

First order ode linear in y' . Assume expansion is around x_0 .

$$y'(x) = f(x, y)$$

Assuming initial condition is $y(x_0) = y_0$

is $f(x, y)$ analytic at x_0 ?

YES

NO

The easy case. Apply Taylor series definition directly to find the series expansion. Let

$$y = y_0 + \sum_{n=1}^{\infty} \frac{x^n}{n!} f_n(x, y) \Big|_{\substack{x=x_0 \\ y=y_0}}$$

Where

$$f_1 = f(x, y)$$

$$f_{n+1} = \frac{\partial f_n}{\partial x} + \left(\frac{\partial f_n}{\partial y} \right) f_1$$

Write the ode as $y' + p(x)y = q(x)$. This is only possible if ode is linear in y .

$\lim_{x \rightarrow x_0} (x - x_0)p(x)$ exists?

YES

NO

Regular singular point. Use Frobenius series
 $y_h = \sum_{n=0}^{\infty} a_n x^{n+r}$

irregular singular point. Asymptotic expansion. Not supported.

Find the recurrence relation for $y' + p(x)y = 0$ and determine all a_n coefficients. Also determine the balance equation.

$q(x) = 0$?

NO

YES

$q(x)$ has series expansion at x_0 ?

Finish

NO

YES

Generate the balance equation from $n = 0$ from the Frobenius series $y = \sum_{n=0}^{\infty} a_n x^{n+r}$ which will give equation that looks like $y' + p(x)y = mc_0 x^m$ and then solve for m, c_0 from $mc_0 x^m = q(x)$ for each sum term in $q(x)$,

Expand $q(x)$ in series around x_0 and match each power of x with the corresponding n term in the recurrence relation to solve for the a_n

All a_n found. Solution is $y = x^r \sum_{n=0}^{\infty} a_n x^n$

Solve for m, c_0 from the balance equation

Not solved

Solved

No series solution exist

Find $y_p = \sum_{n=0}^{\infty} c_n x^{n+m}$ using same recurrence relation for the main ode above.

$$y = y_h + y_p$$

$$= \sum_{n=0}^{\infty} a_n x^{n+r} + \sum_{n=0}^{\infty} c_n x^{n+m}$$