bx+a)}\,(bx+a)}{3c^3x^2\sqrt{cx}\,a}$	33
default	$-\frac{2\sqrt{b\,x^3+a\,x^2}\,(bx+a)}{3x^2c^3\sqrt{cx}\,a}$	35

```
int((b*x^3+a*x^2)^(1/2)/(c*x)^(7/2),x,method=_RETURNVERBOSE)
```

```
-2/3*x*(b*x+a)/a*(b*x^3+a*x^2)^(1/2)/(c*x)^(7/2)
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 34, normalized size of antiderivative = 1.13

$$\int \frac{\sqrt{ax^2+bx^3}}{(cx)^{7/2}}\,dx = -\frac{2\sqrt{bx^3+ax^2}(bx+a)\sqrt{cx}}{3\,ac^4x^3}$$

```
integrate((b*x^3+a*x^2)^(1/2)/(c*x)^(7/2),x, algorithm="fricas")
```

```
-2/3*sqrt(b*x^3 + a*x^2)*(b*x + a)*sqrt(c*x)/(a*c^4*x^3)
```

Sympy [F]

$$\int \frac{\sqrt{ax^2+bx^3}}{(cx)^{7/2}}\,dx = \int \frac{\sqrt{x^2\,(a+bx)}}{(cx)^{\frac{7}{2}}}\,dx$$

```
integrate((b*x**3+a*x**2)**(1/2)/(c*x)**(7/2),x)
```

```
Integral(sqrt(x**2*(a + b*x))/(c*x)**(7/2), x)
```

Maxima [F]

$$\int \frac{\sqrt{ax^2+bx^3}}{(cx)^{7/2}}\,dx = \int \frac{\sqrt{bx^3+ax^2}}{(cx)^{\frac{7}{2}}}\,dx$$

```
integrate((b*x^3+a*x^2)^(1/2)/(c*x)^(7/2),x, algorithm="maxima")
```

```
integrate(sqrt(b*x^3 + a*x^2)/(c*x)^(7/2), x)
```

Giac [A] (verification not implemented)

Time = 0.17 (sec) , antiderivative size = 40, normalized size of antiderivative = 1.33

$$\int \frac{\sqrt{ax^2+bx^3}}{(cx)^{7/2}}\,dx = -\frac{2\,(bx+a)^{\frac{3}{2}}b^4\mathrm{sgn}(x)}{3\,((bx+a)bc-abc)^{\frac{3}{2}}ac^2|b|}$$

```
integrate((b*x^3+a*x^2)^(1/2)/(c*x)^(7/2),x, algorithm="giac")
```

```
-2/3*(b*x + a)^(3/2)*b^4*sgn(x)/(((b*x + a)*b*c - a*b*c)^(3/2)*a*c^2*abs(b
))
```

Mupad [B] (verification not implemented)

Time = 8.83 (sec) , antiderivative size = 39, normalized size of antiderivative = 1.30

$$\int \frac{\sqrt{ax^2+bx^3}}{(cx)^{7/2}}\,dx = -\frac{\left(\frac{2}{3\,c^3}+\frac{2\,b\,x}{3\,a\,c^3}\right)\,\sqrt{b\,x^3+a\,x^2}}{x^2\,\sqrt{c\,x}}$$

```
int((a*x^2 + b*x^3)^(1/2)/(c*x)^(7/2),x)
```

```
-((2/(3*c^3) + (2*b*x)/(3*a*c^3))*(a*x^2 + b*x^3)^(1/2))/(x^2*(c*x)^(1/2))
```

Reduce [B] (verification not implemented)

Time = 0.57 (sec) , antiderivative size = 42, normalized size of antiderivative = 1.40

$$\int \frac{\sqrt{ax^2+bx^3}}{(cx)^{7/2}}\,dx = -\frac{2\sqrt{c}\left(\sqrt{x}\,\sqrt{bx+a}\,a+\sqrt{x}\,\sqrt{bx+a}\,bx+\sqrt{b}\,b\,x^2\right)}{3a\,c^4x^2}$$

```
int((b*x^3+a*x^2)^(1/2)/(c*x)^(7/2),x)
```

```
( - 2*sqrt(c)*(sqrt(x)*sqrt(a + b*x)*a + sqrt(x)*sqrt(a + b*x)*b*x + sqrt(
b)*b*x**2))/(3*a*c**4*x**2)
```

3.332 $\int \frac{\sqrt{ax^2+bx^3}}{(cx)^{9/2}}\,dx$

Optimal result

Integrand size = 23, antiderivative size = 61

$$\int \frac{\sqrt{ax^2+bx^3}}{(cx)^{9/2}}\,dx = -\frac{2c(ax^2+bx^3)^{3/2}}{5a(cx)^{11/2}}+\frac{4b(ax^2+bx^3)^{3/2}}{15a^2(cx)^{9/2}}$$

```
-2/5*c*(b*x^3+a*x^2)^(3/2)/a/(c*x)^(11/2)+4/15*b*(b*x^3+a*x^2)^(3/2)/a^2/(
c*x)^(9/2)
```

Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 46, normalized size of antiderivative = 0.75

$$\int \frac{\sqrt{ax^2+bx^3}}{(cx)^{9/2}}\,dx = -\frac{2x\sqrt{x^2(a+bx)}(3a^2+abx-2b^2x^2)}{15a^2(cx)^{9/2}}$$

```
Integrate[Sqrt[a*x^2 + b*x^3]/(c*x)^(9/2),x]
```

```
(-2*x*Sqrt[x^2*(a + b*x)]*(3*a^2 + a*b*x - 2*b^2*x^2))/(15*a^2*(c*x)^(9/2)
)
```

Rubi [A] (verified)

Time = 0.32 (sec) , antiderivative size = 61, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.087$, Rules used = {1922, 1920}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sqrt{ax^2+bx^3}}{(cx)^{9/2}}\,dx$$

$$\Big\downarrow \text{1922}$$

$$-\frac{2b\int \frac{\sqrt{bx^3+ax^2}}{(cx)^{7/2}}dx}{5ac}-\frac{2c\left(ax^2+bx^3\right)^{3/2}}{5a(cx)^{11/2}}$$

$$\Big\downarrow \text{1920}$$

$$\frac{4b\left(ax^2+bx^3\right)^{3/2}}{15a^2(cx)^{9/2}}-\frac{2c\left(ax^2+bx^3\right)^{3/2}}{5a(cx)^{11/2}}$$

```
Int[Sqrt[a*x^2 + b*x^3]/(c*x)^(9/2),x]
```

```
(-2*c*(a*x^2 + b*x^3)^(3/2))/(5*a*(c*x)^(11/2)) + (4*b*(a*x^2 + b*x^3)^(3/
2))/(15*a^2*(c*x)^(9/2))
```

Defintions of rubi rules used

```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> Simp[(-c^(j - 1))*(c*x)^(m - j + 1)*((a*x^j + b*x^n)^(p + 1)/(a*(n - j
)*(p + 1))), x] /; FreeQ[{a, b, c, j, m, n, p}, x] && !IntegerQ[p] && NeQ[
n, j] && EqQ[m + n*p + n - j + 1, 0] && (IntegerQ[j] || GtQ[c, 0])
```

```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> Simp[c^(j - 1)*(c*x)^(m - j + 1)*((a*x^j + b*x^n)^(p + 1)/(a*(m + j*p
+ 1))), x] - Simp[b*((m + n*p + n - j + 1)/(a*c^(n - j)*(m + j*p + 1)))   I
nt[(c*x)^(m + n - j)*(a*x^j + b*x^n)^p, x], x] /; FreeQ[{a, b, c, j, m, n,
p}, x] &&  !IntegerQ[p] && NeQ[n, j] && ILtQ[Simplify[(m + n*p + n - j + 1)
/(n - j)], 0] && NeQ[m + j*p + 1, 0] && (IntegersQ[j, n] || GtQ[c, 0])
```

Maple [A] (verified)

Time = 0.34 (sec) , antiderivative size = 38, normalized size of antiderivative = 0.62

method	result	size
gosper	$-\frac{2x(bx+a)(-2bx+3a)\sqrt{b\,x^{3}+a\,x^{2}}}{15a^{2}(cx)^{\frac{9}{2}}}$	38
orering	$-\frac{2x(bx+a)(-2bx+3a)\sqrt{b\,x^{3}+a\,x^{2}}}{15a^{2}(cx)^{\frac{9}{2}}}$	38
default	$-\frac{2\sqrt{b\,x^{3}+a\,x^{2}}\,(bx+a)(-2bx+3a)}{15x^{3}c^{4}\sqrt{cx}\,a^{2}}$	43
risch	$-\frac{2\sqrt{x^{2}(bx+a)}\,(-2b^{2}x^{2}+abx+3a^{2})}{15c^{4}x^{3}\sqrt{cx}\,a^{2}}$	46

```
int((b*x^3+a*x^2)^(1/2)/(c*x)^(9/2),x,method=_RETURNVERBOSE)
```

```
-2/15*x*(b*x+a)*(-2*b*x+3*a)*(b*x^3+a*x^2)^(1/2)/a^2/(c*x)^(9/2)
```

Fricas [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 48, normalized size of antiderivative = 0.79

$$\int \frac{\sqrt{ax^2+bx^3}}{(cx)^{9/2}}\,dx = \frac{2\,(2\,b^2x^2-abx-3\,a^2)\sqrt{bx^3+ax^2}\sqrt{cx}}{15\,a^2c^5x^4}$$

```
integrate((b*x^3+a*x^2)^(1/2)/(c*x)^(9/2),x, algorithm="fricas")
```

```
2/15*(2*b^2*x^2 - a*b*x - 3*a^2)*sqrt(b*x^3 + a*x^2)*sqrt(c*x)/(a^2*c^5*x^
4)
```

Sympy [F]

$$\int \frac{\sqrt{ax^2 + bx^3}}{(cx)^{9/2}}\,dx = \int \frac{\sqrt{x^2\,(a + bx)}}{(cx)^{\frac{9}{2}}}\,dx$$

```
integrate((b*x**3+a*x**2)**(1/2)/(c*x)**(9/2),x)
```

```
Integral(sqrt(x**2*(a + b*x))/(c*x)**(9/2), x)
```

Maxima [F]

$$\int \frac{\sqrt{ax^2 + bx^3}}{(cx)^{9/2}}\,dx = \int \frac{\sqrt{bx^3 + ax^2}}{(cx)^{\frac{9}{2}}}\,dx$$

```
integrate((b*x^3+a*x^2)^(1/2)/(c*x)^(9/2),x, algorithm="maxima")
```

```
integrate(sqrt(b*x^3 + a*x^2)/(c*x)^(9/2), x)
```

Giac [A] (verification not implemented)

Time = 0.18 (sec) , antiderivative size = 65, normalized size of antiderivative = 1.07

$$\int \frac{\sqrt{ax^2 + bx^3}}{(cx)^{9/2}}\,dx = \frac{2\left(\frac{2\,(bx+a)b^5c^2\mathrm{sgn}(x)}{a^2} - \frac{5\,b^5c^2\mathrm{sgn}(x)}{a}\right)(bx + a)^{\frac{3}{2}}b}{15\,((bx + a)bc - abc)^{\frac{5}{2}}c^4|b|}$$

```
integrate((b*x^3+a*x^2)^(1/2)/(c*x)^(9/2),x, algorithm="giac")
```

```
2/15*(2*(b*x + a)*b^5*c^2*sgn(x)/a^2 - 5*b^5*c^2*sgn(x)/a)*(b*x + a)^(3/2)
*b/(((b*x + a)*b*c - a*b*c)^(5/2)*c^4*abs(b))
```

Mupad [B] (verification not implemented)

Time = 8.96 (sec) , antiderivative size = 53, normalized size of antiderivative = 0.87

$$\int \frac{\sqrt{ax^2+bx^3}}{(cx)^{9/2}}\,dx = -\frac{\sqrt{b\,x^3+a\,x^2}\,\left(\frac{2}{5\,c^4}-\frac{4\,b^2\,x^2}{15\,a^2\,c^4}+\frac{2\,b\,x}{15\,a\,c^4}\right)}{x^3\,\sqrt{c\,x}}$$

```
int((a*x^2 + b*x^3)^(1/2)/(c*x)^(9/2),x)
```

```
-((a*x^2 + b*x^3)^(1/2)*(2/(5*c^4) - (4*b^2*x^2)/(15*a^2*c^4) + (2*b*x)/(1
5*a*c^4)))/(x^3*(c*x)^(1/2))
```

Reduce [B] (verification not implemented)

Time = 0.51 (sec) , antiderivative size = 66, normalized size of antiderivative = 1.08

$$\int \frac{\sqrt{ax^2+bx^3}}{(cx)^{9/2}}\,dx = \frac{2\sqrt{c}\left(-3\sqrt{x}\,\sqrt{bx+a}\,a^2-\sqrt{x}\,\sqrt{bx+a}\,abx+2\sqrt{x}\,\sqrt{bx+a}\,b^2x^2-2\sqrt{b}\,b^2x^3\right)}{15a^2c^5x^3}$$

```
int((b*x^3+a*x^2)^(1/2)/(c*x)^(9/2),x)
```

```
(2*sqrt(c)*( - 3*sqrt(x)*sqrt(a + b*x)*a**2 - sqrt(x)*sqrt(a + b*x)*a*b*x
+ 2*sqrt(x)*sqrt(a + b*x)*b**2*x**2 - 2*sqrt(b)*b**2*x**3))/(15*a**2*c**5*
x**3)
```

3.333 $\int \frac{\sqrt{ax^2+bx^3}}{(cx)^{11/2}}\,dx$

Optimal result

Integrand size = 23, antiderivative size = 96

$$\int \frac{\sqrt{ax^2+bx^3}}{(cx)^{11/2}}\,dx = -\frac{2c(ax^2+bx^3)^{3/2}}{7a(cx)^{13/2}}+\frac{8b(ax^2+bx^3)^{3/2}}{35a^2(cx)^{11/2}}-\frac{16b^2(ax^2+bx^3)^{3/2}}{105a^3c(cx)^{9/2}}$$

```
-2/7*c*(b*x^3+a*x^2)^(3/2)/a/(c*x)^(13/2)+8/35*b*(b*x^3+a*x^2)^(3/2)/a^2/(
c*x)^(11/2)-16/105*b^2*(b*x^3+a*x^2)^(3/2)/a^3/c/(c*x)^(9/2)
```

Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 58, normalized size of antiderivative = 0.60

$$\int \frac{\sqrt{ax^2+bx^3}}{(cx)^{11/2}}\,dx = -\frac{2x\sqrt{x^2(a+bx)}(15a^3+3a^2bx-4ab^2x^2+8b^3x^3)}{105a^3(cx)^{11/2}}$$

```
Integrate[Sqrt[a*x^2 + b*x^3]/(c*x)^(11/2),x]
```

```
(-2*x*Sqrt[x^2*(a + b*x)]*(15*a^3 + 3*a^2*b*x - 4*a*b^2*x^2 + 8*b^3*x^3))/
(105*a^3*(c*x)^(11/2))
```

Rubi [A] (verified)

Time = 0.40 (sec) , antiderivative size = 103, normalized size of antiderivative = 1.07, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.130, Rules used = {1922, 1922, 1920}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sqrt{ax^2+bx^3}}{(cx)^{11/2}}dx$$

$$\Big\downarrow \text{ 1922}$$

$$-\frac{4b\int \frac{\sqrt{bx^3+ax^2}}{(cx)^{9/2}}dx}{7ac}-\frac{2c\left(ax^2+bx^3\right)^{3/2}}{7a(cx)^{13/2}}$$

$$\Big\downarrow \text{ 1922}$$

$$-\frac{4b\left(-\frac{2b\int \frac{\sqrt{bx^3+ax^2}}{(cx)^{7/2}}dx}{5ac}-\frac{2c\left(ax^2+bx^3\right)^{3/2}}{5a(cx)^{11/2}}\right)}{7ac}-\frac{2c\left(ax^2+bx^3\right)^{3/2}}{7a(cx)^{13/2}}$$

$$\Big\downarrow \text{ 1920}$$

$$-\frac{4b\left(\frac{4b\left(ax^2+bx^3\right)^{3/2}}{15a^2(cx)^{9/2}}-\frac{2c\left(ax^2+bx^3\right)^{3/2}}{5a(cx)^{11/2}}\right)}{7ac}-\frac{2c\left(ax^2+bx^3\right)^{3/2}}{7a(cx)^{13/2}}$$

```
Int[Sqrt[a*x^2 + b*x^3]/(c*x)^(11/2),x]
```

```
(-2*c*(a*x^2 + b*x^3)^(3/2))/(7*a*(c*x)^(13/2)) - (4*b*((-2*c*(a*x^2 + b*x
^3)^(3/2))/(5*a*(c*x)^(11/2)) + (4*b*(a*x^2 + b*x^3)^(3/2))/(15*a^2*(c*x)^
(9/2))))/(7*a*c)
```

Defintions of rubi rules used

```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> Simp[(-c^(j - 1))*(c*x)^(m - j + 1)*((a*x^j + b*x^n)^(p + 1)/(a*(n - j
)*(p + 1))), x] /; FreeQ[{a, b, c, j, m, n, p}, x] &&  !IntegerQ[p] && NeQ[
n, j] && EqQ[m + n*p + n - j + 1, 0] && (IntegerQ[j] || GtQ[c, 0])
```

```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> Simp[c^(j - 1)*(c*x)^(m - j + 1)*((a*x^j + b*x^n)^(p + 1)/(a*(m + j*p
+ 1))), x] - Simp[b*((m + n*p + n - j + 1)/(a*c^(n - j)*(m + j*p + 1)))   I
nt[(c*x)^(m + n - j)*(a*x^j + b*x^n)^p, x], x] /; FreeQ[{a, b, c, j, m, n,
p}, x] &&  !IntegerQ[p] && NeQ[n, j] && ILtQ[Simplify[(m + n*p + n - j + 1)
/(n - j)], 0] && NeQ[m + j*p + 1, 0] && (IntegersQ[j, n] || GtQ[c, 0])
```

Maple [A] (verified)

Time = 0.36 (sec) , antiderivative size = 49, normalized size of antiderivative = 0.51

method	result	size
gosper	$-\frac{2x(bx+a)\left(8b^{2}x^{2}-12abx+15a^{2}\right)\sqrt{b\,x^{3}+a\,x^{2}}}{105a^{3}\left(cx\right)^{\frac{11}{2}}}$	49
orering	$-\frac{2x(bx+a)\left(8b^{2}x^{2}-12abx+15a^{2}\right)\sqrt{b\,x^{3}+a\,x^{2}}}{105a^{3}\left(cx\right)^{\frac{11}{2}}}$	49
default	$-\frac{2\sqrt{b\,x^{3}+a\,x^{2}}\,(bx+a)\left(8b^{2}x^{2}-12abx+15a^{2}\right)}{105x^{4}c^{5}\sqrt{cx}\,a^{3}}$	54
risch	$-\frac{2\sqrt{x^{2}(bx+a)}\,\left(8b^{3}x^{3}-4a\,b^{2}x^{2}+3a^{2}bx+15a^{3}\right)}{105c^{5}x^{4}\sqrt{cx}\,a^{3}}$	58

```
int((b*x^3+a*x^2)^(1/2)/(c*x)^(11/2),x,method=_RETURNVERBOSE)
```

```
-2/105*x*(b*x+a)*(8*b^2*x^2-12*a*b*x+15*a^2)*(b*x^3+a*x^2)^(1/2)/a^3/(c*x)
^(11/2)
```

Fricas [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 59, normalized size of antiderivative = 0.61

$$\int \frac{\sqrt{ax^2+bx^3}}{(cx)^{11/2}}\,dx = -\frac{2\,(8\,b^3x^3-4\,ab^2x^2+3\,a^2bx+15\,a^3)\sqrt{bx^3+ax^2}\sqrt{cx}}{105\,a^3c^6x^5}$$

```
integrate((b*x^3+a*x^2)^(1/2)/(c*x)^(11/2),x, algorithm="fricas")
```

```
-2/105*(8*b^3*x^3 - 4*a*b^2*x^2 + 3*a^2*b*x + 15*a^3)*sqrt(b*x^3 + a*x^2)*
sqrt(c*x)/(a^3*c^6*x^5)
```

Sympy [F]

$$\int \frac{\sqrt{ax^2+bx^3}}{(cx)^{11/2}}\,dx = \int \frac{\sqrt{x^2\,(a+bx)}}{(cx)^{\frac{11}{2}}}\,dx$$

```
integrate((b*x**3+a*x**2)**(1/2)/(c*x)**(11/2),x)
```

```
Integral(sqrt(x**2*(a + b*x))/(c*x)**(11/2), x)
```

Maxima [F]

$$\int \frac{\sqrt{ax^2+bx^3}}{(cx)^{11/2}}\,dx = \int \frac{\sqrt{bx^3+ax^2}}{(cx)^{\frac{11}{2}}}\,dx$$

```
integrate((b*x^3+a*x^2)^(1/2)/(c*x)^(11/2),x, algorithm="maxima")
```

```
integrate(sqrt(b*x^3 + a*x^2)/(c*x)^(11/2), x)
```

Giac [A] (verification not implemented)

Time = 0.15 (sec) , antiderivative size = 88, normalized size of antiderivative = 0.92

$$\int \frac{\sqrt{ax^2+bx^3}}{(cx)^{11/2}}\,dx = -\frac{2\left(\frac{35\,b^3c^3\mathrm{sgn}(x)}{a}+4\left(\frac{2\,(bx+a)b^3c^3\mathrm{sgn}(x)}{a^3}-\frac{7\,b^3c^3\mathrm{sgn}(x)}{a^2}\right)(bx+a)\right)(bx+a)^{\frac{3}{2}}b^5}{105\,((bx+a)bc-abc)^{\frac{7}{2}}c^5|b|}$$

```
integrate((b*x^3+a*x^2)^(1/2)/(c*x)^(11/2),x, algorithm="giac")
```

```
-2/105*(35*b^3*c^3*sgn(x)/a + 4*(2*(b*x + a)*b^3*c^3*sgn(x)/a^3 - 7*b^3*c^
3*sgn(x)/a^2)*(b*x + a))*(b*x + a)^(3/2)*b^5/(((b*x + a)*b*c - a*b*c)^(7/2
)*c^5*abs(b))
```

Mupad [B] (verification not implemented)

Time = 9.60 (sec) , antiderivative size = 67, normalized size of antiderivative = 0.70

$$\int \frac{\sqrt{ax^2+bx^3}}{(cx)^{11/2}}\,dx = -\frac{\sqrt{b\,x^3+a\,x^2}\left(\frac{2}{7\,c^5}-\frac{8\,b^2\,x^2}{105\,a^2\,c^5}+\frac{16\,b^3\,x^3}{105\,a^3\,c^5}+\frac{2\,b\,x}{35\,a\,c^5}\right)}{x^4\,\sqrt{c\,x}}$$

```
int((a*x^2 + b*x^3)^(1/2)/(c*x)^(11/2),x)
```

```
-((a*x^2 + b*x^3)^(1/2)*(2/(7*c^5) - (8*b^2*x^2)/(105*a^2*c^5) + (16*b^3*x
^3)/(105*a^3*c^5) + (2*b*x)/(35*a*c^5)))/(x^4*(c*x)^(1/2))
```

Reduce [B] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 85, normalized size of antiderivative = 0.89

$$\int \frac{\sqrt{ax^2+bx^3}}{(cx)^{11/2}}\,dx = \frac{2\sqrt{c}\left(-15\sqrt{x}\,\sqrt{bx+a}\,a^3-3\sqrt{x}\,\sqrt{bx+a}\,a^2bx+4\sqrt{x}\,\sqrt{bx+a}\,a\,b^2x^2-8\sqrt{x}\,\sqrt{bx+a}\right.}{105a^3c^6x^4}$$

```
int((b*x^3+a*x^2)^(1/2)/(c*x)^(11/2),x)
```

```
(2*sqrt(c)*( - 15*sqrt(x)*sqrt(a + b*x)*a**3 - 3*sqrt(x)*sqrt(a + b*x)*a**
2*b*x + 4*sqrt(x)*sqrt(a + b*x)*a*b**2*x**2 - 8*sqrt(x)*sqrt(a + b*x)*b**3
*x**3 + 8*sqrt(b)*b**3*x**4))/(105*a**3*c**6*x**4)
```

3.334 $\int \frac{\sqrt{ax^2+bx^3}}{(cx)^{13/2}}\,dx$

Optimal result

Integrand size = 23, antiderivative size = 131

$$\int \frac{\sqrt{ax^2+bx^3}}{(cx)^{13/2}}\,dx = -\frac{2c(ax^2+bx^3)^{3/2}}{9a(cx)^{15/2}}+\frac{4b(ax^2+bx^3)^{3/2}}{21a^2(cx)^{13/2}} -\frac{16b^2(ax^2+bx^3)^{3/2}}{105a^3c(cx)^{11/2}}+\frac{32b^3(ax^2+bx^3)^{3/2}}{315a^4c^2(cx)^{9/2}}$$

```
-2/9*c*(b*x^3+a*x^2)^(3/2)/a/(c*x)^(15/2)+4/21*b*(b*x^3+a*x^2)^(3/2)/a^2/(
c*x)^(13/2)-16/105*b^2*(b*x^3+a*x^2)^(3/2)/a^3/c/(c*x)^(11/2)+32/315*b^3*(
b*x^3+a*x^2)^(3/2)/a^4/c^2/(c*x)^(9/2)
```

Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 69, normalized size of antiderivative = 0.53

$$\int \frac{\sqrt{ax^2+bx^3}}{(cx)^{13/2}}\,dx = -\frac{2x\sqrt{x^2(a+bx)}(35a^4+5a^3bx-6a^2b^2x^2+8ab^3x^3-16b^4x^4)}{315a^4(cx)^{13/2}}$$

```
Integrate[Sqrt[a*x^2 + b*x^3]/(c*x)^(13/2),x]
```

```
(-2*x*Sqrt[x^2*(a + b*x)]*(35*a^4 + 5*a^3*b*x - 6*a^2*b^2*x^2 + 8*a*b^3*x^
3 - 16*b^4*x^4))/(315*a^4*(c*x)^(13/2))
```

Rubi [A] (verified)

Time = 0.50 (sec) , antiderivative size = 145, normalized size of antiderivative = 1.11, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.174, Rules used = {1922, 1922, 1922, 1920}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sqrt{ax^2+bx^3}}{(cx)^{13/2}}dx$$

$$\Big\downarrow \; 1922$$

$$-\frac{2b\int \frac{\sqrt{bx^3+ax^2}}{(cx)^{11/2}}dx}{3ac}-\frac{2c\left(ax^2+bx^3\right)^{3/2}}{9a(cx)^{15/2}}$$

$$\Big\downarrow \; 1922$$

$$-\frac{2b\left(-\frac{4b\int \frac{\sqrt{bx^3+ax^2}}{(cx)^{9/2}}dx}{7ac}-\frac{2c\left(ax^2+bx^3\right)^{3/2}}{7a(cx)^{13/2}}\right)}{3ac}-\frac{2c\left(ax^2+bx^3\right)^{3/2}}{9a(cx)^{15/2}}$$

$$\Big\downarrow \; 1922$$

$$-\frac{2b\left(-\frac{4b\left(-\frac{2b\int \frac{\sqrt{bx^3+ax^2}}{(cx)^{7/2}}dx}{5ac}-\frac{2c\left(ax^2+bx^3\right)^{3/2}}{5a(cx)^{11/2}}\right)}{7ac}-\frac{2c\left(ax^2+bx^3\right)^{3/2}}{7a(cx)^{13/2}}\right)}{3ac}-\frac{2c\left(ax^2+bx^3\right)^{3/2}}{9a(cx)^{15/2}}$$

$$\Big\downarrow \; 1920$$

$$-\frac{2b\left(-\frac{4b\left(\frac{4b\left(ax^2+bx^3\right)^{3/2}}{15a^2(cx)^{9/2}}-\frac{2c\left(ax^2+bx^3\right)^{3/2}}{5a(cx)^{11/2}}\right)}{7ac}-\frac{2c\left(ax^2+bx^3\right)^{3/2}}{7a(cx)^{13/2}}\right)}{3ac}-\frac{2c\left(ax^2+bx^3\right)^{3/2}}{9a(cx)^{15/2}}$$

```
Int[Sqrt[a*x^2 + b*x^3]/(c*x)^(13/2),x]
```

```
(-2*c*(a*x^2 + b*x^3)^(3/2))/(9*a*(c*x)^(15/2)) - (2*b*((-2*c*(a*x^2 + b*x
^3)^(3/2))/(7*a*(c*x)^(13/2)) - (4*b*((-2*c*(a*x^2 + b*x^3)^(3/2))/(5*a*(c
*x)^(11/2)) + (4*b*(a*x^2 + b*x^3)^(3/2))/(15*a^2*(c*x)^(9/2))))/(7*a*c))
/(3*a*c)
```

Defintions of rubi rules used

```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> Simp[(-c^(j - 1))*(c*x)^(m - j + 1)*((a*x^j + b*x^n)^(p + 1)/(a*(n - j
)*(p + 1))), x] /; FreeQ[{a, b, c, j, m, n, p}, x] && !IntegerQ[p] && NeQ[
n, j] && EqQ[m + n*p + n - j + 1, 0] && (IntegerQ[j] || GtQ[c, 0])
```

```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> Simp[c^(j - 1)*(c*x)^(m - j + 1)*((a*x^j + b*x^n)^(p + 1)/(a*(m + j*p
+ 1))), x] - Simp[b*((m + n*p + n - j + 1)/(a*c^(n - j)*(m + j*p + 1)))   I
nt[(c*x)^(m + n - j)*(a*x^j + b*x^n)^p, x], x] /; FreeQ[{a, b, c, j, m, n,
p}, x] && !IntegerQ[p] && NeQ[n, j] && ILtQ[Simplify[(m + n*p + n - j + 1)
/(n - j)], 0] && NeQ[m + j*p + 1, 0] && (IntegersQ[j, n] || GtQ[c, 0])
```

Maple [A] (verified)

Time = 0.35 (sec) , antiderivative size = 60, normalized size of antiderivative = 0.46

method	result	size
gosper	$-\frac{2x(bx+a)\left(-16b^{3}x^{3}+24a\,b^{2}x^{2}-30a^{2}bx+35a^{3}\right)\sqrt{b\,x^{3}+a\,x^{2}}}{315a^{4}\left(cx\right)^{\frac{13}{2}}}$	60
orering	$-\frac{2x(bx+a)\left(-16b^{3}x^{3}+24a\,b^{2}x^{2}-30a^{2}bx+35a^{3}\right)\sqrt{b\,x^{3}+a\,x^{2}}}{315a^{4}\left(cx\right)^{\frac{13}{2}}}$	60
default	$-\frac{2\sqrt{b\,x^{3}+a\,x^{2}}\,(bx+a)\left(-16b^{3}x^{3}+24a\,b^{2}x^{2}-30a^{2}bx+35a^{3}\right)}{315x^{5}c^{6}\sqrt{cx}\,a^{4}}$	65
risch	$-\frac{2\sqrt{x^{2}(bx+a)}\,\left(-16b^{4}x^{4}+8a\,b^{3}x^{3}-6a^{2}b^{2}x^{2}+5a^{3}bx+35a^{4}\right)}{315c^{6}x^{5}\sqrt{cx}\,a^{4}}$	69

```
int((b*x^3+a*x^2)^(1/2)/(c*x)^(13/2),x,method=_RETURNVERBOSE)
```

```
-2/315*x*(b*x+a)*(-16*b^3*x^3+24*a*b^2*x^2-30*a^2*b*x+35*a^3)*(b*x^3+a*x^2
)^(1/2)/a^4/(c*x)^(13/2)
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 70, normalized size of antiderivative = 0.53

$$\int \frac{\sqrt{ax^2+bx^3}}{(cx)^{13/2}}\,dx = \frac{2\,(16\,b^4x^4-8\,ab^3x^3+6\,a^2b^2x^2-5\,a^3bx-35\,a^4)\sqrt{bx^3+ax^2}\sqrt{cx}}{315\,a^4c^7x^6}$$

```
integrate((b*x^3+a*x^2)^(1/2)/(c*x)^(13/2),x, algorithm="fricas")
```

```
2/315*(16*b^4*x^4 - 8*a*b^3*x^3 + 6*a^2*b^2*x^2 - 5*a^3*b*x - 35*a^4)*sqrt
(b*x^3 + a*x^2)*sqrt(c*x)/(a^4*c^7*x^6)
```

Sympy [F]

$$\int \frac{\sqrt{ax^2+bx^3}}{(cx)^{13/2}}\,dx = \int \frac{\sqrt{x^2\,(a+bx)}}{(cx)^{\frac{13}{2}}}\,dx$$

```
integrate((b*x**3+a*x**2)**(1/2)/(c*x)**(13/2),x)
```

```
Integral(sqrt(x**2*(a + b*x))/(c*x)**(13/2), x)
```

Maxima [F]

$$\int \frac{\sqrt{ax^2+bx^3}}{(cx)^{13/2}}\,dx = \int \frac{\sqrt{bx^3+ax^2}}{(cx)^{\frac{13}{2}}}\,dx$$

```
integrate((b*x^3+a*x^2)^(1/2)/(c*x)^(13/2),x, algorithm="maxima")
```

```
integrate(sqrt(b*x^3 + a*x^2)/(c*x)^(13/2), x)
```

Giac [A] (verification not implemented)

Time = 0.17 (sec) , antiderivative size = 107, normalized size of antiderivative = 0.82

$$\int \frac{\sqrt{ax^2+bx^3}}{(cx)^{13/2}}\,dx = -\frac{2\left(\frac{105\,b^9c^4\mathrm{sgn}(x)}{a}-2\left(\frac{63\,b^9c^4\mathrm{sgn}(x)}{a^2}+4\left(\frac{2\,(bx+a)b^9c^4\mathrm{sgn}(x)}{a^4}-\frac{9\,b^9c^4\mathrm{sgn}(x)}{a^3}\right)(bx+a)\right)(bx+a)\right)(bx+a)^{\frac{3}{2}}b}{315\left((bx+a)bc-abc\right)^{\frac{9}{2}}c^6|b|}$$

```
integrate((b*x^3+a*x^2)^(1/2)/(c*x)^(13/2),x, algorithm="giac")
```

```
-2/315*(105*b^9*c^4*sgn(x)/a - 2*(63*b^9*c^4*sgn(x)/a^2 + 4*(2*(b*x + a)*b
^9*c^4*sgn(x)/a^4 - 9*b^9*c^4*sgn(x)/a^3)*(b*x + a))*(b*x + a))*(b*x + a)^
(3/2)*b/(((b*x + a)*b*c - a*b*c)^(9/2)*c^6*abs(b))
```

Mupad [B] (verification not implemented)

Time = 9.23 (sec) , antiderivative size = 81, normalized size of antiderivative = 0.62

$$\int \frac{\sqrt{ax^2+bx^3}}{(cx)^{13/2}}\,dx = -\frac{\sqrt{b\,x^3+a\,x^2}\left(\frac{2}{9\,c^6}-\frac{4\,b^2\,x^2}{105\,a^2\,c^6}+\frac{16\,b^3\,x^3}{315\,a^3\,c^6}-\frac{32\,b^4\,x^4}{315\,a^4\,c^6}+\frac{2\,b\,x}{63\,a\,c^6}\right)}{x^5\,\sqrt{c\,x}}$$

```
int((a*x^2 + b*x^3)^(1/2)/(c*x)^(13/2),x)
```

```
-((a*x^2 + b*x^3)^(1/2)*(2/(9*c^6) - (4*b^2*x^2)/(105*a^2*c^6) + (16*b^3*x
^3)/(315*a^3*c^6) - (32*b^4*x^4)/(315*a^4*c^6) + (2*b*x)/(63*a*c^6)))/(x^5
*(c*x)^(1/2))
```

Reduce [B] (verification not implemented)

Time = 0.50 (sec) , antiderivative size = 104, normalized size of antiderivative = 0.79

$$\int \frac{\sqrt{ax^2+bx^3}}{(cx)^{13/2}}\,dx = \frac{2\sqrt{c}\left(-35\sqrt{x}\,\sqrt{bx+a}\,a^4-5\sqrt{x}\,\sqrt{bx+a}\,a^3bx+6\sqrt{x}\,\sqrt{bx+a}\,a^2b^2x^2-8\sqrt{x}\,\sqrt{bx+a}\right.}{315a^4c^7x^5}$$

```
int((b*x^3+a*x^2)^(1/2)/(c*x)^(13/2),x)
```

```
(2*sqrt(c)*( - 35*sqrt(x)*sqrt(a + b*x)*a**4 - 5*sqrt(x)*sqrt(a + b*x)*a**
3*b*x + 6*sqrt(x)*sqrt(a + b*x)*a**2*b**2*x**2 - 8*sqrt(x)*sqrt(a + b*x)*a
*b**3*x**3 + 16*sqrt(x)*sqrt(a + b*x)*b**4*x**4 - 16*sqrt(b)*b**4*x**5))/(
315*a**4*c**7*x**5)
```

3.335 $\int (cx)^{3/2} \left(ax^2+bx^3\right)^{3/2} \, dx$

Optimal result

Integrand size = 23, antiderivative size = 282

$$\int (cx)^{3/2} \left(ax^2+bx^3\right)^{3/2} \, dx = \frac{9a^6c^2\sqrt{ax^2+bx^3}}{1024b^5\sqrt{cx}} - \frac{3a^5c\sqrt{cx}\sqrt{ax^2+bx^3}}{512b^4} + \frac{3a^4(cx)^{3/2}\sqrt{ax^2+bx^3}}{640b^3} - \frac{9a^3(cx)^{5/2}\sqrt{ax^2+bx^3}}{2240b^2c} + \frac{a^2(cx)^{7/2}\sqrt{ax^2+bx^3}}{280bc^2} + \frac{5a(cx)^{9/2}\sqrt{ax^2+bx^3}}{28c^3} + \frac{b(cx)^{11/2}\sqrt{ax^2+bx^3}}{7c^4} - \frac{9a^7c^{3/2}\text{arctanh}\left(\frac{\sqrt{b}(cx)^{3/2}}{c^{3/2}\sqrt{ax^2+bx^3}}\right)}{1024b^{11/2}}$$

```
9/1024*a^6*c^2*(b*x^3+a*x^2)^(1/2)/b^5/(c*x)^(1/2)-3/512*a^5*c*(c*x)^(1/2)
*(b*x^3+a*x^2)^(1/2)/b^4+3/640*a^4*(c*x)^(3/2)*(b*x^3+a*x^2)^(1/2)/b^3-9/2
240*a^3*(c*x)^(5/2)*(b*x^3+a*x^2)^(1/2)/b^2/c+1/280*a^2*(c*x)^(7/2)*(b*x^3
+a*x^2)^(1/2)/b/c^2+5/28*a*(c*x)^(9/2)*(b*x^3+a*x^2)^(1/2)/c^3+1/7*b*(c*x)
^(11/2)*(b*x^3+a*x^2)^(1/2)/c^4-9/1024*a^7*c^(3/2)*arctanh(b^(1/2)*(c*x)^(
3/2)/c^(3/2)/(b*x^3+a*x^2)^(1/2))/b^(11/2)
```

Mathematica [A] (verified)

Time = 0.75 (sec) , antiderivative size = 163, normalized size of antiderivative = 0.58

$$\int (cx)^{3/2} \left(ax^2 +bx^3\right)^{3/2} dx = \frac{(cx)^{3/2}\sqrt{x^2(a+bx)}\left(\sqrt{b}\sqrt{x}\sqrt{a+bx}(315a^6-210a^5bx+168a^4b^2x^2-144a^3b^3x^3+128a^2b^4x^4+6400ab^5x^5+5120b^6x^6)+630a^7\text{arctanh}\left(\frac{\sqrt{b}\sqrt{x}}{\sqrt{a}-\sqrt{a+bx}}\right)\right)}{35840b^{11/2}x^{5/2}\sqrt{a+bx}}$$

```
Integrate[(c*x)^(3/2)*(a*x^2 + b*x^3)^(3/2),x]
```

```
((c*x)^(3/2)*Sqrt[x^2*(a + b*x)]*(Sqrt[b]*Sqrt[x]*Sqrt[a + b*x]*(315*a^6 -
 210*a^5*b*x + 168*a^4*b^2*x^2 - 144*a^3*b^3*x^3 + 128*a^2*b^4*x^4 + 6400*
a*b^5*x^5 + 5120*b^6*x^6) + 630*a^7*ArcTanh[(Sqrt[b]*Sqrt[x])/(Sqrt[a] - S
qrt[a + b*x])]))/(35840*b^(11/2)*x^(5/2)*Sqrt[a + b*x])
```

Rubi [A] (verified)

Time = 1.07 (sec) , antiderivative size = 322, normalized size of antiderivative = 1.14, number of steps used = 11, number of rules used = 10, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.435, Rules used = {1927, 1927, 1930, 1930, 1930, 1930, 1930, 1937, 1935, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (cx)^{3/2} \left(ax^2+bx^3\right)^{3/2} dx$$

$$\Big\downarrow \text{1927}$$

$$\frac{3a\int (cx)^{7/2}\sqrt{bx^3+ax^2}dx}{14c^2}+\frac{(cx)^{5/2}\left(ax^2+bx^3\right)^{3/2}}{7c}$$

$$\Big\downarrow \text{1927}$$

$$\frac{3a\left(\frac{a\int \frac{(cx)^{11/2}}{\sqrt{bx^3+ax^2}}dx}{12c^2}+\frac{(cx)^{9/2}\sqrt{ax^2+bx^3}}{6c}\right)}{14c^2}+\frac{(cx)^{5/2}\left(ax^2+bx^3\right)^{3/2}}{7c}$$

$$\Big\downarrow \text{1930}$$

$$\frac{3a\left(\frac{a\left(\frac{c^2(cx)^{7/2}\sqrt{ax^2+bx^3}}{5b}-\frac{9ac\int\frac{(cx)^{9/2}}{\sqrt{bx^3+ax^2}}dx}{10b}\right)}{12c^2}+\frac{(cx)^{9/2}\sqrt{ax^2+bx^3}}{6c}\right)}{14c^2}+\frac{(cx)^{5/2}\left(ax^2+bx^3\right)^{3/2}}{7c}$$

$$\Big\downarrow \text{1930}$$

$$\frac{3a\left(\frac{a\left(\frac{c^2(cx)^{7/2}\sqrt{ax^2+bx^3}}{5b}-\frac{9ac\left(\frac{c^2(cx)^{5/2}\sqrt{ax^2+bx^3}}{4b}-\frac{7ac\int\frac{(cx)^{7/2}}{\sqrt{bx^3+ax^2}}dx}{8b}\right)}{10b}\right)}{12c^2}+\frac{(cx)^{9/2}\sqrt{ax^2+bx^3}}{6c}\right)}{14c^2}+\frac{(cx)^{5/2}\left(ax^2+bx^3\right)^{3/2}}{7c}$$

$$\Big\downarrow \text{1930}$$

$$\frac{3a\left(\frac{a\left(\frac{c^2(cx)^{7/2}\sqrt{ax^2+bx^3}}{5b}-\frac{9ac\left(\frac{c^2(cx)^{5/2}\sqrt{ax^2+bx^3}}{4b}-\frac{7ac\left(\frac{c^2(cx)^{3/2}\sqrt{ax^2+bx^3}}{3b}-\frac{5ac\int\frac{(cx)^{5/2}}{\sqrt{bx^3+ax^2}}dx}{6b}\right)}{8b}\right)}{10b}\right)}{12c^2}+\frac{(cx)^{9/2}\sqrt{ax^2+bx^3}}{6c}\right)}{14c^2}+\frac{(cx)^{5/2}\left(ax^2+bx^3\right)^{3/2}}{7c}$$

$$\Big\downarrow \color{blue}{1930}$$

$$\frac{3a\left(\frac{a\left(\frac{c^2(cx)^{7/2}\sqrt{ax^2+bx^3}}{5b}-\frac{9ac\left(\frac{c^2(cx)^{5/2}\sqrt{ax^2+bx^3}}{4b}-\frac{7ac\left(\frac{c^2(cx)^{3/2}\sqrt{ax^2+bx^3}}{3b}-\frac{5ac\left(\frac{c^2\sqrt{cx}\sqrt{ax^2+bx^3}}{2b}-\frac{3ac\int\frac{(cx)^{3/2}}{\sqrt{bx^3+ax^2}}dx}{4b}\right)}{6b}\right)}{8b}\right)}{10b}\right)}{12c^2}+\frac{(cx)^{9/\cdots}}{\cdots}\right)}{14c^2}$$

$$\frac{(cx)^{5/2}\left(ax^2+bx^3\right)^{3/2}}{7c}$$

$$\Big\downarrow \; 1930$$

$$3a\left(\frac{a\left(\frac{c^2(cx)^{7/2}\sqrt{ax^2+bx^3}}{5b}-\frac{9ac\left(\frac{c^2(cx)^{5/2}\sqrt{ax^2+bx^3}}{4b}-\frac{7ac\left(\frac{c^2(cx)^{3/2}\sqrt{ax^2+bx^3}}{3b}-\frac{5ac\left(\frac{c^2\sqrt{cx}\sqrt{ax^2+bx^3}}{2b}-\frac{3ac\left(\frac{c^2\sqrt{ax^2+bx^3}}{b\sqrt{cx}}-\frac{ac\int\frac{\sqrt{cx}}{\sqrt{bx^3+ax^2}}dx}{2b}\cdots\right)}{4b}\cdots\right)}{6b}\cdots\right)}{8b}\cdots\right)}{10b}\cdots\right)}{12c^2}\cdots\right.$$

↓ 1937

$$3a\left(\frac{a\left(\frac{c^2(cx)^{7/2}\sqrt{ax^2+bx^3}}{5b}-\frac{9ac\left(\frac{c^2(cx)^{5/2}\sqrt{ax^2+bx^3}}{4b}-\frac{7ac\left(\frac{c^2(cx)^{3/2}\sqrt{ax^2+bx^3}}{3b}-\frac{5ac\left(\frac{c^2\sqrt{cx}\sqrt{ax^2+bx^3}}{2b}-\frac{3ac\left(\frac{c^2\sqrt{ax^2+bx^3}}{b\sqrt{cx}}-\frac{ac\sqrt{cx}\int\frac{\sqrt{x}}{\sqrt{bx^3+ax}}}{2b\sqrt{x}}\right.}{4b}\right.}{6b}\right.}{8b}\right.}{10b}\right.}{12c^2}\right.$$

↓ 1935

$$a\left(\frac{c^2(cx)^{7/2}\sqrt{ax^2+bx^3}}{5b}-\frac{9ac\left(\frac{c^2(cx)^{5/2}\sqrt{ax^2+bx^3}}{4b}-\frac{7ac\left(\frac{c^2(cx)^{3/2}\sqrt{ax^2+bx^3}}{3b}-\frac{5ac\left(\frac{c^2\sqrt{cx}\sqrt{ax^2+bx^3}}{2b}-\frac{3ac\left(\frac{c^2\sqrt{ax^2+bx^3}}{b\sqrt{cx}}-\frac{ac\sqrt{cx}\int\frac{1}{1-\frac{bx^3}{bx^3+a\cdots}}\cdots}{b\sqrt{\cdots}}\cdots\right.}{4b}\cdots\right.}{6b}\cdots\right.}{8b}\cdots\right.}{10b}\cdots\right.$$

$\downarrow$ 219

$$3a\left(\frac{a\left(\frac{c^2(cx)^{7/2}\sqrt{ax^2+bx^3}}{5b}-\frac{9ac\left(\frac{c^2(cx)^{5/2}\sqrt{ax^2+bx^3}}{4b}-\frac{7ac\left(\frac{c^2(cx)^{3/2}\sqrt{ax^2+bx^3}}{3b}-\frac{5ac\left(\frac{c^2\sqrt{cx}\sqrt{ax^2+bx^3}}{2b}-\frac{3ac\left(\frac{c^2\sqrt{ax^2+bx^3}}{b\sqrt{cx}}-\frac{ac\sqrt{cx}\,\mathrm{arctanh}\left(\cdots\right.}{b^{3/2}\sqrt{\cdots}}\right.}{4b}\right.}{6b}\right.}{8b}\right.}{10b}\right.}{12c^2}\right.$$

```
Int[(c*x)^(3/2)*(a*x^2 + b*x^3)^(3/2),x]
```

```
((c*x)^(5/2)*(a*x^2 + b*x^3)^(3/2))/(7*c) + (3*a*(((c*x)^(9/2)*Sqrt[a*x^2
+ b*x^3])/(6*c) + (a*((c^2*(c*x)^(7/2)*Sqrt[a*x^2 + b*x^3])/(5*b) - (9*a*c
*((c^2*(c*x)^(5/2)*Sqrt[a*x^2 + b*x^3])/(4*b) - (7*a*c*((c^2*(c*x)^(3/2)*S
qrt[a*x^2 + b*x^3])/(3*b) - (5*a*c*((c^2*Sqrt[c*x]*Sqrt[a*x^2 + b*x^3])/(2
*b) - (3*a*c*((c^2*Sqrt[a*x^2 + b*x^3])/(b*Sqrt[c*x]) - (a*c*Sqrt[c*x]*Arc
Tanh[(Sqrt[b]*x^(3/2))/Sqrt[a*x^2 + b*x^3]])/(b^(3/2)*Sqrt[x])))/(4*b)))/(
6*b)))/(8*b)))/(10*b)))/(12*c^2)))/(14*c^2)
```

Defintions of rubi rules used

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> Simp[(c*x)^(m + 1)*((a*x^j + b*x^n)^p/(c*(m + n*p + 1))), x] + Simp[a*
(n - j)*(p/(c^j*(m + n*p + 1)))   Int[(c*x)^(m + j)*(a*x^j + b*x^n)^(p - 1)
, x], x] /; FreeQ[{a, b, c, m}, x] &&  !IntegerQ[p] && LtQ[0, j, n] && (Int
egersQ[j, n] || GtQ[c, 0]) && GtQ[p, 0] && NeQ[m + n*p + 1, 0]
```

```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> Simp[c^(n - 1)*(c*x)^(m - n + 1)*((a*x^j + b*x^n)^(p + 1)/(b*(m + n*p
+ 1))), x] - Simp[a*c^(n - j)*((m + j*p - n + j + 1)/(b*(m + n*p + 1)))   I
nt[(c*x)^(m - (n - j))*(a*x^j + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, p},
x] &&  !IntegerQ[p] && LtQ[0, j, n] && (IntegersQ[j, n] || GtQ[c, 0]) && Gt
Q[m + j*p - n + j + 1, 0] && NeQ[m + n*p + 1, 0]
```

```
Int[(x_)^(m_.)/Sqrt[(a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.)], x_Symbol] :> Simp
[-2/(n - j)   Subst[Int[1/(1 - a*x^2), x], x, x^(j/2)/Sqrt[a*x^j + b*x^n]],
 x] /; FreeQ[{a, b, j, n}, x] && EqQ[m, j/2 - 1] && NeQ[n, j]
```

```
Int[((c_)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_))^(p_), x_Symbol]
:> Simp[c^IntPart[m]*((c*x)^FracPart[m]/x^FracPart[m])   Int[x^m*(a*x^j + b
*x^n)^p, x], x] /; FreeQ[{a, b, c, j, m, n, p}, x] && IntegerQ[p + 1/2] &&
NeQ[n, j] && EqQ[Simplify[m + j*p + 1], 0]
```

Maple [A] (verified)

Time = 0.36 (sec) , antiderivative size = 171, normalized size of antiderivative = 0.61

method	result
risch	$\frac{\left(5120 b^{6} x^{6}+6400 a\,b^{5} x^{5}+128 a^{2} b^{4} x^{4}-144 a^{3} b^{3} x^{3}+168 a^{4} b^{2} x^{2}-210 a^{5} b x+315 a^{6}\right) c^{2} \sqrt{x^{2} \left(b x+a\right)}}{35840 b^{5} \sqrt{c x}}-\frac{9 a^{7} \ln\left(\frac{\frac{1}{2} a c+c b x}{\sqrt{b c}}+\sqrt{b c\,x^{2}+a c x}\right.}{2048 b^{5} \sqrt{b c}\, x \left(b\right.}$
default	$\frac{\left(b\,x^{3}+a\,x^{2}\right)^{\frac{3}{2}} \sqrt{c x}\, c \left(10240 b^{6} x^{6} \sqrt{b c}\, \sqrt{c x \left(b x+a\right)}+12800 a\,b^{5} x^{5} \sqrt{b c}\, \sqrt{c x \left(b x+a\right)}+256 a^{2} b^{4} x^{4} \sqrt{b c}\, \sqrt{c x \left(b x+a\right)}-288 a^{3} b^{3} x^{3} \sqrt{b c}\, \sqrt{c x (\right.}{71680 x^{3} \left(b x+a\right) b^{5}}$

```
int((c*x)^(3/2)*(b*x^3+a*x^2)^(3/2),x,method=_RETURNVERBOSE)
```

```
1/35840*(5120*b^6*x^6+6400*a*b^5*x^5+128*a^2*b^4*x^4-144*a^3*b^3*x^3+168*a
^4*b^2*x^2-210*a^5*b*x+315*a^6)/b^5*c^2*(x^2*(b*x+a))^(1/2)/(c*x)^(1/2)-9/
2048*a^7/b^5*ln((1/2*a*c+c*b*x)/(b*c)^(1/2)+(b*c*x^2+a*c*x)^(1/2))/(b*c)^(
1/2)*c^2*(x^2*(b*x+a))^(1/2)/x/(b*x+a)*(c*x*(b*x+a))^(1/2)/(c*x)^(1/2)
```

Fricas [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 314, normalized size of antiderivative = 1.11

$$\int (cx)^{3/2} \left(ax^2+bx^3\right)^{3/2} dx = \left[\frac{315\, a^7 cx \sqrt{\frac{c}{b}} \log\left(\frac{2\, bcx^2+acx-2\, \sqrt{bx^3+ax^2}\sqrt{cx}b\sqrt{\frac{c}{b}}}{x}\right) + 2\,(5120\, b^6 cx^6 + 6400\, ab^5 cx^5 + 128\, a^2 b^4 cx^4}{71680\, b^5 x}\right.$$

```
integrate((c*x)^(3/2)*(b*x^3+a*x^2)^(3/2),x, algorithm="fricas")
```

```
[1/71680*(315*a^7*c*x*sqrt(c/b)*log((2*b*c*x^2 + a*c*x - 2*sqrt(b*x^3 + a*
x^2)*sqrt(c*x)*b*sqrt(c/b))/x) + 2*(5120*b^6*c*x^6 + 6400*a*b^5*c*x^5 + 12
8*a^2*b^4*c*x^4 - 144*a^3*b^3*c*x^3 + 168*a^4*b^2*c*x^2 - 210*a^5*b*c*x +
315*a^6*c)*sqrt(b*x^3 + a*x^2)*sqrt(c*x))/(b^5*x), 1/35840*(315*a^7*c*x*sq
rt(-c/b)*arctan(sqrt(b*x^3 + a*x^2)*sqrt(c*x)*b*sqrt(-c/b)/(b*c*x^2 + a*c*
x)) + (5120*b^6*c*x^6 + 6400*a*b^5*c*x^5 + 128*a^2*b^4*c*x^4 - 144*a^3*b^3
*c*x^3 + 168*a^4*b^2*c*x^2 - 210*a^5*b*c*x + 315*a^6*c)*sqrt(b*x^3 + a*x^2
)*sqrt(c*x))/(b^5*x)]
```

Sympy [F]

$$\int (cx)^{3/2} \left(ax^2 + bx^3\right)^{3/2} \, dx = \int (cx)^{\frac{3}{2}} \left(x^2(a + bx)\right)^{\frac{3}{2}} \, dx$$

```
integrate((c*x)**(3/2)*(b*x**3+a*x**2)**(3/2),x)
```

```
Integral((c*x)**(3/2)*(x**2*(a + b*x))**(3/2), x)
```

Maxima [F]

$$\int (cx)^{3/2} \left(ax^2 + bx^3\right)^{3/2} \, dx = \int \left(bx^3 + ax^2\right)^{\frac{3}{2}} (cx)^{\frac{3}{2}} \, dx$$

```
integrate((c*x)^(3/2)*(b*x^3+a*x^2)^(3/2),x, algorithm="maxima")
```

```
integrate((b*x^3 + a*x^2)^(3/2)*(c*x)^(3/2), x)
```

Giac [A] (verification not implemented)

Time = 0.18 (sec) , antiderivative size = 347, normalized size of antiderivative = 1.23

$$\int (cx)^{3/2} \left(ax^2+bx^3\right)^{3/2} dx =$$

$$-\frac{1}{215040}\left(\frac{945\,a^7|c|\log\left(c^2|a|\right)\operatorname{sgn}(x)}{\sqrt{bc}b^5} - \frac{28\left(\frac{315\,a^6c^2\log\left(\left|-\sqrt{bc}\sqrt{cx}+\sqrt{bc^2x+ac^2}\right|\right)}{\sqrt{bc}b^5} + \sqrt{bc^2x+ac^2}\left(2\left(4\left(2\left(8\,cx\right.\right.\right.\right.\right.}{}\right.$$

```
integrate((c*x)^(3/2)*(b*x^3+a*x^2)^(3/2),x, algorithm="giac")
```

```
-1/215040*(945*a^7*abs(c)*log(c^2*abs(a))*sgn(x)/(sqrt(b*c)*b^5) - 28*(315
*a^6*c^2*log(abs(-sqrt(b*c)*sqrt(c*x) + sqrt(b*c^2*x + a*c^2)))/(sqrt(b*c)
*b^5) + sqrt(b*c^2*x + a*c^2)*(2*(4*(2*(8*c*x*(10*x/c^4 + a/(b*c^4)) - 9*a
^2/(b^2*c^3))*c*x + 21*a^3/(b^3*c^2))*c*x - 105*a^4/(b^4*c))*c*x + 315*a^5
/b^5)*sqrt(c*x))*a*abs(c)*sgn(x)/c^2 + 2*(3465*a^7*c^8*log(abs(-sqrt(b*c)*
sqrt(c*x) + sqrt(b*c^2*x + a*c^2)))/(sqrt(b*c)*b^6) + (3465*a^6*c^6/b^6 -
2*(1155*a^5*c^5/b^5 - 4*(231*a^4*c^4/b^4 - 2*(99*a^3*c^3/b^3 + 8*(10*(12*c
*x + a*c/b)*c*x - 11*a^2*c^2/b^2)*c*x)*c*x)*c*x)*c*x)*sqrt(b*c^2*x + a*c^2
)*sqrt(c*x))*b*abs(c)*sgn(x)/c^8)*c
```

Mupad [F(-1)]

Timed out.

$$\int (cx)^{3/2} \left(ax^2+bx^3\right)^{3/2} dx = \int (c\,x)^{3/2}\,\left(b\,x^3+a\,x^2\right)^{3/2} \,dx$$

```
int((c*x)^(3/2)*(a*x^2 + b*x^3)^(3/2),x)
```

```
int((c*x)^(3/2)*(a*x^2 + b*x^3)^(3/2), x)
```

Reduce [B] (verification not implemented)

Time = 0.47 (sec) , antiderivative size = 155, normalized size of antiderivative = 0.55

$$\int (cx)^{3/2} \left(ax^2 +bx^3\right)^{3/2} dx = \frac{\sqrt{c}\, c\Big(315\sqrt{x}\,\sqrt{bx+a}\,a^6 b - 210\sqrt{x}\,\sqrt{bx+a}\,a^5 b^2 x + 168\sqrt{x}\,\sqrt{bx+a}\,a^4 b^3 x^2 - 144\sqrt{x}\,\sqrt{bx-}$$

```
int((c*x)^(3/2)*(b*x^3+a*x^2)^(3/2),x)
```

```
(sqrt(c)*c*(315*sqrt(x)*sqrt(a + b*x)*a**6*b - 210*sqrt(x)*sqrt(a + b*x)*a
**5*b**2*x + 168*sqrt(x)*sqrt(a + b*x)*a**4*b**3*x**2 - 144*sqrt(x)*sqrt(a
 + b*x)*a**3*b**4*x**3 + 128*sqrt(x)*sqrt(a + b*x)*a**2*b**5*x**4 + 6400*s
qrt(x)*sqrt(a + b*x)*a*b**6*x**5 + 5120*sqrt(x)*sqrt(a + b*x)*b**7*x**6 -
315*sqrt(b)*log((sqrt(a + b*x) + sqrt(x)*sqrt(b))/sqrt(a))*a**7))/(35840*b
**6)
```

3.336 $\int \sqrt{cx}(ax^2+bx^3)^{3/2}\,dx$

Optimal result

Integrand size = 23, antiderivative size = 247

$$\int \sqrt{cx}\left(ax^2+bx^3\right)^{3/2}\,dx = -\frac{7a^5c\sqrt{ax^2+bx^3}}{512b^4\sqrt{cx}}+\frac{7a^4\sqrt{cx}\sqrt{ax^2+bx^3}}{768b^3} - \frac{7a^3(cx)^{3/2}\sqrt{ax^2+bx^3}}{960b^2c}+\frac{a^2(cx)^{5/2}\sqrt{ax^2+bx^3}}{160bc^2}+\frac{13a(cx)^{7/2}\sqrt{ax^2+bx^3}}{60c^3} + \frac{b(cx)^{9/2}\sqrt{ax^2+bx^3}}{6c^4}+\frac{7a^6\sqrt{c}\operatorname{arctanh}\left(\frac{\sqrt{b}(cx)^{3/2}}{c^{3/2}\sqrt{ax^2+bx^3}}\right)}{512b^{9/2}}$$

```
-7/512*a^5*c*(b*x^3+a*x^2)^(1/2)/b^4/(c*x)^(1/2)+7/768*a^4*(c*x)^(1/2)*(b*
x^3+a*x^2)^(1/2)/b^3-7/960*a^3*(c*x)^(3/2)*(b*x^3+a*x^2)^(1/2)/b^2/c+1/160
*a^2*(c*x)^(5/2)*(b*x^3+a*x^2)^(1/2)/b/c^2+13/60*a*(c*x)^(7/2)*(b*x^3+a*x^
2)^(1/2)/c^3+1/6*b*(c*x)^(9/2)*(b*x^3+a*x^2)^(1/2)/c^4+7/512*a^6*c^(1/2)*a
rctanh(b^(1/2)*(c*x)^(3/2)/c^(3/2)/(b*x^3+a*x^2)^(1/2))/b^(9/2)
```

Mathematica [A] (verified)

Time = 0.09 (sec) , antiderivative size = 152, normalized size of antiderivative = 0.62

$$\int \sqrt{cx}\left(ax^2+bx^3\right)^{3/2}\,dx=\frac{\sqrt{x}\sqrt{cx}\sqrt{a+bx}\Big(\sqrt{b}\sqrt{x}\sqrt{a+bx}(-105a^5+70a^4bx-56a^3b^2x^2+48a^2b^3x^3+1664ab^4x^4+1}{7680b^{9/2}\sqrt{x^2(a+bx)}}$$

```
Integrate[Sqrt[c*x]*(a*x^2 + b*x^3)^(3/2),x]
```

```
(Sqrt[x]*Sqrt[c*x]*Sqrt[a + b*x]*(Sqrt[b]*Sqrt[x]*Sqrt[a + b*x]*(-105*a^5
+ 70*a^4*b*x - 56*a^3*b^2*x^2 + 48*a^2*b^3*x^3 + 1664*a*b^4*x^4 + 1280*b^5
*x^5) + 210*a^6*ArcTanh[(Sqrt[b]*Sqrt[x])/(-Sqrt[a] + Sqrt[a + b*x])]))/(7
680*b^(9/2)*Sqrt[x^2*(a + b*x)])
```

Rubi [A] (verified)

Time = 0.92 (sec) , antiderivative size = 280, normalized size of antiderivative = 1.13, number of steps used = 10, number of rules used = 9, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.391, Rules used = {1927, 1927, 1930, 1930, 1930, 1930, 1937, 1935, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sqrt{cx}\left(ax^2+bx^3\right)^{3/2}\,dx$$

$$\Big\downarrow \text{1927}$$

$$\frac{a\int (cx)^{5/2}\sqrt{bx^3+ax^2}dx}{4c^2}+\frac{(cx)^{3/2}\left(ax^2+bx^3\right)^{3/2}}{6c}$$

$$\Big\downarrow \text{1927}$$

$$\frac{a\left(\frac{a\int \frac{(cx)^{9/2}}{\sqrt{bx^3+ax^2}}dx}{10c^2}+\frac{(cx)^{7/2}\sqrt{ax^2+bx^3}}{5c}\right)}{4c^2}+\frac{(cx)^{3/2}\left(ax^2+bx^3\right)^{3/2}}{6c}$$

$$\Big\downarrow \text{1930}$$

$$\frac{a\left(\frac{a\left(\frac{c^2(cx)^{5/2}\sqrt{ax^2+bx^3}}{4b}-\frac{7ac\int\frac{(cx)^{7/2}}{\sqrt{bx^3+ax^2}}dx}{8b}\right)}{10c^2}+\frac{(cx)^{7/2}\sqrt{ax^2+bx^3}}{5c}\right)}{4c^2}+\frac{(cx)^{3/2}\left(ax^2+bx^3\right)^{3/2}}{6c}$$

$$\Big\downarrow \text{1930}$$

$$\frac{a\left(\frac{a\left(\frac{c^2(cx)^{5/2}\sqrt{ax^2+bx^3}}{4b}-\frac{7ac\left(\frac{c^2(cx)^{3/2}\sqrt{ax^2+bx^3}}{3b}-\frac{5ac\int\frac{(cx)^{5/2}}{\sqrt{bx^3+ax^2}}dx}{6b}\right)}{8b}\right)}{10c^2}+\frac{(cx)^{7/2}\sqrt{ax^2+bx^3}}{5c}\right)}{4c^2}+\frac{(cx)^{3/2}\left(ax^2+bx^3\right)^{3/2}}{6c}$$

$$\Big\downarrow \text{1930}$$

$$\frac{a\left(\frac{a\left(\frac{c^2(cx)^{5/2}\sqrt{ax^2+bx^3}}{4b}-\frac{7ac\left(\frac{c^2(cx)^{3/2}\sqrt{ax^2+bx^3}}{3b}-\frac{5ac\left(\frac{c^2\sqrt{cx}\sqrt{ax^2+bx^3}}{2b}-\frac{3ac\int\frac{(cx)^{3/2}}{\sqrt{bx^3+ax^2}}dx}{4b}\right)}{6b}\right)}{8b}\right)}{10c^2}+\frac{(cx)^{7/2}\sqrt{ax^2+bx^3}}{5c}\right)}{4c^2}+\frac{(cx)^{3/2}\left(ax^2+bx^3\right)^{3/2}}{6c}$$

$$\Big\downarrow \text{ 1930}$$

$$\frac{a\left(\frac{a\left(\frac{c^2(cx)^{5/2}\sqrt{ax^2+bx^3}}{4b}-\frac{7ac\left(\frac{c^2(cx)^{3/2}\sqrt{ax^2+bx^3}}{3b}-\frac{5ac\left(\frac{c^2\sqrt{cx}\sqrt{ax^2+bx^3}}{2b}-\frac{3ac\left(\frac{c^2\sqrt{ax^2+bx^3}}{b\sqrt{cx}}-\frac{ac\int\frac{\sqrt{cx}}{\sqrt{bx^3+ax^2}}dx}{2b}\right)}{4b}\right)}{6b}\right)}{8b}\right)}{10c^2}+\frac{(cx)^{7/2}\sqrt{ax^2+b}}{5c}\right.}{4c^2}$$

$$\frac{(cx)^{3/2}\left(ax^2+bx^3\right)^{3/2}}{6c}$$

$$\Big\downarrow \; 1937$$

$$\frac{a\left(\frac{a\left(\frac{c^2(cx)^{5/2}\sqrt{ax^2+bx^3}}{4b}-\frac{7ac\left(\frac{c^2(cx)^{3/2}\sqrt{ax^2+bx^3}}{3b}-\frac{5ac\left(\frac{c^2\sqrt{cx}\sqrt{ax^2+bx^3}}{2b}-\frac{3ac\left(\frac{c^2\sqrt{ax^2+bx^3}}{b\sqrt{cx}}-\frac{ac\sqrt{cx}\int\frac{\sqrt{x}}{\sqrt{bx^3+ax^2}}dx}{2b\sqrt{x}}\right)}{4b}\right)}{6b}\right)}{8b}\right)}{10c^2}+\frac{(cx)^{7/2}\sqrt{ax}}{5c}}{4c^2}$$

$$\frac{(cx)^{3/2}\left(ax^2+bx^3\right)^{3/2}}{6c}$$

$$\Big\downarrow \ 1935$$

$$a\left(\frac{a\left(\frac{c^2(cx)^{5/2}\sqrt{ax^2+bx^3}}{4b}-\frac{7ac\left(\frac{c^2(cx)^{3/2}\sqrt{ax^2+bx^3}}{3b}-\frac{5ac\left(\frac{c^2\sqrt{cx}\sqrt{ax^2+bx^3}}{2b}-\frac{3ac\left(\frac{c^2\sqrt{ax^2+bx^3}}{b\sqrt{cx}}-\frac{ac\sqrt{cx}\int\frac{1}{1-\frac{bx^3}{bx^3+ax^2}}d\frac{x^{3/2}}{\sqrt{bx^3+ax^2}}}{b\sqrt{x}}\right)}{4b}\right)}{6b}\right)}{8b}\right)}{10c^2}+\right.$$

$\downarrow$ 219

$$\frac{a\left(\frac{a\left(\frac{c^2(cx)^{5/2}\sqrt{ax^2+bx^3}}{4b}-\frac{7ac\left(\frac{c^2(cx)^{3/2}\sqrt{ax^2+bx^3}}{3b}-\frac{5ac\left(\frac{c^2\sqrt{cx}\sqrt{ax^2+bx^3}}{2b}-\frac{3ac\left(\frac{c^2\sqrt{ax^2+bx^3}}{b\sqrt{cx}}-\frac{ac\sqrt{cx}\operatorname{arctanh}\left(\frac{\sqrt{b}x^{3/2}}{\sqrt{ax^2+bx^3}}\right)}{b^{3/2}\sqrt{x}}\right)}{4b}\right)}{6b}\right)}{8b}\right)}{10c^2}+\frac{(cx\cdots}{\cdots}\right)}{4c^2}$$

$$\frac{(cx)^{3/2}\left(ax^2+bx^3\right)^{3/2}}{6c}$$

```
Int[Sqrt[c*x]*(a*x^2 + b*x^3)^(3/2),x]
```

```
((c*x)^(3/2)*(a*x^2 + b*x^3)^(3/2))/(6*c) + (a*(((c*x)^(7/2)*Sqrt[a*x^2 +
b*x^3])/(5*c) + (a*((c^2*(c*x)^(5/2)*Sqrt[a*x^2 + b*x^3])/(4*b) - (7*a*c*(
(c^2*(c*x)^(3/2)*Sqrt[a*x^2 + b*x^3])/(3*b) - (5*a*c*((c^2*Sqrt[c*x]*Sqrt[
a*x^2 + b*x^3])/(2*b) - (3*a*c*((c^2*Sqrt[a*x^2 + b*x^3])/(b*Sqrt[c*x]) -
(a*c*Sqrt[c*x]*ArcTanh[(Sqrt[b]*x^(3/2))/Sqrt[a*x^2 + b*x^3]])/(b^(3/2)*Sq
rt[x])))/(4*b)))/(6*b)))/(8*b)))/(10*c^2)))/(4*c^2)
```

Defintions of rubi rules used

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> Simp[(c*x)^(m + 1)*((a*x^j + b*x^n)^p/(c*(m + n*p + 1))), x] + Simp[a*
(n - j)*(p/(c^j*(m + n*p + 1)))   Int[(c*x)^(m + j)*(a*x^j + b*x^n)^(p - 1)
, x], x] /; FreeQ[{a, b, c, m}, x] &&  !IntegerQ[p] && LtQ[0, j, n] && (Int
egersQ[j, n] || GtQ[c, 0]) && GtQ[p, 0] && NeQ[m + n*p + 1, 0]
```

```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> Simp[c^(n - 1)*(c*x)^(m - n + 1)*((a*x^j + b*x^n)^(p + 1)/(b*(m + n*p
+ 1))), x] - Simp[a*c^(n - j)*((m + j*p - n + j + 1)/(b*(m + n*p + 1)))   I
nt[(c*x)^(m - (n - j))*(a*x^j + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, p},
x] &&  !IntegerQ[p] && LtQ[0, j, n] && (IntegersQ[j, n] || GtQ[c, 0]) && Gt
Q[m + j*p - n + j + 1, 0] && NeQ[m + n*p + 1, 0]
```

```
Int[(x_)^(m_.)/Sqrt[(a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.)], x_Symbol] :> Simp
[-2/(n - j)   Subst[Int[1/(1 - a*x^2), x], x, x^(j/2)/Sqrt[a*x^j + b*x^n]],
 x] /; FreeQ[{a, b, j, n}, x] && EqQ[m, j/2 - 1] && NeQ[n, j]
```

```
Int[((c_)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_))^(p_), x_Symbol]
:> Simp[c^IntPart[m]*((c*x)^FracPart[m]/x^FracPart[m])   Int[x^m*(a*x^j + b
*x^n)^p, x], x] /; FreeQ[{a, b, c, j, m, n, p}, x] && IntegerQ[p + 1/2] &&
NeQ[n, j] && EqQ[Simplify[m + j*p + 1], 0]
```

Maple [A] (verified)

Time = 0.35 (sec) , antiderivative size = 156, normalized size of antiderivative = 0.63

method	result
risch	$-\frac{\left(-1280 b^{5} x^{5}-1664 a\, b^{4} x^{4}-48 a^{2} b^{3} x^{3}+56 a^{3} b^{2} x^{2}-70 a^{4} b x+105 a^{5}\right) c \sqrt{x^{2} \left(b x+a\right)}}{7680 b^{4} \sqrt{c x}}+\frac{7 a^{6} \ln \left(\frac{\frac{1}{2} a c+c b x}{\sqrt{b c}}+\sqrt{b c\, x^{2}+a c x}\right) c \sqrt{x^{2} \left(b x+a\right.}}{1024 b^{4} \sqrt{b c}\, x \left(b x+a\right) \sqrt{c x}}$
default	$-\frac{\left(b\, x^{3}+a\, x^{2}\right)^{\frac{3}{2}} \sqrt{c x}\, \left(-2560 b^{5} x^{5} \sqrt{c x \left(b x+a\right)}\, \sqrt{b c}-3328 a\, b^{4} x^{4} \sqrt{c x \left(b x+a\right)}\, \sqrt{b c}-96 a^{2} b^{3} x^{3} \sqrt{c x \left(b x+a\right)}\, \sqrt{b c}+112 a^{3} b^{2} x^{2} \sqrt{c x \left(b x+c\right.}\right.}{15360 x^{3} \left(b x+a\right) b^{4} \sqrt{c x \left(b x+a\right)}\, \sqrt{}}$

```
int((c*x)^(1/2)*(b*x^3+a*x^2)^(3/2),x,method=_RETURNVERBOSE)
```

```
-1/7680*(-1280*b^5*x^5-1664*a*b^4*x^4-48*a^2*b^3*x^3+56*a^3*b^2*x^2-70*a^4
*b*x+105*a^5)/b^4*c*(x^2*(b*x+a))^(1/2)/(c*x)^(1/2)+7/1024*a^6/b^4*ln((1/2
*a*c+c*b*x)/(b*c)^(1/2)+(b*c*x^2+a*c*x)^(1/2))/(b*c)^(1/2)*c*(x^2*(b*x+a))
^(1/2)/x/(b*x+a)*(c*x*(b*x+a))^(1/2)/(c*x)^(1/2)
```

Fricas [A] (verification not implemented)

Time = 0.10 (sec) , antiderivative size = 277, normalized size of antiderivative = 1.12

$$\int \sqrt{cx}\left(ax^2+bx^3\right)^{3/2}\,dx = \left[\frac{105\,a^6 x\sqrt{\frac{c}{b}}\log\left(\frac{2\,bcx^2+acx+2\,\sqrt{bx^3+ax^2}\sqrt{cx}b\sqrt{\frac{c}{b}}}{x}\right)+2\left(1280\,b^5x^5+1664\,ab^4x^4+48\,a^2b^3x^3-56\right.}{15360\,b^4x}\right.$$

$$-\frac{105\,a^6 x\sqrt{-\frac{c}{b}}\arctan\left(\frac{\sqrt{bx^3+ax^2}\sqrt{cx}b\sqrt{-\frac{c}{b}}}{bcx^2+acx}\right)-\left(1280\,b^5x^5+1664\,ab^4x^4+48\,a^2b^3x^3-56\,a^3b^2x^2+70\,a^4bx-\right.}{7680\,b^4x}$$

```
integrate((c*x)^(1/2)*(b*x^3+a*x^2)^(3/2),x, algorithm="fricas")
```

```
[1/15360*(105*a^6*x*sqrt(c/b)*log((2*b*c*x^2 + a*c*x + 2*sqrt(b*x^3 + a*x^
2)*sqrt(c*x)*b*sqrt(c/b))/x) + 2*(1280*b^5*x^5 + 1664*a*b^4*x^4 + 48*a^2*b
^3*x^3 - 56*a^3*b^2*x^2 + 70*a^4*b*x - 105*a^5)*sqrt(b*x^3 + a*x^2)*sqrt(c
*x))/(b^4*x), -1/7680*(105*a^6*x*sqrt(-c/b)*arctan(sqrt(b*x^3 + a*x^2)*sqr
t(c*x)*b*sqrt(-c/b)/(b*c*x^2 + a*c*x)) - (1280*b^5*x^5 + 1664*a*b^4*x^4 +
48*a^2*b^3*x^3 - 56*a^3*b^2*x^2 + 70*a^4*b*x - 105*a^5)*sqrt(b*x^3 + a*x^2
)*sqrt(c*x))/(b^4*x)]
```

Sympy [F]

$$\int \sqrt{cx}\left(ax^2+bx^3\right)^{3/2}\,dx=\int \sqrt{cx}\left(x^2(a+bx)\right)^{\frac{3}{2}}\,dx$$

```
integrate((c*x)**(1/2)*(b*x**3+a*x**2)**(3/2),x)
```

```
Integral(sqrt(c*x)*(x**2*(a + b*x))**(3/2), x)
```

Maxima [F]

$$\int \sqrt{cx}\left(ax^2+bx^3\right)^{3/2}\,dx=\int \left(bx^3+ax^2\right)^{\frac{3}{2}}\sqrt{cx}\,dx$$

```
integrate((c*x)^(1/2)*(b*x^3+a*x^2)^(3/2),x, algorithm="maxima")
```

```
integrate((b*x^3 + a*x^2)^(3/2)*sqrt(c*x), x)
```

Giac [A] (verification not implemented)

Time = 0.19 (sec) , antiderivative size = 312, normalized size of antiderivative = 1.26

$$\int \sqrt{cx}\left(ax^2+bx^3\right)^{3/2}\,dx=\frac{7\,a^6|c|\log\left(c^2|a|\right)\mathrm{sgn}(x)}{1024\,\sqrt{bc}b^4}$$
$$+\frac{\left(\frac{315\,a^6c^2\log\left(\left|-\sqrt{bc}\sqrt{cx}+\sqrt{bc^2x+ac^2}\right|\right)}{\sqrt{bc}b^5}+\sqrt{bc^2x+ac^2}\left(2\left(4\left(2\left(8\,cx\left(\frac{10\,x}{c^4}+\frac{a}{bc^4}\right)-\frac{9\,a^2}{b^2c^3}\right)cx+\frac{21\,a^3}{b^3c^2}\right)cx-\frac{105\,a^4}{b^4c}\right)c\right.\right.}{7680\,c^2}$$
$$-\frac{\left(\frac{105\,a^5c^6\log\left(\left|-\sqrt{bc}\sqrt{cx}+\sqrt{bc^2x+ac^2}\right|\right)}{\sqrt{bc}b^4}+\left(\frac{105\,a^4c^4}{b^4}-2\left(\frac{35\,a^3c^3}{b^3}+4\left(6\left(8\,cx+\frac{ac}{b}\right)cx-\frac{7\,a^2c^2}{b^2}\right)cx\right)cx\right)\sqrt{bc^2x+ac^2}\right.}{1920\,c^6}$$

```
integrate((c*x)^(1/2)*(b*x^3+a*x^2)^(3/2),x, algorithm="giac")
```

```
7/1024*a^6*abs(c)*log(c^2*abs(a))*sgn(x)/(sqrt(b*c)*b^4) + 1/7680*(315*a^6
*c^2*log(abs(-sqrt(b*c)*sqrt(c*x) + sqrt(b*c^2*x + a*c^2)))/(sqrt(b*c)*b^5
) + sqrt(b*c^2*x + a*c^2)*(2*(4*(2*(8*c*x*(10*x/c^4 + a/(b*c^4)) - 9*a^2/(
b^2*c^3))*c*x + 21*a^3/(b^3*c^2))*c*x - 105*a^4/(b^4*c))*c*x + 315*a^5/b^5
)*sqrt(c*x))*b*abs(c)*sgn(x)/c^2 - 1/1920*(105*a^5*c^6*log(abs(-sqrt(b*c)*
sqrt(c*x) + sqrt(b*c^2*x + a*c^2)))/(sqrt(b*c)*b^4) + (105*a^4*c^4/b^4 - 2
*(35*a^3*c^3/b^3 + 4*(6*(8*c*x + a*c/b)*c*x - 7*a^2*c^2/b^2)*c*x)*c*x)*sqr
t(b*c^2*x + a*c^2)*sqrt(c*x))*a*abs(c)*sgn(x)/c^6
```

Mupad [F(-1)]

Timed out.

$$\int \sqrt{cx}\left(ax^2+bx^3\right)^{3/2}\,dx=\int \sqrt{c\,x}\,\left(b\,x^3+a\,x^2\right)^{3/2}\,dx$$

```
int((c*x)^(1/2)*(a*x^2 + b*x^3)^(3/2),x)
```

```
int((c*x)^(1/2)*(a*x^2 + b*x^3)^(3/2), x)
```

Reduce [B] (verification not implemented)

Time = 0.32 (sec) , antiderivative size = 135, normalized size of antiderivative = 0.55

$$\int \sqrt{cx}\left(ax^2+bx^3\right)^{3/2}\,dx = \frac{\sqrt{c}\left(-105\sqrt{x}\,\sqrt{bx+a}\,a^5b+70\sqrt{x}\,\sqrt{bx+a}\,a^4b^2x-56\sqrt{x}\,\sqrt{bx+a}\,a^3b^3x^2+48\sqrt{x}\,\sqrt{bx+a}\right.}{7680}$$

```
int((c*x)^(1/2)*(b*x^3+a*x^2)^(3/2),x)
```

```
(sqrt(c)*( - 105*sqrt(x)*sqrt(a + b*x)*a**5*b + 70*sqrt(x)*sqrt(a + b*x)*a
**4*b**2*x - 56*sqrt(x)*sqrt(a + b*x)*a**3*b**3*x**2 + 48*sqrt(x)*sqrt(a +
 b*x)*a**2*b**4*x**3 + 1664*sqrt(x)*sqrt(a + b*x)*a*b**5*x**4 + 1280*sqrt(
x)*sqrt(a + b*x)*b**6*x**5 + 105*sqrt(b)*log((sqrt(a + b*x) + sqrt(x)*sqrt
(b))/sqrt(a))*a**6))/(7680*b**5)
```

3.337 $\int \frac{(ax^2+bx^3)^{3/2}}{\sqrt{cx}}\,dx$

Optimal result

Integrand size = 23, antiderivative size = 214

$$\int \frac{(ax^2+bx^3)^{3/2}}{\sqrt{cx}}\,dx = \frac{3a^4\sqrt{ax^2+bx^3}}{128b^3\sqrt{cx}} - \frac{a^3\sqrt{cx}\sqrt{ax^2+bx^3}}{64b^2c} + \frac{a^2(cx)^{3/2}\sqrt{ax^2+bx^3}}{80bc^2} + \frac{11a(cx)^{5/2}\sqrt{ax^2+bx^3}}{40c^3} + \frac{b(cx)^{7/2}\sqrt{ax^2+bx^3}}{5c^4} - \frac{3a^5\operatorname{arctanh}\left(\frac{\sqrt{b}(cx)^{3/2}}{c^{3/2}\sqrt{ax^2+bx^3}}\right)}{128b^{7/2}\sqrt{c}}$$

```
3/128*a^4*(b*x^3+a*x^2)^(1/2)/b^3/(c*x)^(1/2)-1/64*a^3*(c*x)^(1/2)*(b*x^3+
a*x^2)^(1/2)/b^2/c+1/80*a^2*(c*x)^(3/2)*(b*x^3+a*x^2)^(1/2)/b/c^2+11/40*a*
(c*x)^(5/2)*(b*x^3+a*x^2)^(1/2)/c^3+1/5*b*(c*x)^(7/2)*(b*x^3+a*x^2)^(1/2)/
c^4-3/128*a^5*arctanh(b^(1/2)*(c*x)^(3/2)/c^(3/2)/(b*x^3+a*x^2)^(1/2))/b^(
7/2)/c^(1/2)
```

Mathematica [A] (verified)

Time = 0.09 (sec) , antiderivative size = 141, normalized size of antiderivative = 0.66

$$\int \frac{(ax^2+bx^3)^{3/2}}{\sqrt{cx}}\,dx = \frac{x^{3/2}\sqrt{a+bx}\left(\sqrt{b}\sqrt{x}\sqrt{a+bx}(15a^4-10a^3bx+8a^2b^2x^2+176ab^3x^3+128b^4x^4)+30\right.}{640b^{7/2}\sqrt{cx}\sqrt{x^2(a+bx)}}$$

```
Integrate[(a*x^2 + b*x^3)^(3/2)/Sqrt[c*x],x]
```

```
(x^(3/2)*Sqrt[a + b*x]*(Sqrt[b]*Sqrt[x]*Sqrt[a + b*x]*(15*a^4 - 10*a^3*b*x
 + 8*a^2*b^2*x^2 + 176*a*b^3*x^3 + 128*b^4*x^4) + 30*a^5*ArcTanh[(Sqrt[b]*
Sqrt[x])/(Sqrt[a] - Sqrt[a + b*x])]))/(640*b^(7/2)*Sqrt[c*x]*Sqrt[x^2*(a +
 b*x)])
```

Rubi [A] (verified)

Time = 0.80 (sec) , antiderivative size = 238, normalized size of antiderivative = 1.11, number of steps used = 9, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}} = 0.348$, Rules used = {1927, 1927, 1930, 1930, 1930, 1937, 1935, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(ax^2+bx^3)^{3/2}}{\sqrt{cx}}\,dx$$

$$\Big\downarrow \text{ 1927}$$

$$\frac{3a\int (cx)^{3/2}\sqrt{bx^3+ax^2}dx}{10c^2}+\frac{\sqrt{cx}(ax^2+bx^3)^{3/2}}{5c}$$

$$\Big\downarrow \text{ 1927}$$

$$\frac{3a\left(\frac{a\int \frac{(cx)^{7/2}}{\sqrt{bx^3+ax^2}}dx}{8c^2}+\frac{(cx)^{5/2}\sqrt{ax^2+bx^3}}{4c}\right)}{10c^2}+\frac{\sqrt{cx}(ax^2+bx^3)^{3/2}}{5c}$$

$$\Big\downarrow \text{ 1930}$$

$$\frac{3a\left(\frac{a\left(\frac{c^2(cx)^{3/2}\sqrt{ax^2+bx^3}}{3b}-\frac{5ac\int\frac{(cx)^{5/2}}{\sqrt{bx^3+ax^2}}dx}{6b}\right)}{8c^2}+\frac{(cx)^{5/2}\sqrt{ax^2+bx^3}}{4c}\right)}{10c^2}+\frac{\sqrt{cx}\left(ax^2+bx^3\right)^{3/2}}{5c}$$

$$\Big\downarrow \; 1930$$

$$\frac{3a\left(\frac{a\left(\frac{c^2(cx)^{3/2}\sqrt{ax^2+bx^3}}{3b}-\frac{5ac\left(\frac{c^2\sqrt{cx}\sqrt{ax^2+bx^3}}{2b}-\frac{3ac\int\frac{(cx)^{3/2}}{\sqrt{bx^3+ax^2}}dx}{4b}\right)}{6b}\right)}{8c^2}+\frac{(cx)^{5/2}\sqrt{ax^2+bx^3}}{4c}\right)}{10c^2}+\frac{\sqrt{cx}\left(ax^2+bx^3\right)^{3/2}}{5c}$$

$$\Big\downarrow \; 1930$$

$$\frac{3a\left(\frac{a\left(\frac{c^2(cx)^{3/2}\sqrt{ax^2+bx^3}}{3b}-\frac{5ac\left(\frac{c^2\sqrt{cx}\sqrt{ax^2+bx^3}}{2b}-\frac{3ac\left(\frac{c^2\sqrt{ax^2+bx^3}}{b\sqrt{cx}}-\frac{ac\int\frac{\sqrt{cx}}{\sqrt{bx^3+ax^2}}dx}{2b}\right)}{4b}\right)}{6b}\right)}{8c^2}+\frac{(cx)^{5/2}\sqrt{ax^2+bx^3}}{4c}\right)}{10c^2}+\frac{\sqrt{cx}\left(ax^2+bx^3\right)^{3/2}}{5c}$$

$$\Big\downarrow \text{1937}$$

$$\frac{3a\left(\frac{a\left(\frac{c^2(cx)^{3/2}\sqrt{ax^2+bx^3}}{3b}-\frac{5ac\left(\frac{c^2\sqrt{cx}\sqrt{ax^2+bx^3}}{2b}-\frac{3ac\left(\frac{c^2\sqrt{ax^2+bx^3}}{b\sqrt{cx}}-\frac{ac\sqrt{cx}\int\frac{\sqrt{x}}{\sqrt{bx^3+ax^2}}dx}{2b\sqrt{x}}\right)}{4b}\right)}{6b}\right)}{8c^2}+\frac{(cx)^{5/2}\sqrt{ax^2+bx^3}}{4c}\right)}{10c^2}+\frac{\sqrt{cx}\left(ax^2+bx^3\right)^{3/2}}{5c}$$

$$\Big\downarrow \text{1935}$$

$$\frac{3a\left(\frac{a\left(\frac{c^2(cx)^{3/2}\sqrt{ax^2+bx^3}}{3b}-\frac{5ac\left(\frac{c^2\sqrt{cx}\sqrt{ax^2+bx^3}}{2b}-\frac{3ac\left(\frac{c^2\sqrt{ax^2+bx^3}}{b\sqrt{cx}}-\frac{ac\sqrt{cx}\int\frac{1}{1-\frac{bx^3}{bx^3+ax^2}}d\frac{x^{3/2}}{\sqrt{bx^3+ax^2}}}{b\sqrt{x}}\right)}{4b}\right)}{6b}\right)}{8c^2}+\frac{(cx)^{5/2}\sqrt{ax^2+bx^3}}{4c}\right)}{10c^2}+\frac{\sqrt{cx}\left(ax^2+bx^3\right)^{3/2}}{5c}$$

$$\Big\downarrow \; 219$$

$$\frac{3a\left(\frac{a\left(\frac{c^2(cx)^{3/2}\sqrt{ax^2+bx^3}}{3b}-\frac{5ac\left(\frac{c^2\sqrt{cx}\sqrt{ax^2+bx^3}}{2b}-\frac{3ac\left(\frac{c^2\sqrt{ax^2+bx^3}}{b\sqrt{cx}}-\frac{ac\sqrt{cx}\,\mathrm{arctanh}\left(\frac{\sqrt{b}x^{3/2}}{\sqrt{ax^2+bx^3}}\right)}{b^{3/2}\sqrt{x}}\right)}{4b}\right)}{6b}\right)}{8c^2}+\frac{(cx)^{5/2}\sqrt{ax^2+bx^3}}{4c}\right)}{10c^2}+\frac{\sqrt{cx}\left(ax^2+bx^3\right)^{3/2}}{5c}$$

```
Int[(a*x^2 + b*x^3)^(3/2)/Sqrt[c*x],x]
```

```
(Sqrt[c*x]*(a*x^2 + b*x^3)^(3/2))/(5*c) + (3*a*(((c*x)^(5/2)*Sqrt[a*x^2 +
b*x^3])/(4*c) + (a*((c^2*(c*x)^(3/2)*Sqrt[a*x^2 + b*x^3])/(3*b) - (5*a*c*(
(c^2*Sqrt[c*x]*Sqrt[a*x^2 + b*x^3])/(2*b) - (3*a*c*((c^2*Sqrt[a*x^2 + b*x^
3])/(b*Sqrt[c*x]) - (a*c*Sqrt[c*x]*ArcTanh[(Sqrt[b]*x^(3/2))/Sqrt[a*x^2 +
b*x^3]])/(b^(3/2)*Sqrt[x])))/(4*b)))/(6*b)))/(8*c^2)))/(10*c^2)
```

Defintions of rubi rules used

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> Simp[(c*x)^(m + 1)*((a*x^j + b*x^n)^p/(c*(m + n*p + 1))), x] + Simp[a*
(n - j)*(p/(c^j*(m + n*p + 1)))   Int[(c*x)^(m + j)*(a*x^j + b*x^n)^(p - 1)
, x], x] /; FreeQ[{a, b, c, m}, x] &&  !IntegerQ[p] && LtQ[0, j, n] && (Int
egersQ[j, n] || GtQ[c, 0]) && GtQ[p, 0] && NeQ[m + n*p + 1, 0]
```

```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> Simp[c^(n - 1)*(c*x)^(m - n + 1)*((a*x^j + b*x^n)^(p + 1)/(b*(m + n*p
+ 1))), x] - Simp[a*c^(n - j)*((m + j*p - n + j + 1)/(b*(m + n*p + 1)))   I
nt[(c*x)^(m - (n - j))*(a*x^j + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, p},
x] &&  !IntegerQ[p] && LtQ[0, j, n] && (IntegersQ[j, n] || GtQ[c, 0]) && Gt
Q[m + j*p - n + j + 1, 0] && NeQ[m + n*p + 1, 0]
```

```
Int[(x_)^(m_.)/Sqrt[(a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.)], x_Symbol] :> Simp
[-2/(n - j)   Subst[Int[1/(1 - a*x^2), x], x, x^(j/2)/Sqrt[a*x^j + b*x^n]],
 x] /; FreeQ[{a, b, j, n}, x] && EqQ[m, j/2 - 1] && NeQ[n, j]
```

```
Int[((c_)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_))^(p_), x_Symbol]
:> Simp[c^IntPart[m]*((c*x)^FracPart[m]/x^FracPart[m])   Int[x^m*(a*x^j + b
*x^n)^p, x], x] /; FreeQ[{a, b, c, j, m, n, p}, x] && IntegerQ[p + 1/2] &&
NeQ[n, j] && EqQ[Simplify[m + j*p + 1], 0]
```

Maple [A] (verified)

Time = 0.34 (sec) , antiderivative size = 143, normalized size of antiderivative = 0.67

method	result
risch	$\frac{\left(128b^{4}x^{4}+176a\,b^{3}x^{3}+8a^{2}b^{2}x^{2}-10a^{3}bx+15a^{4}\right)\sqrt{x^{2}\left(bx+a\right)}}{640b^{3}\sqrt{cx}}-\frac{3a^{5}\ln\left(\frac{\frac{1}{2}ac+cbx}{\sqrt{bc}}+\sqrt{bc\,x^{2}+acx}\right)\sqrt{x^{2}\left(bx+a\right)}\,\sqrt{cx\left(bx+a\right)}}{256b^{3}\sqrt{bc}\,x\left(bx+a\right)\sqrt{cx}}$
default	$\frac{\left(b\,x^{3}+a\,x^{2}\right)^{\frac{3}{2}}\left(256b^{4}x^{4}\sqrt{bc}\,\sqrt{cx\left(bx+a\right)}+352a\,b^{3}x^{3}\sqrt{bc}\,\sqrt{cx\left(bx+a\right)}+16a^{2}b^{2}x^{2}\sqrt{bc}\,\sqrt{cx\left(bx+a\right)}-15\ln\left(\frac{2cbx+2\sqrt{cx\left(bx+a\right)}\,\sqrt{bc}+ac}{2\sqrt{bc}}\right.\right.}{1280x^{2}\left(bx+a\right)b^{3}\sqrt{cx}\,\sqrt{cx\left(bx+a\right)}\,\sqrt{bc}}$

```
int((b*x^3+a*x^2)^(3/2)/(c*x)^(1/2),x,method=_RETURNVERBOSE)
```

```
1/640*(128*b^4*x^4+176*a*b^3*x^3+8*a^2*b^2*x^2-10*a^3*b*x+15*a^4)/b^3*(x^2
*(b*x+a))^(1/2)/(c*x)^(1/2)-3/256*a^5/b^3*ln((1/2*a*c+c*b*x)/(b*c)^(1/2)+(
b*c*x^2+a*c*x)^(1/2))/(b*c)^(1/2)*(x^2*(b*x+a))^(1/2)/x/(b*x+a)*(c*x*(b*x+
a))^(1/2)/(c*x)^(1/2)
```

Fricas [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 256, normalized size of antiderivative = 1.20

$$\int \frac{\left(ax^2+bx^3\right)^{3/2}}{\sqrt{cx}}\,dx=\left[\frac{15\,\sqrt{bc}a^5x\log\left(\frac{2\,bcx^2+acx-2\,\sqrt{bx^3+ax^2}\sqrt{bc}\sqrt{cx}}{x}\right)+2\,(128\,b^5x^4+176\,ab^4x^3+8\,a^2b^3x^2-}{1280\,b^4cx}\right.$$

```
integrate((b*x^3+a*x^2)^(3/2)/(c*x)^(1/2),x, algorithm="fricas")
```

```
[1/1280*(15*sqrt(b*c)*a^5*x*log((2*b*c*x^2 + a*c*x - 2*sqrt(b*x^3 + a*x^2)
*sqrt(b*c)*sqrt(c*x))/x) + 2*(128*b^5*x^4 + 176*a*b^4*x^3 + 8*a^2*b^3*x^2
- 10*a^3*b^2*x + 15*a^4*b)*sqrt(b*x^3 + a*x^2)*sqrt(c*x))/(b^4*c*x), 1/640
*(15*sqrt(-b*c)*a^5*x*arctan(sqrt(b*x^3 + a*x^2)*sqrt(-b*c)*sqrt(c*x)/(b*c
*x^2 + a*c*x)) + (128*b^5*x^4 + 176*a*b^4*x^3 + 8*a^2*b^3*x^2 - 10*a^3*b^2
*x + 15*a^4*b)*sqrt(b*x^3 + a*x^2)*sqrt(c*x))/(b^4*c*x)]
```

Sympy [F]

$$\int \frac{\left(ax^2+bx^3\right)^{3/2}}{\sqrt{cx}}\,dx = \int \frac{\left(x^2(a+bx)\right)^{\frac{3}{2}}}{\sqrt{cx}}\,dx$$

```
integrate((b*x**3+a*x**2)**(3/2)/(c*x)**(1/2),x)
```

```
Integral((x**2*(a + b*x))**(3/2)/sqrt(c*x), x)
```

Maxima [F]

$$\int \frac{\left(ax^2+bx^3\right)^{3/2}}{\sqrt{cx}}\,dx = \int \frac{\left(bx^3+ax^2\right)^{\frac{3}{2}}}{\sqrt{cx}}\,dx$$

```
integrate((b*x^3+a*x^2)^(3/2)/(c*x)^(1/2),x, algorithm="maxima")
```

```
integrate((b*x^3 + a*x^2)^(3/2)/sqrt(c*x), x)
```

Giac [A] (verification not implemented)

Time = 0.17 (sec) , antiderivative size = 200, normalized size of antiderivative = 0.93

$$\int \frac{\left(ax^2+bx^3\right)^{3/2}}{\sqrt{cx}}\,dx = -\frac{3\,a^5\log\left(\sqrt{bc}\sqrt{a}\right)\operatorname{sgn}(x)}{128\,\sqrt{bc}b^2|b|} + \frac{\left(\frac{15\,a^5\log\left(\left|-\sqrt{bc}\sqrt{bx+a}+\sqrt{(bx+a)bc-abc}\right|\right)\operatorname{sgn}(x)}{\sqrt{bc}b^3} + \sqrt{(bx+a)bc-abc}\left(2\left(4\,(bx+a)\left(2\,(bx+a)\left(\frac{8\,(bx+a)\operatorname{sgn}(x)}{b^4c} - \frac{2}{}\right.\right.\right.\right.\right.}{640\,|b|}$$

```
integrate((b*x^3+a*x^2)^(3/2)/(c*x)^(1/2),x, algorithm="giac")
```

```
-3/128*a^5*log(sqrt(b*c)*sqrt(a))*sgn(x)/(sqrt(b*c)*b^2*abs(b)) + 1/640*(1
5*a^5*log(abs(-sqrt(b*c)*sqrt(b*x + a) + sqrt((b*x + a)*b*c - a*b*c)))*sgn
(x)/(sqrt(b*c)*b^3) + sqrt((b*x + a)*b*c - a*b*c)*(2*(4*(b*x + a)*(2*(b*x
+ a)*(8*(b*x + a)*sgn(x)/(b^4*c) - 21*a*sgn(x)/(b^4*c)) + 31*a^2*sgn(x)/(b
^4*c)) - 5*a^3*sgn(x)/(b^4*c))*(b*x + a) - 15*a^4*sgn(x)/(b^4*c))*sqrt(b*x
 + a))*b/abs(b)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(ax^2+bx^3)^{3/2}}{\sqrt{cx}}\,dx=\int \frac{(b\,x^3+a\,x^2)^{3/2}}{\sqrt{c\,x}}\,dx$$

```
int((a*x^2 + b*x^3)^(3/2)/(c*x)^(1/2),x)
```

```
int((a*x^2 + b*x^3)^(3/2)/(c*x)^(1/2), x)
```

Reduce [B] (verification not implemented)

Time = 0.23 (sec) , antiderivative size = 119, normalized size of antiderivative = 0.56

$$\int \frac{(ax^2+bx^3)^{3/2}}{\sqrt{cx}}\,dx=\frac{\sqrt{c}\left(15\sqrt{x}\,\sqrt{bx+a}\,a^4b-10\sqrt{x}\,\sqrt{bx+a}\,a^3b^2x+8\sqrt{x}\,\sqrt{bx+a}\,a^2b^3x^2+176\sqrt{x}\,\sqrt{b}\right.}{640b^4c}$$

```
int((b*x^3+a*x^2)^(3/2)/(c*x)^(1/2),x)
```

```
(sqrt(c)*(15*sqrt(x)*sqrt(a + b*x)*a**4*b - 10*sqrt(x)*sqrt(a + b*x)*a**3*
b**2*x + 8*sqrt(x)*sqrt(a + b*x)*a**2*b**3*x**2 + 176*sqrt(x)*sqrt(a + b*x
)*a*b**4*x**3 + 128*sqrt(x)*sqrt(a + b*x)*b**5*x**4 - 15*sqrt(b)*log((sqrt
(a + b*x) + sqrt(x)*sqrt(b))/sqrt(a))*a**5))/(640*b**4*c)
```

3.338 $\int \frac{(ax^2+bx^3)^{3/2}}{(cx)^{3/2}}\,dx$

Optimal result

Integrand size = 23, antiderivative size = 182

$$\int \frac{(ax^2+bx^3)^{3/2}}{(cx)^{3/2}}\,dx = -\frac{3a^3\sqrt{ax^2+bx^3}}{64b^2c\sqrt{cx}}+\frac{a^2\sqrt{cx}\sqrt{ax^2+bx^3}}{32bc^2} + \frac{3a(cx)^{3/2}\sqrt{ax^2+bx^3}}{8c^3}+\frac{b(cx)^{5/2}\sqrt{ax^2+bx^3}}{4c^4}+\frac{3a^4\operatorname{arctanh}\left(\frac{\sqrt{b}(cx)^{3/2}}{c^{3/2}\sqrt{ax^2+bx^3}}\right)}{64b^{5/2}c^{3/2}}$$

```
-3/64*a^3*(b*x^3+a*x^2)^(1/2)/b^2/c/(c*x)^(1/2)+1/32*a^2*(c*x)^(1/2)*(b*x^
3+a*x^2)^(1/2)/b/c^2+3/8*a*(c*x)^(3/2)*(b*x^3+a*x^2)^(1/2)/c^3+1/4*b*(c*x)
^(5/2)*(b*x^3+a*x^2)^(1/2)/c^4+3/64*a^4*arctanh(b^(1/2)*(c*x)^(3/2)/c^(3/2
)/(b*x^3+a*x^2)^(1/2))/b^(5/2)/c^(3/2)
```

Mathematica [A] (verified)

Time = 0.05 (sec) , antiderivative size = 130, normalized size of antiderivative = 0.71

$$\int \frac{(ax^2+bx^3)^{3/2}}{(cx)^{3/2}}\,dx = \frac{\sqrt{x}\sqrt{x^2(a+bx)}\left(\sqrt{b}\sqrt{x}\sqrt{a+bx}(-3a^3+2a^2bx+24ab^2x^2+16b^3x^3)+6a^4\operatorname{arctanh}\right.}{64b^{5/2}(cx)^{3/2}\sqrt{a+bx}}$$

```
Integrate[(a*x^2 + b*x^3)^(3/2)/(c*x)^(3/2),x]
```

```
(Sqrt[x]*Sqrt[x^2*(a + b*x)]*(Sqrt[b]*Sqrt[x]*Sqrt[a + b*x]*(-3*a^3 + 2*a^
2*b*x + 24*a*b^2*x^2 + 16*b^3*x^3) + 6*a^4*ArcTanh[(Sqrt[b]*Sqrt[x])/(-Sqr
t[a] + Sqrt[a + b*x])]))/(64*b^(5/2)*(c*x)^(3/2)*Sqrt[a + b*x])
```

Rubi [A] (verified)

Time = 0.68 (sec) , antiderivative size = 196, normalized size of antiderivative = 1.08, number of steps used = 8, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}} = 0.304$, Rules used = {1927, 1927, 1930, 1930, 1937, 1935, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\left(ax^2+bx^3\right)^{3/2}}{(cx)^{3/2}}\,dx$$

$$\Big\downarrow \ 1927$$

$$\frac{3a\int \sqrt{cx}\sqrt{bx^3+ax^2}dx}{8c^2}+\frac{\left(ax^2+bx^3\right)^{3/2}}{4c\sqrt{cx}}$$

$$\Big\downarrow \ 1927$$

$$\frac{3a\left(\frac{a\int \frac{(cx)^{5/2}}{\sqrt{bx^3+ax^2}}dx}{6c^2}+\frac{(cx)^{3/2}\sqrt{ax^2+bx^3}}{3c}\right)}{8c^2}+\frac{\left(ax^2+bx^3\right)^{3/2}}{4c\sqrt{cx}}$$

$$\Big\downarrow \ 1930$$

$$\frac{3a\left(\frac{a\left(\frac{c^2\sqrt{cx}\sqrt{ax^2+bx^3}}{2b}-\frac{3ac\int \frac{(cx)^{3/2}}{\sqrt{bx^3+ax^2}}dx}{4b}\right)}{6c^2}+\frac{(cx)^{3/2}\sqrt{ax^2+bx^3}}{3c}\right)}{8c^2}+\frac{\left(ax^2+bx^3\right)^{3/2}}{4c\sqrt{cx}}$$

$$\Big\downarrow \ 1930$$

$$\frac{3a\left(\frac{a\left(\frac{c^2\sqrt{cx}\sqrt{ax^2+bx^3}}{2b}-\frac{3ac\left(\frac{c^2\sqrt{ax^2+bx^3}}{b\sqrt{cx}}-\frac{ac\int\frac{\sqrt{cx}}{\sqrt{bx^3+ax^2}}dx}{2b}\right)}{4b}\right)}{6c^2}+\frac{(cx)^{3/2}\sqrt{ax^2+bx^3}}{3c}\right)}{8c^2}+\frac{\left(ax^2+bx^3\right)^{3/2}}{4c\sqrt{cx}}$$

$$\Big\downarrow \text{1937}$$

$$\frac{3a\left(\frac{a\left(\frac{c^2\sqrt{cx}\sqrt{ax^2+bx^3}}{2b}-\frac{3ac\left(\frac{c^2\sqrt{ax^2+bx^3}}{b\sqrt{cx}}-\frac{ac\sqrt{cx}\int\frac{\sqrt{x}}{\sqrt{bx^3+ax^2}}dx}{2b\sqrt{x}}\right)}{4b}\right)}{6c^2}+\frac{(cx)^{3/2}\sqrt{ax^2+bx^3}}{3c}\right)}{8c^2}+\frac{\left(ax^2+bx^3\right)^{3/2}}{4c\sqrt{cx}}$$

$$\Big\downarrow \text{1935}$$

$$\frac{3a\left(\frac{a\left(\frac{c^2\sqrt{cx}\sqrt{ax^2+bx^3}}{2b}-\frac{3ac\left(\frac{c^2\sqrt{ax^2+bx^3}}{b\sqrt{cx}}-\frac{ac\sqrt{cx}\int\frac{1}{1-\frac{bx^3}{bx^3+ax^2}}d\frac{x^{3/2}}{\sqrt{bx^3+ax^2}}}{b\sqrt{x}}\right)}{4b}\right)}{6c^2}+\frac{(cx)^{3/2}\sqrt{ax^2+bx^3}}{3c}\right)}{8c^2}+\frac{\left(ax^2+bx^3\right)^{3/2}}{4c\sqrt{cx}}$$

$$\Big\downarrow \text{219}$$

$$\frac{3a\left(\frac{a\left(\frac{c^2\sqrt{cx}\sqrt{ax^2+bx^3}}{2b}-\frac{3ac\left(\frac{c^2\sqrt{ax^2+bx^3}}{b\sqrt{cx}}-\frac{ac\sqrt{cx}\operatorname{arctanh}\left(\frac{\sqrt{b}x^{3/2}}{\sqrt{ax^2+bx^3}}\right)}{b^{3/2}\sqrt{x}}\right)}{4b}\right)}{6c^2}+\frac{(cx)^{3/2}\sqrt{ax^2+bx^3}}{3c}\right)}{8c^2}+\frac{\left(ax^2+bx^3\right)^{3/2}}{4c\sqrt{cx}}$$

```
Int[(a*x^2 + b*x^3)^(3/2)/(c*x)^(3/2),x]
```

```
(a*x^2 + b*x^3)^(3/2)/(4*c*Sqrt[c*x]) + (3*a*(((c*x)^(3/2)*Sqrt[a*x^2 + b*
x^3])/(3*c) + (a*((c^2*Sqrt[c*x]*Sqrt[a*x^2 + b*x^3])/(2*b) - (3*a*c*((c^2
*Sqrt[a*x^2 + b*x^3])/(b*Sqrt[c*x]) - (a*c*Sqrt[c*x]*ArcTanh[(Sqrt[b]*x^(3
/2))/Sqrt[a*x^2 + b*x^3]])/(b^(3/2)*Sqrt[x])))/(4*b)))/(6*c^2)))/(8*c^2)
```

Defintions of rubi rules used

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> Simp[(c*x)^(m + 1)*((a*x^j + b*x^n)^p/(c*(m + n*p + 1))), x] + Simp[a*
(n - j)*(p/(c^j*(m + n*p + 1)))   Int[(c*x)^(m + j)*(a*x^j + b*x^n)^(p - 1)
, x], x] /; FreeQ[{a, b, c, m}, x] &&  !IntegerQ[p] && LtQ[0, j, n] && (Int
egersQ[j, n] || GtQ[c, 0]) && GtQ[p, 0] && NeQ[m + n*p + 1, 0]
```

```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> Simp[c^(n - 1)*(c*x)^(m - n + 1)*((a*x^j + b*x^n)^(p + 1)/(b*(m + n*p
+ 1))), x] - Simp[a*c^(n - j)*((m + j*p - n + j + 1)/(b*(m + n*p + 1)))   I
nt[(c*x)^(m - (n - j))*(a*x^j + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, p},
x] &&  !IntegerQ[p] && LtQ[0, j, n] && (IntegersQ[j, n] || GtQ[c, 0]) && Gt
Q[m + j*p - n + j + 1, 0] && NeQ[m + n*p + 1, 0]
```

```
Int[(x_)^(m_.)/Sqrt[(a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.)], x_Symbol] :> Simp
[-2/(n - j)   Subst[Int[1/(1 - a*x^2), x], x, x^(j/2)/Sqrt[a*x^j + b*x^n]],
 x] /; FreeQ[{a, b, j, n}, x] && EqQ[m, j/2 - 1] && NeQ[n, j]
```

```
Int[((c_)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_))^(p_), x_Symbol]
:> Simp[c^IntPart[m]*((c*x)^FracPart[m]/x^FracPart[m])   Int[x^m*(a*x^j + b
*x^n)^p, x], x] /; FreeQ[{a, b, c, j, m, n, p}, x] && IntegerQ[p + 1/2] &&
NeQ[n, j] && EqQ[Simplify[m + j*p + 1], 0]
```

Maple [A] (verified)

Time = 0.37 (sec) , antiderivative size = 138, normalized size of antiderivative = 0.76

method	result
risch	$-\frac{\left(-16 b^{3} x^{3}-24 a \,b^{2} x^{2}-2 a^{2} b x +3 a^{3}\right) \sqrt{x^{2} \left(b x +a \right)}}{64 b^{2} c \sqrt{c x}}+\frac{3 a^{4} \ln \left(\frac{\frac{1}{2} a c +c b x}{\sqrt{b c}}+\sqrt{b c \,x^{2}+a c x}\right) \sqrt{x^{2} \left(b x +a \right)}\, \sqrt{c x \left(b x +a \right)}}{128 b^{2} \sqrt{b c}\, c x \left(b x +a \right) \sqrt{c x}}$
default	$-\frac{\left(b \,x^{3}+a \,x^{2}\right)^{\frac{3}{2}} \left(-32 b^{3} x^{3} \sqrt{c x \left(b x +a \right)}\, \sqrt{b c}-48 a \,b^{2} x^{2} \sqrt{c x \left(b x +a \right)}\, \sqrt{b c}-3 \ln \left(\frac{2 c b x +2 \sqrt{c x \left(b x +a \right)}\, \sqrt{b c}+a c}{2 \sqrt{b c}}\right) a^{4} c -4 \sqrt{b c}\, \sqrt{c x \left(b x +a \right)}\, a^{2}}{128 x^{2} \left(b x +a \right) c \,b^{2} \sqrt{c x}\, \sqrt{c x \left(b x +a \right)}\, \sqrt{b c}}$

```
int((b*x^3+a*x^2)^(3/2)/(c*x)^(3/2),x,method=_RETURNVERBOSE)
```

```
-1/64*(-16*b^3*x^3-24*a*b^2*x^2-2*a^2*b*x+3*a^3)/b^2/c*(x^2*(b*x+a))^(1/2)
/(c*x)^(1/2)+3/128*a^4/b^2*ln((1/2*a*c+c*b*x)/(b*c)^(1/2)+(b*c*x^2+a*c*x)^
(1/2))/(b*c)^(1/2)/c*(x^2*(b*x+a))^(1/2)/x/(b*x+a)*(c*x*(b*x+a))^(1/2)/(c*
x)^(1/2)
```

Fricas [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 235, normalized size of antiderivative = 1.29

$$\int \frac{(ax^2+bx^3)^{3/2}}{(cx)^{3/2}}\,dx = \left[\frac{3\sqrt{bc}a^4x\log\left(\frac{2\,bcx^2+acx+2\,\sqrt{bx^3+ax^2}\sqrt{bc}\sqrt{cx}}{x}\right)+2\,(16\,b^4x^3+24\,ab^3x^2+2\,a^2b^2x-3\,a^3}{128\,b^3c^2x}\right.$$

```
integrate((b*x^3+a*x^2)^(3/2)/(c*x)^(3/2),x, algorithm="fricas")
```

```
[1/128*(3*sqrt(b*c)*a^4*x*log((2*b*c*x^2 + a*c*x + 2*sqrt(b*x^3 + a*x^2)*s
qrt(b*c)*sqrt(c*x))/x) + 2*(16*b^4*x^3 + 24*a*b^3*x^2 + 2*a^2*b^2*x - 3*a^
3*b)*sqrt(b*x^3 + a*x^2)*sqrt(c*x))/(b^3*c^2*x), -1/64*(3*sqrt(-b*c)*a^4*x
*arctan(sqrt(b*x^3 + a*x^2)*sqrt(-b*c)*sqrt(c*x)/(b*c*x^2 + a*c*x)) - (16*
b^4*x^3 + 24*a*b^3*x^2 + 2*a^2*b^2*x - 3*a^3*b)*sqrt(b*x^3 + a*x^2)*sqrt(c
*x))/(b^3*c^2*x)]
```

Sympy [F]

$$\int \frac{(ax^2+bx^3)^{3/2}}{(cx)^{3/2}}\,dx = \int \frac{(x^2(a+bx))^{\frac{3}{2}}}{(cx)^{\frac{3}{2}}}\,dx$$

```
integrate((b*x**3+a*x**2)**(3/2)/(c*x)**(3/2),x)
```

```
Integral((x**2*(a + b*x))**(3/2)/(c*x)**(3/2), x)
```

Maxima [F]

$$\int \frac{(ax^2+bx^3)^{3/2}}{(cx)^{3/2}}\,dx = \int \frac{(bx^3+ax^2)^{\frac{3}{2}}}{(cx)^{\frac{3}{2}}}\,dx$$

```
integrate((b*x^3+a*x^2)^(3/2)/(c*x)^(3/2),x, algorithm="maxima")
```

```
integrate((b*x^3 + a*x^2)^(3/2)/(c*x)^(3/2), x)
```

Giac [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 184, normalized size of antiderivative = 1.01

$$\int \frac{(ax^2+bx^3)^{3/2}}{(cx)^{3/2}}\,dx = \frac{\frac{3\,a^4\log\left(\sqrt{bc}\sqrt{a}\right)\operatorname{sgn}(x)}{\sqrt{bc}b|b|} - \frac{\Bigg(\frac{3\,a^4\log\left(\left|-\sqrt{bc}\sqrt{bx+a}+\sqrt{(bx+a)bc-abc}\right|\right)\operatorname{sgn}(x)}{\sqrt{bc}b^2} - \sqrt{(bx+a)bc-abc}\Bigg(2\,(bx+a)\Bigg(4\,(bx+a \cdots}{|b|}}{64\,c}$$

```
integrate((b*x^3+a*x^2)^(3/2)/(c*x)^(3/2),x, algorithm="giac")
```

```
1/64*(3*a^4*log(sqrt(b*c)*sqrt(a))*sgn(x)/(sqrt(b*c)*b*abs(b)) - (3*a^4*lo
g(abs(-sqrt(b*c)*sqrt(b*x + a) + sqrt((b*x + a)*b*c - a*b*c)))*sgn(x)/(sqr
t(b*c)*b^2) - sqrt((b*x + a)*b*c - a*b*c)*(2*(b*x + a)*(4*(b*x + a)*(2*(b*
x + a)*sgn(x)/(b^3*c) - 3*a*sgn(x)/(b^3*c)) + a^2*sgn(x)/(b^3*c)) + 3*a^3*
sgn(x)/(b^3*c))*sqrt(b*x + a))*b/abs(b))/c
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(ax^2+bx^3)^{3/2}}{(cx)^{3/2}}\,dx = \int \frac{(b\,x^3+a\,x^2)^{3/2}}{(c\,x)^{3/2}}\,dx$$

```
int((a*x^2 + b*x^3)^(3/2)/(c*x)^(3/2),x)
```

```
int((a*x^2 + b*x^3)^(3/2)/(c*x)^(3/2), x)
```

Reduce [B] (verification not implemented)

Time = 0.42 (sec) , antiderivative size = 100, normalized size of antiderivative = 0.55

$$\int \frac{(ax^2+bx^3)^{3/2}}{(cx)^{3/2}}\,dx = \frac{\sqrt{c}\left(-3\sqrt{x}\,\sqrt{bx+a}\,a^3b+2\sqrt{x}\,\sqrt{bx+a}\,a^2b^2x+24\sqrt{x}\,\sqrt{bx+a}\,a\,b^3x^2+16\sqrt{x}\,\sqrt{bx\right.}{64b^3c^2}$$

```
int((b*x^3+a*x^2)^(3/2)/(c*x)^(3/2),x)
```

```
(sqrt(c)*( - 3*sqrt(x)*sqrt(a + b*x)*a**3*b + 2*sqrt(x)*sqrt(a + b*x)*a**2
*b**2*x + 24*sqrt(x)*sqrt(a + b*x)*a*b**3*x**2 + 16*sqrt(x)*sqrt(a + b*x)*
b**4*x**3 + 3*sqrt(b)*log((sqrt(a + b*x) + sqrt(x)*sqrt(b))/sqrt(a))*a**4)
)/(64*b**3*c**2)
```

3.339 $\int \frac{(ax^2+bx^3)^{3/2}}{(cx)^{5/2}}\,dx$

Optimal result

Integrand size = 23, antiderivative size = 147

$$\int \frac{(ax^2+bx^3)^{3/2}}{(cx)^{5/2}}\,dx = \frac{a^2\sqrt{ax^2+bx^3}}{8bc^2\sqrt{cx}} + \frac{7a\sqrt{cx}\sqrt{ax^2+bx^3}}{12c^3} + \frac{b(cx)^{3/2}\sqrt{ax^2+bx^3}}{3c^4} - \frac{a^3\operatorname{arctanh}\left(\frac{\sqrt{b}(cx)^{3/2}}{c^{3/2}\sqrt{ax^2+bx^3}}\right)}{8b^{3/2}c^{5/2}}$$

```
1/8*a^2*(b*x^3+a*x^2)^(1/2)/b/c^2/(c*x)^(1/2)+7/12*a*(c*x)^(1/2)*(b*x^3+a*
x^2)^(1/2)/c^3+1/3*b*(c*x)^(3/2)*(b*x^3+a*x^2)^(1/2)/c^4-1/8*a^3*arctanh(b
^(1/2)*(c*x)^(3/2)/c^(3/2)/(b*x^3+a*x^2)^(1/2))/b^(3/2)/c^(5/2)
```

Mathematica [A] (verified)

Time = 0.05 (sec) , antiderivative size = 119, normalized size of antiderivative = 0.81

$$\int \frac{(ax^2+bx^3)^{3/2}}{(cx)^{5/2}}\,dx = \frac{x^{3/2}\sqrt{x^2(a+bx)}\left(\sqrt{b}\sqrt{x}\sqrt{a+bx}(3a^2+14abx+8b^2x^2)+6a^3\operatorname{arctanh}\left(\frac{\sqrt{b}\sqrt{x}}{\sqrt{a}-\sqrt{a+bx}}\right)\right)}{24b^{3/2}(cx)^{5/2}\sqrt{a+bx}}$$

```
Integrate[(a*x^2 + b*x^3)^(3/2)/(c*x)^(5/2),x]
```

```
(x^(3/2)*Sqrt[x^2*(a + b*x)]*(Sqrt[b]*Sqrt[x]*Sqrt[a + b*x]*(3*a^2 + 14*a*
b*x + 8*b^2*x^2) + 6*a^3*ArcTanh[(Sqrt[b]*Sqrt[x])/(Sqrt[a] - Sqrt[a + b*x
])]))/(24*b^(3/2)*(c*x)^(5/2)*Sqrt[a + b*x])
```

Rubi [A] (verified)

Time = 0.58 (sec) , antiderivative size = 154, normalized size of antiderivative = 1.05, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.261$, Rules used = {1927, 1927, 1930, 1937, 1935, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\left(ax^2+bx^3\right)^{3/2}}{(cx)^{5/2}}dx$$

$$\Big\downarrow \text{1927}$$

$$\frac{a\int \frac{\sqrt{bx^3+ax^2}}{\sqrt{cx}}dx}{2c^2}+\frac{\left(ax^2+bx^3\right)^{3/2}}{3c(cx)^{3/2}}$$

$$\Big\downarrow \text{1927}$$

$$\frac{a\left(\frac{a\int \frac{(cx)^{3/2}}{\sqrt{bx^3+ax^2}}dx}{4c^2}+\frac{\sqrt{cx}\sqrt{ax^2+bx^3}}{2c}\right)}{2c^2}+\frac{\left(ax^2+bx^3\right)^{3/2}}{3c(cx)^{3/2}}$$

$$\Big\downarrow \text{1930}$$

$$\frac{a\left(\frac{a\left(\frac{c^2\sqrt{ax^2+bx^3}}{b\sqrt{cx}}-\frac{ac\int \frac{\sqrt{cx}}{\sqrt{bx^3+ax^2}}dx}{2b}\right)}{4c^2}+\frac{\sqrt{cx}\sqrt{ax^2+bx^3}}{2c}\right)}{2c^2}+\frac{\left(ax^2+bx^3\right)^{3/2}}{3c(cx)^{3/2}}$$

$$\Big\downarrow \text{1937}$$

$$\frac{a\left(\frac{a\left(\frac{c^2\sqrt{ax^2+bx^3}}{b\sqrt{cx}}-\frac{ac\sqrt{cx}\int \frac{\sqrt{x}}{\sqrt{bx^3+ax^2}}dx}{2b\sqrt{x}}\right)}{4c^2}+\frac{\sqrt{cx}\sqrt{ax^2+bx^3}}{2c}\right)}{2c^2}+\frac{\left(ax^2+bx^3\right)^{3/2}}{3c(cx)^{3/2}}$$

$$\Big\downarrow \textcolor{blue}{1935}$$

$$\frac{a\left(\frac{a\left(\frac{c^2\sqrt{ax^2+bx^3}}{b\sqrt{cx}}-\frac{ac\sqrt{cx}\int\frac{1}{1-\frac{bx^3}{bx^3+ax^2}}d\frac{x^{3/2}}{\sqrt{bx^3+ax^2}}}{b\sqrt{x}}\right)}{4c^2}+\frac{\sqrt{cx}\sqrt{ax^2+bx^3}}{2c}\right)}{2c^2}+\frac{\left(ax^2+bx^3\right)^{3/2}}{3c(cx)^{3/2}}$$

$$\Big\downarrow \textcolor{blue}{219}$$

$$\frac{a\left(\frac{a\left(\frac{c^2\sqrt{ax^2+bx^3}}{b\sqrt{cx}}-\frac{ac\sqrt{cx}\,\text{arctanh}\left(\frac{\sqrt{b}x^{3/2}}{\sqrt{ax^2+bx^3}}\right)}{b^{3/2}\sqrt{x}}\right)}{4c^2}+\frac{\sqrt{cx}\sqrt{ax^2+bx^3}}{2c}\right)}{2c^2}+\frac{\left(ax^2+bx^3\right)^{3/2}}{3c(cx)^{3/2}}$$

```
Int[(a*x^2 + b*x^3)^(3/2)/(c*x)^(5/2),x]
```

```
(a*x^2 + b*x^3)^(3/2)/(3*c*(c*x)^(3/2)) + (a*((Sqrt[c*x]*Sqrt[a*x^2 + b*x^
3])/(2*c) + (a*((c^2*Sqrt[a*x^2 + b*x^3])/(b*Sqrt[c*x]) - (a*c*Sqrt[c*x]*A
rcTanh[(Sqrt[b]*x^(3/2))/Sqrt[a*x^2 + b*x^3]])/(b^(3/2)*Sqrt[x])))/(4*c^2)
))/(2*c^2)
```

Defintions of rubi rules used

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> Simp[(c*x)^(m + 1)*((a*x^j + b*x^n)^p/(c*(m + n*p + 1))), x] + Simp[a*
(n - j)*(p/(c^j*(m + n*p + 1)))   Int[(c*x)^(m + j)*(a*x^j + b*x^n)^(p - 1)
, x], x] /; FreeQ[{a, b, c, m}, x] &&  !IntegerQ[p] && LtQ[0, j, n] && (Int
egersQ[j, n] || GtQ[c, 0]) && GtQ[p, 0] && NeQ[m + n*p + 1, 0]
```

```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> Simp[c^(n - 1)*(c*x)^(m - n + 1)*((a*x^j + b*x^n)^(p + 1)/(b*(m + n*p
+ 1))), x] - Simp[a*c^(n - j)*((m + j*p - n + j + 1)/(b*(m + n*p + 1)))   I
nt[(c*x)^(m - (n - j))*(a*x^j + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, p},
x] &&  !IntegerQ[p] && LtQ[0, j, n] && (IntegersQ[j, n] || GtQ[c, 0]) && Gt
Q[m + j*p - n + j + 1, 0] && NeQ[m + n*p + 1, 0]
```

```
Int[(x_)^(m_.)/Sqrt[(a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.)], x_Symbol] :> Simp
[-2/(n - j)   Subst[Int[1/(1 - a*x^2), x], x, x^(j/2)/Sqrt[a*x^j + b*x^n]],
 x] /; FreeQ[{a, b, j, n}, x] && EqQ[m, j/2 - 1] && NeQ[n, j]
```

```
Int[((c_)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_))^(p_), x_Symbol]
:> Simp[c^IntPart[m]*((c*x)^FracPart[m]/x^FracPart[m])   Int[x^m*(a*x^j + b
*x^n)^p, x], x] /; FreeQ[{a, b, c, j, m, n, p}, x] && IntegerQ[p + 1/2] &&
NeQ[n, j] && EqQ[Simplify[m + j*p + 1], 0]
```

Maple [A] (verified)

Time = 0.41 (sec) , antiderivative size = 127, normalized size of antiderivative = 0.86

method	result	si
risch	$\frac{(8b^2x^2+14abx+3a^2)\sqrt{x^2(bx+a)}}{24b\,c^2\sqrt{cx}}-\frac{a^3\ln\left(\frac{\frac{1}{2}ac+cbx}{\sqrt{bc}}+\sqrt{bc\,x^2+acx}\right)\sqrt{x^2(bx+a)}\,\sqrt{cx(bx+a)}}{16b\sqrt{bc}\,c^2x(bx+a)\sqrt{cx}}$	1
default	$\frac{(b\,x^3+a\,x^2)^{\frac{3}{2}}\left(16b^2x^2\sqrt{cx(bx+a)}\,\sqrt{bc}-3\ln\left(\frac{2cbx+2\sqrt{cx(bx+a)}\,\sqrt{bc}+ac}{2\sqrt{bc}}\right)a^3c+28\sqrt{bc}\,\sqrt{cx(bx+a)}\,abx+6\sqrt{bc}\,\sqrt{cx(bx+a)}\,a^2\right)}{48x^2(bx+a)c^2b\sqrt{cx}\,\sqrt{cx(bx+a)}\,\sqrt{bc}}$	1

```
int((b*x^3+a*x^2)^(3/2)/(c*x)^(5/2),x,method=_RETURNVERBOSE)
```

```
1/24*(8*b^2*x^2+14*a*b*x+3*a^2)/b/c^2*(x^2*(b*x+a))^(1/2)/(c*x)^(1/2)-1/16
*a^3/b*ln((1/2*a*c+c*b*x)/(b*c)^(1/2)+(b*c*x^2+a*c*x)^(1/2))/(b*c)^(1/2)/c
^2*(x^2*(b*x+a))^(1/2)/x/(b*x+a)*(c*x*(b*x+a))^(1/2)/(c*x)^(1/2)
```

Fricas [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 212, normalized size of antiderivative = 1.44

$$\int \frac{(ax^2+bx^3)^{3/2}}{(cx)^{5/2}}\,dx = \left[\frac{3\sqrt{bc}a^3x\log\left(\frac{2\,bcx^2+acx-2\,\sqrt{bx^3+ax^2}\sqrt{bc}\sqrt{cx}}{x}\right)+2\,(8\,b^3x^2+14\,ab^2x+3\,a^2b)\sqrt{bx^3+a}}{48\,b^2c^3x}\right.$$

```
integrate((b*x^3+a*x^2)^(3/2)/(c*x)^(5/2),x, algorithm="fricas")
```

```
[1/48*(3*sqrt(b*c)*a^3*x*log((2*b*c*x^2 + a*c*x - 2*sqrt(b*x^3 + a*x^2)*sq
rt(b*c)*sqrt(c*x))/x) + 2*(8*b^3*x^2 + 14*a*b^2*x + 3*a^2*b)*sqrt(b*x^3 +
a*x^2)*sqrt(c*x))/(b^2*c^3*x), 1/24*(3*sqrt(-b*c)*a^3*x*arctan(sqrt(b*x^3
+ a*x^2)*sqrt(-b*c)*sqrt(c*x)/(b*c*x^2 + a*c*x)) + (8*b^3*x^2 + 14*a*b^2*x
 + 3*a^2*b)*sqrt(b*x^3 + a*x^2)*sqrt(c*x))/(b^2*c^3*x)]
```

Sympy [F]

$$\int \frac{(ax^2+bx^3)^{3/2}}{(cx)^{5/2}}\,dx = \int \frac{(x^2(a+bx))^{\frac{3}{2}}}{(cx)^{\frac{5}{2}}}\,dx$$

```
integrate((b*x**3+a*x**2)**(3/2)/(c*x)**(5/2),x)
```

```
Integral((x**2*(a + b*x))**(3/2)/(c*x)**(5/2), x)
```

Maxima [F]

$$\int \frac{(ax^2+bx^3)^{3/2}}{(cx)^{5/2}}\,dx = \int \frac{(bx^3+ax^2)^{\frac{3}{2}}}{(cx)^{\frac{5}{2}}}\,dx$$

```
integrate((b*x^3+a*x^2)^(3/2)/(c*x)^(5/2),x, algorithm="maxima")
```

```
integrate((b*x^3 + a*x^2)^(3/2)/(c*x)^(5/2), x)
```

Giac [A] (verification not implemented)

Time = 0.18 (sec) , antiderivative size = 161, normalized size of antiderivative = 1.10

$$\int \frac{(ax^2+bx^3)^{3/2}}{(cx)^{5/2}}\,dx = -\frac{a^3\log\left(\sqrt{bc}\sqrt{a}\right)\operatorname{sgn}(x)}{8\sqrt{bc}c^2|b|} + \frac{\left(\frac{3a^3\log\left(\left|-\sqrt{bc}\sqrt{bx+a}+\sqrt{(bx+a)bc-abc}\right|\right)\operatorname{sgn}(x)}{\sqrt{bc}b} + \sqrt{(bx+a)bc-abc}\sqrt{bx+a}\left(2(bx+a)\left(\frac{4(bx+a)\operatorname{sgn}(x)}{b^2c} - \frac{a\operatorname{sgn}(x)}{b^2c}\right)\right.\right.}{24c^2|b|}$$

```
integrate((b*x^3+a*x^2)^(3/2)/(c*x)^(5/2),x, algorithm="giac")
```

```
-1/8*a^3*log(sqrt(b*c)*sqrt(a))*sgn(x)/(sqrt(b*c)*c^2*abs(b)) + 1/24*(3*a^
3*log(abs(-sqrt(b*c)*sqrt(b*x + a) + sqrt((b*x + a)*b*c - a*b*c)))*sgn(x)/
(sqrt(b*c)*b) + sqrt((b*x + a)*b*c - a*b*c)*sqrt(b*x + a)*(2*(b*x + a)*(4*
(b*x + a)*sgn(x)/(b^2*c) - a*sgn(x)/(b^2*c)) - 3*a^2*sgn(x)/(b^2*c)))*b/(c
^2*abs(b))
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(ax^2+bx^3)^{3/2}}{(cx)^{5/2}}\,dx = \int \frac{{\left(b\,x^3+a\,x^2\right)}^{3/2}}{{\left(c\,x\right)}^{5/2}}\,dx$$

```
int((a*x^2 + b*x^3)^(3/2)/(c*x)^(5/2),x)
```

```
int((a*x^2 + b*x^3)^(3/2)/(c*x)^(5/2), x)
```

Reduce [B] (verification not implemented)

Time = 0.50 (sec) , antiderivative size = 81, normalized size of antiderivative = 0.55

$$\int \frac{(ax^2+bx^3)^{3/2}}{(cx)^{5/2}}\,dx = \frac{\sqrt{c}\left(3\sqrt{x}\,\sqrt{bx+a}\,a^2b+14\sqrt{x}\,\sqrt{bx+a}\,a\,b^2x+8\sqrt{x}\,\sqrt{bx+a}\,b^3x^2-3\sqrt{b}\log\left(\frac{\sqrt{bx+}}{}\right.\right.}{24b^2c^3}$$

```
int((b*x^3+a*x^2)^(3/2)/(c*x)^(5/2),x)
```

```
(sqrt(c)*(3*sqrt(x)*sqrt(a + b*x)*a**2*b + 14*sqrt(x)*sqrt(a + b*x)*a*b**2
*x + 8*sqrt(x)*sqrt(a + b*x)*b**3*x**2 - 3*sqrt(b)*log((sqrt(a + b*x) + sq
rt(x)*sqrt(b))/sqrt(a))*a**3))/(24*b**2*c**3)
```

3.340 $\int \frac{(ax^2+bx^3)^{3/2}}{(cx)^{7/2}}\,dx$

Optimal result

Integrand size = 23, antiderivative size = 112

$$\int \frac{(ax^2+bx^3)^{3/2}}{(cx)^{7/2}}\,dx=\frac{5a\sqrt{ax^2+bx^3}}{4c^3\sqrt{cx}}+\frac{b\sqrt{cx}\sqrt{ax^2+bx^3}}{2c^4}+\frac{3a^2\operatorname{arctanh}\left(\frac{\sqrt{b}(cx)^{3/2}}{c^{3/2}\sqrt{ax^2+bx^3}}\right)}{4\sqrt{b}c^{7/2}}$$

```
5/4*a*(b*x^3+a*x^2)^(1/2)/c^3/(c*x)^(1/2)+1/2*b*(c*x)^(1/2)*(b*x^3+a*x^2)^
(1/2)/c^4+3/4*a^2*arctanh(b^(1/2)*(c*x)^(3/2)/c^(3/2)/(b*x^3+a*x^2)^(1/2))
/b^(1/2)/c^(7/2)
```

Mathematica [A] (verified)

Time = 0.03 (sec) , antiderivative size = 103, normalized size of antiderivative = 0.92

$$\int \frac{(ax^2+bx^3)^{3/2}}{(cx)^{7/2}}\,dx=\frac{\sqrt{x}\sqrt{x^2(a+bx)}\left(\sqrt{b}\sqrt{x}\sqrt{a+bx}(5a+2bx)-3a^2\log\left(-\sqrt{b}\sqrt{x}+\sqrt{a+bx}\right)\right)}{4\sqrt{b}c^2(cx)^{3/2}\sqrt{a+bx}}$$

```
Integrate[(a*x^2 + b*x^3)^(3/2)/(c*x)^(7/2),x]
```

```
(Sqrt[x]*Sqrt[x^2*(a + b*x)]*(Sqrt[b]*Sqrt[x]*Sqrt[a + b*x]*(5*a + 2*b*x)
- 3*a^2*Log[-(Sqrt[b]*Sqrt[x]) + Sqrt[a + b*x]]))/(4*Sqrt[b]*c^2*(c*x)^(3/
2)*Sqrt[a + b*x])
```

Rubi [A] (verified)

Time = 0.47 (sec) , antiderivative size = 114, normalized size of antiderivative = 1.02, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.217$, Rules used = {1927, 1927, 1937, 1935, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\left(ax^2+bx^3\right)^{3/2}}{(cx)^{7/2}}dx$$

$$\Big\downarrow \text{1927}$$

$$\frac{3a\int \frac{\sqrt{bx^3+ax^2}}{(cx)^{3/2}}dx}{4c^2}+\frac{\left(ax^2+bx^3\right)^{3/2}}{2c(cx)^{5/2}}$$

$$\Big\downarrow \text{1927}$$

$$\frac{3a\left(\frac{a\int \frac{\sqrt{cx}}{\sqrt{bx^3+ax^2}}dx}{2c^2}+\frac{\sqrt{ax^2+bx^3}}{c\sqrt{cx}}\right)}{4c^2}+\frac{\left(ax^2+bx^3\right)^{3/2}}{2c(cx)^{5/2}}$$

$$\Big\downarrow \text{1937}$$

$$\frac{3a\left(\frac{a\sqrt{cx}\int \frac{\sqrt{x}}{\sqrt{bx^3+ax^2}}dx}{2c^2\sqrt{x}}+\frac{\sqrt{ax^2+bx^3}}{c\sqrt{cx}}\right)}{4c^2}+\frac{\left(ax^2+bx^3\right)^{3/2}}{2c(cx)^{5/2}}$$

$$\Big\downarrow \text{1935}$$

$$\frac{3a\left(\frac{a\sqrt{cx}\int \frac{1}{1-\frac{bx^3}{bx^3+ax^2}}d\frac{x^{3/2}}{\sqrt{bx^3+ax^2}}}{c^2\sqrt{x}}+\frac{\sqrt{ax^2+bx^3}}{c\sqrt{cx}}\right)}{4c^2}+\frac{\left(ax^2+bx^3\right)^{3/2}}{2c(cx)^{5/2}}$$

$$\Big\downarrow \text{219}$$

$$\frac{3a\left(\frac{a\sqrt{cx}\,\text{arctanh}\left(\frac{\sqrt{b}x^{3/2}}{\sqrt{ax^2+bx^3}}\right)}{\sqrt{b}c^2\sqrt{x}}+\frac{\sqrt{ax^2+bx^3}}{c\sqrt{cx}}\right)}{4c^2}+\frac{(ax^2+bx^3)^{3/2}}{2c(cx)^{5/2}}$$

```
Int[(a*x^2 + b*x^3)^(3/2)/(c*x)^(7/2),x]
```

```
(a*x^2 + b*x^3)^(3/2)/(2*c*(c*x)^(5/2)) + (3*a*(Sqrt[a*x^2 + b*x^3]/(c*Sqr
t[c*x]) + (a*Sqrt[c*x]*ArcTanh[(Sqrt[b]*x^(3/2))/Sqrt[a*x^2 + b*x^3]])/(Sq
rt[b]*c^2*Sqrt[x])))/(4*c^2)
```

Defintions of rubi rules used

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> Simp[(c*x)^(m + 1)*((a*x^j + b*x^n)^p/(c*(m + n*p + 1))), x] + Simp[a*
(n - j)*(p/(c^j*(m + n*p + 1)))   Int[(c*x)^(m + j)*(a*x^j + b*x^n)^(p - 1)
, x], x] /; FreeQ[{a, b, c, m}, x] &&  !IntegerQ[p] && LtQ[0, j, n] && (Int
egersQ[j, n] || GtQ[c, 0]) && GtQ[p, 0] && NeQ[m + n*p + 1, 0]
```

```
Int[(x_)^(m_.)/Sqrt[(a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.)], x_Symbol] :> Simp
[-2/(n - j)   Subst[Int[1/(1 - a*x^2), x], x, x^(j/2)/Sqrt[a*x^j + b*x^n]],
 x] /; FreeQ[{a, b, j, n}, x] && EqQ[m, j/2 - 1] && NeQ[n, j]
```

```
Int[((c_)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_))^(p_), x_Symbol]
:> Simp[c^IntPart[m]*((c*x)^FracPart[m]/x^FracPart[m])   Int[x^m*(a*x^j + b
*x^n)^p, x], x] /; FreeQ[{a, b, c, j, m, n, p}, x] && IntegerQ[p + 1/2] &&
NeQ[n, j] && EqQ[Simplify[m + j*p + 1], 0]
```

Maple [A] (verified)

Time = 0.38 (sec) , antiderivative size = 110, normalized size of antiderivative = 0.98

method	result	size
risch	$\frac{(2bx+5a)\sqrt{x^2(bx+a)}}{4c^3\sqrt{cx}}+\frac{3a^2\ln\left(\frac{\frac{1}{2}ac+cbx}{\sqrt{bc}}+\sqrt{bc\,x^2+acx}\right)\sqrt{x^2(bx+a)}\,\sqrt{cx(bx+a)}}{8\sqrt{bc}\,c^3x(bx+a)\sqrt{cx}}$	110
default	$\frac{(b\,x^3+a\,x^2)^{\frac{3}{2}}\left(4\sqrt{bc}\,\sqrt{cx(bx+a)}\,bx+3a^2c\ln\left(\frac{2cbx+2\sqrt{cx(bx+a)}\,\sqrt{bc}+ac}{2\sqrt{bc}}\right)+10\sqrt{bc}\,\sqrt{cx(bx+a)}\,a\right)}{8x^2(bx+a)c^3\sqrt{cx}\,\sqrt{cx(bx+a)}\,\sqrt{bc}}$	127

```
int((b*x^3+a*x^2)^(3/2)/(c*x)^(7/2),x,method=_RETURNVERBOSE)
```

```
1/4*(2*b*x+5*a)/c^3*(x^2*(b*x+a))^(1/2)/(c*x)^(1/2)+3/8*a^2*ln((1/2*a*c+c*
b*x)/(b*c)^(1/2)+(b*c*x^2+a*c*x)^(1/2))/(b*c)^(1/2)/c^3*(x^2*(b*x+a))^(1/2
)/x/(b*x+a)*(c*x*(b*x+a))^(1/2)/(c*x)^(1/2)
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 191, normalized size of antiderivative = 1.71

$$\int \frac{(ax^2+bx^3)^{3/2}}{(cx)^{7/2}}\,dx = \left[\frac{3\sqrt{bc}a^2x\log\left(\frac{2\,bcx^2+acx+2\,\sqrt{bx^3+ax^2}\sqrt{bc}\sqrt{cx}}{x}\right)+2\,\sqrt{bx^3+ax^2}(2\,b^2x+5\,ab)\sqrt{cx}}{8\,bc^4x}, -\frac{3\,\sqrt{}}{}\right.$$

```
integrate((b*x^3+a*x^2)^(3/2)/(c*x)^(7/2),x, algorithm="fricas")
```

```
[1/8*(3*sqrt(b*c)*a^2*x*log((2*b*c*x^2 + a*c*x + 2*sqrt(b*x^3 + a*x^2)*sqr
t(b*c)*sqrt(c*x))/x) + 2*sqrt(b*x^3 + a*x^2)*(2*b^2*x + 5*a*b)*sqrt(c*x))/
(b*c^4*x), -1/4*(3*sqrt(-b*c)*a^2*x*arctan(sqrt(b*x^3 + a*x^2)*sqrt(-b*c)*
sqrt(c*x)/(b*c*x^2 + a*c*x)) - sqrt(b*x^3 + a*x^2)*(2*b^2*x + 5*a*b)*sqrt(
c*x))/(b*c^4*x)]
```

Sympy [F]

$$\int \frac{(ax^2+bx^3)^{3/2}}{(cx)^{7/2}}\,dx = \int \frac{(x^2(a+bx))^{\frac{3}{2}}}{(cx)^{\frac{7}{2}}}\,dx$$

```
integrate((b*x**3+a*x**2)**(3/2)/(c*x)**(7/2),x)
```

```
Integral((x**2*(a + b*x))**(3/2)/(c*x)**(7/2), x)
```

Maxima [F]

$$\int \frac{(ax^2+bx^3)^{3/2}}{(cx)^{7/2}}\,dx = \int \frac{(bx^3+ax^2)^{\frac{3}{2}}}{(cx)^{\frac{7}{2}}}\,dx$$

```
integrate((b*x^3+a*x^2)^(3/2)/(c*x)^(7/2),x, algorithm="maxima")
```

```
integrate((b*x^3 + a*x^2)^(3/2)/(c*x)^(7/2), x)
```

Giac [A] (verification not implemented)

Time = 0.43 (sec) , antiderivative size = 139, normalized size of antiderivative = 1.24

$$\int \frac{(ax^2+bx^3)^{3/2}}{(cx)^{7/2}}\,dx = \frac{3\,a^2b\log\left(\sqrt{bc}\sqrt{a}\right)\mathrm{sgn}(x)}{4\,\sqrt{bc}c^3|b|} - \frac{\left(\frac{3\,a^2\log\left(\left|-\sqrt{bc}\sqrt{bx+a}+\sqrt{(bx+a)bc-abc}\right|\right)\mathrm{sgn}(x)}{\sqrt{bc}} - \sqrt{(bx+a)bc-abc}\sqrt{bx+a}\left(\frac{2\,(bx+a)\mathrm{sgn}(x)}{bc}+\frac{3\,a\mathrm{sgn}(x)}{bc}\right)\right)b}{4\,c^3|b|}$$

```
integrate((b*x^3+a*x^2)^(3/2)/(c*x)^(7/2),x, algorithm="giac")
```

```
3/4*a^2*b*log(sqrt(b*c)*sqrt(a))*sgn(x)/(sqrt(b*c)*c^3*abs(b)) - 1/4*(3*a^
2*log(abs(-sqrt(b*c)*sqrt(b*x + a) + sqrt((b*x + a)*b*c - a*b*c)))*sgn(x)/
sqrt(b*c) - sqrt((b*x + a)*b*c - a*b*c)*sqrt(b*x + a)*(2*(b*x + a)*sgn(x)/
(b*c) + 3*a*sgn(x)/(b*c)))*b/(c^3*abs(b))
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(ax^2+bx^3)^{3/2}}{(cx)^{7/2}}\,dx=\int \frac{(b\,x^3+a\,x^2)^{3/2}}{(c\,x)^{7/2}}\,dx$$

```
int((a*x^2 + b*x^3)^(3/2)/(c*x)^(7/2),x)
```

```
int((a*x^2 + b*x^3)^(3/2)/(c*x)^(7/2), x)
```

Reduce [B] (verification not implemented)

Time = 0.35 (sec) , antiderivative size = 62, normalized size of antiderivative = 0.55

$$\int \frac{(ax^2+bx^3)^{3/2}}{(cx)^{7/2}}\,dx=\frac{\sqrt{c}\left(5\sqrt{x}\,\sqrt{bx+a}\,ab+2\sqrt{x}\,\sqrt{bx+a}\,b^2x+3\sqrt{b}\log\left(\frac{\sqrt{bx+a}+\sqrt{x}\,\sqrt{b}}{\sqrt{a}}\right)a^2\right)}{4b\,c^4}$$

```
int((b*x^3+a*x^2)^(3/2)/(c*x)^(7/2),x)
```

```
(sqrt(c)*(5*sqrt(x)*sqrt(a + b*x)*a*b + 2*sqrt(x)*sqrt(a + b*x)*b**2*x + 3
*sqrt(b)*log((sqrt(a + b*x) + sqrt(x)*sqrt(b))/sqrt(a))*a**2))/(4*b*c**4)
```

3.341 $\int \frac{(ax^2+bx^3)^{3/2}}{(cx)^{9/2}}\,dx$

Optimal result

Integrand size = 23, antiderivative size = 103

$$\int \frac{(ax^2+bx^3)^{3/2}}{(cx)^{9/2}}\,dx=-\frac{2a\sqrt{ax^2+bx^3}}{c^3(cx)^{3/2}}+\frac{b\sqrt{ax^2+bx^3}}{c^4\sqrt{cx}}+\frac{3a\sqrt{b}\text{arctanh}\left(\frac{\sqrt{b}(cx)^{3/2}}{c^{3/2}\sqrt{ax^2+bx^3}}\right)}{c^{9/2}}$$

```
-2*a*(b*x^3+a*x^2)^(1/2)/c^3/(c*x)^(3/2)+b*(b*x^3+a*x^2)^(1/2)/c^4/(c*x)^(
1/2)+3*a*b^(1/2)*arctanh(b^(1/2)*(c*x)^(3/2)/c^(3/2)/(b*x^3+a*x^2)^(1/2))/
c^(9/2)
```

Mathematica [A] (verified)

Time = 0.03 (sec) , antiderivative size = 95, normalized size of antiderivative = 0.92

$$\int \frac{(ax^2+bx^3)^{3/2}}{(cx)^{9/2}}\,dx=\frac{\sqrt{x^2(a+bx)}\left((-2a+bx)\sqrt{a+bx}+6a\sqrt{b}\sqrt{x}\text{arctanh}\left(\frac{\sqrt{b}\sqrt{x}}{-\sqrt{a}+\sqrt{a+bx}}\right)\right)}{c^3(cx)^{3/2}\sqrt{a+bx}}$$

```
Integrate[(a*x^2 + b*x^3)^(3/2)/(c*x)^(9/2),x]
```

```
(Sqrt[x^2*(a + b*x)]*((-2*a + b*x)*Sqrt[a + b*x] + 6*a*Sqrt[b]*Sqrt[x]*Arc
Tanh[(Sqrt[b]*Sqrt[x])/(-Sqrt[a] + Sqrt[a + b*x])]))/(c^3*(c*x)^(3/2)*Sqrt
[a + b*x])
```

Rubi [A] (verified)

Time = 0.45 (sec) , antiderivative size = 110, normalized size of antiderivative = 1.07, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.217$, Rules used = {1926, 1927, 1937, 1935, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\left(ax^2+bx^3\right)^{3/2}}{(cx)^{9/2}}dx$$

$$\Big\downarrow \text{1926}$$

$$\frac{3b\int \frac{\sqrt{bx^3+ax^2}}{(cx)^{3/2}}dx}{c^3}-\frac{2\left(ax^2+bx^3\right)^{3/2}}{c(cx)^{7/2}}$$

$$\Big\downarrow \text{1927}$$

$$\frac{3b\left(\frac{a\int \frac{\sqrt{cx}}{\sqrt{bx^3+ax^2}}dx}{2c^2}+\frac{\sqrt{ax^2+bx^3}}{c\sqrt{cx}}\right)}{c^3}-\frac{2\left(ax^2+bx^3\right)^{3/2}}{c(cx)^{7/2}}$$

$$\Big\downarrow \text{1937}$$

$$\frac{3b\left(\frac{a\sqrt{cx}\int \frac{\sqrt{x}}{\sqrt{bx^3+ax^2}}dx}{2c^2\sqrt{x}}+\frac{\sqrt{ax^2+bx^3}}{c\sqrt{cx}}\right)}{c^3}-\frac{2\left(ax^2+bx^3\right)^{3/2}}{c(cx)^{7/2}}$$

$$\Big\downarrow \text{1935}$$

$$\frac{3b\left(\frac{a\sqrt{cx}\int \frac{1}{1-\frac{bx^3}{bx^3+ax^2}}d\frac{x^{3/2}}{\sqrt{bx^3+ax^2}}}{c^2\sqrt{x}}+\frac{\sqrt{ax^2+bx^3}}{c\sqrt{cx}}\right)}{c^3}-\frac{2\left(ax^2+bx^3\right)^{3/2}}{c(cx)^{7/2}}$$

$$\Big\downarrow \text{219}$$

$$\frac{3b\left(\frac{a\sqrt{cx}\text{arctanh}\left(\frac{\sqrt{b}x^{3/2}}{\sqrt{ax^2+bx^3}}\right)}{\sqrt{b}c^2\sqrt{x}}+\frac{\sqrt{ax^2+bx^3}}{c\sqrt{cx}}\right)}{c^3}-\frac{2\left(ax^2+bx^3\right)^{3/2}}{c(cx)^{7/2}}$$

```
Int[(a*x^2 + b*x^3)^(3/2)/(c*x)^(9/2),x]
```

```
(-2*(a*x^2 + b*x^3)^(3/2))/(c*(c*x)^(7/2)) + (3*b*(Sqrt[a*x^2 + b*x^3]/(c*
Sqrt[c*x]) + (a*Sqrt[c*x]*ArcTanh[(Sqrt[b]*x^(3/2))/Sqrt[a*x^2 + b*x^3]])/
(Sqrt[b]*c^2*Sqrt[x])))/c^3
```

Defintions of rubi rules used

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

```
Int[((c_.)*(x_))^(m_)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol]
 :> Simp[(c*x)^(m + 1)*((a*x^j + b*x^n)^p/(c*(m + j*p + 1))), x] - Simp[b*p
*((n - j)/(c^n*(m + j*p + 1)))   Int[(c*x)^(m + n)*(a*x^j + b*x^n)^(p - 1),
 x], x] /; FreeQ[{a, b, c}, x] &&  !IntegerQ[p] && LtQ[0, j, n] && (Integer
sQ[j, n] || GtQ[c, 0]) && GtQ[p, 0] && LtQ[m + j*p + 1, 0]
```

```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> Simp[(c*x)^(m + 1)*((a*x^j + b*x^n)^p/(c*(m + n*p + 1))), x] + Simp[a*
(n - j)*(p/(c^j*(m + n*p + 1)))   Int[(c*x)^(m + j)*(a*x^j + b*x^n)^(p - 1)
, x], x] /; FreeQ[{a, b, c, m}, x] &&  !IntegerQ[p] && LtQ[0, j, n] && (Int
egersQ[j, n] || GtQ[c, 0]) && GtQ[p, 0] && NeQ[m + n*p + 1, 0]
```

```
Int[(x_)^(m_.)/Sqrt[(a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.)], x_Symbol] :> Simp
[-2/(n - j)   Subst[Int[1/(1 - a*x^2), x], x, x^(j/2)/Sqrt[a*x^j + b*x^n]],
 x] /; FreeQ[{a, b, j, n}, x] && EqQ[m, j/2 - 1] && NeQ[n, j]
```

```
Int[((c_)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_))^(p_), x_Symbol]
:> Simp[c^IntPart[m]*((c*x)^FracPart[m]/x^FracPart[m])   Int[x^m*(a*x^j + b
*x^n)^p, x], x] /; FreeQ[{a, b, c, j, m, n, p}, x] && IntegerQ[p + 1/2] &&
NeQ[n, j] && EqQ[Simplify[m + j*p + 1], 0]
```

Maple [A] (verified)

Time = 0.38 (sec) , antiderivative size = 112, normalized size of antiderivative = 1.09

method	result	size
risch	$-\frac{(-bx+2a)\sqrt{x^{2}(bx+a)}}{c^{4}x\sqrt{cx}}+\frac{3ab\ln\left(\frac{\frac{1}{2}ac+cbx}{\sqrt{bc}}+\sqrt{bc\,x^{2}+acx}\right)\sqrt{x^{2}(bx+a)}\,\sqrt{cx(bx+a)}}{2\sqrt{bc}\,c^{4}x(bx+a)\sqrt{cx}}$	112
default	$\frac{\left(b\,x^{3}+a\,x^{2}\right)^{\frac{3}{2}}\left(3\ln\left(\frac{2cbx+2\sqrt{cx(bx+a)}\,\sqrt{bc}+ac}{2\sqrt{bc}}\right)abcx+2\sqrt{bc}\,\sqrt{cx(bx+a)}\,bx-4\sqrt{bc}\,\sqrt{cx(bx+a)}\,a\right)}{2x^{3}(bx+a)c^{4}\sqrt{cx}\,\sqrt{cx(bx+a)}\,\sqrt{bc}}$	127

```
int((b*x^3+a*x^2)^(3/2)/(c*x)^(9/2),x,method=_RETURNVERBOSE)
```

```
-(-b*x+2*a)/c^4*(x^2*(b*x+a))^(1/2)/x/(c*x)^(1/2)+3/2*a*b*ln((1/2*a*c+c*b*
x)/(b*c)^(1/2)+(b*c*x^2+a*c*x)^(1/2))/(b*c)^(1/2)/c^4*(x^2*(b*x+a))^(1/2)/
x/(b*x+a)*(c*x*(b*x+a))^(1/2)/(c*x)^(1/2)
```

Fricas [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 183, normalized size of antiderivative = 1.78

$$\int \frac{(ax^2+bx^3)^{3/2}}{(cx)^{9/2}}\,dx = \left[\frac{3\,acx^2\sqrt{\frac{b}{c}}\log\left(\frac{2\,bx^2+ax+2\,\sqrt{bx^3+ax^2}\sqrt{cx}\sqrt{\frac{b}{c}}}{x}\right)+2\,\sqrt{bx^3+ax^2}(bx-2\,a)\sqrt{cx}}{2\,c^5x^2}, -\frac{3\,acx^2}{}\right.$$

```
integrate((b*x^3+a*x^2)^(3/2)/(c*x)^(9/2),x, algorithm="fricas")
```

```
[1/2*(3*a*c*x^2*sqrt(b/c)*log((2*b*x^2 + a*x + 2*sqrt(b*x^3 + a*x^2)*sqrt(
c*x)*sqrt(b/c))/x) + 2*sqrt(b*x^3 + a*x^2)*(b*x - 2*a)*sqrt(c*x))/(c^5*x^2
), -(3*a*c*x^2*sqrt(-b/c)*arctan(sqrt(b*x^3 + a*x^2)*sqrt(c*x)*sqrt(-b/c)/
(b*x^2 + a*x)) - sqrt(b*x^3 + a*x^2)*(b*x - 2*a)*sqrt(c*x))/(c^5*x^2)]
```

Sympy [F]

$$\int \frac{(ax^2 + bx^3)^{3/2}}{(cx)^{9/2}}\, dx = \int \frac{(x^2(a + bx))^{\frac{3}{2}}}{(cx)^{\frac{9}{2}}}\, dx$$

```
integrate((b*x**3+a*x**2)**(3/2)/(c*x)**(9/2),x)
```

```
Integral((x**2*(a + b*x))**(3/2)/(c*x)**(9/2), x)
```

Maxima [F]

$$\int \frac{(ax^2 + bx^3)^{3/2}}{(cx)^{9/2}}\, dx = \int \frac{(bx^3 + ax^2)^{\frac{3}{2}}}{(cx)^{\frac{9}{2}}}\, dx$$

```
integrate((b*x^3+a*x^2)^(3/2)/(c*x)^(9/2),x, algorithm="maxima")
```

```
integrate((b*x^3 + a*x^2)^(3/2)/(c*x)^(9/2), x)
```

Giac [A] (verification not implemented)

Time = 0.22 (sec) , antiderivative size = 95, normalized size of antiderivative = 0.92

$$\int \frac{(ax^2+bx^3)^{3/2}}{(cx)^{9/2}}\,dx = -\frac{\left(\frac{3\,a\log\left(\left|-\sqrt{bc}\sqrt{bx+a}+\sqrt{(bx+a)bc-abc}\right|\right)\mathrm{sgn}(x)}{\sqrt{bc}}-\frac{\sqrt{bx+a}((bx+a)\mathrm{sgn}(x)-3\,a\mathrm{sgn}(x))}{\sqrt{(bx+a)bc-abc}}\right)b^2}{c^4|b|}$$

```
integrate((b*x^3+a*x^2)^(3/2)/(c*x)^(9/2),x, algorithm="giac")
```

```
-(3*a*log(abs(-sqrt(b*c)*sqrt(b*x + a) + sqrt((b*x + a)*b*c - a*b*c)))*sgn
(x)/sqrt(b*c) - sqrt(b*x + a)*((b*x + a)*sgn(x) - 3*a*sgn(x))/sqrt((b*x +
a)*b*c - a*b*c))*b^2/(c^4*abs(b))
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(ax^2+bx^3)^{3/2}}{(cx)^{9/2}}\,dx = \int \frac{(b\,x^3+a\,x^2)^{3/2}}{(c\,x)^{9/2}}\,dx$$

```
int((a*x^2 + b*x^3)^(3/2)/(c*x)^(9/2),x)
```

```
int((a*x^2 + b*x^3)^(3/2)/(c*x)^(9/2), x)
```

Reduce [B] (verification not implemented)

Time = 0.21 (sec) , antiderivative size = 64, normalized size of antiderivative = 0.62

$$\int \frac{(ax^2+bx^3)^{3/2}}{(cx)^{9/2}}\,dx = \frac{\sqrt{c}\left(-8\sqrt{x}\,\sqrt{bx+a}\,a+4\sqrt{x}\,\sqrt{bx+a}\,bx+12\sqrt{b}\log\left(\frac{\sqrt{bx+a}+\sqrt{x}\,\sqrt{b}}{\sqrt{a}}\right)ax-9\sqrt{b}\,ax\right)}{4c^5x}$$

```
int((b*x^3+a*x^2)^(3/2)/(c*x)^(9/2),x)
```

```
(sqrt(c)*( - 8*sqrt(x)*sqrt(a + b*x)*a + 4*sqrt(x)*sqrt(a + b*x)*b*x + 12*
sqrt(b)*log((sqrt(a + b*x) + sqrt(x)*sqrt(b))/sqrt(a))*a*x - 9*sqrt(b)*a*x
))/(4*c**5*x)
```

3.342 $\int \frac{(ax^2+bx^3)^{3/2}}{(cx)^{11/2}}\,dx$

Optimal result

Integrand size = 23, antiderivative size = 107

$$\int \frac{(ax^2+bx^3)^{3/2}}{(cx)^{11/2}}\,dx = -\frac{2a\sqrt{ax^2+bx^3}}{3c^3(cx)^{5/2}} - \frac{8b\sqrt{ax^2+bx^3}}{3c^4(cx)^{3/2}} + \frac{2b^{3/2}\operatorname{arctanh}\left(\frac{\sqrt{b}(cx)^{3/2}}{c^{3/2}\sqrt{ax^2+bx^3}}\right)}{c^{11/2}}$$

```
-2/3*a*(b*x^3+a*x^2)^(1/2)/c^3/(c*x)^(5/2)-8/3*b*(b*x^3+a*x^2)^(1/2)/c^4/(
c*x)^(3/2)+2*b^(3/2)*arctanh(b^(1/2)*(c*x)^(3/2)/c^(3/2)/(b*x^3+a*x^2)^(1/
2))/c^(11/2)
```

Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 88, normalized size of antiderivative = 0.82

$$\int \frac{(ax^2+bx^3)^{3/2}}{(cx)^{11/2}}\,dx = -\frac{2\sqrt{x^2(a+bx)}\left(\sqrt{a+bx}(a+4bx)+3b^{3/2}x^{3/2}\log\left(-\sqrt{b}\sqrt{x}+\sqrt{a+bx}\right)\right)}{3c^3(cx)^{5/2}\sqrt{a+bx}}$$

```
Integrate[(a*x^2 + b*x^3)^(3/2)/(c*x)^(11/2),x]
```

```
(-2*Sqrt[x^2*(a + b*x)]*(Sqrt[a + b*x]*(a + 4*b*x) + 3*b^(3/2)*x^(3/2)*Log
[-(Sqrt[b]*Sqrt[x]) + Sqrt[a + b*x]]))/(3*c^3*(c*x)^(5/2)*Sqrt[a + b*x])
```

Rubi [A] (verified)

Time = 0.47 (sec) , antiderivative size = 112, normalized size of antiderivative = 1.05, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.217, Rules used = {1926, 1926, 1937, 1935, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\left(ax^2+bx^3\right)^{3/2}}{(cx)^{11/2}}dx$$

$$\Big\downarrow \text{1926}$$

$$\frac{b\int \frac{\sqrt{bx^3+ax^2}}{(cx)^{5/2}}dx}{c^3}-\frac{2\left(ax^2+bx^3\right)^{3/2}}{3c(cx)^{9/2}}$$

$$\Big\downarrow \text{1926}$$

$$\frac{b\left(\frac{b\int \frac{\sqrt{cx}}{\sqrt{bx^3+ax^2}}dx}{c^3}-\frac{2\sqrt{ax^2+bx^3}}{c(cx)^{3/2}}\right)}{c^3}-\frac{2\left(ax^2+bx^3\right)^{3/2}}{3c(cx)^{9/2}}$$

$$\Big\downarrow \text{1937}$$

$$\frac{b\left(\frac{b\sqrt{cx}\int \frac{\sqrt{x}}{\sqrt{bx^3+ax^2}}dx}{c^3\sqrt{x}}-\frac{2\sqrt{ax^2+bx^3}}{c(cx)^{3/2}}\right)}{c^3}-\frac{2\left(ax^2+bx^3\right)^{3/2}}{3c(cx)^{9/2}}$$

$$\Big\downarrow \text{1935}$$

$$\frac{b\left(\frac{2b\sqrt{cx}\int \frac{1}{1-\frac{bx^3}{bx^3+ax^2}}d\frac{x^{3/2}}{\sqrt{bx^3+ax^2}}}{c^3\sqrt{x}}-\frac{2\sqrt{ax^2+bx^3}}{c(cx)^{3/2}}\right)}{c^3}-\frac{2\left(ax^2+bx^3\right)^{3/2}}{3c(cx)^{9/2}}$$

$$\Big\downarrow \text{219}$$

$$\frac{b\left(\frac{2\sqrt{b}\sqrt{cx}\operatorname{arctanh}\left(\frac{\sqrt{b}x^{3/2}}{\sqrt{ax^2+bx^3}}\right)}{c^3\sqrt{x}}-\frac{2\sqrt{ax^2+bx^3}}{c(cx)^{3/2}}\right)}{c^3}-\frac{2\left(ax^2+bx^3\right)^{3/2}}{3c(cx)^{9/2}}$$

```
Int[(a*x^2 + b*x^3)^(3/2)/(c*x)^(11/2),x]
```

```
(-2*(a*x^2 + b*x^3)^(3/2))/(3*c*(c*x)^(9/2)) + (b*((-2*Sqrt[a*x^2 + b*x^3]
)/(c*(c*x)^(3/2)) + (2*Sqrt[b]*Sqrt[c*x]*ArcTanh[(Sqrt[b]*x^(3/2))/Sqrt[a*
x^2 + b*x^3]])/(c^3*Sqrt[x])))/c^3
```

Defintions of rubi rules used

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

```
Int[((c_.)*(x_))^(m_)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol]
 :> Simp[(c*x)^(m + 1)*((a*x^j + b*x^n)^p/(c*(m + j*p + 1))), x] - Simp[b*p
*((n - j)/(c^n*(m + j*p + 1)))   Int[(c*x)^(m + n)*(a*x^j + b*x^n)^(p - 1),
 x], x] /; FreeQ[{a, b, c}, x] &&  !IntegerQ[p] && LtQ[0, j, n] && (Integer
sQ[j, n] || GtQ[c, 0]) && GtQ[p, 0] && LtQ[m + j*p + 1, 0]
```

```
Int[(x_)^(m_.)/Sqrt[(a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.)], x_Symbol] :> Simp
[-2/(n - j)   Subst[Int[1/(1 - a*x^2), x], x, x^(j/2)/Sqrt[a*x^j + b*x^n]],
 x] /; FreeQ[{a, b, j, n}, x] && EqQ[m, j/2 - 1] && NeQ[n, j]
```

```
Int[((c_)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_))^(p_), x_Symbol]
:> Simp[c^IntPart[m]*((c*x)^FracPart[m]/x^FracPart[m])   Int[x^m*(a*x^j + b
*x^n)^p, x], x] /; FreeQ[{a, b, c, j, m, n, p}, x] && IntegerQ[p + 1/2] &&
NeQ[n, j] && EqQ[Simplify[m + j*p + 1], 0]
```

Maple [A] (verified)

Time = 0.38 (sec) , antiderivative size = 110, normalized size of antiderivative = 1.03

method	result	size
risch	$-\frac{2(4bx+a)\sqrt{x^{2}(bx+a)}}{3x^{2}c^{5}\sqrt{cx}}+\frac{b^{2}\ln\left(\frac{\frac{1}{2}ac+cbx}{\sqrt{bc}}+\sqrt{bc\,x^{2}+acx}\right)\sqrt{x^{2}(bx+a)}\,\sqrt{cx(bx+a)}}{\sqrt{bc}\,c^{5}x(bx+a)\sqrt{cx}}$	110
default	$\frac{\left(b\,x^{3}+a\,x^{2}\right)^{\frac{3}{2}}\left(3\ln\left(\frac{2cbx+2\sqrt{cx(bx+a)}\,\sqrt{bc}+ac}{2\sqrt{bc}}\right)b^{2}c\,x^{2}-8\sqrt{bc}\,\sqrt{cx(bx+a)}\,bx-2\sqrt{bc}\,\sqrt{cx(bx+a)}\,a\right)}{3x^{4}(bx+a)c^{5}\sqrt{cx}\,\sqrt{cx(bx+a)}\,\sqrt{bc}}$	130

```
int((b*x^3+a*x^2)^(3/2)/(c*x)^(11/2),x,method=_RETURNVERBOSE)
```

```
-2/3*(4*b*x+a)/x^2/c^5*(x^2*(b*x+a))^(1/2)/(c*x)^(1/2)+b^2*ln((1/2*a*c+c*b
*x)/(b*c)^(1/2)+(b*c*x^2+a*c*x)^(1/2))/(b*c)^(1/2)/c^5*(x^2*(b*x+a))^(1/2)
/x/(b*x+a)*(c*x*(b*x+a))^(1/2)/(c*x)^(1/2)
```

Fricas [A] (verification not implemented)

Time = 0.10 (sec) , antiderivative size = 180, normalized size of antiderivative = 1.68

$$\int \frac{(ax^2+bx^3)^{3/2}}{(cx)^{11/2}}\,dx = \left[\frac{3\,bcx^3\sqrt{\frac{b}{c}}\log\left(\frac{2\,bx^2+ax+2\,\sqrt{bx^3+ax^2}\sqrt{cx}\sqrt{\frac{b}{c}}}{x}\right)-2\,\sqrt{bx^3+ax^2}(4\,bx+a)\sqrt{cx}}{3\,c^6x^3},-\frac{2\left(3\,bc\right.}{}\right.$$

```
integrate((b*x^3+a*x^2)^(3/2)/(c*x)^(11/2),x, algorithm="fricas")
```

```
[1/3*(3*b*c*x^3*sqrt(b/c)*log((2*b*x^2 + a*x + 2*sqrt(b*x^3 + a*x^2)*sqrt(
c*x)*sqrt(b/c))/x) - 2*sqrt(b*x^3 + a*x^2)*(4*b*x + a)*sqrt(c*x))/(c^6*x^3
), -2/3*(3*b*c*x^3*sqrt(-b/c)*arctan(sqrt(b*x^3 + a*x^2)*sqrt(c*x)*sqrt(-b
/c)/(b*x^2 + a*x)) + sqrt(b*x^3 + a*x^2)*(4*b*x + a)*sqrt(c*x))/(c^6*x^3)]
```

Sympy [F]

$$\int \frac{(ax^2+bx^3)^{3/2}}{(cx)^{11/2}}\,dx = \int \frac{(x^2(a+bx))^{\frac{3}{2}}}{(cx)^{\frac{11}{2}}}\,dx$$

```
integrate((b*x**3+a*x**2)**(3/2)/(c*x)**(11/2),x)
```

```
Integral((x**2*(a + b*x))**(3/2)/(c*x)**(11/2), x)
```

Maxima [F]

$$\int \frac{(ax^2+bx^3)^{3/2}}{(cx)^{11/2}}\,dx = \int \frac{(bx^3+ax^2)^{\frac{3}{2}}}{(cx)^{\frac{11}{2}}}\,dx$$

```
integrate((b*x^3+a*x^2)^(3/2)/(c*x)^(11/2),x, algorithm="maxima")
```

```
integrate((b*x^3 + a*x^2)^(3/2)/(c*x)^(11/2), x)
```

Giac [A] (verification not implemented)

Time = 0.47 (sec) , antiderivative size = 98, normalized size of antiderivative = 0.92

$$\int \frac{(ax^2+bx^3)^{3/2}}{(cx)^{11/2}}\,dx = -\frac{2\left(\frac{3\log\left(\left|-\sqrt{bc}\sqrt{bx+a}+\sqrt{(bx+a)bc-abc}\right|\right)\mathrm{sgn}(x)}{\sqrt{bc}}+\frac{(4\,(bx+a)bc\mathrm{sgn}(x)-3\,abc\mathrm{sgn}(x))\sqrt{bx+a}}{((bx+a)bc-abc)^{\frac{3}{2}}}\right)b^3}{3\,c^5|b|}$$

```
integrate((b*x^3+a*x^2)^(3/2)/(c*x)^(11/2),x, algorithm="giac")
```

```
-2/3*(3*log(abs(-sqrt(b*c)*sqrt(b*x + a) + sqrt((b*x + a)*b*c - a*b*c)))*s
gn(x)/sqrt(b*c) + (4*(b*x + a)*b*c*sgn(x) - 3*a*b*c*sgn(x))*sqrt(b*x + a)/
((b*x + a)*b*c - a*b*c)^(3/2))*b^3/(c^5*abs(b))
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(ax^2+bx^3)^{3/2}}{(cx)^{11/2}}\,dx=\int \frac{(b\,x^3+a\,x^2)^{3/2}}{(c\,x)^{11/2}}\,dx$$

```
int((a*x^2 + b*x^3)^(3/2)/(c*x)^(11/2),x)
```

```
int((a*x^2 + b*x^3)^(3/2)/(c*x)^(11/2), x)
```

Reduce [B] (verification not implemented)

Time = 0.57 (sec) , antiderivative size = 60, normalized size of antiderivative = 0.56

$$\int \frac{(ax^2+bx^3)^{3/2}}{(cx)^{11/2}}\,dx=\frac{2\sqrt{c}\left(-\sqrt{x}\,\sqrt{bx+a}\,a-4\sqrt{x}\,\sqrt{bx+a}\,bx+3\sqrt{b}\log\left(\frac{\sqrt{bx+a}+\sqrt{x}\,\sqrt{b}}{\sqrt{a}}\right)b\,x^2\right)}{3c^6x^2}$$

```
int((b*x^3+a*x^2)^(3/2)/(c*x)^(11/2),x)
```

```
(2*sqrt(c)*( - sqrt(x)*sqrt(a + b*x)*a - 4*sqrt(x)*sqrt(a + b*x)*b*x + 3*s
qrt(b)*log((sqrt(a + b*x) + sqrt(x)*sqrt(b))/sqrt(a))*b*x**2))/(3*c**6*x**
2)
```

3.343 $\int \frac{(ax^2+bx^3)^{3/2}}{(cx)^{13/2}}\,dx$

Optimal result

Integrand size = 23, antiderivative size = 30

$$\int \frac{(ax^2+bx^3)^{3/2}}{(cx)^{13/2}}\,dx = -\frac{2c(ax^2+bx^3)^{5/2}}{5a(cx)^{15/2}}$$

```
-2/5*c*(b*x^3+a*x^2)^(5/2)/a/(c*x)^(15/2)
```

Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 28, normalized size of antiderivative = 0.93

$$\int \frac{(ax^2+bx^3)^{3/2}}{(cx)^{13/2}}\,dx = -\frac{2c(x^2(a+bx))^{5/2}}{5a(cx)^{15/2}}$$

```
Integrate[(a*x^2 + b*x^3)^(3/2)/(c*x)^(13/2),x]
```

```
(-2*c*(x^2*(a + b*x))^(5/2))/(5*a*(c*x)^(15/2))
```

Rubi [A] (verified)

Time = 0.25 (sec) , antiderivative size = 30, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.043, Rules used = {1920}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\left(ax^2+bx^3\right)^{3/2}}{(cx)^{13/2}}\,dx$$
$$\Big\downarrow \; 1920$$
$$-\frac{2c\left(ax^2+bx^3\right)^{5/2}}{5a(cx)^{15/2}}$$

```
Int[(a*x^2 + b*x^3)^(3/2)/(c*x)^(13/2),x]
```

```
(-2*c*(a*x^2 + b*x^3)^(5/2))/(5*a*(c*x)^(15/2))
```

Defintions of rubi rules used

```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> Simp[(-c^(j - 1))*(c*x)^(m - j + 1)*((a*x^j + b*x^n)^(p + 1)/(a*(n - j
)*(p + 1))), x] /; FreeQ[{a, b, c, j, m, n, p}, x] && !IntegerQ[p] && NeQ[
n, j] && EqQ[m + n*p + n - j + 1, 0] && (IntegerQ[j] || GtQ[c, 0])
```

Maple [A] (verified)

Time = 0.38 (sec) , antiderivative size = 30, normalized size of antiderivative = 1.00

method	result	size
gosper	$-\frac{2x(bx+a)\left(b\,x^{3}+a\,x^{2}\right)^{\frac{3}{2}}}{5a\left(cx\right)^{\frac{13}{2}}}$	30
orering	$-\frac{2x(bx+a)\left(b\,x^{3}+a\,x^{2}\right)^{\frac{3}{2}}}{5a\left(cx\right)^{\frac{13}{2}}}$	30
default	$-\frac{2\left(b\,x^{3}+a\,x^{2}\right)^{\frac{3}{2}}(bx+a)}{5x^{5}a\,c^{6}\sqrt{cx}}$	35
risch	$-\frac{2\sqrt{x^{2}(bx+a)}\,\left(b^{2}x^{2}+2abx+a^{2}\right)}{5c^{6}x^{3}\sqrt{cx}\,a}$	44

```
int((b*x^3+a*x^2)^(3/2)/(c*x)^(13/2),x,method=_RETURNVERBOSE)
```

```
-2/5*x*(b*x+a)/a*(b*x^3+a*x^2)^(3/2)/(c*x)^(13/2)
```

Fricas [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 45, normalized size of antiderivative = 1.50

$$\int \frac{(ax^2+bx^3)^{3/2}}{(cx)^{13/2}}\,dx=-\frac{2\,(b^2x^2+2\,abx+a^2)\sqrt{bx^3+ax^2}\sqrt{cx}}{5\,ac^7x^4}$$

```
integrate((b*x^3+a*x^2)^(3/2)/(c*x)^(13/2),x, algorithm="fricas")
```

```
-2/5*(b^2*x^2 + 2*a*b*x + a^2)*sqrt(b*x^3 + a*x^2)*sqrt(c*x)/(a*c^7*x^4)
```

Sympy [F(-1)]

Timed out.

$$\int \frac{(ax^2+bx^3)^{3/2}}{(cx)^{13/2}}\,dx=\text{Timed out}$$

```
integrate((b*x**3+a*x**2)**(3/2)/(c*x)**(13/2),x)
```

```
Timed out
```

Maxima [F]

$$\int \frac{(ax^2+bx^3)^{3/2}}{(cx)^{13/2}}\,dx = \int \frac{(bx^3+ax^2)^{\frac{3}{2}}}{(cx)^{\frac{13}{2}}}\,dx$$

```
integrate((b*x^3+a*x^2)^(3/2)/(c*x)^(13/2),x, algorithm="maxima")
```

```
integrate((b*x^3 + a*x^2)^(3/2)/(c*x)^(13/2), x)
```

Giac [A] (verification not implemented)

Time = 0.18 (sec) , antiderivative size = 40, normalized size of antiderivative = 1.33

$$\int \frac{(ax^2+bx^3)^{3/2}}{(cx)^{13/2}}\,dx = -\frac{2\,(bx+a)^{\frac{5}{2}}b^6\mathrm{sgn}(x)}{5\,((bx+a)bc-abc)^{\frac{5}{2}}ac^4|b|}$$

```
integrate((b*x^3+a*x^2)^(3/2)/(c*x)^(13/2),x, algorithm="giac")
```

```
-2/5*(b*x + a)^(5/2)*b^6*sgn(x)/(((b*x + a)*b*c - a*b*c)^(5/2)*a*c^4*abs(b
))
```

Mupad [B] (verification not implemented)

Time = 8.88 (sec) , antiderivative size = 51, normalized size of antiderivative = 1.70

$$\int \frac{(ax^2+bx^3)^{3/2}}{(cx)^{13/2}}\,dx = -\frac{\sqrt{b\,x^3+a\,x^2}\,\left(\frac{2\,a}{5\,c^6}+\frac{4\,b\,x}{5\,c^6}+\frac{2\,b^2\,x^2}{5\,a\,c^6}\right)}{x^3\,\sqrt{c\,x}}$$

```
int((a*x^2 + b*x^3)^(3/2)/(c*x)^(13/2),x)
```

```
-((a*x^2 + b*x^3)^(1/2)*((2*a)/(5*c^6) + (4*b*x)/(5*c^6) + (2*b^2*x^2)/(5*
a*c^6)))/(x^3*(c*x)^(1/2))
```

Reduce [B] (verification not implemented)

Time = 0.50 (sec) , antiderivative size = 66, normalized size of antiderivative = 2.20

$$\int \frac{(ax^2+bx^3)^{3/2}}{(cx)^{13/2}}\,dx = \frac{2\sqrt{c}\left(-\sqrt{x}\,\sqrt{bx+a}\,a^2-2\sqrt{x}\,\sqrt{bx+a}\,abx-\sqrt{x}\,\sqrt{bx+a}\,b^2x^2-\sqrt{b}\,b^2x^3\right)}{5a\,c^7x^3}$$

```
int((b*x^3+a*x^2)^(3/2)/(c*x)^(13/2),x)
```

```
(2*sqrt(c)*( - sqrt(x)*sqrt(a + b*x)*a**2 - 2*sqrt(x)*sqrt(a + b*x)*a*b*x
- sqrt(x)*sqrt(a + b*x)*b**2*x**2 - sqrt(b)*b**2*x**3))/(5*a*c**7*x**3)
```

3.344 $\int \frac{(ax^2+bx^3)^{3/2}}{(cx)^{15/2}}\,dx$

Optimal result

Integrand size = 23, antiderivative size = 61

$$\int \frac{(ax^2+bx^3)^{3/2}}{(cx)^{15/2}}\,dx = -\frac{2c(ax^2+bx^3)^{5/2}}{7a(cx)^{17/2}} + \frac{4b(ax^2+bx^3)^{5/2}}{35a^2(cx)^{15/2}}$$

```
-2/7*c*(b*x^3+a*x^2)^(5/2)/a/(c*x)^(17/2)+4/35*b*(b*x^3+a*x^2)^(5/2)/a^2/(
c*x)^(15/2)
```

Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 36, normalized size of antiderivative = 0.59

$$\int \frac{(ax^2+bx^3)^{3/2}}{(cx)^{15/2}}\,dx = -\frac{2c(5a-2bx)\left(x^2(a+bx)\right)^{5/2}}{35a^2(cx)^{17/2}}$$

```
Integrate[(a*x^2 + b*x^3)^(3/2)/(c*x)^(15/2),x]
```

```
(-2*c*(5*a - 2*b*x)*(x^2*(a + b*x))^(5/2))/(35*a^2*(c*x)^(17/2))
```

Rubi [A] (verified)

Time = 0.33 (sec) , antiderivative size = 61, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.087$, Rules used = {1922, 1920}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(ax^2+bx^3)^{3/2}}{(cx)^{15/2}}dx$$
$$\Big\downarrow \text{1922}$$
$$-\frac{2b\int \frac{(bx^3+ax^2)^{3/2}}{(cx)^{13/2}}dx}{7ac}-\frac{2c(ax^2+bx^3)^{5/2}}{7a(cx)^{17/2}}$$
$$\Big\downarrow \text{1920}$$
$$\frac{4b(ax^2+bx^3)^{5/2}}{35a^2(cx)^{15/2}}-\frac{2c(ax^2+bx^3)^{5/2}}{7a(cx)^{17/2}}$$

```
Int[(a*x^2 + b*x^3)^(3/2)/(c*x)^(15/2),x]
```

```
(-2*c*(a*x^2 + b*x^3)^(5/2))/(7*a*(c*x)^(17/2)) + (4*b*(a*x^2 + b*x^3)^(5/
2))/(35*a^2*(c*x)^(15/2))
```

Defintions of rubi rules used

```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> Simp[(-c^(j - 1))*(c*x)^(m - j + 1)*((a*x^j + b*x^n)^(p + 1)/(a*(n - j
)*(p + 1))), x] /; FreeQ[{a, b, c, j, m, n, p}, x] &&  !IntegerQ[p] && NeQ[
n, j] && EqQ[m + n*p + n - j + 1, 0] && (IntegerQ[j] || GtQ[c, 0])
```

```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> Simp[c^(j - 1)*(c*x)^(m - j + 1)*((a*x^j + b*x^n)^(p + 1)/(a*(m + j*p
+ 1))), x] - Simp[b*((m + n*p + n - j + 1)/(a*c^(n - j)*(m + j*p + 1)))   I
nt[(c*x)^(m + n - j)*(a*x^j + b*x^n)^p, x], x] /; FreeQ[{a, b, c, j, m, n,
p}, x] &&  !IntegerQ[p] && NeQ[n, j] && ILtQ[Simplify[(m + n*p + n - j + 1)
/(n - j)], 0] && NeQ[m + j*p + 1, 0] && (IntegersQ[j, n] || GtQ[c, 0])
```

Maple [A] (verified)

Time = 0.38 (sec) , antiderivative size = 38, normalized size of antiderivative = 0.62

method	result	size
gosper	$-\frac{2x(bx+a)(-2bx+5a)\left(b\,x^{3}+a\,x^{2}\right)^{\frac{3}{2}}}{35a^{2}(cx)^{\frac{15}{2}}}$	38
orering	$-\frac{2x(bx+a)(-2bx+5a)\left(b\,x^{3}+a\,x^{2}\right)^{\frac{3}{2}}}{35a^{2}(cx)^{\frac{15}{2}}}$	38
default	$-\frac{2\left(b\,x^{3}+a\,x^{2}\right)^{\frac{3}{2}}\left(-2b^{2}x^{2}+3abx+5a^{2}\right)}{35x^{6}a^{2}c^{7}\sqrt{cx}}$	49
risch	$-\frac{2\sqrt{x^{2}(bx+a)}\,\left(-2b^{3}x^{3}+a\,b^{2}x^{2}+8a^{2}bx+5a^{3}\right)}{35c^{7}x^{4}\sqrt{cx}\,a^{2}}$	57

```
int((b*x^3+a*x^2)^(3/2)/(c*x)^(15/2),x,method=_RETURNVERBOSE)
```

```
-2/35*x*(b*x+a)*(-2*b*x+5*a)*(b*x^3+a*x^2)^(3/2)/a^2/(c*x)^(15/2)
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 59, normalized size of antiderivative = 0.97

$$\int \frac{\left(ax^2+bx^3\right)^{3/2}}{(cx)^{15/2}}\,dx = \frac{2\left(2\,b^3x^3-ab^2x^2-8\,a^2bx-5\,a^3\right)\sqrt{bx^3+ax^2}\sqrt{cx}}{35\,a^2c^8x^5}$$

```
integrate((b*x^3+a*x^2)^(3/2)/(c*x)^(15/2),x, algorithm="fricas")
```

```
2/35*(2*b^3*x^3 - a*b^2*x^2 - 8*a^2*b*x - 5*a^3)*sqrt(b*x^3 + a*x^2)*sqrt(
c*x)/(a^2*c^8*x^5)
```

Sympy [F(-1)]

Timed out.

$$\int \frac{(ax^2+bx^3)^{3/2}}{(cx)^{15/2}}\,dx = \text{Timed out}$$

```
integrate((b*x**3+a*x**2)**(3/2)/(c*x)**(15/2),x)
```

```
Timed out
```

Maxima [F]

$$\int \frac{(ax^2+bx^3)^{3/2}}{(cx)^{15/2}}\,dx = \int \frac{(bx^3+ax^2)^{\frac{3}{2}}}{(cx)^{\frac{15}{2}}}\,dx$$

```
integrate((b*x^3+a*x^2)^(3/2)/(c*x)^(15/2),x, algorithm="maxima")
```

```
integrate((b*x^3 + a*x^2)^(3/2)/(c*x)^(15/2), x)
```

Giac [A] (verification not implemented)

Time = 0.18 (sec) , antiderivative size = 67, normalized size of antiderivative = 1.10

$$\int \frac{(ax^2+bx^3)^{3/2}}{(cx)^{15/2}}\,dx = \frac{2\left(\frac{2\,(bx+a)b^3c^3\mathrm{sgn}(x)}{a^2} - \frac{7\,b^3c^3\mathrm{sgn}(x)}{a}\right)(bx+a)^{\frac{5}{2}}b^5}{35\,((bx+a)bc-abc)^{\frac{7}{2}}c^7|b|}$$

```
integrate((b*x^3+a*x^2)^(3/2)/(c*x)^(15/2),x, algorithm="giac")
```

```
2/35*(2*(b*x + a)*b^3*c^3*sgn(x)/a^2 - 7*b^3*c^3*sgn(x)/a)*(b*x + a)^(5/2)
*b^5/(((b*x + a)*b*c - a*b*c)^(7/2)*c^7*abs(b))
```

Mupad [B] (verification not implemented)

Time = 8.74 (sec) , antiderivative size = 65, normalized size of antiderivative = 1.07

$$\int \frac{(ax^2+bx^3)^{3/2}}{(cx)^{15/2}}\,dx = -\frac{\sqrt{b\,x^3+a\,x^2}\,\left(\frac{2\,a}{7\,c^7}+\frac{16\,b\,x}{35\,c^7}+\frac{2\,b^2\,x^2}{35\,a\,c^7}-\frac{4\,b^3\,x^3}{35\,a^2\,c^7}\right)}{x^4\,\sqrt{c\,x}}$$

```
int((a*x^2 + b*x^3)^(3/2)/(c*x)^(15/2),x)
```

```
-((a*x^2 + b*x^3)^(1/2)*((2*a)/(7*c^7) + (16*b*x)/(35*c^7) + (2*b^2*x^2)/(
35*a*c^7) - (4*b^3*x^3)/(35*a^2*c^7)))/(x^4*(c*x)^(1/2))
```

Reduce [B] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 85, normalized size of antiderivative = 1.39

$$\int \frac{(ax^2+bx^3)^{3/2}}{(cx)^{15/2}}\,dx = \frac{2\sqrt{c}\left(-5\sqrt{x}\,\sqrt{bx+a}\,a^3-8\sqrt{x}\,\sqrt{bx+a}\,a^2bx-\sqrt{x}\,\sqrt{bx+a}\,a\,b^2x^2+2\sqrt{x}\,\sqrt{bx+a}\right.}{35a^2c^8x^4}$$

```
int((b*x^3+a*x^2)^(3/2)/(c*x)^(15/2),x)
```

```
(2*sqrt(c)*( - 5*sqrt(x)*sqrt(a + b*x)*a**3 - 8*sqrt(x)*sqrt(a + b*x)*a**2
*b*x - sqrt(x)*sqrt(a + b*x)*a*b**2*x**2 + 2*sqrt(x)*sqrt(a + b*x)*b**3*x*
*3 - 2*sqrt(b)*b**3*x**4))/(35*a**2*c**8*x**4)
```

3.345 $\int \frac{(ax^2+bx^3)^{3/2}}{(cx)^{17/2}}\,dx$

Optimal result

Integrand size = 23, antiderivative size = 96

$$\int \frac{(ax^2+bx^3)^{3/2}}{(cx)^{17/2}}\,dx = -\frac{2c(ax^2+bx^3)^{5/2}}{9a(cx)^{19/2}}+\frac{8b(ax^2+bx^3)^{5/2}}{63a^2(cx)^{17/2}}-\frac{16b^2(ax^2+bx^3)^{5/2}}{315a^3c(cx)^{15/2}}$$

```
-2/9*c*(b*x^3+a*x^2)^(5/2)/a/(c*x)^(19/2)+8/63*b*(b*x^3+a*x^2)^(5/2)/a^2/(
c*x)^(17/2)-16/315*b^2*(b*x^3+a*x^2)^(5/2)/a^3/c/(c*x)^(15/2)
```

Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 47, normalized size of antiderivative = 0.49

$$\int \frac{(ax^2+bx^3)^{3/2}}{(cx)^{17/2}}\,dx = -\frac{2c(x^2(a+bx))^{5/2}(35a^2-20abx+8b^2x^2)}{315a^3(cx)^{19/2}}$$

```
Integrate[(a*x^2 + b*x^3)^(3/2)/(c*x)^(17/2),x]
```

```
(-2*c*(x^2*(a + b*x))^(5/2)*(35*a^2 - 20*a*b*x + 8*b^2*x^2))/(315*a^3*(c*x
)^(19/2))
```

Rubi [A] (verified)

Time = 0.42 (sec) , antiderivative size = 103, normalized size of antiderivative = 1.07, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.130, Rules used = {1922, 1922, 1920}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(ax^2+bx^3)^{3/2}}{(cx)^{17/2}}\,dx$$

$$\Big\downarrow \ \textcolor{blue}{1922}$$

$$-\frac{4b\int \frac{(bx^3+ax^2)^{3/2}}{(cx)^{15/2}}dx}{9ac}-\frac{2c(ax^2+bx^3)^{5/2}}{9a(cx)^{19/2}}$$

$$\Big\downarrow \ \textcolor{blue}{1922}$$

$$-\frac{4b\left(-\frac{2b\int \frac{(bx^3+ax^2)^{3/2}}{(cx)^{13/2}}dx}{7ac}-\frac{2c(ax^2+bx^3)^{5/2}}{7a(cx)^{17/2}}\right)}{9ac}-\frac{2c(ax^2+bx^3)^{5/2}}{9a(cx)^{19/2}}$$

$$\Big\downarrow \ \textcolor{blue}{1920}$$

$$-\frac{4b\left(\frac{4b(ax^2+bx^3)^{5/2}}{35a^2(cx)^{15/2}}-\frac{2c(ax^2+bx^3)^{5/2}}{7a(cx)^{17/2}}\right)}{9ac}-\frac{2c(ax^2+bx^3)^{5/2}}{9a(cx)^{19/2}}$$

```
Int[(a*x^2 + b*x^3)^(3/2)/(c*x)^(17/2),x]
```

```
(-2*c*(a*x^2 + b*x^3)^(5/2))/(9*a*(c*x)^(19/2)) - (4*b*((-2*c*(a*x^2 + b*x
^3)^(5/2))/(7*a*(c*x)^(17/2)) + (4*b*(a*x^2 + b*x^3)^(5/2))/(35*a^2*(c*x)^
(15/2))))/(9*a*c)
```

Defintions of rubi rules used

```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> Simp[(-c^(j - 1))*(c*x)^(m - j + 1)*((a*x^j + b*x^n)^(p + 1)/(a*(n - j
)*(p + 1))), x] /; FreeQ[{a, b, c, j, m, n, p}, x] &&  !IntegerQ[p] && NeQ[
n, j] && EqQ[m + n*p + n - j + 1, 0] && (IntegerQ[j] || GtQ[c, 0])
```

```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> Simp[c^(j - 1)*(c*x)^(m - j + 1)*((a*x^j + b*x^n)^(p + 1)/(a*(m + j*p
+ 1))), x] - Simp[b*((m + n*p + n - j + 1)/(a*c^(n - j)*(m + j*p + 1)))   I
nt[(c*x)^(m + n - j)*(a*x^j + b*x^n)^p, x], x] /; FreeQ[{a, b, c, j, m, n,
p}, x] &&  !IntegerQ[p] && NeQ[n, j] && ILtQ[Simplify[(m + n*p + n - j + 1)
/(n - j)], 0] && NeQ[m + j*p + 1, 0] && (IntegersQ[j, n] || GtQ[c, 0])
```

Maple [A] (verified)

Time = 0.38 (sec) , antiderivative size = 49, normalized size of antiderivative = 0.51

method	result	size
gosper	$-\frac{2x(bx+a)\left(8b^{2}x^{2}-20abx+35a^{2}\right)\left(b\,x^{3}+a\,x^{2}\right)^{\frac{3}{2}}}{315a^{3}\left(cx\right)^{\frac{17}{2}}}$	49
orering	$-\frac{2x(bx+a)\left(8b^{2}x^{2}-20abx+35a^{2}\right)\left(b\,x^{3}+a\,x^{2}\right)^{\frac{3}{2}}}{315a^{3}\left(cx\right)^{\frac{17}{2}}}$	49
default	$-\frac{2\left(b\,x^{3}+a\,x^{2}\right)^{\frac{3}{2}}\left(8b^{3}x^{3}-12a\,b^{2}x^{2}+15a^{2}bx+35a^{3}\right)}{315x^{7}a^{3}c^{8}\sqrt{cx}}$	60
risch	$-\frac{2\sqrt{x^{2}(bx+a)}\,\left(8b^{4}x^{4}-4a\,b^{3}x^{3}+3a^{2}b^{2}x^{2}+50a^{3}bx+35a^{4}\right)}{315c^{8}x^{5}\sqrt{cx}\,a^{3}}$	69

```
int((b*x^3+a*x^2)^(3/2)/(c*x)^(17/2),x,method=_RETURNVERBOSE)
```

```
-2/315*x*(b*x+a)*(8*b^2*x^2-20*a*b*x+35*a^2)*(b*x^3+a*x^2)^(3/2)/a^3/(c*x)
^(17/2)
```

Fricas [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 70, normalized size of antiderivative = 0.73

$$\int \frac{(ax^2+bx^3)^{3/2}}{(cx)^{17/2}}\,dx = -\frac{2\,(8\,b^4x^4-4\,ab^3x^3+3\,a^2b^2x^2+50\,a^3bx+35\,a^4)\sqrt{bx^3+ax^2}\sqrt{cx}}{315\,a^3c^9x^6}$$

```
integrate((b*x^3+a*x^2)^(3/2)/(c*x)^(17/2),x, algorithm="fricas")
```

```
-2/315*(8*b^4*x^4 - 4*a*b^3*x^3 + 3*a^2*b^2*x^2 + 50*a^3*b*x + 35*a^4)*sqr
t(b*x^3 + a*x^2)*sqrt(c*x)/(a^3*c^9*x^6)
```

Sympy [F(-1)]

Timed out.

$$\int \frac{(ax^2+bx^3)^{3/2}}{(cx)^{17/2}}\,dx = \text{Timed out}$$

```
integrate((b*x**3+a*x**2)**(3/2)/(c*x)**(17/2),x)
```

```
Timed out
```

Maxima [F]

$$\int \frac{(ax^2+bx^3)^{3/2}}{(cx)^{17/2}}\,dx = \int \frac{(bx^3+ax^2)^{\frac{3}{2}}}{(cx)^{\frac{17}{2}}}\,dx$$

```
integrate((b*x^3+a*x^2)^(3/2)/(c*x)^(17/2),x, algorithm="maxima")
```

```
integrate((b*x^3 + a*x^2)^(3/2)/(c*x)^(17/2), x)
```

Giac [A] (verification not implemented)

Time = 0.18 (sec) , antiderivative size = 86, normalized size of antiderivative = 0.90

$$\int \frac{(ax^2+bx^3)^{3/2}}{(cx)^{17/2}}\,dx = -\frac{2\left(\frac{63\,b^9c^4\mathrm{sgn}(x)}{a}+4\left(\frac{2\,(bx+a)b^9c^4\mathrm{sgn}(x)}{a^3}-\frac{9\,b^9c^4\mathrm{sgn}(x)}{a^2}\right)(bx+a)\right)(bx+a)^{\frac{5}{2}}b}{315\,((bx+a)bc-abc)^{\frac{9}{2}}c^8|b|}$$

```
integrate((b*x^3+a*x^2)^(3/2)/(c*x)^(17/2),x, algorithm="giac")
```

```
-2/315*(63*b^9*c^4*sgn(x)/a + 4*(2*(b*x + a)*b^9*c^4*sgn(x)/a^3 - 9*b^9*c^
4*sgn(x)/a^2)*(b*x + a))*(b*x + a)^(5/2)*b/(((b*x + a)*b*c - a*b*c)^(9/2)*
c^8*abs(b))
```

Mupad [B] (verification not implemented)

Time = 8.79 (sec) , antiderivative size = 79, normalized size of antiderivative = 0.82

$$\int \frac{(ax^2+bx^3)^{3/2}}{(cx)^{17/2}}\,dx = -\frac{\sqrt{b\,x^3+a\,x^2}\left(\frac{2\,a}{9\,c^8}+\frac{20\,b\,x}{63\,c^8}+\frac{2\,b^2\,x^2}{105\,a\,c^8}-\frac{8\,b^3\,x^3}{315\,a^2\,c^8}+\frac{16\,b^4\,x^4}{315\,a^3\,c^8}\right)}{x^5\,\sqrt{c\,x}}$$

```
int((a*x^2 + b*x^3)^(3/2)/(c*x)^(17/2),x)
```

```
-((a*x^2 + b*x^3)^(1/2)*((2*a)/(9*c^8) + (20*b*x)/(63*c^8) + (2*b^2*x^2)/(
105*a*c^8) - (8*b^3*x^3)/(315*a^2*c^8) + (16*b^4*x^4)/(315*a^3*c^8)))/(x^5
*(c*x)^(1/2))
```

Reduce [B] (verification not implemented)

Time = 0.46 (sec) , antiderivative size = 104, normalized size of antiderivative = 1.08

$$\int \frac{(ax^2+bx^3)^{3/2}}{(cx)^{17/2}}\,dx = \frac{2\sqrt{c}\left(-35\sqrt{x}\,\sqrt{bx+a}\,a^4-50\sqrt{x}\,\sqrt{bx+a}\,a^3bx-3\sqrt{x}\,\sqrt{bx+a}\,a^2b^2x^2+4\sqrt{x}\,\sqrt{bx\right.}{315a^3c^9x^5}$$

```
int((b*x^3+a*x^2)^(3/2)/(c*x)^(17/2),x)
```

```
(2*sqrt(c)*( - 35*sqrt(x)*sqrt(a + b*x)*a**4 - 50*sqrt(x)*sqrt(a + b*x)*a*
*3*b*x - 3*sqrt(x)*sqrt(a + b*x)*a**2*b**2*x**2 + 4*sqrt(x)*sqrt(a + b*x)*
a*b**3*x**3 - 8*sqrt(x)*sqrt(a + b*x)*b**4*x**4 + 8*sqrt(b)*b**4*x**5))/(3
15*a**3*c**9*x**5)
```

3.346 $\int \frac{(ax^2+bx^3)^{3/2}}{(cx)^{19/2}}\,dx$

Optimal result

Integrand size = 23, antiderivative size = 131

$$\int \frac{(ax^2+bx^3)^{3/2}}{(cx)^{19/2}}\,dx = -\frac{2c(ax^2+bx^3)^{5/2}}{11a(cx)^{21/2}}+\frac{4b(ax^2+bx^3)^{5/2}}{33a^2(cx)^{19/2}} -\frac{16b^2(ax^2+bx^3)^{5/2}}{231a^3c(cx)^{17/2}}+\frac{32b^3(ax^2+bx^3)^{5/2}}{1155a^4c^2(cx)^{15/2}}$$

```
-2/11*c*(b*x^3+a*x^2)^(5/2)/a/(c*x)^(21/2)+4/33*b*(b*x^3+a*x^2)^(5/2)/a^2/
(c*x)^(19/2)-16/231*b^2*(b*x^3+a*x^2)^(5/2)/a^3/c/(c*x)^(17/2)+32/1155*b^3
*(b*x^3+a*x^2)^(5/2)/a^4/c^2/(c*x)^(15/2)
```

Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 58, normalized size of antiderivative = 0.44

$$\int \frac{(ax^2+bx^3)^{3/2}}{(cx)^{19/2}}\,dx = -\frac{2c(x^2(a+bx))^{5/2}\,(105a^3-70a^2bx+40ab^2x^2-16b^3x^3)}{1155a^4(cx)^{21/2}}$$

```
Integrate[(a*x^2 + b*x^3)^(3/2)/(c*x)^(19/2),x]
```

```
(-2*c*(x^2*(a + b*x))^(5/2)*(105*a^3 - 70*a^2*b*x + 40*a*b^2*x^2 - 16*b^3*
x^3))/(1155*a^4*(c*x)^(21/2))
```

Rubi [A] (verified)

Time = 0.52 (sec) , antiderivative size = 145, normalized size of antiderivative = 1.11, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.174$, Rules used = {1922, 1922, 1922, 1920}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\left(ax^2+bx^3\right)^{3/2}}{(cx)^{19/2}}dx$$

$$\Big\downarrow \text{1922}$$

$$-\frac{6b\int \frac{\left(bx^3+ax^2\right)^{3/2}}{(cx)^{17/2}}dx}{11ac}-\frac{2c\left(ax^2+bx^3\right)^{5/2}}{11a(cx)^{21/2}}$$

$$\Big\downarrow \text{1922}$$

$$-\frac{6b\left(-\frac{4b\int \frac{\left(bx^3+ax^2\right)^{3/2}}{(cx)^{15/2}}dx}{9ac}-\frac{2c\left(ax^2+bx^3\right)^{5/2}}{9a(cx)^{19/2}}\right)}{11ac}-\frac{2c\left(ax^2+bx^3\right)^{5/2}}{11a(cx)^{21/2}}$$

$$\Big\downarrow \text{1922}$$

$$-\frac{6b\left(-\frac{4b\left(-\frac{2b\int \frac{\left(bx^3+ax^2\right)^{3/2}}{(cx)^{13/2}}dx}{7ac}-\frac{2c\left(ax^2+bx^3\right)^{5/2}}{7a(cx)^{17/2}}\right)}{9ac}-\frac{2c\left(ax^2+bx^3\right)^{5/2}}{9a(cx)^{19/2}}\right)}{11ac}-\frac{2c\left(ax^2+bx^3\right)^{5/2}}{11a(cx)^{21/2}}$$

$$\Big\downarrow \text{1920}$$

$$-\frac{6b\left(-\frac{4b\left(\frac{4b\left(ax^2+bx^3\right)^{5/2}}{35a^2(cx)^{15/2}}-\frac{2c\left(ax^2+bx^3\right)^{5/2}}{7a(cx)^{17/2}}\right)}{9ac}-\frac{2c\left(ax^2+bx^3\right)^{5/2}}{9a(cx)^{19/2}}\right)}{11ac}-\frac{2c\left(ax^2+bx^3\right)^{5/2}}{11a(cx)^{21/2}}$$

```
Int[(a*x^2 + b*x^3)^(3/2)/(c*x)^(19/2),x]
```

```
(-2*c*(a*x^2 + b*x^3)^(5/2))/(11*a*(c*x)^(21/2)) - (6*b*((-2*c*(a*x^2 + b*
x^3)^(5/2))/(9*a*(c*x)^(19/2)) - (4*b*((-2*c*(a*x^2 + b*x^3)^(5/2))/(7*a*(
c*x)^(17/2)) + (4*b*(a*x^2 + b*x^3)^(5/2))/(35*a^2*(c*x)^(15/2))))/(9*a*c)
))/(11*a*c)
```

Defintions of rubi rules used

```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> Simp[(-c^(j - 1))*(c*x)^(m - j + 1)*((a*x^j + b*x^n)^(p + 1)/(a*(n - j
)*(p + 1))), x] /; FreeQ[{a, b, c, j, m, n, p}, x] &&  !IntegerQ[p] && NeQ[
n, j] && EqQ[m + n*p + n - j + 1, 0] && (IntegerQ[j] || GtQ[c, 0])
```

```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> Simp[c^(j - 1)*(c*x)^(m - j + 1)*((a*x^j + b*x^n)^(p + 1)/(a*(m + j*p
+ 1))), x] - Simp[b*((m + n*p + n - j + 1)/(a*c^(n - j)*(m + j*p + 1)))   I
nt[(c*x)^(m + n - j)*(a*x^j + b*x^n)^p, x], x] /; FreeQ[{a, b, c, j, m, n,
p}, x] &&  !IntegerQ[p] && NeQ[n, j] && ILtQ[Simplify[(m + n*p + n - j + 1)
/(n - j)], 0] && NeQ[m + j*p + 1, 0] && (IntegersQ[j, n] || GtQ[c, 0])
```

Maple [A] (verified)

Time = 0.39 (sec) , antiderivative size = 60, normalized size of antiderivative = 0.46

method	result	size
gosper	$-\frac{2x(bx+a)\left(-16b^3x^3+40a\,b^2x^2-70a^2bx+105a^3\right)\left(b\,x^3+a\,x^2\right)^{\frac{3}{2}}}{1155a^4(cx)^{\frac{19}{2}}}$	60
orering	$-\frac{2x(bx+a)\left(-16b^3x^3+40a\,b^2x^2-70a^2bx+105a^3\right)\left(b\,x^3+a\,x^2\right)^{\frac{3}{2}}}{1155a^4(cx)^{\frac{19}{2}}}$	60
default	$-\frac{2\left(b\,x^3+a\,x^2\right)^{\frac{3}{2}}\left(-16b^4x^4+24a\,b^3x^3-30a^2b^2x^2+35a^3bx+105a^4\right)}{1155x^8a^4c^9\sqrt{cx}}$	71
risch	$-\frac{2\sqrt{x^2(bx+a)}\left(-16b^5x^5+8a\,b^4x^4-6a^2b^3x^3+5a^3b^2x^2+140a^4bx+105a^5\right)}{1155c^9x^6\sqrt{cx}\,a^4}$	80

```
int((b*x^3+a*x^2)^(3/2)/(c*x)^(19/2),x,method=_RETURNVERBOSE)
```

```
-2/1155*x*(b*x+a)*(-16*b^3*x^3+40*a*b^2*x^2-70*a^2*b*x+105*a^3)*(b*x^3+a*x
^2)^(3/2)/a^4/(c*x)^(19/2)
```

Fricas [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 81, normalized size of antiderivative = 0.62

$$\int \frac{\left(ax^2+bx^3\right)^{3/2}}{(cx)^{19/2}}\,dx = \frac{2\left(16\,b^5x^5-8\,ab^4x^4+6\,a^2b^3x^3-5\,a^3b^2x^2-140\,a^4bx-105\,a^5\right)\sqrt{bx^3+ax^2}\sqrt{cx}}{1155\,a^4c^{10}x^7}$$

```
integrate((b*x^3+a*x^2)^(3/2)/(c*x)^(19/2),x, algorithm="fricas")
```

```
2/1155*(16*b^5*x^5 - 8*a*b^4*x^4 + 6*a^2*b^3*x^3 - 5*a^3*b^2*x^2 - 140*a^4
*b*x - 105*a^5)*sqrt(b*x^3 + a*x^2)*sqrt(c*x)/(a^4*c^10*x^7)
```

Sympy [F(-1)]

Timed out.

$$\int \frac{(ax^2 + bx^3)^{3/2}}{(cx)^{19/2}}\, dx = \text{Timed out}$$

```
integrate((b*x**3+a*x**2)**(3/2)/(c*x)**(19/2),x)
```

```
Timed out
```

Maxima [F]

$$\int \frac{(ax^2 + bx^3)^{3/2}}{(cx)^{19/2}}\, dx = \int \frac{(bx^3 + ax^2)^{\frac{3}{2}}}{(cx)^{\frac{19}{2}}}\, dx$$

```
integrate((b*x^3+a*x^2)^(3/2)/(c*x)^(19/2),x, algorithm="maxima")
```

```
integrate((b*x^3 + a*x^2)^(3/2)/(c*x)^(19/2), x)
```

Giac [A] (verification not implemented)

Time = 0.17 (sec) , antiderivative size = 109, normalized size of antiderivative = 0.83

$$\int \frac{(ax^2 + bx^3)^{3/2}}{(cx)^{19/2}}\, dx = -\frac{2\left(\frac{231\,b^5c^5\mathrm{sgn}(x)}{a} - 2\left(\frac{99\,b^5c^5\mathrm{sgn}(x)}{a^2} + 4\left(\frac{2\,(bx+a)b^5c^5\mathrm{sgn}(x)}{a^4} - \frac{11\,b^5c^5\mathrm{sgn}(x)}{a^3}\right)(bx+a)\right)(bx+a)\right)(bx+a)^{\frac{5}{2}}b^7}{1155\,((bx+a)bc - abc)^{\frac{11}{2}}c^9|b|}$$

```
integrate((b*x^3+a*x^2)^(3/2)/(c*x)^(19/2),x, algorithm="giac")
```

```
-2/1155*(231*b^5*c^5*sgn(x)/a - 2*(99*b^5*c^5*sgn(x)/a^2 + 4*(2*(b*x + a)*
b^5*c^5*sgn(x)/a^4 - 11*b^5*c^5*sgn(x)/a^3)*(b*x + a))*(b*x + a))*(b*x + a
)^(5/2)*b^7/(((b*x + a)*b*c - a*b*c)^(11/2)*c^9*abs(b))
```

Mupad [B] (verification not implemented)

Time = 9.07 (sec) , antiderivative size = 93, normalized size of antiderivative = 0.71

$$\int \frac{(ax^2+bx^3)^{3/2}}{(cx)^{19/2}}\,dx = -\frac{\sqrt{b\,x^3+a\,x^2}\,\left(\frac{2\,a}{11\,c^9}+\frac{8\,b\,x}{33\,c^9}+\frac{2\,b^2\,x^2}{231\,a\,c^9}-\frac{4\,b^3\,x^3}{385\,a^2\,c^9}+\frac{16\,b^4\,x^4}{1155\,a^3\,c^9}-\frac{32\,b^5\,x^5}{1155\,a^4\,c^9}\right)}{x^6\,\sqrt{c\,x}}$$

```
int((a*x^2 + b*x^3)^(3/2)/(c*x)^(19/2),x)
```

```
-((a*x^2 + b*x^3)^(1/2)*((2*a)/(11*c^9) + (8*b*x)/(33*c^9) + (2*b^2*x^2)/(
231*a*c^9) - (4*b^3*x^3)/(385*a^2*c^9) + (16*b^4*x^4)/(1155*a^3*c^9) - (32
*b^5*x^5)/(1155*a^4*c^9)))/(x^6*(c*x)^(1/2))
```

Reduce [B] (verification not implemented)

Time = 0.49 (sec) , antiderivative size = 123, normalized size of antiderivative = 0.94

$$\int \frac{(ax^2+bx^3)^{3/2}}{(cx)^{19/2}}\,dx = \frac{2\sqrt{c}\left(-105\sqrt{x}\,\sqrt{bx+a}\,a^5-140\sqrt{x}\,\sqrt{bx+a}\,a^4bx-5\sqrt{x}\,\sqrt{bx+a}\,a^3b^2x^2+6\sqrt{x}\,\sqrt{\quad}\right.}{1155a^4c^{10}x}$$

```
int((b*x^3+a*x^2)^(3/2)/(c*x)^(19/2),x)
```

```
(2*sqrt(c)*( - 105*sqrt(x)*sqrt(a + b*x)*a**5 - 140*sqrt(x)*sqrt(a + b*x)*
a**4*b*x - 5*sqrt(x)*sqrt(a + b*x)*a**3*b**2*x**2 + 6*sqrt(x)*sqrt(a + b*x
)*a**2*b**3*x**3 - 8*sqrt(x)*sqrt(a + b*x)*a*b**4*x**4 + 16*sqrt(x)*sqrt(a
 + b*x)*b**5*x**5 - 16*sqrt(b)*b**5*x**6))/(1155*a**4*c**10*x**6)
```

3.347 $\int \frac{(cx)^{7/2}}{\sqrt{ax^2+bx^3}}\,dx$

Optimal result

Integrand size = 23, antiderivative size = 152

$$\int \frac{(cx)^{7/2}}{\sqrt{ax^2+bx^3}}\,dx = \frac{5a^2c^4\sqrt{ax^2+bx^3}}{8b^3\sqrt{cx}} - \frac{5ac^3\sqrt{cx}\sqrt{ax^2+bx^3}}{12b^2} + \frac{c^2(cx)^{3/2}\sqrt{ax^2+bx^3}}{3b} - \frac{5a^3c^{7/2}\operatorname{arctanh}\left(\frac{\sqrt{b}(cx)^{3/2}}{c^{3/2}\sqrt{ax^2+bx^3}}\right)}{8b^{7/2}}$$

```
5/8*a^2*c^4*(b*x^3+a*x^2)^(1/2)/b^3/(c*x)^(1/2)-5/12*a*c^3*(c*x)^(1/2)*(b*
x^3+a*x^2)^(1/2)/b^2+1/3*c^2*(c*x)^(3/2)*(b*x^3+a*x^2)^(1/2)/b-5/8*a^3*c^(
7/2)*arctanh(b^(1/2)*(c*x)^(3/2)/c^(3/2)/(b*x^3+a*x^2)^(1/2))/b^(7/2)
```

Mathematica [A] (verified)

Time = 0.05 (sec) , antiderivative size = 124, normalized size of antiderivative = 0.82

$$\int \frac{(cx)^{7/2}}{\sqrt{ax^2+bx^3}}\,dx = \frac{c^3\sqrt{x}\sqrt{cx}\left(\sqrt{b}\sqrt{x}(15a^3+5a^2bx-2ab^2x^2+8b^3x^3)+30a^3\sqrt{a+bx}\operatorname{arctanh}\left(\frac{\sqrt{b}\sqrt{x}}{\sqrt{a}-\sqrt{a+bx}}\right.\right.}{24b^{7/2}\sqrt{x^2(a+bx)}}$$

```
Integrate[(c*x)^(7/2)/Sqrt[a*x^2 + b*x^3],x]
```

```
(c^3*Sqrt[x]*Sqrt[c*x]*(Sqrt[b]*Sqrt[x]*(15*a^3 + 5*a^2*b*x - 2*a*b^2*x^2
+ 8*b^3*x^3) + 30*a^3*Sqrt[a + b*x]*ArcTanh[(Sqrt[b]*Sqrt[x])/(Sqrt[a] - S
qrt[a + b*x])]))/(24*b^(7/2)*Sqrt[x^2*(a + b*x)])
```

Rubi [A] (verified)

Time = 0.60 (sec) , antiderivative size = 162, normalized size of antiderivative = 1.07, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.261, Rules used = {1930, 1930, 1930, 1937, 1935, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(cx)^{7/2}}{\sqrt{ax^2+bx^3}}dx$$

$$\Big\downarrow \text{1930}$$

$$\frac{c^2(cx)^{3/2}\sqrt{ax^2+bx^3}}{3b}-\frac{5ac\int \frac{(cx)^{5/2}}{\sqrt{bx^3+ax^2}}dx}{6b}$$

$$\Big\downarrow \text{1930}$$

$$\frac{c^2(cx)^{3/2}\sqrt{ax^2+bx^3}}{3b}-\frac{5ac\left(\frac{c^2\sqrt{cx}\sqrt{ax^2+bx^3}}{2b}-\frac{3ac\int \frac{(cx)^{3/2}}{\sqrt{bx^3+ax^2}}dx}{4b}\right)}{6b}$$

$$\Big\downarrow \text{1930}$$

$$\frac{c^2(cx)^{3/2}\sqrt{ax^2+bx^3}}{3b}-\frac{5ac\left(\frac{c^2\sqrt{cx}\sqrt{ax^2+bx^3}}{2b}-\frac{3ac\left(\frac{c^2\sqrt{ax^2+bx^3}}{b\sqrt{cx}}-\frac{ac\int \frac{\sqrt{cx}}{\sqrt{bx^3+ax^2}}dx}{2b}\right)}{4b}\right)}{6b}$$

$$\Big\downarrow \text{1937}$$

$$\frac{c^2(cx)^{3/2}\sqrt{ax^2+bx^3}}{3b}-\frac{5ac\left(\frac{c^2\sqrt{cx}\sqrt{ax^2+bx^3}}{2b}-\frac{3ac\left(\frac{c^2\sqrt{ax^2+bx^3}}{b\sqrt{cx}}-\frac{ac\sqrt{cx}\int \frac{\sqrt{x}}{\sqrt{bx^3+ax^2}}dx}{2b\sqrt{x}}\right)}{4b}\right)}{6b}$$

$$\Big\downarrow \text{1935}$$

$$\frac{c^2(cx)^{3/2}\sqrt{ax^2+bx^3}}{3b}-\frac{5ac\left(\frac{c^2\sqrt{cx}\sqrt{ax^2+bx^3}}{2b}-\frac{3ac\left(\frac{c^2\sqrt{ax^2+bx^3}}{b\sqrt{cx}}-\frac{ac\sqrt{cx}\int\frac{1}{1-\frac{bx^3}{bx^3+ax^2}}d\frac{x^{3/2}}{\sqrt{bx^3+ax^2}}}{b\sqrt{x}}\right)}{4b}\right)}{6b}$$

$$\Big\downarrow \text{219}$$

$$\frac{c^2(cx)^{3/2}\sqrt{ax^2+bx^3}}{3b}-\frac{5ac\left(\frac{c^2\sqrt{cx}\sqrt{ax^2+bx^3}}{2b}-\frac{3ac\left(\frac{c^2\sqrt{ax^2+bx^3}}{b\sqrt{cx}}-\frac{ac\sqrt{cx}\,\text{arctanh}\left(\frac{\sqrt{b}x^{3/2}}{\sqrt{ax^2+bx^3}}\right)}{b^{3/2}\sqrt{x}}\right)}{4b}\right)}{6b}$$

```
Int[(c*x)^(7/2)/Sqrt[a*x^2 + b*x^3],x]
```

```
(c^2*(c*x)^(3/2)*Sqrt[a*x^2 + b*x^3])/(3*b) - (5*a*c*((c^2*Sqrt[c*x]*Sqrt[
a*x^2 + b*x^3])/(2*b) - (3*a*c*((c^2*Sqrt[a*x^2 + b*x^3])/(b*Sqrt[c*x]) -
(a*c*Sqrt[c*x]*ArcTanh[(Sqrt[b]*x^(3/2))/Sqrt[a*x^2 + b*x^3]])/(b^(3/2)*Sq
rt[x])))/(4*b)))/(6*b)
```

Defintions of rubi rules used

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> Simp[c^(n - 1)*(c*x)^(m - n + 1)*((a*x^j + b*x^n)^(p + 1)/(b*(m + n*p
+ 1))), x] - Simp[a*c^(n - j)*((m + j*p - n + j + 1)/(b*(m + n*p + 1)))   I
nt[(c*x)^(m - (n - j))*(a*x^j + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, p},
x] &&  !IntegerQ[p] && LtQ[0, j, n] && (IntegersQ[j, n] || GtQ[c, 0]) && Gt
Q[m + j*p - n + j + 1, 0] && NeQ[m + n*p + 1, 0]
```

```
Int[(x_)^(m_.)/Sqrt[(a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.)], x_Symbol] :> Simp
[-2/(n - j)   Subst[Int[1/(1 - a*x^2), x], x, x^(j/2)/Sqrt[a*x^j + b*x^n]],
 x] /; FreeQ[{a, b, j, n}, x] && EqQ[m, j/2 - 1] && NeQ[n, j]
```

```
Int[((c_)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_))^(p_), x_Symbol]
:> Simp[c^IntPart[m]*((c*x)^FracPart[m]/x^FracPart[m])   Int[x^m*(a*x^j + b
*x^n)^p, x], x] /; FreeQ[{a, b, c, j, m, n, p}, x] && IntegerQ[p + 1/2] &&
NeQ[n, j] && EqQ[Simplify[m + j*p + 1], 0]
```

Maple [A] (verified)

Time = 0.40 (sec) , antiderivative size = 126, normalized size of antiderivative = 0.83

method	result
risch	$\frac{\left(8 b^{2} x^{2}-10 a b x+15 a^{2}\right) x^{2} \left(b x+a\right) c^{4}}{24 b^{3} \sqrt{x^{2} \left(b x+a\right)}\, \sqrt{c x}}-\frac{5 a^{3} \ln\left(\frac{\frac{1}{2} a c+c b x}{\sqrt{b c}}+\sqrt{b c\,x^{2}+a c x}\right) c^{4} x \sqrt{c x \left(b x+a\right)}}{16 b^{3} \sqrt{b c}\, \sqrt{x^{2} \left(b x+a\right)}\, \sqrt{c x}}$
default	$-\frac{x \left(b x+a\right) c^{3} \sqrt{c x}\, \left(-16 b^{2} x^{2} \sqrt{c x \left(b x+a\right)}\, \sqrt{b c}+15 \ln\left(\frac{2 c b x+2 \sqrt{c x \left(b x+a\right)}\, \sqrt{b c}+a c}{2 \sqrt{b c}}\right) a^{3} c+20 \sqrt{b c}\, \sqrt{c x \left(b x+a\right)}\, a b x-30 \sqrt{b c}\, \sqrt{c x \left(b x+a\right)}\, a}{48 \sqrt{b\,x^{3}+a\,x^{2}}\, \sqrt{c x \left(b x+a\right)}\, b^{3} \sqrt{b c}}$

```
int((c*x)^(7/2)/(b*x^3+a*x^2)^(1/2),x,method=_RETURNVERBOSE)
```

```
1/24*(8*b^2*x^2-10*a*b*x+15*a^2)*x^2*(b*x+a)/b^3*c^4/(x^2*(b*x+a))^(1/2)/(
c*x)^(1/2)-5/16*a^3/b^3*ln((1/2*a*c+c*b*x)/(b*c)^(1/2)+(b*c*x^2+a*c*x)^(1/
2))/(b*c)^(1/2)*c^4/(x^2*(b*x+a))^(1/2)*x*(c*x*(b*x+a))^(1/2)/(c*x)^(1/2)
```

Fricas [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 234, normalized size of antiderivative = 1.54

$$\int \frac{(cx)^{7/2}}{\sqrt{ax^2+bx^3}}\,dx = \left[\frac{15\,a^3c^3x\sqrt{\frac{c}{b}}\log\left(\frac{2\,bcx^2+acx-2\,\sqrt{bx^3+ax^2}\sqrt{cx}b\sqrt{\frac{c}{b}}}{x}\right)+2\,(8\,b^2c^3x^2-10\,abc^3x+15\,a^2c^3)\sqrt{b}}{48\,b^3x}\right.$$

```
integrate((c*x)^(7/2)/(b*x^3+a*x^2)^(1/2),x, algorithm="fricas")
```

```
[1/48*(15*a^3*c^3*x*sqrt(c/b)*log((2*b*c*x^2 + a*c*x - 2*sqrt(b*x^3 + a*x^
2)*sqrt(c*x)*b*sqrt(c/b))/x) + 2*(8*b^2*c^3*x^2 - 10*a*b*c^3*x + 15*a^2*c^
3)*sqrt(b*x^3 + a*x^2)*sqrt(c*x))/(b^3*x), 1/24*(15*a^3*c^3*x*sqrt(-c/b)*a
rctan(sqrt(b*x^3 + a*x^2)*sqrt(c*x)*b*sqrt(-c/b)/(b*c*x^2 + a*c*x)) + (8*b
^2*c^3*x^2 - 10*a*b*c^3*x + 15*a^2*c^3)*sqrt(b*x^3 + a*x^2)*sqrt(c*x))/(b^
3*x)]
```

Sympy [F]

$$\int \frac{(cx)^{7/2}}{\sqrt{ax^2 + bx^3}}\,dx = \int \frac{(cx)^{\frac{7}{2}}}{\sqrt{x^2\,(a + bx)}}\,dx$$

```
integrate((c*x)**(7/2)/(b*x**3+a*x**2)**(1/2),x)
```

```
Integral((c*x)**(7/2)/sqrt(x**2*(a + b*x)), x)
```

Maxima [F]

$$\int \frac{(cx)^{7/2}}{\sqrt{ax^2 + bx^3}}\,dx = \int \frac{(cx)^{\frac{7}{2}}}{\sqrt{bx^3 + ax^2}}\,dx$$

```
integrate((c*x)^(7/2)/(b*x^3+a*x^2)^(1/2),x, algorithm="maxima")
```

```
integrate((c*x)^(7/2)/sqrt(b*x^3 + a*x^2), x)
```

Giac [A] (verification not implemented)

Time = 0.14 (sec) , antiderivative size = 141, normalized size of antiderivative = 0.93

$$\int \frac{(cx)^{7/2}}{\sqrt{ax^2+bx^3}}\,dx = -\frac{5\,a^3c^5\log\left(c^2|a|\right)\operatorname{sgn}(x)}{16\,\sqrt{bc}b^3|c|} + \frac{\left(\sqrt{bc^2x+ac^2}\left(2\,cx\left(\frac{4\,x}{bc^3}-\frac{5\,a}{b^2c^3}\right)+\frac{15\,a^2}{b^3c^2}\right)\sqrt{cx}+\frac{15\,a^3\log\left(\left|-\sqrt{bc}\sqrt{cx}+\sqrt{bc^2x+ac^2}\right|\right)}{\sqrt{bc}b^3}\right)c^5}{24\,|c|\operatorname{sgn}\left(x\right)}$$

```
integrate((c*x)^(7/2)/(b*x^3+a*x^2)^(1/2),x, algorithm="giac")
```

```
-5/16*a^3*c^5*log(c^2*abs(a))*sgn(x)/(sqrt(b*c)*b^3*abs(c)) + 1/24*(sqrt(b
*c^2*x + a*c^2)*(2*c*x*(4*x/(b*c^3) - 5*a/(b^2*c^3)) + 15*a^2/(b^3*c^2))*s
qrt(c*x) + 15*a^3*log(abs(-sqrt(b*c)*sqrt(c*x) + sqrt(b*c^2*x + a*c^2)))/(
sqrt(b*c)*b^3))*c^5/(abs(c)*sgn(x))
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(cx)^{7/2}}{\sqrt{ax^2+bx^3}}\,dx = \int \frac{{\left(c\,x\right)}^{7/2}}{\sqrt{b\,x^3+a\,x^2}}\,dx$$

```
int((c*x)^(7/2)/(a*x^2 + b*x^3)^(1/2),x)
```

```
int((c*x)^(7/2)/(a*x^2 + b*x^3)^(1/2), x)
```

Reduce [B] (verification not implemented)

Time = 0.45 (sec) , antiderivative size = 81, normalized size of antiderivative = 0.53

$$\int \frac{(cx)^{7/2}}{\sqrt{ax^2+bx^3}}\,dx = \frac{\sqrt{c}\,c^3\Big(15\sqrt{x}\,\sqrt{bx+a}\,a^2b - 10\sqrt{x}\,\sqrt{bx+a}\,a\,b^2x + 8\sqrt{x}\,\sqrt{bx+a}\,b^3x^2 - 15\sqrt{b}\log\Big(\frac{\sqrt{bx}}{}}{24b^4}$$

```
int((c*x)^(7/2)/(b*x^3+a*x^2)^(1/2),x)
```

```
(sqrt(c)*c**3*(15*sqrt(x)*sqrt(a + b*x)*a**2*b - 10*sqrt(x)*sqrt(a + b*x)*
a*b**2*x + 8*sqrt(x)*sqrt(a + b*x)*b**3*x**2 - 15*sqrt(b)*log((sqrt(a + b*
x) + sqrt(x)*sqrt(b))/sqrt(a))*a**3))/(24*b**4)
```

3.348 $\int \frac{(cx)^{5/2}}{\sqrt{ax^2+bx^3}}\,dx$

Optimal result

Integrand size = 23, antiderivative size = 117

$$\int \frac{(cx)^{5/2}}{\sqrt{ax^2+bx^3}}\,dx = -\frac{3ac^3\sqrt{ax^2+bx^3}}{4b^2\sqrt{cx}} + \frac{c^2\sqrt{cx}\sqrt{ax^2+bx^3}}{2b} + \frac{3a^2c^{5/2}\operatorname{arctanh}\left(\frac{\sqrt{b}(cx)^{3/2}}{c^{3/2}\sqrt{ax^2+bx^3}}\right)}{4b^{5/2}}$$

```
-3/4*a*c^3*(b*x^3+a*x^2)^(1/2)/b^2/(c*x)^(1/2)+1/2*c^2*(c*x)^(1/2)*(b*x^3+
a*x^2)^(1/2)/b+3/4*a^2*c^(5/2)*arctanh(b^(1/2)*(c*x)^(3/2)/c^(3/2)/(b*x^3+
a*x^2)^(1/2))/b^(5/2)
```

Mathematica [A] (verified)

Time = 0.04 (sec) , antiderivative size = 110, normalized size of antiderivative = 0.94

$$\int \frac{(cx)^{5/2}}{\sqrt{ax^2+bx^3}}\,dx = \frac{(cx)^{5/2}\left(\sqrt{b}\sqrt{x}(-3a^2-abx+2b^2x^2)+6a^2\sqrt{a+bx}\operatorname{arctanh}\left(\frac{\sqrt{b}\sqrt{x}}{-\sqrt{a}+\sqrt{a+bx}}\right)\right)}{4b^{5/2}x^{3/2}\sqrt{x^2(a+bx)}}$$

```
Integrate[(c*x)^(5/2)/Sqrt[a*x^2 + b*x^3],x]
```

```
((c*x)^(5/2)*(Sqrt[b]*Sqrt[x]*(-3*a^2 - a*b*x + 2*b^2*x^2) + 6*a^2*Sqrt[a
+ b*x]*ArcTanh[(Sqrt[b]*Sqrt[x])/(-Sqrt[a] + Sqrt[a + b*x])]))/(4*b^(5/2)*
x^(3/2)*Sqrt[x^2*(a + b*x)])
```

Rubi [A] (verified)

Time = 0.49 (sec) , antiderivative size = 120, normalized size of antiderivative = 1.03, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.217$, Rules used = {1930, 1930, 1937, 1935, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(cx)^{5/2}}{\sqrt{ax^2+bx^3}}dx$$

$$\Big\downarrow \text{1930}$$

$$\frac{c^2\sqrt{cx}\sqrt{ax^2+bx^3}}{2b}-\frac{3ac\int \frac{(cx)^{3/2}}{\sqrt{bx^3+ax^2}}dx}{4b}$$

$$\Big\downarrow \text{1930}$$

$$\frac{c^2\sqrt{cx}\sqrt{ax^2+bx^3}}{2b}-\frac{3ac\left(\frac{c^2\sqrt{ax^2+bx^3}}{b\sqrt{cx}}-\frac{ac\int \frac{\sqrt{cx}}{\sqrt{bx^3+ax^2}}dx}{2b}\right)}{4b}$$

$$\Big\downarrow \text{1937}$$

$$\frac{c^2\sqrt{cx}\sqrt{ax^2+bx^3}}{2b}-\frac{3ac\left(\frac{c^2\sqrt{ax^2+bx^3}}{b\sqrt{cx}}-\frac{ac\sqrt{cx}\int \frac{\sqrt{x}}{\sqrt{bx^3+ax^2}}dx}{2b\sqrt{x}}\right)}{4b}$$

$$\Big\downarrow \text{1935}$$

$$\frac{c^2\sqrt{cx}\sqrt{ax^2+bx^3}}{2b}-\frac{3ac\left(\frac{c^2\sqrt{ax^2+bx^3}}{b\sqrt{cx}}-\frac{ac\sqrt{cx}\int \frac{1}{1-\frac{bx^3}{bx^3+ax^2}}d\frac{x^{3/2}}{\sqrt{bx^3+ax^2}}}{b\sqrt{x}}\right)}{4b}$$

$$\Big\downarrow \text{219}$$

$$\frac{c^2\sqrt{cx}\sqrt{ax^2+bx^3}}{2b} - \frac{3ac\left(\frac{c^2\sqrt{ax^2+bx^3}}{b\sqrt{cx}} - \frac{ac\sqrt{cx}\,\mathrm{arctanh}\left(\frac{\sqrt{b}x^{3/2}}{\sqrt{ax^2+bx^3}}\right)}{b^{3/2}\sqrt{x}}\right)}{4b}$$

```
Int[(c*x)^(5/2)/Sqrt[a*x^2 + b*x^3],x]
```

```
(c^2*Sqrt[c*x]*Sqrt[a*x^2 + b*x^3])/(2*b) - (3*a*c*((c^2*Sqrt[a*x^2 + b*x^
3])/(b*Sqrt[c*x]) - (a*c*Sqrt[c*x]*ArcTanh[(Sqrt[b]*x^(3/2))/Sqrt[a*x^2 +
b*x^3]])/(b^(3/2)*Sqrt[x])))/(4*b)
```

Defintions of rubi rules used

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> Simp[c^(n - 1)*(c*x)^(m - n + 1)*((a*x^j + b*x^n)^(p + 1)/(b*(m + n*p
+ 1))), x] - Simp[a*c^(n - j)*((m + j*p - n + j + 1)/(b*(m + n*p + 1)))   I
nt[(c*x)^(m - (n - j))*(a*x^j + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, p},
x] &&  !IntegerQ[p] && LtQ[0, j, n] && (IntegersQ[j, n] || GtQ[c, 0]) && Gt
Q[m + j*p - n + j + 1, 0] && NeQ[m + n*p + 1, 0]
```

```
Int[(x_)^(m_.)/Sqrt[(a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.)], x_Symbol] :> Simp
[-2/(n - j)   Subst[Int[1/(1 - a*x^2), x], x, x^(j/2)/Sqrt[a*x^j + b*x^n]],
 x] /; FreeQ[{a, b, j, n}, x] && EqQ[m, j/2 - 1] && NeQ[n, j]
```

```
Int[((c_)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_))^(p_), x_Symbol]
:> Simp[c^IntPart[m]*((c*x)^FracPart[m]/x^FracPart[m])   Int[x^m*(a*x^j + b
*x^n)^p, x], x] /; FreeQ[{a, b, c, j, m, n, p}, x] && IntegerQ[p + 1/2] &&
NeQ[n, j] && EqQ[Simplify[m + j*p + 1], 0]
```

Maple [A] (verified)

Time = 0.39 (sec) , antiderivative size = 115, normalized size of antiderivative = 0.98

method	result	size
risch	$-\frac{(-2bx+3a)x^2(bx+a)c^3}{4b^2\sqrt{x^2(bx+a)}\sqrt{cx}}+\frac{3a^2\ln\left(\frac{\frac{1}{2}ac+cbx}{\sqrt{bc}}+\sqrt{bc\,x^2+acx}\right)c^3x\sqrt{cx(bx+a)}}{8b^2\sqrt{bc}\,\sqrt{x^2(bx+a)}\,\sqrt{cx}}$	115
default	$-\frac{x(bx+a)c^2\sqrt{cx}\left(-4\sqrt{bc}\,\sqrt{cx(bx+a)}\,bx-3a^2c\ln\left(\frac{2cbx+2\sqrt{cx(bx+a)}\,\sqrt{bc}+ac}{2\sqrt{bc}}\right)+6\sqrt{bc}\,\sqrt{cx(bx+a)}\,a\right)}{8\sqrt{b\,x^3+a\,x^2}\,\sqrt{cx(bx+a)}\,b^2\sqrt{bc}}$	126

```
int((c*x)^(5/2)/(b*x^3+a*x^2)^(1/2),x,method=_RETURNVERBOSE)
```

```
-1/4*(-2*b*x+3*a)*x^2*(b*x+a)/b^2*c^3/(x^2*(b*x+a))^(1/2)/(c*x)^(1/2)+3/8*
a^2/b^2*ln((1/2*a*c+c*b*x)/(b*c)^(1/2)+(b*c*x^2+a*c*x)^(1/2))/(b*c)^(1/2)*
c^3/(x^2*(b*x+a))^(1/2)*x*(c*x*(b*x+a))^(1/2)/(c*x)^(1/2)
```

Fricas [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 207, normalized size of antiderivative = 1.77

$$\int\frac{(cx)^{5/2}}{\sqrt{ax^2+bx^3}}\,dx=\left[\frac{3\,a^2c^2x\sqrt{\frac{c}{b}}\log\left(\frac{2\,bcx^2+acx+2\,\sqrt{bx^3+ax^2}\sqrt{cx}b\sqrt{\frac{c}{b}}}{x}\right)+2\,(2\,bc^2x-3\,ac^2)\sqrt{bx^3+ax^2}\sqrt{cx}}{8\,b^2x},\right.$$
$$\left.-\frac{3\,a^2c^2x\sqrt{-\frac{c}{b}}\arctan\left(\frac{\sqrt{bx^3+ax^2}\sqrt{cx}b\sqrt{-\frac{c}{b}}}{bcx^2+acx}\right)-(2\,bc^2x-3\,ac^2)\sqrt{bx^3+ax^2}\sqrt{cx}}{4\,b^2x}\right]$$

```
integrate((c*x)^(5/2)/(b*x^3+a*x^2)^(1/2),x, algorithm="fricas")
```

```
[1/8*(3*a^2*c^2*x*sqrt(c/b)*log((2*b*c*x^2 + a*c*x + 2*sqrt(b*x^3 + a*x^2)
*sqrt(c*x)*b*sqrt(c/b))/x) + 2*(2*b*c^2*x - 3*a*c^2)*sqrt(b*x^3 + a*x^2)*s
qrt(c*x))/(b^2*x), -1/4*(3*a^2*c^2*x*sqrt(-c/b)*arctan(sqrt(b*x^3 + a*x^2)
*sqrt(c*x)*b*sqrt(-c/b)/(b*c*x^2 + a*c*x)) - (2*b*c^2*x - 3*a*c^2)*sqrt(b*
x^3 + a*x^2)*sqrt(c*x))/(b^2*x)]
```

Sympy [F]

$$\int \frac{(cx)^{5/2}}{\sqrt{ax^2+bx^3}}\,dx = \int \frac{(cx)^{\frac{5}{2}}}{\sqrt{x^2\,(a+bx)}}\,dx$$

```
integrate((c*x)**(5/2)/(b*x**3+a*x**2)**(1/2),x)
```

```
Integral((c*x)**(5/2)/sqrt(x**2*(a + b*x)), x)
```

Maxima [F]

$$\int \frac{(cx)^{5/2}}{\sqrt{ax^2+bx^3}}\,dx = \int \frac{(cx)^{\frac{5}{2}}}{\sqrt{bx^3+ax^2}}\,dx$$

```
integrate((c*x)^(5/2)/(b*x^3+a*x^2)^(1/2),x, algorithm="maxima")
```

```
integrate((c*x)^(5/2)/sqrt(b*x^3 + a*x^2), x)
```

Giac [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 123, normalized size of antiderivative = 1.05

$$\int \frac{(cx)^{5/2}}{\sqrt{ax^2+bx^3}}\,dx = \frac{3\,a^2c^4\log\left(c^2|a|\right)\mathrm{sgn}(x)}{8\,\sqrt{bc}b^2|c|} - \frac{\left(\frac{3\,a^2c^2\log\left(\left|-\sqrt{bc}\sqrt{cx}+\sqrt{bc^2x+ac^2}\right|\right)}{\sqrt{bc}b^2} - \sqrt{bc^2x+ac^2}\sqrt{cx}\left(\frac{2\,x}{b}-\frac{3\,a}{b^2}\right)\right)c^2}{4\,|c|\mathrm{sgn}\left(x\right)}$$

```
integrate((c*x)^(5/2)/(b*x^3+a*x^2)^(1/2),x, algorithm="giac")
```

```
3/8*a^2*c^4*log(c^2*abs(a))*sgn(x)/(sqrt(b*c)*b^2*abs(c)) - 1/4*(3*a^2*c^2
*log(abs(-sqrt(b*c)*sqrt(c*x) + sqrt(b*c^2*x + a*c^2)))/(sqrt(b*c)*b^2) -
sqrt(b*c^2*x + a*c^2)*sqrt(c*x)*(2*x/b - 3*a/b^2))*c^2/(abs(c)*sgn(x))
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(cx)^{5/2}}{\sqrt{ax^2+bx^3}}\,dx = \int \frac{(c\,x)^{5/2}}{\sqrt{b\,x^3+a\,x^2}}\,dx$$

```
int((c*x)^(5/2)/(a*x^2 + b*x^3)^(1/2),x)
```

```
int((c*x)^(5/2)/(a*x^2 + b*x^3)^(1/2), x)
```

Reduce [B] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 62, normalized size of antiderivative = 0.53

$$\int \frac{(cx)^{5/2}}{\sqrt{ax^2+bx^3}}\,dx = \frac{\sqrt{c}\,c^2\left(-3\sqrt{x}\,\sqrt{bx+a}\,ab+2\sqrt{x}\,\sqrt{bx+a}\,b^2x+3\sqrt{b}\log\left(\frac{\sqrt{bx+a}+\sqrt{x}\,\sqrt{b}}{\sqrt{a}}\right)a^2\right)}{4b^3}$$

```
int((c*x)^(5/2)/(b*x^3+a*x^2)^(1/2),x)
```

```
(sqrt(c)*c**2*( - 3*sqrt(x)*sqrt(a + b*x)*a*b + 2*sqrt(x)*sqrt(a + b*x)*b*
*2*x + 3*sqrt(b)*log((sqrt(a + b*x) + sqrt(x)*sqrt(b))/sqrt(a))*a**2))/(4*
b**3)
```

3.349 $\int \frac{(cx)^{3/2}}{\sqrt{ax^2+bx^3}}\,dx$

Optimal result

Integrand size = 23, antiderivative size = 77

$$\int \frac{(cx)^{3/2}}{\sqrt{ax^2+bx^3}}\,dx=\frac{c^2\sqrt{ax^2+bx^3}}{b\sqrt{cx}}-\frac{ac^{3/2}\text{arctanh}\left(\frac{\sqrt{b}(cx)^{3/2}}{c^{3/2}\sqrt{ax^2+bx^3}}\right)}{b^{3/2}}$$

```
c^2*(b*x^3+a*x^2)^(1/2)/b/(c*x)^(1/2)-a*c^(3/2)*arctanh(b^(1/2)*(c*x)^(3/2
)/c^(3/2)/(b*x^3+a*x^2)^(1/2))/b^(3/2)
```

Mathematica [A] (verified)

Time = 0.03 (sec) , antiderivative size = 92, normalized size of antiderivative = 1.19

$$\int \frac{(cx)^{3/2}}{\sqrt{ax^2+bx^3}}\,dx=\frac{c\sqrt{x}\sqrt{cx}\left(\sqrt{b}\sqrt{x}(a+bx)+2a\sqrt{a+bx}\text{arctanh}\left(\frac{\sqrt{b}\sqrt{x}}{\sqrt{a}-\sqrt{a+bx}}\right)\right)}{b^{3/2}\sqrt{x^2(a+bx)}}$$

```
Integrate[(c*x)^(3/2)/Sqrt[a*x^2 + b*x^3],x]
```

```
(c*Sqrt[x]*Sqrt[c*x]*(Sqrt[b]*Sqrt[x]*(a + b*x) + 2*a*Sqrt[a + b*x]*ArcTan
h[(Sqrt[b]*Sqrt[x])/(Sqrt[a] - Sqrt[a + b*x])]))/(b^(3/2)*Sqrt[x^2*(a + b*
x)])
```

Rubi [A] (verified)

Time = 0.40 (sec) , antiderivative size = 78, normalized size of antiderivative = 1.01, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.174, Rules used = {1930, 1937, 1935, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(cx)^{3/2}}{\sqrt{ax^2+bx^3}}dx$$

$$\Big\downarrow \text{ 1930}$$

$$\frac{c^2\sqrt{ax^2+bx^3}}{b\sqrt{cx}}-\frac{ac\int \frac{\sqrt{cx}}{\sqrt{bx^3+ax^2}}dx}{2b}$$

$$\Big\downarrow \text{ 1937}$$

$$\frac{c^2\sqrt{ax^2+bx^3}}{b\sqrt{cx}}-\frac{ac\sqrt{cx}\int \frac{\sqrt{x}}{\sqrt{bx^3+ax^2}}dx}{2b\sqrt{x}}$$

$$\Big\downarrow \text{ 1935}$$

$$\frac{c^2\sqrt{ax^2+bx^3}}{b\sqrt{cx}}-\frac{ac\sqrt{cx}\int \frac{1}{1-\frac{bx^3}{bx^3+ax^2}}d\frac{x^{3/2}}{\sqrt{bx^3+ax^2}}}{b\sqrt{x}}$$

$$\Big\downarrow \text{ 219}$$

$$\frac{c^2\sqrt{ax^2+bx^3}}{b\sqrt{cx}}-\frac{ac\sqrt{cx}\text{arctanh}\left(\frac{\sqrt{b}x^{3/2}}{\sqrt{ax^2+bx^3}}\right)}{b^{3/2}\sqrt{x}}$$

```
Int[(c*x)^(3/2)/Sqrt[a*x^2 + b*x^3],x]
```

```
(c^2*Sqrt[a*x^2 + b*x^3])/(b*Sqrt[c*x]) - (a*c*Sqrt[c*x]*ArcTanh[(Sqrt[b]*
x^(3/2))/Sqrt[a*x^2 + b*x^3]])/(b^(3/2)*Sqrt[x])
```

Defintions of rubi rules used

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> Simp[c^(n - 1)*(c*x)^(m - n + 1)*((a*x^j + b*x^n)^(p + 1)/(b*(m + n*p
+ 1))), x] - Simp[a*c^(n - j)*((m + j*p - n + j + 1)/(b*(m + n*p + 1)))   I
nt[(c*x)^(m - (n - j))*(a*x^j + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, p},
x] &&  !IntegerQ[p] && LtQ[0, j, n] && (IntegersQ[j, n] || GtQ[c, 0]) && Gt
Q[m + j*p - n + j + 1, 0] && NeQ[m + n*p + 1, 0]
```

```
Int[(x_)^(m_.)/Sqrt[(a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.)], x_Symbol] :> Simp
[-2/(n - j)   Subst[Int[1/(1 - a*x^2), x], x, x^(j/2)/Sqrt[a*x^j + b*x^n]],
 x] /; FreeQ[{a, b, j, n}, x] && EqQ[m, j/2 - 1] && NeQ[n, j]
```

```
Int[((c_)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_))^(p_), x_Symbol]
:> Simp[c^IntPart[m]*((c*x)^FracPart[m]/x^FracPart[m])   Int[x^m*(a*x^j + b
*x^n)^p, x], x] /; FreeQ[{a, b, c, j, m, n, p}, x] && IntegerQ[p + 1/2] &&
NeQ[n, j] && EqQ[Simplify[m + j*p + 1], 0]
```

Maple [A] (verified)

Time = 0.39 (sec) , antiderivative size = 101, normalized size of antiderivative = 1.31

method	result	size
default	$-\frac{x(bx+a)c\sqrt{cx}\left(ac\ln\left(\frac{2cbx+2\sqrt{cx(bx+a)}\,\sqrt{bc}+ac}{2\sqrt{bc}}\right)-2\sqrt{cx(bx+a)}\,\sqrt{bc}\right)}{2\sqrt{b\,x^{3}+a\,x^{2}}\,\sqrt{cx(bx+a)}\,b\sqrt{bc}}$	101
risch	$\frac{x^{2}(bx+a)c^{2}}{b\sqrt{x^{2}(bx+a)}\,\sqrt{cx}}-\frac{a\ln\left(\frac{\frac{1}{2}ac+cbx}{\sqrt{bc}}+\sqrt{bc\,x^{2}+acx}\right)c^{2}x\sqrt{cx(bx+a)}}{2b\sqrt{bc}\,\sqrt{x^{2}(bx+a)}\,\sqrt{cx}}$	104

```
int((c*x)^(3/2)/(b*x^3+a*x^2)^(1/2),x,method=_RETURNVERBOSE)
```

```
-1/2/(b*x^3+a*x^2)^(1/2)*x*(b*x+a)*c*(c*x)^(1/2)*(a*c*ln(1/2*(2*c*b*x+2*(c
*x*(b*x+a))^(1/2)*(b*c)^(1/2)+a*c)/(b*c)^(1/2))-2*(c*x*(b*x+a))^(1/2)*(b*c
)^(1/2))/(c*x*(b*x+a))^(1/2)/b/(b*c)^(1/2)
```

Fricas [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 169, normalized size of antiderivative = 2.19

$$\int \frac{(cx)^{3/2}}{\sqrt{ax^2+bx^3}}\,dx = \left[\frac{acx\sqrt{\frac{c}{b}}\log\left(\frac{2\,bcx^2+acx-2\,\sqrt{bx^3+ax^2}\sqrt{cx}b\sqrt{\frac{c}{b}}}{x}\right)+2\,\sqrt{bx^3+ax^2}\sqrt{cx}c}{2\,bx}, \frac{acx\sqrt{-\frac{c}{b}}\arctan\left(\frac{\sqrt{}}{-}\right.}{}\right.$$

```
integrate((c*x)^(3/2)/(b*x^3+a*x^2)^(1/2),x, algorithm="fricas")
```

```
[1/2*(a*c*x*sqrt(c/b)*log((2*b*c*x^2 + a*c*x - 2*sqrt(b*x^3 + a*x^2)*sqrt(
c*x)*b*sqrt(c/b))/x) + 2*sqrt(b*x^3 + a*x^2)*sqrt(c*x)*c)/(b*x), (a*c*x*sq
rt(-c/b)*arctan(sqrt(b*x^3 + a*x^2)*sqrt(c*x)*b*sqrt(-c/b)/(b*c*x^2 + a*c*
x)) + sqrt(b*x^3 + a*x^2)*sqrt(c*x)*c)/(b*x)]
```

Sympy [F]

$$\int \frac{(cx)^{3/2}}{\sqrt{ax^2+bx^3}}\,dx = \int \frac{(cx)^{\frac{3}{2}}}{\sqrt{x^2\,(a+bx)}}\,dx$$

```
integrate((c*x)**(3/2)/(b*x**3+a*x**2)**(1/2),x)
```

```
Integral((c*x)**(3/2)/sqrt(x**2*(a + b*x)), x)
```

Maxima [F]

$$\int \frac{(cx)^{3/2}}{\sqrt{ax^2+bx^3}}\,dx = \int \frac{(cx)^{\frac{3}{2}}}{\sqrt{bx^3+ax^2}}\,dx$$

```
integrate((c*x)^(3/2)/(b*x^3+a*x^2)^(1/2),x, algorithm="maxima")
```

```
integrate((c*x)^(3/2)/sqrt(b*x^3 + a*x^2), x)
```

Giac [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 108, normalized size of antiderivative = 1.40

$$\int \frac{(cx)^{3/2}}{\sqrt{ax^2+bx^3}}\,dx = -\frac{1}{2}\left(\frac{ac^2\log\left(c^2|a|\right)\operatorname{sgn}(x)}{\sqrt{bc}b|c|} - \frac{2\left(\frac{ac\log\left(\left|-\sqrt{bc}\sqrt{cx}+\sqrt{bc^2x+ac^2}\right|\right)}{\sqrt{bc}b} + \frac{\sqrt{bc^2x+ac^2}\sqrt{cx}}{bc}\right)c}{|c|\operatorname{sgn}(x)}\right)c$$

```
integrate((c*x)^(3/2)/(b*x^3+a*x^2)^(1/2),x, algorithm="giac")
```

```
-1/2*(a*c^2*log(c^2*abs(a))*sgn(x)/(sqrt(b*c)*b*abs(c)) - 2*(a*c*log(abs(-
sqrt(b*c)*sqrt(c*x) + sqrt(b*c^2*x + a*c^2)))/(sqrt(b*c)*b) + sqrt(b*c^2*x
 + a*c^2)*sqrt(c*x)/(b*c))*c/(abs(c)*sgn(x)))*c
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(cx)^{3/2}}{\sqrt{ax^2+bx^3}}\,dx = \int \frac{(c\,x)^{3/2}}{\sqrt{b\,x^3+a\,x^2}}\,dx$$

```
int((c*x)^(3/2)/(a*x^2 + b*x^3)^(1/2),x)
```

```
int((c*x)^(3/2)/(a*x^2 + b*x^3)^(1/2), x)
```

Reduce [B] (verification not implemented)

Time = 0.51 (sec) , antiderivative size = 41, normalized size of antiderivative = 0.53

$$\int \frac{(cx)^{3/2}}{\sqrt{ax^2+bx^3}}\,dx = \frac{\sqrt{c}\,c\left(\sqrt{x}\,\sqrt{bx+a}\,b-\sqrt{b}\log\left(\frac{\sqrt{bx+a}+\sqrt{x}\,\sqrt{b}}{\sqrt{a}}\right)a\right)}{b^2}$$

```
int((c*x)^(3/2)/(b*x^3+a*x^2)^(1/2),x)
```

```
(sqrt(c)*c*(sqrt(x)*sqrt(a + b*x)*b - sqrt(b)*log((sqrt(a + b*x) + sqrt(x)
*sqrt(b))/sqrt(a))*a))/b**2
```

3.350 $\int \frac{\sqrt{cx}}{\sqrt{ax^2+bx^3}}\,dx$

Optimal result

Integrand size = 23, antiderivative size = 46

$$\int \frac{\sqrt{cx}}{\sqrt{ax^2+bx^3}}\,dx=\frac{2\sqrt{c}\,\mathrm{arctanh}\left(\frac{\sqrt{b}(cx)^{3/2}}{c^{3/2}\sqrt{ax^2+bx^3}}\right)}{\sqrt{b}}$$

```
2*c^(1/2)*arctanh(b^(1/2)*(c*x)^(3/2)/c^(3/2)/(b*x^3+a*x^2)^(1/2))/b^(1/2)
```

Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 64, normalized size of antiderivative = 1.39

$$\int \frac{\sqrt{cx}}{\sqrt{ax^2+bx^3}}\,dx=-\frac{2\sqrt{x}\sqrt{cx}\sqrt{a+bx}\log\left(-\sqrt{b}\sqrt{x}+\sqrt{a+bx}\right)}{\sqrt{b}\sqrt{x^2(a+bx)}}$$

```
Integrate[Sqrt[c*x]/Sqrt[a*x^2 + b*x^3],x]
```

```
(-2*Sqrt[x]*Sqrt[c*x]*Sqrt[a + b*x]*Log[-(Sqrt[b]*Sqrt[x]) + Sqrt[a + b*x]
])/(Sqrt[b]*Sqrt[x^2*(a + b*x)])
```

Rubi [A] (verified)

Time = 0.31 (sec) , antiderivative size = 46, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.130, Rules used = {1937, 1935, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sqrt{cx}}{\sqrt{ax^2+bx^3}}dx$$

$$\Big\downarrow \text{ 1937}$$

$$\frac{\sqrt{cx}\int \frac{\sqrt{x}}{\sqrt{bx^3+ax^2}}dx}{\sqrt{x}}$$

$$\Big\downarrow \text{ 1935}$$

$$\frac{2\sqrt{cx}\int \frac{1}{1-\frac{bx^3}{bx^3+ax^2}}d\frac{x^{3/2}}{\sqrt{bx^3+ax^2}}}{\sqrt{x}}$$

$$\Big\downarrow \text{ 219}$$

$$\frac{2\sqrt{cx}\text{arctanh}\left(\frac{\sqrt{b}x^{3/2}}{\sqrt{ax^2+bx^3}}\right)}{\sqrt{b}\sqrt{x}}$$

```
Int[Sqrt[c*x]/Sqrt[a*x^2 + b*x^3],x]
```

```
(2*Sqrt[c*x]*ArcTanh[(Sqrt[b]*x^(3/2))/Sqrt[a*x^2 + b*x^3]])/(Sqrt[b]*Sqrt
[x])
```

Defintions of rubi rules used

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

```
Int[(x_)^(m_.)/Sqrt[(a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.)], x_Symbol] :> Simp
[-2/(n - j)   Subst[Int[1/(1 - a*x^2), x], x, x^(j/2)/Sqrt[a*x^j + b*x^n]],
 x] /; FreeQ[{a, b, j, n}, x] && EqQ[m, j/2 - 1] && NeQ[n, j]
```

```
Int[((c_)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_))^(p_), x_Symbol]
:> Simp[c^IntPart[m]*((c*x)^FracPart[m]/x^FracPart[m])   Int[x^m*(a*x^j + b
*x^n)^p, x], x] /; FreeQ[{a, b, c, j, m, n, p}, x] && IntegerQ[p + 1/2] &&
NeQ[n, j] && EqQ[Simplify[m + j*p + 1], 0]
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 75 vs. $2(34) = 68$.

Time = 0.38 (sec) , antiderivative size = 76, normalized size of antiderivative = 1.65

method	result	size
default	$\frac{x(bx+a)\sqrt{cx}\,c\ln\left(\frac{2cbx+2\sqrt{cx(bx+a)}\,\sqrt{bc}+ac}{2\sqrt{bc}}\right)}{\sqrt{b\,x^{3}+a\,x^{2}}\,\sqrt{cx(bx+a)}\,\sqrt{bc}}$	76

```
int((c*x)^(1/2)/(b*x^3+a*x^2)^(1/2),x,method=_RETURNVERBOSE)
```

```
1/(b*x^3+a*x^2)^(1/2)*x*(b*x+a)*(c*x)^(1/2)*c*ln(1/2*(2*c*b*x+2*(c*x*(b*x+
a))^(1/2)*(b*c)^(1/2)+a*c)/(b*c)^(1/2))/(c*x*(b*x+a))^(1/2)/(b*c)^(1/2)
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 106, normalized size of antiderivative = 2.30

$$\int \frac{\sqrt{cx}}{\sqrt{ax^2+bx^3}}\,dx = \left[\sqrt{\frac{c}{b}}\log\left(\frac{2\,bcx^2+acx+2\,\sqrt{bx^3+ax^2}\sqrt{cx}b\sqrt{\frac{c}{b}}}{x}\right), -2\,\sqrt{-\frac{c}{b}}\arctan\left(\frac{\sqrt{bx^3+ax^2}\sqrt{cx}b\sqrt{-\frac{c}{b}}}{bcx^2+acx}\right)\right]$$

```
integrate((c*x)^(1/2)/(b*x^3+a*x^2)^(1/2),x, algorithm="fricas")
```

```
[sqrt(c/b)*log((2*b*c*x^2 + a*c*x + 2*sqrt(b*x^3 + a*x^2)*sqrt(c*x)*b*sqrt
(c/b))/x), -2*sqrt(-c/b)*arctan(sqrt(b*x^3 + a*x^2)*sqrt(c*x)*b*sqrt(-c/b)
/(b*c*x^2 + a*c*x))]
```

Sympy [F]

$$\int \frac{\sqrt{cx}}{\sqrt{ax^2+bx^3}}\,dx = \int \frac{\sqrt{cx}}{\sqrt{x^2\,(a+bx)}}\,dx$$

```
integrate((c*x)**(1/2)/(b*x**3+a*x**2)**(1/2),x)
```

```
Integral(sqrt(c*x)/sqrt(x**2*(a + b*x)), x)
```

Maxima [F]

$$\int \frac{\sqrt{cx}}{\sqrt{ax^2+bx^3}}\,dx = \int \frac{\sqrt{cx}}{\sqrt{bx^3+ax^2}}\,dx$$

```
integrate((c*x)^(1/2)/(b*x^3+a*x^2)^(1/2),x, algorithm="maxima")
```

```
integrate(sqrt(c*x)/sqrt(b*x^3 + a*x^2), x)
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 70 vs. $2(34) = 68$.

Time = 0.14 (sec) , antiderivative size = 70, normalized size of antiderivative = 1.52

$$\int \frac{\sqrt{cx}}{\sqrt{ax^2+bx^3}}\,dx = \frac{c^2\log\left(c^2|a|\right)\operatorname{sgn}(x)}{\sqrt{bc}|c|} - \frac{2\,c^2\log\left(\left|-\sqrt{bc}\sqrt{cx}+\sqrt{bc^2x+ac^2}\right|\right)}{\sqrt{bc}|c|\operatorname{sgn}(x)}$$

```
integrate((c*x)^(1/2)/(b*x^3+a*x^2)^(1/2),x, algorithm="giac")
```

```
c^2*log(c^2*abs(a))*sgn(x)/(sqrt(b*c)*abs(c)) - 2*c^2*log(abs(-sqrt(b*c)*s
qrt(c*x) + sqrt(b*c^2*x + a*c^2)))/(sqrt(b*c)*abs(c)*sgn(x))
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\sqrt{cx}}{\sqrt{ax^2+bx^3}}\,dx = \int \frac{\sqrt{c\,x}}{\sqrt{b\,x^3+a\,x^2}}\,dx$$

```
int((c*x)^(1/2)/(a*x^2 + b*x^3)^(1/2),x)
```

```
int((c*x)^(1/2)/(a*x^2 + b*x^3)^(1/2), x)
```

Reduce [B] (verification not implemented)

Time = 0.46 (sec) , antiderivative size = 27, normalized size of antiderivative = 0.59

$$\int \frac{\sqrt{cx}}{\sqrt{ax^2+bx^3}}\,dx = \frac{2\sqrt{c}\,\sqrt{b}\log\left(\frac{\sqrt{bx+a}+\sqrt{x}\,\sqrt{b}}{\sqrt{a}}\right)}{b}$$

```
int((c*x)^(1/2)/(b*x^3+a*x^2)^(1/2),x)
```

```
(2*sqrt(c)*sqrt(b)*log((sqrt(a + b*x) + sqrt(x)*sqrt(b))/sqrt(a)))/b
```

3.351 $\int \frac{1}{\sqrt{cx}\sqrt{ax^2+bx^3}}\,dx$

Optimal result

Integrand size = 23, antiderivative size = 28

$$\int \frac{1}{\sqrt{cx}\sqrt{ax^2+bx^3}}\,dx = -\frac{2c\sqrt{ax^2+bx^3}}{a(cx)^{3/2}}$$

```
-2*c*(b*x^3+a*x^2)^(1/2)/a/(c*x)^(3/2)
```

Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 26, normalized size of antiderivative = 0.93

$$\int \frac{1}{\sqrt{cx}\sqrt{ax^2+bx^3}}\,dx = -\frac{2c\sqrt{x^2(a+bx)}}{a(cx)^{3/2}}$$

```
Integrate[1/(Sqrt[c*x]*Sqrt[a*x^2 + b*x^3]),x]
```

```
(-2*c*Sqrt[x^2*(a + b*x)])/(a*(c*x)^(3/2))
```

Rubi [A] (verified)

Time = 0.25 (sec) , antiderivative size = 28, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}} = 0.043$, Rules used = {1920}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{\sqrt{cx}\sqrt{ax^2+bx^3}}\,dx$$

$$\Big\downarrow \; 1920$$

$$-\frac{2c\sqrt{ax^2+bx^3}}{a(cx)^{3/2}}$$

```
Int[1/(Sqrt[c*x]*Sqrt[a*x^2 + b*x^3]),x]
```

```
(-2*c*Sqrt[a*x^2 + b*x^3])/(a*(c*x)^(3/2))
```

Defintions of rubi rules used

```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> Simp[(-c^(j - 1))*(c*x)^(m - j + 1)*((a*x^j + b*x^n)^(p + 1)/(a*(n - j
)*(p + 1))), x] /; FreeQ[{a, b, c, j, m, n, p}, x] &&  !IntegerQ[p] && NeQ[
n, j] && EqQ[m + n*p + n - j + 1, 0] && (IntegerQ[j] || GtQ[c, 0])
```

Maple [A] (verified)

Time = 0.37 (sec) , antiderivative size = 28, normalized size of antiderivative = 1.00

method	result	size
risch	$-\frac{2x(bx+a)}{\sqrt{x^2(bx+a)}\,\sqrt{cx}\,a}$	28
gosper	$-\frac{2x(bx+a)}{a\sqrt{cx}\,\sqrt{b\,x^3+a\,x^2}}$	30
default	$-\frac{2x(bx+a)}{a\sqrt{cx}\,\sqrt{b\,x^3+a\,x^2}}$	30
orering	$-\frac{2x(bx+a)}{a\sqrt{cx}\,\sqrt{b\,x^3+a\,x^2}}$	30

```
int(1/(c*x)^(1/2)/(b*x^3+a*x^2)^(1/2),x,method=_RETURNVERBOSE)
```

```
-2/(x^2*(b*x+a))^(1/2)*x/(c*x)^(1/2)/a*(b*x+a)
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 29, normalized size of antiderivative = 1.04

$$\int \frac{1}{\sqrt{cx}\sqrt{ax^2+bx^3}}\,dx = -\frac{2\,\sqrt{bx^3+ax^2}\sqrt{cx}}{acx^2}$$

```
integrate(1/(c*x)^(1/2)/(b*x^3+a*x^2)^(1/2),x, algorithm="fricas")
```

```
-2*sqrt(b*x^3 + a*x^2)*sqrt(c*x)/(a*c*x^2)
```

Sympy [F]

$$\int \frac{1}{\sqrt{cx}\sqrt{ax^2+bx^3}}\,dx = \int \frac{1}{\sqrt{cx}\,\sqrt{x^2\,(a+bx)}}\,dx$$

```
integrate(1/(c*x)**(1/2)/(b*x**3+a*x**2)**(1/2),x)
```

```
Integral(1/(sqrt(c*x)*sqrt(x**2*(a + b*x))), x)
```

Maxima [F]

$$\int \frac{1}{\sqrt{cx}\sqrt{ax^2+bx^3}}\,dx = \int \frac{1}{\sqrt{bx^3+ax^2}\sqrt{cx}}\,dx$$

```
integrate(1/(c*x)^(1/2)/(b*x^3+a*x^2)^(1/2),x, algorithm="maxima")
```

```
integrate(1/(sqrt(b*x^3 + a*x^2)*sqrt(c*x)), x)
```

Giac [F(-1)]

Timed out.

$$\int \frac{1}{\sqrt{cx}\sqrt{ax^2+bx^3}}\,dx = \text{Timed out}$$

```
integrate(1/(c*x)^(1/2)/(b*x^3+a*x^2)^(1/2),x, algorithm="giac")
```

```
Timed out
```

Mupad [B] (verification not implemented)

Time = 9.73 (sec) , antiderivative size = 22, normalized size of antiderivative = 0.79

$$\int \frac{1}{\sqrt{cx}\sqrt{ax^2+bx^3}}\,dx = -\frac{2\,|x|\,\sqrt{a+b\,x}}{a\,x\,\sqrt{c\,x}}$$

```
int(1/((c*x)^(1/2)*(a*x^2 + b*x^3)^(1/2)),x)
```

```
-(2*abs(x)*(a + b*x)^(1/2))/(a*x*(c*x)^(1/2))
```

Reduce [B] (verification not implemented)

Time = 0.24 (sec) , antiderivative size = 27, normalized size of antiderivative = 0.96

$$\int \frac{1}{\sqrt{cx}\sqrt{ax^2+bx^3}}\,dx = -\frac{2\sqrt{c}\left(\sqrt{x}\,\sqrt{bx+a}+\sqrt{b}\,x\right)}{acx}$$

```
int(1/(c*x)^(1/2)/(b*x^3+a*x^2)^(1/2),x)
```

```
( - 2*sqrt(c)*(sqrt(x)*sqrt(a + b*x) + sqrt(b)*x))/(a*c*x)
```

3.352 $\int \frac{1}{(cx)^{3/2}\sqrt{ax^2+bx^3}}\,dx$

Optimal result

Integrand size = 23, antiderivative size = 61

$$\int \frac{1}{(cx)^{3/2}\sqrt{ax^2+bx^3}}\,dx = -\frac{2c\sqrt{ax^2+bx^3}}{3a(cx)^{5/2}} + \frac{4b\sqrt{ax^2+bx^3}}{3a^2(cx)^{3/2}}$$

```
-2/3*c*(b*x^3+a*x^2)^(1/2)/a/(c*x)^(5/2)+4/3*b*(b*x^3+a*x^2)^(1/2)/a^2/(c*
x)^(3/2)
```

Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 34, normalized size of antiderivative = 0.56

$$\int \frac{1}{(cx)^{3/2}\sqrt{ax^2+bx^3}}\,dx = -\frac{2c(a-2bx)\sqrt{x^2(a+bx)}}{3a^2(cx)^{5/2}}$$

```
Integrate[1/((c*x)^(3/2)*Sqrt[a*x^2 + b*x^3]),x]
```

```
(-2*c*(a - 2*b*x)*Sqrt[x^2*(a + b*x)])/(3*a^2*(c*x)^(5/2))
```

Rubi [A] (verified)

Time = 0.32 (sec) , antiderivative size = 61, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.087, Rules used = {1922, 1920}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{(cx)^{3/2}\sqrt{ax^2+bx^3}}\,dx$$

$$\Big\downarrow \text{1922}$$

$$-\frac{2b\int \frac{1}{\sqrt{cx}\sqrt{bx^3+ax^2}}dx}{3ac}-\frac{2c\sqrt{ax^2+bx^3}}{3a(cx)^{5/2}}$$

$$\Big\downarrow \text{1920}$$

$$\frac{4b\sqrt{ax^2+bx^3}}{3a^2(cx)^{3/2}}-\frac{2c\sqrt{ax^2+bx^3}}{3a(cx)^{5/2}}$$

```
Int[1/((c*x)^(3/2)*Sqrt[a*x^2 + b*x^3]),x]
```

```
(-2*c*Sqrt[a*x^2 + b*x^3])/(3*a*(c*x)^(5/2)) + (4*b*Sqrt[a*x^2 + b*x^3])/(
3*a^2*(c*x)^(3/2))
```

Defintions of rubi rules used

```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> Simp[(-c^(j - 1))*(c*x)^(m - j + 1)*((a*x^j + b*x^n)^(p + 1)/(a*(n - j
)*(p + 1))), x] /; FreeQ[{a, b, c, j, m, n, p}, x] && !IntegerQ[p] && NeQ[
n, j] && EqQ[m + n*p + n - j + 1, 0] && (IntegerQ[j] || GtQ[c, 0])
```

```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> Simp[c^(j - 1)*(c*x)^(m - j + 1)*((a*x^j + b*x^n)^(p + 1)/(a*(m + j*p
+ 1))), x] - Simp[b*((m + n*p + n - j + 1)/(a*c^(n - j)*(m + j*p + 1)))   I
nt[(c*x)^(m + n - j)*(a*x^j + b*x^n)^p, x], x] /; FreeQ[{a, b, c, j, m, n,
p}, x] &&  !IntegerQ[p] && NeQ[n, j] && ILtQ[Simplify[(m + n*p + n - j + 1)
/(n - j)], 0] && NeQ[m + j*p + 1, 0] && (IntegersQ[j, n] || GtQ[c, 0])
```

Maple [A] (verified)

Time = 0.39 (sec) , antiderivative size = 36, normalized size of antiderivative = 0.59

method	result	size
gosper	$-\frac{2x(bx+a)(-2bx+a)}{3a^{2}(cx)^{\frac{3}{2}}\sqrt{b\,x^{3}+a\,x^{2}}}$	36
risch	$-\frac{2(bx+a)(-2bx+a)}{3c\sqrt{x^{2}(bx+a)}\,\sqrt{cx}\,a^{2}}$	36
orering	$-\frac{2x(bx+a)(-2bx+a)}{3a^{2}(cx)^{\frac{3}{2}}\sqrt{b\,x^{3}+a\,x^{2}}}$	36
default	$-\frac{2(bx+a)(-2bx+a)}{3\sqrt{b\,x^{3}+a\,x^{2}}\,a^{2}c\sqrt{cx}}$	38

```
int(1/(c*x)^(3/2)/(b*x^3+a*x^2)^(1/2),x,method=_RETURNVERBOSE)
```

```
-2/3*x*(b*x+a)*(-2*b*x+a)/a^2/(c*x)^(3/2)/(b*x^3+a*x^2)^(1/2)
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 37, normalized size of antiderivative = 0.61

$$\int \frac{1}{(cx)^{3/2}\sqrt{ax^2+bx^3}}\,dx = \frac{2\,\sqrt{bx^3+ax^2}(2\,bx-a)\sqrt{cx}}{3\,a^2c^2x^3}$$

```
integrate(1/(c*x)^(3/2)/(b*x^3+a*x^2)^(1/2),x, algorithm="fricas")
```

```
2/3*sqrt(b*x^3 + a*x^2)*(2*b*x - a)*sqrt(c*x)/(a^2*c^2*x^3)
```

Sympy [F]

$$\int \frac{1}{(cx)^{3/2}\sqrt{ax^2+bx^3}}\,dx = \int \frac{1}{(cx)^{\frac{3}{2}}\sqrt{x^2(a+bx)}}\,dx$$

```
integrate(1/(c*x)**(3/2)/(b*x**3+a*x**2)**(1/2),x)
```

```
Integral(1/((c*x)**(3/2)*sqrt(x**2*(a + b*x))), x)
```

Maxima [F]

$$\int \frac{1}{(cx)^{3/2}\sqrt{ax^2+bx^3}}\,dx = \int \frac{1}{\sqrt{bx^3+ax^2}\,(cx)^{\frac{3}{2}}}\,dx$$

```
integrate(1/(c*x)^(3/2)/(b*x^3+a*x^2)^(1/2),x, algorithm="maxima")
```

```
integrate(1/(sqrt(b*x^3 + a*x^2)*(c*x)^(3/2)), x)
```

Giac [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 59, normalized size of antiderivative = 0.97

$$\int \frac{1}{(cx)^{3/2}\sqrt{ax^2+bx^3}}\,dx = \frac{2\sqrt{bx+a}b^3\left(\frac{2(bx+a)bc}{a^2}-\frac{3bc}{a}\right)}{3\left((bx+a)bc-abc\right)^{\frac{3}{2}}c|b|\mathrm{sgn}\left(x\right)}$$

```
integrate(1/(c*x)^(3/2)/(b*x^3+a*x^2)^(1/2),x, algorithm="giac")
```

```
2/3*sqrt(b*x + a)*b^3*(2*(b*x + a)*b*c/a^2 - 3*b*c/a)/(((b*x + a)*b*c - a*
b*c)^(3/2)*c*abs(b)*sgn(x))
```

Mupad [B] (verification not implemented)

Time = 9.95 (sec) , antiderivative size = 42, normalized size of antiderivative = 0.69

$$\int \frac{1}{(cx)^{3/2}\sqrt{ax^2+bx^3}}\,dx = -\frac{\sqrt{b\,x^3+a\,x^2}\,\left(\frac{2}{3\,a\,c}-\frac{4\,b\,x}{3\,a^2\,c}\right)}{x^2\,\sqrt{c\,x}}$$

```
int(1/((c*x)^(3/2)*(a*x^2 + b*x^3)^(1/2)),x)
```

```
-((a*x^2 + b*x^3)^(1/2)*(2/(3*a*c) - (4*b*x)/(3*a^2*c)))/(x^2*(c*x)^(1/2))
```

Reduce [B] (verification not implemented)

Time = 0.30 (sec) , antiderivative size = 45, normalized size of antiderivative = 0.74

$$\int \frac{1}{(cx)^{3/2}\sqrt{ax^2+bx^3}}\,dx = \frac{2\sqrt{c}\left(-\sqrt{x}\,\sqrt{bx+a}\,a+2\sqrt{x}\,\sqrt{bx+a}\,bx-2\sqrt{b}\,b\,x^2\right)}{3a^2c^2x^2}$$

```
int(1/(c*x)^(3/2)/(b*x^3+a*x^2)^(1/2),x)
```

```
(2*sqrt(c)*( - sqrt(x)*sqrt(a + b*x)*a + 2*sqrt(x)*sqrt(a + b*x)*b*x - 2*s
qrt(b)*b*x**2))/(3*a**2*c**2*x**2)
```

3.353 $\int \frac{1}{(cx)^{5/2}\sqrt{ax^2+bx^3}}\,dx$

Optimal result

Integrand size = 23, antiderivative size = 96

$$\int \frac{1}{(cx)^{5/2}\sqrt{ax^2+bx^3}}\,dx = -\frac{2c\sqrt{ax^2+bx^3}}{5a(cx)^{7/2}}+\frac{8b\sqrt{ax^2+bx^3}}{15a^2(cx)^{5/2}}-\frac{16b^2\sqrt{ax^2+bx^3}}{15a^3c(cx)^{3/2}}$$

```
-2/5*c*(b*x^3+a*x^2)^(1/2)/a/(c*x)^(7/2)+8/15*b*(b*x^3+a*x^2)^(1/2)/a^2/(c
*x)^(5/2)-16/15*b^2*(b*x^3+a*x^2)^(1/2)/a^3/c/(c*x)^(3/2)
```

Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 47, normalized size of antiderivative = 0.49

$$\int \frac{1}{(cx)^{5/2}\sqrt{ax^2+bx^3}}\,dx = -\frac{2c\sqrt{x^2(a+bx)}(3a^2-4abx+8b^2x^2)}{15a^3(cx)^{7/2}}$$

```
Integrate[1/((c*x)^(5/2)*Sqrt[a*x^2 + b*x^3]),x]
```

```
(-2*c*Sqrt[x^2*(a + b*x)]*(3*a^2 - 4*a*b*x + 8*b^2*x^2))/(15*a^3*(c*x)^(7/
2))
```

Rubi [A] (verified)

Time = 0.39 (sec) , antiderivative size = 103, normalized size of antiderivative = 1.07, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.130, Rules used = {1922, 1922, 1920}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{(cx)^{5/2}\sqrt{ax^2+bx^3}}\,dx$$

$$\Big\downarrow \ \color{blue}{1922}$$

$$-\frac{4b\int \frac{1}{(cx)^{3/2}\sqrt{bx^3+ax^2}}dx}{5ac}-\frac{2c\sqrt{ax^2+bx^3}}{5a(cx)^{7/2}}$$

$$\Big\downarrow \ \color{blue}{1922}$$

$$-\frac{4b\left(-\frac{2b\int \frac{1}{\sqrt{cx}\sqrt{bx^3+ax^2}}dx}{3ac}-\frac{2c\sqrt{ax^2+bx^3}}{3a(cx)^{5/2}}\right)}{5ac}-\frac{2c\sqrt{ax^2+bx^3}}{5a(cx)^{7/2}}$$

$$\Big\downarrow \ \color{blue}{1920}$$

$$-\frac{4b\left(\frac{4b\sqrt{ax^2+bx^3}}{3a^2(cx)^{3/2}}-\frac{2c\sqrt{ax^2+bx^3}}{3a(cx)^{5/2}}\right)}{5ac}-\frac{2c\sqrt{ax^2+bx^3}}{5a(cx)^{7/2}}$$

```
Int[1/((c*x)^(5/2)*Sqrt[a*x^2 + b*x^3]),x]
```

```
(-2*c*Sqrt[a*x^2 + b*x^3])/(5*a*(c*x)^(7/2)) - (4*b*((-2*c*Sqrt[a*x^2 + b*
x^3])/(3*a*(c*x)^(5/2)) + (4*b*Sqrt[a*x^2 + b*x^3])/(3*a^2*(c*x)^(3/2))))/
(5*a*c)
```

Defintions of rubi rules used

```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> Simp[(-c^(j - 1))*(c*x)^(m - j + 1)*((a*x^j + b*x^n)^(p + 1)/(a*(n - j
)*(p + 1))), x] /; FreeQ[{a, b, c, j, m, n, p}, x] && !IntegerQ[p] && NeQ[
n, j] && EqQ[m + n*p + n - j + 1, 0] && (IntegerQ[j] || GtQ[c, 0])
```

```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> Simp[c^(j - 1)*(c*x)^(m - j + 1)*((a*x^j + b*x^n)^(p + 1)/(a*(m + j*p
+ 1))), x] - Simp[b*((m + n*p + n - j + 1)/(a*c^(n - j)*(m + j*p + 1)))   I
nt[(c*x)^(m + n - j)*(a*x^j + b*x^n)^p, x], x] /; FreeQ[{a, b, c, j, m, n,
p}, x] && !IntegerQ[p] && NeQ[n, j] && ILtQ[Simplify[(m + n*p + n - j + 1)
/(n - j)], 0] && NeQ[m + j*p + 1, 0] && (IntegersQ[j, n] || GtQ[c, 0])
```

Maple [A] (verified)

Time = 0.38 (sec) , antiderivative size = 49, normalized size of antiderivative = 0.51

method	result	size
gosper	$-\frac{2x(bx+a)\left(8b^{2}x^{2}-4abx+3a^{2}\right)}{15a^{3}\left(cx\right)^{\frac{5}{2}}\sqrt{b\,x^{3}+a\,x^{2}}}$	49
orering	$-\frac{2x(bx+a)\left(8b^{2}x^{2}-4abx+3a^{2}\right)}{15a^{3}\left(cx\right)^{\frac{5}{2}}\sqrt{b\,x^{3}+a\,x^{2}}}$	49
risch	$-\frac{2(bx+a)\left(8b^{2}x^{2}-4abx+3a^{2}\right)}{15c^{2}\sqrt{x^{2}\left(bx+a\right)}\,x\sqrt{cx}\,a^{3}}$	52
default	$-\frac{2(bx+a)\left(8b^{2}x^{2}-4abx+3a^{2}\right)}{15\sqrt{b\,x^{3}+a\,x^{2}}\,x\,a^{3}c^{2}\sqrt{cx}}$	54

```
int(1/(c*x)^(5/2)/(b*x^3+a*x^2)^(1/2),x,method=_RETURNVERBOSE)
```

```
-2/15*x*(b*x+a)*(8*b^2*x^2-4*a*b*x+3*a^2)/a^3/(c*x)^(5/2)/(b*x^3+a*x^2)^(1
/2)
```

Fricas [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 48, normalized size of antiderivative = 0.50

$$\int \frac{1}{(cx)^{5/2}\sqrt{ax^2+bx^3}}\,dx = -\frac{2\,(8\,b^2x^2-4\,abx+3\,a^2)\sqrt{bx^3+ax^2}\sqrt{cx}}{15\,a^3c^3x^4}$$

```
integrate(1/(c*x)^(5/2)/(b*x^3+a*x^2)^(1/2),x, algorithm="fricas")
```

```
-2/15*(8*b^2*x^2 - 4*a*b*x + 3*a^2)*sqrt(b*x^3 + a*x^2)*sqrt(c*x)/(a^3*c^3
*x^4)
```

Sympy [F]

$$\int \frac{1}{(cx)^{5/2}\sqrt{ax^2+bx^3}}\,dx = \int \frac{1}{(cx)^{\frac{5}{2}}\sqrt{x^2\,(a+bx)}}\,dx$$

```
integrate(1/(c*x)**(5/2)/(b*x**3+a*x**2)**(1/2),x)
```

```
Integral(1/((c*x)**(5/2)*sqrt(x**2*(a + b*x))), x)
```

Maxima [F]

$$\int \frac{1}{(cx)^{5/2}\sqrt{ax^2+bx^3}}\,dx = \int \frac{1}{\sqrt{bx^3+ax^2}\,(cx)^{\frac{5}{2}}}\,dx$$

```
integrate(1/(c*x)^(5/2)/(b*x^3+a*x^2)^(1/2),x, algorithm="maxima")
```

```
integrate(1/(sqrt(b*x^3 + a*x^2)*(c*x)^(5/2)), x)
```

Giac [A] (verification not implemented)

Time = 33.84 (sec) , antiderivative size = 137, normalized size of antiderivative = 1.43

$$\int \frac{1}{(cx)^{5/2}\sqrt{ax^2+bx^3}}\,dx = -\frac{32\left(a^2c^4-5\left(\sqrt{bc}\sqrt{cx}-\sqrt{bc^2x+ac^2}\right)^2ac^2+10\left(\sqrt{bc}\sqrt{cx}-\sqrt{bc^2x+ac^2}\right)^4\right)\sqrt{bc}b^2c^4}{15\left(ac^2-\left(\sqrt{bc}\sqrt{cx}-\sqrt{bc^2x+ac^2}\right)^2\right)^5|c|\mathrm{sgn}\left(x\right)}$$

```
integrate(1/(c*x)^(5/2)/(b*x^3+a*x^2)^(1/2),x, algorithm="giac")
```

```
-32/15*(a^2*c^4 - 5*(sqrt(b*c)*sqrt(c*x) - sqrt(b*c^2*x + a*c^2))^2*a*c^2
+ 10*(sqrt(b*c)*sqrt(c*x) - sqrt(b*c^2*x + a*c^2))^4)*sqrt(b*c)*b^2*c^4/((
a*c^2 - (sqrt(b*c)*sqrt(c*x) - sqrt(b*c^2*x + a*c^2))^2)^5*abs(c)*sgn(x))
```

Mupad [B] (verification not implemented)

Time = 9.76 (sec) , antiderivative size = 56, normalized size of antiderivative = 0.58

$$\int \frac{1}{(cx)^{5/2}\sqrt{ax^2+bx^3}}\,dx = -\frac{\sqrt{b\,x^3+a\,x^2}\,\left(\frac{2}{5\,a\,c^2}+\frac{16\,b^2\,x^2}{15\,a^3\,c^2}-\frac{8\,b\,x}{15\,a^2\,c^2}\right)}{x^3\,\sqrt{c\,x}}$$

```
int(1/((c*x)^(5/2)*(a*x^2 + b*x^3)^(1/2)),x)
```

```
-((a*x^2 + b*x^3)^(1/2)*(2/(5*a*c^2) + (16*b^2*x^2)/(15*a^3*c^2) - (8*b*x)
/(15*a^2*c^2)))/(x^3*(c*x)^(1/2))
```

Reduce [B] (verification not implemented)

Time = 0.47 (sec) , antiderivative size = 66, normalized size of antiderivative = 0.69

$$\int \frac{1}{(cx)^{5/2}\sqrt{ax^2+bx^3}}\,dx = \frac{2\sqrt{c}\left(-3\sqrt{x}\,\sqrt{bx+a}\,a^2+4\sqrt{x}\,\sqrt{bx+a}\,abx-8\sqrt{x}\,\sqrt{bx+a}\,b^2x^2+8\sqrt{b}\,b^2x^3\right)}{15a^3c^3x^3}$$

```
int(1/(c*x)^(5/2)/(b*x^3+a*x^2)^(1/2),x)
```

```
(2*sqrt(c)*( - 3*sqrt(x)*sqrt(a + b*x)*a**2 + 4*sqrt(x)*sqrt(a + b*x)*a*b*
x - 8*sqrt(x)*sqrt(a + b*x)*b**2*x**2 + 8*sqrt(b)*b**2*x**3))/(15*a**3*c**
3*x**3)
```

3.354 $\int \frac{1}{(cx)^{7/2}\sqrt{ax^2+bx^3}}\,dx$

Optimal result

Integrand size = 23, antiderivative size = 131

$$\int \frac{1}{(cx)^{7/2}\sqrt{ax^2+bx^3}}\,dx = -\frac{2c\sqrt{ax^2+bx^3}}{7a(cx)^{9/2}} + \frac{12b\sqrt{ax^2+bx^3}}{35a^2(cx)^{7/2}} - \frac{16b^2\sqrt{ax^2+bx^3}}{35a^3c(cx)^{5/2}} + \frac{32b^3\sqrt{ax^2+bx^3}}{35a^4c^2(cx)^{3/2}}$$

```
-2/7*c*(b*x^3+a*x^2)^(1/2)/a/(c*x)^(9/2)+12/35*b*(b*x^3+a*x^2)^(1/2)/a^2/(
c*x)^(7/2)-16/35*b^2*(b*x^3+a*x^2)^(1/2)/a^3/c/(c*x)^(5/2)+32/35*b^3*(b*x^
3+a*x^2)^(1/2)/a^4/c^2/(c*x)^(3/2)
```

Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 58, normalized size of antiderivative = 0.44

$$\int \frac{1}{(cx)^{7/2}\sqrt{ax^2+bx^3}}\,dx = -\frac{2c\sqrt{x^2(a+bx)}(5a^3-6a^2bx+8ab^2x^2-16b^3x^3)}{35a^4(cx)^{9/2}}$$

```
Integrate[1/((c*x)^(7/2)*Sqrt[a*x^2 + b*x^3]),x]
```

```
(-2*c*Sqrt[x^2*(a + b*x)]*(5*a^3 - 6*a^2*b*x + 8*a*b^2*x^2 - 16*b^3*x^3))/
(35*a^4*(c*x)^(9/2))
```

Rubi [A] (verified)

Time = 0.48 (sec) , antiderivative size = 145, normalized size of antiderivative = 1.11, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.174$, Rules used = {1922, 1922, 1922, 1920}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{(cx)^{7/2}\sqrt{ax^2+bx^3}}dx$$

$$\Big\downarrow \text{1922}$$

$$-\frac{6b\int \frac{1}{(cx)^{5/2}\sqrt{bx^3+ax^2}}dx}{7ac}-\frac{2c\sqrt{ax^2+bx^3}}{7a(cx)^{9/2}}$$

$$\Big\downarrow \text{1922}$$

$$-\frac{6b\left(-\frac{4b\int \frac{1}{(cx)^{3/2}\sqrt{bx^3+ax^2}}dx}{5ac}-\frac{2c\sqrt{ax^2+bx^3}}{5a(cx)^{7/2}}\right)}{7ac}-\frac{2c\sqrt{ax^2+bx^3}}{7a(cx)^{9/2}}$$

$$\Big\downarrow \text{1922}$$

$$-\frac{6b\left(-\frac{4b\left(-\frac{2b\int \frac{1}{\sqrt{cx}\sqrt{bx^3+ax^2}}dx}{3ac}-\frac{2c\sqrt{ax^2+bx^3}}{3a(cx)^{5/2}}\right)}{5ac}-\frac{2c\sqrt{ax^2+bx^3}}{5a(cx)^{7/2}}\right)}{7ac}-\frac{2c\sqrt{ax^2+bx^3}}{7a(cx)^{9/2}}$$

$$\Big\downarrow \text{1920}$$

$$-\frac{6b\left(-\frac{4b\left(\frac{4b\sqrt{ax^2+bx^3}}{3a^2(cx)^{3/2}}-\frac{2c\sqrt{ax^2+bx^3}}{3a(cx)^{5/2}}\right)}{5ac}-\frac{2c\sqrt{ax^2+bx^3}}{5a(cx)^{7/2}}\right)}{7ac}-\frac{2c\sqrt{ax^2+bx^3}}{7a(cx)^{9/2}}$$

```
Int[1/((c*x)^(7/2)*Sqrt[a*x^2 + b*x^3]),x]
```

```
(-2*c*Sqrt[a*x^2 + b*x^3])/(7*a*(c*x)^(9/2)) - (6*b*((-2*c*Sqrt[a*x^2 + b*
x^3])/(5*a*(c*x)^(7/2)) - (4*b*((-2*c*Sqrt[a*x^2 + b*x^3])/(3*a*(c*x)^(5/2
)) + (4*b*Sqrt[a*x^2 + b*x^3])/(3*a^2*(c*x)^(3/2))))/(5*a*c)))/(7*a*c)
```

Defintions of rubi rules used

```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> Simp[(-c^(j - 1))*(c*x)^(m - j + 1)*((a*x^j + b*x^n)^(p + 1)/(a*(n - j
)*(p + 1))), x] /; FreeQ[{a, b, c, j, m, n, p}, x] && !IntegerQ[p] && NeQ[
n, j] && EqQ[m + n*p + n - j + 1, 0] && (IntegerQ[j] || GtQ[c, 0])
```

```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> Simp[c^(j - 1)*(c*x)^(m - j + 1)*((a*x^j + b*x^n)^(p + 1)/(a*(m + j*p
+ 1))), x] - Simp[b*((m + n*p + n - j + 1)/(a*c^(n - j)*(m + j*p + 1)))   I
nt[(c*x)^(m + n - j)*(a*x^j + b*x^n)^p, x], x] /; FreeQ[{a, b, c, j, m, n,
p}, x] && !IntegerQ[p] && NeQ[n, j] && ILtQ[Simplify[(m + n*p + n - j + 1)
/(n - j)], 0] && NeQ[m + j*p + 1, 0] && (IntegersQ[j, n] || GtQ[c, 0])
```

Maple [A] (verified)

Time = 0.39 (sec) , antiderivative size = 60, normalized size of antiderivative = 0.46

method	result	size
gosper	$-\frac{2x(bx+a)\left(-16b^{3}x^{3}+8a\,b^{2}x^{2}-6a^{2}bx+5a^{3}\right)}{35a^{4}(cx)^{\frac{7}{2}}\sqrt{b\,x^{3}+a\,x^{2}}}$	60
orering	$-\frac{2x(bx+a)\left(-16b^{3}x^{3}+8a\,b^{2}x^{2}-6a^{2}bx+5a^{3}\right)}{35a^{4}(cx)^{\frac{7}{2}}\sqrt{b\,x^{3}+a\,x^{2}}}$	60
risch	$-\frac{2(bx+a)\left(-16b^{3}x^{3}+8a\,b^{2}x^{2}-6a^{2}bx+5a^{3}\right)}{35c^{3}\sqrt{x^{2}(bx+a)}\,x^{2}\sqrt{cx}\,a^{4}}$	63
default	$-\frac{2(bx+a)\left(-16b^{3}x^{3}+8a\,b^{2}x^{2}-6a^{2}bx+5a^{3}\right)}{35\sqrt{b\,x^{3}+a\,x^{2}}\,x^{2}a^{4}c^{3}\sqrt{cx}}$	65

```
int(1/(c*x)^(7/2)/(b*x^3+a*x^2)^(1/2),x,method=_RETURNVERBOSE)
```

```
-2/35*x*(b*x+a)*(-16*b^3*x^3+8*a*b^2*x^2-6*a^2*b*x+5*a^3)/a^4/(c*x)^(7/2)/
(b*x^3+a*x^2)^(1/2)
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 59, normalized size of antiderivative = 0.45

$$\int \frac{1}{(cx)^{7/2}\sqrt{ax^2+bx^3}}\,dx = \frac{2\,(16\,b^3x^3-8\,ab^2x^2+6\,a^2bx-5\,a^3)\sqrt{bx^3+ax^2}\sqrt{cx}}{35\,a^4c^4x^5}$$

```
integrate(1/(c*x)^(7/2)/(b*x^3+a*x^2)^(1/2),x, algorithm="fricas")
```

```
2/35*(16*b^3*x^3 - 8*a*b^2*x^2 + 6*a^2*b*x - 5*a^3)*sqrt(b*x^3 + a*x^2)*sq
rt(c*x)/(a^4*c^4*x^5)
```

Sympy [F]

$$\int \frac{1}{(cx)^{7/2}\sqrt{ax^2+bx^3}}\,dx = \int \frac{1}{(cx)^{\frac{7}{2}}\sqrt{x^2\,(a+bx)}}\,dx$$

```
integrate(1/(c*x)**(7/2)/(b*x**3+a*x**2)**(1/2),x)
```

```
Integral(1/((c*x)**(7/2)*sqrt(x**2*(a + b*x))), x)
```

Maxima [F]

$$\int \frac{1}{(cx)^{7/2}\sqrt{ax^2+bx^3}}\,dx = \int \frac{1}{\sqrt{bx^3+ax^2}\,(cx)^{\frac{7}{2}}}\,dx$$

```
integrate(1/(c*x)^(7/2)/(b*x^3+a*x^2)^(1/2),x, algorithm="maxima")
```

```
integrate(1/(sqrt(b*x^3 + a*x^2)*(c*x)^(7/2)), x)
```

Giac [A] (verification not implemented)

Time = 37.29 (sec) , antiderivative size = 175, normalized size of antiderivative = 1.34

$$\int \frac{1}{(cx)^{7/2}\sqrt{ax^2+bx^3}}\,dx = \frac{64\left(a^3c^6-7\left(\sqrt{bc}\sqrt{cx}-\sqrt{bc^2x+ac^2}\right)^2a^2c^4+21\left(\sqrt{bc}\sqrt{cx}-\sqrt{bc^2x+ac^2}\right)^4\right.}{35\left(ac^2-\left(\sqrt{bc}\sqrt{cx}-\sqrt{bc^2x+ac^2}\right)^2\right)^7}$$

```
integrate(1/(c*x)^(7/2)/(b*x^3+a*x^2)^(1/2),x, algorithm="giac")
```

```
64/35*(a^3*c^6 - 7*(sqrt(b*c)*sqrt(c*x) - sqrt(b*c^2*x + a*c^2))^2*a^2*c^4
 + 21*(sqrt(b*c)*sqrt(c*x) - sqrt(b*c^2*x + a*c^2))^4*a*c^2 - 35*(sqrt(b*c
)*sqrt(c*x) - sqrt(b*c^2*x + a*c^2))^6)*sqrt(b*c)*b^3*c^5/((a*c^2 - (sqrt(
b*c)*sqrt(c*x) - sqrt(b*c^2*x + a*c^2))^2)^7*abs(c)*sgn(x))
```

Mupad [B] (verification not implemented)

Time = 9.51 (sec) , antiderivative size = 70, normalized size of antiderivative = 0.53

$$\int \frac{1}{(cx)^{7/2}\sqrt{ax^2+bx^3}}\,dx = -\frac{\sqrt{b\,x^3+a\,x^2}\left(\frac{2}{7\,a\,c^3}+\frac{16\,b^2\,x^2}{35\,a^3\,c^3}-\frac{32\,b^3\,x^3}{35\,a^4\,c^3}-\frac{12\,b\,x}{35\,a^2\,c^3}\right)}{x^4\,\sqrt{c\,x}}$$

```
int(1/((c*x)^(7/2)*(a*x^2 + b*x^3)^(1/2)),x)
```

```
-((a*x^2 + b*x^3)^(1/2)*(2/(7*a*c^3) + (16*b^2*x^2)/(35*a^3*c^3) - (32*b^3
*x^3)/(35*a^4*c^3) - (12*b*x)/(35*a^2*c^3)))/(x^4*(c*x)^(1/2))
```

Reduce [B] (verification not implemented)

Time = 0.50 (sec) , antiderivative size = 85, normalized size of antiderivative = 0.65

$$\int \frac{1}{(cx)^{7/2}\sqrt{ax^2+bx^3}}\,dx = \frac{2\sqrt{c}\left(-5\sqrt{x}\,\sqrt{bx+a}\,a^3+6\sqrt{x}\,\sqrt{bx+a}\,a^2bx-8\sqrt{x}\,\sqrt{bx+a}\,a\,b^2x^2+16\sqrt{x}\,\sqrt{}\right.}{35a^4c^4x^4}$$

```
int(1/(c*x)^(7/2)/(b*x^3+a*x^2)^(1/2),x)
```

```
(2*sqrt(c)*( - 5*sqrt(x)*sqrt(a + b*x)*a**3 + 6*sqrt(x)*sqrt(a + b*x)*a**2
*b*x - 8*sqrt(x)*sqrt(a + b*x)*a*b**2*x**2 + 16*sqrt(x)*sqrt(a + b*x)*b**3
*x**3 - 16*sqrt(b)*b**3*x**4))/(35*a**4*c**4*x**4)
```

3.355 $\int \frac{(cx)^{11/2}}{\left(ax^2+bx^3\right)^{3/2}}\,dx$

Optimal result

Integrand size = 23, antiderivative size = 150

$$\int \frac{(cx)^{11/2}}{\left(ax^2+bx^3\right)^{3/2}}\,dx = -\frac{2a^2c^4(cx)^{3/2}}{b^3\sqrt{ax^2+bx^3}} - \frac{7ac^6\sqrt{ax^2+bx^3}}{4b^3\sqrt{cx}} + \frac{c^5\sqrt{cx}\sqrt{ax^2+bx^3}}{2b^2} + \frac{15a^2c^{11/2}\operatorname{arctanh}\left(\frac{\sqrt{b}(cx)^{3/2}}{c^{3/2}\sqrt{ax^2+bx^3}}\right)}{4b^{7/2}}$$

```
-2*a^2*c^4*(c*x)^(3/2)/b^3/(b*x^3+a*x^2)^(1/2)-7/4*a*c^6*(b*x^3+a*x^2)^(1/
2)/b^3/(c*x)^(1/2)+1/2*c^5*(c*x)^(1/2)*(b*x^3+a*x^2)^(1/2)/b^2+15/4*a^2*c^
(11/2)*arctanh(b^(1/2)*(c*x)^(3/2)/c^(3/2)/(b*x^3+a*x^2)^(1/2))/b^(7/2)
```

Mathematica [A] (verified)

Time = 0.05 (sec) , antiderivative size = 113, normalized size of antiderivative = 0.75

$$\int \frac{(cx)^{11/2}}{\left(ax^2+bx^3\right)^{3/2}}\,dx = \frac{c^5\sqrt{x}\sqrt{cx}\left(\sqrt{b}\sqrt{x}(-15a^2-5abx+2b^2x^2)+30a^2\sqrt{a+bx}\operatorname{arctanh}\left(\frac{\sqrt{b}\sqrt{x}}{-\sqrt{a}+\sqrt{a+bx}}\right)\right)}{4b^{7/2}\sqrt{x^2(a+bx)}}$$

```
Integrate[(c*x)^(11/2)/(a*x^2 + b*x^3)^(3/2),x]
```

```
(c^5*Sqrt[x]*Sqrt[c*x]*(Sqrt[b]*Sqrt[x]*(-15*a^2 - 5*a*b*x + 2*b^2*x^2) +
30*a^2*Sqrt[a + b*x]*ArcTanh[(Sqrt[b]*Sqrt[x])/(-Sqrt[a] + Sqrt[a + b*x])]
))/(4*b^(7/2)*Sqrt[x^2*(a + b*x)])
```

Rubi [A] (verified)

Time = 0.58 (sec) , antiderivative size = 159, normalized size of antiderivative = 1.06, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.261$, Rules used = {1928, 1930, 1930, 1937, 1935, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(cx)^{11/2}}{(ax^2+bx^3)^{3/2}}dx$$

$$\Big\downarrow \text{1928}$$

$$\frac{5c^3\int \frac{(cx)^{5/2}}{\sqrt{bx^3+ax^2}}dx}{b}-\frac{2c^2(cx)^{7/2}}{b\sqrt{ax^2+bx^3}}$$

$$\Big\downarrow \text{1930}$$

$$\frac{5c^3\left(\frac{c^2\sqrt{cx}\sqrt{ax^2+bx^3}}{2b}-\frac{3ac\int \frac{(cx)^{3/2}}{\sqrt{bx^3+ax^2}}dx}{4b}\right)}{b}-\frac{2c^2(cx)^{7/2}}{b\sqrt{ax^2+bx^3}}$$

$$\Big\downarrow \text{1930}$$

$$\frac{5c^3\left(\frac{c^2\sqrt{cx}\sqrt{ax^2+bx^3}}{2b}-\frac{3ac\left(\frac{c^2\sqrt{ax^2+bx^3}}{b\sqrt{cx}}-\frac{ac\int \frac{\sqrt{cx}}{\sqrt{bx^3+ax^2}}dx}{2b}\right)}{4b}\right)}{b}-\frac{2c^2(cx)^{7/2}}{b\sqrt{ax^2+bx^3}}$$

$$\Big\downarrow \text{1937}$$

$$\frac{5c^3\left(\frac{c^2\sqrt{cx}\sqrt{ax^2+bx^3}}{2b}-\frac{3ac\left(\frac{c^2\sqrt{ax^2+bx^3}}{b\sqrt{cx}}-\frac{ac\sqrt{cx}\int \frac{\sqrt{x}}{\sqrt{bx^3+ax^2}}dx}{2b\sqrt{x}}\right)}{4b}\right)}{b}-\frac{2c^2(cx)^{7/2}}{b\sqrt{ax^2+bx^3}}$$

$$\Big\downarrow \text{1935}$$

$$\frac{5c^3\left(\frac{c^2\sqrt{cx}\sqrt{ax^2+bx^3}}{2b}-\frac{3ac\left(\frac{c^2\sqrt{ax^2+bx^3}}{b\sqrt{cx}}-\frac{ac\sqrt{cx}\int\frac{1}{1-\frac{bx^3}{bx^3+ax^2}}d\frac{x^{3/2}}{\sqrt{bx^3+ax^2}}}{b\sqrt{x}}\right)}{4b}\right)}{b}-\frac{2c^2(cx)^{7/2}}{b\sqrt{ax^2+bx^3}}$$

$$\Big\downarrow \text{219}$$

$$\frac{5c^3\left(\frac{c^2\sqrt{cx}\sqrt{ax^2+bx^3}}{2b}-\frac{3ac\left(\frac{c^2\sqrt{ax^2+bx^3}}{b\sqrt{cx}}-\frac{ac\sqrt{cx}\text{arctanh}\left(\frac{\sqrt{b}x^{3/2}}{\sqrt{ax^2+bx^3}}\right)}{b^{3/2}\sqrt{x}}\right)}{4b}\right)}{b}-\frac{2c^2(cx)^{7/2}}{b\sqrt{ax^2+bx^3}}$$

```
Int[(c*x)^(11/2)/(a*x^2 + b*x^3)^(3/2),x]
```

```
(-2*c^2*(c*x)^(7/2))/(b*Sqrt[a*x^2 + b*x^3]) + (5*c^3*((c^2*Sqrt[c*x]*Sqrt
[a*x^2 + b*x^3])/(2*b) - (3*a*c*((c^2*Sqrt[a*x^2 + b*x^3])/(b*Sqrt[c*x]) -
 (a*c*Sqrt[c*x]*ArcTanh[(Sqrt[b]*x^(3/2))/Sqrt[a*x^2 + b*x^3]])/(b^(3/2)*S
qrt[x])))/(4*b)))/b
```

Defintions of rubi rules used

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> Simp[c^(n - 1)*(c*x)^(m - n + 1)*((a*x^j + b*x^n)^(p + 1)/(b*(n - j)*(
p + 1))), x] - Simp[c^n*((m + j*p - n + j + 1)/(b*(n - j)*(p + 1)))   Int[(
c*x)^(m - n)*(a*x^j + b*x^n)^(p + 1), x], x] /; FreeQ[{a, b, c}, x] &&  !In
tegerQ[p] && LtQ[0, j, n] && (IntegersQ[j, n] || GtQ[c, 0]) && LtQ[p, -1] &
& GtQ[m + j*p + 1, n - j]
```

```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> Simp[c^(n - 1)*(c*x)^(m - n + 1)*((a*x^j + b*x^n)^(p + 1)/(b*(m + n*p
+ 1))), x] - Simp[a*c^(n - j)*((m + j*p - n + j + 1)/(b*(m + n*p + 1)))   I
nt[(c*x)^(m - (n - j))*(a*x^j + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, p},
x] &&  !IntegerQ[p] && LtQ[0, j, n] && (IntegersQ[j, n] || GtQ[c, 0]) && Gt
Q[m + j*p - n + j + 1, 0] && NeQ[m + n*p + 1, 0]
```

```
Int[(x_)^(m_.)/Sqrt[(a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.)], x_Symbol] :> Simp
[-2/(n - j)   Subst[Int[1/(1 - a*x^2), x], x, x^(j/2)/Sqrt[a*x^j + b*x^n]],
 x] /; FreeQ[{a, b, j, n}, x] && EqQ[m, j/2 - 1] && NeQ[n, j]
```

```
Int[((c_)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_))^(p_), x_Symbol]
:> Simp[c^IntPart[m]*((c*x)^FracPart[m]/x^FracPart[m])   Int[x^m*(a*x^j + b
*x^n)^p, x], x] /; FreeQ[{a, b, c, j, m, n, p}, x] && IntegerQ[p + 1/2] &&
NeQ[n, j] && EqQ[Simplify[m + j*p + 1], 0]
```

Maple [A] (verified)

Time = 0.40 (sec) , antiderivative size = 163, normalized size of antiderivative = 1.09

method	result
risch	$-\frac{(-2bx+7a)x^2(bx+a)c^6}{4b^3\sqrt{x^2(bx+a)}\,\sqrt{cx}}+\frac{\left(\frac{15a^2\ln\left(\frac{\frac{1}{2}ac+cbx}{\sqrt{bc}}+\sqrt{bc\,x^2+acx}\right)}{8b^3\sqrt{bc}}-\frac{2a^2\sqrt{bc\left(x+\frac{a}{b}\right)^2-ac\left(x+\frac{a}{b}\right)}}{b^4c\left(x+\frac{a}{b}\right)}\right)c^6x\sqrt{cx(bx+a)}}{\sqrt{x^2(bx+a)}\,\sqrt{cx}}$
default	$-\frac{x^3(bx+a)\left(-4b^2x^2\sqrt{cx(bx+a)}\,\sqrt{bc}-15\ln\left(\frac{2cbx+2\sqrt{cx(bx+a)}\,\sqrt{bc}+ac}{2\sqrt{bc}}\right)a^2bcx+10\sqrt{bc}\,\sqrt{cx(bx+a)}\,abx-15\ln\left(\frac{2cbx+2\sqrt{cx(bx+a)}}{2\sqrt{bc}}\right.\right.}{8(b\,x^3+a\,x^2)^{\frac{3}{2}}\sqrt{bc}\,\sqrt{cx(bx+a)}\,b^3}$

```
int((c*x)^(11/2)/(b*x^3+a*x^2)^(3/2),x,method=_RETURNVERBOSE)
```

```
-1/4*(-2*b*x+7*a)*x^2*(b*x+a)/b^3*c^6/(x^2*(b*x+a))^(1/2)/(c*x)^(1/2)+(15/
8/b^3*a^2*ln((1/2*a*c+c*b*x)/(b*c)^(1/2)+(b*c*x^2+a*c*x)^(1/2))/(b*c)^(1/2
)-2/b^4*a^2/c/(x+a/b)*(b*c*(x+a/b)^2-a*c*(x+a/b))^(1/2))*c^6/(x^2*(b*x+a))
^(1/2)*x*(c*x*(b*x+a))^(1/2)/(c*x)^(1/2)
```

Fricas [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 281, normalized size of antiderivative = 1.87

$$\int \frac{(cx)^{11/2}}{(ax^2+bx^3)^{3/2}}\,dx = \left[\frac{15\,(a^2bc^5x^2+a^3c^5x)\sqrt{\frac{c}{b}}\log\left(\frac{2\,bcx^2+acx+2\,\sqrt{bx^3+ax^2}\sqrt{cx}b\sqrt{\frac{c}{b}}}{x}\right)+2\,(2\,b^2c^5x^2-5\,abc^5x}{8\,(b^4x^2+ab^3x)}\right.$$

```
integrate((c*x)^(11/2)/(b*x^3+a*x^2)^(3/2),x, algorithm="fricas")
```

```
[1/8*(15*(a^2*b*c^5*x^2 + a^3*c^5*x)*sqrt(c/b)*log((2*b*c*x^2 + a*c*x + 2*
sqrt(b*x^3 + a*x^2)*sqrt(c*x)*b*sqrt(c/b))/x) + 2*(2*b^2*c^5*x^2 - 5*a*b*c
^5*x - 15*a^2*c^5)*sqrt(b*x^3 + a*x^2)*sqrt(c*x))/(b^4*x^2 + a*b^3*x), -1/
4*(15*(a^2*b*c^5*x^2 + a^3*c^5*x)*sqrt(-c/b)*arctan(sqrt(b*x^3 + a*x^2)*sq
rt(c*x)*b*sqrt(-c/b)/(b*c*x^2 + a*c*x)) - (2*b^2*c^5*x^2 - 5*a*b*c^5*x - 1
5*a^2*c^5)*sqrt(b*x^3 + a*x^2)*sqrt(c*x))/(b^4*x^2 + a*b^3*x)]
```

Sympy [F]

$$\int \frac{(cx)^{11/2}}{(ax^2+bx^3)^{3/2}}\,dx = \int \frac{(cx)^{\frac{11}{2}}}{(x^2\,(a+bx))^{\frac{3}{2}}}\,dx$$

```
integrate((c*x)**(11/2)/(b*x**3+a*x**2)**(3/2),x)
```

```
Integral((c*x)**(11/2)/(x**2*(a + b*x))**(3/2), x)
```

Maxima [F]

$$\int \frac{(cx)^{11/2}}{(ax^2+bx^3)^{3/2}}\,dx = \int \frac{(cx)^{\frac{11}{2}}}{(bx^3+ax^2)^{\frac{3}{2}}}\,dx$$

```
integrate((c*x)^(11/2)/(b*x^3+a*x^2)^(3/2),x, algorithm="maxima")
```

```
integrate((c*x)^(11/2)/(b*x^3 + a*x^2)^(3/2), x)
```

Giac [A] (verification not implemented)

Time = 0.19 (sec) , antiderivative size = 163, normalized size of antiderivative = 1.09

$$\int \frac{(cx)^{11/2}}{(ax^2+bx^3)^{3/2}}\,dx = \frac{15\,a^2c^7\log\left(c^2|a|\right)\operatorname{sgn}(x)}{8\,\sqrt{bc}b^3|c|} + \frac{1}{4}\left(\frac{\left(cx\left(\frac{2\,cx}{b|c|\mathrm{sgn}(x)}-\frac{5\,ac}{b^2|c|\mathrm{sgn}(x)}\right)-\frac{15\,a^2c^2}{b^3|c|\mathrm{sgn}(x)}\right)\sqrt{cx}}{\sqrt{bc^2x+ac^2}} - \frac{15\,a^2c^2\log\left(\left|-\sqrt{bc}\sqrt{cx}+\sqrt{bc^2x+ac^2}\right|\right)}{\sqrt{bc}b^3|c|\mathrm{sgn}\,(x)}\right)c^5$$

```
integrate((c*x)^(11/2)/(b*x^3+a*x^2)^(3/2),x, algorithm="giac")
```

```
15/8*a^2*c^7*log(c^2*abs(a))*sgn(x)/(sqrt(b*c)*b^3*abs(c)) + 1/4*((c*x*(2*
c*x/(b*abs(c)*sgn(x)) - 5*a*c/(b^2*abs(c)*sgn(x))) - 15*a^2*c^2/(b^3*abs(c
)*sgn(x)))*sqrt(c*x)/sqrt(b*c^2*x + a*c^2) - 15*a^2*c^2*log(abs(-sqrt(b*c)
*sqrt(c*x) + sqrt(b*c^2*x + a*c^2)))/(sqrt(b*c)*b^3*abs(c)*sgn(x)))*c^5
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(cx)^{11/2}}{(ax^2+bx^3)^{3/2}}\,dx = \int \frac{(c\,x)^{11/2}}{(b\,x^3+a\,x^2)^{3/2}}\,dx$$

```
int((c*x)^(11/2)/(a*x^2 + b*x^3)^(3/2),x)
```

```
int((c*x)^(11/2)/(a*x^2 + b*x^3)^(3/2), x)
```

Reduce [B] (verification not implemented)

Time = 0.24 (sec) , antiderivative size = 90, normalized size of antiderivative = 0.60

$$\int \frac{(cx)^{11/2}}{(ax^2+bx^3)^{3/2}}\,dx = \frac{\sqrt{c}\,c^5\left(15\sqrt{b}\,\sqrt{bx+a}\log\left(\frac{\sqrt{bx+a}+\sqrt{x}\,\sqrt{b}}{\sqrt{a}}\right)a^2-10\sqrt{b}\,\sqrt{bx+a}\,a^2-15\sqrt{x}\,a^2b-5\sqrt{x}\,a}{4\sqrt{bx+a}\,b^4}$$

```
int((c*x)^(11/2)/(b*x^3+a*x^2)^(3/2),x)
```

```
(sqrt(c)*c**5*(15*sqrt(b)*sqrt(a + b*x)*log((sqrt(a + b*x) + sqrt(x)*sqrt(
b))/sqrt(a))*a**2 - 10*sqrt(b)*sqrt(a + b*x)*a**2 - 15*sqrt(x)*a**2*b - 5*
sqrt(x)*a*b**2*x + 2*sqrt(x)*b**3*x**2))/(4*sqrt(a + b*x)*b**4)
```

3.356 $\int \frac{(cx)^{9/2}}{(ax^2+bx^3)^{3/2}}\,dx$

Optimal result

Integrand size = 23, antiderivative size = 108

$$\int \frac{(cx)^{9/2}}{(ax^2+bx^3)^{3/2}}\,dx=\frac{2ac^3(cx)^{3/2}}{b^2\sqrt{ax^2+bx^3}}+\frac{c^5\sqrt{ax^2+bx^3}}{b^2\sqrt{cx}}-\frac{3ac^{9/2}\text{arctanh}\left(\frac{\sqrt{b}(cx)^{3/2}}{c^{3/2}\sqrt{ax^2+bx^3}}\right)}{b^{5/2}}$$

```
2*a*c^3*(c*x)^(3/2)/b^2/(b*x^3+a*x^2)^(1/2)+c^5*(b*x^3+a*x^2)^(1/2)/b^2/(c
*x)^(1/2)-3*a*c^(9/2)*arctanh(b^(1/2)*(c*x)^(3/2)/c^(3/2)/(b*x^3+a*x^2)^(1
/2))/b^(5/2)
```

Mathematica [A] (verified)

Time = 0.04 (sec) , antiderivative size = 96, normalized size of antiderivative = 0.89

$$\int \frac{(cx)^{9/2}}{(ax^2+bx^3)^{3/2}}\,dx=\frac{c^4\sqrt{x}\sqrt{cx}\left(\sqrt{b}\sqrt{x}(3a+bx)+6a\sqrt{a+bx}\text{arctanh}\left(\frac{\sqrt{b}\sqrt{x}}{\sqrt{a}-\sqrt{a+bx}}\right)\right)}{b^{5/2}\sqrt{x^2(a+bx)}}$$

```
Integrate[(c*x)^(9/2)/(a*x^2 + b*x^3)^(3/2),x]
```

```
(c^4*Sqrt[x]*Sqrt[c*x]*(Sqrt[b]*Sqrt[x]*(3*a + b*x) + 6*a*Sqrt[a + b*x]*Ar
cTanh[(Sqrt[b]*Sqrt[x])/(Sqrt[a] - Sqrt[a + b*x])]))/(b^(5/2)*Sqrt[x^2*(a
+ b*x)])
```

Rubi [A] (verified)

Time = 0.49 (sec) , antiderivative size = 117, normalized size of antiderivative = 1.08, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.217$, Rules used = {1928, 1930, 1937, 1935, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(cx)^{9/2}}{(ax^2+bx^3)^{3/2}}dx$$

$$\Big\downarrow \text{1928}$$

$$\frac{3c^3\int \frac{(cx)^{3/2}}{\sqrt{bx^3+ax^2}}dx}{b}-\frac{2c^2(cx)^{5/2}}{b\sqrt{ax^2+bx^3}}$$

$$\Big\downarrow \text{1930}$$

$$\frac{3c^3\left(\frac{c^2\sqrt{ax^2+bx^3}}{b\sqrt{cx}}-\frac{ac\int \frac{\sqrt{cx}}{\sqrt{bx^3+ax^2}}dx}{2b}\right)}{b}-\frac{2c^2(cx)^{5/2}}{b\sqrt{ax^2+bx^3}}$$

$$\Big\downarrow \text{1937}$$

$$\frac{3c^3\left(\frac{c^2\sqrt{ax^2+bx^3}}{b\sqrt{cx}}-\frac{ac\sqrt{cx}\int \frac{\sqrt{x}}{\sqrt{bx^3+ax^2}}dx}{2b\sqrt{x}}\right)}{b}-\frac{2c^2(cx)^{5/2}}{b\sqrt{ax^2+bx^3}}$$

$$\Big\downarrow \text{1935}$$

$$\frac{3c^3\left(\frac{c^2\sqrt{ax^2+bx^3}}{b\sqrt{cx}}-\frac{ac\sqrt{cx}\int \frac{1}{1-\frac{bx^3}{bx^3+ax^2}}d\frac{x^{3/2}}{\sqrt{bx^3+ax^2}}}{b\sqrt{x}}\right)}{b}-\frac{2c^2(cx)^{5/2}}{b\sqrt{ax^2+bx^3}}$$

$$\Big\downarrow \text{219}$$

$$\frac{3c^3\left(\frac{c^2\sqrt{ax^2+bx^3}}{b\sqrt{cx}}-\frac{ac\sqrt{cx}\operatorname{arctanh}\left(\frac{\sqrt{b}x^{3/2}}{\sqrt{ax^2+bx^3}}\right)}{b^{3/2}\sqrt{x}}\right)}{b}-\frac{2c^2(cx)^{5/2}}{b\sqrt{ax^2+bx^3}}$$

```
Int[(c*x)^(9/2)/(a*x^2 + b*x^3)^(3/2),x]
```

```
(-2*c^2*(c*x)^(5/2))/(b*Sqrt[a*x^2 + b*x^3]) + (3*c^3*((c^2*Sqrt[a*x^2 + b
*x^3])/(b*Sqrt[c*x]) - (a*c*Sqrt[c*x]*ArcTanh[(Sqrt[b]*x^(3/2))/Sqrt[a*x^2
 + b*x^3]])/(b^(3/2)*Sqrt[x])))/b
```

Defintions of rubi rules used

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> Simp[c^(n - 1)*(c*x)^(m - n + 1)*((a*x^j + b*x^n)^(p + 1)/(b*(n - j)*(
p + 1))), x] - Simp[c^n*((m + j*p - n + j + 1)/(b*(n - j)*(p + 1)))   Int[(
c*x)^(m - n)*(a*x^j + b*x^n)^(p + 1), x], x] /; FreeQ[{a, b, c}, x] &&  !In
tegerQ[p] && LtQ[0, j, n] && (IntegersQ[j, n] || GtQ[c, 0]) && LtQ[p, -1] &
& GtQ[m + j*p + 1, n - j]
```

```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> Simp[c^(n - 1)*(c*x)^(m - n + 1)*((a*x^j + b*x^n)^(p + 1)/(b*(m + n*p
+ 1))), x] - Simp[a*c^(n - j)*((m + j*p - n + j + 1)/(b*(m + n*p + 1)))   I
nt[(c*x)^(m - (n - j))*(a*x^j + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, p},
x] &&  !IntegerQ[p] && LtQ[0, j, n] && (IntegersQ[j, n] || GtQ[c, 0]) && Gt
Q[m + j*p - n + j + 1, 0] && NeQ[m + n*p + 1, 0]
```

```
Int[(x_)^(m_.)/Sqrt[(a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.)], x_Symbol] :> Simp
[-2/(n - j)   Subst[Int[1/(1 - a*x^2), x], x, x^(j/2)/Sqrt[a*x^j + b*x^n]],
 x] /; FreeQ[{a, b, j, n}, x] && EqQ[m, j/2 - 1] && NeQ[n, j]
```

```
Int[((c_)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_))^(p_), x_Symbol]
:> Simp[c^IntPart[m]*((c*x)^FracPart[m]/x^FracPart[m])   Int[x^m*(a*x^j + b
*x^n)^p, x], x] /; FreeQ[{a, b, c, j, m, n, p}, x] && IntegerQ[p + 1/2] &&
NeQ[n, j] && EqQ[Simplify[m + j*p + 1], 0]
```

Maple [A] (verified)

Time = 0.41 (sec) , antiderivative size = 150, normalized size of antiderivative = 1.39

method	result
risch	$\frac{x^{2}(bx+a)c^{5}}{b^{2}\sqrt{x^{2}(bx+a)}\,\sqrt{cx}}+\frac{\left(-\frac{3a\ln\left(\frac{\frac{1}{2}ac+cbx}{\sqrt{bc}}+\sqrt{bc\,x^{2}+acx}\right)}{2b^{2}\sqrt{bc}}+\frac{2a\sqrt{bc\left(x+\frac{a}{b}\right)^{2}-ac\left(x+\frac{a}{b}\right)}}{b^{3}c\left(x+\frac{a}{b}\right)}\right)c^{5}x\sqrt{cx(bx+a)}}{\sqrt{x^{2}(bx+a)}\,\sqrt{cx}}$
default	$\frac{x^{3}(bx+a)\left(-3\ln\left(\frac{2cbx+2\sqrt{cx(bx+a)}\,\sqrt{bc}+ac}{2\sqrt{bc}}\right)abcx+2\sqrt{bc}\,\sqrt{cx(bx+a)}\,bx-3a^{2}c\ln\left(\frac{2cbx+2\sqrt{cx(bx+a)}\,\sqrt{bc}+ac}{2\sqrt{bc}}\right)+6\sqrt{bc}\,\sqrt{cx(bx+a)}\,a\right.}{2(b\,x^{3}+a\,x^{2})^{\frac{3}{2}}\sqrt{bc}\,\sqrt{cx(bx+a)}\,b^{2}}$

```
int((c*x)^(9/2)/(b*x^3+a*x^2)^(3/2),x,method=_RETURNVERBOSE)
```

```
1/b^2*x^2*(b*x+a)*c^5/(x^2*(b*x+a))^(1/2)/(c*x)^(1/2)+(-3/2*a/b^2*ln((1/2*
a*c+c*b*x)/(b*c)^(1/2)+(b*c*x^2+a*c*x)^(1/2))/(b*c)^(1/2)+2*a/b^3/c/(x+a/b
)*(b*c*(x+a/b)^2-a*c*(x+a/b))^(1/2))*c^5/(x^2*(b*x+a))^(1/2)*x*(c*x*(b*x+a
))^(1/2)/(c*x)^(1/2)
```

Fricas [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 245, normalized size of antiderivative = 2.27

$$\int \frac{(cx)^{9/2}}{(ax^2+bx^3)^{3/2}}\,dx = \left[\frac{3\,(abc^4x^2+a^2c^4x)\sqrt{\frac{c}{b}}\log\left(\frac{2\,bcx^2+acx-2\,\sqrt{bx^3+ax^2}\sqrt{cx}b\sqrt{\frac{c}{b}}}{x}\right)+2\,(bc^4x+3\,ac^4)\sqrt{bx^3+}}{2\,(b^3x^2+ab^2x)}\right.$$

```
integrate((c*x)^(9/2)/(b*x^3+a*x^2)^(3/2),x, algorithm="fricas")
```

```
[1/2*(3*(a*b*c^4*x^2 + a^2*c^4*x)*sqrt(c/b)*log((2*b*c*x^2 + a*c*x - 2*sqr
t(b*x^3 + a*x^2)*sqrt(c*x)*b*sqrt(c/b))/x) + 2*(b*c^4*x + 3*a*c^4)*sqrt(b*
x^3 + a*x^2)*sqrt(c*x))/(b^3*x^2 + a*b^2*x), (3*(a*b*c^4*x^2 + a^2*c^4*x)*
sqrt(-c/b)*arctan(sqrt(b*x^3 + a*x^2)*sqrt(c*x)*b*sqrt(-c/b)/(b*c*x^2 + a*
c*x)) + (b*c^4*x + 3*a*c^4)*sqrt(b*x^3 + a*x^2)*sqrt(c*x))/(b^3*x^2 + a*b^
2*x)]
```

Sympy [F]

$$\int \frac{(cx)^{9/2}}{(ax^2+bx^3)^{3/2}}\,dx = \int \frac{(cx)^{\frac{9}{2}}}{(x^2(a+bx))^{\frac{3}{2}}}\,dx$$

```
integrate((c*x)**(9/2)/(b*x**3+a*x**2)**(3/2),x)
```

```
Integral((c*x)**(9/2)/(x**2*(a + b*x))**(3/2), x)
```

Maxima [F]

$$\int \frac{(cx)^{9/2}}{(ax^2+bx^3)^{3/2}}\,dx = \int \frac{(cx)^{\frac{9}{2}}}{(bx^3+ax^2)^{\frac{3}{2}}}\,dx$$

```
integrate((c*x)^(9/2)/(b*x^3+a*x^2)^(3/2),x, algorithm="maxima")
```

```
integrate((c*x)^(9/2)/(b*x^3 + a*x^2)^(3/2), x)
```

Giac [A] (verification not implemented)

Time = 0.17 (sec) , antiderivative size = 138, normalized size of antiderivative = 1.28

$$\int \frac{(cx)^{9/2}}{(ax^2+bx^3)^{3/2}}\,dx = -\frac{3\,ac^6\log\left(c^2|a|\right)\operatorname{sgn}(x)}{2\,\sqrt{bc}b^2|c|} + \left(\frac{3\,ac^4\log\left(\left|-\sqrt{bc}\sqrt{cx}+\sqrt{bc^2x+ac^2}\right|\right)}{\sqrt{bc}b^2|c|\operatorname{sgn}(x)} + \frac{\sqrt{cx}\left(\frac{c^4x}{b|c|\operatorname{sgn}(x)}+\frac{3\,ac^4}{b^2|c|\operatorname{sgn}(x)}\right)}{\sqrt{bc^2x+ac^2}}\right)c^2$$

```
integrate((c*x)^(9/2)/(b*x^3+a*x^2)^(3/2),x, algorithm="giac")
```

```
-3/2*a*c^6*log(c^2*abs(a))*sgn(x)/(sqrt(b*c)*b^2*abs(c)) + (3*a*c^4*log(ab
s(-sqrt(b*c)*sqrt(c*x) + sqrt(b*c^2*x + a*c^2)))/(sqrt(b*c)*b^2*abs(c)*sgn
(x)) + sqrt(c*x)*(c^4*x/(b*abs(c)*sgn(x)) + 3*a*c^4/(b^2*abs(c)*sgn(x)))/s
qrt(b*c^2*x + a*c^2))*c^2
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(cx)^{9/2}}{(ax^2+bx^3)^{3/2}}\,dx = \int \frac{(c\,x)^{9/2}}{(b\,x^3+a\,x^2)^{3/2}}\,dx$$

```
int((c*x)^(9/2)/(a*x^2 + b*x^3)^(3/2),x)
```

```
int((c*x)^(9/2)/(a*x^2 + b*x^3)^(3/2), x)
```

Reduce [B] (verification not implemented)

Time = 0.41 (sec) , antiderivative size = 73, normalized size of antiderivative = 0.68

$$\int \frac{(cx)^{9/2}}{(ax^2+bx^3)^{3/2}}\,dx = \frac{\sqrt{c}\,c^4\left(-12\sqrt{b}\,\sqrt{bx+a}\log\left(\frac{\sqrt{bx+a}+\sqrt{x}\,\sqrt{b}}{\sqrt{a}}\right)a+9\sqrt{b}\,\sqrt{bx+a}\,a+12\sqrt{x}\,ab+4\sqrt{x}\,b^2x\right)}{4\sqrt{bx+a}\,b^3}$$

```
int((c*x)^(9/2)/(b*x^3+a*x^2)^(3/2),x)
```

```
(sqrt(c)*c**4*( - 12*sqrt(b)*sqrt(a + b*x)*log((sqrt(a + b*x) + sqrt(x)*sq
rt(b))/sqrt(a))*a + 9*sqrt(b)*sqrt(a + b*x)*a + 12*sqrt(x)*a*b + 4*sqrt(x)
*b**2*x))/(4*sqrt(a + b*x)*b**3)
```

3.357 $\int \frac{(cx)^{7/2}}{(ax^2+bx^3)^{3/2}}\,dx$

Optimal result

Integrand size = 23, antiderivative size = 77

$$\int \frac{(cx)^{7/2}}{(ax^2+bx^3)^{3/2}}\,dx = -\frac{2c^2(cx)^{3/2}}{b\sqrt{ax^2+bx^3}}+\frac{2c^{7/2}\operatorname{arctanh}\left(\frac{\sqrt{b}(cx)^{3/2}}{c^{3/2}\sqrt{ax^2+bx^3}}\right)}{b^{3/2}}$$

```
-2*c^2*(c*x)^(3/2)/b/(b*x^3+a*x^2)^(1/2)+2*c^(7/2)*arctanh(b^(1/2)*(c*x)^(
3/2)/c^(3/2)/(b*x^3+a*x^2)^(1/2))/b^(3/2)
```

Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 89, normalized size of antiderivative = 1.16

$$\int \frac{(cx)^{7/2}}{(ax^2+bx^3)^{3/2}}\,dx = -\frac{2c^3\sqrt{x}\sqrt{cx}\left(\sqrt{b}\sqrt{x}+2\sqrt{a+bx}\operatorname{arctanh}\left(\frac{\sqrt{b}\sqrt{x}}{\sqrt{a}-\sqrt{a+bx}}\right)\right)}{b^{3/2}\sqrt{x^2(a+bx)}}$$

```
Integrate[(c*x)^(7/2)/(a*x^2 + b*x^3)^(3/2),x]
```

```
(-2*c^3*Sqrt[x]*Sqrt[c*x]*(Sqrt[b]*Sqrt[x] + 2*Sqrt[a + b*x]*ArcTanh[(Sqrt
[b]*Sqrt[x])/(Sqrt[a] - Sqrt[a + b*x])]))/(b^(3/2)*Sqrt[x^2*(a + b*x)])
```

Rubi [A] (verified)

Time = 0.38 (sec) , antiderivative size = 80, normalized size of antiderivative = 1.04, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.174, Rules used = {1928, 1937, 1935, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(cx)^{7/2}}{(ax^2+bx^3)^{3/2}} \, dx$$

$$\Big\downarrow \text{1928}$$

$$\frac{c^3 \int \frac{\sqrt{cx}}{\sqrt{bx^3+ax^2}}dx}{b} - \frac{2c^2(cx)^{3/2}}{b\sqrt{ax^2+bx^3}}$$

$$\Big\downarrow \text{1937}$$

$$\frac{c^3\sqrt{cx} \int \frac{\sqrt{x}}{\sqrt{bx^3+ax^2}}dx}{b\sqrt{x}} - \frac{2c^2(cx)^{3/2}}{b\sqrt{ax^2+bx^3}}$$

$$\Big\downarrow \text{1935}$$

$$\frac{2c^3\sqrt{cx} \int \frac{1}{1-\frac{bx^3}{bx^3+ax^2}}d\frac{x^{3/2}}{\sqrt{bx^3+ax^2}}}{b\sqrt{x}} - \frac{2c^2(cx)^{3/2}}{b\sqrt{ax^2+bx^3}}$$

$$\Big\downarrow \text{219}$$

$$\frac{2c^3\sqrt{cx}\text{arctanh}\left(\frac{\sqrt{b}x^{3/2}}{\sqrt{ax^2+bx^3}}\right)}{b^{3/2}\sqrt{x}} - \frac{2c^2(cx)^{3/2}}{b\sqrt{ax^2+bx^3}}$$

```
Int[(c*x)^(7/2)/(a*x^2 + b*x^3)^(3/2),x]
```

```
(-2*c^2*(c*x)^(3/2))/(b*Sqrt[a*x^2 + b*x^3]) + (2*c^3*Sqrt[c*x]*ArcTanh[(S
qrt[b]*x^(3/2))/Sqrt[a*x^2 + b*x^3]])/(b^(3/2)*Sqrt[x])
```

Defintions of rubi rules used

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> Simp[c^(n - 1)*(c*x)^(m - n + 1)*((a*x^j + b*x^n)^(p + 1)/(b*(n - j)*(
p + 1))), x] - Simp[c^n*((m + j*p - n + j + 1)/(b*(n - j)*(p + 1)))   Int[(
c*x)^(m - n)*(a*x^j + b*x^n)^(p + 1), x], x] /; FreeQ[{a, b, c}, x] &&  !In
tegerQ[p] && LtQ[0, j, n] && (IntegersQ[j, n] || GtQ[c, 0]) && LtQ[p, -1] &
& GtQ[m + j*p + 1, n - j]
```

```
Int[(x_)^(m_.)/Sqrt[(a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.)], x_Symbol] :> Simp
[-2/(n - j)   Subst[Int[1/(1 - a*x^2), x], x, x^(j/2)/Sqrt[a*x^j + b*x^n]],
 x] /; FreeQ[{a, b, j, n}, x] && EqQ[m, j/2 - 1] && NeQ[n, j]
```

```
Int[((c_)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_))^(p_), x_Symbol]
:> Simp[c^IntPart[m]*((c*x)^FracPart[m]/x^FracPart[m])   Int[x^m*(a*x^j + b
*x^n)^p, x], x] /; FreeQ[{a, b, c, j, m, n, p}, x] && IntegerQ[p + 1/2] &&
NeQ[n, j] && EqQ[Simplify[m + j*p + 1], 0]
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 144 vs. $2(61) = 122$.

Time = 0.38 (sec) , antiderivative size = 145, normalized size of antiderivative = 1.88

method	result	size
default	$-\frac{x^{3}\left(b x+a\right)\left(-\ln\left(\frac{2 c b x+2 \sqrt{c x \left(b x+a\right)}\, \sqrt{b c}+a c}{2 \sqrt{b c}}\right) b c x-a c \ln\left(\frac{2 c b x+2 \sqrt{c x \left(b x+a\right)}\, \sqrt{b c}+a c}{2 \sqrt{b c}}\right)+2 \sqrt{c x \left(b x+a\right)}\, \sqrt{b c}\right) \sqrt{c x}\, c^{3}}{\left(b \,x^{3}+a \,x^{2}\right)^{\frac{3}{2}} \sqrt{b c}\, \sqrt{c x \left(b x+a\right)}\, b}$	145

```
int((c*x)^(7/2)/(b*x^3+a*x^2)^(3/2),x,method=_RETURNVERBOSE)
```

```
-x^3*(b*x+a)*(-ln(1/2*(2*c*b*x+2*(c*x*(b*x+a))^(1/2)*(b*c)^(1/2)+a*c)/(b*c
)^(1/2))*b*c*x-a*c*ln(1/2*(2*c*b*x+2*(c*x*(b*x+a))^(1/2)*(b*c)^(1/2)+a*c)/
(b*c)^(1/2))+2*(c*x*(b*x+a))^(1/2)*(b*c)^(1/2))*(c*x)^(1/2)*c^3/(b*x^3+a*x
^2)^(3/2)/(b*c)^(1/2)/(c*x*(b*x+a))^(1/2)/b
```

Fricas [A] (verification not implemented)

Time = 0.10 (sec) , antiderivative size = 215, normalized size of antiderivative = 2.79

$$\int \frac{(cx)^{7/2}}{(ax^2+bx^3)^{3/2}}\,dx = \left[-\frac{2\sqrt{bx^3+ax^2}\sqrt{cx}c^3-(bc^3x^2+ac^3x)\sqrt{\frac{c}{b}}\log\left(\frac{2\,bcx^2+acx+2\,\sqrt{bx^3+ax^2}\sqrt{cx}b\sqrt{\frac{c}{b}}}{x}\right)}{b^2x^2+abx}, -\frac{2}{}\right.$$

```
integrate((c*x)^(7/2)/(b*x^3+a*x^2)^(3/2),x, algorithm="fricas")
```

```
[-(2*sqrt(b*x^3 + a*x^2)*sqrt(c*x)*c^3 - (b*c^3*x^2 + a*c^3*x)*sqrt(c/b)*l
og((2*b*c*x^2 + a*c*x + 2*sqrt(b*x^3 + a*x^2)*sqrt(c*x)*b*sqrt(c/b))/x))/(
b^2*x^2 + a*b*x), -2*(sqrt(b*x^3 + a*x^2)*sqrt(c*x)*c^3 + (b*c^3*x^2 + a*c
^3*x)*sqrt(-c/b)*arctan(sqrt(b*x^3 + a*x^2)*sqrt(c*x)*b*sqrt(-c/b)/(b*c*x^
2 + a*c*x)))/(b^2*x^2 + a*b*x)]
```

Sympy [F]

$$\int \frac{(cx)^{7/2}}{(ax^2+bx^3)^{3/2}}\,dx = \int \frac{(cx)^{\frac{7}{2}}}{(x^2\,(a+bx))^{\frac{3}{2}}}\,dx$$

```
integrate((c*x)**(7/2)/(b*x**3+a*x**2)**(3/2),x)
```

```
Integral((c*x)**(7/2)/(x**2*(a + b*x))**(3/2), x)
```

Maxima [F]

$$\int \frac{(cx)^{7/2}}{(ax^2+bx^3)^{3/2}}\,dx = \int \frac{(cx)^{\frac{7}{2}}}{(bx^3+ax^2)^{\frac{3}{2}}}\,dx$$

```
integrate((c*x)^(7/2)/(b*x^3+a*x^2)^(3/2),x, algorithm="maxima")
```

```
integrate((c*x)^(7/2)/(b*x^3 + a*x^2)^(3/2), x)
```

Giac [A] (verification not implemented)

Time = 0.18 (sec) , antiderivative size = 115, normalized size of antiderivative = 1.49

$$\int \frac{(cx)^{7/2}}{(ax^2+bx^3)^{3/2}}\,dx = \frac{c^5\log\left(c^2|a|\right)\operatorname{sgn}(x)}{\sqrt{bc}b|c|} - 2\,c^2\left(\frac{c^3\log\left(\left|-\sqrt{bc}\sqrt{cx}+\sqrt{bc^2x+ac^2}\right|\right)}{\sqrt{bc}b|c|\operatorname{sgn}(x)} + \frac{\sqrt{cx}c^3}{\sqrt{bc^2x+ac^2}b|c|\operatorname{sgn}(x)}\right)$$

```
integrate((c*x)^(7/2)/(b*x^3+a*x^2)^(3/2),x, algorithm="giac")
```

```
c^5*log(c^2*abs(a))*sgn(x)/(sqrt(b*c)*b*abs(c)) - 2*c^2*(c^3*log(abs(-sqrt
(b*c)*sqrt(c*x) + sqrt(b*c^2*x + a*c^2)))/(sqrt(b*c)*b*abs(c)*sgn(x)) + sq
rt(c*x)*c^3/(sqrt(b*c^2*x + a*c^2)*b*abs(c)*sgn(x)))
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(cx)^{7/2}}{(ax^2+bx^3)^{3/2}}\,dx = \int \frac{(c\,x)^{7/2}}{(b\,x^3+a\,x^2)^{3/2}}\,dx$$

```
int((c*x)^(7/2)/(a*x^2 + b*x^3)^(3/2),x)
```

```
int((c*x)^(7/2)/(a*x^2 + b*x^3)^(3/2), x)
```

Reduce [B] (verification not implemented)

Time = 0.58 (sec) , antiderivative size = 61, normalized size of antiderivative = 0.79

$$\int \frac{(cx)^{7/2}}{\left(ax^2+bx^3\right)^{3/2}}\,dx=\frac{2\sqrt{c}\,c^3\left(\sqrt{b}\,\sqrt{bx+a}\,\log\left(\frac{\sqrt{bx+a}+\sqrt{x}\,\sqrt{b}}{\sqrt{a}}\right)-\sqrt{b}\,\sqrt{bx+a}-\sqrt{x}\,b\right)}{\sqrt{bx+a}\,b^2}$$

```
int((c*x)^(7/2)/(b*x^3+a*x^2)^(3/2),x)
```

```
(2*sqrt(c)*c**3*(sqrt(b)*sqrt(a + b*x)*log((sqrt(a + b*x) + sqrt(x)*sqrt(b
))/sqrt(a)) - sqrt(b)*sqrt(a + b*x) - sqrt(x)*b))/(sqrt(a + b*x)*b**2)
```

3.358 $\int \frac{(cx)^{5/2}}{(ax^2+bx^3)^{3/2}}\,dx$

Optimal result

Integrand size = 23, antiderivative size = 28

$$\int \frac{(cx)^{5/2}}{(ax^2+bx^3)^{3/2}}\,dx = \frac{2c(cx)^{3/2}}{a\sqrt{ax^2+bx^3}}$$

```
2*c*(c*x)^(3/2)/a/(b*x^3+a*x^2)^(1/2)
```

Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 26, normalized size of antiderivative = 0.93

$$\int \frac{(cx)^{5/2}}{(ax^2+bx^3)^{3/2}}\,dx = \frac{2c(cx)^{3/2}}{a\sqrt{x^2(a+bx)}}$$

```
Integrate[(c*x)^(5/2)/(a*x^2 + b*x^3)^(3/2),x]
```

```
(2*c*(c*x)^(3/2))/(a*Sqrt[x^2*(a + b*x)])
```

Rubi [A] (verified)

Time = 0.25 (sec) , antiderivative size = 28, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}} = 0.043$, Rules used = $\{1920\}$

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(cx)^{5/2}}{(ax^2+bx^3)^{3/2}} \, dx$$
$$\Big\downarrow \; 1920$$
$$\frac{2c(cx)^{3/2}}{a\sqrt{ax^2+bx^3}}$$

```
Int[(c*x)^(5/2)/(a*x^2 + b*x^3)^(3/2),x]
```

```
(2*c*(c*x)^(3/2))/(a*Sqrt[a*x^2 + b*x^3])
```

Defintions of rubi rules used

```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> Simp[(-c^(j - 1))*(c*x)^(m - j + 1)*((a*x^j + b*x^n)^(p + 1)/(a*(n - j
)*(p + 1))), x] /; FreeQ[{a, b, c, j, m, n, p}, x] &&  !IntegerQ[p] && NeQ[
n, j] && EqQ[m + n*p + n - j + 1, 0] && (IntegerQ[j] || GtQ[c, 0])
```

Maple [A] (verified)

Time = 0.37 (sec) , antiderivative size = 30, normalized size of antiderivative = 1.07

method	result	size
gosper	$\frac{2x(bx+a)(cx)^{\frac{5}{2}}}{a(b\,x^{3}+a\,x^{2})^{\frac{3}{2}}}$	30
orering	$\frac{2x(bx+a)(cx)^{\frac{5}{2}}}{a(b\,x^{3}+a\,x^{2})^{\frac{3}{2}}}$	30
default	$\frac{2x^{3}(bx+a)c^{2}\sqrt{cx}}{(b\,x^{3}+a\,x^{2})^{\frac{3}{2}}a}$	35

```
int((c*x)^(5/2)/(b*x^3+a*x^2)^(3/2),x,method=_RETURNVERBOSE)
```

```
2*x*(b*x+a)/a*(c*x)^(5/2)/(b*x^3+a*x^2)^(3/2)
```

Fricas [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 37, normalized size of antiderivative = 1.32

$$\int \frac{(cx)^{5/2}}{(ax^2+bx^3)^{3/2}}\,dx = \frac{2\sqrt{bx^3+ax^2}\sqrt{cx}c^2}{abx^2+a^2x}$$

```
integrate((c*x)^(5/2)/(b*x^3+a*x^2)^(3/2),x, algorithm="fricas")
```

```
2*sqrt(b*x^3 + a*x^2)*sqrt(c*x)*c^2/(a*b*x^2 + a^2*x)
```

Sympy [F]

$$\int \frac{(cx)^{5/2}}{(ax^2+bx^3)^{3/2}}\,dx = \int \frac{(cx)^{\frac{5}{2}}}{(x^2(a+bx))^{\frac{3}{2}}}\,dx$$

```
integrate((c*x)**(5/2)/(b*x**3+a*x**2)**(3/2),x)
```

```
Integral((c*x)**(5/2)/(x**2*(a + b*x))**(3/2), x)
```

Maxima [F]

$$\int \frac{(cx)^{5/2}}{(ax^2+bx^3)^{3/2}}\,dx = \int \frac{(cx)^{\frac{5}{2}}}{(bx^3+ax^2)^{\frac{3}{2}}}\,dx$$

```
integrate((c*x)^(5/2)/(b*x^3+a*x^2)^(3/2),x, algorithm="maxima")
```

```
integrate((c*x)^(5/2)/(b*x^3 + a*x^2)^(3/2), x)
```

Giac [A] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 35, normalized size of antiderivative = 1.25

$$\int \frac{(cx)^{5/2}}{(ax^2+bx^3)^{3/2}}\,dx = \frac{2\sqrt{cx}c^4}{\sqrt{bc^2x+ac^2}a|c|\mathrm{sgn}\,(x)}$$

```
integrate((c*x)^(5/2)/(b*x^3+a*x^2)^(3/2),x, algorithm="giac")
```

```
2*sqrt(c*x)*c^4/(sqrt(b*c^2*x + a*c^2)*a*abs(c)*sgn(x))
```

Mupad [B] (verification not implemented)

Time = 8.61 (sec) , antiderivative size = 37, normalized size of antiderivative = 1.32

$$\int \frac{(cx)^{5/2}}{(ax^2+bx^3)^{3/2}}\,dx = \frac{2\,c^2\,\sqrt{c\,x}\,\sqrt{b\,x^3+a\,x^2}}{a^2\,x+b\,a\,x^2}$$

```
int((c*x)^(5/2)/(a*x^2 + b*x^3)^(3/2),x)
```

```
(2*c^2*(c*x)^(1/2)*(a*x^2 + b*x^3)^(1/2))/(a^2*x + a*b*x^2)
```

Reduce [B] (verification not implemented)

Time = 0.34 (sec) , antiderivative size = 35, normalized size of antiderivative = 1.25

$$\int \frac{(cx)^{5/2}}{(ax^2+bx^3)^{3/2}}\,dx = \frac{2\sqrt{c}\,c^2\left(\sqrt{b}\,\sqrt{bx+a}+\sqrt{x}\,b\right)}{\sqrt{bx+a}\,ab}$$

```
int((c*x)^(5/2)/(b*x^3+a*x^2)^(3/2),x)
```

```
(2*sqrt(c)*c**2*(sqrt(b)*sqrt(a + b*x) + sqrt(x)*b))/(sqrt(a + b*x)*a*b)
```

3.359 $\int \frac{(cx)^{3/2}}{(ax^2+bx^3)^{3/2}}\,dx$

Optimal result

Integrand size = 23, antiderivative size = 59

$$\int \frac{(cx)^{3/2}}{(ax^2+bx^3)^{3/2}}\,dx=\frac{2c\sqrt{cx}}{a\sqrt{ax^2+bx^3}}-\frac{4c^3\sqrt{ax^2+bx^3}}{a^2(cx)^{3/2}}$$

```
2*c*(c*x)^(1/2)/a/(b*x^3+a*x^2)^(1/2)-4*c^3*(b*x^3+a*x^2)^(1/2)/a^2/(c*x)^
(3/2)
```

Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 32, normalized size of antiderivative = 0.54

$$\int \frac{(cx)^{3/2}}{(ax^2+bx^3)^{3/2}}\,dx=-\frac{2c\sqrt{cx}(a+2bx)}{a^2\sqrt{x^2(a+bx)}}$$

```
Integrate[(c*x)^(3/2)/(a*x^2 + b*x^3)^(3/2),x]
```

```
(-2*c*Sqrt[c*x]*(a + 2*b*x))/(a^2*Sqrt[x^2*(a + b*x)])
```

Rubi [A] (verified)

Time = 0.32 (sec) , antiderivative size = 59, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.087, Rules used = {1921, 1920}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(cx)^{3/2}}{(ax^2+bx^3)^{3/2}}\,dx$$
$$\Big\downarrow \; 1921$$
$$\frac{2c^2\int \frac{1}{\sqrt{cx}\sqrt{bx^3+ax^2}}dx}{a}+\frac{2c\sqrt{cx}}{a\sqrt{ax^2+bx^3}}$$
$$\Big\downarrow \; 1920$$
$$\frac{2c\sqrt{cx}}{a\sqrt{ax^2+bx^3}}-\frac{4c^3\sqrt{ax^2+bx^3}}{a^2(cx)^{3/2}}$$

```
Int[(c*x)^(3/2)/(a*x^2 + b*x^3)^(3/2),x]
```

```
(2*c*Sqrt[c*x])/(a*Sqrt[a*x^2 + b*x^3]) - (4*c^3*Sqrt[a*x^2 + b*x^3])/(a^2
*(c*x)^(3/2))
```

Defintions of rubi rules used

```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> Simp[(-c^(j - 1))*(c*x)^(m - j + 1)*((a*x^j + b*x^n)^(p + 1)/(a*(n - j
)*(p + 1))), x] /; FreeQ[{a, b, c, j, m, n, p}, x] &&  !IntegerQ[p] && NeQ[
n, j] && EqQ[m + n*p + n - j + 1, 0] && (IntegerQ[j] || GtQ[c, 0])
```

```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> Simp[(-c^(j - 1))*(c*x)^(m - j + 1)*((a*x^j + b*x^n)^(p + 1)/(a*(n - j
)*(p + 1))), x] + Simp[c^j*((m + n*p + n - j + 1)/(a*(n - j)*(p + 1)))   In
t[(c*x)^(m - j)*(a*x^j + b*x^n)^(p + 1), x], x] /; FreeQ[{a, b, c, j, m, n}
, x] &&  !IntegerQ[p] && NeQ[n, j] && ILtQ[Simplify[(m + n*p + n - j + 1)/(
n - j)], 0] && LtQ[p, -1] && (IntegerQ[j] || GtQ[c, 0])
```

Maple [A] (verified)

Time = 0.38 (sec) , antiderivative size = 36, normalized size of antiderivative = 0.61

method	result	size
gosper	$-\frac{2x(bx+a)(2bx+a)(cx)^{\frac{3}{2}}}{a^{2}(b\,x^{3}+a\,x^{2})^{\frac{3}{2}}}$	36
orering	$-\frac{2x(bx+a)(2bx+a)(cx)^{\frac{3}{2}}}{a^{2}(b\,x^{3}+a\,x^{2})^{\frac{3}{2}}}$	36
default	$-\frac{2x^{2}(bx+a)c\sqrt{cx}\,(2bx+a)}{(b\,x^{3}+a\,x^{2})^{\frac{3}{2}}a^{2}}$	39
risch	$-\frac{2(bx+a)c^{2}x}{a^{2}\sqrt{x^{2}(bx+a)}\,\sqrt{cx}}-\frac{2b\,x^{2}c^{2}}{a^{2}\sqrt{x^{2}(bx+a)}\,\sqrt{cx}}$	60

```
int((c*x)^(3/2)/(b*x^3+a*x^2)^(3/2),x,method=_RETURNVERBOSE)
```

```
-2*x*(b*x+a)*(2*b*x+a)*(c*x)^(3/2)/a^2/(b*x^3+a*x^2)^(3/2)
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 47, normalized size of antiderivative = 0.80

$$\int \frac{(cx)^{3/2}}{(ax^2+bx^3)^{3/2}}\,dx=-\frac{2\sqrt{bx^3+ax^2}(2\,bcx+ac)\sqrt{cx}}{a^2bx^3+a^3x^2}$$

```
integrate((c*x)^(3/2)/(b*x^3+a*x^2)^(3/2),x, algorithm="fricas")
```

```
-2*sqrt(b*x^3 + a*x^2)*(2*b*c*x + a*c)*sqrt(c*x)/(a^2*b*x^3 + a^3*x^2)
```

Sympy [F]

$$\int \frac{(cx)^{3/2}}{(ax^2+bx^3)^{3/2}}\,dx = \int \frac{(cx)^{\frac{3}{2}}}{\left(x^2\,(a+bx)\right)^{\frac{3}{2}}}\,dx$$

```
integrate((c*x)**(3/2)/(b*x**3+a*x**2)**(3/2),x)
```

```
Integral((c*x)**(3/2)/(x**2*(a + b*x))**(3/2), x)
```

Maxima [F]

$$\int \frac{(cx)^{3/2}}{(ax^2+bx^3)^{3/2}}\,dx = \int \frac{(cx)^{\frac{3}{2}}}{(bx^3+ax^2)^{\frac{3}{2}}}\,dx$$

```
integrate((c*x)^(3/2)/(b*x^3+a*x^2)^(3/2),x, algorithm="maxima")
```

```
integrate((c*x)^(3/2)/(b*x^3 + a*x^2)^(3/2), x)
```

Giac [F(-1)]

Timed out.

$$\int \frac{(cx)^{3/2}}{(ax^2+bx^3)^{3/2}}\,dx = \text{Timed out}$$

```
integrate((c*x)^(3/2)/(b*x^3+a*x^2)^(3/2),x, algorithm="giac")
```

```
Timed out
```

Mupad [B] (verification not implemented)

Time = 8.59 (sec) , antiderivative size = 56, normalized size of antiderivative = 0.95

$$\int \frac{(cx)^{3/2}}{(ax^2+bx^3)^{3/2}}\,dx = -\frac{\sqrt{b\,x^3+a\,x^2}\,\left(\frac{4\,c\,x\,\sqrt{c\,x}}{a^2}+\frac{2\,c\,\sqrt{c\,x}}{a\,b}\right)}{x^3+\frac{a\,x^2}{b}}$$

```
int((c*x)^(3/2)/(a*x^2 + b*x^3)^(3/2),x)
```

```
-((a*x^2 + b*x^3)^(1/2)*((4*c*x*(c*x)^(1/2))/a^2 + (2*c*(c*x)^(1/2))/(a*b)
))/(x^3 + (a*x^2)/b)
```

Reduce [B] (verification not implemented)

Time = 0.21 (sec) , antiderivative size = 42, normalized size of antiderivative = 0.71

$$\int \frac{(cx)^{3/2}}{(ax^2+bx^3)^{3/2}}\,dx = \frac{2\sqrt{c}\,c\left(-2\sqrt{b}\,\sqrt{bx+a}\,x-\sqrt{x}\,a-2\sqrt{x}\,bx\right)}{\sqrt{bx+a}\,a^2x}$$

```
int((c*x)^(3/2)/(b*x^3+a*x^2)^(3/2),x)
```

```
(2*sqrt(c)*c*( - 2*sqrt(b)*sqrt(a + b*x)*x - sqrt(x)*a - 2*sqrt(x)*b*x))/(
sqrt(a + b*x)*a**2*x)
```

3.360 $\int \frac{\sqrt{cx}}{(ax^2+bx^3)^{3/2}}\,dx$

Optimal result

Integrand size = 23, antiderivative size = 94

$$\int \frac{\sqrt{cx}}{(ax^2+bx^3)^{3/2}}\,dx = \frac{2c}{a\sqrt{cx}\sqrt{ax^2+bx^3}} - \frac{8c^3\sqrt{ax^2+bx^3}}{3a^2(cx)^{5/2}} + \frac{16bc^2\sqrt{ax^2+bx^3}}{3a^3(cx)^{3/2}}$$

```
2*c/a/(c*x)^(1/2)/(b*x^3+a*x^2)^(1/2)-8/3*c^3*(b*x^3+a*x^2)^(1/2)/a^2/(c*x
)^(5/2)+16/3*b*c^2*(b*x^3+a*x^2)^(1/2)/a^3/(c*x)^(3/2)
```

Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 47, normalized size of antiderivative = 0.50

$$\int \frac{\sqrt{cx}}{(ax^2+bx^3)^{3/2}}\,dx = \frac{2c(-a^2+4abx+8b^2x^2)}{3a^3\sqrt{cx}\sqrt{x^2(a+bx)}}$$

```
Integrate[Sqrt[c*x]/(a*x^2 + b*x^3)^(3/2),x]
```

```
(2*c*(-a^2 + 4*a*b*x + 8*b^2*x^2))/(3*a^3*Sqrt[c*x]*Sqrt[x^2*(a + b*x)])
```

Rubi [A] (verified)

Time = 0.40 (sec) , antiderivative size = 98, normalized size of antiderivative = 1.04, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.130, Rules used = {1921, 1922, 1920}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sqrt{cx}}{(ax^2+bx^3)^{3/2}}\,dx$$

$$\Big\downarrow \ 1921$$

$$\frac{4c^2\int \frac{1}{(cx)^{3/2}\sqrt{bx^3+ax^2}}dx}{a}+\frac{2c}{a\sqrt{cx}\sqrt{ax^2+bx^3}}$$

$$\Big\downarrow \ 1922$$

$$\frac{4c^2\left(-\frac{2b\int \frac{1}{\sqrt{cx}\sqrt{bx^3+ax^2}}dx}{3ac}-\frac{2c\sqrt{ax^2+bx^3}}{3a(cx)^{5/2}}\right)}{a}+\frac{2c}{a\sqrt{cx}\sqrt{ax^2+bx^3}}$$

$$\Big\downarrow \ 1920$$

$$\frac{4c^2\left(\frac{4b\sqrt{ax^2+bx^3}}{3a^2(cx)^{3/2}}-\frac{2c\sqrt{ax^2+bx^3}}{3a(cx)^{5/2}}\right)}{a}+\frac{2c}{a\sqrt{cx}\sqrt{ax^2+bx^3}}$$

```
Int[Sqrt[c*x]/(a*x^2 + b*x^3)^(3/2),x]
```

```
(2*c)/(a*Sqrt[c*x]*Sqrt[a*x^2 + b*x^3]) + (4*c^2*((-2*c*Sqrt[a*x^2 + b*x^3
])/(3*a*(c*x)^(5/2)) + (4*b*Sqrt[a*x^2 + b*x^3])/(3*a^2*(c*x)^(3/2))))/a
```

Defintions of rubi rules used

```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> Simp[(-c^(j - 1))*(c*x)^(m - j + 1)*((a*x^j + b*x^n)^(p + 1)/(a*(n - j
)*(p + 1))), x] /; FreeQ[{a, b, c, j, m, n, p}, x] &&  !IntegerQ[p] && NeQ[
n, j] && EqQ[m + n*p + n - j + 1, 0] && (IntegerQ[j] || GtQ[c, 0])
```

```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> Simp[(-c^(j - 1))*(c*x)^(m - j + 1)*((a*x^j + b*x^n)^(p + 1)/(a*(n - j
)*(p + 1))), x] + Simp[c^j*((m + n*p + n - j + 1)/(a*(n - j)*(p + 1)))   In
t[(c*x)^(m - j)*(a*x^j + b*x^n)^(p + 1), x], x] /; FreeQ[{a, b, c, j, m, n}
, x] &&  !IntegerQ[p] && NeQ[n, j] && ILtQ[Simplify[(m + n*p + n - j + 1)/(
n - j)], 0] && LtQ[p, -1] && (IntegerQ[j] || GtQ[c, 0])
```

```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> Simp[c^(j - 1)*(c*x)^(m - j + 1)*((a*x^j + b*x^n)^(p + 1)/(a*(m + j*p
+ 1))), x] - Simp[b*((m + n*p + n - j + 1)/(a*c^(n - j)*(m + j*p + 1)))   I
nt[(c*x)^(m + n - j)*(a*x^j + b*x^n)^p, x], x] /; FreeQ[{a, b, c, j, m, n,
p}, x] &&  !IntegerQ[p] && NeQ[n, j] && ILtQ[Simplify[(m + n*p + n - j + 1)
/(n - j)], 0] && NeQ[m + j*p + 1, 0] && (IntegersQ[j, n] || GtQ[c, 0])
```

Maple [A] (verified)

Time = 0.38 (sec) , antiderivative size = 47, normalized size of antiderivative = 0.50

method	result	size
gosper	$-\frac{2x(bx+a)\left(-8b^{2}x^{2}-4abx+a^{2}\right)\sqrt{cx}}{3a^{3}\left(b\,x^{3}+a\,x^{2}\right)^{\frac{3}{2}}}$	47
default	$-\frac{2x(bx+a)\left(-8b^{2}x^{2}-4abx+a^{2}\right)\sqrt{cx}}{3a^{3}\left(b\,x^{3}+a\,x^{2}\right)^{\frac{3}{2}}}$	47
orering	$-\frac{2x(bx+a)\left(-8b^{2}x^{2}-4abx+a^{2}\right)\sqrt{cx}}{3a^{3}\left(b\,x^{3}+a\,x^{2}\right)^{\frac{3}{2}}}$	47
risch	$-\frac{2(bx+a)(-5bx+a)c}{3a^{3}\sqrt{x^{2}(bx+a)}\,\sqrt{cx}}+\frac{2b^{2}x^{2}c}{a^{3}\sqrt{x^{2}(bx+a)}\,\sqrt{cx}}$	63

```
int((c*x)^(1/2)/(b*x^3+a*x^2)^(3/2),x,method=_RETURNVERBOSE)
```

```
-2/3*x*(b*x+a)*(-8*b^2*x^2-4*a*b*x+a^2)*(c*x)^(1/2)/a^3/(b*x^3+a*x^2)^(3/2
)
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 57, normalized size of antiderivative = 0.61

$$\int \frac{\sqrt{cx}}{(ax^2+bx^3)^{3/2}}\,dx = \frac{2\,(8\,b^2x^2+4\,abx-a^2)\sqrt{bx^3+ax^2}\sqrt{cx}}{3\,(a^3bx^4+a^4x^3)}$$

```
integrate((c*x)^(1/2)/(b*x^3+a*x^2)^(3/2),x, algorithm="fricas")
```

```
2/3*(8*b^2*x^2 + 4*a*b*x - a^2)*sqrt(b*x^3 + a*x^2)*sqrt(c*x)/(a^3*b*x^4 +
 a^4*x^3)
```

Sympy [F]

$$\int \frac{\sqrt{cx}}{(ax^2+bx^3)^{3/2}}\,dx = \int \frac{\sqrt{cx}}{(x^2\,(a+bx))^{\frac{3}{2}}}\,dx$$

```
integrate((c*x)**(1/2)/(b*x**3+a*x**2)**(3/2),x)
```

```
Integral(sqrt(c*x)/(x**2*(a + b*x))**(3/2), x)
```

Maxima [F]

$$\int \frac{\sqrt{cx}}{(ax^2+bx^3)^{3/2}}\,dx = \int \frac{\sqrt{cx}}{(bx^3+ax^2)^{\frac{3}{2}}}\,dx$$

```
integrate((c*x)^(1/2)/(b*x^3+a*x^2)^(3/2),x, algorithm="maxima")
```

```
integrate(sqrt(c*x)/(b*x^3 + a*x^2)^(3/2), x)
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 190 vs. 2(78) = 156.

Time = 36.93 (sec) , antiderivative size = 190, normalized size of antiderivative = 2.02

$$\int \frac{\sqrt{cx}}{(ax^2+bx^3)^{3/2}}\,dx = \frac{2\sqrt{cx}b^2c^2}{\sqrt{bc^2x+ac^2}a^3|c|\mathrm{sgn}\,(x)} + \frac{4\left(5\sqrt{bc}a^2bc^7-12\sqrt{bc}\left(\sqrt{bc}\sqrt{cx}-\sqrt{bc^2x+ac^2}\right)^2abc^5+3\sqrt{bc}\left(\sqrt{bc}\sqrt{cx}-\sqrt{bc^2x+ac^2}\right)^4bc^3\right)}{3\left(ac^2-\left(\sqrt{bc}\sqrt{cx}-\sqrt{bc^2x+ac^2}\right)^2\right)^3a^2|c|\mathrm{sgn}\,(x)}$$

```
integrate((c*x)^(1/2)/(b*x^3+a*x^2)^(3/2),x, algorithm="giac")
```

```
2*sqrt(c*x)*b^2*c^2/(sqrt(b*c^2*x + a*c^2)*a^3*abs(c)*sgn(x)) + 4/3*(5*sqr
t(b*c)*a^2*b*c^7 - 12*sqrt(b*c)*(sqrt(b*c)*sqrt(c*x) - sqrt(b*c^2*x + a*c^
2))^2*a*b*c^5 + 3*sqrt(b*c)*(sqrt(b*c)*sqrt(c*x) - sqrt(b*c^2*x + a*c^2))^
4*b*c^3)/((a*c^2 - (sqrt(b*c)*sqrt(c*x) - sqrt(b*c^2*x + a*c^2))^2)^3*a^2*
abs(c)*sgn(x))
```

Mupad [B] (verification not implemented)

Time = 8.89 (sec) , antiderivative size = 67, normalized size of antiderivative = 0.71

$$\int \frac{\sqrt{cx}}{(ax^2+bx^3)^{3/2}}\,dx = \frac{\sqrt{b\,x^3+a\,x^2}\left(\frac{8\,x\,\sqrt{c\,x}}{3\,a^2}-\frac{2\,\sqrt{c\,x}}{3\,a\,b}+\frac{16\,b\,x^2\,\sqrt{c\,x}}{3\,a^3}\right)}{x^4+\frac{a\,x^3}{b}}$$

```
int((c*x)^(1/2)/(a*x^2 + b*x^3)^(3/2),x)
```

```
((a*x^2 + b*x^3)^(1/2)*((8*x*(c*x)^(1/2))/(3*a^2) - (2*(c*x)^(1/2))/(3*a*b
) + (16*b*x^2*(c*x)^(1/2))/(3*a^3)))/(x^4 + (a*x^3)/b)
```

Reduce [B] (verification not implemented)

Time = 0.46 (sec) , antiderivative size = 57, normalized size of antiderivative = 0.61

$$\int \frac{\sqrt{cx}}{(ax^2+bx^3)^{3/2}}\,dx = \frac{2\sqrt{c}\left(-8\sqrt{b}\,\sqrt{bx+a}\,b\,x^2-\sqrt{x}\,a^2+4\sqrt{x}\,abx+8\sqrt{x}\,b^2x^2\right)}{3\sqrt{bx+a}\,a^3x^2}$$

```
int((c*x)^(1/2)/(b*x^3+a*x^2)^(3/2),x)
```

```
(2*sqrt(c)*( - 8*sqrt(b)*sqrt(a + b*x)*b*x**2 - sqrt(x)*a**2 + 4*sqrt(x)*a
*b*x + 8*sqrt(x)*b**2*x**2))/(3*sqrt(a + b*x)*a**3*x**2)
```

3.361 $\int \frac{1}{\sqrt{cx}\left(ax^2+bx^3\right)^{3/2}}\,dx$

Optimal result

Integrand size = 23, antiderivative size = 127

$$\int \frac{1}{\sqrt{cx}\left(ax^2+bx^3\right)^{3/2}}\,dx = \frac{2c}{a(cx)^{3/2}\sqrt{ax^2+bx^3}} - \frac{12c^3\sqrt{ax^2+bx^3}}{5a^2(cx)^{7/2}} + \frac{16bc^2\sqrt{ax^2+bx^3}}{5a^3(cx)^{5/2}} - \frac{32b^2c\sqrt{ax^2+bx^3}}{5a^4(cx)^{3/2}}$$

```
2*c/a/(c*x)^(3/2)/(b*x^3+a*x^2)^(1/2)-12/5*c^3*(b*x^3+a*x^2)^(1/2)/a^2/(c*
x)^(7/2)+16/5*b*c^2*(b*x^3+a*x^2)^(1/2)/a^3/(c*x)^(5/2)-32/5*b^2*c*(b*x^3+
a*x^2)^(1/2)/a^4/(c*x)^(3/2)
```

Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 56, normalized size of antiderivative = 0.44

$$\int \frac{1}{\sqrt{cx}\left(ax^2+bx^3\right)^{3/2}}\,dx = -\frac{2c(a^3-2a^2bx+8ab^2x^2+16b^3x^3)}{5a^4(cx)^{3/2}\sqrt{x^2(a+bx)}}$$

```
Integrate[1/(Sqrt[c*x]*(a*x^2 + b*x^3)^(3/2)),x]
```

```
(-2*c*(a^3 - 2*a^2*b*x + 8*a*b^2*x^2 + 16*b^3*x^3))/(5*a^4*(c*x)^(3/2)*Sqr
t[x^2*(a + b*x)])
```

Rubi [A] (verified)

Time = 0.48 (sec) , antiderivative size = 140, normalized size of antiderivative = 1.10, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.174$, Rules used = {1921, 1922, 1922, 1920}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{\sqrt{cx}\,(ax^2+bx^3)^{3/2}}dx$$

$$\Big\downarrow \text{1921}$$

$$\frac{6c^2\int \frac{1}{(cx)^{5/2}\sqrt{bx^3+ax^2}}dx}{a}+\frac{2c}{a(cx)^{3/2}\sqrt{ax^2+bx^3}}$$

$$\Big\downarrow \text{1922}$$

$$\frac{6c^2\left(-\frac{4b\int \frac{1}{(cx)^{3/2}\sqrt{bx^3+ax^2}}dx}{5ac}-\frac{2c\sqrt{ax^2+bx^3}}{5a(cx)^{7/2}}\right)}{a}+\frac{2c}{a(cx)^{3/2}\sqrt{ax^2+bx^3}}$$

$$\Big\downarrow \text{1922}$$

$$\frac{6c^2\left(-\frac{4b\left(-\frac{2b\int \frac{1}{\sqrt{cx}\sqrt{bx^3+ax^2}}dx}{3ac}-\frac{2c\sqrt{ax^2+bx^3}}{3a(cx)^{5/2}}\right)}{5ac}-\frac{2c\sqrt{ax^2+bx^3}}{5a(cx)^{7/2}}\right)}{a}+\frac{2c}{a(cx)^{3/2}\sqrt{ax^2+bx^3}}$$

$$\Big\downarrow \text{1920}$$

$$\frac{6c^2\left(-\frac{4b\left(\frac{4b\sqrt{ax^2+bx^3}}{3a^2(cx)^{3/2}}-\frac{2c\sqrt{ax^2+bx^3}}{3a(cx)^{5/2}}\right)}{5ac}-\frac{2c\sqrt{ax^2+bx^3}}{5a(cx)^{7/2}}\right)}{a}+\frac{2c}{a(cx)^{3/2}\sqrt{ax^2+bx^3}}$$

```
Int[1/(Sqrt[c*x]*(a*x^2 + b*x^3)^(3/2)),x]
```

```
(2*c)/(a*(c*x)^(3/2)*Sqrt[a*x^2 + b*x^3]) + (6*c^2*((-2*c*Sqrt[a*x^2 + b*x
^3])/(5*a*(c*x)^(7/2)) - (4*b*((-2*c*Sqrt[a*x^2 + b*x^3])/(3*a*(c*x)^(5/2)
) + (4*b*Sqrt[a*x^2 + b*x^3])/(3*a^2*(c*x)^(3/2))))/(5*a*c)))/a
```

Defintions of rubi rules used

```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> Simp[(-c^(j - 1))*(c*x)^(m - j + 1)*((a*x^j + b*x^n)^(p + 1)/(a*(n - j
)*(p + 1))), x] /; FreeQ[{a, b, c, j, m, n, p}, x] &&  !IntegerQ[p] && NeQ[
n, j] && EqQ[m + n*p + n - j + 1, 0] && (IntegerQ[j] || GtQ[c, 0])
```

```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> Simp[(-c^(j - 1))*(c*x)^(m - j + 1)*((a*x^j + b*x^n)^(p + 1)/(a*(n - j
)*(p + 1))), x] + Simp[c^j*((m + n*p + n - j + 1)/(a*(n - j)*(p + 1)))   In
t[(c*x)^(m - j)*(a*x^j + b*x^n)^(p + 1), x], x] /; FreeQ[{a, b, c, j, m, n}
, x] &&  !IntegerQ[p] && NeQ[n, j] && ILtQ[Simplify[(m + n*p + n - j + 1)/(
n - j)], 0] && LtQ[p, -1] && (IntegerQ[j] || GtQ[c, 0])
```

```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> Simp[c^(j - 1)*(c*x)^(m - j + 1)*((a*x^j + b*x^n)^(p + 1)/(a*(m + j*p
+ 1))), x] - Simp[b*((m + n*p + n - j + 1)/(a*c^(n - j)*(m + j*p + 1)))   I
nt[(c*x)^(m + n - j)*(a*x^j + b*x^n)^p, x], x] /; FreeQ[{a, b, c, j, m, n,
p}, x] &&  !IntegerQ[p] && NeQ[n, j] && ILtQ[Simplify[(m + n*p + n - j + 1)
/(n - j)], 0] && NeQ[m + j*p + 1, 0] && (IntegersQ[j, n] || GtQ[c, 0])
```

Maple [A] (verified)

Time = 0.39 (sec) , antiderivative size = 58, normalized size of antiderivative = 0.46

method	result	size
gosper	$-\frac{2x(bx+a)\left(16b^{3}x^{3}+8a\,b^{2}x^{2}-2a^{2}bx+a^{3}\right)}{5a^{4}\sqrt{cx}\,\left(b\,x^{3}+a\,x^{2}\right)^{\frac{3}{2}}}$	58
default	$-\frac{2x(bx+a)\left(16b^{3}x^{3}+8a\,b^{2}x^{2}-2a^{2}bx+a^{3}\right)}{5a^{4}\sqrt{cx}\,\left(b\,x^{3}+a\,x^{2}\right)^{\frac{3}{2}}}$	58
orering	$-\frac{2x(bx+a)\left(16b^{3}x^{3}+8a\,b^{2}x^{2}-2a^{2}bx+a^{3}\right)}{5a^{4}\sqrt{cx}\,\left(b\,x^{3}+a\,x^{2}\right)^{\frac{3}{2}}}$	58
risch	$-\frac{2(bx+a)\left(11b^{2}x^{2}-3abx+a^{2}\right)}{5a^{4}x\sqrt{x^{2}(bx+a)}\,\sqrt{cx}}-\frac{2b^{3}x^{2}}{a^{4}\sqrt{x^{2}(bx+a)}\,\sqrt{cx}}$	75

```
int(1/(c*x)^(1/2)/(b*x^3+a*x^2)^(3/2),x,method=_RETURNVERBOSE)
```

```
-2/5*x*(b*x+a)*(16*b^3*x^3+8*a*b^2*x^2-2*a^2*b*x+a^3)/a^4/(c*x)^(1/2)/(b*x
^3+a*x^2)^(3/2)
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 68, normalized size of antiderivative = 0.54

$$\int \frac{1}{\sqrt{cx}\,(ax^2+bx^3)^{3/2}}\,dx = -\frac{2\,(16\,b^3x^3+8\,ab^2x^2-2\,a^2bx+a^3)\sqrt{bx^3+ax^2}\sqrt{cx}}{5\,(a^4bcx^5+a^5cx^4)}$$

```
integrate(1/(c*x)^(1/2)/(b*x^3+a*x^2)^(3/2),x, algorithm="fricas")
```

```
-2/5*(16*b^3*x^3 + 8*a*b^2*x^2 - 2*a^2*b*x + a^3)*sqrt(b*x^3 + a*x^2)*sqrt
(c*x)/(a^4*b*c*x^5 + a^5*c*x^4)
```

Sympy [F]

$$\int \frac{1}{\sqrt{cx}\,(ax^2+bx^3)^{3/2}}\,dx = \int \frac{1}{\sqrt{cx}\,(x^2\,(a+bx))^{\frac{3}{2}}}\,dx$$

```
integrate(1/(c*x)**(1/2)/(b*x**3+a*x**2)**(3/2),x)
```

```
Integral(1/(sqrt(c*x)*(x**2*(a + b*x))**(3/2)), x)
```

Maxima [F]

$$\int \frac{1}{\sqrt{cx}\,(ax^2+bx^3)^{3/2}}\,dx = \int \frac{1}{(bx^3+ax^2)^{\frac{3}{2}}\sqrt{cx}}\,dx$$

```
integrate(1/(c*x)^(1/2)/(b*x^3+a*x^2)^(3/2),x, algorithm="maxima")
```

```
integrate(1/((b*x^3 + a*x^2)^(3/2)*sqrt(c*x)), x)
```

Giac [F(-1)]

Timed out.

$$\int \frac{1}{\sqrt{cx}\,(ax^2+bx^3)^{3/2}}\,dx = \text{Timed out}$$

```
integrate(1/(c*x)^(1/2)/(b*x^3+a*x^2)^(3/2),x, algorithm="giac")
```

```
Timed out
```

Mupad [B] (verification not implemented)

Time = 9.39 (sec) , antiderivative size = 75, normalized size of antiderivative = 0.59

$$\int \frac{1}{\sqrt{cx}\,(ax^2+bx^3)^{3/2}}\,dx = -\frac{\sqrt{b\,x^3+a\,x^2}\,\left(\frac{2}{5\,a\,b}-\frac{4\,x}{5\,a^2}+\frac{16\,b\,x^2}{5\,a^3}+\frac{32\,b^2\,x^3}{5\,a^4}\right)}{x^4\,\sqrt{c\,x}+\frac{a\,x^3\,\sqrt{c\,x}}{b}}$$

```
int(1/((c*x)^(1/2)*(a*x^2 + b*x^3)^(3/2)),x)
```

```
-((a*x^2 + b*x^3)^(1/2)*(2/(5*a*b) - (4*x)/(5*a^2) + (16*b*x^2)/(5*a^3) +
(32*b^2*x^3)/(5*a^4)))/(x^4*(c*x)^(1/2) + (a*x^3*(c*x)^(1/2))/b)
```

Reduce [B] (verification not implemented)

Time = 0.56 (sec) , antiderivative size = 75, normalized size of antiderivative = 0.59

$$\int \frac{1}{\sqrt{cx}\,(ax^2+bx^3)^{3/2}}\,dx = \frac{2\sqrt{c}\left(16\sqrt{b}\,\sqrt{bx+a}\,b^2x^3-\sqrt{x}\,a^3+2\sqrt{x}\,a^2bx-8\sqrt{x}\,a\,b^2x^2-16\sqrt{x}\,b^3x^3\right)}{5\sqrt{bx+a}\,a^4c\,x^3}$$

```
int(1/(c*x)^(1/2)/(b*x^3+a*x^2)^(3/2),x)
```

```
(2*sqrt(c)*(16*sqrt(b)*sqrt(a + b*x)*b**2*x**3 - sqrt(x)*a**3 + 2*sqrt(x)*
a**2*b*x - 8*sqrt(x)*a*b**2*x**2 - 16*sqrt(x)*b**3*x**3))/(5*sqrt(a + b*x)
*a**4*c*x**3)
```

3.362 $\int \frac{1}{(cx)^{3/2}(ax^2+bx^3)^{3/2}}\,dx$

Optimal result

Integrand size = 23, antiderivative size = 159

$$\int \frac{1}{(cx)^{3/2}\left(ax^2+bx^3\right)^{3/2}}\,dx=\frac{2c}{a(cx)^{5/2}\sqrt{ax^2+bx^3}}-\frac{16c^3\sqrt{ax^2+bx^3}}{7a^2(cx)^{9/2}}+\frac{96bc^2\sqrt{ax^2+bx^3}}{35a^3(cx)^{7/2}}-\frac{128b^2c\sqrt{ax^2+bx^3}}{35a^4(cx)^{5/2}}+\frac{256b^3\sqrt{ax^2+bx^3}}{35a^5(cx)^{3/2}}$$

```
2*c/a/(c*x)^(5/2)/(b*x^3+a*x^2)^(1/2)-16/7*c^3*(b*x^3+a*x^2)^(1/2)/a^2/(c*
x)^(9/2)+96/35*b*c^2*(b*x^3+a*x^2)^(1/2)/a^3/(c*x)^(7/2)-128/35*b^2*c*(b*x
^3+a*x^2)^(1/2)/a^4/(c*x)^(5/2)+256/35*b^3*(b*x^3+a*x^2)^(1/2)/a^5/(c*x)^(
3/2)
```

Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 69, normalized size of antiderivative = 0.43

$$\int \frac{1}{(cx)^{3/2}\left(ax^2+bx^3\right)^{3/2}}\,dx=\frac{2c\left(-5a^4+8a^3bx-16a^2b^2x^2+64ab^3x^3+128b^4x^4\right)}{35a^5(cx)^{5/2}\sqrt{x^2(a+bx)}}$$

```
Integrate[1/((c*x)^(3/2)*(a*x^2 + b*x^3)^(3/2)),x]
```

```
(2*c*(-5*a^4 + 8*a^3*b*x - 16*a^2*b^2*x^2 + 64*a*b^3*x^3 + 128*b^4*x^4))/(
35*a^5*(c*x)^(5/2)*Sqrt[x^2*(a + b*x)])
```

Rubi [A] (verified)

Time = 0.59 (sec) , antiderivative size = 182, normalized size of antiderivative = 1.14, number of steps used = 5, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.217, Rules used = {1921, 1922, 1922, 1922, 1920}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{(cx)^{3/2}\left(ax^2+bx^3\right)^{3/2}}dx$$

$$\Big\downarrow \text{1921}$$

$$\frac{8c^2\int \frac{1}{(cx)^{7/2}\sqrt{bx^3+ax^2}}dx}{a}+\frac{2c}{a(cx)^{5/2}\sqrt{ax^2+bx^3}}$$

$$\Big\downarrow \text{1922}$$

$$\frac{8c^2\left(-\frac{6b\int \frac{1}{(cx)^{5/2}\sqrt{bx^3+ax^2}}dx}{7ac}-\frac{2c\sqrt{ax^2+bx^3}}{7a(cx)^{9/2}}\right)}{a}+\frac{2c}{a(cx)^{5/2}\sqrt{ax^2+bx^3}}$$

$$\Big\downarrow \text{1922}$$

$$\frac{8c^2\left(-\frac{6b\left(-\frac{4b\int \frac{1}{(cx)^{3/2}\sqrt{bx^3+ax^2}}dx}{5ac}-\frac{2c\sqrt{ax^2+bx^3}}{5a(cx)^{7/2}}\right)}{7ac}-\frac{2c\sqrt{ax^2+bx^3}}{7a(cx)^{9/2}}\right)}{a}+\frac{2c}{a(cx)^{5/2}\sqrt{ax^2+bx^3}}$$

$$\Big\downarrow \text{1922}$$

$$\frac{8c^2\left(-\frac{6b\left(-\frac{4b\left(-\frac{2b\int \frac{1}{\sqrt{cx}\sqrt{bx^3+ax^2}}dx}{3ac}-\frac{2c\sqrt{ax^2+bx^3}}{3a(cx)^{5/2}}\right)}{5ac}-\frac{2c\sqrt{ax^2+bx^3}}{5a(cx)^{7/2}}\right)}{7ac}-\frac{2c\sqrt{ax^2+bx^3}}{7a(cx)^{9/2}}\right)}{a}+\frac{2c}{a(cx)^{5/2}\sqrt{ax^2+bx^3}}$$

$$\Big\downarrow \text{1920}$$

$$\frac{8c^2\left(-\frac{6b\left(-\frac{4b\left(\frac{4b\sqrt{ax^2+bx^3}}{3a^2(cx)^{3/2}}-\frac{2c\sqrt{ax^2+bx^3}}{3a(cx)^{5/2}}\right)}{5ac}-\frac{2c\sqrt{ax^2+bx^3}}{5a(cx)^{7/2}}\right)}{7ac}-\frac{2c\sqrt{ax^2+bx^3}}{7a(cx)^{9/2}}\right)}{a}+\frac{2c}{a(cx)^{5/2}\sqrt{ax^2+bx^3}}$$

```
Int[1/((c*x)^(3/2)*(a*x^2 + b*x^3)^(3/2)),x]
```

```
(2*c)/(a*(c*x)^(5/2)*Sqrt[a*x^2 + b*x^3]) + (8*c^2*((-2*c*Sqrt[a*x^2 + b*x
^3])/(7*a*(c*x)^(9/2)) - (6*b*((-2*c*Sqrt[a*x^2 + b*x^3])/(5*a*(c*x)^(7/2)
) - (4*b*((-2*c*Sqrt[a*x^2 + b*x^3])/(3*a*(c*x)^(5/2)) + (4*b*Sqrt[a*x^2 +
 b*x^3])/(3*a^2*(c*x)^(3/2))))/(5*a*c)))/(7*a*c)))/a
```

Defintions of rubi rules used

```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> Simp[(-c^(j - 1))*(c*x)^(m - j + 1)*((a*x^j + b*x^n)^(p + 1)/(a*(n - j
)*(p + 1))), x] /; FreeQ[{a, b, c, j, m, n, p}, x] &&  !IntegerQ[p] && NeQ[
n, j] && EqQ[m + n*p + n - j + 1, 0] && (IntegerQ[j] || GtQ[c, 0])
```

```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> Simp[(-c^(j - 1))*(c*x)^(m - j + 1)*((a*x^j + b*x^n)^(p + 1)/(a*(n - j
)*(p + 1))), x] + Simp[c^j*((m + n*p + n - j + 1)/(a*(n - j)*(p + 1)))   In
t[(c*x)^(m - j)*(a*x^j + b*x^n)^(p + 1), x], x] /; FreeQ[{a, b, c, j, m, n}
, x] &&  !IntegerQ[p] && NeQ[n, j] && ILtQ[Simplify[(m + n*p + n - j + 1)/(
n - j)], 0] && LtQ[p, -1] && (IntegerQ[j] || GtQ[c, 0])
```

```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> Simp[c^(j - 1)*(c*x)^(m - j + 1)*((a*x^j + b*x^n)^(p + 1)/(a*(m + j*p
+ 1))), x] - Simp[b*((m + n*p + n - j + 1)/(a*c^(n - j)*(m + j*p + 1)))   I
nt[(c*x)^(m + n - j)*(a*x^j + b*x^n)^p, x], x] /; FreeQ[{a, b, c, j, m, n,
p}, x] &&  !IntegerQ[p] && NeQ[n, j] && ILtQ[Simplify[(m + n*p + n - j + 1)
/(n - j)], 0] && NeQ[m + j*p + 1, 0] && (IntegersQ[j, n] || GtQ[c, 0])
```

Maple [A] (verified)

Time = 0.40 (sec) , antiderivative size = 71, normalized size of antiderivative = 0.45

method	result	size
gosper	$-\frac{2x(bx+a)\left(-128b^{4}x^{4}-64a\,b^{3}x^{3}+16a^{2}b^{2}x^{2}-8a^{3}bx+5a^{4}\right)}{35a^{5}\left(cx\right)^{\frac{3}{2}}\left(b\,x^{3}+a\,x^{2}\right)^{\frac{3}{2}}}$	71
orering	$-\frac{2x(bx+a)\left(-128b^{4}x^{4}-64a\,b^{3}x^{3}+16a^{2}b^{2}x^{2}-8a^{3}bx+5a^{4}\right)}{35a^{5}\left(cx\right)^{\frac{3}{2}}\left(b\,x^{3}+a\,x^{2}\right)^{\frac{3}{2}}}$	71
default	$-\frac{2(bx+a)\left(-128b^{4}x^{4}-64a\,b^{3}x^{3}+16a^{2}b^{2}x^{2}-8a^{3}bx+5a^{4}\right)}{35\left(b\,x^{3}+a\,x^{2}\right)^{\frac{3}{2}}\sqrt{cx}\,c\,a^{5}}$	73
risch	$-\frac{2(bx+a)\left(-93b^{3}x^{3}+29a\,b^{2}x^{2}-13a^{2}bx+5a^{3}\right)}{35a^{5}x^{2}c\sqrt{x^{2}(bx+a)}\,\sqrt{cx}}+\frac{2b^{4}x^{2}}{a^{5}c\sqrt{x^{2}(bx+a)}\,\sqrt{cx}}$	94

```
int(1/(c*x)^(3/2)/(b*x^3+a*x^2)^(3/2),x,method=_RETURNVERBOSE)
```

```
-2/35*x*(b*x+a)*(-128*b^4*x^4-64*a*b^3*x^3+16*a^2*b^2*x^2-8*a^3*b*x+5*a^4)
/a^5/(c*x)^(3/2)/(b*x^3+a*x^2)^(3/2)
```

Fricas [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 85, normalized size of antiderivative = 0.53

$$\int \frac{1}{(cx)^{3/2}\left(ax^2+bx^3\right)^{3/2}}\,dx = \frac{2\,(128\,b^4x^4+64\,ab^3x^3-16\,a^2b^2x^2+8\,a^3bx-5\,a^4)\sqrt{bx^3+ax^2}\sqrt{cx}}{35\,(a^5bc^2x^6+a^6c^2x^5)}$$

```
integrate(1/(c*x)^(3/2)/(b*x^3+a*x^2)^(3/2),x, algorithm="fricas")
```

```
2/35*(128*b^4*x^4 + 64*a*b^3*x^3 - 16*a^2*b^2*x^2 + 8*a^3*b*x - 5*a^4)*sqr
t(b*x^3 + a*x^2)*sqrt(c*x)/(a^5*b*c^2*x^6 + a^6*c^2*x^5)
```

Sympy [F]

$$\int \frac{1}{(cx)^{3/2}\left(ax^2+bx^3\right)^{3/2}}\,dx = \int \frac{1}{(cx)^{\frac{3}{2}}\left(x^2\left(a+bx\right)\right)^{\frac{3}{2}}}\,dx$$

```
integrate(1/(c*x)**(3/2)/(b*x**3+a*x**2)**(3/2),x)
```

```
Integral(1/((c*x)**(3/2)*(x**2*(a + b*x))**(3/2)), x)
```

Maxima [F]

$$\int \frac{1}{(cx)^{3/2}\left(ax^2+bx^3\right)^{3/2}}\,dx = \int \frac{1}{\left(bx^3+ax^2\right)^{\frac{3}{2}}(cx)^{\frac{3}{2}}}\,dx$$

```
integrate(1/(c*x)^(3/2)/(b*x^3+a*x^2)^(3/2),x, algorithm="maxima")
```

```
integrate(1/((b*x^3 + a*x^2)^(3/2)*(c*x)^(3/2)), x)
```

Giac [F(-1)]

Timed out.

$$\int \frac{1}{(cx)^{3/2}\left(ax^2+bx^3\right)^{3/2}}\,dx=\text{Timed out}$$

```
integrate(1/(c*x)^(3/2)/(b*x^3+a*x^2)^(3/2),x, algorithm="giac")
```

```
Timed out
```

Mupad [B] (verification not implemented)

Time = 9.34 (sec) , antiderivative size = 70, normalized size of antiderivative = 0.44

$$\int \frac{1}{(cx)^{3/2}\left(ax^2+bx^3\right)^{3/2}}\,dx=\frac{-10\,a^4+16\,a^3\,b\,x-32\,a^2\,b^2\,x^2+128\,a\,b^3\,x^3+256\,b^4\,x^4}{35\,a^5\,c\,x^2\,\sqrt{c\,x}\,\sqrt{b\,x^3+a\,x^2}}$$

```
int(1/((c*x)^(3/2)*(a*x^2 + b*x^3)^(3/2)),x)
```

```
(256*b^4*x^4 - 10*a^4 + 128*a*b^3*x^3 - 32*a^2*b^2*x^2 + 16*a^3*b*x)/(35*a
^5*c*x^2*(c*x)^(1/2)*(a*x^2 + b*x^3)^(1/2))
```

Reduce [B] (verification not implemented)

Time = 0.30 (sec) , antiderivative size = 88, normalized size of antiderivative = 0.55

$$\int \frac{1}{(cx)^{3/2}\left(ax^2+bx^3\right)^{3/2}}\,dx=\frac{2\sqrt{c}\left(-128\sqrt{b}\,\sqrt{bx+a}\,b^3x^4-5\sqrt{x}\,a^4+8\sqrt{x}\,a^3bx-16\sqrt{x}\,a^2b^2x^2+64\sqrt{x}\,a\right.}{35\sqrt{bx+a}\,a^5c^2x^4}$$

```
int(1/(c*x)^(3/2)/(b*x^3+a*x^2)^(3/2),x)
```

```
(2*sqrt(c)*( - 128*sqrt(b)*sqrt(a + b*x)*b**3*x**4 - 5*sqrt(x)*a**4 + 8*sq
rt(x)*a**3*b*x - 16*sqrt(x)*a**2*b**2*x**2 + 64*sqrt(x)*a*b**3*x**3 + 128*
sqrt(x)*b**4*x**4))/(35*sqrt(a + b*x)*a**5*c**2*x**4)
```

3.363 $\int \frac{(cx)^{15/2}}{(ax^2+bx^3)^{5/2}}\,dx$

Optimal result

Integrand size = 23, antiderivative size = 145

$$\int \frac{(cx)^{15/2}}{(ax^2+bx^3)^{5/2}}\,dx = -\frac{2a^2c^4(cx)^{7/2}}{3b^3(ax^2+bx^3)^{3/2}} + \frac{14ac^6(cx)^{3/2}}{3b^3\sqrt{ax^2+bx^3}} + \frac{c^8\sqrt{ax^2+bx^3}}{b^3\sqrt{cx}} - \frac{5ac^{15/2}\operatorname{arctanh}\left(\frac{\sqrt{b}(cx)^{3/2}}{c^{3/2}\sqrt{ax^2+bx^3}}\right)}{b^{7/2}}$$

```
-2/3*a^2*c^4*(c*x)^(7/2)/b^3/(b*x^3+a*x^2)^(3/2)+14/3*a*c^6*(c*x)^(3/2)/b^
3/(b*x^3+a*x^2)^(1/2)+c^8*(b*x^3+a*x^2)^(1/2)/b^3/(c*x)^(1/2)-5*a*c^(15/2)
*arctanh(b^(1/2)*(c*x)^(3/2)/c^(3/2)/(b*x^3+a*x^2)^(1/2))/b^(7/2)
```

Mathematica [A] (verified)

Time = 0.05 (sec) , antiderivative size = 111, normalized size of antiderivative = 0.77

$$\int \frac{(cx)^{15/2}}{(ax^2+bx^3)^{5/2}}\,dx = \frac{c^7x^{5/2}\sqrt{cx}\left(\sqrt{b}\sqrt{x}(15a^2+20abx+3b^2x^2)+30a(a+bx)^{3/2}\operatorname{arctanh}\left(\frac{\sqrt{b}\sqrt{x}}{\sqrt{a}-\sqrt{a+bx}}\right)\right)}{3b^{7/2}\left(x^2(a+bx)\right)^{3/2}}$$

```
Integrate[(c*x)^(15/2)/(a*x^2 + b*x^3)^(5/2),x]
```

```
(c^7*x^(5/2)*Sqrt[c*x]*(Sqrt[b]*Sqrt[x]*(15*a^2 + 20*a*b*x + 3*b^2*x^2) +
30*a*(a + b*x)^(3/2)*ArcTanh[(Sqrt[b]*Sqrt[x])/(Sqrt[a] - Sqrt[a + b*x])])
)/(3*b^(7/2)*(x^2*(a + b*x))^(3/2))
```

Rubi [A] (verified)

Time = 0.59 (sec) , antiderivative size = 160, normalized size of antiderivative = 1.10, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.261$, Rules used = {1928, 1928, 1930, 1937, 1935, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(cx)^{15/2}}{(ax^2+bx^3)^{5/2}}\,dx$$

$$\Big\downarrow \text{ 1928}$$

$$\frac{5c^3\int \frac{(cx)^{9/2}}{(bx^3+ax^2)^{3/2}}dx}{3b}-\frac{2c^2(cx)^{11/2}}{3b\left(ax^2+bx^3\right)^{3/2}}$$

$$\Big\downarrow \text{ 1928}$$

$$\frac{5c^3\left(\frac{3c^3\int \frac{(cx)^{3/2}}{\sqrt{bx^3+ax^2}}dx}{b}-\frac{2c^2(cx)^{5/2}}{b\sqrt{ax^2+bx^3}}\right)}{3b}-\frac{2c^2(cx)^{11/2}}{3b\left(ax^2+bx^3\right)^{3/2}}$$

$$\Big\downarrow \text{ 1930}$$

$$\frac{5c^3\left(\frac{3c^3\left(\frac{c^2\sqrt{ax^2+bx^3}}{b\sqrt{cx}}-\frac{ac\int \frac{\sqrt{cx}}{\sqrt{bx^3+ax^2}}dx}{2b}\right)}{b}-\frac{2c^2(cx)^{5/2}}{b\sqrt{ax^2+bx^3}}\right)}{3b}-\frac{2c^2(cx)^{11/2}}{3b\left(ax^2+bx^3\right)^{3/2}}$$

$$\Big\downarrow \text{ 1937}$$

$$\frac{5c^3\left(\frac{3c^3\left(\frac{c^2\sqrt{ax^2+bx^3}}{b\sqrt{cx}}-\frac{ac\sqrt{cx}\int\frac{\sqrt{x}}{\sqrt{bx^3+ax^2}}dx}{2b\sqrt{x}}\right)}{b}-\frac{2c^2(cx)^{5/2}}{b\sqrt{ax^2+bx^3}}\right)}{3b}-\frac{2c^2(cx)^{11/2}}{3b\left(ax^2+bx^3\right)^{3/2}}$$

$$\Big\downarrow \text{1935}$$

$$\frac{5c^3\left(\frac{3c^3\left(\frac{c^2\sqrt{ax^2+bx^3}}{b\sqrt{cx}}-\frac{ac\sqrt{cx}\int\frac{1}{1-\frac{bx^3}{bx^3+ax^2}}d\frac{x^{3/2}}{\sqrt{bx^3+ax^2}}}{b\sqrt{x}}\right)}{b}-\frac{2c^2(cx)^{5/2}}{b\sqrt{ax^2+bx^3}}\right)}{3b}-\frac{2c^2(cx)^{11/2}}{3b\left(ax^2+bx^3\right)^{3/2}}$$

$$\Big\downarrow \text{219}$$

$$\frac{5c^3\left(\frac{3c^3\left(\frac{c^2\sqrt{ax^2+bx^3}}{b\sqrt{cx}}-\frac{ac\sqrt{cx}\,\text{arctanh}\left(\frac{\sqrt{b}x^{3/2}}{\sqrt{ax^2+bx^3}}\right)}{b^{3/2}\sqrt{x}}\right)}{b}-\frac{2c^2(cx)^{5/2}}{b\sqrt{ax^2+bx^3}}\right)}{3b}-\frac{2c^2(cx)^{11/2}}{3b\left(ax^2+bx^3\right)^{3/2}}$$

```
Int[(c*x)^(15/2)/(a*x^2 + b*x^3)^(5/2),x]
```

```
(-2*c^2*(c*x)^(11/2))/(3*b*(a*x^2 + b*x^3)^(3/2)) + (5*c^3*((-2*c^2*(c*x)^
(5/2))/(b*Sqrt[a*x^2 + b*x^3]) + (3*c^3*((c^2*Sqrt[a*x^2 + b*x^3])/(b*Sqrt
[c*x]) - (a*c*Sqrt[c*x]*ArcTanh[(Sqrt[b]*x^(3/2))/Sqrt[a*x^2 + b*x^3]])/(b
^(3/2)*Sqrt[x])))/b))/(3*b)
```

Defintions of rubi rules used

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> Simp[c^(n - 1)*(c*x)^(m - n + 1)*((a*x^j + b*x^n)^(p + 1)/(b*(n - j)*(
p + 1))), x] - Simp[c^n*((m + j*p - n + j + 1)/(b*(n - j)*(p + 1)))   Int[(
c*x)^(m - n)*(a*x^j + b*x^n)^(p + 1), x], x] /; FreeQ[{a, b, c}, x] &&  !In
tegerQ[p] && LtQ[0, j, n] && (IntegersQ[j, n] || GtQ[c, 0]) && LtQ[p, -1] &
& GtQ[m + j*p + 1, n - j]
```

```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> Simp[c^(n - 1)*(c*x)^(m - n + 1)*((a*x^j + b*x^n)^(p + 1)/(b*(m + n*p
+ 1))), x] - Simp[a*c^(n - j)*((m + j*p - n + j + 1)/(b*(m + n*p + 1)))   I
nt[(c*x)^(m - (n - j))*(a*x^j + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, p},
x] &&  !IntegerQ[p] && LtQ[0, j, n] && (IntegersQ[j, n] || GtQ[c, 0]) && Gt
Q[m + j*p - n + j + 1, 0] && NeQ[m + n*p + 1, 0]
```

```
Int[(x_)^(m_.)/Sqrt[(a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.)], x_Symbol] :> Simp
[-2/(n - j)   Subst[Int[1/(1 - a*x^2), x], x, x^(j/2)/Sqrt[a*x^j + b*x^n]],
 x] /; FreeQ[{a, b, j, n}, x] && EqQ[m, j/2 - 1] && NeQ[n, j]
```

```
Int[((c_)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_))^(p_), x_Symbol]
:> Simp[c^IntPart[m]*((c*x)^FracPart[m]/x^FracPart[m])   Int[x^m*(a*x^j + b
*x^n)^p, x], x] /; FreeQ[{a, b, c, j, m, n, p}, x] && IntegerQ[p + 1/2] &&
NeQ[n, j] && EqQ[Simplify[m + j*p + 1], 0]
```

Maple [A] (verified)

Time = 0.41 (sec) , antiderivative size = 196, normalized size of antiderivative = 1.35

method	result
risch	$\frac{x^{2}(bx+a)c^{8}}{b^{3}\sqrt{x^{2}(bx+a)}\,\sqrt{cx}}+\frac{\left(-\frac{5a\ln\left(\frac{\frac{1}{2}ac+cbx}{\sqrt{bc}}+\sqrt{bc\,x^{2}+acx}\right)}{2b^{3}\sqrt{bc}}+\frac{14a\sqrt{bc\left(x+\frac{a}{b}\right)^{2}-ac\left(x+\frac{a}{b}\right)}}{3b^{4}c\left(x+\frac{a}{b}\right)}-\frac{2a^{2}\sqrt{bc\left(x+\frac{a}{b}\right)^{2}-ac\left(x+\frac{a}{b}\right)}}{3b^{5}c\left(x+\frac{a}{b}\right)^{2}}\right)c^{8}x\sqrt{cx(bx+a)}}{\sqrt{x^{2}(bx+a)}\,\sqrt{cx}}$
default	$\frac{x^{5}(bx+a)\left(-15\ln\left(\frac{2cbx+2\sqrt{cx(bx+a)}\,\sqrt{bc}+ac}{2\sqrt{bc}}\right)a\,b^{2}c\,x^{2}+6b^{2}x^{2}\sqrt{cx(bx+a)}\,\sqrt{bc}-30\ln\left(\frac{2cbx+2\sqrt{cx(bx+a)}\,\sqrt{bc}+ac}{2\sqrt{bc}}\right)a^{2}bcx+40\sqrt{bc}\,\sqrt{}\right.}{6(b\,x^{3}+a\,x^{2})^{\frac{5}{2}}\sqrt{bc}\,\sqrt{cx(bx+a)}\,b^{3}}$

```
int((c*x)^(15/2)/(b*x^3+a*x^2)^(5/2),x,method=_RETURNVERBOSE)
```

```
1/b^3*x^2*(b*x+a)*c^8/(x^2*(b*x+a))^(1/2)/(c*x)^(1/2)+(-5/2*a/b^3*ln((1/2*
a*c+c*b*x)/(b*c)^(1/2)+(b*c*x^2+a*c*x)^(1/2))/(b*c)^(1/2)+14/3*a/b^4/c/(x+
a/b)*(b*c*(x+a/b)^2-a*c*(x+a/b))^(1/2)-2/3*a^2/b^5/c/(x+a/b)^2*(b*c*(x+a/b
)^2-a*c*(x+a/b))^(1/2))*c^8/(x^2*(b*x+a))^(1/2)*x*(c*x*(b*x+a))^(1/2)/(c*x
)^(1/2)
```

Fricas [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 326, normalized size of antiderivative = 2.25

$$\int \frac{(cx)^{15/2}}{(ax^2+bx^3)^{5/2}}\,dx = \left[\frac{15\,(ab^2c^7x^3+2\,a^2bc^7x^2+a^3c^7x)\sqrt{\frac{c}{b}}\log\left(\frac{2\,bcx^2+acx-2\,\sqrt{bx^3+ax^2}\sqrt{cx}b\sqrt{\frac{c}{b}}}{x}\right)+2\,(3\,b^2c^7x}{6\,(b^5x^3+2\,ab^4x^2+a^2b^3x)}\right.$$

```
integrate((c*x)^(15/2)/(b*x^3+a*x^2)^(5/2),x, algorithm="fricas")
```

```
[1/6*(15*(a*b^2*c^7*x^3 + 2*a^2*b*c^7*x^2 + a^3*c^7*x)*sqrt(c/b)*log((2*b*
c*x^2 + a*c*x - 2*sqrt(b*x^3 + a*x^2)*sqrt(c*x)*b*sqrt(c/b))/x) + 2*(3*b^2
*c^7*x^2 + 20*a*b*c^7*x + 15*a^2*c^7)*sqrt(b*x^3 + a*x^2)*sqrt(c*x))/(b^5*
x^3 + 2*a*b^4*x^2 + a^2*b^3*x), 1/3*(15*(a*b^2*c^7*x^3 + 2*a^2*b*c^7*x^2 +
 a^3*c^7*x)*sqrt(-c/b)*arctan(sqrt(b*x^3 + a*x^2)*sqrt(c*x)*b*sqrt(-c/b)/(
b*c*x^2 + a*c*x)) + (3*b^2*c^7*x^2 + 20*a*b*c^7*x + 15*a^2*c^7)*sqrt(b*x^3
 + a*x^2)*sqrt(c*x))/(b^5*x^3 + 2*a*b^4*x^2 + a^2*b^3*x)]
```

Sympy [F(-1)]

Timed out.

$$\int \frac{(cx)^{15/2}}{(ax^2 + bx^3)^{5/2}}\, dx = \text{Timed out}$$

```
integrate((c*x)**(15/2)/(b*x**3+a*x**2)**(5/2),x)
```

```
Timed out
```

Maxima [F]

$$\int \frac{(cx)^{15/2}}{(ax^2 + bx^3)^{5/2}}\, dx = \int \frac{(cx)^{\frac{15}{2}}}{(bx^3 + ax^2)^{\frac{5}{2}}}\, dx$$

```
integrate((c*x)^(15/2)/(b*x^3+a*x^2)^(5/2),x, algorithm="maxima")
```

```
integrate((c*x)^(15/2)/(b*x^3 + a*x^2)^(5/2), x)
```

Giac [A] (verification not implemented)

Time = 0.18 (sec) , antiderivative size = 163, normalized size of antiderivative = 1.12

$$\int \frac{(cx)^{15/2}}{(ax^2+bx^3)^{5/2}}\,dx = \frac{1}{3}c^7\left(\frac{\left(cx\left(\frac{3c^3x}{b|c|\mathrm{sgn}(x)}+\frac{20ac^3}{b^2|c|\mathrm{sgn}(x)}\right)+\frac{15a^2c^4}{b^3|c|\mathrm{sgn}(x)}\right)\sqrt{cx}}{(bc^2x+ac^2)^{\frac{3}{2}}}+\frac{15ac^2\log\left(\left|-\sqrt{bc}\sqrt{cx}+\sqrt{bc^2x+ac^2}\right|\right)}{\sqrt{bc}b^3|c|\mathrm{sgn}(x)}\right) - \frac{5ac^9\log(c^2|a|)\,\mathrm{sgn}(x)}{2\sqrt{bc}b^3|c|}$$

```
integrate((c*x)^(15/2)/(b*x^3+a*x^2)^(5/2),x, algorithm="giac")
```

```
1/3*c^7*((c*x*(3*c^3*x/(b*abs(c)*sgn(x)) + 20*a*c^3/(b^2*abs(c)*sgn(x))) +
 15*a^2*c^4/(b^3*abs(c)*sgn(x)))*sqrt(c*x)/(b*c^2*x + a*c^2)^(3/2) + 15*a*
c^2*log(abs(-sqrt(b*c)*sqrt(c*x) + sqrt(b*c^2*x + a*c^2)))/(sqrt(b*c)*b^3*
abs(c)*sgn(x))) - 5/2*a*c^9*log(c^2*abs(a))*sgn(x)/(sqrt(b*c)*b^3*abs(c))
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(cx)^{15/2}}{(ax^2+bx^3)^{5/2}}\,dx = \int \frac{(c\,x)^{15/2}}{(b\,x^3+a\,x^2)^{5/2}}\,dx$$

```
int((c*x)^(15/2)/(a*x^2 + b*x^3)^(5/2),x)
```

```
int((c*x)^(15/2)/(a*x^2 + b*x^3)^(5/2), x)
```

Reduce [B] (verification not implemented)

Time = 0.21 (sec) , antiderivative size = 141, normalized size of antiderivative = 0.97

$$\int \frac{(cx)^{15/2}}{(ax^2+bx^3)^{5/2}}\,dx = \frac{\sqrt{c}\,c^7\left(-30\sqrt{b}\,\sqrt{bx+a}\log\left(\frac{\sqrt{bx+a}+\sqrt{x}\,\sqrt{b}}{\sqrt{a}}\right)a^2-30\sqrt{b}\,\sqrt{bx+a}\log\left(\frac{\sqrt{bx+a}+\sqrt{x}\,\sqrt{b}}{\sqrt{a}}\right)abx\right.}{6\sqrt{bx+a}\,b^4}$$

```
int((c*x)^(15/2)/(b*x^3+a*x^2)^(5/2),x)
```

```
(sqrt(c)*c**7*( - 30*sqrt(b)*sqrt(a + b*x)*log((sqrt(a + b*x) + sqrt(x)*sq
rt(b))/sqrt(a))*a**2 - 30*sqrt(b)*sqrt(a + b*x)*log((sqrt(a + b*x) + sqrt(
x)*sqrt(b))/sqrt(a))*a*b*x - 5*sqrt(b)*sqrt(a + b*x)*a**2 - 5*sqrt(b)*sqrt
(a + b*x)*a*b*x + 30*sqrt(x)*a**2*b + 40*sqrt(x)*a*b**2*x + 6*sqrt(x)*b**3
*x**2))/(6*sqrt(a + b*x)*b**4*(a + b*x))
```

3.364 $\int \frac{(cx)^{13/2}}{(ax^2+bx^3)^{5/2}}\,dx$

Optimal result

Integrand size = 23, antiderivative size = 112

$$\int \frac{(cx)^{13/2}}{(ax^2+bx^3)^{5/2}}\,dx = \frac{2ac^3(cx)^{7/2}}{3b^2(ax^2+bx^3)^{3/2}} - \frac{8c^5(cx)^{3/2}}{3b^2\sqrt{ax^2+bx^3}} + \frac{2c^{13/2}\operatorname{arctanh}\left(\frac{\sqrt{b}(cx)^{3/2}}{c^{3/2}\sqrt{ax^2+bx^3}}\right)}{b^{5/2}}$$

```
2/3*a*c^3*(c*x)^(7/2)/b^2/(b*x^3+a*x^2)^(3/2)-8/3*c^5*(c*x)^(3/2)/b^2/(b*x
^3+a*x^2)^(1/2)+2*c^(13/2)*arctanh(b^(1/2)*(c*x)^(3/2)/c^(3/2)/(b*x^3+a*x^
2)^(1/2))/b^(5/2)
```

Mathematica [A] (verified)

Time = 0.05 (sec) , antiderivative size = 99, normalized size of antiderivative = 0.88

$$\int \frac{(cx)^{13/2}}{(ax^2+bx^3)^{5/2}}\,dx = -\frac{2c^6x^{5/2}\sqrt{cx}\left(\sqrt{b}\sqrt{x}(3a+4bx)+6(a+bx)^{3/2}\operatorname{arctanh}\left(\frac{\sqrt{b}\sqrt{x}}{\sqrt{a}-\sqrt{a+bx}}\right)\right)}{3b^{5/2}\left(x^2(a+bx)\right)^{3/2}}$$

```
Integrate[(c*x)^(13/2)/(a*x^2 + b*x^3)^(5/2),x]
```

```
(-2*c^6*x^(5/2)*Sqrt[c*x]*(Sqrt[b]*Sqrt[x]*(3*a + 4*b*x) + 6*(a + b*x)^(3/
2)*ArcTanh[(Sqrt[b]*Sqrt[x])/(Sqrt[a] - Sqrt[a + b*x])]))/(3*b^(5/2)*(x^2*
(a + b*x))^(3/2))
```

Rubi [A] (verified)

Time = 0.48 (sec) , antiderivative size = 120, normalized size of antiderivative = 1.07, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.217$, Rules used = {1928, 1928, 1937, 1935, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(cx)^{13/2}}{(ax^2+bx^3)^{5/2}}dx$$

$$\Big\downarrow \text{1928}$$

$$\frac{c^3\int \frac{(cx)^{7/2}}{(bx^3+ax^2)^{3/2}}dx}{b}-\frac{2c^2(cx)^{9/2}}{3b(ax^2+bx^3)^{3/2}}$$

$$\Big\downarrow \text{1928}$$

$$\frac{c^3\left(\frac{c^3\int \frac{\sqrt{cx}}{\sqrt{bx^3+ax^2}}dx}{b}-\frac{2c^2(cx)^{3/2}}{b\sqrt{ax^2+bx^3}}\right)}{b}-\frac{2c^2(cx)^{9/2}}{3b(ax^2+bx^3)^{3/2}}$$

$$\Big\downarrow \text{1937}$$

$$\frac{c^3\left(\frac{c^3\sqrt{cx}\int \frac{\sqrt{x}}{\sqrt{bx^3+ax^2}}dx}{b\sqrt{x}}-\frac{2c^2(cx)^{3/2}}{b\sqrt{ax^2+bx^3}}\right)}{b}-\frac{2c^2(cx)^{9/2}}{3b(ax^2+bx^3)^{3/2}}$$

$$\Big\downarrow \text{1935}$$

$$\frac{c^3\left(\frac{2c^3\sqrt{cx}\int \frac{1}{1-\frac{bx^3}{bx^3+ax^2}}d\frac{x^{3/2}}{\sqrt{bx^3+ax^2}}}{b\sqrt{x}}-\frac{2c^2(cx)^{3/2}}{b\sqrt{ax^2+bx^3}}\right)}{b}-\frac{2c^2(cx)^{9/2}}{3b(ax^2+bx^3)^{3/2}}$$

$$\Big\downarrow \text{219}$$

$$\frac{c^3\left(\frac{2c^3\sqrt{cx}\operatorname{arctanh}\left(\frac{\sqrt{b}x^{3/2}}{\sqrt{ax^2+bx^3}}\right)}{b^{3/2}\sqrt{x}}-\frac{2c^2(cx)^{3/2}}{b\sqrt{ax^2+bx^3}}\right)}{b}-\frac{2c^2(cx)^{9/2}}{3b\left(ax^2+bx^3\right)^{3/2}}$$

```
Int[(c*x)^(13/2)/(a*x^2 + b*x^3)^(5/2),x]
```

```
(-2*c^2*(c*x)^(9/2))/(3*b*(a*x^2 + b*x^3)^(3/2)) + (c^3*((-2*c^2*(c*x)^(3/
2))/(b*Sqrt[a*x^2 + b*x^3]) + (2*c^3*Sqrt[c*x]*ArcTanh[(Sqrt[b]*x^(3/2))/S
qrt[a*x^2 + b*x^3]])/(b^(3/2)*Sqrt[x])))/b
```

Defintions of rubi rules used

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> Simp[c^(n - 1)*(c*x)^(m - n + 1)*((a*x^j + b*x^n)^(p + 1)/(b*(n - j)*(
p + 1))), x] - Simp[c^n*((m + j*p - n + j + 1)/(b*(n - j)*(p + 1)))   Int[(
c*x)^(m - n)*(a*x^j + b*x^n)^(p + 1), x], x] /; FreeQ[{a, b, c}, x] &&  !In
tegerQ[p] && LtQ[0, j, n] && (IntegersQ[j, n] || GtQ[c, 0]) && LtQ[p, -1] &
& GtQ[m + j*p + 1, n - j]
```

```
Int[(x_)^(m_.)/Sqrt[(a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.)], x_Symbol] :> Simp
[-2/(n - j)   Subst[Int[1/(1 - a*x^2), x], x, x^(j/2)/Sqrt[a*x^j + b*x^n]],
 x] /; FreeQ[{a, b, j, n}, x] && EqQ[m, j/2 - 1] && NeQ[n, j]
```

```
Int[((c_)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_))^(p_), x_Symbol]
:> Simp[c^IntPart[m]*((c*x)^FracPart[m]/x^FracPart[m])   Int[x^m*(a*x^j + b
*x^n)^p, x], x] /; FreeQ[{a, b, c, j, m, n, p}, x] && IntegerQ[p + 1/2] &&
NeQ[n, j] && EqQ[Simplify[m + j*p + 1], 0]
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 210 vs. $2(88) = 176$.

Time = 0.38 (sec) , antiderivative size = 211, normalized size of antiderivative = 1.88

method	result
default	$-\frac{x^{5}(bx+a)\left(-3\ln\left(\frac{2cbx+2\sqrt{cx(bx+a)}\,\sqrt{bc}+ac}{2\sqrt{bc}}\right)b^{2}c\,x^{2}-6\ln\left(\frac{2cbx+2\sqrt{cx(bx+a)}\,\sqrt{bc}+ac}{2\sqrt{bc}}\right)abcx+8\sqrt{bc}\,\sqrt{cx(bx+a)}\,bx-3a^{2}c\ln\left(\frac{2cbx-}{}\right.\right.}{3(b\,x^{3}+a\,x^{2})^{\frac{5}{2}}\sqrt{bc}\,\sqrt{cx(bx+a)}\,b^{2}}$

```
int((c*x)^(13/2)/(b*x^3+a*x^2)^(5/2),x,method=_RETURNVERBOSE)
```

```
-1/3*x^5*(b*x+a)*(-3*ln(1/2*(2*c*b*x+2*(c*x*(b*x+a))^(1/2)*(b*c)^(1/2)+a*c
)/(b*c)^(1/2))*b^2*c*x^2-6*ln(1/2*(2*c*b*x+2*(c*x*(b*x+a))^(1/2)*(b*c)^(1/
2)+a*c)/(b*c)^(1/2))*a*b*c*x+8*(b*c)^(1/2)*(c*x*(b*x+a))^(1/2)*b*x-3*a^2*c
*ln(1/2*(2*c*b*x+2*(c*x*(b*x+a))^(1/2)*(b*c)^(1/2)+a*c)/(b*c)^(1/2))+6*(b*
c)^(1/2)*(c*x*(b*x+a))^(1/2)*a)*(c*x)^(1/2)*c^6/(b*x^3+a*x^2)^(5/2)/(b*c)^
(1/2)/(c*x*(b*x+a))^(1/2)/b^2
```

Fricas [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 292, normalized size of antiderivative = 2.61

$$\int \frac{(cx)^{13/2}}{(ax^2+bx^3)^{5/2}}\,dx = \left[\frac{3\,(b^2c^6x^3+2\,abc^6x^2+a^2c^6x)\sqrt{\frac{c}{b}}\log\left(\frac{2\,bcx^2+acx+2\,\sqrt{bx^3+ax^2}\sqrt{cx}b\sqrt{\frac{c}{b}}}{x}\right)-2\,(4\,bc^6x+3}{3\,(b^4x^3+2\,ab^3x^2+a^2b^2x)}\right.$$

```
integrate((c*x)^(13/2)/(b*x^3+a*x^2)^(5/2),x, algorithm="fricas")
```

```
[1/3*(3*(b^2*c^6*x^3 + 2*a*b*c^6*x^2 + a^2*c^6*x)*sqrt(c/b)*log((2*b*c*x^2
 + a*c*x + 2*sqrt(b*x^3 + a*x^2)*sqrt(c*x)*b*sqrt(c/b))/x) - 2*(4*b*c^6*x
+ 3*a*c^6)*sqrt(b*x^3 + a*x^2)*sqrt(c*x))/(b^4*x^3 + 2*a*b^3*x^2 + a^2*b^2
*x), -2/3*(3*(b^2*c^6*x^3 + 2*a*b*c^6*x^2 + a^2*c^6*x)*sqrt(-c/b)*arctan(s
qrt(b*x^3 + a*x^2)*sqrt(c*x)*b*sqrt(-c/b)/(b*c*x^2 + a*c*x)) + (4*b*c^6*x
+ 3*a*c^6)*sqrt(b*x^3 + a*x^2)*sqrt(c*x))/(b^4*x^3 + 2*a*b^3*x^2 + a^2*b^2
*x)]
```

Sympy [F(-1)]

Timed out.

$$\int \frac{(cx)^{13/2}}{(ax^2+bx^3)^{5/2}}\,dx = \text{Timed out}$$

```
integrate((c*x)**(13/2)/(b*x**3+a*x**2)**(5/2),x)
```

```
Timed out
```

Maxima [F]

$$\int \frac{(cx)^{13/2}}{(ax^2+bx^3)^{5/2}}\,dx = \int \frac{(cx)^{\frac{13}{2}}}{(bx^3+ax^2)^{\frac{5}{2}}}\,dx$$

```
integrate((c*x)^(13/2)/(b*x^3+a*x^2)^(5/2),x, algorithm="maxima")
```

```
integrate((c*x)^(13/2)/(b*x^3 + a*x^2)^(5/2), x)
```

Giac [A] (verification not implemented)

Time = 0.15 (sec) , antiderivative size = 137, normalized size of antiderivative = 1.22

$$\int \frac{(cx)^{13/2}}{(ax^2+bx^3)^{5/2}}\,dx = \frac{c^8\log\left(c^2|a|\right)\operatorname{sgn}(x)}{\sqrt{bc}b^2|c|} - \frac{2}{3}c^4\left(\frac{3c^4\log\left(\left|-\sqrt{bc}\sqrt{cx}+\sqrt{bc^2x+ac^2}\right|\right)}{\sqrt{bc}b^2|c|\operatorname{sgn}(x)} + \frac{\left(\frac{4c^6x}{b|c|\operatorname{sgn}(x)}+\frac{3ac^6}{b^2|c|\operatorname{sgn}(x)}\right)\sqrt{cx}}{(bc^2x+ac^2)^{\frac{3}{2}}}\right)$$

```
integrate((c*x)^(13/2)/(b*x^3+a*x^2)^(5/2),x, algorithm="giac")
```

```
c^8*log(c^2*abs(a))*sgn(x)/(sqrt(b*c)*b^2*abs(c)) - 2/3*c^4*(3*c^4*log(abs
(-sqrt(b*c)*sqrt(c*x) + sqrt(b*c^2*x + a*c^2)))/(sqrt(b*c)*b^2*abs(c)*sgn(
x)) + (4*c^6*x/(b*abs(c)*sgn(x)) + 3*a*c^6/(b^2*abs(c)*sgn(x)))*sqrt(c*x)/
(b*c^2*x + a*c^2)^(3/2))
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(cx)^{13/2}}{(ax^2+bx^3)^{5/2}}\,dx = \int \frac{(c\,x)^{13/2}}{(b\,x^3+a\,x^2)^{5/2}}\,dx$$

```
int((c*x)^(13/2)/(a*x^2 + b*x^3)^(5/2),x)
```

```
int((c*x)^(13/2)/(a*x^2 + b*x^3)^(5/2), x)
```

Reduce [B] (verification not implemented)

Time = 0.55 (sec) , antiderivative size = 99, normalized size of antiderivative = 0.88

$$\int \frac{(cx)^{13/2}}{(ax^2+bx^3)^{5/2}}\,dx = \frac{2\sqrt{c}\,c^6\left(3\sqrt{b}\,\sqrt{bx+a}\log\left(\frac{\sqrt{bx+a}+\sqrt{x}\,\sqrt{b}}{\sqrt{a}}\right)a+3\sqrt{b}\,\sqrt{bx+a}\log\left(\frac{\sqrt{bx+a}+\sqrt{x}\,\sqrt{b}}{\sqrt{a}}\right)bx-3\cdots\right.}{3\sqrt{bx+a}\,b^3\,(bx+a)}$$

```
int((c*x)^(13/2)/(b*x^3+a*x^2)^(5/2),x)
```

```
(2*sqrt(c)*c**6*(3*sqrt(b)*sqrt(a + b*x)*log((sqrt(a + b*x) + sqrt(x)*sqrt
(b))/sqrt(a))*a + 3*sqrt(b)*sqrt(a + b*x)*log((sqrt(a + b*x) + sqrt(x)*sqr
t(b))/sqrt(a))*b*x - 3*sqrt(x)*a*b - 4*sqrt(x)*b**2*x))/(3*sqrt(a + b*x)*b
**3*(a + b*x))
```

3.365 $\int \frac{(cx)^{11/2}}{(ax^2+bx^3)^{5/2}}\,dx$

Optimal result

Integrand size = 23, antiderivative size = 30

$$\int \frac{(cx)^{11/2}}{(ax^2+bx^3)^{5/2}}\,dx=\frac{2c(cx)^{9/2}}{3a\left(ax^2+bx^3\right)^{3/2}}$$

```
2/3*c*(c*x)^(9/2)/a/(b*x^3+a*x^2)^(3/2)
```

Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 28, normalized size of antiderivative = 0.93

$$\int \frac{(cx)^{11/2}}{(ax^2+bx^3)^{5/2}}\,dx=\frac{2c(cx)^{9/2}}{3a\left(x^2(a+bx)\right)^{3/2}}$$

```
Integrate[(c*x)^(11/2)/(a*x^2 + b*x^3)^(5/2),x]
```

```
(2*c*(c*x)^(9/2))/(3*a*(x^2*(a + b*x))^(3/2))
```

Rubi [A] (verified)

Time = 0.26 (sec) , antiderivative size = 30, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.043, Rules used = {1920}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(cx)^{11/2}}{(ax^2+bx^3)^{5/2}}\,dx$$
$$\Big\downarrow \; 1920$$
$$\frac{2c(cx)^{9/2}}{3a\,(ax^2+bx^3)^{3/2}}$$

```
Int[(c*x)^(11/2)/(a*x^2 + b*x^3)^(5/2),x]
```

```
(2*c*(c*x)^(9/2))/(3*a*(a*x^2 + b*x^3)^(3/2))
```

Defintions of rubi rules used

```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> Simp[(-c^(j - 1))*(c*x)^(m - j + 1)*((a*x^j + b*x^n)^(p + 1)/(a*(n - j
)*(p + 1))), x] /; FreeQ[{a, b, c, j, m, n, p}, x] &&  !IntegerQ[p] && NeQ[
n, j] && EqQ[m + n*p + n - j + 1, 0] && (IntegerQ[j] || GtQ[c, 0])
```

Maple [A] (verified)

Time = 0.37 (sec) , antiderivative size = 30, normalized size of antiderivative = 1.00

method	result	size
gosper	$\frac{2x(bx+a)(cx)^{\frac{11}{2}}}{3a(b\,x^{3}+a\,x^{2})^{\frac{5}{2}}}$	30
orering	$\frac{2x(bx+a)(cx)^{\frac{11}{2}}}{3a(b\,x^{3}+a\,x^{2})^{\frac{5}{2}}}$	30
default	$\frac{2x^{6}(bx+a)c^{5}\sqrt{cx}}{3(b\,x^{3}+a\,x^{2})^{\frac{5}{2}}a}$	35

```
int((c*x)^(11/2)/(b*x^3+a*x^2)^(5/2),x,method=_RETURNVERBOSE)
```

```
2/3*x*(b*x+a)/a*(c*x)^(11/2)/(b*x^3+a*x^2)^(5/2)
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 44, normalized size of antiderivative = 1.47

$$\int \frac{(cx)^{11/2}}{(ax^2+bx^3)^{5/2}}\,dx = \frac{2\sqrt{bx^3+ax^2}\sqrt{cx}c^5}{3\,(ab^2x^2+2\,a^2bx+a^3)}$$

```
integrate((c*x)^(11/2)/(b*x^3+a*x^2)^(5/2),x, algorithm="fricas")
```

```
2/3*sqrt(b*x^3 + a*x^2)*sqrt(c*x)*c^5/(a*b^2*x^2 + 2*a^2*b*x + a^3)
```

Sympy [F]

$$\int \frac{(cx)^{11/2}}{(ax^2+bx^3)^{5/2}}\,dx = \int \frac{(cx)^{\frac{11}{2}}}{(x^2\,(a+bx))^{\frac{5}{2}}}\,dx$$

```
integrate((c*x)**(11/2)/(b*x**3+a*x**2)**(5/2),x)
```

```
Integral((c*x)**(11/2)/(x**2*(a + b*x))**(5/2), x)
```

Maxima [F]

$$\int \frac{(cx)^{11/2}}{(ax^2+bx^3)^{5/2}}\,dx = \int \frac{(cx)^{\frac{11}{2}}}{(bx^3+ax^2)^{\frac{5}{2}}}\,dx$$

```
integrate((c*x)^(11/2)/(b*x^3+a*x^2)^(5/2),x, algorithm="maxima")
```

```
integrate((c*x)^(11/2)/(b*x^3 + a*x^2)^(5/2), x)
```

Giac [A] (verification not implemented)

Time = 0.17 (sec) , antiderivative size = 36, normalized size of antiderivative = 1.20

$$\int \frac{(cx)^{11/2}}{(ax^2+bx^3)^{5/2}}\,dx = \frac{2\sqrt{cx}c^9x}{3\,(bc^2x+ac^2)^{\frac{3}{2}}a|c|\mathrm{sgn}\,(x)}$$

```
integrate((c*x)^(11/2)/(b*x^3+a*x^2)^(5/2),x, algorithm="giac")
```

```
2/3*sqrt(c*x)*c^9*x/((b*c^2*x + a*c^2)^(3/2)*a*abs(c)*sgn(x))
```

Mupad [B] (verification not implemented)

Time = 8.53 (sec) , antiderivative size = 47, normalized size of antiderivative = 1.57

$$\int \frac{(cx)^{11/2}}{(ax^2+bx^3)^{5/2}}\,dx = \frac{2\,c^5\,\sqrt{c\,x}\,\sqrt{b\,x^3+a\,x^2}}{3\,\left(a^3+2\,a^2\,b\,x+a\,b^2\,x^2\right)}$$

```
int((c*x)^(11/2)/(a*x^2 + b*x^3)^(5/2),x)
```

```
(2*c^5*(c*x)^(1/2)*(a*x^2 + b*x^3)^(1/2))/(3*(a^3 + a*b^2*x^2 + 2*a^2*b*x)
)
```

Reduce [B] (verification not implemented)

Time = 0.55 (sec) , antiderivative size = 57, normalized size of antiderivative = 1.90

$$\int \frac{(cx)^{11/2}}{(ax^2+bx^3)^{5/2}}\,dx = \frac{2\sqrt{c}\,c^5\left(\sqrt{b}\,\sqrt{bx+a}\,a+\sqrt{b}\,\sqrt{bx+a}\,bx+\sqrt{x}\,b^2x\right)}{3\sqrt{bx+a}\,a\,b^2\,(bx+a)}$$

```
int((c*x)^(11/2)/(b*x^3+a*x^2)^(5/2),x)
```

```
(2*sqrt(c)*c**5*(sqrt(b)*sqrt(a + b*x)*a + sqrt(b)*sqrt(a + b*x)*b*x + sqr
t(x)*b**2*x))/(3*sqrt(a + b*x)*a*b**2*(a + b*x))
```

3.366 $\int \frac{(cx)^{9/2}}{(ax^2+bx^3)^{5/2}}\,dx$

Optimal result

Integrand size = 23, antiderivative size = 63

$$\int \frac{(cx)^{9/2}}{(ax^2+bx^3)^{5/2}}\,dx = \frac{2c(cx)^{7/2}}{3a\,(ax^2+bx^3)^{3/2}}+\frac{4c^3(cx)^{3/2}}{3a^2\sqrt{ax^2+bx^3}}$$

```
2/3*c*(c*x)^(7/2)/a/(b*x^3+a*x^2)^(3/2)+4/3*c^3*(c*x)^(3/2)/a^2/(b*x^3+a*x
^2)^(1/2)
```

Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 36, normalized size of antiderivative = 0.57

$$\int \frac{(cx)^{9/2}}{(ax^2+bx^3)^{5/2}}\,dx = \frac{2c(cx)^{7/2}(3a+2bx)}{3a^2\,(x^2(a+bx))^{3/2}}$$

```
Integrate[(c*x)^(9/2)/(a*x^2 + b*x^3)^(5/2),x]
```

```
(2*c*(c*x)^(7/2)*(3*a + 2*b*x))/(3*a^2*(x^2*(a + b*x))^(3/2))
```

Rubi [A] (verified)

Time = 0.33 (sec) , antiderivative size = 63, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.087, Rules used = {1921, 1920}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(cx)^{9/2}}{(ax^2+bx^3)^{5/2}} \, dx$$

$$\Big\downarrow \; 1921$$

$$\frac{2c^2 \int \frac{(cx)^{5/2}}{(bx^3+ax^2)^{3/2}}dx}{3a}+\frac{2c(cx)^{7/2}}{3a\left(ax^2+bx^3\right)^{3/2}}$$

$$\Big\downarrow \; 1920$$

$$\frac{4c^3(cx)^{3/2}}{3a^2\sqrt{ax^2+bx^3}}+\frac{2c(cx)^{7/2}}{3a\left(ax^2+bx^3\right)^{3/2}}$$

```
Int[(c*x)^(9/2)/(a*x^2 + b*x^3)^(5/2),x]
```

```
(2*c*(c*x)^(7/2))/(3*a*(a*x^2 + b*x^3)^(3/2)) + (4*c^3*(c*x)^(3/2))/(3*a^2
*Sqrt[a*x^2 + b*x^3])
```

Defintions of rubi rules used

```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> Simp[(-c^(j - 1))*(c*x)^(m - j + 1)*((a*x^j + b*x^n)^(p + 1)/(a*(n - j
)*(p + 1))), x] /; FreeQ[{a, b, c, j, m, n, p}, x] &&  !IntegerQ[p] && NeQ[
n, j] && EqQ[m + n*p + n - j + 1, 0] && (IntegerQ[j] || GtQ[c, 0])
```

```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> Simp[(-c^(j - 1))*(c*x)^(m - j + 1)*((a*x^j + b*x^n)^(p + 1)/(a*(n - j
)*(p + 1))), x] + Simp[c^j*((m + n*p + n - j + 1)/(a*(n - j)*(p + 1)))   In
t[(c*x)^(m - j)*(a*x^j + b*x^n)^(p + 1), x], x] /; FreeQ[{a, b, c, j, m, n}
, x] &&  !IntegerQ[p] && NeQ[n, j] && ILtQ[Simplify[(m + n*p + n - j + 1)/(
n - j)], 0] && LtQ[p, -1] && (IntegerQ[j] || GtQ[c, 0])
```

Maple [A] (verified)

Time = 0.38 (sec) , antiderivative size = 38, normalized size of antiderivative = 0.60

method	result	size
gosper	$\frac{2x(bx+a)(2bx+3a)(cx)^{\frac{9}{2}}}{3a^{2}(b\,x^{3}+a\,x^{2})^{\frac{5}{2}}}$	38
orering	$\frac{2x(bx+a)(2bx+3a)(cx)^{\frac{9}{2}}}{3a^{2}(b\,x^{3}+a\,x^{2})^{\frac{5}{2}}}$	38
default	$\frac{2x^{5}(bx+a)(2bx+3a)\sqrt{cx}\,c^{4}}{3(b\,x^{3}+a\,x^{2})^{\frac{5}{2}}a^{2}}$	43

```
int((c*x)^(9/2)/(b*x^3+a*x^2)^(5/2),x,method=_RETURNVERBOSE)
```

```
2/3*x*(b*x+a)*(2*b*x+3*a)*(c*x)^(9/2)/a^2/(b*x^3+a*x^2)^(5/2)
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 61, normalized size of antiderivative = 0.97

$$\int \frac{(cx)^{9/2}}{(ax^2+bx^3)^{5/2}}\,dx = \frac{2\,(2\,bc^4x+3\,ac^4)\sqrt{bx^3+ax^2}\sqrt{cx}}{3\,(a^2b^2x^3+2\,a^3bx^2+a^4x)}$$

```
integrate((c*x)^(9/2)/(b*x^3+a*x^2)^(5/2),x, algorithm="fricas")
```

```
2/3*(2*b*c^4*x + 3*a*c^4)*sqrt(b*x^3 + a*x^2)*sqrt(c*x)/(a^2*b^2*x^3 + 2*a
^3*b*x^2 + a^4*x)
```

Sympy [F]

$$\int \frac{(cx)^{9/2}}{(ax^2+bx^3)^{5/2}}\,dx = \int \frac{(cx)^{\frac{9}{2}}}{\left(x^2\left(a+bx\right)\right)^{\frac{5}{2}}}\,dx$$

```
integrate((c*x)**(9/2)/(b*x**3+a*x**2)**(5/2),x)
```

```
Integral((c*x)**(9/2)/(x**2*(a + b*x))**(5/2), x)
```

Maxima [F]

$$\int \frac{(cx)^{9/2}}{(ax^2+bx^3)^{5/2}}\,dx = \int \frac{(cx)^{\frac{9}{2}}}{\left(bx^3+ax^2\right)^{\frac{5}{2}}}\,dx$$

```
integrate((c*x)^(9/2)/(b*x^3+a*x^2)^(5/2),x, algorithm="maxima")
```

```
integrate((c*x)^(9/2)/(b*x^3 + a*x^2)^(5/2), x)
```

Giac [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 59, normalized size of antiderivative = 0.94

$$\int \frac{(cx)^{9/2}}{(ax^2+bx^3)^{5/2}}\,dx = \frac{2\sqrt{cx}\left(\frac{2\,bc^4x}{a^2|c|\mathrm{sgn}(x)}+\frac{3\,c^4}{a|c|\mathrm{sgn}(x)}\right)c^4}{3\left(bc^2x+ac^2\right)^{\frac{3}{2}}}$$

```
integrate((c*x)^(9/2)/(b*x^3+a*x^2)^(5/2),x, algorithm="giac")
```

```
2/3*sqrt(c*x)*(2*b*c^4*x/(a^2*abs(c)*sgn(x)) + 3*c^4/(a*abs(c)*sgn(x)))*c^
4/(b*c^2*x + a*c^2)^(3/2)
```

Mupad [B] (verification not implemented)

Time = 8.70 (sec) , antiderivative size = 71, normalized size of antiderivative = 1.13

$$\int \frac{(cx)^{9/2}}{(ax^2+bx^3)^{5/2}}\,dx = \frac{\left(\frac{2\,c^4\,\sqrt{c\,x}}{a\,b^2}+\frac{4\,c^4\,x\,\sqrt{c\,x}}{3\,a^2\,b}\right)\,\sqrt{b\,x^3+a\,x^2}}{x^3+\frac{2\,a\,x^2}{b}+\frac{a^2\,x}{b^2}}$$

```
int((c*x)^(9/2)/(a*x^2 + b*x^3)^(5/2),x)
```

```
(((2*c^4*(c*x)^(1/2))/(a*b^2) + (4*c^4*x*(c*x)^(1/2))/(3*a^2*b))*(a*x^2 +
b*x^3)^(1/2))/(x^3 + (2*a*x^2)/b + (a^2*x)/b^2)
```

Reduce [B] (verification not implemented)

Time = 0.23 (sec) , antiderivative size = 66, normalized size of antiderivative = 1.05

$$\int \frac{(cx)^{9/2}}{(ax^2+bx^3)^{5/2}}\,dx = \frac{2\sqrt{c}\,c^4\left(-2\sqrt{b}\,\sqrt{bx+a}\,a-2\sqrt{b}\,\sqrt{bx+a}\,bx+3\sqrt{x}\,ab+2\sqrt{x}\,b^2x\right)}{3\sqrt{bx+a}\,a^2b\,(bx+a)}$$

```
int((c*x)^(9/2)/(b*x^3+a*x^2)^(5/2),x)
```

```
(2*sqrt(c)*c**4*( - 2*sqrt(b)*sqrt(a + b*x)*a - 2*sqrt(b)*sqrt(a + b*x)*b*
x + 3*sqrt(x)*a*b + 2*sqrt(x)*b**2*x))/(3*sqrt(a + b*x)*a**2*b*(a + b*x))
```

3.367 $\int \frac{(cx)^{7/2}}{\left(ax^2+bx^3\right)^{5/2}}\,dx$

Optimal result

Integrand size = 23, antiderivative size = 95

$$\int \frac{(cx)^{7/2}}{\left(ax^2+bx^3\right)^{5/2}}\,dx = \frac{2c(cx)^{5/2}}{3a\left(ax^2+bx^3\right)^{3/2}}+\frac{8c^3\sqrt{cx}}{3a^2\sqrt{ax^2+bx^3}}-\frac{16c^5\sqrt{ax^2+bx^3}}{3a^3(cx)^{3/2}}$$

```
2/3*c*(c*x)^(5/2)/a/(b*x^3+a*x^2)^(3/2)+8/3*c^3*(c*x)^(1/2)/a^2/(b*x^3+a*x
^2)^(1/2)-16/3*c^5*(b*x^3+a*x^2)^(1/2)/a^3/(c*x)^(3/2)
```

Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 47, normalized size of antiderivative = 0.49

$$\int \frac{(cx)^{7/2}}{\left(ax^2+bx^3\right)^{5/2}}\,dx = -\frac{2c(cx)^{5/2}\left(3a^2+12abx+8b^2x^2\right)}{3a^3\left(x^2(a+bx)\right)^{3/2}}$$

```
Integrate[(c*x)^(7/2)/(a*x^2 + b*x^3)^(5/2),x]
```

```
(-2*c*(c*x)^(5/2)*(3*a^2 + 12*a*b*x + 8*b^2*x^2))/(3*a^3*(x^2*(a + b*x))^(
3/2))
```

Rubi [A] (verified)

Time = 0.41 (sec) , antiderivative size = 100, normalized size of antiderivative = 1.05, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.130$, Rules used = {1921, 1921, 1920}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(cx)^{7/2}}{(ax^2+bx^3)^{5/2}}\,dx$$

$$\Big\downarrow \text{ 1921}$$

$$\frac{4c^2\int \frac{(cx)^{3/2}}{(bx^3+ax^2)^{3/2}}dx}{3a}+\frac{2c(cx)^{5/2}}{3a\,(ax^2+bx^3)^{3/2}}$$

$$\Big\downarrow \text{ 1921}$$

$$\frac{4c^2\left(\frac{2c^2\int \frac{1}{\sqrt{cx}\sqrt{bx^3+ax^2}}dx}{a}+\frac{2c\sqrt{cx}}{a\sqrt{ax^2+bx^3}}\right)}{3a}+\frac{2c(cx)^{5/2}}{3a\,(ax^2+bx^3)^{3/2}}$$

$$\Big\downarrow \text{ 1920}$$

$$\frac{4c^2\left(\frac{2c\sqrt{cx}}{a\sqrt{ax^2+bx^3}}-\frac{4c^3\sqrt{ax^2+bx^3}}{a^2(cx)^{3/2}}\right)}{3a}+\frac{2c(cx)^{5/2}}{3a\,(ax^2+bx^3)^{3/2}}$$

```
Int[(c*x)^(7/2)/(a*x^2 + b*x^3)^(5/2),x]
```

```
(2*c*(c*x)^(5/2))/(3*a*(a*x^2 + b*x^3)^(3/2)) + (4*c^2*((2*c*Sqrt[c*x])/(a
*Sqrt[a*x^2 + b*x^3]) - (4*c^3*Sqrt[a*x^2 + b*x^3])/(a^2*(c*x)^(3/2))))/(3
*a)
```

Defintions of rubi rules used

```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> Simp[(-c^(j - 1))*(c*x)^(m - j + 1)*((a*x^j + b*x^n)^(p + 1)/(a*(n - j
)*(p + 1))), x] /; FreeQ[{a, b, c, j, m, n, p}, x] &&  !IntegerQ[p] && NeQ[
n, j] && EqQ[m + n*p + n - j + 1, 0] && (IntegerQ[j] || GtQ[c, 0])
```

```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> Simp[(-c^(j - 1))*(c*x)^(m - j + 1)*((a*x^j + b*x^n)^(p + 1)/(a*(n - j
)*(p + 1))), x] + Simp[c^j*((m + n*p + n - j + 1)/(a*(n - j)*(p + 1)))   In
t[(c*x)^(m - j)*(a*x^j + b*x^n)^(p + 1), x], x] /; FreeQ[{a, b, c, j, m, n}
, x] &&  !IntegerQ[p] && NeQ[n, j] && ILtQ[Simplify[(m + n*p + n - j + 1)/(
n - j)], 0] && LtQ[p, -1] && (IntegerQ[j] || GtQ[c, 0])
```

Maple [A] (verified)

Time = 0.39 (sec) , antiderivative size = 49, normalized size of antiderivative = 0.52

method	result	size
gosper	$-\frac{2x(bx+a)\left(8b^{2}x^{2}+12abx+3a^{2}\right)(cx)^{\frac{7}{2}}}{3a^{3}\left(b\,x^{3}+a\,x^{2}\right)^{\frac{5}{2}}}$	49
orering	$-\frac{2x(bx+a)\left(8b^{2}x^{2}+12abx+3a^{2}\right)(cx)^{\frac{7}{2}}}{3a^{3}\left(b\,x^{3}+a\,x^{2}\right)^{\frac{5}{2}}}$	49
default	$-\frac{2x^{4}(bx+a)\left(8b^{2}x^{2}+12abx+3a^{2}\right)\sqrt{cx}\,c^{3}}{3\left(b\,x^{3}+a\,x^{2}\right)^{\frac{5}{2}}a^{3}}$	54
risch	$-\frac{2(bx+a)c^{4}x}{a^{3}\sqrt{x^{2}(bx+a)}\,\sqrt{cx}}-\frac{2b(5bx+6a)x^{2}c^{4}}{3(bx+a)a^{3}\sqrt{x^{2}(bx+a)}\,\sqrt{cx}}$	75

```
int((c*x)^(7/2)/(b*x^3+a*x^2)^(5/2),x,method=_RETURNVERBOSE)
```

```
-2/3*x*(b*x+a)*(8*b^2*x^2+12*a*b*x+3*a^2)*(c*x)^(7/2)/a^3/(b*x^3+a*x^2)^(5
/2)
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 77, normalized size of antiderivative = 0.81

$$\int \frac{(cx)^{7/2}}{(ax^2+bx^3)^{5/2}}\,dx = -\frac{2\,(8\,b^2c^3x^2+12\,abc^3x+3\,a^2c^3)\sqrt{bx^3+ax^2}\sqrt{cx}}{3\,(a^3b^2x^4+2\,a^4bx^3+a^5x^2)}$$

```
integrate((c*x)^(7/2)/(b*x^3+a*x^2)^(5/2),x, algorithm="fricas")
```

```
-2/3*(8*b^2*c^3*x^2 + 12*a*b*c^3*x + 3*a^2*c^3)*sqrt(b*x^3 + a*x^2)*sqrt(c
*x)/(a^3*b^2*x^4 + 2*a^4*b*x^3 + a^5*x^2)
```

Sympy [F]

$$\int \frac{(cx)^{7/2}}{(ax^2+bx^3)^{5/2}}\,dx = \int \frac{(cx)^{\frac{7}{2}}}{(x^2\,(a+bx))^{\frac{5}{2}}}\,dx$$

```
integrate((c*x)**(7/2)/(b*x**3+a*x**2)**(5/2),x)
```

```
Integral((c*x)**(7/2)/(x**2*(a + b*x))**(5/2), x)
```

Maxima [F]

$$\int \frac{(cx)^{7/2}}{(ax^2+bx^3)^{5/2}}\,dx = \int \frac{(cx)^{\frac{7}{2}}}{(bx^3+ax^2)^{\frac{5}{2}}}\,dx$$

```
integrate((c*x)^(7/2)/(b*x^3+a*x^2)^(5/2),x, algorithm="maxima")
```

```
integrate((c*x)^(7/2)/(b*x^3 + a*x^2)^(5/2), x)
```

Giac [F(-1)]

Timed out.

$$\int \frac{(cx)^{7/2}}{(ax^2+bx^3)^{5/2}}\,dx = \text{Timed out}$$

```
integrate((c*x)^(7/2)/(b*x^3+a*x^2)^(5/2),x, algorithm="giac")
```

```
Timed out
```

Mupad [B] (verification not implemented)

Time = 9.02 (sec) , antiderivative size = 90, normalized size of antiderivative = 0.95

$$\int \frac{(cx)^{7/2}}{(ax^2+bx^3)^{5/2}}\,dx = -\frac{\sqrt{b\,x^3+a\,x^2}\,\left(\frac{2\,c^3\,\sqrt{c\,x}}{a\,b^2}+\frac{16\,c^3\,x^2\,\sqrt{c\,x}}{3\,a^3}+\frac{8\,c^3\,x\,\sqrt{c\,x}}{a^2\,b}\right)}{x^4+\frac{2\,a\,x^3}{b}+\frac{a^2\,x^2}{b^2}}$$

```
int((c*x)^(7/2)/(a*x^2 + b*x^3)^(5/2),x)
```

```
-((a*x^2 + b*x^3)^(1/2)*((2*c^3*(c*x)^(1/2))/(a*b^2) + (16*c^3*x^2*(c*x)^(
1/2))/(3*a^3) + (8*c^3*x*(c*x)^(1/2))/(a^2*b)))/(x^4 + (2*a*x^3)/b + (a^2*
x^2)/b^2)
```

Reduce [B] (verification not implemented)

Time = 0.36 (sec) , antiderivative size = 79, normalized size of antiderivative = 0.83

$$\int \frac{(cx)^{7/2}}{(ax^2+bx^3)^{5/2}}\,dx = \frac{2\sqrt{c}\,c^3\left(8\sqrt{b}\,\sqrt{bx+a}\,ax+8\sqrt{b}\,\sqrt{bx+a}\,b\,x^2-3\sqrt{x}\,a^2-12\sqrt{x}\,abx-8\sqrt{x}\,b^2x^2\right)}{3\sqrt{bx+a}\,a^3x\,(bx+a)}$$

```
int((c*x)^(7/2)/(b*x^3+a*x^2)^(5/2),x)
```

```
(2*sqrt(c)*c**3*(8*sqrt(b)*sqrt(a + b*x)*a*x + 8*sqrt(b)*sqrt(a + b*x)*b*x
**2 - 3*sqrt(x)*a**2 - 12*sqrt(x)*a*b*x - 8*sqrt(x)*b**2*x**2))/(3*sqrt(a
+ b*x)*a**3*x*(a + b*x))
```

3.368 $\int \frac{(cx)^{5/2}}{(ax^2+bx^3)^{5/2}}\,dx$

Optimal result

Integrand size = 23, antiderivative size = 126

$$\int \frac{(cx)^{5/2}}{(ax^2+bx^3)^{5/2}}\,dx = \frac{2c(cx)^{3/2}}{3a\,(ax^2+bx^3)^{3/2}} + \frac{4c^3}{a^2\sqrt{cx}\sqrt{ax^2+bx^3}} - \frac{16c^5\sqrt{ax^2+bx^3}}{3a^3(cx)^{5/2}} + \frac{32bc^4\sqrt{ax^2+bx^3}}{3a^4(cx)^{3/2}}$$

```
2/3*c*(c*x)^(3/2)/a/(b*x^3+a*x^2)^(3/2)+4*c^3/a^2/(c*x)^(1/2)/(b*x^3+a*x^2
)^(1/2)-16/3*c^5*(b*x^3+a*x^2)^(1/2)/a^3/(c*x)^(5/2)+32/3*b*c^4*(b*x^3+a*x
^2)^(1/2)/a^4/(c*x)^(3/2)
```

Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 58, normalized size of antiderivative = 0.46

$$\int \frac{(cx)^{5/2}}{(ax^2+bx^3)^{5/2}}\,dx = \frac{2c(cx)^{3/2}\,(-a^3+6a^2bx+24ab^2x^2+16b^3x^3)}{3a^4\,(x^2(a+bx))^{3/2}}$$

```
Integrate[(c*x)^(5/2)/(a*x^2 + b*x^3)^(5/2),x]
```

```
(2*c*(c*x)^(3/2)*(-a^3 + 6*a^2*b*x + 24*a*b^2*x^2 + 16*b^3*x^3))/(3*a^4*(x
^2*(a + b*x))^(3/2))
```

Rubi [A] (verified)

Time = 0.49 (sec) , antiderivative size = 137, normalized size of antiderivative = 1.09, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.174$, Rules used = {1921, 1921, 1922, 1920}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(cx)^{5/2}}{(ax^2+bx^3)^{5/2}}\,dx$$

$$\Big\downarrow \text{1921}$$

$$\frac{2c^2\int \frac{\sqrt{cx}}{(bx^3+ax^2)^{3/2}}dx}{a}+\frac{2c(cx)^{3/2}}{3a(ax^2+bx^3)^{3/2}}$$

$$\Big\downarrow \text{1921}$$

$$\frac{2c^2\left(\frac{4c^2\int \frac{1}{(cx)^{3/2}\sqrt{bx^3+ax^2}}dx}{a}+\frac{2c}{a\sqrt{cx}\sqrt{ax^2+bx^3}}\right)}{a}+\frac{2c(cx)^{3/2}}{3a(ax^2+bx^3)^{3/2}}$$

$$\Big\downarrow \text{1922}$$

$$\frac{2c^2\left(\frac{4c^2\left(-\frac{2b\int \frac{1}{\sqrt{cx}\sqrt{bx^3+ax^2}}dx}{3ac}-\frac{2c\sqrt{ax^2+bx^3}}{3a(cx)^{5/2}}\right)}{a}+\frac{2c}{a\sqrt{cx}\sqrt{ax^2+bx^3}}\right)}{a}+\frac{2c(cx)^{3/2}}{3a(ax^2+bx^3)^{3/2}}$$

$$\Big\downarrow \text{1920}$$

$$\frac{2c^2\left(\frac{4c^2\left(\frac{4b\sqrt{ax^2+bx^3}}{3a^2(cx)^{3/2}}-\frac{2c\sqrt{ax^2+bx^3}}{3a(cx)^{5/2}}\right)}{a}+\frac{2c}{a\sqrt{cx}\sqrt{ax^2+bx^3}}\right)}{a}+\frac{2c(cx)^{3/2}}{3a(ax^2+bx^3)^{3/2}}$$

```
Int[(c*x)^(5/2)/(a*x^2 + b*x^3)^(5/2),x]
```

```
(2*c*(c*x)^(3/2))/(3*a*(a*x^2 + b*x^3)^(3/2)) + (2*c^2*((2*c)/(a*Sqrt[c*x]
*Sqrt[a*x^2 + b*x^3]) + (4*c^2*((-2*c*Sqrt[a*x^2 + b*x^3])/(3*a*(c*x)^(5/2
)) + (4*b*Sqrt[a*x^2 + b*x^3])/(3*a^2*(c*x)^(3/2))))/a))/a
```

Defintions of rubi rules used

```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> Simp[(-c^(j - 1))*(c*x)^(m - j + 1)*((a*x^j + b*x^n)^(p + 1)/(a*(n - j
)*(p + 1))), x] /; FreeQ[{a, b, c, j, m, n, p}, x] &&  !IntegerQ[p] && NeQ[
n, j] && EqQ[m + n*p + n - j + 1, 0] && (IntegerQ[j] || GtQ[c, 0])
```

```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> Simp[(-c^(j - 1))*(c*x)^(m - j + 1)*((a*x^j + b*x^n)^(p + 1)/(a*(n - j
)*(p + 1))), x] + Simp[c^j*((m + n*p + n - j + 1)/(a*(n - j)*(p + 1)))   In
t[(c*x)^(m - j)*(a*x^j + b*x^n)^(p + 1), x], x] /; FreeQ[{a, b, c, j, m, n}
, x] &&  !IntegerQ[p] && NeQ[n, j] && ILtQ[Simplify[(m + n*p + n - j + 1)/(
n - j)], 0] && LtQ[p, -1] && (IntegerQ[j] || GtQ[c, 0])
```

```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> Simp[c^(j - 1)*(c*x)^(m - j + 1)*((a*x^j + b*x^n)^(p + 1)/(a*(m + j*p
+ 1))), x] - Simp[b*((m + n*p + n - j + 1)/(a*c^(n - j)*(m + j*p + 1)))   I
nt[(c*x)^(m + n - j)*(a*x^j + b*x^n)^p, x], x] /; FreeQ[{a, b, c, j, m, n,
p}, x] &&  !IntegerQ[p] && NeQ[n, j] && ILtQ[Simplify[(m + n*p + n - j + 1)
/(n - j)], 0] && NeQ[m + j*p + 1, 0] && (IntegersQ[j, n] || GtQ[c, 0])
```

Maple [A] (verified)

Time = 0.40 (sec) , antiderivative size = 58, normalized size of antiderivative = 0.46

method	result	size
gosper	$-\frac{2x(bx+a)(-16b^3x^3-24a\,b^2x^2-6a^2bx+a^3)(cx)^{\frac{5}{2}}}{3a^4(b\,x^3+a\,x^2)^{\frac{5}{2}}}$	58
orering	$-\frac{2x(bx+a)(-16b^3x^3-24a\,b^2x^2-6a^2bx+a^3)(cx)^{\frac{5}{2}}}{3a^4(b\,x^3+a\,x^2)^{\frac{5}{2}}}$	58
default	$-\frac{2x^3(bx+a)c^2\sqrt{cx}\,(-16b^3x^3-24a\,b^2x^2-6a^2bx+a^3)}{3(b\,x^3+a\,x^2)^{\frac{5}{2}}a^4}$	63
risch	$-\frac{2(bx+a)(-8bx+a)c^3}{3a^4\sqrt{x^2(bx+a)}\,\sqrt{cx}}+\frac{2b^2(8bx+9a)x^2c^3}{3(bx+a)a^4\sqrt{x^2(bx+a)}\,\sqrt{cx}}$	82

```
int((c*x)^(5/2)/(b*x^3+a*x^2)^(5/2),x,method=_RETURNVERBOSE)
```

```
-2/3*x*(b*x+a)*(-16*b^3*x^3-24*a*b^2*x^2-6*a^2*b*x+a^3)*(c*x)^(5/2)/a^4/(b
*x^3+a*x^2)^(5/2)
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 91, normalized size of antiderivative = 0.72

$$\int \frac{(cx)^{5/2}}{(ax^2+bx^3)^{5/2}}\,dx = \frac{2\,(16\,b^3c^2x^3+24\,ab^2c^2x^2+6\,a^2bc^2x-a^3c^2)\sqrt{bx^3+ax^2}\sqrt{cx}}{3\,(a^4b^2x^5+2\,a^5bx^4+a^6x^3)}$$

```
integrate((c*x)^(5/2)/(b*x^3+a*x^2)^(5/2),x, algorithm="fricas")
```

```
2/3*(16*b^3*c^2*x^3 + 24*a*b^2*c^2*x^2 + 6*a^2*b*c^2*x - a^3*c^2)*sqrt(b*x
^3 + a*x^2)*sqrt(c*x)/(a^4*b^2*x^5 + 2*a^5*b*x^4 + a^6*x^3)
```

Sympy [F]

$$\int \frac{(cx)^{5/2}}{(ax^2+bx^3)^{5/2}}\,dx = \int \frac{(cx)^{\frac{5}{2}}}{\left(x^2\,(a+bx)\right)^{\frac{5}{2}}}\,dx$$

```
integrate((c*x)**(5/2)/(b*x**3+a*x**2)**(5/2),x)
```

```
Integral((c*x)**(5/2)/(x**2*(a + b*x))**(5/2), x)
```

Maxima [F]

$$\int \frac{(cx)^{5/2}}{(ax^2+bx^3)^{5/2}}\,dx = \int \frac{(cx)^{\frac{5}{2}}}{(bx^3+ax^2)^{\frac{5}{2}}}\,dx$$

```
integrate((c*x)^(5/2)/(b*x^3+a*x^2)^(5/2),x, algorithm="maxima")
```

```
integrate((c*x)^(5/2)/(b*x^3 + a*x^2)^(5/2), x)
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 213 vs. $2(104) = 208$.

Time = 33.45 (sec) , antiderivative size = 213, normalized size of antiderivative = 1.69

$$\int \frac{(cx)^{5/2}}{(ax^2+bx^3)^{5/2}}\,dx = \frac{2}{3}\,c^2\left(\frac{\sqrt{cx}\left(\frac{8\,b^3c^2x|c|}{a^4\mathrm{sgn}(x)}+\frac{9\,b^2c^2|c|}{a^3\mathrm{sgn}(x)}\right)}{(bc^2x+ac^2)^{\frac{3}{2}}}+\frac{4\left(4\sqrt{bc}a^2bc^7-9\sqrt{bc}\left(\sqrt{bc}\sqrt{cx}-\sqrt{bc^2x+ac^2}\right)\right.}{\left(ac^2-\left(\sqrt{bc}\sqrt{cx}-\sqrt{bc^2}\right.\right.}\right.$$

```
integrate((c*x)^(5/2)/(b*x^3+a*x^2)^(5/2),x, algorithm="giac")
```

```
2/3*c^2*(sqrt(c*x)*(8*b^3*c^2*x*abs(c)/(a^4*sgn(x)) + 9*b^2*c^2*abs(c)/(a^
3*sgn(x)))/(b*c^2*x + a*c^2)^(3/2) + 4*(4*sqrt(b*c)*a^2*b*c^7 - 9*sqrt(b*c
)*(sqrt(b*c)*sqrt(c*x) - sqrt(b*c^2*x + a*c^2))^2*a*b*c^5 + 3*sqrt(b*c)*(s
qrt(b*c)*sqrt(c*x) - sqrt(b*c^2*x + a*c^2))^4*b*c^3)/((a*c^2 - (sqrt(b*c)*
sqrt(c*x) - sqrt(b*c^2*x + a*c^2))^2)^3*a^3*abs(c)*sgn(x)))
```

Mupad [B] (verification not implemented)

Time = 8.90 (sec) , antiderivative size = 106, normalized size of antiderivative = 0.84

$$\int \frac{(cx)^{5/2}}{(ax^2+bx^3)^{5/2}}\,dx = \frac{\sqrt{b\,x^3+a\,x^2}\,\left(\frac{16\,c^2\,x^2\,\sqrt{c\,x}}{a^3}-\frac{2\,c^2\,\sqrt{c\,x}}{3\,a\,b^2}+\frac{4\,c^2\,x\,\sqrt{c\,x}}{a^2\,b}+\frac{32\,b\,c^2\,x^3\,\sqrt{c\,x}}{3\,a^4}\right)}{x^5+\frac{2\,a\,x^4}{b}+\frac{a^2\,x^3}{b^2}}$$

```
int((c*x)^(5/2)/(a*x^2 + b*x^3)^(5/2),x)
```

```
((a*x^2 + b*x^3)^(1/2)*((16*c^2*x^2*(c*x)^(1/2))/a^3 - (2*c^2*(c*x)^(1/2))
/(3*a*b^2) + (4*c^2*x*(c*x)^(1/2))/(a^2*b) + (32*b*c^2*x^3*(c*x)^(1/2))/(3
*a^4)))/(x^5 + (2*a*x^4)/b + (a^2*x^3)/b^2)
```

Reduce [B] (verification not implemented)

Time = 0.54 (sec) , antiderivative size = 97, normalized size of antiderivative = 0.77

$$\int \frac{(cx)^{5/2}}{(ax^2+bx^3)^{5/2}}\,dx = \frac{2\sqrt{c}\,c^2\left(-16\sqrt{b}\,\sqrt{bx+a}\,ab\,x^2-16\sqrt{b}\,\sqrt{bx+a}\,b^2x^3-\sqrt{x}\,a^3+6\sqrt{x}\,a^2bx+24\sqrt{x}\,a\right.}{3\sqrt{bx+a}\,a^4x^2\,(bx+a)}$$

```
int((c*x)^(5/2)/(b*x^3+a*x^2)^(5/2),x)
```

```
(2*sqrt(c)*c**2*( - 16*sqrt(b)*sqrt(a + b*x)*a*b*x**2 - 16*sqrt(b)*sqrt(a
+ b*x)*b**2*x**3 - sqrt(x)*a**3 + 6*sqrt(x)*a**2*b*x + 24*sqrt(x)*a*b**2*x
**2 + 16*sqrt(x)*b**3*x**3))/(3*sqrt(a + b*x)*a**4*x**2*(a + b*x))
```

3.369 $\int \frac{(cx)^{3/2}}{(ax^2+bx^3)^{5/2}}\,dx$

Optimal result

Integrand size = 23, antiderivative size = 163

$$\int \frac{(cx)^{3/2}}{(ax^2+bx^3)^{5/2}}\,dx = \frac{2c\sqrt{cx}}{3a\,(ax^2+bx^3)^{3/2}}+\frac{16c^3}{3a^2(cx)^{3/2}\sqrt{ax^2+bx^3}} -\frac{32c^5\sqrt{ax^2+bx^3}}{5a^3(cx)^{7/2}}+\frac{128bc^4\sqrt{ax^2+bx^3}}{15a^4(cx)^{5/2}}-\frac{256b^2c^3\sqrt{ax^2+bx^3}}{15a^5(cx)^{3/2}}$$

```
2/3*c*(c*x)^(1/2)/a/(b*x^3+a*x^2)^(3/2)+16/3*c^3/a^2/(c*x)^(3/2)/(b*x^3+a*
x^2)^(1/2)-32/5*c^5*(b*x^3+a*x^2)^(1/2)/a^3/(c*x)^(7/2)+128/15*b*c^4*(b*x^
3+a*x^2)^(1/2)/a^4/(c*x)^(5/2)-256/15*b^2*c^3*(b*x^3+a*x^2)^(1/2)/a^5/(c*x
)^(3/2)
```

Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 69, normalized size of antiderivative = 0.42

$$\int \frac{(cx)^{3/2}}{(ax^2+bx^3)^{5/2}}\,dx = -\frac{2c\sqrt{cx}(3a^4-8a^3bx+48a^2b^2x^2+192ab^3x^3+128b^4x^4)}{15a^5\,(x^2(a+bx))^{3/2}}$$

```
Integrate[(c*x)^(3/2)/(a*x^2 + b*x^3)^(5/2),x]
```

```
(-2*c*Sqrt[c*x]*(3*a^4 - 8*a^3*b*x + 48*a^2*b^2*x^2 + 192*a*b^3*x^3 + 128*
b^4*x^4))/(15*a^5*(x^2*(a + b*x))^(3/2))
```

Rubi [A] (verified)

Time = 0.59 (sec) , antiderivative size = 181, normalized size of antiderivative = 1.11, number of steps used = 5, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.217$, Rules used = {1921, 1921, 1922, 1922, 1920}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(cx)^{3/2}}{(ax^2+bx^3)^{5/2}}\,dx$$

$$\Big\downarrow \text{ 1921}$$

$$\frac{8c^2\int \frac{1}{\sqrt{cx}(bx^3+ax^2)^{3/2}}dx}{3a}+\frac{2c\sqrt{cx}}{3a\,(ax^2+bx^3)^{3/2}}$$

$$\Big\downarrow \text{ 1921}$$

$$\frac{8c^2\left(\frac{6c^2\int \frac{1}{(cx)^{5/2}\sqrt{bx^3+ax^2}}dx}{a}+\frac{2c}{a(cx)^{3/2}\sqrt{ax^2+bx^3}}\right)}{3a}+\frac{2c\sqrt{cx}}{3a\,(ax^2+bx^3)^{3/2}}$$

$$\Big\downarrow \text{ 1922}$$

$$\frac{8c^2\left(\frac{6c^2\left(-\frac{4b\int \frac{1}{(cx)^{3/2}\sqrt{bx^3+ax^2}}dx}{5ac}-\frac{2c\sqrt{ax^2+bx^3}}{5a(cx)^{7/2}}\right)}{a}+\frac{2c}{a(cx)^{3/2}\sqrt{ax^2+bx^3}}\right)}{3a}+\frac{2c\sqrt{cx}}{3a\,(ax^2+bx^3)^{3/2}}$$

$$\Big\downarrow \text{ 1922}$$

$$\frac{8c^2\left(\frac{6c^2\left(-\frac{4b\left(-\frac{2b\int \frac{1}{\sqrt{cx}\sqrt{bx^3+ax^2}}dx}{3ac}-\frac{2c\sqrt{ax^2+bx^3}}{3a(cx)^{5/2}}\right)}{5ac}-\frac{2c\sqrt{ax^2+bx^3}}{5a(cx)^{7/2}}\right)}{a}+\frac{2c}{a(cx)^{3/2}\sqrt{ax^2+bx^3}}\right)}{3a}+\frac{2c\sqrt{cx}}{3a\left(ax^2+bx^3\right)^{3/2}}$$

$$\Big\downarrow \textit{1920}$$

$$\frac{8c^2\left(\frac{6c^2\left(-\frac{4b\left(\frac{4b\sqrt{ax^2+bx^3}}{3a^2(cx)^{3/2}}-\frac{2c\sqrt{ax^2+bx^3}}{3a(cx)^{5/2}}\right)}{5ac}-\frac{2c\sqrt{ax^2+bx^3}}{5a(cx)^{7/2}}\right)}{a}+\frac{2c}{a(cx)^{3/2}\sqrt{ax^2+bx^3}}\right)}{3a}+\frac{2c\sqrt{cx}}{3a\left(ax^2+bx^3\right)^{3/2}}$$

```
Int[(c*x)^(3/2)/(a*x^2 + b*x^3)^(5/2),x]
```

```
(2*c*Sqrt[c*x])/(3*a*(a*x^2 + b*x^3)^(3/2)) + (8*c^2*((2*c)/(a*(c*x)^(3/2)
*Sqrt[a*x^2 + b*x^3]) + (6*c^2*((-2*c*Sqrt[a*x^2 + b*x^3])/(5*a*(c*x)^(7/2
)) - (4*b*((-2*c*Sqrt[a*x^2 + b*x^3])/(3*a*(c*x)^(5/2)) + (4*b*Sqrt[a*x^2
+ b*x^3])/(3*a^2*(c*x)^(3/2))))/(5*a*c)))/a))/(3*a)
```

Defintions of rubi rules used

```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> Simp[(-c^(j - 1))*(c*x)^(m - j + 1)*((a*x^j + b*x^n)^(p + 1)/(a*(n - j
)*(p + 1))), x] /; FreeQ[{a, b, c, j, m, n, p}, x] &&  !IntegerQ[p] && NeQ[
n, j] && EqQ[m + n*p + n - j + 1, 0] && (IntegerQ[j] || GtQ[c, 0])
```

```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> Simp[(-c^(j - 1))*(c*x)^(m - j + 1)*((a*x^j + b*x^n)^(p + 1)/(a*(n - j
)*(p + 1))), x] + Simp[c^j*((m + n*p + n - j + 1)/(a*(n - j)*(p + 1)))   In
t[(c*x)^(m - j)*(a*x^j + b*x^n)^(p + 1), x], x] /; FreeQ[{a, b, c, j, m, n}
, x] &&  !IntegerQ[p] && NeQ[n, j] && ILtQ[Simplify[(m + n*p + n - j + 1)/(
n - j)], 0] && LtQ[p, -1] && (IntegerQ[j] || GtQ[c, 0])
```

```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> Simp[c^(j - 1)*(c*x)^(m - j + 1)*((a*x^j + b*x^n)^(p + 1)/(a*(m + j*p
+ 1))), x] - Simp[b*((m + n*p + n - j + 1)/(a*c^(n - j)*(m + j*p + 1)))   I
nt[(c*x)^(m + n - j)*(a*x^j + b*x^n)^p, x], x] /; FreeQ[{a, b, c, j, m, n,
p}, x] &&  !IntegerQ[p] && NeQ[n, j] && ILtQ[Simplify[(m + n*p + n - j + 1)
/(n - j)], 0] && NeQ[m + j*p + 1, 0] && (IntegersQ[j, n] || GtQ[c, 0])
```

Maple [A] (verified)

Time = 0.40 (sec) , antiderivative size = 71, normalized size of antiderivative = 0.44

method	result	size
gosper	$-\frac{2x(bx+a)\left(128b^{4}x^{4}+192a\,b^{3}x^{3}+48a^{2}b^{2}x^{2}-8a^{3}bx+3a^{4}\right)(cx)^{\frac{3}{2}}}{15a^{5}\left(b\,x^{3}+a\,x^{2}\right)^{\frac{5}{2}}}$	71
orering	$-\frac{2x(bx+a)\left(128b^{4}x^{4}+192a\,b^{3}x^{3}+48a^{2}b^{2}x^{2}-8a^{3}bx+3a^{4}\right)(cx)^{\frac{3}{2}}}{15a^{5}\left(b\,x^{3}+a\,x^{2}\right)^{\frac{5}{2}}}$	71
default	$-\frac{2x^{2}(bx+a)\left(128b^{4}x^{4}+192a\,b^{3}x^{3}+48a^{2}b^{2}x^{2}-8a^{3}bx+3a^{4}\right)\sqrt{cx}\,c}{15\left(b\,x^{3}+a\,x^{2}\right)^{\frac{5}{2}}a^{5}}$	74
risch	$-\frac{2(bx+a)\left(73b^{2}x^{2}-14abx+3a^{2}\right)c^{2}}{15a^{5}x\sqrt{x^{2}(bx+a)}\,\sqrt{cx}}-\frac{2b^{3}(11bx+12a)x^{2}c^{2}}{3(bx+a)a^{5}\sqrt{x^{2}(bx+a)}\,\sqrt{cx}}$	98

```
int((c*x)^(3/2)/(b*x^3+a*x^2)^(5/2),x,method=_RETURNVERBOSE)
```

```
-2/15*x*(b*x+a)*(128*b^4*x^4+192*a*b^3*x^3+48*a^2*b^2*x^2-8*a^3*b*x+3*a^4)
*(c*x)^(3/2)/a^5/(b*x^3+a*x^2)^(5/2)
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 95, normalized size of antiderivative = 0.58

$$\int \frac{(cx)^{3/2}}{(ax^2+bx^3)^{5/2}}\,dx = -\frac{2\,(128\,b^4cx^4+192\,ab^3cx^3+48\,a^2b^2cx^2-8\,a^3bcx+3\,a^4c)\sqrt{bx^3+ax^2}\sqrt{cx}}{15\,(a^5b^2x^6+2\,a^6bx^5+a^7x^4)}$$

```
integrate((c*x)^(3/2)/(b*x^3+a*x^2)^(5/2),x, algorithm="fricas")
```

```
-2/15*(128*b^4*c*x^4 + 192*a*b^3*c*x^3 + 48*a^2*b^2*c*x^2 - 8*a^3*b*c*x +
3*a^4*c)*sqrt(b*x^3 + a*x^2)*sqrt(c*x)/(a^5*b^2*x^6 + 2*a^6*b*x^5 + a^7*x^
4)
```

Sympy [F]

$$\int \frac{(cx)^{3/2}}{(ax^2+bx^3)^{5/2}}\,dx = \int \frac{(cx)^{\frac{3}{2}}}{(x^2\,(a+bx))^{\frac{5}{2}}}\,dx$$

```
integrate((c*x)**(3/2)/(b*x**3+a*x**2)**(5/2),x)
```

```
Integral((c*x)**(3/2)/(x**2*(a + b*x))**(5/2), x)
```

Maxima [F]

$$\int \frac{(cx)^{3/2}}{(ax^2+bx^3)^{5/2}}\,dx = \int \frac{(cx)^{\frac{3}{2}}}{(bx^3+ax^2)^{\frac{5}{2}}}\,dx$$

```
integrate((c*x)^(3/2)/(b*x^3+a*x^2)^(5/2),x, algorithm="maxima")
```

```
integrate((c*x)^(3/2)/(b*x^3 + a*x^2)^(5/2), x)
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 310 vs. $2(133) = 266$.

Time = 104.84 (sec) , antiderivative size = 310, normalized size of antiderivative = 1.90

$$\int \frac{(cx)^{3/2}}{(ax^2+bx^3)^{5/2}}\,dx =$$

$$-\frac{2}{15}c\left(\frac{5\sqrt{cx}\left(\frac{11\,b^4c^2x|c|}{a^5\mathrm{sgn}(x)}+\frac{12\,b^3c^2|c|}{a^4\mathrm{sgn}(x)}\right)}{(bc^2x+ac^2)^{\frac{3}{2}}}+\frac{2\left(73\sqrt{bc}a^4b^2c^{11}-320\sqrt{bc}\left(\sqrt{bc}\sqrt{cx}-\sqrt{bc^2x+ac^2}\right)^2a^3b^2c^9+49\right.}{}\right.$$

```
integrate((c*x)^(3/2)/(b*x^3+a*x^2)^(5/2),x, algorithm="giac")
```

```
-2/15*c*(5*sqrt(c*x)*(11*b^4*c^2*x*abs(c)/(a^5*sgn(x)) + 12*b^3*c^2*abs(c)
/(a^4*sgn(x)))/(b*c^2*x + a*c^2)^(3/2) + 2*(73*sqrt(b*c)*a^4*b^2*c^11 - 32
0*sqrt(b*c)*(sqrt(b*c)*sqrt(c*x) - sqrt(b*c^2*x + a*c^2))^2*a^3*b^2*c^9 +
490*sqrt(b*c)*(sqrt(b*c)*sqrt(c*x) - sqrt(b*c^2*x + a*c^2))^4*a^2*b^2*c^7
- 240*sqrt(b*c)*(sqrt(b*c)*sqrt(c*x) - sqrt(b*c^2*x + a*c^2))^6*a*b^2*c^5
+ 45*sqrt(b*c)*(sqrt(b*c)*sqrt(c*x) - sqrt(b*c^2*x + a*c^2))^8*b^2*c^3)/((
a*c^2 - (sqrt(b*c)*sqrt(c*x) - sqrt(b*c^2*x + a*c^2))^2)^5*a^4*abs(c)*sgn(
x)))
```

Mupad [B] (verification not implemented)

Time = 8.99 (sec) , antiderivative size = 116, normalized size of antiderivative = 0.71

$$\int \frac{(cx)^{3/2}}{(ax^2+bx^3)^{5/2}}\,dx = -\frac{\sqrt{b\,x^3+a\,x^2}\,\left(\frac{2\,c\,\sqrt{c\,x}}{5\,a\,b^2}+\frac{32\,c\,x^2\,\sqrt{c\,x}}{5\,a^3}-\frac{16\,c\,x\,\sqrt{c\,x}}{15\,a^2\,b}+\frac{128\,b\,c\,x^3\,\sqrt{c\,x}}{5\,a^4}+\frac{256\,b^2\,c\,x^4\,\sqrt{c\,x}}{15\,a^5}\right)}{x^6+\frac{2\,a\,x^5}{b}+\frac{a^2\,x^4}{b^2}}$$

```
int((c*x)^(3/2)/(a*x^2 + b*x^3)^(5/2),x)
```

```
-((a*x^2 + b*x^3)^(1/2)*((2*c*(c*x)^(1/2))/(5*a*b^2) + (32*c*x^2*(c*x)^(1/
2))/(5*a^3) - (16*c*x*(c*x)^(1/2))/(15*a^2*b) + (128*b*c*x^3*(c*x)^(1/2))/
(5*a^4) + (256*b^2*c*x^4*(c*x)^(1/2))/(15*a^5)))/(x^6 + (2*a*x^5)/b + (a^2
*x^4)/b^2)
```

Reduce [B] (verification not implemented)

Time = 0.39 (sec) , antiderivative size = 110, normalized size of antiderivative = 0.67

$$\int \frac{(cx)^{3/2}}{\left(ax^2+bx^3\right)^{5/2}}\,dx = \frac{2\sqrt{c}\,c\Big(128\sqrt{b}\,\sqrt{bx+a}\,a\,b^2x^3+128\sqrt{b}\,\sqrt{bx+a}\,b^3x^4-3\sqrt{x}\,a^4+8\sqrt{x}\,a^3bx-48\sqrt{x}}{15\sqrt{bx+a}\,a^5x^3\,(bx+a)}$$

```
int((c*x)^(3/2)/(b*x^3+a*x^2)^(5/2),x)
```

```
(2*sqrt(c)*c*(128*sqrt(b)*sqrt(a + b*x)*a*b**2*x**3 + 128*sqrt(b)*sqrt(a +
 b*x)*b**3*x**4 - 3*sqrt(x)*a**4 + 8*sqrt(x)*a**3*b*x - 48*sqrt(x)*a**2*b*
*2*x**2 - 192*sqrt(x)*a*b**3*x**3 - 128*sqrt(x)*b**4*x**4))/(15*sqrt(a + b
*x)*a**5*x**3*(a + b*x))
```

3.370 $\int \frac{\sqrt{cx}}{(ax^2+bx^3)^{5/2}}\,dx$

Optimal result

Integrand size = 23, antiderivative size = 198

$$\begin{aligned}\int \frac{\sqrt{cx}}{(ax^2+bx^3)^{5/2}}\,dx &= \frac{2c}{3a\sqrt{cx}\,(ax^2+bx^3)^{3/2}}+\frac{20c^3}{3a^2(cx)^{5/2}\sqrt{ax^2+bx^3}}\\ &\quad-\frac{160c^5\sqrt{ax^2+bx^3}}{21a^3(cx)^{9/2}}+\frac{64bc^4\sqrt{ax^2+bx^3}}{7a^4(cx)^{7/2}}-\frac{256b^2c^3\sqrt{ax^2+bx^3}}{21a^5(cx)^{5/2}}+\frac{512b^3c^2\sqrt{ax^2+bx^3}}{21a^6(cx)^{3/2}}\end{aligned}$$

```
2/3*c/a/(c*x)^(1/2)/(b*x^3+a*x^2)^(3/2)+20/3*c^3/a^2/(c*x)^(5/2)/(b*x^3+a*
x^2)^(1/2)-160/21*c^5*(b*x^3+a*x^2)^(1/2)/a^3/(c*x)^(9/2)+64/7*b*c^4*(b*x^
3+a*x^2)^(1/2)/a^4/(c*x)^(7/2)-256/21*b^2*c^3*(b*x^3+a*x^2)^(1/2)/a^5/(c*x
)^(5/2)+512/21*b^3*c^2*(b*x^3+a*x^2)^(1/2)/a^6/(c*x)^(3/2)
```

Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 80, normalized size of antiderivative = 0.40

$$\int \frac{\sqrt{cx}}{(ax^2+bx^3)^{5/2}}\,dx = \frac{2c(-3a^5+6a^4bx-16a^3b^2x^2+96a^2b^3x^3+384ab^4x^4+256b^5x^5)}{21a^6\sqrt{cx}\,(x^2(a+bx))^{3/2}}$$

```
Integrate[Sqrt[c*x]/(a*x^2 + b*x^3)^(5/2),x]
```

```
(2*c*(-3*a^5 + 6*a^4*b*x - 16*a^3*b^2*x^2 + 96*a^2*b^3*x^3 + 384*a*b^4*x^4
 + 256*b^5*x^5))/(21*a^6*Sqrt[c*x]*(x^2*(a + b*x))^(3/2))
```

Rubi [A] (verified)

Time = 0.68 (sec) , antiderivative size = 223, normalized size of antiderivative = 1.13, number of steps used = 6, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.261$, Rules used = {1921, 1921, 1922, 1922, 1922, 1920}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sqrt{cx}}{(ax^2+bx^3)^{5/2}}\,dx$$

$$\Big\downarrow \text{1921}$$

$$\frac{10c^2\int \frac{1}{(cx)^{3/2}(bx^3+ax^2)^{3/2}}dx}{3a}+\frac{2c}{3a\sqrt{cx}\,(ax^2+bx^3)^{3/2}}$$

$$\Big\downarrow \text{1921}$$

$$\frac{10c^2\left(\frac{8c^2\int \frac{1}{(cx)^{7/2}\sqrt{bx^3+ax^2}}dx}{a}+\frac{2c}{a(cx)^{5/2}\sqrt{ax^2+bx^3}}\right)}{3a}+\frac{2c}{3a\sqrt{cx}\,(ax^2+bx^3)^{3/2}}$$

$$\Big\downarrow \text{1922}$$

$$\frac{10c^2\left(\frac{8c^2\left(-\frac{6b\int \frac{1}{(cx)^{5/2}\sqrt{bx^3+ax^2}}dx}{7ac}-\frac{2c\sqrt{ax^2+bx^3}}{7a(cx)^{9/2}}\right)}{a}+\frac{2c}{a(cx)^{5/2}\sqrt{ax^2+bx^3}}\right)}{3a}+\frac{2c}{3a\sqrt{cx}\,(ax^2+bx^3)^{3/2}}$$

$$\Big\downarrow \text{1922}$$

$$\frac{10c^2\left(\frac{8c^2\left(-\frac{6b\left(-\frac{4b\int\frac{1}{(cx)^{3/2}\sqrt{bx^3+ax^2}}dx}{5ac}-\frac{2c\sqrt{ax^2+bx^3}}{5a(cx)^{7/2}}\right)}{7ac}-\frac{2c\sqrt{ax^2+bx^3}}{7a(cx)^{9/2}}\right)}{a}+\frac{2c}{a(cx)^{5/2}\sqrt{ax^2+bx^3}}\right)}{3a}+\frac{2c}{3a\sqrt{cx}\,(ax^2+bx^3)^{3/2}}$$

$$\Big\downarrow \; 1922$$

$$\frac{10c^2\left(\frac{8c^2\left(-\frac{6b\left(-\frac{4b\left(-\frac{2b\int\frac{1}{\sqrt{cx}\sqrt{bx^3+ax^2}}dx}{3ac}-\frac{2c\sqrt{ax^2+bx^3}}{3a(cx)^{5/2}}\right)}{5ac}-\frac{2c\sqrt{ax^2+bx^3}}{5a(cx)^{7/2}}\right)}{7ac}-\frac{2c\sqrt{ax^2+bx^3}}{7a(cx)^{9/2}}\right)}{a}+\frac{2c}{a(cx)^{5/2}\sqrt{ax^2+bx^3}}\right)}{3a}+\frac{2c}{3a\sqrt{cx}\,(ax^2+bx^3)^{3/2}}$$

$$\Big\downarrow \; 1920$$

$$\frac{10c^2\left(\frac{8c^2\left(-\frac{6b\left(-\frac{4b\left(\frac{4b\sqrt{ax^2+bx^3}}{3a^2(cx)^{3/2}}-\frac{2c\sqrt{ax^2+bx^3}}{3a(cx)^{5/2}}\right)}{5ac}-\frac{2c\sqrt{ax^2+bx^3}}{5a(cx)^{7/2}}\right)}{7ac}-\frac{2c\sqrt{ax^2+bx^3}}{7a(cx)^{9/2}}\right)}{a}+\frac{2c}{a(cx)^{5/2}\sqrt{ax^2+bx^3}}\right)}{3a}+\frac{2c}{3a\sqrt{cx}\,(ax^2+bx^3)^{3/2}}$$

```
Int[Sqrt[c*x]/(a*x^2 + b*x^3)^(5/2),x]
```

```
(2*c)/(3*a*Sqrt[c*x]*(a*x^2 + b*x^3)^(3/2)) + (10*c^2*((2*c)/(a*(c*x)^(5/2
)*Sqrt[a*x^2 + b*x^3]) + (8*c^2*((-2*c*Sqrt[a*x^2 + b*x^3])/(7*a*(c*x)^(9/
2)) - (6*b*((-2*c*Sqrt[a*x^2 + b*x^3])/(5*a*(c*x)^(7/2)) - (4*b*((-2*c*Sqr
t[a*x^2 + b*x^3])/(3*a*(c*x)^(5/2)) + (4*b*Sqrt[a*x^2 + b*x^3])/(3*a^2*(c*
x)^(3/2))))/(5*a*c)))/(7*a*c)))/a))/(3*a)
```

Defintions of rubi rules used

```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> Simp[(-c^(j - 1))*(c*x)^(m - j + 1)*((a*x^j + b*x^n)^(p + 1)/(a*(n - j
)*(p + 1))), x] /; FreeQ[{a, b, c, j, m, n, p}, x] &&  !IntegerQ[p] && NeQ[
n, j] && EqQ[m + n*p + n - j + 1, 0] && (IntegerQ[j] || GtQ[c, 0])
```

```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> Simp[(-c^(j - 1))*(c*x)^(m - j + 1)*((a*x^j + b*x^n)^(p + 1)/(a*(n - j
)*(p + 1))), x] + Simp[c^j*((m + n*p + n - j + 1)/(a*(n - j)*(p + 1)))   In
t[(c*x)^(m - j)*(a*x^j + b*x^n)^(p + 1), x], x] /; FreeQ[{a, b, c, j, m, n}
, x] &&  !IntegerQ[p] && NeQ[n, j] && ILtQ[Simplify[(m + n*p + n - j + 1)/(
n - j)], 0] && LtQ[p, -1] && (IntegerQ[j] || GtQ[c, 0])
```

```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> Simp[c^(j - 1)*(c*x)^(m - j + 1)*((a*x^j + b*x^n)^(p + 1)/(a*(m + j*p
+ 1))), x] - Simp[b*((m + n*p + n - j + 1)/(a*c^(n - j)*(m + j*p + 1)))   I
nt[(c*x)^(m + n - j)*(a*x^j + b*x^n)^p, x], x] /; FreeQ[{a, b, c, j, m, n,
p}, x] &&  !IntegerQ[p] && NeQ[n, j] && ILtQ[Simplify[(m + n*p + n - j + 1)
/(n - j)], 0] && NeQ[m + j*p + 1, 0] && (IntegersQ[j, n] || GtQ[c, 0])
```

Maple [A] (verified)

Time = 0.40 (sec) , antiderivative size = 82, normalized size of antiderivative = 0.41

method	result	size
gosper	$-\frac{2x(bx+a)\left(-256b^{5}x^{5}-384a\,b^{4}x^{4}-96a^{2}b^{3}x^{3}+16a^{3}b^{2}x^{2}-6a^{4}bx+3a^{5}\right)\sqrt{cx}}{21a^{6}\left(b\,x^{3}+a\,x^{2}\right)^{\frac{5}{2}}}$	82
default	$-\frac{2x(bx+a)\left(-256b^{5}x^{5}-384a\,b^{4}x^{4}-96a^{2}b^{3}x^{3}+16a^{3}b^{2}x^{2}-6a^{4}bx+3a^{5}\right)\sqrt{cx}}{21a^{6}\left(b\,x^{3}+a\,x^{2}\right)^{\frac{5}{2}}}$	82
orering	$-\frac{2x(bx+a)\left(-256b^{5}x^{5}-384a\,b^{4}x^{4}-96a^{2}b^{3}x^{3}+16a^{3}b^{2}x^{2}-6a^{4}bx+3a^{5}\right)\sqrt{cx}}{21a^{6}\left(b\,x^{3}+a\,x^{2}\right)^{\frac{5}{2}}}$	82
risch	$-\frac{2(bx+a)\left(-158b^{3}x^{3}+37a\,b^{2}x^{2}-12a^{2}bx+3a^{3}\right)c}{21a^{6}x^{2}\sqrt{x^{2}(bx+a)}\,\sqrt{cx}}+\frac{2b^{4}(14bx+15a)x^{2}c}{3(bx+a)a^{6}\sqrt{x^{2}(bx+a)}\,\sqrt{cx}}$	105

```
int((c*x)^(1/2)/(b*x^3+a*x^2)^(5/2),x,method=_RETURNVERBOSE)
```

```
-2/21*x*(b*x+a)*(-256*b^5*x^5-384*a*b^4*x^4-96*a^2*b^3*x^3+16*a^3*b^2*x^2-
6*a^4*b*x+3*a^5)*(c*x)^(1/2)/a^6/(b*x^3+a*x^2)^(5/2)
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 101, normalized size of antiderivative = 0.51

$$\int \frac{\sqrt{cx}}{(ax^2+bx^3)^{5/2}}\,dx = \frac{2\,(256\,b^5x^5+384\,ab^4x^4+96\,a^2b^3x^3-16\,a^3b^2x^2+6\,a^4bx-3\,a^5)\sqrt{bx^3+ax^2}\sqrt{cx}}{21\,(a^6b^2x^7+2\,a^7bx^6+a^8x^5)}$$

```
integrate((c*x)^(1/2)/(b*x^3+a*x^2)^(5/2),x, algorithm="fricas")
```

```
2/21*(256*b^5*x^5 + 384*a*b^4*x^4 + 96*a^2*b^3*x^3 - 16*a^3*b^2*x^2 + 6*a^
4*b*x - 3*a^5)*sqrt(b*x^3 + a*x^2)*sqrt(c*x)/(a^6*b^2*x^7 + 2*a^7*b*x^6 +
a^8*x^5)
```

Sympy [F]

$$\int \frac{\sqrt{cx}}{(ax^2 + bx^3)^{5/2}}\,dx = \int \frac{\sqrt{cx}}{(x^2\,(a + bx))^{\frac{5}{2}}}\,dx$$

```
integrate((c*x)**(1/2)/(b*x**3+a*x**2)**(5/2),x)
```

```
Integral(sqrt(c*x)/(x**2*(a + b*x))**(5/2), x)
```

Maxima [F]

$$\int \frac{\sqrt{cx}}{(ax^2 + bx^3)^{5/2}}\,dx = \int \frac{\sqrt{cx}}{(bx^3 + ax^2)^{\frac{5}{2}}}\,dx$$

```
integrate((c*x)^(1/2)/(b*x^3+a*x^2)^(5/2),x, algorithm="maxima")
```

```
integrate(sqrt(c*x)/(b*x^3 + a*x^2)^(5/2), x)
```

Giac [F(-1)]

Timed out.

$$\int \frac{\sqrt{cx}}{(ax^2 + bx^3)^{5/2}}\,dx = \text{Timed out}$$

```
integrate((c*x)^(1/2)/(b*x^3+a*x^2)^(5/2),x, algorithm="giac")
```

```
Timed out
```

Mupad [B] (verification not implemented)

Time = 8.79 (sec) , antiderivative size = 126, normalized size of antiderivative = 0.64

$$\int \frac{\sqrt{cx}}{(ax^2+bx^3)^{5/2}}\,dx = \frac{\sqrt{b\,x^3+a\,x^2}\,\left(\frac{256\,b^2\,x^4\,\sqrt{c\,x}}{7\,a^5}-\frac{32\,x^2\,\sqrt{c\,x}}{21\,a^3}-\frac{2\,\sqrt{c\,x}}{7\,a\,b^2}+\frac{512\,b^3\,x^5\,\sqrt{c\,x}}{21\,a^6}+\frac{4\,x\,\sqrt{c\,x}}{7\,a^2\,b}+\frac{64\,b\,x^3\,\sqrt{c\,x}}{7\,a^4}\right)}{x^7+\frac{2\,a\,x^6}{b}+\frac{a^2\,x^5}{b^2}}$$

```
int((c*x)^(1/2)/(a*x^2 + b*x^3)^(5/2),x)
```

```
((a*x^2 + b*x^3)^(1/2)*((256*b^2*x^4*(c*x)^(1/2))/(7*a^5) - (32*x^2*(c*x)^
(1/2))/(21*a^3) - (2*(c*x)^(1/2))/(7*a*b^2) + (512*b^3*x^5*(c*x)^(1/2))/(2
1*a^6) + (4*x*(c*x)^(1/2))/(7*a^2*b) + (64*b*x^3*(c*x)^(1/2))/(7*a^4)))/(x
^7 + (2*a*x^6)/b + (a^2*x^5)/b^2)
```

Reduce [B] (verification not implemented)

Time = 0.23 (sec) , antiderivative size = 122, normalized size of antiderivative = 0.62

$$\int \frac{\sqrt{cx}}{(ax^2+bx^3)^{5/2}}\,dx = \frac{2\sqrt{c}\left(-256\sqrt{b}\,\sqrt{bx+a}\,a\,b^3x^4-256\sqrt{b}\,\sqrt{bx+a}\,b^4x^5-3\sqrt{x}\,a^5+6\sqrt{x}\,a^4bx-16\sqrt{}\right.}{21\sqrt{bx+a}\,a^6x^4\,(bx+a)}$$

```
int((c*x)^(1/2)/(b*x^3+a*x^2)^(5/2),x)
```

```
(2*sqrt(c)*( - 256*sqrt(b)*sqrt(a + b*x)*a*b**3*x**4 - 256*sqrt(b)*sqrt(a
+ b*x)*b**4*x**5 - 3*sqrt(x)*a**5 + 6*sqrt(x)*a**4*b*x - 16*sqrt(x)*a**3*b
**2*x**2 + 96*sqrt(x)*a**2*b**3*x**3 + 384*sqrt(x)*a*b**4*x**4 + 256*sqrt(
x)*b**5*x**5))/(21*sqrt(a + b*x)*a**6*x**4*(a + b*x))
```

3.371 $\int \frac{1}{\sqrt{cx}\left(ax^2+bx^3\right)^{5/2}}\,dx$

Optimal result

Integrand size = 23, antiderivative size = 229

$$\begin{aligned}\int \frac{1}{\sqrt{cx}\,(ax^2+bx^3)^{5/2}}\,dx &= \frac{2c}{3a(cx)^{3/2}\,(ax^2+bx^3)^{3/2}} \\ &+ \frac{8c^3}{a^2(cx)^{7/2}\sqrt{ax^2+bx^3}} - \frac{80c^5\sqrt{ax^2+bx^3}}{9a^3(cx)^{11/2}} + \frac{640bc^4\sqrt{ax^2+bx^3}}{63a^4(cx)^{9/2}} \\ &- \frac{256b^2c^3\sqrt{ax^2+bx^3}}{21a^5(cx)^{7/2}} + \frac{1024b^3c^2\sqrt{ax^2+bx^3}}{63a^6(cx)^{5/2}} - \frac{2048b^4c\sqrt{ax^2+bx^3}}{63a^7(cx)^{3/2}}\end{aligned}$$

```
2/3*c/a/(c*x)^(3/2)/(b*x^3+a*x^2)^(3/2)+8*c^3/a^2/(c*x)^(7/2)/(b*x^3+a*x^2
)^(1/2)-80/9*c^5*(b*x^3+a*x^2)^(1/2)/a^3/(c*x)^(11/2)+640/63*b*c^4*(b*x^3+
a*x^2)^(1/2)/a^4/(c*x)^(9/2)-256/21*b^2*c^3*(b*x^3+a*x^2)^(1/2)/a^5/(c*x)^
(7/2)+1024/63*b^3*c^2*(b*x^3+a*x^2)^(1/2)/a^6/(c*x)^(5/2)-2048/63*b^4*c*(b
*x^3+a*x^2)^(1/2)/a^7/(c*x)^(3/2)
```

Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 91, normalized size of antiderivative = 0.40

$$\int \frac{1}{\sqrt{cx}\,(ax^2+bx^3)^{5/2}}\,dx = -\frac{2c(7a^6-12a^5bx+24a^4b^2x^2-64a^3b^3x^3+384a^2b^4x^4+1536ab^5x^5+1024b^6x^6)}{63a^7(cx)^{3/2}\,(x^2(a+bx))^{3/2}}$$

```
Integrate[1/(Sqrt[c*x]*(a*x^2 + b*x^3)^(5/2)),x]
```

```
(-2*c*(7*a^6 - 12*a^5*b*x + 24*a^4*b^2*x^2 - 64*a^3*b^3*x^3 + 384*a^2*b^4*
x^4 + 1536*a*b^5*x^5 + 1024*b^6*x^6))/(63*a^7*(c*x)^(3/2)*(x^2*(a + b*x))^
(3/2))
```

Rubi [A] (verified)

Time = 0.82 (sec) , antiderivative size = 263, normalized size of antiderivative = 1.15, number of steps used = 7, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}} = 0.304$, Rules used = {1921, 1921, 1922, 1922, 1922, 1922, 1920}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{\sqrt{cx}\,(ax^2+bx^3)^{5/2}}\,dx$$

$$\Big\downarrow \text{1921}$$

$$\frac{4c^2\int \frac{1}{(cx)^{5/2}(bx^3+ax^2)^{3/2}}dx}{a}+\frac{2c}{3a(cx)^{3/2}\,(ax^2+bx^3)^{3/2}}$$

$$\Big\downarrow \text{1921}$$

$$\frac{4c^2\left(\frac{10c^2\int \frac{1}{(cx)^{9/2}\sqrt{bx^3+ax^2}}dx}{a}+\frac{2c}{a(cx)^{7/2}\sqrt{ax^2+bx^3}}\right)}{a}+\frac{2c}{3a(cx)^{3/2}\,(ax^2+bx^3)^{3/2}}$$

$$\Big\downarrow \text{1922}$$

$$\frac{4c^2\left(\frac{10c^2\left(-\frac{8b\int\frac{1}{(cx)^{7/2}\sqrt{bx^3+ax^2}}dx}{9ac}-\frac{2c\sqrt{ax^2+bx^3}}{9a(cx)^{11/2}}\right)}{a}+\frac{2c}{a(cx)^{7/2}\sqrt{ax^2+bx^3}}\right)}{a}+\frac{2c}{3a(cx)^{3/2}\left(ax^2+bx^3\right)^{3/2}}$$

$$\Big\downarrow \text{1922}$$

$$\frac{4c^2\left(\frac{10c^2\left(-\frac{8b\left(-\frac{6b\int\frac{1}{(cx)^{5/2}\sqrt{bx^3+ax^2}}dx}{7ac}-\frac{2c\sqrt{ax^2+bx^3}}{7a(cx)^{9/2}}\right)}{9ac}-\frac{2c\sqrt{ax^2+bx^3}}{9a(cx)^{11/2}}\right)}{a}+\frac{2c}{a(cx)^{7/2}\sqrt{ax^2+bx^3}}\right)}{a}+\frac{2c}{3a(cx)^{3/2}\left(ax^2+bx^3\right)^{3/2}}$$

$$\Big\downarrow \text{1922}$$

$$\frac{4c^2\left(\frac{10c^2\left(-\frac{8b\left(-\frac{6b\left(-\frac{4b\int\frac{1}{(cx)^{3/2}\sqrt{bx^3+ax^2}}dx}{5ac}-\frac{2c\sqrt{ax^2+bx^3}}{5a(cx)^{7/2}}\right)}{7ac}-\frac{2c\sqrt{ax^2+bx^3}}{7a(cx)^{9/2}}\right)}{9ac}-\frac{2c\sqrt{ax^2+bx^3}}{9a(cx)^{11/2}}\right)}{a}+\frac{2c}{a(cx)^{7/2}\sqrt{ax^2+bx^3}}\right)}{a}+\frac{2c}{3a(cx)^{3/2}\left(ax^2+bx^3\right)^{3/2}}$$

$$\Big\downarrow \text{1922}$$

$$\frac{4c^2\left(\frac{10c^2\left(-\frac{8b\left(-\frac{6b\left(-\frac{4b\left(-\frac{2b\int\frac{1}{\sqrt{cx}\sqrt{bx^3+ax^2}}\,dx}{3ac}-\frac{2c\sqrt{ax^2+bx^3}}{3a(cx)^{5/2}}\right)}{5ac}-\frac{2c\sqrt{ax^2+bx^3}}{5a(cx)^{7/2}}\right)}{7ac}-\frac{2c\sqrt{ax^2+bx^3}}{7a(cx)^{9/2}}\right)}{9ac}-\frac{2c\sqrt{ax^2+bx^3}}{9a(cx)^{11/2}}\right)}{a}+\frac{2c}{a(cx)^{7/2}\sqrt{ax^2+bx^3}}\right)}{a}$$

$$\frac{2c}{3a(cx)^{3/2}\left(ax^2+bx^3\right)^{3/2}}$$

$$\downarrow 1920$$

$$\frac{4c^2\left(\frac{10c^2\left(-\frac{8b\left(-\frac{6b\left(-\frac{4b\left(\frac{4b\sqrt{ax^2+bx^3}}{3a^2(cx)^{3/2}}-\frac{2c\sqrt{ax^2+bx^3}}{3a(cx)^{5/2}}\right)}{5ac}-\frac{2c\sqrt{ax^2+bx^3}}{5a(cx)^{7/2}}\right)}{7ac}-\frac{2c\sqrt{ax^2+bx^3}}{7a(cx)^{9/2}}\right)}{9ac}-\frac{2c\sqrt{ax^2+bx^3}}{9a(cx)^{11/2}}\right)}{a}+\frac{2c}{a(cx)^{7/2}\sqrt{ax^2+bx^3}}\right)}{a}+\frac{2c}{3a(cx)^{3/2}\left(ax^2+bx^3\right)^{3/2}}$$

```
Int[1/(Sqrt[c*x]*(a*x^2 + b*x^3)^(5/2)),x]
```

```
(2*c)/(3*a*(c*x)^(3/2)*(a*x^2 + b*x^3)^(3/2)) + (4*c^2*((2*c)/(a*(c*x)^(7/
2)*Sqrt[a*x^2 + b*x^3]) + (10*c^2*((-2*c*Sqrt[a*x^2 + b*x^3])/(9*a*(c*x)^(
11/2)) - (8*b*((-2*c*Sqrt[a*x^2 + b*x^3])/(7*a*(c*x)^(9/2)) - (6*b*((-2*c*
Sqrt[a*x^2 + b*x^3])/(5*a*(c*x)^(7/2)) - (4*b*((-2*c*Sqrt[a*x^2 + b*x^3])/
(3*a*(c*x)^(5/2)) + (4*b*Sqrt[a*x^2 + b*x^3])/(3*a^2*(c*x)^(3/2))))/(5*a*c
)))/(7*a*c)))/(9*a*c)))/a))/a
```

Defintions of rubi rules used

```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> Simp[(-c^(j - 1))*(c*x)^(m - j + 1)*((a*x^j + b*x^n)^(p + 1)/(a*(n - j
)*(p + 1))), x] /; FreeQ[{a, b, c, j, m, n, p}, x] && !IntegerQ[p] && NeQ[
n, j] && EqQ[m + n*p + n - j + 1, 0] && (IntegerQ[j] || GtQ[c, 0])
```

```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> Simp[(-c^(j - 1))*(c*x)^(m - j + 1)*((a*x^j + b*x^n)^(p + 1)/(a*(n - j
)*(p + 1))), x] + Simp[c^j*((m + n*p + n - j + 1)/(a*(n - j)*(p + 1)))   In
t[(c*x)^(m - j)*(a*x^j + b*x^n)^(p + 1), x], x] /; FreeQ[{a, b, c, j, m, n}
, x] && !IntegerQ[p] && NeQ[n, j] && ILtQ[Simplify[(m + n*p + n - j + 1)/(
n - j)], 0] && LtQ[p, -1] && (IntegerQ[j] || GtQ[c, 0])
```

```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> Simp[c^(j - 1)*(c*x)^(m - j + 1)*((a*x^j + b*x^n)^(p + 1)/(a*(m + j*p
+ 1))), x] - Simp[b*((m + n*p + n - j + 1)/(a*c^(n - j)*(m + j*p + 1)))   I
nt[(c*x)^(m + n - j)*(a*x^j + b*x^n)^p, x], x] /; FreeQ[{a, b, c, j, m, n,
p}, x] && !IntegerQ[p] && NeQ[n, j] && ILtQ[Simplify[(m + n*p + n - j + 1)
/(n - j)], 0] && NeQ[m + j*p + 1, 0] && (IntegersQ[j, n] || GtQ[c, 0])
```

Maple [A] (verified)

Time = 0.40 (sec) , antiderivative size = 93, normalized size of antiderivative = 0.41

method	result	size
gosper	$-\frac{2x(bx+a)\left(1024b^{6}x^{6}+1536a\,b^{5}x^{5}+384a^{2}b^{4}x^{4}-64a^{3}b^{3}x^{3}+24a^{4}b^{2}x^{2}-12a^{5}bx+7a^{6}\right)}{63a^{7}\sqrt{cx}\,\left(b\,x^{3}+a\,x^{2}\right)^{\frac{5}{2}}}$	93
default	$-\frac{2x(bx+a)\left(1024b^{6}x^{6}+1536a\,b^{5}x^{5}+384a^{2}b^{4}x^{4}-64a^{3}b^{3}x^{3}+24a^{4}b^{2}x^{2}-12a^{5}bx+7a^{6}\right)}{63a^{7}\sqrt{cx}\,\left(b\,x^{3}+a\,x^{2}\right)^{\frac{5}{2}}}$	93
orering	$-\frac{2x(bx+a)\left(1024b^{6}x^{6}+1536a\,b^{5}x^{5}+384a^{2}b^{4}x^{4}-64a^{3}b^{3}x^{3}+24a^{4}b^{2}x^{2}-12a^{5}bx+7a^{6}\right)}{63a^{7}\sqrt{cx}\,\left(b\,x^{3}+a\,x^{2}\right)^{\frac{5}{2}}}$	93
risch	$-\frac{2(bx+a)\left(667b^{4}x^{4}-176a\,b^{3}x^{3}+69a^{2}b^{2}x^{2}-26a^{3}bx+7a^{4}\right)}{63a^{7}x^{3}\sqrt{x^{2}(bx+a)}\,\sqrt{cx}}-\frac{2b^{5}(17bx+18a)x^{2}}{3(bx+a)a^{7}\sqrt{x^{2}(bx+a)}\,\sqrt{cx}}$	114

```
int(1/(c*x)^(1/2)/(b*x^3+a*x^2)^(5/2),x,method=_RETURNVERBOSE)
```

```
-2/63*x*(b*x+a)*(1024*b^6*x^6+1536*a*b^5*x^5+384*a^2*b^4*x^4-64*a^3*b^3*x^
3+24*a^4*b^2*x^2-12*a^5*b*x+7*a^6)/a^7/(c*x)^(1/2)/(b*x^3+a*x^2)^(5/2)
```

Fricas [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 115, normalized size of antiderivative = 0.50

$$\int \frac{1}{\sqrt{cx}\,(ax^2+bx^3)^{5/2}}\,dx = -\frac{2\,(1024\,b^6x^6+1536\,ab^5x^5+384\,a^2b^4x^4-64\,a^3b^3x^3+24\,a^4b^2x^2-12\,a^5bx+7\,a^6)\sqrt{bx^3+ax^2}\sqrt{cx}}{63\,(a^7b^2cx^8+2\,a^8bcx^7+a^9cx^6)}$$

```
integrate(1/(c*x)^(1/2)/(b*x^3+a*x^2)^(5/2),x, algorithm="fricas")
```

```
-2/63*(1024*b^6*x^6 + 1536*a*b^5*x^5 + 384*a^2*b^4*x^4 - 64*a^3*b^3*x^3 +
24*a^4*b^2*x^2 - 12*a^5*b*x + 7*a^6)*sqrt(b*x^3 + a*x^2)*sqrt(c*x)/(a^7*b^
2*c*x^8 + 2*a^8*b*c*x^7 + a^9*c*x^6)
```

Sympy [F]

$$\int \frac{1}{\sqrt{cx}\,(ax^2+bx^3)^{5/2}}\,dx = \int \frac{1}{\sqrt{cx}\,(x^2\,(a+bx))^{\frac{5}{2}}}\,dx$$

```
integrate(1/(c*x)**(1/2)/(b*x**3+a*x**2)**(5/2),x)
```

```
Integral(1/(sqrt(c*x)*(x**2*(a + b*x))**(5/2)), x)
```

Maxima [F]

$$\int \frac{1}{\sqrt{cx}\,(ax^2+bx^3)^{5/2}}\,dx = \int \frac{1}{(bx^3+ax^2)^{\frac{5}{2}}\sqrt{cx}}\,dx$$

```
integrate(1/(c*x)^(1/2)/(b*x^3+a*x^2)^(5/2),x, algorithm="maxima")
```

```
integrate(1/((b*x^3 + a*x^2)^(5/2)*sqrt(c*x)), x)
```

Giac [A] (verification not implemented)

Time = 2.56 (sec) , antiderivative size = 311, normalized size of antiderivative = 1.36

$$\int \frac{1}{\sqrt{cx}\,(ax^2+bx^3)^{5/2}}\,dx =$$

$$-\frac{2\left(\frac{945\,b^{10}c^4}{a^3|b|\mathrm{sgn}(x)}-\left(\frac{3360\,b^{10}c^4}{a^4|b|\mathrm{sgn}(x)}-\left(\frac{4599\,b^{10}c^4}{a^5|b|\mathrm{sgn}(x)}+\left(\frac{667\,(bx+a)b^{10}c^4}{a^7|b|\mathrm{sgn}(x)}-\frac{2844\,b^{10}c^4}{a^6|b|\mathrm{sgn}(x)}\right)(bx+a)\right)(bx+a)\right)(bx+a)\right)\sqrt{bx+}}{63\left((bx+a)bc-abc\right)^{\frac{9}{2}}}$$

$$-\frac{4\left(17\sqrt{bc}a^2b^8c^2+36\sqrt{bc}\left(\sqrt{bc}\sqrt{bx+a}-\sqrt{(bx+a)bc-abc}\right)^2ab^7c+15\sqrt{bc}\left(\sqrt{bc}\sqrt{bx+a}-\sqrt{(bx+}\right.\right.}{3\left(abc+\left(\sqrt{bc}\sqrt{bx+a}-\sqrt{(bx+a)bc-abc}\right)^2\right)^3a^6|b|\mathrm{sgn}\left(x\right)}$$

```
integrate(1/(c*x)^(1/2)/(b*x^3+a*x^2)^(5/2),x, algorithm="giac")
```

```
-2/63*(945*b^10*c^4/(a^3*abs(b)*sgn(x)) - (3360*b^10*c^4/(a^4*abs(b)*sgn(x
)) - (4599*b^10*c^4/(a^5*abs(b)*sgn(x)) + (667*(b*x + a)*b^10*c^4/(a^7*abs
(b)*sgn(x)) - 2844*b^10*c^4/(a^6*abs(b)*sgn(x)))*(b*x + a))*(b*x + a))*(b*
x + a))*sqrt(b*x + a)/((b*x + a)*b*c - a*b*c)^(9/2) - 4/3*(17*sqrt(b*c)*a^
2*b^8*c^2 + 36*sqrt(b*c)*(sqrt(b*c)*sqrt(b*x + a) - sqrt((b*x + a)*b*c - a
*b*c))^2*a*b^7*c + 15*sqrt(b*c)*(sqrt(b*c)*sqrt(b*x + a) - sqrt((b*x + a)*
b*c - a*b*c))^4*b^6)/((a*b*c + (sqrt(b*c)*sqrt(b*x + a) - sqrt((b*x + a)*b
*c - a*b*c))^2)^3*a^6*abs(b)*sgn(x))
```

Mupad [B] (verification not implemented)

Time = 9.06 (sec) , antiderivative size = 124, normalized size of antiderivative = 0.54

$$\int \frac{1}{\sqrt{cx}\,(ax^2+bx^3)^{5/2}}\,dx = -\frac{\sqrt{b\,x^3+a\,x^2}\,\left(\frac{2}{9\,a\,b^2}+\frac{16\,x^2}{21\,a^3}-\frac{8\,x}{21\,a^2\,b}-\frac{128\,b\,x^3}{63\,a^4}+\frac{256\,b^2\,x^4}{21\,a^5}+\frac{1024\,b^3\,x^5}{21\,a^6}+\frac{2048\,b^4\,x^6}{63\,a^7}\right)}{x^7\,\sqrt{c\,x}+\frac{a^2\,x^5\,\sqrt{c\,x}}{b^2}+\frac{2\,a\,x^6\,\sqrt{c\,x}}{b}}$$

```
int(1/((c*x)^(1/2)*(a*x^2 + b*x^3)^(5/2)),x)
```

```
-((a*x^2 + b*x^3)^(1/2)*(2/(9*a*b^2) + (16*x^2)/(21*a^3) - (8*x)/(21*a^2*b
) - (128*b*x^3)/(63*a^4) + (256*b^2*x^4)/(21*a^5) + (1024*b^3*x^5)/(21*a^6
) + (2048*b^4*x^6)/(63*a^7)))/(x^7*(c*x)^(1/2) + (a^2*x^5*(c*x)^(1/2))/b^2
 + (2*a*x^6*(c*x)^(1/2))/b)
```

Reduce [B] (verification not implemented)

Time = 0.51 (sec) , antiderivative size = 138, normalized size of antiderivative = 0.60

$$\int \frac{1}{\sqrt{cx}\,(ax^2+bx^3)^{5/2}}\,dx = \frac{2\sqrt{c}\left(1024\sqrt{b}\,\sqrt{bx+a}\,a\,b^4x^5+1024\sqrt{b}\,\sqrt{bx+a}\,b^5x^6-7\sqrt{x}\,a^6+12\sqrt{x}\,a^5bx\right.}{63\sqrt{bx+a}}$$

```
int(1/(c*x)^(1/2)/(b*x^3+a*x^2)^(5/2),x)
```

```
(2*sqrt(c)*(1024*sqrt(b)*sqrt(a + b*x)*a*b**4*x**5 + 1024*sqrt(b)*sqrt(a +
 b*x)*b**5*x**6 - 7*sqrt(x)*a**6 + 12*sqrt(x)*a**5*b*x - 24*sqrt(x)*a**4*b
**2*x**2 + 64*sqrt(x)*a**3*b**3*x**3 - 384*sqrt(x)*a**2*b**4*x**4 - 1536*s
qrt(x)*a*b**5*x**5 - 1024*sqrt(x)*b**6*x**6))/(63*sqrt(a + b*x)*a**7*c*x**
5*(a + b*x))
```

3.372 $\int \frac{x^3}{(ax^2+bx^3)^{2/3}}\,dx$

Optimal result

Integrand size = 19, antiderivative size = 238

$$\int \frac{x^3}{(ax^2+bx^3)^{2/3}}\,dx = -\frac{5a\sqrt[3]{ax^2+bx^3}}{6b^2}+\frac{x\sqrt[3]{ax^2+bx^3}}{2b} - \frac{5a^2x^{4/3}(a+bx)^{2/3}\arctan\left(\frac{1}{\sqrt{3}}+\frac{2\sqrt[3]{b}\sqrt[3]{x}}{\sqrt{3}\sqrt[3]{a+bx}}\right)}{3\sqrt{3}b^{8/3}(ax^2+bx^3)^{2/3}} - \frac{5a^2x^{4/3}(a+bx)^{2/3}\log(a+bx)}{18b^{8/3}(ax^2+bx^3)^{2/3}} - \frac{5a^2x^{4/3}(a+bx)^{2/3}\log\left(1-\frac{\sqrt[3]{b}\sqrt[3]{x}}{\sqrt[3]{a+bx}}\right)}{6b^{8/3}(ax^2+bx^3)^{2/3}}$$

```
-5/6*a*(b*x^3+a*x^2)^(1/3)/b^2+1/2*x*(b*x^3+a*x^2)^(1/3)/b-5/9*a^2*x^(4/3)
*(b*x+a)^(2/3)*arctan(1/3*3^(1/2)+2/3*b^(1/3)*x^(1/3)*3^(1/2)/(b*x+a)^(1/3
))*3^(1/2)/b^(8/3)/(b*x^3+a*x^2)^(2/3)-5/18*a^2*x^(4/3)*(b*x+a)^(2/3)*ln(b
*x+a)/b^(8/3)/(b*x^3+a*x^2)^(2/3)-5/6*a^2*x^(4/3)*(b*x+a)^(2/3)*ln(1-b^(1/
3)*x^(1/3)/(b*x+a)^(1/3))/b^(8/3)/(b*x^3+a*x^2)^(2/3)
```

Mathematica [A] (verified)

Time = 0.07 (sec) , antiderivative size = 222, normalized size of antiderivative = 0.93

$$\int \frac{x^3}{(ax^2+bx^3)^{2/3}}\,dx = \frac{x^{4/3}\left(-15a^2b^{2/3}x^{2/3}-6ab^{5/3}x^{5/3}+9b^{8/3}x^{8/3}-10\sqrt{3}a^2(a+bx)^{2/3}\arctan\left(\frac{\sqrt{3}}{\sqrt[3]{b}\sqrt[3]{x}\ldots}\right.\right.}{\ldots}$$

```
Integrate[x^3/(a*x^2 + b*x^3)^(2/3),x]
```

```
(x^(4/3)*(-15*a^2*b^(2/3)*x^(2/3) - 6*a*b^(5/3)*x^(5/3) + 9*b^(8/3)*x^(8/3
) - 10*Sqrt[3]*a^2*(a + b*x)^(2/3)*ArcTan[(Sqrt[3]*b^(1/3)*x^(1/3))/(b^(1/
3)*x^(1/3) + 2*(a + b*x)^(1/3))] - 10*a^2*(a + b*x)^(2/3)*Log[-(b^(1/3)*x^
(1/3)) + (a + b*x)^(1/3)] + 5*a^2*(a + b*x)^(2/3)*Log[b^(2/3)*x^(2/3) + b^
(1/3)*x^(1/3)*(a + b*x)^(1/3) + (a + b*x)^(2/3)]))/(18*b^(8/3)*(x^2*(a + b
*x))^(2/3))
```

Rubi [A] (verified)

Time = 0.48 (sec) , antiderivative size = 182, normalized size of antiderivative = 0.76, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.211$, Rules used = {1930, 1930, 1938, 71}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^3}{(ax^2+bx^3)^{2/3}}\,dx$$

$$\Big\downarrow \text{1930}$$

$$\frac{x\sqrt[3]{ax^2+bx^3}}{2b}-\frac{5a\int \frac{x^2}{(bx^3+ax^2)^{2/3}}dx}{6b}$$

$$\Big\downarrow \text{1930}$$

$$\frac{x\sqrt[3]{ax^2+bx^3}}{2b}-\frac{5a\left(\frac{\sqrt[3]{ax^2+bx^3}}{b}-\frac{2a\int \frac{x}{(bx^3+ax^2)^{2/3}}dx}{3b}\right)}{6b}$$

$$\Big\downarrow \text{1938}$$

$$\frac{x\sqrt[3]{ax^2+bx^3}}{2b}-\frac{5a\left(\frac{\sqrt[3]{ax^2+bx^3}}{b}-\frac{2ax^{4/3}(a+bx)^{2/3}\int\frac{1}{\sqrt[3]{x}(a+bx)^{2/3}}dx}{3b(ax^2+bx^3)^{2/3}}\right)}{6b}$$

$$\Big\downarrow \text{71}$$

$$\frac{x\sqrt[3]{ax^2+bx^3}}{2b}-\frac{5a\left(\frac{\sqrt[3]{ax^2+bx^3}}{b}-\frac{2ax^{4/3}(a+bx)^{2/3}\left(-\frac{\sqrt{3}\arctan\left(\frac{2\sqrt[3]{b}\sqrt[3]{x}}{\sqrt{3}\sqrt[3]{a+bx}}+\frac{1}{\sqrt{3}}\right)}{b^{2/3}}-\frac{\log(a+bx)}{2b^{2/3}}-\frac{3\log\left(\frac{\sqrt[3]{b}\sqrt[3]{x}}{\sqrt[3]{a+bx}}-1\right)}{2b^{2/3}}\right)}{3b(ax^2+bx^3)^{2/3}}\right)}{6b}$$

```
Int[x^3/(a*x^2 + b*x^3)^(2/3),x]
```

```
(x*(a*x^2 + b*x^3)^(1/3))/(2*b) - (5*a*((a*x^2 + b*x^3)^(1/3)/b - (2*a*x^(
4/3)*(a + b*x)^(2/3)*(-((Sqrt[3]*ArcTan[1/Sqrt[3] + (2*b^(1/3)*x^(1/3))/(S
qrt[3]*(a + b*x)^(1/3))])/b^(2/3)) - Log[a + b*x]/(2*b^(2/3)) - (3*Log[-1
+ (b^(1/3)*x^(1/3))/(a + b*x)^(1/3)])/(2*b^(2/3))))/(3*b*(a*x^2 + b*x^3)^(
2/3))))/(6*b)
```

Defintions of rubi rules used

```
Int[1/(((a_.) + (b_.)*(x_))^(1/3)*((c_.) + (d_.)*(x_))^(2/3)), x_Symbol] :>
 With[{q = Rt[d/b, 3]}, Simp[(-Sqrt[3])*(q/d)*ArcTan[2*q*((a + b*x)^(1/3)/(
Sqrt[3]*(c + d*x)^(1/3))) + 1/Sqrt[3]], x] + (-Simp[3*(q/(2*d))*Log[q*((a +
 b*x)^(1/3)/(c + d*x)^(1/3)) - 1], x] - Simp[(q/(2*d))*Log[c + d*x], x])] /
; FreeQ[{a, b, c, d}, x] && PosQ[d/b]
```

```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> Simp[c^(n - 1)*(c*x)^(m - n + 1)*((a*x^j + b*x^n)^(p + 1)/(b*(m + n*p
+ 1))), x] - Simp[a*c^(n - j)*((m + j*p - n + j + 1)/(b*(m + n*p + 1)))   I
nt[(c*x)^(m - (n - j))*(a*x^j + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, p},
x] &&  !IntegerQ[p] && LtQ[0, j, n] && (IntegersQ[j, n] || GtQ[c, 0]) && Gt
Q[m + j*p - n + j + 1, 0] && NeQ[m + n*p + 1, 0]
```

```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> Simp[c^IntPart[m]*(c*x)^FracPart[m]*((a*x^j + b*x^n)^FracPart[p]/(x^(F
racPart[m] + j*FracPart[p])*(a + b*x^(n - j))^FracPart[p]))   Int[x^(m + j*
p)*(a + b*x^(n - j))^p, x], x] /; FreeQ[{a, b, c, j, m, n, p}, x] &&  !Inte
gerQ[p] && NeQ[n, j] && PosQ[n - j]
```

Maple [A] (verified)

Time = 0.51 (sec) , antiderivative size = 153, normalized size of antiderivative = 0.64

method	result
pseudoelliptic	$\frac{9x\left(x^{2}(bx+a)\right)^{\frac{1}{3}} b^{\frac{5}{3}}+10a^{2}\sqrt{3}\,\arctan\left(\frac{\sqrt{3}\left(b^{\frac{1}{3}}x+2\left(x^{2}(bx+a)\right)^{\frac{1}{3}}\right)}{3b^{\frac{1}{3}}x}\right)-15a\left(x^{2}(bx+a)\right)^{\frac{1}{3}} b^{\frac{2}{3}}-10a^{2}\ln\left(\frac{-b^{\frac{1}{3}}x+\left(x^{2}(bx+a)\right)^{\frac{1}{3}}}{x}\right)}{18b^{\frac{8}{3}}}$

```
int(x^3/(b*x^3+a*x^2)^(2/3),x,method=_RETURNVERBOSE)
```

```
1/18/b^(8/3)*(9*x*(x^2*(b*x+a))^(1/3)*b^(5/3)+10*a^2*3^(1/2)*arctan(1/3*3^
(1/2)*(b^(1/3)*x+2*(x^2*(b*x+a))^(1/3))/b^(1/3)/x)-15*a*(x^2*(b*x+a))^(1/3
)*b^(2/3)-10*a^2*ln((-b^(1/3)*x+(x^2*(b*x+a))^(1/3))/x)+5*a^2*ln((b^(2/3)*
x^2+b^(1/3)*(x^2*(b*x+a))^(1/3)*x+(x^2*(b*x+a))^(2/3))/x^2))
```

Fricas [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 194, normalized size of antiderivative = 0.82

$$\int \frac{x^3}{(ax^2+bx^3)^{2/3}}\,dx = \frac{30\sqrt{\frac{1}{3}}a^2(b^2)^{\frac{1}{6}}b\arctan\left(\frac{\sqrt{\frac{1}{3}}\left((b^2)^{\frac{1}{3}}bx+2\,(bx^3+ax^2)^{\frac{1}{3}}(b^2)^{\frac{2}{3}}\right)(b^2)^{\frac{1}{6}}}{b^2x}\right)-10\,a^2(b^2)^{\frac{2}{3}}\log\left(-\frac{(b^2)}{}\right.}{}$$

```
integrate(x^3/(b*x^3+a*x^2)^(2/3),x, algorithm="fricas")
```

```
1/18*(30*sqrt(1/3)*a^2*(b^2)^(1/6)*b*arctan(sqrt(1/3)*((b^2)^(1/3)*b*x + 2
*(b*x^3 + a*x^2)^(1/3)*(b^2)^(2/3))*(b^2)^(1/6)/(b^2*x)) - 10*a^2*(b^2)^(2
/3)*log(-((b^2)^(2/3)*x - (b*x^3 + a*x^2)^(1/3)*b)/x) + 5*a^2*(b^2)^(2/3)*
log(((b^2)^(1/3)*b*x^2 + (b*x^3 + a*x^2)^(1/3)*(b^2)^(2/3)*x + (b*x^3 + a*
x^2)^(2/3)*b)/x^2) + 3*(3*b^3*x - 5*a*b^2)*(b*x^3 + a*x^2)^(1/3))/b^4
```

Sympy [F]

$$\int \frac{x^3}{(ax^2+bx^3)^{2/3}}\,dx = \int \frac{x^3}{(x^2\,(a+bx))^{\frac{2}{3}}}\,dx$$

```
integrate(x**3/(b*x**3+a*x**2)**(2/3),x)
```

```
Integral(x**3/(x**2*(a + b*x))**(2/3), x)
```

Maxima [F]

$$\int \frac{x^3}{(ax^2+bx^3)^{2/3}}\,dx = \int \frac{x^3}{(bx^3+ax^2)^{\frac{2}{3}}}\,dx$$

```
integrate(x^3/(b*x^3+a*x^2)^(2/3),x, algorithm="maxima")
```

```
integrate(x^3/(b*x^3 + a*x^2)^(2/3), x)
```

Giac [A] (verification not implemented)

Time = 4.29 (sec) , antiderivative size = 142, normalized size of antiderivative = 0.60

$$\int \frac{x^3}{(ax^2+bx^3)^{2/3}}\,dx = \frac{\frac{10\sqrt{3}a^3\arctan\left(\frac{\sqrt{3}\left(2\left(b+\frac{a}{x}\right)^{\frac{1}{3}}+b^{\frac{1}{3}}\right)}{3b^{\frac{1}{3}}}\right)}{b^{\frac{8}{3}}}+\frac{5a^3\log\left(\left(b+\frac{a}{x}\right)^{\frac{2}{3}}+\left(b+\frac{a}{x}\right)^{\frac{1}{3}}b^{\frac{1}{3}}+b^{\frac{2}{3}}\right)}{b^{\frac{8}{3}}}-\frac{10a^3\log\left(\left|\left(b+\frac{a}{x}\right)^{\frac{1}{3}}-b^{\frac{1}{3}}\right|\right)}{b^{\frac{8}{3}}}}{18a}$$

```
integrate(x^3/(b*x^3+a*x^2)^(2/3),x, algorithm="giac")
```

```
1/18*(10*sqrt(3)*a^3*arctan(1/3*sqrt(3)*(2*(b + a/x)^(1/3) + b^(1/3))/b^(1
/3))/b^(8/3) + 5*a^3*log((b + a/x)^(2/3) + (b + a/x)^(1/3)*b^(1/3) + b^(2/
3))/b^(8/3) - 10*a^3*log(abs((b + a/x)^(1/3) - b^(1/3)))/b^(8/3) - 3*(5*a^
3*(b + a/x)^(4/3) - 8*a^3*(b + a/x)^(1/3)*b)*x^2/(a^2*b^2))/a
```

Mupad [F(-1)]

Timed out.

$$\int \frac{x^3}{(ax^2+bx^3)^{2/3}}\,dx = \int \frac{x^3}{{\left(b\,x^3+a\,x^2\right)}^{2/3}}\,dx$$

```
int(x^3/(a*x^2 + b*x^3)^(2/3),x)
```

```
int(x^3/(a*x^2 + b*x^3)^(2/3), x)
```

Reduce [F]

$$\int \frac{x^3}{(ax^2 + bx^3)^{2/3}}\, dx = \int \frac{x^{\frac{5}{3}}}{(bx + a)^{\frac{2}{3}}} dx$$

```
int(x^3/(b*x^3+a*x^2)^(2/3),x)
```

```
int(x**2/(x**(1/3)*(a + b*x)**(2/3)),x)
```

3.373 $\int \frac{x^2}{(ax^2+bx^3)^{2/3}}\,dx$

Optimal result

Integrand size = 19, antiderivative size = 200

$$\int \frac{x^2}{(ax^2+bx^3)^{2/3}}\,dx=\frac{\sqrt[3]{ax^2+bx^3}}{b}+\frac{2ax^{4/3}(a+bx)^{2/3}\arctan\left(\frac{1}{\sqrt{3}}+\frac{2\sqrt[3]{b}\sqrt[3]{x}}{\sqrt{3}\sqrt[3]{a+bx}}\right)}{\sqrt{3}b^{5/3}\left(ax^2+bx^3\right)^{2/3}}+\frac{ax^{4/3}(a+bx)^{2/3}\log(a+bx)}{3b^{5/3}\left(ax^2+bx^3\right)^{2/3}}+\frac{ax^{4/3}(a+bx)^{2/3}\log\left(1-\frac{\sqrt[3]{b}\sqrt[3]{x}}{\sqrt[3]{a+bx}}\right)}{b^{5/3}\left(ax^2+bx^3\right)^{2/3}}$$

```
(b*x^3+a*x^2)^(1/3)/b+2/3*a*x^(4/3)*(b*x+a)^(2/3)*arctan(1/3*3^(1/2)+2/3*b
^(1/3)*x^(1/3)*3^(1/2)/(b*x+a)^(1/3))*3^(1/2)/b^(5/3)/(b*x^3+a*x^2)^(2/3)+
1/3*a*x^(4/3)*(b*x+a)^(2/3)*ln(b*x+a)/b^(5/3)/(b*x^3+a*x^2)^(2/3)+a*x^(4/3
)*(b*x+a)^(2/3)*ln(1-b^(1/3)*x^(1/3)/(b*x+a)^(1/3))/b^(5/3)/(b*x^3+a*x^2)^
(2/3)
```

Mathematica [A] (verified)

Time = 0.03 (sec) , antiderivative size = 201, normalized size of antiderivative = 1.00

$$\int \frac{x^2}{(ax^2+bx^3)^{2/3}}\,dx = \frac{x^{4/3}\left(3ab^{2/3}x^{2/3}+3b^{5/3}x^{5/3}+2\sqrt{3}a(a+bx)^{2/3}\arctan\left(\frac{\sqrt{3}\sqrt[3]{b}\sqrt[3]{x}}{\sqrt[3]{b}\sqrt[3]{x}+2\sqrt[3]{a+bx}}\right)+2a(\right.}{3b}$$

```
Integrate[x^2/(a*x^2 + b*x^3)^(2/3),x]
```

```
(x^(4/3)*(3*a*b^(2/3)*x^(2/3) + 3*b^(5/3)*x^(5/3) + 2*Sqrt[3]*a*(a + b*x)^
(2/3)*ArcTan[(Sqrt[3]*b^(1/3)*x^(1/3))/(b^(1/3)*x^(1/3) + 2*(a + b*x)^(1/3
))] + 2*a*(a + b*x)^(2/3)*Log[-(b^(1/3)*x^(1/3)) + (a + b*x)^(1/3)] - a*(a
 + b*x)^(2/3)*Log[b^(2/3)*x^(2/3) + b^(1/3)*x^(1/3)*(a + b*x)^(1/3) + (a +
 b*x)^(2/3)]))/(3*b^(5/3)*(x^2*(a + b*x))^(2/3))
```

Rubi [A] (verified)

Time = 0.41 (sec) , antiderivative size = 150, normalized size of antiderivative = 0.75, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.158$, Rules used = {1930, 1938, 71}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^2}{(ax^2+bx^3)^{2/3}}dx$$

$$\Big\downarrow \text{ 1930}$$

$$\frac{\sqrt[3]{ax^2+bx^3}}{b}-\frac{2a\int \frac{x}{(bx^3+ax^2)^{2/3}}dx}{3b}$$

$$\Big\downarrow \text{ 1938}$$

$$\frac{\sqrt[3]{ax^2+bx^3}}{b}-\frac{2ax^{4/3}(a+bx)^{2/3}\int \frac{1}{\sqrt[3]{x}(a+bx)^{2/3}}dx}{3b\left(ax^2+bx^3\right)^{2/3}}$$

$$\Big\downarrow \text{ 71}$$

$$\frac{\sqrt[3]{ax^2+bx^3}}{b} - \frac{2ax^{4/3}(a+bx)^{2/3}\left(-\frac{\sqrt{3}\arctan\left(\frac{2\sqrt[3]{b}\sqrt[3]{x}}{\sqrt{3}\sqrt[3]{a+bx}}+\frac{1}{\sqrt{3}}\right)}{b^{2/3}} - \frac{\log(a+bx)}{2b^{2/3}} - \frac{3\log\left(\frac{\sqrt[3]{b}\sqrt[3]{x}}{\sqrt[3]{a+bx}}-1\right)}{2b^{2/3}}\right)}{3b\left(ax^2+bx^3\right)^{2/3}}$$

```
Int[x^2/(a*x^2 + b*x^3)^(2/3),x]
```

```
(a*x^2 + b*x^3)^(1/3)/b - (2*a*x^(4/3)*(a + b*x)^(2/3)*(-((Sqrt[3]*ArcTan[
1/Sqrt[3] + (2*b^(1/3)*x^(1/3))/(Sqrt[3]*(a + b*x)^(1/3))])/b^(2/3)) - Log
[a + b*x]/(2*b^(2/3)) - (3*Log[-1 + (b^(1/3)*x^(1/3))/(a + b*x)^(1/3)])/(2
*b^(2/3))))/(3*b*(a*x^2 + b*x^3)^(2/3))
```

Defintions of rubi rules used

```
Int[1/(((a_.) + (b_.)*(x_))^(1/3)*((c_.) + (d_.)*(x_))^(2/3)), x_Symbol] :>
 With[{q = Rt[d/b, 3]}, Simp[(-Sqrt[3])*(q/d)*ArcTan[2*q*((a + b*x)^(1/3)/(
Sqrt[3]*(c + d*x)^(1/3))) + 1/Sqrt[3]], x] + (-Simp[3*(q/(2*d))*Log[q*((a +
 b*x)^(1/3)/(c + d*x)^(1/3)) - 1], x] - Simp[(q/(2*d))*Log[c + d*x], x])] /
; FreeQ[{a, b, c, d}, x] && PosQ[d/b]
```

```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> Simp[c^(n - 1)*(c*x)^(m - n + 1)*((a*x^j + b*x^n)^(p + 1)/(b*(m + n*p
+ 1))), x] - Simp[a*c^(n - j)*((m + j*p - n + j + 1)/(b*(m + n*p + 1)))   I
nt[(c*x)^(m - (n - j))*(a*x^j + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, p},
x] &&  !IntegerQ[p] && LtQ[0, j, n] && (IntegersQ[j, n] || GtQ[c, 0]) && Gt
Q[m + j*p - n + j + 1, 0] && NeQ[m + n*p + 1, 0]
```

```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> Simp[c^IntPart[m]*(c*x)^FracPart[m]*((a*x^j + b*x^n)^FracPart[p]/(x^(F
racPart[m] + j*FracPart[p])*(a + b*x^(n - j))^FracPart[p]))   Int[x^(m + j*
p)*(a + b*x^(n - j))^p, x], x] /; FreeQ[{a, b, c, j, m, n, p}, x] &&  !Inte
gerQ[p] && NeQ[n, j] && PosQ[n - j]
```

Maple [A] (verified)

Time = 0.48 (sec) , antiderivative size = 129, normalized size of antiderivative = 0.64

method	result
pseudoelliptic	$\frac{-2\sqrt{3}\,\arctan\left(\frac{\sqrt{3}\left(b^{\frac{1}{3}}x+2\left(x^{2}(bx+a)\right)^{\frac{1}{3}}\right)}{3b^{\frac{1}{3}}x}\right)a+3\left(x^{2}(bx+a)\right)^{\frac{1}{3}}b^{\frac{2}{3}}+2\ln\left(\frac{-b^{\frac{1}{3}}x+\left(x^{2}(bx+a)\right)^{\frac{1}{3}}}{x}\right)a-\ln\left(\frac{b^{\frac{2}{3}}x^{2}+b^{\frac{1}{3}}\left(x^{2}(bx+a)\right)}{x^{2}}\right.}{3b^{\frac{5}{3}}}$

```
int(x^2/(b*x^3+a*x^2)^(2/3),x,method=_RETURNVERBOSE)
```

```
1/3*(-2*3^(1/2)*arctan(1/3*3^(1/2)*(b^(1/3)*x+2*(x^2*(b*x+a))^(1/3))/b^(1/
3)/x)*a+3*(x^2*(b*x+a))^(1/3)*b^(2/3)+2*ln((-b^(1/3)*x+(x^2*(b*x+a))^(1/3)
)/x)*a-ln((b^(2/3)*x^2+b^(1/3)*(x^2*(b*x+a))^(1/3)*x+(x^2*(b*x+a))^(2/3))/
x^2)*a)/b^(5/3)
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 177, normalized size of antiderivative = 0.88

$$\int \frac{x^2}{(ax^2+bx^3)^{2/3}}\,dx =$$

$$-\frac{6\sqrt{\frac{1}{3}}a(b^2)^{\frac{1}{6}}b\arctan\left(\frac{\sqrt{\frac{1}{3}}\left((b^2)^{\frac{1}{3}}bx+2\,(bx^3+ax^2)^{\frac{1}{3}}(b^2)^{\frac{2}{3}}\right)(b^2)^{\frac{1}{6}}}{b^2x}\right)-2\,a(b^2)^{\frac{2}{3}}\log\left(-\frac{(b^2)^{\frac{2}{3}}x-(bx^3+ax^2)^{\frac{1}{3}}b}{x}\right)+a(b^2)^{\frac{2}{3}}}{3\,b^3}$$

```
integrate(x^2/(b*x^3+a*x^2)^(2/3),x, algorithm="fricas")
```

```
-1/3*(6*sqrt(1/3)*a*(b^2)^(1/6)*b*arctan(sqrt(1/3)*((b^2)^(1/3)*b*x + 2*(b
*x^3 + a*x^2)^(1/3)*(b^2)^(2/3))*(b^2)^(1/6)/(b^2*x)) - 2*a*(b^2)^(2/3)*lo
g(-((b^2)^(2/3)*x - (b*x^3 + a*x^2)^(1/3)*b)/x) + a*(b^2)^(2/3)*log(((b^2)
^(1/3)*b*x^2 + (b*x^3 + a*x^2)^(1/3)*(b^2)^(2/3)*x + (b*x^3 + a*x^2)^(2/3)
*b)/x^2) - 3*(b*x^3 + a*x^2)^(1/3)*b^2)/b^3
```

Sympy [F]

$$\int \frac{x^2}{(ax^2+bx^3)^{2/3}}\,dx = \int \frac{x^2}{\left(x^2\,(a+bx)\right)^{\frac{2}{3}}}\,dx$$

```
integrate(x**2/(b*x**3+a*x**2)**(2/3),x)
```

```
Integral(x**2/(x**2*(a + b*x))**(2/3), x)
```

Maxima [F]

$$\int \frac{x^2}{(ax^2+bx^3)^{2/3}}\,dx = \int \frac{x^2}{(bx^3+ax^2)^{\frac{2}{3}}}\,dx$$

```
integrate(x^2/(b*x^3+a*x^2)^(2/3),x, algorithm="maxima")
```

```
integrate(x^2/(b*x^3 + a*x^2)^(2/3), x)
```

Giac [A] (verification not implemented)

Time = 4.16 (sec) , antiderivative size = 107, normalized size of antiderivative = 0.54

$$\int \frac{x^2}{(ax^2+bx^3)^{2/3}}\,dx =$$

$$-\frac{1}{3}\,a\left(\frac{2\,\sqrt{3}\arctan\left(\frac{\sqrt{3}\left(2\,\left(b+\frac{a}{x}\right)^{\frac{1}{3}}+b^{\frac{1}{3}}\right)}{3\,b^{\frac{1}{3}}}\right)}{b^{\frac{5}{3}}}-\frac{3\,\left(b+\frac{a}{x}\right)^{\frac{1}{3}}x}{ab}+\frac{\log\left(\left(b+\frac{a}{x}\right)^{\frac{2}{3}}+\left(b+\frac{a}{x}\right)^{\frac{1}{3}}b^{\frac{1}{3}}+b^{\frac{2}{3}}\right)}{b^{\frac{5}{3}}}-\frac{2\,\log\left(\left|(b\right.\right.}{}\right.$$

```
integrate(x^2/(b*x^3+a*x^2)^(2/3),x, algorithm="giac")
```

```
-1/3*a*(2*sqrt(3)*arctan(1/3*sqrt(3)*(2*(b + a/x)^(1/3) + b^(1/3))/b^(1/3)
)/b^(5/3) - 3*(b + a/x)^(1/3)*x/(a*b) + log((b + a/x)^(2/3) + (b + a/x)^(1
/3)*b^(1/3) + b^(2/3))/b^(5/3) - 2*log(abs((b + a/x)^(1/3) - b^(1/3)))/b^(
5/3))
```

Mupad [F(-1)]

Timed out.

$$\int \frac{x^2}{(ax^2+bx^3)^{2/3}}\,dx = \int \frac{x^2}{(b\,x^3+a\,x^2)^{2/3}}\,dx$$

```
int(x^2/(a*x^2 + b*x^3)^(2/3),x)
```

```
int(x^2/(a*x^2 + b*x^3)^(2/3), x)
```

Reduce [F]

$$\int \frac{x^2}{(ax^2+bx^3)^{2/3}}\,dx = \int \frac{x^{\frac{2}{3}}}{(bx+a)^{\frac{2}{3}}}dx$$

```
int(x^2/(b*x^3+a*x^2)^(2/3),x)
```

```
int(x/(x**(1/3)*(a + b*x)**(2/3)),x)
```

3.374 $\int \frac{x}{\left(ax^2+bx^3\right)^{2/3}}\,dx$

Optimal result

Integrand size = 17, antiderivative size = 181

$$\int \frac{x}{\left(ax^2+bx^3\right)^{2/3}}\,dx = -\frac{\sqrt{3}x^{4/3}(a+bx)^{2/3}\arctan\left(\frac{1}{\sqrt{3}}+\frac{2\sqrt[3]{b}\sqrt[3]{x}}{\sqrt{3}\sqrt[3]{a+bx}}\right)}{b^{2/3}\left(ax^2+bx^3\right)^{2/3}} - \frac{x^{4/3}(a+bx)^{2/3}\log(a+bx)}{2b^{2/3}\left(ax^2+bx^3\right)^{2/3}} - \frac{3x^{4/3}(a+bx)^{2/3}\log\left(1-\frac{\sqrt[3]{b}\sqrt[3]{x}}{\sqrt[3]{a+bx}}\right)}{2b^{2/3}\left(ax^2+bx^3\right)^{2/3}}$$

```
-3^(1/2)*x^(4/3)*(b*x+a)^(2/3)*arctan(1/3*3^(1/2)+2/3*b^(1/3)*x^(1/3)*3^(1
/2)/(b*x+a)^(1/3))/b^(2/3)/(b*x^3+a*x^2)^(2/3)-1/2*x^(4/3)*(b*x+a)^(2/3)*l
n(b*x+a)/b^(2/3)/(b*x^3+a*x^2)^(2/3)-3/2*x^(4/3)*(b*x+a)^(2/3)*ln(1-b^(1/3
)*x^(1/3)/(b*x+a)^(1/3))/b^(2/3)/(b*x^3+a*x^2)^(2/3)
```

Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 153, normalized size of antiderivative = 0.85

$$\int \frac{x}{\left(ax^2+bx^3\right)^{2/3}}\,dx = \frac{x^{4/3}(a+bx)^{2/3}\left(-2\sqrt{3}\arctan\left(\frac{\sqrt{3}\sqrt[3]{b}\sqrt[3]{x}}{\sqrt[3]{b}\sqrt[3]{x}+2\sqrt[3]{a+bx}}\right)-2\log\left(-\sqrt[3]{b}\sqrt[3]{x}+\sqrt[3]{a+bx}\right)\right.}{2b^{2/3}\left(x^2(a+bx)\right)^{2/3}}$$

```
Integrate[x/(a*x^2 + b*x^3)^(2/3),x]
```

```
(x^(4/3)*(a + b*x)^(2/3)*(-2*Sqrt[3]*ArcTan[(Sqrt[3]*b^(1/3)*x^(1/3))/(b^(
1/3)*x^(1/3) + 2*(a + b*x)^(1/3))] - 2*Log[-(b^(1/3)*x^(1/3)) + (a + b*x)^
(1/3)] + Log[b^(2/3)*x^(2/3) + b^(1/3)*x^(1/3)*(a + b*x)^(1/3) + (a + b*x)
^(2/3)]))/(2*b^(2/3)*(x^2*(a + b*x))^(2/3))
```

Rubi [A] (verified)

Time = 0.33 (sec) , antiderivative size = 123, normalized size of antiderivative = 0.68, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.118, Rules used = {1938, 71}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x}{\left(ax^2+bx^3\right)^{2/3}}dx$$

$$\Big\downarrow \text{1938}$$

$$\frac{x^{4/3}(a+bx)^{2/3}\int \frac{1}{\sqrt[3]{x}(a+bx)^{2/3}}dx}{\left(ax^2+bx^3\right)^{2/3}}$$

$$\Big\downarrow \text{71}$$

$$\frac{x^{4/3}(a+bx)^{2/3}\left(-\frac{\sqrt{3}\arctan\left(\frac{2\sqrt[3]{b}\sqrt[3]{x}}{\sqrt{3}\sqrt[3]{a+bx}}+\frac{1}{\sqrt{3}}\right)}{b^{2/3}}-\frac{\log(a+bx)}{2b^{2/3}}-\frac{3\log\left(\frac{\sqrt[3]{b}\sqrt[3]{x}}{\sqrt[3]{a+bx}}-1\right)}{2b^{2/3}}\right)}{\left(ax^2+bx^3\right)^{2/3}}$$

```
Int[x/(a*x^2 + b*x^3)^(2/3),x]
```

```
(x^(4/3)*(a + b*x)^(2/3)*(-((Sqrt[3]*ArcTan[1/Sqrt[3] + (2*b^(1/3)*x^(1/3)
)/(Sqrt[3]*(a + b*x)^(1/3))])/b^(2/3)) - Log[a + b*x]/(2*b^(2/3)) - (3*Log
[-1 + (b^(1/3)*x^(1/3))/(a + b*x)^(1/3)])/(2*b^(2/3))))/(a*x^2 + b*x^3)^(2
/3)
```

Defintions of rubi rules used

```
Int[1/(((a_.) + (b_.)*(x_))^(1/3)*((c_.) + (d_.)*(x_))^(2/3)), x_Symbol] :>
 With[{q = Rt[d/b, 3]}, Simp[(-Sqrt[3])*(q/d)*ArcTan[2*q*((a + b*x)^(1/3)/(
Sqrt[3]*(c + d*x)^(1/3))) + 1/Sqrt[3]], x] + (-Simp[3*(q/(2*d))*Log[q*((a +
 b*x)^(1/3)/(c + d*x)^(1/3)) - 1], x] - Simp[(q/(2*d))*Log[c + d*x], x])] /
; FreeQ[{a, b, c, d}, x] && PosQ[d/b]
```

```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> Simp[c^IntPart[m]*(c*x)^FracPart[m]*((a*x^j + b*x^n)^FracPart[p]/(x^(F
racPart[m] + j*FracPart[p])*(a + b*x^(n - j))^FracPart[p]))   Int[x^(m + j*
p)*(a + b*x^(n - j))^p, x], x] /; FreeQ[{a, b, c, j, m, n, p}, x] &&  !Inte
gerQ[p] && NeQ[n, j] && PosQ[n - j]
```

Maple [A] (verified)

Time = 0.45 (sec) , antiderivative size = 108, normalized size of antiderivative = 0.60

method	result
pseudoelliptic	$\frac{2\sqrt{3}\,\arctan\left(\frac{\sqrt{3}\left(b^{\frac{1}{3}} x+2\left(x^{2}(bx+a)\right)^{\frac{1}{3}}\right)}{3 b^{\frac{1}{3}} x}\right)-2\ln\left(\frac{-b^{\frac{1}{3}} x+\left(x^{2}(bx+a)\right)^{\frac{1}{3}}}{x}\right)+\ln\left(\frac{b^{\frac{2}{3}} x^{2}+b^{\frac{1}{3}}\left(x^{2}(bx+a)\right)^{\frac{1}{3}} x+\left(x^{2}(bx+a)\right)^{\frac{2}{3}}}{x^{2}}\right)}{2 b^{\frac{2}{3}}}$

```
int(x/(b*x^3+a*x^2)^(2/3),x,method=_RETURNVERBOSE)
```

```
1/2*(2*3^(1/2)*arctan(1/3*3^(1/2)*(b^(1/3)*x+2*(x^2*(b*x+a))^(1/3))/b^(1/3
)/x)-2*ln((-b^(1/3)*x+(x^2*(b*x+a))^(1/3))/x)+ln((b^(2/3)*x^2+b^(1/3)*(x^2
*(b*x+a))^(1/3)*x+(x^2*(b*x+a))^(2/3))/x^2))/b^(2/3)
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 157, normalized size of antiderivative = 0.87

$$\int \frac{x}{(ax^2+bx^3)^{2/3}}\,dx = \frac{2\sqrt{3}(b^2)^{\frac{1}{6}}b\arctan\left(\frac{\sqrt{3}\left((b^2)^{\frac{1}{3}}bx+2\,(bx^3+ax^2)^{\frac{1}{3}}(b^2)^{\frac{2}{3}}\right)(b^2)^{\frac{1}{6}}}{3\,b^2x}\right)-2\,(b^2)^{\frac{2}{3}}\log\left(-\frac{(b^2)^{\frac{2}{3}}x-(bx^3+}{x}\right.}{2\,b^2}$$

```
integrate(x/(b*x^3+a*x^2)^(2/3),x, algorithm="fricas")
```

```
1/2*(2*sqrt(3)*(b^2)^(1/6)*b*arctan(1/3*sqrt(3)*((b^2)^(1/3)*b*x + 2*(b*x^
3 + a*x^2)^(1/3)*(b^2)^(2/3))*(b^2)^(1/6)/(b^2*x)) - 2*(b^2)^(2/3)*log(-((
b^2)^(2/3)*x - (b*x^3 + a*x^2)^(1/3)*b)/x) + (b^2)^(2/3)*log(((b^2)^(1/3)*
b*x^2 + (b*x^3 + a*x^2)^(1/3)*(b^2)^(2/3)*x + (b*x^3 + a*x^2)^(2/3)*b)/x^2
))/b^2
```

Sympy [F]

$$\int \frac{x}{(ax^2+bx^3)^{2/3}}\,dx = \int \frac{x}{(x^2\,(a+bx))^{\frac{2}{3}}}\,dx$$

```
integrate(x/(b*x**3+a*x**2)**(2/3),x)
```

```
Integral(x/(x**2*(a + b*x))**(2/3), x)
```

Maxima [F]

$$\int \frac{x}{(ax^2+bx^3)^{2/3}}\,dx = \int \frac{x}{(bx^3+ax^2)^{\frac{2}{3}}}\,dx$$

```
integrate(x/(b*x^3+a*x^2)^(2/3),x, algorithm="maxima")
```

```
integrate(x/(b*x^3 + a*x^2)^(2/3), x)
```

Giac [A] (verification not implemented)

Time = 3.49 (sec) , antiderivative size = 86, normalized size of antiderivative = 0.48

$$\int \frac{x}{(ax^2+bx^3)^{2/3}}\,dx = \frac{\sqrt{3}\arctan\left(\frac{\sqrt{3}\left(2\left(b+\frac{a}{x}\right)^{\frac{1}{3}}+b^{\frac{1}{3}}\right)}{3\,b^{\frac{1}{3}}}\right)}{b^{\frac{2}{3}}} + \frac{\log\left(\left(b+\frac{a}{x}\right)^{\frac{2}{3}}+\left(b+\frac{a}{x}\right)^{\frac{1}{3}}b^{\frac{1}{3}}+b^{\frac{2}{3}}\right)}{2\,b^{\frac{2}{3}}} - \frac{\log\left(\left|\left(b+\frac{a}{x}\right)^{\frac{1}{3}}-b^{\frac{1}{3}}\right|\right)}{b^{\frac{2}{3}}}$$

```
integrate(x/(b*x^3+a*x^2)^(2/3),x, algorithm="giac")
```

```
sqrt(3)*arctan(1/3*sqrt(3)*(2*(b + a/x)^(1/3) + b^(1/3))/b^(1/3))/b^(2/3)
+ 1/2*log((b + a/x)^(2/3) + (b + a/x)^(1/3)*b^(1/3) + b^(2/3))/b^(2/3) - l
og(abs((b + a/x)^(1/3) - b^(1/3)))/b^(2/3)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{x}{(ax^2+bx^3)^{2/3}}\,dx = \int \frac{x}{(b\,x^3+a\,x^2)^{2/3}}\,dx$$

```
int(x/(a*x^2 + b*x^3)^(2/3),x)
```

```
int(x/(a*x^2 + b*x^3)^(2/3), x)
```

Reduce [F]

$$\int \frac{x}{(ax^2+bx^3)^{2/3}}\,dx = \int \frac{1}{x^{\frac{1}{3}}\,(bx+a)^{\frac{2}{3}}}dx$$

```
int(x/(b*x^3+a*x^2)^(2/3),x)
```

```
int(1/(x**(1/3)*(a + b*x)**(2/3)),x)
```

3.375 $\int \frac{1}{\left(ax^2+bx^3\right)^{2/3}}\, dx$

Optimal result

Integrand size = 15, antiderivative size = 23

$$\int \frac{1}{(ax^2+bx^3)^{2/3}}\, dx = -\frac{3\sqrt[3]{ax^2+bx^3}}{ax}$$

```
-3*(b*x^3+a*x^2)^(1/3)/a/x
```

Mathematica [A] (verified)

Time = 0.00 (sec) , antiderivative size = 21, normalized size of antiderivative = 0.91

$$\int \frac{1}{(ax^2+bx^3)^{2/3}}\, dx = -\frac{3\sqrt[3]{x^2(a+bx)}}{ax}$$

```
Integrate[(a*x^2 + b*x^3)^(-2/3),x]
```

```
(-3*(x^2*(a + b*x))^(1/3))/(a*x)
```

Rubi [A] (verified)

Time = 0.24 (sec) , antiderivative size = 23, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}} = 0.067$, Rules used = $\{1906\}$

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{(ax^2+bx^3)^{2/3}}\,dx$$
$$\Big\downarrow \ 1906$$
$$-\frac{3\sqrt[3]{ax^2+bx^3}}{ax}$$

```
Int[(a*x^2 + b*x^3)^(-2/3),x]
```

```
(-3*(a*x^2 + b*x^3)^(1/3))/(a*x)
```

Defintions of rubi rules used

```
Int[((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol] :> Simp[(a*x^j +
b*x^n)^(p + 1)/(b*(n - j)*(p + 1)*x^(n - 1)), x] /; FreeQ[{a, b, j, n, p},
x] &&  !IntegerQ[p] && NeQ[n, j] && EqQ[j*p - n + j + 1, 0]
```

Maple [A] (verified)

Time = 0.40 (sec) , antiderivative size = 20, normalized size of antiderivative = 0.87

method	result	size
pseudoelliptic	$-\frac{3\left(x^{2}(bx+a)\right)^{\frac{1}{3}}}{ax}$	20
trager	$-\frac{3\left(b\,x^{3}+a\,x^{2}\right)^{\frac{1}{3}}}{ax}$	22
risch	$-\frac{3x(bx+a)}{\left(x^{2}(bx+a)\right)^{\frac{2}{3}}a}$	23
gosper	$-\frac{3x(bx+a)}{a\left(b\,x^{3}+a\,x^{2}\right)^{\frac{2}{3}}}$	25
orering	$-\frac{3x(bx+a)}{a\left(b\,x^{3}+a\,x^{2}\right)^{\frac{2}{3}}}$	25

```
int(1/(b*x^3+a*x^2)^(2/3),x,method=_RETURNVERBOSE)
```

```
-3*(x^2*(b*x+a))^(1/3)/a/x
```

Fricas [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 21, normalized size of antiderivative = 0.91

$$\int \frac{1}{(ax^2+bx^3)^{2/3}}\,dx=-\frac{3\,(bx^3+ax^2)^{\frac{1}{3}}}{ax}$$

```
integrate(1/(b*x^3+a*x^2)^(2/3),x, algorithm="fricas")
```

```
-3*(b*x^3 + a*x^2)^(1/3)/(a*x)
```

Sympy [F]

$$\int \frac{1}{(ax^2 + bx^3)^{2/3}}\,dx = \int \frac{1}{(ax^2 + bx^3)^{\frac{2}{3}}}\,dx$$

```
integrate(1/(b*x**3+a*x**2)**(2/3),x)
```

```
Integral((a*x**2 + b*x**3)**(-2/3), x)
```

Maxima [F]

$$\int \frac{1}{(ax^2 + bx^3)^{2/3}}\,dx = \int \frac{1}{(bx^3 + ax^2)^{\frac{2}{3}}}\,dx$$

```
integrate(1/(b*x^3+a*x^2)^(2/3),x, algorithm="maxima")
```

```
integrate((b*x^3 + a*x^2)^(-2/3), x)
```

Giac [A] (verification not implemented)

Time = 0.14 (sec) , antiderivative size = 14, normalized size of antiderivative = 0.61

$$\int \frac{1}{(ax^2 + bx^3)^{2/3}}\,dx = -\frac{3\left(b + \frac{a}{x}\right)^{\frac{1}{3}}}{a}$$

```
integrate(1/(b*x^3+a*x^2)^(2/3),x, algorithm="giac")
```

```
-3*(b + a/x)^(1/3)/a
```

Mupad [B] (verification not implemented)

Time = 9.45 (sec) , antiderivative size = 21, normalized size of antiderivative = 0.91

$$\int \frac{1}{(ax^2 + bx^3)^{2/3}}\,dx = -\frac{3\,(b\,x^3 + a\,x^2)^{1/3}}{a\,x}$$

```
int(1/(a*x^2 + b*x^3)^(2/3),x)
```

```
-(3*(a*x^2 + b*x^3)^(1/3))/(a*x)
```

Reduce [F]

$$\int \frac{1}{(ax^2 + bx^3)^{2/3}}\,dx = \int \frac{1}{x^{\frac{4}{3}}\,(bx + a)^{\frac{2}{3}}}dx$$

```
int(1/(b*x^3+a*x^2)^(2/3),x)
```

```
int(1/(x**(1/3)*(a + b*x)**(2/3)*x),x)
```

3.376 $\int \frac{1}{x(ax^2+bx^3)^{2/3}}\,dx$

Optimal result

Integrand size = 19, antiderivative size = 52

$$\int \frac{1}{x\left(ax^2+bx^3\right)^{2/3}}\,dx=-\frac{3\sqrt[3]{ax^2+bx^3}}{4ax^2}+\frac{9b\sqrt[3]{ax^2+bx^3}}{4a^2x}$$

```
-3/4*(b*x^3+a*x^2)^(1/3)/a/x^2+9/4*b*(b*x^3+a*x^2)^(1/3)/a^2/x
```

Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 31, normalized size of antiderivative = 0.60

$$\int \frac{1}{x\left(ax^2+bx^3\right)^{2/3}}\,dx=-\frac{3(a-3bx)(a+bx)}{4a^2\left(x^2(a+bx)\right)^{2/3}}$$

```
Integrate[1/(x*(a*x^2 + b*x^3)^(2/3)),x]
```

```
(-3*(a - 3*b*x)*(a + b*x))/(4*a^2*(x^2*(a + b*x))^(2/3))
```

Rubi [A] (verified)

Time = 0.31 (sec) , antiderivative size = 52, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.105, Rules used = {1922, 1906}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{x\left(ax^2+bx^3\right)^{2/3}}dx$$

$$\Big\downarrow \text{ 1922}$$

$$-\frac{3b\int \frac{1}{(bx^3+ax^2)^{2/3}}dx}{4a}-\frac{3\sqrt[3]{ax^2+bx^3}}{4ax^2}$$

$$\Big\downarrow \text{ 1906}$$

$$\frac{9b\sqrt[3]{ax^2+bx^3}}{4a^2x}-\frac{3\sqrt[3]{ax^2+bx^3}}{4ax^2}$$

```
Int[1/(x*(a*x^2 + b*x^3)^(2/3)),x]
```

```
(-3*(a*x^2 + b*x^3)^(1/3))/(4*a*x^2) + (9*b*(a*x^2 + b*x^3)^(1/3))/(4*a^2*
x)
```

Defintions of rubi rules used

```
Int[((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol] :> Simp[(a*x^j +
b*x^n)^(p + 1)/(b*(n - j)*(p + 1)*x^(n - 1)), x] /; FreeQ[{a, b, j, n, p},
x] &&  !IntegerQ[p] && NeQ[n, j] && EqQ[j*p - n + j + 1, 0]
```

```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> Simp[c^(j - 1)*(c*x)^(m - j + 1)*((a*x^j + b*x^n)^(p + 1)/(a*(m + j*p
+ 1))), x] - Simp[b*((m + n*p + n - j + 1)/(a*c^(n - j)*(m + j*p + 1)))   I
nt[(c*x)^(m + n - j)*(a*x^j + b*x^n)^p, x], x] /; FreeQ[{a, b, c, j, m, n,
p}, x] &&  !IntegerQ[p] && NeQ[n, j] && ILtQ[Simplify[(m + n*p + n - j + 1)
/(n - j)], 0] && NeQ[m + j*p + 1, 0] && (IntegersQ[j, n] || GtQ[c, 0])
```

Maple [A] (verified)

Time = 0.41 (sec) , antiderivative size = 26, normalized size of antiderivative = 0.50

method	result	size
pseudoelliptic	$-\frac{3(-3bx+a)\left(x^{2}(bx+a)\right)^{\frac{1}{3}}}{4a^{2}x^{2}}$	26
trager	$-\frac{3(-3bx+a)\left(b\,x^{3}+a\,x^{2}\right)^{\frac{1}{3}}}{4a^{2}x^{2}}$	28
risch	$-\frac{3(bx+a)(-3bx+a)}{4\left(x^{2}(bx+a)\right)^{\frac{2}{3}}a^{2}}$	28
gosper	$-\frac{3(bx+a)(-3bx+a)}{4a^{2}\left(b\,x^{3}+a\,x^{2}\right)^{\frac{2}{3}}}$	30
orering	$-\frac{3(bx+a)(-3bx+a)}{4a^{2}\left(b\,x^{3}+a\,x^{2}\right)^{\frac{2}{3}}}$	30

```
int(1/x/(b*x^3+a*x^2)^(2/3),x,method=_RETURNVERBOSE)
```

```
-3/4*(-3*b*x+a)/a^2/x^2*(x^2*(b*x+a))^(1/3)
```

Fricas [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 29, normalized size of antiderivative = 0.56

$$\int \frac{1}{x\,(ax^2+bx^3)^{2/3}}\,dx = \frac{3\,(bx^3+ax^2)^{\frac{1}{3}}(3\,bx-a)}{4\,a^2x^2}$$

```
integrate(1/x/(b*x^3+a*x^2)^(2/3),x, algorithm="fricas")
```

```
3/4*(b*x^3 + a*x^2)^(1/3)*(3*b*x - a)/(a^2*x^2)
```

Sympy [F]

$$\int \frac{1}{x\,(ax^2+bx^3)^{2/3}}\,dx = \int \frac{1}{x\,(x^2\,(a+bx))^{\frac{2}{3}}}\,dx$$

```
integrate(1/x/(b*x**3+a*x**2)**(2/3),x)
```

```
Integral(1/(x*(x**2*(a + b*x))**(2/3)), x)
```

Maxima [F]

$$\int \frac{1}{x\,(ax^2+bx^3)^{2/3}}\,dx = \int \frac{1}{(bx^3+ax^2)^{\frac{2}{3}}x}\,dx$$

```
integrate(1/x/(b*x^3+a*x^2)^(2/3),x, algorithm="maxima")
```

```
integrate(1/((b*x^3 + a*x^2)^(2/3)*x), x)
```

Giac [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 34, normalized size of antiderivative = 0.65

$$\int \frac{1}{x\,(ax^2+bx^3)^{2/3}}\,dx = -\frac{3\left(\frac{\left(b+\frac{a}{x}\right)^{\frac{4}{3}}}{a} - \frac{4\left(b+\frac{a}{x}\right)^{\frac{1}{3}}b}{a}\right)}{4\,a}$$

```
integrate(1/x/(b*x^3+a*x^2)^(2/3),x, algorithm="giac")
```

```
-3/4*((b + a/x)^(4/3)/a - 4*(b + a/x)^(1/3)*b/a)/a
```

Mupad [B] (verification not implemented)

Time = 9.41 (sec) , antiderivative size = 42, normalized size of antiderivative = 0.81

$$\int \frac{1}{x\,(ax^2+bx^3)^{2/3}}\,dx = -\frac{3\,a\,(b\,x^3+a\,x^2)^{1/3} - 9\,b\,x\,(b\,x^3+a\,x^2)^{1/3}}{4\,a^2\,x^2}$$

```
int(1/(x*(a*x^2 + b*x^3)^(2/3)),x)
```

```
-(3*a*(a*x^2 + b*x^3)^(1/3) - 9*b*x*(a*x^2 + b*x^3)^(1/3))/(4*a^2*x^2)
```

Reduce [F]

$$\int \frac{1}{x\,(ax^2+bx^3)^{2/3}}\,dx = \int \frac{1}{x^{\frac{7}{3}}\,(bx+a)^{\frac{2}{3}}}dx$$

```
int(1/x/(b*x^3+a*x^2)^(2/3),x)
```

```
int(1/(x**(1/3)*(a + b*x)**(2/3)*x**2),x)
```

3.377 $\int \frac{1}{x^2(ax^2+bx^3)^{2/3}}\,dx$

Optimal result

Integrand size = 19, antiderivative size = 80

$$\int \frac{1}{x^2\left(ax^2+bx^3\right)^{2/3}}\,dx=-\frac{3\sqrt[3]{ax^2+bx^3}}{7ax^3}+\frac{9b\sqrt[3]{ax^2+bx^3}}{14a^2x^2}-\frac{27b^2\sqrt[3]{ax^2+bx^3}}{14a^3x}$$

```
-3/7*(b*x^3+a*x^2)^(1/3)/a/x^3+9/14*b*(b*x^3+a*x^2)^(1/3)/a^2/x^2-27/14*b^
2*(b*x^3+a*x^2)^(1/3)/a^3/x
```

Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 42, normalized size of antiderivative = 0.52

$$\int \frac{1}{x^2\left(ax^2+bx^3\right)^{2/3}}\,dx=-\frac{3\sqrt[3]{x^2(a+bx)}(2a^2-3abx+9b^2x^2)}{14a^3x^3}$$

```
Integrate[1/(x^2*(a*x^2 + b*x^3)^(2/3)),x]
```

```
(-3*(x^2*(a + b*x))^(1/3)*(2*a^2 - 3*a*b*x + 9*b^2*x^2))/(14*a^3*x^3)
```

Rubi [A] (verified)

Time = 0.38 (sec) , antiderivative size = 86, normalized size of antiderivative = 1.08, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.158, Rules used = {1922, 1922, 1906}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{x^2 (ax^2+bx^3)^{2/3}} \, dx$$

$$\Big\downarrow \text{1922}$$

$$-\frac{6b \int \frac{1}{x(bx^3+ax^2)^{2/3}}dx}{7a}-\frac{3\sqrt[3]{ax^2+bx^3}}{7ax^3}$$

$$\Big\downarrow \text{1922}$$

$$-\frac{6b\left(-\frac{3b\int \frac{1}{\left(bx^3+ax^2\right)^{2/3}}dx}{4a}-\frac{3\sqrt[3]{ax^2+bx^3}}{4ax^2}\right)}{7a}-\frac{3\sqrt[3]{ax^2+bx^3}}{7ax^3}$$

$$\Big\downarrow \text{1906}$$

$$-\frac{6b\left(\frac{9b\sqrt[3]{ax^2+bx^3}}{4a^2x}-\frac{3\sqrt[3]{ax^2+bx^3}}{4ax^2}\right)}{7a}-\frac{3\sqrt[3]{ax^2+bx^3}}{7ax^3}$$

```
Int[1/(x^2*(a*x^2 + b*x^3)^(2/3)),x]
```

```
(-3*(a*x^2 + b*x^3)^(1/3))/(7*a*x^3) - (6*b*((-3*(a*x^2 + b*x^3)^(1/3))/(4
*a*x^2) + (9*b*(a*x^2 + b*x^3)^(1/3))/(4*a^2*x)))/(7*a)
```

Defintions of rubi rules used

```
Int[((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol] :> Simp[(a*x^j +
b*x^n)^(p + 1)/(b*(n - j)*(p + 1)*x^(n - 1)), x] /; FreeQ[{a, b, j, n, p},
x] &&  !IntegerQ[p] && NeQ[n, j] && EqQ[j*p - n + j + 1, 0]
```

```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> Simp[c^(j - 1)*(c*x)^(m - j + 1)*((a*x^j + b*x^n)^(p + 1)/(a*(m + j*p
+ 1))), x] - Simp[b*((m + n*p + n - j + 1)/(a*c^(n - j)*(m + j*p + 1)))   I
nt[(c*x)^(m + n - j)*(a*x^j + b*x^n)^p, x], x] /; FreeQ[{a, b, c, j, m, n,
p}, x] &&  !IntegerQ[p] && NeQ[n, j] && ILtQ[Simplify[(m + n*p + n - j + 1)
/(n - j)], 0] && NeQ[m + j*p + 1, 0] && (IntegersQ[j, n] || GtQ[c, 0])
```

Maple [A] (verified)

Time = 0.43 (sec) , antiderivative size = 39, normalized size of antiderivative = 0.49

method	result	size
pseudoelliptic	$-\frac{3\left(9 b^{2} x^{2}-3 a b x+2 a^{2}\right) \left(x^{2} \left(b x+a \right)\right)^{\frac{1}{3}}}{14 a^{3} x^{3}}$	39
trager	$-\frac{3\left(9 b^{2} x^{2}-3 a b x+2 a^{2}\right) \left(b \,x^{3}+a \,x^{2}\right)^{\frac{1}{3}}}{14 a^{3} x^{3}}$	41
risch	$-\frac{3\left(b x+a \right) \left(9 b^{2} x^{2}-3 a b x+2 a^{2}\right)}{14 x \left(x^{2} \left(b x+a \right)\right)^{\frac{2}{3}} a^{3}}$	44
gosper	$-\frac{3\left(b x+a \right) \left(9 b^{2} x^{2}-3 a b x+2 a^{2}\right)}{14 x \,a^{3} \left(b \,x^{3}+a \,x^{2}\right)^{\frac{2}{3}}}$	46
orering	$-\frac{3\left(b x+a \right) \left(9 b^{2} x^{2}-3 a b x+2 a^{2}\right)}{14 x \,a^{3} \left(b \,x^{3}+a \,x^{2}\right)^{\frac{2}{3}}}$	46

```
int(1/x^2/(b*x^3+a*x^2)^(2/3),x,method=_RETURNVERBOSE)
```

```
-3/14*(9*b^2*x^2-3*a*b*x+2*a^2)/a^3/x^3*(x^2*(b*x+a))^(1/3)
```

Fricas [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 40, normalized size of antiderivative = 0.50

$$\int \frac{1}{x^2 (ax^2 + bx^3)^{2/3}} \, dx = -\frac{3 (9 b^2 x^2 - 3 abx + 2 a^2)(bx^3 + ax^2)^{\frac{1}{3}}}{14 a^3 x^3}$$

```
integrate(1/x^2/(b*x^3+a*x^2)^(2/3),x, algorithm="fricas")
```

```
-3/14*(9*b^2*x^2 - 3*a*b*x + 2*a^2)*(b*x^3 + a*x^2)^(1/3)/(a^3*x^3)
```

Sympy [F]

$$\int \frac{1}{x^2 (ax^2 + bx^3)^{2/3}} \, dx = \int \frac{1}{x^2 (x^2 (a + bx))^{\frac{2}{3}}} \, dx$$

```
integrate(1/x**2/(b*x**3+a*x**2)**(2/3),x)
```

```
Integral(1/(x**2*(x**2*(a + b*x))**(2/3)), x)
```

Maxima [F]

$$\int \frac{1}{x^2 (ax^2 + bx^3)^{2/3}} \, dx = \int \frac{1}{(bx^3 + ax^2)^{\frac{2}{3}} x^2} \, dx$$

```
integrate(1/x^2/(b*x^3+a*x^2)^(2/3),x, algorithm="maxima")
```

```
integrate(1/((b*x^3 + a*x^2)^(2/3)*x^2), x)
```

Giac [A] (verification not implemented)

Time = 0.40 (sec) , antiderivative size = 43, normalized size of antiderivative = 0.54

$$\int \frac{1}{x^2 (ax^2 + bx^3)^{2/3}}\,dx = -\frac{3\left(2\left(b+\frac{a}{x}\right)^{\frac{7}{3}} - 7\left(b+\frac{a}{x}\right)^{\frac{4}{3}}b + 14\left(b+\frac{a}{x}\right)^{\frac{1}{3}}b^2\right)}{14\,a^3}$$

```
integrate(1/x^2/(b*x^3+a*x^2)^(2/3),x, algorithm="giac")
```

```
-3/14*(2*(b + a/x)^(7/3) - 7*(b + a/x)^(4/3)*b + 14*(b + a/x)^(1/3)*b^2)/a
^3
```

Mupad [B] (verification not implemented)

Time = 9.55 (sec) , antiderivative size = 66, normalized size of antiderivative = 0.82

$$\int \frac{1}{x^2 (ax^2 + bx^3)^{2/3}}\,dx = -\frac{6\,a^2\,(b\,x^3+a\,x^2)^{1/3} + 27\,b^2\,x^2\,(b\,x^3+a\,x^2)^{1/3} - 9\,a\,b\,x\,(b\,x^3+a\,x^2)^{1/3}}{14\,a^3\,x^3}$$

```
int(1/(x^2*(a*x^2 + b*x^3)^(2/3)),x)
```

```
-(6*a^2*(a*x^2 + b*x^3)^(1/3) + 27*b^2*x^2*(a*x^2 + b*x^3)^(1/3) - 9*a*b*x
*(a*x^2 + b*x^3)^(1/3))/(14*a^3*x^3)
```

Reduce [F]

$$\int \frac{1}{x^2 (ax^2 + bx^3)^{2/3}}\,dx = \int \frac{1}{x^{\frac{10}{3}}\,(bx+a)^{\frac{2}{3}}}dx$$

```
int(1/x^2/(b*x^3+a*x^2)^(2/3),x)
```

```
int(1/(x**(1/3)*(a + b*x)**(2/3)*x**3),x)
```

3.378 $\int \frac{x^5}{\left(ax^2+bx^3\right)^{4/3}}\,dx$

Optimal result

Integrand size = 19, antiderivative size = 260

$$\begin{aligned}\int \frac{x^5}{\left(ax^2+bx^3\right)^{4/3}}\,dx &= -\frac{3a^2x}{b^3\sqrt[3]{ax^2+bx^3}}+\frac{\left(ax^2+bx^3\right)^{2/3}}{2b^2}\\ &\quad-\frac{5a\left(ax^2+bx^3\right)^{2/3}}{3b^3x}-\frac{14a^2x^{2/3}\sqrt[3]{a+bx}\arctan\left(\frac{1}{\sqrt{3}}+\frac{2\sqrt[3]{a+bx}}{\sqrt{3}\sqrt[3]{b}\sqrt[3]{x}}\right)}{3\sqrt{3}b^{10/3}\sqrt[3]{ax^2+bx^3}}\\ &\quad-\frac{7a^2x^{2/3}\sqrt[3]{a+bx}\log(x)}{9b^{10/3}\sqrt[3]{ax^2+bx^3}}-\frac{7a^2x^{2/3}\sqrt[3]{a+bx}\log\left(1-\frac{\sqrt[3]{a+bx}}{\sqrt[3]{b}\sqrt[3]{x}}\right)}{3b^{10/3}\sqrt[3]{ax^2+bx^3}}\end{aligned}$$

```
-3*a^2*x/b^3/(b*x^3+a*x^2)^(1/3)+1/2*(b*x^3+a*x^2)^(2/3)/b^2-5/3*a*(b*x^3+
a*x^2)^(2/3)/b^3/x-14/9*a^2*x^(2/3)*(b*x+a)^(1/3)*arctan(1/3*3^(1/2)+2/3*(
b*x+a)^(1/3)*3^(1/2)/b^(1/3)/x^(1/3))*3^(1/2)/b^(10/3)/(b*x^3+a*x^2)^(1/3)
-7/9*a^2*x^(2/3)*(b*x+a)^(1/3)*ln(x)/b^(10/3)/(b*x^3+a*x^2)^(1/3)-7/3*a^2*
x^(2/3)*(b*x+a)^(1/3)*ln(1-(b*x+a)^(1/3)/b^(1/3)/x^(1/3))/b^(10/3)/(b*x^3+
a*x^2)^(1/3)
```

Mathematica [A] (verified)

Time = 0.58 (sec) , antiderivative size = 222, normalized size of antiderivative = 0.85

$$\int \frac{x^5}{(ax^2+bx^3)^{4/3}}\,dx = \frac{x^{2/3}\left(-84a^2\sqrt[3]{b}\sqrt[3]{x}-21ab^{4/3}x^{4/3}+9b^{7/3}x^{7/3}+28\sqrt{3}a^2\sqrt[3]{a+bx}\arctan\left(\frac{\sqrt{3}\sqrt[3]{b}\sqrt[3]{x}}{\sqrt[3]{b}\sqrt[3]{x}+2\sqrt[3]{a+bx}}\right)\right.}{}$$

```
Integrate[x^5/(a*x^2 + b*x^3)^(4/3),x]
```

```
(x^(2/3)*(-84*a^2*b^(1/3)*x^(1/3) - 21*a*b^(4/3)*x^(4/3) + 9*b^(7/3)*x^(7/
3) + 28*Sqrt[3]*a^2*(a + b*x)^(1/3)*ArcTan[(Sqrt[3]*b^(1/3)*x^(1/3))/(b^(1
/3)*x^(1/3) + 2*(a + b*x)^(1/3))] - 28*a^2*(a + b*x)^(1/3)*Log[-(b^(1/3)*x
^(1/3)) + (a + b*x)^(1/3)] + 14*a^2*(a + b*x)^(1/3)*Log[b^(2/3)*x^(2/3) +
b^(1/3)*x^(1/3)*(a + b*x)^(1/3) + (a + b*x)^(2/3)]))/(18*b^(10/3)*(x^2*(a
+ b*x))^(1/3))
```

Rubi [A] (verified)

Time = 0.55 (sec) , antiderivative size = 209, normalized size of antiderivative = 0.80, number of steps used = 5, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.263$, Rules used = {1928, 1930, 1930, 1917, 71}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^5}{(ax^2+bx^3)^{4/3}}\,dx$$

$$\Big\downarrow \text{ 1928}$$

$$\frac{7\int \frac{x^2}{\sqrt[3]{bx^3+ax^2}}dx}{b}-\frac{3x^3}{b\sqrt[3]{ax^2+bx^3}}$$

$$\Big\downarrow \text{ 1930}$$

$$\frac{7\left(\frac{(ax^2+bx^3)^{2/3}}{2b}-\frac{2a\int\frac{x}{\sqrt[3]{bx^3+ax^2}}dx}{3b}\right)}{b}-\frac{3x^3}{b\sqrt[3]{ax^2+bx^3}}$$

$$\Big\downarrow \text{1930}$$

$$\frac{7\left(\frac{(ax^2+bx^3)^{2/3}}{2b}-\frac{2a\left(\frac{(ax^2+bx^3)^{2/3}}{bx}-\frac{a\int\frac{1}{\sqrt[3]{bx^3+ax^2}}dx}{3b}\right)}{3b}\right)}{b}-\frac{3x^3}{b\sqrt[3]{ax^2+bx^3}}$$

$$\Big\downarrow \text{1917}$$

$$\frac{7\left(\frac{(ax^2+bx^3)^{2/3}}{2b}-\frac{2a\left(\frac{(ax^2+bx^3)^{2/3}}{bx}-\frac{ax^{2/3}\sqrt[3]{a+bx}\int\frac{1}{x^{2/3}\sqrt[3]{a+bx}}dx}{3b\sqrt[3]{ax^2+bx^3}}\right)}{3b}\right)}{b}-\frac{3x^3}{b\sqrt[3]{ax^2+bx^3}}$$

$$\Big\downarrow \text{71}$$

$$\frac{7\left(\frac{(ax^2+bx^3)^{2/3}}{2b}-\frac{2a\left(\frac{(ax^2+bx^3)^{2/3}}{bx}-\frac{ax^{2/3}\sqrt[3]{a+bx}\left(-\frac{\sqrt{3}\arctan\left(\frac{2\sqrt[3]{a+bx}}{\sqrt{3}\sqrt[3]{b}\sqrt[3]{x}}+\frac{1}{\sqrt{3}}\right)}{\sqrt[3]{b}}-\frac{3\log\left(\frac{\sqrt[3]{a+bx}}{\sqrt[3]{b}\sqrt[3]{x}}-1\right)}{2\sqrt[3]{b}}-\frac{\log(x)}{2\sqrt[3]{b}}\right)}{3b\sqrt[3]{ax^2+bx^3}}\right)}{3b}\right)}{b}-\frac{3x^3}{b\sqrt[3]{ax^2+bx^3}}$$

```
Int[x^5/(a*x^2 + b*x^3)^(4/3),x]
```

```
(-3*x^3)/(b*(a*x^2 + b*x^3)^(1/3)) + (7*((a*x^2 + b*x^3)^(2/3)/(2*b) - (2*
a*((a*x^2 + b*x^3)^(2/3)/(b*x) - (a*x^(2/3)*(a + b*x)^(1/3)*(-((Sqrt[3]*Ar
cTan[1/Sqrt[3] + (2*(a + b*x)^(1/3))/(Sqrt[3]*b^(1/3)*x^(1/3))])/b^(1/3))
- Log[x]/(2*b^(1/3)) - (3*Log[-1 + (a + b*x)^(1/3)/(b^(1/3)*x^(1/3))])/(2*
b^(1/3))))/(3*b*(a*x^2 + b*x^3)^(1/3))))/(3*b)))/b
```

Defintions of rubi rules used

```
Int[1/(((a_.) + (b_.)*(x_))^(1/3)*((c_.) + (d_.)*(x_))^(2/3)), x_Symbol] :>
 With[{q = Rt[d/b, 3]}, Simp[(-Sqrt[3])*(q/d)*ArcTan[2*q*((a + b*x)^(1/3)/(
Sqrt[3]*(c + d*x)^(1/3))) + 1/Sqrt[3]], x] + (-Simp[3*(q/(2*d))*Log[q*((a +
 b*x)^(1/3)/(c + d*x)^(1/3)) - 1], x] - Simp[(q/(2*d))*Log[c + d*x], x])] /
; FreeQ[{a, b, c, d}, x] && PosQ[d/b]
```

```
Int[((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol] :> Simp[(a*x^j +
b*x^n)^FracPart[p]/(x^(j*FracPart[p])*(a + b*x^(n - j))^FracPart[p])   Int[
x^(j*p)*(a + b*x^(n - j))^p, x], x] /; FreeQ[{a, b, j, n, p}, x] &&  !Integ
erQ[p] && NeQ[n, j] && PosQ[n - j]
```

```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> Simp[c^(n - 1)*(c*x)^(m - n + 1)*((a*x^j + b*x^n)^(p + 1)/(b*(n - j)*(
p + 1))), x] - Simp[c^n*((m + j*p - n + j + 1)/(b*(n - j)*(p + 1)))   Int[(
c*x)^(m - n)*(a*x^j + b*x^n)^(p + 1), x], x] /; FreeQ[{a, b, c}, x] &&  !In
tegerQ[p] && LtQ[0, j, n] && (IntegersQ[j, n] || GtQ[c, 0]) && LtQ[p, -1] &
& GtQ[m + j*p + 1, n - j]
```

```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> Simp[c^(n - 1)*(c*x)^(m - n + 1)*((a*x^j + b*x^n)^(p + 1)/(b*(m + n*p
+ 1))), x] - Simp[a*c^(n - j)*((m + j*p - n + j + 1)/(b*(m + n*p + 1)))   I
nt[(c*x)^(m - (n - j))*(a*x^j + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, p},
x] &&  !IntegerQ[p] && LtQ[0, j, n] && (IntegersQ[j, n] || GtQ[c, 0]) && Gt
Q[m + j*p - n + j + 1, 0] && NeQ[m + n*p + 1, 0]
```

Maple [A] (verified)

Time = 0.75 (sec) , antiderivative size = 187, normalized size of antiderivative = 0.72

method	result
pseudoelliptic	$-\frac{14\left(\sqrt{3}\,\arctan\left(\frac{\sqrt{3}\left(b^{\frac{1}{3}}x+2\left(x^{2}(bx+a)\right)^{\frac{1}{3}}\right)}{3b^{\frac{1}{3}}x}\right)a^{2}\left(x^{2}(bx+a)\right)^{\frac{1}{3}}+\ln\left(\frac{-b^{\frac{1}{3}}x+\left(x^{2}(bx+a)\right)^{\frac{1}{3}}}{x}\right)a^{2}\left(x^{2}(bx+a)\right)^{\frac{1}{3}}-\frac{\ln\left(\frac{b^{\frac{2}{3}}x^{2}+b\cdots}{\cdots}\right)\cdots}{\cdots}\cdots}{9\left(x^{2}(bx+a)\right)^{\frac{1}{3}}b^{\frac{10}{3}}}$

```
int(x^5/(b*x^3+a*x^2)^(4/3),x,method=_RETURNVERBOSE)
```

```
-14/9/(x^2*(b*x+a))^(1/3)*(3^(1/2)*arctan(1/3*3^(1/2)*(b^(1/3)*x+2*(x^2*(b
*x+a))^(1/3))/b^(1/3)/x)*a^2*(x^2*(b*x+a))^(1/3)+ln((-b^(1/3)*x+(x^2*(b*x+
a))^(1/3))/x)*a^2*(x^2*(b*x+a))^(1/3)-1/2*ln((b^(2/3)*x^2+b^(1/3)*(x^2*(b*
x+a))^(1/3)*x+(x^2*(b*x+a))^(2/3))/x^2)*a^2*(x^2*(b*x+a))^(1/3)+3*a^2*x*b^
(1/3)+3/4*a*b^(4/3)*x^2-9/28*b^(7/3)*x^3)/b^(10/3)
```

Fricas [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 506, normalized size of antiderivative = 1.95

$$\int \frac{x^5}{(ax^2+bx^3)^{4/3}}\,dx = \left[\frac{42\sqrt{\frac{1}{3}}(a^2b^2x^2+a^3bx)\sqrt{-\frac{1}{b^{\frac{2}{3}}}}\log\left(\frac{3\,bx^2+2\,ax-3\,(bx^3+ax^2)^{\frac{1}{3}}b^{\frac{2}{3}}x+3\sqrt{\frac{1}{3}}\left(b^{\frac{4}{3}}x^2+(bx^3+ax^2)^{\frac{1}{3}}bx\cdots\right.}{x}\cdots\right.}{\cdots}\right.$$

$$-\frac{28\,(a^2bx^2+a^3x)b^{\frac{2}{3}}\log\left(-\frac{b^{\frac{1}{3}}x-(bx^3+ax^2)^{\frac{1}{3}}}{x}\right)-14\,(a^2bx^2+a^3x)b^{\frac{2}{3}}\log\left(\frac{b^{\frac{2}{3}}x^2+(bx^3+ax^2)^{\frac{1}{3}}b^{\frac{1}{3}}x+(bx^3+ax^2)^{\frac{2}{3}}}{x^2}\right)+\cdots}{18\,(b^5x^2+ab^4x)\cdots}$$

```
integrate(x^5/(b*x^3+a*x^2)^(4/3),x, algorithm="fricas")
```

```
[1/18*(42*sqrt(1/3)*(a^2*b^2*x^2 + a^3*b*x)*sqrt(-1/b^(2/3))*log((3*b*x^2
+ 2*a*x - 3*(b*x^3 + a*x^2)^(1/3)*b^(2/3)*x + 3*sqrt(1/3)*(b^(4/3)*x^2 + (
b*x^3 + a*x^2)^(1/3)*b*x - 2*(b*x^3 + a*x^2)^(2/3)*b^(2/3))*sqrt(-1/b^(2/3
)))/x) - 28*(a^2*b*x^2 + a^3*x)*b^(2/3)*log(-(b^(1/3)*x - (b*x^3 + a*x^2)^
(1/3))/x) + 14*(a^2*b*x^2 + a^3*x)*b^(2/3)*log((b^(2/3)*x^2 + (b*x^3 + a*x
^2)^(1/3)*b^(1/3)*x + (b*x^3 + a*x^2)^(2/3))/x^2) + 3*(3*b^3*x^2 - 7*a*b^2
*x - 28*a^2*b)*(b*x^3 + a*x^2)^(2/3))/(b^5*x^2 + a*b^4*x), -1/18*(28*(a^2*
b*x^2 + a^3*x)*b^(2/3)*log(-(b^(1/3)*x - (b*x^3 + a*x^2)^(1/3))/x) - 14*(a
^2*b*x^2 + a^3*x)*b^(2/3)*log((b^(2/3)*x^2 + (b*x^3 + a*x^2)^(1/3)*b^(1/3)
*x + (b*x^3 + a*x^2)^(2/3))/x^2) + 84*sqrt(1/3)*(a^2*b^2*x^2 + a^3*b*x)*ar
ctan(sqrt(1/3)*(b^(1/3)*x + 2*(b*x^3 + a*x^2)^(1/3))/(b^(1/3)*x))/b^(1/3)
- 3*(3*b^3*x^2 - 7*a*b^2*x - 28*a^2*b)*(b*x^3 + a*x^2)^(2/3))/(b^5*x^2 + a
*b^4*x)]
```

Sympy [F]

$$\int \frac{x^5}{(ax^2+bx^3)^{4/3}}\,dx = \int \frac{x^5}{\left(x^2\,(a+bx)\right)^{\frac{4}{3}}}\,dx$$

```
integrate(x**5/(b*x**3+a*x**2)**(4/3),x)
```

```
Integral(x**5/(x**2*(a + b*x))**(4/3), x)
```

Maxima [F]

$$\int \frac{x^5}{(ax^2+bx^3)^{4/3}}\,dx = \int \frac{x^5}{\left(bx^3+ax^2\right)^{\frac{4}{3}}}\,dx$$

```
integrate(x^5/(b*x^3+a*x^2)^(4/3),x, algorithm="maxima")
```

```
integrate(x^5/(b*x^3 + a*x^2)^(4/3), x)
```

Giac [A] (verification not implemented)

Time = 7.32 (sec) , antiderivative size = 154, normalized size of antiderivative = 0.59

$$\int \frac{x^5}{(ax^2+bx^3)^{4/3}}\,dx = -\frac{14\,\sqrt{3}a^2\arctan\left(\frac{\sqrt{3}\left(2\,\left(b+\frac{a}{x}\right)^{\frac{1}{3}}+b^{\frac{1}{3}}\right)}{3\,b^{\frac{1}{3}}}\right)}{9\,b^{\frac{10}{3}}} + \frac{7\,a^2\log\left(\left(b+\frac{a}{x}\right)^{\frac{2}{3}}+\left(b+\frac{a}{x}\right)^{\frac{1}{3}}b^{\frac{1}{3}}+b^{\frac{2}{3}}\right)}{9\,b^{\frac{10}{3}}} - \frac{14\,a^2\log\left(\left|\left(b+\frac{a}{x}\right)^{\frac{1}{3}}-b^{\frac{1}{3}}\right|\right)}{9\,b^{\frac{10}{3}}} - \frac{3\,a^2}{\left(b+\frac{a}{x}\right)^{\frac{1}{3}}b^3} - \frac{\left(10\,a^2\left(b+\frac{a}{x}\right)^{\frac{5}{3}}-13\,a^2\left(b+\frac{a}{x}\right)^{\frac{2}{3}}b\right)x^2}{6\,a^2b^3}$$

```
integrate(x^5/(b*x^3+a*x^2)^(4/3),x, algorithm="giac")
```

```
-14/9*sqrt(3)*a^2*arctan(1/3*sqrt(3)*(2*(b + a/x)^(1/3) + b^(1/3))/b^(1/3)
)/b^(10/3) + 7/9*a^2*log((b + a/x)^(2/3) + (b + a/x)^(1/3)*b^(1/3) + b^(2/
3))/b^(10/3) - 14/9*a^2*log(abs((b + a/x)^(1/3) - b^(1/3)))/b^(10/3) - 3*a
^2/((b + a/x)^(1/3)*b^3) - 1/6*(10*a^2*(b + a/x)^(5/3) - 13*a^2*(b + a/x)^
(2/3)*b)*x^2/(a^2*b^3)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{x^5}{(ax^2+bx^3)^{4/3}}\,dx = \int \frac{x^5}{\left(b\,x^3+a\,x^2\right)^{4/3}}\,dx$$

```
int(x^5/(a*x^2 + b*x^3)^(4/3),x)
```

```
int(x^5/(a*x^2 + b*x^3)^(4/3), x)
```

Reduce [F]

$$\int \frac{x^5}{(ax^2+bx^3)^{4/3}}\,dx = \int \frac{x^3}{x^{\frac{2}{3}}\,(bx+a)^{\frac{1}{3}}\,a + x^{\frac{5}{3}}\,(bx+a)^{\frac{1}{3}}\,b}dx$$

```
int(x^5/(b*x^3+a*x^2)^(4/3),x)
```

```
int(x**3/(x**(2/3)*(a + b*x)**(1/3)*a + x**(2/3)*(a + b*x)**(1/3)*b*x),x)
```

3.379 $\int \frac{x^4}{\left(ax^2+bx^3\right)^{4/3}}\,dx$

Optimal result

Integrand size = 19, antiderivative size = 222

$$
\begin{aligned}
\int \frac{x^4}{(ax^2+bx^3)^{4/3}}\,dx &= \frac{3ax}{b^2\sqrt[3]{ax^2+bx^3}}+\frac{(ax^2+bx^3)^{2/3}}{b^2x} \\
&+\frac{4ax^{2/3}\sqrt[3]{a+bx}\arctan\left(\frac{1}{\sqrt{3}}+\frac{2\sqrt[3]{a+bx}}{\sqrt{3}\sqrt[3]{b}\sqrt[3]{x}}\right)}{\sqrt{3}b^{7/3}\sqrt[3]{ax^2+bx^3}} \\
&+\frac{2ax^{2/3}\sqrt[3]{a+bx}\log(x)}{3b^{7/3}\sqrt[3]{ax^2+bx^3}}+\frac{2ax^{2/3}\sqrt[3]{a+bx}\log\left(1-\frac{\sqrt[3]{a+bx}}{\sqrt[3]{b}\sqrt[3]{x}}\right)}{b^{7/3}\sqrt[3]{ax^2+bx^3}}
\end{aligned}
$$

```
3*a*x/b^2/(b*x^3+a*x^2)^(1/3)+(b*x^3+a*x^2)^(2/3)/b^2/x+4/3*a*x^(2/3)*(b*x
+a)^(1/3)*arctan(1/3*3^(1/2)+2/3*(b*x+a)^(1/3)*3^(1/2)/b^(1/3)/x^(1/3))*3^
(1/2)/b^(7/3)/(b*x^3+a*x^2)^(1/3)+2/3*a*x^(2/3)*(b*x+a)^(1/3)*ln(x)/b^(7/3
)/(b*x^3+a*x^2)^(1/3)+2*a*x^(2/3)*(b*x+a)^(1/3)*ln(1-(b*x+a)^(1/3)/b^(1/3)
/x^(1/3))/b^(7/3)/(b*x^3+a*x^2)^(1/3)
```

Mathematica [A] (verified)

Time = 0.40 (sec) , antiderivative size = 201, normalized size of antiderivative = 0.91

$$\int \frac{x^4}{(ax^2+bx^3)^{4/3}}\,dx = \frac{x^{2/3}\left(12a\sqrt[3]{b}\sqrt[3]{x}+3b^{4/3}x^{4/3}-4\sqrt{3}a\sqrt[3]{a+bx}\arctan\left(\frac{\sqrt{3}\sqrt[3]{b}\sqrt[3]{x}}{\sqrt[3]{b}\sqrt[3]{x}+2\sqrt[3]{a+bx}}\right)+4a\sqrt[3]{a}\right.}{3b^{7/}}$$

```
Integrate[x^4/(a*x^2 + b*x^3)^(4/3),x]
```

```
(x^(2/3)*(12*a*b^(1/3)*x^(1/3) + 3*b^(4/3)*x^(4/3) - 4*Sqrt[3]*a*(a + b*x)
^(1/3)*ArcTan[(Sqrt[3]*b^(1/3)*x^(1/3))/(b^(1/3)*x^(1/3) + 2*(a + b*x)^(1/
3))] + 4*a*(a + b*x)^(1/3)*Log[-(b^(1/3)*x^(1/3)) + (a + b*x)^(1/3)] - 2*a
*(a + b*x)^(1/3)*Log[b^(2/3)*x^(2/3) + b^(1/3)*x^(1/3)*(a + b*x)^(1/3) + (
a + b*x)^(2/3)]))/(3*b^(7/3)*(x^2*(a + b*x))^(1/3))
```

Rubi [A] (verified)

Time = 0.47 (sec) , antiderivative size = 178, normalized size of antiderivative = 0.80, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.211$, Rules used = {1928, 1930, 1917, 71}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^4}{(ax^2+bx^3)^{4/3}}dx$$

$$\Big\downarrow \text{1928}$$

$$\frac{4\int \frac{x}{\sqrt[3]{bx^3+ax^2}}dx}{b}-\frac{3x^2}{b\sqrt[3]{ax^2+bx^3}}$$

$$\Big\downarrow \text{1930}$$

$$\frac{4\left(\frac{(ax^2+bx^3)^{2/3}}{bx}-\frac{a\int \frac{1}{\sqrt[3]{bx^3+ax^2}}dx}{3b}\right)}{b}-\frac{3x^2}{b\sqrt[3]{ax^2+bx^3}}$$

$$\Big\downarrow \text{1917}$$

$$\frac{4\left(\frac{(ax^2+bx^3)^{2/3}}{bx}-\frac{ax^{2/3}\sqrt[3]{a+bx}\int\frac{1}{x^{2/3}\sqrt[3]{a+bx}}dx}{3b\sqrt[3]{ax^2+bx^3}}\right)}{b}-\frac{3x^2}{b\sqrt[3]{ax^2+bx^3}}$$

$$\Big\downarrow \text{71}$$

$$\frac{4\left(\frac{(ax^2+bx^3)^{2/3}}{bx}-\frac{ax^{2/3}\sqrt[3]{a+bx}\left(-\frac{\sqrt{3}\arctan\left(\frac{2\sqrt[3]{a+bx}}{\sqrt{3}\sqrt[3]{b}\sqrt[3]{x}}+\frac{1}{\sqrt{3}}\right)}{\sqrt[3]{b}}-\frac{3\log\left(\frac{\sqrt[3]{a+bx}}{\sqrt[3]{b}\sqrt[3]{x}}-1\right)}{2\sqrt[3]{b}}-\frac{\log(x)}{2\sqrt[3]{b}}\right)}{3b\sqrt[3]{ax^2+bx^3}}\right)}{b}-\frac{3x^2}{b\sqrt[3]{ax^2+bx^3}}$$

```
Int[x^4/(a*x^2 + b*x^3)^(4/3),x]
```

```
(-3*x^2)/(b*(a*x^2 + b*x^3)^(1/3)) + (4*((a*x^2 + b*x^3)^(2/3)/(b*x) - (a*
x^(2/3)*(a + b*x)^(1/3)*(-((Sqrt[3]*ArcTan[1/Sqrt[3] + (2*(a + b*x)^(1/3))
/(Sqrt[3]*b^(1/3)*x^(1/3))])/b^(1/3)) - Log[x]/(2*b^(1/3)) - (3*Log[-1 + (
a + b*x)^(1/3)/(b^(1/3)*x^(1/3))])/(2*b^(1/3))))/(3*b*(a*x^2 + b*x^3)^(1/3
))))/b
```

Defintions of rubi rules used

```
Int[1/(((a_.) + (b_.)*(x_))^(1/3)*((c_.) + (d_.)*(x_))^(2/3)), x_Symbol] :>
 With[{q = Rt[d/b, 3]}, Simp[(-Sqrt[3])*(q/d)*ArcTan[2*q*((a + b*x)^(1/3)/(
Sqrt[3]*(c + d*x)^(1/3))) + 1/Sqrt[3]], x] + (-Simp[3*(q/(2*d))*Log[q*((a +
 b*x)^(1/3)/(c + d*x)^(1/3)) - 1], x] - Simp[(q/(2*d))*Log[c + d*x], x])] /
; FreeQ[{a, b, c, d}, x] && PosQ[d/b]
```

```
Int[((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol] :> Simp[(a*x^j +
b*x^n)^FracPart[p]/(x^(j*FracPart[p])*(a + b*x^(n - j))^FracPart[p])   Int[
x^(j*p)*(a + b*x^(n - j))^p, x], x] /; FreeQ[{a, b, j, n, p}, x] &&  !Integ
erQ[p] && NeQ[n, j] && PosQ[n - j]
```

```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> Simp[c^(n - 1)*(c*x)^(m - n + 1)*((a*x^j + b*x^n)^(p + 1)/(b*(n - j)*(
p + 1))), x] - Simp[c^n*((m + j*p - n + j + 1)/(b*(n - j)*(p + 1)))   Int[(
c*x)^(m - n)*(a*x^j + b*x^n)^(p + 1), x], x] /; FreeQ[{a, b, c}, x] &&  !In
tegerQ[p] && LtQ[0, j, n] && (IntegersQ[j, n] || GtQ[c, 0]) && LtQ[p, -1] &
& GtQ[m + j*p + 1, n - j]
```

```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> Simp[c^(n - 1)*(c*x)^(m - n + 1)*((a*x^j + b*x^n)^(p + 1)/(b*(m + n*p
+ 1))), x] - Simp[a*c^(n - j)*((m + j*p - n + j + 1)/(b*(m + n*p + 1)))   I
nt[(c*x)^(m - (n - j))*(a*x^j + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, p},
x] &&  !IntegerQ[p] && LtQ[0, j, n] && (IntegersQ[j, n] || GtQ[c, 0]) && Gt
Q[m + j*p - n + j + 1, 0] && NeQ[m + n*p + 1, 0]
```

Maple [A] (verified)

Time = 0.75 (sec) , antiderivative size = 170, normalized size of antiderivative = 0.77

method	result
pseudoelliptic	$\frac{\frac{4\arctan\left(\frac{\sqrt{3}\left(b^{\frac{1}{3}}x+2\left(x^{2}(bx+a)\right)^{\frac{1}{3}}\right)}{3b^{\frac{1}{3}}x}\right)\sqrt{3}\,a\left(x^{2}(bx+a)\right)^{\frac{1}{3}}}{3}+\frac{4\ln\left(\frac{-b^{\frac{1}{3}}x+\left(x^{2}(bx+a)\right)^{\frac{1}{3}}}{x}\right)a\left(x^{2}(bx+a)\right)^{\frac{1}{3}}}{3}-\frac{2\ln\left(\frac{b^{\frac{2}{3}}x^{2}+b^{\frac{1}{3}}\left(x^{2}(bx+\cdots\right.}{\cdots}\right.}{\cdots}}{b^{\frac{7}{3}}\left(x^{2}(bx+a)\right)^{\frac{1}{3}}}$

```
int(x^4/(b*x^3+a*x^2)^(4/3),x,method=_RETURNVERBOSE)
```

```
4/3*(arctan(1/3*3^(1/2)*(b^(1/3)*x+2*(x^2*(b*x+a))^(1/3))/b^(1/3)/x)*3^(1/
2)*a*(x^2*(b*x+a))^(1/3)+ln((-b^(1/3)*x+(x^2*(b*x+a))^(1/3))/x)*a*(x^2*(b*
x+a))^(1/3)-1/2*ln((b^(2/3)*x^2+b^(1/3)*(x^2*(b*x+a))^(1/3)*x+(x^2*(b*x+a)
)^(2/3))/x^2)*a*(x^2*(b*x+a))^(1/3)+3*a*x*b^(1/3)+3/4*b^(4/3)*x^2)/b^(7/3)
/(x^2*(b*x+a))^(1/3)
```

Fricas [A] (verification not implemented)

Time = 0.10 (sec) , antiderivative size = 470, normalized size of antiderivative = 2.12

$$\int \frac{x^4}{(ax^2+bx^3)^{4/3}}\,dx = \left[\frac{6\sqrt{\frac{1}{3}}(ab^2x^2+a^2bx)\sqrt{-\frac{1}{b^{\frac{2}{3}}}}\log\left(\frac{3\,bx^2+2\,ax-3\,(bx^3+ax^2)^{\frac{1}{3}}b^{\frac{2}{3}}x-3\sqrt{\frac{1}{3}}\left(b^{\frac{4}{3}}x^2+(bx^3+ax^2)^{\frac{1}{3}}bx-2\right.}{x}\right.}{}\right.$$

```
integrate(x^4/(b*x^3+a*x^2)^(4/3),x, algorithm="fricas")
```

```
[1/3*(6*sqrt(1/3)*(a*b^2*x^2 + a^2*b*x)*sqrt(-1/b^(2/3))*log((3*b*x^2 + 2*
a*x - 3*(b*x^3 + a*x^2)^(1/3)*b^(2/3)*x - 3*sqrt(1/3)*(b^(4/3)*x^2 + (b*x^
3 + a*x^2)^(1/3)*b*x - 2*(b*x^3 + a*x^2)^(2/3)*b^(2/3))*sqrt(-1/b^(2/3)))/
x) + 4*(a*b*x^2 + a^2*x)*b^(2/3)*log(-(b^(1/3)*x - (b*x^3 + a*x^2)^(1/3))/
x) - 2*(a*b*x^2 + a^2*x)*b^(2/3)*log((b^(2/3)*x^2 + (b*x^3 + a*x^2)^(1/3)*
b^(1/3)*x + (b*x^3 + a*x^2)^(2/3))/x^2) + 3*(b*x^3 + a*x^2)^(2/3)*(b^2*x +
 4*a*b))/(b^4*x^2 + a*b^3*x), 1/3*(4*(a*b*x^2 + a^2*x)*b^(2/3)*log(-(b^(1/
3)*x - (b*x^3 + a*x^2)^(1/3))/x) - 2*(a*b*x^2 + a^2*x)*b^(2/3)*log((b^(2/3
)*x^2 + (b*x^3 + a*x^2)^(1/3)*b^(1/3)*x + (b*x^3 + a*x^2)^(2/3))/x^2) + 12
*sqrt(1/3)*(a*b^2*x^2 + a^2*b*x)*arctan(sqrt(1/3)*(b^(1/3)*x + 2*(b*x^3 +
a*x^2)^(1/3))/(b^(1/3)*x))/b^(1/3) + 3*(b*x^3 + a*x^2)^(2/3)*(b^2*x + 4*a*
b))/(b^4*x^2 + a*b^3*x)]
```

Sympy [F]

$$\int \frac{x^4}{(ax^2+bx^3)^{4/3}}\,dx = \int \frac{x^4}{(x^2(a+bx))^{\frac{4}{3}}}\,dx$$

```
integrate(x**4/(b*x**3+a*x**2)**(4/3),x)
```

```
Integral(x**4/(x**2*(a + b*x))**(4/3), x)
```

Maxima [F]

$$\int \frac{x^4}{(ax^2+bx^3)^{4/3}}\,dx = \int \frac{x^4}{(bx^3+ax^2)^{\frac{4}{3}}}\,dx$$

```
integrate(x^4/(b*x^3+a*x^2)^(4/3),x, algorithm="maxima")
```

```
integrate(x^4/(b*x^3 + a*x^2)^(4/3), x)
```

Giac [A] (verification not implemented)

Time = 7.41 (sec) , antiderivative size = 133, normalized size of antiderivative = 0.60

$$\int \frac{x^4}{(ax^2+bx^3)^{4/3}}\,dx = \frac{4\sqrt{3}a\arctan\left(\frac{\sqrt{3}\left(2\left(b+\frac{a}{x}\right)^{\frac{1}{3}}+b^{\frac{1}{3}}\right)}{3b^{\frac{1}{3}}}\right)}{3b^{\frac{7}{3}}} - \frac{2a\log\left(\left(b+\frac{a}{x}\right)^{\frac{2}{3}}+\left(b+\frac{a}{x}\right)^{\frac{1}{3}}b^{\frac{1}{3}}+b^{\frac{2}{3}}\right)}{3b^{\frac{7}{3}}} + \frac{4a\log\left(\left|\left(b+\frac{a}{x}\right)^{\frac{1}{3}}-b^{\frac{1}{3}}\right|\right)}{3b^{\frac{7}{3}}} + \frac{4a\left(b+\frac{a}{x}\right)-3ab}{\left(\left(b+\frac{a}{x}\right)^{\frac{4}{3}}-\left(b+\frac{a}{x}\right)^{\frac{1}{3}}b\right)b^2}$$

```
integrate(x^4/(b*x^3+a*x^2)^(4/3),x, algorithm="giac")
```

```
4/3*sqrt(3)*a*arctan(1/3*sqrt(3)*(2*(b + a/x)^(1/3) + b^(1/3))/b^(1/3))/b^
(7/3) - 2/3*a*log((b + a/x)^(2/3) + (b + a/x)^(1/3)*b^(1/3) + b^(2/3))/b^(
7/3) + 4/3*a*log(abs((b + a/x)^(1/3) - b^(1/3)))/b^(7/3) + (4*a*(b + a/x)
- 3*a*b)/(((b + a/x)^(4/3) - (b + a/x)^(1/3)*b)*b^2)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{x^4}{(ax^2+bx^3)^{4/3}}\,dx=\int \frac{x^4}{\left(b\,x^3+a\,x^2\right)^{4/3}}\,dx$$

```
int(x^4/(a*x^2 + b*x^3)^(4/3),x)
```

```
int(x^4/(a*x^2 + b*x^3)^(4/3), x)
```

Reduce [F]

$$\int \frac{x^4}{(ax^2+bx^3)^{4/3}}\,dx=\int \frac{x^2}{x^{\frac{2}{3}}\,(bx+a)^{\frac{1}{3}}\,a+x^{\frac{5}{3}}\,(bx+a)^{\frac{1}{3}}\,b}dx$$

```
int(x^4/(b*x^3+a*x^2)^(4/3),x)
```

```
int(x**2/(x**(2/3)*(a + b*x)**(1/3)*a + x**(2/3)*(a + b*x)**(1/3)*b*x),x)
```

3.380 $\int \frac{x^3}{\left(ax^2+bx^3\right)^{4/3}}\,dx$

Optimal result

Integrand size = 19, antiderivative size = 198

$$\int \frac{x^3}{(ax^2+bx^3)^{4/3}}\,dx = -\frac{3x}{b\sqrt[3]{ax^2+bx^3}} - \frac{\sqrt{3}x^{2/3}\sqrt[3]{a+bx}\arctan\left(\frac{1}{\sqrt{3}}+\frac{2\sqrt[3]{a+bx}}{\sqrt{3}\sqrt[3]{b}\sqrt[3]{x}}\right)}{b^{4/3}\sqrt[3]{ax^2+bx^3}}$$
$$-\frac{x^{2/3}\sqrt[3]{a+bx}\log(x)}{2b^{4/3}\sqrt[3]{ax^2+bx^3}} - \frac{3x^{2/3}\sqrt[3]{a+bx}\log\left(1-\frac{\sqrt[3]{a+bx}}{\sqrt[3]{b}\sqrt[3]{x}}\right)}{2b^{4/3}\sqrt[3]{ax^2+bx^3}}$$

```
-3*x/b/(b*x^3+a*x^2)^(1/3)-3^(1/2)*x^(2/3)*(b*x+a)^(1/3)*arctan(1/3*3^(1/2
)+2/3*(b*x+a)^(1/3)*3^(1/2)/b^(1/3)/x^(1/3))/b^(4/3)/(b*x^3+a*x^2)^(1/3)-1
/2*x^(2/3)*(b*x+a)^(1/3)*ln(x)/b^(4/3)/(b*x^3+a*x^2)^(1/3)-3/2*x^(2/3)*(b*
x+a)^(1/3)*ln(1-(b*x+a)^(1/3)/b^(1/3)/x^(1/3))/b^(4/3)/(b*x^3+a*x^2)^(1/3)
```

Mathematica [A] (verified)

Time = 0.30 (sec) , antiderivative size = 184, normalized size of antiderivative = 0.93

$$\int \frac{x^3}{(ax^2+bx^3)^{4/3}}\,dx = \frac{x^{2/3}\left(-6\sqrt[3]{b}\sqrt[3]{x}+2\sqrt{3}\sqrt[3]{a+bx}\arctan\left(\frac{\sqrt{3}\sqrt[3]{b}\sqrt[3]{x}}{\sqrt[3]{b}\sqrt[3]{x}+2\sqrt[3]{a+bx}}\right)-2\sqrt[3]{a+bx}\log\left(-\sqrt[3]{\cdots}\right.\right.}{2b^{4/3}\sqrt[3]{x^2(a\cdots}}$$

```
Integrate[x^3/(a*x^2 + b*x^3)^(4/3),x]
```

```
(x^(2/3)*(-6*b^(1/3)*x^(1/3) + 2*Sqrt[3]*(a + b*x)^(1/3)*ArcTan[(Sqrt[3]*b
^(1/3)*x^(1/3))/(b^(1/3)*x^(1/3) + 2*(a + b*x)^(1/3))] - 2*(a + b*x)^(1/3)
*Log[-(b^(1/3)*x^(1/3)) + (a + b*x)^(1/3)] + (a + b*x)^(1/3)*Log[b^(2/3)*x
^(2/3) + b^(1/3)*x^(1/3)*(a + b*x)^(1/3) + (a + b*x)^(2/3)]))/(2*b^(4/3)*(
x^2*(a + b*x))^(1/3))
```

Rubi [A] (verified)

Time = 0.38 (sec) , antiderivative size = 144, normalized size of antiderivative = 0.73, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.158$, Rules used = {1928, 1917, 71}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^3}{(ax^2+bx^3)^{4/3}}dx$$

$$\Big\downarrow \text{ 1928}$$

$$\frac{\int \frac{1}{\sqrt[3]{bx^3+ax^2}}dx}{b}-\frac{3x}{b\sqrt[3]{ax^2+bx^3}}$$

$$\Big\downarrow \text{ 1917}$$

$$\frac{x^{2/3}\sqrt[3]{a+bx}\int \frac{1}{x^{2/3}\sqrt[3]{a+bx}}dx}{b\sqrt[3]{ax^2+bx^3}}-\frac{3x}{b\sqrt[3]{ax^2+bx^3}}$$

$$\Big\downarrow \text{ 71}$$

$$\frac{x^{2/3}\sqrt[3]{a+bx}\left(-\frac{\sqrt{3}\arctan\left(\frac{2\sqrt[3]{a+bx}}{\sqrt{3}\sqrt[3]{b}\sqrt[3]{x}}+\frac{1}{\sqrt{3}}\right)}{\sqrt[3]{b}}-\frac{3\log\left(\frac{\sqrt[3]{a+bx}}{\sqrt[3]{b}\sqrt[3]{x}}-1\right)}{2\sqrt[3]{b}}-\frac{\log(x)}{2\sqrt[3]{b}}\right)}{b\sqrt[3]{ax^2+bx^3}}-\frac{3x}{b\sqrt[3]{ax^2+bx^3}}$$

```
Int[x^3/(a*x^2 + b*x^3)^(4/3),x]
```

```
(-3*x)/(b*(a*x^2 + b*x^3)^(1/3)) + (x^(2/3)*(a + b*x)^(1/3)*(-((Sqrt[3]*Ar
cTan[1/Sqrt[3] + (2*(a + b*x)^(1/3))/(Sqrt[3]*b^(1/3)*x^(1/3))])/b^(1/3))
- Log[x]/(2*b^(1/3)) - (3*Log[-1 + (a + b*x)^(1/3)/(b^(1/3)*x^(1/3))])/(2*
b^(1/3))))/(b*(a*x^2 + b*x^3)^(1/3))
```

Defintions of rubi rules used

```
Int[1/(((a_.) + (b_.)*(x_))^(1/3)*((c_.) + (d_.)*(x_))^(2/3)), x_Symbol] :>
 With[{q = Rt[d/b, 3]}, Simp[(-Sqrt[3])*(q/d)*ArcTan[2*q*((a + b*x)^(1/3)/(
Sqrt[3]*(c + d*x)^(1/3))) + 1/Sqrt[3]], x] + (-Simp[3*(q/(2*d))*Log[q*((a +
 b*x)^(1/3)/(c + d*x)^(1/3)) - 1], x] - Simp[(q/(2*d))*Log[c + d*x], x])] /
; FreeQ[{a, b, c, d}, x] && PosQ[d/b]
```

```
Int[((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol] :> Simp[(a*x^j +
b*x^n)^FracPart[p]/(x^(j*FracPart[p])*(a + b*x^(n - j))^FracPart[p])   Int[
x^(j*p)*(a + b*x^(n - j))^p, x], x] /; FreeQ[{a, b, j, n, p}, x] &&  !Integ
erQ[p] && NeQ[n, j] && PosQ[n - j]
```

```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> Simp[c^(n - 1)*(c*x)^(m - n + 1)*((a*x^j + b*x^n)^(p + 1)/(b*(n - j)*(
p + 1))), x] - Simp[c^n*((m + j*p - n + j + 1)/(b*(n - j)*(p + 1)))   Int[(
c*x)^(m - n)*(a*x^j + b*x^n)^(p + 1), x], x] /; FreeQ[{a, b, c}, x] &&  !In
tegerQ[p] && LtQ[0, j, n] && (IntegersQ[j, n] || GtQ[c, 0]) && LtQ[p, -1] &
& GtQ[m + j*p + 1, n - j]
```

Maple [A] (verified)

Time = 0.49 (sec) , antiderivative size = 158, normalized size of antiderivative = 0.80

method	result
pseudoelliptic	$-\frac{\sqrt{3}\,\arctan\left(\frac{\sqrt{3}\left(b^{\frac{1}{3}}x+2\left(x^{2}(bx+a)\right)^{\frac{1}{3}}\right)}{3b^{\frac{1}{3}}x}\right)\left(x^{2}(bx+a)\right)^{\frac{1}{3}}+\ln\left(\frac{-b^{\frac{1}{3}}x+\left(x^{2}(bx+a)\right)^{\frac{1}{3}}}{x}\right)\left(x^{2}(bx+a)\right)^{\frac{1}{3}}-\frac{\ln\left(\frac{b^{\frac{2}{3}}x^{2}+b^{\frac{1}{3}}\left(x^{2}(bx-\right.}{}\right.}{}}{b^{\frac{4}{3}}\left(x^{2}(bx+a)\right)^{\frac{1}{3}}}$

```
int(x^3/(b*x^3+a*x^2)^(4/3),x,method=_RETURNVERBOSE)
```

```
-1/b^(4/3)*(3^(1/2)*arctan(1/3*3^(1/2)*(b^(1/3)*x+2*(x^2*(b*x+a))^(1/3))/b
^(1/3)/x)*(x^2*(b*x+a))^(1/3)+ln((-b^(1/3)*x+(x^2*(b*x+a))^(1/3))/x)*(x^2*
(b*x+a))^(1/3)-1/2*ln((b^(2/3)*x^2+b^(1/3)*(x^2*(b*x+a))^(1/3)*x+(x^2*(b*x
+a))^(2/3))/x^2)*(x^2*(b*x+a))^(1/3)+3*b^(1/3)*x)/(x^2*(b*x+a))^(1/3)
```

Fricas [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 432, normalized size of antiderivative = 2.18

$$\int \frac{x^3}{(ax^2+bx^3)^{4/3}}\,dx = \left[\frac{\sqrt{3}(b^2x^2+abx)\sqrt{-\frac{1}{b^{\frac{2}{3}}}}\log\left(\frac{3\,bx^2+2\,ax-3\,(bx^3+ax^2)^{\frac{1}{3}}b^{\frac{2}{3}}x+\sqrt{3}\left(b^{\frac{4}{3}}x^2+(bx^3+ax^2)^{\frac{1}{3}}bx-2\,(bx^3+a\right.}{x}\right.}{}\right.$$

$$-\frac{2\,(bx^2+ax)b^{\frac{2}{3}}\log\left(-\frac{b^{\frac{1}{3}}x-(bx^3+ax^2)^{\frac{1}{3}}}{x}\right)-(bx^2+ax)b^{\frac{2}{3}}\log\left(\frac{b^{\frac{2}{3}}x^2+(bx^3+ax^2)^{\frac{1}{3}}b^{\frac{1}{3}}x+(bx^3+ax^2)^{\frac{2}{3}}}{x^2}\right)+\frac{2\sqrt{3}(b^2x^2+ab}{}}{2\,(b^3x^2+ab^2x)}$$

```
integrate(x^3/(b*x^3+a*x^2)^(4/3),x, algorithm="fricas")
```

```
[1/2*(sqrt(3)*(b^2*x^2 + a*b*x)*sqrt(-1/b^(2/3))*log((3*b*x^2 + 2*a*x - 3*
(b*x^3 + a*x^2)^(1/3)*b^(2/3)*x + sqrt(3)*(b^(4/3)*x^2 + (b*x^3 + a*x^2)^(
1/3)*b*x - 2*(b*x^3 + a*x^2)^(2/3)*b^(2/3))*sqrt(-1/b^(2/3)))/x) - 2*(b*x^
2 + a*x)*b^(2/3)*log(-(b^(1/3)*x - (b*x^3 + a*x^2)^(1/3))/x) + (b*x^2 + a*
x)*b^(2/3)*log((b^(2/3)*x^2 + (b*x^3 + a*x^2)^(1/3)*b^(1/3)*x + (b*x^3 + a
*x^2)^(2/3))/x^2) - 6*(b*x^3 + a*x^2)^(2/3)*b)/(b^3*x^2 + a*b^2*x), -1/2*(
2*(b*x^2 + a*x)*b^(2/3)*log(-(b^(1/3)*x - (b*x^3 + a*x^2)^(1/3))/x) - (b*x
^2 + a*x)*b^(2/3)*log((b^(2/3)*x^2 + (b*x^3 + a*x^2)^(1/3)*b^(1/3)*x + (b*
x^3 + a*x^2)^(2/3))/x^2) + 2*sqrt(3)*(b^2*x^2 + a*b*x)*arctan(1/3*sqrt(3)*
(b^(1/3)*x + 2*(b*x^3 + a*x^2)^(1/3))/(b^(1/3)*x))/b^(1/3) + 6*(b*x^3 + a*
x^2)^(2/3)*b)/(b^3*x^2 + a*b^2*x)]
```

Sympy [F]

$$\int \frac{x^3}{(ax^2+bx^3)^{4/3}}\,dx = \int \frac{x^3}{(x^2(a+bx))^{\frac{4}{3}}}\,dx$$

```
integrate(x**3/(b*x**3+a*x**2)**(4/3),x)
```

```
Integral(x**3/(x**2*(a + b*x))**(4/3), x)
```

Maxima [F]

$$\int \frac{x^3}{(ax^2+bx^3)^{4/3}}\,dx = \int \frac{x^3}{(bx^3+ax^2)^{\frac{4}{3}}}\,dx$$

```
integrate(x^3/(b*x^3+a*x^2)^(4/3),x, algorithm="maxima")
```

```
integrate(x^3/(b*x^3 + a*x^2)^(4/3), x)
```

Giac [A] (verification not implemented)

Time = 8.32 (sec) , antiderivative size = 101, normalized size of antiderivative = 0.51

$$\int \frac{x^3}{(ax^2+bx^3)^{4/3}}\,dx = -\frac{\sqrt{3}\arctan\left(\frac{\sqrt{3}\left(2\left(b+\frac{a}{x}\right)^{\frac{1}{3}}+b^{\frac{1}{3}}\right)}{3\,b^{\frac{1}{3}}}\right)}{b^{\frac{4}{3}}} + \frac{\log\left(\left(b+\frac{a}{x}\right)^{\frac{2}{3}}+\left(b+\frac{a}{x}\right)^{\frac{1}{3}}b^{\frac{1}{3}}+b^{\frac{2}{3}}\right)}{2\,b^{\frac{4}{3}}} - \frac{\log\left(\left|\left(b+\frac{a}{x}\right)^{\frac{1}{3}}-b^{\frac{1}{3}}\right|\right)}{b^{\frac{4}{3}}} - \frac{3}{\left(b+\frac{a}{x}\right)^{\frac{1}{3}}b}$$

```
integrate(x^3/(b*x^3+a*x^2)^(4/3),x, algorithm="giac")
```

```
-sqrt(3)*arctan(1/3*sqrt(3)*(2*(b + a/x)^(1/3) + b^(1/3))/b^(1/3))/b^(4/3)
 + 1/2*log((b + a/x)^(2/3) + (b + a/x)^(1/3)*b^(1/3) + b^(2/3))/b^(4/3) -
log(abs((b + a/x)^(1/3) - b^(1/3)))/b^(4/3) - 3/((b + a/x)^(1/3)*b)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{x^3}{(ax^2+bx^3)^{4/3}}\,dx = \int \frac{x^3}{\left(b\,x^3+a\,x^2\right)^{4/3}}\,dx$$

```
int(x^3/(a*x^2 + b*x^3)^(4/3),x)
```

```
int(x^3/(a*x^2 + b*x^3)^(4/3), x)
```

Reduce [F]

$$\int \frac{x^3}{(ax^2+bx^3)^{4/3}}\,dx = \int \frac{x}{x^{\frac{2}{3}}(bx+a)^{\frac{1}{3}}a + x^{\frac{5}{3}}(bx+a)^{\frac{1}{3}}b}dx$$

```
int(x^3/(b*x^3+a*x^2)^(4/3),x)
```

```
int(x/(x**(2/3)*(a + b*x)**(1/3)*a + x**(2/3)*(a + b*x)**(1/3)*b*x),x)
```

3.381 $\int \frac{x^2}{\left(ax^2+bx^3\right)^{4/3}}\,dx$

Optimal result

Integrand size = 19, antiderivative size = 21

$$\int \frac{x^2}{(ax^2+bx^3)^{4/3}}\,dx = \frac{3x}{a\sqrt[3]{ax^2+bx^3}}$$

```
3*x/a/(b*x^3+a*x^2)^(1/3)
```

Mathematica [A] (verified)

Time = 0.03 (sec) , antiderivative size = 19, normalized size of antiderivative = 0.90

$$\int \frac{x^2}{(ax^2+bx^3)^{4/3}}\,dx = \frac{3x}{a\sqrt[3]{x^2(a+bx)}}$$

```
Integrate[x^2/(a*x^2 + b*x^3)^(4/3),x]
```

```
(3*x)/(a*(x^2*(a + b*x))^(1/3))
```

Rubi [A] (verified)

Time = 0.25 (sec) , antiderivative size = 21, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}} = 0.053$, Rules used = {1920}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^2}{(ax^2+bx^3)^{4/3}}\,dx$$
$$\Big\downarrow \text{ 1920}$$
$$\frac{3x}{a\sqrt[3]{ax^2+bx^3}}$$

```
Int[x^2/(a*x^2 + b*x^3)^(4/3),x]
```

```
(3*x)/(a*(a*x^2 + b*x^3)^(1/3))
```

Defintions of rubi rules used

```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> Simp[(-c^(j - 1))*(c*x)^(m - j + 1)*((a*x^j + b*x^n)^(p + 1)/(a*(n - j
)*(p + 1))), x] /; FreeQ[{a, b, c, j, m, n, p}, x] &&  !IntegerQ[p] && NeQ[
n, j] && EqQ[m + n*p + n - j + 1, 0] && (IntegerQ[j] || GtQ[c, 0])
```

Maple [A] (verified)

Time = 0.42 (sec) , antiderivative size = 18, normalized size of antiderivative = 0.86

method	result	size
pseudoelliptic	$\frac{3x}{a\left(x^2(bx+a)\right)^{\frac{1}{3}}}$	18
gosper	$\frac{3x^3(bx+a)}{a\left(b\,x^3+a\,x^2\right)^{\frac{4}{3}}}$	27
orering	$\frac{3x^3(bx+a)}{a\left(b\,x^3+a\,x^2\right)^{\frac{4}{3}}}$	27
trager	$\frac{3\left(b\,x^3+a\,x^2\right)^{\frac{2}{3}}}{(bx+a)ax}$	29

```
int(x^2/(b*x^3+a*x^2)^(4/3),x,method=_RETURNVERBOSE)
```

```
3*x/a/(x^2*(b*x+a))^(1/3)
```

Fricas [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 29, normalized size of antiderivative = 1.38

$$\int \frac{x^2}{(ax^2+bx^3)^{4/3}}\,dx = \frac{3\left(bx^3+ax^2\right)^{\frac{2}{3}}}{abx^2+a^2x}$$

```
integrate(x^2/(b*x^3+a*x^2)^(4/3),x, algorithm="fricas")
```

```
3*(b*x^3 + a*x^2)^(2/3)/(a*b*x^2 + a^2*x)
```

Sympy [F]

$$\int \frac{x^2}{(ax^2+bx^3)^{4/3}}\,dx = \int \frac{x^2}{\left(x^2\,(a+bx)\right)^{\frac{4}{3}}}\,dx$$

```
integrate(x**2/(b*x**3+a*x**2)**(4/3),x)
```

```
Integral(x**2/(x**2*(a + b*x))**(4/3), x)
```

Maxima [F]

$$\int \frac{x^2}{(ax^2+bx^3)^{4/3}}\,dx = \int \frac{x^2}{(bx^3+ax^2)^{\frac{4}{3}}}\,dx$$

```
integrate(x^2/(b*x^3+a*x^2)^(4/3),x, algorithm="maxima")
```

```
integrate(x^2/(b*x^3 + a*x^2)^(4/3), x)
```

Giac [A] (verification not implemented)

Time = 3.33 (sec) , antiderivative size = 14, normalized size of antiderivative = 0.67

$$\int \frac{x^2}{(ax^2+bx^3)^{4/3}}\,dx = \frac{3}{a\left(b+\frac{a}{x}\right)^{\frac{1}{3}}}$$

```
integrate(x^2/(b*x^3+a*x^2)^(4/3),x, algorithm="giac")
```

```
3/(a*(b + a/x)^(1/3))
```

Mupad [B] (verification not implemented)

Time = 9.00 (sec) , antiderivative size = 28, normalized size of antiderivative = 1.33

$$\int \frac{x^2}{(ax^2+bx^3)^{4/3}}\,dx = \frac{3\,(b\,x^3+a\,x^2)^{2/3}}{a\,x\,(a+b\,x)}$$

```
int(x^2/(a*x^2 + b*x^3)^(4/3),x)
```

```
(3*(a*x^2 + b*x^3)^(2/3))/(a*x*(a + b*x))
```

Reduce [F]

$$\int \frac{x^2}{(ax^2 + bx^3)^{4/3}}\,dx = \int \frac{1}{x^{\frac{2}{3}}\,(bx+a)^{\frac{1}{3}}\,a + x^{\frac{5}{3}}\,(bx+a)^{\frac{1}{3}}\,b}dx$$

```
int(x^2/(b*x^3+a*x^2)^(4/3),x)
```

```
int(1/(x**(2/3)*(a + b*x)**(1/3)*a + x**(2/3)*(a + b*x)**(1/3)*b*x),x)
```

3.382 $\int \frac{x}{\left(ax^2+bx^3\right)^{4/3}}\,dx$

Optimal result

Integrand size = 17, antiderivative size = 46

$$\int \frac{x}{\left(ax^2+bx^3\right)^{4/3}}\,dx=\frac{3}{a\sqrt[3]{ax^2+bx^3}}-\frac{9\left(ax^2+bx^3\right)^{2/3}}{2a^2x^2}$$

```
3/a/(b*x^3+a*x^2)^(1/3)-9/2*(b*x^3+a*x^2)^(2/3)/a^2/x^2
```

Mathematica [A] (verified)

Time = 0.22 (sec) , antiderivative size = 26, normalized size of antiderivative = 0.57

$$\int \frac{x}{\left(ax^2+bx^3\right)^{4/3}}\,dx=-\frac{3(a+3bx)}{2a^2\sqrt[3]{x^2(a+bx)}}$$

```
Integrate[x/(a*x^2 + b*x^3)^(4/3),x]
```

```
(-3*(a + 3*b*x))/(2*a^2*(x^2*(a + b*x))^(1/3))
```

Rubi [A] (verified)

Time = 0.30 (sec) , antiderivative size = 46, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.118, Rules used = {1921, 1920}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x}{(ax^2+bx^3)^{4/3}}dx$$
$$\Big\downarrow \text{1921}$$
$$\frac{3\int \frac{1}{x\sqrt[3]{bx^3+ax^2}}dx}{a}+\frac{3}{a\sqrt[3]{ax^2+bx^3}}$$
$$\Big\downarrow \text{1920}$$
$$\frac{3}{a\sqrt[3]{ax^2+bx^3}}-\frac{9\left(ax^2+bx^3\right)^{2/3}}{2a^2x^2}$$

```
Int[x/(a*x^2 + b*x^3)^(4/3),x]
```

```
3/(a*(a*x^2 + b*x^3)^(1/3)) - (9*(a*x^2 + b*x^3)^(2/3))/(2*a^2*x^2)
```

Defintions of rubi rules used

```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> Simp[(-c^(j - 1))*(c*x)^(m - j + 1)*((a*x^j + b*x^n)^(p + 1)/(a*(n - j
)*(p + 1))), x] /; FreeQ[{a, b, c, j, m, n, p}, x] && !IntegerQ[p] && NeQ[
n, j] && EqQ[m + n*p + n - j + 1, 0] && (IntegerQ[j] || GtQ[c, 0])
```

```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> Simp[(-c^(j - 1))*(c*x)^(m - j + 1)*((a*x^j + b*x^n)^(p + 1)/(a*(n - j
)*(p + 1))), x] + Simp[c^j*((m + n*p + n - j + 1)/(a*(n - j)*(p + 1)))   In
t[(c*x)^(m - j)*(a*x^j + b*x^n)^(p + 1), x], x] /; FreeQ[{a, b, c, j, m, n}
, x] &&  !IntegerQ[p] && NeQ[n, j] && ILtQ[Simplify[(m + n*p + n - j + 1)/(
n - j)], 0] && LtQ[p, -1] && (IntegerQ[j] || GtQ[c, 0])
```

Maple [A] (verified)

Time = 0.44 (sec) , antiderivative size = 23, normalized size of antiderivative = 0.50

method	result	size
pseudoelliptic	$-\frac{3(3bx+a)}{2(x^2(bx+a))^{\frac{1}{3}}a^2}$	23
gosper	$-\frac{3x^2(bx+a)(3bx+a)}{2a^2(b\,x^3+a\,x^2)^{\frac{4}{3}}}$	33
orering	$-\frac{3x^2(bx+a)(3bx+a)}{2a^2(b\,x^3+a\,x^2)^{\frac{4}{3}}}$	33
trager	$-\frac{3(3bx+a)(b\,x^3+a\,x^2)^{\frac{2}{3}}}{2(bx+a)a^2x^2}$	35
risch	$-\frac{3(bx+a)}{2a^2(x^2(bx+a))^{\frac{1}{3}}}-\frac{3bx}{(x^2(bx+a))^{\frac{1}{3}}a^2}$	41

```
int(x/(b*x^3+a*x^2)^(4/3),x,method=_RETURNVERBOSE)
```

```
-3/2/(x^2*(b*x+a))^(1/3)*(3*b*x+a)/a^2
```

Fricas [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 39, normalized size of antiderivative = 0.85

$$\int \frac{x}{(ax^2+bx^3)^{4/3}}\,dx = -\frac{3\,(bx^3+ax^2)^{\frac{2}{3}}(3\,bx+a)}{2\,(a^2bx^3+a^3x^2)}$$

```
integrate(x/(b*x^3+a*x^2)^(4/3),x, algorithm="fricas")
```

```
-3/2*(b*x^3 + a*x^2)^(2/3)*(3*b*x + a)/(a^2*b*x^3 + a^3*x^2)
```

Sympy [F]

$$\int \frac{x}{(ax^2+bx^3)^{4/3}}\,dx = \int \frac{x}{\left(x^2\left(a+bx\right)\right)^{\frac{4}{3}}}\,dx$$

```
integrate(x/(b*x**3+a*x**2)**(4/3),x)
```

```
Integral(x/(x**2*(a + b*x))**(4/3), x)
```

Maxima [F]

$$\int \frac{x}{(ax^2+bx^3)^{4/3}}\,dx = \int \frac{x}{\left(bx^3+ax^2\right)^{\frac{4}{3}}}\,dx$$

```
integrate(x/(b*x^3+a*x^2)^(4/3),x, algorithm="maxima")
```

```
integrate(x/(b*x^3 + a*x^2)^(4/3), x)
```

Giac [A] (verification not implemented)

Time = 3.94 (sec) , antiderivative size = 34, normalized size of antiderivative = 0.74

$$\int \frac{x}{(ax^2+bx^3)^{4/3}}\,dx = -\frac{3\left(\frac{\left(b+\frac{a}{x}\right)^{\frac{2}{3}}}{a}+\frac{2\,b}{a\left(b+\frac{a}{x}\right)^{\frac{1}{3}}}\right)}{2\,a}$$

```
integrate(x/(b*x^3+a*x^2)^(4/3),x, algorithm="giac")
```

```
-3/2*((b + a/x)^(2/3)/a + 2*b/(a*(b + a/x)^(1/3)))/a
```

Mupad [B] (verification not implemented)

Time = 9.18 (sec) , antiderivative size = 26, normalized size of antiderivative = 0.57

$$\int \frac{x}{(ax^2+bx^3)^{4/3}}\,dx = -\frac{3\,a+9\,b\,x}{2\,a^2\,\left(b\,x^3+a\,x^2\right)^{1/3}}$$

```
int(x/(a*x^2 + b*x^3)^(4/3),x)
```

```
-(3*a + 9*b*x)/(2*a^2*(a*x^2 + b*x^3)^(1/3))
```

Reduce [F]

$$\int \frac{x}{(ax^2+bx^3)^{4/3}}\,dx = \int \frac{1}{x^{\frac{5}{3}}\,(bx+a)^{\frac{1}{3}}\,a+x^{\frac{8}{3}}\,(bx+a)^{\frac{1}{3}}\,b}dx$$

```
int(x/(b*x^3+a*x^2)^(4/3),x)
```

```
int(1/(x**(2/3)*(a + b*x)**(1/3)*a*x + x**(2/3)*(a + b*x)**(1/3)*b*x**2),x
)
```

3.383 $\int \frac{1}{(ax^2+bx^3)^{4/3}}\,dx$

Optimal result

Integrand size = 15, antiderivative size = 75

$$\int \frac{1}{(ax^2+bx^3)^{4/3}}\,dx = \frac{3}{ax\sqrt[3]{ax^2+bx^3}} - \frac{18(ax^2+bx^3)^{2/3}}{5a^2x^3} + \frac{27b(ax^2+bx^3)^{2/3}}{5a^3x^2}$$

```
3/a/x/(b*x^3+a*x^2)^(1/3)-18/5*(b*x^3+a*x^2)^(2/3)/a^2/x^3+27/5*b*(b*x^3+a
*x^2)^(2/3)/a^3/x^2
```

Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 43, normalized size of antiderivative = 0.57

$$\int \frac{1}{(ax^2+bx^3)^{4/3}}\,dx = -\frac{3x(a+bx)\left(a^2-3abx-9b^2x^2\right)}{5a^3\left(x^2(a+bx)\right)^{4/3}}$$

```
Integrate[(a*x^2 + b*x^3)^(-4/3),x]
```

```
(-3*x*(a + b*x)*(a^2 - 3*a*b*x - 9*b^2*x^2))/(5*a^3*(x^2*(a + b*x))^(4/3))
```

Rubi [A] (verified)

Time = 0.38 (sec) , antiderivative size = 81, normalized size of antiderivative = 1.08, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.200$, Rules used = {1907, 1922, 1920}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{(ax^2+bx^3)^{4/3}}dx$$

$$\Big\downarrow \text{ 1907}$$

$$\frac{6\int \frac{1}{x^2\sqrt[3]{bx^3+ax^2}}dx}{a}+\frac{3}{ax\sqrt[3]{ax^2+bx^3}}$$

$$\Big\downarrow \text{ 1922}$$

$$\frac{6\left(-\frac{3b\int \frac{1}{x\sqrt[3]{bx^3+ax^2}}dx}{5a}-\frac{3(ax^2+bx^3)^{2/3}}{5ax^3}\right)}{a}+\frac{3}{ax\sqrt[3]{ax^2+bx^3}}$$

$$\Big\downarrow \text{ 1920}$$

$$\frac{6\left(\frac{9b(ax^2+bx^3)^{2/3}}{10a^2x^2}-\frac{3(ax^2+bx^3)^{2/3}}{5ax^3}\right)}{a}+\frac{3}{ax\sqrt[3]{ax^2+bx^3}}$$

```
Int[(a*x^2 + b*x^3)^(-4/3),x]
```

```
3/(a*x*(a*x^2 + b*x^3)^(1/3)) + (6*((-3*(a*x^2 + b*x^3)^(2/3))/(5*a*x^3) +
 (9*b*(a*x^2 + b*x^3)^(2/3))/(10*a^2*x^2)))/a
```

Defintions of rubi rules used

```
Int[((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol] :> Simp[-(a*x^j +
 b*x^n)^(p + 1)/(a*(n - j)*(p + 1)*x^(j - 1)), x] + Simp[(n*p + n - j + 1)/
(a*(n - j)*(p + 1))   Int[(a*x^j + b*x^n)^(p + 1)/x^j, x], x] /; FreeQ[{a,
b, j, n}, x] &&  !IntegerQ[p] && NeQ[n, j] && ILtQ[Simplify[(n*p + n - j +
1)/(n - j)], 0] && LtQ[p, -1]
```

```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> Simp[(-c^(j - 1))*(c*x)^(m - j + 1)*((a*x^j + b*x^n)^(p + 1)/(a*(n - j
)*(p + 1))), x] /; FreeQ[{a, b, c, j, m, n, p}, x] &&  !IntegerQ[p] && NeQ[
n, j] && EqQ[m + n*p + n - j + 1, 0] && (IntegerQ[j] || GtQ[c, 0])
```

```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> Simp[c^(j - 1)*(c*x)^(m - j + 1)*((a*x^j + b*x^n)^(p + 1)/(a*(m + j*p
+ 1))), x] - Simp[b*((m + n*p + n - j + 1)/(a*c^(n - j)*(m + j*p + 1)))   I
nt[(c*x)^(m + n - j)*(a*x^j + b*x^n)^p, x], x] /; FreeQ[{a, b, c, j, m, n,
p}, x] &&  !IntegerQ[p] && NeQ[n, j] && ILtQ[Simplify[(m + n*p + n - j + 1)
/(n - j)], 0] && NeQ[m + j*p + 1, 0] && (IntegersQ[j, n] || GtQ[c, 0])
```

Maple [A] (verified)

Time = 0.42 (sec) , antiderivative size = 37, normalized size of antiderivative = 0.49

method	result	size
pseudoelliptic	$-\frac{3\left(-9 b^{2} x^{2}-3 a b x+a^{2}\right)}{5 x \left(x^{2} \left(b x+a\right)\right)^{\frac{1}{3}} a^{3}}$	37
gosper	$-\frac{3 x \left(b x+a\right) \left(-9 b^{2} x^{2}-3 a b x+a^{2}\right)}{5 a^{3} \left(b \,x^{3}+a \,x^{2}\right)^{\frac{4}{3}}}$	42
orering	$-\frac{3 x \left(b x+a\right) \left(-9 b^{2} x^{2}-3 a b x+a^{2}\right)}{5 a^{3} \left(b \,x^{3}+a \,x^{2}\right)^{\frac{4}{3}}}$	42
trager	$-\frac{3 \left(-9 b^{2} x^{2}-3 a b x+a^{2}\right) \left(b \,x^{3}+a \,x^{2}\right)^{\frac{2}{3}}}{5 \left(b x+a\right) a^{3} x^{3}}$	46
risch	$-\frac{3 \left(b x+a\right) \left(-4 b x+a\right)}{5 a^{3} x \left(x^{2} \left(b x+a\right)\right)^{\frac{1}{3}}}+\frac{3 b^{2} x}{\left(x^{2} \left(b x+a\right)\right)^{\frac{1}{3}} a^{3}}$	52

```
int(1/(b*x^3+a*x^2)^(4/3),x,method=_RETURNVERBOSE)
```

```
-3/5/x*(-9*b^2*x^2-3*a*b*x+a^2)/(x^2*(b*x+a))^(1/3)/a^3
```

Fricas [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 52, normalized size of antiderivative = 0.69

$$\int \frac{1}{(ax^2+bx^3)^{4/3}}\,dx = \frac{3\,(9\,b^2x^2+3\,abx-a^2)(bx^3+ax^2)^{\frac{2}{3}}}{5\,(a^3bx^4+a^4x^3)}$$

```
integrate(1/(b*x^3+a*x^2)^(4/3),x, algorithm="fricas")
```

```
3/5*(9*b^2*x^2 + 3*a*b*x - a^2)*(b*x^3 + a*x^2)^(2/3)/(a^3*b*x^4 + a^4*x^3
)
```

Sympy [F]

$$\int \frac{1}{(ax^2+bx^3)^{4/3}}\,dx = \int \frac{1}{(ax^2+bx^3)^{\frac{4}{3}}}\,dx$$

```
integrate(1/(b*x**3+a*x**2)**(4/3),x)
```

```
Integral((a*x**2 + b*x**3)**(-4/3), x)
```

Maxima [F]

$$\int \frac{1}{(ax^2+bx^3)^{4/3}}\,dx = \int \frac{1}{(bx^3+ax^2)^{\frac{4}{3}}}\,dx$$

```
integrate(1/(b*x^3+a*x^2)^(4/3),x, algorithm="maxima")
```

```
integrate((b*x^3 + a*x^2)^(-4/3), x)
```

Giac [A] (verification not implemented)

Time = 3.48 (sec) , antiderivative size = 57, normalized size of antiderivative = 0.76

$$\int \frac{1}{(ax^2+bx^3)^{4/3}}\,dx = \frac{3\left(\frac{5\,b^2}{a\left(b+\frac{a}{x}\right)^{\frac{1}{3}}}-\frac{a^4\left(b+\frac{a}{x}\right)^{\frac{5}{3}}-5\,a^4\left(b+\frac{a}{x}\right)^{\frac{2}{3}}b}{a^5}\right)}{5\,a^2}$$

```
integrate(1/(b*x^3+a*x^2)^(4/3),x, algorithm="giac")
```

```
3/5*(5*b^2/(a*(b + a/x)^(1/3)) - (a^4*(b + a/x)^(5/3) - 5*a^4*(b + a/x)^(2
/3)*b)/a^5)/a^2
```

Mupad [B] (verification not implemented)

Time = 9.04 (sec) , antiderivative size = 47, normalized size of antiderivative = 0.63

$$\int \frac{1}{(ax^2+bx^3)^{4/3}}\,dx = \frac{3\left(b\,x^3+a\,x^2\right)^{2/3}\left(-a^2+3\,a\,b\,x+9\,b^2\,x^2\right)}{5\,a^3\,x^3\,\left(a+b\,x\right)}$$

```
int(1/(a*x^2 + b*x^3)^(4/3),x)
```

```
(3*(a*x^2 + b*x^3)^(2/3)*(9*b^2*x^2 - a^2 + 3*a*b*x))/(5*a^3*x^3*(a + b*x)
)
```

Reduce [F]

$$\int \frac{1}{(ax^2+bx^3)^{4/3}}\,dx = \int \frac{1}{x^{\frac{8}{3}}\left(bx+a\right)^{\frac{1}{3}}a+x^{\frac{11}{3}}\left(bx+a\right)^{\frac{1}{3}}b}dx$$

```
int(1/(b*x^3+a*x^2)^(4/3),x)
```

```
int(1/(x**(2/3)*(a + b*x)**(1/3)*a*x**2 + x**(2/3)*(a + b*x)**(1/3)*b*x**3
),x)
```

3.384 $\int \frac{1}{x(ax^2+bx^3)^{4/3}}\,dx$

Optimal result

Integrand size = 19, antiderivative size = 103

$$\int \frac{1}{x\left(ax^2+bx^3\right)^{4/3}}\,dx = \frac{3}{ax^2\sqrt[3]{ax^2+bx^3}} - \frac{27(ax^2+bx^3)^{2/3}}{8a^2x^4} + \frac{81b(ax^2+bx^3)^{2/3}}{20a^3x^3} - \frac{243b^2(ax^2+bx^3)^{2/3}}{40a^4x^2}$$

```
3/a/x^2/(b*x^3+a*x^2)^(1/3)-27/8*(b*x^3+a*x^2)^(2/3)/a^2/x^4+81/20*b*(b*x^
3+a*x^2)^(2/3)/a^3/x^3-243/40*b^2*(b*x^3+a*x^2)^(2/3)/a^4/x^2
```

Mathematica [A] (verified)

Time = 0.32 (sec) , antiderivative size = 55, normalized size of antiderivative = 0.53

$$\int \frac{1}{x\left(ax^2+bx^3\right)^{4/3}}\,dx = -\frac{3(a+bx)\left(5a^3-9a^2bx+27ab^2x^2+81b^3x^3\right)}{40a^4\left(x^2(a+bx)\right)^{4/3}}$$

```
Integrate[1/(x*(a*x^2 + b*x^3)^(4/3)),x]
```

```
(-3*(a + b*x)*(5*a^3 - 9*a^2*b*x + 27*a*b^2*x^2 + 81*b^3*x^3))/(40*a^4*(x^
2*(a + b*x))^(4/3))
```

Rubi [A] (verified)

Time = 0.46 (sec) , antiderivative size = 115, normalized size of antiderivative = 1.12, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.211$, Rules used = {1921, 1922, 1922, 1920}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{x\left(ax^2+bx^3\right)^{4/3}}dx$$

$$\Big\downarrow \text{ 1921}$$

$$\frac{9\int \frac{1}{x^3\sqrt[3]{bx^3+ax^2}}dx}{a}+\frac{3}{ax^2\sqrt[3]{ax^2+bx^3}}$$

$$\Big\downarrow \text{ 1922}$$

$$\frac{9\left(-\frac{3b\int \frac{1}{x^2\sqrt[3]{bx^3+ax^2}}dx}{4a}-\frac{3\left(ax^2+bx^3\right)^{2/3}}{8ax^4}\right)}{a}+\frac{3}{ax^2\sqrt[3]{ax^2+bx^3}}$$

$$\Big\downarrow \text{ 1922}$$

$$\frac{9\left(-\frac{3b\left(-\frac{3b\int \frac{1}{x\sqrt[3]{bx^3+ax^2}}dx}{5a}-\frac{3\left(ax^2+bx^3\right)^{2/3}}{5ax^3}\right)}{4a}-\frac{3\left(ax^2+bx^3\right)^{2/3}}{8ax^4}\right)}{a}+\frac{3}{ax^2\sqrt[3]{ax^2+bx^3}}$$

$$\Big\downarrow \text{ 1920}$$

$$\frac{9\left(-\frac{3b\left(\frac{9b\left(ax^2+bx^3\right)^{2/3}}{10a^2x^2}-\frac{3\left(ax^2+bx^3\right)^{2/3}}{5ax^3}\right)}{4a}-\frac{3\left(ax^2+bx^3\right)^{2/3}}{8ax^4}\right)}{a}+\frac{3}{ax^2\sqrt[3]{ax^2+bx^3}}$$

```
Int[1/(x*(a*x^2 + b*x^3)^(4/3)),x]
```

```
3/(a*x^2*(a*x^2 + b*x^3)^(1/3)) + (9*((-3*(a*x^2 + b*x^3)^(2/3))/(8*a*x^4)
 - (3*b*((-3*(a*x^2 + b*x^3)^(2/3))/(5*a*x^3) + (9*b*(a*x^2 + b*x^3)^(2/3)
)/(10*a^2*x^2)))/(4*a)))/a
```

Defintions of rubi rules used

```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> Simp[(-c^(j - 1))*(c*x)^(m - j + 1)*((a*x^j + b*x^n)^(p + 1)/(a*(n - j
)*(p + 1))), x] /; FreeQ[{a, b, c, j, m, n, p}, x] &&  !IntegerQ[p] && NeQ[
n, j] && EqQ[m + n*p + n - j + 1, 0] && (IntegerQ[j] || GtQ[c, 0])
```

```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> Simp[(-c^(j - 1))*(c*x)^(m - j + 1)*((a*x^j + b*x^n)^(p + 1)/(a*(n - j
)*(p + 1))), x] + Simp[c^j*((m + n*p + n - j + 1)/(a*(n - j)*(p + 1)))   In
t[(c*x)^(m - j)*(a*x^j + b*x^n)^(p + 1), x], x] /; FreeQ[{a, b, c, j, m, n}
, x] &&  !IntegerQ[p] && NeQ[n, j] && ILtQ[Simplify[(m + n*p + n - j + 1)/(
n - j)], 0] && LtQ[p, -1] && (IntegerQ[j] || GtQ[c, 0])
```

```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> Simp[c^(j - 1)*(c*x)^(m - j + 1)*((a*x^j + b*x^n)^(p + 1)/(a*(m + j*p
+ 1))), x] - Simp[b*((m + n*p + n - j + 1)/(a*c^(n - j)*(m + j*p + 1)))   I
nt[(c*x)^(m + n - j)*(a*x^j + b*x^n)^p, x], x] /; FreeQ[{a, b, c, j, m, n,
p}, x] &&  !IntegerQ[p] && NeQ[n, j] && ILtQ[Simplify[(m + n*p + n - j + 1)
/(n - j)], 0] && NeQ[m + j*p + 1, 0] && (IntegersQ[j, n] || GtQ[c, 0])
```

Maple [A] (verified)

Time = 0.45 (sec) , antiderivative size = 50, normalized size of antiderivative = 0.49

method	result	size
pseudoelliptic	$\frac{-\frac{243}{40}b^3x^3-\frac{81}{40}a\,b^2x^2+\frac{27}{40}a^2bx-\frac{3}{8}a^3}{x^2\left(x^2(bx+a)\right)^{\frac{1}{3}}a^4}$	50
gosper	$-\frac{3(bx+a)\left(81b^3x^3+27a\,b^2x^2-9a^2bx+5a^3\right)}{40a^4\left(b\,x^3+a\,x^2\right)^{\frac{4}{3}}}$	54
orering	$-\frac{3(bx+a)\left(81b^3x^3+27a\,b^2x^2-9a^2bx+5a^3\right)}{40a^4\left(b\,x^3+a\,x^2\right)^{\frac{4}{3}}}$	54
trager	$-\frac{3\left(81b^3x^3+27a\,b^2x^2-9a^2bx+5a^3\right)\left(b\,x^3+a\,x^2\right)^{\frac{2}{3}}}{40(bx+a)a^4x^4}$	59
risch	$-\frac{3(bx+a)\left(41b^2x^2-14abx+5a^2\right)}{40a^4x^2\left(x^2(bx+a)\right)^{\frac{1}{3}}}-\frac{3b^3x}{\left(x^2(bx+a)\right)^{\frac{1}{3}}a^4}$	65

```
int(1/x/(b*x^3+a*x^2)^(4/3),x,method=_RETURNVERBOSE)
```

```
3/40*(-81*b^3*x^3-27*a*b^2*x^2+9*a^2*b*x-5*a^3)/x^2/(x^2*(b*x+a))^(1/3)/a^
4
```

Fricas [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 63, normalized size of antiderivative = 0.61

$$\int \frac{1}{x\left(ax^2+bx^3\right)^{4/3}}\,dx=-\frac{3\left(81\,b^3x^3+27\,ab^2x^2-9\,a^2bx+5\,a^3\right)\left(bx^3+ax^2\right)^{\frac{2}{3}}}{40\left(a^4bx^5+a^5x^4\right)}$$

```
integrate(1/x/(b*x^3+a*x^2)^(4/3),x, algorithm="fricas")
```

```
-3/40*(81*b^3*x^3 + 27*a*b^2*x^2 - 9*a^2*b*x + 5*a^3)*(b*x^3 + a*x^2)^(2/3
)/(a^4*b*x^5 + a^5*x^4)
```

Sympy [F]

$$\int \frac{1}{x\left(ax^2+bx^3\right)^{4/3}}\,dx=\int \frac{1}{x\left(x^2\left(a+bx\right)\right)^{\frac{4}{3}}}\,dx$$

```
integrate(1/x/(b*x**3+a*x**2)**(4/3),x)
```

```
Integral(1/(x*(x**2*(a + b*x))**(4/3)), x)
```

Maxima [F]

$$\int \frac{1}{x\left(ax^2+bx^3\right)^{4/3}}\,dx=\int \frac{1}{\left(bx^3+ax^2\right)^{\frac{4}{3}}x}\,dx$$

```
integrate(1/x/(b*x^3+a*x^2)^(4/3),x, algorithm="maxima")
```

```
integrate(1/((b*x^3 + a*x^2)^(4/3)*x), x)
```

Giac [A] (verification not implemented)

Time = 3.64 (sec) , antiderivative size = 70, normalized size of antiderivative = 0.68

$$\int \frac{1}{x\left(ax^2+bx^3\right)^{4/3}}\,dx=-\frac{3\,b^3}{a^4\left(b+\frac{a}{x}\right)^{\frac{1}{3}}}-\frac{3\left(5\,a^{28}\left(b+\frac{a}{x}\right)^{\frac{8}{3}}-24\,a^{28}\left(b+\frac{a}{x}\right)^{\frac{5}{3}}b+60\,a^{28}\left(b+\frac{a}{x}\right)^{\frac{2}{3}}b^2\right)}{40\,a^{32}}$$

```
integrate(1/x/(b*x^3+a*x^2)^(4/3),x, algorithm="giac")
```

```
-3*b^3/(a^4*(b + a/x)^(1/3)) - 3/40*(5*a^28*(b + a/x)^(8/3) - 24*a^28*(b +
 a/x)^(5/3)*b + 60*a^28*(b + a/x)^(2/3)*b^2)/a^32
```

Mupad [B] (verification not implemented)

Time = 8.51 (sec) , antiderivative size = 58, normalized size of antiderivative = 0.56

$$\int \frac{1}{x\left(ax^2+bx^3\right)^{4/3}}\,dx=-\frac{3\left(b\,x^3+a\,x^2\right)^{2/3}\left(5\,a^3-9\,a^2\,b\,x+27\,a\,b^2\,x^2+81\,b^3\,x^3\right)}{40\,a^4\,x^4\,\left(a+b\,x\right)}$$

```
int(1/(x*(a*x^2 + b*x^3)^(4/3)),x)
```

```
-(3*(a*x^2 + b*x^3)^(2/3)*(5*a^3 + 81*b^3*x^3 + 27*a*b^2*x^2 - 9*a^2*b*x))
/(40*a^4*x^4*(a + b*x))
```

Reduce [F]

$$\int \frac{1}{x\left(ax^2+bx^3\right)^{4/3}}\,dx=\int \frac{1}{x^{\frac{11}{3}}\left(bx+a\right)^{\frac{1}{3}}a+x^{\frac{14}{3}}\left(bx+a\right)^{\frac{1}{3}}b}dx$$

```
int(1/x/(b*x^3+a*x^2)^(4/3),x)
```

```
int(1/(x**(2/3)*(a + b*x)**(1/3)*a*x**3 + x**(2/3)*(a + b*x)**(1/3)*b*x**4
),x)
```

3.385 $\int \frac{1}{x^2(ax^2+bx^3)^{4/3}}\,dx$

Optimal result

Integrand size = 19, antiderivative size = 131

$$\int \frac{1}{x^2\left(ax^2+bx^3\right)^{4/3}}\,dx=\frac{3}{ax^3\sqrt[3]{ax^2+bx^3}}-\frac{36(ax^2+bx^3)^{2/3}}{11a^2x^5}+\frac{81b(ax^2+bx^3)^{2/3}}{22a^3x^4}-\frac{243b^2(ax^2+bx^3)^{2/3}}{55a^4x^3}+\frac{729b^3(ax^2+bx^3)^{2/3}}{110a^5x^2}$$

```
3/a/x^3/(b*x^3+a*x^2)^(1/3)-36/11*(b*x^3+a*x^2)^(2/3)/a^2/x^5+81/22*b*(b*x
^3+a*x^2)^(2/3)/a^3/x^4-243/55*b^2*(b*x^3+a*x^2)^(2/3)/a^4/x^3+729/110*b^3
*(b*x^3+a*x^2)^(2/3)/a^5/x^2
```

Mathematica [A] (verified)

Time = 0.42 (sec) , antiderivative size = 64, normalized size of antiderivative = 0.49

$$\int \frac{1}{x^2\left(ax^2+bx^3\right)^{4/3}}\,dx=\frac{3(-10a^4+15a^3bx-27a^2b^2x^2+81ab^3x^3+243b^4x^4)}{110a^5x^3\sqrt[3]{x^2(a+bx)}}$$

```
Integrate[1/(x^2*(a*x^2 + b*x^3)^(4/3)),x]
```

```
(3*(-10*a^4 + 15*a^3*b*x - 27*a^2*b^2*x^2 + 81*a*b^3*x^3 + 243*b^4*x^4))/(
110*a^5*x^3*(x^2*(a + b*x))^(1/3))
```

Rubi [A] (verified)

Time = 0.54 (sec) , antiderivative size = 149, normalized size of antiderivative = 1.14, number of steps used = 5, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.263$, Rules used = {1921, 1922, 1922, 1922, 1920}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{x^2 \left(ax^2+bx^3\right)^{4/3}} \, dx$$

$$\Big\downarrow \ 1921$$

$$\frac{12 \int \frac{1}{x^4 \sqrt[3]{bx^3+ax^2}}dx}{a}+\frac{3}{ax^3 \sqrt[3]{ax^2+bx^3}}$$

$$\Big\downarrow \ 1922$$

$$\frac{12 \left(-\frac{9b \int \frac{1}{x^3 \sqrt[3]{bx^3+ax^2}}dx}{11a}-\frac{3\left(ax^2+bx^3\right)^{2/3}}{11ax^5}\right)}{a}+\frac{3}{ax^3 \sqrt[3]{ax^2+bx^3}}$$

$$\Big\downarrow \ 1922$$

$$\frac{12 \left(-\frac{9b \left(-\frac{3b \int \frac{1}{x^2 \sqrt[3]{bx^3+ax^2}}dx}{4a}-\frac{3\left(ax^2+bx^3\right)^{2/3}}{8ax^4}\right)}{11a}-\frac{3\left(ax^2+bx^3\right)^{2/3}}{11ax^5}\right)}{a}+\frac{3}{ax^3 \sqrt[3]{ax^2+bx^3}}$$

$$\Big\downarrow \ 1922$$

$$\frac{12\left(-\frac{9b\left(-\frac{3b\left(-\frac{3b\int\frac{1}{x\sqrt[3]{bx^3+ax^2}}dx}{5a}-\frac{3\left(ax^2+bx^3\right)^{2/3}}{5ax^3}\right)}{4a}-\frac{3\left(ax^2+bx^3\right)^{2/3}}{8ax^4}\right)}{11a}-\frac{3\left(ax^2+bx^3\right)^{2/3}}{11ax^5}\right)}{a}+\frac{3}{ax^3\sqrt[3]{ax^2+bx^3}}$$

$$\Big\downarrow \ 1920$$

$$\frac{12\left(-\frac{9b\left(-\frac{3b\left(\frac{9b\left(ax^2+bx^3\right)^{2/3}}{10a^2x^2}-\frac{3\left(ax^2+bx^3\right)^{2/3}}{5ax^3}\right)}{4a}-\frac{3\left(ax^2+bx^3\right)^{2/3}}{8ax^4}\right)}{11a}-\frac{3\left(ax^2+bx^3\right)^{2/3}}{11ax^5}\right)}{a}+\frac{3}{ax^3\sqrt[3]{ax^2+bx^3}}$$

```
Int[1/(x^2*(a*x^2 + b*x^3)^(4/3)),x]
```

```
3/(a*x^3*(a*x^2 + b*x^3)^(1/3)) + (12*((-3*(a*x^2 + b*x^3)^(2/3))/(11*a*x^
5) - (9*b*((-3*(a*x^2 + b*x^3)^(2/3))/(8*a*x^4) - (3*b*((-3*(a*x^2 + b*x^3
)^(2/3))/(5*a*x^3) + (9*b*(a*x^2 + b*x^3)^(2/3))/(10*a^2*x^2)))/(4*a)))/(1
1*a)))/a
```

Defintions of rubi rules used

```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> Simp[(-c^(j - 1))*(c*x)^(m - j + 1)*((a*x^j + b*x^n)^(p + 1)/(a*(n - j
)*(p + 1))), x] /; FreeQ[{a, b, c, j, m, n, p}, x] &&  !IntegerQ[p] && NeQ[
n, j] && EqQ[m + n*p + n - j + 1, 0] && (IntegerQ[j] || GtQ[c, 0])
```

```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> Simp[(-c^(j - 1))*(c*x)^(m - j + 1)*((a*x^j + b*x^n)^(p + 1)/(a*(n - j
)*(p + 1))), x] + Simp[c^j*((m + n*p + n - j + 1)/(a*(n - j)*(p + 1)))   In
t[(c*x)^(m - j)*(a*x^j + b*x^n)^(p + 1), x], x] /; FreeQ[{a, b, c, j, m, n}
, x] &&  !IntegerQ[p] && NeQ[n, j] && ILtQ[Simplify[(m + n*p + n - j + 1)/(
n - j)], 0] && LtQ[p, -1] && (IntegerQ[j] || GtQ[c, 0])
```

```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> Simp[c^(j - 1)*(c*x)^(m - j + 1)*((a*x^j + b*x^n)^(p + 1)/(a*(m + j*p
+ 1))), x] - Simp[b*((m + n*p + n - j + 1)/(a*c^(n - j)*(m + j*p + 1)))   I
nt[(c*x)^(m + n - j)*(a*x^j + b*x^n)^p, x], x] /; FreeQ[{a, b, c, j, m, n,
p}, x] &&  !IntegerQ[p] && NeQ[n, j] && ILtQ[Simplify[(m + n*p + n - j + 1)
/(n - j)], 0] && NeQ[m + j*p + 1, 0] && (IntegersQ[j, n] || GtQ[c, 0])
```

Maple [A] (verified)

Time = 0.47 (sec) , antiderivative size = 61, normalized size of antiderivative = 0.47

method	result	size
pseudoelliptic	$\frac{\frac{729}{110} b^{4} x^{4}+\frac{243}{110} a \,b^{3} x^{3}-\frac{81}{110} a^{2} b^{2} x^{2}+\frac{9}{22} a^{3} b x -\frac{3}{11} a^{4}}{x^{3} \left(x^{2} \left(b x +a \right)\right)^{\frac{1}{3}} a^{5}}$	61
gosper	$-\frac{3 \left(b x +a \right) \left(-243 b^{4} x^{4}-81 a \,b^{3} x^{3}+27 a^{2} b^{2} x^{2}-15 a^{3} b x +10 a^{4}\right)}{110 x \,a^{5} \left(b \,x^{3}+a \,x^{2}\right)^{\frac{4}{3}}}$	68
orering	$-\frac{3 \left(b x +a \right) \left(-243 b^{4} x^{4}-81 a \,b^{3} x^{3}+27 a^{2} b^{2} x^{2}-15 a^{3} b x +10 a^{4}\right)}{110 x \,a^{5} \left(b \,x^{3}+a \,x^{2}\right)^{\frac{4}{3}}}$	68
trager	$-\frac{3 \left(-243 b^{4} x^{4}-81 a \,b^{3} x^{3}+27 a^{2} b^{2} x^{2}-15 a^{3} b x +10 a^{4}\right) \left(b \,x^{3}+a \,x^{2}\right)^{\frac{2}{3}}}{110 \left(b x +a \right) a^{5} x^{5}}$	70
risch	$-\frac{3 \left(b x +a \right) \left(-133 b^{3} x^{3}+52 a \,b^{2} x^{2}-25 a^{2} b x +10 a^{3}\right)}{110 a^{5} x^{3} \left(x^{2} \left(b x +a \right)\right)^{\frac{1}{3}}}+\frac{3 b^{4} x}{\left(x^{2} \left(b x +a \right)\right)^{\frac{1}{3}} a^{5}}$	76

```
int(1/x^2/(b*x^3+a*x^2)^(4/3),x,method=_RETURNVERBOSE)
```

```
3/110*(243*b^4*x^4+81*a*b^3*x^3-27*a^2*b^2*x^2+15*a^3*b*x-10*a^4)/x^3/(x^2
*(b*x+a))^(1/3)/a^5
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 74, normalized size of antiderivative = 0.56

$$\int \frac{1}{x^2 (ax^2 + bx^3)^{4/3}} \, dx = \frac{3 \left(243 \, b^4 x^4 + 81 \, a b^3 x^3 - 27 \, a^2 b^2 x^2 + 15 \, a^3 b x - 10 \, a^4\right) (bx^3 + ax^2)^{\frac{2}{3}}}{110 \left(a^5 b x^6 + a^6 x^5\right)}$$

```
integrate(1/x^2/(b*x^3+a*x^2)^(4/3),x, algorithm="fricas")
```

```
3/110*(243*b^4*x^4 + 81*a*b^3*x^3 - 27*a^2*b^2*x^2 + 15*a^3*b*x - 10*a^4)*
(b*x^3 + a*x^2)^(2/3)/(a^5*b*x^6 + a^6*x^5)
```

Sympy [F]

$$\int \frac{1}{x^2 (ax^2 + bx^3)^{4/3}} \, dx = \int \frac{1}{x^2 \left(x^2 (a + bx)\right)^{\frac{4}{3}}} \, dx$$

```
integrate(1/x**2/(b*x**3+a*x**2)**(4/3),x)
```

```
Integral(1/(x**2*(x**2*(a + b*x))**(4/3)), x)
```

Maxima [F]

$$\int \frac{1}{x^2\left(ax^2+bx^3\right)^{4/3}}\,dx = \int \frac{1}{\left(bx^3+ax^2\right)^{\frac{4}{3}}x^2}\,dx$$

```
integrate(1/x^2/(b*x^3+a*x^2)^(4/3),x, algorithm="maxima")
```

```
integrate(1/((b*x^3 + a*x^2)^(4/3)*x^2), x)
```

Giac [A] (verification not implemented)

Time = 3.31 (sec) , antiderivative size = 92, normalized size of antiderivative = 0.70

$$\int \frac{1}{x^2\left(ax^2+bx^3\right)^{4/3}}\,dx = \frac{3\left(\frac{110\,b^4}{a\left(b+\frac{a}{x}\right)^{\frac{1}{3}}} - \frac{10\,a^{10}\left(b+\frac{a}{x}\right)^{\frac{11}{3}} - 55\,a^{10}\left(b+\frac{a}{x}\right)^{\frac{8}{3}}b + 132\,a^{10}\left(b+\frac{a}{x}\right)^{\frac{5}{3}}b^2 - 220\,a^{10}\left(b+\frac{a}{x}\right)^{\frac{2}{3}}b^3}{a^{11}}\right)}{110\,a^4}$$

```
integrate(1/x^2/(b*x^3+a*x^2)^(4/3),x, algorithm="giac")
```

```
3/110*(110*b^4/(a*(b + a/x)^(1/3)) - (10*a^10*(b + a/x)^(11/3) - 55*a^10*(
b + a/x)^(8/3)*b + 132*a^10*(b + a/x)^(5/3)*b^2 - 220*a^10*(b + a/x)^(2/3)
*b^3)/a^11)/a^4
```

Mupad [B] (verification not implemented)

Time = 8.56 (sec) , antiderivative size = 110, normalized size of antiderivative = 0.84

$$\int \frac{1}{x^2\left(ax^2+bx^3\right)^{4/3}}\,dx = \frac{\left(b\,x^3+a\,x^2\right)^{2/3}\left(\frac{399\,b^3}{110\,a^4}+\frac{729\,b^4\,x}{110\,a^5}\right)}{x^2\,\left(a+b\,x\right)} - \frac{3\left(b\,x^3+a\,x^2\right)^{2/3}}{11\,a^2\,x^5} + \frac{15\,b\left(b\,x^3+a\,x^2\right)^{2/3}}{22\,a^3\,x^4} - \frac{78\,b^2\left(b\,x^3+a\,x^2\right)^{2/3}}{55\,a^4\,x^3}$$

```
int(1/(x^2*(a*x^2 + b*x^3)^(4/3)),x)
```

```
((a*x^2 + b*x^3)^(2/3)*((399*b^3)/(110*a^4) + (729*b^4*x)/(110*a^5)))/(x^2
*(a + b*x)) - (3*(a*x^2 + b*x^3)^(2/3))/(11*a^2*x^5) + (15*b*(a*x^2 + b*x^
3)^(2/3))/(22*a^3*x^4) - (78*b^2*(a*x^2 + b*x^3)^(2/3))/(55*a^4*x^3)
```

Reduce [F]

$$\int \frac{1}{x^2 \left(ax^2+bx^3\right)^{4/3}}\,dx = \int \frac{1}{x^{\frac{14}{3}} (bx+a)^{\frac{1}{3}} a + x^{\frac{17}{3}} (bx+a)^{\frac{1}{3}} b} dx$$

```
int(1/x^2/(b*x^3+a*x^2)^(4/3),x)
```

```
int(1/(x**(2/3)*(a + b*x)**(1/3)*a*x**4 + x**(2/3)*(a + b*x)**(1/3)*b*x**5
),x)
```

3.386 $\int \frac{x^3}{\sqrt[4]{ax^2+bx^3}}\,dx$

Optimal result

Integrand size = 19, antiderivative size = 173

$$\int \frac{x^3}{\sqrt[4]{ax^2+bx^3}}\,dx = -\frac{16a^3x}{39b^3\sqrt[4]{ax^2+bx^3}}+\frac{8a^2x^2}{117b^2\sqrt[4]{ax^2+bx^3}}-\frac{4ax^3}{117b\sqrt[4]{ax^2+bx^3}}+\frac{4x^4}{13\sqrt[4]{ax^2+bx^3}}+\frac{32a^{7/2}\sqrt{x}\sqrt[4]{\frac{a+bx}{a}}E\left(\frac{1}{2}\arctan\left(\frac{\sqrt{b}\sqrt{x}}{\sqrt{a}}\right)\middle|2\right)}{39b^{7/2}\sqrt[4]{ax^2+bx^3}}$$

```
-16/39*a^3*x/b^3/(b*x^3+a*x^2)^(1/4)+8/117*a^2*x^2/b^2/(b*x^3+a*x^2)^(1/4)
-4/117*a*x^3/b/(b*x^3+a*x^2)^(1/4)+4/13*x^4/(b*x^3+a*x^2)^(1/4)+32/39*a^(7
/2)*x^(1/2)*((b*x+a)/a)^(1/4)*EllipticE(sin(1/2*arctan(b^(1/2)*x^(1/2)/a^(
1/2))),2^(1/2))/b^(7/2)/(b*x^3+a*x^2)^(1/4)
```

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 10.02 (sec) , antiderivative size = 49, normalized size of antiderivative = 0.28

$$\int \frac{x^3}{\sqrt[4]{ax^2+bx^3}}\,dx = \frac{2x^4\sqrt[4]{1+\frac{bx}{a}}\,\text{Hypergeometric2F1}\left(\frac{1}{4},\frac{7}{2},\frac{9}{2},-\frac{bx}{a}\right)}{7\sqrt[4]{x^2(a+bx)}}$$

```
Integrate[x^3/(a*x^2 + b*x^3)^(1/4),x]
```

```
(2*x^4*(1 + (b*x)/a)^(1/4)*Hypergeometric2F1[1/4, 7/2, 9/2, -((b*x)/a)])/(
7*(x^2*(a + b*x))^(1/4))
```

Rubi [A] (verified)

Time = 0.84 (sec) , antiderivative size = 248, normalized size of antiderivative = 1.43, number of steps used = 13, number of rules used = 12, $\frac{\text{number of rules}}{\text{integrand size}} = 0.632$, Rules used = {1930, 1930, 1930, 1917, 73, 836, 27, 765, 762, 1390, 1389, 327}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^3}{\sqrt[4]{ax^2+bx^3}}dx$$

$$\Big\downarrow \text{1930}$$

$$\frac{4x\left(ax^2+bx^3\right)^{3/4}}{13b}-\frac{10a\int \frac{x^2}{\sqrt[4]{bx^3+ax^2}}dx}{13b}$$

$$\Big\downarrow \text{1930}$$

$$\frac{4x\left(ax^2+bx^3\right)^{3/4}}{13b}-\frac{10a\left(\frac{4\left(ax^2+bx^3\right)^{3/4}}{9b}-\frac{2a\int \frac{x}{\sqrt[4]{bx^3+ax^2}}dx}{3b}\right)}{13b}$$

$$\Big\downarrow \text{1930}$$

$$\frac{4x\left(ax^2+bx^3\right)^{3/4}}{13b}-\frac{10a\left(\frac{4\left(ax^2+bx^3\right)^{3/4}}{9b}-\frac{2a\left(\frac{4\left(ax^2+bx^3\right)^{3/4}}{5bx}-\frac{2a\int \frac{1}{\sqrt[4]{bx^3+ax^2}}dx}{5b}\right)}{3b}\right)}{13b}$$

$$\Big\downarrow \text{1917}$$

$$\frac{4x\left(ax^2+bx^3\right)^{3/4}}{13b}-\frac{10a\left(\frac{4\left(ax^2+bx^3\right)^{3/4}}{9b}-\frac{2a\left(\frac{4\left(ax^2+bx^3\right)^{3/4}}{5bx}-\frac{2a\sqrt{x}\sqrt[4]{a+bx}\int\frac{1}{\sqrt{x}\sqrt[4]{a+bx}}dx}{5b\sqrt[4]{ax^2+bx^3}}\right)}{3b}\right)}{13b}$$

$$\Big\downarrow \text{73}$$

$$\frac{4x\left(ax^2+bx^3\right)^{3/4}}{13b}-\frac{10a\left(\frac{4\left(ax^2+bx^3\right)^{3/4}}{9b}-\frac{2a\left(\frac{4\left(ax^2+bx^3\right)^{3/4}}{5bx}-\frac{8a\sqrt{x}\sqrt[4]{a+bx}\int\frac{\sqrt{a+bx}}{\sqrt{\frac{a+bx}{b}-\frac{a}{b}}}d\sqrt[4]{a+bx}}{5b^2\sqrt[4]{ax^2+bx^3}}\right)}{3b}\right)}{13b}$$

$$\Big\downarrow \text{836}$$

$$\frac{4x\left(ax^2+bx^3\right)^{3/4}}{13b}-\frac{10a\left(\frac{4\left(ax^2+bx^3\right)^{3/4}}{9b}-\frac{2a\left(\frac{4\left(ax^2+bx^3\right)^{3/4}}{5bx}-\frac{8a\sqrt{x}\sqrt[4]{a+bx}\left(\sqrt{a}\int\frac{\sqrt{a}+\sqrt{a+bx}}{\sqrt{a}\sqrt{\frac{a+bx}{b}-\frac{a}{b}}}d\sqrt[4]{a+bx}-\sqrt{a}\int\frac{1}{\sqrt{\frac{a+bx}{b}-\frac{a}{b}}}d\sqrt[4]{a+bx}\right)}{5b^2\sqrt[4]{ax^2+bx^3}}\right)}{3b}\right)}{13b}$$

$$\Big\downarrow \text{27}$$

$$\frac{4x\left(ax^2+bx^3\right)^{3/4}}{13b}-\frac{10a\left(\frac{4\left(ax^2+bx^3\right)^{3/4}}{9b}-\frac{2a\left(\frac{4\left(ax^2+bx^3\right)^{3/4}}{5bx}-\frac{8a\sqrt{x}\sqrt[4]{a+bx}\left(\int\frac{\sqrt{a}+\sqrt{a+bx}}{\sqrt{\frac{a+bx}{b}-\frac{a}{b}}}d\sqrt[4]{a+bx}-\sqrt{a}\int\frac{1}{\sqrt{\frac{a+bx}{b}-\frac{a}{b}}}d\sqrt[4]{a+bx}\right)}{5b^2\sqrt[4]{ax^2+bx^3}}\right)}{3b}\right)}{13b}$$

$$\Big\downarrow \text{765}$$

$$\frac{\frac{4x\left(ax^2+bx^3\right)^{3/4}}{13b}-\frac{10a\left(\frac{4\left(ax^2+bx^3\right)^{3/4}}{9b}-\frac{2a\left(\frac{4\left(ax^2+bx^3\right)^{3/4}}{5bx}-\frac{8a\sqrt{x}\sqrt[4]{a+bx}\left(\int\frac{\sqrt{a}+\sqrt{a+bx}}{\sqrt{\frac{a+bx}{b}-\frac{a}{b}}}d\sqrt[4]{a+bx}-\frac{\sqrt{a}\sqrt{1-\frac{a+bx}{a}}\int\frac{1}{\sqrt{1-\frac{a+bx}{a}}}d\sqrt[4]{a+bx}}{\sqrt{\frac{a+bx}{b}-\frac{a}{b}}}\right)}{5b^2\sqrt[4]{ax^2+bx^3}}\right)}{3b}\right)}{13b}}{}$$

$\Big\downarrow$ 762

$$\frac{4x\left(ax^2+bx^3\right)^{3/4}}{13b}-\frac{10a\left(\frac{4\left(ax^2+bx^3\right)^{3/4}}{9b}-\frac{2a\left(\frac{4\left(ax^2+bx^3\right)^{3/4}}{5bx}-\frac{8a\sqrt{x}\sqrt[4]{a+bx}\left(\int\frac{\sqrt{a}+\sqrt{a+bx}}{\sqrt{\frac{a+bx}{b}-\frac{a}{b}}}d\sqrt[4]{a+bx}-\frac{a^{3/4}\sqrt{1-\frac{a+bx}{a}}\operatorname{EllipticF}\left(\arcsin\left(\frac{\sqrt[4]{a+bx}}{\sqrt[4]{a}}\right),-1\right)}{\sqrt{\frac{a+bx}{b}-\frac{a}{b}}}\right.}{5b^2\sqrt[4]{ax^2+bx^3}}\right.}{3b}\right.}{13b}$$

$\Big\downarrow$ 1390

$$\frac{4x\left(ax^2+bx^3\right)^{3/4}}{13b}-\frac{10a\left(\frac{4\left(ax^2+bx^3\right)^{3/4}}{9b}-\frac{2a\left(\frac{4\left(ax^2+bx^3\right)^{3/4}}{5bx}-\frac{8a\sqrt{x}\sqrt[4]{a+bx}\left(\frac{\sqrt{1-\frac{a+bx}{a}}\int\frac{\sqrt{a}+\sqrt{a+bx}}{\sqrt{1-\frac{a+bx}{a}}}d\sqrt[4]{a+bx}}{\sqrt{\frac{a+bx}{b}-\frac{a}{b}}}-\frac{a^{3/4}\sqrt{1-\frac{a+bx}{a}}\,\text{EllipticF}\left(\arcsin\left(\frac{\sqrt[4]{a+\cdots}}{\sqrt[4]{\cdots}}\right)\cdots\right)}{\sqrt{\frac{a+bx}{b}-\frac{a}{b}}}\right)}{5b^2\sqrt[4]{ax^2+bx^3}}\right)}{3b}\right)}{13b}$$

$\Big\downarrow$ 1389

$$\frac{4x\left(ax^2+bx^3\right)^{3/4}}{13b}-\frac{10a\left(\frac{4\left(ax^2+bx^3\right)^{3/4}}{9b}-\frac{2a\left(\frac{4\left(ax^2+bx^3\right)^{3/4}}{5bx}-\frac{8a\sqrt{x}\sqrt[4]{a+bx}\left(\frac{\sqrt{a}\sqrt{1-\frac{a+bx}{a}}\int\frac{\sqrt{\frac{\sqrt{a+bx}}{\sqrt{a}}+1}}{\sqrt{1-\frac{\sqrt{a+bx}}{\sqrt{a}}}}d\sqrt[4]{a+bx}}{\sqrt{\frac{a+bx}{b}-\frac{a}{b}}}-\frac{a^{3/4}\sqrt{1-\frac{a+bx}{a}}\,\text{EllipticF}\left(\arcsin\left(\frac{\sqrt[4]{\cdots}}{\cdots}\right)\cdots\right)}{\sqrt{\frac{a+bx}{b}-\frac{a}{b}}}\right)}{5b^2\sqrt[4]{ax^2+bx^3}}\right)}{3b}\right)}{13b}$$

$\Big\downarrow$ 327

$$\frac{4x\left(ax^2+bx^3\right)^{3/4}}{13b}-\frac{10a\left(\frac{4\left(ax^2+bx^3\right)^{3/4}}{9b}-\frac{2a\left(\frac{4\left(ax^2+bx^3\right)^{3/4}}{5bx}-\frac{8a\sqrt{x}\sqrt[4]{a+bx}\left(\frac{a^{3/4}\sqrt{1-\frac{a+bx}{a}}E\left(\arcsin\left(\frac{\sqrt[4]{a+bx}}{\sqrt[4]{a}}\right)\middle|-1\right)}{\sqrt{\frac{a+bx}{b}-\frac{a}{b}}}-\frac{a^{3/4}\sqrt{1-\frac{a+bx}{a}}\operatorname{EllipticF}\left(\arcsin\left(\right.\right.}{\sqrt{\frac{a+bx}{b}-\frac{a}{b}}}\right.}{5b^2\sqrt[4]{ax^2+bx^3}}\right.}{3b}\right.}{13b}$$

```
Int[x^3/(a*x^2 + b*x^3)^(1/4),x]
```

```
(4*x*(a*x^2 + b*x^3)^(3/4))/(13*b) - (10*a*((4*(a*x^2 + b*x^3)^(3/4))/(9*b
) - (2*a*((4*(a*x^2 + b*x^3)^(3/4))/(5*b*x) - (8*a*Sqrt[x]*(a + b*x)^(1/4)
*((a^(3/4)*Sqrt[1 - (a + b*x)/a]*EllipticE[ArcSin[(a + b*x)^(1/4)/a^(1/4)]
, -1])/Sqrt[-(a/b) + (a + b*x)/b] - (a^(3/4)*Sqrt[1 - (a + b*x)/a]*Ellipti
cF[ArcSin[(a + b*x)^(1/4)/a^(1/4)], -1])/Sqrt[-(a/b) + (a + b*x)/b]))/(5*b
^2*(a*x^2 + b*x^3)^(1/4))))/(3*b)))/(13*b)
```

Defintions of rubi rules used

```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Simp[p/b   Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
 d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && Lt
Q[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntL
inearQ[a, b, c, d, m, n, x]
```

```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Simp[
(Sqrt[a]/(Sqrt[c]*Rt[-d/c, 2]))*EllipticE[ArcSin[Rt[-d/c, 2]*x], b*(c/(a*d)
)], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a, 0]
```

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> Simp[(1/(Sqrt[a]*Rt[-b/a, 4])
)*EllipticF[ArcSin[Rt[-b/a, 4]*x], -1], x] /; FreeQ[{a, b}, x] && NegQ[b/a]
 && GtQ[a, 0]
```

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> Simp[Sqrt[1 + b*(x^4/a)]/Sqrt
[a + b*x^4]   Int[1/Sqrt[1 + b*(x^4/a)], x], x] /; FreeQ[{a, b}, x] && NegQ
[b/a] &&  !GtQ[a, 0]
```

```
Int[(x_)^2/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[-b/a, 2]},
Simp[-q^(-1)   Int[1/Sqrt[a + b*x^4], x], x] + Simp[1/q   Int[(1 + q*x^2)/S
qrt[a + b*x^4], x], x]] /; FreeQ[{a, b}, x] && NegQ[b/a]
```

```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (c_.)*(x_)^4], x_Symbol] :> Simp[d/Sq
rt[a]   Int[Sqrt[1 + e*(x^2/d)]/Sqrt[1 - e*(x^2/d)], x], x] /; FreeQ[{a, c,
 d, e}, x] && EqQ[c*d^2 + a*e^2, 0] && NegQ[c/a] && GtQ[a, 0]
```

```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (c_.)*(x_)^4], x_Symbol] :> Simp[Sqrt
[1 + c*(x^4/a)]/Sqrt[a + c*x^4]   Int[(d + e*x^2)/Sqrt[1 + c*(x^4/a)], x],
x] /; FreeQ[{a, c, d, e}, x] && EqQ[c*d^2 + a*e^2, 0] && NegQ[c/a] &&  !GtQ
[a, 0] &&  !(LtQ[a, 0] && GtQ[c, 0])
```

```
Int[((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol] :> Simp[(a*x^j +
b*x^n)^FracPart[p]/(x^(j*FracPart[p])*(a + b*x^(n - j))^FracPart[p])   Int[
x^(j*p)*(a + b*x^(n - j))^p, x], x] /; FreeQ[{a, b, j, n, p}, x] &&  !Integ
erQ[p] && NeQ[n, j] && PosQ[n - j]
```

```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> Simp[c^(n - 1)*(c*x)^(m - n + 1)*((a*x^j + b*x^n)^(p + 1)/(b*(m + n*p
+ 1))), x] - Simp[a*c^(n - j)*((m + j*p - n + j + 1)/(b*(m + n*p + 1)))   I
nt[(c*x)^(m - (n - j))*(a*x^j + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, p},
x] &&  !IntegerQ[p] && LtQ[0, j, n] && (IntegersQ[j, n] || GtQ[c, 0]) && Gt
Q[m + j*p - n + j + 1, 0] && NeQ[m + n*p + 1, 0]
```

Maple [F]

$$\int \frac{x^{3}}{\left(b\,x^{3}+a\,x^{2}\right)^{\frac{1}{4}}}dx$$

```
int(x^3/(b*x^3+a*x^2)^(1/4),x)
```

```
int(x^3/(b*x^3+a*x^2)^(1/4),x)
```

Fricas [F]

$$\int \frac{x^3}{\sqrt[4]{ax^2+bx^3}}\,dx=\int \frac{x^{3}}{\left(bx^{3}+ax^{2}\right)^{\frac{1}{4}}}\,dx$$

```
integrate(x^3/(b*x^3+a*x^2)^(1/4),x, algorithm="fricas")
```

```
integral((b*x^3 + a*x^2)^(3/4)*x/(b*x + a), x)
```

Sympy [F]

$$\int \frac{x^3}{\sqrt[4]{ax^2+bx^3}}\,dx = \int \frac{x^3}{\sqrt[4]{x^2\,(a+bx)}}\,dx$$

```
integrate(x**3/(b*x**3+a*x**2)**(1/4),x)
```

```
Integral(x**3/(x**2*(a + b*x))**(1/4), x)
```

Maxima [F]

$$\int \frac{x^3}{\sqrt[4]{ax^2+bx^3}}\,dx = \int \frac{x^3}{(bx^3+ax^2)^{\frac{1}{4}}}\,dx$$

```
integrate(x^3/(b*x^3+a*x^2)^(1/4),x, algorithm="maxima")
```

```
integrate(x^3/(b*x^3 + a*x^2)^(1/4), x)
```

Giac [F]

$$\int \frac{x^3}{\sqrt[4]{ax^2+bx^3}}\,dx = \int \frac{x^3}{(bx^3+ax^2)^{\frac{1}{4}}}\,dx$$

```
integrate(x^3/(b*x^3+a*x^2)^(1/4),x, algorithm="giac")
```

```
integrate(x^3/(b*x^3 + a*x^2)^(1/4), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{x^3}{\sqrt[4]{ax^2+bx^3}}\,dx = \int \frac{x^3}{\left(b\,x^3+a\,x^2\right)^{1/4}}\,dx$$

```
int(x^3/(a*x^2 + b*x^3)^(1/4),x)
```

```
int(x^3/(a*x^2 + b*x^3)^(1/4), x)
```

Reduce [F]

$$\int \frac{x^3}{\sqrt[4]{ax^2+bx^3}}\,dx$$
$$= \frac{\frac{16\sqrt{x}\,(bx+a)^{\frac{1}{4}}a^3}{231} - \frac{8\sqrt{x}\,(bx+a)^{\frac{1}{4}}a^2bx}{231} + \frac{4\sqrt{x}\,(bx+a)^{\frac{1}{4}}a\,b^2x^2}{165} + \frac{4\sqrt{x}\,(bx+a)^{\frac{1}{4}}b^3x^3}{15} - \frac{8\sqrt{bx+a}\left(\int \frac{\sqrt{x}\,(bx+a)^{\frac{3}{4}}}{b^2x^3+2ab\,x^2+a^2x}dx\right)a^4}{231}}{\sqrt{bx+a}\,b^3}$$

```
int(x^3/(b*x^3+a*x^2)^(1/4),x)
```

```
(4*(20*sqrt(x)*(a + b*x)**(1/4)*a**3 - 10*sqrt(x)*(a + b*x)**(1/4)*a**2*b*
x + 7*sqrt(x)*(a + b*x)**(1/4)*a*b**2*x**2 + 77*sqrt(x)*(a + b*x)**(1/4)*b
**3*x**3 - 10*sqrt(a + b*x)*int((sqrt(x)*(a + b*x)**(3/4))/(a**2*x + 2*a*b
*x**2 + b**2*x**3),x)*a**4))/(1155*sqrt(a + b*x)*b**3)
```

3.387 $\int \frac{x^2}{\sqrt[4]{ax^2+bx^3}}\,dx$

Optimal result

Integrand size = 19, antiderivative size = 145

$$\int \frac{x^2}{\sqrt[4]{ax^2+bx^3}}\,dx=\frac{8a^2x}{15b^2\sqrt[4]{ax^2+bx^3}}-\frac{4ax^2}{45b\sqrt[4]{ax^2+bx^3}}+\frac{4x^3}{9\sqrt[4]{ax^2+bx^3}}-\frac{16a^{5/2}\sqrt{x}\sqrt[4]{\frac{a+bx}{a}}E\left(\frac{1}{2}\arctan\left(\frac{\sqrt{b}\sqrt{x}}{\sqrt{a}}\right)\middle|2\right)}{15b^{5/2}\sqrt[4]{ax^2+bx^3}}$$

```
8/15*a^2*x/b^2/(b*x^3+a*x^2)^(1/4)-4/45*a*x^2/b/(b*x^3+a*x^2)^(1/4)+4/9*x^
3/(b*x^3+a*x^2)^(1/4)-16/15*a^(5/2)*x^(1/2)*((b*x+a)/a)^(1/4)*EllipticE(si
n(1/2*arctan(b^(1/2)*x^(1/2)/a^(1/2))),2^(1/2))/b^(5/2)/(b*x^3+a*x^2)^(1/4
)
```

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 10.02 (sec) , antiderivative size = 49, normalized size of antiderivative = 0.34

$$\int \frac{x^2}{\sqrt[4]{ax^2+bx^3}}\,dx=\frac{2x^3\sqrt[4]{1+\frac{bx}{a}}\,\text{Hypergeometric2F1}\left(\frac{1}{4},\frac{5}{2},\frac{7}{2},-\frac{bx}{a}\right)}{5\sqrt[4]{x^2(a+bx)}}$$

```
Integrate[x^2/(a*x^2 + b*x^3)^(1/4),x]
```

```
(2*x^3*(1 + (b*x)/a)^(1/4)*Hypergeometric2F1[1/4, 5/2, 7/2, -((b*x)/a)])/(
5*(x^2*(a + b*x))^(1/4))
```

Rubi [A] (verified)

Time = 0.74 (sec) , antiderivative size = 216, normalized size of antiderivative = 1.49, number of steps used = 12, number of rules used = 11, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.579, Rules used = {1930, 1930, 1917, 73, 836, 27, 765, 762, 1390, 1389, 327}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^2}{\sqrt[4]{ax^2+bx^3}}dx$$

$$\Big\downarrow \text{1930}$$

$$\frac{4(ax^2+bx^3)^{3/4}}{9b}-\frac{2a\int \frac{x}{\sqrt[4]{bx^3+ax^2}}dx}{3b}$$

$$\Big\downarrow \text{1930}$$

$$\frac{4(ax^2+bx^3)^{3/4}}{9b}-\frac{2a\left(\frac{4(ax^2+bx^3)^{3/4}}{5bx}-\frac{2a\int \frac{1}{\sqrt[4]{bx^3+ax^2}}dx}{5b}\right)}{3b}$$

$$\Big\downarrow \text{1917}$$

$$\frac{4(ax^2+bx^3)^{3/4}}{9b}-\frac{2a\left(\frac{4(ax^2+bx^3)^{3/4}}{5bx}-\frac{2a\sqrt{x}\sqrt[4]{a+bx}\int \frac{1}{\sqrt{x}\sqrt[4]{a+bx}}dx}{5b\sqrt[4]{ax^2+bx^3}}\right)}{3b}$$

$$\Big\downarrow \text{73}$$

$$\frac{4(ax^2+bx^3)^{3/4}}{9b}-\frac{2a\left(\frac{4(ax^2+bx^3)^{3/4}}{5bx}-\frac{8a\sqrt{x}\sqrt[4]{a+bx}\int \frac{\sqrt{a+bx}}{\sqrt{\frac{a+bx}{b}-\frac{a}{b}}}d\sqrt[4]{a+bx}}{5b^2\sqrt[4]{ax^2+bx^3}}\right)}{3b}$$

$$\Big\downarrow \text{836}$$

$$\frac{4\left(ax^2+bx^3\right)^{3/4}}{9b}-\frac{2a\left(\frac{4\left(ax^2+bx^3\right)^{3/4}}{5bx}-\frac{8a\sqrt{x}\sqrt[4]{a+bx}\left(\sqrt{a}\int\frac{\sqrt{a}+\sqrt{a+bx}}{\sqrt{a}\sqrt{\frac{a+bx}{b}-\frac{a}{b}}}d\sqrt[4]{a+bx}-\sqrt{a}\int\frac{1}{\sqrt{\frac{a+bx}{b}-\frac{a}{b}}}d\sqrt[4]{a+bx}\right)}{5b^2\sqrt[4]{ax^2+bx^3}}\right)}{3b}$$

$$\Big\downarrow \text{27}$$

$$\frac{4\left(ax^2+bx^3\right)^{3/4}}{9b}-\frac{2a\left(\frac{4\left(ax^2+bx^3\right)^{3/4}}{5bx}-\frac{8a\sqrt{x}\sqrt[4]{a+bx}\left(\int\frac{\sqrt{a}+\sqrt{a+bx}}{\sqrt{\frac{a+bx}{b}-\frac{a}{b}}}d\sqrt[4]{a+bx}-\sqrt{a}\int\frac{1}{\sqrt{\frac{a+bx}{b}-\frac{a}{b}}}d\sqrt[4]{a+bx}\right)}{5b^2\sqrt[4]{ax^2+bx^3}}\right)}{3b}$$

$$\Big\downarrow \text{765}$$

$$\frac{4\left(ax^2+bx^3\right)^{3/4}}{9b}-\frac{2a\left(\frac{4\left(ax^2+bx^3\right)^{3/4}}{5bx}-\frac{8a\sqrt{x}\sqrt[4]{a+bx}\left(\int\frac{\sqrt{a}+\sqrt{a+bx}}{\sqrt{\frac{a+bx}{b}-\frac{a}{b}}}d\sqrt[4]{a+bx}-\frac{\sqrt{a}\sqrt{1-\frac{a+bx}{a}}\int\frac{1}{\sqrt{1-\frac{a+bx}{a}}}d\sqrt[4]{a+bx}}{\sqrt{\frac{a+bx}{b}-\frac{a}{b}}}\right)}{5b^2\sqrt[4]{ax^2+bx^3}}\right)}{3b}$$

$$\Big\downarrow \text{762}$$

$$\frac{4\left(ax^2+bx^3\right)^{3/4}}{9b}-\frac{2a\left(\frac{4\left(ax^2+bx^3\right)^{3/4}}{5bx}-\frac{8a\sqrt{x}\sqrt[4]{a+bx}\left(\int\frac{\sqrt{a}+\sqrt{a+bx}}{\sqrt{\frac{a+bx}{b}-\frac{a}{b}}}d\sqrt[4]{a+bx}-\frac{a^{3/4}\sqrt{1-\frac{a+bx}{a}}\text{EllipticF}\left(\arcsin\left(\frac{\sqrt[4]{a+bx}}{\sqrt[4]{a}}\right),-1\right)}{\sqrt{\frac{a+bx}{b}-\frac{a}{b}}}\right)}{5b^2\sqrt[4]{ax^2+bx^3}}\right)}{3b}$$

$$\Big\downarrow \text{1390}$$

$$\frac{2a\left(\frac{4(ax^2+bx^3)^{3/4}}{5bx}-\frac{8a\sqrt{x}\sqrt[4]{a+bx}\left(\frac{\sqrt{1-\frac{a+bx}{a}}\int\frac{\sqrt{a}+\sqrt{a+bx}}{\sqrt{1-\frac{a+bx}{a}}}d\sqrt[4]{a+bx}}{\sqrt{\frac{a+bx}{b}-\frac{a}{b}}}-\frac{a^{3/4}\sqrt{1-\frac{a+bx}{a}}\,\text{EllipticF}\left(\arcsin\left(\frac{\sqrt[4]{a+bx}}{\sqrt[4]{a}}\right),-1\right)}{\sqrt{\frac{a+bx}{b}-\frac{a}{b}}}\right)}{5b^2\sqrt[4]{ax^2+bx^3}}\right)}{3b}$$

$\frac{4(ax^2+bx^3)^{3/4}}{9b}-$

$$\Big\downarrow \text{1389}$$

$$\frac{4(ax^2+bx^3)^{3/4}}{9b}-$$

$$\frac{2a\left(\frac{4(ax^2+bx^3)^{3/4}}{5bx}-\frac{8a\sqrt{x}\sqrt[4]{a+bx}\left(\frac{\sqrt{a}\sqrt{1-\frac{a+bx}{a}}\int\frac{\sqrt{\frac{\sqrt{a+bx}}{\sqrt{a}}+1}}{\sqrt{1-\frac{\sqrt{a+bx}}{\sqrt{a}}}}d\sqrt[4]{a+bx}}{\sqrt{\frac{a+bx}{b}-\frac{a}{b}}}-\frac{a^{3/4}\sqrt{1-\frac{a+bx}{a}}\,\text{EllipticF}\left(\arcsin\left(\frac{\sqrt[4]{a+bx}}{\sqrt[4]{a}}\right),-1\right)}{\sqrt{\frac{a+bx}{b}-\frac{a}{b}}}\right)}{5b^2\sqrt[4]{ax^2+bx^3}}\right)}{3b}$$

$$\Big\downarrow \text{327}$$

$$\frac{4(ax^2+bx^3)^{3/4}}{9b}-$$

$$\frac{2a\left(\frac{4(ax^2+bx^3)^{3/4}}{5bx}-\frac{8a\sqrt{x}\sqrt[4]{a+bx}\left(\frac{a^{3/4}\sqrt{1-\frac{a+bx}{a}}E\left(\arcsin\left(\frac{\sqrt[4]{a+bx}}{\sqrt[4]{a}}\right)\middle|-1\right)}{\sqrt{\frac{a+bx}{b}-\frac{a}{b}}}-\frac{a^{3/4}\sqrt{1-\frac{a+bx}{a}}\,\text{EllipticF}\left(\arcsin\left(\frac{\sqrt[4]{a+bx}}{\sqrt[4]{a}}\right),-1\right)}{\sqrt{\frac{a+bx}{b}-\frac{a}{b}}}\right)}{5b^2\sqrt[4]{ax^2+bx^3}}\right)}{3b}$$

```
Int[x^2/(a*x^2 + b*x^3)^(1/4),x]
```

```
(4*(a*x^2 + b*x^3)^(3/4))/(9*b) - (2*a*((4*(a*x^2 + b*x^3)^(3/4))/(5*b*x)
- (8*a*Sqrt[x]*(a + b*x)^(1/4)*((a^(3/4)*Sqrt[1 - (a + b*x)/a]*EllipticE[A
rcSin[(a + b*x)^(1/4)/a^(1/4)], -1])/Sqrt[-(a/b) + (a + b*x)/b] - (a^(3/4)
*Sqrt[1 - (a + b*x)/a]*EllipticF[ArcSin[(a + b*x)^(1/4)/a^(1/4)], -1])/Sqr
t[-(a/b) + (a + b*x)/b]))/(5*b^2*(a*x^2 + b*x^3)^(1/4))))/(3*b)
```

Defintions of rubi rules used

```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Simp[p/b   Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
 d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && Lt
Q[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntL
inearQ[a, b, c, d, m, n, x]
```

```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Simp[
(Sqrt[a]/(Sqrt[c]*Rt[-d/c, 2]))*EllipticE[ArcSin[Rt[-d/c, 2]*x], b*(c/(a*d)
)], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a, 0]
```

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> Simp[(1/(Sqrt[a]*Rt[-b/a, 4])
)*EllipticF[ArcSin[Rt[-b/a, 4]*x], -1], x] /; FreeQ[{a, b}, x] && NegQ[b/a]
 && GtQ[a, 0]
```

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> Simp[Sqrt[1 + b*(x^4/a)]/Sqrt
[a + b*x^4]   Int[1/Sqrt[1 + b*(x^4/a)], x], x] /; FreeQ[{a, b}, x] && NegQ
[b/a] &&  !GtQ[a, 0]
```

```
Int[(x_)^2/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[-b/a, 2]},
Simp[-q^(-1)   Int[1/Sqrt[a + b*x^4], x], x] + Simp[1/q   Int[(1 + q*x^2)/S
qrt[a + b*x^4], x], x]] /; FreeQ[{a, b}, x] && NegQ[b/a]
```

```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (c_.)*(x_)^4], x_Symbol] :> Simp[d/Sq
rt[a]   Int[Sqrt[1 + e*(x^2/d)]/Sqrt[1 - e*(x^2/d)], x], x] /; FreeQ[{a, c,
 d, e}, x] && EqQ[c*d^2 + a*e^2, 0] && NegQ[c/a] && GtQ[a, 0]
```

```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (c_.)*(x_)^4], x_Symbol] :> Simp[Sqrt
[1 + c*(x^4/a)]/Sqrt[a + c*x^4]   Int[(d + e*x^2)/Sqrt[1 + c*(x^4/a)], x],
x] /; FreeQ[{a, c, d, e}, x] && EqQ[c*d^2 + a*e^2, 0] && NegQ[c/a] &&  !GtQ
[a, 0] &&  !(LtQ[a, 0] && GtQ[c, 0])
```

```
Int[((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol] :> Simp[(a*x^j +
b*x^n)^FracPart[p]/(x^(j*FracPart[p])*(a + b*x^(n - j))^FracPart[p])   Int[
x^(j*p)*(a + b*x^(n - j))^p, x], x] /; FreeQ[{a, b, j, n, p}, x] &&  !Integ
erQ[p] && NeQ[n, j] && PosQ[n - j]
```

```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> Simp[c^(n - 1)*(c*x)^(m - n + 1)*((a*x^j + b*x^n)^(p + 1)/(b*(m + n*p
+ 1))), x] - Simp[a*c^(n - j)*((m + j*p - n + j + 1)/(b*(m + n*p + 1)))   I
nt[(c*x)^(m - (n - j))*(a*x^j + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, p},
x] &&  !IntegerQ[p] && LtQ[0, j, n] && (IntegersQ[j, n] || GtQ[c, 0]) && Gt
Q[m + j*p - n + j + 1, 0] && NeQ[m + n*p + 1, 0]
```

Maple [F]

$$\int \frac{x^2}{\left(b\,x^3+a\,x^2\right)^{\frac{1}{4}}}dx$$

```
int(x^2/(b*x^3+a*x^2)^(1/4),x)
```

```
int(x^2/(b*x^3+a*x^2)^(1/4),x)
```

Fricas [F]

$$\int \frac{x^2}{\sqrt[4]{ax^2+bx^3}}\,dx = \int \frac{x^2}{(bx^3+ax^2)^{\frac{1}{4}}}\,dx$$

```
integrate(x^2/(b*x^3+a*x^2)^(1/4),x, algorithm="fricas")
```

```
integral((b*x^3 + a*x^2)^(3/4)/(b*x + a), x)
```

Sympy [F]

$$\int \frac{x^2}{\sqrt[4]{ax^2+bx^3}}\,dx = \int \frac{x^2}{\sqrt[4]{x^2\,(a+bx)}}\,dx$$

```
integrate(x**2/(b*x**3+a*x**2)**(1/4),x)
```

```
Integral(x**2/(x**2*(a + b*x))**(1/4), x)
```

Maxima [F]

$$\int \frac{x^2}{\sqrt[4]{ax^2+bx^3}}\,dx = \int \frac{x^2}{(bx^3+ax^2)^{\frac{1}{4}}}\,dx$$

```
integrate(x^2/(b*x^3+a*x^2)^(1/4),x, algorithm="maxima")
```

```
integrate(x^2/(b*x^3 + a*x^2)^(1/4), x)
```

Giac [F]

$$\int \frac{x^2}{\sqrt[4]{ax^2+bx^3}}\,dx = \int \frac{x^2}{(bx^3+ax^2)^{\frac{1}{4}}}\,dx$$

```
integrate(x^2/(b*x^3+a*x^2)^(1/4),x, algorithm="giac")
```

```
integrate(x^2/(b*x^3 + a*x^2)^(1/4), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{x^2}{\sqrt[4]{ax^2+bx^3}}\,dx = \int \frac{x^2}{(b\,x^3+a\,x^2)^{1/4}}\,dx$$

```
int(x^2/(a*x^2 + b*x^3)^(1/4),x)
```

```
int(x^2/(a*x^2 + b*x^3)^(1/4), x)
```

Reduce [F]

$$\int \frac{x^2}{\sqrt[4]{ax^2+bx^3}}\,dx$$

$$= \frac{-\frac{8\sqrt{x}\,(bx+a)^{\frac{1}{4}}a^2}{77}+\frac{4\sqrt{x}\,(bx+a)^{\frac{1}{4}}abx}{77}+\frac{4\sqrt{x}\,(bx+a)^{\frac{1}{4}}b^2x^2}{11}+\frac{4\sqrt{bx+a}\left(\int\frac{\sqrt{x}\,(bx+a)^{\frac{3}{4}}}{b^2x^3+2ab\,x^2+a^2x}dx\right)a^3}{77}}{\sqrt{bx+a}\,b^2}$$

```
int(x^2/(b*x^3+a*x^2)^(1/4),x)
```

```
(4*( - 2*sqrt(x)*(a + b*x)**(1/4)*a**2 + sqrt(x)*(a + b*x)**(1/4)*a*b*x +
7*sqrt(x)*(a + b*x)**(1/4)*b**2*x**2 + sqrt(a + b*x)*int((sqrt(x)*(a + b*x
)**(3/4))/(a**2*x + 2*a*b*x**2 + b**2*x**3),x)*a**3))/(77*sqrt(a + b*x)*b*
*2)
```

3.388 $\int \frac{x}{\sqrt[4]{ax^2+bx^3}}\,dx$

Optimal result

Integrand size = 17, antiderivative size = 117

$$\int \frac{x}{\sqrt[4]{ax^2+bx^3}}\,dx = -\frac{4ax}{5b\sqrt[4]{ax^2+bx^3}}+\frac{4x^2}{5\sqrt[4]{ax^2+bx^3}} + \frac{8a^{3/2}\sqrt{x}\sqrt[4]{\frac{a+bx}{a}}E\left(\frac{1}{2}\arctan\left(\frac{\sqrt{b}\sqrt{x}}{\sqrt{a}}\right)\middle|2\right)}{5b^{3/2}\sqrt[4]{ax^2+bx^3}}$$

```
-4/5*a*x/b/(b*x^3+a*x^2)^(1/4)+4/5*x^2/(b*x^3+a*x^2)^(1/4)+8/5*a^(3/2)*x^(
1/2)*((b*x+a)/a)^(1/4)*EllipticE(sin(1/2*arctan(b^(1/2)*x^(1/2)/a^(1/2))),
2^(1/2))/b^(3/2)/(b*x^3+a*x^2)^(1/4)
```

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 10.01 (sec) , antiderivative size = 49, normalized size of antiderivative = 0.42

$$\int \frac{x}{\sqrt[4]{ax^2+bx^3}}\,dx = \frac{2x^2\sqrt[4]{1+\frac{bx}{a}}\,\text{Hypergeometric2F1}\left(\frac{1}{4},\frac{3}{2},\frac{5}{2},-\frac{bx}{a}\right)}{3\sqrt[4]{x^2(a+bx)}}$$

```
Integrate[x/(a*x^2 + b*x^3)^(1/4),x]
```

```
(2*x^2*(1 + (b*x)/a)^(1/4)*Hypergeometric2F1[1/4, 3/2, 5/2, -((b*x)/a)])/(
3*(x^2*(a + b*x))^(1/4))
```

Rubi [A] (verified)

Time = 0.61 (sec) , antiderivative size = 185, normalized size of antiderivative = 1.58, number of steps used = 11, number of rules used = 10, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.588, Rules used = {1930, 1917, 73, 836, 27, 765, 762, 1390, 1389, 327}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x}{\sqrt[4]{ax^2+bx^3}}dx$$

$$\Big\downarrow \text{1930}$$

$$\frac{4(ax^2+bx^3)^{3/4}}{5bx}-\frac{2a\int \frac{1}{\sqrt[4]{bx^3+ax^2}}dx}{5b}$$

$$\Big\downarrow \text{1917}$$

$$\frac{4(ax^2+bx^3)^{3/4}}{5bx}-\frac{2a\sqrt{x}\sqrt[4]{a+bx}\int \frac{1}{\sqrt{x}\sqrt[4]{a+bx}}dx}{5b\sqrt[4]{ax^2+bx^3}}$$

$$\Big\downarrow \text{73}$$

$$\frac{4(ax^2+bx^3)^{3/4}}{5bx}-\frac{8a\sqrt{x}\sqrt[4]{a+bx}\int \frac{\sqrt{a+bx}}{\sqrt{\frac{a+bx}{b}-\frac{a}{b}}}d\sqrt[4]{a+bx}}{5b^2\sqrt[4]{ax^2+bx^3}}$$

$$\Big\downarrow \text{836}$$

$$\frac{4(ax^2+bx^3)^{3/4}}{5bx}-\frac{8a\sqrt{x}\sqrt[4]{a+bx}\left(\sqrt{a}\int \frac{\sqrt{a}+\sqrt{a+bx}}{\sqrt{a}\sqrt{\frac{a+bx}{b}-\frac{a}{b}}}d\sqrt[4]{a+bx}-\sqrt{a}\int \frac{1}{\sqrt{\frac{a+bx}{b}-\frac{a}{b}}}d\sqrt[4]{a+bx}\right)}{5b^2\sqrt[4]{ax^2+bx^3}}$$

$$\Big\downarrow \text{27}$$

$$\frac{4\left(ax^2+bx^3\right)^{3/4}}{5bx}-\frac{8a\sqrt{x}\sqrt[4]{a+bx}\left(\int \frac{\sqrt{a}+\sqrt{a+bx}}{\sqrt{\frac{a+bx}{b}-\frac{a}{b}}}d\sqrt[4]{a+bx}-\sqrt{a}\int \frac{1}{\sqrt{\frac{a+bx}{b}-\frac{a}{b}}}d\sqrt[4]{a+bx}\right)}{5b^2\sqrt[4]{ax^2+bx^3}}$$

$$\Big\downarrow \text{765}$$

$$\frac{4\left(ax^2+bx^3\right)^{3/4}}{5bx}-\frac{8a\sqrt{x}\sqrt[4]{a+bx}\left(\int \frac{\sqrt{a}+\sqrt{a+bx}}{\sqrt{\frac{a+bx}{b}-\frac{a}{b}}}d\sqrt[4]{a+bx}-\frac{\sqrt{a}\sqrt{1-\frac{a+bx}{a}}\int \frac{1}{\sqrt{1-\frac{a+bx}{a}}}d\sqrt[4]{a+bx}}{\sqrt{\frac{a+bx}{b}-\frac{a}{b}}}\right)}{5b^2\sqrt[4]{ax^2+bx^3}}$$

$$\Big\downarrow \text{762}$$

$$\frac{4\left(ax^2+bx^3\right)^{3/4}}{5bx}-\frac{8a\sqrt{x}\sqrt[4]{a+bx}\left(\int \frac{\sqrt{a}+\sqrt{a+bx}}{\sqrt{\frac{a+bx}{b}-\frac{a}{b}}}d\sqrt[4]{a+bx}-\frac{a^{3/4}\sqrt{1-\frac{a+bx}{a}}\,\text{EllipticF}\left(\arcsin\left(\frac{\sqrt[4]{a+bx}}{\sqrt[4]{a}}\right),-1\right)}{\sqrt{\frac{a+bx}{b}-\frac{a}{b}}}\right)}{5b^2\sqrt[4]{ax^2+bx^3}}$$

$$\Big\downarrow \text{1390}$$

$$\frac{4\left(ax^2+bx^3\right)^{3/4}}{5bx}-\frac{8a\sqrt{x}\sqrt[4]{a+bx}\left(\frac{\sqrt{1-\frac{a+bx}{a}}\int \frac{\sqrt{a}+\sqrt{a+bx}}{\sqrt{1-\frac{a+bx}{a}}}d\sqrt[4]{a+bx}}{\sqrt{\frac{a+bx}{b}-\frac{a}{b}}}-\frac{a^{3/4}\sqrt{1-\frac{a+bx}{a}}\,\text{EllipticF}\left(\arcsin\left(\frac{\sqrt[4]{a+bx}}{\sqrt[4]{a}}\right),-1\right)}{\sqrt{\frac{a+bx}{b}-\frac{a}{b}}}\right)}{5b^2\sqrt[4]{ax^2+bx^3}}$$

$$\Big\downarrow \text{1389}$$

$$\frac{4\left(ax^2+bx^3\right)^{3/4}}{5bx}-\frac{8a\sqrt{x}\sqrt[4]{a+bx}\left(\frac{\sqrt{a}\sqrt{1-\frac{a+bx}{a}}\int \frac{\sqrt{\frac{\sqrt{a+bx}}{\sqrt{a}}+1}}{\sqrt{1-\frac{\sqrt{a+bx}}{\sqrt{a}}}}d\sqrt[4]{a+bx}}{\sqrt{\frac{a+bx}{b}-\frac{a}{b}}}-\frac{a^{3/4}\sqrt{1-\frac{a+bx}{a}}\,\text{EllipticF}\left(\arcsin\left(\frac{\sqrt[4]{a+bx}}{\sqrt[4]{a}}\right),-1\right)}{\sqrt{\frac{a+bx}{b}-\frac{a}{b}}}\right)}{5b^2\sqrt[4]{ax^2+bx^3}}$$

$$\Big\downarrow \text{327}$$

$$\frac{4(ax^2+bx^3)^{3/4}}{5bx} - \frac{8a\sqrt{x}\sqrt[4]{a+bx}\left(\frac{a^{3/4}\sqrt{1-\frac{a+bx}{a}}E\left(\arcsin\left(\frac{\sqrt[4]{a+bx}}{\sqrt[4]{a}}\right)\middle|-1\right)}{\sqrt{\frac{a+bx}{b}-\frac{a}{b}}} - \frac{a^{3/4}\sqrt{1-\frac{a+bx}{a}}\,\text{EllipticF}\left(\arcsin\left(\frac{\sqrt[4]{a+bx}}{\sqrt[4]{a}}\right),-1\right)}{\sqrt{\frac{a+bx}{b}-\frac{a}{b}}}\right)}{5b^2\sqrt[4]{ax^2+bx^3}}$$

```
Int[x/(a*x^2 + b*x^3)^(1/4),x]
```

```
(4*(a*x^2 + b*x^3)^(3/4))/(5*b*x) - (8*a*Sqrt[x]*(a + b*x)^(1/4)*((a^(3/4)
*Sqrt[1 - (a + b*x)/a]*EllipticE[ArcSin[(a + b*x)^(1/4)/a^(1/4)], -1])/Sqr
t[-(a/b) + (a + b*x)/b] - (a^(3/4)*Sqrt[1 - (a + b*x)/a]*EllipticF[ArcSin[
(a + b*x)^(1/4)/a^(1/4)], -1])/Sqrt[-(a/b) + (a + b*x)/b]))/(5*b^2*(a*x^2
+ b*x^3)^(1/4))
```

Defintions of rubi rules used

```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Simp[p/b   Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
 d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && Lt
Q[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntL
inearQ[a, b, c, d, m, n, x]
```

```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Simp[
(Sqrt[a]/(Sqrt[c]*Rt[-d/c, 2]))*EllipticE[ArcSin[Rt[-d/c, 2]*x], b*(c/(a*d)
)], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a, 0]
```

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> Simp[(1/(Sqrt[a]*Rt[-b/a, 4])
)*EllipticF[ArcSin[Rt[-b/a, 4]*x], -1], x] /; FreeQ[{a, b}, x] && NegQ[b/a]
 && GtQ[a, 0]
```

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> Simp[Sqrt[1 + b*(x^4/a)]/Sqrt
[a + b*x^4]   Int[1/Sqrt[1 + b*(x^4/a)], x], x] /; FreeQ[{a, b}, x] && NegQ
[b/a] &&  !GtQ[a, 0]
```

```
Int[(x_)^2/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[-b/a, 2]},
Simp[-q^(-1)   Int[1/Sqrt[a + b*x^4], x], x] + Simp[1/q   Int[(1 + q*x^2)/S
qrt[a + b*x^4], x], x]] /; FreeQ[{a, b}, x] && NegQ[b/a]
```

```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (c_.)*(x_)^4], x_Symbol] :> Simp[d/Sq
rt[a]   Int[Sqrt[1 + e*(x^2/d)]/Sqrt[1 - e*(x^2/d)], x], x] /; FreeQ[{a, c,
 d, e}, x] && EqQ[c*d^2 + a*e^2, 0] && NegQ[c/a] && GtQ[a, 0]
```

```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (c_.)*(x_)^4], x_Symbol] :> Simp[Sqrt
[1 + c*(x^4/a)]/Sqrt[a + c*x^4]   Int[(d + e*x^2)/Sqrt[1 + c*(x^4/a)], x],
x] /; FreeQ[{a, c, d, e}, x] && EqQ[c*d^2 + a*e^2, 0] && NegQ[c/a] &&  !GtQ
[a, 0] &&  !(LtQ[a, 0] && GtQ[c, 0])
```

```
Int[((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol] :> Simp[(a*x^j +
b*x^n)^FracPart[p]/(x^(j*FracPart[p])*(a + b*x^(n - j))^FracPart[p])   Int[
x^(j*p)*(a + b*x^(n - j))^p, x], x] /; FreeQ[{a, b, j, n, p}, x] &&  !Integ
erQ[p] && NeQ[n, j] && PosQ[n - j]
```

```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> Simp[c^(n - 1)*(c*x)^(m - n + 1)*((a*x^j + b*x^n)^(p + 1)/(b*(m + n*p
+ 1))), x] - Simp[a*c^(n - j)*((m + j*p - n + j + 1)/(b*(m + n*p + 1)))   I
nt[(c*x)^(m - (n - j))*(a*x^j + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, p},
x] &&  !IntegerQ[p] && LtQ[0, j, n] && (IntegersQ[j, n] || GtQ[c, 0]) && Gt
Q[m + j*p - n + j + 1, 0] && NeQ[m + n*p + 1, 0]
```

Maple [F]

$$\int \frac{x}{\left(b\,x^{3}+a\,x^{2}\right)^{\frac{1}{4}}}dx$$

```
int(x/(b*x^3+a*x^2)^(1/4),x)
```

```
int(x/(b*x^3+a*x^2)^(1/4),x)
```

Fricas [F]

$$\int \frac{x}{\sqrt[4]{ax^2+bx^3}}\,dx=\int \frac{x}{\left(bx^3+ax^2\right)^{\frac{1}{4}}}\,dx$$

```
integrate(x/(b*x^3+a*x^2)^(1/4),x, algorithm="fricas")
```

```
integral((b*x^3 + a*x^2)^(3/4)/(b*x^2 + a*x), x)
```

Sympy [F]

$$\int \frac{x}{\sqrt[4]{ax^2+bx^3}}\,dx=\int \frac{x}{\sqrt[4]{x^2\left(a+bx\right)}}\,dx$$

```
integrate(x/(b*x**3+a*x**2)**(1/4),x)
```

```
Integral(x/(x**2*(a + b*x))**(1/4), x)
```

Maxima [F]

$$\int \frac{x}{\sqrt[4]{ax^2+bx^3}}\,dx = \int \frac{x}{(bx^3+ax^2)^{\frac{1}{4}}}\,dx$$

```
integrate(x/(b*x^3+a*x^2)^(1/4),x, algorithm="maxima")
```

```
integrate(x/(b*x^3 + a*x^2)^(1/4), x)
```

Giac [F]

$$\int \frac{x}{\sqrt[4]{ax^2+bx^3}}\,dx = \int \frac{x}{(bx^3+ax^2)^{\frac{1}{4}}}\,dx$$

```
integrate(x/(b*x^3+a*x^2)^(1/4),x, algorithm="giac")
```

```
integrate(x/(b*x^3 + a*x^2)^(1/4), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{x}{\sqrt[4]{ax^2+bx^3}}\,dx = \int \frac{x}{(b\,x^3+a\,x^2)^{1/4}}\,dx$$

```
int(x/(a*x^2 + b*x^3)^(1/4),x)
```

```
int(x/(a*x^2 + b*x^3)^(1/4), x)
```

Reduce [F]

$$\int \frac{x}{\sqrt[4]{ax^2+bx^3}}\,dx = \frac{\frac{4\sqrt{x}\,(bx+a)^{\frac{1}{4}}a}{21}+\frac{4\sqrt{x}\,(bx+a)^{\frac{1}{4}}bx}{7}-\frac{2\sqrt{bx+a}\left(\int \frac{\sqrt{x}\,(bx+a)^{\frac{3}{4}}}{b^2x^3+2ab\,x^2+a^2x}dx\right)a^2}{21}}{\sqrt{bx+a}\,b}$$

```
int(x/(b*x^3+a*x^2)^(1/4),x)
```

```
(2*(2*sqrt(x)*(a + b*x)**(1/4)*a + 6*sqrt(x)*(a + b*x)**(1/4)*b*x - sqrt(a
 + b*x)*int((sqrt(x)*(a + b*x)**(3/4))/(a**2*x + 2*a*b*x**2 + b**2*x**3),x
)*a**2))/(21*sqrt(a + b*x)*b)
```

3.389 $\int \frac{1}{\sqrt[4]{ax^2+bx^3}}\,dx$

Optimal result

Integrand size = 15, antiderivative size = 87

$$\int \frac{1}{\sqrt[4]{ax^2+bx^3}}\,dx=\frac{4x}{\sqrt[4]{ax^2+bx^3}}-\frac{4\sqrt{a}\sqrt{x}\sqrt[4]{\frac{a+bx}{a}}E\left(\frac{1}{2}\arctan\left(\frac{\sqrt{b}\sqrt{x}}{\sqrt{a}}\right)\middle|2\right)}{\sqrt{b}\sqrt[4]{ax^2+bx^3}}$$

```
4*x/(b*x^3+a*x^2)^(1/4)-4*a^(1/2)*x^(1/2)*((b*x+a)/a)^(1/4)*EllipticE(sin(
1/2*arctan(b^(1/2)*x^(1/2)/a^(1/2))),2^(1/2))/b^(1/2)/(b*x^3+a*x^2)^(1/4)
```

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 0.01 (sec) , antiderivative size = 45, normalized size of antiderivative = 0.52

$$\int \frac{1}{\sqrt[4]{ax^2+bx^3}}\,dx=\frac{2x\sqrt[4]{1+\frac{bx}{a}}\,\text{Hypergeometric2F1}\left(\frac{1}{4},\frac{1}{2},\frac{3}{2},-\frac{bx}{a}\right)}{\sqrt[4]{x^2(a+bx)}}$$

```
Integrate[(a*x^2 + b*x^3)^(-1/4),x]
```

```
(2*x*(1 + (b*x)/a)^(1/4)*Hypergeometric2F1[1/4, 1/2, 3/2, -((b*x)/a)])/(x^
2*(a + b*x))^(1/4)
```

Rubi [A] (verified)

Time = 0.57 (sec) , antiderivative size = 156, normalized size of antiderivative = 1.79, number of steps used = 10, number of rules used = 9, $\frac{\text{number of rules}}{\text{integrand size}} = 0.600$, Rules used = {1917, 73, 836, 27, 765, 762, 1390, 1389, 327}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{\sqrt[4]{ax^2+bx^3}}dx$$

$$\Big\downarrow \text{1917}$$

$$\frac{\sqrt{x}\sqrt[4]{a+bx}\int \frac{1}{\sqrt{x}\sqrt[4]{a+bx}}dx}{\sqrt[4]{ax^2+bx^3}}$$

$$\Big\downarrow \text{73}$$

$$\frac{4\sqrt{x}\sqrt[4]{a+bx}\int \frac{\sqrt{a+bx}}{\sqrt{\frac{a+bx}{b}-\frac{a}{b}}}d\sqrt[4]{a+bx}}{b\sqrt[4]{ax^2+bx^3}}$$

$$\Big\downarrow \text{836}$$

$$\frac{4\sqrt{x}\sqrt[4]{a+bx}\left(\sqrt{a}\int \frac{\sqrt{a}+\sqrt{a+bx}}{\sqrt{a}\sqrt{\frac{a+bx}{b}-\frac{a}{b}}}d\sqrt[4]{a+bx}-\sqrt{a}\int \frac{1}{\sqrt{\frac{a+bx}{b}-\frac{a}{b}}}d\sqrt[4]{a+bx}\right)}{b\sqrt[4]{ax^2+bx^3}}$$

$$\Big\downarrow \text{27}$$

$$\frac{4\sqrt{x}\sqrt[4]{a+bx}\left(\int \frac{\sqrt{a}+\sqrt{a+bx}}{\sqrt{\frac{a+bx}{b}-\frac{a}{b}}}d\sqrt[4]{a+bx}-\sqrt{a}\int \frac{1}{\sqrt{\frac{a+bx}{b}-\frac{a}{b}}}d\sqrt[4]{a+bx}\right)}{b\sqrt[4]{ax^2+bx^3}}$$

$$\Big\downarrow \text{765}$$

$$\frac{4\sqrt{x}\sqrt[4]{a+bx}\left(\int \frac{\sqrt{a}+\sqrt{a+bx}}{\sqrt{\frac{a+bx}{b}-\frac{a}{b}}}d\sqrt[4]{a+bx}-\frac{\sqrt{a}\sqrt{1-\frac{a+bx}{a}}\int \frac{1}{\sqrt{1-\frac{a+bx}{a}}}d\sqrt[4]{a+bx}}{\sqrt{\frac{a+bx}{b}-\frac{a}{b}}}\right)}{b\sqrt[4]{ax^2+bx^3}}$$

$$\Big\downarrow \ 762$$

$$\frac{4\sqrt{x}\sqrt[4]{a+bx}\left(\int \frac{\sqrt{a}+\sqrt{a+bx}}{\sqrt{\frac{a+bx}{b}-\frac{a}{b}}}d\sqrt[4]{a+bx}-\frac{a^{3/4}\sqrt{1-\frac{a+bx}{a}}\,\text{EllipticF}\left(\arcsin\left(\frac{\sqrt[4]{a+bx}}{\sqrt[4]{a}}\right),-1\right)}{\sqrt{\frac{a+bx}{b}-\frac{a}{b}}}\right)}{b\sqrt[4]{ax^2+bx^3}}$$

$$\Big\downarrow \ 1390$$

$$\frac{4\sqrt{x}\sqrt[4]{a+bx}\left(\frac{\sqrt{1-\frac{a+bx}{a}}\int \frac{\sqrt{a}+\sqrt{a+bx}}{\sqrt{1-\frac{a+bx}{a}}}d\sqrt[4]{a+bx}}{\sqrt{\frac{a+bx}{b}-\frac{a}{b}}}-\frac{a^{3/4}\sqrt{1-\frac{a+bx}{a}}\,\text{EllipticF}\left(\arcsin\left(\frac{\sqrt[4]{a+bx}}{\sqrt[4]{a}}\right),-1\right)}{\sqrt{\frac{a+bx}{b}-\frac{a}{b}}}\right)}{b\sqrt[4]{ax^2+bx^3}}$$

$$\Big\downarrow \ 1389$$

$$\frac{4\sqrt{x}\sqrt[4]{a+bx}\left(\frac{\sqrt{a}\sqrt{1-\frac{a+bx}{a}}\int \frac{\sqrt{\frac{\sqrt{a+bx}}{\sqrt{a}}+1}}{\sqrt{1-\frac{\sqrt{a+bx}}{\sqrt{a}}}}d\sqrt[4]{a+bx}}{\sqrt{\frac{a+bx}{b}-\frac{a}{b}}}-\frac{a^{3/4}\sqrt{1-\frac{a+bx}{a}}\,\text{EllipticF}\left(\arcsin\left(\frac{\sqrt[4]{a+bx}}{\sqrt[4]{a}}\right),-1\right)}{\sqrt{\frac{a+bx}{b}-\frac{a}{b}}}\right)}{b\sqrt[4]{ax^2+bx^3}}$$

$$\Big\downarrow \ 327$$

$$\frac{4\sqrt{x}\sqrt[4]{a+bx}\left(\frac{a^{3/4}\sqrt{1-\frac{a+bx}{a}}E\left(\arcsin\left(\frac{\sqrt[4]{a+bx}}{\sqrt[4]{a}}\right)\middle|-1\right)}{\sqrt{\frac{a+bx}{b}-\frac{a}{b}}}-\frac{a^{3/4}\sqrt{1-\frac{a+bx}{a}}\,\text{EllipticF}\left(\arcsin\left(\frac{\sqrt[4]{a+bx}}{\sqrt[4]{a}}\right),-1\right)}{\sqrt{\frac{a+bx}{b}-\frac{a}{b}}}\right)}{b\sqrt[4]{ax^2+bx^3}}$$

```
Int[(a*x^2 + b*x^3)^(-1/4),x]
```

```
(4*Sqrt[x]*(a + b*x)^(1/4)*((a^(3/4)*Sqrt[1 - (a + b*x)/a]*EllipticE[ArcSi
n[(a + b*x)^(1/4)/a^(1/4)], -1])/Sqrt[-(a/b) + (a + b*x)/b] - (a^(3/4)*Sqr
t[1 - (a + b*x)/a]*EllipticF[ArcSin[(a + b*x)^(1/4)/a^(1/4)], -1])/Sqrt[-(
a/b) + (a + b*x)/b]))/(b*(a*x^2 + b*x^3)^(1/4))
```

Defintions of rubi rules used

```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Simp[p/b   Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
 d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && Lt
Q[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntL
inearQ[a, b, c, d, m, n, x]
```

```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Simp[
(Sqrt[a]/(Sqrt[c]*Rt[-d/c, 2]))*EllipticE[ArcSin[Rt[-d/c, 2]*x], b*(c/(a*d)
)], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a, 0]
```

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> Simp[(1/(Sqrt[a]*Rt[-b/a, 4])
)*EllipticF[ArcSin[Rt[-b/a, 4]*x], -1], x] /; FreeQ[{a, b}, x] && NegQ[b/a]
 && GtQ[a, 0]
```

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> Simp[Sqrt[1 + b*(x^4/a)]/Sqrt
[a + b*x^4]   Int[1/Sqrt[1 + b*(x^4/a)], x], x] /; FreeQ[{a, b}, x] && NegQ
[b/a] &&  !GtQ[a, 0]
```

```
Int[(x_)^2/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[-b/a, 2]},
Simp[-q^(-1)   Int[1/Sqrt[a + b*x^4], x], x] + Simp[1/q   Int[(1 + q*x^2)/S
qrt[a + b*x^4], x], x]] /; FreeQ[{a, b}, x] && NegQ[b/a]
```

```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (c_.)*(x_)^4], x_Symbol] :> Simp[d/Sq
rt[a]   Int[Sqrt[1 + e*(x^2/d)]/Sqrt[1 - e*(x^2/d)], x], x] /; FreeQ[{a, c,
 d, e}, x] && EqQ[c*d^2 + a*e^2, 0] && NegQ[c/a] && GtQ[a, 0]
```

```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (c_.)*(x_)^4], x_Symbol] :> Simp[Sqrt
[1 + c*(x^4/a)]/Sqrt[a + c*x^4]   Int[(d + e*x^2)/Sqrt[1 + c*(x^4/a)], x],
x] /; FreeQ[{a, c, d, e}, x] && EqQ[c*d^2 + a*e^2, 0] && NegQ[c/a] &&  !GtQ
[a, 0] &&  !(LtQ[a, 0] && GtQ[c, 0])
```

```
Int[((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol] :> Simp[(a*x^j +
b*x^n)^FracPart[p]/(x^(j*FracPart[p])*(a + b*x^(n - j))^FracPart[p])   Int[
x^(j*p)*(a + b*x^(n - j))^p, x], x] /; FreeQ[{a, b, j, n, p}, x] &&  !Integ
erQ[p] && NeQ[n, j] && PosQ[n - j]
```

Maple [F]

$$\int \frac{1}{\left(b\,x^{3}+a\,x^{2}\right)^{\frac{1}{4}}}dx$$

```
int(1/(b*x^3+a*x^2)^(1/4),x)
```

```
int(1/(b*x^3+a*x^2)^(1/4),x)
```

Fricas [F]

$$\int \frac{1}{\sqrt[4]{ax^2+bx^3}}\,dx = \int \frac{1}{(bx^3+ax^2)^{\frac{1}{4}}}\,dx$$

```
integrate(1/(b*x^3+a*x^2)^(1/4),x, algorithm="fricas")
```

```
integral((b*x^3 + a*x^2)^(-1/4), x)
```

Sympy [F]

$$\int \frac{1}{\sqrt[4]{ax^2 + bx^3}} \, dx = \int \frac{1}{\sqrt[4]{ax^2 + bx^3}} \, dx$$

```
integrate(1/(b*x**3+a*x**2)**(1/4),x)
```

```
Integral((a*x**2 + b*x**3)**(-1/4), x)
```

Maxima [F]

$$\int \frac{1}{\sqrt[4]{ax^2 + bx^3}} \, dx = \int \frac{1}{(bx^3 + ax^2)^{\frac{1}{4}}} \, dx$$

```
integrate(1/(b*x^3+a*x^2)^(1/4),x, algorithm="maxima")
```

```
integrate((b*x^3 + a*x^2)^(-1/4), x)
```

Giac [F]

$$\int \frac{1}{\sqrt[4]{ax^2 + bx^3}} \, dx = \int \frac{1}{(bx^3 + ax^2)^{\frac{1}{4}}} \, dx$$

```
integrate(1/(b*x^3+a*x^2)^(1/4),x, algorithm="giac")
```

```
integrate((b*x^3 + a*x^2)^(-1/4), x)
```

Mupad [B] (verification not implemented)

Time = 9.03 (sec) , antiderivative size = 38, normalized size of antiderivative = 0.44

$$\int \frac{1}{\sqrt[4]{ax^2+bx^3}}\,dx=\frac{2\,x\,{\left(\frac{b\,x}{a}+1\right)}^{1/4}\,{}_2\mathrm{F}_1\left(\frac{1}{4},\frac{1}{2};\ \frac{3}{2};\ -\frac{b\,x}{a}\right)}{{\left(b\,x^3+a\,x^2\right)}^{1/4}}$$

```
int(1/(a*x^2 + b*x^3)^(1/4),x)
```

```
(2*x*((b*x)/a + 1)^(1/4)*hypergeom([1/4, 1/2], 3/2, -(b*x)/a))/(a*x^2 + b*
x^3)^(1/4)
```

Reduce [F]

$$\int \frac{1}{\sqrt[4]{ax^2+bx^3}}\,dx=\frac{4\sqrt{x}\,(bx+a)^{\frac{1}{4}}+\sqrt{bx+a}\left(\int \frac{\sqrt{x}\,(bx+a)^{\frac{3}{4}}}{b^2x^3+2ab\,x^2+a^2x}dx\right)a}{3\sqrt{bx+a}}$$

```
int(1/(b*x^3+a*x^2)^(1/4),x)
```

```
(4*sqrt(x)*(a + b*x)**(1/4) + sqrt(a + b*x)*int((sqrt(x)*(a + b*x)**(3/4))
/(a**2*x + 2*a*b*x**2 + b**2*x**3),x)*a)/(3*sqrt(a + b*x))
```

3.390 $\int \frac{1}{x\sqrt[4]{ax^2+bx^3}}\,dx$

Optimal result

Integrand size = 19, antiderivative size = 86

$$\int \frac{1}{x\sqrt[4]{ax^2+bx^3}}\,dx = -\frac{2}{\sqrt[4]{ax^2+bx^3}} - \frac{2\sqrt{b}\sqrt{x}\sqrt[4]{\frac{a+bx}{a}}E\left(\frac{1}{2}\arctan\left(\frac{\sqrt{b}\sqrt{x}}{\sqrt{a}}\right)\middle|2\right)}{\sqrt{a}\sqrt[4]{ax^2+bx^3}}$$

```
-2/(b*x^3+a*x^2)^(1/4)-2*b^(1/2)*x^(1/2)*((b*x+a)/a)^(1/4)*EllipticE(sin(1
/2*arctan(b^(1/2)*x^(1/2)/a^(1/2))),2^(1/2))/a^(1/2)/(b*x^3+a*x^2)^(1/4)
```

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 10.01 (sec) , antiderivative size = 44, normalized size of antiderivative = 0.51

$$\int \frac{1}{x\sqrt[4]{ax^2+bx^3}}\,dx = -\frac{2\sqrt[4]{1+\frac{bx}{a}}\,\text{Hypergeometric2F1}\left(-\frac{1}{2},\frac{1}{4},\frac{1}{2},-\frac{bx}{a}\right)}{\sqrt[4]{x^2(a+bx)}}$$

```
Integrate[1/(x*(a*x^2 + b*x^3)^(1/4)),x]
```

```
(-2*(1 + (b*x)/a)^(1/4)*Hypergeometric2F1[-1/2, 1/4, 1/2, -((b*x)/a)])/(x^
2*(a + b*x))^(1/4)
```

Rubi [B] (verified)

Leaf count is larger than twice the leaf count of optimal. 180 vs. $2(86) = 172$.

Time = 0.66 (sec) , antiderivative size = 180, normalized size of antiderivative = 2.09, number of steps used = 11, number of rules used = 10, $\frac{\text{number of rules}}{\text{integrand size}} = 0.526$, Rules used = {1931, 1917, 73, 836, 27, 765, 762, 1390, 1389, 327}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{x\sqrt[4]{ax^2+bx^3}}dx$$

$$\Big\downarrow \text{1931}$$

$$\frac{b\int \frac{1}{\sqrt[4]{bx^3+ax^2}}dx}{2a}-\frac{2\left(ax^2+bx^3\right)^{3/4}}{ax^2}$$

$$\Big\downarrow \text{1917}$$

$$\frac{b\sqrt{x}\sqrt[4]{a+bx}\int \frac{1}{\sqrt{x}\sqrt[4]{a+bx}}dx}{2a\sqrt[4]{ax^2+bx^3}}-\frac{2\left(ax^2+bx^3\right)^{3/4}}{ax^2}$$

$$\Big\downarrow \text{73}$$

$$\frac{2\sqrt{x}\sqrt[4]{a+bx}\int \frac{\sqrt{a+bx}}{\sqrt{\frac{a+bx}{b}-\frac{a}{b}}}d\sqrt[4]{a+bx}}{a\sqrt[4]{ax^2+bx^3}}-\frac{2\left(ax^2+bx^3\right)^{3/4}}{ax^2}$$

$$\Big\downarrow \text{836}$$

$$\frac{2\sqrt{x}\sqrt[4]{a+bx}\left(\sqrt{a}\int \frac{\sqrt{a}+\sqrt{a+bx}}{\sqrt{a}\sqrt{\frac{a+bx}{b}-\frac{a}{b}}}d\sqrt[4]{a+bx}-\sqrt{a}\int \frac{1}{\sqrt{\frac{a+bx}{b}-\frac{a}{b}}}d\sqrt[4]{a+bx}\right)}{a\sqrt[4]{ax^2+bx^3}}-\frac{2\left(ax^2+bx^3\right)^{3/4}}{ax^2}$$

$$\Big\downarrow \text{27}$$

$$\frac{2\sqrt{x}\sqrt[4]{a+bx}\left(\int \frac{\sqrt{a}+\sqrt{a+bx}}{\sqrt{\frac{a+bx}{b}-\frac{a}{b}}}d\sqrt[4]{a+bx}-\sqrt{a}\int \frac{1}{\sqrt{\frac{a+bx}{b}-\frac{a}{b}}}d\sqrt[4]{a+bx}\right)}{a\sqrt[4]{ax^2+bx^3}}-\frac{2\left(ax^2+bx^3\right)^{3/4}}{ax^2}$$

$$\Big\downarrow \; 765$$

$$\frac{2\sqrt{x}\sqrt[4]{a+bx}\left(\int \frac{\sqrt{a}+\sqrt{a+bx}}{\sqrt{\frac{a+bx}{b}-\frac{a}{b}}}d\sqrt[4]{a+bx}-\frac{\sqrt{a}\sqrt{1-\frac{a+bx}{a}}\int \frac{1}{\sqrt{1-\frac{a+bx}{a}}}d\sqrt[4]{a+bx}}{\sqrt{\frac{a+bx}{b}-\frac{a}{b}}}\right)}{a\sqrt[4]{ax^2+bx^3}}-\frac{2\left(ax^2+bx^3\right)^{3/4}}{ax^2}$$

$$\Big\downarrow \; 762$$

$$\frac{2\sqrt{x}\sqrt[4]{a+bx}\left(\int \frac{\sqrt{a}+\sqrt{a+bx}}{\sqrt{\frac{a+bx}{b}-\frac{a}{b}}}d\sqrt[4]{a+bx}-\frac{a^{3/4}\sqrt{1-\frac{a+bx}{a}}\,\text{EllipticF}\left(\arcsin\left(\frac{\sqrt[4]{a+bx}}{\sqrt[4]{a}}\right),-1\right)}{\sqrt{\frac{a+bx}{b}-\frac{a}{b}}}\right)}{a\sqrt[4]{ax^2+bx^3}}-\frac{2\left(ax^2+bx^3\right)^{3/4}}{ax^2}$$

$$\Big\downarrow \; 1390$$

$$\frac{2\sqrt{x}\sqrt[4]{a+bx}\left(\frac{\sqrt{1-\frac{a+bx}{a}}\int \frac{\sqrt{a}+\sqrt{a+bx}}{\sqrt{1-\frac{a+bx}{a}}}d\sqrt[4]{a+bx}}{\sqrt{\frac{a+bx}{b}-\frac{a}{b}}}-\frac{a^{3/4}\sqrt{1-\frac{a+bx}{a}}\,\text{EllipticF}\left(\arcsin\left(\frac{\sqrt[4]{a+bx}}{\sqrt[4]{a}}\right),-1\right)}{\sqrt{\frac{a+bx}{b}-\frac{a}{b}}}\right)}{a\sqrt[4]{ax^2+bx^3}}-\frac{2\left(ax^2+bx^3\right)^{3/4}}{ax^2}$$

$$\Big\downarrow \; 1389$$

$$\frac{2\sqrt{x}\sqrt[4]{a+bx}\left(\frac{\sqrt{a}\sqrt{1-\frac{a+bx}{a}}\int \frac{\sqrt{\frac{\sqrt{a+bx}}{\sqrt{a}}+1}}{\sqrt{1-\frac{\sqrt{a+bx}}{\sqrt{a}}}}d\sqrt[4]{a+bx}}{\sqrt{\frac{a+bx}{b}-\frac{a}{b}}}-\frac{a^{3/4}\sqrt{1-\frac{a+bx}{a}}\,\text{EllipticF}\left(\arcsin\left(\frac{\sqrt[4]{a+bx}}{\sqrt[4]{a}}\right),-1\right)}{\sqrt{\frac{a+bx}{b}-\frac{a}{b}}}\right)}{a\sqrt[4]{ax^2+bx^3}}-\frac{2\left(ax^2+bx^3\right)^{3/4}}{ax^2}$$

$$\Big\downarrow \; 327$$

$$\frac{2\sqrt{x}\sqrt[4]{a+bx}\left(\frac{a^{3/4}\sqrt{1-\frac{a+bx}{a}}E\left(\arcsin\left(\frac{\sqrt[4]{a+bx}}{\sqrt[4]{a}}\right)\middle|-1\right)}{\sqrt{\frac{a+bx}{b}-\frac{a}{b}}}-\frac{a^{3/4}\sqrt{1-\frac{a+bx}{a}}\operatorname{EllipticF}\left(\arcsin\left(\frac{\sqrt[4]{a+bx}}{\sqrt[4]{a}}\right),-1\right)}{\sqrt{\frac{a+bx}{b}-\frac{a}{b}}}\right)}{a\sqrt[4]{ax^2+bx^3}}-\frac{2(ax^2+bx^3)^{3/4}}{ax^2}$$

```
Int[1/(x*(a*x^2 + b*x^3)^(1/4)),x]
```

```
(-2*(a*x^2 + b*x^3)^(3/4))/(a*x^2) + (2*Sqrt[x]*(a + b*x)^(1/4)*((a^(3/4)*
Sqrt[1 - (a + b*x)/a]*EllipticE[ArcSin[(a + b*x)^(1/4)/a^(1/4)], -1])/Sqrt
[-(a/b) + (a + b*x)/b] - (a^(3/4)*Sqrt[1 - (a + b*x)/a]*EllipticF[ArcSin[(
a + b*x)^(1/4)/a^(1/4)], -1])/Sqrt[-(a/b) + (a + b*x)/b]))/(a*(a*x^2 + b*x
^3)^(1/4))
```

Defintions of rubi rules used

```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Simp[p/b   Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
 d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && Lt
Q[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntL
inearQ[a, b, c, d, m, n, x]
```

```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Simp[
(Sqrt[a]/(Sqrt[c]*Rt[-d/c, 2]))*EllipticE[ArcSin[Rt[-d/c, 2]*x], b*(c/(a*d)
)], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a, 0]
```

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> Simp[(1/(Sqrt[a]*Rt[-b/a, 4])
)*EllipticF[ArcSin[Rt[-b/a, 4]*x], -1], x] /; FreeQ[{a, b}, x] && NegQ[b/a]
 && GtQ[a, 0]
```

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> Simp[Sqrt[1 + b*(x^4/a)]/Sqrt
[a + b*x^4]   Int[1/Sqrt[1 + b*(x^4/a)], x], x] /; FreeQ[{a, b}, x] && NegQ
[b/a] &&  !GtQ[a, 0]
```

```
Int[(x_)^2/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[-b/a, 2]},
Simp[-q^(-1)   Int[1/Sqrt[a + b*x^4], x], x] + Simp[1/q   Int[(1 + q*x^2)/S
qrt[a + b*x^4], x], x]] /; FreeQ[{a, b}, x] && NegQ[b/a]
```

```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (c_.)*(x_)^4], x_Symbol] :> Simp[d/Sq
rt[a]   Int[Sqrt[1 + e*(x^2/d)]/Sqrt[1 - e*(x^2/d)], x], x] /; FreeQ[{a, c,
 d, e}, x] && EqQ[c*d^2 + a*e^2, 0] && NegQ[c/a] && GtQ[a, 0]
```

```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (c_.)*(x_)^4], x_Symbol] :> Simp[Sqrt
[1 + c*(x^4/a)]/Sqrt[a + c*x^4]   Int[(d + e*x^2)/Sqrt[1 + c*(x^4/a)], x],
x] /; FreeQ[{a, c, d, e}, x] && EqQ[c*d^2 + a*e^2, 0] && NegQ[c/a] &&  !GtQ
[a, 0] &&  !(LtQ[a, 0] && GtQ[c, 0])
```

```
Int[((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol] :> Simp[(a*x^j +
b*x^n)^FracPart[p]/(x^(j*FracPart[p])*(a + b*x^(n - j))^FracPart[p])   Int[
x^(j*p)*(a + b*x^(n - j))^p, x], x] /; FreeQ[{a, b, j, n, p}, x] &&  !Integ
erQ[p] && NeQ[n, j] && PosQ[n - j]
```

```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> Simp[c^(j - 1)*(c*x)^(m - j + 1)*((a*x^j + b*x^n)^(p + 1)/(a*(m + j*p
+ 1))), x] - Simp[b*((m + n*p + n - j + 1)/(a*c^(n - j)*(m + j*p + 1)))   I
nt[(c*x)^(m + n - j)*(a*x^j + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, p}, x]
 &&  !IntegerQ[p] && LtQ[0, j, n] && (IntegersQ[j, n] || GtQ[c, 0]) && LtQ[
m + j*p + 1, 0]
```

Maple [F]

$$\int \frac{1}{x\left(b\,x^{3}+a\,x^{2}\right)^{\frac{1}{4}}}dx$$

```
int(1/x/(b*x^3+a*x^2)^(1/4),x)
```

```
int(1/x/(b*x^3+a*x^2)^(1/4),x)
```

Fricas [F]

$$\int \frac{1}{x\sqrt[4]{ax^2+bx^3}}\,dx = \int \frac{1}{(bx^3+ax^2)^{\frac{1}{4}}x}\,dx$$

```
integrate(1/x/(b*x^3+a*x^2)^(1/4),x, algorithm="fricas")
```

```
integral((b*x^3 + a*x^2)^(3/4)/(b*x^4 + a*x^3), x)
```

Sympy [F]

$$\int \frac{1}{x\sqrt[4]{ax^2+bx^3}}\,dx = \int \frac{1}{x\sqrt[4]{x^2\,(a+bx)}}\,dx$$

```
integrate(1/x/(b*x**3+a*x**2)**(1/4),x)
```

```
Integral(1/(x*(x**2*(a + b*x))**(1/4)), x)
```

Maxima [F]

$$\int \frac{1}{x\sqrt[4]{ax^2+bx^3}}\,dx=\int \frac{1}{(bx^3+ax^2)^{\frac{1}{4}}x}\,dx$$

```
integrate(1/x/(b*x^3+a*x^2)^(1/4),x, algorithm="maxima")
```

```
integrate(1/((b*x^3 + a*x^2)^(1/4)*x), x)
```

Giac [F]

$$\int \frac{1}{x\sqrt[4]{ax^2+bx^3}}\,dx=\int \frac{1}{(bx^3+ax^2)^{\frac{1}{4}}x}\,dx$$

```
integrate(1/x/(b*x^3+a*x^2)^(1/4),x, algorithm="giac")
```

```
integrate(1/((b*x^3 + a*x^2)^(1/4)*x), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{x\sqrt[4]{ax^2+bx^3}}\,dx=\int \frac{1}{x\,{\left(b\,x^3+a\,x^2\right)}^{1/4}}\,dx$$

```
int(1/(x*(a*x^2 + b*x^3)^(1/4)),x)
```

```
int(1/(x*(a*x^2 + b*x^3)^(1/4)), x)
```

Reduce [F]

$$\int \frac{1}{x\sqrt[4]{ax^2+bx^3}}\,dx = \frac{-4\sqrt{x}\,(bx+a)^{\frac{1}{4}} - \sqrt{bx+a}\left(\int \frac{\sqrt{x}\,(bx+a)^{\frac{3}{4}}}{b^2x^4+2ab\,x^3+a^2x^2}dx\right)ax}{\sqrt{bx+a}\,x}$$

```
int(1/x/(b*x^3+a*x^2)^(1/4),x)
```

```
( - 4*sqrt(x)*(a + b*x)**(1/4) - sqrt(a + b*x)*int((sqrt(x)*(a + b*x)**(3/
4))/(a**2*x**2 + 2*a*b*x**3 + b**2*x**4),x)*a*x)/(sqrt(a + b*x)*x)
```

3.391 $\int \frac{1}{x^2 \sqrt[4]{ax^2+bx^3}}\,dx$

Optimal result

Integrand size = 19, antiderivative size = 113

$$\int \frac{1}{x^2 \sqrt[4]{ax^2+bx^3}}\,dx = \frac{b}{3a\sqrt[4]{ax^2+bx^3}} - \frac{2}{3x\sqrt[4]{ax^2+bx^3}} + \frac{b^{3/2}\sqrt{x}\sqrt[4]{\frac{a+bx}{a}}\,E\left(\frac{1}{2}\arctan\left(\frac{\sqrt{b}\sqrt{x}}{\sqrt{a}}\right)\middle|2\right)}{a^{3/2}\sqrt[4]{ax^2+bx^3}}$$

```
1/3*b/a/(b*x^3+a*x^2)^(1/4)-2/3/x/(b*x^3+a*x^2)^(1/4)+b^(3/2)*x^(1/2)*((b*
x+a)/a)^(1/4)*EllipticE(sin(1/2*arctan(b^(1/2)*x^(1/2)/a^(1/2))),2^(1/2))/
a^(3/2)/(b*x^3+a*x^2)^(1/4)
```

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 10.01 (sec) , antiderivative size = 49, normalized size of antiderivative = 0.43

$$\int \frac{1}{x^2 \sqrt[4]{ax^2+bx^3}}\,dx = -\frac{2\sqrt[4]{1+\frac{bx}{a}}\,\text{Hypergeometric2F1}\left(-\frac{3}{2},\frac{1}{4},-\frac{1}{2},-\frac{bx}{a}\right)}{3x\sqrt[4]{x^2(a+bx)}}$$

```
Integrate[1/(x^2*(a*x^2 + b*x^3)^(1/4)),x]
```

```
(-2*(1 + (b*x)/a)^(1/4)*Hypergeometric2F1[-3/2, 1/4, -1/2, -((b*x)/a)])/(3
*x*(x^2*(a + b*x))^(1/4))
```

Rubi [A] (verified)

Time = 0.76 (sec) , antiderivative size = 214, normalized size of antiderivative = 1.89, number of steps used = 12, number of rules used = 11, $\frac{\text{number of rules}}{\text{integrand size}} = 0.579$, Rules used = {1931, 1931, 1917, 73, 836, 27, 765, 762, 1390, 1389, 327}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{x^2\sqrt[4]{ax^2+bx^3}}dx$$

$$\Big\downarrow \text{1931}$$

$$-\frac{b\int \frac{1}{x\sqrt[4]{bx^3+ax^2}}dx}{2a}-\frac{2\left(ax^2+bx^3\right)^{3/4}}{3ax^3}$$

$$\Big\downarrow \text{1931}$$

$$-\frac{b\left(\frac{b\int \frac{1}{\sqrt[4]{bx^3+ax^2}}dx}{2a}-\frac{2\left(ax^2+bx^3\right)^{3/4}}{ax^2}\right)}{2a}-\frac{2\left(ax^2+bx^3\right)^{3/4}}{3ax^3}$$

$$\Big\downarrow \text{1917}$$

$$-\frac{b\left(\frac{b\sqrt{x}\sqrt[4]{a+bx}\int \frac{1}{\sqrt{x}\sqrt[4]{a+bx}}dx}{2a\sqrt[4]{ax^2+bx^3}}-\frac{2\left(ax^2+bx^3\right)^{3/4}}{ax^2}\right)}{2a}-\frac{2\left(ax^2+bx^3\right)^{3/4}}{3ax^3}$$

$$\Big\downarrow \text{73}$$

$$-\frac{b\left(\frac{2\sqrt{x}\sqrt[4]{a+bx}\int \frac{\sqrt{a+bx}}{\sqrt{\frac{a+bx}{b}-\frac{a}{b}}}d\sqrt[4]{a+bx}}{a\sqrt[4]{ax^2+bx^3}}-\frac{2\left(ax^2+bx^3\right)^{3/4}}{ax^2}\right)}{2a}-\frac{2\left(ax^2+bx^3\right)^{3/4}}{3ax^3}$$

$$\Big\downarrow \ 836$$

$$-\frac{b\left(\frac{2\sqrt{x}\sqrt[4]{a+bx}\left(\sqrt{a}\int \frac{\sqrt{a}+\sqrt{a+bx}}{\sqrt{a}\sqrt{\frac{a+bx}{b}-\frac{a}{b}}}d\sqrt[4]{a+bx}-\sqrt{a}\int \frac{1}{\sqrt{\frac{a+bx}{b}-\frac{a}{b}}}d\sqrt[4]{a+bx}\right)}{a\sqrt[4]{ax^2+bx^3}}-\frac{2\left(ax^2+bx^3\right)^{3/4}}{ax^2}\right)}{2a}-\frac{2\left(ax^2+bx^3\right)^{3/4}}{3ax^3}$$

$$\Big\downarrow \ 27$$

$$-\frac{b\left(\frac{2\sqrt{x}\sqrt[4]{a+bx}\left(\int \frac{\sqrt{a}+\sqrt{a+bx}}{\sqrt{\frac{a+bx}{b}-\frac{a}{b}}}d\sqrt[4]{a+bx}-\sqrt{a}\int \frac{1}{\sqrt{\frac{a+bx}{b}-\frac{a}{b}}}d\sqrt[4]{a+bx}\right)}{a\sqrt[4]{ax^2+bx^3}}-\frac{2\left(ax^2+bx^3\right)^{3/4}}{ax^2}\right)}{2a}-\frac{2\left(ax^2+bx^3\right)^{3/4}}{3ax^3}$$

$$\Big\downarrow \ 765$$

$$-\frac{b\left(\frac{2\sqrt{x}\sqrt[4]{a+bx}\left(\int \frac{\sqrt{a}+\sqrt{a+bx}}{\sqrt{\frac{a+bx}{b}-\frac{a}{b}}}d\sqrt[4]{a+bx}-\frac{\sqrt{a}\sqrt{1-\frac{a+bx}{a}}\int \frac{1}{\sqrt{1-\frac{a+bx}{a}}}d\sqrt[4]{a+bx}}{\sqrt{\frac{a+bx}{b}-\frac{a}{b}}}\right)}{a\sqrt[4]{ax^2+bx^3}}-\frac{2\left(ax^2+bx^3\right)^{3/4}}{ax^2}\right)}{2a}-\frac{2\left(ax^2+bx^3\right)^{3/4}}{3ax^3}$$

$$\Big\downarrow \ 762$$

$$-\frac{b\left(\frac{2\sqrt{x}\sqrt[4]{a+bx}\left(\int \frac{\sqrt{a}+\sqrt{a+bx}}{\sqrt{\frac{a+bx}{b}-\frac{a}{b}}}d\sqrt[4]{a+bx}-\frac{a^{3/4}\sqrt{1-\frac{a+bx}{a}}\,\text{EllipticF}\left(\arcsin\left(\frac{\sqrt[4]{a+bx}}{\sqrt[4]{a}}\right),-1\right)}{\sqrt{\frac{a+bx}{b}-\frac{a}{b}}}\right)}{a\sqrt[4]{ax^2+bx^3}}-\frac{2\left(ax^2+bx^3\right)^{3/4}}{ax^2}\right)}{2a}-\frac{2\left(ax^2+bx^3\right)^{3/4}}{3ax^3}$$

$$\Big\downarrow \ 1390$$

$$-\frac{b\left(\frac{2\sqrt{x}\sqrt[4]{a+bx}\left(\frac{\sqrt{1-\frac{a+bx}{a}}\int\frac{\sqrt{a}+\sqrt{a+bx}}{\sqrt{1-\frac{a+bx}{a}}}d\sqrt[4]{a+bx}}{\sqrt{\frac{a+bx}{b}-\frac{a}{b}}}-\frac{a^{3/4}\sqrt{1-\frac{a+bx}{a}}\,\text{EllipticF}\left(\arcsin\left(\frac{\sqrt[4]{a+bx}}{\sqrt[4]{a}}\right),-1\right)}{\sqrt{\frac{a+bx}{b}-\frac{a}{b}}}\right)}{a\sqrt[4]{ax^2+bx^3}}-\frac{2(ax^2+bx^3)^{3/4}}{ax^2}\right)}{2a}-\frac{2(ax^2+bx^3)^{3/4}}{3ax^3}$$

$$\Big\downarrow \; 1389$$

$$-\frac{b\left(\frac{2\sqrt{x}\sqrt[4]{a+bx}\left(\frac{\sqrt{a}\sqrt{1-\frac{a+bx}{a}}\int\frac{\sqrt{\frac{\sqrt{a+bx}}{\sqrt{a}}+1}}{\sqrt{1-\frac{\sqrt{a+bx}}{\sqrt{a}}}}d\sqrt[4]{a+bx}}{\sqrt{\frac{a+bx}{b}-\frac{a}{b}}}-\frac{a^{3/4}\sqrt{1-\frac{a+bx}{a}}\,\text{EllipticF}\left(\arcsin\left(\frac{\sqrt[4]{a+bx}}{\sqrt[4]{a}}\right),-1\right)}{\sqrt{\frac{a+bx}{b}-\frac{a}{b}}}\right)}{a\sqrt[4]{ax^2+bx^3}}-\frac{2(ax^2+bx^3)^{3/4}}{ax^2}\right)}{2a}-\frac{2(ax^2+bx^3)^{3/4}}{3ax^3}$$

$$\Big\downarrow \; 327$$

$$-\frac{b\left(\frac{2\sqrt{x}\sqrt[4]{a+bx}\left(\frac{a^{3/4}\sqrt{1-\frac{a+bx}{a}}E\left(\arcsin\left(\frac{\sqrt[4]{a+bx}}{\sqrt[4]{a}}\right)\middle|-1\right)}{\sqrt{\frac{a+bx}{b}-\frac{a}{b}}}-\frac{a^{3/4}\sqrt{1-\frac{a+bx}{a}}\,\text{EllipticF}\left(\arcsin\left(\frac{\sqrt[4]{a+bx}}{\sqrt[4]{a}}\right),-1\right)}{\sqrt{\frac{a+bx}{b}-\frac{a}{b}}}\right)}{a\sqrt[4]{ax^2+bx^3}}-\frac{2(ax^2+bx^3)^{3/4}}{ax^2}\right)}{2a}-\frac{2(ax^2+bx^3)^{3/4}}{3ax^3}$$

```
Int[1/(x^2*(a*x^2 + b*x^3)^(1/4)),x]
```

```
(-2*(a*x^2 + b*x^3)^(3/4))/(3*a*x^3) - (b*((-2*(a*x^2 + b*x^3)^(3/4))/(a*x
^2) + (2*Sqrt[x]*(a + b*x)^(1/4)*((a^(3/4)*Sqrt[1 - (a + b*x)/a]*EllipticE
[ArcSin[(a + b*x)^(1/4)/a^(1/4)], -1])/Sqrt[-(a/b) + (a + b*x)/b] - (a^(3/
4)*Sqrt[1 - (a + b*x)/a]*EllipticF[ArcSin[(a + b*x)^(1/4)/a^(1/4)], -1])/S
qrt[-(a/b) + (a + b*x)/b]))/(a*(a*x^2 + b*x^3)^(1/4))))/(2*a)
```

Defintions of rubi rules used

```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Simp[p/b   Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
 d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && Lt
Q[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntL
inearQ[a, b, c, d, m, n, x]
```

```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Simp[
(Sqrt[a]/(Sqrt[c]*Rt[-d/c, 2]))*EllipticE[ArcSin[Rt[-d/c, 2]*x], b*(c/(a*d)
)], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a, 0]
```

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> Simp[(1/(Sqrt[a]*Rt[-b/a, 4])
)*EllipticF[ArcSin[Rt[-b/a, 4]*x], -1], x] /; FreeQ[{a, b}, x] && NegQ[b/a]
 && GtQ[a, 0]
```

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> Simp[Sqrt[1 + b*(x^4/a)]/Sqrt
[a + b*x^4]   Int[1/Sqrt[1 + b*(x^4/a)], x], x] /; FreeQ[{a, b}, x] && NegQ
[b/a] &&  !GtQ[a, 0]
```

```
Int[(x_)^2/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[-b/a, 2]},
Simp[-q^(-1)   Int[1/Sqrt[a + b*x^4], x], x] + Simp[1/q   Int[(1 + q*x^2)/S
qrt[a + b*x^4], x], x]] /; FreeQ[{a, b}, x] && NegQ[b/a]
```

```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (c_.)*(x_)^4], x_Symbol] :> Simp[d/Sq
rt[a]   Int[Sqrt[1 + e*(x^2/d)]/Sqrt[1 - e*(x^2/d)], x], x] /; FreeQ[{a, c,
 d, e}, x] && EqQ[c*d^2 + a*e^2, 0] && NegQ[c/a] && GtQ[a, 0]
```

```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (c_.)*(x_)^4], x_Symbol] :> Simp[Sqrt
[1 + c*(x^4/a)]/Sqrt[a + c*x^4]   Int[(d + e*x^2)/Sqrt[1 + c*(x^4/a)], x],
x] /; FreeQ[{a, c, d, e}, x] && EqQ[c*d^2 + a*e^2, 0] && NegQ[c/a] &&  !GtQ
[a, 0] &&  !(LtQ[a, 0] && GtQ[c, 0])
```

```
Int[((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol] :> Simp[(a*x^j +
b*x^n)^FracPart[p]/(x^(j*FracPart[p])*(a + b*x^(n - j))^FracPart[p])   Int[
x^(j*p)*(a + b*x^(n - j))^p, x], x] /; FreeQ[{a, b, j, n, p}, x] &&  !Integ
erQ[p] && NeQ[n, j] && PosQ[n - j]
```

```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> Simp[c^(j - 1)*(c*x)^(m - j + 1)*((a*x^j + b*x^n)^(p + 1)/(a*(m + j*p
+ 1))), x] - Simp[b*((m + n*p + n - j + 1)/(a*c^(n - j)*(m + j*p + 1)))   I
nt[(c*x)^(m + n - j)*(a*x^j + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, p}, x]
 &&  !IntegerQ[p] && LtQ[0, j, n] && (IntegersQ[j, n] || GtQ[c, 0]) && LtQ[
m + j*p + 1, 0]
```

Maple [F]

$$\int \frac{1}{x^{2} \left(b\,x^{3}+a\,x^{2}\right)^{\frac{1}{4}}}dx$$

```
int(1/x^2/(b*x^3+a*x^2)^(1/4),x)
```

```
int(1/x^2/(b*x^3+a*x^2)^(1/4),x)
```

Fricas [F]

$$\int \frac{1}{x^2\sqrt[4]{ax^2+bx^3}}\,dx=\int \frac{1}{(bx^3+ax^2)^{\frac{1}{4}}x^2}\,dx$$

```
integrate(1/x^2/(b*x^3+a*x^2)^(1/4),x, algorithm="fricas")
```

```
integral((b*x^3 + a*x^2)^(3/4)/(b*x^5 + a*x^4), x)
```

Sympy [F]

$$\int \frac{1}{x^2\sqrt[4]{ax^2+bx^3}}\,dx=\int \frac{1}{x^2\sqrt[4]{x^2\,(a+bx)}}\,dx$$

```
integrate(1/x**2/(b*x**3+a*x**2)**(1/4),x)
```

```
Integral(1/(x**2*(x**2*(a + b*x))**(1/4)), x)
```

Maxima [F]

$$\int \frac{1}{x^2\sqrt[4]{ax^2+bx^3}}\,dx=\int \frac{1}{(bx^3+ax^2)^{\frac{1}{4}}x^2}\,dx$$

```
integrate(1/x^2/(b*x^3+a*x^2)^(1/4),x, algorithm="maxima")
```

```
integrate(1/((b*x^3 + a*x^2)^(1/4)*x^2), x)
```

Giac [F]

$$\int \frac{1}{x^2\sqrt[4]{ax^2+bx^3}}\,dx = \int \frac{1}{(bx^3+ax^2)^{\frac{1}{4}}x^2}\,dx$$

```
integrate(1/x^2/(b*x^3+a*x^2)^(1/4),x, algorithm="giac")
```

```
integrate(1/((b*x^3 + a*x^2)^(1/4)*x^2), x)
```

Mupad [B] (verification not implemented)

Time = 9.41 (sec) , antiderivative size = 44, normalized size of antiderivative = 0.39

$$\int \frac{1}{x^2\sqrt[4]{ax^2+bx^3}}\,dx = -\frac{4\left(\frac{a}{b\,x}+1\right)^{1/4}{}_2\mathrm{F}_1\left(\frac{1}{4},\frac{7}{4};\ \frac{11}{4};\ -\frac{a}{b\,x}\right)}{7\,x\,\left(b\,x^3+a\,x^2\right)^{1/4}}$$

```
int(1/(x^2*(a*x^2 + b*x^3)^(1/4)),x)
```

```
-(4*(a/(b*x) + 1)^(1/4)*hypergeom([1/4, 7/4], 11/4, -a/(b*x)))/(7*x*(a*x^2
 + b*x^3)^(1/4))
```

Reduce [F]

$$\int \frac{1}{x^2\sqrt[4]{ax^2+bx^3}}\,dx = \frac{-4\sqrt{x}\,(bx+a)^{\frac{1}{4}}-\sqrt{bx+a}\left(\int \frac{\sqrt{x}\,(bx+a)^{\frac{3}{4}}}{b^2x^5+2ab\,x^4+a^2x^3}dx\right)a\,x^2}{5\sqrt{bx+a}\,x^2}$$

```
int(1/x^2/(b*x^3+a*x^2)^(1/4),x)
```

```
( - 4*sqrt(x)*(a + b*x)**(1/4) - sqrt(a + b*x)*int((sqrt(x)*(a + b*x)**(3/
4))/(a**2*x**3 + 2*a*b*x**4 + b**2*x**5),x)*a*x**2)/(5*sqrt(a + b*x)*x**2)
```

3.392 $\int \frac{1}{x^3 \sqrt[4]{ax^2+bx^3}}\,dx$

Optimal result

Integrand size = 19, antiderivative size = 144

$$\int \frac{1}{x^3 \sqrt[4]{ax^2+bx^3}}\,dx = -\frac{7b^2}{30a^2\sqrt[4]{ax^2+bx^3}} - \frac{2}{5x^2\sqrt[4]{ax^2+bx^3}} + \frac{b}{15ax\sqrt[4]{ax^2+bx^3}} - \frac{7b^{5/2}\sqrt{x}\sqrt[4]{\frac{a+bx}{a}}E\left(\frac{1}{2}\arctan\left(\frac{\sqrt{b}\sqrt{x}}{\sqrt{a}}\right)\middle|2\right)}{10a^{5/2}\sqrt[4]{ax^2+bx^3}}$$

```
-7/30*b^2/a^2/(b*x^3+a*x^2)^(1/4)-2/5/x^2/(b*x^3+a*x^2)^(1/4)+1/15*b/a/x/(
b*x^3+a*x^2)^(1/4)-7/10*b^(5/2)*x^(1/2)*((b*x+a)/a)^(1/4)*EllipticE(sin(1/
2*arctan(b^(1/2)*x^(1/2)/a^(1/2))),2^(1/2))/a^(5/2)/(b*x^3+a*x^2)^(1/4)
```

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 10.01 (sec) , antiderivative size = 49, normalized size of antiderivative = 0.34

$$\int \frac{1}{x^3 \sqrt[4]{ax^2+bx^3}}\,dx = -\frac{2\sqrt[4]{1+\frac{bx}{a}}\,\text{Hypergeometric2F1}\left(-\frac{5}{2},\frac{1}{4},-\frac{3}{2},-\frac{bx}{a}\right)}{5x^2\sqrt[4]{x^2(a+bx)}}$$

```
Integrate[1/(x^3*(a*x^2 + b*x^3)^(1/4)),x]
```

```
(-2*(1 + (b*x)/a)^(1/4)*Hypergeometric2F1[-5/2, 1/4, -3/2, -((b*x)/a)])/(5
*x^2*(x^2*(a + b*x))^(1/4))
```

Rubi [A] (verified)

Time = 0.86 (sec) , antiderivative size = 248, normalized size of antiderivative = 1.72, number of steps used = 13, number of rules used = 12, $\frac{\text{number of rules}}{\text{integrand size}} = 0.632$, Rules used = {1931, 1931, 1931, 1917, 73, 836, 27, 765, 762, 1390, 1389, 327}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{x^3 \sqrt[4]{ax^2+bx^3}}dx$$

$$\Big\downarrow \text{1931}$$

$$-\frac{7b\int \frac{1}{x^2 \sqrt[4]{bx^3+ax^2}}dx}{10a}-\frac{2\left(ax^2+bx^3\right)^{3/4}}{5ax^4}$$

$$\Big\downarrow \text{1931}$$

$$-\frac{7b\left(-\frac{b\int \frac{1}{x \sqrt[4]{bx^3+ax^2}}dx}{2a}-\frac{2\left(ax^2+bx^3\right)^{3/4}}{3ax^3}\right)}{10a}-\frac{2\left(ax^2+bx^3\right)^{3/4}}{5ax^4}$$

$$\Big\downarrow \text{1931}$$

$$-\frac{7b\left(-\frac{b\left(\frac{b\int \frac{1}{\sqrt[4]{bx^3+ax^2}}dx}{2a}-\frac{2\left(ax^2+bx^3\right)^{3/4}}{ax^2}\right)}{2a}-\frac{2\left(ax^2+bx^3\right)^{3/4}}{3ax^3}\right)}{10a}-\frac{2\left(ax^2+bx^3\right)^{3/4}}{5ax^4}$$

$$\Big\downarrow \text{1917}$$

$$-\frac{7b\left(-\frac{b\left(\frac{b\sqrt{x}\sqrt[4]{a+bx}\int\frac{1}{\sqrt{x}\sqrt[4]{a+bx}}dx}{2a\sqrt[4]{ax^2+bx^3}}-\frac{2\left(ax^2+bx^3\right)^{3/4}}{ax^2}\right)}{2a}-\frac{2\left(ax^2+bx^3\right)^{3/4}}{3ax^3}\right)}{10a}-\frac{2\left(ax^2+bx^3\right)^{3/4}}{5ax^4}$$

$$\Big\downarrow \text{73}$$

$$-\frac{7b\left(-\frac{b\left(\frac{2\sqrt{x}\sqrt[4]{a+bx}\int\frac{\sqrt{a+bx}}{\sqrt{\frac{a+bx}{b}-\frac{a}{b}}}d\sqrt[4]{a+bx}}{a\sqrt[4]{ax^2+bx^3}}-\frac{2\left(ax^2+bx^3\right)^{3/4}}{ax^2}\right)}{2a}-\frac{2\left(ax^2+bx^3\right)^{3/4}}{3ax^3}\right)}{10a}-\frac{2\left(ax^2+bx^3\right)^{3/4}}{5ax^4}$$

$$\Big\downarrow \text{836}$$

$$-\frac{7b\left(-\frac{b\left(\frac{2\sqrt{x}\sqrt[4]{a+bx}\left(\sqrt{a}\int\frac{\sqrt{a}+\sqrt{a+bx}}{\sqrt{a}\sqrt{\frac{a+bx}{b}-\frac{a}{b}}}d\sqrt[4]{a+bx}-\sqrt{a}\int\frac{1}{\sqrt{\frac{a+bx}{b}-\frac{a}{b}}}d\sqrt[4]{a+bx}\right)}{a\sqrt[4]{ax^2+bx^3}}-\frac{2\left(ax^2+bx^3\right)^{3/4}}{ax^2}\right)}{2a}-\frac{2\left(ax^2+bx^3\right)^{3/4}}{3ax^3}\right)}{10a}-\frac{2\left(ax^2+bx^3\right)^{3/4}}{5ax^4}$$

$$\Big\downarrow \text{27}$$

$$-\frac{7b\left(-\frac{b\left(\frac{2\sqrt{x}\sqrt[4]{a+bx}\left(\int\frac{\sqrt{a}+\sqrt{a+bx}}{\sqrt{\frac{a+bx}{b}-\frac{a}{b}}}d\sqrt[4]{a+bx}-\sqrt{a}\int\frac{1}{\sqrt{\frac{a+bx}{b}-\frac{a}{b}}}d\sqrt[4]{a+bx}\right)}{a\sqrt[4]{ax^2+bx^3}}-\frac{2\left(ax^2+bx^3\right)^{3/4}}{ax^2}\right)}{2a}-\frac{2\left(ax^2+bx^3\right)^{3/4}}{3ax^3}\right)}{10a}-\frac{2\left(ax^2+bx^3\right)^{3/4}}{5ax^4}$$

$$\Big\downarrow \text{765}$$

$$-\frac{7b\left(-\frac{b\left(\frac{2\sqrt{x}\sqrt[4]{a+bx}\left(\int \frac{\sqrt{a}+\sqrt{a+bx}}{\sqrt{\frac{a+bx}{b}-\frac{a}{b}}}d\sqrt[4]{a+bx}-\frac{\sqrt{a}\sqrt{1-\frac{a+bx}{a}}\int \frac{1}{\sqrt{1-\frac{a+bx}{a}}}d\sqrt[4]{a+bx}}{\sqrt{\frac{a+bx}{b}-\frac{a}{b}}}\right)}{a\sqrt[4]{ax^2+bx^3}}-\frac{2\left(ax^2+bx^3\right)^{3/4}}{ax^2}\right)}{2a}-\frac{2\left(ax^2+bx^3\right)^{3/4}}{3ax^3}\right)}{10a}-\frac{2\left(ax^2+bx^3\right)^{3/4}}{5ax^4}$$

$$\Big\downarrow \ 762$$

$$-\frac{7b\left(-\frac{b\left(\frac{2\sqrt{x}\sqrt[4]{a+bx}\left(\int \frac{\sqrt{a}+\sqrt{a+bx}}{\sqrt{\frac{a+bx}{b}-\frac{a}{b}}}d\sqrt[4]{a+bx}-\frac{a^{3/4}\sqrt{1-\frac{a+bx}{a}}\,\text{EllipticF}\left(\arcsin\left(\frac{\sqrt[4]{a+bx}}{\sqrt[4]{a}}\right),-1\right)}{\sqrt{\frac{a+bx}{b}-\frac{a}{b}}}\right)}{a\sqrt[4]{ax^2+bx^3}}-\frac{2\left(ax^2+bx^3\right)^{3/4}}{ax^2}\right)}{2a}-\frac{2\left(ax^2+bx^3\right)^{3/4}}{3ax^3}\right)}{10a}-\frac{2\left(ax^2+bx^3\right)^{3/4}}{5ax^4}$$

$$\Big\downarrow \ 1390$$

$$-\frac{7b\left(-\frac{b\left(\frac{2\sqrt{x}\sqrt[4]{a+bx}\left(\frac{\sqrt{1-\frac{a+bx}{a}}\int\frac{\sqrt{a}+\sqrt{a+bx}}{\sqrt{1-\frac{a+bx}{a}}}d\sqrt[4]{a+bx}}{\sqrt{\frac{a+bx}{b}-\frac{a}{b}}}-\frac{a^{3/4}\sqrt{1-\frac{a+bx}{a}}\,\text{EllipticF}\left(\arcsin\left(\frac{\sqrt[4]{a+bx}}{\sqrt[4]{a}}\right),-1\right)}{\sqrt{\frac{a+bx}{b}-\frac{a}{b}}}\right)}{a\sqrt[4]{ax^2+bx^3}}-\frac{2\left(ax^2+bx^3\right)^{3/4}}{ax^2}\right)}{2a}-\right)}{10a}$$

$$\frac{2\left(ax^2+bx^3\right)^{3/4}}{5ax^4}$$

$$\Big\downarrow \text{1389}$$

$$-\frac{7b\left(-\frac{b\left(\frac{2\sqrt{x}\sqrt[4]{a+bx}\left(\frac{\sqrt{a}\sqrt{1-\frac{a+bx}{a}}\int\frac{\sqrt{\frac{\sqrt{a+bx}}{\sqrt{a}}+1}}{\sqrt{1-\frac{\sqrt{a+bx}}{\sqrt{a}}}}d\sqrt[4]{a+bx}}{\sqrt{\frac{a+bx}{b}-\frac{a}{b}}}-\frac{a^{3/4}\sqrt{1-\frac{a+bx}{a}}\,\text{EllipticF}\left(\arcsin\left(\frac{\sqrt[4]{a+bx}}{\sqrt[4]{a}}\right),-1\right)}{\sqrt{\frac{a+bx}{b}-\frac{a}{b}}}\right)}{a\sqrt[4]{ax^2+bx^3}}-\frac{2\left(ax^2+bx^3\right)^{3/4}}{ax^2}\right)}{2a}\right)}{10a}$$

$$\frac{2\left(ax^2+bx^3\right)^{3/4}}{5ax^4}$$

$$\Big\downarrow \text{327}$$

$$-\frac{7b\left(-\frac{b\left(\frac{2\sqrt{x}\sqrt[4]{a+bx}\left(\frac{a^{3/4}\sqrt{1-\frac{a+bx}{a}}E\left(\arcsin\left(\frac{\sqrt[4]{a+bx}}{\sqrt[4]{a}}\right)\middle|-1\right)}{\sqrt{\frac{a+bx}{b}-\frac{a}{b}}}-\frac{a^{3/4}\sqrt{1-\frac{a+bx}{a}}\text{EllipticF}\left(\arcsin\left(\frac{\sqrt[4]{a+bx}}{\sqrt[4]{a}}\right),-1\right)}{\sqrt{\frac{a+bx}{b}-\frac{a}{b}}}\right)}{a\sqrt[4]{ax^2+bx^3}}-\frac{2\left(ax^2+bx^3\right)^{3/}}{ax^2}}{2a}}{10a}$$

$$\frac{2\left(ax^2+bx^3\right)^{3/4}}{5ax^4}$$

```
Int[1/(x^3*(a*x^2 + b*x^3)^(1/4)),x]
```

```
(-2*(a*x^2 + b*x^3)^(3/4))/(5*a*x^4) - (7*b*((-2*(a*x^2 + b*x^3)^(3/4))/(3
*a*x^3) - (b*((-2*(a*x^2 + b*x^3)^(3/4))/(a*x^2) + (2*Sqrt[x]*(a + b*x)^(1
/4)*((a^(3/4)*Sqrt[1 - (a + b*x)/a]*EllipticE[ArcSin[(a + b*x)^(1/4)/a^(1/
4)], -1])/Sqrt[-(a/b) + (a + b*x)/b] - (a^(3/4)*Sqrt[1 - (a + b*x)/a]*Elli
pticF[ArcSin[(a + b*x)^(1/4)/a^(1/4)], -1])/Sqrt[-(a/b) + (a + b*x)/b]))/(
a*(a*x^2 + b*x^3)^(1/4))))/(2*a)))/(10*a)
```

Defintions of rubi rules used

```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Simp[p/b   Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
 d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && Lt
Q[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntL
inearQ[a, b, c, d, m, n, x]
```

```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Simp[
(Sqrt[a]/(Sqrt[c]*Rt[-d/c, 2]))*EllipticE[ArcSin[Rt[-d/c, 2]*x], b*(c/(a*d)
)], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a, 0]
```

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> Simp[(1/(Sqrt[a]*Rt[-b/a, 4])
)*EllipticF[ArcSin[Rt[-b/a, 4]*x], -1], x] /; FreeQ[{a, b}, x] && NegQ[b/a]
 && GtQ[a, 0]
```

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> Simp[Sqrt[1 + b*(x^4/a)]/Sqrt
[a + b*x^4]   Int[1/Sqrt[1 + b*(x^4/a)], x], x] /; FreeQ[{a, b}, x] && NegQ
[b/a] &&  !GtQ[a, 0]
```

```
Int[(x_)^2/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[-b/a, 2]},
Simp[-q^(-1)   Int[1/Sqrt[a + b*x^4], x], x] + Simp[1/q   Int[(1 + q*x^2)/S
qrt[a + b*x^4], x], x]] /; FreeQ[{a, b}, x] && NegQ[b/a]
```

```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (c_.)*(x_)^4], x_Symbol] :> Simp[d/Sq
rt[a]   Int[Sqrt[1 + e*(x^2/d)]/Sqrt[1 - e*(x^2/d)], x], x] /; FreeQ[{a, c,
 d, e}, x] && EqQ[c*d^2 + a*e^2, 0] && NegQ[c/a] && GtQ[a, 0]
```

```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (c_.)*(x_)^4], x_Symbol] :> Simp[Sqrt
[1 + c*(x^4/a)]/Sqrt[a + c*x^4]   Int[(d + e*x^2)/Sqrt[1 + c*(x^4/a)], x],
x] /; FreeQ[{a, c, d, e}, x] && EqQ[c*d^2 + a*e^2, 0] && NegQ[c/a] &&  !GtQ
[a, 0] &&  !(LtQ[a, 0] && GtQ[c, 0])
```

```
Int[((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol] :> Simp[(a*x^j +
b*x^n)^FracPart[p]/(x^(j*FracPart[p])*(a + b*x^(n - j))^FracPart[p])   Int[
x^(j*p)*(a + b*x^(n - j))^p, x], x] /; FreeQ[{a, b, j, n, p}, x] &&  !Integ
erQ[p] && NeQ[n, j] && PosQ[n - j]
```

```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> Simp[c^(j - 1)*(c*x)^(m - j + 1)*((a*x^j + b*x^n)^(p + 1)/(a*(m + j*p
+ 1))), x] - Simp[b*((m + n*p + n - j + 1)/(a*c^(n - j)*(m + j*p + 1)))   I
nt[(c*x)^(m + n - j)*(a*x^j + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, p}, x]
 &&  !IntegerQ[p] && LtQ[0, j, n] && (IntegersQ[j, n] || GtQ[c, 0]) && LtQ[
m + j*p + 1, 0]
```

Maple [F]

$$\int \frac{1}{x^{3} \left(b\,x^{3}+a\,x^{2}\right)^{\frac{1}{4}}}dx$$

```
int(1/x^3/(b*x^3+a*x^2)^(1/4),x)
```

```
int(1/x^3/(b*x^3+a*x^2)^(1/4),x)
```

Fricas [F]

$$\int \frac{1}{x^3\sqrt[4]{ax^2+bx^3}}\,dx = \int \frac{1}{(bx^3+ax^2)^{\frac{1}{4}}x^3}\,dx$$

```
integrate(1/x^3/(b*x^3+a*x^2)^(1/4),x, algorithm="fricas")
```

```
integral((b*x^3 + a*x^2)^(3/4)/(b*x^6 + a*x^5), x)
```

Sympy [F]

$$\int \frac{1}{x^3 \sqrt[4]{ax^2+bx^3}}\,dx = \int \frac{1}{x^3 \sqrt[4]{x^2\,(a+bx)}}\,dx$$

```
integrate(1/x**3/(b*x**3+a*x**2)**(1/4),x)
```

```
Integral(1/(x**3*(x**2*(a + b*x))**(1/4)), x)
```

Maxima [F]

$$\int \frac{1}{x^3 \sqrt[4]{ax^2+bx^3}}\,dx = \int \frac{1}{(bx^3+ax^2)^{\frac{1}{4}}x^3}\,dx$$

```
integrate(1/x^3/(b*x^3+a*x^2)^(1/4),x, algorithm="maxima")
```

```
integrate(1/((b*x^3 + a*x^2)^(1/4)*x^3), x)
```

Giac [F]

$$\int \frac{1}{x^3 \sqrt[4]{ax^2+bx^3}}\,dx = \int \frac{1}{(bx^3+ax^2)^{\frac{1}{4}}x^3}\,dx$$

```
integrate(1/x^3/(b*x^3+a*x^2)^(1/4),x, algorithm="giac")
```

```
integrate(1/((b*x^3 + a*x^2)^(1/4)*x^3), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{x^3 \sqrt[4]{ax^2+bx^3}}\,dx = \int \frac{1}{x^3\,{\left(b\,x^3+a\,x^2\right)}^{1/4}}\,dx$$

```
int(1/(x^3*(a*x^2 + b*x^3)^(1/4)),x)
```

```
int(1/(x^3*(a*x^2 + b*x^3)^(1/4)), x)
```

Reduce [F]

$$\int \frac{1}{x^3 \sqrt[4]{ax^2+bx^3}}\,dx = \frac{-4\sqrt{x}\,(bx+a)^{\frac{1}{4}}-\sqrt{bx+a}\left(\int \frac{\sqrt{x}\,(bx+a)^{\frac{3}{4}}}{b^2x^6+2ab\,x^5+a^2x^4}dx\right)a\,x^3}{9\sqrt{bx+a}\,x^3}$$

```
int(1/x^3/(b*x^3+a*x^2)^(1/4),x)
```

```
( - 4*sqrt(x)*(a + b*x)**(1/4) - sqrt(a + b*x)*int((sqrt(x)*(a + b*x)**(3/
4))/(a**2*x**4 + 2*a*b*x**5 + b**2*x**6),x)*a*x**3)/(9*sqrt(a + b*x)*x**3)
```

3.393 $\int \frac{x^4}{\left(ax^2+bx^3\right)^{3/4}}\,dx$

Optimal result

Integrand size = 19, antiderivative size = 144

$$\int \frac{x^4}{\left(ax^2+bx^3\right)^{3/4}}\,dx=\frac{80a^2\sqrt[4]{ax^2+bx^3}}{77b^3}-\frac{40ax\sqrt[4]{ax^2+bx^3}}{77b^2}+\frac{4x^2\sqrt[4]{ax^2+bx^3}}{11b}-\frac{160a^{7/2}x^{3/2}\left(1+\frac{bx}{a}\right)^{3/4}\operatorname{EllipticF}\left(\frac{1}{2}\arctan\left(\frac{\sqrt{b}\sqrt{x}}{\sqrt{a}}\right),2\right)}{77b^{7/2}\left(ax^2+bx^3\right)^{3/4}}$$

```
80/77*a^2*(b*x^3+a*x^2)^(1/4)/b^3-40/77*a*x*(b*x^3+a*x^2)^(1/4)/b^2+4/11*x
^2*(b*x^3+a*x^2)^(1/4)/b-160/77*a^(7/2)*x^(3/2)*(1+b*x/a)^(3/4)*InverseJac
obiAM(1/2*arctan(b^(1/2)*x^(1/2)/a^(1/2)),2^(1/2))/b^(7/2)/(b*x^3+a*x^2)^(
3/4)
```

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 10.01 (sec) , antiderivative size = 49, normalized size of antiderivative = 0.34

$$\int \frac{x^4}{\left(ax^2+bx^3\right)^{3/4}}\,dx=\frac{2x^5\left(1+\frac{bx}{a}\right)^{3/4}\text{Hypergeometric2F1}\left(\frac{3}{4},\frac{7}{2},\frac{9}{2},-\frac{bx}{a}\right)}{7\left(x^2(a+bx)\right)^{3/4}}$$

```
Integrate[x^4/(a*x^2 + b*x^3)^(3/4),x]
```

```
(2*x^5*(1 + (b*x)/a)^(3/4)*Hypergeometric2F1[3/4, 7/2, 9/2, -((b*x)/a)])/(
7*(x^2*(a + b*x))^(3/4))
```

Rubi [A] (verified)

Time = 0.61 (sec) , antiderivative size = 184, normalized size of antiderivative = 1.28, number of steps used = 8, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}} = 0.368$, Rules used = {1930, 1930, 1930, 1938, 73, 765, 762}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^4}{(ax^2+bx^3)^{3/4}}dx$$

$$\Big\downarrow \text{1930}$$

$$\frac{4x^2\sqrt[4]{ax^2+bx^3}}{11b}-\frac{10a\int \frac{x^3}{(bx^3+ax^2)^{3/4}}dx}{11b}$$

$$\Big\downarrow \text{1930}$$

$$\frac{4x^2\sqrt[4]{ax^2+bx^3}}{11b}-\frac{10a\left(\frac{4x\sqrt[4]{ax^2+bx^3}}{7b}-\frac{6a\int \frac{x^2}{(bx^3+ax^2)^{3/4}}dx}{7b}\right)}{11b}$$

$$\Big\downarrow \text{1930}$$

$$\frac{4x^2\sqrt[4]{ax^2+bx^3}}{11b}-\frac{10a\left(\frac{4x\sqrt[4]{ax^2+bx^3}}{7b}-\frac{6a\left(\frac{4\sqrt[4]{ax^2+bx^3}}{3b}-\frac{2a\int \frac{x}{(bx^3+ax^2)^{3/4}}dx}{3b}\right)}{7b}\right)}{11b}$$

$$\Big\downarrow \text{1938}$$

$$\frac{4x^2\sqrt[4]{ax^2+bx^3}}{11b}-\frac{10a\left(\frac{4x\sqrt[4]{ax^2+bx^3}}{7b}-\frac{6a\left(\frac{4\sqrt[4]{ax^2+bx^3}}{3b}-\frac{2ax^{3/2}(a+bx)^{3/4}\int\frac{1}{\sqrt{x}(a+bx)^{3/4}}dx}{3b\left(ax^2+bx^3\right)^{3/4}}\right)}{7b}\right)}{11b}$$

$$\Big\downarrow \text{73}$$

$$\frac{4x^2\sqrt[4]{ax^2+bx^3}}{11b}-\frac{10a\left(\frac{4x\sqrt[4]{ax^2+bx^3}}{7b}-\frac{6a\left(\frac{4\sqrt[4]{ax^2+bx^3}}{3b}-\frac{8ax^{3/2}(a+bx)^{3/4}\int\frac{1}{\sqrt{\frac{a+bx}{b}-\frac{a}{b}}}d\sqrt[4]{a+bx}}{3b^2\left(ax^2+bx^3\right)^{3/4}}\right)}{7b}\right)}{11b}$$

$$\Big\downarrow \text{765}$$

$$\frac{4x^2\sqrt[4]{ax^2+bx^3}}{11b}-\frac{10a\left(\frac{4x\sqrt[4]{ax^2+bx^3}}{7b}-\frac{6a\left(\frac{4\sqrt[4]{ax^2+bx^3}}{3b}-\frac{8ax^{3/2}(a+bx)^{3/4}\sqrt{1-\frac{a+bx}{a}}\int\frac{1}{\sqrt{1-\frac{a+bx}{a}}}d\sqrt[4]{a+bx}}{3b^2\left(ax^2+bx^3\right)^{3/4}\sqrt{\frac{a+bx}{b}-\frac{a}{b}}}\right)}{7b}\right)}{11b}$$

$$\Big\downarrow \text{762}$$

$$\frac{4x^2\sqrt[4]{ax^2+bx^3}}{11b}-\frac{10a\left(\frac{4x\sqrt[4]{ax^2+bx^3}}{7b}-\frac{6a\left(\frac{4\sqrt[4]{ax^2+bx^3}}{3b}-\frac{8a^{5/4}x^{3/2}(a+bx)^{3/4}\sqrt{1-\frac{a+bx}{a}}\,\text{EllipticF}\left(\arcsin\left(\frac{\sqrt[4]{a+bx}}{\sqrt[4]{a}}\right),-1\right)}{3b^2\left(ax^2+bx^3\right)^{3/4}\sqrt{\frac{a+bx}{b}-\frac{a}{b}}}\right)}{7b}\right)}{11b}$$

```
Int[x^4/(a*x^2 + b*x^3)^(3/4),x]
```

```
(4*x^2*(a*x^2 + b*x^3)^(1/4))/(11*b) - (10*a*((4*x*(a*x^2 + b*x^3)^(1/4))/
(7*b) - (6*a*((4*(a*x^2 + b*x^3)^(1/4))/(3*b) - (8*a^(5/4)*x^(3/2)*(a + b*
x)^(3/4)*Sqrt[1 - (a + b*x)/a]*EllipticF[ArcSin[(a + b*x)^(1/4)/a^(1/4)],
-1])/(3*b^2*(a*x^2 + b*x^3)^(3/4)*Sqrt[-(a/b) + (a + b*x)/b])))/(7*b)))/(1
1*b)
```

Defintions of rubi rules used

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Simp[p/b   Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
 d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && Lt
Q[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntL
inearQ[a, b, c, d, m, n, x]
```

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> Simp[(1/(Sqrt[a]*Rt[-b/a, 4])
)*EllipticF[ArcSin[Rt[-b/a, 4]*x], -1], x] /; FreeQ[{a, b}, x] && NegQ[b/a]
 && GtQ[a, 0]
```

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> Simp[Sqrt[1 + b*(x^4/a)]/Sqrt
[a + b*x^4]   Int[1/Sqrt[1 + b*(x^4/a)], x], x] /; FreeQ[{a, b}, x] && NegQ
[b/a] &&  !GtQ[a, 0]
```

```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> Simp[c^(n - 1)*(c*x)^(m - n + 1)*((a*x^j + b*x^n)^(p + 1)/(b*(m + n*p
+ 1))), x] - Simp[a*c^(n - j)*((m + j*p - n + j + 1)/(b*(m + n*p + 1)))   I
nt[(c*x)^(m - (n - j))*(a*x^j + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, p},
x] &&  !IntegerQ[p] && LtQ[0, j, n] && (IntegersQ[j, n] || GtQ[c, 0]) && Gt
Q[m + j*p - n + j + 1, 0] && NeQ[m + n*p + 1, 0]
```

```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> Simp[c^IntPart[m]*(c*x)^FracPart[m]*((a*x^j + b*x^n)^FracPart[p]/(x^(F
racPart[m] + j*FracPart[p])*(a + b*x^(n - j))^FracPart[p]))   Int[x^(m + j*
p)*(a + b*x^(n - j))^p, x], x] /; FreeQ[{a, b, c, j, m, n, p}, x] &&  !Inte
gerQ[p] && NeQ[n, j] && PosQ[n - j]
```

Maple [F]

$$\int \frac{x^4}{(b\,x^3 + a\,x^2)^{\frac{3}{4}}}dx$$

```
int(x^4/(b*x^3+a*x^2)^(3/4),x)
```

```
int(x^4/(b*x^3+a*x^2)^(3/4),x)
```

Fricas [F]

$$\int \frac{x^4}{(ax^2 + bx^3)^{3/4}}\,dx = \int \frac{x^4}{(bx^3 + ax^2)^{\frac{3}{4}}}\,dx$$

```
integrate(x^4/(b*x^3+a*x^2)^(3/4),x, algorithm="fricas")
```

```
integral((b*x^3 + a*x^2)^(1/4)*x^2/(b*x + a), x)
```

Sympy [F]

$$\int \frac{x^4}{(ax^2 + bx^3)^{3/4}}\,dx = \int \frac{x^4}{(x^2\,(a + bx))^{\frac{3}{4}}}\,dx$$

```
integrate(x**4/(b*x**3+a*x**2)**(3/4),x)
```

```
Integral(x**4/(x**2*(a + b*x))**(3/4), x)
```

Maxima [F]

$$\int \frac{x^4}{(ax^2+bx^3)^{3/4}}\,dx = \int \frac{x^4}{(bx^3+ax^2)^{\frac{3}{4}}}\,dx$$

```
integrate(x^4/(b*x^3+a*x^2)^(3/4),x, algorithm="maxima")
```

```
integrate(x^4/(b*x^3 + a*x^2)^(3/4), x)
```

Giac [F]

$$\int \frac{x^4}{(ax^2+bx^3)^{3/4}}\,dx = \int \frac{x^4}{(bx^3+ax^2)^{\frac{3}{4}}}\,dx$$

```
integrate(x^4/(b*x^3+a*x^2)^(3/4),x, algorithm="giac")
```

```
integrate(x^4/(b*x^3 + a*x^2)^(3/4), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{x^4}{(ax^2+bx^3)^{3/4}}\,dx = \int \frac{x^4}{(b\,x^3+a\,x^2)^{3/4}}\,dx$$

```
int(x^4/(a*x^2 + b*x^3)^(3/4),x)
```

```
int(x^4/(a*x^2 + b*x^3)^(3/4), x)
```

Reduce [F]

$$\int \frac{x^4}{(ax^2+bx^3)^{3/4}}\,dx = \frac{\frac{16\sqrt{x}\,(bx+a)^{\frac{3}{4}}a^2}{39} - \frac{40\sqrt{x}\,(bx+a)^{\frac{3}{4}}abx}{117} + \frac{4\sqrt{x}\,(bx+a)^{\frac{3}{4}}b^2x^2}{13} - \frac{8\sqrt{bx+a}\left(\int \frac{\sqrt{x}\,(bx+a)^{\frac{5}{4}}}{b^2x^3+2ab\,x^2+a^2x}dx\right)a^3}{39}}{\sqrt{bx+a}\,b^3}$$

```
int(x^4/(b*x^3+a*x^2)^(3/4),x)
```

```
(4*(12*sqrt(x)*(a + b*x)**(3/4)*a**2 - 10*sqrt(x)*(a + b*x)**(3/4)*a*b*x +
 9*sqrt(x)*(a + b*x)**(3/4)*b**2*x**2 - 6*sqrt(a + b*x)*int((sqrt(x)*(a +
b*x)**(5/4))/(a**2*x + 2*a*b*x**2 + b**2*x**3),x)*a**3))/(117*sqrt(a + b*x
)*b**3)
```

3.394 $\int \frac{x^3}{(ax^2+bx^3)^{3/4}}\,dx$

Optimal result

Integrand size = 19, antiderivative size = 116

$$\int \frac{x^3}{(ax^2+bx^3)^{3/4}}\,dx = -\frac{8a\sqrt[4]{ax^2+bx^3}}{7b^2}+\frac{4x\sqrt[4]{ax^2+bx^3}}{7b} + \frac{16a^{5/2}x^{3/2}\left(1+\frac{bx}{a}\right)^{3/4}\operatorname{EllipticF}\left(\frac{1}{2}\arctan\left(\frac{\sqrt{b}\sqrt{x}}{\sqrt{a}}\right),2\right)}{7b^{5/2}(ax^2+bx^3)^{3/4}}$$

```
-8/7*a*(b*x^3+a*x^2)^(1/4)/b^2+4/7*x*(b*x^3+a*x^2)^(1/4)/b+16/7*a^(5/2)*x^
(3/2)*(1+b*x/a)^(3/4)*InverseJacobiAM(1/2*arctan(b^(1/2)*x^(1/2)/a^(1/2)),
2^(1/2))/b^(5/2)/(b*x^3+a*x^2)^(3/4)
```

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 10.01 (sec) , antiderivative size = 49, normalized size of antiderivative = 0.42

$$\int \frac{x^3}{(ax^2+bx^3)^{3/4}}\,dx = \frac{2x^4\left(1+\frac{bx}{a}\right)^{3/4}\text{Hypergeometric2F1}\left(\frac{3}{4},\frac{5}{2},\frac{7}{2},-\frac{bx}{a}\right)}{5\left(x^2(a+bx)\right)^{3/4}}$$

```
Integrate[x^3/(a*x^2 + b*x^3)^(3/4),x]
```

```
(2*x^4*(1 + (b*x)/a)^(3/4)*Hypergeometric2F1[3/4, 5/2, 7/2, -((b*x)/a)])/(
5*(x^2*(a + b*x))^(3/4))
```

Rubi [A] (verified)

Time = 0.50 (sec) , antiderivative size = 150, normalized size of antiderivative = 1.29, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.316, Rules used = {1930, 1930, 1938, 73, 765, 762}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^3}{(ax^2+bx^3)^{3/4}}dx$$

$$\Big\downarrow \text{1930}$$

$$\frac{4x\sqrt[4]{ax^2+bx^3}}{7b}-\frac{6a\int \frac{x^2}{(bx^3+ax^2)^{3/4}}dx}{7b}$$

$$\Big\downarrow \text{1930}$$

$$\frac{4x\sqrt[4]{ax^2+bx^3}}{7b}-\frac{6a\left(\frac{4\sqrt[4]{ax^2+bx^3}}{3b}-\frac{2a\int \frac{x}{(bx^3+ax^2)^{3/4}}dx}{3b}\right)}{7b}$$

$$\Big\downarrow \text{1938}$$

$$\frac{4x\sqrt[4]{ax^2+bx^3}}{7b}-\frac{6a\left(\frac{4\sqrt[4]{ax^2+bx^3}}{3b}-\frac{2ax^{3/2}(a+bx)^{3/4}\int \frac{1}{\sqrt{x}(a+bx)^{3/4}}dx}{3b(ax^2+bx^3)^{3/4}}\right)}{7b}$$

$$\Big\downarrow \text{73}$$

$$\frac{4x\sqrt[4]{ax^2+bx^3}}{7b}-\frac{6a\left(\frac{4\sqrt[4]{ax^2+bx^3}}{3b}-\frac{8ax^{3/2}(a+bx)^{3/4}\int \frac{1}{\sqrt{\frac{a+bx}{b}-\frac{a}{b}}}d\sqrt[4]{a+bx}}{3b^2(ax^2+bx^3)^{3/4}}\right)}{7b}$$

$$\Big\downarrow \text{765}$$

$$\frac{4x\sqrt[4]{ax^2+bx^3}}{7b}-\frac{6a\left(\frac{4\sqrt[4]{ax^2+bx^3}}{3b}-\frac{8ax^{3/2}(a+bx)^{3/4}\sqrt{1-\frac{a+bx}{a}}\int\frac{1}{\sqrt{1-\frac{a+bx}{a}}}d\sqrt[4]{a+bx}}{3b^2(ax^2+bx^3)^{3/4}\sqrt{\frac{a+bx}{b}-\frac{a}{b}}}\right)}{7b}$$

$$\Big\downarrow \text{762}$$

$$\frac{4x\sqrt[4]{ax^2+bx^3}}{7b}-\frac{6a\left(\frac{4\sqrt[4]{ax^2+bx^3}}{3b}-\frac{8a^{5/4}x^{3/2}(a+bx)^{3/4}\sqrt{1-\frac{a+bx}{a}}\operatorname{EllipticF}\left(\arcsin\left(\frac{\sqrt[4]{a+bx}}{\sqrt[4]{a}}\right),-1\right)}{3b^2(ax^2+bx^3)^{3/4}\sqrt{\frac{a+bx}{b}-\frac{a}{b}}}\right)}{7b}$$

```
Int[x^3/(a*x^2 + b*x^3)^(3/4),x]
```

```
(4*x*(a*x^2 + b*x^3)^(1/4))/(7*b) - (6*a*((4*(a*x^2 + b*x^3)^(1/4))/(3*b)
- (8*a^(5/4)*x^(3/2)*(a + b*x)^(3/4)*Sqrt[1 - (a + b*x)/a]*EllipticF[ArcSi
n[(a + b*x)^(1/4)/a^(1/4)], -1])/(3*b^2*(a*x^2 + b*x^3)^(3/4)*Sqrt[-(a/b)
+ (a + b*x)/b])))/(7*b)
```

Defintions of rubi rules used

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Simp[p/b   Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
 d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && Lt
Q[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntL
inearQ[a, b, c, d, m, n, x]
```

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> Simp[(1/(Sqrt[a]*Rt[-b/a, 4])
)*EllipticF[ArcSin[Rt[-b/a, 4]*x], -1], x] /; FreeQ[{a, b}, x] && NegQ[b/a]
 && GtQ[a, 0]
```

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> Simp[Sqrt[1 + b*(x^4/a)]/Sqrt
[a + b*x^4]   Int[1/Sqrt[1 + b*(x^4/a)], x], x] /; FreeQ[{a, b}, x] && NegQ
[b/a] &&  !GtQ[a, 0]
```

```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> Simp[c^(n - 1)*(c*x)^(m - n + 1)*((a*x^j + b*x^n)^(p + 1)/(b*(m + n*p
+ 1))), x] - Simp[a*c^(n - j)*((m + j*p - n + j + 1)/(b*(m + n*p + 1)))   I
nt[(c*x)^(m - (n - j))*(a*x^j + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, p},
x] &&  !IntegerQ[p] && LtQ[0, j, n] && (IntegersQ[j, n] || GtQ[c, 0]) && Gt
Q[m + j*p - n + j + 1, 0] && NeQ[m + n*p + 1, 0]
```

```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> Simp[c^IntPart[m]*(c*x)^FracPart[m]*((a*x^j + b*x^n)^FracPart[p]/(x^(F
racPart[m] + j*FracPart[p])*(a + b*x^(n - j))^FracPart[p]))   Int[x^(m + j*
p)*(a + b*x^(n - j))^p, x], x] /; FreeQ[{a, b, c, j, m, n, p}, x] &&  !Inte
gerQ[p] && NeQ[n, j] && PosQ[n - j]
```

Maple [F]

$$\int \frac{x^{3}}{\left(b\,x^{3}+a\,x^{2}\right)^{\frac{3}{4}}}dx$$

```
int(x^3/(b*x^3+a*x^2)^(3/4),x)
```

```
int(x^3/(b*x^3+a*x^2)^(3/4),x)
```

Fricas [F]

$$\int \frac{x^3}{(ax^2+bx^3)^{3/4}}\,dx = \int \frac{x^{3}}{{\left(bx^{3}+ax^{2}\right)}^{\frac{3}{4}}}\,dx$$

```
integrate(x^3/(b*x^3+a*x^2)^(3/4),x, algorithm="fricas")
```

```
integral((b*x^3 + a*x^2)^(1/4)*x/(b*x + a), x)
```

Sympy [F]

$$\int \frac{x^3}{(ax^2+bx^3)^{3/4}}\,dx = \int \frac{x^3}{(x^2(a+bx))^{\frac{3}{4}}}\,dx$$

```
integrate(x**3/(b*x**3+a*x**2)**(3/4),x)
```

```
Integral(x**3/(x**2*(a + b*x))**(3/4), x)
```

Maxima [F]

$$\int \frac{x^3}{(ax^2+bx^3)^{3/4}}\,dx = \int \frac{x^3}{(bx^3+ax^2)^{\frac{3}{4}}}\,dx$$

```
integrate(x^3/(b*x^3+a*x^2)^(3/4),x, algorithm="maxima")
```

```
integrate(x^3/(b*x^3 + a*x^2)^(3/4), x)
```

Giac [F]

$$\int \frac{x^3}{(ax^2+bx^3)^{3/4}}\,dx = \int \frac{x^3}{(bx^3+ax^2)^{\frac{3}{4}}}\,dx$$

```
integrate(x^3/(b*x^3+a*x^2)^(3/4),x, algorithm="giac")
```

```
integrate(x^3/(b*x^3 + a*x^2)^(3/4), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{x^3}{(ax^2+bx^3)^{3/4}}\,dx = \int \frac{x^3}{(b\,x^3+a\,x^2)^{3/4}}\,dx$$

```
int(x^3/(a*x^2 + b*x^3)^(3/4),x)
```

```
int(x^3/(a*x^2 + b*x^3)^(3/4), x)
```

Reduce [F]

$$\int \frac{x^3}{(ax^2+bx^3)^{3/4}}\,dx = \frac{-\frac{8\sqrt{x}\,(bx+a)^{\frac{3}{4}}a}{15}+\frac{4\sqrt{x}\,(bx+a)^{\frac{3}{4}}bx}{9}+\frac{4\sqrt{bx+a}\left(\int \frac{\sqrt{x}\,(bx+a)^{\frac{5}{4}}}{b^2x^3+2ab\,x^2+a^2x}dx\right)a^2}{15}}{\sqrt{bx+a}\,b^2}$$

```
int(x^3/(b*x^3+a*x^2)^(3/4),x)
```

```
(4*( - 6*sqrt(x)*(a + b*x)**(3/4)*a + 5*sqrt(x)*(a + b*x)**(3/4)*b*x + 3*s
qrt(a + b*x)*int((sqrt(x)*(a + b*x)**(5/4))/(a**2*x + 2*a*b*x**2 + b**2*x*
*3),x)*a**2))/(45*sqrt(a + b*x)*b**2)
```

3.395 $\int \frac{x^2}{\left(ax^2+bx^3\right)^{3/4}}\,dx$

Optimal result

Integrand size = 19, antiderivative size = 92

$$\int \frac{x^2}{\left(ax^2+bx^3\right)^{3/4}}\,dx = \frac{4\sqrt[4]{ax^2+bx^3}}{3b} - \frac{8a^{3/2}x^{3/2}\left(1+\frac{bx}{a}\right)^{3/4}\operatorname{EllipticF}\left(\frac{1}{2}\arctan\left(\frac{\sqrt{b}\sqrt{x}}{\sqrt{a}}\right),2\right)}{3b^{3/2}\left(ax^2+bx^3\right)^{3/4}}$$

```
4/3*(b*x^3+a*x^2)^(1/4)/b-8/3*a^(3/2)*x^(3/2)*(1+b*x/a)^(3/4)*InverseJacob
iAM(1/2*arctan(b^(1/2)*x^(1/2)/a^(1/2)),2^(1/2))/b^(3/2)/(b*x^3+a*x^2)^(3/
4)
```

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 10.01 (sec) , antiderivative size = 49, normalized size of antiderivative = 0.53

$$\int \frac{x^2}{\left(ax^2+bx^3\right)^{3/4}}\,dx = \frac{2x^3\left(1+\frac{bx}{a}\right)^{3/4}\operatorname{Hypergeometric2F1}\left(\frac{3}{4},\frac{3}{2},\frac{5}{2},-\frac{bx}{a}\right)}{3\left(x^2(a+bx)\right)^{3/4}}$$

```
Integrate[x^2/(a*x^2 + b*x^3)^(3/4),x]
```

```
(2*x^3*(1 + (b*x)/a)^(3/4)*Hypergeometric2F1[3/4, 3/2, 5/2, -((b*x)/a)])/(
3*(x^2*(a + b*x))^(3/4))
```

Rubi [A] (verified)

Time = 0.44 (sec) , antiderivative size = 118, normalized size of antiderivative = 1.28, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.263, Rules used = {1930, 1938, 73, 765, 762}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^2}{(ax^2+bx^3)^{3/4}}\,dx$$
$$\Big\downarrow \text{1930}$$
$$\frac{4\sqrt[4]{ax^2+bx^3}}{3b}-\frac{2a\int \frac{x}{(bx^3+ax^2)^{3/4}}dx}{3b}$$
$$\Big\downarrow \text{1938}$$
$$\frac{4\sqrt[4]{ax^2+bx^3}}{3b}-\frac{2ax^{3/2}(a+bx)^{3/4}\int \frac{1}{\sqrt{x}(a+bx)^{3/4}}dx}{3b\,(ax^2+bx^3)^{3/4}}$$
$$\Big\downarrow \text{73}$$
$$\frac{4\sqrt[4]{ax^2+bx^3}}{3b}-\frac{8ax^{3/2}(a+bx)^{3/4}\int \frac{1}{\sqrt{\frac{a+bx}{b}-\frac{a}{b}}}d\sqrt[4]{a+bx}}{3b^2\,(ax^2+bx^3)^{3/4}}$$
$$\Big\downarrow \text{765}$$
$$\frac{4\sqrt[4]{ax^2+bx^3}}{3b}-\frac{8ax^{3/2}(a+bx)^{3/4}\sqrt{1-\frac{a+bx}{a}}\int \frac{1}{\sqrt{1-\frac{a+bx}{a}}}d\sqrt[4]{a+bx}}{3b^2\,(ax^2+bx^3)^{3/4}\sqrt{\frac{a+bx}{b}-\frac{a}{b}}}$$
$$\Big\downarrow \text{762}$$
$$\frac{4\sqrt[4]{ax^2+bx^3}}{3b}-\frac{8a^{5/4}x^{3/2}(a+bx)^{3/4}\sqrt{1-\frac{a+bx}{a}}\,\text{EllipticF}\left(\arcsin\left(\frac{\sqrt[4]{a+bx}}{\sqrt[4]{a}}\right),-1\right)}{3b^2\,(ax^2+bx^3)^{3/4}\sqrt{\frac{a+bx}{b}-\frac{a}{b}}}$$

```
Int[x^2/(a*x^2 + b*x^3)^(3/4),x]
```

```
(4*(a*x^2 + b*x^3)^(1/4))/(3*b) - (8*a^(5/4)*x^(3/2)*(a + b*x)^(3/4)*Sqrt[
1 - (a + b*x)/a]*EllipticF[ArcSin[(a + b*x)^(1/4)/a^(1/4)], -1])/(3*b^2*(a
*x^2 + b*x^3)^(3/4)*Sqrt[-(a/b) + (a + b*x)/b])
```

Defintions of rubi rules used

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Simp[p/b   Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
 d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && Lt
Q[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntL
inearQ[a, b, c, d, m, n, x]
```

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> Simp[(1/(Sqrt[a]*Rt[-b/a, 4])
)*EllipticF[ArcSin[Rt[-b/a, 4]*x], -1], x] /; FreeQ[{a, b}, x] && NegQ[b/a]
 && GtQ[a, 0]
```

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> Simp[Sqrt[1 + b*(x^4/a)]/Sqrt
[a + b*x^4]   Int[1/Sqrt[1 + b*(x^4/a)], x], x] /; FreeQ[{a, b}, x] && NegQ
[b/a] &&  !GtQ[a, 0]
```

```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> Simp[c^(n - 1)*(c*x)^(m - n + 1)*((a*x^j + b*x^n)^(p + 1)/(b*(m + n*p
+ 1))), x] - Simp[a*c^(n - j)*((m + j*p - n + j + 1)/(b*(m + n*p + 1)))   I
nt[(c*x)^(m - (n - j))*(a*x^j + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, p},
x] &&  !IntegerQ[p] && LtQ[0, j, n] && (IntegersQ[j, n] || GtQ[c, 0]) && Gt
Q[m + j*p - n + j + 1, 0] && NeQ[m + n*p + 1, 0]
```

```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> Simp[c^IntPart[m]*(c*x)^FracPart[m]*((a*x^j + b*x^n)^FracPart[p]/(x^(F
racPart[m] + j*FracPart[p])*(a + b*x^(n - j))^FracPart[p]))   Int[x^(m + j*
p)*(a + b*x^(n - j))^p, x], x] /; FreeQ[{a, b, c, j, m, n, p}, x] &&  !Inte
gerQ[p] && NeQ[n, j] && PosQ[n - j]
```

Maple [F]

$$\int \frac{x^{2}}{\left(b\,x^{3}+a\,x^{2}\right)^{\frac{3}{4}}}dx$$

```
int(x^2/(b*x^3+a*x^2)^(3/4),x)
```

```
int(x^2/(b*x^3+a*x^2)^(3/4),x)
```

Fricas [F]

$$\int \frac{x^2}{(ax^2 + bx^3)^{3/4}}\,dx = \int \frac{x^{2}}{{\left(bx^{3} + ax^{2}\right)}^{\frac{3}{4}}}\,dx$$

```
integrate(x^2/(b*x^3+a*x^2)^(3/4),x, algorithm="fricas")
```

```
integral((b*x^3 + a*x^2)^(1/4)/(b*x + a), x)
```

Sympy [F]

$$\int \frac{x^2}{(ax^2 + bx^3)^{3/4}}\,dx = \int \frac{x^{2}}{\left(x^{2} \left(a + bx\right)\right)^{\frac{3}{4}}}\,dx$$

```
integrate(x**2/(b*x**3+a*x**2)**(3/4),x)
```

```
Integral(x**2/(x**2*(a + b*x))**(3/4), x)
```

Maxima [F]

$$\int \frac{x^2}{(ax^2+bx^3)^{3/4}}\,dx = \int \frac{x^2}{(bx^3+ax^2)^{\frac{3}{4}}}\,dx$$

```
integrate(x^2/(b*x^3+a*x^2)^(3/4),x, algorithm="maxima")
```

```
integrate(x^2/(b*x^3 + a*x^2)^(3/4), x)
```

Giac [F]

$$\int \frac{x^2}{(ax^2+bx^3)^{3/4}}\,dx = \int \frac{x^2}{(bx^3+ax^2)^{\frac{3}{4}}}\,dx$$

```
integrate(x^2/(b*x^3+a*x^2)^(3/4),x, algorithm="giac")
```

```
integrate(x^2/(b*x^3 + a*x^2)^(3/4), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{x^2}{(ax^2+bx^3)^{3/4}}\,dx = \int \frac{x^2}{(b\,x^3+a\,x^2)^{3/4}}\,dx$$

```
int(x^2/(a*x^2 + b*x^3)^(3/4),x)
```

```
int(x^2/(a*x^2 + b*x^3)^(3/4), x)
```

Reduce [F]

$$\int \frac{x^2}{\left(ax^2+bx^3\right)^{3/4}}\,dx = \frac{\frac{4\sqrt{x}\,(bx+a)^{\frac{3}{4}}}{5} - \frac{2\sqrt{bx+a}\left(\int \frac{\sqrt{x}\,(bx+a)^{\frac{5}{4}}}{b^2x^3+2ab\,x^2+a^2x}dx\right)a}{5}}{\sqrt{bx+a}\,b}$$

```
int(x^2/(b*x^3+a*x^2)^(3/4),x)
```

```
(2*(2*sqrt(x)*(a + b*x)**(3/4) - sqrt(a + b*x)*int((sqrt(x)*(a + b*x)**(5/
4))/(a**2*x + 2*a*b*x**2 + b**2*x**3),x)*a))/(5*sqrt(a + b*x)*b)
```

3.396 $\int \frac{x}{\left(ax^2+bx^3\right)^{3/4}}\,dx$

Optimal result

Integrand size = 17, antiderivative size = 67

$$\int \frac{x}{\left(ax^2+bx^3\right)^{3/4}}\,dx=\frac{4\sqrt{a}x^{3/2}\left(1+\frac{bx}{a}\right)^{3/4}\text{EllipticF}\left(\frac{1}{2}\arctan\left(\frac{\sqrt{b}\sqrt{x}}{\sqrt{a}}\right),2\right)}{\sqrt{b}\left(ax^2+bx^3\right)^{3/4}}$$

```
4*a^(1/2)*x^(3/2)*(1+b*x/a)^(3/4)*InverseJacobiAM(1/2*arctan(b^(1/2)*x^(1/
2)/a^(1/2)),2^(1/2))/b^(1/2)/(b*x^3+a*x^2)^(3/4)
```

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 10.01 (sec) , antiderivative size = 47, normalized size of antiderivative = 0.70

$$\int \frac{x}{\left(ax^2+bx^3\right)^{3/4}}\,dx=\frac{2x^2\left(1+\frac{bx}{a}\right)^{3/4}\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{3}{4},\frac{3}{2},-\frac{bx}{a}\right)}{\left(x^2(a+bx)\right)^{3/4}}$$

```
Integrate[x/(a*x^2 + b*x^3)^(3/4),x]
```

```
(2*x^2*(1 + (b*x)/a)^(3/4)*Hypergeometric2F1[1/2, 3/4, 3/2, -((b*x)/a)])/(
x^2*(a + b*x))^(3/4)
```

Rubi [A] (verified)

Time = 0.37 (sec) , antiderivative size = 93, normalized size of antiderivative = 1.39, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.235$, Rules used = {1938, 73, 765, 762}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x}{(ax^2+bx^3)^{3/4}}\,dx$$

$$\Big\downarrow \text{1938}$$

$$\frac{x^{3/2}(a+bx)^{3/4}\int \frac{1}{\sqrt{x}(a+bx)^{3/4}}dx}{(ax^2+bx^3)^{3/4}}$$

$$\Big\downarrow \text{73}$$

$$\frac{4x^{3/2}(a+bx)^{3/4}\int \frac{1}{\sqrt{\frac{a+bx}{b}-\frac{a}{b}}}d\sqrt[4]{a+bx}}{b\,(ax^2+bx^3)^{3/4}}$$

$$\Big\downarrow \text{765}$$

$$\frac{4x^{3/2}(a+bx)^{3/4}\sqrt{1-\frac{a+bx}{a}}\int \frac{1}{\sqrt{1-\frac{a+bx}{a}}}d\sqrt[4]{a+bx}}{b\,(ax^2+bx^3)^{3/4}\sqrt{\frac{a+bx}{b}-\frac{a}{b}}}$$

$$\Big\downarrow \text{762}$$

$$\frac{4\sqrt[4]{a}x^{3/2}(a+bx)^{3/4}\sqrt{1-\frac{a+bx}{a}}\,\text{EllipticF}\left(\arcsin\left(\frac{\sqrt[4]{a+bx}}{\sqrt[4]{a}}\right),-1\right)}{b\,(ax^2+bx^3)^{3/4}\sqrt{\frac{a+bx}{b}-\frac{a}{b}}}$$

```
Int[x/(a*x^2 + b*x^3)^(3/4),x]
```

```
(4*a^(1/4)*x^(3/2)*(a + b*x)^(3/4)*Sqrt[1 - (a + b*x)/a]*EllipticF[ArcSin[
(a + b*x)^(1/4)/a^(1/4)], -1])/(b*(a*x^2 + b*x^3)^(3/4)*Sqrt[-(a/b) + (a +
 b*x)/b])
```

Defintions of rubi rules used

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Simp[p/b   Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
 d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && Lt
Q[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntL
inearQ[a, b, c, d, m, n, x]
```

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> Simp[(1/(Sqrt[a]*Rt[-b/a, 4])
)*EllipticF[ArcSin[Rt[-b/a, 4]*x], -1], x] /; FreeQ[{a, b}, x] && NegQ[b/a]
 && GtQ[a, 0]
```

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> Simp[Sqrt[1 + b*(x^4/a)]/Sqrt
[a + b*x^4]   Int[1/Sqrt[1 + b*(x^4/a)], x], x] /; FreeQ[{a, b}, x] && NegQ
[b/a] &&  !GtQ[a, 0]
```

```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> Simp[c^IntPart[m]*(c*x)^FracPart[m]*((a*x^j + b*x^n)^FracPart[p]/(x^(F
racPart[m] + j*FracPart[p])*(a + b*x^(n - j))^FracPart[p]))   Int[x^(m + j*
p)*(a + b*x^(n - j))^p, x], x] /; FreeQ[{a, b, c, j, m, n, p}, x] &&  !Inte
gerQ[p] && NeQ[n, j] && PosQ[n - j]
```

Maple [F]

$$\int \frac{x}{\left(b\,x^{3}+a\,x^{2}\right)^{\frac{3}{4}}}dx$$

```
int(x/(b*x^3+a*x^2)^(3/4),x)
```

```
int(x/(b*x^3+a*x^2)^(3/4),x)
```

Fricas [F]

$$\int \frac{x}{\left(ax^2+bx^3\right)^{3/4}}\,dx=\int \frac{x}{\left(bx^3+ax^2\right)^{\frac{3}{4}}}\,dx$$

```
integrate(x/(b*x^3+a*x^2)^(3/4),x, algorithm="fricas")
```

```
integral((b*x^3 + a*x^2)^(1/4)/(b*x^2 + a*x), x)
```

Sympy [F]

$$\int \frac{x}{\left(ax^2+bx^3\right)^{3/4}}\,dx=\int \frac{x}{\left(x^2\left(a+bx\right)\right)^{\frac{3}{4}}}\,dx$$

```
integrate(x/(b*x**3+a*x**2)**(3/4),x)
```

```
Integral(x/(x**2*(a + b*x))**(3/4), x)
```

Maxima [F]

$$\int \frac{x}{\left(ax^2+bx^3\right)^{3/4}}\,dx=\int \frac{x}{\left(bx^3+ax^2\right)^{\frac{3}{4}}}\,dx$$

```
integrate(x/(b*x^3+a*x^2)^(3/4),x, algorithm="maxima")
```

```
integrate(x/(b*x^3 + a*x^2)^(3/4), x)
```

Giac [F]

$$\int \frac{x}{(ax^2+bx^3)^{3/4}}\,dx=\int \frac{x}{(bx^3+ax^2)^{\frac{3}{4}}}\,dx$$

```
integrate(x/(b*x^3+a*x^2)^(3/4),x, algorithm="giac")
```

```
integrate(x/(b*x^3 + a*x^2)^(3/4), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{x}{(ax^2+bx^3)^{3/4}}\,dx=\int \frac{x}{(b\,x^3+a\,x^2)^{3/4}}\,dx$$

```
int(x/(a*x^2 + b*x^3)^(3/4),x)
```

```
int(x/(a*x^2 + b*x^3)^(3/4), x)
```

Reduce [F]

$$\int \frac{x}{(ax^2+bx^3)^{3/4}}\,dx=\int \frac{\sqrt{x}\,(bx+a)^{\frac{3}{4}}}{\sqrt{bx+a}\,ax+\sqrt{bx+a}\,b\,x^2}dx$$

```
int(x/(b*x^3+a*x^2)^(3/4),x)
```

```
int((sqrt(x)*(a + b*x)**(3/4))/(sqrt(a + b*x)*a*x + sqrt(a + b*x)*b*x**2),
x)
```

3.397 $\int \frac{1}{(ax^2+bx^3)^{3/4}}\,dx$

Optimal result

Integrand size = 15, antiderivative size = 91

$$\int \frac{1}{(ax^2+bx^3)^{3/4}}\,dx = -\frac{2\sqrt[4]{ax^2+bx^3}}{ax} - \frac{2\sqrt{b}x^{3/2}\left(1+\frac{bx}{a}\right)^{3/4}\operatorname{EllipticF}\left(\frac{1}{2}\arctan\left(\frac{\sqrt{b}\sqrt{x}}{\sqrt{a}}\right),2\right)}{\sqrt{a}\,(ax^2+bx^3)^{3/4}}$$

```
-2*(b*x^3+a*x^2)^(1/4)/a/x-2*b^(1/2)*x^(3/2)*(1+b*x/a)^(3/4)*InverseJacobi
AM(1/2*arctan(b^(1/2)*x^(1/2)/a^(1/2)),2^(1/2))/a^(1/2)/(b*x^3+a*x^2)^(3/4
)
```

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 0.01 (sec) , antiderivative size = 45, normalized size of antiderivative = 0.49

$$\int \frac{1}{(ax^2+bx^3)^{3/4}}\,dx = -\frac{2x\left(1+\frac{bx}{a}\right)^{3/4}\operatorname{Hypergeometric2F1}\left(-\frac{1}{2},\frac{3}{4},\frac{1}{2},-\frac{bx}{a}\right)}{(x^2(a+bx))^{3/4}}$$

```
Integrate[(a*x^2 + b*x^3)^(-3/4),x]
```

```
(-2*x*(1 + (b*x)/a)^(3/4)*Hypergeometric2F1[-1/2, 3/4, 1/2, -((b*x)/a)])/(
x^2*(a + b*x))^(3/4)
```

Rubi [A] (verified)

Time = 0.39 (sec) , antiderivative size = 111, normalized size of antiderivative = 1.22, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.333$, Rules used = {1917, 61, 73, 765, 762}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{(ax^2+bx^3)^{3/4}}dx$$

$$\Big\downarrow \text{1917}$$

$$\frac{x^{3/2}(a+bx)^{3/4}\int \frac{1}{x^{3/2}(a+bx)^{3/4}}dx}{(ax^2+bx^3)^{3/4}}$$

$$\Big\downarrow \text{61}$$

$$\frac{x^{3/2}(a+bx)^{3/4}\left(-\frac{b\int \frac{1}{\sqrt{x}(a+bx)^{3/4}}dx}{2a}-\frac{2\sqrt[4]{a+bx}}{a\sqrt{x}}\right)}{(ax^2+bx^3)^{3/4}}$$

$$\Big\downarrow \text{73}$$

$$\frac{x^{3/2}(a+bx)^{3/4}\left(-\frac{2\int \frac{1}{\sqrt{\frac{a+bx}{b}-\frac{a}{b}}}d\sqrt[4]{a+bx}}{a}-\frac{2\sqrt[4]{a+bx}}{a\sqrt{x}}\right)}{(ax^2+bx^3)^{3/4}}$$

$$\Big\downarrow \text{765}$$

$$\frac{x^{3/2}(a+bx)^{3/4}\left(-\frac{2\sqrt{1-\frac{a+bx}{a}}\int \frac{1}{\sqrt{1-\frac{a+bx}{a}}}d\sqrt[4]{a+bx}}{a\sqrt{\frac{a+bx}{b}-\frac{a}{b}}}-\frac{2\sqrt[4]{a+bx}}{a\sqrt{x}}\right)}{(ax^2+bx^3)^{3/4}}$$

$$\Big\downarrow \text{762}$$

$$\frac{x^{3/2}(a+bx)^{3/4}\left(-\frac{2\sqrt{1-\frac{a+bx}{a}}\,\text{EllipticF}\left(\arcsin\left(\frac{\sqrt[4]{a+bx}}{\sqrt[4]{a}}\right),-1\right)}{a^{3/4}\sqrt{\frac{a+bx}{b}-\frac{a}{b}}}-\frac{2\sqrt[4]{a+bx}}{a\sqrt{x}}\right)}{(ax^2+bx^3)^{3/4}}$$

```
Int[(a*x^2 + b*x^3)^(-3/4),x]
```

```
(x^(3/2)*(a + b*x)^(3/4)*((-2*(a + b*x)^(1/4))/(a*Sqrt[x]) - (2*Sqrt[1 - (
a + b*x)/a]*EllipticF[ArcSin[(a + b*x)^(1/4)/a^(1/4)], -1])/(a^(3/4)*Sqrt[
-(a/b) + (a + b*x)/b])))/(a*x^2 + b*x^3)^(3/4)
```

Defintions of rubi rules used

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] - Simp[d*((
m + n + 2)/((b*c - a*d)*(m + 1)))   Int[(a + b*x)^(m + 1)*(c + d*x)^n, x],
x] /; FreeQ[{a, b, c, d, n}, x] && LtQ[m, -1] &&  !(LtQ[n, -1] && (EqQ[a, 0
] || (NeQ[c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && IntLinearQ[a, b, c, d
, m, n, x]
```

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Simp[p/b   Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
 d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && Lt
Q[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntL
inearQ[a, b, c, d, m, n, x]
```

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> Simp[(1/(Sqrt[a]*Rt[-b/a, 4])
)*EllipticF[ArcSin[Rt[-b/a, 4]*x], -1], x] /; FreeQ[{a, b}, x] && NegQ[b/a]
 && GtQ[a, 0]
```

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> Simp[Sqrt[1 + b*(x^4/a)]/Sqrt
[a + b*x^4]   Int[1/Sqrt[1 + b*(x^4/a)], x], x] /; FreeQ[{a, b}, x] && NegQ
[b/a] &&  !GtQ[a, 0]
```

```
Int[((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol] :> Simp[(a*x^j +
b*x^n)^FracPart[p]/(x^(j*FracPart[p])*(a + b*x^(n - j))^FracPart[p])   Int[
x^(j*p)*(a + b*x^(n - j))^p, x], x] /; FreeQ[{a, b, j, n, p}, x] &&  !Integ
erQ[p] && NeQ[n, j] && PosQ[n - j]
```

Maple [F]

$$\int \frac{1}{\left(b\,x^{3}+a\,x^{2}\right)^{\frac{3}{4}}}dx$$

```
int(1/(b*x^3+a*x^2)^(3/4),x)
```

```
int(1/(b*x^3+a*x^2)^(3/4),x)
```

Fricas [F]

$$\int \frac{1}{(ax^2+bx^3)^{3/4}}\,dx = \int \frac{1}{(bx^3+ax^2)^{\frac{3}{4}}}\,dx$$

```
integrate(1/(b*x^3+a*x^2)^(3/4),x, algorithm="fricas")
```

```
integral((b*x^3 + a*x^2)^(-3/4), x)
```

Sympy [F]

$$\int \frac{1}{(ax^2+bx^3)^{3/4}}\,dx = \int \frac{1}{(ax^2+bx^3)^{\frac{3}{4}}}\,dx$$

```
integrate(1/(b*x**3+a*x**2)**(3/4),x)
```

```
Integral((a*x**2 + b*x**3)**(-3/4), x)
```

Maxima [F]

$$\int \frac{1}{(ax^2+bx^3)^{3/4}}\,dx = \int \frac{1}{(bx^3+ax^2)^{\frac{3}{4}}}\,dx$$

```
integrate(1/(b*x^3+a*x^2)^(3/4),x, algorithm="maxima")
```

```
integrate((b*x^3 + a*x^2)^(-3/4), x)
```

Giac [F]

$$\int \frac{1}{(ax^2+bx^3)^{3/4}}\,dx = \int \frac{1}{(bx^3+ax^2)^{\frac{3}{4}}}\,dx$$

```
integrate(1/(b*x^3+a*x^2)^(3/4),x, algorithm="giac")
```

```
integrate((b*x^3 + a*x^2)^(-3/4), x)
```

Mupad [B] (verification not implemented)

Time = 9.05 (sec) , antiderivative size = 38, normalized size of antiderivative = 0.42

$$\int \frac{1}{(ax^2+bx^3)^{3/4}}\,dx = -\frac{2\,x\,\left(\frac{b\,x}{a}+1\right)^{3/4}\,{}_2\mathrm{F}_1\left(-\frac{1}{2},\frac{3}{4};\ \frac{1}{2};\ -\frac{b\,x}{a}\right)}{\left(b\,x^3+a\,x^2\right)^{3/4}}$$

```
int(1/(a*x^2 + b*x^3)^(3/4),x)
```

```
-(2*x*((b*x)/a + 1)^(3/4)*hypergeom([-1/2, 3/4], 1/2, -(b*x)/a))/(a*x^2 +
b*x^3)^(3/4)
```

Reduce [F]

$$\int \frac{1}{(ax^2+bx^3)^{3/4}}\,dx = \int \frac{\sqrt{x}\,(bx+a)^{\frac{3}{4}}}{\sqrt{bx+a}\,a\,x^2+\sqrt{bx+a}\,b\,x^3}dx$$

```
int(1/(b*x^3+a*x^2)^(3/4),x)
```

```
int((sqrt(x)*(a + b*x)**(3/4))/(sqrt(a + b*x)*a*x**2 + sqrt(a + b*x)*b*x**
3),x)
```

3.398 $\int \frac{1}{x(ax^2+bx^3)^{3/4}}\,dx$

Optimal result

Integrand size = 19, antiderivative size = 121

$$\int \frac{1}{x\left(ax^2+bx^3\right)^{3/4}}\,dx = -\frac{2\sqrt[4]{ax^2+bx^3}}{3ax^2}+\frac{5b\sqrt[4]{ax^2+bx^3}}{3a^2x} + \frac{5b^{3/2}x^{3/2}\left(1+\frac{bx}{a}\right)^{3/4}\operatorname{EllipticF}\left(\frac{1}{2}\arctan\left(\frac{\sqrt{b}\sqrt{x}}{\sqrt{a}}\right),2\right)}{3a^{3/2}\left(ax^2+bx^3\right)^{3/4}}$$

```
-2/3*(b*x^3+a*x^2)^(1/4)/a/x^2+5/3*b*(b*x^3+a*x^2)^(1/4)/a^2/x+5/3*b^(3/2)
*x^(3/2)*(1+b*x/a)^(3/4)*InverseJacobiAM(1/2*arctan(b^(1/2)*x^(1/2)/a^(1/2
)),2^(1/2))/a^(3/2)/(b*x^3+a*x^2)^(3/4)
```

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 10.01 (sec) , antiderivative size = 46, normalized size of antiderivative = 0.38

$$\int \frac{1}{x\left(ax^2+bx^3\right)^{3/4}}\,dx = -\frac{2\left(1+\frac{bx}{a}\right)^{3/4}\text{Hypergeometric2F1}\left(-\frac{3}{2},\frac{3}{4},-\frac{1}{2},-\frac{bx}{a}\right)}{3\left(x^2(a+bx)\right)^{3/4}}$$

```
Integrate[1/(x*(a*x^2 + b*x^3)^(3/4)),x]
```

```
(-2*(1 + (b*x)/a)^(3/4)*Hypergeometric2F1[-3/2, 3/4, -1/2, -((b*x)/a)])/(3
*(x^2*(a + b*x))^(3/4))
```

Rubi [A] (verified)

Time = 0.46 (sec) , antiderivative size = 144, normalized size of antiderivative = 1.19, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.316, Rules used = {1931, 1917, 61, 73, 765, 762}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{x\left(ax^2+bx^3\right)^{3/4}}dx$$

$$\Big\downarrow \text{1931}$$

$$-\frac{5b\int \frac{1}{(bx^3+ax^2)^{3/4}}dx}{6a}-\frac{2\sqrt[4]{ax^2+bx^3}}{3ax^2}$$

$$\Big\downarrow \text{1917}$$

$$-\frac{5bx^{3/2}(a+bx)^{3/4}\int \frac{1}{x^{3/2}(a+bx)^{3/4}}dx}{6a\left(ax^2+bx^3\right)^{3/4}}-\frac{2\sqrt[4]{ax^2+bx^3}}{3ax^2}$$

$$\Big\downarrow \text{61}$$

$$-\frac{5bx^{3/2}(a+bx)^{3/4}\left(-\frac{b\int \frac{1}{\sqrt{x}(a+bx)^{3/4}}dx}{2a}-\frac{2\sqrt[4]{a+bx}}{a\sqrt{x}}\right)}{6a\left(ax^2+bx^3\right)^{3/4}}-\frac{2\sqrt[4]{ax^2+bx^3}}{3ax^2}$$

$$\Big\downarrow \text{73}$$

$$-\frac{5bx^{3/2}(a+bx)^{3/4}\left(-\frac{2\int \frac{1}{\sqrt{\frac{a+bx}{b}-\frac{a}{b}}}d\sqrt[4]{a+bx}}{a}-\frac{2\sqrt[4]{a+bx}}{a\sqrt{x}}\right)}{6a\left(ax^2+bx^3\right)^{3/4}}-\frac{2\sqrt[4]{ax^2+bx^3}}{3ax^2}$$

$$\Big\downarrow \text{765}$$

$$-\frac{5bx^{3/2}(a+bx)^{3/4}\left(-\frac{2\sqrt{1-\frac{a+bx}{a}}\int \frac{1}{\sqrt{1-\frac{a+bx}{a}}}d\sqrt[4]{a+bx}}{a\sqrt{\frac{a+bx}{b}-\frac{a}{b}}}-\frac{2\sqrt[4]{a+bx}}{a\sqrt{x}}\right)}{6a\left(ax^2+bx^3\right)^{3/4}}-\frac{2\sqrt[4]{ax^2+bx^3}}{3ax^2}$$

$$\Big\downarrow 762$$

$$-\frac{5bx^{3/2}(a+bx)^{3/4}\left(-\frac{2\sqrt{1-\frac{a+bx}{a}}\,\text{EllipticF}\left(\arcsin\left(\frac{\sqrt[4]{a+bx}}{\sqrt[4]{a}}\right),-1\right)}{a^{3/4}\sqrt{\frac{a+bx}{b}-\frac{a}{b}}}-\frac{2\sqrt[4]{a+bx}}{a\sqrt{x}}\right)}{6a\left(ax^2+bx^3\right)^{3/4}}-\frac{2\sqrt[4]{ax^2+bx^3}}{3ax^2}$$

```
Int[1/(x*(a*x^2 + b*x^3)^(3/4)),x]
```

```
(-2*(a*x^2 + b*x^3)^(1/4))/(3*a*x^2) - (5*b*x^(3/2)*(a + b*x)^(3/4)*((-2*(
a + b*x)^(1/4))/(a*Sqrt[x]) - (2*Sqrt[1 - (a + b*x)/a]*EllipticF[ArcSin[(a
 + b*x)^(1/4)/a^(1/4)], -1])/(a^(3/4)*Sqrt[-(a/b) + (a + b*x)/b])))/(6*a*(
a*x^2 + b*x^3)^(3/4))
```

Defintions of rubi rules used

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] - Simp[d*((
m + n + 2)/((b*c - a*d)*(m + 1)))   Int[(a + b*x)^(m + 1)*(c + d*x)^n, x],
x] /; FreeQ[{a, b, c, d, n}, x] && LtQ[m, -1] &&  !(LtQ[n, -1] && (EqQ[a, 0
] || (NeQ[c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && IntLinearQ[a, b, c, d
, m, n, x]
```

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Simp[p/b   Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
 d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && Lt
Q[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntL
inearQ[a, b, c, d, m, n, x]
```

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> Simp[(1/(Sqrt[a]*Rt[-b/a, 4])
)*EllipticF[ArcSin[Rt[-b/a, 4]*x], -1], x] /; FreeQ[{a, b}, x] && NegQ[b/a]
 && GtQ[a, 0]
```

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> Simp[Sqrt[1 + b*(x^4/a)]/Sqrt
[a + b*x^4]   Int[1/Sqrt[1 + b*(x^4/a)], x], x] /; FreeQ[{a, b}, x] && NegQ
[b/a] &&  !GtQ[a, 0]
```

```
Int[((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol] :> Simp[(a*x^j +
b*x^n)^FracPart[p]/(x^(j*FracPart[p])*(a + b*x^(n - j))^FracPart[p])   Int[
x^(j*p)*(a + b*x^(n - j))^p, x], x] /; FreeQ[{a, b, j, n, p}, x] &&  !Integ
erQ[p] && NeQ[n, j] && PosQ[n - j]
```

```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> Simp[c^(j - 1)*(c*x)^(m - j + 1)*((a*x^j + b*x^n)^(p + 1)/(a*(m + j*p
+ 1))), x] - Simp[b*((m + n*p + n - j + 1)/(a*c^(n - j)*(m + j*p + 1)))   I
nt[(c*x)^(m + n - j)*(a*x^j + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, p}, x]
 &&  !IntegerQ[p] && LtQ[0, j, n] && (IntegersQ[j, n] || GtQ[c, 0]) && LtQ[
m + j*p + 1, 0]
```

Maple [F]

$$\int \frac{1}{x\,(b\,x^{3}+a\,x^{2})^{\frac{3}{4}}}dx$$

```
int(1/x/(b*x^3+a*x^2)^(3/4),x)
```

```
int(1/x/(b*x^3+a*x^2)^(3/4),x)
```

Fricas [F]

$$\int \frac{1}{x\,(ax^2+bx^3)^{3/4}}\,dx = \int \frac{1}{(bx^3+ax^2)^{\frac{3}{4}}x}\,dx$$

```
integrate(1/x/(b*x^3+a*x^2)^(3/4),x, algorithm="fricas")
```

```
integral((b*x^3 + a*x^2)^(1/4)/(b*x^4 + a*x^3), x)
```

Sympy [F]

$$\int \frac{1}{x\left(ax^2+bx^3\right)^{3/4}}\,dx=\int \frac{1}{x\left(x^2\left(a+bx\right)\right)^{\frac{3}{4}}}\,dx$$

```
integrate(1/x/(b*x**3+a*x**2)**(3/4),x)
```

```
Integral(1/(x*(x**2*(a + b*x))**(3/4)), x)
```

Maxima [F]

$$\int \frac{1}{x\left(ax^2+bx^3\right)^{3/4}}\,dx=\int \frac{1}{\left(bx^3+ax^2\right)^{\frac{3}{4}}x}\,dx$$

```
integrate(1/x/(b*x^3+a*x^2)^(3/4),x, algorithm="maxima")
```

```
integrate(1/((b*x^3 + a*x^2)^(3/4)*x), x)
```

Giac [F]

$$\int \frac{1}{x\left(ax^2+bx^3\right)^{3/4}}\,dx=\int \frac{1}{\left(bx^3+ax^2\right)^{\frac{3}{4}}x}\,dx$$

```
integrate(1/x/(b*x^3+a*x^2)^(3/4),x, algorithm="giac")
```

```
integrate(1/((b*x^3 + a*x^2)^(3/4)*x), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{x\left(ax^2+bx^3\right)^{3/4}}\,dx=\int \frac{1}{x\,{\left(b\,x^3+a\,x^2\right)}^{3/4}}\,dx$$

```
int(1/(x*(a*x^2 + b*x^3)^(3/4)),x)
```

```
int(1/(x*(a*x^2 + b*x^3)^(3/4)), x)
```

Reduce [F]

$$\int \frac{1}{x\left(ax^2+bx^3\right)^{3/4}}\,dx=\int \frac{\sqrt{x}\,(bx+a)^{\frac{3}{4}}}{\sqrt{bx+a}\,a\,x^3+\sqrt{bx+a}\,b\,x^4}dx$$

```
int(1/x/(b*x^3+a*x^2)^(3/4),x)
```

```
int((sqrt(x)*(a + b*x)**(3/4))/(sqrt(a + b*x)*a*x**3 + sqrt(a + b*x)*b*x**
4),x)
```

3.399 $\int \frac{1}{x^2\left(ax^2+bx^3\right)^{3/4}}\,dx$

Optimal result

Integrand size = 19, antiderivative size = 149

$$\int \frac{1}{x^2\left(ax^2+bx^3\right)^{3/4}}\,dx = -\frac{2\sqrt[4]{ax^2+bx^3}}{5ax^3}+\frac{3b\sqrt[4]{ax^2+bx^3}}{5a^2x^2} - \frac{3b^2\sqrt[4]{ax^2+bx^3}}{2a^3x}-\frac{3b^{5/2}x^{3/2}\left(1+\frac{bx}{a}\right)^{3/4}\text{EllipticF}\left(\frac{1}{2}\arctan\left(\frac{\sqrt{b}\sqrt{x}}{\sqrt{a}}\right),2\right)}{2a^{5/2}\left(ax^2+bx^3\right)^{3/4}}$$

```
-2/5*(b*x^3+a*x^2)^(1/4)/a/x^3+3/5*b*(b*x^3+a*x^2)^(1/4)/a^2/x^2-3/2*b^2*(
b*x^3+a*x^2)^(1/4)/a^3/x-3/2*b^(5/2)*x^(3/2)*(1+b*x/a)^(3/4)*InverseJacobi
AM(1/2*arctan(b^(1/2)*x^(1/2)/a^(1/2)),2^(1/2))/a^(5/2)/(b*x^3+a*x^2)^(3/4
)
```

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 10.01 (sec) , antiderivative size = 49, normalized size of antiderivative = 0.33

$$\int \frac{1}{x^2\left(ax^2+bx^3\right)^{3/4}}\,dx = -\frac{2\left(1+\frac{bx}{a}\right)^{3/4}\text{Hypergeometric2F1}\left(-\frac{5}{2},\frac{3}{4},-\frac{3}{2},-\frac{bx}{a}\right)}{5x\left(x^2(a+bx)\right)^{3/4}}$$

```
Integrate[1/(x^2*(a*x^2 + b*x^3)^(3/4)),x]
```

```
(-2*(1 + (b*x)/a)^(3/4)*Hypergeometric2F1[-5/2, 3/4, -3/2, -((b*x)/a)])/(5
*x*(x^2*(a + b*x))^(3/4))
```

Rubi [A] (verified)

Time = 0.55 (sec) , antiderivative size = 178, normalized size of antiderivative = 1.19, number of steps used = 8, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}} = 0.368$, Rules used = {1931, 1931, 1917, 61, 73, 765, 762}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{x^2 (ax^2+bx^3)^{3/4}} \, dx$$

$$\Big\downarrow \text{1931}$$

$$-\frac{9b \int \frac{1}{x(bx^3+ax^2)^{3/4}}dx}{10a}-\frac{2\sqrt[4]{ax^2+bx^3}}{5ax^3}$$

$$\Big\downarrow \text{1931}$$

$$-\frac{9b\left(-\frac{5b\int \frac{1}{\left(bx^3+ax^2\right)^{3/4}}dx}{6a}-\frac{2\sqrt[4]{ax^2+bx^3}}{3ax^2}\right)}{10a}-\frac{2\sqrt[4]{ax^2+bx^3}}{5ax^3}$$

$$\Big\downarrow \text{1917}$$

$$-\frac{9b\left(-\frac{5bx^{3/2}(a+bx)^{3/4}\int \frac{1}{x^{3/2}(a+bx)^{3/4}}dx}{6a(ax^2+bx^3)^{3/4}}-\frac{2\sqrt[4]{ax^2+bx^3}}{3ax^2}\right)}{10a}-\frac{2\sqrt[4]{ax^2+bx^3}}{5ax^3}$$

$$\Big\downarrow \text{61}$$

$$-\frac{9b\left(-\frac{5bx^{3/2}(a+bx)^{3/4}\left(-\frac{b\int \frac{1}{\sqrt{x}(a+bx)^{3/4}}dx}{2a}-\frac{2\sqrt[4]{a+bx}}{a\sqrt{x}}\right)}{6a(ax^2+bx^3)^{3/4}}-\frac{2\sqrt[4]{ax^2+bx^3}}{3ax^2}\right)}{10a}-\frac{2\sqrt[4]{ax^2+bx^3}}{5ax^3}$$

$$\Big\downarrow \textcolor{blue}{73}$$

$$-\frac{9b\left(-\frac{5bx^{3/2}(a+bx)^{3/4}\left(-\frac{2\int\frac{1}{\sqrt{\frac{a+bx}{b}-\frac{a}{b}}}d\sqrt[4]{a+bx}}{a}-\frac{2\sqrt[4]{a+bx}}{a\sqrt{x}}\right)}{6a(ax^2+bx^3)^{3/4}}-\frac{2\sqrt[4]{ax^2+bx^3}}{3ax^2}\right)}{10a}-\frac{2\sqrt[4]{ax^2+bx^3}}{5ax^3}$$

$$\Big\downarrow \textcolor{blue}{765}$$

$$-\frac{9b\left(-\frac{5bx^{3/2}(a+bx)^{3/4}\left(-\frac{2\sqrt{1-\frac{a+bx}{a}}\int\frac{1}{\sqrt{1-\frac{a+bx}{a}}}d\sqrt[4]{a+bx}}{a\sqrt{\frac{a+bx}{b}-\frac{a}{b}}}-\frac{2\sqrt[4]{a+bx}}{a\sqrt{x}}\right)}{6a(ax^2+bx^3)^{3/4}}-\frac{2\sqrt[4]{ax^2+bx^3}}{3ax^2}\right)}{10a}-\frac{2\sqrt[4]{ax^2+bx^3}}{5ax^3}$$

$$\Big\downarrow \textcolor{blue}{762}$$

$$-\frac{9b\left(-\frac{5bx^{3/2}(a+bx)^{3/4}\left(-\frac{2\sqrt{1-\frac{a+bx}{a}}\,\text{EllipticF}\left(\arcsin\left(\frac{\sqrt[4]{a+bx}}{\sqrt[4]{a}}\right),-1\right)}{a^{3/4}\sqrt{\frac{a+bx}{b}-\frac{a}{b}}}-\frac{2\sqrt[4]{a+bx}}{a\sqrt{x}}\right)}{6a(ax^2+bx^3)^{3/4}}-\frac{2\sqrt[4]{ax^2+bx^3}}{3ax^2}\right)}{10a}-\frac{2\sqrt[4]{ax^2+bx^3}}{5ax^3}$$

```
Int[1/(x^2*(a*x^2 + b*x^3)^(3/4)),x]
```

```
(-2*(a*x^2 + b*x^3)^(1/4))/(5*a*x^3) - (9*b*((-2*(a*x^2 + b*x^3)^(1/4))/(3
*a*x^2) - (5*b*x^(3/2)*(a + b*x)^(3/4)*((-2*(a + b*x)^(1/4))/(a*Sqrt[x]) -
 (2*Sqrt[1 - (a + b*x)/a]*EllipticF[ArcSin[(a + b*x)^(1/4)/a^(1/4)], -1])/
(a^(3/4)*Sqrt[-(a/b) + (a + b*x)/b])))/(6*a*(a*x^2 + b*x^3)^(3/4))))/(10*a
)
```

Defintions of rubi rules used

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] - Simp[d*((
m + n + 2)/((b*c - a*d)*(m + 1)))   Int[(a + b*x)^(m + 1)*(c + d*x)^n, x],
x] /; FreeQ[{a, b, c, d, n}, x] && LtQ[m, -1] &&  !(LtQ[n, -1] && (EqQ[a, 0
] || (NeQ[c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && IntLinearQ[a, b, c, d
, m, n, x]
```

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Simp[p/b   Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
 d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && Lt
Q[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntL
inearQ[a, b, c, d, m, n, x]
```

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> Simp[(1/(Sqrt[a]*Rt[-b/a, 4])
)*EllipticF[ArcSin[Rt[-b/a, 4]*x], -1], x] /; FreeQ[{a, b}, x] && NegQ[b/a]
 && GtQ[a, 0]
```

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> Simp[Sqrt[1 + b*(x^4/a)]/Sqrt
[a + b*x^4]   Int[1/Sqrt[1 + b*(x^4/a)], x], x] /; FreeQ[{a, b}, x] && NegQ
[b/a] &&  !GtQ[a, 0]
```

```
Int[((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol] :> Simp[(a*x^j +
b*x^n)^FracPart[p]/(x^(j*FracPart[p])*(a + b*x^(n - j))^FracPart[p])   Int[
x^(j*p)*(a + b*x^(n - j))^p, x], x] /; FreeQ[{a, b, j, n, p}, x] &&  !Integ
erQ[p] && NeQ[n, j] && PosQ[n - j]
```

```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> Simp[c^(j - 1)*(c*x)^(m - j + 1)*((a*x^j + b*x^n)^(p + 1)/(a*(m + j*p
+ 1))), x] - Simp[b*((m + n*p + n - j + 1)/(a*c^(n - j)*(m + j*p + 1)))   I
nt[(c*x)^(m + n - j)*(a*x^j + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, p}, x]
 &&  !IntegerQ[p] && LtQ[0, j, n] && (IntegersQ[j, n] || GtQ[c, 0]) && LtQ[
m + j*p + 1, 0]
```

Maple [F]

$$\int \frac{1}{x^{2} \left(b\,x^{3}+a\,x^{2}\right)^{\frac{3}{4}}}dx$$

```
int(1/x^2/(b*x^3+a*x^2)^(3/4),x)
```

```
int(1/x^2/(b*x^3+a*x^2)^(3/4),x)
```

Fricas [F]

$$\int \frac{1}{x^2 (ax^2 + bx^3)^{3/4}} dx = \int \frac{1}{(bx^3 + ax^2)^{\frac{3}{4}} x^2} dx$$

```
integrate(1/x^2/(b*x^3+a*x^2)^(3/4),x, algorithm="fricas")
```

```
integral((b*x^3 + a*x^2)^(1/4)/(b*x^5 + a*x^4), x)
```

Sympy [F]

$$\int \frac{1}{x^2 (ax^2 + bx^3)^{3/4}} dx = \int \frac{1}{x^2 (x^2 (a + bx))^{\frac{3}{4}}} dx$$

```
integrate(1/x**2/(b*x**3+a*x**2)**(3/4),x)
```

```
Integral(1/(x**2*(x**2*(a + b*x))**(3/4)), x)
```

Maxima [F]

$$\int \frac{1}{x^2 (ax^2 + bx^3)^{3/4}} \, dx = \int \frac{1}{(bx^3 + ax^2)^{\frac{3}{4}} x^2} \, dx$$

```
integrate(1/x^2/(b*x^3+a*x^2)^(3/4),x, algorithm="maxima")
```

```
integrate(1/((b*x^3 + a*x^2)^(3/4)*x^2), x)
```

Giac [F]

$$\int \frac{1}{x^2 (ax^2 + bx^3)^{3/4}} \, dx = \int \frac{1}{(bx^3 + ax^2)^{\frac{3}{4}} x^2} \, dx$$

```
integrate(1/x^2/(b*x^3+a*x^2)^(3/4),x, algorithm="giac")
```

```
integrate(1/((b*x^3 + a*x^2)^(3/4)*x^2), x)
```

Mupad [B] (verification not implemented)

Time = 8.89 (sec) , antiderivative size = 44, normalized size of antiderivative = 0.30

$$\int \frac{1}{x^2 (ax^2 + bx^3)^{3/4}} \, dx = -\frac{4 \left(\frac{a}{b\,x} + 1\right)^{3/4} {}_2\mathrm{F}_1\left(\frac{3}{4}, \frac{13}{4};\ \frac{17}{4};\ -\frac{a}{b\,x}\right)}{13\,x \left(b\,x^3 + a\,x^2\right)^{3/4}}$$

```
int(1/(x^2*(a*x^2 + b*x^3)^(3/4)),x)
```

```
-(4*(a/(b*x) + 1)^(3/4)*hypergeom([3/4, 13/4], 17/4, -a/(b*x)))/(13*x*(a*x
^2 + b*x^3)^(3/4))
```

Reduce [F]

$$\int \frac{1}{x^2 (ax^2+bx^3)^{3/4}}\,dx = \int \frac{\sqrt{x}\,(bx+a)^{\frac{3}{4}}}{\sqrt{bx+a}\,a\,x^4+\sqrt{bx+a}\,b\,x^5}dx$$

```
int(1/x^2/(b*x^3+a*x^2)^(3/4),x)
```

```
int((sqrt(x)*(a + b*x)**(3/4))/(sqrt(a + b*x)*a*x**4 + sqrt(a + b*x)*b*x**
5),x)
```

3.400 $\int \frac{x^5}{\left(ax^2+bx^3\right)^{5/4}}\,dx$

Optimal result

Integrand size = 19, antiderivative size = 148

$$\int \frac{x^5}{(ax^2+bx^3)^{5/4}}\,dx = \frac{16a^2x}{3b^3\sqrt[4]{ax^2+bx^3}} - \frac{8ax^2}{9b^2\sqrt[4]{ax^2+bx^3}} + \frac{4x^3}{9b\sqrt[4]{ax^2+bx^3}} - \frac{32a^{5/2}\sqrt{x}\sqrt[4]{\frac{a+bx}{a}}E\left(\frac{1}{2}\arctan\left(\frac{\sqrt{b}\sqrt{x}}{\sqrt{a}}\right)\middle|2\right)}{3b^{7/2}\sqrt[4]{ax^2+bx^3}}$$

```
16/3*a^2*x/b^3/(b*x^3+a*x^2)^(1/4)-8/9*a*x^2/b^2/(b*x^3+a*x^2)^(1/4)+4/9*x
^3/b/(b*x^3+a*x^2)^(1/4)-32/3*a^(5/2)*x^(1/2)*((b*x+a)/a)^(1/4)*EllipticE(
sin(1/2*arctan(b^(1/2)*x^(1/2)/a^(1/2))),2^(1/2))/b^(7/2)/(b*x^3+a*x^2)^(1
/4)
```

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 10.01 (sec) , antiderivative size = 52, normalized size of antiderivative = 0.35

$$\int \frac{x^5}{(ax^2+bx^3)^{5/4}}\,dx = \frac{2x^4\sqrt[4]{1+\frac{bx}{a}}\,\text{Hypergeometric2F1}\left(\frac{5}{4},\frac{7}{2},\frac{9}{2},-\frac{bx}{a}\right)}{7a\sqrt[4]{x^2(a+bx)}}$$

```
Integrate[x^5/(a*x^2 + b*x^3)^(5/4),x]
```

```
(2*x^4*(1 + (b*x)/a)^(1/4)*Hypergeometric2F1[5/4, 7/2, 9/2, -((b*x)/a)])/(
7*a*(x^2*(a + b*x))^(1/4))
```

Rubi [A] (verified)

Time = 0.86 (sec) , antiderivative size = 245, normalized size of antiderivative = 1.66, number of steps used = 13, number of rules used = 12, $\frac{\text{number of rules}}{\text{integrand size}} = 0.632$, Rules used = {1928, 1930, 1930, 1917, 73, 836, 27, 765, 762, 1390, 1389, 327}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^5}{(ax^2+bx^3)^{5/4}}dx$$

$$\Big\downarrow \text{ 1928}$$

$$\frac{10\int \frac{x^2}{\sqrt[4]{bx^3+ax^2}}dx}{b}-\frac{4x^3}{b\sqrt[4]{ax^2+bx^3}}$$

$$\Big\downarrow \text{ 1930}$$

$$\frac{10\left(\frac{4(ax^2+bx^3)^{3/4}}{9b}-\frac{2a\int \frac{x}{\sqrt[4]{bx^3+ax^2}}dx}{3b}\right)}{b}-\frac{4x^3}{b\sqrt[4]{ax^2+bx^3}}$$

$$\Big\downarrow \text{ 1930}$$

$$\frac{10\left(\frac{4(ax^2+bx^3)^{3/4}}{9b}-\frac{2a\left(\frac{4\left(ax^2+bx^3\right)^{3/4}}{5bx}-\frac{2a\int \frac{1}{\sqrt[4]{bx^3+ax^2}}dx}{5b}\right)}{3b}\right)}{b}-\frac{4x^3}{b\sqrt[4]{ax^2+bx^3}}$$

$$\Big\downarrow \text{ 1917}$$

$$\frac{10\left(\frac{4\left(ax^2+bx^3\right)^{3/4}}{9b}-\frac{2a\left(\frac{4\left(ax^2+bx^3\right)^{3/4}}{5bx}-\frac{2a\sqrt{x}\sqrt[4]{a+bx}\int\frac{1}{\sqrt{x}\sqrt[4]{a+bx}}dx}{5b\sqrt[4]{ax^2+bx^3}}\right)}{3b}\right)}{b}-\frac{4x^3}{b\sqrt[4]{ax^2+bx^3}}$$

$\Big\downarrow$ 73

$$\frac{10\left(\frac{4\left(ax^2+bx^3\right)^{3/4}}{9b}-\frac{2a\left(\frac{4\left(ax^2+bx^3\right)^{3/4}}{5bx}-\frac{8a\sqrt{x}\sqrt[4]{a+bx}\int\frac{\sqrt{a+bx}}{\sqrt{\frac{a+bx}{b}-\frac{a}{b}}}d\sqrt[4]{a+bx}}{5b^2\sqrt[4]{ax^2+bx^3}}\right)}{3b}\right)}{b}-\frac{4x^3}{b\sqrt[4]{ax^2+bx^3}}$$

$\Big\downarrow$ 836

$$\frac{10\left(\frac{4\left(ax^2+bx^3\right)^{3/4}}{9b}-\frac{2a\left(\frac{4\left(ax^2+bx^3\right)^{3/4}}{5bx}-\frac{8a\sqrt{x}\sqrt[4]{a+bx}\left(\sqrt{a}\int\frac{\sqrt{a}+\sqrt{a+bx}}{\sqrt{a}\sqrt{\frac{a+bx}{b}-\frac{a}{b}}}d\sqrt[4]{a+bx}-\sqrt{a}\int\frac{1}{\sqrt{\frac{a+bx}{b}-\frac{a}{b}}}d\sqrt[4]{a+bx}\right)}{5b^2\sqrt[4]{ax^2+bx^3}}\right)}{3b}\right)}{b}-\frac{4x^3}{b\sqrt[4]{ax^2+bx^3}}$$

$\Big\downarrow$ 27

$$\frac{10\left(\frac{4\left(ax^2+bx^3\right)^{3/4}}{9b}-\frac{2a\left(\frac{4\left(ax^2+bx^3\right)^{3/4}}{5bx}-\frac{8a\sqrt{x}\sqrt[4]{a+bx}\left(\int\frac{\sqrt{a}+\sqrt{a+bx}}{\sqrt{\frac{a+bx}{b}-\frac{a}{b}}}d\sqrt[4]{a+bx}-\sqrt{a}\int\frac{1}{\sqrt{\frac{a+bx}{b}-\frac{a}{b}}}d\sqrt[4]{a+bx}\right)}{5b^2\sqrt[4]{ax^2+bx^3}}\right)}{3b}\right)}{b}-\frac{4x^3}{b\sqrt[4]{ax^2+bx^3}}$$

$\Big\downarrow$ 765

$$\frac{10\left(\frac{4\left(ax^2+bx^3\right)^{3/4}}{9b}-\frac{2a\left(\frac{4\left(ax^2+bx^3\right)^{3/4}}{5bx}-\frac{8a\sqrt{x}\sqrt[4]{a+bx}\left(\int \frac{\sqrt{a}+\sqrt{a+bx}}{\sqrt{\frac{a+bx}{b}-\frac{a}{b}}}d\sqrt[4]{a+bx}-\frac{\sqrt{a}\sqrt{1-\frac{a+bx}{a}}\int \frac{1}{\sqrt{1-\frac{a+bx}{a}}}d\sqrt[4]{a+bx}}{\sqrt{\frac{a+bx}{b}-\frac{a}{b}}}\right)}{5b^2\sqrt[4]{ax^2+bx^3}}\right)}{3b}\right)}{b}-\frac{4x^3}{b\sqrt[4]{ax^2+bx^3}}$$

$$\Big\downarrow \; 762$$

$$\frac{10\left(\frac{4\left(ax^2+bx^3\right)^{3/4}}{9b}-\frac{2a\left(\frac{4\left(ax^2+bx^3\right)^{3/4}}{5bx}-\frac{8a\sqrt{x}\sqrt[4]{a+bx}\left(\int \frac{\sqrt{a}+\sqrt{a+bx}}{\sqrt{\frac{a+bx}{b}-\frac{a}{b}}}d\sqrt[4]{a+bx}-\frac{a^{3/4}\sqrt{1-\frac{a+bx}{a}}\operatorname{EllipticF}\left(\arcsin\left(\frac{\sqrt[4]{a+bx}}{\sqrt[4]{a}}\right),-1\right)}{\sqrt{\frac{a+bx}{b}-\frac{a}{b}}}\right)}{5b^2\sqrt[4]{ax^2+bx^3}}\right)}{3b}\right)}{b}-\frac{4x^3}{b\sqrt[4]{ax^2+bx^3}}$$

$$\Big\downarrow \; 1390$$

$$\frac{10\left(\frac{4\left(ax^2+bx^3\right)^{3/4}}{9b}-\frac{2a\left(\frac{4\left(ax^2+bx^3\right)^{3/4}}{5bx}-\frac{8a\sqrt{x}\sqrt[4]{a+bx}\left(\frac{\sqrt{1-\frac{a+bx}{a}}\int\frac{\sqrt{a}+\sqrt{a+bx}}{\sqrt{1-\frac{a+bx}{a}}}d\sqrt[4]{a+bx}}{\sqrt{\frac{a+bx}{b}-\frac{a}{b}}}-\frac{a^{3/4}\sqrt{1-\frac{a+bx}{a}}\,\text{EllipticF}\left(\arcsin\left(\frac{\sqrt[4]{a+\cdots}}{\sqrt[4]{a}\cdots}\right)\cdots\right)}{\sqrt{\frac{a+bx}{b}-\frac{a}{b}}}\cdots\right)}{5b^2\sqrt[4]{ax^2+bx^3}}\cdots\right)}{3b}\cdots\right)}{b}\cdots$$

$$\frac{4x^3}{b\sqrt[4]{ax^2+bx^3}}$$

$$\Big\downarrow \; 1389$$

$$\frac{10\left(\frac{4\left(ax^2+bx^3\right)^{3/4}}{9b}-\frac{2a\left(\frac{4\left(ax^2+bx^3\right)^{3/4}}{5bx}-\frac{8a\sqrt{x}\sqrt[4]{a+bx}\left(\frac{\sqrt{a}\sqrt{1-\frac{a+bx}{a}}\int\frac{\sqrt{\frac{\sqrt{a+bx}}{\sqrt{a}}+1}}{\sqrt{1-\frac{\sqrt{a+bx}}{\sqrt{a}}}}d\sqrt[4]{a+bx}}{\sqrt{\frac{a+bx}{b}-\frac{a}{b}}}-\frac{a^{3/4}\sqrt{1-\frac{a+bx}{a}}\,\text{EllipticF}\left(\arcsin\left(\frac{\sqrt[4]{a\cdots}}{\sqrt[4]{\cdots}}\right)\cdots\right)}{\sqrt{\frac{a+bx}{b}-\frac{a}{b}}}\cdots\right)}{5b^2\sqrt[4]{ax^2+bx^3}}\cdots\right)}{3b}\cdots\right)}{b}\cdots$$

$$\frac{4x^3}{b\sqrt[4]{ax^2+bx^3}}$$

$$\Big\downarrow \; 327$$

$$\frac{10\left(\frac{4(ax^2+bx^3)^{3/4}}{9b}-\frac{2a\left(\frac{4\left(ax^2+bx^3\right)^{3/4}}{5bx}-\frac{8a\sqrt{x}\sqrt[4]{a+bx}\left(\frac{a^{3/4}\sqrt{1-\frac{a+bx}{a}}E\left(\arcsin\left(\frac{\sqrt[4]{a+bx}}{\sqrt[4]{a}}\right)\middle|-1\right)}{\sqrt{\frac{a+bx}{b}-\frac{a}{b}}}-\frac{a^{3/4}\sqrt{1-\frac{a+bx}{a}}\,\text{EllipticF}\left(\arcsin\left(\frac{\sqrt[4]{a+bx}}{\sqrt[4]{a}}\right),-1\right)}{\sqrt{\frac{a+bx}{b}-\frac{a}{b}}}\right)}{5b^2\sqrt[4]{ax^2+bx^3}}\right)}{3b}\right)}{b}$$

$$\frac{4x^3}{b\sqrt[4]{ax^2+bx^3}}$$

```
Int[x^5/(a*x^2 + b*x^3)^(5/4),x]
```

```
(-4*x^3)/(b*(a*x^2 + b*x^3)^(1/4)) + (10*((4*(a*x^2 + b*x^3)^(3/4))/(9*b)
- (2*a*((4*(a*x^2 + b*x^3)^(3/4))/(5*b*x) - (8*a*Sqrt[x]*(a + b*x)^(1/4)*(
(a^(3/4)*Sqrt[1 - (a + b*x)/a]*EllipticE[ArcSin[(a + b*x)^(1/4)/a^(1/4)],
-1])/Sqrt[-(a/b) + (a + b*x)/b] - (a^(3/4)*Sqrt[1 - (a + b*x)/a]*EllipticF
[ArcSin[(a + b*x)^(1/4)/a^(1/4)], -1])/Sqrt[-(a/b) + (a + b*x)/b]))/(5*b^2
*(a*x^2 + b*x^3)^(1/4))))/(3*b)))/b
```

Defintions of rubi rules used

```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Simp[p/b   Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
 d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && Lt
Q[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntL
inearQ[a, b, c, d, m, n, x]
```

```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Simp[
(Sqrt[a]/(Sqrt[c]*Rt[-d/c, 2]))*EllipticE[ArcSin[Rt[-d/c, 2]*x], b*(c/(a*d)
)], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a, 0]
```

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> Simp[(1/(Sqrt[a]*Rt[-b/a, 4])
)*EllipticF[ArcSin[Rt[-b/a, 4]*x], -1], x] /; FreeQ[{a, b}, x] && NegQ[b/a]
 && GtQ[a, 0]
```

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> Simp[Sqrt[1 + b*(x^4/a)]/Sqrt
[a + b*x^4]   Int[1/Sqrt[1 + b*(x^4/a)], x], x] /; FreeQ[{a, b}, x] && NegQ
[b/a] &&  !GtQ[a, 0]
```

```
Int[(x_)^2/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[-b/a, 2]},
Simp[-q^(-1)   Int[1/Sqrt[a + b*x^4], x], x] + Simp[1/q   Int[(1 + q*x^2)/S
qrt[a + b*x^4], x], x]] /; FreeQ[{a, b}, x] && NegQ[b/a]
```

```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (c_.)*(x_)^4], x_Symbol] :> Simp[d/Sq
rt[a]   Int[Sqrt[1 + e*(x^2/d)]/Sqrt[1 - e*(x^2/d)], x], x] /; FreeQ[{a, c,
 d, e}, x] && EqQ[c*d^2 + a*e^2, 0] && NegQ[c/a] && GtQ[a, 0]
```

```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (c_.)*(x_)^4], x_Symbol] :> Simp[Sqrt
[1 + c*(x^4/a)]/Sqrt[a + c*x^4]   Int[(d + e*x^2)/Sqrt[1 + c*(x^4/a)], x],
x] /; FreeQ[{a, c, d, e}, x] && EqQ[c*d^2 + a*e^2, 0] && NegQ[c/a] &&  !GtQ
[a, 0] &&  !(LtQ[a, 0] && GtQ[c, 0])
```

```
Int[((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol] :> Simp[(a*x^j +
b*x^n)^FracPart[p]/(x^(j*FracPart[p])*(a + b*x^(n - j))^FracPart[p])   Int[
x^(j*p)*(a + b*x^(n - j))^p, x], x] /; FreeQ[{a, b, j, n, p}, x] &&  !Integ
erQ[p] && NeQ[n, j] && PosQ[n - j]
```

```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> Simp[c^(n - 1)*(c*x)^(m - n + 1)*((a*x^j + b*x^n)^(p + 1)/(b*(n - j)*(
p + 1))), x] - Simp[c^n*((m + j*p - n + j + 1)/(b*(n - j)*(p + 1)))   Int[(
c*x)^(m - n)*(a*x^j + b*x^n)^(p + 1), x], x] /; FreeQ[{a, b, c}, x] &&  !In
tegerQ[p] && LtQ[0, j, n] && (IntegersQ[j, n] || GtQ[c, 0]) && LtQ[p, -1] &
& GtQ[m + j*p + 1, n - j]
```

```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> Simp[c^(n - 1)*(c*x)^(m - n + 1)*((a*x^j + b*x^n)^(p + 1)/(b*(m + n*p
+ 1))), x] - Simp[a*c^(n - j)*((m + j*p - n + j + 1)/(b*(m + n*p + 1)))   I
nt[(c*x)^(m - (n - j))*(a*x^j + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, p},
x] &&  !IntegerQ[p] && LtQ[0, j, n] && (IntegersQ[j, n] || GtQ[c, 0]) && Gt
Q[m + j*p - n + j + 1, 0] && NeQ[m + n*p + 1, 0]
```

Maple [F]

$$\int \frac{x^{5}}{\left(b\,x^{3}+a\,x^{2}\right)^{\frac{5}{4}}}dx$$

```
int(x^5/(b*x^3+a*x^2)^(5/4),x)
```

```
int(x^5/(b*x^3+a*x^2)^(5/4),x)
```

Fricas [F]

$$\int \frac{x^5}{(ax^2+bx^3)^{5/4}}\,dx = \int \frac{x^5}{(bx^3+ax^2)^{\frac{5}{4}}}\,dx$$

```
integrate(x^5/(b*x^3+a*x^2)^(5/4),x, algorithm="fricas")
```

```
integral((b*x^3 + a*x^2)^(3/4)*x/(b^2*x^2 + 2*a*b*x + a^2), x)
```

Sympy [F]

$$\int \frac{x^5}{(ax^2 + bx^3)^{5/4}} \, dx = \int \frac{x^5}{(x^2 (a + bx))^{\frac{5}{4}}} \, dx$$

```
integrate(x**5/(b*x**3+a*x**2)**(5/4),x)
```

```
Integral(x**5/(x**2*(a + b*x))**(5/4), x)
```

Maxima [F]

$$\int \frac{x^5}{(ax^2 + bx^3)^{5/4}} \, dx = \int \frac{x^5}{(bx^3 + ax^2)^{\frac{5}{4}}} \, dx$$

```
integrate(x^5/(b*x^3+a*x^2)^(5/4),x, algorithm="maxima")
```

```
integrate(x^5/(b*x^3 + a*x^2)^(5/4), x)
```

Giac [F]

$$\int \frac{x^5}{(ax^2 + bx^3)^{5/4}} \, dx = \int \frac{x^5}{(bx^3 + ax^2)^{\frac{5}{4}}} \, dx$$

```
integrate(x^5/(b*x^3+a*x^2)^(5/4),x, algorithm="giac")
```

```
integrate(x^5/(b*x^3 + a*x^2)^(5/4), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{x^5}{(ax^2+bx^3)^{5/4}}\,dx = \int \frac{x^5}{(b\,x^3+a\,x^2)^{5/4}}\,dx$$

```
int(x^5/(a*x^2 + b*x^3)^(5/4),x)
```

```
int(x^5/(a*x^2 + b*x^3)^(5/4), x)
```

Reduce [F]

$$\int \frac{x^5}{(ax^2+bx^3)^{5/4}}\,dx = \frac{\frac{80\sqrt{x}\,(bx+a)^{\frac{1}{4}}a^2}{77} - \frac{40\sqrt{x}\,(bx+a)^{\frac{1}{4}}abx}{77} + \frac{4\sqrt{x}\,(bx+a)^{\frac{1}{4}}b^2x^2}{11} - \frac{40\sqrt{bx+a}\left(\int \frac{\sqrt{x}\,(bx+a)^{\frac{3}{4}}}{b^2x^3+2ab\,x^2+a^2x}dx\right)a^3}{77}}{\sqrt{bx+a}\,b^3}$$

```
int(x^5/(b*x^3+a*x^2)^(5/4),x)
```

```
(4*(20*sqrt(x)*(a + b*x)**(1/4)*a**2 - 10*sqrt(x)*(a + b*x)**(1/4)*a*b*x +
 7*sqrt(x)*(a + b*x)**(1/4)*b**2*x**2 - 10*sqrt(a + b*x)*int((sqrt(x)*(a +
 b*x)**(3/4))/(a**2*x + 2*a*b*x**2 + b**2*x**3),x)*a**3))/(77*sqrt(a + b*x
)*b**3)
```

3.401 $\int \frac{x^4}{(ax^2+bx^3)^{5/4}}\,dx$

Optimal result

Integrand size = 19, antiderivative size = 120

$$\int \frac{x^4}{(ax^2+bx^3)^{5/4}}\,dx = -\frac{24ax}{5b^2\sqrt[4]{ax^2+bx^3}}+\frac{4x^2}{5b\sqrt[4]{ax^2+bx^3}} + \frac{48a^{3/2}\sqrt{x}\sqrt[4]{\frac{a+bx}{a}}E\left(\frac{1}{2}\arctan\left(\frac{\sqrt{b}\sqrt{x}}{\sqrt{a}}\right)\middle|2\right)}{5b^{5/2}\sqrt[4]{ax^2+bx^3}}$$

```
-24/5*a*x/b^2/(b*x^3+a*x^2)^(1/4)+4/5*x^2/b/(b*x^3+a*x^2)^(1/4)+48/5*a^(3/
2)*x^(1/2)*((b*x+a)/a)^(1/4)*EllipticE(sin(1/2*arctan(b^(1/2)*x^(1/2)/a^(1
/2))),2^(1/2))/b^(5/2)/(b*x^3+a*x^2)^(1/4)
```

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 10.01 (sec) , antiderivative size = 52, normalized size of antiderivative = 0.43

$$\int \frac{x^4}{(ax^2+bx^3)^{5/4}}\,dx = \frac{2x^3\sqrt[4]{1+\frac{bx}{a}}\,\text{Hypergeometric2F1}\left(\frac{5}{4},\frac{5}{2},\frac{7}{2},-\frac{bx}{a}\right)}{5a\sqrt[4]{x^2(a+bx)}}$$

```
Integrate[x^4/(a*x^2 + b*x^3)^(5/4),x]
```

```
(2*x^3*(1 + (b*x)/a)^(1/4)*Hypergeometric2F1[5/4, 5/2, 7/2, -((b*x)/a)])/(
5*a*(x^2*(a + b*x))^(1/4))
```

Rubi [A] (verified)

Time = 0.75 (sec) , antiderivative size = 214, normalized size of antiderivative = 1.78, number of steps used = 12, number of rules used = 11, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.579, Rules used = {1928, 1930, 1917, 73, 836, 27, 765, 762, 1390, 1389, 327}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^4}{(ax^2+bx^3)^{5/4}}dx$$

$$\Big\downarrow \text{1928}$$

$$\frac{6\int \frac{x}{\sqrt[4]{bx^3+ax^2}}dx}{b}-\frac{4x^2}{b\sqrt[4]{ax^2+bx^3}}$$

$$\Big\downarrow \text{1930}$$

$$\frac{6\left(\frac{4(ax^2+bx^3)^{3/4}}{5bx}-\frac{2a\int \frac{1}{\sqrt[4]{bx^3+ax^2}}dx}{5b}\right)}{b}-\frac{4x^2}{b\sqrt[4]{ax^2+bx^3}}$$

$$\Big\downarrow \text{1917}$$

$$\frac{6\left(\frac{4(ax^2+bx^3)^{3/4}}{5bx}-\frac{2a\sqrt{x}\sqrt[4]{a+bx}\int \frac{1}{\sqrt{x}\sqrt[4]{a+bx}}dx}{5b\sqrt[4]{ax^2+bx^3}}\right)}{b}-\frac{4x^2}{b\sqrt[4]{ax^2+bx^3}}$$

$$\Big\downarrow \text{73}$$

$$\frac{6\left(\frac{4(ax^2+bx^3)^{3/4}}{5bx}-\frac{8a\sqrt{x}\sqrt[4]{a+bx}\int \frac{\sqrt{a+bx}}{\sqrt{\frac{a+bx}{b}-\frac{a}{b}}}d\sqrt[4]{a+bx}}{5b^2\sqrt[4]{ax^2+bx^3}}\right)}{b}-\frac{4x^2}{b\sqrt[4]{ax^2+bx^3}}$$

$$\Big\downarrow \text{836}$$

$$\frac{6\left(\frac{4(ax^2+bx^3)^{3/4}}{5bx}-\frac{8a\sqrt{x}\sqrt[4]{a+bx}\left(\sqrt{a}\int\frac{\sqrt{a}+\sqrt{a+bx}}{\sqrt{a}\sqrt{\frac{a+bx}{b}-\frac{a}{b}}}d\sqrt[4]{a+bx}-\sqrt{a}\int\frac{1}{\sqrt{\frac{a+bx}{b}-\frac{a}{b}}}d\sqrt[4]{a+bx}\right)}{5b^2\sqrt[4]{ax^2+bx^3}}\right)}{b}-\frac{4x^2}{b\sqrt[4]{ax^2+bx^3}}$$

$$\Big\downarrow \text{27}$$

$$\frac{6\left(\frac{4(ax^2+bx^3)^{3/4}}{5bx}-\frac{8a\sqrt{x}\sqrt[4]{a+bx}\left(\int\frac{\sqrt{a}+\sqrt{a+bx}}{\sqrt{\frac{a+bx}{b}-\frac{a}{b}}}d\sqrt[4]{a+bx}-\sqrt{a}\int\frac{1}{\sqrt{\frac{a+bx}{b}-\frac{a}{b}}}d\sqrt[4]{a+bx}\right)}{5b^2\sqrt[4]{ax^2+bx^3}}\right)}{b}-\frac{4x^2}{b\sqrt[4]{ax^2+bx^3}}$$

$$\Big\downarrow \text{765}$$

$$\frac{6\left(\frac{4(ax^2+bx^3)^{3/4}}{5bx}-\frac{8a\sqrt{x}\sqrt[4]{a+bx}\left(\int\frac{\sqrt{a}+\sqrt{a+bx}}{\sqrt{\frac{a+bx}{b}-\frac{a}{b}}}d\sqrt[4]{a+bx}-\frac{\sqrt{a}\sqrt{1-\frac{a+bx}{a}}\int\frac{1}{\sqrt{1-\frac{a+bx}{a}}}d\sqrt[4]{a+bx}}{\sqrt{\frac{a+bx}{b}-\frac{a}{b}}}\right)}{5b^2\sqrt[4]{ax^2+bx^3}}\right)}{b}-\frac{4x^2}{b\sqrt[4]{ax^2+bx^3}}$$

$$\Big\downarrow \text{762}$$

$$\frac{6\left(\frac{4(ax^2+bx^3)^{3/4}}{5bx}-\frac{8a\sqrt{x}\sqrt[4]{a+bx}\left(\int\frac{\sqrt{a}+\sqrt{a+bx}}{\sqrt{\frac{a+bx}{b}-\frac{a}{b}}}d\sqrt[4]{a+bx}-\frac{a^{3/4}\sqrt{1-\frac{a+bx}{a}}\,\text{EllipticF}\left(\arcsin\left(\frac{\sqrt[4]{a+bx}}{\sqrt[4]{a}}\right),-1\right)}{\sqrt{\frac{a+bx}{b}-\frac{a}{b}}}\right)}{5b^2\sqrt[4]{ax^2+bx^3}}\right)}{b}-\frac{4x^2}{b\sqrt[4]{ax^2+bx^3}}$$

$$\Big\downarrow \text{1390}$$

$$\frac{6\left(\frac{4\left(ax^2+bx^3\right)^{3/4}}{5bx}-\frac{8a\sqrt{x}\sqrt[4]{a+bx}\left(\frac{\sqrt{1-\frac{a+bx}{a}}\int\frac{\sqrt{a}+\sqrt{a+bx}}{\sqrt{1-\frac{a+bx}{a}}}d\sqrt[4]{a+bx}}{\sqrt{\frac{a+bx}{b}-\frac{a}{b}}}-\frac{a^{3/4}\sqrt{1-\frac{a+bx}{a}}\,\text{EllipticF}\left(\arcsin\left(\frac{\sqrt[4]{a+bx}}{\sqrt[4]{a}}\right),-1\right)}{\sqrt{\frac{a+bx}{b}-\frac{a}{b}}}\right)}{5b^2\sqrt[4]{ax^2+bx^3}}\right)}{b}-\frac{4x^2}{b\sqrt[4]{ax^2+bx^3}}$$

$$\Big\downarrow \textbf{1389}$$

$$\frac{6\left(\frac{4\left(ax^2+bx^3\right)^{3/4}}{5bx}-\frac{8a\sqrt{x}\sqrt[4]{a+bx}\left(\frac{\sqrt{a}\sqrt{1-\frac{a+bx}{a}}\int\frac{\sqrt{\frac{\sqrt{a+bx}}{\sqrt{a}}+1}}{\sqrt{1-\frac{\sqrt{a+bx}}{\sqrt{a}}}}d\sqrt[4]{a+bx}}{\sqrt{\frac{a+bx}{b}-\frac{a}{b}}}-\frac{a^{3/4}\sqrt{1-\frac{a+bx}{a}}\,\text{EllipticF}\left(\arcsin\left(\frac{\sqrt[4]{a+bx}}{\sqrt[4]{a}}\right),-1\right)}{\sqrt{\frac{a+bx}{b}-\frac{a}{b}}}\right)}{5b^2\sqrt[4]{ax^2+bx^3}}\right)}{b}-\frac{4x^2}{b\sqrt[4]{ax^2+bx^3}}$$

$$\Big\downarrow \textbf{327}$$

$$\frac{6\left(\frac{4\left(ax^2+bx^3\right)^{3/4}}{5bx}-\frac{8a\sqrt{x}\sqrt[4]{a+bx}\left(\frac{a^{3/4}\sqrt{1-\frac{a+bx}{a}}E\left(\arcsin\left(\frac{\sqrt[4]{a+bx}}{\sqrt[4]{a}}\right)\middle|-1\right)}{\sqrt{\frac{a+bx}{b}-\frac{a}{b}}}-\frac{a^{3/4}\sqrt{1-\frac{a+bx}{a}}\,\text{EllipticF}\left(\arcsin\left(\frac{\sqrt[4]{a+bx}}{\sqrt[4]{a}}\right),-1\right)}{\sqrt{\frac{a+bx}{b}-\frac{a}{b}}}\right)}{5b^2\sqrt[4]{ax^2+bx^3}}\right)}{b}-\frac{4x^2}{b\sqrt[4]{ax^2+bx^3}}$$

```
Int[x^4/(a*x^2 + b*x^3)^(5/4),x]
```

```
(-4*x^2)/(b*(a*x^2 + b*x^3)^(1/4)) + (6*((4*(a*x^2 + b*x^3)^(3/4))/(5*b*x)
 - (8*a*Sqrt[x]*(a + b*x)^(1/4)*((a^(3/4)*Sqrt[1 - (a + b*x)/a]*EllipticE[
ArcSin[(a + b*x)^(1/4)/a^(1/4)], -1])/Sqrt[-(a/b) + (a + b*x)/b] - (a^(3/4
)*Sqrt[1 - (a + b*x)/a]*EllipticF[ArcSin[(a + b*x)^(1/4)/a^(1/4)], -1])/Sq
rt[-(a/b) + (a + b*x)/b]))/(5*b^2*(a*x^2 + b*x^3)^(1/4))))/b
```

Defintions of rubi rules used

```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Simp[p/b   Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
 d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && Lt
Q[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntL
inearQ[a, b, c, d, m, n, x]
```

```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Simp[
(Sqrt[a]/(Sqrt[c]*Rt[-d/c, 2]))*EllipticE[ArcSin[Rt[-d/c, 2]*x], b*(c/(a*d)
)], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a, 0]
```

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> Simp[(1/(Sqrt[a]*Rt[-b/a, 4])
)*EllipticF[ArcSin[Rt[-b/a, 4]*x], -1], x] /; FreeQ[{a, b}, x] && NegQ[b/a]
 && GtQ[a, 0]
```

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> Simp[Sqrt[1 + b*(x^4/a)]/Sqrt
[a + b*x^4]   Int[1/Sqrt[1 + b*(x^4/a)], x], x] /; FreeQ[{a, b}, x] && NegQ
[b/a] &&  !GtQ[a, 0]
```

```
Int[(x_)^2/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[-b/a, 2]},
Simp[-q^(-1)   Int[1/Sqrt[a + b*x^4], x], x] + Simp[1/q   Int[(1 + q*x^2)/S
qrt[a + b*x^4], x], x]] /; FreeQ[{a, b}, x] && NegQ[b/a]
```

```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (c_.)*(x_)^4], x_Symbol] :> Simp[d/Sq
rt[a]   Int[Sqrt[1 + e*(x^2/d)]/Sqrt[1 - e*(x^2/d)], x], x] /; FreeQ[{a, c,
 d, e}, x] && EqQ[c*d^2 + a*e^2, 0] && NegQ[c/a] && GtQ[a, 0]
```

```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (c_.)*(x_)^4], x_Symbol] :> Simp[Sqrt
[1 + c*(x^4/a)]/Sqrt[a + c*x^4]   Int[(d + e*x^2)/Sqrt[1 + c*(x^4/a)], x],
x] /; FreeQ[{a, c, d, e}, x] && EqQ[c*d^2 + a*e^2, 0] && NegQ[c/a] &&  !GtQ
[a, 0] &&  !(LtQ[a, 0] && GtQ[c, 0])
```

```
Int[((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol] :> Simp[(a*x^j +
b*x^n)^FracPart[p]/(x^(j*FracPart[p])*(a + b*x^(n - j))^FracPart[p])   Int[
x^(j*p)*(a + b*x^(n - j))^p, x], x] /; FreeQ[{a, b, j, n, p}, x] &&  !Integ
erQ[p] && NeQ[n, j] && PosQ[n - j]
```

```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> Simp[c^(n - 1)*(c*x)^(m - n + 1)*((a*x^j + b*x^n)^(p + 1)/(b*(n - j)*(
p + 1))), x] - Simp[c^n*((m + j*p - n + j + 1)/(b*(n - j)*(p + 1)))   Int[(
c*x)^(m - n)*(a*x^j + b*x^n)^(p + 1), x], x] /; FreeQ[{a, b, c}, x] &&  !In
tegerQ[p] && LtQ[0, j, n] && (IntegersQ[j, n] || GtQ[c, 0]) && LtQ[p, -1] &
& GtQ[m + j*p + 1, n - j]
```

```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> Simp[c^(n - 1)*(c*x)^(m - n + 1)*((a*x^j + b*x^n)^(p + 1)/(b*(m + n*p
+ 1))), x] - Simp[a*c^(n - j)*((m + j*p - n + j + 1)/(b*(m + n*p + 1)))   I
nt[(c*x)^(m - (n - j))*(a*x^j + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, p},
x] &&  !IntegerQ[p] && LtQ[0, j, n] && (IntegersQ[j, n] || GtQ[c, 0]) && Gt
Q[m + j*p - n + j + 1, 0] && NeQ[m + n*p + 1, 0]
```

Maple [F]

$$\int \frac{x^4}{(b\,x^3+a\,x^2)^{\frac{5}{4}}}dx$$

```
int(x^4/(b*x^3+a*x^2)^(5/4),x)
```

```
int(x^4/(b*x^3+a*x^2)^(5/4),x)
```

Fricas [F]

$$\int \frac{x^4}{(ax^2+bx^3)^{5/4}}\,dx = \int \frac{x^4}{(bx^3+ax^2)^{\frac{5}{4}}}\,dx$$

```
integrate(x^4/(b*x^3+a*x^2)^(5/4),x, algorithm="fricas")
```

```
integral((b*x^3 + a*x^2)^(3/4)/(b^2*x^2 + 2*a*b*x + a^2), x)
```

Sympy [F]

$$\int \frac{x^4}{(ax^2+bx^3)^{5/4}}\,dx = \int \frac{x^4}{(x^2\,(a+bx))^{\frac{5}{4}}}\,dx$$

```
integrate(x**4/(b*x**3+a*x**2)**(5/4),x)
```

```
Integral(x**4/(x**2*(a + b*x))**(5/4), x)
```

Maxima [F]

$$\int \frac{x^4}{(ax^2+bx^3)^{5/4}}\,dx = \int \frac{x^4}{(bx^3+ax^2)^{\frac{5}{4}}}\,dx$$

```
integrate(x^4/(b*x^3+a*x^2)^(5/4),x, algorithm="maxima")
```

```
integrate(x^4/(b*x^3 + a*x^2)^(5/4), x)
```

Giac [F]

$$\int \frac{x^4}{(ax^2+bx^3)^{5/4}}\,dx = \int \frac{x^4}{(bx^3+ax^2)^{\frac{5}{4}}}\,dx$$

```
integrate(x^4/(b*x^3+a*x^2)^(5/4),x, algorithm="giac")
```

```
integrate(x^4/(b*x^3 + a*x^2)^(5/4), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{x^4}{(ax^2+bx^3)^{5/4}}\,dx = \int \frac{x^4}{(b\,x^3+a\,x^2)^{5/4}}\,dx$$

```
int(x^4/(a*x^2 + b*x^3)^(5/4),x)
```

```
int(x^4/(a*x^2 + b*x^3)^(5/4), x)
```

Reduce [F]

$$\int \frac{x^4}{\left(ax^2+bx^3\right)^{5/4}}\,dx = \frac{-\frac{8\sqrt{x}\,(bx+a)^{\frac{1}{4}}a}{7}+\frac{4\sqrt{x}\,(bx+a)^{\frac{1}{4}}bx}{7}+\frac{4\sqrt{bx+a}\left(\int \frac{\sqrt{x}\,(bx+a)^{\frac{3}{4}}}{b^2x^3+2ab\,x^2+a^2x}dx\right)a^2}{7}}{\sqrt{bx+a}\,b^2}$$

```
int(x^4/(b*x^3+a*x^2)^(5/4),x)
```

```
(4*( - 2*sqrt(x)*(a + b*x)**(1/4)*a + sqrt(x)*(a + b*x)**(1/4)*b*x + sqrt(
a + b*x)*int((sqrt(x)*(a + b*x)**(3/4))/(a**2*x + 2*a*b*x**2 + b**2*x**3),
x)*a**2))/(7*sqrt(a + b*x)*b**2)
```

3.402 $\int \frac{x^3}{\left(ax^2+bx^3\right)^{5/4}}\,dx$

Optimal result

Integrand size = 19, antiderivative size = 90

$$\int \frac{x^3}{\left(ax^2+bx^3\right)^{5/4}}\,dx=\frac{4x}{b\sqrt[4]{ax^2+bx^3}}-\frac{8\sqrt{a}\sqrt{x}\sqrt[4]{\frac{a+bx}{a}}E\left(\frac{1}{2}\arctan\left(\frac{\sqrt{b}\sqrt{x}}{\sqrt{a}}\right)\middle|2\right)}{b^{3/2}\sqrt[4]{ax^2+bx^3}}$$

```
4*x/b/(b*x^3+a*x^2)^(1/4)-8*a^(1/2)*x^(1/2)*((b*x+a)/a)^(1/4)*EllipticE(si
n(1/2*arctan(b^(1/2)*x^(1/2)/a^(1/2))),2^(1/2))/b^(3/2)/(b*x^3+a*x^2)^(1/4
)
```

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 10.01 (sec) , antiderivative size = 52, normalized size of antiderivative = 0.58

$$\int \frac{x^3}{\left(ax^2+bx^3\right)^{5/4}}\,dx=\frac{2x^2\sqrt[4]{1+\frac{bx}{a}}\,\text{Hypergeometric2F1}\left(\frac{5}{4},\frac{3}{2},\frac{5}{2},-\frac{bx}{a}\right)}{3a\sqrt[4]{x^2(a+bx)}}$$

```
Integrate[x^3/(a*x^2 + b*x^3)^(5/4),x]
```

```
(2*x^2*(1 + (b*x)/a)^(1/4)*Hypergeometric2F1[5/4, 3/2, 5/2, -((b*x)/a)])/(
3*a*(x^2*(a + b*x))^(1/4))
```

Rubi [A] (verified)

Time = 0.64 (sec) , antiderivative size = 178, normalized size of antiderivative = 1.98, number of steps used = 11, number of rules used = 10, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.526, Rules used = {1928, 1917, 73, 836, 27, 765, 762, 1390, 1389, 327}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^3}{(ax^2+bx^3)^{5/4}}\,dx$$

$$\Big\downarrow \text{1928}$$

$$\frac{2\int \frac{1}{\sqrt[4]{bx^3+ax^2}}dx}{b}-\frac{4x}{b\sqrt[4]{ax^2+bx^3}}$$

$$\Big\downarrow \text{1917}$$

$$\frac{2\sqrt{x}\sqrt[4]{a+bx}\int \frac{1}{\sqrt{x}\sqrt[4]{a+bx}}dx}{b\sqrt[4]{ax^2+bx^3}}-\frac{4x}{b\sqrt[4]{ax^2+bx^3}}$$

$$\Big\downarrow \text{73}$$

$$\frac{8\sqrt{x}\sqrt[4]{a+bx}\int \frac{\sqrt{a+bx}}{\sqrt{\frac{a+bx}{b}-\frac{a}{b}}}d\sqrt[4]{a+bx}}{b^2\sqrt[4]{ax^2+bx^3}}-\frac{4x}{b\sqrt[4]{ax^2+bx^3}}$$

$$\Big\downarrow \text{836}$$

$$\frac{8\sqrt{x}\sqrt[4]{a+bx}\left(\sqrt{a}\int \frac{\sqrt{a}+\sqrt{a+bx}}{\sqrt{a}\sqrt{\frac{a+bx}{b}-\frac{a}{b}}}d\sqrt[4]{a+bx}-\sqrt{a}\int \frac{1}{\sqrt{\frac{a+bx}{b}-\frac{a}{b}}}d\sqrt[4]{a+bx}\right)}{b^2\sqrt[4]{ax^2+bx^3}}-\frac{4x}{b\sqrt[4]{ax^2+bx^3}}$$

$$\Big\downarrow \text{27}$$

$$\frac{8\sqrt{x}\sqrt[4]{a+bx}\left(\int \frac{\sqrt{a}+\sqrt{a+bx}}{\sqrt{\frac{a+bx}{b}-\frac{a}{b}}}d\sqrt[4]{a+bx}-\sqrt{a}\int \frac{1}{\sqrt{\frac{a+bx}{b}-\frac{a}{b}}}d\sqrt[4]{a+bx}\right)}{b^2\sqrt[4]{ax^2+bx^3}}-\frac{4x}{b\sqrt[4]{ax^2+bx^3}}$$

$$\Big\downarrow \; 765$$

$$\frac{8\sqrt{x}\sqrt[4]{a+bx}\left(\int \frac{\sqrt{a}+\sqrt{a+bx}}{\sqrt{\frac{a+bx}{b}-\frac{a}{b}}}d\sqrt[4]{a+bx}-\frac{\sqrt{a}\sqrt{1-\frac{a+bx}{a}}\int \frac{1}{\sqrt{1-\frac{a+bx}{a}}}d\sqrt[4]{a+bx}}{\sqrt{\frac{a+bx}{b}-\frac{a}{b}}}\right)}{b^2\sqrt[4]{ax^2+bx^3}}-\frac{4x}{b\sqrt[4]{ax^2+bx^3}}$$

$$\Big\downarrow \; 762$$

$$\frac{8\sqrt{x}\sqrt[4]{a+bx}\left(\int \frac{\sqrt{a}+\sqrt{a+bx}}{\sqrt{\frac{a+bx}{b}-\frac{a}{b}}}d\sqrt[4]{a+bx}-\frac{a^{3/4}\sqrt{1-\frac{a+bx}{a}}\,\text{EllipticF}\left(\arcsin\left(\frac{\sqrt[4]{a+bx}}{\sqrt[4]{a}}\right),-1\right)}{\sqrt{\frac{a+bx}{b}-\frac{a}{b}}}\right)}{b^2\sqrt[4]{ax^2+bx^3}}-\frac{4x}{b\sqrt[4]{ax^2+bx^3}}$$

$$\Big\downarrow \; 1390$$

$$\frac{8\sqrt{x}\sqrt[4]{a+bx}\left(\frac{\sqrt{1-\frac{a+bx}{a}}\int \frac{\sqrt{a}+\sqrt{a+bx}}{\sqrt{1-\frac{a+bx}{a}}}d\sqrt[4]{a+bx}}{\sqrt{\frac{a+bx}{b}-\frac{a}{b}}}-\frac{a^{3/4}\sqrt{1-\frac{a+bx}{a}}\,\text{EllipticF}\left(\arcsin\left(\frac{\sqrt[4]{a+bx}}{\sqrt[4]{a}}\right),-1\right)}{\sqrt{\frac{a+bx}{b}-\frac{a}{b}}}\right)}{b^2\sqrt[4]{ax^2+bx^3}}-\frac{4x}{b\sqrt[4]{ax^2+bx^3}}$$

$$\Big\downarrow \; 1389$$

$$\frac{8\sqrt{x}\sqrt[4]{a+bx}\left(\frac{\sqrt{a}\sqrt{1-\frac{a+bx}{a}}\int \frac{\sqrt{\frac{\sqrt{a+bx}}{\sqrt{a}}+1}}{\sqrt{1-\frac{\sqrt{a+bx}}{\sqrt{a}}}}d\sqrt[4]{a+bx}}{\sqrt{\frac{a+bx}{b}-\frac{a}{b}}}-\frac{a^{3/4}\sqrt{1-\frac{a+bx}{a}}\,\text{EllipticF}\left(\arcsin\left(\frac{\sqrt[4]{a+bx}}{\sqrt[4]{a}}\right),-1\right)}{\sqrt{\frac{a+bx}{b}-\frac{a}{b}}}\right)}{b^2\sqrt[4]{ax^2+bx^3}}-\frac{4x}{b\sqrt[4]{ax^2+bx^3}}$$

$$\Big\downarrow \; 327$$

$$\frac{8\sqrt{x}\sqrt[4]{a+bx}\left(\frac{a^{3/4}\sqrt{1-\frac{a+bx}{a}}E\left(\arcsin\left(\frac{\sqrt[4]{a+bx}}{\sqrt[4]{a}}\right)\middle|-1\right)}{\sqrt{\frac{a+bx}{b}-\frac{a}{b}}}-\frac{a^{3/4}\sqrt{1-\frac{a+bx}{a}}\,\text{EllipticF}\left(\arcsin\left(\frac{\sqrt[4]{a+bx}}{\sqrt[4]{a}}\right),-1\right)}{\sqrt{\frac{a+bx}{b}-\frac{a}{b}}}\right)}{b^2\sqrt[4]{ax^2+bx^3}}-\frac{4x}{b\sqrt[4]{ax^2+bx^3}}$$

```
Int[x^3/(a*x^2 + b*x^3)^(5/4),x]
```

```
(-4*x)/(b*(a*x^2 + b*x^3)^(1/4)) + (8*Sqrt[x]*(a + b*x)^(1/4)*((a^(3/4)*Sq
rt[1 - (a + b*x)/a]*EllipticE[ArcSin[(a + b*x)^(1/4)/a^(1/4)], -1])/Sqrt[-
(a/b) + (a + b*x)/b] - (a^(3/4)*Sqrt[1 - (a + b*x)/a]*EllipticF[ArcSin[(a
+ b*x)^(1/4)/a^(1/4)], -1])/Sqrt[-(a/b) + (a + b*x)/b]))/(b^2*(a*x^2 + b*x
^3)^(1/4))
```

Defintions of rubi rules used

```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Simp[p/b   Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
 d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && Lt
Q[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntL
inearQ[a, b, c, d, m, n, x]
```

```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Simp[
(Sqrt[a]/(Sqrt[c]*Rt[-d/c, 2]))*EllipticE[ArcSin[Rt[-d/c, 2]*x], b*(c/(a*d)
)], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a, 0]
```

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> Simp[(1/(Sqrt[a]*Rt[-b/a, 4])
)*EllipticF[ArcSin[Rt[-b/a, 4]*x], -1], x] /; FreeQ[{a, b}, x] && NegQ[b/a]
 && GtQ[a, 0]
```

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> Simp[Sqrt[1 + b*(x^4/a)]/Sqrt
[a + b*x^4]   Int[1/Sqrt[1 + b*(x^4/a)], x], x] /; FreeQ[{a, b}, x] && NegQ
[b/a] &&  !GtQ[a, 0]
```

```
Int[(x_)^2/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[-b/a, 2]},
Simp[-q^(-1)   Int[1/Sqrt[a + b*x^4], x], x] + Simp[1/q   Int[(1 + q*x^2)/S
qrt[a + b*x^4], x], x]] /; FreeQ[{a, b}, x] && NegQ[b/a]
```

```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (c_.)*(x_)^4], x_Symbol] :> Simp[d/Sq
rt[a]   Int[Sqrt[1 + e*(x^2/d)]/Sqrt[1 - e*(x^2/d)], x], x] /; FreeQ[{a, c,
 d, e}, x] && EqQ[c*d^2 + a*e^2, 0] && NegQ[c/a] && GtQ[a, 0]
```

```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (c_.)*(x_)^4], x_Symbol] :> Simp[Sqrt
[1 + c*(x^4/a)]/Sqrt[a + c*x^4]   Int[(d + e*x^2)/Sqrt[1 + c*(x^4/a)], x],
x] /; FreeQ[{a, c, d, e}, x] && EqQ[c*d^2 + a*e^2, 0] && NegQ[c/a] &&  !GtQ
[a, 0] &&  !(LtQ[a, 0] && GtQ[c, 0])
```

```
Int[((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol] :> Simp[(a*x^j +
b*x^n)^FracPart[p]/(x^(j*FracPart[p])*(a + b*x^(n - j))^FracPart[p])   Int[
x^(j*p)*(a + b*x^(n - j))^p, x], x] /; FreeQ[{a, b, j, n, p}, x] &&  !Integ
erQ[p] && NeQ[n, j] && PosQ[n - j]
```

```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> Simp[c^(n - 1)*(c*x)^(m - n + 1)*((a*x^j + b*x^n)^(p + 1)/(b*(n - j)*(
p + 1))), x] - Simp[c^n*((m + j*p - n + j + 1)/(b*(n - j)*(p + 1)))   Int[(
c*x)^(m - n)*(a*x^j + b*x^n)^(p + 1), x], x] /; FreeQ[{a, b, c}, x] &&  !In
tegerQ[p] && LtQ[0, j, n] && (IntegersQ[j, n] || GtQ[c, 0]) && LtQ[p, -1] &
& GtQ[m + j*p + 1, n - j]
```

Maple [F]

$$\int \frac{x^3}{(b\,x^3 + a\,x^2)^{\frac{5}{4}}}dx$$

```
int(x^3/(b*x^3+a*x^2)^(5/4),x)
```

```
int(x^3/(b*x^3+a*x^2)^(5/4),x)
```

Fricas [F]

$$\int \frac{x^3}{(ax^2+bx^3)^{5/4}}\,dx = \int \frac{x^3}{(bx^3+ax^2)^{\frac{5}{4}}}\,dx$$

```
integrate(x^3/(b*x^3+a*x^2)^(5/4),x, algorithm="fricas")
```

```
integral((b*x^3 + a*x^2)^(3/4)/(b^2*x^3 + 2*a*b*x^2 + a^2*x), x)
```

Sympy [F]

$$\int \frac{x^3}{(ax^2+bx^3)^{5/4}}\,dx = \int \frac{x^3}{(x^2\,(a+bx))^{\frac{5}{4}}}\,dx$$

```
integrate(x**3/(b*x**3+a*x**2)**(5/4),x)
```

```
Integral(x**3/(x**2*(a + b*x))**(5/4), x)
```

Maxima [F]

$$\int \frac{x^3}{(ax^2+bx^3)^{5/4}}\,dx = \int \frac{x^3}{(bx^3+ax^2)^{\frac{5}{4}}}\,dx$$

```
integrate(x^3/(b*x^3+a*x^2)^(5/4),x, algorithm="maxima")
```

```
integrate(x^3/(b*x^3 + a*x^2)^(5/4), x)
```

Giac [F]

$$\int \frac{x^3}{(ax^2+bx^3)^{5/4}}\,dx = \int \frac{x^3}{(bx^3+ax^2)^{\frac{5}{4}}}\,dx$$

```
integrate(x^3/(b*x^3+a*x^2)^(5/4),x, algorithm="giac")
```

```
integrate(x^3/(b*x^3 + a*x^2)^(5/4), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{x^3}{(ax^2+bx^3)^{5/4}}\,dx = \int \frac{x^3}{(b\,x^3+a\,x^2)^{5/4}}\,dx$$

```
int(x^3/(a*x^2 + b*x^3)^(5/4),x)
```

```
int(x^3/(a*x^2 + b*x^3)^(5/4), x)
```

Reduce [F]

$$\int \frac{x^3}{(ax^2+bx^3)^{5/4}}\,dx = \frac{\frac{4\sqrt{x}\,(bx+a)^{\frac{1}{4}}}{3} - \frac{2\sqrt{bx+a}\left(\int \frac{\sqrt{x}\,(bx+a)^{\frac{3}{4}}}{b^2x^3+2ab\,x^2+a^2x}dx\right)a}{3}}{\sqrt{bx+a}\,b}$$

```
int(x^3/(b*x^3+a*x^2)^(5/4),x)
```

```
(2*(2*sqrt(x)*(a + b*x)**(1/4) - sqrt(a + b*x)*int((sqrt(x)*(a + b*x)**(3/
4))/(a**2*x + 2*a*b*x**2 + b**2*x**3),x)*a))/(3*sqrt(a + b*x)*b)
```

3.403 $\int \frac{x^2}{(ax^2+bx^3)^{5/4}}\,dx$

Optimal result

Integrand size = 19, antiderivative size = 68

$$\int \frac{x^2}{(ax^2+bx^3)^{5/4}}\,dx=\frac{4\sqrt{x}\sqrt[4]{\frac{a+bx}{a}}E\left(\frac{1}{2}\arctan\left(\frac{\sqrt{b}\sqrt{x}}{\sqrt{a}}\right)\middle|2\right)}{\sqrt{a}\sqrt{b}\sqrt[4]{ax^2+bx^3}}$$

```
4*x^(1/2)*((b*x+a)/a)^(1/4)*EllipticE(sin(1/2*arctan(b^(1/2)*x^(1/2)/a^(1/
2))),2^(1/2))/a^(1/2)/b^(1/2)/(b*x^3+a*x^2)^(1/4)
```

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 10.01 (sec) , antiderivative size = 48, normalized size of antiderivative = 0.71

$$\int \frac{x^2}{(ax^2+bx^3)^{5/4}}\,dx=\frac{2x\sqrt[4]{1+\frac{bx}{a}}\,\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{5}{4},\frac{3}{2},-\frac{bx}{a}\right)}{a\sqrt[4]{x^2(a+bx)}}$$

```
Integrate[x^2/(a*x^2 + b*x^3)^(5/4),x]
```

```
(2*x*(1 + (b*x)/a)^(1/4)*Hypergeometric2F1[1/2, 5/4, 3/2, -((b*x)/a)])/(a*
(x^2*(a + b*x))^(1/4))
```

Rubi [B] (verified)

Leaf count is larger than twice the leaf count of optimal. 181 vs. $2(68) = 136$.

Time = 0.65 (sec) , antiderivative size = 181, normalized size of antiderivative = 2.66, number of steps used = 11, number of rules used = 10, $\frac{\text{number of rules}}{\text{integrand size}} = 0.526$, Rules used = {1929, 1917, 73, 836, 27, 765, 762, 1390, 1389, 327}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^2}{(ax^2+bx^3)^{5/4}}\,dx$$

$$\Big\downarrow \text{1929}$$

$$\frac{4x}{a\sqrt[4]{ax^2+bx^3}}-\frac{\int \frac{1}{\sqrt[4]{bx^3+ax^2}}dx}{a}$$

$$\Big\downarrow \text{1917}$$

$$\frac{4x}{a\sqrt[4]{ax^2+bx^3}}-\frac{\sqrt{x}\sqrt[4]{a+bx}\int \frac{1}{\sqrt{x}\sqrt[4]{a+bx}}dx}{a\sqrt[4]{ax^2+bx^3}}$$

$$\Big\downarrow \text{73}$$

$$\frac{4x}{a\sqrt[4]{ax^2+bx^3}}-\frac{4\sqrt{x}\sqrt[4]{a+bx}\int \frac{\sqrt{a+bx}}{\sqrt{\frac{a+bx}{b}-\frac{a}{b}}}d\sqrt[4]{a+bx}}{ab\sqrt[4]{ax^2+bx^3}}$$

$$\Big\downarrow \text{836}$$

$$\frac{4x}{a\sqrt[4]{ax^2+bx^3}}-\frac{4\sqrt{x}\sqrt[4]{a+bx}\left(\sqrt{a}\int \frac{\sqrt{a}+\sqrt{a+bx}}{\sqrt{a}\sqrt{\frac{a+bx}{b}-\frac{a}{b}}}d\sqrt[4]{a+bx}-\sqrt{a}\int \frac{1}{\sqrt{\frac{a+bx}{b}-\frac{a}{b}}}d\sqrt[4]{a+bx}\right)}{ab\sqrt[4]{ax^2+bx^3}}$$

$$\Big\downarrow \text{27}$$

$$\frac{4x}{a\sqrt[4]{ax^2+bx^3}}-\frac{4\sqrt{x}\sqrt[4]{a+bx}\left(\int\frac{\sqrt{a}+\sqrt{a+bx}}{\sqrt{\frac{a+bx}{b}-\frac{a}{b}}}d\sqrt[4]{a+bx}-\sqrt{a}\int\frac{1}{\sqrt{\frac{a+bx}{b}-\frac{a}{b}}}d\sqrt[4]{a+bx}\right)}{ab\sqrt[4]{ax^2+bx^3}}$$

$$\Big\downarrow \text{765}$$

$$\frac{4x}{a\sqrt[4]{ax^2+bx^3}}-\frac{4\sqrt{x}\sqrt[4]{a+bx}\left(\int\frac{\sqrt{a}+\sqrt{a+bx}}{\sqrt{\frac{a+bx}{b}-\frac{a}{b}}}d\sqrt[4]{a+bx}-\frac{\sqrt{a}\sqrt{1-\frac{a+bx}{a}}\int\frac{1}{\sqrt{1-\frac{a+bx}{a}}}d\sqrt[4]{a+bx}}{\sqrt{\frac{a+bx}{b}-\frac{a}{b}}}\right)}{ab\sqrt[4]{ax^2+bx^3}}$$

$$\Big\downarrow \text{762}$$

$$\frac{4x}{a\sqrt[4]{ax^2+bx^3}}-\frac{4\sqrt{x}\sqrt[4]{a+bx}\left(\int\frac{\sqrt{a}+\sqrt{a+bx}}{\sqrt{\frac{a+bx}{b}-\frac{a}{b}}}d\sqrt[4]{a+bx}-\frac{a^{3/4}\sqrt{1-\frac{a+bx}{a}}\,\text{EllipticF}\left(\arcsin\left(\frac{\sqrt[4]{a+bx}}{\sqrt[4]{a}}\right),-1\right)}{\sqrt{\frac{a+bx}{b}-\frac{a}{b}}}\right)}{ab\sqrt[4]{ax^2+bx^3}}$$

$$\Big\downarrow \text{1390}$$

$$\frac{4x}{a\sqrt[4]{ax^2+bx^3}}-\frac{4\sqrt{x}\sqrt[4]{a+bx}\left(\frac{\sqrt{1-\frac{a+bx}{a}}\int\frac{\sqrt{a}+\sqrt{a+bx}}{\sqrt{1-\frac{a+bx}{a}}}d\sqrt[4]{a+bx}}{\sqrt{\frac{a+bx}{b}-\frac{a}{b}}}-\frac{a^{3/4}\sqrt{1-\frac{a+bx}{a}}\,\text{EllipticF}\left(\arcsin\left(\frac{\sqrt[4]{a+bx}}{\sqrt[4]{a}}\right),-1\right)}{\sqrt{\frac{a+bx}{b}-\frac{a}{b}}}\right)}{ab\sqrt[4]{ax^2+bx^3}}$$

$$\Big\downarrow \text{1389}$$

$$\frac{4x}{a\sqrt[4]{ax^2+bx^3}}-\frac{4\sqrt{x}\sqrt[4]{a+bx}\left(\frac{\sqrt{a}\sqrt{1-\frac{a+bx}{a}}\int\frac{\sqrt{\frac{\sqrt{a+bx}}{\sqrt{a}}+1}}{\sqrt{1-\frac{\sqrt{a+bx}}{\sqrt{a}}}}d\sqrt[4]{a+bx}}{\sqrt{\frac{a+bx}{b}-\frac{a}{b}}}-\frac{a^{3/4}\sqrt{1-\frac{a+bx}{a}}\,\text{EllipticF}\left(\arcsin\left(\frac{\sqrt[4]{a+bx}}{\sqrt[4]{a}}\right),-1\right)}{\sqrt{\frac{a+bx}{b}-\frac{a}{b}}}\right)}{ab\sqrt[4]{ax^2+bx^3}}$$

$$\Big\downarrow \text{327}$$

$$\frac{\frac{4x}{a\sqrt[4]{ax^2+bx^3}} - 4\sqrt{x}\sqrt[4]{a+bx}\left(\frac{a^{3/4}\sqrt{1-\frac{a+bx}{a}}E\left(\arcsin\left(\frac{\sqrt[4]{a+bx}}{\sqrt[4]{a}}\right)\middle|-1\right)}{\sqrt{\frac{a+bx}{b}-\frac{a}{b}}} - \frac{a^{3/4}\sqrt{1-\frac{a+bx}{a}}\,\text{EllipticF}\left(\arcsin\left(\frac{\sqrt[4]{a+bx}}{\sqrt[4]{a}}\right),-1\right)}{\sqrt{\frac{a+bx}{b}-\frac{a}{b}}}\right)}{ab\sqrt[4]{ax^2+bx^3}}$$

```
Int[x^2/(a*x^2 + b*x^3)^(5/4),x]
```

```
(4*x)/(a*(a*x^2 + b*x^3)^(1/4)) - (4*Sqrt[x]*(a + b*x)^(1/4)*((a^(3/4)*Sqr
t[1 - (a + b*x)/a]*EllipticE[ArcSin[(a + b*x)^(1/4)/a^(1/4)], -1])/Sqrt[-(
a/b) + (a + b*x)/b] - (a^(3/4)*Sqrt[1 - (a + b*x)/a]*EllipticF[ArcSin[(a +
 b*x)^(1/4)/a^(1/4)], -1])/Sqrt[-(a/b) + (a + b*x)/b]))/(a*b*(a*x^2 + b*x^
3)^(1/4))
```

Defintions of rubi rules used

```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Simp[p/b   Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
 d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && Lt
Q[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntL
inearQ[a, b, c, d, m, n, x]
```

```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Simp[
(Sqrt[a]/(Sqrt[c]*Rt[-d/c, 2]))*EllipticE[ArcSin[Rt[-d/c, 2]*x], b*(c/(a*d)
)], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a, 0]
```

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> Simp[(1/(Sqrt[a]*Rt[-b/a, 4])
)*EllipticF[ArcSin[Rt[-b/a, 4]*x], -1], x] /; FreeQ[{a, b}, x] && NegQ[b/a]
 && GtQ[a, 0]
```

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> Simp[Sqrt[1 + b*(x^4/a)]/Sqrt
[a + b*x^4]   Int[1/Sqrt[1 + b*(x^4/a)], x], x] /; FreeQ[{a, b}, x] && NegQ
[b/a] &&  !GtQ[a, 0]
```

```
Int[(x_)^2/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[-b/a, 2]},
Simp[-q^(-1)   Int[1/Sqrt[a + b*x^4], x], x] + Simp[1/q   Int[(1 + q*x^2)/S
qrt[a + b*x^4], x], x]] /; FreeQ[{a, b}, x] && NegQ[b/a]
```

```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (c_.)*(x_)^4], x_Symbol] :> Simp[d/Sq
rt[a]   Int[Sqrt[1 + e*(x^2/d)]/Sqrt[1 - e*(x^2/d)], x], x] /; FreeQ[{a, c,
 d, e}, x] && EqQ[c*d^2 + a*e^2, 0] && NegQ[c/a] && GtQ[a, 0]
```

```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (c_.)*(x_)^4], x_Symbol] :> Simp[Sqrt
[1 + c*(x^4/a)]/Sqrt[a + c*x^4]   Int[(d + e*x^2)/Sqrt[1 + c*(x^4/a)], x],
x] /; FreeQ[{a, c, d, e}, x] && EqQ[c*d^2 + a*e^2, 0] && NegQ[c/a] &&  !GtQ
[a, 0] &&  !(LtQ[a, 0] && GtQ[c, 0])
```

```
Int[((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol] :> Simp[(a*x^j +
b*x^n)^FracPart[p]/(x^(j*FracPart[p])*(a + b*x^(n - j))^FracPart[p])   Int[
x^(j*p)*(a + b*x^(n - j))^p, x], x] /; FreeQ[{a, b, j, n, p}, x] &&  !Integ
erQ[p] && NeQ[n, j] && PosQ[n - j]
```

```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> Simp[(-c^(j - 1))*(c*x)^(m - j + 1)*((a*x^j + b*x^n)^(p + 1)/(a*(n - j
)*(p + 1))), x] + Simp[c^j*((m + n*p + n - j + 1)/(a*(n - j)*(p + 1)))   In
t[(c*x)^(m - j)*(a*x^j + b*x^n)^(p + 1), x], x] /; FreeQ[{a, b, c, m}, x] &
&  !IntegerQ[p] && LtQ[0, j, n] && (IntegersQ[j, n] || GtQ[c, 0]) && LtQ[p,
 -1]
```

Maple [F]

$$\int \frac{x^2}{\left(b\,x^3+a\,x^2\right)^{\frac{5}{4}}}dx$$

```
int(x^2/(b*x^3+a*x^2)^(5/4),x)
```

```
int(x^2/(b*x^3+a*x^2)^(5/4),x)
```

Fricas [F]

$$\int \frac{x^2}{\left(ax^2+bx^3\right)^{5/4}}\,dx=\int \frac{x^2}{\left(bx^3+ax^2\right)^{\frac{5}{4}}}\,dx$$

```
integrate(x^2/(b*x^3+a*x^2)^(5/4),x, algorithm="fricas")
```

```
integral((b*x^3 + a*x^2)^(3/4)/(b^2*x^4 + 2*a*b*x^3 + a^2*x^2), x)
```

Sympy [F]

$$\int \frac{x^2}{\left(ax^2+bx^3\right)^{5/4}}\,dx=\int \frac{x^2}{\left(x^2\left(a+bx\right)\right)^{\frac{5}{4}}}\,dx$$

```
integrate(x**2/(b*x**3+a*x**2)**(5/4),x)
```

```
Integral(x**2/(x**2*(a + b*x))**(5/4), x)
```

Maxima [F]

$$\int \frac{x^2}{(ax^2+bx^3)^{5/4}}\,dx = \int \frac{x^2}{(bx^3+ax^2)^{\frac{5}{4}}}\,dx$$

```
integrate(x^2/(b*x^3+a*x^2)^(5/4),x, algorithm="maxima")
```

```
integrate(x^2/(b*x^3 + a*x^2)^(5/4), x)
```

Giac [F]

$$\int \frac{x^2}{(ax^2+bx^3)^{5/4}}\,dx = \int \frac{x^2}{(bx^3+ax^2)^{\frac{5}{4}}}\,dx$$

```
integrate(x^2/(b*x^3+a*x^2)^(5/4),x, algorithm="giac")
```

```
integrate(x^2/(b*x^3 + a*x^2)^(5/4), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{x^2}{(ax^2+bx^3)^{5/4}}\,dx = \int \frac{x^2}{(b\,x^3+a\,x^2)^{5/4}}\,dx$$

```
int(x^2/(a*x^2 + b*x^3)^(5/4),x)
```

```
int(x^2/(a*x^2 + b*x^3)^(5/4), x)
```

Reduce [F]

$$\int \frac{x^2}{(ax^2 + bx^3)^{5/4}}\, dx = \int \frac{\sqrt{x}\,(bx + a)^{\frac{1}{4}}}{\sqrt{bx + a}\, ax + \sqrt{bx + a}\, b\, x^2} dx$$

```
int(x^2/(b*x^3+a*x^2)^(5/4),x)
```

```
int((sqrt(x)*(a + b*x)**(1/4))/(sqrt(a + b*x)*a*x + sqrt(a + b*x)*b*x**2),
x)
```

3.404 $\int \frac{x}{(ax^2+bx^3)^{5/4}}\,dx$

Optimal result

Integrand size = 17, antiderivative size = 89

$$\int \frac{x}{(ax^2+bx^3)^{5/4}}\,dx = -\frac{2}{a\sqrt[4]{ax^2+bx^3}} - \frac{6\sqrt{b}\sqrt{x}\sqrt[4]{\frac{a+bx}{a}}E\left(\frac{1}{2}\arctan\left(\frac{\sqrt{b}\sqrt{x}}{\sqrt{a}}\right)\middle|2\right)}{a^{3/2}\sqrt[4]{ax^2+bx^3}}$$

```
-2/a/(b*x^3+a*x^2)^(1/4)-6*b^(1/2)*x^(1/2)*((b*x+a)/a)^(1/4)*EllipticE(sin
(1/2*arctan(b^(1/2)*x^(1/2)/a^(1/2))),2^(1/2))/a^(3/2)/(b*x^3+a*x^2)^(1/4)
```

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 10.01 (sec) , antiderivative size = 47, normalized size of antiderivative = 0.53

$$\int \frac{x}{(ax^2+bx^3)^{5/4}}\,dx = -\frac{2\sqrt[4]{1+\frac{bx}{a}}\,\text{Hypergeometric2F1}\left(-\frac{1}{2},\frac{5}{4},\frac{1}{2},-\frac{bx}{a}\right)}{a\sqrt[4]{x^2(a+bx)}}$$

```
Integrate[x/(a*x^2 + b*x^3)^(5/4),x]
```

```
(-2*(1 + (b*x)/a)^(1/4)*Hypergeometric2F1[-1/2, 5/4, 1/2, -((b*x)/a)])/(a*
(x^2*(a + b*x))^(1/4))
```

Rubi [B] (verified)

Leaf count is larger than twice the leaf count of optimal. 206 vs. $2(89) = 178$.

Time = 0.74 (sec) , antiderivative size = 206, normalized size of antiderivative = 2.31, number of steps used = 12, number of rules used = 11, $\frac{\text{number of rules}}{\text{integrand size}} = 0.647$, Rules used = {1929, 1931, 1917, 73, 836, 27, 765, 762, 1390, 1389, 327}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x}{(ax^2+bx^3)^{5/4}}\,dx$$

$$\Big\downarrow \text{1929}$$

$$\frac{3\int \frac{1}{x\sqrt[4]{bx^3+ax^2}}dx}{a}+\frac{4}{a\sqrt[4]{ax^2+bx^3}}$$

$$\Big\downarrow \text{1931}$$

$$\frac{3\left(\frac{b\int \frac{1}{\sqrt[4]{bx^3+ax^2}}dx}{2a}-\frac{2(ax^2+bx^3)^{3/4}}{ax^2}\right)}{a}+\frac{4}{a\sqrt[4]{ax^2+bx^3}}$$

$$\Big\downarrow \text{1917}$$

$$\frac{3\left(\frac{b\sqrt{x}\sqrt[4]{a+bx}\int \frac{1}{\sqrt{x}\sqrt[4]{a+bx}}dx}{2a\sqrt[4]{ax^2+bx^3}}-\frac{2(ax^2+bx^3)^{3/4}}{ax^2}\right)}{a}+\frac{4}{a\sqrt[4]{ax^2+bx^3}}$$

$$\Big\downarrow \text{73}$$

$$\frac{3\left(\frac{2\sqrt{x}\sqrt[4]{a+bx}\int \frac{\sqrt{a+bx}}{\sqrt{\frac{a+bx}{b}-\frac{a}{b}}}d\sqrt[4]{a+bx}}{a\sqrt[4]{ax^2+bx^3}}-\frac{2(ax^2+bx^3)^{3/4}}{ax^2}\right)}{a}+\frac{4}{a\sqrt[4]{ax^2+bx^3}}$$

$$\Big\downarrow \text{836}$$

$$\frac{3\left(\frac{2\sqrt{x}\sqrt[4]{a+bx}\left(\sqrt{a}\int \frac{\sqrt{a}+\sqrt{a+bx}}{\sqrt{a}\sqrt{\frac{a+bx}{b}-\frac{a}{b}}}d\sqrt[4]{a+bx}-\sqrt{a}\int \frac{1}{\sqrt{\frac{a+bx}{b}-\frac{a}{b}}}d\sqrt[4]{a+bx}\right)}{a\sqrt[4]{ax^2+bx^3}}-\frac{2(ax^2+bx^3)^{3/4}}{ax^2}\right)}{\frac{a^4}{a\sqrt[4]{ax^2+bx^3}}}+$$

$$\Big\downarrow \text{27}$$

$$\frac{3\left(\frac{2\sqrt{x}\sqrt[4]{a+bx}\left(\int \frac{\sqrt{a}+\sqrt{a+bx}}{\sqrt{\frac{a+bx}{b}-\frac{a}{b}}}d\sqrt[4]{a+bx}-\sqrt{a}\int \frac{1}{\sqrt{\frac{a+bx}{b}-\frac{a}{b}}}d\sqrt[4]{a+bx}\right)}{a\sqrt[4]{ax^2+bx^3}}-\frac{2(ax^2+bx^3)^{3/4}}{ax^2}\right)}{\frac{a^4}{a\sqrt[4]{ax^2+bx^3}}}+$$

$$\Big\downarrow \text{765}$$

$$\frac{3\left(\frac{2\sqrt{x}\sqrt[4]{a+bx}\left(\int \frac{\sqrt{a}+\sqrt{a+bx}}{\sqrt{\frac{a+bx}{b}-\frac{a}{b}}}d\sqrt[4]{a+bx}-\frac{\sqrt{a}\sqrt{1-\frac{a+bx}{a}}\int \frac{1}{\sqrt{1-\frac{a+bx}{a}}}d\sqrt[4]{a+bx}}{\sqrt{\frac{a+bx}{b}-\frac{a}{b}}}\right)}{a\sqrt[4]{ax^2+bx^3}}-\frac{2(ax^2+bx^3)^{3/4}}{ax^2}\right)}{\frac{a^4}{a\sqrt[4]{ax^2+bx^3}}}+$$

$$\Big\downarrow \text{762}$$

$$\frac{3\left(\frac{2\sqrt{x}\sqrt[4]{a+bx}\left(\int \frac{\sqrt{a}+\sqrt{a+bx}}{\sqrt{\frac{a+bx}{b}-\frac{a}{b}}}d\sqrt[4]{a+bx}-\frac{a^{3/4}\sqrt{1-\frac{a+bx}{a}}\,\text{EllipticF}\left(\arcsin\left(\frac{\sqrt[4]{a+bx}}{\sqrt[4]{a}}\right),-1\right)}{\sqrt{\frac{a+bx}{b}-\frac{a}{b}}}\right)}{a\sqrt[4]{ax^2+bx^3}}-\frac{2(ax^2+bx^3)^{3/4}}{ax^2}\right)}{\frac{4^a}{a\sqrt[4]{ax^2+bx^3}}}+$$

$$\Big\downarrow \text{1390}$$

$$\frac{3\left(\frac{2\sqrt{x}\sqrt[4]{a+bx}\left(\frac{\sqrt{1-\frac{a+bx}{a}}\int\frac{\sqrt{a}+\sqrt{a+bx}}{\sqrt{1-\frac{a+bx}{a}}}d\sqrt[4]{a+bx}}{\sqrt{\frac{a+bx}{b}-\frac{a}{b}}}-\frac{a^{3/4}\sqrt{1-\frac{a+bx}{a}}\,\text{EllipticF}\left(\arcsin\left(\frac{\sqrt[4]{a+bx}}{\sqrt[4]{a}}\right),-1\right)}{\sqrt{\frac{a+bx}{b}-\frac{a}{b}}}\right)}{a\sqrt[4]{ax^2+bx^3}}-\frac{2(ax^2+bx^3)^{3/4}}{ax^2}\right)}{a}+\frac{4}{a\sqrt[4]{ax^2+bx^3}}$$

$$\Big\downarrow \; 1389$$

$$\frac{3\left(\frac{2\sqrt{x}\sqrt[4]{a+bx}\left(\frac{\sqrt{a}\sqrt{1-\frac{a+bx}{a}}\int\frac{\sqrt{\frac{\sqrt{a+bx}}{\sqrt{a}}+1}}{\sqrt{1-\frac{\sqrt{a+bx}}{\sqrt{a}}}}d\sqrt[4]{a+bx}}{\sqrt{\frac{a+bx}{b}-\frac{a}{b}}}-\frac{a^{3/4}\sqrt{1-\frac{a+bx}{a}}\,\text{EllipticF}\left(\arcsin\left(\frac{\sqrt[4]{a+bx}}{\sqrt[4]{a}}\right),-1\right)}{\sqrt{\frac{a+bx}{b}-\frac{a}{b}}}\right)}{a\sqrt[4]{ax^2+bx^3}}-\frac{2(ax^2+bx^3)^{3/4}}{ax^2}\right)}{a}+\frac{4}{a\sqrt[4]{ax^2+bx^3}}$$

$$\Big\downarrow \; 327$$

$$\frac{3\left(\frac{2\sqrt{x}\sqrt[4]{a+bx}\left(\frac{a^{3/4}\sqrt{1-\frac{a+bx}{a}}E\left(\arcsin\left(\frac{\sqrt[4]{a+bx}}{\sqrt[4]{a}}\right)\middle|-1\right)}{\sqrt{\frac{a+bx}{b}-\frac{a}{b}}}-\frac{a^{3/4}\sqrt{1-\frac{a+bx}{a}}\,\text{EllipticF}\left(\arcsin\left(\frac{\sqrt[4]{a+bx}}{\sqrt[4]{a}}\right),-1\right)}{\sqrt{\frac{a+bx}{b}-\frac{a}{b}}}\right)}{a\sqrt[4]{ax^2+bx^3}}-\frac{2(ax^2+bx^3)^{3/4}}{ax^2}\right)}{a}+\frac{4}{a\sqrt[4]{ax^2+bx^3}}$$

```
Int[x/(a*x^2 + b*x^3)^(5/4),x]
```

```
4/(a*(a*x^2 + b*x^3)^(1/4)) + (3*((-2*(a*x^2 + b*x^3)^(3/4))/(a*x^2) + (2*
Sqrt[x]*(a + b*x)^(1/4)*((a^(3/4)*Sqrt[1 - (a + b*x)/a]*EllipticE[ArcSin[(
a + b*x)^(1/4)/a^(1/4)], -1])/Sqrt[-(a/b) + (a + b*x)/b] - (a^(3/4)*Sqrt[1
 - (a + b*x)/a]*EllipticF[ArcSin[(a + b*x)^(1/4)/a^(1/4)], -1])/Sqrt[-(a/b
) + (a + b*x)/b]))/(a*(a*x^2 + b*x^3)^(1/4))))/a
```

Defintions of rubi rules used

```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Simp[p/b   Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
 d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && Lt
Q[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntL
inearQ[a, b, c, d, m, n, x]
```

```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Simp[
(Sqrt[a]/(Sqrt[c]*Rt[-d/c, 2]))*EllipticE[ArcSin[Rt[-d/c, 2]*x], b*(c/(a*d)
)], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a, 0]
```

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> Simp[(1/(Sqrt[a]*Rt[-b/a, 4])
)*EllipticF[ArcSin[Rt[-b/a, 4]*x], -1], x] /; FreeQ[{a, b}, x] && NegQ[b/a]
 && GtQ[a, 0]
```

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> Simp[Sqrt[1 + b*(x^4/a)]/Sqrt
[a + b*x^4]   Int[1/Sqrt[1 + b*(x^4/a)], x], x] /; FreeQ[{a, b}, x] && NegQ
[b/a] &&  !GtQ[a, 0]
```

```
Int[(x_)^2/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[-b/a, 2]},
Simp[-q^(-1)   Int[1/Sqrt[a + b*x^4], x], x] + Simp[1/q   Int[(1 + q*x^2)/S
qrt[a + b*x^4], x], x]] /; FreeQ[{a, b}, x] && NegQ[b/a]
```

```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (c_.)*(x_)^4], x_Symbol] :> Simp[d/Sq
rt[a]   Int[Sqrt[1 + e*(x^2/d)]/Sqrt[1 - e*(x^2/d)], x], x] /; FreeQ[{a, c,
 d, e}, x] && EqQ[c*d^2 + a*e^2, 0] && NegQ[c/a] && GtQ[a, 0]
```

```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (c_.)*(x_)^4], x_Symbol] :> Simp[Sqrt
[1 + c*(x^4/a)]/Sqrt[a + c*x^4]   Int[(d + e*x^2)/Sqrt[1 + c*(x^4/a)], x],
x] /; FreeQ[{a, c, d, e}, x] && EqQ[c*d^2 + a*e^2, 0] && NegQ[c/a] &&  !GtQ
[a, 0] &&  !(LtQ[a, 0] && GtQ[c, 0])
```

```
Int[((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol] :> Simp[(a*x^j +
b*x^n)^FracPart[p]/(x^(j*FracPart[p])*(a + b*x^(n - j))^FracPart[p])   Int[
x^(j*p)*(a + b*x^(n - j))^p, x], x] /; FreeQ[{a, b, j, n, p}, x] &&  !Integ
erQ[p] && NeQ[n, j] && PosQ[n - j]
```

```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> Simp[(-c^(j - 1))*(c*x)^(m - j + 1)*((a*x^j + b*x^n)^(p + 1)/(a*(n - j
)*(p + 1))), x] + Simp[c^j*((m + n*p + n - j + 1)/(a*(n - j)*(p + 1)))   In
t[(c*x)^(m - j)*(a*x^j + b*x^n)^(p + 1), x], x] /; FreeQ[{a, b, c, m}, x] &
&  !IntegerQ[p] && LtQ[0, j, n] && (IntegersQ[j, n] || GtQ[c, 0]) && LtQ[p,
 -1]
```

```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> Simp[c^(j - 1)*(c*x)^(m - j + 1)*((a*x^j + b*x^n)^(p + 1)/(a*(m + j*p
+ 1))), x] - Simp[b*((m + n*p + n - j + 1)/(a*c^(n - j)*(m + j*p + 1)))   I
nt[(c*x)^(m + n - j)*(a*x^j + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, p}, x]
 &&  !IntegerQ[p] && LtQ[0, j, n] && (IntegersQ[j, n] || GtQ[c, 0]) && LtQ[
m + j*p + 1, 0]
```

Maple [F]

$$\int \frac{x}{\left(b\,x^{3}+a\,x^{2}\right)^{\frac{5}{4}}}dx$$

```
int(x/(b*x^3+a*x^2)^(5/4),x)
```

```
int(x/(b*x^3+a*x^2)^(5/4),x)
```

Fricas [F]

$$\int \frac{x}{(ax^2+bx^3)^{5/4}}\,dx = \int \frac{x}{(bx^3+ax^2)^{\frac{5}{4}}}\,dx$$

```
integrate(x/(b*x^3+a*x^2)^(5/4),x, algorithm="fricas")
```

```
integral((b*x^3 + a*x^2)^(3/4)/(b^2*x^5 + 2*a*b*x^4 + a^2*x^3), x)
```

Sympy [F]

$$\int \frac{x}{(ax^2+bx^3)^{5/4}}\,dx = \int \frac{x}{(x^2\,(a+bx))^{\frac{5}{4}}}\,dx$$

```
integrate(x/(b*x**3+a*x**2)**(5/4),x)
```

```
Integral(x/(x**2*(a + b*x))**(5/4), x)
```

Maxima [F]

$$\int \frac{x}{(ax^2+bx^3)^{5/4}}\,dx = \int \frac{x}{(bx^3+ax^2)^{\frac{5}{4}}}\,dx$$

```
integrate(x/(b*x^3+a*x^2)^(5/4),x, algorithm="maxima")
```

```
integrate(x/(b*x^3 + a*x^2)^(5/4), x)
```

Giac [F]

$$\int \frac{x}{(ax^2+bx^3)^{5/4}}\,dx = \int \frac{x}{(bx^3+ax^2)^{\frac{5}{4}}}\,dx$$

```
integrate(x/(b*x^3+a*x^2)^(5/4),x, algorithm="giac")
```

```
integrate(x/(b*x^3 + a*x^2)^(5/4), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{x}{(ax^2+bx^3)^{5/4}}\,dx = \int \frac{x}{(b\,x^3+a\,x^2)^{5/4}}\,dx$$

```
int(x/(a*x^2 + b*x^3)^(5/4),x)
```

```
int(x/(a*x^2 + b*x^3)^(5/4), x)
```

Reduce [F]

$$\int \frac{x}{(ax^2+bx^3)^{5/4}}\,dx = \int \frac{\sqrt{x}\,(bx+a)^{\frac{1}{4}}}{\sqrt{bx+a}\,a\,x^2+\sqrt{bx+a}\,b\,x^3}dx$$

```
int(x/(b*x^3+a*x^2)^(5/4),x)
```

```
int((sqrt(x)*(a + b*x)**(1/4))/(sqrt(a + b*x)*a*x**2 + sqrt(a + b*x)*b*x**
3),x)
```

3.405 $\int \frac{1}{\left(ax^2+bx^3\right)^{5/4}}\,dx$

Optimal result

Integrand size = 15, antiderivative size = 117

$$\int \frac{1}{(ax^2+bx^3)^{5/4}}\,dx = \frac{7b}{3a^2\sqrt[4]{ax^2+bx^3}}-\frac{2}{3ax\sqrt[4]{ax^2+bx^3}} + \frac{7b^{3/2}\sqrt{x}\sqrt[4]{\frac{a+bx}{a}}E\left(\frac{1}{2}\arctan\left(\frac{\sqrt{b}\sqrt{x}}{\sqrt{a}}\right)\middle|2\right)}{a^{5/2}\sqrt[4]{ax^2+bx^3}}$$

```
7/3*b/a^2/(b*x^3+a*x^2)^(1/4)-2/3/a/x/(b*x^3+a*x^2)^(1/4)+7*b^(3/2)*x^(1/2
)*((b*x+a)/a)^(1/4)*EllipticE(sin(1/2*arctan(b^(1/2)*x^(1/2)/a^(1/2))),2^(
1/2))/a^(5/2)/(b*x^3+a*x^2)^(1/4)
```

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 0.01 (sec) , antiderivative size = 52, normalized size of antiderivative = 0.44

$$\int \frac{1}{(ax^2+bx^3)^{5/4}}\,dx = -\frac{2\sqrt[4]{1+\frac{bx}{a}}\,\text{Hypergeometric2F1}\left(-\frac{3}{2},\frac{5}{4},-\frac{1}{2},-\frac{bx}{a}\right)}{3ax\sqrt[4]{x^2(a+bx)}}$$

```
Integrate[(a*x^2 + b*x^3)^(-5/4),x]
```

```
(-2*(1 + (b*x)/a)^(1/4)*Hypergeometric2F1[-3/2, 5/4, -1/2, -((b*x)/a)])/(3
*a*x*(x^2*(a + b*x))^(1/4))
```

Rubi [B] (verified)

Leaf count is larger than twice the leaf count of optimal. 243 vs. 2(117) = 234.

Time = 0.82 (sec) , antiderivative size = 243, normalized size of antiderivative = 2.08, number of steps used = 13, number of rules used = 12, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.800, Rules used = {1912, 1931, 1931, 1917, 73, 836, 27, 765, 762, 1390, 1389, 327}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{(ax^2+bx^3)^{5/4}}dx$$

$$\Big\downarrow \text{1912}$$

$$\frac{7\int \frac{1}{x^2\sqrt[4]{bx^3+ax^2}}dx}{a}+\frac{4}{ax\sqrt[4]{ax^2+bx^3}}$$

$$\Big\downarrow \text{1931}$$

$$\frac{7\left(-\frac{b\int \frac{1}{x\sqrt[4]{bx^3+ax^2}}dx}{2a}-\frac{2(ax^2+bx^3)^{3/4}}{3ax^3}\right)}{a}+\frac{4}{ax\sqrt[4]{ax^2+bx^3}}$$

$$\Big\downarrow \text{1931}$$

$$\frac{7\left(-\frac{b\left(\frac{b\int \frac{1}{\sqrt[4]{bx^3+ax^2}}dx}{2a}-\frac{2(ax^2+bx^3)^{3/4}}{ax^2}\right)}{2a}-\frac{2(ax^2+bx^3)^{3/4}}{3ax^3}\right)}{a}+\frac{4}{ax\sqrt[4]{ax^2+bx^3}}$$

$$\Big\downarrow \text{1917}$$

$$\frac{7\left(-\frac{b\left(\frac{b\sqrt{x}\sqrt[4]{a+bx}\int\frac{1}{\sqrt{x}\sqrt[4]{a+bx}}dx}{2a\sqrt[4]{ax^2+bx^3}}-\frac{2\left(ax^2+bx^3\right)^{3/4}}{ax^2}\right)}{2a}-\frac{2\left(ax^2+bx^3\right)^{3/4}}{3ax^3}\right)}{a}+\frac{4}{ax\sqrt[4]{ax^2+bx^3}}$$

$$\Big\downarrow \textbf{73}$$

$$\frac{7\left(-\frac{b\left(\frac{2\sqrt{x}\sqrt[4]{a+bx}\int\frac{\sqrt{a+bx}}{\sqrt{\frac{a+bx}{b}-\frac{a}{b}}}d\sqrt[4]{a+bx}}{a\sqrt[4]{ax^2+bx^3}}-\frac{2\left(ax^2+bx^3\right)^{3/4}}{ax^2}\right)}{2a}-\frac{2\left(ax^2+bx^3\right)^{3/4}}{3ax^3}\right)}{a}+\frac{4}{ax\sqrt[4]{ax^2+bx^3}}$$

$$\Big\downarrow \textbf{836}$$

$$\frac{7\left(-\frac{b\left(\frac{2\sqrt{x}\sqrt[4]{a+bx}\left(\sqrt{a}\int\frac{\sqrt{a}+\sqrt{a+bx}}{\sqrt{a}\sqrt{\frac{a+bx}{b}-\frac{a}{b}}}d\sqrt[4]{a+bx}-\sqrt{a}\int\frac{1}{\sqrt{\frac{a+bx}{b}-\frac{a}{b}}}d\sqrt[4]{a+bx}\right)}{a\sqrt[4]{ax^2+bx^3}}-\frac{2\left(ax^2+bx^3\right)^{3/4}}{ax^2}\right)}{2a}-\frac{2\left(ax^2+bx^3\right)^{3/4}}{3ax^3}\right)}{a}+\frac{4}{ax\sqrt[4]{ax^2+bx^3}}$$

$$\Big\downarrow \textbf{27}$$

$$\frac{7\left(-\frac{b\left(\frac{2\sqrt{x}\sqrt[4]{a+bx}\left(\int\frac{\sqrt{a}+\sqrt{a+bx}}{\sqrt{\frac{a+bx}{b}-\frac{a}{b}}}d\sqrt[4]{a+bx}-\sqrt{a}\int\frac{1}{\sqrt{\frac{a+bx}{b}-\frac{a}{b}}}d\sqrt[4]{a+bx}\right)}{a\sqrt[4]{ax^2+bx^3}}-\frac{2\left(ax^2+bx^3\right)^{3/4}}{ax^2}\right)}{2a}-\frac{2\left(ax^2+bx^3\right)^{3/4}}{3ax^3}\right)}{a}+\frac{4}{ax\sqrt[4]{ax^2+bx^3}}$$

$$\Big\downarrow \textbf{765}$$

$$\frac{7\left(-\frac{b\left(\frac{2\sqrt{x}\sqrt[4]{a+bx}\left(\int \frac{\sqrt{a}+\sqrt{a+bx}}{\sqrt{\frac{a+bx}{b}-\frac{a}{b}}}d\sqrt[4]{a+bx}-\frac{\sqrt{a}\sqrt{1-\frac{a+bx}{a}}\int \frac{1}{\sqrt{1-\frac{a+bx}{a}}}d\sqrt[4]{a+bx}}{\sqrt{\frac{a+bx}{b}-\frac{a}{b}}}\right)}{a\sqrt[4]{ax^2+bx^3}}-\frac{2\left(ax^2+bx^3\right)^{3/4}}{ax^2}\right)}{2a}-\frac{2\left(ax^2+bx^3\right)^{3/4}}{3ax^3}\right)}{a}+\frac{4}{ax\sqrt[4]{ax^2+bx^3}}$$

$$\Big\downarrow \text{762}$$

$$\frac{7\left(-\frac{b\left(\frac{2\sqrt{x}\sqrt[4]{a+bx}\left(\int \frac{\sqrt{a}+\sqrt{a+bx}}{\sqrt{\frac{a+bx}{b}-\frac{a}{b}}}d\sqrt[4]{a+bx}-\frac{a^{3/4}\sqrt{1-\frac{a+bx}{a}}\operatorname{EllipticF}\left(\arcsin\left(\frac{\sqrt[4]{a+bx}}{\sqrt[4]{a}}\right),-1\right)}{\sqrt{\frac{a+bx}{b}-\frac{a}{b}}}\right)}{a\sqrt[4]{ax^2+bx^3}}-\frac{2\left(ax^2+bx^3\right)^{3/4}}{ax^2}\right)}{2a}-\frac{2\left(ax^2+bx^3\right)^{3/}}{3ax^3}}{a}$$

$$\frac{4}{ax\sqrt[4]{ax^2+bx^3}}$$

$$\Big\downarrow \text{1390}$$

$$\frac{7\left(-\frac{b\left(\frac{2\sqrt{x}\sqrt[4]{a+bx}\left(\frac{\sqrt{1-\frac{a+bx}{a}}\int\frac{\sqrt{a}+\sqrt{a+bx}}{\sqrt{1-\frac{a+bx}{a}}}d\sqrt[4]{a+bx}}{\sqrt{\frac{a+bx}{b}-\frac{a}{b}}}-\frac{a^{3/4}\sqrt{1-\frac{a+bx}{a}}\text{EllipticF}\left(\arcsin\left(\frac{\sqrt[4]{a+bx}}{\sqrt[4]{a}}\right),-1\right)}{\sqrt{\frac{a+bx}{b}-\frac{a}{b}}}\right)}{a\sqrt[4]{ax^2+bx^3}}-\frac{2\left(ax^2+bx^3\right)^{3/4}}{ax^2}\right)}{2a}-\frac{2(a}{}\right.}{a}$$

$$\frac{4}{ax\sqrt[4]{ax^2+bx^3}}$$

$$\Big\downarrow \text{1389}$$

$$\frac{7\left(-\frac{b\left(\frac{2\sqrt{x}\sqrt[4]{a+bx}\left(\frac{\sqrt{a}\sqrt{1-\frac{a+bx}{a}}\int\frac{\sqrt{\frac{\sqrt{a+bx}}{\sqrt{a}}+1}}{\sqrt{1-\frac{\sqrt{a+bx}}{\sqrt{a}}}}d\sqrt[4]{a+bx}}{\sqrt{\frac{a+bx}{b}-\frac{a}{b}}}-\frac{a^{3/4}\sqrt{1-\frac{a+bx}{a}}\text{EllipticF}\left(\arcsin\left(\frac{\sqrt[4]{a+bx}}{\sqrt[4]{a}}\right),-1\right)}{\sqrt{\frac{a+bx}{b}-\frac{a}{b}}}\right)}{a\sqrt[4]{ax^2+bx^3}}-\frac{2\left(ax^2+bx^3\right)^{3/4}}{ax^2}\right)}{2a}-\right.}{a}$$

$$\frac{4}{ax\sqrt[4]{ax^2+bx^3}}$$

$$\Big\downarrow \text{327}$$

$$\frac{7\left(-\frac{b\left(\frac{2\sqrt{x}\sqrt[4]{a+bx}\left(\frac{a^{3/4}\sqrt{1-\frac{a+bx}{a}}E\left(\arcsin\left(\frac{\sqrt[4]{a+bx}}{\sqrt[4]{a}}\right)\middle|-1\right)}{\sqrt{\frac{a+bx}{b}-\frac{a}{b}}}-\frac{a^{3/4}\sqrt{1-\frac{a+bx}{a}}\operatorname{EllipticF}\left(\arcsin\left(\frac{\sqrt[4]{a+bx}}{\sqrt[4]{a}}\right),-1\right)}{\sqrt{\frac{a+bx}{b}-\frac{a}{b}}}\right)}{a\sqrt[4]{ax^2+bx^3}}-\frac{2\left(ax^2+bx^3\right)^{3/4}}{ax^2}\right)}{2a}\right.}{a}$$

$$\frac{4}{ax\sqrt[4]{ax^2+bx^3}}$$

```
Int[(a*x^2 + b*x^3)^(-5/4),x]
```

```
4/(a*x*(a*x^2 + b*x^3)^(1/4)) + (7*((-2*(a*x^2 + b*x^3)^(3/4))/(3*a*x^3) -
 (b*((-2*(a*x^2 + b*x^3)^(3/4))/(a*x^2) + (2*Sqrt[x]*(a + b*x)^(1/4)*((a^(
3/4)*Sqrt[1 - (a + b*x)/a]*EllipticE[ArcSin[(a + b*x)^(1/4)/a^(1/4)], -1])
/Sqrt[-(a/b) + (a + b*x)/b] - (a^(3/4)*Sqrt[1 - (a + b*x)/a]*EllipticF[Arc
Sin[(a + b*x)^(1/4)/a^(1/4)], -1])/Sqrt[-(a/b) + (a + b*x)/b]))/(a*(a*x^2
+ b*x^3)^(1/4))))/(2*a)))/a
```

Defintions of rubi rules used

```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Simp[p/b   Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
 d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && Lt
Q[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntL
inearQ[a, b, c, d, m, n, x]
```

```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Simp[
(Sqrt[a]/(Sqrt[c]*Rt[-d/c, 2]))*EllipticE[ArcSin[Rt[-d/c, 2]*x], b*(c/(a*d)
)], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a, 0]
```

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> Simp[(1/(Sqrt[a]*Rt[-b/a, 4])
)*EllipticF[ArcSin[Rt[-b/a, 4]*x], -1], x] /; FreeQ[{a, b}, x] && NegQ[b/a]
 && GtQ[a, 0]
```

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> Simp[Sqrt[1 + b*(x^4/a)]/Sqrt
[a + b*x^4]   Int[1/Sqrt[1 + b*(x^4/a)], x], x] /; FreeQ[{a, b}, x] && NegQ
[b/a] &&  !GtQ[a, 0]
```

```
Int[(x_)^2/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[-b/a, 2]},
Simp[-q^(-1)   Int[1/Sqrt[a + b*x^4], x], x] + Simp[1/q   Int[(1 + q*x^2)/S
qrt[a + b*x^4], x], x]] /; FreeQ[{a, b}, x] && NegQ[b/a]
```

```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (c_.)*(x_)^4], x_Symbol] :> Simp[d/Sq
rt[a]   Int[Sqrt[1 + e*(x^2/d)]/Sqrt[1 - e*(x^2/d)], x], x] /; FreeQ[{a, c,
 d, e}, x] && EqQ[c*d^2 + a*e^2, 0] && NegQ[c/a] && GtQ[a, 0]
```

```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (c_.)*(x_)^4], x_Symbol] :> Simp[Sqrt
[1 + c*(x^4/a)]/Sqrt[a + c*x^4]   Int[(d + e*x^2)/Sqrt[1 + c*(x^4/a)], x],
x] /; FreeQ[{a, c, d, e}, x] && EqQ[c*d^2 + a*e^2, 0] && NegQ[c/a] &&  !GtQ
[a, 0] &&  !(LtQ[a, 0] && GtQ[c, 0])
```

```
Int[((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol] :> Simp[-(a*x^j +
 b*x^n)^(p + 1)/(a*(n - j)*(p + 1)*x^(j - 1)), x] + Simp[(n*p + n - j + 1)/
(a*(n - j)*(p + 1))   Int[(a*x^j + b*x^n)^(p + 1)/x^j, x], x] /; FreeQ[{a,
b}, x] &&  !IntegerQ[p] && LtQ[0, j, n] && LtQ[p, -1]
```

```
Int[((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol] :> Simp[(a*x^j +
b*x^n)^FracPart[p]/(x^(j*FracPart[p])*(a + b*x^(n - j))^FracPart[p])   Int[
x^(j*p)*(a + b*x^(n - j))^p, x], x] /; FreeQ[{a, b, j, n, p}, x] &&  !Integ
erQ[p] && NeQ[n, j] && PosQ[n - j]
```

```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> Simp[c^(j - 1)*(c*x)^(m - j + 1)*((a*x^j + b*x^n)^(p + 1)/(a*(m + j*p
+ 1))), x] - Simp[b*((m + n*p + n - j + 1)/(a*c^(n - j)*(m + j*p + 1)))   I
nt[(c*x)^(m + n - j)*(a*x^j + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, p}, x]
 &&  !IntegerQ[p] && LtQ[0, j, n] && (IntegersQ[j, n] || GtQ[c, 0]) && LtQ[
m + j*p + 1, 0]
```

Maple [F]

$$\int \frac{1}{\left(b\,x^{3}+a\,x^{2}\right)^{\frac{5}{4}}}dx$$

```
int(1/(b*x^3+a*x^2)^(5/4),x)
```

```
int(1/(b*x^3+a*x^2)^(5/4),x)
```

Fricas [F]

$$\int \frac{1}{(ax^2+bx^3)^{5/4}}\,dx=\int \frac{1}{(bx^3+ax^2)^{\frac{5}{4}}}\,dx$$

```
integrate(1/(b*x^3+a*x^2)^(5/4),x, algorithm="fricas")
```

```
integral((b*x^3 + a*x^2)^(3/4)/(b^2*x^6 + 2*a*b*x^5 + a^2*x^4), x)
```

Sympy [F]

$$\int \frac{1}{(ax^2+bx^3)^{5/4}}\,dx = \int \frac{1}{(ax^2+bx^3)^{\frac{5}{4}}}\,dx$$

```
integrate(1/(b*x**3+a*x**2)**(5/4),x)
```

```
Integral((a*x**2 + b*x**3)**(-5/4), x)
```

Maxima [F]

$$\int \frac{1}{(ax^2+bx^3)^{5/4}}\,dx = \int \frac{1}{(bx^3+ax^2)^{\frac{5}{4}}}\,dx$$

```
integrate(1/(b*x^3+a*x^2)^(5/4),x, algorithm="maxima")
```

```
integrate((b*x^3 + a*x^2)^(-5/4), x)
```

Giac [F]

$$\int \frac{1}{(ax^2+bx^3)^{5/4}}\,dx = \int \frac{1}{(bx^3+ax^2)^{\frac{5}{4}}}\,dx$$

```
integrate(1/(b*x^3+a*x^2)^(5/4),x, algorithm="giac")
```

```
integrate((b*x^3 + a*x^2)^(-5/4), x)
```

Mupad [B] (verification not implemented)

Time = 8.92 (sec) , antiderivative size = 38, normalized size of antiderivative = 0.32

$$\int \frac{1}{(ax^2+bx^3)^{5/4}}\,dx = -\frac{2\,x\,\left(\frac{b\,x}{a}+1\right)^{5/4}\,{}_2\mathrm{F}_1\left(-\frac{3}{2},\frac{5}{4};\ -\frac{1}{2};\ -\frac{b\,x}{a}\right)}{3\,\left(b\,x^3+a\,x^2\right)^{5/4}}$$

```
int(1/(a*x^2 + b*x^3)^(5/4),x)
```

```
-(2*x*((b*x)/a + 1)^(5/4)*hypergeom([-3/2, 5/4], -1/2, -(b*x)/a))/(3*(a*x^
2 + b*x^3)^(5/4))
```

Reduce [F]

$$\int \frac{1}{(ax^2+bx^3)^{5/4}}\,dx = \int \frac{\sqrt{x}\,(bx+a)^{\frac{1}{4}}}{\sqrt{bx+a}\,a\,x^3+\sqrt{bx+a}\,b\,x^4}dx$$

```
int(1/(b*x^3+a*x^2)^(5/4),x)
```

```
int((sqrt(x)*(a + b*x)**(1/4))/(sqrt(a + b*x)*a*x**3 + sqrt(a + b*x)*b*x**
4),x)
```

3.406 $\int \frac{1}{x(ax^2+bx^3)^{5/4}}\,dx$

Optimal result

Integrand size = 19, antiderivative size = 147

$$\int \frac{1}{x\left(ax^2+bx^3\right)^{5/4}}\,dx=-\frac{77b^2}{30a^3\sqrt[4]{ax^2+bx^3}}-\frac{2}{5ax^2\sqrt[4]{ax^2+bx^3}}+\frac{11b}{15a^2x\sqrt[4]{ax^2+bx^3}}-\frac{77b^{5/2}\sqrt{x}\sqrt[4]{\frac{a+bx}{a}}E\left(\frac{1}{2}\arctan\left(\frac{\sqrt{b}\sqrt{x}}{\sqrt{a}}\right)\middle|2\right)}{10a^{7/2}\sqrt[4]{ax^2+bx^3}}$$

```
-77/30*b^2/a^3/(b*x^3+a*x^2)^(1/4)-2/5/a/x^2/(b*x^3+a*x^2)^(1/4)+11/15*b/a
^2/x/(b*x^3+a*x^2)^(1/4)-77/10*b^(5/2)*x^(1/2)*((b*x+a)/a)^(1/4)*EllipticE
(sin(1/2*arctan(b^(1/2)*x^(1/2)/a^(1/2))),2^(1/2))/a^(7/2)/(b*x^3+a*x^2)^(
1/4)
```

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 10.02 (sec) , antiderivative size = 54, normalized size of antiderivative = 0.37

$$\int \frac{1}{x\left(ax^2+bx^3\right)^{5/4}}\,dx=-\frac{2(a+bx)\sqrt[4]{1+\frac{bx}{a}}\,\text{Hypergeometric2F1}\left(-\frac{5}{2},\frac{5}{4},-\frac{3}{2},-\frac{bx}{a}\right)}{5a\left(x^2(a+bx)\right)^{5/4}}$$

```
Integrate[1/(x*(a*x^2 + b*x^3)^(5/4)),x]
```

```
(-2*(a + b*x)*(1 + (b*x)/a)^(1/4)*Hypergeometric2F1[-5/2, 5/4, -3/2, -((b*
x)/a)])/(5*a*(x^2*(a + b*x))^(5/4))
```

Rubi [A] (verified)

Time = 0.97 (sec) , antiderivative size = 277, normalized size of antiderivative = 1.88, number of steps used = 14, number of rules used = 13, $\frac{\text{number of rules}}{\text{integrand size}} = 0.684$, Rules used = {1929, 1931, 1931, 1931, 1917, 73, 836, 27, 765, 762, 1390, 1389, 327}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{x\left(ax^2+bx^3\right)^{5/4}}dx$$

$$\Big\downarrow \text{1929}$$

$$\frac{11\int \frac{1}{x^3\sqrt[4]{bx^3+ax^2}}dx}{a}+\frac{4}{ax^2\sqrt[4]{ax^2+bx^3}}$$

$$\Big\downarrow \text{1931}$$

$$\frac{11\left(-\frac{7b\int \frac{1}{x^2\sqrt[4]{bx^3+ax^2}}dx}{10a}-\frac{2\left(ax^2+bx^3\right)^{3/4}}{5ax^4}\right)}{a}+\frac{4}{ax^2\sqrt[4]{ax^2+bx^3}}$$

$$\Big\downarrow \text{1931}$$

$$\frac{11\left(-\frac{7b\left(-\frac{b\int \frac{1}{x\sqrt[4]{bx^3+ax^2}}dx}{2a}-\frac{2\left(ax^2+bx^3\right)^{3/4}}{3ax^3}\right)}{10a}-\frac{2\left(ax^2+bx^3\right)^{3/4}}{5ax^4}\right)}{a}+\frac{4}{ax^2\sqrt[4]{ax^2+bx^3}}$$

$$\Big\downarrow \text{1931}$$

$$\frac{\frac{11\left(-\frac{7b\left(-\frac{b\left(\frac{b\int \frac{1}{\sqrt[4]{bx^3+ax^2}}dx}{2a}-\frac{2\left(ax^2+bx^3\right)^{3/4}}{ax^2}\right)}{2a}-\frac{2\left(ax^2+bx^3\right)^{3/4}}{3ax^3}\right)}{10a}-\frac{2\left(ax^2+bx^3\right)^{3/4}}{5ax^4}\right)}{4}}{ax^2\sqrt[4]{ax^2+bx^3}}+$$

$$\Big\downarrow \; 1917$$

$$\frac{\frac{11\left(-\frac{7b\left(-\frac{b\left(\frac{b\sqrt{x}\sqrt[4]{a+bx}\int \frac{1}{\sqrt{x}\sqrt[4]{a+bx}}dx}{2a\sqrt[4]{ax^2+bx^3}}-\frac{2\left(ax^2+bx^3\right)^{3/4}}{ax^2}\right)}{2a}-\frac{2\left(ax^2+bx^3\right)^{3/4}}{3ax^3}\right)}{10a}-\frac{2\left(ax^2+bx^3\right)^{3/4}}{5ax^4}\right)}{4}}{ax^2\sqrt[4]{ax^2+bx^3}}+$$

$$\Big\downarrow \; 73$$

$$\frac{11\left(-\frac{7b\left(-\frac{b\left(\frac{2\sqrt{x}\sqrt[4]{a+bx}\int \frac{\sqrt{a+bx}}{\sqrt{\frac{a+bx}{b}-\frac{a}{b}}}d\sqrt[4]{a+bx}}{a\sqrt[4]{ax^2+bx^3}}-\frac{2\left(ax^2+bx^3\right)^{3/4}}{ax^2}\right)}{2a}-\frac{2\left(ax^2+bx^3\right)^{3/4}}{3ax^3}\right)}{10a}-\frac{2\left(ax^2+bx^3\right)^{3/4}}{5ax^4}\right)}{a}+\frac{4}{ax^2\sqrt[4]{ax^2+bx^3}}$$

$$\Big\downarrow \ 836$$

$$\frac{11\left(-\frac{7b\left(-\frac{b\left(\frac{2\sqrt{x}\sqrt[4]{a+bx}\left(\sqrt{a}\int \frac{\sqrt{a}+\sqrt{a+bx}}{\sqrt{a}\sqrt{\frac{a+bx}{b}-\frac{a}{b}}}d\sqrt[4]{a+bx}-\sqrt{a}\int \frac{1}{\sqrt{\frac{a+bx}{b}-\frac{a}{b}}}d\sqrt[4]{a+bx}\right)}{a\sqrt[4]{ax^2+bx^3}}-\frac{2\left(ax^2+bx^3\right)^{3/4}}{ax^2}\right)}{2a}-\frac{2\left(ax^2+bx^3\right)^{3/4}}{3ax^3}\right)}{10a}-2(a}{a}+\frac{4}{ax^2\sqrt[4]{ax^2+bx^3}}$$

$$\Big\downarrow \ 27$$

$$\frac{11\left(-\frac{7b\left(-\frac{b\left(\frac{2\sqrt{x}\sqrt[4]{a+bx}\left(\int\frac{\sqrt{a}+\sqrt{a+bx}}{\sqrt{\frac{a+bx}{b}-\frac{a}{b}}}d\sqrt[4]{a+bx}-\sqrt{a}\int\frac{1}{\sqrt{\frac{a+bx}{b}-\frac{a}{b}}}d\sqrt[4]{a+bx}\right)}{a\sqrt[4]{ax^2+bx^3}}-\frac{2\left(ax^2+bx^3\right)^{3/4}}{ax^2}\right)}{2a}-\frac{2\left(ax^2+bx^3\right)^{3/4}}{3ax^3}\right)}{10a}-\frac{2\left(ax^2+bx\right.}{5ax}\right.}{a}$$

$$\frac{4}{ax^2\sqrt[4]{ax^2+bx^3}}$$

$$\Big\downarrow 765$$

$$\frac{11\left(-\frac{7b\left(-\frac{b\left(\frac{2\sqrt{x}\sqrt[4]{a+bx}\left(\int\frac{\sqrt{a}+\sqrt{a+bx}}{\sqrt{\frac{a+bx}{b}-\frac{a}{b}}}d\sqrt[4]{a+bx}-\frac{\sqrt{a}\sqrt{1-\frac{a+bx}{a}}\int\frac{1}{\sqrt{1-\frac{a+bx}{a}}}d\sqrt[4]{a+bx}}{\sqrt{\frac{a+bx}{b}-\frac{a}{b}}}\right)}{a\sqrt[4]{ax^2+bx^3}}-\frac{2\left(ax^2+bx^3\right)^{3/4}}{ax^2}\right)}{2a}-\frac{2\left(ax^2+bx^3\right)^{3/4}}{3ax^3}\right)}{10a}-\right.}{a}$$

$$\frac{4}{ax^2\sqrt[4]{ax^2+bx^3}}$$

$$\Big\downarrow \; 762$$

$$\frac{11\left(-\frac{7b\left(-\frac{b\left(\frac{2\sqrt{x}\sqrt[4]{a+bx}\left(\int\frac{\sqrt{a}+\sqrt{a+bx}}{\sqrt{\frac{a+bx}{b}-\frac{a}{b}}}d\sqrt[4]{a+bx}-\frac{a^{3/4}\sqrt{1-\frac{a+bx}{a}}\,\text{EllipticF}\left(\arcsin\left(\frac{\sqrt[4]{a+bx}}{\sqrt[4]{a}}\right),-1\right)}{\sqrt{\frac{a+bx}{b}-\frac{a}{b}}}\right)}{a\sqrt[4]{ax^2+bx^3}}\right)-\frac{2\left(ax^2+bx^3\right)^{3/4}}{ax^2}}{2a}-\frac{2\left(ax^2+bx\right.}{3ax^3}}{10a}\right.}{a}$$

$$\frac{4}{ax^2\sqrt[4]{ax^2+bx^3}}$$

$$\Big\downarrow \; 1390$$

$$\frac{11\left(-\frac{7b\left(-\frac{b\left(\frac{2\sqrt{x}\sqrt[4]{a+bx}\left(\frac{\sqrt{1-\frac{a+bx}{a}}\int\frac{\sqrt{a}+\sqrt{a+bx}}{\sqrt{1-\frac{a+bx}{a}}}d\sqrt[4]{a+bx}}{\sqrt{\frac{a+bx}{b}-\frac{a}{b}}}-\frac{a^{3/4}\sqrt{1-\frac{a+bx}{a}}\operatorname{EllipticF}\left(\arcsin\left(\frac{\sqrt[4]{a+bx}}{\sqrt[4]{a}}\right),-1\right)}{\sqrt{\frac{a+bx}{b}-\frac{a}{b}}}\right)}{a\sqrt[4]{ax^2+bx^3}}-\frac{2\left(ax^2+bx^3\right)^{3/4}}{ax^2}\right)}{2a}-\cdots\right)}{10a}\cdots\right)}{a}$$

$$\frac{4}{ax^2\sqrt[4]{ax^2+bx^3}}$$

$$\Big\downarrow 1389$$

$$\frac{11\left(-\frac{7b\left(-\frac{b\left(\frac{2\sqrt{x}\sqrt[4]{a+bx}\left(\frac{\sqrt{a}\sqrt{1-\frac{a+bx}{a}}\int\frac{\sqrt{\frac{\sqrt{a+bx}}{\sqrt{a}}+1}}{\sqrt{1-\frac{\sqrt{a+bx}}{\sqrt{a}}}}d\sqrt[4]{a+bx}}{\sqrt{\frac{a+bx}{b}-\frac{a}{b}}}-\frac{a^{3/4}\sqrt{1-\frac{a+bx}{a}}\,\text{EllipticF}\left(\arcsin\left(\frac{\sqrt[4]{a+bx}}{\sqrt[4]{a}}\right),-1\right)}{\sqrt{\frac{a+bx}{b}-\frac{a}{b}}}\right)}{a\sqrt[4]{ax^2+bx^3}}-\frac{2\left(ax^2+bx^3\right)^{3/4}}{ax^2}\cdots}{2a}\cdots}{10a}\cdots\right)}{a}\cdots$$

$$\frac{4}{ax^2\sqrt[4]{ax^2+bx^3}}$$

$$\downarrow 327$$

$$\frac{11\left(-\frac{7b\left(-\frac{b\left(\frac{2\sqrt{x}\sqrt[4]{a+bx}\left(\frac{a^{3/4}\sqrt{1-\frac{a+bx}{a}}E\left(\arcsin\left(\frac{\sqrt[4]{a+bx}}{\sqrt[4]{a}}\right)\middle|-1\right)}{\sqrt{\frac{a+bx}{b}-\frac{a}{b}}}-\frac{a^{3/4}\sqrt{1-\frac{a+bx}{a}}\,\mathrm{EllipticF}\left(\arcsin\left(\frac{\sqrt[4]{a+bx}}{\sqrt[4]{a}}\right),-1\right)}{\sqrt{\frac{a+bx}{b}-\frac{a}{b}}}\right)}{a\sqrt[4]{ax^2+bx^3}}-\frac{2\left(ax^2+bx^3\right.}{ax^2}}{2a}}{10a}}{a}$$

$$\frac{4}{ax^2\sqrt[4]{ax^2+bx^3}}$$

```
Int[1/(x*(a*x^2 + b*x^3)^(5/4)),x]
```

```
4/(a*x^2*(a*x^2 + b*x^3)^(1/4)) + (11*((-2*(a*x^2 + b*x^3)^(3/4))/(5*a*x^4
) - (7*b*((-2*(a*x^2 + b*x^3)^(3/4))/(3*a*x^3) - (b*((-2*(a*x^2 + b*x^3)^(
3/4))/(a*x^2) + (2*Sqrt[x]*(a + b*x)^(1/4)*((a^(3/4)*Sqrt[1 - (a + b*x)/a]
*EllipticE[ArcSin[(a + b*x)^(1/4)/a^(1/4)], -1])/Sqrt[-(a/b) + (a + b*x)/b
] - (a^(3/4)*Sqrt[1 - (a + b*x)/a]*EllipticF[ArcSin[(a + b*x)^(1/4)/a^(1/4
)], -1])/Sqrt[-(a/b) + (a + b*x)/b]))/(a*(a*x^2 + b*x^3)^(1/4))))/(2*a)))/
(10*a)))/a
```

Defintions of rubi rules used

```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Simp[p/b   Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
 d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && Lt
Q[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntL
inearQ[a, b, c, d, m, n, x]
```

```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Simp[
(Sqrt[a]/(Sqrt[c]*Rt[-d/c, 2]))*EllipticE[ArcSin[Rt[-d/c, 2]*x], b*(c/(a*d)
)], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a, 0]
```

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> Simp[(1/(Sqrt[a]*Rt[-b/a, 4])
)*EllipticF[ArcSin[Rt[-b/a, 4]*x], -1], x] /; FreeQ[{a, b}, x] && NegQ[b/a]
 && GtQ[a, 0]
```

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> Simp[Sqrt[1 + b*(x^4/a)]/Sqrt
[a + b*x^4]   Int[1/Sqrt[1 + b*(x^4/a)], x], x] /; FreeQ[{a, b}, x] && NegQ
[b/a] &&  !GtQ[a, 0]
```

```
Int[(x_)^2/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[-b/a, 2]},
Simp[-q^(-1)   Int[1/Sqrt[a + b*x^4], x], x] + Simp[1/q   Int[(1 + q*x^2)/S
qrt[a + b*x^4], x], x]] /; FreeQ[{a, b}, x] && NegQ[b/a]
```

```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (c_.)*(x_)^4], x_Symbol] :> Simp[d/Sq
rt[a]   Int[Sqrt[1 + e*(x^2/d)]/Sqrt[1 - e*(x^2/d)], x], x] /; FreeQ[{a, c,
 d, e}, x] && EqQ[c*d^2 + a*e^2, 0] && NegQ[c/a] && GtQ[a, 0]
```

```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (c_.)*(x_)^4], x_Symbol] :> Simp[Sqrt
[1 + c*(x^4/a)]/Sqrt[a + c*x^4]   Int[(d + e*x^2)/Sqrt[1 + c*(x^4/a)], x],
x] /; FreeQ[{a, c, d, e}, x] && EqQ[c*d^2 + a*e^2, 0] && NegQ[c/a] &&  !GtQ
[a, 0] &&  !(LtQ[a, 0] && GtQ[c, 0])
```

```
Int[((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol] :> Simp[(a*x^j +
b*x^n)^FracPart[p]/(x^(j*FracPart[p])*(a + b*x^(n - j))^FracPart[p])   Int[
x^(j*p)*(a + b*x^(n - j))^p, x], x] /; FreeQ[{a, b, j, n, p}, x] &&  !Integ
erQ[p] && NeQ[n, j] && PosQ[n - j]
```

```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> Simp[(-c^(j - 1))*(c*x)^(m - j + 1)*((a*x^j + b*x^n)^(p + 1)/(a*(n - j
)*(p + 1))), x] + Simp[c^j*((m + n*p + n - j + 1)/(a*(n - j)*(p + 1)))   In
t[(c*x)^(m - j)*(a*x^j + b*x^n)^(p + 1), x], x] /; FreeQ[{a, b, c, m}, x] &
&  !IntegerQ[p] && LtQ[0, j, n] && (IntegersQ[j, n] || GtQ[c, 0]) && LtQ[p,
 -1]
```

```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> Simp[c^(j - 1)*(c*x)^(m - j + 1)*((a*x^j + b*x^n)^(p + 1)/(a*(m + j*p
+ 1))), x] - Simp[b*((m + n*p + n - j + 1)/(a*c^(n - j)*(m + j*p + 1)))   I
nt[(c*x)^(m + n - j)*(a*x^j + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, p}, x]
 &&  !IntegerQ[p] && LtQ[0, j, n] && (IntegersQ[j, n] || GtQ[c, 0]) && LtQ[
m + j*p + 1, 0]
```

Maple [F]

$$\int \frac{1}{x\left(b\,x^{3}+a\,x^{2}\right)^{\frac{5}{4}}}dx$$

```
int(1/x/(b*x^3+a*x^2)^(5/4),x)
```

```
int(1/x/(b*x^3+a*x^2)^(5/4),x)
```

Fricas [F]

$$\int \frac{1}{x\,(ax^2+bx^3)^{5/4}}\,dx = \int \frac{1}{(bx^3+ax^2)^{\frac{5}{4}}x}\,dx$$

```
integrate(1/x/(b*x^3+a*x^2)^(5/4),x, algorithm="fricas")
```

```
integral((b*x^3 + a*x^2)^(3/4)/(b^2*x^7 + 2*a*b*x^6 + a^2*x^5), x)
```

Sympy [F]

$$\int \frac{1}{x\,(ax^2+bx^3)^{5/4}}\,dx = \int \frac{1}{x\,(x^2\,(a+bx))^{\frac{5}{4}}}\,dx$$

```
integrate(1/x/(b*x**3+a*x**2)**(5/4),x)
```

```
Integral(1/(x*(x**2*(a + b*x))**(5/4)), x)
```

Maxima [F]

$$\int \frac{1}{x\,(ax^2+bx^3)^{5/4}}\,dx = \int \frac{1}{(bx^3+ax^2)^{\frac{5}{4}}x}\,dx$$

```
integrate(1/x/(b*x^3+a*x^2)^(5/4),x, algorithm="maxima")
```

```
integrate(1/((b*x^3 + a*x^2)^(5/4)*x), x)
```

Giac [F]

$$\int \frac{1}{x\,(ax^2+bx^3)^{5/4}}\,dx=\int \frac{1}{(bx^3+ax^2)^{\frac{5}{4}}x}\,dx$$

```
integrate(1/x/(b*x^3+a*x^2)^(5/4),x, algorithm="giac")
```

```
integrate(1/((b*x^3 + a*x^2)^(5/4)*x), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{x\,(ax^2+bx^3)^{5/4}}\,dx=\int \frac{1}{x\,(b\,x^3+a\,x^2)^{5/4}}\,dx$$

```
int(1/(x*(a*x^2 + b*x^3)^(5/4)),x)
```

```
int(1/(x*(a*x^2 + b*x^3)^(5/4)), x)
```

Reduce [F]

$$\int \frac{1}{x\,(ax^2+bx^3)^{5/4}}\,dx=\int \frac{\sqrt{x}\,(bx+a)^{\frac{1}{4}}}{\sqrt{bx+a}\,a\,x^4+\sqrt{bx+a}\,b\,x^5}dx$$

```
int(1/x/(b*x^3+a*x^2)^(5/4),x)
```

```
int((sqrt(x)*(a + b*x)**(1/4))/(sqrt(a + b*x)*a*x**4 + sqrt(a + b*x)*b*x**
5),x)
```

3.407 $\int (cx)^m \left(ax^2+bx^3\right)^3 \, dx$

Optimal result

Integrand size = 19, antiderivative size = 81

$$\int (cx)^m \left(ax^2+bx^3\right)^3 \, dx = \frac{a^3(cx)^{7+m}}{c^7(7+m)} + \frac{3a^2b(cx)^{8+m}}{c^8(8+m)} + \frac{3ab^2(cx)^{9+m}}{c^9(9+m)} + \frac{b^3(cx)^{10+m}}{c^{10}(10+m)}$$

```
a^3*(c*x)^(7+m)/c^7/(7+m)+3*a^2*b*(c*x)^(8+m)/c^8/(8+m)+3*a*b^2*(c*x)^(9+m
)/c^9/(9+m)+b^3*(c*x)^(10+m)/c^10/(10+m)
```

Mathematica [A] (verified)

Time = 0.07 (sec) , antiderivative size = 57, normalized size of antiderivative = 0.70

$$\int (cx)^m \left(ax^2+bx^3\right)^3 \, dx = x^7(cx)^m \left(\frac{a^3}{7+m} + \frac{3a^2bx}{8+m} + \frac{3ab^2x^2}{9+m} + \frac{b^3x^3}{10+m}\right)$$

```
Integrate[(c*x)^m*(a*x^2 + b*x^3)^3,x]
```

```
x^7*(c*x)^m*(a^3/(7 + m) + (3*a^2*b*x)/(8 + m) + (3*a*b^2*x^2)/(9 + m) + (
b^3*x^3)/(10 + m))
```

Rubi [A] (verified)

Time = 0.39 (sec) , antiderivative size = 85, normalized size of antiderivative = 1.05, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.158$, Rules used = {9, 53, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (ax^2+bx^3)^3 (cx)^m \, dx$$

$$\Big\downarrow \text{ 9}$$

$$\frac{\int (cx)^{m+6}(a+bx)^3dx}{c^6}$$

$$\Big\downarrow \text{ 53}$$

$$\frac{\int \left(a^3(cx)^{m+6}+\frac{3a^2b(cx)^{m+7}}{c}+\frac{3ab^2(cx)^{m+8}}{c^2}+\frac{b^3(cx)^{m+9}}{c^3}\right)dx}{c^6}$$

$$\Big\downarrow \text{ 2009}$$

$$\frac{\frac{a^3(cx)^{m+7}}{c(m+7)}+\frac{3a^2b(cx)^{m+8}}{c^2(m+8)}+\frac{3ab^2(cx)^{m+9}}{c^3(m+9)}+\frac{b^3(cx)^{m+10}}{c^4(m+10)}}{c^6}$$

```
Int[(c*x)^m*(a*x^2 + b*x^3)^3,x]
```

```
((a^3*(c*x)^(7 + m))/(c*(7 + m)) + (3*a^2*b*(c*x)^(8 + m))/(c^2*(8 + m)) +
 (3*a*b^2*(c*x)^(9 + m))/(c^3*(9 + m)) + (b^3*(c*x)^(10 + m))/(c^4*(10 + m
)))/c^6
```

Defintions of rubi rules used

```
Int[(u_.)*(Px_)^(p_.)*((e_.)*(x_))^(m_.), x_Symbol] :> With[{r = Expon[Px,
x, Min]}, Simp[1/e^(p*r)   Int[u*(e*x)^(m + p*r)*ExpandToSum[Px/x^r, x]^p,
x], x] /; IGtQ[r, 0]] /; FreeQ[{e, m}, x] && PolyQ[Px, x] && IntegerQ[p] &&
  !MonomialQ[Px, x]
```

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && LeQ[7*m + 4*n + 4, 0])
|| LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 172 vs. $2(81) = 162$.

Time = 0.36 (sec) , antiderivative size = 173, normalized size of antiderivative = 2.14

method	result
gosper	$\frac{(cx)^m (b^3m^3x^3+3a\,b^2m^3x^2+24b^3m^2x^3+3a^2b\,m^3x+75a\,b^2m^2x^2+191m\,x^3b^3+a^3m^3+78a^2b\,m^2x+618a\,b^2m\,x^2+504b^3x^3+}{(10+m)(9+m)(8+m)(7+m)}$
risch	$\frac{(cx)^m (b^3m^3x^3+3a\,b^2m^3x^2+24b^3m^2x^3+3a^2b\,m^3x+75a\,b^2m^2x^2+191m\,x^3b^3+a^3m^3+78a^2b\,m^2x+618a\,b^2m\,x^2+504b^3x^3+}{(10+m)(9+m)(8+m)(7+m)}$
orering	$\frac{(b^3m^3x^3+3a\,b^2m^3x^2+24b^3m^2x^3+3a^2b\,m^3x+75a\,b^2m^2x^2+191m\,x^3b^3+a^3m^3+78a^2b\,m^2x+618a\,b^2m\,x^2+504b^3x^3+27a^3m}{(10+m)(9+m)(8+m)(7+m)(bx+a)^3}$
parallelrisch	$x^{10}(cx)^m b^3m^3+24x^{10}(cx)^m b^3m^2+3x^9(cx)^m a\,b^2m^3+191x^{10}(cx)^m b^3m+75x^9(cx)^m a\,b^2m^2+3x^8(cx)^m a^2b\,m^3+504x^{10}(cx)$

```
int((c*x)^m*(b*x^3+a*x^2)^3,x,method=_RETURNVERBOSE)
```

```
(c*x)^m*(b^3*m^3*x^3+3*a*b^2*m^3*x^2+24*b^3*m^2*x^3+3*a^2*b*m^3*x+75*a*b^2
*m^2*x^2+191*b^3*m*x^3+a^3*m^3+78*a^2*b*m^2*x+618*a*b^2*m*x^2+504*b^3*x^3+
27*a^3*m^2+669*a^2*b*m*x+1680*a*b^2*x^2+242*a^3*m+1890*a^2*b*x+720*a^3)*x^
7/(10+m)/(9+m)/(8+m)/(7+m)
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 161, normalized size of antiderivative = 1.99

$$\int (cx)^m \left(ax^2+bx^3\right)^3 \, dx$$
$$= \frac{((b^3m^3 + 24\,b^3m^2 + 191\,b^3m + 504\,b^3)x^{10} + 3\,(ab^2m^3 + 25\,ab^2m^2 + 206\,ab^2m + 560\,ab^2)x^9 + 3\,(a^2bm^3 +}{m^4 + 34\,m^3 + 431\,m^2 + 2414}$$

```
integrate((c*x)^m*(b*x^3+a*x^2)^3,x, algorithm="fricas")
```

```
((b^3*m^3 + 24*b^3*m^2 + 191*b^3*m + 504*b^3)*x^10 + 3*(a*b^2*m^3 + 25*a*b
^2*m^2 + 206*a*b^2*m + 560*a*b^2)*x^9 + 3*(a^2*b*m^3 + 26*a^2*b*m^2 + 223*
a^2*b*m + 630*a^2*b)*x^8 + (a^3*m^3 + 27*a^3*m^2 + 242*a^3*m + 720*a^3)*x^
7)*(c*x)^m/(m^4 + 34*m^3 + 431*m^2 + 2414*m + 5040)
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 711 vs. 2(73) = 146.

Time = 0.53 (sec) , antiderivative size = 711, normalized size of antiderivative = 8.78

$$\int (cx)^m \left(ax^2+bx^3\right)^3 \, dx = \text{Too large to display}$$

```
integrate((c*x)**m*(b*x**3+a*x**2)**3,x)
```

```
Piecewise(((-a**3/(3*x**3) - 3*a**2*b/(2*x**2) - 3*a*b**2/x + b**3*log(x))
/c**10, Eq(m, -10)), ((-a**3/(2*x**2) - 3*a**2*b/x + 3*a*b**2*log(x) + b**
3*x)/c**9, Eq(m, -9)), ((-a**3/x + 3*a**2*b*log(x) + 3*a*b**2*x + b**3*x**
2/2)/c**8, Eq(m, -8)), ((a**3*log(x) + 3*a**2*b*x + 3*a*b**2*x**2/2 + b**3
*x**3/3)/c**7, Eq(m, -7)), (a**3*m**3*x**7*(c*x)**m/(m**4 + 34*m**3 + 431*
m**2 + 2414*m + 5040) + 27*a**3*m**2*x**7*(c*x)**m/(m**4 + 34*m**3 + 431*m
**2 + 2414*m + 5040) + 242*a**3*m*x**7*(c*x)**m/(m**4 + 34*m**3 + 431*m**2
 + 2414*m + 5040) + 720*a**3*x**7*(c*x)**m/(m**4 + 34*m**3 + 431*m**2 + 24
14*m + 5040) + 3*a**2*b*m**3*x**8*(c*x)**m/(m**4 + 34*m**3 + 431*m**2 + 24
14*m + 5040) + 78*a**2*b*m**2*x**8*(c*x)**m/(m**4 + 34*m**3 + 431*m**2 + 2
414*m + 5040) + 669*a**2*b*m*x**8*(c*x)**m/(m**4 + 34*m**3 + 431*m**2 + 24
14*m + 5040) + 1890*a**2*b*x**8*(c*x)**m/(m**4 + 34*m**3 + 431*m**2 + 2414
*m + 5040) + 3*a*b**2*m**3*x**9*(c*x)**m/(m**4 + 34*m**3 + 431*m**2 + 2414
*m + 5040) + 75*a*b**2*m**2*x**9*(c*x)**m/(m**4 + 34*m**3 + 431*m**2 + 241
4*m + 5040) + 618*a*b**2*m*x**9*(c*x)**m/(m**4 + 34*m**3 + 431*m**2 + 2414
*m + 5040) + 1680*a*b**2*x**9*(c*x)**m/(m**4 + 34*m**3 + 431*m**2 + 2414*m
 + 5040) + b**3*m**3*x**10*(c*x)**m/(m**4 + 34*m**3 + 431*m**2 + 2414*m +
5040) + 24*b**3*m**2*x**10*(c*x)**m/(m**4 + 34*m**3 + 431*m**2 + 2414*m +
5040) + 191*b**3*m*x**10*(c*x)**m/(m**4 + 34*m**3 + 431*m**2 + 2414*m + 50
40) + 504*b**3*x**10*(c*x)**m/(m**4 + 34*m**3 + 431*m**2 + 2414*m + 504...
```

Maxima [A] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 77, normalized size of antiderivative = 0.95

$$\int (cx)^m \left(ax^2+bx^3\right)^3 \, dx = \frac{b^3c^mx^{10}x^m}{m+10} + \frac{3\,ab^2c^mx^9x^m}{m+9} + \frac{3\,a^2bc^mx^8x^m}{m+8} + \frac{a^3c^mx^7x^m}{m+7}$$

```
integrate((c*x)^m*(b*x^3+a*x^2)^3,x, algorithm="maxima")
```

```
b^3*c^m*x^10*x^m/(m + 10) + 3*a*b^2*c^m*x^9*x^m/(m + 9) + 3*a^2*b*c^m*x^8*
x^m/(m + 8) + a^3*c^m*x^7*x^m/(m + 7)
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 264 vs. $2(81) = 162$.

Time = 0.12 (sec) , antiderivative size = 264, normalized size of antiderivative = 3.26

$$\int (cx)^m \left(ax^2+bx^3\right)^3 dx$$
$$= \frac{(cx)^m b^3m^3x^{10} + 3(cx)^m ab^2m^3x^9 + 24(cx)^m b^3m^2x^{10} + 3(cx)^m a^2bm^3x^8 + 75(cx)^m ab^2m^2x^9 + 191(cx}{}$$

```
integrate((c*x)^m*(b*x^3+a*x^2)^3,x, algorithm="giac")
```

```
((c*x)^m*b^3*m^3*x^10 + 3*(c*x)^m*a*b^2*m^3*x^9 + 24*(c*x)^m*b^3*m^2*x^10
+ 3*(c*x)^m*a^2*b*m^3*x^8 + 75*(c*x)^m*a*b^2*m^2*x^9 + 191*(c*x)^m*b^3*m*x
^10 + (c*x)^m*a^3*m^3*x^7 + 78*(c*x)^m*a^2*b*m^2*x^8 + 618*(c*x)^m*a*b^2*m
*x^9 + 504*(c*x)^m*b^3*x^10 + 27*(c*x)^m*a^3*m^2*x^7 + 669*(c*x)^m*a^2*b*m
*x^8 + 1680*(c*x)^m*a*b^2*x^9 + 242*(c*x)^m*a^3*m*x^7 + 1890*(c*x)^m*a^2*b
*x^8 + 720*(c*x)^m*a^3*x^7)/(m^4 + 34*m^3 + 431*m^2 + 2414*m + 5040)
```

Mupad [B] (verification not implemented)

Time = 9.07 (sec) , antiderivative size = 171, normalized size of antiderivative = 2.11

$$\int (cx)^m \left(ax^2+bx^3\right)^3 dx = (c\,x)^m \left(\frac{a^3\,x^7\,(m^3+27\,m^2+242\,m+720)}{m^4+34\,m^3+431\,m^2+2414\,m+5040} + \frac{b^3\,x^{10}\,(m^3+24\,m^2+191\,m+504)}{m^4+34\,m^3+431\,m^2+2414\,m+5040} + \frac{3\,a\,b^2\,x^9\,(m^3+25\,m^2+206\,m+560)}{m^4+34\,m^3+431\,m^2+2414\,m+5040} + \frac{3\,a^2\,b\,x^8\,(m^3+26\,m^2+223\,m+630)}{m^4+34\,m^3+431\,m^2+2414\,m+5040}\right)$$

```
int((c*x)^m*(a*x^2 + b*x^3)^3,x)
```

```
(c*x)^m*((a^3*x^7*(242*m + 27*m^2 + m^3 + 720))/(2414*m + 431*m^2 + 34*m^3
 + m^4 + 5040) + (b^3*x^10*(191*m + 24*m^2 + m^3 + 504))/(2414*m + 431*m^2
 + 34*m^3 + m^4 + 5040) + (3*a*b^2*x^9*(206*m + 25*m^2 + m^3 + 560))/(2414
*m + 431*m^2 + 34*m^3 + m^4 + 5040) + (3*a^2*b*x^8*(223*m + 26*m^2 + m^3 +
 630))/(2414*m + 431*m^2 + 34*m^3 + m^4 + 5040))
```

Reduce [B] (verification not implemented)

Time = 0.47 (sec) , antiderivative size = 173, normalized size of antiderivative = 2.14

$$\int (cx)^m \left(ax^2+bx^3\right)^3 \, dx$$
$$= \frac{x^m c^m x^7 (b^3 m^3 x^3 + 3a\,b^2 m^3 x^2 + 24 b^3 m^2 x^3 + 3a^2 b\,m^3 x + 75a\,b^2 m^2 x^2 + 191 b^3 m\,x^3 + a^3 m^3 + 78 a^2 b\,m^2 x +}{m^4 + 34m^3 + 431m^2 +}$$

```
int((c*x)^m*(b*x^3+a*x^2)^3,x)
```

```
(x**m*c**m*x**7*(a**3*m**3 + 27*a**3*m**2 + 242*a**3*m + 720*a**3 + 3*a**2
*b*m**3*x + 78*a**2*b*m**2*x + 669*a**2*b*m*x + 1890*a**2*b*x + 3*a*b**2*m
**3*x**2 + 75*a*b**2*m**2*x**2 + 618*a*b**2*m*x**2 + 1680*a*b**2*x**2 + b*
*3*m**3*x**3 + 24*b**3*m**2*x**3 + 191*b**3*m*x**3 + 504*b**3*x**3))/(m**4
 + 34*m**3 + 431*m**2 + 2414*m + 5040)
```

3.408 $\int (cx)^m \left(ax^2+bx^3\right)^2 \, dx$

Optimal result

Integrand size = 19, antiderivative size = 58

$$\int (cx)^m \left(ax^2+bx^3\right)^2 \, dx = \frac{a^2(cx)^{5+m}}{c^5(5+m)}+\frac{2ab(cx)^{6+m}}{c^6(6+m)}+\frac{b^2(cx)^{7+m}}{c^7(7+m)}$$

```
a^2*(c*x)^(5+m)/c^5/(5+m)+2*a*b*(c*x)^(6+m)/c^6/(6+m)+b^2*(c*x)^(7+m)/c^7/
(7+m)
```

Mathematica [A] (verified)

Time = 0.05 (sec) , antiderivative size = 41, normalized size of antiderivative = 0.71

$$\int (cx)^m \left(ax^2+bx^3\right)^2 \, dx = x^5(cx)^m \left(\frac{a^2}{5+m}+\frac{2abx}{6+m}+\frac{b^2x^2}{7+m}\right)$$

```
Integrate[(c*x)^m*(a*x^2 + b*x^3)^2,x]
```

```
x^5*(c*x)^m*(a^2/(5 + m) + (2*a*b*x)/(6 + m) + (b^2*x^2)/(7 + m))
```

Rubi [A] (verified)

Time = 0.35 (sec) , antiderivative size = 62, normalized size of antiderivative = 1.07, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.158$, Rules used = {9, 53, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \left(ax^2+bx^3\right)^2 (cx)^m \, dx$$

$$\Big\downarrow \; 9$$

$$\frac{\int (cx)^{m+4}(a+bx)^2dx}{c^4}$$

$$\Big\downarrow \; 53$$

$$\frac{\int \left(a^2(cx)^{m+4}+\frac{2ab(cx)^{m+5}}{c}+\frac{b^2(cx)^{m+6}}{c^2}\right)dx}{c^4}$$

$$\Big\downarrow \; 2009$$

$$\frac{\frac{a^2(cx)^{m+5}}{c(m+5)}+\frac{2ab(cx)^{m+6}}{c^2(m+6)}+\frac{b^2(cx)^{m+7}}{c^3(m+7)}}{c^4}$$

```
Int[(c*x)^m*(a*x^2 + b*x^3)^2,x]
```

```
((a^2*(c*x)^(5 + m))/(c*(5 + m)) + (2*a*b*(c*x)^(6 + m))/(c^2*(6 + m)) + (
b^2*(c*x)^(7 + m))/(c^3*(7 + m)))/c^4
```

Defintions of rubi rules used

```
Int[(u_.)*(Px_)^(p_.)*((e_.)*(x_))^(m_.), x_Symbol] :> With[{r = Expon[Px,
x, Min]}, Simp[1/e^(p*r)   Int[u*(e*x)^(m + p*r)*ExpandToSum[Px/x^r, x]^p,
x], x] /; IGtQ[r, 0]] /; FreeQ[{e, m}, x] && PolyQ[Px, x] && IntegerQ[p] &&
  !MonomialQ[Px, x]
```

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && LeQ[7*m + 4*n + 4, 0])
|| LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 0.32 (sec) , antiderivative size = 59, normalized size of antiderivative = 1.02

method	result
norman	$\frac{a^{2} x^{5} {\mathrm e}^{m \ln \left(c x \right)}}{5+m}+\frac{b^{2} x^{7} {\mathrm e}^{m \ln \left(c x \right)}}{7+m}+\frac{2 a b \,x^{6} {\mathrm e}^{m \ln \left(c x \right)}}{6+m}$
gosper	$\frac{\left(c x \right)^{m} \left(b^{2} m^{2} x^{2}+2 a b \,m^{2} x +11 m \,x^{2} b^{2}+a^{2} m^{2}+24 m x a b +30 b^{2} x^{2}+13 a^{2} m +70 a b x +42 a^{2}\right) x^{5}}{\left(7+m \right) \left(6+m \right) \left(5+m \right)}$
risch	$\frac{\left(c x \right)^{m} \left(b^{2} m^{2} x^{2}+2 a b \,m^{2} x +11 m \,x^{2} b^{2}+a^{2} m^{2}+24 m x a b +30 b^{2} x^{2}+13 a^{2} m +70 a b x +42 a^{2}\right) x^{5}}{\left(7+m \right) \left(6+m \right) \left(5+m \right)}$
orering	$\frac{\left(b^{2} m^{2} x^{2}+2 a b \,m^{2} x +11 m \,x^{2} b^{2}+a^{2} m^{2}+24 m x a b +30 b^{2} x^{2}+13 a^{2} m +70 a b x +42 a^{2}\right) x \left(c x \right)^{m} \left(b \,x^{3}+a \,x^{2}\right)^{2}}{\left(7+m \right) \left(6+m \right) \left(5+m \right) \left(b x +a \right)^{2}}$
parallelrisch	$\frac{x^{7} \left(c x \right)^{m} b^{2} m^{2}+11 x^{7} \left(c x \right)^{m} b^{2} m +2 x^{6} \left(c x \right)^{m} a b \,m^{2}+30 x^{7} \left(c x \right)^{m} b^{2}+24 x^{6} \left(c x \right)^{m} a b m +x^{5} \left(c x \right)^{m} a^{2} m^{2}+70 x^{6} \left(c x \right)^{m} a b +13 x^{5} \left(c x \right)^{m}}{\left(7+m \right) \left(6+m \right) \left(5+m \right)}$

```
int((c*x)^m*(b*x^3+a*x^2)^2,x,method=_RETURNVERBOSE)
```

```
a^2/(5+m)*x^5*exp(m*ln(c*x))+b^2/(7+m)*x^7*exp(m*ln(c*x))+2*a*b/(6+m)*x^6*
exp(m*ln(c*x))
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 89, normalized size of antiderivative = 1.53

$$\int (cx)^m \left(ax^2+bx^3\right)^2 \, dx$$
$$= \frac{((b^2m^2+11\,b^2m+30\,b^2)x^7+2\,(abm^2+12\,abm+35\,ab)x^6+(a^2m^2+13\,a^2m+42\,a^2)x^5)(cx)^m}{m^3+18\,m^2+107\,m+210}$$

```
integrate((c*x)^m*(b*x^3+a*x^2)^2,x, algorithm="fricas")
```

```
((b^2*m^2 + 11*b^2*m + 30*b^2)*x^7 + 2*(a*b*m^2 + 12*a*b*m + 35*a*b)*x^6 +
 (a^2*m^2 + 13*a^2*m + 42*a^2)*x^5)*(c*x)^m/(m^3 + 18*m^2 + 107*m + 210)
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 330 vs. $2(51) = 102$.

Time = 0.35 (sec) , antiderivative size = 330, normalized size of antiderivative = 5.69

$$\int (cx)^m \left(ax^2 + bx^3\right)^2 \, dx$$
$$= \begin{cases} \frac{-\frac{a^2}{2x^2} - \frac{2ab}{x} + b^2 \log(x)}{c^7} \\ \frac{-\frac{a^2}{x} + 2ab \log(x) + b^2 x}{c^6} \\ \frac{a^2 \log(x) + 2abx + \frac{b^2 x^2}{2}}{c^5} \\ \frac{a^2 m^2 x^5 (cx)^m}{m^3 + 18m^2 + 107m + 210} + \frac{13a^2 m x^5 (cx)^m}{m^3 + 18m^2 + 107m + 210} + \frac{42a^2 x^5 (cx)^m}{m^3 + 18m^2 + 107m + 210} + \frac{2abm^2 x^6 (cx)^m}{m^3 + 18m^2 + 107m + 210} + \frac{24abm x^6 (cx)^m}{m^3 + 18m^2 + 107m + 210} + \end{cases}$$

```
integrate((c*x)**m*(b*x**3+a*x**2)**2,x)
```

```
Piecewise(((-a**2/(2*x**2) - 2*a*b/x + b**2*log(x))/c**7, Eq(m, -7)), ((-a
**2/x + 2*a*b*log(x) + b**2*x)/c**6, Eq(m, -6)), ((a**2*log(x) + 2*a*b*x +
 b**2*x**2/2)/c**5, Eq(m, -5)), (a**2*m**2*x**5*(c*x)**m/(m**3 + 18*m**2 +
 107*m + 210) + 13*a**2*m*x**5*(c*x)**m/(m**3 + 18*m**2 + 107*m + 210) + 4
2*a**2*x**5*(c*x)**m/(m**3 + 18*m**2 + 107*m + 210) + 2*a*b*m**2*x**6*(c*x
)**m/(m**3 + 18*m**2 + 107*m + 210) + 24*a*b*m*x**6*(c*x)**m/(m**3 + 18*m*
*2 + 107*m + 210) + 70*a*b*x**6*(c*x)**m/(m**3 + 18*m**2 + 107*m + 210) +
b**2*m**2*x**7*(c*x)**m/(m**3 + 18*m**2 + 107*m + 210) + 11*b**2*m*x**7*(c
*x)**m/(m**3 + 18*m**2 + 107*m + 210) + 30*b**2*x**7*(c*x)**m/(m**3 + 18*m
**2 + 107*m + 210), True))
```

Maxima [A] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 55, normalized size of antiderivative = 0.95

$$\int (cx)^m \left(ax^2+bx^3\right)^2 \, dx = \frac{b^2c^m x^7 x^m}{m+7} + \frac{2\,abc^m x^6 x^m}{m+6} + \frac{a^2c^m x^5 x^m}{m+5}$$

```
integrate((c*x)^m*(b*x^3+a*x^2)^2,x, algorithm="maxima")
```

```
b^2*c^m*x^7*x^m/(m + 7) + 2*a*b*c^m*x^6*x^m/(m + 6) + a^2*c^m*x^5*x^m/(m +
 5)
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 141 vs. $2(58) = 116$.

Time = 0.29 (sec) , antiderivative size = 141, normalized size of antiderivative = 2.43

$$\int (cx)^m \left(ax^2+bx^3\right)^2 \, dx$$
$$= \frac{(cx)^m\, b^2m^2x^7 + 2\,(cx)^m\, abm^2x^6 + 11\,(cx)^m\, b^2mx^7 + (cx)^m\, a^2m^2x^5 + 24\,(cx)^m\, abmx^6 + 30\,(cx)^m\, b^2x^7 +}{m^3 + 18\,m^2 + 107\,m + 210}$$

```
integrate((c*x)^m*(b*x^3+a*x^2)^2,x, algorithm="giac")
```

```
((c*x)^m*b^2*m^2*x^7 + 2*(c*x)^m*a*b*m^2*x^6 + 11*(c*x)^m*b^2*m*x^7 + (c*x
)^m*a^2*m^2*x^5 + 24*(c*x)^m*a*b*m*x^6 + 30*(c*x)^m*b^2*x^7 + 13*(c*x)^m*a
^2*m*x^5 + 70*(c*x)^m*a*b*x^6 + 42*(c*x)^m*a^2*x^5)/(m^3 + 18*m^2 + 107*m
 + 210)
```

Mupad [B] (verification not implemented)

Time = 8.82 (sec) , antiderivative size = 97, normalized size of antiderivative = 1.67

$$\int (cx)^m \left(ax^2+bx^3\right)^2 \, dx = (c\,x)^m \left(\frac{a^2\,x^5\,\left(m^2+13\,m+42\right)}{m^3+18\,m^2+107\,m+210} + \frac{b^2\,x^7\,\left(m^2+11\,m+30\right)}{m^3+18\,m^2+107\,m+210} + \frac{2\,a\,b\,x^6\,\left(m^2+12\,m+35\right)}{m^3+18\,m^2+107\,m+210}\right)$$

```
int((c*x)^m*(a*x^2 + b*x^3)^2,x)
```

```
(c*x)^m*((a^2*x^5*(13*m + m^2 + 42))/(107*m + 18*m^2 + m^3 + 210) + (b^2*x
^7*(11*m + m^2 + 30))/(107*m + 18*m^2 + m^3 + 210) + (2*a*b*x^6*(12*m + m^
2 + 35))/(107*m + 18*m^2 + m^3 + 210))
```

Reduce [B] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 90, normalized size of antiderivative = 1.55

$$\int (cx)^m \left(ax^2+bx^3\right)^2 \, dx = \frac{x^m c^m x^5 (b^2 m^2 x^2 + 2ab\,m^2 x + 11b^2 m\,x^2 + a^2 m^2 + 24abmx + 30b^2 x^2 + 13a^2 m + 70abx + 42a^2)}{m^3+18m^2+107m+210}$$

```
int((c*x)^m*(b*x^3+a*x^2)^2,x)
```

```
(x**m*c**m*x**5*(a**2*m**2 + 13*a**2*m + 42*a**2 + 2*a*b*m**2*x + 24*a*b*m
*x + 70*a*b*x + b**2*m**2*x**2 + 11*b**2*m*x**2 + 30*b**2*x**2))/(m**3 + 1
8*m**2 + 107*m + 210)
```

3.409 $\int (cx)^m \left(ax^2+bx^3\right) dx$

Optimal result

Integrand size = 17, antiderivative size = 35

$$\int (cx)^m \left(ax^2+bx^3\right) dx = \frac{a(cx)^{3+m}}{c^3(3+m)}+\frac{b(cx)^{4+m}}{c^4(4+m)}$$

```
a*(c*x)^(3+m)/c^3/(3+m)+b*(c*x)^(4+m)/c^4/(4+m)
```

Mathematica [A] (verified)

Time = 0.04 (sec) , antiderivative size = 25, normalized size of antiderivative = 0.71

$$\int (cx)^m \left(ax^2+bx^3\right) dx = x^3(cx)^m\left(\frac{a}{3+m}+\frac{bx}{4+m}\right)$$

```
Integrate[(c*x)^m*(a*x^2 + b*x^3),x]
```

```
x^3*(c*x)^m*(a/(3 + m) + (b*x)/(4 + m))
```

Rubi [A] (verified)

Time = 0.30 (sec) , antiderivative size = 39, normalized size of antiderivative = 1.11, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.176, Rules used = {9, 53, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \left(ax^2+bx^3\right)(cx)^m\,dx$$

$$\Big\downarrow \ 9$$

$$\frac{\int (cx)^{m+2}(a+bx)dx}{c^2}$$

$$\Big\downarrow \ 53$$

$$\frac{\int \left(a(cx)^{m+2}+\frac{b(cx)^{m+3}}{c}\right)dx}{c^2}$$

$$\Big\downarrow \ 2009$$

$$\frac{\frac{a(cx)^{m+3}}{c(m+3)}+\frac{b(cx)^{m+4}}{c^2(m+4)}}{c^2}$$

```
Int[(c*x)^m*(a*x^2 + b*x^3),x]
```

```
((a*(c*x)^(3 + m))/(c*(3 + m)) + (b*(c*x)^(4 + m))/(c^2*(4 + m)))/c^2
```

Defintions of rubi rules used

```
Int[(u_.)*(Px_)^(p_.)*((e_.)*(x_))^(m_.), x_Symbol] :> With[{r = Expon[Px,
x, Min]}, Simp[1/e^(p*r)   Int[u*(e*x)^(m + p*r)*ExpandToSum[Px/x^r, x]^p,
x], x] /; IGtQ[r, 0]] /; FreeQ[{e, m}, x] && PolyQ[Px, x] && IntegerQ[p] &&
  !MonomialQ[Px, x]
```

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && LeQ[7*m + 4*n + 4, 0])
|| LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 0.05 (sec) , antiderivative size = 35, normalized size of antiderivative = 1.00

method	result	size
gosper	$\frac{(cx)^{m}(bmx+am+3bx+4a)x^{3}}{(4+m)(3+m)}$	35
risch	$\frac{(cx)^{m}(bmx+am+3bx+4a)x^{3}}{(4+m)(3+m)}$	35
norman	$\frac{a\,x^{3}{\mathrm e}^{m\ln(cx)}}{3+m}+\frac{b\,x^{4}{\mathrm e}^{m\ln(cx)}}{4+m}$	36
orering	$\frac{(bmx+am+3bx+4a)x(cx)^{m}(b\,x^{3}+a\,x^{2})}{(4+m)(3+m)(bx+a)}$	51
parallelrisch	$\frac{x^{4}(cx)^{m}bm+3x^{4}(cx)^{m}b+x^{3}(cx)^{m}am+4x^{3}(cx)^{m}a}{(4+m)(3+m)}$	57

```
int((c*x)^m*(b*x^3+a*x^2),x,method=_RETURNVERBOSE)
```

```
(c*x)^m*(b*m*x+a*m+3*b*x+4*a)*x^3/(4+m)/(3+m)
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 39, normalized size of antiderivative = 1.11

$$\int (cx)^m\left(ax^2+bx^3\right)\,dx=\frac{((bm+3\,b)x^4+(am+4\,a)x^3)(cx)^m}{m^2+7\,m+12}$$

```
integrate((c*x)^m*(b*x^3+a*x^2),x, algorithm="fricas")
```

```
((b*m + 3*b)*x^4 + (a*m + 4*a)*x^3)*(c*x)^m/(m^2 + 7*m + 12)
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 105 vs. $2(29) = 58$.

Time = 0.23 (sec) , antiderivative size = 105, normalized size of antiderivative = 3.00

$$\int (cx)^m \left(ax^2+bx^3\right) dx = \begin{cases} \frac{-\frac{a}{x}+b\log(x)}{c^4} & \text{for } m=-4 \\ \frac{a\log(x)+bx}{c^3} & \text{for } m=-3 \\ \frac{amx^3(cx)^m}{m^2+7m+12} + \frac{4ax^3(cx)^m}{m^2+7m+12} + \frac{bmx^4(cx)^m}{m^2+7m+12} + \frac{3bx^4(cx)^m}{m^2+7m+12} & \text{otherwise} \end{cases}$$

```
integrate((c*x)**m*(b*x**3+a*x**2),x)
```

```
Piecewise(((-a/x + b*log(x))/c**4, Eq(m, -4)), ((a*log(x) + b*x)/c**3, Eq(
m, -3)), (a*m*x**3*(c*x)**m/(m**2 + 7*m + 12) + 4*a*x**3*(c*x)**m/(m**2 +
7*m + 12) + b*m*x**4*(c*x)**m/(m**2 + 7*m + 12) + 3*b*x**4*(c*x)**m/(m**2
+ 7*m + 12), True))
```

Maxima [A] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 33, normalized size of antiderivative = 0.94

$$\int (cx)^m \left(ax^2+bx^3\right) dx = \frac{bc^m x^4 x^m}{m+4} + \frac{ac^m x^3 x^m}{m+3}$$

```
integrate((c*x)^m*(b*x^3+a*x^2),x, algorithm="maxima")
```

```
b*c^m*x^4*x^m/(m + 4) + a*c^m*x^3*x^m/(m + 3)
```

Giac [A] (verification not implemented)

Time = 0.22 (sec) , antiderivative size = 56, normalized size of antiderivative = 1.60

$$\int (cx)^m \left(ax^2+bx^3\right) \, dx = \frac{(cx)^m bmx^4 + (cx)^m amx^3 + 3\,(cx)^m bx^4 + 4\,(cx)^m ax^3}{m^2+7\,m+12}$$

```
integrate((c*x)^m*(b*x^3+a*x^2),x, algorithm="giac")
```

```
((c*x)^m*b*m*x^4 + (c*x)^m*a*m*x^3 + 3*(c*x)^m*b*x^4 + 4*(c*x)^m*a*x^3)/(m
^2 + 7*m + 12)
```

Mupad [B] (verification not implemented)

Time = 8.99 (sec) , antiderivative size = 34, normalized size of antiderivative = 0.97

$$\int (cx)^m \left(ax^2+bx^3\right) \, dx = \frac{x^3\,{(c\,x)}^m\,(4\,a+a\,m+3\,b\,x+b\,m\,x)}{m^2+7\,m+12}$$

```
int((c*x)^m*(a*x^2 + b*x^3),x)
```

```
(x^3*(c*x)^m*(4*a + a*m + 3*b*x + b*m*x))/(7*m + m^2 + 12)
```

Reduce [B] (verification not implemented)

Time = 0.24 (sec) , antiderivative size = 35, normalized size of antiderivative = 1.00

$$\int (cx)^m \left(ax^2+bx^3\right) \, dx = \frac{x^m c^m x^3 (bmx+am+3bx+4a)}{m^2+7m+12}$$

```
int((c*x)^m*(b*x^3+a*x^2),x)
```

```
(x**m*c**m*x**3*(a*m + 4*a + b*m*x + 3*b*x))/(m**2 + 7*m + 12)
```

3.410 $\int \frac{(cx)^m}{ax^2+bx^3} \, dx$

Optimal result

Integrand size = 19, antiderivative size = 33

$$\int \frac{(cx)^m}{ax^2+bx^3} \, dx = -\frac{c(cx)^{-1+m} \text{Hypergeometric2F1}\left(1,-1+m,m,-\frac{bx}{a}\right)}{a(1-m)}$$

```
-c*(c*x)^(-1+m)*hypergeom([1, -1+m],[m],-b*x/a)/a/(1-m)
```

Mathematica [A] (verified)

Time = 0.04 (sec) , antiderivative size = 30, normalized size of antiderivative = 0.91

$$\int \frac{(cx)^m}{ax^2+bx^3} \, dx = \frac{(cx)^m \text{Hypergeometric2F1}\left(1,-1+m,m,-\frac{bx}{a}\right)}{a(-1+m)x}$$

```
Integrate[(c*x)^m/(a*x^2 + b*x^3),x]
```

```
((c*x)^m*Hypergeometric2F1[1, -1 + m, m, -((b*x)/a)])/(a*(-1 + m)*x)
```

Rubi [A] (verified)

Time = 0.26 (sec) , antiderivative size = 33, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.105$, Rules used = {9, 74}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(cx)^m}{ax^2+bx^3}\,dx$$

$$\Big\downarrow \; 9$$

$$c^2\int \frac{(cx)^{m-2}}{a+bx}dx$$

$$\Big\downarrow \; 74$$

$$-\frac{c(cx)^{m-1}\,\text{Hypergeometric2F1}\left(1,m-1,m,-\frac{bx}{a}\right)}{a(1-m)}$$

```
Int[(c*x)^m/(a*x^2 + b*x^3),x]
```

```
-((c*(c*x)^(-1 + m)*Hypergeometric2F1[1, -1 + m, m, -((b*x)/a)])/(a*(1 - m
)))
```

Defintions of rubi rules used

```
Int[(u_.)*(Px_)^(p_.)*((e_.)*(x_))^(m_.), x_Symbol] :> With[{r = Expon[Px,
x, Min]}, Simp[1/e^(p*r)   Int[u*(e*x)^(m + p*r)*ExpandToSum[Px/x^r, x]^p,
x], x] /; IGtQ[r, 0]] /; FreeQ[{e, m}, x] && PolyQ[Px, x] && IntegerQ[p] &&
  !MonomialQ[Px, x]
```

```
Int[((b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[c^n*((b*x
)^(m + 1)/(b*(m + 1)))*Hypergeometric2F1[-n, m + 1, m + 2, (-d)*(x/c)], x]
/; FreeQ[{b, c, d, m, n}, x] &&  !IntegerQ[m] && (IntegerQ[n] || (GtQ[c, 0]
 &&  !(EqQ[n, -2^(-1)] && EqQ[c^2 - d^2, 0] && GtQ[-d/(b*c), 0])))
```

Maple [F]

$$\int \frac{(cx)^m}{b\,x^3+a\,x^2}dx$$

```
int((c*x)^m/(b*x^3+a*x^2),x)
```

```
int((c*x)^m/(b*x^3+a*x^2),x)
```

Fricas [F]

$$\int \frac{(cx)^m}{ax^2+bx^3}\,dx=\int \frac{(cx)^m}{bx^3+ax^2}\,dx$$

```
integrate((c*x)^m/(b*x^3+a*x^2),x, algorithm="fricas")
```

```
integral((c*x)^m/(b*x^3 + a*x^2), x)
```

Sympy [F]

$$\int \frac{(cx)^m}{ax^2+bx^3}\,dx=\int \frac{(cx)^m}{x^2\,(a+bx)}\,dx$$

```
integrate((c*x)**m/(b*x**3+a*x**2),x)
```

```
Integral((c*x)**m/(x**2*(a + b*x)), x)
```

Maxima [F]

$$\int \frac{(cx)^m}{ax^2 + bx^3} \, dx = \int \frac{(cx)^m}{bx^3 + ax^2} \, dx$$

```
integrate((c*x)^m/(b*x^3+a*x^2),x, algorithm="maxima")
```

```
integrate((c*x)^m/(b*x^3 + a*x^2), x)
```

Giac [F]

$$\int \frac{(cx)^m}{ax^2 + bx^3} \, dx = \int \frac{(cx)^m}{bx^3 + ax^2} \, dx$$

```
integrate((c*x)^m/(b*x^3+a*x^2),x, algorithm="giac")
```

```
integrate((c*x)^m/(b*x^3 + a*x^2), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(cx)^m}{ax^2 + bx^3} \, dx = \int \frac{(c\,x)^m}{b\,x^3 + a\,x^2} \, dx$$

```
int((c*x)^m/(a*x^2 + b*x^3),x)
```

```
int((c*x)^m/(a*x^2 + b*x^3), x)
```

Reduce [F]

$$\int \frac{(cx)^m}{ax^2 + bx^3}\,dx = c^m\left(\int \frac{x^m}{b\,x^3 + a\,x^2}dx\right)$$

```
int((c*x)^m/(b*x^3+a*x^2),x)
```

```
c**m*int(x**m/(a*x**2 + b*x**3),x)
```

3.411 $\int \frac{(cx)^m}{(ax^2+bx^3)^2} dx$

Optimal result

Integrand size = 19, antiderivative size = 37

$$\int \frac{(cx)^m}{(ax^2+bx^3)^2} dx = -\frac{c^3(cx)^{-3+m}\text{Hypergeometric2F1}\left(2,-3+m,-2+m,-\frac{bx}{a}\right)}{a^2(3-m)}$$

```
-c^3*(c*x)^(-3+m)*hypergeom([2, -3+m],[-2+m],-b*x/a)/a^2/(3-m)
```

Mathematica [A] (verified)

Time = 0.03 (sec) , antiderivative size = 32, normalized size of antiderivative = 0.86

$$\int \frac{(cx)^m}{(ax^2+bx^3)^2} dx = \frac{(cx)^m\text{Hypergeometric2F1}\left(2,-3+m,-2+m,-\frac{bx}{a}\right)}{a^2(-3+m)x^3}$$

```
Integrate[(c*x)^m/(a*x^2 + b*x^3)^2,x]
```

```
((c*x)^m*Hypergeometric2F1[2, -3 + m, -2 + m, -((b*x)/a)])/(a^2*(-3 + m)*x
^3)
```

Rubi [A] (verified)

Time = 0.26 (sec) , antiderivative size = 37, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.105$, Rules used = {9, 74}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(cx)^m}{(ax^2+bx^3)^2}dx$$
$$\Big\downarrow \; 9$$
$$c^4 \int \frac{(cx)^{m-4}}{(a+bx)^2}dx$$
$$\Big\downarrow \; 74$$
$$-\frac{c^3(cx)^{m-3}\,\text{Hypergeometric2F1}\left(2,m-3,m-2,-\frac{bx}{a}\right)}{a^2(3-m)}$$

```
Int[(c*x)^m/(a*x^2 + b*x^3)^2,x]
```

```
-((c^3*(c*x)^(-3 + m)*Hypergeometric2F1[2, -3 + m, -2 + m, -((b*x)/a)])/(a
^2*(3 - m)))
```

Defintions of rubi rules used

```
Int[(u_.)*(Px_)^(p_.)*((e_.)*(x_))^(m_.), x_Symbol] :> With[{r = Expon[Px,
x, Min]}, Simp[1/e^(p*r)   Int[u*(e*x)^(m + p*r)*ExpandToSum[Px/x^r, x]^p,
x], x] /; IGtQ[r, 0]] /; FreeQ[{e, m}, x] && PolyQ[Px, x] && IntegerQ[p] &&
  !MonomialQ[Px, x]
```

```
Int[((b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[c^n*((b*x
)^(m + 1)/(b*(m + 1)))*Hypergeometric2F1[-n, m + 1, m + 2, (-d)*(x/c)], x]
/; FreeQ[{b, c, d, m, n}, x] &&  !IntegerQ[m] && (IntegerQ[n] || (GtQ[c, 0]
 &&  !(EqQ[n, -2^(-1)] && EqQ[c^2 - d^2, 0] && GtQ[-d/(b*c), 0])))
```

Maple [F]

$$\int \frac{(cx)^m}{(b\,x^3+a\,x^2)^2}dx$$

```
int((c*x)^m/(b*x^3+a*x^2)^2,x)
```

```
int((c*x)^m/(b*x^3+a*x^2)^2,x)
```

Fricas [F]

$$\int \frac{(cx)^m}{(ax^2+bx^3)^2}\,dx = \int \frac{(cx)^m}{(bx^3+ax^2)^2}\,dx$$

```
integrate((c*x)^m/(b*x^3+a*x^2)^2,x, algorithm="fricas")
```

```
integral((c*x)^m/(b^2*x^6 + 2*a*b*x^5 + a^2*x^4), x)
```

Sympy [F]

$$\int \frac{(cx)^m}{(ax^2+bx^3)^2}\,dx = \int \frac{(cx)^m}{x^4\,(a+bx)^2}\,dx$$

```
integrate((c*x)**m/(b*x**3+a*x**2)**2,x)
```

```
Integral((c*x)**m/(x**4*(a + b*x)**2), x)
```

Maxima [F]

$$\int \frac{(cx)^m}{(ax^2+bx^3)^2}\,dx = \int \frac{(cx)^m}{(bx^3+ax^2)^2}\,dx$$

```
integrate((c*x)^m/(b*x^3+a*x^2)^2,x, algorithm="maxima")
```

```
integrate((c*x)^m/(b*x^3 + a*x^2)^2, x)
```

Giac [F]

$$\int \frac{(cx)^m}{(ax^2+bx^3)^2}\,dx = \int \frac{(cx)^m}{(bx^3+ax^2)^2}\,dx$$

```
integrate((c*x)^m/(b*x^3+a*x^2)^2,x, algorithm="giac")
```

```
integrate((c*x)^m/(b*x^3 + a*x^2)^2, x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(cx)^m}{(ax^2+bx^3)^2}\,dx = \int \frac{(c\,x)^m}{(b\,x^3+a\,x^2)^2}\,dx$$

```
int((c*x)^m/(a*x^2 + b*x^3)^2,x)
```

```
int((c*x)^m/(a*x^2 + b*x^3)^2, x)
```

Reduce [F]

$$\int \frac{(cx)^m}{(ax^2+bx^3)^2}\,dx = c^m\left(\int \frac{x^m}{b^2x^6+2ab\,x^5+a^2x^4}dx\right)$$

```
int((c*x)^m/(b*x^3+a*x^2)^2,x)
```

```
c**m*int(x**m/(a**2*x**4 + 2*a*b*x**5 + b**2*x**6),x)
```

3.412 $\int \frac{(cx)^m}{(ax^2+bx^3)^3} \, dx$

Optimal result

Integrand size = 19, antiderivative size = 37

$$\int \frac{(cx)^m}{(ax^2+bx^3)^3} \, dx = -\frac{c^5(cx)^{-5+m} \text{Hypergeometric2F1}\left(3,-5+m,-4+m,-\frac{bx}{a}\right)}{a^3(5-m)}$$

```
-c^5*(c*x)^(-5+m)*hypergeom([3, -5+m],[-4+m],-b*x/a)/a^3/(5-m)
```

Mathematica [A] (verified)

Time = 0.04 (sec) , antiderivative size = 32, normalized size of antiderivative = 0.86

$$\int \frac{(cx)^m}{(ax^2+bx^3)^3} \, dx = \frac{(cx)^m \text{Hypergeometric2F1}\left(3,-5+m,-4+m,-\frac{bx}{a}\right)}{a^3(-5+m)x^5}$$

```
Integrate[(c*x)^m/(a*x^2 + b*x^3)^3,x]
```

```
((c*x)^m*Hypergeometric2F1[3, -5 + m, -4 + m, -((b*x)/a)])/(a^3*(-5 + m)*x
^5)
```

Rubi [A] (verified)

Time = 0.26 (sec) , antiderivative size = 37, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.105, Rules used = {9, 74}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(cx)^m}{(ax^2+bx^3)^3}\,dx$$
$$\Big\downarrow \text{ 9}$$
$$c^6\int \frac{(cx)^{m-6}}{(a+bx)^3}dx$$
$$\Big\downarrow \text{ 74}$$
$$-\frac{c^5(cx)^{m-5}\,\text{Hypergeometric2F1}\left(3,m-5,m-4,-\frac{bx}{a}\right)}{a^3(5-m)}$$

```
Int[(c*x)^m/(a*x^2 + b*x^3)^3,x]
```

```
-((c^5*(c*x)^(-5 + m)*Hypergeometric2F1[3, -5 + m, -4 + m, -((b*x)/a)])/(a
^3*(5 - m)))
```

Defintions of rubi rules used

```
Int[(u_.)*(Px_)^(p_.)*((e_.)*(x_))^(m_.), x_Symbol] :> With[{r = Expon[Px,
x, Min]}, Simp[1/e^(p*r)   Int[u*(e*x)^(m + p*r)*ExpandToSum[Px/x^r, x]^p,
x], x] /; IGtQ[r, 0]] /; FreeQ[{e, m}, x] && PolyQ[Px, x] && IntegerQ[p] &&
  !MonomialQ[Px, x]
```

```
Int[((b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[c^n*((b*x
)^(m + 1)/(b*(m + 1)))*Hypergeometric2F1[-n, m + 1, m + 2, (-d)*(x/c)], x]
/; FreeQ[{b, c, d, m, n}, x] &&  !IntegerQ[m] && (IntegerQ[n] || (GtQ[c, 0]
 &&  !(EqQ[n, -2^(-1)] && EqQ[c^2 - d^2, 0] && GtQ[-d/(b*c), 0])))
```

Maple [F]

$$\int \frac{(cx)^m}{(b\,x^3+a\,x^2)^3}dx$$

```
int((c*x)^m/(b*x^3+a*x^2)^3,x)
```

```
int((c*x)^m/(b*x^3+a*x^2)^3,x)
```

Fricas [F]

$$\int \frac{(cx)^m}{(ax^2+bx^3)^3}\,dx = \int \frac{(cx)^m}{(bx^3+ax^2)^3}\,dx$$

```
integrate((c*x)^m/(b*x^3+a*x^2)^3,x, algorithm="fricas")
```

```
integral((c*x)^m/(b^3*x^9 + 3*a*b^2*x^8 + 3*a^2*b*x^7 + a^3*x^6), x)
```

Sympy [F]

$$\int \frac{(cx)^m}{(ax^2+bx^3)^3}\,dx = \int \frac{(cx)^m}{x^6\,(a+bx)^3}\,dx$$

```
integrate((c*x)**m/(b*x**3+a*x**2)**3,x)
```

```
Integral((c*x)**m/(x**6*(a + b*x)**3), x)
```

Maxima [F]

$$\int \frac{(cx)^m}{(ax^2+bx^3)^3}\,dx = \int \frac{(cx)^m}{(bx^3+ax^2)^3}\,dx$$

```
integrate((c*x)^m/(b*x^3+a*x^2)^3,x, algorithm="maxima")
```

```
integrate((c*x)^m/(b*x^3 + a*x^2)^3, x)
```

Giac [F]

$$\int \frac{(cx)^m}{(ax^2+bx^3)^3}\,dx = \int \frac{(cx)^m}{(bx^3+ax^2)^3}\,dx$$

```
integrate((c*x)^m/(b*x^3+a*x^2)^3,x, algorithm="giac")
```

```
integrate((c*x)^m/(b*x^3 + a*x^2)^3, x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(cx)^m}{(ax^2+bx^3)^3}\,dx = \int \frac{(c\,x)^m}{(b\,x^3+a\,x^2)^3}\,dx$$

```
int((c*x)^m/(a*x^2 + b*x^3)^3,x)
```

```
int((c*x)^m/(a*x^2 + b*x^3)^3, x)
```

Reduce [F]

$$\int \frac{(cx)^m}{(ax^2+bx^3)^3}\,dx = c^m\left(\int \frac{x^m}{b^3x^9+3a\,b^2x^8+3a^2b\,x^7+a^3x^6}dx\right)$$

```
int((c*x)^m/(b*x^3+a*x^2)^3,x)
```

```
c**m*int(x**m/(a**3*x**6 + 3*a**2*b*x**7 + 3*a*b**2*x**8 + b**3*x**9),x)
```

3.413 $\int (cx)^m \left(ax^2+bx^3\right)^{3/2}\,dx$

Optimal result

Integrand size = 21, antiderivative size = 68

$$\int (cx)^m \left(ax^2+bx^3\right)^{3/2}\,dx = -\frac{2b^4\left(-\frac{bx}{a}\right)^{-8-m}(cx)^{3+m}\left(ax^2+bx^3\right)^{5/2}\text{Hypergeometric2F1}\left(\frac{5}{2},-3-m,\frac{7}{2},1+\frac{bx}{a}\right)}{5a^5c^3}$$

```
-2/5*b^4*(-b*x/a)^(-8-m)*(c*x)^(3+m)*(b*x^3+a*x^2)^(5/2)*hypergeom([5/2, -
3-m],[7/2],1+b*x/a)/a^5/c^3
```

Mathematica [A] (verified)

Time = 0.16 (sec) , antiderivative size = 68, normalized size of antiderivative = 1.00

$$\int (cx)^m \left(ax^2+bx^3\right)^{3/2}\,dx = \frac{2a^2\left(-\frac{bx}{a}\right)^{-1-m}(cx)^m(a+bx)^2\sqrt{x^2(a+bx)}\,\text{Hypergeometric2F1}\left(\frac{5}{2},-3-m,\frac{7}{2},1+\frac{bx}{a}\right)}{5b^3}$$

```
Integrate[(c*x)^m*(a*x^2 + b*x^3)^(3/2),x]
```

```
(2*a^2*(-((b*x)/a))^(-1 - m)*(c*x)^m*(a + b*x)^2*Sqrt[x^2*(a + b*x)]*Hyper
geometric2F1[5/2, -3 - m, 7/2, 1 + (b*x)/a])/(5*b^3)
```

Rubi [B] (verified)

Leaf count is larger than twice the leaf count of optimal. 158 vs. $2(68) = 136$.

Time = 0.55 (sec) , antiderivative size = 158, normalized size of antiderivative = 2.32, number of steps used = 5, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.238$, Rules used = {1927, 1927, 1938, 77, 75}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \left(ax^2+bx^3\right)^{3/2} (cx)^m \, dx$$

$$\Big\downarrow \text{1927}$$

$$\frac{3a \int (cx)^{m+2} \sqrt{bx^3+ax^2} dx}{c^2 (2m+11)} + \frac{2 \left(ax^2+bx^3\right)^{3/2} (cx)^{m+1}}{c(2m+11)}$$

$$\Big\downarrow \text{1927}$$

$$\frac{3a \left(\frac{a \int \frac{(cx)^{m+4}}{\sqrt{bx^3+ax^2}} dx}{c^2 (2m+9)} + \frac{2\sqrt{ax^2+bx^3} (cx)^{m+3}}{c(2m+9)}\right)}{c^2 (2m+11)} + \frac{2 \left(ax^2+bx^3\right)^{3/2} (cx)^{m+1}}{c(2m+11)}$$

$$\Big\downarrow \text{1938}$$

$$\frac{3a \left(\frac{ac^2 x^{1-m} \sqrt{a+bx} (cx)^m \int \frac{x^{m+3}}{\sqrt{a+bx}} dx}{(2m+9)\sqrt{ax^2+bx^3}} + \frac{2\sqrt{ax^2+bx^3} (cx)^{m+3}}{c(2m+9)}\right)}{c^2 (2m+11)} + \frac{2 \left(ax^2+bx^3\right)^{3/2} (cx)^{m+1}}{c(2m+11)}$$

$$\Big\downarrow \text{77}$$

$$\frac{3a \left(\frac{2\sqrt{ax^2+bx^3} (cx)^{m+3}}{c(2m+9)} - \frac{a^4 c^2 x \sqrt{a+bx} (cx)^m \left(-\frac{bx}{a}\right)^{-m} \int \frac{\left(-\frac{bx}{a}\right)^{m+3}}{\sqrt{a+bx}} dx}{b^3 (2m+9)\sqrt{ax^2+bx^3}}\right)}{c^2 (2m+11)} + \frac{2 \left(ax^2+bx^3\right)^{3/2} (cx)^{m+1}}{c(2m+11)}$$

$$\Big\downarrow \text{75}$$

$$\frac{3a\left(\frac{2\sqrt{ax^2+bx^3}(cx)^{m+3}}{c(2m+9)}-\frac{2a^4c^2x(a+bx)(cx)^m\left(-\frac{bx}{a}\right)^{-m}\text{Hypergeometric2F1}\left(\frac{1}{2},-m-3,\frac{3}{2},\frac{bx}{a}+1\right)}{b^4(2m+9)\sqrt{ax^2+bx^3}}\right)}{c^2(2m+11)}+\frac{2\left(ax^2+bx^3\right)^{3/2}(cx)^{m+1}}{c(2m+11)}$$

```
Int[(c*x)^m*(a*x^2 + b*x^3)^(3/2),x]
```

```
(2*(c*x)^(1 + m)*(a*x^2 + b*x^3)^(3/2))/(c*(11 + 2*m)) + (3*a*((2*(c*x)^(3
 + m)*Sqrt[a*x^2 + b*x^3])/(c*(9 + 2*m)) - (2*a^4*c^2*x*(c*x)^m*(a + b*x)*
Hypergeometric2F1[1/2, -3 - m, 3/2, 1 + (b*x)/a])/(b^4*(9 + 2*m)*(-((b*x)/
a))^m*Sqrt[a*x^2 + b*x^3])))/(c^2*(11 + 2*m))
```

Defintions of rubi rules used

```
Int[((b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[((c + d*x
)^(n + 1)/(d*(n + 1)*(-d/(b*c))^m))*Hypergeometric2F1[-m, n + 1, n + 2, 1 +
 d*(x/c)], x] /; FreeQ[{b, c, d, m, n}, x] &&  !IntegerQ[n] && (IntegerQ[m]
 || GtQ[-d/(b*c), 0])
```

```
Int[((b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[((-b)*(c/
d))^IntPart[m]*((b*x)^FracPart[m]/((-d)*(x/c))^FracPart[m])   Int[((-d)*(x/
c))^m*(c + d*x)^n, x], x] /; FreeQ[{b, c, d, m, n}, x] &&  !IntegerQ[m] &&
 !IntegerQ[n] &&  !GtQ[c, 0] &&  !GtQ[-d/(b*c), 0]
```

```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> Simp[(c*x)^(m + 1)*((a*x^j + b*x^n)^p/(c*(m + n*p + 1))), x] + Simp[a*
(n - j)*(p/(c^j*(m + n*p + 1)))   Int[(c*x)^(m + j)*(a*x^j + b*x^n)^(p - 1)
, x], x] /; FreeQ[{a, b, c, m}, x] &&  !IntegerQ[p] && LtQ[0, j, n] && (Int
egersQ[j, n] || GtQ[c, 0]) && GtQ[p, 0] && NeQ[m + n*p + 1, 0]
```

```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> Simp[c^IntPart[m]*(c*x)^FracPart[m]*((a*x^j + b*x^n)^FracPart[p]/(x^(F
racPart[m] + j*FracPart[p])*(a + b*x^(n - j))^FracPart[p]))   Int[x^(m + j*
p)*(a + b*x^(n - j))^p, x], x] /; FreeQ[{a, b, c, j, m, n, p}, x] &&  !Inte
gerQ[p] && NeQ[n, j] && PosQ[n - j]
```

Maple [F]

$$\int (cx)^m \left(b\,x^3+a\,x^2\right)^{\frac{3}{2}}dx$$

```
int((c*x)^m*(b*x^3+a*x^2)^(3/2),x)
```

```
int((c*x)^m*(b*x^3+a*x^2)^(3/2),x)
```

Fricas [F]

$$\int (cx)^m \left(ax^2+bx^3\right)^{3/2}\,dx=\int \left(bx^3+ax^2\right)^{\frac{3}{2}}(cx)^m\,dx$$

```
integrate((c*x)^m*(b*x^3+a*x^2)^(3/2),x, algorithm="fricas")
```

```
integral((b*x^3 + a*x^2)^(3/2)*(c*x)^m, x)
```

Sympy [F]

$$\int (cx)^m \left(ax^2+bx^3\right)^{3/2}\,dx=\int (cx)^m \left(x^2(a+bx)\right)^{\frac{3}{2}}\,dx$$

```
integrate((c*x)**m*(b*x**3+a*x**2)**(3/2),x)
```

```
Integral((c*x)**m*(x**2*(a + b*x))**(3/2), x)
```

Maxima [F]

$$\int (cx)^m \left(ax^2+bx^3\right)^{3/2} dx = \int \left(bx^3+ax^2\right)^{\frac{3}{2}} (cx)^m \, dx$$

```
integrate((c*x)^m*(b*x^3+a*x^2)^(3/2),x, algorithm="maxima")
```

```
integrate((b*x^3 + a*x^2)^(3/2)*(c*x)^m, x)
```

Giac [F]

$$\int (cx)^m \left(ax^2+bx^3\right)^{3/2} dx = \int \left(bx^3+ax^2\right)^{\frac{3}{2}} (cx)^m \, dx$$

```
integrate((c*x)^m*(b*x^3+a*x^2)^(3/2),x, algorithm="giac")
```

```
integrate((b*x^3 + a*x^2)^(3/2)*(c*x)^m, x)
```

Mupad [F(-1)]

Timed out.

$$\int (cx)^m \left(ax^2+bx^3\right)^{3/2} dx = \int (c\,x)^m \left(b\,x^3+a\,x^2\right)^{3/2} dx$$

```
int((c*x)^m*(a*x^2 + b*x^3)^(3/2),x)
```

```
int((c*x)^m*(a*x^2 + b*x^3)^(3/2), x)
```

Reduce [F]

$$\int (cx)^m \left(ax^2+bx^3\right)^{3/2} dx = \text{too large to display}$$

```
int((c*x)^m*(b*x^3+a*x^2)^(3/2),x)
```

```
(2*c**m*( - 24*x**m*sqrt(a + b*x)*a**5*m**3 - 144*x**m*sqrt(a + b*x)*a**5*
m**2 - 264*x**m*sqrt(a + b*x)*a**5*m - 144*x**m*sqrt(a + b*x)*a**5 + 24*x*
*m*sqrt(a + b*x)*a**4*b*m**3*x + 132*x**m*sqrt(a + b*x)*a**4*b*m**2*x + 20
4*x**m*sqrt(a + b*x)*a**4*b*m*x + 72*x**m*sqrt(a + b*x)*a**4*b*x - 24*x**m
*sqrt(a + b*x)*a**3*b**2*m**3*x**2 - 120*x**m*sqrt(a + b*x)*a**3*b**2*m**2
*x**2 - 162*x**m*sqrt(a + b*x)*a**3*b**2*m*x**2 - 54*x**m*sqrt(a + b*x)*a*
*3*b**2*x**2 + 24*x**m*sqrt(a + b*x)*a**2*b**3*m**3*x**3 + 108*x**m*sqrt(a
 + b*x)*a**2*b**3*m**2*x**3 + 138*x**m*sqrt(a + b*x)*a**2*b**3*m*x**3 + 45
*x**m*sqrt(a + b*x)*a**2*b**3*x**3 + 32*x**m*sqrt(a + b*x)*a*b**4*m**5*x**
4 + 448*x**m*sqrt(a + b*x)*a*b**4*m**4*x**4 + 2224*x**m*sqrt(a + b*x)*a*b*
*4*m**3*x**4 + 4832*x**m*sqrt(a + b*x)*a*b**4*m**2*x**4 + 4434*x**m*sqrt(a
 + b*x)*a*b**4*m*x**4 + 1260*x**m*sqrt(a + b*x)*a*b**4*x**4 + 32*x**m*sqrt
(a + b*x)*b**5*m**5*x**5 + 400*x**m*sqrt(a + b*x)*b**5*m**4*x**5 + 1840*x*
*m*sqrt(a + b*x)*b**5*m**3*x**5 + 3800*x**m*sqrt(a + b*x)*b**5*m**2*x**5 +
 3378*x**m*sqrt(a + b*x)*b**5*m*x**5 + 945*x**m*sqrt(a + b*x)*b**5*x**5 +
1536*int((x**m*sqrt(a + b*x))/(64*a*m**6*x + 1152*a*m**5*x + 8080*a*m**4*x
 + 27840*a*m**3*x + 48556*a*m**2*x + 39048*a*m*x + 10395*a*x + 64*b*m**6*x
**2 + 1152*b*m**5*x**2 + 8080*b*m**4*x**2 + 27840*b*m**3*x**2 + 48556*b*m*
*2*x**2 + 39048*b*m*x**2 + 10395*b*x**2),x)*a**6*m**10 + 36864*int((x**m*s
qrt(a + b*x))/(64*a*m**6*x + 1152*a*m**5*x + 8080*a*m**4*x + 27840*a*m*...
```

3.414 $\int (cx)^m \sqrt{ax^2+bx^3}\,dx$

Optimal result

Integrand size = 21, antiderivative size = 68

$$\int (cx)^m \sqrt{ax^2+bx^3}\,dx = -\frac{2b^2\left(-\frac{bx}{a}\right)^{-4-m}(cx)^{1+m}\left(ax^2+bx^3\right)^{3/2}\text{Hypergeometric2F1}\left(\frac{3}{2},-1-m,\frac{5}{2},1+\frac{bx}{a}\right)}{3a^3c}$$

```
-2/3*b^2*(-b*x/a)^(-4-m)*(c*x)^(1+m)*(b*x^3+a*x^2)^(3/2)*hypergeom([3/2, -
1-m],[5/2],1+b*x/a)/a^3/c
```

Mathematica [A] (verified)

Time = 0.10 (sec) , antiderivative size = 60, normalized size of antiderivative = 0.88

$$\int (cx)^m \sqrt{ax^2+bx^3}\,dx = -\frac{2a\left(-\frac{bx}{a}\right)^{-m}(cx)^m\left(x^2(a+bx)\right)^{3/2}\text{Hypergeometric2F1}\left(\frac{3}{2},-1-m,\frac{5}{2},1+\frac{bx}{a}\right)}{3b^2x^3}$$

```
Integrate[(c*x)^m*Sqrt[a*x^2 + b*x^3],x]
```

```
(-2*a*(c*x)^m*(x^2*(a + b*x))^(3/2)*Hypergeometric2F1[3/2, -1 - m, 5/2, 1
+ (b*x)/a])/(3*b^2*x^3*(-((b*x)/a))^m)
```

Rubi [A] (verified)

Time = 0.42 (sec) , antiderivative size = 107, normalized size of antiderivative = 1.57, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.190$, Rules used = {1927, 1938, 77, 75}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sqrt{ax^2+bx^3}(cx)^m\,dx$$

$$\Big\downarrow \ 1927$$

$$\frac{a\int \frac{(cx)^{m+2}}{\sqrt{bx^3+ax^2}}dx}{c^2(2m+5)}+\frac{2\sqrt{ax^2+bx^3}(cx)^{m+1}}{c(2m+5)}$$

$$\Big\downarrow \ 1938$$

$$\frac{ax^{1-m}\sqrt{a+bx}(cx)^m\int \frac{x^{m+1}}{\sqrt{a+bx}}dx}{(2m+5)\sqrt{ax^2+bx^3}}+\frac{2\sqrt{ax^2+bx^3}(cx)^{m+1}}{c(2m+5)}$$

$$\Big\downarrow \ 77$$

$$\frac{2\sqrt{ax^2+bx^3}(cx)^{m+1}}{c(2m+5)}-\frac{a^2x\sqrt{a+bx}(cx)^m\left(-\frac{bx}{a}\right)^{-m}\int \frac{\left(-\frac{bx}{a}\right)^{m+1}}{\sqrt{a+bx}}dx}{b(2m+5)\sqrt{ax^2+bx^3}}$$

$$\Big\downarrow \ 75$$

$$\frac{2\sqrt{ax^2+bx^3}(cx)^{m+1}}{c(2m+5)}-\frac{2a^2x(a+bx)(cx)^m\left(-\frac{bx}{a}\right)^{-m}\text{Hypergeometric2F1}\left(\frac{1}{2},-m-1,\frac{3}{2},\frac{bx}{a}+1\right)}{b^2(2m+5)\sqrt{ax^2+bx^3}}$$

```
Int[(c*x)^m*Sqrt[a*x^2 + b*x^3],x]
```

```
(2*(c*x)^(1 + m)*Sqrt[a*x^2 + b*x^3])/(c*(5 + 2*m)) - (2*a^2*x*(c*x)^m*(a
+ b*x)*Hypergeometric2F1[1/2, -1 - m, 3/2, 1 + (b*x)/a])/(b^2*(5 + 2*m)*(-
((b*x)/a))^m*Sqrt[a*x^2 + b*x^3])
```

Defintions of rubi rules used

```
Int[((b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[((c + d*x
)^(n + 1)/(d*(n + 1)*(-d/(b*c))^m))*Hypergeometric2F1[-m, n + 1, n + 2, 1 +
 d*(x/c)], x] /; FreeQ[{b, c, d, m, n}, x] &&  !IntegerQ[n] && (IntegerQ[m]
 || GtQ[-d/(b*c), 0])
```

```
Int[((b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[((-b)*(c/
d))^IntPart[m]*((b*x)^FracPart[m]/((-d)*(x/c))^FracPart[m])   Int[((-d)*(x/
c))^m*(c + d*x)^n, x], x] /; FreeQ[{b, c, d, m, n}, x] &&  !IntegerQ[m] &&
 !IntegerQ[n] &&  !GtQ[c, 0] &&  !GtQ[-d/(b*c), 0]
```

```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> Simp[(c*x)^(m + 1)*((a*x^j + b*x^n)^p/(c*(m + n*p + 1))), x] + Simp[a*
(n - j)*(p/(c^j*(m + n*p + 1)))   Int[(c*x)^(m + j)*(a*x^j + b*x^n)^(p - 1)
, x], x] /; FreeQ[{a, b, c, m}, x] &&  !IntegerQ[p] && LtQ[0, j, n] && (Int
egersQ[j, n] || GtQ[c, 0]) && GtQ[p, 0] && NeQ[m + n*p + 1, 0]
```

```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> Simp[c^IntPart[m]*(c*x)^FracPart[m]*((a*x^j + b*x^n)^FracPart[p]/(x^(F
racPart[m] + j*FracPart[p])*(a + b*x^(n - j))^FracPart[p]))   Int[x^(m + j*
p)*(a + b*x^(n - j))^p, x], x] /; FreeQ[{a, b, c, j, m, n, p}, x] &&  !Inte
gerQ[p] && NeQ[n, j] && PosQ[n - j]
```

Maple [F]

$$\int (cx)^m \sqrt{b\,x^3+a\,x^2}dx$$

```
int((c*x)^m*(b*x^3+a*x^2)^(1/2),x)
```

```
int((c*x)^m*(b*x^3+a*x^2)^(1/2),x)
```

Fricas [F]

$$\int (cx)^m\sqrt{ax^2+bx^3}\,dx = \int \sqrt{bx^3+ax^2}(cx)^m\,dx$$

```
integrate((c*x)^m*(b*x^3+a*x^2)^(1/2),x, algorithm="fricas")
```

```
integral(sqrt(b*x^3 + a*x^2)*(c*x)^m, x)
```

Sympy [F]

$$\int (cx)^m\sqrt{ax^2+bx^3}\,dx = \int (cx)^m\sqrt{x^2\,(a+bx)}\,dx$$

```
integrate((c*x)**m*(b*x**3+a*x**2)**(1/2),x)
```

```
Integral((c*x)**m*sqrt(x**2*(a + b*x)), x)
```

Maxima [F]

$$\int (cx)^m \sqrt{ax^2+bx^3}\,dx = \int \sqrt{bx^3+ax^2}(cx)^m\,dx$$

```
integrate((c*x)^m*(b*x^3+a*x^2)^(1/2),x, algorithm="maxima")
```

```
integrate(sqrt(b*x^3 + a*x^2)*(c*x)^m, x)
```

Giac [F]

$$\int (cx)^m \sqrt{ax^2+bx^3}\,dx = \int \sqrt{bx^3+ax^2}(cx)^m\,dx$$

```
integrate((c*x)^m*(b*x^3+a*x^2)^(1/2),x, algorithm="giac")
```

```
integrate(sqrt(b*x^3 + a*x^2)*(c*x)^m, x)
```

Mupad [F(-1)]

Timed out.

$$\int (cx)^m \sqrt{ax^2+bx^3}\,dx = \int (c\,x)^m\,\sqrt{b\,x^3+a\,x^2}\,dx$$

```
int((c*x)^m*(a*x^2 + b*x^3)^(1/2),x)
```

```
int((c*x)^m*(a*x^2 + b*x^3)^(1/2), x)
```

Reduce [F]

$$\int (cx)^m\sqrt{ax^2+bx^3}\,dx$$
$$= \frac{2c^m\left(-2x^m\sqrt{bx+a}\,a^2m-2x^m\sqrt{bx+a}\,a^2+2x^m\sqrt{bx+a}\,abmx+x^m\sqrt{bx+a}\,abx+4x^m\sqrt{bx+a}\,b^2m^2\right.}{}$$

```
int((c*x)^m*(b*x^3+a*x^2)^(1/2),x)
```

```
(2*c**m*( - 2*x**m*sqrt(a + b*x)*a**2*m - 2*x**m*sqrt(a + b*x)*a**2 + 2*x*
*m*sqrt(a + b*x)*a*b*m*x + x**m*sqrt(a + b*x)*a*b*x + 4*x**m*sqrt(a + b*x)
*b**2*m**2*x**2 + 8*x**m*sqrt(a + b*x)*b**2*m*x**2 + 3*x**m*sqrt(a + b*x)*
b**2*x**2 + 16*int((x**m*sqrt(a + b*x))/(8*a*m**3*x + 36*a*m**2*x + 46*a*m
*x + 15*a*x + 8*b*m**3*x**2 + 36*b*m**2*x**2 + 46*b*m*x**2 + 15*b*x**2),x)
*a**3*m**5 + 88*int((x**m*sqrt(a + b*x))/(8*a*m**3*x + 36*a*m**2*x + 46*a*
m*x + 15*a*x + 8*b*m**3*x**2 + 36*b*m**2*x**2 + 46*b*m*x**2 + 15*b*x**2),x
)*a**3*m**4 + 164*int((x**m*sqrt(a + b*x))/(8*a*m**3*x + 36*a*m**2*x + 46*
a*m*x + 15*a*x + 8*b*m**3*x**2 + 36*b*m**2*x**2 + 46*b*m*x**2 + 15*b*x**2)
,x)*a**3*m**3 + 122*int((x**m*sqrt(a + b*x))/(8*a*m**3*x + 36*a*m**2*x + 4
6*a*m*x + 15*a*x + 8*b*m**3*x**2 + 36*b*m**2*x**2 + 46*b*m*x**2 + 15*b*x**
2),x)*a**3*m**2 + 30*int((x**m*sqrt(a + b*x))/(8*a*m**3*x + 36*a*m**2*x +
46*a*m*x + 15*a*x + 8*b*m**3*x**2 + 36*b*m**2*x**2 + 46*b*m*x**2 + 15*b*x*
*2),x)*a**3*m))/(b**2*(8*m**3 + 36*m**2 + 46*m + 15))
```

3.415 $\int \frac{(cx)^m}{\sqrt{ax^2+bx^3}}\,dx$

Optimal result

Integrand size = 21, antiderivative size = 59

$$\int \frac{(cx)^m}{\sqrt{ax^2+bx^3}}\,dx = -\frac{2c\left(-\frac{bx}{a}\right)^{-m}(cx)^{-1+m}\sqrt{ax^2+bx^3}\,\text{Hypergeometric2F1}\left(\frac{1}{2},1-m,\frac{3}{2},1+\frac{bx}{a}\right)}{a}$$

```
-2*c*(c*x)^(-1+m)*(b*x^3+a*x^2)^(1/2)*hypergeom([1/2, 1-m],[3/2],1+b*x/a)/
a/((-b*x/a)^m)
```

Mathematica [A] (verified)

Time = 0.10 (sec) , antiderivative size = 57, normalized size of antiderivative = 0.97

$$\int \frac{(cx)^m}{\sqrt{ax^2+bx^3}}\,dx = \frac{2b\left(-\frac{bx}{a}\right)^{-1-m}(cx)^m\sqrt{x^2(a+bx)}\,\text{Hypergeometric2F1}\left(\frac{1}{2},1-m,\frac{3}{2},1+\frac{bx}{a}\right)}{a^2}$$

```
Integrate[(c*x)^m/Sqrt[a*x^2 + b*x^3],x]
```

```
(2*b*(-((b*x)/a))^(-1 - m)*(c*x)^m*Sqrt[x^2*(a + b*x)]*Hypergeometric2F1[1
/2, 1 - m, 3/2, 1 + (b*x)/a])/a^2
```

Rubi [A] (verified)

Time = 0.31 (sec) , antiderivative size = 62, normalized size of antiderivative = 1.05, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.143$, Rules used = {1938, 77, 75}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(cx)^m}{\sqrt{ax^2+bx^3}}dx$$

$$\Big\downarrow \text{1938}$$

$$\frac{x^{1-m}\sqrt{a+bx}(cx)^m\int \frac{x^{m-1}}{\sqrt{a+bx}}dx}{\sqrt{ax^2+bx^3}}$$

$$\Big\downarrow \text{77}$$

$$-\frac{bx\sqrt{a+bx}(cx)^m\left(-\frac{bx}{a}\right)^{-m}\int \frac{\left(-\frac{bx}{a}\right)^{m-1}}{\sqrt{a+bx}}dx}{a\sqrt{ax^2+bx^3}}$$

$$\Big\downarrow \text{75}$$

$$-\frac{2x(a+bx)(cx)^m\left(-\frac{bx}{a}\right)^{-m}\text{Hypergeometric2F1}\left(\frac{1}{2},1-m,\frac{3}{2},\frac{bx}{a}+1\right)}{a\sqrt{ax^2+bx^3}}$$

```
Int[(c*x)^m/Sqrt[a*x^2 + b*x^3],x]
```

```
(-2*x*(c*x)^m*(a + b*x)*Hypergeometric2F1[1/2, 1 - m, 3/2, 1 + (b*x)/a])/(
a*(-((b*x)/a))^m*Sqrt[a*x^2 + b*x^3])
```

Defintions of rubi rules used

```
Int[((b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[((c + d*x
)^(n + 1)/(d*(n + 1)*(-d/(b*c))^m))*Hypergeometric2F1[-m, n + 1, n + 2, 1 +
 d*(x/c)], x] /; FreeQ[{b, c, d, m, n}, x] &&  !IntegerQ[n] && (IntegerQ[m]
 || GtQ[-d/(b*c), 0])
```

```
Int[((b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[((-b)*(c/
d))^IntPart[m]*((b*x)^FracPart[m]/((-d)*(x/c))^FracPart[m])   Int[((-d)*(x/
c))^m*(c + d*x)^n, x], x] /; FreeQ[{b, c, d, m, n}, x] &&  !IntegerQ[m] &&
 !IntegerQ[n] &&  !GtQ[c, 0] &&  !GtQ[-d/(b*c), 0]
```

```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> Simp[c^IntPart[m]*(c*x)^FracPart[m]*((a*x^j + b*x^n)^FracPart[p]/(x^(F
racPart[m] + j*FracPart[p])*(a + b*x^(n - j))^FracPart[p]))   Int[x^(m + j*
p)*(a + b*x^(n - j))^p, x], x] /; FreeQ[{a, b, c, j, m, n, p}, x] &&  !Inte
gerQ[p] && NeQ[n, j] && PosQ[n - j]
```

Maple [F]

$$\int \frac{(cx)^m}{\sqrt{b\,x^3+a\,x^2}}dx$$

```
int((c*x)^m/(b*x^3+a*x^2)^(1/2),x)
```

```
int((c*x)^m/(b*x^3+a*x^2)^(1/2),x)
```

Fricas [F]

$$\int \frac{(cx)^m}{\sqrt{ax^2+bx^3}}\,dx = \int \frac{(cx)^m}{\sqrt{bx^3+ax^2}}\,dx$$

```
integrate((c*x)^m/(b*x^3+a*x^2)^(1/2),x, algorithm="fricas")
```

```
integral((c*x)^m/sqrt(b*x^3 + a*x^2), x)
```

Sympy [F]

$$\int \frac{(cx)^m}{\sqrt{ax^2 + bx^3}}\,dx = \int \frac{(cx)^m}{\sqrt{x^2\,(a + bx)}}\,dx$$

```
integrate((c*x)**m/(b*x**3+a*x**2)**(1/2),x)
```

```
Integral((c*x)**m/sqrt(x**2*(a + b*x)), x)
```

Maxima [F]

$$\int \frac{(cx)^m}{\sqrt{ax^2 + bx^3}}\,dx = \int \frac{(cx)^m}{\sqrt{bx^3 + ax^2}}\,dx$$

```
integrate((c*x)^m/(b*x^3+a*x^2)^(1/2),x, algorithm="maxima")
```

```
integrate((c*x)^m/sqrt(b*x^3 + a*x^2), x)
```

Giac [F]

$$\int \frac{(cx)^m}{\sqrt{ax^2 + bx^3}}\,dx = \int \frac{(cx)^m}{\sqrt{bx^3 + ax^2}}\,dx$$

```
integrate((c*x)^m/(b*x^3+a*x^2)^(1/2),x, algorithm="giac")
```

```
integrate((c*x)^m/sqrt(b*x^3 + a*x^2), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(cx)^m}{\sqrt{ax^2+bx^3}}\,dx = \int \frac{(c\,x)^m}{\sqrt{b\,x^3+a\,x^2}}\,dx$$

```
int((c*x)^m/(a*x^2 + b*x^3)^(1/2),x)
```

```
int((c*x)^m/(a*x^2 + b*x^3)^(1/2), x)
```

Reduce [F]

$$\int \frac{(cx)^m}{\sqrt{ax^2+bx^3}}\,dx = c^m \left(\int \frac{x^m\sqrt{bx+a}}{b\,x^2+ax}dx\right)$$

```
int((c*x)^m/(b*x^3+a*x^2)^(1/2),x)
```

```
c**m*int((x**m*sqrt(a + b*x))/(a*x + b*x**2),x)
```

3.416 $\int \frac{(cx)^m}{(ax^2+bx^3)^{3/2}}\,dx$

Optimal result

Integrand size = 21, antiderivative size = 64

$$\int \frac{(cx)^m}{(ax^2+bx^3)^{3/2}}\,dx = \frac{2ac^3\left(-\frac{bx}{a}\right)^{4-m}(cx)^{-3+m}\text{Hypergeometric2F1}\left(-\frac{1}{2},3-m,\frac{1}{2},1+\frac{bx}{a}\right)}{b^2\sqrt{ax^2+bx^3}}$$

```
2*a*c^3*(-b*x/a)^(4-m)*(c*x)^(-3+m)*hypergeom([-1/2, 3-m],[1/2],1+b*x/a)/b
^2/(b*x^3+a*x^2)^(1/2)
```

Mathematica [A] (verified)

Time = 0.14 (sec) , antiderivative size = 58, normalized size of antiderivative = 0.91

$$\int \frac{(cx)^m}{(ax^2+bx^3)^{3/2}}\,dx = \frac{2b^2x\left(-\frac{bx}{a}\right)^{-m}(cx)^m\,\text{Hypergeometric2F1}\left(-\frac{1}{2},3-m,\frac{1}{2},1+\frac{bx}{a}\right)}{a^3\sqrt{x^2(a+bx)}}$$

```
Integrate[(c*x)^m/(a*x^2 + b*x^3)^(3/2),x]
```

```
(2*b^2*x*(c*x)^m*Hypergeometric2F1[-1/2, 3 - m, 1/2, 1 + (b*x)/a])/(a^3*(-
((b*x)/a))^m*Sqrt[x^2*(a + b*x)])
```

Rubi [A] (verified)

Time = 0.40 (sec) , antiderivative size = 99, normalized size of antiderivative = 1.55, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.190, Rules used = {1929, 1938, 77, 75}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(cx)^m}{(ax^2+bx^3)^{3/2}}\,dx$$

$$\Big\downarrow \; 1929$$

$$\frac{c^2(5-2m)\int \frac{(cx)^{m-2}}{\sqrt{bx^3+ax^2}}dx}{a}+\frac{2c(cx)^{m-1}}{a\sqrt{ax^2+bx^3}}$$

$$\Big\downarrow \; 1938$$

$$\frac{(5-2m)x^{1-m}\sqrt{a+bx}(cx)^m\int \frac{x^{m-3}}{\sqrt{a+bx}}dx}{a\sqrt{ax^2+bx^3}}+\frac{2c(cx)^{m-1}}{a\sqrt{ax^2+bx^3}}$$

$$\Big\downarrow \; 77$$

$$\frac{2c(cx)^{m-1}}{a\sqrt{ax^2+bx^3}}-\frac{b^3(5-2m)x\sqrt{a+bx}(cx)^m\left(-\frac{bx}{a}\right)^{-m}\int \frac{\left(-\frac{bx}{a}\right)^{m-3}}{\sqrt{a+bx}}dx}{a^4\sqrt{ax^2+bx^3}}$$

$$\Big\downarrow \; 75$$

$$\frac{2c(cx)^{m-1}}{a\sqrt{ax^2+bx^3}}-\frac{2b^2(5-2m)x(a+bx)(cx)^m\left(-\frac{bx}{a}\right)^{-m}\text{Hypergeometric2F1}\left(\frac{1}{2},3-m,\frac{3}{2},\frac{bx}{a}+1\right)}{a^4\sqrt{ax^2+bx^3}}$$

```
Int[(c*x)^m/(a*x^2 + b*x^3)^(3/2),x]
```

```
(2*c*(c*x)^(-1 + m))/(a*Sqrt[a*x^2 + b*x^3]) - (2*b^2*(5 - 2*m)*x*(c*x)^m*
(a + b*x)*Hypergeometric2F1[1/2, 3 - m, 3/2, 1 + (b*x)/a])/(a^4*(-((b*x)/a
))^m*Sqrt[a*x^2 + b*x^3])
```

Defintions of rubi rules used

```
Int[((b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[((c + d*x
)^(n + 1)/(d*(n + 1)*(-d/(b*c))^m))*Hypergeometric2F1[-m, n + 1, n + 2, 1 +
 d*(x/c)], x] /; FreeQ[{b, c, d, m, n}, x] &&  !IntegerQ[n] && (IntegerQ[m]
 || GtQ[-d/(b*c), 0])
```

```
Int[((b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[((-b)*(c/
d))^IntPart[m]*((b*x)^FracPart[m]/((-d)*(x/c))^FracPart[m])   Int[((-d)*(x/
c))^m*(c + d*x)^n, x], x] /; FreeQ[{b, c, d, m, n}, x] &&  !IntegerQ[m] &&
 !IntegerQ[n] &&  !GtQ[c, 0] &&  !GtQ[-d/(b*c), 0]
```

```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> Simp[(-c^(j - 1))*(c*x)^(m - j + 1)*((a*x^j + b*x^n)^(p + 1)/(a*(n - j
)*(p + 1))), x] + Simp[c^j*((m + n*p + n - j + 1)/(a*(n - j)*(p + 1)))   In
t[(c*x)^(m - j)*(a*x^j + b*x^n)^(p + 1), x], x] /; FreeQ[{a, b, c, m}, x] &
&  !IntegerQ[p] && LtQ[0, j, n] && (IntegersQ[j, n] || GtQ[c, 0]) && LtQ[p,
 -1]
```

```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> Simp[c^IntPart[m]*(c*x)^FracPart[m]*((a*x^j + b*x^n)^FracPart[p]/(x^(F
racPart[m] + j*FracPart[p])*(a + b*x^(n - j))^FracPart[p]))   Int[x^(m + j*
p)*(a + b*x^(n - j))^p, x], x] /; FreeQ[{a, b, c, j, m, n, p}, x] &&  !Inte
gerQ[p] && NeQ[n, j] && PosQ[n - j]
```

Maple [F]

$$\int \frac{(cx)^m}{(b\,x^3+a\,x^2)^{\frac{3}{2}}}dx$$

```
int((c*x)^m/(b*x^3+a*x^2)^(3/2),x)
```

```
int((c*x)^m/(b*x^3+a*x^2)^(3/2),x)
```

Fricas [F]

$$\int \frac{(cx)^m}{(ax^2+bx^3)^{3/2}}\,dx = \int \frac{(cx)^m}{(bx^3+ax^2)^{\frac{3}{2}}}\,dx$$

```
integrate((c*x)^m/(b*x^3+a*x^2)^(3/2),x, algorithm="fricas")
```

```
integral(sqrt(b*x^3 + a*x^2)*(c*x)^m/(b^2*x^6 + 2*a*b*x^5 + a^2*x^4), x)
```

Sympy [F]

$$\int \frac{(cx)^m}{(ax^2+bx^3)^{3/2}}\,dx = \int \frac{(cx)^m}{(x^2\,(a+bx))^{\frac{3}{2}}}\,dx$$

```
integrate((c*x)**m/(b*x**3+a*x**2)**(3/2),x)
```

```
Integral((c*x)**m/(x**2*(a + b*x))**(3/2), x)
```

Maxima [F]

$$\int \frac{(cx)^m}{(ax^2+bx^3)^{3/2}}\,dx = \int \frac{(cx)^m}{(bx^3+ax^2)^{\frac{3}{2}}}\,dx$$

```
integrate((c*x)^m/(b*x^3+a*x^2)^(3/2),x, algorithm="maxima")
```

```
integrate((c*x)^m/(b*x^3 + a*x^2)^(3/2), x)
```

Giac [F]

$$\int \frac{(cx)^m}{(ax^2+bx^3)^{3/2}}\,dx = \int \frac{(cx)^m}{(bx^3+ax^2)^{\frac{3}{2}}}\,dx$$

```
integrate((c*x)^m/(b*x^3+a*x^2)^(3/2),x, algorithm="giac")
```

```
integrate((c*x)^m/(b*x^3 + a*x^2)^(3/2), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(cx)^m}{(ax^2+bx^3)^{3/2}}\,dx = \int \frac{(c\,x)^m}{(b\,x^3+a\,x^2)^{3/2}}\,dx$$

```
int((c*x)^m/(a*x^2 + b*x^3)^(3/2),x)
```

```
int((c*x)^m/(a*x^2 + b*x^3)^(3/2), x)
```

Reduce [F]

$$\int \frac{(cx)^m}{(ax^2+bx^3)^{3/2}}\,dx = c^m\left(\int \frac{x^m\sqrt{bx+a}}{b^2x^5+2ab\,x^4+a^2x^3}dx\right)$$

```
int((c*x)^m/(b*x^3+a*x^2)^(3/2),x)
```

```
c**m*int((x**m*sqrt(a + b*x))/(a**2*x**3 + 2*a*b*x**4 + b**2*x**5),x)
```

3.417 $\int \frac{(cx)^m}{(ax^2+bx^3)^{5/2}}\,dx$

Optimal result

Integrand size = 21, antiderivative size = 68

$$\int \frac{(cx)^m}{(ax^2+bx^3)^{5/2}}\,dx=\frac{2a^3c^5\left(-\frac{bx}{a}\right)^{8-m}(cx)^{-5+m}\,\text{Hypergeometric2F1}\left(-\frac{3}{2},5-m,-\frac{1}{2},1+\frac{bx}{a}\right)}{3b^4\,(ax^2+bx^3)^{3/2}}$$

```
2/3*a^3*c^5*(-b*x/a)^(8-m)*(c*x)^(-5+m)*hypergeom([-3/2, 5-m],[-1/2],1+b*x
/a)/b^4/(b*x^3+a*x^2)^(3/2)
```

Mathematica [A] (verified)

Time = 0.15 (sec) , antiderivative size = 62, normalized size of antiderivative = 0.91

$$\int \frac{(cx)^m}{(ax^2+bx^3)^{5/2}}\,dx=\frac{2b^4x^3\left(-\frac{bx}{a}\right)^{-m}(cx)^m\,\text{Hypergeometric2F1}\left(-\frac{3}{2},5-m,-\frac{1}{2},1+\frac{bx}{a}\right)}{3a^5\,(x^2(a+bx))^{3/2}}$$

```
Integrate[(c*x)^m/(a*x^2 + b*x^3)^(5/2),x]
```

```
(2*b^4*x^3*(c*x)^m*Hypergeometric2F1[-3/2, 5 - m, -1/2, 1 + (b*x)/a])/(3*a
^5*(-((b*x)/a))^m*(x^2*(a + b*x))^(3/2))
```

Rubi [B] (verified)

Leaf count is larger than twice the leaf count of optimal. 148 vs. $2(68) = 136$.

Time = 0.51 (sec) , antiderivative size = 148, normalized size of antiderivative = 2.18, number of steps used = 5, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.238$, Rules used = {1929, 1929, 1938, 77, 75}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(cx)^m}{(ax^2+bx^3)^{5/2}}\,dx$$

$$\Big\downarrow \text{1929}$$

$$\frac{c^2(11-2m)\int \frac{(cx)^{m-2}}{(bx^3+ax^2)^{3/2}}dx}{3a}+\frac{2c(cx)^{m-1}}{3a\left(ax^2+bx^3\right)^{3/2}}$$

$$\Big\downarrow \text{1929}$$

$$\frac{c^2(11-2m)\left(\frac{c^2(9-2m)\int \frac{(cx)^{m-4}}{\sqrt{bx^3+ax^2}}dx}{a}+\frac{2c(cx)^{m-3}}{a\sqrt{ax^2+bx^3}}\right)}{3a}+\frac{2c(cx)^{m-1}}{3a\left(ax^2+bx^3\right)^{3/2}}$$

$$\Big\downarrow \text{1938}$$

$$\frac{c^2(11-2m)\left(\frac{(9-2m)x^{1-m}\sqrt{a+bx}(cx)^m\int \frac{x^{m-5}}{\sqrt{a+bx}}dx}{ac^2\sqrt{ax^2+bx^3}}+\frac{2c(cx)^{m-3}}{a\sqrt{ax^2+bx^3}}\right)}{3a}+\frac{2c(cx)^{m-1}}{3a\left(ax^2+bx^3\right)^{3/2}}$$

$$\Big\downarrow \text{77}$$

$$\frac{c^2(11-2m)\left(\frac{2c(cx)^{m-3}}{a\sqrt{ax^2+bx^3}}-\frac{b^5(9-2m)x\sqrt{a+bx}(cx)^m\left(-\frac{bx}{a}\right)^{-m}\int \frac{\left(-\frac{bx}{a}\right)^{m-5}}{\sqrt{a+bx}}dx}{a^6c^2\sqrt{ax^2+bx^3}}\right)}{3a}+\frac{2c(cx)^{m-1}}{3a\left(ax^2+bx^3\right)^{3/2}}$$

$$\Big\downarrow \text{75}$$

$$\frac{c^2(11-2m)\left(\frac{2c(cx)^{m-3}}{a\sqrt{ax^2+bx^3}}-\frac{2b^4(9-2m)x(a+bx)(cx)^m\left(-\frac{bx}{a}\right)^{-m}\text{Hypergeometric2F1}\left(\frac{1}{2},5-m,\frac{3}{2},\frac{bx}{a}+1\right)}{a^6c^2\sqrt{ax^2+bx^3}}\right)}{3a}+\frac{2c(cx)^{m-1}}{3a\left(ax^2+bx^3\right)^{3/2}}$$

```
Int[(c*x)^m/(a*x^2 + b*x^3)^(5/2),x]
```

```
(2*c*(c*x)^(-1 + m))/(3*a*(a*x^2 + b*x^3)^(3/2)) + (c^2*(11 - 2*m)*((2*c*(
c*x)^(-3 + m))/(a*Sqrt[a*x^2 + b*x^3]) - (2*b^4*(9 - 2*m)*x*(c*x)^m*(a + b
*x)*Hypergeometric2F1[1/2, 5 - m, 3/2, 1 + (b*x)/a])/(a^6*c^2*(-((b*x)/a))
^m*Sqrt[a*x^2 + b*x^3])))/(3*a)
```

Defintions of rubi rules used

```
Int[((b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[((c + d*x
)^(n + 1)/(d*(n + 1)*(-d/(b*c))^m))*Hypergeometric2F1[-m, n + 1, n + 2, 1 +
 d*(x/c)], x] /; FreeQ[{b, c, d, m, n}, x] &&  !IntegerQ[n] && (IntegerQ[m]
 || GtQ[-d/(b*c), 0])
```

```
Int[((b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[((-b)*(c/
d))^IntPart[m]*((b*x)^FracPart[m]/((-d)*(x/c))^FracPart[m])   Int[((-d)*(x/
c))^m*(c + d*x)^n, x], x] /; FreeQ[{b, c, d, m, n}, x] &&  !IntegerQ[m] &&
 !IntegerQ[n] &&  !GtQ[c, 0] &&  !GtQ[-d/(b*c), 0]
```

```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> Simp[(-c^(j - 1))*(c*x)^(m - j + 1)*((a*x^j + b*x^n)^(p + 1)/(a*(n - j
)*(p + 1))), x] + Simp[c^j*((m + n*p + n - j + 1)/(a*(n - j)*(p + 1)))   In
t[(c*x)^(m - j)*(a*x^j + b*x^n)^(p + 1), x], x] /; FreeQ[{a, b, c, m}, x] &
&  !IntegerQ[p] && LtQ[0, j, n] && (IntegersQ[j, n] || GtQ[c, 0]) && LtQ[p,
 -1]
```

```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> Simp[c^IntPart[m]*(c*x)^FracPart[m]*((a*x^j + b*x^n)^FracPart[p]/(x^(F
racPart[m] + j*FracPart[p])*(a + b*x^(n - j))^FracPart[p]))   Int[x^(m + j*
p)*(a + b*x^(n - j))^p, x], x] /; FreeQ[{a, b, c, j, m, n, p}, x] &&  !Inte
gerQ[p] && NeQ[n, j] && PosQ[n - j]
```

Maple [F]

$$\int \frac{(cx)^m}{(b\,x^3 + a\,x^2)^{\frac{5}{2}}}dx$$

```
int((c*x)^m/(b*x^3+a*x^2)^(5/2),x)
```

```
int((c*x)^m/(b*x^3+a*x^2)^(5/2),x)
```

Fricas [F]

$$\int \frac{(cx)^m}{(ax^2 + bx^3)^{5/2}}\,dx = \int \frac{(cx)^m}{(bx^3 + ax^2)^{\frac{5}{2}}}\,dx$$

```
integrate((c*x)^m/(b*x^3+a*x^2)^(5/2),x, algorithm="fricas")
```

```
integral(sqrt(b*x^3 + a*x^2)*(c*x)^m/(b^3*x^9 + 3*a*b^2*x^8 + 3*a^2*b*x^7
+ a^3*x^6), x)
```

Sympy [F]

$$\int \frac{(cx)^m}{(ax^2 + bx^3)^{5/2}}\,dx = \int \frac{(cx)^m}{(x^2\,(a + bx))^{\frac{5}{2}}}\,dx$$

```
integrate((c*x)**m/(b*x**3+a*x**2)**(5/2),x)
```

```
Integral((c*x)**m/(x**2*(a + b*x))**(5/2), x)
```

Maxima [F]

$$\int \frac{(cx)^m}{(ax^2+bx^3)^{5/2}}\,dx = \int \frac{(cx)^m}{(bx^3+ax^2)^{\frac{5}{2}}}\,dx$$

```
integrate((c*x)^m/(b*x^3+a*x^2)^(5/2),x, algorithm="maxima")
```

```
integrate((c*x)^m/(b*x^3 + a*x^2)^(5/2), x)
```

Giac [F]

$$\int \frac{(cx)^m}{(ax^2+bx^3)^{5/2}}\,dx = \int \frac{(cx)^m}{(bx^3+ax^2)^{\frac{5}{2}}}\,dx$$

```
integrate((c*x)^m/(b*x^3+a*x^2)^(5/2),x, algorithm="giac")
```

```
integrate((c*x)^m/(b*x^3 + a*x^2)^(5/2), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(cx)^m}{(ax^2+bx^3)^{5/2}}\,dx = \int \frac{(c\,x)^m}{(b\,x^3+a\,x^2)^{5/2}}\,dx$$

```
int((c*x)^m/(a*x^2 + b*x^3)^(5/2),x)
```

```
int((c*x)^m/(a*x^2 + b*x^3)^(5/2), x)
```

Reduce [F]

$$\int \frac{(cx)^m}{(ax^2+bx^3)^{5/2}}\,dx = c^m\left(\int \frac{x^m\sqrt{bx+a}}{b^3x^8+3a\,b^2x^7+3a^2b\,x^6+a^3x^5}dx\right)$$

```
int((c*x)^m/(b*x^3+a*x^2)^(5/2),x)
```

```
c**m*int((x**m*sqrt(a + b*x))/(a**3*x**5 + 3*a**2*b*x**6 + 3*a*b**2*x**7 +
 b**3*x**8),x)
```

3.418 $\int x^2(ax^2+bx^3)^p\,dx$

Optimal result

Integrand size = 17, antiderivative size = 46

$$\int x^2(ax^2+bx^3)^p\,dx=\frac{x(ax^2+bx^3)^{1+p}\,\text{Hypergeometric2F1}\left(1,4+3p,2(2+p),-\frac{bx}{a}\right)}{a(3+2p)}$$

```
x*(b*x^3+a*x^2)^(p+1)*hypergeom([1, 4+3*p],[4+2*p],-b*x/a)/a/(3+2*p)
```

Mathematica [A] (verified)

Time = 0.05 (sec) , antiderivative size = 55, normalized size of antiderivative = 1.20

$$\begin{aligned}&\int x^2(ax^2+bx^3)^p\,dx\\&=\frac{x^3(x^2(a+bx))^p\left(1+\frac{bx}{a}\right)^{-p}\text{Hypergeometric2F1}\left(-p,3+2p,4+2p,-\frac{bx}{a}\right)}{3+2p}\end{aligned}$$

```
Integrate[x^2*(a*x^2 + b*x^3)^p,x]
```

```
(x^3*(x^2*(a + b*x))^p*Hypergeometric2F1[-p, 3 + 2*p, 4 + 2*p, -((b*x)/a)]
)/((3 + 2*p)*(1 + (b*x)/a)^p)
```

Rubi [A] (verified)

Time = 0.32 (sec) , antiderivative size = 57, normalized size of antiderivative = 1.24, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.176, Rules used = {1938, 76, 74}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^2 (ax^2+bx^3)^p \, dx$$

$$\Big\downarrow \text{1938}$$

$$x^{-2p}(a+bx)^{-p} (ax^2+bx^3)^p \int x^{2(p+1)}(a+bx)^p dx$$

$$\Big\downarrow \text{76}$$

$$x^{-2p}\left(\frac{bx}{a}+1\right)^{-p} (ax^2+bx^3)^p \int x^{2(p+1)}\left(\frac{bx}{a}+1\right)^p dx$$

$$\Big\downarrow \text{74}$$

$$\frac{x^3 \left(\frac{bx}{a}+1\right)^{-p} (ax^2+bx^3)^p \text{Hypergeometric2F1}\left(-p,2p+3,2(p+2),-\frac{bx}{a}\right)}{2p+3}$$

```
Int[x^2*(a*x^2 + b*x^3)^p,x]
```

```
(x^3*(a*x^2 + b*x^3)^p*Hypergeometric2F1[-p, 3 + 2*p, 2*(2 + p), -((b*x)/a
)])/((3 + 2*p)*(1 + (b*x)/a)^p)
```

Defintions of rubi rules used

```
Int[((b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[c^n*((b*x
)^(m + 1)/(b*(m + 1)))*Hypergeometric2F1[-n, m + 1, m + 2, (-d)*(x/c)], x]
/; FreeQ[{b, c, d, m, n}, x] &&  !IntegerQ[m] && (IntegerQ[n] || (GtQ[c, 0]
 &&  !(EqQ[n, -2^(-1)] && EqQ[c^2 - d^2, 0] && GtQ[-d/(b*c), 0])))
```

```
Int[((b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[c^IntPart
[n]*((c + d*x)^FracPart[n]/(1 + d*(x/c))^FracPart[n])   Int[(b*x)^m*(1 + d*
(x/c))^n, x], x] /; FreeQ[{b, c, d, m, n}, x] &&  !IntegerQ[m] &&  !Integer
Q[n] &&  !GtQ[c, 0] &&  !GtQ[-d/(b*c), 0] && ((RationalQ[m] &&  !(EqQ[n, -2
^(-1)] && EqQ[c^2 - d^2, 0])) ||  !RationalQ[n])
```

```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> Simp[c^IntPart[m]*(c*x)^FracPart[m]*((a*x^j + b*x^n)^FracPart[p]/(x^(F
racPart[m] + j*FracPart[p])*(a + b*x^(n - j))^FracPart[p]))   Int[x^(m + j*
p)*(a + b*x^(n - j))^p, x], x] /; FreeQ[{a, b, c, j, m, n, p}, x] &&  !Inte
gerQ[p] && NeQ[n, j] && PosQ[n - j]
```

Maple [F]

$$\int x^2 (b\,x^3 + a\,x^2)^p\,dx$$

```
int(x^2*(b*x^3+a*x^2)^p,x)
```

```
int(x^2*(b*x^3+a*x^2)^p,x)
```

Fricas [F]

$$\int x^2 (ax^2 + bx^3)^p\ dx = \int (bx^3 + ax^2)^p x^2\,dx$$

```
integrate(x^2*(b*x^3+a*x^2)^p,x, algorithm="fricas")
```

```
integral((b*x^3 + a*x^2)^p*x^2, x)
```

Sympy [F]

$$\int x^2 (ax^2 + bx^3)^p \, dx = \int x^2 (x^2(a + bx))^p \, dx$$

```
integrate(x**2*(b*x**3+a*x**2)**p,x)
```

```
Integral(x**2*(x**2*(a + b*x))**p, x)
```

Maxima [F]

$$\int x^2 (ax^2 + bx^3)^p \, dx = \int (bx^3 + ax^2)^p x^2 \, dx$$

```
integrate(x^2*(b*x^3+a*x^2)^p,x, algorithm="maxima")
```

```
integrate((b*x^3 + a*x^2)^p*x^2, x)
```

Giac [F]

$$\int x^2 (ax^2 + bx^3)^p \, dx = \int (bx^3 + ax^2)^p x^2 \, dx$$

```
integrate(x^2*(b*x^3+a*x^2)^p,x, algorithm="giac")
```

```
integrate((b*x^3 + a*x^2)^p*x^2, x)
```

Mupad [F(-1)]

Timed out.

$$\int x^2 (ax^2 + bx^3)^p \, dx = \int x^2 \left(b x^3 + a x^2\right)^p dx$$

```
int(x^2*(a*x^2 + b*x^3)^p,x)
```

```
int(x^2*(a*x^2 + b*x^3)^p, x)
```

Reduce [F]

$$\int x^2 (ax^2 + bx^3)^p \, dx$$
$$= \frac{4(b x^3 + a x^2)^p a^3 p^2 + 6(b x^3 + a x^2)^p a^3 p + 2(b x^3 + a x^2)^p a^3 - 6(b x^3 + a x^2)^p a^2 b p^2 x - 6(b x^3 + a x^2)^p}{}$$

```
int(x^2*(b*x^3+a*x^2)^p,x)
```

```
(4*(a*x**2 + b*x**3)**p*a**3*p**2 + 6*(a*x**2 + b*x**3)**p*a**3*p + 2*(a*x
**2 + b*x**3)**p*a**3 - 6*(a*x**2 + b*x**3)**p*a**2*b*p**2*x - 6*(a*x**2 +
 b*x**3)**p*a**2*b*p*x + 9*(a*x**2 + b*x**3)**p*a*b**2*p**2*x**2 + 3*(a*x*
*2 + b*x**3)**p*a*b**2*p*x**2 + 27*(a*x**2 + b*x**3)**p*b**3*p**2*x**3 + 2
7*(a*x**2 + b*x**3)**p*b**3*p*x**3 + 6*(a*x**2 + b*x**3)**p*b**3*x**3 - 72
*int((a*x**2 + b*x**3)**p/(9*a*p**2*x + 9*a*p*x + 2*a*x + 9*b*p**2*x**2 +
9*b*p*x**2 + 2*b*x**2),x)*a**4*p**5 - 180*int((a*x**2 + b*x**3)**p/(9*a*p*
*2*x + 9*a*p*x + 2*a*x + 9*b*p**2*x**2 + 9*b*p*x**2 + 2*b*x**2),x)*a**4*p*
*4 - 160*int((a*x**2 + b*x**3)**p/(9*a*p**2*x + 9*a*p*x + 2*a*x + 9*b*p**2
*x**2 + 9*b*p*x**2 + 2*b*x**2),x)*a**4*p**3 - 60*int((a*x**2 + b*x**3)**p/
(9*a*p**2*x + 9*a*p*x + 2*a*x + 9*b*p**2*x**2 + 9*b*p*x**2 + 2*b*x**2),x)*
a**4*p**2 - 8*int((a*x**2 + b*x**3)**p/(9*a*p**2*x + 9*a*p*x + 2*a*x + 9*b
*p**2*x**2 + 9*b*p*x**2 + 2*b*x**2),x)*a**4*p)/(9*b**3*(9*p**3 + 18*p**2 +
 11*p + 2))
```

3.419 $\int x(ax^2+bx^3)^p\,dx$

Optimal result

Integrand size = 15, antiderivative size = 46

$$\int x\left(ax^2+bx^3\right)^p\,dx=\frac{\left(ax^2+bx^3\right)^{1+p}\text{Hypergeometric2F1}\left(1,3(1+p),3+2p,-\frac{bx}{a}\right)}{2a(1+p)}$$

```
1/2*(b*x^3+a*x^2)^(p+1)*hypergeom([1, 3*p+3],[3+2*p],-b*x/a)/a/(p+1)
```

Mathematica [A] (verified)

Time = 0.05 (sec) , antiderivative size = 55, normalized size of antiderivative = 1.20

$$\int x\left(ax^2+bx^3\right)^p\,dx = \frac{x^2(x^2(a+bx))^p\left(1+\frac{bx}{a}\right)^{-p}\text{Hypergeometric2F1}\left(-p,2+2p,3+2p,-\frac{bx}{a}\right)}{2+2p}$$

```
Integrate[x*(a*x^2 + b*x^3)^p,x]
```

```
(x^2*(x^2*(a + b*x))^p*Hypergeometric2F1[-p, 2 + 2*p, 3 + 2*p, -((b*x)/a)]
)/((2 + 2*p)*(1 + (b*x)/a)^p)
```

Rubi [A] (verified)

Time = 0.33 (sec) , antiderivative size = 66, normalized size of antiderivative = 1.43, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.200, Rules used = {1938, 76, 74}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x\left(ax^2+bx^3\right)^p\,dx$$

$$\Big\downarrow \text{1938}$$

$$x^{-2p}(a+bx)^{-p}\left(ax^2+bx^3\right)^p\int x^{2p+1}(a+bx)^pdx$$

$$\Big\downarrow \text{76}$$

$$x^{-2p}\left(\frac{bx}{a}+1\right)^{-p}\left(ax^2+bx^3\right)^p\int x^{2p+1}\left(\frac{bx}{a}+1\right)^pdx$$

$$\Big\downarrow \text{74}$$

$$\frac{x^{2(p+1)-2p}\left(\frac{bx}{a}+1\right)^{-p}\left(ax^2+bx^3\right)^p\text{Hypergeometric2F1}\left(-p,2(p+1),2p+3,-\frac{bx}{a}\right)}{2(p+1)}$$

```
Int[x*(a*x^2 + b*x^3)^p,x]
```

```
(x^(-2*p + 2*(1 + p))*(a*x^2 + b*x^3)^p*Hypergeometric2F1[-p, 2*(1 + p), 3
 + 2*p, -((b*x)/a)])/(2*(1 + p)*(1 + (b*x)/a)^p)
```

Defintions of rubi rules used

```
Int[((b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[c^n*((b*x
)^(m + 1)/(b*(m + 1)))*Hypergeometric2F1[-n, m + 1, m + 2, (-d)*(x/c)], x]
/; FreeQ[{b, c, d, m, n}, x] && !IntegerQ[m] && (IntegerQ[n] || (GtQ[c, 0]
 && !(EqQ[n, -2^(-1)] && EqQ[c^2 - d^2, 0] && GtQ[-d/(b*c), 0])))
```

```
Int[((b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[c^IntPart
[n]*((c + d*x)^FracPart[n]/(1 + d*(x/c))^FracPart[n])   Int[(b*x)^m*(1 + d*
(x/c))^n, x], x] /; FreeQ[{b, c, d, m, n}, x] && !IntegerQ[m] && !Integer
Q[n] && !GtQ[c, 0] && !GtQ[-d/(b*c), 0] && ((RationalQ[m] && !(EqQ[n, -2
^(-1)] && EqQ[c^2 - d^2, 0])) || !RationalQ[n])
```

```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> Simp[c^IntPart[m]*(c*x)^FracPart[m]*((a*x^j + b*x^n)^FracPart[p]/(x^(F
racPart[m] + j*FracPart[p])*(a + b*x^(n - j))^FracPart[p]))   Int[x^(m + j*
p)*(a + b*x^(n - j))^p, x], x] /; FreeQ[{a, b, c, j, m, n, p}, x] && !Inte
gerQ[p] && NeQ[n, j] && PosQ[n - j]
```

Maple [F]

$$\int x\left(b\,x^{3}+a\,x^{2}\right)^{p}dx$$

```
int(x*(b*x^3+a*x^2)^p,x)
```

```
int(x*(b*x^3+a*x^2)^p,x)
```

Fricas [F]

$$\int x\left(ax^{2}+bx^{3}\right)^{p}\,dx=\int\left(bx^{3}+ax^{2}\right)^{p}x\,dx$$

```
integrate(x*(b*x^3+a*x^2)^p,x, algorithm="fricas")
```

```
integral((b*x^3 + a*x^2)^p*x, x)
```

Sympy [F]

$$\int x\left(ax^2+bx^3\right)^p\,dx=\int x\left(x^2(a+bx)\right)^p\,dx$$

```
integrate(x*(b*x**3+a*x**2)**p,x)
```

```
Integral(x*(x**2*(a + b*x))**p, x)
```

Maxima [F]

$$\int x\left(ax^2+bx^3\right)^p\,dx=\int \left(bx^3+ax^2\right)^p x\,dx$$

```
integrate(x*(b*x^3+a*x^2)^p,x, algorithm="maxima")
```

```
integrate((b*x^3 + a*x^2)^p*x, x)
```

Giac [F]

$$\int x\left(ax^2+bx^3\right)^p\,dx=\int \left(bx^3+ax^2\right)^p x\,dx$$

```
integrate(x*(b*x^3+a*x^2)^p,x, algorithm="giac")
```

```
integrate((b*x^3 + a*x^2)^p*x, x)
```

Mupad [F(-1)]

Timed out.

$$\int x(ax^2+bx^3)^p\,dx=\int x\,(b\,x^3+a\,x^2)^p\,dx$$

```
int(x*(a*x^2 + b*x^3)^p,x)
```

```
int(x*(a*x^2 + b*x^3)^p, x)
```

Reduce [F]

$$\int x(ax^2+bx^3)^p\,dx$$
$$=\frac{-2(b\,x^3+a\,x^2)^p\,a^2p-(b\,x^3+a\,x^2)^p\,a^2+3(b\,x^3+a\,x^2)^p\,abpx+9(b\,x^3+a\,x^2)^p\,b^2p\,x^2+3(b\,x^3+a\,x^2)^p\,b}{}$$

```
int(x*(b*x^3+a*x^2)^p,x)
```

```
( - 2*(a*x**2 + b*x**3)**p*a**2*p - (a*x**2 + b*x**3)**p*a**2 + 3*(a*x**2
+ b*x**3)**p*a*b*p*x + 9*(a*x**2 + b*x**3)**p*b**2*p*x**2 + 3*(a*x**2 + b*
x**3)**p*b**2*x**2 + 36*int((a*x**2 + b*x**3)**p/(9*a*p**2*x + 9*a*p*x + 2
*a*x + 9*b*p**2*x**2 + 9*b*p*x**2 + 2*b*x**2),x)*a**3*p**4 + 54*int((a*x**
2 + b*x**3)**p/(9*a*p**2*x + 9*a*p*x + 2*a*x + 9*b*p**2*x**2 + 9*b*p*x**2
+ 2*b*x**2),x)*a**3*p**3 + 26*int((a*x**2 + b*x**3)**p/(9*a*p**2*x + 9*a*p
*x + 2*a*x + 9*b*p**2*x**2 + 9*b*p*x**2 + 2*b*x**2),x)*a**3*p**2 + 4*int((
a*x**2 + b*x**3)**p/(9*a*p**2*x + 9*a*p*x + 2*a*x + 9*b*p**2*x**2 + 9*b*p*
x**2 + 2*b*x**2),x)*a**3*p)/(3*b**2*(9*p**2 + 9*p + 2))
```

3.420 $\int (ax^2 + bx^3)^p \, dx$

Optimal result

Integrand size = 13, antiderivative size = 48

$$\int (ax^2 + bx^3)^p \, dx = \frac{(ax^2 + bx^3)^{1+p} \text{Hypergeometric2F1}\left(1, 2 + 3p, 2(1 + p), -\frac{bx}{a}\right)}{a(1 + 2p)x}$$

```
(b*x^3+a*x^2)^(p+1)*hypergeom([1, 2+3*p],[2*p+2],-b*x/a)/a/(1+2*p)/x
```

Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 53, normalized size of antiderivative = 1.10

$$\int (ax^2 + bx^3)^p \, dx = \frac{x(x^2(a + bx))^p \left(1 + \frac{bx}{a}\right)^{-p} \text{Hypergeometric2F1}\left(-p, 1 + 2p, 2 + 2p, -\frac{bx}{a}\right)}{1 + 2p}$$

```
Integrate[(a*x^2 + b*x^3)^p,x]
```

```
(x*(x^2*(a + b*x))^p*Hypergeometric2F1[-p, 1 + 2*p, 2 + 2*p, -((b*x)/a)])/
((1 + 2*p)*(1 + (b*x)/a)^p)
```

Rubi [A] (verified)

Time = 0.31 (sec) , antiderivative size = 55, normalized size of antiderivative = 1.15, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.231, Rules used = {1917, 76, 74}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (ax^2+bx^3)^p\,dx$$

$$\Big\downarrow \ 1917$$

$$x^{-2p}(a+bx)^{-p}(ax^2+bx^3)^p\int x^{2p}(a+bx)^pdx$$

$$\Big\downarrow \ 76$$

$$x^{-2p}\left(\frac{bx}{a}+1\right)^{-p}(ax^2+bx^3)^p\int x^{2p}\left(\frac{bx}{a}+1\right)^p dx$$

$$\Big\downarrow \ 74$$

$$\frac{x\left(\frac{bx}{a}+1\right)^{-p}(ax^2+bx^3)^p\,\text{Hypergeometric2F1}\left(-p,2p+1,2(p+1),-\frac{bx}{a}\right)}{2p+1}$$

```
Int[(a*x^2 + b*x^3)^p,x]
```

```
(x*(a*x^2 + b*x^3)^p*Hypergeometric2F1[-p, 1 + 2*p, 2*(1 + p), -((b*x)/a)]
)/((1 + 2*p)*(1 + (b*x)/a)^p)
```

Defintions of rubi rules used

```
Int[((b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[c^n*((b*x
)^(m + 1)/(b*(m + 1)))*Hypergeometric2F1[-n, m + 1, m + 2, (-d)*(x/c)], x]
/; FreeQ[{b, c, d, m, n}, x] && !IntegerQ[m] && (IntegerQ[n] || (GtQ[c, 0]
 && !(EqQ[n, -2^(-1)] && EqQ[c^2 - d^2, 0] && GtQ[-d/(b*c), 0])))
```

```
Int[((b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[c^IntPart
[n]*((c + d*x)^FracPart[n]/(1 + d*(x/c))^FracPart[n])   Int[(b*x)^m*(1 + d*
(x/c))^n, x], x] /; FreeQ[{b, c, d, m, n}, x] &&  !IntegerQ[m] &&  !Integer
Q[n] &&  !GtQ[c, 0] &&  !GtQ[-d/(b*c), 0] && ((RationalQ[m] &&  !(EqQ[n, -2
^(-1)] && EqQ[c^2 - d^2, 0])) ||  !RationalQ[n])
```

```
Int[((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol] :> Simp[(a*x^j +
b*x^n)^FracPart[p]/(x^(j*FracPart[p])*(a + b*x^(n - j))^FracPart[p])   Int[
x^(j*p)*(a + b*x^(n - j))^p, x], x] /; FreeQ[{a, b, j, n, p}, x] &&  !Integ
erQ[p] && NeQ[n, j] && PosQ[n - j]
```

Maple [F]

$$\int \left(b\,x^{3}+a\,x^{2}\right)^{p} dx$$

```
int((b*x^3+a*x^2)^p,x)
```

```
int((b*x^3+a*x^2)^p,x)
```

Fricas [F]

$$\int \left(ax^{2}+bx^{3}\right)^{p}\,dx=\int \left(bx^{3}+ax^{2}\right)^{p}dx$$

```
integrate((b*x^3+a*x^2)^p,x, algorithm="fricas")
```

```
integral((b*x^3 + a*x^2)^p, x)
```

Sympy [F]

$$\int \left(ax^2 + bx^3\right)^p \, dx = \int \left(ax^2 + bx^3\right)^p \, dx$$

```
integrate((b*x**3+a*x**2)**p,x)
```

```
Integral((a*x**2 + b*x**3)**p, x)
```

Maxima [F]

$$\int \left(ax^2 + bx^3\right)^p \, dx = \int \left(bx^3 + ax^2\right)^p \, dx$$

```
integrate((b*x^3+a*x^2)^p,x, algorithm="maxima")
```

```
integrate((b*x^3 + a*x^2)^p, x)
```

Giac [F]

$$\int \left(ax^2 + bx^3\right)^p \, dx = \int \left(bx^3 + ax^2\right)^p \, dx$$

```
integrate((b*x^3+a*x^2)^p,x, algorithm="giac")
```

```
integrate((b*x^3 + a*x^2)^p, x)
```

Mupad [B] (verification not implemented)

Time = 8.74 (sec) , antiderivative size = 56, normalized size of antiderivative = 1.17

$$\int \left(ax^2+bx^3\right)^p\,dx=\frac{x\,{\left(b\,x^3+a\,x^2\right)}^p\,{{}}_2{\mathrm{F}}_1\left(2\,p+1,-p;\ 2\,p+2;\ -\frac{b\,x}{a}\right)}{\left(2\,p+1\right)\,{\left(\frac{b\,x}{a}+1\right)}^p}$$

```
int((a*x^2 + b*x^3)^p,x)
```

```
(x*(a*x^2 + b*x^3)^p*hypergeom([2*p + 1, -p], 2*p + 2, -(b*x)/a))/((2*p +
1)*((b*x)/a + 1)^p)
```

Reduce [F]

$$\int \left(ax^2+bx^3\right)^p\,dx$$
$$=\frac{\left(b\,x^3+a\,x^2\right)^p a+3\left(b\,x^3+a\,x^2\right)^p bx-6\left(\int \frac{\left(b\,x^3+a\,x^2\right)^p}{3bp\,x^2+3apx+b\,x^2+ax}dx\right)a^2p^2-2\left(\int \frac{\left(b\,x^3+a\,x^2\right)^p}{3bp\,x^2+3apx+b\,x^2+ax}dx\right)a^2p}{3b\left(3p+1\right)}$$

```
int((b*x^3+a*x^2)^p,x)
```

```
((a*x**2 + b*x**3)**p*a + 3*(a*x**2 + b*x**3)**p*b*x - 6*int((a*x**2 + b*x
**3)**p/(3*a*p*x + a*x + 3*b*p*x**2 + b*x**2),x)*a**2*p**2 - 2*int((a*x**2
 + b*x**3)**p/(3*a*p*x + a*x + 3*b*p*x**2 + b*x**2),x)*a**2*p)/(3*b*(3*p +
 1))
```

3.421 $\int \frac{(ax^2+bx^3)^p}{x} dx$

Optimal result

Integrand size = 17, antiderivative size = 47

$$\int \frac{(ax^2+bx^3)^p}{x} dx = \frac{(ax^2+bx^3)^{1+p} \text{Hypergeometric2F1}\left(1, 1+3p, 1+2p, -\frac{bx}{a}\right)}{2apx^2}$$

```
1/2*(b*x^3+a*x^2)^(p+1)*hypergeom([1, 1+3*p],[1+2*p],-b*x/a)/a/p/x^2
```

Mathematica [A] (verified)

Time = 0.03 (sec) , antiderivative size = 49, normalized size of antiderivative = 1.04

$$\int \frac{(ax^2+bx^3)^p}{x} dx = \frac{(x^2(a+bx))^p \left(1+\frac{bx}{a}\right)^{-p} \text{Hypergeometric2F1}\left(-p, 2p, 1+2p, -\frac{bx}{a}\right)}{2p}$$

```
Integrate[(a*x^2 + b*x^3)^p/x,x]
```

```
((x^2*(a + b*x))^p*Hypergeometric2F1[-p, 2*p, 1 + 2*p, -((b*x)/a)])/(2*p*(
1 + (b*x)/a)^p)
```

Rubi [A] (verified)

Time = 0.31 (sec) , antiderivative size = 51, normalized size of antiderivative = 1.09, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.176, Rules used = {1938, 76, 74}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(ax^2+bx^3)^p}{x}dx$$

$$\Big\downarrow \text{1938}$$

$$x^{-2p}(a+bx)^{-p}\left(ax^2+bx^3\right)^p\int x^{2p-1}(a+bx)^pdx$$

$$\Big\downarrow \text{76}$$

$$x^{-2p}\left(\frac{bx}{a}+1\right)^{-p}\left(ax^2+bx^3\right)^p\int x^{2p-1}\left(\frac{bx}{a}+1\right)^pdx$$

$$\Big\downarrow \text{74}$$

$$\frac{\left(\frac{bx}{a}+1\right)^{-p}\left(ax^2+bx^3\right)^p\text{Hypergeometric2F1}\left(-p,2p,2p+1,-\frac{bx}{a}\right)}{2p}$$

```
Int[(a*x^2 + b*x^3)^p/x,x]
```

```
((a*x^2 + b*x^3)^p*Hypergeometric2F1[-p, 2*p, 1 + 2*p, -((b*x)/a)])/(2*p*(
1 + (b*x)/a)^p)
```

Defintions of rubi rules used

```
Int[((b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[c^n*((b*x
)^(m + 1)/(b*(m + 1)))*Hypergeometric2F1[-n, m + 1, m + 2, (-d)*(x/c)], x]
/; FreeQ[{b, c, d, m, n}, x] &&  !IntegerQ[m] && (IntegerQ[n] || (GtQ[c, 0]
 &&  !(EqQ[n, -2^(-1)] && EqQ[c^2 - d^2, 0] && GtQ[-d/(b*c), 0])))
```

```
Int[((b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[c^IntPart
[n]*((c + d*x)^FracPart[n]/(1 + d*(x/c))^FracPart[n])   Int[(b*x)^m*(1 + d*
(x/c))^n, x], x] /; FreeQ[{b, c, d, m, n}, x] &&  !IntegerQ[m] &&  !Integer
Q[n] &&  !GtQ[c, 0] &&  !GtQ[-d/(b*c), 0] && ((RationalQ[m] &&  !(EqQ[n, -2
^(-1)] && EqQ[c^2 - d^2, 0])) ||  !RationalQ[n])
```

```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> Simp[c^IntPart[m]*(c*x)^FracPart[m]*((a*x^j + b*x^n)^FracPart[p]/(x^(F
racPart[m] + j*FracPart[p])*(a + b*x^(n - j))^FracPart[p]))   Int[x^(m + j*
p)*(a + b*x^(n - j))^p, x], x] /; FreeQ[{a, b, c, j, m, n, p}, x] &&  !Inte
gerQ[p] && NeQ[n, j] && PosQ[n - j]
```

Maple [F]

$$\int \frac{(b\,x^3 + a\,x^2)^p}{x}dx$$

```
int((b*x^3+a*x^2)^p/x,x)
```

```
int((b*x^3+a*x^2)^p/x,x)
```

Fricas [F]

$$\int \frac{(ax^2 + bx^3)^p}{x}\,dx = \int \frac{(bx^3 + ax^2)^p}{x}\,dx$$

```
integrate((b*x^3+a*x^2)^p/x,x, algorithm="fricas")
```

```
integral((b*x^3 + a*x^2)^p/x, x)
```

Sympy [F]

$$\int \frac{(ax^2+bx^3)^p}{x}\,dx = \int \frac{(x^2(a+bx))^p}{x}\,dx$$

```
integrate((b*x**3+a*x**2)**p/x,x)
```

```
Integral((x**2*(a + b*x))**p/x, x)
```

Maxima [F]

$$\int \frac{(ax^2+bx^3)^p}{x}\,dx = \int \frac{(bx^3+ax^2)^p}{x}\,dx$$

```
integrate((b*x^3+a*x^2)^p/x,x, algorithm="maxima")
```

```
integrate((b*x^3 + a*x^2)^p/x, x)
```

Giac [F]

$$\int \frac{(ax^2+bx^3)^p}{x}\,dx = \int \frac{(bx^3+ax^2)^p}{x}\,dx$$

```
integrate((b*x^3+a*x^2)^p/x,x, algorithm="giac")
```

```
integrate((b*x^3 + a*x^2)^p/x, x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(ax^2+bx^3)^p}{x}\,dx = \int \frac{(b\,x^3+a\,x^2)^p}{x}\,dx$$

```
int((a*x^2 + b*x^3)^p/x,x)
```

```
int((a*x^2 + b*x^3)^p/x, x)
```

Reduce [F]

$$\int \frac{(ax^2+bx^3)^p}{x}\,dx = \frac{(b\,x^3+a\,x^2)^p + \left(\int \frac{(b\,x^3+a\,x^2)^p}{b\,x^2+ax}dx\right)ap}{3p}$$

```
int((b*x^3+a*x^2)^p/x,x)
```

```
((a*x**2 + b*x**3)**p + int((a*x**2 + b*x**3)**p/(a*x + b*x**2),x)*a*p)/(3
*p)
```

3.422 $\int \frac{(ax^2+bx^3)^p}{x^2} dx$

Optimal result

Integrand size = 17, antiderivative size = 45

$$\int \frac{(ax^2+bx^3)^p}{x^2} dx = -\frac{(ax^2+bx^3)^{1+p} \text{Hypergeometric2F1}\left(1,3p,2p,-\frac{bx}{a}\right)}{a(1-2p)x^3}$$

```
-(b*x^3+a*x^2)^(p+1)*hypergeom([1, 3*p],[2*p],-b*x/a)/a/(1-2*p)/x^3
```

Mathematica [A] (verified)

Time = 0.03 (sec) , antiderivative size = 53, normalized size of antiderivative = 1.18

$$\int \frac{(ax^2+bx^3)^p}{x^2} dx = \frac{(x^2(a+bx))^p \left(1+\frac{bx}{a}\right)^{-p} \text{Hypergeometric2F1}\left(-p,-1+2p,2p,-\frac{bx}{a}\right)}{(-1+2p)x}$$

```
Integrate[(a*x^2 + b*x^3)^p/x^2,x]
```

```
((x^2*(a + b*x))^p*Hypergeometric2F1[-p, -1 + 2*p, 2*p, -((b*x)/a)])/((-1
+ 2*p)*x*(1 + (b*x)/a)^p)
```

Rubi [A] (verified)

Time = 0.32 (sec) , antiderivative size = 56, normalized size of antiderivative = 1.24, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.176, Rules used = {1938, 76, 74}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(ax^2+bx^3)^p}{x^2}dx$$

$$\Big\downarrow \text{1938}$$

$$x^{-2p}(a+bx)^{-p}\left(ax^2+bx^3\right)^p\int x^{-2(1-p)}(a+bx)^pdx$$

$$\Big\downarrow \text{76}$$

$$x^{-2p}\left(\frac{bx}{a}+1\right)^{-p}\left(ax^2+bx^3\right)^p\int x^{-2(1-p)}\left(\frac{bx}{a}+1\right)^pdx$$

$$\Big\downarrow \text{74}$$

$$-\frac{\left(\frac{bx}{a}+1\right)^{-p}\left(ax^2+bx^3\right)^p\text{Hypergeometric2F1}\left(-p,2p-1,2p,-\frac{bx}{a}\right)}{(1-2p)x}$$

```
Int[(a*x^2 + b*x^3)^p/x^2,x]
```

```
-(((a*x^2 + b*x^3)^p*Hypergeometric2F1[-p, -1 + 2*p, 2*p, -((b*x)/a)])/((1
 - 2*p)*x*(1 + (b*x)/a)^p))
```

Defintions of rubi rules used

```
Int[((b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[c^n*((b*x
)^(m + 1)/(b*(m + 1)))*Hypergeometric2F1[-n, m + 1, m + 2, (-d)*(x/c)], x]
/; FreeQ[{b, c, d, m, n}, x] &&  !IntegerQ[m] && (IntegerQ[n] || (GtQ[c, 0]
 &&  !(EqQ[n, -2^(-1)] && EqQ[c^2 - d^2, 0] && GtQ[-d/(b*c), 0])))
```

```
Int[((b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[c^IntPart
[n]*((c + d*x)^FracPart[n]/(1 + d*(x/c))^FracPart[n])   Int[(b*x)^m*(1 + d*
(x/c))^n, x], x] /; FreeQ[{b, c, d, m, n}, x] &&  !IntegerQ[m] &&  !Integer
Q[n] &&  !GtQ[c, 0] &&  !GtQ[-d/(b*c), 0] && ((RationalQ[m] &&  !(EqQ[n, -2
^(-1)] && EqQ[c^2 - d^2, 0])) ||  !RationalQ[n])
```

```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> Simp[c^IntPart[m]*(c*x)^FracPart[m]*((a*x^j + b*x^n)^FracPart[p]/(x^(F
racPart[m] + j*FracPart[p])*(a + b*x^(n - j))^FracPart[p]))   Int[x^(m + j*
p)*(a + b*x^(n - j))^p, x], x] /; FreeQ[{a, b, c, j, m, n, p}, x] &&  !Inte
gerQ[p] && NeQ[n, j] && PosQ[n - j]
```

Maple [F]

$$\int \frac{(b\,x^3 + a\,x^2)^p}{x^2}dx$$

```
int((b*x^3+a*x^2)^p/x^2,x)
```

```
int((b*x^3+a*x^2)^p/x^2,x)
```

Fricas [F]

$$\int \frac{(ax^2 + bx^3)^p}{x^2}\,dx = \int \frac{(bx^3 + ax^2)^p}{x^2}\,dx$$

```
integrate((b*x^3+a*x^2)^p/x^2,x, algorithm="fricas")
```

```
integral((b*x^3 + a*x^2)^p/x^2, x)
```

Sympy [F]

$$\int \frac{(ax^2 + bx^3)^p}{x^2} \, dx = \int \frac{(x^2(a + bx))^p}{x^2} \, dx$$

```
integrate((b*x**3+a*x**2)**p/x**2,x)
```

```
Integral((x**2*(a + b*x))**p/x**2, x)
```

Maxima [F]

$$\int \frac{(ax^2 + bx^3)^p}{x^2} \, dx = \int \frac{(bx^3 + ax^2)^p}{x^2} \, dx$$

```
integrate((b*x^3+a*x^2)^p/x^2,x, algorithm="maxima")
```

```
integrate((b*x^3 + a*x^2)^p/x^2, x)
```

Giac [F]

$$\int \frac{(ax^2 + bx^3)^p}{x^2} \, dx = \int \frac{(bx^3 + ax^2)^p}{x^2} \, dx$$

```
integrate((b*x^3+a*x^2)^p/x^2,x, algorithm="giac")
```

```
integrate((b*x^3 + a*x^2)^p/x^2, x)
```

Mupad [B] (verification not implemented)

Time = 8.79 (sec) , antiderivative size = 63, normalized size of antiderivative = 1.40

$$\int \frac{(ax^2+bx^3)^p}{x^2}\,dx = \frac{\left(x^3\left(b+\frac{a}{x}\right)\right)^p {}_2F_1\left(1-3\,p,-p;\ 2-3\,p;\ -\frac{a}{b\,x}\right)}{x\,(3\,p-1)\left(\frac{b+\frac{a}{x}}{b}\right)^p}$$

```
int((a*x^2 + b*x^3)^p/x^2,x)
```

```
((x^3*(b + a/x))^p*hypergeom([1 - 3*p, -p], 2 - 3*p, -a/(b*x)))/(x*(3*p -
1)*((b + a/x)/b)^p)
```

Reduce [F]

$$\int \frac{(ax^2+bx^3)^p}{x^2}\,dx = \frac{(b\,x^3+a\,x^2)^p+3\left(\int \frac{(b\,x^3+a\,x^2)^p}{3bp\,x^3+3ap\,x^2-b\,x^3-a\,x^2}dx\right)a\,p^2x-\left(\int \frac{(b\,x^3+a\,x^2)^p}{3bp\,x^3+3ap\,x^2-b\,x^3-a\,x^2}dx\right)apx}{x\,(3p-1)}$$

```
int((b*x^3+a*x^2)^p/x^2,x)
```

```
((a*x**2 + b*x**3)**p + 3*int((a*x**2 + b*x**3)**p/(3*a*p*x**2 - a*x**2 +
3*b*p*x**3 - b*x**3),x)*a*p**2*x - int((a*x**2 + b*x**3)**p/(3*a*p*x**2 -
a*x**2 + 3*b*p*x**3 - b*x**3),x)*a*p*x)/(x*(3*p - 1))
```

3.423 $\int (cx)^{3/2} (ax^2 + bx^3)^p \, dx$

Optimal result

Integrand size = 21, antiderivative size = 58

$$\int (cx)^{3/2} (ax^2 + bx^3)^p \, dx = \frac{2c\sqrt{cx}(ax^2+bx^3)^{1+p}\,\text{Hypergeometric2F1}\left(1, \frac{7}{2}+3p, \frac{7}{2}+2p, -\frac{bx}{a}\right)}{a(5+4p)}$$

```
2*c*(c*x)^(1/2)*(b*x^3+a*x^2)^(p+1)*hypergeom([1, 7/2+3*p],[7/2+2*p],-b*x/
a)/a/(5+4*p)
```

Mathematica [A] (verified)

Time = 0.03 (sec) , antiderivative size = 66, normalized size of antiderivative = 1.14

$$\int (cx)^{3/2} (ax^2 + bx^3)^p \, dx = \frac{x(cx)^{3/2} (x^2(a+bx))^p \left(1+\frac{bx}{a}\right)^{-p} \text{Hypergeometric2F1}\left(-p, \frac{5}{2}+2p, \frac{7}{2}+2p, -\frac{bx}{a}\right)}{\frac{5}{2}+2p}$$

```
Integrate[(c*x)^(3/2)*(a*x^2 + b*x^3)^p,x]
```

```
(x*(c*x)^(3/2)*(x^2*(a + b*x))^p*Hypergeometric2F1[-p, 5/2 + 2*p, 7/2 + 2*
p, -((b*x)/a)])/((5/2 + 2*p)*(1 + (b*x)/a)^p)
```

Rubi [A] (verified)

Time = 0.34 (sec) , antiderivative size = 70, normalized size of antiderivative = 1.21, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.143, Rules used = {1938, 76, 74}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (cx)^{3/2} \left(ax^2+bx^3\right)^p \, dx$$

$$\Big\downarrow \text{1938}$$

$$c\sqrt{cx}x^{-2p-\frac{1}{2}}(a+bx)^{-p}\left(ax^2+bx^3\right)^p\int x^{2p+\frac{3}{2}}(a+bx)^pdx$$

$$\Big\downarrow \text{76}$$

$$c\sqrt{cx}x^{-2p-\frac{1}{2}}\left(\frac{bx}{a}+1\right)^{-p}\left(ax^2+bx^3\right)^p\int x^{2p+\frac{3}{2}}\left(\frac{bx}{a}+1\right)^pdx$$

$$\Big\downarrow \text{74}$$

$$\frac{2cx^2\sqrt{cx}\left(\frac{bx}{a}+1\right)^{-p}\left(ax^2+bx^3\right)^p \text{Hypergeometric2F1}\left(-p,2p+\frac{5}{2},2p+\frac{7}{2},-\frac{bx}{a}\right)}{4p+5}$$

```
Int[(c*x)^(3/2)*(a*x^2 + b*x^3)^p,x]
```

```
(2*c*x^2*Sqrt[c*x]*(a*x^2 + b*x^3)^p*Hypergeometric2F1[-p, 5/2 + 2*p, 7/2
+ 2*p, -((b*x)/a)])/((5 + 4*p)*(1 + (b*x)/a)^p)
```

Defintions of rubi rules used

```
Int[((b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[c^n*((b*x
)^(m + 1)/(b*(m + 1)))*Hypergeometric2F1[-n, m + 1, m + 2, (-d)*(x/c)], x]
/; FreeQ[{b, c, d, m, n}, x] &&  !IntegerQ[m] && (IntegerQ[n] || (GtQ[c, 0]
 &&  !(EqQ[n, -2^(-1)] && EqQ[c^2 - d^2, 0] && GtQ[-d/(b*c), 0])))
```

```
Int[((b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[c^IntPart
[n]*((c + d*x)^FracPart[n]/(1 + d*(x/c))^FracPart[n])   Int[(b*x)^m*(1 + d*
(x/c))^n, x], x] /; FreeQ[{b, c, d, m, n}, x] &&  !IntegerQ[m] &&  !Integer
Q[n] &&  !GtQ[c, 0] &&  !GtQ[-d/(b*c), 0] && ((RationalQ[m] &&  !(EqQ[n, -2
^(-1)] && EqQ[c^2 - d^2, 0])) ||  !RationalQ[n])
```

```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> Simp[c^IntPart[m]*(c*x)^FracPart[m]*((a*x^j + b*x^n)^FracPart[p]/(x^(F
racPart[m] + j*FracPart[p])*(a + b*x^(n - j))^FracPart[p]))   Int[x^(m + j*
p)*(a + b*x^(n - j))^p, x], x] /; FreeQ[{a, b, c, j, m, n, p}, x] &&  !Inte
gerQ[p] && NeQ[n, j] && PosQ[n - j]
```

Maple [F]

$$\int (cx)^{\frac{3}{2}} \left(b\,x^{3}+a\,x^{2}\right)^{p} dx$$

```
int((c*x)^(3/2)*(b*x^3+a*x^2)^p,x)
```

```
int((c*x)^(3/2)*(b*x^3+a*x^2)^p,x)
```

Fricas [F]

$$\int (cx)^{3/2} \left(ax^2+bx^3\right)^p \, dx = \int (cx)^{\frac{3}{2}} \left(bx^3+ax^2\right)^p \, dx$$

```
integrate((c*x)^(3/2)*(b*x^3+a*x^2)^p,x, algorithm="fricas")
```

```
integral(sqrt(c*x)*(b*x^3 + a*x^2)^p*c*x, x)
```

Sympy [F]

$$\int (cx)^{3/2} \left(ax^2+bx^3\right)^p \, dx = \int (cx)^{\frac{3}{2}} \left(x^2(a+bx)\right)^p \, dx$$

```
integrate((c*x)**(3/2)*(b*x**3+a*x**2)**p,x)
```

```
Integral((c*x)**(3/2)*(x**2*(a + b*x))**p, x)
```

Maxima [F]

$$\int (cx)^{3/2} \left(ax^2+bx^3\right)^p \, dx = \int (cx)^{\frac{3}{2}} \left(bx^3+ax^2\right)^p \, dx$$

```
integrate((c*x)^(3/2)*(b*x^3+a*x^2)^p,x, algorithm="maxima")
```

```
integrate((c*x)^(3/2)*(b*x^3 + a*x^2)^p, x)
```

Giac [F]

$$\int (cx)^{3/2} \left(ax^2+bx^3\right)^p \, dx = \int (cx)^{\frac{3}{2}} \left(bx^3+ax^2\right)^p \, dx$$

```
integrate((c*x)^(3/2)*(b*x^3+a*x^2)^p,x, algorithm="giac")
```

```
integrate((c*x)^(3/2)*(b*x^3 + a*x^2)^p, x)
```

Mupad [F(-1)]

Timed out.

$$\int (cx)^{3/2} \left(ax^2+bx^3\right)^p \, dx = \int (c\,x)^{3/2}\,\left(b\,x^3+a\,x^2\right)^p \, dx$$

```
int((c*x)^(3/2)*(a*x^2 + b*x^3)^p,x)
```

```
int((c*x)^(3/2)*(a*x^2 + b*x^3)^p, x)
```

Reduce [F]

$$\int (cx)^{3/2} \left(ax^2 +bx^3\right)^p \, dx = \frac{2\sqrt{c}\,c\Big(-8\sqrt{x}\,(b\,x^3+a\,x^2)^p\,a^2p^2-6\sqrt{x}\,(b\,x^3+a\,x^2)^p\,a^2p+12\sqrt{x}\,(b\,x^3+a\,x^2)^p\,ab\,p^2x+2\sqrt{x}}{}$$

```
int((c*x)^(3/2)*(b*x^3+a*x^2)^p,x)
```

```
(2*sqrt(c)*c*( - 8*sqrt(x)*(a*x**2 + b*x**3)**p*a**2*p**2 - 6*sqrt(x)*(a*x
**2 + b*x**3)**p*a**2*p + 12*sqrt(x)*(a*x**2 + b*x**3)**p*a*b*p**2*x + 2*s
qrt(x)*(a*x**2 + b*x**3)**p*a*b*p*x + 36*sqrt(x)*(a*x**2 + b*x**3)**p*b**2
*p**2*x**2 + 24*sqrt(x)*(a*x**2 + b*x**3)**p*b**2*p*x**2 + 3*sqrt(x)*(a*x*
*2 + b*x**3)**p*b**2*x**2 + 1152*int((sqrt(x)*(a*x**2 + b*x**3)**p)/(72*a*
p**3*x + 108*a*p**2*x + 46*a*p*x + 5*a*x + 72*b*p**3*x**2 + 108*b*p**2*x**
2 + 46*b*p*x**2 + 5*b*x**2),x)*a**3*p**6 + 2880*int((sqrt(x)*(a*x**2 + b*x
**3)**p)/(72*a*p**3*x + 108*a*p**2*x + 46*a*p*x + 5*a*x + 72*b*p**3*x**2 +
 108*b*p**2*x**2 + 46*b*p*x**2 + 5*b*x**2),x)*a**3*p**5 + 2680*int((sqrt(x
)*(a*x**2 + b*x**3)**p)/(72*a*p**3*x + 108*a*p**2*x + 46*a*p*x + 5*a*x + 7
2*b*p**3*x**2 + 108*b*p**2*x**2 + 46*b*p*x**2 + 5*b*x**2),x)*a**3*p**4 + 1
140*int((sqrt(x)*(a*x**2 + b*x**3)**p)/(72*a*p**3*x + 108*a*p**2*x + 46*a*
p*x + 5*a*x + 72*b*p**3*x**2 + 108*b*p**2*x**2 + 46*b*p*x**2 + 5*b*x**2),x
)*a**3*p**3 + 218*int((sqrt(x)*(a*x**2 + b*x**3)**p)/(72*a*p**3*x + 108*a*
p**2*x + 46*a*p*x + 5*a*x + 72*b*p**3*x**2 + 108*b*p**2*x**2 + 46*b*p*x**2
 + 5*b*x**2),x)*a**3*p**2 + 15*int((sqrt(x)*(a*x**2 + b*x**3)**p)/(72*a*p*
*3*x + 108*a*p**2*x + 46*a*p*x + 5*a*x + 72*b*p**3*x**2 + 108*b*p**2*x**2
+ 46*b*p*x**2 + 5*b*x**2),x)*a**3*p))/(3*b**2*(72*p**3 + 108*p**2 + 46*p +
 5))
```

3.424 $\int \sqrt{cx}(ax^2+bx^3)^p\,dx$

Optimal result

Integrand size = 21, antiderivative size = 58

$$\int \sqrt{cx}\left(ax^2+bx^3\right)^p\,dx=\frac{2c\left(ax^2+bx^3\right)^{1+p}\operatorname{Hypergeometric2F1}\left(1,\frac{5}{2}+3p,\frac{5}{2}+2p,-\frac{bx}{a}\right)}{a(3+4p)\sqrt{cx}}$$

```
2*c*(b*x^3+a*x^2)^(p+1)*hypergeom([1, 5/2+3*p],[5/2+2*p],-b*x/a)/a/(3+4*p)
/(c*x)^(1/2)
```

Mathematica [A] (verified)

Time = 0.03 (sec) , antiderivative size = 66, normalized size of antiderivative = 1.14

$$\begin{aligned}&\int \sqrt{cx}\left(ax^2+bx^3\right)^p\,dx\\&=\frac{x\sqrt{cx}\left(x^2(a+bx)\right)^p\left(1+\frac{bx}{a}\right)^{-p}\operatorname{Hypergeometric2F1}\left(-p,\frac{3}{2}+2p,\frac{5}{2}+2p,-\frac{bx}{a}\right)}{\frac{3}{2}+2p}\end{aligned}$$

```
Integrate[Sqrt[c*x]*(a*x^2 + b*x^3)^p,x]
```

```
(x*Sqrt[c*x]*(x^2*(a + b*x))^p*Hypergeometric2F1[-p, 3/2 + 2*p, 5/2 + 2*p,
 -((b*x)/a)])/((3/2 + 2*p)*(1 + (b*x)/a)^p)
```

Rubi [A] (verified)

Time = 0.33 (sec) , antiderivative size = 67, normalized size of antiderivative = 1.16, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.143$, Rules used = {1938, 76, 74}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sqrt{cx}(ax^2+bx^3)^p\,dx$$

$$\Big\downarrow \text{ 1938}$$

$$\sqrt{cx}x^{-2p-\frac{1}{2}}(a+bx)^{-p}\left(ax^2+bx^3\right)^p\int x^{2p+\frac{1}{2}}(a+bx)^pdx$$

$$\Big\downarrow \text{ 76}$$

$$\sqrt{cx}x^{-2p-\frac{1}{2}}\left(\frac{bx}{a}+1\right)^{-p}\left(ax^2+bx^3\right)^p\int x^{2p+\frac{1}{2}}\left(\frac{bx}{a}+1\right)^pdx$$

$$\Big\downarrow \text{ 74}$$

$$\frac{2x\sqrt{cx}\left(\frac{bx}{a}+1\right)^{-p}\left(ax^2+bx^3\right)^p\text{Hypergeometric2F1}\left(-p,2p+\frac{3}{2},2p+\frac{5}{2},-\frac{bx}{a}\right)}{4p+3}$$

```
Int[Sqrt[c*x]*(a*x^2 + b*x^3)^p,x]
```

```
(2*x*Sqrt[c*x]*(a*x^2 + b*x^3)^p*Hypergeometric2F1[-p, 3/2 + 2*p, 5/2 + 2*
p, -((b*x)/a)])/((3 + 4*p)*(1 + (b*x)/a)^p)
```

Defintions of rubi rules used

```
Int[((b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[c^n*((b*x
)^(m + 1)/(b*(m + 1)))*Hypergeometric2F1[-n, m + 1, m + 2, (-d)*(x/c)], x]
/; FreeQ[{b, c, d, m, n}, x] &&  !IntegerQ[m] && (IntegerQ[n] || (GtQ[c, 0]
 &&  !(EqQ[n, -2^(-1)] && EqQ[c^2 - d^2, 0] && GtQ[-d/(b*c), 0])))
```

```
Int[((b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[c^IntPart
[n]*((c + d*x)^FracPart[n]/(1 + d*(x/c))^FracPart[n])   Int[(b*x)^m*(1 + d*
(x/c))^n, x], x] /; FreeQ[{b, c, d, m, n}, x] && !IntegerQ[m] && !Integer
Q[n] && !GtQ[c, 0] && !GtQ[-d/(b*c), 0] && ((RationalQ[m] && !(EqQ[n, -2
^(-1)] && EqQ[c^2 - d^2, 0])) || !RationalQ[n])
```

```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> Simp[c^IntPart[m]*(c*x)^FracPart[m]*((a*x^j + b*x^n)^FracPart[p]/(x^(F
racPart[m] + j*FracPart[p])*(a + b*x^(n - j))^FracPart[p]))   Int[x^(m + j*
p)*(a + b*x^(n - j))^p, x], x] /; FreeQ[{a, b, c, j, m, n, p}, x] && !Inte
gerQ[p] && NeQ[n, j] && PosQ[n - j]
```

Maple [F]

$$\int \sqrt{cx}\,\left(b\,x^{3}+a\,x^{2}\right)^{p}dx$$

```
int((c*x)^(1/2)*(b*x^3+a*x^2)^p,x)
```

```
int((c*x)^(1/2)*(b*x^3+a*x^2)^p,x)
```

Fricas [F]

$$\int \sqrt{cx}\left(ax^{2}+bx^{3}\right)^{p}\,dx=\int \sqrt{cx}\left(bx^{3}+ax^{2}\right)^{p}dx$$

```
integrate((c*x)^(1/2)*(b*x^3+a*x^2)^p,x, algorithm="fricas")
```

```
integral(sqrt(c*x)*(b*x^3 + a*x^2)^p, x)
```

Sympy [F]

$$\int \sqrt{cx}\left(ax^2+bx^3\right)^p\,dx=\int \sqrt{cx}\left(x^2(a+bx)\right)^p\,dx$$

```
integrate((c*x)**(1/2)*(b*x**3+a*x**2)**p,x)
```

```
Integral(sqrt(c*x)*(x**2*(a + b*x))**p, x)
```

Maxima [F]

$$\int \sqrt{cx}\left(ax^2+bx^3\right)^p\,dx=\int \sqrt{cx}\left(bx^3+ax^2\right)^p\,dx$$

```
integrate((c*x)^(1/2)*(b*x^3+a*x^2)^p,x, algorithm="maxima")
```

```
integrate(sqrt(c*x)*(b*x^3 + a*x^2)^p, x)
```

Giac [F]

$$\int \sqrt{cx}\left(ax^2+bx^3\right)^p\,dx=\int \sqrt{cx}\left(bx^3+ax^2\right)^p\,dx$$

```
integrate((c*x)^(1/2)*(b*x^3+a*x^2)^p,x, algorithm="giac")
```

```
integrate(sqrt(c*x)*(b*x^3 + a*x^2)^p, x)
```

Mupad [F(-1)]

Timed out.

$$\int \sqrt{cx}\left(ax^2+bx^3\right)^p\,dx=\int \sqrt{c\,x}\,\left(b\,x^3+a\,x^2\right)^p\,dx$$

```
int((c*x)^(1/2)*(a*x^2 + b*x^3)^p,x)
```

```
int((c*x)^(1/2)*(a*x^2 + b*x^3)^p, x)
```

Reduce [F]

$$\int \sqrt{cx}\left(ax^2+bx^3\right)^p\,dx$$
$$=\frac{2\sqrt{c}\left(2\sqrt{x}\,(b\,x^3+a\,x^2)^p\,ap+6\sqrt{x}\,(b\,x^3+a\,x^2)^p\,bpx+\sqrt{x}\,(b\,x^3+a\,x^2)^p\,bx-48\left(\int \frac{\sqrt{x}\,(b\,x^3+a\,x^2}{12b\,p^2x^2+12a\,p^2x+8bp\,x^2-}\right.\right.}{}$$

```
int((c*x)^(1/2)*(b*x^3+a*x^2)^p,x)
```

```
(2*sqrt(c)*(2*sqrt(x)*(a*x**2 + b*x**3)**p*a*p + 6*sqrt(x)*(a*x**2 + b*x**
3)**p*b*p*x + sqrt(x)*(a*x**2 + b*x**3)**p*b*x - 48*int((sqrt(x)*(a*x**2 +
 b*x**3)**p)/(12*a*p**2*x + 8*a*p*x + a*x + 12*b*p**2*x**2 + 8*b*p*x**2 +
b*x**2),x)*a**2*p**4 - 44*int((sqrt(x)*(a*x**2 + b*x**3)**p)/(12*a*p**2*x
+ 8*a*p*x + a*x + 12*b*p**2*x**2 + 8*b*p*x**2 + b*x**2),x)*a**2*p**3 - 12*
int((sqrt(x)*(a*x**2 + b*x**3)**p)/(12*a*p**2*x + 8*a*p*x + a*x + 12*b*p**
2*x**2 + 8*b*p*x**2 + b*x**2),x)*a**2*p**2 - int((sqrt(x)*(a*x**2 + b*x**3
)**p)/(12*a*p**2*x + 8*a*p*x + a*x + 12*b*p**2*x**2 + 8*b*p*x**2 + b*x**2)
,x)*a**2*p))/(3*b*(12*p**2 + 8*p + 1))
```

3.425 $\int \frac{(ax^2+bx^3)^p}{\sqrt{cx}}\,dx$

Optimal result

Integrand size = 21, antiderivative size = 58

$$\int \frac{(ax^2+bx^3)^p}{\sqrt{cx}}\,dx = \frac{2c(ax^2+bx^3)^{1+p}\,\text{Hypergeometric2F1}\left(1,\frac{3}{2}+3p,\frac{3}{2}+2p,-\frac{bx}{a}\right)}{a(1+4p)(cx)^{3/2}}$$

```
2*c*(b*x^3+a*x^2)^(p+1)*hypergeom([1, 3/2+3*p],[3/2+2*p],-b*x/a)/a/(1+4*p)
/(c*x)^(3/2)
```

Mathematica [A] (verified)

Time = 0.05 (sec) , antiderivative size = 66, normalized size of antiderivative = 1.14

$$\int \frac{(ax^2+bx^3)^p}{\sqrt{cx}}\,dx = \frac{x(x^2(a+bx))^p\left(1+\frac{bx}{a}\right)^{-p}\text{Hypergeometric2F1}\left(-p,\frac{1}{2}+2p,\frac{3}{2}+2p,-\frac{bx}{a}\right)}{\left(\frac{1}{2}+2p\right)\sqrt{cx}}$$

```
Integrate[(a*x^2 + b*x^3)^p/Sqrt[c*x],x]
```

```
(x*(x^2*(a + b*x))^p*Hypergeometric2F1[-p, 1/2 + 2*p, 3/2 + 2*p, -((b*x)/a
)])/((1/2 + 2*p)*Sqrt[c*x]*(1 + (b*x)/a)^p)
```

Rubi [A] (verified)

Time = 0.33 (sec) , antiderivative size = 67, normalized size of antiderivative = 1.16, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.143$, Rules used = {1938, 76, 74}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(ax^2+bx^3)^p}{\sqrt{cx}}\,dx$$

$$\Big\downarrow \text{1938}$$

$$\frac{x^{\frac{1}{2}-2p}(a+bx)^{-p}\left(ax^2+bx^3\right)^p\int x^{2p-\frac{1}{2}}(a+bx)^pdx}{\sqrt{cx}}$$

$$\Big\downarrow \text{76}$$

$$\frac{x^{\frac{1}{2}-2p}\left(\frac{bx}{a}+1\right)^{-p}\left(ax^2+bx^3\right)^p\int x^{2p-\frac{1}{2}}\left(\frac{bx}{a}+1\right)^pdx}{\sqrt{cx}}$$

$$\Big\downarrow \text{74}$$

$$\frac{2x\left(\frac{bx}{a}+1\right)^{-p}\left(ax^2+bx^3\right)^p\text{Hypergeometric2F1}\left(-p,2p+\frac{1}{2},2p+\frac{3}{2},-\frac{bx}{a}\right)}{(4p+1)\sqrt{cx}}$$

```
Int[(a*x^2 + b*x^3)^p/Sqrt[c*x],x]
```

```
(2*x*(a*x^2 + b*x^3)^p*Hypergeometric2F1[-p, 1/2 + 2*p, 3/2 + 2*p, -((b*x)
/a)])/((1 + 4*p)*Sqrt[c*x]*(1 + (b*x)/a)^p)
```

Defintions of rubi rules used

```
Int[((b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[c^n*((b*x
)^(m + 1)/(b*(m + 1)))*Hypergeometric2F1[-n, m + 1, m + 2, (-d)*(x/c)], x]
/; FreeQ[{b, c, d, m, n}, x] &&  !IntegerQ[m] && (IntegerQ[n] || (GtQ[c, 0]
 &&  !(EqQ[n, -2^(-1)] && EqQ[c^2 - d^2, 0] && GtQ[-d/(b*c), 0])))
```

```
Int[((b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[c^IntPart
[n]*((c + d*x)^FracPart[n]/(1 + d*(x/c))^FracPart[n])   Int[(b*x)^m*(1 + d*
(x/c))^n, x], x] /; FreeQ[{b, c, d, m, n}, x] &&  !IntegerQ[m] &&  !Integer
Q[n] &&  !GtQ[c, 0] &&  !GtQ[-d/(b*c), 0] && ((RationalQ[m] &&  !(EqQ[n, -2
^(-1)] && EqQ[c^2 - d^2, 0])) ||  !RationalQ[n])
```

```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> Simp[c^IntPart[m]*(c*x)^FracPart[m]*((a*x^j + b*x^n)^FracPart[p]/(x^(F
racPart[m] + j*FracPart[p])*(a + b*x^(n - j))^FracPart[p]))   Int[x^(m + j*
p)*(a + b*x^(n - j))^p, x], x] /; FreeQ[{a, b, c, j, m, n, p}, x] &&  !Inte
gerQ[p] && NeQ[n, j] && PosQ[n - j]
```

Maple [F]

$$\int \frac{(b\,x^3 + a\,x^2)^p}{\sqrt{cx}}dx$$

```
int((b*x^3+a*x^2)^p/(c*x)^(1/2),x)
```

```
int((b*x^3+a*x^2)^p/(c*x)^(1/2),x)
```

Fricas [F]

$$\int \frac{(ax^2 + bx^3)^p}{\sqrt{cx}} \, dx = \int \frac{(bx^3 + ax^2)^p}{\sqrt{cx}} \, dx$$

```
integrate((b*x^3+a*x^2)^p/(c*x)^(1/2),x, algorithm="fricas")
```

```
integral(sqrt(c*x)*(b*x^3 + a*x^2)^p/(c*x), x)
```

Sympy [F]

$$\int \frac{(ax^2 + bx^3)^p}{\sqrt{cx}} \, dx = \int \frac{(x^2(a + bx))^p}{\sqrt{cx}} \, dx$$

```
integrate((b*x**3+a*x**2)**p/(c*x)**(1/2),x)
```

```
Integral((x**2*(a + b*x))**p/sqrt(c*x), x)
```

Maxima [F]

$$\int \frac{(ax^2 + bx^3)^p}{\sqrt{cx}} \, dx = \int \frac{(bx^3 + ax^2)^p}{\sqrt{cx}} \, dx$$

```
integrate((b*x^3+a*x^2)^p/(c*x)^(1/2),x, algorithm="maxima")
```

```
integrate((b*x^3 + a*x^2)^p/sqrt(c*x), x)
```

Giac [F]

$$\int \frac{(ax^2+bx^3)^p}{\sqrt{cx}}\,dx=\int \frac{(bx^3+ax^2)^p}{\sqrt{cx}}\,dx$$

```
integrate((b*x^3+a*x^2)^p/(c*x)^(1/2),x, algorithm="giac")
```

```
integrate((b*x^3 + a*x^2)^p/sqrt(c*x), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(ax^2+bx^3)^p}{\sqrt{cx}}\,dx=\int \frac{(b\,x^3+a\,x^2)^p}{\sqrt{c\,x}}\,dx$$

```
int((a*x^2 + b*x^3)^p/(c*x)^(1/2),x)
```

```
int((a*x^2 + b*x^3)^p/(c*x)^(1/2), x)
```

Reduce [F]

$$\int \frac{(ax^2+bx^3)^p}{\sqrt{cx}}\,dx$$
$$=\frac{2\sqrt{c}\left(\sqrt{x}\,(b\,x^3+a\,x^2)^p+6\left(\int \frac{\sqrt{x}\,(b\,x^3+a\,x^2)^p}{6bp\,x^2+6apx+b\,x^2+ax}dx\right)a\,p^2+\left(\int \frac{\sqrt{x}\,(b\,x^3+a\,x^2)^p}{6bp\,x^2+6apx+b\,x^2+ax}dx\right)ap\right)}{c\,(6p+1)}$$

```
int((b*x^3+a*x^2)^p/(c*x)^(1/2),x)
```

```
(2*sqrt(c)*(sqrt(x)*(a*x**2 + b*x**3)**p + 6*int((sqrt(x)*(a*x**2 + b*x**3
)**p)/(6*a*p*x + a*x + 6*b*p*x**2 + b*x**2),x)*a*p**2 + int((sqrt(x)*(a*x*
*2 + b*x**3)**p)/(6*a*p*x + a*x + 6*b*p*x**2 + b*x**2),x)*a*p))/(c*(6*p +
1))
```

3.426 $\int \frac{(ax^2+bx^3)^p}{(cx)^{3/2}} dx$

Optimal result

Integrand size = 21, antiderivative size = 58

$$\int \frac{(ax^2+bx^3)^p}{(cx)^{3/2}} dx = -\frac{2c(ax^2+bx^3)^{1+p}\operatorname{Hypergeometric2F1}\left(1,\frac{1}{2}+3p,\frac{1}{2}+2p,-\frac{bx}{a}\right)}{a(1-4p)(cx)^{5/2}}$$

```
-2*c*(b*x^3+a*x^2)^(p+1)*hypergeom([1, 1/2+3*p],[1/2+2*p],-b*x/a)/a/(1-4*p
)/(c*x)^(5/2)
```

Mathematica [A] (verified)

Time = 0.04 (sec) , antiderivative size = 66, normalized size of antiderivative = 1.14

$$\int \frac{(ax^2+bx^3)^p}{(cx)^{3/2}} dx = \frac{x(x^2(a+bx))^p\left(1+\frac{bx}{a}\right)^{-p}\operatorname{Hypergeometric2F1}\left(-p,-\frac{1}{2}+2p,\frac{1}{2}+2p,-\frac{bx}{a}\right)}{\left(-\frac{1}{2}+2p\right)(cx)^{3/2}}$$

```
Integrate[(a*x^2 + b*x^3)^p/(c*x)^(3/2),x]
```

```
(x*(x^2*(a + b*x))^p*Hypergeometric2F1[-p, -1/2 + 2*p, 1/2 + 2*p, -((b*x)/
a)])/((-1/2 + 2*p)*(c*x)^(3/2)*(1 + (b*x)/a)^p)
```

Rubi [A] (verified)

Time = 0.33 (sec) , antiderivative size = 69, normalized size of antiderivative = 1.19, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.143, Rules used = {1938, 76, 74}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(ax^2+bx^3)^p}{(cx)^{3/2}} \, dx$$
$$\Big\downarrow \text{1938}$$
$$\frac{x^{\frac{1}{2}-2p}(a+bx)^{-p}\left(ax^2+bx^3\right)^p \int x^{2p-\frac{3}{2}}(a+bx)^p dx}{c\sqrt{cx}}$$
$$\Big\downarrow \text{76}$$
$$\frac{x^{\frac{1}{2}-2p}\left(\frac{bx}{a}+1\right)^{-p}\left(ax^2+bx^3\right)^p \int x^{2p-\frac{3}{2}}\left(\frac{bx}{a}+1\right)^p dx}{c\sqrt{cx}}$$
$$\Big\downarrow \text{74}$$
$$-\frac{2\left(\frac{bx}{a}+1\right)^{-p}\left(ax^2+bx^3\right)^p \text{Hypergeometric2F1}\left(-p,2p-\frac{1}{2},2p+\frac{1}{2},-\frac{bx}{a}\right)}{c(1-4p)\sqrt{cx}}$$

```
Int[(a*x^2 + b*x^3)^p/(c*x)^(3/2),x]
```

```
(-2*(a*x^2 + b*x^3)^p*Hypergeometric2F1[-p, -1/2 + 2*p, 1/2 + 2*p, -((b*x)
/a)])/(c*(1 - 4*p)*Sqrt[c*x]*(1 + (b*x)/a)^p)
```

Defintions of rubi rules used

```
Int[((b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[c^n*((b*x
)^(m + 1)/(b*(m + 1)))*Hypergeometric2F1[-n, m + 1, m + 2, (-d)*(x/c)], x]
/; FreeQ[{b, c, d, m, n}, x] &&  !IntegerQ[m] && (IntegerQ[n] || (GtQ[c, 0]
 &&  !(EqQ[n, -2^(-1)] && EqQ[c^2 - d^2, 0] && GtQ[-d/(b*c), 0])))
```

```
Int[((b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[c^IntPart
[n]*((c + d*x)^FracPart[n]/(1 + d*(x/c))^FracPart[n])   Int[(b*x)^m*(1 + d*
(x/c))^n, x], x] /; FreeQ[{b, c, d, m, n}, x] &&  !IntegerQ[m] &&  !Integer
Q[n] &&  !GtQ[c, 0] &&  !GtQ[-d/(b*c), 0] && ((RationalQ[m] &&  !(EqQ[n, -2
^(-1)] && EqQ[c^2 - d^2, 0])) ||  !RationalQ[n])
```

```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> Simp[c^IntPart[m]*(c*x)^FracPart[m]*((a*x^j + b*x^n)^FracPart[p]/(x^(F
racPart[m] + j*FracPart[p])*(a + b*x^(n - j))^FracPart[p]))   Int[x^(m + j*
p)*(a + b*x^(n - j))^p, x], x] /; FreeQ[{a, b, c, j, m, n, p}, x] &&  !Inte
gerQ[p] && NeQ[n, j] && PosQ[n - j]
```

Maple [F]

$$\int \frac{(b\,x^{3}+a\,x^{2})^{p}}{(cx)^{\frac{3}{2}}}dx$$

```
int((b*x^3+a*x^2)^p/(c*x)^(3/2),x)
```

```
int((b*x^3+a*x^2)^p/(c*x)^(3/2),x)
```

Fricas [F]

$$\int \frac{(ax^2+bx^3)^p}{(cx)^{3/2}}\,dx = \int \frac{(bx^3+ax^2)^p}{(cx)^{\frac{3}{2}}}\,dx$$

```
integrate((b*x^3+a*x^2)^p/(c*x)^(3/2),x, algorithm="fricas")
```

```
integral(sqrt(c*x)*(b*x^3 + a*x^2)^p/(c^2*x^2), x)
```

Sympy [F]

$$\int \frac{(ax^2 + bx^3)^p}{(cx)^{3/2}}\,dx = \int \frac{(x^2(a + bx))^p}{(cx)^{\frac{3}{2}}}\,dx$$

```
integrate((b*x**3+a*x**2)**p/(c*x)**(3/2),x)
```

```
Integral((x**2*(a + b*x))**p/(c*x)**(3/2), x)
```

Maxima [F]

$$\int \frac{(ax^2 + bx^3)^p}{(cx)^{3/2}}\,dx = \int \frac{(bx^3 + ax^2)^p}{(cx)^{\frac{3}{2}}}\,dx$$

```
integrate((b*x^3+a*x^2)^p/(c*x)^(3/2),x, algorithm="maxima")
```

```
integrate((b*x^3 + a*x^2)^p/(c*x)^(3/2), x)
```

Giac [F]

$$\int \frac{(ax^2 + bx^3)^p}{(cx)^{3/2}}\,dx = \int \frac{(bx^3 + ax^2)^p}{(cx)^{\frac{3}{2}}}\,dx$$

```
integrate((b*x^3+a*x^2)^p/(c*x)^(3/2),x, algorithm="giac")
```

```
integrate((b*x^3 + a*x^2)^p/(c*x)^(3/2), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(ax^2+bx^3)^p}{(cx)^{3/2}}\,dx = \int \frac{(b\,x^3+a\,x^2)^p}{(c\,x)^{3/2}}\,dx$$

```
int((a*x^2 + b*x^3)^p/(c*x)^(3/2),x)
```

```
int((a*x^2 + b*x^3)^p/(c*x)^(3/2), x)
```

Reduce [F]

$$\int \frac{(ax^2+bx^3)^p}{(cx)^{3/2}}\,dx = \frac{2\sqrt{c}\left((b\,x^3+a\,x^2)^p + 6\sqrt{x}\left(\int \frac{\sqrt{x}\,(b\,x^3+a\,x^2)^p}{6bp\,x^3+6ap\,x^2-b\,x^3-a\,x^2}dx\right)a\,p^2 - \sqrt{x}\left(\int \frac{\sqrt{x}\,(b\,x^3+a\,x^2)^p}{6bp\,x^3+6ap\,x^2-b\,x^3-}\right.\right.}{\sqrt{x}\,c^2\,(6p-1)}$$

```
int((b*x^3+a*x^2)^p/(c*x)^(3/2),x)
```

```
(2*sqrt(c)*((a*x**2 + b*x**3)**p + 6*sqrt(x)*int((sqrt(x)*(a*x**2 + b*x**3
)**p)/(6*a*p*x**2 - a*x**2 + 6*b*p*x**3 - b*x**3),x)*a*p**2 - sqrt(x)*int(
(sqrt(x)*(a*x**2 + b*x**3)**p)/(6*a*p*x**2 - a*x**2 + 6*b*p*x**3 - b*x**3)
,x)*a*p))/(sqrt(x)*c**2*(6*p - 1))
```

3.427 $\int (cx)^m \left(ax^2+bx^3\right)^p \, dx$

Optimal result

Integrand size = 19, antiderivative size = 56

$$\int (cx)^m \left(ax^2+bx^3\right)^p \, dx = \frac{c(cx)^{-1+m} \left(ax^2+bx^3\right)^{1+p} \text{Hypergeometric2F1}\left(1,2+m+3p,2+m+2p,-\frac{bx}{a}\right)}{a(1+m+2p)}$$

```
c*(c*x)^(-1+m)*(b*x^3+a*x^2)^(p+1)*hypergeom([1, 2+m+3*p],[2+m+2*p],-b*x/a
)/a/(1+m+2*p)
```

Mathematica [A] (verified)

Time = 0.03 (sec) , antiderivative size = 61, normalized size of antiderivative = 1.09

$$\int (cx)^m \left(ax^2+bx^3\right)^p \, dx = \frac{x(cx)^m \left(x^2(a+bx)\right)^p \left(1+\frac{bx}{a}\right)^{-p} \text{Hypergeometric2F1}\left(-p,1+m+2p,2+m+2p,-\frac{bx}{a}\right)}{1+m+2p}$$

```
Integrate[(c*x)^m*(a*x^2 + b*x^3)^p,x]
```

```
(x*(c*x)^m*(x^2*(a + b*x))^p*Hypergeometric2F1[-p, 1 + m + 2*p, 2 + m + 2*
p, -((b*x)/a)])/((1 + m + 2*p)*(1 + (b*x)/a)^p)
```

Rubi [A] (verified)

Time = 0.33 (sec) , antiderivative size = 63, normalized size of antiderivative = 1.12, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.158, Rules used = {1938, 76, 74}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (cx)^m \left(ax^2+bx^3\right)^p \, dx$$

$$\Big\downarrow \text{1938}$$

$$(cx)^m x^{-m-2p}(a+bx)^{-p} \left(ax^2+bx^3\right)^p \int x^{m+2p}(a+bx)^p dx$$

$$\Big\downarrow \text{76}$$

$$(cx)^m x^{-m-2p}\left(\frac{bx}{a}+1\right)^{-p} \left(ax^2+bx^3\right)^p \int x^{m+2p}\left(\frac{bx}{a}+1\right)^p dx$$

$$\Big\downarrow \text{74}$$

$$\frac{x(cx)^m \left(\frac{bx}{a}+1\right)^{-p} \left(ax^2+bx^3\right)^p \text{Hypergeometric2F1}\left(-p,m+2p+1,m+2p+2,-\frac{bx}{a}\right)}{m+2p+1}$$

```
Int[(c*x)^m*(a*x^2 + b*x^3)^p,x]
```

```
(x*(c*x)^m*(a*x^2 + b*x^3)^p*Hypergeometric2F1[-p, 1 + m + 2*p, 2 + m + 2*
p, -((b*x)/a)])/((1 + m + 2*p)*(1 + (b*x)/a)^p)
```

Defintions of rubi rules used

```
Int[((b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[c^n*((b*x
)^(m + 1)/(b*(m + 1)))*Hypergeometric2F1[-n, m + 1, m + 2, (-d)*(x/c)], x]
/; FreeQ[{b, c, d, m, n}, x] &&  !IntegerQ[m] && (IntegerQ[n] || (GtQ[c, 0]
 &&  !(EqQ[n, -2^(-1)] && EqQ[c^2 - d^2, 0] && GtQ[-d/(b*c), 0])))
```

```
Int[((b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[c^IntPart
[n]*((c + d*x)^FracPart[n]/(1 + d*(x/c))^FracPart[n])   Int[(b*x)^m*(1 + d*
(x/c))^n, x], x] /; FreeQ[{b, c, d, m, n}, x] &&  !IntegerQ[m] &&  !Integer
Q[n] &&  !GtQ[c, 0] &&  !GtQ[-d/(b*c), 0] && ((RationalQ[m] &&  !(EqQ[n, -2
^(-1)] && EqQ[c^2 - d^2, 0])) ||  !RationalQ[n])
```

```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> Simp[c^IntPart[m]*(c*x)^FracPart[m]*((a*x^j + b*x^n)^FracPart[p]/(x^(F
racPart[m] + j*FracPart[p])*(a + b*x^(n - j))^FracPart[p]))   Int[x^(m + j*
p)*(a + b*x^(n - j))^p, x], x] /; FreeQ[{a, b, c, j, m, n, p}, x] &&  !Inte
gerQ[p] && NeQ[n, j] && PosQ[n - j]
```

Maple [F]

$$\int (cx)^m \left(b\,x^3+a\,x^2\right)^p\,dx$$

```
int((c*x)^m*(b*x^3+a*x^2)^p,x)
```

```
int((c*x)^m*(b*x^3+a*x^2)^p,x)
```

Fricas [F]

$$\int (cx)^m \left(ax^2+bx^3\right)^p \, dx = \int \left(bx^3+ax^2\right)^p (cx)^m \, dx$$

```
integrate((c*x)^m*(b*x^3+a*x^2)^p,x, algorithm="fricas")
```

```
integral((b*x^3 + a*x^2)^p*(c*x)^m, x)
```

Sympy [F]

$$\int (cx)^m \left(ax^2+bx^3\right)^p \, dx = \int (cx)^m \left(x^2(a+bx)\right)^p \, dx$$

```
integrate((c*x)**m*(b*x**3+a*x**2)**p,x)
```

```
Integral((c*x)**m*(x**2*(a + b*x))**p, x)
```

Maxima [F]

$$\int (cx)^m \left(ax^2+bx^3\right)^p \, dx = \int \left(bx^3+ax^2\right)^p (cx)^m \, dx$$

```
integrate((c*x)^m*(b*x^3+a*x^2)^p,x, algorithm="maxima")
```

```
integrate((b*x^3 + a*x^2)^p*(c*x)^m, x)
```

Giac [F]

$$\int (cx)^m \left(ax^2+bx^3\right)^p \, dx = \int \left(bx^3+ax^2\right)^p (cx)^m \, dx$$

```
integrate((c*x)^m*(b*x^3+a*x^2)^p,x, algorithm="giac")
```

```
integrate((b*x^3 + a*x^2)^p*(c*x)^m, x)
```

Mupad [F(-1)]

Timed out.

$$\int (cx)^m \left(ax^2+bx^3\right)^p \, dx = \int (c\,x)^m \left(b\,x^3+a\,x^2\right)^p \, dx$$

```
int((c*x)^m*(a*x^2 + b*x^3)^p,x)
```

```
int((c*x)^m*(a*x^2 + b*x^3)^p, x)
```

Reduce [F]

$$\int (cx)^m \left(ax^2+bx^3\right)^p \, dx$$
$$= \frac{c^m \left(x^m (b\,x^3+a\,x^2)^p\,ap + x^m (b\,x^3+a\,x^2)^p\,bmx + 3x^m (b\,x^3+a\,x^2)^p\,bpx - \left(\int \frac{x^m}{b\,m^2x^2+6bmp\,x^2+9b\,p^2x^2+a\,m^2x\cdots}\right.\right.}{}$$

```
int((c*x)^m*(b*x^3+a*x^2)^p,x)
```

```
(c**m*(x**m*(a*x**2 + b*x**3)**p*a*p + x**m*(a*x**2 + b*x**3)**p*b*m*x + 3
*x**m*(a*x**2 + b*x**3)**p*b*p*x - int((x**m*(a*x**2 + b*x**3)**p)/(a*m**2
*x + 6*a*m*p*x + a*m*x + 9*a*p**2*x + 3*a*p*x + b*m**2*x**2 + 6*b*m*p*x**2
 + b*m*x**2 + 9*b*p**2*x**2 + 3*b*p*x**2),x)*a**2*m**3*p - 8*int((x**m*(a*
x**2 + b*x**3)**p)/(a*m**2*x + 6*a*m*p*x + a*m*x + 9*a*p**2*x + 3*a*p*x +
b*m**2*x**2 + 6*b*m*p*x**2 + b*m*x**2 + 9*b*p**2*x**2 + 3*b*p*x**2),x)*a**
2*m**2*p**2 - int((x**m*(a*x**2 + b*x**3)**p)/(a*m**2*x + 6*a*m*p*x + a*m*
x + 9*a*p**2*x + 3*a*p*x + b*m**2*x**2 + 6*b*m*p*x**2 + b*m*x**2 + 9*b*p**
2*x**2 + 3*b*p*x**2),x)*a**2*m**2*p - 21*int((x**m*(a*x**2 + b*x**3)**p)/(
a*m**2*x + 6*a*m*p*x + a*m*x + 9*a*p**2*x + 3*a*p*x + b*m**2*x**2 + 6*b*m*
p*x**2 + b*m*x**2 + 9*b*p**2*x**2 + 3*b*p*x**2),x)*a**2*m*p**3 - 5*int((x*
*m*(a*x**2 + b*x**3)**p)/(a*m**2*x + 6*a*m*p*x + a*m*x + 9*a*p**2*x + 3*a*
p*x + b*m**2*x**2 + 6*b*m*p*x**2 + b*m*x**2 + 9*b*p**2*x**2 + 3*b*p*x**2),
x)*a**2*m*p**2 - 18*int((x**m*(a*x**2 + b*x**3)**p)/(a*m**2*x + 6*a*m*p*x
+ a*m*x + 9*a*p**2*x + 3*a*p*x + b*m**2*x**2 + 6*b*m*p*x**2 + b*m*x**2 + 9
*b*p**2*x**2 + 3*b*p*x**2),x)*a**2*p**4 - 6*int((x**m*(a*x**2 + b*x**3)**p
)/(a*m**2*x + 6*a*m*p*x + a*m*x + 9*a*p**2*x + 3*a*p*x + b*m**2*x**2 + 6*b
*m*p*x**2 + b*m*x**2 + 9*b*p**2*x**2 + 3*b*p*x**2),x)*a**2*p**3))/(b*(m**2
 + 6*m*p + m + 9*p**2 + 3*p))
```

3.428 $\int (cx)^{-5-3p} \left(ax^2+bx^3\right)^p \, dx$

Optimal result

Integrand size = 23, antiderivative size = 181

$$\int (cx)^{-5-3p} \left(ax^2+bx^3\right)^p \, dx = \frac{3b(cx)^{-5-3p}\left(ax^2+bx^3\right)^{1+p}}{a^2(3+p)(4+p)} - \frac{6b^2(cx)^{-4-3p}\left(ax^2+bx^3\right)^{1+p}}{a^3c(2+p)(3+p)(4+p)} + \frac{6b^3(cx)^{-3(1+p)}\left(ax^2+bx^3\right)^{1+p}}{a^4c^2(1+p)(2+p)(3+p)(4+p)} - \frac{c(cx)^{-3(2+p)}\left(ax^2+bx^3\right)^{1+p}}{a(4+p)}$$

```
3*b*(c*x)^(-5-3*p)*(b*x^3+a*x^2)^(p+1)/a^2/(3+p)/(4+p)-6*b^2*(c*x)^(-4-3*p
)*(b*x^3+a*x^2)^(p+1)/a^3/c/(2+p)/(3+p)/(4+p)+6*b^3*(b*x^3+a*x^2)^(p+1)/a^
4/c^2/(p+1)/(2+p)/(3+p)/(4+p)/((c*x)^(3*p+3))-c*(b*x^3+a*x^2)^(p+1)/a/(4+p
)/((c*x)^(6+3*p))
```

Mathematica [A] (verified)

Time = 0.05 (sec) , antiderivative size = 104, normalized size of antiderivative = 0.57

$$\int (cx)^{-5-3p} \left(ax^2+bx^3\right)^p \, dx = -\frac{(cx)^{-3p} \left(x^2(a+bx)\right)^{1+p} \left(a^3(6+11p+6p^2+p^3)-3a^2b(2+3p+p^2)\, x+6ab^2(1+p)x^2-6b^3x^3\right)}{a^4c^5(1+p)(2+p)(3+p)(4+p)x^6}$$

```
Integrate[(c*x)^(-5 - 3*p)*(a*x^2 + b*x^3)^p,x]
```

```
-(((x^2*(a + b*x))^(1 + p)*(a^3*(6 + 11*p + 6*p^2 + p^3) - 3*a^2*b*(2 + 3*
p + p^2)*x + 6*a*b^2*(1 + p)*x^2 - 6*b^3*x^3))/(a^4*c^5*(1 + p)*(2 + p)*(3
 + p)*(4 + p)*x^6*(c*x)^(3*p)))
```

Rubi [A] (verified)

Time = 0.67 (sec) , antiderivative size = 175, normalized size of antiderivative = 0.97, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.174$, Rules used = {1922, 1922, 1922, 1920}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (cx)^{-3p-5} \left(ax^2+bx^3\right)^p dx$$

$$\Big\downarrow \text{ 1922}$$

$$-\frac{3b\int (cx)^{-3p-4} \left(bx^3+ax^2\right)^p dx}{ac(p+4)}-\frac{c(cx)^{-3(p+2)} \left(ax^2+bx^3\right)^{p+1}}{a(p+4)}$$

$$\Big\downarrow \text{ 1922}$$

$$-\frac{3b\left(-\frac{2b\int (cx)^{-3(p+1)} \left(bx^3+ax^2\right)^p dx}{ac(p+3)}-\frac{c(cx)^{-3p-5} \left(ax^2+bx^3\right)^{p+1}}{a(p+3)}\right)}{ac(p+4)}-\frac{c(cx)^{-3(p+2)} \left(ax^2+bx^3\right)^{p+1}}{a(p+4)}$$

$$\Big\downarrow \text{ 1922}$$

$$-\frac{3b\left(-\frac{2b\left(-\frac{b\int(cx)^{-3p-2}\left(bx^3+ax^2\right)^pdx}{ac(p+2)}-\frac{c(cx)^{-3p-4}\left(ax^2+bx^3\right)^{p+1}}{a(p+2)}\right)}{ac(p+3)}-\frac{c(cx)^{-3p-5}\left(ax^2+bx^3\right)^{p+1}}{a(p+3)}\right)}{ac(p+4)}-\frac{c(cx)^{-3(p+2)}\left(ax^2+bx^3\right)^{p+1}}{a(p+4)}$$

$$\Big\downarrow \text{1920}$$

$$-\frac{3b\left(-\frac{2b\left(\frac{b(cx)^{-3(p+1)}\left(ax^2+bx^3\right)^{p+1}}{a^2(p+1)(p+2)}-\frac{c(cx)^{-3p-4}\left(ax^2+bx^3\right)^{p+1}}{a(p+2)}\right)}{ac(p+3)}-\frac{c(cx)^{-3p-5}\left(ax^2+bx^3\right)^{p+1}}{a(p+3)}\right)}{ac(p+4)}-\frac{c(cx)^{-3(p+2)}\left(ax^2+bx^3\right)^{p+1}}{a(p+4)}$$

```
Int[(c*x)^(-5 - 3*p)*(a*x^2 + b*x^3)^p,x]
```

```
-((c*(a*x^2 + b*x^3)^(1 + p))/(a*(4 + p)*(c*x)^(3*(2 + p)))) - (3*b*(-((c*
(c*x)^(-5 - 3*p)*(a*x^2 + b*x^3)^(1 + p))/(a*(3 + p))) - (2*b*(-((c*(c*x)^
(-4 - 3*p)*(a*x^2 + b*x^3)^(1 + p))/(a*(2 + p))) + (b*(a*x^2 + b*x^3)^(1 +
 p))/(a^2*(1 + p)*(2 + p)*(c*x)^(3*(1 + p)))))/(a*c*(3 + p))))/(a*c*(4 + p
))
```

Defintions of rubi rules used

```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> Simp[(-c^(j - 1))*(c*x)^(m - j + 1)*((a*x^j + b*x^n)^(p + 1)/(a*(n - j
)*(p + 1))), x] /; FreeQ[{a, b, c, j, m, n, p}, x] &&  !IntegerQ[p] && NeQ[
n, j] && EqQ[m + n*p + n - j + 1, 0] && (IntegerQ[j] || GtQ[c, 0])
```

```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> Simp[c^(j - 1)*(c*x)^(m - j + 1)*((a*x^j + b*x^n)^(p + 1)/(a*(m + j*p
+ 1))), x] - Simp[b*((m + n*p + n - j + 1)/(a*c^(n - j)*(m + j*p + 1)))   I
nt[(c*x)^(m + n - j)*(a*x^j + b*x^n)^p, x], x] /; FreeQ[{a, b, c, j, m, n,
p}, x] &&  !IntegerQ[p] && NeQ[n, j] && ILtQ[Simplify[(m + n*p + n - j + 1)
/(n - j)], 0] && NeQ[m + j*p + 1, 0] && (IntegersQ[j, n] || GtQ[c, 0])
```

Maple [A] (verified)

Time = 0.52 (sec) , antiderivative size = 133, normalized size of antiderivative = 0.73

method	result	size
gosper	$-\frac{(cx)^{-5-3p}\left(b\,x^{3}+a\,x^{2}\right)^{p}\left(a^{3}p^{3}-3a^{2}b\,p^{2}x+6a\,b^{2}p\,x^{2}-6b^{3}x^{3}+6a^{3}p^{2}-9a^{2}bpx+6a\,b^{2}x^{2}+11a^{3}p-6a^{2}bx+6a^{3}\right)x(bx+a)}{(4+p)(3+p)(2+p)(p+1)a^{4}}$	133
orering	$-\frac{(cx)^{-5-3p}\left(b\,x^{3}+a\,x^{2}\right)^{p}\left(a^{3}p^{3}-3a^{2}b\,p^{2}x+6a\,b^{2}p\,x^{2}-6b^{3}x^{3}+6a^{3}p^{2}-9a^{2}bpx+6a\,b^{2}x^{2}+11a^{3}p-6a^{2}bx+6a^{3}\right)x(bx+a)}{(4+p)(3+p)(2+p)(p+1)a^{4}}$	133

```
int((c*x)^(-5-3*p)*(b*x^3+a*x^2)^p,x,method=_RETURNVERBOSE)
```

```
-(c*x)^(-5-3*p)*(b*x^3+a*x^2)^p*(a^3*p^3-3*a^2*b*p^2*x+6*a*b^2*p*x^2-6*b^3
*x^3+6*a^3*p^2-9*a^2*b*p*x+6*a*b^2*x^2+11*a^3*p-6*a^2*b*x+6*a^3)*x*(b*x+a)
/(4+p)/(3+p)/(2+p)/(p+1)/a^4
```

Fricas [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 162, normalized size of antiderivative = 0.90

$$\int (cx)^{-5-3p}\left(ax^2+bx^3\right)^p\,dx = -\frac{(6\,ab^3px^4-6\,b^4x^5-3\,(a^2b^2p^2+a^2b^2p)x^3+(a^3bp^3+3\,a^3bp^2+2\,a^3bp)x^2+(a^4p^3+6\,a^4p^2+11\,a^4p+}{a^4p^4+10\,a^4p^3+35\,a^4p^2+50\,a^4p+24\,a^4}$$

```
integrate((c*x)^(-5-3*p)*(b*x^3+a*x^2)^p,x, algorithm="fricas")
```

```
-(6*a*b^3*p*x^4 - 6*b^4*x^5 - 3*(a^2*b^2*p^2 + a^2*b^2*p)*x^3 + (a^3*b*p^3
 + 3*a^3*b*p^2 + 2*a^3*b*p)*x^2 + (a^4*p^3 + 6*a^4*p^2 + 11*a^4*p + 6*a^4)
*x)*(b*x^3 + a*x^2)^p*(c*x)^(-3*p - 5)/(a^4*p^4 + 10*a^4*p^3 + 35*a^4*p^2
+ 50*a^4*p + 24*a^4)
```

Sympy [F]

$$\int (cx)^{-5-3p} \left(ax^2+bx^3\right)^p \, dx = \int (cx)^{-3p-5} \left(x^2(a+bx)\right)^p \, dx$$

```
integrate((c*x)**(-5-3*p)*(b*x**3+a*x**2)**p,x)
```

```
Integral((c*x)**(-3*p - 5)*(x**2*(a + b*x))**p, x)
```

Maxima [F]

$$\int (cx)^{-5-3p} \left(ax^2+bx^3\right)^p \, dx = \int (bx^3+ax^2)^p (cx)^{-3p-5} \, dx$$

```
integrate((c*x)^(-5-3*p)*(b*x^3+a*x^2)^p,x, algorithm="maxima")
```

```
integrate((b*x^3 + a*x^2)^p*(c*x)^(-3*p - 5), x)
```

Giac [F]

$$\int (cx)^{-5-3p} \left(ax^2+bx^3\right)^p \, dx = \int (bx^3+ax^2)^p (cx)^{-3p-5} \, dx$$

```
integrate((c*x)^(-5-3*p)*(b*x^3+a*x^2)^p,x, algorithm="giac")
```

```
integrate((b*x^3 + a*x^2)^p*(c*x)^(-3*p - 5), x)
```

Mupad [B] (verification not implemented)

Time = 8.87 (sec) , antiderivative size = 241, normalized size of antiderivative = 1.33

$$\int (cx)^{-5-3p} \left(ax^2+bx^3\right)^p \, dx$$
$$= -\left(b\,x^3+a\,x^2\right)^p \left(\frac{x\,\left(p^3+6\,p^2+11\,p+6\right)}{{\left(c\,x\right)}^{3\,p+5}\,\left(p^4+10\,p^3+35\,p^2+50\,p+24\right)} - \frac{6\,b^4\,x^5}{a^4\,{\left(c\,x\right)}^{3\,p+5}\,\left(p^4+10\,p^3+35\,p^2+50\,p+24\right)} + \frac{6\,b^3\,p\,x^4}{a^3\,{\left(c\,x\right)}^{3\,p+5}\,\left(p^4+10\,p^3+35\,p^2+50\,p+24\right)} + \frac{b\,p\,x^2\,\left(p^2+3\,p+2\right)}{a\,{\left(c\,x\right)}^{3\,p+5}\,\left(p^4+10\,p^3+35\,p^2+50\,p+24\right)} - \frac{3\,b^2\,p\,x^3\,\left(p+1\right)}{a^2\,{\left(c\,x\right)}^{3\,p+5}\,\left(p^4+10\,p^3+35\,p^2+50\,p+24\right)}\right)$$

```
int((a*x^2 + b*x^3)^p/(c*x)^(3*p + 5),x)
```

```
-(a*x^2 + b*x^3)^p*((x*(11*p + 6*p^2 + p^3 + 6))/((c*x)^(3*p + 5)*(50*p +
35*p^2 + 10*p^3 + p^4 + 24)) - (6*b^4*x^5)/(a^4*(c*x)^(3*p + 5)*(50*p + 35
*p^2 + 10*p^3 + p^4 + 24)) + (6*b^3*p*x^4)/(a^3*(c*x)^(3*p + 5)*(50*p + 35
*p^2 + 10*p^3 + p^4 + 24)) + (b*p*x^2*(3*p + p^2 + 2))/(a*(c*x)^(3*p + 5)*
(50*p + 35*p^2 + 10*p^3 + p^4 + 24)) - (3*b^2*p*x^3*(p + 1))/(a^2*(c*x)^(3
*p + 5)*(50*p + 35*p^2 + 10*p^3 + p^4 + 24)))
```

Reduce [B] (verification not implemented)

Time = 0.49 (sec) , antiderivative size = 157, normalized size of antiderivative = 0.87

$$\int (cx)^{-5-3p} \left(ax^2+bx^3\right)^p \, dx$$
$$= \frac{\left(b\,x^3+a\,x^2\right)^p \left(-a^3 b\,p^3 x+3a^2 b^2 p^2 x^2-6a\,b^3 p\,x^3+6b^4 x^4-a^4 p^3-3a^3 b\,p^2 x+3a^2 b^2 p\,x^2-6a^4 p^2-2a^3 bp\right.}{x^{3p} c^{3p} a^4 c^5 x^4 \left(p^4+10p^3+35p^2+50p+24\right)}$$

```
int((c*x)^(-5-3*p)*(b*x^3+a*x^2)^p,x)
```

```
((a*x**2 + b*x**3)**p*( - a**4*p**3 - 6*a**4*p**2 - 11*a**4*p - 6*a**4 - a
**3*b*p**3*x - 3*a**3*b*p**2*x - 2*a**3*b*p*x + 3*a**2*b**2*p**2*x**2 + 3*
a**2*b**2*p*x**2 - 6*a*b**3*p*x**3 + 6*b**4*x**4))/(x**(3*p)*c**(3*p)*a**4
*c**5*x**4*(p**4 + 10*p**3 + 35*p**2 + 50*p + 24))
```

3.429 $\int (cx)^{-4-3p}\left(ax^2+bx^3\right)^p\,dx$

Optimal result

Integrand size = 23, antiderivative size = 126

$$\int (cx)^{-4-3p}\left(ax^2+bx^3\right)^p\,dx=-\frac{c(cx)^{-5-3p}\left(ax^2+bx^3\right)^{1+p}}{a(3+p)}+\frac{2b(cx)^{-4-3p}\left(ax^2+bx^3\right)^{1+p}}{a^2(2+p)(3+p)}-\frac{2b^2(cx)^{-3(1+p)}\left(ax^2+bx^3\right)^{1+p}}{a^3c(1+p)(2+p)(3+p)}$$

```
-c*(c*x)^(-5-3*p)*(b*x^3+a*x^2)^(p+1)/a/(3+p)+2*b*(c*x)^(-4-3*p)*(b*x^3+a*
x^2)^(p+1)/a^2/(2+p)/(3+p)-2*b^2*(b*x^3+a*x^2)^(p+1)/a^3/c/(p+1)/(2+p)/(3+
p)/((c*x)^(3*p+3))
```

Mathematica [A] (verified)

Time = 0.05 (sec) , antiderivative size = 75, normalized size of antiderivative = 0.60

$$\int (cx)^{-4-3p}\left(ax^2+bx^3\right)^p\,dx=-\frac{(cx)^{-3p}\left(x^2(a+bx)\right)^{1+p}\left(a^2\left(2+3p+p^2\right)-2ab(1+p)x+2b^2x^2\right)}{a^3c^4(1+p)(2+p)(3+p)x^5}$$

```
Integrate[(c*x)^(-4 - 3*p)*(a*x^2 + b*x^3)^p,x]
```

```
-(((x^2*(a + b*x))^(1 + p)*(a^2*(2 + 3*p + p^2) - 2*a*b*(1 + p)*x + 2*b^2*
x^2))/(a^3*c^4*(1 + p)*(2 + p)*(3 + p)*x^5*(c*x)^(3*p)))
```

Rubi [A] (verified)

Time = 0.53 (sec) , antiderivative size = 125, normalized size of antiderivative = 0.99, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.130$, Rules used = {1922, 1922, 1920}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (cx)^{-3p-4} \left(ax^2+bx^3\right)^p \, dx$$

$$\Big\downarrow \text{1922}$$

$$-\frac{2b\int (cx)^{-3(p+1)} \left(bx^3+ax^2\right)^p dx}{ac(p+3)}-\frac{c(cx)^{-3p-5} \left(ax^2+bx^3\right)^{p+1}}{a(p+3)}$$

$$\Big\downarrow \text{1922}$$

$$-\frac{2b\left(-\frac{b\int (cx)^{-3p-2} \left(bx^3+ax^2\right)^p dx}{ac(p+2)}-\frac{c(cx)^{-3p-4} \left(ax^2+bx^3\right)^{p+1}}{a(p+2)}\right)}{ac(p+3)}-\frac{c(cx)^{-3p-5} \left(ax^2+bx^3\right)^{p+1}}{a(p+3)}$$

$$\Big\downarrow \text{1920}$$

$$-\frac{2b\left(\frac{b(cx)^{-3(p+1)} \left(ax^2+bx^3\right)^{p+1}}{a^2(p+1)(p+2)}-\frac{c(cx)^{-3p-4} \left(ax^2+bx^3\right)^{p+1}}{a(p+2)}\right)}{ac(p+3)}-\frac{c(cx)^{-3p-5} \left(ax^2+bx^3\right)^{p+1}}{a(p+3)}$$

```
Int[(c*x)^(-4 - 3*p)*(a*x^2 + b*x^3)^p,x]
```

```
-((c*(c*x)^(-5 - 3*p)*(a*x^2 + b*x^3)^(1 + p))/(a*(3 + p))) - (2*b*(-((c*(
c*x)^(-4 - 3*p)*(a*x^2 + b*x^3)^(1 + p))/(a*(2 + p))) + (b*(a*x^2 + b*x^3)
^(1 + p))/(a^2*(1 + p)*(2 + p)*(c*x)^(3*(1 + p)))))/(a*c*(3 + p))
```

Defintions of rubi rules used

```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> Simp[(-c^(j - 1))*(c*x)^(m - j + 1)*((a*x^j + b*x^n)^(p + 1)/(a*(n - j
)*(p + 1))), x] /; FreeQ[{a, b, c, j, m, n, p}, x] &&  !IntegerQ[p] && NeQ[
n, j] && EqQ[m + n*p + n - j + 1, 0] && (IntegerQ[j] || GtQ[c, 0])
```

```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> Simp[c^(j - 1)*(c*x)^(m - j + 1)*((a*x^j + b*x^n)^(p + 1)/(a*(m + j*p
+ 1))), x] - Simp[b*((m + n*p + n - j + 1)/(a*c^(n - j)*(m + j*p + 1)))   I
nt[(c*x)^(m + n - j)*(a*x^j + b*x^n)^p, x], x] /; FreeQ[{a, b, c, j, m, n,
p}, x] &&  !IntegerQ[p] && NeQ[n, j] && ILtQ[Simplify[(m + n*p + n - j + 1)
/(n - j)], 0] && NeQ[m + j*p + 1, 0] && (IntegersQ[j, n] || GtQ[c, 0])
```

Maple [A] (verified)

Time = 0.52 (sec) , antiderivative size = 87, normalized size of antiderivative = 0.69

method	result	size
gosper	$-\frac{(bx+a)x\left(a^{2}p^{2}-2abpx+2b^{2}x^{2}+3a^{2}p-2abx+2a^{2}\right)\left(b\,x^{3}+a\,x^{2}\right)^{p}(cx)^{-4-3p}}{(3+p)(2+p)(p+1)a^{3}}$	87
orering	$-\frac{(bx+a)x\left(a^{2}p^{2}-2abpx+2b^{2}x^{2}+3a^{2}p-2abx+2a^{2}\right)\left(b\,x^{3}+a\,x^{2}\right)^{p}(cx)^{-4-3p}}{(3+p)(2+p)(p+1)a^{3}}$	87

```
int((c*x)^(-4-3*p)*(b*x^3+a*x^2)^p,x,method=_RETURNVERBOSE)
```

```
-(b*x+a)*x*(a^2*p^2-2*a*b*p*x+2*b^2*x^2+3*a^2*p-2*a*b*x+2*a^2)*(b*x^3+a*x^
2)^p*(c*x)^(-4-3*p)/(3+p)/(2+p)/(p+1)/a^3
```

Fricas [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 113, normalized size of antiderivative = 0.90

$$\int (cx)^{-4-3p} \left(ax^2+bx^3\right)^p \, dx = \frac{(2\,ab^2px^3-2\,b^3x^4-(a^2bp^2+a^2bp)x^2-(a^3p^2+3\,a^3p+2\,a^3)x)(bx^3+ax^2)^p(cx)^{-3\,p-4}}{a^3p^3+6\,a^3p^2+11\,a^3p+6\,a^3}$$

```
integrate((c*x)^(-4-3*p)*(b*x^3+a*x^2)^p,x, algorithm="fricas")
```

```
(2*a*b^2*p*x^3 - 2*b^3*x^4 - (a^2*b*p^2 + a^2*b*p)*x^2 - (a^3*p^2 + 3*a^3*
p + 2*a^3)*x)*(b*x^3 + a*x^2)^p*(c*x)^(-3*p - 4)/(a^3*p^3 + 6*a^3*p^2 + 11
*a^3*p + 6*a^3)
```

Sympy [F]

$$\int (cx)^{-4-3p} \left(ax^2+bx^3\right)^p \, dx = \int (cx)^{-3p-4} \left(x^2(a+bx)\right)^p \, dx$$

```
integrate((c*x)**(-4-3*p)*(b*x**3+a*x**2)**p,x)
```

```
Integral((c*x)**(-3*p - 4)*(x**2*(a + b*x))**p, x)
```

Maxima [F]

$$\int (cx)^{-4-3p} \left(ax^2+bx^3\right)^p \, dx = \int (bx^3+ax^2)^p (cx)^{-3\,p-4} \, dx$$

```
integrate((c*x)^(-4-3*p)*(b*x^3+a*x^2)^p,x, algorithm="maxima")
```

```
integrate((b*x^3 + a*x^2)^p*(c*x)^(-3*p - 4), x)
```

Giac [F]

$$\int (cx)^{-4-3p}\left(ax^2+bx^3\right)^p\,dx=\int (bx^3+ax^2)^p(cx)^{-3\,p-4}\,dx$$

```
integrate((c*x)^(-4-3*p)*(b*x^3+a*x^2)^p,x, algorithm="giac")
```

```
integrate((b*x^3 + a*x^2)^p*(c*x)^(-3*p - 4), x)
```

Mupad [B] (verification not implemented)

Time = 8.80 (sec) , antiderivative size = 165, normalized size of antiderivative = 1.31

$$\int (cx)^{-4-3p}\left(ax^2+bx^3\right)^p\,dx=-\left(b\,x^3+a\,x^2\right)^p\left(\frac{x\,(p^2+3\,p+2)}{(c\,x)^{3\,p+4}\,(p^3+6\,p^2+11\,p+6)}+\frac{2\,b^3\,x^4}{a^3\,(c\,x)^{3\,p+4}\,(p^3+6\,p^2+11\,p+6)}-\frac{2\,b^2\,p\,x^3}{a^2\,(c\,x)^{3\,p+4}\,(p^3+6\,p^2+11\,p+6)}+\frac{b\,p\,x^2\,(p+1)}{a\,(c\,x)^{3\,p+4}\,(p^3+6\,p^2+11\,p+6)}\right)$$

```
int((a*x^2 + b*x^3)^p/(c*x)^(3*p + 4),x)
```

```
-(a*x^2 + b*x^3)^p*((x*(3*p + p^2 + 2))/((c*x)^(3*p + 4)*(11*p + 6*p^2 + p
^3 + 6)) + (2*b^3*x^4)/(a^3*(c*x)^(3*p + 4)*(11*p + 6*p^2 + p^3 + 6)) - (2
*b^2*p*x^3)/(a^2*(c*x)^(3*p + 4)*(11*p + 6*p^2 + p^3 + 6)) + (b*p*x^2*(p +
 1))/(a*(c*x)^(3*p + 4)*(11*p + 6*p^2 + p^3 + 6)))
```

Reduce [B] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 108, normalized size of antiderivative = 0.86

$$
\int (cx)^{-4-3p} \left(ax^2+bx^3\right)^p \, dx
= \frac{(b\,x^3+a\,x^2)^p\,(-a^2b\,p^2x+2a\,b^2p\,x^2-2b^3x^3-a^3p^2-a^2bpx-3a^3p-2a^3)}{x^{3p}c^{3p}a^3c^4x^3\,(p^3+6p^2+11p+6)}
$$

```
int((c*x)^(-4-3*p)*(b*x^3+a*x^2)^p,x)
```

```
((a*x**2 + b*x**3)**p*( - a**3*p**2 - 3*a**3*p - 2*a**3 - a**2*b*p**2*x -
a**2*b*p*x + 2*a*b**2*p*x**2 - 2*b**3*x**3))/(x**(3*p)*c**(3*p)*a**3*c**4*
x**3*(p**3 + 6*p**2 + 11*p + 6))
```

3.430 $\int (cx)^{-3-3p} \left(ax^2+bx^3\right)^p \, dx$

Optimal result

Integrand size = 23, antiderivative size = 75

$$\int (cx)^{-3-3p} \left(ax^2+bx^3\right)^p \, dx = -\frac{c(cx)^{-4-3p}\left(ax^2+bx^3\right)^{1+p}}{a(2+p)} + \frac{b(cx)^{-3(1+p)}\left(ax^2+bx^3\right)^{1+p}}{a^2(1+p)(2+p)}$$

```
-c*(c*x)^(-4-3*p)*(b*x^3+a*x^2)^(p+1)/a/(2+p)+b*(b*x^3+a*x^2)^(p+1)/a^2/(p
+1)/(2+p)/((c*x)^(3*p+3))
```

Mathematica [A] (verified)

Time = 0.05 (sec) , antiderivative size = 50, normalized size of antiderivative = 0.67

$$\int (cx)^{-3-3p} \left(ax^2+bx^3\right)^p \, dx = -\frac{(cx)^{-3p}(a+ap-bx)\left(x^2(a+bx)\right)^{1+p}}{a^2c^3(1+p)(2+p)x^4}$$

```
Integrate[(c*x)^(-3 - 3*p)*(a*x^2 + b*x^3)^p,x]
```

```
-(((a + a*p - b*x)*(x^2*(a + b*x))^(1 + p))/(a^2*c^3*(1 + p)*(2 + p)*x^4*(
c*x)^(3*p)))
```

Rubi [A] (verified)

Time = 0.38 (sec) , antiderivative size = 75, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.087$, Rules used = {1922, 1920}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (cx)^{-3p-3} \left(ax^2+bx^3\right)^p \, dx$$

$$\Big\downarrow \text{1922}$$

$$-\frac{b \int (cx)^{-3p-2} \left(bx^3+ax^2\right)^p dx}{ac(p+2)}-\frac{c(cx)^{-3p-4} \left(ax^2+bx^3\right)^{p+1}}{a(p+2)}$$

$$\Big\downarrow \text{1920}$$

$$\frac{b(cx)^{-3(p+1)} \left(ax^2+bx^3\right)^{p+1}}{a^2(p+1)(p+2)}-\frac{c(cx)^{-3p-4} \left(ax^2+bx^3\right)^{p+1}}{a(p+2)}$$

```
Int[(c*x)^(-3 - 3*p)*(a*x^2 + b*x^3)^p,x]
```

```
-((c*(c*x)^(-4 - 3*p)*(a*x^2 + b*x^3)^(1 + p))/(a*(2 + p))) + (b*(a*x^2 +
b*x^3)^(1 + p))/(a^2*(1 + p)*(2 + p)*(c*x)^(3*(1 + p)))
```

Defintions of rubi rules used

```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> Simp[(-c^(j - 1))*(c*x)^(m - j + 1)*((a*x^j + b*x^n)^(p + 1)/(a*(n - j
)*(p + 1))), x] /; FreeQ[{a, b, c, j, m, n, p}, x] &&  !IntegerQ[p] && NeQ[
n, j] && EqQ[m + n*p + n - j + 1, 0] && (IntegerQ[j] || GtQ[c, 0])
```

```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> Simp[c^(j - 1)*(c*x)^(m - j + 1)*((a*x^j + b*x^n)^(p + 1)/(a*(m + j*p
+ 1))), x] - Simp[b*((m + n*p + n - j + 1)/(a*c^(n - j)*(m + j*p + 1)))   I
nt[(c*x)^(m + n - j)*(a*x^j + b*x^n)^p, x], x] /; FreeQ[{a, b, c, j, m, n,
p}, x] &&  !IntegerQ[p] && NeQ[n, j] && ILtQ[Simplify[(m + n*p + n - j + 1)
/(n - j)], 0] && NeQ[m + j*p + 1, 0] && (IntegersQ[j, n] || GtQ[c, 0])
```

Maple [A] (verified)

Time = 0.51 (sec) , antiderivative size = 53, normalized size of antiderivative = 0.71

method	result	size
gosper	$-\frac{\left(b\,x^{3}+a\,x^{2}\right)^{p} \left(c x \right)^{-3-3 p} \left(a p -b x +a \right) x \left(b x +a \right)}{\left(2+p \right) \left(p +1\right) a^{2}}$	53
orering	$-\frac{\left(b\,x^{3}+a\,x^{2}\right)^{p} \left(c x \right)^{-3-3 p} \left(a p -b x +a \right) x \left(b x +a \right)}{\left(2+p \right) \left(p +1\right) a^{2}}$	53

```
int((c*x)^(-3-3*p)*(b*x^3+a*x^2)^p,x,method=_RETURNVERBOSE)
```

```
-(b*x^3+a*x^2)^p*(c*x)^(-3-3*p)*(a*p-b*x+a)*x*(b*x+a)/(2+p)/(p+1)/a^2
```

Fricas [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 72, normalized size of antiderivative = 0.96

$$\int (cx)^{-3-3p} \left(ax^2+bx^3\right)^p \, dx = -\frac{\left(abpx^2 - b^2x^3 + (a^2p + a^2)x\right)(bx^3 + ax^2)^p (cx)^{-3\,p-3}}{a^2p^2 + 3\,a^2p + 2\,a^2}$$

```
integrate((c*x)^(-3-3*p)*(b*x^3+a*x^2)^p,x, algorithm="fricas")
```

```
-(a*b*p*x^2 - b^2*x^3 + (a^2*p + a^2)*x)*(b*x^3 + a*x^2)^p*(c*x)^(-3*p - 3
)/(a^2*p^2 + 3*a^2*p + 2*a^2)
```

Sympy [F]

$$\int (cx)^{-3-3p} \left(ax^2+bx^3\right)^p \, dx = \int (cx)^{-3p-3} \left(x^2(a+bx)\right)^p \, dx$$

```
integrate((c*x)**(-3-3*p)*(b*x**3+a*x**2)**p,x)
```

```
Integral((c*x)**(-3*p - 3)*(x**2*(a + b*x))**p, x)
```

Maxima [F]

$$\int (cx)^{-3-3p} \left(ax^2+bx^3\right)^p \, dx = \int \left(bx^3+ax^2\right)^p (cx)^{-3p-3} \, dx$$

```
integrate((c*x)^(-3-3*p)*(b*x^3+a*x^2)^p,x, algorithm="maxima")
```

```
integrate((b*x^3 + a*x^2)^p*(c*x)^(-3*p - 3), x)
```

Giac [F]

$$\int (cx)^{-3-3p} \left(ax^2+bx^3\right)^p \, dx = \int \left(bx^3+ax^2\right)^p (cx)^{-3p-3} \, dx$$

```
integrate((c*x)^(-3-3*p)*(b*x^3+a*x^2)^p,x, algorithm="giac")
```

```
integrate((b*x^3 + a*x^2)^p*(c*x)^(-3*p - 3), x)
```

Mupad [B] (verification not implemented)

Time = 9.12 (sec) , antiderivative size = 104, normalized size of antiderivative = 1.39

$$\int (cx)^{-3-3p} \left(ax^2+bx^3\right)^p \, dx = -\left(b\,x^3+a\,x^2\right)^p \left(\frac{x\,(p+1)}{(c\,x)^{3\,p+3}\,(p^2+3\,p+2)} - \frac{b^2\,x^3}{a^2\,(c\,x)^{3\,p+3}\,(p^2+3\,p+2)} + \frac{b\,p\,x^2}{a\,(c\,x)^{3\,p+3}\,(p^2+3\,p+2)}\right)$$

```
int((a*x^2 + b*x^3)^p/(c*x)^(3*p + 3),x)
```

```
-(a*x^2 + b*x^3)^p*((x*(p + 1))/((c*x)^(3*p + 3)*(3*p + p^2 + 2)) - (b^2*x
^3)/(a^2*(c*x)^(3*p + 3)*(3*p + p^2 + 2)) + (b*p*x^2)/(a*(c*x)^(3*p + 3)*(
3*p + p^2 + 2)))
```

Reduce [B] (verification not implemented)

Time = 0.25 (sec) , antiderivative size = 72, normalized size of antiderivative = 0.96

$$\int (cx)^{-3-3p} \left(ax^2+bx^3\right)^p \, dx = \frac{(b\,x^3+a\,x^2)^p\,(-abpx+b^2x^2-a^2p-a^2)}{x^{3p}c^{3p}a^2c^3x^2\,(p^2+3p+2)}$$

```
int((c*x)^(-3-3*p)*(b*x^3+a*x^2)^p,x)
```

```
((a*x**2 + b*x**3)**p*( - a**2*p - a**2 - a*b*p*x + b**2*x**2))/(x**(3*p)*
c**(3*p)*a**2*c**3*x**2*(p**2 + 3*p + 2))
```

3.431 $\int (cx)^{-2-3p} \left(ax^2+bx^3\right)^p \, dx$

Optimal result

Integrand size = 23, antiderivative size = 35

$$\int (cx)^{-2-3p} \left(ax^2+bx^3\right)^p \, dx = -\frac{c(cx)^{-3(1+p)} \left(ax^2+bx^3\right)^{1+p}}{a(1+p)}$$

```
-c*(b*x^3+a*x^2)^(p+1)/a/(p+1)/((c*x)^(3*p+3))
```

Mathematica [A] (verified)

Time = 0.03 (sec) , antiderivative size = 37, normalized size of antiderivative = 1.06

$$\int (cx)^{-2-3p} \left(ax^2+bx^3\right)^p \, dx = \frac{x(cx)^{-2-3p}(a+bx) \left(x^2(a+bx)\right)^p}{a(-1-p)}$$

```
Integrate[(c*x)^(-2 - 3*p)*(a*x^2 + b*x^3)^p,x]
```

```
(x*(c*x)^(-2 - 3*p)*(a + b*x)*(x^2*(a + b*x))^p)/(a*(-1 - p))
```

Rubi [A] (verified)

Time = 0.28 (sec) , antiderivative size = 35, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}} = 0.043$, Rules used = {1920}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (cx)^{-3p-2} \left(ax^2+bx^3\right)^p \, dx$$
$$\Big\downarrow \textcolor{blue}{1920}$$
$$-\frac{c(cx)^{-3(p+1)} \left(ax^2+bx^3\right)^{p+1}}{a(p+1)}$$

```
Int[(c*x)^(-2 - 3*p)*(a*x^2 + b*x^3)^p,x]
```

```
-((c*(a*x^2 + b*x^3)^(1 + p))/(a*(1 + p)*(c*x)^(3*(1 + p))))
```

Defintions of rubi rules used

```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> Simp[(-c^(j - 1))*(c*x)^(m - j + 1)*((a*x^j + b*x^n)^(p + 1)/(a*(n - j
)*(p + 1))), x] /; FreeQ[{a, b, c, j, m, n, p}, x] && !IntegerQ[p] && NeQ[
n, j] && EqQ[m + n*p + n - j + 1, 0] && (IntegerQ[j] || GtQ[c, 0])
```

Maple [A] (verified)

Time = 0.47 (sec) , antiderivative size = 39, normalized size of antiderivative = 1.11

method	result	size
gosper	$-\frac{x(bx+a)(cx)^{-3p-2}(b\,x^{3}+a\,x^{2})^{p}}{a(p+1)}$	39
orering	$-\frac{x(bx+a)(cx)^{-3p-2}(b\,x^{3}+a\,x^{2})^{p}}{a(p+1)}$	39
parallelrisch	$-\frac{x^{2}(cx)^{-3p-2}(x^{2}(bx+a))^{p}b+x(cx)^{-3p-2}(x^{2}(bx+a))^{p}a}{a(p+1)}$	60

```
int((c*x)^(-3*p-2)*(b*x^3+a*x^2)^p,x,method=_RETURNVERBOSE)
```

```
-x*(b*x+a)/a/(p+1)*(c*x)^(-3*p-2)*(b*x^3+a*x^2)^p
```

Fricas [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 40, normalized size of antiderivative = 1.14

$$\int (cx)^{-2-3p}\left(ax^2+bx^3\right)^p\,dx=-\frac{(bx^2+ax)(bx^3+ax^2)^p(cx)^{-3\,p-2}}{ap+a}$$

```
integrate((c*x)^(-2-3*p)*(b*x^3+a*x^2)^p,x, algorithm="fricas")
```

```
-(b*x^2 + a*x)*(b*x^3 + a*x^2)^p*(c*x)^(-3*p - 2)/(a*p + a)
```

Sympy [F]

$$\int (cx)^{-2-3p}\left(ax^2+bx^3\right)^p\,dx=\int (cx)^{-3p-2}\left(x^2(a+bx)\right)^p\,dx$$

```
integrate((c*x)**(-2-3*p)*(b*x**3+a*x**2)**p,x)
```

```
Integral((c*x)**(-3*p - 2)*(x**2*(a + b*x))**p, x)
```

Maxima [F]

$$\int (cx)^{-2-3p} \left(ax^2+bx^3\right)^p \, dx = \int (bx^3+ax^2)^p (cx)^{-3\,p-2} \, dx$$

```
integrate((c*x)^(-2-3*p)*(b*x^3+a*x^2)^p,x, algorithm="maxima")
```

```
integrate((b*x^3 + a*x^2)^p*(c*x)^(-3*p - 2), x)
```

Giac [F]

$$\int (cx)^{-2-3p} \left(ax^2+bx^3\right)^p \, dx = \int (bx^3+ax^2)^p (cx)^{-3\,p-2} \, dx$$

```
integrate((c*x)^(-2-3*p)*(b*x^3+a*x^2)^p,x, algorithm="giac")
```

```
integrate((b*x^3 + a*x^2)^p*(c*x)^(-3*p - 2), x)
```

Mupad [B] (verification not implemented)

Time = 9.09 (sec) , antiderivative size = 43, normalized size of antiderivative = 1.23

$$\int (cx)^{-2-3p} \left(ax^2+bx^3\right)^p \, dx = -\frac{\left(b\,x^3+a\,x^2\right)^p\,\left(a+b\,x\right)}{a\,c^2\,x\,{\left(c\,x\right)}^{3\,p}\,\left(p+1\right)}$$

```
int((a*x^2 + b*x^3)^p/(c*x)^(3*p + 2),x)
```

```
-((a*x^2 + b*x^3)^p*(a + b*x))/(a*c^2*x*(c*x)^(3*p)*(p + 1))
```

Reduce [B] (verification not implemented)

Time = 0.22 (sec) , antiderivative size = 48, normalized size of antiderivative = 1.37

$$\int (cx)^{-2-3p} \left(ax^2+bx^3\right)^p \, dx = -\frac{\left(b\,x^3+a\,x^2\right)^p (bx+a)}{x^{3p}c^{3p}a\,c^2x\,(p+1)}$$

```
int((c*x)^(-2-3*p)*(b*x^3+a*x^2)^p,x)
```

```
( - (a*x**2 + b*x**3)**p*(a + b*x))/(x**(3*p)*c**(3*p)*a*c**2*x*(p + 1))
```

3.432 $\int (cx)^{-1-3p} \left(ax^2+bx^3\right)^p \, dx$

Optimal result

Integrand size = 23, antiderivative size = 48

$$\int (cx)^{-1-3p} \left(ax^2+bx^3\right)^p \, dx = -\frac{c(cx)^{-2-3p} \left(ax^2+bx^3\right)^{1+p} \text{Hypergeometric2F1}\left(1,1,1-p,-\frac{bx}{a}\right)}{ap}$$

```
-c*(c*x)^(-2-3*p)*(b*x^3+a*x^2)^(p+1)*hypergeom([1, 1],[1-p],-b*x/a)/a/p
```

Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 57, normalized size of antiderivative = 1.19

$$\int (cx)^{-1-3p} \left(ax^2+bx^3\right)^p \, dx = -\frac{x(cx)^{-1-3p} \left(x^2(a+bx)\right)^p \left(1+\frac{bx}{a}\right)^{-p} \text{Hypergeometric2F1}\left(-p,-p,1-p,-\frac{bx}{a}\right)}{p}$$

```
Integrate[(c*x)^(-1 - 3*p)*(a*x^2 + b*x^3)^p,x]
```

```
-((x*(c*x)^(-1 - 3*p)*(x^2*(a + b*x))^p*Hypergeometric2F1[-p, -p, 1 - p, -
((b*x)/a)])/(p*(1 + (b*x)/a)^p))
```

Rubi [A] (verified)

Time = 0.32 (sec) , antiderivative size = 59, normalized size of antiderivative = 1.23, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.130, Rules used = {1938, 76, 74}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (cx)^{-3p-1} \left(ax^2+bx^3\right)^p \, dx$$

$$\Big\downarrow \text{1938}$$

$$\frac{x^p (cx)^{-3p} (a+bx)^{-p} \left(ax^2+bx^3\right)^p \int x^{-p-1} (a+bx)^p dx}{c}$$

$$\Big\downarrow \text{76}$$

$$\frac{x^p (cx)^{-3p} \left(\frac{bx}{a}+1\right)^{-p} \left(ax^2+bx^3\right)^p \int x^{-p-1} \left(\frac{bx}{a}+1\right)^p dx}{c}$$

$$\Big\downarrow \text{74}$$

$$-\frac{(cx)^{-3p} \left(\frac{bx}{a}+1\right)^{-p} \left(ax^2+bx^3\right)^p \text{Hypergeometric2F1}\left(-p,-p,1-p,-\frac{bx}{a}\right)}{cp}$$

```
Int[(c*x)^(-1 - 3*p)*(a*x^2 + b*x^3)^p,x]
```

```
-(((a*x^2 + b*x^3)^p*Hypergeometric2F1[-p, -p, 1 - p, -((b*x)/a)])/(c*p*(c
*x)^(3*p)*(1 + (b*x)/a)^p))
```

Defintions of rubi rules used

```
Int[((b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[c^n*((b*x
)^(m + 1)/(b*(m + 1)))*Hypergeometric2F1[-n, m + 1, m + 2, (-d)*(x/c)], x]
/; FreeQ[{b, c, d, m, n}, x] && !IntegerQ[m] && (IntegerQ[n] || (GtQ[c, 0]
 && !(EqQ[n, -2^(-1)] && EqQ[c^2 - d^2, 0] && GtQ[-d/(b*c), 0])))
```

```
Int[((b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[c^IntPart
[n]*((c + d*x)^FracPart[n]/(1 + d*(x/c))^FracPart[n])   Int[(b*x)^m*(1 + d*
(x/c))^n, x], x] /; FreeQ[{b, c, d, m, n}, x] && !IntegerQ[m] && !Integer
Q[n] && !GtQ[c, 0] && !GtQ[-d/(b*c), 0] && ((RationalQ[m] && !(EqQ[n, -2
^(-1)] && EqQ[c^2 - d^2, 0])) || !RationalQ[n])
```

```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> Simp[c^IntPart[m]*(c*x)^FracPart[m]*((a*x^j + b*x^n)^FracPart[p]/(x^(F
racPart[m] + j*FracPart[p])*(a + b*x^(n - j))^FracPart[p]))   Int[x^(m + j*
p)*(a + b*x^(n - j))^p, x], x] /; FreeQ[{a, b, c, j, m, n, p}, x] && !Inte
gerQ[p] && NeQ[n, j] && PosQ[n - j]
```

Maple [F]

$$\int (cx)^{-1-3p} \left(b\,x^3+a\,x^2\right)^p dx$$

```
int((c*x)^(-1-3*p)*(b*x^3+a*x^2)^p,x)
```

```
int((c*x)^(-1-3*p)*(b*x^3+a*x^2)^p,x)
```

Fricas [F]

$$\int (cx)^{-1-3p} \left(ax^2+bx^3\right)^p \, dx = \int (bx^3+ax^2)^p (cx)^{-3\,p-1} \, dx$$

```
integrate((c*x)^(-1-3*p)*(b*x^3+a*x^2)^p,x, algorithm="fricas")
```

```
integral((b*x^3 + a*x^2)^p*(c*x)^(-3*p - 1), x)
```

Sympy [F]

$$\int (cx)^{-1-3p} \left(ax^2+bx^3\right)^p \, dx = \int (cx)^{-3p-1} \left(x^2(a+bx)\right)^p \, dx$$

```
integrate((c*x)**(-1-3*p)*(b*x**3+a*x**2)**p,x)
```

```
Integral((c*x)**(-3*p - 1)*(x**2*(a + b*x))**p, x)
```

Maxima [F]

$$\int (cx)^{-1-3p} \left(ax^2+bx^3\right)^p \, dx = \int (bx^3+ax^2)^p (cx)^{-3\,p-1} \, dx$$

```
integrate((c*x)^(-1-3*p)*(b*x^3+a*x^2)^p,x, algorithm="maxima")
```

```
integrate((b*x^3 + a*x^2)^p*(c*x)^(-3*p - 1), x)
```

Giac [F]

$$\int (cx)^{-1-3p} \left(ax^2+bx^3\right)^p \, dx = \int (bx^3+ax^2)^p(cx)^{-3\,p-1}\, dx$$

```
integrate((c*x)^(-1-3*p)*(b*x^3+a*x^2)^p,x, algorithm="giac")
```

```
integrate((b*x^3 + a*x^2)^p*(c*x)^(-3*p - 1), x)
```

Mupad [F(-1)]

Timed out.

$$\int (cx)^{-1-3p} \left(ax^2+bx^3\right)^p \, dx = \int \frac{\left(b\,x^3+a\,x^2\right)^p}{\left(c\,x\right)^{3\,p+1}}\, dx$$

```
int((a*x^2 + b*x^3)^p/(c*x)^(3*p + 1),x)
```

```
int((a*x^2 + b*x^3)^p/(c*x)^(3*p + 1), x)
```

Reduce [F]

$$\int (cx)^{-1-3p} \left(ax^2+bx^3\right)^p \, dx = \frac{\int \frac{(b\,x^3+a\,x^2)^p}{x^{3p}x}dx}{c^{3p}c}$$

```
int((c*x)^(-1-3*p)*(b*x^3+a*x^2)^p,x)
```

```
int((a*x**2 + b*x**3)**p/(x**(3*p)*x),x)/(c**(3*p)*c)
```

3.433 $\int (cx)^{-3p} \left(ax^2+bx^3\right)^p \, dx$

Optimal result

Integrand size = 21, antiderivative size = 51

$$\int (cx)^{-3p} \left(ax^2+bx^3\right)^p \, dx = \frac{c(cx)^{-1-3p}\left(ax^2+bx^3\right)^{1+p} \text{Hypergeometric2F1}\left(1,2,2-p,-\frac{bx}{a}\right)}{a(1-p)}$$

```
c*(c*x)^(-1-3*p)*(b*x^3+a*x^2)^(p+1)*hypergeom([1, 2],[2-p],-b*x/a)/a/(1-p
)
```

Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 60, normalized size of antiderivative = 1.18

$$\int (cx)^{-3p} \left(ax^2+bx^3\right)^p \, dx = \frac{x(cx)^{-3p}\left(x^2(a+bx)\right)^p \left(1+\frac{bx}{a}\right)^{-p} \text{Hypergeometric2F1}\left(1-p,-p,2-p,-\frac{bx}{a}\right)}{1-p}$$

```
Integrate[(a*x^2 + b*x^3)^p/(c*x)^(3*p),x]
```

```
(x*(x^2*(a + b*x))^p*Hypergeometric2F1[1 - p, -p, 2 - p, -((b*x)/a)])/((1
- p)*(c*x)^(3*p)*(1 + (b*x)/a)^p)
```

Rubi [A] (verified)

Time = 0.31 (sec) , antiderivative size = 62, normalized size of antiderivative = 1.22, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.143$, Rules used = {1938, 76, 74}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (cx)^{-3p} \left(ax^2+bx^3\right)^p dx$$

$$\Big\downarrow \text{1938}$$

$$x^p(cx)^{-3p}(a+bx)^{-p} \left(ax^2+bx^3\right)^p \int x^{-p}(a+bx)^p dx$$

$$\Big\downarrow \text{76}$$

$$x^p(cx)^{-3p} \left(\frac{bx}{a}+1\right)^{-p} \left(ax^2+bx^3\right)^p \int x^{-p} \left(\frac{bx}{a}+1\right)^p dx$$

$$\Big\downarrow \text{74}$$

$$\frac{x(cx)^{-3p} \left(\frac{bx}{a}+1\right)^{-p} \left(ax^2+bx^3\right)^p \text{Hypergeometric2F1}\left(1-p,-p,2-p,-\frac{bx}{a}\right)}{1-p}$$

```
Int[(a*x^2 + b*x^3)^p/(c*x)^(3*p),x]
```

```
(x*(a*x^2 + b*x^3)^p*Hypergeometric2F1[1 - p, -p, 2 - p, -((b*x)/a)])/((1
- p)*(c*x)^(3*p)*(1 + (b*x)/a)^p)
```

Defintions of rubi rules used

```
Int[((b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[c^n*((b*x
)^(m + 1)/(b*(m + 1)))*Hypergeometric2F1[-n, m + 1, m + 2, (-d)*(x/c)], x]
/; FreeQ[{b, c, d, m, n}, x] &&  !IntegerQ[m] && (IntegerQ[n] || (GtQ[c, 0]
 &&  !(EqQ[n, -2^(-1)] && EqQ[c^2 - d^2, 0] && GtQ[-d/(b*c), 0])))
```

```
Int[((b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[c^IntPart
[n]*((c + d*x)^FracPart[n]/(1 + d*(x/c))^FracPart[n])   Int[(b*x)^m*(1 + d*
(x/c))^n, x], x] /; FreeQ[{b, c, d, m, n}, x] &&  !IntegerQ[m] &&  !Integer
Q[n] &&  !GtQ[c, 0] &&  !GtQ[-d/(b*c), 0] && ((RationalQ[m] &&  !(EqQ[n, -2
^(-1)] && EqQ[c^2 - d^2, 0])) ||  !RationalQ[n])
```

```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> Simp[c^IntPart[m]*(c*x)^FracPart[m]*((a*x^j + b*x^n)^FracPart[p]/(x^(F
racPart[m] + j*FracPart[p])*(a + b*x^(n - j))^FracPart[p]))   Int[x^(m + j*
p)*(a + b*x^(n - j))^p, x], x] /; FreeQ[{a, b, c, j, m, n, p}, x] &&  !Inte
gerQ[p] && NeQ[n, j] && PosQ[n - j]
```

Maple [F]

$$\int \left(b\,x^{3}+a\,x^{2}\right)^{p} \left(c x \right)^{-3 p}d x$$

```
int((b*x^3+a*x^2)^p/((c*x)^(3*p)),x)
```

```
int((b*x^3+a*x^2)^p/((c*x)^(3*p)),x)
```

Fricas [F]

$$\int (cx)^{-3p} \left(ax^2+bx^3\right)^p \, dx = \int \frac{(bx^3+ax^2)^p}{(cx)^{3\,p}} \, dx$$

```
integrate((b*x^3+a*x^2)^p/((c*x)^(3*p)),x, algorithm="fricas")
```

```
integral((b*x^3 + a*x^2)^p/(c*x)^(3*p), x)
```

Sympy [F]

$$\int (cx)^{-3p} \left(ax^2+bx^3\right)^p \, dx = \int (cx)^{-3p} \left(x^2(a+bx)\right)^p \, dx$$

```
integrate((b*x**3+a*x**2)**p/((c*x)**(3*p)),x)
```

```
Integral((x**2*(a + b*x))**p/(c*x)**(3*p), x)
```

Maxima [F]

$$\int (cx)^{-3p} \left(ax^2+bx^3\right)^p \, dx = \int \frac{(bx^3+ax^2)^p}{(cx)^{3\,p}} \, dx$$

```
integrate((b*x^3+a*x^2)^p/((c*x)^(3*p)),x, algorithm="maxima")
```

```
integrate((b*x^3 + a*x^2)^p/(c*x)^(3*p), x)
```

Giac [F]

$$\int (cx)^{-3p} \left(ax^2+bx^3\right)^p \, dx = \int \frac{(bx^3+ax^2)^p}{(cx)^{3p}} \, dx$$

```
integrate((b*x^3+a*x^2)^p/((c*x)^(3*p)),x, algorithm="giac")
```

```
integrate((b*x^3 + a*x^2)^p/(c*x)^(3*p), x)
```

Mupad [F(-1)]

Timed out.

$$\int (cx)^{-3p} \left(ax^2+bx^3\right)^p \, dx = \int \frac{(b\,x^3+a\,x^2)^p}{(c\,x)^{3\,p}} \, dx$$

```
int((a*x^2 + b*x^3)^p/(c*x)^(3*p),x)
```

```
int((a*x^2 + b*x^3)^p/(c*x)^(3*p), x)
```

Reduce [F]

$$\int (cx)^{-3p} \left(ax^2+bx^3\right)^p \, dx = \frac{(b\,x^3+a\,x^2)^p\,x + x^{3p}\left(\int \frac{(b\,x^3+a\,x^2)^p}{x^{3p}a+x^{3p}bx}dx\right)ap}{x^{3p}c^{3p}}$$

```
int((b*x^3+a*x^2)^p/((c*x)^(3*p)),x)
```

```
((a*x**2 + b*x**3)**p*x + x**(3*p)*int((a*x**2 + b*x**3)**p/(x**(3*p)*a +
x**(3*p)*b*x),x)*a*p)/(x**(3*p)*c**(3*p))
```

3.434 $\int (cx)^{1-3p} (ax^2 + bx^3)^p \, dx$

Optimal result

Integrand size = 23, antiderivative size = 49

$$\int (cx)^{1-3p} (ax^2 + bx^3)^p \, dx = \frac{c(cx)^{-3p} (ax^2 + bx^3)^{1+p} \text{Hypergeometric2F1}\left(1, 3, 3-p, -\frac{bx}{a}\right)}{a(2-p)}$$

```
c*(b*x^3+a*x^2)^(p+1)*hypergeom([1, 3],[3-p],-b*x/a)/a/(2-p)/((c*x)^(3*p))
```

Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 63, normalized size of antiderivative = 1.29

$$\int (cx)^{1-3p} (ax^2 + bx^3)^p \, dx = \frac{cx^2 (cx)^{-3p} (x^2(a+bx))^p \left(1 + \frac{bx}{a}\right)^{-p} \text{Hypergeometric2F1}\left(2-p, -p, 3-p, -\frac{bx}{a}\right)}{2-p}$$

```
Integrate[(c*x)^(1 - 3*p)*(a*x^2 + b*x^3)^p,x]
```

```
(c*x^2*(x^2*(a + b*x))^p*Hypergeometric2F1[2 - p, -p, 3 - p, -((b*x)/a)])/
((2 - p)*(c*x)^(3*p)*(1 + (b*x)/a)^p)
```

Rubi [A] (verified)

Time = 0.32 (sec) , antiderivative size = 65, normalized size of antiderivative = 1.33, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.130, Rules used = {1938, 76, 74}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (cx)^{1-3p} \left(ax^2+bx^3\right)^p dx$$

$$\Big\downarrow \text{1938}$$

$$cx^p(cx)^{-3p}(a+bx)^{-p} \left(ax^2+bx^3\right)^p \int x^{1-p}(a+bx)^p dx$$

$$\Big\downarrow \text{76}$$

$$cx^p(cx)^{-3p} \left(\frac{bx}{a}+1\right)^{-p} \left(ax^2+bx^3\right)^p \int x^{1-p} \left(\frac{bx}{a}+1\right)^p dx$$

$$\Big\downarrow \text{74}$$

$$\frac{cx^2(cx)^{-3p} \left(\frac{bx}{a}+1\right)^{-p} \left(ax^2+bx^3\right)^p \text{Hypergeometric2F1}\left(2-p,-p,3-p,-\frac{bx}{a}\right)}{2-p}$$

```
Int[(c*x)^(1 - 3*p)*(a*x^2 + b*x^3)^p,x]
```


```
(c*x^2*(a*x^2 + b*x^3)^p*Hypergeometric2F1[2 - p, -p, 3 - p, -((b*x)/a)])/
((2 - p)*(c*x)^(3*p)*(1 + (b*x)/a)^p)
```

Defintions of rubi rules used

```
Int[((b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[c^n*((b*x
)^(m + 1)/(b*(m + 1)))*Hypergeometric2F1[-n, m + 1, m + 2, (-d)*(x/c)], x]
/; FreeQ[{b, c, d, m, n}, x] &&  !IntegerQ[m] && (IntegerQ[n] || (GtQ[c, 0]
 &&  !(EqQ[n, -2^(-1)] && EqQ[c^2 - d^2, 0] && GtQ[-d/(b*c), 0])))
```

```
Int[((b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[c^IntPart
[n]*((c + d*x)^FracPart[n]/(1 + d*(x/c))^FracPart[n])   Int[(b*x)^m*(1 + d*
(x/c))^n, x], x] /; FreeQ[{b, c, d, m, n}, x] &&  !IntegerQ[m] &&  !Integer
Q[n] &&  !GtQ[c, 0] &&  !GtQ[-d/(b*c), 0] && ((RationalQ[m] &&  !(EqQ[n, -2
^(-1)] && EqQ[c^2 - d^2, 0])) ||  !RationalQ[n])
```

```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> Simp[c^IntPart[m]*(c*x)^FracPart[m]*((a*x^j + b*x^n)^FracPart[p]/(x^(F
racPart[m] + j*FracPart[p])*(a + b*x^(n - j))^FracPart[p]))   Int[x^(m + j*
p)*(a + b*x^(n - j))^p, x], x] /; FreeQ[{a, b, c, j, m, n, p}, x] &&  !Inte
gerQ[p] && NeQ[n, j] && PosQ[n - j]
```

Maple [F]

$$\int (cx)^{1-3p} \left(b\,x^{3}+a\,x^{2}\right)^{p}dx$$

```
int((c*x)^(1-3*p)*(b*x^3+a*x^2)^p,x)
```



```
int((c*x)^(1-3*p)*(b*x^3+a*x^2)^p,x)
```

Fricas [F]

$$\int (cx)^{1-3p} \left(ax^2+bx^3\right)^p \, dx = \int \left(bx^3+ax^2\right)^p (cx)^{-3\,p+1} \, dx$$

```
integrate((c*x)^(1-3*p)*(b*x^3+a*x^2)^p,x, algorithm="fricas")
```

```
integral((b*x^3 + a*x^2)^p*(c*x)^(-3*p + 1), x)
```

Sympy [F]

$$\int (cx)^{1-3p} \left(ax^2+bx^3\right)^p \, dx = \int (cx)^{1-3p} \left(x^2(a+bx)\right)^p \, dx$$

```
integrate((c*x)**(1-3*p)*(b*x**3+a*x**2)**p,x)
```

```
Integral((c*x)**(1 - 3*p)*(x**2*(a + b*x))**p, x)
```

Maxima [F]

$$\int (cx)^{1-3p} \left(ax^2+bx^3\right)^p \, dx = \int \left(bx^3+ax^2\right)^p (cx)^{-3\,p+1} \, dx$$

```
integrate((c*x)^(1-3*p)*(b*x^3+a*x^2)^p,x, algorithm="maxima")
```

```
integrate((b*x^3 + a*x^2)^p*(c*x)^(-3*p + 1), x)
```

Giac [F]

$$\int (cx)^{1-3p} \left(ax^2+bx^3\right)^p \, dx = \int \left(bx^3+ax^2\right)^p (cx)^{-3\,p+1} \, dx$$

```
integrate((c*x)^(1-3*p)*(b*x^3+a*x^2)^p,x, algorithm="giac")
```

```
integrate((b*x^3 + a*x^2)^p*(c*x)^(-3*p + 1), x)
```

Mupad [F(-1)]

Timed out.

$$\int (cx)^{1-3p} \left(ax^2+bx^3\right)^p \, dx = \int (c\,x)^{1-3\,p} \left(b\,x^3+a\,x^2\right)^p \, dx$$

```
int((c*x)^(1 - 3*p)*(a*x^2 + b*x^3)^p,x)
```

```
int((c*x)^(1 - 3*p)*(a*x^2 + b*x^3)^p, x)
```

Reduce [F]

$$\int (cx)^{1-3p} \left(ax^2+bx^3\right)^p \, dx = \frac{c\Big(\left(b\,x^3+a\,x^2\right)^p apx+\left(b\,x^3+a\,x^2\right)^p b\,x^2+x^{3p}\left(\int \frac{(b\,x^3+a\,x^2)^p}{x^{3p}a+x^{3p}bx}dx\right)a^2p^2-x^{3p}\left(\int \frac{(b\,x^3+a\,x^2)^p}{x^{3p}a+x^{3p}bx}dx\right)a^2p\Big)}{2x^{3p}c^{3p}b}$$

```
int((c*x)^(1-3*p)*(b*x^3+a*x^2)^p,x)
```

```
(c*((a*x**2 + b*x**3)**p*a*p*x + (a*x**2 + b*x**3)**p*b*x**2 + x**(3*p)*in
t((a*x**2 + b*x**3)**p/(x**(3*p)*a + x**(3*p)*b*x),x)*a**2*p**2 - x**(3*p)
*int((a*x**2 + b*x**3)**p/(x**(3*p)*a + x**(3*p)*b*x),x)*a**2*p))/(2*x**(3
*p)*c**(3*p)*b)
```

3.435 $\int (cx)^m \left(ax^n+bx^{1+n}\right)^3 \, dx$

Optimal result

Integrand size = 21, antiderivative size = 101

$$\int (cx)^m \left(ax^n+bx^{1+n}\right)^3 \, dx = \frac{3ab^2x^{3(1+n)}(cx)^m}{3+m+3n}+\frac{a^3x^{1+3n}(cx)^m}{1+m+3n}+\frac{3a^2bx^{2+3n}(cx)^m}{2+m+3n}+\frac{b^3x^{4+3n}(cx)^m}{4+m+3n}$$

```
3*a*b^2*x^(3+3*n)*(c*x)^m/(3+m+3*n)+a^3*x^(1+3*n)*(c*x)^m/(1+m+3*n)+3*a^2*
b*x^(2+3*n)*(c*x)^m/(2+m+3*n)+b^3*x^(4+3*n)*(c*x)^m/(4+m+3*n)
```

Mathematica [A] (verified)

Time = 0.07 (sec) , antiderivative size = 73, normalized size of antiderivative = 0.72

$$\int (cx)^m \left(ax^n+bx^{1+n}\right)^3 \, dx = x^{1+3n}(cx)^m\left(\frac{a^3}{1+m+3n}+\frac{3a^2bx}{2+m+3n}+\frac{3ab^2x^2}{3+m+3n}+\frac{b^3x^3}{4+m+3n}\right)$$

```
Integrate[(c*x)^m*(a*x^n + b*x^(1 + n))^3,x]
```

```
x^(1 + 3*n)*(c*x)^m*(a^3/(1 + m + 3*n) + (3*a^2*b*x)/(2 + m + 3*n) + (3*a*
b^2*x^2)/(3 + m + 3*n) + (b^3*x^3)/(4 + m + 3*n))
```

Rubi [A] (verified)

Time = 0.42 (sec) , antiderivative size = 96, normalized size of antiderivative = 0.95, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.190, Rules used = {1939, 10, 53, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (cx)^m \left(ax^n+bx^{n+1}\right)^3 \, dx$$
$$\Big\downarrow \text{1939}$$
$$x^{-m}(cx)^m \int x^m \left(ax^n+bx^{n+1}\right)^3 dx$$
$$\Big\downarrow \text{10}$$
$$x^{-m}(cx)^m \int x^{m+3n}(a+bx)^3 dx$$
$$\Big\downarrow \text{53}$$
$$x^{-m}(cx)^m \int \left(a^3x^{m+3n}+3a^2bx^{m+3n+1}+3ab^2x^{m+3n+2}+b^3x^{m+3n+3}\right) dx$$
$$\Big\downarrow \text{2009}$$
$$x^{-m}(cx)^m \left(\frac{a^3x^{m+3n+1}}{m+3n+1}+\frac{3a^2bx^{m+3n+2}}{m+3n+2}+\frac{3ab^2x^{m+3n+3}}{m+3n+3}+\frac{b^3x^{m+3n+4}}{m+3n+4}\right)$$

```
Int[(c*x)^m*(a*x^n + b*x^(1 + n))^3,x]
```

```
((c*x)^m*((a^3*x^(1 + m + 3*n))/(1 + m + 3*n) + (3*a^2*b*x^(2 + m + 3*n))/
(2 + m + 3*n) + (3*a*b^2*x^(3 + m + 3*n))/(3 + m + 3*n) + (b^3*x^(4 + m +
3*n))/(4 + m + 3*n)))/x^m
```

Defintions of rubi rules used

```
Int[(u_.)*((e_.)*(x_))^(m_.)*((a_.)*(x_)^(r_.) + (b_.)*(x_)^(s_.))^(p_.), x
_Symbol] :> Simp[1/e^(p*r)   Int[u*(e*x)^(m + p*r)*(a + b*x^(s - r))^p, x],
 x] /; FreeQ[{a, b, e, m, r, s}, x] && IntegerQ[p] && (IntegerQ[p*r] || GtQ
[e, 0]) && PosQ[s - r]
```

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && LeQ[7*m + 4*n + 4, 0])
|| LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

```
Int[(u_)^(m_.)*((a_.)*(v_)^(j_.) + (b_.)*(v_)^(n_.))^(p_.), x_Symbol] :> Si
mp[u^m/(Coefficient[v, x, 1]*v^m)   Subst[Int[x^m*(a*x^j + b*x^n)^p, x], x,
 v], x] /; FreeQ[{a, b, j, m, n, p}, x] && LinearPairQ[u, v, x]
```

```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 446 vs. $2(101) = 202$.

Time = 1.68 (sec) , antiderivative size = 447, normalized size of antiderivative = 4.43

method	result
orering	$\frac{(b^3m^3x^3+9b^3m^2n\,x^3+27b^3m\,n^2x^3+27b^3n^3x^3+3a\,b^2m^3x^2+27a\,b^2m^2n\,x^2+81a\,b^2m\,n^2x^2+81a\,b^2n^3x^2+6b^3m^2x^3+36b^3mn}{}$
risch	$\frac{x(b^3m^3x^3+9b^3m^2n\,x^3+27b^3m\,n^2x^3+27b^3n^3x^3+3a\,b^2m^3x^2+27a\,b^2m^2n\,x^2+81a\,b^2m\,n^2x^2+81a\,b^2n^3x^2+6b^3m^2x^3+36b^3m}{}$
parallelrisch	$\frac{24x\,x^{3n}(cx)^m a^3+6x\,x^{3+3n}(cx)^m b^3+3x\,x^{2n}x^{1+n}(cx)^m a^2b\,m^3+9x\,x^{3n}(cx)^m a^3m^2n+27x\,x^{3n}(cx)^m a^3m\,n^2+9x\,x^{3+3n}(cx)^m}{}$

```
int((c*x)^m*(a*x^n+b*x^(1+n))^3,x,method=_RETURNVERBOSE)
```

```
(b^3*m^3*x^3+9*b^3*m^2*n*x^3+27*b^3*m*n^2*x^3+27*b^3*n^3*x^3+3*a*b^2*m^3*x
^2+27*a*b^2*m^2*n*x^2+81*a*b^2*m*n^2*x^2+81*a*b^2*n^3*x^2+6*b^3*m^2*x^3+36
*b^3*m*n*x^3+54*b^3*n^2*x^3+3*a^2*b*m^3*x+27*a^2*b*m^2*n*x+81*a^2*b*m*n^2*
x+81*a^2*b*n^3*x+21*a*b^2*m^2*x^2+126*a*b^2*m*n*x^2+189*a*b^2*n^2*x^2+11*b
^3*m*x^3+33*b^3*n*x^3+a^3*m^3+9*a^3*m^2*n+27*a^3*m*n^2+27*a^3*n^3+24*a^2*b
*m^2*x+144*a^2*b*m*n*x+216*a^2*b*n^2*x+42*a*b^2*m*x^2+126*a*b^2*n*x^2+6*b^
3*x^3+9*a^3*m^2+54*a^3*m*n+81*a^3*n^2+57*a^2*b*m*x+171*a^2*b*n*x+24*a*b^2*
x^2+26*a^3*m+78*a^3*n+36*a^2*b*x+24*a^3)/(1+m+3*n)/(2+m+3*n)/(3+m+3*n)/(4+
m+3*n)/(b*x+a)^3*x*(c*x)^m*(a*x^n+b*x^(1+n))^3
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 419 vs. $2(101) = 202$.

Time = 0.18 (sec) , antiderivative size = 419, normalized size of antiderivative = 4.15

$$\int (cx)^m \left(ax^n + bx^{1+n}\right)^3 dx$$
$$= \frac{(a^3m^3 + 27\,a^3n^3 + 9\,a^3m^2 + 26\,a^3m + (b^3m^3 + 27\,b^3n^3 + 6\,b^3m^2 + 11\,b^3m + 6\,b^3 + 27\,(b^3m + 2\,b^3)n^2 + \cdots}{}$$

```
integrate((c*x)^m*(a*x^n+b*x^(1+n))^3,x, algorithm="fricas")
```

```
(a^3*m^3 + 27*a^3*n^3 + 9*a^3*m^2 + 26*a^3*m + (b^3*m^3 + 27*b^3*n^3 + 6*b
^3*m^2 + 11*b^3*m + 6*b^3 + 27*(b^3*m + 2*b^3)*n^2 + 3*(3*b^3*m^2 + 12*b^3
*m + 11*b^3)*n)*x^3 + 24*a^3 + 27*(a^3*m + 3*a^3)*n^2 + 3*(a*b^2*m^3 + 27*
a*b^2*n^3 + 7*a*b^2*m^2 + 14*a*b^2*m + 8*a*b^2 + 9*(3*a*b^2*m + 7*a*b^2)*n
^2 + 3*(3*a*b^2*m^2 + 14*a*b^2*m + 14*a*b^2)*n)*x^2 + 3*(3*a^3*m^2 + 18*a^
3*m + 26*a^3)*n + 3*(a^2*b*m^3 + 27*a^2*b*n^3 + 8*a^2*b*m^2 + 19*a^2*b*m +
 12*a^2*b + 9*(3*a^2*b*m + 8*a^2*b)*n^2 + 3*(3*a^2*b*m^2 + 16*a^2*b*m + 19
*a^2*b)*n)*x)*x^(3*n + 3)*e^(m*log(c) + m*log(x))/((m^4 + 54*(2*m + 5)*n^3
 + 81*n^4 + 10*m^3 + 9*(6*m^2 + 30*m + 35)*n^2 + 35*m^2 + 6*(2*m^3 + 15*m^
2 + 35*m + 25)*n + 50*m + 24)*x^2)
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 4519 vs. $2(94) = 188$.

Time = 38.09 (sec) , antiderivative size = 4519, normalized size of antiderivative = 44.74

$$\int (cx)^m \left(ax^n + bx^{1+n}\right)^3 \, dx = \text{Too large to display}$$

```
integrate((c*x)**m*(a*x**n+b*x**(1+n))**3,x)
```

```
Piecewise((-a**3*x*x**(3*n)*(c*x)**(-3*n - 4)/3 - 3*a**2*b*x*x**(2*n)*x**(
n + 1)*(c*x)**(-3*n - 4)/2 - 3*a*b**2*x*x**n*x**(2*n + 2)*(c*x)**(-3*n - 4
) + b**3*x*x**(3*n + 3)*(c*x)**(-3*n - 4)*log(x), Eq(m, -3*n - 4)), (-a**3
*x*x**(3*n)*(c*x)**(-3*n - 3)/2 - 3*a**2*b*x*x**(2*n)*x**(n + 1)*(c*x)**(-
3*n - 3) + 3*a*b**2*x*x**n*x**(2*n + 2)*(c*x)**(-3*n - 3)*log(x) + b**3*x*
x**(3*n + 3)*(c*x)**(-3*n - 3), Eq(m, -3*n - 3)), (-a**3*x*x**(3*n)*(c*x)*
*(-3*n - 2) + 3*a**2*b*x*x**(2*n)*x**(n + 1)*(c*x)**(-3*n - 2)*log(x) + 3*
a*b**2*x*x**n*x**(2*n + 2)*(c*x)**(-3*n - 2) + b**3*x*x**(3*n + 3)*(c*x)**
(-3*n - 2)/2, Eq(m, -3*n - 2)), (a**3*x*x**(3*n)*(c*x)**(-3*n - 1)*log(x)
+ 3*a**2*b*x*x**(2*n)*x**(n + 1)*(c*x)**(-3*n - 1) + 3*a*b**2*x*x**n*x**(2
*n + 2)*(c*x)**(-3*n - 1)/2 + b**3*x*x**(3*n + 3)*(c*x)**(-3*n - 1)/3, Eq(
m, -3*n - 1)), (a**3*m**3*x*x**(3*n)*(c*x)**m/(m**4 + 12*m**3*n + 10*m**3
+ 54*m**2*n**2 + 90*m**2*n + 35*m**2 + 108*m*n**3 + 270*m*n**2 + 210*m*n +
 50*m + 81*n**4 + 270*n**3 + 315*n**2 + 150*n + 24) + 9*a**3*m**2*n*x*x**(
3*n)*(c*x)**m/(m**4 + 12*m**3*n + 10*m**3 + 54*m**2*n**2 + 90*m**2*n + 35*
m**2 + 108*m*n**3 + 270*m*n**2 + 210*m*n + 50*m + 81*n**4 + 270*n**3 + 315
*n**2 + 150*n + 24) + 9*a**3*m**2*x*x**(3*n)*(c*x)**m/(m**4 + 12*m**3*n +
10*m**3 + 54*m**2*n**2 + 90*m**2*n + 35*m**2 + 108*m*n**3 + 270*m*n**2 + 2
10*m*n + 50*m + 81*n**4 + 270*n**3 + 315*n**2 + 150*n + 24) + 27*a**3*m*n*
*2*x*x**(3*n)*(c*x)**m/(m**4 + 12*m**3*n + 10*m**3 + 54*m**2*n**2 + 90*...
```

Maxima [A] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 119, normalized size of antiderivative = 1.18

$$\int (cx)^m \left(ax^n + bx^{1+n}\right)^3 \, dx = \frac{b^3c^m x^4 e^{(m\log(x)+3\,n\log(x))}}{m+3\,n+4} + \frac{3\,ab^2c^m x^3 e^{(m\log(x)+3\,n\log(x))}}{m+3\,n+3} + \frac{3\,a^2bc^m x^2 e^{(m\log(x)+3\,n\log(x))}}{m+3\,n+2} + \frac{a^3c^m x e^{(m\log(x)+3\,n\log(x))}}{m+3\,n+1}$$

```
integrate((c*x)^m*(a*x^n+b*x^(1+n))^3,x, algorithm="maxima")
```

```
b^3*c^m*x^4*e^(m*log(x) + 3*n*log(x))/(m + 3*n + 4) + 3*a*b^2*c^m*x^3*e^(m
*log(x) + 3*n*log(x))/(m + 3*n + 3) + 3*a^2*b*c^m*x^2*e^(m*log(x) + 3*n*lo
g(x))/(m + 3*n + 2) + a^3*c^m*x*e^(m*log(x) + 3*n*log(x))/(m + 3*n + 1)
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1090 vs. 2(101) = 202.

Time = 0.14 (sec) , antiderivative size = 1090, normalized size of antiderivative = 10.79

$$\int (cx)^m \left(ax^n + bx^{1+n}\right)^3 \, dx = \text{Too large to display}$$

```
integrate((c*x)^m*(a*x^n+b*x^(1+n))^3,x, algorithm="giac")
```

```
(b^3*m^3*x^4*x^(3*n)*e^(m*log(c) + m*log(x)) + 9*b^3*m^2*n*x^4*x^(3*n)*e^(
m*log(c) + m*log(x)) + 27*b^3*m*n^2*x^4*x^(3*n)*e^(m*log(c) + m*log(x)) +
27*b^3*n^3*x^4*x^(3*n)*e^(m*log(c) + m*log(x)) + 3*a*b^2*m^3*x^3*x^(3*n)*e
^(m*log(c) + m*log(x)) + 27*a*b^2*m^2*n*x^3*x^(3*n)*e^(m*log(c) + m*log(x)
) + 81*a*b^2*m*n^2*x^3*x^(3*n)*e^(m*log(c) + m*log(x)) + 81*a*b^2*n^3*x^3*
x^(3*n)*e^(m*log(c) + m*log(x)) + 6*b^3*m^2*x^4*x^(3*n)*e^(m*log(c) + m*lo
g(x)) + 36*b^3*m*n*x^4*x^(3*n)*e^(m*log(c) + m*log(x)) + 54*b^3*n^2*x^4*x^
(3*n)*e^(m*log(c) + m*log(x)) + 3*a^2*b*m^3*x^2*x^(3*n)*e^(m*log(c) + m*lo
g(x)) + 27*a^2*b*m^2*n*x^2*x^(3*n)*e^(m*log(c) + m*log(x)) + 81*a^2*b*m*n^
2*x^2*x^(3*n)*e^(m*log(c) + m*log(x)) + 81*a^2*b*n^3*x^2*x^(3*n)*e^(m*log(
c) + m*log(x)) + 21*a*b^2*m^2*x^3*x^(3*n)*e^(m*log(c) + m*log(x)) + 126*a*
b^2*m*n*x^3*x^(3*n)*e^(m*log(c) + m*log(x)) + 189*a*b^2*n^2*x^3*x^(3*n)*e^
(m*log(c) + m*log(x)) + 11*b^3*m*x^4*x^(3*n)*e^(m*log(c) + m*log(x)) + 33*
b^3*n*x^4*x^(3*n)*e^(m*log(c) + m*log(x)) + a^3*m^3*x*x^(3*n)*e^(m*log(c)
+ m*log(x)) + 9*a^3*m^2*n*x*x^(3*n)*e^(m*log(c) + m*log(x)) + 27*a^3*m*n^2
*x*x^(3*n)*e^(m*log(c) + m*log(x)) + 27*a^3*n^3*x*x^(3*n)*e^(m*log(c) + m*
log(x)) + 24*a^2*b*m^2*x^2*x^(3*n)*e^(m*log(c) + m*log(x)) + 144*a^2*b*m*n
*x^2*x^(3*n)*e^(m*log(c) + m*log(x)) + 216*a^2*b*n^2*x^2*x^(3*n)*e^(m*log(
c) + m*log(x)) + 42*a*b^2*m*x^3*x^(3*n)*e^(m*log(c) + m*log(x)) + 126*a*b^
2*n*x^3*x^(3*n)*e^(m*log(c) + m*log(x)) + 6*b^3*x^4*x^(3*n)*e^(m*log(c)...
```

Mupad [B] (verification not implemented)

Time = 9.29 (sec) , antiderivative size = 541, normalized size of antiderivative = 5.36

$$\int (cx)^m \left(ax^n+bx^{1+n}\right)^3 \, dx$$
$$= \frac{a^3\,x\,x^{3\,n}\,{\left(c\,x\right)}^m\,\left(m^3+9\,m^2\,n+9\,m^2+27\,m\,n^2+54\,m\,n+26\,m+27\,n^3+81\,n^2+\right.}{m^4+12\,m^3\,n+10\,m^3+54\,m^2\,n^2+90\,m^2\,n+35\,m^2+108\,m\,n^3+270\,m\,n^2+210\,m\,n+50\,m+81\,n^4}$$
$$+\frac{b^3\,x\,x^{3\,n+3}\,{\left(c\,x\right)}^m\,\left(m^3+9\,m^2\,n+6\,m^2+27\,m\,n^2+36\,m\,n+11\,m+27\,n^3+54\,n\right.}{m^4+12\,m^3\,n+10\,m^3+54\,m^2\,n^2+90\,m^2\,n+35\,m^2+108\,m\,n^3+270\,m\,n^2+210\,m\,n+50\,m+81\,n}$$
$$+\frac{3\,a\,b^2\,x\,x^{2\,n+2}\,x^n\,{\left(c\,x\right)}^m\,\left(m^3+9\,m^2\,n+7\,m^2+27\,m\,n^2+42\,m\,n+14\,m+27\,n^3+6\right.}{m^4+12\,m^3\,n+10\,m^3+54\,m^2\,n^2+90\,m^2\,n+35\,m^2+108\,m\,n^3+270\,m\,n^2+210\,m\,n+50\,m+81\,n}$$
$$+\frac{3\,a^2\,b\,x\,x^{2\,n}\,x^{n+1}\,{\left(c\,x\right)}^m\,\left(m^3+9\,m^2\,n+8\,m^2+27\,m\,n^2+48\,m\,n+19\,m+27\,n^3+72\right.}{m^4+12\,m^3\,n+10\,m^3+54\,m^2\,n^2+90\,m^2\,n+35\,m^2+108\,m\,n^3+270\,m\,n^2+210\,m\,n+50\,m+81\,n}$$

```
int((c*x)^m*(a*x^n + b*x^(n + 1))^3,x)
```

```
(a^3*x*x^(3*n)*(c*x)^m*(26*m + 78*n + 54*m*n + 27*m*n^2 + 9*m^2*n + 9*m^2
+ m^3 + 81*n^2 + 27*n^3 + 24))/(50*m + 150*n + 210*m*n + 270*m*n^2 + 90*m^
2*n + 108*m*n^3 + 12*m^3*n + 35*m^2 + 10*m^3 + m^4 + 315*n^2 + 270*n^3 + 8
1*n^4 + 54*m^2*n^2 + 24) + (b^3*x*x^(3*n + 3)*(c*x)^m*(11*m + 33*n + 36*m*
n + 27*m*n^2 + 9*m^2*n + 6*m^2 + m^3 + 54*n^2 + 27*n^3 + 6))/(50*m + 150*n
 + 210*m*n + 270*m*n^2 + 90*m^2*n + 108*m*n^3 + 12*m^3*n + 35*m^2 + 10*m^3
 + m^4 + 315*n^2 + 270*n^3 + 81*n^4 + 54*m^2*n^2 + 24) + (3*a*b^2*x*x^(2*n
 + 2)*x^n*(c*x)^m*(14*m + 42*n + 42*m*n + 27*m*n^2 + 9*m^2*n + 7*m^2 + m^3
 + 63*n^2 + 27*n^3 + 8))/(50*m + 150*n + 210*m*n + 270*m*n^2 + 90*m^2*n +
108*m*n^3 + 12*m^3*n + 35*m^2 + 10*m^3 + m^4 + 315*n^2 + 270*n^3 + 81*n^4
+ 54*m^2*n^2 + 24) + (3*a^2*b*x*x^(2*n)*x^(n + 1)*(c*x)^m*(19*m + 57*n + 4
8*m*n + 27*m*n^2 + 9*m^2*n + 8*m^2 + m^3 + 72*n^2 + 27*n^3 + 12))/(50*m +
150*n + 210*m*n + 270*m*n^2 + 90*m^2*n + 108*m*n^3 + 12*m^3*n + 35*m^2 + 1
0*m^3 + m^4 + 315*n^2 + 270*n^3 + 81*n^4 + 54*m^2*n^2 + 24)
```

Reduce [B] (verification not implemented)

Time = 0.23 (sec) , antiderivative size = 471, normalized size of antiderivative = 4.66

$$\int (cx)^m \left(ax^n + bx^{1+n}\right)^3 \, dx$$
$$= \frac{x^{m+3n} c^m x(b^3 m^3 x^3 + 9b^3 m^2 n\, x^3 + 27b^3 m\, n^2 x^3 + 27b^3 n^3 x^3 + 3a\, b^2 m^3 x^2 + 27a\, b^2 m^2 n\, x^2 + 81a\, b^2 m\, n^2 x^2 +}{}$$

```
int((c*x)^m*(a*x^n+b*x^(1+n))^3,x)
```

```
(x**(m + 3*n)*c**m*x*(a**3*m**3 + 9*a**3*m**2*n + 9*a**3*m**2 + 27*a**3*m*
n**2 + 54*a**3*m*n + 26*a**3*m + 27*a**3*n**3 + 81*a**3*n**2 + 78*a**3*n +
 24*a**3 + 3*a**2*b*m**3*x + 27*a**2*b*m**2*n*x + 24*a**2*b*m**2*x + 81*a*
*2*b*m*n**2*x + 144*a**2*b*m*n*x + 57*a**2*b*m*x + 81*a**2*b*n**3*x + 216*
a**2*b*n**2*x + 171*a**2*b*n*x + 36*a**2*b*x + 3*a*b**2*m**3*x**2 + 27*a*b
**2*m**2*n*x**2 + 21*a*b**2*m**2*x**2 + 81*a*b**2*m*n**2*x**2 + 126*a*b**2
*m*n*x**2 + 42*a*b**2*m*x**2 + 81*a*b**2*n**3*x**2 + 189*a*b**2*n**2*x**2
+ 126*a*b**2*n*x**2 + 24*a*b**2*x**2 + b**3*m**3*x**3 + 9*b**3*m**2*n*x**3
 + 6*b**3*m**2*x**3 + 27*b**3*m*n**2*x**3 + 36*b**3*m*n*x**3 + 11*b**3*m*x
**3 + 27*b**3*n**3*x**3 + 54*b**3*n**2*x**3 + 33*b**3*n*x**3 + 6*b**3*x**3
))/(m**4 + 12*m**3*n + 10*m**3 + 54*m**2*n**2 + 90*m**2*n + 35*m**2 + 108*
m*n**3 + 270*m*n**2 + 210*m*n + 50*m + 81*n**4 + 270*n**3 + 315*n**2 + 150
*n + 24)
```

3.436 $\int (cx)^m \left(ax^n + bx^{1+n}\right)^2 \, dx$

Optimal result

Integrand size = 21, antiderivative size = 73

$$\int (cx)^m \left(ax^n + bx^{1+n}\right)^2 \, dx = \frac{2abx^{2(1+n)}(cx)^m}{2+m+2n} + \frac{a^2x^{1+2n}(cx)^m}{1+m+2n} + \frac{b^2x^{3+2n}(cx)^m}{3+m+2n}$$

```
2*a*b*x^(2+2*n)*(c*x)^m/(2+m+2*n)+a^2*x^(1+2*n)*(c*x)^m/(1+m+2*n)+b^2*x^(3
+2*n)*(c*x)^m/(3+m+2*n)
```

Mathematica [A] (verified)

Time = 0.05 (sec) , antiderivative size = 54, normalized size of antiderivative = 0.74

$$\int (cx)^m \left(ax^n + bx^{1+n}\right)^2 \, dx = x^{1+2n}(cx)^m \left(\frac{a^2}{1+m+2n} + \frac{2abx}{2+m+2n} + \frac{b^2x^2}{3+m+2n}\right)$$

```
Integrate[(c*x)^m*(a*x^n + b*x^(1 + n))^2,x]
```

```
x^(1 + 2*n)*(c*x)^m*(a^2/(1 + m + 2*n) + (2*a*b*x)/(2 + m + 2*n) + (b^2*x^
2)/(3 + m + 2*n))
```

Rubi [A] (verified)

Time = 0.38 (sec) , antiderivative size = 72, normalized size of antiderivative = 0.99, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.190$, Rules used = {1939, 10, 53, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (cx)^m \left(ax^n + bx^{n+1}\right)^2 dx$$

$$\Big\downarrow \text{ 1939}$$

$$x^{-m}(cx)^m \int x^m \left(ax^n + bx^{n+1}\right)^2 dx$$

$$\Big\downarrow \text{ 10}$$

$$x^{-m}(cx)^m \int x^{m+2n}(a+bx)^2dx$$

$$\Big\downarrow \text{ 53}$$

$$x^{-m}(cx)^m \int \left(a^2x^{m+2n} + 2abx^{m+2n+1} + b^2x^{m+2n+2}\right) dx$$

$$\Big\downarrow \text{ 2009}$$

$$x^{-m}(cx)^m \left(\frac{a^2x^{m+2n+1}}{m+2n+1} + \frac{2abx^{m+2n+2}}{m+2n+2} + \frac{b^2x^{m+2n+3}}{m+2n+3}\right)$$

```
Int[(c*x)^m*(a*x^n + b*x^(1 + n))^2,x]
```

```
((c*x)^m*((a^2*x^(1 + m + 2*n))/(1 + m + 2*n) + (2*a*b*x^(2 + m + 2*n))/(2
 + m + 2*n) + (b^2*x^(3 + m + 2*n))/(3 + m + 2*n)))/x^m
```

Defintions of rubi rules used

```
Int[(u_.)*((e_.)*(x_))^(m_.)*((a_.)*(x_)^(r_.) + (b_.)*(x_)^(s_.))^(p_.), x
_Symbol] :> Simp[1/e^(p*r)   Int[u*(e*x)^(m + p*r)*(a + b*x^(s - r))^p, x],
 x] /; FreeQ[{a, b, e, m, r, s}, x] && IntegerQ[p] && (IntegerQ[p*r] || GtQ
[e, 0]) && PosQ[s - r]
```

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && LeQ[7*m + 4*n + 4, 0])
|| LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

```
Int[(u_)^(m_.)*((a_.)*(v_)^(j_.) + (b_.)*(v_)^(n_.))^(p_.), x_Symbol] :> Si
mp[u^m/(Coefficient[v, x, 1]*v^m)   Subst[Int[x^m*(a*x^j + b*x^n)^p, x], x,
 v], x] /; FreeQ[{a, b, j, m, n, p}, x] && LinearPairQ[u, v, x]
```

```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 190 vs. $2(73) = 146$.

Time = 0.63 (sec) , antiderivative size = 191, normalized size of antiderivative = 2.62

method	result
orering	$\frac{(b^2m^2x^2+4b^2mn\,x^2+4b^2n^2x^2+2ab\,m^2x+8abmnx+8ab\,n^2x+3b^2x^2m+6b^2x^2n+a^2m^2+4a^2mn+4a^2n^2+8mxab+16abnx+}{(1+m+2n)(2+m+2n)(3+m+2n)(bx+a)^2}$
risch	$\frac{x(b^2m^2x^2+4b^2mn\,x^2+4b^2n^2x^2+2ab\,m^2x+8abmnx+8ab\,n^2x+3b^2x^2m+6b^2x^2n+a^2m^2+4a^2mn+4a^2n^2+8mxab+16abnx}{(1+m+2n)(2+m+2n)(3+m}$
parallelrisch	$\frac{10x\,x^{2n}(cx)^m a^2n+3x\,x^{2+2n}(cx)^m b^2m+6x\,x^{2+2n}(cx)^m b^2n+2x\,x^n x^{1+n}(cx)^m ab\,m^2+8x\,x^n x^{1+n}(cx)^m ab\,n^2+16x\,x^n x^{1+n}(}{}$

```
int((c*x)^m*(a*x^n+b*x^(1+n))^2,x,method=_RETURNVERBOSE)
```

```
(b^2*m^2*x^2+4*b^2*m*n*x^2+4*b^2*n^2*x^2+2*a*b*m^2*x+8*a*b*m*n*x+8*a*b*n^2
*x+3*b^2*m*x^2+6*b^2*n*x^2+a^2*m^2+4*a^2*m*n+4*a^2*n^2+8*a*b*m*x+16*a*b*n*
x+2*b^2*x^2+5*a^2*m+10*a^2*n+6*a*b*x+6*a^2)/(1+m+2*n)/(2+m+2*n)/(3+m+2*n)/
(b*x+a)^2*x*(c*x)^m*(a*x^n+b*x^(1+n))^2
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 188 vs. $2(73) = 146$.

Time = 0.14 (sec) , antiderivative size = 188, normalized size of antiderivative = 2.58

$$\int (cx)^m \left(ax^n + bx^{1+n}\right)^2 dx$$
$$= \frac{(a^2m^2 + 4\,a^2n^2 + 5\,a^2m + (b^2m^2 + 4\,b^2n^2 + 3\,b^2m + 2\,b^2 + 2\,(2\,b^2m + 3\,b^2)n)x^2 + 6\,a^2 + 2\,(2\,a^2m + 5\,a^2}{(m^3 + 12\,(m+2)n^2 + 8\,n^3 + 6\,m^2 + 2\,(3\,m^2 + }$$

```
integrate((c*x)^m*(a*x^n+b*x^(1+n))^2,x, algorithm="fricas")
```

```
(a^2*m^2 + 4*a^2*n^2 + 5*a^2*m + (b^2*m^2 + 4*b^2*n^2 + 3*b^2*m + 2*b^2 +
2*(2*b^2*m + 3*b^2)*n)*x^2 + 6*a^2 + 2*(2*a^2*m + 5*a^2)*n + 2*(a*b*m^2 +
4*a*b*n^2 + 4*a*b*m + 3*a*b + 4*(a*b*m + 2*a*b)*n)*x)*x^(2*n + 2)*e^(m*log
(c) + m*log(x))/((m^3 + 12*(m + 2)*n^2 + 8*n^3 + 6*m^2 + 2*(3*m^2 + 12*m +
 11)*n + 11*m + 6)*x)
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1428 vs. $2(66) = 132$.

Time = 10.91 (sec) , antiderivative size = 1428, normalized size of antiderivative = 19.56

$$\int (cx)^m \left(ax^n + bx^{1+n}\right)^2 dx = \text{Too large to display}$$

```
integrate((c*x)**m*(a*x**n+b*x**(1+n))**2,x)
```

```
Piecewise((-a**2*x*x**(2*n)*(c*x)**(-2*n - 3)/2 - 2*a*b*x*x**n*x**(n + 1)*
(c*x)**(-2*n - 3) + b**2*x*x**(2*n + 2)*(c*x)**(-2*n - 3)*log(x), Eq(m, -2
*n - 3)), (-a**2*x*x**(2*n)*(c*x)**(-2*n - 2) + 2*a*b*x*x**n*x**(n + 1)*(c
*x)**(-2*n - 2)*log(x) + b**2*x*x**(2*n + 2)*(c*x)**(-2*n - 2), Eq(m, -2*n
 - 2)), (a**2*x*x**(2*n)*(c*x)**(-2*n - 1)*log(x) + 2*a*b*x*x**n*x**(n + 1
)*(c*x)**(-2*n - 1) + b**2*x*x**(2*n + 2)*(c*x)**(-2*n - 1)/2, Eq(m, -2*n
- 1)), (a**2*m**2*x*x**(2*n)*(c*x)**m/(m**3 + 6*m**2*n + 6*m**2 + 12*m*n**
2 + 24*m*n + 11*m + 8*n**3 + 24*n**2 + 22*n + 6) + 4*a**2*m*n*x*x**(2*n)*(
c*x)**m/(m**3 + 6*m**2*n + 6*m**2 + 12*m*n**2 + 24*m*n + 11*m + 8*n**3 + 2
4*n**2 + 22*n + 6) + 5*a**2*m*x*x**(2*n)*(c*x)**m/(m**3 + 6*m**2*n + 6*m**
2 + 12*m*n**2 + 24*m*n + 11*m + 8*n**3 + 24*n**2 + 22*n + 6) + 4*a**2*n**2
*x*x**(2*n)*(c*x)**m/(m**3 + 6*m**2*n + 6*m**2 + 12*m*n**2 + 24*m*n + 11*m
 + 8*n**3 + 24*n**2 + 22*n + 6) + 10*a**2*n*x*x**(2*n)*(c*x)**m/(m**3 + 6*
m**2*n + 6*m**2 + 12*m*n**2 + 24*m*n + 11*m + 8*n**3 + 24*n**2 + 22*n + 6)
 + 6*a**2*x*x**(2*n)*(c*x)**m/(m**3 + 6*m**2*n + 6*m**2 + 12*m*n**2 + 24*m
*n + 11*m + 8*n**3 + 24*n**2 + 22*n + 6) + 2*a*b*m**2*x*x**n*x**(n + 1)*(c
*x)**m/(m**3 + 6*m**2*n + 6*m**2 + 12*m*n**2 + 24*m*n + 11*m + 8*n**3 + 24
*n**2 + 22*n + 6) + 8*a*b*m*n*x*x**n*x**(n + 1)*(c*x)**m/(m**3 + 6*m**2*n
+ 6*m**2 + 12*m*n**2 + 24*m*n + 11*m + 8*n**3 + 24*n**2 + 22*n + 6) + 8*a*
b*m*x*x**n*x**(n + 1)*(c*x)**m/(m**3 + 6*m**2*n + 6*m**2 + 12*m*n**2 + ...
```

Maxima [A] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 86, normalized size of antiderivative = 1.18

$$\int (cx)^m \left(ax^n + bx^{1+n}\right)^2 \, dx = \frac{b^2c^mx^3e^{(m\log(x)+2\,n\log(x))}}{m+2\,n+3} + \frac{2\,abc^mx^2e^{(m\log(x)+2\,n\log(x))}}{m+2\,n+2} + \frac{a^2c^mxe^{(m\log(x)+2\,n\log(x))}}{m+2\,n+1}$$

```
integrate((c*x)^m*(a*x^n+b*x^(1+n))^2,x, algorithm="maxima")
```

```
b^2*c^m*x^3*e^(m*log(x) + 2*n*log(x))/(m + 2*n + 3) + 2*a*b*c^m*x^2*e^(m*l
og(x) + 2*n*log(x))/(m + 2*n + 2) + a^2*c^m*x*e^(m*log(x) + 2*n*log(x))/(m
 + 2*n + 1)
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 470 vs. $2(73) = 146$.

Time = 0.14 (sec) , antiderivative size = 470, normalized size of antiderivative = 6.44

$$\int (cx)^m \left(ax^n + bx^{1+n}\right)^2 dx = \frac{b^2m^2x^3x^{2n}e^{(m\log(c)+m\log(x))} + 4b^2mnx^3x^{2n}e^{(m\log(c)+m\log(x))} + 4b^2n^2x^3x^{2n}e^{(m\log(c)+m\log(x))} + 2abm^2x^2x}{}$$

```
integrate((c*x)^m*(a*x^n+b*x^(1+n))^2,x, algorithm="giac")
```

```
(b^2*m^2*x^3*x^(2*n)*e^(m*log(c) + m*log(x)) + 4*b^2*m*n*x^3*x^(2*n)*e^(m*
log(c) + m*log(x)) + 4*b^2*n^2*x^3*x^(2*n)*e^(m*log(c) + m*log(x)) + 2*a*b
*m^2*x^2*x^(2*n)*e^(m*log(c) + m*log(x)) + 8*a*b*m*n*x^2*x^(2*n)*e^(m*log(
c) + m*log(x)) + 8*a*b*n^2*x^2*x^(2*n)*e^(m*log(c) + m*log(x)) + 3*b^2*m*x
^3*x^(2*n)*e^(m*log(c) + m*log(x)) + 6*b^2*n*x^3*x^(2*n)*e^(m*log(c) + m*l
og(x)) + a^2*m^2*x*x^(2*n)*e^(m*log(c) + m*log(x)) + 4*a^2*m*n*x*x^(2*n)*e
^(m*log(c) + m*log(x)) + 4*a^2*n^2*x*x^(2*n)*e^(m*log(c) + m*log(x)) + 8*a
*b*m*x^2*x^(2*n)*e^(m*log(c) + m*log(x)) + 16*a*b*n*x^2*x^(2*n)*e^(m*log(c
) + m*log(x)) + 2*b^2*x^3*x^(2*n)*e^(m*log(c) + m*log(x)) + 5*a^2*m*x*x^(2
*n)*e^(m*log(c) + m*log(x)) + 10*a^2*n*x*x^(2*n)*e^(m*log(c) + m*log(x)) +
 6*a*b*x^2*x^(2*n)*e^(m*log(c) + m*log(x)) + 6*a^2*x*x^(2*n)*e^(m*log(c) +
 m*log(x)))/(m^3 + 6*m^2*n + 12*m*n^2 + 8*n^3 + 6*m^2 + 24*m*n + 24*n^2 +
11*m + 22*n + 6)
```

Mupad [B] (verification not implemented)

Time = 9.06 (sec) , antiderivative size = 243, normalized size of antiderivative = 3.33

$$\begin{aligned} \int (cx)^m \left(ax^n + bx^{1+n}\right)^2 dx &= \frac{a^2\,x\,x^{2\,n}\,{\left(c\,x\right)}^m\,\left(m^2+4\,m\,n+5\,m+4\,n^2+10\,n+6\right)}{m^3+6\,m^2\,n+6\,m^2+12\,m\,n^2+24\,m\,n+11\,m+8\,n^3+24\,n^2+22\,n+6} \\ &\quad + \frac{b^2\,x\,x^{2\,n+2}\,{\left(c\,x\right)}^m\,\left(m^2+4\,m\,n+3\,m+4\,n^2+6\,n+2\right)}{m^3+6\,m^2\,n+6\,m^2+12\,m\,n^2+24\,m\,n+11\,m+8\,n^3+24\,n^2+22\,n+6} \\ &\quad + \frac{2\,a\,b\,x\,x^n\,x^{n+1}\,{\left(c\,x\right)}^m\,\left(m^2+4\,m\,n+4\,m+4\,n^2+8\,n+3\right)}{m^3+6\,m^2\,n+6\,m^2+12\,m\,n^2+24\,m\,n+11\,m+8\,n^3+24\,n^2+22\,n+6} \end{aligned}$$

```
int((c*x)^m*(a*x^n + b*x^(n + 1))^2,x)
```

```
(a^2*x*x^(2*n)*(c*x)^m*(5*m + 10*n + 4*m*n + m^2 + 4*n^2 + 6))/(11*m + 22*
n + 24*m*n + 12*m*n^2 + 6*m^2*n + 6*m^2 + m^3 + 24*n^2 + 8*n^3 + 6) + (b^2
*x*x^(2*n + 2)*(c*x)^m*(3*m + 6*n + 4*m*n + m^2 + 4*n^2 + 2))/(11*m + 22*n
 + 24*m*n + 12*m*n^2 + 6*m^2*n + 6*m^2 + m^3 + 24*n^2 + 8*n^3 + 6) + (2*a*
b*x*x^n*x^(n + 1)*(c*x)^m*(4*m + 8*n + 4*m*n + m^2 + 4*n^2 + 3))/(11*m + 2
2*n + 24*m*n + 12*m*n^2 + 6*m^2*n + 6*m^2 + m^3 + 24*n^2 + 8*n^3 + 6)
```

Reduce [B] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 193, normalized size of antiderivative = 2.64

$$\int (cx)^m \left(ax^n + bx^{1+n}\right)^2 \, dx$$
$$= \frac{x^{m+2n}c^m x(b^2m^2x^2 + 4b^2mn\,x^2 + 4b^2n^2x^2 + 2ab\,m^2x + 8abmnx + 8ab\,n^2x + 3b^2m\,x^2 + 6b^2n\,x^2 + a^2m^2}{m^3 + 6m^2n + 12m\,n^2 + 8n^3 + 6m^2 + 24mn + 2}$$

```
int((c*x)^m*(a*x^n+b*x^(1+n))^2,x)
```

```
(x**(m + 2*n)*c**m*x*(a**2*m**2 + 4*a**2*m*n + 5*a**2*m + 4*a**2*n**2 + 10
*a**2*n + 6*a**2 + 2*a*b*m**2*x + 8*a*b*m*n*x + 8*a*b*m*x + 8*a*b*n**2*x +
 16*a*b*n*x + 6*a*b*x + b**2*m**2*x**2 + 4*b**2*m*n*x**2 + 3*b**2*m*x**2 +
 4*b**2*n**2*x**2 + 6*b**2*n*x**2 + 2*b**2*x**2))/(m**3 + 6*m**2*n + 6*m**
2 + 12*m*n**2 + 24*m*n + 11*m + 8*n**3 + 24*n**2 + 22*n + 6)
```

3.437 $\int (cx)^m \left(ax^n + bx^{1+n}\right) dx$

Optimal result

Integrand size = 19, antiderivative size = 37

$$\int (cx)^m \left(ax^n + bx^{1+n}\right) dx = \frac{ax^{1+n}(cx)^m}{1+m+n} + \frac{bx^{2+n}(cx)^m}{2+m+n}$$

```
a*x^(1+n)*(c*x)^m/(1+m+n)+b*x^(2+n)*(c*x)^m/(2+m+n)
```

Mathematica [A] (verified)

Time = 0.03 (sec) , antiderivative size = 29, normalized size of antiderivative = 0.78

$$\int (cx)^m \left(ax^n + bx^{1+n}\right) dx = x^{1+n}(cx)^m \left(\frac{a}{1+m+n} + \frac{bx}{2+m+n}\right)$$

```
Integrate[(c*x)^m*(a*x^n + b*x^(1 + n)),x]
```

```
x^(1 + n)*(c*x)^m*(a/(1 + m + n) + (b*x)/(2 + m + n))
```

Rubi [A] (verified)

Time = 0.33 (sec) , antiderivative size = 40, normalized size of antiderivative = 1.08, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.211, Rules used = {1939, 10, 53, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (cx)^m \left(ax^n+bx^{n+1}\right) \, dx$$

$$\Big\downarrow \ 1939$$

$$x^{-m}(cx)^m \int x^m \left(ax^n+bx^{n+1}\right)dx$$

$$\Big\downarrow \ 10$$

$$x^{-m}(cx)^m \int x^{m+n}(a+bx)dx$$

$$\Big\downarrow \ 53$$

$$x^{-m}(cx)^m \int \left(ax^{m+n}+bx^{m+n+1}\right)dx$$

$$\Big\downarrow \ 2009$$

$$x^{-m}(cx)^m \left(\frac{ax^{m+n+1}}{m+n+1}+\frac{bx^{m+n+2}}{m+n+2}\right)$$

```
Int[(c*x)^m*(a*x^n + b*x^(1 + n)),x]
```

```
((c*x)^m*((a*x^(1 + m + n))/(1 + m + n) + (b*x^(2 + m + n))/(2 + m + n)))/
x^m
```

Defintions of rubi rules used

```
Int[(u_.)*((e_.)*(x_))^(m_.)*((a_.)*(x_)^(r_.) + (b_.)*(x_)^(s_.))^(p_.), x
_Symbol] :> Simp[1/e^(p*r)   Int[u*(e*x)^(m + p*r)*(a + b*x^(s - r))^p, x],
 x] /; FreeQ[{a, b, e, m, r, s}, x] && IntegerQ[p] && (IntegerQ[p*r] || GtQ
[e, 0]) && PosQ[s - r]
```

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && LeQ[7*m + 4*n + 4, 0])
|| LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

```
Int[(u_)^(m_.)*((a_.)*(v_)^(j_.) + (b_.)*(v_)^(n_.))^(p_.), x_Symbol] :> Si
mp[u^m/(Coefficient[v, x, 1]*v^m)   Subst[Int[x^m*(a*x^j + b*x^n)^p, x], x,
 v], x] /; FreeQ[{a, b, j, m, n, p}, x] && LinearPairQ[u, v, x]
```

```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 0.12 (sec) , antiderivative size = 61, normalized size of antiderivative = 1.65

method	result	size
orering	$\frac{(bmx+nxb+am+an+bx+2a)x(cx)^{m}(a\,x^{n}+b\,x^{1+n})}{(1+m+n)(2+m+n)(bx+a)}$	61
risch	$\frac{x(bmx+nxb+am+an+bx+2a)x^{n}x^{m}c^{m}\mathrm{e}^{\frac{i\pi\ \mathrm{csgn}(icx)m(\mathrm{csgn}(icx)-\mathrm{csgn}(ix))(-\ \mathrm{csgn}(icx)+\mathrm{csgn}(ic))}{2}}}{(1+m+n)(2+m+n)}$	85
parallelrisch	$\frac{x\,x^{n}(cx)^{m}am+x\,x^{n}(cx)^{m}an+x\,x^{1+n}(cx)^{m}bm+x\,x^{1+n}(cx)^{m}bn+2x\,x^{n}(cx)^{m}a+x\,x^{1+n}(cx)^{m}b}{(1+m+n)(2+m+n)}$	92

```
int((c*x)^m*(a*x^n+b*x^(1+n)),x,method=_RETURNVERBOSE)
```

```
(b*m*x+b*n*x+a*m+a*n+b*x+2*a)/(1+m+n)/(2+m+n)/(b*x+a)*x*(c*x)^m*(a*x^n+b*x
^(1+n))
```

Fricas [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 56, normalized size of antiderivative = 1.51

$$\int (cx)^m \left(ax^n+bx^{1+n}\right) \,dx=\frac{(am+an+(bm+bn+b)x+2\,a)x^{n+1}e^{(m\log(c)+m\log(x))}}{m^2+(2\,m+3)n+n^2+3\,m+2}$$

```
integrate((c*x)^m*(a*x^n+b*x^(1+n)),x, algorithm="fricas")
```

```
(a*m + a*n + (b*m + b*n + b)*x + 2*a)*x^(n + 1)*e^(m*log(c) + m*log(x))/(m
^2 + (2*m + 3)*n + n^2 + 3*m + 2)
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 282 vs. 2(32) = 64.

Time = 2.88 (sec) , antiderivative size = 282, normalized size of antiderivative = 7.62

$$\int (cx)^m \left(ax^n+bx^{1+n}\right) \,dx$$
$$=\begin{cases} -axx^n(cx)^{-n-2}+bxx^{n+1}(cx)^{-n-2}\log{(x)} \\ axx^n(cx)^{-n-1}\log{(x)}+bxx^{n+1}(cx)^{-n-1} \\ \frac{amxx^n(cx)^m}{m^2+2mn+3m+n^2+3n+2}+\frac{anxx^n(cx)^m}{m^2+2mn+3m+n^2+3n+2}+\frac{2axx^n(cx)^m}{m^2+2mn+3m+n^2+3n+2}+\frac{bmxx^{n+1}(cx)^m}{m^2+2mn+3m+n^2+3n+2}+\frac{bnxx^{n+1}}{m^2+2mn+3m} \end{cases}$$

```
integrate((c*x)**m*(a*x**n+b*x**(1+n)),x)
```

```
Piecewise((-a*x*x**n*(c*x)**(-n - 2) + b*x*x**(n + 1)*(c*x)**(-n - 2)*log(
x), Eq(m, -n - 2)), (a*x*x**n*(c*x)**(-n - 1)*log(x) + b*x*x**(n + 1)*(c*x
)**(-n - 1), Eq(m, -n - 1)), (a*m*x*x**n*(c*x)**m/(m**2 + 2*m*n + 3*m + n*
*2 + 3*n + 2) + a*n*x*x**n*(c*x)**m/(m**2 + 2*m*n + 3*m + n**2 + 3*n + 2)
+ 2*a*x*x**n*(c*x)**m/(m**2 + 2*m*n + 3*m + n**2 + 3*n + 2) + b*m*x*x**(n
+ 1)*(c*x)**m/(m**2 + 2*m*n + 3*m + n**2 + 3*n + 2) + b*n*x*x**(n + 1)*(c*
x)**m/(m**2 + 2*m*n + 3*m + n**2 + 3*n + 2) + b*x*x**(n + 1)*(c*x)**m/(m**
2 + 2*m*n + 3*m + n**2 + 3*n + 2), True))
```

Maxima [A] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 47, normalized size of antiderivative = 1.27

$$\int (cx)^m \left(ax^n + bx^{1+n}\right) \, dx = \frac{bc^m x^2 e^{(m \log(x) + n \log(x))}}{m+n+2} + \frac{ac^m x e^{(m \log(x) + n \log(x))}}{m+n+1}$$

```
integrate((c*x)^m*(a*x^n+b*x^(1+n)),x, algorithm="maxima")
```

```
b*c^m*x^2*e^(m*log(x) + n*log(x))/(m + n + 2) + a*c^m*x*e^(m*log(x) + n*lo
g(x))/(m + n + 1)
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 129 vs. 2(37) = 74.

Time = 0.13 (sec) , antiderivative size = 129, normalized size of antiderivative = 3.49

$$\int (cx)^m \left(ax^n + bx^{1+n}\right) \, dx$$
$$= \frac{bmx^2x^ne^{(m \log(c) + m \log(x))} + bnx^2x^ne^{(m \log(c) + m \log(x))} + amxx^ne^{(m \log(c) + m \log(x))} + anxx^ne^{(m \log(c) + m \log(x))} +}{m^2 + 2\,mn + n^2 + 3\,m + 3\,n + 2}$$

```
integrate((c*x)^m*(a*x^n+b*x^(1+n)),x, algorithm="giac")
```

```
(b*m*x^2*x^n*e^(m*log(c) + m*log(x)) + b*n*x^2*x^n*e^(m*log(c) + m*log(x))
 + a*m*x*x^n*e^(m*log(c) + m*log(x)) + a*n*x*x^n*e^(m*log(c) + m*log(x)) +
 b*x^2*x^n*e^(m*log(c) + m*log(x)) + 2*a*x*x^n*e^(m*log(c) + m*log(x)))/(m
^2 + 2*m*n + n^2 + 3*m + 3*n + 2)
```

Mupad [B] (verification not implemented)

Time = 9.07 (sec) , antiderivative size = 73, normalized size of antiderivative = 1.97

$$\int (cx)^m \left(ax^n + bx^{1+n}\right) \, dx = \frac{a\,x^{n+1}\,(c\,x)^m\,(m+n+2)}{m^2+2\,m\,n+3\,m+n^2+3\,n+2} + \frac{b\,x^{n+2}\,(c\,x)^m\,(m+n+1)}{m^2+2\,m\,n+3\,m+n^2+3\,n+2}$$

```
int((c*x)^m*(a*x^n + b*x^(n + 1)),x)
```

```
(a*x^(n + 1)*(c*x)^m*(m + n + 2))/(3*m + 3*n + 2*m*n + m^2 + n^2 + 2) + (b
*x^(n + 2)*(c*x)^m*(m + n + 1))/(3*m + 3*n + 2*m*n + m^2 + n^2 + 2)
```

Reduce [B] (verification not implemented)

Time = 0.23 (sec) , antiderivative size = 51, normalized size of antiderivative = 1.38

$$\int (cx)^m \left(ax^n + bx^{1+n}\right) \, dx = \frac{x^{m+n}c^m x(bmx+bnx+am+an+bx+2a)}{m^2+2mn+n^2+3m+3n+2}$$

```
int((c*x)^m*(a*x^n+b*x^(1+n)),x)
```

```
(x**(m + n)*c**m*x*(a*m + a*n + 2*a + b*m*x + b*n*x + b*x))/(m**2 + 2*m*n
+ 3*m + n**2 + 3*n + 2)
```

3.438 $\int \frac{(cx)^m}{ax^n+bx^{1+n}}\,dx$

Optimal result

Integrand size = 21, antiderivative size = 45

$$\int \frac{(cx)^m}{ax^n+bx^{1+n}}\,dx = \frac{x^{1-n}(cx)^m\,\text{Hypergeometric2F1}\left(1,1+m-n,2+m-n,-\frac{bx}{a}\right)}{a(1+m-n)}$$

```
x^(1-n)*(c*x)^m*hypergeom([1, 1+m-n],[2+m-n],-b*x/a)/a/(1+m-n)
```

Mathematica [A] (verified)

Time = 0.04 (sec) , antiderivative size = 45, normalized size of antiderivative = 1.00

$$\int \frac{(cx)^m}{ax^n+bx^{1+n}}\,dx = \frac{x^{1-n}(cx)^m\,\text{Hypergeometric2F1}\left(1,1+m-n,2+m-n,-\frac{bx}{a}\right)}{a(1+m-n)}$$

```
Integrate[(c*x)^m/(a*x^n + b*x^(1 + n)),x]
```

```
(x^(1 - n)*(c*x)^m*Hypergeometric2F1[1, 1 + m - n, 2 + m - n, -((b*x)/a)])
/(a*(1 + m - n))
```

Rubi [A] (verified)

Time = 0.33 (sec) , antiderivative size = 45, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.143, Rules used = {1939, 10, 74}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(cx)^m}{ax^n+bx^{n+1}}\,dx$$
$$\Big\downarrow \text{1939}$$
$$x^{-m}(cx)^m\int \frac{x^m}{ax^n+bx^{n+1}}dx$$
$$\Big\downarrow \text{10}$$
$$x^{-m}(cx)^m\int \frac{x^{m-n}}{a+bx}dx$$
$$\Big\downarrow \text{74}$$
$$\frac{x^{1-n}(cx)^m \text{Hypergeometric2F1}\left(1,m-n+1,m-n+2,-\frac{bx}{a}\right)}{a(m-n+1)}$$

```
Int[(c*x)^m/(a*x^n + b*x^(1 + n)),x]
```

```
(x^(1 - n)*(c*x)^m*Hypergeometric2F1[1, 1 + m - n, 2 + m - n, -((b*x)/a)])
/(a*(1 + m - n))
```

Defintions of rubi rules used

```
Int[(u_.)*((e_.)*(x_))^(m_.)*((a_.)*(x_)^(r_.) + (b_.)*(x_)^(s_.))^(p_.), x
_Symbol] :> Simp[1/e^(p*r)   Int[u*(e*x)^(m + p*r)*(a + b*x^(s - r))^p, x],
 x] /; FreeQ[{a, b, e, m, r, s}, x] && IntegerQ[p] && (IntegerQ[p*r] || GtQ
[e, 0]) && PosQ[s - r]
```

```
Int[((b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[c^n*((b*x
)^(m + 1)/(b*(m + 1)))*Hypergeometric2F1[-n, m + 1, m + 2, (-d)*(x/c)], x]
/; FreeQ[{b, c, d, m, n}, x] &&  !IntegerQ[m] && (IntegerQ[n] || (GtQ[c, 0]
 &&  !(EqQ[n, -2^(-1)] && EqQ[c^2 - d^2, 0] && GtQ[-d/(b*c), 0])))
```

```
Int[(u_)^(m_.)*((a_.)*(v_)^(j_.) + (b_.)*(v_)^(n_.))^(p_.), x_Symbol] :> Si
mp[u^m/(Coefficient[v, x, 1]*v^m)   Subst[Int[x^m*(a*x^j + b*x^n)^p, x], x,
 v], x] /; FreeQ[{a, b, j, m, n, p}, x] && LinearPairQ[u, v, x]
```

Maple [F]

$$\int \frac{(cx)^m}{a\,x^n+b\,x^{1+n}}dx$$

```
int((c*x)^m/(a*x^n+b*x^(1+n)),x)
```

```
int((c*x)^m/(a*x^n+b*x^(1+n)),x)
```

Fricas [F]

$$\int \frac{(cx)^m}{ax^n+bx^{1+n}}\,dx=\int \frac{(cx)^m}{bx^{n+1}+ax^n}\,dx$$

```
integrate((c*x)^m/(a*x^n+b*x^(1+n)),x, algorithm="fricas")
```

```
integral((c*x)^m/(b*x^(n + 1) + a*x^n), x)
```

Sympy [F]

$$\int \frac{(cx)^m}{ax^n + bx^{1+n}}\,dx = \int \frac{(cx)^m}{ax^n + bx^{n+1}}\,dx$$

```
integrate((c*x)**m/(a*x**n+b*x**(1+n)),x)
```

```
Integral((c*x)**m/(a*x**n + b*x**(n + 1)), x)
```

Maxima [F]

$$\int \frac{(cx)^m}{ax^n + bx^{1+n}}\,dx = \int \frac{(cx)^m}{bx^{n+1} + ax^n}\,dx$$

```
integrate((c*x)^m/(a*x^n+b*x^(1+n)),x, algorithm="maxima")
```

```
integrate((c*x)^m/(b*x^(n + 1) + a*x^n), x)
```

Giac [F]

$$\int \frac{(cx)^m}{ax^n + bx^{1+n}}\,dx = \int \frac{(cx)^m}{bx^{n+1} + ax^n}\,dx$$

```
integrate((c*x)^m/(a*x^n+b*x^(1+n)),x, algorithm="giac")
```

```
integrate((c*x)^m/(b*x^(n + 1) + a*x^n), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(cx)^m}{ax^n + bx^{1+n}}\,dx = \int \frac{(c\,x)^m}{a\,x^n + b\,x^{n+1}}\,dx$$

```
int((c*x)^m/(a*x^n + b*x^(n + 1)),x)
```

```
int((c*x)^m/(a*x^n + b*x^(n + 1)), x)
```

Reduce [F]

$$\int \frac{(cx)^m}{ax^n + bx^{1+n}}\,dx = \frac{c^m \left(x^m - x^n \left(\int \frac{x^m}{x^n a x + x^n b\,x^2}dx\right) am + x^n \left(\int \frac{x^m}{x^n a x + x^n b\,x^2}dx\right) an\right)}{x^n b\,(m-n)}$$

```
int((c*x)^m/(a*x^n+b*x^(1+n)),x)
```

```
(c**m*(x**m - x**n*int(x**m/(x**n*a*x + x**n*b*x**2),x)*a*m + x**n*int(x**
m/(x**n*a*x + x**n*b*x**2),x)*a*n))/(x**n*b*(m - n))
```

3.439 $\int \frac{(cx)^m}{(ax^n+bx^{1+n})^2}\,dx$

Optimal result

Integrand size = 21, antiderivative size = 45

$$\int \frac{(cx)^m}{(ax^n+bx^{1+n})^2}\,dx = \frac{x^{1-2n}(cx)^m \operatorname{Hypergeometric2F1}\left(2,1+m-2n,2+m-2n,-\frac{bx}{a}\right)}{a^2(1+m-2n)}$$

```
x^(1-2*n)*(c*x)^m*hypergeom([2, 1+m-2*n],[2+m-2*n],-b*x/a)/a^2/(1+m-2*n)
```

Mathematica [A] (verified)

Time = 0.04 (sec) , antiderivative size = 45, normalized size of antiderivative = 1.00

$$\int \frac{(cx)^m}{(ax^n+bx^{1+n})^2}\,dx = \frac{x^{1-2n}(cx)^m \operatorname{Hypergeometric2F1}\left(2,1+m-2n,2+m-2n,-\frac{bx}{a}\right)}{a^2(1+m-2n)}$$

```
Integrate[(c*x)^m/(a*x^n + b*x^(1 + n))^2,x]
```

```
(x^(1 - 2*n)*(c*x)^m*Hypergeometric2F1[2, 1 + m - 2*n, 2 + m - 2*n, -((b*x
)/a)])/(a^2*(1 + m - 2*n))
```

Rubi [A] (verified)

Time = 0.31 (sec) , antiderivative size = 45, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.143$, Rules used = {1939, 10, 74}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(cx)^m}{(ax^n+bx^{n+1})^2}dx$$

$$\Big\downarrow \text{1939}$$

$$x^{-m}(cx)^m\int \frac{x^m}{(ax^n+bx^{n+1})^2}dx$$

$$\Big\downarrow \text{10}$$

$$x^{-m}(cx)^m\int \frac{x^{m-2n}}{(a+bx)^2}dx$$

$$\Big\downarrow \text{74}$$

$$\frac{x^{1-2n}(cx)^m \text{Hypergeometric2F1}\left(2,m-2n+1,m-2n+2,-\frac{bx}{a}\right)}{a^2(m-2n+1)}$$

```
Int[(c*x)^m/(a*x^n + b*x^(1 + n))^2,x]
```


```
(x^(1 - 2*n)*(c*x)^m*Hypergeometric2F1[2, 1 + m - 2*n, 2 + m - 2*n, -((b*x
)/a)])/(a^2*(1 + m - 2*n))
```

Defintions of rubi rules used

```
Int[(u_.)*((e_.)*(x_))^(m_.)*((a_.)*(x_)^(r_.) + (b_.)*(x_)^(s_.))^(p_.), x
_Symbol] :> Simp[1/e^(p*r)   Int[u*(e*x)^(m + p*r)*(a + b*x^(s - r))^p, x],
 x] /; FreeQ[{a, b, e, m, r, s}, x] && IntegerQ[p] && (IntegerQ[p*r] || GtQ
[e, 0]) && PosQ[s - r]
```

```
Int[((b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[c^n*((b*x
)^(m + 1)/(b*(m + 1)))*Hypergeometric2F1[-n, m + 1, m + 2, (-d)*(x/c)], x]
/; FreeQ[{b, c, d, m, n}, x] &&  !IntegerQ[m] && (IntegerQ[n] || (GtQ[c, 0]
 &&  !(EqQ[n, -2^(-1)] && EqQ[c^2 - d^2, 0] && GtQ[-d/(b*c), 0])))
```

```
Int[(u_)^(m_.)*((a_.)*(v_)^(j_.) + (b_.)*(v_)^(n_.))^(p_.), x_Symbol] :> Si
mp[u^m/(Coefficient[v, x, 1]*v^m)   Subst[Int[x^m*(a*x^j + b*x^n)^p, x], x,
 v], x] /; FreeQ[{a, b, j, m, n, p}, x] && LinearPairQ[u, v, x]
```

Maple [F]

$$\int \frac{(cx)^m}{(a\,x^n+b\,x^{1+n})^2}dx$$

```
int((c*x)^m/(a*x^n+b*x^(1+n))^2,x)
```

```
int((c*x)^m/(a*x^n+b*x^(1+n))^2,x)
```

Fricas [F]

$$\int \frac{(cx)^m}{(ax^n+bx^{1+n})^2}\,dx = \int \frac{(cx)^m}{(bx^{n+1}+ax^n)^2}\,dx$$

```
integrate((c*x)^m/(a*x^n+b*x^(1+n))^2,x, algorithm="fricas")
```

```
integral((c*x)^m/(2*a*b*x^(n + 1)*x^n + a^2*x^(2*n) + b^2*x^(2*n + 2)), x)
```

Sympy [F]

$$\int \frac{(cx)^m}{(ax^n + bx^{1+n})^2} \, dx = \int \frac{(cx)^m}{(ax^n + bx^{n+1})^2} \, dx$$

```
integrate((c*x)**m/(a*x**n+b*x**(1+n))**2,x)
```

```
Integral((c*x)**m/(a*x**n + b*x**(n + 1))**2, x)
```

Maxima [F]

$$\int \frac{(cx)^m}{(ax^n + bx^{1+n})^2} \, dx = \int \frac{(cx)^m}{(bx^{n+1} + ax^n)^2} \, dx$$

```
integrate((c*x)^m/(a*x^n+b*x^(1+n))^2,x, algorithm="maxima")
```

```
integrate((c*x)^m/(b*x^(n + 1) + a*x^n)^2, x)
```

Giac [F]

$$\int \frac{(cx)^m}{(ax^n + bx^{1+n})^2} \, dx = \int \frac{(cx)^m}{(bx^{n+1} + ax^n)^2} \, dx$$

```
integrate((c*x)^m/(a*x^n+b*x^(1+n))^2,x, algorithm="giac")
```

```
integrate((c*x)^m/(b*x^(n + 1) + a*x^n)^2, x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(cx)^m}{(ax^n + bx^{1+n})^2} \, dx = \int \frac{(c\,x)^m}{(a\,x^n + b\,x^{n+1})^2} \, dx$$

```
int((c*x)^m/(a*x^n + b*x^(n + 1))^2,x)
```

```
int((c*x)^m/(a*x^n + b*x^(n + 1))^2, x)
```

Reduce [F]

$$\int \frac{(cx)^m}{(ax^n + bx^{1+n})^2} \, dx = \text{Too large to display}$$

```
int((c*x)^m/(a*x^n+b*x^(1+n))^2,x)
```

```
(c**m*(x**m*x - x**(2*n)*int((x**m*x)/(x**(2*n)*a**2*m - 2*x**(2*n)*a**2*n
 + x**(2*n)*a**2 + 2*x**(2*n)*a*b*m*x - 4*x**(2*n)*a*b*n*x + 2*x**(2*n)*a*
b*x + x**(2*n)*b**2*m*x**2 - 2*x**(2*n)*b**2*n*x**2 + x**(2*n)*b**2*x**2),
x)*a*b*m**2 + 4*x**(2*n)*int((x**m*x)/(x**(2*n)*a**2*m - 2*x**(2*n)*a**2*n
 + x**(2*n)*a**2 + 2*x**(2*n)*a*b*m*x - 4*x**(2*n)*a*b*n*x + 2*x**(2*n)*a*
b*x + x**(2*n)*b**2*m*x**2 - 2*x**(2*n)*b**2*n*x**2 + x**(2*n)*b**2*x**2),
x)*a*b*m*n - x**(2*n)*int((x**m*x)/(x**(2*n)*a**2*m - 2*x**(2*n)*a**2*n +
x**(2*n)*a**2 + 2*x**(2*n)*a*b*m*x - 4*x**(2*n)*a*b*n*x + 2*x**(2*n)*a*b*x
 + x**(2*n)*b**2*m*x**2 - 2*x**(2*n)*b**2*n*x**2 + x**(2*n)*b**2*x**2),x)*
a*b*m - 4*x**(2*n)*int((x**m*x)/(x**(2*n)*a**2*m - 2*x**(2*n)*a**2*n + x**
(2*n)*a**2 + 2*x**(2*n)*a*b*m*x - 4*x**(2*n)*a*b*n*x + 2*x**(2*n)*a*b*x +
x**(2*n)*b**2*m*x**2 - 2*x**(2*n)*b**2*n*x**2 + x**(2*n)*b**2*x**2),x)*a*b
*n**2 + 2*x**(2*n)*int((x**m*x)/(x**(2*n)*a**2*m - 2*x**(2*n)*a**2*n + x**
(2*n)*a**2 + 2*x**(2*n)*a*b*m*x - 4*x**(2*n)*a*b*n*x + 2*x**(2*n)*a*b*x +
x**(2*n)*b**2*m*x**2 - 2*x**(2*n)*b**2*n*x**2 + x**(2*n)*b**2*x**2),x)*a*b
*n - x**(2*n)*int((x**m*x)/(x**(2*n)*a**2*m - 2*x**(2*n)*a**2*n + x**(2*n)
*a**2 + 2*x**(2*n)*a*b*m*x - 4*x**(2*n)*a*b*n*x + 2*x**(2*n)*a*b*x + x**(2
*n)*b**2*m*x**2 - 2*x**(2*n)*b**2*n*x**2 + x**(2*n)*b**2*x**2),x)*b**2*m**
2*x + 4*x**(2*n)*int((x**m*x)/(x**(2*n)*a**2*m - 2*x**(2*n)*a**2*n + x**(2
*n)*a**2 + 2*x**(2*n)*a*b*m*x - 4*x**(2*n)*a*b*n*x + 2*x**(2*n)*a*b*x +...
```

3.440 $\int \frac{(cx)^m}{(ax^n+bx^{1+n})^3}\,dx$

Optimal result

Integrand size = 21, antiderivative size = 45

$$\int \frac{(cx)^m}{(ax^n+bx^{1+n})^3}\,dx = \frac{x^{1-3n}(cx)^m \,\text{Hypergeometric2F1}\left(3,1+m-3n,2+m-3n,-\frac{bx}{a}\right)}{a^3(1+m-3n)}$$

```
x^(1-3*n)*(c*x)^m*hypergeom([3, 1+m-3*n],[2+m-3*n],-b*x/a)/a^3/(1+m-3*n)
```

Mathematica [A] (verified)

Time = 0.03 (sec) , antiderivative size = 45, normalized size of antiderivative = 1.00

$$\int \frac{(cx)^m}{(ax^n+bx^{1+n})^3}\,dx = \frac{x^{1-3n}(cx)^m \,\text{Hypergeometric2F1}\left(3,1+m-3n,2+m-3n,-\frac{bx}{a}\right)}{a^3(1+m-3n)}$$

```
Integrate[(c*x)^m/(a*x^n + b*x^(1 + n))^3,x]
```

```
(x^(1 - 3*n)*(c*x)^m*Hypergeometric2F1[3, 1 + m - 3*n, 2 + m - 3*n, -((b*x
)/a)])/(a^3*(1 + m - 3*n))
```

Rubi [A] (verified)

Time = 0.31 (sec) , antiderivative size = 45, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.143$, Rules used = $\{1939, 10, 74\}$

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(cx)^m}{(ax^n+bx^{n+1})^3} \, dx$$

$$\Big\downarrow \ 1939$$

$$x^{-m}(cx)^m \int \frac{x^m}{(ax^n+bx^{n+1})^3}dx$$

$$\Big\downarrow \ 10$$

$$x^{-m}(cx)^m \int \frac{x^{m-3n}}{(a+bx)^3}dx$$

$$\Big\downarrow \ 74$$

$$\frac{x^{1-3n}(cx)^m \text{Hypergeometric2F1}\left(3, m-3n+1, m-3n+2, -\frac{bx}{a}\right)}{a^3(m-3n+1)}$$

```
Int[(c*x)^m/(a*x^n + b*x^(1 + n))^3,x]
```


```
(x^(1 - 3*n)*(c*x)^m*Hypergeometric2F1[3, 1 + m - 3*n, 2 + m - 3*n, -((b*x
)/a)])/(a^3*(1 + m - 3*n))
```

Defintions of rubi rules used

```
Int[(u_.)*((e_.)*(x_))^(m_.)*((a_.)*(x_)^(r_.) + (b_.)*(x_)^(s_.))^(p_.), x
_Symbol] :> Simp[1/e^(p*r)   Int[u*(e*x)^(m + p*r)*(a + b*x^(s - r))^p, x],
 x] /; FreeQ[{a, b, e, m, r, s}, x] && IntegerQ[p] && (IntegerQ[p*r] || GtQ
[e, 0]) && PosQ[s - r]
```

```
Int[((b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[c^n*((b*x
)^(m + 1)/(b*(m + 1)))*Hypergeometric2F1[-n, m + 1, m + 2, (-d)*(x/c)], x]
/; FreeQ[{b, c, d, m, n}, x] &&  !IntegerQ[m] && (IntegerQ[n] || (GtQ[c, 0]
 &&  !(EqQ[n, -2^(-1)] && EqQ[c^2 - d^2, 0] && GtQ[-d/(b*c), 0])))
```

```
Int[(u_)^(m_.)*((a_.)*(v_)^(j_.) + (b_.)*(v_)^(n_.))^(p_.), x_Symbol] :> Si
mp[u^m/(Coefficient[v, x, 1]*v^m)   Subst[Int[x^m*(a*x^j + b*x^n)^p, x], x,
 v], x] /; FreeQ[{a, b, j, m, n, p}, x] && LinearPairQ[u, v, x]
```

Maple [F]

$$\int \frac{(cx)^m}{(a\,x^n+b\,x^{1+n})^3}dx$$

```
int((c*x)^m/(a*x^n+b*x^(1+n))^3,x)
```

```
int((c*x)^m/(a*x^n+b*x^(1+n))^3,x)
```

Fricas [F]

$$\int \frac{(cx)^m}{(ax^n+bx^{1+n})^3}\,dx=\int \frac{(cx)^m}{(bx^{n+1}+ax^n)^3}\,dx$$

```
integrate((c*x)^m/(a*x^n+b*x^(1+n))^3,x, algorithm="fricas")
```

```
integral((c*x)^m/(3*a^2*b*x^(2*n)*x^(n + 1) + a^3*x^(3*n) + (b^3*x^(n + 1)
 + 3*a*b^2*x^n)*x^(2*n + 2)), x)
```

Sympy [F]

$$\int \frac{(cx)^m}{(ax^n + bx^{1+n})^3}\,dx = \int \frac{(cx)^m}{(ax^n + bx^{n+1})^3}\,dx$$

```
integrate((c*x)**m/(a*x**n+b*x**(1+n))**3,x)
```

```
Integral((c*x)**m/(a*x**n + b*x**(n + 1))**3, x)
```

Maxima [F]

$$\int \frac{(cx)^m}{(ax^n + bx^{1+n})^3}\,dx = \int \frac{(cx)^m}{(bx^{n+1} + ax^n)^3}\,dx$$

```
integrate((c*x)^m/(a*x^n+b*x^(1+n))^3,x, algorithm="maxima")
```

```
integrate((c*x)^m/(b*x^(n + 1) + a*x^n)^3, x)
```

Giac [F]

$$\int \frac{(cx)^m}{(ax^n + bx^{1+n})^3}\,dx = \int \frac{(cx)^m}{(bx^{n+1} + ax^n)^3}\,dx$$

```
integrate((c*x)^m/(a*x^n+b*x^(1+n))^3,x, algorithm="giac")
```

```
integrate((c*x)^m/(b*x^(n + 1) + a*x^n)^3, x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(cx)^m}{(ax^n + bx^{1+n})^3} \, dx = \int \frac{(c\,x)^m}{(a\,x^n + b\,x^{n+1})^3} \, dx$$

```
int((c*x)^m/(a*x^n + b*x^(n + 1))^3,x)
```

```
int((c*x)^m/(a*x^n + b*x^(n + 1))^3, x)
```

Reduce [F]

$$\int \frac{(cx)^m}{(ax^n + bx^{1+n})^3} \, dx = \text{too large to display}$$

```
int((c*x)^m/(a*x^n+b*x^(1+n))^3,x)
```

```
(c**m*(x**m*x - x**(3*n)*int((x**m*x)/(x**(3*n)*a**3*m - 3*x**(3*n)*a**3*n
 + x**(3*n)*a**3 + 3*x**(3*n)*a**2*b*m*x - 9*x**(3*n)*a**2*b*n*x + 3*x**(3
*n)*a**2*b*x + 3*x**(3*n)*a*b**2*m*x**2 - 9*x**(3*n)*a*b**2*n*x**2 + 3*x**
(3*n)*a*b**2*x**2 + x**(3*n)*b**3*m*x**3 - 3*x**(3*n)*b**3*n*x**3 + x**(3*
n)*b**3*x**3),x)*a**2*b*m**2 + 6*x**(3*n)*int((x**m*x)/(x**(3*n)*a**3*m -
3*x**(3*n)*a**3*n + x**(3*n)*a**3 + 3*x**(3*n)*a**2*b*m*x - 9*x**(3*n)*a**
2*b*n*x + 3*x**(3*n)*a**2*b*x + 3*x**(3*n)*a*b**2*m*x**2 - 9*x**(3*n)*a*b*
*2*n*x**2 + 3*x**(3*n)*a*b**2*x**2 + x**(3*n)*b**3*m*x**3 - 3*x**(3*n)*b**
3*n*x**3 + x**(3*n)*b**3*x**3),x)*a**2*b*m*n - 9*x**(3*n)*int((x**m*x)/(x*
*(3*n)*a**3*m - 3*x**(3*n)*a**3*n + x**(3*n)*a**3 + 3*x**(3*n)*a**2*b*m*x
- 9*x**(3*n)*a**2*b*n*x + 3*x**(3*n)*a**2*b*x + 3*x**(3*n)*a*b**2*m*x**2 -
 9*x**(3*n)*a*b**2*n*x**2 + 3*x**(3*n)*a*b**2*x**2 + x**(3*n)*b**3*m*x**3
- 3*x**(3*n)*b**3*n*x**3 + x**(3*n)*b**3*x**3),x)*a**2*b*n**2 + x**(3*n)*i
nt((x**m*x)/(x**(3*n)*a**3*m - 3*x**(3*n)*a**3*n + x**(3*n)*a**3 + 3*x**(3
*n)*a**2*b*m*x - 9*x**(3*n)*a**2*b*n*x + 3*x**(3*n)*a**2*b*x + 3*x**(3*n)*
a*b**2*m*x**2 - 9*x**(3*n)*a*b**2*n*x**2 + 3*x**(3*n)*a*b**2*x**2 + x**(3*
n)*b**3*m*x**3 - 3*x**(3*n)*b**3*n*x**3 + x**(3*n)*b**3*x**3),x)*a**2*b -
2*x**(3*n)*int((x**m*x)/(x**(3*n)*a**3*m - 3*x**(3*n)*a**3*n + x**(3*n)*a*
*3 + 3*x**(3*n)*a**2*b*m*x - 9*x**(3*n)*a**2*b*n*x + 3*x**(3*n)*a**2*b*x +
 3*x**(3*n)*a*b**2*m*x**2 - 9*x**(3*n)*a*b**2*n*x**2 + 3*x**(3*n)*a*b**...
```

3.441 $\int (cx)^m \left(ax^n+bx^{1+n}\right)^{3/2} \, dx$

Optimal result

Integrand size = 23, antiderivative size = 75

$$\int (cx)^m \left(ax^n+bx^{1+n}\right)^{3/2} \, dx=\frac{2x^{-n}\left(-\frac{bx}{a}\right)^{-m-\frac{3n}{2}} (cx)^m \left(ax^n+bx^{1+n}\right)^{5/2} \text{Hypergeometric2F1}\left(\frac{5}{2},-m-\frac{3n}{2},\frac{7}{2},1+\frac{bx}{a}\right)}{5b}$$

```
2/5*(-b*x/a)^(-m-3/2*n)*(c*x)^m*(a*x^n+b*x^(1+n))^(5/2)*hypergeom([5/2, -m
-3/2*n],[7/2],1+b*x/a)/b/(x^n)
```

Mathematica [A] (verified)

Time = 0.17 (sec) , antiderivative size = 71, normalized size of antiderivative = 0.95

$$\int (cx)^m \left(ax^n+bx^{1+n}\right)^{3/2} \, dx=\frac{2\left(-\frac{bx}{a}\right)^{-m-\frac{3n}{2}} (cx)^m(a+bx) \left(x^n(a+bx)\right)^{3/2} \text{Hypergeometric2F1}\left(\frac{5}{2},-m-\frac{3n}{2},\frac{7}{2},1+\frac{bx}{a}\right)}{5b}$$

```
Integrate[(c*x)^m*(a*x^n + b*x^(1 + n))^(3/2),x]
```

```
(2*(-((b*x)/a))^(-m - (3*n)/2)*(c*x)^m*(a + b*x)*(x^n*(a + b*x))^(3/2)*Hyp
ergeometric2F1[5/2, -m - (3*n)/2, 7/2, 1 + (b*x)/a])/(5*b)
```

Rubi [A] (verified)

Time = 0.35 (sec) , antiderivative size = 80, normalized size of antiderivative = 1.07, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.130$, Rules used = {1938, 77, 75}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (cx)^m \left(ax^n+bx^{n+1}\right)^{3/2} \, dx$$

$$\Big\downarrow \text{1938}$$

$$\frac{(cx)^m x^{-m-\frac{n}{2}} \sqrt{ax^n+bx^{n+1}} \int x^{m+\frac{3n}{2}} (a+bx)^{3/2}dx}{\sqrt{a+bx}}$$

$$\Big\downarrow \text{77}$$

$$\frac{x^n (cx)^m \sqrt{ax^n+bx^{n+1}} \left(-\frac{bx}{a}\right)^{-m-\frac{3n}{2}} \int \left(-\frac{bx}{a}\right)^{m+\frac{3n}{2}} (a+bx)^{3/2}dx}{\sqrt{a+bx}}$$

$$\Big\downarrow \text{75}$$

$$\frac{2x^n (a+bx)^2 (cx)^m \sqrt{ax^n+bx^{n+1}} \left(-\frac{bx}{a}\right)^{-m-\frac{3n}{2}} \text{Hypergeometric2F1}\left(\frac{5}{2},-m-\frac{3n}{2},\frac{7}{2},\frac{bx}{a}+1\right)}{5b}$$

```
Int[(c*x)^m*(a*x^n + b*x^(1 + n))^(3/2),x]
```

```
(2*x^n*(-((b*x)/a))^(-m - (3*n)/2)*(c*x)^m*(a + b*x)^2*Sqrt[a*x^n + b*x^(1
 + n)]*Hypergeometric2F1[5/2, -m - (3*n)/2, 7/2, 1 + (b*x)/a])/(5*b)
```

Defintions of rubi rules used

```
Int[((b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[((c + d*x
)^(n + 1)/(d*(n + 1)*(-d/(b*c))^m))*Hypergeometric2F1[-m, n + 1, n + 2, 1 +
 d*(x/c)], x] /; FreeQ[{b, c, d, m, n}, x] &&  !IntegerQ[n] && (IntegerQ[m]
 || GtQ[-d/(b*c), 0])
```

```
Int[((b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[((-b)*(c/
d))^IntPart[m]*((b*x)^FracPart[m]/((-d)*(x/c))^FracPart[m])   Int[((-d)*(x/
c))^m*(c + d*x)^n, x], x] /; FreeQ[{b, c, d, m, n}, x] &&  !IntegerQ[m] &&
 !IntegerQ[n] &&  !GtQ[c, 0] &&  !GtQ[-d/(b*c), 0]
```

```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> Simp[c^IntPart[m]*(c*x)^FracPart[m]*((a*x^j + b*x^n)^FracPart[p]/(x^(F
racPart[m] + j*FracPart[p])*(a + b*x^(n - j))^FracPart[p]))   Int[x^(m + j*
p)*(a + b*x^(n - j))^p, x], x] /; FreeQ[{a, b, c, j, m, n, p}, x] &&  !Inte
gerQ[p] && NeQ[n, j] && PosQ[n - j]
```

Maple [F]

$$\int (cx)^m \left(a\,x^n + b\,x^{1+n}\right)^{\frac{3}{2}} dx$$

```
int((c*x)^m*(a*x^n+b*x^(1+n))^(3/2),x)
```

```
int((c*x)^m*(a*x^n+b*x^(1+n))^(3/2),x)
```

Fricas [F(-2)]

Exception generated.

$$\int (cx)^m \left(ax^n + bx^{1+n}\right)^{3/2} dx = \text{Exception raised: TypeError}$$

```
integrate((c*x)^m*(a*x^n+b*x^(1+n))^(3/2),x, algorithm="fricas")
```

```
Exception raised: TypeError >> Error detected within library code: inte
grate: implementation incomplete (has polynomial part)
```

Sympy [F]

$$\int (cx)^m \left(ax^n + bx^{1+n}\right)^{3/2} dx = \int (cx)^m \left(ax^n + bx^{n+1}\right)^{\frac{3}{2}} dx$$

```
integrate((c*x)**m*(a*x**n+b*x**(1+n))**(3/2),x)
```

```
Integral((c*x)**m*(a*x**n + b*x**(n + 1))**(3/2), x)
```

Maxima [F]

$$\int (cx)^m \left(ax^n + bx^{1+n}\right)^{3/2} dx = \int \left(bx^{n+1} + ax^n\right)^{\frac{3}{2}} (cx)^m dx$$

```
integrate((c*x)^m*(a*x^n+b*x^(1+n))^(3/2),x, algorithm="maxima")
```

```
integrate((b*x^(n + 1) + a*x^n)^(3/2)*(c*x)^m, x)
```

Giac [F]

$$\int (cx)^m \left(ax^n + bx^{1+n}\right)^{3/2} dx = \int \left(bx^{n+1} + ax^n\right)^{\frac{3}{2}} (cx)^m dx$$

```
integrate((c*x)^m*(a*x^n+b*x^(1+n))^(3/2),x, algorithm="giac")
```

```
integrate((b*x^(n + 1) + a*x^n)^(3/2)*(c*x)^m, x)
```

Mupad [F(-1)]

Timed out.

$$\int (cx)^m \left(ax^n+bx^{1+n}\right)^{3/2}\,dx=\int (c\,x)^m\,{\left(a\,x^n+b\,x^{n+1}\right)}^{3/2}\,dx$$

```
int((c*x)^m*(a*x^n + b*x^(n + 1))^(3/2),x)
```

```
int((c*x)^m*(a*x^n + b*x^(n + 1))^(3/2), x)
```

Reduce [F]

$$\int (cx)^m (ax^n + bx^{1+n})^{3/2}\,dx = c^m\left(\left(\int x^{m+\frac{3n}{2}}\sqrt{bx+a}\,x dx\right)b+\left(\int x^{m+\frac{3n}{2}}\sqrt{bx+a}dx\right)a\right)$$

```
int((c*x)^m*(a*x^n+b*x^(1+n))^(3/2),x)
```

```
c**m*(int(x**((2*m + 3*n)/2)*sqrt(a + b*x)*x,x)*b + int(x**((2*m + 3*n)/2)
*sqrt(a + b*x),x)*a)
```

3.442 $\int (cx)^m \sqrt{ax^n + bx^{1+n}}\, dx$

Optimal result

Integrand size = 23, antiderivative size = 75

$$\int (cx)^m \sqrt{ax^n + bx^{1+n}}\, dx = \frac{2x^{-n}\left(-\frac{bx}{a}\right)^{-m-\frac{n}{2}}(cx)^m\left(ax^n + bx^{1+n}\right)^{3/2}\text{Hypergeometric2F1}\left(\frac{3}{2}, -m-\frac{n}{2}, \frac{5}{2}, 1+\frac{bx}{a}\right)}{3b}$$

```
2/3*(-b*x/a)^(-m-1/2*n)*(c*x)^m*(a*x^n+b*x^(1+n))^(3/2)*hypergeom([3/2, -m
-1/2*n],[5/2],1+b*x/a)/b/(x^n)
```

Mathematica [A] (verified)

Time = 0.11 (sec) , antiderivative size = 71, normalized size of antiderivative = 0.95

$$\int (cx)^m \sqrt{ax^n + bx^{1+n}}\, dx = \frac{2\left(-\frac{bx}{a}\right)^{-m-\frac{n}{2}}(cx)^m(a+bx)\sqrt{x^n(a+bx)}\,\text{Hypergeometric2F1}\left(\frac{3}{2}, -m-\frac{n}{2}, \frac{5}{2}, 1+\frac{bx}{a}\right)}{3b}$$

```
Integrate[(c*x)^m*Sqrt[a*x^n + b*x^(1 + n)],x]
```

```
(2*(-((b*x)/a))^(-m - n/2)*(c*x)^m*(a + b*x)*Sqrt[x^n*(a + b*x)]*Hypergeom
etric2F1[3/2, -m - n/2, 5/2, 1 + (b*x)/a])/(3*b)
```

Rubi [A] (verified)

Time = 0.33 (sec) , antiderivative size = 75, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.130, Rules used = {1938, 77, 75}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (cx)^m\sqrt{ax^n+bx^{n+1}}\,dx$$

$$\Big\downarrow \ 1938$$

$$\frac{(cx)^m x^{-m-\frac{n}{2}}\sqrt{ax^n+bx^{n+1}}\int x^{m+\frac{n}{2}}\sqrt{a+bx}dx}{\sqrt{a+bx}}$$

$$\Big\downarrow \ 77$$

$$\frac{(cx)^m\sqrt{ax^n+bx^{n+1}}\left(-\frac{bx}{a}\right)^{-m-\frac{n}{2}}\int \left(-\frac{bx}{a}\right)^{m+\frac{n}{2}}\sqrt{a+bx}dx}{\sqrt{a+bx}}$$

$$\Big\downarrow \ 75$$

$$\frac{2(a+bx)(cx)^m\sqrt{ax^n+bx^{n+1}}\left(-\frac{bx}{a}\right)^{-m-\frac{n}{2}}\text{Hypergeometric2F1}\left(\frac{3}{2},-m-\frac{n}{2},\frac{5}{2},\frac{bx}{a}+1\right)}{3b}$$

```
Int[(c*x)^m*Sqrt[a*x^n + b*x^(1 + n)],x]
```

```
(2*(-((b*x)/a))^(-m - n/2)*(c*x)^m*(a + b*x)*Sqrt[a*x^n + b*x^(1 + n)]*Hyp
ergeometric2F1[3/2, -m - n/2, 5/2, 1 + (b*x)/a])/(3*b)
```

Defintions of rubi rules used

```
Int[((b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[((c + d*x
)^(n + 1)/(d*(n + 1)*(-d/(b*c))^m))*Hypergeometric2F1[-m, n + 1, n + 2, 1 +
 d*(x/c)], x] /; FreeQ[{b, c, d, m, n}, x] &&  !IntegerQ[n] && (IntegerQ[m]
 || GtQ[-d/(b*c), 0])
```

```
Int[((b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[((-b)*(c/
d))^IntPart[m]*((b*x)^FracPart[m]/((-d)*(x/c))^FracPart[m])   Int[((-d)*(x/
c))^m*(c + d*x)^n, x], x] /; FreeQ[{b, c, d, m, n}, x] &&  !IntegerQ[m] &&
 !IntegerQ[n] &&  !GtQ[c, 0] &&  !GtQ[-d/(b*c), 0]
```

```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> Simp[c^IntPart[m]*(c*x)^FracPart[m]*((a*x^j + b*x^n)^FracPart[p]/(x^(F
racPart[m] + j*FracPart[p])*(a + b*x^(n - j))^FracPart[p]))   Int[x^(m + j*
p)*(a + b*x^(n - j))^p, x], x] /; FreeQ[{a, b, c, j, m, n, p}, x] &&  !Inte
gerQ[p] && NeQ[n, j] && PosQ[n - j]
```

Maple [F]

$$\int (cx)^m \sqrt{a\,x^n+b\,x^{1+n}}dx$$

```
int((c*x)^m*(a*x^n+b*x^(1+n))^(1/2),x)
```

```
int((c*x)^m*(a*x^n+b*x^(1+n))^(1/2),x)
```

Fricas [F(-2)]

Exception generated.

$$\int (cx)^m\sqrt{ax^n+bx^{1+n}}\,dx = \text{Exception raised: TypeError}$$

```
integrate((c*x)^m*(a*x^n+b*x^(1+n))^(1/2),x, algorithm="fricas")
```

```
Exception raised: TypeError >> Error detected within library code: inte
grate: implementation incomplete (has polynomial part)
```

Sympy [F]

$$\int (cx)^m\sqrt{ax^n+bx^{1+n}}\,dx=\int (cx)^m\,\sqrt{ax^n+bx^{n+1}}\,dx$$

```
integrate((c*x)**m*(a*x**n+b*x**(1+n))**(1/2),x)
```

```
Integral((c*x)**m*sqrt(a*x**n + b*x**(n + 1)), x)
```

Maxima [F]

$$\int (cx)^m\sqrt{ax^n+bx^{1+n}}\,dx=\int \sqrt{bx^{n+1}+ax^n}(cx)^m\,dx$$

```
integrate((c*x)^m*(a*x^n+b*x^(1+n))^(1/2),x, algorithm="maxima")
```

```
integrate(sqrt(b*x^(n + 1) + a*x^n)*(c*x)^m, x)
```

Giac [F]

$$\int (cx)^m\sqrt{ax^n+bx^{1+n}}\,dx=\int \sqrt{bx^{n+1}+ax^n}(cx)^m\,dx$$

```
integrate((c*x)^m*(a*x^n+b*x^(1+n))^(1/2),x, algorithm="giac")
```

```
integrate(sqrt(b*x^(n + 1) + a*x^n)*(c*x)^m, x)
```

Mupad [F(-1)]

Timed out.

$$\int (cx)^m \sqrt{ax^n + bx^{1+n}}\, dx = \int (c\,x)^m\,\sqrt{a\,x^n + b\,x^{n+1}}\, dx$$

```
int((c*x)^m*(a*x^n + b*x^(n + 1))^(1/2),x)
```

```
int((c*x)^m*(a*x^n + b*x^(n + 1))^(1/2), x)
```

Reduce [F]

$$\int (cx)^m \sqrt{ax^n + bx^{1+n}}\, dx = c^m \left(\int x^{m+\frac{n}{2}} \sqrt{bx + a}dx\right)$$

```
int((c*x)^m*(a*x^n+b*x^(1+n))^(1/2),x)
```

```
c**m*int(x**((2*m + n)/2)*sqrt(a + b*x),x)
```

3.443 $\int \frac{(cx)^m}{\sqrt{ax^n+bx^{1+n}}}\,dx$

Optimal result

Integrand size = 23, antiderivative size = 73

$$\int \frac{(cx)^m}{\sqrt{ax^n+bx^{1+n}}}\,dx = \frac{2x^{-n}\left(-\frac{bx}{a}\right)^{\frac{1}{2}(-2m+n)}(cx)^m\sqrt{ax^n+bx^{1+n}}\,\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{1}{2}(-2m+n),\frac{3}{2},1+\frac{bx}{a}\right)}{b}$$

```
2*(-b*x/a)^(-m+1/2*n)*(c*x)^m*(a*x^n+b*x^(1+n))^(1/2)*hypergeom([1/2, -m+1
/2*n],[3/2],1+b*x/a)/b/(x^n)
```

Mathematica [A] (verified)

Time = 0.11 (sec) , antiderivative size = 69, normalized size of antiderivative = 0.95

$$\int \frac{(cx)^m}{\sqrt{ax^n+bx^{1+n}}}\,dx = \frac{2\left(-\frac{bx}{a}\right)^{\frac{1}{2}(-2m+n)}(cx)^m(a+bx)\,\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{1}{2}(-2m+n),\frac{3}{2},1+\frac{bx}{a}\right)}{b\sqrt{x^n(a+bx)}}$$

```
Integrate[(c*x)^m/Sqrt[a*x^n + b*x^(1 + n)],x]
```

```
(2*(-((b*x)/a))^((-2*m + n)/2)*(c*x)^m*(a + b*x)*Hypergeometric2F1[1/2, (-
2*m + n)/2, 3/2, 1 + (b*x)/a])/(b*Sqrt[x^n*(a + b*x)])
```

Rubi [A] (verified)

Time = 0.35 (sec) , antiderivative size = 91, normalized size of antiderivative = 1.25, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.130, Rules used = {1938, 77, 75}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(cx)^m}{\sqrt{ax^n+bx^{n+1}}}dx$$

$$\Big\downarrow \text{1938}$$

$$\frac{\sqrt{a+bx}(cx)^m x^{\frac{1}{2}(n-2m)}\int \frac{x^{m-\frac{n}{2}}}{\sqrt{a+bx}}dx}{\sqrt{ax^n+bx^{n+1}}}$$

$$\Big\downarrow \text{77}$$

$$\frac{\sqrt{a+bx}(cx)^m x^{\frac{1}{2}(n-2m)+m-\frac{n}{2}}\left(-\frac{bx}{a}\right)^{\frac{1}{2}(n-2m)}\int \frac{\left(-\frac{bx}{a}\right)^{m-\frac{n}{2}}}{\sqrt{a+bx}}dx}{\sqrt{ax^n+bx^{n+1}}}$$

$$\Big\downarrow \text{75}$$

$$\frac{2(a+bx)(cx)^m x^{\frac{1}{2}(n-2m)+m-\frac{n}{2}}\left(-\frac{bx}{a}\right)^{\frac{1}{2}(n-2m)}\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{1}{2}(n-2m),\frac{3}{2},\frac{bx}{a}+1\right)}{b\sqrt{ax^n+bx^{n+1}}}$$

```
Int[(c*x)^m/Sqrt[a*x^n + b*x^(1 + n)],x]
```

```
(2*x^(m - n/2 + (-2*m + n)/2)*(-((b*x)/a))^((-2*m + n)/2)*(c*x)^m*(a + b*x
)*Hypergeometric2F1[1/2, (-2*m + n)/2, 3/2, 1 + (b*x)/a])/(b*Sqrt[a*x^n +
b*x^(1 + n)])
```

Defintions of rubi rules used

```
Int[((b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[((c + d*x
)^(n + 1)/(d*(n + 1)*(-d/(b*c))^m))*Hypergeometric2F1[-m, n + 1, n + 2, 1 +
 d*(x/c)], x] /; FreeQ[{b, c, d, m, n}, x] && !IntegerQ[n] && (IntegerQ[m]
 || GtQ[-d/(b*c), 0])
```

```
Int[((b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[((-b)*(c/
d))^IntPart[m]*((b*x)^FracPart[m]/((-d)*(x/c))^FracPart[m])   Int[((-d)*(x/
c))^m*(c + d*x)^n, x], x] /; FreeQ[{b, c, d, m, n}, x] && !IntegerQ[m] &&
 !IntegerQ[n] && !GtQ[c, 0] && !GtQ[-d/(b*c), 0]
```

```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> Simp[c^IntPart[m]*(c*x)^FracPart[m]*((a*x^j + b*x^n)^FracPart[p]/(x^(F
racPart[m] + j*FracPart[p])*(a + b*x^(n - j))^FracPart[p]))   Int[x^(m + j*
p)*(a + b*x^(n - j))^p, x], x] /; FreeQ[{a, b, c, j, m, n, p}, x] && !Inte
gerQ[p] && NeQ[n, j] && PosQ[n - j]
```

Maple [F]

$$\int \frac{(cx)^m}{\sqrt{a\,x^n+b\,x^{1+n}}}dx$$

```
int((c*x)^m/(a*x^n+b*x^(1+n))^(1/2),x)
```

```
int((c*x)^m/(a*x^n+b*x^(1+n))^(1/2),x)
```

Fricas [F(-2)]

Exception generated.

$$\int \frac{(cx)^m}{\sqrt{ax^n+bx^{1+n}}}\,dx = \text{Exception raised: TypeError}$$

```
integrate((c*x)^m/(a*x^n+b*x^(1+n))^(1/2),x, algorithm="fricas")
```

```
Exception raised: TypeError >> Error detected within library code: inte
grate: implementation incomplete (has polynomial part)
```

Sympy [F]

$$\int \frac{(cx)^m}{\sqrt{ax^n + bx^{1+n}}}\,dx = \int \frac{(cx)^m}{\sqrt{ax^n + bx^{n+1}}}\,dx$$

```
integrate((c*x)**m/(a*x**n+b*x**(1+n))**(1/2),x)
```

```
Integral((c*x)**m/sqrt(a*x**n + b*x**(n + 1)), x)
```

Maxima [F]

$$\int \frac{(cx)^m}{\sqrt{ax^n + bx^{1+n}}}\,dx = \int \frac{(cx)^m}{\sqrt{bx^{n+1} + ax^n}}\,dx$$

```
integrate((c*x)^m/(a*x^n+b*x^(1+n))^(1/2),x, algorithm="maxima")
```

```
integrate((c*x)^m/sqrt(b*x^(n + 1) + a*x^n), x)
```

Giac [F]

$$\int \frac{(cx)^m}{\sqrt{ax^n + bx^{1+n}}}\,dx = \int \frac{(cx)^m}{\sqrt{bx^{n+1} + ax^n}}\,dx$$

```
integrate((c*x)^m/(a*x^n+b*x^(1+n))^(1/2),x, algorithm="giac")
```

```
integrate((c*x)^m/sqrt(b*x^(n + 1) + a*x^n), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(cx)^m}{\sqrt{ax^n + bx^{1+n}}} \, dx = \int \frac{(c\,x)^m}{\sqrt{a\,x^n + b\,x^{n+1}}} \, dx$$

```
int((c*x)^m/(a*x^n + b*x^(n + 1))^(1/2),x)
```

```
int((c*x)^m/(a*x^n + b*x^(n + 1))^(1/2), x)
```

Reduce [F]

$$\int \frac{(cx)^m}{\sqrt{ax^n + bx^{1+n}}} \, dx = c^m \left(\int \frac{x^m}{x^{\frac{n}{2}} \sqrt{bx + a}} dx \right)$$

```
int((c*x)^m/(a*x^n+b*x^(1+n))^(1/2),x)
```

```
c**m*int(x**m/(x**(n/2)*sqrt(a + b*x)),x)
```

3.444 $\int \frac{(cx)^m}{\left(ax^n+bx^{1+n}\right)^{3/2}}\,dx$

Optimal result

Integrand size = 23, antiderivative size = 73

$$\int \frac{(cx)^m}{(ax^n+bx^{1+n})^{3/2}}\,dx = -\frac{2x^{-n}\left(-\frac{bx}{a}\right)^{-m+\frac{3n}{2}}(cx)^m \text{Hypergeometric2F1}\left(-\frac{1}{2},-m+\frac{3n}{2},\frac{1}{2},1+\frac{bx}{a}\right)}{b\sqrt{ax^n+bx^{1+n}}}$$

```
-2*(-b*x/a)^(-m+3/2*n)*(c*x)^m*hypergeom([-1/2, -m+3/2*n],[1/2],1+b*x/a)/b
/(x^n)/(a*x^n+b*x^(1+n))^(1/2)
```

Mathematica [A] (verified)

Time = 0.16 (sec) , antiderivative size = 69, normalized size of antiderivative = 0.95

$$\int \frac{(cx)^m}{(ax^n+bx^{1+n})^{3/2}}\,dx = -\frac{2\left(-\frac{bx}{a}\right)^{-m+\frac{3n}{2}}(cx)^m(a+bx) \text{Hypergeometric2F1}\left(-\frac{1}{2},-m+\frac{3n}{2},\frac{1}{2},1+\frac{bx}{a}\right)}{b\left(x^n(a+bx)\right)^{3/2}}$$

```
Integrate[(c*x)^m/(a*x^n + b*x^(1 + n))^(3/2),x]
```

```
(-2*(-((b*x)/a))^(-m + (3*n)/2)*(c*x)^m*(a + b*x)*Hypergeometric2F1[-1/2,
-m + (3*n)/2, 1/2, 1 + (b*x)/a])/(b*(x^n*(a + b*x))^(3/2))
```

Rubi [A] (verified)

Time = 0.35 (sec) , antiderivative size = 86, normalized size of antiderivative = 1.18, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.130, Rules used = {1938, 77, 75}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(cx)^m}{(ax^n+bx^{n+1})^{3/2}}\,dx$$
$$\Big\downarrow \ 1938$$
$$\frac{\sqrt{a+bx}(cx)^m x^{\frac{1}{2}(n-2m)}\int \frac{x^{m-\frac{3n}{2}}}{(a+bx)^{3/2}}dx}{\sqrt{ax^n+bx^{n+1}}}$$
$$\Big\downarrow \ 77$$
$$\frac{\sqrt{a+bx}(cx)^m x^{\frac{1}{2}(n-2m)+m-\frac{3n}{2}}\left(-\frac{bx}{a}\right)^{\frac{3n}{2}-m}\int \frac{\left(-\frac{bx}{a}\right)^{m-\frac{3n}{2}}}{(a+bx)^{3/2}}dx}{\sqrt{ax^n+bx^{n+1}}}$$
$$\Big\downarrow \ 75$$
$$-\frac{2(cx)^m x^{\frac{1}{2}(n-2m)+m-\frac{3n}{2}}\left(-\frac{bx}{a}\right)^{\frac{3n}{2}-m}\text{Hypergeometric2F1}\left(-\frac{1}{2},\frac{3n}{2}-m,\frac{1}{2},\frac{bx}{a}+1\right)}{b\sqrt{ax^n+bx^{n+1}}}$$

```
Int[(c*x)^m/(a*x^n + b*x^(1 + n))^(3/2),x]
```

```
(-2*x^(m - (3*n)/2 + (-2*m + n)/2)*(-((b*x)/a))^(-m + (3*n)/2)*(c*x)^m*Hyp
ergeometric2F1[-1/2, -m + (3*n)/2, 1/2, 1 + (b*x)/a])/(b*Sqrt[a*x^n + b*x^
(1 + n)])
```

Defintions of rubi rules used

```
Int[((b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[((c + d*x
)^(n + 1)/(d*(n + 1)*(-d/(b*c))^m))*Hypergeometric2F1[-m, n + 1, n + 2, 1 +
 d*(x/c)], x] /; FreeQ[{b, c, d, m, n}, x] &&  !IntegerQ[n] && (IntegerQ[m]
 || GtQ[-d/(b*c), 0])
```

```
Int[((b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[((-b)*(c/
d))^IntPart[m]*((b*x)^FracPart[m]/((-d)*(x/c))^FracPart[m])   Int[((-d)*(x/
c))^m*(c + d*x)^n, x], x] /; FreeQ[{b, c, d, m, n}, x] &&  !IntegerQ[m] &&
 !IntegerQ[n] &&  !GtQ[c, 0] &&  !GtQ[-d/(b*c), 0]
```

```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> Simp[c^IntPart[m]*(c*x)^FracPart[m]*((a*x^j + b*x^n)^FracPart[p]/(x^(F
racPart[m] + j*FracPart[p])*(a + b*x^(n - j))^FracPart[p]))   Int[x^(m + j*
p)*(a + b*x^(n - j))^p, x], x] /; FreeQ[{a, b, c, j, m, n, p}, x] &&  !Inte
gerQ[p] && NeQ[n, j] && PosQ[n - j]
```

Maple [F]

$$\int \frac{(cx)^m}{(a\,x^n+b\,x^{1+n})^{\frac{3}{2}}}dx$$

```
int((c*x)^m/(a*x^n+b*x^(1+n))^(3/2),x)
```

```
int((c*x)^m/(a*x^n+b*x^(1+n))^(3/2),x)
```

Fricas [F(-2)]

Exception generated.

$$\int \frac{(cx)^m}{(ax^n + bx^{1+n})^{3/2}}\,dx = \text{Exception raised: TypeError}$$

```
integrate((c*x)^m/(a*x^n+b*x^(1+n))^(3/2),x, algorithm="fricas")
```

```
Exception raised: TypeError >>  Error detected within library code:   inte
grate: implementation incomplete (has polynomial part)
```

Sympy [F]

$$\int \frac{(cx)^m}{(ax^n + bx^{1+n})^{3/2}}\,dx = \int \frac{(cx)^m}{(ax^n + bx^{n+1})^{\frac{3}{2}}}\,dx$$

```
integrate((c*x)**m/(a*x**n+b*x**(1+n))**(3/2),x)
```

```
Integral((c*x)**m/(a*x**n + b*x**(n + 1))**(3/2), x)
```

Maxima [F]

$$\int \frac{(cx)^m}{(ax^n + bx^{1+n})^{3/2}}\,dx = \int \frac{(cx)^m}{(bx^{n+1} + ax^n)^{\frac{3}{2}}}\,dx$$

```
integrate((c*x)^m/(a*x^n+b*x^(1+n))^(3/2),x, algorithm="maxima")
```

```
integrate((c*x)^m/(b*x^(n + 1) + a*x^n)^(3/2), x)
```

Giac [F]

$$\int \frac{(cx)^m}{(ax^n + bx^{1+n})^{3/2}}\, dx = \int \frac{(cx)^m}{(bx^{n+1} + ax^n)^{\frac{3}{2}}}\, dx$$

```
integrate((c*x)^m/(a*x^n+b*x^(1+n))^(3/2),x, algorithm="giac")
```

```
integrate((c*x)^m/(b*x^(n + 1) + a*x^n)^(3/2), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(cx)^m}{(ax^n + bx^{1+n})^{3/2}}\, dx = \int \frac{(c\,x)^m}{(a\,x^n + b\,x^{n+1})^{3/2}}\, dx$$

```
int((c*x)^m/(a*x^n + b*x^(n + 1))^(3/2),x)
```

```
int((c*x)^m/(a*x^n + b*x^(n + 1))^(3/2), x)
```

Reduce [F]

$$\int \frac{(cx)^m}{(ax^n + bx^{1+n})^{3/2}}\, dx = c^m \left(\int \frac{x^m}{x^{\frac{3n}{2}}\sqrt{bx+a}\,a + x^{\frac{3n}{2}}\sqrt{bx+a}\,bx} dx \right)$$

```
int((c*x)^m/(a*x^n+b*x^(1+n))^(3/2),x)
```

```
c**m*int(x**m/(x**((3*n)/2)*sqrt(a + b*x)*a + x**((3*n)/2)*sqrt(a + b*x)*b
*x),x)
```

3.445 $\int \frac{(cx)^m}{(ax^n+bx^{1+n})^{5/2}}\,dx$

Optimal result

Integrand size = 23, antiderivative size = 75

$$\int \frac{(cx)^m}{(ax^n+bx^{1+n})^{5/2}}\,dx = -\frac{2x^{-n}\left(-\frac{bx}{a}\right)^{-m+\frac{5n}{2}}(cx)^m \text{Hypergeometric2F1}\left(-\frac{3}{2},-m+\frac{5n}{2},-\frac{1}{2},1+\frac{bx}{a}\right)}{3b\,(ax^n+bx^{1+n})^{3/2}}$$

```
-2/3*(-b*x/a)^(-m+5/2*n)*(c*x)^m*hypergeom([-3/2, -m+5/2*n],[-1/2],1+b*x/a
)/b/(x^n)/(a*x^n+b*x^(1+n))^(3/2)
```

Mathematica [A] (verified)

Time = 0.17 (sec) , antiderivative size = 71, normalized size of antiderivative = 0.95

$$\int \frac{(cx)^m}{(ax^n+bx^{1+n})^{5/2}}\,dx = -\frac{2\left(-\frac{bx}{a}\right)^{-m+\frac{5n}{2}}(cx)^m(a+bx)\,\text{Hypergeometric2F1}\left(-\frac{3}{2},-m+\frac{5n}{2},-\frac{1}{2},1+\frac{bx}{a}\right)}{3b\,(x^n(a+bx))^{5/2}}$$

```
Integrate[(c*x)^m/(a*x^n + b*x^(1 + n))^(5/2),x]
```

```
(-2*(-((b*x)/a))^(-m + (5*n)/2)*(c*x)^m*(a + b*x)*Hypergeometric2F1[-3/2,
-m + (5*n)/2, -1/2, 1 + (b*x)/a])/(3*b*(x^n*(a + b*x))^(5/2))
```

Rubi [A] (verified)

Time = 0.36 (sec) , antiderivative size = 95, normalized size of antiderivative = 1.27, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.130, Rules used = {1938, 77, 75}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(cx)^m}{(ax^n+bx^{n+1})^{5/2}}\,dx$$

$$\Big\downarrow \text{1938}$$

$$\frac{\sqrt{a+bx}(cx)^m x^{\frac{1}{2}(n-2m)}\int \frac{x^{m-\frac{5n}{2}}}{(a+bx)^{5/2}}dx}{\sqrt{ax^n+bx^{n+1}}}$$

$$\Big\downarrow \text{77}$$

$$\frac{\sqrt{a+bx}(cx)^m x^{\frac{1}{2}(n-2m)+m-\frac{5n}{2}}\left(-\frac{bx}{a}\right)^{\frac{5n}{2}-m}\int \frac{\left(-\frac{bx}{a}\right)^{m-\frac{5n}{2}}}{(a+bx)^{5/2}}dx}{\sqrt{ax^n+bx^{n+1}}}$$

$$\Big\downarrow \text{75}$$

$$-\frac{2(cx)^m x^{\frac{1}{2}(n-2m)+m-\frac{5n}{2}}\left(-\frac{bx}{a}\right)^{\frac{5n}{2}-m}\text{Hypergeometric2F1}\left(-\frac{3}{2},\frac{5n}{2}-m,-\frac{1}{2},\frac{bx}{a}+1\right)}{3b(a+bx)\sqrt{ax^n+bx^{n+1}}}$$

```
Int[(c*x)^m/(a*x^n + b*x^(1 + n))^(5/2),x]
```

```
(-2*x^(m - (5*n)/2 + (-2*m + n)/2)*(-((b*x)/a))^(-m + (5*n)/2)*(c*x)^m*Hyp
ergeometric2F1[-3/2, -m + (5*n)/2, -1/2, 1 + (b*x)/a])/(3*b*(a + b*x)*Sqrt
[a*x^n + b*x^(1 + n)])
```

Defintions of rubi rules used

```
Int[((b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[((c + d*x
)^(n + 1)/(d*(n + 1)*(-d/(b*c))^m))*Hypergeometric2F1[-m, n + 1, n + 2, 1 +
 d*(x/c)], x] /; FreeQ[{b, c, d, m, n}, x] &&  !IntegerQ[n] && (IntegerQ[m]
 || GtQ[-d/(b*c), 0])
```

```
Int[((b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[((-b)*(c/
d))^IntPart[m]*((b*x)^FracPart[m]/((-d)*(x/c))^FracPart[m])   Int[((-d)*(x/
c))^m*(c + d*x)^n, x], x] /; FreeQ[{b, c, d, m, n}, x] &&  !IntegerQ[m] &&
 !IntegerQ[n] &&  !GtQ[c, 0] &&  !GtQ[-d/(b*c), 0]
```

```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> Simp[c^IntPart[m]*(c*x)^FracPart[m]*((a*x^j + b*x^n)^FracPart[p]/(x^(F
racPart[m] + j*FracPart[p])*(a + b*x^(n - j))^FracPart[p]))   Int[x^(m + j*
p)*(a + b*x^(n - j))^p, x], x] /; FreeQ[{a, b, c, j, m, n, p}, x] &&  !Inte
gerQ[p] && NeQ[n, j] && PosQ[n - j]
```

Maple [F]

$$\int \frac{(cx)^m}{\left(a\,x^n + b\,x^{1+n}\right)^{\frac{5}{2}}}dx$$

```
int((c*x)^m/(a*x^n+b*x^(1+n))^(5/2),x)
```



```
int((c*x)^m/(a*x^n+b*x^(1+n))^(5/2),x)
```

Fricas [F(-2)]

Exception generated.

$$\int \frac{(cx)^m}{(ax^n + bx^{1+n})^{5/2}}\, dx = \text{Exception raised: TypeError}$$

```
integrate((c*x)^m/(a*x^n+b*x^(1+n))^(5/2),x, algorithm="fricas")
```

```
Exception raised: TypeError >>  Error detected within library code:   inte
grate: implementation incomplete (has polynomial part)
```

Sympy [F]

$$\int \frac{(cx)^m}{(ax^n + bx^{1+n})^{5/2}}\, dx = \int \frac{(cx)^m}{(ax^n + bx^{n+1})^{\frac{5}{2}}}\, dx$$

```
integrate((c*x)**m/(a*x**n+b*x**(1+n))**(5/2),x)
```

```
Integral((c*x)**m/(a*x**n + b*x**(n + 1))**(5/2), x)
```

Maxima [F]

$$\int \frac{(cx)^m}{(ax^n + bx^{1+n})^{5/2}}\, dx = \int \frac{(cx)^m}{(bx^{n+1} + ax^n)^{\frac{5}{2}}}\, dx$$

```
integrate((c*x)^m/(a*x^n+b*x^(1+n))^(5/2),x, algorithm="maxima")
```

```
integrate((c*x)^m/(b*x^(n + 1) + a*x^n)^(5/2), x)
```

Giac [F]

$$\int \frac{(cx)^m}{(ax^n + bx^{1+n})^{5/2}}\,dx = \int \frac{(cx)^m}{(bx^{n+1} + ax^n)^{\frac{5}{2}}}\,dx$$

```
integrate((c*x)^m/(a*x^n+b*x^(1+n))^(5/2),x, algorithm="giac")
```

```
integrate((c*x)^m/(b*x^(n + 1) + a*x^n)^(5/2), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(cx)^m}{(ax^n + bx^{1+n})^{5/2}}\,dx = \int \frac{(c\,x)^m}{(a\,x^n + b\,x^{n+1})^{5/2}}\,dx$$

```
int((c*x)^m/(a*x^n + b*x^(n + 1))^(5/2),x)
```

```
int((c*x)^m/(a*x^n + b*x^(n + 1))^(5/2), x)
```

Reduce [F]

$$\int \frac{(cx)^m}{(ax^n + bx^{1+n})^{5/2}}\,dx = c^m \left(\int \frac{x^m}{x^{\frac{5n}{2}}\sqrt{bx+a}\,a^2 + 2x^{\frac{5n}{2}}\sqrt{bx+a}\,abx + x^{\frac{5n}{2}}\sqrt{bx+a}\,b^2x^2}dx\right)$$

```
int((c*x)^m/(a*x^n+b*x^(1+n))^(5/2),x)
```

```
c**m*int(x**m/(x**((5*n)/2)*sqrt(a + b*x)*a**2 + 2*x**((5*n)/2)*sqrt(a + b
*x)*a*b*x + x**((5*n)/2)*sqrt(a + b*x)*b**2*x**2),x)
```

3.446 $\int x^2(ax^n+bx^{1+n})^p\,dx$

Optimal result

Integrand size = 19, antiderivative size = 55

$$\int x^2\left(ax^n+bx^{1+n}\right)^p\,dx = \frac{x^{3-n}\left(ax^n+bx^{1+n}\right)^{1+p}\text{Hypergeometric2F1}\left(1,4+p+np,4+np,-\frac{bx}{a}\right)}{a(3+np)}$$

```
x^(3-n)*(a*x^n+b*x^(1+n))^(p+1)*hypergeom([1, n*p+p+4],[n*p+4],-b*x/a)/a/(
n*p+3)
```

Mathematica [A] (verified)

Time = 0.04 (sec) , antiderivative size = 55, normalized size of antiderivative = 1.00

$$\int x^2\left(ax^n+bx^{1+n}\right)^p\,dx = \frac{x^3(x^n(a+bx))^p\left(1+\frac{bx}{a}\right)^{-p}\text{Hypergeometric2F1}\left(-p,3+np,4+np,-\frac{bx}{a}\right)}{3+np}$$

```
Integrate[x^2*(a*x^n + b*x^(1 + n))^p,x]
```

```
(x^3*(x^n*(a + b*x))^p*Hypergeometric2F1[-p, 3 + n*p, 4 + n*p, -((b*x)/a)]
)/((3 + n*p)*(1 + (b*x)/a)^p)
```

Rubi [A] (verified)

Time = 0.32 (sec) , antiderivative size = 59, normalized size of antiderivative = 1.07, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.158, Rules used = {1938, 76, 74}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^2 \left(a x^n+b x^{n+1}\right)^p \, dx$$

$$\Big\downarrow \text{1938}$$

$$x^{-n p} (a+b x)^{-p} \left(a x^n+b x^{n+1}\right)^p \int x^{n p+2} (a+b x)^p dx$$

$$\Big\downarrow \text{76}$$

$$x^{-n p} \left(\frac{b x}{a}+1\right)^{-p} \left(a x^n+b x^{n+1}\right)^p \int x^{n p+2} \left(\frac{b x}{a}+1\right)^p dx$$

$$\Big\downarrow \text{74}$$

$$\frac{x^3 \left(\frac{b x}{a}+1\right)^{-p} \left(a x^n+b x^{n+1}\right)^p \text{Hypergeometric2F1}\left(-p,n p+3,n p+4,-\frac{b x}{a}\right)}{n p+3}$$

```
Int[x^2*(a*x^n + b*x^(1 + n))^p,x]
```

```
(x^3*(a*x^n + b*x^(1 + n))^p*Hypergeometric2F1[-p, 3 + n*p, 4 + n*p, -((b*
x)/a)])/((3 + n*p)*(1 + (b*x)/a)^p)
```

Defintions of rubi rules used

```
Int[((b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[c^n*((b*x
)^(m + 1)/(b*(m + 1)))*Hypergeometric2F1[-n, m + 1, m + 2, (-d)*(x/c)], x]
/; FreeQ[{b, c, d, m, n}, x] && !IntegerQ[m] && (IntegerQ[n] || (GtQ[c, 0]
 && !(EqQ[n, -2^(-1)] && EqQ[c^2 - d^2, 0] && GtQ[-d/(b*c), 0])))
```

```
Int[((b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[c^IntPart
[n]*((c + d*x)^FracPart[n]/(1 + d*(x/c))^FracPart[n])   Int[(b*x)^m*(1 + d*
(x/c))^n, x], x] /; FreeQ[{b, c, d, m, n}, x] && !IntegerQ[m] && !Integer
Q[n] && !GtQ[c, 0] && !GtQ[-d/(b*c), 0] && ((RationalQ[m] && !(EqQ[n, -2
^(-1)] && EqQ[c^2 - d^2, 0])) || !RationalQ[n])
```

```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> Simp[c^IntPart[m]*(c*x)^FracPart[m]*((a*x^j + b*x^n)^FracPart[p]/(x^(F
racPart[m] + j*FracPart[p])*(a + b*x^(n - j))^FracPart[p]))   Int[x^(m + j*
p)*(a + b*x^(n - j))^p, x], x] /; FreeQ[{a, b, c, j, m, n, p}, x] && !Inte
gerQ[p] && NeQ[n, j] && PosQ[n - j]
```

Maple [F]

$$\int x^2 \left(a\,x^n + b\,x^{1+n}\right)^p dx$$

```
int(x^2*(a*x^n+b*x^(1+n))^p,x)
```

```
int(x^2*(a*x^n+b*x^(1+n))^p,x)
```

Fricas [F]

$$\int x^2 \left(ax^n + bx^{1+n}\right)^p \, dx = \int \left(bx^{n+1} + ax^n\right)^p x^2 \, dx$$

```
integrate(x^2*(a*x^n+b*x^(1+n))^p,x, algorithm="fricas")
```

```
integral((b*x^(n + 1) + a*x^n)^p*x^2, x)
```

Sympy [F]

$$\int x^2 \left(ax^n + bx^{1+n}\right)^p \, dx = \int x^2 \left(ax^n + bx^{n+1}\right)^p \, dx$$

```
integrate(x**2*(a*x**n+b*x**(1+n))**p,x)
```

```
Integral(x**2*(a*x**n + b*x**(n + 1))**p, x)
```

Maxima [F]

$$\int x^2 \left(ax^n + bx^{1+n}\right)^p \, dx = \int \left(bx^{n+1} + ax^n\right)^p x^2 \, dx$$

```
integrate(x^2*(a*x^n+b*x^(1+n))^p,x, algorithm="maxima")
```

```
integrate((b*x^(n + 1) + a*x^n)^p*x^2, x)
```

Giac [F]

$$\int x^2 \left(ax^n + bx^{1+n}\right)^p \, dx = \int \left(bx^{n+1} + ax^n\right)^p x^2 \, dx$$

```
integrate(x^2*(a*x^n+b*x^(1+n))^p,x, algorithm="giac")
```

```
integrate((b*x^(n + 1) + a*x^n)^p*x^2, x)
```

Mupad [F(-1)]

Timed out.

$$\int x^2 \left(ax^n + bx^{1+n}\right)^p \, dx = \int x^2 \left(a\,x^n + b\,x^{n+1}\right)^p \, dx$$

```
int(x^2*(a*x^n + b*x^(n + 1))^p,x)
```

```
int(x^2*(a*x^n + b*x^(n + 1))^p, x)
```

Reduce [F]

$$\int x^2 \left(ax^n + bx^{1+n}\right)^p \, dx = \text{too large to display}$$

```
int(x^2*(a*x^n+b*x^(1+n))^p,x)
```

```
((x**n*a + x**n*b*x)**p*a**3*n**2*p**2 + 3*(x**n*a + x**n*b*x)**p*a**3*n*p
 + 2*(x**n*a + x**n*b*x)**p*a**3 - (x**n*a + x**n*b*x)**p*a**2*b*n**2*p**2
*x - (x**n*a + x**n*b*x)**p*a**2*b*n*p**2*x - 2*(x**n*a + x**n*b*x)**p*a**
2*b*n*p*x - 2*(x**n*a + x**n*b*x)**p*a**2*b*p*x + (x**n*a + x**n*b*x)**p*a
*b**2*n**2*p**2*x**2 + 2*(x**n*a + x**n*b*x)**p*a*b**2*n*p**2*x**2 + (x**n
*a + x**n*b*x)**p*a*b**2*n*p*x**2 + (x**n*a + x**n*b*x)**p*a*b**2*p**2*x**
2 + (x**n*a + x**n*b*x)**p*a*b**2*p*x**2 + (x**n*a + x**n*b*x)**p*b**3*n**
3*p**2*x**3 + 3*(x**n*a + x**n*b*x)**p*b**3*n**2*p**2*x**3 + 3*(x**n*a + x
**n*b*x)**p*b**3*n**2*p*x**3 + 3*(x**n*a + x**n*b*x)**p*b**3*n*p**2*x**3 +
 6*(x**n*a + x**n*b*x)**p*b**3*n*p*x**3 + 2*(x**n*a + x**n*b*x)**p*b**3*n*
x**3 + (x**n*a + x**n*b*x)**p*b**3*p**2*x**3 + 3*(x**n*a + x**n*b*x)**p*b*
*3*p*x**3 + 2*(x**n*a + x**n*b*x)**p*b**3*x**3 - int((x**n*a + x**n*b*x)**
p/(a*n**4*p**3*x + 4*a*n**3*p**3*x + 6*a*n**3*p**2*x + 6*a*n**2*p**3*x + 1
8*a*n**2*p**2*x + 11*a*n**2*p*x + 4*a*n*p**3*x + 18*a*n*p**2*x + 22*a*n*p*
x + 6*a*n*x + a*p**3*x + 6*a*p**2*x + 11*a*p*x + 6*a*x + b*n**4*p**3*x**2
+ 4*b*n**3*p**3*x**2 + 6*b*n**3*p**2*x**2 + 6*b*n**2*p**3*x**2 + 18*b*n**2
*p**2*x**2 + 11*b*n**2*p*x**2 + 4*b*n*p**3*x**2 + 18*b*n*p**2*x**2 + 22*b*
n*p*x**2 + 6*b*n*x**2 + b*p**3*x**2 + 6*b*p**2*x**2 + 11*b*p*x**2 + 6*b*x*
*2),x)*a**4*n**7*p**6 - 4*int((x**n*a + x**n*b*x)**p/(a*n**4*p**3*x + 4*a*
n**3*p**3*x + 6*a*n**3*p**2*x + 6*a*n**2*p**3*x + 18*a*n**2*p**2*x + 11...
```

3.447 $\int x(ax^n+bx^{1+n})^p\,dx$

Optimal result

Integrand size = 17, antiderivative size = 55

$$\int x\left(ax^n+bx^{1+n}\right)^p\,dx = \frac{x^{2-n}(ax^n+bx^{1+n})^{1+p}\,\text{Hypergeometric2F1}\left(1,3+p+np,3+np,-\frac{bx}{a}\right)}{a(2+np)}$$

```
x^(2-n)*(a*x^n+b*x^(1+n))^(p+1)*hypergeom([1, n*p+p+3],[n*p+3],-b*x/a)/a/(
n*p+2)
```

Mathematica [A] (verified)

Time = 0.03 (sec) , antiderivative size = 55, normalized size of antiderivative = 1.00

$$\int x\left(ax^n+bx^{1+n}\right)^p\,dx = \frac{x^2(x^n(a+bx))^p\left(1+\frac{bx}{a}\right)^{-p}\text{Hypergeometric2F1}\left(-p,2+np,3+np,-\frac{bx}{a}\right)}{2+np}$$

```
Integrate[x*(a*x^n + b*x^(1 + n))^p,x]
```

```
(x^2*(x^n*(a + b*x))^p*Hypergeometric2F1[-p, 2 + n*p, 3 + n*p, -((b*x)/a)]
)/((2 + n*p)*(1 + (b*x)/a)^p)
```

Rubi [A] (verified)

Time = 0.31 (sec) , antiderivative size = 59, normalized size of antiderivative = 1.07, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.176, Rules used = {1938, 76, 74}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x\left(ax^n+bx^{n+1}\right)^p\,dx$$

$$\Big\downarrow \text{ 1938}$$

$$x^{-np}(a+bx)^{-p}\left(ax^n+bx^{n+1}\right)^p\int x^{np+1}(a+bx)^pdx$$

$$\Big\downarrow \text{ 76}$$

$$x^{-np}\left(\frac{bx}{a}+1\right)^{-p}\left(ax^n+bx^{n+1}\right)^p\int x^{np+1}\left(\frac{bx}{a}+1\right)^pdx$$

$$\Big\downarrow \text{ 74}$$

$$\frac{x^2\left(\frac{bx}{a}+1\right)^{-p}\left(ax^n+bx^{n+1}\right)^p\text{Hypergeometric2F1}\left(-p,np+2,np+3,-\frac{bx}{a}\right)}{np+2}$$

```
Int[x*(a*x^n + b*x^(1 + n))^p,x]
```

```
(x^2*(a*x^n + b*x^(1 + n))^p*Hypergeometric2F1[-p, 2 + n*p, 3 + n*p, -((b*
x)/a)])/((2 + n*p)*(1 + (b*x)/a)^p)
```

Defintions of rubi rules used

```
Int[((b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[c^n*((b*x
)^(m + 1)/(b*(m + 1)))*Hypergeometric2F1[-n, m + 1, m + 2, (-d)*(x/c)], x]
/; FreeQ[{b, c, d, m, n}, x] &&  !IntegerQ[m] && (IntegerQ[n] || (GtQ[c, 0]
 &&  !(EqQ[n, -2^(-1)] && EqQ[c^2 - d^2, 0] && GtQ[-d/(b*c), 0])))
```

```
Int[((b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[c^IntPart
[n]*((c + d*x)^FracPart[n]/(1 + d*(x/c))^FracPart[n])   Int[(b*x)^m*(1 + d*
(x/c))^n, x], x] /; FreeQ[{b, c, d, m, n}, x] &&  !IntegerQ[m] &&  !Integer
Q[n] &&  !GtQ[c, 0] &&  !GtQ[-d/(b*c), 0] && ((RationalQ[m] &&  !(EqQ[n, -2
^(-1)] && EqQ[c^2 - d^2, 0])) ||  !RationalQ[n])
```

```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> Simp[c^IntPart[m]*(c*x)^FracPart[m]*((a*x^j + b*x^n)^FracPart[p]/(x^(F
racPart[m] + j*FracPart[p])*(a + b*x^(n - j))^FracPart[p]))   Int[x^(m + j*
p)*(a + b*x^(n - j))^p, x], x] /; FreeQ[{a, b, c, j, m, n, p}, x] &&  !Inte
gerQ[p] && NeQ[n, j] && PosQ[n - j]
```

Maple [F]

$$\int x\left(a\,x^{n}+b\,x^{1+n}\right)^{p}dx$$

```
int(x*(a*x^n+b*x^(1+n))^p,x)
```

```
int(x*(a*x^n+b*x^(1+n))^p,x)
```

Fricas [F]

$$\int x(ax^n + bx^{1+n})^p \, dx = \int (bx^{n+1} + ax^n)^p x \, dx$$

```
integrate(x*(a*x^n+b*x^(1+n))^p,x, algorithm="fricas")
```

```
integral((b*x^(n + 1) + a*x^n)^p*x, x)
```

Sympy [F]

$$\int x(ax^n + bx^{1+n})^p \, dx = \int x(ax^n + bx^{n+1})^p \, dx$$

```
integrate(x*(a*x**n+b*x**(1+n))**p,x)
```

```
Integral(x*(a*x**n + b*x**(n + 1))**p, x)
```

Maxima [F]

$$\int x(ax^n + bx^{1+n})^p \, dx = \int (bx^{n+1} + ax^n)^p x \, dx$$

```
integrate(x*(a*x^n+b*x^(1+n))^p,x, algorithm="maxima")
```

```
integrate((b*x^(n + 1) + a*x^n)^p*x, x)
```

Giac [F]

$$\int x(ax^n + bx^{1+n})^p \, dx = \int (bx^{n+1} + ax^n)^p x \, dx$$

```
integrate(x*(a*x^n+b*x^(1+n))^p,x, algorithm="giac")
```

```
integrate((b*x^(n + 1) + a*x^n)^p*x, x)
```

Mupad [F(-1)]

Timed out.

$$\int x(ax^n + bx^{1+n})^p \, dx = \int x \left(a\,x^n + b\,x^{n+1}\right)^p \, dx$$

```
int(x*(a*x^n + b*x^(n + 1))^p,x)
```

```
int(x*(a*x^n + b*x^(n + 1))^p, x)
```

Reduce [F]

$$\int x(ax^n + bx^{1+n})^p \, dx = \text{too large to display}$$

```
int(x*(a*x^n+b*x^(1+n))^p,x)
```

```
( - (x**n*a + x**n*b*x)**p*a**2*n*p - (x**n*a + x**n*b*x)**p*a**2 + (x**n*
a + x**n*b*x)**p*a*b*n*p*x + (x**n*a + x**n*b*x)**p*a*b*p*x + (x**n*a + x*
*n*b*x)**p*b**2*n**2*p*x**2 + 2*(x**n*a + x**n*b*x)**p*b**2*n*p*x**2 + (x*
*n*a + x**n*b*x)**p*b**2*n*x**2 + (x**n*a + x**n*b*x)**p*b**2*p*x**2 + (x*
*n*a + x**n*b*x)**p*b**2*x**2 + int((x**n*a + x**n*b*x)**p/(a*n**3*p**2*x
+ 3*a*n**2*p**2*x + 3*a*n**2*p*x + 3*a*n*p**2*x + 6*a*n*p*x + 2*a*n*x + a*
p**2*x + 3*a*p*x + 2*a*x + b*n**3*p**2*x**2 + 3*b*n**2*p**2*x**2 + 3*b*n**
2*p*x**2 + 3*b*n*p**2*x**2 + 6*b*n*p*x**2 + 2*b*n*x**2 + b*p**2*x**2 + 3*b
*p*x**2 + 2*b*x**2),x)*a**3*n**5*p**4 + 3*int((x**n*a + x**n*b*x)**p/(a*n*
*3*p**2*x + 3*a*n**2*p**2*x + 3*a*n**2*p*x + 3*a*n*p**2*x + 6*a*n*p*x + 2*
a*n*x + a*p**2*x + 3*a*p*x + 2*a*x + b*n**3*p**2*x**2 + 3*b*n**2*p**2*x**2
 + 3*b*n**2*p*x**2 + 3*b*n*p**2*x**2 + 6*b*n*p*x**2 + 2*b*n*x**2 + b*p**2*
x**2 + 3*b*p*x**2 + 2*b*x**2),x)*a**3*n**4*p**4 + 4*int((x**n*a + x**n*b*x
)**p/(a*n**3*p**2*x + 3*a*n**2*p**2*x + 3*a*n**2*p*x + 3*a*n*p**2*x + 6*a*
n*p*x + 2*a*n*x + a*p**2*x + 3*a*p*x + 2*a*x + b*n**3*p**2*x**2 + 3*b*n**2
*p**2*x**2 + 3*b*n**2*p*x**2 + 3*b*n*p**2*x**2 + 6*b*n*p*x**2 + 2*b*n*x**2
 + b*p**2*x**2 + 3*b*p*x**2 + 2*b*x**2),x)*a**3*n**4*p**3 + 3*int((x**n*a
+ x**n*b*x)**p/(a*n**3*p**2*x + 3*a*n**2*p**2*x + 3*a*n**2*p*x + 3*a*n*p**
2*x + 6*a*n*p*x + 2*a*n*x + a*p**2*x + 3*a*p*x + 2*a*x + b*n**3*p**2*x**2
+ 3*b*n**2*p**2*x**2 + 3*b*n**2*p*x**2 + 3*b*n*p**2*x**2 + 6*b*n*p*x**2...
```

3.448 $\int (ax^n + bx^{1+n})^p \, dx$

Optimal result

Integrand size = 15, antiderivative size = 55

$$\int (ax^n + bx^{1+n})^p \, dx = \frac{x^{1-n}(ax^n + bx^{1+n})^{1+p} \text{Hypergeometric2F1}\left(1, 2+p+np, 2+np, -\frac{bx}{a}\right)}{a(1+np)}$$

```
x^(1-n)*(a*x^n+b*x^(1+n))^(p+1)*hypergeom([1, n*p+p+2],[n*p+2],-b*x/a)/a/(
n*p+1)
```

Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 53, normalized size of antiderivative = 0.96

$$\int (ax^n + bx^{1+n})^p \, dx = \frac{x(x^n(a+bx))^p \left(1+\frac{bx}{a}\right)^{-p} \text{Hypergeometric2F1}\left(-p, 1+np, 2+np, -\frac{bx}{a}\right)}{1+np}$$

```
Integrate[(a*x^n + b*x^(1 + n))^p,x]
```

```
(x*(x^n*(a + b*x))^p*Hypergeometric2F1[-p, 1 + n*p, 2 + n*p, -((b*x)/a)])/
((1 + n*p)*(1 + (b*x)/a)^p)
```

Rubi [A] (verified)

Time = 0.31 (sec) , antiderivative size = 57, normalized size of antiderivative = 1.04, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.200$, Rules used = {1917, 76, 74}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \left(ax^n + bx^{n+1}\right)^p \, dx$$

$$\Big\downarrow \ 1917$$

$$x^{-np}(a+bx)^{-p}\left(ax^n + bx^{n+1}\right)^p \int x^{np}(a+bx)^p dx$$

$$\Big\downarrow \ 76$$

$$x^{-np}\left(\frac{bx}{a}+1\right)^{-p}\left(ax^n + bx^{n+1}\right)^p \int x^{np}\left(\frac{bx}{a}+1\right)^p dx$$

$$\Big\downarrow \ 74$$

$$\frac{x\left(\frac{bx}{a}+1\right)^{-p}\left(ax^n + bx^{n+1}\right)^p \text{Hypergeometric2F1}\left(-p, np+1, np+2, -\frac{bx}{a}\right)}{np+1}$$

```
Int[(a*x^n + b*x^(1 + n))^p,x]
```

```
(x*(a*x^n + b*x^(1 + n))^p*Hypergeometric2F1[-p, 1 + n*p, 2 + n*p, -((b*x)
/a)])/((1 + n*p)*(1 + (b*x)/a)^p)
```

Defintions of rubi rules used

```
Int[((b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[c^n*((b*x
)^(m + 1)/(b*(m + 1)))*Hypergeometric2F1[-n, m + 1, m + 2, (-d)*(x/c)], x]
/; FreeQ[{b, c, d, m, n}, x] &&  !IntegerQ[m] && (IntegerQ[n] || (GtQ[c, 0]
 &&  !(EqQ[n, -2^(-1)] && EqQ[c^2 - d^2, 0] && GtQ[-d/(b*c), 0])))
```

```
Int[((b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[c^IntPart
[n]*((c + d*x)^FracPart[n]/(1 + d*(x/c))^FracPart[n])   Int[(b*x)^m*(1 + d*
(x/c))^n, x], x] /; FreeQ[{b, c, d, m, n}, x] &&  !IntegerQ[m] &&  !Integer
Q[n] &&  !GtQ[c, 0] &&  !GtQ[-d/(b*c), 0] && ((RationalQ[m] &&  !(EqQ[n, -2
^(-1)] && EqQ[c^2 - d^2, 0])) ||  !RationalQ[n])
```

```
Int[((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol] :> Simp[(a*x^j +
b*x^n)^FracPart[p]/(x^(j*FracPart[p])*(a + b*x^(n - j))^FracPart[p])   Int[
x^(j*p)*(a + b*x^(n - j))^p, x], x] /; FreeQ[{a, b, j, n, p}, x] &&  !Integ
erQ[p] && NeQ[n, j] && PosQ[n - j]
```

Maple [F]

$$\int \left(a\,x^{n}+b\,x^{1+n}\right)^{p}dx$$

```
int((a*x^n+b*x^(1+n))^p,x)
```

```
int((a*x^n+b*x^(1+n))^p,x)
```

Fricas [F]

$$\int (ax^n + bx^{1+n})^p \, dx = \int (bx^{n+1} + ax^n)^p \, dx$$

```
integrate((a*x^n+b*x^(1+n))^p,x, algorithm="fricas")
```

```
integral((b*x^(n + 1) + a*x^n)^p, x)
```

Sympy [F]

$$\int \left(ax^n + bx^{1+n}\right)^p \, dx = \int \left(ax^n + bx^{n+1}\right)^p \, dx$$

```
integrate((a*x**n+b*x**(1+n))**p,x)
```

```
Integral((a*x**n + b*x**(n + 1))**p, x)
```

Maxima [F]

$$\int \left(ax^n + bx^{1+n}\right)^p \, dx = \int \left(bx^{n+1} + ax^n\right)^p \, dx$$

```
integrate((a*x^n+b*x^(1+n))^p,x, algorithm="maxima")
```

```
integrate((b*x^(n + 1) + a*x^n)^p, x)
```

Giac [F]

$$\int \left(ax^n + bx^{1+n}\right)^p \, dx = \int \left(bx^{n+1} + ax^n\right)^p \, dx$$

```
integrate((a*x^n+b*x^(1+n))^p,x, algorithm="giac")
```

```
integrate((b*x^(n + 1) + a*x^n)^p, x)
```

Mupad [B] (verification not implemented)

Time = 8.47 (sec) , antiderivative size = 58, normalized size of antiderivative = 1.05

$$\int \left(ax^n+bx^{1+n}\right)^p\,dx=\frac{x\,\left(a\,x^n+b\,x^{n+1}\right)^p\,{}_2\mathrm{F}_1\left(n\,p+1,-p;\ n\,p+2;\ -\frac{b\,x}{a}\right)}{\left(\frac{b\,x}{a}+1\right)^p\,\left(n\,p+1\right)}$$

```
int((a*x^n + b*x^(n + 1))^p,x)
```

```
(x*(a*x^n + b*x^(n + 1))^p*hypergeom([n*p + 1, -p], n*p + 2, -(b*x)/a))/((
(b*x)/a + 1)^p*(n*p + 1))
```

Reduce [F]

$$\int \left(ax^n+bx^{1+n}\right)^p\,dx$$
$$=\frac{(x^na+x^nbx)^p\,a+(x^na+x^nbx)^p\,bnx+(x^na+x^nbx)^p\,bx-\left(\int\frac{(x^na+x^nbx)^p}{b\,n^2p\,x^2+a\,n^2px+2bnp\,x^2+2anpx+bn\,x^2+bp\,x^2+anx-}\right.}{}$$

```
int((a*x^n+b*x^(1+n))^p,x)
```

```
((x**n*a + x**n*b*x)**p*a + (x**n*a + x**n*b*x)**p*b*n*x + (x**n*a + x**n*
b*x)**p*b*x - int((x**n*a + x**n*b*x)**p/(a*n**2*p*x + 2*a*n*p*x + a*n*x +
 a*p*x + a*x + b*n**2*p*x**2 + 2*b*n*p*x**2 + b*n*x**2 + b*p*x**2 + b*x**2
),x)*a**2*n**3*p**2 - 2*int((x**n*a + x**n*b*x)**p/(a*n**2*p*x + 2*a*n*p*x
 + a*n*x + a*p*x + a*x + b*n**2*p*x**2 + 2*b*n*p*x**2 + b*n*x**2 + b*p*x**
2 + b*x**2),x)*a**2*n**2*p**2 - int((x**n*a + x**n*b*x)**p/(a*n**2*p*x + 2
*a*n*p*x + a*n*x + a*p*x + a*x + b*n**2*p*x**2 + 2*b*n*p*x**2 + b*n*x**2 +
 b*p*x**2 + b*x**2),x)*a**2*n**2*p - int((x**n*a + x**n*b*x)**p/(a*n**2*p*
x + 2*a*n*p*x + a*n*x + a*p*x + a*x + b*n**2*p*x**2 + 2*b*n*p*x**2 + b*n*x
**2 + b*p*x**2 + b*x**2),x)*a**2*n*p**2 - int((x**n*a + x**n*b*x)**p/(a*n*
*2*p*x + 2*a*n*p*x + a*n*x + a*p*x + a*x + b*n**2*p*x**2 + 2*b*n*p*x**2 +
b*n*x**2 + b*p*x**2 + b*x**2),x)*a**2*n*p)/(b*(n**2*p + 2*n*p + n + p + 1)
)
```

3.449 $\int \frac{(ax^n+bx^{1+n})^p}{x} \, dx$

Optimal result

Integrand size = 19, antiderivative size = 52

$$\int \frac{(ax^n+bx^{1+n})^p}{x} \, dx = \frac{x^{-n}(ax^n+bx^{1+n})^{1+p} \text{Hypergeometric2F1}\left(1,1+p+np,1+np,-\frac{bx}{a}\right)}{anp}$$

```
(a*x^n+b*x^(1+n))^(p+1)*hypergeom([1, n*p+p+1],[n*p+1],-b*x/a)/a/n/p/(x^n)
```

Mathematica [A] (verified)

Time = 0.04 (sec) , antiderivative size = 49, normalized size of antiderivative = 0.94

$$\int \frac{(ax^n+bx^{1+n})^p}{x} \, dx = \frac{(x^n(a+bx))^p \left(1+\frac{bx}{a}\right)^{-p} \text{Hypergeometric2F1}\left(-p,np,1+np,-\frac{bx}{a}\right)}{np}$$

```
Integrate[(a*x^n + b*x^(1 + n))^p/x,x]
```

```
((x^n*(a + b*x))^p*Hypergeometric2F1[-p, n*p, 1 + n*p, -((b*x)/a)])/(n*p*(
1 + (b*x)/a)^p)
```

Rubi [A] (verified)

Time = 0.31 (sec) , antiderivative size = 53, normalized size of antiderivative = 1.02, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.158$, Rules used = {1938, 76, 74}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(ax^n + bx^{n+1})^p}{x} \, dx$$

$$\Big\downarrow \text{1938}$$

$$x^{-np}(a+bx)^{-p}\left(ax^n+bx^{n+1}\right)^p \int x^{np-1}(a+bx)^p dx$$

$$\Big\downarrow \text{76}$$

$$x^{-np}\left(\frac{bx}{a}+1\right)^{-p}\left(ax^n+bx^{n+1}\right)^p \int x^{np-1}\left(\frac{bx}{a}+1\right)^p dx$$

$$\Big\downarrow \text{74}$$

$$\frac{\left(\frac{bx}{a}+1\right)^{-p}\left(ax^n+bx^{n+1}\right)^p \text{Hypergeometric2F1}\left(-p,np,np+1,-\frac{bx}{a}\right)}{np}$$

```
Int[(a*x^n + b*x^(1 + n))^p/x,x]
```

```
((a*x^n + b*x^(1 + n))^p*Hypergeometric2F1[-p, n*p, 1 + n*p, -((b*x)/a)])/
(n*p*(1 + (b*x)/a)^p)
```

Defintions of rubi rules used

```
Int[((b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[c^n*((b*x
)^(m + 1)/(b*(m + 1)))*Hypergeometric2F1[-n, m + 1, m + 2, (-d)*(x/c)], x]
/; FreeQ[{b, c, d, m, n}, x] &&  !IntegerQ[m] && (IntegerQ[n] || (GtQ[c, 0]
 &&  !(EqQ[n, -2^(-1)] && EqQ[c^2 - d^2, 0] && GtQ[-d/(b*c), 0])))
```

```
Int[((b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[c^IntPart
[n]*((c + d*x)^FracPart[n]/(1 + d*(x/c))^FracPart[n])   Int[(b*x)^m*(1 + d*
(x/c))^n, x], x] /; FreeQ[{b, c, d, m, n}, x] &&  !IntegerQ[m] &&  !Integer
Q[n] &&  !GtQ[c, 0] &&  !GtQ[-d/(b*c), 0] && ((RationalQ[m] &&  !(EqQ[n, -2
^(-1)] && EqQ[c^2 - d^2, 0])) ||  !RationalQ[n])
```

```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> Simp[c^IntPart[m]*(c*x)^FracPart[m]*((a*x^j + b*x^n)^FracPart[p]/(x^(F
racPart[m] + j*FracPart[p])*(a + b*x^(n - j))^FracPart[p]))   Int[x^(m + j*
p)*(a + b*x^(n - j))^p, x], x] /; FreeQ[{a, b, c, j, m, n, p}, x] &&  !Inte
gerQ[p] && NeQ[n, j] && PosQ[n - j]
```

Maple [F]

$$\int \frac{(a\,x^{n}+b\,x^{1+n})^{p}}{x}dx$$

```
int((a*x^n+b*x^(1+n))^p/x,x)
```

```
int((a*x^n+b*x^(1+n))^p/x,x)
```

Fricas [F]

$$\int \frac{(ax^n + bx^{1+n})^p}{x}\,dx = \int \frac{(bx^{n+1} + ax^n)^p}{x}\,dx$$

```
integrate((a*x^n+b*x^(1+n))^p/x,x, algorithm="fricas")
```

```
integral((b*x^(n + 1) + a*x^n)^p/x, x)
```

Sympy [F]

$$\int \frac{(ax^n + bx^{1+n})^p}{x}\,dx = \int \frac{(ax^n + bx^{n+1})^p}{x}\,dx$$

```
integrate((a*x**n+b*x**(1+n))**p/x,x)
```

```
Integral((a*x**n + b*x**(n + 1))**p/x, x)
```

Maxima [F]

$$\int \frac{(ax^n + bx^{1+n})^p}{x}\,dx = \int \frac{(bx^{n+1} + ax^n)^p}{x}\,dx$$

```
integrate((a*x^n+b*x^(1+n))^p/x,x, algorithm="maxima")
```

```
integrate((b*x^(n + 1) + a*x^n)^p/x, x)
```

Giac [F]

$$\int \frac{(ax^n + bx^{1+n})^p}{x}\,dx = \int \frac{(bx^{n+1} + ax^n)^p}{x}\,dx$$

```
integrate((a*x^n+b*x^(1+n))^p/x,x, algorithm="giac")
```

```
integrate((b*x^(n + 1) + a*x^n)^p/x, x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(ax^n + bx^{1+n})^p}{x}\,dx = \int \frac{(a\,x^n + b\,x^{n+1})^p}{x}\,dx$$

```
int((a*x^n + b*x^(n + 1))^p/x,x)
```

```
int((a*x^n + b*x^(n + 1))^p/x, x)
```

Reduce [F]

$$\int \frac{(ax^n + bx^{1+n})^p}{x}\,dx = \frac{(x^n a + x^n bx)^p + \left(\int \frac{(x^n a+x^n bx)^p}{bn\,x^2+anx+b\,x^2+ax}dx\right) anp + \left(\int \frac{(x^n a+x^n bx)^p}{bn\,x^2+anx+b\,x^2+ax}dx\right) ap}{p\,(n+1)}$$

```
int((a*x^n+b*x^(1+n))^p/x,x)
```

```
((x**n*a + x**n*b*x)**p + int((x**n*a + x**n*b*x)**p/(a*n*x + a*x + b*n*x**
*2 + b*x**2),x)*a*n*p + int((x**n*a + x**n*b*x)**p/(a*n*x + a*x + b*n*x**2
 + b*x**2),x)*a*p)/(p*(n + 1))
```

3.450 $\int \frac{(ax^n+bx^{1+n})^p}{x^2}\,dx$

Optimal result

Integrand size = 19, antiderivative size = 54

$$\int \frac{(ax^n+bx^{1+n})^p}{x^2}\,dx = -\frac{x^{-1-n}(ax^n+bx^{1+n})^{1+p}\,\text{Hypergeometric2F1}\left(1,(1+n)p,np,-\frac{bx}{a}\right)}{a(1-np)}$$

```
-x^(-1-n)*(a*x^n+b*x^(1+n))^(p+1)*hypergeom([1, (1+n)*p],[n*p],-b*x/a)/a/(
-n*p+1)
```

Mathematica [A] (verified)

Time = 0.03 (sec) , antiderivative size = 53, normalized size of antiderivative = 0.98

$$\int \frac{(ax^n+bx^{1+n})^p}{x^2}\,dx = \frac{(x^n(a+bx))^p\left(1+\frac{bx}{a}\right)^{-p}\text{Hypergeometric2F1}\left(-p,-1+np,np,-\frac{bx}{a}\right)}{(-1+np)x}$$

```
Integrate[(a*x^n + b*x^(1 + n))^p/x^2,x]
```

```
((x^n*(a + b*x))^p*Hypergeometric2F1[-p, -1 + n*p, n*p, -((b*x)/a)])/((-1
+ n*p)*x*(1 + (b*x)/a)^p)
```

Rubi [A] (verified)

Time = 0.31 (sec) , antiderivative size = 59, normalized size of antiderivative = 1.09, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.158, Rules used = {1938, 76, 74}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(ax^n + bx^{n+1})^p}{x^2} dx$$

$$\Big\downarrow \text{1938}$$

$$x^{-np}(a+bx)^{-p}\left(ax^n+bx^{n+1}\right)^p \int x^{np-2}(a+bx)^p dx$$

$$\Big\downarrow \text{76}$$

$$x^{-np}\left(\frac{bx}{a}+1\right)^{-p}\left(ax^n+bx^{n+1}\right)^p \int x^{np-2}\left(\frac{bx}{a}+1\right)^p dx$$

$$\Big\downarrow \text{74}$$

$$-\frac{\left(\frac{bx}{a}+1\right)^{-p}\left(ax^n+bx^{n+1}\right)^p \text{Hypergeometric2F1}\left(-p,np-1,np,-\frac{bx}{a}\right)}{x(1-np)}$$

```
Int[(a*x^n + b*x^(1 + n))^p/x^2,x]
```

```
-(((a*x^n + b*x^(1 + n))^p*Hypergeometric2F1[-p, -1 + n*p, n*p, -((b*x)/a)
])/((1 - n*p)*x*(1 + (b*x)/a)^p))
```

Defintions of rubi rules used

```
Int[((b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[c^n*((b*x
)^(m + 1)/(b*(m + 1)))*Hypergeometric2F1[-n, m + 1, m + 2, (-d)*(x/c)], x]
/; FreeQ[{b, c, d, m, n}, x] &&  !IntegerQ[m] && (IntegerQ[n] || (GtQ[c, 0]
 &&  !(EqQ[n, -2^(-1)] && EqQ[c^2 - d^2, 0] && GtQ[-d/(b*c), 0])))
```

```
Int[((b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[c^IntPart
[n]*((c + d*x)^FracPart[n]/(1 + d*(x/c))^FracPart[n])   Int[(b*x)^m*(1 + d*
(x/c))^n, x], x] /; FreeQ[{b, c, d, m, n}, x] &&  !IntegerQ[m] &&  !Integer
Q[n] &&  !GtQ[c, 0] &&  !GtQ[-d/(b*c), 0] && ((RationalQ[m] &&  !(EqQ[n, -2
^(-1)] && EqQ[c^2 - d^2, 0])) ||  !RationalQ[n])
```

```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> Simp[c^IntPart[m]*(c*x)^FracPart[m]*((a*x^j + b*x^n)^FracPart[p]/(x^(F
racPart[m] + j*FracPart[p])*(a + b*x^(n - j))^FracPart[p]))   Int[x^(m + j*
p)*(a + b*x^(n - j))^p, x], x] /; FreeQ[{a, b, c, j, m, n, p}, x] &&  !Inte
gerQ[p] && NeQ[n, j] && PosQ[n - j]
```

Maple [F]

$$\int \frac{(a\,x^n + b\,x^{1+n})^p}{x^2}dx$$

```
int((a*x^n+b*x^(1+n))^p/x^2,x)
```

```
int((a*x^n+b*x^(1+n))^p/x^2,x)
```

Fricas [F]

$$\int \frac{(ax^n + bx^{1+n})^p}{x^2}\,dx = \int \frac{(bx^{n+1} + ax^n)^p}{x^2}\,dx$$

```
integrate((a*x^n+b*x^(1+n))^p/x^2,x, algorithm="fricas")
```

```
integral((b*x^(n + 1) + a*x^n)^p/x^2, x)
```

Sympy [F]

$$\int \frac{(ax^n + bx^{1+n})^p}{x^2}\,dx = \int \frac{(ax^n + bx^{n+1})^p}{x^2}\,dx$$

```
integrate((a*x**n+b*x**(1+n))**p/x**2,x)
```

```
Integral((a*x**n + b*x**(n + 1))**p/x**2, x)
```

Maxima [F]

$$\int \frac{(ax^n + bx^{1+n})^p}{x^2}\,dx = \int \frac{(bx^{n+1} + ax^n)^p}{x^2}\,dx$$

```
integrate((a*x^n+b*x^(1+n))^p/x^2,x, algorithm="maxima")
```

```
integrate((b*x^(n + 1) + a*x^n)^p/x^2, x)
```

Giac [F]

$$\int \frac{(ax^n + bx^{1+n})^p}{x^2} \, dx = \int \frac{(bx^{n+1} + ax^n)^p}{x^2} \, dx$$

```
integrate((a*x^n+b*x^(1+n))^p/x^2,x, algorithm="giac")
```

```
integrate((b*x^(n + 1) + a*x^n)^p/x^2, x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(ax^n + bx^{1+n})^p}{x^2} \, dx = \int \frac{(a\,x^n + b\,x^{n+1})^p}{x^2} \, dx$$

```
int((a*x^n + b*x^(n + 1))^p/x^2,x)
```

```
int((a*x^n + b*x^(n + 1))^p/x^2, x)
```

Reduce [F]

$$\int \frac{(ax^n + bx^{1+n})^p}{x^2} \, dx$$
$$= \frac{(x^n a + x^n bx)^p + \left(\int \frac{(x^n a+x^n bx)^p}{bnp\,x^3+anp\,x^2+bp\,x^3+ap\,x^2-b\,x^3-a\,x^2} dx\right) an\,p^2 x + \left(\int \frac{(x^n a+x^n bx)^p}{bnp\,x^3+anp\,x^2+bp\,x^3+ap\,x^2-b\,x^3-a\,x^2} dx\right) a}{x\,(np+p-1)}$$

```
int((a*x^n+b*x^(1+n))^p/x^2,x)
```

```
((x**n*a + x**n*b*x)**p + int((x**n*a + x**n*b*x)**p/(a*n*p*x**2 + a*p*x**
2 - a*x**2 + b*n*p*x**3 + b*p*x**3 - b*x**3),x)*a*n*p**2*x + int((x**n*a +
 x**n*b*x)**p/(a*n*p*x**2 + a*p*x**2 - a*x**2 + b*n*p*x**3 + b*p*x**3 - b*
x**3),x)*a*p**2*x - int((x**n*a + x**n*b*x)**p/(a*n*p*x**2 + a*p*x**2 - a*
x**2 + b*n*p*x**3 + b*p*x**3 - b*x**3),x)*a*p*x)/(x*(n*p + p - 1))
```

3.451 $\int (cx)^{3/2} \left(ax^n+bx^{1+n}\right)^p \, dx$

Optimal result

Integrand size = 23, antiderivative size = 69

$$\int (cx)^{3/2} \left(ax^n+bx^{1+n}\right)^p \, dx = \frac{2cx^{2-n}\sqrt{cx}(ax^n+bx^{1+n})^{1+p}\,\text{Hypergeometric2F1}\left(1,\frac{7}{2}+p+np,\frac{7}{2}+np,-\frac{bx}{a}\right)}{a(5+2np)}$$

```
2*c*x^(2-n)*(c*x)^(1/2)*(a*x^n+b*x^(1+n))^(p+1)*hypergeom([1, 7/2+p+n*p],[
7/2+n*p],-b*x/a)/a/(2*n*p+5)
```

Mathematica [A] (verified)

Time = 0.05 (sec) , antiderivative size = 66, normalized size of antiderivative = 0.96

$$\int (cx)^{3/2} \left(ax^n+bx^{1+n}\right)^p \, dx = \frac{x(cx)^{3/2}\left(x^n(a+bx)\right)^p \left(1+\frac{bx}{a}\right)^{-p}\text{Hypergeometric2F1}\left(-p,\frac{5}{2}+np,\frac{7}{2}+np,-\frac{bx}{a}\right)}{\frac{5}{2}+np}$$

```
Integrate[(c*x)^(3/2)*(a*x^n + b*x^(1 + n))^p,x]
```

```
(x*(c*x)^(3/2)*(x^n*(a + b*x))^p*Hypergeometric2F1[-p, 5/2 + n*p, 7/2 + n*
p, -((b*x)/a)])/((5/2 + n*p)*(1 + (b*x)/a)^p)
```

Rubi [A] (verified)

Time = 0.33 (sec) , antiderivative size = 73, normalized size of antiderivative = 1.06, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.130, Rules used = {1938, 76, 74}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (cx)^{3/2} \left(ax^n+bx^{n+1}\right)^p \, dx$$

$$\Big\downarrow \text{1938}$$

$$c\sqrt{cx}x^{-np-\frac{1}{2}}(a+bx)^{-p} \left(ax^n+bx^{n+1}\right)^p \int x^{np+\frac{3}{2}}(a+bx)^p dx$$

$$\Big\downarrow \text{76}$$

$$c\sqrt{cx}x^{-np-\frac{1}{2}}\left(\frac{bx}{a}+1\right)^{-p} \left(ax^n+bx^{n+1}\right)^p \int x^{np+\frac{3}{2}}\left(\frac{bx}{a}+1\right)^p dx$$

$$\Big\downarrow \text{74}$$

$$\frac{2cx^2\sqrt{cx}\left(\frac{bx}{a}+1\right)^{-p} \left(ax^n+bx^{n+1}\right)^p \text{Hypergeometric2F1}\left(-p,np+\frac{5}{2},np+\frac{7}{2},-\frac{bx}{a}\right)}{2np+5}$$

```
Int[(c*x)^(3/2)*(a*x^n + b*x^(1 + n))^p,x]
```

```
(2*c*x^2*Sqrt[c*x]*(a*x^n + b*x^(1 + n))^p*Hypergeometric2F1[-p, 5/2 + n*p
, 7/2 + n*p, -((b*x)/a)])/((5 + 2*n*p)*(1 + (b*x)/a)^p)
```

Defintions of rubi rules used

```
Int[((b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[c^n*((b*x
)^(m + 1)/(b*(m + 1)))*Hypergeometric2F1[-n, m + 1, m + 2, (-d)*(x/c)], x]
/; FreeQ[{b, c, d, m, n}, x] &&  !IntegerQ[m] && (IntegerQ[n] || (GtQ[c, 0]
 &&  !(EqQ[n, -2^(-1)] && EqQ[c^2 - d^2, 0] && GtQ[-d/(b*c), 0])))
```

```
Int[((b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[c^IntPart
[n]*((c + d*x)^FracPart[n]/(1 + d*(x/c))^FracPart[n])   Int[(b*x)^m*(1 + d*
(x/c))^n, x], x] /; FreeQ[{b, c, d, m, n}, x] &&  !IntegerQ[m] &&  !Integer
Q[n] &&  !GtQ[c, 0] &&  !GtQ[-d/(b*c), 0] && ((RationalQ[m] &&  !(EqQ[n, -2
^(-1)] && EqQ[c^2 - d^2, 0])) ||  !RationalQ[n])
```

```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> Simp[c^IntPart[m]*(c*x)^FracPart[m]*((a*x^j + b*x^n)^FracPart[p]/(x^(F
racPart[m] + j*FracPart[p])*(a + b*x^(n - j))^FracPart[p]))   Int[x^(m + j*
p)*(a + b*x^(n - j))^p, x], x] /; FreeQ[{a, b, c, j, m, n, p}, x] &&  !Inte
gerQ[p] && NeQ[n, j] && PosQ[n - j]
```

Maple [F]

$$\int (cx)^{\frac{3}{2}} \left(a\,x^{n}+b\,x^{1+n}\right)^{p} dx$$

```
int((c*x)^(3/2)*(a*x^n+b*x^(1+n))^p,x)
```

```
int((c*x)^(3/2)*(a*x^n+b*x^(1+n))^p,x)
```

Fricas [F]

$$\int (cx)^{3/2} \left(ax^n + bx^{1+n}\right)^p \, dx = \int (cx)^{\frac{3}{2}} \left(bx^{n+1} + ax^n\right)^p \, dx$$

```
integrate((c*x)^(3/2)*(a*x^n+b*x^(1+n))^p,x, algorithm="fricas")
```

```
integral(sqrt(c*x)*(b*x^(n + 1) + a*x^n)^p*c*x, x)
```

Sympy [F(-1)]

Timed out.

$$\int (cx)^{3/2} \left(ax^n + bx^{1+n}\right)^p \, dx = \text{Timed out}$$

```
integrate((c*x)**(3/2)*(a*x**n+b*x**(1+n))**p,x)
```

```
Timed out
```

Maxima [F]

$$\int (cx)^{3/2} \left(ax^n + bx^{1+n}\right)^p \, dx = \int (cx)^{\frac{3}{2}} \left(bx^{n+1} + ax^n\right)^p \, dx$$

```
integrate((c*x)^(3/2)*(a*x^n+b*x^(1+n))^p,x, algorithm="maxima")
```

```
integrate((c*x)^(3/2)*(b*x^(n + 1) + a*x^n)^p, x)
```

Giac [F]

$$\int (cx)^{3/2} \left(ax^n + bx^{1+n}\right)^p \, dx = \int (cx)^{\frac{3}{2}} \left(bx^{n+1} + ax^n\right)^p \, dx$$

```
integrate((c*x)^(3/2)*(a*x^n+b*x^(1+n))^p,x, algorithm="giac")
```

```
integrate((c*x)^(3/2)*(b*x^(n + 1) + a*x^n)^p, x)
```

Mupad [F(-1)]

Timed out.

$$\int (cx)^{3/2} \left(ax^n + bx^{1+n}\right)^p \, dx = \int (c\,x)^{3/2} \left(a\,x^n + b\,x^{n+1}\right)^p \, dx$$

```
int((c*x)^(3/2)*(a*x^n + b*x^(n + 1))^p,x)
```

```
int((c*x)^(3/2)*(a*x^n + b*x^(n + 1))^p, x)
```

Reduce [F]

$$\int (cx)^{3/2} \left(ax^n + bx^{1+n}\right)^p \, dx = \text{too large to display}$$

```
int((c*x)^(3/2)*(a*x^n+b*x^(1+n))^p,x)
```

```
(2*sqrt(c)*c*( - 4*sqrt(x)*(x**n*a + x**n*b*x)**p*a**2*n*p**2 - 6*sqrt(x)*
(x**n*a + x**n*b*x)**p*a**2*p + 4*sqrt(x)*(x**n*a + x**n*b*x)**p*a*b*n*p**
2*x + 4*sqrt(x)*(x**n*a + x**n*b*x)**p*a*b*p**2*x + 2*sqrt(x)*(x**n*a + x*
*n*b*x)**p*a*b*p*x + 4*sqrt(x)*(x**n*a + x**n*b*x)**p*b**2*n**2*p**2*x**2
+ 8*sqrt(x)*(x**n*a + x**n*b*x)**p*b**2*n*p**2*x**2 + 8*sqrt(x)*(x**n*a +
x**n*b*x)**p*b**2*n*p*x**2 + 4*sqrt(x)*(x**n*a + x**n*b*x)**p*b**2*p**2*x*
*2 + 8*sqrt(x)*(x**n*a + x**n*b*x)**p*b**2*p*x**2 + 3*sqrt(x)*(x**n*a + x*
*n*b*x)**p*b**2*x**2 + 32*int((sqrt(x)*(x**n*a + x**n*b*x)**p)/(8*a*n**3*p
**3*x + 24*a*n**2*p**3*x + 36*a*n**2*p**2*x + 24*a*n*p**3*x + 72*a*n*p**2*
x + 46*a*n*p*x + 8*a*p**3*x + 36*a*p**2*x + 46*a*p*x + 15*a*x + 8*b*n**3*p
**3*x**2 + 24*b*n**2*p**3*x**2 + 36*b*n**2*p**2*x**2 + 24*b*n*p**3*x**2 +
72*b*n*p**2*x**2 + 46*b*n*p*x**2 + 8*b*p**3*x**2 + 36*b*p**2*x**2 + 46*b*p
*x**2 + 15*b*x**2),x)*a**3*n**5*p**6 + 96*int((sqrt(x)*(x**n*a + x**n*b*x)
**p)/(8*a*n**3*p**3*x + 24*a*n**2*p**3*x + 36*a*n**2*p**2*x + 24*a*n*p**3*
x + 72*a*n*p**2*x + 46*a*n*p*x + 8*a*p**3*x + 36*a*p**2*x + 46*a*p*x + 15*
a*x + 8*b*n**3*p**3*x**2 + 24*b*n**2*p**3*x**2 + 36*b*n**2*p**2*x**2 + 24*
b*n*p**3*x**2 + 72*b*n*p**2*x**2 + 46*b*n*p*x**2 + 8*b*p**3*x**2 + 36*b*p*
*2*x**2 + 46*b*p*x**2 + 15*b*x**2),x)*a**3*n**4*p**6 + 208*int((sqrt(x)*(x
**n*a + x**n*b*x)**p)/(8*a*n**3*p**3*x + 24*a*n**2*p**3*x + 36*a*n**2*p**2
*x + 24*a*n*p**3*x + 72*a*n*p**2*x + 46*a*n*p*x + 8*a*p**3*x + 36*a*p**...
```

3.452 $\int \sqrt{cx}(ax^n + bx^{1+n})^p \, dx$

Optimal result

Integrand size = 23, antiderivative size = 68

$$\int \sqrt{cx}\left(ax^n+bx^{1+n}\right)^p \, dx = \frac{2x^{1-n}\sqrt{cx}\left(ax^n+bx^{1+n}\right)^{1+p} \text{Hypergeometric2F1}\left(1,\frac{5}{2}+p+np,\frac{5}{2}+np,-\frac{bx}{a}\right)}{a(3+2np)}$$

```
2*x^(1-n)*(c*x)^(1/2)*(a*x^n+b*x^(1+n))^(p+1)*hypergeom([1, 5/2+p+n*p],[5/
2+n*p],-b*x/a)/a/(2*n*p+3)
```

Mathematica [A] (verified)

Time = 0.04 (sec) , antiderivative size = 66, normalized size of antiderivative = 0.97

$$\int \sqrt{cx}\left(ax^n+bx^{1+n}\right)^p \, dx = \frac{x\sqrt{cx}(x^n(a+bx))^p \left(1+\frac{bx}{a}\right)^{-p} \text{Hypergeometric2F1}\left(-p,\frac{3}{2}+np,\frac{5}{2}+np,-\frac{bx}{a}\right)}{\frac{3}{2}+np}$$

```
Integrate[Sqrt[c*x]*(a*x^n + b*x^(1 + n))^p,x]
```

```
(x*Sqrt[c*x]*(x^n*(a + b*x))^p*Hypergeometric2F1[-p, 3/2 + n*p, 5/2 + n*p,
 -((b*x)/a)])/((3/2 + n*p)*(1 + (b*x)/a)^p)
```

Rubi [A] (verified)

Time = 0.34 (sec) , antiderivative size = 70, normalized size of antiderivative = 1.03, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.130, Rules used = {1938, 76, 74}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sqrt{cx}\left(ax^n+bx^{n+1}\right)^p\,dx$$

$$\Big\downarrow \text{ 1938}$$

$$\sqrt{cx}x^{-np-\frac{1}{2}}(a+bx)^{-p}\left(ax^n+bx^{n+1}\right)^p\int x^{np+\frac{1}{2}}(a+bx)^pdx$$

$$\Big\downarrow \text{ 76}$$

$$\sqrt{cx}x^{-np-\frac{1}{2}}\left(\frac{bx}{a}+1\right)^{-p}\left(ax^n+bx^{n+1}\right)^p\int x^{np+\frac{1}{2}}\left(\frac{bx}{a}+1\right)^pdx$$

$$\Big\downarrow \text{ 74}$$

$$\frac{2x\sqrt{cx}\left(\frac{bx}{a}+1\right)^{-p}\left(ax^n+bx^{n+1}\right)^p\text{Hypergeometric2F1}\left(-p,np+\frac{3}{2},np+\frac{5}{2},-\frac{bx}{a}\right)}{2np+3}$$

```
Int[Sqrt[c*x]*(a*x^n + b*x^(1 + n))^p,x]
```

```
(2*x*Sqrt[c*x]*(a*x^n + b*x^(1 + n))^p*Hypergeometric2F1[-p, 3/2 + n*p, 5/
2 + n*p, -((b*x)/a)])/((3 + 2*n*p)*(1 + (b*x)/a)^p)
```

Defintions of rubi rules used

```
Int[((b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[c^n*((b*x
)^(m + 1)/(b*(m + 1)))*Hypergeometric2F1[-n, m + 1, m + 2, (-d)*(x/c)], x]
/; FreeQ[{b, c, d, m, n}, x] &&  !IntegerQ[m] && (IntegerQ[n] || (GtQ[c, 0]
 &&  !(EqQ[n, -2^(-1)] && EqQ[c^2 - d^2, 0] && GtQ[-d/(b*c), 0])))
```

```
Int[((b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[c^IntPart
[n]*((c + d*x)^FracPart[n]/(1 + d*(x/c))^FracPart[n])   Int[(b*x)^m*(1 + d*
(x/c))^n, x], x] /; FreeQ[{b, c, d, m, n}, x] &&  !IntegerQ[m] &&  !Integer
Q[n] &&  !GtQ[c, 0] &&  !GtQ[-d/(b*c), 0] && ((RationalQ[m] &&  !(EqQ[n, -2
^(-1)] && EqQ[c^2 - d^2, 0])) ||  !RationalQ[n])
```

```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> Simp[c^IntPart[m]*(c*x)^FracPart[m]*((a*x^j + b*x^n)^FracPart[p]/(x^(F
racPart[m] + j*FracPart[p])*(a + b*x^(n - j))^FracPart[p]))   Int[x^(m + j*
p)*(a + b*x^(n - j))^p, x], x] /; FreeQ[{a, b, c, j, m, n, p}, x] &&  !Inte
gerQ[p] && NeQ[n, j] && PosQ[n - j]
```

Maple [F]

$$\int \sqrt{cx}\,\left(a\,x^{n}+b\,x^{1+n}\right)^{p}dx$$

```
int((c*x)^(1/2)*(a*x^n+b*x^(1+n))^p,x)
```

```
int((c*x)^(1/2)*(a*x^n+b*x^(1+n))^p,x)
```

Fricas [F]

$$\int \sqrt{cx}\left(ax^n + bx^{1+n}\right)^p\,dx = \int \sqrt{cx}\left(bx^{n+1} + ax^n\right)^p\,dx$$

```
integrate((c*x)^(1/2)*(a*x^n+b*x^(1+n))^p,x, algorithm="fricas")
```

```
integral(sqrt(c*x)*(b*x^(n + 1) + a*x^n)^p, x)
```

Sympy [F]

$$\int \sqrt{cx}\left(ax^n + bx^{1+n}\right)^p\,dx = \int \sqrt{cx}\left(ax^n + bx^{n+1}\right)^p\,dx$$

```
integrate((c*x)**(1/2)*(a*x**n+b*x**(1+n))**p,x)
```

```
Integral(sqrt(c*x)*(a*x**n + b*x**(n + 1))**p, x)
```

Maxima [F]

$$\int \sqrt{cx}\left(ax^n + bx^{1+n}\right)^p\,dx = \int \sqrt{cx}\left(bx^{n+1} + ax^n\right)^p\,dx$$

```
integrate((c*x)^(1/2)*(a*x^n+b*x^(1+n))^p,x, algorithm="maxima")
```

```
integrate(sqrt(c*x)*(b*x^(n + 1) + a*x^n)^p, x)
```

Giac [F]

$$\int \sqrt{cx}\left(ax^n + bx^{1+n}\right)^p \, dx = \int \sqrt{cx}\left(bx^{n+1} + ax^n\right)^p \, dx$$

```
integrate((c*x)^(1/2)*(a*x^n+b*x^(1+n))^p,x, algorithm="giac")
```

```
integrate(sqrt(c*x)*(b*x^(n + 1) + a*x^n)^p, x)
```

Mupad [F(-1)]

Timed out.

$$\int \sqrt{cx}\left(ax^n + bx^{1+n}\right)^p \, dx = \int \sqrt{c\,x}\,\left(a\,x^n + b\,x^{n+1}\right)^p \, dx$$

```
int((c*x)^(1/2)*(a*x^n + b*x^(n + 1))^p,x)
```

```
int((c*x)^(1/2)*(a*x^n + b*x^(n + 1))^p, x)
```

Reduce [F]

$$\int \sqrt{cx}\left(ax^n + bx^{1+n}\right)^p \, dx = \text{Too large to display}$$

```
int((c*x)^(1/2)*(a*x^n+b*x^(1+n))^p,x)
```

```
(2*sqrt(c)*(2*sqrt(x)*(x**n*a + x**n*b*x)**p*a*p + 2*sqrt(x)*(x**n*a + x**
n*b*x)**p*b*n*p*x + 2*sqrt(x)*(x**n*a + x**n*b*x)**p*b*p*x + sqrt(x)*(x**n
*a + x**n*b*x)**p*b*x - 8*int((sqrt(x)*(x**n*a + x**n*b*x)**p)/(4*a*n**2*p
**2*x + 8*a*n*p**2*x + 8*a*n*p*x + 4*a*p**2*x + 8*a*p*x + 3*a*x + 4*b*n**2
*p**2*x**2 + 8*b*n*p**2*x**2 + 8*b*n*p*x**2 + 4*b*p**2*x**2 + 8*b*p*x**2 +
 3*b*x**2),x)*a**2*n**3*p**4 - 16*int((sqrt(x)*(x**n*a + x**n*b*x)**p)/(4*
a*n**2*p**2*x + 8*a*n*p**2*x + 8*a*n*p*x + 4*a*p**2*x + 8*a*p*x + 3*a*x +
4*b*n**2*p**2*x**2 + 8*b*n*p**2*x**2 + 8*b*n*p*x**2 + 4*b*p**2*x**2 + 8*b*
p*x**2 + 3*b*x**2),x)*a**2*n**2*p**4 - 20*int((sqrt(x)*(x**n*a + x**n*b*x)
**p)/(4*a*n**2*p**2*x + 8*a*n*p**2*x + 8*a*n*p*x + 4*a*p**2*x + 8*a*p*x +
3*a*x + 4*b*n**2*p**2*x**2 + 8*b*n*p**2*x**2 + 8*b*n*p*x**2 + 4*b*p**2*x**
2 + 8*b*p*x**2 + 3*b*x**2),x)*a**2*n**2*p**3 - 8*int((sqrt(x)*(x**n*a + x*
*n*b*x)**p)/(4*a*n**2*p**2*x + 8*a*n*p**2*x + 8*a*n*p*x + 4*a*p**2*x + 8*a
*p*x + 3*a*x + 4*b*n**2*p**2*x**2 + 8*b*n*p**2*x**2 + 8*b*n*p*x**2 + 4*b*p
**2*x**2 + 8*b*p*x**2 + 3*b*x**2),x)*a**2*n*p**4 - 24*int((sqrt(x)*(x**n*a
 + x**n*b*x)**p)/(4*a*n**2*p**2*x + 8*a*n*p**2*x + 8*a*n*p*x + 4*a*p**2*x
+ 8*a*p*x + 3*a*x + 4*b*n**2*p**2*x**2 + 8*b*n*p**2*x**2 + 8*b*n*p*x**2 +
4*b*p**2*x**2 + 8*b*p*x**2 + 3*b*x**2),x)*a**2*n*p**3 - 14*int((sqrt(x)*(x
**n*a + x**n*b*x)**p)/(4*a*n**2*p**2*x + 8*a*n*p**2*x + 8*a*n*p*x + 4*a*p*
*2*x + 8*a*p*x + 3*a*x + 4*b*n**2*p**2*x**2 + 8*b*n*p**2*x**2 + 8*b*n*p...
```

3.453 $\int \frac{(ax^n+bx^{1+n})^p}{\sqrt{cx}}\,dx$

Optimal result

Integrand size = 23, antiderivative size = 68

$$\int \frac{(ax^n+bx^{1+n})^p}{\sqrt{cx}}\,dx = \frac{2x^{1-n}(ax^n+bx^{1+n})^{1+p}\,\text{Hypergeometric2F1}\left(1,\frac{3}{2}+p+np,\frac{3}{2}+np,-\frac{bx}{a}\right)}{a(1+2np)\sqrt{cx}}$$

```
2*x^(1-n)*(a*x^n+b*x^(1+n))^(p+1)*hypergeom([1, 3/2+p+n*p],[3/2+n*p],-b*x/
a)/a/(2*n*p+1)/(c*x)^(1/2)
```

Mathematica [A] (verified)

Time = 0.05 (sec) , antiderivative size = 66, normalized size of antiderivative = 0.97

$$\int \frac{(ax^n+bx^{1+n})^p}{\sqrt{cx}}\,dx = \frac{x(x^n(a+bx))^p\left(1+\frac{bx}{a}\right)^{-p}\text{Hypergeometric2F1}\left(-p,\frac{1}{2}+np,\frac{3}{2}+np,-\frac{bx}{a}\right)}{\left(\frac{1}{2}+np\right)\sqrt{cx}}$$

```
Integrate[(a*x^n + b*x^(1 + n))^p/Sqrt[c*x],x]
```

```
(x*(x^n*(a + b*x))^p*Hypergeometric2F1[-p, 1/2 + n*p, 3/2 + n*p, -((b*x)/a
)])/((1/2 + n*p)*Sqrt[c*x]*(1 + (b*x)/a)^p)
```

Rubi [A] (verified)

Time = 0.33 (sec) , antiderivative size = 70, normalized size of antiderivative = 1.03, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.130$, Rules used = $\{1938, 76, 74\}$

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(ax^n+bx^{n+1})^p}{\sqrt{cx}}dx$$

$$\Big\downarrow \text{1938}$$

$$\frac{x^{\frac{1}{2}-np}(a+bx)^{-p}(ax^n+bx^{n+1})^p\int x^{np-\frac{1}{2}}(a+bx)^pdx}{\sqrt{cx}}$$

$$\Big\downarrow \text{76}$$

$$\frac{x^{\frac{1}{2}-np}\left(\frac{bx}{a}+1\right)^{-p}(ax^n+bx^{n+1})^p\int x^{np-\frac{1}{2}}\left(\frac{bx}{a}+1\right)^pdx}{\sqrt{cx}}$$

$$\Big\downarrow \text{74}$$

$$\frac{2x\left(\frac{bx}{a}+1\right)^{-p}(ax^n+bx^{n+1})^p\,\text{Hypergeometric2F1}\left(-p,np+\frac{1}{2},np+\frac{3}{2},-\frac{bx}{a}\right)}{\sqrt{cx}(2np+1)}$$

```
Int[(a*x^n + b*x^(1 + n))^p/Sqrt[c*x],x]
```

```
(2*x*(a*x^n + b*x^(1 + n))^p*Hypergeometric2F1[-p, 1/2 + n*p, 3/2 + n*p, -
((b*x)/a)])/((1 + 2*n*p)*Sqrt[c*x]*(1 + (b*x)/a)^p)
```

Defintions of rubi rules used

```
Int[((b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[c^n*((b*x
)^(m + 1)/(b*(m + 1)))*Hypergeometric2F1[-n, m + 1, m + 2, (-d)*(x/c)], x]
/; FreeQ[{b, c, d, m, n}, x] && !IntegerQ[m] && (IntegerQ[n] || (GtQ[c, 0]
 && !(EqQ[n, -2^(-1)] && EqQ[c^2 - d^2, 0] && GtQ[-d/(b*c), 0])))
```

```
Int[((b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[c^IntPart
[n]*((c + d*x)^FracPart[n]/(1 + d*(x/c))^FracPart[n])   Int[(b*x)^m*(1 + d*
(x/c))^n, x], x] /; FreeQ[{b, c, d, m, n}, x] && !IntegerQ[m] && !Integer
Q[n] && !GtQ[c, 0] && !GtQ[-d/(b*c), 0] && ((RationalQ[m] && !(EqQ[n, -2
^(-1)] && EqQ[c^2 - d^2, 0])) || !RationalQ[n])
```

```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> Simp[c^IntPart[m]*(c*x)^FracPart[m]*((a*x^j + b*x^n)^FracPart[p]/(x^(F
racPart[m] + j*FracPart[p])*(a + b*x^(n - j))^FracPart[p]))   Int[x^(m + j*
p)*(a + b*x^(n - j))^p, x], x] /; FreeQ[{a, b, c, j, m, n, p}, x] && !Inte
gerQ[p] && NeQ[n, j] && PosQ[n - j]
```

Maple [F]

$$\int \frac{(a\,x^{n} + b\,x^{1+n})^{p}}{\sqrt{cx}}dx$$

```
int((a*x^n+b*x^(1+n))^p/(c*x)^(1/2),x)
```

```
int((a*x^n+b*x^(1+n))^p/(c*x)^(1/2),x)
```

Fricas [F]

$$\int \frac{(ax^n + bx^{1+n})^p}{\sqrt{cx}}\,dx = \int \frac{(bx^{n+1} + ax^n)^p}{\sqrt{cx}}\,dx$$

```
integrate((a*x^n+b*x^(1+n))^p/(c*x)^(1/2),x, algorithm="fricas")
```

```
integral(sqrt(c*x)*(b*x^(n + 1) + a*x^n)^p/(c*x), x)
```

Sympy [F]

$$\int \frac{(ax^n + bx^{1+n})^p}{\sqrt{cx}}\,dx = \int \frac{(ax^n + bx^{n+1})^p}{\sqrt{cx}}\,dx$$

```
integrate((a*x**n+b*x**(1+n))**p/(c*x)**(1/2),x)
```

```
Integral((a*x**n + b*x**(n + 1))**p/sqrt(c*x), x)
```

Maxima [F]

$$\int \frac{(ax^n + bx^{1+n})^p}{\sqrt{cx}}\,dx = \int \frac{(bx^{n+1} + ax^n)^p}{\sqrt{cx}}\,dx$$

```
integrate((a*x^n+b*x^(1+n))^p/(c*x)^(1/2),x, algorithm="maxima")
```

```
integrate((b*x^(n + 1) + a*x^n)^p/sqrt(c*x), x)
```

Giac [F]

$$\int \frac{(ax^n + bx^{1+n})^p}{\sqrt{cx}}\,dx = \int \frac{(bx^{n+1} + ax^n)^p}{\sqrt{cx}}\,dx$$

```
integrate((a*x^n+b*x^(1+n))^p/(c*x)^(1/2),x, algorithm="giac")
```

```
integrate((b*x^(n + 1) + a*x^n)^p/sqrt(c*x), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(ax^n + bx^{1+n})^p}{\sqrt{cx}}\,dx = \int \frac{(a\,x^n + b\,x^{n+1})^p}{\sqrt{c\,x}}\,dx$$

```
int((a*x^n + b*x^(n + 1))^p/(c*x)^(1/2),x)
```

```
int((a*x^n + b*x^(n + 1))^p/(c*x)^(1/2), x)
```

Reduce [F]

$$\int \frac{(ax^n + bx^{1+n})^p}{\sqrt{cx}}\,dx$$
$$= \frac{2\sqrt{c}\left(\sqrt{x}\,(x^n a + x^n bx)^p + 2\left(\int \frac{\sqrt{x}\,(x^n a+x^n bx)^p}{2bnp\,x^2+2anpx+2bp\,x^2+2apx+b\,x^2+ax}dx\right) an\,p^2 + 2\left(\int \frac{\sqrt{x}\,(x^n a+x^n bx)^p}{2bnp\,x^2+2anpx+2bp\,x^2+2apx+b}\right.\right.}{c\,(2np+2p+1)}$$

```
int((a*x^n+b*x^(1+n))^p/(c*x)^(1/2),x)
```

```
(2*sqrt(c)*(sqrt(x)*(x***n*a + x***n*b*x)**p + 2*int((sqrt(x)*(x***n*a + x**n
*b*x)**p)/(2*a*n*p*x + 2*a*p*x + a*x + 2*b*n*p*x***2 + 2*b*p*x***2 + b*x**2)
,x)*a*n*p**2 + 2*int((sqrt(x)*(x***n*a + x***n*b*x)**p)/(2*a*n*p*x + 2*a*p*x
 + a*x + 2*b*n*p*x***2 + 2*b*p*x***2 + b*x**2),x)*a*p**2 + int((sqrt(x)*(x**
n*a + x***n*b*x)**p)/(2*a*n*p*x + 2*a*p*x + a*x + 2*b*n*p*x***2 + 2*b*p*x**2
 + b*x**2),x)*a*p))/(c*(2*n*p + 2*p + 1))
```

3.454 $\int \frac{(ax^n+bx^{1+n})^p}{(cx)^{3/2}}\,dx$

Optimal result

Integrand size = 23, antiderivative size = 69

$$\int \frac{(ax^n+bx^{1+n})^p}{(cx)^{3/2}}\,dx = -\frac{2x^{-n}(ax^n+bx^{1+n})^{1+p}\,\text{Hypergeometric2F1}\left(1,\frac{1}{2}+p+np,\frac{1}{2}+np,-\frac{bx}{a}\right)}{ac(1-2np)\sqrt{cx}}$$

```
-2*(a*x^n+b*x^(1+n))^(p+1)*hypergeom([1, 1/2+p+n*p],[1/2+n*p],-b*x/a)/a/c/
(-2*n*p+1)/(x^n)/(c*x)^(1/2)
```

Mathematica [A] (verified)

Time = 0.04 (sec) , antiderivative size = 66, normalized size of antiderivative = 0.96

$$\int \frac{(ax^n+bx^{1+n})^p}{(cx)^{3/2}}\,dx = \frac{x(x^n(a+bx))^p\left(1+\frac{bx}{a}\right)^{-p}\text{Hypergeometric2F1}\left(-p,-\frac{1}{2}+np,\frac{1}{2}+np,-\frac{bx}{a}\right)}{\left(-\frac{1}{2}+np\right)(cx)^{3/2}}$$

```
Integrate[(a*x^n + b*x^(1 + n))^p/(c*x)^(3/2),x]
```

```
(x*(x^n*(a + b*x))^p*Hypergeometric2F1[-p, -1/2 + n*p, 1/2 + n*p, -((b*x)/
a)])/((-1/2 + n*p)*(c*x)^(3/2)*(1 + (b*x)/a)^p)
```

Rubi [A] (verified)

Time = 0.33 (sec) , antiderivative size = 72, normalized size of antiderivative = 1.04, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.130, Rules used = {1938, 76, 74}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(ax^n+bx^{n+1})^p}{(cx)^{3/2}}\,dx$$

$$\Big\downarrow \text{ 1938}$$

$$\frac{x^{\frac{1}{2}-np}(a+bx)^{-p}\left(ax^n+bx^{n+1}\right)^p\int x^{np-\frac{3}{2}}(a+bx)^pdx}{c\sqrt{cx}}$$

$$\Big\downarrow \text{ 76}$$

$$\frac{x^{\frac{1}{2}-np}\left(\frac{bx}{a}+1\right)^{-p}\left(ax^n+bx^{n+1}\right)^p\int x^{np-\frac{3}{2}}\left(\frac{bx}{a}+1\right)^pdx}{c\sqrt{cx}}$$

$$\Big\downarrow \text{ 74}$$

$$-\frac{2\left(\frac{bx}{a}+1\right)^{-p}\left(ax^n+bx^{n+1}\right)^p\text{Hypergeometric2F1}\left(-p,np-\frac{1}{2},np+\frac{1}{2},-\frac{bx}{a}\right)}{c\sqrt{cx}(1-2np)}$$

```
Int[(a*x^n + b*x^(1 + n))^p/(c*x)^(3/2),x]
```

```
(-2*(a*x^n + b*x^(1 + n))^p*Hypergeometric2F1[-p, -1/2 + n*p, 1/2 + n*p, -
((b*x)/a)])/(c*(1 - 2*n*p)*Sqrt[c*x]*(1 + (b*x)/a)^p)
```

Defintions of rubi rules used

```
Int[((b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[c^n*((b*x
)^(m + 1)/(b*(m + 1)))*Hypergeometric2F1[-n, m + 1, m + 2, (-d)*(x/c)], x]
/; FreeQ[{b, c, d, m, n}, x] &&  !IntegerQ[m] && (IntegerQ[n] || (GtQ[c, 0]
 &&  !(EqQ[n, -2^(-1)] && EqQ[c^2 - d^2, 0] && GtQ[-d/(b*c), 0])))
```

```
Int[((b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[c^IntPart
[n]*((c + d*x)^FracPart[n]/(1 + d*(x/c))^FracPart[n])   Int[(b*x)^m*(1 + d*
(x/c))^n, x], x] /; FreeQ[{b, c, d, m, n}, x] &&  !IntegerQ[m] &&  !Integer
Q[n] &&  !GtQ[c, 0] &&  !GtQ[-d/(b*c), 0] && ((RationalQ[m] &&  !(EqQ[n, -2
^(-1)] && EqQ[c^2 - d^2, 0])) ||  !RationalQ[n])
```

```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> Simp[c^IntPart[m]*(c*x)^FracPart[m]*((a*x^j + b*x^n)^FracPart[p]/(x^(F
racPart[m] + j*FracPart[p])*(a + b*x^(n - j))^FracPart[p]))   Int[x^(m + j*
p)*(a + b*x^(n - j))^p, x], x] /; FreeQ[{a, b, c, j, m, n, p}, x] &&  !Inte
gerQ[p] && NeQ[n, j] && PosQ[n - j]
```

Maple [F]

$$\int \frac{(a\,x^{n}+b\,x^{1+n})^{p}}{(cx)^{\frac{3}{2}}}dx$$

```
int((a*x^n+b*x^(1+n))^p/(c*x)^(3/2),x)
```

```
int((a*x^n+b*x^(1+n))^p/(c*x)^(3/2),x)
```

Fricas [F]

$$\int \frac{(ax^n + bx^{1+n})^p}{(cx)^{3/2}}\,dx = \int \frac{(bx^{n+1} + ax^n)^p}{(cx)^{\frac{3}{2}}}\,dx$$

```
integrate((a*x^n+b*x^(1+n))^p/(c*x)^(3/2),x, algorithm="fricas")
```

```
integral(sqrt(c*x)*(b*x^(n + 1) + a*x^n)^p/(c^2*x^2), x)
```

Sympy [F]

$$\int \frac{(ax^n + bx^{1+n})^p}{(cx)^{3/2}}\,dx = \int \frac{(ax^n + bx^{n+1})^p}{(cx)^{\frac{3}{2}}}\,dx$$

```
integrate((a*x**n+b*x**(1+n))**p/(c*x)**(3/2),x)
```

```
Integral((a*x**n + b*x**(n + 1))**p/(c*x)**(3/2), x)
```

Maxima [F]

$$\int \frac{(ax^n + bx^{1+n})^p}{(cx)^{3/2}}\,dx = \int \frac{(bx^{n+1} + ax^n)^p}{(cx)^{\frac{3}{2}}}\,dx$$

```
integrate((a*x^n+b*x^(1+n))^p/(c*x)^(3/2),x, algorithm="maxima")
```

```
integrate((b*x^(n + 1) + a*x^n)^p/(c*x)^(3/2), x)
```

Giac [F]

$$\int \frac{(ax^n + bx^{1+n})^p}{(cx)^{3/2}}\,dx = \int \frac{(bx^{n+1} + ax^n)^p}{(cx)^{\frac{3}{2}}}\,dx$$

```
integrate((a*x^n+b*x^(1+n))^p/(c*x)^(3/2),x, algorithm="giac")
```

```
integrate((b*x^(n + 1) + a*x^n)^p/(c*x)^(3/2), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(ax^n + bx^{1+n})^p}{(cx)^{3/2}}\,dx = \int \frac{(a\,x^n + b\,x^{n+1})^p}{(c\,x)^{3/2}}\,dx$$

```
int((a*x^n + b*x^(n + 1))^p/(c*x)^(3/2),x)
```

```
int((a*x^n + b*x^(n + 1))^p/(c*x)^(3/2), x)
```

Reduce [F]

$$\int \frac{(ax^n + bx^{1+n})^p}{(cx)^{3/2}}\,dx = \frac{2\sqrt{c}\left((x^n a + x^n bx)^p + 2\sqrt{x}\left(\int \frac{\sqrt{x}\,(x^n a + x^n bx)^p}{2bnp\,x^3 + 2anp\,x^2 + 2bp\,x^3 + 2ap\,x^2 - b\,x^3 - a\,x^2}dx\right) an\,p^2 + 2\sqrt{x}\right.}{\sqrt{x}}$$

```
int((a*x^n+b*x^(1+n))^p/(c*x)^(3/2),x)
```

```
(2*sqrt(c)*((x**n*a + x**n*b*x)**p + 2*sqrt(x)*int((sqrt(x)*(x**n*a + x**n
*b*x)**p)/(2*a*n*p*x**2 + 2*a*p*x**2 - a*x**2 + 2*b*n*p*x**3 + 2*b*p*x**3
- b*x**3),x)*a*n*p**2 + 2*sqrt(x)*int((sqrt(x)*(x**n*a + x**n*b*x)**p)/(2*
a*n*p*x**2 + 2*a*p*x**2 - a*x**2 + 2*b*n*p*x**3 + 2*b*p*x**3 - b*x**3),x)*
a*p**2 - sqrt(x)*int((sqrt(x)*(x**n*a + x**n*b*x)**p)/(2*a*n*p*x**2 + 2*a*
p*x**2 - a*x**2 + 2*b*n*p*x**3 + 2*b*p*x**3 - b*x**3),x)*a*p))/(sqrt(x)*c*
*2*(2*n*p + 2*p - 1))
```

3.455 $\int (cx)^m \left(ax^n+bx^{1+n}\right)^p \, dx$

Optimal result

Integrand size = 21, antiderivative size = 63

$$\int (cx)^m \left(ax^n+bx^{1+n}\right)^p \, dx = \frac{x^{1-n}(cx)^m \left(ax^n+bx^{1+n}\right)^{1+p} \text{Hypergeometric2F1}\left(1,2+m+p+np,2+m+np,-\frac{bx}{a}\right)}{a(1+m+np)}$$

```
x^(1-n)*(c*x)^m*(a*x^n+b*x^(1+n))^(p+1)*hypergeom([1, n*p+m+p+2],[n*p+m+2]
,-b*x/a)/a/(n*p+m+1)
```

Mathematica [A] (verified)

Time = 0.04 (sec) , antiderivative size = 61, normalized size of antiderivative = 0.97

$$\int (cx)^m \left(ax^n+bx^{1+n}\right)^p \, dx = \frac{x(cx)^m (x^n(a+bx))^p \left(1+\frac{bx}{a}\right)^{-p} \text{Hypergeometric2F1}\left(-p,1+m+np,2+m+np,-\frac{bx}{a}\right)}{1+m+np}$$

```
Integrate[(c*x)^m*(a*x^n + b*x^(1 + n))^p,x]
```

```
(x*(c*x)^m*(x^n*(a + b*x))^p*Hypergeometric2F1[-p, 1 + m + n*p, 2 + m + n*
p, -((b*x)/a)])/((1 + m + n*p)*(1 + (b*x)/a)^p)
```

Rubi [A] (verified)

Time = 0.33 (sec) , antiderivative size = 65, normalized size of antiderivative = 1.03, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.143, Rules used = {1938, 76, 74}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (cx)^m \left(ax^n+bx^{n+1}\right)^p \, dx$$

$$\Big\downarrow \text{ 1938}$$

$$(cx)^m(a+bx)^{-p}x^{-m-np}\left(ax^n+bx^{n+1}\right)^p\int x^{m+np}(a+bx)^pdx$$

$$\Big\downarrow \text{ 76}$$

$$(cx)^m\left(\frac{bx}{a}+1\right)^{-p}x^{-m-np}\left(ax^n+bx^{n+1}\right)^p\int x^{m+np}\left(\frac{bx}{a}+1\right)^pdx$$

$$\Big\downarrow \text{ 74}$$

$$\frac{x(cx)^m\left(\frac{bx}{a}+1\right)^{-p}\left(ax^n+bx^{n+1}\right)^p\text{Hypergeometric2F1}\left(-p,m+np+1,m+np+2,-\frac{bx}{a}\right)}{m+np+1}$$

```
Int[(c*x)^m*(a*x^n + b*x^(1 + n))^p,x]
```

```
(x*(c*x)^m*(a*x^n + b*x^(1 + n))^p*Hypergeometric2F1[-p, 1 + m + n*p, 2 +
m + n*p, -((b*x)/a)])/((1 + m + n*p)*(1 + (b*x)/a)^p)
```

Defintions of rubi rules used

```
Int[((b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[c^n*((b*x
)^(m + 1)/(b*(m + 1)))*Hypergeometric2F1[-n, m + 1, m + 2, (-d)*(x/c)], x]
/; FreeQ[{b, c, d, m, n}, x] &&  !IntegerQ[m] && (IntegerQ[n] || (GtQ[c, 0]
 &&  !(EqQ[n, -2^(-1)] && EqQ[c^2 - d^2, 0] && GtQ[-d/(b*c), 0])))
```

```
Int[((b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[c^IntPart
[n]*((c + d*x)^FracPart[n]/(1 + d*(x/c))^FracPart[n])   Int[(b*x)^m*(1 + d*
(x/c))^n, x], x] /; FreeQ[{b, c, d, m, n}, x] &&  !IntegerQ[m] &&  !Integer
Q[n] &&  !GtQ[c, 0] &&  !GtQ[-d/(b*c), 0] && ((RationalQ[m] &&  !(EqQ[n, -2
^(-1)] && EqQ[c^2 - d^2, 0])) ||  !RationalQ[n])
```

```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> Simp[c^IntPart[m]*(c*x)^FracPart[m]*((a*x^j + b*x^n)^FracPart[p]/(x^(F
racPart[m] + j*FracPart[p])*(a + b*x^(n - j))^FracPart[p]))   Int[x^(m + j*
p)*(a + b*x^(n - j))^p, x], x] /; FreeQ[{a, b, c, j, m, n, p}, x] &&  !Inte
gerQ[p] && NeQ[n, j] && PosQ[n - j]
```

Maple [F]

$$\int (cx)^m \left(a\,x^n+b\,x^{1+n}\right)^p dx$$

```
int((c*x)^m*(a*x^n+b*x^(1+n))^p,x)
```

```
int((c*x)^m*(a*x^n+b*x^(1+n))^p,x)
```

Fricas [F]

$$\int (cx)^m \left(ax^n + bx^{1+n}\right)^p \, dx = \int (bx^{n+1} + ax^n)^p (cx)^m \, dx$$

```
integrate((c*x)^m*(a*x^n+b*x^(1+n))^p,x, algorithm="fricas")
```

```
integral((b*x^(n + 1) + a*x^n)^p*(c*x)^m, x)
```

Sympy [F]

$$\int (cx)^m \left(ax^n + bx^{1+n}\right)^p \, dx = \int (cx)^m \left(ax^n + bx^{n+1}\right)^p \, dx$$

```
integrate((c*x)**m*(a*x**n+b*x**(1+n))**p,x)
```

```
Integral((c*x)**m*(a*x**n + b*x**(n + 1))**p, x)
```

Maxima [F]

$$\int (cx)^m \left(ax^n + bx^{1+n}\right)^p \, dx = \int (bx^{n+1} + ax^n)^p (cx)^m \, dx$$

```
integrate((c*x)^m*(a*x^n+b*x^(1+n))^p,x, algorithm="maxima")
```

```
integrate((b*x^(n + 1) + a*x^n)^p*(c*x)^m, x)
```

Giac [F]

$$\int (cx)^m \left(ax^n + bx^{1+n}\right)^p \, dx = \int (bx^{n+1} + ax^n)^p (cx)^m \, dx$$

```
integrate((c*x)^m*(a*x^n+b*x^(1+n))^p,x, algorithm="giac")
```

```
integrate((b*x^(n + 1) + a*x^n)^p*(c*x)^m, x)
```

Mupad [F(-1)]

Timed out.

$$\int (cx)^m \left(ax^n + bx^{1+n}\right)^p \, dx = \int (c\,x)^m \left(a\,x^n + b\,x^{n+1}\right)^p \, dx$$

```
int((c*x)^m*(a*x^n + b*x^(n + 1))^p,x)
```

```
int((c*x)^m*(a*x^n + b*x^(n + 1))^p, x)
```

Reduce [F]

$$\int (cx)^m \left(ax^n + bx^{1+n}\right)^p \, dx = \text{too large to display}$$

```
int((c*x)^m*(a*x^n+b*x^(1+n))^p,x)
```

```
(c**m*(x**m*(x**n*a + x**n*b*x)**p*a*p + x**m*(x**n*a + x**n*b*x)**p*b*m*x
 + x**m*(x**n*a + x**n*b*x)**p*b*n*p*x + x**m*(x**n*a + x**n*b*x)**p*b*p*x
 - int((x**m*(x**n*a + x**n*b*x)**p)/(a*m**2*x + 2*a*m*n*p*x + 2*a*m*p*x +
 a*m*x + a*n**2*p**2*x + 2*a*n*p**2*x + a*n*p*x + a*p**2*x + a*p*x + b*m**
2*x**2 + 2*b*m*n*p*x**2 + 2*b*m*p*x**2 + b*m*x**2 + b*n**2*p**2*x**2 + 2*b
*n*p**2*x**2 + b*n*p*x**2 + b*p**2*x**2 + b*p*x**2),x)*a**2*m**3*p - 3*int
((x**m*(x**n*a + x**n*b*x)**p)/(a*m**2*x + 2*a*m*n*p*x + 2*a*m*p*x + a*m*x
 + a*n**2*p**2*x + 2*a*n*p**2*x + a*n*p*x + a*p**2*x + a*p*x + b*m**2*x**2
 + 2*b*m*n*p*x**2 + 2*b*m*p*x**2 + b*m*x**2 + b*n**2*p**2*x**2 + 2*b*n*p**
2*x**2 + b*n*p*x**2 + b*p**2*x**2 + b*p*x**2),x)*a**2*m**2*n*p**2 - 2*int(
(x**m*(x**n*a + x**n*b*x)**p)/(a*m**2*x + 2*a*m*n*p*x + 2*a*m*p*x + a*m*x
+ a*n**2*p**2*x + 2*a*n*p**2*x + a*n*p*x + a*p**2*x + a*p*x + b*m**2*x**2
+ 2*b*m*n*p*x**2 + 2*b*m*p*x**2 + b*m*x**2 + b*n**2*p**2*x**2 + 2*b*n*p**2
*x**2 + b*n*p*x**2 + b*p**2*x**2 + b*p*x**2),x)*a**2*m**2*p**2 - int((x**m
*(x**n*a + x**n*b*x)**p)/(a*m**2*x + 2*a*m*n*p*x + 2*a*m*p*x + a*m*x + a*n
**2*p**2*x + 2*a*n*p**2*x + a*n*p*x + a*p**2*x + a*p*x + b*m**2*x**2 + 2*b
*m*n*p*x**2 + 2*b*m*p*x**2 + b*m*x**2 + b*n**2*p**2*x**2 + 2*b*n*p**2*x**2
 + b*n*p*x**2 + b*p**2*x**2 + b*p*x**2),x)*a**2*m**2*p - 3*int((x**m*(x**n
*a + x**n*b*x)**p)/(a*m**2*x + 2*a*m*n*p*x + 2*a*m*p*x + a*m*x + a*n**2*p*
*2*x + 2*a*n*p**2*x + a*n*p*x + a*p**2*x + a*p*x + b*m**2*x**2 + 2*b*m*...
```

3.456 $\int (cx)^{-5-(1+n)p} \left(ax^n+bx^{1+n}\right)^p \, dx$

Optimal result

Integrand size = 28, antiderivative size = 226

$$
\begin{aligned}
\int (cx)^{-5-(1+n)p} \left(ax^n+bx^{1+n}\right)^p \, dx = & -\frac{x^{-4-n}(cx)^{-((1+n)p)}\left(ax^n+bx^{1+n}\right)^{1+p}}{ac^5(4+p)} \\
& + \frac{3bx^{-3-n}(cx)^{-((1+n)p)}\left(ax^n+bx^{1+n}\right)^{1+p}}{a^2c^5(3+p)(4+p)} \\
& - \frac{6b^2x^{-2-n}(cx)^{-((1+n)p)}\left(ax^n+bx^{1+n}\right)^{1+p}}{a^3c^5(2+p)(3+p)(4+p)} \\
& + \frac{6b^3x^{-1-n}(cx)^{-((1+n)p)}\left(ax^n+bx^{1+n}\right)^{1+p}}{a^4c^5(1+p)(2+p)(3+p)(4+p)}
\end{aligned}
$$

```
-x^(-4-n)*(a*x^n+b*x^(1+n))^(p+1)/a/c^5/(4+p)/((c*x)^((1+n)*p))+3*b*x^(-3-
n)*(a*x^n+b*x^(1+n))^(p+1)/a^2/c^5/(3+p)/(4+p)/((c*x)^((1+n)*p))-6*b^2*x^(
-2-n)*(a*x^n+b*x^(1+n))^(p+1)/a^3/c^5/(2+p)/(3+p)/(4+p)/((c*x)^((1+n)*p))+
6*b^3*x^(-1-n)*(a*x^n+b*x^(1+n))^(p+1)/a^4/c^5/(p+1)/(2+p)/(3+p)/(4+p)/((c
*x)^((1+n)*p))
```

Mathematica [A] (verified)

Time = 0.14 (sec) , antiderivative size = 110, normalized size of antiderivative = 0.49

$$\int (cx)^{-5-(1+n)p} \left(ax^n+bx^{1+n}\right)^p \, dx = -\frac{(cx)^{-((1+n)p)}(a+bx)\,(x^n(a+bx))^p\,(a^3(6+11p+6p^2+p^3)-3a^2b(2+3p+p^2)\,x+6ab^2(1+p)x^2-}{a^4c^5(1+p)(2+p)(3+p)(4+p)x^4}$$

```
Integrate[(c*x)^(-5 - (1 + n)*p)*(a*x^n + b*x^(1 + n))^p,x]
```

```
-(((a + b*x)*(x^n*(a + b*x))^p*(a^3*(6 + 11*p + 6*p^2 + p^3) - 3*a^2*b*(2
+ 3*p + p^2)*x + 6*a*b^2*(1 + p)*x^2 - 6*b^3*x^3))/(a^4*c^5*(1 + p)*(2 + p
)*(3 + p)*(4 + p)*x^4*(c*x)^((1 + n)*p)))
```

Rubi [A] (verified)

Time = 0.75 (sec) , antiderivative size = 208, normalized size of antiderivative = 0.92, number of steps used = 5, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.179$, Rules used = {1923, 1922, 1922, 1922, 1920}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (cx)^{-((n+1)p)-5} \left(ax^n+bx^{n+1}\right)^p \, dx$$

$$\Big\downarrow \text{1923}$$

$$\frac{x^{(n+1)p}(cx)^{-((n+1)p)} \int x^{-((n+1)p)-5} \left(ax^n+bx^{n+1}\right)^p dx}{c^5}$$

$$\Big\downarrow \text{1922}$$

$$\frac{x^{(n+1)p}(cx)^{-((n+1)p)} \left(-\frac{3b\int x^{-((n+1)p)-4}\left(ax^n+bx^{n+1}\right)^p dx}{a(p+4)}-\frac{x^{-n(p+1)-p-4}\left(ax^n+bx^{n+1}\right)^{p+1}}{a(p+4)}\right)}{c^5}$$

$$\Big\downarrow \text{1922}$$

$$\frac{x^{(n+1)p}(cx)^{-((n+1)p)}\left(-\frac{3b\left(-\frac{2b\int x^{-((n+1)p)-3}\left(ax^n+bx^{n+1}\right)^p dx}{a(p+3)}-\frac{x^{-n(p+1)-p-3}\left(ax^n+bx^{n+1}\right)^{p+1}}{a(p+3)}\right)}{a(p+4)}-\frac{x^{-n(p+1)-p-4}\left(ax^n+bx^{n+1}\right)^p}{a(p+4)}\right.}{c^5}$$

$$\Big\downarrow \text{ 1922}$$

$$\frac{x^{(n+1)p}(cx)^{-((n+1)p)}\left(-\frac{3b\left(-\frac{2b\left(-\frac{b\int x^{-((n+1)p)-2}\left(ax^n+bx^{n+1}\right)^p dx}{a(p+2)}-\frac{x^{-n(p+1)-p-2}\left(ax^n+bx^{n+1}\right)^{p+1}}{a(p+2)}\right)}{a(p+3)}-\frac{x^{-n(p+1)-p-3}\left(ax^n+bx^{n+1}\right)}{a(p+3)}\right.}{a(p+4)}\right.}{c^5}$$

$$\Big\downarrow \text{ 1920}$$

$$\frac{x^{(n+1)p}(cx)^{-((n+1)p)}\left(-\frac{3b\left(-\frac{2b\left(\frac{bx^{-((n+1)(p+1))}\left(ax^n+bx^{n+1}\right)^{p+1}}{a^2(p+1)(p+2)}-\frac{x^{-n(p+1)-p-2}\left(ax^n+bx^{n+1}\right)^{p+1}}{a(p+2)}\right)}{a(p+3)}-\frac{x^{-n(p+1)-p-3}\left(ax^n+bx^{n+1}\right)^{p}}{a(p+3)}\right.}{a(p+4)}\right.}{c^5}$$

```
Int[(c*x)^(-5 - (1 + n)*p)*(a*x^n + b*x^(1 + n))^p,x]
```

```
(x^((1 + n)*p)*(-((x^(-4 - p - n*(1 + p))*(a*x^n + b*x^(1 + n))^(1 + p))/(
a*(4 + p))) - (3*b*(-((x^(-3 - p - n*(1 + p))*(a*x^n + b*x^(1 + n))^(1 + p
))/(a*(3 + p))) - (2*b*((b*(a*x^n + b*x^(1 + n))^(1 + p))/(a^2*(1 + p)*(2
+ p)*x^((1 + n)*(1 + p))) - (x^(-2 - p - n*(1 + p))*(a*x^n + b*x^(1 + n))^
(1 + p))/(a*(2 + p))))/(a*(3 + p))))/(a*(4 + p))))/(c^5*(c*x)^((1 + n)*p))
```

Defintions of rubi rules used

```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> Simp[(-c^(j - 1))*(c*x)^(m - j + 1)*((a*x^j + b*x^n)^(p + 1)/(a*(n - j
)*(p + 1))), x] /; FreeQ[{a, b, c, j, m, n, p}, x] &&  !IntegerQ[p] && NeQ[
n, j] && EqQ[m + n*p + n - j + 1, 0] && (IntegerQ[j] || GtQ[c, 0])
```

```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> Simp[c^(j - 1)*(c*x)^(m - j + 1)*((a*x^j + b*x^n)^(p + 1)/(a*(m + j*p
+ 1))), x] - Simp[b*((m + n*p + n - j + 1)/(a*c^(n - j)*(m + j*p + 1)))   I
nt[(c*x)^(m + n - j)*(a*x^j + b*x^n)^p, x], x] /; FreeQ[{a, b, c, j, m, n,
p}, x] &&  !IntegerQ[p] && NeQ[n, j] && ILtQ[Simplify[(m + n*p + n - j + 1)
/(n - j)], 0] && NeQ[m + j*p + 1, 0] && (IntegersQ[j, n] || GtQ[c, 0])
```

```
Int[((c_)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol]
 :> Simp[c^IntPart[m]*((c*x)^FracPart[m]/x^FracPart[m])   Int[x^m*(a*x^j +
b*x^n)^p, x], x] /; FreeQ[{a, b, c, j, m, n, p}, x] &&  !IntegerQ[p] && NeQ
[n, j] && ILtQ[Simplify[(m + n*p + n - j + 1)/(n - j)], 0]
```

Maple [A] (verified)

Time = 2.45 (sec) , antiderivative size = 138, normalized size of antiderivative = 0.61

method	result	s
orering	$-\frac{\left(a^{3} p^{3}-3 a^{2} b\,p^{2} x+6 a\,b^{2} p\,x^{2}-6 b^{3} x^{3}+6 a^{3} p^{2}-9 a^{2} b p x+6 a\,b^{2} x^{2}+11 a^{3} p-6 a^{2} b x+6 a^{3}\right) x \left(b x+a\right) \left(c x\right)^{-5-\left(1+n\right) p} \left(a\,x^{n}+b\,x^{1+n}\right)^{p}}{\left(4+p\right) \left(3+p\right) \left(2+p\right) \left(p+1\right) a^{4}}$	1

```
int((c*x)^(-5-(1+n)*p)*(a*x^n+b*x^(1+n))^p,x,method=_RETURNVERBOSE)
```

```
-(a^3*p^3-3*a^2*b*p^2*x+6*a*b^2*p*x^2-6*b^3*x^3+6*a^3*p^2-9*a^2*b*p*x+6*a*
b^2*x^2+11*a^3*p-6*a^2*b*x+6*a^3)/(4+p)/(3+p)/(2+p)/(p+1)/a^4*x*(b*x+a)*(c
*x)^(-5-(1+n)*p)*(a*x^n+b*x^(1+n))^p
```

Fricas [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 180, normalized size of antiderivative = 0.80

$$\int (cx)^{-5-(1+n)p} \left(ax^n + bx^{1+n}\right)^p \, dx =$$
$$-\frac{\left(6\,ab^3px^4 - 6\,b^4x^5 - 3\,(a^2b^2p^2 + a^2b^2p)x^3 + (a^3bp^3 + 3\,a^3bp^2 + 2\,a^3bp)x^2 + (a^4p^3 + 6\,a^4p^2 + 11\,a^4p + \right.}{a^4p^4 + 10\,a^4p^3 + 35\,a^4p^2 + 50\,a^4p + 24\,a^4}$$

```
integrate((c*x)^(-5-(1+n)*p)*(a*x^n+b*x^(1+n))^p,x, algorithm="fricas")
```

```
-(6*a*b^3*p*x^4 - 6*b^4*x^5 - 3*(a^2*b^2*p^2 + a^2*b^2*p)*x^3 + (a^3*b*p^3
 + 3*a^3*b*p^2 + 2*a^3*b*p)*x^2 + (a^4*p^3 + 6*a^4*p^2 + 11*a^4*p + 6*a^4)
*x)*((b*x + a)*x^(n + 1)/x)^p*e^(-((n + 1)*p + 5)*log(c) - ((n + 1)*p + 5)
*log(x))/(a^4*p^4 + 10*a^4*p^3 + 35*a^4*p^2 + 50*a^4*p + 24*a^4)
```

Sympy [F]

$$\int (cx)^{-5-(1+n)p} \left(ax^n + bx^{1+n}\right)^p \, dx = \int (cx)^{-p(n+1)-5} \left(ax^n + bx^{n+1}\right)^p \, dx$$

```
integrate((c*x)**(-5-(1+n)*p)*(a*x**n+b*x**(1+n))**p,x)
```

```
Integral((c*x)**(-p*(n + 1) - 5)*(a*x**n + b*x**(n + 1))**p, x)
```

Maxima [F]

$$\int (cx)^{-5-(1+n)p} \left(ax^n + bx^{1+n}\right)^p \, dx = \int (bx^{n+1} + ax^n)^p (cx)^{-(n+1)p-5} \, dx$$

```
integrate((c*x)^(-5-(1+n)*p)*(a*x^n+b*x^(1+n))^p,x, algorithm="maxima")
```

```
integrate((b*x^(n + 1) + a*x^n)^p*(c*x)^(-(n + 1)*p - 5), x)
```

Giac [F]

$$\int (cx)^{-5-(1+n)p} \left(ax^n+bx^{1+n}\right)^p \,dx = \int \left(bx^{n+1}+ax^n\right)^p (cx)^{-(n+1)p-5} \,dx$$

```
integrate((c*x)^(-5-(1+n)*p)*(a*x^n+b*x^(1+n))^p,x, algorithm="giac")
```

```
integrate((b*x^(n + 1) + a*x^n)^p*(c*x)^(-(n + 1)*p - 5), x)
```

Mupad [F(-1)]

Timed out.

$$\int (cx)^{-5-(1+n)p} \left(ax^n+bx^{1+n}\right)^p \,dx = \int \frac{\left(a\,x^n+b\,x^{n+1}\right)^p}{\left(c\,x\right)^{p\,(n+1)+5}} \,dx$$

```
int((a*x^n + b*x^(n + 1))^p/(c*x)^(p*(n + 1) + 5),x)
```

```
int((a*x^n + b*x^(n + 1))^p/(c*x)^(p*(n + 1) + 5), x)
```

Reduce [B] (verification not implemented)

Time = 0.21 (sec) , antiderivative size = 162, normalized size of antiderivative = 0.72

$$\int (cx)^{-5-(1+n)p} \left(ax^n+bx^{1+n}\right)^p \,dx$$
$$= \frac{(x^n a+x^n bx)^p \left(-a^3 b\,p^3 x+3a^2b^2p^2x^2-6a\,b^3 p\,x^3+6b^4x^4-a^4p^3-3a^3b\,p^2x+3a^2b^2p\,x^2-6a^4p^2-2a^3b\right.}{x^{np+p}c^{np+p}a^4c^5x^4\left(p^4+10p^3+35p^2+50p+24\right)}$$

```
int((c*x)^(-5-(1+n)*p)*(a*x^n+b*x^(1+n))^p,x)
```

```
((x**n*a + x**n*b*x)**p*( - a**4*p**3 - 6*a**4*p**2 - 11*a**4*p - 6*a**4 -
 a**3*b*p**3*x - 3*a**3*b*p**2*x - 2*a**3*b*p*x + 3*a**2*b**2*p**2*x**2 +
3*a**2*b**2*p*x**2 - 6*a*b**3*p*x**3 + 6*b**4*x**4))/(x**(n*p + p)*c**(n*p
 + p)*a**4*c**5*x**4*(p**4 + 10*p**3 + 35*p**2 + 50*p + 24))
```

3.457 $\int (cx)^{-4-(1+n)p} \left(ax^n+bx^{1+n}\right)^p \, dx$

Optimal result

Integrand size = 28, antiderivative size = 161

$$\int (cx)^{-4-(1+n)p} \left(ax^n+bx^{1+n}\right)^p \, dx = -\frac{x^{-3-n}(cx)^{-((1+n)p)} \left(ax^n+bx^{1+n}\right)^{1+p}}{ac^4(3+p)} + \frac{2bx^{-2-n}(cx)^{-((1+n)p)} \left(ax^n+bx^{1+n}\right)^{1+p}}{a^2c^4(2+p)(3+p)} - \frac{2b^2x^{-1-n}(cx)^{-((1+n)p)} \left(ax^n+bx^{1+n}\right)^{1+p}}{a^3c^4(1+p)(2+p)(3+p)}$$

```
-x^(-3-n)*(a*x^n+b*x^(1+n))^(p+1)/a/c^4/(3+p)/((c*x)^((1+n)*p))+2*b*x^(-2-
n)*(a*x^n+b*x^(1+n))^(p+1)/a^2/c^4/(2+p)/(3+p)/((c*x)^((1+n)*p))-2*b^2*x^(
-1-n)*(a*x^n+b*x^(1+n))^(p+1)/a^3/c^4/(p+1)/(2+p)/(3+p)/((c*x)^((1+n)*p))
```

Mathematica [A] (verified)

Time = 0.13 (sec) , antiderivative size = 81, normalized size of antiderivative = 0.50

$$\int (cx)^{-4-(1+n)p} \left(ax^n+bx^{1+n}\right)^p \, dx = -\frac{(cx)^{-((1+n)p)}(a+bx)\left(x^n(a+bx)\right)^p \left(a^2(2+3p+p^2)-2ab(1+p)x+2b^2x^2\right)}{a^3c^4(1+p)(2+p)(3+p)x^3}$$

```
Integrate[(c*x)^(-4 - (1 + n)*p)*(a*x^n + b*x^(1 + n))^p,x]
```

```
-(((a + b*x)*(x^n*(a + b*x))^p*(a^2*(2 + 3*p + p^2) - 2*a*b*(1 + p)*x + 2*
b^2*x^2))/(a^3*c^4*(1 + p)*(2 + p)*(3 + p)*x^3*(c*x)^((1 + n)*p)))
```

Rubi [A] (verified)

Time = 0.56 (sec) , antiderivative size = 156, normalized size of antiderivative = 0.97, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.143$, Rules used = {1923, 1922, 1922, 1920}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (cx)^{-((n+1)p)-4} \left(ax^n+bx^{n+1}\right)^p \, dx$$

$$\Big\downarrow \text{1923}$$

$$\frac{x^{(n+1)p}(cx)^{-((n+1)p)} \int x^{-((n+1)p)-4}\left(ax^n+bx^{n+1}\right)^p dx}{c^4}$$

$$\Big\downarrow \text{1922}$$

$$\frac{x^{(n+1)p}(cx)^{-((n+1)p)} \left(-\frac{2b\int x^{-((n+1)p)-3}\left(ax^n+bx^{n+1}\right)^p dx}{a(p+3)}-\frac{x^{-n(p+1)-p-3}\left(ax^n+bx^{n+1}\right)^{p+1}}{a(p+3)}\right)}{c^4}$$

$$\Big\downarrow \text{1922}$$

$$\frac{x^{(n+1)p}(cx)^{-((n+1)p)} \left(-\frac{2b\left(-\frac{b\int x^{-((n+1)p)-2}\left(ax^n+bx^{n+1}\right)^p dx}{a(p+2)}-\frac{x^{-n(p+1)-p-2}\left(ax^n+bx^{n+1}\right)^{p+1}}{a(p+2)}\right)}{a(p+3)}-\frac{x^{-n(p+1)-p-3}\left(ax^n+bx^{n+1}\right)^{p+}}{a(p+3)}\right.}{c^4}$$

$$\Big\downarrow \text{1920}$$

$$\frac{x^{(n+1)p}(cx)^{-((n+1)p)} \left(-\frac{2b\left(\frac{bx^{-((n+1)(p+1))}\left(ax^n+bx^{n+1}\right)^{p+1}}{a^2(p+1)(p+2)}-\frac{x^{-n(p+1)-p-2}\left(ax^n+bx^{n+1}\right)^{p+1}}{a(p+2)}\right)}{a(p+3)}-\frac{x^{-n(p+1)-p-3}\left(ax^n+bx^{n+1}\right)^{p+1}}{a(p+3)}\right.}{c^4}$$

```
Int[(c*x)^(-4 - (1 + n)*p)*(a*x^n + b*x^(1 + n))^p,x]
```

```
(x^((1 + n)*p)*(-((x^(-3 - p - n*(1 + p))*(a*x^n + b*x^(1 + n))^(1 + p))/(
a*(3 + p))) - (2*b*((b*(a*x^n + b*x^(1 + n))^(1 + p))/(a^2*(1 + p)*(2 + p)
*x^((1 + n)*(1 + p))) - (x^(-2 - p - n*(1 + p))*(a*x^n + b*x^(1 + n))^(1 +
 p))/(a*(2 + p))))/(a*(3 + p))))/(c^4*(c*x)^((1 + n)*p))
```

Defintions of rubi rules used

```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> Simp[(-c^(j - 1))*(c*x)^(m - j + 1)*((a*x^j + b*x^n)^(p + 1)/(a*(n - j
)*(p + 1))), x] /; FreeQ[{a, b, c, j, m, n, p}, x] && !IntegerQ[p] && NeQ[
n, j] && EqQ[m + n*p + n - j + 1, 0] && (IntegerQ[j] || GtQ[c, 0])
```

```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> Simp[c^(j - 1)*(c*x)^(m - j + 1)*((a*x^j + b*x^n)^(p + 1)/(a*(m + j*p
+ 1))), x] - Simp[b*((m + n*p + n - j + 1)/(a*c^(n - j)*(m + j*p + 1)))   I
nt[(c*x)^(m + n - j)*(a*x^j + b*x^n)^p, x], x] /; FreeQ[{a, b, c, j, m, n,
p}, x] &&  !IntegerQ[p] && NeQ[n, j] && ILtQ[Simplify[(m + n*p + n - j + 1)
/(n - j)], 0] && NeQ[m + j*p + 1, 0] && (IntegersQ[j, n] || GtQ[c, 0])
```

```
Int[((c_)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol]
 :> Simp[c^IntPart[m]*((c*x)^FracPart[m]/x^FracPart[m])   Int[x^m*(a*x^j +
b*x^n)^p, x], x] /; FreeQ[{a, b, c, j, m, n, p}, x] &&  !IntegerQ[p] && NeQ
[n, j] && ILtQ[Simplify[(m + n*p + n - j + 1)/(n - j)], 0]
```

Maple [A] (verified)

Time = 2.43 (sec) , antiderivative size = 92, normalized size of antiderivative = 0.57

method	result	size
orering	$-\frac{\left(a^{2} p^{2}-2 a b p x+2 b^{2} x^{2}+3 a^{2} p-2 a b x+2 a^{2}\right) x \left(b x+a\right) \left(c x\right)^{-4-\left(1+n\right) p} \left(a\,x^{n}+b\,x^{1+n}\right)^{p}}{\left(3+p\right) \left(2+p\right) a^{3} \left(p+1\right)}$	92

```
int((c*x)^(-4-(1+n)*p)*(a*x^n+b*x^(1+n))^p,x,method=_RETURNVERBOSE)
```

```
-(a^2*p^2-2*a*b*p*x+2*b^2*x^2+3*a^2*p-2*a*b*x+2*a^2)/(3+p)/(2+p)/a^3/(p+1)
*x*(b*x+a)*(c*x)^(-4-(1+n)*p)*(a*x^n+b*x^(1+n))^p
```

Fricas [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 131, normalized size of antiderivative = 0.81

$$\int (cx)^{-4-(1+n)p}\left(ax^n+bx^{1+n}\right)^p\,dx$$
$$=\frac{\left(2\,ab^2px^3-2\,b^3x^4-(a^2bp^2+a^2bp)x^2-(a^3p^2+3\,a^3p+2\,a^3)x\right)\left(\frac{(bx+a)x^{n+1}}{x}\right)^p e^{(-((n+1)p+4)\log(c)-((n+1)p+4}}{a^3p^3+6\,a^3p^2+11\,a^3p+6\,a^3}$$

```
integrate((c*x)^(-4-(1+n)*p)*(a*x^n+b*x^(1+n))^p,x, algorithm="fricas")
```

```
(2*a*b^2*p*x^3 - 2*b^3*x^4 - (a^2*b*p^2 + a^2*b*p)*x^2 - (a^3*p^2 + 3*a^3*
p + 2*a^3)*x)*((b*x + a)*x^(n + 1)/x)^p*e^(-((n + 1)*p + 4)*log(c) - ((n +
 1)*p + 4)*log(x))/(a^3*p^3 + 6*a^3*p^2 + 11*a^3*p + 6*a^3)
```

Sympy [F]

$$\int (cx)^{-4-(1+n)p}\left(ax^n+bx^{1+n}\right)^p\,dx=\int (cx)^{-p(n+1)-4}\left(ax^n+bx^{n+1}\right)^p\,dx$$

```
integrate((c*x)**(-4-(1+n)*p)*(a*x**n+b*x**(1+n))**p,x)
```

```
Integral((c*x)**(-p*(n + 1) - 4)*(a*x**n + b*x**(n + 1))**p, x)
```

Maxima [F]

$$\int (cx)^{-4-(1+n)p} \left(ax^n + bx^{1+n}\right)^p \, dx = \int \left(bx^{n+1} + ax^n\right)^p (cx)^{-(n+1)p-4} \, dx$$

```
integrate((c*x)^(-4-(1+n)*p)*(a*x^n+b*x^(1+n))^p,x, algorithm="maxima")
```

```
integrate((b*x^(n + 1) + a*x^n)^p*(c*x)^(-(n + 1)*p - 4), x)
```

Giac [F]

$$\int (cx)^{-4-(1+n)p} \left(ax^n + bx^{1+n}\right)^p \, dx = \int \left(bx^{n+1} + ax^n\right)^p (cx)^{-(n+1)p-4} \, dx$$

```
integrate((c*x)^(-4-(1+n)*p)*(a*x^n+b*x^(1+n))^p,x, algorithm="giac")
```

```
integrate((b*x^(n + 1) + a*x^n)^p*(c*x)^(-(n + 1)*p - 4), x)
```

Mupad [F(-1)]

Timed out.

$$\int (cx)^{-4-(1+n)p} \left(ax^n + bx^{1+n}\right)^p \, dx = \int \frac{\left(a\,x^n + b\,x^{n+1}\right)^p}{(c\,x)^{p\,(n+1)+4}} \, dx$$

```
int((a*x^n + b*x^(n + 1))^p/(c*x)^(p*(n + 1) + 4),x)
```

```
int((a*x^n + b*x^(n + 1))^p/(c*x)^(p*(n + 1) + 4), x)
```

Reduce [B] (verification not implemented)

Time = 0.24 (sec) , antiderivative size = 113, normalized size of antiderivative = 0.70

$$\int (cx)^{-4-(1+n)p} \left(ax^n + bx^{1+n}\right)^p \, dx$$
$$= \frac{(x^n a + x^n bx)^p \left(-a^2 b\, p^2 x + 2a\, b^2 p\, x^2 - 2b^3 x^3 - a^3 p^2 - a^2 bpx - 3a^3 p - 2a^3\right)}{x^{np+p} c^{np+p} a^3 c^4 x^3 \left(p^3 + 6p^2 + 11p + 6\right)}$$

```
int((c*x)^(-4-(1+n)*p)*(a*x^n+b*x^(1+n))^p,x)
```

```
((x**n*a + x**n*b*x)**p*( - a**3*p**2 - 3*a**3*p - 2*a**3 - a**2*b*p**2*x
- a**2*b*p*x + 2*a*b**2*p*x**2 - 2*b**3*x**3))/(x**(n*p + p)*c**(n*p + p)*
a**3*c**4*x**3*(p**3 + 6*p**2 + 11*p + 6))
```

3.458 $\int (cx)^{-3-(1+n)p} \left(ax^n+bx^{1+n}\right)^p \, dx$

Optimal result

Integrand size = 28, antiderivative size = 100

$$\int (cx)^{-3-(1+n)p} \left(ax^n+bx^{1+n}\right)^p \, dx = -\frac{x^{-2-n}(cx)^{-((1+n)p)}\left(ax^n+bx^{1+n}\right)^{1+p}}{ac^3(2+p)} + \frac{bx^{-1-n}(cx)^{-((1+n)p)}\left(ax^n+bx^{1+n}\right)^{1+p}}{a^2c^3(1+p)(2+p)}$$

```
-x^(-2-n)*(a*x^n+b*x^(1+n))^(p+1)/a/c^3/(2+p)/((c*x)^((1+n)*p))+b*x^(-1-n)
*(a*x^n+b*x^(1+n))^(p+1)/a^2/c^3/(p+1)/(2+p)/((c*x)^((1+n)*p))
```

Mathematica [A] (verified)

Time = 0.08 (sec) , antiderivative size = 56, normalized size of antiderivative = 0.56

$$\int (cx)^{-3-(1+n)p} \left(ax^n+bx^{1+n}\right)^p \, dx = -\frac{(cx)^{-((1+n)p)}(a+ap-bx)(a+bx)\left(x^n(a+bx)\right)^p}{a^2c^3(1+p)(2+p)x^2}$$

```
Integrate[(c*x)^(-3 - (1 + n)*p)*(a*x^n + b*x^(1 + n))^p,x]
```

```
-(((a + a*p - b*x)*(a + b*x)*(x^n*(a + b*x))^p)/(a^2*c^3*(1 + p)*(2 + p)*x
^2*(c*x)^((1 + n)*p)))
```

Rubi [A] (verified)

Time = 0.43 (sec) , antiderivative size = 104, normalized size of antiderivative = 1.04, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.107$, Rules used = {1923, 1922, 1920}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (cx)^{-((n+1)p)-3} \left(ax^n+bx^{n+1}\right)^p dx$$
$$\Big\downarrow \text{1923}$$
$$\frac{x^{(n+1)p}(cx)^{-((n+1)p)} \int x^{-((n+1)p)-3}\left(ax^n+bx^{n+1}\right)^p dx}{c^3}$$
$$\Big\downarrow \text{1922}$$
$$\frac{x^{(n+1)p}(cx)^{-((n+1)p)} \left(-\frac{b\int x^{-((n+1)p)-2}\left(ax^n+bx^{n+1}\right)^p dx}{a(p+2)}-\frac{x^{-n(p+1)-p-2}\left(ax^n+bx^{n+1}\right)^{p+1}}{a(p+2)}\right)}{c^3}$$
$$\Big\downarrow \text{1920}$$
$$\frac{x^{(n+1)p}(cx)^{-((n+1)p)} \left(\frac{bx^{-((n+1)(p+1))}\left(ax^n+bx^{n+1}\right)^{p+1}}{a^2(p+1)(p+2)}-\frac{x^{-n(p+1)-p-2}\left(ax^n+bx^{n+1}\right)^{p+1}}{a(p+2)}\right)}{c^3}$$

```
Int[(c*x)^(-3 - (1 + n)*p)*(a*x^n + b*x^(1 + n))^p,x]
```

```
(x^((1 + n)*p)*((b*(a*x^n + b*x^(1 + n))^(1 + p))/(a^2*(1 + p)*(2 + p)*x^(
(1 + n)*(1 + p))) - (x^(-2 - p - n*(1 + p))*(a*x^n + b*x^(1 + n))^(1 + p))
/(a*(2 + p))))/(c^3*(c*x)^((1 + n)*p))
```

Defintions of rubi rules used

```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> Simp[(-c^(j - 1))*(c*x)^(m - j + 1)*((a*x^j + b*x^n)^(p + 1)/(a*(n - j
)*(p + 1))), x] /; FreeQ[{a, b, c, j, m, n, p}, x] && !IntegerQ[p] && NeQ[
n, j] && EqQ[m + n*p + n - j + 1, 0] && (IntegerQ[j] || GtQ[c, 0])
```

```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> Simp[c^(j - 1)*(c*x)^(m - j + 1)*((a*x^j + b*x^n)^(p + 1)/(a*(m + j*p
+ 1))), x] - Simp[b*((m + n*p + n - j + 1)/(a*c^(n - j)*(m + j*p + 1)))   I
nt[(c*x)^(m + n - j)*(a*x^j + b*x^n)^p, x], x] /; FreeQ[{a, b, c, j, m, n,
p}, x] && !IntegerQ[p] && NeQ[n, j] && ILtQ[Simplify[(m + n*p + n - j + 1)
/(n - j)], 0] && NeQ[m + j*p + 1, 0] && (IntegersQ[j, n] || GtQ[c, 0])
```

```
Int[((c_)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol]
 :> Simp[c^IntPart[m]*((c*x)^FracPart[m]/x^FracPart[m])   Int[x^m*(a*x^j +
b*x^n)^p, x], x] /; FreeQ[{a, b, c, j, m, n, p}, x] && !IntegerQ[p] && NeQ
[n, j] && ILtQ[Simplify[(m + n*p + n - j + 1)/(n - j)], 0]
```

Maple [A] (verified)

Time = 2.42 (sec) , antiderivative size = 58, normalized size of antiderivative = 0.58

method	result	size
orering	$-\frac{(ap-bx+a)x(bx+a)(cx)^{-3-(1+n)p}\left(a\,x^{n}+b\,x^{1+n}\right)^{p}}{(2+p)a^{2}(p+1)}$	58

```
int((c*x)^(-3-(1+n)*p)*(a*x^n+b*x^(1+n))^p,x,method=_RETURNVERBOSE)
```

```
-(a*p-b*x+a)/(2+p)/a^2/(p+1)*x*(b*x+a)*(c*x)^(-3-(1+n)*p)*(a*x^n+b*x^(1+n)
)^p
```

Fricas [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 90, normalized size of antiderivative = 0.90

$$\int (cx)^{-3-(1+n)p} \left(ax^n + bx^{1+n}\right)^p \, dx$$
$$= -\frac{(abpx^2 - b^2x^3 + (a^2p + a^2)x)\left(\frac{(bx+a)x^{n+1}}{x}\right)^p e^{(-((n+1)p+3)\log(c) - ((n+1)p+3)\log(x))}}{a^2p^2 + 3\,a^2p + 2\,a^2}$$

```
integrate((c*x)^(-3-(1+n)*p)*(a*x^n+b*x^(1+n))^p,x, algorithm="fricas")
```

```
-(a*b*p*x^2 - b^2*x^3 + (a^2*p + a^2)*x)*((b*x + a)*x^(n + 1)/x)^p*e^(-((n
 + 1)*p + 3)*log(c) - ((n + 1)*p + 3)*log(x))/(a^2*p^2 + 3*a^2*p + 2*a^2)
```

Sympy [F]

$$\int (cx)^{-3-(1+n)p} \left(ax^n + bx^{1+n}\right)^p \, dx = \int (cx)^{-p(n+1)-3} \left(ax^n + bx^{n+1}\right)^p \, dx$$

```
integrate((c*x)**(-3-(1+n)*p)*(a*x**n+b*x**(1+n))**p,x)
```

```
Integral((c*x)**(-p*(n + 1) - 3)*(a*x**n + b*x**(n + 1))**p, x)
```

Maxima [F]

$$\int (cx)^{-3-(1+n)p} \left(ax^n + bx^{1+n}\right)^p \, dx = \int \left(bx^{n+1} + ax^n\right)^p (cx)^{-(n+1)p-3} \, dx$$

```
integrate((c*x)^(-3-(1+n)*p)*(a*x^n+b*x^(1+n))^p,x, algorithm="maxima")
```

```
integrate((b*x^(n + 1) + a*x^n)^p*(c*x)^(-(n + 1)*p - 3), x)
```

Giac [F]

$$\int (cx)^{-3-(1+n)p} \left(ax^n + bx^{1+n}\right)^p \, dx = \int \left(bx^{n+1} + ax^n\right)^p (cx)^{-(n+1)p-3} \, dx$$

```
integrate((c*x)^(-3-(1+n)*p)*(a*x^n+b*x^(1+n))^p,x, algorithm="giac")
```

```
integrate((b*x^(n + 1) + a*x^n)^p*(c*x)^(-(n + 1)*p - 3), x)
```

Mupad [F(-1)]

Timed out.

$$\int (cx)^{-3-(1+n)p} \left(ax^n + bx^{1+n}\right)^p \, dx = \int \frac{\left(a\,x^n + b\,x^{n+1}\right)^p}{\left(c\,x\right)^{p\,(n+1)+3}} \, dx$$

```
int((a*x^n + b*x^(n + 1))^p/(c*x)^(p*(n + 1) + 3),x)
```

```
int((a*x^n + b*x^(n + 1))^p/(c*x)^(p*(n + 1) + 3), x)
```

Reduce [B] (verification not implemented)

Time = 0.21 (sec) , antiderivative size = 77, normalized size of antiderivative = 0.77

$$\int (cx)^{-3-(1+n)p} \left(ax^n + bx^{1+n}\right)^p \, dx = \frac{\left(x^n a + x^n bx\right)^p \left(-abpx + b^2x^2 - a^2p - a^2\right)}{x^{np+p}c^{np+p}a^2c^3x^2 \left(p^2 + 3p + 2\right)}$$

```
int((c*x)^(-3-(1+n)*p)*(a*x^n+b*x^(1+n))^p,x)
```

```
((x**n*a + x**n*b*x)**p*( - a**2*p - a**2 - a*b*p*x + b**2*x**2))/(x**(n*p
 + p)*c**(n*p + p)*a**2*c**3*x**2*(p**2 + 3*p + 2))
```

3.459 $\int (cx)^{-2-(1+n)p} \left(ax^n + bx^{1+n}\right)^p \, dx$

Optimal result

Integrand size = 28, antiderivative size = 47

$$\int (cx)^{-2-(1+n)p} \left(ax^n + bx^{1+n}\right)^p \, dx = -\frac{x^{-1-n}(cx)^{-((1+n)p)} \left(ax^n + bx^{1+n}\right)^{1+p}}{ac^2(1+p)}$$

```
-x^(-1-n)*(a*x^n+b*x^(1+n))^(p+1)/a/c^2/(p+1)/((c*x)^((1+n)*p))
```

Mathematica [A] (verified)

Time = 0.06 (sec) , antiderivative size = 39, normalized size of antiderivative = 0.83

$$\int (cx)^{-2-(1+n)p} \left(ax^n + bx^{1+n}\right)^p \, dx = -\frac{x(cx)^{-2-(1+n)p}(a+bx) \left(x^n(a+bx)\right)^p}{a(1+p)}$$

```
Integrate[(c*x)^(-2 - (1 + n)*p)*(a*x^n + b*x^(1 + n))^p,x]
```

```
-((x*(c*x)^(-2 - (1 + n)*p)*(a + b*x)*(x^n*(a + b*x))^p)/(a*(1 + p)))
```

Rubi [A] (verified)

Time = 0.29 (sec) , antiderivative size = 46, normalized size of antiderivative = 0.98, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.071, Rules used = {1938, 48}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (cx)^{-((n+1)p)-2} \left(ax^n+bx^{n+1}\right)^p \, dx$$

$$\Big\downarrow \text{ 1938}$$

$$\frac{x^p(a+bx)^{-p}(cx)^{-((n+1)p)} \left(ax^n+bx^{n+1}\right)^p \int x^{-p-2}(a+bx)^p dx}{c^2}$$

$$\Big\downarrow \text{ 48}$$

$$-\frac{(a+bx)(cx)^{-((n+1)p)} \left(ax^n+bx^{n+1}\right)^p}{ac^2(p+1)x}$$

```
Int[(c*x)^(-2 - (1 + n)*p)*(a*x^n + b*x^(1 + n))^p,x]
```

```
-(((a + b*x)*(a*x^n + b*x^(1 + n))^p)/(a*c^2*(1 + p)*x*(c*x)^((1 + n)*p)))
```

Defintions of rubi rules used

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp
[(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] /; FreeQ[{
a, b, c, d, m, n}, x] && EqQ[m + n + 2, 0] && NeQ[m, -1]
```

```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> Simp[c^IntPart[m]*(c*x)^FracPart[m]*((a*x^j + b*x^n)^FracPart[p]/(x^(F
racPart[m] + j*FracPart[p])*(a + b*x^(n - j))^FracPart[p]))   Int[x^(m + j*
p)*(a + b*x^(n - j))^p, x], x] /; FreeQ[{a, b, c, j, m, n, p}, x] &&  !Inte
gerQ[p] && NeQ[n, j] && PosQ[n - j]
```

Maple [A] (verified)

Time = 2.44 (sec) , antiderivative size = 44, normalized size of antiderivative = 0.94

method	result	size
orering	$-\frac{x(bx+a)(cx)^{-2-(1+n)p}\left(a\,x^{n}+b\,x^{1+n}\right)^{p}}{a(p+1)}$	44

```
int((c*x)^(-2-(1+n)*p)*(a*x^n+b*x^(1+n))^p,x,method=_RETURNVERBOSE)
```

```
-1/a/(p+1)*x*(b*x+a)*(c*x)^(-2-(1+n)*p)*(a*x^n+b*x^(1+n))^p
```

Fricas [A] (verification not implemented)

Time = 0.14 (sec) , antiderivative size = 58, normalized size of antiderivative = 1.23

$$\int (cx)^{-2-(1+n)p}\left(ax^n+bx^{1+n}\right)^p\,dx$$
$$=-\frac{(bx^2+ax)\left(\frac{(bx+a)x^{n+1}}{x}\right)^pe^{(-((n+1)p+2)\log(c)-((n+1)p+2)\log(x))}}{ap+a}$$

```
integrate((c*x)^(-2-(1+n)*p)*(a*x^n+b*x^(1+n))^p,x, algorithm="fricas")
```

```
-(b*x^2 + a*x)*((b*x + a)*x^(n + 1)/x)^p*e^(-((n + 1)*p + 2)*log(c) - ((n
+ 1)*p + 2)*log(x))/(a*p + a)
```

Sympy [F]

$$\int (cx)^{-2-(1+n)p}\left(ax^n+bx^{1+n}\right)^p\,dx=\int (cx)^{-p(n+1)-2}\left(ax^n+bx^{n+1}\right)^p\,dx$$

```
integrate((c*x)**(-2-(1+n)*p)*(a*x**n+b*x**(1+n))**p,x)
```

```
Integral((c*x)**(-p*(n + 1) - 2)*(a*x**n + b*x**(n + 1))**p, x)
```

Maxima [F]

$$\int (cx)^{-2-(1+n)p} \left(ax^n + bx^{1+n}\right)^p \, dx = \int \left(bx^{n+1} + ax^n\right)^p (cx)^{-(n+1)p-2} \, dx$$

```
integrate((c*x)^(-2-(1+n)*p)*(a*x^n+b*x^(1+n))^p,x, algorithm="maxima")
```

```
integrate((b*x^(n + 1) + a*x^n)^p*(c*x)^(-(n + 1)*p - 2), x)
```

Giac [F]

$$\int (cx)^{-2-(1+n)p} \left(ax^n + bx^{1+n}\right)^p \, dx = \int \left(bx^{n+1} + ax^n\right)^p (cx)^{-(n+1)p-2} \, dx$$

```
integrate((c*x)^(-2-(1+n)*p)*(a*x^n+b*x^(1+n))^p,x, algorithm="giac")
```

```
integrate((b*x^(n + 1) + a*x^n)^p*(c*x)^(-(n + 1)*p - 2), x)
```

Mupad [F(-1)]

Timed out.

$$\int (cx)^{-2-(1+n)p} \left(ax^n + bx^{1+n}\right)^p \, dx = \int \frac{\left(a\,x^n + b\,x^{n+1}\right)^p}{(c\,x)^{p\,(n+1)+2}} \, dx$$

```
int((a*x^n + b*x^(n + 1))^p/(c*x)^(p*(n + 1) + 2),x)
```

```
int((a*x^n + b*x^(n + 1))^p/(c*x)^(p*(n + 1) + 2), x)
```

Reduce [B] (verification not implemented)

Time = 0.20 (sec) , antiderivative size = 53, normalized size of antiderivative = 1.13

$$\int (cx)^{-2-(1+n)p} \left(ax^n + bx^{1+n}\right)^p \, dx = -\frac{(x^n a + x^n bx)^p (bx + a)}{x^{np+p} c^{np+p} a \, c^2 x (p + 1)}$$

```
int((c*x)^(-2-(1+n)*p)*(a*x^n+b*x^(1+n))^p,x)
```

```
( - (x**n*a + x**n*b*x)**p*(a + b*x))/(x**(n*p + p)*c**(n*p + p)*a*c**2*x*
(p + 1))
```

3.460 $\int (cx)^{-1-(1+n)p} \left(ax^n + bx^{1+n}\right)^p \, dx$

Optimal result

Integrand size = 28, antiderivative size = 58

$$\int (cx)^{-1-(1+n)p} \left(ax^n + bx^{1+n}\right)^p \, dx = -\frac{x^{-n}(cx)^{-((1+n)p)} \left(ax^n + bx^{1+n}\right)^{1+p} \text{Hypergeometric2F1}\left(1, 1, 1-p, -\frac{bx}{a}\right)}{acp}$$

```
-(a*x^n+b*x^(1+n))^(p+1)*hypergeom([1, 1],[1-p],-b*x/a)/a/c/p/(x^n)/((c*x)
^((1+n)*p))
```

Mathematica [A] (verified)

Time = 0.06 (sec) , antiderivative size = 60, normalized size of antiderivative = 1.03

$$\int (cx)^{-1-(1+n)p} \left(ax^n + bx^{1+n}\right)^p \, dx = -\frac{(cx)^{-((1+n)p)} (x^n(a+bx))^p \left(1+\frac{bx}{a}\right)^{-p} \text{Hypergeometric2F1}\left(-p, -p, 1-p, -\frac{bx}{a}\right)}{cp}$$

```
Integrate[(c*x)^(-1 - (1 + n)*p)*(a*x^n + b*x^(1 + n))^p,x]
```

```
-(((x^n*(a + b*x))^p*Hypergeometric2F1[-p, -p, 1 - p, -((b*x)/a)])/(c*p*(c
*x)^((1 + n)*p)*(1 + (b*x)/a)^p))
```

Rubi [A] (verified)

Time = 0.33 (sec) , antiderivative size = 64, normalized size of antiderivative = 1.10, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.107, Rules used = {1938, 76, 74}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (cx)^{-((n+1)p)-1} \left(ax^n+bx^{n+1}\right)^p \, dx$$

$$\Big\downarrow \text{1938}$$

$$\frac{x^p(a+bx)^{-p}(cx)^{-((n+1)p)} \left(ax^n+bx^{n+1}\right)^p \int x^{-p-1}(a+bx)^p dx}{c}$$

$$\Big\downarrow \text{76}$$

$$\frac{x^p\left(\frac{bx}{a}+1\right)^{-p}(cx)^{-((n+1)p)} \left(ax^n+bx^{n+1}\right)^p \int x^{-p-1}\left(\frac{bx}{a}+1\right)^p dx}{c}$$

$$\Big\downarrow \text{74}$$

$$-\frac{\left(\frac{bx}{a}+1\right)^{-p}(cx)^{-((n+1)p)} \text{Hypergeometric2F1}\left(-p,-p,1-p,-\frac{bx}{a}\right) \left(ax^n+bx^{n+1}\right)^p}{cp}$$

```
Int[(c*x)^(-1 - (1 + n)*p)*(a*x^n + b*x^(1 + n))^p,x]
```

```
-(((a*x^n + b*x^(1 + n))^p*Hypergeometric2F1[-p, -p, 1 - p, -((b*x)/a)])/(
c*p*(c*x)^((1 + n)*p)*(1 + (b*x)/a)^p))
```

Defintions of rubi rules used

```
Int[((b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[c^n*((b*x
)^(m + 1)/(b*(m + 1)))*Hypergeometric2F1[-n, m + 1, m + 2, (-d)*(x/c)], x]
/; FreeQ[{b, c, d, m, n}, x] && !IntegerQ[m] && (IntegerQ[n] || (GtQ[c, 0]
 && !(EqQ[n, -2^(-1)] && EqQ[c^2 - d^2, 0] && GtQ[-d/(b*c), 0])))
```

```
Int[((b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[c^IntPart
[n]*((c + d*x)^FracPart[n]/(1 + d*(x/c))^FracPart[n])   Int[(b*x)^m*(1 + d*
(x/c))^n, x], x] /; FreeQ[{b, c, d, m, n}, x] && !IntegerQ[m] && !Integer
Q[n] && !GtQ[c, 0] && !GtQ[-d/(b*c), 0] && ((RationalQ[m] && !(EqQ[n, -2
^(-1)] && EqQ[c^2 - d^2, 0])) || !RationalQ[n])
```

```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> Simp[c^IntPart[m]*(c*x)^FracPart[m]*((a*x^j + b*x^n)^FracPart[p]/(x^(F
racPart[m] + j*FracPart[p])*(a + b*x^(n - j))^FracPart[p]))   Int[x^(m + j*
p)*(a + b*x^(n - j))^p, x], x] /; FreeQ[{a, b, c, j, m, n, p}, x] && !Inte
gerQ[p] && NeQ[n, j] && PosQ[n - j]
```

Maple [F]

$$\int (cx)^{-1-(1+n)p} \left(a\,x^{n}+b\,x^{1+n}\right)^{p} dx$$

```
int((c*x)^(-1-(1+n)*p)*(a*x^n+b*x^(1+n))^p,x)
```

```
int((c*x)^(-1-(1+n)*p)*(a*x^n+b*x^(1+n))^p,x)
```

Fricas [F]

$$\int (cx)^{-1-(1+n)p} \left(ax^n+bx^{1+n}\right)^p \,dx=\int \left(bx^{n+1}+ax^n\right)^p (cx)^{-(n+1)p-1} \,dx$$

```
integrate((c*x)^(-1-(1+n)*p)*(a*x^n+b*x^(1+n))^p,x, algorithm="fricas")
```

```
integral((b*x^(n + 1) + a*x^n)^p*(c*x)^(-(n + 1)*p - 1), x)
```

Sympy [F]

$$\int (cx)^{-1-(1+n)p} \left(ax^n+bx^{1+n}\right)^p \,dx=\int (cx)^{-p(n+1)-1} \left(ax^n+bx^{n+1}\right)^p \,dx$$

```
integrate((c*x)**(-1-(1+n)*p)*(a*x**n+b*x**(1+n))**p,x)
```

```
Integral((c*x)**(-p*(n + 1) - 1)*(a*x**n + b*x**(n + 1))**p, x)
```

Maxima [F]

$$\int (cx)^{-1-(1+n)p} \left(ax^n+bx^{1+n}\right)^p \,dx=\int \left(bx^{n+1}+ax^n\right)^p (cx)^{-(n+1)p-1} \,dx$$

```
integrate((c*x)^(-1-(1+n)*p)*(a*x^n+b*x^(1+n))^p,x, algorithm="maxima")
```

```
integrate((b*x^(n + 1) + a*x^n)^p*(c*x)^(-(n + 1)*p - 1), x)
```

Giac [F]

$$\int (cx)^{-1-(1+n)p} \left(ax^n+bx^{1+n}\right)^p \, dx = \int \left(bx^{n+1}+ax^n\right)^p (cx)^{-(n+1)p-1} \, dx$$

```
integrate((c*x)^(-1-(1+n)*p)*(a*x^n+b*x^(1+n))^p,x, algorithm="giac")
```

```
integrate((b*x^(n + 1) + a*x^n)^p*(c*x)^(-(n + 1)*p - 1), x)
```

Mupad [F(-1)]

Timed out.

$$\int (cx)^{-1-(1+n)p} \left(ax^n+bx^{1+n}\right)^p \, dx = \int \frac{\left(a\,x^n+b\,x^{n+1}\right)^p}{\left(c\,x\right)^{p\,(n+1)+1}} \, dx$$

```
int((a*x^n + b*x^(n + 1))^p/(c*x)^(p*(n + 1) + 1),x)
```

```
int((a*x^n + b*x^(n + 1))^p/(c*x)^(p*(n + 1) + 1), x)
```

Reduce [F]

$$\int (cx)^{-1-(1+n)p} \left(ax^n+bx^{1+n}\right)^p \, dx = \frac{\int \frac{(x^n a+x^n bx)^p}{x^{np+p}x}dx}{c^{np+p}c}$$

```
int((c*x)^(-1-(1+n)*p)*(a*x^n+b*x^(1+n))^p,x)
```

```
int((x**n*a + x**n*b*x)**p/(x**(n*p + p)*x),x)/(c**(n*p + p)*c)
```

3.461 $\int (cx)^{-((1+n)p)} \left(ax^n+bx^{1+n}\right)^p \, dx$

Optimal result

Integrand size = 26, antiderivative size = 60

$$\int (cx)^{-((1+n)p)} \left(ax^n+bx^{1+n}\right)^p \, dx = \frac{x^{1-n}(cx)^{-((1+n)p)} \left(ax^n+bx^{1+n}\right)^{1+p} \text{Hypergeometric2F1}\left(1,2,2-p,-\frac{bx}{a}\right)}{a(1-p)}$$

```
x^(1-n)*(a*x^n+b*x^(1+n))^(p+1)*hypergeom([1, 2],[2-p],-b*x/a)/a/(1-p)/((c
*x)^((1+n)*p))
```

Mathematica [A] (verified)

Time = 0.05 (sec) , antiderivative size = 62, normalized size of antiderivative = 1.03

$$\int (cx)^{-((1+n)p)} \left(ax^n+bx^{1+n}\right)^p \, dx = -\frac{x(cx)^{-((1+n)p)} \left(x^n(a+bx)\right)^p \left(1+\frac{bx}{a}\right)^{-p} \text{Hypergeometric2F1}\left(1-p,-p,2-p,-\frac{bx}{a}\right)}{-1+p}$$

```
Integrate[(a*x^n + b*x^(1 + n))^p/(c*x)^((1 + n)*p),x]
```

```
-((x*(x^n*(a + b*x))^p*Hypergeometric2F1[1 - p, -p, 2 - p, -((b*x)/a)])/((
-1 + p)*(c*x)^((1 + n)*p)*(1 + (b*x)/a)^p))
```

Rubi [A] (verified)

Time = 0.32 (sec) , antiderivative size = 67, normalized size of antiderivative = 1.12, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.115$, Rules used = $\{1938, 76, 74\}$

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (cx)^{-((n+1)p)} \left(ax^n + bx^{n+1}\right)^p \, dx$$

$$\Big\downarrow \text{1938}$$

$$x^p(a+bx)^{-p}(cx)^{-((n+1)p)} \left(ax^n + bx^{n+1}\right)^p \int x^{-p}(a+bx)^p dx$$

$$\Big\downarrow \text{76}$$

$$x^p \left(\frac{bx}{a}+1\right)^{-p} (cx)^{-((n+1)p)} \left(ax^n + bx^{n+1}\right)^p \int x^{-p} \left(\frac{bx}{a}+1\right)^p dx$$

$$\Big\downarrow \text{74}$$

$$\frac{x\left(\frac{bx}{a}+1\right)^{-p} (cx)^{-((n+1)p)} \text{Hypergeometric2F1}\left(1-p,-p,2-p,-\frac{bx}{a}\right) \left(ax^n + bx^{n+1}\right)^p}{1-p}$$

```
Int[(a*x^n + b*x^(1 + n))^p/(c*x)^((1 + n)*p),x]
```

```
(x*(a*x^n + b*x^(1 + n))^p*Hypergeometric2F1[1 - p, -p, 2 - p, -((b*x)/a)]
)/((1 - p)*(c*x)^((1 + n)*p)*(1 + (b*x)/a)^p)
```

Defintions of rubi rules used

```
Int[((b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[c^n*((b*x
)^(m + 1)/(b*(m + 1)))*Hypergeometric2F1[-n, m + 1, m + 2, (-d)*(x/c)], x]
/; FreeQ[{b, c, d, m, n}, x] && !IntegerQ[m] && (IntegerQ[n] || (GtQ[c, 0]
 && !(EqQ[n, -2^(-1)] && EqQ[c^2 - d^2, 0] && GtQ[-d/(b*c), 0])))
```

```
Int[((b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[c^IntPart
[n]*((c + d*x)^FracPart[n]/(1 + d*(x/c))^FracPart[n])   Int[(b*x)^m*(1 + d*
(x/c))^n, x], x] /; FreeQ[{b, c, d, m, n}, x] && !IntegerQ[m] && !Integer
Q[n] && !GtQ[c, 0] && !GtQ[-d/(b*c), 0] && ((RationalQ[m] && !(EqQ[n, -2
^(-1)] && EqQ[c^2 - d^2, 0])) || !RationalQ[n])
```

```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> Simp[c^IntPart[m]*(c*x)^FracPart[m]*((a*x^j + b*x^n)^FracPart[p]/(x^(F
racPart[m] + j*FracPart[p])*(a + b*x^(n - j))^FracPart[p]))   Int[x^(m + j*
p)*(a + b*x^(n - j))^p, x], x] /; FreeQ[{a, b, c, j, m, n, p}, x] && !Inte
gerQ[p] && NeQ[n, j] && PosQ[n - j]
```

Maple [F]

$$\int \left(a\,x^{n}+b\,x^{1+n}\right)^{p} \left(cx\right)^{-\left(1+n\right)p}\,dx$$

```
int((a*x^n+b*x^(1+n))^p/((c*x)^((1+n)*p)),x)
```

```
int((a*x^n+b*x^(1+n))^p/((c*x)^((1+n)*p)),x)
```

Fricas [F]

$$\int (cx)^{-((1+n)p)} \left(ax^n + bx^{1+n}\right)^p \, dx = \int \frac{\left(bx^{n+1} + ax^n\right)^p}{(cx)^{(n+1)p}} \, dx$$

```
integrate((a*x^n+b*x^(1+n))^p/((c*x)^((1+n)*p)),x, algorithm="fricas")
```

```
integral((b*x^(n + 1) + a*x^n)^p/(c*x)^((n + 1)*p), x)
```

Sympy [F]

$$\int (cx)^{-((1+n)p)} \left(ax^n + bx^{1+n}\right)^p \, dx = \int (cx)^{-p(n+1)} \left(ax^n + bx^{n+1}\right)^p \, dx$$

```
integrate((a*x**n+b*x**(1+n))**p/((c*x)**((1+n)*p)),x)
```

```
Integral((a*x**n + b*x**(n + 1))**p/(c*x)**(p*(n + 1)), x)
```

Maxima [F]

$$\int (cx)^{-((1+n)p)} \left(ax^n + bx^{1+n}\right)^p \, dx = \int \frac{\left(bx^{n+1} + ax^n\right)^p}{(cx)^{(n+1)p}} \, dx$$

```
integrate((a*x^n+b*x^(1+n))^p/((c*x)^((1+n)*p)),x, algorithm="maxima")
```

```
integrate((b*x^(n + 1) + a*x^n)^p/(c*x)^((n + 1)*p), x)
```

Giac [F]

$$\int (cx)^{-((1+n)p)}\left(ax^n+bx^{1+n}\right)^p\,dx=\int \frac{(bx^{n+1}+ax^n)^p}{(cx)^{(n+1)p}}\,dx$$

```
integrate((a*x^n+b*x^(1+n))^p/((c*x)^((1+n)*p)),x, algorithm="giac")
```

```
integrate((b*x^(n + 1) + a*x^n)^p/(c*x)^((n + 1)*p), x)
```

Mupad [F(-1)]

Timed out.

$$\int (cx)^{-((1+n)p)}\left(ax^n+bx^{1+n}\right)^p\,dx=\int \frac{(a\,x^n+b\,x^{n+1})^p}{(c\,x)^{p\,(n+1)}}\,dx$$

```
int((a*x^n + b*x^(n + 1))^p/(c*x)^(p*(n + 1)),x)
```

```
int((a*x^n + b*x^(n + 1))^p/(c*x)^(p*(n + 1)), x)
```

Reduce [F]

$$\int (cx)^{-((1+n)p)}\left(ax^n+bx^{1+n}\right)^p\,dx=\frac{(x^na+x^nbx)^p\,x+x^{np+p}\left(\int \frac{(x^na+x^nbx)^p}{x^{np+p}a+x^{np+p}bx}dx\right)ap}{x^{np+p}c^{np+p}}$$

```
int((a*x^n+b*x^(1+n))^p/((c*x)^((1+n)*p)),x)
```

```
((x**n*a + x**n*b*x)**p*x + x**(n*p + p)*int((x**n*a + x**n*b*x)**p/(x**(n
*p + p)*a + x**(n*p + p)*b*x),x)*a*p)/(x**(n*p + p)*c**(n*p + p))
```

3.462 $\int (cx)^{1-(1+n)p} \left(ax^n+bx^{1+n}\right)^p \, dx$

Optimal result

Integrand size = 28, antiderivative size = 61

$$\int (cx)^{1-(1+n)p} \left(ax^n+bx^{1+n}\right)^p \, dx = \frac{cx^{2-n}(cx)^{-((1+n)p)} \left(ax^n+bx^{1+n}\right)^{1+p} \text{Hypergeometric2F1}\left(1,3,3-p,-\frac{bx}{a}\right)}{a(2-p)}$$

```
c*x^(2-n)*(a*x^n+b*x^(1+n))^(p+1)*hypergeom([1, 3],[3-p],-b*x/a)/a/(2-p)/(
(c*x)^((1+n)*p))
```

Mathematica [A] (verified)

Time = 0.06 (sec) , antiderivative size = 65, normalized size of antiderivative = 1.07

$$\int (cx)^{1-(1+n)p} \left(ax^n+bx^{1+n}\right)^p \, dx = -\frac{cx^2(cx)^{-((1+n)p)} (x^n(a+bx))^p \left(1+\frac{bx}{a}\right)^{-p} \text{Hypergeometric2F1}\left(2-p,-p,3-p,-\frac{bx}{a}\right)}{-2+p}$$

```
Integrate[(c*x)^(1 - (1 + n)*p)*(a*x^n + b*x^(1 + n))^p,x]
```

```
-((c*x^2*(x^n*(a + b*x))^p*Hypergeometric2F1[2 - p, -p, 3 - p, -((b*x)/a)]
)/((-2 + p)*(c*x)^((1 + n)*p)*(1 + (b*x)/a)^p))
```

Rubi [A] (verified)

Time = 0.33 (sec) , antiderivative size = 70, normalized size of antiderivative = 1.15, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.107$, Rules used = {1938, 76, 74}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (cx)^{1-(n+1)p} \left(ax^n+bx^{n+1}\right)^p \, dx$$

$$\Big\downarrow \text{1938}$$

$$cx^p(a+bx)^{-p}(cx)^{-((n+1)p)} \left(ax^n+bx^{n+1}\right)^p \int x^{1-p}(a+bx)^p dx$$

$$\Big\downarrow \text{76}$$

$$cx^p\left(\frac{bx}{a}+1\right)^{-p}(cx)^{-((n+1)p)} \left(ax^n+bx^{n+1}\right)^p \int x^{1-p}\left(\frac{bx}{a}+1\right)^p dx$$

$$\Big\downarrow \text{74}$$

$$\frac{cx^2\left(\frac{bx}{a}+1\right)^{-p}(cx)^{-((n+1)p)}\text{Hypergeometric2F1}\left(2-p,-p,3-p,-\frac{bx}{a}\right)\left(ax^n+bx^{n+1}\right)^p}{2-p}$$

```
Int[(c*x)^(1 - (1 + n)*p)*(a*x^n + b*x^(1 + n))^p,x]
```

```
(c*x^2*(a*x^n + b*x^(1 + n))^p*Hypergeometric2F1[2 - p, -p, 3 - p, -((b*x)
/a)])/((2 - p)*(c*x)^((1 + n)*p)*(1 + (b*x)/a)^p)
```

Defintions of rubi rules used

```
Int[((b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[c^n*((b*x
)^(m + 1)/(b*(m + 1)))*Hypergeometric2F1[-n, m + 1, m + 2, (-d)*(x/c)], x]
/; FreeQ[{b, c, d, m, n}, x] &&  !IntegerQ[m] && (IntegerQ[n] || (GtQ[c, 0]
 &&  !(EqQ[n, -2^(-1)] && EqQ[c^2 - d^2, 0] && GtQ[-d/(b*c), 0])))
```

```
Int[((b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[c^IntPart
[n]*((c + d*x)^FracPart[n]/(1 + d*(x/c))^FracPart[n])   Int[(b*x)^m*(1 + d*
(x/c))^n, x], x] /; FreeQ[{b, c, d, m, n}, x] &&  !IntegerQ[m] &&  !Integer
Q[n] &&  !GtQ[c, 0] &&  !GtQ[-d/(b*c), 0] && ((RationalQ[m] &&  !(EqQ[n, -2
^(-1)] && EqQ[c^2 - d^2, 0])) ||  !RationalQ[n])
```

```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> Simp[c^IntPart[m]*(c*x)^FracPart[m]*((a*x^j + b*x^n)^FracPart[p]/(x^(F
racPart[m] + j*FracPart[p])*(a + b*x^(n - j))^FracPart[p]))   Int[x^(m + j*
p)*(a + b*x^(n - j))^p, x], x] /; FreeQ[{a, b, c, j, m, n, p}, x] &&  !Inte
gerQ[p] && NeQ[n, j] && PosQ[n - j]
```

Maple [F]

$$\int (cx)^{1-(1+n)p} \left(a\,x^n+b\,x^{1+n}\right)^p dx$$

```
int((c*x)^(1-(1+n)*p)*(a*x^n+b*x^(1+n))^p,x)
```


```
int((c*x)^(1-(1+n)*p)*(a*x^n+b*x^(1+n))^p,x)
```

Fricas [F]

$$\int (cx)^{1-(1+n)p} \left(ax^n + bx^{1+n}\right)^p \, dx = \int \left(bx^{n+1} + ax^n\right)^p (cx)^{-(n+1)p+1} \, dx$$

```
integrate((c*x)^(1-(1+n)*p)*(a*x^n+b*x^(1+n))^p,x, algorithm="fricas")
```

```
integral((b*x^(n + 1) + a*x^n)^p*(c*x)^(-(n + 1)*p + 1), x)
```

Sympy [F]

$$\int (cx)^{1-(1+n)p} \left(ax^n + bx^{1+n}\right)^p \, dx = \int (cx)^{-p(n+1)+1} \left(ax^n + bx^{n+1}\right)^p \, dx$$

```
integrate((c*x)**(1-(1+n)*p)*(a*x**n+b*x**(1+n))**p,x)
```

```
Integral((c*x)**(-p*(n + 1) + 1)*(a*x**n + b*x**(n + 1))**p, x)
```

Maxima [F]

$$\int (cx)^{1-(1+n)p} \left(ax^n + bx^{1+n}\right)^p \, dx = \int \left(bx^{n+1} + ax^n\right)^p (cx)^{-(n+1)p+1} \, dx$$

```
integrate((c*x)^(1-(1+n)*p)*(a*x^n+b*x^(1+n))^p,x, algorithm="maxima")
```

```
integrate((b*x^(n + 1) + a*x^n)^p*(c*x)^(-(n + 1)*p + 1), x)
```

Giac [F]

$$\int (cx)^{1-(1+n)p} \left(ax^n + bx^{1+n}\right)^p \, dx = \int \left(bx^{n+1} + ax^n\right)^p (cx)^{-(n+1)p+1} \, dx$$

```
integrate((c*x)^(1-(1+n)*p)*(a*x^n+b*x^(1+n))^p,x, algorithm="giac")
```

```
integrate((b*x^(n + 1) + a*x^n)^p*(c*x)^(-(n + 1)*p + 1), x)
```

Mupad [F(-1)]

Timed out.

$$\int (cx)^{1-(1+n)p} \left(ax^n + bx^{1+n}\right)^p \, dx = \int (c\,x)^{1-p\,(n+1)} \left(a\,x^n + b\,x^{n+1}\right)^p \, dx$$

```
int((c*x)^(1 - p*(n + 1))*(a*x^n + b*x^(n + 1))^p,x)
```

```
int((c*x)^(1 - p*(n + 1))*(a*x^n + b*x^(n + 1))^p, x)
```

Reduce [F]

$$\int (cx)^{1-(1+n)p} \left(ax^n + bx^{1+n}\right)^p \, dx$$
$$= \frac{c\left((x^n a + x^n bx)^p\, apx + (x^n a + x^n bx)^p\, b\,x^2 + x^{np+p}\left(\int \frac{(x^n a+x^n bx)^p}{x^{np+p}a+x^{np+p}bx}dx\right) a^2p^2 - x^{np+p}\left(\int \frac{(x^n a+x^n bx)^p}{x^{np+p}a+x^{np+p}bx}d\right.\right.}{2x^{np+p}c^{np+p}b}$$

```
int((c*x)^(1-(1+n)*p)*(a*x^n+b*x^(1+n))^p,x)
```

```
(c*((x**n*a + x**n*b*x)**p*a*p*x + (x**n*a + x**n*b*x)**p*b*x**2 + x**(n*p
 + p)*int((x**n*a + x**n*b*x)**p/(x**(n*p + p)*a + x**(n*p + p)*b*x),x)*a*
*2*p**2 - x**(n*p + p)*int((x**n*a + x**n*b*x)**p/(x**(n*p + p)*a + x**(n*
p + p)*b*x),x)*a**2*p))/(2*x**(n*p + p)*c**(n*p + p)*b)
```

CHAPTER 4

Appendix

4.1 Listing of Grading functions

The following are the current version of the grading functions used for grading the quality of the antiderivative with reference to the optimal antiderivative included in the test suite.

There is a version for Maple and for Mathematica/Rubi. There is a version for grading Sympy and version for use with Sagemath.

The following are links to the current source code.

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